Point particle in general background fields
and generalized equivalence principle

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Abstract

The model of point particle in general external fields is considered and the generalized equivalence principle is suggested identifying all backgrounds which give rise to equivalent particle dynamics. The equivalence transformations for external fields are interpreted as gauge ones. The gauge group appears to be a semidirect product of all phase space canonical transformations to an abelian ideal of "hyper-Weyl" transformations and includes $U(1)$ and general coordinate symmetries as a subgroup. The implications of this gauge symmetry are considered and a connection of general backgrounds to the infinite collection of Fronsdal gauge fields is studied. Although the result is negative and no direct connection arises, it is discussed how higher spin fields could be found among general external fields if one relaxes somehow the equivalence principle. Besides, the particle action in general backgrounds is shown to reproduce the De Wit-Freedman point particle – symmetric tensors first order interaction suggested many years ago, and generalizes their result to all orders in interaction.

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1 Introduction.

Efim Samoilovich Fradkin was a brilliant scientist who had been always interested in the most important directions of theoretical physics and made many significant contributions to modern high energy physics.

One of the topics he studied in depth is the higher spin problem which presents itself the issue of constructing consistent interacting lagrangian theories of massless fields of arbitrary spin and, particularly, of coupling the arbitrary spin massless fields to gravity.

Generally speaking, the free higher spin models are gauge theories constructed in terms of symmetric tensors, so the higher spin problem is intimately related to the construction of the interacting theory of symmetric tensor gauge fields (including gravity). In the present paper, we try to put forward a new approach to the problem which uses the covariance of point particle – external fields interaction to provide the full nonlinearized gauge transformations for exterior fields being a collection of symmetric tensors $H = \{H(x), H^m(x), H^{m_1m_2}(x), \ldots, H^{m_1\ldots m_s}(x), \ldots\}$ (every rank appears once), the low 0,1,2-rank tensors correspond to low spin dilaton, electromagnetic and gravitational fields. Despite the overall result will be negative in the sense the gauge transformations appear to be too restrictive to associate $H$'s with higher spin gauge fields, our study provides the hope the connection to higher spin fields may be found after some modifications of gauge transformations or fields content.

Before passing to the main text we recall basic features of of higher spin theories and provide a brief historical survey. This will give us additional grounds in favour of our program. (The reader may skip this part of the text and pass directly to Sec. (2) where the program starts).

The lagrangians models of free arbitrary spin massless fields were constructed by Fronsdal and Fang and Fronsdal in $4D$ Minkowski and anti-de Sitter spaces, and then reformulated in the "gauge form" by Vasiliev and, independently, by Aragone and Deser for fermion case. Later on, the actions for arbitrary spin massless fields in flat $d$-dimensional space were built by Labastida, and for symmetric Young tableaux bosons in $d$-dimensional $AdS$ space by Lopatin and Vasiliev and for fermions by Vasiliev. E.g., the theories of totally symmetric spin-$s$ boson fields in Minkowski space of any dimensions are formulated in terms of symmetric rank-$s$ tensor fields $\varphi_{m_1\ldots m_s}(x)$ satisfying the $double$-$tracelessness$ constraint

$$\varphi_{kl}^{m_5\ldots m_s} = 0,$$

where the contraction is performed by the background metric. For $s = 0,1,2,3$ this constraint is satisfied automatically.

For $s \geq 1$, the action is unambiguously determined by the requirement of absence of higher derivatives and invariance w.r.t. gauge transformations (the round brackets stand for symmetrization)

$$\delta \varphi_{m_1\ldots m_s} = \partial_{(m_1} \varepsilon_{m_2\ldots m_s)}$$

whit $traceless$ gauge parameter:

$$\varepsilon_{km_3\ldots m_{s-1}} = 0.$$

For $s = 1,2$, this constraint is automatically fulfilled.
The invariant action reads
\[
A_s[\varphi_s] = \frac{1}{2} (-)^s \int d^d x \left\{ \partial_n \varphi_{m_1 \ldots m_s} \partial^n \varphi^{m_1 \ldots m_s} - \frac{1}{2} s (s - 1) \partial_n \varphi^k_{km_1 \ldots m_{s-2}} \partial_m \varphi^k_{m_1 \ldots m_{s-2}} + s (s - 1) \partial_n \varphi^k_{km_1 \ldots m_{s-2}} \partial^l \varphi^{lm_1 \ldots m_{s-1}} - \frac{1}{4} s (s - 1) (s - 2) \partial_n \varphi^k_{knm_1 \ldots m_{s-3}} \partial^l \varphi^{lm_1 \ldots m_{s-3}} \right\}.
\] (4)

For \( s = 0 \) there are no gauge parameters at all, i.e. a theory is not a gauged one. Nevertheless, the formulas (4) make sense in this case and provide the massless scalar theory. Though the actions (4) were originally constructed in 4D [1] they do not contain explicitly the dimension \( d \) of spacetime and describe consistently massless higher spin dynamics for any \( d \). We shall refer to them as to Fronsdal actions.

It is not easy to preserve the gauge symmetries (2) upon introducing interaction. Particularly, already the minimal coupling to gravity is inconsistent as it breaks the linearized gauge invariance by terms proportional to the full Riemann curvature: \( \delta A_s \sim \int d^d x \varphi R \varepsilon \) which could not be cancelled by contributions from any nonminimal terms. This inconsistency is a core of the higher spin problem [7], [8].

The way out was found by Fradkin and Vasiliev who noticed that a good starting point for perturbative analysis is anti-de Sitter vacuum for metric instead of Minkowski one [9]. It became possible to avoid the higher spin "no-go" restrictions at least in the first nontrivial order in interaction. Furthermore, the all-orders interaction had been conjectured to be consistent only provided all higher spin fields are used altogether.

A few years later, having invented the powerful unfolded formulation technology Vasiliev had constructed consistent nonlinear equations of motion describing a supersymmetric interacting system of higher spin massless fields (every spin appears twice) in 4D space-time [10]. The equations of motion of interacting 3D higher spin - matter system were constructed [11], [12] and have been shown to admit (for the massless matter case) a nontrivial action principle [13]. Besides, 2D actions for higher spin interactions of matter fields have been given [11], [14]. The reviews of this activity may be found in [15]. One may pick up the main properties of higher spin theories that seem to persist in any model describing consistent interactions of higher spin massless fields:

1) the model is invariant under infinite-dimensional gauge symmetry which includes in particular Yang-Mills and general coordinate transformations;

2) there is an infinite number of massless fields of all spins including low spin dilaton, Yang-Mills and gravity fluctuations;

3) a natural background for metric is a nonzero curvature space rather than flat one; the straightforward flat limit is singular as interaction contains terms proportional to the inverse powers of cosmological constant;

4) the theory is unitary at the quadratic level and the linearized equations of motion do not contain higher derivatives, but interaction necessarily involves higher derivatives, so that a full theory may even be nonlocal.

By now, the higher spin problem is elaborated by many authors and there is a lot of interesting results [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28].
It is likely that a desicive solution of higher spin problem in any dimension which should provide a consistent lagrangian description of higher spin interactions may exist but it still requires a lot of study. If such a fundamental theory exists indeed, much probably it is based on deep and simple geometrical principles associated with its huge gauge symmetry group. It is tempting to anticipate these principles may appear to be as clear as gauge principles of Einstein or Yang-Mills theories and determine the action functional unambiguously up to a well controlled arbitrariness and fields redefinition.

One has another hint of existence of higher spin theories. The off-shell fields content, the gauge transformations (2) and the form of actions (4) are the same in any dimension. This fact can be thought about as a manifestation of a geometry which can be developed uniformly in any dimension, this hypothetical geometry has to provide the invariant actions for arbitrary spin fields which after the linearization over some vacuum solution give Fronsdal actions, just like $U(1)$ geometry gives the Maxwell action, and like Riemann geometry underlies the Einstein action.

The basic input governing Einstein gravity is the equivalence principle, similarly, the electrodynamics action may be considered as a realization of the "$U(1)$ equivalence principle". In this paper, we propose a higher-spin extension of the equivalence principle, which unifies $U(1)$ invariance, general-coordinate invariance and a huge amount of another gauge symmetries into one simple gauge law associated with the geometry of point particle phase space.

We study the classical point particle in general external fields, and postulate the generalized equivalence principle: two external fields configurations are equivalent if the particle dynamics in these configurations are equivalent (for the careful definition of equivalent particle dynamics, see the main text), it gives rise to nonlinear, background independent gauge transformations, which contain $U(1)$ and general coordinate transformations subgroups, and of course contain much more. On the other hand, general background $H$ is given by the infinite set of symmetric tensors, every rank appears once $H = \{H(x), H^m(x), H^{m_1m_2}(x), ..., H^{m_1...m_s}(x), ...\}$ the low 0,1,2-rank tensors correspond to low spin dilaton, electromagnetic and gravitational fields. For higher rank fields, the gauge invariance appears to be too restrictive to associate them with Fronsdal gauge fields. We analyze why it happens by studying the linearization of gauge transformations implied by the generalized equivalence principle around natural vacuum solution corresponding to the flat metric and constant dilaton and discuss some modifications which may cure the situation.

As a byproduct of our approach it appears possible to reproduce an old result by De Wit and Freedman [34] on the point particle – symmetric tensors first-order interaction and generalize it to all orders.

In Conclusion we briefly discuss the results and some perspectives.

2 Point particle in general external fields and the generalized equivalence principle.

The basic observations come from the consideration of the dynamics of ordinary point particle in $d$-dimensional spacetime $\mathcal{M}^d$. The worldlines $x^m(\tau), m = 0, ..., d-1$ describe
evolution of the particle. Clearly, the worldlines differing by \( \tau \)-reparametrization
\[
x'^m(\tau'(\tau)) = x^m(\tau)
\] (5)
present equivalent space-time evolution, therefore the particle action
\[
S = \int d\tau L\left(x(\tau), \dot{x}(\tau), x^{(2)}(\tau), \ldots, x^{(n)}(\tau), \ldots\right)
\] (6)
should be reparametrization invariant. From now on we suppose that Lagrangian \( L \) does
not depend on higher \( \tau \)-derivatives \( x^{(2)}, \ldots, x^{(n)}, \ldots \), then \( L \) should be a homogeneous
function of \( \dot{x}^m \) of a first order. The theory admits covariant constrained hamiltonian
formulation [33], which is built as usual by introducing the \( x^m \)-conjugated momenta \( p_n \) and
making the Legendre transform. As a consequence of reparametrization symmetry,
the model is determined by a single first class constraint \( H(x^m, p_n) \approx 0 \). In summary, the
hamiltonian action reads
\[
S_H[x(\tau), p(\tau), \lambda(\tau)] = \int d\tau \{ p_m \dot{x}^m - \lambda H(p, q) \},
\] (7)
\( \lambda \) is a Lagrange multiplier to the unique first class constraint \( H \approx 0 \) which we shall call
Hamiltonian.

Varying the action (7) w.r.t. \( x, p, \lambda \) one gets the dynamics equivalent to that derived
from the Lagrangian, in the form of Hamilton-like equations with the Hamiltonian \( H \) and
the canonical Poisson bracket
\[
\{ x^m, p_n \} = \delta^m_n,
\]
\[
\dot{x}^m = \lambda \{ x^m, H \}, \quad \dot{p}_n = \lambda \{ p_n, H \}
\] (8)
plus the condition
\[
H(x, p) = 0.
\] (9)
The last equation implies that in order to set a nontrivial particle dynamics \( H \) have to
possess zeroes as otherwise there are no solutions for the classical equations of motions.

In the hamiltonian picture, the worldline reparametrization symmetry (5) becomes
the local symmetry of the action (7)
\[
\delta x^m(\tau) = \mu(\tau) \{ x^m, H \}(\tau), \quad \delta p_m(\tau) = \mu(\tau) \{ p_m, H \}(\tau), \quad \delta \lambda = -\dot{\mu}.
\] (10)
This local symmetry may be fixed by implying the gauge condition
\[
\dot{\lambda} = 0 \quad \Rightarrow \quad \dot{\mu} = 0
\] (11)
after that only global modes of gauge transformations survive \( \mu(\tau) = \nu_1 \tau + \nu_2 \).

In fact, the hamiltonian action (7) with general Hamiltonian \( H(x, p) \) describes all possi-
ble scalar particle dynamics on \( \mathcal{M}^d \) with Lagrangians independent on higher \( \tau \)-derivatives.
A natural question is which Hamiltonians are equivalent in the sense they set equivalent
particle dynamics. To answer the question, first of all it is necessary to specify which
particle dynamics are considered as equivalent ones. This topic is rather subtle, e.g. it
is well known any finite dimensional hamiltonian dynamics may be represented locally in appropriate action-angle coordinates as $\dot{X}_m(x,p,\tau) = A_n, \dot{P}_n(x,p,\tau) = 0$, where $X_m(x,p,\tau), P_n(x,p,\tau)$ is some $\tau$-dependent canonical transformation. Thus, all finite dimensional models with the same dimension seem to be locally isomorphic. Our definition (the generalized equivalence principle) is:

**Definition 1** Two particle dynamics determined by the actions (7) with Hamiltonians $H'$ and $H$ are called equivalent if there exists a $\tau$-local, $\tau$-independent, continuous change of variables $x'(x,p,\lambda), p'(x,p,\lambda), \lambda'(x,p,\lambda)$ such that the actions are equal up to an integral of a total $\tau$-derivative:

$$S_{H'}[x',p',\lambda'] = S_H[x,p,\lambda]$$

(12)

In this case, the Hamiltonians $H'$ and $H$ are called equivalent: $H' \sim H$.

Let us analyze the definition. First, it is clear that Hamiltonians differing by a canonical transformation are equivalent:

$$(\text{the transform } (x'(x,p), p'(x,p)) \text{ is canonical }, \lambda' = \lambda, H'(x',p') = H(x,p)) \Rightarrow \Rightarrow H' \sim H,$$

(13)

since the kinetic term $\int d\tau p_m \dot{x}_m$ is invariant.

Another equivalence of Hamiltonians comes from the transformation

$$x'^m = x^m, p'_n = p_n, \lambda' = A^{-1}(x,p)\lambda, H' = A(x,p)H,$$

(14)

$$\Rightarrow H' \sim H,$$

where $A(x,p)$ is a nonzero function on the phase space. We call this equivalence hyperWeyl transformations, while ordinary Weyl dilations are associated with $p$-independent $A$.

The action of infinitesimal equivalence transformations (13,14) on $H$ may be represented in the form

$$\delta H(x,p) = \{\epsilon(x,p), H(x,p)\} + a(x,p)H(x,p)$$

(15)

where $\epsilon$ is a generating function of the canonical transformations while $a$ corresponds to the infinitesimal form of (14) after the substitution $A = e^a$. These equivalence transformations form the infinite-dimensional Lie algebra $G$, being the semidirect product of canonical and hyperWeyl ones:

$$[\delta_{(\epsilon_1,a_1)}, \delta_{(\epsilon_2,a_2)}]H = \delta_{(\epsilon_3,a_3)}H$$

$$\epsilon_3 = \{\epsilon_1,\epsilon_2\}, \ a_3 = \{\epsilon_1,a_2\} - \{\epsilon_2,a_1\}.$$  

(16)

The equivalence transformations (13) may seem to be strong enough to identify all Hamiltonians at all, in this case our definition could be meaningless. However, this is not the case. Formally one could say any two Hamiltonians are connected by some canonical change of variables (13). However, in general case the change is highly singular and can not be considered as an equivalence transformation. Below we will return to this issue in
more detail by finding some natural invariant of (15) (Sec (4)), now we concentrate on hyperWeyl transformations (14). It is clear the Hamiltonians $H_1$ and $H_2$ are not identified by hyperWeyl transformations (14) if the surfaces $H_1 = 0$ and $H_2 = 0$ do not coincide, as the transformations (15) leave them intact. On the other hand, the surfaces $H_{1,2} = 0$ are just the ones the particle dynamics is concentrated on (the constraint surfaces), therefore, one may say that the particle dynamics fills the invariance w.r.t. hyperWeyl transformations (14). Besides, due to the hyperWeyl symmetry the unphysical Hamiltonians which have no zeroes are equivalent to a constant. Thus, at least infinitesimal equivalence transformations do not identify all hamiltonians.

Still, the simple comment that any two Hamiltonians are connected by a (rather singular) canonical map has hard consequences for dynamical content of a theory we will try to construct in terms of $H$. The final result will be that in order to allow a nontrivial dynamics for $H$ the generalized equivalence principle should be either modified or relaxed, otherwise no sensible dynamical theory constructed in terms of $H$ exists.

3 An example: gravity+electromagnetic+scalar background.

Our consideration has been general till this moment. Let us consider the important example to illustrate that our definition (1) of equivalence has a direct physical interpretation.

We will say that a function $f(x, p)$ is of a $k$-th order, if it possesses the following decomposition in momenta:

$$f = \sum_{i=0}^{k} f^{m_1...m_i}(x)p_{m_1}...p_{m_i}$$

(17)

The example deals with the second order Hamiltonians

$$H = D(x) + C^m(x)p_m + \frac{1}{2}g^{mn}(x)p_mp_n,$$

(18)

It is easy to see that these Hamiltonians describe a point particle in a general gravity-electromagnetism-dilaton background\footnote{In this paper, ”dilaton” stands just for a scalar field, with no other limitations on its dynamics, like the connection to string theory etc.}. Indeed, the lagrangian action of such a particle with nonzero mass is

$$S = \int d\tau \{-m\phi(x)\sqrt{-g_{mn}\dot{x}^m\dot{x}^n} + eA_m\dot{x}^m\}$$

(19)

where $g_{mn}(x), A_m(x), \phi(x)$ are the gravitational, electromagnetic and dilaton fields, and the parameters $m$ and $e$ are the particle’s mass and electric charge, respectively. Carrying out the hamiltonization, one finds the first class constraint

$$H = \frac{1}{2}g^{mn}(x)\Pi_m\Pi_n + \frac{m^2}{2}\phi^2(x), \quad \Pi_m = p_m - eA_m(x),$$

(20)
$g^{mn}$ is an inverse metric, $p_m$ are the momenta and $\Pi_m$ are the extended momenta. Now it is seen that a general second order Hamiltonian (18) is equal to (20) after the identification

$$D = \frac{m^2}{2} \phi^2 + \frac{e^2}{2} g^{mn} A_m A_n, \quad C^m = - e g^{mn} A_n.$$  \hspace{1cm} (21)

In fact, the Hamiltonian description is more general than the one via ”square root” Lagrangian (19), as it includes massless limit ($m = 0$ case).

Consider the equivalence transformations (15) which do not change the second order of Hamiltonians and therefore may be interpreted as the equivalence transformations for the Hamiltonian coefficients, i.e. for metric, electromagnetic potential and dilaton. To this end, it is sufficient to make the following choice:

$$\epsilon(x, p) = \epsilon(x) + \xi^m(x)p_m, \quad a(x, p) = a(x),$$  \hspace{1cm} (22)

so that the generator of canonical transformations $\epsilon(x, p)$ is taken to be a general first order function while $a(x, p)$ does not depend on momenta at all (zero order). The transformations (15,22) form the infinite-dimensional Lie algebra $G_0$

$$[\delta_{(\epsilon_1, \alpha_1)}, \delta_{(\epsilon_2, \alpha_2)}]H = \delta_{(\epsilon_3, \alpha_3)}H$$  \hspace{1cm} (23)

which is unambiguously interpreted as semidirect sum of spacetime diffeomorphisms $\xi_m$ and two $U(1)$ gauge symmetries corresponding to the phase canonical transformations $\epsilon(x)$ and ordinary Weyl dilations $\alpha(x)$.

It is easy to see that (15,22) give the following gauge transformations for the background fields:

$$\delta g_{mn} = -\xi^k g_{m,k} - \xi^k_{,m} g_{kn} - \xi^k_{,m} g_{km} + \alpha g_{mn}$$

$$\delta e A_m = -e (\xi^k A_{m,k} + \xi^k_{,m} A_k) + \epsilon_{,m}$$

$$\delta \phi = -\xi^k \phi_{,k} + \frac{1}{2} \epsilon \phi$$  \hspace{1cm} (24)

Here one easily recognizes again the standard $U(1)$ and general-coordinate transformations associated with $\epsilon$ and $\xi^m$, correspondingly, and the Weyl dilations associated with $\alpha$. Therefore, the equivalence transformations (15) are interpreted as gauge ones for the coefficients of Hamiltonian. It is clear that the gauge equivalent backgrounds lead to the particle dynamics which are physically equivalent. Indeed, the invariance related to $\alpha$ is automatically accounted for in the Lagrangian (19) as all fields appear in the invariant combinations $\phi^2 g_{mn}, A_m$. It is also well known that the Lagrangian (19) is covariant w.r.t. general coordinate transformations of background fields and changes by total derivative if electromagnetic field transforms under $U(1)$ symmetry. It means that gauge transformations of metric, electromagnetic field and dilaton may be compensated by transferring to new coordinates in the space of worldlines $x^m(\tau)(x^n(\tau))$. Needless to say, this map is induced by the canonical transformation with generating function $\epsilon(x, p)$ (24).
Given the gauge laws (24) one may wonder if there exists an invariant action $\mathcal{A}[g_{mn}, A_m, \phi]$. Clearly the answer is positive and is given by the action of Weyl-invariant dilaton-Maxwell gravity

$$\mathcal{A}[g_{mn}, A_m, \phi] =$$

$$= \frac{1}{4} \int d^d x \sqrt{-\text{det}(g)} \phi^d \left\{ \Lambda + a_1((d + 2)g^{pq}\phi_p \phi_q + \frac{d-2}{d-1} R(g)\phi^2) + a_2 F_{pq} F^{pq}\phi^{-4} + \cdots \right\}$$

(25)

where $a_{1,2}$ are some constants, $\Lambda$ is a cosmological constant, $F_{mn}$ is the electromagnetic field strength, $R$ is the scalar curvature and “...” stand for higher derivative terms. If one drops the higher derivative terms, the action describes the dynamics equivalent to ordinary Einstein gravity (possibly, with cosmological term) plus Maxwell theory. This can be seen by gauging the dilaton to a nonzero constant by Weyl transformations. The special singular case arises when the dilaton is allowed to take zero value. The most simple way to deal with this limiting value seems to be the redefinition of the dilaton according to the rule

$$\Phi = \phi^\frac{d+2}{2}$$

(26)

to get

$$\mathcal{A}[g_{mn}, A_m, \Phi] =$$

$$= \int d^d x \sqrt{-\text{det}(g)} \left\{ \Lambda \Phi^\frac{d+2}{2} + a_1(g^{pq}\Phi_p \Phi_q + \frac{1}{4} \frac{d-2}{d-1} R(g)\Phi^2) + a_2 F_{pq} F^{pq}\Phi^{-4} + \cdots \right\},$$

(27)

wherefrom it is seen that in the limit $\Phi \rightarrow 0$ only dilaton’s kinetic term survives.

Let us summarize the results of this section: the second order Hamiltonians describe scalar particle in general gravity, electromagnetic and dilaton backgrounds. The standard background fields gauge transformations: general coordinate, $U(1)$ and Weyl dilations, do not affect the particle dynamics in the sense they do not change the action after compensation by a canonical transformation with generating function of a first order. Needless to say, these properties are nothing but a paraphrase of the equivalence principle. There exists the standard action (25) invariant w.r.t. equivalence transformations of background fields which appear to describe (at general values of dilaton) Maxwell + Einstein gravity system. As far as the form of the action (25) is known to be fixed by the requirements of gauge invariance (24) and absence of higher derivatives one may conclude that the equivalence principle applied to the second order ansatz (18), (22) determines uniquely the theory action and thus sets up the whole Einstein + Maxwell theory.

On the other hand the standard low-spin gauge fields saturates only quadratic ansatz in momenta (18), (21), while it is clear one will get a consistent particle dynamics on $\mathcal{M}^d$ for more general Hamiltonians than the quadratic ones. Therefore, the generalized equivalence principle will work for general case either and it may happen to determine some interesting gauge theory with infinite number of gauge fields and gauge invariances. This possibility is studied in the next section.
4 General backgrounds

The facts exposed in the previous section are well known. Point out once again the affinity between the geometry of point particle phase space and the geometry of gauge fields – in fact the full (nonlinearized, background-independent) general coordinate and $U(1)$ transformations of gravity and electromagnetism are induced by the canonical transformations of the scalar particle phase space.

Here we extend these observations by working out general (in an appropriate class of functions) Hamiltonians and general equivalence transformations. It will provide us with a huge algebra of gauge transformations acting on the coefficients of Hamiltonian identified with an infinite set of space-time fields. Proceeding in this way one gets the gauge transformation laws for the infinite system of symmetric tensor fields. The infinitesimal gauge transformations (15) read

$$\delta H(x, p) = \{\epsilon(x, p), H(x, p)\} + a(x, p)H(x, p)$$

Consider the Hamiltonians which are the formal series in momenta

$$H = \sum_{k=0}^{\infty} H^{m_1...m_k}(x)p_{m_1}...p_{m_k} = \sum_{k=0}^{\infty} H_k$$

where $H_k$ denotes the polynomial of $k$-th degree in momenta. Similarly, the equivalence transformations (15) are generated by the parameters $\epsilon(x, p)$ and $a(x, p)$, which belong to the same class

$$\epsilon = \sum_{k=0}^{\infty} \epsilon^{m_1...m_k}(x)p_{m_1}...p_{m_k}, \quad a = \sum_{k=0}^{\infty} a^{m_1...m_k}(x)p_{m_1}...p_{m_k}$$

and required to have a compact support in space-time. $x$-diffeomorphisms, $U(1)$ and Weyl dilations (22,23) form the subalgebra $G_0$. Note that the full gauge algebra $G$ gets broken to $G_0$ if one restricts the sums in (29,30) by some highest $k$. Therefore, the infinite number of component fields $H^{m_1...m_k}$ is necessary to have not only $G_0$ but also its infinite extension. This property naturally corresponds to the requirement (2) of infinite number of higher spin fields mentioned in the Introduction among the conditions necessary for consistent interactions.

The coefficients $H^{m_1...m_k}(x)$ are interpreted as background space-time gauge fields. One may set the question whether it is possible to construct an action $A[H_0, H_1, ..., H_k, ...]$ invariant under the gauge transformations (28). Unfortunately, these transformations appear to prohibit the construction of nontrivial invariant action functionals.

Indeed, the $\epsilon$-variations of $H$ are ordinary canonical transformations with an arbitrary generating function $\epsilon$. The invariants of these transformations seem to be only the integrals over total phase space

$$I_F[H] = \int d^d x d^d p F(H)$$

where $F$ is some function of $H$ such that the integral converges.
Furthermore the hyperWeyl invariance with parameter $a$ implies that the function $F[H]$ should be dilation invariant:

$$F(AH) = F(H)$$

for all $A(x,p) \neq 0$. Then the unique possibility for $F(\sigma)$ is

$$F(\sigma) = \theta(\sigma)$$

where $\theta(\sigma)$ is the "step" $\theta$-function:

$$\theta(\sigma) = \begin{cases} 0 & , \sigma < 0 \\ \frac{1}{2} & , \sigma = 0 \\ 1 & , \sigma > 0 \end{cases}$$

Suppose that the surface $H = 0$ is a boundary of a compact domain $U$ in the phase space, and $H > 0$ inside the domain while $H < 0$ outside, e.g. $H = \frac{1}{2}(-p^2 - \alpha^2 x^2 + m^2)$ with Euclidean metric. Then the action (31, 33) equals $\frac{1}{2}$ times the volume of this compact domain ($2n - 1$ dimensional sphere in the last example). An analogous situation arises when the surface $H = 0$ contains a number of nested disconnected pieces (e.g. $H = \prod_{i=1}^{N} H_i$, $H_i = -p^2 - \alpha x^2 + m_i^2$, $m_1 < m_2 < \ldots < m_N$), then the volume between $H_i$ and $H_j$ surfaces is invariant w.r.t. (28) and may be taken as an invariant "action".

Clearly, such type of actions is inappropriate as they do not contain space-time derivatives of dynamical fields and therefore do not set a nontrivial dynamics. One may wonder why the full gauge transformations are too restrictive while for the quadratic ansatz of the previous section the equivalence transformations provide standard gauge laws for gravity+Maxwell+dilaton fields which are known to provide good space-time actions.

A possible answer which we do not dwell here is that the equivalence transformations should be deformed in order to allow for actions with derivatives. In fact, there exist a very natural deformation related to the covariant quantization of the point particle in general background fields, in that case one easily constructs the actions of the type

$$A_H = Tr F(\hat{H}(\hat{x},\hat{\mathbf{p}})),$$

where $\hat{H}$ is a quantized Hamiltonian and the trace is performed over the quantized particle states space. It is possible to show that the functionals of this kind possess natural quasiclassical expansion by the powers of $\hbar$, and the $\hbar^k$-order terms have exactly $k$ space-time derivatives of the $\hat{H}$ components while $\hbar^0$-terms does not contain derivatives and coincides with the classical result (31). So, in this scheme, the terms with derivatives appear as quantum corrections to the classical "cosmological" term. We plan to present these matters in a separate publication.

Here we try to give another answer which makes it evident from the physical viewpoint that the generalized equivalence transformations are too restrictive. To provide the
evidence we have to analyze the linearization of transformations (28). We expand the Hamiltonian \( H(x, p) \) around a natural background,

\[
H = H_v + \frac{1}{2}(g^{mn} p_m p_n + m^2) \equiv \frac{1}{2}(p^2 + m^2).
\] (36)

which presents a configuration with nonzero metric, constant dilaton and all other fields being zeroes. Let us suppose there exists an action \( \tilde{\mathcal{A}}[H] \) which possesses the vacuum solution \( H_v \) and describes well-defined space-time dynamics of \( H \). Introduce the fluctuation \( h \) near the vacuum \( H_v \),

\[
H = H_v + h.
\] (37)

and extract the \( h \)-quadratic part from \( \tilde{\mathcal{A}}[H] \), i.e. construct the linearized action \( \tilde{\mathcal{A}}_2[H] = hP h \), where \( P \) is some gauge-invariant operator.

Recall a general proposition: if one has a nonlinear action (\( \tilde{\mathcal{A}}[H] \)) with infinitesimal gauge invariance \( \delta H = R_\varepsilon[H] \), then the linearized action \( hP h \) is gauge invariant w.r.t. linearized inhomogeneous, \( h \)-independent gauge transformations given by gauge variation of the vacuum solution: \( \delta h = R_\varepsilon[H_v] \). We suppose the gauge invariance of the action differs from (28) only at interaction level while the difference may be neglected in the linearized approximation. Then the linearization of gauge transformations (28) reads

\[
\delta h = \{ \varepsilon, H_v \} + a H_v,
\] (38)

Note that for \( h \) satisfying the second order ansatz of the previous section (18) with first order \( \varepsilon \) and zero order \( a \) (22), these gauge transformations reproduce the linearization of standard general coordinate, \( U(1) \) and Weyl symmetries (24) around the vacuum (36):

\[
\delta h^{mn} = \xi^{k;m} + \xi^{k;n} + \frac{1}{2}\alpha g^{mn},
\delta h^m = \varepsilon^m
\] (39)

\[
\delta h = \frac{1}{2}\alpha m^2
\]

where ";'" denotes covariant differentiation w.r.t. vacuum metric. For \( m^2 \neq 0 \) these transformations imply that the invariant quadratic actions should be the standard spin-1 and spin-2 ones. The "spin-0" fluctuation of dilaton \( h \) is gauged away by linearized Weyl transformations. Another way to see it is to observe that due to the Weyl symmetry \( h \) combines with the trace of metric fluctuation \( h^m_m \) into the single invariant combination

\[
h_{m^2} = h - \frac{m^2}{d} h^m_m\]

which then merely serves as the counterpart of another Weyl-invariant object: traceless part of \( h^{mn} \) in building the standard linearized gravity action. So the spin-0 is absent here and this is in the complete agreement with nonlinear situation discussed in the previous section.

If \( m^2 = 0 \) the fluctuation of dilaton \( h \) is gauge invariant and therefore it decouples from the linearized action describing fluctuations of rank 1 and 2 fields. On the other
hand, the Weyl transformations in gravity fluctuations sector gauge away the trace $h^m_m$ thus the invariant action has to depend only on the traceless part of $h^{mn}$. It results in Weyl-invariant higher derivative gravity theories without zero and second-order terms (e.g. in 4D the Lagrangian is the linearization of $\sqrt{-g} C_{mnpq}^2$ for $C_{mnpq}$ being the Weyl tensor) + further higher derivatives terms. One concludes that the fluctuations around the $m^2 = 0$ vacuum are nonunitary and thus this case does not provide a possibility to interpret the fluctuations in terms of particles.

Let us study the general case (38) and try to find a relationship of general fluctuations around the vacuum $H_v$ to the Fronsdal double-traceless gauge fields (1),(2). Despite the overall result will be negative both for $m^2 \neq 0$ and $m^2 = 0$ cases it will be seen how the fluctuations of Fronsdal fields can be embedded into general fluctuations of Hamiltonian $h(x,p)$. For simplicity, we take the vacuum metric to be flat.

Let us start with the gauge invariance (36,38) which reads

$$\delta h = \frac{1}{2} a(p^2 + m^2) + p^m \partial_m \epsilon,$$

where $\partial_m$ is a derivative w.r.t. $x^m$. The linearized hyperWeyl transformations parameters $a(x,p)$ enter the gauge laws (41) without derivatives and therefore they just eliminate some auxiliary fields. If an invariant action $\tilde{A}^2[H] \propto h P h$ exists, the invariance w.r.t. $a$ implies these auxiliary fields do not enter action at all and the action depends only on the invariants of $a$-transformations. So let us find these invariants. It can be done as follows. Given an arbitrary power series in momenta $f$ of the form (29) one may expand each coefficient $f^{m_1...m_i}$ in terms of its traceless components in the manner

$$f^{m_1...m_i} = f_0^{m_1...m_i} + g^{(m_1m_2} f_1^{m_3...m_i)} + g^{(m_1m_2} g^{m_3m_4} f_2^{m_5...m_i)} + ...,$$

where all $f_k$ are traceless, so any $f$ may be represented as

$$f = f_0 + f_1 p^2 + f_2 (p^2)^2 + ... = \sum_{k=0}^{\infty} f_k (p^2)^k,$$

where $f_k$ are the power series of the form (29), but with traceless coefficients. The expansion (43) may be considered as a function $f(p^2)$ of one variable $p^2$ with coefficients $f_k$ taking values in the power series (29) with traceless coefficients, then (43) provides the Taylor series of the function $f(p^2)$ at the point $p^2 = 0$. Decomposing the function at another point, say, $p^2 = -m^2$, one gets

$$f = f^{(m^2)}_0 + f^{(m^2)}_1 (p^2 + m^2) + f^{(m^2)}_2 (p^2 + m^2)^2 + ... = \sum_{k=0}^{\infty} f_k^{(m^2)} (p^2 + m^2)^k$$

Then it is clear the unique invariant of hyperWeyl transformations (11) is given by

$$h_{m^2} \equiv h_0^{(m^2)} = \sum_{k=0}^{\infty} (-m^2)^k h_k,$$

or, formally, $h_{m^2} = h_{|p^2+m^2=0}$, where the reduction to the ”constraint surface” $p^2 + m^2 = 0$ is carried out only in the $p^2$-factors of (13). Indeed, representing $h$ and the hyperWeyl
variation of \( h \) in the form (44) one observes that \( a \) gauges away all \( h_k^{(m^2)} \) components except \( h_0^{(m^2)} \).

\( h_{m^2} \) is a power series comprising only *traceless* coefficients

\[
h_{m^2} = \sum_{k=0}^{\infty} \chi^{a_1 \ldots a_k} p_{a_1} \ldots p_{a_k} \ ; \ 2 \chi_a^{ba_3 \ldots a_k} = 0. \tag{46}
\]

Now analyze how the linearized canonical transformations \( \epsilon \) act on \( h_{m^2} \). To this end one decomposes \( p \partial_x \epsilon \) in power series (44) and then substitutes \(-m^2\) instead of \( p^2\). The parameter \( \epsilon \) may be taken traceless

\[
\epsilon = \epsilon_0^{(m^2)} = \sum_{k=0}^{\infty} \epsilon^{a_1 \ldots a_k} p_{a_1} \ldots p_{a_k} \ ; \ \epsilon_b^{ba_3 \ldots a_k} = 0 \tag{47}
\]

as its traces \( \epsilon = (p^2 + m^2) \epsilon_1^{(m^2)} + \ldots \) lead to transformations equivalent to hyperWeyl ones and already projected out automatically in the variation of \( h_{m^2} \). One easily finds

\[
\delta h_{m^2} = (p \partial_x \epsilon)(-m^2) =
\sum_{k=0}^{\infty} \left( \partial^{a_1} \epsilon^{a_2 \ldots a_k} - \frac{k-1}{d+2k-4} g^{a_1 a_2} \partial_b \epsilon^{ba_3 \ldots a_k} - m^2 \frac{k+1}{d+2k} \partial_b \epsilon^{ba_1 a_2 \ldots a_k} \right) p_{a_1} p_{a_2} \ldots p_{a_k} \tag{48}
\]

or equivalently

\[
\delta \chi^{a_1 \ldots a_k} = \text{Traceless part of } \partial^{(a_1} \epsilon^{a_2 \ldots a_k)} - m^2 \frac{k+1}{d+2k} \partial_b \epsilon^{ba_1 a_2 \ldots a_k} \tag{49}
\]

where the round brackets denote the symmetrization.

For \( m^2 \neq 0 \), each tensor of rank \( k \) is transformed by two different gauge parameters: \( \epsilon \) of rank \( (k-1) \) via the “Traceless part of ...” in (43) and of rank \((k+1)\) via the divergence-like term in (49).

Note that restricting (43) to the sector of \( \chi, \chi^a, \chi^{a_1 a_2} \) and choosing the situation with the only nonzero parameters \( \epsilon, \epsilon^a \) one finds for \( m^2 \neq 0 \) exactly the linearized gauge transformations for Maxwell+Einstein system, where \( \chi, \chi^{a_1 a_2} \) describe the off-shell graviton and \( \chi^a \) the off-shell photon. Thus we come back to the low-spin analysis carried out above which showed how gravity +Maxwell fields arise by virtue of the equivalence principle. However, the higher rank gauge symmetries break good low-spin picture. In particular, the graviton transforms not only w.r.t. linearized diffeomorphisms \( \epsilon^a \) but also w.r.t. \( \epsilon^{a_1 a_2 a_3} \):

\[
\delta \chi^{a_1 a_2} = \text{Traceless part of } \partial^{(a_1} \epsilon^{a_2)} - \frac{3m^2}{d+4} \partial_b \epsilon^{ba_1 a_2} \tag{50}
\]

So, despite the analogy with the low-spin case was the main motivation for the consideration of generalized equivalence principle, the results of its promotion to general case (43) are not very appealing. It may be traced back to the simple fact noted above that, from the very beginning, the number of canonical gauge parameters \( \epsilon \) coincides with the number of \( H \) components and therefore any two Hamiltonians are identified by a singular canonical transformation.
On the other hand, let us compare the gauge laws (49) with those of Fronsdal theories (1) – (4). Recall the Fronsdal gauge transformations for true higher spin fields (1) – (3).

Decomposing $\phi^{a_1...a_k}$ as a sum of its traceless part $\rho^{a_1...a_k}$ and the trace $\sigma^{a_1...a_{k-2}}$

$$\phi^{a_1...a_k} = \rho^{a_1...a_k} + g^{(a_1a_2}\sigma^{a_3...a_k)}$$ (51)

one gets

$$\delta \rho^{a_1...a_k} = \text{Traceless part of } \partial^{(a_1} \varepsilon^{a_2...a_k)} , \quad \delta \sigma^{a_3...a_k} = \frac{k - 1}{d + 2k - 4} \partial_b \varepsilon^{b a_3...a_k}. \quad (52)$$

In Fronsdal theories, the "physical" field describing the dynamics is $\rho^{a_1...a_k}$ while the trace $\sigma^{a_1...a_{k-2}}$ serves as a compensator designed to ensure the constraint $\partial_b \varepsilon^{b a_3...a_k} = 0$ after $\sigma$ is eliminated by gauge transformations. Comparing this situation to (49), (50) one concludes that each traceless component $\chi^{a_1...a_k}$ plays the double role: first, it transforms as "physical" field $\rho^{a_1...a_k}$ w.r.t. $\varepsilon^{a_1...a_{k-1}}$, second, it transforms like the compensator $\sigma^{a_1...a_k}$ w.r.t. $\varepsilon^{a_1...a_{k+1}}$.

As a consequence, the gauge laws (49) do not determine any sensible free theory. Possibly, the situation with the gauge transformations (49) may be improved if one relaxes the generalized equivalence principle, e.g. by relaxing somehow hyper Weyl transformations or finding some constraints on $\epsilon$ and $a$ (15) which restrict the gauge transformations in such a way they allow nontrivial dynamics for $H$. Still, this modification has to possess good features of generalized equivalence principle: background independence, inclusion of general coordinate, $U(1)$ and higher gauge symmetries, clear physical interpretation etc.

On the other hand, our study brings the idea of doubling: if one replaces the fluctuation $\chi$ by two fields $\chi_1, \chi_2$ transforming by the laws (13) with two different mass values

$$\delta \chi_1^{a_1...a_k} = \text{Traceless part of } \partial^{(a_1} \varepsilon^{a_2...a_k)} - m_1^2 \frac{k + 1}{d + 2k} \partial_b \varepsilon^{b a_1 a_2...a_k}$$

$$\delta \chi_2^{a_1...a_k} = \text{Traceless part of } \partial^{(a_1} \varepsilon^{a_2...a_k)} - m_2^2 \frac{k + 1}{d + 2k} \partial_b \varepsilon^{b a_1 a_2...a_k},$$ (53)

then these gauge laws determine the system exactly equivalent to the infinite collection of Fronsdal fields, every spin enters once, where the "physical" $\rho$ and compensator $\sigma$ components of Fronsdal fields are expressed via linear combinations of $\chi_1, \chi_2$. The derivation of the system (53) as a linearization of some sensible nonlinear transformations analogous to the derivation of (13) from (13) is an interesting issue we plan to study in a separate paper.

Now let $m^2 = 0$. Then the gauge laws (49) simplify:

$$\delta \chi^{a_1...a_k} = \text{Traceless part of } \partial^{(a_1} \varepsilon^{a_2...a_k)}.$$(54)

Here $\chi$ transforms as "physical" Fronsdal field and the low spin situation is not broken by higher spin gauge symmetries. However, the "compensators" are absent and therefore the gauge laws (54) has no relation to Fronsdal theories. The low spin case was shown above to lead, in 4D, to the linearization of (higher derivative) conformal gravity. There may exist some nonunitary higher derivative actions invariant w.r.t. the symmetry (54) which provide the "higher spin" generalizations of linearized conformal gravity.
5  De Wit - Freedman point particle - symmetric tensors first order interaction.

Concluding the analysis of classical particle model it is worth mentioning that our approach allows to reproduce and generalize some old results. Recall that an attempt to find a point particle-symmetric tensor fields interaction in the lagrangian approach had been undertaken in [34] by De Wit and Freedman who succeeded in constructing consistent first-order interaction linear in background fields fluctuations. Here we show that their results are identically reproduced if one extracts linear in the fluctuations of external fields terms from the Lagrangian derived from the full action (7), while our construction appears to generalize the De Wit - Freedman results to all orders in fluctuations.

Consider the full hamiltonian action (7). Represent the Hamiltonian in the manner

$$H = \frac{1}{2}p^2 + \frac{1}{2}m^2 + eh(x, p)$$

where $\frac{1}{2}(p^2 + m^2)$ corresponds to the vacuum configuration of background fields (with arbitrary metric) and $eh$ is a fluctuation over this vacuum with $e$ being the expansion parameter. The action takes the form

$$S_H[x(\tau), p(\tau), \lambda(\tau)] = \int d\tau \{p_m \dot{x}^m - \lambda \left(\frac{1}{2}p^2 + \frac{1}{2}m^2 + eh(x, p)\right)\}.$$ 

(56)

In accordance with (10) this action is invariant under the gauge transformations

$$\delta x^m(\tau) = (p^m + e \frac{\partial}{\partial p_m} h(\mu(\tau), \lambda(\tau) - 1 \dot{x}^2 - e\lambda p \frac{\partial}{\partial p_m} h(\mu(\tau), \lambda(\tau) - 1 \dot{x}^2) + o(e^2))$$

$$\delta p_m(\tau) = -e(\frac{\partial}{\partial x^m} h(\mu(\tau), \lambda(\tau) - 1 \dot{x}^2) + o(e^2))$$

wherefrom it follows that

$$p \dot{x} = \lambda^{-1} \dot{x}^2 - e\lambda p \frac{\partial}{\partial p_m} h(\lambda^{-1} \dot{x}) + o(e^2)$$

$$p^2 = \lambda^{-2} \dot{x}^2 - 2e\lambda p \frac{\partial}{\partial p_m} h(\lambda^{-1} \dot{x}) + o(e^2)$$

$$eh = eh(\lambda^{-1} \dot{x}) + o(e^2).$$

(61)
Substituting these expressions into (56) we obtain the lagrangian action
\[ S_L[x(\tau), \lambda(\tau)] = \int d\tau \left\{ \frac{1}{2} \left( \lambda^{-1} \dot{x}^2 - m^2 \lambda \right) - e \lambda \dot{\lambda} \left( \lambda^{-1} \dot{x} \right) \right\} + o(e^2 h^2). \] (62)

The \( e \)-independent terms give the well known scalar particle action in the gravitational background, while \( e \)-linear ones present higher rank corrections. To make contact with De Wit -Freedman Lagrangian one excludes \( \lambda \) by means of it’s equations of motions
\[ \delta S_L / \delta \lambda = -\frac{1}{2} \left[ \frac{m^2}{x^2} - \frac{1}{2} \lambda^{-2} \dot{x}^2 - e \partial_x h(\lambda^{-1} \dot{x}) + o(e^2) \right] = 0. \] (63)

Like in the case of momenta, these equations are solved by iterations in \( e \). Expanding \( \lambda^{-1} \equiv y \) in the manner
\[ y = y(0) + e y(1) + e^2 y(2) + \ldots \] (64)

one gets
\[ y(1) = m^{-2} \sum_{k=0}^{\infty} (1 - k) h_{m_1 \ldots m_k} \dot{x}^{m_1} \ldots \dot{x}^{m_k} \left( -\frac{m^2}{x^2} \right)^{k+1}/2. \] (65)

In fact, the explicit form of \( y_1 \) is inessential as in our first order approximation its contribution drops out from the action. Substituting (65) in (62) we arrive at the result
\[ -S_L[x(\tau)] = \int d\tau \left\{ \sqrt{-m^2 \dot{x}^2} \left( 1 + \frac{e}{m^2} \sum_{k=0}^{\infty} h_{m_1 \ldots m_k} \dot{x}^{m_1} \ldots \dot{x}^{m_k} \left( -\frac{m^2}{x^2} \right)^{k+1}/2 \right) \right\} + o(e^2 h^2). \] (66)

This action coincides with the De Wit-Freedman (DF) one [34] after the identification
\[ \varphi_{DF}^{m_1 \ldots m_k} = -h_{m_1 \ldots m_k} \] and setting \( m^2 = -1 \) (the negative sign of \( m^2 \) just accounts the difference in metric’s signature, \( \sqrt{\dot{x}^2}_{DF} = \sqrt{-\dot{x}^2}_{our} \)).

In their work it was observed that, besides being explicitly reparametrization invariant, this lagrangian action is invariant w.r.t. simultaneous gauge transformation of external fields
\[ \delta \varphi_{DF}^{m_1 \ldots m_k} = k \xi_{(m_2 \ldots m_k; m_1)} \] (67)

(; denotes the covariant derivative) and point particle worldlines
\[ \delta x^m(\tau) = -e k (k-1) \dot{x}^{1-2k/2} \xi_{m_3 \ldots m_k} \dot{x}^{m_3} \ldots \dot{x}^{m_k} \] (68)

In our formulation, this invariance is a particular manifestation of the generalized equivalence principle [12], corresponding to the canonical equivalence transformations [13] written up to \( o(e^2) \) terms. Explicitly, writing down the infinitesimal canonical \( \epsilon \)-transformations
\[ \delta x^m = -e \{ \epsilon, x^m \} \] (69)

and substituting the expressions for momenta and Lagrange multipliers [53, 60, 61, 65], one gets in the \( o(e^2) \) approximation the De Wit-Freedman transformations (67), (68) after the
identification \( \epsilon_{m_2...m_k} = -k \xi_{m_2...m_k} \), while the invariance of the action is guaranteed by the very definition (12).

Doing further orders of perturbation procedure for Eqs. (58), (63) and substituting the solutions for \( p_m \) and \( \lambda \) into the action (56) one obtains the Lagrangian up to an arbitrary order in \( e \hbar \). Then the generalized equivalence transformations (69) present the generalization of (67), (68) to all orders in \( e \hbar \).

6 Conclusion

We have considered the model of point particle in general external fields and postulated the generalized equivalence principle (Definition (1)). This principle identifies background fields which set up equivalent particle dynamics, with constraint surfaces \( H(x, p) = 0 \) differing by a canonical transformation. The Hamiltonians \( H(x, p) \) have a form of formal power series in momenta \( p \) and coefficients of power series are identified with an infinite collection of space-time symmetric tensors.

It is shown that second order Hamiltonians which saturate the quadratic ansatz in momenta correspond to general gravitational+Maxwell+dilaton background. In this case, the equivalence transformations which do not spoil the ansatz give exactly the general coordinate, \( U(1) \) and Weyl transformations.

The generalization to arbitrary Hamiltonians and equivalence transformations is worked out to some depth. The general equivalence transformations appear to prohibit the construction of invariant actions with space-time-derivatives. It is not too surprising as the equivalence transformations contain as many parameters as Hamiltonian. The invariants of full equivalence transformations seem to be only the integrals over the total phase space (31) which do not contain space-time derivatives of \( H(x, p) \) and therefore do not set a nontrivial dynamics. This defect could be cured after quantizing the particle and considering the functionals of the form (35), we plan to consider this possibility in a separate paper.

On the other hand we tried to see explicitly why the generalized equivalence principle does not allow for good space-time actions while the ordinary equivalence principle for low spin fields does. To this end we have undertaken the analysis of fluctuations over a natural (Minkowski) vacuum and found that the surviving degrees of freedom are described by the infinite collection of traceless symmetric tensors (every rank enters once), and linearized gauge transformations mix the components of all ranks. Moreover, even low spin Maxwell+gravity system is broken as graviton may be gauged away by gauge transformations with third-rank gauge parameter. Comparing the situation with the Fronsdal theory where a fixed spin-\( s \) system is described off-shell by two traceless fields – rank-\( s \) ”physical” and rank \( (s-2) \) ”compensator”, we observed that the linearized equivalence transformations imply too hard duties on the fluctuations: every component of a fixed rank \( s \) transforms as a ”physical” field w.r.t. rank-(\( s-1 \)) parameters and as a ”compensator” w.r.t. rank-(\( s+1 \)) ones.

Summing up, the generalized equivalence principle is too restrictive and need to be relaxed either by finding appropriate constraints on the canonical and hyperWeyl transformations or by modification of the original gauge law (15). This hypothetical relaxed
equivalence principle has to possess background independence, contain general-coordinate and $U(1)$ transformations as well as infinity of higher spin symmetries, and has a clear physical interpretation.

On the other hand, the situation could be improved after introducing some additional degrees of freedom which should have some clear physical origin and may be incorporated naturally in the framework of generalized equivalence. Particularly interesting is the "doubled" linearized system (53) which is exactly equivalent to the infinite collection of Fronsdal gauge fields, and the question is whether this system may be obtained by linearization from some background independent gauge transformations, in that case the full transformations could present the nonlinear higher spin gauge symmetries. We leave this topic for future study.

We also have shown that the De Wit – Freedman first order point particle- symmetric tensors lagrangian interaction [34] is reproduced naturally in our approach if one derives it from the Hamiltonian action (7) excluding the momenta and Lagrange multiplier by means of their equations of motion in the framework of simple perturbative procedure, this way the invariance of the action w.r.t. simultaneous gauge transformations of background fields and worldlines of the particle found in [34] arises as a particular manifestation of generalized equivalence principle (which is satisfied by construction in all orders of perturbative procedure).

We hope our observations may get important developments in the future.

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