Valleys in Non-Critical String Foam Suppress Quantum Coherence

John Ellis\textsuperscript{a}, N.E. Mavromatos\textsuperscript{b} and D.V. Nanopoulos\textsuperscript{a,c}

Abstract

As an example of our non-critical string approach to microscopic black hole dynamics, we exhibit some string contributions to the $S$ matrix relating in- and out- state density matrices that do not factorize as a product of $S$ and $S^\dagger$ matrices. They are associated with valley trajectories between topological defects on the string world sheet, that appear as quantum fluctuations in the space-time foam. Through their ultraviolet renormalization scale dependences these valleys cause non-Hamiltonian time evolution and suppress off-diagonal entries in the density matrix at large times. Our approach is a realization of previous formulations of non-equilibrium quantum statistical mechanics with an arrow of time.

\textsuperscript{a} Theory Division, CERN, CH-1211, Geneva, Switzerland,
\textsuperscript{b} Laboratoire de Physique Théorique ENSLAPP (URA 14-36 du CNRS, associée à l’ E.N.S de Lyon, et au LAPP (IN2P3-CNRS) d’Annecy-le-Vieux), Chemin de Bellevue, BP 110, F-74941 Annecy-le-Vieux Cedex, France, on leave from P.P.A.R.C. Advanced Fellowship, Dept. of Physics (Theoretical Physics), University of Oxford, 1 Keble Road, Oxford OX1 3NP, U.K.
\textsuperscript{c} Center for Theoretical Physics, Dept. of Physics, Texas A & M University, College Station, TX 77843-4242, USA, and Astroparticle Physics Group, Houston Advanced Research Center (HARC), The Mitchell Campus, The Woodlands, TX 77381, USA
1 Introduction

Studies of quantum gravity suggest [1, 2] that a pure state description cannot be maintained in the context of point-like field theories. It has been proposed [3, 4] that quantum states be described by density matrices within a framework that allows pure states to evolve into mixed states with entropy increasing monotonically. The transitions between asymptotic in- and out-states would be governed by a superscattering matrix $S$ that does not factorize as a product of $S$ and $S^\dagger$ matrices as in point-like field theory [3]. The corresponding evolution of density matrices through intermediate times would include [4] a non-Hamiltonian term $\delta H$:

$$\partial_t \rho = i[\rho, H] + \delta H \rho$$

(1)

due to quantum fluctuations in the space-time foam.

Although their failures to solve the problem of quantum gravity suggest the need to modify quantum field theory and quantum mechanics, point-like field theories are inadequate to prove this, or model $S$ or $\delta H$. For this one needs a consistent quantum theory of gravity, for which the only available candidate is string theory. No one questions the validity of quantum field theory and quantum mechanics on the world sheet. Moreover, we have identified [5] an infinite set of intrinsically stringy symmetries sufficient to preserve quantum coherence in the presence of a space-time background with a horizon and/or a singularity, such as a two-dimensional or (we conjecture) spherically-symmetric four-dimensional black hole. These $W$ symmetries were first identified on the world sheet, couple string states at different string levels, and are elevated to become local space-time gauge symmetries with an infinite set of associated commuting conserved charges available to encode information [5]. The $W$ charges in target space are defined by non-local integrals over space-time. For this reason, and because laboratory experiments observe only light particles from the lowest string level, these cannot take $W$ symmetries and extended massive string modes into account. Therefore we claim [6] that also in string theory the quantum field theory and quantum mechanics of the effective light-particle theory must indeed be modified so as to accommodate transitions from pure to mixed states, where the information associated with massive modes is lost.

The form of the modifications can be derived [6] by observing that, although the full string theory is finite, the truncated light-particle theory can be defined only by introducing a renormalization scale cut-off, that we identify with a negative-metric Liouville field that can be interpreted as the target time [6, 7]. Thus we arrive at a non-critical string description of the effective light-particle theory, whose time (renormalization scale) -dependence in a fixed space-time background includes the Hamiltonian dynamics of conventional quantum mechanics, which yields asymptotically the $S$ matrix of conventional quantum field theory, as shown in ref. [8, 9]. However, when fluctuations in the space-time background, such as back-reaction
and quantum fluctuations in the space-time foam, are taken into account, additional scale (time) -dependences appear that we claim contribute non-Hamiltonian terms $\delta H$ in (1) and non-factorizable terms in $\delta S$.

In a previous paper [6], we used the language of world-sheet $\sigma$-models and considered dynamics in the $(g^i, p^j)$ phase space, where the $g^i$ denote $\sigma$-model couplings, that can be regarded as coordinates in a space endowed with the Zamolodchikov metric $G_{ij}$, the $p^j$ are the corresponding conjugate momenta, and the $\beta^i$ are the corresponding renormalization group $\beta$-functions. We found that this dynamics was described by an equation of the form (1), with a Hamiltonian $H(g^i, p_j)$ derived from the Zamolodchikov $C$-function [10] that serves as an effective $\sigma$-model action by a Legendre transformation, and derived an expression for $\delta H$:

$$\delta H = \beta^i G_{ij} \frac{\partial \rho}{\partial p^j}$$

Our modified Liouville equation (1,2) is an explicit realization [11] of previous approaches to non-equilibrium quantum statistical mechanics. It exemplifies the general $\Lambda$-transformation theory of ref. [12], with an arrow of time, and has a Lie-admissible structure [13], as a result [14] of the symmetry of the Zamolodchikov metric $G_{ij}$ [11]. This also guarantees [8] its consistency with canonical quantization conditions [15].

The formalism leading to (2) is analogous to that of the Drude model of quantum friction that was used in refs. [16, 17] to discuss decoherence in open quantum-mechanical systems due to couplings to ‘environmental oscillators’. In our approach, the corresponding couplings are required by $W$-symmetry, the unobservable extended massive string modes take the place of the ‘environmental oscillators’, and the resulting loss of quantum coherence is regarded as an inevitable consequence of fluctuations in the space-time background.

In this paper, we exhibit explicit contributions to $\delta H$ (2) associated with two distinct classes of space-time fluctuations: the quantum creation and annihilation of a microscopic black hole, and the back-reaction of light matter particles on the space-time foam. We work in the context of the $SL(2, R)/U(1)$ coset Wess-Zumino model on the world-sheet, that describes a two-dimensional (spherically-symmetric four-dimensional) black hole [18]. We have argued previously that the quantum creation and annihilation of microscopic black holes is modelled by the dipole phase of a monopole-antimonopole gas [19, 20] on the world-sheet, which therefore provides suitable framework for analyzing the corresponding contribution to $\delta H$. As we discuss in section 2, the back-reaction of light particles on the black-hole metric is described by instantons [21] in the $SL(2, R)/U(1)$ model that capture the effects of couplings to the unobservable extended massive string modes [8].
In section 3, we evaluate monopole and instanton contributions to $\mathcal{S}$ and $\delta H$ by considering absorptive parts \cite{22} of world-sheet correlation functions that are dominated \cite{23} by valley trajectories \cite{24} in monopole-antimonopole and instanton-anti-instanton configurations respectively. We exhibit explicit valley trajectories, and show that they make contributions to the scale (time) -dependences of transitions between density matrices that are linear in small anomalous dimensions, contributing terms to $\delta H$ and hence $\mathcal{S}$. As we discuss in section 4, these have the effect of suppressing off-diagonal terms in the target configuration space representation of the effective light-particle density matrix at large times.

2 String Black Hole Monopoles and Instantons

The action of $SL(2, R)/U(1)$ coset Wess-Zumino model \cite{18} describing a Euclidean black hole can be written in the form

$$S = \frac{k}{4\pi} \int d^2 z \frac{1}{1 + |w|^2} \partial \bar{w} \partial w + \ldots$$

(3)

where the conventional radial and angular coordinates $(r, \theta)$ are given by $w = \sinh re^{-i\theta}$ and the target space $(r, \theta)$ line element is

$$ds^2 = \frac{dw d\bar{w}}{1 + w\bar{w}} = dr^2 + \tanh^2 rd\theta^2$$

(4)

The Euclidean black hole can be written as a vortex-antivortex pair \cite{20}, which is a solution of the following Green function equations on a spherical world sheet:

$$\partial_z \partial_{\bar{z}} X_v = i\pi \frac{q_v}{2} [\delta(z - z_1) - \delta(z - z_2)]$$

(5)

The world-sheet can also accommodate monopole-antimonopole pairs \cite{20}, which are solutions of:

$$\partial_z \partial_{\bar{z}} X_m = -\frac{q_m \pi}{2} [\delta(z - z_1) - \delta(z - z_2)]$$

(6)

These are related to Minkowski black holes with masses $\propto q_m$. We believe that some essential features of four-dimensional space-time foam can be captured by studying an analogue two-dimensional model, in which the above vortex and monopole configurations are both regarded as sine-Gordon deformations of the effective action for the field $X \equiv \beta^{-\frac{1}{2}} \bar{X}$, where $\beta^{-1}$ is an effective ‘pseudo-temperature’: $\beta = \frac{3}{\pi(c-25)}$ in Liouville theory. The partition function \cite{19}

$$Z = \int D\bar{X} \exp(-\beta S_{eff}(\bar{X}))$$

$$\beta S_{eff} = \int d^2 z [2\partial \bar{X} \partial X + \frac{1}{4\pi} [\gamma \omega \frac{1}{2}(2|g(z)|1 - \gamma) : \cos(\sqrt{2\pi \alpha}[X(z) + \bar{X}(\bar{z})]) : + (\gamma, \alpha, \bar{X}(z) + \bar{X}(\bar{z})) \rightarrow (\gamma, \alpha', \bar{X}(z) + \bar{X}(\bar{z}))]]$$

(7)
requires for its specification an angular ultraviolet cut-off $\omega$ on the world-sheet with metric $g(z, \bar{z})$. Here $\gamma_{v,m}$ are the fugacities for vortices and spikes respectively, and $\frac{\alpha}{\Gamma}$ is the conformal dimension $\Delta$. This deformed sine-Gordon theory has a low-temperature phase modelling four-dimensional space-time foam, in which monopole-antimonopole pairs are bound in dipoles as irrelevant deformations with the conformal dimension

$$\Delta_m = \frac{\alpha_m}{4} = \frac{\pi \beta}{2} d_m > 1$$  \hspace{1cm} (8)

A monopole-antimonopole pair corresponds to the creation and anihilation of a microscopic black hole in the space-time foam.

As shown in ref. [21], the $SL(2, R)/U(1)$ Wess-Zumino coset model describing a Euclidean black hole also has instantons given by the holomorphic function

$$w(z) = \frac{\rho}{z - z_0}$$  \hspace{1cm} (9)

with topological charge

$$Q = \frac{1}{\pi} \int d^2 z \frac{1}{1 + |w|^2} [\partial w \partial w - h.c.] = -2 \ln(a) + \text{const}$$  \hspace{1cm} (10)

where $a$ is an ultraviolet cut-off discussed later. The instanton action on the world-sheet also depends logarithmically on the ultraviolet cut-off. As in the case of the more familiar vortex configuration in the Kosterlitz-Thouless model, this logarithmic divergence does not prevent the instanton from having important dynamical effects. The instanton-anti-instanton vertices take the form [21]

$$V_{II} \propto -\frac{d}{2\pi} \int d^2 z d^2 \rho e^{-S_0} \left( e^{\frac{k |\rho| \partial w + h.c. + ...}{\text{h.c.}}} + e^{\frac{k |\rho| \partial w + h.c. + ...}{\text{h.c.}}} f(|w|) \right)$$  \hspace{1cm} (11)

introducing a new term into the effective action. Making a derivative expansion of the instanton vertex and taking the large-$k$ limit, i.e. restricting our attention to instanton sizes $\rho \simeq a$, this new term has the same form as the kinetic term in (3), and hence corresponds to a renormalization of the effective level parameter in the large $k$ limit:

$$k \to k - 2\pi k^2 d' \quad : \quad d' \equiv d \int \frac{d|\rho|}{|\rho|^3} \frac{a^2}{[(\rho/a)^2 + 1]^2}$$  \hspace{1cm} (12)

If other perturbations are ignored, the instantons are irrelevant deformations and conformal invariance is maintained. However, in the presence of “tachyon” deformations, $T_0 \int d^2 z \mathcal{F}_{-\frac{1}{2}, 0, 0}^c$ in the $SL(2, R)$ notation of ref. [22], there are extra logarithmic infinities in the shift (12), that are visible in the dilute gas and weak-“tachyon”-field approximations. In this case, there is a contribution to the effective action of the form

$$T_0 \int d^2 z d^2 z' \mathcal{F}_{-\frac{1}{2}, 0, 0}^c(z, \bar{z}) V_{II}(z', \bar{z'})$$  \hspace{1cm} (13)
Using the explicit form of the “tachyon” vertex $F$

$$F_{c,c}^{\frac{1}{2},0,0} = \frac{1}{\sqrt{1 + |w|^2}} \frac{1}{\Gamma\left(\frac{1}{2}\right)^2} \sum_{n=0}^{\infty} \left\{2\psi(n + 1) - 2\psi(n + \frac{1}{2}) + \ln(1 + |w|^2)\right\}(\sqrt{1 + |w|^2})^{-n} \tag{14}$$

given by $SL(2, R)$ symmetry [25], it is straightforward to isolate a logarithmically-infinite contribution to the kinetic term in (3), associated with infrared infinities on the world-sheet expressible in terms of the world-sheet area $\Omega/a^2$, the latter being measured in units of the ultraviolet cut-off $a [8, 9]$, 

$$\propto T_0 \int d^2 z' \int \frac{d\rho}{\rho} \left(\frac{a^2}{a^2 + \rho^2}\right)^{\frac{1}{2}} \int d^2 z \frac{1}{|z - z'|^2} \frac{1}{1 + |w|^2} \partial_{z'} w(z') \partial_{\bar{z}'} \bar{w}(z') + \ldots$$

$$\propto T_0 \ln\frac{\Omega}{a^2} \int d^2 z' \frac{1}{1 + |w|^2} \partial_{z'} w(z') \partial_{\bar{z}'} \bar{w}(z') \tag{15}$$

Such covariant-scale-dependent contributions can be attributed to Liouville field dynamics, through the “fixed-area constraint” in the Liouville path integral [26, 27]. The zero-mode part can be absorbed in a scale-dependent shift of $k[8]$, which for large $k >> 1$ may be assumed to exponentiate:

$$k_R \propto \left(\frac{\Omega}{a^2}\right)^{(\text{const}) \beta I T_0} \tag{16}$$

where $\beta I$ is the instanton $\beta$-function [21]. In ref. [8] we gave general arguments and verified to lowest order that instantons represent higher mode effects, enabling us to identify $\beta I = -\beta T$, where $\beta T$ is the renormalization-group $\beta$-function of a matter deformation of the black hole [1]. Notice that in (15) both the infrared and the ultraviolet cut-off scales enter. In the following we shall not distinguish between infrared and ultraviolet cut-offs. The physical scale of the system, which varies along a renormalization group trajectory, is the dimensionless ratio of the two, which is identified with the Liouville field.

The separation of the full string theory into an effective light-particle theory and higher modes described here by instantons entails a non-critical description of the former. This we achieve using the Liouville string formalism, with the Liouville field identified not only as a renormalization scale, but also as time, as we discuss in more detail later.

The change in $k$ and the associated change in the central charge $c = \frac{3k}{k^2 - 2} - 1$ and the black-hole mass $M_{bh} \propto (k - 2)^{-\frac{1}{2}}$ do not conflict with any general theorems. An analogous instanton renormalization of $\theta$ has been demonstrated [28] in related

\[1\] Notice that this implies that the matter $\beta$-function has to be computed in a non-perturbative way, which is consistent with the exact conformal field theory analysis of ref. [25].
σ-models that describe the Integer Quantum Hall Effect (IQHE), discussed further in section 4. Instanton renormalization of $k$ can also be seen in the Minkowski black hole model of ref. [18], defined on a non-compact manifold $SL(2, R)/O(1, 1)^2$. In our case, as we have seen, the instantons reflect a shift of the central charge between the matter and background sectors of a combined matter + black hole theory, in which the total central charge is unchanged. They correspond to a combination of world-sheet deformation operators in Wess-Zumino model [18]: the exactly marginal operator $L_0^2$ and the irrelevant part of the exactly marginal deformation $L_0^1$, which involves an infinite sum of higher-level string operators [23]. It is well known that the light matter (‘tachyon’) perturbation is not by itself exactly marginal, but this property can be enforced by including these higher-level string state operators [23]. The previously-mentioned fact that, at large $k$, these operators rescale the black hole metric, can also be seen from their contributions to the action of the deformed Wess-Zumino σ-model after the gauge field integration [25].

$$S_\sigma \ni \int d^2z \{ \partial r \partial r(1 - 2g\text{csc}^2r - 2g\text{sech}^2r) + $$

$$\partial \theta \partial \theta (\sinh^2r + 2g - \frac{(\sinh^2r + 2g)^2}{\cosh^2r + 2g}) \}$$

(17)

where $g$ is the coupling of the $L_0^2$ deformation. Changing variables $\cosh^2r + 2g \rightarrow \cosh^2r$ in (17) one finds that to $O(g)$ the target space metric is rescaled by an overall constant, and thus such perturbations have the same effect as the instanton. Thus the instanton represents the effects of higher string modes that are related to each other and to massless excitations by a $\hat{W}$ symmetry. Matrix elements of the full exactly marginal light matter + instanton operator have no dependence on the ultraviolet cut-off $a$, but the separate matter and instanton parts do depend on $a$, as we have seen above.

3 Valleys in String Foam

We now consider the contributions of monopoles and instantons to $\sigma$ matrix elements giving transitions between a generic initial-state density matrix $\rho_B^{A(\text{in})}$ and final-state density matrix $\rho_D^{C(\text{out})}$. This is described by an absorptive part of a world-sheet correlation function

$$\sum_{X_{\text{out}}} \langle A|D, X >_{\text{out}} < X, C|B >_{\text{in}}$$

$$= \sum_{X_{\text{out}}} \langle 0|T(\phi(z_A)\phi(z_D))|X >_{\text{out}} < X|T(\phi(z_C)\phi(z_B))|0 >_{\text{in}}$$

$\sigma$-model action of such a theory contains [21], in addition to the action [8], a total-derivative $\theta$-term which can be thought of as a deformation of the black hole by an “antisymmetric tensor” background, which in two dimensions is a discrete mode as a result of the abelian gauge symmetry. Its Euclideanized version has also instanton solutions of the form [8], but with finite action, which induce “Liouville”-time-dependent shifts to $k$, prior to matter couplings.
\[ T(\phi(z_A)\phi(z_D))T(\phi(z_C)\phi(z_B))|0 >_{in} \]

Here we have used the optical theorem \[22\] on the world sheet, which is valid because conventional quantum field theory, and indeed quantum mechanics, remain valid on the world sheet, to replace the sum over unseen states \( X \) by unity. Next, we use dilute-gas approximations to estimate the leading monopole-antimonopole and instanton-anti-instanton contributions to the absorptive part \((18)\). We expect these to be dominated \[23\] by valley configurations in the Euclidean functional integral, so that in a semi-classical approximation

\[ \$ \propto \text{Abs} \int D\phi_c \exp(-S_v(\phi_c))F_{\text{kin}} \]

where the integral is over the collective coordinates \( \phi_c \) of the valley, whose action is \( S_v(\phi_c) \). The function \( F_{\text{kin}} \) depends on kinematic factors, taking generically the form

\[ F_{\text{kin}} = \exp(E\Delta R) \]

in the case of a four-point function for large \( E\Delta R \), where \( E \) is the centre-of-mass energy and \( \Delta R \) is the valley separation parameter. This enables us to make a saddle-point approximation to the integral \((19)\), which we then continue back to Minkowski space.

Valley trajectories \( \psi_v \) have a homotopic parameter \( \mu \) and obey an equation of the form

\[ \left. \frac{\partial S_0}{\partial \psi} \right|_{\psi = \psi_v} = W_\psi(\mu) \frac{\partial \psi_v}{\partial \mu} \]

where \( W_\psi(\mu) \) is a weight function that is positive definite and decays rapidly at large distances \[24, 23\]. We adapt techniques used in the \( O(3) \) \( \sigma \)-model \[29, 30\] to find the monopole-antimonopole and instanton-anti-instanton valleys in a reduced version of the \( SL(2, R)/U(1) \) model. The separations of the topological defects and anti-defects are well-defined in the presence of conformal symmetry breaking, which is provided in our case by the dilaton field \[8\]. Valleys can be found by using the analogy \[29\] between \( \mu \) and a ‘time’ variable for defect-anti-defect scattering. We do not discuss here the details of their construction, but record the results.

The monopole-antimonopole valley function, expressed in terms of the original world-sheet variables, reads

\[ w(z, \bar{z}) = \frac{(v - 1/v)\bar{z}}{1 + |z|^2} \]

where \( v \) denotes the separation in the \( \sigma \)-model framework. Eq. \((22)\) represents a concentric valley, which can then be mapped into an ordinary valley by applying appropriate conformal transformations. The function \((22)\) interpolates between a far-separated monopole-antimonopole pair \( (v \to \infty) \) and the trivial vacuum \( (v = 1) \).
For large but finite separations the corresponding valley action leads to the action of a monopole-antimonopole pair interacting via dipole interactions. The action of the monopole-antimonopole valley depends on the angular ultraviolet cut-off $w$ introduced in section 2:

$$S_m = 8\pi q^2 \ln(2)\sqrt{e^\gamma} + 2\pi q^2 \ln \frac{2R}{\omega} + 2\pi q^2 \ln \left[ \frac{|z_1 - z_2|}{(4R^2 + |z_1|^2)^{\frac{1}{2}}} \left( \frac{4}{(4R^2 + |z_2|^2)^{\frac{1}{2}}} \right) \right]$$

(23)

for a monopole and antimonopole pair of equal and opposite charges $q$, which we treat as a collective coordinate over which we must integrate, where $\gamma$ is Euler’s constant, the second term in (23) is a logarithmically-divergent self-energy term on a spherical world sheet of radius $R$, and the last term in (23) is a dipole interaction energy. For finite separations $0 < |z_1 - z_2| < \infty$ and very small world-sheets $R = O(a \to 0)$, the action (23) yields

$$S_m = 2\pi q^2 \ln \frac{a}{\omega} + \text{finite parts}$$

(24)

where the ultraviolet cut-off dependence is apparent.

To construct the instanton valley, we notice that in the reduced model used for the construction of the monopole valley (22) the solution for an instanton-anti-instanton pair is derived from the corresponding monopole case via a conformal transformation in the $(\mu, \ln|z|)$-plane. In ref. [8] we give arguments why this construction is true for finite separations as well, thereby leading to an expression of the instanton valley as an (approximate) conformal transform of the monopole valley (22). The action of the instanton-anti-instanton valley in the large-separation limit of the dilute-gas approximation is

$$S_{IT} = k\ln(1 + |\rho|^2/a^2) + O\left( \frac{\rho \bar{\rho}}{(\Delta R)^2} \right)$$

(25)

where $\Delta R$ is the separation of an instanton of size $\rho$ and an anti-instanton of size $\bar{\rho}$, and we find a dependence on the ultraviolet cut-off $a$. The actions (24,25) substituted into the general expression (19) make non-trivial contributions to the $S$ matrix that do not factorize as a product of $S$ and $S^\dagger$ matrix elements.

### 4 Suppression of Coherence

To understand how the above non-perturbative contributions suppress quantum coherence at large times, we now review our interpretation [6, 8, 7] of target time as a renormalization group scale parameter in the effective light-particle theory with an ultraviolet cut-off (cf, $\omega$ in the monopole case, $a$ in the instanton case). We use the concept of the local (on the world sheet) renormalization group equation [32, 33], according to which perturbative $\beta$-functions constitute a gradient flow of the string effective action in target space [34], and identify the local cut-off as the Liouville mode, whose kinetic term has a temporal signature for supercritical strings ($c > 26$).
The inclusion of topological fluctuations such as monopole-antimonomopole or instanton-anti-instanton pairs in a critical string theory makes it supercritical \textit{locally}, necessitating the introduction of such a time-like Liouville field, which we interpret as target time \cite{6, 8, 4}. We regard the two-dimensional black hole model of ref. \cite{18} as a toy laboratory that gives us insight into the nature of time in string theory and contributes to the physical effects that will be of interest to us here. The action of the model is

\[ S_0 = \frac{k}{2\pi} \int d^2z [\partial r \partial r - \text{tanh}^2 r \partial t \partial t] + \frac{1}{8\pi} \int d^2z R^{(2)} \Phi(r) \] (26)

where \( r \) is a space-like coordinate and \( t \) is time-like, \( R^{(2)} \) is the scalar curvature, and \( \Phi \) is the dilaton field. The customary interpretation of (26) is as a string model with \( c = 1 \) matter, represented by the \( t \) field, interacting with a Liouville mode, represented by the \( r \) field, which has \( c < 1 \) and is correspondingly space-like \cite{35, 36, 37}. As an illustration of the approach outlined above, however, we re-interpret (26) as a fixed point of the renormalization group flow in the local scale variable \( t \). In our interpretation, the “matter” sector is defined by the spatial coordinate \( r \), and has central charge \( c_m = 25 \) when \( k = 9/4 \) \cite{18}. Thus the model (26) describes a critical string in a dilaton/graviton background. The fact that this is static, i.e. independent of \( t \), reflects the fact that one is at a fixed point of the renormalization group flow \cite{8, 4}.

We now outline how one can use the machinery of the renormalization group in curved space, with \( t \) introduced as a local renormalization scale on the world sheet, to derive the model (26). A detailed technical description is given in \cite{8, 7, 9}. There are two contributions to the kinetic term for \( t \) in our approach, one associated with the Jacobian of the path integration over the world-sheet metrics, and the other with fluctuations in the background metric.

To exhibit the former, we first choose the conformal gauge \( \gamma_{\alpha\beta} = e^\rho \bar{\gamma}_{\alpha\beta} \) \cite{34}, where \( \rho \) represents the Liouville mode. One must identify \( \rho \) with \( 2\alpha' \phi \), where \( \phi(z, \bar{z}) \equiv \ln \mu(z, \bar{z}) \) is a local scale on the world-sheet, in order to reproduce the critical string results of ref. \cite{18}. This makes the local scale \( \phi \) a dynamical \( \sigma \)-model field. Ref. \cite{36} contains an explicit computation of the Jacobian using heat-kernel regularization, which yields

\[-\frac{1}{48\pi} \left[ \frac{1}{2} \partial_\alpha \rho \partial^\alpha \rho + R^{(2)} \rho + \frac{\mu}{\epsilon} e^\rho + S'_G \right] \] (27)

where the counterterms \( S'_G \) are needed to remove the non-logarithmic divergences associated with the induced world-sheet cosmological constant term \( \frac{\mu}{\epsilon} e^\rho \), and depend on the background fields.
To find the contributions to the effective action from the background fields we recall that the renormalization of composite operators in $\sigma$-models formulated on curved world sheets is achieved by allowing an arbitrary dependence of the couplings $g^i$ on the world-sheet variables $z, \bar{z}$ \cite{32, 33}. This induces counterterms of “tachyonic” form, which, in dimensional regularization with $d = 2 - \epsilon$, include the following simple pole at one loop, for a $\sigma$-model propagating in a graviton background \cite{33}:

$$Y^{(1)} = \frac{\lambda}{16\pi\epsilon} \partial_\alpha G_{MN} \partial^\alpha G^{MN}$$

where $\lambda \equiv 4\pi\alpha'$ is a loop-counting parameter. In ref. \cite{33} $G_{MN}$ was allowed to depend arbitrarily on the world-sheet variables, and all world-sheet derivatives of the couplings were set to zero at the end of the calculation. In our Liouville mode interpretation, we assume that such dependence occurs only through the local scale $\mu(z, \bar{z})$, so that

$$\partial_\alpha g^i = \hat{\beta}^i \partial_\alpha \phi(z, \bar{z})$$

where $\hat{\beta}^i = \epsilon g^i + \beta^i(g)$ and $\phi = \ln\mu(z, \bar{z})$. Taking the $\epsilon \to 0$ limit, separating the finite and $O(1/\epsilon)$ terms, and taking into account that the one-loop graviton $\beta$-function is

$$\beta^G_{MN} = \frac{\lambda}{2\pi} R_{MN}$$

and that in the case of the Minkowski black hole model of ref. \cite{18}, the Lorentzian target-space curvature is

$$R = \frac{4}{\cosh^2 r} = 4 - 4\tanh^2 r,$$

we recover, upon combining the world-sheet metric Jacobian term in \cite{27} with the gravitational background fluctuation terms, the critical string $\sigma$-model action \cite{26} for the Minkowski black-hole. Dilaton counterterms are incorporated in a similar way, yielding the dilaton background of \cite{18}. In addition, as standard in stringy $\sigma$-models, one also obtains the necessary counterterms that guarantee target-space diffeomorphism invariance of the Weyl-anomaly coefficients \cite{32}. Details are given in ref. \cite{9}.

It should be noticed that the renormalization group yields automatically the Minkowski signature, due to the $c_m = 25$ value of the matter central charge \cite{35, 37}. However, the relevant conformal field theory is more developed in the Euclidean case \cite{23}. As we remarked in ref. \cite{3, 4, 8}, one can switch over to the Euclidean black hole model, and still maintain the identification of the compact time with some appropriate function of the Liouville scale $\phi$ that takes into account the compactness of $t$ in that case. In ref. \cite{23} it was argued that the exactly marginal deformation that turned on a static tachyon background for the black hole of ref. \cite{18} necessarily involved the higher-level topological string modes, which are non-propagating
extended states analogous to the de-phasons of the Hall model \[38\]. This is a consequence of the operator product expansion of the tachyon zero-mode operator \( \mathcal{F}_{-\frac{1}{2},0} \):

\[
\mathcal{F}_{-\frac{1}{2},0} \circ \mathcal{F}_{-\frac{1}{2},0} = \mathcal{F}_{-\frac{1}{2},0} + W_{h,w}^{1,0} + W_{l,w}^{1,0} + \ldots
\]

(32)

where we only exhibit the appropriate holomorphic part for reasons of economy of space. The \( W \) operators and the \( \ldots \) denote level-one and higher string states. The latter cannot be detected in local scattering experiments, due to their extended character. From a formal field-theoretic point of view such states cannot exist as asymptotic states to define scattering, and also cannot be integrated out in a local path-integral. They can only exist as marginal deformations in a string theory. An ‘experimentalist’ therefore sees necessarily a truncated matter theory, where the only deformation is the tachyon \( \mathcal{F}_{-\frac{1}{2},0} \), which is a (1,1) operator in the black hole \( \sigma \)-model \[26\], but is not exactly marginal. This implies additional Liouville dressing, due to the non-vanishing operator-product-expansion coefficients. The latter contribute to the quadratic, and higher order, parts of the Wilson Renormalization Group \( \beta \)-functions of the pertinent \( \sigma \)-model which are non-zero. Hence, this truncated theory is non-critical, and the additional Liouville dressing in the sense is essential, implies time-dependence of the matter background. Due to the fact that the appropriate exactly-marginal deformation associated with the tachyon in these models involves all higher-level string states, one can conclude that in this picture the ensuing non-equilibrium time-dependent backgrounds are a consequence of information carried off by the unobserved topological string modes. The world-sheet instantons studied in section 2, which in the presence of tachyons lead to violations of conformal invariance, is a way to represent collectively, in a semi-classical approximation, the effects of these topological string modes on the low-energy field theory. The rôle of the space-time singularity\[3\] was crucial for this argument. Indeed, in flat target-space matrix models \[38\] the tachyon zero-mode operator \( \mathcal{F} \) is exactly marginal. As we argue in ref. \[5, 7\], these flat models can be regarded as an asymptotic ultraviolet limit in time of the Wess-Zumino black hole. Hence, any time-dependence of the matter disappears in the vacuum, which is thus viewed as an ‘equilibrium’ situation for the string.

There is a very important aspect of Liouville theory that makes it distinct from any other approach with a local renormalization group cut-off. When evaluating correlation functions among Liouville-dressed vertex operators, the result is proportional \[40\] to \( \Gamma(-s) \) factors, where \( s \) is the sum of the respective Liouville energies, including the terms of the background charge at infinity that represent the Liouville mode itself. Among the physically-interesting values of \( s \) are those corresponding to positive integers, which for instance is the case of light-matter scattering off a

\[\text{We would like to stress that the notion of ‘singularity’ is clearly a low-energy effective-theory concept. The existence of infinite-dimensional stringy symmetries associated with higher-level string states (\( W_{\infty} \)-symmetries \[8\]) ‘smooth out’ the singularity, and render the full string theory finite.}\]
string black hole, that involves an excitation of the latter to a discrete massive state. Such values of $\Gamma(-|n|)$ call for regularization via analytic continuation [41] along a saddle-point contour of the form illustrated in the figure. The result is

$$\Gamma(-|n|) = \int_C dAA^{-|n|-1} e^{-A}$$  \hspace{1cm} \text{(33)}$$

implying the existence of imaginary parts, which are then interpreted as indicating the renormalization group instability. By inspection of the figure and applying the logic of ref. [42], we observe that the effective flow of target time is from an infrared fixed point with large apparent world-sheet area in target space, to an ultraviolet fixed point with small world-sheet area. This flow is analogous to the decay of a metastable vacuum in conventional field theory. It is characterized by energy conservation [43] and a monotonic increase in entropy [3].

In the dilute-gas approximation introduced in section 3, the topologically trivial zero monopole-antimonopole, zero instanton-anti-instanton sector in the unitary sum in (18) provides the usual $S$-matrix description of scattering in a fixed background, with no back reaction of the light matter on the metric. This result is well-known in the conformal field theory approach to critical string theory, and is discussed explicitly in the present context in section 6 of ref. [8]. This $S$-matrix contribution corresponds to the usual Hamiltonian description of quantum mechanics, via the representation $S = 1 + iT : T = \int_\infty^- dt H(t)$. Any topologically non-trivial contribution to the unitarity sum in (18) goes beyond the usual treatment of conformal field theory in critical strings, and makes a contribution to the non-factorization of the $S$-matrix : $S = SS^\dagger + \ldots$. Two such contributions that we have identified above come from the monopole-antimonopole and instanton-anti-instanton sectors discussed above, which we expect to be dominated by the valley actions (24) and (25) respectively.

The dependence of the monopole-antimonopole valley action (24) on the ultraviolet cutoff $\omega$, which we identify with the target-space time $t = -ln\omega$, and of the instanton-anti-instanton valley action (25) on the local scale-dependent level parameter $k$ (12, 16) where $t = -lna$, tell us that both valleys contribute to the non-Hamiltonian term in the modified quantum Liouville equation (1). From the monopole-antimonopole valley (24) we find [8]

$$\Delta S \simeq e^{-2(\Delta_m-1)t} + \ldots$$  \hspace{1cm} \text{(34)}$$

where $\Delta_m - 1$ is the anomalous dimension (8) of an irrelevant dipole-like monopole-antimonopole pair, and from the instanton-anti-instanton valley (25) we find [8]

$$\Delta S \simeq e^{-2\gamma_0 t} + \ldots$$  \hspace{1cm} \text{(35)}$$

where $\gamma_0$ is the anomalous dimension (14) of a matter deformation. Both of these time-dependences apply in limits of far-separated defect and anti-defect. Differentiating (34, 35) with respect to time, we find contributions to $\delta H$ that are proportional
to the anomalous dimensions $(\Delta_m - 1, \gamma_0)$. As we discussed in ref. [9], it is a feature of our non-critical Liouville string approach that correlation functions in general, and $S$ in particular, receive area- and hence time-dependences proportional to the anomalous dimension of any deformation.

The time-irreversibility inherent in (34,35) reflects the fact that our modified Liouville equation (1,2) possesses an arrow of time [11]. Moreover, it has a Lie-admissible structure [13], as a consequence [14] of the symmetry of the Zamolodchikov metric $G_{ij}$ [11], and is consistent [9] with canonical quantization [15], again as a result of the symmetry of $G_{ij}$ and the gradient property of the renormalization group flow.

The resulting formalism for the time evolution of the density matrix is analogous to the Drude model of quantum friction [16, 17], with the massive string modes playing the rôles of ‘environmental oscillators’. In the language of world-sheet $\sigma$-model couplings $\{g\}$, the reduced density matrix of the observable states is given, relative to that evaluated in conventional Schrödinger quantum mechanics, by an expression of the form

$$\rho(g, g', t)/\rho_S(g, g', t) \sim e^{-\eta \int_0^t d\tau \int_{\tau}^t d\tau' G_{ij} \beta^i \beta^j} \sim e^{-D \frac{(g - g')^2}{2} + ...}$$

(36)

where $\eta$ is a calculable proportionality coefficient, and $G_{ij}$ is the Zamolodchikov metric [10] in the space of couplings. In string theory, the identification of the target-space action with the Zamolodchikov c-function $C(\{g\})$ [10] enables the Drude exponent to be written in the form

$$\beta^i G_{ij} \beta^j = \partial_i C(\{g\})$$

(37)

which also determines the rate of increase of entropy

$$\dot{S} = \beta^i G_{ij} \beta^j S$$

(38)

In the string analogue [34] of the Drude model [33] the rôle of the coordinates in (real) space is played by the $\sigma$-model couplings $g^i$ that are target-space background fields. Relevant for us is the tachyon field $T(X)$, leading us to interpret $(g - g')^2$ in (36) as

$$\nabla T)^2(X - X')^2$$

(39)

for small target separations $(X, X')$.

The effect of the time-dependences (34,35, 36,39) is to suppress off-diagonal elements in the target configuration space representation [14] of the out-state density matrix:

$$\rho_{\text{out}}(x, x') = \hat{\rho}(x) \delta(x - x')$$

(40)

This behaviour can be understood intuitively as being related to the apparent shrinking of the string world sheet in target space, which destroys interferences between
strings localized at different points in target configuration space, c.f. the de-phasons in the Hall model \[38\]. This behaviour is generic for string contributions to the space-time foam, which make the theory supercritical locally, inducing renormalization group (target time) flow. The two specific contributions \((34,35)\) to this suppression \((40)\) of space-time coherence that we have identified in this paper correspond \((34)\) to microscopic black hole formation and \((35)\) to the back-reaction of matter on a microscopic black hole, entailing in each case information loss across an event horizon.

**Acknowledgements**

The work of N.E.M. is supported by a EC Research Fellowship, Proposal Nr. ERB4001GT922259. That of D.V.N. is partially supported by DOE grant DE-FG05-91-GR-40633. J.E. would like to thank LAPP (Annecy-le-Vieux, France) for its hospitality while part of this work was being done. We would also like to thank Tim Hollowood and A.V. Yung for useful discussions.
References

[1] J. Bekenstein, Phys. Rev. D12 (1975), 3077.

[2] S. Hawking, Comm. Math. Phys. 43 (1975), 199.

[3] S. Hawking, Comm. Math. Phys. 87 (1982), 395.

[4] J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, Nucl. Phys. B241 (1984), 381.

[5] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B267 (1991), 465; ibid B272 (1991), 261.

[6] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992), 37.

[7] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Proc. HARC workshop on “Recent Advances in the Superworld”, The Woodlands, Texas (USA), April 14-16 1993, eds. J. Lopez and D.V. Nanopoulos, (World Sci., Singapore, 1994), p. 3; hep-th/9311148.

[8] J. Ellis, N.E. Mavromatos, and D.V. Nanopoulos, CERN and Texas A & M Univ. preprint, CERN-TH.6897/93; CTP-TAMU-30/93; ACT-10/93 (1993).

[9] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, preprint CERN-TH.7480/94, ENS-LAPP-A-492/94, ACT–/94, CTP-TAMU—/94.

[10] A.B. Zamolodchikov, JETP Lett. 43 (1986), 730; Sov. J. Nucl. Phys. 46 (1987), 1090.

[11] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, preprint CERN-TH.7195/94, ENS-LAPP-A-463/94, ACT-5/94, CTP-TAMU-13/94, lectures presented at the Erice Summer School, 31st Course: From Supersymmetry to the Origin of Space-Time, Ettore Majorana Centre, Erice, July 4-12 1993.

[12] B. Misra, I. Prigogine and M. Courbage, Physica A98 (1979), 1;
   I. Prigogine, Entropy, Time, and Kinetic Description, in Order and Fluctuations in Equilibrium and Non-Equilibrium Statistical Mechanics, ed G. Nicolis et al. (Wiley, New York 1981);
   B. Misra and I. Prigogine, Time, Probability and Dynamics, in Long-Time Prediction in Dynamics, ed G. W. Horton, L. E. Reichl and A.G. Szebehely (Wiley, New York 1983);
   B. Misra, Proc. Nat. Acad. Sci. U.S.A. 75 (1978), 1627.

[13] R. M. Santilli, Hadronic J. 1 (1978), 223, 574 and 1279; Foundations of Theoretical Mechanics, Vol. I (1978) and II (1983) (Springer-Verlag, Heidelberg-New York);
For an application of this approach to dissipative statistical systems, which is directly relevant to our work here, see: J. Fronteau, A. Tellez-Arenas and R.M. Santilli, Hadronic J. 3 (1979), 130; J. Fronteau, Hadronic J. 4 (1981), 742.

[14] J.P. Constantopoulos and C.N. Ktorides, J. Phys. A17 (1984), L29.
[15] S.A. Hojman and L.C. Shepley, Journ. Math. Phys. 31, (1991), 142.
[16] R.P. Feynman and F.L. Vernon Jr., Ann. Phys. (NY) 94 (1963), 118.
[17] A.O. Caldeira and A.J. Leggett, Ann. Phys. 149 (1983), 374.
[18] E. Witten, Phys. Rev. D44 (1991), 314.
[19] B.A. Ovrut and S. Thomas, Phys. Rev. D43 (1991), 1314.
[20] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B289 (1992), 25; ibid B296 (1992), 40.
[21] A.V. Yung, Int. J. Mod. Phys. A9 (1994), 591.
[22] A. Mueller, Phys. Rep. 73 (1981), 237.
[23] V.V. Khoze and A. Ringwald, Nucl. Phys. B355 (1991), 351.
[24] I.I. Balitsky and A.V. Yung, Phys. Lett. B168 (1986), 113; A.V. Yung, Nucl. Phys. B297 (1988), 47.
[25] S. Chaudhuri and J. Lykken, Nucl. Phys B396 (1993), 270.
[26] F. David, Mod. Phys. Lett. A3 (1988), 1651; J. Distler and H. Kawai, Nucl. Phys. B321 (1989), 509.
[27] D. Kutasov, Mod. Phys. Lett. A7 (1992), 2943.
[28] A. Pruisken, Nucl. Phys. B290[FS20] (1987), 61.
[29] N. Dorey, Los Alamos National Lab. preprint, LA-UR-92-1380 (1992), Phys. Rev. D to be published.
[30] J. Bossart and Ch. Wiesendanger, Phys. Rev. D46 (1992), 1820.
[31] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B288 (1992), 23.
[32] G. Shore, Nucl. Phys. B286 (1987), 349.
[33] H. Osborn, Nucl. Phys. B294 (1987), 595; ibid B308 (1988), 629; Phys. Lett. B222 (1989), 97.
[34] N.E. Mavromatos, J.L. Miramontes and J.M. Sanchez de Santos, Phys. Rev. D40 (1989), 535.

[35] I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, Phys. Lett. B211 (1988), 393; Nucl. Phys. B328 (1989), 117; Phys. Lett. B257 (1991), 278.

[36] N.E. Mavromatos and J.L. Miramontes, Mod. Phys. Lett. A4 (1989), 1847.

[37] J. Polchinski, Nucl. Phys. B324 (1989), 123;

D.V. Nanopoulos, in *Proc. Int. School of Astroparticle Physics*, HARC-Houston (World Scientific, Singapore, 1991), p. 183.

[38] A. Pruisken, Nucl. Phys. B235[FS11] (1984), 277;

H. Levine, S. Libby and A. Pruisken, Phys. Rev. Lett. 51 (1983), 1915; Nucl. Phys. B240[FS12] (1984), 30;49;71.

[39] E. Brézin and V.A. Kazakov, Phys. Lett. B236 (1990), 144;

M. Douglas and A. Shenker, Nucl. Phys. B335 (1990), 635;

D. Gross and A.A. Migdal, Phys. Rev. Lett. 64 (1990), 127;

For a recent review see, e.g., I. Klebanov, in *String Theory and Quantum Gravity*, Proc. Trieste Spring School 1991, ed. by J. Harvey et al. (World Scientific, Singapore, 1991), and references therein.

[40] M. Goulian and M. Li, Phys. Rev. Lett. 66 (1991), 2051.

[41] I. Kogan, Phys. Lett. B265 (1991), 269.

[42] S. Coleman, Phys. Rev. D15 (1977), 2929; 1248 (E) (1977).

C. Callan and S. Coleman, Phys. Rev. D16 91977), 1762.

[43] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992), 142;

CERN and Texas A & M Univ. preprint CERN-TH. 6755/92; ACT-24/92;CTP-TAMU-83/92.

[44] J. Ellis, S. Mohanty and D.V. Nanopoulos, Phys. Lett. B221 (1989), 113.
Figure Caption

Contour of integration in the analytically-continued (regularized) version of $\Gamma(-s)$ for $s \in \mathbb{Z}^+$. 