Models for evaluating the performance of complex information and communication systems

I Ya Lvovich¹, Ya E Lvovich², A P Preobrazhenskiy¹, Yu P Preobrazhenskiy¹ and O N Choporov²

¹Information Systems and Technologies Department, Voronezh institute of high technologies, 73a, Lenina st, Voronezh, 394043, Russia
²Information Security Department, Voronezh state technical university, 14, Moscow dist., Voronezh, 394026, Russia

E-mail: komkovvivt@yandex.ru

Abstract. Information and telecommunication systems are one of the key components of modern companies. The effectiveness of the company depends on their performance. The paper considers several approaches related to the assessment of information and telecommunication systems: probabilistic method, series decomposition method, worst case method, statistical test method. An algorithm for analyzing the stability of an information and telecommunication system when reversible and irreversible changes in parameters are observed is considered. The results of evaluating the performance of information and telecommunication system. The comparison of methods is carried out.

1. Introduction

If we consider the components of complex information and communication systems [1, 2], they are characterized by a large number of parameters \( x_j (j = 1, n) \). The analysis shows that such parameters will be random \( \tilde{x}_j = (j = 1, n) \). When taking into account the features of the functioning of information and telecommunication systems, it is required to take into account the joint distribution density \( \varphi(\tilde{x}_1, ..., \tilde{x}_n) \). Information and telecommunication systems performance

\[
y_i = f_i(x_1, ..., x_j, ..., x_n), i = 1, m
\]

depends on the following main parameters: available memory, processor load, information transfer rate, including via wireless communication channels. For performance, we have a random variable \( \tilde{y} \). Moreover, it is characterized by the corresponding distribution density \( \varphi(\tilde{y}) \). It is necessary to ensure the stable functioning of the information and telecommunication system [3, 4]. In solving this problem, the following approaches can be applied: the probabilistic method based on series expansion, the worst-case method, and the statistical test method [5]. In this paper, we consider the features of these approaches and show some results on evaluating the performance of information and telecommunication systems.
2. Probabilistic method
In order to implement such an approach, it is necessary to form a mathematical model (1). In addition, it is important to know the mathematical expectation \( M(\bar{x}_j) \), variance \( D(\bar{x}_j) \), pair correlation coefficients \( R_{jk} (j = 1, n; k = 1, n) \) for parameters \( x_j \). For the performance of the information and telecommunication system, it is necessary to describe the statistical properties of \( \bar{y}_i \) [5].

Based on the central limit theorem, we can state that the performance of complex information and communication systems \( \bar{y} \) obeys the normal distribution [6]. Based on experimental data for random variables, it is necessary to determine the mathematical expectation \( M(\bar{x}_j) \) and variance \( D(\bar{x}_j) \).

3. Series decomposition method
The performance function of information and communication systems (1) is expanded in a Taylor series. In this case, the neighborhood \( \Delta x = (\Delta x_1, ..., \Delta x_n) \) is used for the average values of the analyzed parameters \( x_1^0, ..., x_n^0 \). In this case, the following notation is used: \( x_j^0 = M(\bar{x}_j) \), \( \Delta x_j = x_j - M(\bar{x}_j), j = 1, n \). As a result, we have the expression:

\[
y = f(x_1^0, ..., x_n^0) + \sum_{j=1}^{n} \frac{\partial f}{\partial x_j} x_j = x_j^0, \Delta x_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \frac{\partial^2 f}{\partial x_j \partial x_k} \Delta x_j \Delta x_k + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \frac{\partial^2 f}{\partial x_j^2} \Delta x_j^2 \ldots \quad (2)
\]

In the indicated expansion, it is sufficient to take into account only the terms of the second order expansion [7]. In this case, the sensitivity coefficients:

\[
A_j = \left. \frac{\partial f}{\partial x_j} \right|_{x_j = x_j^0, j = 1, n}; \quad A_{jk} = \left. \frac{\partial^2 f}{\partial x_j \partial x_k} \right|_{x_j = x_j^0, j = 1, n, k \neq j}
\]

For performance, a sensitivity model is formed

\[
y = A_0 + \sum_{j=1}^{n} A_j \Delta x_j + \sum_{j=1}^{n} \sum_{k=1}^{n} A_{jk} \Delta x_j \Delta x_k + \sum_{j=1}^{n} A_{j} \Delta x_j^2 . \quad (3)
\]

The deviations \( \Delta x_j (j = 1, n) \) in (3) will be random. This is because the parameters \( \bar{x}_j (j = 1, n) \) vary randomly

\[
\Delta \bar{x}_j = \bar{x}_j - M(\bar{x}_j); \quad M(\Delta \bar{x}_j) = 0, D(\Delta \bar{x}_j) = D(\bar{x}_j); \quad M(\Delta \bar{x}_j \Delta \bar{x}_k) = R_{jk} \sigma(\bar{x}_j) \sigma(\bar{x}_k) . \quad (5)
\]

Based on (2) and (5), we obtain

\[
M(\bar{y}) = f(x_1^0, ..., x_n^0) + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} A_{jk} R_{jk} \sigma(\bar{x}_j) \sigma(\bar{x}_k) + \frac{1}{2} \sum_{j=1}^{n} A_{j} D(\bar{x}_j) . \quad (6)
\]
In expression (6), the first term corresponds to the performance of the information and telecommunication system when average values of parameters are considered. The correlation of parameters is taken into account in the second term. Nonlinearity of performance is taken into account in the third term. If we consider the linear terms in the expansion, as well as the transformations for the variance, then we can write

\[ D(\bar{y}) = \sum_{j=1}^{n} A_j^2 D(\bar{x}_j) + \sum_{j=1}^{n} \sum_{j \neq k} A_j A_k R_{jk} \sigma(\bar{x}_j) \sigma(\bar{x}_k). \]  

(7)

Performance stability is described based on characteristics (6) and (7). That is, the parameter \( \bar{y} \) will be within the limits \( y_{\text{min}}, y_{\text{max}} \) with some probability parameters:

\[ P(y_{\text{min}} \leq \bar{y} \leq y_{\text{max}}) = \Phi \left( \frac{y_{\text{max}} - M(\bar{y})}{\sigma(\bar{y})} \right) - \Phi \left( \frac{y_{\text{min}} - M(\bar{y})}{\sigma(\bar{y})} \right). \]

(8)

In the indicated expression, \( \Phi(\cdot) \) is the normalized Laplace function.

4. Worst case method

A similar approach is applied if a simplified calculation is carried out. Absolute \( \Delta x_j = x_j - x_j^0, \Delta y = y - y^0 \) or relative \( \delta x_j = (x_j - x_j^0)/x_j^0, \delta y (y - y^0)/y^0 \) parameters are taken into account. In this case, a linear approximation [8, 9] of expression (1) is carried out. If absolute deviations are considered, we obtain

\[ \Delta y = \sum_{j=1}^{n} A_j \Delta x_j. \]  

(9)

In case relative deviations are considered:

\[ \delta y = \sum_{j=1}^{n} a_j \delta x_j \left( a_j = \frac{A_j x_j^0}{y^0} \right). \]  

(10)

we take into account the condition

\[ A_j > 0 \forall j = 1, n_1, A_j < 0 \forall j = n_1 + 1, n. \]  

(11)

Then for the worst performance deviations of information and telecommunication systems, taking into account the deviations of the parameters, we will have the expression:

\[ \Delta y_{\text{max}} = \sum_{j=1}^{n_1} A_j \Delta x_j^{\text{max}} + \sum_{j=n_1+1}^{n} A_j \Delta x_j^{\text{max}}, \Delta y_{\text{min}} = \sum_{j=1}^{n_1} A_j \Delta x_j^{\text{min}} + \sum_{j=n_1+1}^{n} A_j \Delta x_j^{\text{max}}. \]  

(12)

Relative deviations can be taken into account in a similar way [10, 11]. If the parameters are characterized by symmetrical deviations

\[ \Delta x_j^{\lim} = \Delta x_j^{\text{max}} - \Delta x_j^{\text{min}}, \]  

(13)

then productivity is characterized by marginal deviation

\[ \Delta y^{\lim} = \sum_{j=1}^{n} |A_j| \Delta x_j^{\lim}. \]  

(14)

5. Statistical test method

Using this approach to evaluate the performance of information and telecommunication systems for parameters \( x_j \), pseudo-random sequences for values will be generated in a computer way. Moreover, the frequency of their implementation will be characterized by the distribution density of the random variable \( \bar{x}_j \) [5]. In order to generate, it is necessary to specify a sequence of random numbers \( \xi \). It is described on the basis of a uniform distribution law corresponding to the interval \((0,1)\). The specified sequence must be converted to a sequence of random numbers. They will be described using the distribution function \( F(x) \). To determine it, you need to make a choice of a random number \( \xi \) from a sequence of random numbers and then find a solution to the equation
\[ F(\eta) = \xi. \]  

(15)

In it, \( \eta \) will be unknown. When solving (15) we get a random number. It refers to a collection of random numbers. This set is related to the distribution function \( F(\chi) \). In practical problems, random variables are independent. Table 1 shows the transformations for various distribution laws.

### Table 1. Transformations for various distribution laws.

| Type of distribution | Parameters, describing the distribution | The method based on which we receive a random number \( \eta \) | Algorithm allowing calculate random number \( \eta \) |
|----------------------|----------------------------------------|-------------------------------------------------|------------------------------------------------|
| The uniform distribution \( \xi \) corresponding to the interval \((0,1)\) | \( a = 0 \) \( b = 1 \) | N.M. Korobov method [12] | - |
| Uniform distribution corresponding to the interval \((a,b)\) | \( a \) \( b \) | The random variable \( \xi \) undergoes a functional transformation | \( a + (b - a) \xi \) |
| Exponential | \( \lambda \) | The same method | \(-\frac{1}{\lambda} \ln \xi \) |
| Weibull’s distribution | \( \lambda \) \( k \) | The same method | \( k\sqrt{-\frac{1}{\lambda} \ln \xi} \) |
| Normal | \( M \) \( \sigma \) | Method of summation | \( M + \sigma \left( \sum_{i=1}^{12} \xi_i - 6 \right) \) |
| \( \chi^2 \)-distribution | \( n \)-number of degrees of freedom | The same method | \( \sum_{j=1}^{n} \gamma_j^2 \), where \( \gamma_j = \left( \sum_{i=1}^{12} \xi_i - 6 \right) \) |
| Erlang distribution | \( \lambda \) \( \nu \) | Distribution density will be placed in a unit square | - |

The uniform law will correspond to random numbers \( \xi \). They will go over to some values of the input parameters \( \tilde{x}_j \). For them, the distribution densities \( \phi(\tilde{x}_j) \) are observed. If such values are known, then using (1), the performance of the information and telecommunication system \( y \) is calculated. It is necessary to carry out the calculation process \( N \) times. After these procedures, a sample will be generated based on \( N \) values.

These will be the values of the random variable \( \tilde{y}_j \). By analyzing this random variable, one can determine the estimates of the mathematical expectation, variance, and also the probability that the analyzed value will be within a given interval. There are possibilities for determining the distribution density \( \phi(\tilde{x}_j) \).
6. An algorithm for analyzing the stability of an information and telecommunication system when reversible changes in parameters are observed

The information and telecommunication system may be in a stable condition. This will correspond to the cases when the parameters will be inside the given intervals, taking into account that there are external influences. For reversible changes in the parameters, you can specify the dependence

\[ x_j = \Psi(z_t). \]  

(16)

In an information and telecommunication system, random implementations corresponding to \( \Psi(z_t) \) can be observed. The linearization of the function \( \Psi(z_t) \) is carried out, which makes it possible for the estimated calculations to be implemented.

The designation of the intervals that will correspond to the external impact \( z_B \leq z \leq z_U \). For them, the process of approximation of dependence (16) occurs using the linear function \( \Delta x_j = K_x \Delta z_t \). In the indicated expression, the coefficient \( K_x \), which is linear, is a random variable. For it there are corresponding characteristics \( M(K_x) \) and \( D(K_x) \). They will vary depending on what interval we have: \( \Delta z_t = z_t - z_{IB}; \Delta x_j = x_j - \Psi(z_{IB}) \).

The mathematical expectation and variance of the parameters associated with the fact that there is an external influence \( z \) are determined on the basis of expressions

\[ M_x(\Delta x) = M(\bar{K}_x)\Delta z; \quad D(\Delta x) = D(\bar{K}_x)\Delta z^2. \]  

(17)

It can be seen that they depend on the linear coefficient. The change in the performance of the information and telecommunication system depends on its initial value \( y_0 \). In this case, the nature of how \( \Delta zM_x(\Delta y) \) and \( D_x(\Delta y) \) depend on changes in external influence is taken into account. We must use the formulas:

\[ M_x(\Delta y) = \sum_{j=1}^n A_j M_x(\Delta \bar{y}_j) + \frac{1}{2} \sum_{j=1}^n \sum_{k=1, j \neq k}^n A_{jk} R_{jk} \sigma_x(\Delta \bar{y}_j) \sigma_x(\Delta \bar{y}_k) + \frac{1}{2} \sum_{j=1}^n A_{jj} D_x(\Delta \bar{y}_j). \]  

(18)

\[ D_x(\Delta y) = \sum_{j=1}^n A_j^2 D_x(\Delta \bar{y}_j) + \sum_{j=1}^n \sum_{k=1, j \neq k}^n A_{jk} A_{kj} \sigma_x(\Delta \bar{y}_j) \sigma_x(\Delta \bar{y}_k). \]  

(19)

7. An algorithm for analyzing the stability of an information and telecommunication system when irreversible changes in parameters are observed

Changes in the parameters of information and telecommunication systems during their long work may be unsteady [13, 14]. If we analyze the functioning of such systems, then the smoothed implementations \( \bar{x}_{sm}(t), \bar{x}_{sm}^2(t), \bar{x}_{sm}^3(t) \), as well as fluctuations \( x(t) = x(t) - \bar{x}_{sm}(t) \).

The components are considered characterized by randomness. As a result of studies of information and telecommunication systems, it was shown that for many cases, fluctuations have a very weak effect.

The processes will be random due to the fact that the main process under consideration is due to the fact that there is randomness in smoothed implementations. It should be noted that they may not have fluctuations [15].

This is due to the fact that the smoothed implementations for each of the instances will be different. A quasi-deterministic model is used to describe a similar process.

If irreversible changes simply accumulate, then a similar linear model corresponding to the analyzed time range of work will be

\[ \bar{x}'(t) = a_0 + \bar{a}t. \]  

(20)

The notation is introduced here: \( \bar{a} = \frac{a_0 - x(t)}{t}; \quad a_0 = M(\bar{x}(0)) \).

To carry out the assessment, it is necessary to apply linear coefficients of change \( \beta = a/x^0 \). In this case, the designation \( x^0 \) was introduced, indicating the nominal value of the parameter. When we know \( M(\bar{\beta}_j), D(\bar{\beta}_j) \) by the coefficient \( \bar{\beta}_j \), then for any parameter \( x_j \) we calculate
Also we calculate time characteristics

\[ M_t(\Delta \tilde{x}_j) = M(\tilde{a}_j) t, \quad D_t(\Delta x_j) = D(\tilde{a}_j)t^2. \]  

(22)

The notation \( \Delta \tilde{x}_j = \tilde{x}_j(t) - x_j^0 \) is introduced here.

Formulas (21) and (22) provide opportunities for calculating the numerical characteristics of errors that relate to changes in productivity over time: \( M_t(\Delta \tilde{y}), D_t = (\Delta \tilde{y}). \)

8. Results

Figure 1 shows the example of experimentally obtained dependence of wireless data rate from signal-to-noise ratio.

The possibility of linear approximation of this dependence is demonstrated. Figure 2 shows a non-stationary process of changing the amount of free memory over time \( x(t) \). We can approximate the indicated dependence based on a polynomial \( (x^\alpha(t)) \) of degree 4. Table 2 show the comparison between several pointed out methods for some parameters in information and communication systems when we estimate errors.

![Figure 1](image1.png)  
**Figure 1.** The dependence of wireless data rate from the signal-to-noise ratio.

![Figure 2](image2.png)  
**Figure 2.** Non-stationary process of changing amount of free memory from time.

|                          | Probabilistic method | Series decomposition method | Worst case method | Statistical test method |
|--------------------------|----------------------|-----------------------------|-------------------|------------------------|
| Available memory         | Good                 | Excellent                   | Excellent         | Excellent              |
| Processor load           | Good                 | Excellent                   | Good              | Excellent              |
| Information transfer rate| Excellent            | Good                        | Good              | Excellent              |

9. Conclusion

The paper considers the features of performance estimates of information and telecommunication systems, which are part of the structure of modern organizations. It is shown how external factors affect the main parameters of systems, how the parameters change in time, the methods used are compared with each other in terms of possible accuracy of the results. The comparison of methods is carried out.
References

[1] Lvovich I, Preobrazhenskiy A, Preobrazhenskiy Y, Lvovich Y and Choporov O 2019 Managing developing internet of things systems based on models and algorithms of multi-alternative aggregation Int. Seminar on Electron Devices Design and Production, SED-2019 - Proceedings p 8798413

[2] Lvovich I Ya, Lvovich Ya E, Preobrazhenskiy A P, Preobrazhenskiy Yu P and Choporov O N 2019 Modelling of information systems with increased efficiency with application of optimization-expert evaluation Journal of Physics: Conf. Ser. Krasnoyarsk Science and Technology City Hall of the Russian Union of Scientific and Engineering Associations Polytechnical Institute of Siberian Federal University. Bristol, United Kingdom 33079

[3] Orlova D E 2018 Stability of solutions in ensuring the functioning of organizational and technical systems Modeling Optimization and Information Technologies 6(1) 325-36

[4] Sorokin S O 2018 Optimization modeling of the functioning of the system of homogeneous objects in a multidimensional digital environment Modeling optimization and information technologies 6(3) 153-64

[5] Parmezan A R S, Souza V M A and Batista G E Evaluation of statistical and machine learning models for time series prediction: Identifying the state-of-the-art and the best conditions for the use of each model Information Sciences 484 302-37

[6] Zhang J, Lee G M and Wang J 2016 A comparative analysis of univariate time series methods forestimating and forecasting daily spam in United States Twenty-second Americas Conference on Information Systems, San Diego, USA 1-10

[7] Golden RM, Henley SS, White H et al. 2016 Generalized information matrix tests for detecting model misspecification. Econometrics 4(4) 46

[8] Pekka Neittaanmki, Sergey Repin and Tero Tuovinen (Eds.) 2016 Mathematical Modeling and Optimization of Complex Structures Series: Computational Methods in Applied Sciences Springer International Publishing AG, Switzerland

[9] Rios L M and Sahinidis N V 2013 Derivative-free optimization: a review of algorithms and comparison of software implementations Journal of Global Optimization 54 1247-93

[10] Hugo Morais, Péter Kádár, Pedro Faria, Zita A Vale and Khodr H M 2010 Optimal Scheduling of a Renewable Micro-Grid in an Isolated Load Area Using Mixed-Integer Linear Programming Elsevier Editorial System(tm) for Renewable Energy Magazine 35(1) 151-6

[11] Odu G O and Charles-Owaba O E 2013 Review of Multi-criteria Optimization Methods Theory and Applications 3 1-14

[12] Korobov N M 1960 Properties and calculation of optimal coefficients Dokl. Akad. Nauk SSSR 132(5) 1009-12

[13] Talluri S, Kim M K and Schoenherr T 2013 The relationship between operating efficiency and service quality: are they compatible? Int. J Prod Res 51 2548-67

[14] Groefsema H and van Beest N R T P 2015 Design-time compliance of service compositions in dynamic service environments Int. Conf. on Service Oriented Computing & Applications 108-15

[15] Lee S, Liao Y, Seo M H, and Shin Y 2018 Oracle estimation of a change point in highdimensional quantile regression Journal of the American Statistical Association 113:523 1184-94