A damage constitutive model and its characterization parameters identification for elastic-plastic materials

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Abstract. This paper proposes a simple damage constitutive model for elastic-plastic materials on the basis of the classical elastic-plastic constitutive model and damage mechanics theory. Based on the simple damage constitutive model, ABAQUS built-in Python software was used to develop the virtual experimental platform, and the whole process simulation of the specimens stretching until the fracture was carried out. The correspondence between the damage characterization parameters and the performance indexes such as the fracture strain and the percentage reduction of area was studied. According to the comparison of the virtual/real test results of ZL114A, the situation shows that the simple damage constitutive model can realize the high fidelity simulation of the damage behavior of elastic-plastic materials. The relative accuracy of the important material property parameters such as the ultimate strength, the percentage total elongation at fracture and the percentage reduction of area reaches 5%.

1. Introduction
Historically, the use of structural analysis in aircraft design has been focussed on finite element analysis for the calculation of internal stress distributions and comparing these values with the allowable values of the materials, so as to draw the conclusion that whether the aircraft structure meets the requirement of safety. This stressing approach has no ability to predict damage and failure behavior of structures, and load bearing capacity of the structure can only be obtained through test. Due to the high cost and long cycle of test, and the backward arrangement of many conditions and restrictions, the design defects cannot be exposed in time and the problems are relatively volatile.

The development of progressive damage analysis technology and the improvement of high performance computing capability are expected to fundamentally solve the problem of high fidelity simulation of structural load bearing, so that the designer can master the virtual test evaluation ability in addition to the physical test. However, the theoretical models of classical damage mechanics tend to be ideal [1], and it is difficult to obtain the characterization parameters. In the early stage, there are few applications of theoretical models for complex project, but in a recent period, there are obviously more applications in this area. For example, Lockheed Martin has developed F-35 project based on its special damage analysis tools to effectively reduce test [4]. A user-defined 3D damage model (VUMAT) with solid elements was developed and implemented into the finite element code ABAQUS/Explicit to predict the type and extent of damage through the laminate thickness [7]. In order to assess the seismic vulnerability of historical buildings, the concrete damage plasticity (CDP) material model in ABAQUS was used in numerical analysis [8]. When using ABAQUS for damage analysis, it is necessary to select an available damage model, such as concrete damaged plasticity,
concrete smeared cracking etc. If there is no available damage model in ABAQUS, a user-defined damage model is necessary in analysis. There are also applications of damage mechanics for the design of composite cabin section of reentry aircraft [7].

From the traditional check to the high fidelity simulation, the core is to develop a simple and practical material damage constitutive model, and put forward a simple and feasible method to identify the characterization parameters. It is already to implement the monotonic tensile loading test in ABAQUS, but the conventional model in ABAQUS cannot accurately simulate the engineering stress-strain curves and the fracture morphology. The Comparison of the conventional model and the damage model of engineering stress-strain curves and shrinkage of tensile fracture section of specimens is shown in figure 1. The results show that the high fidelity simulation of engineering stress-strain curves, necking phenomenon and fracture morphology is realized based on the damage model.

![Figure 1](image1.png)

**Figure 1.** Comparison of the conventional model and the damage model of engineering stress-strain curves and shrinkage of tensile fracture section of specimens. (a) Comparison of engineering stress-strain curves, (b) The damage model, (c) The conventional model, (e) Real test result and (f) Real test result.

In this paper, the whole process of mechanical failure of elastic-plastic materials is divided into two stages: pre-damage and post-damage. The characterization parameters can be identified only by using the percentage reduction of area in conventional performance parameters. Based on the simple damage constitutive model, the digital virtual test capability of structural strength can be developed, which can realize the high fidelity simulation of physical test, and make the traditional finite element analysis level realize technical span.

### 2. Elastic-plastic simple damage constitutive model

The typical stress-strain curve of elastic-plastic materials (epm) is shown in figure 2. The ultimate stress $\sigma_b$ is defined as the damage cut-off point. Through the damage cut-off point, the whole process of deformation and failure of elastic-plastic materials is divided into two stages: pre-damage stage and post-damage stage. The constitutive model is established according to the different damage behavior in the pre-damage and post-damage stages.

In figure 2, $\sigma_y$ is the static yield strength; $\varepsilon^{ie}$ is the inelastic strain; $\sigma_b$ and $\varepsilon_b^{ie}$ are the ultimate stress and inelastic strain of the damage cut-off point; $\sigma_f$ and $\varepsilon_f^{ie}$ are the fracture stress and inelastic strain of the fracture point; the damage factor variable $d_h$ reflects the softening degree of the material stiffness, and set the cut-off point $d_h = d_0 = 0$, the fracture point $d_h = d_c$, and after fracture $d_h = 1.0$. 

![Figure 2](image2.png)
2.1. Pre-damage constitutive model
The pre-damage stage can be divided into linear elastic section and plastic deformation section according to whether plastic deformation occurs or not. There is no damage in the linear elastic section of materials. After entering the plastic deformation section, materials harden and the stiffness begins to damage. According to the classical elastic-plastic theory, the stress-strain relationship should be expressed in an incremental form [11]:

\[
\{d\sigma\} = [D]\{de\}
\]  

where \([D]\) is the stiffness of plastic hardening deformation.

The hardening law and flow law of elastic-plastic materials are the same, which are described by the following yield surface equation [11]:

\[
\sigma_{eq} - \left[\sigma_s + R(e^p)\right] = 0
\]

where \(\sigma_{eq}\) is the Von-Mises equivalent stress, \(\sigma_s\) is the static yield strength and \(e^p\) is the equivalent plastic strain. Additionally, the terms in the bracket denotes the hardening yield strength and \(R(e^p)\) denotes the hardening function.

According the classical damage mechanics, one can derive the following formula [11]:

\[
[D] = [C](1 - d_h)
\]

where \([C]\) is the linear elastic stiffness of materials; \(d_h\) is the stiffness damage coefficient caused by plastic hardening.

2.2. Post-damage constitutive model
The continuous deformation of the post-damage stage will inevitably lead to further accumulation of damage, resulting in the reduction of unloading stiffness and the occurrence of stiffness softening and loss of load. The damage factor variable \(d_h\) is introduced to reflect the softening degree of the relative ideal plastic state of the material stiffness. According to the hypothesis of Lemaitre equivalent strain, the stress-strain relationship in the post-damage stage can be expressed as follows [2]:

\[
\{d\sigma\} = [C](1 - d_h)\{de^e\}
\]

where \(\{de^e\}\) is the elastic strain increment.

In papers [12] and [13]. The integral expression of damage law under the condition of neglecting elastic deformation under proportional loading is given:
where \( d_0 \) and \( d_c \) are the initial and critical damage values, \( \alpha \) is the index reflecting the damage accumulation. The R in equation (2) and the R in equation (5) are different. In equation (2), \( R(\varepsilon^p) \) is the hardening function and in equation (5), \( R_i \) is the stress triaxiality factor [1]:

\[
R_i = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_m}{\sigma_{eq}}\right)^2
\]

(6)

where \( \sigma_m \) is the hydrostatic pressure, \( \nu \) is Poisson's ratio.

The damage factor variable \( d_h \) in the equation (5) is directly related to the plastic strain, and the problem is small when the degree of stiffness softening is not obvious, but the proportion of the plastic strain in the whole strain is reduced with the increase of the softening degree of the stiffness, and the applicability of the equation (5) will be limited. The concept of inelastic strain is introduced, and the inelastic strain at any point of stress and strain curve is defined as [1]:

\[
\varepsilon^{ne} = \varepsilon - \varepsilon^e
\]

(7)

where \( \varepsilon^e \) is the elastic strain.

Based on the division of the material shown in figure 1, the damage law of equation (5) is further developed. Set the cut-off point \( d_h = d_0 = 0 \), the fracture point \( d_h = d_c \), and after fracture \( d_h = 1.0 \). Assuming that the damage factor varies monotonically and continuously with the inelastic strain in the range of \([0,d_c]\), a general equation of exponential decay type is proposed to describe the variation rule of damage factor with strain in a simpler form:

\[
d_h = R_i \left[ 1 - \exp \left( \frac{\varepsilon^{ne} - \varepsilon^{ne}_b}{\varepsilon^i - \varepsilon^{ne}_b} \right)^k \right] \ln(1-d_c)
\]

(8)

where \( \varepsilon^{ne}_b \) is the inelastic strain of damage cut-off point; \( k \) is the damage shape factor, as shown in figure 3 to control the shape of damage factor curve.

![Figure 3. Shape curve of post-damage factor under different \( k \) value.](image)

In equation (8), \( R_i \), \( \varepsilon^{ne} \) is the process variable, \( \varepsilon^{ne}_b \) can be easily calculated from equation...
(7), according to the ultimate stress (material strength limit $\sigma_b$), which are not characterization parameters. $\varepsilon^{ne}_f$ and $d_c$ are related to the stress and strain when the material fracture, together with the damage shape factor, they are the key parameters used to characterize the fracture behavior of the simple damage constitutive model.

3. Algorithm and program realization of simple damage constitutive model

The pre-damage stage of simple damage constitutive model has no difference from that of the ordinary elastic-plastic material. The elastic modulus and stress-plastic strain data of material can be inputted simply. The core of the algorithm is to establish the linkage mechanism of the change of elastic modulus with post-damage factor. In each step of calculation in the post-damage stage, the post-damage factor is calculated in real-time according to the current value of inelastic strain, and the elastic modulus of the material is adjusted in real-time. The material definition in ABAQUS software can associate elastic modulus with field variables, and provide interface of user subroutine to redefine field variables at material point (USDFLD), which provides a convenient condition for the realization of simple damage constitutive program.

Assuming that the field variable FIELD(1) is used to store $d_h$ and is related to the elastic modulus of the material, the state variable STATEV(1) is used to transfer $d_h$ among the calculation steps. Based on the simple damage constitutive model of this paper, the user subroutine to redefine field variables of ABAQUS is developed. The main steps of the program are as follows:

- Call the internal function GETVRM(‘SINV’,……) to get $\sigma_{eq}$ and $\sigma_m$.
- Call GETVRM(‘E’,……), get the strain component, calculate the equivalent strain $\varepsilon$.
- The inelastic strain is calculated from equation (7).
- Compare $\varepsilon^{ne}$ and $\varepsilon^{ne}_b$ to judge the specific damage of the material.
  - if $\varepsilon^{ne} \leq \varepsilon^{ne}_b$, $d_h = 0$;
  - if $\varepsilon^{ne}_b < \varepsilon^{ne} < \varepsilon^{ne}_f$, calculate $d_h$ according to equation (8);
  - if $\varepsilon^{ne} > \varepsilon^{ne}_f$, $d_h = 1$.
- Compare $d_h$ and STATEV(1), if $d_h >$ STATEV(1), re-assign a value to STATEV(1) with $d_h$.
- Re-assign a value to FIELD(1) with STATEV(1).

Based on the ABAQUS script language python, the relatively simple elastic-plastic constitutive model generation process in the pre-damage stage and the user-defined program generation and compilation process in the post-damage stage are built-in and integrated. Further a special plug-in for damage constitutive creation of elastic-plastic materials is developed. Such a plug-in can help design a damage material model that is as simple and efficient as creating a conventional material model.

4. Identification and verification of characterization parameters for simple damage constitutive model

4.1. Identification method of damage constitutive parameters

If additional performance test requirements are put forward because of the characteristic parameters of damage constitutive model, the engineering application of damage constitutive model will undoubtedly be hindered. According to the classical elastic-plastic and damage mechanics theory, the inelastic strain $\varepsilon^{ne}_f$ of the fracture point can be estimated directly from the percentage reduction of area according to the following equation [14]:

$$\varepsilon^{ne}_f \equiv -\ln(1-\psi)$$ (9)
where $\psi$ is the percentage reduction of area.

The critical damage factor can also be predicted by equation (10) [1]:

$$ d_c \equiv 1 - \frac{\sigma_f}{\sigma_b} $$

(10)

Fracture stress $\sigma_f$ is not a conventional performance parameter, so equation (10) is not the same as equation (9), which has obvious guiding significance for engineering selection of characterization parameters. However, the material damage constitutive model is created by taking the $\varepsilon_{fr}^{ne}$ estimated by equation (9) and $d_c$ selected by a certain range (usually within 0.2-0.5) as the initial condition. Taking the virtual test plug-in of tensile/compressive strength of metal test rod based on ABAQUS software python language, the virtual test of tensile strength of specimens is carried out, and the virtual/real test results are compared. According to the stress/strain curve, the percentage total elongation at fracture, the percentage reduction of area, the appropriate characterization parameters, and a few rendezvous can identify the specific values of the representation parameters that meet the precision requirements.

4.2. Identification and verification of characterization parameters of ZL114A

Taking ZL114A as an example, the damage constitutive parameters of ZL114A are identified based on the methods and tools presented in chapter 4.1, and the damage constitutive parameters of ZL114A are identified: $k = 0.8$, $d_c = 0.05$, $\varepsilon_{fr}^{ne} = 0.09$. The specimen tensile virtual test was carried out according to the identified material damage constitutive model. The virtual results of the percentage reduction of area and the percentage total elongation at fracture were compared with the experimental results as shown in table 1. The comparison of the virtual/real test results of engineering stress-strain curves in specimen tensile is shown in figure 4. The results show that the high fidelity simulation of engineering stress-strain curves is realized based on simple damage constitutive model, and the relative accuracy of the percentage reduction of area and the percentage total elongation at fracture is up to 5%.

| Sample   | Real test | Virtual test | Error/% |
|----------|-----------|--------------|---------|
| Sample 1 | 12.94     | 12.32        | 3.40    |
| Sample 2 | 12.43     |              |         |
| Sample 3 | 12.89     |              |         |
| Average  | 12.75     |              |         |
| Sample 1 | 7.52      | 7.40         | 1.16    |
| Sample 2 | 7.21      |              |         |
| Sample 3 | 7.69      |              |         |
| Average  | 7.47      |              |         |

4.3. Analysis of the influence of the characterization parameters on the fracture behavior

On the basis of the simple damage constitutive identification results, the damage parameters are deflected, and the fracture behavior of the material is further studied. Taking ZL114A as an example, the fracture behavior of material is affected by the characterization parameters. Comparison of tensile behavior of materials with different values of damage characterization parameters can be found in figures 5-7.
Figure 4. Comparison of virtual/real test results of engineering stress-strain curves.

Figure 5. Effect of $\varepsilon_{ef}^*$ value change on tensile behavior of material.

Figure 6. Effect of $d_c$ value change on tensile behavior of material.

Figure 7. Effect of $k$ value change on tensile behavior of material.

The results show that:

- $\varepsilon_{ef}^*$ can affect the macro fracture behavior of ZL114A most significantly, and large $\varepsilon_{ef}^*$ means large percentage total elongation at fracture and large percentage reduction of area.
- The change of $d_c$ has no significant effect on the macro fracture behavior of ZL114A, but it can play a fine-tuning role in the percentage total elongation at fracture.
- The difference of $k$ value in the range of 0.8-1.5 does not lead to significant change in fracture behavior of ZL114A, and has sufficient coverage, which is consistent with the conclusion given in reference [6] that $k = 0.8$ can be used as a typical value.

5. Conclusion
In this paper, the classical theory of elastic-plastic and damage mechanics is developed to meet requirements of virtual test construction for high fidelity simulation of structural strength. A simple damage constitutive model of elastic-plastic materials is proposed to solve the core technical problem of high fidelity simulation of structural load bearing failure.

The intrinsic relationship between damage characterization parameters and the percentage total elongation at fracture and the percentage reduction of area is studied. The identification methods and verification tools for characterization parameters is given. The identification verification of typical materials shows: Based on the identification results of simple damage constitutive model and characterization parameters, the mechanical behavior of specimen deformation up to fracture can be simulated with high fidelity. The relative accuracy of the percentage total elongation at fracture and...
the percentage reduction of area can reach 5%.

The damage parameters were deflected and the influence of the characterization parameters on the fracture mechanical behavior of the materials was studied with ZL114A. The results show that the change of $\varepsilon_{\text{eff}}^e$ has the most significant effect on the macro-fracture behavior of the material. Large $\varepsilon_{\text{eff}}^e$ means large percentage total elongation at fracture and large percentage reduction of area. Changing $d_c$ can play a fine-tuning role in the percentage total elongation at fracture. The difference of $k$ value in the range of 0.8-1.5 does not lead to significant change in fracture behavior of materials.

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