A Mathematical Evaluation of Mitral Regurgitation Severity with EROA

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Abstract  It is to be investigated and verified that vena contracta width (VCW) obtained from color Doppler images and effective regurgitant orifice area (EROA) obtained from the VCW is capable of quantifying the severity of mitral regurgitation (MR). A mathematical expression has been developed between VCW and EROA. Vena contracta width is an efficient method for grading the severity of MR, but it is very difficult to locate this narrowest part exactly in transthoracic echocardiography as well as in transesophageal echocardiography. So, indeed, there is a need to develop a method which is capable of finding out the vena contracta width. A comparison has also been made between the results obtained using the proposed computing method and the results obtained by the clinicians, manually. For this analysis have been done from the results obtained with automatic VCW finding method and with clinical data for severity grading.

Keywords  Vena Contracta Width, Effective Regurgitant Orifice Area, Mitral Regurgitation, Severity Grading, Transthoracic Echocardiography, Transesophageal Echocardiography

1. Introduction

At present interest in color Doppler imaging for vena contracta for the assessment of mitral regurgitation is increasing progressively [1], [2] and [3]. The vena contracta is the narrowest part of the regurgitant jet, which can also be called as ‘neck’ of the jet [4] and having location just downstream from the orifice and before the jet area [5].

The cross sectional area is directly related to the regurgitant orifice area. Due to the complex and the unpredictable shape, it is very difficult to measure the regurgitant orifice area. For such a measurement jet is to be imaged in the short axis plane. In clinical settings, it is very difficult to localize the narrowest area from the jet. So a correlation is measured between the VCW which is measured from the zoomed view of the parasternal long axis and the apical views, and the EROA, by quantitative Doppler [3]. Relationship between the EROA and the VCW given at many places in the literatures [6], [7] and [8]. This relationship shows good results even in eccentric MR [9].

The VCW is the one dimensional measurement whereas EROA is the two dimensional so there are some limitations also. But in [10] it shown that VCW is the much better tool for measurement of severity of MR. Such as VCW less than 0.3cm indicates the mild MR. If this value is in between 0.3 and 0.5 then it is the case of moderate MR. But if the VCW is more than 0.5cm then it indicates the severe MR.

In any view the localization of the VCW is very difficult. So an experienced clinician is required for the measurement of such a narrowest portion. This gets very time consuming also to locate the exact VC. Hence as discussion in our research paper [14] an automatic segmentation have been done of echocardiographic images to get VCW. In this paper, a mathematical solution is derived which gives a relation between VCW and EROA.

2. Mathematical Treatment for EROA Calculation

Mathematically, blood flow can be described by Darcy's law (which can be viewed as the fluid equivalent of Ohm's law) and approximately by Hagen-Poiseuille equation:

\[ \text{Darcy's law} \quad F = \frac{\Delta P}{R} \]  \hspace{1cm} (1)

\[ \text{Hagen–Poiseuille Equation} \quad R = \frac{\nu L}{\pi r^4} \]  \hspace{1cm} (2)

Where \( F \) is blood flow, \( P \) is pressure, \( R \) is resistance, \( \nu \) is blood viscosity, \( L \) is length of tube, and \( r \) is radius of tube.

Here we are trying to calculate the vena contracta width according to the explanation given in [11]. However they used their formulation for the fluid flow. Here motive is to find out the vena contracta width formulation for the mitral regurgitation jet.
When blood passes through the mitral orifice then the momentum flux is given as:

\[
\frac{dp}{dt} = \rho \left( v_{\text{vena}}^3 - v_{\text{vent}}^2 \right) v_{\text{vena}} \approx \rho v_{\text{vena}}^2 A_{\text{vena}} \approx 2 P_{\text{vent}} A_{\text{vena}}
\]

(3)

Where \( A_{\text{vent}}, A_{\text{atr}} \) and \( A_{\text{vena}} \) are the area of jet in ventricle, atrium and vena contracta, \( v_{\text{vena}}, v_{\text{vent}}, v_{\text{vent}} \) and \( v_{\text{vena}} \) are the velocities of the blood in the ventricle, atrium and vena contracta, and \( P_{\text{vent}}, P_{\text{at}} \) and \( P_{\text{vena}} \) are the pressures in the respective regions.

According to Bernoulli’s equation in the limit that \( P_{\text{vena}} \ll P_{\text{vent}} \). The force that causes this momentum change is:

\[
F \approx P_{\text{vent}} A_{\text{vent}} \left[ P_{\text{vent}} (A_{\text{vent}} - A_{\text{at}}) + P_{\text{vena}} A_{\text{vena}} \right]
\]

(4)

Vena contracta can be estimated as:

\[
A_{\text{vena}} = \frac{A_{\text{at}}}{2}
\]

(5)

K.T McDonald [11] refer the comment given by Maxwell [12] on the vena contracta, he suggest that if vena contracta of the jet is to be measured in two dimensional potential flow through the orifice (in present case mitral orifice) then a more accurate analysis could be done based on the velocity potential \( \phi \) that exists when the flow is irrotational.

Any analytic function \( w(z) = \phi + i \psi \) of the complex variable \( z = x + iy \), obeys the Cauchy-Riemann equations,

\[
\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \quad (6i)
\]

and

\[
\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \quad (6ii)
\]

Which imply that \( \nabla^2 \phi = 0 = \nabla^2 \psi \). That is, any analytic function of a complex variable can, in principle, be related to the velocity potential of the blood flow. Furthermore, the curves \( \psi(x, y) \) constant follow streamlines of the Potential \( \phi(x, y) \) (and vice versa).

According to Kirchhoff [13] if the blood flow includes a free surface, on which the pressure is constant, then Bernoulli’s equation implies that the velocity of the blood flow is constant on this surface.

This velocity can always be scaled to unity. Then, since, \( V = -\nabla \phi = -\left( \frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y} \right) \) the scaled flow on a free surface obeys

\[
\left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 = 1
\]

(7)

Furthermore, according to the Jacobian transformation of area elements

\[
dx dy = J d\phi d\psi = J' dx dy,
\]

(8)

So, \( J' = 1 \). The Cauchy-Riemann equations for \( \phi + i \psi = w(x + iy) \) and \( x + iy = w^{-1}(\phi + i \psi) \) allow us to write the Jacobian determinants as

\[
J = \left( \frac{\partial \phi}{\partial x}, -\frac{\partial \psi}{\partial x} \right) \left( \frac{\partial \psi}{\partial y}, \frac{\partial \phi}{\partial y} \right) = \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2
\]

(9i)

and

\[
J' = \left( \frac{\partial \phi}{\partial x \phi}, \frac{\partial \phi}{\partial y \phi} \right) \left( \frac{\partial \phi}{\partial x \psi}, -\frac{\partial \phi}{\partial y \psi} \right) = \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2
\]

(9ii)

Then, the scaled flow on a free surface also obeys

\[
\left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 = 1
\]

(10)

Kirchhoff [3], extended Helmholtz’ technique of functions \( w(z) \), that are defined implicitly via knowledge of \( z(w) \) by consideration of the derivative, \( \frac{dz}{dw} \) defined as

\[
\frac{dz}{dw} = -f(w) - \sqrt{f^2(w) - 1},
\]

(11)

arguing that if along some portion of a streamline \( \psi = \text{constant} \) the function \( f \) is real and \( |f| \) less than 1, then \( \frac{dz}{dw} = \text{Re} \left( \frac{dz}{dw} \right) = -f \) and \( \frac{dy}{d\phi} = \text{Im} \left( \frac{dz}{dw} \right) = -\sqrt{1 - f^2} \), such that eq. (10) is satisfied, and this portion of the streamline corresponds to a free surface of the blood flow. In particular, the function

\[
f = e^{\phi} = e^x \cos y - ie^{\phi} \sin y
\]

(12)

obeys eq. (10) for \( \psi = 0 \) and \( \phi > 0 \). The free surface is defined by the equations

\[
\frac{dx}{d\phi} = -f = e^{-\phi}
\]

(13i)

and

\[
\frac{dy}{d\phi} = -\sqrt{1 - f^2} = -\sqrt{1 - e^{-2\phi}} (\phi > 0, \psi = 0)
\]

(13ii)

which integrate to

\[
\frac{x}{\phi} = e^{-\phi} - 1
\]

(14i)

and

\[
y = \sqrt{1 - e^{-2\phi}} - \ln \left( e^\phi \pm \sqrt{e^{2\phi} - 1} \right) (\phi > 0, \psi = 0)
\]

(14ii)

Similarly, the streamline \( \psi = \pi \) also obeys eq. (10) for
\( \phi > 0, \) such that

\[
\frac{dx}{d\phi} = e^{-\phi} \tag{15i}
\]

and

\[
\frac{dy}{d\phi} = -\sqrt{1 - e^{-2\phi}} \left( \phi > 0, \psi = \pi \right) \tag{15ii}
\]

which integrate to

\[
x = -x_0 + 1 - e^{-\phi} \tag{16i}
\]

and

\[
y = \sqrt{1 - e^{-2\phi}} - \ln \left( e^\phi + \sqrt{e^{2\phi} - 1} \right) \left( \phi > 0, \psi = \pi \right) \tag{16ii}
\]

On this streamline \( \frac{dz}{d\phi} \) is real and negative for \( \phi < 0 \), so \( y \) is again constant at 0 while \( x \) decreases from \( -x_0 \) to \( -\infty \).

Furthermore, on the streamline \( \psi = \pi/2, \frac{dz}{d\phi} \) is purely imaginary so that \( x \) is constant (at \( -x_0/2 \)), and this streamline is the symmetry axis of the flow.

The constant \( x_0 \) can be determined by noting that for large negative \( y \) the streamlines \( 0 \leq \psi \leq \pi \) are all in the \( y \) direction, so the corresponding equipotentials of \( \phi \) are at constant \( y \). Then, the velocity \( V = -\nabla \phi \) is constant across the jet, with value unity, such that integrating across the jet at large negative \( y \) find, using the Cauchy-Riemann eq.(6)

\[
\Delta \psi = \pi = \int_{-x_0 + 1}^{-x_0 + 1} \frac{\partial \psi}{\partial x} dx = -\int_{-x_0 + 1}^{-x_0 + 1} \frac{\partial \phi}{\partial y} dx = \int_{-x_0 + 1}^{-x_0 + 1} dx = x_0 - 2 \tag{17}
\]

and hence the width of the mitral orifice is \( x_0 = \pi + 2 \). Thus, we determine the vena contracta to be

\[
A_{vena} = \frac{\pi}{\pi + 2} A_{mitral orifice} = 0.61 A_{mitral orifice} \tag{18}
\]

\( A_{mitral orifice} \), is effective regurgitant orifice area (EROA), because this is the case when the orifice is considered as a circular aperture but actually mitral orifice is not circular.

If VCW is represented as \( W_{vena} \) and mitral orifice diameter is \( d_{mitral orifice} \) and we know that

Table 2. MR Severity Checking On the Basis of EROA

| Patient | Effective Regurgitant Orifice Area (EROA) in cm² (by Proposed method) | MR Severity by Proposed EROA method | MR Severity by Clinicians |
|---------|---------------------------------------------------------------|------------------------------------|---------------------------|
| 1       | 0.43                                                          | severe                             | severe                    |
| 2       | 0.47                                                          | severe                             | severe                    |
| 3       | 0.26                                                          | moderate                           | moderate                  |
| 4       | 0.39                                                          | moderate-severe                    | severe                    |
| 5       | 0.26                                                          | moderate                           | moderate                  |
| 6       | 0.39                                                          | moderate-severe                    | severe                    |
| 7       | 0.37                                                          | moderate-severe                    | severe                    |
| 8       | 0.37                                                          | moderate-severe                    | severe                    |
| 9       | 0.20                                                          | moderate                           | moderate                  |

\[ \pi W_{vena}^2 = 0.61 \pi d_{mitral orifice}^2 \tag{19} \]

\[ W_{vena}^2 = \left(0.78 d_{mitral orifice}\right)^2 \tag{20} \]

\[ d_{mitral orifice} = \frac{W_{vena}}{0.78} \tag{21} \]

3. Results

Table 1 shows the values of vena contracta width of different echocardiographic images which are taken from the patients suffering from mitral regurgitation. These results are obtained from automatic vena contracta width detection method. The results corroborate quite closely with the severity grades demined by the clinicians. If the cases 1, 2, 4, 6, 7 and 8 are viewed, the values of vena contracta width are greater than 0.5 cm suggesting that the MR is of the severe grade. Whereas, in cases 3, 5, and 9, the VCWs observed are within 0.4 to 0.5 cm wide. This range of VCW suggests that the MR is of the moderate severity grade. The MR severity evaluation using proposed method corroborates 100% with MR severity gradation carried out by the clinicians.

Table 1. MR Severity Checking On the Basis of VCW

| Patient | Vena Contracta width (in pixels) by Automatic VC detection method | Vena Contracta width (cm) by Automatic VC detection method | MR Severity (Automatic VC width method) | MR Severity (by Clinicians) |
|---------|---------------------------------------------------------------|---------------------------------------------------------------|------------------------------------|---------------------------|
| 1       | 13                                                           | 0.58                                                          | severe                             | severe                    |
| 2       | 14                                                           | 0.61                                                          | severe                             | severe                    |
| 3       | 8                                                            | 0.46                                                          | moderate                           | moderate                  |
| 4       | 12                                                           | 0.56                                                          | severe                             | severe                    |
| 5       | 8                                                            | 0.46                                                          | moderate                           | moderate                  |
| 6       | 12                                                           | 0.56                                                          | severe                             | severe                    |
| 7       | 11                                                           | 0.54                                                          | severe                             | severe                    |
| 8       | 11                                                           | 0.54                                                          | severe                             | severe                    |
| 9       | 9                                                            | 0.39                                                          | moderate                           | moderate                  |

\[ A_{vena} = \frac{\pi}{\pi + 2} A_{mitral orifice} = 0.61 A_{mitral orifice} \tag{18} \]

Figure 1. Grading result by VCW. The criteria for grading severity are: Mild VCW < 0.3 cm, Moderate 0.3 cm to 0.49 cm, Severe VCW ≥ 0.5 cm
Grading of results by EROA. The criterion for severity grading are, mild EROA<0.2cm², moderate EROA 0.2-0.29, 0.3-0.39cm², Severe EROA ≥ 0.4 cm²

From the values of vena contract a width obtained from the table 1, EROA can be calculated using our proposed method. Results are shown in table 2. We know that EROA below 0.2cm² shows the mid MR, however between 0.2cm² to 0.4cm² MR is categorized as moderate MR. If this value goes beyond 0.4cm², than MR will be sever MR. here we create a more section for severity grading is moderate-severe.

4. Statistical Analysis of Results

The grading result of MR, by VCW is shown by Fig.1. The grading result is that out of nine patients 3 patients are graded as moderate (0.437±0.023cm) and 6 patients as severe (0.565±0.045cm).

The grading result of MR, by EROA is shown by Fig.2. The grading result is that out of nine patients 3 patients are graded as moderate (0.24±0.02cm) and 4 patients as moderate-severe (0.38±0.01cm), and 2 patients as severe (0.45±0.02cm).grading by clinician and severity grading by automatic effective regurgitant orifice area. The result reveals that severity grading by EROA correlates significantly to severity grading by clinician (r² =0.7273).

Fig.5 shows the correlation between vena contract a width and effective regurgitant orifice area. There is a good correlation between EROA and vena contract width (r² =0.9874).

5. Conclusion

Vena contract a diameter is directly related to the effective regurgitant orifice area (EROA). Since, the algorithm of the proposed method is quit fast, the multiple images during the systole can be images and dynamicity of EROA can be examined. Statistical results shows good correlation between severity grading proposed by clinicians and by the automatic VCW method and EROA method.

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