A Bayesian Particle Filtering Method For Brain Source Localisation

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Abstract
In this paper, we explore the multiple source localisation problem in brain cortical area using MEG data. We model neural currents as point-wise dipolar sources which dynamically evolve over time. We model the dipole dynamics using a probabilistic state space model (i.e., a hidden Markov model, HMM) which is strictly constrained within the brain cortical area. Based on the proposed model, we develop a Bayesian particle filtering algorithm for localisation of both the known and unknown number of dipoles. The algorithm contains a prior estimation step for initial dipole number estimation, a multiple particle filters step for individual dipole state estimation, and a heuristic selection criterion step for estimates selection. The estimated result from prior estimation is also used to adaptively adjust particle filter’s sample size and sample range to reduce the computational cost. The proposed model and algorithm are tested using both synthetic and real data. The results are accessed with existing particle filtering methods and they show that the method achieves a better dipole number estimation performance regarding estimation accuracy and robustness.

Keywords: Source localisation, MEG, Multi-dipole, Bayesian method, Particle filter

1. Introduction

In recent years, the development of non-invasive brain signal measuring techniques such as magnetoencephalography (MEG) and electroencephalography (EEG) have seen rapid progress. These techniques are helpful in clinical research and in mental disease diagnosis such as epilepsy, Alzheimer’s...
and Parkinson’s disease [1]. In non-invasive brain signal processing, we are particularly interested in the signal generated from the cerebral cortex which is a 2–4 mm thick sheet of grey matter located in the uppermost layer of the brain [2]. Brain neurons or dendrites from different cortical regions (such as somatosensory, visual, motor or auditory cortex) can be activated by suitable stimulus (such as an image or a piece of song). A single active neuron or dendrite is too weak to be measured, thus tens of thousands of synchronously active neurons or dendrites are needed to produce a measurable brain signal. For modelling purposes, many spatially neighbouring active neurons can be summarised and modelled as a current dipolar source, which can be simply named as a “dipole”. The electromagnetic field generated by such a dipolar source is measurable for M/EEG devices. Our goal is to localise the active brain dipolar sources using the measured M/EEG data.

1.1. Related work

Brain source localisation is fundamentally an ill-posed inverse problem. We aim to accurately localise the spatio-temporal brain sources using the electromagnetic signals collected outside the surface of the head. The main barrier of the problem is that there may exist many different solutions for the same set of data. It is an active research field that a significant amount of work has been done in the past two decades (see, e.g., [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] and the references therein).

There are mainly two types of methods to model the M/EEG inverse problem: distributed source approaches and point-wise dipole estimation approaches [2]. The distributed source methods identify the potential active brain sources that are distributed with fixed locations throughout the whole brain cortex (or the whole brain volume if under a looser constraint). Since the number of unknown sources is larger than the number of the sensors of the MEG/EEG devices, mathematical assumptions or constraints are required in order to obtain an unique solution. Some existing methods include (but not limit to) least square minimum norm estimation (MNE) [2], dynamic statistical parametric mapping (dSPM) [14], standardised low-resolution electromagnetic tomography (sLORETA) [3], and Kalman filter related approaches [6, 4].

The point-wise dipole estimation approaches treat the brain currents as the point-wise dipole sources and estimate the states (this may include dipole location, moment and orientation) of the point-wise dipoles. In this type of modelling, the state of a dipole source remains unknown. A number of work
has been done under this type of modelling, these include (but not limit to) multiple signal classification (MUSIC) related approaches [15], Markov chain Monte Carlo related approaches [16, 5], and sequential Monte Carlo (or particle filtering) related approaches [7, 10, 11, 13, 12].

1.2. Bayesian particle filtering methods for dipole localisation problem

Among various methods developed, Bayesian particle filtering is one of the promising approaches for the dipole localisation problem. In this paper, we focus on the point-wise dipolar sources localisation approaches using Bayesian particle filtering methods. Some research work has been done in last decade. Somersalo et al. [7] applied a standard sequential importance resampling (SIR) particle filter for the dipole localisation problem using artificial planar/3D geometry. Results of a two dipoles localisation was shown using an ideal spherical head model. Campi et al. [10] proposed a Rao-Blackwellised particle filter (RBPF) for dipole tracking with single dipole and two dipoles. It was shown in the work that RBPF provided better localisation result with a low computational cost when compared with the standard particle filter. Sorrentino et al. [11] integrated a random finite set scheme into the particle filtering using MEG data. The proposed method was able to track the time-varying number of dipoles with the maximum dipole number specified in advance.

Recently, Sorrentino et al. [13] suggested to model the problem using a static dipole setup. The work employed a resample-move particle filter to recursively estimate the dipole moment. Chen et al. [17, 18] integrated a minimum norm estimation method into a multiple particle filter method to address the problem of tracking an unknown number of dipoles. The estimation of the dipole number relied on both the minimum norm estimation and the previous tracking history. Miao et al. [12] adopted a multiple particle filter method for dynamic number of dipole tracking. The work also used a probability hypothesis density (PHD) filter to perform the estimation of the unknown/time-varying dipole number. The algorithm was implemented and assessed in a real-time FPGA board. However, it modelled the brain under the ideal spherical head model, which cannot provide a good description of the human brain.

1.3. Our work

In this paper, we propose an adaptive Bayesian particle filtering algorithm that can simultaneously estimate the multiple dipolar sources with a
relatively small computational cost. The work is developed based on our previous work [17, 18].

Firstly, a continuous head model which forces the state dynamics to strictly remain on the brain cortex is presented. To fit with real world applications, we adopt a 1-layer realistic head model, the Nolte model [19]. Although this head model is quite realistic, the off-the-shelf software implementations of it can only be used to evaluate the model at a discrete set of points (the mesh nodes). For distributed source implementations this is all that is needed. However, in our case we need a smooth manifold which defines the cortex surface and hence the discrete set of points is not enough. For this purpose, in this work we adopt a nearest-neighbour (NN) interpolation method to form an approximate continuous cortical manifold. This allows us to formulate the particle filter state directly in terms of the location on the continuous cortex surface.

Secondly, we apply a computational adaptive scheme to adjust the sample number and the state transition range in every particle filtering run. Since it is difficult to know the number of dipoles before the algorithm run, we integrate a standard noise normalised MNE method [2] and a spatial clustering method [20] to gain prior knowledge of the active cortical region. These prior information are used to evaluate the localisation accuracy at every time instant. The particle number and the particle transition range are adjusted in next time step depending on the estimation accuracy at current time step.

Thirdly, we develop a dipole number dynamic model along with the multiple particle filtering (MPF) method [21, 22] for localisation with an unknown dynamic number of dipoles. The model generates three potential dipole numbers based on the dipole number estimation from the previous time step. All three potential cases are examined and their corresponding dipole number estimation are generated at each time step. We then apply a heuristic selection criterion (HSC) method to obtain the optimal solution for next particle filtering run. This approach improves the estimation accuracy for the number of dipoles, and thereby improves the localisation performance of the proposed algorithm.

The rest of the paper is organised as follows. Section 2 introduces the data modelling procedure. A discrete / continuous head model, a dipole state transition model and a dipole number dynamic model are described in this section. The localisation algorithm is proposed in Section 3. Both the models and the algorithms are evaluated in Section 4. Section 5 concludes the article.
2. Data Model

We consider a clinical application using an MEG system with \( M = 204 \) magnetometers (sensors). All sensors are placed outside the brain surface to obtain non-invasive measurement. We are interested in the neural activities in the brain cortical region. The state space is constrained within the brain cortex and is denoted as \( \Omega \). A three dimensional Cartesian coordinate system is established to calibrate the brain cortex.

For MEG data, a 1-layer realistic head model is introduced (see below) to generate the lead-field matrix (the forward matrix). This lead-field matrix contains \( G \) fixed vertices. An NN interpolation method is used to interpolate the state space between these vertices.

2.1. Discrete head model

The head model contains \( G \) vertices, \( \{g_1 \cdots g_\nu \cdots g_G \} \); and \( F \) triangular faces on the surface of the cortex, created assuming a 1-shell Nolte model for MEG. The lead-field matrix \( L \) was generated using the Statistical Parametric Mapping (SPM) software [23]. Although the lead-field matrix provides a relatively accurate approximation for the source distribution in the cortical space, it is discretised artificially to a limited number of fixed-location points. The neural current density is in reality a continuous spatial flow. For this reason, we propose an interpolated realistic head model for point-wise dipole localisation.

Figure 1 shows the triangulation of the cortex. The blue dots are the pre-defined vertices on the brain cortex, and the 5 coloured small areas are example sub-planes that represent the individual triangular faces on the cortex. To better fit real world applications, we strictly force the trajectory of a point-wise dipolar source to be within the brain cortex. Each individual dipolar source may only move within a single triangular cortical region defined by the fixed vertices and the triangular faces. Thus we model each dipole as semi-static within a small spatial volume for the whole observation interval.

For a point-wise dipolar source, we define its state as a vector \( \mathbf{X}_k \in \mathbb{R}^{(6N)\times 1} \), where \( k \) is the time instant and \( N \) is the number of dipoles. We have the joint state matrix:

\[
\mathbf{X}_k = [\mathbf{x}_k^1 \cdots \mathbf{x}_k^n \cdots \mathbf{x}_k^N]^T.
\]

(1)
Figure 1: Vertices (blue dots) and individual triangular faces (the five coloured small area) in a brain cortex

Each $x^n_k$ is a state vector for a single dipole defined as $x^n_k = [r^n_k, q^n_k]^T$, where $r^n_k$ is the 3D location and $q^n_k$ is the dipole moment (the dipole amplitude with orientation). The dipole orientation is set as normal to the cortical sub-plane surface.

A general measurement model that describes the relationship between the MEG measurement and the dipole state is defined as:

$$Y_k = h(X_k, N_k) + \zeta_k,$$

(2)

where $N_k$ is the dipole number, $Y_k \in \mathbb{R}^{M \times 1}$ is the measurement vector at time $k$, $Y_k = \begin{bmatrix} y_1^k \\
\vdots \\
y_M^k \end{bmatrix}^T$. $\zeta_k$ is the measurement noise, an independent Gaussian random variable with zero mean and variance $\sigma^2_{\zeta}$.

For the discrete head model, the measurement function $h(\cdot)$ is simply the discrete lead-field matrix $L$. The model can be rewritten, assuming a linear structure, as:

$$Y_k = L X^L_k + \zeta^L_k,$$

(3)

where $X^L_k \in \mathbb{R}^{G \times 1}$ is the amplitude vector that represents $G$ fixed location vertices in the cortex, $X^L_k = [x^{q_1}_k \cdots x^{q_\nu}_k \cdots x^{q_G}_k]^T$, where $\nu$ is the vertex index. For the discrete model, $x^{q_\nu}_k = q^{q_\nu}_k$, the dipole location $r^n_k$ is specified from the fixed spatial grids with their location denoted as $[g_1 \cdots g_\nu \cdots g_G]^T$.

2.2. Continuous head model

We apply a simple NN interpolation method [24] onto the $G$ fixed location anchor vertices from $L$. As shown in Figure 2 for every three neighbouring anchor vertices (denoted as $g_1$, $g_2$ and $g_3$ in the example), we obtain $Y^{q_1}_k$, $Y^{q_2}_k$.
and $Y_{g_k}^{\nu}$ from $L$. $Y_{g_k}^{\nu}$ represents the unit response measured by MEG when we place a unit dipole at the $\nu$th anchor vertex with the location $g_\nu$. The triangular cortical region is too small that it is reasonable to treat it as a flat sub-plane in which all the interpolated points have the same orientation. In Figure 2, the orientation that normal to the sub-plane is denoted as $e_p$. $e_1$, $e_2$ and $e_3$ are the orientations of the three anchor vertices respectively when there is a unit dipole placed on the vertex. We define $\theta$ as the angle between $e_p$ and the orientation of the anchor vertex. We have $\theta_1$, $\theta_2$ and $\theta_3$ for vertex $g_1$, $g_2$ and $g_3$ respectively.

![A sub-plane in the cortex](image)

Figure 2: Nearest-neighbour interpolation for three neighbouring vertices.

For a random dipole with state $x_{n_k}^\nu$, we can compute its unit response by:

$$Y_{k}^{x_n^\nu} = (1 - \phi - \varphi) \tilde{Y}_{k}^{x_n^\nu g_3} + \phi \tilde{Y}_{k}^{x_n^\nu g_2} + \varphi \tilde{Y}_{k}^{x_n^\nu g_1},$$

(4)

where $Y_{k}^{x_n^\nu}$ is the MEG measurement. $\tilde{Y}_{k}^{x_n^\nu}$ is the unit response after orientation mapping. $\tilde{Y}_{k}^{x_n^\nu g_\nu}$ represents the response for vertex $g_\nu$. $\phi$ and $\varphi$ are interpolation coefficients drawn from the standard uniform distribution, with $\phi \geq 0$, $\varphi \geq 0$ and $\phi + \varphi \leq 1$. These two coefficients describe the relationship between the location of a dipole and the three anchor vertices.

In this paper, we perform the multiple dipolar sources localisation by estimating the number of dipoles and the location of dipoles. The dipole amplitude is set as the unit length in continuous head model.

To obtain the relationship between $\phi$, $\varphi$ and 3D location $r_{k}^n$, $g_1$, $g_2$, $g_3$, we define $\alpha = g_1 - g_3$, $\beta = g_2 - g_3$ and $\gamma = r_{k}^n - g_3$. The relationship is
described as follows [24]:

\[
\phi = \frac{(\alpha \cdot \beta)(\gamma \cdot \beta) - (\beta \cdot \beta)(\gamma \cdot \alpha)}{(\alpha \cdot \beta)^2 - (\alpha \cdot \alpha)(\beta \cdot \beta)},
\]

(5)

\[
\varphi = \frac{(\alpha \cdot \beta)(\gamma \cdot \alpha) - (\alpha \cdot \alpha)(\gamma \cdot \beta)}{(\alpha \cdot \beta)^2 - (\alpha \cdot \alpha)(\beta \cdot \beta)}.
\]

(6)

Because the total measured response at sensor level is additive, it can be obtained by summing up all of the unit response from different individual dipoles: \(Y_k = \sum_{n=1}^{N} Y_{x_n}^k\). The interpolation is executed in the following steps:

- Find the triangular sub-planes that the dipoles \(X_k\) are located in.
- For each individual dipole \(x_n^k\), identify the anchor vertices \(g_\nu\) and compute their corresponding orientations.
- Calculate the angles \(\theta_\nu\) to obtain \(\tilde{Y}_{x_n^k}^{g_\nu}\).
- Calculate \(\phi\) and \(\varphi\) using Equation (5) and (6), then compute \(Y_{x_n^k}^k\).
- Sum up all \(Y_{x_n^k}^k\) to obtain the predicted measurement \(Y_k\).

### 2.3. Individual dipole dynamic model

For an individual dipole which exists from time instant \(k - 1\) to \(k\), we define the following individual dipole transition model:

\[
x_n^k = f(S_\kappa^n, \phi^n, \varphi^n),
\]

(7)

where \(f(\cdot)\) can be a linear or nonlinear function. \(S_\kappa\) is the sub-plane notation, \(\kappa\) is the face indice and \(n\) is the dipole indice. For each of the \(F\) sub-planes in the 3D cortical space, it can be treated as a two dimensional plane. Thus we adopt a two dimensional random walk model as our transition function \(f(\cdot)\).

As we have stated above, the location of a dipole is modelled as semi-static in the brain cortex. The state space is constrained accurately on the brain cortical surface \(\Omega\), thereby we divide the dipole dynamic into two phases: the transition between different triangular faces and the transition within an individual face.

For the transition within a single face, we define \(\vartheta(\cdot)\) as a neighbouring sub-plane set. It stores all the neighbouring triangular faces of the face
where the dipole $x_{n,k-1}^n$ is located at the previous time step. In practice, this is computed and stored in a lookup table using the grid information provided by the lead-field matrix.

For the transition within a face, we draw values for the coefficients $\phi^n$ and $\varphi^n$, and randomly select a position within the triangular sub-plane, $\phi^n \sim U[0, 1]$ and $\varphi^n \sim U[0, 1]$ with constraint described in Equation (1).

The procedure to perform the dipole transition is as follows:

- For a dipole with state $x_{n,k-1}^n$, find the neighbour sub-plane set $\theta(x_{n,k-1}^n)$.
- For those $F_n^\kappa$ identified sub-planes $\{S_1^n \cdots S_\kappa^n \cdots S_{F_n^\kappa}^n\}; S_\kappa^n \in \theta(x_{n,k-1}^n)$, randomly select one sub-plane with equal probability $p_n^\kappa = \frac{1}{F_n^\kappa}$.
- Randomly choose a position using $\phi^n$ and $\varphi^n$ in the selected sub-plane.
- Calculate $x_{n,k}^n = f(S_\kappa^n, \phi^n, \varphi^n)$ to obtain the new dipole state prediction.

$F_n^\kappa$ is the total number of the identified neighbour faces.

In the initialisation step, dipole states are randomly selected from the $G$ fixed location grid points. Since a grid point connects several sub-planes, it is difficult to define which sub-plane it belongs to. In practice, we compute the distance between the grid point and all its neighbour sub-planes. We select the sub-plane with the shortest distance to the grid point as its sub-plane.

2.4. Dipole number dynamic model

In real world applications, the number of the active dipolar sources does not change dramatically between adjacent time steps. For scenarios with more than one dipole, we can model the dipole number transition process by allowing an individual dipole to appear or disappear at a single time instant.

Given the dipole number $N_{k-1}$ from the previous time instant, the current dipole number $N_k$ can be obtained from the following dipole number dynamic model:

$$p(N_k | N_{k-1}) = \begin{cases} p_{k,+1} & \text{if } N_k = N_{k-1} + 1 \\ p_{k,0} & \text{if } N_k = N_{k-1} \\ p_{k,-1} & \text{if } N_k = N_{k-1} - 1 \end{cases} \quad (8)$$

with $p_{k,+1} + p_{k,0} + p_{k,-1} = 1$. $j = \{-1, 0, +1\}$ denotes three pre-defined cases: a new dipole appears, the number of dipoles remains the same or an existing dipole disappears. The dynamic probability $p_{k,(j)}$ is predefined. A detailed choice of the dynamic probability is described in the results section.
2.4.1. Dipole birth-death move

We adopt a simple birth-death move for cases when a new dipole appears or an existing dipole disappears from time $k - 1$ to $k$.

For the dipole birth process, a randomly selected initial state within $\Omega$ is assigned for the new birth dipole. The initialisation is assisted by the results from the prior estimation (detailed in the next section). The prior estimation provides us knowledge about the amplitude of the $G$ grid points in the lead-field matrix. A threshold is set to pick up those grid points with higher amplitude. The selected point set is defined as $\Psi_k$. We randomly select one of the grid points in $\Psi_k$ and assign its location to the new birth dipole. $\Psi_k$ is also used to generate the region of interests (ROIs) by applying a spatial clustering algorithm [20] to the selected point set.

For the dipole death process, the algorithm randomly selects and deletes one of the existing dipoles. In the particle filtering algorithm, many samples are drawn from the importance distribution; samples with lower weights are filtered out to improve the estimation accuracy.

The dipole association problem is addressed by computing and comparing the mean square error (in spatial distance) between the estimated dipole and the identified ROIs generated from $\Psi_k$. The spatial clustering algorithm [20] automatically clusters those geographically neighbouring points. We define the generated clusters as the ROIs. We can compute the centre of each ROI by taking the mean location of all clustered points in each of the ROIs, please see [18] for a more detailed description.

3. Bayesian sequential Monte Carlo algorithm

In this section, we develop a Bayesian particle filter algorithm to deal with the localisation with a known/unknown dipole number. The dipole localisation problem is cast as a general multi-target tracking problem in the Bayesian framework.

3.1. Algorithm execution

We first describe the algorithm execution mechanism, followed by a detailed description for each component in the proposed algorithm. The algorithm contains three different interactive components: the prior estimation, the multiple particle filtering, and the heuristic selection criterion. Figure 3 illustrates the relationship between the components and the corresponding variables. $\hat{N}$ is the estimation for the number of the ROIs.
The algorithm executes in three steps:

1. A prior estimation step uses the noise normalised MNE method and the spatial clustering algorithm to obtain the point set $\Psi_k$. Therefore we can obtain an estimation of the number of ROIs. The ROIs are used for both the new birth dipole initialisation and the error computation in the dipole association. Given the estimated dipole number $N_{k-1}$ at the previous time step $k - 1$, three potential dipole numbers $N_k$ (with their corresponding dynamic probability) can be obtained from (8).

2. For the fixed dipole number $N_k^{(j)}$, we employ a modified MPF algorithm to perform the dipole localisation. The algorithm assigns an individual particle filter (iPF) for each of the identified targets. In a single MPF run, each iPF is executed to generate the estimate $x_{k}^n$ for its corresponding dipole. We can finally obtain the estimated dipole state by sequentially updating the estimates from each of the iPFs.

3. We then obtain the estimation for the three potential ($j$) cases. We use a heuristic selection criterion scheme to compute the likelihood for each of the cases. An estimation for both the dipole number $\tilde{N}_k$ and the dipole state $\tilde{X}_k$ can then be obtained.

In addition, the discrete point set $\Psi_k$ from the MNE is used to adaptively control the number of particles and the state transition range in the particle
filtering.

In the remainder of this section, we give a more detailed description.

3.2. Prior estimation

A noise-normalised MNE approach is employed to obtain knowledge of the ROIs.

The linear discrete head model from Equation (3) is \( Y_k = L X_k^L + \zeta_k^L \). A typical noise-normalised MNE solution [2] can be obtained as follows:

\[
X_k^L = L^T (LL^T + \lambda I)^{-1} Y_k,
\]

where \( X_k^L \) is a amplitude estimation vector for the \( G \) grid points. \( \lambda \) is a noise normalised regularisation parameter.

We define an amplitude threshold \( q_{th} \) and obtain the point set \( \Psi_k \) by filtering out those points with lower amplitude in \( X_k^L \). We then employ a hierarchical spatial clustering method [20] to cluster the selected points in \( \Psi_k \) with respect to their corresponding geographical positions. We define these clusters as the ROIs. We can obtain from this the number and extent of the ROIs \( \Psi_k(n), n = 1, 2, \cdots, N_k \), where \( N_k \) is the number of the ROIs.

\( \Psi_k(n) = \{g_{k,1}, \cdots g_{k,i}, \cdots g_{k,G^n}\} \) is the subset of the grid points for active region \( n \), where \( G^n \) is the total number of points in that subset and \( i \) is the grid point index. We compute the location of the \( n_k \)th ROI by taking the mean of the points in \( \Psi_k(n) \) so that the centre of the ROI is defined as:

\[
c^n_k = \frac{1}{G^n} \sum_{i=1}^{G^n} g_{k,i}.
\]

The centre location of each ROI will be used in the adaptive filtering subsection.

3.3. Multiple particle filtering

Three potential dipole numbers are passed from the prior detection step to the MPF step. For each of the \( N_k^{(j)} \), the problem turns to tracking a fixed number of dipoles. We want to investigate the pdf \( p(X_k^{(j)} | Y_{1:k}, \tilde{N}_{1:k-1}, N_k^{(j)}) \), where \( \tilde{N}_{1:k-1} \) are the estimated dipole number from time steps 1 to \( k - 1 \), and \( N_k^{(j)} \) is the potential dipole number from the current time step. The state vector of each MPF being denoted by \( X_k^{(j)} \). We assign an MPF for each \( N_k^{(j)} \); all three MPFs operate in parallel and are independent of each other.
For each of the three $N_n^{(j)}$, there are $N_k^{(j)}$ dipoles and therefore $N_k^{(j)}$ iPFs are assigned, as shown in Figure 3. The state vector can be further expanded as $X_k^{(j)} = \{x_k^1, x_k^2, \cdots x_k^{N_k^{(j)}}, \cdots \}$, where $x_k^{n_k^{(j)}}$ denotes the state of the $n_k^{(j)}$th individual dipole out of $N_k^{(j)}$. To simplify the notation, we rewrite the state $x_k^{n_k^{(j)}}$ as $x_{n,k}^{(j)}$.

### 3.3.1. Gibbs Sampling updating step for conditional pdf

In each single time step, the iPFs are executed from the first iPF to the $N_k^{(j)}$th iPF sequentially. The updating procedure acts in a similar way as that in a standard Gibbs Sampler: once a new state $x_{n,k}^{(j)}$ is generated, it is immediately used to assist other iPFs by updating the corresponding conditional pdf.

The conditional pdf of the individual dipole state in an iPF can be written as $p(x_{n,k}^{(j)}|y_{1:k}^{(j)}, \tilde{N}_{1:k-1}^{(j)}, N_k^{(j)}, x_{-n,k}^{(j)})$, where $x_{-n,k}^{(j)}$ describes a state vector excluding its own state $x_{n,k}^{(j)}$:

$$x_{-n,k}^{(j)} = \{x_{1,k}^{(j)}, \cdots x_{n-1,k}^{(j)} , x_{n+1,k}^{(j)}, \cdots x_{N,k}^{(j)}\}. \quad (11)$$

$x_{n,k}^{(j)}$ is used in the estimation of the $n$th iPF, its first $n-1$ elements are the estimates updated at time $k$ while the rest elements are the estimates at time $k-1$. Once a new state $x_{n,k}^{(j)}$ is generated, we update the corresponding $x_{n,k}^{(j)}$ and use it in the $n+1$th iPF. In order to obtain a good estimate $X_k^{(j)}$, this Gibbs Sampling manner is run for several iterations in every single time step, we denote the number of iteration by the iteration index $\{1, 2, \cdots \}$. Therefore, the final estimate at time step $k$ can be written as $X_k^{(j)[l]} = \{\tilde{x}_{1,k}^{(j)}, \tilde{x}_{2,k}^{(j)}, \cdots \tilde{x}_{n,k}^{(j)}, \cdots \tilde{x}_{N,k}^{(j)}\}[l]$, where $\tilde{x}_{n,k}^{(j)}$ denotes the final estimate at time step $k$.

### 3.3.2. Individual particle filtering

Since $N_k^{(j)}, \tilde{N}_{1:k-1}$ and $x_{-n,k}^{(j)}$ are available terms at this stage, we only need to estimate the unknown state $x_{n,k}^{(j)}$ in each iPF. We define $\psi_k^{(j)} = \{\tilde{N}_{1:k-1}, N_k^{(j)}, x_{-n,k}^{(j)}\}$ as the known evidences, the conditional pdf can be rewritten as $p(x_{n,k}^{(j)}|y_{1:k}^{(j)}, \psi_k^{(j)})$. We define $I_k$ as the number of particles for each iPF at time $k$. A particular sample in an iPF is denoted as $x_{n,k}^{(j)}$. 

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We aim to use the conditional pdf to estimate the hidden state $x_{n,k}^{(j)}$ in a single iPF. For some function of interest $\varrho_k(.)$, its expectation can be obtained:

$$\mathbb{E}(\varrho_k) = \int \varrho_k(x_{n,k}^{(j)}) p(x_{n,k}^{(j)}|Y_{1:k}, \psi_{k}^{(j)}) dx_{n,k}^{(j)}. \tag{12}$$

Since it is difficult to draw samples directly from the conditional pdf, we introduce some known distribution that samples can be easily drawn from. This particular distribution is named as importance density $q(.)$. The above equation can be rewritten as

$$\mathbb{E}(\varrho_k) = \int \varrho_k(x_{n,k}^{(j)}) w_{n,k}^{(j)} q(x_{n,k}^{(j)}|Y_{1:k}, \psi_{k}^{(j)}) dx_{n,k}^{(j)}, \tag{13}$$

where $w_{n,k}^{(j)}$ is a weight term that evaluates the similarity between the adopted importance density and the unknown pdf, it is defined as

$$w_{n,k}^{(j)} = \frac{p(x_{n,k}^{(j)}|Y_{1:k}, \psi_{k}^{(j)})}{q(x_{n,k}^{(j)}|Y_{1:k}, \psi_{k}^{(j)})}. \tag{14}$$

In the Monte Carlo methods, a limited number of samples can be drawn from the importance density, the unknown identity may be approximated using the samples as follows:

$$\hat{\mathbb{E}}(\varrho_k) = \frac{1}{I_k} \sum_{i=1}^{I_k} w_{n,k}^{(j)} \varrho_k(x_{n,k}^{(ij)}). \tag{15}$$

The conditional pdf can be expanded in two steps: an updating step and a prediction step, the updating step can be expressed as:

$$p(x_{n,k}^{(ij)}|Y_{1:k}, \psi_{k}^{(j)}) = \frac{p(Y_{k}^{(j)}|x_{n,k}^{(ij)}, \psi_{k}^{(j)}) p(x_{n,k}^{(ij)}|Y_{1:k-1}, \psi_{k-1}^{(j)})}{p(Y_{k}^{(j)}|Y_{1:k-1}, \psi_{k-1}^{(j)})}, \tag{16}$$

where

$$p(x_{n,k}^{(ij)}|Y_{1:k-1}, \psi_{k-1}^{(j)}) = \int p(x_{n,k}^{(ij)}|x_{n,k-1}^{(ij)}, \psi_{k-1}^{(j)}) p(x_{n,k-1}^{(ij)}|Y_{1:k-1}, \psi_{k-1}^{(j)}) dx_{n,k-1}^{(ij)}$$
is the prediction step. This term can be obtained directly by integrating out the variable $x_{n-1,k}^{(ij)}$. Therefore we can rewrite the conditional pdf expression by ignoring the known evidence terms as

$$p(x_{n,k}^{(ij)} | Y_{1:k}, \psi_k^{(j)}) \propto p(Y_k | x_{n,k}^{(ij)}, \psi_k^{(j)}) p(x_{n,k}^{(ij)} | x_{n-1,k}^{(ij)}, \psi_{k-1}^{(j)}) p(x_{n-1,k}^{(ij)} | Y_{1:k-1}, \psi_{k-1}^{(j)}).$$

If the importance density is properly chosen and factorized, we could obtain the weight as

$$w_{n,k}^{(ij)} \propto w_{n,k-1}^{(ij)} p(Y_k | x_{n,k}^{(ij)}, \psi_k^{(j)}) p(x_{n,k}^{(ij)} | x_{n-1,k}^{(ij)}, \psi_{k-1}^{(j)}) / q(x_{n,k}^{(ij)} | x_{n-1,k}^{(ij)} Y_{1:k}, \psi_{k-1}^{(j)}). \quad (17)$$

We choose the prior as the importance distribution, so that we have the following simplification:

$$w_{n,k}^{(ij)} \propto w_{n,k-1}^{(ij)} p(Y_k | x_{n,k}^{(ij)}, \psi_k^{(j)}), \quad (18)$$

We normalise $w_{n,k}^{(ij)}$ to obtain $\tilde{w}_{n,k}^{(ij)}$. We adopt a residual resampling step [25] to avoid the so-called “degeneracy problem” [26].

For an iPF, we employ the standard SIR particle filter as described in Algorithm [1]

**Algorithm 1: Individual particle filter (iPF)**

```plaintext
// At time k for the n_{ik}^{(j)}th dipole of a total N_{ik}^{(j)} dipoles, at the lth Gibbs Sampling iteration
Assign I_k particles in this iPF.
for i = 1, ..., I_k do
    // Prediction
    • Draw samples $x_{n,k}^{(ij)} \sim p(x_{n,k}^{(ij)} | x_{n-1,k}^{(ij)}, \psi_k^{(j)}).
    • Compute weights: $w_{n,k}^{(ij)} \propto w_{n,k-1}^{(ij)} p(Y_k | x_{n,k}^{(ij)}, \psi_k^{(j)}).
    • Normalise weights to obtain $\tilde{w}_{n,k}^{(ij)}$.
    // Resample
    $\{x_{n,k}^{(ij)}, \tilde{w}_{n,k}^{(ij)}\}_{i=1}^{I_k} \to \{x_{n,k}^{(i')j}, \tilde{w}_{n,k}^{(i')j}\}_{i'=1}^{I_k}$.
    // Gibbs Sampling choice
    • Update the estimate $\tilde{x}_{n,k}^{(j)}$ by randomly sampling from $x_{n,k}^{(i')j}$.
    • Update the nth entry of the lth Gibbs Sampling iteration in the state $X_{k}^{(j)[l]}$.
end
```

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\( \mathbf{x}_{n,k-1}^{(j)} \) is the prediction update from all other iPFs in the same MPF run at time \( k \).

Since it is impossible to obtain the ground-truth state \( \mathbf{x}_{n,k}^{(j)} \) for each \( n_k^{(j)} \)th dipole source directly, we randomly pick up the state estimate from the resampled state \( \mathbf{x}_{n,k}^{(j)} \). When the particle weights at the current time step are not available, we randomly select a sample from \( \mathbf{x}_{n,k}^{(j)} \) according to the weight \( \tilde{w}_{n,k-1}^{(ij)} \) from the previous time step. Once the current particle weights are obtained, we have the estimation update \( \tilde{\mathbf{x}}_{n,k}^{(j)} \) by randomly selecting a sample from the resampled particles \( \{ \mathbf{x}_{n,k}^{(ij)} \} \) with the equal probability \( p^{(i)} = \frac{1}{I_k} \), where \( i' \) represents the particle index after resampling.

The proposed algorithm is described as follows in Algorithm 2.

**Algorithm 2: Bayesian multiple dipole localisation algorithm**

Initialisation: assign \( N_0 \) at time \( k = 0 \).

for \( k = 1, \ldots, K \) do

// Prior estimation
Compute \( \mathbf{X}_k^L \) through Equation (9), obtain \( \bar{N}_k \) and \( \Psi_k \).
Propagate \( N_k \) to obtain \( N_k^{(j)} \) through Equation (8).

for \( j = (-), (0), (+) \) three cases do

Initialise \( \mathbf{X}_k^{(j)} \), uniformly draw from \( \Omega \).

// Multiple particle filtering
for \( l' = 1, 2 \ldots l \) do

// individual particle filtering
Follow Algorithm 1
Update \( \mathbf{X}_k^{(j)}[l'] = \{ \tilde{\mathbf{x}}_{1,k}^{(j)}, \ldots, \tilde{\mathbf{x}}_{n,k}^{(j)}, \ldots, \tilde{\mathbf{x}}_{N,k}^{(j)} \}[l'] \).

end

end

Obtain \( \{ N_k^{(j)}, \mathbf{X}_k^{(j)}[l] \} \).

// Heuristic selection criterion
Obtain \( \{ \tilde{N}_k, \tilde{\mathbf{X}}_k \}^{HSC} \) through Equation (20).

// Adaptive filtering
Adjust \( I_{k+1} \) w.r.t. evaluation result from Equation (21).

end
To sum up, we modify the original MPF algorithm in the following three aspects:

1. We integrate an iterated Gibbs Sampling procedure to generate the individual state estimate in each iPF. This enables us to obtain more reliable estimations in each $x_{n,k}^{(j)}$ assisted iPF run. The number of iterations is controlled by the parameter $l$.

2. Instead of dividing the state space into several subspaces, samples of each iPF are drawn from the same state space. In the dipole initialisation step, the samples are drawn from the $G$ vertices; they are propagated using the individual dipole dynamic model in $\Omega$.

3. Rather than using a weighted mean method, the estimation $x_{n,k}^{(j)}$ in the $n$th iPF is selected randomly from all the samples $x_{n,k}^{(ij)}$, where $i$ is the sample index. In practice, we randomly pick up one of the particle filter samples $x_{n,k}^{(ij)}$ with the equal probability $p(i) = \frac{1}{I_k}$.

### 3.4. Heuristic selection criterion

We then obtain the three estimations of the dipole number $N_k^{(j)}$ and their corresponding states $X_k^{(j)}$ from MPF. We apply a heuristic selection criterion scheme to find the optimal values amongst all the available estimates. We can obtain the probability:

\[ p(N_k, X_k | Y_{1:k}) \propto p(Y_k | N_k, X_k) p(N_k | N_{k-1}) . \]  

We can obtain the estimate:

\[ \{ \tilde{N}_k, \tilde{X}_k \}^{HSC} = \arg \max p(Y_k | N_k, X_k) p(N_k | N_{k-1}) , \]  

where $p(N_k)$ is known from the dipole number dynamic model. $p(Y_{1:k} | N_k, X_k)$ is the likelihood term that can be computed using the proposed head model.

### 3.5. Adaptive filtering

As shown in Figure 3, the discrete amplitude matrix $X_k^{T}$ from MNE is used in MPF to assist the sampling procedure in every iPF run. The identified ROIs and their point sub-set $\Psi_k(n)$ are used to control the particle number and particle transition range. In practice, we calculate the localisation root mean squared error (RMSE) between the centre of each ROI and the dipole state estimation $\tilde{X}_k$ at each time step $k$. The localisation RMSE $e_k$ can be obtained from:

\[ e_k = D(\Psi_k(n), \tilde{X}_k) , \]  

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where function $D(\cdot)$ computes the localisation RMSE between all elements in $\Psi_k(n)$ and $\tilde{X}_k$.

In practice, the centre points of the ROIs $c^n_k$ are compared with the localisation estimation $\tilde{X}_k$, we compute the spatial distance $||\tilde{x}_k - c^n_k||$ between each of the the individual dipole state and the ROIs. We then obtain a $N_k \times G^n_k$ matrix $C_k$ that contains all pairs of the localisation RMSE. We execute the target association according to the RMSE matrix $C_k$. We identifies and couples those pairs with the smallest RMSE. $e_k$ is then used as a reference criteria to adaptively adjust the particle number and the particle transition range in particle filtering at time $k+1$.

$e_k$ assists adaptive filtering mainly in two aspects: (1) the number of samples $I_{k+1}$ is modified with respect to the RMSE level $e_k$, a smaller $I_{k+1}$ is assigned when we obtain a lower $e_k$ and vice versa. (2) In the individual dipole propagation, the dipole dynamic range for the sample $x^{ij}_k$ depends on the value of $e_k$; a lower $e_k$ results in a smaller dipole dynamic range and vice versa.

4. Numerical Results

In this section, we present numerical results for both the simulated and the real MEG data. Since the ground-truth dipole location from the real data remains unknown, the performance evaluation relies on the results from the synthetic data. We adopted typical examples with both the known and unknown number of dipoles. The number of dipoles varied between one and five. The ground-truth dipoles had unit amplitude in our simulation. The orientation of each dipole was set as normal to its corresponding sub-plane. The results are shown in both numerical tables and visualised in figures. Visualisations were carried out with tools further developed from those published in Helsinki BEM Library [27].

In order to mimic empirical results, the simulation contained up to five simultaneously evolving dipoles. We generated the MEG data using a 204-magnetometer sensor setup. All magnetometers were distributed around the surface of the head. The state space $\Omega$ was strictly constrained within the pre-defined 1-layer real head cortex. The width of the brain was 136 mm in our simulation. According to empirical observations, a brain current source often appears and disappears in the same region, and the centre of the current source evolves within a small volume in the cortex. Therefore, it was reasonable for us to assume that all the dipoles were identical and
independent of each other, and that each individual dipole might move within a pre-defined triangular sub-plane. We set the measurement SNR (signal to noise ratio) as 10. The measurement noise in the head model was a Gaussian distribution with zero mean and variance \( \sigma_z^2 \), where \( \sigma_z \) was two times larger than that of the ground-truth noise. We tested each of the algorithms and the model with more than 30 repeated identical experiments. The number of dipoles for each individual particle filter at \( k = 0 \) was set as \( I_0 = 10000 \).

In the remainder of this section, we first show the results using the simulated data. We compare the performance of different head models in an example with a known number of dipoles. An example with five known numbers of dipoles was used to evaluate the localisation algorithms using the proposed continuous head model. For localisation with an unknown number of dipoles, we tested and compared the performance of different particle filtering algorithms in an example with three and dynamic numbers of dipoles. We also present an evaluation result particularly for the estimation of the dipole number. Finally we show the results of localisation performance using real MEG data.

4.1. Localization with the known number of dipoles

The performance of the known number of dipoles in terms of their localisation is easier and more accurate than the localisation with the unknown number of dipoles. In this section we present two examples, one with three dipoles (two on the left and one on the right hemisphere) and the other with five dipoles (two on the left and three on the right hemisphere). The three dipole example is used to compare three different head models.

4.1.1. Head model comparison

We compared the model performance between the spherical head model, discrete real head model and the proposed continuous real head model. The spherical head model is a relatively older model that assumes the human head is a perfect spherical shape. The discrete head model is the 1-layer real head model generated using the BEM method. In this paper we adopt the discrete model which contains \( G = 8196 \) discretised potential source points, all of which have a fixed location. A lead-field matrix with \( G \) columns is then generated. We employ the NN interpolation method to convert the discrete model into the continuous model, which was presented in the Data Model section.
Table 1: Localisation RMSE comparison of three head models using three particle filter algorithms, all unit in [mm]. GTM: ground-truth model, MM: means measurement model, S: spherical head model, D: discrete head model, C: continuous head model, SIR: SIR particle filter, MPF: multiple particle filter, AMPF: the proposed adaptive multiple particle filter, the number in column one represent the particle number

|             | SIR/2000 | MPF/2000 | AMPF/2000 | SIR/5000 | MPF/5000 | AMPF/5000 | SIR/10000 | MPF/10000 | AMPF/10000 |
|-------------|----------|----------|-----------|----------|----------|-----------|-----------|-----------|-----------|
| S/S         | 17.32    | 13.33    | 13.20     | 15.28    | 11.57    | 11.77     | 13.05     | 9.36      | 9.28      |
| S/D         | 29.37    | 19.29    | 19.17     | 28.22    | 16.32    | 16.28     | 25.81     | 15.78     | 15.90     |
| S/C         | 22.22    | 19.11    | 18.16     | 21.35    | 18.06    | 18.24     | 25.81     | 15.13     | 14.84     |
| D/S         | 35.61    | 23.65    | 22.99     | 35.06    | 21.39    | 20.12     | 29.77     | 20.22     | 20.35     |
| D/D         | 13.01    | 11.31    | 11.68     | 10.73    | 10.25    | 10.25     | 9.88      | 9.36      | 9.33      |
| D/C         | 15.55    | 13.77    | 13.94     | 12.89    | 12.45    | 12.45     | 11.29     | 10.97     | 10.92     |
| C/S         | 21.28    | 16.34    | 16.28     | 16.45    | 16.99    | 16.99     | 17.53     | 15.68     | 15.71     |
| C/D         | 22.78    | 13.46    | 12.67     | 17.44    | 12.08    | 12.08     | 14.02     | 9.52      | 9.48      |
| C/C         | 16.24    | 9.22     | 9.38      | 15.36    | 7.10     | 7.53      | 12.74     | 5.70      | 5.64      |

In Table 1, we present the numerical result with three dipoles with known locations. Three models are tested using a simple standard SIR particle filter, a standard MPF and the proposed algorithm. We vary the particle number from 2000 to 10000. The simulated data were generated and tested using all three models, for example, S/D in the table means we generate the data using a spherical head model, and the measurement model we use in particle filtering is a discrete head model.

Regarding the RMSE performance between different particle filtering algorithms, we can find that both MPF and AMPF perform better than the standard SIR. MPF and AMPF demonstrated similar performance. This is not surprising since the proposed AMPF algorithm is executed in a similar way to that of MPF in the fixed and known dipole number scenario. We also find that the RMSE performance improves with an increase in particle number. For those few entries with a similar performance between $I = 2000$ and $I = 5000$ (e.g. column C/S of MPF and AMPF), a similar RMSE may occur due to model mismatching. In terms of head model comparison, it is as expected that S/S, D/D and C/C achieve better performance. Amongst all model pairs for the same algorithm, the continuous head model performs better than the others, as shown in bold. We also find that the spherical model can only perform well when the ground-truth model is the same, while the discrete head model and the continuous head model are more robust to
different data.

Figure 4: RMSE performance using C/C model pair and AMPF algorithm, (a) varying particle number, (b) particle number is 10000

In Figure 4 we show the detailed RMSE performance using the continuous head model and the proposed AMPF algorithm. For the initial particle numbers larger than 10000, the RMSE stays at the same level. This phenomenon is as expected, according to the individual dipole dynamic model, a dipole only moves within the triangular sub-plane.

4.1.2. Five known number of dipole example

Figure 5 shows the tracking results of five known dipoles using the continuous head model and the proposed algorithm. We can see from Figure 5(a) that the average RMSE over all five dipoles remains at the level of 5 mm. The box-plot ranges between 2-9 mm. In Figure 5(b) we show the changes of particle number of the 30 identical repeated experiments. The initial particle number $I_0 = 10000$. We can see that $I_k$ in most experiments dramatically decreases to around 1500 at $k = 4$. If we compare the two figures, one can observe that the proposed algorithm is able to achieve a good RMSE performance while adaptively eliminating the number of particles. This greatly saves the computational cost for multiple particle filter types of algorithms. The total particle number is $I_k$ multiplied by the number of dipoles.
Figure 5: 5 known dipole localisation using C/C model pair and AMPF algorithm, (a) average RMSE from AMPF (b) particle number over time (c) average RMSE from MPF (d) RMSE of an individual dipole in a single experiment

| Alg / Dipole | 1      | 2      | 3      | 4      | 5      | Processing Time |
|--------------|--------|--------|--------|--------|--------|-----------------|
| MPF          | 8.72   | 9.85   | 9.14   | 10.68  | 9.57   | 56 s            |
| AMPF         | 7.02   | 9.33   | 7.58   | 10.24  | 9.17   | 28 s            |

Table 2: Detailed RMSE table and processing time for MPF and AMPF algorithm with 10000 particles

From Figure 5 (a) and (c), we can see that the average RMSE over five dipoles is at the same level for both MPF and AMPF. This can also be seen in Table 2. The middle 5 columns are the dipole index in the five dipole localisation example. This shows the average RMSE for each dipole over 30
identical experiments. We implemented the algorithms in Matlab using a computer with intel Core i7 CPU @ 3.7 GHz. The processing time for an iteration (with 50 time steps) of the MPF algorithm is 56 seconds and that of AMPF is 28 seconds.

If we compare the average RMSE performance of PF and AMPF with 10000 particles in the three known dipoles case and the five known dipoles case, we can find that the average RMSE for the three dipole case is smaller than that of the five dipole case. From a series of experiments with known multiple dipoles, we found that the greater the number of dipoles, the lower the localisation accuracy.

4.2. Localization with the unknown number of dipoles

In real world applications, we are not able to obtain prior information of the dipole number, thus an example with an unknown dipole number needs to be tested to access the proposed model and algorithm. The algorithm is tested using two examples: an unknown three dipoles localisation problem and an unknown dynamic dipole numbers localisation. The dipole number in the latter example varies between three and five. The three unknown dipoles example is used to evaluate the performance of the proposed algorithm.

As we stated in Equation (8), the dipole number dynamic is modelled and estimated through $p(N_k|N_{k-1}, \mathcal{N}_k)$. In other words, the new estimated dipole number at time $k$ depends on the previous time step estimates $N_{k-1}$. For the initial dipole number at time $k = 0$ where there is no historical data available, we assign $N_k$ with the clustered MNE estimation result $\mathcal{N}_0$.

4.2.1. Evaluation of dipole number estimation

The new dipole number estimates for $k > 0$ highly depend on the estimation at its previous time step. In our HSC step, there are three candidate pairs $\{N_k^{(j)}, X_k^{(j)}\}$ where $j = (-), (0), (+)$ as stated previously. In this paper, we multiply the dipole number dynamic probability $p_k(\tau_j)$ from Equation (8) with the probability $p(N_k, X_k|Y_{1:k})$ computed in the HSC step as the candidate likelihood probability. We assign $p_k(\tau_0) = 0.5$, $p_k(\tau_-) = 0.25$ and $p_k(\tau_+) = 0.25$. The dipole number dynamic probability can be assigned using other designed schemes. Here we simply give the algorithm a prior knowledge (which comes from the observation using real examples) that the dipole number evolves slowly and it is more likely that the dipole number stays the same between two time steps. $N_k$ is assigned using the candidate pair with the highest normalised candidate likelihood probability.
Here we use a three unknown dipoles example to test the proposed approach. The example setting is the same as that in the known dipole example except that no prior dipole number knowledge is given to the algorithm. We apply both the MPF algorithm in paper [18] and the proposed AMPF algorithm to the example using a range of particle numbers. Figure 6 shows the histogram of the dipole number estimation. Each plot is accumulated from 30 identical experiments, each with 50 time steps. Figure 6(a) is the estimation result from the MPF algorithm in paper [18], since its estimation does not depend on the state estimates from the particle filter, we only show one with 10000 particles. We apply our HSC assisted AMPF algorithm with a range of particle numbers.

![Histograms of dipole number estimation](image)

Figure 6: Histogram of dipole number over 30 identical experiments using different number of particles. (a) MPF with 10000 particles (b) AMPF with 7000 particles (c) AMPF with 10000 particles (d) AMPF with 13000 particles (e) AMPF with 16000 particles (f) AMPF with 19000 particles

| AMPF Particle Number | 7000   | 10000  | 13000  | 16000  | 19000  | MPF 10000 |
|----------------------|--------|--------|--------|--------|--------|------------|
| Avg. Dipole Number   | 3.568  | 3.442  | 3.260  | 3.282  | 3.292  | 3.562      |

Table 3: Average estimated dipole number table

Table 3 shows the average estimated dipole number over 30 iterations. Since the ground-truth dipole number is 3, we can observe that with an
increase in the particle number from 7000 to 19000, the average estimated dipole number improves slightly. The numbers between AMPF 7000 and MPF 10000 are similar; however if we check Figure 6(a) and Figure 6(b) for their corresponding histogram distribution, we can see there are more successful estimates (dipole estimate equal to 3) in AMPF 7000 than MPF 10000. From the histogram we can also observe that, with an increase in the particle number using AMPF, the result does not improve much for particles larger than 13000.

4.2.2. Dynamic unknown number of dipole example

We use a dynamic particle number setting in this example. The ground-truth dipole number varies between 3 and 5 during the 50 time steps. The number of dipoles is shown in the 'True' row in Table 4. We use an AMPF algorithm with 10000 particles to perform the localisation task.

In Figure 7(a) we show the plot of the ground truth dipole number (the blue solid line) versus the average estimated dipole number (the red dashed line) over 50 time steps. The black dashed line is the estimate in the prior estimation step. Figure 7(a) shows the corresponding average RMSE for localisation. We discard the estimated dipole with the highest RMSE when the dipole number is different in some of the time steps. Amongst the 30 iterations, there are eight at which we lost track (we define the average RMSE larger than 30 mm as a lost track). The following calculations are based on the rest of the 22 iteration results. We can find that the estimated dipole number follows that of the prior estimation at the first time step. It evolves to catch the ground-truth with a delay afterwards. The average RMSE decreases from 13.5 mm to around 11 mm after the algorithm run.
Figure 7: (a) Unknown dipole number estimation results, average dipole number estimates using AMPF versus ground-truth dipole number for 50 time steps. (b) The corresponding average localisation RMSE

| Time k | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|--------|---|----|----|----|----|----|----|----|----|----|
| True   | 3 | 3  | 4  | 4  | 4  | 5  | 5  | 4  | 4  | 4  |
| ROIs   | 4.4| 4.6| 4.6| 4.9| 5.2| 5.8| 6.4| 4.9| 4.8| 4.8|
| Avg. $\tilde{N}_k$ | 3.7| 3.5| 3.9| 4.2| 4.1| 4.7| 5.0| 4.2| 4.1| 4.1|
| RMSE   | 11.40| 11.11| 10.88| 10.96| 11.25| 10.55| 10.87| 11.04| 10.74| 11.16|

Table 4: Comparison between the ground-truth dipole number and estimated dipole number (averaged over 30 iterations), ROIs is the number of ROIs estimated using prior estimation, Avg. $\tilde{N}_k$ is the average estimated dipole number after HSC step, the unit of row Avg. RMSE is in units [mm].

Table 4 shows the numerical results of the same example for every 5 time steps. Both the estimation of the dipole number and the dipole location are acceptable given the random walk noise around 3–4 mm. If we compare the result with that achieved by the known dipole number tracking, the localisation error increases, and the average RMSE is 2 times larger than that in the known dipole number tracking. This is as expected since we have no prior knowledge of the dipole number.

4.3. Real MEG data

In this section we show the algorithm performance using the real MEG data. The data was collected using both the 102 magnetometers and the 204 planar gradiometers. The data was epoched from $-100\text{ms}$ to $+400\text{ms}$ with
a baseline correction between $-100\text{ms}$ to $0\text{ms}$. The data was then sampled with $1000\text{ Hz}$ and processed by a low-pass filter with a cutoff frequency of $40\text{ Hz}$. In order to compensate for subject movement before epoching, a “MaxFilter” was applied in the data denoising step. The experiment used an auditory stimuli for both the left and right side of the human head and the data was averaged over more than $100\text{ events}$. The time step unit is one millisecond.

In practice, the first $100\text{ ms}$ data was processed and was treated as background noise. The noise component in the continuous head model was modified to be comparable to the background noise. The AMPF algorithm assisted by the prior estimation and the heuristic selection criterion was implemented in this real data event.

Again, we employed $I=10000$ particles for each iPF to run the AMPF algorithm. As shown in Figure 8, we plot both the standard MNE results and the results from the proposed algorithm together in the same graph. We show the localization performance from three view angles (top, right hemisphere and left hemisphere) in column one, two and three respectively. For the colorbar of MNE, deep red and blue areas identifies the active regions. In this example, there are seven ROIs identified at the prior detection step, the average number of ROIs over 30 repeated algorithm run is $7.3$.

In the Figure 8, the 12 graphs represent the algorithm performance during the period between $190\text{ ms}$ and $220\text{ ms}$. The estimated dipole number remains as five during this period in this example. The average estimated dipole number over 30 repeated algorithm run is $5.5$. As we can observe in the figure, most of the identified dipoles are located in the left hemisphere. Since it is difficult to obtain the ground-truth, we take the relative focal area in the MNE result as a reference. Figure 8(f) clearly portrays both the intensive areas from the MNE and the identified dipoles from the proposed algorithm. The algorithm fails to find one of the ROIs in the right hemisphere. However, the dipole localisation result is more focal than the estimation result from the standard MNE in the left hemisphere.
Figure 8: Real data dipole number estimation and dipole localisation results. The colorbar shows the minimum norm estimation result and the blue stars are the identified dipoles by AMPF algorithm.
The locations of the brain active areas evolve slowly as we can observe from this 30 ms length period. This observation matches our assumption in the modelling step that the dipolar sources may only move within a small cortical region and they can be treated as semi-static targets. The computational time for a single experiment (in a same computer as described before) is 148 seconds for the algorithm integrated with the adaptive filtering scheme, while it is 312 s for the same algorithm without the adaptive filtering. The average RMSE between the estimated locations and the ROIs locations for the five identified dipoles is 12.6 mm. This result shows the localisation difference between the MNE and the proposed algorithm.

5. Discussion and Conclusion

In this paper, we proposed a continuous real head model and an adaptive Bayesian SMC algorithm to perform multiple dipolar sources localisation using MEG data. The proposed algorithm integrated a prior estimation step and a heuristic selection criterion step to assist in the estimation of the dipole number and the adjustment of the particle number.

The dipole number estimation in the multiple dipole tracking problem remains a difficult problem as there is no prior knowledge that can be obtained. For point-wise particle filtering tracking, the dimensionality is another issue that needs to be considered. We adopted a multiple particle filtering scheme to avoid the dimensionality problem by assigning each target an individual particle filter. We also incorporated a prior estimation step using the minimum norm estimation method to provide the tracking step with prior information.

However, this method highly depends on the deterministic estimates from MNE and the historical data. It does not perform well when the dipole number changes over time. In order to address this problem, we modified the deterministic prior estimation step to a partly probabilistic estimation: we modified the dipole number dynamic model and allowed three potential dipole number guesses at each time step. The three candidate pairs are compared at the end of each time step and we employed a heuristic selection criterion method to select the pair with the highest likelihood. As we have shown in the results section, this approach achieves better localisation and dipole number estimation performance than the previously proposed algorithms.
In addition, we also proposed a new continuous real head model strictly constrained in the brain cortical area. This model provides an alternative way to achieve 1-layer real head modelling. The comparison result demonstrates that this model fits well with the discrete real head model. For data not generated using the pre-defined discrete head model, the continuous model achieves better performance in terms of dipole localisation. Due to the properties of the multiple particle filter, the computational cost is equal to that of a single particle filter multiplied by the number of identified dipoles. However, we found that it is not necessary to keep a large volume of particles for every time step when the tracking estimate is close enough to the ground-truth. We proposed an adaptive method to reduce the computational cost by diminishing particle number when the localisation error is negligible.

We are currently working on a fully probabilistic dipole number estimation method and we will incorporate it into our current localisation algorithm. As the brain current source is a continuous state space in reality, we will explore a better continuous modelling scheme to interpret the brain’s current source. Further research will also focus on decreasing the computational time. This may be addressed by using the parallel computing technique and by implementing the Rao–Blackwelisation methods in the linear part of the model.

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