Flavour symmetries and SUSY soft breaking in the LHC era

Oscar Vives
Departament de Física Teòrica and IFIC, Universitat de Valencia-CSIC, 46100 Valencia, Spain
E-mail: oscar.vives@uv.es

Abstract. The so-called supersymmetric flavour problem does not exist in isolation to the Standard Model flavour problem. We show that a realistic flavour symmetry can simultaneously solve both problems without ad hoc modifications of the SUSY model. Furthermore, departures from the SM expectations in these models can be used to discriminate among different possibilities. In particular we present the expected values for the electron EDM in a flavour model solving the supersymmetric flavour and CP problems.

1. Introduction
The SUSY flavour problem has been traditionally used to justify different departures from the “natural” gravity-mediated MSSM setting. However, in this talk we will take a different point of view and we will show that the so-called supersymmetric flavour problem does not really exist, or more exactly, it can not be detached from the Standard Model flavour problem. In fact a correct solution to the Standard Model flavour problem will probably pass unscathed all the stringent constraints on flavour changing neutral currents after the inclusion of the MSSM soft sector.

The supersymmetric flavour problem is usually stated as follows: The SUSY soft-breaking terms and they have a completely different origin from the Yukawa couplings in the superpotential and we have no information on their structure. In principle, we could expect that all the entries in the soft breaking matrices were $O(1)$ in any basis and in particular in the basis where the Yukawa couplings are diagonal. In this situation FCNC and $CP$ violation observables would receive too large contributions from loops involving SUSY particles and this disagrees strongly with the stringent phenomenological bounds on these processes. As formulated above we can only agree with this statement, however, it is trivial to reformulate this statement in terms of the Yukawa couplings of the superpotential: We have no theoretical guidance to build the Yukawa couplings. If we had to write an SM Lagrangian ignoring the measured quark and lepton masses and mixings, any flavour structure would be possible and in fact we would naturally expect all the different entries in the Yukawa matrices to be $O(1)$. Clearly this would never agree with the observed fermion masses and mixing angles. Therefore we have to conclude that there is a much stronger flavour problem in the SM than in the MSSM. The real flavour problem is simply our inability to understand the complicated structures in the quark and lepton Yukawa couplings and likewise soft-breaking flavour structures in the MSSM.

At this point we have to emphasize that the presence of new physics, as for instance supersymmetry, is not a problem for flavour but on the contrary a necessary tool to advance
in our understanding of the flavour problem. In the framework of the Standard Model all the information we can extract on flavour are the Yukawa eigenvalues (quark and lepton masses) and the left-handed misalignment between up and down quarks (CKM matrix) or leptons (MNS matrix) and this is not enough to determine the full structure of the Yukawa matrices. However, in supersymmetric extensions of the SM, the new interactions can provide additional information on the physics of flavour which will be fundamental to improve our knowledge on flavour. In the following we show that finding a solution to the “SM” flavour problem will also solve the so-called “supersymmetric flavour problem” to a sufficient degree.

2. Flavour symmetries

The flavour structure associated to the SM Yukawas is very special: a strong hierarchy in the couplings and a peculiar structure of the mixing matrices. In a truly fundamental theory we would expect all dimensionless couplings to be $O(1)$ and thus these small couplings must be explained. The basic idea of flavour symmetries is to use an spontaneously broken family symmetry in analogy with the gauge sector to generate these couplings. A scalar vev breaking the flavour symmetry normalized with a large mediator mass provides a small expansion parameter that enters in different powers in the fermion Yukawa couplings[1]. In the limit of exact symmetry the Yukawa couplings are forbidden and only when the symmetry is broken these couplings appear as a function of small vevs. Similarly, in a supersymmetric theory, the flavour symmetry applies both to the fermion and sfermion sectors. Therefore, the structures in the soft-breaking matrices and the Yukawa couplings are related. The starting point in our analysis is then the texture in the Yukawa couplings. However, the complete texture of the Yukawa matrices can not be fixed though Standard Model interactions. Still, it is reasonable to assume that the smallness of CKM mixing angles is due to the smallness of the off-diagonal elements in the Yukawa matrices with respect to the corresponding diagonal elements. Then we can fix the elements above the diagonal, corresponding to the left-handed mixings, but not the elements below the diagonal [2]. Therefore, we can consider two complementary situations that we call symmetric and asymmetric Yukawa textures. In the symmetric textures we make the additional simplifying assumption of choosing the matrices to be symmetric. Note that this situation is not unusual in many flavour models [3, 4, 5] as well as in GUT theories. Asymmetric textures are also common in simple Abelian flavour symmetries with a single flavon field [6, 7].

The simplest example is provided by a $U(1)$ flavour symmetry, which generates an asymmetric texture. As an example we can assign the three generations of SM fields the charges: $Q_i = (3, 2, 0)$, $d_i^c = (0, 0, 1)$, $u_i^c = (3, 2, 0)$ with a single flavon field of charge $-1$. The vev of the flavon field normalized to the mass of the heavy mediator fields $M_f$, is $\epsilon = v/M_f \ll 1$. The superpotential of this model is $W_{\text{Yukawa}} = Q_i d_j^c H_1 \left( \frac{\theta}{M_n} \right)^{q_i+d_j} + Q_i u_j^c H_2 \left( \frac{\theta}{M_n} \right)^{q_i+u_j}$ where unknown $O(1)$ coefficients have been suppressed for clarity. Then we have,

$$Y_u = \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \quad Y_d = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}.$$ (1)

The soft masses are couplings $\phi^i \phi$, clearly invariant under any symmetry, and therefore always allowed. Hence, diagonal soft masses are allowed in the limit of unbroken symmetry and unsuppressed. Assuming diagonal masses of different generations are equal in the symmetric limit$^1$, the universality is then broken by the flavon vevs. Any combination of two MSSM scalar fields $\phi_i$ and an arbitrary number of flavon vevs invariant under the symmetry will contribute to

$^1$ Unlike in the case of non-Abelian symmetries, this is not guaranteed by symmetry, but it is still possible in some cases like dilaton domination in gravity mediation models.
the soft masses: \( \mathcal{L}_{\text{soft}} = m_{\theta}^2 (\phi_i^1 \phi_4^1 + \phi_i^2 \phi_2^2 + \phi_i^3 \phi_3^3 + (\frac{q_i}{M}) \eta_i^j \phi_i^j + \text{h.c.}) \). Thus, the structure of the right-handed down squark mass matrix we would have in this model is:

\[
M_{D_R}^2 \simeq \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} m_{\theta}^2. \tag{2}
\]

In this case, we would expect large mixings in the second and third generations of right-handed down sfermions. Notice however, that this simple model is already ruled out by the stringent constraints in the 1–2 sector unless sfermions are very heavy.

Symmetric textures are obtained, for instance, from a spontaneously broken \( SU(3) \) family symmetry. The basic features of this symmetry are the following. All left-handed fermions (\( \psi_i \) and \( \psi_i^c \)) are triplets under \( SU(3)_f \). To allow for the spontaneous symmetry breaking of \( SU(3) \) it is necessary to add several new scalar fields which are either triplets (\( \overline{\theta}_3, \overline{\theta}_{23}, \bar{\theta}_3 \)) or antitriplets (\( \theta_3, \theta_{23} \)). We assume that \( SU(3)_f \) is broken in two steps. The first step occurs when \( \theta_3 \) and \( \bar{\theta}_3 \) get a large vev breaking \( SU(3) \) to \( SU(2) \). Subsequently a smaller vev of \( \theta_{23} \) and \( \overline{\theta}_{23} \) breaks the remaining symmetry. After this breaking we obtain the effective Yukawa couplings through the Froggatt-Nielsen mechanism [1] integrating out heavy fields. In fact, to reproduce measured masses and mixings, the large third generation Yukawa couplings require a \( \theta_{23}, \theta_3 \) vev of the order of the mediator scale, \( M_f \), while \( \theta_{23}/M_f, \bar{\theta}_{23}/M_f \) have vevs of order \( \varepsilon = 0.05 \) in the up sector and \( \bar{\varepsilon} = 0.15 \) in the down sector with different mediator scales in both sectors. Moreover in the minimization of the scalar potential it is possible to ensure that the fields \( \theta_{23} \) and \( \bar{\theta}_{23} \) get equal vevs in the second and third components. In this model, CP is spontaneously broken by the flavon vevs that are complex generating the observed CP violation in the CKM matrix. The basic structure of the Yukawa superpotential is then given by:

\[
W_Y = H \psi_i \psi_i^c \theta_3^i \theta_3^j + \theta_{23}^i \bar{\theta}_{23}^j + \epsilon^{ikl} \bar{\theta}_{23,k} \theta_3^j \theta_{23}^l + \ldots \]

This structure is quite general for the different \( SU(3) \) models we can build, for additional details refer to [5, 3, 4]. The Yukawa textures are then symmetric and suppressing \( O(1) \) coefficients:

\[
Y_d \propto \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}, \quad Y_u \propto \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}. \tag{3}
\]

In the same way after \( SU(3) \) breaking the scalar soft masses deviate from exact universality. In first place we must notice that a mass term \( \psi_i^1 \psi_i^1 \) is invariant under any symmetry and hence gives rise to a common contribution for the family triplet. However, \( SU(3) \) breaking terms give rise to important corrections [5, 8]. Any invariant combination of flavon fields can also contribute to the sfermion masses. Including these corrections the leading contributions to the sfermion mass matrices are: \( (M_f^2)^{ij} = m_{\theta}^2 (\delta^{ij} + \frac{1}{M_f^2} [\theta_3^i \theta_3^j + \theta_{23}^i \theta_{23}^j] + \frac{1}{M_f^2} (\epsilon^{ikl} \bar{\theta}_{23,k} \theta_3^j \theta_{23}^l]) \)

where \( f \) represents the \( SU(2) \) doublet or the up and down singlets with \( M_f = M_L, M_u, M_d \). For instance, the charged slepton mass matrices after running to the electroweak scale and in the basis of diagonal charged lepton Yukawas are,

\[
M_{E_R}^{SCKM} \simeq 0.15 \, M_{1/2}^2 \, \begin{pmatrix} 1 + \varepsilon^3 & \varepsilon^3 e^{-i\alpha} & \varepsilon^3 e^{-i\beta} \\ \varepsilon^3 e^{i\alpha} & 1 + \varepsilon^2 & \varepsilon^2 e^{i\omega} \\ \varepsilon^3 e^{i\beta} & \varepsilon^2 e^{i\omega} & 1 + \varepsilon \end{pmatrix} m_{\theta}^2
\]

\[
M_{E_L}^2 \simeq 0.5 \, M_{1/2}^2 \, \begin{pmatrix} 1 + \varepsilon^3 & \varepsilon^3 + c_{\text{run}} \varepsilon^3 \\ \varepsilon^3 \varepsilon + c_{\text{run}} \varepsilon^3 & 1 + \varepsilon^2 & \varepsilon^2 + 3 c_{\text{run}} \varepsilon^2 \\ \varepsilon^2 + c_{\text{run}} \varepsilon^3 & \varepsilon^2 + 2 c_{\text{run}} \varepsilon^2 & 1 + \varepsilon \end{pmatrix} m_{\theta}^2, \tag{4}
\]
with similar textures for down squarks. The structures in the symmetric textures are different from the asymmetric case and this provides a chance to distinguish the two Yukawa structures.

As said above, in this SU(3) flavour model CP violation is only broken spontaneously by the flavon vevs below the Planck scale. In this way all terms in the Kähler potential, giving rise to the soft masses and the $\mu$ term by the Giudice-Masiero mechanism are real before the breaking of the flavour symmetry. After breaking the flavour symmetry phases $O(1)$ will appear in the Yukawa matrices and the off-diagonal elements of the soft mass matrices. In this way $\mu$ is real to a very good approximation and similarly, diagonal elements in the trilinear terms are also real at leading order in the SCKM basis. In this way electric dipole moments (EDMs) are under control and the SUSY CP problem is solved. Nevertheless off-diagonal phases in the soft mass matrices contribute to the EDMs. For instance we have a contribution to the electron EDM as $d_e \propto m_{\tau} \mu \tan \beta \cdot \Im[\delta_{13}^{\ell} \cdot \delta_{31}^{\ell}]$. In figure 1 we show the expected contributions to the electron EDM assuming that the phases in the off-diagonal elements are $O(1)$ and the lepton Yukawas have CKM-like mixings [9]. We can see here that in this model, reaching a sensitivity of $10^{-29}$ e cm in the electron EDM will allow us to explore a significant region of the parameter space even for intermediate values of $\tan \beta$ [10].

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