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Dynamics of the quantum vacuum: Cosmology as relaxation to the equilibrium state

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Abstract. The behavior of the gravitating vacuum energy density in an expanding universe is discussed. A scenario is presented with a step-wise relaxation of the vacuum energy density. The vacuum energy density moves from plateau to plateau and follows, on average, the steadily decreasing matter energy density. The current plateau with a small positive value of the vacuum energy density (effective cosmological constant) may result from a still not equilibrated contribution of the light massive neutrinos to the quantum vacuum.

1. Introduction

There are many different contributions to the vacuum energy density. They can be separated into sub-Planckian and trans-Planckian contributions with respect to the energy scale $E_{\text{Planck}} = [\hbar c^5/(8\pi G_N)]^{1/2} \approx 2.44 \times 10^{18}$ GeV. The sub-Planckian contributions are described by relativistic quantum fields propagating over a classical spacetime manifold. The trans-Planckian contributions come from the fundamental microscopic degrees of freedom of the “deep vacuum.” In the perfect equilibrium Minkowski vacuum [1], the trans-Planckian degrees of freedom compensate the contribution of the sub-Planckian quantum fields to the gravitating vacuum energy density $\rho_{\text{vac}}$ (effective cosmological constant $\Lambda_{\text{eff}}$).

The above discussion adopts the condensed-matter point of view, where the effective quantum fields emerge only at low energies and where the high-energy phenomena are determined by the fundamental (atomic) degrees of freedom. In a self-sustained equilibrium vacuum, the full compensation of the vacuum energy density occurs without fine-tuning [1, 2]: the microscopic degrees of freedom are automatically adjusted to the equilibrium state. The self-tuned nullification of the energy density in the ground state of an arbitrary equilibrium system, including the relativistic quantum vacuum, suggests a possible solution [3] of the main cosmological constant problem [4]. A brief review of this so-called $q$–theory approach to the cosmological constant problem is presented in Appendix A.

The $q$–theory approach to the cosmological constant problem differs from those approaches which consider only relativistic quantum fields and where the contributions of the different quantum fields, fermionic and bosonic, compensate each other. This fermion-boson compensation can be due to supersymmetry or to a special choice of the fermionic and bosonic content.
of the vacuum, relying on special relations between the masses of the scalar, spinor, and vector fields (see, e.g., Refs. [5, 6, 7, 8, 9]).

In the perfect equilibrium state, the individual contributions to the vacuum energy density cannot be distinguished. But the situation changes dramatically for the dynamical vacuum of an expanding universe [2]. Cosmology, then, corresponds to the process of relaxation towards the equilibrium vacuum state, with the vacuum energy density dropping from its initial large value [which can be of order $\rho_{\text{vac}} \sim (E_{\text{Planck}})^4$] to its present small value [$\rho_{\text{vac}} \approx (2 \text{meV})^4$]. During the expansion of the Universe, the energy hierarchy of the different contributions to the vacuum energy density from the different quantum fields gives rise to a time sequence, since each epoch is characterized by one dominant contribution to the vacuum energy density and, thus, by one particular value of the effective cosmological constant. This suggests a step-wise relaxation of the effective cosmological constant, which, on average, follows the matter energy density.

The contribution of a quantum matter field to the vacuum energy density is given by the following expression (see, e.g., Refs. [5, 6, 7, 8, 9, 10, 11, 12]):

$$\rho_{\text{vac}}^{(\text{matter})}(M, E_{uv}) = c_4 \left( E_{uv} \right)^4 + c_2 \left( E_{uv} \right)^2 M^2 + c_0 M^4 \ln \left[ \frac{\left( E_{uv} \right)^2}{M^2} \right] + O(M^4) ,$$

where $M$ is the mass of the matter field considered and $E_{uv}$ is the ultraviolet energy cutoff, which is typically assumed to be of the order of the energy scale $E_{\text{Planck}}$ defined above. Different regularization schemes have been suggested, in order to obtain the coefficients $c_n$ appearing in (1). Here, and in the following, natural units are used with $\hbar = c = k_B = 1$.

Cosmological phase transitions or crossovers lead to time-varying masses of the fermionic and bosonic fields and, thereby, affect the contributions of these fields to the effective cosmological constant. In this article, the focus will be on the fermionic degrees of freedom, whose number is larger than the one of the bosonic degrees of freedom in the standard model of elementary particle physics (cf. Sec. 19.3.2 of Ref. [13]).

It is natural to suppose that, for higher temperatures, the number of fermionic quantum fields with nonzero masses is smaller, thereby removing the corresponding negative contributions to the vacuum energy density (see Secs. 2–4 and Appendix B for details, in particular, for the reason of having a negative contribution). As a result, the vacuum energy density from fermionic mass contributions is typically large for high temperature and small for low temperature. In an expanding universe, this vacuum energy density (effective cosmological constant) then relaxes in a step-wise manner, following the appearance of masses of the fermionic matter fields (see Secs. 5–7 for further discussion). The step-wise relaxation continues until the vacuum energy density reaches the zero value of the perfect equilibrium state or until the vacuum energy density is frozen at a stage where the relaxation process becomes ineffective.

Several scenarios have been proposed for this final (frozen) stage and the corresponding remnant vacuum energy density. These scenarios rely, for example, on new TeV–scale electroweak physics [14, 15, 16] or nonperturbative QCD dynamics [17, 18, 19, 20, 21] with a remnant vacuum energy density given by, respectively, $\rho_{\text{vac,\infty}} \sim (E_{\text{ew}})^8 / (E_{\text{Planck}})^4$ or $\rho_{\text{vac,\infty}} \sim (E_{\text{QCD}})^6 / (E_{\text{Planck}})^2$. In Sec. 5, we add a scenario related to neutrino mass $M_\nu$, which may give a remnant vacuum energy density of order $(M_\nu)^4$; see, e.g., Refs. [22, 23, 24] for related discussions. All these scenarios do not exclude each other. Different contributions to the vacuum energy density produce successive plateaux in the process of the relaxation of the effective cosmological constant and the one which survives (or is frozen) is responsible for the current plateau of the effective cosmological constant at the meV–scale level.

2. Fermion propagator in an interacting system and vacuum energy density $\rho_{\text{vac}}$

As a start, consider the contribution of a fermionic quantum field to the vacuum energy density. The discussion will be based on certain properties of the mass function in the fermion propagator.
This fermion propagator must apply to the virtual fermions of the quantum vacuum (the ground state in condensed-matter-physics language) which is a dense interacting system with Planck-scale energies.

The general form of the Green’s function of a fermionic particle is

\[ G(p) = \frac{Z(p^2)}{i \gamma \mu p_\mu + M(p^2)} , \]

for a Euclidean metric, \( p^2 = p_\mu p_\nu \delta^{\mu\nu} = |p|^2 + (p_0)^2 \), and appropriate Dirac matrices, \( \gamma^\mu \gamma ^\nu + \gamma ^\nu \gamma ^\mu = 2 \delta^{\mu\nu} \). The mass function \( M(p^2) \) in (2) must disappear at large \( p^2 \), where interaction effects can be neglected. The simplest form of \( M(p^2) \) satisfying that condition is

\[ M(p^2) = \frac{a}{b + p^2} , \]

in term of parameters \( a > 0 \) and \( b \geq 0 \) with mass dimensions 3 and 2, respectively.

This two-parameter Ansatz reflects two important physical quantities of the mass function, which depend on the interaction strength characterizing the system. First, the parameter \( b^{1/2} \) from model (3) gives the energy scale of the effective ultraviolet cutoff for mass-dependent effects. Second, the ratio of the parameters \( a \) and \( b \) represents the mass parameter at zero momentum, \( M(0) = a/b = (a b^{-3/2}) b^{1/2} \). The fermion mass \( M \) at the pole of the Green’s function (i.e., on mass shell, temporarily reverting to a Lorentzian metric) is determined by the equation \( M^2 \left(b-M^2\right)=M^2(0)b^2 \). For \( a^2 \ll b^2 \), the on-shell mass is simply given by \( M \approx M(0) = a/b \).

For charged leptons, the natural value of the cutoff is the electroweak energy scale, \( b \sim (E_{\text{ew}})^2 \). For neutrinos, the cutoff may be comparable with the neutrino mass itself (a heuristic argument based on the see-saw mechanism is given in Appendix B). For the analytic Ansatz (2), this gives \( a^2 \sim b^3 \) and the on-shell neutrino mass \( M_\nu \) typically differs from \( M_\nu(0) \) but is still of order \( M_\nu(0) \).

The fermionic contribution to the vacuum energy density is obtained from the following Euclidean effective action:

\[ S_E = -V_4 \int \frac{d^4 p}{(2\pi)^4} \text{tr} \ln [G(p) \tilde{m}] , \]

where \( V_4 \) is the Euclidean spacetime volume considered and \( \tilde{m} \) a fixed reference mass. The zeroth-order term in the gradient expansion gives the contribution of the fermionic field to the cosmological constant term,

\[ S = -\int d^4x \sqrt{-\text{det} [g(x)]} \rho^{(\text{fermion})} + \ldots , \]

where, now, the action has been given for an arbitrary Lorentzian metric \( g_{\mu\nu}(x) \) and the conventions of Ref. [3] have been used.

The fermionic contribution to the cosmological constant can be calculated by introducing the vierbein field in (4), \( \gamma^\mu e_\mu^\nu p_\nu \), and taking the functional derivative of \( S_E \) with respect to \( e_\mu^\nu \). Consider the contribution from the denominator of the Green’s function (2). In order to obtain a simple estimate of the mass effects, take the derivative of (4) with respect to \( a \),

\[ \frac{d S_E}{d a} \sim \int \frac{d^4 p}{(2\pi)^4} \text{tr} \left[ (b + p^2)^{-1} G \right] \sim \int \frac{d^4 p}{(2\pi)^4} \frac{a}{a^2 + p^2 (b + p^2)^2} , \]
leaving out $V_3$ for the sake of brevity. The case $b^3 \lesssim a^2$ gives $dS_E/da \sim a^{1/3}$ and $S_E \sim a^{4/3}$. The case $a^2 \lesssim b^3$ gives $S_E \sim a^2/b$. Combined, there is thus the following simple estimate of the fermion contribution to the vacuum energy density:

$$\rho_{\text{vac}}^{(\text{fermion})} \sim \begin{cases} -a^{1/3} & \text{for } b^3 \lesssim a^2, \\ -a^2/b & \text{for } b^3 \gtrsim a^2, \end{cases}$$ \hspace{1cm} (7)

where the overall minus sign will be verified independently in the next section.

3. Fermion contributions to $\rho_{\text{vac}}$ at the electroweak energy scale

In the electroweak standard model, the natural choice for the Green’s function of quarks and leptons with masses $M \lesssim E_{\text{ew}}$ gives parameters $b \sim (E_{\text{ew}})^2$ and $a = M(0) \lesssim (E_{\text{ew}})^3$ for the model mass function (3). According to (7), the contribution to the vacuum energy density from light fermions with $M \ll E_{\text{ew}}$ is of order $M^2 (E_{\text{ew}})^2$ and the one of heavy fermions with $M \sim E_{\text{ew}}$ of order $(E_{\text{ew}})^4$. Hence, the general expectation is that the contribution of a charged fermionic field with mass $M \lesssim E_{\text{ew}}$ to the vacuum energy density is

$$\rho_{\text{vac}}^{(\text{fermion})}(M) \sim -M^2 (E_{\text{ew}})^2,$$ \hspace{1cm} (8)

taking over the minus sign from (7).

The previous estimates (7) and (8) are obtained from an analysis of the denominator of the Green’s function (2), but the numerator gives similar results. The residue $Z(p^2)$ approaches unity as $p^2 \to \infty$. For finite values of $p^2$, however, the residue $Z(p^2)$ is again determined by two energy scales, $M(0)$ and $E_{\text{ew}}$.

Estimate (8), including the minus sign, can also be obtained by direct calculation of the mass contribution to the zero-point energy from a spin-$\frac{1}{2}$ Dirac field:

$$\int (E_{\text{cutoff}}) \frac{d^3p}{(2\pi)^3} \left( -\sqrt{|p|^2 + M^2} + |p| \right) \sim -M^2 (E_{\text{cutoff}})^2,$$ \hspace{1cm} (9)

where $E_{\text{cutoff}}$ is the ultraviolet cutoff of this quadratically divergent integral (see, e.g., Ref. [5]). Since the Green’s function of a massive standard-model fermion differs from the Green’s function of a massless fermion only at energies below $E_{\text{ew}}$, it is the electroweak scale $E_{\text{ew}}$ which provides the natural ultraviolet cutoff in (9) rather than the Planck scale: $E_{\text{cutoff}} = E_{\text{ew}}$.

A Planck-scale cutoff $E_{\text{cutoff}} = E_{\text{Planck}}$ in (9) would be relevant only if the mass of the fermion were fundamental or generated at the Planck energy scale, i.e., with $b \sim (E_{\text{Planck}})^2$ in the mass function (3). However, if the Green’s function of a massive fermion differs from the Green’s function of a massless fermions only at energies of order of mass scale (as discussed in the fourth paragraph of Sec. 2), then the cutoff in (9) is determined by the fermion mass itself, leading to $\rho_{\text{vac}}^{(\text{fermion})}(M) \sim -M^4$. We suggest that the last estimate holds for the neutrino contribution (see Sec. 5 for possible cosmological implications).

4. Fermion contributions to $\rho_{\text{vac}}$ at the QCD energy scale

For the quarks of quantum chromodynamics (QCD), the natural choice of parameters in model (3) is $a \sim (E_{\text{QCD}})^3$ and $b = 0$. Such a choice effectively corresponds to the phenomenon of confinement, which leads to a singular behavior of the effective quark mass in the infrared limit $p \to 0$. Actually, this Green’s function at $p_0 = 0$ has been used in Refs. [18, 19] to justify the presence of a running term $|H(t)|(E_{\text{QCD}})^3$ in the vacuum energy density [here, $H(t)$ is the Hubble parameter of the expanding universe] and to calculate the corresponding remnant vacuum energy density $\rho_{\text{vac,\infty}} \sim (E_{\text{QCD}})^6/(E_{\text{Planck}})^2$. 

4
\[ \rho_{\text{vac}}(t) \sim (E_{\text{Planck}})^4/t^2 \text{ from Ref. [2].} \]

According to this estimate, the vacuum energy density at the present time passes through a value close to the one of the observed cosmological "constant":
\[ \rho_{\text{vac}}(t_{\text{present}}) \sim (E_{\text{Planck}})^2/t_{\text{present}}^2 \approx \Lambda_{\text{present}}^{(\text{obs})} \approx (2 \text{ meV})^4. \]

Full curve: dissipative processes and cosmological phase transitions lead to a modified behavior, with a step-wise decrease of the vacuum energy density towards zero. The vacuum energy density follows the matter energy density on average, but locally, on each plateau, the vacuum behaves as a medium with equation-of-state parameter \( w \approx -1 \).

5. Fermion contributions to \( \rho_{\text{vac}} \) at the mass scale of light neutrinos

At finite temperature \( T \), the mass function \( M(p^2) \) in (2) can be expected to depend also on \( T \). Moreover, temperature may provide the relevant energy scale for the ultraviolet cutoff in (9), \( E_{\text{cutoff}} \sim T \). As a result, the mass-dependent contribution becomes part of the thermal energy rather than the vacuum energy. This suggests a natural form of energy exchange between thermal fermions and vacuum fermions, i.e., between matter and vacuum. In any case, it can be expected that, with increasing temperature, the mass-dependent contributions to the vacuum energy density will be reduced or even disappear.

As the temperature of the adiabatically expanding universe decreases, more and more fermions become massive and add extra negative contributions (9) to the vacuum energy density. This implies that the total vacuum energy density \( \rho_{\text{vac}} \) decreases with time and that this change occurs in a step-like manner (Fig. 1), following, on the whole, the matter energy density.

According to this scenario, one of the plateaux in \( \rho_{\text{vac}}(t) \) for \( t \sim T \) results from the lack of the
neutrino-mass contribution to the vacuum energy density, \( \rho_{\text{vac}}^{(\text{neutrino})} (M_\nu) \sim -(M_\nu)^4 \). The lack of a negative neutrino contribution to the vacuum energy density corresponds, of course, to a positive value of \( \rho_{\text{vac}}(T) \) at the time scale \( T \) considered. Contributions of fermionic matter fields with larger vacuum energy densities have already been released at higher temperatures and have been relaxed due to the fast oscillations of the microscopic \( q \)-type fields (see Appendix A.2).

Restrict the discussion temporarily to one neutrino flavor. Then, the neutrino contribution to the vacuum energy density can only be released after the temperature \( T_\nu \) of the relic neutrino drops below the neutrino mass \( M_\nu \) [which happens at a redshift of order \( M_\nu/(3 \times 10^{-4} \text{ eV}) \sim 10^2 \) for \( M_\nu \sim 0.05 \text{ eV} \)]. It is, however, very well possible that the contribution from the light neutrino is still not released at the present moment, because of the lack of an efficient equilibration mechanism between neutrino matter and neutrino vacuum in the current epoch. In that case, the excess of the vacuum energy density remains frozen at a positive value of order \( (M_\nu)^4 \). The lowest plateau in Fig. 1 can be expected to extend far beyond the present age of the Universe.

Turning to three neutrino flavors with substantial (near-maximal) mixing [13], a heuristic argument suggests the following expression for the frozen value of the vacuum energy density:

\[
\rho_{\text{vac}}(t_{\text{present}}) \overset{t_{\text{present}}}{=} c_\nu \left( \text{tr} \left[ (M_\nu)^2 \right] \right) \left( \text{det} \left[ (M_\nu)^2 \right] \right)^{1/3},
\]

(10)

where \( M_\nu \) is the \( 3 \times 3 \) neutrino mass matrix with eigenvalues \( \{m_{\nu1}, m_{\nu2}, m_{\nu3}\} \) and the positive coefficient \( c_\nu \) is assumed to be of order 1. Additional terms, possibly depending on the mixing angles, can be expected in (10). Note that neutrino contributions to the effective cosmological constant have been discussed in a number of recent articles (see, e.g., Refs. [22, 23, 24]).

Given the measured value \( \rho_{\text{vac}}(t_{\text{present}}) \approx (2.3 \text{ meV})^4 \) for the left-hand side of (10) and the neutrino-oscillation data [13], there are three equations for the three neutrino masses. Take the neutrino-oscillation input to be \( (m_{\nu3})^2 - (m_{\nu1})^2 = \pm 2.4 \times 10^{-3} \text{ eV}^2 \) and \( (m_{\nu2})^2 - (m_{\nu1})^2 = 7.7 \times 10^{-5} \text{ eV}^2 \). Then, formula (10) with \( c_\nu = 1 \) gives the following neutrino mass spectra:

\[
(m_{\nu1}, m_{\nu2}, m_{\nu3}) \overset{c_\nu=1}{=} \left\{ \begin{array}{c}
2.793 \times 10^{-6}, 8.775, 48.99 \times \text{meV} ,
48.99, 49.77, 1.783 \times 10^{-7} \times \text{meV} .
\end{array} \right.
\]

(11)

These neutrino masses (close to the minimum values needed for the neutrino oscillations) lie outside the reach of the KATRIN tritium beta-decay detector with a 0.2-eV design sensitivity [25]. The same conclusion holds for the neutrino mass spectra resulting from formula (10) with \( c_\nu \geq 10^{-8} \) [the \( c_\nu = 10^{-8} \) spectra for both “normal” and “inverted” mass hierarchies have \( m_{\nu1} \sim m_{\nu2} \sim m_{\nu3} \sim 0.2 \text{ eV} \)]. See Sec. 6 for additional comments.

6. Discussion

The main message of this article is that there is a hierarchy of different contributions to the vacuum energy density from different matter fields. In the context of an expanding universe, these contributions are step-by-step released with decreasing temperature of the Universe. Note that the few big steps of the vacuum energy density in Fig. 1 are only schematic, there may be many additional small steps. Some of these (small) steps of the vacuum energy density may even increase the vacuum energy density, if they result from the mass effects of bosonic fields (corresponding to (9) multiplied by \(-1\)). But, as mentioned in Sec. 1, the present article focusses on the mass effects of fermionic fields, which typically give decreasing steps of the vacuum energy density as the Universe expands and cools.

The vacuum energy density roughly follows the matter energy density, but in a stepwise manner. This suggests a possible solution of the cosmic coincidence puzzle (cf. Ref. [14]), namely, why the observed cosmological “constant” is precisely of the order of the present matter energy density. Indeed, the step-wise relaxation of the vacuum energy density may solve the cosmic coincidence puzzle, because, during each epoch, the effective cosmological constant is related to the energy scale characterizing the given epoch and, thus, to the matter energy density.
It has been shown [2] that the huge initial vacuum energy density may continuously relax to a zero value as the age of the Universe goes to infinity. Moreover, it was found that this relaxing vacuum energy density passes through the observed numerical value ($\rho_{\text{vac},0} \sim 10^{-11}$ eV$^4$) for an age of the model universe of the order of the observed value ($t_0 \sim 10$ Gyr); see, in particular, Eq. (5.16) of Ref. [2]. This scenario gives a reasonable estimate of the present value of the vacuum energy density but contradicts the more detailed astronomical observations which are in favor of a genuine cosmological constant, at least, in the current epoch. The reason for the failure of the continuous-relaxation scenario may be that it is oversimplified, since it does not take into consideration the processes of radiation, dissipation, and cosmological phase transitions or crossovers. The new scenario with a step-wise decrease of the vacuum energy density may remove this discrepancy: the vacuum energy density follows the matter energy density on average, but locally, on each plateau, behaves as a medium with equation-of-state parameter $w \approx -1$.

The overall picture for the evolution of the vacuum energy density is, then, as follows. The initial relaxation dynamics of the vacuum energy density is dominated by microscopic (trans-Planckian) degrees of freedom [2], leading to $1/t^2$ decay from ultrafast oscillations or to exponential decay if radiation of matter fields or gravitational waves is taken into account. (This behavior is similar to that of Starobinsky inflation [26, 27], where the role of the microscopic degrees of freedom is played by heavy Planck-mass fields.) For later times, probably starting at the electroweak epoch, the vacuum dynamics and the energy exchange between vacuum and matter is dominated by the low-energy contributions of quantum fields and can be described by standard-model physics. The dynamics of the trans-Planckian degrees of freedom (e.g., the dynamics of the vacuum variable $q$ from Ref. [2]) becomes irrelevant at late times, providing only a tiny response to perturbations occurring during the crossovers [15, 16]. The microscopic degrees of freedom have already played their main role: to fully compensate the vacuum energy density of quantum fields in the final equilibrium state, that is, the Minkowski vacuum.

In order to clarify this point about the main role of the microscopic degrees of freedom, consider, for example, the large vacuum energy density released by an intermediate cosmological phase transition. It is assumed that the chemical potential $\mu$ for the microscopic degrees of freedom takes its natural value $\mu_0$, which corresponds to the true quantum vacuum of a global equilibrium (in the cosmological context, the final Minkowski state with massive fermions at zero temperature), and not a different value $\tilde{\mu} \neq \mu_0$, which would correspond to a nonequilibrium state (in the cosmological context, the intermediate false-vacuum state with massless fermions at temperatures above the phase transition temperature considered). This value $\mu_0$ either is fixed globally as an integration constant being conserved throughout the history of the Universe [1, 2] or emerges dynamically from an attractor-type solution [3]. This is the reason why the full curve in Fig. 1 (taking dissipative effects to have been operative at $t > t_{\text{Planck}}$) does not approach a zero value of the vacuum energy density at times $t_{\text{Planck}} \ll t < t_{\text{ew}}$ but a finite positive value. Later, this vacuum energy density is reduced (released) at the transition to the next plateau.

If the vacuum energy density in the current epoch results from the lack of the neutrino contribution due to non-equilibration (as discussed in Sec. 5) and formula (10) holds with $c_\nu \sim 1$, then the 0.2-eV sensitivity of the KATRIN tritium beta-decay detector [25] does not suffice to detect the predicted nonzero (electron-type) neutrino mass $\leq 0.05$ eV from (11).

If, on the other hand, KATRIN does find a neutrino mass $\gtrsim 0.4$ eV, then this implies that the neutrino contribution to the vacuum energy density (10) [assuming $c_\nu \gtrsim 10^{-6}$] exceeds by several orders of magnitude the observed value of the current effective cosmological constant ($\rho_{\text{vac},0} \approx 3 \times 10^{-11}$ eV$^4$) and that this part of the vacuum energy density cannot be responsible for the presently observed value $\rho_{\text{vac},0}$. In this case, the neutrino contribution to $\rho_{\text{vac}}(t)$ must have dominated in one of the previous epochs and must have been released by now.

Assuming that the neutrino contribution has already equilibrated (as would be suggested by a positive result from KATRIN with $M_\nu \gtrsim 0.4$ eV), the remnant vacuum energy density $\rho_{\text{vac},\infty}$
must be associated with other possible contributions such as \((E_{\text{ew}})^8/(E_{\text{Planck}})^4\) from Refs. [15, 16] or \((E_{\text{QCD}})^6/(E_{\text{Planck}})^2\) from Refs. [18, 19]. However, the last two explanations of \(\rho_{\text{vac,} \infty}\) may fare differently when compared with astronomical observations: the QCD explanation corresponds to a modified gravity theory and may already be ruled out by the astronomical observations, whereas the electroweak explanation corresponds to a standard \(\Lambda\text{CDM}\) model (now, with a calculated value of \(\Lambda\)) and appears to fit the current astronomical data well. Still, the electroweak explanation requires nonstandard physics at the TeV energy scale, which remains to be confirmed by particle-collider experiments.

All this also suggests that, in the past, there were additional plateaux in the effective cosmological constant associated with, for example, the mass \(M_e\) of the electron, the masses \(M_{u,d}\) of the light (stable) quarks, the QCD energy scale, and the electroweak energy scale. These contributions to \(\rho_{\text{vac}}\) may be of order \((M_e)^2(E_{\text{ew}})^2\), \((M_{u,d})^2(E_{\text{ew}})^2\), \((E_{\text{QCD}})^4\), and \((E_{\text{ew}})^4\).

7. Conclusion

It has been argued, in this article, that cosmology can be viewed as the relaxation of the Universe towards the equilibrium vacuum state, with the vacuum energy density dropping from its initial Planck-scale value to its present meV-scale value. The initial relaxation dynamics of the vacuum is dominated by microscopic (trans-Planckian) degrees of freedom [1, 2], leading to \(1/t^2\) decay of the average vacuum energy density (or to exponential decay from dissipative effects).

The dynamics of the vacuum energy density at later times is governed by contributions to the vacuum energy density from the relativistic quantum fields of the standard model. The energy hierarchy of the contributions of different quantum fields to the vacuum energy density leads to a step-wise relaxation with time of the effective cosmological constant (Fig. 1), which, on average, follows the matter energy density. Such a step-wise behavior of the vacuum energy density may solve the cosmic coincidence puzzle [14], because, in each epoch, the effective cosmological constant is related to the energy scale characterizing the given epoch and, thus, to the matter energy density. There are several scenarios for the origin of the latest plateau of the effective cosmological constant, including a possible contribution (10) from the masses of the neutrinos (more precisely, the positive effective cosmological constant would result from the lack of the negative mass-effect contribution of the neutrino fields, if these fields of the quantum vacuum still have not reached their equilibrium state).

Returning to a crucial point, the hierarchical structure of the quantum vacuum implies a complicated spectral function of the vacuum energy density. (The spectral function of the vacuum energy density has been introduced by Zeldovich [5] and its relation to the standard model of elementary particle physics has been discussed in, e.g., Refs. [7, 28, 29,].) The individual contributions to the spectral function of the vacuum energy density cannot be resolved in the static Minkowski vacuum, since, in equilibrium, the contributions of the bosonic and fermionic quantum fields to the low-energy part of the spectral function are compensated by the trans-Planckian part of the spectrum [29]. The trans-Planckian degrees of freedom, which are responsible for the automatic nullification of the cosmological constant in the equilibrium Minkowski vacuum, play also an important role in the dynamics at the early (Planckian) stage of expansion of the Universe. At later stages, the dynamics of the quantum vacuum is primarily determined by the low-energy tail of the spectral function. The various hierarchies of low-energy scales give rise to different plateaux in the vacuum energy density, each of which resembles a genuine cosmological constant but has a dynamic origin nevertheless.

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Appendix A. Résumé of q–theory
One route to understanding the gravitational effects of the vacuum energy density goes under the name of “q–theory” [1, 2, 3]. The idea is to consider the macroscopic equations of a conserved microscopic variable q, whose precise nature need not be known. The goal of this appendix is to explain the basic logic of q–theory as an approach to the cosmological constant problem [4].

Appendix A.1. Statics
For a special type of vacuum variable q introduced in Ref. [1], the vacuum energy density ρvac(q) entering the gravitational field equations has been found to differ from the vacuum energy density ε(q) appearing in the action:

\[ \rho_{\text{vac}}(q) = \epsilon(q) - q \frac{d\epsilon(q)}{dq} = \epsilon(q) - q \epsilon'(q) . \] (A.1)

Here, q is a microscopic variable describing the physics of the deep (ultraviolet) vacuum, but its thermodynamics and dynamics are governed by macroscopic equations, because q is a conserved quantity. This quantity q is similar to the mass density in liquids, which describes a microscopic quantity – the number density of atoms – but obeys the macroscopic equations of hydrodynamics, because of particle-number conservation.

However, different from known liquids, the quantum vacuum is Lorentz invariant. The quantity q must, therefore, be Lorentz invariant, at least, in the equilibrium state,

\[ q = q_0 = \text{constant} . \] (A.2a)

The variable q naturally allows for a vanishing gravitating vacuum energy density (cosmological constant)

\[ \rho_{\text{vac}}(q_0) = \epsilon(q_0) - q_0 \epsilon'(q_0) = 0 . \] (A.2b)

Even though both terms in the middle expression of (A.2b) can be of order \(E_{\text{Planck}}^4\), they cancel exactly in the equilibrium state corresponding to Minkowski spacetime. As such, q–theory provides a possible solution [3] of the main cosmological constant problem [4], namely, why the energy scale of the observed cosmological constant is essentially zero compared to the known energy scales of elementary particle physics (e.g., \(E_{\text{QCD}} \sim 10^2\) MeV and \(E_{\text{ew}} \sim 1\) TeV).

Appendix A.2. Dynamics
For a particular realization of the q variable and a nontrivial q–dependence of the gravitational coupling parameter, it has been shown [2] that q becomes spacetime-dependent and so does the vacuum energy density \(\rho_{\text{vac}}(q) = \frac{1}{2}(q - q_0)^2 + O((q - q_0)^3)\).

In a spatially flat, isotropic, and homogeneous Robertson–Walker universe, the field equations then give a rapidly oscillating vacuum energy density \(\rho_{\text{vac}}(t)\), which drops from a Planck-scale value to zero as \(t \rightarrow \infty\). Specifically, the following behavior has been found [2]:

\[ q(\tau)/q_0 - 1 \sim \tau^{-1} \sin \tau, \quad r_{\text{vac}}(\tau) \sim \tau^{-2} \sin^2 \tau, \] (A.3)

where the dimensionless cosmic time \(\tau\) and the dimensionless vacuum energy density \(r_{\text{vac}}\) have been obtained by appropriate scalings with microscopic (Planckian) quantities such as \(q_0\).

Appendix B. Neutrino mass and effective momentum cutoff
In this appendix, a heuristic argument is given for a low effective momentum cutoff of the mass effects from neutrino fields in an interacting system (in fact, the system relevant to the virtual particles populating the quantum vacuum up to Planck-scale energies).

Start from the see-saw mechanism for a single neutrino flavor (see, e.g., Ref. [30] and references therein), which gives a light neutrino mass, \(M_\nu(0) = (M_D)^2/M_R\), for a right-handed Majorana scale \(M_R\) and a Dirac scale \(M_D \sim E_{\text{ew}}\) with hierarchy \(M_R \gg M_D\).
Adding linear momentum contributions to the standard see-saw mass matrix gives the following effective mass matrix:

$$M_{\nu}^{\text{lin}}(p^2) = \begin{pmatrix} c_{11} p & M_D + c_{12} p \\ M_D + c_{12} p & M_R + c_{22} p \end{pmatrix},$$

with $p \equiv \sqrt{p^2} \geq 0$ for a Euclidean metric [see below (2)] and nonnegative numerical coefficients $c_{ij}$ of order 1. The $c_{11}$ term in (B.1) corresponds to an effective Higgs-triplet contribution from the interactions, independent from mass terms. One eigenvalue of (B.1) is of order $M_R$. The absolute value of the other eigenvalue is

$$M_\nu(p^2) = (M_D)^2/M_R - c_{11} p + O(p^2),$$

for $p \ll M_D \ll M_R$. Observe that the two terms on the right-hand side of (B.2) have a relative minus sign, which is not the case for the individual entries of the matrix (B.1).

Equation (B.2) for $c_{11} \sim 1$ shows an effective momentum cutoff for mass effects of the order of the neutrino mass itself, as discussed in the fourth paragraph of Sec. 2. For the simple example considered in this appendix and small enough $p^2$, the Ansatz for the mass function would be nonanalytic in $p^2$, namely, $M_{\nu}(p^2) = \alpha/(\beta + p)$ with $\alpha \sim \beta^2$.

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