The Great Wall of SDSS Galaxies

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Abstract

An enhancement in the number of galaxies as function of the redshift is visible on the SDSS Photometric Catalogue DR 12 at $z = 0.383$. This over-density of galaxies is named the Great Wall. This variable number of galaxies as a function of the redshift can be explained in the framework of the luminosity function for galaxies. The differential of the luminosity distance in respect to the redshift is evaluated in the framework of the LCDM cosmology.

Keywords

Galaxy Groups, Clusters, and Superclusters, Large Scale Structure of the Universe Cosmology

1. Introduction

We review some early works on the “CfA2 Great Wall”, which is the name that was introduced by [1] to classify an enhancement in the number of galaxies as a function of the redshift that is visible in the Center for Astrophysics (CfA) redshift survey [2]. The evaluation of the two point correlation function was done by [3] on the three slices of the CfA redshift survey. A careful analysis was performed on Sloan Digital Sky Survey (SDSS) DR4 galaxies by [4]: The great wall was detected in the range $0.07 < z < 0.09$. The substructures, the morphology and the galaxy contents were analyzed by [5] [6] and the luminosities and masses of galaxies were discussed in the framework of SDSS-III’s Baryon Oscillation Spectroscopic Survey (BOSS) [7]. The theoretical explanations for the Great Wall include an analysis of the peculiar velocities, see [8] [9], the non-linear fields of the Zel’dovich approximation, see [10] [11] [12], and the cosmology of the Great Attractor, see [13]. The layout of the rest of this paper is as follows. In Section 2, we introduce the LCDM cosmology and the luminosity function for galaxies. In Section 3, we introduce the adopted catalog for galaxies and the theoretical basis of the maximum for galaxies as function of the redshift.
2. Preliminaries

This section introduces an approximate luminosity distance as a function of the redshift in LCDM and derives the connected differential. The Schechter luminosity function for galaxies is reviewed.

2.1. Adopted Cosmology

Some useful formulae in ΛCDM cosmology can be expressed in terms of a Padé approximant. The basic parameters are: the Hubble constant, \( H_0 \), expressed in km s\(^{-1}\)·Mpc\(^{-1}\), the velocity of light, \( c \), expressed in km s\(^{-1}\), and the three numbers \( \Omega_M \), \( \Omega_K \), and \( \Omega_\Lambda \), see [14] for more details. In the case of the Union 2.1 compilation, see [15], the parameters are \( H_0 = 69.81 \text{ km s}^{-1} \cdot \text{Mpc}^{-1} \), \( \Omega_M = 0.239 \) and \( \Omega_\Lambda = 0.651 \). To have the luminosity distance, \( D_L(z; H_0, c, \Omega_M, \Omega_\Lambda) \), as a function of the redshift only, we apply the minimax rational approximation, which is characterized by the two parameters \( p \) and \( q \). We find a simplified expression for the luminosity distance, \( D_{L,6,2} \), when \( p = 6 \) and \( q = 2 \)

\[
D_{L,6,2} = \frac{ND}{0.284483 + 0.153266z + 0.0681615z^2}
\text{ for } 0.001 < z < 4, \tag{1}
\]

where

\[
ND = -0.0017 + 1221.80z + 1592.35z^2 + 504.386z^3
+ 85.8574z^4 + 0.41684z^5 + 0.186189z^6. \tag{2}
\]

The inverse of the above function, i.e. the redshift \( z_{6,2} \) as function of the luminosity distance, is

\[
z_{6,2} = 3.3754 \times 10^{-5} D_L - 0.46438 + 2.1625 \times 10^{-14} \times \sqrt{2.4363 \times 10^{13} D_L^2 + 3.9538 \times 10^{20} D_L + 4.6114 \times 10^{26}}. \tag{3}
\]

2.2. Luminosity Function for Galaxies

We used the Schechter function, see [16], as a luminosity function (LF) for galaxies

\[
\Phi(L) dL = \left( \frac{\Phi^*}{L^*} \right) \left( \frac{L}{L^*} \right)^\alpha \exp \left( -\frac{L}{L^*} \right) dL, \tag{4}
\]

here \( \alpha \) sets the slope for low values of luminosity, \( L, L^* \) is the characteristic luminosity and \( \Phi^* \) is the normalisation. The equivalent distribution in absolute magnitude is

\[
\Phi(M) dM = 0.921 \Phi^* 10^{0.4(M-M^*)} \exp \left( -10^{0.4(M-M^*)} \right) dM, \tag{5}
\]

where \( M^* \) is the characteristic magnitude as derived from the data. The scaling with \( h \) is \( M^* - 5 \log_{10} h \) and \( \Phi^* h^3 \text{ [Mpc}^{-3} \text{]} \).

3. The Photometric Maximum

This section models the Great Wall that is visible on the SDSS Photometric Ca-
talogue DR 12. It also evaluates the theoretical number of galaxies as a function of the redshift.

3.1. The SDSS Data

We processed the SDSS Photometric Catalogue DR 12, see [17], which contains 10,450,256 galaxies (elliptical + spiral) with redshift. In the following we will use the generic term galaxies without distinction between the two types, elliptical and spiral. The number of galaxies for an area in redshift of $0.025 \times 0.025$ of the u-band is reported in Figure 1 as a contour plot and in Figure 2 as a cut along a line.

3.2. The Theory

The flux, $f$, is

$$f = \frac{L}{4\pi r^2},$$

where $r$ is the luminosity distance. The luminosity distance is

$$r = D_{L,6,2},$$

and the relationship between $dr$ and $dz$ is

$$dr = \frac{N}{D} dz,$$

where

$$N = 74813.67 + 10.9263z^2 + 49.05768z^6 + 2642.64259z^8 + 16024.51314z^{10}$$

$$+ 54307.16663z^{12} + 127258.486z^{14} + 195005.8564z^{16},$$

and

![Figure 1. Contour for the number of galaxies (u-band) for areas of 0.025 × 0.025.](image)
Figure 2. The number of galaxies (u-band) along a line for areas of $0.025 \times 0.025$.

\[
D = \left( z^2 + 2.248575472z + 4.173664398 \right)^2. \tag{10}
\]

The joint distribution in $z$ and $f$ for the number of galaxies is

\[
\frac{dN}{d\Omega dz df} = \frac{1}{4\pi} \int_0^\infty 4\pi r^2 dr \Phi(L; L', \sigma) \delta(z - (z_{6.2})) \delta \left( f - \frac{L}{4\pi r^2} \right), \tag{11}
\]

where $\delta$ is the Dirac delta function, $\Phi(L; L', \sigma)$ has been defined in Equation (4) and $z_{6.2}$ has been defined in Equation (3). The explicit version is

\[
\frac{dN}{d\Omega dz df} = \frac{NN}{DD}. \tag{12}
\]

where

\[
NN = 644.3(z + 1.058)^4 \left( z - 0.000001412 \right)^4 \left( z^2 + 4.889z + 13.34 \right)^4 \times \left( z^2 - 3.708z + 464.6 \right)^4 e^{AA} \frac{CC}{BB^2 L'} \Phi' (z + 0.5328) \tag{13}
\]

\[
\times \left( z^2 + 5.047z + 7.9141 \right) \left( z^2 + 0.7z + 7.011 \right) \left( z^2 - 1.79z + 231.58 \right),
\]

\[
CC = f(z + 1.058)^2 \left( z - 0.0000014124 \right)^2 \left( z^2 + 4.889z + 13.343 \right)^2 \times \left( z^2 - 3.708z + 464.6 \right)^2
\]

\[
DD = \left( z^2 + 2.248z + 4.173 \right)^6 L'. \tag{15}
\]

\[
AA = -93.76 f_2^{12} - 419.8 f_2^{11} - 8694.5 f_2^{10} - 701622.4 f_2^9
\]

\[
-2267941 f_2^8 - 239083307 f_2^7 - 1430432291 f_2^6
\]

\[
-4912205831 f_2^5 + (4.540788\alpha L' - 10191896880f) z^4
\]

\[
+ (20.4206\alpha L' - 10524532830f) z^3 + \left( 60.862\alpha L' - 4037704632f \right) z^2
\]

\[
+ (85.22\alpha L' + 11406.2f) z + 79.098\alpha L' - 0.008055 f, \tag{16}
\]
\[ BB = z^2 + 2.2485z + 4.173. \] (17)

**Figure 3** presents the number of galaxies that are observed in SDSS DR 12 as a function of the redshift for a given window in flux, in addition to the theoretical curve. The theoretical number of galaxies is reported in **Figure 4** as a function of the flux and redshift, and is reported in **Figure 5** as a function of \( \alpha \) and redshift.

The total number of galaxies comprised between a minimum value of flux, \( f_{\text{min}} \), and a maximum value of flux \( f_{\text{max}} \), for the Schechter LF can be computed through the integral

\[
\frac{dN}{d\Omega dz} = \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{NN}{DD} df. \] (18)

This integral has a complicated analytical solution in terms of the Whittaker function \( M_{\kappa,\mu}(z) \), see [18]. **Figure 6** reports all of the galaxies of SDSS DR12 and also the theoretical curve.

A theoretical surface/contour of the Great Wall is displayed in **Figure 7**

### 4. Conclusions

1) **\( \Lambda \)CDM cosmology**

In this paper, we use the framework of \( \Lambda \)CDM cosmology with parameters 
\[ H_0 = 69.81 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad \Omega_m = 0.239 \quad \text{and} \quad \Omega_\Lambda = 0.651. \] A relationship for
Figure 4. The theoretical number of galaxies divided by 1000 as a function of redshift and flux is expressed in $L_\odot/Mpc^3$. The parameters are $L' = 2.038 \times 10^{10} L_\odot$, $\alpha = -0.9$ and $\Phi' = 0.038 \text{Mpc}^{-3}$.

Figure 5. The theoretical number of galaxies divided by 100,000 as a function of $\alpha$ and redshift when $L' = 2.038 \times 10^{10} L_\odot$, $f = 1000 L_\odot/Mpc^3$, and $\Phi' = 0.038 \text{Mpc}^{-3}$. 
Figure 6. All of the galaxies of the SDSS DR12 catalog (u-band) are organized in frequencies versus spectroscopic redshift. The error bar is given by the square root of the frequency (Poisson distribution). The maximum frequency of all observed galaxies is at $z = 0.383$. The full line is the theoretical curve generated by \( \frac{dN}{d\Omega dz}(z) \) as given by the numerical integration of Equation (18) with $L^* = 2.038 \times 10^6 L_\odot$, $\alpha = -0.9$ and $\Phi^* = 0.038$.

Figure 7. Theoretical surface/contour of the number of all the galaxies divided by 1,000,000 as a function of redshift. Parameters as in Figure 6.
the luminosity distance is derived using the method of the minimax approximation when \( p = 6 \) and \( q = 2 \), see Equation (2). The inverse relationship, the redshift as function of the luminosity function is derived in Equation (3).

2) The Great Wall

The enhancement in the number of galaxies as a function of the redshift for the SDSS Photometric Catalogue DR 12, which is at \( z = 0.383 \), is here modeled by the theoretical Equation (12) that is derived in the framework of the Schechter LF for galaxies and the \( \Lambda \) CDM cosmology. Figure 6 reports the observed maximum in the number of galaxies and also the theoretical curve. These results are in agreement with a catalog of photometric redshift of \( \approx 3,000,000 \) SDSS DR8 galaxies made by [19]: their Figure 9 bottom reports that the count of elliptical galaxies has a peak at \( z \approx 0.37 \) when the spirals galaxies conversely peaks at \( z \approx 0.08 \).

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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