$O(\alpha_s)$ Corrections to $B \to X_s e^+ e^-$ Decay in the 2HDM.

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Abstract

$O(\alpha_s)$ QCD corrections to the inclusive $B \to X_s e^+ e^-$ decay are investigated within the two - Higgs doublet extension of the standard model (2HDM). The analysis is performed in the so - called off -resonance region; the dependence of the obtained results on the choice of the renormalization scale is examined in details. It is shown that $O(\alpha_s)$ corrections can suppress the $B \to X_s e^+ e^-$ decay width up to $1.5 \div 3$ times (depending on the choice of the dilepton invariant mass $s$ and the low - energy scale $\mu$). As a result, in the experimentally allowed range of the parameter space, the relations between the $B \to X_s e^+ e^-$ branching ratio and the new physics parameters are strongly affected. It is found also that though the renormalization scale dependence of the $B \to X_s e^+ e^-$ branching is significantly reduced, higher order effects in the perturbation theory can still be nonnegligible.
1. Rare B - meson decays can serve as an important source of information on new physics beyond the standard model. The first experimental evidence for these decays has been observed by CLEO \cite{1} for the exclusive \( B \rightarrow K^*\gamma \) channel. Later on the branching ratio of the inclusive \( B \rightarrow X_s\gamma \) decay has been measured by CLEO, ALEPH and Belle collaborations \cite{2}-\cite{4}. The weighted average for this decay branching is \cite{5}

\[
B^{\text{exp}}(B \rightarrow X_s\gamma) = (2.96 \pm 0.35) \times 10^{-4}
\]

Recently the evidence for the rare exclusive channel \( B \rightarrow K\mu^+\mu^- \) has been also observed \cite{6}.

The experimental result for the \( B \rightarrow X_s\gamma \) branching is in a good agreement with the SM predictions (see \cite{6} and references therein). The new physics contribution to this decay width must be small enough to avoid the contradiction with the experiment, hence studying \( B \rightarrow X_s\gamma \) decay one can get some constraints on the new physics parameters.

The another popular inclusive rare B decay mode, \( B \rightarrow X_s\mu^+\mu^- \), has not been observed yet (only upper bound on its branching ratio exists \cite{6}), however it is expected to be measured during the forthcoming experiments at the B - factories. Then, analogously to \( B \rightarrow X_s\gamma \), the study of \( B \rightarrow X_s\mu^+\mu^- \) can provide some information on the physics, which occur above the scale \( \sim 100\text{GeV} \).

2. The \( B \rightarrow X_s\mu^+\mu^- \) decay has been studied within the standard model and its extensions in \cite{7}-\cite{9} and \cite{10}-\cite{12} respectively. In the latter works it has been shown that new physics contribution can make \( B \rightarrow X_s\mu^+\mu^- \) branching ratio two and more times larger than in the SM. These calculations have been performed in the next-to-leading order (NLO) of the perturbation theory, which includes \( O(\alpha_s^{-1}) \) and \( O(1) \) contribution to \( B \rightarrow X_s\mu^+\mu^- \) decay. On the other hand, it has been proven in \cite{6} that to this order the obtained results may suffer from the uncertainties, connected with those in the choice of the low-energy scale \( \mu \sim m_b \) and the heavy mass (matching) scale \( \mu_W \sim M_W, m_t \).

Recently the \( O(\alpha_s) \) corrections to the \( B \rightarrow X_s\mu^+\mu^- \) decay branching have been calculated in \cite{13}-\cite{14}. The calculations have been performed in the off-resonance region, corresponding to \( 0.05 < \hat{s} < 0.25 \), where \( \hat{s} \) is the dilepton invariant mass, normalized over the b - quark mass. It has been found that within the standard model these corrections reduce the above-mentioned uncertainties about two times.

The aim of the present paper is studying the impact of the \( O(\alpha_s) \) corrections on the \( B \rightarrow X_s\mu^+\mu^- \) decay rate in the two-Higgs doublet extension of the standard model (2HDM). The investigation is carried out for the most general version of the 2HDM (so-called Model III), where sizable deviations from the SM result are possible. The deviations from the SM predictions (called new physics effects) occur due to diagrams with the charged Higgs boson mediated loops. It is shown that when the contribution of these diagrams is sizable, \( O(\alpha_s) \) contribution suppresses the \( B \rightarrow X_s\mu^+\mu^- \) decay width up to \( 1.5 \div 3 \) times, depending on the choice of the dilepton invariant mass and the low - energy scale. As a result, in the experimentally allowed region of the parameter space the behavior of the \( B \rightarrow X_s\mu^+\mu^- \) branching ratio with the new physics parameters is changed drastically. It is also pointed out that the dependence of \( B(B \rightarrow X_s\mu^+\mu^-) \) on the parameters of theory can further be modified. While due to the \( O(\alpha_s) \) corrections low - energy scale dependence of the obtained results is very small (or even negligible), the matching scale dependence remains large enough. This indicates that higher order effects in the perturbation theory can be numerically relevant as well.

3. It is known that in the SM the decay \( B \rightarrow X_s\mu^+\mu^- \) is loop-induced: in the lowest order it proceeds via exchange of the up-type quarks and \( W^\pm \) boson in the loops. In the 2HDM there are additional diagrams with \( W \)-boson replaced by the charged Higgs boson (\( H^\pm \)). The interaction of
the charged Higgs boson with quarks may be written in the following form [13]:

\[
\frac{g}{\sqrt{2}M_W} \left( \xi_t m_t H^- (d, s, b)_{L} \left( \begin{array}{c} V_{ts} \\ V_{ts}^* \\ V_{tb} \\ \end{array} \right) t_R + \xi_b m_b H^+ (u, c, t)_{L} \left( \begin{array}{c} V_{td} \\ V_{ts} \\ V_{tb} \end{array} \right) b_R + h.c \right)
\]

(2)

where \( V \) is the CKM matrix, \( g \) is the weak coupling constant, \( m_t, m_b \) are the running top and bottom masses (for the relation between the pole and running quark masses see e.g. [16]), and the parameters \( \xi_t \) and \( \xi_b \) are the functions of the Higgs doublet vev’s, top and bottom masses and the couplings of Yukawa interaction of t- and b- quarks with the Higgs doublets [17]. Due to the Higgs doublet vacuum phase, \( \xi_t \) and \( \xi_b \) are complex in general. Notice also that while \( |\xi_t| \sim 1 \) or smaller, \( |\xi_b| \) can be much larger than unity, unless it contradicts with the experimental constraints on the \( B \to X_{s}\gamma \) branching.

To avoid flavor changing neutral currents (FCNC) in the Lagrangian, one usually considers the simplified versions of the 2HDM: Model I, where only one of the Higgs doublets interacts with quarks and the Model II, where one of the Higgs doublets interacts with the up-type quarks and the second one does with the down-type quarks. In these versions of the 2HDM, \( \xi_t = \xi_b = \cot \beta \) and \( \xi_t = -\cot \beta, \xi_b = \tan \beta \) respectively (\( \tan \beta \) is the Higgs vev’s ratio). Notice however that due to the stringent constraints on the new physics parameters, in the Models I and II the predictions for the \( B \to X_{s}e^+e^- \) decay branching coincide (with \( (10 \div 15)\% \) accuracy) to those of the standard model.

In the present paper the most general version of the 2HDM (Model III), where both of the Higgs doublets interact with both the up-type and down-type quarks, is considered (neglecting possible FCNC’s in the Lagrangian). In this case, due to larger parameter space, \( B \to X_{s}e^+e^- \) branching can be up to three times larger than in the SM. It is worth also to mention that the results derived for the Model III may be also considered valid for multi-Higgs doublet models with only one light charged Higgs boson.

4. The \( B \to X_{s}e^+e^- \) decay is studied, using the effective theory with five quarks obtained by integrating out the heavy degrees of freedom which are W and Z bosons, t-quark and the charged Higgs boson. The effective Hamiltonian for the decay \( B \to X_{s}e^+e^- \) can be written as

\[
H_{eff}(b \to se^+e^-(+g)) = -\frac{4G_F}{\sqrt{2}} \left[ \lambda_t^0 \sum_{i=1}^{10} C_i(\mu)O_i(\mu) - \lambda_u^2 \sum_{i=1}^{2} C_i(\mu)\left(O_i^{u}(\mu) - O_i(\mu)\right) \right]
\]

(3)

where \( \lambda_t^0 = V_{ts}^* V_{tb}, \lambda_u^2 = V_{us}^* V_{ub}, C_i \) are the coefficients of the Wilson expansion and the full set of operators \( O_i \) can be found elsewhere [13, 14]. As it was mentioned above, current study includes only so-called off-resonance region, where \( 0.05 < \hat{s} < 0.25 \). In this case \( B \to X_{s}e^+e^- \) decay is well described by \( b \to se^+e^- \) and \( b \to se^+e^-g \) partonic transitions. The calculation of these partonic transitions includes the following three steps:

1. The Wilson coefficients \( C_i \) at the heavy mass scale, \( \mu_W \sim M_W, m_t, \) must be calculated, matching the effective and full theories. In the next-to-next-to-leading order (NNLO) the matching has to be done at the \( O(\alpha_s) \) level, i.e. \( C_i(\mu_W) = C_i^{(0)}(\mu_W) + \alpha_s/(4\pi)C_i^{(1)}(\mu_W) \). When determining matching conditions for the Wilson coefficients one must, generally speaking, take into account the difference between the electroweak breaking scale and new physics scale. In the 2HDM, Model III, such a problem occurs when \( m_{H^+} \gg \mu_W \). Such values of the charged Higgs mass are out of the scope of this paper: here one takes \( m_{H^+} = 100\text{GeV}, 200\text{GeV and } 400\text{GeV} \).
2. The renormalization group equations (RGE) must be used to obtain the Wilson coefficients at the low-energy scale $\mu \sim m_b$. In the next-to-next-to-leading order this step requires the knowledge of the anomalous dimension matrix up to the order $\alpha_s^2$.

3. The matrix elements of the operators $O_i$ for the processes $b \to s e^+ e^-$, $b \to s e^+ e^- g$ have to be calculated.

Using this procedure, one finds that in the standard model the differential branching ratio of $B \to X_s e^+ e^-$ decay is given by the following expression \[^1\] [14]:

$$
\frac{dB(B \to X_s e^+ e^-)}{d\hat{s}} = \frac{\alpha_{em}^2}{4\pi^2} \frac{|\lambda_t|^2}{|V_{cb}|^2} (1 - \hat{s})^2 \times \left(1 + 2\hat{s}\right) \left(|C_{9,\text{new}}^{\text{eff}}(\mu)|^2 + |C_{10,\text{new}}^{\text{eff}}(\mu)|^2\right) + 4 \left(1 + \frac{2}{s}\right) |C_{7,\text{new}}^{\text{eff}}(\mu)|^2 + 12\Re\left(C_{9,\text{new}}^{\text{eff}}(\mu)C_{7,\text{new}}^{\text{eff}}(\mu)\right) B_{sl} \tag{4}
$$

where $\alpha_{em}$ is the electromagnetic coupling constant, $B_{sl}$ is the $B \to X_s e^+ \nu$ (experimental) branching ratio, $z$ is the ratio of $c$- and $b$-quark masses squared and the functions $g(z)$ and $k(z)$ are given in [13]. The effective new Wilson coefficients are related with the old ones, given e.g. in [8, 13, 18], as:

$$
C_{9,\text{new}}^{\text{eff}} = C_9^{(0)\text{eff}} + \frac{\alpha_s}{4\pi} C_9^{(1)\text{eff}} + \frac{\alpha_s}{4\pi} \left(4C_9^{(0)\text{eff}} \omega_9(\hat{s}) - C_1^{(0)} F_1^{(9)} - C_2^{(0)} F_2^{(9)} - C_8^{(0)\text{eff}} F_8^{(9)}\right) \tag{5}
$$

$$
C_{7,\text{new}}^{\text{eff}} = C_7^{(0)\text{eff}} + \frac{\alpha_s}{4\pi} C_7^{(1)\text{eff}} + \frac{\alpha_s}{4\pi} \left(4C_7^{(0)\text{eff}} \omega_7(\hat{s}) - C_1^{(0)} F_1^{(7)} - C_2^{(0)} F_2^{(7)} - C_8^{(0)\text{eff}} F_8^{(7)}\right) \tag{5}
$$

$$
C_{10,\text{new}}^{\text{eff}} = C_{10}^{(0)} + \frac{\alpha_s}{4\pi} C_{10}^{(1)} + \frac{\alpha_s}{\pi} C_{10}^{(0)} \omega_9(\hat{s}) \tag{5}
$$

$$
C_{9,\text{new}}^{\text{eff},*} C_{7,\text{new}}^{\text{eff}} = C_9^{(0)\text{eff},*} C_7^{(0)\text{eff}} + \frac{\alpha_s}{4\pi} \left(C_9^{(0)\text{eff},*} C_7^{(1)\text{eff}} + C_9^{(1)\text{eff},*} C_7^{(0)\text{eff}}\right) \tag{5}
$$

$$
+ \frac{\alpha_s}{4\pi} C_9^{(0)\text{eff},*} \left(4C_9^{(0)\text{eff}} \omega_9(\hat{s}) - C_1^{(0)} F_1^{(9)\text{*}} - C_2^{(0)} F_2^{(9)\text{*}} - C_8^{(0)\text{eff}} F_8^{(9)\text{*}}\right) \tag{5}
$$

where the function $F_i$, $\omega_i, \omega_9, \omega_9$ are given in [14, 8] and the relevant dimension six operators $O_i$ are the following:

$$
O_1 = \bar{s}_L \gamma^\mu T^a c_L \bar{c}_L \gamma^\mu T^a b_L, \quad O_{1u} = \bar{s}_L \gamma^\mu T^a u_L \bar{u}_L \gamma^\mu T^a b_L, \tag{6}
$$

$$
O_2 = \bar{s}_L \gamma^\mu c_L \bar{c}_L \gamma^\mu b_L, \quad O_{2u} = \bar{s}_L \gamma^\mu u_L \bar{u}_L \gamma^\mu b_L, \tag{6}
$$

$$
O_7 = \frac{e}{16\pi^2} m_b(\mu) \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, \quad O_8 = \frac{q_s}{16\pi^2} m_b(\mu) \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a, \tag{6}
$$

$$
O_9 = \frac{e^2}{16\pi^2} \bar{s}_L \gamma^\mu b_L \bar{c}_L \gamma^\mu e, \quad O_{10} = \frac{e^2}{16\pi^2} \bar{s}_L \gamma^\mu b_L \bar{c}_L \gamma^\mu e \tag{6}
$$

In the expression for $C_{9,\text{new}}^{\text{eff},*} C_{7,\text{new}}^{\text{eff}}$ the $O(\alpha_s^2)$ terms have been discarded. Similarly, in (4) for $|C_{1,\text{new}}^{\text{eff}}|^2$, i=7,9,10, only the terms linear in $\alpha_s$ are retained, when performing numerical calculations.

$O(\alpha_s)$ terms in (6) arise due to the $O(\alpha_s)$ corrections to the Wilson coefficients (the terms proportional to $C_1^{(1)\text{eff}}$ [13]), due to the $O(\alpha_s)$ corrections to $<e^+e^-|O_i|b>$ matrix elements and due

\[^1\]Our notation for the Wilson coefficients is different from that in ref. [13]: here the superscript (0) denotes $O(\alpha_s^{-1})$ and $O(1)$ contribution to $C_i$, whereas (1) always denotes $O(\alpha_s)$ contribution.
to the singular graphs connected with the gluon bremsstrahlung \( b \rightarrow se^+e^-g \) effects \[14\]. The contribution of nonsingular bremsstrahlung diagrams and \( O(\alpha_s) \) corrections to the \( O_{1u} \) \( O_{2u} \) matrix elements are still unknown. It is expected however for them to be small.

Both for the SM and the 2HDM the content of the operators \( O_i \) is the same, so is the anomalous dimension matrix. The new physics effects enter only through the matching conditions. In other words, formulae (4) and (5) are valid also in the 2HDM, however for the Wilson coefficients one has now \[13, 14, 20, 21\]

\[
C^{(0)}_{12}(\mu) = C^{(0)_{SM}}_{12}(\mu) \quad (7)
\]

\[
C^{eff}_{7,8}(\mu) = C^{eff_{SM}}_{7,8}(\mu) - \xi_t\xi_bC^{effH}_{7,8}(\mu) + |\xi_t|^2C^{effR}_{7,8}(\mu) \quad (8)
\]

\[
C^{eff}_{9,10}(\mu) = C^{eff_{SM}}_{9,10}(\mu) + |\xi_t|^2C^{effR}_{9,10}(\mu) \quad (9)
\]

The full set of formulae for \( C^{eff}_{7,8}(\mu) \) (so is for \( C^{(0)}_{12}(\mu) \) in the chosen operator basis) is given in \[19\]. Here it is worth to notify only that the contribution of the second term in r.h.s. of (8) dominates over the last term\[2\]. For \( C^{eff_{SM}}_{9,10}(\mu) \) one has

\[
C^{eff}_{9,10}(\mu) = \tilde{C}^{eff}_{9,10}(\hat{s}) - \tilde{C}^{effc}_{9,10}(\hat{s}) + \frac{\lambda_s}{\lambda_t} \Delta C^{eff}_{9,10}(\hat{s})
\]

\[
C^{eff}_{10}(\mu) = \tilde{C}^{eff}_{10}(\hat{s}) - \tilde{C}^{effc}_{10}(\hat{s}) \quad (10)
\]

where \( \tilde{C}^{eff}_{9,10}(\hat{s}) \), \( \tilde{C}^{effc}_{9,10}(\hat{s}) \) and \( \Delta C^{eff}_{9,10}(\hat{s}) \) are given in \[13\]. For \( C^{(0)_{effH}}_{9,10}(\mu) \) one can easy deduce that

\[
C^{(0)_{effH}}_{9,10}(\mu) = C^{(0)_{H}}_{9,10}(\mu_W) + C^{(1)_{H}}_{4}(\mu_W) \sum_{i=5}^{9} q_i^{t(+)} \tilde{\eta} \alpha_{i+1}
\]

\[
C^{(0)_{effH}}_{10}(\mu) = C^{(0)_{H}}_{10}(\mu_W) \quad (11)
\]

where \( \tilde{\eta} = \alpha_s(\mu_W)/\alpha_s(\mu) \), \( C^{(0)_{H}}_{9,10}(\mu_W) \) are given in \[20, 21\], \( C^{(1)_{H}}_{4}(\mu_W) \) is given in \[19\] and the ”magic numbers” \( q_i^{t(+)} \) can be found in \[13\].

Unfortunately the matching conditions for \( C^{(1)_{effH}}_{9,10} \) are unknown yet. For this reason here the SM values \( C^{(1)_{effH}}_{9,10}(\mu) \) will be used.

5. The effective Wilson coefficients\[3\] deviate from their SM values differently. Thus \( C^{eff}_{9} \) deviates for \( C^{eff_{SM}}_{9} \) only by few percents. On the contrary, \( |C^{eff}_{10}| \) can be about 1.4 times larger than in the SM\[4\] (Table 1).

The largest deviations from the SM interval occur for \( C^{eff}_{7} \). It is known that in the standard model extensions \( Re(C^{eff}_{7}) \) may have the sign opposite to that in the SM and furthermore unlike the SM the dispersive part of \( C^{eff}_{7} \) can be complex (recall that in the SM \( C^{(0)_{eff}}_{7} \) is real and the imaginary part

\[2\]Moreover, the last term in (8) is about one order smaller than the SM contribution.

\[3\]Because of the smallness of \( C^{(0)_{eff}}_{8} \) and condition (7) the qualitative discussion throughout the paper is valid both for the old and the new Wilson coefficients, unless the difference is specially notified.

\[4\]One may subsequently expect that omitted here \( O(\alpha_s) \) corrections to \( C^{eff}_{10} \) will not exceed \((10 \div 15)\% \) and those to \( C^{eff}_{9} \) will be negligible.
Table 1: \( R_{10} = C_{10}^{(0)\text{d}} / C_{10}^{(0)\text{SM}} \).

| \( m_{H^+} \) | 100GeV | 200GeV | 400GeV |
|------------|--------|--------|--------|
| \( R_{10} \) | 0.36 \(÷\) 0.39 | 0.23 \(÷\) 0.26 | 0.12 \(÷\) 0.14 |

Table 2: The restrictions on \( |C_{7,\text{new}}^{\text{eff}}(\mu)|^2 \) for \( m_{H^+} = 100\text{GeV} \), coming from the condition \( B(B \rightarrow X_s\gamma) = (2.96 \pm 0.35) \times 10^{-4} \).

| \( \mu_W = M_W \) | \( \mu = m_b/2 \) | \( \mu = 2m_b \) | \( \mu = m_b/2 \) | \( \mu = m_b/2 \) | \( \mu = 2m_b \) |
|------------------|------------------|------------------|------------------|------------------|------------------|
| \( |C_{7,\text{new}}^{\text{eff}}|^2 \) | \(-0.04 \pm 0.13\) | \(-0.03 \pm 0.12\) | \(-0.05 \pm 0.12\) | \(-0.05 \pm 0.13\) | \(-0.06 \pm 0.12\) |

of \( C_{7,\text{new}}^{\text{eff}} \) arises only due to absorptive parts of the \( O_1 \) and \( O_2 \) matrix elements). As for \( |C_7^{\text{eff}}| \), it is bounded due to the experimental constraints on the \( B(B \rightarrow X_s\gamma) \) branching. In this paper the numerical calculations are performed both including and neglecting \( O(\alpha_s) \) effects, and the derived results are compared to each other. When neglecting \( O(\alpha_s) \) corrections to \( B(B \rightarrow X_s e^+ e^-) \) is it reasonable to do the same also for \( B(B \rightarrow X_s\gamma) \). In this case more conservative bound than (1) should be used: one takes therefore \( [10] \) \( 1 \times 10^{-4} < B(B \rightarrow X_s\gamma) < 4.2 \times 10^{-4} \). This gives \( 0.04 \leq |C_7^{(0)\text{eff}}|^2 \leq 0.18 \) in the next-to-leading order. When including \( O(\alpha_s) \) corrections, the condition (1) puts some constraint on \( |C_{7,\text{new}}^{\text{eff}}|^2 \) at the point \( \hat{s} = 0 \). Though this point is out of the consideration, due to the weak dependence of \( C_{7,\text{new}}^{\text{eff}} \) on \( \hat{s} \) (see formulae of \( [14] \)) the aforesaid constraint is essential also for the considered range of the dilepton invariant mass.

Because of discarding \( O(\alpha_s^2) \) terms in the expression for \( |C_{7,\text{new}}^{\text{eff}}|^2 \), there are some problems, connected with the bounds on this quantity. As one can see from the Table 2, for \( \mu_W = m_W \) the restrictions on \( |C_{7,\text{new}}^{\text{eff}}|^2 \) are highly sensitive to the choice of the low-energy scale. Moreover, for \( \mu = m_b/2 \) the condition (1) allows negative values of \( |C_{7,\text{new}}^{\text{eff}}|^2 \). The unnatural negative values of \( |C_{7,\text{new}}^{\text{eff}}|^2 \) indicate on the fact that in some regions of the 2HDM parameter space the discarded \( O(\alpha_s^2) \) terms are numerically relevant and may not be neglected. Such a situation occurs in particular in the case when \( |C_7^{(0)\text{eff}}| \sim 1 \), however due to the \( O(\alpha_s) \) corrections \( |C_{7,\text{new}}^{\text{eff}}| \sim |C_{7,\text{new}}^{\text{eff}^s_{SM}}| \) or even smaller. Then it is possible that although \( \Gamma(b \rightarrow s\gamma) \sim |C_{7,\text{new}}^{\text{eff}}|^2 < 0 \), the long-distance \( O(1/m_b^2) \) corrections (which to the considered order are taken proportional to \( |C_7^{(0)\text{eff}}|^2 \) \( [19] \)) drive \( B \rightarrow X_s\gamma \) branching to the experimentally allowed interval.

Of course there is no sense to consider such unphysical possibilities, which arise only due to neglecting of higher order corrections to the \( B \rightarrow X_s\gamma \) branching. One can easy avoid to consider such unfavorable regions of the parameter space. As one can see from the Table 2, for \( \mu_W = m_t \) the reliability of the restrictions on \( |C_{7,\text{new}}^{\text{eff}}|^2 \) increases. In particular, the allowed interval of \( |C_{7,\text{new}}^{\text{eff}}|^2 \) is weakly sensitive to the choice of the low-energy scale and one may take now \( 0.05 \leq |C_{7,\text{new}}^{\text{eff}}|^2 |_{\hat{s}=0} \leq 0.13 \) (this bound turns to be valid also for \( m_{H^+} = 200\text{GeV} \) and \( m_{H^+} = 400\text{GeV} \)). This result is not surprising: it is known that for \( \mu_W = m_t \), \( O(\alpha_s) \) corrections to the \( B \rightarrow X_s\gamma \) branching are minimized, as compared to the case of \( \mu_W = M_W \) \( [19] \). Thus the above-mentioned problem is avoided, if calcula-
tions are performed only for $\mu_W = m_t$. This strategy will be used throughout this paper in the most of cases. However, the estimation of possible inaccuracy in the obtained results, connected with the uncertainty in the choice of the matching scale, will be done as well.

6. Let us proceed to the numerical results. During the numerical analysis we use $M_W = 80\text{GeV}$, $m_t^{\text{pole}} = (174 \pm 5)\text{GeV}$, $\alpha_s(M_Z) = 0.119 \pm 0.002$ ([23]), $m_h^{\text{pole}} = (4.8 \pm 0.15)\text{GeV}$, $m_c/m_b = 0.29 \pm 0.02$, $\alpha_{em} = 1/(130.3 \pm 2.3)$ and $B_{sl} = (10.49 \pm 0.46)\%$ (see [17] and references therein). The low-energy scale $\mu$ is varied as $\mu = m_b/2$, $\mu = m_b$, $\mu = 2m_b$. As it was noted already, the matching scale is identified here with the top quark mass. Only when the heavy mass scale dependence is examined, the matching scale is chosen as $\mu_W = M_W$ as well.

The charged Higgs mass is varied as $m_{H^+} = 100\text{GeV}$, $200\text{GeV}$, $400\text{GeV}$. The restrictions on $|\xi_t|$ and $|\xi_b|$ are derived from the requirement for the top and bottom Yukawa couplings to be in the perturbativity range in a whole energy interval between the electroweak breaking scale and unification scales, and using the experimental constraints coming from the measurements of $B \rightarrow X_s\gamma$ branching ratio and $B - \bar{B}$ mixing effects (see the discussion in previous section and [17] for more details).

At the low-energy scale $\alpha_s$ is computed, using its two-loop renormalization group equation [19]. However, when neglecting $O(\alpha_s)$ corrections to $B \rightarrow X_s e^+ e^-$ and $B \rightarrow X_s \gamma$ branching ratios, one-loop result for $\alpha_s(\mu)$ is used.

In the Wolfenstein parameterization the necessary CKM-factors are given by [23]:

$$\frac{\lambda_i^s}{|V_{cb}|} = -(1 - \lambda^2/2 + \lambda^2(\rho - i\eta)), \quad \frac{\lambda_i^u}{|V_{ub}|} = \lambda^2(\rho - i\eta),$$

(12)

where $\lambda = \sin \theta_C \approx 0.22$ and the unitarity triangle parameters $\rho$ and $\eta$ can be obtained from the unitarity fits, which yield [24, 25]

$$\sqrt{\rho^2 + \eta^2} = 0.423 \pm 0.064, \quad 38^\circ \leq \tilde{\gamma} = \arctan \frac{\eta}{\rho} \leq 81^\circ$$

(13)

The numerical calculations consist of two steps. At first one investigates the renormalization scale dependence of the $B \rightarrow X_s e^+ e^-$ normalized differential width:

$$R(\hat{s}) = \frac{1}{\Gamma(B \rightarrow X_s e^+ \nu)} \frac{d\Gamma(B \rightarrow X_s e^+ e^-)}{d\hat{s}} = \frac{1}{B_{sl}} \frac{dB(B \rightarrow X_s e^+ e^-)}{d\hat{s}}$$

The use of $R(\hat{s})$ would allow one to compare our results with those of ref.’s [13, 14]. The renormalization scale dependence is examined for some fixed characteristic values of the new physics parameters, for the ”best fit” values of the CKM parameters ($\rho = 0.19$, $\eta = 0.37$) and for the central values of the remaining parameters of theory.

The next step involves complete investigation for the differential and partially integrated branching ratios of $B \rightarrow X_s e^+ e^-$ in the 2HDM and comparison of the obtained results with those in the SM. The parameters of theory are varied now in the intervals specified above.

Let me briefly recall how the situation with the low-energy and matching scale dependence of $R(\hat{s})$ in the SM looks. On the absence of the $O(\alpha_s)$ contribution (then $C^{\text{eff}}_{i,\text{new}}$ in (4) are replaced by $C_i^{(0)\text{eff}}$, $i=7,9,10$), the $\mu$-dependence of $R(\hat{s})$ (in the naive scheme) is $\sim 6\%$ or smaller. However, such a weak sensitivity of the $B \rightarrow X_s e^+ e^-$ decay width to the choice of the low-energy scale in the next-to-leading order is accidental [9]. The error from the $\mu$-dependence of $R(\hat{s})$ grows up to
Table 3: The matching scale dependence of $C_{10}(\mu) = C_{10}^{(0)}(\mu) + \alpha_s(\mu)/(4\pi) C_{10}^{(1)}(\mu)$ in the SM and in the 2HDM for $|\xi_t| = 1$ and $m_{H^+} = 100$GeV.

|                           | with $O(\alpha_s)$ corrections | without $O(\alpha_s)$ corrections |
|---------------------------|---------------------------------|-----------------------------------|
|                           | $\mu_W = M_W$                  | $\mu_W = m_t$                     |
| $C_{10}^{eff,SM}(\mu)$   | -4.37 $\div$ -4.02             | -4.50 $\div$ -4.10               |
| $C_{10}^{eff}(\mu)$      | -6.24 $\div$ -5.64             | -6.17 $\div$ -5.54               |

13%, when $O(\alpha_s)$ corrections to the Wilson coefficients are taken into account \[13\]. Only after $O(\alpha_s)$ corrections to the matrix elements of the operators $O_j$, $j=1,2,7,\ldots,10$, are also included, this error is reduced down to 6.5\% \[14\]. As for the error, connected with the uncertainty of the heavy mass scale, it is reduced from $\sim 15\%$ in the NLO to few percents in the NNLO.

The difference between the renormalization scale behavior of $R(\hat{s})$ in the SM and 2HDM occurs predominantly due to $C_{7}^{eff}$. The $\mu$- dependence of $C_{7}^{eff}$ originates only at $O(\alpha_s)$ order and is small therefore. There is no essential difference between the $\mu_W$ - dependence of $C_{10}$ in the SM and 2HDM even for $|\xi_t| \sim 1$. As one can see from the Table III, in both of models the $\mu_W$ - error of $C_{10}$ is $\sim 10\%$ in the NLO and $\sim (2-3)\%$ in the NNLO.

When investigating the renormalization scale behavior of $R(\hat{s})$, the new physics parameters are chosen as $m_{H^+} = 100$GeV, $|\xi_t|^2 = 0.5$ and a) $\xi_t\xi_b = 6.4$, b) $\xi_t\xi_b = 2.8 \pm 2.8i$. In the case a), the dispersive part of $C_{7}^{eff}$ is real and has the sign opposite to that in the standard model. In the case b), the imaginary part of $C_{7}^{eff}$ dominates over the real one.

For the case a), the behavior of the $B \to X_s e^+ e^-$ normalized differential width as a function of the dilepton invariant mass for different choices of the low-energy scale and for $\mu_W = m_t$ is presented in the Fig. 1. As one can see from this figure, in the next-to-leading order the uncertainty of $R(\hat{s})$ connected with that of the low-energy scale is large enough: it reaches 25\% for lower values of $\hat{s}$ and 17\% for larger values of $\hat{s}$. $O(\alpha_s)$ corrections reduce the $\mu$-error of $R(\hat{s})$ significantly: in the naive scheme (where the errors of different terms in (4) are simply summed) it is $\sim 10\%$ for lower values of $\hat{s}$ and its sign is flipped, it is only about few percents for larger values of $\hat{s}$, and it almost disappears for the intermediate values of $\hat{s}$. Such a dependence of $R(\hat{s})$ on the choice of the low-energy scale indicates on the cancellation of $\mu$-errors of the terms in (4). In such cases one usually uses so-called Kagan-Neubert method \[21, 9\]: in context of the present calculations this approach implies the calculation of $\mu$-errors of different blocks of the (effective) Wilson coefficients separately and then adding them in quadrature. The result derived in the Kagan-Neubert scheme does not differ essentially from that in the naive scheme: now the uncertainty of $R(\hat{s})$, connected with that of the low-energy scale, is $\sim 12\%$ for lower values of $\hat{s}$ and $\sim 6\%$ for large values of $\hat{s}$. This means that weak $\mu$-dependence of $R(\hat{s})$ in the NNLO is not accidental.

The investigations in the Kagan-Neubert scheme show also that in the next-to-next-to-leading order the main source of the $\mu$-dependence is the term proportional to $4 \left(1 + \frac{2}{s}\right) |C_{7,new}^{eff}(\mu)|^2$ (in the

\[5\] Although this result has been derived in \[13\] considering $O(\alpha_s)$ corrections only to the Wilson coefficients, it is easy to check that it remains valid also after taking into account $O(\alpha_s)$ corrections to the operators matrix elements.

\[6\] For the cases a) and b) the parameters of theory are within or close to the experimentally allowed values only in the NNLO. As it is shown in the section 8, the experimentally allowed intervals of the new physics parameters are strongly different in the NLO and the NNLO.
Figure 1: Low energy scale dependence of $R(\hat{s})$ in the NNLO (solid lines) and NLO (dashed and dotted lines) in the 2HDM, case a), for $\mu_W = m_t$. The analysis is performed both in the naive and the Kagan-Neubert schemes. Unless notified, lower dashed (dotted) and solid lines correspond to $\mu = m_b/2$, middle lines correspond to $\mu = m_b$ and upper lines correspond to $\mu = 2m_b$.

NLO there is also sizable contribution from the term proportional to $\text{Re} \left( C^{eff*}_{9,\text{new}}(\mu)C^{eff}_{7,\text{new}}(\mu) \right)$. This explains why in the NNLO the low-energy scale dependence of $R(\hat{s})$ for larger values of $\hat{s}$ is especially small.

It is important to stress that in the case a), $O(\alpha_s)$ corrections suppress the $B \to X_s e^+ e^-$ decay width about $1.5 \div 3$ times, depending on $\hat{s}$ and $\mu$. In other words, NNLO corrections to $R(\hat{s})$ are about $(35 \div 65)\%$ of the leading and next-to-leading order terms. It is reasonable therefore to expect that $O(\alpha_s^2)$ contribution to the $B \to X_s e^+ e^-$ decay width will be nonnegligible as well. This is argued also by the investigation of the matching scale dependence of $R(\hat{s})$. As one can see from the Fig. 2, in the next-to-leading order the uncertainty of $R(\hat{s})$ is $\sim 15\%$, when varying the matching scale from $m_t$ to $M_W$. This uncertainty is almost independent on $\hat{s}$. In the next-to-next-to-leading order the situation is quite different: while for larger values of $\hat{s}$, $\mu_W$-error of $R(\hat{s})$ decreases down to $\sim 10\%$, for lower values of $\hat{s}$ it increases up to $\sim 25\%$. Such a large error can be reduced only by higher order corrections to the $B \to X_s e^+ e^-$ decay width.

The obtained results for the case b) are presented in the Fig. 3. As one can see from this figure, the situation is similar to that of the case a). Due to the $O(\alpha_s)$ corrections, the low-energy scale dependence of $R(\hat{s})$ becomes negligible, so is the matching scale dependence for larger values of $\hat{s}$. 
Figure 2: The matching scale dependence of $R(\hat{s})$ in the NNLO (solid lines) and NLO (dashed and dotted lines) in the case a) for $\mu = m_b/2$. The fat (thin) solid and dashed (dotted) lines correspond to $\mu_W = m_t$ ($\mu_W = M_W$).

However for lower values of $\hat{s}$, the $\mu_W$-error of $R(\hat{s})$ is of the same order ($\sim 14\%$) as without $O(\alpha_s)$ corrections.

Thus, although in the 2HDM $O(\alpha_s)$ corrections reduce significantly the low-energy scale dependence of the $B\to X_s e^+ e^-$ decay width, higher order terms in the perturbation theory can still be nonnegligible, when the new physics contribution is sizable. The importance of higher order corrections is manifested by the sensitivity of the obtained results to the choice of the heavy mass scale.

7. Let us compare now the predictions of the 2HDM for the $B\to X_s e^+ e^-$ branching ratio to those of the SM (whole allowed range of the 2HDM parameter space is considered now). The deviation of $B(B\to X_s e^+ e^-)$ from the SM results can take place due to the change of the sign of $Re(C_{t7}^{eff})$ (the source I), due to the imaginary part of $C_{t7}^{eff}$, connected with the Higgs doublet vacuum phase (the source II), and due to the deviation of $C_{10}^{eff}$ from $C_{10}^{effSM}$ (the source III). It is easy deduce from the formula (4) that all aforementioned three sources increase the $B\to X_s e^+ e^-$ branching, once $Re(C_{t7}^{effSM}) < 0$ and $|C_{10}^{eff}| \geq |C_{10}^{effSM}|$.

The contribution of the source II is expected to be small: it is proportional to $[Im(C_{9,new}^{eff})Im(C_{7,new}^{eff})]$. 

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7During the numerical calculation the terms which make the difference between the $B\to X_s e^+ e^-$ decay and the...
and one has $|\text{Im}(C_{9,\text{new}}^{\text{eff}})| \ll |\text{Re}(C_{9,\text{new}}^{\text{eff}})|$. The effects connected with the source III are strongly correlated with the sign of $\text{Re}(C_{9,\text{new}}^{\text{eff}})$. In the SM-like case the terms of (4), proportional to $|C_{9,\text{new}}^{\text{eff}}|^2$ and $\text{Re}(C_{9,\text{new}}^{\text{eff}}) C_{7,\text{new}}^{\text{eff}}$, have opposite signs so that their contribution is partially canceled. Consequently, the partial weight of the the term proportional to the $|C_{10,\text{new}}^{\text{eff}}|^2$ and hence the contribution of the source III is significant. Respectively, when $\text{Re}(C_{7,\text{new}}^{\text{eff}})$ has the sign, opposite to that in the standard model, the effects of the source III are minimized. It is straightforward to deduce from current discussion that the sources I and III do not interfere in fact.

The maximum value of the $B \to X_s e^+ e^-$ differential branching ratio as a function of $\hat{s}$ is presented in the Fig. 4 for particular cases, when only one of the aforementioned sources is actual. As it was expected, the contribution of the source II is not large: if only this source is actual, $dB(B \to X_s e^+ e^-)/d\hat{s}$ deviates from its SM maximum value at most 1.4 times. The source III can make $dB(B \to X_s e^+ e^-)/d\hat{s}$ 1.6 ÷ 1.9 times larger than in the SM (respectively for $\hat{s}$ varying from 0.05 to 0.25). The deviations from the SM results, connected with the source III, are most perceptible when $|\xi_b| \ll 1$ and $|\xi_t| \sim 1$. The effects of the source III are rapidly minimized with the increasing of the charged Higgs mass: for $m_{H^+} = 400\text{GeV}$ they are already of the same order as those connected with the source II.

Figure 3: The same as in Fig.'s 1,2 (respectively the left- and right-hand-side graphs) but for the case b).
Figure 4: Maximum value of $d\mathcal{B}(B \to X_s e^+ e^-)/d\hat{s}$ in the SM (dashed line) and 2HDM (solid lines). The following regions of the 2HDM parameter space are considered: $\text{Im}(\xi_t \xi_b) = 0$, $|C_{10,\text{new}}^{\text{eff}}|^2 < 1.15|C_{10,\text{new}}^{\text{eff} \Delta m}|^2$ (line 1, only the source I is actual); $\text{sign}(\text{Re}(C_{7,\text{new}}^{\text{eff}})) = \text{sign}(\text{Re}(C_{7,\text{new}}^{\text{eff} \Delta m})), |C_{10,\text{new}}^{\text{eff}}|^2 < 1.15|C_{10,\text{new}}^{\text{eff} \Delta m}|^2$ (line 2, only the source II is actual); $\text{Im}(\xi_t \xi_b) = 0$, $\text{sign}(\text{Re}(C_{7,\text{new}}^{\text{eff}})) = \text{sign}(\text{Re}(C_{7,\text{new}}^{\text{eff} \Delta m})), m_{H^+} = 100\text{GeV}, 400\text{GeV}$ (lines 3, 3' respectively, only the source III is actual).

The most sizable deviations from the SM predictions occur due to the source I. Due to this source, maximum value of $d\mathcal{B}(B \to X_s e^+ e^-)/d\hat{s}$ can be up to 2.6 times larger than in the standard model. The effects connected with the source I are almost insensitive to the charged Higgs mass. Large deviations from the SM result are possible even for $m_{H^+} \sim 1\text{TeV}$ (until $\text{Re}(C_{7,\text{new}}^{\text{eff}})$ becomes positive, when $|\xi_b| \gg 1$). However the consideration of such large values of the charged Higgs mass is out of the scope of the present paper, otherwise one should take into account the difference between the electroweak breaking scale and the charged Higgs mass scale.

It is worth to make here some digression on the situation in the Models I and II. If one applies the restrictions on the parameters $\xi_t$ and $\xi_b$ notified in the section 6, only the source III will be relevant in these versions of the 2HDM. But the effects of this source are very small, because one has $|\xi_t|^2 < 1/4$ for $m_{H^+} = 100\text{GeV}$ in the Model I and $m_{H^+} > 400\text{GeV}$ in the Model II. As a result, the predictions of these models for $B \to X_s e^+ e^-$ branching are close to those of the SM.

Let us return back to the Model III. The results derived for the most general case (when all of three sources are actual) are presented in the Fig. 5. As one can see from this figure, while the
Figure 5: Minimum and maximum values of $dB(B \to X_s e^+e^-)/d\hat{s}$ in the SM (dashed lines) and 2HDM (solid lines) a) in the NNLO, b) in the NLO.

The minimum value of $dB(B \to X_s e^+e^-)/d\hat{s}$ coincides (within 10% accuracy) with that in the SM, the maximum value is $2.5 \div 3$ times larger than in the standard model. Note that $O(\alpha_s)$ corrections do not enhance or suppress the deviation from the SM predictions. This is because in both the NLO and the NNLO the experimental constraints on the $B \to X_s \gamma$ branching fix $|C_7^{eff}|$ approximately in the same range (at least for the choice of the heavy mass scale $\mu_W = m_t$).

For the partially integrated branching ratio

$$\Delta B(B \to X_s e^+e^-) = \int_{0.05}^{0.25} d\hat{s} \frac{dB(B \to X_s e^+e^-)}{d\hat{s}}$$

(14)
on one gets:

- SM, NLO: $\Delta B(B \to X_s e^+e^-) = (1.3 \div 2.1) \times 10^{-6}$
- 2HDM, NLO: $\Delta B(B \to X_s e^+e^-) = (1.3 \div 5.6) \times 10^{-6}$
- SM, NLO: $\Delta B(B \to X_s e^+e^-) = (1.2 \div 1.9) \times 10^{-6}$
- 2HDM, NLO: $\Delta B(B \to X_s e^+e^-) = (1.1 \div 5.3) \times 10^{-6}$

As it follows from these results, in the 2HDM partially integrated branching ratio of $B \to X_s e^+e^-$ decay can be up to 2.8 times larger than in the standard model. Again, the deviations from the SM result are not affected by $O(\alpha_s)$ corrections.

Thus, if both $B \to X_s \gamma$ and $B \to X_s e^+e^-$ branching ratios are computed with the same accuracy, then in any order of the perturbation theory (unless in some numerically relevant order $C_7^{eff}$ becomes
highly sensitive to $\xi$) one will derive that in the 2HDM, maximum value of the $B \to X_s e^+ e^-$ branching ratio is $2.5 \div 3$ times larger than in the standard model.

8. When considering whole allowed range of the 2HDM parameter space, one can find out the difference between the NLO and NNLO predictions for the $B \to X_s e^+ e^-$ branching, examining the dependence of this variable on the new physics parameters. Once in the 2HDM $O(\alpha_s)$ terms are important primarily for $C_{7,nw}$, it is reasonable to take $m_{H^+} = 400 GeV$, to minimize the contribution of the source III, and investigate the dependence of the $B \to X_s e^+ e^-$ branching ratio on the product $|\xi_t \xi_b|$ (see formula (8) and the comment beneath). The minimum and maximum values of the partially integrated $B \to X_s e^+ e^-$ branching ratio as functions of $|\xi_t \xi_b|$ are presented in the Fig. 6. One can see that the dependence of $\Delta B$ on $|\xi_t \xi_b|$ is quite different in the NLO and the NNLO. Thus, in the next-to-leading order maximum value of $\Delta B(B \to X_s e^+ e^-)$ is two and more times larger than in the standard model, when taking $8 \leq |\xi_t \xi_b| \leq 15$. In the next-to-next-to-leading this occurs for $17 \leq |\xi_t \xi_b| \leq 30$. In other words, after including $O(\alpha_s)$ corrections, the dependence of $\Delta B(B \to X_s e^+ e^-)$ on $|\xi_t \xi_b|$ is changed drastically.
Generally speaking, the behavior of $\Delta B(B \to X_s e^+e^-)$ with $|\xi_0\xi_b|$ can further be modified by higher order effects in the perturbation theory. On the other hand, it has been shown that in the next-to-next-to-leading order, when choosing the matching scale as $\mu_W = m_t$, the low-energy scale dependence of the $B \to X_s e^+e^-$ decay width is weak. In order to preserve the weak sensitivity of $R(\hat{s})$ to $\mu$, higher order corrections to the $B \to X_s e^+e^-$ branching must somehow cancel each other. This allows one to expect that higher order effects in the perturbation theory will not modify the obtained result so drastically, as it is the case in the Fig. 6. In other words, one may suppose that the dependence of $\Delta B(B \to X_s e^+e^-)$ on $|\xi_0\xi_b|$, derived in the NNLO, is close enough to the proper one. However, it is necessary to emphasize that such a suggestion may be done only for specific choices of the heavy mass scale (like above). More generally (for instance for $\mu_W = M_W$) the NNLO results for the $B \to X_s e^+e^-$ branching can be highly unreliable and be largely modified by higher order effects in the perturbation theory.

Summarizing the discussion of this section one may conclude that including the $O(\alpha_s)$ effects is an important step on the way of establishing the proper relations between the $B \to X_s e^+e^-$ branching and the new physics parameters.

9. Thus, $O(\alpha_s)$ corrections to $B \to X_s e^+e^-$ decay have been examined in the two-Higgs doublet extension of the standard model. The investigations have been performed for the most general version of the 2HDM (Model III) in the so-called off-resonance region of the dilepton invariant mass ($0.05 < \hat{s} < 0.25$).

It has been shown that in the case when the new physics effects are sizable, $O(\alpha_s)$ corrections (for the fixed values of the 2HDM parameters) suppress the $B \to X_s e^+e^-$ decay width $1.5 \div 3$ times. It is natural to suppose that $O(\alpha_s^2)$ corrections to the $B \to X_s e^+e^-$ branching will be numerically relevant as well.

Last suggestion is confirmed by the fact that the obtained results are still sensitive to the choice of the heavy mass scale, when the new physics effects are sizable. Even after $O(\alpha_s)$ corrections are included, the uncertainty in the $B \to X_s e^+e^-$ decay width connected with the choice of the matching scale reaches 25%.

On the other hand, $O(\alpha_s)$ corrections reduce significantly the low-energy scale dependence of the $B \to X_s e^+e^-$ branching. The $\mu$-error of the obtained results in the NNLO is $\sim 10\%$ or smaller (compared to $\sim 25\%$ in the NLO). This means that in the next-to-next-to leading order the reliability of the predictions for the $B \to X_s e^+e^-$ branching ratio increases.

When using the experimental constraints on the $B \to X_s \gamma$ branching and calculating $B \to X_s e^+e^-$ decays with the same accuracy, one will probably get in all orders of the perturbation theory that in the 2HDM $B \to X_s e^+e^-$ branching ratio can be about $2.5 \div 3$ times larger than in the standard model. However only after taking into account $O(\alpha_s)$ corrections and probably those of higher orders, one is able to derive the proper relations between the new physics parameters and the $B \to X_s e^+e^-$ branching ratio.

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8 When taking $\mu_W = M_W$, one can examine the dependence of the $B \to X_s e^+e^-$ branching on the new physics parameters, requiring that $0 < |C_{\gamma,\text{new}}(\mu)|_z = 0 \leq 0.13$. 

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