Research on bat algorithms for inversion of particle size distribution in spectral extinction method

Li Wang¹,², Qianghuang Huang¹,³, Su Wang¹,⁴ and Huiming Yu¹,⁵, Fuxin Xu¹*

¹College of Physics and Electronics, Central South University, ChangSha, HuNan, 410012, China.
²First author’s e-mail: 924811684@qq.com
³Second author’s e-mail: 1493808491@qq.com
⁴Third author’s e-mail: 1691547178@qq.com
⁵Fourth author’s e-mail: Yhm734917945@csu.edu.cn
*Corresponding author’s e-mail: xfx.300@163.com

Abstract: The development of rapid and effective inversion algorithm is an important subject in the field of particle size distribution (PSD) measurement. An intelligent bat algorithm has been proposed to solve the problem of independent mode inversion in particle size distribution reconstruction based on spectral extinction method. The improved algorithm is more suitable for particle size distribution inversion under extinction method by enriching its behavior mode, improving local search mode and adding greedy strategy. In simulations, uniform spherical particles that obey the unimodal, bimodal and multimodal R-R distribution were used, and the scattering intensity values of the target function was added to the random noise of ±0%, ±3%, ±5%, ±8% respectively. In the inversion calculation of the improved bat algorithm, 400, 800 and 1000 iterations were performed for different distributions, respectively. Compared with modified pattern search method, the improved bat algorithm has strong noise resistance, moderate inversion time, independent of initial solution and high inversion accuracy for complex distribution. Compared with genetic algorithm, the improved bat algorithm has fast convergence speed, high accuracy and short time-consuming. Considering the practical application, the algorithm is suitable for field measurement and has a wide application prospect.

1. Introduction

With the development of science and technology, a large number of technical problems related to particle size distribution (PSD) measurement has appeared in many areas. Light scattering method is relatively advanced and suitable for modern measurement technology. Based on the optical scattering principle, the light scattering method can retrieve the information of particle size distribution by detecting the change of the emitted light signal relative to the incident light signal. Light scattering method can be divided into angular scattering method, diffraction scattering method, spectral extinction method, photon correlation spectroscopy method and laser holographic measurement method. Spectral extinction method has the advantages of simple principle, short time-consuming, high accuracy, simple operation, safety and reliability. It can realize the measurement of nanometer and micron-sized particles, and has great development space and application potential. The on-line measurement of particle size distribution requires high anti-noise ability of the inversion algorithm, and the research
of fast and effective inversion algorithm has become one of the hot research topics of scholars at home and abroad at this stage. The reconstruction methods of particle size distribution are mature, but these inversion algorithms have some limitations. Traditional algorithms, which include Chahine iteration method, Phillips-Twomey method and Projection method, generally have some problems, such as poor anti-noise ability, inaccurate reconstruction of multi-peak distribution and complex objective function. In recent years, the research of inversion algorithms mainly focuses on the introduction of intelligent algorithms such as genetic algorithm and simulated annealing algorithm. Compared with the traditional algorithm, the intelligent algorithm has strong anti-noise and global search ability, but it still has some shortcomings such as slow evolution speed, long calculation time and inadequate precision. In order to overcome the shortcomings of the intelligent algorithm, the new intelligent algorithm bat algorithm \(^{[11]}\) is used to reconstruct the particle size distribution of the spectral extinction method in the independent mode for the first time in this paper. The behavior strategy, local search strategy and optimization process of the bat algorithm are improved, and a greedy selection mode is added to ensure the algorithm converges quickly and accurately. The outstanding advantages of the improved bat algorithm are strong global search ability, fast convergence speed, high accuracy and strong anti-noise ability. It is suitable for practical application.

2. Reconstruction principle of particle size distribution
Spectral extinction method is based on Beer-Lamber law. For the multidisperse system satisfying the condition of uncorrelated single scattering, the equation is as follows\(^{[8]}\):

\[
\ln \left( I_0 \right) \lambda_i = -\frac{3}{2} L N_D \int_{D_{\text{MIN}}}^{D_{\text{MAX}}} \frac{Q_{\text{ext}}(\lambda_i, m, D) f(D)}{dD} dD
\]

Where \( Q_{\text{ext}}(\lambda_i, m, D) \) represents the extinction efficiency of a single spherical particle, which is a complex function of particle diameter \( D \), relative refractive index \( m \) and wavelength \( \lambda \). \( L \) is the optical path, \( n \) is wavelengths’ number, \( N_D \) is the number of particles. Respectively, \( D_{\text{MIN}} \) and \( D_{\text{MAX}} \) are the lower and upper integration limits. Here, \( f(D) \) is the volume frequency distribution function. Finally, \((I/I_0)_{\lambda_i}\) is the extinction value of monochromatic light at \( \lambda_i \).

By means of numerical integration, equation (3) can be discretized as follows\(^{[8]}\):

\[
\ln \left( I_0 \right) \lambda_i = -\frac{3}{2} L N_D \sum_{i=1}^{n} \frac{Q_{\text{ext}}(\lambda_i, m, D_i)}{D_i} f(D_i)
\]

Matrix representation:

\[
E = Af
\]

Where \( E = (\ln(I/I_0)_{\lambda_1}, \ln(I/I_0)_{\lambda_2}, ..., \ln(I/I_0)_{\lambda_n}) \), \( A \) is weight matrix, whose elements can be given by \( A_{ij} = -3 L N_D C_i Q_{\text{ext}}(\lambda_i, m, D_i)/(2D_i), (i = 1, 2, 3, ..., n; j = 1, 2, 3, ..., N) \), where \( C_j \) is the coefficient of numerical integration, \( f = [f(D_1), f(D_2), ... f(D_N)]^T \) is the vector of the size distribution function.

3. Bat algorithm

3.1. Standard bat algorithm.
The update equation of the standard bat’s speed and position are as follows:

\[
v_{i}^{t+1} = v_{i}^{t} + (x_{i}^{t} - x^*)f_i
\]

\[
x_{i}^{t+1} = x_{i}^{t} + v_{i}^{t+1}
\]

Where \( v_{i}^{t} \) and \( v_{i}^{t+1} \) denote the flying speed of individual bat \( i \) at time \( t \) and at time \( t+1 \), respectively. \( x_{i}^{t} \) and \( x_{i}^{t+1} \) denote the position of individual bat \( i \) at time \( t \) and at time \( t+1 \), respectively. \( x^* \) represents the global optimal position. \( f_i \) is the pulse frequency emitted by bat \( i \) during search process. It is defined as follows:

\[
f_i = f_{\text{min}} + (f_{\text{max}} - f_{\text{min}}) * \beta
\]

Where \( \beta \) is a random number between \([0,1]\). \( f_{\text{min}} \) and \( f_{\text{max}} \) are the lower
pulse frequency, respectively. Bat algorithm designs the following local search methods as well.

\[ x_i^t = x^* + \varepsilon \overline{A}^t \]

Where \( \varepsilon \) is a random number between \([-1, 1]\), \( \overline{A}^t \) is the average loudness of bats at time \( t \).

Equation (7) and (8) are used to simulate the changes of loudness and frequency during bat search process.

\[ r_i^{t+1} = r_i^0 [1 - \exp(-\gamma t)] \]
\[ A_i^{t+1} = \alpha A_i^t \] (6)

Where \( r_i^{t+1} \) and \( A_i^{t+1} \) are the pulse frequency and pulse sound intensity emitted by bat \( i \) at time \( t+1 \). \( r_i^0 \) is the maximum pulse frequency. \( \gamma \) and \( \alpha \) are the pulse frequency increase coefficient and the pulse loudness attenuation coefficient, respectively, which are both constant greater than zero.

The reconstruction of particle size distribution can be attributed to the least square problem of the measured and calculated spectra, so the objective function of limit. Define the upper and lower bound of \( r \) as \( \gamma \) and \( \alpha \), respectively, which are both const Laden greater than zero.

3.2. The flow of bat algorithm.

The flow of bat algorithm is as follows.

Step1: Set control parameters such as the colony size, the maximum number of cycles and the value of limit. Define the upper and lower bound of characteristic parameters to be optimized for the volume frequency distribution function of PSD.

Step2: Initialization the feasible position \( x_i^0 \) and velocity \( v_i^0 \) of each bat according to equation (5), initialization the pulse emission frequency \( r_i^0 \) and pulse loudness \( A_i^0 \), and calculate fitness value according to equation (8) to find the optimal individual of the population.

Step3: Create new velocity \( v_i^t \) and position \( x_i^t \) according to equation (3) and (4);

Step4: For each bat individual, a random number \( \text{rand1} \) is generated. If \( \text{rand1} > r_i^t \), a new position is regenerated near the current optimal individual according to equation (2.2.4).

Step5: Calculate the fitness of all bats.

Step6: \( \text{rand2} \) is generated for each individual bat. If \( \text{rand2} < A_i^t \& f(x_i^t) < f(x^*) \), the new solution is accepted and the \( r_i^t \) and \( A_i^t \) are adjusted according to the equation (7) and (8).

Step7: Update the global optimal solution and determine the termination condition. If the condition is satisfactory, output \( x^* \), otherwise return to Step3.

4. Improved bat algorithms

4.1. Defects of standard bat algorithms.

The standard bat algorithm has three drawbacks, which are listed below.

(1) Random numbers have a great influence on bats

According to equation (3) and (4), the position of bats at \( t+1 \) is determined by \( x_i^t \), \( (x_i^t - x_i^{t-1}) \) and \( (x_i^t - x^*)f_i \). Bats are greatly affected by random factors \( (x_i^t - x^*)f_i \), which guarantees the bat’s global search ability, but can not guarantee the bat to approach the optimal location. That is to say, \( t+1 \) generation may be better or worse, which leads to the ineffective convergence of the algorithm.

(2) Population characteristics are not utilized

According to equation (3) and (4), the next position of a bat individual is just related to its historical position and the current position of the best individual. That is to say, there is no communication between individual bats. This will make it difficult for the group to gather and greatly reduce the search efficiency.

(3) Low efficiency of local search algorithm

In high-dimensional space, it is often difficult for local optimal solutions to jump out to better solutions. Equation (2.2.4) generates a solution randomly, which may be better or worse. This leads to the possibility that each bat will become worse and thus fail to converge effectively.
4.2. Algorithmic improvement scheme.
According to the analysis of the defects of BA algorithm, the BA algorithm has been improved effectively. The improved algorithm is called GBA. The improvement is as follows:\(^{(1)}\):

1. Enriching Bat Flight Patterns
   - Free flight mode
     Random free search is carried out according to equation (3) and (4).
   - Following Flight Mode
     Current bats locate themselves by referring to the positions of three other random bats. The location update equation is as follows:
     \[
     x_{i,j}^{t+1} = x_{i,j}^t + \alpha (x_{i,z,j}^t - x_{i,b,j}^t)
     \]
     \(\alpha\) is the contraction coefficient, \(0 < \alpha < 1\).
   - Group Flight Mode
     Bat individuals fly into the best bat population at present. That is to say, bats are divided into three groups according to the fitness value. In the optimal population, \(m\) bats are randomly selected, and the average position of \(M\) bats is taken as the next position of bat individuals.

2. Adding Greedy Strategy and Improving Local Search
   Fitness evaluation was carried out when individual bats selected flight modes for position updating. Fitness evaluation was carried out when individual bats selected flight modes for position updating. According to the greedy strategy, if the position is better than the previous one, replace the previous position and adjust \(r_i^t\) and \(A_i^t\) according to the equation (2.2.5) and (2.2.6), otherwise the position will remain unchanged. When the global optimal individual is not updated for three successive generations, the worst individual I is found. If \(\text{rand1} > r_i^t\), then \(x_i^t\) is regenerated near the current optimal individual according to equation (2.2.4). Calculate the fitness value of \(x_i^t\); if \(\text{rand2} < A_i^t \& f(x_i^t) < f(x_i^t')\), accept the new solution and adjust \(r_i^t\) and \(A_i^t\) according to the equation (2.2.5) and (2.2.6), otherwise stay unchanged. This ensures that individual bats will only fly to better positions, not worse positions. It is also more in line with the biological characteristics of bats when they hunt in nature.

4.3. The flow of GBA algorithm.
The flow of GBA is as follows.

Step1: Set control parameters such as the colony size, the maximum number of cycles and the value of limit. Define the upper and lower bound of characteristic parameters to be optimized for the volume frequency distribution function of PSD.

Step2: Initialization the feasible position \(x_i^0\) and velocity \(v_i^0\), generate pulse frequency \(f_i\) of each bat according to equation (5), initialization the pulse emission frequency \(r_i^0\) and pulse loudness \(A_i^0\), and calculate fitness value according to equation (8) to find the optimal individual of the population.

Step3: Behavior patterns are selected by probability. Create new velocity \(v_i^t\) and position \(x_i^t\). Calculate the fitness value, If the fitness value is better than the original position, the original position is replaced and the \(r_i^t\) and \(A_i^t\) are adjusted according to the equation (7) and (8), otherwise stay in situ.

Step4: Calculate the fitness values of all bats and find out the best and worst individual at present.

Step5: If the current optimal position is unchanged for three consecutive generations, a random number \(\text{rand1}\) is generated for the current worst bat individual i. If \(\text{rand1} > r_i^t\), then \(x_i^t\) is regenerated near the current optimal individual according to equation (6). Calculate the fitness value of \(x_i^t\);

Step6: If \(\text{rand2} < A_i^t \& f(x_i^t) < f(x_i^t')\), accept the new solution and adjust \(r_i^t\) and \(A_i^t\) according to the equation (7) and (8), otherwise stay unchanged.

Step7: Update the global optimal solution and determine the termination condition. If the condition is satisfactory, output \(x^*\), otherwise return to Step3.

5. Numerical results and discussions
In order to verify the feasibility and reliability of the algorithm, simulation experiments were carried out
for particle systems with unimodal R-R distribution, bimodal R-R distribution and multimodal R-R distribution. At the same time, the inversion results of pattern search algorithm\(^8\)(MSSS) and genetic algorithm\(^1\) (GA) are listed for comparison. In order to approach the actual measurement, a certain random noise is introduced into the extinction value at each wavelength. The R-R distribution is expressed as follows:

\[
f(D)_{RR-S} = k \frac{D}{\bar{D}} \times \left( \frac{D}{\bar{D}} \right)^{k-1} \times \exp \left[ -\left( \frac{D}{\bar{D}} \right)^k \right]
\]

\[
f(D)_{RR-b} = n \left[ k_1 \frac{D}{\bar{D}_1} \times \left( \frac{D}{\bar{D}_1} \right)^{k_1-1} \times \exp \left[ -\left( \frac{D}{\bar{D}_1} \right)^{k_1} \right] \right] + (1 - n) \left[ k_2 \frac{D}{\bar{D}_2} \times \left( \frac{D}{\bar{D}_2} \right)^{k_2-1} \times \exp \left[ -\left( \frac{D}{\bar{D}_2} \right)^{k_2} \right] \right]
\]

\[
f(D)_{RR-t} = n \left[ k_3 \frac{D}{\bar{D}_3} \times \left( \frac{D}{\bar{D}_3} \right)^{k_3-1} \times \exp \left[ -\left( \frac{D}{\bar{D}_3} \right)^{k_3} \right] \right]
\]

Where \(f(D)_{RR-S}\) is unimodal R-R distribution, \(f(D)_{RR-b}\) is bimodal R-R distribution and \(f(D)_{RR-t}\) is multimodal R-R distribution. \(\bar{D}, k, \bar{D}_1, k_1, \bar{D}_2, k_2, \bar{D}_3, k_3, n, n_1\) are the characteristic parameters. \(0 \leq n \leq 1, 0 \leq n_1 \leq 1, 0 \leq n + n_1 \leq 1\). Forty-two wavelengths (0.35μm~2.0μm) are selected as the wavelength of incident light. Standard particles (polystyrene\(^{11}\)) were selected for simulation, the particle size range was limited to 0.1~10μm, and the relative complex refractive index was \(m = (1.596 - 0.1i)/1.33\). The objective functions are equation (4) and (5). The 2.5GHZ CPU is used to compute the inversion. The parameters of three R-R distributions are set as follows:

\((\bar{D}, k) = (2.70, 6.90) ; (\bar{D}_1, k_1, \bar{D}_2, k_2, n) = (2.9, 6.1, 7.2, 9.8, 0.3) ; (\bar{D}_1, k_1, \bar{D}_2, k_2, \bar{D}_3, k_3, n, n_1) = (1.5, 3.1, 4.6, 7.9, 7.9, 7.2, 0.3, 0.3)\);

GBA algorithm and GA algorithm have the characteristics of random search, which results in different inversion consequences. Under the same conditions, the error of each result of GBA algorithm is generally less than ±5%; Under the same number of iterations, the results of GA algorithm differ greatly from each other. So, the average values of 10 calculations were taken for comparison.

Figure 1, figure 2 and figure 3 are the inversion results of the improved bat algorithm (GBA) which iterates 400, 800 and 1000 times for three kinds of distributions under different noises. The population size is 50, and the initial position of each bat is randomly generated by the system. It can be seen that the GBA algorithm can reconstruct three R-R distribution particle systems well under the noiseless circumstance. The position and height of the peak can be reconstructed accurately. The inversion results with less burr and deviation are satisfactory. With the increase of noise, the results of GBA algorithm can still accurately reconstruct the position and height of the peak, but a slight jitter appears in the large particle size range. This is especially evident in the inversion of bimodal distribution with ±8% random noise. In the inversion of bimodal distribution, the difficulty of inversion increases with the approaching of the two peaks. Satisfactory results can be obtained by increasing the number of iterations of GBA algorithm.
Figure 1. Inversion results of particle with unimodal R-R distribution using GBA.
(a): No random noise; (b): ±3% random noise; (c): ±5% random noise; (d): ±8% random noise;

Figure 2. Inversion results of particle with bimodal R-R distribution using GBA.
(a): No random noise; (b): ±3% random noise; (c): ±5% random noise; (d): ±8% random noise;

Figure 3. Inversion results of particle with multimodal R-R distribution using GBA.
(a): No random noise; (b): ±3% random noise; (c): ±5% random noise; (d): ±8% random noise;

The inversion results of the pattern search algorithm (MSSS) have been shown in the figure 4, figure 5 and figure 6. The algorithm iterated 5000, 10000 and 15000 times for three kinds of distributions under different noises. It can be seen that the PSD can be well retrieved under the condition of no noise. With the increase of noise, burrs and deviations occur sharply, resulting in large fluctuations and large errors. In the absence of noise, the inversion results of the three-peak distribution show jitter and deviation at the first peak position. Although adding Tikhonov smoothing function to the algorithm to construct a new objective function\cite{6} can effectively improve the anti-noise performance of the algorithm, the algorithm still has the shortcomings of depending on the initial solution and having difficult to reconstruct the complex distribution. Compared with MSSS algorithm, GBA algorithm takes a little longer time, but it has higher accuracy in noisy environment.
Figure 4. Inversion results of particle with unimodal R-R distribution using MSSS. 
(a): No random noise; (b): ±3% random noise; (c): ±5% random noise; (d): ±8% random noise;

Figure 5. Inversion results of particle with bimodal R-R distribution using MSSS. 
(a): No random noise; (b): ±3% random noise; (c): ±5% random noise; (d): ±8% random noise;

Figure 6. Inversion results of particle with multimodal R-R distribution using MSSS. 
(a): No random noise; (b): ±3% random noise; (c): ±5% random noise; (d): ±8% random noise;

The results of genetic algorithm (GA) have been shown in figure 7, figure 8 and figure 9. The related parameter settings are the same as those of GBA algorithm. As it can be seen in figure. 7, 8 and 9, GA algorithm can roughly invert the location and height of distribution peak in noiseless environment, but with the increase of noise, it will produce relatively large fluctuations. In the inversion of bimodal distribution and multimodal distribution, with the increase of noise, the fluctuation in the large particle size range becomes more serious and the inversion accuracy is poor. Because the large particle size has a great influence on the scattered light intensity, the relative position of each large particle size range can not be well locked in the case of insufficient iterations. By increasing the number of iterations, the inversion accuracy can be effectively improved, the false peaks and burrs can be reduced as well. Compared with GBA algorithm, GA algorithm has poor inversion accuracy under the same number of iterations. In conclusion, GBA algorithm converges faster and has higher accuracy than GA algorithm.
The expression of the inversion quality is as follows:

slow convergence rate. Increasing the number of iterations appropriately can effectively improve the converge to the optimal position in time.

This is mainly because the small particle size has little influence on the transmission intensity. With the increase of noise, the algorithm will gradually increase the proportion of small particle size to balance the error. This phenomenon generally exists in these three algorithms, and the algorithm still needs to be improved. In the large particle size range, the inversion results of the three algorithms always show abnormal jitter. In the pattern search algorithm, the main reason is that the algorithm is sensitive to noise. In the GBA and GA algorithms, the reason is that the search space dimension of the algorithm is large, and the large particle size interval which has a great influence on the scattering light intensity can not converge to the optimal position in time. This phenomenon is particularly evident in GA algorithm with slow convergence rate. Increasing the number of iterations appropriately can effectively improve the fluctuation of results in large particle size range.

The inversion quality can be evaluated by comparing the calculated PSD with the presupposed PSD. The expression of the inversion quality is as follows:

\[
\text{RMS} = \left( \frac{\sum_{i=1}^{N} \left| f_{\text{pre}}(D_i) - f_{\text{inv}}(D_i) \right|^2}{\left( \sum_{i=1}^{N} \left| f_{\text{pre}}(D_i) \right|^2 \right)^{1/2}} \right)^{1/2} \times 100\%
\]  

(14)

Where \( f_{\text{pre}}(D_i) \) is the preset particle size distribution, \( f_{\text{inv}}(D_i) \) is the inversion particle size distribution.
distribution, and $S$ is the number of incident wavelengths.

| Presupposed Distribution | Comparison item | Random noise /% | GBA       | MSSS       | GA        |
|--------------------------|-----------------|-----------------|-----------|------------|-----------|
| Unimodal R-R distribution(2.70,6.90) | RMS             | ±0               | 0.1481    | 0.1143     | 0.2888    |
|                         |                 | ±3               | 0.0981    | 0.6139     | 0.2289    |
|                         |                 | ±5               | 0.2334    | 0.3911     | 0.5010    |
|                         |                 | ±8               | 0.1946    | 1.6777     | 0.2800    |
|                         | Inversion time /s| ±0               | 31.03     | 8.58       | 55.27     |
|                         |                 | ±3               | 0.1146    | 0.0842     | 0.1792    |
| Bimodal R-R distribution(2.9,6.1,7.2,9.8,0.3) | RMS             | ±0               | 0.1983    | 0.6399     | 0.4234    |
|                         |                 | ±3               | 0.1416    | 0.4750     | 0.5137    |
|                         |                 | ±5               | 0.4122    | 1.3498     | 0.5366    |
|                         | Inversion time /s| ±0               | 64.60     | 16.73      | 102.30    |
|                         |                 | ±3               | 0.1934    | 0.5173     | 0.3057    |
| Multimodal R-R distribution(1.5,3.1,4.6,7.9,7.9,7.2,0.3) | RMS             | ±0               | 0.2267    | 0.3045     | 0.2585    |
|                         |                 | ±3               | 0.2189    | 0.7376     | 0.3253    |
|                         |                 | ±5               | 0.1984    | 0.6560     | 0.3042    |
|                         | Inversion time /s| ±0               | 78.30     | 25.28      | 131.56    |

The errors and time of the three algorithms in different cases are listed in Table 1 for visualization. As can be seen from Table 1, the error of GBA algorithm is generally less than 20%, the inversion time is short and the inversion accuracy is high. With the increase of iteration times, the error can be further reduced, but it will also consume more time. The errors are mainly caused by the phenomenon of tail warping and "burr", which is especially evident in the inversion of bimodal distribution with ±8% random noise. From the data in the table, it can be seen that under the same iteration times, GBA algorithm takes less time and has less error than GA algorithm; As a direct algorithm, MSSS algorithm has the characteristics of short time-consuming and high inversion accuracy, but it also reflects the shortcomings of being sensitive to noise, depending on the initial solution, and low precision in complex distribution reconstruction. Although Tikhonov smoothing function can effectively improve its anti-noise performance, it is still difficult to accurately reconstruct multi-peak distribution particle system; In summary, GBA shows good anti-noise characteristics, high inversion accuracy, fast convergence rate and short running time.

6. Conclusion

In this paper, a new intelligent algorithm (bat algorithm) is introduced into the reconstruction of particle size distribution by spectral extinction method. The disadvantages and advantages of bat algorithm are analyzed, different behavior patterns are designed, local search pattern is improved, and the algorithm flow is adjusted. In order to ensure the fast and accurate convergence of bat algorithm, greedy selection strategy is added. In simulations, uniform spherical particles that obey the unimodal, bimodal and multimodal R-R distribution were used, and the scattering intensity values of the target function was added to the random noise of ±0%, ±3%, ±5%, ±8% respectively. The relative error of the improved bat algorithm is less than 20%. It still has high precision when adding ±8% random noise. The inversion time of unimodal R-R distribution is less than 40 seconds, bimodal R-R distribution is less than 65 seconds and multimodal R-R distribution is less than 80 seconds. Compared with the model search algorithm, the improved algorithm has better robustness; Compared with genetic algorithm, the improved algorithm has faster convergence speed and higher accuracy. In summary, GBA shows good anti-noise characteristics, high inversion accuracy, fast convergence rate and short time-consumption. Considering practical factors, GBA algorithm has advantages in particle size distribution reconstruction, which is suitable for field measurement and has good application prospects.

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