Global QCD analysis of pion parton distributions with threshold resummation

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Global QCD Analysis of Pion Parton Distributions with Threshold Resummation

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Motivation

• QCD allows us to study the **structure of hadrons** in terms of **partons** (quarks, antiquarks, and gluons)

• Use **factorization theorems** to separate hard partonic physics out of soft, non-perturbative objects to quantify structure
Pions

• Pion presents itself as a **dichotomy**

1. It is the **Goldstone boson** associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry

2. Made up of **quark and antiquark constituents**
Experiments to probe pion structure

Drell-Yan (DY)

Leading Neutron (LN)

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Large-$x_{\pi}$ behavior

• Generally, the parametrization lends a behavior as $x \to 1$ of the valence quark PDF of $q_v(x) \propto (1 - x)^\beta$

• For a fixed order analysis, analyses find $\beta \approx 1$

• Aicher, Schaefer Vogelsang (ASV) found $\beta = 2$ with threshold resummation

ASV valence PDF
Phys. Rev. Lett. 105, 114023 (2011).
Threshold Resummation

Significant contributions to cross section occur in soft gluon emissions and follow the pattern

\[ \frac{d\hat{\sigma}^{q\bar{q}}_{N^k LO}}{\hat{s}} \propto \alpha_S^k \frac{\ln^{2k-1} (1 - z)}{1 - z} + \ldots \]

\[ z = \frac{Q^2}{\hat{s}} \]
Methods of resummation – Mellin-Fourier

• Threshold resummation is done in conjugate space

\[ \sigma_{MF}(N, M) \equiv \int_0^1 d\tau \tau^{N-1} \int_{\log \sqrt{\tau}}^{\log \frac{1}{\sqrt{\tau}}} dY e^{iMY} \frac{d^2\sigma}{d\tau dY}, \]

Two choices occur when isolating the hard part

\[ \hat{\sigma}_{MF}(N, M) = \int_0^1 dz z^{N-1} \cos \left( \frac{M}{2} \log z \right) \frac{d^2\hat{\sigma}}{d\tau dY}(z) \]

Keep cosine intact – “cosine” method

Keep the first order term in the expansion – \( \cos \left( \frac{M}{2} \log z \right) \approx 1 \) “expansion” method

\[ \tau = \frac{Q^2}{S} \]
Method of resummation – double Mellin

• Alternatively, perform a double Mellin transform

\[ \sigma_{DM}(N, M) \equiv \int_0^1 dx_\pi^0 (x_\pi^0)^{N-1} \int_0^1 dx_A^0 (x_A^0)^{M-1} \frac{d^2 \sigma}{d\tau dY}. \]

where

\[ x_\pi^0 = \sqrt{\tau} e^Y, \quad x_A^0 = \sqrt{\tau} e^{-Y} \]

• Double Mellin transform is theoretically cleaner and sums up terms appropriately
Data and theory comparison

• Cosine method tends to overpredict the data at very large $x_F$

• Double Mellin method is qualitatively very similar to NLO

Current data do not distinguish between NLO and NLO+NLL

| Method                        | $\chi^2$/npts |
|-------------------------------|---------------|
| NLO                           | 0.85          |
| NLO+NLL cosine                 | 1.29          |
| NLO+NLL expansion              | 0.95          |
| NLO+NLL double Mellin          | 0.80          |

Slightly disfavored
Resulting PDFs

- Large $x$ behavior of $q_v$ **highly sensitive** to method of resummation
Effective $\beta_v$ parameter

- $q_v(x) \sim (1 - x)^{\beta_v^{\text{eff}}}$ as $x \to 1$
- Threshold resummation does not give universal behavior of $\beta_v^{\text{eff}}$
- NLO and double Mellin give $\beta_v^{\text{eff}} \approx 1$ – theoretically cleaner
- Cosine and Expansion give $\beta_v^{\text{eff}} > 2$
Conclusions

• Threshold resummation does not give universal value for $\beta^\text{eff}_V$

• Large-$x$ behavior of valence quark distribution highly dependent on treatment of large logarithms

• Using the trustworthy double Mellin approach, we achieve a similar linear falloff at large momentum fraction

• Need to include more observables and data to further constrain pion PDFs
  • Lattice QCD – see my talk in WG1 on Thursday at 10:20am
  • Transverse momentum dependent data – see my talk in WG5 on Tuesday at 10:00am

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Backup Slides
Datasets -- Kinematics

\[ Q^2 (\text{GeV}^2) \]

\[ x^\pi \]

- E615
- NA10
- H1
- ZEUS

\[ \text{DY} \]

\[ \text{LN} \]
Deriving resummation expressions – MF

Claim: yellow terms give rise to the resummation expressions

\[
\frac{C_{q\bar{q}}}{e_q^2} = \delta(1 - z) \left\{ \frac{\delta(y) + \delta(1 - y)}{2} \right\} \left[ 1 + \frac{C_F \alpha_s}{\pi} \left( \frac{3}{2} \ln \frac{M^2}{\mu_f^2} + \frac{2\pi^2}{3} - 4 \right) \right]
\]

\[
+ \frac{C_F \alpha_s}{\pi} \left\{ \frac{\delta(y) + \delta(1 - y)}{2} \right\} \left( 1 + z^2 \right) \left[ \frac{1}{1 - z} \ln \frac{M^2(1 - z)^2}{\mu_f^2 z} \right] + 1 - z
\]

\[
+ \frac{1}{2} \left[ 1 + \frac{(1 - z)^2}{z} y(1 - y) \right] \left[ \frac{1 + z^2}{1 - z} \left( \left[ \frac{1}{y} \right]_+ + \left[ \frac{1}{1 - y} \right]_+ \right) - 2(1 - z) \right]
\]

Claim: Red terms are power suppressed in \((1 - z)\) and wouldn’t contribute to the same order as the yellow terms
Generalized Threshold resummation

• Write the \((z, y)\) coefficients in terms of \((z_a, z_b)\), and for the red terms, you get:

\[
dz \, dy \, \frac{1}{1-z} \left( \frac{1}{y} + \frac{1}{1-y} \right) = dz_a \, dz_b \left( 1 \ominus \frac{1}{(1-z_a)(1-z_b)} \right) [1 + \mathcal{O}(1-z_a, 1-z_b)].
\]

• This is *not* power suppressed in \((1 - z_a)\) or \((1 - z_b)\) but instead the same order as the leading power in the soft limit

• Generalized threshold resummation in the soft limit does not agree with the MF methods

G. Lustermans, J. K. L. Michel, and F. J. Tackmann, arXiv:1908.00985 [hep-ph].
Critiques suggested \((1 - x)^2\) is a fact of QCD

\[ u^\pi(x; \zeta) \xrightarrow{x \to 1} (1 - x)^\beta = 2 + \gamma(\zeta). \]

T1: If QCD describes the pion, then at any scale for which an analysis of data using known techniques is valid, the form extracted for the pion’s valence-quark DF must behave as \((1 - x)^\beta, \beta > 2, \) on \( x \gtrsim 0.9 \) \([10, 59, 73, 74]..\)

- **T1:** There is no proof of this in QCD
- **[a]** The double Mellin method is more rigorous than Mellin-Fourier
- **[b]** We carefully apply factorization; lattice QCD data prefer a linear falloff; there is no evidence to suggest these data are wrong
- **[c]** There is no indication to insinuate QCD is not the theory of strong interactions
Wide-Angle Scattering in Softened Field Theory

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(riccuito il 25 Marzo 1974)

Summary. — The picture of Brodsky and Farrar for scattering processes at large transverse momentum is formulated in softened field theory. A modest softening of the quark-quark-gluon vertex is introduced to suppress unwanted logarithms in the formalism. It is shown that the electromagnetic form factors of the proton and the pion yield asymptotically behaviours which agree with the result of simple dimensional counting. The threshold behaviours of the deep inelastic structure functions are calculated for the proton and the pion to give $\sim (1 - \omega)^3$ and $\sim (1 - \omega)^4$, respectively. Thus the Drell-Yan-West relation holds in the case of the proton target but is violated in the case of the pion target. It is also proved that the asymptotic behaviours of wide-angle elastic $\pi\pi$ and $p\bar{p}$ scattering naively predicted by dimensional counting and conjectured by Brodsky and Farrar on the basis of simple Born diagrams are actually the next-to-leading-order terms. The highest-order terms come from a certain set of diagrams that Landshoff studied.

- No explicit proof of nonperturbative $q_v^\pi(x \to 1) \sim (1 - x)^2$
- Assumes one hard gluon exchange dominance
Assumption made that the below diagram dominates the structure

![Diagram](image)

**Assumption**

In a colored-quark and vector-gluon model of hadrons we show that a quark carrying nearly all the momentum of a nucleon \( (x \approx 1) \) must have the same helicity as the nucleon; consequently \( n W_2^x / n W_3^x \rightarrow \frac{1}{4} \) as \( x \rightarrow 1 \), not \( \frac{1}{4} \) as might naively have been expected. Furthermore as \( x \rightarrow 1 \), \( n W_2^x (1 - x)^2 \) and \( \alpha_s / \pi \sim \mu Q^{-2} (1 - x)^2 + O(g^2) \); the resulting angular dependence for \( e^+ e^- \rightarrow h^+ + X \) is consistent with present data and has a distinctive form which can be easily tested when better data are available.

This is a **perturbative** assumption – we cannot say that higher order terms or soft gluons do not contribute to the **nonperturbative** structure of the hadron in QCD.

First principles QCD does not **prove** this behavior for PDF.
Not necessary to have \((1 - x)^\beta\) behavior

• A recent work by Collins, Rogers, and Sato proved that \(\overline{\text{MS}}\) PDFs were not necessarily positive as long as cross section was positive.

• PDFs do not have to have a large-\(x\) behavior associated with the counting rules
QCD does not fail if $\beta^\pi_\nu \neq 2$

• The perturbative expansion performed in Ezawa and Farrar & Jackson does not capture nonperturbative effects

• Like in threshold resummation, the buildup of very soft gluon exchanges between quark states may be non-negligible contributions to the perturbation

• When $(1 - x) \to 0$, the light front zero mode could play a non-trivial role, which cannot be calculated perturbatively