Achromatic acoustic generalized phase-reversal zone plates

Gaokun Yu*, Xinyu Zou and Peifeng Wang
College of Information Science and Engineering, Ocean University of China, Qingdao 266100, People’s Republic of China

* Author to whom any correspondence should be addressed.
E-mail: gkyu@ouc.edu.cn

Keywords: achromatic focusing, Fano resonance, phase-reversal zone plate

Abstract
We report an achromatic acoustic generalized phase-reversal zone plate by harnessing the response of dipole and monopole, which eliminate the chromatic aberration of conventional zone plates. The focusing properties of the proposed metalens are compared with that of the conventional Soret-type Fresnel zone plate (FZP) in both experiments and simulations. Due to the combination of the phase-reversal characteristic and the tunable transmission phase induced by dipole and monopole, an achromatic high efficient focusing is confirmed by experiment in the frequency range from 3350 to 3950 Hz, with the focal intensity of achromatic metalens being approximately twice that of Soret-type FZP. The proposed achromatic metalens has potential applications in the broad field of acoustics, such as imaging and energy harvesting.

1. Introduction

Metamaterials and metasurfaces have attracted much attention from researchers [1–6], by which light and acoustic properties can be locally tailored. Up to now, a number of potential applications related to wave focusing have been demonstrated [7–17], including the nondiffracting beams [9, 15], the self-bending beam [11], achromatic metasurfaces for full-color detection and imaging [18–21], etc. However, to obtain achromatic focusing in acoustics [22–28] is still a challenge, especially for the design of transmitted metalens. Although an achromatic acoustic metasurface is recently proposed for broadband focusing [28] using a bottom-up inverse-design paradigm, considering its size (thickness being on the order of a wavelength) and its focusing efficiency (transmission coefficient $|t_0| > 0.6$ for each unit cell), it is interesting to achieve an achromatic high efficient focusing by a thinner metasurface. The success of achromatic metalenses in optics [18–21] is attributed to the combination of a frequency-dependent transmission phase by resonance and a frequency-independent phase (the geometric phase), unfortunately, there exists certain difficulty in exploiting the geometric phase in acoustics.

Zone plate as another type of planar diffractive lens has found many applications in optics [29], x-ray microscopy [30, 31], THz optics [32] and microwaves [33], and it can also be used to focus sound waves [34–38]. To overcome the high chromatic aberration of Fresnel zone plate (FZP), several aperiodic zone plates such as fractal zone plates [39–42] and Thue–Morse zone plates [43, 44] have been proposed, which produce an extended depth of field by the fractal structure of their foci. However, their focal intensity is lower than that of FZPs because part of the energy is distributed among the multiple subsidiary foci [42].

In this work, we propose a concept of achromatic acoustic generalized phase-reversal zone plate in figure 1(b), which combines the advantages of metasurface (locally tailoring phase) and the phase-reversal FZPs (without the requirement of the full $2\pi$ transmission phase tuning). Different from the conventional phase-reversal zone plates, in which a $\pi$-phase jump occurs abruptly from 0 to $\pi$ and from $\pi$ to 0, here, we manipulate the transmission phase gradually from 0 (or $2\pi$) to $\pi$ by a combination of dipole and monopole, and then an approximate $\pi$-phase jump occurs from $\pi$ to 0 (or $2\pi$). It is stressed that the frequency-dependent response of dipole and monopole plays a role to reduce chromatic aberration. On the other hand, different from the full $2\pi$ transmission phase tuning in optics to achieve achromatic focusing, the proposed achromatic metalens requires only the $\pi$ transmission phase tuning, which can be satisfied by a transmitted metalens in acoustics.
Figure 1. (a) The Soret-type FZP is comprised of zones that alternate between acoustically transparent and opaque. (b) Achromatic metalens consists of a series of complex zones, in each zone the response of unit cell changes gradually from the dipole to monopole. (c) Dipole: the first unit cell marked in (b), where the width of unit cell is $\Lambda$, the thickness of lens being $l = l_1 + l_2 + l_3 + l_4 = 1.4\Lambda$ with $l_1 = 0.2\Lambda$, $l_2 = 0.5\Lambda$, $l_3 = 0.5\Lambda$ and $l_4 = 0.2\Lambda$, and for each Helmholtz resonator, the following parameters are fixed: the size of throat $a = 0.125\Lambda$, the length of throat $b = 0.05\Lambda$ and the height of cavity $h = 0.3\Lambda$. (d) Monopole: the seventh unit cell marked in (b), with $l = 1.4\Lambda$. Note that the global coordinate system $xOz$ is marked in (a) and (b) for the Soret-type FZP and achromatic metalens, respectively. In addition, the incident wave ($p_i$) comes from the negative direction of the $z$-axis, which has been marked in (c) as an example.

| Unit cell | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12   | 13   | 14   | 15   | 16   |
|-----------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| $w_0(\Lambda)$ | 0.2371 | 0.2858 | 0.3831 | 0.5589 | 0.6061 | 0.6093 | 0.6093 | 0.8990 | 0.8990 | 0.2536 | 0.4661 | 0.6347 | 0.8990 | 0.2585 | 0.3140 | 0.8441 | 0.8990 |
| $w_1(\Lambda)$ | 0.2987 | 0.3087 | 0.3017 | 0.3010 | 0.2330 | 0.2907 | — — | — — | — — | 0.2996 | 0.2650 | 0.2653 | — — | 0.2484 | 0.2515 | 0.0558 | — — |
| $w_2(\Lambda)$ | 0.2847 | 0.2873 | 0.2900 | 0.2888 | 0.2876 | — — | — — | — — | — — | 0.2851 | 0.3115 | — — | — — | 0.0986 | 0.1751 | — — | — — |
| $w_3(\Lambda)$ | 0.2888 | 0.2966 | 0.3169 | 0.3158 | 0.2938 | — — | — — | — — | — — | 0.2923 | 0.2675 | — — | — — | 0.2489 | 0.2525 | 0.0559 | — — |

2. The physical principle to design an achromatic metalens

For an achromatic transmitted metalens in figure 1(b), where only half of metalens is given due to symmetry, the transmission phases of the $n$th unit cell and the first unit, $\phi_n$ and $\phi_1$, follow a simple relation,

$$\phi_n + k\sqrt{x_{cn}^2 + F^2} = \phi_1 + k\sqrt{x_{c1}^2 + F^2}, \quad (1)$$

where $F$ is the focal length, $x_{cn}$ is the center position of the $n$th unit cell, and $k = \omega/c_0 = 2\pi/\lambda$ is the wavenumber in air. However, for the conventional Soret-type FZP in figure 1(a), the radius of each zone is determined by $r_n = \sqrt{n\Lambda F + \left(\frac{L_n}{2}\right)^2}$, $(n = 1, 2, \ldots)$, and the focal length $F$ has a dependence on frequency for the given radius $r_n$. To reduce the chromatic aberration, a dipole in figure 1(c) and a monopole in figure 1(d) are substituted for the transparent zone and the opaque zone, respectively, where the dipole is
Figure 2. (a)–(c) Dependence of transmission phase $\phi$ on frequency $f$ for 16 unit cells marked in figure 1(b), while (d)–(f) give the corresponding dependence of the volume velocity ratio between dipole and monopole on frequency, in which the peak point (') and valley point (') represent the dominant contribution of dipole and monopole, respectively. Note that the frequencies of dipole and monopole marked in (d)–(f) are also marked correspondingly in (a)–(c).

achieved by Fano resonances [11, 45, 46], and it can also be obtained by coiled-up space [47]. By the position of the first opaque zone in figure 1(a), the seventh unit cell in figure 1(b) is set to be the monopole, and its transmission phase is approximate to $\pi$ when Fabry–Pérot resonance is satisfied, $l = c_0/2f_0$, with $l$ being the thickness of lens, the sound speed $c_0 = 343$ m s$^{-1}$, $f_0 = 0.355c_0/\Lambda$ a reference frequency, and $\Lambda$ the width of unit cell. Once the seventh unit cell is given, the required transmission phases of all unit cells are determined by equation (1) with the given focal length $F = 4c_0/f_0 = 4\Lambda/0.355$. For detailed parameters of each unit cell, equation (1) is adopted by a particle swarm optimization algorithm to obtain the width of slit and widths of cavities of three Helmholtz resonators, which are marked by $w_0$, $w_1$, $w_2$, and $w_3$ in figure 1(c), respectively. Through manipulating these four widths, the response of dipole can be smoothly transferred to the response of monopole, and the corresponding optimized values for each unit cell are listed in table 1. Note that a high transmission coefficient $|t_0| > 0.85$ is achieved for each unit cell with a relative bandwidth of 16.9%, larger than the bandwidth (11.5%) of achromatic metalens in the visible [48]. Therefore, the designed metalens is called as achromatic acoustic metalens.

By the theoretical derivations given in appendix A, figures 2(a)–(c) gives the theoretical transmission phases $\phi$ of 16 unit cells, which are extracted from the transmission coefficient $t_0$ of a normally incident plane wave. It is found that the transmission phase gradually changes from $2\pi$ (or 0) to $\pi$ for the number of unit cells from 1 to 8 in (a), from 9 to 12 in (b), and from 13 to 16 in (c), which look like the phase-reversal zone plates, but the transmission phase has a frequency-dependent characteristic, resulting in the reduction of chromatic aberration. By calculating the volume velocity $U_0$ at the inlet ($z = 0$) of each unit and the
volume velocity $U_l$ at the outlet ($z = l$), the contribution of monopole and dipole can be described by two quantities $\frac{U_l - U_0}{2}$ and $\frac{U_l + U_0}{2}$, respectively, and we then define a volume velocity ratio between the dipole and monopole $R = \frac{\frac{U_l + U_0}{2}}{\frac{U_l - U_0}{2}}$. It is seen in figures 2(d)–(f) that the volume velocity ratios have peak points or valley points for different unit cells. The peak point represents that the contribution of dipole dominates the transmission, having a phase near $2\pi$. However, for the valley point, the monopole plays a role in tuning the transmission phase close to $\pi$. Therefore, the proposed achromatic metalens generalizes the conventional phase-reversal zone plates by introducing the dipole and monopole as a substitute, which not only has the phase-reversal characteristic, but also has abilities to tune the frequency dependence of transmission phase, resulting in an achromatic focusing.

To validate theoretical predictions, the finite element simulation is carried out by using the software COMSOL MULTIPHYSICS, where the viscous friction on the wall of apertures is modeled using the thermoviscous acoustic module. It is shown in figure 3(a) that the theoretical transmission coefficients $t_0$ for the first unit cell and the seventh unit cell are consistent with the numerical simulation, except for the simulated transmission peak of the first unit cell at the frequency near the bandgap induced by Helmholtz resonances. The corresponding volume velocity ratios between the dipole and monopole are also compared in figure 3(b). It is illustrated that the peak point (dipole) and the valley point (monopole), adapted to design the metalens, are robust to the thermoviscous effect. In addition, the simulated distributions of normalized pressure and velocity for the dipole and monopole are given in figures 3(c) and (d), respectively. It is shown that the vibration of dipole at inlet and outlet of the unit cell are in phase, while an anti-phase motion is observed for the monopole.

### 3. The performance of achromatic focusing

With the optimized parameters in table 1, it is illustrated in figure 4(a) that the designed metalens can focus sound waves at the predetermined focusing position. To demonstrate the performance of achromatic focusing, the transmitted intensity along the axis of lens ($x = 0$ m) is given in figure 4(b) for a normally incident plane wave of unit amplitude. For comparison, the Soret-type FZP (shown in figure 1(a)) is also designed with its size and the focusing position being the same as the metalens. Figure 4(c) shows that the focusing intensity of Soret-type FZP is lower than that of metalens in figure 4(a) at the reference frequency $f_0 = 3550$ Hz, and its focusing position is slightly deviated from the predetermined value. This is because
that the value of the phase retardation difference has a little deviation from $2\pi$ for sound waves propagating from the center of the neighbouring transparent zone of the Soret-type FZP to the predetermined focal point. In addition, an obvious chromatic aberration is observed in figure 4(d) when the frequency $f$ is away from its reference value $f_0$. By comparing between the transmitted intensities along the axis of lens in figures 4(b) and (d) (the same color bar range), it is concluded that an achromatic high efficient focusing can be obtained by the proposed metalens.

Figure 5 shows the focusing performance comparison under the oblique incidence. It is found that the achromatic focusing can also be maintained at the incident angle $\theta_i = 10^\circ$ and $\theta_i = 30^\circ$ in figures 5(a) and (c), respectively. Owing to the design of achromatic metalens at normal incidence, the focusing intensity decreases with the incident angle (see the difference between the color bar range in (a) and (c)), which means that the proposed achromatic metalens cannot be worked at very large incident angles. For comparison, the transmitted intensities through FZP at the incident angles $\theta_i = 10^\circ$ and $\theta_i = 30^\circ$ are also illustrated in figures 5(b) and (d), respectively, where an obvious chromatic aberration and a lower focusing efficiency are observed.

4. Experimental measurement

Then an experiment is carried out to confirm the theoretical prediction. Figure 6(b) shows a photo of achromatic metalens, which is fabricated by 3D printing with the optimized parameters given in table 1. This metalens consists of 32 unit cells ($\Lambda = 3.43$ cm), where the aperture of the lens is $L = 1.0976$ m (along the $x$-axis), its thickness $l \approx 4.8$ cm (along the $z$-axis), and its height 3 cm (along the $y$-axis). In addition, two walls perpendicular to the $y$-axis are added to the fabricated lens and their thickness is 2 mm. Figure 6(a) illustrates the fabricated Soret-type FZP with its size and the predetermined focusing position being the same as the achromatic metalens. Figure 6(c) shows a schematic diagram of experimental measurement, where the lens is sandwiched in between two Plexiglas plates of dimensions $4$ m $\times$ $2$ m $\times$ $8$ mm, the edge of which is closed by anechoic cottons. A loudspeaker is placed at a distance $2.17$ m away from the lens to radiate sine waves in the frequency range from 3300 to 4000 Hz. A probe microphone is moved in 1 cm step by a stepping motor to detect acoustic field along the measurement line marked in figure 6(c). All the receiving acoustic signals are achieved by the National Instruments PXI-6733 and PXI-4496.
Figure 5. The transmitted intensity $|p(x,z)|^2$ for a plane wave of unit amplitude incident on the lens at an angle $\theta_i$. (a) For achromatic metalens at $\theta_i = 10^\circ$, in which transmitted intensities at eight frequencies ($f = 3200 : 100 : 3900$ Hz) are shown. (b) For FZP at $\theta_i = 10^\circ$. (c) for achromatic metalens at $\theta_i = 30^\circ$, and (d) for FZP at $\theta_i = 30^\circ$. Note that the dashed lines represent the predetermined focusing position $z - l = 4\Lambda/0.355$ with $\Lambda = 3.43$ cm. The range along x-axis in each subfigure varies from $-0.5$ m to $0.5$ m. In addition, for the identical incident angle, the same color bar range is used for the comparison between the achromatic metalens and FZP.

Since sound radiation from a point source in the two-dimensional free space is attenuated with the distance away from it, the method of insertion-loss measurement is always adopted to eliminate the effect of distance attenuation, where the insertion loss is defined as the ratio of sound pressure being measured with and without sample in dB [49]. Similar to the concept of insertion loss, the normalized transmitted intensity $|p(x,z)|^2/|p_{\text{free}}(x,z)|^2$ is introduced here to describe the focusing performance, where $p(x,z)$ and $p_{\text{free}}(x,z)$ represent the measured sound pressure with and without the lens, respectively. Since $p_{\text{free}}(x,z)$ represents the pressure of the incident wave, the ratio $|p(x,z)|^2/|p_{\text{free}}(x,z)|^2$ can be considered as the power transmission coefficient, which eliminates the effect of distance attenuation, especially when the point source is located in the near-field zone. On the other hand, the introduction of this ratio has an advantage that we do not need to precisely control the amplitude of incident wave. Figure 7(a) gives the experimental normalized transmitted intensity of achromatic metalens. It is found that the focal length for the incidence of a point source is larger than that of plane wave incidence. This is because that the point source is located in the near-field zone, $L^2/\lambda > 2.17$ m. For the Soret-type FZP in figure 7(c), an obvious chromatic aberration for the focusing of acoustic field is observed. For clarity, the range of color bar for Soret-type FZP is half that for the achromatic metalens, which means that the focal intensity of achromatic metalens is approximately twice that of Soret-type FZP. In addition, the simulated normalized transmitted intensities of achromatic metalens and FZP are given in figures 7(b) and (d), respectively. By comparison, it is concluded that the experimental measurement is consistent with the theoretical prediction.
Figure 6. (a) Photograph of the fabricated FZP. (b) Photograph of the fabricated achromatic metalens. (c) Schematic diagram of experimental measurement.

Figure 7. Frequency dependence of normalized transmitted intensity $|p(x,z)|^2/|p_{\text{free}}(x,z)|^2$ along $x = 0$ m, where $p_{\text{free}}(x,z)$ represents the sound pressure in free space. Experimental results for a point source located at position (0, −2.17 m) incident on an achromatic lens in (a) and a Soret-type FZP in (c), and the corresponding simulated results are given in (b) and (d). Note that the dashed lines represent the predetermined focusing position under a normally incident plane wave, and the range of color bar for (a) and (b) is twice that for (c) and (d).
5. Conclusion

In summary, we have proposed an achromatic acoustic generalized phase-reversal zone plate, the focusing performance of which is demonstrated numerically and confirmed experimentally. Although the working bandwidth of the proposed achromatic metalens is narrower compared to that designed by using a bottom-up inverse-design paradigm [28], the proposed metalens is designed based on an intuitive and physical approach, and it has the advantages of higher efficient focusing (transmission coefficient $|t_0| > 0.85$ for each unit cell) and thinner thickness (one half of central wavelength). Therefore, the proposed metalens has potential application in acoustic energy harvesting and imaging.

Acknowledgments

We wish to acknowledge the support of the National Science Foundation of China under Grant No. 11674293.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

Appendix A. Theoretical method

By the transfer matrix method and Floquet’s theory [50], we derive the volume velocities $U_0$ and $U_I$ at the inlet ($z = 0$) and outlet ($z = l$) of a metasurface, respectively. Then, the transmission coefficient $t_0$ is obtained. Without loss of generality, a metasurface made up of unit cells shown in figure 1(c) is adopted in theoretical derivation. Note that we will always omit the temporally harmonic factor $e^{-j\omega t}$ as understood throughout this paper, and since the higher-order modes in the cavity of Helmholtz resonator are being adopted, the viscothermal loss is neglected in the theoretical derivations for simplification, however, the viscothermal loss is considered in FEM simulations.

A.1. Input acoustic impedance of a Helmholtz resonator grafted to the wall of a slit

In this subsection, with consideration of acoustic end correction, the input acoustic impedance of a Helmholtz resonator grafted to the wall of a slit is derived. For convenience, a local coordinate system $x'O'y'$ is introduced in figure 8.

Let the sound pressure in the throat ($\frac{c_0}{2} < x' < \frac{c_0}{2} + b, -\frac{h_0}{2} < z' < \frac{h_0}{2}$) be expressed as,

$$p(x', z') = A e^{j(k(x' - \frac{c_0}{2}) + B e^{-j(k(x' - \frac{c_0}{2} + b)),}}$$ (A1)

and the sound pressure in the cavity ($\frac{c_0}{2} + b < x' < \frac{c_0}{2} + b + w, -\frac{b}{2} < z' < \frac{b}{2}$) be expanded as,

$$p(x', z') = \sum_{n=0}^{\infty} 2B\varphi_n(z') \cos \left[ \sqrt{k_n^2 - k_{in}^2} \left( x' - \frac{w_0}{2} - b - w \right) \right],$$ (A2)

where the eigenmode $\varphi_n(z') = \sqrt{2 - \delta_{0n}} \cos (\pi n (z' + \frac{b}{2}))$ satisfies the rigid boundary condition at $z' = \pm b/2$, and $k_{in} = \frac{\pi}{h}$. By the continuity of sound pressure and normal velocity at the boundary between the throat and cavity ($x' = \frac{c_0}{2} + b$), we obtain a relationship between the coefficients $A$ and $B$ in equation (A1),

$$A = B e^{-jkb} \chi,$$ (A3)

with $\chi = \left( 1 - \sum_{n=0}^{\infty} \frac{8 \cos \left( 2 \sqrt{k_n^2 - k_{in}^2} \right)}{\sqrt{k_n^2 - k_{in}^2} \sin \left( 2 \sqrt{k_n^2 - k_{in}^2} \right)} \left( \sqrt{2 - \delta_{0n}} \cos (\pi n) \right)^2 \right)^{-1} \times \left( 1 - \sum_{n=0}^{\infty} \frac{8 \cos \left( 2 \sqrt{k_n^2 - k_{in}^2} \right)}{\sqrt{k_n^2 - k_{in}^2} \sin \left( 2 \sqrt{k_n^2 - k_{in}^2} \right)} \left( \sqrt{2 - \delta_{0n}} \cos (\pi n) \right)^2 \right)^{-1}$.

Substituting equation (A3) into equation (A1), and using the equation of motion $jk\rho_0 c_0 u_n(x', z') = \partial p/\partial x'$, we obtain the input acoustic impedance at $x' = \frac{c_0}{2} + b$,

$$Z_{in} \bigg|_{x' = \frac{c_0}{2} + b} = \left. \frac{p(x', z')}{{\partial}_x u_n(x', z')} \right|_{x' = \frac{c_0}{2} + b} = \frac{\rho_0 c_0 (\chi + 1)}{\rho_0 c_0 (\chi - 1)},$$ (A4)
where $\rho_0 G_0$ is the characteristic acoustic impedance of air, and $a$ the breadth of throat. $Z_a |_{x' = \mp a + b}$ in equation (A4) can be transferred to the inlet of Helmholtz resonator ($x' = \mp a$),

$$Z_a |_{x' = \mp a + b} = \frac{\rho_0 G_0}{a} \tan(kb) + \frac{j \rho_0 k^2}{\tan(kb)} \frac{a}{\tan(kb)} \tan(kb) \tag{A5}$$

Note that the effect of higher-order modes in the cavity becomes obvious with the increase of frequency, and it is illustrated in figure 3(a) that the existence of the higher-order modes affects the transmission in the working frequency range of the designed achromatic metalens.

In order to describe the effect of acoustic end correction, Green’s theorem is used to describe the sound field in the slit [45],

$$p(x', z') = p_{inc}(x', z') + \int_{-\frac{a}{2}}^{\frac{a}{2}} G(x', z', x_0', z_0') \left. \frac{\partial p(x_0', z_0')}{\partial x_0'} \right|_{x_0' = \mp a} dz_0', \tag{A6}$$

where $p_{inc}(x', z')$ represents the sound pressure of incident wave (marked in figure 8), and the chosen Green’s function $G(x', z', x_0', z_0') = \sum_{n=0}^{\infty} \frac{\alpha_n(x') \phi_n(x_0')}{2j\sqrt{\kappa_n^2 - k_n^2}} \cos[k_n(x' - z') + \beta_n(x_0' - z_0')]$ satisfies the rigid boundary condition at the wall of slits, where the transverse eigenmode is $\phi_n(x') = \sqrt{2} - \frac{\partial \ln \cos(\kappa_n(x' + \frac{w}{2}))}{\partial x' + \frac{w}{2}}$ and $k_n = \frac{\omega_n}{c_0}$.

Using the equation of motion $jk\rho_0 c_0 \phi_n(x', z') = \partial p/\partial x'$, and introducing the average velocity $\bar{u}_x = \frac{1}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} u_n(x_0', z_0') |_{x_0' = \mp a} dz_0'$, equation (A6) can be approximated as,

$$p(x', z') \approx p_{inc}(x', z') + jk\rho_0 c_0 \bar{u}_x \int_{-\frac{a}{2}}^{\frac{a}{2}} G(x', z', x_0', z_0') |_{x_0' = \mp a} dz_0'. \tag{A7}$$

By making an average of pressure $p(x', z')$ in equation (A7) along the inlet of Helmholtz resonator ($x' = \mp a$, $-\frac{a}{2} < z' < \frac{a}{2}$), we obtain

$$\left( Z_a |_{x' = \mp a} + Z_{duc} \right) \bar{u}_x \approx \frac{1}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} p_{inc}(x', z') |_{x_0' = \mp a} dz', \tag{A8}$$

where $\frac{1}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} p(x', z') |_{x_0' = \mp a} dz' = Z_a |_{x' = \mp a} (\bar{u}_x)$ is used according to the definition of acoustic impedance, and $Z_{duc} = -j k \rho_0 c_0 \frac{1}{a} \int_{-\frac{a}{2}}^{\frac{a}{2}} G(x', z', x_0', z_0') |_{x_0' = \mp a} dz_0' dz'$. Then, the input acoustic impedance of a Helmholtz resonator grafted to the wall of a slit ($Z_b$) can be expressed as,

$$Z_b = Z_a |_{x' = \mp a} + j \text{Im}(Z_{duc}), \tag{A9}$$

where $\text{Im}(\cdot)$ means the imaginary parts.
A.2. The transfer matrix between the input and output of a metasurface

By considering Helmholtz resonators as point-like resonators [51], the transfer matrix is derived in this subsection.

For a metasurface made up of unit cells shown in figure 1(c) under a global coordinate system xOz, let the sound pressure in the slit (0 < x < w₀, 0 < z < l₁) be expressed as,

\[ p(x, z) = A₁ e^{j k x} + B₁ e^{-j k x}. \]  \hspace{1cm} (A10)

Using the equation of motion along the z-axis, \( j k ρ₀ c₀ u_1(x, z) = \partial p(x, z)/\partial z \), we obtain

\[
\begin{bmatrix}
    p|_{z=0} \\
    u_z|_{z=0}
\end{bmatrix} = M₁ \begin{bmatrix}
    p|_{z=l₁} \\
    u_z|_{z=l₁}
\end{bmatrix},
\]  \hspace{1cm} (A11)

where the matrix is \( M₁ = \begin{bmatrix}
    \cos(kl₁) & -j ρ₀ c₀ \sin(kl₁) \\
    -j ρ₀ c₀ \sin(kl₁) & \cos(kl₁)
\end{bmatrix} \), and \( u_z \) the velocity along the z-axis.

At the position of the first Helmholtz resonator (x = w₀, z = l₁), we have the continuity of sound pressure,

\[ p|_{z=l₁} = p|_{z=l₁+}. \]  \hspace{1cm} (A12)

and the continuity of volume velocity

\[ w₀ u_z|_{z=l₁} = w₀ u_z|_{z=l₁+} + a uₖ = w₀ u_z|_{z=l₁+} + \frac{p|_{z=l₁+}}{Zₜ₁}, \]  \hspace{1cm} (A13)

where \( a uₖ \) represents the volume velocity at the inlet of the first Helmholtz resonator, and \( Zₜ₁ \) denotes the input acoustic impedance of the first Helmholtz resonator being grafted to the wall of a slit, given by equation (A9). Equations (A12) and (A13) can be written in a matrix form as

\[
\begin{bmatrix}
    p|_{z=l₁} \\
    u_z|_{z=l₁}
\end{bmatrix} = N₁ \begin{bmatrix}
    p|_{z=l₁+} \\
    u_z|_{z=l₁+}
\end{bmatrix},
\]  \hspace{1cm} (A14)

with the matrix \( N₁ = \begin{bmatrix}
    1 & 0 \\
    \frac{1}{w₀ Zₜ₁} & 1
\end{bmatrix} \).

Similar to the derivations adopted in equations (A10)–(A14), we finally get a transfer matrix between the input and output of the metasurface as,

\[
\begin{bmatrix}
    p|_{z=0} \\
    u_z|_{z=0}
\end{bmatrix} = M₁ N₁ M₂ N₂ M₃ N₃ M₄ \begin{bmatrix}
    p|_{z=l} \\
    u_z|_{z=l}
\end{bmatrix},
\]  \hspace{1cm} (A15)

with the matrices \( M₂ = \begin{bmatrix}
    \cos(kl₂) & -j ρ₀ c₀ \sin(kl₂) \\
    -j ρ₀ c₀ \sin(kl₂) & \cos(kl₂)
\end{bmatrix} \), \( M₃ = \begin{bmatrix}
    \cos(kl₃) & -j ρ₀ c₀ \sin(kl₃) \\
    -j ρ₀ c₀ \sin(kl₃) & \cos(kl₃)
\end{bmatrix} \),

\[
M₄ = \begin{bmatrix}
    \cos(kl₄) & -j ρ₀ c₀ \sin(kl₄) \\
    -j ρ₀ c₀ \sin(kl₄) & \cos(kl₄)
\end{bmatrix},
\]

\( N₂ = \begin{bmatrix}
    1 & 0 \\
    \frac{1}{w₀ Zₜ₂} & 1
\end{bmatrix} \) and \( N₃ = \begin{bmatrix}
    1 & 0 \\
    \frac{1}{w₀ Zₜ₃} & 1
\end{bmatrix} \). Note that the corresponding parameters of unit cell are marked in figure 1(c). Owing to the assumption of plane waves propagating in the slit, we introduce the average pressures \( \bar{p}|_{z=0} = \frac{1}{w₀} \int₀^w p|_{z=0} dx \) and \( \bar{p}|_{z=l} = \frac{1}{w₀} \int₀^w p|_{z=l} dx \) at the inlet and outlet of metasurface, respectively. Similarly, the volume velocities \( U₀ = \int₀^w u_z|_{z=0} dx \) and \( Uₜ = \int₀^w u_z|_{z=l} dx \) are defined at the inlet and outlet of metasurface, respectively. We then rewrite equation (A15) as,

\[
\begin{bmatrix}
    \bar{p}|_{z=0} \\
    w₀⁻¹ U₀
\end{bmatrix} = M₁ N₁ M₂ N₂ M₃ N₃ M₄ \begin{bmatrix}
    \bar{p}|_{z=l} \\
    w₀⁻¹ Uₜ
\end{bmatrix}.
\]  \hspace{1cm} (A16)

A.3. The derivation of transmission coefficient \( t₀ \)

We assume a unit incident sound pressure,

\[ p_i = \exp(j k \sin \theta x + j k \cos \theta z), \]  \hspace{1cm} (A17)
where $\theta_i$ represents the angle of incident waves. According to Floquet’s theory, sound fields on the side $z < 0$ can be expanded in series,

$$p(x, z) = p_i + p_t = \exp(jk \sin \theta x + jk \cos \theta z) + \sum_{n=-\infty}^{\infty} r_n \exp(jk\beta_n x + k\alpha_n z), \quad (A18)$$

where $r_n$ is the reflection coefficient of the $n$th diffraction order, $k\beta_n = k \sin \theta_i + 2\pi n / \Lambda$ the horizontal component of wave number along the $x$-axis, and $jk\alpha_n = k \sqrt{1 - \beta_n^2}$ the vertical component of wave number along the $z$-axis. Similarly, the transmitted pressure on the side $z > l$ is expanded as,

$$p_t = \sum_{n=-\infty}^{\infty} t_n \exp(jk\beta_n x - k\alpha_n(z - l)), \quad (A19)$$

where $t_n$ is the transmission coefficient of the $n$th diffraction order.

Using the equation of motion along the $z$-axis, $jk\rho_0 u_c(x, z) = \partial p(x, z) / \partial z$, and equation (A18), the vertical velocity component $u_c(x, z)$ at the boundary $z = 0$ can be described as,

$$jk\rho_0 u_c(x, z) = -k\alpha_0 \exp(jk\beta_0 x) + \sum_{n=-\infty}^{\infty} r_n k\alpha_n \exp(jk\beta_n x). \quad (A20)$$

By the continuity of normal velocity and the orthogonality of function $\exp(jk\beta_n x)$, we obtain from equation (A20) the reflection coefficient,

$$r_n = \delta_{0n} - (j\alpha_n \Lambda)^{-1} \rho_0 \Phi_n(\theta_i) U_0, \quad (A21)$$

where $U_0$ represents the volume velocity defined in equation (A16), and $\Phi_n(\theta_i) = \int_{0}^{\omega} \exp(-jk\beta_n x)dx$. Similar to the derivation adopted in equations (A20) and (A21), the transmission coefficient can be determined by the volume velocity at the outlet of the metasurface ($z = l$),

$$t_n = (j\alpha_n \Lambda)^{-1} \rho_0 \Phi_n(\theta_i) U_l. \quad (A22)$$

By substituting equation (A21) into equation (A18), and making an average of pressure $p(x, z)$ along the inlet of the metasurface ($0 < x < w_0, z = 0$), we obtain

$$\bar{p}|_{z=0} = 2\Phi_0(\theta_i) - \sum_{n=-\infty}^{\infty} \left(j\alpha_n \Lambda\right)^{-1} \rho_0 \Phi_n(\theta_i)^2 U_0. \quad (A23)$$

Similarly, substituting equation (A22) into equation (A19), and making an average of pressure $p(x, z)$ along the outlet of the metasurface ($0 < x < w_0, z = l$), we obtain

$$\bar{p}|_{z=l} = \sum_{n=-\infty}^{\infty} \left(j\alpha_n \Lambda\right)^{-1} \rho_0 \Phi_n(\theta_i)^2 U_l. \quad (A24)$$

With the combination of equations (A16), (A23) and (A24), we obtain the volume velocities $U_0$ and $U_l$, and then the transmission coefficient $t_0$ can be determined by using equation (A22).

References

[1] Yu N, Genevet P, Kats M A, Aieta F, Capasso F and Gaburro Z 2011 Science 334 339–7
[2] Chen Y, Zhou C, Yuan B G, Wu D J, Wei Q and Liu X J 2015 Nat. Mater. 14 1013
[3] Ma G C and Sheng P 2016 Sci. Adv. 2 e1501595
[4] Assouar B, Liang B, Wu Y, Li Y, Cheng J-C and Jing Y 2018 Acoustic metasurfaces Nat. Rev. Mater. 3 460
[5] Li J, Shen C, Diaz-Rubio A, Tretyakov S A and Cummer S A 2018 Nat. Commun. 9 1342
[6] Fu Y, Shen C, Zhu X, Lu J, Liu Y, Cummer S A and Xu Y 2020 Sci. Adv. 6 eaax9876
[7] Climente A, Torrent D and Sánchez-Dehesa J 2010 Appl. Phys. Lett. 97 104103
[8] Romero-García V, Cebrecos A, Picó R, Sánchez-Morcillo V J, García-Raffi L M and Sánchez-Pérez J V 2013 Appl. Phys. Lett. 103 264106
[9] Jiménez N, Romero-García V, Picó R, Cebrecos A, Sánchez-Morcillo V J, García-Raffi L M, Sánchez-Pérez J V and Stalinius K 2014 Europhys. Lett. 106 24005
[10] Wang W, Xie Y, Konneker A, Popa B-I and Cummer S A 2014 Appl. Phys. Lett. 105 101904
[11] Li Y, Jiang X, Liang B, Cheng J and Zhang L 2015 Phys. Rev. Appl. 4 024003
[12] Hyun J, Kim Y T, Doh I, Ahn B, Balk K and Kim S-H 2018 Sci. Rep. 8 9131
[13] Chen J, Rao J, Lisevych D and Fan Z 2019 Appl. Phys. Lett. 114 104101
[14] Zhao Y, Subramanian S and Memoli G 2021 Appl. Phys. Lett. 119 141907
[15] Lu Y-J, Zou H-Y, Qian J, Wang Y, Ge Y, Yuan S-Q, Sun H-x and Liu X-J 2021 Appl. Phys. Lett. 119 173501
[16] Xie H and Hou Z 2021 Phys. Rev. Appl. **15** 034054
[17] Gao H, Gu Z, Liang S, Liu T, Zhu J and Su Z 2022 Appl. Phys. Lett. **120** 111701
[18] Wang S et al 2017 Nat. Commun. **8** 187
[19] Shrestha S, Overvig A C, Lu M, Stein A and Yu N 2018 Light Sci. Appl. **7** 85
[20] Wang S et al 2018 Nat. Nanotechnol. **13** 227–32
[21] Lin R J et al 2019 Nat. Nanotechnol. **14** 227–31
[22] Park C M and Lee S H 2015 J. Appl. Phys. **117** 034904
[23] Zhu Y-F, Zou X-Y, Li R-Q, Jiang X, Tu J, Liang B and Cheng J-C 2015 Sci. Rep. **5** 10966
[24] Park C M, Kim C H, Park H T and Lee S H 2016 Appl. Phys. Lett. **120** 111701
[25] Zhu Y and Assouar B 2019 Phys. Rev. **B** 99 174109
[26] Wang P, Yu G, Li Y, Wang X and Wang N 2020 New J. Phys. **22** 023006
[27] Hyun J, Cho W-H, Park C-S, Chang J and Kim M 2020 Appl. Phys. Lett. **116** 234102
[28] Dong H W et al 2022 Nan. Sci. Rev. https://doi.org/10.1093/nss/nvac001
[29] Ribeiro R R, Dahal P, Guerreiro A, Jorge P A S and Viegas J 2017 Sci. Rep. **7** 4485
[30] Wang Y, Yun W and Jacobsen C 2003 Nature **424** 50–3
[31] Jefimovs K, Bunk O, Pfeiffer F, Grollimund D, van der Veen J F and David C 2007 Microelectron. Eng. **84** 1467–70
[32] Wang S, Zhang X-C, Maley M P, Hundley M F, Bulavskii L N, Koshelev A E and Taylor A J 2002 Opt. Photon. News **13** 58
[33] Hristov H D and Herben M H A J 1995 IEEE Trans. Microwave Theory Techn. **43** 2779–85
[34] Calvo D G, Thangawng A L, Nicholas M and Layman C N 2015 Appl. Phys. Lett. **107** 014103
[35] Clement G, Nomura H and Kamakura T 2015 IEEE Trans. Utron. Ferroelect. Freq. Control **62** 350–9
[36] Pérez-López S, Fuster J, Candelas P, Rubio C and Belmar F 2018 Appl. Phys. Lett. **112** 264102
[37] Tarrazó-Serrano D, Pérez-López S, Candelas P, Uris A and Rubio C 2019 Sci. Rep. **9** 7067
[38] Xia X, Li Y, Cai F, Zhou H, Ma T and Zheng H 2020 Appl. Phys. Lett. **117** 021904
[39] Saavedra G, Furlan W D and Monsoriu J A 2003 Opt. Lett. **28** 971–3
[40] Furlan W D, Saavedra G and Monsoriu J A 2007 Opt. Lett. **32** 2109–11
[41] Mendoza-Yero O, Fernández-Alonso M, Mínguez-Vega G, Lancis J, Climent V and Monsoriu J A 2009 J. Opt. Soc. Am. A **26** 1161–6
[42] Pérez-López S, Fuster J M, Candelas P and Rubio C 2019 Ultrasonics **99** 105967
[43] Ferrando V, Giménez F, Furlan W D and Monsoriu J A 2015 Opt. Express **23** 19846–53
[44] Qi L, Yu G, Wang X, Wang G and Wang N 2014 J. Appl. Phys. **116** 234506
[45] Yang X, Yin J, Yu G, Peng L and Wang N 2015 Appl. Phys. Lett. **107** 193905
[46] Liang Z and Li J 2012 Phys. Rev. Lett. **108** 114301
[47] Khorasaniejad M, Shi Z, Zhu A Y, Chen W T, Sanjeev V, Zaidi A and Capasso F 2017 Nano Lett. **17** 1819–24
[48] Fahy F and Gardonio P 2007 Sound and Structural Vibration: Radiation, Transmission and Response (Amsterdam: Academic)
[49] Craster R V and Guenneau S 2013 Acoustic Metamaterials: Negative Refraction, Imaging, Lensing and Cloaking (Springer Series in Materials Science) (Dordrecht: Springer)
[50] Richoux O, Tournat V and Le Van Suu T 2007 Phys. Rev. E **75** 026615