Many-body exchange-correlation effects in the lowest subband of semiconductor quantum wires

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We consider theoretically the electron–electron interaction induced exchange-correlation effects in the lowest subband of a quasi-one-dimensional GaAs quantum wire structures. We calculate, within the leading order dynamical screening approximation (*i.e.* the so-called $GW$ approximation of the electron gas theory), the electron self-energy, spectral function, momentum distribution function, inelastic scattering rate, band gap renormalization, and, the many-body renormalization factor both at zero and finite temperatures, and, both with and without impurity scattering effects.

We also calculate the effects of finite temperatures and finite impurity scattering on the many-body properties. We propose the possibility of a hot-electron transistor device with a large negative differential resistance which is based on the sudden onset of plasmon emission by energetic ballistic electrons in one dimension. The issue of the existence or non-existence of the Fermi surface among the interacting one-dimensional quantum wire electrons is critically discussed based on our numerical results.

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I. INTRODUCTION

Recently, there has been a great deal of interest in ultranarrow confined semiconductor systems, called quantum wire structures, where the motion of the electrons is essentially restricted to be one-dimensional. These systems are usually fabricated on high-quality two-dimensional electron systems by confining the electrons in one of the remaining free directions through ultrafine nanolithographic patterning. In addition, other methods for fabricating wires, such as growth of wires on tilted superlattices and vicinal substrates have also been reported. These developments have opened up possibilities for novel and exciting experiments, and stimulated considerable theoretical activity. At present, quantum wires with active widths as small as approximately 300 Å and of negligible thickness have been fabricated, with further reductions in size and improvement in quality in the offing. Quantum wires have generated much interest both for the potential for practical applications in solid-state devices such as high-speed transistors, efficient photodetectors and lasers, and because they have afforded us for the first time an experimental opportunity to study real one-dimensional Fermi gases in a relatively controlled manner (much in the same way that, in the past two decades or so, semiconductor inversion layers, heterojunctions and quantum wells have generated considerable activity in pure and applied research of the two-dimensional electron gas). Thus, both from the fundamental and applied physics viewpoints, there is interest in understanding the electronic properties of quasi-one-dimensional quantum wires.

Much of the fundamental theoretical understanding of electrons in one-dimensional systems have come from work on the Tomonaga-Luttinger model. The Tomonaga-Luttinger model makes some drastic simplifying assumptions which allow one to solve the interacting problem completely. One surprising result that is obtained from the solution of this model is that even the smallest interaction results in a disappearance of the Fermi surface, leading to a system which is non-Fermi liquid (in the sense that the elementary excitations are very different from those of the non-interacting system). Therefore, one would
expect that the experimental properties of the semiconductor quantum wires should be very different from any predictions based on the assumption that the one-dimensional electron gases are Fermi liquids.

Recently, quantum wires have been fabricated in which the one-dimensional quantum limit has been attained, in the sense that only one quantum subband is populated by the electrons in the quantum wire, so that a one-dimensional interacting Fermi gas model is valid. Contrary to what one might expect based on the well-established theoretical results, the experimental data obtained from these experiments on these structures, such as Raman scattering and photoluminescence, can be successfully explained on the basis of standard Fermi liquid theory. This fact is also mirrored by the success of other theories, such as the Landauer-Büttiker formula, which treat electrons in mesoscopic quasi-one-dimensional systems as noninteracting entities. Thus we are posed with the question, “Why are experimental results from real quantum wires explicable on the basis of Fermi liquid theories?” This is one of the points we attempt to shed light on in this paper. We show that the interplay between plasmons and impurities play an important role in determining the physical characteristics of the system. In particular, we show that the physical mechanism responsible for the disappearance of a Fermi surface in a clean system is the virtual plasmon emissions, and that in an impure system these virtual plasmons emissions are damped, leading to a restoration of the Fermi surface. (The plasmons in our model are equivalent to the Tomonaga bosons of the Tomonaga-Luttinger model.)

We also present calculations for experimentally measurable properties for quantum wires. For the quantum wire to be in the quantum limit, the doping must necessarily be low \( \lesssim 10^6 \text{ cm}^{-1} \) (otherwise, higher subbands are filled) and hence the Fermi energy and other energy scales pertinent to the system are also small. Therefore, temperature effects are important, even at relatively low temperatures. We have therefore presented the calculations for both \( T = 0 \) and finite temperatures. To perform the \( T \neq 0 \) calculations, we have developed a formalism for the efficient calculation of the finite-temperature self-energy (valid for arbitrary dimensionality), which we describe in this paper.
The many body properties of one-dimensional systems have unique properties, one of which is the singular behavior of the mean free path as a function of electron energy, which occurs when electrons cross the plasmon emission threshold. We discuss the possibility that this singular behavior could in principle be applied to produce a device with large negative differential resistivity. Finally, since the the one-dimensional plasmons are prominent protagonists in the physics of interacting one-dimensional system, we conclude the paper with a discussion of the plasmons in the presence of impurities and at finite temperature.

This paper is organized as follows. In Sec. II, we study the many-body properties of quantum wires at $T = 0$. In Sec. III, we develop the formalism (for arbitrary dimensionality) necessary for efficient calculation of the self-energy at finite temperatures, which we apply to calculate the many-body properties of a quantum wire. In Sec. IV, we discuss the possible device applications of quantum wires. In Sec. V, we discuss the plasmon spectrum of quantum wires, in clean and dirty systems, and at finite temperature. Sec. VI contains our summary and conclusions. Some of the results in this papers have been published previously in brief communications. \[9,12,13\] Also note that in this paper, $\hbar = 1$ and $k_B = 1$.

II. MANY-BODY PROPERTIES OF A QUANTUM WIRE AT $T = 0$

In this section, we present calculations of the many-body properties of a one-dimensional quantum wire, using the so-called $GW$ approximation. We begin with a brief review of the formalism, and then we describe how we put effects of impurities into the system. We then give results and conclude this section with a discussion.

We assume that the quantum wire is formed by confining the electrons to the two-dimensional plane $x-y$ plane (by, say, modulation doping an AlGaAs-GaAs-AlGaAs heterostructure), and then further confining the electrons along the $y$-direction. We assume that the confinement in the $z$-direction is much stronger than the confinement in the $y$-direction, which is a reasonable approximation because, at this stage, the technology for confining electrons to two-dimensions is much more advanced the technology for confining
the electrons along an additional direction. For example, by modulation doping, the electrons can be confined in the $z$-direction on the order of less than 100 Å, whereas currently the best confinement in the $y$-direction is approximately 300 Å, leading to at least an order of magnitude difference in the energy level spacings of the $y$- and $z$-directions. Thus, in this paper, we assume that the electron gas has zero thickness in the $z$-direction, but has a finite width in the $y$-direction. (Note that it is necessary to assume a nonzero width so that the one-dimensional Coulomb interaction is finite. [20]) The confinement of the electrons in the $y$–$z$ plane leads to the quantization of energy levels into different subbands, depending on what the wavefunction in the $y$–$z$ plane is. We assume that the energy separation between the lowest energy and higher energy subbands is so much larger than all other energy scales in the problem that the higher subbands can be ignored. Therefore, all quantities in the main part of this paper refer to the lowest energy subband. In Appendix A, we give the formalism in the case of multiple subbands and briefly discuss why, for processes with energies much less than the subband energy separation, the higher subbands have negligible contribution.

A. The Hamiltonian

The Hamiltonian of the system is given by $\hat{H} = \hat{H}_0 + \hat{H}_{e-e}$ where $\hat{H}_0$ is the bare kinetic energy term and $\hat{H}_{e-e}$ is the electron–electron interaction term, given by

$$\hat{H}_0 = \sum_{k\sigma} \xi(k) \hat{c}_{k\sigma}^\dagger \hat{c}_{k\sigma}$$

$$\hat{H}_{e-e} = \frac{1}{2L} \sum_{k,k',q} \sum_{\sigma,\sigma'} V_c(q) \hat{c}_{k+q,\sigma}^\dagger \hat{c}_{k',-q,\sigma'}^\dagger \hat{c}_{k',\sigma'} \hat{c}_{k\sigma},$$

Here $\hat{c}_{k\sigma}^\dagger$ ($\hat{c}_{k\sigma}$) are fermion creation (annihilation) operators for for states with wavevector $k$ in the $x$-direction and electron spin $\sigma$, $\xi(k)$ is the kinetic energy relative to the chemical potential $\mu$, and $L$ is the length of the system. The $V_c(q)$ are the matrix elements of the Coulomb interaction (screened by the static lattice dielectric constant $\epsilon_0$),
\[ V_c(q) = \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dx \frac{e^{-iqx}}{\epsilon_0 \left[(x-x')^2 + (y-y')^2\right]^{1/2}} |\phi(y)|^2 |\phi(y')|^2 \]
\[ = \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy' v(q, y - y')|\phi(y)|^2 |\phi(y')|^2, \]

where \( \phi(y) \) is the transverse wavefunction and
\[ v(q, y - y') = \frac{2e^2}{\epsilon_0 K_0(|q(y - y')|)} \]
is the one-dimensional Fourier transformation of the Coulomb interaction. Here, \( K_0(x) \) is the zeroth-order modified Bessel function of the second kind.

For an external confining potential that is parabolic, the self-consistent confining potential (that is, the external confining potential plus the self-consistent Hartree term) gives approximately a square well potential. This turns out to be a reasonable approximation for modeling quantum wire confinement. Therefore, we approximate the self-consistent potential by a square well with infinite barriers at \( y = -a/2 \) to \( y = a/2 \), so that
\[ \phi(y) = \begin{cases} \frac{2}{a} \cos \left(\frac{\pi y}{a}\right), & \text{if } -a/2 \leq y \leq a/2; \\ 0, & \text{otherwise}; \end{cases} \]
and the Coulomb matrix element in this square-well case is
\[ V_c(q) = \frac{2e^2}{\epsilon_0} \int_{-a/2}^{a/2} dy \int_{-a/2}^{a/2} dy' K_0(|q(y - y')|) \cos^2 \left(\frac{\pi y}{a}\right) \cos^2 \left(\frac{\pi y'}{a}\right) \]
\[ = \frac{2e^2}{\epsilon_0} \int_{0}^{1} dx K_0(|qa|x) \left[(2 - (1 - x) \cos(2\pi x) + \frac{3}{2\pi} \sin(2\pi x)\right] \]
whose asymptotic forms are
\[ V_c(q) = \begin{cases} \frac{3\pi e^2}{\epsilon_0 |qa|}, & \text{for } |qa| \to \infty; \\ \frac{2e^2}{\epsilon_0} [K_0(|qa|) + 1.9726917...], & \text{for } |qa| \to 0. \end{cases} \]
Note that $K_0(x) \sim -\ln(x)$ as $x \to 0$, so that the Coulomb matrix element diverges as $-2e^2 \ln(|qa|)/\epsilon_0$ small $q$. To speed up numerical computation, we fitted $V_c(q)$ with a function which extrapolated between the $q \to 0$ and $q \to \infty$ forms and which deviated from the true function by no more than 0.6%.

Throughout this paper, we assume that the band for the electrons in the unconfined $x$-direction is parabolic, so that $\xi(k) = \hbar^2 k^2/(2m^*) - \mu$, where $m^*$ is the effective electron band mass. This is an excellent assumption for GaAs quantum wires because the relevant electron densities are so low ($k_F \sim 10^6$ cm$^{-1}$) that the nonparabolicity in the band for $k < k_F$ is insignificant.

B. The electron self-energy $\Sigma(k, \omega)$ within the GW and RPA approximation

A quantity which provides a substantial amount of information on an interacting many-electron system is the electron Green’s function $G(k, \omega)$, or equivalently, the electron self-energy, $\Sigma(k, \omega) = G_0^{-1}(k, \omega) - G^{-1}(k, \omega)$ (where $G_0(k, \omega)$ is the bare noninteracting Green’s function). The self-energy is roughly the correction to the noninteracting electron single-particle energy due to the interactions. Once $G(k, \omega)$ or $\Sigma(k, \omega)$ is known, the one-electron properties of a system, such as the spectral density function, electron distribution function and band-gap renormalizations, can be calculated.

Generally, it is impossible to calculate the $\Sigma(k, \omega)$ exactly for interacting systems, and one must resort to various approximation schemes. However, in one dimension, the self-energy of the Tomonaga-Luttinger model, which uses somewhat artificial assumptions (specifically (1) two completely linearly dispersing bands of electrons with an infinite bandwidth, populated by an infinite density of electrons and (2) a short-ranged interaction), can be calculated exactly. [16,17] As alluded to in the introduction, this models predicts that any interaction, no matter how small, will drive the system away from Fermi liquid behavior. However, the connection between the exactly solvable but artificial Tomonaga-Luttinger model and real semiconductor quantum wires is somewhat tenuous, and there is a need for calculations on
more realistic models which can be compared to experiment. The model we employ assumes that the electron dispersion in the free direction of the quantum wire is parabolic, and the electrons interact via the exact Coulomb interaction. Unfortunately, with these more realistic assumptions, the problem is no longer exactly soluble and therefore, as in higher dimensions, $\Sigma(k, \omega)$ can only be calculated approximately. In this paper, we use the so-called $GW$ approximation to calculate the self-energy, \cite{24} where the Green’s function is dressed by the dynamically screened Coulomb interaction, which is calculated within the random phase approximation (RPA). The Feynman diagram for the self-energy and the screened interaction in these approximations are shown in Fig. 1(a) and 1(b). This approximation, which sums up the largest diagrams in the self-energy perturbation series, has long been employed to successfully calculate properties of three and two dimensional electron systems. \cite{25,26} We ignore diagrams which are higher order in the screened interaction, such the diagram shown in Fig. 1(c), because we expect them to have a smaller contribution. In fact, in Appendix B we show that at $k = k_F$ and $\omega = \xi_{k_F}$, and for a short-ranged interaction, Fig. 1(c) gives zero contribution.

The self-energy within the $GW$ approximation at $T = 0$ is

$$\Sigma(k, \omega) = i \int_{-\infty}^{\infty} \frac{dq}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} W(q, \omega') G_0(k - q, \omega - \omega').$$

(2.7)

where, $G_0(k, \omega) = [w - \xi_k + i0^+ \text{sgn}(k - k_F)]^{-1}$ and $W(q, \omega)$ is the screened Coulomb interaction, which given by

$$W(q, \omega) = \frac{V_c(q)}{\epsilon(q, \omega)}.$$

(2.8)

Here, $\epsilon(q, \omega)$ is the dielectric function, which describes the screening properties of the electron gas. The self-energy can be separated into the frequency-independent exchange and correlation parts

$$\Sigma(k; \omega) \equiv \Sigma_{\text{ex}}(k) + \Sigma_{\text{cor}}(k, \omega).$$

(2.9)

The exchange part is given by \cite{18}
\[ \Sigma_{\text{ex}}(k) = - \int \frac{dq}{2\pi} n_F(k + q)V_c(q), \quad (2.10) \]

where \( n_F(k + q) = \theta(k_F - |k + q|) \) is the Fermi function at \( T = 0 \), and \( \Sigma_{\text{cor}}(k, \omega) \) is defined to be the part of \( \Sigma(k, \omega) \) not included in \( \Sigma_{\text{ex}}(k) \). In the \( GW \) approximation, the \( \Sigma_{\text{cor}}(k, \omega) \) can be written in the line and pole decomposition \[ \Sigma_{\text{cor}}(k, \omega) = \Sigma_{\text{line}}(k, \omega) + \Sigma_{\text{pole}}(k, \omega), \quad (2.11a) \]

\[ \Sigma_{\text{line}}(k, \omega) = - \int_{-\infty}^{\infty} \frac{dq}{2\pi} V_c(q) \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{1}{(\xi_{k+q} - \omega) - i\omega'} \left[ \frac{1}{\epsilon(q, i\omega') - 1} \right], \quad (2.11b) \]

\[ \Sigma_{\text{pole}}(k, \omega) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} \left[ \theta(\omega - \xi_{k+q}) - \theta(-\xi_{k+q}) \right] V_c(q) \left( \frac{1}{\epsilon(q, \xi_{k+q} - \omega) - 1} \right). \quad (2.11c) \]

The \( \Sigma_{\text{line}}(k, \omega) \) is completely real because \( \epsilon(q, i\omega') \) is real and even with respect to \( \omega' \). Thus, \( \text{Im}[\Sigma_{\text{pole}}] \) gives the total contribution to the imaginary part of the self-energy. In evaluating the self-energies, we assume that the frequency \( \omega \) has an infinitesimal positive imaginary part; this gives us the \emph{retarded} self-energy. \[ \text{[18]} \] (Unless otherwise stated, all real frequency self-energies in this paper are retarded.)

Within the RPA, \( \epsilon(q, \omega) \) is given by \( \epsilon(q, \omega) = 1 - V_c(q)\Pi_0(q, \omega) \), where \( \Pi_0(q, \omega) \) is the irreducible polarizability, given diagrammatically by the particle-hole bubble. In a pure system, \[ \text{[21,20,28]} \]

\[ \Pi_0(q, z) = \frac{m}{\pi q} \ln \left[ \frac{z^2 - ((q^2/2m) - qv_F)^2}{z^2 - ((q^2/2m) + qv_F)^2} \right], \quad (2.12) \]

where the principal value of logarithm (\(|\text{Im}[\ln]| < \pi\)) should be taken. In evaluating \( \Pi_0(q, \omega) \) for real frequencies, the limit \( z = \omega + i0^+ \) should be taken. The RPA form of \( \epsilon(q, \omega) \) has recently been shown \[ \text{[29]} \] to \emph{exactly} reproduce the plasmon dispersion of one-dimensional systems, in the sense that the long wavelength RPA correctly gives the exact Tomonaga boson dispersion of the Tomonaga-Luttinger model.

The \( \Pi_0(q, \omega) \) given in Eq. \[ (2.12) \] assumes that the system is free from any defects. While modulation doping techniques result in quantum wires are generally of high quality, impurities and other imperfections always exist. In general, the effect of these impurities is to cause the electrons to diffuse instead of moving ballistically. This diffusive motion of the
electrons affects the polarizability (since it is the response of the electrons to an applied potential) and hence electronic screening, generally tending to make screening less effective since electrons cannot move as quickly to their screening positions. Impurity effects are usually introduced diagrammatically into the screening in the RPA by including impurity ladder diagrams into the electron-hole bubble, \[30\] which yields a polarizability \(\Pi_\gamma(q, \omega)\) which has a diffusive regime for small \(q\) and \(\omega\). Since the exact expression for \(\Pi_\gamma(q, \omega)\) within this diagramatic approach is complicated, we use a particle-conserving expression, given by Mermin, \[31\] which captures the essential physics of impurity collisions on \(\Pi_\gamma(q, \omega)\).

For an impurity scattering rate \(\gamma\),

\[
\Pi_\gamma(q, \omega) = \frac{(\omega + i\gamma)\Pi_0(q, \omega + i\gamma)}{\omega + i\gamma[\Pi_0(q, \omega + i\gamma)/\Pi_0(q, 0)]}.
\] (2.13)

The form given in Eq. (2.13) has the correct diffusive behavior at small \(q\) and \(\omega\), which is absent in the \(\Pi_0(q, \omega)\) for the pure case. In particular, for \(q, \omega \to 0\),

\[
\Pi_\gamma(q, \omega) \approx -\frac{2me}{\pi k_F h^2} \frac{Dq^2}{Dq^2 + i\omega},
\] (2.14)

\[
\Pi_0(q, \omega) \approx \frac{nq^2}{m^*\omega^2} \quad \text{(for } q \ll \omega/v_F)\),
\]

where \(D = v_F^2/\gamma\) is the one-dimensional diffusion constant. The modification of the behavior of the polarizability due to impurities has important consequences for long-wavelength plasmons, which in turn affect the many-body properties of the one-dimensional electron systems significantly.

For frequencies on the imaginary axis (as is the case in the integrand of the line component of the self-energy), the polarizability is given by

\[
\Pi_\gamma(q, i\omega') = \frac{(|\omega'| + \gamma)\Pi_0(q, i|\omega'| + i\gamma)}{|\omega'| + \gamma[\Pi_0(q, i|\omega'| + i\gamma)/\Pi_0(q, 0)]}.
\] (2.15)

C. Calculation and Results

Using the expressions given above, we calculate the self-energy for a quantum wire for both pure and impure cases. We briefly discuss some subtleties involved in the numerical
calculation, and then we show our results.

Both the $\Sigma_{\text{ex}}(k)$ and $\Sigma_{\text{line}}(k, \omega)$ are comparatively straightforward to evaluate numerically. However, evaluation of $\Sigma_{\text{pole}}(k, \omega)$ for the clean case ($\gamma = 0$) is complicated by the singularities present in the integrand.

For $\text{Im}[\Sigma_{\text{pole}}(k, \omega)]$, in addition to the contribution from single-particle excitations ($\text{Im}[\epsilon] \neq 0$), there are also contributions from the plasmon excitations (Re$[\epsilon] = 0$ and $|\text{Im}[\epsilon]| = 0^+$), which produce $\delta$-functions in the integrand of $\text{Im}[\Sigma_{\text{pole}}(k, \omega)]$ (see Fig. 2). Care must be taken to pick up this $\delta$-function in a numerical integration. To numerically evaluate the plasmon contribution to $\text{Im}[\Sigma]$, we performed a search along $q$ for Re$[\epsilon(q, \xi_{k+q} - \omega)] = 0$, in the regions Im$[\Pi_0(q, \xi_{k+q} - \omega)] = 0$. When a zero of Re$[\epsilon]$ was found, we evaluated the integral by two methods: (1) introducing a small damping term $\gamma$ in $\epsilon$ and numerically integrating around the Re$[\epsilon] = 0$ region and (2) numerically evaluating $\partial \epsilon(q, \xi_{k+q} - \omega)/\partial q$, which is proportional to the weight of the $\delta$-function. Both methods gave the same results. Since the danger of missing the plasmon peak in the case of small $\gamma$ is always present, we routinely searched for the Re$[\epsilon] = 0$ even when $\gamma \neq 0$.

In the integrand for Re$[\Sigma_{\text{pole}}(k, \omega)]$, at the $q_0$ where Re$[\epsilon] = 0$, there is a $(q - q_0)^{-1}$ principal part divergence, which also can cause numerical problems if it is not handled carefully. We treat this the problem either by cutting off the integrand around $q_0$ when its absolute value exceeded a certain value or by putting in a small damping term $\gamma$, and paying special attention to the integration around $q_0$. Again, both methods gave the same results.

From $\Sigma(k, \omega)$, we can obtain the single-particle spectral function $[8,32]$

$$A(k, \omega) = \frac{2|\text{Im}[\Sigma(k, \omega)]|}{(\omega - \xi_k - \text{Re}[\Sigma(k, \omega)])^2 + (\text{Im}[\Sigma(k, \omega)])^2}.$$  

(2.16)

(Note that Im$[\Sigma]$ is assumed to contain an infinitesimal negative part.) The spectral function $A(k, \omega)$ can roughly be interpreted as the probability density of the different energy eigenstates required to make up a particular $k$-state. It satisfies the sum rule

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} A(k, \omega) = 1,$$  

(2.17)
which is generally satisfied to within less than a percent in all our numerical calculations. We pay particular attention to the self-energy and spectral function at \( k = k_F \), since the behavior of these functions at the Fermi surface determine the low energy properties of the system. The exact solution of the Luttinger model indicates that interactions produce a non-Fermi liquid, the so-called Luttinger liquid. We reproduce this result in clean systems, and show that virtual plasmon emission is responsible for this behavior. We then show that impurities suppress virtual plasmon emission, and the Fermi surface reappears.

A system is a Fermi liquid if it possesses a Fermi surface (i.e., a discontinuity in \( n_k \)) whose presence is indicated by a \( \delta \)-function in \( A(k_F, \omega) \) at \( \omega = 0 \). The existence of a \( \delta(\omega) \) in \( A(k_F, \omega) \) depends crucially on the behavior of \( \text{Im}[\Sigma(k_F, \omega)] \) as \( \omega \to 0 \). If \( |\text{Im}[\Sigma(k_F, \omega)]| \) goes to zero faster than \( |\omega| \), and

\[
\omega - \xi_{k_F} - \text{Re}[\Sigma(k_F, \omega)] \approx Z_F^{-1} \omega,
\]

(2.18)

where \( Z_F \) is a constant called the renormalization factor. [18] then as \( \omega \to 0 \), \( |\text{Re}[\Sigma(k_F, \omega)]| \gg |\text{Im}[\Sigma]| \), and hence

\[
A(k_F, \omega) \approx \lim_{\varepsilon \to 0} \frac{2\varepsilon}{Z_F^{-1} \omega^2 + \varepsilon^2} = 2\pi Z_F \delta(\omega).
\]

(2.19)

Thus, if \( |\text{Im}[\Sigma(k_F, \omega)]| \) goes to zero faster than \( |\omega| \), then there is a discontinuity of magnitude \( Z_F \) in \( n_k \) at \( k_F \). Since \( Z_F \) is the linear coefficient of the expansion in \( \omega \) of the left hand side of Eq. (2.18), it is given by

\[
Z_F = \left| 1 - \left[ \frac{\partial \text{Re}[\Sigma(k_F, \omega)]}{\partial \omega} \right]_{\omega=0} \right|^{-1}.
\]

(2.20)

In contrast, if \( |\text{Im}[\Sigma(k_F, \omega)]| \) goes to zero slower than \( |\omega| \), then the spectral function \( A(k_F, \omega) \) does not take the form Eq. (2.19) as \( \omega \to 0 \), and hence there is no \( \delta \)-function in \( A(k_F, \omega) \), implying that the system is not a Fermi liquid. [16,17,33] In general, interacting three-dimensional systems with and without disorder, and, interacting pure two-dimensional systems are Fermi liquids. [34,35,36] Through a study of \( \text{Im}[\Sigma(k_F, \omega)] \), we show that in one
dimension within the GW approximation, the system is not a Fermi liquid in the absence of impurity scattering, but becomes a Fermi liquid in the presence of scattering.

The imaginary part of the self-energy in the $GW$ approximation, from Eq. (2.11c), can be written as

$$\text{Im}[\Sigma(k,\omega)] = \frac{1}{2} \int_{-\infty}^{\infty} \frac{dq}{2\pi} \left[ \theta(\omega - \xi_{k+q}) - \theta(-\xi_{k+q}) \right] P(q, \xi_{k+q} - \omega), \quad (2.21)$$

where $P(q, E) = 2V_c(q)\text{Im}[\epsilon^{-1}(q, E)]$ which is the Born approximation transition rate for a momentum change $q$ and energy change $E$. Thus, $\text{Im}[\Sigma(k,\omega)]$ can be interpreted as the total scattering rate for an electron with momentum $k$ and energy $\omega$. By setting $\omega = \xi_k$, the actual electron kinetic energy, and one obtains the total Born approximation scattering rate. For $\omega \neq \xi_k$, $\text{Im}[\Sigma(k,\omega)]$ corresponds to the total scattering rate due to virtual transitions, since the transitions violate energy conservation.

At low energies in two and three dimensions, single-particle scattering is far more important than plasmon scattering because the single-particle excitation spectrum is gapless and the phase-space available for single-particle scattering extends around the entire Fermi surface, whereas the plasmon dispersion either rises quickly or has a gap at $q = 0$. Therefore, for small $\omega$, the major contribution to $\text{Im}[\Sigma(k_F,\omega)]$ in two and three dimensions comes from virtual single particle excitations. In contrast, in one dimension, the single-particle excitation spectrum has a gap except at $|q| = 0, 2k_F$, and the phase-space available for single-particle scattering is severely restricted, while the plasmon dispersion is gapless at $q = 0$. Hence, in one dimension, $\text{Im}[\Sigma(k_F,\omega)]$ at small $\omega$ is dominated not by virtual single-particle excitations but by the virtual excitation of plasmons. It is the domination of the plasmon excitations at low energies that give interacting one-dimensional systems their unique non-Fermi liquid behavior.

In the case of a clean quantum wire ($\gamma = 0$), as $\omega \to 0$, the contribution to $\text{Im}[\Sigma(k_F,\omega)]$ of the low energy plasmon excitations goes as $|\omega|/|\ln(|\omega|)|^{1/2}$, whereas the contribution of the single-particle excitations goes as $\omega/(\ln|\omega|)^2$. Thus the plasmon contribution dominates, and since $\text{Im}[\Sigma(k_F,\omega)]$ goes to zero slower than $|\omega|$, the Fermi surface does not exist (in
agreement with Luttinger liquid theory). The Fermi surface is smeared out to the extent that a momentum-space discontinuity in the distribution function does not exist because of the ease with which the particles at the Fermi surface can emit virtual plasmons. In contrast, in two dimensions, the contribution to $\text{Im}[\Sigma(k_F, \omega)]$ of low energy plasmons goes as $\omega^2$, whereas that of the single-particle excitations goes as $\omega^2|\ln(|\omega|)|$ and therefore in the two-dimensional case, the single-particle excitations dominate, and since the self-energy goes to zero faster than $|\omega|$, the two-dimensional interacting electron gas is a Fermi liquid.

The inclusion of impurity scattering causes the electrons to diffuse at long wavelengths which damps out the plasmons at small $q$ (see section V). Hence, the plasmon contribution to $\text{Im}[\Sigma(k_F, \omega)]$ at small $|\omega|$ is removed, and the entire contribution comes from single-particle excitations. In one dimension, it modifies the behavior to $|\text{Im}[\Sigma(k_F, \omega)]| \sim \omega^2|\ln(|\omega|)|^{3/2}$ as $|\omega| \to 0$, which implies that the Fermi surface is restored. This result indicates that the Fermi surface is resurrected in dirty systems because the low energy virtual plasmon emission responsible for its destruction in clean systems has been suppressed by impurity scattering. In two dimensions, however, the situation is reversed. The plasmon contribution was not dominant in the first place, so its removal is not significant. However, inclusion of impurity scattering enhances the single particle scattering rate (because of phase space effects) and it results in a $|\text{Im}[\Sigma(k_F, \omega)]| \sim |\omega|$. This agrees with the notion that an interacting disordered two-dimensional system is not a Fermi liquid. Table I summarizes the behavior of $[\Sigma(k_F, \omega)]$ resulting from various contributions in both the clean and dirty cases, in both one and two dimensions. The details of the calculations are given in Appendices C and D.

In Fig. 3, we show the real and the imaginary parts of the self-energy for $k = 0$ (bandedge) and $k = k_F$ (Fermi surface), for $E = 0, E_F$, for $k_F a = 0.9$ and $r_s = 4e^2/(\pi\epsilon_0 v_F) = 1.4$. In the $k = k_F$, clean ($\gamma = 0$) case, the $A(k_F, \omega)$ has no $\delta$-function at $\omega = 0$. The spectral function goes as $A(k_F, \omega) \sim (|\omega||\ln(|\omega|)|^{3/2})^{-1}$. In the dirty case $\gamma \neq 0$, we have $A(k_F, \omega) \sim |\ln(|\omega|)|^3 + 2\pi Z_F \delta(\omega)$. We can see by comparison of the $\gamma = 0$ and $\gamma = E_F$ lines in Figs. 2(c) that some of the spectral weight around the origin in the $\gamma = 0$ case has been transferred to the $\delta$-function in the $\gamma \neq 0$ case.
In Fig. 4, we show our calculated Fermi distribution function \[18\]
\[n_k = \int_{-\infty}^{0} \frac{d\omega}{2\pi} A(k, \omega) \quad (2.22)\]
for various values of the impurity scattering rate $\gamma$. We emphasize that $\gamma$ was included only in the dynamical screening function and not in the single-electron Green’s function because we wanted to determine if the suppression of the emission of low energy virtual plasmons produces a discontinuity in $n_k$. In the next subsection, we discuss the effects of introducing disorder into the bare Green’s function. Fig. 4(a) clearly shows a discontinuity in $n_k$ at $k = k_F$ for $\gamma/E_F \neq 0$. In Fig. 4(b), we show the calculated $Z_F$ as a function of the impurity scattering rate. For $\gamma = 0$, $Z_F = 0$ indicating that there is no Fermi surface, but as scattering is increased $Z_F$ also increases until it saturates at very large $\gamma$ (where our results should not be trusted because our treatment ignores localization). Note that $Z_F$ goes to zero slowly as $\gamma \to 0$, implying that even a small amount of impurity scattering results in a fairly pronounced discontinuity in $n_k$ at $k_F$. In Fig. 4(c), we show the density of states (per spin)
\[\rho(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{dk}{2\pi} A(k, \omega) \quad (2.23)\]
for two different impurity scattering rates. For a clean wire, there is a slow (inverse logarithmic) disappearance in $\rho(\omega)$ as $\omega \to 0$, indicating the existence of a gap in the density of states at the Fermi surface. For non-zero $\gamma$, our numerical results indicate that $\rho(\omega)$ initially decays as $\omega \to 0$, but then flattens out to a finite value, indicating the absence of a gap (in accordance with Fermi liquid theory). Furthermore, note that in this calculation, we have not included the effects of scattering in the Green’s function in the self-energy (see subsection II D), which would smear the density of states further and also remove the gap associated with the singular behavior of a Luttinger liquid at the Fermi surface.

Fig. 5(a) shows the inelastic scattering rates of quasiparticles in the conduction band $\Gamma(k) = 2 |\text{Im}[\Sigma(k, \omega = \xi_k)]|$ for parameters corresponding to $a = 100 \, \text{Å}$ and a density of $n = 0.56 \times 10^6 \, \text{cm}^{-1}$ in GaAs. Fig. 5(b) shows the corresponding inelastic mean free path,
\( l = v(k)/\Gamma(k) \), where \( v \) is the electron velocity. For \( \gamma = 0 \), below a threshold wavevector \( k_c \), there is no electron-electron scattering (within the RPA) because in a strictly one-dimensional system, conservation of energy and momentum restricts electron–electron scattering to an exchange of particles, which is not a randomizing process because electrons are indistinguishable. \[14\] (Our treatment ignores multiparticle excitations, which will give rise to a nonzero scattering rate for \( k < k_c \).) For \( k > k_c \), a new scattering channel opens in which electrons genuinely emit plasmons (as opposed to the virtual plasmon excitations at the Fermi surface). The inelastic scattering rate diverges as \( \sim (k-k_c)^{-1/2} \) as one approaches \( k_c \) from above, due to the divergence in the density of states available for scattering right at the plasmon emission threshold, analogous to the divergence in the optical-scattering rate in one-dimensional systems. \[12\] In section \[\text{IV}\], we discuss this singular behavior in the scattering rate in greater detail and propose that this, in principle, can be used to fabricate a novel device. For \( \gamma \neq 0 \), the inelastic scattering rate remains finite because the plasmon line is broadened. Furthermore, the breaking of translational invariance relaxes momentum conservation, permitting inelastic scattering via single particle excitations for \( k < k_c \).

In Fig. \[3\], we show the results of the calculation of the the bandgap renormalization, which is the sum of \( \text{Re} \Sigma(k = 0, \omega = \xi_{k=0}) \) of the conduction band electrons and the valence band holes. These results should be useful in explaining photoluminescence experiments in quantum wires, even though our calculation takes into account the screening effects of electrons only, while a photoexcited semiconductor in fact contains a finite density of both electrons and holes. This is justified because we expect the screening effect of the holes to have a negligible effect on the bandgap renormalization, since their large mass prevents them from screening effectively.

Table \[\text{II}\] summarizes the properties of a quasi one-dimensional quantum wire, for both clean and dirty systems.
D. Impurity effects on the bare Green function

So far, we have ignored the effect of the impurity potential on the non-interacting Green function $G_0(k, \omega)$; i.e., we have used the $G_0(k, \omega)$ for the impurity-free case to calculate the many-body properties of impure, disordered quantum wires. In fact, the disorder caused by the impurity potential in one-dimensional systems has a very significant effect, tending to localize all the eigenstates, [43,44] thus significantly changing $G_0(k, \omega)$ from the impurity-free case. In this subsection, we address the question whether using the non-disordered $G_0(k, \omega)$ for the disordered case is justified. We also calculate $G_0(k, \omega)$ for the disordered case for some simple models.

For a particular impurity configuration, assuming the exact non-interacting electron eigenstates $\psi_i(x)$ with eigenenergies $\xi_i$ (with respect to the chemical potential), the non-interacting Green’s function would be diagonal in the basis of these eigenstates, $G_0(ij, \omega) = \delta_{ij} (\omega - \xi_i + i0^+ \text{sign}(\xi_i))^{-1}$. Thus, for the non-interacting case, the distribution function at $T = 0$ would be $n_i = \theta(-\xi_i)$; i.e., the momentum index $k$ in the pure case is simply replaced by the indices enumerating the exact eigenstates. Hence, our calculation corresponds to the situation where the $k$ labels in the self-energy correspond not exactly to the momentum, but to the indices of the exact eigenstates of the disordered noninteracting system. Thus, the discontinuity at the Fermi surface is not a discontinuity in the distribution function in momentum space, but in the representation of the exact eigenstates of the disordered system, and our results should be interpreted in the basis of the disordered eigenstates. The arguments regarding the suppression of plasmons in the disordered system should still hold, since the fact that there are no low energy collective oscillations in disordered systems is independent of whatever basis is used.

If one wanted to calculate $\Sigma(k, \omega)$ in the basis of the momentum eigenstates then the bare Green’s function would have to include the effects of the impurity potential. In this subsection, we calculate the distribution function $n_k$, which is the occupation of the disorder-free momentum eigenstates, for varying amounts of disorder, using two different methods. In
the first, we use the standard impurity averaged scheme \cite{30,45} to calculate, to lowest order in the impurity scattering potential and in the absence of interactions between the electrons, the Green’s function and the momentum distribution function \( n_k \). The Feynman diagram for this approximation is shown in Fig. 7. We show that, as in higher dimensions, the presence of impurities broadens \( n_k \) because the exact eigenstates are superpositions of different \( k \)-states. Since there are no interactions between the electrons, the electrons still form a Fermi liquid, in the sense that there is a sharp discontinuity between the occupation of states above and below the Fermi energy. The second method we use to calculate \( n_k \) involves putting in a term representing impurity scattering in the expression for the exact Green function of the Luttinger liquid. We show that the impurity scattering tends to remove the singularities associated with the non-Fermi liquid nature of the Luttinger liquid. A comparison of both momentum distribution functions indicates that for large enough impurity scattering rate, the noninteracting and interacting distribution functions are essentially indistinguishable, which implies that any singularities in \( n_k \) associated with the interaction is masked by impurity effects.

For the standard impurity self-energy for impurity scattering potentials of the form \( U_0 \delta (r - r_i) \),

\[
\Sigma(k, \omega) = \frac{N_i |U_0|^2}{2\pi} \int_{-\infty}^{\infty} \frac{dk}{\omega - \xi_k - i0^+} = -C \left\{ \begin{array}{ll}
|\omega + \mu|^{-1/2}, & \text{if } \omega + \mu < 0; \\
i(\omega + \mu)^{-1/2}, & \text{if } \omega + \mu > 0;
\end{array} \right.
\]

\begin{equation}
(2.24)
\end{equation}

where \( C = N_i U_0^2 \left( \frac{m}{2} \right)^{\frac{3}{2}} = \gamma_{imp} \sqrt{E_F}/2 \), where \( \gamma_{imp} \) is the impurity Born approximation scattering rate for an electron at the Fermi surface. Defining \( \omega' = \omega + \mu \), the spectral function, from Eq. (2.16) and (2.24), is

\[
A(k, \omega = \omega' - \mu) = \frac{2\pi}{1 + \frac{C}{2} |\omega'|^{3/2}} \delta(\omega' - \omega_0) + \frac{C/\sqrt{\omega'}}{\omega'(\omega' - E_k)^2 + C^2}
\]

\begin{equation}
(2.25)
\end{equation}

where \( \omega_0 \) is the solution of

\[
\omega_0 - E_k = -\frac{C}{\sqrt{\omega_0}}.
\]

\begin{equation}
(2.26)
\end{equation}
The first term on the right hand side of the above equation is from a \( \delta \)-function in the spectral function below the band-edge, corresponding to a bound state.

The momentum distribution function, from Eq. (2.22), is

\[
n_k = \frac{1}{1 + C|\omega_0|^{3/2}/2} + \int_0^\mu \frac{d\omega'}{\pi} \frac{C\sqrt{\omega'}}{\omega'(\omega' - E_k)^2 + C^2}.
\]

(2.27)

The chemical potential \( \mu \) is calculated self-consistently so that, as \( \gamma_{\text{imp}} \) is changed, the total density \( n = \int dk \, n_k/\pi \) is kept constant. Fig. 8 shows \( \mu \) as a function of \( \gamma_{\text{imp}} \), and the corresponding momentum distribution functions. Note that, by keeping only the first order impurity scattering diagram, we are assuming weak scattering. Therefore, the results are unreliable in the large \( \gamma_{\text{imp}} \) (i.e., large disorder) limit, multiple scattering events are ignored. For example, if we included all the diagrams corresponding to repeated scattering off a single impurity, each such diagram with \( n \) impurity lines for a short-ranged interaction contributes a term \( N_i U_0 (iU_0 \sqrt{m/(2\omega + \mu)})^n \), and therefore the entire series can be summed up to give

\[
\Sigma(k, \omega) = \frac{N_i U_0^2}{-i\sqrt{2\omega/m - U_0}}.
\]

(2.28)

Therefore for large \( U_0 \), the \( \Sigma(k, \omega) \) is modified significantly from its lowest order form. There will also be significant contributions from scattering events off two or more impurities.

We also investigate how disorder affects the distribution function \( n_k \) of the Tomonaga-Luttinger model, where the kinetic energy of the electrons is given by the completely linear dispersion \( \xi_k = v_F k \). The exact real-space Green function for the interacting Luttinger model where the interaction term is

\[
\hat{H}_{e-e} = \frac{1}{2} \sum_{q,k,k'} g \frac{\pi v_F}{2} \hat{a}_k^\dagger \hat{a}_{k'}^\dagger \hat{a}_{k'} \hat{a}_k q
\]

(2.29)

and where the interaction \( g \) cuts off for momenta larger that \( \Lambda \) (which would physically correspond to the band-width cutoff in a real system, and which we set equal to \( k_F \)) can be found exactly, and it is of the form

\[
G(x,t) = G_0(x,t) F(x,t),
\]

(2.30)
where $F(x, t)$ is dependent on the interaction strength and bandwidth cutoff.

The standard method for introducing impurities into the noninteracting Green’s function is to introduce an electron lifetime $\tau$ to the Green’s function,

$$G(k, \omega) = \frac{1}{\omega - \xi_k + \frac{i}{2\tau} \text{sign}(\omega)}$$

which gives a real-space representation of $G(x, t)$ of [30]

$$G_{\text{imp}}(x, t) = \frac{1}{2\pi x - v_F t + i\theta^+ \text{sign}(t)} \exp\left(-\frac{|x|}{2v_0 \tau}\right).$$

Therefore, for the Green function in the case where both interaction and disorder exist, we make the plausible replacement of $G_0(x, t)$ by $G_{\text{imp}}(x, t)$ in Eq. (2.30), yielding $G(x, t) = G_{\text{imp}}(x, t)F(x, t)$. From this, the momentum distribution function is computed to be

$$n_k = \frac{1}{2} - \frac{1}{2\pi} \int_{-\infty}^{\infty} dx \frac{\sin(kx)}{x} \exp\left(-\frac{|x|}{2v_F \tau}\right) \left(1 + \Lambda^2 x^2\right)^{-\alpha},$$

where

$$\alpha = ((1 + g)^{1/2} - 1)^2/(8(1 + g)^{1/2}).$$

For $g \ll 1$, $\alpha \approx g^2/32$.

The exact theories assume interactions which are constant (in momentum space) and short-ranged (in coordinate-space), which is clearly not the case for the bare Coulomb interaction in a quasi-one-dimensional quantum wire. However, in many experimental situations, the quantum wires are surrounded by mobile charges from objects such as metallic gates or adjacent quantum wires which would tend to screen the Coulomb interaction between electrons within the same wire. This screening would reduce the range of the interactions, resulting in an effective intrawell interaction $V_{\text{eff}}(q)$ which is finite as $q \to 0$. The effective interaction for interaction for a wire screened by mobile carriers in an adjacent wire, for experimental parameters given in references [4], is on the order of $g = 7.5$, which gives $\alpha = 0.15$ (see Appendix E).

In Fig. 9 we show the the distribution function given by Eq. (2.33), for $\alpha = 0.15$, and various values of $\gamma$. For $\gamma = 0$, the distribution function has an inflection point,
\[ |n_k - 1/2| \sim |k - k_F|^{2\alpha} \] as \( k \to k_F \), which takes the place of the discontinuity of the non-interacting Fermi liquid. The introduction of disorder smears this inflection point out, as it would with a Fermi surface in higher dimensions, and with increasing disorder it becomes indistinguishable from the \( n_k \) shown in Fig. 8 for a non-interacting disordered system. Thus, impurity effects tend to mask the singular behavior of a Luttinger liquid, which would tend to complicate any experimental attempt to measure the the Luttinger liquid singularity in the \( n_k \).

**E. Discussion of the zero-temperature self-energy**

In this subsection, we compare the relative merits of the Tomonaga-Luttinger model and our approach, and we discuss the justification for treating electrons in one-dimensional semiconductor quantum wires as Fermi liquids.

As mentioned previously, interacting one-dimensional systems have been shown to be non-Fermi liquids, through the solution of the Tomonaga-Luttinger model. With the model’s assumptions of infinite density of negative energy electrons in a completely linear dispersion and short-ranged interactions, all the vertex corrections in the self-energy are included and the solution is exact. On the other hand we assume a finite density of electrons in a parabolic energy dispersion and we use the actual Coulomb interaction [20] between electrons for a rectangular well, but we carry out only the leading order self-energy calculation in the dynamically screened interaction. Thus, previous work has concentrated on the exact solution of an artificial model, whereas our work is an approximate solution of a realistic model.

While the solution of the Tomonaga-Luttinger model is mathematically exact, it is difficult to see from the exact solution which physical processes are responsible for the drastic changes in the low-energy behavior of the system and the disappearance of the Fermi surface. Furthermore, since the model linearizes the spectrum about the Fermi surface, it is not reliable in describing physics at energies far away from the Fermi energy, when the band parabolicity becomes important. While our calculation is only approximate, we can use it
to isolate the processes that have a decisive role in determining the low-energy properties of the one-dimensional interacting electron gas. We have shown that, physically, the virtual plasmon emission at low energies is responsible for the disappearance of the Fermi surface. In Appendix F, we show that inclusion of leading order vertex corrections in the form used by Mahan and Sernelius [47] does not change our results at the Fermi surface. We can also explore the high energy processes, such as inelastic scattering of hot electrons, because we have included band parabolicity and we can calculate the band-gap renormalization effects, because unlike the Tomonaga-Luttinger model, our model has a band minimum.

We conclude this section by discussing the viability of treating one-dimensional electron systems as Fermi liquids. One-dimensional electron systems in solids have theoretically strikingly different behavior from their higher dimensional counterparts. In addition to the Luttinger liquid behavior, there are other effects, such as the Anderson [43,44] localization in the presence of disorder and the Peierls lattice distortion in the presence of electron-phonon coupling, which theoretically should drive a one-dimensional electron system away from Fermi liquid behavior. It would seem to be completely inappropriate to think about one-dimensional electron systems as Fermi liquids. We argue, however, that in real semiconductor quantum wires, various factors may serve to stabilize Fermi-liquid behavior.

In one dimension, for non-interacting electrons in the presence of any disorder (again, invariably present in real wires), all states are localized and the concept of a one-dimensional electron gas does not (except in the unrealistic no-impurity idealization) apply. Therefore, in the absence of electron-electron interactions, all the quantum wire electronic states are exponentially Anderson-localized. The situation is again fundamentally different from higher dimensional electron systems where finite disorder strength is required to strongly localize the electron states. We argue, however, that in the state of the art semiconductor quantum wires the effect of Anderson localization is negligibly small because typically the localization lengths are very long (many microns) due to the high quality of the sample and the modulation doping technique. Furthermore, the effects of electron-electron interaction on the localized states of a disordered one-dimensional system are by no means completely under-
stood, and theories exist which indicate that small amounts of disorder may not localize an interacting one-dimensional electron gas. In any case, for weakly disordered quantum wires, the screening effect of the electrons would reduce the effect of the impurity potential and increase the localization length. Thus, so long as the localization length is much longer than the length scale of the experimental probe (e.g., in Raman scattering, that would be the wavelength of the photons), the electrons in the quantum wires may be considered to be extended for all practical purposes.

The presence of coupling of electrons to the acoustic-phonons produces, in the ideal zero-temperature situation, a Peierls distortion, in which the lattice distorts to lower the energy of the electron gas at the expense of a smaller increase in the lattice potential energy. The Peierls distorted state has a bandgap at the Fermi wavevector, and so theoretically quasi-one-dimensional semiconductor quantum wires should be insulators at $T = 0$. However, the electron-phonon interaction via the deformation potential coupling is so small that the Peierls transition temperature is on the order of $10^{-3}$ K, or much less, depending on the density. Furthermore, the transition is suppressed altogether if the impurity scattering rate is larger than the transition temperature, and therefore in semiconductor quantum wires, since the Peierls transition temperature is so low, even a small amount of disorder will remove the transition.

Finally, we have shown that the impurities serve to remove the mechanism (virtual plasmon emission) responsible for destroying the Fermi surface. Therefore, as with the Peierls distortion, the presence of impurities seems to prevent the system from evolving away from Fermi liquid behavior. Thus, for all practical purposes, real semiconductor quantum wires can be considered to be normal one-dimensional Fermi liquid systems.

III. MANY-BODY PROPERTIES OF QUANTUM WIRES
AT FINITE TEMPERATURE

In general, many-body calculations in metals [52] have used the zero-temperature formalism because the energy scales intrinsic to the problem (Fermi energy, plasmon energy, etc) are usually much larger than the temperature $T$. In contrast, in semiconductors, especially in artificial structures of reduced dimensionality, because of the low electron densities and large lattice dielectric constants involved, the experimental temperature can be comparable to the intrinsic energy scales of the electron gas. For example, for a quantum wire density of $5 \times 10^5$ cm$^{-1}$, the Fermi energies are of order $5$ meV $\sim 50$ K. Furthermore, the one-dimensional plasmons are gapless and have a dispersion that is almost linear, implying that there is a large density of low-energy collective excitations. Thus, even at relatively low temperatures, the intrinsic energy scales of the electrons and of collective excitations can be comparable to the temperature. Therefore, the zero-temperature formalism may not provide an adequate description of the system, and the finite-temperature formalism is needed. [53] In this section, we calculate the finite temperature self-energy, again using the $GW$ approximation (i.e., the Feynman diagram shown in Fig. 1(a)).

In the $T = 0$ case, by obeying the Feynman rules, one obtains an integral expression for $\Sigma(k,\omega)$ in the $GW$ approximation. To numerically evaluate $\Sigma(k,\omega)$, we used the so-called “line and pole” decomposition, which is obtained by a contour deformation of the original integral expression for $\Sigma(k,\omega)$. [18,27] The “line and pole” decomposition is more efficient for the purpose of numerical computation than the original expression. However, to the best of our knowledge, this “line and pole” decomposition has not been generalized, in the published literature, to finite-temperature self-energies. In this section, we introduce a general method for obtaining expressions for the real-frequency finite-temperature self-energies, based on analytic continuation of the temperature (Matsubara) Green’s function, which, in the case of the $GW$ approximation, yields the finite-temperature generalization of the “line and pole” decomposition. This method is valid for arbitrary dimensionality. We present the results of our calculations for a semiconductor quantum wire using this formalism.
A. Formalism

Matsubara [54] first introduced a many-body formalism for finite-temperature systems, which is formally identical to the many-body zero-temperature formalism. While the finite-temperature formalism is easier to handle than the zero-temperature formalism in some ways, there is one added complication. With the finite-temperature formalism, one obtains an expression for the electron self-energy $\sigma(i\nu_n)$ that is only valid at discrete points on the complex frequency plane, $i\nu_n \equiv i(2n+1)\pi T$, where $n$ is an integer. However, the quantity that is of interest for physical properties of a system is the self-energy at real frequencies. Therefore, the $\sigma(i\nu_n)$ must be analytically continued to the complex plane to obtain the self-energy $\Sigma(z)$ that is valid for all complex frequencies, [34] (for convenience, at this stage, we have suppressed all arguments of the self-energy except for frequency) from which the real-frequency (retarded) self-energy, the quantity relevant to experiments, can be obtained by setting $z = \omega + i0^+$. Note that in this section, we use the convention that functions denoted by upper case characters are analytic in the frequency variable, while those denoted by lower case characters may be nonanalytic.

In principle, from $\sigma(i\nu_n)$, one can obtain a formal expression for $\Sigma(z)$ in terms of integrals over spectral representations. [18] For example, in the GW approximation, the expression for the real-frequency retarded self-energy is

$$\Sigma(k, \omega) = \int \frac{dq}{(2\pi)^d} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{B(q, \omega')}{\omega + \omega' - \xi_{k+q} - i0^+ \left( n_B(\omega') + n_F(\xi_{k+q}) \right)}$$

(3.1)

where $d$ is the dimensionality of the system, $B(q, \omega)$ is the spectral function of the screened Coulomb interaction, given by

$$B(q, \omega) = 2\text{Re}[W(q, \omega)] = -i \left( W(q, \omega + i0^+) - W(q, \omega - i0^+) \right),$$

(3.2)

and

$$n_F(x) = \frac{1}{\exp(x/T) + 1}$$

(3.3a)

$$n_B(x) = \frac{1}{\exp(x/T) - 1}$$

(3.3b)
are the bose and fermi functions. Expressions for \( \Sigma(\omega) \) obtained in this manner, however, involve integration over one or more frequency variables, which makes them inefficient for use in numerical computations. On previous occasions various approximations such as the plasmon-pole approximation were used to obtain the finite-temperature self-energy. \[53\] The plasmon-pole approximation assumes that the spectral function of the screened Coulomb interaction, \( B(q, \omega) \), can be replaced by a \( \delta \)-function, whose weight and position are given by sum rules. The \( \delta \)-function removes the frequency integration, considerably simplifying the numerical computation. Of course, this is an uncontrolled approximation, and it has been shown that on some occasions it is quantitatively inaccurate, \[55\] and it is desirable to evaluate the expression Eq. (3.1) efficiently, without making any approximations. In this paper, we present a more direct method of analytically continuing the \( \sigma(i\nu_n) \) to \( \Sigma(z) \) which yields an exact expression which is much more amenable to numerical calculation than the integrals over spectral functions. We then apply this method to calculate various many-body properties of electrons in a quantum wire.

This method is based on the principle that the the analytic continuation of \( \sigma(i\nu_n) \) satisfies certain necessary and sufficient conditions, and once these conditions are satisfied, one is ensured that one has obtained the unique \[56\] analytic continuation, \( \Sigma(z) \). These conditions are as follows. (1) \( \Sigma(z) \) is analytic on the entire complex frequency plane, with the exception of possible branch cuts on the real axis \[34\] (henceforth, when we say a function is “analytic” it is with the implicit understanding that it could have branch cuts on the real axis); (2) \( \Sigma(z = i\nu_n) = \sigma(i\nu_n) \) for all \( i\nu_n \); and (3) \( \Sigma(z) \) goes to a constant as \( |z| \to \infty \). The method is based on systematically fulfilling each of the above conditions one by one, leading us directly from \( \sigma(i\nu_n) \) to its analytic continuation \( \Sigma(z) \).

To expound this method, we discuss how it can be applied to a simple example, that of the self-energy within the \( GW \) approximation (see Fig. 1(a)) to obtain the finite-temperature generalization of the “line and pole” decomposition. From the Feynman rules, the expression for the Matsubara self-energy for imaginary frequencies is \[18\]
\[
\sigma(k, i\nu_n) = -\int \frac{d\mathbf{q}}{(2\pi)^d} \sum_{i\omega_n} \frac{V(q, i\omega_n)}{\epsilon(q, i\omega_n)} \frac{1}{i\nu_n + i\omega_n - \xi_{k+q}},
\]

where the summation is over the boson frequencies \(i\omega_n = i2\pi nT\) \((n\) are integers). This expression can be written as a sum of a frequency-independent (and hence analytic with respect to frequency) exchange and a frequency-dependent correlation part, [18]

\[
\sigma(k, i\nu_n) = \sigma_{\text{ex}}(k) + \sigma_{\text{cor}}(k, i\nu_n); \tag{3.5a}
\]

\[
\sigma_{\text{ex}}(k) = -\int \frac{d\mathbf{q}}{2\pi} V_c(q) n_F(\xi_{k+q}), \tag{3.5b}
\]

\[
\sigma_{\text{cor}}(k, i\nu_n) = -\int \frac{d\mathbf{q}}{(2\pi)^d} h_{k,q}(i\nu_n), \tag{3.5c}
\]

where

\[
h_{k,q}(i\nu_n) = T \sum_{i\omega_n} \frac{w(q, i\omega_n)}{i\nu_n + i\omega_n - \xi_{k+q}}. \tag{3.6}
\]

Here,

\[
w(q, i\omega_n) = V_c(q) \left( \frac{1}{\epsilon(q, i\omega_n)} - 1 \right) \tag{3.7}
\]

is the difference between the screened and bare Coulomb interactions. The problem is to analytically continue \(\sigma_{\text{cor}}(k, i\nu_n)\) to \(\Sigma_{\text{cor}}(k, z)\). The most obvious attempt, the simple substitution \(i\nu_n \rightarrow z\) in Eq. (3.6), does not give the desired analytic continuation because there are poles in the integrand \(h_{k,q}(z)\) at \(z = \xi_{k+q} - i\omega_n\), which translate into branch cuts in \(\sigma_{\text{cor}}(k, z)\) at \(\text{Im}[z] = 2\pi nT\) for all \(n\). This clearly violates the condition (1) stated above.

The \(\sigma_{\text{cor}}(k, i\nu_n)\) is obtained by an integral over \(q\) of an imaginary-frequency-dependent integrand \(h_{k,q}(i\nu_n)\), as shown in Eq. (3.5c). Thus, if we obtain analytic continuation \(H_{k,q}(z)\) of \(h_{k,q}(i\nu_n)\), then

\[
\Sigma_{\text{cor}}(k, z) = -\int \frac{d\mathbf{q}}{(2\pi)^d} H_{k,q}(z), \tag{3.8}
\]

automatically satisfies conditions (1) and (2) above for the analytic continuation of \(\Sigma(k, z)\).

Hence, our problem is reduced to constructing function \(H_{k,q}(z)\) such that (1) it is analytic (in the sense mentioned above, \textit{i.e.}, analytic off the real frequency axis), and (2) \(H_{k,q}(z = i\nu_n) = h_{k,q}(i\nu_n)\).
The outline of the procedure for obtaining the function \( H_{k,q}(z) \) which satisfies conditions (1) and (2) is as follows. First, we write down a function \( H_{k,q}^A(z) = h_{k,q}(z) + \tilde{h}_{k,q}(z) \), where \( \tilde{h}_{k,q}(z) \) is chosen so that it cancels all the singularities in \( h_{k,q}(z) \) on the complex plane. Below, we describe a systematic method of obtaining \( H_{k,q}^A(z) \) from \( h_{k,q}(z) \). Thus, \( H_{k,q}^A(z) \) is analytic, fulfilling condition (2). However, \( \tilde{h}_{k,q}(z) \) is in general nonzero at \( z = i\nu_n \), and hence \( H_{k,q}^A(i\nu_n) \neq h_{k,q}(i\nu_n) \), violating condition (2). The second step is therefore to add an additional analytic term \( H_{k,q}'(z) \) which cancels \( \tilde{h}_{k,q}(z) \) at all \( z = i\nu_n \). Since the function \( H_{k,q}^A(z) + H_{k,q}'(z) \) is analytic and equals \( h_{k,q}(z) \) for all \( z = i\nu_n \), fulfilling both conditions (1) and (2), it is the desired analytic continuation \( H_{k,q}(z) \). Condition (3) can be checked in the end; in the case of the GW approximation (and in other cases we have studied) it is satisfied.

In the case of the GW self-energy, the expression obtained from the Feynman rules is

\[
h_{k,q}(z) = T \sum_{i\omega_n} \frac{w(q, i\omega_n)}{z + i\omega_n - \xi_{k+q}}. \tag{3.9}
\]

One can construct an analytic function which contains \( h_{k,q}(z) \) as one of its terms by writing a contour integral

\[
H_{k,q}^A(z) = \int_C \frac{d\omega}{2\pi i} \frac{w(q, \omega)n_B(\omega)}{z + \omega - \xi_{k+q}}, \tag{3.10}
\]

where the contour of integration \( C \) is shown in Fig. 10. \( H_{k,q}^A(z) \) is clearly analytic (off the real axis) in the variable \( z \), since the function \( (z + \omega - \xi_{k+q})^{-1} \) has no poles off the real axis. By the residue theorem, the integral Eq. (3.10) can be evaluated in terms of the residues of the poles contained within the contour \( C \). This gives \( H_{k,q}^A(z) = h_{k,q}(z) + \tilde{h}_{k,q}(z) \). The term \( h_{k,q}(z) \), comes from the poles of \( n_B(\omega) \), as can be seen more clearly by writing \( n_B(\omega) \) as the sum of its poles,

\[
n_B(\zeta) = \frac{1}{2} + T \sum_{i\omega_n} \frac{1}{\zeta - i\omega_n}. \tag{3.11}
\]

In addition to the poles of the bose function, there is also a pole due to the denominator of Eq. (3.9) which gives the second term.

28
\[ \tilde{h}_{k,q}(z) = w(q, \xi_{k+q} - z) n_B(\xi_{k+q} - z). \]  

(3.12)

(Note that \( w(q, \omega) \) is analytic everywhere except for a branch cut on the real axis, and so it does not have poles which contribute to \( H_{k,q}^A(z) \).) Despite being the sum of two nonanalytic functions, \( H_{k,q}^A(z) \) is analytic because the singularities present in both the functions precisely cancel each other. This can be seen clearly if we use Eq. (3.11) to expand the bose function of \( \tilde{h}_{k,q}(z) \), yielding

\[ H_{k,q}^A(z) = h_{k,q}(z) + \tilde{h}_{k,q}(z) \]
\[ = -\frac{1}{2} w(q, \xi_{k+q} - z) + T \sum_{\omega_n} \frac{w(q, i\omega_n) - w(q, \xi_{k+q} - z)}{z + i\omega_n - \xi_{k+q}}, \]  

which explicitly shows that both the denominator and numerator vanish at \( z = \xi_{k+q} - i\omega_n \).

However, because \( \tilde{h}_{k,q}(i\nu_n) \neq 0, H_{k+q}(i\nu_n) \neq h_{k,q}(i\nu_n) \) and hence \( H_{k,q}^A(z) \) fails to fulfil condition (2). This can be remedied by adding an analytic function which cancels \( \tilde{h}_{k,q}(z) \) at all \( z = i\nu_n \). Noting that \( i\nu_n = i(2n+1)\pi T \), for all integers \( n \),

\[ n_B(\xi_{k+q} - i\nu_n) = \left[ \exp(\xi_{k+q}/T) \exp(-(2n+1)\pi) - 1 \right]^{-1} \]
\[ = -\left[ \exp(\xi_{k+q}/T) + 1 \right]^{-1} = -n_F(\xi_{k+q}) \]  

(3.14)

and thus \( \tilde{h}_{k,q}(i\nu_n) = -w(q, \xi_{k+q} - i\nu_n) n_F(\xi_{k+q}) \), implying that the analytic term needed to cancel \( \tilde{h}_{k,q}(i\nu_n) \) is

\[ H'_{k,q}(z) = w(q, \xi_{k+q} - z)n_F(\xi_{k+q}). \]  

(3.15)

Hence, \( H_{k,q}(z) = H_{k,q}^A(z) + H'_{k,q}(z) \) is given by Eqs. (3.13) and (3.15), and the correlation self-energy in the GW approximation is

\[ \Sigma_{cor}(k, z) = -\int \frac{d^d q}{(2\pi)^d} \left[ H_{k,q}^A(z) + H'_{k,q}(z) \right] \]
\[ = -\int \frac{d^d q}{(2\pi)^d} T \sum_{\omega_n} \frac{w(q, i\omega_n)}{z + i\omega_n - \xi_{k+q}} \]
\[ - \int \frac{d^d q}{(2\pi)^d} w(q, \xi_{k+q} - z) \left( n_B(\xi_{k+q} - z) + n_F(\xi_{k+q}) \right). \]  

(3.16)
The retarded self-energy is obtained by setting $z \rightarrow \omega + i 0^+$. The first and second terms on the right hand side of the last equality of Eq. (3.16) are, respectively, the finite-temperature generalization of the so-called “line” and “pole” components of the GW approximation of the $T = 0$ correlation self-energy. As in the zero-temperature case, the line contribution is completely real because $w(q, -i \omega_n) = w^*(q, i \omega_n)$ and hence the total contribution to the imaginary part of $\Sigma(k, \omega)$ comes from the pole part.

As in the $T = 0$ case, in the GW approximation the “on-shell” imaginary part of the self-energy, $|\text{Im} [\Sigma(k, \omega = \xi_k)]|$, is half the sum of the Born-approximation electron and hole scattering rates. The Born-approximation rate for a particle to be scattered with change in momentum $q$ and energy $E$ in the particle is

$$P(q, E) = -2V_c(q) \text{Im}[\epsilon^{-1}(q, E)] n_B(E)$$

Using the identity

$$n_B(\xi_k + q - \omega) + n_F(\xi_k + q) \equiv n_B(\xi_k + q - \omega) [1 - n_F(\xi_k + q)] - n_B(\omega - \xi_k + q) n_F(\xi_k + q),$$

we can write

$$|\text{Im} [\Sigma(k, \omega)]| = \gamma_e(k) + \gamma_h(k)$$

where

$$\gamma_e(k) = - \int \frac{dq}{(2\pi)^d} V_c(q) \text{Im}[\epsilon^{-1}(q, \xi_k + q - \xi_k)] n_B(\xi_k + q - \xi_k) [1 - n_F(\xi_k + q)],$$

$$\gamma_h(k) = \int \frac{dq}{(2\pi)^d} P(q, \xi_k + q) [1 - n_F(\xi_k + q)],$$

are the total Born approximation electron and hole scattering rates, respectively. The factors of $1 - n_F(\xi_k + q)$ and $n_F(\xi_k + q)$ that multiply the Born approximation scattering rates in Eq. (3.19b) are due to the Pauli exclusion principle. An electron (hole) can scatter to a final state $k + q$ only if it is unoccupied (occupied), and the factor $1 - n_F(\xi_k + q)$ ($n_F(\xi_k + q)$) is the probability that the final state is unoccupied (occupied) by an electron from the system.

The method outlined above which we have applied to the GW approximation can also be applied to higher-order diagrams, although the procedure quickly becomes considerably more
complicated. In a higher-order self-energy with an external Matsubara Fermi frequencies $i\nu_m$, the Feynman rules result in an expression with a summation of a function $f(i\nu_m, \omega_{n_1}, \ldots, \omega_{n_N})$, over several boson frequencies $\omega_{n_1}, \ldots, \omega_{n_N},$

$$h(i\nu_m) = \sum_{i\omega_{n_1}} \ldots \sum_{i\omega_{n_N}} f(i\nu_m, \omega_{n_1}, \ldots, \omega_{n_N}) \quad (3.20)$$

In $f(i\nu_m, \omega_{n_1}, \ldots, \omega_{n_N})$, there are several bare Green’s function which contain both $i\nu_m$ and one (or more) internal Bose frequency $i\omega_{n_i}$; e.g. terms of the form $(i\omega_{n_i} + i\nu_m - \xi_{k+q})^{-1}$. (One can always write down the self-energy in a form such that the external frequency occurs only in the bare Green’s functions, and not in the screened Coulomb interactions.) To obtain the analytic function $H^A(z)$ which contains $h(z)$ as one of its term, one writes down an integral

$$H^A(z) = \int_{C_1} d\omega_1 \ldots \int_{C_N} d\omega_N n_B(\omega_1) \ldots n_B(\omega_N) f(z, \omega_1, \ldots, \omega_N), \quad (3.21)$$

where every summation over $i\omega_{n_i}$, has been replaced replaced by an integral over $\omega_i$, $i\nu_m$ is replaced by $z$ and $f$ is multiplied by the a product of bose factors. Since $z$ only appears in the integrand in the form of factors $(\omega + z - \xi_{k+q})^{-1}$, $H^A(z)$ is analytic off the real axis. By using the residue theorem, one can evaluate the integral in terms the poles of the integrand. The $n_B(\omega_i)$ terms give summations over Bose functions, which correspond exactly to the original expression, Eq. (3.20), with $i\nu_m$ replaced by $z$. There are also other terms generated which serve to cancel the singularities of Eq. (3.20) on the complex plane. As in the example cited above for the $GW$ approximation, this function fails condition (2), and therefore analytic functions must be found that cancel the unwanted terms on the points $z = i\nu_m$. Generally, this can be done by inspection of the resulting term, and by use of Eq. (3.14). We have used this method to obtain an expression for the self-energy which is second-order in the screened interaction, which we give in Appendix G.

Alternatively, one can also derive the finite-temperature “line and pole” expression of self-energy from the spectral representation of the $GW$ approximation of the self-energy, given by Eq. (3.1). The integral over the spectral function for the Coulomb interaction can be rewritten as a contour integration over the contour indicated in Fig. 10, with the spectral
function in the integrand replaced by $i^{-1}w(q, \omega)$. Then, by the residue theorem, one obtains precisely the expression obtained above in Eq. (3.16). This procedure also works for higher order diagrams. One first obtains the expression for the self-energy in terms of integrals over spectral functions, and then by writing these integration of spectral functions as contours of the form shown in Fig. [10] and evaluating the contributions due to the poles within the contour, one obtains an expression involving sums over imaginary frequencies.

The main advantage of the expressions generated by the method outlined above is that the frequency integrals in the spectral representation method are generally replaced by imaginary frequency summations, which are numerically more efficient to perform, especially at high temperatures. Unfortunately, with the higher order diagrams, there are terms that still involve frequency integrations, but the number of these frequency integrations is reduced from the spectral representation expression of the self-energy.

B. Results for quantum wire

We applied the formalism developed above to electrons in a semiconductor quantum wire. As mentioned previously, the densities of current quantum wires are necessarily low, and the Fermi energies are on the order of 5 meV $\sim$ 50 K. Thus, even small temperatures may affect their many-body properties significantly. We applied Eq. (3.16) to the calculation of the self-energy, spectral function and band-gap renormalization of electrons in a one-dimensional quantum wire in the extreme quantum limit (i.e., assuming that the electrons only occupy the lowest energy subband).

We used the RPA form for the dielectric function $\epsilon(q, z) = 1 − V_c(q)\Pi_0(q, z)$. Using the familiar expression for $\Pi_0(q, z)$, given by Lindhard, [60] to numerically compute the real part of the polarizability would have been complicated by a principal part divergence in the integrand. Therefore, instead, we used
\[ \Pi_0(q, z; T, \mu) = \begin{cases} 
\int_0^1 \frac{dx}{(x+1)^2} \Pi_0(q, z; T = 0, \mu + T \ln(x)) 
+ \int_0^1 \frac{dx}{(x+1)^2} \Pi_0(q, z; T = 0, \mu - T \ln(x)), & \text{if } \mu > 0; \\
\int_0^{e^{(\mu/T)}} \frac{dx}{(x+1)^2} \Pi_0(q, z; T = 0, \mu - T \ln(x)), & \text{if } \mu < 0; 
\end{cases} \] (3.22)

which is a computationally efficient version of the expression given by Maldague [61] for the finite-temperature RPA polarizability. In performing the summations over imaginary frequencies of the “line” part of \( \Sigma(k, \omega) \), we used the fact that the asymptotic behavior of the components go as \( n^{-5/2} \) as \( n \to \infty \) (the asymptotic form for these components for an infinitesimally thin \( d = 2 \) electron gas is \( n^{-2} \) and for \( d = 3 \) is \( n^{-5/2} \)), which allowed us to use a slightly modified form of the Richardson extrapolation procedure [62] to obtain rapid convergence of the sum. Also, since there are mutually canceling principal part divergences (in the \( q \)-integration) in the \( n = 0 \) term of the “line” part and the real part of the “pole” term, it is advantageous to numerically evaluate both these terms together.

We calculated the real and imaginary parts of the self-energy and the spectral function of a quantum wire as a function of frequency, for several temperatures. The density was kept constant by adjusting the non-interacting electron gas chemical potential \( \mu_0(T) \) so that the integral \( \int dk \ n_F(E_k - \mu_0(T))/\pi \) was kept constant. We show our results in Fig. [11]. The discontinuities and kinks in \( \text{Im}[\Sigma(\omega)] \) at \( T = 0 \), which arise from virtual plasmon and single-particle emission thresholds, broaden with increasing temperature because the plasmon peaks are broadened by Landau damping, and the boundaries of the single-particle continuum are thermally smeared. At non-zero temperatures, a logarithmic divergence develops in the imaginary part of the self-energy at \( \omega = \xi_k[\equiv E_k - \mu_0(T)] \), and is accompanied by a discontinuity in \( \text{Re}[\Sigma(k, \xi_k)] \), since the real and imaginary parts of \( \Sigma(k, \omega) \) are related by the Kramers-Kronig relations. This divergence is due to the non-integrable \( |q|^{-1} \) singularity as \( q \to 0 \) in the integrand Eq. (3.10), which arises from the product of the bose factor, which goes as \( q^{-1} \), and the \( \text{Im}[e^{-1}(q, \xi_{k+q} - \xi_k)] \), which goes as \( \text{sign}(q) \). Physically, this singularity corresponds to the singular divergence in the Born-approximation electron–electron scatter-
ing at small momentum transfer in $d = 1$, and is unique to one-dimensional systems, since in higher dimensions, the singularity in the integral is removed by the phase-space factor of $q^{d-1}$. It is not obvious whether this singularity exists in the exact $\Sigma(k, \omega)$ or is an artifact of the approximations we have used.

In order to calculate the spectral function, one needs to know the finite temperature chemical potential for the interacting system. At $T = 0$, the chemical potential $\mu$ of the interacting system is simply given by Re$[\Sigma(k_F, \omega = 0)]$ (as is required to form a discontinuity or a singularity at the Fermi surface). Unfortunately, at finite temperatures, there is no such obvious prescription to find $\mu(T)$ for the interacting system. In principle, one could search for the $\mu(T)$ which allows the sum rule Eq. (2.17) to be satisfied for all $k$ (we expect Eq. (2.17) to be satisfied because we are using consistent particle-conserving approximations [63]). However, in practice, the integrated spectral weights are not particularly sensitive to changes in the chemical potential and we were not able to determine $\mu$ accurately in the attempts we made. By the same token, the spectral functions themselves are not very sensitive to the choice of chemical potential. We used the $T = 0$ value of the chemical potential of the interacting system, with a temperature correction based on the temperature dependence of $\mu_0(T)$ for the non-interacting system. As expected, the spectral functions broaden with increasing temperature, corresponding to an increase in the quasiparticle decay rates caused by the presence of thermal excitations. The dips at $\omega = \xi_k$, which go as $\sim |\ln(\omega - \xi_k)|^{-1}$, which are brought about by the logarithmic divergences of Im$[\Sigma(k, \omega)]$.

In Fig. 12, we show the electron and hole band-gap renormalization due to the presence of conduction electrons alone. We note again that holes are not expected to change the results significantly because holes screen weakly. Due to the discontinuity in Re$[\Sigma(k, \xi_k)]$, we take $\frac{1}{2}$ Re$[\Sigma(k, \xi_k + 0^+) + \Sigma(k, \xi_k - 0^+)]$ at $k = 0$ to be the band-gap renormalization. We find that for very low densities, where the Fermi temperature is low, the band-gap renormalization can change by approximately an order of magnitude when the temperature increases from $T = 0$ K to $T = 300$ K. Such changes should be measurable in photoluminescence experiments.
Part of the reason why so much excitement has been generated by quantum wires is the possibility that they might have novel device applications. In the early eighties, Sakaki [14] argued that electrons in quantum wires could have extremely large mobilities because elastic collisions with small momentum transfer are suppressed in one-dimensional systems, which would make them very attractive for use in semiconductor devices. More recently, electronic devices based on the quantum nature of electrons [54] and on the single-particle Coulomb charging energies in mesoscopic systems [55] have been proposed. [56] In addition, various proposals for uses of quantum wires as lasers and optical switches have been advanced. [67]

In this section, we propose another interesting device principle of a one-dimensional quantum wire, based on the many-body properties which are peculiar to quasi-one-dimensional systems.

The device principle we propose is based on the many-body properties of an interacting one-dimensional Fermi system. We show that it may be possible to obtain a device with large and sudden onset of negative differential resistance (NDR) (i.e., \( dI/dV < 0 \)). This sudden onset of NDR could be exploited to produce a transistor, while the NDR itself suggests that this device might be used as an oscillator (e.g., in analogy with the Gunn oscillator [68] or, more recently, the resonant tunneling diode [69]). In the proposed device principle, the predicted NDR is associated with a sharp change in the inelastic mean free path, due to electron–electron interactions (or more specifically, electron–plasmon scattering) of the injected electrons at a specific injection energy — in the ideal system at \( T = 0 \), the mean free path changes from being infinite below the threshold energy to being zero above it.

The device principle which we propose may be experimentally observed in the quasi-one-dimensional version of the tunneling hot electron transistor amplifier (THETA), shown schematically in Fig. 13(a), which has been fabricated successfully in three and two dimensions. [70] As in previous sections, we assume that the quasi-one-dimensional device is in the extreme quantum limit; i.e., that all the electrons are in the lowest energy subband.
in the device. Electrons are injected from an emitter at energies above the Fermi energy $E_F$ into a base region which contains (either through doping or electrostatic confinement) a one-dimensional electron gas, and the injected electrons that travel through the base region enter the collector on the opposite side of the base. The fraction of electrons $\alpha$ that reach the collector goes as $\alpha \sim e^{-d/l}$, where $d$ is the width of the base region and $l$ is the electron mean free path. The $l$ is equal to $v_k \Gamma(k)$, where $v_k$ is the electron velocity and $\Gamma(k)$ is the inelastic scattering rate, which is a strong function of the injection energy of the electrons and the electron density in the base region. Thus, by varying the scattering rate by changing the injection energy, one can achieve a significant change in the electron mean free path and hence the emitter–collector current.

In two and three dimensions, the main scattering mechanism for these electrons in the THETA devices are the coupled plasmon–optic-phonon modes. [71] However, in the semiconductor quantum wires in the extreme quantum-limit that are currently being fabricated, the densities of the electrons in the base are so low that all the energy scales associated with the electron gas and operation of the device ($E_F$, plasmon energy and electron injection energy) are much smaller than the optic-phonon energy, and therefore the optic-phonons play a negligible rôle. Acoustic phonons couple very weakly to electrons in III-V semiconductors, [72] and the associated scattering rates are on the order of $10^{10} \, \text{s}^{-1}$ and can be ignored when compared to the scattering mechanism discussed in this paper. We assume, for the purpose of this paper, that impurity scattering in the wires is negligible, which is not unreasonable given the excellent and continually improving techniques for fabricating these mesoscopic systems. This last assumption is equivalent to assuming that the elastic mean free paths are much longer than the inelastic mean free path to be calculated in this paper — given that our calculated inelastic mean free paths are generally a few thousand Å or less, and in good quality quantum wires, elastic mean free paths are many microns, the neglect of impurity scattering is a good approximation for our purposes. Furthermore, the restriction of the scattering phase-space in one-dimension further reduces the scattering rates of both impurity and acoustic phonons. [14] Thus the main scattering mechanism for an injected
electron is the interaction with the electron gas in the base.

We use the Born approximation in our calculation of the scattering rate of the injected electron with the electron gas. The Born approximation calculation of the scattering rate is equivalent to the calculation of $\text{Im}[\Sigma]$ in the $GW$ approximation described in earlier sections of this paper. This approximation includes single-particle scattering and scattering with collective modes of the plasma, but ignores multi-particle excitations, which are expected to be smaller because they are higher order in the screened interaction. However, as we have mentioned in the previous section, in one dimension and at finite temperature, a straightforward integration of the Born approximation scattering probability over all wavevectors yields a total Born scattering rate that is \textit{infinite}, due to the presence of a non-integrable divergence in the number of small wavevector scattering events. These small-wavevector scattering events are associated with single-particle scatterings (in which the injected particle scatters with an individual electron in the electron gas in the base). The problem is that the Born approximation treats the injected particle as an \textit{external distinguishable particle}, and gives the rate at which this particle is scattered out of its initial state. However, in actual fact, the injected electron is indistinguishable from the other electrons in the base, which is very significant in one-dimensional systems, because a single-particle electron–electron scattering event involves an exchange of electrons, implying that the final state after the scattering is exactly the same as the initial state. Therefore, a scattering rate associated with single-particle electron–electron collisions in one-dimension is spurious because these “collisions” do not change the state of the system. It is the transfer of momentum of the injected the particle to the \textit{plasmons} of the system that degrades the current that flows from the emitter to the collector. The plasmons are collective modes associated with the electrons in the base and cannot exist outside the base region, and hence cannot carry current into the collector. Therefore, to determine the mean free path of the nonequilibrium distribution of electrons injected into the base, we should only include the scattering rate of the injected electron with plasmons. Unfortunately, within the Born approximation at finite temperatures, there is no way of separating the spurious single-particle scattering from the plasmon scattering,
since the sharp boundaries and line-widths of the single-particle continuum and plasmon dispersion which exist at $T = 0$ both become thermally smeared and merge into each other as the temperature increases.\[73\]

Our solution to this problem is to calculate the momentum scattering rate, which is the integral of the Born approximation probabilities\[37\] weighted by the loss of momentum,

$$
\Gamma_t(k) = 2 \int_{-\infty}^{\infty} dq \frac{q}{k} V_c(q) \text{Im} \left[ \epsilon^{-1}(q, \omega_k(q)) \right] n_B(\omega_k(q))[1 - n_F(\xi_k+q)],
$$

(4.1)

where $k$ is the momentum of the injected electron, $\omega_k(q) = \xi_k - \xi_{k-q}$ is the energy lost by the electron when it loses momentum $q$, and $\epsilon(q,\omega)$ is the dielectric function calculated within the RPA. $\Gamma_t(k)$ is generally the quantity that is relevant for transport, since it measures the degradation of the electron current. The form Eq. (4.1) negates the effect of the spurious small $q$ single-particle scattering events by giving them small weights and by canceling momentum loss against momentum gain. The main contribution to $\Gamma_t(k)$ should come from coherent peaks associated with the scattering of plasmons. In Fig. 14, we show the results of our calculation of $\Gamma_t(k)$, and the corresponding inelastic mean free path, $l_k = v_k/\Gamma_t(k)$, for various temperatures.

We concentrate on the zero-temperature $\Gamma_t(k)$, the simplest case. At $T = 0$, there are no thermally excited particles that contribute to the spurious single-particle scattering rate, and hence the only contribution to $\Gamma_t(k)$ comes from the plasmon emission. However, not all injected electrons can emit plasmons. Because the plasmon dispersion in quasi-one-dimensional systems for small $q$ goes as $\omega(q) \sim q|\log(qa)|^{1/2}$, where $a$ is the width of the wire, only injected electrons with large enough kinetic energies can emit plasmons (see Fig. 13(b)). For a given density $n$, there is therefore a threshold wavevector $k_c(n)$ below which no plasmon emission can take place. Within the approximations we have used and at $T = 0$, as $k$ is increased through $k_c$, the scattering rate jumps from zero to infinity (equivalently, the mean free path falls from infinity to zero). The divergence in $\Gamma_t(k)$ as $k \to k_c^+$, is $(k - k_c)^{-1/2}$. Below, we derive this form of the divergence, and we show that under very special circumstances, this divergence may be stronger.
At $T = 0$, the electrons can only scatter by plasmon emission if momentum and energy conservation of the system is obeyed (and the final state of the electron $k - q$ is initially unoccupied). One can graphically determine if it is possible to emit plasmons by plotting, on the same graph, the plasmon dispersion curve, $\omega_p(q)$ [given by $\epsilon(q, \omega_p(q)) = 0$], and the energy-loss vs. momentum-loss curve for an electron with initial momentum $k$, $\omega_k(q) = \xi_k - \xi_{k-q}$. Energy and momentum conservation of the system is obeyed and plasmon emission is allowed only when the curves $\omega_p(q)$ and $\omega_k(q)$ intersect. The wavevectors $q^*$ at which $\omega_p(q)$ and $\omega_k(q)$ intersect correspond to wavevectors of plasmons which the electrons are permitted to emit. The scattering rate at $T = 0$ due to plasmon emission is given by

$$\Gamma_p(k) = \sum_{q^*} V_c(q^*(k)) \left| \frac{\partial \epsilon(q, \omega_k(q))}{\partial q} \right|_{q=q^*(k)}^{-1},$$

where the summation is over all intersections $q^*$. We are interested in finding the $T = 0$ behavior of $\Gamma_p(k)$ near $k = k_c$, the minimum injected electron momentum at which the intersection between $\omega_k(q)$ and $\omega_p(q)$ occurs.

Defining $q_c$ and $\omega_c$ as the wavevector and energy of the at which the intersection of $\omega_p(q)$ and $\omega_k(q)$ occurs for $k = k_c$, the following relations hold:

$$\omega_c = \frac{1}{m} \left( k_c q_c - \frac{1}{2} q_c^2 \right),$$

$$\epsilon(q_c, \omega_c) = 0,$$

$$\frac{\partial \omega_k(q)}{\partial q} \bigg|_{q=q_c} = \frac{\partial \omega_p(q)}{\partial q} \bigg|_{q=q_c}.$$  

The first two equations are immediate consequences of the above definitions. The last equality, Eq. (4.3c), results from the fact that, at $k = k_c$, when $\omega_k(q)$ just impinges on the $\omega_p(q)$, the slopes of the two curves are equal (as can be seen by an inspection of Fig. 13(b)).

Let $\omega_1$, $q_1$ and $k_1$ be the deviations away from $\omega_c$, $q_c$ and $k_c$,

$$\omega = \omega_c + \omega_1,$$

$$q = q_c + q_1,$$

$$k = k_c + k_1.$$  

39
In terms of $q_1$ and $k_1$, the energy vs. momentum loss curve is

$$\omega_k(q) = \omega_c + \frac{1}{m}(q_0 k_1 + (k_0 - q_0)q_1 + k_1 q_1 - \frac{1}{2}q_1^2). \quad (4.5)$$

The dielectric function can be expanded around $q_c$ and $\omega_c$ in the form

$$\epsilon(q, \omega) \approx -aq_1 + b\omega_1 + O(q_1^2, \omega_1^2, q_1, \omega_1), \quad (4.6)$$

where $a, b > 0$ (this inequality can be deduced from the asymptotic behavior of $\epsilon(q, \omega)$). Eq. (4.6) implies that the slope is at $\omega_p(q_c)$ is $a/b$ and hence, by Eq. (4.3c),

$$\frac{k_0 - q_0}{m} = \frac{a}{b}. \quad (4.7)$$

Substituting the expression for $\omega_k(q)$ given by Eq. (4.5) into Eq. (4.6) yields

$$\epsilon(q, \omega_k(q)) = -aq_1 + \frac{b}{m}[q_0 k_1 + (k_0 - q_0)q_1] + O(q_1^2, \omega_1^2, \omega_1 q_1)$$

$$= \frac{b}{m}q_0 k_1 + Ak_1 q_1 - Bq_1^2 + \text{cubic order terms}, \quad (4.8)$$

where $A$ and $B$ are coefficients determined by the form of the higher-order expansion of $\epsilon(q, \omega_k(q))$. Note that, by Eq. (4.7), the leading order terms in $q_1$ in Eq. (4.8) have cancelled, giving the following leading order behavior in $k_1$ and $q_1$:

$$\frac{\partial \epsilon(q, \omega_k(q))}{\partial q} \approx Ak_1 - 2Bq_1 \quad (4.9)$$

$$q_1^* \propto \pm k_1^{1/2}.$$

From Eq. (4.2) and Eq. (4.9), the asymptotic form of the Born approximation plasmon scattering rate, as $k \to k_c$, is

$$\Gamma_p(k) \propto \frac{V_c(q_c)}{q_1} \propto k_c^{-1/2} = (k - k_c)^{-1/2}. \quad (4.10)$$

Note that if up to the $(n - 1)$th order term in $q_1$ also cancels away in Eq. (4.8) (which would occur if all derivatives up to $(n - 1)$th order of $\omega_k(q)$ and $\omega_p(q)$ at $q = q_c$ are equal), then $\epsilon(q, \omega) \sim Ak + Bq^2$ which leads to a divergence of the plasmon scattering rate which
goes as $\propto k_1^{(n-1)/n}$. For example, if the curvatures of the curves $\omega_p(q)$ and $\omega_k(q)$ at $q = q_c$ were the same, then one would have a stronger divergence $(k - k_c)^{-2/3}$ in $\Gamma_p(k)$.

This one-sided divergence in the scattering rate indicates that as the bias voltage between the emitter and the base, $V_{eb}$, is increased so that the $k$ of the injected electrons increases above $k_c$ (or alternatively, if $n$ is decreased so that $k_c$ falls below $k$), the jump in the scattering rate should be spectacular. Thus, when $V_{eb}$ is increased through this scattering threshold, the current passing from emitter to collector should fall dramatically. Thermal and impurity effects will broaden the divergence in the scattering rate, but, as can be seen in Fig. 14, this effect persists up to relatively high temperatures.

A divergence in the scattering rate at the optic-phonon emission threshold has also been predicted in undoped one-dimension systems, due to the bandedge $E^{-1/2}$ divergence of the density of final states to which the electrons are scattered. In the case we have discussed, the density of final states is finite (because the electrons are not scattered to the bottom of the band), but there is an inverse square-root (or possibly stronger) divergence in the joint density of states caused by the coupling of the initial and final states via plasmon emission. The advantage of a device based on plasmon emission over one based on optic-phonon emission is that the threshold energy in the former can be tuned by changing the density of the doping in the base region, whereas in the latter it is fixed. In GaAs, optic-phonon emission threshold energy is 36 meV, but at present, the narrowest quantum wires have an energy separation between the first and second subbands on the order of about 5 meV. It will take considerable technical innovation before one has good quality quantum wires in which the subband separation is larger than the optic phonon energy. One more problem with utilizing the optic-phonon principle is that the final states at the bottom of the band must be unoccupied, and therefore one cannot dope the system heavily, making it difficult to make contacts and use it as a three-terminal device.

As the temperature is increased, the divergence becomes a finite peak. This is because the plasmon line broadens due to Landau damping (the collective excitations of the system
can have a finite life-time because they can lose energy to the single particles of the system.

The shift of the peak is due to an upward shift in energy of the plasmon dispersion curve with increasing temperature, which is a well-known phenomenon in plasma physics. \[74\] In one-dimension, using the Vlasov formulation \[74\] (which reduces to the classical limit of the RPA for equilibrium plasmas), the explicitly upward shift in the plasmon dispersion with increasing temperature is given, for small \(q\), by

\[
\omega^2(q) \approx \frac{nq^2V_c(q)}{m}(1 + \frac{3m<v^2>}{nV_c(q)}),
\]

\[4.11\]

\[
<v^2> = \frac{v_F^2}{3} \left[1 + \frac{\pi^2}{4}\left(\frac{k_B T}{E_F}\right)^2\right] + O(T^4),
\]

where \(<v^2>\) denotes the average \(v^2\) over the distribution of the electron gas in the base.

The sharp drop in the mean free path persists to relatively high temperatures (here on the order of tens of degrees for the parameters chosen), and therefore should be experimentally observable. We believe that this sharp drop in the inelastic mean free path should produce a large NDR in quantum wires as the injected electrons pass through the threshold energy.

We conclude by mentioning that in higher dimensional electron systems there is a plasmon threshold as well at the onset of plasmon emission. The effect in higher dimensions, however, is not dramatic because the ideal mean free path does not change from being infinite below the threshold to zero above (as it does in the one-dimensional system) since single particle scattering contributes in higher dimensions, in contrast to one dimension. Thus, our proposed NDR in quantum wires is a specific one-dimensional many-body property. We also mention that a doping density of approximately \(10^6\) cm\(^{-1}\) implies that a base region of, say, 3000 Å has only 30 electrons, which may cast doubt on the validity of our theory. However, various simulations of very small number of particles (on the order of 10) in interacting systems (for example, in the fractional quantum Hall regime or the Hubbard model) are sufficient to show collective effects, and therefore it is entirely plausible that the predicted effects will be seen even with a relatively small number of electrons in the base.

Furthermore, with further improvements in technology, the width of the wires will decrease,
leading to an increase in the allowable doping density and the number of particles in the base region.

V. PLASMONS IN QUANTUM WIRES

It is apparent from the foregoing discussions that the behavior of the plasmons, especially in the presence of scattering and at finite temperatures, play an important rôle in the many-body physics of electrons in quantum wires. Therefore, it seems appropriate for us to conclude this paper with a brief discussion within the random phase approximation of the effects of impurities and finite temperature on the plasmon dispersion in quantum wires.

The RPA plasmon dispersion is given by [18,37] the vanishing of the dielectric function \( \epsilon(q,\omega) = 1 - V_c(q)\Pi_0(q,\omega) \). The full dispersion for a pure one-dimensional system at \( T = 0 \) within the RPA is [21,29]

\[
\omega_p(q) = A(q)\left[\frac{\omega^2(q) - \omega^2(q)}{A(q) - 1}\right]^{1/2},
\]
where \( A(q) = \exp[q\pi/mV_c(q)] \). An expansion [29] to second order in \( q/k_F \) gives,

\[
\omega_p(q) = |q|\left[v_F^2 + \frac{2\pi v_F V_c(q)}{|\ln(qa)}\right]^{1/2} + O(q^3),
\]
which agrees exactly with the dispersion of the elementary excitations in the Tomonaga-Luttinger model, a point which we return to later. For any reasonable confinement model \( V_c(q \to 0) \sim |\ln(qa)| \) and, therefore, \( \omega_p(q) \sim |q| |\ln(qa)|^{1/2} \) in one dimension. The fact that these one-dimensional plasmons are ungapped at \( \omega \to 0 \) is simply a consequence that the Coulomb interaction does not provide long-ranged restoring forces for charge density perturbations in one-dimension.

In this section, we calculate the plasmon dispersion for a one-dimensional quantum wire for an impure system and at finite temperatures, by numerically finding the zeros of the dielectric function on the complex frequency plane, i.e., the complex frequencies \( \omega_p(q) \) such that \( \epsilon(q,\omega_p(q)) = 0 \). The \( \text{Re}[\omega_p(q)] \) gives the frequency of plasma oscillation, while the
Im[\omega_p(q)] gives the damping rate of the mode. As in the previous sections, we use the \( V(q) \) for electrons confined in an infinite square well of width \( a \), and the RPA form of the dielectric function, modified by the Mermin formula to include effects of impurities. Since the modes oscillate as \( e^{-i\omega_p(q)t} \), the condition of stability of the system implies that \( \text{Im}[\omega_p(q)] \leq 0 \), and thus we search for the zeros of \( \epsilon(q, z) \) on the lower-half complex frequency plane. Note that for \( \text{Im}[z] < 0 \), \( \epsilon(q, z) \) is given by the analytic continuation from the upper-half to the lower-half complex frequency plane. Thus, the polarizability in the RPA goes as

\[
\Pi_0(q, z) = \frac{m}{(\hbar^2)q\pi} \left[ \int_{-\infty}^{\infty} dk \frac{f_{\text{eq}}(k + q/2) - f_{\text{eq}}(k - q/2)}{k - z/q} \right. \\
+ 2\pi i \left( f_{\text{eq}}(z/q + q/2) - f_{\text{eq}}(z/q - q/2) \right) \theta(-\text{Im}[z]) \right],
\]

where the second term in Eq. (5.3) is required for the analytic continuation for \( \Pi_0(q, z) \) on the lower-half complex plane.

As mentioned in section II, the presence of impurity scattering within the wire causes electrons to be diffusive at long times and large distances. This behavior of the electrons overdamps the plasmons and causes them to disappear at small \( q \), as in the two dimensional case, and this disappearance of the plasmon spectral weight at small \( q \) has important consequences for the characteristics of the Fermi surface of a one-dimensional electron gas, as we have shown in section II. Here, we explicitly show in Fig. 15(a) this disappearance by calculating the plasmon dispersion for a one-dimensional quantum wire in the presence of impurity scattering. In the inset of Fig. 15(a), we show the experimental one-dimensional plasmon dispersion compared with the RPA theory. We trapped the zeros of the dielectric function by using the simplex method to search for the minima of \( |\epsilon(q, z)|^2 \). The real and imaginary parts of \( \omega_p(q) \) are plotted above and below the \( x \)-axis, respectively. One can clearly see the overdamping of the plasmon mode \( i.e., \text{Re}[\omega_p(q)] = 0 \) as \( q \) goes to zero at a critical wavevector \( q_d \). With increasing impurity scattering, \( q_d \) increases, as can be seen from the expansion of the dielectric function. For small \( q \) and \( \omega \), the polarizability within the Mermin formula can be expanded to yield
\[ \Pi(q, \omega) \approx \frac{nq^2}{m\omega(\omega + i\gamma)}. \] (5.4)

This form can also be obtained by the standard replacement of \( \omega^2 \) by \( \omega(\omega + i\gamma) \) in the denominator of the usual high-frequency result for the pure system. \[37\] With this form of Eq. (5.4), the plasmon dispersion is given by

\[ \omega(q) \sim -i\frac{\gamma}{2} \pm \sqrt{-\frac{\gamma^2}{4} + \frac{nq^2}{m} V(q)}. \] (5.5)

From Eq. (5.4), the critical \( q_d \) at which \( \omega \) becomes completely imaginary is

\[ q_d = \frac{K\gamma}{|\ln(Ka\gamma)|}, \] (5.6)

where \( K = \sqrt{m\epsilon_0/(8ne^2)} \). In the overdamped region, \( \text{Im}[\omega_p(q)] \) decreases with increasing \( \gamma \) because a larger \( \gamma \) suppresses the electron diffusion rate, slowing the relaxation of the long-wavelength density perturbations.

We next examine the effect of finite temperature on the plasmon dispersion. Finite temperature effects are included by calculating \( \Pi_0(q, \omega) \) at \( T \neq 0 \), using Eq. (5.3) above, where the \( f_{eq}(k) \) is given by the Fermi functions at temperature \( T \). The results of our calculations are shown in Fig. 15(b). In contrast to the case where impurity scattering exists, at finite temperatures, the plasmon mode at low \( q \) remains undamped. This is because Landau damping is caused by the transfer of energy from the collective mode to particles traveling at the same velocity as the (phase) velocity of the collective mode, and, therefore, significant Landau damping at a wavevector \( q \) can occur only if \( f_{eq}(\omega_p(q)/q) \) is non-negligible; i.e., \( \omega_p(q)/q \) must be less than or on the order of the thermal or Fermi velocity (whichever is larger). At small \( q \), the plasmon phase velocity \( \omega_p(q)/q \sim |\ln(qa)|^{1/2} \to \infty \), and hence Landau damping is negligible in this limit. As \( q \) increases, the phase velocity of the plasmon decreases and hence Landau damping increases.

Before we conclude this section, we argue that the plasmons that we have described above (which are associated with the vanishing of the dielectric function) are precisely equivalent to the Tomonaga-Luttinger bosons which are the exact elementary excitations of the
Tomonaga-Luttinger model. In fact, as mentioned earlier, the dispersion of the Tomonaga-Luttinger bosons to order $q^3$ is exactly the same as the plasmons calculated within the RPA, a fact which may be attributed to the fact that only RPA-type Feynman diagrams (i.e. the bubble or ring diagrams) do not cancel out in the Tomonaga-Luttinger model. As further evidence of the equivalence of the Tomonaga-Luttinger bosons and what we call plasmons, we note the similarity of the underlying physical notions of both concepts. Plasmons are simply long-lived charge density excitations of the one-dimensional electron gas. Since Tomonaga-Luttinger bosons are created theoretically by acting on the ground-state wavefunction by charge density operators, they are also elementary excitations caused by charge density perturbations. Therefore, both “Tomonaga-Luttinger bosons” and “plasmons” are simply different names ascribed to the same physical phenomenon. This may explain why the experimental results for one-dimensional plasmon dispersion (Fig. 15(a) inset) agree so well with the RPA theory, which generally is regarded to be a poor approximation in lower dimensions.

A similar correspondence between the spin excitations of the Tomonaga-Luttinger model and that of the single-particle spin density continuum can also be made. In the Tomonaga-Luttinger model, with no spin-dependent interactions, the spin-density excitations have a linear dispersion $\omega = v_F q$. The spin-density excitations (described by the imaginary part of the spin–spin correlation function, which, within the RPA, is exactly equivalent to the irreducible charge-density polarizability $\Pi_0(q, \omega)$) also have a dispersion of $\omega = v_F q$ in the limit that $q \to 0$. Unlike the single-particle charge excitations, which have a vanishing spectral weight as $q \to 0$ for the RPA in one-dimension (all the spectral weight is pushed up to the plasmon [Tomonaga boson] excitations), the spin-density excitations have a nonvanishing spectral weight in the small $q$ limit. This is because the spectral weight of the charge excitations is given by the screened (reducible) polarizability, $\Pi_0(q, \omega)/\epsilon(q, \omega)$, which is screened out at small $q$ by the dielectric function, whereas the spectral weight of the spin excitations is given by the unscreened (irreducible) polarizability $\Pi_0(q, \omega)$, which is not suppressed by the screening at small $q$. 

46
VI. SUMMARY

In this paper, we have studied the many-body properties of semiconductor quantum wires by utilizing a model that is more realistic than the exactly solvable Tomonaga-Luttinger model, but which we have solved approximately using the $GW$ approximation, a renormalized perturbative scheme that has been employed with success in higher dimensions. Our approximate calculation provides us an opportunity to isolate the physical processes which are responsible for the unique properties of quasi-one-dimensional interacting systems. In particular, we find that the well-known result that inter-particle interactions cause the Fermi surface to disappear in one dimension is due to the virtual excitation of the plasmons at the Fermi surface. When impurity scattering is included, the Fermi surface *reappears* because the low energy virtual plasmon excitations which are responsible for the disappearance of the Fermi surface are suppressed by impurity scattering. This is consistent with experimental findings which seem to indicate the existence of a sharp Fermi surface in semiconductor quantum wires through large Fermi-edge singularities. Furthermore, theoretical studies indicate that for a pure Luttinger liquid which is incompressible (*i.e.*, where the velocity of the plasmons goes to infinity as $q$ goes to zero, as is the case studied here) there is no enhancement in the absorption cross section at the Fermi surface.

While both our model (for zero impurity scattering) and the Tomonaga-Luttinger model exhibit non-Fermi liquid behavior with the interacting system having no real Fermi surface, there are important differences between the two models. In our model, $\text{Im}[\Sigma(k_F, \omega)] \sim |\omega| |\ln |\omega||^{1/2}$ for small $\omega$ whereas in the Tomonaga-Luttinger model, $\text{Im}[\Sigma(k_F, \omega)] \sim |\omega|^\alpha$, where the parameter $\alpha (< 1)$ depends on the short-range interaction constant of the model. The momentum distribution function $n_k$ in our model has a logarithmic inflection point at $k_F$ whereas in the Tomonaga-Luttinger model the inflection point in $n_k$ is a power law. We emphasize that our model is an approximate (leading-order perturbative) solution of a realistic model of one-dimensional electrons interacting via the long-range Coulomb interaction whereas the Tomonaga-Luttinger model is an exact solution of an artificial model of
electrons with linear energy dispersion and infinite band width interacting via a short-range interaction (in particular, an interaction which is finite at $q = 0$). An important result of our calculation is that in the presence of impurity scattering the two models produce essentially numerically indistinguishable results.

We have also calculated the finite-temperature self-energy of a semiconductor quantum wire. To perform this calculation in a numerically efficient manner, we have developed a formalism for directly analytically continuing the thermal Matsubara self-energy to real frequencies. Applying this method to the $GW$ approximation, we obtain the finite-temperature generalization of the “line and pole” decomposition, which in which the real-frequency integration in the spectral representation is converted into a more computationally efficient imaginary-frequency sum.

In the absence of impurity scattering and at $T = 0$, the leading order inelastic electron–electron scattering rate is zero for electrons with energies below the threshold energy for emission of plasmons but diverges at the plasmon threshold itself. The divergence as one approaches the threshold from above is generally inverse square-root, and is due to the fact that the joint density of states of the electron states connected by the plasmon emission has the inverse square-root divergence (there are very special circumstances when the joint density of states is larger, and the divergence is in fact stronger). This sudden onset of a very large $\Gamma(k)$, which survives both the effects of impurity scattering and finite temperature, can, in principle, be used in a one-dimensional THETA-type device with the possibility of a large negative differential resistance. This characteristic has potential applications in switching devices or oscillators.

We have calculated the plasmon dispersion, including the imaginary part which describes the damping of the plasmons, for impure wires and at finite temperatures. Impurity scattering causes the plasmons to overdamp and disappear at small $q$, but finite temperature does not because the phase velocity of the plasmons diverges as $q \to 0$.

We also calculated the bandgap renormalization at zero and finite temperatures. It is found to be on the order of $10 – 20\text{ meV}$ for typical experimental parameters, which are
consistent with the currently available experimental results (which have, albeit, multiple-subband occupancy). We find that the band-gap renormalization can change significantly between $T = 0$ K and $300$ K for wires with very low electron densities.

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APPENDIX A: FORMALISM FOR SELF-ENERGY $\Sigma_{ij}(k, i\nu_n)$

FOR THE CASE OF MANY SUBBANDS

In this appendix, we give the formalism for calculating the self-energy $\Sigma_{ij}(k, i\nu_n)$ for the case where several subbands are relevant. In this case, the interaction matrix elements and the Green’s functions are labeled by energy subband indices corresponding to the incoming and outgoing electron lines, in addition to having the momentum label for the $x$-direction. (We employ the thermal Green function formalism in this appendix.)

When several subbands are relevant, the Coulomb matrix element has indices relating to the transitions from different subbands. We define $V_{ij lm}(q)$ as the scattering matrix element for $i \to j$ and $l \to m$, given by

$$V_{ij lm}(q) = \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy' v(q, y - y') \phi_i(y) \phi_j^*(y) \phi_l^*(y') \phi_m^*(y'),$$  \hspace{1cm} (A1)

where $v(q, y - y')$ is given by Eq. (2.3).

The Dyson equation is written as a matrix equation

$$G(k, i\nu_n) = G_0(k, i\nu_n) + G_0(k, i\nu_n) \Sigma(k, i\nu_n) G(k, i\nu_n)$$  \hspace{1cm} (A2)

where the underline indicates the fact the the Green functions and self-energies are matrices and the products are matrix multiplications. The $G_0^{ij}$ is the bare noninteracting Green function.
\[
G_{ij}^{0}(k, i\nu_n) = \frac{\delta_{ij}}{i\nu_n - \xi_k - E_j}, \quad (A3)
\]

where \(E_j\) is the subband energy. Solving Dyson’s equation gives

\[
\mathbf{G} = (\mathbf{G}^{0})^{-1} - \mathbf{\Sigma}^{-1}. \quad (A4)
\]

The quasiparticle energies are then given by the poles of \(\mathbf{G}\): \textit{i.e.}, at the frequencies such that

\[
\text{det}[(\mathbf{G}^{0})^{-1} - \mathbf{\Sigma}] = 0.
\]

The self-energy in the \textit{GW} approximation [24] which is attached to external electron lines from subbands \(i\) and \(j\) is given by

\[
\Sigma_{ij}(k, i\nu_n) = -T \sum_l \sum_m \int \frac{dq}{2\pi} W_{ilj}(q, i\nu_m) \mathbf{G}^{0}_{ll}(k + q, i\nu_n + i\nu_m) \quad (A5)
\]

where \(W_{ijlm} = \langle il | \hat{W} | jm \rangle\) is the screened interaction between scattering states \(i \rightarrow j\), and \(l \rightarrow m\), and the frequency summation is over the Bose frequencies \(i\nu_m = i2\pi mT\).

To calculate the components of the self-energy one must know the components of the screened Coulomb interaction \(W\). Dyson’s equation for the screened interaction, for the random phase approximation (shown in Fig. [1(b)]), which translates to

\[
\mathbf{W} = \mathbf{V} + \mathbf{V} \mathbf{P} \mathbf{W}, \quad (A6)
\]

where \(\mathbf{P}\) is the bare polarizability (given by the bubble in the random phase approximation).

[Note that in Eq. (A6), the matrix multiplication implies summation over two of the four indices, \textit{i.e.}, in \(W_{ijlm}\) the “row” index is \(i\) and \(j\), and the “column” index is \(m\) and \(n\), and therefore the multiplication implied by, say, \(X = \mathbf{W} \mathbf{V}\) is \(X_{mn} = \sum_{\mu\nu} W_{ij\mu\nu} V_{\mu\nu mn}\).]

We can solve for \(\mathbf{W}\) from Eq. (A6), yielding

\[
\mathbf{W} = \mathbf{\xi}^{-1} \mathbf{V}, \quad (A7)
\]

where

\[
\mathbf{\xi} = 1 - \mathbf{V} \mathbf{P} \quad (A8)
\]

is the dielectric matrix. Hence, to find \(\mathbf{W}\), we must invert the dielectric matrix.
The components of the bare polarizability matrix are given by

$$\mathcal{P}_{ijlm}(q, i\nu_n) = \delta_{ii}\delta_{jm}\Pi_{lm}(q, i\nu_n)$$

(A9)

where, in the random phase approximation,

$$\Pi_{lm}(q, i\nu_n) = 2\int\frac{dk}{2\pi} \frac{f_l(k+q) - f_m(k)}{\xi_{k+q} + E_l - \xi_k - E_m - i\nu_n}$$

(A10)

where $f_m(k)$ is the equilibrium Fermi distribution in subband $m$. Therefore, the components of the dielectric matrix are

$$\epsilon_{ijlm}(q, i\nu_n) = \delta_{ii}\delta_{jm} - V_{ijlm}(q)\Pi_{lm}(q, i\nu_n).$$

(A11)

Thus, all the ingredients for obtaining $W$ are available, and computing its components is simply a matter of algebra.

We now give of the expressions for the $W_{ijlm}(q, i\nu_n)$ for cases where (i) there is one relevant subband, and, (ii) there are two relevant subbands in a symmetric wire. In a symmetric wire, the transverse wavefunctions $\phi_n(y)$ are symmetric (antisymmetric) about $y = 0$ for even (odd) $n$, and since the Coulomb potential $v(q, y)$ is symmetric about $y$, Eq. (A1) leads to the following symmetry relationships:

$$V_{ijlm} = 0 \text{ if } i + j + l + m = \text{odd}$$

(A12)

and

$$V_{ijlm} = V_{jilm} = V_{ijml} = V_{jiml} = V_{lmij} = V_{mlij} = V_{mlji} = V_{lmji}$$

(A13)

(i.e., the ordering of the indices in the first two or the last two, or the ordering of the pair of indices, does not matter). These symmetry relations simplify computations significantly.

In the first case, when the only one relevant subband, as in the case discussed in the main text, then all equations are scalar equation, and the screened interaction is (as in the translationally invariant 3-dimensional case)

$$W_{1111}(q, i\nu_n) = \epsilon_{1111}^{-1}(q, i\nu_n) V_{1111}(q) = \frac{V_{1111}(q)}{1 - V_{1111}(q)\Pi_{11}(q, i\nu_n)}.$$
In the second case, where there are two relevant subbands for a wire that symmetric about the $y$-axis, the dielectric matrix is

$$
\epsilon = \begin{pmatrix}
11 & 22 & 12 & 21 \\
11 & 1 - V_{1111} \Pi_{11} & -V_{1122} \Pi_{22} & 0 & 0 \\
22 & -V_{1122} \Pi_{11} & 1 - V_{2222} \Pi_{22} & 0 & 0 \\
12 & 0 & 0 & 1 - V_{1212} \Pi_{12} & -V_{1212} \Pi_{21} \\
21 & 0 & 0 & -V_{1212} \Pi_{12} & 1 - V_{1212} \Pi_{21}
\end{pmatrix}, \quad (A15)
$$

where we have used the symmetry relations of Eqs. (A12) and (A13).

Taking the inverse of $\epsilon$ gives

$$
\epsilon^{-1} = \begin{pmatrix}
11 & 22 & 12 & 21 \\
11 & \frac{1 - V_{1122} \Pi_{22}}{\epsilon_{\text{intra}}} & \frac{V_{1122} \Pi_{22}}{\epsilon_{\text{intra}}} & 0 & 0 \\
22 & \frac{V_{1122} \Pi_{11}}{\epsilon_{\text{intra}}} & \frac{1 - V_{1111} \Pi_{11}}{\epsilon_{\text{intra}}} & 0 & 0 \\
12 & 0 & 0 & \frac{1 - V_{1212} \Pi_{21}}{\epsilon_{\text{inter}}} & \frac{V_{1212} \Pi_{21}}{\epsilon_{\text{inter}}} \\
21 & 0 & 0 & \frac{V_{1212} \Pi_{12}}{\epsilon_{\text{inter}}} & \frac{1 - V_{1212} \Pi_{12}}{\epsilon_{\text{inter}}}
\end{pmatrix}, \quad (A16)
$$

where

$$
\epsilon_{\text{intra}}(q, i \nu_n) = \left(1 - V_{1111}(q) \Pi_{11}(q, i \nu_n)\right)\left(1 - V_{2222}(q) \Pi_{22}(q, i \nu_n)\right)
$$
$$
- V_{1122}^2(q) \Pi_{11}(q, i \nu_n) \Pi_{22}(q, i \nu_n) \quad (A17)
$$
$$
\epsilon_{\text{inter}}(q, i \nu_n) = 1 - V_{1212}(q) \left(\Pi_{12}(q, i \nu_n) + \Pi_{21}(q, i \nu_n)\right)
$$

correspond to the intra-level and inter-level dielectric functions, respectively (the zeros of which give the collective modes of the systems. [21]) The screened interaction matrix is given by

$$
W = \epsilon^{-1} V = \begin{pmatrix}
11 & 22 & 12 & 21 \\
11 & \frac{V_{1111}(1 - V_{2222} \Pi_{22}) + V_{1122} \Pi_{22}}{\epsilon_{\text{intra}}} & \frac{V_{1122} \Pi_{22}}{\epsilon_{\text{intra}}} & 0 & 0 \\
22 & \frac{V_{1122}}{\epsilon_{\text{intra}}} & \frac{V_{2222}(1 - V_{1111} \Pi_{11}) + V_{1111} \Pi_{11}}{\epsilon_{\text{intra}}} & 0 & 0 \\
12 & 0 & 0 & \frac{V_{1212}}{\epsilon_{\text{inter}}} & \frac{V_{1212}}{\epsilon_{\text{inter}}} \\
21 & 0 & 0 & \frac{V_{1212}}{\epsilon_{\text{inter}}} & \frac{V_{1212}}{\epsilon_{\text{inter}}}
\end{pmatrix}, \quad (A18)
$$
Eq. (A5) is then used to obtain the self-energy $\Sigma_{ij}(k, i\nu_n)$.

We now briefly discuss the effect of the second subband in the case where only the first subband is occupied. Then $\Pi_{22} = 0$, and

$$\Sigma_{11}(k, i\nu_n) = -T \sum_m \int dq \left( \frac{V_{1111}(q)G^0_{11}(k+q, i\nu_n+i\nu_m)}{1-V_{1111}(q)\Pi_{11}(q, i\nu_m)} + \frac{V_{1212}(q)G^0_{22}(k+q, i\nu_n+i\nu_m)}{1-V_{1212}(q)(\Pi_{12}(q, i\nu_m)+\Pi_{21}(q, i\nu_m))} \right).$$  \hspace{1cm} (A19)

The first term in the integrand of Eq. (A19) is exactly the same as the expression we used in the main text to calculate the self-energy, when we ignored the higher subband contribution. The second term is the additional contribution of the second subband. The denominator of the second term is the intersubband dielectric function, \[21\] which only becomes significant for energies on the order of the intersubband energy separation. Thus, for the processes in which the energy is much less than the subband energy separation, as we have assumed in this paper, the higher subband contribution is negligible and can be ignored.

**APPENDIX B: CALCULATION FOR $\Sigma(k_F, \omega = \xi_{k_F})$ FOR SECOND ORDER DIAGRAM**

We show that for a short-ranged interaction, the second order diagram shown in Fig. [I(c) evaluated at $k_F$ and $\omega = 0$ is identically zero.

For $V_c(q) = V_0$ expression for the self-energy is \[18\]

$$\Sigma(k, \omega) = -\frac{V_0^2}{(2\pi)^2} \int dq \int dq' \frac{1}{qq'} \left( n_F(\xi_{k+q})[n_F(\xi_{k+q}) - n_F(\xi_{k+q+q'})] + n_F(\xi_{k+q+q'})(1 - n_F(\xi_{k+q})) \right)$$

\hspace{1cm} (B1)

The Fermi functions restrict the four integration regions to the $q, q'$ space shown in Fig. [I].

We calculate the contribution for each region.

For regions I, II, and III, we obtain

$$I = \lim_{\epsilon \to 0} \int_0^\epsilon \frac{dq}{q} \int_{-\epsilon}^\epsilon dq' \frac{dq'}{q'} = \int_0^1 \frac{dx}{x} \int_{1-x}^1 \frac{dx'}{x'}$$

53
\[ \text{II} = \int_0^{2k_F} \frac{dq}{q} \int_0^{2k_F} \frac{dq'}{q'} = \int_0^1 \frac{dx}{x} \int_{1-x}^1 \frac{dx'}{x'} = \frac{\pi^2}{6} \]  

\[ \text{III} = \int_{-\infty}^{-2k_F} \frac{dq}{q} \int_{-q}^{-2k_F} \frac{dq'}{q'} = \int_{-\infty}^{-1} \frac{dy}{y} \int_{-y-1}^{-y} \frac{dy'}{y'} \]

\[ = \int_{-\infty}^{-1} \frac{dy}{y} \ln \left( \frac{y}{1+y} \right) = \int_0^1 \frac{dx}{x} \ln(1-x) = -\frac{\pi^2}{6} \]  

\[ (B2) \]

A calculation similar to that of region III also gives a contribution of \(-\pi^2/6\) from region IV. Therefore, the second order contribution to \(\Sigma(k_F, \omega = 0)\), which is given by the sum of the contributions from I, II, III and IV, is identically zero.

**APPENDIX C: FORM OF \( \Sigma(k_F, \omega) \) AS \( \omega \to 0 \) IN THE GW AND RPA APPROXIMATION IN ONE DIMENSION**

In this appendix, we give the details of the calculation yielding the \(\omega \to 0\) forms of the imaginary part of the retarded self-energy in the one-dimensional electron gas, within the GW approximation, both with and without impurities.

First, we define two useful energy scales

\[ E_F = \frac{k_F^2}{2m}, \quad E_{\text{pot}} = \frac{ne^2}{\epsilon_0} = \frac{2e^2k_F}{(\pi\epsilon_0)}. \]  

\[ (C1) \]

\(E_F\) is the Fermi energy, and \(E_{\text{pot}}\) is the potential energy for two electrons separated by the average distance between electrons at density \(n\). As in higher dimensions, we define the ratio of these energies as

\[ r_s = \frac{4e^2}{\pi\epsilon_0 v_F} = \frac{E_{\text{pot}}}{E_F}, \]  

\[ (C2) \]

where \(v_F = k_F/m\) is the Fermi velocity.

As mentioned previously, the only contribution to the imaginary part in the GW approximation comes from \(\Sigma_{\text{pole}}\). The contribution to \(\text{Im}[\Sigma]\) comes from two sources: (i) the
single particle continuum where $\text{Im}[\Pi(q,\omega)] \neq 0$, and from (ii) the plasmon contribution (i.e., when the $\text{Re}[\epsilon(q,\omega)] = 0$ and $|\text{Im}[\epsilon(q,\omega)]| = 0^+$. The $\theta$-functions in Eq. (2.11d) dictate that the only contribution to $\text{Im}[\Sigma]$ occurs when $\omega - \xi_{k+q}$ and $\xi_{k+q}$ have different signs.

For $k = k_F$, $\xi_{kF+q} = q^2/(2m) + qv_F$, and hence for small $\omega$, where one can linearize the energy-momentum curves about $q = 0$ and $q = -2k_F$, yielding

$$\text{Im}[\Sigma(k_F,\omega)] \approx \int_{0}^{\omega/v_F} \frac{dq}{2\pi} \text{Im}\left[\frac{V_c(q)}{\epsilon(q, qv_F - \omega)}\right] + \int_{-\omega/v_F}^{0} \frac{dq'}{2\pi} \text{Im}\left[\frac{V_c(-2k_F + q')}{\epsilon(-2k_F + q', -q'v_F - \omega)}\right].$$

(C3)

The bold lines in Fig. 2 show the energy-momentum curves over which there is a contribution for the self-energy. We show below that, as $\omega \to 0$, the $\text{Im}[\Sigma]$ is dominated by the plasmon contribution in the case of clean wires, and this contribution leads to a limiting form of $|\text{Im}[\Sigma(k_F,\omega)]| \sim |\omega| \log(|\omega|)^{1/2}$.

1. **Plasmon Contribution to $\text{Im}\Sigma(k_F,\omega)$ in a clean system**

The plasmon contribution comes in when the real part of $\epsilon$ goes through zero (corresponding to the energy-momentum curve cutting through the plasmon dispersion curve). The contribution goes as

$$\text{Im}[\Sigma(k,\omega)] = -\frac{1}{2} V_c(q^*) \left| \frac{\partial \epsilon(q, \xi_{k+q} - \omega)}{\partial q} \right|^{-1}_{q=q^*}$$

(C4)

where $q^*$ is the wavevector at the intersection of the energy curve $\xi_{k+q} - \omega$ and the plasmon dispersion curve $\omega_{pl}(q)$.

For small $\omega$, the plasmon intersection $q^*$ occurs at small $q$, and so one can linearize the energy-momentum curve

$$\xi_{kF+q} - \omega \sim \frac{k_F q}{m} - \omega$$

(C5)

which implies that, for an intersection at $q^*$, the corresponding energy at which it occurs is $E^* = k_F q^*/m$. 

55
The dispersion relation for the plasmon, $\omega_{\text{pl}}(q)$, is given by the solution of $\epsilon(q, \omega_{\text{pl}}(q)) = 0$. In the random phase approximation in a quasi-one-dimensional system, the dispersion relation is given by [29]

$$\omega_{\text{pl}}(q) = qv_F \sqrt{1 + \frac{2V_c(q)}{\pi v_F}}. \tag{C6}$$

We now study two different cases, the Coulomb interaction (where $V_c(q) \to \infty$ as $q \to 0$), and the short-ranged interaction (where $V_c(q)$ is finite when $q \to 0$). In both cases, we obtain non-Fermi-liquid behavior from the $GW$ approximation to the self-energy.

**Coulomb interaction** — For the case of the Coulomb interaction where $V_c(q) \approx 2e^2|\ln|qa||/\epsilon_0$, the plasmon dispersion for small $q$ is

$$\omega_{\text{pl}}(q) \approx q\sqrt{\frac{2v_F V_c(q)}{\pi}} \approx qv_F \sqrt{r_s|\ln|qa||}. \tag{C7}$$

The intersection of the energy-momentum curve $\xi_{k_F+q} - \omega$ and the plasmon curve $\omega_{\text{pl}}(q)$, from Eq. (C3) and (C7), occurs at

$$\omega \approx q^*v_F\left(\sqrt{r_s|\ln|q^*a||} + 1\right). \tag{C8}$$

In the limit of small $|\omega|$ and $|q^*(\omega)|$, the 1 on the right-hand side of Eq. (C8) can be ignored, yielding

$$q^* \approx \frac{\omega}{v_F \sqrt{r_s|\ln|q^*a||}} \approx \frac{\omega}{v_F \sqrt{r_s|\ln|a\omega/(\sqrt{r_s}v_F)|| - \ln(\sqrt{|\ln|q^*a||})}}. \tag{C9}$$

Since $|q^*| \ll |\omega|$ in the limit $\omega \to 0$, the second term in the denominator of the last expression in Eq. (C3) is negligible when compared to the first term and hence

$$q^*(\omega) \approx \frac{\omega}{v_F \sqrt{r_s|\ln|a\omega/(\sqrt{r_s}v_F)||}}. \tag{C10}$$

Now the derivative of the dielectric function for $q \to 0$ and $\omega/q \gg v_F$ is
\[
\frac{\partial \epsilon(q, \xi_{k+q} - \omega)}{\partial q} \approx \frac{\partial V_c(q, \xi_{k+q} - \omega)}{\partial q}
\]
\[
\approx \frac{\partial V_c(q)}{\partial q} \Pi(q, \xi_{k+q}) + V_c(q) \frac{\partial \Pi(q, \xi_{k+q})}{\partial q}
\]
\[
= \frac{n}{m} \frac{\partial V_c(q)}{\partial q} \frac{q^2}{(qk_F/m - \omega)^2} + \frac{n}{m} V_c(q) \frac{\partial q^2/(qk_F/m - \omega)^2}{\partial q}
\]
\[
= r_s v_F^2 \left[ \frac{q}{(qk_F/m - \omega)^2} + \ln(|qa|) \left( \frac{2q}{(qk_F/m - \omega)^2} - \frac{2(q^2k_F/m)}{(qk_F/m - \omega)} \right) \right].
\]
(C11)

Given that \( q \sim \omega / |\ln(\omega)| \ll \omega \) as \( \omega \to 0 \), the second term in the last expression of Eq. (C11) dominates, and thus we have

\[
\left[ \frac{\partial \epsilon(q, \xi_{k+q} - \omega)}{\partial q} \right]_{q=q^*} \approx r_s v_F^2 |\ln|q^*a|| \frac{2q^*}{\omega^2}.
\]
(C12)

Substituting Eqs. (C12) and (C11) into Eq. (C4) gives

\[
\text{Im}[\Sigma(k_F, \omega)] = -\frac{\pi \sqrt{r_s}}{8 \omega} |\ln\left( \frac{\omega}{\sqrt{\pi v_F}} \right)|.\]
(C13)

Thus we have shown that in the clean limit, within the GW and RPA approximations the imaginary part of the self-energy goes as

\[
|\text{Im}[\Sigma(k_F, \omega)]| \sim |\omega| \sqrt{|\ln|\omega||},
\]
(C14)

which implies there is no Fermi surface, and we have shown that the physical mechanism for the disappearance of the Fermi surface is the virtual emission of low energy plasmons.

Short-ranged interaction — In the case of a short-ranged interaction (i.e., a delta-function in real space) where \( V_c(q) = U_0 \) is a constant, we show below that \(|\text{Im}[\Sigma(k_F, \omega)]| \sim |\omega|\) for small \( \omega \).

The plasmon dispersion for short-ranged interactions is

\[
\omega_{pl}(q) = q v_F \sqrt{1 + \frac{2U_0}{\pi v_F}}.
\]
(C15)

The dielectric function at small \( q \) for \( z = \xi_{k+q} - \omega \) is given by

\[
\epsilon(q, \xi_{k+q} - \omega) \approx 1 - \frac{m U_0}{\pi q} \ln \left| \frac{1 + \frac{v_F q^2}{m(\omega^2 - 2\omega v_F q)}}{1 - \frac{v_F q^2}{m(\omega^2 - 2\omega v_F q)}} \right|
\]
\[
\approx 1 - \frac{2U_0 v_F q^2}{\pi (\omega^2 - 2\omega v_F q)}.
\]
(C16)
and therefore,

$$
\frac{\partial \epsilon(q, \xi_{k_F+q} - \omega)}{\partial q} \approx -\frac{2U_0v_F}{\pi} \left[ \frac{2q}{(\omega^2 - 2\omega v_F q)} - \frac{2\omega v_F q^2}{(\omega^2 - 2\omega v_F q)^2} \right].
$$  \tag{C17}

Substituting Eq. (C17), into Eq. (C4) yields the plasmon contribution to the self-energy, in the limit of $\omega \to 0$

$$
\text{Im}[\Sigma_{pl}(k_F, \omega)] = -\frac{\pi(\sqrt{1 + 2U_0/(\pi v_F)} - 1)^2\omega}{4\sqrt{1 + 2U_0/(\pi v_F)}}.
$$  \tag{C18}

Thus in the case where the interaction is short-ranged, one obtains the imaginary part of the self-energy being proportional to $\omega$, giving a marginal Fermi liquid. From exact solutions, we know that short-ranged interactions give non-Fermi liquids. We have shown that the $GW$ approximation is sufficient to describe this non-Fermi-liquid behavior for short-ranged interactions. (Note, however, that the functional dependence is different in the exact Luttinger liquid situations, where $\text{Im}[\Sigma(k_F, \omega)] \sim |\omega|^\alpha$, with $\alpha < 1$.)

2. Single particle contribution to $\text{Im}[\Sigma(k_F, \omega)]$ in a clean system

The single-particle excitation contribution to $\text{Im}[\Sigma]$ comes from each of the two terms on the right-hand-side of Eq. (C3). The first term corresponds to virtual electron-hole excitations around $q = 0$. Within the region of integration $0 < q < \omega/v_F$, $\text{Im}[\epsilon] \neq 0$ only in a region the size of $\sim \omega^2$. In this region, both the real and imaginary parts of $\epsilon(q, qv_F - \omega)$ are of the order of $V_c(\omega/v_F)/\omega$, and therefore the integrand $V_c(q) \text{Im}[\epsilon^{-1}(q, qv_F - \omega)]$ is on the order of $\omega$. Thus, the contribution of this first term is on the order of the product of the size of region of integration and the magnitude of the integrand, $\omega^2 \times \omega = \omega^3$.

The contribution from the second term of Eq. (C3) corresponds to virtual electron-hole excitations around $q = 2k_F$. Here, the region where $\text{Im}[\epsilon(2k_F + q', -q'v_F - \omega)] \neq 0$ is of order $\omega$. In this region, $\text{Im}[\epsilon] \sim \text{constant}$, and $\text{Re}[\epsilon] \gtrsim |\ln(\omega)|$, leading to the integrand being on the order of $|\ln(\omega)|^2$. Therefore the magnitude of the contribution from this term is the product of the size of the region of the integration and the magnitude of the integrand,
which is $\omega/|\ln(\omega)|^2$. Note that the $\ln|\omega|$ comes from the form of the RPA $\Pi_0(q,\omega)$ and is independent of the form of $V_c(q)$. This term dominates the $q = 0$ term, and hence the single-particle contribution to the self-energy is

$$|\text{Im}[\Sigma(k_F,\omega)]| \sim \frac{|\omega|}{(\ln|\omega|)^2}. \quad (C19)$$

3. $\text{Im}[\Sigma(k_F,\omega)]$ as $\omega \to 0$ for dirty systems

In the case where there is impurity scattering, electrons are diffusive and, as shown in section V, the plasmons no longer exist at small wavevectors. Thus, for small $\omega$, there is no plasmon contribution to the imaginary part of the self-energy. In this subsection, we show that the remaining contribution goes as $\text{Im}[\Sigma(k_F,\omega)] \sim \omega^2$.

We find it convenient to define the following the following dimensionless variables:

$$K, Q = \frac{k}{k_F}, \frac{q}{k_F}, \quad \Omega = \frac{\omega}{E_F}, \quad \tilde{\gamma} = \frac{\gamma}{E_F}, \quad \tilde{V}(Q) = \frac{V_c(q)}{2e^2/\epsilon_0}, \quad \tilde{\Sigma}(K, \Omega) = \frac{\Sigma(k,\omega)}{E_{pot}} = \frac{\Sigma(k,\omega)}{2e^2k_F/(\pi\epsilon_0)}. \quad (C20)$$

With these definitions, $\tilde{\Pi}(Q \to 0, \Omega = 0) = -1$ and $\epsilon(Q, \Omega) = 1 - r_s\tilde{V}_c(Q)\tilde{\Pi}(Q, \Omega)$. In terms of these dimensionless variables, Eq. (C3) becomes

$$\tilde{\Sigma}(K = 1, \Omega) \approx \int_0^{\Omega/2} \frac{dQ}{2} \tilde{V}_c(Q) \text{Im}[\epsilon^{-1}(Q,2Q-\Omega)]$$
$$+ \int_{-\Omega/2}^0 \frac{dQ'}{2} \tilde{V}_c(-2+Q') \text{Im}[\epsilon^{-1}(-2+Q',-2Q'-\Omega)]. \quad (C21)$$

The polarizability is given by the Mermin form. In the region $Q, \Omega \ll \tilde{\gamma}$, the polarizability is of the form

$$\tilde{\Pi}_\gamma(Q, \Omega') \approx -\frac{Q^2}{Q^2 - i\Omega\gamma/4} \quad (C22)$$

which gives
\[
\text{Im}\left[ \frac{1}{\epsilon(Q, \Omega)} \right] = -\frac{r_s \Omega' \bar{\gamma} Q^2 \bar{V}_c(Q)}{4 Q^4 (1 + r_s \bar{V}_c(Q))^2 + (\Omega' \bar{\gamma} / 4)^2}.
\] (C23)

Therefore, the virtual transitions around \(Q = 0\) [the first term in Eq. (C21)] give a contribution

\[
\tilde{\Sigma}_1(K = 1, \Omega) \approx \frac{r_s \bar{\gamma}}{8} \int_0^{\Omega / 2} dQ \frac{Q^2 (\Omega - 2Q) \bar{V}_c^2(Q)}{Q^4 (1 + r_s \bar{V}_c(Q))^2 + ((\Omega - 2Q) \bar{\gamma} / 4)^2} = \frac{r_s \Omega^2}{2 \bar{\gamma}} \int_0^1 \frac{\bar{V}_c \left( \Omega (1 - y) / 2 \right) (1 - y)^2 y}{(1 - y)^4 ((1 + r_s \bar{V}_c \left( \Omega (1 - y) / 2 \right)) \Omega / \bar{\gamma})^2 + y^2}.
\] (C24)

As \(\Omega \to 0\), the integral in the last line of Eq. (C24) diverges in the region \(y = 0\), and therefore only the behavior of the integrand around that region is relevant. Therefore, the asymptotic form of \(\tilde{\Sigma}\) (using the asymptotic form \(\bar{V}_c(Q) \sim |\ln(Q)|\)) is

\[
\tilde{\Sigma}_1(K = 1, \Omega) \approx \frac{r_s \bar{\gamma}}{8} \int_0^{\Omega / 2} dy \frac{\ln(\Omega)^2 y}{\omega^2 r_s^2 \ln(\omega)^2 / \bar{\gamma}^2} + y^2 = \frac{r_s \Omega^2 \ln(\Omega)^2}{2 \bar{\gamma}} \ln \left( \frac{r_s \Omega^2 \ln(\Omega)^2}{\bar{\gamma}} \right),
\] (C25)

which to leading order, goes as \(\Omega^2 |\ln|\Omega||^3\).

Now we study the contribution from the \(Q = 2\) virtual transitions. The polarizability in this region is

\[
\tilde{\Pi}_\gamma(Q, \Omega) = \Pi_0(Q, \Omega = 0) - i C(Q) \Omega + 0(\Omega^2),
\] (C26)

\[
C = \frac{\left( \Pi_0(Q, i \bar{\gamma}) - \Pi_0(Q, 0) \right) \Pi_0(Q, 0)}{\bar{\gamma} \Pi_0(Q, i \bar{\gamma})}.
\]

Using this expansion, \[\text{C26}\], we obtain

\[
\text{Im}[\epsilon^{-1}(Q, \Omega)] \approx \frac{\text{Im}[\Pi_\gamma(Q, \Omega)]}{(r_s \bar{V}_c(Q) \text{Re}[\Pi_0(Q, 0)])^2},
\] (C27)

which yields a contribution to \(\tilde{\Sigma}(K = 1, \Omega)\), from Eq. (C21), of

\[
\tilde{\Sigma}_2(K = 1, \Omega) \sim \frac{\Omega}{|\ln|\Omega||^2} \int_{-\Omega / 2}^0 dQ \sim \frac{\Omega^2}{|\ln|\Omega||^2}.
\] (C28)

Thus, in this case where electrons diffuse due to impurity scattering, the virtual excitations close to \(Q = 0\) dominate, and

\[
|\text{Im}\Sigma(k_F, \omega)| \sim \omega^2 |\ln|\omega||^3.
\] (C29)
The ω → 0 limit of the real part of the self-energy at \( k = k_F \) is

\[
\text{Re}[\Sigma(k_F, \omega)] \approx \int_0^{\Omega/2} \frac{dQ}{2} V_c(Q) \text{Re}[\epsilon^{-1}(Q, 2Q - \Omega) - 1] + \int_{-\Omega/2}^0 \frac{dQ'}{2} V_c(-2 + Q') \text{Re}[\epsilon^{-1}(-2 + Q', -2Q' - \Omega) - 1].
\]  

(C30)

The real part of \( \epsilon^{-1}(Q, \Omega) \) in the integration region \( Q = 0, \Omega/2 \) is well-behaved (aside from an inconsequential principal part divergence at the plasmon frequency). The \( \epsilon^{-1} - 1 \) term basically is a constant, and the magnitude of the integrand is determined by the \( \tilde{V}_c(Q) \) term.

Since the size of region of integration goes as \( \Omega \) and the integrand goes approximately as \( \tilde{V}_c(\Omega) \sim |\ln |\Omega|| \), the contribution from the first integral on the right-hand side of Eq. (C30) is on the order of \( |\Omega \ln |\Omega|| \). In the second integral, the integrand does not diverge, and hence its contribution is on the order of \( \Omega \) and can be neglected in comparison to the first. Hence

\[
\text{Re}[\Sigma(k_F, \omega)] \sim |\omega \ln |\omega||.
\]  

(C31)

The slope of \( \text{Re}[\Sigma(k_F, \omega)] \) at \( \omega = 0 \) is infinite (in fact, this is necessary for \( A(k_F, \omega) \) to be integrable), implying from Eq. (2.20) that \( Z_F = 0 \), which is consistent with the fact that there is no Fermi surface.

### APPENDIX D: Im[Σ(k_F,ω)] AS ω → 0, IN THE GW AND RPA APPROXIMATION IN TWO DIMENSIONS

In this appendix, we calculate, for the sake of comparison, the imaginary part of the \( \Sigma(k_F, \omega) \), due to plasmon and single particle emission in two dimensions for the case of Coulomb interaction, which is

\[
V_c(q) = \frac{2\pi e^2}{\epsilon_0 q}.
\]  

(D1)

In the GW approximation for a spherically symmetric parabolic band, the imaginary part of the self-energy can be written as (for the case of \( \omega > 0 \))
\[
\text{Im}[\Sigma(k,\omega)] = \frac{2m}{(2\pi)^2} \int_0^\omega d\xi \int_0^\pi d\theta V_c(q(\theta,\xi)) \text{Im}[\epsilon^{-1}(q(\theta,\xi),\xi - \omega)]
\]  
(D2)

where \(\epsilon(q,\omega) = 1 - V_c(q)\Pi(q,\omega)\) and

\[
q^2(\theta,\xi) = k^2 + k_F^2 + 2m\xi - 2k\sqrt{k_F^2 + 2mE}\cos\theta.
\]  
(D3)

The exact form of \(\Pi(q,\omega)\) in RPA can be found in references [81] and [26]. We define the reduced variables

\[
K, Q = \frac{k}{k_F}, \frac{q}{k_F}, \quad \Omega = \frac{\omega}{E_F}, \quad \gamma = \frac{\gamma}{E_F}, \quad \tilde{V}_c(Q) = \frac{V_c(q)}{(2\pi e^2/(q_F\epsilon_0))} = \frac{1}{Q}, \quad R_s = \frac{2e^2}{\epsilon_0 v_F}, \quad \tilde{\Sigma}(k,\omega) = \frac{\Sigma(k,\omega)}{e^2k_F/(\pi\epsilon_0)}, \quad \tilde{\Pi}(Q,\omega) = \frac{\Pi(q,\omega)}{m/\pi}.
\]  
(D4)

With these definitions, \(\tilde{\Pi}(Q \rightarrow 0, \Omega = 0) = -1\) and \(\epsilon(Q,\Omega) = 1 - R_s\tilde{V}_c(Q)\tilde{\Pi}(Q,\Omega)\). For the case \(k = k_F, Q^2 = 2 + y - 2\sqrt{1 + y}\cos\theta\), which yields

\[
d\theta = \frac{Q dQ}{\sin\theta(Q,y) \sqrt{1 + y}},
\]  
(D5)

where,

\[
\sin\theta(Q,y) = \sqrt{\frac{4Q^2 - y^2 + 2yQ^2 - Q^4}{4(1 + y)}}.
\]  
(D6)

Therefore, using Eqs. (D4), (D5) and (D), Eq. (D2) can be rewritten as

\[
\text{Im}[\tilde{\Sigma}(k_F,\omega)] = \int_0^\Omega dy \int_{Q_-(y)}^{Q_+(y)} dQ \text{Im}[\epsilon^{-1}(Q,y - \Omega)] \frac{1}{\sqrt{4Q^2 - y^2 + 2yQ^2 - Q^4}}
\]  
(D7)

where,

\[
Q_-(y) = \left[2 + y - 2\sqrt{1 + y}\right]^{\frac{1}{2}} = \frac{y}{2} + O(y^2),
\]  
(D8)

\[
Q_+(y) = \left[2 + y + 2\sqrt{1 + y}\right]^{\frac{1}{2}} = 2 + \frac{y}{2} + O(y^2).
\]

Within the \(GW\) approximation for two dimensions, Eq. (D7) is an exact result. Now, we calculate the plasmon and single-particle excitation contributions to \(\Sigma(k_F,\omega)\) in the \(\omega/E_F \ll 1\) limit.
1. Plasmon contribution in two dimensions

The plasmon contribution to the imaginary part of $\Sigma(k, \omega)$ occurs when $\text{Re}[\epsilon(Q, y - \Omega)] = 0$ in Eq. (D7). Since we are interested in small frequencies, the dielectric function in two-dimensions is given in the small frequency limit by [37]

$$\epsilon(Q, y - \Omega) = 1 - \frac{2R_s Q}{(y - \Omega)^2}. \quad \text{(D9)}$$

and therefore, the wavevector at which $\text{Re}[\epsilon] = 0$ is

$$Q_p(y) = \frac{(y - \Omega)^2}{2R_s}. \quad \text{(D10)}$$

Hence,

$$\text{Im}[\epsilon^{-1}(Q, y - \Omega)] = \frac{\pi \delta(Q - Q_p(y))}{|\partial \epsilon(Q, y - \Omega)/\partial Q|} = \frac{\pi \delta(Q - Q_p(y))(y - \Omega)^2}{2R_s} \quad \text{(D11)}$$

There is a non-zero plasmon contribution to the integral in Eq. (D7) only if the delta function peaks between $Q_-(y)$ and $Q_+(y)$; that is (since $Q_p(y)$ is small) only if $Q_p(y) > Q_-(y)$. This occurs if the condition

$$\frac{(\Omega - y)^2}{R_s} > y, \quad \text{(D12)}$$

is satisfied, or equivalently, if

$$y < y_{\text{max}} = \frac{R_s}{4} \left( 1 + \frac{2\Omega}{R_s} - \sqrt{1 + \frac{4\Omega}{R_s}} \right) = \frac{\Omega^2}{R_s} + O(\Omega^3). \quad \text{(D13)}$$

Thus, from Eqs. (D7), (D10), (D11) and (D13), the plasmon contribution, to lowest order in $\Omega^2$, is

$$\text{Im}[\Sigma_{pl}(k_F, \omega)] \approx \frac{e^2 k_F}{2\epsilon_0 R_s} \int_0^{\Omega^2/R_s} \frac{dy}{\sqrt{(\Omega^2/R_s)^2 - y^2}} (y - \Omega)^2$$

$$\approx \frac{E_F}{2} \Omega^2 \sqrt{1 - z^2}$$

$$= \frac{\pi \omega^2}{4 E_F}. \quad \text{(D14)}$$
Therefore, the plasmon contribution to the imaginary part of the self-energy goes as as $\omega^2$.

Note that the quantity we calculate here, $\text{Im}[\Sigma(k_F, \omega)]$, is different from the inverse inelastic lifetime of the quasiparticle, $\tau_{ee}^{-1}(k) = 2 \text{Im}[\Sigma(k_F, \xi_{k_F})]$. For $\tau_{ee}^{-1}$ there is a minimum critical $|p - p_F|$ before the plasmon contributes to $\tau_{ee}^{-1}$, whereas for $\text{Im}[\Sigma(k_F, \omega)]$, there is no gap.

2. The single-particle excitation contribution

We show in this subsection that the single-particle excitation contribution to the imaginary part of the two-dimensional self-energy goes as $\omega^2 |\ln(\omega)|$, and hence gives the dominant contribution to the $\text{Im}[\Sigma]$ in a two-dimensional electron gas. This result has been obtained previously, in similar contexts, although details of the actual calculation were never given. Below, we give a quick derivation of this result.

For small $\Omega$, the polarizability goes as $\text{Re}[\tilde{\Pi}(Q, \Omega)] = -1$

$$\text{Im}[\tilde{\Pi}(Q, \Omega)] \approx -\frac{\Omega}{Q\sqrt{4 - Q^2}}$$

and hence, since $|\text{Im}[\Pi(Q, \Omega)]| \ll |\text{Re}[\Pi(Q, \Omega)]|$ in this limit,

$$\text{Im}[\epsilon^{-1}(Q, \Omega)] \approx \frac{\text{Im}[\epsilon(Q, \Omega)]}{(\text{Re}[\epsilon(Q, \Omega)])^2} \approx \frac{R_s \Omega}{\sqrt{4 - Q^2}(Q + R_s)^2}.$$  \hspace{1cm} (D16)

The single-particle contribution to the imaginary part of the self-energy, from Eq. (D8), is

$$\text{Im}[\Sigma_{sp}(k_F, \omega)] \approx \frac{e^2 k_F}{\pi \epsilon_0} \int_0^\Omega dy \int_{Q}^2 dQ \frac{R_s}{Q(Q + R_s)^2(4 - Q^2)}$$  \hspace{1cm} (D17)

where terms higher order in $y$ and $\Omega$ have been ignored. The lower limit for the $Q$-integration is given by a cutoff on the order of $\Omega$, which is where the expansion given in Eq. (D16) starts to fail (the remaining part of the integral is from $\sim \Omega$ to $Q_-$ is well behaved and gives negligible contribution). The leading order contribution to the Eq. (D17) is
\[ \text{Im}[\Sigma(k_F, \omega)] \approx \frac{e^2 k_F}{\pi \epsilon_0} \int_0^\Omega dx \left( \Omega - x \right) \frac{\ln(\Omega)}{4R_s} \approx \frac{\omega^2}{8\pi E_F} \ln |\omega| \quad (D18) \]

Thus, the single-particle contribution to the imaginary part of the self-energy is on the order of \( \omega^2 \ln |\omega| \), and in the small \(|\omega|\) limit, is the dominant term for \( \text{Im}[\Sigma(k_F, \omega)] \).

3. \textbf{Im}[\Sigma](k_F, \omega) for an impure system

In this subsection, we show that in a dirty system, the main contribution to \( \text{Im}[\Sigma(k_F, \omega)] \) comes from the small \( Q \) and \( y \) region in Eq. (D7), which gives a linear contribution in \( \omega \).

Defining \( x = \frac{(Q - Q_-(y))}{y} \), we rewrite Eq. (D7) in the limit of small \( \Omega \) as

\[ \text{Im}[\tilde{\Sigma}(k_F, \Omega)] \approx \int_0^\Omega dy \ y \int_0^\infty dx \frac{\text{Im}\left[\epsilon^{-1}\left(y(x + \frac{1}{2}), y - \Omega\right)\right]}{y(x + \frac{1}{2})} \frac{1 + 2x}{4\sqrt{x + x^2}}. \quad (D19) \]

We first calculate the inner integral of the right hand side of Eq. (D19),

\[ I(y) = \int_0^\infty dx \frac{\text{Im}\left[\epsilon^{-1}\left(y(x + \frac{1}{2}), y - \Omega\right)\right]}{y(x + \frac{1}{2})} \frac{1 + 2x}{4\sqrt{x + x^2}}. \quad (D20) \]

We approximate the dielectric function \( \epsilon(Q, \Omega) \) by using the diffusive form of the polarizability, written as an expansion in powers of \( Q \) and \( \Omega \) of the numerator and denominator of the Mermin formula [31]. The diffusive form is strictly valid only for \( Q, \Omega << \tilde{\gamma} \). However, since the contribution to \( \text{Im}[\Sigma(k_F, \Omega)] \) from the small \( Q \) region goes as \( \Omega \), and the contribution from the large \( Q \) region goes as \( \Omega^2 \) both in the diffusive and the ballistic (\( i.e., \gamma = 0 \)) forms, the error introduced by using the diffusive form at large \( Q \) is insignificant. The diffusive form of the polarizability is

\[ \tilde{\Pi}_\gamma(Q, \Omega') \approx -\frac{Q^2}{Q^2 - i\tilde{\gamma}\Omega/2} \]

\[ \text{Re}\tilde{\Pi}_\gamma(Q, \Omega') \approx -\frac{Q^4}{Q^4 + (\tilde{\gamma}\Omega/2)^2} \quad (D21) \]

\[ \text{Im}\tilde{\Pi}_\gamma(Q, \Omega') \approx -\frac{Q^2 \tilde{\gamma}\Omega}{2(Q^4 + (\tilde{\gamma}\Omega/2)^2} \]

and hence
\[
\frac{\text{Im}[\epsilon^{-1}(Q, \Omega')]}{Q} = \frac{R_s \text{Im}[\Pi]}{(Q - R_s \text{Re}[\Pi])^2 + (R_s \text{Im}[\Pi])^2} \\
\approx \frac{2R_s}{\gamma\Omega} \left[ 1 + \left( \frac{2R_s Q}{\gamma\Omega} \right)^2 \right]^{-1}
\]

where we have ignored terms higher order in \(Q, \Omega\). The inner integral is therefore

\[
I(y) = \frac{r_s}{2\gamma \Omega} \int_0^\infty dx \frac{2x + 1}{(1 + A_y(2x + 1)^2)\sqrt{x^2 + x}}
\]

\[
A_y = \left( \frac{r_s y}{\gamma(y - \Omega)} \right)^2.
\]

By a change of variables \(z^2 = x^2 + x\), we obtain

\[
I(y) = \frac{r_s}{\gamma \Omega} \int_0^\infty dx \frac{dz}{1 + A_y(1 + 4z^2)}
\]

\[
= \frac{r_s}{4\pi \sqrt{A_y(A_y + 1)\gamma \Omega}},
\]

and hence

\[
\tilde{\Sigma} \approx \int_0^\Omega dy yI(y)
\]

\[
= \frac{1}{4\pi \Omega} \int_0^\Omega dy \frac{(y - \Omega)^2}{\sqrt{(r_s y^2 / \gamma) + (y - \Omega)^2}}
\]

\[
= \Omega \int_0^1 d\zeta \frac{(1 - \zeta)^2}{4\pi \sqrt{(r_s \zeta^2 / \gamma) + (1 - \zeta)^2}}.
\]

APPENDIX E: ESTIMATION OF THE COUPLING PARAMETER \(g\)

FOR A QUANTUM WIRE

In this appendix, we estimate the strength of the Tomonaga-Luttinger coupling parameter \(g\) within a quantum wire of width \(a\) which is screened by another quantum wire which is adjacent and parallel to it, a distance \(l\) away. We denote the wires by numbers 1 and 2. We assume that the electrons do not tunnel and are screened within the random phase.
approximation. The effective intrawire interaction (i.e., electrons within the same wire) is
given by the the bare interaction screened by the motion of the electrons in the adjacent wire. The Dyson’s equation for the screened interaction is

\[ V_{\text{eff}}(q, \omega) = V_{c,11}(q) + V_{c,12}(q)\Pi_2(q, \omega)V_{c,21}(q) \]

\[ + V_{c,12}(q)\Pi_2(q, \omega)V_{c,22}(q)\Pi_2(q, \omega)V_{c,21} + \ldots \]

\[ = V_{c,11}(q) + V_{c,12}\Pi_2(q, \omega)V_{c,21}(q) \sum_{n=0}^{\infty} \Pi_2(q, \omega)V_{c,22}(q) \]

\[ = V_{c,11}(q) + \frac{V_{c,12}\Pi_2(q, \omega)V_{c,21}(q)}{1 - V_{c,22}(q)\Pi_2(q, \omega)} \]  

(E1)

where \( V_{c,ij} \) is the bare Coulomb interaction between electrons in wires \( i \) and \( j \), and \( \Pi_i \) is the polarizability of wire \( i \).

Now assume both wires are identical, so that the intrawire interaction \( V_{11} = V_{22} = V_a \), the interwire interaction \( V_{12} = V_{21} = V_e \) and \( \Pi_{11} = \Pi_{22} = \Pi \). Since we are interested in the physics close to the Fermi surface, we take the \( q \to 0 \) asymptotic forms of these potentials. The intrawire potential \( V_a \) is given by Eq. (2.5), and the interwire potential \( V_e \) is given, in the limit of small \( q \), by

\[ V_e(q) = \frac{2e^2}{\epsilon_0} \int_{-a/2}^{a/2} dy \int_{-l-a/2}^{l+a/2} dy' K_0(|q(y - y')|) \cos^2\left(\frac{\pi y}{a}\right) \cos^2\left(\frac{\pi y'}{a}\right) \]

\[ \approx K_0(|ql|) \approx V_a(q) - 2e^2\epsilon_0^{-1} \ln(l/a). \]  

(E2)

This gives a long wavelength effective intrawire interaction of

\[ V_{\text{eff}}(q, \omega) = \frac{V_a(q) - 2e^2\epsilon_0^{-1} \ln(l/a)\left(2V_a(q) - 2e^2\epsilon_0^{-1} \ln(l/a)\right)}{1 - \Pi(q, \omega)V_a(q)} \]  

(E3)

In the limit \( q \to 0 \), \( V_a(q) \to \infty \) and \( \Pi(q, \omega = 0) = -(2/\pi v_F) \), which yields, from Eq. (E3),

\[ V_{\text{eff}}(q \to 0, 0) = \frac{v_F\pi}{2} \left(1 + \frac{8e^2}{\epsilon_0 v_F \pi} \ln(l/a)\right). \]  

(E4)

For \( l/a = 10 \), and for the parameters of Fig. 3, \( 8e^2/(\epsilon_0 v_F \pi) = 2.8 \), we obtain \( g = 2V_{\text{eff}}/(\pi v_F) = \left(1 + 2.8 \ln(10)\right) \approx 7.5. \) and therefore, using Eq. (2.34), \( \alpha \approx 0.15. \)
APPENDIX F: Im[Σ(⟨kF, ω⟩)] AS ω → 0,

IN THE GWT AND RPA APPROXIMATION

In the GWT approximation, \([82,47]\) the Matsubara expression for the self-energy is

\[
\Sigma(k, i\nu_n) = -\frac{1}{\beta} \sum_{i\nu_m} \int \frac{dq}{2\pi} \frac{\epsilon(q, i\nu_m) \Gamma(q, i\nu_n)}{\epsilon(q, i\nu_n) - i\nu_n + i\nu_m - \xi_{k+q}}
\]

where

\[
\epsilon(q, i\nu_n) = 1 - V_c(q) \Pi(q, \omega) \Gamma(q, i\nu_n),
\]

\[
\Gamma(q, i\nu_n) = \frac{1}{1 + G(q)V_c(q)\Pi(q, \omega)}.
\]

Γ gives contributions from the vertex corrections. In the GW and RPA approximations we have used, Γ = 1. We now show that when vertex corrections in the form given in Eq. (F2) are included, the main conclusions of our calculations regarding the existence of Fermi surface do not change.

Substituting Eq. (F2) in Eq. (F1), one can rewrite the expression for the self-energy as

\[
\Sigma(k, i\nu_n) = -\frac{1}{\beta} \sum_{i\nu_m} \int \frac{dq}{2\pi} \frac{\epsilon(q, i\nu_m) \Gamma(q, i\nu_n)}{\epsilon(q, i\nu_n) - i\nu_n + i\nu_m - \xi_{k+q}}
\]

Physically, this equation says that the main effect that the vertex corrections have is that the local-field factor \(G(q)\) (which accounts for the short-range correlation of electrons beyond RPA) is to renormalize the dielectric function, changing the dispersion at large \(q\).

One can make the equivalent of the Hubbard approximation \([83,84]\) for \(G(q)\) in one dimension to obtain \([85]\)

\[
G(q) = \frac{1}{2} \frac{V_c(\sqrt{q^2 + k_F^2})}{V_c(q)}.
\]

In the limit where \(q \rightarrow 0\), \(G(q) \sim |\ln |qa||^{-1} \rightarrow 0\), and therefore, the local-field correction term in Eq. (F4) is negligible for small \(q\). However, the behavior of \(\text{Im}[\Sigma(⟨k_F, ω⟩)]\) for small \(ω\) only depends on the integrand of Eq. (F1) at small \(q\), which the vertex corrections leave
essentially unchanged. Thus the inclusion of vertex corrections of the form Eq. (F2) does not change \( \Sigma(k_F, \omega) \) at small \( \omega \), and hence does not change the conclusions regarding the Fermi surface.

**APPENDIX G: EXPRESSION FOR THE FINITE-TEMPERATURE SECOND-ORDER SELF-ENERGY**

Here, we give the expression obtained for the second order (in the screened interaction \( w \)) self-energy (Fig. 1(c)) using our contour deformation method. We obtained

\[
\Sigma(k, z) = \int \frac{dq}{(2\pi)^d} \int \frac{dq'}{(2\pi)^d} H_{k,q}(z)
\]

where

\[
H_{k,q}(z) = T^2 \sum_{i\omega_n, i\omega_{n'}} \frac{w(q', i\omega_{n'})w(q, i\omega_n)}{(i\omega_n + z - \xi_{k+q})(i\omega_n + i\omega_{n'} + z - \xi_{k+q+q'}) (i\omega_{n'} + z - \xi_{k+q'})}
\]

\[
+ T \sum_{i\omega_n} \frac{w(q, i\omega_n)w(q', \xi_{k+q} - z)R_z(\xi_{k+q})}{(i\omega_n + \xi_{k+q} - \xi_{k+q+q'})(i\omega_n + z - \xi_{k+q})}
\]

\[
+ T \sum_{i\omega_n} \frac{w(q', i\omega_n)w(q, \xi_{k+q} - z)R_z(\xi_{k+q})}{(i\omega_n + \xi_{k+q} - \xi_{k+q+q'})(i\omega_n + z - \xi_{k+q'})}
\]

\[
- T \sum_{i\omega_n} \frac{w(q', i\omega_n)w(q, \xi_{k+q} - z - i\omega_n)R_z(\xi_{k+q+q'})}{(i\omega_n + z - \xi_{k+q+q'})(i\omega + \xi_{k+q} - \xi_{k+q+q'})}
\]

\[
+ \frac{w(q, \xi_{k+q} - z)w(q', \xi_{k+q} - z)R_z(\xi_{k+q})R_z(\xi_{k+q'})}{\xi_{k+q} + \xi_{k+q'} - \xi_{k+q+q'} - z}
\]

\[
- \frac{w(q', \xi_{k+q} - z)W(z, \xi_{k+q+q'} - \xi_{k+q})R_z(\xi_{k+q})n_F(\xi_{k+q+q'})}{\xi_{k+q} + \xi_{k+q'} - \xi_{k+q+q'} - z}
\]

\[
- \int_{-\infty}^{\infty} \frac{dq}{2\pi} B(q, \omega)w(q', \xi_{k+q+q'} - z - \omega)R_z(\xi_{k+q+q'} - \omega)[n_F(\xi_{k+q+q'}) + n_B(\omega)] \frac{1}{(\omega + \xi_{k+q'} - \xi_{k+q}' + \omega)(\omega + \xi_{k+q} - \xi_{k+q})},
\]

where \( R_z(E) = n_B(E - z) + n_F(E) \).
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[59] To avoid being tripped up by sign errors, it is most convenient to deal with the advanced self-energy, which has an imaginary part which is nonnegative. The advanced self-energy is obtained by substituting $z \rightarrow \xi_k - i0^+$, in Eq. (3.16). This gives a factor of $w(q,z = \xi_{k+q} - \xi_k + i0^+) = V_c(q)[(\epsilon(q,\xi_{k+q} - \xi_k)^{-1} - 1]$ in the integrand. (The dielectric function for real frequencies is defined with a small positive infinitesimal, i.e. by $\epsilon(q, E) = 1 - V_c(q)\Pi(q, z = E + i0^+).$) Also note that $\epsilon(q, E) = \epsilon^*(q, -E)$ is used in obtaining $\gamma_h$. 

74
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TABLES

TABLE I. The small $|\omega|$ forms of the real and imaginary parts of $\Sigma$ in one and two dimensions, in the GW and RPA approximations, for clean and dirty systems

|                          | $d = 1$                                           | $d = 2$                                           |
|--------------------------|---------------------------------------------------|---------------------------------------------------|
| $|\text{Im}[\Sigma]|$, clean, (plasmon contribution) | $\omega \sqrt{|\ln(|\omega|)|}$                     | $\omega^2$                                         |
| $|\text{Im}[\Sigma]|$, clean, (single-particle contrib.) | $\omega/|\ln(|\omega|)|^2$                          | $\omega^2 |\ln(|\omega|)|$                         |
| $\text{Re}[\Sigma]$, clean | $\omega \ln(\omega)$                             | $\omega$                                          |
| $|\text{Im}[\Sigma]|$, dirty                  | $\omega^2 |\ln(|\omega|)|^3$                         | $|\omega|$                                         |
| $\text{Re}[\Sigma]$, dirty   | $\omega$                                           | $> \omega$                                        |

TABLE II. Many-body characteristics of one-dimensional quantum wire with and without scattering, in the random phase approximation.

|                          | $\gamma = 0$                                      | $\gamma \neq 0$                                   |
|--------------------------|---------------------------------------------------|---------------------------------------------------|
| Fermi liquid?            | No $(Z_F = 0)$                                    | Yes $(Z_F \neq 0)$                                 |
| $\text{Re}[\Sigma(k_F, \omega)]$ as $\omega \to 0$ | $\sim \omega \log(|\omega|)$                   | $\sim -\omega$                                    |
| $|\text{Im}[\Sigma(k_F, \omega)]|$ as $\omega \to 0$ | $\sim |\omega| |\log(|\omega|)|^{1/2}$                  | $\sim \omega^2 |\log(|\omega|)|^3$                   |
| $A(k_F, \omega)$ as $\omega \to 0$ | $\sim \left(|\omega| |\log(|\omega|)|\right)^{-1}$ | $\sim |\log(|\omega|)|^3 –$                        |
| Inelastic scattering rate $\Gamma$ | $(k - k_c)^{-1/2}$ as $k \to k_c^+$ | Finite for all $k$                                |

0 below plasmon emission threshold Nonzero except at $k_F$
FIGURES

FIG. 1. (a) Electron self-energy in leading order in the effective dynamical screened Coulomb interaction. The straight line denotes the bare Green’s function and the thick wavy line denotes the dynamically screened interaction. (b) Dyson’s equation for the screened interaction, within the random phase approximation (the thin wavy line is the bare Coulomb interaction). Impurity effects are introduced in the electron-hole polarization bubble through the Mermin formula. (c) Second order diagram generally ignored in this paper. In appendix B, we show that this diagram, for $k = k_F$ and $\omega = -\xi_{k_F}$ with a screened interaction which is a delta-function in real-space (i.e. a constant in momentum space), is identically zero.

FIG. 2. Regions of $q - \omega'$ space with non-zero $\text{Im}[\epsilon^{-1}(q, \omega')]$. The grey shaded regions correspond to the single-particle excitations, $\text{Im}[\epsilon] \neq 0$, and the thin line corresponds to the plasmon dispersion $\text{Re}[\epsilon] = 0$. To evaluate $\text{Im}[\Sigma(k, \omega)]$ within the GW approximation, one integrates the function $V(q)\text{Im}[\epsilon^{-1}(q, \omega')]$ over the two bold line segments, which belong to the curve $\omega'(q) = \omega - \xi_{k+q}$ [see Eq. (2.11c)]. The figure shows that both the single-particle excitation region and the plasmon line contribute to $\text{Im}[\Sigma(k_F, \omega)]$.

FIG. 3. (a),(b) Real (bold lines) and imaginary (thin lines) parts of the self-energy $\Sigma(k, \omega)$ and (c), (d) spectral functions $A(k, \omega)$ as functions of the energy $\omega$, for $\gamma = 0$ (solid lines) and $\gamma = E_F$ (dashed lines). Figs. (a) and (c) are for $k = 0$, and (b) and (d) are for $k = k_F$. The parameters used are $k_Fa = 0.9$ and $r_s = (4m_e e^2/\pi\hbar^2k_F\epsilon_0) = 1.4$. [39] (corresponding to $a = 100$ Å, a density of $0.56 \times 10^6$ cm$^{-1}$ and a Fermi energy of $E_F \approx 4.4$ MeV in GaAs). The vertical arrows in (c) denote $\delta$-functions in the spectral function at $\omega = -4.9, 0.9$ with weights $(2\pi) \times 0.32$ and $(2\pi) \times 0.33$, respectively. Note that in (d), $A(k_F, \omega)$ has a $\delta$-function of weight $(2\pi) \times 0.3$ at $\omega = 0$ for $\gamma = E_F$, but not for $\gamma = 0$. The straight lines are given by $\omega - \xi_k - \mu$, and their intersections with $\text{Re}[\Sigma]$ indicate the solutions to Dyson’s equation and correspond to a quasiparticle peak.
FIG. 4. (a) Momentum distribution function $n_k$, around $k/k_F = 1$, (b) Fermi surface renormalization factor $Z_F$, and (c) density of states $\rho(\omega)$ for small $\omega$, of a quasi-one dimensional electron gas for $k_F a = 0.9$ and $r_s = 1.4$ (as in Fig. 3), for various impurity scattering rates $\gamma$. The bold lines refer to $k > k_F$, and the thin lines to $k < k_F$. For $\gamma = 0$, $n_k$ is continuous at $k = k_F$, implying that the system is non-Fermi liquid, but for $\gamma \neq 0$ a discontinuity occurs at $k = k_F$, signalling the presence of a Fermi surface. The magnitude of the discontinuity in $n_{k_F}$ is given by $Z_F$.

FIG. 5. (a) Inelastic scattering rates $\Gamma(k)$, and (b) the corresponding mean free path $l(k) = \Gamma(k)k/m$, as a function of $k$, for various $\gamma$‘s (electron-impurity scattering rates), for $k_F a = 0.9$ and $r_s = 1.4$. Within RPA, for $\gamma = 0$, the $\Gamma(k)$ is identically zero below $k = k_c$ because energy and momentum conservation prohibits single particle excitations and plasmon emissions. Above $k_c$, the scattering rate is caused by plasmon emissions. For $\gamma \neq 0$, the plasmon line broadens and momentum conservation is relaxed, resulting in a nonzero $\Gamma$ for $k < k_c$.

FIG. 6. Total bandgap renormalization ($\text{Re}[\Sigma_e + \Sigma_h]$ at $k = 0$, $\omega = \xi_{k=0}$) as a function of electron density in the quantum wire for various wire widths with parameters corresponding to GaAs.

FIG. 7. Electron self-energy for impurity scattering, to lowest nontrivial order in the impurity potential, used to calculate the momentum distribution function of a disordered one-dimensional electron gas (see Fig. $\Box$). The dashed lines are the screened impurity scattering potential, which we approximate by $\delta$-functions in real space.
FIG. 8. (a) The momentum distribution function, for $\gamma_{\text{imp}}/E_F = 0.02, 0.2$ and 2, and (b) spectral function (bold solid line), and real (thin solid line) and imaginary (dashed line) parts of the self-energy and for $k = k_F$, $\gamma_{\text{imp}} = 2E_F$, of a disordered non-interacting one-dimensional electron gas at $T = 0$, calculated with the self-energy shown in Fig. 6. $\gamma_{\text{imp}} = 2mN_iU_0^2/k_F$ is the Born approximation impurity scattering rate for the electron of energy $E_F$ (the Fermi energy corresponding to the pure system of the same electron density). The inset in Fig. (a) shows the chemical potential $\mu$ for fixed density, as a function of the disorder. In Fig. (b), the arrows correspond to delta functions. The dashed arrow marks the position of the delta function for the pure system, and has weight $2\pi$. The solid arrow has weight $(2\pi) \times 0.39$. The straight dotted line is given by $\omega - \xi_k - \mu$, and its intersections with Re$[\Sigma]$ correspond to a quasiparticle peaks.

FIG. 9. The distribution function of a Luttinger liquid with Fermi surface exponent $\alpha = 0.15$, which corresponds to a dimensionless coupling constant $g \approx 7.5$, for various scattering rates. The Luttinger model linearizes the dispersion around the Fermi surface, and therefore the curves strictly should only be valid around $k = k_F$.

FIG. 10. The contour of integration $C$ for Eq. (3.11). The hatched real axis indicates a branch cut due to $w(q, \omega)$ in the integrand of Eq. (3.10). The crosses mark the poles due to the integrand; the ones on the imaginary axis are due to $n_B(\omega)$, and the isolated pole is due to the denominator. The residues of the poles on the imaginary axis give $h_{k,q}(z)$, while the residue of the isolated pole gives $\tilde{h}_{k,q}(z)$. 

81
FIG. 11. (a), (b) Real and (c), (d) imaginary parts of the self-energies and (e), (f) spectral functions, as a function of frequency, for electrons in a quantum wire, for $k = 0, k_F$, at finite temperatures. As in previous figures, $k_F a = 0.9$ and $r_s = 1.4$, which correspond to $E_F \approx 4.4\text{meV} \approx 50\text{K}$ in GaAs. The discontinuities is $\text{Im}[\Sigma(k, \omega)]$ at $T = 0$, which arise from virtual plasmon emission thresholds, broaden with increasing temperature because the plasmon peaks broaden due to Landau damping. The logarithmic divergence in the $T \neq 0 \text{Im}[\Sigma(k, \omega)]$ at $\omega = E_k - \mu_0(T)$, due to the divergence of the one-dimensional Born-approximation scattering rate, shifts slightly with respect to temperature because of the temperature dependence of $\mu_0$, the non-interacting chemical potential.

FIG. 12. Temperature dependence of the band-gap renormalization due to conduction electrons for a wire width of 100 Å in GaAs, for electron densities of $10^4$ cm$^{-1}$ (solid lines), $10^5$ cm$^{-1}$ (dotted lines) and $10^6$ cm$^{-1}$ (dashed lines). The thin lines are for the electrons $\text{Re}[\Sigma_{\text{electron}}(k = 0, \xi_k=0)]$, the light bold lines are for the holes $\text{Re}[\Sigma_{\text{hole}}(k = 0, \xi_k=0)]$ and the heavy bold lines are for the sum of the two. The densities $n = 10^4$ cm$^{-1}$, $10^5$ cm$^{-1}$ and $10^6$ cm$^{-1}$ correspond to Fermi temperatures of $E_F = 1.6 \times 10^{-2}$ K, 1.6 K and 160 K, respectively.
FIG. 13. (a) A schematic of the band diagram of a one-dimensional tunneling hot electron transistor, where electrons are injected from the emitter into the base (which contains a Fermi sea of electrons) and (b) energy– vs. momentum–loss diagram for the injected electron. In the transistor, the electrons that are scattered below the base–emitter barrier do not reach the collector, and therefore the fraction of the injected electrons reaching the collector depends on the electron scattering rate in the base region. In Fig. (b), the solid (dashed) line indicate the energy- vs. momentum-loss for electrons injected into the base region below (above) the plasmon emission threshold (i.e., the solid line is for $k < k_c$ and the dashed line is for $k > k_c$). The intersections of the energy– vs. momentum–loss curve and the plasmon dispersion curve (bold line) indicates the wavevectors at which plasmons are emitted; if there is no intersection (as with the solid line), plasmon emissions are not allowed. As the energies of the injected electrons is raised above the plasmon emission threshold, the scattering rate increases dramatically (see Fig. 14), drastically reducing the fraction of injected electrons that reach the collector.

FIG. 14. (a) Momentum scattering rate $\Gamma_{t,k}$ and (b) the corresponding mean free path, $l_k = v_k/\Gamma_{t,k}$, of an electron in a doped one-dimensional quantum wire, as a function electron momentum, for various temperatures. As in previous figures, $k_F a = 0.9$ and $r_s = 1.4$.

FIG. 15. Calculated one-dimensional plasmon dispersion in a GaAs quantum wire within RPA for (a) $T = 0$ and various $\gamma$’s, (b) $\gamma = 0$ and various temperatures, calculated by finding the zeros of $\epsilon(q, \omega)$ on the complex frequency plane. The curves with $\omega > 0$ give the real part and those with $\omega < 0$ give the imaginary part. The curves include full $V_c(q)$ for infinite square well confinement. As in previous figures, the system parameters are $k_F a = 0.9$ and $r_s = 1.4$, which, for GaAs, correspond to a Fermi wavevector of $k_F = 0.88 \times 10^6 \text{ cm}^{-1}$. In (b), we show both the long wavelength result, $\omega_{p}^2 = n q^2 V_c(q)/m$, using $V_c(q) = 2e^2 |\ln(qa)|/\kappa$ (dotted line) and the full $V_c(q)$ (long dashed lines). Experimental results of reference [5] compared with RPA theory (of reference [21]) are shown as an inset in Fig. (a).
FIG. 16. Regions of integration for the calculation of the second-order self-energy, for $k = k_F$ and $\omega = 0$. 