Massless and spinning particles as dynamics in one dimensional (super)diffeomorphism groups.

A. Pashnev

JINR–Bogoliubov Laboratory of Theoretical Physics
Dubna, Head Post Office, P.O.Box 79, 101 000 Moscow, Russia

Abstract

It is shown that dynamics of \( D + 2 \) elements of the (super)diffeomorphism group in one \((1+1)\) for the super) dimension describes the \( D \) - dimensional (spinning) massless relativistic particles. The coordinates of this elements \((D + 2 \text{ einbeins}, D + 2 \text{ connections and } 1 \text{ additional common coordinate of higher dimensionality})\) play the role of coordinates, momenta and Lagrange multiplier, needed for the manifestly conformal and reparametrization invariant description of the \( D \) - dimensional (spinning) particle in terms of the \( D + 2 \) - dimensional spacetime.

Submitted to “Classical and Quantum Gravity”
1 Introduction

As is well known there exist several equivalent formulations of the massless relativistic particles. The second order and first order formalisms are examples of them. The essential ingredient of both approaches is einbein - the field describing one dimensional gravity. One more example is the conformally invariant description, which starts from $D + 2$ dimensional spacetime. The existence of alternative approaches always sheds some new light on the nature of the physical system. In particular, the conformally invariant description from the very beginning considers the particle coordinates and einbein on the equal footing. For the extended spinning particle the analogous description shows that the gravitinos of the corresponding one dimensional supergravity are on the same footing with coordinates superpartners as well.

In the present work we consider the natural description of the massless relativistic particle and $N = 1$ spinning particle in terms of nonlinear realization of the infinite dimensional diffeomorphisms group of the one dimensional space. We construct the first order conformally invariant formulation and show that the spacetime coordinates and one dimensional supergravity fields are realized as dilatons of one dimensional diffeomorphisms group. We consider simultaneously the dynamics of $D + 2$ different points in the group space, hence they contain the same number of dilatons. The corresponding $(D + 2)$ components of momentum are connected with the Cristoffel symbols. One more parameter of the group having higher dimension is the same for all $(D + 2)$ points. It plays the role of Lagrange multiplier and effectively reduces the number of spacetime coordinates from $D + 2$ to $D$ ones.

In the second section of the paper we shortly describe the conformally invariant approach to relativistic particles and spinning particles. The third section is devoted to the description of spinless particle in terms of the diffeomorphisms group. In the fourth and fifth sections we construct the reparametrization invariant in the $(1,1)$ superspace worldvolume action for $N = 1$ spinning particle. Some further possibilities of applying the developed formalism are discussed in Conclusions.

2 Conformally invariant description

In this section for convenience of reader we remind the conformally invariant description of the relativistic particle and extended spinning particle.

The action for bosonic massless relativistic particle in $D$ - dimensional spacetime can be written in terms of $D + 2$ coordinates $x_A$, $A = 0, 1, \ldots, D + 1$, of the spacetime with the signature

$$\Sigma_A = (- + + \ldots + + -): \quad (2.1)$$
\[ S = \int d\tau \left( \frac{1}{2} \dot{x}^2 - \frac{1}{2} \lambda x^2 \right). \]  

(2.2)

Besides of the \( SO(D, 2) \) invariance, it is gauge-invariant under the transformations

\[ \delta x = \epsilon \dot{x} - \frac{1}{2} \dot{\epsilon} x, \]

(2.3)

\[ \delta \lambda = \epsilon \dot{\lambda} + 2 \dot{\epsilon} \lambda + \frac{1}{2} \ddot{\epsilon}. \]

(2.4)

The relation of the action (2.2) with the usual \( D \)-dimensional action is established by solving the equation of motion for the Lagrange multiplier \( \lambda \)

\[ x^A x_A \equiv x^a x_a + 2 x_+ x_- = 0; \quad a = 0, \ldots, D - 1; \quad x_\pm = \frac{1}{\sqrt{2}} (x_D \pm x_{D+1}). \]

(2.5)

In terms of new variables

\[ \tilde{x} = \frac{x}{x_+}, \quad e = \frac{1}{x_+}, \quad (x_- = -\frac{x^a x_a}{2x_+}), \]

(2.6)

the Lagrangian becomes

\[ L = \frac{1}{2} \dot{\tilde{x}}^2 e. \]

(2.7)

Its reparametrization invariance

\[ \delta \tilde{x} = \epsilon \ddot{x}, \quad \delta e = \dot{\epsilon} e + \ddot{\epsilon} \]

(2.8)

is the consequence of (2.3)-(2.4).

The modification of the action (2.2) to the case of extended spinning particle \( \mathbb{F} \), \( \mathbb{7} \) is \( \mathbb{3} \)

\[ L = \left( \frac{1}{2} \dot{x}^2 + \frac{1}{2} i \gamma_i \cdot \gamma_i \right) - \left( \frac{1}{2} \lambda x^2 + i \lambda_i \gamma_i \cdot x + \frac{1}{2} i \lambda_{ij} \gamma_i \cdot \gamma_j \right) \]

(2.9)

(for \( N = 1 \) spinning particle the action was constructed in \( \mathbb{1} \)). Here \( \gamma_i, \ i = 1, \ldots, N \), are Grassmann variables which become \( \gamma \) - matrices upon quantization. After the solution of the equations of motion for the Lagrange multipliers \( \lambda \), \( \lambda_i \) and some redefinitions like (2.6) one can derive the usual \( D \)-dimensional action for \( N \)-extended spinning particle in the form of \( \mathbb{3}, \mathbb{7} \).

So, the Lagrange multipliers \( \lambda \) in pure bosonic case and \( \lambda, \lambda_i \) in the case of extended spinning particle play the crucial role in the conversion of the \( D + 2 \)-dimensional actions into \( D \)-dimensional ones. Nevertheless, their geometrical meaning as well as the nature of initial \( D + 2 \) coordinates \( x_A \) is unclear. In the next sections we will show that all this functions of \( \tau \) have an interpretation in terms of parameters of diffeomorphisms groups.
3 Geometrical description of the massless particle

Consider the auxiliary 1-dimensional bosonic space with the coordinate $s$. The generators of the corresponding diffeomorphisms group

$$L_m = is^{m+1}\frac{\partial}{\partial s},$$

form the Virasoro algebra without central charge

$$[L_n, L_m] = -i(n - m)L_{n+m}.$$  

In what follows we will consider the subalgebra of the algebra (3.2) which is formed by the regular at the origin generators $L_m, m \geq -1$.

The most natural is the following parametrization of the group element

$$G = e^{i\tau L_{-1}} \cdot e^{iU^{(1)}L_1} \cdot e^{iU^{(2)}L_2} \cdot e^{iU^{(3)}L_3} \cdots e^{iU L_0},$$

in which all multipliers with the exception of $e^{iU L_0}$, $U \equiv U^{(0)}$, are ordered by the dimensionality of the correspondent generators: $[L_m] = m$. Such structure of the group element simplifies the evaluation of the variations $\delta U^{(m)}$ under the infinitesimal left action

$$G' = (1 + i\epsilon)G,$$

where $\epsilon = \sum_{m=0}^{\infty} \epsilon^{(m)} L_{m-1}$ belongs to the algebra of the diffeomorphisms group. The transformation laws of the coordinates in (3.3) are [10]

$$\delta \tau = \epsilon(\tau) \equiv \epsilon^{(0)} + \epsilon^{(1)}\tau + \epsilon^{(2)}\tau^2 + \ldots,$$

$$\delta U = \dot{\epsilon}(\tau),$$

$$\delta U^{(1)} = -\dot{\epsilon}(\tau)U^{(1)} + \frac{1}{2}\ddot{\epsilon}(\tau),$$

$$\delta U^{(2)} = -2\dot{\epsilon}(\tau)U^{(2)} + \frac{1}{6}\dddot{\epsilon}(\tau).$$

In general $U^{(n)}$ transforms through $\tau$ and $U^{(m)}$, $m < n$. At this stage it is natural to consider all parameters as the fields in one dimensional space parametrized by the coordinate $\tau$. It means the following active form of the transformations of the parameters $U(\tau), U^{(m)}(\tau)$

$$\delta U(\tau) = -\epsilon(\tau)\dot{U}(\tau) + \dot{\epsilon}(\tau),$$

$$\delta U^{(1)}(\tau) = -\epsilon(\tau)\dot{U}^{(1)}(\tau) - \dot{\epsilon}(\tau)U^{(1)}(\tau) + \frac{1}{2}\ddot{\epsilon}(\tau),$$

$$\delta U^{(2)}(\tau) = -\epsilon(\tau)\dot{U}^{(2)}(\tau) - 2\dot{\epsilon}(\tau)U^{(2)}(\tau) + \frac{1}{6}\dddot{\epsilon}(\tau).$$

One can easily verify that the functions $x = e^{U(\tau)/2}$ and $\lambda = -3U^{(2)}(\tau)$ have exactly the transformation laws [2.3]-[2.4] with $\epsilon(\tau) = -\epsilon$. Simultaneously $U^{(1)}(\tau)$ transforms as one dimensional Cristoffel symbol.
The independence of $\delta U^{(2)}$ from $U$ and $U^{(1)}$ means that one can consider more than one group elements

$$G_\mathcal{A} = e^{i\tau L_{-1}} \cdot e^{iU^{(2)} L_2} \cdot e^{iU^{(3)} L_3} \cdots e^{iU^{(1)}_\mathcal{A} L_1} \cdot e^{iU_\mathcal{A} L_0}, \quad \mathcal{A} = 0, 1, \ldots, D + 1,$$

(3.12)

which have identical values of parameters $\tau$ and $U^{(m)}(\tau)$, $m \geq 2$, and differ in the values of the parameters $U^{(1)}_\mathcal{A}$ and $U^{(0)}_\mathcal{A} \equiv U_\mathcal{A}$. This property is valid when all of these group elements are transformed with the same infinitesimal transformation parameter $\varepsilon(\tau)$.

In general it is true for any group which admits the parametrization in the form $G = K \cdot H$, where $H$ is some subgroup of the group $G$ and $K$ parametrizes the corresponding coset $K = G/H$. One can consistently consider the set of group elements

$$G_\mathcal{A} = K \cdot H_\mathcal{A}$$

(3.13)

with equal coset element $K$ and different elements of subgroup $H_\mathcal{A}$. This property (the equality of the coset elements for all $G_\mathcal{A}$) is invariant with respect to the left multiplication

$$G_\mathcal{A} \rightarrow G'_\mathcal{A} = g \cdot G_\mathcal{A}$$

(3.14)

with some group element $g$.

Consider the Cartan’s differential form for each value of the index $\mathcal{A}$

$$\Omega_\mathcal{A} = G^{-1}_\mathcal{A} dG_\mathcal{A} = i\Omega^{(-1)}_\mathcal{A} L_{-1} + i\Omega^{(0)}_\mathcal{A} L_0 + i\Omega^{(1)}_\mathcal{A} L_1 + \ldots.$$  

(3.15)

All their components ($\Omega^{(-1)}_\mathcal{A}, \Omega^{(0)}_\mathcal{A}, \Omega^{(1)}_\mathcal{A}, \ldots$) are invariant with respect to the left transformation (3.14). The explicit expressions for the components of the $\Omega$-form are:

$$\Omega^{(-1)}_\mathcal{A} = e^{-U^{(1)}_\mathcal{A}} d\tau,$$

(3.16)

$$\Omega^{(0)}_\mathcal{A} = dU_\mathcal{A} - 2d\tau U^{(1)}_\mathcal{A},$$

(3.17)

$$\Omega^{(1)}_\mathcal{A} = (dU^{(1)}_\mathcal{A} + d\tau (U^{(1)}_\mathcal{A})^2 - 3d\tau U^{(2)}_\mathcal{A}) e^{U_\mathcal{A}}, \ldots.$$  

(3.18)

The first of these forms is differential one-form einbein. The covariant derivatives (carrying the external index $\mathcal{A}$) calculated with its help are

$$D_{\tau, \mathcal{A}} = e^{U_\mathcal{A}} \frac{d}{d\tau}.$$  

(3.19)

The most interesting is the form $\Omega^{(1)}_\mathcal{A}$. The following expression for the action

$$S = -\frac{1}{2} \int \sum_\mathcal{A} \Sigma_\mathcal{A} \Omega^{(1)}_\mathcal{A} =$$

(3.20)

$$= -\frac{1}{2} \int d\tau \sum_\mathcal{A} \Sigma_\mathcal{A} e^{U_\mathcal{A}} (\dot{U}^{(1)}_\mathcal{A} + (U^{(1)}_\mathcal{A})^2 - 3U^{(2)}_\mathcal{A}),$$

4
where $\Sigma_A$ is the signature (2.1), is invariant under the transformation (3.14) and corresponds to the first order formalism for the action (2.2). Indeed, after the integration by parts in the first term and change of variables

$$x_A = e^{U_A(\tau)/2}, \quad p_A = e^{U_A(\tau)/2}U_A^{(1)}, \quad \lambda = -3U^{(2)}(\tau)$$

it becomes (omitting summation over indices $A$ with signature (2.1))

$$S_f = \int d\tau(\dot{x}p - \frac{1}{2}p^2 - \frac{1}{2}\lambda x^2).$$

This action is invariant under the gauge transformations

$$\delta x = \epsilon \dot{x} - \frac{1}{2} \dot{\epsilon} x,$$

$$\delta \lambda = \epsilon \dot{\lambda} + 2\epsilon \lambda + \frac{1}{2} \ddot{\epsilon}.$$

$$\delta p = \epsilon \dot{p} + \frac{1}{2} \ddot{\epsilon} p - \frac{1}{2} \dddot{\epsilon} x.$$

After the elimination of $p_A$ with the help of its equation of motion $p_A = \dot{x}_A$ the action (3.22) coincides with the action (2.2).

### 4 N = 1 spinning particle in a superconformal gauge

To generalize the approach on the spinning particles we firstly consider more simple example of the $N = 1$ Superconformal Algebra (SCA)

$$[L_m, L_n] = -i(m - n)L_{m+n}$$

$$[L_m, G_s] = -i(\frac{m}{2} - s)G_{m+s}$$

$$\{G_r, G_s\} = 2L_{r+s}.$$

The indices $m, n \geq -1$ are integer and $r, s \geq -1/2$ -halfinteger. Following the considerations of the previous Chapter and [10] we write the group element as

$$G_A = e^{i\tau L_{-1}} \cdot e^{i\theta G_{-1/2}} \cdot e^{i\Theta(3/2)G_{3/2}} \cdot e^{iU^{(2)}L_2} \ldots$$

$$e^{i\Theta A G_{1/2}} \cdot e^{iU^{(1)}_{A}L_1} \cdot e^{iU_A L_0}, \quad A = 0, 1 \ldots, D + 1.$$

Last three multipliers in this expression form the subgroup of the whole superconformal group and they consistently can carry external index $A$, as discussed in the previous Chapter.

All parameters (Grassmann $\Theta$-s and commuting $U$-s) are considered as superfunctions of $\tau$ and $\theta$ which parametrize the $(1, 1)$ superspace. The variation of
superspace coordinates under the left action of infinitesimal superconformal transformation can be written in terms of one bosonic superfunction $\Lambda$

$$\delta \tau = \Lambda - \frac{1}{2} \theta D_\theta \Lambda, \quad (4.5)$$

$$\delta \theta = -\frac{i}{2} D_\theta \Lambda, \quad (4.6)$$

where

$$D_\theta = \frac{\partial}{\partial \theta} + i \theta \frac{\partial}{\partial \tau}, \quad (4.7)$$

is the flat supercovariant derivative.

To calculate the invariant differential $\Omega$-forms one should take into account that Grassmann parity of differential of any variable is opposite to its own Grassmann parity, i.e. $d\tau$ is odd and $d\theta$ is even [11]. The general expression for $\Omega$-form is

$$\Omega^A = G^{-1}_A dG_A = i\Omega^{(-1)}_A L_{-1} + i\Omega^{(-1/2)} A G_{-1/2} + i\Omega^{(0)} A L_0 + i\Omega^{(1/2)} A G_{1/2} + i\Omega^{(1)} A L_1 + \ldots, \quad (4.8)$$

where two first components

$$\Omega^\tau_A \equiv \Omega^{(-1)}_A = (d\tau - id\theta \theta)e^{-U_A} = dx^M E^\tau_M A, \quad (4.9)$$

$$\Omega^\theta_A \equiv \Omega^{(-1/2)} A = \{d\theta - (d\tau - id\theta \theta)\Theta_A\}e^{-U_A/2} = dx^M E^\theta_M A \quad (4.10)$$

define supervielbein $(x^1 \equiv \tau, x^2 \equiv \theta)$:

$$E^A_M A = \begin{vmatrix} e^{-U_A} & -\Theta_A \cdot e^{-U_A/2} \\ -ie^{-U_A} \cdot \theta & e^{-U_A/2}(1-i\Theta_A \cdot \theta) \end{vmatrix}$$

The covariant derivatives $D_B A \equiv E^M_B A \partial_M, \quad (B = \tau, \theta)$, are defined with the help of inverse supervielbein

$$E^M_A A = \begin{vmatrix} e^{U_A}(1+i\Theta_A \cdot \theta) & \Theta_A \cdot e^{U_A} \\ ie^{U_A/2} \cdot \theta & e^{U_A/2} \end{vmatrix}$$

As a result

$$D_\theta A = e^{U_A/2} D_\theta, \quad D_\tau A = e^{U_A}(D_\tau + \Theta_A D_\theta), \quad (4.11)$$

where $D_\tau \equiv \frac{\partial}{\partial \tau}$ and $D_\theta$ are flat covariant derivatives. The invariant integration measure is

$$dV_A = d\tau d\theta Ber(E^A_M A), \quad (4.12)$$

where $d\theta$ is the Berezin differential and

$$Ber(E^A_M A) = e^{-U_A/2}. \quad (4.13)$$

Note, that all considered quantities – supervielbein, covariant derivatives and integration measure, depend on the external index $A$. 

6
To construct the action for $N = 1$ spinning particle consider the component $\Omega^{(1)}_A$ and express it in terms of the full system of invariant differential forms $\Omega^\tau_A$ and $\Omega^\theta_A$

$$\Omega^1_A = \Omega^\tau_A Y_A + \Omega^\theta_A \Gamma_A.$$ (4.14)

The coefficients are also invariant. In particular, $\Gamma_A$ is odd and can be used for the construction of invariant action

$$S = \frac{i}{2} \int \sum_A dV_A \Sigma_A \Gamma_A = \frac{i}{2} \int d\tau d\theta \sum_A \Sigma_A e^{U_A}(D_\theta U^{(1)}_A - iD_\theta \Theta_A \Theta_A + 2i\Theta_A U^{(1)}_A - 2i\Theta^{(3/2)}).$$ (4.16)

After the introduction of new variables

$$X_A = e^{U_A/2}, \quad \Pi_A = e^{U_A/2} U^{(1)}_A, \quad \Xi_A = e^{U_A/2} \Theta_A,$$

integration by parts in first term and omitting the index $A$ the action becomes

$$S = \frac{i}{2} \int d\tau d\theta (-2\Pi D_\theta X - iD_\theta \Xi \cdot \Xi + 2i\Xi \Pi - 2i\Theta^{3/2} \cdot X^2).$$ (4.17)

The equation of motion for $\Pi$ gives $\Xi = -iD_\theta X$. Making use of this equation and identity $D_\theta^2 = i\partial_\tau$ one can find the final result for the action in terms of even superfields

$$X_A = x_A + i\theta \gamma_A$$ (4.18)

and odd ones

$$\Theta^{3/2} = \frac{1}{2}(\lambda_{\text{odd}} - \theta \lambda),$$ (4.19)

$$S = \frac{i}{2} \int d\tau d\theta (\dot{X}D_\theta X + 2\Theta^{3/2} X^2)$$ (4.20)

After the Berezin integration over $\theta$ it coincides with the manifestly conformal component action for the $N = 1$ spinning particle (2.9).

5 Reparametrization invariant $N = 1$ spinning particle

The diffeomorphisms group of the superspace with one even and one odd coordinates $s$ and $\eta$ is generated by two families of even operators $N_n$, $n \geq -1$, and $M_m$, $m \geq 0$

$$N_n = is^{n+1} \frac{\partial}{\partial s}, \quad M_n = is^n \eta \frac{\partial}{\partial \eta}$$ (5.1)

and two families of odd operators $P_r$, $Q_r$, $r \geq -1/2$

$$P_{n-1/2} = is^n \frac{\partial}{\partial \eta}, \quad Q_{n-1/2} = is^n \eta \frac{\partial}{\partial s}.$$ (5.2)
Their algebra is

\[
[N_m, N_n] = -i(m-n)N_{m+n}, \quad (5.3)
\]

\[
[N_m, M_n] = i n M_{m+n}, \quad (5.4)
\]

\[
[N_m, P_s] = i(s + \frac{1}{2})P_{m+s}, \quad (5.5)
\]

\[
[N_m, Q_s] = -i(m-s + \frac{1}{2})Q_{m+s}, \quad (5.6)
\]

\[
[M_m, P_s] = -iP_{m+s}, \quad (5.7)
\]

\[
[M_m, Q_s] = iQ_{m+s}, \quad (5.8)
\]

\[
\{P_r, Q_s\} = iN_{r+s} + i(r + \frac{1}{2})M_{r+s}. \quad (5.9)
\]

The superconformal algebra as subalgebra is generated by

\[
L_n = N_n + \frac{n+1}{2}M_n, \quad G_r = P_r - iQ_r. \quad (5.10)
\]

One can take the rest of linearly independent generators in the form

\[
M_n, \quad F_r = P_r + iQ_r. \quad (5.11)
\]

It is convenient to write the group element as

\[
G_A = e^{i\tau L_{-1}} \cdot e^{i\theta P_{-1/2}} \cdot e^{i\psi Q_{1/2}} \cdot e^{iV(1)M_1} \cdot e^{i\Theta_{3/2}P_{3/2}} \cdot e^{i\Psi_{3/2}Q_{3/2}} \cdots \quad (5.12)
\]

\[
e^{i\Theta A P_{1/2}} \cdot e^{i\Psi A Q_{1/2}} \cdot e^{iU_A^{(1)}L_1} \cdot e^{iU_A L_0} \cdot e^{iV_A M_0}, \quad A = 0, 1, \ldots, D + 1.
\]

The last five multipliers in this expression form the subgroup and corresponding parameters \(\Theta_A, \Psi_A, U_A^{(1)}, U_A, V_A\) carry additional external index \(A\). All parameters (odd \(\Theta, \Psi\) and even \(U, V\)) again are considered as superfunctions of \(\tau\) and \(\theta\) which parametrize the \((1, 1)\) superspace. The left infinitesimal transformation leads to the reparametrization of superspace coordinates

\[
\delta \tau = a(\tau, \theta), \quad \delta \theta = \xi(\tau, \theta) \quad (5.13)
\]

and to the following variation of \(\psi\)

\[
\delta \psi = -\partial_\theta a + \dot{a}\psi - \partial_\theta \xi \psi. \quad (5.14)
\]

One can show that such gauge freedom is enough to choose gauge

\[
\psi = -i\theta. \quad (5.15)
\]

Before going to such gauge one can calculate all invariant quantities - supervielbein, covariant derivatives and integration measure:

\[
E_{M\dot{A}} = \begin{pmatrix}
e^{-U_A} & -\Theta_A \cdot e^{-V_A} \\
e^{-U_A} \cdot \psi & e^{-V_A}(1 + \Theta_A \cdot \psi)
\end{pmatrix}
\]
\[ D_{\theta A} = e^{V_A}D_\theta, \quad D_\theta = \partial_\theta - \psi \partial_\tau, \quad (5.16) \]
\[ D_{\tau A} = e^{U_A}(\partial_\tau + \Theta_A D_\theta), \quad (5.17) \]
\[ Ber(E_{M, A}) = e^{-U_A+V_A}. \quad (5.18) \]

As in the case of superconformal algebra, described in the previous section, consider the component of \( \Omega \)-forms corresponding to the generator \( L_1 \)
\[ \Omega(L_1)_A = \Omega^\tau_A \cdot \tilde{Y}_A + \Omega^\theta_A \cdot \tilde{\Gamma}_A, \quad (5.19) \]
and write the invariant action in the form
\[ S = \frac{i}{2} \int d\tau d\theta \cdot e^{-U_A+V_A} \sum_A \tilde{\Gamma}_A = \quad (5.20) \]
\[ \frac{i}{2} \int d\tau d\theta \sum_A \sum_A \cdot e^{-U_A+2V_A} \{ D_\theta U_A^1 + D_\theta \Theta_A \cdot \Psi_A + \Psi^{3/2} + D_\theta \psi \cdot \Theta^{3/2} - U_A^1(\Psi_A \cdot + D_\theta \psi \cdot \Theta_A) - V^{(1)}(\Psi_A \cdot - D_\theta \psi \cdot \Theta_A) \}. \]

Obviously, the combination \( \Psi^{3/2} + D_\theta \psi \cdot \Theta^{3/2} \) should be considered as one independent field. The field \( V^{(1)} \) in the action plays the role of Lagrange multiplier which leads to the equation
\[ \sum_A \sum_A \cdot e^{-U_A+2V_A} \cdot (\Psi_A - D_\theta \psi \cdot \Theta_A) = 0. \quad (5.21) \]

Note, that in the superconformal subgroup, generated by (5.10), takes place more strong equation for each value of the index \( A \)
\[ (\Psi_A + i \Theta_A) = 0 \]

The equation (5.21), in contrast, contains the summation over the index \( A \). Nevertheless, one can solve the constraint (5.21) and substitute the solution for \( \sum_A \sum_A \cdot e^{-U_A+2V_A} \cdot \Psi_A \) back into the action (5.20). In the resulting action without loss of any information one can choose the gauge (5.13). Indeed, when we calculate the equation of motion for \( \psi \) and choose this gauge, they are consequence of the equations of motion, which follow from the gauge fixed action.

One easily can see that the gauge fixing reduces the action (5.20) to the action for \( N = 1 \) superconformal group (1.13). So, the action (5.20), which is invariant under the transformations of the whole diffeomorphism group of the (1, 1) superspace describes the \( N = 1 \) spinning particle.

### 6 Conclusions

In the framework of nonlinear realizations of infinite-dimensional diffeomorphism groups of one dimensional bosonic space and (1, 1) superspace we have constructed
the conformally and reparametrization invariant actions for massless particle and $N = 1$ spinning particle in arbitrary dimension $D$. It is achieved by simultaneous consideration of several group elements. The parameters of corresponding group points include simultaneously the coordinates and momenta. The interaction between coordinates is obliged to parameters with higher dimensions, which are the same for all considered points on the group space.

It would be interesting to apply the method developed here and in [10] to other infinite dimensional symmetries, such as diffeomorphism groups of extended superspaces and higher dimensional spaces, W-algebras and so on

**Acknowledgments.** I would like to thank Ch. Preitshopf for useful discussions.

This investigation has been supported in part by the Russian Foundation of Fundamental Research, grant 99-02-18417, joint grant RFFR-DFG 96-02-04022, and INTAS, grants 93-127-ext, 96-0308, 96-0538, 94-2317 and grant of the Dutch NWO organization.

**References**

[1] R. Marnelius. Phys.Rev., **D20** (1979) 2091
[2] A. Barducci, R. Casalbuoni and L. Lusanna. Nuovo Cimento **35A** (1976) 377
[3] V.D. Gershun and V.I. Tkach. JETP Lett., **29** (1979) 320
[4] L. Brink, S Deser, B. Zumino, P. DiVecchia and P. Hove. Phys.Lett., **64B** (1976) 435
[5] P.A. Collins and R.W. Tucker. Nucl.Phys. **B121** (1977) 307
[6] L. Brink, P. DiVecchia and P.S. Howe. Nucl.Phys. **B118** (1977) 76
[7] P. Howe, S. Penati, M. Pernici and P. Townsend. Phys.Lett., **B215** (1988) 555
[8] A. Pashnev and D. Sorokin. Phys.Lett. **B253** (1991) 301
[9] W. Siegel. Int.J.Mod.Phys., **A3** (1988) 2713
[10] A. Pashnev. Nonlinear realizations of the (super)diffeomorphism groups, geometrical objects and integral invariants in the superspace. Preprint JINR E2-97-122.  
    e-Print archive [hep-th/9704203](http://arxiv.org/abs/hep-th/9704203)
[11] F.A. Berezin. Introduction to the algebra and analysis with anticommuting variables. Moscow Univ., 1983