QCD Propagators at non-vanishing temperatures

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Abstract

We investigate the behaviour of the gluon and ghost propagators, especially their infrared properties, at non-vanishing temperatures. To this end we solve their Dyson-Schwinger equations on a torus and find an infrared enhanced ghost propagator and an infrared vanishing gluon propagator.

Lattice simulations provide evidence that zero-flavour QCD undergoes a “deconfining” phase transition at some critical temperature. The properties of the gluon and the ghost propagator at finite temperature can provide insights into the mechanisms of this phase transition and they are related to the issue of confinement and deconfinement.

We study the Dyson-Schwinger equations (DSEs) of these propagators in Landau gauge for which a well understood truncation scheme at zero temperature is available \cite{1}. The infrared behaviour of the gluon and ghost propagators can be determined analytically and we obtain power-laws. This is consistent with the Kugo-Ojima confinement picture for Landau gauge, which requires the ghost propagator to diverge stronger than a free propagator at zero momentum. On the other hand the gluon propagator is infrared suppressed ($D_{\mu\nu}(k \to 0) = 0$). Thus it violates positivity and the gluons are removed from the physical spectrum \cite{2}. Since this infrared singular structure is expected to persist for non-vanishing temperatures a continuum based method for studying the deconfinement phase transition is desirable.

At non-vanishing temperatures the analytical determination of the infrared behaviour has not been achieved yet, thus we need an infrared regulator. A possible one is to compactify space-time to a four dimensional torus by imposing periodic boundary conditions on the fields. This has been done already at zero temperature \cite{3}.

The truncation scheme at zero temperature described in \cite{3} is extended to finite temperatures in the Matsubara-Formalism (imaginary time formalism) for thermodynamical equilibrium. The DSEs in pure Yang-Mills-Theory for the gluon and the ghost propagator are considered. Since the heat bath defines a preferred reference frame, the full gluon-propagator in Landau gauge now depends on two independent transverse tensor structures, a heat-bath transverse and longitudinal part, leading to the following decomposition \cite{4}:

\begin{equation}
D_{\mu\nu}^{ab}(k) := \frac{\delta^{ab}}{k^2} \left( P_{T\mu\nu}(k)Z_T(k_0, |\vec{k}|) + P_{L\mu\nu}(k)Z_L(k_0, |\vec{k}|) \right), \quad P_{Tij}(k) = \delta_{ij} - \frac{k_i k_j}{k^2},
\end{equation}

\begin{equation}
P_{T00} = P_{Ti0} = P_{T0i} = 0, \quad P_{L\mu\nu}(k) = P_{\mu\nu}(k) - P_{T\mu\nu}(k), \quad P_{\mu\nu} = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}
\end{equation}

with $i, j = 1, 2, 3; \mu, \nu = 1, 4$ and $k^2 = k_0^2 + \vec{k}^2$. The ghost propagator being a Lorentz scalar reads: $D_{G}^{ab}(k) := -\frac{\delta^{ab}}{k^2}G(k_0, |\vec{k}|)$. Note, that the dressing functions of the gluon ($Z_T$ and $Z_L$) and the ghost
(G) propagators depend on the Matsubara-frequency \( k_0 = 2\pi n T \) \((n = \ldots, -2, -1, 0, 1, 2, \ldots)\) and the three momentum \(|\vec{k}|\) separately. So the DSEs for the propagators lead to a coupled system of three coupled integral equations for three scalar functions [5]. In compactified space-time non-vanishing temperatures render the torus asymmetric with a "shorter" time direction:

\[
T \sum_{n=-\infty}^{+\infty} \int \frac{d^3q}{(2\pi)^3} \to \frac{T}{L^3} \sum_{n=-\infty}^{+\infty} \sum_{j_1, j_2, j_3} \tag{3}
\]

with \( L_0 := \beta = 1/T \) and \( L_0 \ll L \). The numerical results from an asymmetric torus for the dressing functions at small temperatures indicate a rather weak temperature dependence. With increasing temperature the heat-bath transverse dressing \( Z_T \) of the gluon gets suppressed (fig. 1) while the longitudinal part \( Z_L \) gets enhanced (fig. 2). However, the power law resists, especially for the ghost propagator.

In the \( T \to \infty \) limit the four dimensional Yang-Mills theory is reduced to a three dimensional one with an additional Higgs field. Starting from the respective DSEs [5] the infrared analysis for the system of gluon, ghost and "Higgs" has been performed and first numerical results have been obtained. The full "Higgs" propagator is close to a massive tree-level propagator indicating the existence of a chromoelectric screening mass at high temperatures. However, in the chromomagnetic sector an overscreening of soft modes occurs and the gluon propagator vanishes at zero momentum.

The further aim of these studies is to provide a dynamical description of the expected phase transition from the two different regions of the phase diagram by use of the methods sketched here. In addition the studies at low temperatures will be supplemented by more complete and accurate numerical solutions of the DSEs. Thereby we aim at a dynamical description of the physical excitations across the phase transition.

References

[1] C. S. Fischer and R. Alkofer, Phys. Rev. D 67 (2003) 094020 [arXiv:hep-ph/0301094].
L. von Smekal, R. Alkofer and A. Hauck, Phys. Rev. Lett. 79 (1997) 3591 [arXiv:hep-ph/9705242].
[2] R. Alkofer, W. Detmold, C. S. Fischer and P. Maris, arXiv:hep-ph/0309078.
[3] C. S. Fischer, R. Alkofer and H. Reinhardt, Phys. Rev. D 65 (2002) 094008 [arXiv:hep-ph/0202195].
[4] J. I. Kapusta, Finite-temperature field theory (Cambridge University Press, 1993)
[5] A. Maas, B. Grütter, R. Alkofer, J. Wambach, Proceedings of the 40th International School of
Subnuclear Physics, August 29th - September 7th, Erice, Italy; arXiv:hep-ph/0210178.