Supernova Explosions in Accretion Disks in Active Galactic Nuclei: Three-dimensional Models

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Abstract

Supernova (SN) explosions can potentially affect the structure and evolution of circumnuclear disks in active galactic nuclei (AGN). Some previous studies have suggested that a relatively low rate of SN explosions can provide an effective value of alpha viscosity between 0.1 and 1 in AGN accretion disks within a 1 pc scale. In order to test this possibility, we provide some analytic scalings of the evolution of an SN remnant embedded in a differentially rotating smooth disk. We calibrate our estimates using three-dimensional hydrodynamical simulations where the gas is modeled as adiabatic with index γ. Our simulations are suited to include the fact that a fraction of the momentum injected by the SN escapes from the disk into the corona. Based on these results, we calculate the contribution of SN explosions to the effective alpha viscosity, denoted by α_{SNe}, in a model AGN accretion disk, where accretion is driven by the local viscosity α. We find that for AGN galaxies with a central black hole of \( \sim 10^8 M_\odot \) and a disk with viscosity \( \alpha = 0.1 \), the contribution of SN explosions may be as large as \( \alpha_{SNe} \lesssim 0.02 \), provided that \( \gamma \gtrsim 1.1 \). On the other hand, in the momentum conservation limit, which is valid when the push by the internal pressure of the SN remnant is negligible, we find \( \alpha_{SNe} \lesssim 6 \times 10^{-4} \).

Unified Astronomy Thesaurus concepts: Accretion (14); Galaxy accretion disks (562); Black hole physics (159); Hydrodynamics (1963); Active galaxies (17); Quasars (1319)

1. Introduction

It is well established that the luminosity of the active galactic nuclei (AGN) is the result of gas being accreted by the central supermassive black hole (SMBH). In order to reach the central SMBH, the interstellar medium (ISM) gas must be transported from galactic scales down to the last stable orbit. At galactic scales, gravitational torques produced by bars and arms can bring the gas into the central subkiloparsec region (e.g., Shlosman et al. 1989, 1990; Hopkins & Quataert 2011). At intermediate radii (scales 1–100 pc), the circumnuclear disk or torus is gravitationally unstable. As a result, part of the gas turns into stars. It has been suggested that energy feedback from supernovae (SNe) inside the circumnuclear disks may support a geometrically thick, turbulent circumnuclear disk (e.g., Wada & Norman 2002; Kawakatu & Wada 2008). Hobbs et al. (2011) suggested that feedback from stellar winds and SNe within a kiloparsec-scale disk can make the gas highly turbulent at “intermediate scales,” and this turbulence can promote accretion (see also Hopkins et al. 2012).

At parsec and subparsec scales from the nucleus, models predict the formation of a thin accretion disk surrounding the SMBH. Interestingly, reverberation mapping indicates that broad-line regions (BLRs) in AGN have a characteristic radial distance from 0.01 to 1 pc (e.g., Kaspi et al. 2007; Bentz et al. 2009; Du & Wang 2019), implying that BLRs overlap with the outer parts of the accretion disks. Understanding the interplay between the underlying accretion disk, the BLR, and the inner parts of the dusty torus is crucial in order to have a complete picture of the physical processes.

Models suggest that the accretion disk is gravitationally unstable between \( \sim 0.01 \) pc and a few parsecs from the SMBH; thus, it is likely to fragment into clouds leading to a star-forming disk (e.g., Paczynski 1978; Shlosman & Begelman 1987; Collin & Zahn 1999; Collin & Huré 2001; Goodman 2003). Vigorous star formation could raise material from the surface of the underlying disk, feeding the BLR with metal-rich gas (Wang et al. 2012; Czerny et al. 2016) if so, the BLR can trace the metallicity of the accretion disk (Wang et al. 2011).

Emission-line flux ratios have been used to estimate the BLR metallicity. It has been found that, in many quasars, the BLR metallicity is very high, with typical values of four to five times solar (e.g., Hamann & Ferland 1993; Baldwin et al. 2003; Dietrich et al. 2003; Warner et al. 2003; Nagao et al. 2006; Kurk et al. 2007; Jiang et al. 2007; Juarez et al. 2009). The almost constant BLRs over the redshift range suggest that such high metallicities were reached by rapid and intense chemical enrichment at the cores of the host galaxies. However, the fact that the metallicity of the host galaxies is generally lower than it is in BLRs indicates that the BLRs have been enriched locally within the star-forming disk, rather than by the stellar population of the host galaxy (Wang et al. 2011).

A major long-standing puzzle is to explain how gas can power the central SMBH if a significant fraction of the gas in the radial inflow is consumed in forming stars. The problem of the consumption of gas can be alleviated if the angular momentum transport in the disk is efficient, i.e., if the effective \( \alpha \) viscosity is \( \gtrsim 0.1 \) (Shlosman et al. 1989; Goodman 2003; Chen et al. 2009; Wang et al. 2010, 2011). Some authors have suggested that repeated SN explosions in the star-forming disk produce an effective viscosity \( \alpha \sim 0.1 \) (Rózycka et al. 1995; Collin & Zahn 1999, 2008; Chen et al. 2009; Wang et al. 2010, 2011). However, there is still no consensus on the rate of SN explosions in the accretion disk required to provide \( \alpha \sim 0.1 \). The most detailed study of the amount of angular momentum redistributed by a single SN explosion in the accretion disk is given in Rózycka et al. (1995). Based on two-
dimensional (2D) simulations, Różycka et al. (1995) showed that the blast wave driven by an SN explosion deflects the trajectory of disk gas elements, resulting in an outward angular momentum flux, because the mixing between elements that have acquired angular momentum and those that have lost angular momentum is small. They argued that an SN rate as low as $10^{-4}$ yr$^{-1}$ in a disk rotating around a black hole of $(10^8 - 10^9)M_\odot$ provides a radial flux of angular momentum corresponding to a viscosity parameter $\alpha \sim 0.1$.

Collin & Zahn (1999, 2008) built a steady-state accretion disk, including star formation feedback. They argued that the angular momentum redistributed by one SN is lower than the value derived by Różycka et al. (1995). Therefore, they required a larger number of SN explosions to have the same rate of angular momentum transport. The difference between the prescriptions of Różycka et al. (1995) and Collin & Zahn (1999, 2008) is related to the uncertainties on the amount of momentum that escapes from the disk carried by the outflowing gas when the SN remnant (SNR) breaks out of the disk.

In the present paper, we reconsider the angular momentum transport in the AGN accretion disks provided by SN explosions. In Section 2, we describe the physical model of the disk. Section 3 gives general insight into the evolution of SNRs in accretion disks around SMBHs. In particular, we provide estimates about conditions for the breakout of the disk and give scaling laws for the radial width of the cavity opened by a single SN explosion and the redistribution of angular momentum. In Section 4, we present the results of three-dimensional (3D) simulations, which take into account that some fraction of the mass, energy, and momentum can be carried outside the disk by the vertical outflow induced by the explosion. In Section 5, we apply our results to estimate the effective $\alpha$ viscosity induced by SN explosions. Finally, a summary of the main conclusions is given in Section 6.

### 2. Model and Disk Parameters

We consider an accretion disk in the gravitational field of a central SMBH with mass $M_{BH}$ and Schwarzschild radius $R_\text{Sch}$. In the outer regions of accretion disks, i.e., beyond $10^3R_\text{Sch}$, the dust sublimes and the opacity drops. Consequently, these outer regions are expected to be gravitationally unstable, and fragmentation of the disk into clouds seems unavoidable (e.g., Goodman 2003; Rafikov 2009; Jiang & Goodman 2011). It is usually assumed that feedback from newly formed stars can keep the outer parts of accretion disks marginally stable, so that the Toomre $Q$-parameter remains close to 1, inhibiting further star formation (e.g., Collin & Zahn 1999; Gammie 2001; Goodman 2003; Sikirko & Goodman 2003, hereafter SG; Thompson et al. 2005, hereafter TQM; Rafikov 2009; Begelman & Silk 2017). Radiation pressure from massive stars and from the accretion of gas onto stellar black holes in the disk, momentum injection from SN explosions, thermal pressure, and magnetic fields can provide the support necessary to maintain $Q \approx 1$.

There is a wide variety of plausible theoretical models to describe the structure of quasi-stationary self-gravitating accretion disks. They differ on the assumed speed of the radial inflow, the adopted disk opacities, and the importance of the magnetic support. For instance, in the SG model, accretion is driven by local turbulent viscosity, which is parameterized by the Shakura–Sunyaev viscosity parameter $\alpha$ (Shakura & Sunyaev 1973). It was presumed by TQM that a global torque could drive a larger inflow speed than local viscosity. As a result, at distances from the SMBH between $10^2R_\text{Sch}$ and $10^5R_\text{Sch}$, the disk surface density and thickness are smaller in the TQM model than they are in the SG model (a comparison between the outcomes of these models is given in Figure 1 in Bellovary et al. 2016, assuming plausible parameters). Other models suggest that magnetic fields play an important role in the gas dynamics. For example, in magnetically elevated disks, the inflow, which is carried by low-density gas at large heights, is driven mainly by stress due to the large-scale component of the magnetic field (Mishra et al. 2020).

The angular momentum transport due to local turbulence by magnetorotational instabilities (MRIs) and gravitational instabilities (GIs) can be accommodated in the $\alpha$ model. The effect of SN explosions on the disk can also contribute to enhancing the effective viscosity, which can also be described by a viscosity parameter $\alpha_{\text{SN}}$. The $\alpha$ parameter will be the sum of all of the above contributions,

$$\alpha = \alpha_{\text{MRI}} + \alpha_{\text{GI}} + \alpha_{\text{SN}} + \alpha_{\text{others}},$$

where $\alpha_{\text{others}}$ represents the viscosity due to other potential sources, e.g., star–disk collisions (Pariev et al. 2003). The viscosity coefficients on the right-hand side of Equation (1) are not independent. In the present work, we estimate the contribution of SN explosions to the angular momentum transport, i.e., $\alpha_{\text{SN}}$, in a disk model where the local viscosity $\alpha$ drives the inflow. Self-consistent models require $\alpha_{\text{SN}} \lesssim \alpha$. In those models where $\alpha_{\text{SN}} \sim \alpha$, SN explosions by themselves can account for the rate of angular momentum transfer.

In the SG model, accretion is driven by local viscosity. In this model, the inflow mass rate $\dot{M}_{\text{acc}}$ is constant with radius and assumed to be $\dot{M}_{\text{Edd}}L_{\text{Edd}}/(\eta c_s^3)$, where $\dot{M}_{\text{Edd}}$ is the Eddington ratio and $\eta$ is the radiative efficiency. In terms of the dimensionless parameters $\alpha_{0.1} = \alpha/0.1$ and $\xi = L_{\text{Edd}}/(\alpha_{0.1}\eta c_s^3)$, it can be written as

$$\dot{M}_{\text{acc}} = 2.2\alpha_{0.1}f_{\text{M}}M_8 M_{\odot} \text{yr}^{-1},$$

where $M_8 = M_{BH}/(10^8 M_{\odot})$. Typical ranges for the values of these parameters are $L_{\text{Edd}} = 0.1 - 0.5$, $\eta_{0.1} \approx 1$, and $\alpha_{0.1} = 0.1 - 3$.

Between a radius $\sim 10^3R_\text{Sch}$ and the outer radius $10^5R_\text{Sch}$, Equation (2), together with the condition $Q = Q_{\text{m}}$ (where $Q_{\text{m}} \approx 1$), determines the surface density $\Sigma(R)$, scale height $H(R)$, midplane total pressure $p_0(R)$, and effective sound speed $c_s(R)$ (including thermal, radiation, and turbulent pressure). More specifically, the Toomre parameter in a Keplerian disk can be expressed as

$$Q = \frac{H\Theta^2}{\pi G\Sigma} = \frac{\Omega^2}{2\pi^3 G\rho_0},$$

where $\Theta(R)$ is the angular velocity of the disk. Note that in the second equality of Equation (3), we have used a midplane density in a vertically isothermal disk of $\rho_0 = \Sigma/(\sqrt{2\pi} H)$. The condition $Q \approx Q_{\text{m}}$ implies $H = \pi Q_{\text{m}} G\Sigma/\Omega^2$, $c_s = \Omega H = \pi Q_{\text{m}} G\Sigma/\Theta$, and

$$\rho_0 = \frac{\Omega^2}{2\pi^3 Q_{\text{m}} G}.$$
By imposing the inflow mass rate given in Equation (2),
the SG model predicts
\[
\Sigma(R) = 1.6 \times 10^8 \xi^{1/3} Q_m^{-2/3} M_8^{-2/3} R_3^{-1/3} M_8 \, \text{pc}^{-2},
\]
\[
H(R) = 4.6 \times 10^{-5} \xi^{1/3} Q_m^{1/3} M_8^{4/3} R_3^{3/2} \, \text{pc},
\]
and
\[
c_5 = 32 \xi^{1/3} Q_m^{1/3} M_8^{1/3} \, \text{km s}^{-1},
\]
where \( R_3 = R/(10^3 R_{\text{Sch}}) \). The aspect ratio of the disk,
\[ h(R) = H/R, \] is
\[
4.6 \times 10^{-5} \xi^{1/3} Q_m^{1/3} M_8^{1/3} R_3^{1/2}.
\]
For \( Q_m = 1, M_8 = 1, \) and \( \xi = 0.3, \) the disk is very thin,
with an aspect ratio \( h = 3.1 \times 10^{-3} \) at \( R_3 = 1 \). If we take \( \xi = 5 \) as
in SG, \( h = 8 \times 10^{-3} \) at \( R_3 = 1 \), in agreement with the value plotted
in Figure 2 in SG. These values for the aspect ratio are
intermediate between those found in the TQM model
(\( h \sim 10^{-3} \) at \( R_3 \sim 1 \)) and in magnetically elevated disks
(\( h \sim 0.05 \); Mishra et al. 2020).

The scale height given in Equation (6) includes all of
the components that provide support to the disk: the thermal
pressure \( (p_{th}) \), radiation pressure \( (p_{rad}) \), and turbulent pressure \( (p_{tu}) \).
The latter is thought to be mostly produced by random
motions in the gas excited by SN explosions and stellar winds.
Following TQM (see Appendix A for details), we have
computed the different contributions \( (p_{th}, p_{rad}, \) and \( p_{tu}) \) to
the total pressure. The approach of TQM rests on the
assumption that star formation is the main feedback
mechanism. In that scenario, the radiation-to-turbulent pressure ratio
is \( \approx \tau/2 \), where \( \tau \) is the midpoint optical depth. Figure 1 shows
the pressure distribution in a disk with \( Q_m = 1, \alpha_{0.1} = 1, \)
\( \xi = 0.3, \) and \( M_8 = 1. \) In this figure, we can see that the inner
region \( (R_3 \lesssim 2.5) \) is dominated by thermal pressure, an
intermediate region dominated by radiation pressure
\( (2.5 \lesssim R_3 \lesssim 15) \), and a region dominated by turbulent pressure
at \( 15 \lesssim R_3 \lesssim 100 \).

As noted by TQM, a potential problem of disk models driven
by local viscosity is that the star formation rate is so large that it
consumes almost all of the gas, and little gas is left to fuel the
central black hole (see also Section 5.2). However, there are
many uncertainties regarding the feedback. In a scenario where
irradiation of the disk can also occur from the accretion of gas
onto intermediate-mass black holes embedded in the disk
(Dittmann & Miller 2020), the ratio of radiation to turbulent
pressure could be much larger than \( \tau/2 \). The SG disk is
considered a plausible model and has been adopted to study the
migration of stellar-mass black holes within AGN disks
(Mckernan et al. 2012; Bellovary et al. 2016; Secunda et al.
2019; Yang et al. 2020).

The discussion throughout this paper is highly specific to the
canonical model of SG (Equations (6)–(7)). But, since we
provide analytical formulae for calculating the impact of SN
explosions on the disk, the interested reader can apply them to
other disk models.

3. SNe in AGN Accretion Disks: Evolution of a Single
Explosion

For simplicity, we consider the evolution of an SN that
exploses in the midplane of the accretion disk, at a radial
distance \( R_{\text{SNe}} \) from the SMBH, in the range
\( 10^3 R_{\text{Sch}} \lesssim R_{\text{SNe}} \lesssim 10^8 R_{\text{Sch}} \). The explosion has an energy \( E_{\text{SNe}} \)
and a mass ejecta \( M_{\text{SNe}} \). The momentum associated with this
energy and mass is \( p_{\text{SNe}} = (2M_{\text{SNe}} E_{\text{SNe}})^{1/2} \). The SN explosion
will drive a shock in the disk and carve a cavity of low-density
gas surrounded by a shell of swept material. As a first approach
to the evolution of an SN explosion in the disk, we do not
include the physics of radiation pressure, radiative transport,
and self-gravity. Although our hydrodynamical approach is
clearly simplistic, we believe that it captures much of the
physics of interest.

Given that the problem is multidimensional, it is useful to
have analytical estimates of the key quantities. In this section,
we make predictions regarding the radial extent of the cavity
and the redistribution of the angular momentum of the disk by
a single SN explosion in an initially unperturbed disk with a
surface density and half-thickness as given in Equations (5)
and (6). As will become clear in Section 5, we need these quantities
to characterize the contribution of SN explosions to the
effective viscosity coefficient.

3.1. First Stage: Free Expansion Phase

An initial free expansion phase will take place until a mass
comparable to the SN ejecta has been swept up. Let \( \Delta \) denote
the radius of a cylinder perpendicular to the disk at the position
\( R_{\text{SNe}} \) that contains a mass \( M_{\text{SNe}} \). By its definition,
\( \Delta^2 \equiv M_{\text{SNe}}/(\pi \Sigma) \), where the tilde over a quantity indicates
evaluation at \( R_{\text{SNe}} \), i.e., \( \tilde{\Sigma} \equiv \Sigma(R_{\text{SNe}}) \). In terms of \( \tilde{H}, \) it is
\[
\Delta = \tilde{H} = \xi^{-1/2} M_8^{-1/2} R_3^{-1/2},
\]
where \( \tilde{R}_3 = R_{\text{SNe}}/(10^3 R_{\text{Sch}}) \) and \( \tilde{M}_0 = M_{\text{SNe}}/(10 M_8) \). Thus, for
\[
\tilde{R}_3 > \xi^{-2/3} M_8^{-4/3} \tilde{M}_0^{2/3},
\]
we have \( \Delta \lesssim \tilde{H} \). This condition implies that when the SNR
has a radius \( \sim \tilde{H} \), the SNR has swept an amount of matter comparable
to or larger than the SN ejecta. In particular, for
\( M_8 \geq 1, \tilde{M}_0 = 1, \) and \( \xi = 0.3, \) this condition is met if the SN
explosion occurs at a radius \( \tilde{R}_3 \) larger than 2.2.
3.2. Conditions for Breakout of the Disk

A fraction of the momentum and energy released by the SN vents into the corona if the SNR can break out of the disk. In this subsection, we consider the conditions in which breakout of the disk occurs. For simplicity, we will implicitly assume that $\Delta > H$. Kompaneets (1960) studied the propagation of the shock wave in a plane-parallel stratified medium. It was shown that if the explosion occurs at the midplane, the SNR is prolate in shape because the expansion velocity of the shell will be higher in the vertical direction than in the radial direction. Let us denote $Z_{sh}(t)$ as the vertical distance of the topmost point of the shell (i.e., the semimajor axis in the $z$-direction of the prolate SNR) and $R_{sh}(t)$ as the radius of the SNR in the $z = 0$ plane. Here $Z_{sh}$ decreases with $z$ up to a certain height $z_b$, and then it is reaccelerated (e.g., Kompaneets 1960; mac Low et al. 1989; Ferrara & Tolstoy 2000). The SN explosion will break out of the disk if $v_b > \hat{c}_s$, where $v_b$ is the velocity of the shock $Z_{sh}$ at $z_b$ (e.g., Ferrara & Tolstoy 2000).

In order to calculate $v_b$, we consider two limiting scenarios. In scenario A, we assume that the SNR reaches the reaccelerating height $z_b$ with negligible radiative cooling. Thus, we may use the Kompaneets approximation to evaluate the $z$-component of the velocity of the topmost point of the shell as

$$Z_{sh}(z) \approx \frac{\rho_{SNR}/(\rho c^2)}{2^{1/2}}$$

where $\rho_{SNR} = \rho_0 \exp(-z^2/(2H^2))$ (e.g., Ferrara & Tolstoy 2000; Olano 2009). Scenario B assumes that cooling is already important at the end of the free expansion phase so that the pressure interior to the shell is negligible. In such a case, momentum conservation implies $Z_{sh}(z) \approx 3\rho_{SNR}/(4\pi c^2)$, again if $Z_{sh} < z_b$. Other possibilities that may be considered more realistic, such as a pressure-modified snowplow phase, lie between these two limiting scenarios.

Assuming that the disk has a vertical Gaussian profile, we find $z_b = \sqrt[3]{3H}$ in both scenarios. In scenario A and for our disk model (Equations (5) and (6)), we get

$$v_b = 5.5 \times 10^5 \xi^{-1/3} M_8^{-1/2} E_{51}^{1/2} R_{sh}^{3/4} \text{km s}^{-1},$$

where $E_{51} = E_{SNR}/(10^{51} \text{erg})$. The breakout condition $v_b > \hat{c}_s$ implies that the SNR can punch a hole in the disk if the explosion occurs within a radius $R_{sh}^{(A)}$, given by

$$R_{sh}^{(A)} \equiv 1.0 \times 10^3 \xi^{-10/9} M_8^{14/9} E_{51}^{2/3}.$$  

For the reference values $\xi = 0.3$ and $E_{51} = 1$, we find that $R_{sh}^{(A)} \geq 20$ if $M_8 \leq 30$. In particular, for $M_8 = 1$, we obtain that $R_{sh}^{(A)} = 3.8 \times 10^3$, which is much larger than the outer edge of the accretion disk.

In scenario B, we get

$$v_b \approx 4.5 \times 10^3 \xi^{-1} M_8^{-2} \hat{M}_{10}^{1/2} E_{51}^{1/2} R_{sh}^{3/2} \text{km s}^{-1}.$$  

The breakout condition is fulfilled if

$$R_{sh} \leq R_b^{(B)} \equiv 30 \xi^{-8/9} M_8^{-14/9} \hat{M}_{10}^{1/3} E_{51}^{1/3}.$$  

For $\xi = 0.3$ and $M_8 = 1$, an SN explosion with $E_{51} = \hat{M}_{10} = 1$ is capable of breaking out of the disk in the range of interest $1 \leq R_{sh} \leq 100$. On the contrary, if $M_8 \geq 18$, SN explosions with $E_{51} = \hat{M}_{10} = 1$ will never break out of the disk at $R_{sh} > 1$.

3.3. Radial Width of the SNR

In the $z = 0$ plane, the otherwise circular SNR will be deformed by the Coriolis forces in the first stage, when the expansion velocity of the SNR $R_{sh}$ is still much larger than the shear velocity. In the second stage, when $R_{sh}$ becomes comparable to the shear velocity, the SNR will become elongated along the azimuthal direction, resembling an ellipse in shape, due mainly to differential rotation induced by the ram pressure with the ambient medium (Olof 1982; Tenorio-Tagle & Palous 1987; Palous et al. 1990; Silich 1992; Różycka et al. 1995). In the final stage, shear dominates, and the radial width of the cavity $W$ may decrease over time (e.g., Tenorio-Tagle & Palous 1987). In this section, we evaluate the maximum radial width of the SNR, denoted by $W_{\text{max}}$, in scenario A. An analog derivation but for scenario B can be found in Appendix C.

Let us denote $W_{\text{max}}^{\text{trans}}$ as the width of the SNR when the shock velocity drops to a value similar to the effective sound speed of the external medium (i.e., the shock velocity becomes transonic) and $W_{\text{max}}^{\text{shear}}$ as the radial width of the SNR when the shock velocity is comparable to the shear velocity $\approx (3/4)\Omega R_{sh}$, where $\Omega$ is the angular velocity. The maximum width of the SNR will be given by $W_{\text{max}} = \min(W_{\text{max}}^{\text{trans}}, W_{\text{max}}^{\text{shear}})$.

The expansion velocity $R_{sh}$ of the SNR depends on the thermodynamics of the gas and the fraction of momentum that is transferred into the disk. In order to compute $R_{sh}$, we will consider the adiabatic phase and the momentum-driven snowplow phase. Therefore, we implicitly assume that a mass of gas larger than the SN ejecta has been swept up. As a consequence, our results are valid only if $\rho_{\text{sh}} W_{\text{sh}} > 8 M_{SNR}$, i.e., $W_{\text{max}} > W_{\text{lim}} \equiv (2 M_{SNR}/\rho_0)^{1/3}$.

Consider first the case where the SN explosion occurs at a distance $R_{sh} < R_b^{(A)}$ from the SMBH. If so, the SN can make a hole in the disk. At short times after the explosion, before breakout of the disk, when $R_{sh} + Z_{sh} \ll H$, the shell is spherical because the SNR evolves as if the medium were homogeneous, so that $R_{sh} = Z_{sh} \approx (2/5)(E_{SNR}/[\rho_0 R_{sh}^2])^{1/2}$. The interior pressure of the SNR continues pushing the shell in both the vertical and radial directions while the SNR is embedded within the disk, i.e., while $R_{sh} \sim Z_{sh} \ll H$. At some point, the flow accelerates in the vertical direction, leading to a rapid depressurization of the cavity. This phase of depressurization starts when $R_{sh} \approx H$ and $Z_{sh} \approx \sqrt{3}H$ (see Section 3.2). We recall here that $H$ is the scale height of the disk prior to breakout, which includes the vertical support provided by the turbulent pressure. Anticipating what we observe in the simulations, the SNR forms an overdense ringlike structure in the disk with expansion velocity $R_{sh}$. If we assume that after depressurization of the cavity, the ringlike SNR enters a snowplow phase, momentum conservation may be expressed as

$$R_{sh}^2 R_{sh} \approx \chi_4 H^2 R_{sh}(H),$$  

where $\chi_4$ is a dimensionless correction factor to account for the push by the interior pressure during the pressure-driven phase. Equation (15) encapsulates the fact that the interior pressure of the cavity can push the shell for a longer time if the disk (prior to breakout) is thick, imparting more momentum to the shell than in thinner disks (e.g., Tenorio-Tagle & Palous 1987).
From the Kompaneets approximation, we have $R_{sh}(H) \approx 0.4(E_{SNec}/[p_0H^2])^{1/2}$. Using this value in Equation (15), we can derive $R_{sh}$. The transonic condition implies

$$W_{A,\text{trans}}^{\text{max}} \approx 1.2 \left( \frac{x_A E_{SNec} \tilde{H}}{\rho_0 \tilde{c}_s^2} \right)^{1/4}. \quad (16)$$

In our disk model,

$$W_{A,\text{trans}}^{\text{max}} \approx 7.5 \times 10^{-4} \chi_A^{1/2} \xi^{1/12} \frac{Q_m^1}{E_{51}} \frac{M_8^{2/3}}{R_{51}^{9/8}} \text{pc.} \quad (17)$$

The condition $R_{sh} \approx (3/4) \tilde{H} \tilde{R}_{sh}$ leads to

$$W_{A,\text{shear}}^{\text{max}} \approx 1.6 \left( \frac{x_A E_{SNec} \tilde{H}}{\rho_0 \tilde{c}_s^2} \right)^{1/6}. \quad (18)$$

The dependence of $W_{A,\text{shear}}^{\text{max}}$ on $E_{SNec}$, $\rho_0$, and $\tilde{H}$ is weak. This agrees with the fitting formula found in Palous et al. (1990) for the SNR minor axis in the $z = 0$ plane. Using a model in 1.5 dimensions that describes the propagation of the shell from a strong explosion in a rotating disk, they found that the semiminor axis is proportional to $E_{SNec} \rho_0^{-0.22} R_{0.1}^1$ (their Equation (4)). In terms of our independent variables and for our disk model, Equation (18) becomes

$$W_{A,\text{shear}}^{\text{max}} \approx 4 \times 10^{-4} \chi_A^{1/3} \xi^{1/18} Q_m^{3/9} M_8^{8/9} E_{51}^{1/6} R_{51}^{5/4} \text{pc.} \quad (19)$$

If the explosion occurs at $\tilde{R}_3 > \tilde{R}_{sh}^{(A)}$, the disk is not perforated. Thus, the shell is confined to the disk. When the explosion occurs at the midplane, Kompaneets’ approximation implies that $R_{sh} \approx 0.4(E_{SNec}/[\rho_0 \tilde{R}_{sh}^4])^{1/2}$ if $Z_{sh} \leq Z_b = \sqrt{3} \tilde{H}$ (see Figure 2 in Olano 2009). The transonic condition $R_{sh} \approx \tilde{c}_s$ leads to

$$W_{A,\text{trans}}^{\text{max}} \approx \left( \frac{E_{SNec}}{\rho_0 \tilde{c}_s^2} \right)^{1/3}. \quad (20)$$

For our accretion disk model (Equations (5)–(7)), we can recast $W_{A,\text{trans}}^{\text{max}}$ in terms of $M_8$ and $\tilde{R}_3$ as

$$W_{A,\text{trans}}^{\text{max}} \approx 1.5 \times 10^{-3} \chi_A^{2/9} \xi^{1/9} Q_m^{3/9} E_{51}^{1/3} M_8^{4/9} R_{3} \text{pc.} \quad (21)$$

On other hand, if the radial expansion of the SNR is limited by shear, the condition $R_{sh} \approx (3/4) \tilde{H} \tilde{R}_{sh}$ implies

$$W_{A,\text{shear}}^{\text{max}} \approx 0.8 \left( \frac{E_{SNec}}{\rho_0 \tilde{c}_s^2} \right)^{1/5}. \quad (22)$$

For our disk model,

$$W_{A,\text{shear}}^{\text{max}} \approx 6 \times 10^{-4} Q_m^{1/5} M_8^{4/5} E_{51}^{1/5} \tilde{R}_3 \tilde{c}_s^{6/5} \text{pc.} \quad (23)$$

The corresponding values of $W_{max}$ in scenario B are given in Appendix C (see Equations (C9)–(C10)). Figure 2 compares $W_{max}$ in scenarios A and B, assuming $Q_m = 1$, $M_8 = 1$, and $E_{51} = 1$. The values of the width $W_{max}$ are larger than $\tilde{H}$ because of the SNR breakout of the disk for the range of values of $\tilde{R}_3$ under consideration. Since the SNR in scenario B evolves as a pure momentum-driven snowplow, the values of $W_{B,\text{trans}}^{\text{max}}$ and $W_{B,\text{shear}}^{\text{max}}$ are smaller than the corresponding values in scenario A. We should note that we have assumed $\chi_A = \chi_B = 1$ for convenience.

Figure 3 shows the same quantities as Figure 2 but for $M_8 = 100$. The SNRs formed within the range $10^3 R_{Sch}$ and $10^5 R_{Sch}$ and $E_{51} = 1$ do not have not enough energy to break out of the disk. In addition, we see that the curve of $W_{max}$ is very close to the curve for $W_{lim}$. This indicates that the values of $W_{max}$ should be taken with caution. In scenarios A and B, the values of $W_{max}$ are similar. In fact, the curves for scenarios A and B overlap.

There have been other attempts to estimate $W_{max}$ in the literature. Collin & Zahn (1999) suggested that for SN explosions powerful enough to produce breakout of the disk, the momentum transferred to the disk is $P_{SNe} W_{max}/(2\tilde{H})$. As a
result, they found
\[ W^{\text{CZ,shear}}_{\text{max}} \approx 1.1 \left( \frac{P_{\text{SNe}}}{\pi \rho_0 \Omega} \right)^{1/4}. \]  
(24)

For the parameters of Figure 2, \( W^{\text{CZ,shear}}_{\text{max}} \) is roughly a factor of 2–3 smaller than \( W^{B}_{\text{max}} \). This implies that we would need \( \chi_B \approx 0.1 \) to have a match between both estimates. Simulations in 3D can shed light on the appropriate value for the fraction of momentum that is absorbed by the disk.

### 3.4. Angular Momentum Redistribution in the Disk by SN Explosions

The angular momentum of the disk may be redistributed by SN explosions. As discussed in Różycka et al. (1995), disk gas elements that enter the shock facing the inner disk lose angular momentum, whereas gas elements that cross the shock facing the outer disk gain angular momentum. The net effect is a transport of angular momentum radially outward. Denote \( \mathcal{J}_{\text{SNe}} \) as the total amount of angular momentum that is transported from the inner disk to the outer disk by just one SN explosion. Różycka et al. (1995) showed that \( \mathcal{J}_{\text{SNe}} \) is given by
\[ \mathcal{J}_{\text{SNe}} = \mu \bar{R}_{\text{sh}} \bar{R}^2_{\text{sh}} R_{\text{SNe}}, \]  
(25)

where \( R_{\text{sh}} \) and \( \bar{R}_{\text{sh}} \) should be evaluated once the SNR has broken out of the disk (see Section 4.2 in Różycka et al. 1995). In Equation (25), we have introduced the fudge factor \( \mu \) to be fixed through our numerical simulations in Section 4. This factor may depend on the scenario, so we will refer to them as \( \mu_A \) and \( \mu_B \).

In scenario A, we can use Equations (15) and (25), plus the disk scaling laws in Equations (5) and (6), to obtain
\[ \mathcal{J}^{(A)}_{\text{SNe}} = 0.6 \beta_A (E_{\text{SNe}} \Sigma)^{1/2} \mathcal{H} R_{\text{SNe}} = 75 \beta_A \varepsilon^{1/2} E_{\text{SNe}}^{1/2} M_r^2 R_{\text{sh}}^{7/4}, \]  
(26)

where \( \beta \equiv \mu \chi \), and \( \mathcal{J}^{(A)}_{\text{SNe}} \) is in units of \( \dot{M}_\odot \) pc km \(^{-1}\) s \(^{-1}\).

In scenario B, momentum conservation implies \( \bar{R}_{\text{sh}} R_{\text{sh}}^2 = \beta_R P_{\text{SNe}} / (\pi \Sigma) \) (see Equation (C6)). Thereby, we find
\[ \mathcal{J}^{(B)}_{\text{SNe}} = \frac{\beta_R}{\pi} P_{\text{SNe}} R_{\text{SNe}} = 100 \beta_R M_r \varepsilon^{1/2} E_{\text{SNe}}^{1/2} \bar{R}_{\text{sh}}, \]  
(27)

again in units of \( \dot{M}_\odot \) pc km \(^{-1}\) s \(^{-1}\). The next section is devoted to simulating the 3D evolution of an SNR in a disk, and inferences of \( \beta \) will be provided.

### 4. Simulations

Our 3D simulations of the evolution of an SN in a disk in Keplerian rotation were performed using the code FARGO3D\(^5\) (Benítez-Llambay & Masset 2016) in a spherical coordinate system \((r, \theta, \phi)\). Magnetic fields and self-gravity of the disk were ignored.

We placed the site of the SN explosion at the midplane of the disk. Given the symmetry of the problem, we simulated only the upper half of the disk. We chose a system of reference that rotates with the angular velocity at \( R_{\text{SNe}} \), so that the explosion site does not change over time. We took \( M_r = 1 \) and \( \bar{R}_r = 20 \), which corresponds to \( R_{\text{SNe}} = 0.2 \) pc. In this model, the circular velocity of a test particle with orbital radius \( R_{\text{SNe}} \) is 1467 km s \(^{-1}\), and its orbital period \( P_{\text{sh}} \) is 838 yr.

The initial surface density is given in Equation (5). At \( R_{\text{SNe}} \), it is \( 1.2 \times 10^5 \rho_0 \) pc \(^{-2}\). The initial vertical profile of the density was derived by assuming that the temperature of the gas is independent of \( \theta \) and imposing hydrostatic equilibrium with an aspect ratio \( h = 0.01375 (R/R_{\text{SNe}})^{1/2} \). Thus, the isothermal sound speed is 21.4 km s \(^{-1}\), constant along the disk.

The mass ejecta is \( M_{\text{SNe}} = 10 M_\odot \), and the explosion has an energy \( E_{\text{SNe}} = 2 \times 10^{51} \text{erg} \). As we are modeling only half of the disk, we deposit \( 5M_\odot \) and \( 10^{51} \text{erg} \) in our domain. At \( t = 0 \), this mass and energy is injected by increasing the density and thermal energy of the gas into a region with a radius of 6.5 \times 10^{-3} \text{pc} \). The equation of energy is solved by assuming the equation of ideal gas \( p = (\gamma - 1) e \), where \( p \) is the gas pressure and \( e \) is the internal energy density. For the adiabatic index, we take \( \gamma = 5/3 \) (model 1) and 1.1 (model 2).

The azimuthal angle ranges from zero to \( \pi/2 \). Hence, we only simulate an octant of the disk. The explosion center is placed at \( \phi = \pi/4 \). The latitude, \( \pi/2 - \theta \), ranges from zero (midplane of the disk) to 6.2 \( h \) (in radians), where \( h \) is the aspect ratio of the disk at the explosion center. In the radial direction, the domain extends from \( r_{\text{in}} = 0.15 \) to \( r_{\text{out}} = 0.25 \) pc. In the upper tap of the disk, we employ open boundary conditions. Damping boundary conditions for the radial component of the velocity have been used at \( r_{\text{in}} \) and \( r_{\text{out}} \) (de Val-Borro et al. 2006). The number of zones in each direction is \( N_r = 768 \), \( N_\theta = 128 \), and \( N_\phi = 2304 \).

#### 4.1. Evolution of the SNR

We will first focus on model 1. Model 2 will be discussed at the end of this section. Since model 1 is adiabatic with \( \gamma = 5/3 \), we will use scenario A, which ignores cooling, to make predictions.

For the parameters in model 1, the condition for breakout of the disk (Equation (12)) is satisfied. Figure 4 shows vertical

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\(^5\) The code is publicly available at \texttt{http://fargo.in2p3.fr}.

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![Figure 4](image-url)
cuts of the volume density along the radial direction, passing through the explosion center, in model 1. At \( t = 20.9 \) yr, the disk is in the process of breakout; the blast wave in the \( z \)-direction is just leaving the disk. The shape of the SNR in this vertical plane is prolate (\( \tilde{R}_H = 2.7H \) and \( \tilde{Z}_H = 4.85H \) at \( t = 20.9 \) yr) because the \( z \)-direction presents less resistance. Before the breakout of the disk, the shell in these cuts appears as convex arcs, but after breakout, these arcs become concave. Some of the vented gas escapes from our computational domain. Before presenting a more quantitative analysis of the mass lost through the boundaries of our computational domain, we will look at the evolution of the SNR along different cuts.

The evolution of the volume density in the midplane of the disk is shown in Figure 5. The gas disk rotates counterclockwise. At \( t \leq 20 \) yr, the SNR along a cut through the \( z = 0 \) plane is almost circular, though it is a bit elongated due to the Coriolis force. At \( t = 41.91 \) yr, the SNR has an ellipsoidal shape with an axis ratio of \( \sim 0.72 \). The angle between the major axis of the ellipse and the radial direction is \( \pm 45^\circ \). Due to the differential rotation of the accretion disk, the shear continues stretching the SNR in the azimuthal direction (see panel at \( t = 209.58 \) yr). At this time, the SNR presents a banana-like shape. Between \( t = 209.6 \) (0.25\( P_{\text{orb}} \)) and 335.3 (0.4\( P_{\text{orb}} \)) yr, the major axis of the SNR continues growing, while its minor axis (width along the radial direction) barely changes.

The overall evolution of the shape of the SNR in the disk midplane is similar to that described in Tenorio-Tagle & Palous (1987) in the context of formation of holes in galactic \( H \) disks by the explosion of multiple SNe in evolved OB associations. In their study, they used a 1.5-dimensional model of the SNR and assumed a flat rotation curve (see also Palous et al. 1990). The morphology of the SNR is also similar to the results of Różycka et al. (1995), who made 2D simulations of an SN explosion in a Keplerian disk.

To gain a more physical insight into the evolution of the SNR, Figure 6 shows the volume gas density at the midplane of the disk along a radial cut that passes through the explosion center, i.e., for \( \phi = \pi / 4 \). As expected, at the very center of the SNR, the material has been evacuated efficiently. The maximum radial width of the SNR is \( W_{\text{max}} = 0.026 \) pc or, in terms of the local scale height, \( W_{\text{max}} = 9.5H \). Between 10.2 and 83.8 yr, the peaks in density, which correspond to the position of the shell, are equally spaced, implying an effective radial expansion velocity in kilometers per second,

\[
\dot{W} = 5 \times 10^3 \exp \left( -\frac{W}{0.005} \right),
\]

where \( W \) is in parsecs. The effective velocity \( \dot{W} / 2 \) is supersonic at \( t \leq 100 \) yr. We also see that during the first 100 yr after the explosion, the SNR is able to keep the cavity clean of material,
indicating that the SNR is able to deflect the disk gas entering the SNR.

After \( t \approx 100 \text{ yr} \), the behavior of the SNR changes. Because \( W \) increases very slowly, the expansion of the SNR in the radial direction has almost stalled at \( t = 167 \text{ yr} \) (see Figure 6). After \( t = 167 \text{ yr} \), the density in the cavity starts to increase. The reason is that the disk gas can penetrate into the cavity. In fact, as time goes on, the shock waves weaken and the pitch angle decreases. As a result, the deflection of the disk gas that enters the SNR is much more moderate. The elements inside the cavity are accelerated inward due to the pressure gradient that tries to refill the cavity (see Figure 7). Indeed, after 100 yr, the central cavity is depressurized, and the evolution follows a momentum conservation phase. The disk velocity field at 335.3 yr after the explosion is shown in Figure 8. In this "passive" phase, the SNR only grows along the azimuthal direction.

Since our simulations are 3D, we have information about the redistribution of mass in the vertical direction and the amount of mass that is vented into the corona. Figure 9 shows the mass that has been lost from our computational domain at a given time. The mass-loss rate is approximately constant between \( t = 40 \) and 170 yr. Mass expulsion from our domain is halted at 200 yr after the explosion. Approximately 300\( M_\odot \) is lost through the upper boundary.

Now consider the mass that is contained below a height of \( 3H \) from the midplane, i.e., at \( z < 3H \) (see also Figure 9). We see that the SN explosion is cleaning up material from \( z < 3H \) during the first \( \sim 250 \text{ yr} \) after the explosion. It is interesting to note that a significant fraction of the mass that has been evacuated from regions \( z < 3H \) remains in the region \( 3H < z < 6.2H \). For instance, at \( t = 250 \text{ yr} \), the mass evacuated from \( z < 3H \) is \( \sim 1000M_\odot \), and 70% of this mass remains at altitudes between \( 3H \) and 6.2H. Figure 10 illustrates how the azimuthally averaged volume density, \( \langle \rho \rangle_{\phi} \), is distributed in the upper parts of model 1 at two different times.

The velocity of expansion of the SNR depends on the adopted adiabatic index \( \gamma \). The smaller the value of \( \gamma \), the lower the internal pressure that pushes outward on the shell. Because cooling in the accretion disk is efficient, values of \( \gamma \) close to 1 are thought to be more realistic. In order to quantify the dependence of the flow pattern on the adiabatic index, we carried out a simulation with the same parameters as model 1 but using \( \gamma = 1.1 \) (model 2).

Figure 11 shows cuts of the density through the center of the SN explosion along the radial direction in model 2, whereas Figure 12 shows cuts of the pressure along the azimuthal direction. The SNR acquires a maximum width along the radial direction of \( W_{\text{max}} \approx 0.02 \text{ pc} \), which is a factor of 1.3 smaller than in model 1. The expansion velocity of the SNR in the azimuthal direction is also slower than in model 1. The amount of mass that is carried outside the region \( z < 3H \) by the SNR is a factor of \( \sim 3 \) smaller in model 2 than it is in model 1. The amount of angular momentum that is redistributed by the SNR in models 1 and 2.

**Figure 6.** Volume density at the midplane of the disk as a function of radius \( r \) in the direction of the explosion center \( (\phi = \pi/4) \) at different times in model 1.

**Figure 7.** Cross sections of the gas pressure along the azimuthal direction at \( z = 0 \) and \( R = R_{\text{SN}} \) in model 1. Here \( p_0 \) is the unperturbed pressure. Note that 6° are equivalent to a distance of 0.021 pc.
4.2. Determining $\chi$ and $\beta$

In Sections 3.3 and 3.4, we have introduced some dimensionless factors when deriving the scaling laws that obey $W_{\text{max}}$ and $\mathcal{J}_{\text{SNe}}$ on theoretical grounds. These factors, which are expected to be of the order of unity, can be measured in our simulations.

We have computed $\chi_A$ in our simulations as follows. From Equation (15), the momentum imparted to the disk, $\tilde{p}_{SRR} \Sigma_{sh} R_{sh}$, is 

$$0.4\pi \chi_A \Sigma^{1/2} (E_{\text{SNe}}/\rho_0 R^{3/2})^{1/2} = 2\chi_A H (E_{\text{SNe}} \Sigma)^{1/2}.$$ 

Therefore, we can measure $\chi_A$ in our simulations as

$$\chi_A \simeq \frac{1}{H (E_{\text{SNe}} \Sigma)^{1/2}} \int_{z<3H} \rho \, \delta v \, d^2 r,$$

(29)

where $\delta v = \sqrt{\delta v_x^2 + \delta v_y^2}$ is the planar component of the perturbed velocity field $\delta v = v - v_0$, with $v_0$ as the unperturbed Keplerian velocity of the disk. In model 1, we have computed $\chi_A$ at $t = 41.9$ yr, when the low-density cavity produced by the explosion is still circular in the $z = 0$ plane (see Figure 5), and at $t = 83.8$ yr, when the SNR has evacuated a significant mass of the disk. We find $\chi_A \simeq 0.7$ at both times.

Once we know $\chi_A$, we can evaluate the predicted maximum width of the SNR in scenario A as described in Section 3.3. From Equations (16) and (18) with $\chi_A = 0.7$, $\xi = 0.3$, and $E_{51} = 2$, we find $W_{\text{max}}^{A,\text{trans}} = 0.02$ and $W_{\text{max}}^{A,\text{shear}} = 0.011$ pc. The maximum width measured in model 1 is 0.026 pc. Therefore, our $W_{\text{max}}^{A,\text{shear}}$ underestimates the width by a factor of $\sim 2$. Interestingly, for $\chi_A = 1$, $W_{\text{max}}^{A,\text{trans}} = 0.024$ pc, which is close to the value obtained from our simulations.

A value of $\chi_A \simeq 0.7$ is probably more adequate for simulations with a lower value of $\gamma$. In order to test this idea,
we carried out a simulation with all the same parameters except $E_7 = 78$ and $\gamma = 1.4$. We measured $W_{\text{max}} = 0.04 \text{ pc}$ in our simulation, which agrees with $W_{\text{max,trans}}$. On the other hand, $W_{\text{max,shear}}$ still underestimates the width by a factor of 2.

In the following, we evaluate $\mathcal{J}_{\text{SNe}}$, that is, the amount of angular momentum transported outward across a circle of radius $R_{\text{SNe}}$. More specifically, we give $\beta_A$ (see Section 3.4), which is related to $\mathcal{J}_{\text{SNe}}$ by

$$\beta_A = \frac{\mathcal{J}_{\text{SNe}}}{0.6(E_{\text{SNe}}/\Sigma)^{1/2} R_{\text{SNe}}}$$

(see Equation (26)). In order to determine $\beta_A$, we have measured $\mathcal{J}_{\text{SNe}}(t)$ in our simulations. The remainder of the variables in Equation (30) are known input parameters. Since we are simulating the upper half of the disk, we include a factor of 2 in the calculation of $\mathcal{J}_{\text{SNe}}$. The resultant $\beta_A$ values in models 1 and 2 are shown in Figure 14. We see that $\beta_A$ increases over time following a power law. In an inviscid disk, the value of $\beta_A$ is expected to converge asymptotically to a constant value at large times. The 2D simulations indicate that for the parameters used in these simulations, the angular momentum ceases to drift outward $\sim 1P_{\text{orb}}$ after the explosion. Consistent with this result, we find that, in our 3D simulations, the rms azimuthal component of perturbed velocity $\delta v_\phi$ in the shell decays with a characteristic timescale of $0.7P_{\text{orb}}$. Using the power-law fits shown in Figure 14, we evaluate the value of $\beta_A$ at $t = 838 \text{ yr}$ to obtain $\beta_A \approx 6$ in model 1 and $\beta_A \approx 4$ in model 2. We will use these estimates of $\beta_A$ in Section 5 when we study the effect of repeated SN explosions in the accretion disk.

To illustrate how the angular momentum transport occurs in the midplane of the disk, we calculate

$$a_{\text{inst}} = \frac{\rho \nu_R \delta v_\phi}{p}$$

(31)

where $\rho \nu_R \delta v_\phi$ is the Reynolds stress, $p$ is the gas pressure, and $\langle \cdots \rangle$ indicates averaging over $\phi$. Figure 15 shows the $\langle \alpha_{\text{inst}} \rangle$ coefficient at the midplane for models 1 and 2 at two different times. Integrated over $R$, $\langle \alpha_{\text{inst}} \rangle$ is positive, signifying that the net transport of angular momentum is outward. We also observe that $\langle \alpha_{\text{inst}} \rangle$ at $R = R_{\text{SNe}} = 0.2 \text{ pc}$ decreases over time.

### 5. Repeated SN Explosions: The Viscosity Parameter $\alpha$ in Steady State

In the previous sections, we have calculated the amount of angular momentum that a single SN explosion can transport outward. The effect of many SN explosions can be represented as an effective viscosity $\alpha_{\text{SNe}}$. As stated in Section 2, other agents besides SN explosions may be at work in AGN accretion disks that contribute to the effective viscosity of the disk $\alpha$. The redistribution of angular momentum by SN explosions will be the major contributor to the viscosity if $\alpha_{\text{SNe}} \approx \alpha$. Given the rate of SN explosions per unit area in the disk $\rho_{\text{SNe}}$, we can evaluate $\alpha_{\text{SNe}}$. This will be done in the following sections.
5.1. Contribution of SN Explosions to $\alpha$

We can always express the redistribution of angular momentum through a radial flux $F_j(R, t)$. We will use the convention that $F_j(R, t) > 0$ implies that the transport of angular momentum is outward. If this flux was carried by density waves in a nondissipative medium, then the angular momentum can be transported to infinity without deposition into the disk. However, in our case, the redistribution of angular momentum occurs at scales of the order of $W$. Following Goodman & Rafikov (2001), we may write the flux driven by SN explosions as

$$F_j(R) = 2\pi \int R S_n e \theta S_n e J S_n e \phi (R - R S_n e) d R S_n e,$$  

(32)

where $\phi(x)$ is a dimensionless distribution function with thickness $\sim 2W_{\text{max}}$ and $\phi(0) = 1$; it accounts for the damping length for the deposition of angular momentum. We may approximate the flux as

$$F_j(R) \approx 4\pi \theta S_n e W_{\text{max}} R J S_n e.$$  

(33)

The expected rate of SN explosions can be estimated as $\dot{\theta}_{S_n e} = f_{S_n e} \Sigma_*$, where $\Sigma_*$ is the star formation rate per unit area, and $f_{S_n e}$ is the number of massive stars per solar mass of the star formation. To estimate $f_{S_n e}$, we assume a Salpeter initial mass function with a low-mass cutoff at $0.1M_{\odot}$. We have $f_{S_n e} = 0.01M_{\odot}^{-1}$.

In order to estimate the viscosity parameter, we consider that the angular momentum flux in a steady-state Keplerian disk has the form

$$F_j(R) = 3\pi \alpha_{\text{SNe}} c_s H \Sigma R^2 \Omega.$$  

(34)

By equating Equations (33) and (34), we find

$$\alpha_{\text{SNe}} \simeq \frac{4}{3} f_{\text{SNe}} \left( \frac{W_{\text{max}}}{R} \right) \left( \frac{\Sigma_*}{\Sigma} \right) \left( \frac{J_{\text{SNe}}}{c_s^2} \right).$$  

(35)

In the next section, we estimate $\alpha_{\text{SNe}}$ in the SG model. To do so, we need the star formation rate $\Sigma_*$.  

### 5.2. Application to the SG Model

Assuming that stellar feedback is the main agent to provide vertical support to the disk, we can infer the required star formation rate per unit area, denoted by $\Sigma_*^{\text{exp}}$ (see Appendix A). Figure 16 shows $\Sigma_*^{\text{exp}}$ for our fiducial parameters. As discussed in TQM, the star formation rate has a bump where the opacity is low (in the "opacity gap"). In our case, the disk becomes optically thin at $R_1 > 10$.

As anticipated in Section 2 and discussed in detail in TQM, a large $\Sigma_*^{\text{exp}}$ makes it difficult to fuel the central SMBH, as star formation may halt gas accretion onto the central SMBH (starvation). More specifically, star formation produces starvation if

$$M_{\text{acc}} \leq 2\pi (1 + f_{\text{SNe}} M_{\text{exp}}) \int_{R_{\text{min}}}^{R_{\text{max}}} \Sigma_* R dR,$$  

(36)

where $R_{\text{min}} = 10^3 R_{\text{Sch}}, R_{\text{max}} = 10^7 R_{\text{Sch}},$ and $M_{\text{exp}}$ is the mass expelled from the disk to the corona by one SN explosion.

Consider the phenomenological Kennicutt–Schmidt law $\Sigma = C_{\text{KS}} \Sigma_{7/5}$, where $\Sigma$ is in units of $M_{\odot} \text{ pc}^{-2}$ and $\Sigma_*$ is in $M_{\odot} \text{ pc}^{-2} \text{ yr}^{-1}$ (Kennicutt 1998; Chen et al. 2009). Using Equations (2), (5), and (36), it is easy to show that in order to avoid starvation, we need $C_{\text{KS}} < C_{\text{cr}}$, where

$$C_{\text{cr}} = 1.5 \times 10^{-10} \alpha_{0.1} f^{0.4}_{\text{SNe}} \xi^{0.93} \Omega^{-0.53} M_{\odot}^{-0.07},$$  

(37)

with $f_{\text{SNe}} \equiv 1 + f_{\text{SNe}} M_{\text{exp}}$.

In Figure 16, we can compare $\Sigma_*^{\text{cr}} \equiv C_{\text{cr}} \Sigma_{7/5}^{7/5}$ and $\Sigma_*^{\text{exp}}$. We clearly see that $\Sigma_*^{\text{cr}}$ is not enough to provide support to the disk, and additional sources of feedback/heating are required.
Substituting Equation (26) into Equation (35) and generously assuming \( \Sigma_\star = \Sigma_{\star, \text{eq}} \), we get the upper value

\[
\alpha_{\text{SNe}}^{(a)} \simeq 1 \times 10^{-3} \frac{\rho_{\text{SNe}}}{M_{\star}} \frac{E_{\text{SNe}}^{0.5}}{R_3^{1.5}} \left( \frac{W_{\text{max}}}{R} \right) \tag{38}
\]

in scenario A. In the above equation, we have used \( f_{\text{SNe}} = 0.01 M_{\odot}^{-1} \).

On the other hand, combining Equation (27) and Equation (35), we arrive at the equation

\[
\alpha_{\text{SNe}}^{(B)} \simeq 1.3 \times 10^{-3} \beta_\alpha \frac{\rho_{\text{SNe}}}{M_{\star}} \frac{E_{\text{SNe}}^{1/2}}{R_3^{2/5}} \left( \frac{W_{\text{max}}}{R} \right) \tag{39}
\]

in scenario B. We recall that the equations above are valid as long as \( \alpha_{\text{SNe}} \ll \alpha \).

We see that the upper values for \( \alpha_{\text{SNe}}^{(a)} \) and \( \alpha_{\text{SNe}}^{(B)} \) do not explicitly depend on the precise value adopted for \( Q_\text{m} \), but they depend weakly on \( Q_\text{m} \) through \( W_{\text{max}} \) (see, e.g., Equations (17), (19), (C9), and (C10)).

Now we evaluate \( \alpha_{\text{SNe}}^{(a)} \) for our reference values \( \alpha = 1 \), \( Q_\text{m} = 1 \), \( \xi = 0.3 \), \( E_{51} = 2 \), and \( \gamma = 5/3 \). For these parameters, we have found in Section 4 that \( \beta_\alpha \simeq 6 \), \( M_{\exp} = 300 M_\odot \), and \( W_{\text{max}} \sim 0.1 \) pc. For these values, Equation (38) implies \( \alpha_{\text{SNe}}^{(a)} \simeq 0.02 \); therefore, SN explosions may contribute up to \( \sim 20\% \) to the effective viscosity in this model.

For \( \gamma = 1.1 \), \( \alpha = 1 \), \( M_8 = 1 \), \( \xi = 0.3 \), \( E_{51} = 2 \), \( \beta_\alpha \simeq 4 \), \( M_{\exp} = 100 M_\odot \), and \( W_{\text{max}} \sim 0.075 \) pc, we find a similar value of \( \alpha_{\text{SNe}}^{(a)} \simeq 0.02 \).

Interestingly, \( \alpha_{\text{SNe}}^{(a)} \) increases with \( M_8 \). More specifically, since \( W_{\text{max}}^{\text{shear}} \propto M_8^{8/9} \) (Equation (19)) and \( R \propto M_8 R_3 \), Equation (38) predicts \( \alpha_{\text{SNe}}^{(a)} \propto M_8^{8/9} \). Therefore, \( \alpha_{\text{SNe}}^{(a)} \approx \alpha \) if \( M_8 \approx 5 \).

In scenario B, we do not have an estimate of \( \beta_\alpha \), but certainly \( \beta_\alpha \ll 1 \) (see Appendix C). If we generously take \( \beta_\alpha = 8 \), \( M_8 = 1 \), \( E_{51} = 2 \), \( M_{\exp} = 100 M_\odot \), and \( W_{\text{max}} \sim 0.075 \) pc, and \( \xi = 0.3 \), we find a similar value of \( \alpha_{\text{SNe}}^{(B)} \approx 6 \times 10^{-4} \). This small value of \( \alpha_{\text{SNe}}^{(B)} \) compared to \( \alpha \) means that SN explosions are not efficient enough to drive the effective viscosity in this model, and other sources need to be invoked.

In scenario B, the effective \( \alpha_{\text{SNe}} \) viscosity depends weakly on \( M_8 \). For illustration, consider a disk around a black hole with \( M_8 \ll 1 \). In this case, we are in the limit \( \Delta \gg H \) (Appendix B). From Equation (B6) with \( \chi \propto M_8 \), we have \( W_{\text{max}} \propto M_8^{8/9} \). Since \( R \propto M_8 R_3 \), we get \( \alpha_{\text{SNe}}^{(B)} \propto (W_{\text{max}}/R) \propto M_8^{1/9} \).

Our result that \( \alpha_{\text{SNe}}^{(B)} \approx 6 \times 10^{-4} \) for \( \xi = 0.3 \), \( M_8 = 1 \), and an SN rate of \( 8 \times 10^{-2} \) yr\(^{-1} \) contrasts with the findings of Różycka et al. (1995) that \( \alpha_{\text{SNe}} \approx 0.1 \) for \( M_8 = 1 \) and an SN rate of \( 10^{-4} \) yr\(^{-1} \) in scenario B. Różycka et al. (1995) overestimated \( \alpha_{\text{SNe}} \) because they implicitly assumed that the flow carries the angular momentum without any dissipation. A finite damping length leads to a smaller effective viscosity, since angular momentum is deposited in the disk.

### 6. Summary and Conclusions

We have studied the role of SN explosions on the density structure and angular momentum redistribution in AGN accretion disks within a 1 pc scale. In our models, the AGN accretion disk properties are taken from the accretion disk model derived by SG. An SN explosion drives a shock that sweeps up mass, forms a cavity in the disk, and redistributes disk angular momentum. We have provided some analytical estimates of the width of the cavity and the redistribution of angular momentum induced by a single SN explosion. We have introduced some fudge factors to include deviations from our simple approximations. By means of 3D hydrodynamical simulations, which take into account the lost mass and the momentum carried by the outflow, we have calibrated these fudge factors.

The radial width of the cavity, the mass ejected from the disk, and the amount of angular momentum that is redistributed in the disk (\( J_{\text{SNe}} \)) by a single SN explosion all depend upon the value adopted for the adiabatic index \( \gamma \) of the gas. As a reference number, for \( \gamma = 1.1 \), we find \( J_{\text{SNe}} \approx 8 \times 10^{60} \text{ g cm}^{-2} \text{ s}^{-1} \) for an SN explosion at a radius of 0.2 pc in our disk model with \( M_8 = 1 \).

We have estimated the effective \( \alpha_{\text{SNe}} \) viscosity provided by SN explosions in a steady state, where the rate of SN explosions is determined by adopting a Kennicutt–Schmidt law for the star formation. For \( \gamma \) between 1.1 and 5/3, we find that \( \alpha_{\text{SNe}} \gtrsim 0.1 \) if \( M_8 \gtrsim 5 \).

Some authors adopted the momentum conservation limit to infer \( \alpha_{\text{SNe}} \) (e.g., Różycka et al. 1995; Collin & Zahn 1999). In this limit, which is relevant when cooling is already important in the final stages of the free expansion phase, we find \( \alpha_{\text{SNe}} \approx 6 \times 10^{-3} \). Therefore, the contribution of SN explosions to the effective viscosity is negligible.

We have assumed that SN explosions occur in a smooth disk. However, unless the stellar heating is effective, the disk may fragment into clouds through GIs (e.g., Jiang & Goodman 2011). If SN explosions occur inside dense clouds, part of their initial momentum will be absorbed by their natal clouds. Therefore, our estimates of \( \alpha_{\text{SNe}} \) should be treated as upper limits.

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### Appendix A

#### Pressure Distribution and Star Formation Rate

Following TQM, we can make predictions about the relative importance of thermal, radiation, and turbulent pressure and their dependence with distance. We can also calculate the star formation rate per unit area. The equation of vertical equilibrium (Equation (39) in TQM) becomes

\[
\rho_0 c_s^2 = p_0 + \Sigma_{\text{sup}} \left( \frac{T}{2} + \lambda \right),
\]

where \( E \) is the efficiency with which star formation converts rest mass into radiation, and \( \lambda \) is a dimensionless parameter that measures the amount of the momentum injected by stars that is converted into turbulent motions (\( c \) is the light speed). To compute \( p_0 \), we evaluate the gas temperature \( T \) through

\[
T^4 = \frac{3}{4} T_{\text{eff}}^4 \left( \frac{\tau}{3} + \frac{4}{3} \right),
\]

where \( T_{\text{eff}} \) is the effective temperature of the star.
where $\tau = \kappa \Sigma / 2$, $\kappa$ is the disk opacity, and

$$
\sigma_{SB} T_{\text{eff}}^4 = \frac{1}{2} \epsilon_{\Sigma_{c}^{\text{sup}}} e^2 + \frac{3}{8\pi} M_{\text{acc}} \Omega^2.
$$

(A3)

Here $\sigma_{SB}$ is the Stefan–Boltzmann constant. Assuming $\epsilon = 10^{-3}$, $\lambda = 1$, and the disk opacities given in TQM, which are based on Semenov et al. (2003), we can derive $\Sigma_{c}^{\text{sup}}$, $p_{\text{th}}$, $p_{\text{rad}}$ and $p_{\text{tur}}$ in our disk model. We note that $p_{\text{rad}} = \epsilon \sigma_{SB} T_{\text{eff}}^4 c^2/2$ and $p_{\text{tur}} = \lambda \epsilon \Sigma_{c}^{\text{sup}} c$. For easy comparison, we have taken the same values for $\epsilon$ and $\lambda$ as TQM, but we warn that they are uncertain.

Appendix B

**Maximum Radial Width of the SNR in the Limit $\Delta \gg \hat{H}$**

As pointed out in Section 3.1, in some cases (especially for $M_8 < 1$), the SN explosion can break out of the disk in the free expansion phase. In cases in which $\Delta \gg \hat{H}$, this free expansion phase continues until $R_{\text{sh}} \approx \Delta$. We assume that a momentum conservation phase begins just after the free expansion phase ends. Only the momentum of those elements ejected in a polar angle $\hat{\theta}$ between $\hat{\theta}_{\text{min}} \approx \arccos(\hat{H}/(\sqrt{\Delta^2 + \hat{H}^2}))$ and $\hat{\theta}_{\text{max}} \approx \pi - \arccos(\hat{H}/(\sqrt{\Delta^2 + \hat{H}^2}))$ will be able to push the disk along the planar directions. Thus, the momentum absorbed by the disk will be

$$
P_{\text{abs}} \approx P_{\text{SNe}} \frac{\hat{H}}{4\pi} \int_{\hat{\theta}_{\text{min}}}^{\hat{\theta}_{\text{max}}} \sin^2 \hat{\theta} d\hat{\theta} = P_{\text{SNe}} \frac{\hat{H}}{\sqrt{\Delta^2 + \hat{H}^2}}.
$$

(B1)

It is easy to show that the mass that escapes from the disk is $\approx 0.5 \pi \Sigma \hat{R}_{\text{sh}}$. Momentum conservation implies that $R_{\text{sh}}$ obeys

$$
\pi \Sigma \left(R_{\text{sh}}^2 - \frac{\Delta^2}{2}\right) \hat{R}_{\text{sh}} = \chi P_{\text{SNe}},
$$

(B2)

with $\chi' = \hat{H}/\sqrt{\Delta^2 + \hat{H}^2}$. The transonic condition (see Section 3.3) occurs when the width of the SNR in the radial direction is

$$
W_{\text{max}}^{\text{trans}} \approx 2 \left(\frac{\chi P_{\text{SNe}}}{\pi \Sigma \hat{c}_s} + \frac{\Delta}{2}\right)^{1/2}.
$$

(B3)

On the other hand, $W_{\text{max}}^{\text{shear}}$ satisfies a cubic equation. Here we just provide a lower limit:

$$
W_{\text{max}}^{\text{shear}} > 1.5 \left(\frac{\chi P_{\text{SNe}}}{\Sigma \hat{c}_s}\right)^{1/3}.
$$

(B4)

We notice that $W_{\text{max}}^{\text{trans}}$ and $W_{\text{max}}^{\text{shear}}$ do not depend explicitly on $E_{\text{SNe}}$ because the adiabatic phase never develops.

In our disk model (Equations (5)–(7)), approximating $\chi' \approx \hat{H}/\Delta$, the widths read

$$
W_{\text{max}}^{\text{trans}} \approx 10^{-3} \xi^{-1/2} Q_{m}^{1/4} M_{8}^{2/3} M_{10}^{1/2} E_{51}^{1/4} R_{3}^{9/8}
\times (1 + 4 \times 10^{-3} \xi^{-1/2} Q_{m}^{1/4} M_{8} M_{10}^{2/3} R_{3}^{-3/4})^{1/2} \text{ pc}
$$

(B5)

and

$$
W_{\text{max}}^{\text{shear}} > 4.5 \times 10^{-4} \chi^{1/3} \xi^{-1/9}
\times Q_{m}^{2/3} M_{8}^{5/9} M_{10}^{1/6} E_{51}^{1/6} R_{3} \text{ pc}.
$$

(B6)

Appendix C

**Maximum Radial Width of the SNR in Scenario B**

Scenario B assumes that the internal pressure in the cavity is negligible, and, therefore, the SNR evolves as a pure momentum-driven snowplow. We will distinguish between cases where the SN explosion occurs within the radius $R_{b}^{(B)}$ and beyond $R_{b}^{(B)}$.

Case $R_{3} > R_{b}^{(B)}$. Since the SNR cannot break out of the disk, it will hardly reach a height larger than $\approx \sqrt{3} \hat{H}$. Momentum conservation dictates

$$
P_{\text{SNe}} \approx \frac{4\pi}{3} \hat{R}_{\text{sh}}^3 \hat{R}_{\text{sh}}.
$$

(C1)

The transonic and shear conditions imply

$$
W_{\text{max}}^{\text{trans}} \approx 1.2 \left(\frac{P_{\text{SNe}}}{\hat{c}_s^3}\right)^{1/3}
$$

(C2)

and

$$
W_{\text{max}}^{\text{shear}} \approx 1.5 \left(\frac{P_{\text{SNe}}}{\hat{c}_s^3}\right)^{1/4}.
$$

(C3)

In terms of $M_8$ and $R_{3}$,

$$
W_{\text{max}}^{\text{trans}} = 5 \times 10^{-4} \xi^{-1/9} Q_{m}^{2/3} M_{8}^{5/9} M_{10}^{1/6} E_{51}^{1/6} R_{3} \text{ pc}
$$

(C4)

and

$$
W_{\text{max}}^{\text{shear}} = 2 \times 10^{-4} Q_{m}^{1/4} M_{8}^{3/4} M_{10}^{1/8}
\times E_{51}^{1/8} R_{3}^{9/8} \text{ pc}.
$$

(C5)

Case $R_{3} < R_{b}^{(B)}$. If the explosion site lies at a radius less than $R_{b}^{(B)}$, the SNR is able to break out of the disk. After breakout, i.e., when $R_{\text{sh}} > \sqrt{3} \hat{H}$, the shock velocity is given by

$$
\chi_{B} P_{\text{SNe}} = \pi R_{3}^2 \Sigma \hat{c}_s.
$$

(C6)

Here $\chi_{B}$ is the fraction of momentum that is absorbed by the disk; thus, $\chi_{B} \leq 1$. Imposing the transonic and shear conditions, it follows that

$$
W_{\text{max}}^{\text{trans}} \approx \left(\frac{\chi_{B} P_{\text{SNe}}}{\Sigma \hat{c}_s}\right)^{1/2}
$$

(C7)

and

$$
W_{\text{max}}^{\text{shear}} \approx 1.5 \left(\frac{\chi_{B} P_{\text{SNe}}}{\Sigma \hat{c}_s}\right)^{1/3}.
$$

(C8)

For our accretion disk model (Equations (5)–(7)),

$$
W_{\text{max}}^{\text{trans}} \approx 10^{-3} \chi_{B}^{1/2} \xi^{-1/3} Q_{m}^{1/6}
\times M_{8}^{4/9} M_{10}^{1/6} E_{51}^{1/6} R_{3}^{9/4} \text{ pc}
$$

(C9)
and

\[
W_{\text{max}}^{\text{shear}} \simeq 4.5 \times 10^{-4} \chi_B^{1/3} \epsilon^{-1/9} Q_m^{2/9} \\
\times M_8^{5/6} \dot{M}_1^{1/6} E_1^{1/6} \tilde{R}_3 \text{ pc.}
\] (C10)

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