Spin Polarization in Single Top Events

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We discuss the optimal spin bases for describing angular correlations in single top quark events at the Fermilab Tevatron. We define spin bases that exploit the fact that the top quarks are produced with 100% polarization along the momenta of the $d$-type quarks in these events. For single top production through an $s$-channel $W$ boson, 98% of the top quarks have their spins in the antiproton direction. For single top production via $Wg$-fusion, 96% of the top quarks have their spins in the spectator jet direction. The direction of the top quark spin is reflected in the angular correlations of its decay products.

Until the discovery of the top quark, most studies of spin in high energy physics were formulated in terms of the helicity basis. For ultrarelativistic particles, this is appropriate. However, in general, the direction and degree of polarization of a massive spinning particle depends on how it was produced. Thus, for moderate particle energies, it should not be surprising to find that the optimal axis for studying spin correlations is something other than the particle’s direction of motion.

For the case of single top quark production at the Tevatron, all of the Standard Model diagrams contain a common subdiagram: somewhere there is a $ud$ quark line which is attached to a $tb$ quark line via a $W$ boson. The exact orientation of this subdiagram depends on the process being considered. However, the fact that the $W$ boson couples only to fermions with left-handed chirality leads to a 100% polarization of the produced top quark in the direction of the $d$-type quark. Thus, in studying the spin correlations in single top production at the Tevatron the two key questions are “Where is the $d$-type quark?” and “How well can we know the location of the $d$-type quark?”

Before answering these questions, let us clarify exactly what we mean when we state that the top quark spin points in the direction of the $d$-type quark. As the first step, consider the correlations among the decay products of a spin up top quark. The dominant Standard Model decay chain is

$$t \rightarrow W^+ b \rightarrow ℓ^+ ν b$$

(1)

For concreteness, we will describe the leptonic $W$ decay. However, everything which we say about the charged lepton applies equally to the $d$-type quark in a hadronic decay.

We define the decay angles in the top quark rest frame with respect to top quark spin vector $s$, as shown in Fig. 1. The decay angular distributions are simply linear in the cosine of these decay angles:

$$\frac{1}{Γ} \frac{dΓ}{d(\cos θ_i)} = \frac{1}{2} (1 + α_i \cos θ_i),$$

(2)

where $θ_i$ is the decay angle of the $i$th decay product. The degree to which each decay product is correlated with the spin is encoded in the value of $α_i$. From the listing of $α_i$ values in Table 1, we see that the charged lepton is maximally correlated. Thus, the most distinctive distribution plots the angle between the spin axis and the charged lepton in the top quark rest frame (see Fig. 2).

When we write the decay matrix element in an arbitrary Lorentz frame, we find that the natural 4-vectors...
are not the top quark momentum \( t \) and its spin vector \( s \) (normalized such that \( s \mu s^\mu = -1 \)). Instead, it is more convenient to use the combinations

\[
t_1 \equiv \frac{1}{2}(t + M s) \quad \text{and} \quad t_2 \equiv \frac{1}{2}(t - M s),
\]

where \( M \) is the mass of the top quark. In the top quark rest frame, the spatial parts of \( t_1 \) and \( s \) point in the same direction, since in this frame \( t = (M, 0) \). In some other frame, however, these vectors are not parallel. In this case, the form of the matrix element clearly indicates that the preferred charged lepton emission axis is the spatial part of \( t_1 \). Hence, we should regard \( t_1 \) as the appropriate generalization of the spin axis to an arbitrary reference frame.

Finally, we observe that \( t_1 \) is a massless vector. Thus, we may specify it by using the 4-momentum of any massless particle in the process being studied. In particular, for single top production, we find that the spin down polarized production amplitude contains a factor of \( d \cdot t_1 \), where \( d \) is the 4-momentum of the \( d \)-type quark. Choosing \( t_1 = d \) causes this amplitude to vanish, meaning that the entire production cross section comes from the spin up amplitude. This is what we mean by saying that the top quarks are 100% polarized in the direction of the \( d \)-type quark. Note that the preferred emission direction of the charged lepton from the decaying top quark as viewed in any frame is the same as the direction of the \( d \)-type quark in that frame.

As we shall see shortly, it is not always possible to know the location of the \( d \)-type quark. The best we can do is choose a spin axis that maximizes the likelihood that we have picked the “correct” direction. In this situation, we will have a mixed sample containing \( N_d \) spin-up and \( N_d \) spin-down top quarks, and the angular distribution in Eq. (4) must be replaced by

\[
\frac{1}{\Gamma} \frac{d\Gamma}{d(\cos \theta_l)} = \frac{1}{2} \left( 1 + A_{t+} \alpha_t \cos \theta_l \right),
\]

where \( A_{t+} \) is the spin asymmetry

\[
A_{t+} \equiv \frac{N_t + N_d}{N_t - N_d}.
\]  

Eq. (4) tells us that we obtain the largest correlations by using the charged lepton and choosing the spin axis which maximizes the magnitude of \( A_{t+} \), i.e. the best approximation to the direction of the \( d \)-type quark.

This brings us to the two key questions proposed at the beginning of this talk: “Where is the \( d \)-type quark?” and “How well can we know the location of the \( d \)-type quark?”

We begin with a discussion of the \( W^* \) production mechanism, also known as the \( s \)-channel production mechanism, because the off-shell \( W \) is in the \( s \)-channel (see Fig. 3). Conceptually, this is a very simple process: a \( u \) quark and a \( d \) quark annihilate, forming an off-shell \( W \) which “decays” to a \( t \) quark and a \( b \) quark. Since the proton beam is a copious source of quarks and the antiproton beam a copious source of antiquarks, we intuitively expect that the \( d \)-type quark will come from the antiproton beam most of the time. In fact, we find that the antiproton beam supplies the \( d \)-type quark 98% of the time at a center of mass energy \( \sqrt{s} = 2.0 \) TeV (see Table 2). Thus, we define the antiproton basis to be that basis where the top quark spin is measured along the direction of the antiproton beam.

![Figure 3: Feynman diagram for single top quark production in the \( W^* \) process.](image)

Table 2: Fractional cross sections for single top quark production in the \( W^* \) channel at the Tevatron with \( \sqrt{s} = 2.0 \) TeV, decomposed according to the parton content of the initial state.

| \( p \) | \( \bar{p} \) | Fraction |
|---|---|---|
| \( u \) | \( \bar{d} \) | 98% |
| \( \bar{d} \) | \( u \) | 2% |

Table 3: Dominant spin fractions and asymmetries for the helicity and antiproton bases for single top quark production in the \( W^* \) channel at the Tevatron with \( \sqrt{s} = 2.0 \) TeV.

| Basis | Spin Content | \( A_{t+} \) |
|---|---|---|
| helicity | 83% L | -0.66 |
| antiproton | 98% \( \uparrow \) | +0.96 |
rection of the antiproton beam momentum. We compare this basis to the helicity basis in Table 3. Notice that in the antiproton basis, the spin asymmetry is $A_{1\downarrow} = 0.96$, whereas in the helicity basis $A_{1\downarrow}$ is only $-0.66$. Thus, the correlations are a factor of 1.45 larger in the antiproton basis than in the helicity basis.

The other, and, in fact, dominant production mechanism for single top quarks at the Tevatron at 2.0 TeV is the so-called $W$-gluon fusion process $t\bar{g}\to t\bar{b}d$. For the purposes of determining the spin correlations of these events at lowest order, it is sufficient to consider the two pairs of diagrams in Fig. 4.

In these diagrams, a gluon from one beam splits into a $b\bar{b}$ or $t\bar{t}$ pair, which fuses with a $W$ radiated from a light ($u$ or $d$) spectator quark from the other beam. The final state contains a $b$ jet, the top quark decay products, and the spectator jet.

Since the $d$-type quark is either contained in one of the beams or in the spectator jet, we should choose the spin axis direction (proton momentum, antiproton momentum, or spectator jet momentum) which maximizes the probability that our spin axis is aligned with the $d$-type quark. Now, the $u$ quark content of the proton is greater than the $\bar{d}$ quark content of the antiproton. Furthermore, the gluon content of both is the same. Hence, we expect that the largest share of the cross section comes from $u\bar{g}\to t\bar{b}d$, with the spectator jet containing the $\bar{d}$ quark. As we can see from Table 3, this expectation is correct. In fact, the spectator jet is the $d$ quark 77% of the time. Thus, we define the spectator basis as the basis in which we choose the spin axis to be aligned with the momentum of the spectator jet. In this basis, the top quark is produced in the spin up state 96% of the time, leading to correlations which are a factor of 1.35 larger than in the helicity basis (see Table 3).

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aWe equate spin-up with right-handed helicity and spin-down with left-handed helicity.
of the spectator jet, etc. Of course, what would be measured in a real experiment is the sum of the signal and background distributions, in which case $A$ is further reduced to $-14\%$. Nevertheless, with the $2 \text{ fb}^{-1}$ of integrated luminosity expected at Run II, this asymmetry should be visible at the $3\sigma$ level. To reach the $5\sigma$ level, a total of $5\text{ fb}^{-1}$ is required.

In conclusion, we have found that the natural spin axis in single top events is the direction of the $d$-type quark: the top quarks are $100\%$ polarized in this direction. Since the exact direction of the $d$-type quark is unknown, it is necessary to choose the direction which is most likely to be correct. In particular, for production through an $s$-channel $W$ boson, we define the antiproton basis, where the spin axis is the direction of the antiproton beam, since this is the location of the $d$-type quark $98\%$ of the time. We find that the correlations are a factor of 1.45 larger in the antiproton basis than in the helicity basis. A recent study has shown for the $Wg$-fusion mode that these correlations will be observable at the $3\sigma$ level in Tevatron Run II.

**APPENDIX: $W$ Boson Spin in Top Quark Decay**

It is well-known that the $W$ bosons emitted from decaying top quarks have a specific mixture of helicities in the Standard Model: the right-handed helicity state is absent, while the left-handed and longitudinal states are present in the ratio $2M_W^2 : M_W^2$. This is reflected in the charged lepton decay distribution in the $W$ rest frame

$$\frac{1}{\Gamma} \frac{d\Gamma}{d(\cos \chi_W)} = \frac{3}{4} \frac{m_t^2 \sin^2 \chi_W + \frac{1}{2}(1 - \cos \chi_W)^2}{m_t^2 + 2m_W^2},$$

where $\chi_W$ is defined in Fig. 6. Parke and Shadmi have pointed out an interesting correlation between this angle, $\chi_W$, and the direction that the $W$ was emitted with respect to the top quark spin axis in the top quark rest frame (cf. $\chi_W$, see Fig. 6). This correlation, which is caused by the interference between the two (unobserved) spin states of the $W$ boson, is illustrated in Fig. 7. It is apparent from this distribution that as viewed in the top quark rest frame, the longitudinal $W$ bosons are emitted preferentially in the same direction as the top quark spin, whereas the transverse $W$’s prefer the backwards direction relative to the spin axis.

These correlations may be intuitively understood.
Figure 8: Angular momentum conservation in the decay of a polarized top quark. Since the $b$ quark is effectively massless and couples to a $W$, it is produced with left-handed helicity. (a) $W$ emitted parallel to top quark spin is longitudinal. (b) $W$ emitted antiparallel to top quark spin is left-handed.

from elementary angular momentum conservation arguments and the $V-A$ coupling between the $W$ and quarks. Since the $b$ quark mass is much less than the energy it receives from the decay, the left-handed chirality of the $tbW$ vertex translates into left-handed helicity for the $b$. Suppose first that the $W$ boson is emitted along the top quark spin axis, as in Fig. 8a. Then, the spin of the $b$ points in the same direction as the spin of the original $t$ and we must have zero spin projection for the $W$ boson (i.e. it must be longitudinal). On the other hand, when the $W$ is emitted in the backwards direction (Fig. 8b), the spin of the $b$ is opposite to the spin of the parent $t$. In this case, the $W$ must have left-handed helicity in order to conserve angular momentum.

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