Systematic analysis of doubly charmed baryons $\Xi_{cc}$ and $\Omega_{cc}$

Guo-Liang Yu$^1$, Zhen-Yu Li$^2$, Zhi-Gang Wang$^1$, Lu Jie$^1$, and Yan Meng$^1$

$^1$ Department of Mathematics and Physics, North China Electric Power University, Baoding 071003, People’s Republic of China
$^2$ School of Physics and Electronic Science, Guizhou Education University, Guiyang 550018, People’s Republic of China

(Dated: November 2, 2022)

In this work, we perform a systematic study of the mass spectra, the root mean square(r.m.s.) radii and the radial density distributions of the doubly charmed baryons $\Xi_{cc}$ and $\Omega_{cc}$. The calculations are carried out in the framework of Godfrey-Isgur (GI) relativized quark model, where the baryon is regarded as a real three-body system of quarks. Our results show that the excited energy of doubly charmed baryon with $\rho$-mode is lower than those of the $\lambda$-mode and $\lambda-\rho$ mixing mode, which indicates that the lowest state is dominated by the $\rho$-mode. According to this conclusion, we systematically investigate the mass spectra, the r.m.s. radii of the ground and excited states ($1S \sim 4S$, $1P \sim 4P$, $1D \sim 4D$, $1F \sim 4F$ and $1G \sim 4G$) with $\rho$-mode. Using the wave functions obtained from quark model, we also study the radial density distributions. Finally, with the predicated mass spectra, the Regge trajectories of $\Xi_{cc}$ and $\Omega_{cc}$ in the $(J,M^2)$ plane are constructed, and the slopes, intercepts are determined by linear fitting. It is found that model predicted masses fit nicely to the constructed Regge trajectories.

PACS numbers: 13.25.Ft; 14.40.Lb

1 Introduction

The investigation of doubly heavy baryons is of great interest to experimental and theoretical physicist, as it provides a good opportunity for us to understand the strong interactions and basic QCD theory. Up to now, many single heavy baryons have been well discovered by Belle, BABAR, CLEO and LHCb collaborations$^{[1]}$, and the mass spectra of single heavy baryons have become more and more abundance. However, searching for doubly heavy baryons in experiments ended with no progress for a long time. The first observation of a doubly heavy baryon $\Xi_{cc}^+ (3519)$ was reported by the SELEX collaboration in 2002 in the decay mode $\Xi_{cc}^+ \rightarrow \Lambda^+_c K^- \pi^+$$^{[2]}$. Although SELEX collaboration confirmed this state in another decay mode $pD^+ K^-$, the FOCUS, BaBar, Belle and LHCb collaborations reported no evidence of the production of this doubly charmed baryon$^{[4–7]}$. The breakthrough came...
in 2017 with the discovery of a doubly charmed baryon $\Xi_{cc}^{++}$ by the LHCb collaboration\(^8\). This state was observed in the decay mode $\Xi_{cc}^{++} \rightarrow \Lambda^+ K^- \pi^+ \pi^+$ with a measured mass $3621.40 \pm 0.72 \pm 0.14 \pm 0.27$ MeV and later was confirmed in another decay mode $\Xi_{cc}^{++} \rightarrow \Lambda^+ \pi^+ \pi^+$\(^9,10\).

In theory, the mass spectra of the doubly heavy baryons have been predicted with various methods, such as the relativistic or nonrelativistic quark model\(^11–37\), QCD sum rules\(^38–46\), bag models\(^47–50\), the Bethe-Salpeter equation\(^51–54\), effective field theories\(^55–58\), Lattice QCD\(^59–65\) and the others\(^66–69\). To our knowledge, only Refs.\(^13,31\) focused on the mass spectra of the doubly heavy baryons from the ground states to the high excited states systematically in the quark-diquark picture. Under this picture, the initial three-body problem is reduced to two-step two-body calculations. However, the popular quark-diquark picture of a baryon is not universal and its results needs further confirmation by different methods. Thus, it is necessary for us give a systematic analysis of the properties of ground and excited states of doubly heavy baryons.

The relativized quark model, developed first by Godfrey, Capstick and Isgur\(^70,71\), has been widely used to investigate the properties of the mesons, baryons, and even the tetraquark states\(^72–74\). In this model, the relativistic effects are involved, which may be essential for doubly heavy baryon involving a light quark. Since the baryon is a three-body system, its theory is much more complicated compared to the two-body meson system, especially in the calculations of the matrix elements of the Hamiltonian in quark model. In our previous work, we employed a method of infinitesimally-shifted Gaussian (ISG) basis function in the Godfrey-Isgur (GI) relativized quark model\(^75,76\), where the calculation of the matrix element is simplified and the baryon is treated as a real three-body system.

In the present work, we use the method in Refs.\(^75,76\) to study the mass spectra and r.m.s. radii of the excited doubly charmed baryons up to rather high orbital and radial excitations. With the predicted mass spectra, we construct the Regge trajectories in the $(J,M^2)$ planes and determine their Regge slopes and intercepts. Using the wave functions obtained from the relativized quark model, we also study the radial density distributions of the doubly charmed baryons. The paper is organized as follows. After the introduction, we briefly describe the phenomenological methods adopted in this work in Sec.II. In Sec.III we present our numerical results and discussions about $\Xi_{cc}$ and $\Omega_{cc}$. In Sec.IV the baryon Regge trajectories in the $(J, M^2)$ plane are constructed. And Sec V is reserved for our conclusions.

## 2 Phenomenological methods adopted in this work

### 2.1 Wave function of doubly charmed baryon

The doubly charmed baryon is a three-body system which contains two charmed quarks and one light quark($u$, $d$ or $s$ quark) inside. The inter-quark interaction in this three-body system is commonly described by the three sets of Jacobi coordinates in Fig.1. Each set of Jacobi coordinate is called a channel($c$) and is defined as,
FIG. 1: Jacobi coordinates for the three body system.

\[ r_{\lambda i} = r_i - \frac{m_j r_i + m_k r_k}{m_j + m_k} \quad \text{(1)} \]
\[ r_{\rho i} = r_j - r_k \quad \text{(2)} \]

where \( i, j, k = 1, 2, 3 \) (or replace their positions in turn). \( r_i \) and \( m_i \) denote the position vector and the mass of the \( i \)th quark, respectively.

In the heavy quark limit, one light quark within the doubly charmed baryon is decoupled from two heavy quarks. It can be seen from Fig.1 that channel 3 properly reflects the characteristic of the heavy quark symmetry. Thus, the calculations in this work are performed based on channel 3. Using the transformation of Jacobi coordinates, we can calculate all the matrix elements in channel 3. Under this picture, the degree of freedom between two heavy quarks is commonly called the \( \rho \)-mode, while the degree between the center of mass of two heavy quarks and the light quark is called the \( \lambda \)-mode.

It was indicated by Refs. \[29, 75, 76\] that the lowest state of a single heavy baryon is dominated by the \( \lambda \)-mode. We will see in the following analysis that the doubly charmed baryons are dominated by \( \rho \)-mode.

In this work, we employ Gaussian basis functions to construct the orbital part of the wave function for a three-body system, which can be written as,

\[ \phi_{nlm\lambda}(r) = N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{r}) \quad \text{(3)} \]

with

\[ N_{nl} = \sqrt{\frac{2^{l+2}(2\nu_n)^{l+3/2}}{\pi(2l+1)!}} \quad \text{(4)} \]
\[ \nu_n = \frac{1}{r_n^2}, \quad r_n = r_{a1} \left[ \frac{r_{n_{max}}}{r_{a1}} \right]^{\frac{n_{max}-1}{n_{max}}} \quad \text{(5)} \]

In Eq.(5), \( n_{max} \) is the maximum number of the Gaussian basis functions. The orbital wave function is constructed from the wave functions of the two Jacobi coordinates \( \rho \) and \( \lambda \), and takes the form,

\[ \Phi_{l_\rho l_\lambda L} = [\phi_{n_\rho l_\rho m_\rho}(r_\rho) \phi_{n_\lambda l_\lambda m_\lambda}(r_\lambda)]_L \quad \text{(6)} \]
The spatial part of the wave function includes the spin wave function and orbital part, which can be written as,

\[ \psi_{J,M} = \left[ \left[ \chi_{1/2}(Q) \chi_{1/2}(q), \Phi_{l_\rho,l_\lambda,L} \right] \chi_{1/2}(q) \right]_{JM} \]  

(7)

Thus, the full wave function for a definite state of a baryon can be expressed as,

\[ \Psi_{JM}^{full} = \sum_{n_\rho,n_\lambda} C_{n_\rho,n_\lambda} \Psi_{JM}(r_\rho,r_\lambda) \quad (n_\rho,n_\lambda = 1, \cdots, n_{max}) \]  

(8)

where \( \Psi_{JM}(r_\rho,r_\lambda) \) is the direct product of color wave function, flavor wave function and the spatial wave function

\[ \Psi_{JM}(r_\rho,r_\lambda) = \phi_{color} \otimes \phi_{flavor} \otimes \psi_{JM} \]  

(9)

The state of a doubly charmed baryon can be characterized by a given quantum numbers \( l_\rho, l_\lambda, L, s, j \) and \( J^P \). The flavor wave function and color function of a doubly charmed baryon is symmetric and antisymmetric, respectively. Because the total wave function must be antisymmetric, the total spin \( s \) of two charmed quarks(\( cc \)) and orbital quantum number \( l_\rho \) should satisfy the condition \((-1)^{s+l_\rho} = -1\).

### 2.2 GI relativistic quark model and ISG method

In the relativistic quark model, baryons are formed by three valence(constituent) quarks. They are confined by a confining potential and interact with each other by residual two-body interactions. In the framework of GI quark model, the Hamiltonian for a three-body system can be written as

\[ \hat{H} = 3 \sum_{i=1}^{3} \left( p_i^2 + m_i^2 \right)^{1/2} + \sum_{i<j} \tilde{H}_{ij}^{conf} + \sum_{i<j} \tilde{H}_{ij}^{hyp} + \sum_{i<j} \tilde{H}_{ij}^{so} \]  

(10)

where the first term is the relativistic kinetic energy term, \( \tilde{H}_{ij}^{conf} \) is the spin-independent potential which contains a linear confining potential \( \tilde{S}(r_{ij}) \) and the one-gluon exchange potential \( G'(r_{ij}) \). \( \tilde{H}_{ij}^{hyp} \) and \( \tilde{H}_{ij}^{so} \) are the color-hyperfine interaction and spin-orbit interaction, respectively. If one want to learn more details about the interaction in GI model, please consults Refs. [70,71].

For a three-body system, the calculations of the Hamiltonian matrix elements become laborious even with Gaussian basis functions. This process can be simplified by introducing the ISG basis functions. These new sets of basis functions can be written as

\[ \phi_{nlm_i}(r) = N_{nl} \lim_{\varepsilon \to 0} \frac{1}{(\nu_n\varepsilon)^l} \sum_{k=1}^{k_{max}} C_{l_{m_i}k} e^{-\nu_n(r-\varepsilon D_{l_{m_i}k})^2} \]  

(11)

where \( \varepsilon \) is the shifted distance of the Gaussian basis. Taking the limit \( \varepsilon \to 0 \) is to be carried out after the matrix elements have been calculated analytically. For more details about the calculations of the Hamiltonian matrix elements, one can consults our previous work[75].
After all of the matrix elements are evaluated, the mass spectra can be obtained by solving the generalized eigenvalue problem,

$$\sum_{j=1}^{n_{\text{max}}^2} (H_{ij} - E N_{ij}) C_j = 0, \quad (i = 1 - n_{\text{max}}^2)$$

(12)

Where $H_{ij}$ denotes the matrix element in the total color-flavor-spin-spatial base, $E$ is the eigenvalue, $C_j$ stands for the corresponding eigenvector, and $N_{ij}$ is the overlap matrix elements of the Gaussian functions, which arises from the nonorthogonality of the bases and can be expressed as,

$$N_{ij} \equiv \langle \phi_{n_\rho \lambda \rho_a \lambda a \rho b \lambda b} | \phi_{n_\rho \lambda \rho_a \lambda a \rho b \lambda b} \rangle \times \langle \phi_{n_\lambda \lambda \lambda a \lambda a \lambda b \lambda b} | \phi_{n_\lambda \lambda \lambda a \lambda a \lambda b \lambda b} \rangle$$

$$= \left( \frac{2 \sqrt{\nu_{n_\rho} \nu_{n_\lambda}}} {\nu_{n_\rho} + \nu_{n_\lambda}} \right)^{l_\rho + 3/2} \times \left( \frac{2 \sqrt{\nu_{n_\lambda} \nu_{n_\lambda}}} {\nu_{n_\rho} + \nu_{n_\lambda}} \right)^{l_\lambda + 3/2}$$

(13)

3 Numerical results and discussions

3.1 Numerical stabilities and $\rho$-modes

![FIG. 2: Convergence of the energy of the lowest $\Xi_{cc}$ and $\Omega_{cc}$ for increasing the number of bases functions.](image)

![FIG. 3: Convergence of the r.m.s. radius $\sqrt{\langle r^2 \rangle}$ and $\sqrt{\langle r_\lambda^2 \rangle}$ of the lowest $\Xi_{cc}$ for increasing the number of bases functions.](image)

The parameters used in the Hamiltonian in Eq.(10) are the same as those in our previous work [75, 76] where the experimental masses of single heavy baryons were well reproduced. In order to investigate the convergence and stability of the numerical results, we plot the masses of the lowest lying $\Xi_{cc}(1^{+})$ and $\Omega_{cc}(4^{+})$ baryons in Fig.2 and the r.m.s. radii $\sqrt{\langle r_\rho^2 \rangle}$ and $\sqrt{\langle r_\lambda^2 \rangle}$ of the lowest $\Xi_{cc}(1^{+})$ in Fig.3. We can see that the results decrease with the basis number and converge to a stable value when the $n_{\text{max}}^2 = 10^2$. Thus, it is reliable for us to carry out the calculations with $10^2$ Gaussian bases in present work.

For the orbital excitations of doubly charmed baryons, they can be classified by the orbital angular momentum $l_\rho$ and $l_\lambda$. For example, there are two orbital excitation modes $\lambda$- and $\rho$-mode with
FIG. 4: Quark mass dependence of excited energy of doubly charmed baryons \((\frac{1}{2}^-, \frac{3}{2}^-)\) with \(\lambda\)-mode and \(\rho\)-mode

FIG. 5: Quark mass dependence of excited energy of doubly charmed baryons \((\frac{1}{2}^+, \frac{3}{2}^+)\) with \(\lambda\)-mode, \(\rho\)-mode and \(\lambda-\rho\) mixing mode

\((l_\rho, l_\lambda) = (0,1)\) and \((1,0)\) for \(P\)-wave baryons. While there are three excitation modes for \(D\)-wave baryons with \((l_\rho, l_\lambda) = (0,2), (2,0)\) and \((1,1)\), which are called the \(\lambda\)-mode, \(\rho\)-mode and \(\lambda-\rho\) mixing mode, respectively. For higher orbital excited states, their situations are similar to \(D\)-wave baryons which also have three excitation modes. By changing the mass of light quark (denoted as \(m_3\) in Figs.4-5) from \(0.01 \sim 0.5\) GeV, we plot the excited energy of different excited modes in Figs.4-5. It is shown that the \(\rho\)-mode appears lower in excited energy than both the \(\lambda\)-mode and \(\lambda-\rho\) mixing mode. This indicates the lowest states of doubly charmed baryons are dominated by the \(\rho\)-mode whether for \(P\)- or \(D\)-wave baryons. And this result is consistent with that of Ref. [29].

3.2 Mass spectra of \(\Xi_{cc}\) and \(\Omega_{cc}\)

For \(\Xi_{cc}\) and \(\Omega_{cc}\) baryons with \(\rho\) excited mode, we obtain their r.m.s. radii and complete mass spectra with quantum numbers up to \(n = 4\) and \(L = 4\). Many collaborations have focused on the ground state masses of doubly charmed baryons, which results are listed in Table I together with ours. From Table I, we can see that our predicted mass for the ground state of \(\Xi_{cc}\) is 3640 MeV. Considering the model uncertainties, this result is consistent with the experimental data 3621.40 MeV. In addition, our predictions for \(\Xi_{cc}(\frac{1}{2}^+, \frac{3}{2}^+)\) and \(\Omega_{cc}(\frac{1}{2}^+, \frac{3}{2}^+)\) are close to the results from Refs. [14, 22, 23, 43, 49, 59, 67].

For higher radial and orbital excitations together with the ground states, the predicted masses and the r.m.s. radii are shown in Tables II-III, where each state is characterized by the quantum numbers \((l_\rho, l_\lambda, L, s, j)\) and \(nL(J^P)\) in the first two columns. In order to see the feature of the mass spectra obviously, we show some of the results in Figs.6-7. From Tables II-III and Figs. 6-7, we can see that the structure of the mass spectra of \(\Xi_{cc}\) and \(\Omega_{cc}\) are similar to each other. They have the following features: (1) The \(1P\)-wave doublet \((\frac{1}{2}^-, \frac{3}{2}^-)\) are the lowest excited states, and then is the \(2S\) doublet \((\frac{1}{2}^+, \frac{3}{2}^+)\). This means they have good potentials to be observed in the future experiments. (2) After
TABLE I: Masses (in MeV) of the ground states of \( \Xi^{cc} \) and \( \Omega^{cc} \) heavy baryons

| Baryons      | \( \Xi^{cc}(\frac{3}{2}^+) \) | \( \Xi^{cc}*(\frac{5}{2}^+) \) | \( \Omega^{cc}(\frac{1}{2}^+) \) | \( \Omega^{cc}*(\frac{3}{2}^+) \) |
|--------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Present work | 3640                          | 3695                          | 3750                          | 3799                          |
| [11]         | 3676                          | 3753                          | 3815                          | 3876                          |
| [14]         | 3620                          | 3727                          | 3778                          | 3872                          |
| [13]         | 3478                          | 3610                          | -                             | -                             |
| [47]         | 3520                          | 3630                          | 3619                          | 3721                          |
| [24]         | 3510                          | 3548                          | 3719                          | 3746                          |
| [69]         | 3676                          | 3746                          | 3787                          | 3851                          |
| [23]         | 3613                          | 3707                          | 3712                          | 3795                          |
| [26]         | 3579                          | 3708                          | 3718                          | 3847                          |
| [27]         | 3678                          | 3752                          | -                             | -                             |
| [43]         | -                             | 3690                          | -                             | 3780                          |
| [28]         | 3532                          | 3623                          | 3667                          | 3758                          |
| [29]         | 3685                          | 3754                          | 3832                          | 3883                          |
| [30]         | 3606                          | 3675                          | 3715                          | 3772                          |
| [22]         | 3612                          | 3706                          | 3702                          | 3783                          |
| [52]         | 3547                          | 3719                          | 3648                          | 3770                          |
| [31]         | 3520                          | 3695                          | 3650                          | 3810                          |
| [50]         | 3550                          | 3590                          | 3730                          | 3770                          |
| [32]         | 3679                          | 3763                          | 3830                          | 3891                          |
| [66]         | 3627                          | 3690                          | 3692                          | 3756                          |
| [59]         | 3626                          | 3693                          | 3719                          | 3788                          |
| [67]         | 3615                          | 3747                          | -                             | -                             |
| [41]         | 3630                          | 3750                          | 3750                          | 3850                          |
| [51]         | 3620                          | 3620                          | 3720                          | 3720                          |
| [68]         | 3653                          | 3741                          | -                             | -                             |
| [49]         | 3604                          | 3714                          | 3726                          | 3820                          |
| [33]         | 3620                          | 3653                          | 3798                          | 3831                          |

considering the spin-orbital interaction, there still exist degeneracy for higher orbital excited states with different \( J^P \). For example, the excited states \( 1D(\frac{1}{2}^+)_{j=1} \), \( 1D(\frac{3}{2}^+)_{j=2} \) and \( 1D(\frac{5}{2}^+)_{j=3} \) almost lie in the same energy level. (3) For the spin-doublet states, the energy of the \( J = j - \frac{1}{2} \) state is higher than that of the \( J = j + \frac{1}{2} \) state. (4) The energy difference between two adjacent radial excited states gradually decreases with radial quantum number \( n \) increasing.

### 3.3 The r.m.s. radii and radial density distributions

The r.m.s. radius and radial density distribution of the baryons are important for testing various conjectures about strongly interacting systems. Thus, using the wave functions obtained from quark model, we also study these parameters. The r.m.s. radii for doubly charmed states are also shown in Tables II-III, and the radial density distributions are defined as,

\[
\omega(r_p) = \int |\Psi(r_p, r_\lambda)|^2 dr_\lambda d\Omega_p
\]
\[
\omega(r_\lambda) = \int |\Psi(r_p, r_\lambda)|^2 dr_p d\Omega_\lambda
\]

where \( \Omega_p \) and \( \Omega_\lambda \) are the solid angles spanned by vectors \( r_p \) and \( r_\lambda \), respectively. Some of the results about the radial density distributions of baryons \( \Xi^{cc} \) and \( \Omega^{cc} \) are shown in Figs. 8-11.


### TABLE II: Masses (in MeV) and r.m.s. radii (in fm) of the $\Xi_{cc}$ heavy baryons

| $l_\rho$ $l_\lambda$ $L$ $s$ $j$ | $nL$($J^P$) | $M$ | $\sqrt{\langle r_\rho^2 \rangle}$ | $\sqrt{\langle r_\lambda^2 \rangle}$ | $l_\rho$ $l_\lambda$ $L$ $s$ $j$ | $nL$($J^P$) | $M$ | $\sqrt{\langle r_\rho^2 \rangle}$ | $\sqrt{\langle r_\lambda^2 \rangle}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0 0 0 1 1        | $1S(\frac{1}{2}^+)$ | 3640            | 0.435           | 0.462           | 2 0 2 1 3        | $1D(\frac{1}{2}^+)$ | 4233            | 0.856           | 0.596           |
|                 | $2S(\frac{1}{2}^+)$ | 4069            | 0.759           | 0.585           |                 |                 |                 |                 |                 |
|                 | $3S(\frac{3}{2}^+)$ | 4182            | 0.524           | 0.836           |                 |                 |                 |                 |                 |
|                 | $4S(\frac{1}{2}^+)$ | 4411            | 1.147           | 0.658           |                 |                 |                 |                 |                 |
| 0 0 1 1          | $1S(\frac{3}{2}^+)$ | 3695            | 0.443           | 0.496           | 3 0 3 0 3        | $1F(\frac{1}{2}^-)$ | 4342            | 0.983           | 0.550           |
|                 | $2S(\frac{3}{2}^+)$ | 4111            | 0.764           | 0.623           |                 | $2F(\frac{3}{2}^-)$ | 4663            | 1.347           | 0.629           |
|                 | $3S(\frac{3}{2}^+)$ | 4209            | 0.539           | 0.851           |                 | $3F(\frac{3}{2}^-)$ | 4824            | 1.097           | 0.925           |
|                 | $4S(\frac{3}{2}^+)$ | 4445            | 1.147           | 0.689           |                 | $4F(\frac{3}{2}^-)$ | 4891            | 1.372           | 0.661           |
| 1 1 0 1          | $1P(\frac{1}{2}^-)$ | 3932            | 0.642           | 0.502           | 3 0 3 0 3        | $1G(\frac{3}{2}^+)$ | 4526            | 1.121           | 0.577           |
|                 | $2P(\frac{1}{2}^-)$ | 4289            | 0.946           | 0.589           |                 | $2G(\frac{3}{2}^+)$ | 4843            | 1.600           | 0.672           |
|                 | $3P(\frac{1}{2}^-)$ | 4447            | 0.717           | 0.886           |                 | $3G(\frac{3}{2}^+)$ | 4990            | 1.233           | 0.944           |
|                 | $4P(\frac{1}{2}^-)$ | 4582            | 1.299           | 0.669           |                 | $4G(\frac{3}{2}^+)$ | 5063            | 1.308           | 0.652           |
| 1 0 1 1          | $1P(\frac{3}{2}^-)$ | 3978            | 0.654           | 0.536           | 4 0 4 1 3        | $1G(\frac{5}{2}^+)$ | 4599            | 1.142           | 0.650           |
|                 | $2P(\frac{3}{2}^-)$ | 4328            | 0.963           | 0.622           |                 | $2G(\frac{5}{2}^+)$ | 4905            | 1.669           | 0.752           |
|                 | $3P(\frac{3}{2}^-)$ | 4472            | 0.722           | 0.907           |                 | $3G(\frac{5}{2}^+)$ | 5041            | 1.224           | 1.006           |
|                 | $4P(\frac{3}{2}^-)$ | 4615            | 1.290           | 0.695           |                 | $4G(\frac{5}{2}^+)$ | 5146            | 1.240           | 0.705           |
| 2 0 1 1          | $1D(\frac{1}{2}^-)$ | 4163            | 0.819           | 0.538           | 4 0 4 1 3        | $1G(\frac{3}{2}^-)$ | 4511            | 1.120           | 0.565           |
|                 | $2D(\frac{1}{2}^-)$ | 4490            | 1.121           | 0.614           |                 | $2G(\frac{3}{2}^-)$ | 4829            | 1.599           | 0.660           |
|                 | $3D(\frac{1}{2}^-)$ | 4656            | 0.908           | 0.916           |                 | $3G(\frac{3}{2}^-)$ | 4979            | 1.236           | 0.936           |
|                 | $4D(\frac{1}{2}^-)$ | 4745            | 1.381           | 0.679           |                 | $4G(\frac{3}{2}^-)$ | 5050            | 1.310           | 0.642           |
| 2 0 1 1          | $1D(\frac{3}{2}^-)$ | 4203            | 0.831           | 0.572           | 4 0 4 1 4        | $1G(\frac{5}{2}^-)$ | 4605            | 1.147           | 0.659           |
|                 | $2D(\frac{3}{2}^-)$ | 4526            | 1.148           | 0.648           |                 | $2G(\frac{5}{2}^-)$ | 4909            | 1.683           | 0.762           |
|                 | $3D(\frac{3}{2}^-)$ | 4679            | 0.911           | 0.940           |                 | $3G(\frac{5}{2}^-)$ | 5045            | 1.225           | 1.014           |
|                 | $4D(\frac{3}{2}^-)$ | 4777            | 1.359           | 0.703           |                 | $4G(\frac{5}{2}^-)$ | 5158            | 1.227           | 0.713           |
| 2 0 1 2          | $1D(\frac{5}{2}^-)$ | 4151            | 0.821           | 0.528           | 4 0 4 1 4        | $1G(\frac{7}{2}^-)$ | 4695            | 1.122           | 0.533           |
|                 | $2D(\frac{5}{2}^-)$ | 4481            | 1.124           | 0.604           |                 | $2G(\frac{7}{2}^-)$ | 4916            | 1.603           | 0.649           |
|                 | $3D(\frac{5}{2}^-)$ | 4650            | 0.915           | 0.910           |                 | $3G(\frac{7}{2}^-)$ | 4968            | 1.238           | 0.929           |
|                 | $4D(\frac{5}{2}^-)$ | 4736            | 1.379           | 0.669           |                 | $4G(\frac{7}{2}^-)$ | 5039            | 1.305           | 0.630           |
| 2 0 1 2          | $1D(\frac{7}{2}^-)$ | 4217            | 0.841           | 0.583           | 4 0 4 1 5        | $1G(\frac{9}{2}^-)$ | 4611            | 1.154           | 0.669           |
|                 | $2D(\frac{7}{2}^-)$ | 4540            | 1.169           | 0.660           |                 | $2G(\frac{9}{2}^-)$ | 4914            | 1.700           | 0.773           |
|                 | $3D(\frac{7}{2}^-)$ | 4689            | 0.919           | 0.949           |                 | $3G(\frac{9}{2}^-)$ | 5048            | 1.228           | 1.024           |
|                 | $4D(\frac{7}{2}^-)$ | 4790            | 1.342           | 0.708           |                 | $4G(\frac{9}{2}^-)$ | 5172            | 1.214           | 0.722           |
| 2 0 1 3          | $1D(\frac{9}{2}^-)$ | 4142            | 0.829           | 0.519           | 4 0 4 1 5        | $1G(\frac{11}{2}^-)$ | 4611            | 1.154           | 0.669           |
TABLE III: Masses (in MeV) and r.m.s. radii (in fm) of the Ω_{cc} heavy baryons

| lµ lδ L s j | nL(J^P) | M   | \(\langle r^2 \rangle\) | \(\langle r_s^2 \rangle\) | lµ lδ L s j | nL(J^P) | M   | \(\langle r^2 \rangle\) | \(\langle r_s^2 \rangle\) |
|------------|--------|-----|----------------|----------------|------------|--------|-----|----------------|----------------|
| 0 0 0 1 1   | 1S(\frac{1}{2}^+) | 3750 | 0.426 | 0.427 | 1D(\frac{3}{2}^+) | 4346 | 0.844 | 0.553 |
|            | 2S(\frac{1}{2}^+) | 4182 | 0.731 | 0.567 | 2D(\frac{3}{2}^+) | 4671 | 1.159 | 0.633 |
|            | 3S(\frac{3}{2}^+) | 4291 | 0.540 | 0.775 | 3D(\frac{3}{2}^+) | 4807 | 0.932 | 0.907 |
|            | 4S(\frac{1}{2}^+) | 4531 | 1.120 | 0.650 | 4D(\frac{3}{2}^+) | 4921 | 1.345 | 0.682 |
| 0 0 0 1 1   | 1S(\frac{3}{2}^+) | 3799 | 0.435 | 0.457 | 1F(\frac{3}{2}^+) | 4471 | 0.973 | 0.519 |
|            | 2S(\frac{3}{2}^+) | 4219 | 0.735 | 0.599 | 2F(\frac{3}{2}^-) | 4792 | 1.302 | 0.598 |
|            | 3S(\frac{3}{2}^+) | 4315 | 0.553 | 0.790 | 3F(\frac{3}{2}^-) | 4937 | 1.094 | 0.878 |
|            | 4S(\frac{3}{2}^+) | 4561 | 1.121 | 0.677 | 4F(\frac{3}{2}^-) | 5018 | 1.414 | 0.636 |
| 1 0 1 0 1   | 1P(\frac{1}{2}^-) | 4049 | 0.631 | 0.468 | 1G(\frac{1}{2}^+) | 4658 | 1.112 | 0.545 |
|            | 2P(\frac{1}{2}^-) | 4407 | 0.922 | 0.561 | 2G(\frac{1}{2}^+) | 4976 | 1.551 | 0.639 |
|            | 3P(\frac{1}{2}^-) | 4557 | 0.718 | 0.834 | 3G(\frac{1}{2}^+) | 5049 | 1.098 | 0.932 |
|            | 4P(\frac{1}{2}^-) | 4706 | 1.298 | 0.646 | 4G(\frac{1}{2}^+) | 5083 | 1.343 | 0.681 |
| 1 0 1 0 1   | 1P(\frac{3}{2}^-) | 4089 | 0.642 | 0.497 | 1G(\frac{3}{2}^+) | 4658 | 1.112 | 0.545 |
|            | 2P(\frac{3}{2}^-) | 4441 | 0.936 | 0.589 | 2G(\frac{3}{2}^+) | 4976 | 1.551 | 0.639 |
|            | 3P(\frac{3}{2}^-) | 4579 | 0.723 | 0.854 | 3G(\frac{3}{2}^+) | 5104 | 1.242 | 0.897 |
|            | 4P(\frac{3}{2}^-) | 4734 | 1.292 | 0.667 | 4G(\frac{3}{2}^+) | 5188 | 1.357 | 0.629 |
| 2 0 2 1 1   | 1D(\frac{3}{2}^+) | 4285 | 0.807 | 0.504 | 1G(\frac{3}{2}^+) | 4718 | 1.131 | 0.607 |
|            | 2D(\frac{3}{2}^+) | 4612 | 1.088 | 0.582 | 2G(\frac{3}{2}^+) | 5028 | 1.623 | 0.706 |
|            | 3D(\frac{3}{2}^+) | 4767 | 0.904 | 0.867 | 3G(\frac{3}{2}^+) | 5148 | 1.238 | 0.954 |
|            | 4D(\frac{3}{2}^+) | 4870 | 1.402 | 0.654 | 4G(\frac{3}{2}^+) | 5255 | 1.285 | 0.677 |
| 2 0 2 1 1   | 1D(\frac{3}{2}^-) | 4318 | 0.818 | 0.532 | 1G(\frac{3}{2}^-) | 4645 | 1.112 | 0.536 |
|            | 2D(\frac{3}{2}^-) | 4642 | 1.111 | 0.610 | 2G(\frac{3}{2}^-) | 4965 | 1.550 | 0.629 |
|            | 3D(\frac{3}{2}^-) | 4788 | 0.908 | 0.889 | 3G(\frac{3}{2}^-) | 5093 | 1.244 | 0.889 |
|            | 4D(\frac{3}{2}^-) | 4896 | 1.384 | 0.674 | 4G(\frac{3}{2}^-) | 5177 | 1.358 | 0.620 |
| 2 0 2 1 2   | 1D(\frac{3}{2}^+) | 4276 | 0.810 | 0.496 | 1G(\frac{3}{2}^+) | 4722 | 1.136 | 0.615 |
|            | 2D(\frac{3}{2}^+) | 4604 | 1.091 | 0.573 | 2G(\frac{3}{2}^+) | 5032 | 1.640 | 0.715 |
|            | 3D(\frac{3}{2}^+) | 4762 | 0.910 | 0.861 | 3G(\frac{3}{2}^+) | 5151 | 1.239 | 0.963 |
|            | 4D(\frac{3}{2}^+) | 4863 | 1.400 | 0.645 | 4G(\frac{3}{2}^+) | 5265 | 1.269 | 0.684 |
| 2 0 2 1 2   | 1D(\frac{3}{2}^-) | 4331 | 0.829 | 0.543 | 1G(\frac{3}{2}^-) | 4632 | 1.114 | 0.526 |
|            | 2D(\frac{3}{2}^-) | 4655 | 1.130 | 0.620 | 2G(\frac{3}{2}^-) | 4954 | 1.556 | 0.620 |
|            | 3D(\frac{3}{2}^-) | 4796 | 0.917 | 0.897 | 3G(\frac{3}{2}^-) | 5084 | 1.247 | 0.882 |
|            | 4D(\frac{3}{2}^-) | 4907 | 1.369 | 0.678 | 4G(\frac{3}{2}^-) | 5168 | 1.352 | 0.609 |
| 2 0 2 1 3   | 1D(\frac{3}{2}^+) | 4269 | 0.818 | 0.488 | 1G(\frac{3}{2}^+) | 4727 | 1.142 | 0.622 |
|            | 2D(\frac{3}{2}^+) | 4600 | 1.101 | 0.566 | 2G(\frac{3}{2}^+) | 5036 | 1.660 | 0.725 |
|            | 3D(\frac{3}{2}^+) | 4759 | 0.922 | 0.856 | 3G(\frac{3}{2}^+) | 5154 | 1.242 | 0.971 |
|            | 4D(\frac{3}{2}^+) | 4857 | 1.392 | 0.635 | 4G(\frac{3}{2}^+) | 5277 | 1.252 | 0.692 |
FIG. 6: Mass spectrum of $\Xi_{cc}$ family

FIG. 8: Radial density distributions for some $1S - 1F$ states in the $\Xi_{cc}$ family

FIG. 9: Radial density distributions for some $1S - 1F$ states in the $\Omega_{cc}$ family
FIG. 7: Mass spectrum of \(\Omega_{cc}\) family

FIG. 10: Radial density distributions for \(1S \sim 3S\) states in the \(\Xi_{cc}\) family.
From Tables II-III, we can see the r.m.s. radii $\sqrt{\langle r^2 \rho \rangle}$ and $\sqrt{\langle r^2 \lambda \rangle}$ of the $1S(\frac{1}{2}^+)$ state are 0.435 fm, 0.462 fm for $\Xi_{cc}$, and 0.426 fm, 0.427 fm for $\Omega_{cc}$. In Refs. [77–79], their predicted r.m.s. radii for the ground state of $c\bar{c}$ meson are 0.449 fm, 0.445 fm, and 0.484 fm. These values are comparable with our results about $\sqrt{\langle r^2 \rho \rangle}$ for doubly charmed baryons. For the states with same radial quantum number $n$, the $\sqrt{\langle r^2 \rho \rangle}$ becomes larger obviously when the orbital angular momentum $L$ increases. However, $\sqrt{\langle r^2 \lambda \rangle}$ increases a little with $L$ increasing. This is consistent with the results shown in Figs.8-9, where the peak of $r^2 \omega(r_{\rho})$ shifts outward with $L$ increment and $r^2 \omega(r_{\lambda})$ changes little. For the states with same angular momentum $L$, both $\sqrt{\langle r^2 \rho \rangle}$ and $\sqrt{\langle r^2 \lambda \rangle}$ increase with radial quantum number $n$. We can also see this feature from Figs.10-11. It is shown that the peak of radial density distribution becomes lower from $1S \sim 3S$ states and the peak position shifts outward slightly. Theoretically, the larger the r.m.s. radii become, the looser the baryons will be. We hope these results can help us to estimate the upper limit of the mass spectra and to search for the new doubly charmed baryons in forthcoming experiments.

4 Regge trajectories of doubly charmed baryons $\Xi_{cc}$ and $\Omega_{cc}$

The Regge theory is very successful in studying the strong interaction at high energy and it is an indispensable tool in phenomenological studies for hadrons [80–89]. In our previous work, we have successfully constructed the Regge trajectories for the single heavy baryons [75, 76]. In the present work, we have obtained the $1S \sim 4S$, $1P \sim 4P$, $1D \sim 4D$, $1F \sim 4F$ and $1G \sim 4G$ state masses for doubly charmed baryons. This makes it easy for us to construct their Regge trajectories in $(J,M^2)$ plane. The doubly charmed baryons are classified into two groups which have natural parity $S(\frac{1}{2}^+)_{j=1}$, $P(\frac{3}{2}^-)_{j=1}$, $D(\frac{1}{2}^+)_{j=2}$, $F(\frac{5}{2}^-)_{j=3}$, $G(\frac{9}{2}^+)_{j=4}$ and unnatural parity $P(\frac{1}{2}^-)_{j=1}$, $D(\frac{3}{2}^+)_{j=2}$, $F(\frac{7}{2}^-)_{j=3}$, $G(\frac{11}{2}^+)_{j=4}$ [90]. The Regge trajectories are presented in Figs.12-13 for $\Xi_{cc}$ and in Figs.14-15 for $\Omega_{cc}$, where the predicted masses in quark model are denoted by diamonds. The ground and radial excited states are plotted from bottom to top. We use the following definition about the $(J,M^2)$ Regge trajectories,

$$M^2 = \alpha J + \alpha_0$$  \hspace{1cm} (15)
where $\alpha$ and $\alpha_0$ are slope and intercept. The straight lines in Figs.12-15 are obtained by linear fitting of the predicted values. The fitted slopes and intercepts of the Regge trajectories are listed in Table IV. We can see from these figures that all of the predicted masses in our model fit nicely to the linear trajectories in the $(J, M^2)$ plane. These results can help us to assign an accurate position in the mass spectra for observed doubly charmed baryons in the future.

**FIG. 12**: Parent and daughter Regge trajectories for the $\Xi_{cc}$ baryons with natural parity.

**FIG. 13**: Parent and daughter Regge trajectories for the $\Xi_{cc}$ baryons with unnatural parity.

**FIG. 14**: Parent and daughter Regge trajectories for the $\Omega_{cc}$ baryons with natural parity.

**FIG. 15**: Parent and daughter Regge trajectories for the $\Omega_{cc}$ baryons with unnatural parity.

### 5 Conclusions

In this work, we have systematically investigate the mass spectra, the r.m.s. radii and the radial density distributions of the doubly charmed baryons $\Xi_{cc}$ and $\Omega_{cc}$ in the frame work of GI quark model. In addition, with the predicted mass spectra, we also construct the Regge trajectories in $(J, M^2)$ plane. In summary, we have obtained the followings:
TABLE IV: Fitted parameters $\alpha$ and $\alpha_0$ for the slope and intercept of the $(J,M^2)$ parent and daughter Regge trajectories for $\Xi_{cc}$ and $\Omega_{cc}$.

| Trajectory | $\alpha$(Gev$^2$) | $\alpha_0$(Gev$^2$) | $\alpha$(Gev$^2$) | $\alpha_0$(Gev$^2$) |
|------------|-------------------|----------------------|-------------------|----------------------|
| $\Xi_{cc}(\frac{1}{2}^+)$ parent | 1.975±0.312 | 12.601±1.005 | 1.627±0.192 | 14.725±0.431 |
| 1 daughter | 1.887±0.167 | 15.780±0.420 | 1.646±0.076 | 17.591±0.175 |
| 2 daughter | 1.977±0.297 | 16.821±0.912 | 1.668±0.223 | 19.036±0.512 |
| 3 daughter | 1.810±0.117 | 18.530±0.331 | 1.502±0.101 | 20.217±0.223 |
| $\Omega_{cc}(\frac{3}{2}^+)$ parent | 2.044±0.331 | 13.392±0.982 | 1.725±0.204 | 15.617±0.469 |
| 1 daughter | 1.958±0.158 | 16.669±0.429 | 1.744±0.068 | 18.569±0.156 |
| 2 daughter | 2.017±0.297 | 17.708±0.850 | 1.723±0.234 | 19.992±0.536 |
| 3 daughter | 1.807±0.994 | 19.626±0.285 | 1.551±0.083 | 21.339±0.192 |

The first feature of this work is that a doubly charmed baryon is regarded as a real three-body system of quarks and all quarks contribute fully to the dynamics in the baryon. This is very different with the light-quark-heavy-diquark approximation where the three-body problem is reduced to two-body calculations. Second, all parameters in our calculations such as quark masses and parameters of the interquark potential are consistent with those of our previous work\[75, 76\]. Third, this is the first time that the masses, r.m.s. radii and radial density distributions of the ground, orbital and radial excited states($1S \sim 4S, 1P \sim 4P, 1D \sim 4D, 1F \sim 4F$ and $1G \sim 4G$) are systematically studied(in Tables II-III). It is found that model predicted mass of $\Xi_{cc}^+$ 3640 MeV is in agreement with the experimental data 3621.4 MeV. Finally, for the three orbital excitations $\lambda$-mode, $\rho$-mode and $\lambda$-$\rho$ mixing mode, it is shown that the mixing of these excited modes is suppressed and only $\rho$-mode dominates. We hope these analysis can help us to predict the upper limit of the mass spectra and to search for doubly charmed baryons in future experiments.

[1] R. L. Workmanet et al.(Particle Data Group), Prog.Theor.Exp.Phys.2022, 083C01 (2022)
[2] M. Mattson et al. (SELEX Collaboration), Phys. Rev. Lett. 89, 112001 (2002).
[3] A. Ocherashvili et al. (SELEX Collaboration), Phys. Lett. B 628, 18 (2005).
[4] S. P. Ratti, Nucl. Phys. Proc. Suppl. 115, 33 (2003).
[5] B. Aubert et al. (BaBar Collaboration), Phys. Rev. D 74, 011103 (2006).
[6] R. Chistov et al. (Belle Collaboration), Phys. Rev. Lett. 97,162001 (2006).
[7] R. Aaij et al. (LHCb Collaboration), JHEP 1312, 090 (2013).
[8] R. Aaij et al. (LHCb collaboration), Phys. Rev. Lett. 119, 112001 (2017).
[49] Wen-Xuan Zhang, Hao Xu, Duojie Jia, Phys. Rev. D104, 114011(2021).
[50] Da-Heng He, Ke Qian, Yi-Bing Ding, et al., Phys. Rev. D 70, 094004(2004).
[51] Qi-Xin Yu, Xin-Heng Guo, Nucl. Phys. B 947, 114727(2019).
[52] F. Giannuzzi, Phys. Rev. D79, 094002(2009), arXiv:0902.4624 [hep-ph]
[53] S. P. Tong, Y. B. Ding, X. H. Guo, et al., Phys. Rev. D 62, 054024(2000), arXiv:9910259 [hep-ph].
[54] M.-H. Weng, X.-H. Guo, A.W. Thomas, Phys. Rev. D 83, 056006 (2011), arXiv:1012.2002 [hep-ph].
[55] N. Brambilla, A. Vairo and T. Rosch, Phys. Rev. D 72, 034021 (2005).
[56] J. Hu and T. Mehen, Phys. Rev. D 73, 054003 (2006).
[57] S. Fleming and T. Mehen, Phys. Rev. D 73, 034502 (2006).
[58] M.-H. Weng, X.-H. Guo, A.W. Thomas, Phys. Rev. D 83, 056006 (2011), arXiv:1012.2002 [hep-ph].
[59] N. Brambilla, A. Vairo and T. Rosch, Phys. Rev. D 72, 034021 (2005).
[60] J. Hu and T. Mehen, Phys. Rev. D 73, 054003 (2006).
[61] S. Fleming and T. Mehen, Phys. Rev. D 73, 034502 (2006).
[62] T. Mehen and B. C. Tiburzi, Phys. Rev. D 74, 054505 (2006).
[63] H. Bahtiyar, K. U. Can, G. Erkol, et al., Phys. Rev. D 98, 114505 (2018).
[64] R. Lewis, N. Mathur, and R. M. Woloshyn, Phys. Rev D 64, 094509 (2001).
[65] J. M. Flynn, F. Mescia and A. S. B. Tariq [UKQCD Collaboration], JHEP 0307, 066 (2003).
[66] H. Na and S. A. Gottlieb, arXiv:0710.1422 [hep-lat].
[67] L. Liu, H. W. Lin, K. Orginos, et al., Phys. Rev. D 81, 094505 (2010).
[68] Z. S. Brown, W. Detmold, S. Meinel, et al., Phys. Rev. D 90, 094507 (2014).
[69] M. Padmanath, R. G. Edwards, N. Mathur, et al., Phys. Rev. D 91, 094502 (2015).
[70] V. V. Kiselev, A. V. Berezhnoy, A. K. Likhoded, Phys. Atom. Nucl. 81(3),369(2018).
[71] Marek Karliner and Jonathan L. Rosner, Phys. Rev. D 97, 094006 (2018).
[72] Joan Soto, Jaume Tarrus Castella, Phys. Rev. D 104, 074027 (2021).
[73] D. B. Lichtenberg, R. Roncaglia, and E. Predazzi, Phys. Rev. D 53, 6678 (1996).
[74] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189(1985).
[75] S. Capstick and N. Isgur, Phys. Rev. D 34, 2809 (1986); AIPConf. Proc. 132, 267(1985).
[76] Q. F. Lü, D. Y. Chen, and Y. B. Dong, Eur. Phys. J. C 80, 871 (2020).
[77] Q. F. Lü, D. Y. Chen, and Y. B. Dong, et al., Phys. Rev. D 104, 054026 (2021).
[78] Q. F. Lü, D. Y. Chen, and Y. B. Dong, Phys. Rev. D 102, 074021 (2020).
[79] Guo-Liang Yu, Zhen-Yu Li, Zhi-Gang Wang, et al., arXiv:2206.08128 [hep-ph]
[80] Zhao-Liang Yu, Zhen-Yu Li, Zhi-Gang Wang, et al., arXiv:2207.04167 [hep-ph]
[81] F Karsch, M T Mehr and H Satz, Z. Phys. C 37, 617 (1988).
[82] B Liu, P N Shen and H C Chiang, Phys. Rev. C 55, 3021(1997).
[83] Tapas Das, EJTP 35, 207(2016).
[84] T. Regge, Nuovo Cim. 14, 951(1959).
[85] T. Regge, Nuovo Cim. 18, 947(1960).
[86] G. F. Chew, S. C. Frautschi, Phys. Rev. Lett. 7, 394(1961).
[87] G. F. Chew, S. C. Frautschi, Phys. Rev. Lett. 8, 41 (1962).
[88] G. S. Bali, Phys. Rept. 343, 1, arXiv:hep-ph/0001312(2001).
[89] D. V. Bugg, Four sorts of meson, Phys. Rept. 397, 257, arXiv:hep-ex/0412045 (2004).
[90] E. Klempt, A. Zaitsev, Phys. Rept. 454, 1, arXiv:0708.4016 (2007).
[91] W. Lucha, F. F. Schoberl, D. Gromes, Phys. Rept. 200, 127(1991).
[92] Y. Nambu, Phys. Rev. D 10, 4262(1974).
[89] Y. Nambu, Phys. Lett. B 80, 372 (1979).

[90] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D 84, 014025 (2011).