Polarised and Unpolarised Charmonium Production at Higher Orders in $v$

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We study the unpolarised and polarised hadro-production of charmonium in non-relativistic QCD (NRQCD) at low transverse momentum, including sufficiently higher orders in the relative velocity, $v$, so as to study the ratio of $\chi_{c1}$ and $\chi_{c2}$ production rates.

1. Introduction

Recent progress in the understanding of cross sections for production of heavy quarkonium resonances has come through the NRQCD reformulation of this problem \cite{Bodwin:1994jh}. A Factorisation approach based on NRQCD developed by Bodwin, Braaten and Lepage \cite{Bodwin:1994jh} enables the factorisation of the production cross sections for a quarkonium $H$ (with momentum $P$) into a perturbative part and a non perturbative part—

$$d\sigma = \frac{1}{\Phi} \frac{d^3P}{(2\pi)^3 2E_P} \sum_{ij} C_{ij} \left\langle K_i \Pi(H) K_j^\dagger \right\rangle,$$  \hspace{1cm} (1)

where $\Phi$ is a flux factor and $\Pi(H)$ denotes the hadronic projection operator. The fermion bilinear operators $K_i$ are built out of heavy quark fields sandwiching colour and spin matrices and the covariant derivative $D$. The labels $i, j$ include the colour index, spin $S$, orbital angular momentum $L$ (coupling the N covariant derivatives), the total angular momentum $J$ and helicity $J_z$.

The coefficient function $C_{ij}$ is computable in perturbative QCD and has an expansion in the strong coupling $\alpha_s(m)$ (where $m$ is the mass of heavy quark), whereas the matrix elements are non perturbative. However, in NRQCD, these matrix elements scale as powers of $v$. Hence the resulting cross section is an expansion in powers of $\alpha_s(m)$ and $v$. Often, higher orders in $v$ involves previously neglected colour-octet states of the heavy quark pairs. For charmonium states, a numerical coincidence, $v^2 \sim \alpha_s(m^2)$, makes the higher order terms in $v$ important and the double expansion more complicated. The above non perturbative matrix elements can be reduced to the diagonal form, $\mathcal{O}_\alpha^{(S+1)L_j^N}$ (where $\alpha$ denotes the colour singlet or octet state) and off-diagonal form, $\mathcal{P}_\alpha^{H}(2S+1L_j^N, 2S+1L_j^{N'})$. These matrix elements scale as $v^d$ where $d = 3 + N + N' + 2(E_d + 2M_d)$, $E_d$ and $M_d$ are the number of colour electric and magnetic transitions. This formalism has been successfully applied to large transverse momentum processes \cite{Bodwin:1994jh}. Inclusive production cross sections for charmonium at low energies, dominated by low transverse momenta, also seem to have a good phenomenological description in terms of this approach \cite{Bodwin:1994jh}. The spin asymmetries have also been computed both for the low transverse momentum processes \cite{Bodwin:1994jh} and for those with high transverse momenta \cite{Bodwin:1994jh}.

It was argued in \cite{Bodwin:1994jh} that a better understanding of such cross sections, and asymmetries, can be obtained if the higher order terms in $v$ and $\alpha_s$ are used. This follows from the fact that the total inclusive $J/\psi$ cross sections arise either from direct $J/\psi$ production (which starts at order $\alpha_s v^7$) or through radiative decays of $\chi_J$ states. $\chi_9$ and $\chi_2$ are first produced at order $\alpha_s v^5$, whereas $\chi_1$, which has the largest branching fraction into $J/\psi$, is produced only at order $\alpha_s v^9$. Further phenomenological problem is to explain the $\chi_1/\chi_2$ ratio observed in hadro production \cite{Bodwin:1994jh}. A better understanding of these cross sections requires the NRQCD expansion up to order $\alpha_s v^9$. The unpolarised and polarisation cross sections are defined as

$$\sigma = \sum_{hh'} \sigma(h,h'), \hspace{1cm} \Delta \sigma = \sum_{hh'} hh' \sigma(h,h'),$$  \hspace{1cm} (2)

where $h$ and $h'$ are the helicities of the beam and target respectively, and $\sigma(h,h')$ denotes the cross section for fixed initial helicities. The difference between the polarised and unpolarised case is in the coefficient functions denoted by $\tilde{C}_{ij}$ for the polarised case; the set of non perturbative matrix element are the same. We construct the coeffi-
2. Cross Sections

To lowest order in $\alpha_S$ the contributing parton level cross section are $\bar{q}q \to \bar{Q}Q$ and $gg \to \bar{Q}Q$. The hadron level cross section is obtained by multiplying appropriate parton luminosities—

$$\mathcal{L}_{ab} = a(x_1) \, b(x_2), \quad (3)$$

where $a, b$ runs over quark $q_j$, antiquark $\bar{q}_j$ or gluon $g$ densities depending on the subprocess cross sections. For polarised cross sections, $\Delta \sigma$, the corresponding polarised luminosities $\Delta \mathcal{L}_{ab}$ is obtained by replacing $a, b$ by polarised parton densities $\Delta a$. Data indicates that $\mathcal{L}_{\bar{q}q} \ll \mathcal{L}_{gg}$ for $\sqrt{S} \geq 20$ GeV and $|\Delta \mathcal{L}_{\bar{q}q}| \ll \mathcal{L}_{gg}$. Consequently, the $\bar{q}q$ channel may be neglected for double polarised asymmetries to good precision.

The squared matrix element for the $gg$ process is technically more complicated. The difference between the unpolarised [12] and polarised cases [3] lies solely in the flipped sign of the $J = 2$ part, which arise in the polarisation sum of initial state gluons. The subprocess cross section for the production of a charmonium $H$ can be written as

$$\hat{s}_{gg}(\hat{s}) = \frac{\pi^3 \alpha_s^2}{4m^2} \delta(\hat{s} - 4m^2) \sum_d \left[ \frac{1}{18} \Theta^H_S(d) + \frac{5}{48} \Theta^H_D(d) + \frac{3}{16} \Theta^H_F(d) \right], \quad (4)$$

where $d$ runs over the various matrix elements that contribute to the charmonium $H$ at order $v^d$. The subscript $S, D$ and $F$ denotes the colour singlet, colour octet symmetric and antisymmetric parts respectively. For the polarised case $\hat{s}^H$ is replaced by $\Delta \hat{s}^H$ and the combination of non perturbative matrix elements $\Theta^H$ by $\hat{\Theta}^H$. For the various charmonium states $J/\psi$ and $\chi_J$ the combination of non perturbative matrix elements are listed below. The changes for the polarised case is mentioned appropriately.

Direct $J/\psi$ Production

$$\Theta^{J/\psi}_D(7) = \frac{1}{2m^2} \frac{O^J(1)}{S_0^J}, \quad \Theta^{J/\psi}_D(9) = \frac{1}{2m^2} \left[ 3O^{J/\psi}(3)P_0^J + \frac{4}{5} O^{J/\psi}(3)P_2^J \right],$$

$$\Theta^{J/\psi}_F(9) = \frac{1}{2m^2} \left[ \frac{3}{4} O^{J/\psi}(2)P^J_1 + 2 O^{J/\psi}(2)P^J_2 \right]. \quad (5)$$

$\chi_0$ Production

$$\Theta^{\chi_0}_S(5) = \frac{3}{2m^4} O^{\chi_0}(3)P_0^J, \quad \Theta^{\chi_0}_S(7) = \frac{7}{4 \sqrt{3m^6}} P_1^{\chi_0}(3)P_0^J,$$

$$\Theta^{\chi_0}_S(9) = \frac{1}{8m^8} \left[ 245 \Theta^{\chi_0}_S(3)P_0^J \right] + \frac{149}{10 \sqrt{3}} \Theta^{\chi_0}_S(3)P_0^J + \frac{2}{5m^4} O^{\chi_0}(3)P_2^J, \quad (6)$$

$$\Theta^{\chi_0}_F(9) = \frac{1}{2m^2} \left[ O^{\chi_0}(3)S_0^J \right] + \frac{1}{18m^6} \left[ O^{\chi_0}(3)S_1^J \right] + \frac{5}{6} O^{\chi_0}(3)D_1^J. \quad (7)$$

$\chi_1$ Production

$$\Theta^{\chi_1}_S(9) = \frac{1}{2m^2} \left[ 3O^{\chi_1}(3)P_0^J + \frac{4}{5} O^{\chi_1}(3)P_2^J \right], \quad \Theta^{\chi_1}_S(9) = \frac{1}{2m^2} \left[ 3O^{\chi_1}(3)P_0^J + \frac{4}{5} O^{\chi_1}(3)P_2^J \right],$$

$$\Theta^{\chi_1}_F(9) = \frac{1}{6m^4} O^{\chi_1}(3)P_1^J + \frac{1}{3m^6} \left[ \frac{1}{6} O^{\chi_1}(3)S_1^J \right] + \frac{5}{6} O^{\chi_1}(3)D_1^J - \frac{1}{5} O^{\chi_1}(3)D_2^J. \quad (7)$$

In eqs. (5)-(7) the coefficient of $J = 2$ matrix elements changes sign for polarised cases. $\chi_1$ is produced first at order $v^9$. The large branching ratio for the decay $\chi_1 \to J/\psi$ makes this a phenomenologically important term, and is the main motivation for this work.
3. Discussion

In spite of the large number of unknown non-perturbative matrix elements in the final results, it is possible to make several quantitative and qualitative comments about the polarisation asymmetries by making use of heavy quark spin symmetry and scaling arguments developed in [13]. This scaling argument allows us to make rough estimates. Neglecting possible logarithms of \( m \) and \( v \), dimensional argument can be used to write

\[
\langle \mathcal{K}_i H \mathcal{K}_j^\dagger \rangle = R_H Y_{ij} \Lambda^{D_{ij}} v^d,
\]

where \( D_{ij} \) is the mass dimension of the operator, \( d \) is the velocity scaling exponent in NRQCD, \( \Lambda \) is the cutoff scale below which NRQCD is defined (\( \Lambda \sim m \)), and \( Y_{ij} \) and \( R_H \) are dimensionless numbers. \( R_H \) contains the irreducible minimum non-perturbative information.

Assuming the constancy of \( R_H \) [13] and using heavy-quark symmetry, we find that the non-perturbative matrix elements contribute approximately \( 12 R_H m^2 v^0 \) to the \( \chi_1 \) cross section and about \( (1 + v^2 + 12 v^4) R_H m^2 v^5 \) to the \( \chi_2 \) cross sections. Then we expect

\[
\frac{\sigma(\chi_1)}{\sigma(\chi_2)} \approx \frac{12 v^4}{1 + v^2 + 12 v^4} = 0.45, \tag{10}
\]

independent of \( \sqrt{S} \). This estimate is in reasonable agreement with the measured values in proton-nucleon collisions (see fig. 1) — 0.34 ± 0.16 at \( \sqrt{S} = 38.8 \text{GeV} \) [14] and 0.24 ± 0.28 at \( \sqrt{S} = 19.4 \text{GeV} \) [15]. The measurements are also compatible with a lack of \( \sqrt{S} \) dependence. Estimate of \( O(\alpha_s^3) \) effects in [3] was used to show that the \( \chi_1/\chi_2 \) ratio could be about 0.3. In NRQCD this ratio cannot depend on the beam hadron. It turns out that the estimate in eq. (14) is not very far from the recently measured value in pion-nucleon collisions — 0.57 ± 0.19 at \( \sqrt{S} = 31.1 \text{GeV} \) [16]. However, the experimental situation certainly needs clarification.
A straightforward application of the NRQCD scaling laws would lead us to the conclusion that the asymmetries for $pp \rightarrow \chi_{0,2}$ are given by

$$A^{\chi_{0}}_{pp} \approx -A^{\chi_{2}}_{pp} \approx \frac{\Delta L_{gg}}{L_{gg}} + O(v^4),$$  \hspace{1cm} (11)

where we have neglected the contribution of the $\bar{q}q$ channel. The asymmetry for $\chi_1$ production is

$$A^{\chi_{1}}_{pp} = \frac{\bar{\Theta}^S_{\chi_{1}}(9) + \bar{\Theta}^{L+}_{\chi_{1}}(9) + \bar{\Theta}^{L-}_{\chi_{1}}(9)}{\Theta^S_{\chi_{1}}(9) + \Theta^{L+}_{\chi_{1}}(9) + \Theta^{L-}_{\chi_{1}}(9)} \left[ \frac{\Delta L_{gg}}{L_{gg}} \right].$$ \hspace{1cm} (12)

The ratio of matrix elements can be estimated using heavy quark spin symmetry and the scaling relations in eq. (9). $\Theta^S_{\chi_{1}}(9)$ vanishes in this approximation and the terms $\bar{\Theta}^{L_{\pm}}_{\chi_{1}}(9)$ come with opposite signs. The numerator is positive but small and we expect—

$$A^{\chi_{1}}_{pp} \approx 0.2 \frac{\Delta L_{gg}}{L_{gg}}.$$ \hspace{1cm} (13)

The $\bar{q}q$ channel remains negligible even at $\sqrt{S} = 500$ GeV.

The $J/\psi$ asymmetry seems to be enormously complicated because of the radiative decays of the $\chi$ states. However, a major simplification occurs because of the near vanishing asymmetry in direct $J/\psi$ production. Thus the asymmetry comes entirely from the 20–40% of the cross section due to $\chi$ decays. Taking into account the ratios of the production cross sections of $\chi$ and the branching fractions for their decays into $J/\psi$, we find that the $\chi_1$ and $\chi_2$ states contribute equally to $J/\psi$. Hence the $J/\psi$ polarisation asymmetry is expected to be approximately

$$A^{J/\psi}_{pp} \approx -(0.15 \pm 0.05) \frac{\Delta L_{gg}}{L_{gg}}.$$ \hspace{1cm} (14)

We summarise the predictions made on the basis of the NRQCD scaling in eq. (9) and the assumption of $R_H$ depends only on the hadron $H$—

$$-A^{J/\psi}_{pp} \approx A^{\chi_1}_{pp} < A^{\chi_0}_{pp} = -A^{\chi_2}_{pp} \approx \frac{\Delta L_{gg}}{L_{gg}}.$$ \hspace{1cm} (15)

In conclusion, low energy double polarised asymmetries are a good test-bed for understanding the origin of all observed systematics in fixed target hadro-production of charmonium. The high order computations presented here provides a set of processes which can be used to test aspects of NRQCD factorisation and scaling.

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