Small-$Q^2$ extension of DGLAP-constrained Regge residues

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Abstract

In a previous paper, we have shown that it was possible to use the DGLAP evolution equation to constrain the high-$Q^2$ ($Q^2 \geq 10$ GeV$^2$) behaviour of the residues of a high-energy Regge model, and we applied the developed method to the triple-pole pomeron model. We show here that one can obtain a description of the low-$Q^2$ $\gamma^{(*)}p$ data matching the high-$Q^2$ results at $Q^2 = 10$ GeV$^2$.

We know that one can use Regge theory \cite{1} to describe high-energy hadronic interactions. Particularly, using a triple-pole pomeron model \cite{2,3,4}, one can reproduce the hadronic total cross-sections, the $\gamma p$ and $\gamma\gamma$ cross-sections, and also the proton and photon structure functions $F_2^p$ and $F_2^\gamma$. In the latter case, one must point out that Regge theory is applied at all values of $Q^2$.

On the other hand, it is well known that the high-$Q^2$ behaviour of the proton structure function can be reproduced using the DGLAP evolution equation \cite{5}. Therefore, we would like to find a model compatible both with Regge theory and with DGLAP evolution at high $Q^2$. We have shown \cite{6} that it is possible to extract the behaviour of the triple-pole pomeron residues at high $Q^2$ from DGLAP evolution. In such an analysis, we need information not only on $F_2$ but also on parton distributions. One easily shows that the minimal number of quark distributions needed to reproduce $F_2^p$ is 2: one flavour-non-singlet distribution

$$T(x,Q^2) = x \left[ (u^+ + c^+ + t^+) - (d^+ + s^+ + b^+) \right],$$

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with \( q^+ = q + \bar{q} \), evolving alone with \( xP_{qq} \) as splitting function, and one flavour-singlet distribution

\[
\Sigma(x, Q^2) = x \left[ (u^+ + c^+ + t^+) + (d^+ + s^+ + b^+) \right],
\]

coupled with the gluon distribution \( xg(x, Q^2) \) and evolving with the full splitting matrix. Before going into the main subject of this paper, we shall summarise the techniques developed in this previous paper [6] and show how we can extend the results down to \( Q^2 = 0 \).

First of all, given that \( F_2 \) can be parametrised at small \( x \) by a \( \log^2(1/x) \) term, we have parametrised the quark content of the proton in the most natural way i.e. using a triple-pole pomeron term and an \( f/a_2 \) reggeon terms. After a few manipulations, we end up with the following functions

\[
T(x, Q_0^2) = d^τ x^η (1-x)^{b_2},
\]

\[
Σ(x, Q_0^2) = a_Σ \log^2(1/x) + b_Σ \log(1/x) + c_Σ^*(1-x)^{b_1} + d_Σ x^η (1-x)^{b_2},
\]

\[
xg(x, Q_0^2) = a_G \log^2(1/x) + b_G \log(1/x) + c_G^*(1-x)^{b_1}. \tag{1}
\]

Since Regge theory does not extend up to \( x = 1 \), we used the GRV parametrisation for \( x \geq x_{\text{Regge}} = 0.15 \) and imposed that our distributions match GRV’s at \( x = x_{\text{Regge}} \). This requirement constrains the parameters marked with a superscript * in eq. (1). Thus, the 7 parameters \( a_Σ, b_Σ, d_Σ, a_G, b_G, b_1 \) and \( b_2 \) need to be extracted from DGLAP evolution.

Since DGLAP evolution generates an essential singularity in the complex-\( j \) plane at \( j = 1 \), the only place where we can use the Regge model is in the initial distributions at \( Q^2 = Q_0^2 \). In such a case, we shall not worry about the presence of an essential singularity for \( Q^2 \neq Q_0^2 \) and consider the result of DGLAP evolution as a numerical approximation to a triple-pole pomeron. One can therefore extract the residues of the Regge model at high \( Q^2 \) using the following method:

1. choose an initial scale \( Q_0^2 \),
2. choose a value for the parameters in the initial distribution,
3. compute the parton distributions for \( Q_0^2 \leq Q^2 \leq Q_{\text{max}}^2 \) using forward DGLAP evolution and for \( Q_{\text{min}}^2 \leq Q^2 \leq Q_0^2 \) using backward DGLAP evolution,
4. repeat \( 2 \) and \( 3 \) until the value of the parameters reproducing the \( F_2 \) data for \( Q^2 > Q_{\text{min}}^2 \) and \( x \leq x_{\text{Regge}} \) is found.
5. This gives the residues at the scale $Q_0^2$ and steps 1 to 4 are repeated in order to obtain the residues at all $Q^2$ values.

We have applied this method to the parametrisation (1) within the domain
\[
\begin{cases}
10 \leq Q^2 \leq 1000 \text{ GeV}^2, \\
\cos(\theta_t) = \frac{\sqrt{Q^2}}{2m_p} \geq \frac{49 \text{ GeV}^2}{2m_p},
\end{cases}
\]
(2)
ensuring that both Regge theory and DGLAP evolution can be applied, and required $x < 0.15$. Using the residues of the triple-pole pomeron obtained in this way, we have a description of $F_2^n$ for $Q^2 \geq 10$ GeV$^2$ with a $\chi^2/\text{nop}$ of 1.02 for 560 experimental points.

Since the method explained here gives us the Regge residues at large scales, one may ask if it is possible to extend the results down to $Q^2 = 0$. The main problem here is that, instead of using $x$ and $Q^2$, we must use $\nu$ and $Q^2$ if we want to obtain a relevant expression for the total cross section. Of course, we shall only extend the $F_2^n$ predictions instead of the parton distributions $T$ and $\Sigma$.

As a starting point, we shall not consider the powers of $(1-x)$ since, at low $Q^2$, there are no point inside the Regge domain beyond $x = 0.003$, which means that it is just a correction of a few percents. At low $Q^2$, we require that $F_2^n$ has the same form as used in [2]
\[
F_2(\nu, Q^2) = \frac{Q^2}{4\pi^2 \alpha_e} \left\{ A(Q^2) [\log(2\nu) - B(Q^2)]^2 + C(Q^2) + D(Q^2)(2\nu)^{-\eta} \right\}.
\]
(3)
The total $\gamma p$ cross-section is then
\[
\sigma_{\gamma p} = A(0) [\log(s) - B(0)]^2 + C(0) + D(0)s^{-\eta}.
\]
(4)
At $Q^2 = Q_0^2$, the form factors $A$, $B$, $C$ and $D$ are related to the parametrisation (1) by the relations
\[
\begin{align*}
A(Q_0^2) &= \frac{4\pi^2 \alpha_e}{Q_0^2} a_0, \\
B(Q_0^2) &= \log(Q_0^2) - \frac{b_0}{2a_0}, \\
C(Q_0^2) &= \frac{4\pi^2 \alpha_e}{Q_0^2} \left( c_0 - \frac{b_0^2}{4a_0} \right), \\
D(Q_0^2) &= \frac{4\pi^2 \alpha_e}{Q_0^2} d_0(Q_0^2)^{\eta}.
\end{align*}
\]
(5)
\footnote{This limit is only effective at large $Q^2$.}
where the subscript 0 to refer to the form factors obtained at $Q^2 = Q^2_0$ from
DGLAP evolution.

At small $Q^2$, the unknown functions $A$, $B$, $C$ and $D(Q^2)$ are parametrised
in the same way as in [2]

\begin{align*}
A(Q^2) &= a \left( \frac{Q^2}{Q^2_a + Q^2} \right)^{\varepsilon_a}, \\
B(Q^2) &= b \left( \frac{Q^2}{Q^2 + Q^2_b} \right)^{\varepsilon_b} + A', \\
C(Q^2) &= c \left( \frac{Q^2}{Q^2 + Q^2_c} \right)^{\varepsilon_c}, \\
D(Q^2) &= d \left( \frac{Q^2}{Q^2 + Q^2_d} \right)^{\varepsilon_d}.
\end{align*}

(6)

If we use the relations [3] to fix the parameters $A_a$, $A_b$, $A_c$ and $A_d$ in [6], we find the final form of the small-$Q^2$ form factors:

\begin{align*}
A(Q^2) &= \frac{4\pi^2 \alpha_e}{Q^2_0} a \left( \frac{Q^2_0 + Q^2}{Q^2_a + Q^2} \right)^{\varepsilon_a}, \\
B(Q^2) &= \log(Q^2_0) - \frac{b_0}{2a_0} + A_b \left( \left( \frac{Q^2}{Q^2 + Q^2_b} \right)^{\varepsilon_b} - \left( \frac{Q^2_0}{Q^2_0 + Q^2} \right)^{\varepsilon_b} \right), \\
C(Q^2) &= \frac{4\pi^2 \alpha_e}{Q^2_0} \left( c_0 - \frac{b_0^2}{4a_0} \right) \left( \frac{Q^2_0}{Q^2 + Q^2} \right)^{\varepsilon_c}, \\
D(Q^2) &= \frac{4\pi^2 \alpha_e}{Q^2_0} d_0(Q^2_0)^{\eta} \left( \frac{Q^2_0 + Q^2}{Q^2_a + Q^2} \right)^{\varepsilon_d}.
\end{align*}

(7)

If we now want to reinsert the large-$x$ corrections, we need to multiply
$c$ and $d$ by some power of $(1-x)$. This gives

\begin{align*}
\frac{4\pi^2 \alpha_e}{Q^2} F_2(x, Q^2) &= A(Q^2) \log(1/x) \left\{ \log(1/x) + 2 \left[ \log(Q^2) - B(Q^2) \right] \right\} \\
&\quad + \left\{ A(Q^2) \left[ \log(Q^2) - B(Q^2) \right]^2 + C(Q^2) \right\} (1-x)^{b_1} \\
&\quad + D(Q^2) \left( \frac{Q^2}{x} \right)^{-\eta} (1-x)^{b_2}.
\end{align*}

These large-$x$ corrections do not modify the expression of the total cross
section since, when $Q^2 \to 0$

\begin{align*}
1 - x &= 1 - \frac{2\nu}{Q^2} \to 1.
\end{align*}
| Parameter | value   | error    |
|-----------|---------|----------|
| $A_b$     | 69.151  | 0.055    |
| $Q^2_a$   | 25.099  | 0.088    |
| $Q^2_b$   | 4.943   | 0.086    |
| $Q^2_c$   | 0.002468| 0.000042 |
| $Q^2_d$   | 0.01292 | 0.00074  |
| $\varepsilon_a$ | 1.5745 | 0.0046    |
| $\varepsilon_b$ | 0.08370 | 0.00052   |
| $\varepsilon_c$ | 0.92266 | 0.00019   |
| $\varepsilon_d$ | 0.3336 | 0.0029    |

Table 1: Values of the parameters for the low-$Q^2$ fit ($0 \leq Q^2 \leq Q^2_0$).

| Experiment | $n$ | $\chi^2$ | $\chi^2/n$ |
|-----------|-----|----------|------------|
| E665      | 69  | 59.811   | 0.867      |
| H1        | 99  | 104.924  | 1.060      |
| NMC       | 37  | 28.392   | 0.767      |
| ZEUS      | 216 | 201.790  | 0.934      |
| $F^p_2$   | 421 | 394.916  | 0.938      |
| $\sigma_{\gamma p}$ | 30 | 17.171 | 0.572 |
| Total     | 451 | 412.086  | 0.914      |

Table 2: $\chi^2$ resulting from the small-$Q^2$ Regge fit. The results are given for all $F^p_2$ experiments and for the total cross-section.

Moreover, since the large-$x$ corrections are only a few percents effects, we shall keep the exponents $b_1$ and $b_2$ constant and equal to their value at $Q^2 = Q^2_0$.

Now, we may adjust the parameters in the form factors by fitting $F^p_2$ in the Regge domain

\[
\begin{align*}
\nu & \geq 49 \text{ GeV}^2, \\
\cos(\theta_t) & = \sqrt{\frac{Q^2}{2m_p}} \geq \frac{49 \text{ GeV}^2}{2m_p}, \\
Q^2 & \leq 10 \text{ GeV}^2,
\end{align*}
\]

(8)

together with the total cross-section for $\sqrt{s} \geq 7$ GeV. The resulting parameters are presented in Table 2 and the form factor are plotted in Figure 1.
As we can see from Table 2 and from Figures 2 and 3, this gives a very good extension in the soft region (see Table 2).

To conclude, we have seen that, using a triple-pole-pomeron model, one can obtain a description of the $\gamma^*(p)$ interactions at all values of $Q^2$ compatible with the DGLAP equation at large $Q^2$. It should be interesting, in the future, to test this method with other Regge models and to see if the results are compatible with the $t$-channel unitarity relations obtained in [7] and if they can give useful information on how to link perturbative and non-perturbative QCD.

Acknowledgments
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Figure 3: Fit for the $F_2^b$ at low $Q^2$. Only the most populated $Q^2$ bins are shown.