I. INTRODUCTION

Recently, it was proposed that a K⁻ meson can be deeply bound in light nuclei as a discrete state, such as ³He+K⁻, ⁴He+K⁻ and ⁸Be+K⁻, where the K⁻ meson makes the nucleus shrink drastically to form a dense state. Exotic proton-rich bound systems with K⁻, such as ppK⁻, pppK⁻, pppnK⁻, and ⁹BeK⁻, are expected to be produced in (K⁻π⁻) reactions. In our previous paper, we investigated kaonic nuclei, which are denoted as K⁻ nuclei hereafter. Exotic proton-rich bound systems with K⁻, such as ³He+K⁻ and ⁸Be+K⁻, with a simple version of Antisymmetrized Molecular Dynamics (AMD) [3]. Although our results are similar to those obtained in Ref. [1], a strange property appeared in Λ(1405). In this paper we employ a phenomenological K⁻N interaction, to adequately treat the I = 0 K⁻N interaction, which is essential to study kaonic nuclei. The improved points are 1) pK⁻/nK⁰ mixing and 2) total spin and isospin projections. These improvements enable us to investigate various kaonic nuclei (ppnK⁻, pppK⁻, pppnK⁻, ⁶BeK⁻ and ⁹BK⁻) systematically. We have found that they are deeply bound and extremely dense with a variety of shapes.

Thus, the I = 0 K⁻N interaction is essential for studying K⁻ nuclei.

In this paper we present systematic studies of K⁻ nuclei with AMD. Since AMD treats a system in a fully microscopic way without any assumption concerning the structure of the system, it is suitable for studying K⁻ nuclei, whose structures might be exotic. The simple version of AMD [3], however, has a technical problem in treating the I = 0 KN interaction, which dominates K⁻ nuclear systems: it cannot include the degree of freedom of K⁰, and fails to describe the I = 0 KN state. Therefore, in our previous paper we dealt with the I = 0 KN interaction by incorporating its K⁰ part effectively into the K⁻ p interaction. Of course, the I = 0 KN interaction should be treated as precisely as possible, because it plays an essential role in K⁻ nuclei. In this paper, we improve the framework of AMD so that it can treat the I = 0 KN interaction adequately. We introduce the degree of freedom of K⁰ into the model space of AMD ("pK⁻/nK⁰ mixing"). Since K⁻ nuclear states depend largely on their isospin due to the strong isospin-dependence of the KN interaction, we carry out the isospin projection as well as the angular momentum projection ("J & T projections") of the obtained intrinsic state.

With the new version of AMD, we systematically investigate a variety of K⁻ nuclei. We try to answer the following questions: i) What K⁻ nuclei are deeply bound with narrow widths? ii) Is there any strange structure peculiar to K⁻ nuclei?

This paper is composed as follows: In Section II, we present the improvements of AMD: pK⁻/nK⁰ mixing, J & T projections, and other formalisms. In Section III, we demonstrate the capability of our new framework, and then apply it to various K⁻ nuclei (ppnK⁻, pppK⁻, pppnK⁻, ⁶BeK⁻, and ⁹BK⁻). The results and a discussion are given in Section IV. We summarize our study in Section V.
II. FORMALISM

In the present study, we employ the AMD approach as a means of studying $\bar{K}$ nuclei. It has succeeded in studying the structures of light unstable nuclei $^{3}_H$. In particular, it is powerful for investigating the global properties of many light nuclei systematically. We know that various few-body methods, such as summarized in [8], can treat few-body $K$ nuclei more accurately than AMD. Compared with these usual methods, AMD is more handy and applicable to more complex nuclei. Our aim is a systematic study of a variety of $K$ nuclei.

In the simple version of AMD employed in our previous study, we restricted its model space to the proton, neutron and $K^-$ meson. Due to the lack of $\bar{K}^0$ in the model space, we could not describe the $I = 0$ KN state,

$$|\bar{K}N(I = 0)⟩ = \frac{1}{\sqrt{2}} (|pK^-⟩ + |n\bar{K}^0⟩). \tag{1}$$

In other words, we could not precisely treat the coupling of the $pK^-$ pair with the $n\bar{K}^0$ one through the $I = 0$ KN interaction in the particle basis treatment of AMD. In the previous study, we incorporated all contributions from the $I = 0$ KN interaction into an effective $K^-$ interaction, as follows:

$$V_{K^-p} = α V_{\bar{K}N(I=0)} + β V_{\bar{K}N(I=1)}, \tag{2}$$

where $α$ and $β$ are some constants determined by counting the number of $I = 0$ pairs and $I = 1$ ones in a given state of total isospin $T$. For example, we set $⟨α, β⟩ = (3, 4, 1)_{3}$ in the case of $ppn\bar{K}^0$ ($T = 0$). However, we have to check how this prescription is reliable for various $K$ nuclear cases. For this purpose, we introduce the degree of freedom of $\bar{K}^0$ into the AMD framework to treat “pK$^-$/n$\bar{K}^0$ mixing” directly.

A. pK$^-$/n$\bar{K}^0$ mixing

First, we explain our idea for the simple case of a $\bar{K}$ nucleus $ppn\bar{K}^-$. In this $\bar{K}$ nucleus, the component of $ppn\bar{K}^0$ is mixed because a pair of $pK^-$ is replaced with that of $n\bar{K}^0$ by the $I = 0$ KN interaction. Hereafter, we express this state as $^{3}_H$. An ordinary way to treat it is to perform a coupled channel calculation, preparing several Slater determinants for both channels of $ppn\bar{K}^-$ and $ppn\bar{K}^0$. In the present paper, we deal with such systems where several channels are coupled as follows: In stead of multi Slater determinants, we employ a single Slater determinant with charge-mixed single particle wave functions, i.e.

$$|N_i⟩ = x_i|p⟩ + y_i|n⟩, \tag{3}$$
$$|K⟩ = z|K^-⟩ + w|\bar{K}^0⟩, \tag{4}$$

where $|N_i⟩$ and $|K⟩$ indicate a single nucleon wave function and a $K$ meson wave function, respectively. $|N_i⟩$ can describe the state where a proton and a neutron are mixed, and also $|K⟩$ can describe the state where $K^-$ and $\bar{K}^0$ are mixed. With these wave functions we describe $^{3}_H$ as $|det[N_1N_2N_3]K⟩$. This state contains the component of $|ppn\bar{K}^0⟩$ as well as that of $|ppnK^-⟩$. In this method, since each nucleon has a chance to be a proton or a neutron, the important channel is automatically chosen in the process of the energy variation. In addition, we expect that $|det[N_1N_2N_3]K⟩$ can represent a state in which the contribution of several configurations is coherently additive: for example, if two configurations such as $|ppn\bar{K}^-⟩$ and $|ppn\bar{K}^0⟩$ work coherently, such state is represented as $|p(p + n)n(K^- + \bar{K}^0)⟩$.

However, we remark one point: $|det[N_1N_2N_3]K⟩$ is likely to have incorrect components, for example $ppp\bar{K}^-$, $ppn\bar{K}^0$ and so on, which should not couple with $^{3}_H$. To avoid the mixing of such incorrect components, we project it onto a state whose isospin-$z$ component $T_z$ is equal to that of $^{3}_H$.

Now, we show the details of our wave function based on the concept of pK$^-$/n$\bar{K}^0$ mixing. Our nucleon wave function, $|φ_i⟩$, is represented by the superposition of several Gaussian wave packets [10].

$$|φ_i⟩ = \sum_{α=1}^{N_n} C_α^i \exp \left[ -ν \left( r - \frac{Z_i^α}{\sqrt{v}} \right)^2 \right] |σ_i⟩|τ_α⟩. \tag{5}$$

Namely, the $i$-th nucleon is described by the superposition of $N_n$ Gaussian wave packets whose centers are $\{Z_i^α\}$. $|σ_i⟩$ means a spin wave function, and is $|↑⟩$ or $|↓⟩$. $|τ_α⟩$ means an isospin wave function, and has the following form:

$$|τ_α⟩ = \left( \frac{1}{2} + γ_α^i \right) |p⟩ + \left( \frac{1}{2} - γ_α^i \right) |n⟩, \tag{6}$$

where $γ_α^i$ is a variational parameter. In the usual AMD the isospin of each nucleon does not change, i.e. in the process of energy-variation the protons remain as protons and the neutrons as neutrons. However, in the present paper we make the isospins of all nucleons changeable so that we can treat pK$^-$/n$\bar{K}^0$ mixing. In the same way, a $K$ meson wave function, $|φ_K⟩$, has the form

$$|φ_K⟩ = \sum_{α=1}^{N_K} C_α^K \exp \left[ -ν \left( r - \frac{Z^K_α}{\sqrt{v}} \right)^2 \right] |τ_α^K⟩. \tag{7}$$

Here, the isospin wave function of a $K$, $|τ_α^K⟩$, as well as that of a nucleon, is changeable,

$$|τ_α^K⟩ = \left( \frac{1}{2} + γ_α^K \right) |\bar{K}^0⟩ + \left( \frac{1}{2} - γ_α^K \right) |K^-⟩. \tag{8}$$

Because a nucleon is a fermion, we antisymmetrize the wave function of the nucleon’s system, $|Φ_N⟩ = det[|φ_i⟩(j)]$. Then, the $K$ meson wave function is combined to it, $|Φ⟩ = |Φ_N⟩ ⊗ |φ_K⟩$. Moreover, we project the
total wave function onto the eigen-state of parity,

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}} (|\Phi\rangle \pm P|\Phi\rangle).$$  \hfill (9)

If we perform an energy-variation with a trial wave function \(|\Phi\rangle\), it is likely that the z-component of the isospin \((T_z)\) of the total system is different from that of a K nucleus that we try to calculate originally. To avoid any mixing of components having an incorrect \(T_z\), we project the total system onto an eigen-state of \(T_z\) before the energy-variation,

$$|\hat{P}_M\Phi^\pm\rangle = \int d\theta \exp[-i\theta(T_z - M)]|\Phi^\pm\rangle.$$  \hfill (10)

Thus, we can obtain a wave function containing only the components of \(T_z = M\). We utilize \(|\hat{P}_M\Phi^\pm\rangle\) as a trial wave function.

Our wave function includes complex number parameters \(\{X^i\} = \{C^i_{\alpha}, Z^i_{\alpha}, \gamma^i_{\alpha} : C^K_{\alpha}, Z^K_{\alpha}, \gamma^K_{\alpha}\}\) and a real number parameter \(\nu\). These are determined by the frictional cooling equation, as mentioned in §3.1B.

In the present study, we use a common width parameter \(\nu\) of a Gaussian wave packet for a nucleon and for a \(K\) meson so as to simplify our calculation. However, it seems natural that the spreading width of a nucleon is different from that of a \(K\) meson. We take this point into account by using different numbers of Gaussian wave packets for a nucleon and a \(K\) meson, i.e. \(N_n\) in Eq. (15) is not equal to \(N_K\) in Eq. (17).

B. \(J\) & \(T\) projections

The angular momentum projection (\(J\) projection) is necessary to study \(K\) nuclei as well as that of usual nuclei. In addition, the isospin projection (\(T\) projection) also seems to be important because the \(KN\) interaction has strong isospin-dependence. Therefore, we perform angular-momentum and isospin projections simultaneously. \(J\) projection is done numerically by rotating the system in space, as has often been done. \(T\) projection is performed in quite the same way, but by rotating in isospin space. Our \(J\) & \(T\) projections are as follows:

$$|\hat{P}^T_{MK} \hat{P}^T_{T_z T'_z} \Phi^\pm\rangle = \int d\Omega_{\text{ang}}. D^J_{MK}(\Omega_{\text{ang}}) \hat{R}_{\text{ang}}(\Omega_{\text{ang}})$$
$$\times \int d\Omega_{\text{iso}}. D^{T^z}_{T_z T'_z}(\Omega_{\text{iso}}) \hat{R}_{\text{iso}}(\Omega_{\text{iso}}) |\Phi^\pm\rangle,$$  \hfill (11)

where \(|\Phi^\pm\rangle\) is the intrinsic wave function, which is already determined by the frictional cooling equation, as shown in §3.1D. We calculate various expectation values with \(|\hat{P}^J_{MK} \hat{P}^T_{T_z T'_z} \Phi^\pm\rangle\).

C. Hamiltonian

Our Hamiltonian in AMD calculations,

$$\hat{H} = \hat{T} + \hat{V}_{NN} + \hat{V}_C + \hat{V}_{KN} - \hat{T}_G,$$  \hfill (12)

is composed of the kinetic energy \(\hat{T}\), the effective NN potential \(\hat{V}_{NN}\), the Coulomb force \(\hat{V}_C\), and the effective \(KN\) potential \(\hat{V}_{KN}\). The center-of-mass motion energy, \(\hat{T}_G\), is subtracted. In the kinetic energy and the center-of-mass motion energy, we treat the mass difference between a nucleon and a \(K\) meson correctly. For example, the kinetic energy is

$$\hat{T} = \sum_{i=1}^A \frac{\vec{p}_i^2}{2m_N} + \frac{\vec{p}_K^2}{2m_K},$$  \hfill (13)

where \(m_N\) and \(m_K\) indicate the mass of a nucleon and that of \(K^-\), respectively. The Coulomb force is represented by the superposition of seven-range Gaussians [11].

In the study of \(K\) nuclei, we do not use existing effective interactions which may be justified for studying phenomena around the normal density. Since the system is likely to become extremely dense due to the strong \(K\) attraction, we employ the \(g\)-matrix method [1]. We adopt the Tamagaki potential (OPEG) [12] as a bare NN interaction, and the Akaishi-Yamazaki \(KN\) potential [11] as a bare \(KN\) interaction. Because the Tamagaki potential can reproduce NN phase shifts up to 660 MeV in the lab. [12], we expect that it can be applied to such extremely dense states. The effective NN/\(KN\) interactions constructed from the bare ones are represented by the following ten-range Gaussians:

$$V^K_{NN}(r) = \sum_{a=1}^{10} V^X_{NN,a} \exp[-(r/r_a)^2]$$  \hfill (14)
$$V^K_{KN}(r) = \sum_{a=1}^{10} V^Y_{KN,a} \exp[-(r/r_a)^2]$$  \hfill (15)

“\(X\)” in Eq. (14) is \(1E, 3E, 1O\) or \(3O\), and “\(I\)” in Eq. (15) is 0 or 1. We use these \(V^K_{NN}(r)\) and \(V^K_{KN}(r)\) as effective NN/\(KN\) interactions in our AMD calculation.

Our procedure is as follows: 1) For a given density and starting energy of \(K^-\), we construct a \(g\)-matrix. 2) Using the \(g\)-matrix we carry out the AMD calculation. 3) After the AMD calculation, we check whether or not the obtained density and binding energy of \(K^-\) are consistent with those of the \(g\)-matrix used in the calculation. 4) If no consistency is accomplished, we guess and impose a new density and a new starting energy of \(K^-\) and return to 1). We repeat this cycle until obtaining a consistent result.

D. Frictional cooling equation with constraint

Our wave function contains complex variational parameters, \(\{X^i\} = \{C^i_{\alpha}, Z^i_{\alpha}, \gamma^i_{\alpha} : C^K_{\alpha}, Z^K_{\alpha}, \gamma^K_{\alpha}\}\). They are determined by the energy-variation. In our study, we employ the frictional cooling method as a means of
energy-variation,

$$\dot{X}_o^i = \left( \lambda + i\mu \right) \frac{1}{i\hbar} \left[ \frac{\partial\mathcal{H}}{\partial X_o^i} + \eta \frac{\partial W}{\partial X_o^i} \right]$$

and C.C. (16)

Here, $\mathcal{H}$ is the expectation value of the Hamiltonian and $W$ is a constraint condition. $\eta$ is a Lagrange multiplier, which is determined by $\partial W/\partial \eta = 0$. It is easily proved that, if we assume $\mu < 0$ in Eq. (16) and all of the parameters are developed with time according to Eq. (16), the energy of the system decreases while satisfying the constraint condition $W = 0$. If we use the superposition of several Gaussian wave packets to represent a nucleon and a K meson wave function, we need a constraint condition in order to fix the center of mass of the total system to the origin, and then $W$ is expressed as follows:

$$W = \langle \hat{R}_G \rangle^2 + \langle \hat{P}_G \rangle^2,$$

$$\hat{R}_G = \sum_{i=1}^{A} m_N \hat{r}_i + m_K \hat{r}_K / Am_N + m_K,$$

$$\hat{P}_G = \sum_{i=1}^{A} \hat{p}_i + \hat{p}_K.$$

### III. TESTS OF OUR METHOD

Before applying our method to studies of various K nuclei, we investigate the basic properties of our method.

#### A. Dependence on the number of wave packets

As shown in Eq. (15) and Eq. (17), we represent a single nucleon wave function and a K meson wave function with $N_n$ and $N_K$ Gaussian wave packets, respectively. We investigate how much the solution depends on $N_n$ and $N_K$. First, we perform a test in the case of $\text{ppnK}^-$ without $pK^-/n\bar{K}^0$ mixing for simplicity. Table I shows the results of $\text{ppnK}^-$ for various $N_n$ and $N_K$. From this table, we find that the total binding energy and the central density are almost converged up to $N_n = 4$ and $N_K = 10$. However, we notice that the shape of the system, represented by the deformation parameters $(\beta, \gamma)$, is strongly dependent on $N_n$. This phenomenon can be understood as follows. As mentioned in our previous study [3], the protons distribute compactly near a $K^-$ so as to decrease their total energy by the strongly attractive $K^-\text{p}$ interaction. On the other hand, the neutron is widely spread and its total energy decreases by reducing its kinetic energy. Therefore, the protons stay compactly inside the system, while the neutron remains widely outside of the system. Thus, the neutron contributes to the shape of the total system. In the case of $N_n = 2$, since the neutron is represented by two Gaussian wave packets, it can spread only linearly. Thus, the total system deforms prolately. In the case of $N_n = 3$, it can spread with a triangular shape. Thus, the total system deforms oblately. In the case of $N_n = 4$, it can spread with a tetrahedron shape. The total system is therefore spherical. Thus, the shape of the total system changes as $N_n$ is varied.

Such a dependence of the shape on $N_n$ seems to be peculiar to $\text{ppnK}^-$ where the neutron number is equal to 1 and the proton number is 2. In addition, the total binding energy and the central density do not so strongly depend on $N_n$ and $N_K$. Therefore, taking the cost-performance of calculations into account, we adopt the model space of $N_n = 2$ and $N_K = 5$ in our calculations.

#### B. Solution of $\text{ppnK}^-$

We now check whether our new framework, $pK^-/n\bar{K}^0$ mixing and $J & T$ projections, works correctly or not. We perform a test on a system of $\text{ppnK}^-$. First, we investigate the property of $J & T$ projections. Although only the $J$ projection has often been carried out in the study of light unstable nuclei [3, 10], the present study for the first time makes the $T$ projection. In Table II, we show various quantum numbers of the wave function before projection ($|\hat{P}_M\Phi^\pm\rangle$) and that after projection ($|\hat{P}_M^J \hat{T}_K^T \Phi^\pm\rangle$). Apparently, the ground state of $\text{ppnK}^-$ seems to have quantum numbers of $J^z = \frac{3}{2}^+$ and $T = 0$. We performed $J & T$ projections so that the total system had such quantum numbers. Table II

| $T^2$ | $T_z$ |
|-------|--|
| 0.00 | 0.00 |
| 0.02 | 0.00 |

#### Table II: Quantum numbers before and after projection.

$J_z^z$, $J_N^z$, $L_N^z$, $S_N^z$, $L_K^z$, $T^2$, and $T_z$ of total system. $J_{\bar{K}}^z$, $L_{\bar{K}}^z$, and $S_{\bar{K}}^z$ of nucleon system. $L_N^z$, $L_K^z$, $T^2$, and $T_z$ of total system.
shows that \( \langle J^2 \rangle = 0.75 \) and \( \langle T^2 \rangle = 0.00 \), which agree with \( J(J+1) = \frac{1}{4} \cdot \frac{9}{4} \) and \( T(T+1) = 0 \cdot 1 \), respectively. Therefore, it is found that our \( J \) & \( T \) projections work well.

Next, in Table III various quantities obtained in the present calculation (Present) are compared with our previous result of a simple version of AMD (simple AMD) and the result of a BHF calculation (BHF) \[2\]. This table shows that the present result is almost identical to others. Since the isospin-z component of each particle is changeable in the present framework, we investigated each particle-number. Although we calculated \( \text{ppnK}^- \), the numbers of protons and neutrons after the calculation are both equal to 1.5, while those of \( \text{K}^- \) and \( \text{K}^0 \) are both 0.5. This means that \( \text{ppnK}^- \) and \( \text{ppnK}^0 \) are mixed with a ratio of 1:1 as the result of \( \pi \text{K}^- \) coupling through the \( I = 0 \) \( \text{KN} \) interaction.

Here, we remark on the components of the \( \text{KN} \) interaction. We can separate it into three parts in the particle base: i) \( V_{n,K}^- \) and \( V_{p,K}^0 \), ii) \( V_{p,K}^- \) and \( V_{n,K}^0 \), and iii) \( V_{p,K^-}, \) \( -\text{nK}^0 \). The interactions i) and ii) are working in each channel of \( \text{ppnK}^- \) and \( \text{ppnK}^0 \), and their expectation values are equal to \(-45 \) MeV and \(-255 \) MeV, respectively. Interaction iii) is related to \( \pi \text{K}^- / n\text{K}^0 \) mixing through the \( I = 0 \) \( \text{KN} \) interaction, and its expectation value is equal to \(-88 \) MeV. Thus, we find that the binding of this system is mainly due to the type ii) interaction and is further supported by the type iii) interaction, which causes coupling between the two channels.

### C. Interpretation of the density distribution

The influence of \( \pi \text{K}^- / n\text{K}^0 \) mixing can clearly be seen in the density distribution. Fig. 2 displays the density distribution of protons and neutrons in the \( \text{ppnK}^- \) system calculated by the new framework. We can see that the proton distribution is almost the same as the neutron one, contrary to our previous result \[2\], where protons distribute more compactly than neutrons because of the strong attraction between the \( \text{K}^- \) and the proton. We can solve this contradiction by introducing the concept of an intrinsic state in isospin space, as follows: the calculated expectation value of \( T^0 \) with the state \( |\text{K}^0 \rangle \) in the case of \( \text{ppnK}^- \), is nearly equal to zero. Therefore, this state is the eigen-state of isospin, i.e. \( T = 0 \), and we express it as \( |\text{K}^0 \rangle = |\text{K}^0 \rangle \) hereafter. It is easily found that \( |\text{K}^0 \rangle \) is composed of two configurations concerning the z-component of isospin:

\[
\begin{align*}
|\text{K}^0 \rangle (T = 0) &= \hat{P}_{T_z=0} \left( |\Phi_N \rangle \otimes |\varphi_K \rangle \right) \\
&= \hat{P}_{T_z=0} \left[ \sum_{m=-\infty}^{+\infty} \hat{P}_{T_z=m} |\Phi_N \rangle \otimes \left( \sum_{m=\pm1/2} |\hat{P}_{T_z=m} |\varphi_K \rangle \right) \right] \\
&= |\hat{P}_{T_z=+\frac{1}{2}} \Phi_N \rangle \otimes |\hat{P}_{T_z=-\frac{1}{2}} \varphi_K \rangle + |\hat{P}_{T_z=-\frac{1}{2}} \Phi_N \rangle \otimes |\hat{P}_{T_z=+\frac{1}{2}} \varphi_K \rangle,
\end{align*}
\]

(20)

where \( \hat{P}_{T_z} \) and \( \hat{P}_{T_K} \) are \( T_z \)-projection operators for the nucleon system and the \( \bar{K} \) meson, respectively. According to the values of \( T_z \) and \( T_K \), the first term indicates \( \text{ppnK}^- \), while the second term indicates \( \text{ppnK}^0 \). Hereafter, we express them as \( |\text{ppnK}^- \rangle \) and \( |\text{ppnK}^0 \rangle \), respectively. In addition, the overlap between \( |\text{ppnK}^0 \rangle \) and \( e^{i\pi T_z} |\text{ppnK}^- \rangle \), calculated numerically, is found to be almost one. Since this fact indicates \( |\text{ppnK}^0 \rangle \approx e^{i\pi T_z} |\text{ppnK}^- \rangle \), we can say that our wave function satisfies

\[
|\text{K}^0 \rangle (T = 0) = \sum_{\theta=0,\pi} e^{i\theta \hat{T}_z} |\text{ppnK}^- \rangle
\]

(21)

Namely, the \( |\text{ppnK}^- \rangle \) state rotates in isospin space so that \( |\text{K}^0 \rangle (T = 0) \), which has the good quantum number, \( T = 0 \), is formed. Based on the analogy of an “intrinsic” state in a deformed nucleus, rotating in the space to form the eigen-state of the angular momentum, we can regard the \( |\text{ppnK}^- \rangle \) or equivalent \( |\text{ppnK}^0 \rangle \) as an intrinsic state in the isospin space of \( |\text{K}^0 \rangle (T = 0) \).

Fig. 2 displays the proton and neutron distributions of the intrinsic state \( |\text{ppnK}^- \rangle \). Clearly, the proton distribution is more compact than the neutron one; this fact

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**TABLE III: Results of ppnK\(^-\).** B.E.: binding energy measured from the threshold of ppnK\(^+\). \( \Gamma \): width decaying to \( \Lambda \pi \). \( \rho(0) \): central density. \( R_{rms} \): root-mean-square radius of nucleon system.

|          | B.E. [MeV] | \( \Gamma \) [MeV] | \( \rho(0) \) [fm\(^3\)] | \( R_{rms} \) [fm] |
|----------|------------|-----------------|----------------|----------------|
| Present  | 110.3      | 21.2            | 1.50            | 0.72           |
| simple AMD | 105.2      | 23.7            | 1.39            | 0.72           |
| BHF      | 108        | 20              | —              | 0.97           |


TABLE IV: Summary of present calculations. $J^\pi$ and $T$: spin-parity and isospin of total system. $E(\bar{K})$: binding energy of $K^-$ meson. $\Gamma$: width decaying to $\Sigma\pi$ and $\Lambda\pi$ channels. $\rho(0)$: central density. $R_{rms}$: root-mean-square radius of nucleon system. $(\beta, \gamma)$: deformation parameters.

| $J^\pi$ | $T$ | $E(\bar{K})$ | $\Gamma$ | $\rho(0)$ | $R_{rms}$ | $\beta$ | $\gamma$ |
|---------|-----|-------------|--------|-------|--------|--------|-------|
| ppn$K^-$ | 2 | 110.3 | 21.2 | 1.50 | 0.72 | 0.22 | 9.2 |
| pp$K^-$ | 1 | 96.7 | 12.5 | 1.56 | 0.81 | 0.70 | 11.8 |
| ppp$K^-$ | 1 | 105.0 | 25.9 | 1.29 | 0.97 | 0.54 | 3.8 |
| $^6$Be$K^-$ | 0+ | 104.2 | 33.3 | 0.91 | 1.17 | 0.44 | 0.3 |
| $^9$BK$^-$ | 3− | 118.5 | 33.0 | 0.71 | 1.45 | 0.46 | 20.8 |

TABLE V: Binding energy and number of strongly interacting nucleons near the $K^-$ meson.

| Nucleon | B.E. | Nucleon | B.E. |
|---------|------|---------|------|
| ppn$K^-$ | 110.3 | pp$K^-$ | 96.7 |
| ppp$K^-$ | 105.0 | $^6$Be$K^-$ | 104.2 |
| $^9$BK$^-$ | 118.5 |
FIG. 3: Behavior of the total binding energy and the decay width, from $ppnK^-$ to $^9BK^-$. The decay width is to $\Lambda\pi$ and $\Sigma\pi$ channels. The red point and line indicates the binding energy and width of the $K$ nucleus. The blue-dashed line corresponds to the $\Sigma\pi$ threshold. The thresholds for other decay modes are expressed by the green-dashed lines.

FIG. 4: Density contours of the nucleon distributions of various $\bar{K}$ nuclei. $ppnK^-$, $pppK^-$ and $pppnK^-$: $3$fm $\times$ $3$fm. $^6BeK^-$ and $^9BK^-$: $4$fm $\times$ $4$fm.
In Table V, we show the number of strongly interacting nucleons (“Nucleon”) staying around the $\bar{K}$ in various $K$ nuclei. Except for pppK$^-$, which has a peculiar structure, as mentioned in the later section, about 1.7 to 2.5 nucleons are found to stay near the $K$ meson.

**B. Density distribution**

Fig. 4 displays the nucleon density distributions of pppK$^-$, pppK$^-$, pppnK$^-$, $^6$BeK$^-$ and $^9$BK$^-$. It is found that $K$ nuclei have extremely dense and peculiar nucleon distributions.

$^6$BeK$^-$ has a two-center-like structure similar to $^8$BeK$^-$. Fig. 5 shows proton and neutron distributions separately. We can find that protons have a two-center-like structure and that neutrons stay between two pairs of protons, against our expectation that $^6$Be should have such a structure as $\alpha + 2p$. We note that the $K$ meson's density distribution, which is not displayed here, is very similar to the neutrons' one. The structure of $^9$BK$^-$ is quite similar to that of $^8$BeK$^-$(3).

The most exotic system of pppK$^-$ shows a very peculiar density distribution. Strictly speaking, this system has not only the component of pppK$^-$, but also that of ppuK$^0$ due to the $I = 0$ KN interaction. We express this system as $^3_8^3$He. Fig. 6 shows only proton density distribution extracted. We can see a “satellite” in this figure. Summing up the density of proton in the region of this satellite, we find that the proton number is nearly equal to one. Thus, this satellite is a single proton. In addition, the particle numbers of proton, neutron, K$^-$ and K$^0$ are 2.67, 0.33, 0.67 and 0.33, respectively. We can understand these particle numbers and the density distribution consistently as follows. We regard this system as a single proton combining $^2_8^3$H, following its density distribution (Fig. 6). In isospin space, $^2_8^3$H means the $|T = \frac{1}{2}, T_z = \frac{1}{2}\rangle$ state, which is composed of two nucleons and a $K$ meson. The weight of each component included in it is determined by Clebsch-Gordan coefficients as shown below:

$$\begin{align*}
|\frac{3}{8}^3\text{He}\rangle &= |p\rangle \otimes |\frac{2}{8}^3\text{H}\rangle = |p\rangle \otimes \left| T = \frac{1}{2}, T_z = \frac{1}{2}\right\rangle \\
&= |p\rangle \otimes \left( \sqrt{\frac{2}{3}} \left| T_N = 1, T_z^N = 1; T^K = \frac{1}{2}, T^K_z = -\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}} \left| T_N = 1, T_z^N = 0; T^K = \frac{1}{2}, T^K_z = \frac{1}{2}\right\rangle \right) \\
&= |p\rangle \otimes \left( \sqrt{\frac{2}{3}} |pp \otimes K^-\rangle - \sqrt{\frac{1}{3}} |pn \otimes K^0\rangle \right) \\
\end{align*}$$ (22)

The particle numbers counted according to Eq. 22 are quite the same as the above values. After all, we come to an idea that in $^3_8^3$He one proton keeps its identity and that the residual part is composed of ppK$^-$ and pnK$^0$, which are mixed due to the $I = 0$ KN interaction. Note, however, that the single proton is strongly bound to $^2_8^3$H. In addition, we studied the dependence of this system on the number of wave packets. Even if one nucleon is represented by four Gaussian wave packets (i.e. $N_n = 4$ in Eq. 3), a proton is still pushed out, but a little less clearly. We consider that the third proton is pushed up to the 0p-shell due to Pauli Blocking, so that it forms a satellite-like structure. pppnK$^-$ has an extra neutron compared to pppK$^-$. When a neutron is added to pppK$^-$, such a satellite-like structure disappears and a different structure is formed for pppnK$^-$. 

**V. SUMMARY**

We improved the antisymmetrized molecular dynamics (AMD) regarding two points and applied it to systematic studies of $K$ nuclei. One of our improvements is “pK$^-$/nK$^0$ mixing”, which enables us to treat directly the coupling of pK$^-$ and nK$^0$ through the $I = 0$ KN interaction. The other one is “$J$ & $T$ projections” with which the strong isospin dependence of the KN interaction is expected to be correctly treated. Thus, AMD is capable of treating the $I = 0$ KN interaction adequately, which plays an essential role in K nuclei.

We have investigated the properties of our new framework on pppK$^-$. After $J$ & $T$ projections, the total system is found to possess the quantum numbers $J$ and $T$, which turn out to be equal to those which we had as-
signed beforehand. Namely, we have confirmed that the 
J & T projections work correctly. The new result and the
previous one are very similar to each other, but we can see the influence of pK^-/nK^0 mixing in the density distributions of the protons and neutrons. Owing to the introduction of K^0, the present wave function can form the eigen-state of isospin, i.e. \(|\vec{\rho}_K H(T=0)\rangle\). When we draw the proton and neutron distributions of \(|\vec{\rho}_K H(T=0)\rangle\), they are clearly different from those shown in our previous study. We confirm that the \(|\vec{\rho}_K H(T=0)\rangle\) is formed by the rotation of \(|ppnK^-\rangle\) in isospin space. Namely, \(|ppnK^-\rangle\) is considered to be an “intrinsic state in isospin space” for the \(|\vec{\rho}_K H(T=0)\rangle\). This intrinsic state \(|ppnK^-\rangle\) is found to have quite the same proton and neutron distributions as those in our previous study. The \(|\vec{\rho}_K H(T=0)\rangle\) contains both \(|ppnK^0\rangle\) and \(|ppnK^-\rangle\) components with the same ratio. This is consistent with the fact that the calculated proton and neutron numbers are both equal to 1.5. The coupling of these two components due to the I = 0 KN interaction helps the binding of the total system \(|\vec{\rho}_K H(T=0)\rangle\).

We have studied ppnK^-, pppK^-, ppnK^-, K^-BeK^- and K^-BeK^- with our new framework. All K nuclei that we investigated are bound by about 100 MeV below the threshold of each nucleus+K^-. Except for ppnK^-, they are bound below the Σπ threshold, which is the main decay channel. Since their decay width is 20 to 40 MeV and small compared to their binding energy, they appear to be discrete states. For the strong interaction, ppnK^- is found to be unstable, while K^-BeK^- and K^-BeK^- are stable. We found that they have very different structures. Especially, ppnK^- shows an interesting satellite-like which is composed of a single proton. This proton keeps its identity and is strongly bound by the main body.

According to our present study, we predict various deeply bound K nuclei. They have very peculiar structures with extremely high densities, which we have never seen. In the future, such K nuclei as those investigated in the present paper may be explored experimentally. For instance, ppnK^- can be formed from a \(^4\text{He}\) beam (K^-, n) experiment, which is now under way at KEK [1]. The use of in-flight (K^-, N) reactions is also proposed [14]. Proton-rich exotic K nuclei are expected to be produced by (K^-, π^-) reactions via Λ* doorway states [2].

Finally, we would like to mention possibility of the new framework of AMD. Since it can be extended straightforwardly to the case of multi-K’s, we can investigate multi-K nuclei, which are closely related to kaon condensation and strange quark matter. The success of the new framework of AMD means that we can deal with even more fields of physics, because coupled-channel-like calculation can be carried out in the new version of AMD. For example, we can study Λ-Σ mixing in hypernuclei [12] with this new framework.

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