Forecasting performance comparison of daily maximum temperature using ARMA based methods

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Abstract. Daily maximum temperature of four different regions in Kerala, India, from 01/01/2019 to 31/12/2020, is recorded and is used for modelling and forecasting. The forecasting methods used are Autoregressive integrated moving average (ARIMA), Seasonal Autoregressive integrated moving average (SARIMA) and Autoregressive fractional integrated moving average (ARFIMA). The comparison of forecasting performance was based on percentage accuracy, mean squared error (MSE) and mean absolute error (MAE). The models used can capture the variations of time series data. All the models exhibit reasonably good performance in predicting the daily maximum temperature. ARFIMA model gives the least forecast errors compared to other models.

Keywords: Daily Maximum temperature, ARIMA, SARIMA, ARFIMA, Autocorrelation Function (ACF), Partial Autocorrelation Function (PACF)

1. Introduction

Kerala state of India is located between $8^\circ18'$ N and $12^\circ48'$ N latitude and $74^\circ28'$E and $77^\circ37'$ E longitude. It has humid tropical monsoon climates with good solar radiation and is popularly known as “God’s own country”. Kerala is witnessing great variations and uncertainties in its climatic pattern especially in the amount and timing of rainfall and the temperature variations. According to the Indian Meteorological Department, the state has witnessed an increase of 0.8$^\circ$C in the mean maximum temperature for the last 50 years. Kerala witnesses the hottest days between February and May. Palakkad and Punalur districts of Kerala records the highest of maximum temperatures, at times crossing above 40$^\circ$C. Apart from this, the state has also been witnessing huge variations and unpredictability in rainfall. In the year 2018, Kerala witnessed the worst flood in its history. During 2019 also, several parts of North Kerala suffered from flood and its calamities. These extremes in climatic conditions result in huge agricultural losses. The cash crops and thermo sensitive crops like cardamom, coffee, tea and black pepper of high range areas are threatened by these huge variations in weather conditions.
In this work of forecasting the daily maximum temperature, four stations are selected. They are Thiruvananthapuram, Punalur, Kochi and Palakkad. Thiruvananthapuram and Kochi can be grouped to coastal tracts while Punalur and Palakkad are considered as midlands.

Time series methodologies are based on the analysis of past data on the assumption that past values can be used to forecast future values. ARMA models were brought into light by Box and Jenkins in 1971[1]. ARIMA model is a generalisation of the ARMA model to include the case of nonstationarity. In SARIMA, seasonal differencing is used to remove the nonstationarity. ARFIMA is a generalisation of the ARIMA model which unlike ARIMA allows non-integer values of the differencing parameters.

2. Literature Review

Two popular time series models are Autoregressive (AR) and Moving Average (MA) models [1]. Combining AR and MA is Autoregressive Moving Average (ARMA)[1]. ARMA model is suitable for univariate, stationary time series modelling. Practically seen time series data usually contain both trend and seasonal patterns and are non-stationary [2]. ARIMA model is a generalised ARMA model which includes nonstationary case [1]. Finite differencing is applied to nonstationary data to make it stationary [3]. El-Mallah et.al. in 2016[4] showed that the ARIMA model performed well in making predictions of annual temperature in Libya. Muhammet in 2012 [5] used ARIMA to forecast temperature and precipitation of places in Turkey. Anitha et.al. in 2014[6] used SARIMA to predict the monthly maximum surface temperature of India. Abdul-Aziz et.al. in 2013[7] used the SARIMA model to predict monthly rainfall in Ghana. Time series data that exhibits Long-Range Dependence (LRD) gives algebraically and slowly decaying Autocorrelation Function (ACF). ARFIMA model gives better results for the time series data with LRD property. Hosking in 1981 [8] established the relation between fractional difference and LRD. Sun et.al. [9] applied the ARFIMA model to predict the elevation of Great Salt Lake. Contreras Reyes et.al.[10] in 2013 developed the statistical tools in ‘R’ for analysing ARFIMA.

3. Methodologies

3.1 ARIMA

Autoregressive models are used for forecasting when there is some correlation between present values in a time series and the past values. ARIMA is a type of univariate modelling where a time series is expressed in terms of past values of itself, which forms the AR component and the current and lagged values of the error term, which forms the MA component. ARMA models are used only for stationary time series. But the data like maximum daily temperature shows non-stationary behaviour. It also exhibits trend and seasonal patterns. The non-stationarity is accommodated by applying finite differencing of the data points. The formula of ARIMA (p,d,q) [1] is

\[ \phi(L)(1 - L)^d y_t = \theta(L)e_t \]  
\[ (1) \]

\[ [1 - \sum_{i=1}^{p} \phi_i L^i](1 - L)^d y_t = \left( 1 + \sum_{j=i+1}^{q} \phi_j L^j \right)e_t \]  
\[ (2) \]

p is the order of AR  
d is the level of differencing and  
q is the order of MA  
In ARIMA p,d and q are integers greater than zero. The integer d determines the level of differencing and in this work ‘d’ equals to 1 is sufficient to make the data stationary.
3.1.1 Manual ARIMA implementation

Manual ARIMA implementation is done by choosing the values of \((p,d,q)\) manually. As illustrated in figure 1 the first step of ARIMA implementation is checking for stationarity and is done by plotting and observing its Autocorrelation Function (ACF).

For stationary data, the values of ACF quickly degrade to zero, within ten lags but for non-stationary data, the degradation will happen slowly. Accordingly ACF of original data, first differencing and second differencing are plotted. ACF of original series is decaying very slowly and the ACF of first-order differencing goes to negative values quickly. So the order of differencing is chosen to be one. This is done for all the four stations and the observations and inferences are same for all the cases. Figure 2 shows the ACF of original data, first-order and second-order differencing of Thiruvananthapuram station.

To determine the value of ‘p’ and ‘q’, inspect the Partial Autocorrelation Function (PACF) and ACF of the first differencing respectively. From observations, the value of p is fixed to be ‘6’. The lag is determined by thresholding, which is 10% of the maximum peak. The plot of PACF of Thiruvananthapuram station is illustrated below in figure 3.

Figure 1. Determination of ‘p’, ‘q’ and ‘d’
Figure 2. Plot and ACF of Original series, first-order and second-order differencing of the daily maximum temperature of Thiruvananthapuram station from 01/01/2019 to 31/12/2020.

Figure 3. PACF of the Thiruvananthapuram station.
So the ARIMA (6,1,1) model is chosen for predicting the daily maximum temperature. The observations were similar for all the four stations.

![Finite Differencing](image)

**Figure 4.** Forecasting using the identified ARMA model

As illustrated in Figure 4, the identified model is fitted and the ARIMA (p,d,q) model is used to predict the next day maximum temperature of the station.

### 3.1.2 Auto ARIMA

In Auto ARIMA, the model itself will generate optimum values of p, q and d that is best suited for a particular time series to provide better forecasting. The final values are determined based on the values of the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The models with lower values of AIC and BIC are selected for forecasting. BIC is an index used in Bayesian statistics to choose between two or more alternative models. The formula for AIC and BIC are defined as (11):

\[
AIC = -\frac{2}{N} \times (\log \text{ - likelihood}) + 2 \times k/N \\
BIC = -2 \times (\log \text{ - likelihood}) + \ln(N) \times k
\]

Where k is the number of model parameters, N is the number of examples in the training dataset and log-likelihood is a measure of model fit.

The model obtained for the daily maximum temperature data of Thiruvananthapuram station is shown in figure 5.

![ARIMA Model Results](image)

**Figure 5.** ARIMA model results
As observed in the model, ‘css-mle’ method is used for defining the model parameters. Other commonly used methods are Yule-Walker procedure and Method of moments or Maximum Likelihood estimation (MLE). Here ‘css-mle’ stands for ‘conditional sum of squares-maximum likelihood estimation’. In this method conditional sum of squares likelihood is maximized and its values are used in the estimation of the exact likelihood via Kalman filter [12]. Similar models are obtained for all the stations and these models were used for predicting the daily maximum temperature.

3.2 Auto SARIMA

Seasonal ARIMA includes seasonal parameters in ARIMA. Configuring it requires finding hyper parameters for both trend and seasonal elements of the time series data [13]. The seasonal parameters used in addition to (p,d,q) are [13]:

- P - Seasonal AR ordering
- D - Seasonal difference ordering
- Q - Seasonal MA order
- M - Number of time steps for a single seasoned period.

Mathematical representation of a SARIMA(p,d,q)x(P, D, Q)_M model is:

\[ \phi_p(L^M) \varphi_p(L)(1 - L)^d(1 - L^M)^P y_t = \theta_q(L^M) \theta_q(L) \epsilon_t \]  \tag{5}

In equation 5, \( y_t \) represents the time series value at time t. \( \Phi, \varphi, \theta \) and \( \Theta \) are orders of \( p, P, q \) and \( Q \) respectively. \( L \) is the back shift parameters. \( M \) represents the order of seasonality and is 12 for monthly time series. d and D are non-seasonal and seasonal difference order. \( \epsilon_t \) represents white noise process. The SARIMA model obtained for the daily maximum temperature data of Thiruvananthapuram station is shown in figure 6.

**Figure 6.** SARIMA model results obtained for Thiruvananthapuram Station
Similar models are obtained for all the stations and are used for forecasting.

3.3 ARFIMA

ARFIMA is a generalised form of the ARIMA model with ‘d’ having fractional values. ARFIMA is capable of capturing both short-range and long-range dependence. Long memory property can be defined by ACF. It diverges for long memory time series data [14]. Granjer and Joyeux [15] first proposed the ARFIMA model. ARFIMA models have the flexibility to model many real situations by estimating the parameter d. The mathematical representation of ARFIMA is as follows:

$$\phi(B)y_t = \theta(B)(1 - B)^d\epsilon_t$$  \hspace{1cm} (6)

where d ranges between -0.5 and 0.5 and \((1-B)^d\) are defined as a fractional difference operator.

4. IMPLEMENTATION

In Python ‘statsmodel tsa’ and its associated libraries have functions for implementing ARIMA, Auto ARIMA and Auto SARIMA. The ‘tsa’ has useful functions for univariate time series analysis. It gives acf, pacf, periodogram and other statistical properties. For implementing Auto ARIMA and Auto SARIMA ‘auto-arima’ function of ‘pmdarima’ library is used. For ARFIMA, an R time series library ‘Arfima’ is used to simulate, fit and predict the time series.

5. RESULTS AND DISCUSSION

Daily maximum temperature data recorded in the four stations in Kerala are used to create ARIMA, SARIMA and ARFIMA models. All the programs are written in Python. To compare the forecast performance of different models, the following measures are used:

i) Mean Absolute Error (MAE)- It measures the average absolute deviation of forecasted values from the original ones.

$$MAE = \frac{1}{N} \sum_{n=1}^{N} |y(n) - \hat{y}(n)|$$  \hspace{1cm} (7)

ii) Percentage Accuracy- Its a measure of closeness of predicted value to actual value.

$$PercentageAccuracy = 100 - \frac{100}{N} \sum_{n=1}^{N} \frac{|y(n) - \hat{y}(n)|}{y(n)}$$  \hspace{1cm} (8)

iii) Mean Squared Error (MSE)- It measures the average squared deviation of forecasted values.

$$MSE = \frac{1}{N} \sum_{n=1}^{N} (y(n) - \hat{y}(n))^2$$  \hspace{1cm} (9)

The daily maximum temperature for the four stations is obtained from the website of the Indian Meteorological Department. The data from 1/01/2019 to 31/12/2020 is used for training and testing. Data from 01/01/2019 to 30/09/2020 is used as training dataset and observations from 01/10/2020 to 31/12/2020 are used as the testing data set. Training dataset is applied to fit the statistical models for daily maximum temperature. Testing dataset is used to assess the forecasting accuracy of the fit. In the manual implementation of ARIMA, original time series, it’s first differencing and their ACF and PACF plots are observed to fix the model. From the observations, the model for all the stations is intuitively fixed as ARIMA(6,1,1). Figure 7. shows the forecast diagram obtained for Thiruvananthapuram station.
Auto_Arima library is used to find the optimal order for ARIMA and SARIMA model. This is done by stepwise execution of hyperparameters and fitting models in parallel. At the end of the execution, it returns a fitted ARIMA/SARIMA model. To find ‘d’ and seasonal difference ‘D’ the model conducts differencing tests. Model is fitted within ranges defined by start_p order and max_p for AR order, start_q and max_q for MA order. For the seasonal orders of SARIMA, start_P,start_Q,max_P and max_Q are used. The optimum model is obtained using the Akaike Information Criterion(AIC), Corrected Akaike Information Criterion (AICC), Bayesian Information Criterion(BIC) and Hannan-Quinn Information Criterion(HQIC). Lowest values of these criteria correspond to the best model. Table 1 shows the ARIMA and SARIMA models obtained using Auto_Arima library, for all the four stations. Forecasting is then done using these models and performance matrices are evaluated and compared.

**Table 1.** ARIMA and SARIMA models obtained using Auto_Arima.

| Station       | ARIMA(p,d,q) | SARIMA(p,d,q)x(P,D,Q)M |
|---------------|--------------|------------------------|
| Thiruvananthapuram | (2,1,3)      | (1,1,1)(0,0,0)12       |
| Punalur       | (2,1,1)      | (2,1,1)(1,0,1)12       |
| Kochi         | (1,1,2)      | (1,1,2)(1,0,1)12       |
| Palakkad      | (1,1,1)      | (1,1,1)(1,0,0)12       |

Figure 8 and 9 show the forecasting diagrams for Auto-ARIMA and Auto-SARIMA models, for Thiruvananthapuram station, respectively.
Figure 8. Forecast diagram of Thiruvananthapuram station for ARIMA(2,1,3)

Figure 9. Forecast diagram of Thiruvananthapuram station using SARIMA(2,1,3)(2,0,2)12
For the prediction using ARFIMA, an R time series library package ‘Arfima’ is used to simulate, fit and predict the time series. In this, the model fits by direct optimisation. The fractional difference ‘d’ value that is obtained for Thiruvananthapuram is approximately 0.5. The main arguments that are used are the order c(p,d,q) and ‘numeach’. The first argument in Numeach indicates the number of starts for each AR/MA parameter. The second number indicates the number of starts for the fractional parameter. If it is set to zero, then no fractional noise is set. The number of starts in total is multiplicative. Figure 10 shows the forecast diagram of ARFIMA(1,0.5,1) model for Thiruvananthapuram station.

Figure 10. Forecast diagram of Thiruvananthapuram station using ARFIMA(1,0.5,1)

Table 2 below shows the comparison of forecasting performances based on Accuracy, Mean Absolute Error and Mean Squared error. The comparisons are based on the daily maximum temperatures of four selected stations in Kerala.

Table 2. Forecast performance measures for different forecasting methods for daily maximum temperature of four stations in Kerala.

| Station          | Model                  | Percentage Accuracy (%) | MAE  | MSE  |
|------------------|------------------------|-------------------------|------|------|
| KOCHI            | ARIMA(6,1,1)           | 97.08                   | 0.94 | 1.54 |
|                  | AutoARIMA(1,1,2)       | 97.11                   | 0.93 | 1.5  |
|                  | SARIMA(1,1,2)(1,0,1,12)| 97.15                   | 0.92 | 1.51 |
|                  | ARFIMA(1,0.5,1)        | 97.16                   | 0.92 | 1.48 |
| PALAKKAD         | ARIMA(5,1,1)           | 97.10                   | 0.89 | 1.62 |
|                  | AutoARIMA(1,1,1)       | 97.1                    | 0.89 | 1.61 |
|                  | SARIMA(1,1,1)(1,0,0,12)| 97.15                   | 0.92 | 1.51 |
|                  | ARFIMA(1,0.5,1)        | 97.16                   | 0.92 | 1.48 |
| THIRUVANANTHAPURAM | ARIMA(6,1,1)           | 96.83                   | 1.00 | 1.79 |
|                  | AutoARIMA(2,1,3)       | 96.84                   | 0.99 | 1.76 |
|                  | SARIMA(1,1,1)(0,0,0,12)| 96.87                   | 0.98 | 1.8  |
|                  | ARFIMA(1,0.5,1)        | 96.93                   | 0.97 | 1.75 |
| PUNALUR          | ARIMA(6,1,1)           | 96.79                   | 1.02 | 1.89 |
|                  | AutoARIMA(2,1,1)       | 96.85                   | 0.98 | 1.87 |
|                  | SARIMA(2,1,1)(1,0,1,12)| 96.8                    | 1    | 1.87 |
|                  | ARFIMA(1,0.5,1)        | 96.93                   | 0.97 | 1.81 |
Analysis of daily maximum temperature of four stations in Kerala shows similar dynamics. Here, ARIMA modelling is done using both manual mode and auto mode. Manual modelling of ARIMA is done by observing the ACF and PACF of the time series data. These models give reasonably good forecast values but the performance metrics of Auto ARIMA and Auto SARIMA gives even better values. ARFIMA models exhibit the best performance, even though by a small margin, its MAE and MSE values are the least among all the other models.

6. References

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