On Heavy Carbon Doping of MgB₂

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Heavy carbon doping of MgB₂ is studied by first principles electronic structure studies of two types, an ordered supercell (Mg(B₁₋ₓCₓ)₂, x=0.0833) and also the coherent potential approximation method that incorporates effects of B-C disorder. For the ordered model, the twofold degenerate \( \sigma \)-bands that are the basis of the high temperature superconductivity are split by 60 meV (i.e. 7 meV/\% C) and the \( \sigma \) Fermi cylinders contain 0.070 holes/cell, compared to 0.11 for MgB₂. A virtual crystal treatment tends to overestimate the rate at which \( \sigma \) holes are filled by substitutional carbon. The coherent potential approximation (CPA) calculations give the same rate of band filling as the supercell method. The occupied local density of states of carbon impurities (due to unreacted components and/or oxygen and carbon impurities) the perpendicular and parallel critical fields have reached \( H_{c2}^\parallel \approx 34 \) T, \( H_{c2}^\perp \approx 49 \) T, making MgB₂ a real possibility for a high field conductor. Such application require defects or grain boundaries to pin the vortices whose motion would otherwise lead to energy dissipation and joule heating. Such defects also affect the underlying electronic structure and pairing interaction, at least in some parts of the sample, and often decrease \( T_c \). The simplest defect to understand is the substitutional impurity, which provides a means of varying the intrinsic properties in a continuous and controllable manner and may enhance pinning mechanisms. Although readily synthesizable as a stoichiometric, rather clean compound, the MgB₂ lattice resists most attempts to alloy, both on the Mg and B sites, at more than the very dilute level.

The two established exceptions are Al substitution for Mg, and C substitution for B, both of which lead to a rapid decrease in \( T_c \). Published reports on the behavior vary considerably with synthesis method and sample treatment, but some of the general behavior seems to be established. Al substitution should be the simpler one, since the very strongly bonded honeycomb structure B layers remain intact, and simple rigid band filling of the hole states at first appears to be a likely possibility. Up to \( y = 0.10 \) in Mg₁₋ₓAlₓB₂ that appears to be the case, with \( T_c \) decreasing smoothly. Beyond the concentration \( y = 0.10 \), however, there are signs of two-phase behavior (two \( c \) lattice parameters in diffraction studies) arise. Around \( y = 0.25 \) the system reverts to a single phase very low and finally vanishing \( T_c \). A study of the trends in electron-phonon coupling strength using first principles calculations in the virtual crystal approximation reported a sharp change of behavior at \( y=0.25 \). A thorough study of the energetics of Al substitution revealed a strong tendency for superstructure formation at \( y=0.25 \) (and \( y=0.75 \)). Still, the observed onset of two-phase behavior already at \( y=0.10 \) is unexplained, and nonstoichiometry and microscopic defects must be kept in mind.

Unlike the Al alloying case where there has been at least rough consensus on changes of properties, reports of the change in \( T_c \) and structure with addition of carbon on MgB₂ have varied widely. These differences seem in some cases to reflect real differences in materials due to the various methods of synthesis and heat treatment, and the samples prepared were highly mixed phase, thereby creating additional uncertainty. The very similar xray scattering strength of the B and C atoms has rendered standard xray diffraction ineffective in determining the C content of a sample. Reports early on suggested C miscibilities in Mg(B₁₋ₓCₓ)₂ as small6 as \( x=0.02 \) to as large as \( x=0.3 \). Recent studies of several groups suggest that \( x=0.2 \) can be achieved while larger concentrations are questionable in MgB₂, although reports of larger C concentrations under high pressure growth persist.³¹

With some of the materials questions coming under control, the observation trends are raising some serious questions. The decrease in \( T_c \) follows \( dT_c/dx \sim 1 \) K/\% C,12 with several reports of \( T_c \) \( \approx 21-23 \) K at \( x=0.2 \). However, with carbon adding one electron and the \( \sigma \) bands of MgB₂ holding only \( \sim 0.11 \) holes per unit cell, one might guess that the \( \sigma \) bands would be filled (or nearly so, as

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I. INTRODUCTION

Discovery of superconductivity at \( T_c \approx 40 \) K in MgB₂ has enlivened not only interest in new classes of superconductors with high \( T_c \) and novel two-gap behavior, but also pursuit of new materials for applications that require high critical current densities and high critical fields (H\(_{c2}\)). In highly resistive films (due to unreacted components and/or oxygen and carbon impurities) the perpendicular and parallel critical fields have reached \( H_{c2}^\parallel \approx 34 \) T, \( H_{c2}^\perp \approx 49 \) T, making MgB₂ a real possibility for a high field conductor. Such application require defects or grain boundaries to pin the vortices whose motion would otherwise lead to energy dissipation and joule heating. Such defects also affect the underlying electronic structure and pairing interaction, at least in some parts of the sample, and often decrease \( T_c \). The simplest defect to understand is the substitutional impurity, which provides a means of varying the intrinsic properties in a continuous and controllable manner and may enhance pinning mechanisms. Although readily synthesizable as a stoichiometric, rather clean compound, the MgB₂ lattice resists most attempts to alloy, both on the Mg and B sites, at more than the very dilute level.

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the \( \pi \) bands can also accept carriers) and superconductivity would have vanished. Not only is \( T_c \) still robust at \( x=0.2 \), analysis of tunneling spectra indicates two-band superconductivity is retained\(^{14,15} \) whereas it might also seem likely that disorder scattering should have averaged out the gap to a single value. The upper critical field \( H_{c2} \) initially increases strongly with \( C \) content as \( T_c \) is depressed, reaching a maximum\(^{14,15} \) of \( \sim 33-35 \) T. This critical field is substantially less than that for more disordered films (see above).

Recent studies seem to agree that \( C \) can be substituted substitutionally for \( B \) up to 10% replacement \( x=0.10 \)\(^{15} \). Tunneling spectroscopies indicate that \( \text{Mg(B}_{1-x}\text{C}_x)\text{B}_2 \) remains a two gap superconductor\(^{5,14} \) (i.e. the gap anisotropy is not washed out by scattering) for 10\( \pm \)2% \( C \) substitution for \( B \). This question of how the anisotropy gets washed out has attracted much interest. Similarly, the mechanisms underlying the high critical fields are not understood. These questions are not the focus of the present study, although the knowledge gained from first principles calculations will be useful in the resolution of these topics.

A few first principles calculations have been done for \( C \) substitution of \( B \) in \( \text{MgB}_2 \). Pseudopotential calculations including relaxation in a 27 unit cell supercell confirmed that substitutional \( C \) is energetically favorable to interstitial \( C \), and that the \( B-B \) bondlength is 5-6\% shorter than the bulk \( B-B \) bondlength. A projector-augmented-wave calculation\(^9 \) reported a smaller relaxation, but the difference may be due to constraints related to supercell size. A coherent potential approximation study of the disordered alloy using a Korringa-Kohn-Rostoker multiple scattering method in the atomic sphere approximation\(^20 \) revealed relatively small effects of disorder in the range \( 0 \leq x \leq 0.3 \). The main effect of \( C \) substitution was reported to be the raising of the Fermi energy due to the additional carriers, but the filling of the crucial \( \sigma \) hole band was not quantified.

In this paper, we present a more detailed analysis of the changes in electronic structure, and the effective \( \sigma \) band doping, using both periodic supercells and disordered alloy calculations. Our study lays the groundwork for another issue in the superconductivity of \( \text{MgB}_2 \) alloys that has received no direct attention from experimentalists: the effect of decreasing holes on the strength and character of electron-phonon coupling. Theoretical studies\(^{21,22,23} \) have predicted the following remarkable changes in doped \( \text{MgB}_2 \): as the number of holes decreases (hence the Fermi wavevector \( k_F \) of the cylindrical \( \sigma \) Fermi surfaces decreases), (1) the substantial downward renormalization of the \( E_{2g} \) bond-stretching modes with \( Q \leq 2k_F \) does not change, (2) the coupling strengths of these already strongly coupled modes increases, and (3) the total coupling strength \( \lambda \), and hence \( T_c \), remains unchanged. This scenario assumes two-dimensionality of the \( \sigma \) bands which is only approximately true, and neglects the change of electron-phonon matrix elements which should be reasonable. This argument also neglects the fact that the applicability of conventional electron-phonon (Midgall-Eliashberg) theory, already somewhat suspect in \( \text{MgB}_2 \), definitely becomes inapplicable as the Fermi energy decreases and approaches more closely the bond-stretching mode energy (65 meV). Item (2) definitely portends unusual dynamics related to the \( B-B \) stretch modes. We hope the current paper stimulates experimental investigation into these questions.

II. CALCULATIONAL METHODS

Two methods of assessing the effects of substitutional \( C \) in \( \text{MgB}_2 \) have been used in the work reported here. Our ordered impurity calculations have been performed using the full-potential linearized augmented plane wave code Wien2k\(^{24} \) applying the generalized gradient approximation to the exchange-correlation potential. The basis set is reduced by using the APW+lo method\(^{25} \), retaining the accuracy of the LAPW method. \( \text{Mg} \) was set to 7.00 which is a high quality basis for the \( s-p \) electron systems. The Brillouin zone integration for the supercell was carried out using 432 \( k \) points in the irreducible part of the zone. We have used the experimental results of Avdeev \textit{et al.}\(^{17} \) for the lattice parameters of the 8.33\% doped supercell and the 10\% doped virtual crystal.

For the disordered (randomly substituted) alloy calculations, we have used the full-potential nonorthogonal local-orbital minimum-basis scheme (FPLO)\(^{26} \) applied in the coherent potential approximation (CPA).\(^{20} \) In the calculations, 1152 (45 \( \times 45 \) \( \times 10 \)) irreducible \( k \) points, and valence orbitals \( 2s2p3s3p3d \) for \( \text{Mg} \) and \( 2s2p3d \) for \( B \) and \( C \), were used. The implementation of the CPA in FPLO relies on the Blackman-Esterling-Berk theory\(^{27} \) that includes random off-diagonal matrix elements in the local orbital representation.

III. ORDERED Mg(B\(_{1-x}\)C\(_x\))\(_2\), \( x = 0.0833 \)

Smaller concentrations of ordered \( C \) substitution for \( B \) require larger supercells. Our choice of supercell was guided by the desire for a small enough \( C \) concentration to be relevant to address experimental data, yet not so small as to make it difficult to distinguish the effect of \( C \) addition or to make the calculations unreasonably tedious. Since interlayer hopping is not an issue we will address, the supercell will involve enlargement of only a single \( B \) layer. The shape of supercell should have as small as possible
FIG. 1: (Color online) Top view of the B-C layer of the Mg$_6$B$_{11}$C supercell. Large (blue) circles denote Mg atoms, small light (yellow) circles denote boron, and small dark (red) circles denote carbon. The supercell boundary is outlined.

TABLE I: Lattice parameter values used in the virtual crystal and supercell calculations.

| Lattice Parameter | undoped MgB$_2$ (Å) | 10% doping MgB$_{1.8}$C$_{0.2}$ (Å) | 8.33% doping Mg$_6$B$_{11}$C (Å) |
|-------------------|----------------------|--------------------------------------|-------------------------------|
| a                 | 3.083                | 3.053                                | 3×3.053=9.159                 |
| b                 | 3.083                | 3.053                                | 2×3.053=6.106                 |
| c                 | 3.521                | 3.525                                | 3.525                         |

aspect ratio in order to maximize the separation of C atoms. The compromise we chose was the 8.33% doped supercell (Mg$_6$B$_{11}$C → Mg(B$_{1-x}$C$_x$)$_2$; x = 1/12 = 0.0833). Figure 1 shows the top view of the B$_{11}$C layer in the 2×3 supercell. Each carbon atom has three first, five second, and three third nearest neighbor boron atoms.

A. Band structure

There are two effects of carbon substitution: a change in the average potential in the B-C layer, and the breaking of symmetry by C replacement of B in the supercell. In the ordered supercell the potential difference is kept explicit and symmetry breaking will be clear. In Sec. VI CPA will be used to probe these effects in a different manner. Another effect, B relaxation around the C impurities, has been addressed previously to some extent, but it will not be studied here since the effect cannot be included within the CPA.

The band structure within 4 eV of the Fermi level (E$_F$) is shown in Fig. 2. The primary band in the understanding of MgB$_2$ superconductivity is the (twofold degenerate) $\sigma$ band along the Γ-A direction. The results of supercell band-folding can be distinguished especially for the Γ-A direction, where (for example) the $\sigma$ band is replicated at Γ at -1.8 eV, -2.3 eV, -2.6 eV, and even further lower energies.

One of the most readily apparent effects of C doping is the splitting of the $\sigma$ bands along Γ-A, by 60 meV. It is helpful in understanding this splitting to obtain the atomic characters of each of the split bands, which will in any case involve only the B and C $p\sigma$ ($p_x$, $p_y$) states. The carbon $p$ contribution is emphasized by enlarged symbols (“fatbands”) in Fig. 3. The lower of the two bands has stronger C content (although this is not easy to distinguish in Fig. 3). In addition, it has more first and second B neighbor character than does the upper band. The upper band contains more third neighbor B character, this being the B site farthest from the C atom and representative of the bulk material. The interpretation of this splitting is that lower band has been pulled down due to the stronger potential of the C atom compared to that of B. The 60 meV splitting for $x=1/12$ provides (assuming a linear effect in this concentration range) an energy scale for $\sigma$-band broadening $\gamma_\sigma \approx 7$ meV/% C content.
FIG. 3: Bands of Mg$_6$B$_{11}$C near the Fermi level, with symbol size proportional to the C 2$p$ character. The $\sigma$ bands, split by 60 meV (hardly visible in this figure) lie just above $E_F$ at $\Gamma$ and disperse upward toward the A point.

B. Density of States

Figure 4 shows the total and atom-projected density of states (DOS) of the 8.33% doped system. The C and B DOS are similar at and above $E_F$, but the C DOS is somewhat lower in the interval within 2 eV below $E_F$. The C DOS is larger than that of B in the -8 eV to -3 eV region. Based on charge within spheres of 1.65 Å radius we calculated a charge transfer of about 0.095$e^-$ to the carbon from the three first nearest neighbor boron atoms, with other B sites showing negligible change in charge. This charge transfer also reflects the stronger potential and larger electronegativity of C, and provides substitutional C with definite anionic character.

IV. 10% DOPING

We have also performed a virtual crystal calculation for the 10% doped $x=0.10$ system Mg(B$_{0.9}C_{0.1})_2$. The DOS and band structure are shown in Figs. 5 and 6. By aligning the highest peaks in the DOS at -2 eV and +6 eV (they can be aligned simultaneously), we establish that the differences are (1) an increase in the occupied bandwidth (at the MgB$_2$ band filling) from 12.4 eV to 12.5 eV ($\sim$1%), and (2) the raising of the Fermi level by 0.3 eV to accommodate the extra electrons. Otherwise there is very little difference in the two densities of states. This raising of $E_F$ will be compared to the supercell result in the next section. At this band filling, the VCA gives the $\sigma$ band edge at $\Gamma$ precisely at $E_F$ as shown in Fig. 6. At the point A it is still about 0.3 eV above $E_F$, meaning that both cylindrical Fermi surfaces have shrunk to a point at their “waists.”
V. \( \sigma \) HOLE CONCENTRATION

It is accepted that superconductivity in MgB\(_2\) arises from hole-doping of the \( \sigma \) bonding hole states due to the intrinsic chemistry of MgB\(_2\). The observed decrease in electron-phonon coupling strength, and hence \( T_c \), upon C addition, makes the change in hole concentration one of the major points of interest. We have calculated the hole concentration of both the 8.33\% supercell system and the 10\% virtual system from the volume enclosed by the Fermi surface. Since the radii of the cylindrical Fermi surfaces are only slightly different along the \( \Gamma \rightarrow M \) and the \( \Gamma \rightarrow K \) directions, we calculated the average basal area by considering the value along the \( \Gamma \rightarrow M \) direction only. Also, we assumed a sinusoidal dependence of the Fermi surface along the \( \hat{c} \) axis, allowing analytic evaluation of the Fermi surface volumes. The hole concentrations. The results are presented in Table II. In the process of calculating the number of holes, electronic structure reveals that both the \( \sigma \) band Fermi surfaces are still intact, consistent with the two-band superconductivity with substantial \( \Delta \). Also, we interpolated the number of holes for the 10\% doped system (virtual crystal) to calculate the value for the 8.33\% doped system. Supercell calculation gave 0.070 holes/cell while the extrapolation gave only 0.057 holes/cell. This indicates that the virtual crystal approximation is not very reliable for substitutional carbon, and that C cannot be thought of “boron + an electron.”

| No. of holes | MgB\(_2\) | 8.33\% doping | Mg\(_6\)B\(_{11}\)C supercell | 10% doping | MgB\(_{1.8}\)C\(_{0.2}\) virtual crystal | 8.33\% doping | MgB\(_{1.833}\)C\(_{0.167}\) extrapolation |
|--------------|----------|----------------|----------------|-----------|---------------------------------|---------|-------------------------------|
| (holes/cell) |          | 0.11           | 0.070          | 0.0463    | 0.057                           |        |                               |

VI. COHERENT POTENTIAL APPROXIMATION RESULTS

For further comparison and to assess the effects of disorder, we have performed CPA studies of this system. We have carried out CPA calculations (1) at \( x=0.0001 \) to provide a “perfect crystal” reference for evaluation of the various algorithms in the CPA code, (2) at \( x=0.0833 \) for most direct comparison to the \( x=1/12 \) (ordered) supercell calculations described in the previous sections, and (3) at \( x=0.10 \) and 0.20 as representative of the system toward the upper range of achievable C substitution. The spectral function over the whole energy range of interest, plotted as a “smeared” band structure, is shown for the \( x=0.20 \) case in Fig. 7. The primary points of interest are the filling of the \( \sigma \) band hole states, and the broadening (and potentially splitting) of bands.

The full energy region is presented in Fig. 7 for \( x=0.20 \) (where broadening is more easily seen) to illustrate the strong wavevector (\( k \)) and energy (\( E \)) dependence of the broadening of the spectral function. The largest disorder occurs in the 2s region in the lower valence band, where the C 2s state is noticeably lower in energy leading to increased smearing. The other region of large disorder is from +3 eV upward, but some states remain comparatively sharp. The flat band at 5 eV along \( \Gamma-M \) and \( \Gamma-L \), with strong \( \pi \) character, is the lowest conduction band that is strongly affected by the disorder. The bands around the Fermi level, whether \( \sigma \) or \( \pi \), are among those less affected by the chemical disorder. As expected from the virtual crystal results for \( x=0.10 \) from the previous section, the \( \sigma \) band holes are completely filled at \( x=0.20 \), as revealed by the flat band along the \( \Gamma-A \) line lying entirely below \( E_F \). The band filling behavior versus \( x \) is quantified below.

In the valence (occupied) bands, disorder broad-
FIG. 7: CPA spectral density in the full valence-conduction band region, for \( x = 0.20 \), plotted as a broadened band structure. Disorder broadening is largest below -8 eV in the valence bands and above 4 eV in the conduction bands.

In the conduction bands disorder broadening becomes more prevalent. The antibonding \( \sigma^* \) bands (flat along \( \Gamma - A \) at +6 eV) are much broader than their bonding counterparts (below \( E_F \)), and this band along \( \Gamma - M \) and \( A - L \) with Mg 3s character becomes exceedingly diffuse. The \( \pi \) bands show strong \( k \)-dependence of the broadening, beginning at 2-3 eV around the M point and becoming wider in the 6-10 eV range (and above, not shown in the figure).

**Carbon concentration dependence.** The \( x \) dependence of the broadening can be seen in Fig. 8 for \( x = 0.0001 \) (the CPA equivalent of \( x = 0 \) MgB\(_2\)), for \( x = 0.10 \), and for \( x = 0.20 \) (the latter may not be experimentally accessible). Since in each case \( E_F \) is set to zero, band filling appears as downward shifts of the bands, by roughly \( \delta E_F = -0.4 \) eV for \( x = 0.10 \) and \( \delta E_F = -1.0 \) eV for \( x = 0.20 \). For \( x = 0.0833 \) (not shown) the \( \sigma \) bands lie at the same position (with respect to \( E_F \)) as for the ordered \( x = 1/12 \) case shown in Fig. 3. Hence the degree of band filling in the CPA results is the same as for the ordered \( x = 1/12 \) case that was analyzed in the previous section. Although we will not dwell on it, it can be noticed that the band shift is not entirely rigid: the two valence bands at the L point split apart as well as broaden with increasing C concentration.

The other important aspect of the CPA bands of Fig. 8 is the broadening that increases with carbon concentration. Within the resolution of widths that we are able to extract, the widths of the \( \sigma \) bands can be taken to be proportional to \( x \), but differ considerably between \( \Gamma \) and \( A \) (being about 50% wider at \( \Gamma \)). The full width at half maximum of the spectral density of the \( \sigma \) bands, averaged between \( \Gamma \) and \( A \), is about \( \gamma \approx 0.21 \) eV (to perhaps 10% accuracy). This width corresponds to a width in wavevector given by \( \gamma = v_F \delta k \), and therefore a mean free path of \( \ell_F = 2\pi v_F / \gamma \). The 60 meV splitting of the \( \sigma \) bands (see Sec. III.A) for the \( x = 1/12 \) ordered supercell implies that the 0.21 eV mean width can be
interpreted as 0.05 eV from the B/C on-site energy difference, and thus 0.16 eV from disorder itself.  

Due to the anisotropy of the Fermi velocity \( v_F \), the mean free path may vary considerably over the Fermi surface at \( x=0.10. \) At \( \Gamma \) the cylinder radius has shrunk to a point, and the very small \( z \) component of \( v_F \) (vanishing at \( \Gamma \)) suggests a very small mean free path in the \( z \) direction of the order of the layer spacing \( c \) for \( x=0.10. \)

VII. DISCUSSION AND SUMMARY

In the band structure of the 8.33% doped supercell system, the hole \( \sigma \)-band Fermi surfaces along the \( \Gamma \rightarrow A \) direction are still present, and this degree of band filling is reproduced by the CPA. Even for \( x=0.10 \) the virtual crystal picture (which overestimates the rate of band filling by C) leaves \( \sigma \) Fermi surfaces, just beginning to be pinched off. The CPA calculations (Fig. 8) show the \( \sigma \) band holes begin to disappear rapidly for \( x > 0.10. \) Qualitatively this filling is consistent with most experimental reports.

The questions, and experimental probes, should now be focused on the fact that, as the \( \sigma \) band fills, there will be very strong deviation from ‘business as usual’ in the coupled electron-phonon system. The strength of coupling of bond-stretching modes with \( Q<2k_F \) continues to increase, and conventional Migdal-Eliashberg theory ceases to apply. The dynamics of these ultra-strongly coupled modes is unexplored, with their peculiar character being signaled by the divergence of their linewidth (at least within Migdal-Eliashberg theory). The limiting behavior should not revert to the widely studied polaron limit, however, as there remain the background \( \pi \) electrons, which are weakly coupled to vibrations but provide full metallic conductivity and screening to the system. The present study reveals that C substitution for B provides a similarly favorable system to Al substitution for Mg for studying this evolution. Recent reports indicate that Sc substitution (\( \text{Mg}_{1-x}\text{Sc}_x\text{B}_2 \)) may also provide another such system for study.

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