Fuzzy Probist System Reliability of Weaving Machine in Textile Industry

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Abstract: In this paper, the binary state assumptions have been replaced by a fuzzy state assumption. Probist Reliability of a weaving machine is calculated with the help of fault tree diagram using the pendant triangular fuzzy number and pendant trapezoidal fuzzy numbers. The formula thus created is used to evaluate the failure rate of the machines, which would determine the reliability of the machines and maintain the quality of performance.

Keywords: Probist Reliability, Pendant fuzzy number, triangular Pendant fuzzy number and trapezoidal pendant fuzzy number, Probist series system, Probist parallel System

1. Introduction

The concept of Reliability is old as man himself. From ancient days, man had started his work on failure of the system such as unreliability of the machine and reliability evaluation of a system. Characteristics of system reliability depends on time failure, failure factors, availability of the products and probability measures.

Reliability, in its simplest form, failure does not happen between specific time intervals. The performance of a machine depends on some factors that affect the machine:

1. Capability and capacity of the machine.
2. Quality of the raw materials put into the machine.
3. Technical difficulties occurred can alter the performance and output of the machine.

Zedeh started the brief introduction about fuzzy sets [1] in 1965 to represent the data handled non-statistical certainties. Types of fuzzy numbers are defined and the vague idea about the fuzzy numbers. In [2], Yager explained the clear idea about to solving mathematical relationships. Representation of trapezoidal fuzzy numbers and the multiplication operation introduced, Rezvani.S[3]. In [4], Nagoorgani.A, a triangular fuzzy number based operation for solving linear programing problem. A new fuzzy number named as trident fuzzy numbers are introduced to fuzzy system and clarify its arithmetic operations by Praveen Prakash.A and M. Geetha Lakshmi [5]. Another fuzzy number is introduced by B. Rama and G. Michael Rosario [6], named as quadrant fuzzy number and its arithmetic operations.
explained using types of fuzzy numbers. In [7],[8], Sub-Trident fuzzy numbers are explained and ranking is introduced. Pentagonal fuzzy numbers are explained in [9] by Avinash. J. Kamble. In [10] explained the operations fault tree analysis of a system characterized by fuzzy numbers especially by triangular intuitionistic and deeply explained applications. In [11] factors affecting the failure rate explained and discussed the different failure possibility. In [12] G. Michael Rosario and A. Dhana Lakshmi are discussed the applications of intuitionistic fuzzy equations on reliability evaluation. In [13,14] presented the reliability evaluation of weaving machine in textile industry using trapezoidal intuitionistic fuzzy numbers. In [15], Kai-Yuan Cai explained clear-cut idea about probist fuzzy numbers and series and parallel system with the help of probist reliability.

2. Probit Reliability
In the framework of probist reliability theory a probist system is based on probability assumption and the binary-state assumption:
In probability notion, in the context of probability measures the system failure behavior is characterized. In binary-state notion, there are two crisp states for a system, fully functioning system or fully failed system. The system is of one of the two states at the time. A completely functioning system means that the crisp value is one and a non-functioning system means that the crisp value is zero. However, in the fuzzy case the value is between the binary digits. Therefore, other than the the two crisp states in the fuzzy state the system can work between infinite states between the two binary digits.

3. Definitions

3.1. Probit Series System
Let \( R = R_1 R_2 R_3 \ldots \ldots R_n = \prod_{i=1}^{n} R_i \), be a probist system with the n components are arranged in series mode.
Suppose \( R_1, R_2, R_3 \ldots \ldots R_n \) are triangular pendant fuzzy number. Write it as \( R_i = (a_{i1p}, a_{i2p}, a_{i3p}) \); \( i = 1,2, \ldots \ldots n \)
\[
(R_i)_{\alpha} = [a_{i2p} - \alpha^5(a_{i2p} - a_{i1p}), a_{i2p} + \alpha^5(a_{i3p} - a_{i2p})], \alpha \epsilon [0,1]
\]
\[
(R)_{\alpha} = \prod_{i=1}^{n} (R_i)_{\alpha}
\]
\[
= \prod_{i=1}^{n}(a_{i2p} - \alpha^5(a_{i2p} - a_{i1p})), \prod_{i=1}^{n}(a_{i2p} + \alpha^5(a_{i3p} - a_{i2p})), \alpha \epsilon [0,1]
\]
Also, we denote, \( R_i = (m_i - \alpha, m_i, m_i - \beta) \)
\[
R = \prod_{i=1}^{n} R_i
\]
\[
= \prod_{i=1}^{n}(m_i - \alpha), \prod_{i=1}^{n} m_i, \prod_{i=1}^{n}(m_i - \beta) \tag{1}
\]
Symbolically, we write trapezoidal pendant fuzzy number as, \( R_1, R_2 \ldots \ldots R_n \)
\( R_i = (a_{i1p}, a_{i2p}, a_{i3p}, a_{i4p}) \), \( i = 1,2, \ldots \ldots n \)

3.2. Probit Parallel System
Let \( R = 1 - \prod_{i=1}^{n}(1 - R_i) \), be a probist system with the n components are arranged in parallel mode.
Suppose \( R_1, R_2, R_3 \ldots \ldots R_n \) are triangular pendant fuzzy number. Write it as \( R_i = (a_{i1p}, a_{i2p}, a_{i3p}) \); \( i = 1,2, \ldots \ldots n \)
\[
(R_i)_{\alpha} = [a_{i2p} - \alpha^5(a_{i2p} - a_{i1p}), a_{i3p} + \alpha^5(a_{i3p} - a_{i2p})], \alpha \epsilon [0,1]
\]
\[
(R)_{\alpha} = 1 - \prod_{i=1}^{n} 1 - (R_i)_{\alpha}
\]
\[
\frac{1}{5}x \quad \text{for} \quad x < a_{1p} \\
\frac{1}{5}a_{2p-x} \quad \text{for} \quad a_{1p} \leq x \leq a_{2p} \\
0 \quad \text{for} \quad x = a_{2p} \\
\frac{1}{5}a_{3p-x} \quad \text{for} \quad a_{2p} \leq x \leq a_{3p} \\
\frac{1}{5} \quad \text{for} \quad x > a_{3p}
\]

3.3.1 \(\alpha\)-cut of a Triangular Pendant Fuzzy Number

\(\alpha\)-cut-Triangular Pendant Fuzzy Number is given by
\[A_{\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}), a_{2p} + \alpha^5(a_{3p} - a_{2p})], \alpha \in (0, 1].\] (4)

\[i = 1, 2, \ldots, n\]

3.4. Trapezoidal Pendant Fuzzy Number (TrPn)

A Trapezoidal Pendant Fuzzy Number represented by \(\tilde{A} = (a_{1p}, a_{2p}, a_{3p}, a_{4p})\) where \(a_{1p}, a_{2p}, a_{3p}, a_{4p}\) are real numbers and the membership function, with a structures help,
\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{1}{5} & \text{for} \quad x < a_{1p} \\
\frac{1}{5}a_{2p-x} & \text{for} \quad a_{1p} \leq x \leq a_{2p} \\
0 & \text{for} \quad a_{2p} \leq x \leq a_{3p} \\
\frac{1}{5}a_{3p-x} & \text{for} \quad a_{3p} \leq x \leq a_{4p} \\
\frac{1}{5} & \text{for} \quad x > a_{4p}
\end{cases}
\]

3.4.1 \(\alpha\)-cut-Trapezoidal Pendant Fuzzy Number

\(\alpha\)-cut-Trapezoidal Pendant Fuzzy Number is given by
\[ A_{p\alpha} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}), a_{3p} + \alpha^5(a_{4p} - a_{3p})], \alpha \in (0,1]. \] (6)

4. Failure of weaving machine

![Fault tree diagram of the failure rate of the weaving machine with the corresponding factors](image)

Figure 1 Fault tree diagram of the failure rate of the weaving machine with the corresponding factors

Figure shows the failure rate of the following factors are denoted as follows

- \( \hat{f}_{\text{WM}} \) Weaving machine
- \( \hat{f}_{\text{IF}} \) Internal factors
- \( \hat{f}_{\text{EF}} \) External factors
- \( \hat{f}_{\text{PS}} \) Physical stabilities
- \( \hat{f}_{\text{RM}} \) Raw materials
- \( \hat{f}_{\text{WE}} \) Working energy
- \( \hat{f}_{\text{FO}} \) Flow of lubricants oil
- \( \hat{f}_{\text{HC}} \) Controlled humidity
- \( \hat{f}_{\text{P}} \) balanced pressure
- \( \hat{f}_{\text{T}} \) Quality of the thread
- \( \hat{f}_{\text{Y}} \) Continuity of filament yarn
- \( \hat{f}_{\text{F}} \) Availability of the fuel
- \( \hat{f}_{\text{C}} \) Balanced current flow
- \( \hat{f}_{\text{O}} \) Shortage of lubricants oil
- \( \hat{f}_{N} \) Nozzles blocked

There are eight sub factors \( \hat{f}_{H}, \hat{f}_{P}, \hat{f}_{T}, \hat{f}_{Y}, \hat{f}_{F}, \hat{f}_{C}, \hat{f}_{O}, \hat{f}_{N} \). Here, \( R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8 \) are the corresponding reliabilities of \( \hat{f}_{H}, \hat{f}_{P}, \hat{f}_{T}, \hat{f}_{Y}, \hat{f}_{F}, \hat{f}_{C}, \hat{f}_{O}, \hat{f}_{N} \) respectively.

Take a probist parallel machine, of order 8, that is,

\[ R = 1 - \prod_{i=1}^{8}(1 - R_i) \] (7)

Suppose \( R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8 \) are triangular pendant fuzzy numbers, then so are

\[ 1 - R_1, 1 - R_2, 1 - R_3, 1 - R_4, 1 - R_5, 1 - R_6, 1 - R_7, 1 - R_8. \]

Denote \( R_i = (a_{i1p}, a_{i2p}, a_{i3p}) \); \( i = 1,2,3,4,5,6,7,8 \)
And the $\alpha - cut$ is denoted as $(R)_{\alpha} = [1,1] - \prod_{i=1}^{n}([1,1] - (R_i)_{\alpha})$

$= [1,1] - \prod_{i=1}^{n}[a_{i1p} - \alpha^5(a_{i1p} - a_{i1p}), a_{i2p} + \alpha^5(a_{i1p} - a_{i1p})], \alpha \in (0,1].$

Since, a-cut of a Triangular Pendant Fuzzy Number, given by

$A_{pa} = [a_{2p} - \alpha^5(a_{2p} - a_{1p}), a_{2p} + \alpha^5(a_{2p} - a_{3p})], \alpha \in (0,1].$

Here, Triangular Pendant Fuzzy number can be written as

$R_i = (m_i - a_i, m_i, m_i - \beta_i); i = 1,2,3,4,5,6,7,8$ \hspace{1cm} (8)

Suppose, for example height of a person is $m$, where $a, \beta$ the left-right are spreads of $m$. In general, fuzzy number can be represented $m - \alpha, m, m + \beta = (a_1, a_2, a_3)$, these types of numbers are called triangular pendant fuzzy numbers alternatively represented as $(m, \alpha, \beta)$.

(8) in (7) $\Rightarrow R = 1 - \prod_{i=1}^{8}(1 - R_i)$$\Rightarrow$$1 - \prod_{i=1}^{8}(1 - (m_i - a_i, m_i, m_i - \beta_i))$$\Rightarrow$$1 - \prod_{i=1}^{8}(1 - (m_i - a_i), 1 - m_i, 1 - (m_i - \beta_i))$$\Rightarrow$$1 - \prod_{i=1}^{8}(1 - (m_i - a_i), 1 - m_i, 1 - (m_i - \beta_i))$$\Rightarrow$$1 - \prod_{i=1}^{8}(1 - (m_i - a_i), 1 - m_i, 1 - (m_i - \beta_i))$$\Rightarrow$$[1 - \prod_{i=1}^{8}(1 - (m_i - a_i), 1 - m_i, 1 - (m_i - \beta_i))]

Suppose, $R_1, R_2, R_3, R_4, R_5, R_6, R_7, R_8$ trapezoidal pendant fuzzy numbers, then

$1 - R_1, 1 - R_2, 1 - R_3, 1 - R_4, 1 - R_5, 1 - R_6, 1 - R_7, 1 - R_8$ are trapezoidal pendant fuzzy numbers.

So, denote $R_i = (a_{i1p}, a_{i2p}, a_{i3p}, a_{i4p}); i = 1,2,3,4,5,6,7,8$ then

$1 - R_i = (1 - a_{i1p}, 1 - a_{i2p}, 1 - a_{i3p}, 1 - a_{i4p}); i = 1,2,3,4,5,6,7,8$

Therefore, $\prod_{i=1}^{8}(1 - R_i) = \prod_{i=1}^{8}(1 - a_{i1p}), \prod_{i=1}^{8}(1 - a_{i2p}), \prod_{i=1}^{8}(1 - a_{i3p}), \prod_{i=1}^{8}(1 - a_{i4p})$

So, $R = 1 - [\prod_{i=1}^{8}(1 - a_{i1p}), \prod_{i=1}^{8}(1 - a_{i2p}), \prod_{i=1}^{8}(1 - a_{i3p}), \prod_{i=1}^{8}(1 - a_{i4p})]$ \hspace{1cm} (10)

5. Numerical Evaluation

Let $R_1 = (0.1,0.2,0.3), R_2 = (0.1,0.3,0.4), R_3 = (0.2,0.3,0.4), R_4 = (0.3,0.4,0.5)$, $R_5 = (0.4,0.5,0.6), R_6 = (0.4,0.5,0.7), R_7 = (0.5,0.6,0.7), R_8 = (0.5,0.6,0.8)$

\[
\prod_{i=1}^{8}1 - R_i = \left( \prod_{i=1}^{8}(1 - a_{i1}), \prod_{i=1}^{8}(1 - a_{i2}), \prod_{i=1}^{8}(1 - a_{i3}), \prod_{i=1}^{8}(1 - a_{i4}) \right) \\
= (0.9 \times 0.9 \times 0.8 \times 0.7 \times 0.6 \times 0.6 \times 0.5 \times 0.5, \\
0.8 \times 0.7 \times 0.7 \times 0.6 \times 0.5 \times 0.5 \times 0.4 \times 0.4, \\
0.7 \times 0.6 \times 0.6 \times 0.5 \times 0.4 \times 0.3 \times 0.3 \times 0.2, \\
0.6 \times 0.5 \times 0.5 \times 0.4 \times 0.3 \times 0.2 \times 0.2 \times 0.1)
\]

That is, $1 - R = (0.040824, 0.009408, 0.0009072, 0.0000072)$

Therefore, $R = (0.959176, 0.990592, 0.9990928, 0.9999928)$

6. Conclusions

In this paper, different fuzzy numbers are used to evaluate fuzzy reliability. Fuzzy numbers may vary from time to time and new fuzzy numbers are introduced frequently. This is a comparison study. In this study, the reliability evaluation of a machine is one fuzzy numbers is better than the other. This machine cannot be concluded as the most reliable in the prescribed fuzzy numbers, however, we can conclude that the given fuzzy number is more reliable than the other fuzzy numbers. In this paper two new fuzzy numbers are introduced they are triangular pendant fuzzy numbers and trapezoidal pendant fuzzy numbers. In the previous paper while solving the fuzzy reliability of a machine, this value was found, trapezoidal fuzzy numbers is better than triangular fuzzy numbers with the help of numerical evaluation. Similarly, if we evaluate trapezoidal pendant fuzzy numbers is better than triangular pendant fuzzy numbers.
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