Sigmoidal inflation

E. Díaz 1 and O. Meza-Aldama 2,∗

1 Sibatel Communications, 303 W Lincoln Ave No.140, Anaheim, CA 92805, United States of America
2 Lux Systems, Blvd. Independencia #2002, Colonia Estrella, C.P. 27010, Torreón, Coah., Mexico

∗ Author to whom any correspondence should be addressed.
E-mail: alcmaeon@gmail.com and oscar_meza5@hotmail.com

Keywords: Reheating, Cosmological Inflation, Inflationary Observables

Abstract

In this paper we present a new cosmological inflationary model which is constructed using the Ivanov-Salopek-Bond method with a logistic generating function. We derive the inflationary observables as well as the duration and temperature of the subsequent reheating epoch of our model exactly, with no need to recur to the slow roll approximation. The obtained scalar spectral index and tensor-to-scalar ratio of perturbations fall comfortably within the range of the measurements presented by the Planck collaboration. On the other hand, for the reheating era, our model predicts a relatively small number of e-folds and thus high temperatures, still within range of Planck’s bounds. We then consider a generalization of our model that we refer to as Bernoulli-like functions and examine different scenarios that are encompassed within this generalization.

1. Introduction

The data released by the Planck collaboration [1] has high precision cosmological observations which have discarded and restricted inflationary models; of particular relevance is the spectral index of scalar perturbations $n_s = 0.9649 \pm 0.0042$ at 68% C.L. and the upper limit on the tensor-to-scalar ratio $r < 0.056$. These new measurements allow us to validate new inflationary models, particularly for those which follow slow roll dynamics. However, we find that by using the so called Ivanov-Salopek-Bond (ISB) formalism [2, 3], it is not necessary to use the slow-roll approximation in the construction of the model. Additionally, the work involving the slow-roll parameters is greatly simplified since they are defined purely in terms of the Hubble parameter and its time derivatives. The formalism has been previously employed with generating functions that were of a polynomial form, trigonometric, exponential [4], inverse potential and hyperbolic [5]. While we make use of a logistic generating function for the effective potential during inflation and reheating, this form of the generating function allowed us to construct a potential that complies with current Planck measurements of the tensor-to-scalar ratio and spectral index. Furthermore, the reheating period of the Universe is also considered, since it is important for the subsequent evolution of the Universe that the inflaton, through its decays, gives rise to the Standard Model matter content.

The inflationary period of the Universe is driven by a real scalar field $\phi$ with a canonical kinetic term. Substituting the energy-stress tensor for such a field into Einstein field equations yield the so-called Friedmann equations:

\[
H^2 = \frac{\kappa}{3} \left( \frac{1}{2} \dot{\phi}^2 + V \right),
\]

\[
\ddot{H} = -\frac{\kappa}{2} \dot{\phi}^2,
\]

\[
\ddot{\phi} + 3H\dot{\phi} + V' = 0;
\]

(1)
where $H$ is the Hubble parameter, $V$ is the effective potential for $\phi$, and $\kappa \equiv 1/M_P^2$ where $M_P$ is the reduced Planck mass given by $M_P^2 \equiv 1/8\pi G$, and $G$ is Newton’s gravitational constant. A dot denotes the derivative with respect to (physical) time and a prime denotes derivatives with respect to $\phi$.

Inflation has several observational tests to pass. Quite generally, one requires a number of $e$-folds $\sim 60$ in order to solve the flatness problem [6, 7], this in turn implies that $H$ cannot vary a lot within a lapse of a Hubble time $H^{-1}$. To make this statement quantitative, one defines the first slow roll parameter $\epsilon$ as:

$$
\epsilon \equiv -\frac{\dot{H}}{H^2},
$$

and requires that it remains $\ll 1$. In fact, the condition $\epsilon < 1$ is equivalent to an accelerated expansion [8]. Additionally, we define the second slow roll parameter $\eta$ as

$$
\eta \equiv \frac{\ddot{\epsilon}}{\epsilon H}.
$$

The condition $\eta \ll 1$ is usually sufficient (but not necessary) to ensure that inflation lasts long enough to achieve 60 $e$-folds or so. These two parameters govern most of inflation dynamics, and one can get the main inflationary observables ($r$ and $n_s$) by evaluating them at horizon exit [9, 10]. Calculation of the slow roll parameters would in principle require that one solved equations (1) first. In turn, what one can do is define the potential slow roll indices

$$
\epsilon_V \equiv \frac{1}{2\kappa} \left( \frac{V'}{V} \right)^2, \quad \eta_V \equiv \frac{1}{\kappa} \frac{V''}{V}.
$$

Using Friedmann’s equations, one can show the relations [11]

$$
\epsilon = \epsilon_V, \quad \eta = -2\eta_V + 4\epsilon_V.
$$

Thus the smallness of $\epsilon$ and $\eta$ is equivalent to the smallness of $\epsilon_V$ and $\eta_V$, but given that the latter are given directly in terms of $V$, one does not need to solve equations (1) to calculate them. In fact, the assumption of the slow roll conditions simplifies them, turning them into a set of first order differential equations.

On the other hand, the Hamiltonian formalism [7, 8], referred throughout this paper as the ISB formalism [4, 12] for the reasons discussed in [2], allows one to calculate the exact slow roll parameters $\epsilon$ and $\eta$ in an elegant way [8]. The starting point is defining a generating function $F = F(\phi)$. Then, if one identifies the potential and the Hubble parameter (both in terms of $\phi$) as

$$
V = -\frac{2}{3\kappa} F^2 + F^2, \quad H = \sqrt{\kappa} F,
$$

then the Friedmann equations imply

$$
H' = -\frac{\kappa}{2} \dot{\phi}.
$$

This is a much simpler, non-coupled, differential equation for $\phi(t)$. Hence, using this method one can find exact (i.e., without using the slow roll approximations) solutions to the Friedmann equations. Additionally, one can work with the exact parameters $\epsilon$ and $\eta$, instead of $\epsilon_V$ and $\eta_V$.

The procedure for building an inflationary model is pretty straightforward from here: one calculates the $e$-folds $N$ as a function of $\phi$ as

$$
N = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi,
$$

where $\phi_f$ is the value of $\phi$ at the end of inflation, i.e. when $\epsilon = 1$. Once the integral is done, one can in principle invert the expression to obtain $\phi = \phi(N)$. One then obtains the expressions for $\epsilon$ and $\eta$ in terms of $\phi$, and thus in terms of $N$. Finally, one calculates $r$ and $n_s$ in terms of $\epsilon(N)$ and $\eta(N)$ evaluated at horizon exit, $N \approx 60$. Experimental bounds on $r$ and $n_s$ can then be used to bound the parameters of the model.

In this work we study a sigmoidal generating function which generates a sigmoidal, or ’almost sigmoidal’$^3$, effective potential. Recent work has shown that sigmoidal profiles can arise naturally from certain supergravity scenarios; in particular, in [13] a hyperbolic tangent arises naturally in the parametrization of the fields of the Kähler potential of a non-minimal $\mathcal{N} = 1$ supergravity model. The shape of the potential is sigmoidal which makes it have some peculiar characteristics, such as being able to preheat very efficiently and without oscillations around its minimum, and a kination phase in which the kinetic energy is dominant instead of the potential.

$^3$ We refer to the possibility of our potential to have the same asymptotic behavior as a sigmoidal but developing a vacuum ’in the middle of it’; see below.
With the ISB procedure, we can prove quite generally that kination cannot occur right after the end of inflation (if it ever does). Using equation (8) we get that the kinetic energy of the inflaton is:

\[ K \equiv \frac{1}{2} \dot{\phi}^2 = \frac{2}{3\kappa} \frac{F^2}{F'} . \tag{10} \]

Therefore the ratio \( V/K \) is

\[ \frac{V}{K} = -1 + \frac{3\kappa}{2} \left( \frac{F}{F'} \right)^2 . \tag{11} \]

On the other hand, using the chain rule, \( \dot{H} = H' \dot{\phi} \), the first slow roll parameter is

\[ \epsilon = \frac{2}{\kappa} \frac{F^2}{F'} . \tag{12} \]

Substituting into the previous equation we have:

\[ \frac{V}{K} = -1 + \frac{3}{\epsilon} . \tag{13} \]

Therefore, around the end of inflation we have \( V \approx 2K \), the potential energy still dominates.

In this paper we will show how a sigmoidal potential can arise from the ISB formalism considering a logistic as a generating function. The shape of our potential will resemble that of [13], and in fact, we will obtain extremely similar expressions for the slow-roll parameters \( \epsilon \) and \( \eta \). However, our potential favors values of the parameter space which tend to form an absolute vacuum, therefore giving rise to more typical reheating scenarios in which the inflaton oscillates around its minimum, efficiently producing decay products via resonance [14, 15].

This work is organized as follows: in section 2 we present the results of using a logistic generating function to calculate the effective potential during inflation, and describe its qualitative features. In section 3 we calculate the spectral index, the tensor-to-scalar ratio, and scalar power spectrum produced during inflation. In section 4 we obtain the duration of the reheating period in terms of the number of e-folds, as well as the final temperature of this era. In section 5 we generalize our generating function to a more sophisticated one, adding one free parameter to our model. Finally, we present our conclusions in section 6.

2. A logistic generating function

While developing this work, the authors of [16] built an inflationary model in which the effective potential of the inflaton field has a Woods-Saxon form (i.e. a logistic function); here instead, using the ISB procedure [2], we start with a logistic \textit{generating function} and from it we derive the potential and the Hubble parameter. We therefore define the function

\[ F(\phi) = \frac{A}{1 + \lambda e^{-b\phi}} , \tag{14} \]

where we assume \( A \) and \( \lambda \) are positive constants, while \( b \) is a nonzero constant. Notice that we can rewrite this as

\[ F(\phi) = \frac{A e^{b\phi}}{\lambda + e^{b\phi}} . \tag{15} \]

It is also useful to notice that the generating function \( F \) and its derivative \( F' \) satisfy

\[ F' = \frac{b}{A} F(A - F) . \tag{16} \]

The potential \( V \), the Hubble parameter \( H \) and the inflaton field \( \phi \) are related through [2]:

\[ H(\phi) = \sqrt{\frac{\kappa}{3}} (F(\phi) + c) , \tag{17} \]

\[ V(\phi) = -\frac{2}{3\kappa} (F'(\phi))^2 + (F(\phi) + c)^2 , \tag{18} \]

\[ H'(\phi) = -\frac{\kappa}{2} \dot{\phi} , \tag{19} \]

where \( \kappa \equiv 1/M_\text{Pl}^2 \equiv 8\pi G \) and the constant \( c \) is a parameter of the model which can in principle take any value. To simplify our analysis we will take \( c = 0 \) so we simply have

\[ 4 \text{ Taking } c = 0 \text{ opens the possibility for the potential to develop a false vacuum depending on the values of the other parameters of the model.} \]
Notice that this potential is quite different from a simple logistic function, and in fact opens the possibility for different shapes depending on the value of $b$. Thus, setting the constant $c = 0$, the potential offers two distinct qualitative scenarios: it is easy to prove that for $b^2 < 3\kappa/2$ the potential has a well-known sigmoidal shape, as shown in figure 1; on the other hand, if $b^2 > 3\kappa/2$, the potential develops a small curved well with a global minimum around its center. This scenario can be visualized in figure 2.

Also, notice that if we write the constant $\lambda$ as $\lambda \equiv e^{b\phi_0}$, we can see that its effect on the potential is to make the shift $\phi \rightarrow \phi - \phi_0$, thus amounting solely to a horizontal translation of the plot of $V$, but no change in the actual dynamics of the inflaton. Given this, we expect $\lambda$ not to play a determinant role in our results for inflationary and reheating observables; we will see that indeed this is the case throughout the course of this paper. Notice that in the expression for our potential, the change $\phi \rightarrow -\phi$ is completely equivalent to changing $b \rightarrow -b$, and amounts to reflecting the plot of $V(\phi)$ around the vertical axis. Due to this symmetry we will focus only on positive values for $b$.

We can write the Hubble parameter and the time derivative $\dot{\phi}$ as functions of $\phi$ [2]:

$$H(\phi) = \frac{\kappa}{3} \frac{A}{1 + \lambda e^{-b\phi}}^{1/2},$$

$$\dot{\phi}(\phi) = -\frac{4}{3\kappa} Ab\lambda \frac{e^{-b\phi}}{(1 + \lambda e^{-b\phi})^2}.\quad (21)$$

$$V(\phi) = -\frac{2A^2 b^2 \lambda^2}{3\kappa} \left[ \frac{e^{-b\phi}}{(1 + \lambda e^{-b\phi})^2} \right]^2 + \frac{\lambda^2}{(1 + \lambda e^{-b\phi})^2}.\quad (20)$$

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Figure 1. The sigmoidal profile of the potential presented in equation (20) at $b^2 \lesssim 3\kappa/2$.

Figure 2. The potential develops an absolute minimum when $b^2 > 3\kappa/2$. 

J. Phys. Commun. 4 (2020) 125011 E Díaz and O Meza-Aldama
These two expressions allow us to easily calculate the number of e-folds $N$ in terms of $\phi$ as

$$N \equiv \int_{\phi_i}^{\phi_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi,$$

where $\phi_i$ denotes the value of the inflaton at the end of inflation. Substituting, we get

$$N(\phi) = \frac{\kappa}{2b} \left( \frac{1}{b\lambda} e^{b\phi} + \phi \right) \bigg|_{\phi_i}.$$

(24)

Obviously the exact same answer is obtained if we were to calculate it as $N = \sqrt{\kappa} \int_{\phi_i}^{\phi_f} \frac{d\phi}{\sqrt{2\lambda}}$. If we define

$$\Phi \equiv \frac{2bN}{\kappa} + \frac{e^{b\phi_i}}{b\lambda} + \phi_0,$$

then we can solve for $\phi$ simply as

$$\phi = \Phi - \frac{1}{b} W\left( \frac{e^{b\Phi}}{\lambda} \right),$$

(26)

where $W$ is the Lambert function.

The value of $\phi$ at the end of inflation will be calculated from $H = 1$, and gives:

$$\phi_e = \frac{1}{b} \ln \left( \lambda \left( \frac{2}{b\lambda} b - 1 \right) \right).$$

(27)

In calculating this, we have assumed $b > 0$. Notice that in order for inflation to end we must have $b > \sqrt{\kappa/2}$, it is useful to define the dimensionless parameter $q$ through $b = \sqrt{\kappa/2} (q + 1)$. Thus, to have a well-defined value for $\phi$, we must have $q > 0$. Finally, using the property $W(x)e^{W(x)} = x$, it is straightforward to obtain the useful expression

$$e^{b\phi} = \lambda W(q e^{q(q+1)\sqrt{\kappa/2}}).$$

(28)

### 3. Inflationary observables

Slow roll indices are usually calculated using quotients of $V$ and its derivatives, giving a very good approximation as long as the field is slowly rolling, although that may be violated near the end of inflation. In our approach, it is actually possible (and easier) to work with the exact slow roll parameters, given by

$$\epsilon = -\frac{H}{H^2},$$

$$\eta = \frac{\dot{\epsilon}}{\epsilon H}.$$

(29)

(30)

A straightforward calculation gives us

$$\epsilon = \frac{2b^2 \lambda^2}{\kappa} \frac{1}{(\lambda + e^{b\phi})^2},$$

$$\eta = \frac{4b^2 \lambda}{\kappa} \frac{e^{b\phi}}{(\lambda + e^{b\phi})^2}.$$

(31)

(32)

Plugging equation (28) into these expressions gives us the slow roll parameters in terms of the number of e-folds. To first order in $\epsilon$ and $\eta$ in the slow-roll approximation, the spectral index, tensor-to-scalar ratio and spectrum of scalar perturbations are given respectively as $8^\text{n_s} = 1 - 2\epsilon - \eta$, $r = 16\epsilon$, and $A_s = \kappa H^2_\phi / 8\pi^2 H$. In our model these turn out to be

$$n_s = 1 - \frac{4b^2 \lambda}{\kappa(\lambda + e^{b\phi})},$$

$$r = \frac{32b^2 \lambda^2}{\kappa(\lambda + e^{b\phi})^2},$$

$$A_s = \frac{A^2_k \lambda^3}{48\pi^2 b^2} e^{2b\phi},$$

(33)

(34)

(35)

where the quantities on the right-hand side are to be evaluated at horizon exit. Using equation (28) we can rewrite these as...
Notice that for the spectral index and the tensor-to-scalar ratio, the only parameter that plays a role is $b$ (or equivalently, $q$), while $\lambda$ completely disappears from the expressions; on the other hand, from equation (38) we can get an estimated value on our parameter $A$ for given values of $N_*$, $q$ and using the central value of Planck collaboration’s measurement of $A_\text{P}$. Plots of $n_s$ and $r$ can be seen in figures 3 and 4 respectively. In both of these plots, the blue and purple curves correspond to $N_* = 55$ and $N_* = 60$ e-folds, respectively. For $r$ we can see that even extremely small values of $q$ ($\sim 10^{-16}$ and $\sim 10^{-17}$ for the blue and purple curves, respectively) are in agreement with the upper bound on $r$. A slightly more stringent requirement appears for the spectral index: a value of $q \lesssim 0.2$ is sufficient for the blue curve, while even smaller values are allowed for the purple one. Therefore, a sufficiently ‘large’ value of $q$ is in accordance with the observations of both $n_s$ and $r$.

### 4. Reheating

Although reheating is still a mysterious epoch in the evolution of the Universe, it is an almost essential period in the history of the Universe in order for it to contain the kind of matter that we have today. As remarked in [17], however, the reheating epoch of certain inflationary models can be characterized by the number of e-folds $N_R$.
between the end of inflation and the start of the radiation-dominated era, the temperature \( T_R \) at which thermalization between the inflaton and its decay products occur, and the equation of state \( \omega_R = p/\rho \) during reheating. This \( \omega_R \) must be bounded between \(-\frac{1}{3}\) and 1 in order for inflation to stop and to preserve causality, respectively.

In [17] and [18], generic expressions for \( N_R \) and \( T_R \) are derived assuming an equation of state with constant \( \omega_R \). In particular, assuming conservation of entropy between the reheating era and today, we have [17]

\[
N_R \approx \frac{4}{1 - 3\omega_R} \left[ -\frac{1}{4} \ln \left( \frac{45}{\pi^2 g_\ast} \right) - \frac{1}{3} \ln \left( \frac{11 g_\ast}{43} \right) - \ln \left( \frac{k}{a_0 T_0} \right) - \ln \left( \frac{V_r^{1/4}}{H_R} \right) - N_\ast \right],
\]

for \( \omega_R \approx \frac{1}{3} \), where \( g_\ast \) is the quantity of relativistic species at the end of the reheating phase, \( k \) is a specific pivot scale, \( a_0 \) and \( T_0 \) are the scale factor and temperature at the present day respectively, and \( V_r \) is the value of our potential evaluated at the end of inflation. Taking Planck’s pivot scale of \( k=0.05 \text{ Mpc}^{-1} \) and using the estimated value \( g_\ast \approx 100 \), this simplifies to [17]:

\[
N_R \approx \frac{4}{1 - 3\omega_R} \left[ 61.6 + \ln \left( \frac{H_R}{\sqrt[4]{V_r}} \right) - N_\ast \right],
\]

where only the last two terms depend on our specific model.

One can invert equation (36) to express \( N_\ast \) in terms of \( n_\ast \):

\[
N_\ast \approx \frac{2}{1 - n_\ast} - \frac{1}{q + 1} + \frac{1}{(q + 1)^2} \ln \left( \frac{2(q + 1)^2}{q(1 - n_\ast)} - \frac{1}{q} \right).
\]

On the other hand, we can use equation (38) to solve for \( A \) in terms of \( A_\ast \) and \( n_\ast \), and then obtain \( H_\ast \) and \( V_r \) by simply evaluating the Hubble parameter and the potential at \( \phi_\ast \) and \( \phi_r \), respectively:

\[
H_\ast = \pi \frac{8 A_\ast}{\kappa} \sqrt{q + 1} \sqrt{1 + W(q e^{q_0(q+1)N_\ast})},
\]

\[
V_r = \frac{16 \pi^2 A_\ast}{\kappa^2} \frac{q^2}{W^2(q e^{q_0(q+1)N_\ast})}.
\]

Now we can plug equation (41) into these last two equations and substitute them into equation (40) to obtain \( N_R \) in terms of \( A_\ast, n_\ast \), and \( q \).

Finally, we can use the expression given in [17] for the reheating temperature:

\[
T_R = \left( \frac{43}{118 \ast} \right)^{1/3} \frac{a_0}{k} T_0 H_\ast e^{-N_\ast - N_\ast},
\]

where we have already calculated all of the quantities on the right-hand side.

We find that the value of the parameter \( q \) does not significantly affect the \( N_R \) and \( T_R \) plots so we simply take \( q = 2 \) for both figures presented in figures 5 and 6. Also, in those graphs, the blue, purple, black and gray lines correspond to \( \omega_R = -\frac{1}{3}, 0, \frac{2}{3}, \frac{1}{3} \) respectively. The dotted vertical lines correspond to the limits of the interval of 68% C.L. given by Planck [1].

We feel that it is important to notice that the reheating temperature plots presented in [16] are done by varying their parameter \( V_0 \), which is the analogon of our \( A^2 \) in the sense that they control the ‘amplitude’ of the potential) over a huge range of values, from \( \sim 10^2 \) to \( \sim 10^{-60} \); but since in their model the scalar power spectrum is directly proportional to \( V_0 \), the variation of \( V_0 \) over such a large range would affect their prediction for \( A \), which would be off by several orders of magnitude.

5. Generalization to Bernoulli-like functions

We extend the above calculations to a more general class of generating functions, since the logistic function considered previously is the solution to the logistic differential equation, which is a special case of the Bernoulli equation. We generalize this by taking as a generating function, a solution of the Bernoulli differential equation in the particular form

\[
F' = F(q - F^n),
\]

where \( n \) and \( c_1 \) are constants, and we assume for simplicity that \( n > 0 \). Notice that the case we covered in previous sections corresponds to \( n = 1 \). It is easy to find an integrating factor and write the solution as
We have omitted a constant in front of the exponential because it does not affect our results, just as in the \( n = 1 \) case. Notice that

\[ F' = \frac{b}{nA^0} F(A^n - F^n); \]  

(47)

also it is easy to show that

\[ A^n - F^n = \frac{A^n}{1 + e^{b\phi}}. \]  

(48)

The potential produced by this generating function is

\[ V(\phi) = \frac{A^2}{(1 + e^{-b\phi})^2/n} \left[ 1 - \frac{2b^2}{3\kappa n^2 (1 + e^{b\phi})^2} \right]. \]  

(49)

Now the potential has a sigmoidal-like shape for \( b \lesssim n \sqrt{\frac{3\kappa}{2}} \) and develops an absolute minimum for \( b > n \sqrt{\frac{3\kappa}{2}} \).

The first two slow-roll parameters are

\[ \epsilon = \frac{2b^2}{\kappa n^2 (1 + e^{b\phi})^2}, \]  

(50)
From \( \epsilon(\phi_e) = 1 \), we get that inflation ends at \( \phi_e = \frac{1}{b} \ln q \), where

\[
q \equiv \sqrt{\frac{2}{\kappa b} - 1}.
\]  

The number of e-folds results to be

\[
N = \frac{\kappa H}{2b} \left( \phi - \phi_e + \frac{1}{b} e^{b\phi_e} - \frac{1}{b} e^{b\phi} \right).
\]  

Substituting the expression for \( \phi_e \) and using again the property \( W(x)e^{W(x)} = x \) we get the identity

\[
e^{b\phi} = W(q e^{q(q+1)^2 N}),
\]  

Therefore we get the spectral index and tensor-to-scalar ratio as:

\[
n_s = 1 - \frac{2n(q + 1)^2}{1 + W(q e^{q(q+1)^2 N})},
\]

\[
r = \frac{16n(q + 1)^2}{[1 + W(q e^{q(q+1)^2 N})]^2}.
\]  

The plots of these can be seen in figures 7 and 8 respectively. The blue, purple, black and gray lines correspond to \( n = 0.04, 0.4, 4, \) and 4000 respectively. The choice of these particular values is based on instantaneous reheating considerations (see below). We also give the parametric plot of the tensor-to-scalar ratio against the spectral index in figure 9.
The power spectrum of scalar perturbations is given by $A_s = \frac{\kappa}{8\pi^2} \frac{q + 1}{\sqrt{\kappa}} \left( \frac{q}{q + 1} \right)^{2/n} \left( \frac{q + 1}{W(qe^{q(q+1)^2N_0})} \right)^{\frac{2n}{q+1}}$, in our case it is

$$A_s = \frac{\kappa^2 A^2}{24\pi^2} \frac{W^2(qe^{q(q+1)^2N_0})}{(q+1)^2} \left[ 1 + \frac{W(qe^{q(q+1)^2N_0})}{W(qe^{q(q+1)^2N_0})} \right]^\frac{2n}{q+1}. \tag{57}$$

For the reheating epoch we calculate the following quantities:

$$H_\text{re} = \pi \frac{\sqrt{8A_s}}{\kappa} \frac{q + 1}{1 + W(qe^{q(q+1)^2N_0})}, \tag{58}$$

$$V_\text{re} = \frac{16A_s \pi^2}{\kappa^2} \left( \frac{q}{q + 1} \right)^{2/n} \left( \frac{q + 1}{W^2(qe^{q(q+1)^2N_0})} \right) \left[ \frac{W(qe^{q(q+1)^2N_0})}{1 + W(qe^{q(q+1)^2N_0})} \right]^\frac{2n}{q+1}. \tag{59}$$
Notice that all these expressions reduce to the ones given in sections 2, 3 and 4 when \( n = 1 \).

To determine plausible values for our parameter \( n \) we take into account the duration \( N_R \). In general, larger values of \( n \) result in instantaneous reheating happening at smaller values of \( n_s \), and vice-versa. It is relatively easy to estimate that instantaneous reheating is possible for values of \( n \) in the approximate range \( \sim [0.0345, 8000] \) given Planck bounds of \( n_s = 0.9649 \pm 0.0042 \) (i.e. larger values of \( n \) imply that instantaneous reheating happens at a value of \( n_s \) smaller than 0.9649 – 0.0042, and smaller values of \( n \) imply that instantaneous reheating happens at a value of \( n_s \) larger than 0.9649 + 0.0042). Thus for our plots we select values for \( n \) of 0.04, 0.4, 4, and 4000, all of which lie within the aforementioned range. We present the plots for \( N_R \) and \( T_R \) in figures 10 and 11 respectively. We can see that instantaneous reheating is possible for this whole range of values.

6. Conclusion

Using the ISB formalism, we have analytically calculated the observables and general properties of a new class of inflationary potentials. We considered effective potentials from certain supergravity models which can produce a no-oscillatory reheating period. By analyzing a generating function which has the same qualitative form as these potentials, we obtain a new form of the potential that nevertheless develops a minimum for the most part of its parameter space (large values of \( b \)). In this regime, at the end of inflation, the friction term due to the expansion of the Universe and to the coupling to other particles causes the total energy of the inflaton to diminish, making it oscillate around its minimum and producing other particles in the process, just as in the original analyses of the reheating period. Given that the logistic function we considered at the beginning is a well-known solution to a well-known equation, we then generalize it by adding a second parameter. This allows us to study even more new potentials which have the same basic qualitative behavior but can produce different cosmological observables, as well as different reheating temperatures and durations. For each one of the above models, we analytically calculated the main observables: \( r \), \( n_s \) and \( A_s \), all of which satisfy current experimental bounds. By insisting that instantaneous reheating is possible, we can obtain appropriate ranges for the values of our free parameters. Our model satisfies observations quite comfortably, given the current experimental bounds; these virtues validate this new model of cosmological inflation.

ORCID iDs

E Díaz  @ https://orcid.org/0000-0001-6127-3641
O Meza-Aldama  @ https://orcid.org/0000-0002-5185-1759

Figure 11. Reheating temperature \( T_R \) for different values of \( n \). From top to bottom, left to right, the figures correspond to \( n = 0.04, 0.4, 4, \) and 4000.
References

[1] Akrami Y (Planck Collaboration) et al ActA 641 A10
[2] Chervon S, Fomin I, Yurov V and Yurov A 2011 Physics of the Large and the Small (Singapore: World Scientific) 523–686
[3] Chervon S V, Fomin I V and Beesham A 2018 Eur. Phys. J. C 78 301
[4] Ivanov G 1981 Friedmann cosmological models with non-linear scalar field Gravitaciya i Teoriya Otnositelosti 18 54
[5] Muslimov A 1990 On the scalar field dynamics in a spatially flat Friedman universe Class. Quant. Grav. 7 2317
[6] Dodelson S 2003 Modern Cosmology (New York: Academic)
[7] Liddle A R and Lyth D H 2000 Cosmological Inflation and Large-Scale Structure (Cambridge: Cambridge University Press)
[8] Baumann D arXiv:0907.5424 [hep-th]
[9] Malik K A and Wands D 2009 Phys. Rept. 475 1–51
[10] Bassett B A, Tsujikawa S and Wands D 2006 Rev. Mod. Phys. 78 537–89
[11] Chen X 2010 Adv. Astron. 2010 638979
[12] Salopek D S and Bond J R 1990 Phys. Rev. D 42 3936–62
[13] Ellis J et al arXiv:2008.09099 [hep-ph]
[14] Kofman L, Linde A and Starobinsky A 1997 Phys. Rev. D 56 3258–95
[15] Allahverdi R, Brandenberger R, Cyr-Racine F and Mazumdar A 2010 Ann. Rev. Nucl. Part. Sci. 60 27–51
[16] Oikonomou V K and Chatzarakis N T 2019 Annals Phys. 411 167999
[17] Cook J L, Dimastrogiovanni E, Easson D A and Krauss L M 2015 J. Cosmol. Astropart. Phys. JCAP04(2015)047
[18] Muñoz J B and Kamionkowski M 2015 Phys. Rev. D 91 043521