Spin Issues in $t\bar{t}$ Production and Decay

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We describe the off-diagonal spin basis for observing angular correlations in top quark pair production events at the Fermilab Tevatron. For events initiated by quark-antiquark annihilation, the top and antitop quark spins are 100% correlated in this basis: a spin-up top quark is always accompanied by a spin-down antitop quark and vice versa. Inclusion of the gluon-gluon initial state lowers the fraction of unlike spin events to 92%. Nevertheless, the angular correlations between the top and antitop quark decay products are twice as large in the off-diagonal basis as those in the more traditional helicity basis. We give two brief examples of how the presence of new physics would alter these correlations.

Until the discovery of the top quark, most studies of spin in high energy physics were formulated in terms of the helicity basis. For ultrarelativistic particles, this is appropriate. However, in general, the direction and degree of polarization of a massive spinning particle depends on how it was produced. Thus, for moderate particle energies, it should not be surprising to find that the optimal axis for studying spin correlations is something other than the particle’s direction of motion.

In this talk, we will discuss the spin correlations in top quark pair ($t\bar{t}$) production at the Tevatron with a center-of-mass energy of 2 TeV. The top quarks in these events are only moderately relativistic: they typically have speeds of $\beta \sim 0.6$ in the zero momentum frame (ZMF) of the initial partons (see Fig. 1). Therefore, it is not surprising to learn that the optimal basis for studying the spin correlations in the $t\bar{t}$ system at this energy is not the helicity basis, but rather the off-diagonal basis. As we shall see, the spin correlations in the off-diagonal basis are a factor of two larger than the correlations in the helicity basis.

Before examining $t\bar{t}$ production, let us review a few facts about top quark decay. Because of the enormous width of the top quark ($\Gamma_t = 1.6$ GeV in the Standard Model), its decay occurs before either hadronization (governed by the scale $\Lambda_{QCD}$) or depolarization (governed by the scale $\Lambda^2_{QCD}/M_t$) can take place. The dominant Standard Model decay chain is

$$t \rightarrow W^+ b \begin{pmatrix} l^- \\ \nu \end{pmatrix}$$

(1)

For concreteness, we will describe the leptonic $W$ decay. However, everything which we say about the charged lepton applies equally to the $d$-type quark in a hadronic decay.

We define the decay angles in the top quark rest frame with respect to top quark spin vector $s$, as shown in Fig. 2. The decay angular distributions of a spin-up top quark are simply linear in the cosine of these decay angles:

$$\frac{1}{\Gamma} \frac{d\Gamma}{d(\cos \theta_i)} = \frac{1}{2} (1 + \alpha_i \cos \theta_i),$$

(2)

where $\theta_i$ is the decay angle of the $i$th decay product. The distribution for spin-down top quarks has a minus sign in front of the $\cos \theta_i$ term. The degree to which each decay product is correlated with the spin is encoded in the value of $\alpha_i$ (see Table 1). Notice that the charged lepton (or $d$-type quark) is maximally correlated, with $\alpha_i = 1$. Thus, the most distinctive distribution plots the angle between the spin axis and the charged lepton in the top quark rest frame (see Fig. 3).

When we write the decay matrix element in an arbitrary Lorentz frame, we find that the natural 4-vectors are not the top quark momentum $t$ and its spin vector $s$ (normalized such that $s_\mu s^\mu = -1$). Instead, it is more convenient to use the combinations

$$t_1 \equiv \frac{1}{2} (t + ms) \quad \text{and} \quad t_2 \equiv \frac{1}{2} (t - ms),$$

(3)
considered individually, the spin-up and spin-down top quarks will wash out the correlations, polarized: spin up or spin down (\(\ell^+\), \(d\), \(b\), \(\nu\), and \(u\) are the angle between the spin axis and the particle in the rest frame of the top quark).

Figure 2: Definition of the top quark decay angles in the top quark rest frame. The direction of the top quark spin is indicated by the vector \(s\). Although we have drawn this figure assuming a leptonic \(W\) decay, the same correlations hold in a hadronic decay if we replace the charged lepton by the \(d\)-type quark and the neutral lepton by the \(s\)-type quark.

Table 1: Correlation coefficients \(\alpha_i\) for both semileptonic and hadronic top quark decays. The first two entries are a function of \(m_t^2/M_W^2\), and have been evaluated for \(M_t = 173.8\) GeV and \(M_W = 80.41\) GeV.

| Decay Product | \(\alpha_i\) |
|---------------|-------------|
| \(b\)         | -0.40       |
| \(\nu_\ell, u,\) or \(c\) | -0.33       |
| \(\ell, d,\) or \(\bar{s}\) | 1.00        |

where \(m\) is the mass of the top quark. In the top quark rest frame, the spatial parts of \(t_1\) and \(s\) point in the same direction, since in this frame \(t = (m, 0)\). In some other frame, however, these vectors are not parallel. In this case, the form of the matrix element clearly indicates that the preferred charged lepton emission axis is the spatial part of \(t_1\). Hence, we regard \(t_1\) as the appropriate generalization of the spin axis to an arbitrary reference frame.

Unless there is some type of spin asymmetry in the data, the opposite dependence upon \(\cos \theta_i\) for spin-up and spin-down top quarks will wash out the correlations, leaving a flat distribution. Considered individually, the top quarks in \(t\bar{t}\) pairs at the Tevatron are essentially unpolarized\(^{4}\) spin-up and spin-down top quarks are produced in equal numbers. However, there is an asymmetry when we examine the top and antitop quarks as a pair. In general, the number of pairs where both quarks have spin up or spin down (\(N_\parallel\)) is not equal to the number of pairs where one quark is spin up and the other is spin down (\(N_\perp\)). In this situation, correlations are visible in a joint distribution containing one decay angle from the top side of the event and one decay angle from the antitop side of the event. Denoting these two decay angles by \(\theta_i\) and \(\theta_i\) respectively, we have

\[
\frac{1}{\sigma} \frac{d^2 \sigma}{d(\cos \theta_i) d(\cos \theta_i)} = \frac{1}{4} \left[ 1 + \frac{N_\parallel - N_\perp}{N_\parallel + N_\perp} \alpha_i \bar{\alpha}_i \cos \theta_i \cos \bar{\theta}_i \right]
\]

for the complete production and decay process, \(p\bar{p} \rightarrow t\bar{t} \rightarrow 6\)-body final state. Eq. (4) explicitly exhibits the dependence of the correlations on the production and decay stages of the event. Production is represented by the pairwise spin asymmetry \((N_\parallel - N_\perp)/(N_\parallel + N_\perp)\). This factor depends upon the choice of spin basis and may be maximized by employing the off-diagonal basis\(^{13}\) (see below). Decay is represented by the correlation coefficients \(\alpha_i\) as well as the decay angles \(\theta_i\) (measured in the \(t\) rest frame) and \(\bar{\theta}_i\) (measured in the \(t\) rest frame).\(^{14}\) Our choice of which decay angles we measure determines how well we can see a given production asymmetry. From this point of view, we want to make the \(\alpha\)'s as large as possible – i.e. we should choose to measure the decay angles of the charged leptons. Because 2-dimensional distributions generally require high statistics for accurate mapping, it may be desirable to construct 1-dimensional or even 0-dimensional projections of Eq. (4). Refs. \(^{13}\) and \(^{14}\) contain some suggestions on how to do this.

We now consider how spin issues relate to \(t\bar{t}\) production. Because the majority (\(\sim 90\%\)) of the cross section comes from the quark-antiquark initial state, we will first focus our attention on the process \(q\bar{q} \rightarrow t\bar{t}\), as illustrated in Fig. 4. We describe this event in terms of the ZMF production angle \(\theta^*\) and the ZMF speed of the top quark \(\beta\). The initial quarks are firmly in the ultrarelativistic

\(^a\)This follows trivially from the observation that \(t_1\) is a massless vector, whereas \(s\) has been constructed to be spacelike.

\(^b\)The small QCD loop-induced transverse polarization of the top quarks may be ignored for our purposes.

\(^c\)The experimental challenge of reconstructing the \(t\) and \(\bar{t}\) rest frames with sufficient accuracy is one which must be met no matter what spin basis is employed.
regime since $m_t \gg m_q$. Because the $q\bar{q}$ coupling in QCD is helicity-conserving, we conclude that the initial $q$ and $\bar{q}$ have unlike helicities.

Suppose that the $t\bar{t}$ pair in Fig. 4 is produced very near threshold. Then, the $t$ rest frame and the $\bar{t}$ rest frame both coincide with the ZMF to a good approximation. Knowledge of the $q$ and $\bar{q}$ helicities translates into knowledge of the total angular momentum along the beam axis; i.e. the unlike $q$ and $\bar{q}$ helicities implies unlike $t$ and $\bar{t}$ spins measured along the beam axis. Along any other axis, there will be a superposition of like and unlike $t$ and $\bar{t}$ spins. Thus, at threshold, the helicity basis does not describe the physics most simply.

On the other hand, if the $t\bar{t}$ pair is produced in the ultrarelativistic regime far above threshold, then the picture in the $t$ and $\bar{t}$ rest frames is vastly different from the picture in the ZMF (see Fig. 5). In the rest frame of either top, the momenta of the other top and both light quarks are essentially parallel. The light quarks still have opposite helicities. Knowledge of the $q$ and $\bar{q}$ helicities thus translates into knowledge of the $t$ and $\bar{t}$ helicities. Using any other spin axis, there would be a superposition of like and unlike spins. Hence, we recover the rationale for employing the helicity basis to describe ultrarelativistic fermions.

Note that in both extremes ($\beta \to 0$ and $\beta \to 1$), there is a basis in which the $t$ and $\bar{t}$ spins are 100% correlated: a spin-up $t$ implies a spin-down $\bar{t}$ and vice versa. This suggests that we should seek a basis for which this property holds for arbitrary $\beta$. The authors of Ref. [14] have constructed such a basis, which they call the off-diagonal basis. This basis takes its name from the fact that for this choice of spin axis, the top pairs coming from $q\bar{q} \to t\bar{t}$ are purely in a state of unlike spins independent of their production angle and ZMF speed. The important vectors in this basis are illustrated in Fig. 6. The direction of the spin vector $s$ in the off-diagonal basis is given by the angle $\psi$, where

$$\tan \psi = \frac{\beta^2 \cos \theta^* \sin \theta^*}{1 - \beta^2 \sin \theta^*}.$$ \hspace{1cm} (5)

The vectors $t_1$ and $t_2$ (cf. Eq. (3)) have a much simpler dependence on $\theta^*$ and $\beta$: they are at an angle $\omega$ with respect to the beam, where

$$\sin \omega = \beta \sin \theta^*.$$ \hspace{1cm} (6)

The $\beta \to 0$ and $\beta \to 1$ limits are particularly transparent in Eq. (5): near threshold, $\omega \to 0$ (the beam direction) while at very high energy, $\omega \to \theta^*$ (the helicity direction). Either of (5) or (6) may be taken as the relation defining the off-diagonal basis. In any case, the vectors $\frac{1}{2}(t \pm ms)$ and $\frac{1}{2}(t \pm m\bar{s})$ are special: for the up-down spin configuration the preferred emission directions of the charged leptons are $\frac{1}{2}(t + ms)$ for the $\ell^+$ and $\frac{1}{2}(t + m\bar{s})$ for the $\ell^-$. For the down-up spin configuration, the charged leptons prefer the directions $\frac{1}{2}(t - ms)$ and $\frac{1}{2}(t - m\bar{s})$.

If $q\bar{q}$ were the only initial state at the Tevatron, then we would have $(N_{\parallel} - N_{\perp})/(N_{\parallel} + N_{\perp}) = -1$. However, approximately 10% of the top pair events are initiated by a
There can be no orbital angular momentum at threshold. The spin-1 nature of the gluon translates into different $t\bar{t}$ correlations. For example, near threshold, the gluons must have like helicities to form a $t\bar{t}$ final state, since opposite helicity gluons would have a total spin projection of $\pm 2$ along the beam axis, whereas the maximum spin projection of the $t\bar{t}$ pair is $\pm 1$. Thus, low mass $t\bar{t}$ pairs tend to have like spins along the beam direction. At very high energies, we recover the preference for opposite helicities since in the top rest frame the gluons have parallel momenta. The upshot of this is that the off-diagonal basis, which works well for the $q\bar{q}$ initial state, is not ideally suited to the $gg$ initial state. Nevertheless, since $q\bar{q}$ dominates the total cross section, the off-diagonal basis is still an excellent choice, although the spin-pair asymmetry is degraded a bit. In Fig. 7 we have plotted the distribution in the $t\bar{t}$ invariant mass for both $q\bar{q}$ and $gg$ production mechanisms.

Table 2: Dominant spin fractions and asymmetries for the helicity and off-diagonal bases for top quark pair production at the Tevatron with $\sqrt{s} = 2.0$ TeV.

| Basis       | Spin Content | $N_\parallel - N_\perp$ |
|-------------|--------------|-------------------------|
| helicity    | 70% unlike   | -0.39                   |
| off-diagonal| 92% unlike   | -0.84                   |

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The presence of non-Standard Model physics would alter the correlations we have just described. One interesting possibility involves a new scalar or pseudoscalar state which couples strongly to the top quark. Typically, this kind of new physics will manifest as a bump in the $t\bar{t}$ invariant mass spectrum. Suppose that we are fortunate and such a bump is observed. Then, an analysis of the $t\bar{t}$ spin correlations can tell us about the parity of the new state. In particular, a pseudoscalar coupled to $t\bar{t}$ produces top pairs which have like spins independent of the choice of spin basis. On the other hand, the top pairs formed via an intermediate scalar would have like spins 100% of the time only in the helicity basis. In (almost) any other basis, such as the off-diagonal basis, a scalar would contribute to both the like and unlike spin configurations. The exception is the basis where the scalar couples exclusively to unlike spin top pairs. This basis is obtained by taking the spin vector to be at right angles to the antitop direction of motion in the top rest frame. In the lab frame, the corresponding direction of $\frac{1}{2}(t + ms)$ is

$$\sin \omega = \beta \sin \theta^* + \sqrt{1 - \beta^2} \cos \theta^*. \quad (7)$$

Our point is that should a bump in the $M_{t\bar{t}}$ spectrum be observed, it would be worthwhile to measure the spin correlations for those events within the peak in more than one spin basis. Doing so allows us to distinguish between scalar and pseudoscalar intermediate states, something which could not be done using only the helicity basis.

As a second example of how new physics would alter the $t\bar{t}$ spin correlations, suppose that there is a charged Higgs decay of top, $t \to H^+ b$. Then, the value of $\alpha$ appearing in Eq. (4) would be different. In particular, we would have $\alpha_b = 1$ and $\alpha_\ell = (-\xi^2 + 1 + 2\xi \ln \xi)/(\xi - 1)^2$, where $\xi \equiv M_t^2/M_H^2$, and $j$ is either $H^+$ decay product (independent of whether the $H^+$ goes to $c\bar{s}$ or $\tau^+\nu_\tau$). In a sample containing both Standard Model and charged Higgs decays, the observed correlations would depend upon the relative size of the two branching ratios.

In conclusion, the extremely short top lifetime provides us with the opportunity to study the spin properties of a “free” quark. In general, the number of produced like-spin top quark pairs is not equal to the number of unlike-spin top pairs. For the process $q\bar{q} \to t\bar{t}$, 100% of the top quark pairs have unlike spins with respect to the off-diagonal basis defined in Eqs. (3) and (4). When the $gg \to t\bar{t}$ process is included, some like-spin top quark pairs are produced. However, the spin correlations in the off-diagonal basis are still twice as large as those in the helicity basis. The largest correlations involve the charged leptons (or $d$-type quarks) with respect to the spin axis in the rest frame of the parent top quark. These correlations can be significantly altered from their Standard Model values if the top quark is strongly coupled to new physics.
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