QCD Corrections in Supersymmetric Theories

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I discuss the effects of QCD radiative corrections in Supersymmetric theories. After summarizing the SUSY–QCD lagrangian in the Minimal Supersymmetric extension of the Standard Model, I will discuss the new features introduced by SUSY, and the main complications compared to standard QCD corrections. I will then discuss a few examples of QCD calculations in SUSY theories, for standard processes and for processes involving SUSY particles including the extended Higgs sector. [Talk given at “QCD 97”, Montpellier 3-9 July 1997.]

1. The SUSY–QCD Lagrangian

Supersymmetry (SUSY) predicts the existence of a spin–zero partner for each Standard Model (SM) chiral fermion and a spin–1/2 partner for each gauge boson or Higgs boson \[ \lambda \]. Therefore, as strongly interacting particles one has in addition to gluons and quarks, the gluinos \[ \tilde{g} \] and three generations of left– and right–handed squarks, \[ \tilde{q}_L \] and \[ \tilde{q}_R \]. The interactions between gluon \[ V^\mu \], gluino \[ \lambda \], quark \[ \psi_i \] and scalar quark \[ \phi_i \] fields are dictated by SU(3)C gauge invariance and are given by the Lagrangian

\[ \mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mat}} + \mathcal{L}_{\text{self}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{soft}} \] (1)

There is first the self–interactions of the gauge fields, where in addition to the 3 and 4–gluon vertices that we do not write, there is a term containing the interaction of the gluinos with the gluons \[ \sigma^\mu \] are the Pauli matrices which help to write down things in a two-component notation and \[ f_{abc} \] the structure constants of SU(3)C:

\[ \mathcal{L}_{\text{kin}} = ig f_{abc} \lambda^a \sigma^\mu \lambda^b V^\mu + "3V" + "4V" \] (2)

Then there is a piece describing the interaction of the gauge and matter particles

\[ \mathcal{L}_{\text{mat}} = -g T^a_{ij} V^\mu_{ij} \psi_i \bar{\sigma}^\mu \psi_j 
-ig T^a_{ij} V^\mu_{ij} \phi_i \bar{\sigma}^\mu \phi_j + g^2 (T^a \sigma^b)_{ij} V^\mu_{ij} V^\nu_{ji} \phi_i \phi_j
+ig V \sqrt{2} T^a_{ij} (\lambda^a \phi_i \phi_j - \bar{\lambda}^a \bar{\psi}_i \bar{\psi}_j) \] (3)

Besides the usual term for the gluon–quark interaction and the terms for purely scalar QCD [the derivative term for the gluon–squark interaction and the quartic term for the interaction between two gluons and two squarks] one also has a Yukawa–like term for the interaction of a quark, a squark and a gluino; SUSY imposes that the two coupling constants are the same \( g_v = g \).

There is also a term for the self–interactions between the scalar fields; in the case where squarks have the same helicity and flavor, one has

\[ \mathcal{L}_{\text{self}} = -g^2/3 (\delta^{i j} \delta^{k l} + \delta^{i l} \delta^{j k}) \phi_i \phi_j^* \phi_k \phi_l^* \] (4)

Finally, there are the Yukawa interactions which generate the fermion masses, and the soft–SUSY breaking parameters which give masses to the gaugino and scalar fields and introduce the trilinear couplings \( A_4 \). In the Minimal Supersymmetric Standard Model (MSSM), where two Higgs doublets \( H_1 \) and \( H_2 \) are needed to break the electroweak symmetry \[ \mathcal{L} \], these terms can be written in a simplified way for the first generation as \[ u \] and \( d \) are the left–handed quarks, \( \bar{u} \) and \( \bar{d} \) their partners and \( Q/\tilde{Q} \) the left–handed doublets

\[ \mathcal{L}_{\text{Yuk}} \] = \[ h_u Q H_1 u^c + h_d Q H_2 d^c \] (5)

\[ \mathcal{L}_{\text{soft}} \] = \[-m_\tilde{g}/2 \lambda \lambda + \sum m_\phi^2 \phi_i \phi_i + \cdots \]

\[ + h_u A_u \bar{Q} H_1 \bar{u}^c + h_d A_d \bar{Q} H_2 \bar{d}^c + \cdots \] (6)

Here some assumptions, such as R–parity conservation etc., have been made; for a more detailed [and more rigorous] discussion see Ref. [1].

We are now in a position to discuss the SUSY–QCD corrections to physical processes.
2. New features and complications compared to Standard QCD

When one deals with calculations of QCD corrections in SUSY theories, a few complications compared to standard QCD corrections appear:

- Contrary to their standard partners the gluons, gluinos are massive particles due to the soft breaking of SUSY as discussed previously. In fact, gluinos are rather heavy in most realistic and theoretically interesting models, and from the negative search of these states at the Tevatron a lower bound $m_{\tilde{g}} \geq 150$ GeV has been set on their masses. Light gluinos, which could be produced in 4–jet events at LEP1 seem to be experimentally ruled out. Note also that gluinos are Majorana particles, and some care is needed in handling these states.

- The left– and right–handed current eigenstates $\tilde{q}_L$ and $\tilde{q}_R$, mix to give the mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$. The amount of mixing is proportional to the partner quark mass, and therefore important only in the case of third generation, especially for the top squark. In fact for stops, the mixing can be so large that the lightest $\tilde{t}$ can be much lighter than the $t$ quark and all the other squarks. The mixing can also be important in the $\tilde{b}$ sector for large $\tan \beta$ (the ratio of the vev’s of the two MSSM Higgs doublets) values.

- In standard QCD, the only parameters are the QCD coupling constant $\alpha_s$ as well as the quark masses $m_q$ which in the high–energy limit can be set to zero. In SUSY–QCD, much more parameters are present: besides the $\tilde{q}$ masses [which are different in general] and the $\tilde{g}$ mass, one has the soft–SUSY breaking trilinear couplings $A_q$ as well as the mixing angles $\theta_q$. These parameters are in general related, complicating the renormalisation procedure and making next–to–leading order calculations more involved since one has to deal with loop diagrams involving different particles or with multi–particle final states with several different masses.

- There is also a problem with the regularisation scheme. Indeed, the usual dimensional regularisation scheme which is used in standard QCD, breaks Supersymmetry. For instance the equality between the strong gauge coupling $g$ and the Yukawa coupling $g_Y$ is not automatically maintained at higher orders, and one has to enforce it by adding additional counterterms. In the dimensional reduction scheme, where only the four–vectors and not the Dirac algebra are in $n$–dimension, the equality between the two couplings is maintained automatically and this scheme is therefore more convenient. However, in some cases, gauge invariance can be broken in this scheme and again one has to add extra counterterms to satisfy the Ward identities.

- Finally, there is an additional complication when Higgs bosons are involved. As already mentioned, at least two Higgs doublets are needed in SUSY theories to break the electroweak symmetry, leading to the existence of at least five physical states: two CP–even $h/H$, one CP–odd $A$ and two–charged $H^\pm$ bosons. When calculating QCD corrections for the pseudoscalar Higgs boson $A$, one has to be careful with the treatment of $\gamma_5$ beyond the one–loop level.

3. SUSY–QCD Corrections to Physical Processes

3.1. Standard Processes

I discuss now SUSY–QCD corrections to standard processes, i.e. processes where only standard particles are involved in the initial and final state. Because no direct signal of SUSY particles has been observed directly, it is useful to look indirectly for SUSY in high–precision observables where SUSY loops effects can be important enough to alter the predictions of the SM. Of course, because of the large value of $\alpha_s$, the potentially largest effects are expected to come from corrections involving strong interactions.

One of the simplest cases where SUSY–QCD corrections can be looked for is the cross section for $e^+e^- \rightarrow$ hadrons. In addition to the standard corrections, virtual gluon exchange and gluon emission in the final state, one has also diagrams where squarks and gluinos are exchanged in the loops. Unfortunately, because gluinos and squarks are expected to have masses above 150 GeV, the corrections are rather small at present energies. For instance, for the hadronic width of the $Z$ boson, $\Gamma(Z \rightarrow \tilde{q}\tilde{q})$, the SUSY–QCD correc-
Another measurable where strongly interacting SUSY particles can give large virtual effects is the $\rho$ parameter, defined as the difference between the $W$ and $Z$ boson self-energies at zero momentum transfer. If there is a large splitting between the masses of the squarks which belong to the same weak isodoublet, for instance the $t/b$ doublet, the correction $\Delta \rho$ will grow with the square of the mass of the heaviest particle. This is similar to the SM case, where the $t/b$ doublet generates a correction which grows as $m_{t/b}^2$. $\Delta \rho$ enters all the high-precision measurements such as $\sin^2 \theta_W$ or $M_W$, and one can constrain the masses of squarks by looking at the magnitude of their contributions. To make the constraints more precise, one needs to include QCD corrections. These two-loop corrections consist of diagrams with pure gluon exchange, pure scalar interactions and the gluino–quark–squark exchange diagrams, plus the corresponding counterterms. The calculation has been done recently \cite{12} and it turns out that the QCD correction can be large, reaching the level of 30%. They are in general positive, therefore including the QCD corrections makes the calculation rather involved especially with loops involving gluinos, squarks and quarks large number of Feynman diagrams to consider, and increase with increasing $m_{q\tilde{g}}$; because of the Coulomb singularity, the correction blows up near threshold and non-perturbative effect must be included. For the gluino–exchange contribution, the correction is different for the partners of the light quarks and for the top squarks, because of the large value of $m_t$ and also because of the possible large mixing in the stop sector; it is in general rather small and tend to decrease the cross section.

Finally we have also the SUSY–QCD corrections to the top quark production at hadron colliders, $pp \rightarrow t\bar{t}$ \cite{13}, and for the top quark main decay mode, $t \rightarrow bW^+$ \cite{14}, involving $t\bar{b}g$ virtual exchange. For $t\bar{t}$ production at the Tevatron, the corrections are of $\mathcal{O}(10\%)$ and therefore small; at the LHC, the corrections can be larger but will be difficult to see due the hostile environment at hadronic machines. For the top quark main decay mode, the SUSY–QCD corrections are even smaller, being at best a few percent. Therefore virtual effects of SUSY particles will be also difficult to isolate in this case.

### 3.2. SUSY Processes

Another aspect that I discuss now, is the QCD corrections to processes involving SUSY particles in the final state. As usual, in order to have full control on the theoretical predictions for production cross sections and for decay widths, one needs to include the QCD corrections, which in general turn out to be rather large.

The simplest process in this context, is the SUSY–QCD corrections to the production of scalar quark pairs in $e^+e^-$ annihilation, $e^+e^- \rightarrow \gamma/Z \rightarrow q\bar{q}$. Part of the QCD corrections, the one due to pure gluon exchange and finally gluon emission for equal mass squarks, can be adapted from Schwinger results for scalar QED \cite{15}. But in SUSY theories, one needs to include first the gluino–quark exchange diagrams, and second to consider the case where the two squarks [in both the loops and the final state] have different masses. The calculation has been done by two independent groups \cite{12} and the results can be summarized as follows: for very large $m_{\tilde{g}}$, the gluino decouples and one is left only with the QED–like corrections; the corrections are of $\mathcal{O}(+15\%)$ for small $m_{\tilde{q}}$ [i.e. three times more than for quark pair production], and increase with increasing $m_{\tilde{q}}$; because of the Coulomb singularity, the correction blows up near threshold and non-perturbative effect must be included. For the gluino–exchange contribution, the correction is different for the partners of the light quarks and for the top squarks, because of the large value of $m_t$ and also because of the possible large mixing in the stop sector; it is in general rather small and tend to decrease the cross section.

Another important process where SUSY–QCD corrections are very important is the the production of squark and gluino pairs at hadron colliders, $pp/\bar{p}p \rightarrow g\bar{g}, g\bar{g}, g\tilde{g}$. In this case, one has a large number of Feynman diagrams to consider, with loops involving gluinos, squarks and quarks making the calculation rather involved especially in the case of the $\tilde{t}$ squark where mixing should be included. The calculation has been recently completed \cite{13} and the results are as follows: the theoretical prediction for all processes and for both LHC and Tevatron energies, are nicely stabilized by including the NLO corrections. The
K–factors, \( K = \sigma_{NLO}/\sigma_{LO} \), depend strongly on the considered process. For processes with \( \tilde{q} \) final states, \( pp \to \tilde{g}\tilde{g}, \tilde{q}\tilde{g} \), the corrections are large [up to 90\%] and positive, while for squark pair production they are moderate [up to 30\%]. The corrections exhibit a sizeable dependence on the squark masses. Comparison of the NLO cross sections with those used for experimental studies at the Tevatron reveal that the bounds on gluino and squark masses can be raised by +10 to +30 GeV; for LHC, the shift in mass due to the inclusion of NLO corrections can go up to 50 GeV.

SUSY–QCD corrections to various scalar quark decays are also available. For instance, QCD corrections to the decays of squarks [including \( \tilde{t} \) squarks which needs a special treatment due the mixing and the large value of \( m_{\tilde{t}} \)] into their partners quarks and charginos or neutralinos, \( \tilde{q} \to q\chi^0, q\gamma^\pm \), have been calculated by several groups \(^{14,15}\) and have been found to be rather important since they can reach the level of 30 to 40\%. The SUSY–QCD corrections to the decays of squarks to quarks and gluinos, \( \tilde{q} \to q\tilde{g} \), can be even larger, while they are moderate for the reverse decay \( \tilde{g} \to q\tilde{q} \). All these corrections are positive and increase the decay widths.

Finally, there are also QCD corrections to the SUSY decays of the top quark, \( t \to \tilde{t}_1 \chi^0 \) \(^{16}\), with \( \tilde{t}_1 \) the lightest top squark and \( \chi^0 \) the invisible lightest neutralino. If the decay is kinematically allowed, the QCD corrections increase the decay width significantly.

### 3.3. The Higgs sector

QCD corrections in the MSSM Higgs sector are very important for neutral Higgs boson production at the LHC, and lead to significant effects in the decays of the Higgs bosons into quark or squark pairs, or the decays of top quarks and squarks into Higgs particles. The main production mechanism of the SM neutral Higgs boson at the LHC is the gluon–gluon fusion process, \( gg \to H^0 \), which proceeds mainly through virtual top quark loops \(^{17}\). The two–loop QCD correction leads to a large K–factor, of about \( K \sim 1.6–1.8 \), almost independent of the Higgs mass and stabilizes the theoretical prediction nicely \(^{18}\). In the MSSM, two additional points have to be considered for the production of the neutral Higgs particles in the gluon fusion mechanism.

First, in the standard QCD corrections, one has to include the contribution of the \( b \)-quark whose couplings can be strongly enhanced for large values of \( tg\beta \); one also has to consider the case of the pseudoscalar Higgs boson where subtle problems related to the implementation of \( \gamma_t \) will appear. The K–factors \(^{13}\) also vary little with the Higgs boson mass in general, yet they are strongly dependent on \( tg\beta \): for small \( tg\beta \) their size is approximately as in the SM, \( K \sim 1.7 \), but for large \( tg\beta \) they are in general close to unity except for the \( h \) boson when it is SM–like.

In addition to the standard QCD corrections, one has to include the QCD corrections to the squark [mainly \( \tilde{t} \) loops which could enhance the production cross section significantly for relatively light top squarks, \( m_{\tilde{t}} \lesssim 350 \) GeV. The K–factors in this case \(^{19}\) are almost the same as for the quark contributions and therefore, one can use the standard K–factor to correct the sum of the quark+squark contribution at one–loop. Note that this result is obtained when the gluinos are very heavy and decouple.

These calculations can be applied to the reverse process, the decay of the Higgs bosons into two–gluons \( h, H, A^0 \to gg \). The corrections are very large, increasing the decay widths by approximately 70\%. The SUSY–QCD corrections to the decays of the five MSSM Higgs bosons into quark pairs [the standard QCD corrections have to be supplemented by the gluino-squark exchange contributions] have also been discussed; they turned out to be rather significant, especially for large values of \( tg\beta \); see Ref. \(^{20}\). The QCD corrections to Higgs boson decays into scalar quarks have been also recently completed \(^{21}\): they can be very large, altering the decay widths by an amount which can be larger than 50\%; they are positive and strongly depend on the \( \tilde{g} \) mass.

SUSY–QCD corrections to various decay modes involving MSSM Higgs bosons, such as \( t \to H^+b \) \(^{22}\) and \( \tilde{t}_2 \to \tilde{t}_1 h \) or \( \tilde{t}_1 A \) \(^{22}\), have also been considered and found to be significant.

Last but not least, the QCD corrections to the relations between the MSSM Higgs boson masses
are also very important. These two–loop corrections, for instance, decrease the maximal value of the lightest MSSM $h$ boson mass by an amount of the order of 10 to 20 GeV.

4. Summary

A large theoretical effort has been made in the recent years for the calculation of QCD corrections in Supersymmetric theories. These corrections turned out to be very important for the production and the decays of SUSY particles, including the extended Higgs sector. For processes involving only standard particles, unfortunately, the SUSY–loop effects are in most cases rather small, if squarks and gluinos are too heavy to be directly produced. More work will be still needed in the future on this subject.

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