Auto– and cross–correlation analysis of the QSOs radio wave intensity

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Abstract. We discuss here the Flicker-Noise Spectroscopy approach to studying astrophysical systems, for example the radio wave intensity of quasi-stellar object (QSO) 1641+399 and BL Lacertae (BL Lac) 0215+015 in different frequency ranges. The presented method allows to parameterize the study dynamics using a short set of characteristics. The considering sources have a significant differences in manifesting the non-stationary effects, dynamical intermittency and synchronization. The radio wave intensity dynamics of the BL Lac 0215+015 is characterized by well-defined set of natural frequencies, persistent behavior with low effects of non-stationarity and high level of frequency-phase synchronization. For dynamics of the QSO 1641+399 reverse occurs including the asymmetrical structure of cross-correlator. Our findings show that using the flicker-noise spectroscopy approach to studying astrophysical objects allows to carry out the more detail analysis of their behavior and evolution.

1. Introduction

The main factors determining the evolution of accretive astrophysical systems are nonlinearity, non-stationarity, dynamic intermittency and also the collective effects [1]. The disk accretion of the astrophysical objects is largely determined by processes in high-temperature plasma. The variety of accretive objects and the high number effects realized in plasma of accretion disk, lead to necessity of using all the accessible analytical methods for the further understanding the properties of evolution of these systems [2].

In this work we use the Flicker-Noise Spectroscopy (FNS) [3] which is a phenomenological approach to studying the auto- and cross-correlation dynamics in time series. We study the dynamics of radio wave intensity of the QSOs and BL Lacs in frequency bands 2 and 8 GHz [4]. Experimental data have been provided by N. Tanizuka (Laboratory for Complex Systems Analysis, Osaka Prefecture University). The data registration has been carried out in period from 1979 to 1988 years (3309 days).

QSOs and BL Lacs are the most energetic and distant members of a class of objects called active galactic nuclei (AGN). QSOs and BL Lacs are extremely luminous and were first identified as being high redshift sources of electromagnetic energy, including radio waves and visible light, that appeared to be similar to stars, rather than extended sources similar to galaxies. The QSO’s spectra contain very broad emission lines, unlike any known from stars, hence the name “quasi-stellar”. The BL Lacs – another type of extragalactic radio sources, as distinct from QSOs, are characterized by continuous spectrum. According to the most common hypothesizes, QSOs and BL Lacs are the active nuclei of distant galaxies, in which a super massive black hole absorbs a matter from the gas-dust disk. At moving the layers of falling matter the collective effects and resonance phenomena arise. Active galactic nuclei are some of the most intrigue phenomena in the Universe because of a variety of their peculiar properties: a colossal energy release, a nonthermal character of radiation and its high and rapid variability in whole electromagnetic spectrum, from radio to gamma band. Studying these sources is an important in solving a high number of significant physical problems (for example, variations of fundamental constants over cosmic time [5]).

The prospectivity of using the FNS to study the signals generated by the accretive astrophysical objects is determined by introducing the information quantifiers characterizing the dynamic components in different frequency ranges. The FNS advantage consists in extracting information from the series of distinct irregularities (spikes, jumps, discontinuities of derivatives of various orders) by analyzing the behavior of time, spatial and power dynamic variables at each existential level of the
hierarchical organization of the system. Thus, the most valuable information is obtained by analyzing
the power spectra and the difference moments ("structural functions") of various orders. It is
necessary to point out, that the difference moments are formed exclusively by irregularities of a jump
type. On the other hand, the power spectra are formed by the contributions of two types of
irregularities: peaks and jumps. As distinct from capabilities of linear and non-linear methods, which
are used in studying the radio wave intensity of QSOs and BL Lacs (Hurst exponent, fractal and
correlation dimensions, phase space maps) [2], FNS allows to discover the system – specific
"resonances" and their interferential contributions at lower frequencies, chaotic "random walk"
("irregularity-jump") components at larger frequencies, and chaotic "irregularity-spike" (inertial)
components in the highest frequency range. The introduced cross-correlation expression allows to
describe the collective phenomena in distributed systems [3].

2. Principles of flicker-noise spectroscopy and basic relations

In FNS, introduced parameters for signal \( V(t) \), where \( t \) is time, are related to the autocorrelation
function [3]

\[
\psi(\tau) = \langle V(t)V(t+\tau) \rangle_{T-\tau}, \quad \langle \ldots \rangle_{T-\tau} = \frac{1}{T-\tau} \int_0^{T-\tau} \langle \ldots \rangle \, dt,
\]

where \( \tau \) is the time lag parameter (\( 0 \leq \tau \leq T_M \)) and \( T_M \) is the upper bound of \( \tau \) (\( T_M \leq T/2 \)). The averaging
over interval \( T-\tau \) implies that all the characteristics that can be extracted by analyzing functions \( \psi(\tau) \)
should be regarded as the average values on this interval. To extract the information contained in \( \psi(\tau) \)
(\( \langle V(t) \rangle = 0 \) is assumed), the following transforms, or \"projections\", of this function are analyzed:
cosine transforms (power spectrum estimates) \( S(f) \), where \( f \) is the frequency,

\[
S(f) = \int_0^T \langle V(t)V(t+t_i) \rangle_{T-\tau} \cos(2\pi f t_i) \, dt_i
\]

and its difference moments (Kolmogorov transient structure functions) of the second order \( \Phi^{(2)}(\tau) \):

\[
\Phi^{(2)}(\tau) = \langle [V(t)-V(t+\tau)]^2 \rangle_{T-\tau}.
\]

The information contents of \( S(f) \) and \( \Phi^{(2)}(\tau) \) are generally different, and the parameters for both
functions are needed to solve parameterization problems. By considering the intermittent character of
signals under study, interpolation expressions for the stochastic components \( S(f) \) and \( \Phi^{(2)}(\tau) \) of \( S(f) \)
and \( \Phi^{(2)}(\tau) \), respectively, were derived using the theory of generalized functions [6]. It was shown that
the stochastic components of structure functions \( \Phi^{(2)}(\tau) \) are formed only by jump-like irregularities
("random walks"), and stochastic components of functions \( S(f) \), which characterize the "energy
side" of the process, are formed by spike-like (inertial) and jump-like irregularities [7].

In FNS parameterization, the original signal \( V(t) \) is separated into three components: system-
specific "resonances" and their interferential contributions at lower frequencies, stochastic jump-like
("random walk") component at larger frequencies, and stochastic spike-like (inertial) component in
the highest frequency range. For simplicity, we will further refer to jump-like and spike-like
irregularities as "jumps" and "spikes", respectively.

Let us write the basic interpolation expressions for stochastic components. The parameters
characterizing the dynamic correlations on every level of the evolution hierarchy are assumed to be the
same. Consider the simplest case, in which there is only one characteristic scale in the sequences of
spikes and jumps:

\[
\Phi^{(2)}_j(\tau) \approx 2\sigma^2 \left[ 1 - \Gamma^{-1}(H_1) \cdot \Gamma(H_1, \tau/T) \right]^2, \quad \Gamma(s,x) = \int_x^{\infty} \exp(-t) \cdot t^{s-1} \, dt, \quad \Gamma(s) = \Gamma(s,0),
\]

where \( \Gamma(s) \) is the complete and incomplete gamma functions, respectively (\( x \geq 0 \) and \( s > 0 \)); \( \sigma \)
is the standard deviation of the measured dynamic variable with dimension; \( H_1 \) – is the Hurst exponent,
which describes the rate at which the dynamic variable \"forgets\" its values on the time intervals that
are less than the correlation time \( T_i \).

The interpolating function for power spectrum component formed by spikes can be written as
\[ S_n(f) = \frac{S_i(0)}{1 + (2\pi f T_0)^n}, \]

here \( S_i(0), T_0 \) and \( n \) are the phenomenological parameters introduced to describe the power spectra of experimental data. As a measure of chaotic signal component formed by spikes and jumps in high frequency range, it is useful to choose \( S_i(T_0) \), determined as “spikiness” factor.

The dynamics of complex systems includes both stochastic components, i.e., spikes and jumps, and system-specific slowly varying regular components associated with a set of frequencies. These frequencies correspond to internal and external resonances and their interferences. It should be noted that the whole set of resonance and interferential frequencies may get rearranged during the evolution of an open system. All the specific frequencies and their interferential contributions, which manifest themselves as oscillations in the \( S(f) \), will be further called “resonant”. It is assumed that \( S(f) \) can be presented as a linear superposition of stochastic component \( S_i(f) \) and resonant component \( S_r(f) \):

\[ S(f) = S_i(f) + S_r(f). \]

Here, we assume that the resonant components are statistically stationary (they depend only on time lag \( \tau \)). This allows us to estimate \( \psi_r(\tau) \) as an “incomplete” cosine transform of \( S_i(f) \) by applying the Wiener–Khinchin Theorem:

\[ \psi_r(\tau) \approx 2 \int_{f_d}^{f_g} S_i(f) \cos(2\pi f \tau) df, \]

where \( f_d = 0.5 f_{\text{max}}, f_g \) is the sampling frequency. It should be noted that (7) is an approximation applied to a finite discrete time series assuming the wide-sense stationarity of the resonant signal component. The resonant component \( \Phi_i^{(2)}(\tau) \) in this case is found by

\[ \Phi_i^{(2)}(\tau) = 2[\psi_r(0) - \psi_r(\tau)]. \]

The stochastic component of \( \Phi_i^{(2)}(\tau) \) can then be estimated as

\[ \Phi_i^{(2)}(\tau) = \Phi^{(2)}(\tau) - \Phi_i^{(2)}(\tau). \]

These equations allow one to sequentially separate out resonant and stochastic components of structure functions and power spectrum estimates for experimental time series and perform the parameterization of the components.

The information about the dynamics of correlations in variables \( V_i(t) \) and \( V_j(t) \), measured at different points \( i \) and \( j \), can be extracted by analyzing the temporal variations of various cross-correlation functions. Here, we will use the simplest “two-point” correlation expression characterizing the links between \( V_i(t) \) and \( V_j(t) \) [3, 8]:

\[ q_{ij}(\tau, \theta_j) = \left( \frac{\sqrt{2\sigma_i}}{\sqrt{2\sigma_j}} \right)^\frac{1}{\sigma_j(\tau)} \left( \frac{V_j(t+\tau) - V_j(t+\theta_j + \tau)}{\sqrt{2\sigma_j}} - \frac{V_j(t + \theta_j) - V_j(t + \theta_j + \tau)}{\sqrt{2\sigma_j}} \right)_{\tau - |\theta_j|}^{\tau + |\theta_j|}, \]

where \( \tau \) is the “lag” time (\( \tau > 0 \)), \( \theta_j \) is the “time shift” parameter. The cross-correlation expression \( q_{ij}(\tau, \theta_j) \) is a function of temporal parameters \( \tau \) and \( \theta_j \), which can be represented as a three-dimensional plot. Of most interest for the analysis are the intervals of \( \tau \) and \( \theta_j \) where the cross-correlation function \( q_{ij} \) approaches positive unity (maximum level of positive correlations) or negative unity (maximum level of negative correlations). The value of \( \theta_j \) corresponding to maximum values of cross-correlation \( q_{ij}(\tau, \theta_j) \) characterizes the cause-and-effect relation (“flow direction”) between signals \( V_i(t) \) and \( V_j(t) \). When \( \theta_j > 0 \), the flow moves from point \( i \) to point \( j \), when \( \theta_j < 0 \), from \( j \) to \( i \). When the distance between points \( i \) and \( j \) is fixed, the value of \( \theta_j \) can be used to estimate the rate of information transfer between these two points. The magnitude and behavior of the two-parameter expression (10) may significantly depend on the value of selected averaging interval \( T \) and upper-bound values of \( \tau \) and \( \theta_j \), which we will refer to as \( \tau_{\text{max}} \) and \( \theta_{\text{max}} \). From the statistical reliability point of view, we set a constraint of \( \tau_{\text{max}} + |\theta_{\text{max}}| \leq T/2 \). Analysis of cross-correlators allow to study the collective phenomena in distributed system [8, 9].
3. FNS analysis of QSO and BL Lac radio wave intensity

3.1. FNS parameterization of the radio wave intensity

To demonstrate the FNS approach capabilities we analyze the spectral flux density of the radio wave intensity from two sources: QSO 0215+015 and BL Lac 1641+399 [4]. In figure 1 and in table 1 we show the FNS-parameterization results. Power spectrum of 0215+015 has the peaks corresponding by quasi-periodic processes, dominating in studied dynamics. As distinct from 0215+015, the 1641+399's radio wave intensity is characterized by peaks absence that means the strong dynamic intermittence in disk accretion.

The different moment of the second order for 0215+015 is in satisfactory agreements with the FNS interpolation curve and demonstrates the distinct quasi-oscillating structure reflecting a strong influence of the jumps on studied dynamics. At the same time the influence of these irregularities on 1641+399 dynamics is lower.

**Figure 1.** FNS-dependences for radio wave intensity of 0215+015 (a, c) and 1641+399 (b, d) in frequency band 2 GHz. Power spectrum $S(f)$ (a, b) and difference moment of the second order $\Phi^{(2)}(\tau)$ (c, d): calculated on the basis of the experimental data (1, blue curve); interpolation FNS-dependence (2, green curve) and the resonant component $\Phi_{r}^{(2)}(\tau)$ (3, red curve).

Analysis of findings in table 1 allow us to conclude that the values of "spikiness" factor reflects the strong intermittence effects in dynamics of 1641+399 and low manifestation of these effects in activity of 0215+015. $H_{t}$ parameter shows the high trend stability in accretion processes of 0215+015 and the low trend stability in 1641+399.
Table 1. The FNS-parameters for signals of spectral flux density of the studied radio wave intensity.

| Source   | Frequency | $\sigma$ (rel. un.) | $H_1$ | $S_f(T_{01}^{-1})$ (rel.un.$^2 f_d^{-1}$) | $n$ |
|----------|-----------|---------------------|-------|-----------------------------------|-----|
| 0215+015 | 2 GHz     | 0.28                | 1.82  | 0.61                              | 0.34|
|          | 8 GHz     | 0.42                | 0.70  | 1.52                              | 0.33|
| 1641+399 | 2 GHz     | 2.36                | 0.20  | 36.24                             | 0.80|
|          | 8 GHz     | 2.41                | 0.3   | 133.84                            | 0.76|

3.2. Cross-correlations and frequency phase synchronization in spectral flux density

Figure 2 demonstrates 3D cross-correlators for radio wave intensity of the QSO and BL Lac. For the most visualisation of the cross-correlators we choose the following values of parameters: $T = 1000$ days; $\tau_{\text{max}} = 0.4 \, T$; $\theta_{ij} = 0.1 \, T$. Cross-correlators for 0215+015 radio wave intensity (figure 2a) has a clear oscillating structure and the specific frequency set. It means that the QSO radio wave intensity is characterized by the strong frequency-phase synchronization for signals in bands 2 and 8 GHz. The corresponding dependence for BL Lac 1641+399 (figure 2b) shows the large scale diffuse structure with no the characterized frequency set. It means that the synchronization here is low manifesting in comparison of 0215+015.

![Figure 2. 3D cross-correlation dependences $q_{ij}(\tau, \theta_{ij})$ for radio wave intensity of 0215+015 (a) and 1641+399 (b) in frequency bands 2 and 8 GHz.](image)

4. Conclusion

In this paper we have presented the findings of analysing the radio wave intensity of the two sources by means of the Flicker-Noise Spectroscopy. The basis of FNS is the introducing the describing relations for the different types of irregularities reflecting the resonant and chaotic components of the dynamics. It allows to parameterize a dynamics by using a small set of parameters. The two-point FNS cross-correlators allow to describe the synchronization and collective effects in QSO and BL Lac radio wave intensity.

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