Gate-controlled Kondo screening in graphene: Quantum criticality and electron-hole asymmetry

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Abstract – Magnetic impurities in neutral graphene provide a realization of the pseudogap Kondo model, which displays a quantum phase transition between phases with screened and unscreened impurity moment. Here, we present a detailed study of the pseudogap Kondo model with finite chemical potential $\mu$. While carrier doping restores conventional Kondo screening at lowest energies, properties of the quantum critical fixed point turn out to influence the behavior over a large parameter range. Most importantly, the Kondo temperature $T_K$ shows an extreme asymmetry between electron and hole doping. At criticality, depending on the sign of $\mu$, $T_K$ follows either the scaling prediction $T_K \propto |\mu|$ with a universal prefactor, or $T_K \propto |\mu|^x$ with $x \approx 2.6$. This asymmetry between electron and hole doping extends well outside the quantum critical regime and also implies a qualitative difference in the shape of the tunneling spectra for both signs of $\mu$.

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Introduction. – Electrons in graphene provide an essentially perfect realization of two-dimensional (2d) Dirac fermions [1–3]. Scanning tunneling microscopy (STM) allows to locally study their electronic structure, including the physics of defects and impurity atoms. Recently, Kondo resonances for Co atoms adsorbed on heavily doped graphene have been reported [4]. On the theory side, the study of magnetic impurities coupled to 2d Dirac electrons started in the 1990s in the context of $d$-wave superconductors [5–7]. For a spin-1/2 impurity coupled to electrons with a density of states (DOS) obeying $\rho(\omega) \propto |\omega|^r$ — the so-called pseudogap Kondo problem (with $r=1$ for $d$-wave superconductors and graphene) — it has been established that Kondo screening exists for low temperatures $T$ only if the Kondo coupling $J$ is larger than a critical value $J_c$. The properties of the resulting quantum phase transition (QPT) have been studied extensively [5–11]. Interestingly, the fixed-point structure changes as a function of the bath exponent $r$ [7]. In particular, for $r > r^* \approx 0.375$ the critical fixed point is particle-hole (p-h) asymmetric, and $r=1$ plays the role of an upper critical dimension [10].

For an impurity adsorbed on graphene, its position relative to the underlying carbon honeycomb lattice and the character of its magnetic orbitals determine the electronic hybridization paths. As a result, orbital degrees of freedom are crucial for Kondo screening if the impurity is located at a high-symmetry position [12]. For the case of Co impurities, the most interesting situation has been identified as that of a C atom with spin 1/2 sitting in the center of a C hexagon. Here, a spin-orbit–driven crossover from $SU(4)$ Kondo physics at high energies to conventional $SU(2)$ Kondo physics at low energies is predicted. Hence, the low-temperature behavior is that of a standard Kondo model with a pseudogapped bath DOS. Moreover, the Kondo coupling $J$ for this model has been estimated to be rather close to $J_c$, corresponding to the QPT between screened and unscreened impurity phases in neutral graphene [12].

This opens the exciting possibility to observe Kondo criticality [13] in graphene. Even if a tuning of the coupling constant exactly to its critical value is impossible, quantum critical signatures can still be expected at energies above some crossover scale. The most important new aspect (as compared to $d$-wave superconductors) is the possibility of doping via a gate voltage, which moves the chemical potential $\mu$ away from the DOS minimum.

In this paper, we discuss near-critical Kondo physics in the pseudogap Kondo model with finite $\mu$, using
a combination of analytical and numerical renormalization group (RG) methods, with an eye towards possible experiments. As detailed below, we find an extremely strong asymmetry in the impurity properties between electron and hole doping. This is less related to the p-h asymmetry of the hybridization functions, but instead is dictated by the character of the relevant critical fixed point (dubbed ACR) in the pseudogap Kondo model, which has been shown to have maximal p-h asymmetry near \( r = 1 \) [11]. Scale invariance of the critical fixed point, realized for \( r < 1 \), implies that the Kondo temperature \( T_K \) is proportional to the gate voltage, \( T_K = \kappa |\mu| \), with a universal (and p-h asymmetric) prefactor \( \kappa \) [14]\(^1\), in a regime close to criticality shown in fig. 1. Remarkably, we find that logarithmic corrections at the upper critical dimension \( r = 1 \) conspire such that \( T_K = \kappa |\mu| \) still holds for one sign of \( \mu \), while for the other sign \( T_K \propto |\mu|^2 \) with a universal exponent \( x \approx 2.6 \). Even more importantly, this strong asymmetry in \( T_K \) between electron and hole doping extends over a large parameter range also away from criticality and should thus be easily detectable in graphene experiments. We also show that impurity tunneling spectra are sensitive to the sign of \( \mu \) as well.

While a number of theory works on Kondo impurities and STM in graphene have appeared previously [12,15–19], some of which also discussing aspects of gate-tuned Kondo physics [12,15], the dependence of \( T_K \) on the gate voltage has not been studied systematically, and the connection to quantum criticality has not been made precise. Note here that the often employed methods of poor man’s scaling and slave bosons provide a qualitatively incorrect description of the QPT near \( r = 1 \).

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\(^1\)Universal means that \( \kappa \) does only depend on the bath DOS exponent \( r \), but neither on its prefactor \( \rho_0 \) nor on high-energy properties of the DOS.

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**Model.** – Consider the standard spin-1/2 Kondo model, \( \mathcal{H} = \mathcal{H}_b + \mathcal{H}_{\text{imp}} \), with

\[
\mathcal{H}_b = \sum_{\sigma \sigma'} c_{\sigma}(0)^\dagger \tilde{t}_{\sigma \sigma'} c_{\sigma}(0) + \frac{J}{2} \mathbf{S} \cdot \mathbf{S}
\]

with a bath DOS \( \rho(\omega) \) at site 0 where \( c_{\sigma}(0) = N^{-1/2} \sum_k c_{\sigma,k} \), \( \tilde{t}_{\sigma \sigma'} \) is the vector of Pauli matrices. Complications arising from orbital degeneracies will be discussed towards the end of the paper. For the undoped case, we assume \( \rho(\omega) = \rho_0 |\omega|^r \) for \( |\omega| \) much smaller than some crossover scale \( \Lambda_P \), while we make no assumptions about \( \rho \) for large energies below the band cutoff \( \Lambda \) (where \( \rho(\omega) \) can be p-h asymmetric). A finite chemical potential \( \mu \) shifts the minimum of \( \rho \) away from the Fermi level, \( \rho(\omega) = \rho_0 |\omega + \mu|^r \), such that \( \mu > 0 \) corresponds to electron doping; we focus on \( |\mu| \ll \Lambda_P \). For graphene \( r = 1 \), \( \Lambda_P \approx 0.5 \text{eV} \) [12] and \( \Lambda \approx 8 \text{eV} \). The low-energy p-h asymmetry of the impurity problem arises from asymmetries of both the bath and the impurity (the latter coming from additional potential scattering).

**Chemical-potential-tuned crossover.** – For \( \mu \neq 0 \), the impurity spin will be screened below a temperature \( T_K(\mu) \) for any \( J \). While \( J > J_c \) results in a finite \( T_K \) as \( \mu \to 0 \), \( J < J_c \) yields an exponentially small \( T_K \). For graphene \( r = 1 \), \( \Lambda_P \approx 0.5 \text{eV} \) [12] and \( \Lambda \approx 8 \text{eV} \). The low-energy p-h asymmetry of the impurity problem arises from asymmetries of both the bath and the impurity (the latter coming from additional potential scattering).

**RG analysis.** – We start with the theoretically most interesting case \( J = J_c \). Scale invariance of the critical system at \( \mu = 0 \) entails \( T_K = \kappa |\mu| \) where \( \kappa(r) \) is a universal prefactor [15], because \( \mu \) scales as an energy and is a relevant perturbation at the critical fixed point. We shall show that this scaling prediction is indeed obeyed for \( r < 1 \). Notably, the prefactor is p-h asymmetric, \( T_K = \kappa_+ |\mu| \) for \( \mu > 0 \), for \( r^* < r < 1 \) where the critical behavior is controlled by the ACR fixed point [7,11].

The universal critical theory for the QPT near \( r = 1 \), which is not accessible from weak Kondo coupling, has been worked out in refs. [10,11]. It is the theory of a crossing of singlet and doublet impurity levels minimally coupled to conduction electrons, or, equivalently, a \( U = \infty \) Anderson impurity model. Using the notation of ref. [11], its impurity part can be written as

\[
\mathcal{H}_{\text{imp}} = \varepsilon_0 |\sigma\rangle \langle \sigma | + V_0 |\sigma\rangle \langle \epsilon_0 | 0 \rangle + \text{h.c.},
\]

where \( |\uparrow\rangle \) and \( |\downarrow\rangle \) represent the three allowed impurity states. \( \varepsilon_0 \) is the tuning parameter (“mass”) of the QPT, i.e., the (bare) energy difference between doublet and singlet states. The QPT occurs at some \( \varepsilon_0 = \varepsilon_c \), with screening present for \( \varepsilon_0 > \varepsilon_c \). The hybridization \( V_0 \) is marginal (relevant) at \( r = 1 \) \( \left( r < 1 \right) \), i.e., and \( r = 1 \) plays the role of an upper critical dimension. Due to \( U = \infty \),
the critical fixed point is maximally p-h asymmetric. Note that the pseudogap Anderson and Kondo models at any fixed \( r \) share the same universality class.

Reference [11] presented a field-theoretic RG analysis of the model (2) for \( r \leq 1 \), in a double expansion in \( V_0 \) and \( (1 - r) \). Here we generalize the RG to include a finite \( \mu \), restricting ourselves to the simpler momentum-shell scheme with ultraviolet (UV) cutoff \( D \). We introduce dimensionless couplings \( \varepsilon \), \( v \) according to \( V_0 = D^{(1 - r) / 2} \rho_0^{-1 / 2} v \) and \( \varepsilon_0 = D \varepsilon \) for the hybridization and mass, respectively. The one-loop RG equations read

\[
\frac{dv}{d \ln D} = -\frac{1 - r}{2} v + \frac{v^3}{2} F_1(\mu/D),
\]

\[
\frac{d\varepsilon}{d \ln D} = -\varepsilon + v^2 \varepsilon F_1(\mu/D) + v^4 F_2(\mu/D)
\]

with \( F_{1,2}(y) = |1 + y|^r \pm 2|1 - y|^r \). The last term in \( dv/d \ln D \) simply describes the level shift due to the real part of the bath Green’s function. For \( \mu = 0 \), the RG equations (3) reduce to those of ref. [11], with the flow depicted in fig. 2. They yield a critical point at \( v^* = (1 - r)/3 \) and \( \varepsilon^* = -(1 - r)/(3r) \) for \( r \lesssim 1 \) —this is the ACR fixed point. The fixed points \( \varepsilon = \pm \infty \) correspond to the screened and unscreened phases, ASC and LM, respectively.

For small \( \mu \neq 0 \), the flow at \( \varepsilon_0 = \varepsilon_c \) deviates from the critical fixed point on the scale \( |\mu| \) (see footnote 2). For negative \( \mu \), the flow is directly into the screened (ASC) phase, \( \varepsilon \rightarrow \infty \), implying \( T_K \sim |\mu| \), i.e., \( \kappa_- = 0(1) \). For a more detailed analysis, we can deduce \( T_K \) from the flow of \( m \equiv \varepsilon/v^2 \). From eqs. (3) one finds

\[
\frac{dn}{d \ln D} = -mr + F_2(\mu/D)
\]

cutoff \( D \) is reduced down to \( \mu \), following the flow equations for \( \mu = 0 \). The second stage starts at \( D = \mu \) (and \( m = m^* \) at \( J = J_c \)), and follows eqs. (3), (4). For simplicity, we may approximate here \( F_2(y) \approx -|y|^r \), which allows to integrate eq. (4) analytically. Finally, \( T_K \) may be defined as the value of \( D \) where \( m \) has changed by unity, \( m(T_K) = m^* + 1 \), which yields

\[
T_K = |\mu|[(r - 1)/|r - 1|]^{1/r},
\]

where \( W(x) \) is the Lambert \( W \) (or product log) function and \( e = 2.71828 \ldots \).

In contrast, for small positive \( \mu \) the flow at \( \varepsilon_c \) is driven towards negative \( \varepsilon \), i.e., the doublet (LM) fixed point, which implies that a spin-1/2 degree of freedom is formed on the scale \( |\mu| \). This moment is subsequently screened due to the finite DOS at the Fermi level —this second crossover is akin to conventional Kondo screening. It involves restoration of p-h symmetry and is therefore not described by the RG equations (3). To obtain a rough estimate of \( T_K \) here, we may again envision a second stage of RG, which starts at \( D = \mu \) and now uses the language of poor man’s scaling. This results in a conventional exponential expression for \( T_K \), here to be used with bandwidth \( \mu \) and dimensionless hybridization \( \sim \sqrt{1 - r} \) (taken from the fixed-point value \( v^* \)). Consequently, \( T_K = \kappa_+ |\mu| \), with \( \ln \kappa_+ \sim -1/(1 - r) \), i.e., \( T_K \ll |\mu| \) for \( r \lesssim 1 \).

For \( r = 1 \), the upper critical dimension of the problem, one may expect logarithmic corrections due to the logarithmic critical flow towards \( \varepsilon = v = 0 \), e.g., \( v^2 = V_0^2/|1 - 3V_0^2\ln(D/D_0)| \) where \( D_0 \) is the initial UV cutoff [11]. Remarkably, \( m^* = -1/r \) also applies to this logarithmic flow, and hence logarithmic corrections are absent from the \( T_K \) expression (5) for \( \mu < 0 \) (see footnote 3). For \( \mu > 0 \), the above two-stage RG analysis now suggests \( \ln \kappa_+ \propto \ln(|\mu/D_0|) \) because the relevant dimensionless hybridization is \( v^2 \propto -1/\ln(|\mu/D_0|) \). Hence, \( \kappa_+ \) becomes scale-dependent: the scaling prediction is spoiled and replaced by \( T_K \propto |\mu|/D_0^{1/x} \) where \( x > 1 \) is now a universal exponent. (To obtain an estimate for \( x \) here, a detailed analysis of the crossover between the two RG stages would be required which we do not attempt.) \( T_K \propto |\mu|^2 \) also implies that the low-energy impurity physics for \( \mu > 0 \) is characterized by two distinct scales, \( \mu \) and \( T_K \).

Let us summarize the main physics at this point. The fundamental asymmetry between electron and hole doping arises from the p-h asymmetric character of the critical theory (2). Its phase transition can be driven by varying the real part of the bath Green’s function. Now, varying \( \mu \) leads to a linear variation of this real part, and hence drives the critical system either towards the singlet or the doublet phase on the scale \( |\mu| \). We naturally expect that the electron-hole asymmetry, existing at the critical coupling, pertains to the off-critical situation as well. Our numerical results (fig. 5) show that this is indeed the case.

\footnote{2For \( \mu \neq 0 \), the problem at hand is not scale-invariant. This renders the standard coupling constant RG insufficient for quantitative calculations, because all vertex functions will develop structures on the scale \( \mu \). Our two-stage RG scheme partially accounts for this complication; a more accurate treatment could be achieved using functional RG.}

\footnote{3Weak additive logarithmic corrections may occur upon including higher loop orders. We found no evidence of those in the NRG calculations.}
Fig. 3: (Colour on-line) NRG results\(^4\) for impurity moment and entropy, \(T_{\chi_{\text{imp}}}\) and \(S_{\text{imp}}\), for a U = \(\infty\) Anderson model (2) at the critical point, \(\varepsilon_c = -0.3105935\), for different \(\mu\), with dashed (solid) curves for \(\mu > 0\) (\(\mu < 0\)). The bath DOS is \(\rho(\omega) = |\omega + \mu|\Theta(1 - |\omega + \mu|)\), i.e., \(r = 1\), and the hybridization \(V^2/g = 1/\pi\). The screening process displays an extreme p-h asymmetry.

For completeness, we also mention the result for \(T_K(\mu)\) at criticality for small \(r\). Here, the slave-boson mean-field treatment of (1) \([5]\) gives qualitatively correct results for the p-h symmetric critical (SCR) fixed point \([7,11]\). The mean-field equation determining \(T_K\) can be written as

\[
\frac{1}{\mathcal{J}} = \int d\omega \frac{\rho(\omega)}{\omega} \tanh \frac{\omega}{2T_K}.
\]

At \(\mu = 0\), setting \(T_K = 0\) gives the critical coupling \(J_c\). For a system with \(\rho(\omega) = \rho_0 |\omega + \mu|^r\) and \(J = J_c\), \(T_K(\mu)\) follows from the dimensionless equation

\[
\int dx \left( |x|^{r-1} - \frac{|x|^{r+1}}{x} \tanh \frac{x}{2T_K} \right) = 0,
\]

where \(x = \omega/|\mu|\), \(\tilde{T}_K = T_K/|\mu|\). For \(r < 1\), the ultraviolet cutoff of the integral can be sent to infinity. Note that the non-universal factor \(\rho_0\) has dropped out. The result for \(T_K\) is independent of the sign of \(\mu\), \(T_K = k|\mu|\). Solving eq. (7) yields \(\kappa \propto \exp(-1/r)\) for small \(r\). The same result can be obtained using a two-stage version of poor man’s scaling. In the first stage, \(\mu = 0\), the RG equation for the dimensionless Kondo coupling \(j\) is \(dj/d\ln D = -r j + j^2\) \([11]\). In the second RG stage, starting at the scale \(\mu\), the fixed-point coupling \(j^* = r\) simply enters the conventional exponential expression for \(T_K\), i.e., \(T_K = |\mu|\exp(-1/r)\).

**NRG results.** All the above considerations are well borne out by simulations using Wilson’s numerical RG (NRG) technique \([20]\). In fig. 3 we display NRG results for

\(^4\)The NRG parameters are \(\Lambda = 2\) and \(N_x = 800\).

the impurity magnetic moment \(T_{\chi_{\text{imp}}}\) and entropy \(S_{\text{imp}}\) of a \(U = \infty\) Anderson model with symmetric \(r = 1\) DOS at the critical coupling, \(\varepsilon = \varepsilon_c\), for different \(\mu\). At the critical fixed point \(T_{\chi_{\text{imp}}} = 1/6\) and \(S_{\text{imp}} = \ln 3\) \([10]\). Various features are apparent in fig. 3: The observables deviate from their critical value for \(T < |\mu|\). More importantly, there is a striking asymmetry between the two signs of \(\mu\), e.g., the magnetic moment \(T_{\chi}\) increases (decreases) for \(\mu > 0\) (\(\mu < 0\)) once \(T\) drops below \(|\mu|\). This reflects the fact that the system is driven towards the doublet or singlet phase depending on the sign of \(\mu\). Further, for \(\mu > 0\) a clear two-stage crossover is seen, where a spin-\(1/2\) moment is formed below \(T \sim |\mu|\), with \(T_{\chi_{\text{imp}}} \sim 1/4\) and \(S_{\text{imp}} \sim \ln 2\), which is only screened at very low \(T\). The power law behavior, \(T_K(\mu) \propto \mu^s\) for \(\mu > 0\), is visible, and a fit gives \(s = 2.6 \pm 0.1\).

Figure 4 shows the universal prefactors \(\kappa(r)\), obtained from NRG, of the “critical” relation \(T_K = k_{\pm} |\mu|\). Note that \(T_K\) has been extracted from \(T_{\chi_{\text{imp}}}\) (see footnote \(^3\)); in general, \(T_K\) is only defined up to a prefactor of order unity. We have verified universality of \(\kappa\) by performing calculations for both Kondo and \(U = \infty\) Anderson models and for various power law DOS with different high-energy parts. The results for \(\kappa\) nicely follow the asymptotic forms derived above, i.e., appear well fitted by \(\kappa(r \to 0) \approx \exp(-1/r)\), \(\kappa_{\pm}(r \to 1) \approx \exp[-1.7/(1 - r)]\), and \(\kappa_{\pm}(r \to 1) \approx [W(1 + r)/(1 + r)]^{1/r}\). NRG results away from the critical coupling will be discussed in connection with graphene in the next section.

We finally note that the role of the two signs of \(\mu\) is interchanged if the sign of the model’s p-h asymmetry is switched. Our quoted signs are valid for a \(U = \infty\) Anderson

\(^3\)For convenience, we have extracted \(T_K\) from the NRG data by the crossover criterion \(T_{\chi_{\text{imp}}}(T_K) = 0.1\). For selected parameters, we have checked that this matches the frequently used definition via the \(T \to 0\) specific heat coefficient \(\gamma_{\text{imp}}, T_K = 0.4128(\pi^2/3)\ln 3\) \([21]\), with a deviation less than 20%. Furthermore, NRG discretization effects lead to errors in \(T_K\) of order 10–20%.

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model, and also apply for a Co impurity on graphene, assuming a Kondo model and an orbital E1 configuration of Co [12], see below.

**Application to graphene.** – For Co in the center of a graphene’s hexagon, the hybridization function is strongly asymmetric on the scale of a few eV [12]. We have used a ρ(ω) which resembles the tight-binding fit to the LDA DOS in fig. 2 of ref. [12] (for Co in E1 configuration) to calculate TK in a standard Kondo model as a function of Kondo coupling J and chemical potential µ. Results are displayed in fig. 5. The strong electron-hole asymmetry is apparent, which continues to exist away from the critical coupling. At J = Jc the linear and power law behaviors of TK(µ) are nicely visible. Moreover, the minimum of TK(µ) for J > Jc is found at significant positive bias.

A few comments are in order: i) For simplicity, we have assumed a simple SU(2) spin-1/2 impurity. According to ref. [12], spin-orbit coupling reduces the symmetry of the Co impurity problem from SU(4) to SU(2) on a scale of 60 meV. This implies that our results are experimentally relevant only below a temperature of ~600 K (see footnote 7). Moreover, the Kondo coupling in our SU(2) model has to be taken essentially by a factor of 2 larger compared to the bare Kondo coupling of the Co impurity to obtain the same TK (because the impurity degeneracy multiplies the Kondo coupling within the poor man’s scaling equation.) Hence, we predict a Jc of 2.2 eV for Co which approximately matches the estimate in ref. [12]. ii) The existing STM data, with TK ≈ 15 K at |µ| = 0.2 eV [4], can either be consistent with our results for µ < 0 and J significantly smaller than Jc or with µ > 0 and J ≈ Jc. The two cases differ in the change of TK upon inverting the sign of µ: in the former case TK should become vanishingly small, but huge in the latter (which would still be measurable if µ is reduced). iii) The results in fig. 5 are switched according to µ ↔ −µ if the Co impurity is in an E2 configuration, because the hybridization functions for E1 and E2 symmetry are approximately identical after p-h transformation [12].

**Tunneling spectra.** – The different screening processes for µ ≥ 0 result in distinct spectral functions as well. In fig. 6 we show NRG results for the impurity T-matrix at T = 0, for situations with similar TK, but reached with different combinations of µ and J. Most striking is again the case J = Jc, where the high-energy part of the spectrum follows the critical law ImT(ω) ∝ 1/(ω|Imω|2) [6,11]. For µ < 0, the RG flow from critical to singlet behavior results in a large peak in ImT(ω) of width TK away from the Fermi level, located at ω ≈ TK ∼ −µ. In contrast, the two-stage screening for µ > 0 yields a more conventional Kondo peak at the Fermi level, but in addition a broad peak at a much larger energy ω ∼ −µ (see inset of fig. 6). This behavior persists away from J = Jc, i.e., the Kondo peak has a pronounced

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6. We found the van-Hove singularities of the graphene DOS to cause numerical problems upon conversion into a Wilson chain when µ ≠ 0. Therefore we employed a model DOS with a broadened van Hove peak, but otherwise the same characteristics as that of fig. 2 of ref. [12]. For this model DOS Jc = 4.7 eV while Jc = 4.3 eV for the graphene DOS.

7. For an SU(4)-symmetric impurity, the fundamental p-h asymmetry of the critical fixed point is expected to persist: the critical theory will have a structure similar to eq. (2), owing to the fact that the hybridization is marginal for τ = 1.

8. In our notation of the Kondo Hamiltonian, J is by a factor of 2 larger than that of ref. [12].
asymmetry w.r.t. the Fermi level for $\mu < 0$ (and $J$ not too far from $J_c$).

The impurity $T$-matrix spectrum, fig. 6, is in general not identical to the signal in a STM experiment, due to additional tunneling contributions into the host orbitals. The interference between the two is known to convert Kondo peaks into Fano lineshapes, with the Fano parameter depending upon details of the tunneling process [22]. Nevertheless, the position and width of the Kondo feature are independent of this interference. Hence, we predict that for $J \sim J_c$ the Kondo feature in STM will either be centered at the Fermi level or be located at an energy of order $T_K$ away from the Fermi level with a sign opposite to $\mu$, depending on the sign of $\mu$.

Conclusions. – We have discussed Kondo screening for magnetic impurities in doped pseudogap Fermi systems, with focus on the effects of nearby quantum criticality. For bath exponents $r < 1$, we found that the Kondo temperature follows the universal law $T_K = \kappa_\pm |\mu|$ for $\mu \geq 0$ in the critical regime. For $r = 1$, a subtle interplay of p-h asymmetry and logarithmic corrections at the upper critical dimension leads to an extreme p-h asymmetry in $T_K(\mu)$, with linear or power law behavior being realized depending on the sign of $\mu$.

For a situation corresponding to Co in graphene, we have predicted both the dependence of $T_K$ on $\mu$ and characteristics of low-temperature STM spectra. Most striking is the strong asymmetry between electron and hole doping over a large parameter range. This behavior is surprising because the relevant graphene DOS is weakly p-h asymmetric at low energies and hence a strong asymmetry is not expected from standard weak-coupling approaches. Instead, it is the strong p-h asymmetry of the critical fixed point of the pseudogap Kondo model which is responsible for the physics advertised here (and which is not captured by slave-boson or weak-coupling approaches to the Kondo problem).

Our findings call for systematic measurements of $T_K$ as a function of $\mu$, in particular for gate voltages in the linear DOS regime, i.e., below 0.2 eV. The asymmetry of $T_K(\mu)$, together with the ab initio data of ref. [12], will allow to determine the orbital nature of the Co impurity. Moreover, a linear or power law behavior of $T_K(\mu)$ will be a strong indication of nearby quantum criticality at $\mu = 0$.

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