A Reliability based Multidisciplinary Design Optimization Method with Multi-Source Uncertainties

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Abstract: The complexity of engineering systems is increasing greatly, and more coupled disciplines and multi-source uncertainties are involved in the design and development of complex engineered systems (CESs). Actually, uncertainties are ubiquitous in design of CESs, which can be mainly classified into aleatory uncertainty (AU) and epistemic uncertainty (EU). To gain high reliability and safety of CESs, the Reliability-based MDO (RBMDO) which considers uncertainties of design variables and parameters has become a hot research topic. In this paper, the quantification of aleatory and epistemic uncertainties based on the probability theory and convex set-theory, multidisciplinary comprehensive reliability evaluation index, adaptive collaborative optimization strategy based on the intelligent optimization algorithm, sequential multidisciplinary reliability analysis method based on the concurrent subspace optimization (CSSO) and performance measure approach (PMA), and hierarchical and hybrid sequential optimization and reliability assessment (HSORA) RBMDO strategy under multi-source uncertainties have been researched. All of the researches can expand and improve the RBMDO theoretical system, and also provide the effective method for the design and optimization of CESs under multi-source uncertainties.

1. Introduction
The early Multidisciplinary Design Optimization (MDO) method does not consider the influence of uncertainties, limiting its application in engineering. Therefore, Reliability-based Multidisciplinary Design Optimization (RBMDO) which improves the design quality of complex engineered systems and considers the influence of uncertainties arised in the late 1990s and got rapid development.

Quantification of uncertainties is the key and foundation of RBMDO. At present, the uncertainty is divided into aleatory uncertainty (AU) and Epistemic Uncertainty (EU) [1, 2]. Researches on quantification of uncertainties began in artificial intelligence and information processing field. Probability theory is the most common method to process quantification of aleatory uncertainties and was a huge success. Later, researchers used convex set-theory [3], possibility theory [4] and evidence theory [5] to quantify the epistemic uncertainties. When multi-source uncertainties exist at the same time, the general method is unified quantification after transforming aleatory uncertainties and epistemic uncertainties from each other [6, 7].

Multidisciplinary reliability analysis is general the integration of traditional reliability analysis method with the optimization strategy of MDO. Padmanabhan et al. [8] proposed MPP-CSSO method
for the multidisciplinary reliability analysis based on Concurrent Subspace Optimization (CSSO). Ahn et al. [9] put forward a serialized multidisciplinary reliability analysis method based on BLISS (Bi-Level Integrated System Synthesis). The method decouples reliability analysis and system analysis that improve the efficiency of reliability analysis. In addition, Koch et al. [10] proposed a probability design optimization method supporting multistage parallel execution on the research of optimization framework of RBMDO, which integrates the existing reliability analysis method into the MDO framework, so as to make the process of multidisciplinary probability design more intuitive and convenient to implement. In order to improve the computation efficiency of the RBMDO

Against the above problems and considering the improvement of RBMDO’s efficiency and accuracy, this paper puts forward theory and method that can process RBMDO with multi-source uncertainties. Paper is organized as follows: Section 1 introduces the modeling of reliability-based multidisciplinary design optimization with multi-source uncertainties, laying a foundation for design optimization. Section 2, 3 respectively introduces adaptive collaborative optimization strategy based on the intelligent algorithm and sequential multidisciplinary reliability analysis method with multi-source uncertainties to improve the efficiency and accuracy of RBMDO. Section 4 is the integration of these methods. Section 5 is the validation of the method combined with algorithms.

2. Modeling of reliability-based multidisciplinary design optimization with multi-source uncertainties

Reliability-based multidisciplinary design optimization require to establish a correct and reasonable mathematical model whose premises is the quantification of multi-source uncertainties and the construction of reliability evaluation index.

2.1. The quantification of multi-source uncertainties based on the probability theory and convex model

The traditional method which quantifies after transforming multi-source uncertainties from each other did not make full use of existing data. Studies have shown that the convex model has characteristics of clear concept, simple model and easy to deal with multivariate. Based on the assorting thought, taking the maximum use of comprehensively describing aleatory and epistemic uncertainties as principle this paper integrates the probability theory and convex model and puts forward a method of the quantification of multi-source uncertainties based on the probability theory and convex model. The specific quantitation process is shown in figure 1.

[Figure1 Flowchart of multi-source uncertainties quantification]

The quantitation method first classify the uncertainties according to the abundant degree of
existing data and characterizes the distribution of the uncertainties using probability theory and convex model theory, then converts aleatory and epistemic uncertainties under x space into standard normal space (u space) and equivalent standard unit ellipsoid based on space transform method so as to realize the comprehensive quantification of aleatory and epistemic uncertainties. This method further determines the probability theory’s core position and leading role in the quantification of multi-source uncertainties, and also stresses the convex model as a special case is a beneficial supplement to deal with uncertainty problems at the same time.

2.2 Establishing the comprehensive evaluation index of reliability
In this paper, reliability-based multidisciplinary comprehensive evaluation index with multi-source uncertainties is defined as: In the standard u space, the shortest distance (lower interval limit) from "the most likely failure surface" to the origin of coordinates is used for assessment of reliability value, denoted by (C for Comprehensive, represents comprehensive reliability index, L represents the lower limit of reliability interval), and the farther the most likely failure surface is from the origin, the smaller the failure probability of the limit state function is. In fact, the solution to the comprehensive reliability evaluation index can be given by formula (1):

\[
\begin{align*}
\text{Find } & \ (u,v) \\
\beta_{c}^L & = \min \sqrt{u^T u} \\
\text{s.t. } & \ G(u,v) = 0 \\
& \ v_i^Tv_i \leq 1 \ (i = 1, 2, \cdots, n)
\end{align*}
\]

Obviously, if more data and information about epistemic uncertainties can be get through a lot of testing or other ways, thus improving people's cognition degree to it, we can get the probability distribution of epistemic uncertainty design variables and parameters. The limit state function becomes one which contains only random variable and comprehensive reliability evaluation index defined by formula (1) is degraded to the traditional probabilistic reliability evaluation index. Therefore, reliability evaluation index proposed by this article is of more general significance.

3. Sequential multidisciplinary reliability analysis method with multi-source uncertainties
Studies have shown that, Multidisciplinary Reliability Analysis (MRA) dominates the computing efficiency of the RBMDO process. Traditional multidisciplinary reliability analysis methods simply integrates single discipline reliability analysis method and the MDO strategy which results in that multidisciplinary reliability analysis with multi-source uncertainties becomes a typical three-layer nested loop optimization problems (including multidisciplinary probabilistic reliability analysis, multidisciplinary non-probabilistic reliability analysis and multidisciplinary analysis), therefore how to improve the computing efficiency is the key to the practical engineering application.

This paper mainly researches from two aspects: including integrating the function measure method and the CSSO strategy in order to provide a more efficient multidisciplinary analysis process. and taking into full consideration to the aleatory and epistemic uncertainties existing in the actual engineering design and propose Sequential Multidisciplinary Reliability Analysis (SMRA) based on the decoupling ideas to improve the efficiency of reliability analysis. The decoupling principle of sequential multidisciplinary reliability analysis method with multi-source uncertainties (MU - SMRA) is shown in figure 2.
4. RBMDO with Multi-Source Uncertainties

The traditional RBMDO with multi-source uncertainties integrates deterministic MDO and reliability analysis directly, forming a four-layer nested loop optimization problem. So the calculation efficiency is low. However, during the execution of SORA, each round of optimization needs to analyze the reconstructed deterministic constraint and assess the function value again, which undoubtedly adds to the burden of multidisciplinary design optimization, resulting in a dropped calculation efficiency, especially when constraints is in great numbered highly nonlinear, the problem is particularly prominent.

This paper proposes a hybrid hierarchical optimization strategy (HSORA) to improve SORA from the following two aspects: (1) The traditional RBMDO can be decoupled based on SORA. Namely, in the upper layer DMDO and MRA are decoupled, forming a single recursive loop optimization process. And the lower layer integrate CO and sequence multidisciplinary reliability analysis method (MU-SMRA) presented above to improve the computation efficiency of local module. (2) Using convex linear approximation technique combined with the sensitivity information and MPP values got by last round. approximate the reconstructed deterministic constraint, which avoids the additionally evaluation and analysis in each round of optimization in DMDO and improve the global computation efficiency of SORA.

Figure 3 is the flowchart of HSORA.

The HSORA can be integrated with other MDO strategies and MRA approach easily, due to its better scalability. The RBMDO with multi-source uncertainties based on HSORA includes the following steps:

Step 1: Set the initial value of design variables: 
\[ d_j^{(0)}, d_i^{(0)}, x_s^{M(0)}, x_i^{M(0)}, w_s^{M(0)}, w_i^{M(0)} \], let k = 1.
Step 2: Perform deterministic MDO. Use GASA-ACO strategy for deterministic MDO (DMDO).

Step 3: Implement multidisciplinary reliability analysis. Due to taking random uncertainty and cognitive uncertainty into account, this step includes multidisciplinary probabilistic reliability analysis and multidisciplinary convex model analysis.

Step 3-1: Multidisciplinary probabilities reliability analysis Based on SMRA. Before analysis, solidify the value of the cognitive uncertainty design variable with the MPP value calculated in the last round. Calculate both the MPPs of all limit state functions \( G^{(l)(k)} \) and the function value \( f(x) \).

Step 3-2: Multidisciplinary reliability analysis with convex model. Before convex analysis, we should take the MPP values calculated by step 3-1 to random design parameter. Meanwhile, MRA with multi-source uncertainty degenerate into calculating the minimum of limit state function within the value scope of cognitive uncertainty parameter \( G_{\text{min}}^{(l)(k)} \). After MCRA, we can acquire the value of cognitive uncertainty design parameter. If \( G_{\text{min}}^{(l)(k)} \geq 0 \), it can be considered satisfies the reliability requirement, and stop the loop; otherwise turn to the next analysis loop.

Step 4: Convergence. The calculation go to step 6 if all the reliability constraints meet the design requirements and the optimizing objective function are converging; otherwise, \( k = k + 1 \), turn to step 5.

Step 5: Based on the shift strategy and MPP information, the reliability constraints of the original optimization problem can be refactored into deterministic constraints. This deterministic constraints will be processed with convex linear approximation to rebuild DMDO model. Repeat steps 2 to 4.

Step 6: End.

5. Illustrate example
The simulation example is the design of aviation gear transmission system supplied by Golinski [11], which is a transmission applied between aircraft engine and the screw propeller to make the engine and propeller function the best way of rotation speed. The aim of optimization design of aviation gear transmission system is to make the weight of aviation gear transmission system is the minimum within the constraints of gear and shaft. The objective function of the original optimization model of aviation gear transmission system is:

\[
\min f(x) = 0.7854x_1x_2^2(3.333x_3^2 + 14.9334x_3) \\
-43.0934 - 1.5079x_1(x_6^2 + x_7^2) \\
+7.4769(x_6^2 + x_7^2) + 0.7854(x_3x_5^2 + x_4x_7^2)
\]  

(2)

The optimization example contains two systems: gear subsystem and bearing subsystem (Figure 4). The design variables \( x_1, x_2, x_3 \) and constraintsg1, g2, g3, g4, g5 belong to the gear subsystem. Meanwhile \( x_4, x_5, x_6, x_7 \) and constraintsg6, g7, g8, g9, g10, g11 belong to bearing subsystem.

\[ Y_{12} = (x_1, x_2, x_3, g_1 - g_5) \]
\[ Y_{21} = (x_4 - x_7, g_6 - g_{11}) \]

Figure 4 structure chart of aviation gear transmission

Of which, the module of gear \( x_2 \) and the teeth number of pinion \( x_3 \) are deterministic design
variables, x1, x3, and x4 are random uncertainty design variables. Among them, x1 and x3 follow Gaussian distributions, x4 follow exponential distribution, x5, x6 and x7 are cognitive uncertainty design variables. Besides that, all probabilistic reliability constraints in the example must meet the given reliability $\Phi(3) = 0.9987$.

The results calculated using different methods are given in table 1 and table 2. n is the number of function iterations in the process of the optimization. As there is no coupling state variables in the project examples, IDF method degenerates into MDF method. Therefore, the same optimization results obtained by SORA-MDF methods and SORA-IDF method can be seen in table 1. It is almost identical to the result comes from HSORA-ACO method while which only needs 1502 times function iterations resulting in maximum efficiency. Assuming all the design variables are deterministic, the volume of aviation gear transmission system is 2995.5 cm³ using GASA-ACO for deterministic optimization.

### Table 1 RBMDO results of aviation gear transmission system (1)

| Optimization strategies | Optimization Results |
|-------------------------|----------------------|
|                         | $x_1$/cm | $x_2$/cm | $x_3$ | $x_4$/cm | $x_5$/cm | $x_6$/cm | $x_7$/cm | $f$/cm³ |
| GASA-ACO                | 3.500     | 0.700     | 17.00 | 7.300     | 7.726     | 3.354     | 5.287     | 2995.5   | 128   |
| SORA-MDF               | 3.577     | 0.700     | 17.00 | 7.300     | 7.913     | 3.427     | 5.363     | 3097.9   | 2635  |
| SORA-IDF               | 3.577     | 0.700     | 17.00 | 7.300     | 7.913     | 3.427     | 5.363     | 3097.9   | 2635  |
| HSORA-ACO              | 3.575     | 0.700     | 17.00 | 7.300     | 7.909     | 3.426     | 5.362     | 3095.9   | 1502  |

### Table 2 RBMDO results of aviation gear transmission system (2)

| Optimization methods | The value of probability constraints in the MPP points |
|----------------------|-----------------------------------------------------|
|                      | $g_1$  | $g_2$  | $g_3$  | $g_4$  | $g_5$  | $g_6$  | $g_7$  | $g_8$  | $g_9$  | $g_{10}$ | $g_{11}$ |
| ACO                  | 0.055  | 0.2199 | -0.009 | 1.226  | 1.305  | 0.750  | 5.560  | -0.102 | -0.022 | 0.056     | -0.023   |
| HSORA-ACO            | 0.082  | 0.291  | 0.055  | 1.672  | 2.604  | 0.110  | 6.341  | 0      | 0      | 0.009     | 0.088    |

If we consider the uncertainties in the design using HSORA-ACO, the volume of aviation gear transmission system is 3095.9 cm³ increasing by 100.4 cm³, and design variables $x_1$, $x_5$, $x_6$, $x_7$ increase by different degrees compared to the corresponding deterministic optimization results. Which is ensuring the reliability at the sacrifice of design cost while taking the design uncertainties into consideration. Besides, the computation efficiency for reliability based design optimization is significantly lower than DMDO due to a large number of reliability constraints are added into the optimization process.

The optimization results in table 2 show that the constraints conditions $g_3$, $g_8$, $g_9$, $g_{10}$, $g_{11}$ are negative calculated by reliability analysis in the MPPs of deterministic optimization, which indicates these four constraints conditions cannot meet the requirement. After Optimized by HSORA-ACO, the value of limit state functions calculated by reliability analysis for each probabilistic constraints condition are positive. This shows that RBMDO has promoted the optimization results to the safety area of constraints, which improve the overall reliability of aviation gear transmission system.

### 6. Conclusions

A hierarchically hybrid sequential optimization and reliability assessment strategy (HSORA) for RBMDO is presented based on decoupling theory, hierarchicy concept and convex linearization approximate technique. The main idea, procedures and implementation steps of HSORA are elaborated. The RBMDO under aleatory uncertainty is researched based on the HSORA. And also, the adopted strategies, implementations, procedures, DMDO and MRA models adopted in RBMDO problems are given. Finally, an example is given to show the effectiveness of the method. It is worth noting that the efficiency of current multidisciplinary reliability analysis methods is still insufficient for engineering application. The method multidisciplinary reliability analysis that have more efficient is worth studying.
Acknowledgments
The authors gratefully acknowledge the fund support from the National Key Research and Development Program of China (2018YFB1700800) and funds of (41401030301).

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