Level densities and shell corrections of superheavy nuclei

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Abstract. The intrinsic level densities of superheavy nuclei in the $\alpha$-decay chains of $^{296,298,300}_{120}$ nuclei are calculated using the single-particle spectra obtained with the modified two-center shell model. The level density parameters are extracted and compared with their phenomenological values used in the calculations of the survival of excited heavy nuclei. The dependences of the level density parameters on the mass and charge numbers as well as on the ground-state shell corrections are studied.

1. Introduction

An extension of the nuclear chart to the elements with $Z > 100$ and the study of their properties provide an important insight into the determination of an island of stability. The experimental trend of nuclear properties ($Q_\alpha$ values and half-lives) as well as the production cross sections of superheavy elements (SHE) reveal an increasing stability of nuclei approaching $N = 184$, and indicate quite a large shell effects behind $Z = 114$ [1-6]. This means that the predictions of relativistic and nonrelativistic mean-field models [7-11] seem to be valid. In accordance with these self-consistent models a center of the island of stability is expected at $Z = 120 - 126$ and $N = 172$ or 184. In the $(N;Z)$ plane, the line, along which all new SHE were discovered in the actinide-based reactions with $^{48}$Ca beam, just approaches this region [1].

In Refs. [12, 13] we modified the two-center shell model (TCSM) [14] to describe the quasiparticle structure of well-studied heavy nuclei in the best way. Based on the TCSM single-particle potential and shape parametrization we calculated the ground-state binding energies and transition energies $Q_\alpha$ for $\alpha$ decays of nuclei with $105 \leq Z \leq 126$. The calculated $Q_\alpha$ are found to be in a good agreement with available experimental data. The dependence of $Q_\alpha$ on the neutron number at given $Z$ indicates the strong shell effects at $N = 184$. Also our calculations result in the stronger shell effects at $Z \geq 120$ than at $Z = 114$.

In this work we use the single-particle spectra calculated with the modified TCSM to investigate the nuclear level densities (NLD) of superheavy nuclei. The NLD is an important characteristic both for the nuclear reaction calculations and for the investigation of the stability of the new superheavy elements. At low excitations the single-particle level densities are closely related to the shell structure and could indicate the position of the island of stability of heaviest nuclei. The level density, as a function of excitation energy, is required to calculate the survival...
probability and, correspondingly, the production cross section of heavy nucleus. For nuclei lighter than Cf, the systematically study of NLD was performed in Refs. [15, 16]. In this work we performed the study of the intrinsic level densities of nuclei with 100 \( Z \leq 130 \) whose \( \alpha \)-decay chains contain \(^{296,298,300}120\) which are of interest for future experiments.

2. A few words about performed calculations.

In our work we use the expression for the nuclear single-particle level density obtained in the superfluid model [17, 18]. Employing the saddle-point method, the intrinsic level density \( \rho(U) \) of nucleus with \( Z \) protons, \( N \) neutrons, and excitation energy \( U \) is expressed as

\[
\rho = \frac{\exp[S(\beta, \lambda_Z, \lambda_N)]}{(2\pi)^{3/2}\sqrt{D}}.
\]

Here, \( S \) is the entropy, \( \beta = T^{-1} \) is the inverse temperature, \( \lambda_Z \) and \( \lambda_N \) are the chemical potentials for protons and neutrons, respectively, and \( D \) is the determinant of the matrix comprised of the second derivatives of the entropy. In the superfluid model adopted here, the entropy \( S \) is expressed as

\[
S = 2 \sum_{k=Z,N} \sum_{\nu} \left\{ \ln[1 + \exp(-\beta E_{k\nu})] + \frac{\beta E_{k\nu}}{1 + \exp(\beta E_{k\nu})} \right\},
\]

where proton \((k = Z)\) and neutron \((k = N)\) quasi-particle energies \( E_{k\nu} = \sqrt{\left(\varepsilon_{k\nu} - \lambda_k\right)^2 + \Delta_k^2} \) are calculated using the single-particle levels \( \varepsilon_{k\nu} \) obtained with the TCSM. The details of the calculations with the TCSM as well as the parameters used are described in Refs. [12, 13]. The Fermi energies \( \lambda_k \) and correlation functions \( \Delta_k \) \((k = N, Z)\), are calculated at the thermodynamic equilibrium of nucleus determined by solving the system of equations

\[
Z = \sum_{\nu} \left( 1 - \frac{\varepsilon_{Z\nu} - \lambda_Z}{E_{Z\nu}} \tanh \left[ \frac{1}{2} \beta E_{Z\nu} \right] \right), \quad
N = \sum_{\nu} \left( 1 - \frac{\varepsilon_{N\nu} - \lambda_N}{E_{N\nu}} \tanh \left[ \frac{1}{2} \beta E_{N\nu} \right] \right),
\]

\[
\frac{2}{G_{Z}} = \sum_{\nu} \frac{\tanh[\beta E_{Z\nu}/2]}{E_{Z\nu}^2}, \quad \frac{2}{G_{N}} = \sum_{\nu} \frac{\tanh[\beta E_{N\nu}/2]}{E_{N\nu}},
\]

where \( G_{Z} \) and \( G_{N} \) are the constants of pairing interaction. In Eqs. (3), the sums run over all the single-particle levels considered.

In Fig. 1, the calculated level densities \( \rho_m(U) \) are presented for well-deformed nuclei \(^{162}\text{Dy}, \text{Er}\) together with the experimental data [19]. The agreement with experimental data is rather good, which provides us some confidence in the predictions of \( \rho(U) \) in the region of SHE.

3. Dependence of level density parameter on shell corrections

Intrinsic level density is strongly influenced by nuclear shell structure. For magic or nearly magic nuclei, the level density is smaller than for mid-shell nuclei at the same excitation energy. This effect is related to the large single-particle spacings in the nuclei with the closed shell in the ground state. So, irregularities of the single-particle spectra are responsible for the shell corrections and peculiarities of the intrinsic level density. The calculated intrinsic level densities for the superheavy nuclei are presented in Fig. 2 for two nuclei from each of the considered alpha-decay chains of the elements \(^{296,298,300}120\). The solid lines correspond to the isotopes \(^{296,298,300}120\) and exemplify the behavior of the level density in nuclei with quite a strong shell correction, while the dotted lines are for the isotopes \(^{264,266,268}104\) with smaller shell corrections. As seen, at low excitation energies from 10 to 30 MeV the level densities in \( Z = 120 \) isotopes are smaller within one order of magnitude than those in \( Z = 104 \) isotopes due to the smaller densities of the single-particle states near the Fermi surfaces. At smaller excitation energies
Figure 1. Calculated level densities $\rho_m$ in $^{162}$Dy and $^{166}$Er as functions of excitation energy (lines) are compared with the experimental data (symbols) from Ref. [19]. For the $^{162}$Dy nucleus, the experimental and theoretical results are multiplied by factor 0.1.

$U < 10$ MeV, the effects of pairing are stronger in the nuclei with weaker shell effects and the level densities of nuclei with strong and weak shell effects become comparable. At higher excitation energies, $U > 40 - 50$ MeV, the shell effects are faded and the level densities of all heavy nuclei become similar.

The similar effects as for $Z = 120$ isotopes are found for the isotones with $N = 184$ (Fig. 3). So our calculations specify the shell closure for neutrons at $N = 184$ which is consistent with all previous predictions.

We found that the best fit of the calculated intrinsic level density with the Fermi-gas expression is achieved if one uses the level density parameter calculated as

$$ a = S^2 / (4U), \quad (4) $$

where $S$ and $U$ are entropy and excitation energy of the nucleus, respectively. Since the thermodynamical equilibrium is assumed for each excitation energy, the level density parameter can be written as

$$ a = S_N^2 / 4U_N + S_Z^2 / 4U_Z = a_N + a_Z, \quad (5) $$

where $S_N(S_Z)$ and $E_N(E_Z)$ are the entropies and excitation energies of only neutron (proton) subsystems.

The Fig. 4 presents the dependences of shell corrections $\delta E_{sh}$ and level density parameters $a$ at $U = 10$ MeV and $U = 60$ MeV excitation energies on the atomic number $A$ for three $\alpha$-decay chains containing the SHE $^{296;298;300}120$ which could be synthesized in near future with available projectiles and targets. The data of Ref. [22] are used for $\delta E_{sh}$. One can see the strong correlations between the shell corrections and level density parameter $a$ at excitation energy...
parametrization [20] of the level density parameter:

\[ a(A; U) = \tilde{a}(A) \left[ 1 + \frac{1 - \exp\{-U/E_D'\}}{U} \delta E_{sh} \right], \]

where \( \tilde{a}(A) \) is the parameter smoothly depending on \( A \). It defines \( a \) at large excitations when the shell effects are washed out. By analyzing the level density parameters with the Eq. (4), the value of the damping parameter \( E_D' = 27 \) MeV is found. The corresponding asymptotic level density parameter \( \tilde{a}(A) \) can be fitted with the following functions [20]:

\[ \tilde{a}(A) = \alpha A + \beta A^2, \]
where the constants $\alpha=0.118\text{ MeV}^{-1}$ and $\beta=-0.53 \times 10^{-4}\text{ MeV}^{-1}$ are found with the least square method. These values are close to those proposed in Ref. [20].

The level density parameters for the nuclei with $Z \geq 100$ are of especial interest to look for the position of the next proton and neutron shell closures beyond $Z = 82$. To separate the neutron and proton shell effects one should investigate the proton ($a_Z$) and neutron ($a_N$) level density parameters defined in Eq. (5) (see Fig. 5). At $Z = 108$ and 120 there are minima of $a_Z$ in all chains. This reflects quite strong proton shell effects at $Z = 108$ and 120. At $Z = 120$, the minima of $a$ are the deepest and well pronounced. The similar behavior of $a$ occurs near $Z = 82$. The sub-shell at $Z = 114$ exists but provides weaker shell effect than at $Z = 120$. For nuclei with $Z = 124 - 128$, the minima of $a$ are due to the neutron shell at $N = 184$.

4. Summary

The level density parameters were calculated for the nuclei of alpha-decay chains containing $^{296}120$, $^{298}120$, and $^{300}120$. The strong shell effects at $Z = 120$ and $N = 184$ were demonstrated to be in accordance with our previous calculations and shell-model predictions. The dependencies of the level density parameter on the shell correction and excitation energy were studied. The damping factor in the well-known expression (6) [20] was found to be $E_D' = 27\text{ MeV}$. For superheavy nuclei considered, the level density parameter is approximately $A/(11 - 13)\text{ MeV}$ at

Figure 4. Calculated ground-state shell corrections $\delta E_{sh}$ (lower part), the level density parameter $a$ at $U=10$ (middle part) and 60 MeV (upper part), obtained with Eq. (4), as functions of mass number $A$. The nuclei from alpha-decay chains containing $^{296}120$, $^{298}120$, $^{300}120$ are marked by closed circles, open circles, and stars, respectively.

Figure 5. Calculated level density parameters $a$ ($U=10$ MeV) as a function of neutron number $N$ (upper part) and proton number $Z$ (lower part) for the superheavy nuclei considered. The nuclei from alpha-decay chains containing $^{296}120$, $^{298}120$, $^{300}120$ are marked by closed circles, open circles, and stars, respectively.
excitation energies corresponding to (3-5) neutron evaporation channels.

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References
[1] Oganessian Yu Ts 2007 J Phys. G 34 R165, 2010, Phys. Rev. Lett. 104 142502, 2013, Phys. Rev. C 87 014302.
[2] Hofmann S et al. 2007 Eur. Phys. J. A 32 251, 2009, Lec. Notes Phys. 764 203.
[3] Armbruster P 2008 Eur. Phys. J. A 37 159.
[4] Adamian G G, Antonenko N V and Sargsyan V V 2009 Phys. Rev. C 79 054608.
[5] Adamian G G, Antonenko N V, Sargsyan V V and Scheid W 2010 Nucl. Phys. A834 345c.
[6] Dong J, Zuo W and Scheid W 2011 Phys. Rev. Lett. 107 012501; Li Z, Sun B, Shen C H and Zuo W 2013 Phys. Rev. C 88 057303.
[7] Reinhard P G 1989 Rep. Prog. Phys. 52 439.
[8] Ring P 1996 Prog. Part. Nucl. Phys. 37 193.
[9] Bender M, Heenen P H and Reinhard P G 2003 Rev. Mod. Phys. 75 121.
[10] Meng J, Toki H, Zhou S G, Zhang S Q, Long W H and Geng L S 2006 Prog. Part. Nucl. Phys. 57 470.
[11] Li J J, Long W H, Margueron J and Van Giai N 2013 arXiv: nucl-th 1303.2765v1.
[12] Adamian G G, Antonenko N V and Scheid W 2010 Phys. Rev. C 81 024320.
[13] Adamian G G, Antonenko N V, Kuklin S N and Scheid W 2010 Phys. Rev. C 82 054304.
[14] Maruhn J and Greiner W 1972 Z. Phys. A 251 431.
[15] Iljinov A S et al. 1992 Nucl. Phys. A543 517.
[16] von Egidy T and Bucurescu D 2005 Phys. Rev. C 72 044311.
[17] Decowski P, Grochulski W, Marcinkowski A, Siwek K and Wilhelmi Z 1968 Nucl. Phys. A 110 129.
[18] Adeev G D and Cherdantsev P A 1975 Yadernaya Fizika 21 491.
[19] Melby E, Guttorpsen M, Rekstad J, Schiller A, Siem S and Voinov A 2001 Phys. Rev. C 63 044309.
[20] Ignatyuk A B, Smirenkin G N and Tishin A S 1975 Yadernaya Fizika 21 485.
[21] Shlomo S and Natowitz J B 1991 Phys. Rev. C 44 2878.
[22] Kuzmina A N, Adamian G G, Antonenko N V and Scheid W 2012 Phys. Rev. C 85 014319.