Avoiding BBN Constraints on Mirror Models for Sterile Neutrinos

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We point out that in models that explain the LSND result for neutrino oscillation using the mirror neutrinos, the big bang nucleosynthesis constraint can be avoided by using the late time phase transition that only helps to mix the active and the sterile neutrinos. We discuss the astrophysical as well as cosmological implications of this proposal.

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INTRODUCTION

The existence of neutrino oscillations has now been confirmed for solar and atmospheric neutrinos as well as for reactor and accelerator neutrinos. It is remarkable that all the data from many different experiments can be well understood in terms of only three neutrinos that mix among themselves. They imply very narrow ranges of both the mass difference squares among these neutrinos as well as mixings.

There is however another piece of evidence for oscillations which if confirmed will require severe departure from the successful three neutrino scheme just mentioned. It is the apparent observation of the muon anti-neutrino oscillating to the electron type anti-neutrino in the Los Alamos LSND experiment. An attempt was made to confirm this result by KARMEN collaboration which eliminated a large fraction of the parameter space allowed by LSND. It is hoped that the Mini-BOONE experiment at FERMILAB currently under way will settle the issue in near future.

If the LSND experiment is confirmed, one straightforward way to understand the results would be to postulate the existence of one or more extra neutrinos with mass in the eV range that do not interact with the W boson, the so-called sterile neutrinos and have them mix the known neutrinos. There have been various versions of this suggestion depending on the detailed mass arrangement of the sterile neutrinos with respect to the known ones: they are known in the literature as the $2+2$, $3+1$, $3+2$ models. Of the three, $3+1$ model seems less disfavored than the $2+2$ by the null results of other oscillation experiments. However the more recently proposed $3+2$ scenario that involves two sterile neutrinos is apparently in better agreement with all data than the others.

The major challenge posed by the sterile neutrino for theory is to understand its ultra-lightness despite its being a standard model singlet. A class of particle physics models that successfully answer this challenge are the mirror matter models. The basic assumption of these models is that there is an identical copy of the standard model (both constituents and forces) in nature that co-exists with the familiar standard model matter and forces. It is then clear that the same mechanism that keeps the active neutrinos light, will also keep the mirror neutrinos light which can therefore play the role of the sterile neutrinos. These models are inspired by the superstring theories and have been widely discussed. Phenomenological and astrophysical constraints on these models have also been extensively discussed.

Sterile neutrino models for LSND face two cosmological huddles that we would like to address in this paper. The issues are: how to make them consistent with (i) our understanding of big bang nucleosynthesis (BBN) and (ii) the recent bounds on neutrino masses from WMAP observations. The first problem is that BBN allows the number neutrinos $N_\nu$, in equilibrium when the temperature of the Universe is one MeV is restricted by $^4$He and $D_2$ observations to be very close to three. On the other hand for $\nu_s$ mass in the eV range and mixing in the few per cent range required to explain the LSND data, rapid $\nu_e - \nu_s$ oscillations would lead to $N_\nu = 4$ for the $3+1$ and $2+2$ scenarios and $N_\nu = 5$ for the $3+2$ scenario.

The WMAP constraints are on the sum of all neutrino masses in equilibrium at the epoch of structure formation which corresponds to a temperature around an eV. According to $^{14}$, $\sum m_\nu \leq 1.38$ eV for one sterile and $\sum m_\nu \leq 2.12$ eV for two extra ones assuming that they went into equilibrium at the BBN epoch. These constraints are also quite important since taken at face value, they would seem to rule out the $3+2$ model for LSND.

It is therefore important to look for scenarios that may allow one to avoid both the above constraints while at the same time providing an explanation of the LSND experiment. Recently, it has been suggested by using late time phase transition to generate the masses and mixings of both the active and sterile neutrinos, one can avoid both these constraints. In ref. $^{15}$, it is shown that this can be achieved by endowing two scalar fields $\phi$ with vevs in the 100 keV range so that at the BBN time the sterile as well as the active neutrinos are massless. As a result there is no oscillation among them that can bring the sterile neutrinos into equilibrium. Since the sterile neutrinos decouple from Hubble expansion at very high temperatures, their abundance at the BBN epoch is suppressed leading to concordance with the BBN constraints. Cosmological signatures of generic models of
this type have been given in Ref. [13].

In this paper we propose an alternative way to avoid the cosmological constraints using the same idea of late time phase transitions. We show that if the sterile neutrinos are the mirror neutrinos, we need only generate the mixing between the active and the sterile neutrino (and not masses) by the late time phase transition to avoid the BBN and WMAP constraints. An advantage of this model is that the contribution of the sterile neutrinos to the energy density of the universe at the BBN epoch is governed by a free parameter unlike the model of ref. [15]. We further find a convenient realization of mirror model with the seesaw scale in the TeV range which implies that we must employ the double seesaw mechanism [14] to get small neutrino masses. We construct explicit scenarios with late phase transition and discuss their cosmological and astrophysical implications. The detailed field theoretical models for them can be worked out but we do not discuss them here.

AN EXTENDED MIRROR MODEL

We start by reminding the reader about the generic features of the mirror models where one assumes that the universe consists not only of the observed standard model particles and forces but also coexisting with an identical set of constituents experiencing analogous but different gauge forces. Gravity is however common to all the particles. The forces are dictated by the gauge group $G \otimes G$ where one of the gauge groups $G$ acts in the standard model sector and on its fermions whereas the other acts in the other and on the mirror fermions. The fermion spectra on both sides are identical. Mirror symmetry keeps the gauge couplings equal but the effective strength of various forces in both sectors may be different due to different patterns of symmetry breaking. These models are inspired by the superstring theories as well as M-theory inspired brane-bulk models that have been widely discussed.

As is clear, the neutrinos in the mirror sector do not experience the known weak interactions and will not therefore appear in the Z and W-decays. They can therefore play the role of the sterile neutrinos used in the interpretation of the LSND results. The mixing between the active and mirror neutrinos can arise in a manner consistent with gauge invariance (see below for details). For purposes of notation, we denote all particles and parameters of the mirror sector will by a prime over the corresponding familiar sector symbol—e.g. mirror quarks are $u', d', s'$, etc and mirror Higgs field as $H_{u,d}'$ etc.

Before proceeding further, let us discuss the origin of the masses and mixings for the active and sterile neutrinos. For this purpose, we extend the standard model gauge group in each sector to $SU(2)_L \times U(1)_{B-L}$, which is an anomaly free gauge group in the presence of the right handed neutrino $\nu^c$. All the fermions have obvious quantum numbers under the gauge group. We add a gauge singlet chiral fermion, $S$ in each sector, one per family. Mirror symmetry requires that we do the same in the mirror sector. In order to get the standard model gauge group from the extended group in each sector, we need to add a pair of new Higgs bosons $\Delta(1, +\frac{1}{2}, 1)$ and a conjugate field $\Delta(1, -\frac{1}{2}, +1)$ in the visible sector and two similar fields in the mirror sector. We then add a gauge singlet Higgs field $\chi$ (and a mirror $\chi'$), which gives Majorana mass for the singlet fermions $S$ by an interaction of the form $\lambda_{\alpha\beta}(S_{\alpha}S_{\beta}\chi + S'_{\alpha}S'_{\beta}\chi')$.

The full superpotential relevant for neutrinos in each sector can be written as:

$$ W = h_{\nu} L H_u \nu^c + \lambda_1 \nu^c \Delta S + \lambda' S S \chi; \quad (1) $$

where we have omitted an identical set of terms for the mirror sector and have suppressed the generation index. For three generation case that we will be interested in, $h_{\nu}$, $\lambda_1$ and $\lambda'$ are $3 \times 3$ matrices.

The $U(1)_{B-L}$ part of the gauge symmetry is broken by the vev of field $< \Delta >$ assumed to be in the multi-TeV range. We choose the vev of $\chi$ field to be in the GeV range. It is then clear that this leads to the double seesaw form [14] for the $(\nu, \nu^c, S)$ mass matrix:

$$ M_{\nu} = \begin{pmatrix} 0 & h_{\nu}v & 0 \\ h_{\nu}v & 0 & \lambda_1 v_R \\ 0 & \lambda' v_R & \lambda' < \chi > \end{pmatrix}. \quad (2) $$

There is also a similar matrix for the mirror neutrinos. This leads to the light neutrino mass matrix of the form:

$$ M_{\nu} = h_{\nu} M_{R}^{-1} \chi' < \chi > M_{R}^{-1T} h_{\nu}^T v^2 \quad (3) $$

where $M_{R} = \lambda_1 < \Delta >$. It follows that if we choose $< \chi >$ about a GeV, $h_{\nu} \sim 10^{-1}$ and $\lambda' \sim 10^{-4}$, then for $\frac{\lambda' v}{h_{\nu}^2} \sim \frac{\lambda v}{h_{\nu}'} \sim 10^{-2}$, the neutrino masses are in the $0.1$ eV range as required by observations. Also, typical neutrino mass textures can be built into the coupling matrix $\chi'$. A similar situation will occur in the mirror sector, where we can choose $< \chi' >$ about a factor of 10 higher to get $m_{\nu^c}$ in the eV range to fit LSND (henceforth, we will call the sterile neutrinos $\nu'$ as $\nu_{\ell}$).

In order to generate mixing between the active and sterile neutrinos, we postulate the existence of a scalar field $\phi$ that mixes the two sectors. This can only be done through an interaction of the form $\beta S S \phi$. A simple tree level diagram via the exchange of $\nu^c$, $S$ and $\nu'^c$, $S'$ then leads to an effective coupling of the form:

$$ \lambda'' \frac{L H_{u} U H'_{u} \phi}{M_{\text{field}}}, \text{ where } \lambda'' \sim \beta h_{\nu}^2 \sim 10^{-2}. \text{ We assume as in ref. [14] that the vev of the field } < \phi > \sim 100 \text{ keV, so that at the BBN epoch the active and sterile neutrinos are unmixed}^1. \text{ The resulting } \nu - \nu_{\ell} \text{ mixing is then given}$$

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1 The smallness of the $\phi$ vev can be justified if we embed our theory
by: \( m_{\nu - \nu_s} \simeq \lambda'' \frac{\phi^2}{M_\phi^2} \simeq 10^{-6} < \phi > \) for \( \lambda'' \simeq 10^{-2} \). This gives the right order of mixing for the LSND experiment. One difference between our model and that of ref. [15] is that, late phase transition was used in [15] to generate both masses and mixings whereas in our case, only the mixing need to be generated at a late stage. As we will see, the mirror model has the advantage that contribution of the sterile neutrinos to the energy density of the Universe at the BBN epoch is given by an arbitrary parameter. The model can therefore work even if the BBN constraints on the number of extra neutrino species tightened further.

**BBN, ASYMMETRIC INFLATION AND NEUTRINO MIXINGS**

Before we address the issue of neutrino mixings and BBN, we note that in the mirror model, we have three light neutrinos, a mirror photon and a mirror electron that could be potential contributors to the energy density at BBN epoch and affect the success of BBN. In order to reduce their contribution to a negligible level, the idea of asymmetric inflation was proposed in the second paper of ref. [8], according to which it is assumed that the reheat temperature after inflation in the mirror sector is lower than that in the visible sector by a factor of 10 or so i.e. \( T_R \simeq T_R / 10 \). If the interactions linking the two sectors are such that they are not in thermal contact for \( T \leq T_R \), then Hubble expansion will roughly maintain the ratio of the two temperatures till the BBN epoch apart from minor corrections arising from particle annihilation in both sectors. Thus at the BBN epoch, the total contribution of the light mirror particles to \( \rho_{\text{tot}} \) is at the level of about \( 10^{-3} \rho_r \) which therefore keeps the predictions of standard BBN unchanged. This will also keep the number densities of the light particles such as mirror neutrinos and mirror photons suppressed.

**CONSTRAINTS AND CONSEQUENCES**

The first constraints on the parameters of the model come from the fact that at temperatures below the reheating temperature after inflation, the interaction rates for \( L + H \rightarrow L' + H' + \phi \) should be out of equilibrium, otherwise, the two sectors will be in the same thermal bath and at BBN, the effective \( N_{\nu} \) will far exceed the allowed limit due to the fact that the population of the sterile neutrinos will build up their density to the level of ordinary active neutrinos. Only exception is if the reheat temperature after inflation is below an MeV which we do not invoke here. This translates into the following constraints in the three temperature regimes discussed below.

(i) \( T \geq < H >, < H' >: \) In this region, the condition for being out-of-equilibrium is

\[
T_D \leq \left( g_s^{-1/2} \frac{M_R^4}{M_P^4} \right)^{1/3}
\]

This inequality implies that for \( T \gg \text{TeV} \) (note that the mirror Higgs mass is expected to be in the TeV range), the visible and the mirror sector in our model are in equilibrium. So to be consistent with BBN requirements, we must require that the reheat temperature after inflation be less than about a TeV or so.

(ii) \( 0.1 \text{MeV} \leq T \leq < H >, < H' >: \) For \( T \leq M_{\chi} \sim 1 \) TeV, the Higgs fields decay instantly In this regime, the effective interaction connecting the visible to the mirror sector is the coupling \( \nu_{\nu_s} \phi \) with a strength given by \( g_{\nu_{\nu_s} \phi} \sim \lambda'' \left( \frac{\phi'}{M_R^2 M_R} \right) \approx 10^{-6} \). In discussing whether this interaction is in equilibrium, it has been noted in ref. [15] that the process \( \nu_{\nu_s} \rightarrow \phi \), vanishes in the limit of \( m_{\phi} = 0 \) by energy momentum conservation. The rate for this process must therefore be proportional to \( m_{\phi}^2(T) \).

This effective thermal mass is given by \( \kappa^2 T^2 / 16 \pi^2 \). If we choose the scalar self coupling \( \kappa \) to be of order \( 10^{-2} \), due to the fact that the number density of \( \nu_{\nu_s} \) is down by a factor of \( 10^{-3} \), we expect the rate \( \Gamma(\nu_{\nu_s} \rightarrow \phi) \approx 10^{-21} T \) to be of order \( 10^{-3} \) MeV. Below this temperature the interactions connecting the visible with the mirror sector are out of equilibrium. Thus (i) and (ii) together then help to satisfy the BBN constraints.

(iii) \( T \leq 0.1 \text{MeV}: \)

This regime is below the scale of \( < \phi > \). Therefore, \( \nu - \nu_s \rightarrow \phi \) is kinematically forbidden since \( \text{Im}\phi \) becomes a pseudo-Goldstone boson. The only process that can lead to production of sterile neutrinos is \( \nu \nu \rightarrow \nu_e \nu_{\nu_s} \). The rate for this process is given by \( 10^{-22} T \). This process is in equilibrium below \( T \leq 100 \text{ eV} \). Below this temperature the mirror sector and the standard model neutrinos will thermalize without a significant transfer of energy. When the temperature drops to \( T \simeq m_{\nu_s} \) the sterile neutrinos decay into one of the three active neutrinos and the singlet Higgs and finally, when below the temperature \( \sim 1 \) eV which is the mass of the \( \phi \), it will annihilate via the process \( \phi\phi \rightarrow \nu \nu \) leaving only a bath of active neutrino.

To calculate the final temperature of neutrino bath in terms of the photon temperature, we first remember that the boson \( \phi \) is part of a supersymmetric multiplet whose scalar field part has only one surviving imaginary part and the real part has decoupled at very high temperature. We assume that the imaginary part is also superlight. Then we proceed through the following steps:
Just below $T \simeq m_e$, electron-positron annihilation heats up the photons leading to the relation $T^0 = \left(\frac{4}{\pi}\right)^{1/3} T_\gamma$. The $\nu_s$ and $\phi$ at this stage are not in thermal contact with the active neutrinos. Once the temperature of the universe cools below 100 eV, $\nu - \nu_s\phi$ system comes into full thermal equilibrium. Using energy conservation and taking the effect of the incomplete $\phi$ supermultiplet into account, we get
\begin{equation}
(3 + \frac{11}{7} + n')T^4_{\nu+\phi} = 3T^4_{\nu}
\end{equation}
where $n'$ is the number of sterile neutrinos and the factor $\frac{11}{7}$ take into account the contribution of the fermionic part of singlet Higgs superfield $\phi$. As the universe cools below the mass of $\nu_s$, the $\nu_s$ decay to $\nu + \phi$. Using entropy conservation at this stage, we get
\begin{equation}
(3 + \frac{11}{7})T^3_{\nu+\phi} = (3 + \frac{11}{7} + n')T^3_{\nu+\phi+\nu_s}
\end{equation}
Using the above two equations, we get for the temperature of the $\nu + \phi$ system
\begin{equation}
T^4_{\nu+\phi} = \frac{3}{3 + \frac{11}{7}} \left[1 + \frac{n'}{3 + \frac{11}{7}}\right]^{1/3} \left(\frac{4}{11}\right)^{4/3} T^4_{\gamma}
\end{equation}
Noting that $\rho_\nu \propto (3 + \frac{11}{7})T^4_{\nu+\phi}$, we find the effective number of neutrinos at matter radiation equality is
\begin{equation}
N_\nu = 3\left[1 + \frac{n'}{3 + \frac{11}{7}}\right]^{1/3}
\end{equation}
When the temperature drops to $T \sim m_\phi$ the number of $\phi$'s get depleted via annihilation into standard model neutrinos. In this case the neutrino temperature is
\begin{equation}
\frac{T_\nu}{T_\gamma} = \frac{4}{11} \left[1 + \frac{n' + 11}{7/3}\right]^{1/12}
\end{equation}
As an example if $n' = 3$ the effective number of neutrinos after BBN is $N_\nu = 4.08$ and it is still consistent with CMB data. Future CMB experiments like PLANCK and CMBpol will be able to improve the limit on $N_\nu$ and can provide a test of this model. The contribution of the active neutrinos to the critical energy density is
\begin{equation}
\Omega_\nu = \left(\frac{53}{21}\right)^{1/3} \frac{m_\nu}{92h^2}
\end{equation}
Using the upper recent bound on the neutrino energy density on finds
\begin{equation}
\sum m_\nu < 0.36 \text{ eV}
\end{equation}
We emphasize that this limit is only on the sum of the masses of the active neutrinos since in our model the sterile neutrinos have decayed away.

Incidentally, the same steps can be repeated for theories without supersymmetry. In which case, we will assume that the $\phi$ field has only a real part $\rho$ and an imaginary part $\chi$. The $\phi$ field will have mass of order of the $\phi$-vev or about 100 keV whereas we will assume the $\chi$ mass to be an eV. In this case, below $T \simeq m_e$, first $\rho$ decay dumps into the $\nu + \nu_s$ system giving
\begin{equation}
T_{\nu+\nu_s+\chi} = \left(\frac{n' + 3}{n' + 3 + \frac{8}{7}}\right)^{1/3} T_{\nu+\nu_s+\phi}
\end{equation}
where $T_{\nu+\nu_s+\phi} \simeq \left(\frac{4}{11}\right)^{1/4} T^0_{\gamma}$ due to $\nu - \nu_s - \phi$ equilibrium. Taking entropy conservation at the subsequent $\nu_s$ decay and $\chi\to \nu\nu$ as in the previous case, we get
\begin{equation}
\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11}\right)^{1/3} \left(\frac{3 + \frac{8}{7} + n'}{3}\right)^{1/12}
\end{equation}
This changes the coefficient in the formula in Eq. (10) to $(\frac{50}{21})^{1/3}$ and changes on the limit on the sum of active neutrino masses from 0.36 eV to 0.37 eV.

Our scenario for sterile neutrinos has also interesting astrophysical implications. The first point is to look for any new mechanism for energy loss from the supernova core via emission of $\nu_\tau$ or $\nu_s$. Since $\nu - \nu_s$ mixing arises from spontaneous symmetry breaking at scale $\ll$ MeV, inside hot astrophysical environments such as a supernova, the active and sterile neutrinos remain unmixed. As a result, there is no energy loss via the emission of $\nu_s$. However, there could be energy loss due to the processes $\nu\nu \to \phi\phi, \nu_\tau\nu_\tau$. The rates for these processes are estimated to be: $\sim 10^{-25} T \sim (50 \text{ sec})^{-1}$. Comparing this with typical supernova explosion time scale, we expect this energy loss mechanism not to be significant. Also due to zero mixing between $\nu - \nu_s$ all supernova results based on three active neutrinos remain unaffected. Only in the very outer layers of the supernova explosion when the temperature drops below 100 KeV, will these mixings become operative.

**DISCUSSION AND SUMMARY**

In summary, we have presented a mirror model for the sterile neutrinos that can explain the LSND results and yet be consistent with stringent constraints from big bang nucleosynthesis as well as cosmic microwave background as well as structure formation bounds on neutrino properties. We make predictions for the effective neutrino number to which the next generation CMB measurements are sensitive. An important requirement of this model is that the reheat temperature after inflation must be less than a TeV. The model has also other interesting properties discussed earlier such as the mirror hydrogen being a dark matter, which remain unaffected by our modification.
Similarly, suggestions that the ultra high energy neutrinos could be originating from topological defects in the mirror sector remain unchanged by our extension.

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