Toward an Idiomatic Framework for Cognitive Robotics

Malte R. Damgaard*, Rasmus Pedersen, Thomas Bak

Summary—Inspired by the “Cognitive Hourglass” model presented by the researchers behind the cognitive architecture called Sigma, we propose a framework for developing cognitive architectures aimed at cognitive robotics. The purpose of the proposed framework is foremost to ease the development of cognitive architectures by encouraging and mitigating cooperation and re-use of existing results. This is done by proposing a framework dividing the development of cognitive architectures into a series of layers that can be considered partly in isolation, and some of which directly relate to other research fields. Finally, we give introductions to and review some topics essential to the proposed framework.

1 Introduction

Research in cognitive robotics originates from a need to perform and automate tasks in dynamic environments and in close or direct interaction with humans. Uncertainty about the environment and complexity of the tasks require robots with the ability to reason and plan while being reactive to changes in their environment. To achieve such behavior, robots cannot rely on predefined rules of behavior and inspiration is taken from cognitive architectures.

Cognitive architectures provide a model for information processing that can capture robot functionalities. In combination with acquired sensory data, they can potentially generate intelligent autonomous behavior. Cognitive architectures dates back to the 1950s with a grand goal of implementing a full working cognitive system. From this considerable challenge, an abundance of architectures have evolved, and a recent survey suggests that the number of existing architectures has reached several hundred. Some are aimed towards robotics application, e.g., Robo-Soar, CARACaS and RoboCog. Unfortunately, most of these architectures take wildly different approaches to model cognition and are implemented in different programming languages. Furthermore, most of these architectures are constructed from a diverse set of specialized modules, e.g., special purpose speech, vision, and simultaneous localization and mapping modules, interacting with each other and have taken decades to develop. Thereby, making it non-trivial to expand upon, combine, and re-use parts of these architectures. Again, this makes it potentially harder for other researchers and practitioners to contribute to or adopt the architectures for specific problems and applications. In line with the arguments for developing an interface layer for artificial intelligence put forward by other researchers, we argue that a unifying and standardized framework for developing new cognitive architectures aimed at cognitive robotics could potentially remedy these issues and ease the development of cognitive robotics.

In recent years a community consensus has begun to emerge about a standard model of humanlike minds, i.e., computational entities whose structures and processes are substantially similar to those found in human cognition. While this “Standard Model of the Mind” spans key aspects of structure and processing, memory and content, learning, and perception and motor, it is agnostic to the best practice for modeling and implementing these things. In line with the idea proposed by the researchers behind the cognitive architecture Sigma, we argue that the evolution of the scientific field of cognitive robotics could benefit from anchoring new implementations around a common theoretical elegant base separating a specific model of a part of cognition from the algorithm that implements it. Furthermore, this theoretical base could allow new functionalities to evolve hierarchically just like software libraries build on top of each other. Thereby, allowing the discussions and development to flourish at different levels of abstractions, and allow for direct synergy with other research fields.

To explain the cognitive architecture Sigma, the authors present a Cognitive Hourglass model based on the following four desiderata:

- Grand Unification, spanning all of cognition,
- Generic Cognition, spanning both natural and artificial cognition,
- Functional Elegance, achieving generically cognitive grand unification with simplicity and theoretical elegance,
- Sufficient Efficiency, efficient enough to support the anticipated uses in real-time.

While Grand Unification and Sufficient Efficiency aligns well with the needs of cognitive robotics, the need for Generic Cognition and Functional Elegance is subtle for cognitive robotics. Although the end goal of cognitive robotics might only be to develop functional artificial intelligence, building upon something that potentially is also able to model natural intelligence would allow artificial intelligence to more easily benefit from insights obtained by the modeling of natural intelligence and vice versa. Similarly, Functional Elegance is not a goal of cognitive robotics per se. Still, it could allow researchers and practitioners working on different levels of cognition to obtain a common reference point and understanding at a basic level, potentially easing co-operation and re-use of results and innovative ideas.
In an attempt to obtain all four of these desiderata the so-called “graphical architecture” is placed at the waist of Sigma’s Cognitive Hourglass model as such a potential theoretical elegant base glueing everything together just like the Internet Protocol (IP) in the Internet-hourglass model. Functional elegance is obtained by recognizing and developing general architectural fragments, and based on these defining idioms that can be re-used in modelling different parts of cognition. Having defined sufficiently general idioms, the hope is to be able to develop full models of cognition from a limited set of such idioms and thereby achieve functional elegance, while at the same time achieving the three other desiderata.

With roots in the given desiderata, Sigma’s Cognitive Hourglass model in many ways could constitute a unifying and standardized framework for cognitive robotics. However, as we will elaborate on in Section 2 the model commits to specific architectural decisions, which hinders the utilization of new technology and ideas. E.g., their commitment to the sum-product algorithm prevents the use of new algorithms for efficient probabilistic inference. Therefore, based on the observation that the layers of Sigma’s Cognitive Hourglass model conceptually can be divided into more and generalized layers, in this paper, we propose a new generalized Cognitive Hourglass model based on recent advancements within the machine learning community that makes no such commitments. The presented Generalized Cognitive Hourglass model constitutes a new framework for guiding and discussing the future development of cognitive robotics. As such, with this paper we do not intend to construct a new specific cognitive architecture our-self. Instead, our framework should be viewed as a space of systems, and our intend with this framework and manuscript is

1) to provide a framework for other researchers to expand upon,
2) to ease the development of cognitive architectures for robotics by encouraging and mitigating cooperation and re-use of existing results,
3) and finally to highlight some of the current state-of-the-art technology available to progress this research field.

In Section 2 we briefly introduce the Sigma’s Cognitive Hourglass model in more detail. Based on this, we present our Generalized Cognitive Hourglass model as a framework for developing new cognitive architectures aimed at cognitive robotics in Section 3. In Section 3, we give a brief introduction to probabilistic programs since the presented framework is built around them. Explaining the functionality of probabilistic programs with conventional methods can be difficult, therefore in Section 3.1 we present a new graphical representation of probabilistic programs which we call “generative flow graphs”. We do so in the hope that it will ease the dissemination of new models of parts of cognition developed within the proposed framework. Being fundamental to achieving functional elegance within the proposed framework, we formally introduce the concept of probabilistic programming idioms in Section 3.2 and explain how “generative flow graphs” can aid the identification of such idioms. In Section 3.3 we discuss the intrinsic problem of performing approximate inference in complex probabilistic programs and present some modern algorithms to tackle this problem for cognitive robotics. As probabilistic programming languages form the foundation of the present framework, we give a brief survey of probabilistic programming languages relevant to the framework in Section 3. Finally, in Section 4 we shortly present some preliminary work to support the framework presented.

2 SIGMA’S COGNITIVE HOURGLASS MODEL

Fig. 1 illustrates how the dimensions of the model Sigma’s Cognitive Hourglass Model relate to the four desiderata. The top layer of the hourglass represents all the knowledge and skills implemented by the cognitive system. This includes high level cognitive capabilities such as reasoning, decision making, and meta cognition, as well as low level cognitive capabilities such as perception, attention, and the formation of knowledge and memory that could potentially be inspired by human cognition. But it also includes artificial cognitive capabilities such as, e.g., the creation of a grid-maps common in robotics. As such, the extend of this layer corresponds to the achievable extend of Grand Unification and Generic Cognition.

The “cognitive architecture” layer defines central architectural decisions such as the utilization of the cognitive cycle and tri-level control structure for information processing, and the division of memory into a perceptual buffer, working memory and long-term memory. But also defines other architectural concepts such as “functions”, “structures”, “affect/emotion”, “surprise”, and “attention”. Thereby, the cognitive architecture induces what can be considered an “cognitive programming language” in which all of the knowledge and skills in the top layer can be embodied and learned. As an intermediate layer, cognitive idioms provide design patterns, libraries, and services that ease the implementation of knowledge and skills.

Below the Cognitive architecture and at the waist of the model, they have the graphical architecture constituting a small elegant core of functionality. Functional elegance is thereby obtained by compilation of knowledge and skills through a series of layers into a common representation in the graphical architecture. This graphical architecture primarily consists of probabilistic inference over graphical models, more specifically factor graphs, utilizing the sum-product algorithm plus the following extensions:

1) each variable node is allowed to correspond to one or more function variables,
2) special purpose factor nodes,
3) and the possibility of limiting the direction of influence along a link in the graph.

Of these extensions, the two first are merely special-purpose optimizations for the inference algorithm, i.e., a part of the implementation layer in Fig. 1. According to the authors the third extension has “a less clear status concerning factor graph semantics". Finally, the graphical architecture is implemented in the programming language LISP. In this model, sufficient efficiency is achieved as the cumulative efficiency of all layers. I.e., an efficient implementation in LISP is futile if models of knowledge and skills are inefficient for a given task.
To summarize, the model shown in Fig. 1 commits to multiple more or less restrictive decisions such as the utilization of factor graphs and the sum-product algorithm at its core, the “cognitive cycle”, the tri-level control structure, and LISP as the exclusive implementation language. While these commitments may be suitable for the specific cognitive architecture Sigma mainly targeted human-like intelligences, they would hinder the exploration of new ideas and the utilization of new technologies, making this model less suitable as a general framework.

3 GENERALIZED COGNITIVE HOURGLASS MODEL

While Sigma’s Cognitive Hourglass model has an advantageous structure with roots in highly appropriate desiderata, it is not suitable as a general framework, due to some exclusive structural commitments. We argue that these structural commitments are mostly artefacts of the limited expressibility of factor graphs and the sum-product algorithm.

Consider, for instance, the “cognitive cycle” dividing processing into an elaboration and adaption phase. The elaboration phase performs inference over the factor graph, while the adaption phase modifies the factor graph before further inference. We argue that the need for this two-phase division of processing is caused by the need for the sum-product algorithm to operate on a static factor graph. This cognitive cycle further makes the tri-level control structure necessary to make cognitive branching and recursion possible. Similarly, we argue that the third extension of the factor graph semantics employed in the cognitive architecture Sigma is nothing more than a simple control flow construct over the information flow in the graphical model and inference algorithm. It is straightforward to imagine how other control flow constructs such as recursion, loops, and conditionals could also be advantageous in modeling cognition.

Basically, we believe that special-purpose implementations of architectural constructs such as the two-phase “cognitive cycle” employed by Sigma and similar cognitive architectures have previously been necessary due to the limitations of the available modeling tools. The flexibility of probabilistic programs provided by the possibility of incorporating I/O operations, loops, branching, and recursion into a probabilistic model, should permit representing such constructs as either Probabilistic Programming or Cognitive Idioms instead.

In Fig. 2 we present our proposal for a more general cognitive hourglass model having probabilistic programs at its waist as the theoretical modeling base. Just like the model in Section 2, our model is composed of a series of layers that expands away from the waist of the hourglass. On top of the pure probabilistic program, we might be able to recognize program fragments that are sufficiently general to be considered idioms. From these idioms, it might be possible to construct dedicated programming languages for expressing cognitive behavior, knowledge, and skills, such as the “Cognitive Language” employed in the cognitive architecture Sigma. In this framework, functional elegance above the probabilistic program is obtained via compilation of knowledge and skills through appropriate cognitive programming languages into probabilistic programs. Below the probabilistic program’s different inference algorithms can carry out the necessary inference in the probabilistic program. Different versions of these inference algorithms can potentially be implemented in different probabilistic languages. Both the probabilistic program and probabilistic programming language can be situated in standard deterministic programming languages. Furthermore, one needs not even use the same deterministic programming language for both. Finally, the deterministic programming...
languages allow us to execute a model of cognition on different types of hardware doing the actual computations. When comparing Sigma’s Hourglass model to the Generalized Hourglass model the complexity might seem to have increased. However, this is not the case. The Generalized Hourglass model simply highlights some of the components implicit in Sigma’s Hourglass model.

We expect that this model is sufficiently general to be considered a framework for research in, and development of, cognitive robotics. In fact, as probabilistic programs can be considered an extension of deterministic programs, it should even be possible to situate both emergent, symbolic, and hybrid approaches to cognitive architectures in this framework, thereby covering the full taxonomy considered in. In our framework constructs such as the cognitive cycle and tri-level control structure could potentially be expressed as probabilistic programming idioms rather than special-purpose architectural implementations. Similarly, incorporation of results from other research areas such as deep learning is only limited to the extent that a given probabilistic programming language and corresponding inference algorithms can incorporate essential tools used in these research areas, i.e., automatic differentiation for deep learning. Furthermore, this framework gives a satisfying view on the foundational hypothesis in artificial intelligence about search areas, i.e., automatic differentiation for deep learning.

The Generalized Hourglass model is reduced thus depends upon the technology available in each of the layers of the hourglass.

Even though our generalized cognitive hourglass model has roots in the ideas behind the cognitive architecture Sigma, other evidence for the appropriability of the framework for cognitive or developmental robotics can also be found in the literature. In recent years, the power of utilizing probabilistic models for cognitive robotics has been demonstrated in a lot of studies. For example, multi-layered multimodal latent Dirichlet allocation (mMLDA) models have been used for learning and representing the hierarchical structure of concepts. A complex probabilistic graphical model and an online inference algorithm for simultaneous lexical and spatial concept acquisition have also been demonstrated. Based on this model, another probabilistic graphical model for navigation utilizing the learned concepts has also been proposed. The interaction between multiple probabilistic graphical models, mainly mMLDA, integrated as a cognitive architecture for robot learning of action and language has also been studied. Related to this research two frameworks called SERKET and Neuro-SERKET have been proposed with the goal of connecting multiple probabilistic graphical models on a large scale to construct cognitive architectures for robotics. Thereby, both of these frameworks indirectly acknowledge the advantages of constructing large-scale cognitive models by composing them from smaller fundamental entities. In both frameworks, these fundamental entities are called “modules”. In SERKET these “modules” are limited to fragments of hierarchical probabilistic graphical models with head-to-tail connections. Neuro-SERKET extends the expressibility of SERKET by allowing “modules” to be general probabilistic generative models with special attention to deep generative models. Connections between these “modules” are allowed in both a head-to-tail and head-to-head fashion, but also in a tail-to-tail fashion via a product of expert approximation of such connections. In both frameworks exact message-passing is used to perform inference on models with discrete and finite variables, otherwise sampling importance resampling is used. As such both of these frameworks can be considered special cases of our framework, with “modules” and their connections somehow resembling what we have chosen to call “probabilistic programming idioms”. The difference is that “modules” in SERKET and Neuro-SERKET are supposed to be fully defined and self-contained, whereas our definition of “probabilistic programming idioms” allows for nesting and e.g. class definitions with abstract methods as we will exemplify in Section. Furthermore, being based solely on probabilistic graphical models, SERKET and Neuro-SERKET currently do not seem to incorporate logic into their “modules”. Some of the most influential work arguing that a combination of logic, especially first-order, and pure probabilistic graphical models is necessary to compose a sufficiently general
interface layer between artificial intelligence and the algorithms that implements it, i.e. the waist of the presented framework, is perhaps the work related to Markov Logic and the system called Alchemy. Like SERKET and NeuroSERKET, Alchemy may also be considered one instance of our framework, limiting the probabilistic programs at the waist to Markov logic, and utilizing a combination of Markov chain Monte Carlo and lifted belief propagation for inference.

4 Preliminaries

In this paper, we do not distinguish between probability density functions and probability mass functions, and jointly denote them as probability functions. The symbol \( f \) is used to denote both integrals and summations depending on the context. In general, we use \( z \) to denote latent random variables, \( x \) to denote observed random variables, \( p(...) \) to denote “true” probability functions, \( q(...) \) to denote approximations to “true” probability functions, \( \theta \) to denote parameters of “true” probability functions, \( p(...) \), and \( \phi \) to denote parameters of approximations to “true” probability functions, \( q(...) \). When a probability function directly depends on a parameter we write the parameter in a subscript before the parentheses, e.g., \( p_\theta(...) \) and \( q_\phi(...) \). We use line over a value, parameter, or random variable to denote that it is equal to a specific value, e.g., \( \bar{x} = 1,432 \). We use a breve over a parameter or random variable to denote that it should be considered a fixed parameter or random variable within that equation, e.g., \( \bar{\theta} \) or \( \bar{z} \). For parameters this means that they attain a specific value, \( \bar{\theta} \), i.e., \( \bar{\theta} \) means that \( \theta = \bar{\theta} \).

For random variables it means that the probability functions that this variable is associated with is considered fixed within a given equation. We use capital letters to denote sets, e.g., \( A = \{1, ..., \pi\} \). We use a superscript with curly brackets to denote indexes. E.g., \( z^{(i)} \) would denote the \( i \)'th latent random variable. Similarly, we use a superscript with curly brackets and two numbers separated by a semicolon to denote a set of indexes values, i.e., \( z^{(1:\pi)} = z^{(A)} = \{z^{(1)}, ..., z^{(\pi)}\} \). We use a backslash, \( \setminus \), after a set followed by a value, random variable, or parameter to denote the exclusion of that value, random variable, or parameter from that set, i.e., \( z^{(A)} \setminus z^{(\pi)} = \{z^{(1)}, ..., z^{(\pi-1)}\} \). We use capital \( C \) to denote a collection of latent random variables, observed random variables, and parameters. Furthermore, we will specify such a collection by enclosing variables and parameters with curly brackets around and with a semicolon separating latent random variables, observed random variables, and parameters in that order, e.g., \( C = \{Z; X; \Theta\} \). We will use \( Pa, Ch, An, \) and \( De \) as abbreviations for parent, child, ancestors, and descendants, respectively, and use, e.g., \( Pa(\Theta)(C) \) to denote the set of parameters parent to the collection \( C \), and \( Ch(X)(Z) \) to denote the set of observed variables that are children of the latent random variable \( Z \).

5 Probabilistic Programs

At the heart of our framework, we have chosen to place probabilistic programs. One definition of probabilistic programs is as follows:

“Probabilistic programs are usual functional or imperative programs with two added constructs: (1) the ability to draw values at random from distributions, and (2) the ability to condition values of variables in a program via observations.”

With these two constructs, any functional or imperative program can be turned into a simultaneous representation of a joint distribution, \( p_\Theta(Z, X) \), and conditional distribution, \( p_\Theta(X|Z) \), where \( X \) represent the conditioned/observed random variables, \( Z \) the unconditioned/latent random variables, and \( \Theta \) represents other parameters in the program that are not given a probabilistic treatment. Thereby allowing us to integrate classical control constructs familiar to any programmer such as if/else statement, loops, and recursions into probabilistic models. As such probabilistic programs can express exactly the same functionality as any deterministic programs can and even more. These two constructs are usually provided as extensions to a given programming language through special \texttt{sample} and \texttt{observe} functions or keywords. Thus it would be natural to represent such probabilistic programs by pseudo-code. However, based on experiences it can be hard to follow the generative flow of random variables in such pseudo-code. Alternatively, such generative flows have classically been represented by directed graphical models. Unfortunately, we have also found that the semantics of classical directed graphical models neither provide an appropriate presentation.

5.1 Generative Flow Graphs

We have found that combining the semantics of classical directed graphical models with the semantics of flowcharts into a hybrid representation is a good visual representation. Directed graphical models represent the conditional dependency structure of a model and flowcharts represent the steps in an algorithm or workflow. The hybrid representation illustrates the order in which samples of random variables in a probabilistic program are generated and how these samples influence the distributions used to generate other samples. For this reason, we denote this hybrid representation by the name Generative Flow Graph.

To exemplify the utility of the Generative Flow Graph representation consider the graphical model for a classical Markov Decision Process and the simultaneous localization and mapping (SLAM) problem depicted in Fig. 1. With the classical semantics of directed graphical models, it is often the case that size limitations of figures coerce authors to remove some variables from the figure and represent them indirectly, e.g., dashed arrows as the case in both Fig. 3a and Fig. 3b. Similarly, the classical semantics of directed graphical models does not represent the influence from other parameters or variables that are not given a probabilistic treatment, even though such variables and parameters might have equal importance for a model. This is especially true if they are not fixed and have to be learned, e.g., if one wants to incorporate artificial neural networks into a model. The classical semantics of directed graphical models also cannot represent dependency structures depending on conditionals giving the illusion that a variable always depends on all of its possible parents, and
that all variables in the graph are relevant in all situations. Furthermore, while the semantics of directed graphical models allows us to represent the structure of the joint distribution, \( p(Z, X) \), its ability to explicitly express the structure of the posterior distribution, \( p(Z|X) \), is limited. Finally, there is no standardized way of representing a fragment of a graphical model, which hinders discussions at different levels of abstraction. Probabilistic programs easily allow us to incorporate the above in our models and thus a more appropriate representation is needed. The semantics of generative flow graphs shown in Table 1 in Section 12 alleviate these problems. Utilizing these semantics we can redraw the directed graphical model in Fig. 3a in multiple ways with different levels of information as in Fig. 4. Notice, that the choice of node collections is not unique.

One advantage of the semantics of directed graphical models is that for graphs with no cycles such models represents a specific factorization of the joint probability of all the random variables in the model of the form:

\[
p(x^{(n)}; z^{(m)}) = \prod_{n=1}^{\pi} p(x^{(n)}|PaZ(x^{(n)})) \prod_{m=1}^{\pi} p(z^{(m)}|PaZ(z^{(m)}))
\]  

(1)

where \( x^{(n)} \) and \( z^{(m)} \) are the \( n \)’th observed and the \( m \)’th latent random variable in the model, respectively. This is in principle also true for the generative flow graph representation if it neither contains any cycles, just with the additional explicit representation of dependency on parameters. For generative flow graphs, we can similarly to Eq. 1 write up a factorization by including a factor of the form

\[
P_{Pa\theta(z^{(m)})}(z^{(m)}|PaZ(z^{(m)}))
\]

for each latent random variable node, \( z^{(m)} \), in the graph, and a factor of the form

\[
P_{Pa\theta(x^{(n)})}(x^{(n)}|PaZ(x^{(n)}))
\]
for each observed random variable node, \( x^{(n)} \), in the graph, and finally a factor of the form

\[
P_{\Theta, \phi(C^{(k)})}(Z, X | Pa(Z(C^{(k)})))
\]

for each node collection, \( C^{(k)} = \{Z; X; \Theta \} \). If a parent node of \( y \) is a node collection \( \{Z; X; \Theta \} \) then \( Pa(Z(y)) = Z \) and \( Pa(\phi(y)) = \phi \) unless a subset of the variables or parameters in the node collection is explicitly specified next to the parent link. If the internal structure of a node collection is known from somewhere else, the factor in Eq. (2) can of course be replaced by the corresponding factorization. The catch, however, is that a probabilistic program, and thus also generative flow graphs, potentially can denote models with an unbounded number of random variables and parameters making it impossible to write up the full factorization explicitly. On the other hand, this just emphasizes the need for alternative ways of representing probabilistic programs other than pseudo-code.

Besides the possibility of expressing a factorization of the joint prior distribution, the detached link allows us to express additional structure for the posterior distribution, \( p(z|x) \). Consider the two generative flow graphs in Fig. 5. By applying standard manipulations we can obtain the factorization in Eq. (3) for the graph in Fig. 5a.

\[
p_{\theta_a, \theta_b}(z_a, z_b | x_a, x_b)
= p_{\theta_a, \theta_b}(z_b | z_a, x_a, x_b)p_{\theta_a, \theta_b}(z_a | x_a, x_b)
= p_{\theta_a, \theta_b}(z_b | z_a, x_b)p_{\theta_a, \phi_b}(z_a | x_b)
\]

wheres from the definition of the detached link we can write the factorization in Eq. (4) for the graph in Fig. 5b.

\[
p_{\theta_a, \theta_b}(z_a, z_b | x_a, x_b)
= p_{\theta_a, \theta_b}(z_b | z_a, \bar{x}_a, x_b)p_{\theta_a}(z_a | x_a)
= p_{\theta_a, \phi_b}(z_b | z_a, x_b)p_{\theta_a}(z_a | x_a)
\]

The main difference between these two factorizations is the distribution over the latent variable \( z_a \). In Eq. (3) the distribution over the latent variable \( z_a \) depends on the evidence provided by both observations \( x_a \) and \( x_b \), and is influenced by both parameters \( \theta_a \) and \( \theta_b \). In Eq. (4) the distribution over \( z_a \) depends only on the evidence provided by the observations \( x_a \), and is only influenced by the parameter \( \theta_a \). As such the inference problem of obtaining the posterior distribution over \( z_a \) is independent of the inference problem of obtaining the posterior distribution over \( z_b \), but not conversely. In general, for model consisting of \( n \) node collections, \( C^{(n)} = \{Z^{(n)}, X^{(n)}; \Theta^{(n)}\} \), connected only by detached links we can write the factorization of the posterior as

\[
p_{\theta}(Z|X) = \prod_{a=1}^{n} p_{\phi(a), \Theta^{(a)}}(C^{(a)}) \left(Z^{(a)} | Pa(Z(C^{(a)})), X^{(a)}\right)
\]

where the breves are used to emphasize that the variables and parameters are related through a detached link. The possible benefit of being able to express such structure will become clear in Section 6.

Another added benefit of the generative flow graph representation is to express models by different levels of abstraction. As an example consider the three different factorization of the simultaneous localisation and mapping problem given in Eq. (5), Eq. (6), and Eq. (7).

\[
p\left(z_s^{(0:t-1)}, z_a^{(0:t-1)}, x_p^{(1:t)}; z_m^{(0:t)}\right)
= p\left(z_s^{(0:t)}; z_a^{(0:t-1)}, x_p^{(1:t)} | z_s^{(0)}, z_m^{(0:t)}\right)
= p\left(z_s^{(0)}\right) \prod_{i=1}^{t} p\left(z_i^{(1)}\right)
\]

\[
p\left(z_s^{(0)}\right) \prod_{i=1}^{t} p\left(z_i^{(1)}\right)
= p\left(z_s^{(0)}\right) \prod_{i=1}^{t} p\left(z_i^{(0)}\right)
\]

\[
\prod_{i=1}^{t} p\left(x_p^{(1:t)} | z_s^{(0)}, z_a^{(0:t-1)}, z_m^{(0):i}\right)
\]

The generative flow graphs in Fig. 4a, Fig. 4b, and Fig. 4c corresponds directly to the factorization in Eq. (5), Eq. (6), and Eq. (7), respectively. Thereby, they represents different levels of abstractions for the same model. As such generative flow graphs simply yield better expressibility over their directed graphical model counterparts.

### 5.2 Probabilistic Programming Idioms

We have already discussed how probabilistic programming idioms can be seen as a means to achieve functional elegance. In this section, we describe how such idioms can be discovered by inspecting generative flow graphs. We define probabilistic programming idioms as follows:

"Probabilistic programming idioms are reusable code fragments of probabilistic programs sharing an equivalent
To identify such probabilistic programming idioms, we can look for node collections containing the same nodes and with the same internal structure in at least two different probabilistic programs. Consider for example the node collection \( \{ z_s^\tau, z_a^\tau-1 \} \) highlighted with a green border in the generative flow graph for both the simultaneous localization and mapping problem and Markov Decision Process depicted in Fig. 4 and Fig. 6 respectively. From Fig. 4a and Fig. 6a it is clear that the internal structure of this node collection is identical in both graphs, and that it represents the factorization

\[
p(z_s^\tau | z_a^\tau-1, z_a^\tau-1) p(z_a^\tau-1).\]

Assuming that the distributions \( p(z_s^\tau | z_a^\tau-1, z_a^\tau-1) \) and \( p(z_a^\tau-1) \) are the same in both models, we could possibly create a probabilistic program for this node collection once, and then reuse it in both models. This probabilistic program should then take a sample \( z_s^\tau-1 \) as input. From this input the program could sample both \( z_s^\tau \) and \( z_a^\tau-1 \) from “hard-coded” distributions \( p(z_s^\tau | z_a^\tau-1, z_a^\tau-1) \) and \( p(z_a^\tau-1) \) using the sample function or keyword of the probabilistic programming language. Finally, the program should return both of these samples. While this approach might work perfectly for some applications, the two distributions \( p(z_s^\tau | z_a^\tau-1, z_a^\tau-1) \) and \( p(z_a^\tau-1) \) are usually application specific, limiting the usability for an idiom in which they are “hard-coded”. A far more general approach would be to allow the probabilistic program to instead take the two distributions as input or have these distributions as free variables, allowing us to re-use the code fragment even for problems where these distributions are not necessarily the same. Rather than fully defining a model of a part of cognition, such a probabilistic program would constitute a template method for the generative flow of that part of cognition. Specific utilization of the model could then be done via a function closure specifying the free distributions. While the benefits of the above example arguably might be limited since the internal structure of the node collection is relatively simple, it is not hard to imagine more complex structures. Consider for instance the node collection highlighted with a blue border in Fig. 6b. By constructing an appropriate probabilistic program for this node collection we have defined a probabilistic programming idiom constituting the foundation for optimal control and reinforcement learning.

When we have developed such probabilistic programming idioms it empowers us to mix and match them to construct higher-level intelligence without worrying about all details of the underlying models. E.g. Fig. 7 implies that the output of a specific model for the simultaneous localization and mapping problem is used as the input to a Markov decision process, but leaves out details about their internal structures.

### 6 inference algorithms

As stated in Section 5 a probabilistic program is a simultaneous representation of a joint distribution, \( p_\Theta(Z, X) \), and a conditional distribution, \( p_\Theta(X|Z) \). Having defined a model as such distributions we are usually interested in answering queries about the unconditioned/latent random variables, \( Z \), given information about the conditioned/observed random variables, \( X = \overline{X} \). In the combined navigation problem illustrated in Fig. 7 we are interested in determining which action to take, \( z_s^\tau \), given prior perceived information, \( x_p^\tau \) for \( \tau = 1, ..., t \), and future optimality variables, \( x_o^\tau \) for \( \tau = t+1, ..., T \). Often queries of interest are statistics such as the posterior mean and variance of specific random variables, or the posterior probability of a random variable being within a given set. Still, it could also simply be to sample from the posterior, \( p_\Theta(Z|X = \overline{X}) \). All of these
queries are somehow related to the posterior distribution given by

\[ p_\theta (Z|X = \bar{X}) = \frac{p_\theta (X = \bar{X}, Z)}{p_\theta (X = \bar{X})} = \int p_\theta (X = \bar{X}, Z) \, dZ. \]  

(9)

The marginalization by the integral in the denominator of Eq. (9) in general does not have an analytical solution or is intractable to compute in most realistic problems and approximate inference is therefore necessary.\(^6\)

Through time, an abundance of algorithms has been developed to find an approximation to the posterior in specific problems. Unfortunately, many of these algorithms cannot be applied to general probabilistic programs mainly due to the possible unbounded number of random variables.\(^7\) Possible applicable inference algorithms can roughly be divided into Monte Carlo based algorithms such as Sequential Monte Carlo, Metropolis-Hasting, and Hamiltonian Monte Carlo, and the optimization based Variational Inference algorithms such as Stochastic Variational inference. As the size and complexity of models of cognition increase, the computational efficiency of inference algorithms becomes a paramount necessity to achieve sufficient efficiency of the framework presented in Section 3. While Monte Carlo methods often converge to the true posterior in the limit, convergence can be slow. Conversely, Variational Inference algorithms are often faster even though they can suffer from simplified posterior approximations.\(^2\) Also, as Variational Inference methods are based on optimization they provide a natural synergy with data-driven discriminative techniques such as deep learning. By accepting that robots are not necessarily supposed to behave optimally, but rather should behave as agents with bounded rationality, the above characteristic makes Variational Inference algorithms an especially interesting choice for cognitive robotics. Therefore, Section 6.1 is devoted to giving the reader an introduction to the overall concept of Variational Inference. Section 6.2 and 6.3 present two specific solution approaches commonly used in variational inference, namely Message-passing algorithms and stochastic variational inference, respectively. Both approaches have their weaknesses. Therefore, in Section 6.4 we outline a way of combining these two approaches to overcome their weaknesses. The idea of combining Message-passing with stochastic variational inference, we have presented before, however, we generalize the idea to generative flow graphs.

### 6.1 Variational inference

Variational inference is an optimization based approach to approximate one distribution, \( p(Z) \), by another simpler distribution, \( q(Z) \). \( q(Z) \) is usually called the variational distribution. In general, variational inference is not only used to approximate conditional distribution, \( p(Z|X = \bar{X}) \), as in Eq. (9). However, with the presented framework in mind we will limit our presentation to this case, and focus on variational inference problems on the form

\[ q^\ast (Z) = \arg \min_{q(Z) \in Q} D (p_\theta (Z|X = \bar{X})\|q(Z)). \]  

(10)

where \( D \) is a measure of the similarity between \( p \) and \( q \)

\( q \) often called a divergence measure, and \( Q \) is the family of variational distributions from which the approximation should be found. The notation \( D(p\|q) \) denotes a divergence measure and that the order of the arguments, \( p \) and \( q \), matters. The choice the family of variational distributions, \( Q \), is a compromise between computational efficiency and precise an approximation one wants. Furthermore, \( Q \) should be chosen such that we can easily answer given queries. It is important to stress that any variational inference method is more or less biased via the choice of the family of variational distributions, \( Q \). As a consequence we cannot view the original model in isolation, and have to consider the variational distribution, \( q(Z) \), as an implicit part of the cognitive model. Besides the family of variational distributions, the choice of the divergence measure, \( D \), can substantially impact the properties of the approximation. However, empirical results suggest that for the family of \( \alpha \)-divergences, subsuming the commonly used Kullback–Leibler divergence, all choices will give similar results as long as the approximating family, \( Q \), is a good fit to the true posterior distribution.\(^3\)

### 6.2 Message-Passing

Message-passing algorithms solves a possible complicated variational inference problem as defined by Eq. (10) by it down into a series of more tractable sub-problems.\(^4\) The methods are known as message-passing algorithms due to the way that the solution to one sub-problem is distributed to the other sub-problems. Message-passing algorithms assumes that the model of a problem, \( p(Z|X) \), can be factorized into a product of probability distributions

\[ p(Z|X) = \prod_{a \in A} p^{(a)}(Z|X). \]  

(11)

This factorization need not be unique and each factor, \( p^{(a)}(Z|X) \), can depend on any number of the variables of \( p(Z|X) \). The variational distribution, \( q(Z) \), should furthermore be chosen such that it factorizes into a similar form

\[ q(Z) = \prod_{a \in A} q^{(a)}(Z). \]  

(12)

With these assumptions, define the product of all other than the \( a \)'th factor of \( q(Z) \) and \( p(Z|X) \), respectively as

\[ q^{\backslash a}(Z) = \prod_{b \in A \backslash a} q^{(b)}(Z), \]  

(13)

\[ p^{\backslash a}(Z|X) = \prod_{b \in A \backslash a} p^{(b)}(Z|X). \]  

(14)

With these definitions it is possible to rewrite the problem in Eq. (10) into a series of approximate sub-problems on the form

\[ q^{(\alpha)}(Z) \approx \arg \min_{q^{(\alpha)}(Z) \in Q^{(\alpha)}} \{ D \left( p^{(a)}(Z|X)q^{\backslash a}(Z)\|q^{(\alpha)}(Z)q^{\backslash a}(Z) \right) \}. \]  

(15)

where \( q^{\backslash a}(Z) \) is assumed to be a good approximation and thus is kept fixed. If the factor families, \( Q^{(\alpha)} \), from which \( q^{(\alpha)} \) can be chosen, have been chosen sensible, the problem
1. Initialize \( q^{(a)}(Z) \) for all \( a \in A \)

2. repeat

3. Pick a factor \( a \in A \)

4. Solve Eq. (15) to find \( q^{(a)}(Z) \)

5. until \( q^{(a)}(Z) \) converges for all \( a \in A \)

Fig. 8. The generic message-passing algorithm. The loop in line 2 can potentially be run in parallel and in a distributed fashion.

An unbiased estimate of the gradient, \( \nabla_W L \), of this dual objective \( L \) can be obtained by utilizing the REINFORCE-gradient or the reparameterization trick. The objective can then iteratively be optimized by stochastic gradient ascent via the update equation

\[
W^{(l)} = W^{(l-1)} + \rho^{(l-1)} \nabla_W L^{(l)} \left( W^{(l-1)} \right)
\]

where superscript \( \{l\} \) is used to denote the \( l \)th iteration. Stochastic gradient ascent converges to a maximum of the objective function \( L \) if the sequence of learning rates, \( \rho^{(l-1)} \), follows the Robbins-Monro conditions given by

\[
\sum_{l=1}^{\infty} \rho^{(l)} = \infty, \quad \sum_{l=1}^{\infty} \left( \rho^{(l)} \right)^2 < \infty.
\]

Since Eq. (17) is dual to the original minimization problem, this maximum also provides a solution to the original problem. Although Robbins-Monro conditions are satisfied it is often necessary to apply variance reduction methods to obtain unbiased gradient estimators with sufficiently low variance. Fortunately, reduction methods can often be applied automatically by probabilistic programming libraries/languages such as Pyro. One benefit of solving variational inference problems with stochastic optimization is that noisy gradient estimates are often relatively cheap to compute due to, e.g., subsampling of data. Another benefit is that the use of noisy gradient estimates can cause algorithms to escape shallow local optima of complex objective functions. The downside of Stochastic variational inference is that it is inherently serial and that it requires the parameters to fit in the memory of a single processor. This could potentially be a problem for cognitive robotics where large models with lots of variables and parameters presumably are necessary to obtain a high level of intelligence, and where queries have to be answered within different time scales. I.e. signals to motors have to be updated frequently while high-level decisions can be allowed to take a longer time.

### 6.3 Stochastic Variational Inference

The approach taken by Stochastic Variational inference (SVI) is to reformulate a variational inference problem, e.g., Eq. (10) or Eq. (15), to a dual maximization problem with an objective, \( L \), that can be solved with stochastic optimization. Stochastic Variational inference assumes that the variational distribution, \( q \), is parameterized by some parameters, \( \Phi \). To obtain the dual problem and the objective function, \( L \), of the resulting maximization problem of course the steps and assumptions needed depends on whether we have chosen the use the Kullback–Leibler divergence or the \( \alpha \)-divergences. Nevertheless, the resulting problem ends up using on the form

\[
\Phi^* = \arg \max_{\Phi} \frac{L\left(p_\theta (Z, X = \bar{X}), q_\Phi (Z)\right)}{\int_{Z, \Theta, \Phi} \left[l(Z, \Theta, \Phi)\right]}
\]

This dual objective function, \( L \), does not depend on the posterior, \( p_\theta (Z, X = \bar{X}) \), but only the variational distribution, \( q_\Phi (Z) \) and the unconditional distribution \( p_\theta (Z, X = \bar{X}) \) making the problem much easier to work with. Besides being dual to Eq. (10) it turns out that for the family of alpha-divergences with \( \alpha > 0 \), \( L \) is also an lower bound on the log evidence, \( \log \left(p_\theta (Z)\right) \). Since the log evidence is a measure of how well a model fits the data, we can instead consider the optimization problem

\[
\Theta^*, \Phi^* = \arg \max_{\Theta, \Phi} \frac{L\left(p_\theta (Z, X = \bar{X}), q_\Phi (Z)\right)}{\int_{Z, \Theta, \Phi} \left[l(Z, \Theta, \Phi)\right]}
\]

that allows us to simultaneously fit the posterior approximation, \( q_\Phi \), and model parameters, \( \Theta \), to the data, \( \bar{X} \).
and a set of “global” observed variable nodes, \(X_G\), having more than one node collection as parent, we can write the posterior factorization
\[
p_{\Theta}(Z|X) = p_{\Theta}(Z|A)X(A), X_G
\]
\[
= \frac{1}{p(X_G|X(A))} p_{\Theta}(Z|A), X_G|X(A)
\]
\[
= \frac{p(X_G|Z(A))}{p_{\Theta}(X_G|X(A))} p_{\Theta}(Z|A)|X(A)
\]
\[
= p_{\Theta}(X_G|X(A)) p_{\Theta}(Z|A)|X(A)
\]
\[
= p(X_G|Z(A)) \cdot \prod_{a \in A} p_{\Theta}(a) \left(Z^{(a)}|\Theta^{(a)}(a), X^{(a)}\right)
\]
where Eq. (20) follows from conditional independence between \(X_G\) and \(X^{(A)}\) given \(Z^{(A)}\), and Eq. (21) follows from Eq. (9). Following the procedure of message-passing we choose a variational distribution that factorizes as
\[
q_{\Phi}(Z) = \prod_{a \in A} q_{\Phi^{(a)}}(Z^{(a)})
\]
Notice, that in Eq. (22) we have exactly one factor for each node collection and that this factor only contains the latent variables of that node collection. This is unlike Eq. (12) where a latent variable could be present in multiple factors. By combining Eq. (21) and Eq. (22) we can write an approximate posterior distribution related to the \(a\)th node collection
\[
p_{\tilde{\Theta}^{(a)}}(Z|X) = \frac{p_{\Theta^{(a)}}(Z|A)X(A), X^{(a)}}{p_{\tilde{\Theta}^{(a)}}(X_G|X^{(a)}) \cdot \prod_{b \in A \setminus a} q_{\Phi^{(b)}}(Z^{(b)})}
\]
where \(p_{\tilde{\Theta}^{(a)}}(X_G|X^{(a)})\) is defined in Eq. (25) in Section 12.1. Based on Eq. (15) we can then define approximate sub-problems as
\[
\min_{\Phi^{(a)}} \mathbb{E}_{q_{\Phi^{(a)}}}(Z) \left| p_{\tilde{\Theta}^{(a)}}(Z|X)\right.
\]
Each of these sub-problems can then be solved successively or in parallel potentially on distributed compute instances as outlined in Fig. 8 and utilizing Stochastic variational inference as described in Section 6.3. To see how this choice of factorization affects the posterior approximations and the learning of model parameters, \(\Theta\), consider the KL-divergence as divergence measure. Considering the KL-divergence we can rewrite the objective in Eq. (23) as shown in Eq. (26) through Eq. (33) in Section 12.1 to obtain the following local dual objective for stochastic variational inference
\[
L_{KL}^{(a)}(\Theta^{(a)}, \Phi^{(a)}) =
\frac{E_{Z \sim q_{\Phi^{(a)}}} \left[ LogEvd^{(a)}_{X_G, X^{(a)}}(\Theta^{(a)}) \right] - C}{D_{KL} \left[ q_{\Phi^{(a)}}(Z) || \tilde{\Theta}^{(a)}(Z|X) \right]}
\]
Where \(C\) is a constant with respect to \(\Theta^{(a)}\) and \(\Phi^{(a)}\), and \(LogEvd^{(a)}_{X_G, X^{(a)}}(\Theta^{(a)})\) is the joint log-evidence over global observed variables, \(X_G\), and observed variables, \(X^{(a)}\), local to the \(a\)th node collection. Since the first term on the right-hand side is constant with respect to \(\Phi^{(a)}\), maximizing this local dual objective with respect to \(\Phi^{(a)}\) will minimize the KL-divergence. Furthermore, since \(D_{KL} \left[ q_{\Phi^{(a)}}(Z) || \tilde{\Theta}^{(a)}(Z|X) \right] \geq 0\) by definition it follows from Eq. (24) that
\[
E_{Z \sim q_{\Phi^{(a)}}} \left[ LogEvd^{(a)}_{X_G, X^{(a)}}(\Theta^{(a)}) \right] - C \geq L_{KL}^{(a)}(\Theta^{(a)}, \Phi^{(a)})
\]
Therefore, by maximizing the local dual objective, \(L_{KL}^{(a)}(\Theta^{(a)}, \Phi^{(a)})\), with respect to the local model parameters, \(\Theta^{(a)}\), we will push the expected joint log-evidence over the global, \(X_G\), and the local, \(X^{(a)}\), observed variables higher, where the expectation is taken with respect to the joint variational distribution over latent variables parent to the \(a\)th node collection. In summary, this means that we can simultaneously fit our local model parameters, \(\Theta^{(a)}\), to the evidence and obtain an approximate local posterior distribution, \(q_{\Phi^{(a)}}(Z^{(a)})\). While the above derivations were made for the KL-divergence, similar derivations can be done for the more general family of \(\alpha\)-divergences.
To evaluate this local dual objective, we only need information related to the local node collection, its parents, and other node collections having the same global observed variables as children. Thereby providing substantially computational speedups for generative flow graphs with sparsely connected node collections and global observed variables. To use this procedure with a standard probabilistic programming language, we would have to create a probabilistic program fragment for each node collection, their corresponding variational distribution, and the global observed variables. These fragments would then have to be composed together to form the local objectives potentially in an automated fashion.

So far within this section, we have assumed that all sub-problems are solved through a variational problem as in Eq. (23). However, there are in principle no reasons why we could not use estimates of sub-posteriors, \(q(z^{(b)})\), obtained through other means in Eq. (25), as long as we can sample from these sub-posteriors. Thus, making the outlined method very flexible to combine with other methods, albeit analysis of the results obtained through the combined inference becomes more difficult. It is also important to stress that the factorization used above is not unique. It would be interesting to investigate if other factorizations could be employed, and for which problems these factorizations could be useful.
In summary, if we can divide a generative flow graph representing an overall model of cognition into node collections and global observed variables, then we can utilize the combination of Message-Passing and Stochastic Variational inference presented within this section, to distribute the computational burden of performing inference within this model. At the same time, we can learn local model parameters. Thus, yielding a very flexible tool allowing us
to fully specify the part of a model that we are certain about, and potentially learn the rest.

7 Probabilistic Programming Languages

So far, our focus has been on the representation of models defined by probabilistic programs, and on how to answer queries related to these models via modern probabilistic inference. However, we have not considered how this is made possible by probabilistic programming languages and their relation to deterministic programming languages. Here we will not give a detailed introduction to probabilistic programming and refer interested readers to other sources. Instead, we will give a short overview of languages relevant to modeling cognition.

As already mentioned in Section 5, the main characteristics of a probabilistic program is to construct for sampling randomly from distributions and another construct for condition values of variables in the program. The purpose of probabilistic programming languages is to provide these two constructs and to handle the underlying machinery for implementing inference algorithms and performing inference from these constructs. As with any other programming language, design decisions are not universally applicable or desirable, and different trade-offs are purposefully made to achieve different goals. This fact combined with theoretical advancements has resulted in several different probabilistic programming languages. For an extensive list see.

Some of these are domain-specific aimed at performing inference in a restricted class of probabilistic programs, such as Stan. These restrictions are usually employed to obtain more efficient inference. More interesting for the framework presented in Section 5, however, are languages self-identifying as universal or general-purpose, such as Pyro and Venture. These languages aim at performing inference in arbitrary probabilistic programs. Thus maximizing the flexibility for modeling cognition.

A recent trend has been to build probabilistic programming languages on top of deep-learning libraries such as PyTorch and TensorFlow. This is done both to use the efficient tensor math, automatic differentiation, and hardware acceleration that these libraries provide and to get tighter integration of deep-learning models within probabilistic models. Examples of such languages are Pyro and ProbTorch build upon PyTorch, and Edward build upon TensorFlow. Again, when considering the use within the framework presented in Section 5 the languages based on PyTorch or TensorFlow 2.0 could potentially have the edge over others due to the dynamic approach to constructing computation graphs. This is because the dynamic computation graphs, more easily allow us to define dynamic models which include recursion and unbounded numbers of random choices. Constructs potentially being indispensable for models of higher-level cognition supposed to evolve.

Python as the high-level general-purpose programming language it makes modeling effortless in these languages. However, being based on python the computational efficiency of these languages is potentially limited by the need for interpretation. For this reason the relatively new project called NumPyro is under active development. NumPyro provides a backend to Pyro based on NumPy and JAX, which enables just-in-time compilation, and thus potentially could provide much better computational efficiency, which again is essential for any practical robotic system.

To summarize, the choice of which probabilistic programming language to use depends on the end flexibility needed to model cognition. However, universal or general-purpose languages based on deep-learning libraries possibly with just-in-time compilation and hardware acceleration seem promising for general modeling of cognition, and especially for cognitive robotics.

8 Application Examples

To demonstrate the concepts presented within this paper and the utility of the framework we have begun an initiative to implement some general applicable probabilistic programming idioms with basis in the “Standard Model of the Mind” which is available as a GitHub repository. The repository currently contain one such idiom called “Planning”. The purpose of this idiom is to provide basic functionality to plan future actions of a robot based on concepts of desirability, progress, information gain, and constraints. To keep the idiom generally applicable it is implemented as an abstract python class with methods containing the main functionality, and some abstract methods that needs to be specified on a per application basis. Besides this idiom, the repository also contains two different applications examples utilizing this idiom.

Fig. 9. Results of a simulation of high-level robot motion planning with the goal of exploring an unknown environment with a lidar as the perceptual input. Gray indicate unexplored parts of the environment, white indicate unoccupied areas, black indicates obstacles, the green circle with a black boarder shows the current location of the robot, the green dashed line shows the robots past path, the solid green lines shows samples from the future optimal path distribution, the black stars shows the mean of these samples, and the transparent blue circles illustrates the lidars range at these positions.

The first example demonstrates high-level robot motion planning with the goal of exploring an environment represented by a grid map in the long-term memory as illustrated in Fig. 9.

The second example relies heavily on the Stochastic Message-Passing approach described in Section 6.4 to implement a simplistic form of theory-of-mind. In this application N robots each uses the “Planning” idiom to plan low-level actions towards their goals while avoiding collisions
with the other robots given knowledge about the other robots expected future path as illustrated in Fig. 10. This approach have both been verified through simulations and a real-world experiment.

Although the repository currently do not contain a broad range of cognitive capabilities, the two examples demonstrates most of the concepts presented in Sections 5 through Section 7. Furthermore, the fact that the two vastly different applications are implemented from the same probabilistic programming idiom demonstrates how models developed in the proposed framework can encourage cooperation and re-use of existing results.

9 CONCLUSION

Inspired by Sigma’s Cognitive Hourglass Model[1] we have outlined a new framework for developing cognitive architectures for cognitive robotics. With probabilistic programs at the center, this framework is sufficiently general to span the full spectrum of emergent, symbolic, and hybrid architectures. By dividing cognitive architectures into a series of layers this framework provides levels of abstractions between models of cognition and the algorithms that implement them on computational devices. Some of these layers also directly relate to other fields of research, thereby encouraging better cooperation.

We also presented a novel graphical representation of probabilistic programs which we call generative flow graphs. We showed how such generative flow graphs can help identify important universal fragments of probabilistic programs and models. Fragments that could potentially be re-used in the development of other cognitive architectures. Thereby, again encouraging cooperation and easier re-use of existing results.

Furthermore, we introduced the problem of inference within probabilistic programs. We briefly reviewed possible approaches and argued that variational inference approaches seems interesting for cognitive robotics. We introduced two commonly used approaches called Message-Passing and Stochastic Variational Inference. We also outline the weaknesses of each approach and proposed a combined approach that we call Stochastic Message-Passing. The proposed approach provides a principle way of distributing the computational burden of inference and parameter learning.

To support implementations within the framework we reviewed existing probabilistic programming languages providing the necessary machinery to implement inference algorithms for and perform inference in probabilistic programs.

Finally, we provided a brief introduction to a new initiative that both provide evidence for the applicability of the frameworks and concepts presented within this paper, but also functions as a starting point and a tool for researchers who wants to work within the framework.

The main topics within this paper have been the framework itself, the representation of cognitive models, and the computational burden. These topics are indeed essential ingredients of the framework and pose interesting research directions by themselves.

10 AUTHOR CONTRIBUTIONS

Conceptualization, M.R.D., R.P., and T.B.; Investigation, M.R.D.; Methodology, M.R.D.; Formal Analysis, M.R.D.; Visualization, M.R.D.; Writing – Original Draft, M.R.D.; Writing – Review & Editing, M.R.D., R.P., and T.B.; Supervision, R.P., and T.B.

11 DECLARATION OF INTERESTS

The authors declare no competing interests.

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12 \textbf{SUPPLEMENTAL INFORMATION}

12.1 Rewriting KL-divergence for Stochastic Message-Passing

In this section, we will derive the dual objective for the combination of message-passing and stochastic variational inference presented in Section 6.4. Start by considering

\[
p_{\Theta}(X_G|X^{(A)}) = \int p_{\Theta}(X_G, Z^{(A)}|X^{(A)})dZ
\]
\[
= \int p(X_G|Z^{(A)}, X^{(A)})p_{\Theta}(Z^{(A)}|X^{(A)})dZ
\]
\[
= \int p(X_G|Z^{(A)})p_{\Theta}(Z^{(A)}|X^{(A)})dZ
\]
\[
= \int p(X_G|Z^{(A)}) \prod_{a \in A} p_{\Theta(a), p_a\Theta(c(a))}(Z^{(a)}|p_aZ(C^{(a)}), X^{(a)})dZ
\]

By replacing \( p_{\Theta(a), p_a\Theta(C^{(a)})}(Z^{(b)}|p_aZ(C^{(b)}) , X^{(b)}) \) with their corresponding variational distributions \( q_{\phi(b)}(Z^{(b)}) \) for \( b \in A \setminus a \) we obtain

\[
\tilde{p}^{(a)}(X_G|X^{(a)}) = \int p(X_G|Z^{(A)})p_{\Theta(a), p_a\Theta(C^{(a)})}(Z^{(a)}|p_aZ(C^{(a)}), X^{(a)}) \prod_{b \in A \setminus a} q_{\phi(b)}(Z^{(b)}) dZ \tag{25}
\]

where we have used \( \tilde{p}^{(a)}(X_G|X^{(a)}) \) instead of \( \hat{p}^{(a)}(X_G|X^{(A)}) \) to emphasize the conditional independence between \( X_G \) and \( X^{(b)} \) given \( Z^{(b)} \) for \( b \in A \setminus a \) implicitly assumed by the approximation. Furthermore, also notice that we can rewrite the distribution, \( p(X_G|Z^{(A)}) \), as follows

\[
p(X_G|Z^{(A)}) = p(ChX_G(Z^{(a)})|Z^{(A)})p(X_G|\text{Ch}X_G(Z^{(a)})|Z^{(A)})
\]
\[
= p \left( \text{Ch}X_G(Z^{(a)})|p_aZ(\text{Ch}X_G(Z^{(a)})) \right) p \left( X_G|\text{Ch}X_G(Z^{(a)})|Z^{(A)} \setminus Z^{(a)} \right)
\]

by separating the global observed variables, \( X_G \), into those who are direct children of \( Z^{(a)} \), and those who are not. With the definition above and the ones given in Section 6.4 we can rewrite the KL-divergence as follows:

\[
D_{KL} \left[ q_{\phi^{(a)}}(Z)||\tilde{p}^{(a)}(Z|X) \right]
\]
\[
= \int_Z q_{\phi^{(a)}}(Z) \log \left( \frac{q_{\phi^{(a)}}(Z)}{\tilde{p}^{(a)}(Z|X)} \right) dZ
\]
\[
= \int_Z q_{\phi^{(a)}}(Z) \log \left( \frac{p(X_G|Z^{(A)})}{\tilde{p}^{(a)}(X_G|Z^{(A)})} \prod_{b \in A \setminus a} \frac{p_{\Theta(b), p_b\Theta(C^{(b)})}(Z^{(b)}|p_bZ(C^{(b)}), X^{(a)})}{q_{\phi(b)}(Z^{(b)})} \right) dZ
\]
\[
= \int_Z q_{\phi^{(a)}}(Z) \log \left( \frac{p(X_G|Z^{(A)})}{\tilde{p}^{(a)}(X_G|Z^{(A)})} \prod_{b \in A \setminus a} \frac{p_{\Theta(b), p_b\Theta(C^{(b)})}(Z^{(b)}|p_bZ(C^{(b)}), X^{(a)})}{q_{\phi(b)}(Z^{(b)})} \right) dZ
\]
\[
= \int_Z q_{\phi^{(a)}}(Z) \log \left( \frac{p(X_G|Z^{(A)})}{\tilde{p}^{(a)}(X_G|Z^{(A)})} \prod_{b \in A \setminus a} \frac{p_{\Theta(b), p_b\Theta(C^{(b)})}(Z^{(b)}|p_bZ(C^{(b)}), X^{(a)})}{q_{\phi(b)}(Z^{(b)})} \right) dZ
\]
\[
= \int_Z q_{\phi^{(a)}}(Z) \log \left( \frac{p(X_G(Z^{(a)}))}{\tilde{p}^{(a)}(X_G(Z^{(a)}))} \prod_{b \in A \setminus a} \frac{p_{\Theta(b), p_b\Theta(C^{(b)})}(Z^{(b)}, X^{(a)}|p_bZ(C^{(b)}), X^{(a)})}{q_{\phi(b)}(Z^{(b)})} \right) dZ
\]
\[
= \int_Z q_{\phi^{(a)}}(Z) \log \left( \frac{p(X_G(Z^{(a)}))}{\tilde{p}^{(a)}(X_G(Z^{(a)}))} \prod_{b \in A \setminus a} \frac{p_{\Theta(b), p_b\Theta(C^{(b)})}(Z^{(b)}, X^{(a)}|p_bZ(C^{(b)}), X^{(a)})}{q_{\phi(b)}(Z^{(b)})} \right) dZ
\]
| Symbol | Description | Meaning |
|--------|-------------|---------|
| $z$    | Circle without colored background | A node symbolizing a probabilistic variable, corresponding to a "sample" function or keyword in the probabilistic program. We denote this as a "Latent Variable Node". |
| $x$    | Circle with colored background | A node symbolizing an observed probabilistic variable, corresponding to a "observe" function or keyword in the probabilistic program. We denote this as an "Observed Variable Node". |
| $\theta$ | Square without colored background | A node symbolizing learn-able parameters in the probabilistic program. That is model parameters that can change at run-time. We denote this as a "Variable Parameter Node". |
| $\hat{\theta}$ | Square with colored background | A node symbolizing fixed parameters in the probabilistic program. Use-full for representing parameters that cannot change at run-time such as tuning parameters. We denote this as a "Fixed parameter node". |
| $\rightarrow$ | Simple arrow | Link showing the generative path in a Probabilistic Program. The arrow can start in Latent Variable Nodes and Parameter nodes, and only point towards latent variable nodes and observed variable nodes. In large graphs or in cases where the origin of a link can be uncertain the readability can be improved by adding the name of the node from which the link originates next to the link. We denote this as a "Generative Link". |
| $\leftarrow$ | Half circle on arrow | Symbolizing that operations downstream of this link in the probabilistic program should not influence nodes upstream to this link in the generative path. I.e. information such as accumulated gradients in a backward pass of automatic differentiation should not be propagated back through this link, corresponding to a "detach()" and "stop_gradient" call in PyTorch and TensorFlow, respectively. We denote this as a "Detached Link". |
| $\rightarrow\downarrow$ | Simple arrow pointing towards another simple arrow | Depending on the context, samples from different latent variable nodes may be used to generate the next sample. The generative link from the most recent sampled latent variable node in the generative path is the new active link. Used, e.g., to represent for loops. |
| $\{z;x_1, x_2;\theta\}$ | Polygon with rounded corners | Collection of nodes with internal dependency structure. The full structure of links between nodes can be shown inside the polygon. Alternatively, the names of the nodes can be written between curly brackets, \(\{z:x;\theta\}\), with a semi-colon separating variables nodes, \(z\), observed nodes, \(x\), and parameter nodes, \(\theta\), in that order. Finally, a node collection can also simply be defined somewhere else, \(C = \{z:x;\theta\}\), and referred to by, e.g., a single letter, in which case we encourage the use of capital letters to emphasize the difference from the other types of nodes. In cases where one or more types of nodes are not present in a collection, both semi-colons should still be there. E.g., \(\{x_1,x_2;\}\) for a collection the two observed nodes \(x_1\) and \(x_2\). Such a collection of nodes directly corresponds to the factor \(p_0(x_1,x_2|z)\) in a factorization of the joint distribution over all variables in a model. We denote this as a "Node Collection". A node is only allowed to be within one node collection unless it is within a node collection fully nested within another. |
| $\{x^{(n)},z^{(n)},p^{(n)}\}_{n \in N}$ | Index specification in one corner of a polygon with rounded corners | Collection of nodes with indexed names. When the index is used to name nodes it is good practice to use the index in a superscript and encapsulate it in round brackets to emphasize that it is an index. We denote such a node collection an "Indexed Node Collection". The valid index should be clear from the context. E.g., in the case of a loop around the indexed collection, the index is incremented each time the loop enters the indexed node collection. When no such loops exist around the indexed node collection that could potentially cause ambiguity, it can be used instead of writing all the variables in a node collection with the curly brackets. For an example see Fig. [3]. |
| $\{z^{(n)},x^{(n)},p^{(n)}\}_{n \in N}$ | Stacked polygons with rounded corners | Explicitly representation of multiple identical collections of nodes conditionally independent given their parents or simply independent if there are no parents. Here the one index is used for each of the independent collections. |
| $\quad\quad\quad\quad\rightarrow\uparrow$ | Dashed arrow | A link symbolizing an indirect relation between nodes. Such a link is non-generative, meaning that it is not directly used as a parameter in the generation of other samples, but only influences the generative path of other samples. We denote this as an "Influence Link". |
| $\quad\quad\quad\quad\rightarrow\uparrow\rightarrow\rightarrow\rightarrow\rightarrow$ | A Polygon with generative links connected at vertices, influence links towards the polygon connected at edges, and conditional values nearby generative links pointing away from the polygon | Node representing a condition changing the "direction" of the generative flow in a probabilistic program. We denote this as "conditioned generative branching". |
| $\quad\quad\quad\quad\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow$ | A Polygon with generative links connected at vertices, influence links towards the polygon connected at edges, and conditional values nearby generative links pointing towards the polygon | Node representing a condition selecting one out of two or more possible generative flows from parent nodes. We denote this as "conditioned generative selection". |

**TABLE 1**

Semantics of the Generative Flow Graph representation of probabilistic programs.
\[
+ \int_{Z} q_{\Phi^{(a)}}(Z) \log \left( \hat{p}^{(a)}(X_G|X^{(a)})p_{\Theta^{(a)},Pa\Theta(C^{(a)})}(X^{(a)}|Pa\hat{Z}(C^{(a)})) \right) dZ \\
- \int_{Z} q_{\Phi^{(a)}}(Z) \log \left( p \left( X_G \setminus \text{Ch}X_G \left( Z^{(a)} \right) |Z^{(A)} \setminus Z^{(a)} \right) \right) dZ \\
E_{Z \sim \hat{q}_{\Phi^{(a)}}(Z)} \left[ \log \left( \frac{q_{\Phi^{(a)}}(Z^{(a)})}{p \left( \text{Ch}X_G \left( Z^{(a)} \right) |Pa\hat{Z}(C^{(a)}) \right) p_{\Theta^{(a)},Pa\Theta(C^{(a)})}(Z^{(a)}, X^{(a)}|Pa\hat{Z}(C^{(a)))} \right) \right] \\

\hspace{1cm} - L_{KL}^{(a)}(\Theta^{(a)}, \Phi^{(a)}) \\
+ E_{Z \sim \hat{q}_{\Phi^{(a)}}(Z)} \left[ \log \left( \hat{p}^{(a)}(X_G|X^{(a)})p_{\Theta^{(a)},Pa\Theta(C^{(a)})}(X^{(a)}|Pa\hat{Z}(C^{(a)))} \right) \right] \\
- E_{Z \sim \Pi_{b \in A \setminus a} q_{\Phi(b)}(Z^{(b)})} \left[ \log \left( p \left( X_G \setminus \text{Ch}X_G \left( Z^{(a)} \right) |Z^{(A)} \setminus Z^{(a)} \right) \right) \right] \\
\hspace{1cm} \mathcal{C}
\]

where
\[
\hat{q}_{\Phi^{(a)}}(Z^{(a)}) = q_{\Phi^{(a)}}(Z^{(a)}) \prod_{Z^{(b)} \in \text{Pa}\hat{Z}(C^{(a)}) \setminus \text{Pa}\hat{Z}(\text{Ch}X_G(Z^{(a)))) \setminus Z^{(a)}} q_{\Phi^{(b)}(Z^{(b)})}
\]
is the joint variational distribution over the latent variables, \( Z^{(a)} \), local to the \( a \)'th node collection, and the latent variables parent to the \( a \)'th node collection, \( \text{Pa}\hat{Z}(C^{(a)}) \), or having the same child global observed variables as the \( a \)'th node collection, \( Z^{(b)} \in \text{Pa}\hat{Z}(\text{Ch}X_G(Z^{(a)))) \setminus Z^{(a)} \). Furthermore,
\[
\hat{q}_{\text{Pa}\hat{Z}}(Z^{(a)}) = \prod_{Z^{(b)} \in \text{Pa}\hat{Z}(C^{(a)})} q_{\Phi^{(b)}(Z^{(b)})}
\]
is the joint variational distribution over the latent variables parent to the \( a \)'th node collection, \( \text{Pa}\hat{Z}(C^{(a)}) \). Finally, \( \text{LogEvd}_{X_G, X^{(a)}}(\Theta^{(a)}) \) denotes the joint log-evidence for \( X_G \) and \( X^{(a)} \), and \( \mathcal{C} \) is constant with respect to \( \Theta^{(a)}, \Phi^{(a)} \).

By simple rearranging terms we obtain the dual objective
\[
I_{KL}^{(a)}(\Theta^{(a)}, \Phi^{(a)}) = E_{Z \sim \hat{q}_{\Phi^{(a)}}(Z)} \left[ \text{LogEvd}_{X_G, X^{(a)}}(\Theta^{(a)}) \right] - D_{KL} \left[ q_{\Phi^{(a)}}(Z)|\hat{p}^{(a)}(Z|X) \right] - C
\]