The SUSY CP Problem and the MFV Principle

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We address the SUSY CP problem in the framework of Minimal Flavor Violation (MFV), where the SUSY flavor problem finds a natural solution. By contrast, the MFV principle does not solve the SUSY CP problem as it allows for the presence of new flavor blind CP-violating phases. Then, we generalize the MFV ansatz accounting for a natural solution of it. The phenomenological implications of the generalized MFV ansatz are explored for MFV scenarios defined both at the electroweak (EW) and at the GUT scales.

I. INTRODUCTION

Supersymmetric (SUSY) extensions of the Standard Model (SM) are broadly considered as the most motivated and promising New Physics (NP) theories beyond the SM. The solution of the gauge hierarchy problem, the gauge coupling unification and the possibility of having a natural cold dark matter candidate, constitute the most convincing arguments in favor of SUSY.

On the other hand, a generic SUSY scenario provides many (dangerous) new sources of flavor and CP violation, hence, large non-standard effects in flavor processes would be typically expected.

However, the SM has been very successfully tested by low-energy flavor observables both from the kaon and $B_d$ sectors.

In particular, the two $B$ factories have established that $B_d$ flavor and CP violating processes are well described by the SM up to an accuracy of the $(10 - 20)\%$ level [1].

This immediately implies a tension between the solution of the hierarchy problem, calling for a NP scale below the TeV, and the explanation of the Flavor Physics data requiring a multi-TeV NP scale if the new flavor-violating couplings are generic.

An elegant way to simultaneously solve the above problems is provided by the Minimal Flavor Violation (MFV) hypothesis [2, 3], where flavor and CP violation are assumed to be entirely described by the CKM matrix even in theories beyond the SM.

However, the MFV principle does not provide in itself any restriction to the presence of new CP-violating phases, hence, the assumption that the CKM phase provides the only source for CP violation (CPV) even in NP theories satisfying the MFV principle seems to be not general and thus a restrictive assumption [4] (see also [6, 9]).

In this context, we analyze the most general SUSY scenario, compatible with the MFV principle, allowing for the presence of new CP-violating sources.

In general, a MFV MSSM suffers from the same SUSY CP problem as the ordinary MSSM. In fact, the symmetry principle of the MFV does not forbid the presence of the dangerous flavor blind CP-violating sources such as the $\mu$ parameter in the Higgs potential or the trilinear scalar couplings $A_{\ell}$. When such phases assume natural $O(1)$ values and if the SUSY scale is not far from the EW scale, the bounds on the EDMs of the electron and neutron are violated by orders of magnitude: this is the so-called SUSY CP problem.

Either an extra assumption or a mechanism accounting for a natural suppression of these CPV phases are desirable.

In this work, we assume a flavor blindness for the soft sector, i.e. universality of the soft masses and proportionality of the trilinear terms to the Yukawas, when SUSY is broken. In this limit, we also assume CP conservation and we allow for the breaking of CP only through the MFV compatible terms breaking the flavor blindness.

That is, CP is preserved by the sector responsible for SUSY breaking, while it is broken in the flavor sector.

The generalized MFV scenario naturally solves the SUSY CP problem while leading to specific and testable predictions in low energy CP violating processes.

II. CP VIOLATION IN SUSY MFV SCENARIOS

The hypothesis of MFV states that the SM Yukawa matrices are the only source of flavor breaking, even in NP theories beyond the SM [2, 3]. The MFV ansatz offers a natural way to avoid unobserved large effects in flavor physics and it relies on the observation that, for vanishing Yukawa couplings, the SM enjoys an enhanced global symmetry

$$G_{\ell} = SU(3)_u \times SU(3)_d \times SU(3)_Q \times SU(3)_c \times SU(3)_L.$$ (1)

The SM Yukawa couplings are formally invariant under $G_{\ell}$ if the Yukawa matrices are promoted to spurions transforming in a suitable way under $G_{\ell}$. NP models are then of the MFV type if they are formally invariant under $G_{\ell}$, when treating the SM Yukawa couplings as spurions.

In the MSSM with conserved $R$-parity, the most general expressions for the low-energy soft-breaking terms compatible with the MFV principle and relevant for our analysis read [4]

$$m_Q^2 = m_Q^2 \left[ 1 + r_1 Y_u^u Y_u + r_2 Y_d^d Y_d + (c_1 Y_{d}^d Y_{d}^d Y_u + h.c.) \right],$$ (2)
\[
m^2_D = m^2_D \left[ 1 + Y_d \left( r_3 + r_4 Y_u^t Y_u + r_5 Y_d^t Y_d \right) + \left(c_2 Y_d^t Y_d Y_u^t Y_u + h.c.\right) \right],
\]

\[
A^U = A_U Y_u \left( 1 + c_3 Y_d^t Y_d + c_4 Y_u^t Y_u + \right.
\]
\[
\left. + c_5 Y_d^t Y_d Y_u^t Y_u + c_6 Y_u^t Y_u Y_d^t Y_d \right),
\]

\[
A^D = A_D Y_d \left( 1 + c_7 Y_u^t Y_u + c_8 Y_d^t Y_d + \right.
\]
\[
\left. + c_9 Y_d^t Y_d Y_u^t Y_u + c_{10} Y_u^t Y_u Y_d^t Y_d \right),
\]

where \( m_Q, m_D, A_U \) and \( A_D \) set the mass scale of the soft terms, while \( r_i \) and \( c_i \) are unknown, order one, numerical coefficients.

Notice that, in the above expansions, the SM Yukawa couplings are not assumed to be the only source of CPV as done instead in [3]. In particular, while all the \( r_i \) parameters must be real, as the squark mass matrices are hermitian, the \( c_i \) parameters are generally complex [4].

As in the ordinary MSSM, flavor conserving CP violating sources such as the \( \mu \) parameter in the Higgs potential or the trilinear scalar couplings \( A_f \) are unavoidable also in SUSY MFV frameworks, as they are not forbidden by the symmetry principle of the MFV [4].

Physics observables will then depend only on the phases of the combinations \( M_{\mu}, A_{1\mu} \) and \( A_{\mu}^* \) [11] and it is always possible to choose a basis where only the \( \mu \) and \( A_f \) parameters remain complex \(^1\).

These CP violating phases generally lead to too large effects for the electron and neutron EDMs, which are induced already at the one loop level through the virtual exchange of gauginos and sfermions of the first generation.

In particular, the current experimental bounds on the electron [13] and neutron [14] EDMs imply that

\[
|\sin \phi_\mu| \lesssim 10^{-3} \left( \frac{m_{\text{SUSY}}}{300 \text{ GeV}} \right)^2 \left( \frac{10}{\tan \beta} \right),
\]

\[
|\sin \phi_A| \lesssim 10^{-2} \left( \frac{m_{\text{SUSY}}}{300 \text{ GeV}} \right)^2,
\]

if we impose the bounds on \( \phi_\mu \) and \( \phi_A \) separately. In Eq. [6], \( \tan \beta = \tan \beta \) and a common SUSY mass \( m_{\text{SUSY}} \) has been assumed.

\(^1\) To be precise, such a statement is valid as long as the gaugino masses are universal at some scale. Even in this last case, two loop effects driven by a complex stop trilinear \( A_t \) generate an imaginary component for \( M_{\mu} \) [12] that we systematically take into account in our numerical analysis.

The naturalness problem of so small CP-violating phases, provided a SUSY scale of the order of the EW scale, is commonly referred to as the SUSY CP problem. Hence, either an extra assumption or a mechanism accounting for such a strong suppression in a natural way are desirable.

### III. A GENERALIZED MFV ANSATZ AND THE SUSY CP PROBLEM

The SUSY CP problem is automatically solved in the MFV framework of D’Ambrosio et al. [3], as they assume the extreme situation where the SM Yukawa couplings are the only source of CPV.

However, the MFV symmetry principle allows for the presence of new CPV phases, in particular of flavor blind phases that represent the main source of the SUSY CP problem.

Instead of following the approach of D’Ambrosio et al. [3], we first observe that the assumption of the flavor blindness corresponds to setting all the \( r_i \) and \( c_i \) coefficients to zero.

In this limit, we assume CP conservation and we allow for the breaking of CP only through the terms breaking the flavor blindness.

In this way, \( A_U, A_D \), the gaugino masses and the \( \mu \) term turn out to be real at the scale where the MFV holds while the leading imaginary components of the \( A \) terms, induced by the complex parameters \( c_i \), have a cubic scaling with the Yukawas.

Notice that, after the infinite sum of MFV-compatible terms for Eqs. (2)-(5) is taken into account, the generation of CP-violating phases for \( A_U \) and \( A_D \) is unavoidable [4] [5] [10]. However, we have checked that these phases are at most of order \( \sim y_e^2 \sim 10^{-4} \), hence, safely negligible.

If we now deal with a low scale MFV scenario, the one loop contributions to the electron and neutron EDMs, that depend on the first generation \( A \) terms, are proportional to the cube of light fermion masses, hence safely under control even for order one CPV phases of the \( c_i \) parameters. As a result, the generalized MFV ansatz applied to a low scale SUSY MFV scenario can completely cure the SUSY CP problem.

The situation can drastically change if we define a SUSY MFV scenario at the GUT scale.

In this last case, RGE effects stemming from trilinears of the third generation will unavoidably generate complex trilinears for light generations and a complex \( \mu \) term at the low scale. As a result, the EDMs will receive both one and two loop contributions and the SUSY CP problem might reappear. However, as we will discuss in detail later, if CPV arises only from terms breaking the flavor blindness, it will be still possible to account for the SUSY CP problem in natural ways.

But how natural is the assumption for the origin of CP breaking in the generalized MFV scenarios?
As an attempt to address this question, we make a comparison between the generalized MFV scenario and SUSY flavor models.

In fact, one could envisage the possibility that the peculiar flavor structure of the soft-sector dictated by the MFV principle might be the remnant of an underlying flavor symmetry holding at some high energy scale.

Supersymmetric models with abelian [15, 16] and non-abelian [17, 18] flavor symmetries have been extensively discussed in the literature. They are based on the Froggatt-Nielsen [19] mechanism where the flavor symmetries are spontaneously broken by (generally complex) vacuum expectation values of some “flavon” fields Φ and the hierarchical patterns in the fermion mass matrices can then be explained by suppression factors \((\Phi/M)^n\), where \(M\) is the scale of integrated out physics and the power \(n\) depends on the horizontal group charges of the fermion, Higgs and flavon fields.

Then, such flavor symmetries, while being at the origin of the pattern of fermion masses and mixings, relate, at the same time, the flavor structure of fermion and sfermion mass matrices.

However, the CP violating effects to the EDMs driven by the flavor blind phases \(\phi_A, \phi_\mu\) are, in general, not constrained at all by the flavor symmetry and an additional assumption is required.

The usual assumption employed by SUSY flavor models is that CP is a symmetry of the theory that is spontaneously broken only in the flavor sector as a result of the flavor symmetry breaking [16, 18].

Hence, we believe that the assumption we made on the origin of CPV in MFV scenarios is reminiscent of the usual approach followed in SUSY flavor models.

In the light of these considerations, we proceed now to analyze the phenomenological implications of the generalized MFV ansatz for MFV scenarios defined both at the EW scale and at the GUT scale.

In particular, we want to address the question whether \(O(1)\) phases for the MFV coefficients \(c_i\), which are the only source of CPV in our setup, are phenomenologically allowed.

IV. EW SCALE MFV SCENARIOS

The generalized MFV ansatz described in the previous section, where the \(A_i\) terms are assumed to be the only sources of CPV, implies a hierarchical structure for \(\text{Im } A_i\). In particular, it turns out that \(\text{Im } A_i \gg \text{Im } A_{e,u}\), \(\text{Im } A_i \gg \text{Im } A_{d,c}\) and \(\text{Im } A_i \gg \text{Im } A_{\mu,e}\) (as \(\text{Im } A_i\) scale with the cube of the fermion masses) and this leads to a natural suppression for the one loop SUSY contributions to the EDMs.

Still, a potentially relevant one loop effect for the down quark EDMs, proportional to \(\text{Im } A_i\), is induced by the stop exchange, as shown in the left-hand diagram of Fig. 1. It reads

\[
\left\{ \frac{d_{d_i}}{e} \right\} \simeq \frac{\alpha_2}{16\pi} \frac{3}{18} \frac{m_{d_i}}{m_\chi} m_{\tilde{q}}^2 \text{Im}(A_i\mu)|V_{i\beta}|^2 t_\beta, \quad (6)
\]

leading to \(d_e \sim 3 \times 10^{-29}\) ecm for maximum CPV phases, \(m_{\text{SUSY}} \sim 500\) GeV and \(t_\beta = 10\), still far from \(d_e^{\text{exp}} < 10^{-26}\) ecm. The enhancement to \(d_e\) induced by \(\text{Im } A_i\) is compensated by the strong suppressing factor \(|V_{i\beta}|^2\).

A much more important effect is provided by the two loop Barr-Zee type diagram of Fig. 1 also involving only the third sfermion generation [20]. These diagrams will generate the electron EDM \(d_e\) as well as the EDMs and chromo-EDMs for quarks. In particular, it turns out that \(d_e\) and the Mercury EDM \(d_{\text{Hg}}\) (as induced by the down-quark chromo-EDM) are the most sensitive observables to this scenario. However, the theoretical estimation of \(d_{\text{Hg}}\) passes through some nuclear calculations that unavoidably suffer from sizable uncertainties [11] hence, in the following, we focus on the predictions for \(d_e\), to be conservative.

The induced electron EDM \(d_e\) reads

\[
\frac{d_e}{\text{ecm}} \simeq 10^{-27} \left( \frac{500\text{GeV}}{m_{\text{SUSY}}} \right)^2 \left( \frac{t_\beta}{10} \right)^2 \sin(\phi_\mu + \phi_A), \quad (7)
\]

where in Eq. (7) we have assumed \(m_{\text{SUSY}} = m_A\). Thus, if \(O(1)\) phases are allowed, \(d_e\) can reach the current experimental bound for \(m_{\text{SUSY}} \sim 500\) GeV and \(t_\beta = 10\).

So far, we have not considered the contributions to the EDMs stemming from flavor effects [21]. Indeed, the
MFV flavor structures of Eqs. (2)–(5) provide additional one loop “flavored” effects to the hadronic EDMs.

The off-diagonal terms of Eqs. (2)–(5) can be conveniently parameterized by means of the so-called MI parameters [22] defined as usual as

\[ \delta_{ij}^L = (m_{Q_i}^2/m_{Q_j}^2) \quad \text{and} \quad \delta_{ij}^R = (m_{D_i}^2/m_{D_j}^2) \]

(8)

with \( m_{Q_i}^2 = \sqrt{(m_{Q_i}^2)^2 - (m_{Q_i}^2)^2} \) and \( X = Q, L \). Then, from Eqs. (2)–(5), it follows that

\[ \delta_{ii}^L \approx (r_1 + c_1 y_i^2) V_{li} \]
\[ \delta_{ii}^R \approx y_i y_d (r_1 + c_2 y_i^2) V_{li} \].

(9)

(10)

One of the most important “flavored” effects to the hadronic EDMs arises from the gluino/squark contribution shown in Fig. 1, leading to

\[ \left\{ \frac{d_{ij}}{\epsilon} \right\} \sim \alpha_s m_{\tilde{g}} 4 \pi \frac{4}{135} \text{Im} \left[ \delta_{ij}^L \delta_{3i}^L \delta_{3j}^L \right] \],

(11)

where \( \delta_{ii}^{LR} = m_b (A_b - \mu t_3)/m_{\tilde{g}}^2 \). The apparent bottom Yukawa enhancement of Eq. (11), by means of \( \delta_{ii}^{LR} \), is not effective within a SUSY MFV scenario as the necessary \( \delta^R \) MI turns out to be always proportional to light quark Yukawas, see Eq. (10). In the most favorable situation, where we assume maximum CPV and \( y_b \approx 1 \), we find \( \left| \{d_{ij}\}_\beta \right| \approx 3 \times 10^{-29} \text{ cm} \) for \( m_{\text{SUSY}} = 500 \text{ GeV} \) and \( O(1) \) parameters \( |r_i|, |c_i| \).

Still, the two loop contributions of Fig. 1 are largely dominant. The same conclusion holds for all the other flavored effects to the hadronic EDMs, hence, we will not discuss them here.

Having discussed the dominant contributions to quark and lepton EDMs in the low-scale MFV setup, we proceed now to assess its phenomenological viability in light of the experimental bounds on EDMs. As an illustrative example, we choose a common SUSY mass \( m_{\text{SUSY}} \) and consider separately the two leading terms in the MFV expansion of \( A^U \), assuming purely imaginary coefficients to be fair.

Consequently, in Fig. 2 we show the predictions for the electron EDM \( d_e \), as a function of a common SUSY mass \( m_{\text{SUSY}} \), arising within an EW scale MFV framework in these two cases:

i) \( A^U = A_U Y_u (1 + c_4 Y_u^\dagger Y_u) \) with \( c_4 = i \),

ii) \( A^U = A_U Y_u (1 + c_3 Y_u^\dagger Y_u) \) with \( c_3 = i \).

The most prominent feature of the two scenarios is their different scaling properties with \( t_3 \): in case i) \( d_e \sim t_3 \) while in case ii) \( d_e \sim t_3^2 \). Moreover, the predictions for \( d_e \) in the case ii) are suppressed compared to those of case i) by a factor of \( y_d^2/y_u^2 \). Interestingly, Fig. 2 shows that \( d_e \) is safely under control, but it can reach experimentally visible levels, in both scenarios i) and ii) even for maximum CPV phases and a light SUSY spectrum.

We conclude noting that, within an EW scale MFV scenario, the EDMs receive the dominant effects at the two loop level while CPV effects in B-physics observables arise already at one loop [7]; hence large effects in B-physics can be still expected while being compatible with the EDM constraints. In particular, the phenomenology arising from the scenario discussed in this section is very similar to that discussed in Ref. [7].

V. GUT SCALE MFV SCENARIOS

In the previous section, we have assumed that the MFV expansion for the soft-breaking terms of Eqs. (2), (5) holds at the weak scale.

In contrast, in this section, we address the phenomenological implications for a MFV scenario defined at the high scale [23]. In fact, even if we start with universal soft masses and proportional trilinear terms at the high-energy SUSY breaking scale \( M_X \) (corresponding to setting all the coefficients \( r_i \) and \( c_i \) to zero) RGE effects do not preserve such a universality. The MFV coefficients \( r_i \) are RGE generated and their typical size is \((1/4\pi)^2 \ln M^2_X/m^2_{\text{SUSY}}\), so for sufficiently large values of \( M_X \), the effect can be significant.

Moreover, as already discussed in Sec. [1] it might be possible that the MFV flavor structure of the soft-sector can arise from an underlying flavor symmetry holding at some high energy scale.

In this respect, it seems quite natural to define a MFV scenario at the high scale. As seen in Sec. [11] a remarkable virtue of a low-scale
MFV scenario is its natural solution to the SUSY CP problem by means of hierarchical $A$ terms.

However, generational hierarchies in the trilinear couplings are affected by RG effects since the $A$ terms are not protected by the non-renormalization theorem. Therefore, even if these couplings are assumed to vanish at the GUT scale, they can be regenerated through running effects.

This fact is particularly relevant for the impact of complex trilinears on quark or lepton EDMs. For example, consider the RG equation for the up-squark trilinear; neglecting Yukawa couplings of the two light generations and $U(1)$ gauge couplings, it reads

\[
16\pi^2 \frac{d}{dt} A_u = 6 A_t y_t^2 - 6 y_d^2 M_2 - \frac{32}{3} g_3^2 M_3 ,
\]

(12)

where \( t = \ln(\mu/\mu_0) \). The first term on the right-hand side of Eq. (12) clearly shows that, even if the gaugino mass terms are real, \( A_u \) can receive a sizable imaginary part if the stop trilinear \( A_t \) is complex, with potentially dangerous impact on the one-loop contribution to the neutron EDM.

Approximate numerical expressions accounting for the low energy values of \( A_u(m_Z) \) and \( A_t(m_Z) \) as a function of the high energy input parameters, valid for low to intermediate tan $\beta$, are

\[
A_u(m_Z) \approx A_u(m_G) - 0.41 y_t^2 A_U + 0.03 y_d^2 A_D
- (0.05 y_d^2 c_4 + 0.11 y_t^2 c_4) A_U - 2.8 m_{1/2} (13)
\]

\[
A_t(m_Z) \approx A_t(m_G) - 0.81 y_t^2 A_U - 0.09 y_d^2 A_D
+ (0.04 y_d^2 c_3 + 0.10 y_t^2 c_4) A_U
- (0.03 y_d^2 y_t^2 c_7 + 0.01 y_t^2 c_8) A_D - 2.2 m_{1/2} (14)
\]

where the Yukawa couplings are to be evaluated at the low scale and we have neglected terms of $O(y_d^4)$. Eq. (13) shows that, irrespective of $A_u(m_G)$, a sizable contribution to $A_u(m_Z)$ from a complex $A_t(m_G)$ is unavoidable.

This is well illustrated in the upper plot of Fig. 3 where we show the predictions for the ratio $\text{Im} A_u/\text{Im} A_t$ as a function of the renormalization scale $\mu$ assuming the GUT scale boundary condition $\text{Im} A_u(m_G) = 0$ and $\text{Im} A_t(m_G) \neq 0$. Interestingly, the attained low energy values for $|\text{Im} A_u|$ and $|\text{Im} A_t|$ are very similar in spite of their very different values at the GUT scale, as confirmed by Eqs. (13, 14).

At the same time, huge RGE effects driven by the $SU(3)$ interactions strongly reduce the phase of $A_u(m_Z)$ and $A_t(m_Z)$ (see Eqs. (13, 14), provided the gaugino masses are real, as we assume.

This is illustrated in the lower plot of Fig. 3 where we show the predictions for $\text{Im} A_t/|A_t|$ as a function of the renormalization scale $\mu$ assuming the GUT scale boundary condition i) $A^U = c_4 A_U Y_u \bar{Y}_u Y_u$ setting $c_4 = i$ (upper line), ii) $A^U = c_3 A_U Y_u \bar{Y}_d Y_d$ setting $c_3 = i$ (lower band) and assuming $A_U = A_0 = m_{1/2}$ in both cases. We see that, even if we start with purely imaginary $A_t(m_G)$ at the GUT scale, such that $\text{Im} A_t(m_G)/|A_t(m_G)| = 1$, RGE effects reduce the phase of $A_t$ by more than one order of magnitude in case i) and up to four orders of magnitude in case ii) depending on the tan $\beta$ value.

As a result, the attained values for $A_u(m_Z)$ and $A_t(m_Z)$ lie within an experimentally allowed level for large regions of the parameter space even for $O(1)$ phases, ameliorating significantly the SUSY CP problem.

Another even more dangerous CP violating contribution driven by the RGE effects regards the $\mu$ term. To see this point explicitly, let's consider the one loop RGE for the $\mu$ parameter

\[
\frac{d\mu}{dt} = \frac{\mu}{16\pi^2} (-3 g_2^2 + y_t^2 + 3 y_d^2 + 3 y_t^2) .
\]

(15)

As we can see, the phase of $\mu$ does not run and this is still true at the two loop level. On the other hand, the RGE for the bilinear mass term $B$ is

\[
\frac{dB}{dt} = \frac{1}{8\pi^2} (-3 g_2^2 M_2 + y_t^2 A_v + 3 y_d^2 A_b + 3 y_t^2 A_t) .
\]

(16)

then, in contrast to the $\mu$ term, the phase of the $B$ term is affected, through RGE effects, by the phases of the $A$
terms. To have an idea of where we stand, it is useful to provide a numerical solution to Eq. (16) as a function of the high scale parameters

\[ B \approx B_0 + (0.15 + 0.60 t^2) m_{1/2} \]
\[ - 0.41 g_1^2 A_U - 0.42 g_2 A_D - 0.30 g_3^2 A_E \]
\[ - (0.05 g_1^2 y_t^2 c_t + 0.11 g_1^2 c_t) A_U \]
\[ - (0.12 g_1^2 y_t^2 c_t + 0.05 g_1^4 c_s) A_D . \]

where \( t = \tan \beta / 50 \), the Yukawa couplings are to be evaluated at the low scale and we have again neglected terms of \( O(y_t^4) \).

Recall that, since the overall phase of the \( \mu \) and \( B \mu \) terms can be removed by a Peccei-Quinn transformation, only their overall phase is physical; moreover, the phase and absolute value of the \( B \mu \) term at the low scale is dictated by the EWSB conditions: in fact, in the basis where the Higgs VEVs are real, these conditions require a real \( B \mu \) term at the leading order \(^2\). Thus, the condition that this relative phase vanishes at the high scale implies that the \( \mu \) term must be complex.

This is shown in Fig. 4 where we consider the dependence of the phase of \( \phi_\mu + \phi_B \) on the renormalization scale assuming the GUT scale boundary condition \( A^U = c_4 A_U Y_u Y_u^T Y_u \) with \( c_4 = i \) (upper line) and \( A^U = c_3 A_U Y_u Y_u^T Y_d \) with \( c_3 = i \) (lower band), and assuming \( A_D = B_0 = m_{1/2} = m_{1/2} \) for definiteness.

As we can see, the phase \( \phi_\mu + \phi_B = 0 \) at the high scale as \( \phi_\mu = 0 \) and \( \phi_B = 0 \) singularly. However, at the low scale, \( \phi_\mu + \phi_B \neq 0 \) as the phase \( \phi_B \) is generated by RGE effects, in contrast to the phase \( \phi_\mu \) that remains vanishing as it does not run.

In principle, one can now impose by hand a real \( \mu \) term at the EWSB scale (and hence at all scales), but then the \( B \mu \) term will be complex at the high scale; this is the approach that is commonly assumed e.g. in the CMSSM. However, in our scenario, we assume that CP violation only arises from the soft flavor-breaking terms of the MFV expansion, hence, the \( B \) parameter at the high scale is assumed to be real.

Thus, if we start with a SUSY MFV scenario at the GUT scale, where CP violating sources are confined to the third generation \( A \) terms, at the low scale we unavoidably generate complex trilinears for light generations and a complex \( \mu \) term via RGE effects \(^3\). As a result, the EDMs receive both one and two loop contributions. However, in our MFV scenario defined at the GUT scale, the dominant effects to the EDMs are by far those induced by the one loop effects of Fig. 6 through the phase of the \( \mu \) term.

\(^2\) Beyond the leading order, the \( B \mu \) term acquires a small imaginary part in the presence of CP violation in the \( \mu \) or \( A \) terms, in order to compensate the CP-odd tadpole counterterms \(^{20}\), while its real part is corrected by CP-even tadpoles.

\(^3\) A related study within the CMSSM can be found in Ref. \(^{27}\).

After discussing the RGE effects leading to important contributions to quark and lepton EDMs in the high-scale MFV setup, we proceed now to assess its phenomenological viability.

In Fig. 5 we show the predictions for the electron EDM in a MFV framework defined at the GUT scale assuming the two cases i) and ii) for the trilinears \( A^U \) already discussed in the low-scale scenario; moreover, we set \( A_U = A_D = m_{1/2} = m_{1/2} \) in both cases.

The above findings require some comments. In fact, from a phenomenological perspective, it seems unlikely that the coefficient \( c_4 \) of Eq. 4 can be an \( O(1) \) complex parameter, as it would lead to the problems met in the above scenario 1). Let’s try now to argue which could be the underlying theoretical motivation leading to a real \( c_4 \) making a comparison with the typical situations occurring in SUSY flavor models.

In these last cases, the flavor symmetries are spontaneously broken by the complex vacuum expectation values of some “flavon” fields \( \Phi \) and the hierarchical patterns for the Yukawa matrices are explained in terms of suppressing factors \( (\Phi / M)^n \), as discussed in Sec. II. Clearly, given that the top Yukawa coupling is an order one parameter, it does not require any suppressing factor and it is formally of the zeroth order in the \( (\Phi / M)^n \)
expansion. Concerning the low energy phenomenology of the GUT MFV scenario discussed in this section, we want to stress that the EDMs, arising at one loop level, are the most promising observables and they generally prevent any visible effect in CPV B-physics observables. This is in contrast with the EW MFV scenario where the EDMs arise at the two-loop level (see Fig. 1) in the context of hierarchical sfermions with light third and heavy first generations [28]. Should this be the case, the low-energy flavor data with the LHC data for the SUSY spectrum could enable us to reconstruct the underlying scenario at work.

VI. LEPTONIC DIPOLE MOMENTS: $d_e$, $(g - 2)_\mu$

AND BR($\ell_i \rightarrow \ell_j \gamma$)

In the following, we briefly discuss the correlations arising among dipole transitions in the leptonic sector [29]. In particular, we consider the electric dipole moment of the electron $d_e$, the anomalous magnetic moment of the muon $a_\mu = (g - 2)_\mu / 2$ and the branching ratio of the lepton flavor violating (LFV) decay $\mu \rightarrow e\gamma$ as these observables are highly complementary in shedding light on NP. In fact, while $a_\mu$ and $d_e$ are sensitive to the real and imaginary flavor diagonal dipole amplitude, respectively, BR($\ell_i \rightarrow \ell_j \gamma$) constrains the absolute value of off-diagonal dipole amplitudes.

Interestingly, most recent analyses of the muon $(g - 2)$ point towards a $3\sigma$ discrepancy in the $10^{-9}$ range [30,31]: $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} \approx (3 \pm 1) \times 10^{-9}$. Hence, the question we intend to address now is which are the expected values for $d_e$ and BR($\ell_i \rightarrow \ell_j \gamma$) if we interpret the above discrepancy in terms of NP effects, in particular coming from SUSY.

As an illustrative case, if we consider the limit of a degenerate SUSY spectrum, the SUSY contributions to $\Delta a_\mu$ and $d_e$ (as induced by flavor blind phases) read

$$\Delta a_\mu \approx \frac{\alpha_2}{4\pi} L_\beta \frac{5}{12} \frac{m_\mu^2}{m_{\text{SUSY}}^2},$$

$$\frac{d_e}{e} \approx \frac{\alpha_2}{4\pi} L_\beta \frac{5}{24} \left( \frac{m_e}{m_{\text{SUSY}}} \right) \sin \theta_\mu,$$

leading to

$$\frac{|d_e|}{e \text{ cm}} \approx 10^{-27} \times \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) \frac{|\theta_\mu|}{10^{-3}}.$$

The result of Eq. (19) immediately leads to the conclusion that, as long as SUSY effects account for the $(g - 2)$ anomaly, the prediction for $d_e$ typically exceeds its experimental bound $d_e \lesssim 10^{-27}$ unless $|\theta_\mu| \lesssim 10^{-3}$. An explanation for such a strong suppression of $\theta_\mu$ can naturally arise within the general GUT MFV framework, as discussed in the previous section. In fact, even assuming maximum CP violation in the high-scale trilinear and setting the unknown MFV coefficient $c_3 = 1$, we have found that $|\theta_\mu| \gtrsim 10^{-4}$, as shown in Fig. 4.

Passing to BR($\ell_i \rightarrow \ell_j \gamma$) and assuming again a degenerate SUSY spectrum, it is straightforward to find [32]

$$\text{BR}(\mu \rightarrow e\gamma) \approx 2 \times 10^{-12} \left[ \frac{\Delta a_\mu}{3 \times 10^{-9}} \right]^2 \left( \frac{\delta_{\mu e}}{10^{-4}} \right)^2 .$$

where we have assumed that BR($\mu \rightarrow e\gamma$) is generated only by the flavor structures among left-handed sleptons, i.e. $\delta_{\mu e}^L$, as it happens in SUSY see-saw scenarios.
The main messages from the above relations is that within a GUT MFV SUSY scenario, with generalized MFV ansatz, an explanation for the muon (g−2) anomaly leads to predictions for $d_e$ that are close to the current experimental upper bound $d_e \lesssim 10^{-27} \text{cm}$ while BR($\mu \rightarrow e\gamma$) typically lies within the expected MEG resolutions [39] for values of $\delta_{\mu e}^L$ covering the predictions of many SUSY see-saw scenarios.

VII. CONCLUSIONS

In this work we have addressed the SUSY CP problem in the framework of the MFV, where the SUSY flavor problem finds a natural solution. By contrast, the MFV symmetry principle allows for the presence of new CP-violating phases [4, 5] (see also [6–9]). Hence, the MFV ansatz has to be supplemented either by an extra assumption or by a mechanism accounting for a natural suppression of the flavor blind CPV phases.

In the light of these considerations, we have generalized the MFV ansatz accounting for a natural solution of the SUSY CP problem.

We have assumed flavor blindness, i.e. universality of the soft masses and proportionality of the trilinear terms to the Yukawas, when SUSY is broken.

In this limit, we have assumed CP conservation allowing for the breaking of CP only through the MFV compatible terms breaking the flavor blindness.

That is, CP is preserved by the sector responsible for SUSY breaking, while it is broken in the flavor sector.

We have explored the phenomenological implications of this generalized MFV ansatz for MFV scenarios defined both at the electroweak and at the GUT scales, pointing out the profound differences of the two scenarios and their peculiar and testable predictions in low energy CP violating processes.

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