Precursor of Inflation

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We investigate a nonsingular initial state of the Universe which leads to inflation naturally. The model is described by a scalar field with a quadratic potential in Eddington-inspired Born-Infeld gravity. The curvature of this initial state is given by the mass scale of the scalar field which is much smaller than the Planck scale. Therefore, in this model, quantum gravity is not necessary in understanding this pre-inflationary stage, no matter how large the energy density becomes. The initial state in this model evolves eventually to a long inflationary period which is similar to the usual chaotic inflation.

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“How did the Universe begin?” This is one of the oldest questions of the human being. After the discovery of the Hubble’s law in 1929, people started to believe that the Universe originated from big bang. However, the big-bang scenario soon confronted with obstacles such as the flatness, horizon, and monopole problems. The inflationary scenario was introduced in 1980 to resolve these problems. One of the simplest model of the inflation scenario must be the chaotic inflation described by a scalar field with a quadratic potential,

\[
S_M = \int d^4x \sqrt{-|g_{\mu\nu}|} \left[ -\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi) \right], \quad V(\phi) = \frac{m^2}{2}\phi^2, \quad (1)
\]

where \(|g_{\mu\nu}|\) represents the determinant of \(g_{\mu\nu}\). The Universe undergoes inflation during the slow-roll evolution of the scalar field, during which the scalar field and the scale factor are approximated by

\[
\phi(t) \approx \phi_i + \sqrt{2/3m}t, \quad a(t) \approx a_i e^{\frac{1}{2}[\phi_i^2 - \phi(t)^2]}, \quad (2)
\]

where \(\phi_i < 0\) is the value of the inflaton field in the beginning of inflation. (We assume that \(\phi\) rolls down the potential \(V(\phi)\) in the region of \(\phi < 0\), and set \(8\pi G = 1\) in this letter.) This slow-roll solution is known to be an attractor. For \(N \sim 70\) e-foldings, \(|\phi_i| \gtrsim 10\) is required, and \(m \sim 10^{-5}\) from observational data.

Even with the success of the inflationary scenario, it connotes problems such as the fine-tuning [1] and the low-entropy initial state [2]. In addition, we still do not know how inflation appears and what happened before and at the early stage of inflation. Due to the high-curvature scale in the theory of general relativity (GR), these issues inevitably require the introduction of quantum gravity.

In this letter, we show that the Eddington-inspired Born-Infeld (EiBI) theory of gravity, recently proposed by Banados and Ferreira [3], provides a natural origin of inflation without introducing quantum gravity. For the same matter field in Eq. (1), at the high-energy scale in EiBI theory, the curvature scale remains finite while the Universe undergoes a pre-inflationary exponential expansion, followed by the ordinary chaotic inflationary period.

The EiBI theory of gravity is described by the action

\[
S_{\text{EiBI}} = \frac{1}{\kappa} \int d^4x \left[ \sqrt{-|g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)|} - \lambda \sqrt{|g_{\mu\nu}|} \right] + S_M(g, \Phi), \quad (3)
\]

where \(\lambda\) is a dimensionless parameter related with the cosmological constant by \(\Lambda = (\lambda - 1)/\kappa\), and \(\kappa\) is the only additional parameter of the theory. In this theory the metric \(g_{\mu\nu}\) and the connection \(\Gamma^\rho_{\mu\nu}\) are treated as independent fields (Palatini formalism). The Ricci tensor \(R_{\mu\nu}(\Gamma)\) is evaluated solely by the connection, and the matter field \(\Phi\) is coupled only to the gravitational field \(g_{\mu\nu}\). The merit of EiBI theory is that it is equivalent to GR in the vacuum (or with cosmological constant). Therefore, the results of astronomical experiments such as the light deflection by a star are still viable. In addition, it predicts a singularity-free initial state of the Universe filled with perfect fluid [3, 4]. Subsequent work has been investigated in the cosmological and astrophysical aspects in Refs. [5, 10].

Let us consider the Universe driven the scalar field described by the action (1) in the spatially flat homogeneous and isotropic spacetime,

\[
g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)dx^2. \quad (4)
\]
The scalar-field equation is given by

$$\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0,$$

where the prime denotes the differentiation with respect to $\phi$. The Hubble parameter\textsuperscript{14, 17} is given by

$$H \equiv \frac{\dot{a}}{a} = \frac{1}{(\lambda + V)^2 + \dot{\phi}^2/2} \left\{ -\frac{1}{2} \left( \lambda + V + \frac{\dot{\phi}^2}{2} \right) V'(\phi) \pm \frac{1}{\sqrt{3}} \left( \lambda + V - \frac{\dot{\phi}^2}{2} \right) \times \left[ \left( \lambda + V + \frac{\dot{\phi}^2}{2} \right)^{3/2} \left( \lambda + V - \frac{\dot{\phi}^2}{2} \right)^{3/2} - \frac{1}{\kappa} \left( \lambda + V + \frac{\dot{\phi}^2}{2} \right) \left( \lambda + V - \frac{\dot{\phi}^2}{2} \right) \right]^{1/2} \right\},$$

where $\lambda \equiv \lambda/\kappa$. (We shall consider the case of $\kappa, \lambda > 0$ in this work.) A distinct feature of this equation is that the curvature scale is not simply proportional to the energy scale. One of the consequences of this feature is the existence of a nongravitating-dynamical field solution investigated in Ref.\textsuperscript{16}. The other is what we study in this work, the non-quantum birth of the inflationary Universe without beginning.

If we consider the evolution of the scalar field with the first slow-roll condition, $\dot{\phi}^2 \ll \lambda + V(\phi)$, the Hubble parameter is proportional to the scalar field, $H \approx \sqrt{V(\phi) + \Lambda/3} = (m/\sqrt{6})|\phi|$, which is the same as in GR (we assume $\Lambda = 0$). Then the scalar-field equation without ignoring $\ddot{\phi}$ term becomes

$$\ddot{\phi} + 3H \dot{\phi} + m^2 \phi = 0 \implies \frac{1}{2} \frac{d\dot{\phi}^2}{dt} = \frac{3}{2} \frac{d\phi^2}{dt} \approx \sqrt{3/m} \frac{d\phi^2}{dt}.$$ 

This equation can be integrated to give

$$|\dot{\phi}| + \sqrt{2/3m} \log |\dot{\phi}| - \sqrt{2/3m} \approx \sqrt{3/8m}(\phi^2 - \phi^2_0),$$

where, $\phi_0 \sim \dot{\phi}_1$. If $|\phi| < |\phi_0|$, the $\ddot{\phi}$ term in Eq.\textsuperscript{17}, or the first term in Eq.\textsuperscript{8} is subdominant; this is the second slow-roll condition$, |\phi| \approx \sqrt{2/3m}$ and the Universe evolves along the attractor trajectory. If $|\phi| > |\phi_0|$ (only the first slow-roll condition is satisfied), the first term in Eq.\textsuperscript{8} becomes dominant to the second term, so we have $\dot{\phi} \propto 2\phi$. Therefore, the $\ddot{\phi}$ increases rapidly as we go back in time, while $\phi$ climbs up the potential. The energy scale also increases rapidly due to this dynamical behavior, and arrives at the Planck scale soon. In GR, the quantum gravity is required beyond this point. However, we shall show below that it is not the story in the EiBI theory, in which there exists an upper limit on the value of the field velocity $\dot{\phi}$. In addition, the curvature scale is not simply proportional to the energy scale as we can see from Eq.\textsuperscript{8}.

The upper limit of $\dot{\phi}$ comes from the requirement that $H$ be real-valued. This means from Eq.\textsuperscript{10} that the value of $\lambda - \phi^2/2 + V(\phi) \equiv \lambda - p$ should be non-negative. The maximum value of $\phi^2$ is achieved when

$$\frac{1}{2} \dot{\phi}^2 - V(\phi) - \lambda = 0,$$

which we shall call the maximal pressure condition (MPC). For the perfect fluid investigated in Refs.\textsuperscript{2, 4}, MPC is responsible for the nonsingular initial state of the Universe. Equation\textsuperscript{19} can be rewritten as\textsuperscript{1}

$$\dot{\phi} = U(\phi) \implies \ddot{\phi} = U(\phi)U'(\phi), \quad \text{where} \quad U(\phi) \equiv \sqrt{2/V(\phi) + \lambda}.$$ 

Note that the reality of $\dot{\phi}$ requires $V(\phi) \geq -\lambda$. Then from the scalar-field equation\textsuperscript{15}, the Hubble parameter becomes

$$H = -\frac{2}{3} U'(\phi).$$

It is not difficult to show that Eq.\textsuperscript{16} coincides with Eq.\textsuperscript{11} after plugging Eq.\textsuperscript{10}. Therefore, Eqs.\textsuperscript{10} and\textsuperscript{11} provide the complete set of the field equations with MPC.

\textsuperscript{1} We shall consider only the positive signature since $\dot{\phi} = -U$ can be obtained by making the replacement $t \to -t$ in Eq.\textsuperscript{10}. 

For a given potential $U(\phi)$, in general, one can now obtain the maximal pressure solutions (MPS) for the scalar field and the scale factor. Equation (10) is integrated to give the scalar field,

$$\mathcal{V}(\phi) = \int_{\phi_0}^\phi \frac{d\varphi}{U(\varphi)} = \int^t d\tau \implies \phi(t) = \mathcal{V}^{-1}(t),$$  \hspace{1cm} (12)

and Eq. (11) is integrated to give the scale factor,

$$\frac{\dot{a}}{a} = -\frac{2}{3}U' = -\frac{2U}{3U} \implies a(t) = a_0 [U(\phi)]^{-\frac{2}{3}},$$  \hspace{1cm} (13)

where $a_0$ is an integration constant.

For the massive scalar field that we consider, we obtain the MPS from Eqs. (12) and (13),

$$\phi(t) = \frac{\sqrt{2\lambda}}{m} \sinh[m(t-t_0)], \quad a(t) = \frac{a_0}{(2\lambda)^{1/3}} \cosh^{-2/3}[m(t-t_0)].$$  \hspace{1cm} (14)

The scalar field runs from $-\infty$ to $+\infty$ tracking the symmetric potential. The Universe expands till the bouncing moment $t = t_0$, and then contracts. The evolution is symmetric about the bouncing moment. At the early stage ($t \ll t_0$), the Universe undergoes an exponential expansion and no singularity is accompanied,

$$\phi(t) \approx -\sqrt{\frac{2\lambda}{2m^2}} e^{m(t-t_0)}, \quad a(t) \approx a_0 \left(\frac{2}{3\lambda}\right)^{1/2} e^{\frac{2}{3}m(t-t_0)}. \hspace{1cm} (15)$$

At this stage, the Hubble parameter is constant and is solely determined by the mass of the scalar field, $H = H_{\text{MPS}} \approx 2m/3$. Therefore, the curvature scale of MPS is finite even when the energy density is very high for large $|\phi|$. The value of $H$ is plotted in Fig. 1. In the phase space $(\phi, \dot{\phi})$, the energy scale increases along the diagonal lines. However, the curvature scale does not coincide with the behavior of the energy scale.

Let us discuss the stability of MPS. We introduce the linear perturbations $\psi(t)$ and $h(t)$ for the velocities of the scalar field and the metric,

$$\dot{\psi} = U(\phi) \left[1 + \epsilon \psi(t)\right], \quad H = -\frac{2}{3}U'(\phi) \left[1 + \epsilon h(t)\right],$$  \hspace{1cm} (16)

and consider the field equations in the linear order in $\epsilon$. Plugging Eq. (10) into the Eqs. (5) and (6), we get

$$\ddot{\psi} - 2U'h = 0, \quad h = \left(-\frac{2}{3} + \sqrt{\frac{1}{3mU'}}\right)\psi.$$  \hspace{1cm} (17)

Combining these two equations, we get

$$\ddot{\psi} = \frac{4}{3} U'' + 2\sqrt{3} \epsilon.$$  \hspace{1cm} (18)

Using the relation $UU' = \dot{U}$ from Eq. (11), we finally get

$$\psi = \psi_0 [U(\phi)]^{-\frac{4}{3}} e^{t/t_c}, \quad h = \psi_0 \left[-\frac{2}{3} + \sqrt{\frac{1}{3mU'(\phi)}}\right] [U(\phi)]^{-\frac{4}{3}} e^{t/t_c},$$  \hspace{1cm} (19)

where $t_c = \sqrt{3\kappa/8}$. For MPS of the massive scalar field in Eq. (14), the linear perturbations for $\dot{\psi}$ and $H$ in Eq. (16) become

$$U\psi = \psi_0 (2\lambda)^{-1/6} \cosh^{-1/3}[m(t-t_0)] e^{t/t_c} \propto e^{\frac{4\sqrt{m}}{3\lambda} t} \quad (as \ t \to -\infty),$$  \hspace{1cm} (20)

$$U'h = \psi_0 (2\lambda)^{-2/3} \left\{\frac{2}{3\sqrt{3\kappa}} - \frac{2m}{3} \tanh[m(t-t_0)]\right\} \cosh^{4/3}[m(t-t_0)] e^{t/t_c} \propto e^{\frac{4\sqrt{m}}{3\lambda} t} \quad (as \ t \to -\infty).$$  \hspace{1cm} (21)

Both of the perturbations grow in time, so MPS is unstable. Below, we shall show that the Universe starts from MPS at high energy, and evolves to the usual inflationary stage due to this instability.
FIG. 1: Plot of the Hubble parameter $H$ (left panel) and the phase diagram $\phi, \dot{\phi}$ (right panel) for various initial data with $m = 1/4, \kappa = 1/4$, and $\lambda = 1$. In the left panel, $H = 0$ at the very center. The contours are separated by $\Delta H = 0.1$. The blue color represents small $H$. The reddest part denotes the region of $H > 1$. The intermediate parts denote the region of $0.1 \leq H \leq 1$. The region in white is physically forbidden. In the right panel, the blue-shaded region denotes the physically forbidden region bounded by MPS (black line). The other curves denote the evolution of the perturbed scalar field from MPS with other initial conditions. The solid curves denote the evolution starting from left top, and the dashed curves denote the evolution starting from right bottom. The attractor trajectory is the curve to which the nearby paths converge. The Universe undergoes chaotic inflation along the attractor trajectory until the scalar field becomes small enough. The high-curvature regime is denoted by the shaded region $\text{QG}$.

Now let us discuss the whole picture of the evolution of the Universe (EU) which is plotted in the phase diagram in Fig. 1 based on numerical calculations. Consider EU of a high-energy MPS with a small perturbation. At the early stage of EU, the Universe is closer to MPS. As time elapses, the perturbation grows exponentially and $|\dot{\phi}|$ decreases. Once EU departs from MPS considerably, the Universe enters into the first slow-roll regime where $|\dot{\phi}|$ rapidly drops. Soon after, the Universe settles down to the second slow-roll regime which is described by the attractor solution (2), and undergoes the usual chaotic inflationary expansion. Finally the Universe exits from the inflationary stage, and the scalar field starts to oscillate about the minimum of the potential.

There are a few distinct features in the phase diagram.

(i) There are forbidden regions bounded by the hyperbolic MPC.

(ii) All the evolution paths of the Universe start from left top, or from right bottom in the diagram.

(iii) The high-curvature region is much suppressed than that in GR. (Remember that the curvature scale for MPS is constant.)

(iv) There exists an attractor solution to which the evolution paths departed from MPS converge.

(v) All the paths but MPS settle down to the center of the diagram.

The features (iv) and (v) are nothing but the familiar properties in the chaotic inflation model. The features (i), (ii), and (iii) are new and provide a resolution to various problems of the inflation model.

In GR, the quantum theory of gravity is required when the energy scale is larger than the Planck scale, $3H^2 = \rho = K + V > M_P^2$, where $K = \dot{\phi}^2/2$ and $V = m^2\phi^2/2$. (This quantum-gravity regime would correspond to a region outside an ellipse if it were plotted in the phase diagram.) For the chaotic inflation in GR, the pre-inflationary period is dominated by the kinetic energy (K) as mentioned below Eq. (8). Therefore, the pre-inflationary period inevitably enters the quantum-gravity regime.

However in the EiBI theory, the quantum-gravity regime is considerably suppressed. The kinetic energy is bounded by MPS for a given value of the scalar field. The region of $K - V > \lambda$ in the phase diagram is forbidden. The
The curvature scale of MPS is $\sim O(m)$, so the quantum treatment of gravity is not necessary in the region of $K - V \sim \lambda$. While EU joins the attractor after departing from MPS, the spacetime curvature increases to $H^2 \sim m^2 \phi_0^2$. As long as $|\phi_0| < m^{-1}$, the curvature is not comparable to the Planck scale. In this sense, MPS provides a natural initial state for inflation without quantum gravity. Only in the region of $V > M_p^2$ and $V \gg K$, the curvature scale becomes large and the quantum treatment is required (QG in Fig. 1). However, this region can be avoided for the evolution paths of $\phi$ satisfying $|\phi_0| < m^{-1}$.

In the high-curvature region, the quantum fluctuation of the scalar field is large, $\delta\phi \sim O(H)$. The corresponding quantum fluctuation of the field velocity is even larger, $\delta\dot{\phi} \sim \delta\phi/\delta t \sim O(H^2)$. This large fluctuation pushes the state in the high-curvature region to the near-MPS region in which the classical EiBI gravity plays rather than quantum gravity. Therefore, the quantum fluctuation will lead the Universe to follow the path of MPS rather than that of the quantum theory.

In Ref. [1], the authors showed that the tensor perturbation grows linearly in the early Universe filled with radiation as $h_{ij} \propto A\delta\eta = A/a(t_c) \times \delta t$, where $A$ is a constant, $t_c$ is an early moment, and $\delta\eta = \eta - \eta_c$ is the increment of the conformal time. For MPS, the tensor perturbation grows in a similar manner. However, combined with the exponential growth of the MPS perturbation investigated earlier, the problem of the tensor perturbation is naturally resolved. Consider the evolution of tensor perturbation on EU. For the perturbation to be well defined at $t = t_c$, we need $A/a(t_c) = \epsilon$, where $\epsilon$ is a small parameter of order $O(m)$.

The time scale for the tensor perturbation to destroy EU is given by $\delta t_T \sim 1/\epsilon$. On the other hand, following Eq. (20), the MPS perturbation drives EU to the inflationary stage in the time scale $\delta t_{\text{MPS}} \sim (m/3 + \sqrt{8/3\epsilon})^{-1}$. Once we have $\delta t_T > \delta t_{\text{MPS}}$, which will be satisfied if $m \ll 1/\sqrt{\epsilon}$, the tensor perturbation fails to grow enough simply because EU deviates from MPS exponentially fast. Once EU settles down to the attractor solution, the tensor mode becomes oscillatory. Therefore, the whole picture in Fig. 1 will be kept, and the problem of the tensor instability is resolved.

The study of the scalar perturbation will require a more rigorous elaboration. It is very important to develop the formalism in the EiBI theory, and will be interesting to apply it to investigating this model. First, the growth of the scalar perturbation for MPS needs to be slower than that of the MPS perturbation. Second, the study of the scalar perturbation produced during the inflationary period in this theory is very important in testing the model compared with the observed density perturbation. We will get back to these in near future.

In summary, we investigated the evolution of the Universe driven by a scalar field with a quadratic potential in Eddington-inspired Born-Infeld gravity. When the energy density is high, the maximal pressure condition is achieved, for which the Universe undergoes an exponential expansion from a nonsingular initial state. Although the energy density is high (in fact, is not even bounded from above) during this period, the curvature scale remains constant, $H \sim H_{\text{MPS}} \approx 2m/3$. This state is unstable under perturbations and evolves to the inflationary period.

The succeeding inflation feature is the same with the ordinary chaotic inflation in GR, but it is not quite chaotic since the pre-inflationary stage can have a finite low curvature. Therefore, our model provides a natural precursor of inflation. It is not clear yet whether the EiBI theory is fundamental or effective, and whether or not this theory requires the quantum-gravity regime in the end. However, what we can say is that quantum gravity is not necessary in understanding the pre-inflationary stage in this model.

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