Analysis of the dynamic coordinate system using photoelectric lunar occultations

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Abstract: In this work we propose the determination of the astrophysical dynamic coordinate system’s orientation in relation to HCRF (Hipparcos Celestial Reference Frame) using photoelectric lunar occultation. The photoelectric method of recording an occultation allows obtaining occultations moment with the accuracy up to 0.001s (i.e. 100 times more accurate than by visual method) was found to be the most accurate. Apart from the tasks of space geodesy, photoelectric observations allow carrying out other interesting researches, too. These tasks include determination of amendments to orbital longitude and latitude of the Moon and amendments to ephemeris time. The photoelectric observations of occultation are valuable materials for solving some astrophysical tasks. If one records changes in magnitude at the moment when a star is occultated by the Moon, then one may obtain diameter of the occultated star through the spectrum of those changes. Other important problems are detecting double and multiple systems of stars and measuring angular distances between their components.

1. Introduction
57365 photoelectric observations of occultations (POO) taken during the period from 1960 to 2005 are analyzed in the present paper. The database on POO taken from 1943 to 1980 was built at Greenwich observatory. It contains 4186 POO [1], [2]. We are constructing the full database of POO using the Internet, printed editions, and the data provided by the scientists of National Astronomical Observatory of Japan (NAOJ). As a result of this work, the number of POO is 57365.

2. Analysis of PPO database
Although one can use any optical tool from binoculars to telescope to observe POO, our database contains only those POO which have been taken using professional telescopes [3]. One of the most important things is telescope's magnification which affects an observed phenomenon in direct ratio. When taking POO, great magnifications up to 2d (d – diameter of a telescope's lens) are important. When observing reappearing POO, one should use smaller magnifications to see a significant part of a lunar edge out of which a star is supposed to appear. Otherwise, the latter may appear out of view and a telescope will not see the star at all. Adverse effect of the moonlight can be weakened by imposing a diaphragm on the lens or installing a damper screening the moonlight. In the latter case POO of weaker stars can be observed as well. When observing tangential POO, a star slowly approaches a lunar edge along the tangent line and at some moment suddenly and almost instantly disappears behind the edge of the lunar disc, but in a few seconds appears a bit further down the trajectory depending on the relief of the lunar disc. After that, a few moments later, the star suddenly disappears again behind the lunar surface roughness and then appears. The entire phenomenon of tangential POO lasts no longer than 2-3 minutes. Tangential POO can only be observed in rather narrow line of the Earth's surface, width of 4-5 km, going along the border of POO, i.e. the line behind which the POO does not take place. This is why, to observe tangential POO observers should be in phenomenon's visibility area and take POO using any optical tools. When taking the tangential POO, observers should be on the line perpendicular to the POO's border; half of the observers should be on one side of the border and half – on the other. The largest distance from the border should not exceed 1,5 – 2,0 km. If that is possible, the observers should be located at equal distances depending on their number. The more observers are involved, the more accurately POO is taken. However, even the POO taken by a single observer are of significant scientific interest. Although the desirable accuracy of recoding
POO moments is 0.01°, when dealing with tangential POO this requirement is not so strict and the accuracy of 2-3 seconds is sufficient. Moreover, even the POO without recording time moments, but with noticing only the numbers of appearance and disappearance of a star and their character (slow or instant appearance and disappearance) are valuable. Tangential POO should only be used when studying the polar areas of the Moon, since for other purposes their accuracy is too low.

It should be noted that accurate models of the lunar surface [4] and the use of robust parameters estimation methods [5] are of great importance in reducing occultations.

There is a temporal distribution of POO shown in the figure 1. As we can see, the number of photoelectric occultation observations started to increase in 1967. If Morrison [6] and Soma [7] could reduce only 4186 POO, we work with 57365 observations.

![Figure 1. Temporal distribution of POO](image1)

![Figure 2. Distribution of POO for lunations](image2)
Figure 2 shows the distribution of POO for lunations. As all the timing used in this analysis correspond to disappearances of the lunar dark limb (DD) during the first half of the lunation and reappearances of the lunar dark limb (RD) during the second half, it is expected that the distribution of POO in each lunation is highly correlated with the phase of the Moon and this is evident in figure 2. The dependence of observations accuracy on star brightness was established as well. Very bright stars and the ones with magnitude less than 7.5 were observed with a greater error than the stars of average brightness.

3. Results of determining the orientation of astrophysical dynamical system coordinate relative HCRF

Let us denote axes of the dynamic coordinate system [8] by $X, Y, Z$, axes of the coordinate system of the catalogue studied by $X_{cat}, Y_{cat}, Z_{cat}$, correction to zero point of the catalogue in right ascension by $\Delta A$, correction of the longitude of the Sun by $\Delta L$, correction of the slope of the ecliptic to the equator by $\Delta \epsilon$, zero direction - directory entry by $\hat{A}$. Let us also assign the orientation of the axes of the coordinate system of the $X_{cat}, Y_{cat}, Z_{cat}$ catalog relative to the dynamic coordinate system by the angles of rotation $\xi_x, \xi_y, \xi_z$ around the $X, Y, Z$ axes of the dynamic system. Then for the amendments $\Delta \alpha_{O-C} = \alpha_{cat} - \alpha_{dyn}, \Delta \delta_{O-C} = \delta_{cat} - \delta_{dyn}$ we will have the equations for small turning angles:

$$
\Delta \alpha_{O-C} \cos \delta = \sin \delta \cos \alpha \xi_x + \sin \delta \sin \alpha \xi_y - \cos \delta \xi_z,
\Delta \delta_{O-C} = \sin \alpha \xi_x + \cos \alpha \xi_y.
$$

The rotation angles $\xi_x, \xi_y, \xi_z$ are related to amendments to the Euler rotation angles by the relations

$$
\xi_x = -\Delta \varepsilon, \xi_y = \Delta L \sin \varepsilon, \xi_z = \Delta A - \Delta L \cos \varepsilon.
$$

With the classical scheme for determining the orientation of the catalogue associated with Euler angles, and given that the $\Delta D$ - additive zero correction is the declination point of the catalogue, for $\Delta \alpha_{O-C}$ and $\Delta \delta_{O-C}$ can be written by substituting (2) in (1):

$$
\Delta \alpha_{O-C} = -\Delta A + \Delta L \cos \varepsilon(1 + \tan \varepsilon \tan \delta \sin \alpha) - \Delta \varepsilon \tan \delta \cos \alpha,
\Delta \delta_{O-C} = -\Delta D + \Delta L \sin \varepsilon \cos \alpha + \Delta \varepsilon \sin \alpha.
$$

Equation (3) is a correction equation for determining the $\Delta A, \Delta L, \Delta D, \Delta \varepsilon$ - parameters orientation of the catalogue.

The angle change rate $\xi_{x,t}, \xi_{y,t}, \xi_{z,t}$ are determined by the formulas:

$$
\dot{w}_x = -\Delta \dot{\varepsilon}, \dot{w}_y = \Delta \dot{L} \sin \varepsilon, \dot{w}_z = \Delta \dot{A} - \Delta \dot{L} \cos \varepsilon,
$$

where dots denote derivatives of the corresponding quantities over time.

The expressions for the angles of rotation $\xi_{x,t}, \xi_{y,t}, \xi_{z,t}$ depending on the time will be:

$$
\xi_{x,t} = \xi_x + w_x(t - t_0), \xi_{y,t} = \xi_y + w_y(t - t_0), \xi_{z,t} = \xi_z + w_z(t - t_0).
$$

Substituting (2), (4) and (5) in equation (3), we obtain the following expressions:
\[
\Delta \alpha_{O-C} = -(\Delta A + \Delta \dot{A}(t-t_0)) + \\
+ (\Delta L + \Delta \dot{L}(t-t_0)) \cos \varepsilon (1 + \tan \varepsilon \tan \delta \sin \alpha) - (\Delta \varepsilon + \Delta \dot{\varepsilon}(t-t_0)) \tan \delta \cos \alpha, \\
\Delta \delta_{O-C} = -\Delta D + (\Delta L + \Delta \dot{L}(t-t_0)) \sin \varepsilon \cos \alpha \\
+ (\Delta \varepsilon + \Delta \dot{\varepsilon}(t-t_0)) \sin \alpha.
\]

Taking into account that, in fact, \((\alpha_{cat} - \alpha_{dyn})\) and \((\delta_{cat} - \delta_{dyn})\) are \((\Delta \alpha_{O-C} \cos \delta)_{m}\) and \((\Delta \delta_{O-C})_{m}\), and substituting the latter into equations (6), we obtain a system of 2n correction equations. Having estimated the unknowns by the least squares method we obtain the desired orientation parameters of the dynamic coordinate system relative to HCRF.

\[
\varepsilon_x = 30''10^{-4} \pm 24''10^{-4}, \varepsilon_y = -11''10^{-3} \pm 12''10^{-4}, \varepsilon_z = 16''10^{-3} \pm 27''10^{-4} \\
w_x = 710^{-4} \pm 10^{-4}/day, w_y = -1410^{-4} \pm 1910^{-4}/day, \\
w_z = -1210^{-4} \pm 2610^{-4}/day \\
\Delta A = 7410^{-3} \pm 4810^{-3}, \Delta D = 4510^{-3} \pm 1910^{-3}, \\
\Delta L = 4110^{-3} \pm 3810^{-3}, \Delta \varepsilon = 5010^{-4} \pm 4110^{-4}.
\]

As a result, the orientation parameters of the dynamic coordinate system relative to the HCRF system are determined. Comparison of the obtained orientation parameters with the values obtained in [9] and [10] showed good agreement of the desired parameters within the limits of computation errors.

4. Summary and conclusions
According to the results of the work we may draw the following conclusions. The constructed database can be used when carrying out investigations during the construction of the selenocentric coordinate system [11]. By the method of lunar occultation and using photoelectric recording one may observe double stars 100 times closer than by any other methods. Recording changes of a star's magnitude forms the main task of the photoelectric observation method. If a star has a large diameter, then the curve of magnitude change differs from the curve of a point light source. The value of that difference depends on the diameter of the star in a certain way, and the intensity drop lasts 20-40 milliseconds. Thus, the problem is to record a very quick process of a star's brightness fluctuations. However, when taking photoelectric observations of occultation it is necessary to record changes of small light flow from a star in the presence of the bright Moon. Distinguishing light from the star among the background formed by the Moon is the main task which should be solved when reducing the occultation. Therefore that enables us to use the results of this work for the analysis of topocentric and gravimetric data from modern space missions [12], solving problems of the motion of small bodies [13] and their genetic relationships [14], as well as modeling the theory of lunar rotation [15, 16]. The prospects for further research will be to study the position of the dynamic coordinate system and the accuracy of the proper motions of the catalogue stars: GAIA [17], UCAC [18], URAT [19], USNO [20].

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