Numerical modelling of the liquid filtering process in a porous environment including the mobile boundary of the “oil-water” section

N Ravshanov¹, E Sh Nazirova¹ and V M Pitolin²

¹Tashkent University of Information Technologies, 108 Amir Temur Ave., Tashkent 100200, Uzbekistan
²Voronezh State Technical University, 14, Moskow ave., Voronezh, 394026, Russian Federation

E-mail: ravshanzade-09@mail.ru

Abstract. The paper considers the process of oil-water filtration in a porous medium. To study, predict the main process indicators and make managerial decisions, a mathematical model has been developed, an effective numerical algorithm based on the ideas of differential-difference schemes and the differential sweep method, with the help of which computer experiments can be carried out. When modelling the object of study, the process of displacement of one liquid by another in a porous medium is considered.

1. Introduction

The study of the problems of unsteady filtration of oil and gas in a porous medium with various physical properties has always attracted the attention of scientists and was of great interest for the design and development of oil and gas fields. One of the main methods for developing oil and gas fields is water flooding to maintain reservoir pressure. At the same time, a significant increase in oil and gas production from the reservoir is achieved. Such problems are of interest as peculiar problems with moving oil and water interfaces in a porous medium. However, the intensification of oil production using conventional water flooding methods is supported by natural factors: oil viscosity, bedding conditions, heterogeneity of oil reservoirs, porosity and reservoir filtration coefficient, etc.

Some oil and gas fields are developed under pressure conditions. When designing and developing such fields under water pressure conditions, when oil and gas are displaced to the well by the pressure of regional waters, it is necessary to take into account the nature of the movement of the water circuit in the reservoir when the reservoir pressure drops. Calculations of the advancement of contour or bottom water in deposits are among the most complex tasks, characterized by a moving interface. The problems of displacing one fluid with another under elastic conditions are close in formulation to the classical Stefan problem of the motion of the phase boundary in the theory of thermal conductivity.

The process of co-filtering two or more immiscible liquids in a porous medium is very complex and is characterized by the following features:

- the coefficients of the equation depend on time and spatial coordinates;
- the pressure values at the interface between the two phases are not known in advance;
• the position of the interface between the two phases is determined in the process of solving.

Oil production occurs in the most difficult conditions, the efficiency of the field’s operation depends on the degree of adequacy of the decisions taken on the design and management. The adequacy of the decisions made depends on the degree of conformity of mathematical models, computational algorithms and software tools for the analysis and forecasting of technological indicators of the development of oil and gas fields to modern requirements.

When developing oil fields in a water-pressure regime, there is a progress in the contour or bottom water. In mathematical terms, such processes are formulated as problems with a moving oil-water interface.

Mathematical models of the process of oil and gas filtration in porous media are based on the use of general laws of mechanics and are reduced to systems of partial differential equations with the corresponding initial, boundary, and internal conditions characterizing the variable state of the system. Their analytical solution, as a rule, is not possible. Therefore, in mathematical modeling of filtration flows of multicomponent mixtures, researchers make various simplifications in the physical formulation of the problem, thereby simplifying the mathematical models of the research object, because of which they move away from the studied object itself.

An analytical review of scientific research and literature on the problems of mass transfer in filtration processes showed that significant theoretical and practical results have been obtained over the past 10 years, and a number of recommendations have been developed.

The process of oil displacement by a water-gas mixture, taking into account the formation of microbubbles due to the use of foaming properties of oil, was considered in [1]. The researchers compared the effectiveness of various technologies for injecting a water-gas mixture and obtained the optimal ratio of water to gas in the mixture, which allows to achieve maximum oil displacement in comparison with standard methods.

M.Yu. Zaslavsky, P.Yu. Tomin considered the development of a mathematical model of the flow of a weakly compressible fluid in a porous medium, constructed by analogy with the quasi-gas dynamic system of equations [2]. The model is generalized for the case of a three-phase fluid and supplemented by the energy conservation equation, which allows it to be used in modeling promising thermal methods of oil production.

Modeling the unsteady flow of a multiphase flow of oil-gas-water in the reservoir and well at the pump is considered in [3]. The authors take the flow in the reservoir as single-phase, and take into account the relative motion of the components of the multiphase medium, heat and mass transfer, as well as the flow regime. The authors provide examples of the use of the model to illustrate the non-stationary effects that occur during oil production.

The work [4] is devoted to two models of filtration of two immiscible incompressible liquids of different densities separated by a free boundary in a porous-elastic space. The paper presents the results of a numerical solution of the problem for viscoelastic filtration using precise microscopic models with a free boundary for pore space structures of various geometries.

The paper [5] considers the possibility of numerically solving the two-dimensional problem of unsteady filtration of an elastic fluid in an inhomogeneous formation. The problem of finding the pressure distribution during the operation of the reservoir is reduced to solving a differential equation of the parabolic type with variable coefficients. The problem is solved approximately using the finite difference method.

From the analysis of published works, it follows that the conditions of uniqueness for the tasks of joint filtration of oil and water in a porous medium are diverse. However, taking into account the progress of the oil-water contact requires introducing the function of the oil pressure or its derivative - insolation with the necessary accuracy and taking into account all factors available for formalization. In this regard, the paper proposes mathematical software (a mathematical model, an effective numerical algorithm) that allows you to determine the main indicators of oil and gas fields, that is, the pressure distribution over time in the reservoir, pressure drop in the well, as well as the advancement...
of the oil-water contact interface taking into account filling pores of gel particles as a result of oil filtration, changing the coefficient of the filtration layer of porous media and other hydrodynamic parameters of the object of study.

2. Problem statement
The presented mathematical model is based on the following assumptions:

- liquids in question are non-biased;
- the movement of fluids in a porous medium is straightforward and in each filtration area obeys the linear Darcy law;
- the conductivity coefficients of the formation in the vertical direction are identical;
- the properties of the liquids of both phases are unchanged in time.

Based on these assumptions, we consider in terms of a problem with a moving oil-water interface. It is assumed here that the formation is characterized by a constant thickness $h$, length $L$, porosity $m$ and initial formation pressure $P_n$.

Under these assumptions, the mathematical model of the problem with a moving oil-water interface is described by a system of differential equations of the parabolic type:

$$
\frac{\partial}{\partial x} \left( k \frac{\partial P_1}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial P_1}{\partial y} \right) = a \beta \frac{\partial P_1}{\partial t} \quad \text{at} \quad (x,y) \in G_1,
$$

$$
\frac{\partial}{\partial x} \left( k \frac{\partial P_2}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial P_2}{\partial y} \right) = (1-a_0) \beta \frac{\partial P_2}{\partial t} \quad \text{at} \quad (x,y) \in G_2.
$$

The system of equations (1) is integrated under the following initial, boundary, and internal conditions:

$$
P_i = P_2 = P_H (x,y) \quad \text{at} \quad t = 0, \quad (x,y) \in G_1 + G_2,
$$

$$
q_{H_{i_q}}(t) = \int_{s_q} k \frac{\partial P_1}{\partial n} ds \quad \text{at} \quad (x,y) \in s_q, \quad i_q = 1,N_q,
$$

$$
q_{B_{i_q}}(t) = \int_{s_q} -k \frac{\partial P_2}{\partial n} ds \quad \text{at} \quad (x,y) \in s_q, \quad i_q = 1,M_q,
$$

$$
\frac{\partial P_2}{\partial n} = 0 \quad \text{at} \quad (x,y) \in \Gamma_2,
$$

$$
P_1(x,y) = P_2(x,y), \quad \frac{k}{\mu_1} \frac{\partial P_1}{\partial n} = \frac{k}{\mu_2} \frac{\partial P_2}{\partial n} \quad \text{at} \quad (x,y) \in \Gamma_1,
$$

$$
\frac{\partial l(x,y,t)}{\partial t} = - \frac{k}{\mu_1 m (a-a_0)} \frac{\partial P_1(x,y,t)}{\partial n},
$$

$$
l(x,y,0) = \phi(x,y) \quad \text{at} \quad (x,y) \in \Gamma_1.
$$

Here $P_1, P_2$ - is the pressure in the area of oil and water availability; $k$ - coefficient of permeability of the reservoir; $\mu_1, \mu_2$ - dynamic viscosity coefficients of oil and water; $\beta_n, \beta_c$ - formation elasticity coefficients in oil and water ($\beta_n = m \beta_{nc} + \beta_c$: $\beta_a = m \beta_{ac} + \beta_c$);
3. Methods for solving the problem

From the statement of the problem, it follows that it is difficult to obtain an analytical solution, and for the numerical integration of the problem, the dimensionless variables in (1-8) will use the formulas

\[
\begin{align*}
&x^* = \frac{x}{L}; \quad y^* = \frac{y}{L}; \quad k_p^* = \frac{k_p}{k_s}; \quad P_1^* = \frac{P_1}{P_s}; \quad P_2^* = \frac{P_2}{P_s}; \\
&t = \beta_m a \mu_t L^2 \tau; \quad q_H^* = \frac{q_H}{\mu_t \pi k_s h_y}; \quad q_b^* = \frac{q_b}{\mu_t \pi k_s h_y}; \quad R^* = \frac{(a-a_0)\beta_b}{a\beta_h}.
\end{align*}
\]

Here \( P_s \) - some characteristic pressure values; \( k_s \) - some characteristic values of permeability; \( L \) - characteristic length and width of the reservoir.

After passing to dimensionless variables in system (1) with the corresponding boundary conditions (2) - (8), the problem is solved numerically using the longitudinal-transverse scheme for the differential-difference problem and the differential sweep method to determine the main indicators of the filtration process.

Given the above filtering area \( G_1 + G_2 \) covered with grid area \( \Omega_{n_0,n_0} \), formed by a regular grid of coordinate lines:

\[
\Omega_{n_0,n_0} = \left\{ x_j = i\Delta h, \quad y_j = j\Delta h, \quad \tau_k = k\Delta \tau, \quad i = 1,N_i, \quad j = 1,M_j, \quad k = 0,N_k, \quad \Delta \tau = \frac{1}{N_k} \right\}.
\]

Here \( N_j \), \( M_i \) - the number of nodes on the line \( y_j \) and \( x_i \) respectively; \( \Delta h \) - grid spacing \( x \) and \( y \).

We will use the algorithmic idea of an implicit scheme of variable directions (longitudinal-transverse scheme) to obtain a differential-difference problem. The transition from the time layer \( r \) to the layer \( r+1 \) takes place in two stages with a step \( 0.5\Delta \tau \). Then solution (1) is found by sequentially solving the following system of equations:

\[
\begin{align*}
&\frac{d}{dx} \left( k_{i,j} \frac{dP_{1j}^{(r+0.5)}}{dx}(x) \right) - \frac{1}{0.5\Delta \tau} P_{1j}^{(r+0.5)}(x) = -\frac{P_{1j}^{(r)}}{0.5\Delta \tau} - \Lambda_{y} \left[ k_{i,j} P_{1i,j}^{(r)} \right], \\
&\frac{d}{dy} \left( k_{i,j} \frac{dP_{1i}^{(r+1)}}{dy}(y) \right) - \frac{1}{0.5\Delta \tau} P_{1i}^{(r+1)}(y) = -\frac{P_{1i}^{(r+0.5)}}{0.5\Delta \tau} - \Lambda_{x} \left[ k_{i,j} P_{1i,j}^{(r+0.5)} \right], \\
&\frac{d}{dx} \left( k_{i,j} \frac{dP_{2j}^{(r+0.5)}}{dx}(x) \right) - \frac{R^*}{0.5\Delta \tau} P_{2j}^{(r+0.5)}(x) = -\frac{P_{2j}^{(r)}}{0.5\Delta \tau} - \Lambda_{2x} \left[ k_{i,j} P_{2i,j}^{(r)} \right], \\
&\frac{d}{dy} \left( k_{i,j} \frac{dP_{2i}^{(r+1)}}{dy}(y) \right) - \frac{R^*}{0.5\Delta \tau} P_{2i}^{(r+1)}(y) = -\frac{P_{2i}^{(r+0.5)}}{0.5\Delta \tau} - \Lambda_{2y} \left[ k_{i,j} P_{2i,j}^{(r+0.5)} \right],
\end{align*}
\]

(9)
where
\[
\Lambda_{1y} \left[ k_{i,j} P_{li,j}^{(r)} \right] = \frac{k_{i-0.5,j} P_{li,j}^{(r)} - \left( k_{i-0.5,j} + k_{i+0.5,j} \right) P_{li,j}^{(r)} + k_{i+0.5,j} P_{li+1,j}^{(r)}}{\Delta h^2},
\]
\[
\Lambda_{1x} \left[ k_{i,j} P_{li,j}^{(r+0.5)} \right] = \frac{k_{i,j-0.5} P_{li,j-1}^{(r+0.5)} - \left( k_{i,j-0.5} + k_{i,j+0.5} \right) P_{li,j}^{(r+0.5)} + k_{i,j+0.5} P_{li,j+1}^{(r+0.5)}}{\Delta h^2},
\]
\[
\Lambda_{2y} \left[ k_{i,j} P_{2i,j}^{(r)} \right] = \frac{k_{i-0.5,j} P_{2i,j-1}^{(r)} - \left( k_{i-0.5,j} + k_{i+0.5,j} \right) P_{2i,j}^{(r)} + k_{i+0.5,j} P_{2i,j+1}^{(r)}}{\Delta h^2},
\]
\[
\Lambda_{2x} \left[ k_{i,j} P_{2i,j}^{(r+0.5)} \right] = \frac{k_{i,j-0.5} P_{2i,j-1}^{(r+0.5)} - \left( k_{i,j-0.5} + k_{i,j+0.5} \right) P_{2i,j}^{(r+0.5)} + k_{i,j+0.5} P_{2i,j+1}^{(r+0.5)}}{\Delta h^2}.
\]

The resulting system of differential-difference equations (9) is solved by differential run along each of the \(x\) lines with the initial conditions known at \(\tau = \tau_s\), and then along each of the lines \(y\), where the initial condition is taken just found values corresponding to the \(r+0.5\) th layer.

According to the method of differential run solutions of differential-difference equations (9) on \(r+0.5\) the and \(r+1\) time layers with boundary conditions (2)-(7) are determined by the formulas
\[
P_{1j}^{(r+0.5)}(x) = \frac{\gamma_j(x) u_j(x) - \alpha_j(x) w_j(x)}{\alpha_j(x) v_j(x) - \beta_j(x) u_j(x)},
\]
\[
\frac{dP_{1j}^{(r+0.5)}(x)}{dx} = \frac{1}{k_j(x)} v_j(x) \gamma_j(x) - w_j(x) \beta_j(x),
\]
\[
P_{1j}^{(r+1)}(y) = \frac{\gamma_j(y) u_j(y) - \alpha_j(y) w_j(y)}{\alpha_j(y) v_j(y) - \beta_j(y) u_j(y)},
\]
\[
\frac{dP_{1j}^{(r+1)}(y)}{dy} = \frac{1}{k_j(y)} v_j(y) \gamma_j(y) - w_j(y) \beta_j(y)
\]

where the coefficients of the left and right sweep \(\alpha_j(x), \beta_j(x), \gamma_j(x)\) and \(u_j(x), v_j(x), w_j(x)\) are found from the solution of the following Cauchy problems:
\[
\begin{cases}
  k_j(x) \frac{du_j(x)}{dx} = v_j(x), & u_j(0) = 1, \\
  \frac{dv_j(x)}{dx} = R_j u_j(x), & v_j(0) = 0, \\
  \frac{dw_j(x)}{dx} = F_j u_j(x), & w_j(0) = 0;
\end{cases}
\]
\[
\begin{align*}
&k_j(x) \frac{d\alpha_j(x)}{dx} = \beta_j(x), \quad \alpha_j(1) = 1, \\
&\frac{d\beta_j(x)}{dx} = R_j\alpha_j(x), \quad \beta_j(1) = 0, \\
&\frac{d\gamma_j(x)}{dx} = F_j\alpha_j(x), \quad \gamma_j(1) = 0;
\end{align*}
\]

\[
\begin{align*}
k_i(y) \frac{du_i(y)}{dy} &= v_i(y), \quad u_i(0) = 1, \\
&\frac{dv_i(y)}{dy} = R_iu_i(y), \quad v_i(0) = 0, \\
&\frac{dw_i(y)}{dy} = F_iu_i(y), \quad w_i(0) = 0;
\end{align*}
\]

\[
\begin{align*}
k_i(y) \frac{d\alpha_i(y)}{dy} &= \beta_i(y), \quad \alpha_i(1) = 1, \\
&\frac{d\beta_i(y)}{dy} = R_i\alpha_i(y), \quad \beta_i(1) = 0, \\
&\frac{d\gamma_i(y)}{dy} = F_i\alpha_i(y), \quad \gamma_i(1) = 0.
\end{align*}
\]

Here

\[
F_j = -\frac{p_{j,j}^{(0)}(x)}{0.5\Delta \tau} - \Lambda \left[ k_{ij}p_{h,ij}^{(0)} \right], F_i = -\frac{p_{i,j}^{(0+5)}(y)}{0.5\Delta \tau} - \Lambda \left[ k_{ij}p_{h,ij}^{(r,0+5)} \right], R_{ij} = \frac{1}{0.5\Delta \tau}, \quad R_i = \frac{1}{0.5\Delta \tau}
\]

To solve the Cauchy problems (14) - (17), the initial conditions are determined from the boundary conditions (4)

\[
F_{2j} = -\frac{p_{j,j}^{(0)}(x)}{0.5\Delta \tau} - \Lambda \left[ k_{ij}p_{h,ij}^{(0)} \right], F_{2i} = -\frac{p_{i,j}^{(0+5)}(y)}{0.5\Delta \tau} - \Lambda \left[ k_{ij}p_{h,ij}^{(r,0+5)} \right], R_{2j} = \frac{R^*}{0.5\Delta \tau}, \quad R_{2i} = \frac{R^*}{0.5\Delta \tau}
\]

Approximating differential equations (7) by \(\tau\), we obtain a formula for clarifying the position of the interface on each time layer:

\[
\hat{l}_{ij} = \hat{l}_{ij} - \frac{\Delta \tau k_{ij}}{m \mu_i} \left[ \frac{dP}{dx} \cos \tilde{\alpha} + \frac{dP}{dy} \cos \tilde{\beta} \right],
\]

where \(\hat{l}_{ij}\) – velocity vector directed along the internal normal at the interface;

\(\hat{l}_{ij}\) – velocity vector directed along the internal normal at the interface in the previous time layer;

\(\tilde{\beta}\) – angle between normal and axis oy: \(\tilde{\beta} = \frac{3}{2} \pi + \tilde{\alpha}\).

When specifying the position of the interface, the values \(\hat{l}_{ij}\) taken from the initial condition (8).

At the boundary of the filtration region, one of the following conditions may be satisfied: the first kind; second kind and mixed condition.
If the pressure values are known at the boundary of the filtration region, i.e. the first boundary condition is given, then the initial conditions of the Cauchy problem take the following form, respectively, on the left and right sides of the outer boundary $\Gamma$:

$$u_i = 0, \quad v_i = -1, \quad w_i = P; \quad \alpha_n = 0, \quad \beta_n = -1, \quad \gamma_n = P.$$

If a flow is specified at the outer boundary of the filtration region, i.e. the second boundary condition, then the initial conditions of the Cauchy problem take the following form, respectively, on the left and right sides of the boundary:

$$u_i = 1, \quad v_i = 0, \quad w_i = f_i; \quad \alpha_n = 0, \quad \beta_n = 0, \quad \gamma_n = f_i;$$

$f_i$ - known function. If $f_i = 0$, then the border is impenetrable.

In the case when a condition of the first kind is specified on one part of the boundaries of the filtration region, and a condition of the second kind is defined on the other part, i.e. pressure is set on one part and flow on the other part, the initial conditions of the Cauchy problem are defined similarly. At the internal boundaries of the multiply connected region, the condition of impermeability and continuity of pressure is set. These conditions are satisfied automatically at the transition of the interface between the two phases when applying the differential sweep method in the process of sequentially finding the values of $u_i(x), \, v_i(x), \, w_i(x)$ during the transition from one phase to another, the previous values of these functions are used as initial conditions.

Numerical integration of the Cauchy problem is carried out by the Runge-Kutta method using the normalization procedure of the running coefficients and the coefficients of this method.

In each iterative step in calculating the vector $\vec{U}_{i+1}$ to the right side of the system of equations instead $\vec{U}_i$ the normalized vector is substituted (where $\vec{U} = (u, v, w)$ or $\vec{U} = (\alpha, \beta, \gamma)$). The normalization procedure can be omitted if the chosen method steadily solves the Cauchy problem.

4. Conclusion
A mathematical model has been developed for analysis, research and determination of the main indicators of an oil field during the oil-water filtration process in a porous medium, taking into account the moving oil-water interface.

An effective numerical algorithm is proposed for solving two-dimensional filtration problems in a porous medium, taking into account the moving oil-water interface based on the differential sweep method.

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