A scheme for generating entangled cluster state of atomic ensembles

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It was shown in Ref.[Phys.Rev.A 77,045802(2008)] that the dynamics of a control atom and an atomic sample interacting dispersively with a cavity can be described by the Jaynes-Cummings model and the collective mode of the atomic sample can be analogous with a bosonic mode. Here, by analogizing the behaviour of the atomic sample with the one of the cavity, we propose a scheme to generate cluster states of atomic ensembles by Cavity QED.

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The phenomena of superposition and entanglement in quantum system can provide enhancements on the ability of processing information over what is possible or impossible classically, such as quantum computation [1], quantum teleportation [2], superdense coding [3], and quantum cryptography [4]. Generation of entanglement between atoms, photons or combinations is at the heart of quantum information science. The technology of generation and manipulation of several partite entangled states has been realized in many systems [5]. There still has been much interest in using quantum resource to get more and more subsystems entangled [6] for more applications. It has been shown that there are several inequivalent classes of entangled states [7]. Recently, Briegel and Raussendorf introduced a class of multi-qubit entangled states, i.e. the so-called cluster states [8], which have some interesting features. They have a high persistence of entanglement, and can be regarded as a resource for the GHZ states [9], but they are more immune to decoherence than the GHZ states [10]. The cluster states have been shown to violate a new Bell inequality which is not violated by the GHZ state [10]. And more importantly, the cluster state has been reported to constitute a universal resource for one-way quantum computation with proceeding only by local measurements [11], thereby eliminating the troublesome requirement for dynamically controlled two-qubit operations. Much effort has been devoted to generating the cluster state of single-particle systems [12]. Atomic ensembles have been proposed to be a promising candidate for implementations of quantum information processing by many recently discovered schemes [13]. The schemes based on atomic ensembles have some special advantages compared with the schemes on the control of single particles, such as easier laser manipulation, collectively enhanced coupling [14]. In the last few years, much attention has been paid to generate substantial spin squeezing [15] and continuous variable entanglement [16], realize scalable long-distance quantum communication [17] and prepare many-party entanglement [19]. However, there is no any scheme for generation of the cluster state in this system.

Recently a result has been shown in Ref.[20] that, putting a single control atom driven by an auxiliary classical field and an atomic sample into a nonresonant cavity, under certain conditions the coherent energy exchange between the control atom and the atomic sample can be described by an effective Jaynes-Cummings model, where the collective ensemble atomic spin is treated as a bosonic mode. This dynamics provide us an alternative method to entangle in a single step a control atom with a mesoscopic number of atoms. Based on the dynamics, we describe a scheme of preparing the cluster state between many atomic ensembles in this paper. The scheme involves the microwave cavity

![Diagram](image)

FIG. 1: Schematic setup to generate the cluster states of $K$ atomic samples. $C_i$ ($i = 1 \sim K$) denotes the cavity $i$ including an atomic sample and $R_i$ ($i = 1 \sim K-1$, $E$) the Ramsey zone.

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From the definition it is obvious that $|g\rangle \leftrightarrow |e\rangle$ and $|g\rangle \leftrightarrow |f\rangle$ are resonant at the frequency $\omega_0$ and $\omega_1$, respectively.

The first step of the scheme is to feed the atomic sample and the control atom into the first microwave cavity, the control atom is illuminated by an auxiliary classical field. The cavity frequency is appropriately chosen in a way that only the transition $|g\rangle \leftrightarrow |e\rangle$ is affected by the cavity, the state $|f\rangle$ is not affected during the atom-cavity interaction. So in the first cavity the Hamiltonian for the whole system is (assuming $\hbar = 1$) [20]

$$H = H_0 + H_i,$$

where

$$H_0 = \omega_e a^+a + \omega_0 (S_{z,c} + \sum_{j=1}^{N} S_{z,j}),$$

$$H_i = \Omega (S_c^- e^{i\omega_L t} + S_c^+ e^{-i\omega_L t}) + g(a^+ S^- + a S^+) + g \sum_{j=1}^{N} (a^+ S_j^- + a S_j^+),$$

$a^+$ and $a$ are the creation and annihilation operators for the cavity mode, $S_z$ is $\frac{1}{2}(|e\rangle \langle e| - |g\rangle \langle g|)$, $S^+ = |e\rangle \langle g|$, $S^- = |g\rangle \langle e|$ are the inversion, rising, and lowering operators for the atom, the subscripts $c$ and $j$ denote the control atom and the $j$th atom in the sample, respectively. $\omega_L$ and $\Omega$ are the oscillation frequency and the Rabi frequency of the classical field, respectively. $\omega_c$ is the cavity frequency, $g$ is the atom-cavity coupling strength. Under the detuning condition $\delta_c = \omega_0 - \omega_c \gg g\sqrt{N(\bar{n} + 1)}$, with $\bar{n}$ being the mean photon number of the cavity field, there is no energy exchange between the atomic system and the cavity. The dispersive atom-cavity interaction leads to photon-number-dependent Stark shifts and dipole couplings for the atomic system. And when $\delta_L = \omega_0 - \omega_L \gg \Omega$, the classical field only induces a Stark shift [20]. If the cavity is initially in the vacuum state, it will remain in the state. So in the following state of the cavity is omitted. The effective Hamiltonian of the whole system reduces to [20]

$$H_e = \lambda_L (\langle e|c\rangle \langle e|c\rangle - |g_c\rangle \langle g_c|) + \lambda_c (\langle e|c\rangle \langle e|c\rangle + \sum_{j=1}^{N} |e_j\rangle \langle e_j|)$$

$$+ \lambda_c \sum_{j=1}^{N} (S_{c,j}^- S^- + S_{c,j}^+ S^+) + \sum_{j,k=1,j\neq k}^{N} S_{c,j}^- S_{c,k}^-,$$

where $\lambda_L = \frac{\Omega^2}{\delta_L}$ and $\lambda_c = \frac{\Omega^2}{\delta_L}$. Introduce the operators $b = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} S_j^-$, $b^+ = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} S_j^+$, $n_b = \sum_{j=1}^{N} |e_j\rangle \langle e_j|$. By discarding a constant energy $\frac{\Omega^2}{\delta_L}$, the Hamiltonian of the system can be rewritten as [20]

$$H_e = (2\lambda_L + \lambda_c) S_{z,c} + N\lambda_c b^+ b + \sqrt{N} \lambda_c b S_c^+ + b S_c^- b^+.$$

From the definition it is obvious that $[b, b^+] = 1 - \frac{2}{N} n_b$, $[n_b, b^+] = b^+$, $[n_b, b] = -b$. Therefore, if $N \gg 1, \bar{n}_b$, with $\bar{n}_b$ being the average excitation number of the atomic sample, $b$ and $b^+$ can be regarded as the bosonic operators and the
atomic sample can be taken for a bosonic system. The Hamiltonian $H_c$ in Eq.(5), resembling the Jaynes-Cummings Hamiltonian, describes the oscillatory exchange of an excitation between the control atom and the collective atomic mode.

Suppose that $2\lambda_L = (N - 1)\lambda_c$, the above Hamiltonian gives rise to the evolution of the state of the atomic system as

$$|e_c\rangle|n\rangle \rightarrow \cos(\sqrt{(n + 1)N}\lambda_c t)|e_c\rangle|n\rangle - i\sin(\sqrt{(n + 1)N}\lambda_c t)|g_c\rangle|n + 1\rangle,$$

$$|g_c\rangle|n + 1\rangle \rightarrow \cos(\sqrt{(n + 1)N}\lambda_c t)|g_c\rangle|n + 1\rangle - i\sin(\sqrt{(n + 1)N}\lambda_c t)|e_c\rangle|n\rangle,$$

with $|n\rangle$ denoting the Fock-like state for the collective atomic mode.

Assuming the control atom is initially prepared in the superposition state $|\frac{f_c}{\sqrt{2}} + |e_c\rangle\rangle$, the state of the collective atomic mode is $|0\rangle$, i.e., all the atoms in the sample stay in the state $|g_j\rangle$. That is, the initial state of the atomic system is $|\frac{f_c}{\sqrt{2}} + |e_c\rangle\rangle|0\rangle$. After the control atom passed through the first cavity with an interaction time $t = \frac{\pi}{2\sqrt{N}\lambda_c}$, the whole state of the control atom and the atomic sample becomes

$$\frac{1}{\sqrt{2}}(|f_c\rangle|0\rangle - i|g_c\rangle|1\rangle).$$

Leaving the first cavity, the control atom is sequentially subjected to two classical pulse in the Ramsey zone $R_1$. The first one is tuned to induce the transition $|g_c\rangle \rightarrow i|e_c\rangle$ and the second is for inducing the transformation $|e_c\rangle \rightarrow \frac{1}{\sqrt{2}}(|f_c\rangle - |e_c\rangle)$ and $|f_c\rangle \rightarrow \frac{1}{\sqrt{2}}(|f_c\rangle + |e_c\rangle)$. The state (7) evolves into

$$\frac{1}{2}(|f_c\rangle(|0\rangle + |1\rangle) + |e_c\rangle(|0\rangle - |1\rangle)),$$

which can be rewritten as follows:

$$\frac{1}{2}(|f_c\rangle + |e_c\rangle \sigma)(|0\rangle + |1\rangle),$$

where $\sigma = |0\rangle \langle 0| - |1\rangle \langle 1|$ is acting on the collective mode of the atomic sample when the control atom is in the state $|e_c\rangle$.

Then the control atom is fed into the second cavity including another atomic sample with the same level structure and atomic number. The state of the second cavity is still in the vacuum and the collective mode of the second atomic sample is also in the state $|0\rangle$. Again, the dispersive atom-cavity interaction leads to the oscillatory exchange of an excitation between the control atom and the collective mode of the second atomic sample. With the interaction time $t = \frac{\pi}{2\sqrt{N}\lambda_c}$ in the second cavity, the whole state of the atom system becomes

$$\frac{1}{2}(|f_c\rangle|0\rangle_2 - i|g_c\rangle|1\rangle_2 \sigma_1)(|0\rangle_1 + |1\rangle_1).$$

Here the subscripts 1, 2 are introduced to distinguish the atomic sample in different cavities.

After leaving the second cavity, the control atom passes through the second Ramsey zone $R_2$, where the control atom suffers in turns the transition $|g_c\rangle \rightarrow i|e_c\rangle$ and the transformation $|e_c\rangle \rightarrow \frac{1}{\sqrt{2}}(|f_c\rangle - |e_c\rangle)$ and $|f_c\rangle \rightarrow \frac{1}{\sqrt{2}}(|f_c\rangle + |e_c\rangle)$ by two classical pulses. Thus the state (10) becomes

$$\frac{1}{2\sqrt{2}}(|f_c\rangle + |e_c\rangle \sigma_2)(|0\rangle_2 + |1\rangle_2 \sigma_1)(|0\rangle_1 + |1\rangle_1).$$

Continually, The control atom passes in sequence through the cavities containing the same atomic sample and the Ramsey zones $C_3, R_3, C_4, R_4, ..., C_{K-1}, R_{K-1}$ as illustrated in the Fig. 1. The whole state of the atom system evolves into

$$\frac{1}{\sqrt{2^{K}}}(|f_c\rangle + |e_c\rangle \sigma_{K-1})(|0\rangle_{K-1} + |1\rangle_{K-1} \sigma_{K-2})...(|0\rangle_2 + |1\rangle_2 \sigma_1)(|0\rangle_1 + |1\rangle_1).$$

Then the control atom is subjected to an extra Ramsey zone $R_E$ before entering the $K$th cavity, where a transition $|f\rangle \leftrightarrow -i|g\rangle$ is induced by one classical pulse. The state becomes

$$\frac{1}{\sqrt{2^{K}}}(-i|g_c\rangle + |e_c\rangle \sigma_{K-1})(|0\rangle_{K-1} + |1\rangle_{K-1} \sigma_{K-2})...(|0\rangle_2 + |1\rangle_2 \sigma_1)(|0\rangle_1 + |1\rangle_1).$$
Finally, after the control atom passes through the $K$th cavity holding the $K$th atomic sample with the duration $t = \frac{\pi}{2N\lambda_c}$, the quantum state of the atom system becomes

$$\frac{1}{\sqrt{2^K}} |g_c\rangle (|0\rangle_K + |1\rangle_K \sigma_{K-1})(|0\rangle_{K-1} + |1\rangle_{K-1} \sigma_{K-2})...(|0\rangle_2 + |1\rangle_2 \sigma_1)(|0\rangle_1 + |1\rangle_1),$$

where the control atom is disentangled with the atomic samples. Neglecting the control atom state, we get a one-dimensional cluster state consisting of $K$ atomic samples by encoding the vacuum state and one-excitation state of the collective mode of the atomic sample as the logic zero and one of qubit.

Up to now, one-dimensional cluster state of the atomic sample is generated, yet it is insufficient for a practical task of computation on the one-way quantum computer. We have to create various cluster state with different shapes. Next, we describe a way to combine two one-dimensional cluster states into a new two-dimensional cluster state. The schematic setup for connecting two one-dimensional cluster states is given in Fig. 3, where the atomic sample $i$ ($i = 1, 2$) in the cavity $i$ is a node of a cluster state $|\Phi_i\rangle$ of atomic ensembles.

$$|\Phi_i\rangle = \frac{1}{\sqrt{2}}(|0\rangle_i |\Psi_{i1}\rangle + |1\rangle_i |\Psi_{i2}\rangle),$$

which means the $i$th atomic sample is entangled with other atomic samples, and the $|\Psi_{i1}\rangle$ and $|\Psi_{i2}\rangle$ represent arbitrary normalized states of the others. To connect two cluster states, a control atom $c_1$ is prepared in the quantum state $|\frac{f_1+c_1}{\sqrt{2}}\rangle$, and then is sent sequentially through two cavities 1 and 2 with the same interaction $t = \frac{\pi}{2N\lambda_c}$. After the control atom $c_1$ exits from the cavity 2, following the evolution described in Eq.[6] the state of the atom system becomes

$$\frac{1}{2\sqrt{2}}(|f_{c1}|g_{c1}\rangle \otimes (|0\rangle_1 |0\rangle_2 |\Psi_{11}\rangle |\Psi_{21}\rangle + |1\rangle_1 |1\rangle_2 |\Psi_{12}\rangle |\Psi_{22}\rangle)$$

$$+ (|f_{c1}|g_{c1}\rangle \otimes (|0\rangle_1 |1\rangle_2 |\Psi_{11}\rangle |\Psi_{22}\rangle + |1\rangle_1 |0\rangle_2 |\Psi_{12}\rangle |\Psi_{21}\rangle)).$$

Then the state of the control atom $c_1$ is detected by passing through the atom $c_1$ through the classical microwave field zone and field ionization counters. When the state $|\frac{f_{c1}+g_{c1}}{\sqrt{2}}\rangle$ of the atom $c_1$ is confirmed, the state of two atomic-sample clusters is projected into

$$\frac{1}{\sqrt{2}} (|0\rangle_1 |0\rangle_2 |\Psi_{11}\rangle |\Psi_{21}\rangle + |1\rangle_1 |1\rangle_2 |\Psi_{12}\rangle |\Psi_{22}\rangle).$$

Otherwise, the connection is failed.

The second control atom $c_2$ prepared in the state $|g_{c2}\rangle$ is fed into the cavity 1 in the following step. After the duration time $t = \frac{\pi}{2N\lambda_c}$, the state of the atom system becomes

$$\frac{1}{\sqrt{2}} |0\rangle_1 (|g_{c2}\rangle |0\rangle_2 |\Psi_{11}\rangle |\Psi_{21}\rangle - i |e_{c2}\rangle |1\rangle_2 |\Psi_{12}\rangle |\Psi_{22}\rangle),$$

where the atomic sample 1 is extricated from entanglement with others samples. When the control atom $c_2$ flies out from the cavity 1, one detects its state in the basis of $|\frac{|g_{c2}\rangle+|e_{c2}\rangle}{\sqrt{2}}\rangle$. If the state $|\frac{|g_{c2}\rangle-|e_{c2}\rangle}{\sqrt{2}}\rangle$ is announced, the above state (18) is projected into

$$\frac{1}{\sqrt{2}} (|0\rangle_2 |\Psi_{11}\rangle |\Psi_{21}\rangle + |1\rangle_2 |\Psi_{12}\rangle |\Psi_{22}\rangle).$$
which is the standard form of a two-dimensional cluster state with the atomic sample 2 being the node. While the state \(\frac{|g\rangle_2 + i|e\rangle_2}{\sqrt{2}}\) is announced, the state (18) collapses into

\[
\frac{1}{\sqrt{2}} \left( |0\rangle_2 |\Psi_{11}\rangle - |1\rangle_2 |\Psi_{12}\rangle - |2\rangle_2 |\Psi_{22}\rangle \right),
\]

which can be transformed into the form in Eq.(19) by local operation. Therefore, we connect two one-dimensional cluster state of the atomic samples into a two-dimensional one with the success probability 1/2 and one-qubit loss in the cluster.

In summary, we have taken the idea [20] that the dynamics of a control atom and an atomic sample interacting dispersively with a cavity field can be described by the Jaynes-Cummings model and the collective mode of the atomic sample can be analogous with a bosonic mode. By analogizing the behavior of the atomic sample with the one of the cavity field, we have proposed a scheme to generate the cluster state of the atomic samples in the way similar to that in Ref. [21]. In recent years the manipulation on one Rydberg atom and clouds of Rydberg atoms has been demonstrated in the laboratory by several groups in many remarkable experiments [22, 23, 24, 25].

The three circular levels with principal quantum numbers 51, 50 and 49 to embody the states \(|e\rangle\), \(|g\rangle\), \(|f\rangle\), respectively. The \(|g\rangle \leftrightarrow |e\rangle\) and \(|g\rangle \leftrightarrow |f\rangle\) transitions are at 51.1 and 54.3 GHz, respectively. The radiative lifetimes of \(|e\rangle\), \(|g\rangle\), \(|f\rangle\) is of the order of 30 ns. The coupling constant of the atoms to the cavity is 25 kHz. As analyzed in Ref. [21], the radiative time is enough for the atom-cavity interaction and atom crossing the classical field. And during the whole process the cavities in the proposed scheme remain in the vacuum, the cavity loss can be negligible. Based on the present cavity QED techniques, the proposed scheme to generate cluster state of the atomic ensembles will be realizable in the future.

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