On the Evolution of the Galactic Globular Cluster System

E. Vesperini
Scuola Normale Superiore Piazza dei Cavalieri 7, 56126-I, Pisa
E-mail: vesperin@astro.sns.it

ABSTRACT
By means of a semi-analytical method allowing us to follow the evolution of individual globular clusters spanning a large set of different initial conditions we address the issue of the origin of some observational properties of the galactic globular cluster system.

After a preliminary study of some general properties of the main evolutionary processes by a discussion of the relevant timescales and of a suitable “survival factor”, we investigate the evolution of systems of globular clusters located in a model of the Milky Way starting from different initial conditions for the mass function of the system (power-law and log-normal) and for the distribution of concentrations of individual clusters.

In particular, we study the role of the evolutionary processes in changing the spatial distribution and mass function of the cluster system, in establishing and/or preserving some of the observed correlations and trends between internal properties of globular clusters and between internal properties and location inside the host galaxy and we provide an estimate for the rates of core collapse and disruption of globular clusters.

For the initial conditions considered in this paper a significant fraction of clusters (∼60%) is lost because of disruption; the fraction of those undergoing core collapse is consistent with the present observational limits. The initial mass function and spatial distribution of the cluster system evolve quite significantly in one Hubble time and the evolution is toward a final state similar to the observed one. If the mass function is initially taken to be a log-normal distribution similar to the one currently observed in our galaxy, its shape is not significantly altered during the entire simulation even though a significant number of clusters are disrupted before one Hubble time, which suggests that the present mass function might represent a sort of ‘quasi-equilibrium’ distribution.

Key words: globular clusters: general

1 INTRODUCTION
Recent surveys of the observational properties of the globular clusters of our Galaxy (see, e.g., Djorgovski & Meylan 1994, hereafter DM) have shown the existence of interesting correlations and trends between structural parameters and between structural parameters and location inside the Galaxy, suggesting a close connection between dynamical evolution of clusters and the galactic environment of the host galaxy.

Many of the observed correlations still lack a solid theoretical interpretation; in particular it remains unclear to what extent the present properties of the globular globular cluster system reflect the initial conditions and to what extent they have been determined by evolutionary processes causing the complete evaporation of a fraction of primordial clusters and the modification of the initial properties of the surviving ones. While many theoretical studies suggest that the present cluster system, as the relic of the initial system (see, e.g., Aguilar et al. 1988, Lee & Goodman 1995, van den Bergh 1993, Murali & Weinberg 1996, Gnedin & Ostriker 1997), would have properties substantially different from those determined by formation...
processes, some observational evidence, such as the similarity of the luminosity function in galaxies of different types and different sizes (see, e.g., Harris 1991), is an indication that the local environment might be not so important.

In this paper we provide a simple theoretical framework for the investigation of the evolution of globular clusters. Here we extend and improve some of the tools and arguments considered in previous articles that addressed the issue of the efficiency of evolutionary processes in terms of an analysis of the relevant timescales (Fall & Rees 1977, Caputo & Castellani 1984) or on the same method adopted here (Chernoff, Kochanek & Shapiro 1986, hereafter CKS, Chernoff & Shapiro 1987) and show the results of a set of simulations following the evolution of the properties of a globular cluster system (mass function, spatial distribution, correlations between structural parameters) starting from given initial conditions. The main goal of the analysis is that of establishing the role of initial conditions and evolutionary processes in determining the present observational properties.

In section 2 we set the empirical background for our theoretical analysis. In section 3 we describe the method adopted to follow the evolution of individual globular clusters under the effect of internal relaxation and disk shocking. In section 4 we discuss the behaviour of the time scales associated with the above processes and introduce a survival factor describing the regions of the Galaxy where clusters of a given mass are more likely to survive and those where they are more efficiently destroyed. In section 5 we show the main results of some simulations which we have carried out by the method described in section 3 and by which we investigate the evolution of the main properties of globular cluster systems starting from specific initial conditions. Summary and conclusions are in section 6.

2 EMPIRICAL BACKGROUND AND GOALS OF THE THEORETICAL INVESTIGATION

The results of several observational studies of the Galactic globular cluster system (Chernoff & Djorgovski 1989, Djorgovski 1991, Trager, Djorgovski & King 1993, DM, Bellazzini et al. 1996 and references therein) can be summarized in the following way:

- Core parameters vs. total luminosity
  More luminous clusters have higher concentrations (Djorgovski 1991, DM, van den Bergh 1994), smaller cores (DM) and higher central densities (DM); the large spread present suggests the existence of a second parameter in the above correlations. As to the trend between core radius and luminosity, it is interesting to note that it is opposite to that observed for elliptical galaxies for which the size of the core increases with the luminosity of the system (see, e.g., Lauer 1985).

- Spatial distribution.
  The number density of clusters, \( n \) (number of clusters per cubic kiloparsec), is well fitted for galactocentric distances \( R_g > 3 \) kpc by a power-law \( n(R_g) \propto R_g^{-3.5} \) (see, e.g., figure 2 in Zinn 1985). The distribution shows a flattening in the inner regions whose origin is uncertain: the fact that clusters appear to be missing may be due to obscuration, to an intrinsic flattening of the distribution or to the effects of tidal destruction.

- Core parameters vs. total luminosity
  More luminous clusters have higher concentrations (Djorgovski 1991, DM, van den Bergh 1994), smaller cores (DM) and higher central densities (DM); the large spread present suggests the existence of a second parameter in the above correlations. As to the trend between core radius and luminosity, it is interesting to note that it is opposite to that observed for elliptical galaxies for which the size of the core increases with the luminosity of the system (see, e.g., Lauer 1985).

- Trends of structural parameters with the position in the Galaxy.
  Clusters closer to the galactic center tend to have higher concentrations and smaller cores (see figure 9 in DM). Moreover Chernoff & Djorgovski (1989) (see their figure 9) show that the fraction of post-core-collapse (PCC) clusters is a strong function of the distance from the galactic center with most PCC clusters located in the inner regions of the Galaxy.

- Luminosity function.
  The luminosity function of the Galactic globular cluster system is well fitted by a Gaussian with peak value \( \langle M_V \rangle = -7.24 \pm 0.13 \) and variance \( \sigma = 1.1 \) (Secker 1992; see also Abraham & van den Bergh 1995). Harris & Pudritz (1994) have shown that the luminosity function can be adequately described also by a multiple index power-law in the luminosity \( L \) and have suggested a possible origin for such a shape assuming that clusters have been originated from the cores of supergiant molecular clouds present in the early protogalactic epoch.

In this work we will try to clarify which of the observed correlations and properties of the galactic globular cluster system are likely to be the relic of primordial initial conditions and which instead are likely to be essentially independent of the detailed initial conditions and must have been mostly determined by evolutionary processes.
Special attention will be paid to:
- Characterization of the most important internal and external processes (with respect to a given cluster) that determine dynamical evolution and development of simple and flexible tools for their description, to be incorporated in a study of the evolution of a system of globular clusters from given initial conditions.
- Efficiency of the various evolutionary processes in affecting the results of given initial conditions and in erasing the memory of such conditions.
- Evolution of the shape and of the parameters that characterize the luminosity function of the globular cluster system.
- General correspondence with some observed correlations and trends.

3 EVOLVING A SYSTEM OF GLOBULAR CLUSTERS FROM GIVEN INITIAL CONDITIONS

3.1 Evolution of an individual cluster

The general method followed in the present investigation, which was introduced by King (1966) and later used by Prata (1971a,b), CKS and Chernoff & Shapiro (1987), is based on the assumption that the evolution of individual globular clusters can be approximately described as a sequence of King models with different concentrations, masses and radii.

A King model (King 1965) is identified by three parameters, the total energy $E_{\text{tot}}$, the total mass $M$ and the truncation radius $r_t$ which we shall assume to be determined by the tidal field of the galaxy (see, e.g., Spitzer 1987) and defined as

$$r_t = \left( \frac{M}{3M_g} \right)^{1/3} R_g,$$

where $R_g$ is the radius of the orbit of the globular cluster around the host galaxy and $M_g$ is the mass of the galaxy contained inside that radius. For simplicity, the cluster orbits are taken to be circular. After a given time interval, the change of mass and energy produced by evolutionary processes can be calculated and a new tidal radius can then be determined by the tidal condition (1). The total change in mass and energy is given by the sum of the variations of these quantities due to all the processes included in the investigation; in our investigation we have included the effects of disk shocking and internal relaxation.

The three new values of mass, energy and radius identify a new King model with different concentration and different scale parameters. The evolution of each cluster is followed until one of these three conditions is satisfied:

- $W_0 > 7.4$ ($W_0$ indicates the dimensionless central potential of a King model). Both analytical studies and numerical integrations of the Fokker-Planck equation (Katz 1980, Wiyanto, Kato & Inagaki 1985) indicate that King models with $W_0$ larger than this value are unstable against gravothermal catastrophe. Gravothermal collapse is driven by a thermodynamic instability and the description of the evolution of the cluster parameters during this phase is outside the reach of this method. Moreover the structure of a cluster in the PCC phase and its evolution are well known to depend significantly on the source of energy supporting the PCC expansion (3-body, tidal or primordial binaries), and in the case in which primordial binaries are the relevant source on the fraction and the binding energy distribution of these (Heggie & Aarseth 1992, Vesperini & Chernoff 1994). A reliable modelling of these aspects, as well as of other processes (e.g. gravothermal oscillations), is beyond the possibilities of the method adopted for our investigation.
- $W_0 < 0.05$. Under this value the system is conventionally considered dissolved (Chernoff & Shapiro 1987).
- $t = 1.5 \times 10^{10}$ years (Hubble time). The present era has been reached.

In the simulations discussed in section 5 we have also taken into account the effects of dynamical friction, even though in a very approximate way, by eliminating from the final sample of survived clusters all those for which the orbital decay timescale (see, e.g., Binney & Tremaine 1987) is smaller than one Hubble time. Given the initial conditions chosen the inclusion of dynamical friction does not change significantly the final results.

As a test for the numerical code developed in our work, some runs with the evaporation rate used in CKS have been carried out and the time evolution of the concentration shown in their figure [11a-c] has been reproduced.

It is clear that our final sample of globular clusters will include only those clusters that can still be represented as King models after one Hubble time, while the method cannot predict anything about the state of those which underwent gravothermal catastrophe before that time.

The main limitations of our approach are the impossibility of following the evolution of clusters after the onset of gravothermal catastrophe and the fact that we have restricted our study to circular orbits. The latter point suggests that some caution should be used when comparison is made with the observations, since some of the present results on the trends with galactocentric distance are expected to be affected by orbit mixing. On the other hand, our approach has the advantage of allowing us to single out certain dynamical effects in a model where only a few parameters are varied. The evolution of clusters with a more general distribution of orbits will be considered in a future work but, of course, at the cost of increasing significantly the relevant parameter space of initial conditions.
3.2 Internal relaxation

As a result of repeated weak encounters, stars can gain sufficient energy to escape from the system. Due to the diffusive nature of this process the energy of a single star escaping from the system is equal just to that necessary to reach the tidal radius where the cluster ends. This means that escaping stars do not carry away kinetic energy and that the change in cluster energy due to the evaporation of stars from the cluster driven by internal relaxation is

$$\Delta E_{\text{tot}} = -\frac{GM\Delta M}{r_t}$$

(2)

An estimate of the evaporation rate can be made using the Fokker-Planck equation. The evaporation rate is calculated as the flux of particles through the boundary in the energy space, \(E_t\)

$$\frac{\partial N}{\partial t} = -16\pi^2 \left( 4\pi G m^2 \ln(N/2) + q \int_0^{\tau_{\text{max}}}[2(E - \Phi)]^{3/2} r^2 \, dr \right)$$

(3)

where \(\Gamma \equiv 4\pi G^2 m^2 \ln(N/2); q \equiv \frac{1}{2} \int_0^{\tau_{\text{max}}} [2(E - \Phi)]^{3/2} r^2 \, dr \) \((\Phi(r)\) is the potential energy of a star at a radius \(r\) and \(\tau_{\text{max}}\) is the radius where \(\Phi(\tau_{\text{max}}) = E\) and we take \(f\) equal to the King distribution function. This can be written in the form

$$\frac{\partial N}{\partial t} = -(4\pi G)^2 \sqrt{G} \ln(N/2) \sqrt{M/R_t} F(W_0)$$

(4)

where \(F(W_0)\) depends only on the central dimensionless potential \(W_0\). The mass loss due to internal relaxation during a lapse of time \(\Delta t\) is

$$\Delta M = \langle m \rangle \frac{\partial N}{\partial t} \Delta t.$$

(5)

In the following we will generally refer to \(\langle m \rangle = 0.7 M_\odot\). The evaporation rate derived above is a factor 2-3 (depending on \(W_0\)) smaller than that used in CKS following King (1966) which overestimates the real rate as it is correct only in the central regions of the cluster.

We define the characteristic timescale for evaporation

$$T_{\text{ev}} \equiv \left| \frac{N}{\partial N/\partial t} \right| = \frac{2.27 \times 10^{12} \text{yrs}}{\ln(N/2)} \left( \frac{M}{10^2 M_\odot} \right)^{1/2} \left( \frac{r_t}{50 \text{pc}} \right)^{3/2} T_{\text{ev}}(W_0)$$

(6)

where \(T_{\text{ev}}(W_0)\) depends only on the central dimensionless potential \(W_0\) and it covers a range of values between 0.2 and 0.35.

Replacing \(r_t\) by its expression given by the tidal boundary condition (eq. (1)) and assuming an ‘isothermal’ model for the host galaxy \((M_/R_\odot = \text{const})\) we have

$$T_{\text{ev}} \propto M R_\odot T_{\text{ev}}(W_0).$$

(7)

3.3 Model for the host galaxy and disk shocking

When a globular cluster, during its orbital motion around the host galaxy, crosses the disk, due to the tidal interaction the stars, on average, increase their kinetic energy and some of them gain energy enough to escape from the cluster. If we select the direction of the \(z\) axis as that orthogonal to the galactic disk and approximate the motion of a single star inside the cluster in this direction as that of a harmonic oscillator with frequency \(\omega\) we can write the equation of the motion for the single star along the \(z\) axis, including the perturbation due to the presence of the disk, as

$$\frac{d^2 \delta z}{dt^2} + \omega^2 \delta z = g(t) \delta z$$

(8)

where \(g(t) = \left( \frac{d^2 c_\odot(R, z)}{dt^2} \right)\), \(K_\odot(R, z)\) being the acceleration due to the gravitational field of the disk at a given galactocentric distance \(R\) and at a distance from the disk of the center of the globular cluster \(z = V_t \cos \Theta\) (here \(t = 0\) denotes the instant when the cluster crosses the equatorial plane of the disk), \(\Theta\) is the angle between the direction of motion of the cluster and the \(z\) axis, \(V\) is the orbital velocity of the globular cluster, taken to be constant \(V \simeq 210 \text{km s}^{-1}\) corresponding to a constant ratio \(M_/R_\odot \simeq 10^2 M_\odot / \text{pc}\) and \(\delta z\) is the distance of the star from the center of the globular cluster along the \(z\) axis.

For the potential of the disk of our Galaxy we have used the one obtained by CKS by fitting with a two-component ‘isothermal’ model the acceleration along the \(z\) direction in the solar neighbourhood determined by Bahcall (1984) (taken to be at a galactocentric distance \(R_\odot\) equal to 8 kpc).

The vertical acceleration, \(K_\odot\), varies with the galactocentric distance according to the surface density that, according to the Bahcall-Schmidt-Soneira (1982) model for the Galaxy we will adopt in our investigation, falls off exponentially with an exponential scalelength \(h = 3.5 \text{kpc}\). The change of kinetic energy \(\Delta e\) per unit mass of a single star produced by the interaction with the potential of the disk can be shown to be
The change in energy is

\[
\Delta E = \frac{1}{2} \left( \Delta \nu^2 \right) = \frac{\pi^2}{8} (\Delta z^2) \omega^2 \left[ \frac{K_0}{z_0 \nu_0^2 \sinh(\pi \omega/2 \nu_0)} + \frac{K_1}{z_1 \nu_1^2 \sinh(\pi \omega/2 \nu_1)} \right]^2 \left( \frac{e^{-R_g/h}}{e^{-R_g/h}} \right)^2
\]

(9)

where \( K_0 = 3.47 \times 10^{-9} \text{cm s}^{-2}, z_0 = 175 \text{ pc}, K_1 = 3.96 \times 10^{-9} \text{cm s}^{-2}, z_1 = 550 \text{ pc} \) are the parameters for the disk model obtained by CKS and \( \nu_0 = \frac{\sqrt{3} \cos \Theta}{2 \pi} \) and \( \nu_1 = \frac{\sqrt{3} \cos \Theta}{2 \pi} \) are the characteristic frequencies of disk crossing by the globular cluster. For stars with \( \omega \ll \nu \) (impulsive limit) \( \Delta E \) is maximum, while if \( \omega \gg \nu \) (adiabatic limit) \( \Delta E \) tends to zero. Assuming

\[
\langle \Delta z^2 \rangle = \frac{r^2}{3}
\]

(10)

and

\[
\omega^2 = \frac{GM(r)}{r^3}
\]

(11)

we get that \( \Delta E \) depends only on the distance from the centre of the cluster.

The input of kinetic energy in the cluster due to the interaction with the disk is thus calculated as

\[
\Delta T = 4\pi \int_0^{r_t} \rho(r) \Delta E(r) r^2 dr.
\]

(12)

The integral includes only those stars that do not evaporate from the cluster after the interaction with the disk. The total change in energy is

\[
\Delta E_{ds} = -\frac{GM \Delta M_{ds}}{r_t} + \Delta T.
\]

(13)

where \( \Delta M_{ds} \) is the mass loss due to disk shocking. For the process of disk shocking we can define a characteristic time scale as

\[
T_{ds} = \left| \frac{E}{\partial E/\partial t} \right| \propto \sqrt{\frac{R_g^3}{GM}} \bar{T}_{ds}(W_0, M, r_t, \omega/\nu_1, K_i, R_g, \bar{h})
\]

(14)

where \( \partial E/\partial t \) is calculated as the ratio of the change of energy during one interaction with the disk (eq. [13]) to half of the orbital period \( T_{orb}/2 = \pi \sqrt{R_g^3/GM} \) and in the function \( \bar{T}_{ds} \) we have included both the dependence on the internal structure of the cluster and the structure of the disk. Fixing the structure parameters of the disk and using the tidal boundary relation [6], we find that the disk shocking timescale depends only on the galactocentric distance and cluster concentration

\[
T_{ds} \propto \bar{T}_{ds}(R_g, W_0)
\]

(15)

4 TIME SCALES AND SURVIVAL FACTOR IN THE NATURAL PARAMETER SPACE

The two timescales, \( T_{ev} \) and \( T_{ds} \), introduced in the previous sections are plotted in Figure 1 as functions of the galactocentric distance.

The timescales plotted represent the average of values corresponding to different values \( W_0 \) (\( W_0 = 1, 3, 4, 7 \)). For what concerns the timescale associated with internal relaxation, values corresponding to \( M = 10^4 M_\odot, M = 10^5 M_\odot \), and \( M = 10^6 M_\odot \) have been plotted. It is clear that disk shocking is more important for more massive clusters and particularly at distances close to \( R_g \sim 4 \text{ kpc} \) where disk shocking reaches its maximum efficiency. For galactocentric distances smaller than 4 kpc, as \( \omega \) increases to a larger extent than \( \nu \), the heating from disk shocking is adiabatically suppressed and the disk shocking timescale increases.

The conclusion of CKS and Chernoff & Shapiro (1987) concerning the faster dynamical evolution of clusters near the galactic center is confirmed by the behaviour of timescales. It is also evident that in the inner regions of the Galaxy, survival of less massive clusters is more difficult and we can expect that, as a result of dynamical evolution, massive clusters might become dominant in the inner regions. In the outer regions, all timescales are greater than the Hubble time and the processes driving the dynamical evolution of globular clusters become less and less efficient and we can expect that properties of the globular cluster system will be closer to the primordial conditions.

These qualitative expectations depend, of course, on the initial mass function of the globular cluster system: an initial mass function including many very massive clusters would be unaltered also in the inner regions (except for the effects of dynamical friction), while a ‘weak’ initial mass function including many low-mass clusters would be strongly affected also in the outer regions.

In order to make the above considerations more quantitative we have followed the evolution of clusters at a galactocentric distance between 1 kpc and 20 kpc starting with \( W_0 \) in the range \( 0.6 < W_0 < 7.25 \) and defined a survival factor, \( \xi \), as the ratio of the number of clusters that after a Hubble time have not yet undergone core collapse or dissolution to the total number of initial conditions considered. For this general calculation we have assumed that all the values of \( W_0 \) are equiprobable.
The survival parameter depends on the mass of the cluster and on the galactocentric distance. For each value of the mass of the globular cluster considered we have investigated the dependence of the survival factor on the galactocentric distance. In Figure 2 the survival factor versus \( R_g \) is plotted for six different values of \( M \). For less massive clusters \( \xi \) is different from zero only for the higher values of \( R_g \). For high-mass clusters disk shocking becomes important and the behaviour of \( \xi \) resembles that of disk shocking timescale with a minimum close to \( R_g \sim 4 \) kpc where disk shocking has its maximum efficiency.

We emphasize that the survival factor \( \xi \) we have calculated is intended to give a general quantitative indication of the trends induced by dynamical evolution. The behaviour of the survival factor and of the timescales confirms the conclusion drawn by CKS about the larger efficiency of evolutionary processes in the inner regions of the Galaxy. Inner clusters are destroyed more easily and this might account for at least some of the observed flattening in the number density of clusters and for the trend for more evolved clusters to be located closer to the galactic center; moreover less massive clusters are destroyed in the inner regions to a larger extent than the more massive ones and a trend for clusters in the inner regions of the Galaxy to be on the average more massive than outer clusters might result from the action of evolutionary processes.

Further interesting conclusions can be drawn from Figures 3-4 providing additional information on the fate of clusters of different initial masses located at different galactocentric distances. Evolution of clusters starting with different values of \( W_0 \) and \( R_g \) has been followed: Figure 3 shows only those values of the initial conditions such that clusters survive disruption and do not undergo core collapse. An analogous plot has been recently shown by Weinberg (1994a) (see his figure 9) who has implemented a Fokker-Planck code including accurate estimates of the effects of disk shocking which, moreover, have been modelled by means of a new expression of the adiabatic correction derived in Weinberg (1994b). While Weinberg's numerical model is able to follow the evolution of individual globular clusters in larger detail (see also Murali & Weinberg 1996) than we can do with the method adopted in this work, our method has the advantage of requiring a very small computer time for each model investigated thus allowing the investigation of a much larger set of different initial conditions; the survival region (to disruption) as obtained by Weinberg, for an initial mass of the clusters \( M = 3 \times 10^5 M_\odot \) essentially coincides with the one obtained in our work for the same mass lending support to the reliability of our calculations.

Figure 4 shows the final values of \( W_0 \) versus \( R_g \) for the survived clusters. It is interesting to note that if more massive clusters with low values of \( W_0 \) (low concentrations) were initially present, they are expected to survive with such low values of \( W_0 \) at larger galactocentric distances; some low-concentration massive clusters are also expected to be present in the inner regions due to the effects of disk shocking driving the clusters toward the dissolution. It is important to note also that if some high-concentration, low-mass clusters are present at the beginning they will still be present in the present epoch.

Thus the results summarized in Figure 4 lend support to the hypothesis that the observed correlation between concentration and mass is primordial and cannot be the result of evolutionary processes.

5 EVOLUTION OF A SYSTEM OF GLOBULAR CLUSTERS

5.1 Choice of Initial conditions

The present knowledge of the formation of globular clusters (see, e.g., Fall & Rees 1985, Vietri & Pesce 1995) and of the detailed initial conditions of the galactic globular cluster system is rather poor and it is not clear whether the present globular cluster system keeps some trace of its initial properties or it has largely lost memory of the initial conditions as a result of evolutionary processes. On the other hand, the analysis of the characteristic time scales associated with the evolutionary processes shows that evolutionary effects are likely to be small for clusters located in the outer parts of the Galaxy. Thus a promising procedure could be to assume that the initial properties of globular clusters do not vary with galactocentric distance to get from the current properties of clusters at large galactocentric distances some indication on the initial conditions for the entire system.

For each run carried out in our investigation the evolution of 1000 clusters has been followed by the method described in sect. 3. The set of initial conditions has been produced according to the distributions described below in this section and the model adopted for the Milky Way is that described in section 3.

Whenever we compare our results with the observational data, we will refer to the data of Chernoff & Djorgovski (1989) restricted to clusters with \( R_g < 20 \) kpc and \( c \leq 1.7 \) as for clusters in our theoretical analysis. The mass of clusters is calculated from their luminosity based on a constant mass-to-light ratio \( M/L_V = 2 \) (see, e.g., Mandushev, Spassova & Staneva 1991).

A globular cluster system is characterized once the following properties are specified:

- Initial mass function.

Three different initial mass functions have been considered:

- A truncated power-law mass function \( f(M) \propto M^{-2} \) with lower cut-off \( M_{\text{low}} = 10^{4.5} M_\odot \) and upper cut-off \( M_{\text{up}} = 10^{6} M_\odot \) (POW).
- A Gaussian distribution in the logarithm of the mass with mean value \( \log M > 4.5 \) and variance \( \sigma^2 = 1.0 \) (GAU1).
A Gaussian distribution in the logarithm of the mass with mean value $<\log M> = 5.0$ and variance $\sigma^2 = 0.25$ (GAU2).

The motivation for the choices made in our investigation can be summarized by the following questions:

1) Given a power-law initial mass function how does evolution change it? Can evolution drive it to a multiple-index power law or is this shape the result of formation processes as suggested by Harris & Pudritz (1994)?

2) Given a Gaussian initial mass function, will this shape be preserved during evolution? How does its mean value evolve and in which direction?

3) How much do the final mass functions in the three cases differ from each other?

As for the GAU2 case, we point out that the mass function of this case is very similar to the present mass function: the reason for investigating this case is that we wonder whether it is plausible that the initial mass function was essentially similar to the present one and to what extent it was modified by evolutionary processes.

We stress that, given the very limited number of different initial mass functions investigated, our analysis is not meant to provide any definitive indication on the entire possible set of initial conditions eventually evolving into a state resembling that currently observed for the galactic globular cluster system; a more extensive investigation spanning a larger set of different initial values of the parameters of the initial mass function, aimed to draw more specific conclusions on this issue, is currently in progress (Vesperini 1996, in preparation).

Of course, besides the above points, it will be interesting to investigate how all the final results, involving structural parameters, depend on the initial mass function.

- Spatial distribution.

The number density profile is chosen to be $n(R_q) \propto R_q^{-3.5}$ according to what is observed in our Galaxy between $4 < R_q < 20$ kpc, where the sample of observed globular clusters is likely to be complete (Zinn 1985).

- Distribution of orbital parameters.

For simplicity we have limited our analysis to circular orbits crossing the plane of the galactic disk perpendicularly.

- Initial distribution of $c = \log r_i / r_c$.

An interesting feature of the galactic globular cluster system is the correlation $c - \log M$. The results discussed in sect. 4 (see also Bellazzini et al. 1996) show that evolutionary processes are unlikely to play a dominant role in establishing the $c - \log M$ correlation. Thus we have decided to assume among the initial conditions a relation between $c$ and $\log M$. The relation sets an initial trend on the basis of the properties of the more massive and distant clusters (probably less affected by evolutionary processes)

$$c = -2 + 0.6 \log M$$

Notice that it is not obvious whether such a correlation would be preserved during evolution or not. In order to test the hypothesis that evolutionary processes might play some role in establishing the observed correlation (Djorgovski 1991, DM), we have also carried out two simulations starting from an initial condition with uncorrelated $c$ and $\log M$ (POW-nc and GAU1-nc).

### 5.2 Results

#### 5.2.1 Correlations between structural parameters

Figure 5 shows the final $c - \log M$ plane for the POW run (similar results have been obtained for the GAU1 and the GAU2 runs) with the line showing the initial condition of the simulation. It is interesting to point out that the evolutionary processes give rise to a spread similar to that present in the observational data. It is evident that the effect of evolutionary processes is that of scattering initial conditions: clusters undergo different evolution in the $c - \log M$ plane toward larger or smaller concentration and lose a different fraction of their initial mass depending on the efficiency of disk shocking and internal relaxation. In agreement with the trend emerging from the analysis of observational data (Bellazzini et al. 1996), in all three simulations, the correlation $c - \log M$ is stronger for outer clusters which preserve better the memory of the initial conditions while for those closer to the galactic center evolution is faster and tends to smear more efficiently the initial correlation. Our results give strong support to the hypothesis that the correlation between concentration and mass of clusters, or more in general between core parameters and mass (see Bellazzini et al. 1996), is primordial and not induced by evolutionary processes.

The results of the runs GAU1-nc and POW-nc strengthen this view further: Figure 6 (from the GAU1-nc run; an analogous result is obtained in the POW-nc run) shows that, if $c$ and $\log M$ are initially uncorrelated, also at the end of the simulation there is neither a correlation between these two quantities nor even a trend to create the correlation.
5.2.2 Mass function

Figures 7a-c show the initial and the final histogram of the globular clusters mass function for the three runs, while Figures 8a-c show the initial, the final and the observational cumulative distribution function (CDF) of log $M$. It is worth noting that all the final samples are just the relic of larger initial systems. Of course a 'stronger' initial mass function, including a larger number of high mass globular clusters, would be largely unaffected by evolution and its final state would be much closer to the primeval conditions.

Evolution changes the power-law initial mass function into a bell shaped mass function, resembling a Gaussian mass function in log $M$. Moreover from Figure 9 it is evident that evolution modifies the initial power-law distribution in a two-component power law distribution in $M$ consistent with the analysis of Harris & Pudritz (1994; see also references therein) who show that the mass function of galactic globular clusters is almost flat in the low-mass tail and a power-law with index $n = −1.7$ for masses $M > 0.75 \times 10^5 M_\odot$. The observed modification of the power-law mass function toward multiple power-law is equivalent to the trend for the distribution to assume a bell-shaped form resembling a Gaussian in log $M$ (see also McLaughlin 1994).

In the GAU1 run the gaussian shape is essentially preserved, but shifted to a higher mean value. The low-mass side is almost entirely destroyed by the evolutionary processes. Even though a thorough comparison of our final results with observational data is beyond the scope of this analysis, nevertheless the agreement between the final CDF for the GAU1 run and the observational one is remarkable. The similarity between the two distributions is even more remarkable if one considers how different the initial distribution is from the observed one.

The low-mass tail obtained in the final mass function of the POW run is produced by loss of mass of the less massive clusters present in the initial mass function while for the GAU1 run the low-mass tail is what is left of the much larger low-mass side of the initial mass function.

The result of the GAU2 is extremely intriguing. It is evident from Figures 7c and 8c that, though about 70% of the initial number of clusters undergo disruption or core collapse, the shape of the mass function is essentially preserved during the entire evolution. The result of this simulation shows the surprising result that a significant effect of evolutionary processes does not necessary imply a strong difference between initial and final mass function. It is tempting to suggest that the present mass function represents a sort of "dynamical equilibrium"; this hypothesis seems to be further supported by the following points:

- Figure 10a shows the time evolution of the mean value of log $M$ for the runs GAU1 and GAU2. $\langle\log M\rangle$ for the GAU2 run is approximately constant during the entire evolution, even though, as pointed out earlier, a significant number of clusters undergoes disruption or core collapse and thus are being excluded from the sample; $\langle\log M\rangle$ for the GAU1 undergoes a significant change until $\sim 4−5$ Gyr when it reaches a value close to that of the GAU2 run and after that it is almost constant and equal to the 'equilibrium' value. Figure 10b shows the time evolution of the total number of clusters.
- Figure 11 shows the CDF of log $M$ for the GAU1 run at five different times $t = 0, 2, 4, 9, 15$ Gyr; it is evident that after an initial lapse of time during which the distribution undergoes a significant change, it reaches a quasi-equilibrium state similar to the observational distribution and to the initial and final distribution of the GAU2 run.

The rate of evolution of the mass function of the globular cluster system is determined in part by the properties of the mass function itself and in part by the underlying spatial distribution of the clusters.

The properties of a mass function can be in a state of approximate "dynamic equilibrium", if the number of disrupted clusters is suitably compensated for by an appropriate mass evolution of the surviving ones as it appears to occur in the GAU2 run. On the other hand, as a cluster system evolves, the destruction of clusters at small galactocentric distances and of low-mass clusters in general tends to leave a population of clusters more resistant to evolutionary processes and eventually a sort of equilibrium can be achieved, which is approximately "static", at least on the long Hubble timescale.

Obviously the mass function would also be in equilibrium on the Hubble timescale (a "static equilibrium" right from the beginning, in this case) if all or most of the clusters were initially located at very large galactocentric distances. In this case not only the shape of the mass function would be preserved but also the total number of cluster would be essentially unchanged. Such equilibrium would be a trivial result expected for any initial condition. What we find in the GAU2 run is, instead, a major surprise since the mass function is found to persist not because of a particularly favourable underlying spatial distribution, but as a result of a subtle balance of competing processes. In a sense we have thus identified a "privileged" distribution, since it can persist long both in dynamic and in static (or quasi-static) equilibrium.

It may well be that by an appropriate choice of the initial spatial distribution, the GAU1 initial mass function (or any other possible mass function) would also stay practically unchanged, but this would be not so interesting since, most likely, the required spatial distribution would be less realistic. A systematic investigation of possible options in relation to this issue is currently in progress and the results will be discussed in a following article (Vesperini 1996, in preparation).

As for the difference between the mass function of inner and outer clusters, in all of three simulations evolutionary processes tend to give rise to a trend for inner clusters to be on the average more massive than outer clusters (see van den Bergh 1995 for the observational trend in our Galaxy and Crampton et al. 1985 for the analogous trend in M31), though the extent of
the difference depends on the initial mass function (see Vesperini 1994 and Vesperini 1996, in preparation, for further details on this issue).

5.2.3 Spatial distribution and trends with galactocentric distance

All runs start with a spatial distribution such that the number density of clusters \( n(R_g) \propto R_g^{-3.5} \). Figure 12 shows the histogram of radial position at the beginning and at the end of the simulation for the POW run (the histograms resulting from the runs GAU1 and GAU2 are similar). In all cases almost all globular clusters in the outer regions \( (R_g > 10 \text{kpc}) \) survive and the shape of the number density distribution is essentially unchanged. In the GAU1 run, for which the mass function is ‘weaker’, a slight depletion takes place also at larger galactocentric distances.

Figures 13a-c show the initial and final CDF of galactocentric distances for all the simulations. The CDF of observed galactocentric distances of galactic globulars with \( R_g < 20 \text{kpc} \) and \( c < 1.7 \) (as in our sample) is also shown. Evolutionary processes deplete the central regions and modify the initial CDF making it very similar to the observed one.

As for trends of structural parameters with the position inside the host galaxy, it is interesting to note the result shown in Figure 14 where the fraction of high concentration clusters (conventionally taken as those clusters with \( c > 1 \)) is plotted as a function of the galactocentric distance at the beginning and at the end of the simulation POW (analogous behaviour was obtained for the runs GAU1 and GAU2). It is evident that evolution tends to create a trend for more concentrated clusters to dominate near the galactic center, consistent with the observational evidence (Chernoff & Djorgovski 1989) that PCC clusters are preferentially located at small galactocentric distances (see Bellazzini et al. 1996 for further comments on the trends between core parameters and galactocentric distance).

5.2.4 Rates of core collapse and evaporation

As discussed in sect. 3 the method adopted for our investigation can not follow the evolution of clusters after they have reached the critical concentration for the onset of gravothermal instability and so the code does not provide information about their final fate; in particular, there remains the possibility that a few of these objects completely evaporate before one Hubble time. Nevertheless, it is interesting to give the following estimate of the fraction of the initial number of clusters undergone core collapse and evaporation and of the current rates of core collapse and evaporation of globular clusters. We introduce two quantities \( \xi_D \) and \( \xi_{CC} \), defined as the ratio of the number of clusters undergoing respectively disruption and core collapse before one Hubble time to the total initial number of clusters. Another quantity of interest is the ratio of the number of core collapsed clusters to the total number of clusters (core collapsed clusters plus normal ‘King model’ clusters) at the present epoch, \( f_{CC} \).

In order to take into account the possibility of complete evaporation for clusters undergoing core collapse, we have counted among the disrupted clusters described by the parameter \( \xi_D \) also those undergoing core collapse if their evaporation timescale, \( t_{ev} = M/(dM/dt) \), calculated at the time \( t_{cc} \) of the onset of gravothermal instability, is sufficiently short, i.e. if \( (t_{ev} + t_{cc}) \) is smaller than one Hubble time. The actual value of the evaporation time is likely to be smaller than \( t_{ev} \) defined above (see, e.g., Lee & Goodman 1995 for a detailed analysis of the evaporation rate of PCC clusters) and therefore, the values we supply represent actually an upper limit for \( \xi_{CC} \) and \( f_{CC} \) and a lower limit for \( \xi_D \).

In our theoretical analysis we obtain for the GAU1 run \( \xi_D = 0.610, \xi_{CC} = 0.145, f_{CC} = 0.370 \), for the GAU2 run \( \xi_D = 0.418, \xi_{CC} = 0.188, f_{CC} = 0.323 \) and for the POW run \( \xi_D = 0.587, \xi_{CC} = 0.121, f_{CC} = 0.292 \). In our galaxy the observed value of the frequency of core collapse clusters is \( f_{CC} \simeq 0.2 \) (see, e.g., Trager, Djorgovski & King 1993), in reasonable agreement with the above theoretical values.

By the above calculation it is also possible to estimate the current rate of clusters undergoing core collapse, \( F_{cc} \), and the current rate of cluster destruction, \( F_D \), providing the fraction of the present population of globular clusters undergoing core collapse or destruction per Gyr: \( F_{cc} = 0.05 \) and \( F_D = 0.03 \) for GAU1, \( F_{cc} = 0.03 \) and \( F_D = 0.03 \) for GAU2 and \( F_{cc} = 0.02 \) and \( F_D = 0.05 \) for POW. The above values are in good agreement with those obtained by Hut & Djorgovski (1992) on the basis of an analysis of the distributions of the central and half-mass relaxation times of galactic globular clusters and of a very simple model for their evolution; they estimate \( 2 \pm 1 \) clusters per Gyr to be the current rate of core collapse and \( 5 \pm 3 \) clusters per Gyr the current destruction rate, corresponding to a fraction of the current number of galactic clusters of \( F_{cc} \simeq 0.015 \pm 0.007 \) for the core collapse rate and of \( F_D \simeq 0.038 \pm 0.02 \) for the destruction rate.

Finally we point out that our simulations show that both \( F_{cc} \) and \( F_D \) may undergo a significant time evolution and the extrapolation of the current values back in time might lead to a wrong estimate of the past population of globular clusters.

\( \xi_{CC} \) and \( f_{CC} \) may represent an upper limit for \( t_{ev} \) and a lower limit for \( t_{cc} \). In our galaxy the observed value of the frequency of core collapse clusters is \( f_{CC} \simeq 0.2 \) (see, e.g., Trager, Djorgovski & King 1993), in reasonable agreement with the above theoretical values.
6 SUMMARY AND CONCLUSIONS

After setting the empirical background for our theoretical analysis with a brief summary of the observational properties of the Galactic globular cluster system, we have studied the evolution of globular clusters by means of a simple and flexible semi-analytical model.

A preliminary study of the efficiency of various evolutionary processes has been carried out by investigating the characteristic time scales and following the evolution of individual globular clusters starting from a large set of different initial conditions without making any particular assumption on the initial conditions of the galactic globular cluster system. This has allowed us to draw some general conclusions on the origin (primordial or induced by evolution) of a few interesting observational properties of the globular cluster system of our Galaxy. These conclusions have received further support by the results of a set of simulations following the evolution of globular cluster systems starting from specific choices of initial conditions for the initial mass function of the cluster system, for its spatial distribution and for the initial distribution of concentrations. By these simulations we have investigated how the relationships between structural parameters may change toward the observational results and in which way the initial mass function and spatial distribution are modified as a result of the evolutionary process. Although the main goal of this work was not a thorough comparison with the observations, nevertheless in some cases (spatial distribution, mass function) our results show excellent consistency with the observational data.

The main results of our analysis are the following:

(i) Correlation between structural parameters.
Evolutionary processes are unlikely to be responsible for the observed correlation between concentration and mass. In fact, if low-concentration high-mass clusters were present at the time of cluster formation, many of them should still populate that region of the \( c - \log M \) plane. Some high-concentration low-mass clusters are also expected to be present if concentration and mass were initially uncorrelated.

Our results point to a primordial origin for this correlation with evolutionary processes giving rise to the observed scatter.

(ii) Mass function
Three simulations starting with different initial mass functions have been carried out: a power-law initial mass function (POW), a log-normal with a mean value significantly smaller than the observed value (GAU1) and a log-normal equal to the present mass function of galactic globular clusters (GAU2).

In the POW run the final mass function has a bell shape in the logarithm of the mass resembling a Gaussian; if binned in \( M \) the final mass function can be described by a two-component power-law distribution.

In both the GAU runs the gaussian shape of the mass function is preserved in the final sample. For the GAU1 run, the mean value of \( \log M \) undergoes a significant evolution from the initial value \( \langle \log M \rangle = 4.5 \) to a final value, \( \langle \log M \rangle = 5.05 \), approximately equal to the observed one. Inspection of the time evolution of \( \langle \log M \rangle \) for the GAU1 run shows that after an initial stage during which a significant change is observed, evolution slows down reaching a state of quasi-equilibrium. In this 'quasi-equilibrium' state, although a significant disruption and core collapse of clusters still take place, the value of \( \langle \log M \rangle \) is approximately constant. GAU2 run starts with an initial condition close to this state of quasi-equilibrium and both the shape and the parameters of the initial mass function do not change during the evolution even though 70\% of the clusters of the initial sample undergo disruption or core collapse before one Hubble time and then are not included in the final sample.

(iii) Trends with galactocentric distance.
The observed trend for more concentrated clusters to be located in the inner regions has been shown to be a result of evolution.
A trend for inner clusters to be on average more massive than those located in the outer regions is produced by evolutionary processes. The extent of this effect depends on the initial mass function of the globular cluster system and it is more pronounced if the initial mass function has a larger fraction of low-mass clusters.

(iv) Spatial distribution.
A strong depletion of clusters in the very inner regions of the galaxy has been observed. A good agreement between the spatial distribution of clusters resulting from our simulations and the observational data has been obtained in all the runs done starting with \( c \) and \( \log M \) initially correlated.

Future work will extend this study by investigating the evolution of globular cluster systems starting from a much larger number of different initial conditions (Vesperini 1996, in preparation). Moreover we plan to study the dependence of the above results on the model for the host galaxy (see Vesperini 1994 for some preliminary results) and to include processes, such as bulge-shocking, that are expected to occur in the presence of non-circular orbits.
ACKNOWLEDGMENTS

I wish to thank Giuseppe Bertin for many enlightening discussions. I am very grateful to him for his very careful reading of this paper and for the numerous suggestions and comments.

I wish to thank T.S. van Albada, D.C. Heggie and M. Stiavelli for many useful discussions.

REFERENCES

Abraham R.G., van den Bergh S., 1995, ApJ, 438, 218
Aguilar L., Hut P., Ostriker J.P., 1988, ApJ, 335, 720
Bahcall J.N., 1984, ApJ, 287, 926
Bahcall J.N., Schmidt M., Soneira R.M., 1982, ApJL, 258, L23
Bellazzini M., Vesperini E., Ferraro F.R., Fusi Pecci F., 1996, MNRAS, 279,337
Binggeli B., Sandage A., Tarenghi M., 1984, AJ, 89, 64
Binney J., Tremaine S. 1987, Galactic Dynamics,
Princeton University Press, Princeton, New Jersey
Caputo F., Castellani V., 1984, MNRAS, 207,185
Chernoff D.F., Kochanek C.S., Shapiro S.L., 1986, ApJ, 309, 183 [CKS]
Chernoff D.F., Shapiro S.L., 1987, ApJ, 322, 113
Chernoff D.F., Djorgovski S.G., 1989, ApJ, 339, 904
Crampton D., Cowley A.P., Schade D., Chayer P., 1985, ApJ, 288, 494
Djorgovski S.G., 1991, in ‘Formation and Evolution of Star Clusters’,
ed. K.Janes, ASP, p. 112
Djorgovski S.G., Meylan G., 1994, AJ, 108,1292 [DM]
Fall S.M., Rees M.J., 1977, MNRAS, 181,27p
Fall S.M., Rees M.J., 1985, ApJ, 298, 18
Gnedin O.Y., Ostriker J.P., 1997, ApJ, 474, 223
Harris W.E., 1991, ARAA, 29, 543
Harris W.E., Pudritz R.E., 1994, ApJ, 429, 177
Heggie D.C., Aarseth S.J., 1992, MNRAS, 257, 513
Hut P., Djorgovski S.G., 1992, Nature, 359, 806
Ichikawa S., Wakamatsu K., Okamura S., 1986, ApJS, 60, 475
Katz J., 1980, MNRAS, 190, 497
King I. R., 1965, AJ, 70, 376
King I. R., 1966, AJ, 71, 64
Lauer T. R., 1985, ApJ, 292, 104
Lee H.M., Goodman J., 1995, ApJ, 443, 109
Mandushev G., Sspassova N., Staneva A., 1991, A&A, 252, 94
McLaughlin D.E., 1994, PASP, 106, 47
Murali C., Weinberg M., 1996, MNRAS, submitted
Prata S.W., 1971a, AJ, 76, 1017
Prata S.W., 1971b, AJ, 76, 1029
Secker J., 1992, AJ, 104, 1472
Spitzer L., 1987, 'Dynamical Evolution of Globular Clusters', Princeton
University Press
Trager S.C., Djorgovski S.G., King I., 1993 in : Structure and Dynamics
of Globular Clusters, p.343, eds. S.G. Djorgovski & G. Meylan ASP
van den Bergh S., 1993, in : Structure and Dynamics
of Globular Clusters, p.1, eds. S.G. Djorgovski & G. Meylan ASP
van den Bergh S., 1994, ApJ, 435, 203
van den Bergh S., 1995, AJ, 110, 1171
Vesperini E., 1994, Ph.D. Thesis, Scuola Normale Superiore, Pisa
Vesperini E., Chernoff D.F., 1994, ApJ, 431, 231
Vietri M., Pesce E., 1995, ApJ, 442, 618
Weinberg M., 1994a, AJ, 108, 1414
Weinberg M., 1994b, AJ, 108, 1403
Wiyanto P., Kato S., Inagaki S., 1985, PASJ, 37, 715
Zinn R., 1985, ApJ, 293, 424
FIGURE CAPTIONS

Figure 1 Evaporation (straight lines) and disk shocking timescale versus galactocentric distance. Average of timescales corresponding to $W_0 = 1, 3, 4, 7$ have been plotted.
Figure 2 Survival factor versus galactocentric distance for different initial values of the cluster mass.
Figure 3 Initial conditions in the plane $W_0 - R_g$ of clusters surviving disruption and core collapse. The evolution of clusters with initial conditions in the grid $2 < W_0 < 7, 3 < R_g(\text{kpc}) < 18$ has been followed. The points plotted are those initial conditions that do not lead to the disruption or core collapse of clusters in one Hubble time. Empty regions indicate initial conditions for which the cluster undergoes disruption or core collapse before one Hubble time. Different symbols indicate different values of the initial mass: squares $M = 3 \times 10^4 M_\odot$, crosses $M = 3 \times 10^5 M_\odot$, circles $M = 3 \times 10^6 M_\odot$.
Figure 4 Final $W_0$ versus galactocentric distance for clusters surviving disruption and core collapse whose initial conditions are shown in Figure 3. Symbols are as in Figure 3.
Figure 5 Concentration versus logarithm of the mass of clusters at the end of the POW run; the solid line shows the initial relationship between $c$ and $\log M$ in the simulation.
Figure 6 Concentration versus logarithm of the mass of clusters at the end of the GAU1-nc run.
Figure 7 Histogram of the initial (grey) and final (stripes) values of masses of clusters for the POW run (a), the GAU1 run (b) and the GAU2 run (c).
Figure 8 Initial (short dashed line), final (solid line) and observational (long dashed line) cumulative distribution function of the mass of clusters for the POW run (a), the GAU1 run (b) and the GAU2 run (c).
Figure 9 Initial (open squares) and final (full squares) mass function in $M$ from the POW run; lines show the best fit.
Figure 10 (a) Time evolution of the mean value of the logarithm of the mass of clusters for the GAU1 run (full dots) and for the GAU2 run (full triangles); (b) Time evolution of the number of clusters in the simulations for the GAU1 run (full dots) and for the GAU2 run (full triangles).
Figure 11 Cumulative distribution function of the logarithm of the mass of clusters in the GAU1 run at different times: from the left to the right the curves corresponds to $t=0$ Gyr (thin solid line), $t=2$ Gyr (dot-dashed line), $t=4$ Gyr (dotted line), $t=9$ Gyr (dashed line), $t=15$ Gyr (thick solid line).
Figure 12 Histogram of the galactocentric distances for the initial (grey) and final (stripes) sample of clusters from the POW run. No cluster with $R_g > 8$ Kpc undergoes core collapse or disruption and thus the initial and the final histograms are exactly superimposed.
Figure 13 Initial (short dashed line), final (solid line) and observational (long dashed line) cumulative distribution function of galactocentric distances for the POW run (a), the GAU1 run (b) and the GAU2 run (c).
Figure 14 Ratio of the number of clusters with $c > 1$ to the total number in each radial bin versus galactocentric distance for the POW run. (open squares indicate initial values, full squares indicate final values).