Proton Decay, Neutrino Oscillations And Other Consequences From Supersymmetric SU(6) With Pseudo-Goldstone Higgs

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Abstract

We suggest a new mechanism for naturally suppressing dimension five baryon number violating in supersymmetric SU(5 + N) (N = 0, 1, \cdots) GUTs. The mechanism is realized through suppression of \(qqT\) type couplings, and is implemented by introducing new ‘matter’ multiplets belonging to symmetric representations of SU(5 + N). Together with the suppression of nucleon decay, these multiplets also enable one to avoid the unwanted asymptotic mass relations \(m_s = m_\mu, \frac{m_d}{m_s} = \frac{m_e}{m_\mu}\).

As an example, we consider a SU(6) model with pseudo-Goldstone Higgs. By supplementing the model with an anomalous U(1) flavor symmetry, we also obtain a simple ‘all-order’ solution of the gauge hierarchy problem and natural explanation of charged fermion mass hierarchies and values of the CKM matrix elements. The proton life time \(\tau_p \sim 10^2 \tau_p^{SU(5)}\) yr. is compatible with experiments, with the dominant decay being \(p \to K\nu_\mu,\tau\). Thanks to the SU(6) symmetry, successful unification of the gauge couplings can be retained, and the value of the strong coupling \(\alpha_s(M_Z)\) can be reduced to \(\approx 0.12\).

Finally, we show how to accommodate the solar and atmospheric neutrino data through the bi-maximal neutrino mixing scenario, with maximal vacuum \(\nu_e - \nu_\mu,\tau\) and large angle \(\nu_\mu - \nu_\tau\) oscillations.

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1 Introduction

One of the most outstanding problems of supersymmetric (SUSY) GUTs is the question of nucleon stability. Within the frameworks $SU(5)$ and $SO(10)$ GUTs the nucleon lifetime is about $10^{29 \pm 2}$ yr, to be compared with the latest experimental bound $\tau_{N}^{\text{exp}} \gtrsim 10^{32}$ yr. This result is valid for the simplest versions of SUSY $SU(5)$, $SO(10)$, etc., unless some mechanism for suppressing or eliminating the dangerous baryon number violating $d = 5$ operators is applied. In $SO(10)$, it seems, possible to resolve this problem, through the missing VEV solution, with the VEV of the adjoint 45-plet Higgs in the $T_R$ direction. Attempts to suppress dimension five of nucleon decay in the framework of extended (with 75, 50, 50 multiplets) SUSY $SU(5)$ theory were suggested in [8], while in [9] the different mechanism, tied to the pattern of fermion masses, was proposed.

In this paper we present a new mechanism for the suppression of dimension five nucleon decay in SUSY $SU(5+N)$ ($N = 0, 1, \cdots$) GUTs. The mechanism is realized via symmetry reasons, and a crucial role is played by symmetric pairs of vector-like ‘matter’ $\bar{S}+S$, which we introduce in the theory. Through a special arrangement of the couplings involving these ‘$S$’-plets, the $qqT$-type terms can be suppressed to the needed level. [Here $q$ and $T$ belong to the $(3,2)$ and $(1,3)$ representations of $SU(3)_c \times SU(2)_W$ respectively.] Further, as pointed out in [10], it turns out that these symmetric multiplets are important not only for nucleon stability, but also for obtaining a realistic pattern of fermion masses. Namely, thanks to these additional states, the unwanted asymptotic relations $m_s = m_\mu$, $m_d = m_e$ can be avoided, and a reduced value, compared to $SU(5)$ for $\alpha_s(M_Z)$ can be achieved.

For a transparent demonstration of the proposed mechanism we consider a SUSY $SU(6)$ model, in which the doublet-triplet (DT) hierarchy is achieved through the pseudo-Goldstone boson (PGB) mechanism. An anomalous $U(1)$ symmetry, which we use, leads to an ‘all-order’ DT hierarchy and a desirable pattern of symmetry breaking. We consider $U(1)$ as a flavor symmetry acting on the fermion generations and gain a natural explanation of the hierarchies of charged fermion masses and CKM matrix elements.

The scenario suggests a desirable value (for the $SU(6)$ PGB model) of the MSSM parameter $\tan \beta$ (near unity), retains $b-\tau$ unification and avoids the analogous unwanted asymptotic relations for light generations. Although the PGB $SU(6)$ scenarios considered earlier, lead to other realistic implications, the problem of nucleon stability had not been previously addressed within its framework and was an open question. Applying the proposed mechanism for suppression of nucleon decay, we will see that the $SU(6)$ scenario which we consider does not suffer from this problem. The dominant nucleon decay mode is $p \rightarrow K \nu_{\mu,\tau}$, and the life time is $\sim 10^2$ times larger than the minimal SUSY

\footnote{In SUSY theories such as flipped $SU(5)$, $SU(3)^3$, flipped $SU(6)$ and $SU(4)_c \times SU(2)_L \times SU(2)_R$, the situation is not so hard and it is easier to achieve the suppression of nucleon decay by imposing some continuous or discrete $\mathcal{R}$-symmetries. See [8]–[9] respectively.}
SU(5) value. Due to the peculiarity of SU(6) gauge group and some additional multiplets, the successful unification of three gauge couplings can be retained and a reduced value for \(\alpha_s(M_Z) \simeq 0.12\) can be achieved.

Finally, for accommodating the solar and atmospheric neutrino data, we consider the ‘bi-maximal’ neutrino mixing scenario, in which the solar neutrino puzzle is resolved through maximal \(\nu_e - \nu_{\mu,\tau}\) vacuum oscillations, while the atmospheric neutrino deficit is due to large \(\nu_{\mu} - \nu_{\tau}\) mixing. Alternative scenarios for neutrino oscillations are possible, as was recently discussed in [11].

2 Nucleon Decay and Mechanism for Suppression of \(qqT\) Type Couplings in \(SU(5 + N)\) GUTs

As pointed out some time ago in [12], in supersymmetric GUTs (whose minimal version is the \(SU(5)\) gauge group), there is a new source for baryon number violation. Namely, dimension five \((d = 5)\) operators

\[
O_L = \frac{1}{M_T} qqql, \quad O_R = \frac{1}{M_T} u^c u^c d^c e^c ,
\]

which emerge through the couplings

\[
qqT + qlT + u^c e^c T + u^c d^c \bar{T} + M_T \bar{T} T ,
\]

after integrating out the heavy color triplets \(\bar{T}, T\). For \(M_T \sim 10^{16}\) GeV, the dominant decay mode is \(p \rightarrow K\nu_\mu\)\(^5\), resulting in proton lifetime of \(10^{30 \pm 2}\) yr, which is embarrassingly small in comparison with the latest experimental limit \(\tau_p^{\text{exp}} > 10^{32}\) yr [1].

In this section we propose a new mechanism for suppressing the \(d = 5\) operators in the framework of supersymmetric \(SU(5 + N)\) GUTs, where \(N = 0, 1, \ldots\). For its breaking down to the \(SU(3)_c \times SU(2)_W \times U(1)_Y\), it is sufficient to introduce one adjoint \((\Sigma)\), and \(N\) pairs of fundamental-antifundamental \((N \cdot (H + \bar{H}) \equiv H^{(a)} + \bar{H}^{(a)}, a = 1, \ldots, N)\) scalar supermultiplets.

As far as the fermion sector is concerned, the anomaly free chiral supermultiplets (which unify quark-lepton superfields) of \(SU(5 + N)\) can be taken as follows [6]:

\[
A_{ij} + (N + 1) \cdot \bar{F}^i ,
\]

\(^5\)As it turns out, the dominant contribution to nucleon decay is from the \(O_L\) operator, with the decay modes emerging through \(O_R\) being more suppressed.

\(^6\)For demonstration of the mechanism we will consider the simplest choice of anomaly free chiral ‘matter’. However, other choices are also possible.
(for simplicity we consider only one generation) where \( i,j (= 1, \ldots, N + 5) \) are gauge indices, and \( A \) and \( \bar{F} \) are the antisymmetric and (anti-)fundamental representations respectively. In terms of \( SU(5) \) these multiplets contain, in addition to \( 10 + \bar{5} \) superfields, \( N \) pairs of vector-like \( \bar{5} + 5 \) and \( N(3N + 1)/2 \) singlet states. After the GUT symmetry breaks to \( SU(3)_c \times SU(2)_W \times U(1)_Y \) all the singlet and vector-like ‘matter’ decouples and we are left with the minimal chiral superfields in \( 10 + \bar{5} \):

\[
10 = (q, u^c, e^c), \quad 5 = (d^c, l).
\]

Since \( A \supset 10 \supset q \), and we are interested in \( qqT \)-type couplings, we will consider the relevant couplings involving the superfield \( A \).

Let us introduce a pair of symmetric supermultiplets \( S_{ij} + \bar{S}^{ij} \). Together with other fragments, they contain \( q + \bar{q} \) states respectively. From the superpotential couplings

\[
W(A, S) = A\Sigma \bar{S} + M_S \bar{S}S,
\]

and assuming that

\[
\langle \Sigma \rangle \equiv M_G \gg M_S,
\]

one can easily verify that \( q_A \) decouples by forming a massive state with \( \bar{q}_S \), and the ‘light’ state \( q \) mainly resides in \( S \):

\[
S \supset q, \quad A \gtrsim \frac{M_S}{M_G} q.
\]

The states \( u^c \) and \( e^c \) reside purely in \( A \),

\[
A \supset (u^c, e^c).
\]

The operator which generates mass for up-type quark has the form

\[
\frac{1}{M^{N+1}} (\Sigma S)_{mn} A_{pq} H^{(1)}_{i_1} \cdots H^{(N)}_{i_N} (\Sigma H^{(a)})_{i} \epsilon^{mnpq}_{i_1 \cdots i_N \epsilon},
\]

where convolution of \( SU(5 + N) \) group indices are indicated, and \( M \) is some cut-off mass scale. For the Yukawa coupling of up-type quark we have to substitute the VEVs of appropriate scalar fields and also extract from them the Higgs doublet \( h_u \) (in general the \( h_u \) state can reside, with suitable weights in \( H^{(a)} \) and also in \( \Sigma \), depending on the specifics of the model). Due to (6), (8) the states \( q, u^c \) should be extracted from \( S, A \) respectively, and the corresponding Yukawa coupling will have the form:

\[
\lambda^U qu^ch_u.
\]
For the $qqT$-type coupling, the light $q$ state should be extracted from $A$, and since the latter contains it with a suppressed weight (see (7)), the $qqT$ coupling will also be suppressed:

$$\frac{M_S}{M_G^2} \lambda^qqT,$$

Consequently, the nucleon decay width induced from this coupling (if $qlT$ coupling has the ‘standard’ form), will be suppressed by a factor $\left(\frac{M_S}{M_G}\right)^2$ in comparison to minimal $SU(5)$. To obtain the desired suppression it is enough to have $\frac{M_S}{M_G} \sim 10^{-2}$. Therefore, with the help of $\bar{S} + S$ states we can get a natural suppression of nucleon decay. A crucial assumption for realizing sufficient nucleon stability is (6), which should be satisfied for the mechanism described here to work.

It is worth noting that in addition to their role in the suppression of nucleon decay, the $\bar{S} + S$-plets are crucial also for avoiding the unwanted asymptotic relations $\lambda^q_\alpha = \lambda^e_\alpha$ for the ‘light’ generations (8). This relation is automatically violated if the condition $\langle \Sigma \rangle \gg M_S$ is satisfied and the light $q, e^c$ states reside in different multiplets (in the above example in $S$ and $A$ respectively). If we wish, however, to retain the asymptotic relation $\lambda^q_\beta = \lambda^\tau_\beta$, the above mechanism should be applied only to the first two generations. This means that the coupling $q_3q_3T$ is not suppressed, so that nucleon decay can also emerge from this term through mixing with the ‘light’ generations. Since the mixings are small nucleon decay will still be suppressed. Whether the main contribution to the nucleon decay width arises from this effect, or from the direct couplings (11) depends on the details of the model.

In concluding this section, let us note that since the mass scale $M_S$ of $\bar{S} + S$ states is below $M_G$, additional fragments appearing in the $M_S - M_G$ region can change the running of the gauge couplings, and so care is needed for retaining the unification of the gauge couplings. In the next section we will consider the PGB $SU(6)$ scenario, which neatly meets this constraint while leading to more stable nucleon than in $SU(5)$. As it turns out, the dominant decay $p \rightarrow K\nu_{\mu,\tau}$ occurs through mixing of the ‘light’ families with the third generation.

### 3 Pseudo-Goldstone $SU(6)$ Scenario

For a transparent demonstration of the mechanism described above, we will consider the $SU(6)$ gauge theory which provides a natural solution of the DT splitting problem through the pseudo-Goldstone mechanism. It will turn out that the peculiarities of some representations of $SU(6)$ allow the possibility to retain the successful unification of the three gauge couplings, even though the masses of additional symmetric multiplets lie below the GUT scale.
3.1 Higgs Sector: Symmetry Breaking and ‘All Order’ DT Hierarchy

In addition to $\Sigma(35) + \bar{H}(6) + H(6)$ Higgs supermultiplets (which are necessary for $SU(6)$ breaking to $SU(3)_c \times SU(2)_W \times U(1)_Y \equiv G_{321}$) we introduce an additional singlet superfield $X$, and an anomalous $U(1)$ symmetry. The latter plays a crucial role in achieving an ‘all-order’ DT hierarchy \[15\], and helps provide a natural understanding of the magnitudes of charged fermion masses and mixings. The $U(1)$ charges of these superfields are

$$Q_\Sigma = 0, \quad Q_H = r,$$
$$Q_{\bar{H}} = r', \quad Q_X = 1,$$

where $r, r'$ must be taken in such a way that in the scalar superpotential

$$W = \frac{\Lambda}{2} \Sigma^2 + \frac{\lambda}{3} \Sigma^3 + M_P^3 \left( \frac{X}{M_P} \right)^m \left( \frac{\bar{H}H}{M_P} \right)^n,$$

$$m + 2n = 25,$$

so that

$$r + r' = -\frac{m}{n} < 0.$$

Values for $m, n$ should be chosen in such a way that a lower order term in (13) is not allowed. One desirable choice is $(m, n) = (7, 9)$.

From the first two terms in (13) there exists a non-vanishing SUSY conserving solution for $\langle \Sigma \rangle$ along the direction

$$\langle \Sigma \rangle = \text{Diag}(1, 1, 1, -2, -2, 1) \cdot V$$

with

$$V = \frac{\Lambda}{\chi} \equiv M_P \epsilon_G (\sim 10^{16}\text{GeV}).$$

From the last term in (13) $\langle X \rangle = \langle H \rangle = \langle \bar{H} \rangle = 0$ at this stage.

The situation will be changed if the $U(1)$ symmetry happens to be an anomalous gauge symmetry, arising in effective field theories from strings. The cancellation of anomalies occurs through the Green-Schwarz mechanism \[16\]. Due to the anomaly, the Fayet-Iliopoulos term

$$\xi \int d^4 \theta V_A$$

5
is always generated \[17\] where, in string theory \[18\],
\[
\xi = \frac{g_3^2 M_P^2}{192 \pi^2} \text{Tr} Q .
\] (19)
The $D_A$ term will have the form
\[
\frac{g_A^2}{8} D_A^2 = \frac{g_A^2}{8} \left( \Sigma Q_a |\varphi_a|^2 + \xi \right)^2 ,
\] (20)
where $\varphi_a$ denotes a superfield which carries a nonzero ‘anomalous’ $Q_a$ charge. Assuming that $\xi < 0$ ($\text{Tr} Q < 0$) and taking into account (12), (15), the cancellation of (20) fixes the $\langle X \rangle$ VEV,
\[
\langle X \rangle = \sqrt{-\xi} .
\] (21)
Note that for ensuring a non-zero VEV for $X$, the important thing is not the absolute sign of $\xi$, but the (opposite) relative signs of $Q_X$ and $\xi$. Further, we will take
\[
\frac{\langle X \rangle}{M_P} = \frac{\sqrt{-\xi}}{M_P} \approx 0.22 \approx \left( \frac{m_{3/2}}{M_P} \right)^{1/23} ,
\] (22)
where $m_{3/2}$ is the gravitino mass($\sim 10^3$ GeV) and $M_P = 2.4 \cdot 10^{18}$ GeV.

After SUSY breaking in minimal $N = 1$ SUGRA theory, the soft bilinear terms
\[
V_{soft}^m = m_{3/2}^2 \left( |H|^2 + |\bar{H}|^2 \right)
\] (23)
emerge, and taking into account \[13\], \[22\], one obtains nonzero VEVs $\langle H \rangle$, $\langle \bar{H} \rangle$ (along the $SU(5)$ singlet direction):
\[
\frac{\langle H \rangle}{M_P} \sim \frac{\langle H \rangle}{M_P} \equiv \frac{v}{M_P} \sim \frac{\langle X \rangle}{M_P} \equiv \epsilon \approx \left( \frac{m_{3/2}}{M_P} \right)^{1/23} \approx 0.22 .
\] (24)

It will turn out $\epsilon$ is an important expansion parameter for understanding the hierarchies among the Yukawa couplings and the magnitude of the CKM matrix elements. It is clear that in attempting to express these hierarchies through appropriate powers of $\epsilon$, the expansion operator should be an $SU(6)$ singlet. Without the superfield $X$ we would only have $HH/M_P^2 \sim \epsilon^2$, which is too small. Also, as we will see in sect. (3.2), odd powers of $\epsilon$ are also needed. Therefore, the introduction of the singlet superfield $X$ is crucial in our model.

The superpotential in \[13\] has $SU(6)_\Sigma \times U(6)_{H+\bar{H}}$ global symmetry which, by \[16\], \[24\] is broken down to $[SU(4)_c \times SU(2)_W \times U(1)']_\Sigma \times U(5)_{H+\bar{H}}$, while the $SU(6)$ gauge
symmetry in broken to $SU(3)_c \times SU(2)_W \times U(1)_Y$. It is easy to verify that the pseudo-Goldstone states, ‘massless’ if SUSY is unbroken, are a doublet-antidoublet pair which can be identified with the MSSM Higgs doublets\footnote{For more detailed discussions see [13, 14].}. The states
\[
h_u = \frac{v h_u^{(\Sigma)} - 3 V h_u^{(H)}}{\sqrt{v^2 + 9 V^2}}, \quad h_d = \frac{v h_d^{(\Sigma)} - 3 V h_d^{(H)}}{\sqrt{v^2 + 9 V^2}},
\]
are ‘massless’ pseudo-Goldstones, while their orthogonal superpositions
\[
h_u^G = \frac{v h_u^{(\Sigma)} + 3 V h_u^{(H)}}{\sqrt{v^2 + 9 V^2}}, \quad h_d^G = \frac{v h_d^{(\Sigma)} + 3 V h_d^{(H)}}{\sqrt{v^2 + 9 V^2}},
\]
are genuine Goldstones, ‘eaten up’ by the appropriate gauge fields. From (25), (26), taking into account (17) and (24), one can easily verify that the physical doublets reside in $\Sigma$ and $H, \bar{H}$ as follows:
\[
\Sigma \supset (h_u, h_d), \quad H \supset 3 \frac{\epsilon_G}{\epsilon} h_u, \quad \bar{H} \supset 3 \frac{\epsilon_G}{\epsilon} h_d.
\]

Note that the $SU(6)_\Sigma \times U(6)_{H+\bar{H}}$ global symmetry violating higher order operator $\Sigma X_m (\bar{H} H)^n$ gives the desirable contribution ($\sim 100$ GeV) to the $\mu$-term. In summary, the anomalous $U(1)$ symmetry helps provide an ‘all-order’ solution of the DT splitting problem in the framework of PGB $SU(6)$ model.

### 3.2 Charged Fermion Masses and Mixings

The ‘matter’ sector of $SU(6)$ contains the anomaly free ‘matter’ supermultiplets $(15 + \bar{6} + \bar{6}^\prime)_\alpha (\alpha = 1, 2, 3$ is a family index). For generating the top quark mass at renormalizable level, we also introduce a 20plet of $SU(6)$. In order for the proton decay suppression mechanism (discussed in section 2) to work, we introduce two pairs of symmetric supermultiplets $(\bar{21} + 21)_1, 2$. These fields will be crucial also for getting a realistic pattern of charged fermion masses. In terms of $SU(5)$, the ‘matter’ superfields decompose as:
\[
\bar{6} = \bar{5} + 1, \quad 15 = 10 + 5, \\
20 = 10 + \bar{10}, \quad 21 = 15 + 5 + 1.
\]

The decomposition of $\bar{21}$ is analogous to 21.

Here we will use the anomalous $U(1)$ as a flavor symmetry in order to gain an understanding of hierarchies of Yukawa couplings and CKM matrix elements. [Let us note that anomalous Abelian factors are widely used in the literature for this purpose [20].]
transformation properties of ‘matter’ superfields under $U(1)$ are presented in Table (1). The relevant couplings are:

\[
\begin{align*}
20\Sigma 20 + 20 & \left( 15_3 + \frac{X}{M_P} 15_2 + \frac{X^2}{M_P^2} 15_1 \right) H + \\
+ 20 & \left( \bar{6}_3 + \bar{6}_2 + \frac{X^2}{M_P^2} \bar{6}_1 + (\bar{6}_i \rightarrow \bar{6}'_i) \right) \Sigma \frac{X H^2}{M_P^3} \\
& \left( \begin{array}{ccc}
\bar{6}_1(\bar{6}'_1) & \bar{6}_2(\bar{6}'_2) & \bar{6}_3(\bar{6}'_3) \\
\left( \frac{X}{M_P} \right)^5 & \left( \frac{X}{M_P} \right)^3 & \left( \frac{X}{M_P} \right)^3 \\
\left( \frac{X}{M_P} \right)^4 & \left( \frac{X}{M_P} \right)^2 & \left( \frac{X}{M_P} \right)^2 \\
\left( \frac{X}{M_P} \right)^3 & \frac{X}{M_P} & \frac{X}{M_P} \\
\end{array} \right) \frac{\bar{H} H}{M_P} (1 + \Sigma \frac{M_P}{M_P}) \bar{H},
\end{align*}
\]

(29)

\[
\begin{align*}
21_1 \left( \begin{array}{ccc}
\bar{6}_1(\bar{6}'_1) & \bar{6}_2(\bar{6}'_2) & \bar{6}_3(\bar{6}'_3) \\
\left( \frac{X}{M_P} \right)^5 & \left( \frac{X}{M_P} \right)^3 & \left( \frac{X}{M_P} \right)^3 \\
\left( \frac{X}{M_P} \right)^4 & \left( \frac{X}{M_P} \right)^2 & \left( \frac{X}{M_P} \right)^2 \\
\left( \frac{X}{M_P} \right)^3 & \frac{X}{M_P} & \frac{X}{M_P} \\
\end{array} \right) \frac{\bar{H} H}{M_P} (1 + \Sigma \frac{M_P}{M_P}) \bar{H},
\end{align*}
\]

(30)

\[
\begin{align*}
21_2 \left( \begin{array}{ccc}
\frac{1}{X M_p} & 0 & 0 \\
1 & 0 & 0 \\
\end{array} \right) \Sigma, \\
\bar{21}_1 \left( \begin{array}{ccc}
\frac{1}{X M_p} & 0 & 0 \\
\frac{1}{X M_p} & 1 & 0 \\
\end{array} \right) M_S.
\end{align*}
\]

(32)

Without loss of generality one can choose a basis in which (29) has the form

\[
20\Sigma 20 + 2015_3 H + \frac{X}{M_P^3} 20\bar{6}_3 \Sigma H^2.
\]

(33)

Through this redefinition the hierarchical structures of (30)-(32) will not be changed. Assuming that $\langle \Sigma \rangle \gg M_S$ and taking into account (32), (33), we see that $10_{15_3}$ (after
forming a massive state with $\mathbf{10}_{20}$ decouples. Also the states $q_{15,2} \tilde{\mathbf{15}}_{4,2}$ decouple. The appropriate ‘light’ chiral fragments reside as follows:

$$
\begin{align*}
15_1 & \supset q_1, & 15_2 & \supset q_2, & 20 & \supset (q, u^c, e^c)_3 \\
15_1 \supset M^{(1)} q_1, & & 15_2 \supset M^{(2)} q_2, \\
15_1 \supset (u^c, e^c)_1, & & 15_2 \supset (u^c, e^c)_2,
\end{align*}
$$

where $M^{(1,2)}$ are the eigenvalues of the second matrix in (32).

Let us begin with the masses of charged leptons and down quarks. From (30), three $\bar{6}''_\alpha$ from $\bar{6}_\alpha'$ couple with $\bar{5}_\alpha$ states from $15_\alpha$. To get realistic pattern of fermion masses, the light $(d^c, l)_\alpha$ states should reside in $\bar{6}_\alpha$. On the other hand, from (31) two states of $(d^c, l)_6$ can form massive states with $(d^c, l)_{21,1,2}$, and decouple. This would mean that the light $(d^c, l)_{1,2}$ states reside in $\mathbf{21}_{1,2}$. To avoid this, we introduce two pairs of superfields $[\chi(\bar{6}) + \chi(6)]_{1,2}$ with $U(1)$ charges:

$$
\begin{align*}
Q_{\chi_1} &= 2 + r + r', & Q_{\chi_1} &= -2 - 2r, \\
Q_{\chi_2} &= 1 + r + r', & Q_{\chi_2} &= -1 - 2r.
\end{align*}
$$

Then, from the couplings

$$
W_{21,1} = 21 \chi_i \tilde{H} + 2 \mathbf{15}_1 \chi_i H ,
$$

the states $(\bar{5} + 5)_{1,2}$ (from $(\mathbf{21} + 21)_{1,2}$) became super heavy by coupling with $(5_\chi + \bar{5}_\chi)_{1,2}$ respectively. Therefore, the 21-plets affect only the appropriate $q$ states, and we have

$$
\bar{6}_\alpha \supset (d^c, l)_\alpha.
$$

The couplings (30), (31) are responsible for the generation of charged lepton and down quark masses of the light generations respectively. If $\Sigma$ in (30) is not used, the charged leptons (of light generations) will remain massless since the $e^c$ states will have couplings with superpositions (carrying quantum numbers of $l$) which are already decoupled together with $\bar{l}_{15}$ states. Thus, $\Sigma$ in (30) is crucial, and $h_d$ should be extracted from it. Taking into account all this, from (30), (31) and the last term in (33), for the mass matrices of charged leptons and down quarks we obtain:

\[Without loss of generality we can assume that the light $(d^c, l)_\alpha$ states reside in $\bar{6}_\alpha$, while the fragments from $\bar{6}_\alpha'$ are superheavy.\]
\[
\begin{pmatrix}
  l_1 & l_2 & l_3 \\
l_2 & e^3 & e^3 \\
l_3 & 0 & 0 & 1
\end{pmatrix} \epsilon^3 h_d, \quad (38)
\]

\[
\begin{pmatrix}
  d_1^c & d_2^c & d_3^c \\
  e^5 & e^3 & e^3 \\
  0 & 0 & 1
\end{pmatrix} \epsilon^3 h_d. \quad (39)
\]

Upon diagonalization, we find

\[
\lambda_\tau \sim \epsilon^3, \quad \lambda_e : \lambda_\mu : \lambda_\tau \sim \epsilon^5 : \epsilon^2 : 1, 
\]

\[
\lambda_b \sim \epsilon^3, \quad \lambda_d : \lambda_s : \lambda_b \sim \epsilon^5 : \epsilon^2 : 1. 
\]

Taking into account that the \(\tau\) lepton and \(b\) quark Yukawas are generated from the last term of (33) we will have

\[
\lambda_b = \lambda_\tau \sim \epsilon^3 \simeq 10^{-2}, \quad (40)
\]

which means that the MSSM parameter \(\tan \beta\) is of order unity. This regime is also preferable for the PGB scenario [21]. As far as the light generations are concerned, due to fact that \(e^e\) and \(q\) states come from different \((15\) and \(21\) respectively\) multiplets, the unwanted asymptotic relations \(\lambda_e = \lambda_d\), \(\lambda_\mu = \lambda_s\) are avoided.

Turning to the up quark sector, from the first term in (33) and taking into account (34), we have \(\lambda_t \sim 1\). The \(u\) and \(c\) quark masses are generated through the operators:

\[
\frac{\Sigma^2}{M_1} 211151 \left( \frac{X}{M_P} \right)^4 \frac{H^2}{M_P} + \frac{\Sigma^2}{M_2} 2112152 \left( \frac{X}{M_P} \right)^2 \frac{H^2}{M_P}, \quad (43)
\]

from which, taking into account (33) and assuming \(M_1 \simeq M_G\), \(M_2 \simeq M_G\epsilon\), one obtains \(\lambda_u \sim \epsilon^6\), \(\lambda_c \sim \epsilon^3\). The operators in (43) can be obtained through the exchange of pairs of \([\Psi(15) + \bar{\Psi}(15)]_{1,2}\), with masses \(M_1, M_2\) respectively, and with \(U(1)\) charges

\[
Q_{\Psi_1} = -Q_{\bar{\Psi}_1} = 2 - r, \quad Q_{\Psi_2} = -Q_{\bar{\Psi}_2} = 1 - r. 
\]

The relevant couplings are
For $M_1 \sim M_G$, $M_2 \sim \epsilon M_G$, $M' \ll \epsilon M_G$ integration of $\Psi + \bar{\Psi}$ states (and first term of (33)) yields the following mass matrix

\[
\begin{pmatrix}
\psi_1 \\
\psi_2 \\
(\frac{x}{M_P})^2 \\
1
\end{pmatrix}
\begin{pmatrix}
\frac{x}{M_P} \\
1
\end{pmatrix}
\begin{pmatrix}
\psi_1 \\
\psi_2 \\
\Sigma \\
H
\end{pmatrix}
\]

\[
(M_1 M'_X) \begin{pmatrix}
1 \\
0
\end{pmatrix}
\begin{pmatrix}
\psi_1 \\
\psi_2
\end{pmatrix}
\]

from which one can obtain

\[
\lambda_t \sim 1, \quad \lambda_u : \lambda_c : \lambda_t \sim \epsilon^6 : \epsilon^3 : 1.
\]

(48)

From (39), (47) we have for the CKM matrix elements

\[
V_{us} \sim \epsilon, \quad V_{cb} \sim \epsilon^2, \quad V_{ub} \sim \epsilon^3,
\]

(49)

which have the desired magnitudes!

In summary, by suitably implementing a flavor $U(1)$ symmetry we have obtained a natural understanding of hierarchies of Yukawa couplings and mixing angles of charged fermions in the framework of PGB $SU(6)$ scenario.

### 3.3 Unification and Value of $\alpha_s(M_Z)$

In our scenario we have assumed that $\langle \Sigma \rangle \gg M_S$ ($M_S$ is the mass scale of 21-plets). This assumption is important not only for obtaining the desired pattern of fermion masses, it also is crucial for the suppression of nucleon decay. On the other hand, it turns out that the states

\[
2 \cdot [(6, 1) + (1, 3) + (\bar{6}, 1) + (1, 3)]
\]

(50)

(in terms of $SU(3)_c \times SU(2)_W$) will lie below the GUT scale, and if $M_S$ is much lower than $M_G$, this will ruin the unification at $M_G$ of the three gauge couplings. However, the
PGB $SU(6)$ scenario provides an elegant possibility for retaining unification. For this, we introduce two states $20_1, 20_2$, which do not couple with ‘matter’ superfields and appear in the terms:

$$W_{20} = \Sigma 20_2 20_j + M_{20} 20_1 20_2 ,$$

(note that $20_1 20_1$ and $20_2 20_2$ couplings vanish due to $SU(6)$ symmetry).

Having $\langle \Sigma \rangle$ along the direction given in (16), it is easy to verify from the first coupling in (51) that the fragments $(6, 2)_{1,2}$ (in terms of $SU(4)_c \times SU(2)_W$) remain massless, while the remaining components get masses of order $M_G$. From the last coupling in (51) arises the mass term $M_{20} (6, 2)_{1} \cdot (6, 2)_{2}$. In terms of $SU(3)_c \times SU(2)_W$

$$(6, 2) = (3, 2) + (3, 2) \equiv q' + \bar{q}' .$$

Assuming that $M_{20} \simeq M_S$, it is easy to see that the states $(q' + \bar{q}')_{1,2}$ from $20_{1,2}$, together with fragments in (50), constitute two complete pairs of $SU(5)_{15} + \overline{15}$, and therefore the picture of unification will not be altered in the one loop approximation. As we will see in next section, for an acceptable suppression of nucleon decay it is enough to have $M_S / M_G \sim 10^{-2}$. A factor $10^{-2}$ for the suppression of $(6, 2)_{1,2}$-plets masses relative to $M_G$ is natural in the sense that without the last term (51), they gain mass ($\sim 10^{-2} M_G$) through the coupling $\Sigma 20_1 20_2$.

It is worth pointing out that this scenario even allows the possibility of obtaining a reduced value (compared to $SU(5)$) for $\alpha_s(M_Z)$. Note that for pairs in (54) we have two mass eigenvalues $M_S^{(1,2)}$ (emerging from diagonalization of the second matrix in (52)). Assuming that $M_S^{(1)} \simeq M_{20}, M_S^{(2)} \simeq M_{20}/2$, one pair of $(6, 1) + (6, 1)$ and $(1, 3) + (1, 3)$ will lie below the $M_{20}$ scale. For $\alpha_s(M_Z)$ this gives

$$\alpha_s^{-1} = \left(\alpha_s^{-1}\right)^0 + \frac{3}{2\pi} \ln \frac{M_{20}}{M_S^{(2)}} ,$$

where $\alpha_s^0$ is the value of the strong coupling at $M_Z$ in minimal SUSY $SU(5)$ which is calculated in two loop approximation and includes SUSY and heavy threshold corrections. Taking $\alpha_s^0 = 1/0.126$ from (53), we obtain $\alpha_s \simeq 0.121$, which is in good agreement with the world average value.

### 3.4 Nucleon Decay in PGB $SU(6)$

In this section we will investigate the all important issue of nucleon decay in the PGB $SU(6)$ scenario. It will turn out that the proton lifetime is consistent with the latest
Superkamiokande data, and the dominant decays occur through mixing of the two light families with the third generation. From (31), (33) and also from (34), we observe that the $qBL\bar{T}$ type couplings have the same hierarchical structure as the down quark mass matrix in (39). Taking into account (33), (34) and the assumption $M_S^{(1,2)}/M_G \equiv \eta \sim 10^{-2}$, the integration of $(\psi + \Psi)_{1,2}$ states (see (45), (46)) yields the $qAQ\bar{T}$ type couplings.

\[
\begin{pmatrix}
q_1 \\
q_2 \\
q_3 \\
\end{pmatrix} = 
\begin{pmatrix}
q_1 \\
q_2 \\
q_3 \\
\end{pmatrix} =
\begin{pmatrix}
\eta \epsilon^6 & \eta \epsilon^4 & 0 \\
\eta \epsilon^4 & \eta \epsilon^3 & 0 \\
0 & 0 & 1 \\
\end{pmatrix} T .
\]

Neglecting the third generation for the time being, we see that the nucleon decay amplitude is suppressed by $\eta \sim 10^{-2}$, which would lead to $\tau_p \sim 10^4 \cdot \tau_p^{SU(5)}$ ($\tau_p^{SU(5)}$ denotes the proton lifetime in minimal SUSY $SU(5)$). Since the coupling $q_3q_3\bar{T}$ is not suppressed and there exist mixings between the appropriate light and heavy states, we should investigate proton life time in more detail.

The decay channel involving the neutrino occurs through operator

\[
\mathcal{O} = x \cdot (u_d^{\alpha} (d_{\gamma} \nu_{\beta}) ,
\]

which emerges from the exchange of color triplet higgsinos. Here

\[
x = \alpha \left(- (L_d^{\dagger} B L_e)_{\gamma \beta} (L_u^A L_d^*)_{\delta \rho} \hat{V}_{\delta \rho} \hat{V}_{\rho 1}^{\dagger} + (L_u^A L_d^*)_{\alpha \delta} (L_u^{\dagger} B L_e)_{\delta \beta} \hat{V}_{\delta \gamma} - 
- (L_d^A L_u^*)_{\gamma \delta} (L_d^{\dagger} B L_e)_{\rho \beta} \hat{V}_{\delta \alpha} \hat{V}_{\rho 1}^{\dagger} - (L_d^A L_u^*)_{\alpha \delta} (L_d^{\dagger} B L_e)_{\beta 1} \hat{V}_{\delta \gamma} \right) ,
\]

$\alpha$ is a family independent factor, $\hat{V}$ is the CKM matrix

\[
\hat{V} = L_u^T L_d^* ,
\]

and $L_e$, $L_d$, $L_u$ are unitary matrices which rotate the corresponding left handed states, transforming them from the flavor to the mass eigenstate basis. Substituting (57) in (56), one obtains

\[
x = -2\alpha (L_d^{\dagger} A L_u^*)_{\alpha \gamma} (L_d^{\dagger} B L_e)_{1 \beta} .
\]

Taking into account $A_{33} = \lambda_t$ and $(L_d)_{13} = \hat{V}_{ub}^*$, $(L_d)_{23} = \hat{V}_{cb}^*$ (see forms of (39) and (17)), for $p \rightarrow K \nu_{\mu,\tau}$ decay width we have

\[
\Gamma(p \rightarrow K \nu_{\mu,\tau}) \sim |2\lambda_t \lambda_s | \hat{V}_{ub} | \hat{V}_{cb} e|^2
\]

to be compared with the decay width of the dominant decay mode of minimal SUSY $SU(5)$ [23].
\[ \Gamma(p \to K\nu_\mu)_{SU(5)} \sim [2\lambda_\tau\lambda_s\sin^2\theta]^2, \quad (60) \]

where \( \sin \theta \) is the Cabibbo angle. For \( \epsilon = 0.22, |\hat{V}_{ub}| = 0.0035, |\hat{V}_{cb}| = 0.04 \) (central values of CKM matrix elements [4]), from (59), (60) we have

\[ \frac{\Gamma(p \to K\nu_\mu,\tau)}{\Gamma(p \to K\nu_\mu)_{SU(5)}} \simeq \frac{1}{300}. \quad (61) \]

Therefore, in our model we expect \( \tau_p \sim 10^{2-3} \text{yr} \). The decay modes into the charged leptons are more suppressed.

A crucial role in suppressing \( d = 5 \) nucleon decay is played by the mass scale \( M_S \) of the symmetric 21-plets, whose origin we do not explain. In general, in \( SU(5+N) \) GUTs even the origin of the GUT scale is unknown and only unification of the three gauge couplings give information about its value. As we have seen, the lower masses of additional symmetric plets can lead to greater nucleon stability. The generation of mass scales is beyond the scope of this paper.

Once we have insured that color triplet induced \( d = 5 \) nucleon decay is compatible with experiments, we should check whether the contribution from Planck scale induced operators are significant or not. In \( SU(6) \) scenario \( \frac{1}{M_P}qqql \)-type operators arise from the coupling:

\[ \frac{1}{M_P} \Gamma_{\alpha\beta\gamma\delta} \sum_{21} \frac{1}{M_P} \cdot 21_\alpha \cdot 21_\beta \cdot 21_\gamma \cdot \bar{6}_\delta H, \quad (62) \]

where the \( \Sigma \)s appear due to symmetric 21-plets, and \( \Gamma \) depends on powers of \( X, H, \bar{H} \). For the operator \( q_1q_1q_2l_2 \), which gives the dominant contribution to nucleon decay, \( \Gamma_{1122} = \left( \frac{X}{M_P} \right)^7 \left( \frac{\bar{H}H}{M_P} \right)^2 \). This, together with the suppression factors in (62), gives rise to a suppression \( \epsilon_6^3\epsilon_{12}^4 \), which makes nucleon decay unobservable. Another appropriate operator also can be obtained if instead of 21s we use 15-plets in (62) (this does not require the presence of \( \Sigma \)). However, since \( 15_1, 2 \) contain \( q_{1,2} \) states with weight \( \eta \sim 10^{-2} \), the suppression factor will be same. As far as the ‘right’-handed operators are concerned, as was pointed out in [24], the dominant contribution arises from the combination \( u^c t^c d^c \tau^c \), either through chargino or wino dressings. In our model operator responsible for coupling is

\[ \left( \frac{X}{M_P} \right)^5 \frac{\bar{H}H}{M_P^2} 15_1 \cdot 20 \cdot 20 \cdot \bar{6}_1 H. \quad (63) \]

The diagrams which arise through chargino dressings are irrelevant since there does not exist mixings of \( t^c \) state with the light generations (see [17]). As far as the wino dressing diagrams are concerned, the effective four-fermion \( (u^c d^c)(s\nu_{\mu,\tau}) \) operator will have the coefficient \( \epsilon^8\lambda_\tau V_{cb} \), which provides strong suppression of nucleon decay.
We therefore conclude that due to $U(1)$ symmetry and details of the $SU(6)$ model, Planck scale induced dimension five operators are suppressed well beyond the required level.

### 3.5 Neutrino Oscillations

We now attempt to accommodate the recent solar and atmospheric neutrino data (see [25] and [26] respectively) in the framework of the PGB $SU(6)$ model. The prescription of $U(1)$ charges (see Table (1)), permits us to realize bi-maximal mixings between the active neutrinos, and we will consider this possibility in detail. Our strategy will be to follow the mechanism suggested in [11], in which such a scenario was realized in the framework of $SU(5)$ which also gave a realistic pattern for charged fermion masses and mixings.

The relevant couplings will be those which involve the $(\bar{6} + \bar{6}')_\alpha$-plets. These contain the singlets $(n + n')_\alpha$, and in order for the mechanism for maximal neutrino mixings [27] to work, they should decouple. For this, we introduce the states $(N + N')_\alpha$ with $U(1)$ charges

$$Q_{N_1} = Q_{N'_1} = 2r' - r + 3,$$
$$Q_{N_2} = Q_{N'_2} = Q_{N_3} = Q_{N'_3} = 2r' - r + 1. \quad (64)$$

Through the couplings

$$\bar{6}_\alpha N_\alpha H + \bar{6}'_\alpha N'_\alpha H, \quad (65)$$

the states $(n + n')_\alpha$ form massive states with $(N + N')_\alpha$ respectively and will decouple at low energies.

Since we have assumed that the light $\nu_\alpha$ states reside in $\bar{6}_\alpha$ (see section 3.2), we will only consider terms involving them. Because the $U(1)$ charges of $\bar{6}_2$ and $\bar{6}_3$ are the same, one can expect large $\nu_\mu - \nu_\tau$ mixings, which is desirable for an explanation of the atmospheric anomaly. The $U(1)$ charge of $\bar{6}_1$ state differs from those of $\bar{6}_{2,3}$. This opens up the possibility of obtaining maximal $\nu_e - \nu_{\mu,\tau}$ oscillations. To implement all of this, we introduce three right handed states $N_{1,2,3}$ with the following $U(1)$ transformation properties:

$$Q_{N_1} = -Q_{N_2} = -r - 2, \quad Q_{N_3} = -r, \quad (66)$$

and taking $r' = 0$ in Table (1), the relevant couplings are
\[
\begin{pmatrix}
N_1 & N_2 & N_3 \\
\frac{X}{M_P} & 1 & a \left( \frac{X}{M_P} \right)^2 \\
\frac{X}{M_P}^2 & 0 & b \\
\frac{X}{M_P}^2 & 0 & c
\end{pmatrix}
\left( \frac{XH}{M_P} (1 + \Psi/M_P) \right)
\]

\[
\begin{pmatrix}
N_1 & N_2 & N_3 \\
M' \left( \frac{X}{M_P} \right)^2 & M' & M'' \left( \frac{X}{M_P} \right)^2 \\
M' & 0 & 0 \\
M'' \left( \frac{X}{M_P} \right)^2 & 0 & M
\end{pmatrix}
\left( \frac{H H}{M_P} \right)^2
\]

where \( a, b, c \) are dimensionless couplings of order unity. If for the mass scales in (67) we take \( M' \sim M'' \simeq 3 \cdot 10^{16} \) GeV, \( M \simeq 10^{15} \) GeV, integration of \( N \) states leads to the following mass matrix for the ‘light’ neutrinos

\[
\hat{m}_\nu = \begin{pmatrix}
\alpha^2 & \alpha b c & \alpha c^2 \\
\alpha b c & b^2 & b c \\
\alpha c^2 & b c & c^2
\end{pmatrix} m + \begin{pmatrix}
\epsilon^2 & 1 & 1 \\
1 & 0 & 0 \\
1 & 0 & 0
\end{pmatrix} m'
\]

where

\[
m \equiv \frac{\epsilon^2 h_u^2}{M'} \simeq 4 \cdot 10^{-5} \text{ eV} , \quad m' \equiv \frac{h_u^2}{M} \simeq 3 \cdot 10^{-2} \text{ eV} .
\]

From (68), (69) one can verify that the ‘heaviest’ state has mass

\[
m_{\nu_3} \simeq (b^2 + c^2)m \simeq 3 \cdot 10^{-2} \text{ eV} ,
\]

while the two ‘light’ states are nearly degenerate

\[
m_{\nu_1} \simeq m_{\nu_2} \simeq m' \simeq 4 \cdot 10^{-5} \text{ eV} .
\]

Consequently, for the solar and atmospheric neutrino oscillation parameters respectively, we find (taking into account that \( b \sim c \)):

\[
\Delta m_{21}^2 \sim 2m^2 \epsilon^2 \simeq 10^{-10} \text{ eV}^2 ,
\]

\[
\sin^2 2\theta_{e2,3} = 1 - O(\epsilon^2) ,
\]

and

\[16\]
\[ \Delta m_{32}^2 \simeq m_{\nu_3}^2 \simeq m^2 \sim 10^{-3} \text{ eV}^2 , \]
\[ \sin^2 2\theta_{\mu 3} = \frac{4b^2 c^2}{(b^2 + c^2)^2} \sim 1 . \] 
(73)

These suggest the solution of the solar neutrino puzzle through maximal $\nu_e - \nu_{\mu,\tau}$ vacuum oscillations, while the atmospheric neutrino anomaly is explained via large $\nu_{\mu} - \nu_\tau$ mixings.

Although we have concentrated on the bi-maximal scenario, by proper selection of the right handed neutrino content and their mass scales, it is possible to resolve the solar neutrino puzzle through the small angle MSW oscillations. Also, it is possible to introduce a sterile neutrino state (which can be kept light by the $U(1)$ symmetry [28, 4, 5]) and realize different oscillation scenarios for a simultaneous explanation of the solar and atmospheric neutrino data. For more details about all this we refer the reader to [11], where these issues are discussed.

4 Conclusions

In this paper we have proposed a rather general mechanism for suppressing nucleon decay in SUSY GUTs which embed the MSSM gauge group in $SU(5+N)$. This mechanism can be a powerful tool for realistic model building. One particularly interesting transparent example is the pseudo-Goldstone $SU(6)$ model, which we have investigated in detail. We have observed that, by supplementing $SU(6)$ with an anomalous $U(1)$ flavor symmetry, we can obtain a more stable nucleon, provide an ‘all-order’ resolution of the gauge hierarchy problem, a natural explanation of the charged fermion mass hierarchies and mixings, and obtain an acceptable value for the strong coupling $\alpha_s(M_Z)$. For solving the solar and atmospheric neutrino puzzles, we have presented the ‘bi-maximal’ neutrino mixing scenario, which nicely blends with the pattern of charged fermion masses and mixings.

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