The Financial Analysis of Public Company in Stock Exchange Mergers Ratio

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Abstract. The stock exchange mergers of public companies are the result of the modern market competition. Due to the advantages of the stock exchange mergers such as they are not limited by the scale of mergers, are able to prevent cash flow pressure, enjoy the policy of preferential tax and retain the owners' equity, they are applied by more and more public companies. Based on CAPM (Capital Asset Price Model), the paper designs a model of the common stock in the capital market which integrates stock dividends into the process of bonus sharing. The stock exchange ratio of both companies who are going to merger is calculated by the equilibrium price through this model. It provides the preliminary reference for the operation of stock exchange mergers.

1. Introduction
M&A has always been a hot spot in the theory and practice of capital markets. So far, there have been six waves of M&A around the world [1]. Each wave of M&A has its own distinctive era characteristics. At present, economic development of China has entered a period of in-depth transformation. To some extent, there are problems such as excess capacity, insufficient momentum for the development of traditional industries, low operating efficiency of enterprises, and serious homogeneous competition. Government clearly put forward the five major tasks of capacity reduction, de-stocking, deleveraging, cost reduction, and improving underdeveloped areas. State-owned enterprises have accelerated the pace of M&A. In recent years, many large state-owned enterprises including COSCO and China Shipping (Group) Company, China Minmetals Corporation and MCC have completed M&A. According to statistics, in recent years, the number of M&A of Chinese enterprises has shown a significant upward trend, stock exchange M&A, which are not restricted by the size of mergers, avoid cash flow pressure, enjoy tax benefits, and retain owner's equity, have been used more and more widely.

At the specific operational level, Effect of stock exchange merger and how to determine the ratio of stock exchange has also been one of the hot topics in academic research. At present, the research on M&A performance mainly adopts the event research method. Jensen [2], Buckley and Casson [3] use the event research method to test that M & A can create wealth for shareholders of target companies. Zhang Xin [4] conducted an empirical test on 1,216 M&A events in China Stock Market from 1993 to 2002, and concluded that M&A created value for the target company. Song Xiliang [5] took the stock exchange merger between 1998 and 2007 as the research object, believed that the main company and
the company had obtained excessive returns in the short term, but suffered losses in the long term. In
the research on the conversion ratio, Larson and Gronede [6] proposed the L-G model with the price-
earnings ratio and P/B ratio as parameters to determine the conversion ratio of both parties in the
merger. On the basis of L-G model, Joseph Yagil [7] proposed a model with growth expectation and
dividend growth, and calculated the proportional boundary value. Liu Haidong [8], Lu Yanling [9] and
others analyzed the stock exchange ratio through specific M&A cases. This article starts from the
Capital Asset Pricing Model (CAPM) for the common stock design model circulating in the market,
troduces the dividend factor into the dividend process, and uses the derived equilibrium price to
define the range of the conversion ratio between the two parties in the M&A operation.

2. Market structure assumptions
In the process of stock dividends, we only consider the impact of the dividend factor, and ignore the
difference in investor risk tolerance in the utility function, so that we can pay more attention to the
M&A operation of the two companies. In order to more clearly reflect the main characteristics of
corporate mergers, we consider the M&A of the two companies in the following special markets in the
sense of economics. That is, there are only two types of assets in the market, one is a risky asset
(hereinafter referred to as a stock) and the other is a risk-free asset. Only a limited number of investors
trade in the market. They have the same expectations for the future. They only trade in discrete time
and do not consider transaction costs. The number of risk assets is given. Finally, we introduce the
following five assumptions about the market structure.

Assumption 1: The dividend process of the stock obeys the following rules.

\[ D_t = \alpha_t P_t \quad (0 < \alpha_t < 1) \]  

(1)

\[ D_t \text{ is a sequence of random variables, } \alpha_t = \tau \frac{P_{t+1} - P_t}{P_t}, \quad P_{t+1} - P_t \text{ is stock returns at the } t \text{ time, } \tau \text{ is dividend factor (0 < } \tau < 1) \]. In addition, company information is fully disclosed. In other words, profit
and loss of the company can be fully reflected in the stock price.

Assumption 2: The interest rate on the bond is r (r>0), the supply of bonds is completely elastic, In
other words, the behavior of investors will not affect the price of bonds.

Assumption 3: There are \( h_t \) investors at the \( t \) time, \( h_t \) is a sequence of random variables. For the
convenience of calculation, we introduce a new sequence of random variables \( \eta_t = \frac{1}{h_t} \).

Assumption 4: All random variables are defined in a probability space \( \Omega, F, P \), there are
complete disclosures in the market. At the \( t \) time, all investors know all the information about prices
and dividends before the \( t \) time, and this information set we use as follows \( \sigma \)-algebra to represent:

\[ F_t = \sigma\{P_s, D_s / s \leq t\} \subset F, \]

\( E_t(\cdot) \) expresses conditional expectation of \( F_t \).

Assumption 5: All investors have a constant risk aversion preference, regardless of their risk
tolerance, that is the expected utility of the \( i \)-th investor to maximize at each time \( t \) is

\[ E_t[u(\omega_{j,t+1})] = E_t(-\exp(-\omega_{j,t+1})). \]
3. Analysis of stock exchange M&A

3.1. Basic model analysis

Assume that the number of original shares of the M&A company and the target company are M, N, and the stock price is \( P^{(1)} \), \( P^{(2)} \). The net assets are fully reflected in the stock market value which is \( C_1 = MP^{(1)} \), \( C_2 = NP^{(2)} \). The number of shares in the new company after the merger is R, and the stock price is P. In determining the conversion ratio, the two companies mainly rely on the comparison of the company's estimated net assets obtained after its stocks operate on the capital market and their estimated asset values after the M&A, thus obtaining a conversion acceptable to both companies. The upper and lower limits of the share exchange ratio \( \beta \), and then determine the \( \beta \) by solving the maximum value of the company's utility function.

We first consider the equilibrium price of the stocks of the acquired company. In order to obtain market equilibrium, we first describe the investment opportunities of each investor. To make it easier for us to assume that the investor's wealth at the initial moment is 0, he financed by selling short bonds and buying a unit of stock from the short sale, and the undiscounted cash flow of this portfolio is \( \Phi_t \).

At the \( t+1 \) time:

\[
\Phi_{t+1} = P_t^{(1)} + D_{t+1} - P_t^{(1)} (1 + r) = \hat{e}_{\phi,t} + \nu_{\phi,t} \tag{2}
\]

\( \hat{e}_{\phi,t} = E_t[\Phi_{t+1}] \) is expectations of excess returns in t time, \( \nu_{\phi,t} = \Phi_{t+1} - \hat{e}_{\phi,t} \) is the unpredictable residual at time t, and \( \nu_{\phi,t} \) satisfies Gaussian normal distribution with conditional expectation \( \sigma^2_v \).

We solve the conditional expectation utility maximization problem of the i-th investor, because the investor has a constant risk aversion preference. So the i-th investors face the following optimization problems at t time:

\[
\max_{\lambda_{i,t}} E_t(-\exp(-\omega_{i,t+1})) \tag{3}
\]

Constraint is \( \omega_{i,t+1} = (1 + r)\omega_{i,t} + \lambda_{i,t}\hat{e}_{\phi,t} + \lambda_{i,t}\nu_{\phi,t} \), \( \lambda_{i,t} \) is the stock holding of the i-th investor, \( \omega_{i,t+1} \) is cash flow of the i-th investor.

After solving the above maximization problem, we know that the optimal investment strategy is

\[
\lambda_{i,t} = \frac{\hat{e}_{\phi,t}}{\sigma^2_v}. \tag{4}
\]

Since the number of investors is \( h_i \), market clearing condition is

\[
\sum_{i=1}^{h_i} \lambda_{i,t} = M. \tag{5}
\]

Substituting (5) into(4), we get

\[
\hat{e}_{\phi,t} = \frac{\sigma^2_v M}{h_i}. \tag{6}
\]
Substituting (6) into (2), we get the expression of the equilibrium price $P_t^{(1)}$ of the stock of the M&A company,

$$P_t^{(1)} = (1 + r)^{-1} [E_t (P_{t+1}^{(1)} + D_{t+1}) - \frac{\sigma^2 M}{h_t}].$$

(7)

Using variable substitution and iteration to derive (7), the equilibrium price [10] can be obtained as,

$$P_t^{(1)} = -\frac{\sigma^2 M}{1 + r} \left( \sum_{n=0}^{\infty} x_{t+n} \eta_{t+n+1} + \eta_t \right).$$

(8)

Similarly, we can get the stock equilibrium prices of the target company and the new company as,

$$P_t^{(2)} = -\frac{\sigma^2 N}{1 + r} \left( \sum_{n=0}^{\infty} x_{t+n} \eta_{t+n+1} + \eta_t \right).$$

(9)

$$P_t = -\frac{\sigma^2 R}{1 + r} \left( \sum_{n=0}^{\infty} x_{t+n} \eta_{t+n+1} + \eta_t \right).$$

(10)

and $x_{t+n}$ satisfies

$$x_{t+n} = \left( \frac{A(1 - \tau)}{2} + \frac{A\tau}{2\sqrt{A^2 (1 - \tau)^2 + 4A\tau}} \right) \frac{1}{(t+n)!} \left( \frac{A(1 - \tau) + \sqrt{A^2 (1 - \tau)^2 + 4A\tau}}{2} \right)^{t+n}$$

$$+ \left( \frac{A(1 - \tau)}{2} - \frac{A\tau}{2\sqrt{A^2 (1 - \tau)^2 + 4A\tau}} \right) \frac{1}{(t+n)!} \left( \frac{A(1 - \tau) - \sqrt{A^2 (1 - \tau)^2 + 4A\tau}}{2} \right)^{t+n}$$

$$A = \frac{1}{r + 1}$$

For the acquisition company and the target company, when determining the share exchange ratio, they require that their expected net assets in the new company are not less than the net asset value of the company in the capital market after the merger. The mathematical expression has the following form

$$\beta R P_t \geq M P_t^{(1)}$$

$$\beta R P_t \geq N P_t^{(2)}$$

From this we get the acceptance domain of the share exchange ratio as:

$$\frac{M^2}{R^2} \leq \beta \leq 1 - \frac{N^2}{R^2}$$

(11)
Note: From here we can launch, under this model framework, the number of shares issued by the new company must obey $R \geq \sqrt{M^2 + N^2}$. Otherwise, at least one company on both sides of the merger and acquisition cannot achieve the expected increase in net assets, and therefore will reject mergers and acquisitions. This conclusion gives the minimum lower bound for shares issued by a new company, which provides references for stock exchange M&A.

Below we prove the existence of the share exchange ratio.

**Theorem 1:** Assuming that the utility function of the acquisition company and the target company is $U$, then for any strictly concave utility function $U(\beta R P_t, (1-\beta) R P_t)$, the share exchange ratio $\beta$ is unique which satisfies $\beta \in \left[\frac{M^2}{R^2}, 1 - \frac{N^2}{R^2}\right]$ and makes the utility of the new company reach its maximum.

**Proof:** In fact, the optimization problem facing the new company can be written as

$$\max E[U(\beta R P_t, (1-\beta) R P_t)]$$

Constraint is $\frac{M^2}{R^2} \leq \beta \leq 1 - \frac{N^2}{R^2}$.

Because $U(x, y)$ is a strictly concave utility function, so $\begin{pmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{pmatrix}$ is negative. For any two-dimensional vector $X$, there is $\begin{pmatrix} U_{xx} & U_{xy} \\ U_{yx} & U_{yy} \end{pmatrix} X^T < 0$. When $X = (1, -1)$, we have $U_{xx} - 2U_{xy} + U_{yy} < 0$.

Under the framework of this model, due to

$$U_{\beta\beta}(\beta R P_t, (1-\beta) R P_t) = R P_t U_{xx} - R P_t U_{yy} ,$$

$$U_{\beta\beta}(\beta R P_t, (1-\beta) R P_t) = (U_{xx} - 2U_{xy} + U_{yy})(R P_t)^2 < 0 ,$$

Therefore, $U(\beta R P_t, (1-\beta) R P_t)$ is concave function of $\beta$ and it’s obviously satisfies $E[U_{\beta\beta}] < 0$.

There must be a unique $\beta$ in the closed interval $\left[\frac{M^2}{R^2}, 1 - \frac{N^2}{R^2}\right]$ to maximize the expected utility.

3.2. Other common stock exchange M&A

At present, the most popular methods for determining the stock exchange ratio in the theory and practice of stock exchange M&A are The market price method, the income method, the net asset method, the earnings per share method, and the L-G model method. Below we briefly introduce the advantages and disadvantages of the common methods.

3.2.1. The market price method. This is based on the market price of each share before the merger between the two parties. The stock exchange ratio is

$$\beta = \frac{P^{(1)}}{P^{(2)}}$$
The shortcomings of the market price method are that the current stock price is affected by speculative factors and policy fluctuations, and has short-term volatility, which does not reflect the company's intrinsic value and long-term profitability.

3.2.2. The income method. This is a method of determining the share conversion ratio based on the earnings per share before the merger of the two parties. The stock exchange ratio is

\[ \beta = \frac{EPS_1}{EPS_2} \]

EPS\(_1\) and EPS\(_2\) indicate earnings per share of the company respectively.

The disadvantage of the income method is that it only reflects the profitability of the two companies in the current period and cannot reflect future changes. At the same time, when the two parties in the M&A have losses, the stock exchange ratio cannot be determined.

3.2.3. Net assets method. This is a method of determining the share exchange ratio based on the net assets per share before the merger of the two parties. The stock exchange ratio is

\[ \beta = \frac{C_1}{C_2} \]

C\(_1\) and C\(_2\) indicate net assets per share of the company respectively.

The shortcomings of the net asset method are that it relies too much on the book value of the company, and insufficient consideration of the company's profitability, asset ratio, and time value of money, and cannot accurately reflect the true value of the company.

3.2.4. The earnings per share method. This determines the exchange ratio interval based on the earnings per share remaining unchanged before and after the merger between the two parties. The stock exchange ratio interval is

\[ \frac{EPS_2 \times M}{(E_1 + E_2) \times (1 + I\%) - E_2} \leq \beta \leq \frac{(E_1 + E_2) \times (1 + I\%) - E_1}{EPS_1 \times N} \]

EPS\(_1\) and EPS\(_2\) indicate earnings per share of the company respectively, E\(_1\) and E\(_2\) is net profit respectively, M and N is total shares respectively, I\% is Incremental benefits from merger synergies.

The disadvantage of this method is that the combined synergy incremental income is used as the basis for determining the share conversion ratio interval, but the combined synergistic incremental income can only be estimated with reference to historical data, which is relatively unpredictable. At the same time, it is too dependent on earnings per share.

3.2.5. L-G model method. This is based on the constraint that the wealth of the shareholders of both parties is not reduced, and the synergies of the merger are not considered. Through the factors such as the share price, the yield, the net profit, the total number of shares, and the expected price-earnings ratio after the merger, the exchange ratio \( \beta \) interval is determined. The exchange ratio interval is

\[ \frac{P^{(2)} \times N}{PE_{12} \times (E_1 + E_2) - P^{(2)} \times N} \leq \beta \leq \frac{PE_{12} \times (E_1 + E_2) - M}{P \times N} \]
$P^{(1)}$ and $P^{(2)}$ is current share price respectively, M and N is total shares respectively, $E_1$ and $E_2$ is net profit respectively, $\text{PE}_{12}$ is expected P/E ratio after merger. The shortcoming of the L-G model method is that it takes the price-earnings ratio of the merged company as an important factor for determining the share exchange ratio. It can only refer to the price-earnings ratio of the same type of companies after mergers and acquisitions. The price-earnings ratio as an indicator of the intrinsic value of listed companies is different at home and abroad. This is based on the maturity of the capital market. In the past year, Black Swan Incident Caused by New Coronavirus, the irrational plunge of the fuse mechanism and the huge fluctuations in the broader market have proven that the price-earnings ratio as an important factor in the research of listed companies' share swaps and mergers is questionable.

4. Conclusion

The above research methods only start from current or historical data, or refer to the current stock price, net assets, and net income of both parties in the merger and acquisition, or preset the company's price-earnings ratio and incremental income after the merger. In the framework of the Capital Asset Pricing Model (CAPM), based on the current share price and earnings per share of the M&A parties, this article takes into account the expected returns of the three companies before and after the merger, Calculating the future equilibrium prices of the stocks respectively, and derives the exchange ratio interval which can better reflect the company's true value and long-term profitability, That provides a preliminary reference for the exchange and merger operation of listed companies.

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