PROMPT OPTICAL EMISSION FROM RESIDUAL COLLISIONS IN GAMMA-RAY BURST OUTFLOWS

ZHUO LI1 and ELI WAXMAN1

Received 2007 November 15; accepted 2008 January 2; published 2008 January 14

ABSTRACT

The prompt γ-ray emission in γ-ray bursts is believed to be produced by internal shocks within a relativistic unsteady outflow. The recent detection of prompt optical emission accompanying the prompt γ-ray emission appears to be inconsistent with this model, because the outflowing plasma is expected to be highly optically thick to optical photons. We show here that fluctuations in flow properties on short, ~1 ms, timescales, which drive the γ-ray–producing collisions at small radii, are expected to lead to “residual” collisions at much larger radii, where the optical depth to optical photons is low. The late residual collisions naturally account for the relatively bright optical emission. The apparent simultaneity of γ-ray and optical emission is due to the highly relativistic speed with which the plasma expands. Residual collisions may also account for the X-ray emission during the early “steep decline” phase, where the radius is inferred to be larger than the γ-ray emission radius. Finally, we point out that inverse Compton emission from residual collisions at large radii is expected to contribute significantly to the emission at high energy and may therefore “smear” the pair-production spectral cutoff.

Subject headings: acceleration of particles — gamma rays: bursts — magnetic fields — shock waves

1. INTRODUCTION

In the leading model for gamma-ray bursts (GRBs), the energy source is a compact object that drives a relativistic unsteady outflow with a fluctuating Lorentz factor. Internal shocks within the outflow dissipate the bulk kinetic energy and produce gamma rays (Rees & Mészáros 1994; Paczyński & Xu 1994). In this model, a substantial fraction of the outflow kinetic energy may be dissipated outside the photosphere, allowing one to account for the nonthermal spectra and for the complicated light curves of GRBs (Kobayashi et al. 1997). The internal shocks are expected to generate/amply magnet fields and to accelerate electrons, which produce MeV gamma rays by synchrotron emission (for review of internal shock models, see Waxman 2003).

Due to the relatively short duration of the prompt gamma-ray emission, T ~ 10^3 s, the observation of long-wavelength (optical) prompt emission is a difficult task. GRB 990123 was the first event for which optical emission was detected during the burst (Akerlof et al. 1999). Today, thanks to the rapid localization of GRBs by the Swift satellite (see Zhang 2007 for recent review), a larger number of optical (and longer wavelengths) observations are carried out during the bursting phase (e.g., Blake et al. 2005; Vestrand et al. 2005, 2006; Yost et al. 2007).

As shown in § 2 (see also Li & Song 2004), during the emission of prompt gamma rays, the plasma is expected to be optically thick to optical photons due to strong synchrotron self-absorption. This appears to be inconsistent with the detection of bright optical emission accompanying gamma-ray emission. We point out here that the internal collisions at small radii, which produce the gamma-ray emission, are expected to lead to “residual” collisions at much larger radii, where the optical depth to optical photons is low. These late optical collisions may naturally account for the relatively bright optical emission. The apparent simultaneity of gamma-ray and optical emission is due to the highly relativistic speed with which the plasma expands. The time delay between gamma-ray and optical emission is expected to be shorter than a second, too short to be identified by current optical observations that usually have lower temporal resolution. We discuss in § 3 residual collisions in unsteady outflows, and we derive the long-wavelength emission that they are expected to produce at large radii. The implications of our results are discussed in § 4.

2. STRONG SYNCHROTRON ABSORPTION AT SMALL RADII

We first show that, during the prompt gamma-ray emission phase, the plasma is highly optically thick to optical photons. Consider a relativistic outflow with a Lorentz factor fluctuating over a timescale t_{\sigma}. We denote the mean Lorentz factor by \Gamma, its variance by \sigma^2, and assume \sigma^2 \ll \Gamma. The internal collisions that produce gamma rays occur in this model at a radius R_s, with \Gamma^2 t_{\sigma} a_g \approx 10^{12.2} (T / a_g) \Gamma^2 t_{\sigma} \approx 100 cm, where \Gamma_{s} = \Gamma / 10^{2.5} and t_{\sigma} = t_{\sigma} / 10^{-3} s. The observed variability time implies R_s \leq 10^{12} \Gamma_{s}^3 cm for a large fraction of BATSE bursts (Woods & Loeb 1995). We assume that internal collisions lead to shocks that generate/amply magnetic fields and accelerate electrons to high energy, leading to synchrotron emission that accounts for the prompt gamma rays.

For typical optical parameters, the cooling time of the electrons, t_c, is short compared to the dynamical time, t_d, over which the plasma expands. The dynamical time measured in the plasma frame is t_d \sim R / T = 1 R_s / \Gamma^2 s, where R_s = R / 10^{13} cm. The cooling time of electrons with Lorentz factor \gamma_e, emitting synchrotron photons of frequency \nu, is t_c \approx \gamma_e m_e c^2 \gamma_e B^2 8 \pi and \gamma_e \approx \nu B_{e} / P_{ed} is the ratio between inverse Compton and synchrotron emission, which is roughly given by the ratio of radiation and magnetic field energy densities, U_r / (B^2 / 8 \pi). In order to account for the gamma-ray emission, the magnetic field energy density needs to be close to equipartition with the thermal energy of the plasma. Since a significant fraction of this thermal energy is emitted as gamma rays, we estimate B^2 / 8 \pi \approx U_r. Using U_r = L / 4 \pi R^2 c^2, this gives \nu \sim 10^{15} \Gamma_{s}^2 \Gamma_{s} / \Gamma_{s} \approx 10^{12} \Gamma_{s}^2 / \Gamma_{s} G and \nu \sim 10^{2} \Gamma_{s} / \Gamma_{s} / s, where \nu_{\gamma} \approx L_{\nu} / 10^{31} erg s^{-1}, B_{\Gamma} = B / 10^{5} G, and \nu_{\gamma} = \nu / 10^{23} Hz is the observed frequency, with \nu = \Gamma \nu_e / \Gamma_{e} B / 2 \pi m_e c. Since t_{\nu} (\nu_{\gamma} = 1) \approx \nu, the electrons, which were initially accelerated to high energy at which their synchrotron emission peaks at \sim 1 MeV, rapidly cool down to energies at which their synchrotron emission peaks well below the optical band. Neglecting synchrotron self-absorption, this would have...
lead to a synchrotron spectrum of $F_i \propto \nu^{-1/2}$ extending from the gamma-ray band to below the optical band ($F_i$ stands for the flux per unit frequency).

Self-absorption of photons of frequency $\nu$ is dominated by electrons with Lorentz factor $\gamma_r$, which constitute a fraction $\alpha \approx n_i (\gamma_r/t_c)$ of the electron population. We may therefore approximate the (volume-averaged) absorption coefficient by $\kappa = \alpha \approx n_i (\gamma_r/t_c) \epsilon_i B_i^2 / 2 \gamma_r (\epsilon_i/c^2)$, where the electron density is given by $n_i = L_i / 4 \pi T_i^2 R_i^2 m_i c^3$, with $L_i$ the kinetic luminosity of the GRB outflow. The self-absorption frequency, where the optical depth $\alpha R / \Gamma$ equals unity, is

$$h\nu' \approx 0.3L_{51}^{1/2} \Gamma_{51}^{1/2} R_{13}^{-3/2} (1 + y)^{-1/3} \text{keV},$$  

(1)

independent of $B$, and the corresponding electron Lorentz factor is $\gamma_r' = \gamma_r (\nu') \approx 30L_{51}^{1/6} \Gamma_{51}^{1/3} B_{51}^{1/3} R_{13}^{-1/2} (1 + y)^{-1/8}$. Here $L_{51} = L / 10^{51}$ erg s$^{-1}$. Note, that the electron cooling rate is modified below $\gamma_r$ and that $t_c$ becomes larger than that used for deriving equation (1), due to the absorption of radiation. However, this modification is not large for $y \sim 1$, in which case cooling by inverse Compton emission is comparable to synchrotron cooling.

Examining equation (1), we expect a large optical depth below the X-ray band and hence a strong suppression of the optical flux. This appears to be inconsistent with observations, which typically show $F_\nu \approx F_\nu'$. (e.g., Yost et al. 2007). It should be mentioned here that, within the context of the current model, the constraint $R_c < 10^{14}$ cm, which implies $\nu' > 1$ eV, is obtained not only from the observed variability time, $t_{\text{var}}$, but also from the requirement that the synchrotron emission peaks in the MeV band. The characteristic (plasma frame) Lorentz factor of the $\gamma$-ray–emitting electrons is $\gamma_r \sim m_p / m_e$ (see § 3.2), leading to synchrotron emission peaking at

$$h\nu_p' \approx h\Gamma_{51}^2 \epsilon_i B_{51} R_{13}^{-3} / m_i c \approx 0.3L_{51}^{1/2} \Gamma_{51}^{1/2} R_{13}^{-3} \text{MeV};$$  

(2)

here $h\nu_p' \sim 1$ MeV implies therefore that $R_c \approx 10^{13}$ cm. This constraint may be avoided, for bursts where $R_c \approx 10^{13}$ cm cannot be inferred from $t_{\text{var}}$, in a model where $\gamma$-ray emission is assumed to be produced by inverse Compton scattering of $h\nu_p' \ll 1$ MeV synchrotron photons (assuming a magnetic field well below equipartition). In such a model, the inverse Compton spectrum is expected to be hard, $F \propto \nu^2$, at low frequencies, $h\nu' < 1$ MeV, due to self-absorption of the synchrotron spectrum (e.g., Panaitescu & Meszaros 2000). The observed spectrum is softer for most bursts.

3. LARGE-RADIUS EMISSION FROM RESIDUAL COLLISIONS

The optical depth for optical photons drops below unity at radii $R \approx 10^{13}$ cm [see eq. (1); note that $(1 + y) \propto R^{2/3}$ (see § 3.2)]. We show here that the optical emission could be produced by “residual” collisions at such large radii. Note that the time delay between gamma-ray and optical emission in this model,

$$\tau_{\text{delay}} \approx R_{\text{op}} / 2 \Gamma_{51}^2 c \approx 0.2R_{\text{op},15} \Gamma_{2.5}^{-2} \text{s},$$  

(3)

is expected to be shorter than the characteristic temporal resolution of the optical observations, which is a few seconds. Thus, optical and gamma-ray emission may appear to be simultaneous. However, better temporal resolution may allow one to detect a systematic time delay between the two wave bands.

In addition, one would expect larger observed variability timescales at longer wavelengths, $t_{\text{var,op}} \sim \tau_{\text{delay}}$.

We approximate the outflow by a sequence of $N \gg 1$ equal-mass shells ($i = 1, \ldots, N$) separated by an initial fixed distance $ct_\text{var}$ and expanding with (initial) Lorentz factors $\Gamma_{i,0}$ drawn from a random distribution with an average $\Gamma$ and initial variance $\Gamma_{i,0}^2 < \Gamma^2$. We assume that the radial extent of the outflow $R_{\text{var}}$ is much smaller than the collision radii $R > \Gamma^2 ct_\text{var}$ (i.e., $N \ll \Gamma^3$), which is reasonable given the observed variability (e.g., Fishman & Meegan 1995). The model may, of course, be complicated, e.g., by adding several variability times or by allowing variable mass shells. Adding such degrees of freedom may allow one to control the details of the predicted long-wavelength emission. Our main goal is to demonstrate that the simplest model considered here may naturally account for the observed optical emission.

The dynamics of late residual collisions is discussed in § 3.1, and the radiation that they are expected to generate is discussed in § 3.2.

3.1. Late Residual Collisions

Let us first consider the evolution of the outflow by using the simplifying assumption that shells merge after collisions. This assumption would be approximately valid if all the internal energy generated by a collision of two shells is radiated away. As the flow radius increases, the typical number $n(R)$ of initial shells that merge into one single shell increases, and the variance of the Lorentz factors of the resulting shells decreases. For a group of shells with a small Lorentz factor variance, the velocities $v_i$ of the shells in the shells’ center of momentum frame are not highly relativistic. In this case, conservation of momentum implies that the velocity of a merged group of shells is given by the average of merged shells’ velocities, $v = (1/n) \sum v_i$, and that the variance of the velocities of merged groups of shells is $\sigma(v) = \sigma_{i,0} / \sqrt{n}$, where $\sigma_{i,0}$ is the initial variance. This, in turn, implies that the variance of (observer frame) Lorentz factors, $\sigma(v) / \Gamma \approx \sigma(v)/c$, evolves like $\sigma(v) = \sigma_{i,0} / \sqrt{n}$. Collisions of merged groups of $n$ shells will therefore take place at a radius $R(n) \approx \Gamma^3 c n t_{\text{var}} / \sigma(v)$, which implies

$$n \propto R^{2/3}, \quad \sigma \propto \Gamma \sim R^{-1/3}.$$  

(4)

The outflow energy that may be dissipated and radiated away is the energy associated with the random velocities of the shells (in the outflow rest frame). This energy decreases as

$$E_{\text{frac}} \propto \Gamma_{51}^2 \propto R^{2/3}.$$  

(5)

Let us consider next the evolution of the outflow, dropping the assumption of shell merger. In order to describe the evolution in this case, we carried out a numerical simulation, assuming that, in each collision, one-third of the kinetic energy of the two shells (in the center of momentum frame) is radiated away and that the shells separate after the collision, each carrying half the remaining energy (in the center of momentum frame). Figure 1 shows the evolution of an outflow with the following parameters: $N = 10^3$, $t_{\text{var}} = 1$ ms, and $\Gamma = 10^{1.5} \times 3^i$, with $i$ normally distributed with zero mean and unit standard deviation. As can be seen from the top two panels of the figure, the evolution of $\sigma_i$ and the evolution of the radiated energy are well approximated by the analytic expressions of equations (4) and (5), which were obtained under the shell
merger assumption. We will therefore use these approximate analytic expressions in the next section, where the emitted radiation is discussed.

3.2. Predicted Emission

Let us first consider the energy band into which energy is radiated. We make the common assumptions that internal shocks accelerate electrons and generate or amplify magnetic fields, such that the postshock electrons and magnetic fields carry fixed fractions, \( \epsilon_e \) and \( \epsilon_B \), respectively, of the postshock internal energy. Under these assumptions, the characteristic Lorentz factor of postshock electrons (in the outflow comoving frame) scales as \( \gamma_e \propto \epsilon_e \sigma_T^2 \), and the postshock magnetic field scales as \( B^2 \propto \epsilon_B \sigma_T^2 n_e \) (the particle number density scales as \( n_e \propto R^{-3} \)). Using equation (4), the characteristic (observer frame) frequency of synchrotron photons, \( \nu_i \propto \Gamma \gamma_e^2 B \), scales as

\[
\nu_i \propto \sigma_T^3 R^{-1} \propto R^{-8/3}.
\]

As can be seen in the bottom panel of Figure 1, equation (6), which is based on the analytic approximations of equation (4) for the simplified “merging-shell” model, also describes well the results of the numerical simulation for the nonmerging model.

Next, consider the emitted flux. It is straightforward to show that the cooling time of the electrons is short compared to the dynamical time during the late residual collision phase, up to radii \( R \sim 10^4 R_g \). We therefore assume that electrons radiate away all their energy. During the phase of late residual collisions, the plasma is immersed in the radiation bath of the prompt gamma rays. The radiation energy density dominates the magnetic field energy density, because the photon energy density drops as \( U_e \propto R^{-3} \) and because the ratio \( y = U_e/(B^2/8\pi) \propto \sigma_T^2 \propto R^{2/3} \) increases with \( R \). Therefore, the electrons lose most of their energy by inverse Compton cooling, and only a fraction \( (1+y) \approx y^{-1} \propto R^{-2/3} \) of the radiated energy is emitted as synchrotron radiation. Neglecting synchrotron self-absorption, the observed (time-integrated) spectrum would be

\[
\nu F_{\nu} \propto E_{\text{iso}}^{5/2}\left| \nu_{\text{iso}} \right| \propto R^{-8/3} \propto \nu^{5/2}.
\]

Finally, let us consider the effects of synchrotron self-absorption. Equation (1) is valid for \( \nu > \nu_i \), which implies \( \nu_i \propto R^{-5/3} \). For \( \nu < \nu_i \), we therefore have \( \nu_i / \nu \propto R^{10/3} \), implying that the optical depth to synchrotron photons emitted by electrons with the characteristic Lorentz factor \( \gamma_e \) will exceed unity at sufficiently large radii, \( R > R_{\gamma_e} \). Since, at small radii, \( R \sim R_{\gamma_e} \), \( \nu_i \approx 1 \text{ MeV} \), and \( \nu_i \approx 0.5 \text{ GEV} \) (see eq. [1]), \( \nu_i = \nu_{\gamma_e} \) is obtained at \( R \approx R_{\gamma_e} \). At this radius, \( \nu_i = \nu_{\gamma_e} \) is \( \nu_i \propto \nu_{\gamma_e} \approx 10 \text{ keV} \). Thus, the \( \nu F_{\nu} \propto \nu^{1/2} \) (time-integrated) spectrum obtained above, neglecting self-absorption, does not extend down to the optical band. In order to derive the spectrum at lower frequencies, \( \nu < \nu_{\gamma_e} \), we first derive the evolution of \( \nu_i \) at \( R > R_{\gamma_e} \). For these radii, one needs to consider the electrons accelerated to Lorentz factors larger than the characteristic Lorentz factor \( \gamma_e \). Because these electrons dominate emission and absorption at \( \nu > \nu_i \), Shock acceleration is expected to generate a power-law energy distribution of electrons, \( \nu_i / d\gamma_e \propto \gamma_e^{-7} \) at \( \gamma_e > \gamma_i \). For this energy distribution, the volume-averaged number density of electrons with Lorentz factor \( \gamma_e > \gamma_i \) is

\[
n_e \approx \int_{\gamma_i}^{\gamma_e} \frac{d\gamma}{\gamma^6} = n_i \left( \frac{\gamma_i}{\gamma_e} \right)^5 \approx n_i \left( \frac{\gamma_i}{\gamma_{\gamma_e}} \right) \left( \frac{\nu_i}{\nu_{\gamma_e}} \right)^{1/2}.
\]

Using the same argument leading to equation (1), we find, for \( \nu > \nu_i \),

\[
\nu_i \approx \sigma_T^{1/7} R^{-5/7} \propto R^{-8/7}.
\]

The flat-electron energy distribution, \( \nu_i / d\gamma_e \propto \gamma_e^{-6} \), generates equal amounts of synchrotron energy in logarithmic photon energy intervals, \( \nu F_{\nu} \propto \nu^{5/6} \), for \( \nu > \nu_i \). We therefore obtain for \( \nu < \nu_{\gamma_e} \) (a time-integrated) spectrum given by

\[
\nu F_{\nu} \propto E_{\text{iso}}^{5/2} \left| \nu_{\text{iso}} \right| \nu^{1/2} \propto R^{-8/3} \propto \nu^{7/6}.
\]

Combining the above results, the observed flux at \( \nu < \nu_{\gamma_e} \) is given by

\[
F_{\nu} / F_{\nu_i} \approx \left( \frac{\nu_{\gamma_e}}{\nu} \right)^{-1/2} \left( \frac{\nu}{\nu_{\gamma_e}} \right)^{1/6} \approx 10^2 \left( \frac{h \nu}{1 \text{ eV}} \right)^{1/6}.
\]

Several comments should be made here. The flux ratio given by equation (8) holds only on average. The observed flux ratios in individual GRB events may differ significantly, because, for a small number of shells (and collisions), large variations in the late residual collisions should be expected. It should also be noticed that we have assumed \( \sigma_T < \Gamma \), whereas initial conditions with \( \sigma_T = \Gamma \) may lead to more efficient gamma-ray production at small radii, in which case \( F_{\nu} / F_{\nu_e} \) should be smaller by a factor of a few than the ratio given in equation (8).

4. DISCUSSION

We have shown that late residual collisions, which occur at radii much larger than those where gamma-ray–producing collisions take place, may naturally account for the observed strong optical emission accompanying the prompt GRB. In-ternal collisions at small radii reduce the variance of colliding shell velocities. As a result, the energy available for radiation at large radii and the characteristic frequency of radiated photons decrease with radius. We find that one may expect an
optical–γ-ray energy ratio of \( \sim 10^{-4} \), with large burst-to-burst scatter (see Fig. 1, eq. [8], and discussion at the end of § 3.2). This is consistent with the results of Yost et al. (2007), who find that, during the prompt emission of GRBs, the spectral indices between the optical and the gamma-ray bands are in the range of \( 0 < \beta_{\text{opt}} < 0.5 \), corresponding to \( F_{\text{opt}}/F_{\gamma} \sim 1 \times 10^{3} \) and implying that the optical emission is only a small fraction, \( \sim 10^{-6} \) to \( 10^{-5} \), of the total emitted energy.

Although the optical emission is produced at large radii, where synchrotron self-absorption is avoided, the expected time delay between gamma-ray and optical emission, \( \sim 0.1 \) s (see eq. [3]), is shorter than the characteristic temporal resolution of the optical observations, which is a few seconds (e.g., Blake et al. 2005). Thus, optical and gamma-ray emission may appear to be simultaneous. However, better temporal resolution may allow one to detect a systematic time delay between the two wave bands. In addition, one would expect larger observed variability timescales at longer wavelengths, \( t_{\text{var, opt}} \sim t_{\text{delay}} \).

Wei (2007) has suggested that optical emission may be generated by strong internal shocks at radii \( R/c > 10^{6} \) s, driven by shells emitted with a large time delay, \( \sim 10 \) s, following those producing the main gamma-ray emission (see also Fan et al. 2005). Our model is quite different. We show that optical emission is naturally expected to arise, without postulating the existence of delayed shells, by residual collisions at small radii \( R_{\text{c}} \sim 10^{6} \) s, in which the characteristic emitted photon frequency is low, \( h\nu \sim 1 \) eV, due to the reduction of the Lorentz factor variance in the flow (rather than by the large radius \( R/c > 10^{6} \) s). Moreover, we have shown that the optical luminosity can be estimated from the burst \( \gamma\)-ray properties and that it is consistent with the observations.

The energy released in residual collisions of the relativistic outflow is large. In fact, it would overproduce the optical emission if all the energy is released in the optical band. In the models discussed here, only a small fraction of the energy, \( \sim 10^{-2} \), is released as synchrotron radiation, because electrons accelerated in residual collisions cool mainly by inverse Compton scattering of the prompt GRB gamma rays. This has some important implications for observations at high energy, \( >100 \) MeV. Such observations are expected to be useful in determining the bulk Lorentz factor of GRB outflows and the size of the emitting region, by detecting the high-energy cutoff due to \( \gamma\gamma \) absorption (e.g., Fishman & Meegan 1995; Rees & Meszaros 1994; Zhang 2007).

REFERENCES

Akerlof, C., et al. 1999, Nature, 398, 400
Baron, M. G. 2000, in AIP Conf. Proc. 515, GeV-TeV Gamma Ray Astrophysics Workshop: Towards a Major Cherenkov Detector VI, ed. B. L. Dingus, M. H. Salamon, & D. B. Kieda (Melville: AIP), 238
Blake, C. H., et al. 2005, Nature, 435, 181
Fan, Y. Z., Zhang, B., & Wei, D. M. 2005, ApJ, 628, L25
Fishman, G. J., & Meegan, C. A. 1995, ARA&A, 33, 415
Kobayashi, S., Piran, T., & Sari, R. 1997, ApJ, 490, 92
Kumar, P., & Panaitea, A. 2000, ApJ, 541, L9
Kumar, P., et al. 2007, MNRAS, 376, L57
Lazzati, D., & Begelman, M. C. 2006, ApJ, 641, 972
Li, Z., Dai, Z. G., Lu, T., & Song, L. M. 2003, ApJ, 599, 380
Li, Z., & Song, L. M. 2004, ApJ, 608, L17
Lithwick, Y., & Sari, R. 2001, ApJ, 555, 540
Lyutikov, M. 2006, MNRAS, 369, L5
Paczyński, B., & Xu, G. 1994, ApJ, 427, 708
Panaitea, A., & Mészáros, P. 2000, ApJ, 544, L17
Rees, M. J., & Mészáros, P. 1994, ApJ, 430, L93
Tagliaferri, G., et al. 2005, Nature, 436, 985
Tang, S. M., & Zhang, S. N. 2006, A&A, 456, 141
Vestrand, W. T., et al. 2005, Nature, 435, 178
Waxman, E. 2003, in Supernovae and Gamma-Ray Bursters, ed. K. Weiler (Lecture Notes in Physics 598), 393
Wei, D. M. 2007, MNRAS, 374, 525
Woods, E., & Loeb, A. 1995, ApJ, 453, 583
Yost, S. A., et al. 2007, ApJ, 669, 1107
Zhang, B. 2007, Chinese J. Astron. Astrophys., 7, 1