Double Pion Production Reactions

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Abstract. We report on reactions producing two pions induced by real and virtual photons or nucleons. The role of different resonances in these reactions is emphasized. Novel results on coherent $2\pi$ photoproduction in nuclei are also reported.

1 Double pion photoproduction on the nucleon.

The $\gamma N \to \pi^+\pi^-N$ reaction is attracting attentions of both theoretical and experimental groups and is bound to play a significant role in photnuclear reactions much as the $\gamma N \to \pi N$ played in the past.

Apart from the work of the Valencia group which I will report here, there is work by other groups. A simplified model containing many of the important features of the reaction was worked out in [1] and improved in [2], [3]. The model of [3] contains more mechanisms than the one of [2], presently under revision. On the other hand, the model of [2] incorporates an approximate unitarization prescription which allows one to go to higher energies with the model. In ref. [5] a simplified model is also used incorporating, however, some $\rho$ decay channels. This work has been revised in [6] in view that some mass distribution was in disagreement with the data, and a new parametrization is offered, which relies on a range parameter for the $\rho$ of the order of 200 $MeV$, which would not accommodate easily other known facts of phenomenology as the isovector $\pi N$ s-wave amplitude.

The model of [3] contains parameters determined solely from $\gamma$ and $\pi$ couplings to nucleons and resonances plus known properties of resonance decay.
with some undetermined sign borrowed from quark models.

The \((\gamma, \pi\pi)\) reaction has also been studied at threshold with the aim of testing chiral perturbation theory \([7, 8]\), particularly the \(\gamma p \rightarrow \pi^0\pi^0 p\) reaction where chiral loops are very important.

The \(\gamma p \rightarrow \pi^+\pi^- p\) reaction was studied in ref. \([3]\) using effective Lagrangians, which incorporate the couplings of the photon and pion to the nucleon and resonances. The \(N\) and the \(\Delta(1232), N^*(1440)\) and \(N^*(1520)\) (or \(N^*\)) resonances were taken into account. Furthermore, the \(\rho\) as an intermediate state coupling to two pions was also considered. The model reproduces fairly well the experimental cross section \([9]\). The model is further improved \([10]\) to account for \(s - d\) waves in the \(N^* \rightarrow \Delta\pi\) decay, while at the same time reduces from 67 to 20 the number of Feynman diagrams needed to study the reaction in the range of Mainz energies \(E_\gamma \leq 800\) MeV. In ref. \([10]\) this simplified model is used to evaluate cross sections for all other charge channels: \(\gamma p \rightarrow \pi^+\pi^0 n\), \(\gamma p \rightarrow \pi^0\pi^0 p\), \(\gamma n \rightarrow \pi^+\pi^- n\), \(\gamma n \rightarrow \pi^-\pi^0 p\), \(\gamma n \rightarrow \pi^0\pi^0 n\). The disagreement with the data is overall good but some discrepancies remain in the peak of the \(\gamma p \rightarrow \pi^+\pi^0 n\) reaction and its charge conjugate one, the \(\gamma n \rightarrow \pi^-\pi^0 p\) reaction, recently measured \([11]\).

The relevance of this reaction for the resonance field is the novel information that it provides on the \(N^*(1520)\) resonance, which I try to explain here. In Fig. 1a, I show the dominant diagram in the \(\gamma p \rightarrow \pi^+\pi^- p\) reaction. It is the \(\Delta N\pi\gamma\) Kroll Ruderman or gauge term. On the other hand in Fig. 1b, I show a diagram where the \(N^*(1520, J^\pi = 3/2^-)\) is photoexcited from the nucleon and then it decays into \(\Delta\pi\), the \(\Delta\) decaying later into \(N\pi\).

![Feynman diagrams](image)

**Figure 1.** Feynman diagrams

From the \(1/2\) and \(3/2\) experimental \(N^*(1520)\) helicity amplitudes we can construct an effective Lagrangian from where we obtain a transition operator given by

\[
- i \delta H = i g_\gamma S^\dagger \epsilon + g_\sigma (S^\dagger \times \sigma) \epsilon ,
\]

where \(S^\dagger\) is a spin transition operator from spin \(1/2\) to spin \(3/2\). Furthermore, we write the \(N^* \rightarrow \Delta\pi\) transition operator as

\[
- i \delta H = -[f + \frac{\tilde{g}}{\mu^2} (S^\dagger q) (Sq)] T^{\dagger \lambda} + h.c. ,
\]
where \( T^\dagger \) is the isospin 1/2 to 3/2 transition operator and \( \mu \) the pion mass. The choice of eq. (2) is not arbitrary. It allows \( N^{*\pi} \rightarrow \Delta \pi \) decay in \( s \) and \( d \) waves and provides a \( q \) dependence of the amplitudes (\( q \) is the CM pion momentum) which provides the best agreement with experiment. By means of eq. (2) we can write the \( s \) and \( d \) wave decay amplitudes in \( N^{*\pi} \rightarrow \Delta \pi \). We find

\[
A_s = -\sqrt{4\pi}(\bar{f} + \frac{1}{3}\bar{g}\frac{q^2}{\mu^2}),
\]

\[
A_d = \sqrt{\frac{4\pi}{3}}\bar{g}\frac{q^2}{\mu^2},
\]

and the width is given by

\[
\Gamma_{N^{*\pi} \rightarrow \Delta \pi} = \frac{1}{4\pi^2} \frac{m_{\Delta}}{m_{N^{*\pi}}} q(|A_s|^2 + |A_d|^2).
\]

From the analysis of the \( \pi N \rightarrow \pi \pi N \) reaction of ref. [12] one has information on \( \Gamma_s, \Gamma_d \) plus also another ingredient, the relative sign of \( A_s \) to \( A_d \) which is positive. With this information we obtain \( A_s \) and \( A_d \) up to a global sign (a sign relative to the \( \gamma N \rightarrow N^\ast \) amplitudes). This sign is the first novel thing that the \( \gamma p \rightarrow \pi^+ \pi^- p \) reaction provides. Indeed we can see in Fig. 2 the results with the two signs and we observe that while one of the signs is in good agreement with the experiment (a), the other choice leads to unacceptable results (b).

\[ \text{Figure 2. Total cross section for } \gamma p \rightarrow \pi^+ \pi^- p \text{ reaction for different global sign} \]

The reason for the so different results with the two signs is that the \( N^\ast(1520) \) mechanism of Fig. 1b interferes with the dominant one of Fig. 1a. The two amplitudes (by taking the s-wave part of the \( N^{*\pi} \rightarrow \Delta \pi \) decay) have the same momentum and spin structure, and the \( N^\ast(1520) \) piece can be accounted for by making a simple substitution in the \( N\Delta \pi \gamma \) Kroll Ruderman piece:

\[
ed^{\frac{f^s}{\mu}} \rightarrow -(g_\gamma - g_\sigma)(\bar{f} + \frac{1}{3}\bar{g}\frac{q^2}{\mu^2})D_{N^{*\pi}}(s),
\]
where $D_{N^*}$ is the $N^*(1520)$ propagator. We can see that with the value $g_\gamma - g_\sigma = 0.157 > 0$ and $\hat{f} + \frac{1}{2} \hat{g} \frac{\omega}{m^2} > 0$ one gets a constructive interference before the $N^*$ pole and a destructive one after it. This is what can be observed in Fig. 2.

I shall not discuss here the other channels. Some results and comments can be found in the talk of Krusche in this Workshop [13]. The $\gamma p \rightarrow \pi^0 \pi^0 p$ is well reproduced and here the $N^*(1520)$ term does not show up through the interference but as the main term by itself. On the other hand there are some discrepancies in the $\gamma p \rightarrow \pi^+ \pi^0 n$ channel which we can not explain so far.

2 Repercussion of the $N^*(1520)$ findings on quark models.

With the values of $\hat{f} = 0.911$ and $\hat{g} = -0.552$ obtained from a fit to the $s$ and $d$ wave partial decay widths of $N^*(1520) \rightarrow \Delta \pi$ and the global sign given by the $\gamma p \rightarrow \pi^+ \pi^- p$ experiment, the amplitudes $A_s, A_d$ of eqs. (3) provide a definite $q$ dependence of these amplitudes.

As mentioned, this $q$ dependence is the one providing an optimal fit to the experiment. We have checked that any other $q$ dependence of the $s$-wave amplitude, consistent with the value for the on shell decay width, provides a worse agreement.

At this point it is worth mentioning the repercussion of these results in the quark models. This has been shown recently [15]. In this work a nonrelativistic constituent quark model using the input of Badhuri’s model [16], adapted by Silvestre-Brac to the baryonic sector [17], is employed, and the decay amplitudes $B \rightarrow B' \pi$ are evaluated. For this purpose one starts with a coupling of pions to quarks

$$H_{qq\pi} = \frac{f_{qq\pi}}{\mu} \bar{\psi}_q \gamma^\mu \gamma_5 \tau \psi_q \partial_\mu \phi$$  (6)

and makes a nonrelativistic expansion keeping recoil terms

$$H_{qq\pi} = f_{qq\pi} [\sigma q e^{-iqr} - \frac{\omega_p}{2m_q} \sigma (p e^{-iqr} + e^{-iqr} p)].$$  (7)

Now, when evaluating the $N^*, \Delta$ transition matrix elements, since one has a radial excitation in the $N^*$ state, one needs to expand the exponential in the first term of eq. (7) (direct term) up to order $q$. On the other hand, the second term of eq. (7) (recoil term) already gives a contribution keeping the unity in the expansion of the exponential. Hence, we find

$$\text{DIR} \propto q^2 ; \quad \text{REC} \propto 1$$  (8)

A direct evaluation of the $s$ and $d$-wave amplitudes for the $N^* \rightarrow \Delta \pi$ decay gives

$$\frac{A_d}{A_s} = \frac{\text{DIR}}{2 \text{REC} - \text{DIR}}.$$  (9)
which implies

\[ A_d \propto \text{DIR} \propto q^2 \]
\[ A_s + A_d \propto \text{REC} \propto 1 \]  \hspace{1cm} (10)

Hence, the non relativistic constituent quark model keeping recoil terms makes very clear predictions on the \( q \) dependence of the amplitudes. Now, by looking at the \( q \) dependence demanded by the \( \gamma p \rightarrow \pi^+ \pi^- p \) reaction, expressed in eq. 3, we obtain

\[ A_d = \frac{\sqrt{4\pi}}{3} \hat{q}^2 \mu^2; \ A_s + A_d = -\sqrt{4\pi} \hat{f}, \] \hspace{1cm} (11)

which is the exactly the \( q \) dependence provided by the quark model with recoil terms, eq. (10).

The global sign of these amplitudes preferred by the \( \gamma p \rightarrow \pi^+ \pi^- p \) experimental data is also the one provided by the quark model. This is another accomplishment of these quark models, but one should recall that not all variants of nonrelativistic, or relativized quark models will satisfy these new constraints. This is important to note since problems still remain when one comes to absolute values of these amplitudes. Extra work is needed to explain these discrepancies, but it is important that these improvements are done respecting the new constraints found thanks to the \( \gamma p \rightarrow \pi^+ \pi^- p \) reaction. Actually, a treatment similar to the present one but making an expansion in terms of \( \omega/\pi \) instead of \( \omega/\pi \) seems to lead to very much improved results, while keeping the consistency with the findings discussed in this section.

3 Meson exchange current and coherent 2\( \pi \) photoproduction

Assume the \( \gamma N \rightarrow \pi \pi N \) reaction occurs inside a nucleus and one of the pions, say the \( \pi^- \), is produced off shell and absorbed by a second nucleon. One obtains then meson exchange current mechanisms which contribute to the \( (\gamma, \pi^+) \) reaction in nuclei and which would be represented by diagrams like those in Fig. 1 with the \( \pi \) line attached to a nucleon line. This mechanism has already been explored in where it was found to contribute significantly to the \( \gamma^3 \text{He} \rightarrow t \pi^+ \) reaction at large momentum transfer.

In addition, the coherent 2\( \pi \) photoproduction process in nuclei has been studied in and has shown very interesting features tied to the isospin structure of the amplitudes. A photon coupling to a nucleon can have an isoscalar and isovector component. Assume we have the coherent reaction occurring in isospin \( I = 0 \) nuclei

\[ \gamma + A \rightarrow \pi^+ \pi^- + A_{g.s.} \]
\[ \pi^0 \pi^0 + A_{g.s.} \]  \hspace{1cm} (12)

and let us take the isoscalar part of the amplitude. This will force the \( \pi^+ \pi^- (\pi^0 \pi^0) \) system to have \( I = 0 \) and, because of symmetry, even angular
momentum $L = 0, 2, \ldots$ The isovector part will force the $\pi^+\pi^- (\pi^0\pi^0)$ system into $I = 1$. This is forbidden for the $\pi^0\pi^0$ system, so only the $\pi^+\pi^-$ can go with $I = 1$, which forces $L = 1, 3, \ldots$. The dynamics of the elementary reaction is such that the $\gamma N \rightarrow \pi^+\pi^- N$ reaction is dominated by the diagram of Fig. 1a, where the photon behaves as an isovector, while this mechanism is forbidden for $\pi^0\pi^0$ production. Obvious consequences of that are that the $\pi^+\pi^-$ system is largely suppressed when the pions travel together ($L = 0$ and hence $I = 0$).

Similarly the $\pi^0\pi^0$ system is only produced in $I = 0$ and hence the pions prefer to travel together. On the other hand the strength of the isoscalar part of the $\gamma N \rightarrow \pi\pi N$ amplitude is much smaller than the isovector part in the model of [10], and the consequence of it is that the maximum of the $\pi^0\pi^0$ cross section is about three orders of magnitude smaller than the maximum of the $\pi^+\pi^-$ one. These are very strong tests of the model which should encourage the experimentalists to perform such reactions.

### 4 Two pion electroproduction.

The model of ref. [9] can be extended to virtual photons coming from the $(e, e')$ vertex. These reactions are presently under experimental investigation at TJNAF [24, 23]. We have studied the $2\pi$ production processes where there is a $\Delta\pi$ in the final state, i.e., $ep \rightarrow e'\Delta^{\pm+}\pi^-$ and $ep \rightarrow e'\Delta^0\pi^+$ (with $\Delta^0 \rightarrow \pi^- p$). Only 8 diagrams of the model of [10] contain a $\Delta$ in the final state and we depict these diagrams below in Fig. 3

![Feynman diagrams](image)

**Figure 3.** Feynman diagrams used in the model for $\gamma_vp \rightarrow \pi\Delta$

The evaluation of the amplitudes for these reactions requires the extension of the model of [10] to account for the zeroth component of the electromagnetic current and the implementation of adequate form factors. This task has been undertaken in [20]. In Fig. 4 we show the results obtained for the cross section of the virtual photons, defined in the standard way

$$\frac{d\sigma}{d\Omega'_c dE'_c} = \Gamma(\sigma_T^{\gamma_v} + c\sigma_L^{\gamma_v}) = \Gamma\sigma_{\gamma_v}, \quad (13)$$
with $\Gamma$ and $\epsilon$ the flux factor and the polarization of the virtual photon [27]. The results are shown for the $ep \rightarrow e'\Delta^+\pi^-$ reaction as a function of $Q^2 = -q^2$ and averaged over the range $0.3 < Q^2 < 1.4 GeV^2$ in order to compare with the data. As one can see, the agreement is fair but more precise data are expected to come soon which will impose stronger constraints on the theory.

![Graph](image)

**Figure 4.** Cross sections for $\gamma_e p \rightarrow \Delta^+\pi^-$ as a function of the $\gamma_e p$ center of mass energy

5 **Application of isoscalar $N^*$ excitation in the $NN \rightarrow NN\pi\pi$ reaction.**

We have developed recently a model for the $NN \rightarrow NN\pi\pi$ reaction which contains terms coming from chiral Lagrangians, $\Delta$ excitation and Roper excitation [28]. The model is depicted in Fig. 5. The excitation on the second nucleon and antisymmetry are incorporated in addition. Summarizing the results we find that the $N^*$ excitation terms (4 - 7), where the $N^*$ decays into $N(\pi\pi)^{I=0}_{s-wave}$ are largely dominant close to threshold in the channels where the two pions can be in $I = 0$ in the final state, like the $pp \rightarrow pp\pi^+\pi^-$ reaction. On the other hand in the $pp \rightarrow pn\pi^+\pi^0$ reaction the $\Delta\Delta$ excitation terms are the most important. The comparison of these two channels allows us to appreciate the role played by $N^*$ excitation in some of the isospin channels which, as we can see, is essential to understand the experiment at low energies. In Fig. 6, we show the cross sections for the $pp \rightarrow pp\pi^+\pi^-$ and $pp \rightarrow pn\pi^+\pi^0$ reactions. The calculations are done with plane waves, but the results are increased at lower energies when final state interaction is considered, and the agreement with experiment is improved. In the figures, the total cross sections are given by the solid lines, corresponding to two different options for the $N(\pi\pi)^{I=0}_{s-wave}$ decay [28].

One of the important ingredients in this reaction is that the largest strength for $N^*$ excitation comes from isoscalar exchange. The strength of this transition was obtained from a theoretical analysis [28] of the $(\alpha,\alpha')$ reaction on proton targets exciting the Roper resonance [30].
In conclusion we have seen several reactions involving two pions in the final state. In all of them the $N^*$ resonances play an important role and we have clarified the links between some resonance properties and observables in $2\pi$ production reactions. Further investigations both theoretical and experimental, extending the work at higher energies, look also like a fertile land to extend our knowledge about $N^*$ resonance properties.

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