Improved two-vector model predictive torque control for PMSM

Chaojun Sun1*, Shuxi Liu1,2 and Lan chen1

1School of Electrical and Electronic Engineering, Chongqing University of Technology, Chongqing, 400054, China
2Chongqing Energy Internet Engineering Technology Research Center, Chongqing, 400054, China
*Corresponding author’s e-mail: suncj1997@2019.cqut.edu.cn

Abstract. Aiming at the problems of the traditional model predictive control (MPTC) of permanent magnet synchronous motor (PMSM) that the weight coefficient setting is cumbersome and the torque and flux pulsation are large, an improved PMSM two-vector model is proposed to predict the torque. This method expands the virtual vector to increase the voltage candidate vector, combines the voltage vector to act on the torque and flux linkage principle, selects the voltage vector, and reduces the calculation amount of the candidate vector. The evaluation mechanism of the selected vector is set by combined with the duty cycle method, so the weight factor of cost function in the traditional method is eliminated. The simulation results show that the improved method proposed can not only maintain high dynamic performance, but also improve the control effect of torque and flux linkage in the steady state of the system.

1. Introduction
Permanent magnet synchronous motors (PMSM) are widely used in many fields due to their simple structure, high efficiency, high power density and reliable operation[1]. In the actual PMSM system, rapid dynamic response is the key factor that determines the entire system[2]. Therefore, the model predictive torque control (MPTC) is introduced into the application of controlling high-performance PMSM relying on its good dynamic response capability[3]. The duty cycle MPTC is proposed in[4] for the traditional MPTC, the effective voltage vector is selected and applied to a part of the sampling period. However, this method needs to design appropriate weights when constructing the value function. For the design of weight coefficients, there is no sufficient theoretical basis.

This paper proposes an improved two-vector MPTC of PMSM for above problems. By expanding the virtual vector to increase the voltage candidate vector, combined with the voltage vector acting on the torque and flux linkage principle, the number of candidate vectors is reduced. The torque and flux duty cycles of candidate vectors are calculated respectively by targeting at zero torque error and flux error. The cost function in the traditional method is not necessary in the optimal vector selection. So the weight coefficient tuning is avoided in the proposed method. Finally, the MATLAB/Simulink simulation platform is used for comparison and verification. The simulation results show that the improved method proposed can not only maintain high dynamic performance, but also improve the control effect of torque and flux linkage in the steady state of the system.
2. Mathematical model of IPMSM

This paper takes the interior permanent magnet synchronous motor (IPMSM) as the specific research object. Establish the d-q axis mathematical model of IPMSM with rotor flux orientation, the voltage equations of IPMSM in d-q can be written as

\[
\begin{align*}
\frac{d}{dt} V_d & = R_d i_d + L_d \frac{d}{dt} i_d + \omega_L L_q i_q \\
\frac{d}{dt} V_q & = R_q i_q + L_q \frac{d}{dt} i_q + \omega_L (\psi_r + L_q i_q)
\end{align*}
\]

where \( V_d \) and \( V_q \) are the d-axis and the q-axis components of the stator voltage. \( i_d \) and \( i_q \) are the d-axis and q-axis components of the stator current. \( L_d \) and \( L_q \) are the d-axis and q-axis inductances. \( \psi_r \) is the permanent magnet flux linkage of the rotor. \( R_s \) is the stator resistance. \( \omega_L \) is the electrical angular velocity of the rotor.

The stator flux linkage equation of IPMSM is written as

\[
\begin{align*}
\frac{d}{dt} \psi_d & = L_d i_d + \psi_r \\
\frac{d}{dt} \psi_q & = L_q i_q
\end{align*}
\]

where \( \psi_d \) and \( \psi_q \) are the d-axis and the q-axis components of the stator flux linkage.

The electromagnetic torque equation of IPMSM is written as

\[
T_e = \frac{3p}{2} \left[ \psi_r i_q + (L_d - L_q) i_d i_q \right]
\]

where \( p \) is the number of pole pairs.

3. Traditional MPTC strategy

3.1. Basic principles of the system

The principle block diagram of the traditional MPTC is shown in figure 1. The speed loop generates the torque reference value \( T^*_e \) through the PI regulator, and the stator flux linkage is given as \( \psi_s^* \). The current \( i_s(k+1) \) at the next moment is predicted to compensate for the one-beat control delay. The effective vectors are substituted into the torque and stator flux prediction equations in turn, and the torque and flux linkage under the action of each vector are predicted to obtain \( T_e(k+2) \) and \( \psi_s(k+2) \). The predicted torque and flux linkage are substituted into the evaluation function for calculation, and the vector that minimizes the objective function is selected as the optimal voltage vector.

3.2. Control delay compensation

In an actual digital control system, the voltage vector \( V(k) \) selected in the \( k \)th control cycle will not act on the motor until the \( (k+1) \)th control cycle starts. This will cause the system control to delay one control
cycle, which will affect the motor control effect. Therefore, this delay needs to be effectively compensated.

Discretize the mathematical model of the motor shown in equation (1), we can get

\[
i_q(k+1) = i_q(k) + \frac{1}{L_d}[-R_L i_q(k) + \omega_L (k)L_{d} q(k) + V_d(k)]T_s
\]

\[
i_q(k+1) = i_q(k) + \frac{1}{L_q}[-R_L i_q(k) - \omega_L (k)L_{d} d(k) - \omega_L (k)\psi_f + V_q(k)]T_s
\]

where \(T_s\) represents the sampling period, \(k\) and \(k+1\) represent the \(k\)th and \((k+1)\)th instant respectively, and \(V_d\) and \(V_q\) are the \(d\)-axis and \(q\)-axis components of the candidate voltage vectors.

According to the above equation, substituting the current which is delayed compensation into equation (2) and equation (3) can get the corresponding torque \(T_e(k+1)\) and flux linkage value \(\psi_s(k+1)\), which can be used as the basis for model predictive control.

3.3. Optimal vector selection

The predicted value of the control variable and the corresponding given value are substituted into the evaluation function equation (6), and the voltage vector that minimizes the evaluation function is selected as the optimal voltage vector output by the inverter.

\[
g(n) = |T^*_e - T_e(k+2)| + Q|\psi^*_f - \psi_f(k+2)|
\]

where \(T^*_e\) and \(\psi^*_f\) are the reference value of torque and flux, \(Q\) is a weighting factor.

4. Improved two-vector model predictive torque control

4.1. Virtual vectors and rotating sectors

The spatial distribution position of the basic vector is shown in figure 2. By combining two adjacent vectors for half of the time in a control cycle, six virtual vectors can be obtained. The rotation sector division and candidate vector distribution in the rotating coordinate system are shown in figure 3.

![Figure 2. Schematic diagram of the basic vector space distribution.](image1)

![Figure 3. Schematic diagram of virtual vector and rotating sector.](image2)

Construct a rotating sector oriented by the stator flux linkage, taking the stator flux linkage as 0 angle, and dividing the coordinate system plane into four 90 degrees sectors in a counter clockwise direction. Each sector contains three candidate vectors. As shown in figure 4, each candidate vector \(V_n\) is equivalent to the stator flux linkage \(\psi_s\) decomposed into \(V_{n1}\) and \(V_{n2}\).

According to the principle of direct torque control, a positive \(V_{n1}\) increases \(\psi_s\), and a negative \(V_{n2}\)
decreases $\psi_s$. In the same way, positive $v_n$ increases $T_e$ and negative $v_n$ decreases $T_e$. When the motor is running, the candidate vector in sector 1 can increase the torque and stator flux amplitude, other sectors and so on.

By judging the positive or negative of the torque error $T_e_{\text{error}}$ and the flux error $\psi_{s_{\text{error}}}$, the appropriate sector is selected. For example, when the torque error $T_e_{\text{error}}$ and the flux error $\psi_{s_{\text{error}}}$ are greater than 0, it is necessary to output a candidate vector that can increase the torque and the stator flux linkage at the same time to achieve real-time tracking of the torque reference $T_e^*$ and the flux reference $\psi_s^*$. The voltage vector in sector 1 can be selected to achieve the purpose of increasing torque and flux linkage at the same time. The selection range of candidate vectors is reduced from the original 12 to 3, which greatly reduces the amount of calculation. Other situations are similar. The sector selection rules are shown in table 1. After determining the sector position, the candidate vector can be selected by the flux linkage position angle.

| $T_e_{\text{error}}$ | $\psi_{s_{\text{error}}}$ | Sector |
|----------------------|-------------------------|--------|
| $T_e_{\text{error}} > 0$ | $\psi_{s_{\text{error}}} > 0$ | 1 |
| $T_e_{\text{error}} < 0$ | $\psi_{s_{\text{error}}} < 0$ | 3 |

4.2. Traditional duty cycle calculation

The traditional duty cycle calculation is proposed in[4] which does not work well in two situations:

Case 1: The difference between the vector duty ratios corresponding to the torque and the magnetic flux is too large to balance the control performance of the torque and the flux linkage by adjusting the duty ratio.

Case 2: When the duty cycle is 1, the optimal vector can obtain the best control performance. The torque and magnetic flux are close to the reference value, but not reach the reference value.

In this paper, the duty cycle calculation and optimal vector selection are considered to solve the above problems. In a control cycle, the torque and flux linkage changes are expressed as $S_T$ and $S_{\psi}$. The torque and flux linkage can be expressed as

$$
\begin{align*}
T_e(k+1) &= T_e^* = T_e(k) + S_T D_T \\
\psi_s(k+1) &= \psi_s^* = \psi_s(k) + S_{\psi} D_{\psi}
\end{align*}
$$

The duty cycle of torque and flux can be expressed as

$$
\begin{align*}
D_T &= (T_e^* - T_e(k)) / S_T = T_e_{\text{error}} / S_T \\
D_{\psi} &= (\psi_s^* - \psi_s(k)) / S_{\psi} = \psi_{s_{\text{error}}} / S_{\psi}
\end{align*}
$$

$S_T$ and $S_{\psi}$ are obtained as

$$
\begin{align*}
S_T &= T_e(k+2) - T_e(k+1) \\
S_{\psi} &= \psi_s(k+2) - \psi_s(k+1)
\end{align*}
$$

where $T_e(k+1)$ and $\psi_s(k+1)$ are torque and flux at the $(k+1)$th after adding delay compensation, and the torque $T_e(k+2)$ and flux $\psi_s(k+2)$ at $(k+2)$th can be predicted by equation (1)-(3).
4.3. **Optimal vector selection**

As shown in figure 3, the three candidate vectors are determined by the sector. Torque duty ratio $D_{T1}, D_{T2}, D_{T3}$ and flux linkage duty ratio $D_{\psi1}, D_{\psi2}, D_{\psi3}$ are calculated by equation (8), and their values are limited between 0 and 1. When the main control target is torque, the optimal vector selection rule determined by the duty cycle is as follows:

**Rule 1:** When $D_{T1}=D_{T2}=D_{T3}=1$, none of the three vectors can make the torque reach the reference value in one control cycle. At this time, let the voltage vector with the largest torque change as the optimal vector, and the duty cycle is 1.

**Rule 2:** When one of $D_{T1}, D_{T2}, D_{T3}$ is not equal to 1, the corresponding vector can make the torque reach the reference value. Then select this vector as the optimal vector, and its torque duty cycle is the optimal vector duty cycle.

**Rule 3:** When three of $D_{T1}, D_{T2}, D_{T3}$ are not equal to 1, that all three vectors can make the torque reach the reference value. It is necessary to further compare the difference between the torque duty cycle and the flux linkage duty cycle of the three vectors, and select the vector with the smallest difference as the optimal voltage vector, and its corresponding torque duty cycle as the optimal duty cycle.

**Rule 4:** When two of $D_{T1}, D_{T2}, D_{T3}$ are not equal to 1, that is, these two vectors can make the torque reach the reference. In this time, the selection of the optimal vector is the same as in Rule 3.

The specific rules are shown in table 2. $V_{\text{max}, k}$ is the voltage vector with the maximum torque change value. $V_{\text{min}(x,y)}$ is the smallest difference between torque and flux duty cycle under the action of two candidate vectors $x$ and $y$. $V_{\text{min}(1,2,3)}$ is the smallest difference between the torque duty cycle and the flux linkage duty cycle under the action of the three candidate vectors.

![Figure 4. Voltage vector decomposition diagram.](image)

![Figure 5. Flow chart of proposed algorithm.](image)

The optimal vector selection process introduced above is the core of the improved dual vector model predictive torque control method proposed in this paper. The flow chart is shown in figure 5.

5. **Simulation results and analysis**

In order to verify the correctness and effectiveness of the proposed method, simulation models of different methods were built in the MATLAB/Simulink environment. The duty cycle model predictive torque control proposed in [4] and the improved model prediction proposed in this paper (which are named as MPTC-I and MPTC-II). The motor parameters used in the simulation are shown in table 3.

| Parameter      | Numerical value | Unit | Parameter      | Numerical value | Unit |
|----------------|-----------------|------|----------------|-----------------|------|
| Rate speed $n$ | 3000            | r/min| $d$ axis inductance $\ell_d$ | 0.2            | mH   |
| Rate voltage $U_n$ | 320            | V    | $q$ axis inductance $\ell_q$ | 0.555          | mH   |
| Rate current $I_n$ | 94 | A  | Flux linkage $\psi_i$ | 0.07574 | Wb |
| Rate torque $T_n$ | 64 | N·m | Stator resistance $r_s$ | 11.4 | mΩ |

Figure 6 shows the steady-state performance simulation waveform. The simulation waveforms of the two methods are torque waveform, stator flux linkage waveform and single-phase current waveform from top to bottom. The control frequency is 10 kHz, the speed is 3000 r/min, and the load is 30 N·m.

Figure 7 shows the dynamic simulation waveform. The simulation waveforms of the two methods are torque, stator flux and current waveforms from top to bottom. The motor accelerates from 0 r/min to the rated speed of 3000 r/min, and loads the rated load of 64 N·m to the motor at 1.0 s.

![Figure 6](image1)

(a) Simulation waveform of MPTC-I, (b) Simulation waveform of MPTC-II.

![Figure 7](image2)

(a) Simulation waveform of MPTC-I, (b) Simulation waveform of MPTC-II.

It can be seen from the above waveform that the motor can start stably under the two methods, and the torque and flux can quickly track the reference value. After loading the rated load of 64 N·m, the motor can quickly reach a stable state, which shows that the system has good anti-disturbance ability. But in MPTC-I, the disturbance of torque and flux linkage is greater than MPTC-II. The reason is that the duty cycle in MPTC-I is 1 most of the time. As shown in figure 8, when the duty cycle is 1, the optimal vector may not allow the torque or flux linkage to reach the reference value at the end of a control cycle, and the duty cycle will reach its limit at this time. The vector used in MPTC-II may not be the optimal vector, but due to the existence of duty cycle adjustment, torque and flux linkage can track the reference value well. So it can be seen that the duty cycle in MPTC-II is adjusted between 0 and 1.
It can be seen from figure 6, figure 7 and figure 8 that regardless of the steady state or the dynamic state, the torque and flux linkage waveforms are improved, and the current waveform is more stable. At the same time, the proposed MPTC-II has improved torque and flux control performance while maintaining the same dynamic performance.

![Waveform Comparison](image)

(a) (b)

Figure 8. The duty cycle comparison simulation waveform.
(a) Simulation waveform of MPTC-I, (b) Simulation waveform of MPTC-II.

6. Conclusion
In this paper, an improved two-vector MPTC method is proposed. This method introduces a virtual vector, establishes a rotating coordinate system, uses the torque duty ratio and flux duty ratio of the candidate vector to select the optimal vector, instead of the cost function in the traditional method, and avoids the problem of weight coefficient tuning. Compared with method MPTC-I, this method makes full use of the duty cycle adjustment function and has good torque and flux control performance. The simulation results show that the MPTC-II proposed in this paper is effective.

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