Phonon Emission from a 2D Electron Gas: Evidence of Transition to the Hydrodynamic Regime

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Using as a thermometer the temperature dependent magneto-transport of a two-dimensional electron gas, we find that effective temperature scales with current as $T_e \sim I^a$, where $a = 0.4 \pm 2\%$ in the Shubnikov-de Haas regime, and $0.53 \pm 2\%$ in both the integer and fractional quantum Hall effect. This implies the phonon energy emission rate changes from the expected $P \sim T^3$ to $P \sim T^4$. We explain this, as well as the dramatic enhancement in phonon emission efficiency using a hydrodynamic model.

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The concept of the inelastic scattering time is fundamental to the physics of disordered quantum systems. Elastic scattering produces random interference patterns of the electron waves but does not introduce decoherence. In contrast, inelastic scattering determines the phase coherence time which in turn controls the strength of the quantum interference observed, for example in the Aharonov-Bohm effect in mesoscopic rings [1] and in Anderson localization in disordered metals [2]. Inelastic scattering is also fundamental to questions of energy equilibration and cooling. In order to see Coulomb blockade and related effects in mesoscopic devices it is necessary to go to extremely low temperatures and to isolate the devices from their environment by means of large series resistances ($R > h/e^2$). It turns out to be difficult to keep the electrons transiting these thin film resistors from falling out of equilibrium and heating up [3] due to the long inelastic scattering time at low temperatures [4].

One difficulty in studying the dynamics of inelastic scattering in ordinary metallic films is the problem of making and comparing samples with significantly different characteristics. We overcome this problem here by studying high mobility two-dimensional electron gases both in the low magnetic field Shubnikov-de Haas (SdH) regime and the high field quantum Hall (IQHE and FQHE) regimes within the same sample.

Previous models of cooling by phonon emission have always assumed that the electrons are in plane wave states and invoke momentum conservation. Simple phase space considerations then dictate that the power radiated into phonons is $P \sim T^{d+2}$, where $d$ is the dimension of space seen by the phonons ($d = 3$ in this case). The factor $T^{d}$ arises from the phonon phase space and the \textit{statically screened} phonon matrix element. This applies both to metal films and inversion layers in piezoelectric media. One additional factor of $T$ represents the mean energy per phonon and the final factor of $T$ represents the number of electrons which are sufficiently thermally excited to emit a phonon. Equating the radiated power to the Joule heating gives

$$T_e \sim \left[ \rho_{xx} I^2 \right]^{1/5} \sim I^{0.4} \tag{1}$$

Experimentally we find $T_e \sim I^a$ with $a = 0.40 \pm 2\%$ in the SdH regime.

In the presence of strong disorder or high magnetic field, the reduced mean free path (and magnetic length) can put the system into a new, hydrodynamic regime. Experimentally we find a new exponent $a = 0.53 \pm 2\%$ (corresponding to $P \sim T^4$) and a nearly two order of magnitude enhancement of the emission rate. Mittal [6] has discussed the failure of existing theories to properly describe this regime. We present a new hydrodynamic theory of phonon emission in which \textit{dynamic screening} plays a crucial role and which is in quantitative agreement with these unexpected experimental results.

Current ($I$) or electric field ($E$) dependent transport measurements have been used extensively to study the electron phonon scattering process in 2 DEG’s in separate regimes such as at zero $B$ [5], in the SdH regime [3], in between Hall plateaus in the IQHE regime [8,9] and at the $\nu = \frac{1}{2}$ FQHE plateau [10]. The latter indicated a possible dramatic enhancement in the electron-phonon coupling. However, no experiment has been performed that systematically explores different regimes in the same sample to explicitly investigate the consequences of the QHE, even though it has been suggested that the QHE will affect the electron phonon scattering [11].

The sample used in this work is a GaAs/AlGaAs heterostructure with an electron density $n = 0.65 \times 10^{11} \text{cm}^{-2}$, and a mobility $\mu \approx 500,000 \text{cm}^2/\text{Vs}$ at 0.1 K. The high mobility was chosen to allow the SdH oscillations to be seen at low $B$. The donor spacer thickness is 1800Å, and the Hall bar pattern on the sample has a channel width of 300μm. The large device is used to avoid the influence of edge effects [4], which are known

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to be more pronounced in small devices, and to avoid problems with heat being carried out of the sample directly by electron diffusion. The sample and a ruthenium oxide resistor, to measure the temperature \( T_e \) at the sample position, are mounted in a dilution refrigerator \([10,11(c)]\) for 45 mK \(< T_e < 1 \) K. The transport coefficients are measured by the standard ac lock-in technique. It is known that low-frequency ac and dc current produce the same results for the current dependence \([10,11(b)]\).

The inset of Fig.1 shows \( \rho_{xx} \) as a function of \( B \) for \( 0.06 \) T \(< B < 0.16 \) T (which corresponds to spin-unresolved Landau level filling factors, \( 18 \leq \nu \leq 38 \)) at 3 different excitation currents \( (I) \) at fixed \( T_b = 100 \) mK. We measure the \( T_b \) dependence of these SdH oscillations for \( 100 \) mK \(< T_b < 750 \) mK while applying \( I = 5 \) nA. By fitting the \( T \) dependent \( \rho_{xx} \) amplitudes to Ando’s semi-classical formula \([14]\), we obtain the electron effective mass \( 0.068 \pm 0.001 \) in units of the free electron mass, comparable to that obtained from cyclotron resonance measurements \([7]\).

Using the \( T \) dependent \( \rho_{xx} \) amplitude as a thermometer, we find the same relation between \( T_e \) and \( I \) for all oscillations down to 0.092 T in the inset of Fig.1. Oscillations at lower \( B \) exist only in a very small range of \( I \). We choose to plot, in Fig.1, \( T_e \) vs. \( I \) for the two oscillations pointed to by arrows in the inset of Fig.1, because their amplitudes are measurable in a wider range of \( I \). The closed and open symbols represent the data taken from the peak at \( B = 0.139 \) T and the dip at \( B = 0.133 \) T respectively. Different symbols represent data taken at different \( T_b \)’s which are labeled by the side of the data set. For clarity, the open symbol data set has been offset by scaling by a factor of 1/3. For each \( T_b \), there is a current \( I_0 \), below which \( T_e \) remains constant, and above which \( T_e \) merges into a single power law: \( T_e \sim I^a \) over about one and a half decades in \( I \) with a close to 0.4. The solid line under each data set has a slope of 0.4 and is drawn for reference. For comparison, we also draw a dashed line of slope 0.5. The experimental value of \( a \) is obtained by first collecting all data points which have \( T_e > 2T_b \) for each fixed \( T_b \), and then performing a linear least-squares fit to the resulting data points. We obtain \( a = 0.4 \pm 2\% \), where 2\% sets the statistical 68\% confidence limit.

We also deduce the absolute energy relaxation rate \((1/\tau_{in})\) by equating the input Joule heating to the cooling rate \( P \approx CT_e/5\tau_{in} \), where \( C \) is the specific heat of the 2 DEG at \( B = 0 \). We find \( 1/\tau_{in} \sim 7.3 \times 10^5 \text{sec}^{-1}\text{K}^{-3} \) at \( B = 0.13 \) T. Price has calculated \( 1/\tau_{in} = 2.92 \times 10^5 \text{sec}^{-1}\text{K}^{-3} \) for screened piezoelectric coupling in the absence of disorder. \([13]\) This agreement indicates that statically screened piezoelectric electron-phonon coupling controls the physics in the SdH regime.

In the same sample, we also study the effect of \( I \) on \( \rho_{xy} \) in the FQHE regime \([14]\). The inset of Fig.2 shows \( \rho_{xy} \) as function of \( B \) at 3 different \( I \)’s with fixed \( T_b = 100 \) mK, between well developed FQHE states at \( \nu = \frac{1}{3} \) and \( \frac{2}{5} \). There is no signature of higher order fractional states for \( \frac{1}{3} < \nu < \frac{2}{5} \) in our \( T \) range. We characterize the effect of by assigning a \( T_e \) at each \( I \), where \( T_e \) is obtained by using \((d\rho_{xy}/dB)^{\text{max}}\) as a thermometer. Here, \((d\rho_{xy}/dB)^{\text{max}}\) is the maximum slope of \( \rho_{xy} \) vs. \( B \) for \( \frac{1}{3} < \nu < \frac{2}{5} \) measured with \( I = 3 \) nA. In Fig.2, we plot \( T_e \) vs. \( I \) in this region using open symbols, scaled by a factor of 1/6. Different symbols represent data taken at different \( T_b \)’s or upon different cooldowns of the sample. We also study the IQHE regime for Landau level index \( N = 2 \downarrow \) (5 < \( \nu \) < 6) and 2 \uparrow (4 < \( \nu \) < 5), where \( \uparrow \) and \( \downarrow \) represent spin direction. We do not study the IQHE transition for \( N = 1 \downarrow \) and \( 1 \uparrow \) because there appears to have FQHE structures in \( \rho_{xy} \). The result for \( N = 2 \downarrow \) is plotted in Fig.2 by closed symbols. There are no data points for \( I > 2 \) \( \mu \)A for the FQHE data because \((d\rho_{xy}/dB)^{\text{max}}\) is saturated to the classical Hall resistance. However, it is still clear that in both the IQHE and FQHE regimes \( T_e \) behaves the same and there is a single power law, \( T_e \sim I^a \), over more than one decade in \( I \) with \( a \approx 0.5 \). The solid line of slope 0.5 and the dashed line of slope 0.4 are drawn for reference. We obtain the best fit values \( a = 0.53 \pm 2\% \) in the FQHE regime and \( a = 0.54 \pm 2\% \) and \( 0.53 \pm 4\% \) for \( N = 2 \downarrow \) and \( 2 \uparrow \) respectively. This is significantly different from that \((0.40 \pm 2\%)\) in the SdH regime. Similar results have also been obtained in a second sample. This change in \( a \) can be viewed as a change in the cooling rate from \( P \sim T^5 \) in the SdH to \( P \sim T^4 \) in the QHE.

We note that in our sample the peak values of \( \rho_{xx} \) are 212, 210, 335, and 4620 A/\( \Omega \) for \( B = 0.139 \) T, \( N = 2 \downarrow \), \( N = 2 \uparrow \), and 1/3 < \( \nu \) < 2/5 respectively at \( T = 100 \) mK.
FIG. 2. Electron effective temperature $T_e$ vs. applied current $I$ in the IQHE for $5 < \nu < 6$ (closed symbols), and in the FQHE for $1/3 < \nu < 2/5$ (open symbols). Each curve is labeled by the bath temperature. The dashed reference line under each data set has a slope of 0.5, and the solid line 0.4.

Inset: $\rho_{xy}$ vs. $B$ at $T_b = 100$ mK for three different $I$’s in the FQHE for $1/3 < \nu < 2/5$.

The $\rho_{xx}$ and hence the Joule heating at the fractional transition is 22 times that at the integer transition shown in Fig.2, and yet the two curves are nearly identical. In addition, the current threshold in Fig.2 for seeing heating at $T_b = 100$ mK is about twice as large as for the SdH case in Fig.1. This indicates that, at the integer transition, cooling is about 4 times as efficient as in the SdH regime, and at the fractional transition, fully 80 times as efficient. One possible explanation for this dramatic enhancement and the $P \sim T^4$ power law is outlined in the following hydrodynamic model.

The standard Fermi’s Golden Rule expression for the rate of phonon emission [19,20] can be reexpressed in the following form to yield the power radiated by the 2DEG at temperature $T_e$ into the lattice at $T_b = 0$:

$$P = \sum_{\lambda,Q} \omega_{\lambda}(Q)|M_{\lambda}(Q)|^2 n_B(\beta_e \hbar \omega_{\lambda}(Q)) G(q,\omega_{\lambda}(Q)),$$  \hspace{1cm} (2)

where $G(q,\omega)$ is given by

$$\frac{4\pi\hbar\epsilon_0 q}{2\pi e^2} \left\{ -2\text{Im} \frac{1}{\epsilon(q,\omega)} \right\} = \frac{2\omega(2\kappa\epsilon_0)^2}{e^2} \text{Re} \left( \frac{1}{\sigma_{xx}} \right),$$  \hspace{1cm} (3)

$\omega_{\lambda}(Q)$ is the phonon frequency for polarization $\lambda$, $q$ is the projection of the wavevector $Q$ onto the plane of the 2DEG, $M_{\lambda}(Q)$ is the electron-phonon matrix element, $n_B$ is the Bose-Einstein factor, $\beta_e = 1/k_B T_e$, $\kappa = 12.9$ is the background semiconductor dielectric constant and $\epsilon(q,\omega)$ is the 2DEG dielectric function. The expression for $G$ in terms of the conductivity $\sigma_{xx}$ is the standard one used in the theory of surface acoustic wave absorption.

We assume that at high $B$ and low $T$ the system is in the hydrodynamic limit where typical phonon wave vectors and frequencies are negligible compared to characteristic scales over which the conductivity varies, so that $\sigma_{xx}$ is a real constant. This is justified from direct measurement [22] that at the IQHE critical point the peak value of $\sigma_{xx}$ (at $q \sim 0$) is approximately independent of frequency up to at least 14 GHz ($\hbar \omega \sim 0.7 K$).

At low temperatures, piezoelectric coupling dominates [18,20] so that $|M_{\lambda}(Q)|^2 \sim 1/Q$. Using the fact that $\sigma_{xx}$ is constant, simple power counting in Eq.(2) yields $P \sim T^4$. The physical interpretation of this is the following. In the hydrodynamic limit, momentum conservation is lost because of frequent collisions of the electrons with the disorder potential. The rate energy is absorbed from a phonon mode depends only on the square of its electric field and is independent of wave vector and frequency. Hence the 2DEG acts like a black (more precisely a “gray”) body and emits phonons with the usual $T^4$ spectrum.

In the clean limit, the power counting is different since the conductivity is non-local: $\text{Re}(1/\sigma_{xx}) \approx q/\hbar \tau_T 2\kappa \epsilon_0 v_F$, where $\tau_T$ is the Thomas-Fermi wave vector, $v_F$ is the Fermi velocity. This changes the temperature exponent to $P \sim T^5$ and correctly reproduces the usual Thomas-Fermi static screening result [13] for the prefactor. In the hydrodynamic limit, the static screening approximation fails (since charge fluctuations are no longer high frequency plasmons at $\omega \sim q/\tau_T$ but rather relax slowly with $z = 1$ dynamics: $\omega = i(\sigma_{xx}/2\kappa\epsilon_0)q$). It is not the static compressibility of the electron gas that counts, but rather its limited ability to move charge quickly. The smaller the conductivity is, the poorer the screening is and hence the larger the emitted power. Using $\sigma_{xx} = e^2/k_B T_e$ gives an elastic mean free path of $\ell_e \approx 1.9 \mu m$ at $T = 0$ and so the system enters the hydrodynamic regime at $T_e \sim 10$ mK. If however $\sigma_{xx} \sim e^2/h$, the hydrodynamic regime extends up to about 1K.

Evaluation of Eq.(3) yields

$$P = \frac{e^2}{\hbar \sigma_{xx}} 6.75 \times 10^{-2} \left( \frac{T_e}{1K} \right)^4 \text{Watts/m}^2.$$  \hspace{1cm} (4)

This expression, which contains no adjustable parameters, yields the correct $T$ dependence and (using the measured values of $\rho_{xx}$ and $\rho_{xy}$) lies below the experimentally measured power by only 17% (15%) for the IQHE (FQHE) transition. Considering the simplicity of the model, this level of agreement is quite good. Notice that one of the advantages of this formulation is that, unlike previous formulations, it is not necessary to make any assumptions about the unknown density of states in the interacting 2DEG.

Equating the Joule heating and radiated power we have

$$\epsilon(q,\omega) = \frac{4\pi\hbar\epsilon_0 q}{2\pi e^2} \left\{ -2\text{Im} \frac{1}{\epsilon(q,\omega)} \right\} = \frac{2\omega(2\kappa\epsilon_0)^2}{e^2} \text{Re} \left( \frac{1}{\sigma_{xx}} \right).$$  \hspace{1cm} (3)

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\[ T_c = 24.9(\sigma_{xx} \rho_{xx})^{1/4}(J/A/m)^{1/2}K, \]

which is \( \sim 4\% \) below the observed values. This result explains why the curves for the fractional and integer transitions are nearly identical. The factor \((\sigma_{xx} \rho_{xx})^{1/4}\) (which is unity at \( B = 0 \)) is a weak function of \( B \) in the sense that it is nearly equal for the two plateau transitions studied. [23] This thus the increased Joule heating is precisely balanced by increased electron-phonon coupling due to reduced screening. That is, the threshold for heating occurs, to a good approximation, at fixed current rather than fixed power.

We note that the result \( a = 0.53 \pm 2\% \) is close to the prediction \( a = 0.5 \) by Polyakov and Shklovskii in their theoretical model of \( T \)-scaling [24]. They consider the insulating IQHE plateau regime where Coulomb variable range hopping dominates the transport. The electron-phonon interaction is not directly relevant in that study.

The present hydrodynamic heating model for the metallic critical point assumes that phonon emission is the rate limiting step and that the electrons are effectively in equilibrium at some temperature. An alternative picture can also be developed in which one recognizes that there is a diverging correlation time at the critical point of the quantum Hall transition. [25,26] Assuming that it is this internal time scale rather than phonon emission that controls the dynamics, it can be shown [26] that \( a = z/(1 + z) = 0.5 \), also in agreement with the present experiments (since for Coulomb interactions, one expects the dynamical exponent \( z = 1 \)). More theoretical and experimental work will be needed to distinguish these two models. [26] One caveat for the quantum critical picture is that, because of the high mobility, the sample does not show critical point power-law \( T \)-scaling in the linear response transport in the \( T \) range studied.

Finally we note that in silicon deformation potential coupling yields \( P \sim T^6 \) and hence an exponent \( a = 1/3 \). This may well explain the apparent failure of dynamical scaling in the experiments of Kravchenko et al. [27] on non-linear response in Si inversion layers where \( a = 1/3 \) is observed.

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