Bailey, Robert F.; Hawtin, Daniel R.
On the classification of binary completely transitive codes with almost-simple top-group.
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Summary: A code \( C \) in the Hamming metric, that is, is a subset of the vertex set \( V_\Gamma \) of the Hamming graph \( \Gamma = H(m,q) \), gives rise to a natural distance partition \( \{ C, C_1, \ldots, C_\rho \} \), where \( \rho \) is the covering radius of \( C \). Such a code \( C \) is called completely transitive if the automorphism group \( \text{Aut}(C) \) acts transitively on each of the sets \( C, C_1, \ldots, C_\rho \). A code \( C \) is called 2-neighbour-transitive if \( \rho \geq 2 \) and \( \text{Aut}(C) \) acts transitively on each of \( C, C_1 \) and \( C_2 \).

Let \( C \) be a completely transitive code in a binary \((q = 2)\) Hamming graph having full automorphism group \( \text{Aut}(C) \) and minimum distance \( \delta \geq 5 \). Then it is known that \( \text{Aut}(C) \) induces a 2-homogeneous action on the coordinates of the vertices of the Hamming graph. The main result of this paper classifies those \( C \) for which this induced 2-homogeneous action is not an affine, linear or symplectic group. We find that there are 13 such codes, 4 of which are non-linear codes. Though most of the codes are well-known, we obtain several new results. First, a non-linear completely transitive code that does not explicitly appear in the existing literature is constructed, as well as a related non-linear code that is 2-neighbour-transitive but not completely transitive. Moreover, new proofs of the complete transitivity of several codes are given. Additionally, we consider the question of the existence of distance-regular graphs related to the completely transitive codes appearing in our main result.

MSC:

94Bxx Theory of error-correcting codes and error-detecting codes
20Bxx Permutation groups
05Bxx Designs and configurations

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