Extraction of the Axial Nucleon Form Factor from Neutrino Experiments on Deuterium

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Abstract. We present new parameterizations of vector and axial nucleon form factors. We maintain an excellent description of the form factors at low momentum transfers ($Q^2$), where the spatial structure of the nucleon is important, and use the Nachtman scaling variable $\xi$ to relate elastic and inelastic form factors and impose quark-hadron duality constraints at high $Q^2$ where the quark structure dominates. We use the new vector form factors to re-extract updated values of the axial form factor from $\nu_\mu$ experiments on deuterium. We obtain an updated world average value from $\nu_\mu d$, $\nu_\mu H$ and pion electroproduction experiments of $M_A = 1.014 \pm 0.014 \text{GeV}/c^2$. Our parameterizations are useful in modeling $\nu$ interactions at low energies (e.g. for $\nu_\mu$ oscillations experiments). The predictions for high $Q^2$ can be tested in the next generation electron and $\nu_\mu$ scattering experiments. (Presented by A. Bodek at the European Physical Society Meeting, EPS2007, Manchester, England, July 2007).

The nucleon vector and axial elastic form factors have been measured for more than 50 years in $e^- N$ and $\nu N$ scattering. At low $Q^2$, a reasonable description of the proton and neutron elastic form factors is given by the dipole approximation. The dipole approximation is a lowest-order attempt to incorporate the non-zero size of the proton into the form factors. The approximation assumes that the proton has a simple exponential spatial charge distribution, $\rho(r) = \rho_0 e^{-r/r_0}$, where $r_0$ is the scale of the proton radius. Since the form factors are related in the non-relativistic limit to the Fourier transform of the charge and magnetic moment distribution, the above $\rho(r)$ yields the dipole form defined by: $G_D^{V,A}(Q^2) = C^{V,A}/\left(1 + \frac{Q^2}{M_{V,A}^2}\right)^2$. Here $C^{V,A} = (1,g_A)$, $g_A = -1.267$, $M_V^2 = 0.71 \text{ (GeV}/c^2)^2$, and $M_A = 1.015 \text{ GeV}/c^2$ (see below).

Since $M_A$ is not equal to $M_V$, the distribution of electric and axial charge are different. However, the magnetic moment distributions were assumed to have the same spatial dependence as the charge distribution (i.e., form factor scaling). Recent measurements from Jefferson Lab show that the ratio of $\nu_\mu G_{Ep}/G_{Mp}$ falls at high $Q^2$ challenging the validity of form factor scaling and resulting in new updated parameterizations of the form factors [2]). In this paper we present parameterizations that simultaneously satisfy constraints at low $Q^2$ where the spatial structure of the nucleon is important, and at high $Q^2$ where the quark structure is important. A violation of form-factor scaling is expected from quark-hadron duality. We use our new vector form factors to re-extract updated values of the axial form factor from a re-analysis of previous neutrino scattering data on deuterium and present a new parameterization for the axial form factor within the framework of quark-hadron duality.

The new parameterizations presented in this paper are referred to as the duality based "BBBA07" parameterization. Our updated parameterizations feature the following: (1) Improved functional form that adds an additional $Q^2$ dependence using the Nachtman scaling.
variable ξ to relate elastic and inelastic form factors. For elastic scattering (x = 1) ξ^{p,n,N} = \frac{2}{(1+\sqrt{1+1/\tau_{p,n,N}})}, where \tau_{p,n,N} = Q^2/4M_{p,n,N}^2. Here M_{p,n,N} are the proton (0.9383 GeV/c^2), neutron (0.9396 GeV/c^2), and average nucleon mass (for proton, neutron, and axial form factors, respectively). (2) Yield the same values as Arrington and Sick [3] for Q^2 < 0.64(GeV/c)^2, while satisfying quark-hadron duality constraints at high-Q^2.

For vector form factors our fit functions are \mathcal{A}_N(ξ) (i.e., \mathcal{A}_{Ep}(ξ^p), \mathcal{A}_{Mp}(ξ^p), \mathcal{A}_{En}(ξ^p), \mathcal{A}_{Mn}(ξ^n)) multiplying an updated Kelly [2] type parameterization of one of the proton form factors. The Kelly parameterization is: 

\mathcal{G}^{Kelly}(Q^2) = \sum_{\xi=0}^{n} a_\xi/\tau_\xi + \sum_{m=1}^{\infty} b_\xi/\tau_m,

where a_0 = 1 and m = 1. In our analysis, we use all the datasets used by Kelly [2], updated to include the recent BLAST results, to fit \mathcal{G}_{Ep}, \mathcal{G}_{En}, \mathcal{G}_{Mp}/\mu_p, and \mathcal{G}_{Mn}/\mu_n (\mu_p = 2.7928, \mu_n = -1.9130).

Our parameterization employs the published Kelly functional form to \mathcal{G}_{Ep}, and an updated set of parameters for \mathcal{G}_{Mp}^{Kelly-upd}(Q^2). The parameters \mathcal{A}_N(ξ) is given by

\mathcal{A}_N(ξ) = \sum_{j=1}^{n} p_j \prod_{k=1,k\neq j}^{n} \frac{ξ - ξ_k}{ξ_j - ξ_k}.

The ξ_j are equidistant “nodes” on an interval [0,1] and p_j are the fit parameters that have an additional property \mathcal{A}_N(ξ_j) = p_j. The functional form \mathcal{A}_N(ξ) (for \mathcal{G}_{Ep}, \mathcal{G}_{Mp}, \mathcal{G}_{En}, and \mathcal{G}_{Mn}) is used with seven p_j parameters at ξ_j = 0, 1/6, 1/3, 1/2, 2/3, 5/6, and 1.0. In the fitting procedure the parameters of \mathcal{A}_N(ξ) are constrained to give the same vector form factors as the recent low Q^2 fit of Arrington and Sick [3] for Q^2 < 0.64(GeV/c)^2 (as that analysis includes coulombs corrections which modify \mathcal{G}_{Ep}, and two photon exchange corrections which modify \mathcal{G}_{Mp} and \mathcal{G}_{Mn}). Our fits to the form factors are:

\mathcal{G}_{Mp}(Q^2)/\mu_p = \mathcal{A}_{Mp}(ξ^p) \times \mathcal{G}_{Mp}^{Kelly-upd}(Q^2)

\mathcal{G}_{Ep}(Q^2) = \mathcal{A}_{Ep}(ξ^p) \times \mathcal{G}_{Ep}^{Kelly}(Q^2)

\mathcal{G}_{Mn}(Q^2)/\mu_n = \mathcal{A}_{Mn}^{25,43}(ξ^n) \times \mathcal{G}_{Mp}(Q^2)/\mu_p

\mathcal{G}_{En}(Q^2) = \mathcal{A}_{En}^{25,43}(ξ^n) \times \mathcal{G}_{Ep}(Q^2) \times \left(\frac{a\tau_n}{1 + b\tau_n}\right),

where we use our updated parameters in the Kelly parameterizations. For \mathcal{G}_{En} the parameters a=1.7 and b=3.3 are the same as in the Galster [2] parametrization and ensure that \mathcal{dG}_{En}/dQ^2 at for Q^2 = 0 is in agreement with measurements. The values \mathcal{A}(ξ_j) = p_j at ξ_j = 0 (Q^2 = 0) for \mathcal{G}_{Mp}, \mathcal{G}_{Ep}, \mathcal{G}_{En}, \mathcal{G}_{Mn} are set to to 1.0. The value \mathcal{A}(ξ_j) = p_j at ξ_j = 1 (Q^2 \rightarrow \infty) for \mathcal{G}_{Mp} and \mathcal{G}_{Ep} is set to 1.0. The value \mathcal{A}(ξ_j) = p_j at ξ_j = 1 for \mathcal{G}_{Mn} and \mathcal{G}_{En} are fixed by constraints from quark-hadron duality [1]. The parameters and plots of the new form factors \mathcal{G}_{Ep}, \mathcal{G}_{Mp}/\mu_p, \mathcal{G}_{Mn}/\mu_n, and \mathcal{G}_{En} are given in ref. [1].

Using our updated BBBA2007 form factors and an updated value g_A = -1.267, we perform a complete reanalysis of published ν quasielastic [6] (QE) data on deuterium (ν_μ n → μ^- p) using the procedure described in detail in ref. [4]. We extract new values of M_A (given in Table [1]), and updated values of F_A(Q^2). The average of the corrected measurements of M_A from Table [1] is 1.0137 ± 0.0264 GeV/c^2. This is to be compared to the average value of 1.0140 ± 0.0160 GeV/c^2 extracted from pion electroproduction experiments after corrections for hadronic effects. World average value from ν_μ d, p_μ H and pion electroproduction experiments of M_A = 1.0137 ± 0.0137 GeV/c^2. This is smaller than the recent results [9] from MiniBoone on a carbon target (M_A = 1.23 ± 0.20 GeV/c^2) and K2K [10] on oxygen (M_A = 1.20 ± 0.12 GeV/c^2). Both collaborations use updated vector form factors. Collaborations attribute a difference from deuterium to nuclear effects. However, there is experimental and theoretical evidence [11] that
| Experiment   | QE events | $Q^2$ range GeV/c$^2$ | $M_A$ (published) | $\Delta M_A$ FF | $M_A^{\text{updated}}$ GeV/c$^2$ |
|--------------|-----------|-----------------|-----------------|----------------|--------------------------|
| Mann73       | 166       | .05 - 1.6       | 0.95 ± 0.12     | -0.026         | 0.970 ± 0.05             |
| Barish77     | 500       | .05 - 1.6       | 0.95 ± 0.09     | -0.030         | 0.970 ± 0.05             |
| Miller82,77,73 | 1737     | .05 - 2.5       | 1.00 ± 0.05     | -0.028         | 1.042 ± 0.06             |
| Baker81      | 1138      | .06 - 3.0       | 1.07 ± 0.06     | -0.025         | 1.025 ± 0.06             |
| Kitagaki83   | 362       | .11 - 3.0       | 1.05 ± 0.12     | -0.036         | 1.034 ± 0.045            |
| Kitagaki90   | 2544      | .10 - 3.0       | 1.070 ± 0.040   | -0.036         | 1.034 ± 0.045            |
| Atlas990     | 552       | .1-3.75         | 1.080 ± 0.08    | -0.080         | 1.00 ± 0.08              |
| Av. $\nu_{\mu}$d & above | 5780     |                 | 1.014 ± 0.026   | 1.014 ± 0.016 |
| $\pi$ electprod. |          |                 |                 | 1.014 ± 0.016 |
| $\bar{\nu}_{\mu}$H $\rightarrow \mu^-$ n | 13       | 0-1.0           | 0.9 ± 0.35      | -0.070         | 0.83 ± 0.35              |
| $\bar{\nu}_{\mu}$H $\rightarrow \mu^-$ n | 13       | 0-1.0           | $\sigma_{QE}$   | -0.070         | 1.04 ± 0.40              |
| Average - all |          |                 |                 |               | 1.014 ± 0.014            |

Table 1. $M_A$ (GeV/c$^2$) values published by $\nu_{\mu}$-deuterium experiments\[6\] and updated corrections $\Delta M_A$ when re-extracted with updated $BBBA2007_{25,1}$ form factors, and $g_A = -1.267$. Also shown is updated $M_A$ from $\bar{\nu}_{\mu}$Hydrogen $\rightarrow \mu^-$ n $[7]$.  

![Figure 1](image.png)

Figure 1. (a) $F_A(Q^2)$ re-extracted from neutrino-deuterium data divided by $G^A_D(Q^2)$. (b) $F_A(Q^2)$ from pion electroproduction divided by $G^A_N(Q^2)$, corrected for for hadronic effects. Solid line - duality based fit; Short-dashed line - $F_A(Q^2)_{A2=V1}$. The long-dashed line is $F_A(Q^2)_{A1=V1}$. Dashed-dot line - constituent quark model

$M_A$ in nuclear targets is the same (or smaller) than in deuterium. This discrepancy is important for $\nu$ oscillations experiments since it affects the normalization (at high energies the QE cross section is approximately proportional to $M_A$) and non-linearity of the QE cross section, which is relevant to the extraction of $\nu$ mass difference and mixing angle.

For deep-inelastic scattering, the vector and axial parts of $F_2$ are equal. Local quark-hadron duality at large $Q^2$ implies that the axial and vector parts of $F_2^{\text{elastic}}$ are also equal: 

$$[F_A(Q^2)_{A2=V2}]^2 = (G^V_E(Q^2))^2 + \tau_N(G^V_M(Q^2))^2/(1 + \tau_N),$$

where $G^V_E(Q^2) = G_{Ep}(Q^2) - G_{En}(Q^2)$ and $G^V_M(Q^2) = G_{Mp}(Q^2) - G_{Mn}(Q^2)$.

We extract values of $F_A(Q^2)$ from the differential cross sections using the procedure of
Figure 2. Average $F_A(Q^2)/G_D(M_A = 1.015)$ from (a) $\nu$-deuterium experiments, and (b) from pion electroproduction experiments. Also shown are the average E734 $\nu_\mu$ and $\pi_\mu$ results from carbon. Solid line - duality based fit; Short-dashed line - $F_A(Q^2)_{A2-V2}$; long-dashed)-$G_D(M_A = 1.20)/G_D(M_A = 1.015)$; dashed-dot line - $G_D(M_A = 0.80)/G_D(M_A = 1.015)$.

ref. [1]. The overall normalization is set by the theoretical QE cross section [12]. We then do a duality based fit to $F_A(Q^2)$ (including pion electroproduction data) of the form: $F_A(Q^2) = A_{25}^{\nu A}(\xi^N) \times G_D^A(Q^2)$.

We impose the constraint $A_{25}^{\nu A}(\xi_1 = 0) = p_1 = 1.0$. We also constrain the fit by requiring that $A_{25}^{\nu A}(\xi^N)$ yield $F_A(Q^2) = F_A(Q^2)_{A2-V2}$ by including additional "fake" data points for $\xi > 0.9$ ($Q^2 > 7.2 \text{(GeV/c)}^2$). Figure 1(a) shows $F_A(Q^2)$ extracted from neutrino-deuterium experiments divided by $G_D^A(Q^2)$ [12]. Figure 1(b) shows $F_A(Q^2)$ extracted from pion electroproduction experiments divided by $G_D^A(Q^2)$ [12]. These pion electroproduction values can be directly compared to the neutrino results because they are multiplied by a factor $F_A(Q^2, M_A = 1.014 \text{ GeV/c}^2)/F_A(Q^2, M_A = 1.069 \text{ GeV/c}^2)$ to correct for $\Delta M_A = 0.055 \text{ GeV/c}^2$ originating from hadronic effects. Figure (2) shows the same data averaged over all experiments in bins of $Q^2$. The solid line is our duality based fit. The short-dashed line is $F_A(Q^2)_{A2-V2}$. The dashed-dot line is a constituent-quark model [8] prediction.

In summary, our new parameterizations are useful in modeling $\nu$ interactions for oscillations experiments. Our predictions for $G_{En}(Q^2)$ and $F_A(Q^2)$ can be tested in future $e-N$ and $\nu-N$ experiments.

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