Correlation of the Sunspot Number and the Waiting-time Distribution of Solar Flares, Coronal Mass Ejections, and Solar Wind Switchback Events Observed with the Parker Solar Probe

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Abstract

Waiting-time distributions of solar flares and coronal mass ejections (CMEs) exhibit power-law-like distribution functions with slopes in the range of \( \alpha_r \approx 1.4-3.2 \), as observed in annual data sets during four solar cycles (1974–2012). We find a close correlation between the waiting-time power-law slope \( \alpha_r \) and the sunspot number (SN), i.e., \( \alpha_r = 1.38 + 0.01 \times SN \). The waiting-time distribution can be fitted with a Pareto-type function of the form \( \mathcal{N}(\tau) = N_0 (\tau_0 + \tau)^{-\alpha_r} \), where the offset \( \tau_0 \) depends on the instrumental sensitivity, the detection threshold of events, and pulse pileup effects. The time-dependent power-law slope \( \alpha_r(t) \) of waiting-time distributions depends only on the global solar magnetic flux (quantified by the sunspot number) or flaring rate, which is not predicted by self-organized criticality or magnetohydrodynamic turbulence models. Power-law slopes of \( \alpha_r \approx 1.2-1.6 \) were also found in solar wind switchback events, as observed with the Parker Solar Probe during the solar minimum, while steeper slopes are predicted during the solar maximum. We find that the annual variability of switchback events in the heliospheric solar wind and solar flare and CME rates (originating in the photosphere and lower corona) are highly correlated.

Unified Astronomy Thesaurus concepts: Solar wind (1534); Solar flares (1496)

1. Introduction

Waiting times, also called elapsed times, interoccurrence times, interburst times, or laminar times, are defined by the time interval between two subsequent events, i.e., \( \tau_r = (t_{i+1} - t_i) \), measured from a time series \( t_1, \ldots, t_n \) of events. Simple examples are (i) periodic processes (where the waiting time is constant and is equal to the time period); (ii) random, Poissonian, or Markov point processes (where the distribution of waiting times follows an exponential function); (iii) exponentially growing avalanche processes (where the waiting-time distribution matches a scale-free power-law-like function, as is common in self-organized criticality [SOC] processes), (iv) magnetohydrodynamic (MHD) turbulence processes (where power spectra can be represented by piecewise power-law functions of waiting-time distributions), or (v) sympathetic flaring, which is a clustering (or memory) effect that is not consistent with independent flaring events (Wheatland et al. 1998). Hence, the study of waiting-time distributions, applied to solar flares here, can be a powerful tool to identify and disentangle the relevant physical processes, in particular in connection with physical scaling laws (Aschwanden 2020).

In solar physics, the waiting-time distribution functions of solar events have been found to be dominantly power-law-like (Boffetta et al. 1999; Wheatland 2000a, 2003; Lepreti et al. 2001; Grigolini et al. 2002; Aschwanden & McTiernan 2010), unless the sample of waiting times covers a too small range or is incompletely sampled otherwise, due to selection effects (e.g., in Pearce et al. 1993; Crosby et al. 1996), as demonstrated in comparison with larger and more complete data sets (Aschwanden & McTiernan 2010). The observationally established result of power-law functions in the waiting-time distribution of solar flares rules out a stationary Poisson process and requires an alternative explanation in terms of nonstationary Poisson processes (Wheatland 2000a, 2003), SOC models (Aschwanden & McTiernan 2010; Aschwanden & Freeland 2012; Aschwanden 2014), or MHD turbulence (Boffetta et al. 1999; Lepreti et al. 2001; Grigolini et al. 2002). It was argued that SOC avalanches occur statistically independently (Bak et al. 1987, 1988) and thus would predict an exponential waiting-time distribution (Boffetta et al. 1999). However, inclusion of a driver with time-dependent variations, such as the solar cycle variability (Aschwanden 2011b), leads to a nonstationary Poisson process with power-law behavior (Wheatland 2000a, 2003). While stationary and nonstationary Poisson processes are mutually exclusive, SOC and MHD turbulence are not. In fact, SOC signatures can be a manifestation of intermittent turbulence (Klimas et al. 2010; Sharma et al. 2016, and references therein). In this paper we study the time variability of the power-law slope of waiting times in more detail and find a strong correlation between the level of solar activity in terms of the sunspot number SN(t), the annual flaring rate \( \lambda(t) \), and the power-law slope \( \alpha_r(t) \) of the waiting-time distribution, which can be explained by their common magnetic drivers. Apparently, as we are going to demonstrate in the remainder of the paper, the global magnetic flux (as quantified by the sunspot number) could be the most dominant physical process that modulates the value of the power-law slope \( \alpha_r(t) \) in solar flare waiting-time distributions, an argument that has not received much attention (except in Wheatland & Litvinenko 2002; Wheatland 2003), but it puts previous modeling of waiting-time distributions into a new light.

New results emerge also from the variability of the solar wind, especially from the renewed interest in switchback events, as observed by the Parker Solar Probe (PSP) mission. Switchbacks are sudden deflections of the magnetic field that have been found to be ubiquitous in the inner heliosphere. These events are likely to play an important role in structuring the young solar wind
Table 1
Waiting-time Distributions Measured from Solar Flares’ Hard X-Ray Events, Soft X-Ray Events, Coronal Mass Ejections, and Radio Bursts

| Observation Year of Events | Observation Spacecraft or Instrument | Number Range | Waiting Time $\tau$ | WTD | Power Law | Power Law References |
|---------------------------|-------------------------------------|--------------|---------------------|-----|----------|---------------------|
| 1980–1985                 | HXRBS/SMM                           | 8319         | 0.01–1 hr           | PL  | 0.75 ± 0.1 | Pearce et al. (1993) |
| 1991–2000                 | BATSE/CGRO                          | 6596         | 0.03–7 hr           | E   | Biesecker (1994) |
| 1990–1992                 | WATCH/GRANAT                        | 182          | 0.17–5 hr           | PE  | 0.78 ± 0.13 | Crosby et al. (1996) |
| 1978–1986                 | ICE/ISEE-3                          | 6916         | 0.01–20 hr          | DE  | Wheatland et al. (1998) |
| 1980–1989                 | HXRBS/SMM                           | 12,772       | 0.01–500 hr         | PL  | 2.0       | Aschwanden & McTiernan (2010) |
| 1991–1993                 | BATSE/CGRO                          | 4113         | 0.01–200 hr         | PL  | 2.0       | Aschwanden & McTiernan (2010) |
| 1991–2000                 | BATSE/CGRO                          | 7212         | 1–5000 hr           | PL  | 2.14 ± 0.01 | Grigolini et al. (2002) |
| 2002–2008                 | RHESSI                              | 11,594       | 2–1000 hr           | PL  | 2.0       | Aschwanden & McTiernan (2010) |
| 1975–1999                 | GOES 1–8 Å                          | 32,563       | 1–1000 hr           | PL  | 2.4 ± 0.1  | Boffetta et al. (1999) |
| 1996–2001                 | SOHO/LASCO                          | 4645         | 1–1000 hr           | PL  | 2.16 ± 0.05 | Wheatland (2000a), Lepreti et al. (2001) |
| 1996–1998                 | SOHO/LASCO                          | ...          | 1–1000 hr           | PL  | 2.26 ± 0.11 | Wheatland (2003) |
| 1999–2001                 | SOHO/LASCO                          | ...          | 1–1000 hr           | PL  | 1.75 ± 0.08 | Wheatland (2003) |
| 1975–2001                 | GOES 1–8 Å                          | ...          | 1–1000 hr           | PL  | 3.04 ± 0.19 | Wheatland (2003) |
| Solar min                 | GOES 1–8 Å                          | ...          | 1–1000 hr           | PL  | 2.1 ± 0.01  | Wheatland & Litvinenko (2002) |
| Solar max                 | GOES 1–8 Å                          | ...          | 1–1000 hr           | PL  | 3.2 ± 0.3   | Wheatland & Litvinenko (2002) |
| 1996–2001                 | SOHO/LASCO                          | 4645         | 1–1000 hr           | PL  | 2.36 ± 0.11 | Wheatland (2003) |
| 1996–1998                 | SOHO/LASCO                          | ...          | 1–1000 hr           | PL  | 1.86 ± 0.14 | Wheatland (2003) |
| 1999–2001                 | SOHO/LASCO                          | ...          | 1–1000 hr           | PL  | 2.98 ± 0.20 | Wheatland (2003) |
| 2018                      | PSP                                 | ...          | $10^{-6}$–$10^{7}$ hr | PL  | 1.4–1.6  | Dudok de Wit et al. (2020) |
| 2018                      | PSP                                 | ...          | $10^{-6}$–$10^{7}$ hr | PE  | 1.21 ± 0.01 | This work |

Note. The waiting-time distribution (WTD) functions are abbreviated as follows: PL = power law; E = exponential; PE = power law with exponential cutoff; DE = double exponential.

(Mozzer et al. 2020; Dudok de Wit et al. 2020; Horbury et al. 2020; Tenerani et al. 2020). Their origin, however, remains elusive.

The structure of the paper entails observations and data analysis in Section 2, discussions in Section 3, and conclusions in Section 4.

2. Observations and Data Analysis

2.1. Previous Studies of Solar Flare Hard X-Rays

In Table 1 we compile power-law slopes $\alpha_\tau$ of published waiting-time distributions of solar flares, which we briefly describe in turn.

Pearce et al. (1993) determine a total of 8319 waiting times (within a time range of $\tau = 1–60$ minutes during the observation period of 1980–1985), using data from the Hard X-ray Burst Spectrometer (HXRBS) on board the Solar Maximum Mission (SMM). Similarly, Biesecker (1994) detects a total of 6596 waiting times, using Burst And Transient Source Experiment (BATSE) on board the Compton Gamma Ray Observatory (CGRO). A much smaller data set with 182 waiting times was obtained from the Wide Angle Telescope for Cosmic Hard X-rays (WATCH) on board the GRANAT satellite (Crosby et al. 1996). More extended statistics in hard X-ray wavelengths was gathered from the International Cometary Explorer (ICE) on board the International Sun/Earth Explorer (ISEE-3) (Wheatland et al. 1998), HXRBS/SMM (Aschwanden & McTiernan 2010), BATSE/CGRO (Grigolini et al. 2002; Aschwanden & McTiernan 2010), and RHESSI (Aschwanden & McTiernan 2010).

These studies are all based on hard X-ray solar flare catalogs, where waiting times are found within a range of $\tau \approx 0.01–1000$ hr (see time ranges in Table 1), detected above a typical flux threshold of $F_{\text{HRX}} \gtrsim 100$ counts s$^{-1}$ at hard X-ray energies of $E_{\text{HRX}} \gtrsim 25$ keV. Most waiting-time distributions are self-similar (or scale-free) and consequently exhibit a power-law scaling. We find only two exceptions: one is an exponential case, and another is a double-exponential case; see Column (5) in Table 1. Thus, most of the waiting-time distributions of hard X-ray data can be fitted with a power-law function, with a typical slope of $\alpha_\tau \approx 2.0$ (see Column (6) in Table 1), except for two cases with relatively small waiting-time ranges. These two cases exhibit unusually low values of $\alpha_\tau < 0.8$ (Pearce et al. 1993; Crosby et al. 1996), because the fitted range is too narrow and suffers from incomplete sampling (Aschwanden & McTiernan 2010). Hence, we discard these two dubious cases in the following analysis.

2.2. Previous Studies of Solar Flare Soft X-Rays

More waiting-time distributions of solar flares were sampled in soft X-ray wavelengths, making use of the 1–8 Å flux data observed with the Geostationary Orbiting Earth Satellite (GOES), which yields an uninterrupted time series of up to 47 yr (from 1974 to present). Subsets of GOES time series in different year ranges were analyzed by Boffetta et al. (1999), Wheatland (2000a, 2003), and Lepreti et al. (2001). These data are ideal for waiting-time statistics, since the duty cycle of GOES data is very high (94%) thanks to the geostationary orbit. The number of solar flares above a threshold of the C-class level ($10^{-6}$ W m$^{-2}$) amounts to 35,221 events, while a deep survey with automated flare detection and with $\approx 10$ times higher sensitivity revealed a total of 338,661 events (Aschwanden & Freeland 2012). This data set provides the largest statistics of waiting times and is investigated in Section 2.5. All waiting-time distributions could be fitted with a “thresholded” power-law...
distribution (Aschwanden 2015), also called the Pareto function (Equation (6)), or Lomax function (Lomax 1954; Hosking & Wallis 1987), which yields here power-law slopes in the range of \(\alpha_T \approx 1.4–3.2\) (see Column (6) in Table 1). Note that these results encompass a large range of power-law slopes that needs to be explained by any theoretical model. Moreover, the formal errors or uncertainties in the power-law slopes are typically \(\sigma_T \approx 0.1–0.2\), which is much smaller than the spread of the slope values, \(\alpha_T \approx 1.4–3.2\), and thus cannot be explained with a single theoretical constant.

2.3. Previous Studies of Coronal Mass Ejections

There is a general consensus now that solar flare events and coronal mass ejections (CMEs) have very high mutual association rates, especially for eruptive flare events, so that the event statistics of one event group can be used as a proxy of the other event group. Hence, we expect that the waiting-time distributions of solar flares and CMEs are similar. Wheatland (2003) sampled the waiting-time distributions of CMEs based on Large Angle Solar Spectrometric Coronagraph (LASCO) data from the Solar and Heliospheric Observatory (SOHO) spacecraft. These waiting-time distributions could be fitted with thresholded power-law (or Pareto) functions also, with power-law slopes in the range of \(\alpha_T \approx 1.9–3.0\), sampled in different time phases of the solar cycle (see Column (6) in Table 1). It turns out that the flare waiting times produce similar power-law slopes \(\alpha_T\) to the CME waiting times, if one compares data from the same phase of the solar cycle.

2.4. Solar Cycle Dependence of Waiting Times

Wheatland (2003) examined the distribution of waiting times between subsequent CMEs in the LASCO CME catalog for the years 1996–2001 and found a power-law slope of \(\alpha_T \approx 2.36 \pm 0.11\) for large waiting times (at \(\geq 10\) hr). Wheatland (2003) noted that the power-law index of the waiting-time distribution varies with the solar cycle: for the years 1996–1998 (a period of low activity), the power-law slope is \(\alpha_T \approx 1.86 \pm 0.14\), and for the years 1999–2001 (a period of higher activity), the slope is \(\alpha_T \approx 2.98 \pm 0.20\). Wheatland (2003) concluded that the observed CME waiting-time distribution, as well as its variation with the solar cycle, may be understood in terms of CMEs occurring as a time-dependent Poisson process. The same can be said for solar flares, since flares and CMEs are highly correlated.

In order to prove this hypothesis, we investigate the functional relationship between the waiting-time power-law slope \(\alpha_T\) and the level of solar activity, which we quantify by the sunspot number SN. We list the mean annual sunspot number (from the Royal Observatory in Belgium, www.sidc.be/silso/) in Table 2 for the years of 1974–2012. Then, we average the annual sunspot numbers over the range of years that correspond to each data set given in Table 1: 1980–1989 (HXRBSS/MM; Aschwanden & McTiernan 2010), 1991–1993 (BATSE/CGRO; Aschwanden & McTiernan 2010), 1991–2000 (BATSE/CGRO; Grigoli et al. 2002), 2002–2008 (RHESSI; Aschwanden & McTiernan 2010), 1975–1999 (GOES; Boffetta et al. 1999; Wheatland 2000a; Lepreti et al. 2001), 1996–2001 (GOES, LASCO/SOHO; Wheatland 2003), 1996–1998 (GOES, LASCO/SOHO; Wheatland 2003), 1999–2001 (GOES, LASCO/SOHO; Wheatland 2003), 2018 (PSP; Dudok de Wit et al. 2020). Then, we plot the power-law slopes \(\alpha_T\) as a function of the annually averaged sunspot numbers SN in Figure 1 and find a linear relationship of

\[ \alpha_T = 1.38 + 0.01 \times SN, \]  

(1)
as confirmed by the high value of the Pearson’s cross-correlation coefficient (CCC = 0.987). The sunspot number varies from SN \(\approx 40\) during the solar cycle minimum to SN \(\approx 160\) during the solar cycle maximum. Interestingly, this result is robust in the sense that it predicts the same linear relationship even when different event detection thresholds, different phenomena (flares, CMEs, solar wind switchbacks), and different wavelengths (hard X-rays, soft X-rays) are used. This empirical linear relationship explains a variety of observed power-law slopes for waiting times, covering a range of \(\alpha_T \approx 1.4–3.2\) (Figure 1).

2.5. Annual GOES Waiting-time Statistics

The published power-law slopes mentioned in the previous section mostly cover multiflare ranges. In the following task we examine the degree of correlation between the annual power-law slopes of waiting times and the annual sunspot numbers from the entire 47 yr data set of GOES flares. We break the data sets down into 39 annual groups (from 1974 to 2012), for the same time epoch as we used to perform automated flare detection in a previous study (Aschwanden & Freeeland 2012). For each of the 39 data sets (Figure 2) we fit a thresholded power-law (or Pareto-type) distribution (Equation (6)), parameterized with \((N_0, \tau_0, \alpha_{Tm})\).

The published power-law slopes \(\alpha_T\) are usually fitted in the inertial range \((x_0, x_2)\) and have a mean value that corresponds to the (logarithmic) midpoint \(x_3 \approx (x_0 + x_2)/2\) of the waiting-time distribution. In contrast, the fitting of a power-law function plus an exponential cutoff has a bias to yield steeper power-law slopes \(\alpha_{Tm}\) at the upper end \(x_2\), where the slope is steepest. In order to make the two methods compatible, we can define an empirical correction factor \(q_{mm}\), for which we find an average value of \(q_{mm} \approx 0.62\),

\[ \alpha_T = q_{mm} \alpha_{Tm} \approx 0.62 \alpha_{Tm}. \]  

(2)

Applying this correction to the upper limits \(\alpha_{Tm}\) (crosses in Figure 3(a)), the corrected power-law slopes \(\alpha_T\) (crosses in Figure 3(b)) become fully compatible with the published power-law values \(\alpha_T\) (diamonds in Figures 1, 3(a), and (b)) based on standard power-law fitting methods in the observed inertial ranges.

We present the fits of the Pareto distribution functions (Equation (6)) in Figure 2 and list the years, the number of events \(n_{eve}\), the power-law slope \(\alpha_T\) of the waiting times, the goodness-of-fit \(\chi^2\), the lower \(x_0\) and upper \(x_2\) bounds of the fitted range, the decades of the inertial range \(\log(x_2/x_0)\), and the annual sunspot number SN in Table 2.

Plotting the time evolution of the power-law slope \(\alpha_T(t)\) in annual steps (Figure 4(a)), we see a solar cycle variation that is similar to the time variation of the annual sunspot number SN(t) (Figure 4(b)), or the flaring rate per year (Figure 4(c)). Fitting a linear regression between the two parameters, we find a linear relationship of \(\alpha_T = 1.442 + 0.009 \times SN\) (Figure 5(a)), with a correlation coefficient of CCC = 0.961. This correlation corroborates the result of a linear relationship between the power-law slope and the sunspot number from previous work (Figure 1).

The value of the power-law slope \(\alpha_T\) after the inertial range correction, appears not to depend on event detection thresholds,
efficiency of event detection, and pulse pileup effects. For long-duration flare events, short flares could pile up on long tails, violating the separation of timescales, and possibly steepening the power-law slope of waiting times. Our automated flare detection is about 10 times more sensitive than the standard wind streams within a distance of 50 solar radii from the Sun. The omnipresence of so-called switchback events in Alfvénic solar time distributions are in the inertial range of the solar wind: the sampled waiting-time distribution covers a range of $\tau \approx 10^{-8}$ to $10^{6}$ hr. The power-law slope varies in a range of $\alpha_{\tau} \approx 1.4$–1.6 (Dudok de Wit et al. 2020).

Here we revisit these results by considering a new data set of $n_{ev} = 19,543$ individual switchback events that have been automatically selected by feature recognition using magnetic field data sampled at 0.1 s. Switchback events are defined here as a step function in the deflection of the magnetic field with respect to the orientation of the Parker spiral. These events are required to be aligned for some duration before and after the event. Waiting times with $\tau < 10$ s are believed to be biased by smaller events, and waiting times with $\tau < 2$ s are increasingly polluted by ion cyclotron waves and are discarded anyway. Let us stress that unlike with flare events, in which large flares can...
be followed by smaller ones, no switchbacks can occur during long switchback events. For that reason the long tails of the distributions may differ, because the separation of timescales (waiting times, flare durations) is violated.

We show the waiting-time distribution in Figure 6, fitted with two theoretical models (see Section 3.1), with (i) the Pareto distribution that is close to a power-law function at large waiting times (Figure 6(a)) and (ii) a Pareto plus an exponential cutoff (Figure 6(b)). The latter model yields a superior fit (Figure 6(b)), with a power-law slope of \( \alpha = 1.21 \pm 0.01 \) and a goodness-of-fit \( \chi = 2.21 \), which indicates that a power law with an exponential cutoff is a more realistic description of the waiting-time distribution.

3. Discussion

3.1. Pareto-type Waiting-time Distribution Function

The statistics of waiting times bears information that enables us to discriminate between two statistical distribution functions: (i) random processes with Poissonian noise, and (ii) clustering of events within individual intervals (such as Omori’s law for earthquakes, which exhibits precursors and aftershock events during a major complex earthquake event). A Poissonian process in the time domain is a sequence of randomly distributed and thus statistically independent events, producing a waiting-time distribution of time intervals \( \tau \) that follows an exponential function,

\[
N(\tau) \, d\tau = \lambda_0 \exp(-\lambda_0 \, \tau) \, d\tau, \tag{3}
\]

where \( \lambda_0 = 1/\tau_0 \) is the mean flaring rate or mean reciprocal waiting time and \( N(\tau) \) is the probability density function (or differential occurrence frequency). Therefore, if individual events are produced by a physical random process, the occurrence frequency of waiting times should follow such an exponential-like function (Equation (3)).

In reality, however, almost all observed waiting-time distributions of solar flare events exhibit a power-law-like distribution for the probability function \( N(\tau) \), rather than an exponential function. In an attempt to match this observational constraint, a nonstationary flare occurrence rate \( \lambda(t) \) was defined that varies as a function of time in piecewise time intervals or Bayesian blocks (Wheatland et al. 1998; Wheatland 2000a, 2001, 2003). Some examples of such nonstationary Poisson processes are given in Aschwanden & McTiernan (2010) and Aschwanden (2011a, Section 5.2). For instance, it includes an exponentially growing (or decaying) flare rate \( \lambda(t) \) that produces a Pareto-type distribution for the waiting-time distribution with a power-law slope of 3,

\[
N(\tau) \, d\tau = \frac{2\lambda_0}{(1 + \lambda_0 \tau)^3} \, d\tau. \tag{4}
\]

It includes also a flare rate that varies highly intermittently in the form of \( \delta \)-functions, which produces a Pareto distribution also, but with a different power-law slope of 2,

\[
N(\tau) \, d\tau = \frac{\lambda_0}{(1 + \lambda_0 \tau)^2} \, d\tau. \tag{5}
\]

In order to generalize these two solutions (Equations (4) and (5)) into a single function, we can define a Pareto-type distribution with a variable power-law slope \( \alpha_{\tau} \), as parameterized in Equations (1) and (2),

\[
N(\tau)d\tau = N_0(\tau_0 + \tau)^{-\alpha_{\tau}} \exp\left(-\frac{\tau}{\tau_e}\right) d\tau, \tag{6}
\]

which yields a superior fit, as shown in the case of solar wind switchbacks (Figure 6(b)). This equation fully describes the observed waiting-time distributions (as shown in Figures 2 and 6), expressed by five parameters \( (N_0, \tau_0, \tau_e, \alpha_{\tau}, SN) \). The mean waiting time \( \tau_0 \) approximately represents the lower bound of the inertial range and separates the range of incompletely sampled events (see Figure 6(b)) from the scale-free power-law range of completely sampled events. The waiting time \( \tau_e \) demarcates the \( \delta \)-folding cutoff due to finite system size effects. \( \tau_0 \) is an offset that depends on the instrumental sensitivity, the flux threshold in the automated detection of waiting times, and pulse pileup effects. For long-duration flare events, short flares may break up long waiting times into smaller waiting times, which steepens the power-law slope. Since the sunspot number \( SN \) is an observable that is known from earlier centuries up to today (Table 2), there are only four free variables left \( (N_0, \tau_0, \tau_e, \alpha_{\tau}) \), which can be derived empirically by fitting Equation (7) to an observed data set. Note that the maximum power-law slope \( \alpha_{\tau} \) relates to the mean power-law slope \( \alpha_{SN} \) by a correction factor given in Equation (2).

The fact that most waiting-time distributions exhibit a power-law-like function, rather than an exponential-like function, clearly requires a nonstationary Poisson process (Wheatland et al. 1998; Wheatland 2000a, 2001), which implies that the flaring rate \( \lambda(t) \) has a substantial time variability. The time variability of the mean flaring rate has been found to be highly correlated with the annual sunspot number (Figure 4) and thus is modulated by Hale’s magnetic cycle of solar activity. Apparently, the mean waiting time between solar flares depends on the global magnetic flux (quantified by the sunspot number), but not on the flare size (or...
Figure 2. Annual waiting-time distributions of GOES 1–8 Å soft X-ray fluxes for the years 1974–2012 (histograms), with least-squares fits of Pareto distributions (thick solid curves) and corresponding power-law slopes $a$ (dotted diagonal lines).
peak count rate) according to observations, in contrast to theoretical expectations of energy storage models (Rosner & Vaiana 1978; Wheatland 2000b).

3.2. Flare Model of Waiting Times

All fitted waiting-time distributions shown in Figure 2 are modulated by annual variations of the solar cycle. On shorter (than annual) timescales, the flaring rate varies also, as statistics based on Bayesian-block decomposition reveal (Wheatland et al. 1998; Wheatland 2000b, 2001, 2003; Wheatland & Litvinenko 2002). A flare model that possibly could explain the waiting-time distribution function was proposed (Wheatland & Litvinenko 2002; Wheatland & Craig 2006), based on the assumptions of (i) Alfvénic timescale for crossing the magnetic reconnection region, (ii) 2D geometry of the reconnection region (or separatrix), and (iii) correlation of flare energy build-up (or storage) and flare waiting time. This model, however, is not consistent with observations, which show no correlation between flare sizes and flare waiting times (Crosby et al. 1996; Wheatland 2000b; Georgoulis et al. 2001), not even between subsequent flares of the same active region (Crosby et al. 1996; Wheatland 2000b). Consequently, such theoretical flare energy storage models (Rosner & Vaiana 1978; Lu 1995) have been abandoned. A more likely model involves interchange reconnection between coronal loops and open magnetic fields (Zank et al. 2020).

3.3. Self-organized Criticality Models

The fractal-diffusive avalanche model of a slowly driven self-organized criticality (FD-SOC) system (Aschwanden 2012, 2014; Aschwanden & Freeland 2012; Aschwanden et al. 2016), expanded from the original version of Bak et al. (1987, 1988), is based on a scale-free (power-law) size distribution function of avalanche (or flare) length scales $L$, 

$$N(L) \, dL \propto L^{-d} \, dL,$$

(8)

with $d$ the Euclidean spatial dimension (which can have values of $d = 1, 2, \text{or } 3$). This reciprocal relationship between the
spatial size $L$ of a switchback structure and the occurrence frequency $N(L)$ is visualized in Figure 7, for the case of a space-filling avalanche mechanism, but it holds for rare events in terms of relative probabilities also.

The transport process of an avalanche is described by classical diffusion according to the FD-SOC model, which obeys the scaling law,

$$L \propto T^{3/2},$$

(9)

with $\beta = 1$ for classical diffusion. Substituting the length scale $L \propto T^{3/2}$ with the duration $T$ of an avalanche event, using Equations (8)–(9) and the derivative $dL/dT = T^{3/2-1}$, predicts a power-law distribution function for the size distribution of time durations $T$,

$$N(T) \frac{dT}{d\tau} = N(T[L]) \left( \frac{dL}{dT} \right) dT = T^{-1+(d-1)3/2} dT = T^{-\alpha_T} dT \approx T^{-2} dT,$$

(10)

for $d = 3$ and $\beta = 1$, defining the waiting-time power-law slope $\alpha_T$,

$$\alpha_T = 1 + (d - 1)\frac{\beta}{2} = 2.$$

(11)

We can now estimate the size distribution of waiting times by assuming that the avalanche durations represent upper limits to the waiting times $\tau$ during flaring time intervals, while the waiting times become much larger during quiescent time periods. Such a bimodal size distribution with a power-law slope of $\alpha_{\tau} \leq 2$ at short waiting times ($\tau \leq \tau_c$) and an exponential-like cutoff function at long waiting times ($\tau \geq \tau_c$)

Figure 5. Linear regression fits of the waiting-time power-law slope $\alpha_{\tau}$ vs. (a) the sunspot number $SN$ and (b) the annual flare rate $N_{av}$ (solid line), measured from automated flare detections of GOES flares in annual time intervals (diamonds).

Figure 6. Waiting-time distributions of 19,452 magnetic field switchback events (histograms) observed with PSP. The observed distributions are fitted with two theoretical models: (a) the Pareto distribution model (PM), and (b) the Pareto distribution with an exponential cutoff (PF model). Note that the best fit favors the PF model (panel (b)) with a power-law slope of $\alpha_{\tau} = 1.21 \pm 0.01$ and a goodness-of-fit $\chi^2 = 2.21$.

\[ \frac{dL}{dT} = T^{3/2-1} \]
Thus, this FD-SOC model predicts a power law with a slope of $\alpha_t \approx 2.0$ in the inertial range and a steepening cutoff function at longer waiting times. Observationally, we find a range of $\alpha_t \approx 1.4$–$3.2$, varying as a function of the solar activity, but the solar cycle modulation is not quantified in any SOC model (Aschwanden 2019b).

3.4. MHD Turbulence Processes

Boffetta et al. (1999) argue that the statistics of solar flare (laminar or quiescent) waiting times indicate a physical process with complex dynamics with long correlation times, such as in chaotic models, in contradiction to stationary SOC models that predict Poisson-like statistics. They consider chaotic models that include the destabilization of the laminar phases and subsequent restabilization due to nonlinear dynamics, as invoked in their shell model of MHD turbulence.

Similarly, Lepreti et al. (2001) attribute the origin of the observed waiting-time distribution to the fact that the physical process underlying solar flares is statistically self-similar in time and is characterized by a certain amount of “memory.” They find that the power-law distribution can be modeled by a Lévy function that can explain a power-law exponent of $\alpha_t = 3$ (Equation (4)) in the waiting-time distribution.

Grigolini et al. (2002) develop a technique called the diffusion entropy method to reproduce the observed waiting-time distribution function, which evaluates the entropy of the diffusion process generated by the time series. Note that classical diffusion (Equation (9)) has been employed in SOC models (Aschwanden 2012), which may be related to the diffusion entropy method, since both models produce a similar scaling of short waiting times, with power-law slopes in the range of $\alpha_t \approx 1.4$–$3.2$ (Table 1). The change of the power-law index from $\alpha_t > 3$ to $\alpha_t < 3$ has been interpreted in terms of a phase transition from the Gaussian to the Lévy basin of attraction (Grigolini et al. 2002).

3.5. Switchback Events Observed with the Parker Solar Probe

The novel phenomena of the so-called switchback events were sampled in situ with PSP in the solar wind at a distance of $\approx 36R_\odot = 0.166$ au. Switchback events have durations of less than 1 s to more than an hour. Hallmarks of switchback events are reversals in the radial field component $B_r$ (with respect to the Parker spiral geometry), which can produce deflection angles from a few degrees to nearly $180^\circ$ in the fully antisunward direction (Dudok de Wit et al. 2020). A switchback event can be quantified either by the magnetic potential energy,

$$ E_p = -\mathbf{B} \cdot \mathbf{B}, $$

or by the normalized deflection angle $\mu$,

$$ z = \frac{1}{2} (1 - \cos \mu) \quad \text{for} \quad 0 \leq z \leq 1. $$

Moreover, we can sample the waiting times $\tau_i = (t_{i+1} - t_i)$ of subsequent events, which resulted in a power-law-like inertial range of $\tau \approx 10$–$500$ s, an exponential cutoff at $\tau \approx 500$–$2000$ s,
and an undersampled range at $\tau \approx 0.1\text{--}10$ s (Figure 6(b)). The power-law slope of the waiting-time distribution, $\alpha_f = 1.21 \pm 0.01$ (Figure 6), is measured in the year 2018, close to the minimum of the solar cycle, and follows the same trend as solar flares and CMEs (Figure 1).

This close similarity of the slopes obtained with solar flares and with switchbacks is intriguing and could be the signature of common drivers. However, as of today the origin of switchbacks is unclear. They may be generated either locally in the upper solar corona or by instabilities such as plasma jets occurring much deeper in the corona. In addition, there are also several differences in the way flares and switchbacks are registered: the flaring rate, for example, includes independent and sympathetic flares occurring everywhere on the solar disk and at the limb, while switchback events are recorded at one single point in space only, along the satellite orbit. In addition, the impact of local solar wind conditions and the relative speed of PSP on the rate of switchback events still has to be properly investigated. Therefore, while the similarity of the waiting-time distributions is likely to be deeply rooted in the underlying physical processes, it is premature to conclude about the connection between the two types of events.

Long-term memories, expressed by the residence time of switchback events, are then expected to scale with the flare or CME duration $T$, which is predicted from SOC models to follow a power-law distribution function of $N(T) \propto T^{-2}$ (Equation (10)) and a proportional distribution of $N(\tau) \propto \tau^{-2}$ for short waiting times (Equation (12)). Both the waiting time $\tau$ and the residence time $T$ distributions of these switchback deflections tend to follow a power law and are remarkably similar (Dudok de Wit et al. 2020). The long memory we observe is most likely associated with the strong spatial connection between adjacent magnetic flux tubes and their common photospheric footpoints (Dudok de Wit et al. 2020). Consequently, it has been proposed that switchback events are modulated by impulsive flare (or CME) events in the lower corona (Roberts et al. 2018; Tenerani et al. 2020; Zank et al. 2020).

4. Conclusions

In this study we investigate the statistics of waiting-time distributions of solar flares, CMEs, and solar wind switchback events. The motivation for this type of analysis method is the diagnostics of stationary and nonstationary Poissonian random processes, SOC systems, and MHD turbulence systems. The observational analysis is very simple, since only an event catalog with the starting times $t_i$ of the events is necessary to sample waiting times $\tau = (t_{i+1} - t_i)$. We obtain the following results:

1. Using the statistics of hard X-ray solar flares (using flare catalogs from HXRBS/SMM, BATSE/CGRo, WATCH, ICE/ISEE-3, and RHESSI), we find power-law distribution functions with slopes in the range of $\alpha_f \approx 1.5\text{--}3.2$, following a linear regression fit of $\alpha_{pm} = 1.38 \pm 0.01 \times \text{SN}$ and a cross-correlation coefficient of $\text{CCC} = 0.987$ (Figure 1). This trend clearly indicates that the waiting-time power-law slope $\alpha_f$ is foremost correlated with the sunspot number (or the flaring rate), which is fully consistent with previous findings (Wheatland & Litvinenko 2002; Wheatland 2003).

2. Using the statistics of soft X-ray flares, sampled by GOES over 47 yr in annual intervals, but with a 10 times higher sensitivity, we perform fits with a Pareto-type distribution function (Figure 2), which consists of an inertial (power-law) range, an exponential cutoff range, and a range of undersampling. The fits clearly show power-law slopes that are modulated by the four solar cycles, strongly correlated with the annual sunspot number.
number and the annual flaring rate (Figures 4, 5), consistent with the hard X-ray flare results.

3. We sample 19,452 magnetic field switchback events from data observed with PSP and find a power-law slope of \( \alpha_r = 1.21 \pm 0.01 \). A theoretical value of \( \alpha_r = 2.0 \) is predicted for short waiting times (\( \tau \lesssim 500 \text{ s} \)) by an SOC model during contiguous flaring time episodes, while an exponentially dropping cutoff is expected for long waiting times (\( \tau \approx 500-2000 \text{ s} \)). The theoretical value of \( \alpha_r = 2.0 \), however, applies only to the regimes where flare durations approximately last as long as the flare waiting times, which is the case in slowly driven SOC systems with separation of timescales and negligible pulse pileups (Aschwanden & McTiernan 2010; Aschwanden et al. 2016). Nevertheless, we propose that a realistic waiting-time distribution contains a power-law part for short waiting times and an exponential part for long waiting times (Equation (12)), which reflects the duality between flaring and quiescent episodes, corresponding to a combination of stationary and nonstationary Poissonian components, which can be modeled with piecewise Bayesian time intervals (e.g., Wheatland & Litvinenko 2002).

4. Although the four theoretical models discussed here (nonstationary Poissonian, stationary Poissonian, SOC, and MHD turbulence) can all explain some partial aspects of the observed waiting-time distribution functions (exponential, power law), none of them predict the most dominating parameter, namely, the time-dependent modulation of the magnetic solar cycle, which can be modeled in terms of the flaring rate or sunspot number. This result suggests that the variability observed in the solar wind is intimately related to the variability of the flare rate and CME rate, which are both observed in the lower corona. One possibility, among others, is that the occurrence times and rates of all these events could be preconditioned by the photospheric flows and the associated free energy buildup in the corona (Uritsky & Davila 2012). Further conceptual uncertainties in data modeling include undersampling and thresholding in automated event detection, pulse buildup of small events in the wake of larger events, Bayesian interval selection, distinction of flaring and quiescent episodes, and exponential cutoff at maximum waiting times. A Pareto-type function with an exponential cutoff appears to be the most appropriate choice to fit the observed waiting-time distributions.

Waiting-time statistics has been applied to many nonlinear phenomena. While we deal here with three phenomena only (solar flares, CMEs, solar wind switchback events), other analyzed data include earthquakes (Omori’s law), auroral emission, substorms in magnetosphere, solar radio bursts, stellar flares (Aschwanden et al. 2019a; Aschwanden & Güdel 2021), and black hole accretion disks (see Aschwanden 2011a; Aschwanden et al. 2016, and references therein).

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