Gauge anomalies in the Standard-Model Effective Field Theory

Oscar Catà,¹,² Wolfgang Kilian,¹,² and Nils Kreher¹,²

¹University of Siegen, Department of Physics, D–57068 Siegen, Germany

(Dated: November 20, 2020)

If the Standard Model is understood as the first term of an effective field theory, the anomaly-cancellation conditions have to be worked out and fulfilled order by order in the effective field-theory expansion. We bring attention to this issue and study in detail a subset of the anomalies of the effective field theories at the electroweak scale. The end result is a set of sum rules for the operator coefficients. These conditions, which are necessary for the internal consistency of the theory, lead to a number of phenomenological consequences when implemented in analyses of experimental data. In particular, they not only decrease the number of free parameters in different physical processes but have the potential to relate processes with different flavor content. Conversely, a violation of these conditions would necessarily imply the existence of undetected non-decoupling new physics associated with the electroweak energy scale.

Introduction. Anomalies in quantum field theories occur whenever a classical symmetry does not survive the process of quantization. Such effects are actually not uncommon and are typically accompanied by profound phenomenological consequences. In the strong interactions, the chiral anomaly explains the π⁰ → γγ decay [1], indirectly confirms the number of quark colors, and it also implies the existence of pions as Goldstone bosons [2]. The axial U(1)ₐ anomaly, in turn, justifies the mass range of the η′(958) boson [3]. All of these effects are associated with anomalous global symmetries. Local symmetries have a different status, and the existence of gauge anomalies has more severe consequences for a relativistic quantum field theory, namely the absence of a unitary S matrix.

If the theory at hand is assumed to be complete, gauge anomalies deem it inconsistent. Instead, if we are dealing with an effective field theory (EFT) [4, 5], the presence of an anomaly can be amended with the addition of a Wess-Zumino term, which is associated with heavy integrated-out degrees of freedom. A paradigmatic example is the Standard Model (SM) with the top quark integrated out. In this theory, the top quark contribution to the anomaly is retained by a Wess-Zumino term [6] which cancels the anomaly associated with the active fermions. The term vanishes again in the deep infrared regime, where all fermions are integrated out. In general, EFTs with an anomalous gauge symmetry indicate the existence of non-decoupling physics beyond the SM and must be augmented either by non-local interactions, by extra degrees of freedom, or by non-polynomial terms of Wess-Zumino type [7, 8].

The SM is a formally complete theory, in particular free from gauge anomalies. However, strong indications suggest that it should be considered as the first term of an EFT expansion. An EFT provides a well-defined low-energy approximation to a field theory, organizing amplitudes and observables as a power-series expansion in terms of an inverse heavy mass scale Λ, beyond which the theory is no longer applicable.

The Standard-Model Effective Theory (SMEFT) [9–11] extends the Lagrangian of the SM by gauge-invariant local operators O⁽⁽D⁾⁾ of dimension D > 4, built from its elementary fields. Formally,

\[
\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{D>4} \frac{1}{\Lambda^{D-4}} \sum_{j} c_j^{(D)} O_j^{(D)}
\]

where the sum over j is finite, i.e., each order of this (infinite) dimensional expansion contains a finite number of operators. The SMEFT Lagrangian and the associated perturbative expansion at each order systematically encode the potential effects of unresolved new degrees of freedom beyond the SM, even without knowledge about the actual high-energy theory. The framework is valid if there is a clean separation between the new-physics scale Λ and the electroweak scale v, assuming that the limit v/Λ → 0 is well-behaved within perturbation theory. The number of independent operators of SMEFT for each dimension D can be determined either by explicit construction [11] or by formal methods [12]. By assumption, the theory has no extra infrared degrees of freedom, and non-local and Wess-Zumino terms are also absent.

In the SM, anomaly cancellation holds separately for each fermion generation due to the hypercharge assignments. However, there is no proof that these assignments are enough to render the full SMEFT anomaly-free. Actually, extending the arguments of Ref. [8] to the 1/A² sector, it is logically possible to develop gauge anomalies at NLO in the EFT expansion, while having an anomaly-free SM. Accordingly, in order to have an EFT free of anomalies, one should explicitly check that they cancel at each order of the EFT expansion.

The SMEFT, commonly truncated to contain only dimension-six operators, has been extensively used in data analysis at the LHC [13–19], and further for estimating the physics potential of future experiments [14–19].
Anomalies in EFTs. The problem of identifying the classes of operators that lead to an anomaly can be systematically solved by using well-known methods in algebraic renormalization (for a review, see e.g., [20]). The renormalization of gauge theories requires the introduction of a gauge-fixing condition and ghosts terms. After their introduction, the Lagrangian possesses a BRST symmetry, which translates into a set of Slavnov-Taylor identities (STI) for the correlators of the theory. The STI ensure unitarity of each loop order. The method relies on an implicit assumption that the properties of the SM carry over without modification. As we will show explicitly below for the dimension-six operators, this assumption is not warranted. At the quantum level, the SMEFT generates its own gauge anomalies at each order in $1/\Lambda$. These are local contributions to amplitudes that violate Ward identities, and thus unitarity, which cannot be removed by renormalization. As a consequence, anomaly cancellation can only be achieved through the enforcement of a set of sum rules that relate the different Wilson coefficients. The number of independent parameters is therefore reduced in a model-independent way by a consistency requirement. If the SMEFT is the low-energy limit of an anomaly-free UV model, such sum rules are necessarily satisfied. In a generic bottom-up analysis, they must be imposed by hand as extra conditions on the SMEFT parameter space.

However, to our knowledge there is no proof of perturbative unitarity of its gauge sector. Rather, applications rely on an implicit assumption that the properties of the SM carry over without modification. As we will show explicitly below for the dimension-six operators, this assumption is not warranted. At the quantum level, the SMEFT generates its own gauge anomalies at each order in $1/\Lambda$. These are local contributions to amplitudes that violate Ward identities, and thus unitarity, which cannot be removed by renormalization. As a consequence, anomaly cancellation can only be achieved through the enforcement of a set of sum rules that relate the different Wilson coefficients. The number of independent parameters is therefore reduced in a model-independent way by a consistency requirement. If the SMEFT is the low-energy limit of an anomaly-free UV model, such sum rules are necessarily satisfied. In a generic bottom-up analysis, they must be imposed by hand as extra conditions on the SMEFT parameter space.

In general, this condition is not automatically fulfilled at the quantum level. Instead, the general solution is

$$\mathcal{S}\Gamma^{(n)} = \int d^4x \sum_j Q_j,$$

where $Q_j$ are a set of local operators. The STI implies $Q_j = 0$. Introducing the anticommuting space-time and BRST differentials $\delta$ and $\sigma$, respectively, each of the offending terms can be removed by renormalization if the cohomology is trivial, i.e., all the $Q_j$ are equivalent to a BRST variation up to total derivatives, $Q_j = dP^{(1)}_j + sP^{(2)}_j$, where $P^{(k)}_j$ are local operators. Otherwise, the STI are not satisfied, which signals the existence of an anomaly. As an algebraic structure, the anomaly is the combined cohomology of $\delta$ and $s$ in the space of local operators with fixed operator dimension and unit ghost number. In general, this space is non-empty and there exist local operators which potentially violate the Slavnov-Taylor identities, unless their coefficients vanish.

For standard renormalizable gauge theories, including the SM, this problem was solved long ago. For each single gauge group, there is at most one contribution to the anomaly, namely $Q = \varepsilon_{\mu\nu\lambda\rho} X^{\mu\nu} X^{\lambda\rho} x$, where $X^{\mu\nu}$ is the field strength, and $x$ denotes the associated ghost field. These are the famous ABJ triangle anomalies [1, 22], which were initially discovered by the computation of the graph in Fig. 1. In the SM, there are several of these diagrams, depending on the combinations of external gauge fields, which can only vanish through cancellations of the different fermion loops. The vanishing of all triangles can be written as a set of sum rules involving the different quantum numbers of fermions. All the sum rules are satisfied in the SM, hence the anomaly vanishes and a Fock space can be defined.

The same procedure can be applied to the SMEFT, where now the effective action is a series expansion in inverse powers of $\Lambda$. The STI should be fulfilled at each order in the EFT expansion. The anomaly structures can a priori be more general than the SM ones and, as a result, one expects that the conditions that ensure anomaly-cancellation in the SM are enlarged when the NLO terms in SMEFT are considered.

The formalism to solve the cohomology problem for gauge theories with operators of arbitrary dimension is known [23]. If the gauge group contains $U(1)$ factors, at each order in the EFT expansion there are potential anomaly contributions beyond the ABJ-type one. A basis for the dimension-six anomaly is provided by operators of the form $Q^{(6)} = R^{(6)} b$, where $R^{(6)}$ are operators of the dimension-six SMEFT Lagrangian, and $b$ is the $U(1)_Y$ (hypercharge) ghost.

Triangle anomalies in SMEFT. In this letter we will concentrate on the subset of parity-odd operators with-
out fermion fields, namely
\[
(\phi^\dagger \phi)e^{\mu \nu \rho \sigma} \text{tr} [G_{\mu \nu} G_{\rho \sigma}] b + 2c_{\rho \sigma} = 0
\]  
(3)
\[
(\phi^\dagger \phi)e^{\mu \nu \rho \sigma} B_{\mu \nu} B_{\rho \sigma} b + 2c_{\rho \sigma} = 0
\]  
(4)
\[
(\phi^\dagger \phi)e^{\mu \nu \rho \sigma} [W_{\mu \nu} W_{\rho \sigma}] b + 2c_{\rho \sigma} = 0
\]  
(5)
\[
(\phi^\dagger \phi)e^{\mu \nu \rho \sigma} R_{\mu \nu \alpha \beta}R_{\rho \sigma \alpha \beta} b + 2c_{\rho \sigma} = 0
\]  
(6)
\[
(\phi^\dagger \phi)e^{\mu \nu \rho \sigma} W_{\mu \nu} B_{\rho \sigma} b + 2c_{\rho \sigma} = 0
\]  
(7)
where $G_{\mu \nu}$, $W_{\mu \nu}$ and $B_{\mu \nu}$ are the field strength tensors of $SU(3)$, $SU(2)$ and $U(1)_Y$, respectively, $\phi$ is the Higgs field, $R_{\mu \nu \alpha \beta}$ is the Riemann tensor, and $\text{tr}[]$ denotes the trace over gauge indices. In the broken phase, where the Higgs field is expanded around its vacuum expectation value, the coefficients of these operators can be computed from the triangle graphs of Fig.2 where each diagram has a single dimension-six insertion. The integrands become entirely analogous to the SM case, hence no new explicit loop calculation is necessary. At NLO in SMEFT, one can also build diagrams with three external gauge bosons and scalar loops, but they can be shown not to contribute to the anomalies.

In order to evaluate the diagrams, one needs to identify the NLO corrections to the gauge boson couplings to fermions. The presence of dimension-six operators corrects the SM fields and parameters, which have to be brought back to canonical form (see e.g., [22]). Once this is done, one finds that the gauge interactions to fermions have flavor- and family-dependent corrections, which come from the operators [11]

\begin{align}
Q_{\phi f}^{(1)}_{ij} &= (i \phi^\dagger D_{\mu} \phi)(\bar{\ell}_i \gamma^\mu \ell_j) \\
Q_{\phi f}^{(3)}_{ij} &= (i \phi^\dagger D_{\mu} \phi)(\bar{\ell}_i \gamma^\mu \gamma^\nu \ell_j) \\
Q_{\phi q}^{(1)}_{ij} &= (i \phi^\dagger D_{\mu} \phi)(\bar{q}_i \gamma^\mu q_j) \\
Q_{\phi q}^{(3)}_{ij} &= (i \phi^\dagger D_{\mu} \phi)(\bar{q}_i \gamma^\mu \gamma^\nu q_j)
\end{align}

where $\gamma^a$, $a = 1, 2, 3$ are the Pauli matrices. The fermion generation indices are explicitly shown. There are also universal corrections, associated with the oblique parameters $S$ and $T$ through the operators $Q_{\phi D} = (\phi^\dagger \phi)(\phi^\dagger D_{\mu} \phi)^* (\phi^\dagger D_{\mu} \phi)$ and $Q_{\phi W B} = (\phi^\dagger \tau^a \phi) W_{\mu \nu} B_{\rho \sigma}$. However, these contributions to the triangles cancel automatically once all the fermions are considered, precisely because of their universal character. The operator $Q_{\phi u d}$ does not contribute to the triangles, since one cannot build any diagram with it. The structure of the corrections coming from the remaining operators in eqs. (5)-(15) is SM-like, except for the neutral $W^3_u$ gauge boson, which has interactions not proportional to the identity and to right-handed fermions.

We will perform our analysis in the gauge basis, where generation mixing is only present in the Yukawa terms. Since the anomalies under consideration are independent of the fermion masses, in this basis the anomaly conditions are generation-diagonal. The resulting sum rules are listed in Table I where $R$ stands for the gravitational field. Each of the SMEFT coefficients contains the sum over all the generations, e.g., $c_{\phi u} = \sum_q c_{\phi u}^{(q)}$. The first four conditions correspond to the same triangle topologies as in the SM, whereas the last two are new configurations of SMEFT at NLO. The remaining triangles either trivially vanish or reproduce one of the previous conditions. For instance, the triangles $\langle GGW^3 \rangle$, $\langle W_3 W_3 W_3 \rangle$ or $\langle RWW^3 \rangle$ are nonzero but do not provide additional constraints.

The sum rules above imply that there are just two independent parameters, which we may denote as $c_{\phi f}^{(3)}$ and $c_{\phi f}^{(1)}$. The solution in terms of them is

\begin{align}
Q_{\phi f}^{(3)} &= c_{\phi f}^{(3)} c_{\phi f}^{(1)} , \\
Q_{\phi f}^{(1)} &= \frac{1}{y_q} c_{\phi q}^{(1)} = \frac{1}{y_u} c_{\phi u} = \frac{1}{y_d} c_{\phi d} = \frac{1}{y_e} c_{\phi e} .
\end{align}

Anomaly cancellation requires that the trace (over flavor) of the NLO SMEFT coefficients $c_{\phi q}$ containing singlet $SU(2)$ fermion currents have to scale as the ratio of the hypercharges. In turn, the coefficients of the triplet fermion currents for leptons and quarks have the same trace.

Notice that these conditions open up a very rich spectrum of (anomaly-free) flavor phenomenology. They also

\begin{table}
| Triangle | SMEFT | EWChL |
|----------|-------|-------|
| $\langle GGB \rangle$ | $2c_{\phi u}^{(1)} - c_{\phi u} - c_{\phi d} = 0$ | $2c_{\psi 1} - c_{\psi 4} - c_{\psi 5} = 0$ |
| $\langle WWB \rangle$ | $6y_q c_{\phi q}^{(1)} + 12y_u c_{\phi u}^{(1)} + 2y_d c_{\phi d}^{(1)} + 4y_e c_{\phi e}^{(3)} = 0$ | $12y_q Re [c_{\psi 3}] + 4y_e Re [c_{\psi 9}] - 3c_{\psi 1} - c_{\psi 7} = 0$ |
| $\langle BBB \rangle$ | $A \equiv 6y_q c_{\phi q}^{(1)} + 2y_u c_{\phi u}^{(1)} - 3y_q^2 c_{\phi u} - 3y_u^2 c_{\phi d} - y_q^2 e_{\phi e} = 0$ | $B \equiv 6y_q c_{\psi 1} + 2y_u c_{\psi 1} - 3y_q^2 c_{\psi 4} - 3y_u^2 c_{\psi 4} - y_q^2 c_{\psi 10} = 0$ |
| $\langle BBW^3 \rangle$ | $(12y_q c_{\phi q}^{(3)} + 4y_e c_{\phi e}^{(3)}) y_q + A = 0$ | $3y_q c_{\psi 2} + y_u c_{\psi 8} - B = 0$ |
| $\langle W_3 W_3 B \rangle$ | $6y_q c_{\phi q}^{(1)} + 3c_{\phi u} - 3c_{\phi d} - c_{\phi e} = 0$ | $6c_{\psi 1} + 2c_{\psi 7} - 3c_{\psi 4} - 3c_{\psi 2} - c_{\psi 10} = 0$ |

| Table I: Sum rules enforcing anomaly cancellation for the different triangle diagrams in both the SMEFT and the EWChL. |
\end{table}
entail that fits with generation-universal Wilson coefficients have substantially less free parameters than normal.

We have performed a number of checks of the previous conditions by exploring the matching to SMEFT of some anomaly-free models with an additional $U(1)$ gauge group. In all cases, we have found that the conditions are fulfilled. An independent cross-check also comes from the renormalization evolution of the different coefficients. The anomaly-cancellation sum rules that we have found provide constraints that must be respected in any valid renormalization scheme and for any renormalization scale $\mu$. We took the running of the relevant coefficients from [27], which list the complete one-loop evolution of SMEFT at NLO without imposing anomaly cancellation. The first thing to notice is that the coefficients in eqs. (16) and (17) are not closed under renormalization, but involve other operators. We have checked that by keeping only the operators of eqs. (15), neglecting fermion masses, the constraints in eqs. (16) and (17) are indeed invariant under the renormalization-group flow. With these simplifications in place, this part of the renormalization-group equations reduces to

$$\frac{d}{d\mu}c^{(3)}_{\varepsilon f} = \frac{7}{3}g^2 c^{(3)}_{\varepsilon f}, \quad \frac{d}{d\mu}c^{(1)}_{\varepsilon f} = \frac{41}{3}g^2 c^{(1)}_{\varepsilon f}.$$  

However, the renormalization-group evolution of the coefficients (16, 17) contains, in the general case, contributions from four-fermion and Higgs operators, and indirectly mix with the whole NLO SMEFT Lagrangian. Since anomaly cancellation requires invariance under the renormalization-group flow for the whole set of operators, the corresponding coefficients must be subject to additional sum rules that are not associated with the graphs of Fig. 2. The determination of these sum rules is beyond the scope of the present paper.

So far we have concentrated our attention on the SMEFT. However, at the electroweak scale one can build another EFT, which goes under the name of the Higgs-Effective Field Theory (HEFT) [28, 29]. This EFT is linked to scenarios of new-physics which are strongly-coupled at the electroweak scale (see e.g., the discussion in [28, 29]). As opposed to SMEFT, the leading-order is not the SM, but a more general framework where the Higgs is not necessarily a $SU(2)$ doublet. However, when it comes to the anomaly, the arguments spelled out above carry over unchanged. As with the SMEFT, we will be concerned with the anomalies that can be computed from the diagrams of Fig. 2. The corrections to the gauge interactions of fermions contain universal and flavor-specific components [30, 31]. As before, only the flavor-specific corrections are relevant. In the basis of Ref. [32], there are 10 NLO operators that belong to this class. The resulting conditions for the Wilson coefficients, in the notation of Ref. [33], are summarized in Table I, where, as before, each coefficient implicitly contains the trace over generation indices. We note that the HEWχL has one additional sum rule, which is associated with the anomaly

$$\varepsilon^{\mu\nu\rho\sigma} \text{tr} \left[ U^\dagger W_{\mu\nu} U \tau_3 \right] \text{tr} \left[ U^\dagger W_{\rho\sigma} U \tau_3 \right] b,$$

whose SMEFT counterpart is a dimension-eight operator. The matrix $U$ is a (non-linear) function of the electroweak Goldstone modes. The solutions for the NLO coefficients form

$$c_3 \equiv c V_8 = c V_2,$$

$$c_1 \equiv \text{Re} [c V_3] = \text{Re} [c V_9]$$

$$c_2 \equiv \frac{1}{y_t} c V_1 = \frac{1}{y_u} c V_4 = \frac{1}{y_d} c V_5 = \frac{1}{y_t} c V_7 = \frac{1}{y_c} c V_{10}.$$  

The one-loop renormalization of the HEWχL has been worked out in [34, 35]. As a cross-check of the previous conditions, we have verified that, when the fermion masses are neglected, the relations above are renormalization-group invariant. The reason in this case is that the flavor-specific part of the beta functions for each coefficient is proportional to the hypercharges of the corresponding fermions.

Conclusions. The conditions that render the SM anomaly-free do not automatically carry over to the EFTs at the electroweak scale, namely the SMEFT and the HEWχL. Rather, the cancellation of anomalies has to be implemented order by order in the EFT expansion. In the present work we have concentrated on a subset of NLO anomalies, namely those that can be computed with gauge boson triangle diagrams. This results in a series of sum rules which impose constraints on the gauge-fermion interactions at NLO.

It is interesting to remark that anomaly operators at NLO can also contain matter fields. By inspection, one finds dimension-six operator structures of the generic form

$$\tilde{M}_{\mu \nu}^{(1)} B^{\rho \sigma} b, \quad \text{tr} \left[ \tilde{M}_{\mu \nu}^{(3)} W^{\rho \sigma} \right] b, \quad \text{tr} \left[ \tilde{M}_{\mu \nu}^{(8)} G^{\rho \sigma} \right] b,$$

where $\tilde{M}_{\mu \nu}^{(1,3,8)}$ are parity-odd antisymmetric tensors of dimension four built from matter fields. The superscripts denote the representation of the tensor under the relevant SM gauge group. CP-violating phases should also be taken into account.

The sum rules presented in this paper are universally valid, i.e., they are the infrared manifestation of anomaly-free extensions of the SM and guarantee the consistency of the EFT. Practical advantages are the reduction of parameters in phenomenological analyses, e.g., in global fits to LHC data, or correlations between processes with charged lepton currents and the analogous processes with charged quark currents, e.g., using the condition $\varepsilon_{\varepsilon f}^{(3)} = \varepsilon_{\varepsilon f}^{(3)}$. Interestingly, failure to satisfy the sum rules would be a clear indication of the existence of non-decoupling physics that cannot be described within the current EFT formulations, i.e., new degrees of freedom below the TeV scale. The analysis of the full set of anomaly operators
at NLO and the associated sum rules will be addressed in a future publication [37].

Acknowledgments. We thank Thorsten Ohl and Tilman Plehn for useful discussions. This research was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under grant 396021762 – TRR 257.

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