Orientifold in Conifold and Quantum Deformation

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Abstract

We describe orientifold operation defining O3 plane in the conifold background by deriving it from that of O4 plane in the Type IIA brane construction by T-duality. We find that both O3+ and O3− are at the tip of the cone so that there is no net untwisted RR charge. RG analysis shows that we need two ‘fractional’ branes for the conformal invariance in orientifolded conifold. We argue that the gravity solution is the same as Klebanov and Tseytlin since SUGRA cannot distinguish the orientifolds and D branes in this case. We describe the duality cascade as well as the quantum deformation of the moduli space of the field theory in the presence of the orientifold. The finitely resolved conifold does not allow the orientifold, while deformed conifold leaves us an unresolved issue on supersymmetry.

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1 Introduction

A fruitful generalization of the duality between $\mathcal{N} = 4$ $SU(N)$ gauge theory and type IIB strings on $AdS_5 \times S^5$ \cite{1,2,3} is to consider other backgrounds of type IIB string theory, say on $AdS_5 \times X_5$, where $X_5$ is a positively curved Einstein manifold. We are interested in theories with reduced supersymmetry, variety of gauge groups and matter contents.

The simplest example realizing the $\mathcal{N} = 1$ supersymmetry is provided by D3 branes at the singularity of a Calabi-Yau threefold with singularity known as conifold \cite{4}. The conifold in the AdS/CFT context was first considered in \cite{5,6} and generalized to the case where conifold is deformed by the quantum effect of the fractional branes by Klebanov and Strassler \cite{7}.

To construct more realistic model, gauge theory with $SO(N)/Sp(N)$ is necessary and this can be obtained in brane language by including the orientifolds \cite{8,9}. Since there are so many $Z_2$ symmetry that can act on the conifold \cite{6}, one needs care to determine which symmetry is relevant to the specific orbifolding operation to define the orientifold. One way to fix the notion of the orientifold operation is to try to derive it from that of the type IIA brane construction where things are canonically defined. Under the T-duality the regular D4 branes are mapped to the regular D3 branes and and the fractional D4 branes are mapped to fractional branes which can be considered as D5 branes wrapping the the vanishing a 2-cycle. We use T duality using the prescription given in \cite{15}.

There are discussions on orientifolding the conifold based on O6 plane in type IIA picture \cite{10,11,12,13,14}. Here we present a discussion on the orientifold in the conifold based on O4 brane of type IIA, because that is the one relevant to the physics of fractional branes discussed in Ref.\cite{3}. However it should be kept in mind that the resulting O3 branes has nature of fractional brane, since in type IIA picture O4 branes are between NS branes like D4 branes that is mapped to fractional D3 branes. Therefore our resulting O3 branes has the character of the wrapped O5 branes, although our starting point in type IIA is completely different from the O6 brane configurations.

We will find that both $O3^+$ and $O3^-$ are at the tip of the cone without annihilating each other since they are ‘topologically protected’ from mutual annihilation. But their effect is combine to give zero net untwisted RR charges. The non-perturbative quantum effect deforms the tip of the cone. We will discuss how orientifold response to the deformation or resolution of the conifold. The finitely resolved conifold does not allow the orientifold, while deformed conifold leaves us an unresolved issue on supersymmetry. We will also show that we need two ‘fractional’ branes for the conformal invariance so that the theory should have gauge group
We will give several explanations for this. We then give generalizations of the gauge theory results of ref. [7] to the case with orientifold. We describe the duality cascade and chiral symmetry breaking as well as the quantum deformation of the moduli space of the field theory in the presence of the conifold. We argue that the gravity solution is the same as Klebanov and Tseytlin since it cannot distinguish the orientifolds and D branes.

The rest of paper comes in following order. In section 2, we give a brief review on the relevant background. In section 3, we derive the orientifold operation from that of Type IIA picture. In section 4, we perform renormalization group analysis to fix the that the conformally invariant configuration as well as to compare bulk and boundary theory. We also discuss the duality cascade and chiral symmetry breaking in the presence of the conifold. In section 5, we show that quantum moduli space of the corresponding gauge theory is a deformed conifold. We conclude in section 6.

2 Conifold and its type IIA brane construction: a review

A conifold is a complex submanifold in \( \mathbb{C}^4 \) described by the quadratic equation: \( \sum_{i=1}^{4} z_i^2 = 0 \). Its metric is known [4] to be \( ds^2 = dr^2 + r^2 ds_{T^{1,1}}^2 \) with

\[
 ds_{T^{1,1}}^2 = \frac{1}{9} (d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2)^2 + \frac{1}{6} \sum_{i=1}^{2} (d\theta_i^2 + \sin^2 \theta_i d\phi_i^2). \tag{1}
\]

The base of the conifold, \( T^{1,1} \), is an \( S^1 \) bundle over \( S^2 \times S^2 \), and has the metric \( ds_{T^{1,1}}^2 \) given above. The conifold is a Calabi-Yau manifold and respects \( \mathcal{N} = 2 \) supersymmetry. By putting \( N \) D3 branes at the tip of the conifold we can get \( \mathcal{N} = 1 \) supersymmetric field theory living on the D3. This field theory was constructed by Klebanov and Witten [3]. It is a \( SU(N) \times SU(N) \) gauge theory coupled to two chiral superfields \( A_i, \hat{A}_i, i = 1, 2 \) in the \((N,\bar{N})\) and \( B_j \) in the \((\bar{N},N)\) representation. The most general superpotential which preserves the symmetry of the conifold is

\[
 W = \text{Tr} \epsilon^{ij} \epsilon^{kl} A_i B_k A_j B_l. \tag{2}
\]

One can wrap various D branes over the cycles of \( T^{1,1} \) and identify these with states in the field theory [10].

A very intuitive way to understand the field theory content is to T-dualize the theory and consider brane configurations in type IIA string theory [18, 13, 19, 20]. As we discussed before, \footnote{The reason for the gauge group being a product group is due to the underlying orbifold symmetry \( Z_2 \).}
there are ambiguity which T-duality should we take. We focus the regular D3 branes and take T6 duality. In Ref. [15], N D3 branes on a conifold is shown to be T6-dual to the type IIA brane configuration with NS5(1,2,3,4,5), NS5′(1,2,3,7,8,9) and N D4(1,2,3,6). \(x^6 = \psi\) is periodic, NS brane is at \(\psi = 0\), NS′ is at \(\psi = 2\pi\), N D4 branes wrap the \(x^6\) circle so that the system is an elliptic model. In type IIA picture it is very easy to see the resulting theory is \(SU(N) \times SU(N)\) gauge theory with 2\(N\) flavors \((A_i, B_i), i = 1, 2\). The theory is conformally invariant, as can be checked explicitly from the calculation of beta functions [6].

Introducing \(M\) fractional D3 branes into the picture, the resulting gauge theory is \(SU(N + M) \times SU(N)\) (with gauge couplings \(g_1\) and \(g_2\) respectively) [1], [19]. In type IIA picture, this corresponds to putting \(M\) D4 branes between \(0 \leq \psi \leq 2\pi\) and the theory is no longer conformally invariant. The two gauge couplings are determined as follows [22, 23]:

\[
\frac{1}{g_1^2} + \frac{1}{g_2^2} \sim e^{-\phi}, \quad \frac{1}{g_1^2} - \frac{1}{g_2^2} \sim e^{-\phi} \left[\left(\int_{S^2} B_2\right) - 1/2\right].
\]

(3)

Since RG flow of the coupling constant in \(\mathcal{N} = 1\) gauge theory is logarithmic, the gravity dual is expected to have similar logarithmic behavior in the radial coordinate of \(AdS_5\) space and in fact this is true [24]. In terms of the type IIA brane construction, the two gauge couplings are determined by the positions of the NS5 branes along the \(x^6\) circle. If one of the NS5 branes is located at \(x_6 = 0\) and the other at \(x_6 = a\), then

\[
\frac{1}{g_1^2} = \frac{l_6 - a}{g_s}, \quad \frac{1}{g_2^2} = \frac{a}{g_s},
\]

(4)

where \(l_6\) is the circumference of the \(x^6\) circle [15].

As the NS5 branes approach each other, one of the couplings becomes strong. In fact, the two gauge couplings \(1/g_1^2\) and \(1/g_2^2\) flow in the opposite directions and there is a scale where one of the couplings diverge, which necessitates the use of Seiberg’s dual gauge theory [25]. In the present situation, this corresponds to the moving the NS brane across the NS′ brane. After all the re-connections are made, we get \(SU(N - M) \times SU(N)\) theory [7]. Notice that \(SU(N_f - N_c) = SU(N - M)\).

As we go to the further IR region, the same process repeat until we get \(SU(M + p) \times SU(p)\) with \(p\) less than \(M\). This is so called the cascade of the duality. The simplest case is \(p = 1\). one may consider this as one D3 brane probing the the background of M-fractional branes. Using the ADS superpotential [26], together with the original classical superpotential for the conifold, it was shown that the conifold singularity is resolved into ‘deformed conifold’ [7]. A very interesting phenomenon happens as a consequence, namely the chiral symmetry is broken.
3 Orientifold in conifold

The orientifolding operation is consist of world sheet orientation reversal and reflection of transverse spacetime, together with appropriate projection of the left moving fermion number. We will concentrate on the spacetime reflection part hereon. In the conifold there are many $Z_2$ operations so that it is not clear which is the relevant $Z_2$ for the orientifolding. Therefore we want to derive the orientifold operation in the conifold from the type IIA picture where O4-plane is clearly defined.

We have D4 (01236), O4 (01236), NS5 (012345), NS5' (012389) with $x^6$ compact. Under the T-duality along the $x^6$, the whole configuration is mapped to the conifold. The prescription for the (T-duality) mapping flat $R^6$ to conifold suggested by Dasgupta and Mukhi is:

$$x^4, x^5 \text{ plane } \rightarrow S^2 \text{ described by } \theta_1, \phi_1, 0 \leq \theta_1 < \pi, 0 \leq \phi_1 < 2\pi,$$
$$x^8, x^9 \text{ plane } \rightarrow S^2 \text{ described by } \theta_2, \phi_2,$$
$$x^6 \rightarrow \psi, \quad |x^7| \rightarrow \log 1/r.$$ (5)

The orientifolding in type IIA picture is given by

$$x^i \rightarrow -x^i \text{ for } i = 4, 5, 7, 8, 9, \text{ and } x^i \rightarrow x^i, \text{ otherwise.}$$ (6)

Under T-duality along $x^6$, O4 brane becomes O3 brane. The reflections in $x^4, x^5, x^6, x^8, x^9$ induces an antipodal mapping in $S^2$ and $S^3$ of $T^{1,1}$, the base of the cone. Above reflection in terms of the polar coordinates is

$$\phi_i \rightarrow \phi_i + \pi_i, \quad \theta_i \rightarrow \pi - \theta_i, \quad \psi \rightarrow -\psi.$$ (7)

Now we want to express the above operation in terms of the conifold variables $z_i$’s. For doing that we have to express them in terms of the angular variables. Fortunately this has been done in Ref.[4]. Introducing the variable

$$Z \equiv \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} z_3 + iz_4 & z_1 - iz_2 \\ z_1 + iz_2 & -z_3 + iz_4 \end{pmatrix},$$ (8)

the equation defining the conifold can be rewritten as $\det(z_{ij}) = 0$. The base $T^{1,1}$ is the intersection of the conifold with the $S^5$ sphere $\sum_{i=1}^4 |z_i|^2 = r^2$, and is locally $SU(2) \times SU(2)/U(1)$.

We parametrize $T^{1,1}$ by $SU(2)$ parameters $a_i, b_i, \ i = 1, 2$:

$$\begin{pmatrix} a_i \\ b_i \end{pmatrix} = \begin{pmatrix} \cos \frac{\theta_i}{2}e^{i(\psi+\phi_i)/2} \\ \sin \frac{\theta_i}{2}e^{i(\psi-\phi_i)/2} \end{pmatrix}. $$ (9)
We can naturally map the $S^2$'s, which are compactifications of the 4,5 and 8,9 planes, into two SU(2)'s by the Hopf fibration $\tilde{n} = a^\dagger \sigma a$, where $\tilde{n} \in S^2$, $a = \begin{pmatrix} a \\ b \end{pmatrix}$. With this identification, the antipodal mappings in the two $S^2$ induces a mapping in $T^{1,1}$ hence a mapping in the conifold. From the two SU(2) matrices $L = \begin{pmatrix} a_1 & -b_1 \\ b_1 & \bar{a}_1 \end{pmatrix}$, $R = \begin{pmatrix} a_2 & -b_2 \\ b_2 & \bar{a}_2 \end{pmatrix}$, we may construct the base variable

$$Z/r = LZ_0 R^\dagger = \begin{pmatrix} -a_1 b_2 & a_1 a_2 \\ -b_1 b_2 & b_1 a_2 \end{pmatrix},$$

(10)

with $r^2 = \text{Tr}(ZZ^\dagger) = \sum_i |z_i|^2$.

In terms of SU(2) variables the orientifold operation is

$$a_i \rightarrow i\bar{b}_i, \quad b_i \rightarrow -i\bar{a}_i,$$

(11)

which implies $z_{11} \rightarrow -\bar{z}_{22}$, $z_{12} \rightarrow \bar{z}_{21}$. In terms of original variables $z_i$'s, above operation has a simple expression:

$$z_i \rightarrow \bar{z}_i, \quad i = 1, \cdots, 4.$$

(12)

The conifold is invariant under this $Z_2$ operation and so is the superpotential.

Let $z_j = x_j + iy_j$, $j = 1, \cdots, 4$ with $x_j, y_j$ be real. Then fixed points of this reflection are given by $y_i = 0$, $i = 1, \cdots, 4$. Together with the conifold equation $\sum_i z_i^2 = 0$, we conclude that $z_i = 0$, i.e. the tip of the cone, is the only fixed point. This is consistent with the fact that the O3 is a point in the conifold.

However, O3 branes are like fractional D3 branes, since they are between NS branes rather than wrapping the whole circle of the IIA picture. In type IIB conifold picture, they are O5 branes wrapping the different singular $S^2$ cycles. In fact the base of the $T^{1,1}$ is $\frac{SU(2) \times SU(2)}{U(1)}$ and the $U(1)$ is acting symmetrically on both SU(2)'s by

$$
\begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}.
$$

It is also known that the second homology basis is a combination of two $S^2$'s above, namely, $\Sigma_2 = S^2_2 - S^2_1$ \cite{16, 19}. The charge of the O3 brane is determined according to which $S^2$ the O5 wraps. Therefore both $O3^+$ and $O3^-$ can be considered as a wrapped O5 branes wrapping different vanishing cycles. Since they are wrapping different $S^2$'s (although in the vanishing limit), we can say that they are stable due to the topological reason. However, since they are wrapping vanishing cycles at the same 'point', they are like overlapping charges of opposite charge. So if we measure the net effect, there are no untwisted RR charges and it is an effect of
one object that has only twisted RR charge. This situation is closely related to the orientifold of $C^2/Z_N$ considered by Uranga \[17\]

We now ask what happens to the small resolution of the conifold?

$$|a_1|^2 + |b_1|^2 - |a_2|^2 - |b_2|^2 = \delta. \quad (13)$$

One immediately see that the small resolution of the conifold is invariant under the orientifold operation. However, from Eq.(11) there is no fixed point on the $S^2$ unless its size is zero. Only when the fractional branes are wrapping the ‘vanishing cycle’, we can have orientifold in conifold. This means that the resolved conifold does not admit an super symmetric orientifold.

If there are large number of fractional branes, due to the quantum effect, the conifold background is modified \[7\] to deformed conifold $\sum_i x_i^2 = \mu$. In this case, the fixed points form a manifold $S^3$ given by $\sum_i x_i^2 = \mu$. Does this mean that O6 is created? Then, where is the orientifold in the deformed conifold? Part of the answer also lies in the large $N$ geometric transition \[1, 27\]: when the number of the fractional branes are large, it goes to the large N dual description where branes disappear and only flux remains. Presumably O3 branes disappeared leaving only its flux. Since $O3^-$ comes with extra two D3, $O3^\pm$ contribute the same amount of 3-form flux. Therefore the equality $O3^+ = O3^- + 2D3$ holds as far as supergravity solutions are concerned. The flux of the fractional D3 brane charge resolve the singularity in the $O3^+$ side. Since O6 RR charge can not be created by the deformation process, the most natural answer to above question seems to be that the $O3^+$ charge is smeared uniformly over the fixed manifold, while $O3^-$ is wrapping the vanishing $S^2$. We will be back to this issue in the discussion section for other possibilities. Now we make some remarks.

1. In type IIA, $x^7$ is a spectator. However, under the identification $|x^7| = \log 1/r$, we are abandoning the region $r \geq 1$. We could equally cut out the $r \leq 1$ by the identification $|x^7| = \log r$. We have chosen above convention since we are interested in the near the singularity: $r \to 0$ corresponds to $x^7 \to \infty$.

2. Recently the same operation was considered in Ref.\[13\] in the context of the topological string with O3 brane wrapping $S^3$. Here our interpretation is different: The fixed three sphere of the deformed conifold is due to smearing of the O3 brane charge.

3. It may be worth while to describe the orientifolding induced by the $T_{\phi}$ duality, although we do not follow it. In this scheme, 4,5 and 8,9 plane is consequence of the T-duality along the circle action at the two sphere of the resolved conifold: two fixed points of the circle action under the $T_{\phi}$ duality is mapped to the NS and NS$'$ branes. The orientifold
operation of the type IIA branes does not involve the reflection along $x^6 = \psi$ therefore not a antipodal point mapping. The action is simply given by

$$\phi_i \rightarrow \phi_i + \pi, \quad \theta_i \rightarrow \theta_i, \psi \rightarrow \psi.$$ (14)

In terms of $a_i$ and $b_i$, this is written

$$a_i \rightarrow a_i, \quad b_i \rightarrow -ib_i.$$ (15)

Finally, in terms of the $z_i$’s,

$$z_1 \rightarrow z_1, z_2 \rightarrow z_2, z_3 \rightarrow -z_3, z_4 \rightarrow -z_4.$$ (16)

which also appear in literature as an orientifold operation.

4 The RG analysis and Duality cascade in the presence of the orientifold

If $N$ D3 branes are sitting at the singular point of the conifold, the gauge theory content turned out to be $SU(N) \times SU(N)$ [6]. In this section, we discuss how introducing the orientifold modify the theory.

We use the type IIA brane picture. If we add O4 plane along 01236 direction, we have to change the sign of the RR charge of the O4 plane as we cross the NS brane [30, 29]. The corresponding gauge theory must be of alternating SO and Sp gauge groups. So one may naively expect that the gauge group would be $SO(N) \times Sp(N)$. (Here we use the convention where $Sp(2) = SU(2)$.) However, this is not conformal as we can see from the RG flow:

$$\frac{d}{d \log(\Lambda/\mu)} \left[ \frac{8\pi^2}{g_{so}^2} \right] \sim 3(N - 2) - 2N(1 - \gamma),$$

$$\frac{d}{d \log(\Lambda/\mu)} \left[ \frac{8\pi^2}{g_{sp}^2} \right] \sim 3(N/2 + 1) - (N)(1 - \gamma).$$ (17)

One cannot require two beta functions to vanish simultaneously. We can overcome the difficulty if we replace $SO(N)$ by $SO(N+2)$. In brane language, we should add an extra brane and its image over the negative charged orientifold for the stable configuration.

The presence of the extra two branes can be also understood from the brane dynamics [30]: for stable configuration, RR charge density must be continuous across the NS5. Since $O4^\pm$ has
RR charge ±1, we need extra 2 D4 over the O4−. Then the gauge group is $SO(N+2) \times Sp(N)$. One may interpret the extra 2 branes as the fractional branes in IIB picture \[19, 20\]. So in the presence of the orientifold, the brane configuration with no fractional branes is not stable fixed point.

If we place $N$ D3 branes and $M + 2$ fractional D3 branes on the conifold with an O3 plane, we obtain an $SO(N + M + 2) \times Sp(N)$ or $Sp(N + M) \times SO(N + 2)$ gauge groups. Here, we discuss the first one since the other one is exactly parallel. The two gauge group factors have holomorphic scales $\Lambda_1$ and $\tilde{\Lambda}_1$. The matter consists of two chiral superfields $A_1, A_2$ in the $(N + M + 2, N)$ representation and two fields $B_1, B_2$ in the $(N + M + 2, N)$ representation. The invariants of the theory under the global symmetry $SU(2) \times SU(2) \times U(1)$ are

$$I_1 \sim \lambda_1^{3M+2} \frac{\tilde{\Lambda}_1}{\Lambda_1^{b_{SO}}} \left[ \text{tr} (A_i B_j A_k B_\ell e^{ik \epsilon^j \ell}) \right]^{2M},$$

$$R_1^{(1)} = \frac{\text{tr} [A_i B_j] \text{tr} [A_k B_\ell] e^{ik \epsilon^j \ell}}{\text{tr} (A_i B_j A_k B_\ell e^{ik \epsilon^j \ell})};$$

$$J_1 \equiv \lambda_1^{b_{SO}+2b_{Sp}} \Lambda_1^{b_{SO}} \tilde{\Lambda}_1^{-2b_{Sp}};$$

where $b_{SO}$, $b_{Sp}$ are β function coefficients:

$$b_{SO} = 3(N + M + 2 - 2) - 4 \cdot \frac{N}{2} \cdot 1 = N + 3M,$$

$$b_{Sp} = 3 \left( \frac{N}{2} + 1 \right) - 4 \cdot \frac{N + M + 2}{2} \cdot \frac{1}{2} = N/2 - M + 1.$$ (21)

These statements can easily be verified by the quantum number assignment similar to that given in table 1 in Ref.\[7\]. The only difference is that $\Lambda_1^{3(N+M)-2N}$ and $\tilde{\Lambda}_1^{3(N)-2(N+M)}$ of the for $SU(N) \times SU(N)$ should be replaced by $\Lambda_1^{3(N+M+2)-2N}$ and $\tilde{\Lambda}_1^{3(N+2)-2(N+M+2)}$ of the $SO(N + M + 2) \times Sp(N)$. The superpotential of the model will get multiplicative renormalization depending on $I_1, J_1, R_1$’s.

In the presence of the orientifold, the geometry of the base of the cone is $RP^2 \times S^3$. The 3-form flux breaks the conformal symmetry and $B_2$ gets radial dependence.

$$\int_{RP^2} B_2 \sim (M/2) \ln(r/r_0),$$

which indicate the logarithmic running of $\frac{1}{g_{SO}^2} - \frac{1}{g_{Sp}^2}$ in the $SO(N + M + 2) \times Sp(N)$ gauge theory. To check this bulk result, we look at the β functions of the boundary theory:

$$\frac{d}{d \log(\Lambda/\mu)} \left[ \frac{8\pi^2}{g_{SO}^2} \right] \sim 3(N + M) - 2N(1 - \gamma)$$

$$\frac{d}{d \log(\Lambda/\mu)} \left[ \frac{8\pi^2}{g_{Sp}^2} \right] \sim 3(N/2 + 1) - (N + M + 2)(1 - \gamma)$$

(23)
For the conformal invariance of $M = 0$ case, we require $\gamma = -1/2$. Notice that two flows give the same condition of the anomalous dimension in spite of the difference of the gauge group. The difference of the flow is

$$\frac{8\pi^2}{g_{SO}^2} - \frac{8\pi^2}{g_{Sp}^2} = (4 - \gamma)M \log(\Lambda / \mu).$$  \hspace{1cm} (24)

showing the consistency of bulk and boundary result.

Since $1/g_{SO}^2$ and $1/g_{Sp}^2$ flow in opposite directions, there is a scale where the $SO(N + M + 2)$ coupling, $g_{SO}$, diverges. Using the Seiberg duality transformation, we get the $SO(2N - [N + M + 2] + 4) = SO(N - M + 2)$ gauge group with $2N$ flavors $(a_i, b_i)$ and “meson” bilinears $M_{ij} = A_i B_j$. The fields $a_i$ and $b_i$ are anti-fundamentals and fundamentals of $Sp(N)$, while the mesons are in the adjoint-plus-singlet of $Sp(N)$. The superpotential after the transformation

$$W = \lambda_1 \text{tr} M_{ij} M_{k\ell} e^{i k \ell} F_1(I_1, J_1, R_1^{(s)}) + \frac{1}{\mu} \text{tr} M_{ij} a_i b_j ,$$ \hspace{1cm} (25)

where $\mu$ is the matching scale for the duality transformation [7], shows the $M_{ij}$ are actually massive. Thus we may integrate them out and get the superpotential

$$W = \lambda_2 \text{tr} a_i b_j a_k b_\ell e^{ik\ell} F_2(I_2, J_2, R_2^{(s)}).$$ \hspace{1cm} (26)

Here $F_2$, $\lambda_2$, $I_2$, $J_2$ and $R_2$ are defined similarly as in the original theory. Thus we obtain an $Sp(N) \times SO(N - M + 2)$ theory which resembles closely the theory we started with. This can be shown more carefully using the matching condition as discussed in [7]. The next step is that the $Sp(N)$ gauge group becomes strongly coupled, and under a Seiberg duality transformation the full gauge group becomes $SO(N - 3M + 2) \times Sp(N - 4M)$, and so forth.

## 5 Deformation of the orientifolded conifold

The classical field theory reveals that it represents branes moving on a conifold [6, 23] by having the conifold as its moduli space of the gauge theory. Klebanov and Strassler showed that the vacuum configurations of gauge theory, the conifold, is modified to a deformed conifold by studying non-perturbative quantum corrections [7]. Here we study corresponding phenomena in the presence of the orientifold. In our case, the minimal configuration is $SO(M + 4) \times Sp(2)$, corresponding to a D-brane and its image in the presence of $M + 2$ fractional branes. The fields are

$$A^r_{i,\alpha}, B^a_{j,\alpha} : i = 1, 2, \quad \alpha = 1, ..., N_c, \quad r = 1, ..., N_f.$$  \hspace{1cm} (27)
Define $N_{ij}$ by $N_{ij}^{rs} = \sum_{\alpha} A_{i,\alpha}^r B_{j,\alpha}^s$. Then the classical superpotential can be written as

$$W_c = \frac{1}{2} \lambda \text{Tr} A_r B_j A_k B_l \epsilon^{ik} \epsilon^{jl} = \frac{1}{2} \lambda \text{Tr} N_{ij} N_{kl} \epsilon^{ik} \epsilon^{jl}.$$  \hspace{1cm} (28)

where the trace is over the flavor index. For the gauge theory with $N_c$ colors and $N_f$ flavors, the quantum effect gives the ADS super potential [26]. In our case, the gauge group is $SO(N + M + 2) \times Sp(N)$. So one may expect that we should add up the ADS potential for each gauge group. However, one should notice that we are interested in $M \gg 2 = N$ and the ADS potential is meaningful only for $N_c > N_f$. So we only have to consider the superpotential for the $SO(N + M + 2)$ part. We propose that following is the leading order superpotential for the product group:

$$W_{total} = \lambda W_c + (N_c - 2 - N_f) \left( \frac{\Lambda_{b_0}}{\text{det}_{ir,js} N_{ij}^{rs}} \right)^{1/(N_c - 2 - N_f)}.$$  \hspace{1cm} (29)

Now we consider the completely higgsed configuration $SU(N) \rightarrow U(1)^N$, where the matrix $N_{ij}$ have vacuum expectation values:

$$N_{ij}^{rs} = n_{ij}^{(r)} \delta^{rs}. \hspace{1cm} (30)$$

This is justified since $r$-th and $s$-th fractional (flavor) branes are far separated in higgsed case. So, the determinant is trivially factorized:

$$\text{det}_{ir,js} N_{ij}^{rs} = \prod_{r=1}^{N_f} W^{(r)}, \hspace{1cm} (31)$$

where $W^{(r)} = n_{ij}^{(r)} n_{kl}^{(r)} \epsilon^{ik} \epsilon^{jl}$ Using $W_c = \lambda \sum_r W^{(r)}$, and considering $N_f = 2$ case, the total superpotential becomes

$$W_{total} = \lambda (W^{(1)} + W^{(2)}) + (N_c - 2 - N_f) \left( \frac{\Lambda_{b_0}}{W^{(1)} W^{(2)}} \right)^{1/(N_c - 2 - N_f)}, \hspace{1cm} (32)$$

where $b_0 = 3(N_c - 2) - N_f$. The total potential is minimized if we have

$$(N_{ij} N_{kl} \epsilon^{ik} \epsilon^{jl})^{(N_c - N_f)} = \left( \frac{\Lambda_{b_0}}{\Lambda^{N_c - N_f - 2}} \right). \hspace{1cm} (33)$$

Notice that $N_c - N_f = (M + 2) - 2 = M$ in our case. This is nothing but the equation for the deformed conifold and there are $M$ branches: each of the probe branes move on the deformed conifolds.
The M-theory curve \[28\] for the Type IIA version is given as follows:

\[(vw)^M = \Lambda^{3M}, \quad t = v^{N_c-N_f} = v^M.\] (34)

The curve indicates that if there is more than two fractional branes \((M > 0)\), the whole brane configuration is nicely connected and this indicates the deformation of the conifold. Interesting fact is that the curve is insensitive to the presence of the regular D branes since it contribute to \(N_f\) as well as to \(N_c\). It depends only on the number of fractional branes. Orientifolds does not change the formal behavior at all, either. It just require that \(M\) is even.

6 Conclusion

In this paper, we discussed the string theory in conifold in the presence of the orientifold. We first mapped the orientifolding operation from the type IIA brane picture to the conifold picture, using the T-duality along the \(x^6\) direction. Under the T-duality the fractional D4 branes (D4 branes between NS-NS' branes) are mapped to fractional D3 branes, which is identified as D5 wrapping the vanishing 2-cycles. We showed how the conifold can admit orientifolds of both + and − charges. Blowing up the tip, the base is still \(S^2 \times S^2 \times S^1\). The fractional D3 and \(O3^-\) are D5 and O5 branes respectively wrapping one of the \(S^2\). The \(O3^+\) is O5 brane wrapping the other \(S^2\). Since homology 2-cycle which defines the RR charge is given by the difference of two \(S^2\)'s, it is natural to have both \(O3^+\) and \(O3^-\) simultaneously at the tip of the cone without annihilating each other: they are topologically protected not to be annihilated. We then showed that in the presence of the orientifold, the conformally invariant configuration is \(SO(N+2) \times Sp(N)\) rather than \(SO(N) \times Sp(N)\). This is shown both by field theory and brane dynamics. If we add fractional branes, there are duality cascade as in Ref.[7]. We analyzed the corresponding gauge theory as well as the super gravity solution. When there are fractional branes, the conifold is deformed as is the case of \(SU(N) \times SU(N)\). We showed this by writing quantum corrected superpotential for the product group.

However more detailed discussion of the supergravity part is not discussed. We expect that the gravity solution is the same as Klebanov and Tseytlin since it cannot distinguish the orientifolds and D branes. However we expect that there are minor modification due to torsion part. Also the K-theoretic discussion of the orientifold charge in the conifold is also worthwhile do be discussed in detail. We wish to come back to this issue in near future.

Finally we give a discussion on the deformation of the orientifold. As we have seen in section 6, the gauge theory analysis showed that the conifold is deformed by the quantum mechanical
effect. First we should notice that, once the conifold is deformed, the connection between $z_{ij}$ and $(a_i, b_i)$ is lost. So there is no way to derive the orientifold operation in the deformed conifold from that of the IIA branes. What we have done is to assume that the orientifold operation of the original (singular) conifold $z_i \to \bar{z}_i$ is still valid for the deformed case. Then $S^3$ is the fixed points, which gave us a big conceptual trouble, since the most natural interpretation is to regard $S^3$ as the cycle wrapped by O6. But this is not allowed by the SUSY of IIB. In this situation, we have several options.

1. The identification of orientifold operation $z_i \to \bar{z}_i$, for the deformed case, is not correct.

2. O3 charge is smeared out so that it has O3 charges but have the effect of the O6. In this case, the SUSY is intact.

3. O6 really is created by the deformation. In this case we loose SUSY.

We excluded the first option since $z_i \to \bar{z}_i$ is the only one that leads to the correct one in the limit of zero deformation parameter. We have chosen the second option since gauge theory or SUGRA so far does not show any signal of supersymmetry breaking.

However, before the paper of Strassler and Klebanov [7] appeared, Mukhi and DasGupta in [19] claimed that SUSY is broken when the singularity is resolved. So the situation not entirely clear even in the absence of the orientifold and we believe that it is an interesting subject to study. One may further speculate that the appearance of fixed $S^3$ means appearance of O6 meaning the supersymmetry is broken. If true it can be used as a dynamical supersymmetry breaking mechanism. However, this require more extensive analysis which goes beyond the scope of present work and we hope to settle this issue in later publication.

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References

[1] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231 [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov, and A.M. Polyakov, Phys. Lett. B428 (1998) 105 [hep-th/9802109].

[3] E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253 [hep-th/9802150].

[4] P. Candelas and X. C. de la Ossa, Nucl. Phys. B342 (1990) 246.

[5] A. Kehagias, Phys. Lett. B435 (1998) 337 [hep-th/9805131].

[6] I.R. Klebanov and E. Witten, Nucl. Phys. B536 (1998) 199 [hep-th/9807080].

[7] I.R. Klebanov and M.J. Strassler, JHEP 0008 (2000) 052 [hep-th/0007191].

[8] G. Pradisi and A. Sagnotti, Phys. Lett. B216 (1989) 59; M. Bianchi and A. Sagnotti, Phys. Lett. B247 (1990) 517; M. Bianchi and A. Sagnotti, Nucl. Phys. B361 (1991) 519.

[9] E. G. Gimon and J. Polchinski, Phys. Rev. D54 (1996) 1667 [hep-th/9601038].

[10] J. Park, R. Rabadan, and A. M. Uranga, Nucl. Phys. B570 (2000) 38 [hep-th/9907080].

[11] K. Oh and R. Tatar, JHEP 0005 (2000) 030 [hep-th/0003183].

[12] K. Dasgupta, S. Hyun, K. Oh, and R. Tatar, JHEP 0009 (2000) 043 [hep-th/0008091].

[13] S. Sinha and C. Vafa, “SO and Sp Chern-Simons at large N,” [hep-th/0012130].

[14] S. G. Naculich, H. J. Schnitzer, and N. Wyllard, “1/N corrections to anomalies and the AdS/CFT correspondence for orientifolded N = 2 orbifold models and N = 1 conifold models,” [hep-th/0106020].

[15] K. Dasgupta and S. Mukhi, Nucl. Phys. B551 (1999) 204 [hep-th/9811139].

[16] S. S. Gubser and I. R. Klebanov, Phys. Rev. D58 (1998) 125025 [hep-th/9808073].

[17] Angel M. Uranga, Nucl.Phys. B577 (2000) 73-87 [hep-th/9910153].

[18] A. Karch, D. Lust and D. Smith, Nucl. Phys. B533, 348 (1998) [hep-th/9803232].

[19] K. Dasgupta and S. Mukhi, JHEP 9907 (1999) 008 [hep-th/9904131].
[20] A.M. Uranga, JHEP 9901 (1999) 022 [hep-th/9811004].

[21] H. Ooguri and C. Vafa, Nucl. Phys. B463 (1996) 55 [hep-th/9511164].

[22] A. E. Lawrence, N. Nekrasov and C. Vafa, Nucl. Phys. B533 (1998) 199 [hep-th/9803015].

[23] D. R. Morrison and M. R. Plesser, Adv. Theor. Math. Phys. 3 (1999) 1 [hep-th/9810201].

[24] I.R. Klebanov and N.A. Nekrasov, Nucl. Phys. B574 (2000) 263 [hep-th/9911096].

[25] N. Seiberg, Nucl. Phys. B435 (1995) 129 [hep-th/9411149].

[26] I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. B256 (1985) 557.

[27] C. Vafa, “Superstrings and Topological Strings at Large N,” hep-th/0008142; M. Atiyah, J. Maldacena, C. Vafa, “An M-theory Flop as a Large N Duality,” hep-th/0011256.

[28] E. Witten, Nucl. Phys. B507 (1997) 658 [hep-th/9706109].

[29] A. Giveon and D. Kutasov, Rev. Mod. Phys. 71 (1999) 983 [hep-th/9802067].

[30] N. Evans, C. V. Johnson and A. D. Shapere, Nucl. Phys. B505 (1997) 251 [hep-th/9703210].