Thermodynamics of an Evaporating Schwarzschild Black Hole in Noncommutative Space

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Abstract

We investigate the effects of space noncommutativity and the generalized uncertainty principle on the thermodynamics of a radiating Schwarzschild black hole. We show that evaporation process is in such a way that black hole reaches to a maximum temperature before its final stage of evolution and then cools down to a nonsingular remnant with zero temperature and entropy. We compare our results with more reliable results of string theory. This comparison shows that GUP and space noncommutativity are similar concepts at least from the viewpoint of black hole thermodynamics.

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Key Words: Noncommutative Geometry, Generalized Uncertainty Principle, Black Hole Thermodynamics
1 Introduction

After thirty years of intensive research in the field of radiating black holes[1], various aspects of the problem still remain under debate. For example the last stage of black hole evaporation is not obvious in some respects. The string/black hole correspondence principle [2] suggests that in this extreme regime stringy effects cannot be neglected. In spite of the promising results that string theory has had in quantizing gravity, the actual calculations of the Hawking radiation are currently obtained by means of quantum field theory in curved space [3]. In fact the black hole evaporation occurs in a semiclassical regime, namely when the density of gravitons is lower than that of the matter field quanta. Nevertheless, the divergent behavior of the black hole temperature in the final stage of the evaporation remains rather obscure.

In addition to string theory itself, which provides an elegant framework for incorporation of quantum gravity effects in black hole physics by direct state counting, several alternative approaches to incorporate quantum gravity effects in the calculation of black hole thermodynamics have been proposed. These approaches can be classified as follows:

- **Generalized Uncertainty Principle(GUP)**
  
  Existence of a nonzero minimal length scale (which leads to finite resolution of spacetime structure) can be addressed in GUP(see[4] and references therein). From a heuristic argument, one can use GUP to find modification of Bekenstein-Hawking formalism of black hole thermodynamics[5,6,7,8]. The main consequences of this approach are summarized as follows:
  
  Black hole evaporation ends up with a phase consisting a remnant with zero entropy and there exists a finite temperature that black hole can reach in its final stage of evaporation. This picture differs drastically with Bekenstein-Hawking prescription which accepts the total evaporation of Black holes.

- **Modified Dispersion Relations(MDRs)**
  
  MDRs induced modification of black hole thermodynamics have their origin on loop quantum gravity considerations(MDRs are signature of Lorentz invariance violation at high energy sector of the field theory). Attempts to modify Bekenstein-Hawking formalism based on MDRs show more or less the same behaviors as GUP framework, but now we find severe constraints on the functional form of MDRs when we compare our results with string theory more reliable results[9,10].
• Noncommutative Geometry

Noncommutativity eliminates point-like structures in favor of smeared objects in flat spacetime. Based on this idea, several attempts have been performed to find modification of Bekenstein-Hawking formalism of black hole thermodynamics within noncommutative geometry[11,12]. The consequences of these attempts are as follows:
The end-point of black hole evaporation is a zero temperature extremal remnant with no curvature singularity.

In this paper we are going to proceed one more step in the line of third alternative i.e. Noncommutative Geometry. Our strategy differs with existing literatures in two main respects: we don’t consider smeared picture of objects in noncommutative spacetime(as has been considered in [11,12]), instead we deal with coordinate noncommutativity which results modification of Schwarzschild radius. Also we consider possible generalization of uncertainty principle within a string theory point of view. We calculate entropy-area relation and compare our results with more reliable results of string theory(calculated based on direct state-counting). This comparison shows that GUP and space noncommutativity are essentially similar concepts.

In which follows we suppose $c = \hbar = G = 1$.

2 Black Hole Thermodynamics in GUP Framework

The canonical commutation relations in a commutative spacetime manifold are given as follows

$$[x_i, x_j] = 0, \quad [x_i, p_j] = i\delta_{ij}, \quad [p_i, p_j] = 0.$$  

From a string theory point of view, existence of a minimal length scale can be addressed in the following generalized uncertainty principle

$$\delta x \delta p \geq \frac{1}{2} \left(1 + \beta (\delta p)^2 + \gamma\right),$$  

where $\beta$ is string theory parameter related to minimal length. Since we are dealing with absolutely minimum position uncertainty we set $\gamma = \beta \langle p \rangle^2$ and therefore the corresponding canonical commutation relation becomes

$$[x, p] = i(1 + \beta p^2).$$
The canonical commutation relations in commutative spacetime with GUP become

\[ [x_i, x_j] = 0, \quad [x_i, p_j] = i\delta_{ij}(1 + \beta p^2), \quad [p_i, p_j] = 0. \quad (4) \]

Now consider the geometry of Schwarzschild spacetime with the following metric

\[ ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right), \quad (5) \]

where \( f(r) = 1 - \frac{2M}{r} \). There is a horizon at \( r_s = 2M \) with the following area

\[ A = r_s^2 \int_0^{2\pi} d\phi \int_0^\pi \sin \theta d\theta = 4\pi r_s^2 = 16\pi M^2. \quad (6) \]

Bekenstein-Hawking formalism of black hole thermodynamics gives the following relations for temperature and entropy of black hole

\[ T_H = \frac{1}{8\pi M} \quad (7) \]

and

\[ S = 4\pi M^2 \quad (8) \]

respectively. Within GUP framework, these equations should be modified to incorporate quantum gravity effects. We use Bekenstein’s argument type considerations to find GUP induced modification of black hole thermodynamics. For simplicity, consider the following GUP

\[ \delta x \delta p \geq \frac{1}{2} \left(1 + \beta (\delta p)^2\right). \quad (9) \]

A simple calculation gives,

\[ \delta p \approx \frac{\delta x}{\beta} \left[1 \pm \sqrt{1 - \frac{\beta}{(\delta x)^2}}\right]. \quad (10) \]

Here to achieve correct limiting result we should consider the minus sign in round bracket.

In original Bekenstein approach, from a heuristic argument based on Heisenberg uncertainty relation, one deduces the following equation for Hawking temperature of black hole,

\[ T_H \approx \frac{\delta p}{2\pi}. \quad (11) \]

Therefore, in the framework of generalized uncertainty principle, modified black hole temperature is as follows

\[ T_H^{\text{GUP}} \approx \frac{\delta x}{2\pi \beta} \left[1 - \sqrt{1 - \frac{\beta}{(\delta x)^2}}\right]. \quad (12) \]
Within black hole near horizon geometry, since \( \delta x \sim r_s \) where \( r_s = 2M \), one can write this equation in such a way that can be comparable with equation (7):

\[
T_H^{GUP} \approx \frac{M}{\pi \beta} \left[ 1 - \sqrt{1 - \frac{\beta}{4M^2}} \right],
\]

which leads to the following relation

\[
T_H^{GUP} \approx \frac{1}{8\pi M} \left[ 1 + \frac{\beta}{16M^2} + \frac{\beta^2}{128M^4} \right],
\]

(14)

up to second order in \( \beta \). Obviously, when quantum gravitational effects are negligible, that is when \( \beta \to 0 \), this relation gives (7) as a manifestation of correspondence principle.

Now consider a quantum particle that starts out in the vicinity of an event horizon and then ultimately absorbed by black hole. For a black hole absorbing such a particle with energy \( E \) and size \( R \), the minimal increase in the horizon area can be expressed as

\[
(\Delta A)_{min} \geq 4(\ln 2)ER,
\]

(15)

then one can write

\[
(\Delta A)_{min} \geq 8(\ln 2)\delta p\delta x,
\]

(16)

where \( E \sim c\delta p \) (with \( c = 1 \)) and \( R \sim 2\delta x \). Using equation (10) for \( \delta p \), we find

\[
(\Delta A)_{min} \approx \frac{2(\ln 2)A}{\beta \pi} \left[ 1 - \sqrt{1 - \frac{4\pi \beta}{A}} \right]
\]

(17)

where we have defined \( A = 4\pi(\delta x)^2 \). Now we should determine \( \delta x \). Since our goal is to compute microcanonical entropy of a large black hole, near-horizon geometry considerations suggests the use of inverse surface gravity or simply the Schwarzschild radius for \( \delta x \). Therefore, \( \delta x \sim r_s \) and defining \( 4\pi r_s^2 = A \) and \( (\Delta S)_{min} = b = constant \), then it is easy to show that,

\[
\frac{dS}{dA} \approx \frac{(\Delta S)_{min}}{(\Delta A)_{min}} \approx \frac{b\beta \pi}{2(\ln 2)A \left[ 1 - \sqrt{1 - \frac{4\pi \beta}{A}} \right]},
\]

(18)

Note that \( b \) can be considered as one bit of information since entropy is an extensive quantity. Considering calibration factor of Bekenstein as \( \ln 2 \), the minimum increase of entropy(i.e. \( b \)), should be \( \ln 2 \). Now we should perform integration. There are two possible choices for lower limit of integration, \( A = 0 \) and \( A = A_p \). Existence of a minimal observable length leads to existence of a minimum event horizon area, \( A_p = 4\pi(\delta x_{min})^2 \).
So it is physically reasonable to set $A_p$ as lower limit of integration. Based on these arguments, we can write

$$S \simeq \int_{A_p}^A \frac{\beta \pi}{2A \left[ 1 - \sqrt{1 - \frac{4\pi}{A}} \right]} dA.$$  \hspace{1cm} (19)

An integration gives

$$S \simeq \frac{A}{4} - \frac{\pi \beta}{4} \ln \frac{A}{4} + \sum_{n=1}^{\infty} c_n \left( \frac{4}{A} \right)^n + C,$$  \hspace{1cm} (20)

where $C$ is a constant. This is an interesting result which shows the logarithmic leading order correction plus a power series expansion in terms of inverse of area. Up to third order in $\frac{1}{A}$, we find

$$S \simeq \frac{A}{4} - \frac{\pi \beta}{4} \ln \frac{A}{4} + \left( \frac{\pi \beta}{4} \right)^2 \left( \frac{4}{A} \right) - 3 \left( \frac{\pi \beta}{4} \right)^3 \left( \frac{4}{A} \right)^2 + C,$$  \hspace{1cm} (21)

where

$$C = -\frac{A_p}{4} + \frac{\pi \beta}{4} \ln \frac{A_p}{4} - \left( \frac{\pi \beta}{4} \right)^2 \left( \frac{4}{A_p} \right) - \left( \frac{\pi \beta}{4} \right)^3 \left( \frac{4}{A_p} \right)^2 + 3 \left( \frac{\pi \beta}{4} \right)^4 \left( \frac{4}{A_p} \right)^3.$$  \hspace{1cm} (22)

It is obvious that when $A = A_p$, $S \to 0$ and therefore black hole remnant should have zero entropy. A result which is physically acceptable since small classical fluctuations are not allowed at remnant scales because of existence of minimal observable length.

### 3 Black Hole Thermodynamics in Noncommutative Geometry

A noncommutative space can be realized by the coordinate operators satisfying

$$[\hat{x}_i, \hat{x}_j] = i \theta_{ij}, \quad i, j = 1, 2, 3,$$  \hspace{1cm} (23)

where $\hat{x}$’s are the coordinate operators and $\theta_{ij}$ is the noncommutativity parameter with dimension of $(\text{length})^2$. Canonical commutation relations in noncommutative spaces read

$$[\hat{x}_i, \hat{x}_j] = i \theta_{ij}, \quad [\hat{x}_i, \hat{p}_j] = i \delta_{ij}, \quad [\hat{p}_i, \hat{p}_j] = 0,$$  \hspace{1cm} (24)

Now, we note that there is a new coordinate system

$$x_i = \hat{x}_i + \frac{1}{2} \theta_{ij} \hat{p}_j, \quad p_i = \hat{p}_i$$  \hspace{1cm} (25)
with these new variables, \( x_i \)'s satisfy the usual (commutative) commutation relations

\[
[x_i, x_j] = 0, \quad [x_i, p_j] = i\delta_{ij}, \quad [p_i, p_j] = 0. \tag{26}
\]

Note that noncommutativity is an intrinsic characteristic of underlying manifold. For a noncommutative Schwarzschild black hole, we have \([13, 14]\)

\[
f(r) = \left(1 - \frac{2M}{\sqrt{\hat{r}r}^3}\right), \tag{27}
\]

where \( \hat{r} \) satisfies (25). The horizon of the noncommutative Schwarzschild metric as usual satisfies the condition \( \hat{g}_{00} = 0 \) which leads to

\[
1 - \frac{2M}{\sqrt{\hat{r}r}^3} = 0. \tag{28}
\]

If in this relation we change the variables \( \hat{x}_i \) to \( x_i \), and then using (25), the horizon of the noncommutative Schwarzschild black hole satisfies the following condition

\[
1 - \frac{2M}{\sqrt{(x_i - \frac{\theta_{ij}p_j}{2})(x_i - \frac{\theta_{ik}p_k}{2})}} = 0. \tag{29}
\]

This leads us to the following relation

\[
1 - \frac{2M}{r} \left( 1 + \frac{x_i\theta_{ij}p_j}{2r^2} - \frac{\theta_{ij}\theta_{ik}p_jp_k}{8r^2} \right) + O(\theta^3) + \ldots = 0, \tag{30}
\]

where \( \theta_{ij} = \frac{1}{2}\epsilon_{ijk}\theta_k \). Using the identity \( \epsilon_{ijr}\epsilon_{iks} = \delta_{jk}\delta_{rs} - \delta_{js}\delta_{rk} \), one can rewrite (30) as follows

\[
1 - \frac{2M}{r} - \frac{M}{2r^3} \left[ \hat{L}\hat{\theta} - \frac{1}{8} \left( p^2\theta^2 - (\hat{p}\hat{\theta})^2 \right) \right] + O(\theta^3) + \ldots = 0, \tag{31}
\]

where \( L_k = \epsilon_{ijk}x_ip_j, \quad p^2 = \hat{p}\hat{p} \) and \( \theta^2 = \hat{\theta}\hat{\theta} \). If we set \( \theta_3 = \theta \) and assuming that remaining components of \( \theta \) all vanish (which can be done by a rotation or a re-definition of the coordinates), then \( \hat{L}\hat{\theta} = L\theta \) and \( \hat{p}\hat{\theta} = p\theta \). In this situation equation (31) can be written as

\[
r^3 - 2Mr^2 - \frac{M}{2} \left[ L\theta - \frac{1}{8} \left( p^2 - p_z^2 \right) \theta^2 \right] + O(\theta^3) + \ldots = 0. \tag{32}
\]

Since \( p^2 = p_x^2 + p_y^2 + p_z^2 \), one can write \( (p^2 - p_z^2)\theta^2 = (p_x^2 + p_y^2)\theta^2 \) and therefore (32) can be written as follows

\[
r^3 - 2Mr^2 - \frac{MLz\theta}{2} + \frac{M}{16} \left( p_x^2 + p_y^2 \right) \theta^2 + O(\theta^3) + \ldots = 0. \tag{33}
\]
Since Schwarzschild black hole is non-rotating, we set $\vec{L} = 0$ and therefore $L_z = 0$ (this means that space noncommutativity has no effect on the Schwarzschild geometry up to first order of space noncommutativity parameter). So we find

$$r^3 - 2Mr^2 + \frac{M}{16} \left( p_x^2 + p_y^2 \right) \theta^2 + \mathcal{O}(\theta^3) + ... = 0.$$  

(34)

With the following definitions

$$a \equiv -2M = -r_s, \quad \eta \equiv \frac{M}{16} \left( p_x^2 + p_y^2 \right) \theta^2,$$

(35)

and considering only terms up to second order of $\theta$, the radius of event horizon for non-commutative Schwarzschild black hole becomes

$$\hat{r}_s \equiv \frac{-a}{3} + \left( -\frac{2a^3 - 27\eta + \sqrt{108a^3\eta + 729\eta^2}}{54} \right)^{1/3} + \frac{a^2}{9} \left( \frac{-2a^3 - 27\eta + \sqrt{108a^3\eta + 729\eta^2}}{54} \right)^{-1/3}.$$  

(36)

Two other roots of (34) are not real. In the case of commutative space, $\eta = 0$, and therefore we recover usual Schwarzschild radius, $r_s = 2M$. Since $a \gg \eta$, we can expand equation (36) to find the following relation for Schwarzschild radius in noncommutative space

$$\hat{r}_s = -a - \frac{\eta}{a^2} - \frac{27\eta^2}{2a^5}.$$  

(37)

Since $a = -r_s$ we have considered only the real parts of our equations. One can write $\eta = M\alpha$, where

$$\alpha = \frac{1}{16} \left( p_x^2 + p_y^2 \right) \theta^2,$$

(38)

and therefore $\eta = \frac{p}{2}\alpha$. In this manner, we can write equation (37) as follows

$$\hat{r}_s = r_s - \frac{\alpha}{2r_s} + \frac{27\alpha^2}{8r_s^3}.$$  

(39)

After calculation of Schwarzschild radius of black hole in noncommutative space, we have all prerequisites to calculate thermodynamics of black hole in noncommutative spacetime. First we consider black hole temperature. The Hawking temperature of Schwarzschild black hole in noncommutative space can be given by the following relation

$$\hat{T}_H = \frac{M}{2\pi \hat{r}_s \hat{r}_s},$$

(40)

where substitution of $\hat{r}_s$ leads to the following generalized statement

$$\hat{T}_H = \frac{M}{2\pi} \left( r_s - \frac{\alpha}{2r_s} + \frac{27\alpha^2}{8r_s^3} \right)^{-2}.$$  

(41)
which leads to the following relation

\[ \hat{T}_H \approx \frac{1}{8\pi M} \left[ 1 + \frac{\alpha}{4M^2} - \frac{3\alpha^2}{8M^4} \right]. \]  

(42)

Figure 1 shows the plot of black hole temperature versus its horizon radius in three candidate models. As this figure shows, within GUP and Noncommutative geometry, black hole before its terminal stage of evaporation reaches to a maximum temperature and then cools down to a zero temperature remnant.

Now we calculate entropy of black hole in a noncommutative spacetime. In the standard Bekenstein argument, the relation between energy and position uncertainty of a given particle is given by (see [10] and references therein)

\[ E \geq \frac{1}{\delta x}. \]  

(43)

Within a noncommutative framework, we suppose \( \delta x = \hat{r}_s \). Therefore, we find the following generalization

\[ E \geq \frac{1}{\hat{r}_s}, \]  

(44)

which substitution of \( \hat{r}_s \) from (39) leads to

\[ E \geq \frac{1}{(r_s - \frac{\alpha}{2r_s} + \frac{27\alpha^2}{8r_s^3})}. \]  

(45)

Since \( r_s = 2M \), this relation implicitly shows the modification of standard dispersion relations which has strong support on loop quantum gravity [9]. In this manner, the increase of event horizon area is given by

\[ \Delta \hat{A} \geq 4(\ln 2) \frac{1}{\left(1 - \frac{\alpha}{2r_s^2} + \frac{27\alpha^2}{8r_s^4}\right)}. \]  

(46)

which leads to the following relation

\[ \frac{dS}{dA} \approx \frac{\Delta S_{(min)}}{\Delta \hat{A}_{(min)}} \approx \frac{\ln 2}{4(\ln 2) \frac{1}{\left(1 - \frac{\alpha}{2r_s^2} + \frac{27\alpha^2}{8r_s^4}\right)}}. \]  

(47)

Therefore we can write

\[ \frac{dS}{dA} \approx \frac{1}{4} \left[ 1 - \frac{\alpha}{2r_s^2} + \frac{27\alpha^2}{8r_s^4} \right]. \]  

(48)
Now we should calculate $d\hat{A}$. Since

$$\hat{A} = 4\pi \hat{r}_s \hat{r}_s,$$

we find

$$d\hat{A} = \left[ 1 + \gamma_1 \left( \frac{4\pi \alpha}{A} \right)^2 + \gamma_2 \left( \frac{4\pi \alpha}{A} \right)^3 + \gamma_3 \left( \frac{4\pi \alpha}{A} \right)^4 \right] dA,$$

where $\gamma_i$'s are some constant ($\gamma_1 = -7$, $\gamma_2 = \frac{27}{4}$, $\gamma_3 = -3(\frac{3}{2})^6$) and $A = 4\pi \hat{r}_s^2$. We can integrate (48) to find

$$S \simeq \frac{A}{4} - \frac{\pi \alpha}{2} \ln \frac{A}{4} + \kappa_1 \left( \frac{\pi \alpha}{2} \right)^2 \left( \frac{4}{A} \right) + \kappa_2 \left( \frac{\pi \alpha}{2} \right)^3 \left( \frac{4}{A} \right)^2 +$$

$$\kappa_3 \left( \frac{\pi \alpha}{2} \right)^4 \left( \frac{4}{A} \right)^3 + \kappa_4 \left( \frac{\pi \alpha}{2} \right)^5 \left( \frac{4}{A} \right)^4 + \kappa_5 \left( \frac{\pi \alpha}{2} \right)^6 \left( \frac{4}{A} \right)^5,$$

where $\kappa_i$'s are some constant ($\kappa_1 = \frac{29}{2}$, $\kappa_2 = -41$, $\kappa_3 = \frac{1305}{4}$, $\kappa_4 = -63 \times \left( \frac{3}{2} \right)^4$, $\kappa_5 = \frac{3^{10}}{40}$). Generally, this relation can be written as

$$S \simeq \frac{A}{4} - \frac{\pi \alpha}{2} \ln \frac{A}{4} + \sum_{n=1}^{\infty} c_n \left( \frac{4}{A} \right)^n + C$$

(52)

Where

$$C \simeq -\frac{A_p}{4} + \frac{\pi \alpha}{2} \ln \frac{A_p}{4} - \sum_{n=1}^{\infty} c_n \left( \frac{4}{A_p} \right)^n.$$

(53)

This is an interesting result which shows the modified entropy of black hole within noncommutative geometry. In the case of commutative spaces $\alpha = 0$ and this equation yields the standard Bekenstein entropy,

$$S \simeq \frac{A}{4}.$$

(54)

Equation (52) is very similar to (20). As a result we see that GUP and Noncommutative geometry give the same area dependence to the modified entropy of black hole. This feature may inherently reflect the fact that GUP and spacetime noncommutativity are not different in essence. Figure 2 shows the entropy-area relation for an evaporating black hole in bekenstein-Hawking and the noncommutative geometry view points. Within noncommutative geometry approach black hole in its final stage of evaporation reaches to a zero entropy remnant.
4 Conclusion

There are several approaches to incorporate quantum gravitational effects in thermodynamics of black holes. Here we have developed two of these approaches with details. In another work [10], we have calculated quantum correction of black hole thermodynamics using modified dispersion relations. Our calculations show that overall behavior (functional form) of entropy-area or temperature-mass relations are independent of different approaches. For example, we have shown here that with GUP and Noncommutative geometry NCG one finds the following relations

\[ T_H \approx \frac{1}{8\pi M} \left[ 1 + \frac{\beta}{16M^2} + \frac{\beta^2}{128M^4} \right] \quad \text{in GUP} \]

\[ \hat{T}_H \approx \frac{1}{8\pi M} \left[ 1 + \frac{\alpha}{4M^2} - \frac{3\alpha^2}{8M^4} \right] \quad \text{in NCG} \]

for temperature up to second order of expansion parameter. These two statements are not different in mass dependence. Similarly for entropy-area relation we have found

\[ S \approx \frac{A}{4} - \frac{\pi\beta}{4} \ln \frac{A}{4} + \sum_{n=1}^{\infty} c_n \left( \frac{4}{A} \right)^n + C \quad \text{in GUP} \]

\[ S \approx \frac{A}{4} - \frac{\pi\alpha}{2} \ln \frac{A}{4} + \sum_{n=1}^{\infty} c_n \left( \frac{4}{A} \right)^n + C \quad \text{in NCG}. \]

These similarity may reflect the fact that GUP and space noncommutativity essentially are not different concepts. In this way one can obtain easily the relation between GUP and space noncommutativity parameters by comparison of the corresponding relations for entropy or temperature. On the other hand, from a string theory point of view, one can show the following relations for temperature and entropy of a black hole

\[ T = \frac{1}{8\pi M} \left( 1 - \rho \frac{1}{4M^2} + \frac{1}{4M^4} (\rho^2 + \frac{\lambda}{4}) \right), \]

and

\[ S = \frac{A}{4} + \rho \ln \frac{A}{4} + \lambda \left( \frac{4}{A} \right). \]

These results are more reliable since they are based on direct analysis of quantum behavior of black hole. Comparison with previous results shows that GUP and space noncommutativity are two similar concept of string theory.

In addition, our analysis shows that space noncommutativity has no effect on the structure of a Schwarzschild black hole in the first order approximation in noncommutativity.
parameter. The final stage of black hole evaporation is a remnant with absolute zero temperature and also zero entropy. Before black hole reach its final state as a remnant, it reaches to a maximum temperature and then cools down to zero temperature. A direct calculation of curvature shows that this remnant is not singular[11]. Note that our works differs from previous findings in two main respects: previous works based on GUP have considered final stage of black hole as a remnant with non-zero temperature (see for example[5,6,7]). Here we have shown that actually this remnants should have zero temperature. Secondly, our approach based on space noncommutativity differs from existing works such as [11,12] since we have not used the smeared picture of objects with Gaussian profile as a result of space noncommutativity, instead we have considered direct generalization of Schwarzschild radius in a perturbational framework. In other words, we have considered the effect of noncommutativity on geometric part of Einstein’s equations whereas Nicolini et al have considered the effects of space noncommutativity on matter part of Einstein’s equations. Although these approaches seems to be different in their view point on space noncommutativity, but their results are similar in many respects. one can easily see the close coincidence of these two approaches by comparing for example the temperature of black hole calculated in these two view points; our figure 1 is in complete agreement with figure 4 of Nicolini et al first paper in reference[11].

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Figure 1: Black hole temperature versus its radius of event horizon in three candidate models: a) Bekenstein-Hawking Model, b) NCG and c) GUP.
Figure 2: Black hole entropy versus its radius of event horizon in two candidate models: a) Bekenstein-Hawking Model, b) NCG. GUP result is similar to curve b.