Structure of neutron, quark and exotic stars in Eddington-inspired Born-Infeld gravity

Tiberiu Harko1, Francisco S. N. Lobo2, M. K. Mak3, and Sergey V. Sushkov4

1Department of Mathematics, University College London, Gower Street, London WC1E 6BT, United Kingdom
2Centro de Astronomia e Astrofísica da Universidade de Lisboa, Campo Grande, Ed. C8 1749-016 Lisbon, Portugal
3Department of Computing and Information Management, Hong Kong Institute of Vocational Education, Chai Wan, Hong Kong, P. R. China, and
4Institute of Physics, Kazan Federal University, Kremlevskaya Street 18, Kazan 420008, Russia

(Dated: May 30, 2013)

We consider the structure and physical properties of specific classes of neutron, quark and “exotic” stars in Eddington-inspired Born-Infeld (EiBI) gravity. The latter reduces to standard general relativity in vacuum, but presents a different behavior of the gravitational field in the presence of matter. The equilibrium equations for a spherically symmetric configuration (mass continuity and Tolman-Oppenheimer-Volkoff) are derived, and their solutions are obtained numerically for different equations of state of neutron and quark matter. More specifically, stellar models, described by the stiff fluid, radiation-like, polytropic and the bag model quark equations of state are explicitly constructed in both general relativity and EiBI gravity, thus allowing a comparison between the predictions of these two gravitational models. As a general result it turns out that for all the considered equations of state, EiBI gravity stars are more massive than their general relativistic counterparts. Furthermore, an exact solution of the spherically symmetric field equations in EiBI gravity, describing an “exotic” star, with decreasing pressure but increasing energy density, is also obtained. As a possible astrophysical application of the obtained results we suggest that stellar mass black holes, with masses in the range of $3.8M_\odot$ and $6M_\odot$, respectively, could be in fact EiBI neutron or quark stars.

PACS numbers: 04.50.Kd, 04.20.Cv

I. INTRODUCTION

Despite its remarkable successes, the standard ΛCDM (A Cold Dark Matter) cosmological model faces severe theoretical, interpretational and observational challenges. The most important of these is the explanation of the late accelerated expansion of the Universe, inferred from observations of the expansionary evolution of Type Ia supernovae [1]. Combined with the recent Cosmic Microwave Background observations of the Planck satellite, [2], astronomical and astrophysical data provide compelling evidence that our Universe is dominated by a mysterious and exotic component, whose properties are difficult to be understood in the framework of our present day knowledge. Indeed, the standard model of cosmology has favored a missing energy-momentum component, in particular, the dark energy models. This exotic component can be interpreted theoretically either by assuming that it is a cosmological constant, which would represent an intrinsic curvature of space-time, or a vacuum energy. Alternatively, the dominant component of the Universe can be seen as a dark energy, which would mimic a cosmological constant. One of main dark energy scenarios is based on the so-called quintessence, where dark energy corresponds to a dynamical scalar field $\phi$ [3].

On the other hand, the possibility that general relativity breaks down at cosmological scales cannot be ruled out a priori, and the late-time cosmic acceleration may be due to infra-red modifications of general relativity. Therefore, a second possibility in explaining the observational data is to assume that at large scales the nature of the gravitational interaction is modified, and a new theoretical model of gravity is necessary in order to understand, and interpret, the observational data. Several, essentially geometric, modifications of standard general relativity have been considered, and investigated in detail as alternatives to dark energy. In particular, the $f(R)$ type models [4], where $R$ is the Ricci scalar, $f(R, L_m)$ models with geometry-matter coupling [5], where $L_m$ is the matter Lagrangian, $f(R, T)$ models [6], where $T$ is the trace of the energy-momentum tensor, Weyl-Cartan-Weitzenböck gravity [7], hybrid metric-Palatini $f(X)$-gravity models [8], or the recently proposed $f(R, T; R_{\mu\nu}T^{\mu\nu})$ gravity [9], where $R_{\mu\nu}$ is the Ricci tensor, and $T_{\mu\nu}$ is the matter energy-momentum tensor, are some of the proposed geometric modifications of general relativity that can explain the late de Sitter type expansionary phase in the evolution of the Universe.

In the context of modified theories of gravity, based on the classic work of Eddington [10], and on the non-linear electrodynamics of Born and Infeld [11], an interesting extension of general relativity was introduced in [12], and further developed in [13]. Essentially, in this model, de-
noted Eddington-inspired Born-Infeld gravity (EiBI), the Eddington action is coupled to matter without insisting on a purely affine action, or on a theory equivalent to Einstein gravity. The metric is present in the model, and the gravitational action has a Born-Infeld like structure. The model is based on a Palatini-type formulation, with the metric tensor \( g_{\mu \nu} \) and the connection \( \Gamma^\alpha_{\mu \nu} \) are varied independently. In this model, the Newton-Poisson equation is modified in the presence of matter sources, and the charged black holes are similar with those arising in Born-Infeld electrodynamics coupled to gravity. The cosmological solutions of the model for homogeneous and isotropic space-times show that there is a minimum length (and maximum density) at early times, indicating the possibility of an alternative theory of the Big Bang [13]. For a positive coupling parameter, the field equations have an important impact on the collapse of dust, and do not lead to singularities [14]. The theory supports stable, compact pressureless stars made of perfect fluid, and the existence of relativistic stars imposes a strong, near optimal constraint on the coupling parameter. This constraint can be improved by observations of the moment of inertia of double pulsars.

In [15] it was shown that the EiBI theory coupled to a perfect fluid reduces to general relativity coupled to a nonlinearly modified perfect fluid, leading to an ambiguity between the modified coupling and the modified equation of state. The observational consequences of this degeneracy were discussed, and it was argued that this extension of general relativity is viable from both an experimental and theoretical point of view through the energy conditions, consistency, and singularity-avoidance perspectives [15]. However, in [16] it was shown that the EiBI theory, which is reminiscent of Palatini \( f(R) \) gravity, shares the same pathologies, such as curvature singularities at the surface of polytropic stars and unacceptable Newtonian limit. The singularity avoidance in EiBI gravity was analyzed in [17], by considering the behavior of a homogeneous and isotropic universe filled with phantom energy in addition to the dark and baryonic matter. Unlike the Big Bang singularity that can be avoided in this kind of model, the Big Rip singularity is unavoidable in the EiBI phantom model. The dark matter density profile in EiBI gravity was also considered in [18], and it was found that in this model the dark matter density distribution is described by the Lane-Emden equation with a polytropic index \( n = 1 \), and is non-singular at the galactic center. The tensor perturbations of a homogeneous and isotropic space-time in the Eddington regime, where modifications to Einstein gravity are strong were analyzed, and it was found that the tensor mode is linearly unstable deep in the Eddington regime [19]. Furthermore, it was also argued that EiBI cosmologies may present viable alternatives to the inflationary paradigm as a solution to fundamental problems of the standard cosmological model, and that under specific assumptions the model is free from tensor singularities [20]. Other cosmological and astrophysical aspects of the EiBI model were considered in [21].

In an astrophysical context, the hydrostatic equilibrium structure of compact stars in the EiBI gravity was explored in [22], and a framework to study the radial perturbations and stability of compact stars in this theory was also developed. The standard results of stellar stability still hold in the EiBI theory, with the frequency square of the fundamental oscillation mode vanishing for the maximum-mass stellar configuration. The dependence of the oscillation mode frequencies on the coupling parameter \( \kappa \) of the theory was also investigated. The fundamental mode is insensitive to the value of the coupling constant, while higher order modes depend more strongly on it. However, generic phase transitions taking place in compact stars constructed in the framework of the EiBI gravity can lead to anomalous behavior of these stars [23]. In the case of first-order phase transitions, compact stars in EiBI gravity with a positive coupling parameter \( \kappa \) possess a constant pressure finite region, which is not present in general relativistic stars. For the case of a negative \( \kappa \), an equilibrium stellar configuration cannot be realized. Hence, in EiBI gravity there are stricter constraints on the microphysics of the stellar matter. Besides, in the presence of spatial discontinuities in the speed of sound due to phase transitions, the Ricci scalar is spatially discontinuous, and contains delta-function singularities, proportional to the jump in the speed of sound [22].

It is the purpose of this paper to investigate the properties of relativistic compact stars in the EiBI model. By assuming a spherically symmetric perfect fluid matter, the gravitational field equations of the EiBI model are solved numerically with several prescribed equations of state. As specific examples of stellar models we consider stars described by the causal stiff fluid equation of state, for which the speed of sound equals the speed of light; the radiation-type equation of state, for which the trace of the energy-momentum tensor is zero; the degenerate relativistic neutron matter equation of state, representing a polytrope of index \( n = 3 \); and the quark matter equation of state. For all these models the global astrophysical parameters of the stars (radius and mass) are obtained in both standard general relativity and in the EiBI gravity model, thus allowing a detailed comparison of the two approaches to stellar structure. As a general result of our study it follows that EiBI gravity allows the existence of more massive stars, as compared to general relativity. We also obtain an exact stellar model solution, corresponding to an equation of state of the form \( \rho + 3p = 1/4 \pi \kappa \), where \( \rho \) and \( p \) are the energy density and isotropic pressure, respectively. This model corresponds to an “exotic” EiBI stellar-type object, with decreasing pressure, but increasing energy density.

The present paper is organized as follows. The EiBI gravity theory is briefly presented in Section [II]. The system of gravitational field equations describing the star interior (mass continuity and hydrostatic equilibrium equations) is derived in Section [III]. Stellar models described
by the stiff fluid, radiation, polytropic and MIT bag model equations of state are studied numerically, in both EiBI model and standard general relativity, in Section IV. An exact “exotic” stellar model is obtained in Section V. Finally, we discuss and conclude our results in Section VI.

II. EDDINGTON-INSPIRED BORN-INFELD GRAVITY: FORMALISM

In the present Section, we adopt for simplicity the natural system of units with $G = c = 1$. The EiBI theory, which is based on the Eddington gravitational action and Born-Infeld nonlinear electrodynamics, is obtained from the action $S$ given by

$$S = \frac{1}{16\pi \kappa} \int d^4 x \left( \sqrt{-|g_{\mu\nu} + \kappa R_{\mu\nu}|} - \lambda \sqrt{-g} \right) + S_M [g, \Psi_M],$$

where $g = \text{det}(g_{\mu\nu})$ and $R_{\mu\nu}$ is the symmetric part of the Ricci tensor, which is constructed solely from the connection $\Gamma^\sigma_{\beta\gamma}$. The determinant of the tensor $g_{\mu\nu} + \kappa R_{\mu\nu}$ is denoted by $|g_{\mu\nu} + \kappa R_{\mu\nu}|$. In addition to this, $\lambda \neq 0$ is a dimensionless constant and $\kappa$ is the Eddington parameter with inverse dimension to that of the cosmological constant $\Lambda$.

The matter action $S_M$ depends only on the metric $g_{\mu\nu}$ and the matter fields $\Psi_M$. In the limit $\kappa \to 0$, the action recovers the Einstein-Hilbert action with $\lambda = \Lambda \kappa + 1$. In the present paper, we consider only asymptotic flat solutions, and hence we take $\lambda = 1$. Therefore the cosmological constant vanishes, and the remaining parameter $\kappa$ plays the fundamental role for describing the physical behavior of various cosmological and stellar scenarios. Several constraints on the value and the sign of the parameter $\kappa$ have been obtained from solar observations, big bang nucleosynthesis, and the existence of neutron stars in [13, 14, 24, 25]. In particular, for cases with positive $\kappa$, effective gravitational repulsion prevails, leading to the existence of pressureless stars (stars made of non-interacting particles which provide interesting models for self-gravitating dark matter [18]) and to an increase in the mass limits of compact stars [14, 22].

Note that in the EiBI theory the metric $g_{\mu\nu}$ and the connection $\Gamma^\sigma_{\beta\gamma}$ are treated as independent fields. Variation of the action leads to the following results:

$$g_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu},$$

(2)

$$q_{\mu\nu} = \tau (g_{\mu\nu} - 8\pi \kappa T_{\mu\nu}),$$

(3)

$$\Gamma^\sigma_{\beta\gamma} = \frac{1}{2} q^{\sigma\sigma} (\partial_\gamma q_{\sigma\beta} + \partial_\beta q_{\sigma\gamma} - \partial_\sigma q_{\beta\gamma}),$$

(4)

where $q_{\mu\nu}$ is an auxiliary metric, $q = \text{det}(q_{\mu\nu})$ and we have denoted $\tau$ as $\tau = \sqrt{g/q}$.

In the EiBI model, the energy-momentum tensor $T_{\mu\nu}$, defined as

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{\mu\nu}},$$

(5)

satisfies the standard conservation equations $\nabla_\mu T^{\mu\nu} = 0$, where, as in general relativity, the covariant derivative $\nabla_\mu$ refers to the metric $g_{\mu\nu}$. If the energy-momentum tensor $T_{\mu\nu}$ vanishes in Eq. (5), then the physical metric $g_{\mu\nu}$ is equal to the apparent metric $q_{\mu\nu}$. Hence in vacuum the EiBI theory is completely equivalent to standard general relativity.

Note that Eqs. (2) and (3) may be expressed in the following forms:

$$q^{\mu\alpha} g_{\alpha\nu} = \delta^\mu_\nu - \kappa R^\mu_\nu,$$

(6)

$$q^{\mu\alpha} g_{\alpha\nu} = \tau (\delta^\mu_\nu - 8\pi \kappa T^\mu_\nu),$$

(7)

where $R^\mu_\nu = q^{\mu\alpha} R_{\alpha\nu}$ and $T^\mu_\nu = g^{\mu\alpha} T_{\alpha\nu}$. Now, combining Eqs. (6), (7), yields the following relations

$$R^\mu_\nu = 8\pi \tau T^\mu_\nu + \frac{1 - \tau}{\kappa} \delta^\mu_\nu,$$

(8)

$$R = 8\pi \tau T + \frac{4(1 - \tau)}{\kappa}.$$  

(9)

One may now write the modified Einstein equation as

$$G^\mu_\nu \equiv R^\mu_\nu - \frac{1}{2} R \delta^\mu_\nu = 8\pi \tau T^\mu_\nu - \left[ \frac{1 - \tau}{\kappa} + 4\pi \tau T \right] \delta^\mu_\nu,$$

(10)

where the Einstein tensor $G^\mu_\nu$ is defined in terms of the auxiliary $q$-metric. The factor $\tau$ can be obtained from $T^\mu_\nu$ by the relation

$$\tau = \left[ \text{det}(\delta^\mu_\nu - 8\pi \kappa T^\mu_\nu) \right]^{-\frac{1}{2}}.$$

(11)

Throughout this work we consider that the energy-momentum tensor of the compact object is given by the standard form

$$T_{\mu\nu} = (\rho + p) u^\mu u^\nu + pg_{\mu\nu},$$

(12)

where $\rho$, $p$ and $u^\mu$ are the energy density, the isotropic pressure and the four velocity of the fluid, respectively, with the latter satisfying the normalization condition $u^\mu u^\nu g_{\mu\nu} = -1$. Thus, in terms of physical quantities $\tau$ can be expressed as

$$\tau = \left[ (1 + 8\pi \kappa \rho)(1 - 8\pi \kappa \rho)^3 \right]^{-\frac{1}{2}}.$$  

(13)

With the EiBI gravity theory briefly presented above, we now analyze the structure equations for static and spherically symmetric compact objects below.

III. STRUCTURE EQUATIONS FOR COMPACT OBJECTS IN EDDINGTON-INSPIRED BORN-INFELD GRAVITY

In the following, we shall incorporate the natural system of units $G$ and $c$ to the corresponding equations.
Now, we will investigate the structure of compact static and spherically symmetric objects. The line elements for the physical metric $g_{\mu\nu}$ and for the auxiliary metric $q_{\mu\nu}$ are given by 

\[ g_{\mu\nu}dx^\mu dx^\nu = -e^{\nu(r)}c^2dt^2 + e^{\lambda(r)}dr^2 + f(r) d\Omega^2, \quad q_{\mu\nu}dx^\mu dx^\nu = -e^{\beta(r)}c^2dt^2 + e^{\alpha(r)}dr^2 + r^2d\Omega^2, \]

respectively, where $\nu(r)$, $\lambda(r)$, $\beta(r)$, $\alpha(r)$ and $f(r)$ are arbitrary metric functions of the radial coordinate $r$, and $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$.

Using Eq. (10), the system of gravitational field equations describing the structure of a compact object is given by \[22, 23\]

\[ \frac{d}{dr} (re^{-\alpha}) = 1 - \frac{1}{2\kappa} \left( 2 + \frac{a}{b^3} - \frac{3}{ab} \right) r^2, \]

\[ e^{-\alpha} \left( 1 + \frac{r}{2\kappa} \frac{db}{dr} \right) = 1 + \frac{1}{2\kappa} \left( \frac{1}{ab} + \frac{a}{b^3} - 2 \right) r^2, \]

and Eq. (3) yields the following relations

\[ e^\beta = \frac{e^{\nu}b^3}{a}, \quad e^\alpha = e^\lambda ab, \quad f = \frac{\rho^2}{ab}, \]

where we have defined the arbitrary functions $a(r)$ and $b(r)$ as

\[ a = \sqrt{1 + \frac{8\pi G}{c^2} \kappa \rho}, \]

and

\[ b = \sqrt{1 - \frac{8\pi G}{c^4} \kappa \rho}, \]

respectively.

The conservation of the energy-momentum tensor in the $g$-metric,

\[ \frac{dv}{dr} = -\frac{2}{p + \rho c^2} \frac{dp}{dr} = \frac{4b}{a^2 - b^2} \frac{db}{dr}, \]

provides the following conservation relation in the auxiliary $q$-metric

\[ \frac{d\beta}{dr} = \frac{4b}{a^2 - b^2} \frac{db}{dr} + \frac{3db}{b} - \frac{1}{a} \frac{da}{dr}. \]

The existence of a barotropic equation of state of the dense matter $p = p(\rho)$ imposes a similar equation of state in the $q$-metric, $a = a(b)$. Therefore, by defining $c_i^2 = da/db$, the energy-momentum conservation equation in the $q$-metric can be formulated as

\[ \frac{d\beta}{dr} = \left( \frac{4b}{a^2 - b^2} + \frac{3}{b} - \frac{1}{a} \frac{c_i^2}{c_i^2} \right) \frac{db}{dr}. \]

Note that Eq. (10) can be immediately integrated to give

\[ e^{-\alpha} = 1 - \frac{2Gm(r)}{c^2 r}, \]

where the function $m(r)$ is obtained as

\[ \frac{dm}{dr} = \frac{c^2}{4G\kappa} \left( 2 + \frac{a}{b^3} - \frac{3}{ab} \right) r^2. \]

By substituting Eqs. (23) and (24) into Eq. (17) we obtain the $q$-metric generalization of the standard Tolman-Oppenheimer-Volkoff (TOV) equation of general relativity as

\[ db = \frac{ab(a^2 - b^2)}{r^2} \left[ (1/2\kappa) \left( 1/ab + a/b^3 - 2 \right) r^3 + 2Gm/c^2 \right]. \]

Once the equation of state of matter is known, the mass continuity, Eq. (25), and the generalized hydrostatic equilibrium Eq. (26), describe all the properties of compact objects in EiBI gravity. In order to obtain a dimensionless form of the mass continuity and hydrostatic equilibrium equations we introduce a set of dimensionless variables $(\eta, m_0, \kappa_0, \theta, p_0)$, defined as

\[ r = \frac{c}{\sqrt{2\pi G \rho_c}}, \quad m = \frac{c^3}{\sqrt{2\pi G \rho_c}} m_0, \]

\[ \kappa = \frac{c^2}{8\pi G \rho_c \kappa_0}, \quad \rho = \rho_c \theta, \quad p = \rho_c c^2 p_0, \]

where $\rho_c$ is the central density of the star. These dimensionless quantities will be extremely useful for the numerical analysis carried out below.

Therefore in EiBI gravity the mass continuity and hydrostatic equilibrium equations for compact objects take the dimensionless form

\[ \frac{dm_0}{d\eta} = \frac{1}{\kappa_0} \left( 2 + \frac{a^2 - b^2}{ab} \right) \eta^2, \]

and

\[ \frac{db}{d\eta} = \frac{2 \left( (a^2 + b^2) - 2 \right) (\eta^3/\kappa_0) + m_0}{\eta^2 (1 - 2m_0/\eta) \left[ \frac{4b}{a^2 - b^2} + \frac{3}{b} - \frac{d\ln m_0}{db} \right]}, \]

respectively. The functions $a$ and $b$ are obtained as

\[ a = \sqrt{1 + \kappa_0 \theta}, \quad b = \sqrt{1 - \kappa_0 \rho_0}, \]

respectively and they must satisfy an equation of state of the form $a = a(b)$. The mass continuity and the hydrostatic equilibrium equations must be integrated with the boundary conditions

\[ m(0) = 0, \quad \theta(0) = 1, \]

\[ b(0) = \sqrt{1 - \kappa_0 \rho_0}, \quad b(\eta_S) = 1, \]

where $\rho_0 = p_c/\rho_c c^2$, with $p_c$ the central pressure, while $\eta_S$ determines the radius $R$ of the star through the condition $p(R) = 0$. Once the dimensionless parameters
\((\eta, m_0, \kappa_0, \theta)\) are obtained from the numerical integration of the structure equations of the star, the physical parameters in the \(q\)-metric can be obtained as

\[
\begin{align*}
 r &= 3.276 \times 10^6 \times \left( \frac{\rho}{\rho_n} \right)^{-1/2} \times \eta \, \text{cm}, \\
m &= 22.107 \times \left( \frac{\rho}{\rho_n} \right)^{-1/2} \times m_0 \times M_\odot, \\
\kappa &= 2.684 \times 10^{12} \times \kappa_0 \, \text{cm}^2, \\
\end{align*}
\]

where \(\rho_n = 2 \times 10^{14} \, \text{g/cm}^3\) is the nuclear density, and \(M_\odot = 2 \times 10^{33} \, \text{g}\) is the solar mass.

In the \(g\)-metric the physical mass \(M(r)\) of the star is defined with the help of the metric tensor component \(e^{-\lambda}\) as

\[
e^{-\lambda} = 1 - \frac{2GM(r)}{c^2r}.
\]

Therefore we obtain the following relation between the masses \(M(r)\) and \(m(r)\) in the physical \(g\) and auxiliary \(q\) metrics,

\[
\frac{2GM(r)}{c^2r} = 1 - \left[ 1 - \frac{2Gm(r)}{c^2r} \right] ab.
\]

Taking into account the dimensionless variables introduced in Eqs. \ref{eq:dimensionless}, we have

\[
\frac{2M_0(\eta)}{\eta} = 1 - \left[ 1 - \frac{2m_0(\eta)}{\eta} \right] \sqrt{(1 + \kappa_0\theta)(1 - \kappa_0\rho_0)}.
\]

In the case of true vacuum, \(a = b = 1\), from Eqs. \ref{eq:dimensionless}, we obtain the metric function \(f(r) = r^2\), the \(g\)-metric coefficients

\[
e^{\nu(r)} = e^{-\lambda(r)} = 1 - \frac{2GM}{c^2r},
\]

and the \(q\)-metric coefficients,

\[
e^{\beta(r)} = e^{-\alpha(r)} = 1 - \frac{2Gm}{c^2r},
\]

respectively, which is the Schwarzschild solution. From Eq. \ref{eq:dimensionless}, we obtain the relation \(M = m\). Therefore it follows that the physical \(g\)-metric is identical to the apparent \(q\) metric. Hence the EiBI theory is completely equivalent to standard general relativity in true vacuum.

### IV. High Density Compact Objects in EiBI Gravity

In the present Section, we consider four cases of stellar structures in the EiBI gravity model, corresponding to different choices of the equation of state of dense matter. More specifically, we will consider the structure of high density stars composed of matter obeying the Zeldovich (stiff fluid), the radiation, the polytropic and the MIT bag model equations of state, respectively. In all these cases the properties of the corresponding neutron and quark stars are obtained by numerically integrating the structure equations. We will compare our results with the standard general relativistic ones, in which the structure of the high density compact objects is described by the mass continuity and the TOV equation, given by

\[
\frac{dM}{dr} = 4\pi \rho r^2,
\]

and

\[
\frac{dp}{dr} = -G \left( \frac{\rho + p/c^2}{M + 4\pi r^3 p/c^2} \right) \frac{\rho}{r^2 (1 - 2GM/c^2r)},
\]

respectively. In the dimensionless variables introduced in Eqs. \ref{eq:dimensionless}, the standard general relativistic mass continuity and TOV equations take the form

\[
\frac{dM_\ast}{d\eta} = 2\theta \eta^2,
\]

and

\[
\frac{dp_\ast}{d\eta} = -\left( \frac{\rho_\ast + p_\ast}{\eta^2 (1 - 2M_\ast/\eta)} \right),
\]

respectively, where \(M = c^3/\sqrt{2\pi G^3 \rho_\ast M_\ast}\), and \(p = \rho_\ast c^2 p_\ast\).

#### A. Compact stars in the EiBI model obeying the Zeldovich (Stiff Fluid) EOS

One of the most common equations of state, which has been used extensively to study the properties of compact objects is the linear barotropic equation of state, \(p = (\gamma - 1)\rho c^2\), with \(\gamma = \text{constant} \in [1, 2]\). The Zeldovich (stiff fluid) equation of state, corresponds to the case \(\gamma = 2\). This equation of state is valid for densities significantly higher than nuclear densities, \(\rho > 10\rho_n\). It can be obtained by constructing a relativistic Lagrangian that allows bare nucleons to interact attractively via scalar meson exchange, and repulsively via the exchange of a more massive vector meson \cite{27}. In the non-relativistic limit, in both the quantum and classical theories the interaction is mediated via Yukawa-type potentials. The vector meson exchange dominates at the highest matter densities and, by using a mean field approximation, it follows that in the extreme limit of infinite densities the pressure tends to the energy density, \(p \rightarrow \rho c^2\) \cite{27}. In this case, the speed of sound approaches the velocity of light, i.e., \(c^2_s = dp/d\rho \rightarrow c^2\), and therefore the stiff fluid equation of state satisfies the causality condition, with the speed of sound equal to the speed of light.

In the dimensionless variables given by Eqs. \ref{eq:dimensionless}, we obtain the following expressions

\[
\begin{align*}
 a &= \sqrt{1 + \kappa_0\theta}, \\
b &= \sqrt{1 - \kappa_0\theta}, \\
a^2 - b^2 &= 2(1 - b^2), \\
c_q^2 &= -\frac{b}{\sqrt{2 - b^2}}.
\end{align*}
\]
In order to have a real \( b \), the parameter \( \kappa_0 \) must satisfy the constraint \( \kappa_0 < 1 \).

Then the mass continuity and the hydrostatic equilibrium equation for the stiff fluid star in EiBI gravity become

\[
\frac{dm_0}{d\eta} = \frac{2}{\kappa_0} \left( 1 + 1 - 2b^2 \right) \eta^2,
\]

and

\[
\frac{db}{d\eta} = \frac{(b^2 - 1)}{\kappa_0 b^2} \times \frac{\left[ 2 \left( b^5 - 2b^3 + \sqrt{2 - b^2} \right) \eta^3 - b^3 \left( b^2 - 2 \right) \kappa_0 m_0 \right]}{(2b^2 - 3) \eta (\eta - 2m_0)}.
\]

respectively. The variation of the density and mass profiles of the stiff fluid stars in standard general relativity and EiBI gravity model are represented in Fig. 1.

As one can see from the figures, there is a very good concordance between the general relativistic and the EiBI gravity model predictions. For values of \( \kappa \) so that \( \kappa \leq 0.2 \) basically the predictions of the two models coincide. For values of \( \kappa \) in the range of \( \kappa \in (0.3, 0.9999) \) there are some small quantitative differences in the density and mass profiles, but which do not lead to significant differences in the global astrophysical parameters (mass and radius) of the star. In standard general relativity the maximum mass of neutron stars was obtained in [28], and estimated to be of the order of \( 3.2M_\odot \), by assuming that at densities higher than \( 4.6 \times 10^{14} \text{ g/cm}^3 \) the equation of state of matter is the stiff fluid equation of state. The dimensionless density \( \eta \approx 0 \) at \( \eta_S \approx 2 \), corresponds to a dimensionless mass value of \( M_0 \approx 0.5 \). Hence we obtain the radius and the mass of the star as a function of the central density in the form

\[
R \approx \frac{9.268 \times 10^{13}}{\sqrt{\rho_c}}, \quad M(R) \approx \frac{1.563 \times 10^8}{\sqrt{\rho_c}}.
\]

For central densities of the order of \( \rho_c = 2 \times 10^{15} \text{ g/cm}^3 \), the radius and the mass of the neutron star are \( R = 2.07 \times 10^6 \text{ cm} \), and \( M = 3.49M_\odot \), respectively. Hence the EiBI gravity corrections do not modify significantly the maximum values of the static neutron star masses.

### B. Compact star with a radiation equation of state in EiBI gravity

For a radiation-type high density fluid the equation of state (EOS) is \( p = \rho c^2 / 3 \) [23]. For this case we get

\[
a = \sqrt{1 + \kappa_0 \theta}, \quad b = \sqrt{1 - \frac{\kappa_0 \theta}{3}},
\]

with the parameter \( \kappa_0 \) satisfying the constraint \( \kappa_0 < 3 \), thus we obtain the following results

\[
a^2 = 4 - 3b^2, \quad a^2 - b^2 = 4 \left( 1 - b^2 \right), \quad c_\eta^2 = -\frac{3b}{\sqrt{4 - 3b^2}}.
\]

For a high density star with a radiation-like EOS, the mass continuity and the hydrostatic equilibrium equations take the form

\[
\frac{dm_0}{d\eta} = \frac{2}{\kappa_0} \left( 1 + \frac{2 - 3b^2}{b^3 \sqrt{4 - 3b^2}} \right) \eta^2,
\]

and

\[
\frac{db}{d\eta} = \frac{2\sqrt{4 - 3b^2} (b^2 - 1)}{\kappa_0 b^2} \times \frac{\left[ 2 \left( b^3 \sqrt{4 - 3b^2} + b^2 - 2 \right) \eta^3 - b^3 \left( b^2 - 2 \right) \kappa_0 m_0 \right]}{(3b^4 - 14b^2 + 12) \eta (\eta - 2m_0)}
\]

respectively. The variation of the dimensionless density and mass profiles for the radiation-type equation of state is represented in Fig. 2.

For a radiation fluid like star, the qualitative behavior of the mass and of the density are similar in both general relativity, and EiBI gravity. However, some quantitative differences between the two models do appear for this case. The dimensionless density reaches the value zero at around \( \eta_S = 2, \theta(\eta_S) \approx 0 \), with the corresponding dimensionless masses being given by \( M_0 \approx 0.45 \) in general relativity, and by \( M_0 \approx 0.55 \) in EiBI gravity, corresponding to the radii and masses

\[
R_{GR} \approx R_{EiBI} \approx \frac{9.268 \times 10^{13}}{\sqrt{\rho_c}}, \quad M_{GR}(R) \approx \frac{1.40 \times 10^8}{\sqrt{\rho_c}}, \quad M_{EiBI}(R) \approx \frac{1.719 \times 10^8}{\sqrt{\rho_c}}.
\]

For \( \rho_c = 2 \times 10^{15} \text{ g/cm}^3 \), we obtain \( R_{GR} \approx R_{EiBI} \approx 2.07 \times 10^6 \text{ cm} \), \( M_{GR}(R) \approx 3.24M_\odot \), and \( M_{EiBI}(R) \approx 3.84M_\odot \), representing an increase of around 22% of the high density neutron star mass due to the EiBI gravitational effects.

### C. Polytropic stars in EiBI gravity model

The polytropic equation of state

\[
p = K \rho \Gamma = K \rho^{1 + 1/n},
\]

where \( K, \Gamma \) and \( n \) are usually called the polytropic constant, the polytropic exponent and the polytropic index respectively, has been extensively used in astrophysics for the study of white dwarfs and neutron stars [21]. By introducing the transformation

\[
\rho = \rho_c \theta^n,
\]

the polytropic EOS can be written as

\[
p = K \rho_c^{1 + 1/n} \theta^{n+1}.
\]

Therefore we obtain for the parameters \( a \) and \( b \) the expressions

\[
a = \sqrt{1 + \kappa_0 \theta^n}, \quad b = \sqrt{1 - \kappa_0 \kappa_0 \theta^{n+1}},
\]
FIG. 1: Comparison of the dimensionless density (left figure) and g-metric mass (right figure) profiles for standard general relativistic and EiBI gravity models with stiff fluid equation of state, for different values of the parameter \( \kappa_0 \): \( \kappa_0 \to 0 \) – the general relativistic case – (solid curve), \( \kappa_0 = 0.9999 \) (dotted curve), \( \kappa_0 = 0.8 \) (dashed curve), and \( \kappa_0 = 0.2 \) (long dashed curve), respectively. The initial conditions used for the numerical integration of the mass continuity and hydrostatic equilibrium equations are \( \theta(0) = 1 \), \( M_*(0) = 0 \), \( m_0(0) = 0 \), and \( b(0) = \sqrt{1 - \kappa_0} \), respectively. See the text for more details.

FIG. 2: The plots depict the comparison of the dimensionless density (left figure) and g-metric mass (right figure) profiles for standard general relativistic and EiBI gravity models with radiation fluid equation of state, for different values of the parameter \( \kappa_0 \): \( \kappa_0 \to 0 \) – the general relativistic case – (solid curve), \( \kappa_0 = 2.9999 \) (dotted curve), \( \kappa_0 = 2 \) (dashed curve), and \( \kappa_0 = 1 \) (long dashed curve), respectively. The initial conditions used for the numerical integration of the mass continuity and hydrostatic equilibrium equations are \( \theta(0) = 1 \), \( M_*(0) = 0 \), \( m_0(0) = 0 \), and \( b(0) = \sqrt{1 - \kappa_0} / 3 \), respectively. We refer the reader to the text for further details.

where \( k_0 = K \rho_0^{1/n}/c^2 \). The parameter \( \kappa_0 \) must satisfy the constraint \( \kappa_0 < 1/k_0 \). Therefore we obtain

\[
a = \sqrt{1 + \kappa_0^{1/(n+1)} \left( \frac{1 - b^2}{k_0} \right)^{n/(n+1)}}.
\]

(54)

Hence the mass continuity and the hydrostatic equilibrium equations for polytropic stars in EiBI gravity take the form

\[
\frac{dm_0}{d\eta} = \frac{1}{k_0} \left[ 2 + \frac{1 - 3b^2 + \kappa_0^{1/n} \left( \frac{1 - b^2}{k_0} \right)^{n+1}}{b^3 \sqrt{1 + \kappa_0^{1/n} \left( \frac{1 - b^2}{k_0} \right)^{n+1}}} \right] \eta^2,
\]

(55)
\[
\frac{db}{d\eta} = \frac{2}{\eta^2 \left(1 - \frac{2M_*}{\eta}\right)} \left\{ \frac{b_k M_0}{b_k (n+1) \eta^2 \left(1 - \frac{2M_*}{\eta}\right)} \right\}.
\]

respectively. For the standard general relativistic case, the mass continuity and the hydrostatic equilibrium equations are given by

\[
\frac{dM_*}{d\eta} = 2\theta^n \eta^2, \quad (57)
\]

and

\[
\frac{d\theta}{d\eta} = -\frac{1 + \kappa_0 \theta}{k_0 (n+1) \eta^2 \left(1 - \frac{2M_*}{\eta}\right)}, \quad (58)
\]

respectively. In the following, we restrict our analysis to the case of the degenerate ultra-relativistic neutron gas, with equation of state \( p = (3\pi^2)^{1/3} (hc/4) (\rho/m_n)^{4/3} \), where \( m_n \) is the neutron mass. This equation of state has the polytropic index \( n = 3 \), and \( K = 1.23 \times 10^{15} \). For central star densities of the order of four times the nuclear density, \( \rho_c = 8 \times 10^{14} \text{g/cm}^3 \), the parameter \( k_0 \) has the value \( k_0 = 0.13 \), which is the value we will use for the numerical study of the polytropic stars. For this value of \( k_0 \) we obtain for \( \kappa_0 \) the constraint \( \kappa_0 < 7.692 \). The variation of the dimensionless density and mass of the general relativistic and EiBI stars with \( n = 3 \) polytropic equation of state are represented in Fig. 8.

In the case of the polytropic equation of state, and for the chosen values of the physical parameters, significant differences between the global properties of stars in the two gravitational theories appear. The dimensionless radius \( \eta_S \) of the star in the EiBI model varies in the range \( \eta_S \in (2.2, 2.9) \) for \( \kappa_0 \in (6.5, 7.2) \), while the dimensionless radius of the polytropic general relativistic star is around \( \eta_S \approx 4 \). This shows that polytropic stars are more compact (smaller radius) in EiBI gravity.

D. Structure and properties of quark stars in EiBI gravity

The chemical composition of neutron stars at densities beyond the nuclear saturation remains uncertain, with alternatives ranging from purely nucleonic composition through hyperon or meson condensates, to deconfined quark matter \[30\]. It was suggested that at all pressures strange quark matter (consisting of up \( u \), down \( d \), and strange \( s \) quarks) might be the absolute ground state of hadronic matter \[31, 32\].

Quark matter is formed from a Fermi gas of 3A quarks, constituting a single color singlet baryon with baryon number \( A \). The theory of the equation of state of strange matter is directly based on the fundamental Quantum Chromodynamics (QCD) Lagrangian \[32\]. In first order perturbation theory, by neglecting the quark masses, the equation of state for zero temperature quark matter is given by the MIT Bag model equation of state \[31, 32\]

\[
p = \frac{1}{3} (\rho - 4B) c^2, \quad (59)
\]

where \( B \) is the difference between the energy density of the perturbative and non-perturbative QCD vacuum (the bag constant). Equation (59) is essentially the equation of state of a gas of massless particles with corrections due to the QCD trace anomaly and perturbative interactions. The vacuum pressure \( B \), which holds quark matter together, is a simple model for the long-range, confining interactions in QCD. At the surface of the quark star, as \( p \to 0 \), we have \( \rho \to 4B \). The typical value of the bag constant is of the order \( B \approx 10^{14} \text{g/cm}^3 \, [31] \). After the neutron matter-quark matter phase transition (which is supposed to take place in the dense core of neutron stars) the energy density of strange matter is \( \rho \approx 5 \times 10^{14} \text{g/cm}^3 \). Therefore quark matter always satisfies the condition \( p \geq 0 \). In the dimensionless variables introduced by Eqs. (27) the Bag model equation of state takes the form

\[
p_0 = \frac{1}{3} (\theta - 4B_0), \quad (60)
\]

where \( B_0 = B/\rho_c \). For the parameters \( a \) and \( b \) we obtain

\[
a = \sqrt{1 + \kappa_0 \theta}, \quad b = \sqrt{1 - \frac{\kappa_0}{3} (\theta - 4B_0)}, \quad (61)
\]
which provides the following relationships

\[
\begin{align*}
    a &= \sqrt{4(1 + \kappa_0 B_0) - 3b^2}, \\
    \kappa_0^2 &= -\frac{3b}{\sqrt{4(1 + \kappa_0 B_0) - 3b^2}}, \\
    a^2 - b^2 &= 4(1 + \kappa_0 B_0 - b^2),
\end{align*}
\]

respectively. The parameter \( \kappa_0 \) must satisfy the constraint \( \kappa_0 < 3/(1 - 4B_0) \). Therefore, the gravitational field equations describing the structure of a quark star satisfying the MIT Bag model equation of state in EiBI gravity take the form

\[
\frac{dm_0}{d\eta} = \frac{2}{\kappa_0} \left[ 1 + \frac{2(\kappa_0 B_0 + 1 - 3b^2)}{b^3 \sqrt{4(\kappa_0 B_0 + 1 - 3b^2)}} \right] \eta^2,
\]

and

\[
\frac{db}{d\eta} = \frac{2 \left( b^2 - \kappa_0 B_0 - 1 \right) \sqrt{-3b^2 + 4\kappa_0 B_0 + 4 \left[ -2 (b^2 - 2) \eta^3 + b^3 \sqrt{-3b^2 + 4\kappa_0 B_0 + 4 (\kappa_0 m_0 - 2\eta^3) + 4\kappa_0 B_0 \eta^3} \right]}}{\kappa_0 b^2 \eta^2 \left( 1 - 2m_0/\eta \right) \left[ 3b^4 - 14(1 + \kappa_0 B_0) b^2 + 12(1 + \kappa_0 B_0)^2 \right]}. \tag{66}
\]

For the central density of the quark star we adopt the value \( \rho_c = 4 \times 10^{15} \text{ g/cm}^3 \), leading to \( 4B_0 = 0.1 \), and \( B_0 = 0.025 \), respectively. With these values for \( \kappa_0 \) we have the constraint \( \kappa_0 < 3.33 \). The variations of the density and mass profiles of the quark stars in general relativity and EiBI gravity are presented in Fig. 4.

The quark star models are relatively similar in both general relativity and EiBI gravity. The dimensionless radius of the quark star is \( \eta_S \approx 1.3 \), obtained from the condition \( \theta(\eta_S) = 0.10 \), with the corresponding quark dimensionless mass of \( M_* \approx 0.3 \) and \( M_0 \approx 0.38 \), corresponding to \( \kappa_0 = 3.2 \). The general relativistic quark star radius is \( R = 9.52 \times 10^5 \text{ cm} \), while its mass is around \( M_{GR}(R) = 1.48M_\odot \). The EiBI star has a similar radius, and a mass given by \( M_{EiBI} \approx 1.87M_\odot \). Similarly to the polytropic case, EiBI gravity effects lead to an increase of around 25% of the mass of the quark stars described by the MIT bag model equation of state.

V. AN EXACT STELLAR SOLUTION IN EIBI GRAVITY: THE \( a^2 = 3b^2 \) CASE

Due to the highly nonlinear nature of the EiBI gravitational field equations, it is extremely difficult to obtain exact analytical solutions. However, in this section we present an exact solution for a peculiar compact object. In the following, we adopt for simplicity the natural system of units with \( G = c = 1 \). We consider the specific case

\[
a^2 = 3b^2.
\]
Then, the corresponding relation between the energy density and the pressure is

\[ p = -\frac{1}{3} \theta + \frac{1}{12\pi \kappa}. \tag{68} \]

In this case, the field equations are essentially simplified, and admit an exact solution. In particular, Eq. (16) can be integrated to give

\[ e^{-\alpha} = 1 - \frac{2m(r)}{r}, \tag{69} \]

where

\[ m(r) = M + \frac{r^3}{6\kappa}, \tag{70} \]

and \( M \) is an arbitrary constant of integration. The regularity condition requires that \( m(0) = 0 \), hence we assume that \( M = 0 \). Therefore we obtain the metric function

\[ e^{-\alpha} = 1 - \frac{r^2}{3\kappa}. \tag{71} \]

The conservation relation (22) now yields the result

\[ \frac{d\beta}{dr} = \frac{4}{3\kappa} \frac{db}{dr}. \tag{72} \]

Using the relation \( a^2 = 3b^2 \) and substituting Eqs. (71) and (72) into Eq. (17) yields the following equation for \( b(r) \),

\[ 3\kappa \left( 1 - \frac{r^2}{3\kappa} \right) \frac{db^2}{dr} + rb^2 = \sqrt{3} r, \tag{73} \]

with the general solution given by

\[ b^2(r) = \sqrt{3} - C \sqrt{1 - \frac{r^2}{3\kappa}}, \tag{74} \]

where \( C \) is an arbitrary constant of integration. In order to fix \( C \), we assume that \( p(0) = p_c \), where \( p_c \) is the pressure at the center of the star. Then, using the relation \( b^2 = 1 - 8\pi \kappa p_c \), we verify that the arbitrary constant of integration is given by

\[ C = 8\pi \kappa p_c + \sqrt{3} - 1, \]

and hence

\[ b^2(r) = \sqrt{3} - \left( 8\pi \kappa p_c + \sqrt{3} - 1 \right) \sqrt{1 - \frac{r^2}{3\kappa}}. \tag{75} \]

The corresponding relation for \( p(r) \) takes the form

\[ p(r) = \frac{1}{8\pi \kappa} \left[ \left( 8\pi \kappa p_c + \sqrt{3} - 1 \right) \sqrt{1 - \frac{r^2}{3\kappa}} - \sqrt{3} + 1 \right]. \tag{76} \]

The radius of the star is defined as a sphere \( r = R \) where the pressure is equal to zero, i.e. \( p(R) = 0 \). From Eq. (76) we find

\[ R^2 = \frac{48\pi \kappa^2 p_c (4\pi \kappa p_c + \sqrt{3} - 1)}{9(8\pi \kappa p_c + \sqrt{3} - 1)^2}. \tag{77} \]

Note that \( R \) depends on two parameters \( \kappa \) and \( p_c \), i.e. \( R = R(\kappa, p_c) \). It will be useful to consider various limiting cases. Namely, by fixing the value of \( p_c \), then \( R \to 0 \) if \( \kappa \to 0 \), and \( R \to \infty \) if \( \kappa \to \infty \). Now, by fixing the value of \( \kappa \), then \( R \to 0 \) if \( p_c \to 0 \), and \( R \to R_{\text{max}} = \sqrt{3} \kappa \) if \( p_c \to \infty \). It is worth noticing that the size of a star supported by this exact solution model cannot exceed the maximal size \( R_{\text{max}} = \sqrt{3} \kappa \). Finally, note that \( p(r) \) given by Eq. (76) is monotonically decreasing from \( p_c \) to 0 within the interval \( r \in [0, R] \).
Using Eq. (72) yields the metric function
\[ e^{\beta(r)} = e^{\beta_c} \frac{b^4(r)}{b_c^4}, \] (78)
where \( \beta_c = \beta(0) \) and \( b_c = b(0) \). Then, the explicit expressions for \( \alpha(r) \) and \( b(r) \) are found. Using the relation \( a^2 = 3b^2 \) and Eqs. (18), we can easily obtain the following metric functions
\[ e^\nu(r) = \frac{\sqrt{3} e^{b_c}}{b_c} \left[ \sqrt{3} - \left( 8\pi \kappa p_c + \sqrt{3} - 1 \right) \sqrt{1 - \frac{r^2}{3\kappa}} \right], \] (79)
\[ e^\lambda(r) = \frac{1}{\sqrt{3}} \left( 1 - \frac{r^2}{3\kappa} \right)^{-1} \times \left[ \sqrt{3} - \left( 8\pi \kappa p_c + \sqrt{3} - 1 \right) \sqrt{1 - \frac{r^2}{3\kappa}} \right]^{-1}, \] (80)
\[ f(r) = \frac{r^2}{\sqrt{3}} \left[ \sqrt{3} - \left( 8\pi \kappa p_c + \sqrt{3} - 1 \right) \sqrt{1 - \frac{r^2}{3\kappa}} \right]^{-1}. \] (81)

The resulting g-metric has the following form
\[ ds^2 = \frac{\sqrt{3} e^{b_c} b^2(r)}{b_c^4} dt^2 + \frac{1}{3b^2(r)} \left( \frac{dt^2}{1 - \frac{r^2}{3\kappa}} + r^2 d\Omega^2 \right). \] (82)

We stress that \( r < R_{\text{max}} = \sqrt{3} \kappa \), therefore the term \( 1 - r^2/3\kappa \) in the line element (82) is strictly positive.

Using the equation of state (45) yields
\[ \rho(r) = -3p(r) + \frac{1}{4\pi \kappa}. \] (83)

Since \( p(r) \) is monotonically decreasing from \( p_c \) to 0 within the interval \( r \in [0, R] \), the energy density \( \rho(r) \) is monotonically increasing from \( p_c = -3p_c + 1/4\pi \kappa \) to \( 1/4\pi \kappa \). Demanding the positivity of the energy density, i.e., \( \rho > 0 \) yields the following condition for \( p_c \)
\[ p_c < \frac{1}{12\pi \kappa}. \] (84)

Therefore, the pressure at the center of the star is restricted by the value \( p_{\text{max}} = 1/12\pi \kappa \).

VI. CONCLUSIONS

In the present paper, we have considered the properties of specific stellar models in the recently proposed EiBI gravity model, and we have performed a comparative study of high density compact objects in standard general relativity and EiBI gravity, respectively. Generally, on a qualitative level, the predictions of these two theoretical models do agree, but important quantitative differences also appear. An important and interesting feature of EiBI gravity is that it predicts more massive objects than general relativity, with an equation of state dependent increase in the stellar mass in the range of 22%-26%. In the analysis outlined in this work, we have also restricted our study to positive values of the parameter \( \kappa_0 \), since only in this case more massive stellar configurations than the general relativistic ones are possible. By assuming a linear equation of state of the form \( \rho + 3p = 1/4\pi \kappa \), we have also obtained the complete exact analytical solution of the gravitational field equations describing the interior of an “exotic” star in EiBI gravity. In the limit of large parameter \( \kappa \to \infty \), the obtained solution describes the string gas stellar model, with the equation of state of the form \( \rho + 3p = 0 \) [32]. Note that a short review on the building blocks of the string gas cosmological model has been recently presented in [37].

One of the fundamental property of stellar models is their stability with respect to small perturbations. The stability of EiBI compact stars was considered in [22], where it was shown that the standard results of stellar stability theory still hold in this theory. For the maximum-mass stellar configuration the frequency square of the fundamental oscillation mode vanishes. However, an interesting difference with respect to general relativity is that the criterion \( dM/d\rho_c \) does not guarantee stability.

A very intriguing type of astrophysical objects are stellar mass black holes, with masses in the range of \( 3-6M_\odot \). The stellar mass black holes have been observed in close binary systems, in which transfer of matter from a companion star to the black hole occurs. The energy released in the fall heats up the matter to temperatures of several hundred million degrees, and it is radiated in X-rays, thus allowing the detection of the black hole, whereas the companion star can be observed with optical telescopes. It is estimated that in the Milky Way alone there should be at least 1000 dormant black hole X Ray Transients, while the total number of stellar mass black holes (isolated and in binaries) could be as large as 100 million [34]. Since ordinary neutron or quark stars in EiBI gravity can acquire larger masses than the general relativistic maximum mass of \( 3.2M_\odot \), stellar mass black holes, with masses in the range of \( 3.8M_\odot \) and \( 6M_\odot \), respectively, could in fact be EiBI neutron or quark stars. EiBI stars can achieve much higher masses than standard neutron stars, thus making them possible stellar mass black hole candidates.

Acknowledgments

*Acknowledgments.* FNSL acknowledges financial support of the Fundação para a Ciência e Tecnologia through the grants CERN/FP/123615/2011 and CERN/FP/123618/2011. SVS acknowledges financial support of the Russian Foundation for Basic Research through grants No. 11-02-01162 and 13-02-12093.
