Current noise in a irradiated point contact

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(24 January 1999)

We propose a new approach to calculate current and current correlations in a ballistic quantum point contact interacting with a classical field. The approach is based on the concept of scattering states for a time dependent Hamiltonian neglecting electron-electron interaction. Using this approach we calculated the spectra of the current noise in a biased point contact irradiated by a weak random field. For typical radiation frequencies \( \nu \) less than the temperature \( T \) and the bias voltage \( V \) we find a narrow peak of width \( \nu \) on top of a broad background of width \( \max(T,eV) \).

Current fluctuations in ballistic quantum point contacts (PC) attract now much attention, both in theory [1 2] and experiment [3 4], partly because of the assumed possibility to measure fractional charges in shot noise [1 5] and to probe other non Fermi liquid properties [6]. We are interested in current fluctuations in a ballistic PC biased by applied voltage and irradiated by external field. Time averaged current in microstructures under such conditions (phonon-assisted current) was investigated experimentally in point contacts [11] and quantum dots [13 14].

Current in a PC irradiated by a monochromatic a.c. field was discussed in [19 22]. Current fluctuations under such an irradiation were discussed in [24 27].

We consider a classical field which can be coherent (e.g. microwave radiation) or incoherent (e.g. representing the environment at high enough temperature or a heat phonon pulse). We assume that the field does not irradiate the leads between which the bias is applied. [Which means e.g. modulated gate voltage, not modulated bias voltage.] We model this situation considering a 1D channel with a time-dependent barrier potential \( U(x,t) \). The d.c. part of the potential \( U_0(x) \) is due to the squeezing of the PC while the a.c. part \( \delta U(x,t) \) is due to the field.

Consider the 1D Schrödinger equation \( i\hbar \partial / \partial t \psi = H \psi \) for one particle with a time dependent Hamiltonian \( H = -\nabla^2 / 2m + U(x,t) \), where the barrier potential \( U(x,t) = 0 \) at \( x \rightarrow \pm \infty \) for all \( t \). For any energy \( \epsilon_k \equiv k^2 / 2m > 0 \) (with \( k > 0 \)) we define time dependent scattering states \( \chi^\sigma_k(x,t) \), \( \sigma = \pm \), as solutions of the Schrödinger equation with the following boundary conditions. At \( x \rightarrow -\infty \)

\[
\chi^+_k(x,t) = L^{-1/2} \sum_{k'} t_{kk'} e^{-i\epsilon_{k'} t + ik'x},
\]

and at \( x \rightarrow +\infty \)

\[
\chi^-_k(x,t) = L^{-1/2} \sum_{k'} \tilde{t}_{kk'} e^{i\epsilon_{k'} t - ik'x},
\]

where \( L \) is the normalization length. One obtains time independent reflection and transmission coefficients \( t_{kk'} \) and \( \tilde{t}_{kk'} \) for inelastic scattering since \( U(x) = 0 \) for \( x \rightarrow \pm \infty \) and any outgoing wave can be presented as a superposition of time dependent plane waves with time independent coefficients. For any fixed time \( t \), the states \( \chi^\sigma_k(x,t) \) form an orthonormal and complete basis since the states \( \chi^\sigma_k \) can be obtained from a complete set of the functions \( L^{-1/2} e^{\pm i k x} \) by the unitary transformation corresponding to time evolution with Hamiltonian \( H(x,t) \) from \( t = -\infty \). The solutions are labeled according to the energy of the incoming wave. The sums over \( k' \) represent inelastic scattering in transmission and reflection by the a.c. barrier. They are restricted to a \( k' \) interval defined by \( |\epsilon_{k'} - \epsilon_k| \lesssim \nu \), where \( \nu \) is the typical frequency of the barrier variation. For harmonic oscillations of the barrier these scattering states reduce to those used in [23 27].

For a d.c. barrier the reflection and transmission coefficients are diagonal \( t_{kk} = r_k \delta k \) and \( \tilde{t}_{kk'} \) and \( \tilde{t}_{kk'} \) are the time dependent scattering states are reduced to the usual ones, i.e. \( \chi^\sigma_k(x,t) = e^{-i\epsilon_k t} \chi^\sigma_k(x) \).

The many-particle wave functions of the system, neglecting electron-electron interactions are given by Slater determinants of the scattering states \( \chi^\sigma_k(x,t) \). We define creation (annihilation) operators \( a^\dagger_{k\sigma} (a_{k\sigma}) \) for electrons in scattering states \( \chi^\sigma_k(x,t) \). Employing the usual interpretation of the scattering states as describing the transition amplitudes for a wavepacket approaching the interaction zone from the far left side \( \chi^+ \) or far right side \( \chi^- \) to be transmitted or reflected, we may interpret the operators \( a^\dagger_{k\sigma} \) and \( a_{k\sigma} \) as creating electrons in the left and right leads \( a \) and \( b \), respectively. Assuming the electron system in the leads to be modelled by a free Fermi gas in equilibrium at temperature \( T \) and chemical potentials \( \mu_a \) and \( \mu_b \), the average occupation numbers of the states \( \chi^\sigma_k \) are given by \( \langle a^\dagger_{k\sigma} a_{k\sigma} \rangle = \delta_{kk} \delta_{\sigma\sigma} n_{k\sigma} \), with \( n_{k\sigma} \) being the respective Fermi functions. The bias voltage \( V \)
applied to the point contact between the leads a and b is 
\( eV = -(\mu_a - \mu_b) \) with \( e > 0 \).

The time-dependent electron field operator can then be represented in the following way
\[ \Psi(x, t) = \sum_{k\sigma} a_{k\sigma} \chi_{k\sigma}^*(x, t). \]  
(3)

The representation Eq. (3) assumes that the leads are “black bodies” and do not reflect incoming electron waves. Our presentation of the electron field can be considered as a formalization of the wave-packet approach used in [27] and is similar to that used in [3] for a d.c. barrier.

The time-dependent current operator is
\[ j(x, t) = (ie/2m)\nabla \Psi(x, t) + h.c.. \]
For a d.c. barrier one can see from Eq.(6) that the diagonal combinations \( A_{kk}^{\sigma\sigma'} \) and hence the current Eq.(7) do not depend on \( x \) and \( t \).

In what follows we will concentrate on the case when the time dependent part of the barrier \( \delta U(x, t) \) is a stationary random function of \( t \), defined by a correlator \( \delta U(x, t) \delta U(x', t') \), (assuming \( \delta U(x, t) = 0 \), where (...) means statistical averaging.

For a randomly fluctuating barrier the current given by Eq.(6) to be also statistically averaged, giving
\[ \langle j(x, t) \rangle = \sum_{k, \sigma} n_{k\sigma} A_{kk}^{\sigma\sigma}(x, t). \]
(7)

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For a randomly fluctuating barrier the current given by Eq.(6) has to be also statistically averaged, giving
\[ \langle j \rangle = \sum_{k, \sigma} n_{k\sigma} A_{kk}^{\sigma\sigma}. \]
(8)

In the stationary case this current does not depend on \( t \) and hence on \( x \). Note that for an asymmetric barrier and/or asymmetric irradiation \( \langle j \rangle \neq 0 \) for \( V = 0 \), in general.

The current correlator is defined using full averaging
\[ K(1, 2) = \frac{1}{2} \langle j(1) j(2) + j(2) j(1) \rangle - \langle j(1) \rangle \langle j(2) \rangle. \]
(9)

where the short notation means \( 1 \equiv x_1, t_1 \) and \( 2 \equiv x_2, t_2 \). It is convenient to represent \( K = K_q + K_s \), where the first term is the statistically averaged quantum-mechanical correlator
\[ K_q(1, 2) = \frac{1}{2} \langle j(1) j(2) + j(2) j(1) \rangle - \langle j(1) \rangle \langle j(2) \rangle, \]
(10)
while the second term is the statistical correlator of the quantum-mechanical current,
\[ K_s(1, 2) = \langle j(1) \rangle \langle j(2) \rangle - \langle j(1) \rangle \langle j(2) \rangle. \]
(11)

Note that \( K_q = 0 \) if the current is classical, while \( K_s = 0 \) for a d.c. barrier.

Introducing in Eq.(10) and Eq.(11) the current operator from Eq.(6) one finds
\[ K_q(1, 2) = \frac{1}{2} \sum_{kk'\sigma\sigma'} \left[ n_{k\sigma}(1 - n_{k'\sigma'}) A_{kk}^{\sigma\sigma'} A_{kk'}^{\sigma\sigma'}(2) + c.c. \right] \]
\[ K_s(1, 2) = \sum_{kk'\sigma\sigma'} n_{k\sigma} n_{k'\sigma'} \delta A_{kk}^{\sigma\sigma}(1) \delta A_{kk'}^{\sigma\sigma'}(2), \]
(12)

where \( \delta A_{kk}^{\sigma\sigma} = A_{kk}^{\sigma\sigma} - A_{kk}^{\sigma''\sigma''} \). For a stationary random barrier both correlators \( K_q \) and \( K_s \) depend on \( t_1 - t_2 \) and on \( x_1, x_2 \).

We will be interested in low-frequency ”quasistationary” current fluctuations for which the correlator does not depend on \( x_1, x_2 \). To understand when this situation occurs consider first a d.c. barrier. We assume the barrier is “simple”, i.e. its height is of the order of Fermi energy \( \epsilon_F \) and its length \( d \) is of the order of the Fermi wave length \( 2\pi/k_F \) and there are no other energy or length scales, as e.g. in a double barrier potential. In this simple case the energy scale for \( t_k, r_k, \tilde{r}_k \) is \( \epsilon_F \). The scale for \( A_{kk}^{\sigma\sigma'} \) is the same as if these quantities are calculated for \( x \lesssim d \). At \( x \gg d \) a new smaller scale appears. To see it we calculate \( A_{kk}^{\sigma\sigma'} \) using the Eq.(3) for the scattering states, giving at \( x \gg d \), for example
\[ A_{kk'}^{++} = (e/2mL)e^{i(\epsilon_k' - \epsilon_k)t}. \]
(13)

As we will see later the relevant momenta \( k, k' \) correspond to energies \( \epsilon_k, \epsilon_k' \) within the exchange window between the Fermi distributions in both leads \( |\epsilon_k - \epsilon_F| \lesssim \max(eV, T) \) which is assumed to be narrow compared to \( \epsilon_F \). Hence for a simple barrier one can put \( k = k' = k_F \) everywhere except in the exponentials (since \( x \) and \( t \) can be large). As a result the fast oscillating exponentials (of Friedel type) \( e^{\pm i(k'+k)x} \) disappear. The slow oscillating exponentials \( e^{\pm i(k'-k)x} \) introduce a new energy scale \( v_F/x \) which is smaller than \( \epsilon_F \) if \( x \gg d \). This scale corresponds to the inverse time of flight from the barrier to the point where current correlations are measured.

The correlations are quasistationary if \( (k - k')x \ll 1 \). Since the relevant energy transfers are \( \epsilon_k' - \epsilon_k \approx \omega \), where \( \omega \) is the current fluctuation frequency, the relevant
\[ k - k' \simeq \omega / v_F. \] If \( \omega \ll \epsilon_F \) we can satisfy the quasistationarity condition choosing \( x \) in the interval \( d \ll x \ll v_F / \omega \).

As a result we see that if the current correlations at frequencies \( \omega \) are measured not too far from the barrier, at \( x \ll v_F / \omega \), the current fluctuations are quasistationary and are the same in all across sections of the PC.

With these assumptions one can put \( k = k' = k_F \) everywhere except in the time exponentials yielding the much simpler expressions

\[
A_{k}(1)^* A_{k'}(2) = \Omega_{k}(t_1 - t_2) A_{k'}(2)
\]

where \( \Omega_{k}(t) = (ev_F/L)^2 \exp[i(\epsilon_k - \epsilon_k')(t)] \), \( A_{k'}(2) = |t_F|^4 \), \( A_{k'}(2) = |r_F|^2 \) and \( t_F, r_F \) are the transmission and reflection amplitudes \( t_k, r_k \) at \( \epsilon_k = \epsilon_F \). Using this result in Eq. (15) one obtains the current correlator for a static barrier, calculated in \([2]\), which we quote in the time domain for later comparison:

\[
K(t) = (e^2/4\pi^2) F(t) [ |t_F|^2 + |t_F|^2 |r_F|^2 \cos(eVt) - 1],
\]

where

\[
F(t) = 2 \int_0^t \int d\epsilon' n(\epsilon)[1 - n(\epsilon)] \cos[(\epsilon - \epsilon')t].
\]

One can see from Eq. (13) that \( F(t) \) is (up to a factor) the correlator of equilibrium noise in a non biased PC with \( V = 0 \).

In the case of a time-dependent barrier fluctuating with frequencies \( \nu \) the exchange window is max(\( eV, T, \nu \)). In case the barrier fluctuations are slow, i.e. \( \nu \ll \epsilon_F \), the quasistationarity conditions of the current fluctuations are the same as for a d.c. barrier. For fast barrier fluctuations these conditions can not be satisfied.

In what follows we consider the case when the radiation field is weak, which means in our model that the fluctuating part of the barrier is small compared to the d.c. part, i.e. \( \delta U(x, t) \ll \epsilon_F \). In the lowest order the field induced current noise is proportional to \( \delta U^2 \).

The time-dependent scattering waves can be expanded in powers of \( \delta U \) as \( \chi_k(x, t) = e^{-i\omega t} [\chi_k^1(x) + \chi_k^2(x, t) + \chi_k^2(x, t) + ...]. \) Here \( \chi_k^1(x) \) are the scattering states for the Hamiltonian \( H_0 \) with the average barrier \( U_0 \), while \( \chi_k^2(x, t) \) are slow functions of \( t \). These functions contain only outgoing waves and can be calculated in the Born approximation using the retarded Green function given by \([28]\).

\[
G(x, x', t) = \frac{1}{2\pi} \int dk e^{-i\epsilon_k t} \frac{mL}{ikl_k} \chi_k^+(x) \chi_k(x),
\]

where \( x_\sigma \) and \( x_\pi \) are the larger and smaller of \( x \) and \( x' \).

Using the above expansion one can expand the averages entering Eq. (14) and find after lengthy but straightforward calculation the field induced parts of these averages. Having in mind quasistationary frequencies and a simple barrier one obtains using the properties of the scattering states for a d.c. barrier

\[
A_{k}(1)^* A_{k'}(2) = \Omega_{k}(t_1 - t_2) \Psi_{\sigma \sigma}(t_1 - t_2),
\]

\[
\delta A_{k}(1)\delta A_{k'}(2) = (eV/L)^2 \Psi_{\sigma \sigma}(t_1 - t_2),
\]

where

\[
\Psi_{\sigma \sigma}(t) = (L/v_F)^2 \Psi_{\sigma \sigma}(t) + \Psi_{\sigma \sigma}(t) + \Psi_{\sigma \sigma}(t).
\]

and the effective matrix elements are

\[
\delta U_{\sigma \sigma}(t) = \int dx \delta U(x, t) t_F t_F^* \chi_{k}^+(x) \chi_{k}^+(x) - c.c,
\]

\[
\delta U_{\sigma \sigma}(t) = \int dx \delta U(x, t) t_F t_F^* \chi_{k}^+(x) \chi_{k}^+(x) - c.c.
\]

For later use we define \( \Psi(t) = \Psi_{\sigma \sigma}(t) \) and \( \Phi(t) = \Psi_{\sigma \sigma}(t) = \Phi(-t)^* \).

Now we introduce Eqs. (18) into Eqs. (12) and find the field induced correlator,

\[
K_q^{(2)}(t) = \frac{1}{2} \sum_{kk'\sigma\sigma'} \Omega_{kk'}(t) \delta_{kk'}(1 - \epsilon_{k'}) \Psi_{\sigma \sigma'}(t) + c.c.
\]

\[
K_q^{(2)}(t) = (ev_F/L)^2 \sum_{k} \left( \sum_{k' - n} \right)^2 \Psi(t).
\]

To simplify the expressions we replace \( \sum_k \) by \( (L/2\pi v_F) \int dx \) and shift the arguments in the distribution functions. Using Eq. (16) we find

\[
K_q^{(2)}(t) = (e^2/8\pi^2) F(t)(\Phi(t) e^{iVt} + \Psi(t) + c.c.]
\]

and

\[
K_q^{(2)}(t) = (e^2/8\pi^2)(eV)^2 \Psi(t).
\]

It is obvious from the above derivation that the results Eqs. (22), (23) and (15) are valid only for \( t \gg \epsilon_F^{-1} \), i.e. for Fourier components \( \omega \ll \epsilon_F \). The spectra \( \Phi(\omega) \) and \( \Psi(\omega) \) of the functions \( \Phi(t) \) and \( \Psi(t) \) contain only such frequencies, but this is not the case for \( F(t) \) which has a singularity \( t^{-2} \) at \( t \to 0 \). The Fourier transform of this function can be presented as \( F(\omega) = 2|\omega| \left( \mathcal{N}(|\omega|) + \frac{1}{2} \right) \), where \( \mathcal{N}(\omega) = \exp(\omega/T) - 1 \) is the Planck distribution. Because of the zero-point fluctuations \( F(\omega) \) has no natural cutoff below -\( \epsilon_F \). (A cutoff at \( \epsilon_F \) exists because of fast oscillations of the partial currents Eqs. (11) when \( |\epsilon_k - \epsilon_{k'}| \gtrsim \epsilon_F \).)

The zero-point fluctuations disappear if one calculates the "shot noise" contributions, i.e. subtracts from all correlators the values at \( V = 0 \). For a static PC the shot noise is
\[ K_V^{(0)}(t) \equiv K_V^{(0)}(t) - K_V^{(0)}(t)|_{V=0} = (e^2/4\pi^2)|t_F|^2|r_F|^2F(t)\cos eVt - 1, \]

while the shot-noise induced by a time-dependent field is given by

\[ K_V^{(2)}(t) \equiv K^{(2)}_V(t) - K^{(2)}_V(t)|_{V=0} = (e^2/8\pi^2)(F(t)(e^{eVt} - 1)\Phi(t) + (eV)^2\Psi(t) + c.c.). \]

In Eq. (24) the singularity of \( F(t) \) is compensated by the factor \( \cos eVt - 1 \). The same happens in Eq. (25) since \( \Phi(t) \) is real at small \( t \).

The spectra \( S_V^{(2)}(\omega) \) of the correlator \( K_V^{(2)}(t) \) contain two different contributions. One, due to \( K_q \), is a convolution of \( S_V^{(0)}(\omega) \), the shot noise spectrum in a static PC, with \( \Psi(\omega) \). The presence of such a contribution which contains frequencies \( \omega \approx T \pm eV \pm \nu \), is almost obvious. The less obvious result is the second contribution due to \( K_e \), which is proportional to \( \Psi(\omega) \) and contains only frequencies \( \nu \) of the radiation field. This contribution can be easily separated from the first one, since it is temperature independent and proportional to \( V^2 \). This contribution allows for a spectral resolution of a narrow-band radiation by a device having broad band noise.

To understand more about the nature of the field induced current fluctuations consider some specific cases.

First consider a symmetric barrier \( U_0(-x) = U_0(x) \) exposed to symmetric irradiation \( \delta U(-x,t) = \delta U(x,t) \). Using \( \chi_k(x) = \chi_k(-x) \) we find from Eqs. (22), (23)

\[ K_q^{(2)}(t) = (e^2/4\pi^2)(F(t)\Xi(t)). \]

\[ \left( |t_F|^2 - |r_F|^2 \right)^2 \cos eVt + 4|t_F|^2|r_F|^2 \}

\[ K_e^{(2)}(t) = (e^2/4\pi^2)(eV)^2|t_F|^2|r_F|^2\Xi(t). \]

Here \( \Xi(t) = \langle L/v_F^2g^4g^4 \rangle \) with \( g(t) = \int dx\delta U(x,t)x\chi_k^+(x)\chi_k(-x)^* = g(t)^* \). For \( V = 0 \) one finds the total correlator to be

\[ K^{(2)}(t) = (e^2/4\pi^2)F(t)\Xi(t). \]

Taking here \( V = 0 \) we can compare the nonequilibrium noise in a asymmetrically irradiated non biased PC with the nonequilibrium noise in a biased non irradiated PC given by Eq. (24). One can see that “one side excitation” of the PC does not simulate a bias voltage. Even if we choose the excitation to be quasimonochromatic with frequency \( \nu = eV \), in which case \( \Xi(t) \sim \cos eVt \), the nonequilibrium noise due to irradiation contains the field amplitude.

Both considered examples demonstrate that the nonequilibrium noise excited by irradiation differs essentially from nonequilibrium noise excited by bias.

This work was supported by the Alexander von Humboldt Foundation, Israel Academy of Sciences (Y.L), the German-Israeli Foundation and the Deutsche Forschungsgemeinschaft (P.W.)

[1] V.A.Khlus, Sov.Phys.JETP 66 1243 (1987)
[2] G.B.Lesovik, JETP Lett. 49, 592 (1989)
[3] M.Buttkiker, Phys.Rev.Lett. 65, 2001 (1990)
[4] B.Yurke and G.P.Kochanski, Phys.Rev. B41, 8184 (1990)
[5] S.R Eric Yang, Solid State Comm.81, 375 (1992)
[6] M.Buttkiker, Phys.Rev. B46, 12485 (1992)
[7] Th.Martin and R.Landauer Phys.Rev. B45, 1742 (1992)
[8] M.Reznikov at al, Phys.Rev.Lett. 75, 3340 (1995)
[9] A.Kumar et al, Phys.Rev.Lett. 76, 2778 (1996)
[10] H.Birk, M.J.M. de Jong and C.Schönenberger, Phys.Rev.Lett. 75, 1610 (1995)
[11] M.Reznikov et al, Supperlatt. and Microstr. 23, 901 (1998)
[12] Th.Martin, Supperlatt. and Microstr. 23, 859 (1998)
[13] R.A.Wyss et al, Appl.Phys.Lett. 63, 1522 (1993)
[14] T.J.B.M.Janssen et al, J.Phys.: Condens.Matter 6, L163 (1994)
[15] L.P.Kouwenhoven et al, Phys.Rev.Lett. 73, 3443 (1994)
[16] R.H.Blick et al, Appl.Phys.Lett. 67, 3924 (1995)
[17] T.Fujisawa and S.Tarucha, Supperlatt. and Microstr. 21, 247 (1997), Jpn.J.Appl.Phys. 36, 4000 (1997)
[18] T.H.Oosterkamp et al, Phys.Rev.Lett. 78, 1536 (1997)
[19] S.Datta and M.P.Anantram, Phys.Rev. B45, 13761 (1992)
[20] A.Grinwajg et al, Phys.Rev. B52, 12168 (1995)
[21] A.B.Pashkovskii, JETP 82, 959 (1996)
[22] C.S.Chu and C.S.Tang, Solid State Comm. 97, 119 (1996)
[23] A.Levy Yeyati and F.Flores, Phys.Rev. B44,9020 (1991)
[24] D.A.Ivanov and L.S.Levitov, JETP Lett. 58, 461 (1993)
[25] G.B.Lesovik and L.S.Levitov, Phys.Rev.Lett. 72, 538 (1994)
[26] V.S.Borovikov et al, Fiz.Nizk.Temp.23,313 (1997), [see Low.Temp.Phys.23, 230 (1997)]
[27] R.Lanauer, Physica Scripta, T42, 110 (1992)
[28] P.M.Morse and H.Feshbach, Methods of Theoretical Physics, NY,Toronto, London, McGraw-Hill,1953 (chap.7)