A robust trajectory tracking controller for four-wheel skid-steering mobile robots

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Abstract—A novel dynamic model-based trajectory tracking control law is proposed for a four-wheel differentially driven mobile robot using a backstepping technique that guarantees the Lyapunov stability. The present work improves the work of Caracciolo et al. [1], a dynamic feedback linearization approach, by reducing the number of required assumptions and the number of state terms. We also thoroughly investigate on a gain tuning procedure which is often overlooked for nonlinear controllers. Finally, the performance of the proposed controller is compared with the dynamic feedback linearization approach via simulation results which indicate that our controller is robust even in the presence of measurement noise and control time delay.

I. INTRODUCTION

Skid-steering mobile robots are often used for traversing over uneven terrains because they are mechanically robust due to the reduced number of degrees of freedom and the non-requirement of active steering mechanisms. They steer by creating a differential of the forces generated from the actuators located along the two sides of the longitudinal axis of the robot [2]. This differential of forces generates a non-null lateral velocity causing in turn the effect of side skidding.

The task of following a desired path (or trajectory) by a skid-steering mobile robot involves controlling the amount of the differential of the forces generated from the two sides of the robot and therefore the amount of skidding. However, controlling the amount of skidding is not an easy task. When a skid-steering robot follows a curved path, its heading is not parallel to the tangent of the curved path because it laterally skids. The robot’s instantaneous center of rotation (ICR) is not fixed as in the case of active steering mobile robots with ideal rolling, but it may continuously change, and, in extreme cases, the ICR may be located beyond the dimension of the robot along the longitudinal axis causing the robot’s motion instability.

In the past, there have been several works in estimating the location of the ICR while a four-wheel skid-steering mobile robot [3] or a tracked mobile robot [4] make turns with the purpose to improve in controlling these types of robots. However, estimating the location of the ICR is not straightforward because it depends on the robot’s instantaneous lateral velocity and its instantaneous angular velocity.

A novel dynamic model-based nonlinear controller in the dynamic feedback linearization paradigm that accounts for the fact that the ICR does not lie along the lateral axis of the robot’s center of mass but at a certain fixed distance from the robot’s center of mass along its longitudinal axis. This notion is translated into an operational nonholonomic constraint and is added to the equations of motion in order to “virtually” impose that the robot’s lateral velocity must be proportional to its angular velocity [1]. However, their controller requires that the longitudinal velocity does not vanish at any time instant in order to have a finite-valued control input signal. This implies that the robot should not have a non-null initial velocity, like the simulation example reported by the authors in [1]. Besides, their control input signal requires the measurement of the acceleration term in addition to the position and the velocity terms.

In the present work, on top of the dynamic modeling that Caracciolo et al. developed in [1], we propose a nonlinear control design that preserves the dynamics of the system which does not require nonzero-velocity constraint. In addition, the acceleration term is not necessary, but only the position and the velocity terms suffice to control the robot. For simulation illustration purposes, we emulate the sensory noise by adding a multi-variate white Gaussian noise to the state vector and show that the proposed controller can robustify the system and track tightly a trajectory with the curvature changing continuously (an eight-shaped Lissajous curve). Then, we further introduce to the noisy system a control time delay and a zero-order-hold to hold the control input signal during a certain period of time. Reported simulation results show that the proposed controller is able to robustly track an eight-shaped Lissajous curve whereas the controller based on the dynamic feedback linearization fails to track the reference trajectory.

Recently, Kozłowski and Paderski proposed a controller to obtain practical stabilization in trajectory tracking. This controller can stabilize the trajectory tracking up to certain bounds of the position and orientation errors [5]. However, high gains are required in order to obtain sufficiently small error tracking, bearing in mind that high gains may excessively amplify the destabilizing effects of measurement noise, control discretization and/or time delay.

The remainder of the paper is organized as follows. In Section II, the dynamic modeling of a four-wheel skid-steering mobile robot is recalled and discussed. In Section III, we present the design of a novel controller and investigate on a gain-tuning procedure. In Section IV, simulation results are reported and discussed. Finally, conclusion remarks and perspectives are given in Section V.
and the sign function, respectively. Besides, \( \dot{x}_i \) and \( \dot{y}_i \), with \( i = 1, \ldots, 4 \), are respectively the longitudinal and the lateral wheel velocities, subject to the following relationships with the linear and angular velocities \( (\dot{x}, \dot{y}, \dot{\theta}) \) expressed in the body frame

\[
\begin{align*}
\dot{x}_1 &= \dot{x}_4 = \dot{x} - t\dot{\theta}, \\
\dot{x}_2 &= \dot{x}_3 = \dot{x} + t\dot{\theta}, \\
\dot{y}_1 &= \dot{y}_2 = \dot{y} + a\dot{\theta}, \\
\dot{y}_3 &= \dot{y}_4 = \dot{y} - b\dot{\theta}.
\end{align*}
\]

The velocity in the body frame is related to the velocity in the inertial frame as follows

\[
\begin{bmatrix}
\dot{X} \\ \dot{Y} \\
\dot{\theta}
\end{bmatrix} = \mathbf{R} \begin{bmatrix}
\dot{x} \\ \dot{y} \\
\dot{\theta}
\end{bmatrix},
\]

with \( \mathbf{R} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \) the rotation matrix.

The location \((x_{ICR}, y_{ICR})\), expressed in the body frame, of the instantaneous center of rotation (ICR) should remain inside the robot’s dimension along the longitudinal direction (i.e., \(-b \leq x_{ICR} \leq a\)) at any time instant in order to ensure the robot’s motion stability. If the ICR goes outside the robot’s dimension along the longitudinal direction, then all the resistive lateral forces \(F_{yi}^i\), with \( i = 1, \ldots, 4 \), will have the same sign, and, consequently, there is no way to balance the amount of skidding with the wheel actuators, causing the loss of controllability of the mobile robot [6].

If the location of the ICR is known, then a controller may be designed to track a reference trajectory while ensuring the constraint \(-b \leq x_{ICR} \leq a\) as to avoid instability. However, it is not easy to design such a controller due to the fact that the longitudinal coordinate of ICR, \(x_{ICR} = -\dot{y}/\dot{\theta}\) (see [1]), is a function of the vehicle’s state. A practical solution has been proposed by Caracciolo et al. [1] by imposing a “virtual” constraint \(x_{ICR} = d_0\), with \(0 < d_0 < a\). This yields the following nonholonomic constraint

\[
\dot{y} + d_0 \dot{\theta} = 0,
\]

which implies that the lateral speed and the angular velocity should have a fixed relationship by the constant distance \(d_0\). This “unnatural” constraint is not always satisfied in reality, and a controller should be designed to closely maintain this relationship. Inspired by [1], the control design in the next section is based on the following augmented model (instead of (1))

\[
\mathbf{M}\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{E}(\mathbf{q})\tau + \mathbf{A}(\mathbf{q})^T\lambda,
\]

where \(\lambda\) is a vector of Lagrangian multipliers representing the constrained forces, while the matrix \(\mathbf{A}\) holds the following relationship

\[
\begin{bmatrix}
-\sin \theta & \cos \theta & -d_0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{X} \\ \dot{Y} \\
\dot{\theta}
\end{bmatrix} = \mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = 0.
\]

The admissible generalized velocities \(\dot{\mathbf{q}}\) can be defined as

\[
\dot{\mathbf{q}} = \mathbf{N}(\mathbf{q})\eta,
\]
where $\eta \in \mathbb{R}^2$ is a pseudo-velocity, and the columns of the matrix $N$ are in the null space of $A$, e.g.,

$$N(q) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \\ 0 & -\frac{1}{d_0} \end{bmatrix}.$$  

By differentiating (5) and eliminating $\lambda$ from (4) one obtains

$$\begin{cases} \dot{q} = N\eta, \\
N^T MN \dot{\eta} = N^T (E \tau - MN \eta - c). \end{cases} \quad (6)$$

One verifies that the matrices $N^T M N$ and $N^T E$ are invertible. Thus, by making simple change of control variables

$$\tau = (N^T E)^{-1} \left( N^T M N u + N^T M N \eta + N^T c \right). \quad (7)$$

with $u = [u_1 \ u_2]^T$ the vector of new control variables, then system (6) can be rewritten as

$$\begin{cases} \dot{q} = N\eta, \\
\dot{\eta} = u, \end{cases} \quad (8)$$

which is equivalent to

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \\ \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} \cos \theta \eta_1 - \sin \theta \eta_2 \\ \sin \theta \eta_1 + \cos \theta \eta_2 \\ -\frac{1}{d_0} \eta_2 \\ \eta_1 \\ \eta_2 \end{bmatrix}. \quad (9)$$

Th control design proposed in the next section is based on (9).

III. LYAPUNOV-BASED CONTROL DESIGN

A. Control design

Similar to [11], a control point is chosen on the longitudinal body axis at the distance $d_0$ from the origin of the body frame. The vector of coordinates expressed in the inertial frame of this control point is thus given by

$$\xi = \begin{bmatrix} X + d_0 \cos \theta \\ Y + d_0 \sin \theta \end{bmatrix}.$$  

From (9), one verifies that the time-derivative of $\xi$ satisfies

$$\dot{\xi} = \eta_1 \bar{R} e_1, \quad \text{with} \quad e_1 \triangleq [1 \ 0]^T.$$  

Let $\xi_r \in \mathbb{R}^2$ denote the reference position expressed in the inertial frame for the control point defined up to third-order derivative. Define $\xi = \xi - \xi_r$ and $\dot{\xi} = R^T \dot{\xi}$ as the position errors expressed in the inertial frame and body frame, respectively.

It is straightforward to deduce following equations of the error dynamics

$$\begin{cases} \dot{\xi} = -\omega S \xi + \eta_1 e_1 - R^T \dot{\xi}_r, \\
R = \omega R S, \\
\dot{\eta}_1 = u_1, \\
\dot{\omega} = u_2, \end{cases} \quad (10)$$

with $\bar{u}_2 \triangleq -\frac{1}{d_0} u_2$ the new control variable, $\omega \triangleq \dot{\theta}$ and $S \triangleq \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Then, the control objective can be stated as the asymptotical stabilization of $\dot{\xi}$, or equivalently of $\xi$, about zero using $(u_1, \bar{u}_2)$ as control inputs.

The first equation of (10) indicates that the relation $\ddot{\xi} \equiv 0$ implies that

$$\eta_1 e_1 - R^T \dot{\xi}_r \equiv 0. \quad (11)$$

As long as $|\dot{\xi}|$ is different from zero, one can define a locally unique solution of $R$ (or $\theta$) to equation (11). However, this solution cannot be prolonged by continuity at $\dot{\xi} = 0$. This singularity corresponds to the case when the linearization of system (10) at any equilibrium $(\bar{\xi}, R, \omega) = (0, R^*, 0)$ is not controllable. Moreover, one can verify from the application of the Brockett’s theorem [7] for this case that there does not exist any time-invariant $C^1$ feedback control law that asymptotically stabilizes the system at the equilibrium $(\bar{\xi}, R, \omega) = (0, R^*, 0)$. We thus discard this difficult issue in the present paper by making the following reasonable assumption.

Assumption 1. There exists a positive constants $\delta_r$ and $\alpha_r$ such that $|\dot{\xi}_r(t)| \geq \delta_r$ and $|\dot{\xi}_r(t)| \leq \alpha_r$, $\forall t$.

The following result is obtained based on a Lyapunov function constructed using a backstepping procedure.

Proposition 1. Consider the error system (10). Assume that Assumption 1 holds. Let $\eta_{id}$ and $\omega_d$ denote auxiliary control variables derived from the backstepping procedure and defined as

$$\begin{cases} \eta_{id} \triangleq e_1^T R^T \dot{\xi}_r - k_1 \xi_1, \\
\omega_d \triangleq \omega_r - k_2 |\dot{\xi}_r| \xi_2 + k_3 |\dot{\xi}_r| \left( e_1^T R^T \xi_r \right), \end{cases} \quad (12)$$

where $\omega_r \triangleq \frac{-\xi^T S \xi}{|\xi_r|^2}$, $k_1, k_2$ are positive constant gains, and $k_3$ is a positive gain (not necessarily constant) satisfying $\inf_t k_3(t) > 0$. Apply the following control law

$$\begin{cases} u_1 = \dot{\eta}_{id} - k_4 \xi_1 - k_6 (\eta_1 - \eta_{id}), \\
\bar{u}_2 = \omega_d + \frac{k_5 e_1^T R \xi_r}{k_2 |\xi_r|} + \left( \frac{k_5}{2k_2} - k_7 \right) (\omega - \omega_d), \end{cases} \quad (13)$$

where $k_{4,6}$ are positive constant gains and $k_{5,7}$ are positive gains (not necessarily constant) satisfying $\inf_t k_{5,7}(t) > 0$. Then, the following properties hold:

1) There exist only two equilibria $(\bar{\xi}, R, \omega) = (0, R^*, 0)$, with $R^* e_1 = \frac{\xi}{|\xi_r|}$ and $R^T e_1 = \frac{\xi}{|\xi_r|}$.

2) The equilibrium $(\bar{\xi}, R, \omega) = (0, R^*, \omega_r)$ is almost globally asymptotically stable.

Proof. The first property of Proposition 1 can be straightforwardly deduced from (10) and (11). We prove now the second property. Consider the following storage function

$$S \triangleq \frac{1}{2} |\dot{\xi}|^2 + \frac{1}{k_2} \left( 1 - \frac{e_1^T R^T \xi_r}{|\xi_r|} \right), \quad (k_2 > 0) \quad (14)$$
whose time-derivative along the system’s solutions satisfies (using Lemma 5 in [3])

\[ \dot{S} = \xi^T \left( \eta_1 e_1 - R^T \dot{\xi}_r \right) + \frac{1}{k_2} \left( \omega e_1^T S R^T \dot{\xi}_r - e_1^T R^T \frac{d}{dt} \left( \frac{\dot{\xi}_r}{|\dot{\xi}_r|} \right) \right) \]

\[ = \xi_1 \left( \eta_1 - e_1^T R^T \dot{\xi}_r \right) - \frac{e_1^T R^T \dot{\xi}_r}{k_2 |\dot{\xi}_r|} (\omega - \omega_r + k_2 |\dot{\xi}_r| \xi_2), \]

with \( \omega_r \) defined in Proposition 1. Then, using the expressions (12) of the auxiliary control variables \( \eta_{1d} \) and \( \omega_d \) one deduces

\[ \dot{S} = -k_1 \xi_1^2 - \frac{k_3}{k_2} (e_2^T R^T \dot{\xi}_r)^2 + \xi_1 (\eta_1 - \eta_{1d}) - \frac{e_1^T R^T \dot{\xi}_r}{|\dot{\xi}_r|} (\omega - \omega_d). \]

Now, backstepping procedure can be applied to deduce the real control inputs \((u_1, u_2)\). Consider the following Lyapunov candidate function

\[ L \triangleq \frac{1}{2k_2} (\eta_1 - \eta_{1d})^2 + \frac{1}{2k_5} (\omega - \omega_d)^2, \quad (15) \]

with \( S \) defined by (14). From the system (10) and the control expressions (13), one deduces

\[ \dot{L} = \dot{S} + \frac{1}{k_4} (\eta_1 - \eta_{1d}) (u_1 - \eta_{1d}) + \frac{1}{k_5} (\omega - \omega_d) (u_2 - \omega_d) - \frac{k_5}{2k_5} (\omega - \omega_d)^2 \]

\[ = -k_1 \xi_1^2 - \frac{k_3}{k_2} (e_2^T R^T \dot{\xi}_r)^2 - \frac{k_a}{k_4} (\eta_1 - \eta_{1d})^2 - \frac{k_5}{k_5} (\omega - \omega_d)^2 \]

Since \( \dot{L} \) is negative semi-definite, the terms \( \dot{L}, \eta_1 - \eta_{1d} \) and \( \omega - \omega_d \) remain bounded. From the boundedness of the reference acceleration \( \dot{\xi}_r \) (Assumption 1), one can show that \( \dot{L} \) is bounded which implies that \( L \) is uniformly continuous along every system’s solution. Then, by the application of the Barbital’s lemma [9], one can ensure that \( \dot{L} \) converges to zero. Consequently, one can deduce that

\[ \left( \xi_1, e_1^T R^T \dot{\xi}_r, \eta_1 - \eta_{1d}, \omega - \omega_d \right) \to 0. \quad (17) \]

In addition, one needs to make sure that \( \dot{\xi}_2 \) asymptotically converges to zero. If \( u_1 \) and \( \bar{u}_2 \) are defined as (13), then \( \omega_d \) converges to \( \omega_d \) as indicated in (17). Using this fact and the Lemma 5 of [3], one gets

\[ \frac{d}{dt} \left( \frac{e_2^T R^T \dot{\xi}_r}{|\dot{\xi}_r|} \right) \to -e_2^T R^T \dot{\xi}_r (\omega_d - \omega_r). \quad (18) \]

From (17), \( e_2^T R^T \dot{\xi}_r \) converges to zero. Therefore, the \( \omega_d \) given in (12) converges to

\[ \omega_d \to \omega_r - k_2 |\dot{\xi}_r| \xi_2. \quad (19) \]

Using (19) in (18), one gets

\[ \frac{d}{dt} \left( \frac{e_2^T R^T \dot{\xi}_r}{|\dot{\xi}_r|} \right) \to k_2 (e_1^T R^T \dot{\xi}_r) \xi_2. \quad (20) \]

On the other hand, since \( e_1^T R^T \dot{\xi}_r \to 0 \) holds from (17), \( \frac{d}{dt} \left( \frac{e_2^T R^T \dot{\xi}_r}{|\dot{\xi}_r|} \right) \to 0 \) must be true. Using Assumption 1, \( |\dot{\xi}_r| \neq 0 \). Hence, \( \frac{d}{dt} \left( e_2^T R^T \dot{\xi}_r \right) \to 0 \) must also be true. Therefore, \( e_1^T R^T \dot{\xi}_r \to 0 \). But, \( e_1^T R^T \dot{\xi}_r \to 0 \), and \( k_2 > 0 \). Therefore, \( \xi_2 \to 0 \).

\[ \Box \]

B. Gain tuning

Generally, gain tuning for nonlinear control laws is less obvious than for linear control ones. However, we will show how the gains for our proposed controller can be tuned by using existing tuning techniques in linear control theory. A simple way to determine the control gains consists in using the pole placement technique for the linearization of the system (10) at the equilibrium and for a particular reference trajectory such as a straight line or a circle with constant forward speed. In this case, one deduces that

\[ \dot{\xi} \approx \eta_1 e_1 - R^T \dot{\xi}_r = \begin{bmatrix} \eta_1 - e_1^T R^T \dot{\xi}_r \\ -e_2^T R^T \dot{\xi}_r \end{bmatrix}. \quad (21) \]

Then, defining \( \bar{\eta}_1 \triangleq \eta_1 - \eta_{1d} \) and using the definition of \( \eta_{1d} \) given in (12), one obtains

\[ \dot{\xi} = \begin{bmatrix} \xi_1 \\ \bar{\eta}_1 \end{bmatrix} \approx \begin{bmatrix} \bar{\eta}_d + \bar{\eta}_1 - e_1^T R^T \dot{\xi}_r \\ -e_2^T R^T \dot{\xi}_r \end{bmatrix} = \begin{bmatrix} -k_1 \xi_1 + \bar{\eta}_1 \\ -e_2^T R^T \dot{\xi}_r \end{bmatrix}. \quad (22) \]

On the other hand, by differentiating \( \bar{\eta}_1 \) and by using (13), one deduces

\[ \dot{\bar{\eta}}_1 = \bar{\eta}_1 - \bar{\eta}_{1d} = u_1 - \eta_{1d} = -k_4 \xi_1 - k_6 (\eta_1 - \eta_{1d}). \quad (23) \]

One can regroup the expressions for \( \dot{\xi}_1 \) (from (22)) and \( \dot{\bar{\eta}}_1 \) (from (23)) in matrix form as follows

\[ \begin{bmatrix} \dot{\xi}_1 \\ \dot{\bar{\eta}}_1 \end{bmatrix} = \begin{bmatrix} -k_1 & 1 \\ -k_4 & -k_6 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \bar{\eta}_1 \end{bmatrix} = A_1 \begin{bmatrix} \xi_1 \\ \bar{\eta}_1 \end{bmatrix}. \]

Now, the gains \( k_1, k_4 \) and \( k_6 \) can be chosen such that \( A_1 \) is Hurwitz. One verifies that the characteristic polynomial of \( A_1 \) given by

\[ P_1(\lambda) = \lambda^2 + (k_1 + k_6) \lambda + k_1 k_6 + k_4 \]

is Hurwitz if \((k_1 + k_6)k_1 k_6 > k_4\). For instance, given two negative real numbers \( \lambda_1, \lambda_2 < 0 \) and choosing

\[ \begin{cases} k_1 = -\max(\lambda_1, \lambda_2), \\ k_6 = -\lambda_1 - \lambda_2 - k_1, \\ k_4 = (k_1 + \lambda_1)(k_1 + \lambda_2), \end{cases} \]

one ensures the positivity of \( k_1, k_4 \) and \( k_6 \) and that the matrix \( A_1 \) is Hurwitz with two negative real poles \( \lambda_{1,2} < 0 \).

Now, let \( \theta \) be the angle formed between \( R e_1 \) and \( \dot{\xi}_r \) (i.e., \( \cos \theta = (R e_1)^T \dot{\xi}_r / |\dot{\xi}_r| \)). Then, in the first order approximation, one has \( \dot{\theta} \approx -\frac{e_1^T R^T \dot{\xi}_r}{|\dot{\xi}_r|} \). One can easily verifies that

\[ \dot{\theta} \approx \bar{w} - k_2 |\dot{\xi}_r| \xi_2 - k_3 |\dot{\xi}_r|^2 \bar{\theta}, \]
with \( \dot{\omega} \triangleq \omega - \omega_d \). Besides, by differentiating \( \dot{\omega} \) and using (13) one also verifies that

\[
\dot{\omega} = \dot{u}_2 - \omega_d = \frac{k_5}{k_2} \mathbf{e}_2^T \mathbf{R}^T \mathbf{e}_r - k_T \dot{\omega} = -\frac{k_5}{k_2} \theta - k_T \dot{\omega}.
\]

From here, one deduces the following second linearized sub-system in matrix form

\[
\begin{bmatrix}
\frac{\dot{\xi}}{\dot{\theta}} \\
\frac{\dot{\omega}}{\dot{\theta}}
\end{bmatrix} = \begin{bmatrix} 0 & \frac{|\xi_r|}{\xi} & 0 \\
-k_2 |\xi_r| & -k_3 |\xi_r| & 1 \\
0 & -\frac{k_4}{k_2} & -k_5 \end{bmatrix} \begin{bmatrix} \frac{\dot{\xi}}{\dot{\theta}} \\
\frac{\dot{\omega}}{\dot{\theta}}
\end{bmatrix} = \mathbf{A}_2 \begin{bmatrix} \frac{\dot{\xi}}{\dot{\theta}} \\
\frac{\dot{\omega}}{\dot{\theta}}
\end{bmatrix}.
\]

It matters now to choose the gains \( k_2, k_3, k_5 \) and \( k_7 \) such that the characteristic polynomial of \( \mathbf{A}_2 \) given by

\[
P_2(\lambda) = \lambda^3 + \lambda^2 (k_3 |\xi_r|^2 + k_7) + \lambda \left( k_3 k_7 |\xi_r|^2 + \frac{k_5}{k_2} + k_2 |\xi_r|^2 \right) + |\xi_r|^2 k_2 k_7
\]

is Hurwitz. To simplify the task, let us set

\[
k_3 = \frac{k_4}{|\xi_r|}, \quad k_5 = |\xi_r|^2 k_5, \quad k_7 = |\xi_r| k_7,
\]

with \( \kappa_3, \kappa_5, \kappa_7 \) positive constants. Then, the polynomial \( P_2(\lambda) \) can be factorized as

\[
P_2(\lambda) = \lambda^3 + \lambda^2 |\xi_r|^2 (k_3 + \kappa_7) + \lambda |\xi_r|^2 (k_5 + k_2) + |\xi_r|^3 k_2 k_7.
\]

From the above expression of \( P_2(\lambda) \), one may set poles for this characteristic polynomial depending on the norm of the reference velocity as \( \lambda_{1,2,3} = \frac{1}{\xi} |\xi_r|^2 \lambda_{1,2,3} \) with \( \lambda_{1,2,3} \) negative real numbers. This implies the following relations

\[
\begin{aligned}
\kappa_3 &= -\frac{\lambda_1 \lambda_2 \lambda_3}{k_2}, \\
\kappa_5 &= -\frac{\lambda_1 - \lambda_2 - \lambda_3 - \kappa_7}{k_2}.
\end{aligned}
\]

Then, the values of \( \lambda_{1,2,3} \) \((<0)\) and \( k_2 \) \((>0)\) should be chosen such that \( \kappa_3, \kappa_5 \) and \( \kappa_7 \) computed according to (25) are positive. For instance, by setting \( \lambda_{1,2} \) equal, and choosing \( k_2 \) \((>0)\) and \( \lambda_3 \) \((<0)\) such that

\[
\begin{aligned}
k_2 &< \lambda_2, \\
\lambda_3 &> \frac{2k_1 k_2}{(\lambda_1^2 - k_2^2)},
\end{aligned}
\]

one can verify from (25) that \( \kappa_3, \kappa_5 \) and \( \kappa_7 \) are positive.

IV. RESULTS AND DISCUSSION

In this section, the performance of the controller proposed in the present work using a backstepping procedure that guarantees the Lyapunov stability is compared to that of the controller proposed by Caracciolo et al. in [1] using the dynamic feedback linearization approach. The comparison is performed using the MATLAB/SIMULINK. The system (11) is solved using MATLAB ode-solver of type ode5 (Dormand-Prince) with a fixed time step (5ms).

Fig. 2. For a reasonable comparison of the performance of the controller proposed in the present work and the dynamic feedback linearization controller proposed in [1], the gains are independently tuned for a circular trajectory with 5 m radius in order to achieve similar behaviors in terms of the rising time, maximum peak and the decay ratio, for both the error along the X- and Y- directions as shown in (b) and (d).

In the first place, the considered initial conditions are \( x_0 = 8 \text{ m}, y_0 = 5 \text{ m}, \theta_0 = \pi/2 \text{ rad}, \dot{x}_0 = 0.5 \text{ m/s}, \dot{y}_0 = 0.5 \text{ m/s}, \theta_0 = 0.1 \text{ rad/s}. \) Next, the considered robot dimensions correspond to those of an ATRV-2 mobile robot used in [1] with \( m = 116 \text{ kg}, I = 20 \text{ kgm}^2, \alpha = 0.37 \text{ m}, b = 0.55 \text{ m}, t = 0.315 \text{ m}, d_0 = 0.18 \text{ m}, \text{ and } r = 0.2 \text{ m}. \)

For a reasonable comparison between the two controllers, the gains are independently tuned for tracking a circular trajectory of 5 m radius. The criteria for choosing the gains for each controller are such that similar raising time, maximum peak and decay ratio are obtained for both cases while tracking the considered trajectory. For the controller proposed in the present work, the conditions (24) and (25) given in Section III-B must be also satisfied. The resulting gains for the dynamic-feedback-linearization-based controller are \( k_{v_3} = 131, k_{a_1} = 20, k_{p_3} = 325, k_{a_2} = 210, k_{a_1} = 67, \) and \( k_{p_1} = 580. \) Whereas, for the Lyapunov-based controller, the resulting gains are \( k_3 = 3, k_5 = 15.8, \kappa_3 = 7.95, k_4 = 1, \kappa_5 = 0.0005, k_6 = 5, \) and \( \kappa_7 = 4.05. \)

In effect, Fig. 2 shows the choice of such gains makes both controllers track the circular trajectory in a similar fashion. However, in both cases the errors do not converge to zero but they oscillate. Because both controllers are designed for the reduced dynamical system defined in (9), the error asymptotic convergence occurs for this system. Whereas, when these control laws are used in the full dynamical system defined in (1), oscillatory behaviors can be observed from the results, and this phenomenon might be due to the discrepancy that exists between the desired longitudinal component of the instantaneous center of rotation, \( d_0 \), imposed by the operational
nonholonomic constraint defined in (3) and the actual ICR along the robot’s longitudinal axis, \( x_{ICR} \), as the robot tracks the desired trajectory.

Next, these gains are used to compare the performance of the two controllers in tracking an eight-shaped Lissajous-curve trajectory (shown in Fig. 3), a curve characterized by its curvature that continuously changes. The considered Lissajous curve has the following expression

\[
\xi_r = \begin{cases} 
5 \left( 1 + \sin \left( \sqrt{0.41} t \right) \right), \\
5 \left( 1 + \sin \left( \sqrt{0.41} t/2 \right) \right). 
\end{cases} (27)
\]

Further, a multi-variate white Gaussian noise is added to the state vector to emulate the sensor noise and study the robustness of both controllers. The considered noise has the following mean and standard deviation values: \( \mu_x = 0 \) m, \( \mu_y = 0 \) m, \( \mu_\theta = 0 \) rad, \( \mu_{\dot{x}} = 0 \) m/s, \( \mu_{\dot{y}} = 0 \) m/s, \( \mu_{\dot{\theta}} = 0 \) rad/s, \( \sigma_x = 0.02 \) m, \( \sigma_y = 0.02 \) m, \( \sigma_\theta = 0.01 \) rad, \( \sigma_{\dot{x}} = 0.08 \) m/s, \( \sigma_{\dot{y}} = 0.08 \) m/s, and \( \sigma_{\dot{\theta}} = 0.01 \) rad/s.

Fig. 3 shows the results of tracking the Lissajous curve trajectory with emulated sensory noise for both controllers. In both cases, the controllers are able to track the reference trajectory even in the presence of the described noise. Notice that the error is accentuated when tracking the four corners of the Lissajous curve, where the curvature abruptly changes.

Finally, on top of the additive noise, a control time delay is also considered to further study the robustness of the system controlled by each of the considered controllers. A control time delay of 10 ms is introduced along with a zero-order-holder to hold the control input signal for 10 ms. The results shown in Fig. 4(a) and Fig. 4(b) reveal that the controller proposed in the present work is able to track the desired trajectory, whereas this was not the case for the dynamic feedback linearization approach, as the controller was unable to track the desired trajectory (see Fig. 4(c) and Fig. 4(d)).

V. CONCLUSION AND FUTURE WORK

In the present work, we propose a novel trajectory controller for a four-wheel skid-steering mobile robot using a backstepping technique guaranteeing the Lyapunov stability on top of the dynamic model that Caracciolo et al. proposed in [1]. Their feedback-linearization-based controller requires the acceleration state as well as the non-zero velocity constraint at any instant of time, whereas the proposed controller does not require none of these preconditions. Moreover, the proposed controller is robust in tracking trajectories even in the presence of measurement noise and control time delay.

In the near future, we will experimentally validate the performance of the proposed controller. On the other hand, the error dynamics observed from using both controllers show that the error does not asymptotically vanish. We believe that this effect is observed because the equality operational nonholonomic constraint used in the present work overconstrains the instantaneous center of rotation to be at a fixed distance from the robot’s center of gravity along the longitudinal direction. In the future, we will relax this equality constraint into an inequality constraint with the hope to show asymptotic convergence.

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REFERENCES

[1] L. Caracciolo, A. De Luca, and S. Iannitti, “Trajectory tracking control of a four-wheel differentially driven mobile robot,” in Proceedings of IEEE International Conference on Robotics and Automation, 1999.

[2] J. Y. Wong, Theory of ground vehicles, 4th ed. John Wiley & Sons, Inc., 1999.

[3] A. Mandow, J. L. Martinez, J. Morales, and J. L. Blanco, “Experimental kinematics for wheeled skid-steer mobile robots,” in Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, 2007.

[4] J. Martinez, A. Mandow, J. Morales, S. Pedraza, and A. Garcia-Cerezo, “Approximating kinematics for tracked mobile robots,” International Journal of Robotics Research, vol. 24, pp. 867-878, 2005.

[5] K. Kozlowski and P. Pazderski, “Modeling and control of a 4-wheel skid-steering mobile robot,” International Journal of Applied Mathematics and Computer Science, vol. 14, no. 4, pp. 477-96, 2004.

[6] Z. Shiller, W. Serate, and M. Hua, “Trajectory planning of tracked vehicles,” in Proceedings of IEEE International Conference on Robotics and Automation, 1993.

[7] R. W. Brockett, “Asymptotic stability and feedback stabilization,” Differential Geometric Control Theory, pp. 181-191, 1983.

[8] M.-D. Hua, “Contributions au contrôle automatique de véhicules aériens,” Ph.D. dissertation, Université de Nice-Sophia Antipolis, 2009.

[9] H. K. Khalil, Nonlinear systems, 3rd ed. Prentice-Hall, 2002.