Predictions of the FBD model for the synthesis cross sections of \(Z = 114-120\) elements based on macroscopic-microscopic fission barriers

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A complete set of existing data on hot fusion reactions leading to synthesis of superheavy nuclei of \(Z = 114-118\), obtained in a series of experiments in Dubna and later in GSI Darmstadt and LBNL Berkeley, was analyzed in terms of a new angular-momentum dependent version of the fusion-by-diffusion (FBD) model with fission barriers and ground-state masses taken from the Warsaw macroscopic-microscopic model (involving non-axial shapes) of Kowal et al. The only empirically adjustable parameter of the model, the injection-point distance \((s_{inj})\), has been determined individually for all the reactions. Very regular systematics of this parameter have been established. The regularity of the obtained \(s_{inj}\) systematics indirectly points at the internal consistency of the whole set of fission barriers used in the calculations. (In an attempt to fit the same set of data by using the alternative theoretical fission barriers of Möller et al. we did not obtain such a consistent result.) Having fitted all the experimental excitation functions for elements \(Z = 114-118\), the FBD model was used to predict cross sections for synthesis of elements \(Z = 119\) and 120. Regarding prospects to produce the new element \(Z = 119\), our calculations prefer the \(^{252}\text{Es}(^{48}\text{Ca},xn)\) reaction for which the synthesis cross section of about 0.2 pb in 4n channel at \(E_{c.m.} \approx 220\) MeV is expected. The most favorable reaction to synthesize the element \(Z = 120\) turns out to be \(^{249}\text{Cr}(^{50}\text{Ti},xn)\), but the predicted cross section for this reaction is only 6 fb (for 3n and 4n channels).

PACS numbers: 25.70.Jj, 25.70.Gh

I. INTRODUCTION

Superheavy nuclei of \(Z \geq 104\) were synthesized either in “cold” fusion reactions on closed-shell \(^{208}\text{Pb}\) and \(^{209}\text{Bi}\) target nuclei bombarded by projectiles ranging from Ti to Zn or in “hot” fusion reactions, in which the heaviest available actinide targets were bombarded with the neutron rich \(^{48}\text{Ca}\) projectiles. See review articles ¹, ² and ³, respectively. In the cold fusion reactions only one neutron is emitted from the compound nucleus to form the final compound-residue nucleus in its ground state. In hot fusion reactions more neutrons are emitted. At each step of the deexcitation cascade the neutron evaporation competes with the dominating process of fission. Therefore the synthesis cross section represents only a small part of the fusion cross section.

A characteristic feature of the fusion-evaporation reactions leading to the synthesis of superheavy nuclei is enormous hindrance of the fusion process itself. Consequently, the cross sections for the synthesis of heaviest elements are measured in picobarns or even femtobarns. It is believed that the hindrance is caused by the highly dissipative dynamics of the fusing system in its passage over the saddle point on the way through the multidimensional potential energy surface from the initial configuration of two touching nuclei into the configuration of the compound nucleus. Zagrebaev and Greiner developed a method of solving Langevin equations of motion to describe this stochastic stage of the fusion process ⁴. In spite of very time consuming Langevin trajectory calculations, in which only one of say a million trajectories leads to formation of the compound nucleus, the model is used effectively to calculate synthesis cross sections for various reactions ⁵. Another approach to the process of fusion of a “dinuclear system” (DNS) was proposed in Ref. ⁶. It was assumed in this model that the dinuclear system stays in contact configuration and undergoes successive transfer of all nucleons from the lighter nucleus to the heavier partner (in competition with the quasi-fission processes). Applications of this concept have been used in recent years by several groups. In still another approach, the “fusion-by-diffusion” model ⁷, the stochastic process of shape fluctuations that lead to the overcoming the saddle point was described as the solution of the Smoluchowski diffusion equation in the deformation space along the fission valley.

The cold fusion reactions leading to the synthesis of nuclei of \(Z \leq 113\) were studied systematically with the DNS model in Ref. ⁷, with the fusion-by-diffusion (FBD) model ⁸, ⁹ and with the Langevin dynamics model ⁵. The hot fusion reactions leading to the synthesis of the heaviest nuclei of \(Z \geq 114\) have not been studied so systematically. In Ref. ⁵ excitation functions for some selected reactions were calculated although they were not confronted with experimental cross sections. Most of the publications on this topic concentrated on the predictions concerning possible ways of synthesis of the heaviest elements of \(Z = 119\) and 120 ⁴ ¹⁰ ¹¹ ¹² ¹³ ¹⁷. Only very recently, an extensive study of cold and hot fusion reactions in terms of a phenomenological approach based on the DNS model was reported.
There is one important aspect of all the models of the synthesis of superheavy nuclei that was not treated with proper attention so far. This is the question of the choice of theoretical fission barriers and ground-state masses which have to be adopted for the description of the deexcitation of the compound nucleus. It is well known that calculations of the cross sections for synthesis of superheavy nuclei are extremely sensitive to the height of the fission barrier, especially in case of “hot” fusion reactions, in which three or four neutrons are emitted from the compound nucleus. When the barrier heights are not known precisely, an error in evaluation of the $\Gamma_{n}/\Gamma_{f}$ ratio in each step of the $(xn)$ deexcitation cascade accumulates $x$ times leading to enormous errors in the calculation of the synthesis cross sections. (Here, $\Gamma_{n}$ and $\Gamma_{f}$ denote the neutron decay width and fission width, respectively.) Thus, precise knowledge of theoretical fission barriers and neutron binding energies (ground-state masses) is crucial for reasonable predictions of the synthesis cross sections.

In the last decade the mass tables of Möller et al. [13] have most frequently been used in the field of superheavy nuclei. Unfortunately, fission barriers heights are not given in these tables. Therefore, in most of the mentioned above calculations of the synthesis cross sections the ground-state shell effect of the compound nucleus (that is listed in these tables) was used as the barrier height. In this simplification, both the macroscopic deformation energy and the shell effect at the saddle configuration are neglected. It seems, therefore, that these approximate values of the fission barrier are not sufficiently accurate to guarantee reliable predictions of the synthesis cross sections. (Absolute value of both these neglected effects may be of about 1–2 MeV each, while a 1 MeV-shift of the barrier height may result in a change of the calculated cross section of $3n$ or $4n$ reaction by 2–3 orders of magnitude.)

Only in recent years systematic compilations of theoretical fission barriers of superheavy nuclei (combined with the necessary information on the ground-state masses) have become available in literature. Calculations in framework of the macro-microscopic approach were reported by Muntian et al. [20] and later by Möller at al. [21]. The model [20] has been extended recently by Kowal et al. [22, 23] by the inclusion of nonaxiality as an important new degree of freedom. Fission barriers of superheavy nuclei have been calculated also in a number of other papers within various models (see Ref. [24]. Table IV for a review), however no sufficiently systematic information on the fission barriers and, simultaneously, ground-state masses has been provided.

In the present study we adopt the “fusion-by-diffusion” (FBD) model [8, 9] for calculating the synthesis cross sections of the heaviest nuclei in hot fusion $(xn)$ reactions by using the information on the fission-barriers heights [22, 23] and other properties of the superheavy nuclei obtained within the Warsaw macroscopic-microscopic model [20].

The whole set of experimental data [3, 25–35] on the synthesis of new superheavy elements of $Z = 114–118$ (obtained in Dubna by Oganessian and coworkers and later in a series of confirming experiments at GSI Darmstadt and LBNL Berkeley) was analyzed. Based on this test of the model predictions, the calculations were then performed for experimentally unexplored yet reactions aimed at the synthesis of new elements of $Z = 119$ and 120.

II. REVIEW OF THE FBD MODEL

The fusion-by-diffusion (FBD) model [8, 9] serves for calculating cross sections for the synthesis of superheavy nuclei. Recently the model was modified in order to describe both cold fusion $(1n)$ and hot fusion $(xn)$ reactions. In this extended version [9], for each angular momentum $l$ the partial evaporation-residue cross section $\sigma_{ER}(l)$ for production of a given final nucleus in its ground state is factorized as the product of the partial capture cross section $\sigma_{cap}(l) = \pi \lambda^2 (2l+1)T(l)$, the fusion probability $P_{fus}(l)$ and the survival probability $P_{surv}(l)$:

$$\sigma_{ER} = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1)T(l) \cdot P_{fus}(l) \cdot P_{surv}(l).$$ (1)

The capture transmission coefficients $T(l)$ are calculated in a simple sharp cut-off approximation, where the upper limit $l_{max}$ of full transmission, $T(l) = 1$, is determined by the capture cross section known from the systematics described in Refs. [8, 36]. Here $\lambda$ is the wave length, $\lambda^2 = \hbar^2/2\mu E_{c.m.}$, and $\mu$ is the reduced mass of the colliding system. The fusion probability $P_{fus}(l)$ is the probability that the colliding system, after reaching the capture configuration (sticking), will eventually overcome the saddle point and fuse, thus avoiding reseparation. The other factor in Eq. (1), the survival probability $P_{surv}(l)$, is the probability for the compound nucleus to decay to the ground state of the final residual nucleus via evaporation of light particles and $\gamma$ rays, thus avoiding fission.

The cross sections for the synthesis of superheavy nuclei are dramatically small because the fusion probability $P_{fus}(l)$ is hindered (in some reactions even by several orders of magnitude) due to the fact that the saddle configuration of the heaviest compound nuclei is much more compact than the configuration of two colliding nuclei at sticking. It is assumed in the FBD model that after the contact of the two nuclei, a neck between them grows rapidly at an
approximately fixed mass asymmetry and constant length of the system. This “neck zip” is expected to carry the system towards the bottom of the asymmetric fission valley. This is the “injection point”, from where the system starts its climb uphill over the saddle in the process of thermal fluctuations in the shape degrees of freedom. Theoretical justification of the above picture of fast zipping the neck was given in Ref. [37], where the later stage of the stochastic climb uphill was described by solving the two-dimensional Langevin equation. Theoretical location of an effective injection point can be deduced from this model [37]. Also in a modified fusion-by-diffusion model [38], the location of the injection point was estimated theoretically. In our model we rely, however, on empirical determination of the injection point. Its location in the asymmetric fission valley, $s_{inj}$, is the only adjustable parameter of the FBD model.

By solving the Smoluchowski diffusion equation, it was shown in Ref. [39] that the probability of overcoming a parabolic barrier for the system injected on the outside of the saddle point at an energy $H$ below the saddle is:

$$P_{fus} = \frac{1}{2} \left(1 - \text{erf}\sqrt{H/T}\right),$$

where $T$ is the temperature of the fusing system. The energy threshold $H$ opposing fusion in the diffusion process is thus the difference between the energy of the saddle point $E_{saddle}$ and the energy of the combined system at the injection point $E_{inj}$, where $E_{inj}$ is calculated using algebraic expressions given in Ref. [9] which approximate the potential energy surface along the fission valley. The energy of the saddle point is given by the adopted theoretical value of the fission barrier $B_f$ and the ground-state energy of the compound nucleus. The corresponding values of the rotational energy at the injection point and at the symmetric saddle point are calculated assuming the rigid-body moments of inertia at these configurations [4].

As regards the survival probability $P_{\text{surv}}$, the standard statistical-model calculation were done by applying the Weisskopf formula for the particle (neutron) emission width $\Gamma_n$ and the conventional expression of the transition-state theory for the fission width $\Gamma_f$. The level density parameters $a_n$ and $a_f$ for neutron evaporation and fission channels were calculated as proposed by Reisdorf [40], with shell effects accounted for by the Ignatyuk formula [41]. All details regarding the calculations of the survival probability $P_{\text{surv}}$ can be found in our recent paper [5]. In case of calculating multiple evaporation ($xn$) channels a simplified algorithm avoiding the necessity of using the Monte Carlo method was applied [42].

### III. CALCULATIONS FOR $Z=114-120$ ELEMENTS WITH THE MACROSCOPIC-MICROSCOPIC BARRIERS

As pointed out in the Introduction, calculations of the cross sections for synthesis of superheavy nuclei are extremely sensitive to the height of the fission barrier, especially in case of “hot fusion” reactions because at each step of deexcitation cascade the competition between neutron emission and fission strongly depends on the difference of energy thresholds for these two decay modes. Therefore, in attempts to reasonably calculate the synthesis cross sections, the choice of realistic and consistent theoretical information on the fission barrier heights and the ground-state masses is essential. In our previous applications of the FBD model, devoted mostly to analysis of cold fusion sections, the choice of realistic and consistent theoretical information on the fission barrier heights and the ground-deexcitation cascade the competition between neutron emission and fission strongly depends on the difference of

\begin{equation}
B \text{inj} \leq B \text{CN}
\end{equation}

\begin{equation}
B \text{inj} = \frac{E \text{inj}}{T} = \frac{E \text{saddle}}{T} - \frac{E \text{CN}}{T}
\end{equation}

\begin{equation}
B \text{CN} = B \text{fus} = B \text{g.s.}
\end{equation}

\begin{equation}
B \text{CN} = B \text{fus} = B \text{g.s.}
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In the first stage of calculations a complete set of experimental data on the synthesis of $Z$=114-118 elements in reactions induced by $^{48}$Ca projectiles on $^{242,244}$Pu, $^{243,245}$Am, $^{245,248}$Cm, $^{249}$Bk and $^{249}$Cf targets was analyzed with the aim to determine location of the injection point $s_{inj}$. Here $s_{inj}$ is defined as the excess of the total length of the combined system over the length of the initial system (at the touching configuration) when the neck-zip process brings the system to the asymmetric fission valley.

In order to determine systematics of $s_{inj}$ for the set of hot fusion reactions [3, 25-35], the individual values of $s_{inj}$ were deduced for each reaction and each particular $xn$ channel by adjusting the assumed $s_{inj}$-value to the experimental synthesis cross section at the maximum of a given $xn$ excitation function. The compilation of so deduced $s_{inj}$-values is displayed in Fig. 1 as a function of the kinetic energy excess $E_{c.m.} - B_0$ above the Coulomb barrier $B_0$. (For the definition of $B_0$ see Ref. [4].)

It should be commented here that values of $s_{inj}$ are inferred from the synthesis cross sections in a model-dependent way, assuming particular ground-state masses and fission barriers. Therefore the result of this procedure obviously...
depends to some extent on these theoretical input data used in the calculations. Consequently, the systematics of $s_{inj}$ obtained in calculations employing different sources of the theoretical input data may appear different (cf. the $s_{inj}$ systematics obtained in recent calculations of cold fusion reactions [2] analyzed assuming masses and fission barriers based on the Thomas-Fermi model [43]).

It is clearly seen from Fig. 1 that the injection distance $s_{inj}$ increases with the decreasing energy $E_{c.m.} - B_0$, in agreement with expectations based on the dynamics of nucleus-nucleus collisions, for example the classical trajectory calculations [44]. Very good correlation between the $s_{inj}$-values and the corresponding energies $E_{c.m.} - B_0$ can be viewed as an argument in favor of the fission barriers of Kowal et al. [22, 23] because such a striking correlation would be very unlikely if the theoretical barrier heights were inconsistent with experimental values.

A linear fit to the dependence of $s_{inj}$ on $E_{c.m.} - B_0$ in Fig. 1,

$$s_{inj} \approx 4.09 \text{ fm} - 0.192(E_{c.m.} - B_0) \text{ fm/MeV},$$  

represents the only empirical input to our model and once this systematics of the injection-point distance is determined in form of Eq. (3), one can use the FBD model to calculate excitation functions of fusion-evaporation reactions without any adjustable parameters.

In Fig. 2 we present a comparison of our FBD model predictions of excitation functions for different $xn$ channels with experimental synthesis cross sections (assigned to the corresponding $xn$ channels) in the following hot fusion reactions: $^{244}\text{Pu}^{(18}\text{Ca},xn)^{292−z}114$ [3, 24, 27, 31, 34], $^{243}\text{Am}^{(48}\text{Ca},xn)^{291−z}115$ [3, 24, 25, 245]$^{(48}\text{Ca},xn)^{293−z}116$ [3, 26], $^{248}\text{Cm}^{(48}\text{Ca},xn)^{296−z}116$ [3, 27, 33], $^{249}\text{Bk}^{(48}\text{Ca},xn)^{297−z}117$ [30] and $^{249}\text{Cf}^{(48}\text{Ca},xn)^{297−z}118$ [3, 28]. The largest deviations of our general fit to the data approach a factor of 10 that corresponds effectively to a difference of about 0.5 MeV in the assumed height of the theoretical fission barrier. Given this high sensitivity of the model predictions to the assumed fission barrier heights, the overall agreement between the FBD predictions and measured cross sections is quite satisfactory. (It is rather unlikely that the accuracy of the theoretical predictions of individual fission barriers might be much better than ±0.5 MeV.)

It is instructive to compare results of calculations presented in Figs. 1 and 2 with predictions for an alternative set of theoretical fission barriers. In Fig. 3 we present individual values of the injection distance $s_{inj}$ deduced for the same set of data on hot fusion reactions [3, 22, 32], but obtained assuming fission barriers of Möller et al. [21], the only alternative, complete set of necessary information available in literature. The barriers of Möller et al. are considerably higher than barriers of Kowal et al. [22, 23], thus resulting in larger values of the calculated survival probability $I_{surv}$. Consequently, the procedure of "calibrating" the individual $s_{inj}$ values by fitting the predictions to experimental cross sections resulted in larger values of the determined injection distance $s_{inj}$. Contrary to the consistent systematics of $s_{inj}$ values shown in Fig. 1, Fig. 3 demonstrates the evident inconsistency of the set of $s_{inj}$ values obtained for the barriers of Ref. [21]. It is seen from Fig. 3 that the $s_{inj}$ values range from 5.5 fm to 8.5 fm and are too large to have a reasonable physical meaning. (In most cases, they correspond to the injection distance that exceeds the distance of the scission configuration.) Most importantly, the individual points in Fig. 3 seem to be almost randomly scattered and do not show any correlation with energy.

There is one more inconsistency that can be noticed when the fission barriers of Ref. [21] and the ground-state masses [19] are used. Namely, for these high fission barriers and corresponding $Q$-values, the predicted positions of the maxima of the $xn$ excitation functions are shifted by some 5−7 MeV toward lower energies as compared with the data (and also with respect to the predictions for barriers of Ref. [22, 23]). This effect is illustrated in Fig. 4, where the data for 3$n$ and 4$n$ channels in the $^{243}\text{Am}^{(48}\text{Ca},xn)^{291−z}115$ reaction are compared with the excitation functions calculated for these two reaction channels. This considerable energy shift, seen also for other reactions, stems from the fact that for the Möller’s barriers [21] and the corresponding ground-state masses [19], the fission barrier $B_f$ is larger than the neutron binding energy $B_n$ for all the compound nuclei formed in the studied reactions. Consequently, the $\Gamma_n/\Gamma_f$ ratio rises very fast at low excitation energies thus influencing the position and shape of the $xn$ excitation functions.

From Figs. 1 and 2 it is seen that contrary to the generally higher fission barriers of Ref. [21], the input data of Kowal et al. [22, 23] give a reasonable agreement of the calculated and measured cross sections as well as the very clear correlation between $s_{inj}$ and $E_{c.m.} − B_0$ that “calibrates” the injection distance $s_{inj}$. This entitles us to believe that the set of theoretical fission-barrier heights and ground-state masses [22, 23] is quite adequate for a wide range of the heaviest nuclei considered in this study. Therefore we are going to use them for predictions of cross sections of yet unexplored reactions aimed at the synthesis of new elements $Z = 119$ and 120.

Regarding possibilities to produce the element $Z = 119$ we consider, first of all, the most preferred reactions induced by the favorable beam of $^{48}\text{Ca}$ on two isotopes of einsteinium, $^{252}\text{Es}$ and $^{254}\text{Es}$. These extremely difficult-to-produce targets might possibly be available in the near future. Therefore we present in Figs. 5(a) and 5(b) the predicted energy dependence of the $xn$ cross sections in reactions on these two isotopes. The largest cross section, which turns out to be at the edge of experimental possibilities (about 0.2 pb in $4n$ channel at $E_{c.m.} \approx 220$ MeV), is predicted for the $^{252}\text{Es}^{(48}\text{Ca},xn)^{300−z}119$ reaction. Surprisingly, the cross section in the reaction on a more neutron-rich target,
same reaction, \(5.77 \text{ MeV}, \) \(5.55 \text{ MeV} \) and \(6.03 \text{ MeV} \), respectively. Very recently Zagrebaev et al. \[16\] have reported a prediction for the same reaction was given. \[4\]

values of the fission barrier taken as the pure ground-state shell effect from tables of Ref. \[19\].

recently in Ref. \[15\], however a relatively large cross section (about 0.6 pb) was obtained for probably overestimated barriers \[22, 23\] in the chain of subsequent neutron-emitting nuclei, \(Z\_\text{Es}\text{(}48\text{Ca},\text{xn})^{302-x}\text{Es}119\), is by one order of magnitude lower (only about 15 fb). This is a consequence of lower fission barriers \[22, 23\] in the chain of subsequent neutron-emitting nuclei, \(B_f = 4.87 \text{ MeV}, \) \(4.98 \text{ MeV}, \) \(5.77 \text{ MeV} \) in \(302\text{Es}119, \) \(301\text{Es}119\) and \(300\text{Es}119\), while in a chain of neutron decays starting from the \(300\text{Es}119\) nucleus, the predicted fission barriers are \(5.77 \text{ MeV}, \) \(5.55 \text{ MeV} \) and \(6.03 \text{ MeV} \), respectively. Very recently Zagrebaev et al. \[16\] have reported a prediction for the same reaction, \(254\text{Es(}48\text{Ca},\text{xn})^{302-x}\text{Es}119\) (about 0.3 pb for 3n channel). No prediction for the \(252\text{Es(}48\text{Ca},\text{xn})^{300-x}\text{Es}119\) reaction was given.

In case of inaccessibility of Es targets, the most promising target-projectile combination to synthesize the element \(Z = 119\) is the \(249\text{Bk(}50\text{Ti},\text{xn})^{299-x}\text{Es}119\) reaction. Predictions for this reaction are shown in Fig. 5(c). Both 3n and 4n channels are expected to have comparable cross sections of about 30 fb (at maximum) at \(E_{c.m.} \approx 225 \) and \(232 \text{ MeV} \), respectively. Almost an equally small cross section for the \(248\text{Bk(}50\text{Ti},\text{xn})^{299-x}\text{Es}119\) reaction (about 60 fb) was predicted in Ref. \[3\], and somewhat larger value (about 110 fb) in Ref. \[17\]. Unfortunately, such small cross sections seem to be beyond the reach of present-state experiments. More optimistic predictions for the same reaction appeared recently in Ref. \[13\], however a relatively large cross section (about 0.6 pb) was obtained for probably overestimated values of the fission barrier taken as the pure ground-state shell effect from tables of Ref. \[19\].

Prospects for the synthesis of element \(Z = 120\) are considerably worse than those for \(Z = 119\). First of all, there is no chance to use the favorable beam of \(48\text{Ca}\) because the complementary \(257\text{Fm}\) target cannot be produced. We consider therefore reactions with \(50\text{Ti}\) beam on two available isotopes of californium, \(248\text{Cf(}50\text{Ti},\text{xn})^{299-x}\text{Cf}120\) and \(251\text{Cf(}50\text{Ti},\text{xn})^{301-x}\text{Cf}120\), which seem to be the best choice. Excitation functions for these two reactions are shown in Figs. 5(d) and 5(e). The largest cross section is expected in the former reaction (about 6 fb at maximum in both 3n and 4n channels), in the latter reaction the maximum cross section is about 3 fb for 4n channel. Again, similarly as in case of reactions on two isotopes of einsteinium discussed above, a smaller cross section for more neutron rich compound nucleus is associated with respectively lower fission barriers predicted in Refs. \[22, 23\].

In Fig. 5(f) we present results of calculations for the \(248\text{Cm(}64\text{Ni},\text{xn})^{302-x}\text{Cm}120\) reaction that is a more symmetric combination of even–Z target and projectile, next to \(\text{Ti} + \text{Cf}\). The obtained cross sections of the order of 1 fb for 3n and 4n reaction channels clearly demonstrate that fusion processes are too strongly hindered in more symmetric systems. For completeness, we calculated also cross sections in two reactions of much more symmetric systems, \(238\text{U(}64\text{Ni},\text{xn})^{302-x}\text{U}120\) and \(244\text{Pu(}58\text{Fe},\text{xn})^{302-x}\text{Pu}120\) (not shown in figures), for which attempts to produce the element \(Z = 120\) were done \[10, 17\]. The calculated 3n and 4n cross sections in these two reactions are dramatically small, about 0.3 fb and 0.1 fb, respectively. Note that experimental upper limits for these two reactions had been established at 90 fb \[10\] and 400 fb \[17\], respectively.

Our calculations show that if the fission barriers of Refs. \[22, 23\] were correct, there is no chance to synthesize the element \(Z = 120\), even in the most favorable reaction \(249\text{Cf(}50\text{Ti},\text{xn})^{299-x}\text{Cf}120\), for which the predicted cross section is only 6 fb. Note that other model calculations for the \(249\text{Cf(}50\text{Ti},\text{xn})^{299-x}\text{Cf}120\) reaction, published previously \[5, 10, 12–14, 17\], predicted considerably larger cross sections though also too small to be measurable (typically of the order of 50 fb). The dispersion of these different theoretical results has to be linked, first of all, to different fission barriers and ground-state masses used in these calculations.

We would like to emphasize that our predictions concerning the synthesis of \(Z = 119\) and \(120\) nuclei are based on the consistency of the FBD model calculations with the adopted ground-state masses and fission barriers of Refs. \[22, 23\] and with all the existing experimental data on the synthesis of superheavy nuclei in hot fusion reactions \[3, 25–35\]. Therefore the accuracy of these predictions is expected to be comparable with the accuracy of our overall fit to the data for the synthesis of \(Z = 114–118\) nuclei, shown in Fig. 2.

In summary, we analyzed a complete set of existing data on hot fusion reactions leading to the synthesis of super-heavy nuclei of \(Z = 114–118\) \[3, 25–35\] in terms of a new \(l\)-dependent version of the FBD model with fission barriers and ground-state masses taken from the macroscopic-microscopic model of Kowal et al. \[22, 23\]. By “calibrating” the assumed injection-point distances \(s_{inj}\) to the measured cross sections, perfect systematics of \(s_{inj}\) values have been established for a wide range of hot fusion reactions enabling, hopefully, reliable predictions of the synthesis cross sections for yet unexplored reactions. Regarding prospects to produce the new element \(Z = 119\), our calculations prefer the \(254\text{Es(}48\text{Ca},\text{xn})^{300-x}\text{Es}119\) reaction, for which the synthesis cross section of about 0.2 pb in 4n channel at \(E_{c.m.} \approx 220 \text{ MeV} \) is expected. According to the microscopic-macroscopic model predictions \[22, 23\], fission barriers for heavier isotopes of the element \(Z = 119\) are significantly lower leading to a considerably smaller cross section in the alternative \(254\text{Es(}48\text{Ca},\text{xn})^{302-x}\text{Es}119\) reaction. Also the reaction \(254\text{Bk(}50\text{Ti},\text{xn})^{300-x}\text{Bk}119\) gives little chances for a measurable cross section (the predicted cross section is about 30 fb for both 3n and 4n channels). The most favorable reaction to synthesize the element \(Z = 120\) is the \(249\text{Cf(}50\text{Ti},\text{xn})^{299-x}\text{Cf}120\) reaction, but the predicted cross section is
only 6 fb (for 3n and 4n channels).
FIG. 1: (Color online) Systematics of the injection-point distance $s_{inj}$ as a function of the kinetic energy excess $E_{c.m.} - B_0$ above the Coulomb barrier $B_0$. Values of $s_{inj}$ have been determined for each reaction and each particular $xn$ channel by fitting the theoretical cross section at the maximum of a given $xn$ excitation function to the data. The calculations have been done for the fission barrier heights and ground-state masses of Kowal et al. [22, 23]. Complete list of the analyzed reactions with references is given in the text. Identical symbols for a given $Z$ and a given experiment refer to data for consecutive $xn$ channels.
FIG. 2: (Color online) Energy dependence of the cross section for synthesis of superheavy nuclei in hot fusion reactions. Full circles represent data for 3n, 4n and 5n reaction channels obtained in Dubna experiments for elements $Z = 114–118$ [3, 25–28, 30, 35]; open circles represent data obtained at GSI Darmstadt for $Z = 114$ and 116 [31, 33]. Data are compared with excitation functions for separate $xn$ channels, calculated with the FBD model assuming fission barriers and ground-state masses of Kowal et al. [22, 23] and the systematics of the injection-point distance, Eq. (2).
FIG. 3: (Color online) Dependence of the injection-point distance $s_{inj}$ on the kinetic energy excess $E_{c.m.} - B_0$ above the Coulomb barrier $B_0$, deduced from analysis of experimental data [3, 25–35] the same way as in Fig. 1, but assuming the fission barrier heights [21] and ground-state masses [19] of Möller et al. See text.

FIG. 4: (Color online) Excitation functions for the 3n and 4n channels of the $^{243}$Am($^{48}$Ca,xn)$^{291-x}$115 reaction calculated with the FBD model assuming the fission barriers [21] and ground-state masses [19] of Möller et al. (dashed lines) compared with the experimental cross sections [3, 25, 35] and the predictions for the fission barriers and ground-state masses of Kowal et al. [22, 23] (solid lines). In the absence of clear correlation between $s_{inj}$ and $E_{c.m.} - B_0$ for the barriers of Möller et al. (see Fig. 3), the dashed lines were calculated for a fixed value $s_{inj} = 7.2$ fm (the mean value).
FIG. 5: (Color online) Synthesis cross sections of undiscovered yet superheavy nuclei of $Z = 119$ and 120 predicted by using the fusion-by-diffusion (FBD) model with fission barriers and ground-state masses of Kowal et al. [22, 23] and the systematics of the injection-point distance, Eq. (3). See text.