The upper limit of the $e^+e^-$ partial width of $X(3872)$

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The $e^+e^-$ decay partial width of the recently observed state, $X(3872)$, is estimated using the ISR data collected at $\sqrt{s} = 4.03$ GeV in $e^+e^-$ annihilation experiment by BES at BEPC. It is found that $\Gamma_{e^+e^-} - B_{e^+e^-, J/\psi} < 10$ eV at 90% C. L. if the $J^{PC}$ of $X(3872)$ is $1^{--}$. Together with the potential models and other information, we conclude that $X(3872)$ is very unlikely to be a vector state.

1. Introduction

Belle recently reported a new state at 3872 MeV (denoted as $X(3872)$) in $\pi^+\pi^-J/\psi$ invariant mass spectrum in $B^\pm \to K^\pm(\pi^+\pi^-J/\psi)$, besides the huge signal of $\psi(2S)$ at 3686 MeV [1]. This was soon confirmed by CDF in the inclusive mass spectrum of $\pi^+\pi^-J/\psi$ in $p\bar{p}$ experiment at Tevatron [2].

The small width and the mass very close to the $DD^*$ mass threshold are of great interest and there have been various interpretations of this state, as the $1^3D_2$ state of the charmonium, as the $DD^*$ molecular, as the mixture of the $1^3D_2$ charmonium and $DD^*$ molecular, as the $h'_c(1P_1)$, as the diquark-diquark bound state, or the deuteron and so on [3,4,5,6]. Among these possible interpretations, the $1^3D_2$ state of the charmonium state has gained great weight due to its naturalness, and coincidence with the potential model prediction, and its forbidden decay to $DD^*$ due to parity conservation. The CDF result on production rates of this state and $\psi(2S)$ in $p\bar{p}$ experiment also supports $X(3872)$ being a natural state [4]. However, this interpretation will result in big decay branching fraction to $\gamma\chi_{c1}$, which was found to be in contradiction with the Belle measurement [4].

The possibility of $X(3872)$ being a vector charmonium state is believed to be faint because the typical width of a vector charmonium state at this mass is around a few ten MeV and its decays to charmed mesons will be dominant. However, there is no direct experimental test on this hypothesis. It has also been suggested in Ref. [6] that BES or CLEOc search for this state in $e^+e^-$ annihilation in the vicinity of its mass to rule out this possibility (or very unlikely to establish its $J^{PC}$ as $1^{--}$). While high precision experimental information from BES and CLEOc will certainly improve the situation greatly, the existing experimental result in literature has already given us some information on this, that is, the Initial State Radiation (ISR) events collected at higher energy experiments.

It is of great interest to note that using 22.3 pb$^{-1}$ data at $\sqrt{s} = 4.03$ GeV from BES, through $\pi^+\pi^-J/\psi$ events with $J/\psi$ decays into lepton pairs, an extensive study was made [8], which includes the searching for the possible metastable hybrids ($q\bar{q}g$) produced in $e^+e^-$ annihilation, the searching for the massive charmonium state $\psi(3836)$, the measuring of the $e^+e^-$ partial width of $\psi(2S)$, and so forth. If $X(3872)$ is a $1^{--}$ state, it can be produced in the same final states in this data sample with even larger effective luminosity comparing with $\psi(2S)$, since $X(3872)$ is closer than $\psi(2S)$ to the center of mass energy 4.03 GeV.

In this Letter, the number of detected $X(3872) \to \pi^+\pi^-J/\psi$ events $n^{obs}$ is obtained from the ISR data at $\sqrt{s} = 4.03$ GeV. The production cross section $\sigma^{prod}$ is evaluated theoretically taking into account the ISR and the energy spread of the experiment. Using above two num-
bers, the upper limit of the $e^+e^-$ partial width of $X(3872)$ is obtained with the estimation of the branching fraction of $X(3872) \rightarrow \pi^+\pi^-J/\psi$. At last, possible ways to further refine the result to have a better understanding of the nature of $X(3872)$ state are suggested.

2. Evaluation of the observed number from ISR data

Using ISR data collected by BES detector [9], the final state $\pi^+\pi^-J/\psi$ was studied, where $J/\psi$ resonance is tagged by lepton pairs, either $e^+e^-$ or $\mu^+\mu^-$ [5]. A $J/\psi$ candidate, defined as the dilepton invariant mass between 2.5 and 3.25 GeV, is combined with a pair of oppositely charged tracks, where at least one track should be identified as a pion according to the energy loss ($dE/dx$) in the main drift chamber and the time-of-flight measurements. The difference in invariant mass between $\pi^+\pi^-\ell^+\ell^-$ and $\ell^+\ell^-$ ($\ell = e, \mu$) is shown in Fig. 1 (reproduced from Fig. 1 of Ref. [8]) for the two decay modes. The prominent peaks around 0.6 GeV correspond to $\psi(2S) \rightarrow \pi^+\pi^-J/\psi$, $J/\psi \rightarrow e^+e^-$ and $\mu^+\mu^-$ decays.

For the resonance $X(3872)$, which corresponds to a mass difference from $J/\psi$ of 0.775 GeV, there is no signal in either $e^+e^-$ or $\mu^+\mu^-$ channel, as can be seen in Fig. 1 (the insets are the details of the figure). In the following, we will try to determine the upper limits of the numbers of $X(3872)$ events.

Our fit is performed for both $e^+e^-$ and $\mu^+\mu^-$ modes for the mass differences ranging from 0.65 to 0.9 GeV, with a linear background and a Gaussian smeared Breit-Wigner (BW) for the signal using maximum likelihood method. In the fitting, the resonance mass is fixed at 3.872 GeV according to Ref. [8], and the mass resolution is set to be 9.4 MeV by the measurement at $\psi(2S)$ in Ref. [8]. So far as the total decay width $\Gamma_{tot}$ is concerned, Belle gave a BW width parameter $\Gamma_{tot} = (1.4 \pm 0.7)$ MeV, from which, the upper limit of $\Gamma_{tot} < 2.3$ MeV was inferred at 90% confidence level (C. L.). In our study, the values $\Gamma_{tot} = 2.3$ MeV (the upper limit of the Belle measurement), $\Gamma_{tot} = 1.4$ MeV (the central value of the Belle measurement) and $\Gamma_{tot} = 0.23$ MeV (the typical width of non-$D\bar{D}$ decay charmonium states), are attempted in the following evaluations.

With these parameters, the fits yield no signal events in both $e^+e^-$ and $\mu^+\mu^-$ channels, almost independent on the $\Gamma_{tot}$ used. The upper limits of the numbers of the observed events from $X(3872)$ decays at 90% C. L. are listed in Table I.

| $\Gamma_{tot}$ (MeV) | $e^+e^-$ Mode | $\mu^+\mu^-$ Mode |
|----------------------|----------------|-------------------|
| 2.3                  | 5.98           | 1.92              |
| 1.4                  | 5.91           | 1.90              |
| 0.23                 | 5.81           | 1.86              |

From Table I it can be seen that the effect due to different $\Gamma_{tot}$ is rather small. As a conservative estimation, the largest numbers are used as
the upper limits for the numbers of the observed events, that is, at 90\% C. L.,
\[
n_{\pi^+\pi^-e^+e^-}^{\text{obs}} < n_{e^+e^-}^{\text{up}} = 5.98 ,
\]
and
\[
n_{\pi^+\pi^-\mu^+\mu^-}^{\text{obs}} < n_{\mu^+\mu^-}^{\text{up}} = 1.92 .
\]

3. Theoretical calculation of the production cross section

In $e^+e^-$ annihilation experiment at the center of mass energy $\sqrt{s}$, the cross section of resonance $X(3872)$ at the Born order is expressed by the BW formula
\[
\sigma^{\text{Born}}(s) = \frac{12\pi \Gamma_{e^+e^-} \Gamma_{\text{tot}}}{(s - M^2)^2 + \Gamma_{\text{tot}}^2 M^2} ,
\]
where $M$ and $\Gamma_{\text{tot}}$ are the resonance mass and the total width of $X(3872)$ respectively, and $\Gamma_{e^+e^-}$ is the partial width of $X(3872) \rightarrow e^+e^-$. The production cross section of $X(3872)$ due to ISR from experiment operating at the center of mass energy $\sqrt{s}$ can be calculated by
\[
\sigma^{\text{prod}}(s_0) = \int_{x_{\text{low}}}^{x_{\text{up}}} dx F(x, s_0) \frac{\sigma^{\text{Born}}(s_0(1 - x))}{|1 - \Pi(s_0(1 - x))|^2} ,
\]
where $F(x, s_0)$ has been calculated to an accuracy of 0.1\% [10,11,12], $\Pi(s)$ is the vacuum polarization factor [13], $x_{\text{up}}$ and $x_{\text{low}}$ denote the superior and inferior limits of the integration, which are defined as
\[
x_{\text{up}} = 1 - \frac{s_{\text{low}}}{s_0} ,
\]
and
\[
x_{\text{low}} = 1 - \frac{s_{\text{up}}}{s_0} .
\]
sup and $s_{\text{low}}$ correspond to the fitting range of the experimental data in Fig. [4] that is
\[
\sqrt{s_{\text{up}}} - M_{J/\psi} = 0.9 \text{ GeV} ,
\]
and
\[
\sqrt{s_{\text{low}}} - M_{J/\psi} = 0.65 \text{ GeV} ,
\]
where $M_{J/\psi}$ is the $J/\psi$ resonance mass. In unit of keV,
\[
\Gamma_{e^+e^-} = \alpha \cdot 1 \text{ keV}.
\]
Fix the mass and total width to the values used above (from Belle [1]), the integration gives the production cross section
\[
\sigma^{\text{prod}}([4.03\text{GeV}]^2) = \alpha \cdot 0.61 \text{ nb} .
\]
It should be pointed out that varying $\Gamma_{\text{tot}}$ has little effect on $\sigma^{\text{prod}}$, the integration with different $\Gamma_{\text{tot}}$ actually gives the same value up to the significant digits listed in Eq. [4].

The energy spread effect on cross section is also taken into account. In fact, the energy spread hardly affects the calculated cross section, because the energy spectrum of the ISR photon is already very flat in the expected $X(3872)$ mass region. For example, if the energy spread is 1.5 MeV at 4.03 GeV [14], the difference between the cross sections with and without energy spread is at the level of $10^{-4}$ relatively. So the production cross section given in Eq. [4] without energy spread correction, is accurate enough for our following estimations.

4. Estimation of the $e^+e^-$ partial width

If the number of the produced $X(3872)$ events is denoted as $n^{\text{prod}}$, and the final state $\pi^+\pi^-X^{\ell\ell}$ is used in the experiment observation, the relation between $n^{\text{prod}}$ and $n^{\text{obs}}_{\pi^+\pi^-X^{\ell\ell}}$ is expressed as
\[
\frac{n^{\text{obs}}_{\pi^+\pi^-X^{\ell\ell}}}{n^{\text{prod}}} = \frac{\mathcal{B}^X(3872)_{\pi^+\pi^-J/\psi} \cdot \mathcal{B}^{J/\psi}_{\ell\ell}}{\mathcal{L} \cdot \sigma^{\text{prod}}} ,
\]
where $\mathcal{B}^X(3872)$ is the branching fraction of $X(3872) \rightarrow \pi^+\pi^-J/\psi$, $\mathcal{B}^{J/\psi}_{\ell\ell}$ the branching fraction of $J/\psi \rightarrow \ell^+\ell^-$, and $\varepsilon_{\pi^+\pi^-X^{\ell\ell}}$ the efficiency of detecting $\pi^+\pi^-X^{\ell\ell}$ final state. $n^{\text{prod}}$ can also be expressed by
\[
n^{\text{prod}} = \mathcal{L} \cdot \sigma^{\text{prod}} ,
\]
with $\mathcal{L} = 22.3 \text{ pb}^{-1}$, which is the integrated luminosity of the data taken at 4.03 GeV [8], and $\sigma^{\text{prod}}$ is given in Eq. [4].

With Eqs. [1] and [2], it is obtained
\[
\sigma^{\text{prod}} \cdot \frac{\mathcal{B}^X(3872)_{\pi^+\pi^-J/\psi}}{\mathcal{L} \cdot \mathcal{B}^{J/\psi}_{\ell\ell} \cdot \varepsilon_{\pi^+\pi^-X^{\ell\ell}}} < \frac{n_{\ell\ell}}{\sigma^{\text{prod}}} .
\]
According to PDG [15], $B_{\pi^+\pi^-}^{\rho^0} = (5.93 \pm 0.10)\%$ and $B_{\mu^+\mu^-}^{\rho^0} = (5.88 \pm 0.10)\%$. As an estimation, the efficiency of $\pi^+\pi^-\ell^+\ell^-$ final state from $X(3872)$ decay is treated as the same as that from $\psi(2S)$: $\varepsilon_{\pi^+\pi^-e^+e^-} = (22.9 \pm 0.1)\%$ and $\varepsilon_{\pi^+\pi^-\mu^+\mu^-} = (18.9 \pm 0.1)\%$ [8]. Then the product of $\Gamma_{e^+e^-}$ and $B_{\pi^+\pi^-J/\psi}^{X(3872)}$ is acquired

$$\Gamma_{e^+e^-}B_{\pi^+\pi^-J/\psi}^{X(3872)} < 30 \text{ eV}$$

for $\pi^+\pi^-e^+e^-$ final state, and

$$\Gamma_{e^+e^-}B_{\pi^+\pi^-J/\psi}^{X(3872)} < 10 \text{ eV}$$

for $\pi^+\pi^-\mu^+\mu^-$ final state at 90% C.L.

If we assume that $\Gamma_{\pi^+\pi^-J/\psi}$ of $X(3872)$ is about the same as that of $\psi(2S)$ (85.4 keV [10]), then

$$B_{\pi^+\pi^-J/\psi}^{X(3872)} > \frac{85.4 \text{ keV}}{2.3 \text{ MeV}} = 3.7\%$$

so $\Gamma_{e^+e^-} < 0.82 \text{ keV}$ or $\Gamma_{e^+e^-} < 0.28 \text{ keV}$ for $e^+e^-$ or $\mu^+\mu^-$ mode, respectively.

Taking the more stringent ones as the final results, we get

$$\Gamma_{e^+e^-}B_{\pi^+\pi^-J/\psi} < 10 \text{ eV at } 90\% \text{ C. L.}$$

and

$$\Gamma_{e^+e^-} < 0.28 \text{ keV at } 90\% \text{ C. L.}$$

for $X(3872)$ state.

5. Discussion

A charmonium state with quantum number $J^{PC} = 1^{--}$ is either a $^3S_1$ or a $^3D_1$ state.

In charmonium family, $J/\psi$ and $\psi(2S)$ are well established as $^1S_1$ and $^3S_1$ states. If $X(3872)$ is a $^3S_1$ state, the only place to be filled into is $3^3S_1$. But there are some arguments against this assignment: firstly, there is a relation between the $e^+e^-$ decay partial widths of the $^3S_1$ states of $\psi$ and $\Upsilon$, that is $\Gamma_{ee}(\psi, n^3S_1) \approx 4\Gamma_{ee}(\Upsilon, n^3S_1)$, which holds at least for $n = 1$ and $n = 2$. Extrapolate this relation to $n = 3$, and use $\Gamma_{ee}(\Upsilon, 3^3S_1)$ from PDG [15], it is expected $\Gamma_{ee}(\psi, 3^3S_1) \approx 1.8 \text{ keV}$. This contradicts with the upper limit of $\Gamma_{ee}(X(3872)) < 0.28 \text{ keV}$; secondly, $m_{\psi}(2^3S_1) - m_{\psi}(1^3S_1) \approx m_{\Upsilon}(2^3S_1) - m_{\Upsilon}(1^3S_1)$ (it is 589 MeV for $\psi$ and 563 MeV for $\Upsilon$). If the same spacing between $\psi$ and $\Upsilon$ states is extrapolated to $m_{\psi}(3^3S_1) - m_{\psi}(1^3S_1)$, then the mass of $3^3S_1$ state of charmonium is close to 4 GeV, which is usually assigned to $\psi(4030)$.

If $X(3872)$ is $^3D_1$ state, it is known that $\psi(3770)$ is the $n = 1$ candidate with some mixing of $2^3S_1$ state. The $2^3D_1$ state should be weakly coupled to $e^+e^-$, which is in agreement with the experimental limit of $X(3872)$. However, its mass at 3.872 GeV is too low to accommodate with potential model predictions [17].

One more important argument against the assignment of $X(3872)$ as a vector meson is that $1^{--}$ charmonium state above open charm threshold decays into $DD$ copiously, which makes its total width around a few ten MeV, an order of magnitude greater than the upper limit of the $X(3872)$ width.

In conclusion, $X(3872)$ is very unlikely to be a vector state of charmonium.

There are possible experiments which can further check this. BES or CLEOc can perform fine scan in the vicinity of the state to set a more stringent upper limit on the production rate, independent on the $\pi^+\pi^-J/\psi$ decay branching fraction of $X(3872)$; B-factors can study it using ISR events with higher luminosities. Furthermore, the state can be searched in more decay channels in B decays, while HERA and Tevatron experiments may supply more information on the production mechanism. All these will help to finally establish the nature of $X(3872)$ state.

6. Summary

Using the ISR events from BES data at $\sqrt{s} = 4.03 \text{ GeV}$, the product of the $e^+e^-$ partial width and $X(3872) \to \pi^+\pi^-J/\psi$ decay branching fraction is determined to be

$$\Gamma_{e^+e^-}B_{\pi^+\pi^-J/\psi} < 10 \text{ eV at } 90\% \text{ C. L.}$$

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1 This should be an underestimation of the efficiency since the momentum of the pion tracks from $X(3872)$ decays will be more energetic and the detection efficiency will be larger than in $\psi(2S)$ case. This leads to an overestimation of the upper limit of $\Gamma_{e^+e^-}$, so the numbers we obtained will be conservative.
for $X(3872)$ state if its $J^{PC} = 1^{--}$. With a comparison between $\psi$ and $\Upsilon$ families and predictions of potential models, we conclude that $X(3872)$ is very unlikely to be a vector state.

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