RANS CANOPY CONSTANTS FROM WEAK TURBULENCE REGIME

J. Viana Lopes & J. M. L. M. Palma and A. Silva Lopes
Faculdade de Engenharia da Universidade do Porto,
CEsA - Centre for Wind Energy and Atmospheric Flows,
Rua Dr. Roberto Frias s/n, 4200-465 Porto, Portugal,
E-mail: jelopes@fe.up.pt

Summary A Taylor series expansion was used to model the effect of forest canopies in the $k-\varepsilon$ RaNS model transport equation for the turbulent kinetic energy. The model assumes that the turbulent kinetic energy is smaller than the kinetic energy of the mean flow (weak turbulence regime). Comparisons with large eddy simulations showed that a second-order approximation is able to account for 80% of the total canopy effect, while including third- and forth-order terms increases this value to 98%

INTRODUCTION

The modeling of turbulence over forested areas is a topic of major importance and current scientific interest. The standard approach (e.g., Katul et al., 2004) considers the foliage effect in the averaged momentum equation through a drag force to describe the momentum transfer between the wind and the vegetation, whereas in the $k-\varepsilon$ model equations the vegetation effect is taken into account via an additional source term. This additional term contains two opposite contributions, on the grounds that forest can simultaneously promote the production and the destruction of turbulence. The established phenomenological model is based on the so-called spectral short-cut (cf., Finnigan, 2000) mechanism, that removes energy from large eddies and diverts it to fine scales.

MATHEMATICAL MODEL

Following the standard approach, the canopy effect is introduced in the Navier-Stokes momentum equations through a canopy drag force. The momentum equations are,

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - C(z) |U| U_i$$

(1)

where $U$ and $P$ are the instantaneous velocity and pressure fields. The set of equations is completed with the continuity equation for incompressible flows,

$$\frac{\partial U_i}{\partial x_i} = 0 .$$

Here we followed the common approach, where the canopy is modelled through a drag force valid within the canopy height, $h_{can}$. The drag coefficient ($C_D$) and the leaf area density $a(z)$ are introduced in the definition $C(z) \equiv C_D a(z)$, which in the present case were identical to those as in Shaw and Schumann [1992], where the leaf area index $LAI = \int_0^{h_{can}} a(z) dz$ is equal to 2.

Applying the Reynolds decomposition, $U_i = \overline{U_i} + u_i$, we can split the velocity field ($U_i$) into the mean flow field ($\overline{U_i}$) and the fluctuation ($u_i$) and derive the exact transport equation of the turbulent kinetic energy, $k = \overline{u_i u_i}/2$, the basis of the RaNS $k-\varepsilon$ model,

$$\frac{\partial k}{\partial t} + \overline{U_i} \frac{\partial k}{\partial x_j} = - \overline{u_i u_j} \frac{\partial \overline{U_i}}{\partial x_j} - \frac{\partial u_j(u_j^2/2 + p)}{\partial x_j} + \nu \frac{\partial^2 k}{\partial x_j \partial x_j} - \varepsilon + S_k^{(\text{exact})}$$

where the effect of the canopy is present through the source term,

$$S_k^{(\text{exact})} = -C(z) |\overline{U}| \overline{U_i} u_i$$

(2)
Figure 1. In the left panel it is represented the turbulent kinetic energy \( k \) and mean flow kinetic energy \( K \) from LES simulation. On the right panel we represent the ration \( k/K \) used as perturbation parameter in the weak turbulent regime.

For completeness purposes, the \( k \) canopy source term is modelled by

\[
S_k^{(k-\varepsilon)} = C(z) \left( \beta_d U^3 - \beta_p U k \right)
\]  

\( k / K \approx 1 \)

where \( U \) is the modulus of the mean flow field, \( \beta_d \) and \( \beta_p \) are coefficients that, according with the literature (e.g., Sogachev and Panferov, 2006, Sanz, 2003), can vary between 0.05–1.0 and 0.0–5.1.

The exact canopy source term of the turbulent kinetic energy transport equation could be written as

\[
S_k^{(exact)} = -C(z) \overline{U_i u_i} = -C(z) \overline{(U u_i) + u_i u_i} \sqrt{U^2 + 2(U u_i) + u_i u_i} \]  

\( \beta_d \) and \( \beta_p \) are coefficients that, according with the literature (e.g., Sogachev and Panferov, 2006, Sanz, 2003), can vary between 0.05–1.0 and 0.0–5.1.

where \( u_i U_i / U \) is the projection of the instantaneous fluctuation field in the mean field direction and \( u_i u_i \) is related with the instantaneous turbulent kinetic energy. In fact, the mean field flow imposes in each point of space a special direction, breaking the rotational symmetry. We can define the perpendicular part of the fluctuating field as

\[
u_i^2 = u_i u_i - u_i^2.
\]

This symmetry break introduces a separation of the turbulent kinetic energy in two contributions,

\[
k = k_i + k_i,
\]

the parallel and perpendicular part of the turbulent kinetic energy.

The square root in (4) implies additional difficulties in the calculation of the time average. Nevertheless, this calculation could be performed in terms of fluctuation moments in the weak turbulence regime. In this limit, the turbulence is a small perturbation to the mean flow. Within these conditions, we can define the ratio of the turbulent kinetic energy and the mean flow kinetic energy \( (K = U^2/2) \) as the small parameter to use in the perturbation theory. This is the right regime whenever the condition

\[
\frac{k}{K} \ll 1
\]
is verified. To check the validity of the analytical results or assumptions we used large eddy simulations to compute the time averages of all the relevant analytical terms.

The LES were performed in a horizontal homogeneous canopy with fully developed flow in $x$ orientation. The Navier-Stokes equations were discretized in a non-staggered grid using finite volume approach and the subgrid stresses were accounted using the Lagrangian dynamic model. For further information on the LES code and implementation of the canopy model refer to Silva Lopes et al. [cf. 2007] and Silva Lopes et al. [2011].

In figure 1 we present the comparison of these kinetic energies computed from the LES results within the canopy region. Turbulent kinetic energy is not greater than 30% of the mean kinetic energy.

Applying the perturbation theory, we can express the canopy source term of $k$ equation as a series expansion,

$$S_k^{(exact)} = -C(z)\overline{U|U_iu_i|} \approx -C(z)\overline{U(u_i^2 + u_i^2)} - C(z) \frac{3\bar{u}_i\bar{u}_i^2}{2} - C(z) \frac{\bar{\|u\|}^2}{2U} + \cdots$$

with vanishing zero and first order terms. The second order term dominates the expansion. In figure 2 we discuss the validity of the expansion and show how the orders of the expansion vary with the tree height. Defining the truncated series as

$$S_n = \sum_{i=2}^{n} A_i,$$

we can conclude from figure 2 d) that difference from $S_4$ and $S_k^{(exact)}$ is less than 1%, which reinforce the validity of the assumption of weak turbulence regime.
The RANS $k-\varepsilon$ equations need to be expressed in terms of the resolved fields $\mathbf{U}$, $\varepsilon$, $k$, and $\tau$. The canopy source terms expressed in (5) are expressed as a function of field averages not resolved in $k-\varepsilon$ model. Nevertheless, we could use the standard approximations already used in the derivation of RANS $k-\varepsilon$ model to close the equations set. Using the eddy-viscosity model for the Reynolds stress,

$$u_i u_j = -\nu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) + \frac{2}{3} k \delta_{ij},$$

we can close the RANS equations set using only the second order expansion term,

$$u_i^2 = \frac{2}{3} k - 2\nu \frac{U_i}{U} \frac{\partial U}{\partial x_j},$$

$$u_i^2 = \frac{4}{3} k + 2\nu \frac{U_i}{U} \frac{\partial U}{\partial x_j},$$

and a new formulation alternative to the established approach (3) can be derived

$$S_2 = -C(z)2U(k + k_2) \approx C(z) \left( \frac{8}{3} U k + 2\nu \frac{U_i}{U} \frac{\partial U}{\partial x_j} \right).$$

Comparing this result with the standard model source canopy term in the equation (3), we notice the absence of the positive term ($\propto U^3$) and the introduction of a dependence on the turbulent viscosity. This last dependence is exactly zero when gradient of the fields is perpendicular to the flow direction, which is the case in our LES. Based on this order only, we could obtain the value of the two coefficients, $\beta_d = 8/3$ and $\beta_p = 0$, showing that the foliage does not promote the turbulence production.

**CONCLUSIONS**

The present work exploits an analytical approximation to the canopy source term in $k$ equation in RANS $k-\varepsilon$ model. This approximation suggests that $\beta_p = 0$ which means that the canopy effect is to destroy the kinetic energy. This conclusion agrees with the fit of the $S_k^{(exact)}$ to functional form in equation (3) obtained in Silva Lopes et al. [2011].

**References**

- J. Finnigan. Turbulence in plant canopies. *Annual Review of Fluid Mechanics*, 32(1):519–571, 2000. ISSN 0066-4189. doi: 10.1146/anurev.fluid.32.1.519. URL http://www.annualreviews.org/doi/abs/10.1146/anurev.fluid.32.1.519.
- G. G. Katul, L. Mahrt, D. Poggii, and C. Sanz. One- and two-equation models for canopy turbulence. *Boundary-Layer Meteorology*, 113:81–109, 2004.
- C. Sanz. A note on $k-\varepsilon$ modelling of vegetation canopy air-flows. *Boundary-Layer Meteorology*, 108:191–197, 2003. ISSN 0006-8314. URL http://dx.doi.org/10.1023/A:1023066012766. 10.1023/A:1023066012766.
- A. Silva Lopes, J. M. L. M. Palmia, and F. A. Castro. Simulation of the Askervin flow. Part 2: Large-eddy simulations. *Boundary-Layer Meteorology*, 125(1):85–108, 2007. ISSN 0006-8314. doi: 10.1007/s10546-007-9195-4. URL http://www.springerlink.com/content/s2122073272 Missouri.
- A. Silva Lopes, J. Palmer, and J. V. Lopes. Accounting for turbulence destruction in the flow over forests. To be presented at the 7th International Symposium on Turbulence and Shear Flow Phenomena, Ottawa, Canada, 26–31 July 2011.
- A. Sogachev and O. Panferov. Modification of Two-Equation models to account for plant drag. *Boundary-Layer Meteorology*, 122(2):229–266, 2006. ISSN 0006-8314. doi: 10.1007/s10546-006-9073-5. URL http://www.springerlink.com/content/86364800ju7235np/.