Scheme of 2-dimensional atom localization for a three-level atom via quantum coherence

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Abstract

We present a scheme for two-dimensional (2D) atom localization in a three-level atomic system. The scheme is based on quantum coherence via classical standing wave fields between the two excited levels. Our results show that conditional position probability is significantly phase dependent of the applied field and frequency detuning of spontaneously emitted photons. We obtain a single localization peak having probability close to unity by manipulating the control parameters. The effect of atomic level coherence on the sub-wavelength localization has also been studied. Our scheme may be helpful in systems involving atom-field interaction.

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1 Introduction

Atom localization has been the potentially rich area of research because of a number of important applications and fundamental research [1, 2, 3, 4]. The idea of measuring the position of an atom has been discussed since the beginning of quantum mechanics as proposed by Heisenberg [5] where the resolving power of equipment is limitized by uncertainty principle. Furthermore, typical resolution of measurement is restricted by diffraction condition where the length scale of object to be measured must be of the order of wavelength of light used for measurement [6]. A few of potential applications include laser cooling and

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trapping of neutral atoms [7], nano-lithography [8, 9], atomic wavefunction measurement [10, 11], Bose-Einstein condensation [12, 13] and coherent patterning of matter waves [14].

There are many existing schemes for 1D atom localization that utilizes spatially dependent wave fields carrying advantage of atom to be localized inside standing wave region. Earlier, the schemes were based on absorption of light masks [15, 16, 17], measurement of standing wave field inside the cavity [18, 19, 20, 21] or of atomic dipole [22, 23, 24], Raman gain process [25], entanglement of atom’s position and internal state [26], phase dependence of standing wave field [27] and the localization scheme based on resonance fluorescence [28]. Since atomic coherence and quantum interference results in some interesting phenomenon such as Kerr non-linearity [29, 30], four wave mixing (FWM) [31], electromagnetically induced transparency (EIT) [32], spontaneous emission enhancement or suppression [33, 34] and optical bistability (OB) [35, 36], the idea of sub-wavelength atom localization via atomic coherence and quantum interference has also been acquired considerable attention in recent years.

In past few years, atom localization in two dimensions has been extensively studied for its better prospect in applications. A considerable attention has been given to 2D atom localization in a multi-level atomic system where two interacting orthogonal wave fields are utilized for measuring atom’s location. These studies show that 2D atom localization could be possible by the measurement of populations of atomic states as proposed by Ivanov and Rozhdestvensky [37] and the interaction of double-dark resonances presented by Gao et al. [38]. Other techniques practiced for 2D atom localization includes probe absorption spectrum [39, 40, 41] and controlled spontaneous emission [42, 43, 44].

In this work, we present an efficient scheme for two-dimensional (2D) atom localization based on quantum coherence effects in a three-level atomic system. There exists many proposals based on the atomic coherence and quantum interference for one-dimensional atom localization. For example, the scheme proposed by Herkommer et al. [45] in which they used Autler-Townes spontaneous spectrum. In another scheme, Papalakis and Knight [46] proposed scheme based on quantum interference that induces sub-wavelength atom localization in three-level Lambda (Λ) system traveling through the standing wave field. Furthermore, the scheme proposed by Zubairy and collaborators exploited the phase of driving field for sub-wavelength localization, hence reducing the number of localization peaks from four to two [47, 48]. Gong et al., in their schemes demonstrated atom localization at the nodes of standing-wave field with increased detection probability [49, 50]. In contrast to above schemes that discussed atom localization via single decay channel for spontaneous emission, we have focused on fact that spontaneous emission via two coherent decay channels may potentially improve the localization probability. During atom-field interaction in two dimensions, the spontaneously emitted photon carries information of the atom by creating spatial structures of filter function at various frequencies. Typically such spatial structures deliver lattice-like, crater-like and spike-like patterns which are mainly dependent on dynamic interference between two orthogonal spatially dependent fields. Subsequently, high precision in 2D atom localization...
can be attained by manipulating the system parameters.

Our scheme is based on a three level-ladder configuration, where the atom is initially prepared in the coherent superposition of upper two excited levels. Consequently, we studied the combined effect of relative phase between two applied orthogonal standing wave fields and frequency of emitted photon.

The system is also discussed in the absence of atomic coherence where the result showed radially dependence on initial atomic state preparation. We reported multiple results for two dimensional atom localization including a single localization peak having fairly large conditional position probability.

This paper is organized as follows. Section II presents atomic model followed by theoretical treatment for deriving conditional probability distribution. In section III, we provide detailed analysis and discussion of results regarding two-dimensional atom localization in our proposed scheme. Finally, section IV offers concise conclusion.

2 Model and Equations

We consider an atom moving in $z$-direction passing through two intersecting classical standing fields as shown in Fig. 1 (a). The two fields are assumed to be orthogonal and aligned along $x$ and $y$ axis, respectively. The internal energy levels of three level atomic system is shown in Fig. 1(b). The two excited states $|2\rangle$ and $|1\rangle$ are coupled to ground state $|0\rangle$ by vacuum modes in free space. Further, the transition $|2\rangle \rightarrow |1\rangle$ is driven via classical standing field with Rabi frequency $\Omega(x,y)$. Hence, the interaction between atom and classical standing field is spatial dependent in $x - y$ plane.

Next we assume that the atom (with mass $m$) follows thermal distribution at temperature $T$ so that its energy $k_BT$ is quite large as compared to photon recoil $\hbar^2k^2/2m$. Therefore, the atom moving along $z$-direction is not affected by the interaction fields and we can treat it classically. Neglecting the kinetic energy part of Hamiltonian in Raman-Nath approximation [51], the interaction Hamiltonian of the system under dipole approximation reads as,

$$H_{int} = H_{field} + H_{vacuum},$$

where

$$H_{field} = \Omega(x,y)e^{i(\Delta t + \alpha_c)}X_{12} + \Omega^*(x,y)e^{-i(\Delta t + \alpha_c)}X_{21},$$

and

$$H_{vacuum} = \sum_{k} \left( g_k^{(1)} e^{-i(\omega_k-\omega_{10})t} X_{10}b_k + g_k^{(1)*} e^{i(\omega_k-\omega_{10})t} X_{01}b_k^\dagger \right) + \sum_{k} \left( g_k^{(2)} e^{-i(\omega_k-\omega_{20})t} X_{20}b_k + g_k^{(2)*} e^{i(\omega_k-\omega_{20})t} X_{02}b_k^\dagger \right).$$

Here $X_{ij} = |i\rangle \langle j|$ represents the atomic transition operator for levels $|i\rangle$ and $|j\rangle$ with transitions $|2\rangle \leftrightarrow |0\rangle$ and $|1\rangle \leftrightarrow |0\rangle$ are characterized by frequencies
\( \omega_{20} \) and \( \omega_{10} \), respectively. The frequency of coupling filed between the upper two levels is given by \( \omega_c \) with associated phase \( \alpha_c \) and detuning parameter \( \Delta = \omega_c - \omega_{21} \). The coupling constants \( g^{(n)}_k \) \((n = 1, 2)\) are defined for atom-vacuum field interactions that corresponds to spontaneous decay while \( b_k \) and \( b_k^\dagger \) are the annihilation and creation operators of vacuum modes \( k \).

The wave function of the whole atom-field interaction system at time \( t \) can be represented in terms of state vectors as,

\[
|\psi(x, y; t)\rangle = \int \int dx dy f(x, y) |x\rangle |y\rangle [a_{2,0}(x, y; t) |2, \{0\}\rangle + a_{1,0}(x, y; t) |1, \{0\}\rangle + \sum_k a_{0,1k}(x, y; t) |0, 1_k\rangle],
\]

where \( f(x, y) \) is the center of mass wave function for the atom. The position dependent probability amplitudes \( a_{n,0}(x, y; t) \) \((n = 1, 2)\) represents when there is no photon and \( a_{0,1k}(x, y; t) \) corresponds to a single photon spontaneously emitted in \( k \)th mode of vacuum.

In our scheme, we make use of the fact that the interaction of position dependent Rabi frequency of atom, associated with classical standing field, interacts with the frequency of spontaneously emitted photon \([28, 45, 52]\). Such emitted photon carries information regarding location of the atom. Hence, atom position measurement is subjected to the detection of spontaneously emitted photon. Thus, the probability of atom to be located at any position \((x, y)\) at any time \( t \) can be described by conditional position probability distribution function \( W(x, y) \) defined as,

\[
W(x, y) \equiv W(x, y; t|0, 1_k) = F(x, y; t|0, 1_k) |f(x, y)|^2.
\]

Here \( F(x, y; t|0, 1_k) \) is the filter function that can be defined as,

\[
F(x, y; t|0, 1_k) = |N|^2 |a_{0,1k}(x, y; t \rightarrow \infty)|^2,
\]

with \( N \) being the normalization constant. Eq.(5) shows that the filter function or more generally, the conditional probability distribution is provided by probability amplitude \( a_{0,1k}(x, y; t) \).

In order to find the probability amplitude, we solve Schrodinger wave equation using interaction picture Hamiltonian given by Eq.(1) with state vector as defined in Eq.(4). The time evolution equations of probability amplitudes are then given by,

\[
i\dot{a}_{1,0}(x, y; t) = \Omega(x, y) a_{2,0}(x, y; t) e^{-i(\Delta t + \alpha_c)}
+ \sum_k g^{(1)}_k a_{0,1k}(x, y; t) e^{-i(\omega_k - \omega_{10})t},
\]

\[
i\dot{a}_{2,0}(x, y; t) = \Omega(x, y) a_{1,0}(x, y; t) e^{-i(\Delta t + \alpha_c)}
+ \sum_k g^{(2)}_k a_{0,1k}(x, y; t) e^{-i(\omega_k - \omega_{20})t},
\]

4
\[
\dot{a}_{0,1}(x, y; t) = g_k^{(1)} a_{1,0}(x, y; t) e^{i(\omega_k - \omega_{10})t} \\
+ g_k^{(2)} a_{2,0}(x, y; t) e^{i(\omega_k - \omega_{20})t}.
\]

(9)

Proceeding with regular integration of Eq. (9) and substituting it in Eq. (7) and (8) gives us,

\[
\dot{a}_{1,0}(x, y; t) = \Omega(x, y) a_{2,0}(x, y; t) e^{i(\Delta t + \alpha_c)} \\
- i \int_0^t dt' a_{1,0}(x, y; t') \sum_k |g_k^{(1)}|^2 e^{-i(\omega_k - \omega_{10})t(t-t')}
- i \int_0^t dt' a_{2,0}(x, y; t') \sum_k g_k^{(2)} g_k^{(1)*} e^{-i\omega_k(t-t')} e^{-i\omega_{10}(t-t')},
\]

(10)

\[
\dot{a}_{2,0}(x, y; t) = \Omega(x, y) a_{1,0}(x, y; t) e^{-i(\Delta t + \alpha_c)} \\
- i \int_0^t dt' a_{2,0}(x, y; t') \sum_k |g_k^{(2)}|^2 e^{-i(\omega_k - \omega_{20})t(t-t')}
- i \int_0^t dt' a_{1,0}(x, y; t') \sum_k g_k^{(1)} g_k^{(2)*} e^{-i\omega_k(t-t')} e^{-i\omega_{20}(t-t')}.
\]

(11)

Simplifying the above two equations under Weisskopf-Wigner theory results in coupled differential equations of following form

\[
\dot{a}_{1,0}(x, y; t) = -\frac{\Gamma_1}{2} a_{1,0}(x, y; t) - \left( i \Omega(x, y) e^{i(\Delta t + \alpha_c)} + \frac{p \sqrt{\Gamma_1 \Gamma_2}}{2} e^{-i\omega_{21}t} \right) a_{2,0}(x, y; t),
\]

(12)

\[
\dot{a}_{2,0}(x, y; t) = - \left( i \Omega(x, y) e^{-i(\Delta t + \alpha_c)} + \frac{p \sqrt{\Gamma_1 \Gamma_2}}{2} e^{i\omega_{21}t} \right) a_{1,0}(x, y; t) \\
- \frac{\Gamma_2}{2} a_{2,0}(x, y; t),
\]

(13)

with \( \Gamma_1 \) and \( \Gamma_2 \) are the spontaneous decay rates for \(|1\rangle \leftrightarrow |0\rangle\) and \(|2\rangle \leftrightarrow |0\rangle\) transitions, respectively. These are defined as \( \Gamma_n = 2\pi |g_n|^2 D(\omega_k) \) where \( D(\omega_k) \) represents mode density at frequency \( \omega_k \) for vacuum. Additionally, the term \( p \sqrt{\Gamma_1 \Gamma_2} e^{\pm i\omega_{21}t} \) corresponds to quantum interference whenever the higher energy levels in two spontaneous emissions are very close. The parameter
p in our case is defined as \( p = \frac{\|p_{20}\|}{\|p_{01}\|} \) which provides alignment of two matrix elements. This clearly shows that orthogonal and parallel matrix elements are presented by \( p = 0 \) and \( p = 1 \), respectively. For orthogonal matrix elements i.e., \( p = 0 \), there is no interference and for parallel matrix elements \( p = 1 \), the interference is maximum. In order to solve Eq. (12) and (13) analytically, we assume orthogonal dipole moments that are easy to find in nature [55]. Further, the time dependence in above equations i.e., \( e^{\pm i\omega_{21} t} \) can be ignore by setting energy difference between upper two levels \( \omega_{21} \) very large as compared to the decay rates \( \Gamma_1 \) and \( \Gamma_2 \) [56]. Introducing the transformations as,

\[
\begin{align*}
&b_{1,0}(x, y; t) = a_{1,0}(x, y; t), \\
&b_{2,0}(x, y; t) = a_{2,0}(x, y; t) e^{i\Delta t}, \\
&b_{0,1_\mathbf{k}}(x, y; t) = a_{0,1_\mathbf{k}}(x, y; t),
\end{align*}
\]

with

\[
\delta_\mathbf{k} \equiv \omega_\mathbf{k} - (\omega_{20} + \omega_{10}) / 2,
\]

the rate equations (12), (13) and (9) becomes,

\[
\begin{align*}
\dot{b}_{1,0}(x, y; t) &= -\frac{\Gamma_1}{2} b_{1,0}(x, y; t) - i\Omega(x, y) b_{2,0}(x, y; t), \\
\dot{b}_{2,0}(x, y; t) &= -i\Omega(x, y) e^{-i\alpha_\mathbf{p}} b_{1,0}(x, y; t) \\
&\quad + (i\Delta - \frac{\Gamma_2}{2}) b_{2,0}(x, y; t), \\
\dot{b}_{0,1_\mathbf{k}}(x, y; t) &= -i g_{(1)}^{(1)} b_{1,0}(x, y; t) e^{i(\delta_\mathbf{k} + \frac{\omega_{21}}{2}) t} \\
&\quad -i g_{(2)}^{(2)} b_{2,0}(x, y; t) e^{i(\delta_\mathbf{k} - \frac{\omega_{21}}{2} - \Delta) t}.
\end{align*}
\]

The set of first two differential equations can readily be solved by formal integration techniques. At this moment, we define initial state of the system by superposition of two orthogonal states as,

\[
|\psi(x, y; t = 0)\rangle = e^{i\alpha_\mathbf{p}} \sin(\xi) |2, \{0\}\rangle + \cos(\xi) |1, \{0\}\rangle,
\]

where \( \alpha_\mathbf{p} \) is the phase related to pump filed. Under this initial state with assumption that the pumping phase, \( \alpha_\mathbf{p} = 0 \) and decay rates \( \Gamma_1 = \Gamma_2 = \Gamma \), the solution of Eq. (16) and (17) reads as,

\[
\begin{align*}
&b_{1,0}(x, y; t) = B_1 e^{\lambda_1 t} + B'_1 e^{\lambda_2 t}, \\
&b_{2,0}(x, y; t) = B_2 e^{\lambda_1 t} + B'_2 e^{\lambda_2 t},
\end{align*}
\]

with
\[ \lambda_{1,2} = \frac{i\Delta}{2} - \frac{\Gamma}{2} \pm \frac{i}{2} \sqrt{4 \left( \Omega^2 (x,y) - \left( i\Delta - \frac{\Gamma}{2} \right) \frac{\Gamma}{2} \right) - (i\Delta - \Gamma)^2}, \]  

where \( B_1 = \frac{1}{\lambda_2 - \lambda_1} \left( \left( \lambda_2 + \frac{\Gamma}{2} \right) \sin (\xi) + i\Omega (x,y) e^{i\alpha c} \cos (\xi) \right), \]  

\[ B_1' = \frac{1}{\lambda_1 - \lambda_2} \left( \left( \lambda_1 + \frac{\Gamma}{2} \right) \sin (\xi) + i\Omega (x,y) e^{i\alpha c} \cos (\xi) \right), \]  

\[ B_2 = \frac{1}{\lambda_2 - \lambda_1} \left( \left( \lambda_2 + \frac{\Gamma}{2} - i\Delta \right) \sin (\xi) + i\Omega (x,y) e^{-i\alpha c} \cos (\xi) \right), \]  

\[ B_2' = \frac{1}{\lambda_1 - \lambda_2} \left( \left( \lambda_1 + \frac{\Gamma}{2} - i\Delta \right) \sin (\xi) + i\Omega (x,y) e^{-i\alpha c} \cos (\xi) \right), \]  

for \( \lambda_1 \neq \lambda_2 \).

Substituting Eq. (20) and (21) in Eq. (13) and following formal integration, the probability amplitude \( b_{0,1k} (x,y;t) \) for interaction time much larger than the decay rates \( (\Gamma_1 t, \Gamma_2 t \gg 1) \) offers,

\[ b_{0,1k} (x,y;t \to \infty) = g^{(1)}_k \left( \frac{B_1}{\delta_k + \frac{B_1}{2} - i\lambda_1} + \frac{B_1'}{\delta_k + \frac{B_1'}{2} - i\lambda_2} \right) + g^{(2)}_k \left( \frac{-B_2}{\delta_k - \frac{B_2}{2} - \Delta - i\lambda_1} + \frac{-B_2'}{\delta_k - \frac{B_2'}{2} - \Delta - i\lambda_2} \right). \]  

Incorporating the constants as specified in Eq. (22) with trivial simplifications, we get

\[ b_{0,1k} (x,y;t \to \infty) = g^{(1)}_k \left( \frac{\sin (\xi) - e^{i\alpha c} \cos (\xi)}{\delta_k + \frac{1}{2} + 1 + i\Omega (x,y) + i\frac{\Gamma}{2}} + \frac{\sin (\xi) + e^{i\alpha c} \cos (\xi)}{\delta_k + \frac{1}{2} - 1 + i\Omega (x,y) + i\frac{\Gamma}{2}} \right) + g^{(2)}_k \left( \frac{\cos (\xi) - e^{-i\alpha c} \sin (\xi)}{\delta_k - \frac{1}{2} - \Delta + i\Omega (x,y) + i\frac{\Gamma}{2}} + \frac{\cos (\xi) + e^{-i\alpha c} \sin (\xi)}{\delta_k - \frac{1}{2} - \Delta - i\Omega (x,y) + i\frac{\Gamma}{2}} \right). \]  

Consequently, the required conditional probability distribution of finding the atom in state \( |0\rangle \) with emitted photon of frequency \( \omega_k \) corresponding to reservoir mode \( k \) is depicted by

\[ W(x,y) = |N|^2 \left| f(x,y) \right|^2 \left| b_{0,1k} (x,y; t \to \infty) \right|^2, \]  

where we have used the transformation \( b_{0,1k} (x,y;t) = a_{0,1k} (x,y;t) \) from Eq. (14). Since the center-of-mass wave function \( f(x,y) \) for atom is assumed to be almost constant over many wavelengths of the standing-wave fields in \( x - y \) plane, the conditional probability distribution \( W(x,y) \) for atom localization is determined by filter function as defined in Eq. (8).
3 Numerical Results and Discussion

In this section, we will discuss the conditional probability distribution of an atom employing few numerical results based on filter function $F(x,y)$. We will then swing the system parameter to show how atom localization can be attained via quantum coherence. In our analysis, we have considered two orthogonal standing waves with corresponding Rabi frequency $\Omega(x,y) = \Omega_1 \sin(k_1 x) + \Omega_2 \sin(k_2 y)$ [40]. Further, all the parameters are taken in terms of decay rate $\Gamma$. Apparently, the filter function $F(x,y)$ depends on the parameters of standing wave driving fields and frequency of the emitted photon, rather it also depends on the interference effects [40]. Since the two spontaneous decay channels $|1\rangle \rightarrow |0\rangle$ and $|2\rangle \rightarrow |0\rangle$ interact via same vacuum modes, quantum interference subsists. However, we have neglected quantum interference by setting parameter $p = 0$ in our analysis. Moreover, the dynamically induced interference due to two orthogonal standing wave fields does play a considerable effect. Hence, atom localization in 2D can be manipulated by various parameters. As the filter function $F(x,y)$ depends on $\Omega(x,y)$ which itself comprises of $\sin(k_1 x)$ and $\sin(k_2 y)$, the localization is possible for only those values of $(x,y)$ for which $F(x,y)$ reveals maxima. Here, the analytical form of $F(x,y)$ appears to be quite cumbersome, so we will provide only numerical results for precise atom location in two dimensions.

Since the atom is initially prepared in the superposition of upper two excited states $|1\rangle$ and $|2\rangle$ via Eq.(19), the state is strongly dependent on coupling phase for upper two levels $|1\rangle$ and $|2\rangle$, i.e., $\alpha_c$ which shows quantum coherence is phase dependent. Accordingly, we have considered three values of phase that is $\alpha_c = 0$, $\pi/2$ and $\pi$.

In case of $\alpha_c = \pi/2$, we first provide conditional probability distribution by plotting filter function $F(x,y)$ as a function of $(k_1 x, k_2 y)$ over a single wavelength for different value of detuning of spontaneously emitted photon i.e., $\delta_k$ [57]. From Fig. 2(a)-(d), it is evident that the filter function $F(x,y)$ strongly depends on the detuning of spontaneously emitted photon. When $\delta_k = 9.3\Gamma$, the location of atom is distributed in all four quadrants in $x-y$ plane, as shown in Fig. 2(a). Switching values to $\delta_k = 5.3\Gamma$, the location is restricted to quadrant I and IV with peaks providing crater like pattern [Fig. 2(b)]. On further refining the detuning to $\delta_k = 2.9\Gamma$, the peaks get narrowed, as depicted in Fig. 2(c). Furthermore, Fig. 2(d) illustrate the effect of detuning set to appropriate value i.e., $\delta_k = 0.1\Gamma$. The filter function $F(x,y)$ furnishes two spike like patterns corresponding to maximas located in quadrant I and IV at $(k_1 x, k_2 y) = (\pi, \pi)$ and $(-\pi, -\pi)$ in $x-y$ plane which clearly indicates that high precision and localization in two dimension can be obtained when emitted photon is nearly resonant with the corresponding atomic transition. Consequently, the probability of finding the atom at each location is $1/2$ which is twice as compared to the probability obtained in earlier cases [37, 39, 44, 58].

In Fig. 3, we plot the filter function $F(x,y)$ versus $(k_1 x, k_2 y)$ by modulating the detuning of spontaneously emitted photon in 2D for $\alpha_c = 0$. Fig. 3(a) illustrates the results when detuning is large i.e., $\delta_k = 12.4\Gamma$. The lattice like
structure obtained gives distributed on the diagonal in II and IV quadrants. This specifies that the atom localization peaks are determined by $k_1x + k_2y = 2p\pi$ or $k_1x + k_2y = 2(q + 1)\pi$ where $p$ and $q$ are integers. Refining $\delta_k$ to $9.5\Gamma$, the position probability of atom is rather complicated due to the interference of the two fields and the filter function $F(x, y)$ is dispersed in quadrant II, III and IV. However, the distribution is mainly localized in quadrant III, as presented in Fig. 3(b). Narrowing the detuning parameter to $\delta_k = 6.0\Gamma$, the location is distributed in quadrant III with a crater like structure, as shown in Fig. 3(b). Such crater like structure persists in quadrant III for $\delta_k \in [5.2\Gamma, 6.9\Gamma]$, offering atom localized at the circle [Fig. 3(c)]. Tuning the photon detuning to $\delta_k = 2.4\Gamma$, a single spike is achieved at $(-\frac{\pi}{2}, -\frac{\pi}{2})$ as illustrated in Fig. 3(d) which indicates that probability of finding the atom within single wavelength in 2D, is increased by a factor of 2 (as in Fig.2(d)). Hence, we can say that atom localization is undeniably acquired in 2D.

From Eq. (29) with $\Omega(x,y) = \Omega_1 \sin(k_1x) + \Omega_2 \sin(k_2y)$, we can easily identify that the filter function $F(x, y)$ remains unaltered under transform $0 \leftrightarrow \pi$ and $(k_1x, k_2y) \leftrightarrow (-k_1x, -k_2y)$. Therefore, $F(x, y; \alpha_c = 0) = F(-x, -y; \alpha_c = \pi)$ and we obtain vice versa results as in previous case ($\alpha_c = 0$). However, the localization distribution and peak are shifted in quadrant I, as shown in Fig. 4(a)-(d).

These results identify the strong association of detuning of spontaneously emitted photon to the localization of atom. Furthermore, the peaks of atom localization in all of the above cases are obtained at antinodes of the standing fields with precise localization destroyed for large frequency of the spontaneously emitted photon. Indeed, the localization seems to be possible when emitted photon are near in resonance to the atomic transitions.

Finally we present the significance of initial conditions on atom localization by explicitly preparing the atomic system is single state. Therefore for $\alpha_c = 0$, and setting $\xi = 0$ in Eq. (19), the system is initially in state $|1\rangle$ which provides the distribution of localization peaks takes place in quadrant I and III as depicted in Fig. 5(a). Hence, the probability of finding the atom at a single location, in 2D is decreased. Therefore, the number of peaks increased by a factor of 2, as compared to the case when atom is initially prepared in superposition of states $|1\rangle$ and $|2\rangle$ [see Fig. 3(d)]. The reason behind is the absence of atomic coherence for two decaying states $|1\rangle$ and $|2\rangle$. A similar result can also be obtained for $\xi = \pi$ where again the probability is decreased by a factor of 2 as shown in Fig. 4(d). However, Fig. 5(b) shows sharp peaks by setting $\xi = \frac{\pi}{2}$. This indeed provides us high resolution and precision for atomic localization in 2D in the absence of atomic coherence.

4 Conclusions

In summary, we have proposed and analyzed atom localization for a three level atomic system in two dimensions. The scheme under consideration is based on the phenomenon of spontaneous emission when the atom interacts with spatially
dependent standing orthogonal fields. Following the position dependent atom-field interaction, the precise location of atom in 2D, can be achieved by detecting frequency of spontaneously emitted photon. Consequently, the interaction provides various structures of filter function such as lattice like structure, crater like structure and most importantly, the localization spikes. The phenomenon of quantum coherence originates from coupling of two excited levels to standing wave fields. Our results shows that not only the relative phase between two orthogonal standing wave fields but also the frequency detuning substantially controls fluorescence spectra in conditional probability distribution. The localization pattern generates a single spike for inphase position dependent fields. However, the localization pattern is destroyed with an increase in frequency detuning. Remarkably, the pattern of localization peaks remains unaltered with varying vacuum field detuning which is the major advantage of our scheme. We have also presented the effect of initial state preparation on atom localization. In the absence of atomic coherence, the localization probability decreases with increase in spatial resolution. Our analysis indeed provides efficient way for atom localization in two dimension that may be productive for laser cooling and atom nano-lithography [58].

References

[1] W.D. Phillips, Rev. Mod. Phys. 70, 721 (1998)
[2] P. Rudy, R. Ejnisman, N. P. Bigelow, Phys. Rev. Lett. 78, 4906 (1997)
[3] G. Rempe, Appl. Phys. B 60, 233 (1995)
[4] R. Quadt, M. Collett, D. F. Walls, Phys. Rev. Lett. 74, 351 (1995)
[5] W. Heisenberg, Z. Phys. 43, 172 (1927)
[6] M. Born and E. Wolf, *Principles of Optics* (Cambridge University Press, Cambridge, 1999)
[7] H. Metcalf, P. Van der Straten, Phys. Rep. 244, 203 (1994)
[8] K.S. Johnson, J.H. Thywissen, W.H. Dekker, K.K. Berggren, A.P. Chu, R. Younkin, M. Prentiss, Science, 280, 1583 (1998)
[9] A.N. Boto, P. Kok, D.S. Abrams, S.L. Braunstein, C.P. Williams, J.P. Dowling, Phys. Rev. Lett. 85, 2733 (2000)
[10] K.T. Kapale, S. Qamar, M.S. Zubairy, Phys. Rev. A 67, 023805 (2003)
[11] J. Evers, S. Qamar, M.S. Zubairy, Phys. Rev. A 75, 053809 (2007)
[12] G.P. Collins, Phys. Today 49, 18 (1996)
[13] Y. Wu and R. Côté, Phys. Rev. A 65, 053603 (2002)
[14] J. Mompart, V. Ahufinger, G. Birkl, Phys. Rev. A 79, 053638 (2009)
[15] R. Abfalterer, C. Keller, S. Bernet, M.K. Oberthaler, J. Schmiedmayer, A. Zeilinger, Phys. Rev. A 56, 4365 (1997)
[16] C. Keller, R. Abfalterer, S. Bernet, M.K. Oberthaler, J. Schmiedmayer, A. Zeilinger, J. Vac. Sci. Technol. B 16, 3850 (1998)
[17] K.S. Johnson, J.H. Thywissen, N.H. Dekker, K.K. Berggren, A.P. Chu, R. Younkin, M. Prentiss, Science 280, 1583 (1998)
[18] P. Storey, M. Collett, D. F. Walls, Phys. Rev. Lett. 68, 472 (1992)
[19] M.A.M. Marte, P. Zoller, Appl. Phys. B 54, 477(1992)
[20] P. Storey, M. Collett, D. F. Walls, Phys. Rev. A 47, 405 (1993)
[21] R. Quadt, M. Collett, D. F. Walls, Phys. Rev. Lett. 74, 351 (1995)
[22] S. Kunze, G. Rempe, M. Wilkens, Europhys. Lett. 27, 115 (1994)
[23] S. Kunze, K. Dieckmann, G. Rempe, Phys. Rev. Lett. 78, 2038 (1997)
[24] F. L. Kien, G. Rempe, W. P. Schleich, M. S. Zubairy, Phys. Rev. A 56, 2972 (1997)
[25] S. Qamar, A. Mehmood, S. Qamar, Phys. Rev. A 79, 033848 (2009)
[26] S. Kunze, K. Dieckmann, G. Rempe, Phys. Rev. Lett. 78, 2038 (1997)
[27] F. Ghafoor, Phys. Rev. A 84, 063849 (2011)
[28] S. Qamar, S.Y. Zhu, and M.S. Zubairy, Phys. Rev. A 61, 063806 (2000)
[29] H.Wang, D. Goorskey, M. Xiao, Phys. Rev. Lett. 87, 073601 (2001)
[30] Y. Wu and X. Yang, Appl. Phys. Lett. 91, 094104 (2007)
[31] Y. Wu and X. Yang, Phys. Rev. A 70, 053818 (2004)
[32] Y. Wu, L. L. Wen, Y. F. Zhu, Opt. Lett. 28, 631 (2003)
[33] H. Lee, P. Polynkin, M. O. Scully, S. Y. Zhu, Phys. Rev. A 55, 4454 (1997)
[34] J.H. Wu, A.J. Li, Y. Ding, Y.C. Zhao, J.Y. Gao, Phys. Rev. A 72, 023802 (2005)
[35] W. Harshawardhan, G.S. Agarwal, Phys. Rev. A 53, 1812 (1996)
[36] A. Joshi, M. Xiao, Phys. Rev. Lett. 91, 143904 (2003)
[37] V. Ivanov, Y. Rozhdestvensky, Phys. Rev. A 81, 033809 (2010)
[38] R.G. Wan, J. Kou, L. Jiang, Y. Jiang, J.Y. Gao, J. Opt. Soc. Am. B 28, 622 (2011)
[39] R.G. Wan, J. Kou, L. Jiang, Y. Jiang, J.Y. Gao, Opt. Commun. 284, 985 (2011)
[40] C. Ding, J. Li, Z. Zhan, X. Yang, Phys. Rev. A 83, 063834 (2011)
[41] R.G. Wan, T.Y. Zhang, Opt. Express 29, 25823 (2011)
[42] J. Li, R. Yu, M. Liu, C. Ding, X. Yang, Phys. Lett. A 375, 3978 (2011)
[43] C. Ding, J. Li, X. Yang, D. Zhang, H. Xiong, Phys. Rev. A 84, 043840 (2011)
[44] C.L. Ding, J.H. Li, X.X. Yang, Z.M. Zhang, J.B. Liu, J. Phys. B 44, 145501 (2011)
[45] A.M. Herkommer, W.P. Schleich, M.S. Zubairy, J. Mod. Opt. 44, 2507 (1997)
[46] E. Paspalakis, C. H. Keitel, P.L. Knight, Phys. Rev. A 58, 4868 (1998)
[47] M. Sahrai, H. Tajalli, K.T. Kapale, M.S. Zubairy, Phys. Rev. A 72, 013820 (2005)
[48] K.T. Kapale, M.S. Zubairy, Phys. Rev. A 73, 023813 (2006)
[49] C.P. Liu, S.Q. Gong, D.C. Cheng, X.J. Fan, Z.Z. Xu, Phys. Rev. A 73, 025801 (2006)
[50] D.C. Cheng, Y. P. Niu, R.X. Li, S.Q. Gong, J. Opt. Soc. Am. B 23, 2180 (2006)
[51] P. Meystre, M. Sargent, Elements of Quantum Optics (Springer-Verlag, Berlin, 1999)
[52] S. Qamar, S.Y. Zhu, M.S. Zubairy, Opt. Commun. 176, 409 (2000)
[53] M.O. Scully, M.S Zubairy, Quantum Optics (Cambridge University Press, Cambridge, 1997)
[54] G.S. Agarwal, Quantum Statistical Theories of Spontaneous Emission and their Relation to Other Approaches (Springer-Verlag, Berlin, 1974)
[55] F. Ghafoor, S.Y. Zhu, M.S. Zubairy, Phys. Rev. A 62, 013811 (2000)
[56] S.Y. Zhu, R.C.F. Chan, C.P. Lee, Phys. Rev. A 52, 710 (1995)
[57] Z. Wang, B. Yu, F. Xu, S. Zhen, X. Wu, Appl. Phys. B 108, 479 (2012)
[58] L.L. Jin, H. Sun, Y.P. Niu, S.Q. Jin, S.Q. Gong, J. Mod. Opt. 56, 805 (2009)
5 Figure Captions

Fig-1: Schematic diagram of the system. (a) Atom moving along $z-$axis interacts with two dimensional position dependent field in $x-y$ plane; (b) Atomic model under consideration. The two excited levels $|1\rangle$ and $|2\rangle$ are coupled by two dimensional standing wavefield $\Omega(x,y)$ with level $|2\rangle$ having a finite detuning $\Delta$. Both the excited levels $|1\rangle$ and $|2\rangle$ decay spontaneously to ground state via decay rates $\Gamma_1$ and $\Gamma_2$, respectively.

Fig-2: Filter function $F(x,y)$ as a function of $(k_1x,k_2y)$ for detuning of spontaneously emitted photon, (a) $\delta_k = 9.3$; (b) $\delta_k = 5.3$; (c) $\delta_k = 2.9$; (d) $\delta_k = 0.1$. Other parameters are $\alpha_c = \frac{\pi}{2}$, $\xi = \frac{\pi}{4}$, $\Delta = 2.5$, $\omega_{21} = 20$ and $\Omega_1 = \Omega_2 = 5$. All system parameters are scaled in units of $\Gamma$.

Fig-3: Filter function $F(x,y)$ as a function of $(k_1x,k_2y)$ for detuning of spontaneously emitted photon, (a) $\delta_k = 12.4$; (b) $\delta_k = 9.5$; (c) $\delta_k = 6.0$; (d) $\delta_k = 2.4$. Other parameters are $\alpha_c = 0$, $\xi = \frac{\pi}{4}$, $\Delta = 2.5$, $\omega_{21} = 20$ and $\Omega_1 = \Omega_2 = 5$. All system parameters are scaled in units of $\Gamma$.

Fig-4: Filter function $F(x,y)$ as a function of $(k_1x,k_2y)$ for detuning of spontaneously emitted photon, (a) $\delta_k = 12.4$; (b) $\delta_k = 9.5$; (c) $\delta_k = 6.0$; (d) $\delta_k = 2.4$. Other parameters are $\alpha_c = \pi$, $\xi = \frac{\pi}{4}$, $\Delta = 2.5$, $\omega_{21} = 20$ and $\Omega_1 = \Omega_2 = 5$. All system parameters are scaled in units of $\Gamma$.

Fig-5: Filter function $F(x,y)$ as a function of $(k_1x,k_2y)$ for detuning of spontaneously emitted photon, (a) $\xi = 0$; (b) $\xi = \frac{\pi}{2}$. Other parameters are $\alpha_c = \pi$, $\delta_k = 2.4$, $\Delta = 2.5$, $\omega_{21} = 20$ and $\Omega_1 = \Omega_2 = 5$. All system parameters are scaled in units of $\Gamma$. 