Stable $\text{uudd}s$ pentaquarks in the constituent quark model

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The stability of strange pentaquarks $\text{uudd}s$ is studied in a constituent quark model based on a flavor-spin hyperfine interaction between quarks. With this interaction model, which schematically represents the Goldstone boson exchange interaction between constituent quarks, the lowest lying strange pentaquark is a $p$−shell state with positive parity. The flavor-spin interaction lowers the energy of the lowest $p$−shell state below that of the lowest $s$−shell state, which has negative parity because of the negative parity of the strange antiquark. It is found that the strange pentaquark is stable against strong decay provided that the strange antiquark interacts by a fairly strong spin-spin interaction with $u$ and $d$ quarks. This interaction has a form that corresponds to $\eta$ meson exchange. Its strength may be inferred from the $\pi^0$ decay width of $D^*_s$ mesons.

Renewed interest in the existence of pentaquarks [1,2] has been raised by the recent observation of an $S = 1$ baryon resonance in photo-production of kaon pairs on neutrons: $\gamma n \rightarrow K^+K^-n$ [3]. This resonance has a peak at $1.54 \pm 0.01$ GeV/$c^2$ and a width, which is less than 25 MeV/$c^2$. It may be interpreted as a strange meson-baryon resonance or as a pentaquark of the form $\text{uudd}s$.

The expectation has been that stable pentaquarks should be likely to exist in the heavy flavor sectors [1,2,4,5], but experimental searches have remained inconclusive [6,7]. A constituent quark model study of pentaquark states of the form $qqqqs$, indicates that such states are unstable against strong decay if the only interaction between the strange antiquark and the light flavor quarks is the confining interaction [8]. The prediction of a stable strange pentaquark with positive parity at an the energy close to that of the empirically found resonance was first made with a chiral soliton model, in which it was classified as the lowest state of the baryon antidecuplet multiplet [9]. Here it is shown that once an attractive spin dependent hyperfine interaction between the light flavor quarks and the strange antiquark is introduced, stable positive parity strange pentaquarks may

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also be accomodated within the constituent quark model.

The originally proposed pentaquarks, which were introduced in the context of the conventional one-gluon exchange model for the hyperfine interaction between constituent quarks, had negative parity, as they represented states with all the light flavor quarks as well as the strange antiquark in their lowest $s$-states. Once the chromomagnetic interaction is replaced by a spin and flavor dependent interaction, with the form, which corresponds to a Goldstone boson exchange (GBE) interaction between quarks, the lowest lying pentaquarks will, however, have positive rather than negative parity. \[10\] The parity of the pentaquark is given by $P = (-)^L + 1$. Here, we take $L = 1$ and analyze the case where the subsystem of two $u$ and two $d$ quarks is in a state of orbital symmetry $[31]_O$, which thus carries the angular momentum $L = 1$. Although the kinetic energy of such a state is higher than that of the orbitally symmetric state $[4]_O$, an estimate based on a schematic interaction model \[11\] shows that the $[31]_O$ symmetry should be the most favourable from the point of view of stability against strong decays. In Ref. \[5\] the antiquark was assumed to have heavy $c$ or $b$ flavor, and accordingly the interaction between a light quark and the heavy antiquark was neglected, which is justified in the heavy quark limit. As the constituent mass of the strange quark is not much larger than that of the light flavor quarks, that approximation cannot be invoked for strange pentaquarks. Below it is in fact shown that the stable low lying strange pentaquarks only appear if an interaction between $\bar{s}$ and the light quarks is included explicitly in the constituent quark model.

We shall employ the following schematic flavor-spin interaction between light quarks \[10\]:
\[
V_\chi = -C_\chi \sum_{i < j} \lambda^F_i \cdot \lambda^F_j \vec{\sigma}_i \cdot \vec{\sigma}_j.
\]
Here $\lambda^F_i$ are Gell-Mann matrices for flavor $SU(3)$, and $\vec{\sigma}_i$ are the Pauli spin matrices. The constant $C_\chi$ may be determined from the $\Delta$-N splitting to be $C_\chi \simeq 30$ MeV \[10\]. The interaction (1) is the simplest model for the hyperfine interaction between quarks, which can describe the empirical baryon spectrum in the constituent quark model \[10\]. It may be interpreted as arising from pion and (mainly) two-pion exchange, or more generally from exchange of the octet of light pseudoscalar mesons (“Goldstone bosons”) and vector mesons between the constituent quarks \[11\].

The pion decay $D_s^* \to D_s \pi^0$ implies, by $\pi^0 - \eta$ mixing, that $\eta$ mesons couple to strange quarks and antiquarks \[13\]. It is then natural to assume that there is an $\eta$ meson exchange interaction between $\bar{s}$ antiquarks and light flavor quarks. In particular it should lead to a spin-dependent interaction between the strange antiquark and the 4 light flavor quarks, which is similar to (1). This may be schematically be represented by the interaction:
\[
V_\eta = V_0 \sum_i \vec{\sigma}_i \cdot \vec{\sigma}_\pi.
\]
Here $V_0$ is a constant, which should correspond to the ground state matrix element of the spin-spin part of the $\eta$ exchange interaction, but which here will be taken to be a phenomenological constant. The total hyperfine interaction is then
\[
V = V_\chi + V_\eta.
\]
The quark model values for the pseudovector coupling constant between light flavor and \( \eta \) mesons and strange constituent quarks and \( \eta \) mesons are

\[
f_{\eta qq} = \frac{m_\eta}{2\sqrt{3}f_\eta} g_\lambda^q, \quad f_{\eta ss} = -\frac{m_\eta}{\sqrt{3}f_\eta} g_\lambda^s.
\]

These expressions suggest that \( f_{\eta qq} \) falls within the range 1.25 – 1.4 and that \( f_{\eta ss} \) falls in the range -2.5 and -2.8. Here \( f_\eta = 112 \text{ MeV} \) is the \( \eta \) meson decay constant and \( g_\lambda^q \) is the axial coupling of the quark. The value of the latter is expected to fall within the range 0.75 – 1.0 [14].

The strength of the coupling between \( \eta \) mesons and strange constituent quarks may be derived from the known empirical \( \pi^0 \) decay width of \( D_s \) mesons, which is mediated by \( \eta \) mesons. This suggests that \( f_{\eta ss} \sim -1.66 \) [13].

For pseudoscalar mesons the coupling to antiquarks has the same sign as that of quarks. Because of the negative sign of the coupling of strange quarks to \( \eta \) mesons and the positive sign of the coupling of strange quarks to light flavor constituent quarks, the potential coefficient \( V_0 \) is expected to be positive.

An estimate of the \( \eta \) meson exchange contribution to the strength of \( V_0 \) may be obtained from the expectation value of the radial part of the \( \eta \) meson exchange interaction,

\[
V_0(r) = \frac{f_{\eta qq}f_{\eta ss}}{12\pi} \left\{ \frac{e^{-m_\eta r}}{r} - 4\pi \frac{\delta(\vec{r})}{m_\eta^2} \right\},
\]

in the ground state of a quark-antiquark pair described by a harmonic oscillator wave function

\[
\varphi(\vec{r}) = \frac{\alpha^2}{\pi} e^{-\alpha^2 r^2/2},
\]

where the parameter \( \alpha \) may be adjusted to correspond to a realistic wave function model. This yields:

\[
\langle V_0 \rangle = \frac{m_\eta f_{\eta qq}f_{\eta ss}}{3\pi \sqrt{\pi}} \left( \frac{\alpha}{m_\eta} \right)^3 \left\{ \frac{m_\eta^2}{2\alpha^2} - \sqrt{\pi} \frac{m_\eta^3}{4\alpha^3} e^{m_\eta^2/4\alpha^2} \operatorname{erfc}(\frac{m_\eta^2}{2\alpha}) - 1 \right\}.
\]

With the values of the \( \eta \)-quark couplings above, this expression yields values for \( \langle V_0 \rangle \), which are of the same order of magnitude as that of \( C_\chi \), when the baryon wavefunctions are compact, with matter radii less than \( 1/m_\eta \). This condition is fulfilled for example by the model in [16], for which the ground state wavefunction may be approximated by a product of two oscillator functions of the form (6) of the two Jacobi coordinates, with \( \alpha \simeq 650 \text{ MeV} \) [15]. With that value, and with \( f_{\eta qq} = 1.3 \) and \( f_{\eta ss} = -1.66 \), eqn.(7) yields \( \langle V_0 \rangle \sim 90 \text{ MeV} \). This number would be somewhat reduced by the contribution from singlet pseudoscalar exchange mechanisms like \( \eta' \)-meson exchange [16].

For the construction of the wave functions for the pentaquark it is convenient to first consider the light quark \( q^4 \) subsystem. For this the Pauli principle allows for the following two totally antisymmetric states with \( [31]_O \) symmetry, written in the flavour-spin (FS) coupling scheme [5,17]:

\[
|1\rangle = \left( [31]_O[211]_C[1^4]_{OC} ; [22]_F[22]_S[4]_{FS} \right),
\]

(8)
Asymptotically, a ground state baryon and a meson, into which a pentaquark can split, would give $[3]_O \times [2]_O = [5]_O + [41]_O + [32]_O$. By removing the antiquark, one can make the reduction $[41]_O \rightarrow [31]_O \times [1]_O$ or $[32]_O \rightarrow [31]_O \times [1]_O$. Thus, the symmetry $[31]_O$ of the light quark subsystem is compatible with an $L = 1$ asymptotically separated baryon plus meson system.

Each one of these two states, $\Psi^1$ or $\Psi^2$, has to be coupled to the antiquark state. The total angular momentum $\vec{J} = \vec{L} + \vec{S} + \vec{s}_q$, where $\vec{L}$ and $\vec{S}$ are the angular momentum and spin of the light flavor subsystem respectively and $\vec{s}_q$ the spin of the antiquark $s$, takes the values $J = \frac{1}{2}$ or $\frac{3}{2}$. The resulting pentaquark states mix through the quark-antiquark spin-spin interaction (2). Here we study the lowest case, $J = \frac{1}{2}$.

For the stability problem the relevant quantity is

$$\Delta E = E(q^4\bar{q}) - E(q^3) - E(q\bar{q}),$$

where $E(q^4\bar{q})$, $E(q^3)$ and $E(q\bar{q})$ are the masses of the pentaquark, of the ground state baryon and of the meson into which the pentaquark decays, respectively. The multiquark system Hamiltonian used to calculate $E$ is formed of a kinetic energy term, a confining interaction and the hyperfine interaction (3).

Consider first the contribution of (1) only. In the $q^4$ subsystem the expectation value of (1) is $-28 C_\chi$ for $|1\rangle$ and $-64/3 C_\chi$ for $|2\rangle$. Thus $|1\rangle$ is the lowest state. For the ground state $q^3$ system (the nucleon) the contribution is $-14 C_\chi$. There is no such interaction in the $q\bar{q}$ pair. We assume that the confinement energy roughly cancels out in $\Delta E$. Then, the kinetic energy contribution to $\Delta E$ is $\Delta KE = 5/4 \bar{\hbar} \omega$ in a harmonic oscillator model. It follows that for the state $|1\rangle$ the GBE contribution is $\Delta V_\chi = -14 C_\chi$. With $\bar{\hbar} \omega \approx 250$ MeV, determined from the $N(1440) - N$ splitting [10], this would give

$$\Delta E = \frac{5}{4} \bar{\hbar} \omega - 14 C_\chi = -107.5 \text{ MeV}$$

i.e. a substantial binding [4]. This is to be contrasted with the negative parity pentaquarks studied in Ref. [18] within the same model, but where the lowest state has the orbital symmetry $[4]_O$ so that one has $\Delta E = 3/4 \bar{\hbar} \omega - 2 C_\chi = 127.5$ MeV, i.e. instability, in agreement with the detailed study made in [18].

The estimate (11) is a consequence of the flavor dependence of the chiral interaction (1). For a specific spin state $|f⟩$, a schematic color-spin interaction of type $V_{c,m} = - C_{c,m} \sum \lambda^i_\lambda^j \cdot \bar{\sigma}_i \cdot \sigma_j$, which may represent the one gluon exchange interaction, does not make a distinction between $[4]_O$ and $[31]_O$. Consequently, the $[31]_O$ state would appear to lie above the state $[4]_O$, because of the kinetic energy term. The flavor-spin interaction (11) overcomes the excess of kinetic contribution in $[31]_O$ and generates a lower expectation value for $[31]_O$ than for $[4]_O$.

Consider now the total hyperfine interaction (3). The matrix elements of $V_\eta$ of (2) are calculated with the functions $\psi_1$ and $\psi_2$ given in the Appendix. The interaction (3) now

$$|2\rangle = ([31]_O[211]_C [1^4]_{OC} : [31]_F [31]_S [4]_{FS} ).$$

(9)
leads to the following matrix to be diagonalized:

\[
\begin{pmatrix}
\langle \psi_1 | \psi_1 \rangle & \langle \psi_2 | \psi_2 \rangle \\
\langle \psi_1 | \psi_1 \rangle & -28 C_\chi + \frac{8}{\sqrt{2}} V_0 \\
\langle \psi_2 | \psi_2 \rangle & \frac{8}{\sqrt{2}} V_0 - \frac{64}{3} C_\chi - 4V_0
\end{pmatrix}
\]

Note that the contribution of \( V_\eta \) cancels out for the state \( \psi_1 \) derived from (8). Taking \( C_\chi = 30 \text{ MeV} \), as mentioned above, the eigenvalues of this matrix become

\[\langle V \rangle = -(740 + 2V_0) \pm \sqrt{[10,000 - 400V_0 + 36V_0^2]^{1/2}}.\]

When \( V_0 = 0 \), the lowest solution gives \( \langle V \rangle = \langle V_\chi \rangle = -840 \text{ MeV} \), consistent with Ref. [5].

In Fig. 1 the energy of the lowest solution (13) is plotted as a function of the strength \( V_0 \). One can see that for a value of \( V_0 = C_\chi \) the energy \( E(q^3) \) can be lowered by about 130 MeV with respect to the case \( V_0 = 0 \). This implies a decrease by the same amount in \( \Delta E \) of (10) and hence a substantial increase in the stability of the system \( uudd \). This may nevertheless not be sufficient for ensuring stability. Actually estimates similar to those of Ref. [5] containing only the GBE interaction [19] for which \( E(q^3) = 969 \text{ MeV} \), give \( \Delta E = 287 \text{ MeV} \). To get this value we have used \( E(q^3) = 793.6 \text{ MeV} \), i.e. the average mass \( \langle M + M^* \rangle / 4 \) of the pseudoscalar K-meson mass \( M = 495 \text{ MeV} \) and the vector K-meson mass \( M^* = 893.1 \text{ MeV} \). To obtain a negative \( \Delta E \) one needs \( V_0 \approx 50 \text{ MeV} \), i.e. \( V_0 \approx 5/3 C_\chi \), as one can see from Fig. 1.

The estimate obtained from Eq. (6) above suggests that such strength of the spin-spin interaction between the light flavor quarks and the strange antiquark is quite plausible. While \( \eta \) meson exchange is the most obvious source of such an interaction, other mechanisms as two-kaon exchange and \( \eta' \) exchange should also contribute.

The conclusion is that the stable strange pentaquarks with positive parity can be accommodated by the constituent quark model, provided that: 1) there is a flavor-spin dependent hyperfine interaction between the 4 light flavor quarks, which is sufficiently strong for reversing the order of the lowest states in the \( s- \) and \( p- \)shells and that 2) there is an at least as strong spin-spin interaction between the light flavor and the strange antiquark. The hyperfine chromomagnetic interaction between the quarks would in contrast not lead to stable pentaquarks with positive parity, nor with negative. While the presence of a strongly flavor dependent hyperfine interaction between constituent quarks originally was suggested by phenomenological arguments alone [10], and in particular by the requirement of reversal of normal ordering of the states in the constituent quark model with 3 valence quarks, it has received further indirect support by recent QCD lattice calculations, which show the same reversal of normal ordering for small quark mass values [20].

Acknowledgement
Appendix

To calculate the matrix elements of the interaction (2) first one has to couple the antiquark to the subsystem $q^4$. Then one has to decouple a $q\bar{s}$ pair from the pentaquark system. One can work separately in the orbital, flavor, spin and color spaces. But as the interaction (2) concerns only the spin degree of freedom, the task is quite easy because in the spin space the antiquark is on the same footing with the quarks and the problem reduces to the usual recoupling, via Racah coefficients. The only care must be taken of is the symmetry properties of the states. Here we construct explicitly the flavor-spin part of the wave functions of the pentaquark.

Let us denote by $[f_{q_4}]$, $[f_{q_3}]$, $[f_{q_2}]$ and $[f]$ the partitions corresponding to the $q_4$, $q_3$, $q_2$ and $q\bar{s}$ respectively. The corresponding spins are denoted by $J_{q_4}$, $j_1$, $S$ and $J$. For the two states (8) and (9) one has $[f_{q_4}] = [22]$, $J_q = 0$ and $[f_{q_3}] = [31]$, $J_q = 1$ respectively. The coupling to the antiquark spin must therefore lead to the only common case $[f] = [32]$, $J = 1/2$. The $q\bar{s}$ pair can have of course $[f_{q_2}] = [2]$, $S = 1$ or $[f_{q_2}] = [11]$, $S = 0$. Then the spin part of the wave function of the pentaquark reads

$$\langle f_{J_M} \mid [\chi_{J_q} \chi_{1/2}] \mid f \rangle = \sum_{S} [(2S + 1)(2J_q + 1)]^{1/2} W(j_1 \frac{1}{2} J_q S; J_{q_4}) [\chi_{J_q} \chi_{S}] [f_{J_M}] (14)$$

In the recoupling one has however to keep track of the flavor-spin symmetry of the subsystem of 4 identical quark. This part of the wave function is symmetric, both in (8) and (9). The flavor part of the wave function of $q^4$ should be specified but the recoupling with the antiquark does not have to be explicit, inasmuch as the interaction (2) is flavor independent. However the coupling to the antiquark must give the same quantum numbers $(\lambda \mu) = (11)$ in the flavor space, in both cases, otherwise the scalar product cancels.

The two independent pentaquark flavor states associated with (8) are

$$\phi_1 = (\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \end{array}) \times \phi_{\bar{s}}^{(11)},$$

$$\phi_2 = (\begin{array}{cccc} 1 & 3 & 2 & 4 \\ \end{array}) \times \phi_{\bar{s}}^{(11)},$$

where $\phi_{\bar{s}}$ is the flavor antiquark state. Replacing the corresponding Racah coefficients in the relation (14) the flavor-spin wave function of the pentaquark becomes

$$|\psi_1\rangle = |[22][1][32]\rangle_{1/2M} = \frac{1}{\sqrt{2}} \{ \phi_1 \left[ -\frac{1}{2} [\chi_{1/2}^{[21]} [11] \chi_{1/2}^{[32]}]_{1/2M} + \frac{\sqrt{3}}{2} [\chi_{1/2}^{[21]} [2] \chi_{1/2}^{[32]}]_{1/2M} \right] + \phi_2 \left[ -\frac{1}{2} [\chi_{1/2}^{[21]} [2] \chi_{1/2}^{[32]}]_{1/2M} \right] \} + \frac{\sqrt{3}}{2} \{ [\chi_{1/2}^{[21]} [2] \chi_{1/2}^{[32]}]_{1/2M} \}$$

(17)
where in each row $\chi_{1/2}^{[21]}$ is associated with a different Young tableau.

The flavor-spin pentaquark state constructed from (9) contains the following three independent flavor states

\[ \phi_3 = (\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ \hline 4 & \end{array} \times \phi_{\pi})^{(11)}, \]

(18)

\[ \phi_4 = (\begin{array}{cc|cc} 1 & 2 & 4 & 3 \\ \hline 3 & \end{array} \times \phi_{\pi})^{(11)}, \]

(19)

\[ \phi_5 = (\begin{array}{cc|cc} 1 & 3 & 4 & 2 \\ \hline 2 & \end{array} \times \phi_{\pi})^{(11)}. \]

(20)

Then using these states and the recoupling (14) with corresponding Racah coefficients we obtain the pentaquark flavor-spin state

\[ |\psi_2\rangle = |[31][1]; [32]|_{1/2M} = \frac{1}{\sqrt{3}} \left\{ \phi_3 \left[ \chi_{3/2}^{[3]} \chi_{1}^{[2]} \right]_{1/2M}^{[32]} \right. 

+ \phi_4 \left[ \frac{1}{2} \chi_{1/2}^{[21]} \chi_{1}^{[2]} \right]_{1/2M}^{[32]} + \frac{\sqrt{3}}{2} \left[ \chi_{1/2}^{[21]} \chi_{0}^{[11]} \right]_{1/2M}^{[32]} 

+ \phi_5 \left[ \frac{1}{2} \chi_{1/2}^{[21]} \chi_{1}^{[2]} \right]_{1/2M}^{[32]} + \frac{\sqrt{3}}{2} \left[ \chi_{1/2}^{[21]} \chi_{0}^{[11]} \right]_{1/2M}^{[32]} \right\}. \]

(21)

Again, in each row the function $\chi_{1/2}^{[21]}$ has a distinct Young tableau. The explicit form of $q^3$ and $q^4$ flavor or spin states associated with every Young tableau above can be found for example in Ref. [17].

The wave functions $|\psi_1\rangle$ and $|\psi_2\rangle$ are used to calculate the matrix elements of the interaction (2).

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Figure 1. The lowest solution of (13) as a function of the parameter $V_0$.

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