Momentum of electromagnetic fields, speed of light in moving media, and the photon mass

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Abstract
In both the equations for matter and light wave propagation, the momentum of the electromagnetic fields $P_e$ reflects the relevant $em$ interaction. As a review of some applications of wave propagation properties, an optical experiment which tests the speed of light in moving rarefied gases is described. Moreover, $P_e$ is also the link to the unitary vision of the quantum effects of the Aharonov-Bohm (AB) type, which provide a useful quantum approach for the limit of the photon mass $m_{ph}$. A bench-top experiment based on effects of the AB type that exploit new interferometric techniques, is foreseen to yield the limit $m_{ph} \simeq 10^{-54} g$, a value that improves upon the results achieved with other approaches.

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1 Introduction
The existing formal analogy between the wave equation for light in moving media and that for charged matter waves has been described by Spavieri and
Gillies [1] in the context of a proposed optical experiment for light propagation in transparent moving media, previously discussed by several authors [2]. The link between the two wave equations is the interaction electromagnetic (em) momentum $P_e$, which has attracted physicists’ attention because it arises in different scenarios of modern physics involving em interactions. The first of these scenarios is that of light propagation in slowly moving media [2], [1], and another involves quantum nonlocal effects of the Aharonov-Bohm (AB) type [3] and their unitary view [4]. More commonly, the interaction em momentum $P_e$ appears as a nonvanishing quantity in em experiments involving “open” or convection currents, while $P_e$ vanishes in the common em experiments or interactions with closed currents or circuits [1], [5].

The main purpose of this article is to review the recent advances of physics involving the em momentum $P_e$ and its role in the proposal of new tests or in making other advances, such as setting a new limit on the photon mass. The arguments presented here are also been described in our companion paper of Ref. [6].

In the field of electromagnetism, a growing number of articles questioning the standard interpretation of special relativity have appeared [7]-[9]. Some of the authors of Refs. [7] and [8] adhere to a point of view that assumes the existence of a preferred frame, similar to the historical works of Lorentz and Poincaré. It has been argued that these different formulations of Special Relativity are truly compatible only in vacuum, as differences may appear when light propagates in transparent moving media. Thus, Consoli and Costanzo [9], Cahill and Kitto [10], and Guerra and de Abreu [8], point out that, for the experiments of the Michelson–Morley type, which are often said to have given a null-result, this is not the case and cite the famous work by Miller [11]. The claim of these authors is that the available data point towards a consistency of non-null results when the interferometer is operated in the “gas-mode”, corresponding to light propagating through a gas [9] (as in the case of air or helium, for instance, even in modern maser versions of optical tests).

Furthermore, the only tests involving ”open” or convection currents, so far historically performed, have been reconsidered by Indorato and Masotto [12] who points out that these experiments are not completely reliable and may be inconclusive [1]. As a response to this, physicists have recently proposed experiments about those predictions of the theory that have not been fully tested, or they have formulated untested assumptions that differ from the standard interpretation of Special Relativity [1], [5], [8], [9].
The interesting point is that all the above-mentioned scenarios and polemical hypotheses are linked to the interaction em momentum. Therefore, throughout this article we highlight the role of $P_e$ in each one of these scenarios.

2 Wave equations for matter and light waves

To elucidate the role of em momentum in modern physics, we start by considering the wave equations for matter and light waves and show how the interaction term $Q$ of these equations is related to $P_e$. In general, with $T_{ik}^M$, the Maxwell stress-tensor, the covariant description of the em momentum leads to the four-vector em momentum $P_e^\alpha$ expressed as

$$P_e^\alpha = \gamma \int (c g + T_{ik}^M \beta^i) d^3 \sigma$$
$$c P_e^0 = \gamma \int (u_{em} - v \cdot g) d^3 \sigma$$

(1)

where $\beta = v/c$, and the em energy and momentum are evaluated in a special frame $K^{(0)}$ moving with velocity $v$ with respect to the laboratory frame. Here, $u_{em}$ is the energy density and $S = gc$ is the energy flux or flow.

The analogy between the wave equation for light in moving media and that for charged matter waves has been pointed out by Hannay and later addressed by Cook, Fearn, and Milonni who have suggested that light propagation at a fluid vortex is analogous to the Aharonov-Bohm (AB) effect, where charged matter waves (electrons) encircle a localized magnetic flux. Generally, in quantum effects of the AB type, matter waves undergo an em interaction as if they were propagating in a flow of em origin that acts as a moving medium and modifies the wave velocity. This analogy has led to the formulation of the so-called magnetic model of light propagation.

According to Fresnel, light waves propagating in a transparent, incompressible moving medium with uniform refraction index $n$, are dragged by the medium and develop an interference structure that depends on the velocity $u$ of the fluid ($u << c$). At the time of Fresnel the preferred inertial frame was at rest with the so-called ether, which here may be taken to coincide with the laboratory frame. The speed achieved in the ether frame is

$$v = \frac{c}{n} + (1 - \frac{1}{n^2}) u$$

(2)
as later corroborated by Fizeau [14]. Because of the formal analogy between the wave equation for light in slowly moving media and the Schrödinger equation for charged matter waves in the presence of the external vector potential $\mathbf{A}$ (i.e., the magnetic Aharonov-Bohm effect), both equations contain a term that is generically referred to as the interaction momentum $Q$. Thus, the Schrödinger equation for quantum effects of the AB type (with $\hbar = 1$) [4] and the wave equation for light in moving media can be written [2], [1] as

$$( -i \nabla - Q )^2 \Psi = p^2 \Psi. \quad (3)$$

Eq. (3) describes matter waves if the momentum $p$ is that of a material particle, while, if $p$ is taken to be the momentum $\hbar k$ of light (in units of $\hbar = 1$), Eq. (3) describes light waves.

a) All the effects of the AB type discussed in the literature [3]-[4] can be described by Eq. (3), provided that the interaction momentum $Q$ is related [4], [13] to $P_e$, the momentum of the em fields. The AB term $Q = (e/c) \mathbf{A}$ of the magnetic AB effect is obtained by taking $Q = P_e = \frac{1}{4\pi c} \int (\mathbf{E} \times \mathbf{B}) d^3x'$ where $\mathbf{E}$ is the electric field of the charge and $\mathbf{B}$ the magnetic field of the solenoid. A general proof that this result holds in the natural Coulomb gauge, has been given by several authors [15]. For these quantum effects, the solution to Eq. (3) is given by the matter wave function

$$\Psi = e^{i\phi} \Psi_0 = e^{i \int Q \cdot dx} \Psi_0 = e^{i \int Q \cdot dx} e^{i(p \cdot x - Et)} A \quad (4)$$

where $\Psi_0$ solves the Schrödinger equation with $Q = 0$.

b) Calculations of the quantity $Q = P_e$ [11] for light in slowly moving media show [13] that the interaction term yields the Fresnel-Fizeau momentum [1]

$$Q = -\frac{\omega}{c^2} (n^2-1) u, \quad (5)$$

and that a solution of the type described in [14] may assume the forms

$$\Psi = e^{i\phi} \Psi_0 = e^{i \int Q \cdot dx} e^{i(k \cdot dx - \omega dt)} A; \quad \Psi = e^{i \int (K(x) \cdot dx - \omega dt)} A \quad (6)$$

where $\mathbf{k}$ and $K(x)$ are wave vectors, $\omega = k c/n$ the angular frequency, and $n$ the index of refraction, while $\Psi_0$ solves Eq. (3) with $Q = u = 0$.

The fact that the interaction momentum $Q$ is related to $P_e$ [4], [13] for both matter waves of effects of the AB type [4] and light waves in moving media [13], definitely reinforces the existing analogy between the two wave equations. Two theoretical possibilities arise [1]:
- By incorporating the phase $\phi$ in the term $\int K(x) \cdot dx$, the last expression on the rhs of Eq.(6) keeps the usual invariant form of the solution as required by special relativity and one finds [13] for the speed of light the result $v = (c/n)\mathbf{c} + (1-1/n^2)u = (c/n)\mathbf{c} - Q(c^2/n^2\omega)$ in agreement with Eq.(2) and Special Relativity.

- Maintaining instead the analogy with the AB effect, the solution can be chosen to be represented by the first term of Eq.(6), $\Psi = e^{i\phi}\Psi_0$. In this case, the phase velocity changes but the speed of light (the particle, or photon) may not change [1]. This result is in total agreement with the analogous result for the AB effect where $Q = (e/c)A$ and the particle speed is left unchanged by the interaction with the vector potential $A$.

The established relation (5) will be used in the next sections to tentatively express in a quantitative way the hypothesis of Consoli and Costanzo [9] referring to $v$, the speed of light in a moving rarefied media. With a quantitative expression for $v$, it is then possible to formulate a dedicated experiment that tests Consoli and Costanzo’s hypothesis.

2.1 Propagation of em waves in rarefied moving media

Duffy [16] has noted that the concept of an ether-like preferred frame has always incited controversy, even in modern scientific investigations aimed at exploring the less understood aspects of relativity theory. Within this scenario, Consoli and Costanzo [9], Cahill and Kitto [10], and Guerra and de Abreu [8], after a re-analysis of the optical experiments of the Michelson–Morley type, claim that the available data point towards a consistency of non-null results when light in the arms of the interferometer propagates in a rarefied gas, like the cases of air at normal pressure and temperature. The possibility of maintaining the existence of a preferred frame, and parallel interests in the Michelson-Morley, Trouton-Noble and related effects, arises because the coordinate transformation used, the Tangherlini transformations [17] foresee the same length contraction and time dilation of the Lorentz transformations. However, they contain an arbitrariness in the determination of the time synchronization parameter, with the consequence that there are quantities which eventually cannot be measured, such as the one-way speed of light, its measured value depending on the synchronization procedure adopted [17]. Different synchronization procedures are possible [7]-[9], fully compatible with Einstein’s relativity in practice, but with very different assertions in fundamental and philosophical terms.
The original important assumption made by Consoli et al. to corroborate their claims of a non-null result and open a window for the possible existence of a preferred frame, is that light in a moving rarefied gas of refractive index \( n \) very close to 1 propagates with speed \( c/n \), isotropically, in the preferred frame, as if the medium were not moving. Obviously, this hypothesis is in contrast with special relativity that foresees the speed \( \frac{c}{2} \), but it is not ruled out by the known optical tests. Thus, this assumption needs justification and experimental corroboration.

In the following, we explore possible modifications of the form of the present Fresnel-Fizeau momentum when the moving medium is composed of rarefied gas. It is not unconceivable that the effectiveness of the light delay mechanism in a compact moving medium differs, and perhaps even substantially so, from that of a non-compact moving medium, such as a rarefied gas, even if they have the same index \( n \). As an \textit{ad hoc} hypothesis or a tentative model of a light delay mechanism, it has been supposed \cite{18} that its effectiveness \( e_f \) arises from the relative spatial extension \( V_i \) of the interaction em momentum \( Q(u) \) with respect to the extension \( V \) of the total em momentum. Introducing then the ratio \( e_f = V_i/V \), the effective interaction momentum, to be used in determining the speed of light in a moving medium, will be assumed to be given by the effective Fresnel-Fizeau term \( e_f Q = (V_i/V)Q \), while the resulting velocity of light in moving rarefied media is

\[
\mathbf{v} = \frac{c}{n} \hat{c} - \frac{c^2}{n^2 \omega} e_f Q = \frac{c}{n} \hat{c} + e_f (1 - \frac{1}{n^2}) \mathbf{u},
\]

(7)

The hypothesis of Consoli et al. of the speed \( c/n \) in the preferred frame for moving rarefied gases, will be justified by our model if \( e_f = V_i/V \) turns out to be very small and, in this case, negligible. Calculations leading to a rough estimate of \( V_i/V \) for air at room temperature yield \cite{18} \( e_f = N_a(a^3/R^3) \ 22.9 = 6.1 \times 10^{-3} \), which indeed can be neglected. Thus, our model foresees that the speed of light in moving media is actually not \( c/n \) but, quantitatively, the changes found do not alter significantly the basic hypothesis and resulting analysis by Consoli et al. \cite{9}, \cite{10} and Guerra et al. \cite{8}.

3 Optical test in the first order in \( v/c \)

The main consequence is that, with the present hypothesis of negligible drag-like effect for moving rarefied gases, ether drift experiments of the order
$v/c$ become meaningful again. Let us consider for example the following experiment which is a variant of the Mascart and Jamin experiment of 1874 [19].

A ray of light travels from point A to point B of a segment A—B representing an optical interferometer. The original ray is split into two rays at A, which propagate separately through the two arms (1 and 2) of the interferometer. The rays recombine then at B where the interference pattern is observed. The arms 1 and 2 are made of a transparent rarefied gases or materials with indices of refraction $n_1$ and $n_2$ and wherein the speeds are $c/n_1$ and $c/n_2$ in the preferred frame, respectively, in agreement with Consoli’s et al. hypothesis [9] of the velocity expression (7) with $e_f=0$. The laboratory frame with the interferometer and the rarefied gas is moving with speed $u$ with respect to the preferred frame. We could be using the expressions for the speed in the moving laboratory frame resulting from the Tangherlini transformation, which can be found in [17], [8]. The calculation can also be done using the standard velocity addition from the Lorentz transformation, i.e., using the definition of Einstein speed as detailed in [8]. Both approaches yield the same result. The speed of light in arm 1 in the frame of the interferometer, moving with speed $u$ with respect to the preferred frame, is respectively

$$w_1 = \frac{c/n_1 - u}{1 - u^2/c^2} \quad \text{or} \quad w_1 = \frac{c/n_1 - u}{1 - u/(c n_1)}, \tag{8}$$

and analogously for $w_2$. If $L$ is the length of the arms, the time delay, or optical path difference, for the two rays yields, in the first order in $u/c$,

$$\Delta t(0^\circ) = L \left( \frac{1}{w_1} - \frac{1}{w_2} \right) \simeq \frac{L}{c} (n_1 - n_2) \left[ 1 + \frac{u}{c} (n_1 + n_2) \right]. \tag{9}$$

In order to observe a fringe shift, the interferometer needs to be rotated, typically by 90 or 180 degrees. The time delay for 180 degrees is the same of Eq.(9) with $u$ replaced by $-u$. The observable fringe shift upon rotation of the interferometer does not vanish in the first order in $u/c$ and is related to the time delay variation

$$\delta t = \Delta t(0^\circ) - \Delta t(180^\circ) \simeq 2 \frac{u}{c} (n_1^2 - n_2^2) \frac{L}{c}. \tag{10}$$

Choosing two media with different refractive index such that $n_1^2 - n_2^2$ is not too small ($> 10^{-3}$), the resulting fringe shift should be easily observable.
if the preferred frame exists and its speed $u$ is not too small. Knowing the sensitivity of the apparatus, one could set the lower limit of the observable preferred speed $u$. Interferometers, used in advanced Michelson-Morley’s type of experiments, could detect a speed $u$ as small as 1 km/s (a few m/s for He-Ne maser tests). Thus, this optical experiment, in passing from second order ($u^2/c^2$) to first order tests, should be able to improve the range of detectability of $u$ by a factor $(c/u)(n_1^2 - n_2^2) \approx 3 \times 10^5 \times 10^{-3} = 3 \times 10^2$, i.e., detect with the same interferometer speeds $3 \times 10^2$ smaller.

New, more refined versions of the Michelson-Morley type of experiment (including the tests using He-Ne masers) are not suitable to test the hypothesis of Consoli et al. [9] because of the relatively low sensitivity of these experimental approaches for rarefied gases. However, as shown above, an optical test in the first order in $v/c$ becomes meaningful in this case and can provide important advantages over the second order experiments of the Michelson-Morley type.

4 Effects of the Aharonov-Bohm type and the photon mass

We have shown in the previous sections that all the effects of the AB type can be described in a unified way by the wave equation (3) where, for each one of the effects, the quantity $Q$ represents the em interaction momentum (1). Both the interaction energy and momentum appear in the expression of the phase of the quantum wave function. Through the phenomenon of interference, phase variations can be measured and the observable quantity can be related to variations of the interaction em momentum or energy. In the following sections we show how the photon mass can be determined by measuring its effect on the observable phase variation via the related changes of em momentum or energy.

The possibility that the photon possesses a finite mass and its physical implications have been discussed theoretically and investigated experimentally by several researchers [20, 21]. Originally, the finite photon mass $m_\gamma$ (measured in centimeters$^{-1}$) has been related to the range of validity of Coulomb law [20]. If $m_\gamma \neq 0$ this law is modified by the Yukawa potential $U(r) = e^{-m_\gamma r}/r$, with $m_\gamma^{-1} = h/m_{ph}c = \lambda_C/2\pi$ where $m_{ph}$ is expressed in grams and $\lambda_C$ is the Compton wavelength of the photon.
There are direct and indirect tests for the photon mass, most of them based on classical approaches. Recalling some of the classical tests, we mention the results of Williams, Faller and Hill [20] yielding the range of the photon rest mass $m_{\gamma}^{-1} > 3 \times 10^9 \text{cm}$, and of Luo, Tu, Hu, and Luan [21] yielding the range $m_{\gamma}^{-1} > 1.66 \times 10^{13} \text{cm}$ and corresponding photon mass $m_{ph} < 2.1 \times 10^{-51} \text{g}$.

Several conjectures related to the Aharonov-Bohm (AB) effect have been developed assuming electromagnetic interaction of fields of infinite range, i.e., zero photon mass. The possibility that any associated effects become manifest within the context of finite-range electrodynamics has been discussed by Boulware and Deser (BD) [22]. In their approach, BD consider the coupling of the photon mass $m_{\gamma}$, as predicted by the Proca equation $\partial_{\nu} F^{\mu\nu} + m_{\gamma}^2 A^\mu = J^\mu$, and calculate the resulting magnetic field $\mathbf{B} = \mathbf{B}_0 + \hat{k} m_{\gamma}^2 \Pi(\rho)$, that might be used in a test of the AB effect. Because of the extra mass-dependent term, BD obtained a nontrivial limit on the range of the transverse photon from a table-top experiment yielding $m_{\gamma}^{-1} > 1.4 \times 10^7 \text{cm}$.

After the AB effect, other quantum effects of this type have been developed assuming electromagnetic interaction of fields of infinite range, i.e., zero photon mass. The possibility that any associated effects become manifest within the context of finite-range electrodynamics has been discussed by Boulware and Deser (BD) [22]. In their approach, BD consider the coupling of the photon mass $m_{\gamma}$, as predicted by the Proca equation $\partial_{\nu} F^{\mu\nu} + m_{\gamma}^2 A^\mu = J^\mu$, and calculate the resulting magnetic field $\mathbf{B} = \mathbf{B}_0 + \hat{k} m_{\gamma}^2 \Pi(\rho)$, that might be used in a test of the AB effect. Because of the extra mass-dependent term, BD obtained a nontrivial limit on the range of the transverse photon from a table-top experiment yielding $m_{\gamma}^{-1} > 1.4 \times 10^7 \text{cm}$.

Based on theoretical arguments of gauge invariance, SR point out that, in analogy with the AC effect for a coherent superposition of beams of magnetic dipoles of opposite magnetic moments $\pm \mu$ [25] and the effect for electric dipoles of opposite moments $\pm d$ [26], the Spavieri effect of the AB type for a coherent superposition of beams of charged particles with opposite charge state $\pm q$ is theoretically feasible. Using this effect, SR evaluate its relevance in eventually determining a bound for the photon mass $m_{ph}$. SR consider a coherent superposition of beams of charged particles with opposite charge state $\pm q$ passing near a huge superconducting cyclotron. The $\pm$ charges feel the effect of the vector potential $\mathbf{A}$ created by the intense magnetic field of the cyclotron and the phases of the associated wave function are shifted, leading to an observable phase shift [27]. For a cyclotron of standard size, SR show that the limit

$$m_{\gamma}^{-1} = 10^6 m_{\gamma BD}^{-1} \simeq 2 \times 10^{13} \text{cm}$$

is achievable. With their table-top experiment, BD obtained the value $m_{\gamma BD}^{-1}$.
140Km that is equivalent to $m_{phBD} = 2.5 \times 10^{-45} g$. With SR approach, the new limit of the photon mass is $m_{ph} \simeq 2 \times 10^{-51} g$ which is of the same order of magnitude of that found by Luo et al. [21]. Of course, by increasing the size of the cyclotron a better limit could be obtained. With the standard technology available, we expect that the limit $m_{ph} \simeq 2 \times 10^{-52} g$ is not out of reach.

4.1 The scalar Aharonov-Bohm effect and the photon mass

Having exploited the magnetic AB effect in the previous section, we consider now the scalar AB effect. In this effect charged particles interact with an external scalar potential $V$. The standard phase $\varphi_s$ acquired during the time of interaction is $\varphi_s = \frac{1}{\hbar} \int eV(t) \, dt$.

In the actual test of the scalar AB effect, a conducting cylinder of radius $R$ is set at the potential $V$ during a time $\tau$ while electrons travel inside it. Since no forces act on the charges it is a field-free quantum effect. If the photon mass does not vanish the potential is modified according to Proca equation. Gauss’ law is modified and the potential $\Phi$ obeys the equation $\nabla^2 \Phi - m^2 \gamma \Phi = 0$, with the boundary condition that the potential on the cylinder be $V$. In cylindrical coordinates the solutions are the modified Bessel functions of zero order, $I_0(m \gamma \rho)$ and $K_0(m \gamma \rho)$ which are regular at the origin and infinite, respectively. It follows that the acceptable solution is

$$\Phi(\rho) \approx V \left[ 1 + \frac{m^2 \gamma}{2} \left( \rho^2 - R^2 \right) \right]$$  \hspace{1cm} (11)

where the first two terms of the expansion of $I_0(m \gamma \rho)$ have been considered [28].

For two interfering beams of charges passing through separate cylinders, the relative phase shift is

$$\delta \varphi_s = \frac{1}{\hbar} \int e [V_1(t) - V_2(t)] \, dt$$  \hspace{1cm} (12)

where $V_1(t)$ and $V_2(t)$ are the potentials applied to cylinder 1 and 2, respectively. Consequently, according to (11), the contribution of the photon mass to the relative phase shift is
\[ \delta \varphi = \delta \varphi_s + \Delta \varphi = \delta \varphi_s + \frac{m^2_{\gamma}}{4} (\rho^2 - R^2) \delta \varphi_s. \] (13)

Obviously, this additional phase shift term vanishes if \( m_{\gamma} \) vanishes and the standard result is recovered. The last term of (13) is useful for determining the photon mass in a table-top experiment. We consider the simple case of one beam travelling inside cylinder 1 and the other travelling outside it (\( V_2(t) = 0 \)) for a short time interval \( \tau \). It follows that \( \Delta \varphi = \delta \varphi - \delta \varphi_s \) reads

\[ \Delta \varphi = -\frac{e m_{\gamma}^2}{4} (\rho^2 - R^2) \frac{V \tau}{\hbar} \] (14)

where \( V = V_1(t) - V_2(t) \). This is our main result for determining the photon mass limit. Interferometric experiments may be performed with a precision of up to \( 10^{-4} \), therefore, following the approaches of BD and SR we set \( \Delta \varphi = \varepsilon, \varepsilon = 10^{-4} \). Also, we suppose that the beam 1 travels nearly at the centre of the cylinder (\( \rho \ll R \)) so that

\[ m_{\gamma}^{-1} = \frac{R}{2} \sqrt{\frac{\pi V \tau}{\varepsilon (\hbar/2e)}} \] (15)

The following values may be used to estimate \( m_{\gamma}^{-1} \): \( V = 10^7 V, \ h/2e = 2.067 \times 10^{-15} T m^2, \ \tau = 5 \times 10^{-2} s \) and \( R = 27 cm \). The corresponding range of the photon mass is

\[ m_{\gamma}^{-1} = 3.4 \times 10^{13} cm \] (16)

which yields the improved photon mass limit \( m_{\gamma} = 9.4 \times 10^{-52} g \), but we are left to justify the values used above for \( \tau \) and \( R \), which are both quite high.

It is interesting to compare the strength of the AB phase of the scalar AB effect with that of the magnetic AB effect. The scalar AB phase may be expressed as \( eV \tau/\hbar \), while the magnetic AB phase is \( eAL/(c\hbar) \), and the link between the particle’s classical path is \( L = \tau v \) with \( v \) its speed assumed to be uniform. According to special relativity, magnetism is a second order effect of electricity, therefore in normal conditions the strength of the coupling \( eA/c \) is smaller than the coupling \( eV \). As a consequence of this, the phase variation due to the finite photon mass should be smaller in the magnetic than in the scalar AB effect. In other words, the scalar AB effect should be yielding a better limit for the photon mass than the magnetic AB effect. However, the above consideration is valid if in the actual experiments we have comparable path lengths, i.e., if \( \tau \simeq L/v \). In the table-top experiment by SR \( [27] \) L
is of the order of several meters. Choosing as charged particles heavy ions, for example $^{133}\text{Cs}^+$, their speed could be $27\, m/s$ \cite{29}. With this speed and $L = 1.35\, m$ for the cylinder length, we get $\tau = 5 \times 10^{-2}\, s$ for the time of flight inside the cylinder. Since $\tau \approx L/v$, the improved result (16) obtained by exploiting the scalar AB effect is justified.

However, the high values chosen for $R$ and $L$ imply that the charged particle beams will have to keep their state of coherence through an extended region of space $L = 1.35\, m$ during the interferometric measurement process, while in standard interferometry the path separations are of the order of at most a few cm. Thus, technological advances are needed in this respect, as also mentioned in the article by SR \cite{27} and the references cited therein.

Nevertheless, the feasibility of testing the photon mass with the scalar AB effect has been confirmed by the recent work of Neyenhuis, Christensen, and Durfee \cite{28}, lending support to the quantum approach.

Actually, it is conceivable the possibility of extending to the case of the scalar AB effect the techniques of Refs. \cite{25} and \cite{26} for a coherent superposition of beams of charged particles with opposite charge state $\pm q$, as suggested by SR in Ref. \cite{27}. This may lead to achieve even better limits for the photon mass. In fact, by means of these techniques it is feasible to suppose that the particles paths may be $10^2$ times those considered above. Thus, the time of flight $\tau$ becomes $10^2$ times bigger. Although the technical details will be given elsewhere, we anticipate that further improvement can be achieved by bending the particle path into a circular one, as in the case of ions in a cyclotron. In this case, $\tau$ may be increased $10^4$ times. Thus, we project that a photon mass limit of the order of

$$m_{ph} \simeq 10^{-54}\, g$$

may be achieved.

\section{Conclusions}

We have recalled that the interaction momenta $Q$ of the effects of the AB type and of light in moving media have the same physical origin, i.e., are given by the variation of the momentum of the interaction em fields $P_e$. Expecting that the effectiveness of the light delay mechanism in a rarefied gas differs from that of a compact transparent fluid or solid, we consider a tentative model of light propagation that validates the analysis made by
Consoli et al. [9] and Guerra et al. [8]. As a test of the speed of light in moving rarefied media and of the preferred frame velocity, we propose an improved first order optical experiment that is a variant of the historical Mascart-Jamine experiment.

Finally, we have considered the table-top approach of Boulware and Deser to the photon mass and verified its applicability to other effects of the AB type, concluding that the new effect using beams of charged particles with opposite charge state ±q for the magnetic AB effect, and the scalar AB effect are a good candidates for determining the limit of the photon mass. Using a quantum approach to evaluate the limit of $m_{ph}$ with these effects, and supposing that the recent interferometric techniques can be used, we project that a bench-top experiment may yield the limit $m_{ph} \simeq 10^{-54} g$, an important result that would improve the limits achieved with recent classical and quantum approaches. In any event, advances in this area indicate that quantum approaches the photon mass limit are feasible and may compete with or surpass the traditional classical methods.

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References

[1] G. Spavieri and G. T. Gillies, Chin. J. Phys., 45 (2007) 12.

[2] J. H. Hannay, unpubl., Cambridge Univ. Hamilton prize essay (1976); R. J. Cook, H. Fearn, and P. W. Milonni, Am. J. Phys. 63 (1995) 705.

[3] Y. Aharonov and D. Bohm, Phys. Rev. 115 (1959) 485; Y. Aharonov and A. Casher, Phys. Rev. Lett. 53 (1984) 319; G. Spavieri, Phys. Rev. Lett. 81 (1998) 1533, Phys. Rev. A 59 (1999) 3194; V. M. Tkachuk, Phys. Rev. A 62 (2000) 052112-1.

[4] G. Spavieri, Phys. Rev. Lett. 82, 3932 (1999); Phys. Lett. A, 310, 13 (2003); Eur. J. Phys. D, 37 (2006) 327.
[5] G. Spavieri and G. T. Gillies, Nuovo Cimento, 118 B (2003) 205; G. Spavieri, L. Nieves, M. Rodriguez, and G. T. Gillies. *Has the last word been said on Classical Electrodynamics? New Horizons*, Rinton Press, USA (2004) 255.

[6] G. Spavieri, G. T. Gillies et al., in *Ether, Spacetime & Cosmology*, (2009) in press.

[7] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge Univ. Press, Cambridge, 1988; C. Leubner, K. Aufinger, P. Krumm, Eur. J. Phys. 13 (1992) 170. F. Selleri, Found. Phys. 26 (1996) 641; Found. Phys. Lett. 18 (2005) 325.

[8] R. de Abreu, V. Guerra, Relativity–Einstein’s Lost Frame, 1st ed., Extra[muros[, Lisboa, 2005. V. Guerra and R. de Abreu, Found. Phys. 36 (2006) 91826; V. Guerra and R. de Abreu, Phys. Lett. A 333 (2004) 355.

[9] M. Consoli, E. Costanzo, Phys. Lett. A 333 (2004) 355; astro-ph/0311576. M. Consoli, A. Pagano and L. Pappalardo, Phys. Lett. A 318 (2003) 292; M. Consoli, Phys. Rev. D 65 (2002) 105017; Phys. Lett. B 541 (2002) 307; M. Consoli and E. Costanzo, Phys. Lett. A 361 (2007) 513.

[10] R.T. Cahill, K. Kitto, physics/0205070; Apeiron 10 (2003) 104; R.T. Cahill, Apeiron 11 (2004) 53.

[11] D.C. Miller, Rev. Mod. Phys. 5, 203 (1933).

[12] L. Indorato and G. Masotto, Annals of Science, 46, 117-163 (1989).

[13] G. Spavieri, Eur. Phys. J. D, 39, 157 (2006).

[14] A. J. Fresnel, Ann. Chim. (Phys.) 9, 57 (1818). H. Fizeau, C. R. Acad. Sci. (Paris) 33, 349 (1851).

[15] See: T. H. Boyer, Phys. Rev. D 8 (1973) 1667; X. Zhu and W. C. Henneberger, J. Phys. A 23 (1990) 3983; G. Spavieri, in Refs. [4].

[16] M. Duffy, private comm., Int. Conf. *Physical Interpretation of Relativity Theory* 2006.
[17] F. R. Tangherlini, Supp. Nuovo Cimento 20 (1961) 1; T. Sjodin, Nuovo Cimento, B 51 (1979) 299; T. Sjodin and M. F. Podlaha, Lett. Nuovo Cimento, 31 (1982) 433; R. Mansouri and R. V. Sexl, Gen. Rel. Grav., 8, (1977) 497, 515, 809.

[18] G. Spavieri, G. T. Gillies, V. Guerra, and R. De Abreu, EPJD in press (2008).

[19] E. Mascart and J. Jamine, Ann. Éc. norm. 3 (1874) 336.

[20] E. R. Williams, J. E. Faller and H. A. Hill, Phys. Rev. Lett., 26, 721 (1971); L. Davis, A.S. Goldhaber and M. M. Nieto, Phys. Rev. Lett., 35, 1402 (1975); P. A. Franken and G. W. Ampulski, Phys. Rev. Lett, 26, 115 (1971); J. J. Ryan, F. Accetta, and R. H. Austin, Phys. Rev. D, 32, 802 (1985); R. Lakes, Phys. Rev. Lett. 80, 1826 (1998).

[21] J. Luo, L.-C. Tu, Z. K. Hu, and E.-J. Luan, Phys. Rev. Lett., 90, 081801-1 (2003); L.-C. Tu, J. Luo, and G. T. Gillies, Rep. Prog. Phys. 68, 77 (2005).

[22] D. G. Boulware and S. Deser, Phys. Rev. Lett., 63, 2319 (1989).

[23] G. Spavieri, Phys. Lett. A, 310, 13 (2003).

[24] G. Spavieri, Eur. J. Phys. D, 37, 327 (2006).

[25] K. Sangster, E.A. Hinds, S.M. Barnett, E. Riis, Phys. Rev. Lett. 71, 3641 (1993); K. Sangster, E.A. Hinds, S. M. Barnett, E. Riis, A.G. Sinclair, Phys. Rev. A 51, 1776 (1995); see also R.C. Casella, Phys. Rev. Lett. 65, 2217 (1990).

[26] J.P. Dowling, C.P. Williams, J.D. Franson, Phys. Rev. Lett. 83, 2486 (1999).

[27] G. Spavieri and M. Rodriguez, Phys. Rev. A 75, 052113 (2007).

[28] B. Neyenhuis, D. Christensen, D. S. Durfee, Phys. Rev. Lett. 99, 200401 (2007).

[29] Z. T. Lu, K. L. Corwin, M. J. Renn, M. H. Anderson, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. 77, 3331 (1996).