Abstract We discuss the possibility that the detection of gravitational waves emitted by compact stars may allow to constrain the MIT bag model of quark matter equation of state. Our results show that the combined knowledge of frequency of the emitted gravitational wave and of the mass, or the radiation radius, of the source allows one to discriminate between strange stars and neutron stars and set stringent bounds on the bag constants.

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What is the absolute ground state of matter? A definite answer to this fundamental question is still lacking. Following Bodmer’s seminal paper [1], in 1980s Witten [2] suggested that matter consisting of degenerate up, down and strange quarks, called “strange” quark matter, might be bound and stable at zero temperature and pressure. This hypothesis, while not being directly verifiable in terrestrial laboratories, may be confirmed by the observation of “strange stars”, i.e. compact astrophysical objects entirely made of strange matter except, possibly, for a thin outer crust.

Theoretical studies of structural properties of quark stars, pioneered by Itho over three decades ago [3], have been booming after ROSAT discovered the isolated pulsar RXJ1856.5-3754 [4]. The data reported in Ref. [5] seemed in fact to indicate that its radiation radius should be in the range 3.8–8.2 km, so that the corresponding stellar radius would be far too small compared to that typical of neutron stars. The results of this analysis triggered a number of speculations on the nature of RXJ1856.5-3754, and it was suggested that it may be a strange star, though this hypothesis has now been ruled out [6].

Whether quark stars do exist in nature is an open question, and the astrophysical scenario in which they may form is still poorly understood. For instance, it has been argued that, besides the standard gravitational collapse, strange stars may form in low-mass X-ray binaries when, due to accretion,
matter in a neutron star core reaches sufficiently high densities to undergo a deconfinement phase transition to quark matter [7].

In this letter we discuss the possibility that detection of a gravitational signal emitted by a compact star, oscillating in its fundamental mode with frequency $\nu_f$ and damping time $\tau_f$, will allow to infer whether the source is a neutron star or a strange star and to constrain theoretical models of the quark matter equation of state (EOS).

Due to the complexity of the fundamental theory of strong interactions between quarks (Quantum Chromo-Dynamics, or QCD), theoretical studies of strange stars are necessarily based on models. The most used is the MIT bag model [8], in which the two main elements of QCD, namely color confinement and asymptotic freedom, are implemented through the assumptions that: i) quarks occur in color neutral clusters confined to a finite region of space (the bag), the volume of which is limited by the pressure of the QCD vacuum (the bag constant $B$), and ii) residual interactions between quarks are weak, and can be treated in low order perturbation theory in the color coupling constant $\alpha_s$.

The bag model parameters are the bag constant $B$, the quark masses $m_f$ (the index $f = u, d, s$ labels the three active quark flavors; at the densities relevant to our work heavier quarks do not play a role) and the running coupling constant $\alpha_s$, whose value at the energy scale $\mu$ can be obtained from the renormalization group relation

$$\alpha_s(\mu) = \frac{12\pi}{(33 - 2N_f) \ln(\mu^2/\Lambda_{QCD}^2)}.$$  \hspace{1cm} (1)

In the above equation $N_f = 3$ denotes the number of active flavors, while $\Lambda_{QCD}$ is the QCD scale parameter, whose value is constrained to the range $100 - 250$ MeV by high energy data (see, e.g., [9]). At the scale typical of the quark chemical potentials Eq. (1) yields

$$\alpha_s \in [0.4, 0.6].$$ \hspace{1cm} (2)

As quarks are not observable as individual particle, their masses are not directly measurable. However, they can be inferred from hadron properties, supplemented by theoretical calculations. According to the 2002 Edition of the Review of Particle Physics [10], the masses of up and down quarks do not exceed few MeV, and can therefore be safely neglected, while the mass of the strange quark is much larger, its value being in the range

$$m_s \in [80, 155] \text{ MeV}.$$ \hspace{1cm} (3)

The bag constant is subject to a much larger uncertainty. In early applications of the MIT bag model $B$, $\alpha_s$ and $m_s$ were adjusted to fit the measured properties of light hadrons (spectra, magnetic moments and charge radii). This procedure leads to values of $B$ that differ from one another by as much as a factor of $\sim 6$, ranging from 57.5 MeV/fm$^3$ [11] to 351.7 MeV/fm$^3$ [12], while the corresponding values of $\alpha_s$ turn out to be close to or even larger than unity. Moreover, the strange quark masses resulting from these analyses are typically much larger than the upper limit given by Eq. (3). Using the
parameters determined from fits to light hadron spectra to describe bulk quark matter is questionable, as these spectra are known to be strongly affected by the details of the bag wave-functions, as well as by spurious contributions arising from the center of mass motion.

The requirement that strange quark matter be absolutely stable at zero temperature and pressure implies that \( B \) cannot exceed the maximum value \( B_{\text{max}} \approx 95 \text{ MeV/fm}^3 \) \(^{[13]}\). For values of \( B \) exceeding \( B_{\text{max}} \), a star entirely made of deconfined quarks is not stable. Under these conditions quark matter can only occupy a fraction of the available volume and the star is said to be hybrid \(^{[14]}\). The results of Ref. \(^{[15]}\)\(^{[16]}\), based on a state-of-the art description of the low-density hadronic phase, suggest that, assuming that the transition to quark matter proceeds through the formation of a mixed phase, hybrid stars can only exist for values of the bag constant up to \( \sim 200 \text{ MeV/fm}^3 \) and contain a rather small amount of quark matter. Gravitational emission from the hybrid star models of Ref. \(^{[15]}\), corresponding to \( m_s = 150 \text{ MeV} \), \( \alpha_s = 0.5 \) and \( B = 120 \) and \( 200 \text{ MeV/fm}^3 \), referred to as APRB120 and APRB200, respectively, has been analyzed in Ref. \(^{[17]}\).

In this paper we focus on bare strange stars and consider values of the bag constant in the range

\[
B \in [57, 95] \text{ MeV/fm}^3 .
\]

The problem we shall investigate is the following. Suppose that a gravitational signal is detected, which is emitted by a compact object oscillating in its fundamental mode (\( f \)-mode), i.e. the mode which is known to be the most efficient as far as gravitational radiation is concerned \(^{[18]}\). We do not know whether the source is a neutron star or a strange star. Since the damping time of the \( f \)-mode, \( \tau_f \), is known to be of the order of a fraction of second, the mode excitation would correspond to a sharp peak in the energy spectrum of the detected signal, emitted at the mode frequency \( \nu_f \) which could be identifiable by a suitable data analysis technique. The questions we want to address are:

- Does the knowledge of \( \nu_f \) (and/or of \( \tau_f \)) allow one to say anything about the nature of the source?
- Assuming that we can establish the star is a strange star, would these data allow one to set constraints on the parameters of the MIT bag model?

To answer these questions, we compute frequency and damping time of the fundamental mode of strange stars, letting the parameters of the bag model vary in the range indicated by Eqs. (2), (3) and (4), which covers the parameter space allowed for bare strange stars. We consider masses in the range \( [0.7 M_\odot, M_{\text{max}}] \), where \( M_{\text{max}} \) is the maximum mass allowed by each choice of the model parameters. We consider bare stars without a crust, as the presence of a crust does not affect the fundamental mode frequency in a significant way.

We compare these frequencies with those computed in \(^{[17]}\) for neutron stars and for hybrid stars. In \(^{[17]}\) NS were modeled using a set of modern EOS that describe matter at supranuclear densities; they are obtained within non relativistic nuclear many-body theory and relativistic mean field theory, that
model hadronic interactions in different ways, leading to different composition and dynamics. The hybrid stars were modeled using the EOS APR120 and APR200 of Ref. [15], describing hybrid stars with a rather small admixture of quark matter.

The results of this comparison are shown in Figs. 1 and 2, where we plot $\nu_f$ and the corresponding damping time $\tau_f$, respectively, versus the mass of the star. In both figures the shaded region refers to all values of $\nu_f$ and $\tau_f$ allowed for strange stars, assuming that the parameters of the bag model
The fundamental mode frequency $\nu_f$ is plotted versus the gravitational mass, $M$, for different values of the bag constant and $\alpha_s$ and $m_s$ varying in the range indicated by Eqs. (2) and (3).

The continuous lines refer to the values of $\nu_f$ for the neutron/hybrid star models considered in [17]. Figure 1 shows that:

- Strange stars cannot emit gravitational waves with $\nu_f \lesssim 1.7$ kHz for any values of the mass in the range we consider.
- For masses lower than 1.8 $M_\odot$, above which no stable bare strange star can exist, there is a small range of frequency where neutron/hybrid stars are indistinguishable from strange stars. However, there is a large frequency region where only strange stars can emit. For instance if $M = 1.2$ $M_\odot$, a signal with $\nu_f \gtrsim 1.9$ kHz would belong to a strange star. Note that the fundamental mode frequency and damping time of the hybrid stars considered in [15] and [17] and shown in Figs. 1 and 2 (EOS APR120 and APR200), are basically indistinguishable from that of the neutron star with the same low density EOS, except when the mass is close to the
maximum mass. This is due to the small amount of quark matter these hybrid stars contain.

- Even if we do not know the mass of the star (as it is often the case for isolated pulsars) the knowledge of \( \nu_f \) allows to gain information about the source nature; indeed, if \( \nu_f \gtrsim 2.2 \text{ kHz} \) we can reasonably exclude that the signal is emitted by a neutron star.

Figure 2 contains a complementary information: for strange stars \( \tau_f \) is in general smaller than for neutron/hybrid stars.

Thus, the next question is: assuming that we know the signal has been emitted by a strange star, can we constrain in some way the parameters of the MIT bag model? In Fig. 3 we show to what extent this is possible. We plot the values of \( \nu_f \) allowed for strange stars versus the stellar mass, indicating with the same symbol the points that belong to the same value of the bag constant \( B \). From this picture we see that for a given mass the mode frequency increases with \( B \), and that, knowing \( M \) and \( \nu_f \) we would be able to set constraints on \( B \) much more stringent than those provided by the available experimental data.

Similar information can be derived by the simultaneous knowledge of \( \nu_f \) and the radiation radius

\[
R_\infty = \frac{R}{\sqrt{1 - 2M/R}} \tag{5}
\]

In Fig. 4 we plot \( \nu_f \) as in Fig. 3, but versus \( R_\infty \). In addition we plot as continuous lines the values of \( \nu_f \) for the neutron/hybrid star models considered in \cite{17}. The figure shows that radiation radii smaller than \( \simeq 13 \text{ km} \) should be attributed to strange stars, whereas if \( 13 \lesssim R_\infty \lesssim 15.5 \text{ km} \) the star can either be a strange or a neutron/hybrid star. Higher values of \( R_\infty \) can only belong to neutron/hybrid stars. Figure 4 further shows that the knowledge of \( \nu_f \) constrains the value of \( B \).

The problem whether quark stars can be discriminated from neutron stars using gravitational waves, which we discuss in this letter, has already been addressed in \cite{19}, \cite{20} and \cite{21}.

In \cite{19} \( \nu_f \) and \( \tau_f \) have been computed for strange star models obtained within the MIT bag model for two values of \( B \), i.e. \( B = 56 \) and 67 MeV/fm\(^3\), \( m_s = 150 \text{ MeV} \) and assuming \( \alpha_s \) both vanishing and different from zero.

In \cite{20} strange stars have been modeled putting \( \alpha_s \) and \( m_s \) to zero and choosing \( B = 75 \) and 137 MeV/fm\(^3\).

Finally, in \cite{21} the values of \( \nu_f \) and \( \tau_f \) have been plotted versus the radiation radius for the following values of the parameters: \( \alpha_s = 0.6, \ m_s = 0, 150, 300 \text{ MeV, and } B = 57 \) and 209 MeV/fm\(^3\). These values of the parameters and the star central density were chosen to fit \( R_\infty \) within the range 3.8--8.2 Km. However, this choice implies very small values of the stellar mass (ranging from \( \sim 0.05 \) to \( \sim 0.5 \ M_\odot \)) which are hard to explain within current evolutionary scenarios for neutron stars formation. As a consequence, they obtain values of \( \nu_f \) larger than \( \sim 9 \text{ kHz} \), much higher than those we consider.

The study we propose in this paper differs from the preceeding literature in what it explores the entire range of allowed parameters in a systematic way. Our results show that the detection of a signal emitted by a compact
star pulsating in its fundamental mode, combined with a complementary 
information on the stellar mass or the radiation radius, would allow one to 
discriminate between neutron/hybrid stars and strange stars; in addition, 
we show that it would also be possible to constrain the bag constant to a 
range much smaller than that provided by the available data from terrestrial 

The fundamental mode can be excited in a variety of astrophysical pro-
cesses, like in a glitch, in a close interaction with a companion, or after birth 
in a gravitational collapse. Recent simulations of gravitational collapse show 
that a significant fraction of the total energy emitted in gravitational waves, 
of the order of $10^{-9} - 10^{-8} M_\odot c^2$, is indeed emitted at the frequency of the 
$f$-mode [22], [23]. However, this energy is too low to be detectable by current 
interferometric antennas like VIRGO or LIGO, unless the collapse occurs in 
our galaxy; but we know that, unfortunately, the rate of collapse per galaxy 
is only of a few per hundred years. In order to detect signals emitted by more 
distant sources, we would need very sensitive, high frequency detectors, like 
EURO or EURO-XYLO, which have been considered in a preliminary as-

As discussed in [24], this kind of detectors would be able to see signals emitted 
by oscillating stars up to the distance of the VIRGO cluster if the energy 
stored in the mode is of the order of $10^{-7} - 10^{-8} M_\odot c^2$, which is not too 
far from present estimates. Indeed, current numerical simulations assume ax-
symmetric collapse, but if asymmetries are present the emitted energy may 
be larger. And moreover rotation, which is certainly present in stars, has the 
effect of lowering the mode frequencies, enhancing detection chances.

Thus, we can conclude that asteroseismology will become a branch of 
gravitational wave research when high frequency detectors will be operating, 
and we hope that the EURO project will be reconsidered in a not too far 
future.

It has also to be mentioned that, although somewhat more refined dy-
namical models have been proposed, the MIT bag model appears to provide 
a quite reasonable description of quark matter. The results discussed in Ref. 
[25] show that the EOS obtained from the Nambu Jona-Lasinio (NJL) model, 
in which quark masses are dynamically generated through the appearance of a 
condensate associated with chiral symmetry breaking, is in fact similar to the 
one obtained from the bag model. Using more sophisticated models, such 
as the NJL model, may turn out to be required to describe the possible occurrence 
of the pairing instability induced by the attractive one-gluon exchange 
interaction between quarks, leading to a color superconducting phase [25] 
[26]. However, the relative stability of the different superconducting phases 
discussed in the literature is not yet firmly established, and their occurrence 
is expected to affect mostly transport properties and cooling, rather then the 
stellar structure, and consequently the $f$-mode frequency, on which we focus 
in this paper.

References
1. A.R. Bodmer, Phys. Rev. D 4, (1971) 1601.
2. E. Witten, Phys. Rev. D 30, (1984) 272.
3. N. Itoh, Prog. Teor. Phys. 44, (1970) 291.
4. F.M. Walter, S.J. Wolk and R. Neuhäuser, Nature 379, (1996) 233.
5. J.J. Drake et al. Ap. J. 572. (2002) 996.
6. R. Turolla, S. Zane, J.J. Drake, Ap. J. 603, (2004) 265; J.E. Trümper, V. Burwitz, F. Haberl and V.E. Zavlin, Nucl. Phys. B (Proc. Suppl.) 132, (2004) 560; J.A. Pons, F.M. Walter, J.M. Lattimer, M. Prakash, R. Neuhäuser and P.H. An, Ap. J. 564, (2002) 981.
7. K.S. Cheng and Z. G. Dai, Phys. Rev. Lett. 77, (1996) 1210.
8. A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorne and V.F. Weiskopf, Phys. Rev. D 9, (1974) 3471.
9. R.K. Ellis, W.J. Stirling, and B.R. Webber, QCD and Collider Physics (Cambridge University Press, Cambridge, 1996).
10. K. Hagiwara, et al, Particle Data Group, Phys. Rev. D 66, (2002) 1.
11. T. De Grand, R.L. Jaffe, K. Johnsson and J. Kiskis, Phys. Rev. D 12, (1975) 2060.
12. C.E. Carlson, T.H. Hansson and C. Peterson, Phys. Rev. D 27, (1983) 1556.
13. E. Farhi and R.L. Jaffe, Phys. Rev. D 30, (1984) 2379.
14. N.K. Glendenning, Compact Stars (Springer-Verlag, Berlin, 1997).
15. A. Akmal, V.R. Pandharipande and D.G. Ravenhall, Phys. Rev. C 58 (1998) 1804
16. O. Benhar and R. Rubino, A & A 434, (2005) 247.
17. O. Benhar, V. Ferrari and L. Gualtieri, Phys. Rev. D 70, (2004) 124015.
18. G. Allen, N. Andersson, K.D. Kokkotas and B.F. Schutz, Phys. Rev. D 58, (1998) 124012.
19. C.W. Yip, M.-C.Chu, P.T. Leung, Astroph. J. 513, (1999) 849.
20. Y. Kojima and K. Sakata, Prog. Teor. Phys. 108, (2002) 801.
21. H. Sotani and T. Harada, Phys. Rev. D 68, (2003) 024019; H. Sotani, K. Khor and T. Harada, Phys. Rev. D 69, (2004) 084008.
22. M. Shibata, Y. Sekiguchi, Phys. Rev. D 71 (2005) 024014.
23. H. Dimmelmeier, J. A. Font, E. Müller, A&A 393, (2002) 523.
24. V. Ferrari, G. Miniutti, J. A. Pons, Class. Quant. Grav. 20 n. 17, (2003) S841.
25. M. Buballa, Phys. Rep. 407 (2005) 205.
26. M. Alford and S. Reddy, Phys. Rev. D 67, (2003) 074024.