Research on Effect of Radar Fixing Error in Navigation on Delay of Plane Flow

Yi Zhang¹, Leilei Chang¹, Jiping Cao¹ and, Xiaodong Ling², Wei Wang³

¹ High-Tech Institute of Xi'an, Xi'an, Shaanxi 710025, P. R. China
² China Satellite Maritime Tracking and Control Laboratory, Wuxi 214400, P. R. China
³ Military Representative in Wuxi of the Military Representative Bureau in Nanjing, Wuxi 214400, P. R. China

e-mail address

Abstract. Fixing error of radar introduces some deviation into measuring value from the true. Regular interval between consecutive planes must be extended to accommodate air traffic control regulation. Mathematical relationship between fixing error and buffer time was deduced. Then, the variation of plane flow delay was researched in different condition of fixing error by means of simulation and practical calculation. It is shown that the effect of fixing error on plane flow delay is apparent compared with the determined situation. Also, the amount of delay is not only related to fixing error but also to average arrival rate. So, the fixing error factor should be considered when scheduling and delay analyzing is implemented. At the same time, it is indicated that decreasing the radar detecting error is necessary to reduce the plane flow delay.

1. Introduction

The main purpose of air traffic flow management (ATFM) includes: ensure the aircraft safety, supply accurate information for aircraft, optimize the air traffic flow, decrease the delay, increase the utilization of airport and airspace. The terminal is the critical area in ATFM because the traffic jam often happens in this area. For safety, an interval must be kept in two continuous landing aircraft. The size of the interval is a main factor for delay. There are lots of methods for sequencing in terminal area, such as FCFS algorithm, the time advance algorithm, the restraint position exchange algorithm, fuzzy recognition algorithm[1], genetic algorithm[2-3], sliding time window algorithm[4]. However, these algorithms have a common feature, that is the deviations between measured value and the true value of aircraft position is not considered. They believe that the actual position of the aircraft is the same as the measured position. The effect of fixing error of radar on aircraft flow delays is ignored.

Positioning errors may lead to violations of minimum safe spacing between adjacent aircraft which provided by ATC. In practice, in order to control the violation probability within a certain range, the controller will insert a buffer distance between the adjacent aircrafts. The insertion of a buffer distance is equivalent to an increase of the minimum safe spacing between adjacent aircraft. It is necessary to study the fixing error of radar for aircraft safety spacing and aircraft delays in navigation. Interval in this paper mainly refers to longitudinally interval.

2. The fixing error and buffer interval

2.1. Fixing error and factors

Wake is the main factor affecting the aircraft interval. To avoid the wake of previous aircraft, the landing aircrafts in terminal area required to maintain a safe interval. Under the condition of radar control, radar detects the respective positions of the two adjacent aircrafts, and then calculates the interval based on the detected positions. When the fixing error of radar is ignored, the actual interval between the adjacent aircraft is the same the actual interval. At this point, the aircraft can fly in
accordance with the minimum safe separation. But in practice, since radar is limited by its resolution, there will be error between the detected position and the true position, i.e., positioning error, and the two points as shown in Figure 1. Since the two points are close, it may be beyond its ability to distinguish them. The radar will mistakenly believe that they are the same point and display one point on the screen. Another aspect of this issue, the radar will treat the point within a certain range as the same point, it is the fixing error. Because of the fixing error, spacing between aircraft detected by radar will have a certain bias. The detected interval may be greater than the actual interval may also be less than the actual spacing. In this case, if the flight is scheduled in accordance with interval detected by the radar, there will be danger that the later aircraft is involved by previous aircraft’s wake turbulence.

![Figure 1 Fixing error of radar](image)

In practice, there are many factors that can have an impact on radar fixing error, including[5-6]:

1. Equipment factors: the position indicating error, including airborne navigation equipment error, instrument error and display error, and the time error in air traffic control system.

2. Meteorological factors: weather conditions where the airplane is exposed, mainly refers to wind.

3. The human factor: controllers and pilots, including the reaction time of controllers and pilots, the communication delay, aircraft position estimation error, manipulation error and mental state.

2.2. The insertion of buffer interval

Studies have shown that the actual position obeys the normal distribution[7], and the centre is the detected position, as shown in Figure 2.

![Figure 2 The distribution of aircraft’s actual position](image)

Suppose that there are two adjacent landing aircrafts which are flight i-1 and flight i. The detected positions are X and Y respectively. The position of previous flight i-1 has a certain bias from the actual one because of the fixing error of radar. According to the distribution, X is a normally distributed random variable. It consists of two parts. One part is the detected position, which is a certain value x. Another part is a random value due to the fixing error. That is

\[
X = x + \tilde{X}
\]  

(1)

The position of later flight can be expressed as follow

\[
Y = y + \tilde{Y}
\]  

(2)

The minimal safe separation distance of two aircraft by the rule of ATC is \( S_{i-1,i} \). So

\[
S_{i-1,i} = y - x
\]

(3)
Suppose that the standard deviation of $\bar{x}$ and $\bar{y}$ is $\sigma_{\bar{x}}$ and $\sigma_{\bar{y}}$ respectively. That is $\bar{x} \sim N(0, \sigma_{\bar{x}}^2)$, $\bar{y} \sim N(0, \sigma_{\bar{y}}^2)$.

In order to control the probability of violation of the minimum safe separation within a certain range [8-9], the controller will insert a buffer distance in interval of the adjacent aircraft because of the fixing error. The insertion of buffer distance is shown in Figure 3.

![Figure 3 The insertion of buffer distance](image)

The relation of the variables can be expressed as follow

$$p_r = P[Y - X + DBF_{i-1j} \leq S_{i-1j}] = P(y + \bar{Y} - (x + \bar{X}) + DBF_{i-1j} \leq S_{i-1j}]$$

(4)

The formula (5) can be obtained form the (3) and (4)

$$p_r = P[\bar{Y} - \bar{X} \leq -DBF_{i-1j}]$$

(5)

Form the knowledge of the probability theory, $\bar{y} - \bar{x} \sim N(0, \sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2)$, so the formula (5) is

$$p_r = P[\bar{Y} - \bar{X} \leq -DBF_{i-1j}] = \frac{1}{\sqrt{2\pi(\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2)}} \int_{-\infty}^{DBF_{i-1j}} e^{-\frac{t^2}{2(\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2)}} dt = \Phi\left(\frac{-DBF_{i-1j}}{\sqrt{\sigma_{\bar{x}}^2 + \sigma_{\bar{y}}^2}}\right)$$

(6)

Among the formula (6)

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$$

(7)

When the aircrafts lands in the same airport, they use the navigation equipment and obey the same landing procedures. It follows $\sigma_{\bar{x}}^2 = \sigma_{\bar{y}}^2 = \sigma$. So the formula (7) is

$$p_r = \Phi\left(\frac{-DBF_{i-1j}}{\sqrt{2\sigma}}\right) = 1 - \Phi\left(\frac{DBF_{i-1j}}{\sqrt{2\sigma}}\right)$$

(8)

Then

$$DBF_{i-1j} = \sqrt{2\sigma} \cdot \Phi^{-1}(1 - p_r)$$

(9)

Suppose that the velocity of the flight is $V_{IAS}$, the distance error between detected position and actual position can be converted to time deviation $t_r$ by the velocity $V_{IAS}$. That is

$$DBF_{i-1j} = \sqrt{2} \cdot V_{IAS} \cdot t_r \cdot \Phi^{-1}(1 - p_r)$$

(10)

Form the formula (10), there are three factors which effect the size of buffer distance.

(1) The velocity of the flight
The fixing error of radar

The probability of violation of minimal safety distance.
Suppose that $V_{Air} = 300kn$, the value of required buffer distance can be obtained when the $t_e$ and $p_e$ are different values, which is shown in Figure 4.

![Figure 4 The relation between buffer distance and probability of violation of minimal safety distance](image)

3. The delay effect from the fixing error

3.1. Instance selection and simulation
According to queuing theory, landing aircraft flow in the airport terminal area is Poisson flow. By the characteristics of the Poisson flow, the interval of continuous landing aircraft flow obeys the negative exponential distribution, the average inter-arrival time is the reciprocal of the average arrival rate, which can produce terminal area arrival flow of aircraft is expected to arrive time (Estimated Time of Arrival, ETA).

As the flight of aircraft generates wake, the previous aircraft will affect the later one. International Civil Aviation Organization (ICAO) set a safety interval between different types of aircraft in windless condition. And the corresponding time of the interval is shown in Tab. 1\[10\].

The controller will give each aircraft a Scheduled Time of Arrival(STA) based on each aircraft's ETA by certain rules and an algorithm, to minimize the waiting time of every aircraft. In the process of implementing the calculation and scheduling delays, the most considered factor is adjacent safe separation between aircraft.

| Table 1 The minimal interval time of different aircraft(s) |
|-----------------------------------------------------------|
| previous | Heave | 113 | 135 | 170 |
| Large | 89 | 89 | 110 |
| small | 83 | 83 | 94 |

The insertion of buffer distance representing an increase of safe separation between aircraft. When the arrival rate is low, the controller will not consider the insertion of buffer distance. When the arrival rate is high, the average interval is short, the controller have to consider the insertion of buffer distance. And the impact on aircraft’s waiting time increases. In order to study the effect of buffer distance insertion on the waiting time, the different arrival rate is set.
Because the size of the buffer controller added to all aircraft are the same, in order to facilitate the calculation, STA airplane sequence is calculated using FCFS collation. In simulation, the average arrival rate is set as 10 to 100/hr, the range of ETA time of is 1 hour, the value of the fixing error takes 2-10 seconds. Simulation takes 100 times. According to the ATC, the probability of violation does not exceed 5%.

For the flow containing M aircrafts, the effect of fixing error on the whole flow can be obtained by the next formula

\[
W_{ext} = \frac{\sum_{i=1}^{M} (STA^{IF} - STA)}{\sum_{i=1}^{M} (STA - ETA)}
\]  

In formula (11), \( STA \) contains no error, that is the STA of aircraft I when \( t_e = 0 \), and \( STA^{IF} \) is the STA contains error.

3.2. Delay analysis

By MATLAB Simulation, the delay increment induced by fixing error of radar is shown in Figure 5. As can be seen from Figure 5.

(1) The waiting time increases as the fixing error increases. Meanwhile, the extent of delay is related to average arrival rate too.

(2) When the fixing error is fixed, the waiting time increases as the average arrival rate increases. When the average arrival rate is 30-40/h, this effect is the most obvious; when the average arrival rate is 6/h, this effect is tend to stable, at this time, the waiting time of the flow increase about 1.9 percentage points when value of the fixing error increase 2.

(3) When the arrival rate is fixed, the increment of waiting time of the flow is proportional to the size of fixing error.

![Figure 5](image)

**Figure 5** The effect of fixing error on delay

4. Conclusions

The results show that: with respect to the deterministic case, effect of fixing error of radar on the delay of aircraft is apparent. Also, the delay has a greater relationship with the flow density. When the fixing error is fixed, the waiting time increases as the average arrival rate increases. When the arrival rate is fixed, the increment of waiting time of the flow is proportional to the size of fixing error. It shows that only the determined case is considered is not enough, the fixing error of must be considered.

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