Engaging problems on trigonometry: why were student hard to think critically?

M Aminudin¹,², T Nusantara², I N Parta², S Rahardjo², A R As’ari², and Subanji².

¹ Universitas Islam Sultan Agung, Jl. Kaligawe Km 4, Semarang 50112, Indonesia
¹,²Universitas Negeri Malang, Indonesia, Jl. Semarang 5, Malang 65145, Indonesia

E-mail: aminudin@unissula.ac.id

Abstract. This research study aimed to determine the causes of difficulties for students to think critically in solving trigonometric problem. 29 first semester students of the UNISSULA mathematics education program participated in this study. Data collection techniques used mathematic test and unstructured interviews. The math test given is a trigonometric problem. The result of research indicated that there were four factors that caused the students difficult to think critically. Those four factors are lack of skepticism on math problem, learning of school mathematics that did not emphasize critical thinking, student ignorance about critical thinking, and motivation was limited only to solve math problems.

1. Introduction

There are some reasons for everyone including students to have critical thinking needs to solve problems. Critical thinking is a part of thinking that needs to be developed at every age level. Ability to think critically is urgently needed to compete in the global era. Critical thinking is used to identify useful and useless information. It leads students to focus more on the problems they are facing. It is used as a tool for making accurate, valid, and accountable decisions [1-7].

Ennis [8, 9] defines critical thinking as a logical and reflective process of thinking whose purpose is to make decisions about what to believe and what to do. Ennis [8], and Facione [10] divide critical thinking into two consisting of critical thinking skills and critical thinking dispositions. Critical thinking in order to decide whether to believe a claim, arguments, both written and oral, or not mean referring to the ability to think critically as a cognitive domain. While critical thinking in order to decide whether to do the actions that are ordered or not, refers to a critical thinking disposition as the domain of affection. Students’ tendency to think critically can be seen in critical thinking disposition. Disposition is a person's tendency to perform an action based on certain conditions [8]. Critical thinking disposition is a person's tendency to use logical and reflective thinking in order to determine if he/she should believe in a claim or perform an order [7]. Critical thinking disposition is also a person’s tendency to solve problems by a curiosity to find out the truth and to obtain reliable information [10].

Critical thinking has become an important research topic because it is one of the important goals in the 21st century. Some researchers make critical thinking a learning goal that is developing students' critical thinking, so that they make various innovations. In addition to using learning, critical thinking can be developed by selecting mathematical content that is appropriate and aimed at all students without considering limitations regarding the classification of student performance [11]. Mathematical critical thinking can be developed through innovative learning stages including giving problems, small group discussions independently, discussions guided by the instructor, and then teaching critical thinking skills and mathematical knowledge [12]. In addition, the right strategy can improve the mathematical critical thinking disposition of junior high school students [13]. Critical thinking can be improved through the use of interdisciplinary problems between statistics and algebra [14].
The activity of solving mathematical problems is closely related to critical thinking [4, 15]. In general, the stages of solving mathematical problems follow the Polya stage which consists of understanding the problem, devising a plan that will lead to the solution, carry out the plan, and look back [16]. However, solving mathematical problems using critical thinking is something different. As an example, Sutini et.al [17] identifies the process of critical thinking in solving mathematical problems including four stages consisting of gathering information, evaluating information, making solutions, and evaluating conclusions.

One field of mathematics that is still considered difficult by high school students is trigonometry, although it has been taught starting from the elementary school level [18]. Common mistakes made by students in answering trigonometric questions include the use of improper equations, sequences of operations and misuse of sinuses and cosines, misinterpretations of languages, illogical inferences, distorted definitions, and technical error calculations. In addition, in understanding the concepts of degrees and radians, 60% of the mathematics teacher candidates define the concept of degrees correctly, 40% make several incorrect definitions of the concept including "the angle between two straight lines", and "the size of the angle facing the unit length" [19]. As'ari et.al [15] explained that when a prospective mathematics teacher is given a mathematical problem, then immediately implement what is asked of the questions given without examining the terms, assumptions needed. After the results are found, the prospective teacher also does not check whether the completion process and the final results are correct or not, without checking again. And finally it can be concluded that some prospective mathematics teachers have not become critical thinkers in solving mathematical problems.

In general the focus of previous studies on how to develop critical thinking was realized through learning innovation and the use of interdisciplinary problems. Research on why prospective mathematics teacher students find it difficult to think critically has not been discussed. The problem that will be given to students is trigonometry. This is because trigonometry is a field of mathematics that is still considered very difficult and abstract compared to other mathematical fields. Students often find errors, misconceptions, and obstacles in learning trigonometry [18]. Therefore, this study aims to determine the causes of students having difficulty thinking critically in solving trigonometric problems. This research is important in order to prepare prospective mathematics teacher students to have critical thinking skills, especially in solving trigonometry problems.

2. Method

This study is a qualitative descriptive. This study describes the problem of "why students find it difficult to think critically" from the results of data collection based on test results and interviews [20]. There are four things that will be described are about the process of solving trigonometry problems, critical thinking knowledge, mathematics learning experiences, and student motivation. Purpose of the study was to determine the causes of difficulties for students to think critically in solving trigonometry problems.

The subjects who participated in this study were 29 second semester students of the mathematics education program at the UNISSULA FKIP. Data collection techniques used mathematics tests and unstructured interviews. The unstructured interview was used to find out the causes of students difficult to think critically. There are four questions that the researcher gave to the participants, including "do you understand the problem?", "How do you learn mathematics when you study in school?", "Explain your thinking process in solving the problem?" And "what is your motivation in completing that problem?". The mathematic test in the form of trigonometric problems is used to measure critical thinking dispositions. Trigonometric problems were adopted from As'ari [7] (see figure 1)
A triangle given ABC is a right triangle at C. The CD is perpendicular to
AB and DE is perpendicular to BC. If it is known that AC and BE are \( x \)
and \( x\sqrt{3} \) respectively, and the angle of ACD is 30°. Determine the area of
the BED triangle!

Figure 1. Trigonometric problems given to students.

This research was began by giving trigonometry problem to 29 students and given time for 2 x 50
minutes. Furthermore, the results of student answers are evaluated and categorized based on similarity
of answers yaitu 26 students answered procedurally and 3 students answered procedurally but not
finished. Then, the researcher chooses one student answer from 26 answers that answer procedurally to
be analyzed and students who answer are interviewed to find difficulties in solving problems by
thinking critically.

3. Result and Discussion

These errors certainly had a cause why students in general did not yet have a good critical thinking
disposition. Researchers have suspected the cause of these errors. The allegations are lack of skepticism
on mathematics problems, learning of school mathematics that has not led to critical thinking, ignorance
of students about critical thinking, and motivation in solving mathematics problems.

3.1 Lack of skepticism in solving mathematical problems

Skepticism in solving mathematical problems is needed by students. This attitude will bring
students to think critically. Ennis (1987) identified critical thinking dispositions can be seen from the
ability to demonstrate skepticism and the ability to create logical reasons [6]. Skepticism in critical
thinking means bringing the element of doubt with courtesy before solving the problem, through a prior
examination whether a problem or statement is true and valid. Skepticism in critical thinking also helps
identify hidden information clearly and accurately.

Figure 2. Subject answers in solving trigonometry problems

The trigonometric problems provided include problems in which there is false information. Errors
in problems are deliberately made by researchers to test critical thinking skills that whether students
believe in the problem or first check the truth of the problem. Based on figure 2, students generally
solve trigonometry problems through several stages of determining what was known, understanding
what was being asked, applying some formulas to get answers according to what was asked on the matter, and giving final conclusions. These steps clearly did not show skepticism on the given problem. Students did not check the truth about whether it matches the existing concepts and theorems. This result is in accordance with an interview with one of the following subjects (P = Researcher, S = Student).

R : Did you really understand the first question?
S : I tried to understand but I still felt confused what it means.
R : What did you think about before you did the first question?
S : to find the area of right triangle if known information according to the problem. I was thinking about how to find the length of DE, and the triangular area of the formula was \( \frac{1}{2} \times \text{base} \times \text{high} \).
Information was known only BE length so the length of DE was not yet known.
R : were you thinking about how to find the length of DE?
S : yes sir.

Based on student answers in figure 2, the first step is to represent trigonometric problems in the form of image representation. Next, students write down information that is known that \( AC = x \) dan \( BE = \sqrt{3} \), and the information asked is the area of the BED triangle. The second step is implementing the strategy. The strategies used are using sine comparisons, phytagoras theorems, and triangle area formulas. Comparison of sine is used to find the length of AD and DE. While the phytagorean theorem is used to find the length of the CD after the length of the AD is found. After the length of DE is found, then the student looks for the area of BED by multiplying \( \frac{1}{2} \times BE \times DE = \frac{1}{2} \times x\sqrt{3} \times \frac{3}{4}x \). After the BED triangle area is found to be \( \frac{3\sqrt{3}}{8}x^2 \), the student concluded that the area of the triangle BED = \( \frac{3\sqrt{3}}{8}x^2 \). Students do not check whether there is another way of determining the BED triangle area.

Based on the information on the question that if the triangle ABC is a right triangle in C then it should apply the theorems that satisfy the right triangle ABC. One of the right triangle theorems is the angular ratio of \( A:B:C = 60^0:30^0:90^0 \) or has a ratio of \( BC:AC:AB = x\sqrt{3}:x:2x \). So it is clear that the length of BE should not be equal to \( x\sqrt{3} \). But the length of \( x\sqrt{3} \) is BC or CB. In addition, if the ACD angle is 30\(^0\) (as figure 1) then the angle of BAC is 60\(^0\). And if the ABC triangle is a right triangle in C, then the ABC angle is 30\(^0\). By using tangent rules, the length of DE can be found through the following calculation

\[
\tan 30^0 = \frac{DE}{EB} \\
\sqrt{3} = \frac{DE}{x\sqrt{3}} \\
DE = x
\]

The DE = x length found is contrary to the information that is known that the AC length = x. Even if you look at trigonometric problems in Figure 1, then the AC is longer than DE. So it can be concluded that this trigonometric problem has conflicting information. However, that the subject assumes that the given problem is always correct so the subject focuses on defining a strategy for determining the area of the BED triangle without checking whether the problem has the correct measurements.

The mistake is false in choosing the concept, apparently due to lack of skepticism in mathematical problems. This statement goes against that conceptual errors always occur because of procedural errors [21]. According to the researchers, that procedural errors are possible because of conceptual errors. In the first question, students are required to be more careful about the concept of right triangle. Students focus on how to use procedural knowledge to get the area of a right triangle of BED. Students should analyze the concept of the right segment and skepticism the conformity between the concept of right
triangle and the information on the problem. The absence of skepticism shown through the answers in Figure 2 shows the characteristics of a person having a low critical thinking disposition. Skeptics can encourage someone to reflect so that they produce the right conclusions and make the right decisions. Nugroho et.al [22] stated that there were two causes of skepticism consisting of cognitive conflict and two conflicting resolutions.

3.2 Mathematics learning in schools has not yet fully led to critical thinking

Learning mathematics can not be separated from mathematical functions. Learning mathematics means giving the experience of using mathematics as a tool for understanding or conveying information. Learning mathematics means forming the mindset in understanding definitions and reasoning a relationship between those meanings. Learning mathematics means always seeking the truth and willing to rectify the truth that is temporarily accepted [19].

Therefore, the standard of mathematics learning should not be separated from the function of mathematics and national education goals as set forth in the Graduate Competency Standards (SKL). Basic and secondary education SKL states the dimensions of skills that must be possessed by learners at the level of school that one of them has the skills of thinking and acting critically [23]. As suggested by The Partnership for 21st Century Skills (P21) that performance results on curriculum and assessment should be focused on the thinking aspect, one of which is critical thinking on mathematics [24]. In addition every student needs to have 4Cs, one of them is Critical Thinking [7].

Based on the 29 students’ answers it seems that mathematics learning in schools had not fully led to a good critical thinking disposition. The following interview results of researcher to 16 students about mathematics learning at school (table 1)

| subject | subject’s answers |
|---------|------------------|
| M1      | Teachers explained first, gave formulas and give questions. The teacher gave 2 ways to solve the problem that was the fast way and the usual way. |
| M2      | Once done, each student was asked to explain what they were doing. If there were difficulties, then the teacher will helped. |
| M2      | The teacher explained each chapter of the book. Teacher explained in front of the class. Then the teacher gave practice questions |
| M3      | The teacher conveyed the learning objectives first, then the teacher explained the material. The teacher gave the questions to do together, then discussed it. Finally the teacher gave the problem to be done independently. |
| M4      | The teacher gave the questions in the handbook. |
| M5      | Teachers often gave practice questions, discussed in solving them. |
| M6      | The teacher explained the material and the students were asked to listen without anyone writing. Then the teacher gave the opportunity to ask if there was a less clear explanation. |
| M7      | The teacher gave material notes on the board. Then the teacher gave some practice questions and students were asked to do the problem. |
| M8      | The teacher explained the material, then gave practice questions. Sometimes the teacher asked the students to discuss in groups in solving the exercise problem. |
| M10     | The teacher always explained what was in the book |
| M11     | Provided practice problems from easy to difficult. |
| M12     | The teacher explained the material and then gave examples of problems and solutions. Teachers continued learning by giving practice questions until students understood. |
The teacher asked the students. And then teacher and students discussed it to the answers the questions. Teachers continued learning by explaining the material and giving examples of practice questions.

The teacher taught how to solve mathematics problems quickly.

The teacher always gave mathematical formulas and taught how to use these formulas to solve a mathematics problem.

The teacher explained the material and then gave examples of practice problems. Teachers often gave students the opportunity to ask. Furthermore the teacher provided additional problems to be done in class and at home.

Student answers in table 1 provide important information that the learning received by the students while at school is a lesson in which the teacher often describes the mathematical material first according to the book. The teacher emphasizes how mathematical formulas are applied to solve mathematics problems. Then the teacher gives exercises to students. Sometimes teachers use discussion strategies in their learning. Teachers provide guidance in working on practice questions and provide direction how to solve problems quickly and practically. At the end of the lesson the teacher gives mathematics problems as a homework assignment. Therefore, such learning has not yet led to the development of critical thinking.

It seems that teachers are still oriented to problem solving skills. Measures of mathematical skills that need to be mastered include conceptual understanding, procedural knowledge, and problem solving [25]. These three aspects are an unity and can not be separated. Mathematics learning can not only focus on problem solving skills by ignoring other abilities.

Researchers provided suggestions on how to encourage students to think critically. Swartz & Parks (1994) [13] stated two approaches to teaching critical thinking using science content infusion approaches mean that critical thinking skills are taught explicitly using the content of science, and general or embedded approaches critical thinking is taught in an indirect way without informing students. Teaching critical thinking skills the need for internalization of critical thinking skills. Internalization activities are closely related to the formation of dispositions, so the teaching of critical thinking needs to be taught from early childhood. In teaching critical thinking skills, teachers should use infusion methods.

### 3.3 Students unawareness of how to think critically

Mathematics learning in schools not only provides guidance to improve mathematics skills, but more importantly one of them is to guide students to think critically. This is because critical thinking has many benefits as mentioned in the introduction. Therefore, every student should know and understand how to think critically in solving mathematics problems.

Knowledge of how to think critically in solving mathematics problems seems not yet owned by some students. This can be shown from the answers of mathematics tests which generally still have not given any indication of critical thinking. In addition, the lack of knowledge about critical thinking can be seen from students' critical thinking conceptions in solving mathematics problems. The conceptions the researchers obtain through structured interviews with the question "what does it mean to think critically in solving mathematics problems, do you think?". Table 2 is 16 students' critical thinking concepts.
| Subjek | Subject’s answes |
|--------|-----------------|
| M1     | How did one think by observing, reviewing and considering something (matter / problem) that exists. |
| M2     | How to solve the problem in detail, so it can know the cause of the problem. |
| M3     | Be able to formulate the problem, be able to use which formula to use, be able to work on the problem according to plan, use logical ways to solve it, re-examine the answers that have been made by using different ways and able to draw a logical conclusion. |
| M4     | Thinking hard to solve the problem. We should really think if the existing formulas are not related to the given problem. We should be able to combine formulas or create formulas to get things done. |
| M5     | The ability to respond to or understand problems is logical and can analyze a thing whether it is logical or not, and able to express the truth and find fault with something. |
| M6     | The ability to respond to or understand problems is logical and can analyze a thing whether it is logical or not, and able to express the truth and find fault with something. |
| M7     | Thinking systematically according to the concept by looking at and matching the problem. |
| M8     | The thinking ability that aims to analyze a problem uses multiple comparisons and produces a solution. |
| M9     | Thinking freely means students are allowed to do math problems in various ways not necessarily the same as what the teacher teaches. |
| M10    | Thinking unusually, but thinking using logic, diagnosis, approach, and so forth to get accurate results. |
| M11    | Look at a problem or an idea that has not been discussed or look for what really happened from the problem or idea. |
| M12    | Thinking about something you want to know and raising questions to find out more. |
| M13    | Thinking in a complex way from small to large parts. |
| M14    | How to respond or anticipate something or problems encountered in order to find an answer. |
| M15    | Conscious and deliberate thinking is attributed to existing facts. Then formed an argument or idea and the idea can be compared with the previous idea, from the comparison can be drawn conclusions. |
| M16    | Thinking according to the mind through wishful thinking to get a logical answer. |

Critical thinking conceptions in table 2 can be summarized into six things: logical thinking, systematic thinking, thinking complex, arguing until new conclusions are found, formulating, understanding, and analyzing problems, and using the exact mathematical formula. These conceptions become important to know and prove that students have different conceptions of critical thinking. The concept of critical thinking seems to refer to the process of thinking. As it is known that critical thinking can be viewed as a process of thinking [9, 26, 27], the use of cognitive skills [1, 6], and cognitive ability [9].

Critical thinking was seen as a thinking process that has a definition of logical or reasonable and reflective thinking that was focused on making decisions on what to believe or what to do [9]. Critical thinking requires a person to first check the logic of the given problem, so the decision is taken whether...
to trust the information provided, or do what is ordered. Critical thinking is analytical thinking, synthesis, and evaluative [26]. Critical thinking is thinking by deciphering the information provided into several pieces of information, then the pieces of information are used to search for new information, and then assessing the validity of the new information. Critical thinking is the use of reason with caution in determining the truth of a claim, evaluating the claim, and providing reasonable reasons [27]. The proposed reason begins with consideration of various information obtained from various sources, the information obtained is logically and creatively processed, asking the information truth, analyzing and then summarizing the information that is considered correct. Critical thinking is also related to synthesizing complex ideas or ideas.

The critical thinking conceptions of students seem to have not been properly understood. This is evident from the answers produced by students who show a low critical thinking disposition. So it can be concluded that in general students have a conception of critical thinking is as a process of thinking, but not yet know correctly how the use of critical thinking in solving mathematics problems.

### 3.4 Motivation is limited only to solve mathematics problems.

Motivation is one of the psychological factors in learning that contributes quite importantly. Motivation always provides a foundation and ease in an effort to achieve optimal learning objectives. Motivation also always gives impetus to succeed in learning. Therefore, motivation is influenced by experience and interest factor [20]. So that students who have high motivation in learning mathematics will know what is learned and understand why mathematics needs to be learned.

Motivation is a series of attempts to provide certain conditions, so that one wants and wants to do something, and if one does not like it, it will seek to nullify or circumvent those feelings of dislike [20]. Therefore, motivation has an important role for students in solving mathematics problems. Each student has a different motivation in solving a given mathematical problem. Table 3 is the motivations of 16 students when given mathematics problems:

| Subjects | Subject’s answers |
|----------|------------------|
| M1       | I want to be able to do the problem easily and quickly. |
| M2       | The desire to try and try |
| M3       | Motivation to find the right answer and can solve the problem coherently and neatly. |
| M4       | Motivation to solve problems independently. |
| M5       | I need to be able to resolve in various ways so that the problem is solved. |
| M6       | I believe that by doing the best I can do it. |
| M7       | Motivation to work on the problem through direct practice. |
| M8       | I want to solve the problem correctly and thoroughly. |
| M9       | My curiosity so that if given the matter then I try to find an answer |
| M10      | I will try to work on the problem. |
| M11      | I feel calm if I can solve the problem. |
| M12      | if I am given a problem then I try to find an answer. |
| M13      | There was a doubt in me, if I could do the problem. |
| M14      | I must be able to do the problem. |
| M15      | The problem is a challenge for me, so I must be able to solve the problem. |
| M16      | I must be able to do the matter correctly and on time. |

Based on student answers in table 3 that when students are given mathematics problems to be solved, then the student motivation is the desire to try and find the right answer in various ways. Some students consider that the mathematics problem is a challenge to solve. There is no sense of calm what if students can not do.
Such motivation is a need to achieve results and to overcome difficulties [20]. Motivation because the need to achieve results is demonstrated through the desire to solve a given mathematical problem until it gets the right result. Motivation due to a need to overcome difficulties is demonstrated through the students’ view that a given mathematics problem is a challenge to be solved.

The motivations presented by the students are also objective motivation, intrinsic motivation, and extrinsic motivation [20]. The objective motivation in this case is the need of the student to explore and manipulate his or her ability to solve a given mathematical problem. Intrinsic motivation in this case in the form of encouragement in students to solve the problem given to get the correct results. Extrinsic motivation in this case in the form of a request from a lecturer to do mathematics problems as a matter of test. The interrelations of intrinsic and extrinsic motivation [28]. Teachers need to use extrinsic motivation to encourage the emergence of intrinsic motivation. The motivation of students in solving mathematics problems is also considered as the need for self-esteem and self-actualization based on Maslow's theory [21]. The need for self-actualization is the highest level after physiological needs, safety, social, and appreciation. The need for awards in this case is the need for students to be recognized ability through the results of completion of mathematics problems given is shown by the score given by the lecturer to the students. The need for self-actualization in this case is demonstrated through the full use of cognitive ability to solve a given mathematical problem.

4. Conclusion

Some factors that cause students to think critically low in solving trigonometry problems include: first, lack of skepticism in solving trigonometry problems. Students assume correctly for each given trigonometry problems. Students do not check the truth of the matter so that it impacts on the correctness of the process and the final result. Secondly, mathematical learning in schools has not yet fully led to critical thinking. The experience of learning mathematics in schools owned by student shows that it has not led to the development of critical thinking. Third, the student's lack of understanding about how critical thinking is implemented in solving trigonometry problems, even though the student has a conception of critical thinking. Fourth, student motivation is limited only to solve trigonometry problems. The motivation is based on the need to achieve results and overcome the challenges.

5. Acknowledgment

The Authors would like to thank Kemenristekdikti through the BUDI-DN program and the mathematics education students of Unissula FKIP who had participated in this research.

References

[1] Halpern D F 2002 Thought & Knowledge: An Introduction to Critical Thinking (United States of America: Lawrence Erlbaum Associates) p 2
[2] Biber A C, Tuna A, and Incikabi L 2013 An investigation of critical thinking dispositions of mathematics teacher candidates Journal of International Educational Research 4 109
[3] Chaipichit D, Jantharajit N, and Chookhampaeng S 2015 Development of learning management model based on constructivist theory and reasoning strategies for enhancing the critical thinking of secondary students Educ. Res. Rev. 10 2324
[4] Chukwuyenum A N 2013 Impact of Critical thinking on Performance in Mathematics among Senior Secondary School Students in Lagos State IOSR Journal of Research & Method in Education 3 18
[5] Uarattanaraksa H, Chaijareon S, and Kanjug I 2012 Designing Framework of the Learning Environments Enhancing the Learners’ Critical Thinking and Responsibility Model in Thailand Procedia - Soc. Behav. Sci. 46 3375
[6] Cottrell S 2005 *Critical Thinking Skills: Developing Effective Analysis and Argument* (New York: Palgrave Macmillan) p.2

[7] As’ari, A R 2016 *Variasi Konstruk Dalam Pembelajaran Matematika* ed As’ari A R and Irawan E B (Malang: CV. Bintang Sejahtera) pp 1–21

[8] Ennis R H 1996 *Critical Thinking* (United States of America: Prentice-Hall, Inc) p.1

[9] Ennis R H 2011 *The Nature of Critical Thinking: An Outline of Critical Thinking Dispositions* (New York: University of Illinois) pp 1–8

[10] Facione P A 1990 *Critical Thinking: A Statement of Expert Consensus for Purposes of Educational Assessment and Instruction Executive Summary* *The Delphi Report* 423 pp 0–19

[11] Maričića S and Špijunovićb K 2014 *ScienceDirect Developing Critical Thinking in Elementary Mathematics Education through a Suitable Selection of Content and Overall Student Performance* *Procedia - Soc. Behav. Sci.* 180 653

[12] Aizikovitsh E and Amit M 2009 An Innovative Model for Developing Critical Thinking Skills through Mathematical Education *International Conference of the Mathematics Education into the 21st Century* ed Paditz, L and Rogerson, A (Dresden: University of Applied Sciences) pp 19–22

[13] Aizikovitsh-udi E and Cheng D 2015 Developing Critical Thinking Skills from Dispositions to Abilities: Mathematics Education from Early Childhood to High *Creat. Educ.* 6 455

[14] Korres K and Tsami E 2010 Supporting the development of critical thinking skills in secondary education through the use of interdisciplinary statistics’ and mathematics’ problems *J. Interdiscip. Math.* 13 491

[15] As’ari A R, Mahmudi A, and Nurlaelah E 2017 Our Prospective Mathematic Teacher Are Not Critical Thinkers Yet *Journal on Mathematics Education* 8 145

[16] Polya G 1957 *How to solve it* (New Jersey: Princeton University Press) pp 5–14

[17] Sutini, Sutawidjaja A, and Parta I N 2017 Identification of Critical Thinking Process in Solving Mathematical Problems *IOSR J. Res. Method Educ.* 7 5

[18] Gur H 2009 Trigonometry Learning *New Horizons Educ.* 57 67

[19] Tuna A 2013 A Conceptual Analysis of The Knowledge of Prospective Mathematics Teachers About Degree and Radian *World J. Educ.* 3 1

[20] Creswell J W 2012 *Educational Research* ed. Smith P A (Boston: Pearson Education, Inc) p.16

[21] Connor B R, and Norton S 2016 Investigating Students ’ Mathematical Difficulties with Quadratic Equations *Proceedings of the 39th Annual Conference of the Mathematics Education Research Group of Australasia* ed White B, Chinnappan M, and Trenholm S (Adelaide, South Australia: Merga) pp 552–59

[22] Nugroho P B, Nusantara T, As’ari A R, Sisworo S, Hidayanto E, and Susiswo S 2018 Critical Thinking Disposition : Students Skeptic in Dealing with Ill-Logical Mathematics Problem *Int. J. Instr.* 11 635

[23] Permendikbud 2016 *Peraturan Menteri Pendidikan Dan Kebudayaan Nomor 20 Tahun 2016 Tentang Standar Kompetensi Lulusan Pendidikan Dasar Dan Menengah* (Jakarta: Permendikbud) pp 1–8

[24] Partnership for 21st Century Skills 2008 *21st Century skills, education & competitiveness, a resource and policy guide* (New York: Tucson) p 1

[25] Suratman D 2011 Pemahaman Konseptual dan Pengetahuan Prosedural Materi Pertidaksamaan Linear Satu Variabel Siswa Kelas VII SMP (Studi Kasus di Mts. Ushuluiddin Singkawang *J. Cakrawala Kependidikan*) 9 1

[26] Ennis R H 1993 Critical Thinking Assessment *Theory Pract.* 32 179

[27] Moon J 2008 *Critical Thinking: An Exploration of Theory and Practice* (United States of America: Routledge) p 37

[28] Saeed S and Zyngier D 2012 How Motivation Influences Student Engagement: A Qualitative Case Study *J. Educ. Learn.* 1 252