Anomalous ring-down effects and breakdown of the decay rate concept in optical cavities with negative group delay

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Abstract. The propagation of light pulses through negative group velocity media is known to give rise to a number of paradoxical situations that seem to violate causality. The solution of these paradoxes has triggered the investigation of a number of interesting and unexpected features of light propagation. Here, we report a combined theoretical and experimental study of the ring-down oscillations in optical cavities filled with a medium with a sufficiently negative frequency dispersion to give a negative round-trip group delay time. We theoretically anticipate that causality imposes the existence of additional resonance peaks in the cavity transmission, resulting in a non-exponential decay of the cavity field and in a breakdown of the cavity decay rate concept. Our predictions are validated by simulations and by an experiment using a room-temperature gas of metastable helium atoms in the detuned electromagnetically induced transparency regime as the cavity medium.

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1. Introduction

Since the early works of Sommerfeld [1] and Brillouin [2, 3] on light propagation through a resonant dielectric medium, slow and fast light have been the subject of considerable research efforts. It is now well established that the group velocity of light can change dramatically in a dispersive medium: slow, fast or even negative group velocity light can be observed. Moreover, such effects can, under some conditions, occur without any pulse distortion [4]. This has led to much controversy about Einstein’s causality and the propagation of a signal in such situations, which has been solved by considering the information as carried by non-analyticity points [5–9].

The control of group velocities of light pulses is an active subject of research as slow light schemes have been proposed to enhance nonlinear interactions for applications in quantum information processing [10–12]. In recent years, the use of electromagnetically induced transparency (EIT) in high-finesse cavities has given promising results for coherent control of light and nonlinear optics at low light levels [13–15]. The question of the lifetime of the field in cavities filled with a dispersive medium has consequences also for potential applications such as the increase of the sensitivity of gyroscopes using fast light [16–18]. In this context, we have recently confirmed experimentally that in the case of a slow-light medium inserted inside an optical cavity, the field lifetime is governed by the group velocity [19]. We investigate here some paradoxes arising from the consideration of a negative group velocity medium inserted inside a cavity, and we show that in such a case one is forced to take into account the additional resonance peaks of the cavity imposed by causality, making the concept of a single-cavity decay rate no longer relevant6.

Let us start by illustrating the paradox using the simplest case of a short pulse of light propagating through a dispersive medium of refractive index \( n(\omega) \) and length \( L_{\text{cell}} \). If the dispersion \( dn/d\omega \) is positive (see figure 1), the group velocity \( v_g \) is positive and the pulse experiences a positive group delay \( \tau_g = L_{\text{cell}}/v_g \) during its propagation through the medium (see figure 1(1a)). In contrast, when the dispersion is negative enough, \( v_g \) can become negative, leading to the appearance of a negative group delay \( \tau_g \) through the medium. This can lead to the kind of situation sketched in figure 1(2a), in which the peak of the outgoing pulse leaves the medium before the incident pulse enters it while another wave packet appears at the back interface and propagates in the backward direction inside the medium [21]. In these paradoxical

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6 Note that the situation discussed here is different from other schemes leading to non-exponential cavity decays such as, e.g., atoms strongly coupled to cavities in cavity quantum electrodynamics systems. See [20].
Figure 1. Propagation of a light pulse through (1a) a slow-light and (2a) a negative-light medium (the white pulse in (1a) would correspond to propagation in vacuum), and (1b), (2b) a cavity containing such a medium. Series of pulses exiting (1c) the slow-light cavity when it is excited with an incident pulse and (2c) the cavity in the case when the intracavity medium has a strong negative dispersion leading to a negative round-trip group delay $\tau_{RT}^g < 0$. Note the non-causal behavior of the exit pulses in (2c).

situations, causality has been shown to be ensured by the propagation of non-analyticity points at the speed $c$ of light in vacuum [7–9].

Let us now introduce such a dispersive medium inside a resonant cavity. In the case of positive dispersion of figure 1(1b), i.e. slow light, we have recently shown [19] that the lifetime of the field in the cavity is given as expected by $\tau_{\text{cav}} = \tau_{g}^RT / \Pi$, where $\Pi$ stands for the fractional loss per cavity round trip, and $\tau_{g}^RT = \tau_{g} + L_{\text{vac}} / c$ is the group delay for one round trip inside the cavity with $L_{\text{vac}}$, the length of the empty part of the cavity. In this case, the reduced decay rate for the intracavity intensity can be explained in terms of a simple picture of a pulse propagating at the group velocity inside the cavity and decaying at each round trip because of losses (see the decaying pulses of figure 1(1c)).

This picture no longer holds in the case where the intracavity dispersion is negative and is strong enough not only to make the group delay across the cell $\tau_{g}$ negative, but also the cavity group round-trip time $\tau_{g}^RT$ smaller than zero (figure 1(2b)). In this case, if we follow the same picture as in the case of positive dispersion, the pulse that has made one round trip inside the cavity must exit the cavity before the initial pulse, and is even preceded by the pulse that has undergone two round trips inside the cavity, etc. This should lead to an increase of the intensity with time, as shown in figure 1(2c), which is of course absurd. In this paper, we consider a slightly different configuration where a CW laser beam is incident on the cavity and is suddenly switched off: in spite of the different time sequence, paradoxical situations appear in this case as well.

2. Results

2.1. General theory

We consider a cavity like that in figure 1. The input and output mirrors are identical, with intensity reflection and transmission coefficients given by $R$ and $T$, respectively. The two other
mirrors are perfectly reflecting. We denote as \( L_m \) the length between the input and output mirrors. \( \omega_0 \) is the frequency of the laser and \( \omega_p \) the considered resonant frequency of the cavity. For a generic incident excitation \( E_{in}(t) \), the field at the output of the cavity can be evaluated using simple linear response theory. We first consider the case of a cavity with a positive round-trip group delay \( \tau_g^{RT} \). We suppose that the input laser field is monochromatic and is abruptly turned off at \( t = 0 \). The positive-frequency part of the output field reads

\[
E^{(+)}_{out}(t) = \int_{-\infty}^{t} dt' E^{(+)}_{in}(t') R(t - t'),
\]

where \( E^{(+)}_{in}(t) \) is the positive-frequency part of the input field, and \( R(\tau) \) is the response function of the cavity which is zero for \( \tau < 0 \). We can stress the fact that \( R \) is causal by writing it as \( R(t) = S(t)H(t) \), where \( H \) is the Heaviside step function, and \( S \) is the Fourier transform of the cavity transmission \( \tilde{S}(\omega) \) for a monochromatic incident field. Then equation (1) simply reads

\[
E^{(+)}_{out}(t) = [E^{(+)}_{in} \ast (SH)](t).
\]

The cavity transmission for a monochromatic field of angular frequency \( \omega \) is then given by

\[
\tilde{S}(\omega) = \frac{T \exp[i\omega L_m]}{1 - R \exp[i\omega (L_{vac} + n(\omega)L_{cell})]}.
\]

If we suppose that \( \omega \) is close to a resonance frequency \( \omega_p \) of the cavity, for which \( \exp[i\omega_p (L_{vac} + n(\omega_p)L_{cell})] = 1 \), then, at first order in \( (\omega - \omega_p)/\omega_p \), equation (3) becomes

\[
\tilde{S}(\omega) = \frac{T \exp[i\omega L_m]}{1 - R - iR(\omega - \omega_p)\tau_g^{RT}},
\]

leading to

\[
\tilde{S}(\omega) = \left( \frac{T}{R \tau_g^{RT}} \right) \frac{\exp[i\omega L_m]}{\frac{\gamma_{cav}}{2} - i(\omega - \omega_p)},
\]

where the cavity decay rate is given by

\[
\gamma_{cav} = 2(1 - R)/R \tau_g^{RT} \simeq \Pi/\tau_g^{RT} = 1/\tau_{cav}.
\]

We have assumed that \( 1 - R \ll 1 \). In order to predict what a measurement of the field lifetime should give, we consider the response of this cavity to a laser field at frequency \( \omega_0 \) which is turned off at \( t = 0 \):

\[
E^{(+)}_{in}(t) = E_0[1 - H(t)]e^{-i\omega_0 t}.
\]

Equations (2), (5) and (7) then lead to the output fields

\[
E^{(+)}_{out}(t) = \frac{S_0 E_0 e^{-i\omega_0(t - L_m/c)}}{\frac{\gamma_{cav}}{2} - i(\omega_0 - \omega_p)}, \quad \text{if } t \leq \frac{L_m}{c},
\]

\[
E^{(+)}_{out}(t) = S_0 E_0 e^{-i\omega_0(t - L_m/c)} e^{-\frac{\gamma_{cav}}{2}(t - L_m/c)}, \quad \text{if } t \geq \frac{L_m}{c},
\]
Figure 2. (a) Continuous black line: typical negative dispersion curve. The intersections of this curve with the red dashed ($\tau \tau g = 0$), blue dotted ($\tau \tau g > 0$) and green dot-dashed ($\tau \tau g < 0$) curves determine whether the resonance is (b) single-peaked or (c) multi-peaked.

with $S_0 = T / Rg^{RT}$, which is the standard solution for a decaying cavity, in agreement with the observations of [19]. In contrast, in the case of a negative light cavity for which $\tau g^{RT} < 0$, we obtain

$$E^{(+)}_{out}(t) = \frac{S_0 E_0 (e^{-i\omega t} - e^{-i(\omega + \gamma c) t})}{\frac{\gamma c}{2} - i(\omega_0 - \omega_p)}, \text{ if } t \leq \frac{L_m}{c},$$

(10)

$$E^{(+)}_{out}(t) = 0, \text{ if } t \geq \frac{L_m}{c},$$

(11)

which, once again, clearly violates causality.

In deriving equations (10) and (11), the only hypothesis that we have made is that the cavity transmission could be reduced to a single Lorentzian peak (see equations (3) and (4)). This hypothesis is valid as long as the spectrum of the incident field (given by equation (7)) is contained in a single cavity transmission peak and all frequencies experience the same group index.

In order to examine this condition, let us first consider, as an example, a typical negative dispersion curve as given by the continuous line in figure 2(a). Let us suppose, without any loss of generality, that the inflection point of the dispersion curve occurs at the empty cavity resonance frequency $\omega_p$, meaning that $\frac{\omega_p c}{2} (L_{vac} + n(\omega_p) L_{cell}) = 2 \pi p$, where $p$ is an integer. Let us try to determine whether extra resonance peaks, due to negative dispersion, could occur in the vicinity of the peak at $\omega_p$. If $\omega_p + \delta$ is the angular frequency of such an extra peak, the resonance condition reads

$$\frac{\omega_p + \delta}{c} (L_{vac} + n(\omega_p + \delta) L_{cell}) = 2 \pi p.$$

(12)

To first order in $\delta / \omega_p$, this condition is equivalent to

$$n(\omega_p + \delta) - n(\omega_p) = - \frac{L_{vac} + n(\omega_p) L_{cell}}{\omega_p L_{cell}} \delta.$$

(13)
The left-hand side of equation (13) versus \( \delta \) is the continuous line in figure 2(a). The right-hand side is a straight line, as shown by the dotted, dashed, and dot-dashed lines in figure 2(a). One can see that the shape of the resonance, namely, the existence of no other solution than \( \delta = 0 \), leading to a single peak as in figure 2(b), or the existence of two other resonance frequencies for \( \delta \neq 0 \), leading to two extra resonance peaks as in figure 2(c), depends on the relative values of the slopes of the dispersion curve and the line corresponding to the right-hand side of equation (13). In particular, the condition for the existence of two extra solutions reads, at first order in \( \delta \):

\[
-\left. \frac{dn}{d\omega} \right|_{\omega_p} > \frac{L_{\text{vac}} + n(\omega_p)L_{\text{cell}}}{\omega_p L_{\text{cell}}},
\]

which is equivalent to \( \tau_{g}^{\text{RT}} < 0 \). We thus reach the following conclusion: the fact that the group delay for one round trip inside the cavity is negative leads to the existence of satellite peaks around the resonance considered. This negates the approximation used to obtain the cavity transmission (see equation (4)) and explains why the non-causal situation described above can never actually be reached. Figure 2(c) illustrates how this condition results in the existence of two extra peaks for the cavity resonance labeled by the integer \( p \). Note that in the case of slow light, the slope of the dispersion curve in figure 2(a) would be reversed, allowing only one intersection with the continuous line and thus forbidding the existence of extra resonance peaks.

In reaching the conclusion above, we have assumed a specific form for the negative dispersion curve, namely, the one drawn in figure 2(a). We will now show that this conclusion holds, as a consequence of causality, for any arbitrary negative dispersion curve provided \( \tau_{g}^{\text{RT}} < 0 \) at \( \omega_p \). For this, it is convenient to introduce the following function:

\[
f(\delta) = L_{\text{cell}}[(\omega_p + \delta)n(\omega_p + \delta) - \omega_p n(\omega_p)] + \delta L_{\text{vac}}.
\]

It can then be shown, by a straightforward calculation, that equation (12) is equivalent to \( f(\delta) = 0 \). The resonance peak at \( \omega_p \) corresponds to \( f(0) = 0 \), while the satellite peaks would correspond to non-zero solutions of \( f(\delta) = 0 \). The initial hypothesis \( \tau_{g}^{\text{RT}} < 0 \) at \( \omega_p \) is equivalent to \( \partial f/\partial \delta < 0 \) for \( \delta = 0 \). Moreover, for \( \delta \to +\infty \), causality imposes that \( n(\omega) \to 1 \) [22], which results in \( f \sim (L_{\text{vac}} + L_{\text{cell}})\delta \). To summarize, we have \( f(\delta) = 0 \) and \( \partial f/\partial \delta < 0 \) for \( \delta = 0 \). So \( f(\delta) \) has to go to negative values for small (strictly positive) values of \( \delta \). Also, since \( f(\delta) \to +\infty \) for \( \delta \to +\infty \), it has to go at least once through zero, according to the intermediate value theorem. Let \( \delta_1 \) be the smallest value for which this happens. Then \( \omega_p + \delta_1 \) corresponds to an additional resonance peak of the cavity. Similarly, it is possible to prove the existence of \( \delta_2 < 0 \) such that \( f(\delta_2) = 0 \), which corresponds to another additional resonance peak. This leads to the conclusion that at least two satellite peaks exist around \( \omega_p \) for any arbitrary dispersion curve as soon as \( \tau_{g}^{\text{RT}} < 0 \) at \( \omega_p \).

2.2. Example of a gain doublet

Let us be more specific about the situation where a medium can exhibit a strong negative dispersion. A very popular example of negative dispersion is provided by a gain doublet [6, 7, 23–26]. Figure 3(a) shows the transmission \( |\tilde{S}(\omega)|^2 \) of the cavity versus detuning in that case. The dashed curve corresponds to the empty cavity, which is 2.45 m long with 29% loss per round trip. We now suppose that a gain-doublet medium is inserted inside the cavity.
The two gain peaks are separated by 1.5 MHz. We suppose that they are located symmetrically with respect to the cavity resonance. The gain maxima correspond to 28% per round trip and the full-width at half-maximum of each peak is 800 kHz. In these conditions, the group delay for one round trip inside the cavity is $\tau_{g}^{RT} = -3.3$ ns. The corresponding intensity transmission spectrum of the cavity is reproduced as a continuous line in figure 3(a). One can clearly see the two transmission peaks corresponding to the conjugated effects of the two additional resonance peaks and of the two gain maxima. The main difference with respect to figure 2(c), which was computed using only the real part of the dispersion and by artificially setting the imaginary part to zero (no gain or absorption), is that there is no central transmission peak. This is consistent with the fact that there is no gain peak at zero detuning. One can also note in this spectrum that the two lateral peaks are slightly shifted toward the line center with respect to the positions of the atomic resonances, which is consistent with the fact that these gain peaks are located in a positive dispersion spectral region.

We calculate (using equations (2), (3) and (7)) the temporal evolution of the intensity $|E_{\text{out}}(t)|^2$ at the output of the cavity when the incident field is suddenly turned off at $t = 0$. Such a decay is represented in figure 3(b) on a logarithmic scale. It is clearly non-exponential. It consists of a fast decay by several orders of magnitude, followed by oscillations which correspond to beatnotes between the two peaks of the transmission spectra. It is an illustration of the general principle of figure 2: any intracavity negative dispersion effect which is strong enough to make the round-trip group delay negative will cause secondary transmission peaks to emerge that will make the cavity decay non-exponential, forbidding one to define a field lifetime for this cavity.

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2.3. Experiments with detuned electromagnetically induced transparency

In order to give an experimental illustration, we use another system when a large negative group delay can be achieved: detuned EIT in a hot vapor of metastable $^4\text{He}$ atoms [27]. We use a 6 cm long cell filled with 1 torr of helium at room temperature. Some of these atoms are excited to the $^3\text{S}_1$ metastable state using a radiofrequency (RF) discharge at 27 MHz. Metastable helium is well known for exhibiting a pure three-level $\Lambda$ system when excited at the 1.083 $\mu$m transition between the $^2\text{S}_1$ and $^2\text{P}_1$ energy levels using circularly polarized light. Light at 1.083 $\mu$m is provided by a single-frequency diode laser. The frequencies and Rabi frequencies of the coupling and probe beams used in our experiment are driven by two AOMs. A telescope expands the coupling beam diameter up to 0.5 cm, which is larger than the probe beam diameter. The cell is inserted inside a 2.4 m long triangular ring cavity made of two plane mirrors with 2% transmission and a high-reflectivity concave mirror with a 5 m radius of curvature. The cavity is resonant only for the probe field as two polarization beam-splitters drive the coupling beam inside and outside the cavity [19] (see figure 4).

Let us first consider the transmission of our degenerate three-level $\Lambda$ system taken outside the cavity. A large one-photon detuning transforms the transparency peak typical of EIT into an asymmetric absorption peak [28]. Consequently, in the vicinity of the transmission minimum, the system exhibits strong negative dispersion, which leads to a negative group delay. Indeed, if we introduce such an optical detuning $\Delta_c$ between the coupling field and the maximum of the Doppler profile of the transition, the evolution of the cell transmission versus Raman detuning $\delta_R$ between the two fields exhibits the asymmetric Fano-like profile shown in figure 5(a), which is fitted using the expressions of the susceptibility derived in [27] for a three-level system in detuned EIT. With a coupling power of 5 mW equivalent to a coupling Rabi frequency around 10 MHz, and an optical detuning $\Delta_c$ of about 1.4 GHz, we measure a negative group delay $\tau_g \approx -4 \, \mu$s around the absorption maximum ($\delta_R = 5 \, \text{kHz}$) for our 6 cm long cell when the cavity is not present. The measurement is made by modulating the probe signal amplitude at a 1 kHz frequency, so that the spectrum of the field is fully contained in the absorption dip of a few kHz width.

Once this cell is inserted inside the cavity, we apply the same coupling field to the atoms. The probe field, which is then slightly detuned from the coupling field ($\delta_R = 5 \, \text{kHz}$) in order
Figure 5. (a) Experimental (thick blue) transmission of the cell without optical cavity versus Raman detuning $\delta_R$ with an optical detuning $\Delta_c$ around 1.4 GHz and fit (thin orange) using the theoretical expressions of [27]. (b) Calculated cavity transmission profile as a function of the probe frequency. The frequency reference is taken at $\delta_R = 0$. Inset: zoom around the narrow resonance. The narrow peak is due to EIT, while the broad one comes from the response of the empty cavity.

to take advantage of the large negative group delay of $-4 \mu s$, is incident on the cavity input mirror. We slowly scan the length of the cavity using a piezoelectric actuator that carries one of the mirrors. When the cavity is at resonance with the probe, i.e. when the cavity transmission reaches a given threshold, we abruptly turn off the probe field using an AOM. We then observe the evolution of the intensity at the output of the cavity.

The thick blue curve in figure 6 is the experimentally recorded evolution of the intensity at the output of the cavity when we abruptly turn off the probe field. In this figure, the region labeled I shows the signal before switching off the cavity input field. The fluctuations are due to cavity length instabilities. The interesting part of the signal lies on the region labeled II, which shows that the intensity starts increasing, on a time scale shorter than $1 \mu s$, before decreasing. This evolution is clearly non-exponential, showing once more that a negative cavity round-trip
Figure 6. Thick blue: experimental decay of the intracavity intensity when the incident probe field is turned off at $t = 0$. Thin green: corresponding theoretical cavity decay. Orange: corresponding theoretical cavity decay when the 200 ns response time of the AOM is taken into account in the model. Inset: the same plot on a logarithmic scale. Below: zoom of the first part of the evolution of the experimental decay of the intracavity intensity when the incident probe field is turned off at $t = 0$.

group delay leads to a non-exponential decay of the intracavity field. Note here that the cavity length being equal to 2.4 m, the cavity round-trip group delay $\tau_{\text{RT}}$ is also of the order of $-4 \mu s$.

These experimental results are consistent with the theoretical calculations (based on equations (2), (3) and (7)) using the expression for inhomogeneously broadened detuned EIT as in [27]. The values of the parameters have been obtained by fitting the experimental transmission curve as we can see in figure 5(a). With a coupling Rabi frequency of 11 MHz, an optical detuning $\Delta_1 = 1.3 \text{ GHz}$ and a Raman coherence decay rate of 12 kHz, we obtain the thin green curve of figure 6, which is similar to the experimental result (thick blue): when the incident intensity is turned off, the intensity at the output of the cavity starts increasing before decreasing. This typical non-exponential decay constitutes one more illustration of the behavior described in figure 2.

The small difference between experimental and theoretical results can be explained by four immediate reasons: (i) the photodiode used to detect the signal has a time response of
the order of 200 ns, which does not allow us to detect the fast variations of the signal; (ii) the
time taken to turn off the probe field is not zero but driven by the fall time of the acousto-optic
modulator (AOM), which is smaller than the photodiode response time; (iii) the extra losses of
the polarization optics have been set to 1% in the theoretical model and can be slightly different
in the experiment; and (iv) before the probe signal is shut off the signal is somewhat noisy due
to the fluctuations of the laser frequency and of the cavity length, but this does not play any role
in further evolution of the cavity decay as soon as the incident signal is cut off. In order to check
the influence of the fall time of the AOM, we introduce its response in our model by replacing
the Heavyside function for the incident field by an exponential decay with a 200 ns decay time.
This leads to the orange dashed curve of figure 6, which is in very good agreement with our
experimental results.

Because the temporal response we obtain is directly linked to the spectral response of the
cavity, it is possible to qualitatively understand the origin of the different time scales involved.
Figure 5(b) reproduces the calculated cavity transmission spectrum, taking into account the
negative group velocity medium of our experiment. Of course, narrow spectral features will
drive slow temporal behaviors, while, in contrast, broad spectral features will be related to
fast temporal behaviors. In the particular experimental situation presented in this paper, the
spectral response of the cavity exhibits a single narrow resonance. So we can expect, as observed
experimentally and confirmed by theoretical results, an exponential decay driven by a large time
constant at the end of the temporal behavior. This can be seen in the region labeled III and the
inset of figure 6. The inset of figure 5(b) shows a zoom of this narrow resonance. Its width is
approximately equal to 40 kHz, which corresponds to the 4 μs time constant of the exponential
decay shown in region III of figure 6. In the experiment, we fill the cavity with a light frequency
for which the transmission is very weak (δR = 5 kHz). By turning off the incident laser very
fast, we excite some frequencies for which the cavity is much more resonant. So when the
cavity empties, it starts by reaching a light intensity level larger than that previously reached
in the steady-state regime during its excitation by a monochromatic CW beam. We can then
expect an initial fast increase of the signal as shown in the zoomed part of figure 6. Moreover,
the small oscillations with a time period of a few tens of ns that can be seen in the theoretical
curve correspond to the beatnote between the narrow resonance and the wide resonance of the
cavity transmission profile, which are separated by roughly 9 MHz. In the experimental profile,
the oscillations are smoothed out because of the finite response time of the acousto-optic switch,
as reproduced by the orange dashed line in figure 6.

3. Discussion

In conclusion, we have theoretically demonstrated the fact that it is impossible to obtain a
negative group delay for one round trip inside a resonant cavity while keeping an exponential
decay of the intracavity intensity. We have shown that this result is a consequence of causality in
negative group-delay cavities. We have illustrated this both numerically and experimentally, by
using negative velocity light induced by a gain doublet and detuned EIT in a metastable vapor,
respectively. Our result is consistent with the fact that slow light is usually associated with a
transparency peak, which reduces the bandwidth to be considered. In contrast, fast and negative
group velocity light appears in the case of an absorption peak, leading to the possibility of many
frequencies playing a role, and thus the cavity decay rate in such a cavity can no longer be
simply defined. This should have interesting consequences for the spontaneous emission rate of
atoms placed in such a cavity [29], with application in the spontaneous emission noise of lasers based on such negative light cavities.

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References

[1] Sommerfeld A 1914 Über die Fortpflanzung des Lichtes in dispergierenden Medien Ann. Phys. 44 177
[2] Brillouin L 1914 Über die Fortpflanzung des Lichtes in dispergierenden Medien Ann. Phys. 44 203
[3] Brillouin L 1960 Wave Propagation and Group Velocity (New York: Academic)
[4] Garrett C G B and McCumber D E 1970 Propagation of a Gaussian light pulse through an anomalous dispersion medium Phys. Rev. A 1 305
[5] Garrison J C, Mitchell M W, Chiao R Y and Bolda E L 1998 Superluminal signals: causal loop paradoxes revisited Phys. Lett. A 245 19
[6] Kuzmich A, Dogariu A, Wang L J, Milonni P W and Chiao R Y 2001 Signal velocity, causality and quantum noise in superluminal light pulse propagation Phys. Rev. Lett. 86 3925
[7] Stenner M D, Gauthier D J and Neifeld M A 2003 The speed of information in a ‘fast-light’ optical medium Nature 425 695
[8] Stenner M D, Gauthier D J and Neifeld M A 2005 Fast causal information transmission in a medium with a slow group velocity Phys. Rev. Lett. 94 053902
[9] Milonni P W 2005 Fast Light, Slow Light and Left-Handed Light (New York: Taylor and Francis)
[10] Harris S E and Hau L V 1999 Nonlinear optics at low light levels Phys. Rev. Lett. 82 4611
[11] Lukin M D and Imamoglu A 2001 Controlling photons using electromagnetically induced transparency Nature 413 273
[12] Lyovsky A I, Sanders B C and Tittel W 2009 Optical quantum memories Nature Photon. 3 706
[13] Wu H, Gea-Nanacloche J and Xiao M 2008 Observation of intracavity electromagnetically induced transparency and polariton resonances in a Doppler-broadened medium Phys. Rev. Lett. 100 173602
[14] Mücke M, Figueroa E, Bochmann J, Hahn C, Murr K, Ritter S, Villas-Boas C J and Rempe G 2010 Electromagnetically induced transparency with single atoms in a cavity Nature 465 755
[15] Albert M, Dantan A and Drewsen M 2011 Cavity electromagnetically induced transparency and all-optical switching using ion Coulomb crystals Nature Photon. 5 633
[16] Shahriar M S, Pati G S, Tripathi R, Gopal V, Messall M and Salit K 2007 Ultrahigh enhancement in absolute and relative rotation sensing using fast and slow light Phys. Rev. A 75 053807
[17] Salit M, Pati G S, Salit K and Shahriar M S 2007 Fast-light for astrophysics: super-sensitive gyroscopes and gravitational wave detectors J. Mod. Opt. 54 2425
[18] Smith D D, Chang H, Arissian L and Diels J C 2008 Dispersion-enhanced laser gyroscope Phys. Rev. A 78 053824
[19] Lauprêtre T, Proux C, Ghosh R, Schwartz S, Goldfarb F and Bretenaker F 2011 Photon lifetime in a cavity containing a slow-light medium Opt. Lett. 36 1551
[20] Haroche S and Raimond J-M 2006 Exploring the Quantum: Atoms, Cavities, and Photons (Oxford: Oxford University Press)
[21] Bolda E L, Garrison J C and Chiao R Y 1994 Optical pulse propagation at negative group velocities due to a nearby gain line Phys. Rev. A 49 2938
[22] Jackson J D 1962 Classical Electrodynamics (New York: Wiley)
[23] Steinberg A M and Chao R Y 1994 Dispersionless, highly superluminal propagation in a medium with a gain doublet Phys. Rev. A 49 2071
[24] Wang L J, Kuzmich A and Dogariu A 2000 Gain-assisted superluminal light propagation Nature 406 277
[25] Dogariu A, Kuzmich A and Wang L J 2001 Transparent anomalous dispersion and superluminal light-pulse propagation at a negative group velocity Phys. Rev. A 63 053806
[26] Pati G S, Salit M, Salit K and Shariar M S 2007 Demonstration of a tunable-bandwidth white light interferometer using anomalous dispersion in atomic vapor Phys. Rev. Lett. 99 133601
[27] Goldfarb F, Lauprêtre T, Ruggiero J, Bretenaker F, Ghosh J and Ghosh R 2009 Electromagnetically-induced transparency, slow light and negative group velocities in a room temperature vapor of 4He C. R. Phys. 10 919
[28] Mikhailov E E, Sautenkov V A, Novikova I and Welch G R 2004 Large negative and positive delay of optical pulses in coherently prepared dense Rb vapor with buffer gas Phys. Rev. A 69 063808
[29] Bradshaw D H and Di Rosa M D 2011 Vacuum field energy and spontaneous emission in anomalously dispersive cavities Phys. Rev. A 83 053816