The effect of the random distribution of electronic components in the output characteristics of the Howland current source

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Abstract. When a Howland source is designed, the components are chosen so that the designed source has the desired characteristics. However, the operational amplifier limitations and resistor tolerances causes undesired behaviours. This work proposes to take in account the influence of the random distribution of the commercial resistors in the Howland circuit over the frequency range of 10 Hz to 10 MHz. The probability density function due to small changes over the resistors was calculated by using an analytical model. Results show that both output current and impedance are very sensitive to the resistor tolerances. It is shown that the output impedance is very dependent on the open-loop gain of the Opamp rather than the resistor tolerances, especially at higher frequencies. This might improve the implementations of real current source used in electrical bioimpedance.

1. Introduction
Because of its simplicity, stability and other advantages, the voltage controlled current source (VCCS) has been widely used in many applications, such as single-electrode capacitive sensors [1], electrical impedance tomography (EIT) systems both for industrial and medical applications [2] and bioimpedance analysis (BIA) for tissue characterization [3]. Most VCCS circuits in BIA use the Howland current source (HCS) [3]. The first HSC circuit was proposed by Howland in 1962 [4] for converting a voltage into a current. However, it suffers from output voltage compliance [5]. Therefore, modified version circuit have been proposed and widely used [6].

The most important requirement in EIS systems is to assure that the injecting current, also called source current, has constant amplitude over a wide frequency range, which should be obtained by a high output impedance circuit [7]. However, stray capacitances [7] and non-idealities of the operational amplifiers used for the design [1] reduce the current amplitude and introduce phase shift errors at higher frequencies. However, some of these requirements are essentially in conflict with each other. For example, the frequency of the current source is practically limited to 100 kHz when the output impedance is required to be sufficiently larger, say larger than 1 MΩ [8]. In most publications, the analysis of the circuit is based on either simplified ideal op-amp circuits or using simulation tools, e.g. PSpice or Multisim from NI [8]. In many cases, however, the formulas they used are not suitable because the calculation errors are too large with a high-frequency current source [8]. Furthermore, the formulas do not take into account the mismatch between the resistors used to design such a current source. The HCS circuit is sensitive upon this mismatching [6].
The objective of this work is to investigate the probability density function and the sensibility of a current source over the frequency range of 10 Hz to 10 MHz due to resistor tolerances.

2. Materials and Method

2.1. Modeling of the modified Howland circuit

Figure 1 shows the modified Howland current source (HCS) circuit used for modeling the output characteristics taking into account the effects of the resistors tolerances. If all resistors are perfectly matched and the operational amplifier (Opamp) has got a large gain, then the output current is given by $-\text{Vin}/r$ (assuming $R_2=R_3=R$, $R_4=r$ and $R_1=R+r$). This approximation yields a good result if the Opamp gain is sufficiently high and the load impedance is small. In order to get high output impedance it is necessary to trim the resistors and choose FET inputs amplifiers. On the other hand, the non-ideal characteristics of the Opamps (i.e., input impedance $Z_{\text{in}}$, output resistance $R_o$ and open-loop gain $A$) reduce frequency bandwidth of both output current and impedance.

![Figure 1. Schematic diagram of the modified Howland current source with grounded load [modified from 9].](image)

The circuit can be characterized by the nodal equations (1), where $V_1$ is the voltage at non-inverting input of the Opamp, $V_2$ is the inverting input of the Opamp and $V_3$ is the voltage at the output of the Opamp. After determining the voltages, the output current can be calculated by equation (2). The transconductance gain is calculated by $I_{\text{out}}/V_{\text{in}}$ when $V_{\text{out}}$ is grounded. On the other hand, the modulus of the output impedance $Z_{\text{out}}$ is calculated by $V_{\text{out}}/I_{\text{out}}$ when $V_{\text{in}}$ is grounded.

2.2. Probability Distribution Function

The probability density was computed by using the R programming language, a random were taken randomly by producing different Howland sources. The characteristics of each source were calculated at 50 discrete frequencies per decade in the frequency range from 10 Hz to 10 MHz. The density function of the transconductance gain was estimated by the "$r$" function density and then the output impedance distribution is calculated and plotted as transparent lines for each source, as well as the maximum impedances are also calculated and plotted using a red line.

The MHCS circuit, as shown in figure 1, was designed by using $R=100 \, \text{k}\Omega$, $r=1 \, \text{k}\Omega$, $V_{\text{in}}$ is a sine wave of 1 Vp and two Opamps, such as OPA657 and uA741 (both manufactured by Texas Instruments), where the technical characteristics can be found in the manufacturer website. It was considered random variations of $\pm 1\%$ and $\pm 10\%$ over the resistor values.
The output impedance $Z_{\text{out}}$ can be calculated from equations (1) and (2), which is fully described in [9]. If the open-loop gain $A$ is assumed to be very high, then $Z_{\text{out}}$ can be given by (3).

$$
\begin{align*}
\frac{V_1 - V_{\text{out}}}{R_5} + \frac{V_1}{R_1} + \frac{V_1 - V_2}{Z_{\text{in}}} &= 0 \\
\frac{V_2 - V_{\text{in}}}{R_2} + \frac{V_2 - V_1}{Z_{\text{in}}} + \frac{V_2 - V_3}{R_3} &= 0 \\
\frac{V_3 - V_2}{R_4} + \frac{V_3 - V_{\text{out}}}{R_4} + \frac{V_3 - (V_1 - V_2)A}{R_o} &= 0
\end{align*}
$$

(1)

$$
I_{\text{out}} = \frac{V_3 - V_{\text{out}}}{R_4} + \frac{V_2 - V_{\text{out}}}{R_5}
$$

(2)

The output impedance $Z_{\text{out}}$ can be calculated from equations (1) and (2), which is fully described in [9]. If the open-loop gain $A$ is assumed to be very high, then $Z_{\text{out}}$ can be given by (3).

$$
Z_{\text{out}} |_{A=\infty} = \frac{R_2 R_4 R_5 + R_1 R_2 R_4}{R_2 R_5 + R_2 R_4 - R_1 R_3}
$$

(3)

3. Results

Figure 2 shows the transconductance gain probability density of the current source designed by using the Opamp OPA657. It is shown in terms of a gray scale, darker points are more likely to be the resulting transconductance gain. It can also be observed that the transconductance gain is flat from 10 Hz to approximately 1 MHz and this does not depend on the resistor tolerance.

Figure 3 shows the results of the output impedance modulus for both OPA657 and uA741 Opamps in the frequency range from 10 Hz to 10 MHz. The red curves show the maximum output impedance $Z_{\text{out}}$. It can be observed that the MHCS circuit designed by using the OPA657 has got higher $Z_{\text{out}}$ at over all frequencies. $Z_{\text{out}}$ is approximately 1 kΩ at 1 MHz by using the Opamp uA741 whereas 100 kΩ for the OPA657, however any resistor combination may lead the circuit to have high output impedance in a wider band.

![Figure 2](image1.png)

**Figure 2.** Probability distribution of the transconductance gain upon frequency, using a load of 1 kΩ. **(a)** δ of 10%. **(b)** δ of 1%.

4. Discussions

It was shown in figure 2 that the frequency dependency is more relevant than the ones caused by the resistor tolerances. The transconductance gain can be approximately calculated by the ratio $R_2/(R_3 R_4)$, thus if the tolerances of the resistors are $\delta_1$, $\delta_2$ and $\delta_4$, respectively, the tolerance of the
transconductance gain will have a tolerance of approximately $\delta_1 + \delta_2 + \delta_3$. It implies that the use of a wideband operational amplifier will produce a very stable transconductance gain. On the other hand, the output impedance $Z_{out}$ will be significantly changed by the tolerance of the resistors, as showed in figure 3. Any combination of resistors with a tolerance of $\pm1\%$ may produce either significant reduction in the output impedance or large variation in the transconductance gain, which might explain why the load current variations may also be large. If $\delta$ is the nominal tolerance of each resistors, then the minimum output impedance from (3) can be calculated by the ratio between “$rR$” and “$2\delta(R+r)$”. This means that the minimum $Z_{out}$ is inversely proportional to the resistor tolerances.

![Figure 3. Probability distribution of the output impedance modulus upon frequency, using resistor tolerance of 1%. (a) OPA655. (b) uA741.](image)

5. Conclusion
It was shown that both $I_{out}$ and $Z_{out}$ of the MHCS circuit is significantly sensitivity to the resistor tolerances used to design it. In order to get maximum $Z_{out}$, the resistors cannot be exactly matched as the maximum $Z_{in}$ is limited by the open-loop gain, then it can be concluded that the difficulty of getting high $Z_{out}$ at higher frequencies is due to the reduction of the open-loop gain of the Opamp. On the other hand, at lower frequencies, high $Z_{out}$ may be achieved by matching the resistors in the circuit.

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