A method of constitutive experiment for accurate description of tube performance under rotary-draw bending

Y X Zhu\textsuperscript{1,3}, R Chen\textsuperscript{4}, Y Wang\textsuperscript{4}, W B Tu\textsuperscript{4} and Y L Liu\textsuperscript{2}

\textsuperscript{1} School of Mechanical Engineering, Jiangsu University, Zhenjiang 212013, China
\textsuperscript{2} School of Materials Science and Engineering, Northwestern Polytechnical University, Xi'an 710072, China

E-mail: xia166109@163.com; xia166109@ujs.edu.cn

Abstract. Establishing accurate constitutive relations is an important prerequisite for improving the springback prediction accuracy of rotary-draw bending of tubes. In this paper, the stress loading paths are studied for rectangular tube and composite circular tube in rotary-draw bending processes. It is found that there is reverse loading with tangential/circumferential stress for both the rectangular tube and the base/covered tube of composite circular tube. And the reverse loading is in the plastic deformation range. Two constitutive experimental schemes are designed according to the above research results: (1) The constitutive experiment is designed according to the variation of stress peak points of the tube with the bending process and the yield state of the material. The calculation method of the loading force $F$ is given, too. (2) The displacement of the beam of the tensile tester is determined according to the corresponding strain during stress fluctuations, and an example is given to illustrate how the displacement of the beam is calculated according to the strain corresponding to each trajectory point. The calculation formula of the beam displacement during repeated loading, unloading and reverse loading is developed, too.

1. Introduction

The accuracy of finite element simulation depends largely on the accuracy of material constitutive relations. Establishing accurate constitutive relations is an important prerequisite for improving the accuracy of springback prediction. Therefore, in recent years, material constitutive model has always been a hot research topic. The study of constitutive model of tubes is an important branch of material constitutive model research. For example, authors [1, 2] studied the springback and torsional deformation of asymmetric aluminum tubes during rotational stretching and bending process. The springback prediction results of different constitutive models were compared. The constitutive model describing material anisotropy and the Bauschinger effect was established to improve the simulation accuracy of rotational stretching and bending of the asymmetric aluminum tube. In order to accurately predict the material flow characteristics of TC4 tube during hot rotating bending, TAO et al. [3] compared the improved Arrhenius constitutive model with the artificial neural network constitutive model, and found that the prediction accuracy of artificial neural network constitutive model is higher under certain deformation conditions.

The rotary-draw bending is considered to be one kind of tube forming method of high-efficiency and high-precision [4], and there are many researches about the filling mandrels of rotary-draw bending [5-8]. According to study [9], due to the effect of the filled mandrel, the bending forming process of the rectangular tube has a complex deformation history, such as stress loading, reverse loading, cyclic
loading and unloading, and the reverse loading is within the plastic forming range. Therefore, it is necessary to design a constitutive experiment that is closest to the material stress loading path during the bending process, in order to describe the constitutive relationship more accurately. That is, a uniaxial tension-compression constitutive experimental design must be performed. Moreover, the stress loading paths are also different due to the difference of cross-sectional shape and process conditions [10]. Therefore, this paper studies the bending forming processes of rectangular tube and bimetal composite circular tube, analyzes the stress loading paths under the conditions of filling mandrel, and proposes two reasonable constitutive experimental design schemes.

2. Establishment of Finite element (FE) model

2.1. The FE model of rotary-draw bending of rectangular tube

The rectangular tube with geometry of 24.86 (width) × 12.2 (height) × 1 (thickness) mm and material of H96 is taken as the research object. The FE model of rectangular H96 tube NC bending is established and verified by the experimental results of springback and sectional deformation in [9, 10].

2.2. The FE model of rotary-draw bending of composite circular tube

The TA2/T2 copper-titanium composite circular tube with geometrical dimensions of 20 (outer diameter of covered tube) × 1 (thickness of covered tube) × 14 (inner diameter of base tube) mm is taken as the research object. The FE model of rotary-draw bending of the TA2/T2 composite circular tube is established and verified by the experiment, as shown in figure 1. The detailed comparison of experimental and simulative conditions can be seen in table 1. Result shows that the average prediction error of the springback angle is 4.16%, and the maximum prediction error is 12.79%, so the established FE model is reliable.

Table 1. Forming conditions of composite circular tube in experiment and simulation.

| Processing parameters | Experiment | Simulation |
|-----------------------|------------|------------|
| Bending angular velocity /rad s⁻¹ | 0.5 | 0.5 |
| Boosting velocity /mm s⁻¹ | 30 | 30 |
| Clearances between tube and dies /mm | Clamping or Chucking | 0 or 0.1 |
| Bending radius /mm | 60 | 60 |
| Friction between tube and clamp die | Dry friction | Rough |
| Friction between tube and other dies | Dry friction | Coulomb friction coefficient 0.17 |

Figure 1. FE model of rotary-draw bending of copper-titanium composite tube and its verification: (a) the FE model and (b) the comparison of springback prediction value and experimental value.
3. Results and discussion

3.1. Stress loading path of rotary-draw bending of rectangular tube

The stress loading paths of M and R nodes during bending process are studied under the filling process conditions of rigid mandrel and three cores. M and R nodes are on the longitudinal symmetry line of 75° cross-section, as shown in figure 2. It can be seen from figure 3 that both the tangential stress and the circumferential stress of M and R nodes have repeated loading-unloading process and the final "longest plastic reverse loading" process. It is known that when the equivalent stress $\bar{\sigma}$ has the following relationship with the subsequent yield stress $Y$,

$$\bar{\sigma} = Y$$  \hspace{1cm} (1)

the material is considered to be in the yielding state, that is, the material undergoes plastic deformation. Therefore, the partial loading-unloading process and the "longest plastic reverse loading" process are both within the range of plastic deformation according to figure 3. The loading-unloading process and the reverse loading process of the material have the variable elastic modulus effect, the Bauschinger effect and the permanent softening behavior [11]. Therefore, in order to obtain the material constitutive model of rotary-draw bending of rectangular tube, the repeated loading-unloading experiment and the reverse loading experiment should be designed, according to figure 3.

Figure 2. Illustration of the longitudinal section symmetrical lines AB, CD and the position of the two nodes M and R [9].

Figure 3. Variation of the stresses during the bending stage when the number of the cores is three: (a) variation of the stresses on node M, (b) variation of the stresses on node R [9].
3.2. Stress loading path of rotary-draw bending of bimetal composite circular tube

The cross-sectional shape of the circular tube is different from that of the rectangular tube, so their stress loading paths as well as the uneven deformation during the plastic deformation process are inevitably different. The stress loading of the nodes in the 45° cross-section of the composite circular tube are studied under the filling process conditions of rigid mandrel and three cores. The half section of the 45° cross-section is divided from 0 to 180° from the lower bending layer to the upper bending layer, and the corresponding nodes are named \( n_0, n_1, n_2 \ldots n_{180} \), as shown in figure 4.

**Figure 4.** Half of the 45° cross-section.

Figure 5 shows the variation of stresses in nodes \( n_0, n_{36} \ldots \ldots n_{180} \) of the covered tube with the bending process. It can be seen in figure 5(a) that the tangential stress of the covered tube changes from compressive in the lower bending layer to tensile in the upper one, and the tangential stress in node \( n_{72} \) implies a reverse loading process. Figure 5(b) gives more insight into the equivalent stress and the subsequent yield stress in node \( n_{72} \). It implies that the reverse loading of the tangential stress occurs in the plastic deformation range. It can be seen from figure 5(c) that the circumferential stresses of many nodes on the cover tube have large reverse loading processes. Then the circumferential stress on \( n_{180} \) is selected for further study, and a significant reversal also can be seen, which is within the range of plastic deformation, as seen in figure 5(d).
Figure 5. Variation of the stress on 45° cross-section of the covered tube during bending process: (a) tangential stresses in \( n_0, n_{58}, n_{180} \), (b) tangential stress, equivalent stress and subsequent yield stress in node \( n_{72} \), (c) circumferential stresses in nodes \( n_{72}, n_{90}, n_{180} \), (d) circumferential stress, equivalent stress and subsequent yield stress in node \( n_{180} \).

Figure 6 shows the variation of the stresses with the bending process, which are on the representative nodes \( n_0, n_{58}, \ldots, n_{180} \) of the base tube. It can be seen from figure 6(a) that the tangential stress from the lower bending layer to the upper bending layer changes its sign from compressive to tensile one, which is similar to the covered tube case. But the reverse loading amplitudes of tangential stresses are more obvious for the nodes on base tube, such as \( n_{58}, n_{122} \) and \( n_{180} \). Figure 6(b) depicts the tangential stress, equivalent stress, and subsequent yield stress of \( n_{58} \). It is found that the reverse loading of the tangential stress of \( n_{58} \) is in the plastic deformation range. Figure 6(c) illustrates circumferential stresses of nodes \( n_{58}, n_{122}, n_{180} \), and it also can be seen there are reverse loading with large amplitudes. Figure 6 (d) proves that the reverse loading of circumferential/tangential stresses of partial nodes are plastic.

Based on the analysis results in figures 5 and 6, it is known that there are reverse loading processes of tangential/circumferential stress for both the base tube and the covered tube of the composite circular tube. Therefore, the reverse loading experiment should be designed according to figures 5 and 6 to support the material constitutive experiment on the composite circular tube.
3.3. Constitutive experimental design

There are two loading methods for material constitutive experiments: (1) based on force-time loading and (2) based on displacement-time loading.

According to the above studies, there are many nodes on the tube wall experiencing reverse loading, such as the nodes M and R on rectangular tube, the nodes n_{72}, n_{90}, n_{180} on covered tube, and the nodes n_{58}, n_{86}, n_{122} and n_{180} on base tube. At this time, the nodes with the largest reverse loading amplitude and maximum number of repeated loading-unloading should be selected as the reference object to the constitutive experiment design, such as nodes n_{72}, n_{180} on covered tube and nodes n_{58}, n_{180} on base tube.

3.3.1. Constitutive experimental design according to the stress. The force-time loading method is adopted. And then the loading path of constitutive experiment should be designed according to the variation of the stress peak point with the bending process and the yield state of the material.

Some nodes have experienced a loading path that is stretched first and then compressed, while some nodes have experienced a loading path that is compressed first and then stretched. It is deemed by some literatures [12] that the stress-strain curves obtained by the two loading paths are quite different, so such nodes are usually selected as the comparison objects and the reference points for the design of constitutive experiment.

Taking the rotary-draw bending of rectangular tube in Section 3.1 as an example, the fluctuation of the tangential stress peak with time can be obtained according to figure 3(a), as shown in figure 7(a).

Therefore, the constitutive experiment loading path of the rectangular tube bending can be designed as follows,

\[ a \leftarrow b \leftarrow c \leftarrow d \leftarrow e \leftarrow f \leftarrow g \leftarrow h \leftarrow i \leftarrow j \]  

(2)

where "co, st, rl and ul" represent compression, stretching, reverse loading, and unloading processes, respectively. Points a, b, c,.j are the stress peak points in figure 7. When the reverse loading occurs in the elastic range, its influence on the material state is small, and the material can be considered to be in an unloaded state to reduce the complexity of the experiment.

In addition, some of the nodes in the rectangular tube also undergo the stress loading path shown in figure 7(b). Therefore, the constitutive experiment should also include the following loading path:

\[ a \leftarrow b \leftarrow c \leftarrow d \leftarrow e \leftarrow f \leftarrow g \leftarrow h \leftarrow i \leftarrow j \]  

(3)
The calculation formula for the loading force $F$ is as follows,

$$F = \frac{\sigma_t}{\varepsilon_t} \cdot A_0$$

(4)

where $A_0$ is the original cross-sectional area of the specimen, $\sigma_t$ is the value of tangential stress of the peak points $a$, $b$, $c$...$j$ in figure 7, and $\varepsilon_t$ is the tangential strain corresponding to $\sigma_t$ in figure 7.

**Figure 7.** The stress loading path corresponding to figure 3: (a) in node M and (b) in node R.

3.3.2. **Constitutive experimental design according to the strain.** The displacement-time loading method is adopted, and then the beam displacement of the tensile tester is determined according to the corresponding strain of stress fluctuation. The rotary-draw bending of bimetal composite circular tube in Section 3.2 is taken as the example. Referring to the data of figure 6(d), the tangential stress and strain in node $n_{190}$ of figure 8 have undergone the loading process $1\Delta \varepsilon_{t1} 2\Delta \varepsilon_{t2} 3\Delta \varepsilon_{t3} 4\Delta \varepsilon_{t4} 5\Delta \varepsilon_{t5}$, and the circumferential stress and strain have experienced the loading trajectory $a\Delta \varepsilon_{c1} b\Delta \varepsilon_{c2} c\Delta \varepsilon_{c3} d\Delta \varepsilon_{c4}$. The strains corresponding to each track point such as 1, 2 ... 5 and a, b ... d are listed in table 2. Assuming that $L_0$ is the gauge length of the specimen, $\Delta l$ is the displacement of the beam, the calculation formula for the beam displacement $\Delta l_i$ during the repeated loading, unloading, and reverse loading is as follows:

$$\Delta l_i = \begin{cases} 
(1-e^{\Delta \varepsilon_i})
L_0 + \sum_{i=1}^{i} \Delta l_{i-1} & \text{Stretching} \\
(e^{\Delta \varepsilon_i} - 1)
L_0 + \sum_{i=1}^{i} \Delta l_{i-1} & \text{Compressing}
\end{cases}$$

(5)

where subscript $i$ denotes the $i$-th step loading process, $\Delta \varepsilon_i$ is the strain difference between the $i$-th step and $i-1$-th step. The beam displacement is calculated according to equation (5), and the results are shown in table 2. Thus, the constitutive experiment is designed.
Figure 8. The stress loading path corresponding to figure 6: node n_{180} of base tube.

Table 2. Constitutive experimental design according to the node n_{180} of base tube.

| Experiment design according to the | Strain difference $\Delta \varepsilon_i$ | Beam displacement $\Delta l_i$ |
|-----------------------------------|------------------------------------------|--------------------------------|
| tangent stress in figure 8         |                                          |                                |
| 1—2/i=1                          | 0.042                                    | Stretching to 0.043$L_0$, then unloading; |
| 2—3/i=2                          | 0.026                                    | Compression to -0.027 $L_0$, then unloading; |
| 3—4/i=3                          | 0.035                                    | Stretching to 0.036 $L_0$, then unloading; |
| 4—5/i=4                          | 0.099                                    | Compression to -0.109 $L_0$, then unloading. |
| Experimental design according to the |                                          |                                |
| circumferential stress in figure 8 |                                          |                                |
| a—b/i=1                          | 0.048                                    | Compression to -0.049 $L_0$, then unloading; |
| b—c/i=2                          | 0.037                                    | Stretching to 0.036 $L_0$, then unloading; |
| c—d/i=3                          | 0.046                                    | Compression to -0.046 $L_0$, then unloading. |

4. Conclusions

(1) The representative nodes of the rectangular and composite tubes are selected as the research objects, and the loading paths of both tubes during the bending forming process were studied under the mandrel-core filling condition. Reverse loading phenomena were revealed for both the tangential and circumferential stresses of rectangular and covered/base composite tubes, which occurred in the plastic deformation range. Therefore, the constitutive experiments of rectangular tube and composite tube should be designed according to their reverse loading paths.

(2) Two constitutive experimental schemes, namely (i) force-time and (ii) displacement-time loadings, were designed based on the above research results.

(3) In the first scheme, the loading path of the constitutive experiment should be designed according to the variation of the stress peak point with the bending process and the yield state of the material, which is presented in detail for the example of rectangular tube. The calculation method of loading force $F$ is developed too.

(4) In the second scheme, the beam displacement of the tensile tester is determined according to the corresponding strain of stress fluctuation, which is presented in detail for the example of composite tube. The calculation formula for the beam displacement during repeated loading, unloading, and reverse loading was derived.

Acknowledgments
The research is financially supported by the National Natural Science Foundation of China (No. 51601070), Natural Science Foundation of Jiangsu Province (No. BK20181447), Postdoctoral Science
Foundation of Jiangsu Province (No. 1501099B), and Senior Talent Foundation of Jiangsu University of China (No. 14JDG135).

References
[1] Liao J, Xue X, Lee M, Barlat F and Gracio J 2014 Int. J. Mech. Sci. 89 311-22
[2] Xue X, Liao J, Vincze G and Gracio J 2015 J. Mater. Process. Tech. 216 405-17
[3] Tao Z J, Yang H, Li H, Ma J and Gao P F 2016 Rare Metals 35 162-71
[4] Sozen L, Guler M A, Bekar D and Acar E 2012 J. Mech. Eng. Sci. 226 2967-81
[5] Li H, Xu J, Yang H, Yang H and Li G J 2017 T. Nonferr. Metals Soc. China 27 608-15
[6] Zhan M, Zhu Q, Shi F, Wang X X and Yang H 2014 Precis. Forming Eng. 31-6
[7] Zhang H L, Liu Y L and Yang H 2016 Int. J. Adv. Manuf. Tech. 82 1569-80
[8] Ancellotti S, Benedetti M, Fontanari V, Slaghenaufi S and Tassan M 2016 Int. J. Adv. Manuf. Tech. 85 1089-103
[9] Zhu Y X, Liu Y L and Yang H 2015 Int. J. Adv. Manuf. Tech. 78 351-60
[10] Zhu Y X, Liu Y L, Li H P and Yang H 2013 Int. J. Mech. Sci. 76 132-143
[11] Yoshida F, Uemori T and Fujiwara K 2002 Int. J. Plast. 18 633-659
[12] Huang T, Zhan M and Yang H 2011 Proc. Annual Meeting of Plastic Processing Theory and Digital Technology of Plastic Engineering Society