Understanding masses of $c\bar{s}$ states in Regge phenomenology

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Abstract

In the framework of Regge phenomenology, masses of the charmed states $c\bar{q}$ ($q = u, d, s$) lying on the $1^3S_1$-like trajectories are estimated. The overall agreement between our estimated masses and the recent predictions given by modified quark models [hep-ph/0605019, hep-ph/0608011, hep-ph/0608139] is good. Masses of the observed charmed states $D_{s0}(2317)$, $D_{sJ}(2860)$ and $D_{sJ}(2690)/D_{sJ}(2700)$ can be reasonably reproduced in the picture of these charmed states as simple quark-antiquark configurations. We therefore suggest the $D_{s0}(2317)$ can be identified as the $c\bar{s}(1^3P_0)$ states. The possible assignments of the $D_{sJ}(2860)$ and $D_{sJ}(2690)/D_{sJ}(2700)$ are discussed.

Key words: Regge phenomenology; meson mass

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1 Introduction

Recently, discovery of the new charm-strange state $D_{s0}(2317)$ [1] generated strong interest in charmed meson spectroscopy. The $D_{s0}(2317)$ seems an obvious candidate for the $1^3P_0 \ c\bar{s}$ state since it is the first observed charm-strange $0^+$ resonance. However, the observed mass of the $D_{s0}(2317)$ is more than one hundred MeV than the constituent quark model predictions for the $1^3P_0 \ c\bar{s}$. For example, the measured mass of the $D_{s0}(2317)$ is $2317.3 \pm 0.6$ MeV[2], while the prediction of the $1^3P_0 \ c\bar{s}$ state by Isgur and Godrey is $2.48$ GeV[3] and that by Di Pierro and Eichten is $2.487$ GeV[4]. It is widely accepted that the constituent quark model offers the most complete description of hadron properties and is probably the most successful phenomenological model of hadron structure[5]. Therefore, the substantially small observed mass of the $D_{s0}(2317)$ led to many exotic interpretations on the underlying structure of the $D_{s0}(2317)$, such as the $(DK)$ molecule, four-quark state, $D\pi$ atom or baryonium have been proposed in the literature (For the detailed review see e.g. Refs.[6, 7, 8]).

It should be noted that it is very important to exhaust possible conventional $c\bar{s}$ descriptions of the $D_{s0}(2317)$ before resorting to more exotic models, and the discrepancy between the measured mass of the $D_{s0}(2317)$ and the quark model predictions for the $1^3P_0 \ c\bar{s}$ mass maybe imply that approximations to the constituent quark model are not appropriate. In fact, it has been pointed out by Matsuki et al.[9] that conventional quark models do not completely and consistently respect heavy quark symmetry and the $D_{s0}(2317)$ mass can be reproduced if one treats a bound state equation appropriately. Also, it is found by Lee et al.[10] that with the one loop chiral corrections, the quark model for $c\bar{s}$ mesons can naturally account for the unusually mass of the $D_{s0}(2317)$. More recently, it is shown that a simple modification to standard vector Coulomb plus scalar linear quark potential model maintains good agreement with the charmonium spectrum and agrees remarkably well with the $D$ and $D_s$ spectra[11, 12].

In the present work, we shall show in Regge phenomenology, masses of the recent observed charmed states $D_0(2290)$, $D_{s0}(2317)$, $D_{sJ}(2860)$[13] and $D_{sJ}(2690)/D_{sJ}(2700)$[13, 14] can be reproduced within a simple $c\bar{q}$ picture, and our estimated masses are in good agreement with those predicted by[9, 11, 12]. Therefore, our analysis supports the conclusion that it is not
necessary to introduce more exotic models for understanding masses of these reported charmed states.

2 Regge phenomenology

Regge theory is concerned with the particle spectrum, the forces between particles, and the high energy behavior of scattering amplitudes[15]. One of the most distinctive features of Regge theory is the Regge trajectory by which the mass and the spin of a hadron are related. Knowledge of the Regge trajectories is useful not only for spectral purpose, but also for many non-spectral purpose. The intercepts and slopes of the Regge trajectories are of fundamental importance in hadron physics[16].

A series of recent papers by Anisovich et al.[17] show that meson states fit to the quasi-linear Regge trajectories with sufficiently good accuracy, although some suggestions that the realistic Regge trajectories could be nonlinear exist[18, 19].

The quasi-linear Regge trajectories for a meson multiplet can be parameterized as

\[ J = \alpha (t) = \alpha_i \bar{\psi} (N)(0) + \alpha'_i \bar{\psi} (N) t, \]

where \( i \) (\( \bar{i} \)) refers to the quark (antiquark) flavor, \( i \bar{\psi} (N) \) denotes the meson \( i \bar{\psi} \) with radial quantum number \( N \) (\( N = 1, 2, 3, ... \)), \( t = M^2_{i \bar{\psi}(N)} \), \( J \) and \( M_{i \bar{\psi}(N)} \) are respectively the spin and mass of the \( i \bar{\psi}(N) \) meson, \( \alpha_i \bar{\psi}(N)(0) \) and \( \alpha'_i \bar{\psi}(N) \) are respectively the intercept and slope of the trajectory on which the \( i \bar{\psi}(N) \) meson lies. For a meson multiplet, the parameters for different flavors can be related by the following relations (see Ref.[20] and references therein):

(i) additivity of intercepts,

\[ \alpha_i \bar{\psi}(N)(0) + \alpha_j \bar{\psi}(N)(0) = 2\alpha_{i\bar{j}}(N)(0), \]

(ii) additivity of inverse slopes,

\[ \frac{1}{\alpha'_i \bar{\psi}(N)} + \frac{1}{\alpha'_j \bar{\psi}(N)} = \frac{2}{\alpha'_{i\bar{j}}(N)}. \]

From (1) and (2), we obtain that

\[ M^2_{i \bar{\psi}(N)} \alpha'_i \bar{\psi}(N) + M^2_{j \bar{\psi}(N)} \alpha'_j \bar{\psi}(N) = 2M^2_{i\bar{j}}(N)\alpha'_{i\bar{j}}(N). \]
The main purpose of this work is to discuss whether masses of the recent observed charmed states such as the $D_0(2290)$, $D_{s0}(2317)$, $D_{sJ}(2860)$ and $D_{sJ}(2690)/D_{sJ}(2700)$ can be reproduced correctly in the conventional quark-antiquark picture based on Regge phenomenology. Because the possible quantum numbers of the $D_{sJ}(2860)$ include $0^+, 1^-, 2^+$ and $3^-$[13], $J^P$ of the $D_{sJ}(2700)$ ($D_{sJ}(2690)$) is $1^-[14]^1$, and the $0^+, 1^-, 2^+$ and $3^-$ trajectories are parity partners[19], in the following text, we shall adopt two assumptions: (a) the slopes of the parity partners’ trajectories coincide, as proposed by Ref.[19], and (b) $\alpha_i' \bar{n}_j(N) = \alpha_i' \bar{j}(1)$, as adopted by Ref.[17]. Under these two assumptions, the $0^+, 1^-, 2^+$ and $3^-$ are the $1^3S^1$-like trajectories. In this work, the slopes of the $1^3S^1$-like trajectories used as input are taken from our previous work[20] and these slopes are shown in Table 1.

In the presence of $\alpha'_{c\bar{c}(1)} = 0.4364$ GeV$^{-2}$ and $M_{J/\psi} = 3096.916$ MeV, masses of the $2^3S_1$, $3^3S_1$, and $4^3S_1$ $c\bar{c}$ states predicted by relation (6) are respectively (in MeV) $3447$, $3764$ and $4058$.

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1 In $D^0K^+$ system, BaBar observed a resonance ($D_{sJ}(2690)$) with a mass $2688\pm4\pm2$ MeV and width $112\pm7\pm36$ MeV[13], Belle subsequently observed a resonance ($D_{sJ}(2670)$) with a mass $2715\pm11\pm14$ MeV, width $115\pm20\pm36$ MeV and $J^P = 1^-[14]$, we regard them compatible, and the $D_{sJ}(2690)$ and $D_{sJ}(2700)$ should be the same one resonance with $J^P = 1^-$.

2 It is expected that trajectories may occur in integrally spaced sequences, with a ‘parent’ trajectory $\alpha_1(t)$, and an infinite sequence of ‘daughters’ $\alpha_N(t) = \alpha_1(t) - n_r$, $n_r = 0, 1, 2, 3, ..., N = n_r + 1$, see Ref.[15]. In Ref.[17], the mass relation $M^2_N = M^2_0 + (N - 1)\mu^2$, $\mu^2 = 1/\alpha'$ is presented.
which are several hundreds MeV lower than the corresponding measured masses[2] (in MeV) 3686.093±0.034, 4039±1 and 4421±4. This implies the simplification that \( \alpha_{ij}^{(1)}(0) - \alpha_{ij}^{(N)}(0) \) is flavor-independent may be too rough for the heavy mesons. The phenomenological analysis indicates[21, 22] that \( \alpha_{ij}^{(1)}(0) - \alpha_{ij}^{(N)}(0) \) depends on masses of the constituent quarks, \( m_i \) and \( m_j \), and the functional dependence of \( \alpha_{ij}^{(1)}(0) - \alpha_{ij}^{(N)}(0) \) on quark masses is through the combination \( m_i + m_j \). Furthermore, the quantitative results of Ref.[21, 22] show \( \alpha_{ij}^{(1)}(0) - \alpha_{ij}^{(2)}(0) \approx 1.3-1.6 \). This idea motivates us to introduce a factor \( 1 + f_{ij}(m_i + m_j) \) into relation (6) for incorporating the corrections due to the flavor-dependent spacing between \( \alpha_{ij}^{(1)}(0) \) and \( \alpha_{ij}^{(N)}(0) \):

\[
M_{ij}^2(N) - M_{ij}^2(1) = \frac{(N - 1)}{\alpha'_{ij}(1)} (1 + f_{ij}(m_i + m_j)),
\]

where the parameter \( f_{ij} \) depends on the flavors \( i \) and \( j \), and can be obtained by fitting to the data, masses of the constituent quarks are taken the following values (in GeV)

\[
m_u = m_d = 0.29, m_s = 0.46, m_c = 1.65,
\]

which are typical values used in phenomenological quark models[23].

Inserting masses of \( J/\psi \)(\( 1^3S_1 \) c\( \bar{c} \), mass: 3096.916 MeV), \( \psi (4040) \)(\( 3^3S_1 \) c\( \bar{c} \), mass: 4039 MeV), \( \rho \)(\( 1^3S_1 \) n\( \bar{n} \), mass: 775.5 MeV), \( \rho (1450) \)(\( 2^3S_1 \) n\( \bar{n} \), mass: 1459 MeV), \( K^* (892) \)(\( 1^3S_1 \) n\( \bar{s} \), mass: 896 MeV) and \( K^* (1580) \)(\( 2^3S_1 \) n\( \bar{s} \), mass: 1580 MeV)\(^3\) into the following equations,

\[
M_{\psi (4040)}^2 = M_{\psi/\psi}^2 + (3 - 1)(1 + f_{cc}(m_c + m_c))/\alpha'_{cc(1)},
\]

\[
M_{\rho (1450)}^2 = M_{\rho}^2 + (2 - 1)(1 + f_{nn}(m_u + m_u))/\alpha'_{nn(1)},
\]

\[
M_{K^* (1580)}^2 = M_{K^* (892)}^2 + (2 - 1)(1 + f_{ns}(m_u + m_s))/\alpha'_{ns(1)},
\]

and with the help of the following relations derived from (2), (5) and (7)

\[
2f_{ns}(m_u + m_s) = f_{nn}(m_u + m_u) + f_{ss}(m_s + m_s),
\]

\(^3\)All the masses used as input are taken from PDG[2] except for the \( 2^3S_1 \) kaon mass. The assignment of the \( K^* (1410) \) as the \( 2^3S_1 \) kaon is problematic[24, 25]. Quark model and other phenomenological approaches consistently suggest the \( 2^3S_1 \) kaon has a mass about 1580 MeV[26], here we take 1580 MeV as the mass of \( 2^3S_1 \) n\( \bar{s} \).
\[ 2f_{cn}(m_c + m_u) = f_{nn}(m_u + m_u) + f_{cc}(m_c + m_c), \]  
\[ 2f_{cs}(m_c + m_s) = f_{cc}(m_c + m_c) + f_{ss}(m_s + m_s), \]

one can obtain (in GeV\(^{-1}\))

\[ f_{nn} = 0.601, \quad f_{ns} = 0.584, \quad f_{cn} = 0.210, \quad f_{cs} = 0.235, \quad f_{cc} = 0.141. \]  

Based on the above parameters, we find that \( \alpha_{n\bar{n}(1)}(0) - \alpha_{n\bar{n}(2)}(0) = 1.35 \), \( \alpha_{n\bar{s}(1)}(0) - \alpha_{n\bar{s}(2)}(0) = 1.44 \), \( \alpha_{c\bar{n}(1)}(0) - \alpha_{c\bar{n}(2)}(0) = 1.41 \), \( \alpha_{c\bar{s}(1)}(0) - \alpha_{c\bar{s}(2)}(0) = 1.50 \), and \( \alpha_{c\bar{c}(1)}(0) - \alpha_{c\bar{c}(2)}(0) = 1.47 \), which are in good agreement with the quantitative results given by Ref.\[21, 22\] that \( \alpha_{ij(1)}(0) - \alpha_{ij(2)}(0) \approx 1.3-1.6. \)

The spectrum of \( c\bar{c} \) is well predicted by different theoretical approaches, and excited vector \( c\bar{c} \) states are well established experimentally, these predicted and measured results serve us a good testing ground whether our proposed mass relation (7) can give reliable predictions. The measured \( 1^3S_1 \), \( 1^3D_1 \), \( 1^3P_0 \) and \( 1^3P_2 \) \( c\bar{c} \) masses are used as input\(^4\), from (7), (8) and (15), our predicted masses of the radial excitations of the \( c\bar{c} \) states lying on the \( 1^3S_1 \)-like trajectories are shown in Table 2 and Figure 1. Comparison of results predicted by us and those from experiments and other theoretical approaches is also given in Table 2 and Figure 1. Clearly, masses of the \( c\bar{c} \) states lying on the \( 1^3S_1 \)-like trajectories predicted by the mass relation (7) agree remarkably well with those from measurements and other theoretical approaches.

### 3 Masses of the \( c\bar{q} \) states lying on the \( 1^3S_1 \)-like trajectories

In this section, we shall estimate masses of the ground \( c\bar{q} \) states lying on the \( 1^3S_1 \)-like trajectories using relation (4), then the radial excited \( c\bar{q} \) masses can be given by (7). In order to derive masses of the \( c\bar{n} \) and \( c\bar{s} \) using relation (4), we should know the \( n\bar{n} \) (\( I = 1 \)), \( n\bar{s} \) and \( c\bar{c} \) masses. Experimentally, the \( n\bar{n} \) (\( I = 1 \)), \( n\bar{s} \), \( c\bar{c} \) states for the \( 1^3S_1 \), \( 1^3P_2 \) and \( 1^3D_1 \) multiplets are well established\[2\], however for the \( 1^3P_0 \) \( n\bar{n} \) (\( I = 1 \)), \( n\bar{s} \) and \( c\bar{c} \) states, only \( c\bar{c} \) state, \( \chi_{c0}(1P) \), is well established, and the assignment for the \( n\bar{n} \) (\( I = 1 \)) and \( n\bar{s} \) remains open. In the recent literature, there is not yet a consensus on the \( 1^3P_0 \) \( n\bar{n} \) (\( I = 1 \)) mass given by lattice stimulations.

\(^4\)The \( 1^3D_3 \) \( c\bar{c} \) mass is given by \( \sqrt{\alpha_{c\bar{c}(1)} + M_{J/\psi}^2} \[20\]. \)
| $c\bar{c}$ | Expt.[2] | This work | [3] | [27] | [28] | [29] | [30] |
|---|---|---|---|---|---|---|---|
| $1^3S_1$ | 3.097 | 3.097 | 3.10 | 3.10 | 3.096 | 3.100 | 3.15 |
| $2^3S_1$ | 3.686 | 3.600 | 3.68 | 3.73 | 3.686 | 3.676 | 3.63 |
| $3^3S_1$ | 4.039 | 4.039 | 4.10 | 4.18 | 4.088 | 4.079 | 4.04 |
| $4^3S_1$ | 4.421 | 4.434 | 4.45 | 4.56 | 4.434 | 4.42 |
| $1^3D_1$ | 3.771 | 3.771 | 3.82 | 3.80 | 3.794 | 3.794 | 3.76 |
| $2^3D_1$ | 4.153 | 4.192 | 4.19 | 4.22 | 4.156 | 4.17 |
| $3^3D_1$ | 4.576 | 4.52 | 4.59 | 4.482 | 4.53 |
| $4^3D_1$ | 4.929 | 4.889 | 4.87 |
| $1^3P_0$ | 3.415 | 3.415 | 3.42 | 3.44 | 3.434 | 3.412 | 3.42 |
| $2^3P_0$ | 3.875 | 3.92 | 3.94 | 3.854 | 3.867 | 3.86 |
| $3^3P_0$ | 4.287 | 4.228 | 4.25 |
| $4^3P_0$ | 4.662 | 4.538 | 4.61 |
| $1^3D_3$ | 3.765 | 3.85 | 3.83 | 3.815 |
| $2^3D_3$ | 4.187 | 4.22 | 4.24 |
| $3^3D_3$ | 4.571 | |
| $4^3D_3$ | 4.924 | |
| $1^3P_2$ | 3.556 | 3.556 | 3.55 | 3.54 | 3.556 | 3.552 | 3.56 |
| $2^3P_2$ | 3.929 | 4.000 | 3.98 | 4.02 | 3.972 | 3.986 | 3.98 |
| $3^3P_2$ | 4.400 | 4.350 | 4.36 |
| $4^3P_2$ | 4.767 | 4.786 | 4.72 |

Table 2: Masses of $c\bar{c}$ states lying on the $1^3S_1$-like trajectories. Boldface values stand for masses used as input. All in GeV.

For example, both $M_{a_0(1^3P_0)} \sim 1$ GeV[31, 32, 33] and $M_{a_0(1^3P_0)} \sim 1.4 - 1.6$ GeV[34, 35, 36] are predicted recently. In the present work, we shall take $M_{a_0(1^3P_0)} = (1.0 + 1.04 + 1.01)/3 \approx 1.02$ GeV, the average value of predictions given by the recent lattice QCD calculations[31, 32, 33] considering that the naive quark model predicts that the spin-orbit force makes lighter the $a_0(1^3P_0)$ with respect to the $a_2(1^3P_2)$ ($M_{a_2(1^3P_2)} = 1.3183$ GeV[2]) and the same behavior is evident in the $c\bar{c}$ and $b\bar{b}$ spectra[25]. The lattice studies suggest the mass of $1^3P_0$ kaon would be 100-130 MeV heavier than the $a_0$ mass [33]. This is not easily related to any current experimental candidate, while is consistent with the result $M_{n\bar{s}(1^3P_0)} = 1090 \pm 40$ MeV from the K matrix.
The masses of \( M_{D^0}, M_{D^+}, M_{D^{**0}} \) together with the recent predictions by some modified quark models[9, 11, 12]. The masses of \( M_{c\bar{s}(1^3P_0)} \) and \( M_{c\bar{s}(1^3P_2)} \) are given by[20]

\[
M_{c\bar{s}(1^3P_0)} = 1.3183, \quad M_{c\bar{s}(1^3P_2)} = 1.4324, \quad M_{c\bar{s}(1^3P_0)} = 1.02, \quad M_{c\bar{s}(1^3P_2)} = 1.09, \quad M_{c\bar{s}(1^3P_0)} = 1.701, \quad M_{c\bar{s}(1^3P_2)} = 1.735, \quad M_{c\bar{s}(1^3P_0)} = 3.41476, \quad M_{c\bar{s}(1^3P_2)} = 3.7711,
\]

masses of \( c\bar{n} \) and \( c\bar{s} \) states predicted by (4) and (7) are shown in Table 3 and Figures 2-3, together with the recent predictions by some modified quark models[9, 11, 12]. The masses of the \( 1^3D_3 \) \( c\bar{n} \) and \( c\bar{s} \) are given by[20]

\[
M_{c\bar{n}(1^3D_3)} = \sqrt{\frac{2}{\alpha_{c\bar{n}(1)}} + M_{c\bar{n}(1^3S_1)}^2}, \quad M_{c\bar{s}(1^3D_3)} = \sqrt{\frac{2}{\alpha_{c\bar{s}(1)}} + M_{c\bar{s}(1^3S_1)}^2}.
\]

4 Discussions

From Table 3 and Figures 2-3, it is clear that the agreement between our predicted \( D \) and \( D_s \) masses and the recent predictions by some modified quark model[9, 11, 12] is quite good.
It therefore appears likely that Regge phenomenology is capable of describing the $D$ and $D_s$ masses with reasonable accuracy.

Our predicted $D_0$ meson mass is 2.268 GeV, in good agreement with the preliminary Belle measurement of $2290 \pm 22 \pm 20$ MeV[38] and the current Belle mass of $2308 \pm 17 \pm 32$ MeV[39] within one to two percent of accuracy, while in disagreement with the FOCUS mass of $2407 \pm 21 \pm 35$ MeV[40]. For the $D_{s0}$ mass, our prediction is 2.331 GeV, only 13.7 MeV higher than the experimental result, and in good agreement with the recent result predicted by QCD sum rule that $M_{c\bar{s}(1^3P_0)} = 2.31 \pm 0.03$ GeV[41]. Our analysis supports the conclusion that the $D_{s0}(2317)$ can be identified as a conventional $1^3P_0$ $c\bar{s}$ state.

Based on the measured masses of $n\bar{n}, n\bar{s}$ and $c\bar{c}$ states for the $1^3S_1$ and $1^3P_2$ multiplets, the predicted masses for the $1^3S_1$ and $1^3P_2$ $c\bar{q}$ by relation (4) are in excellent agreement with the measurements. In the presence of $M_{n\bar{n}(1^3P_0)} = 1.02$ GeV and $M_{n\bar{s}(1^3P_0)} = 1.09$ GeV, the relation (4) gives an accurate determination for the $D_0$ and $D_{s0}$ masses, it implies that the $a_0(980)$ could

| $D_s$ | Expt.[2] | This work | [9] | [11] | $D$ | Expt.[2] | This work | [9] | [11] |
|------|---------|-----------|----|-----|-----|---------|-----------|----|-----|
| $1^3S_1$ | 2.112 | 2.100 | 2.110 | 2.105 | 2.112 | $1^3S_1$ | 2.010 | 2.010 | 2.011 | 2.017 |
| $2^3S_1$ | 2.653 | 2.711 | $2^3S_1$ | 2.540 |
| $3^3S_1$ | 3.109 | 3.153 | $3^3S_1$ | 2.976 |
| $1^3D_1$ | 2.775 | 2.817 | 2.784 | $1^3D_1$ | 2.738 | 2.762 |
| $2^3D_1$ | 3.214 | $2^3D_1$ | 3.147 |
| $3^3D_1$ | 3.600 | $3^3D_1$ | 3.509 |
| $1^3P_0$ | 2.3173 | 2.331 | 2.325 | 2.341 | 2.329 | $1^3P_0$ | 2.299† | 2.268 | 2.283 | 2.260 |
| $2^3P_0$ | 2.839 | 2.817 | $2^3P_0$ | 2.748 |
| $3^3P_0$ | 3.269 | 3.219 | $3^3P_0$ | 3.156 |
| $1^3D_3$ | 2.815 | $1^3D_3$ | 2.731 |
| $2^3D_3$ | 3.248 | $2^3D_3$ | 3.141 |
| $3^3D_3$ | 3.630 | $3^3D_3$ | 3.504 |
| $1^3P_2$ | 2.5735 | 2.562 | 2.568 | 2.563 | 2.577 | $1^3P_2$ | 2.459 | 2.457 | 2.468 | 2.493 |
| $2^3P_2$ | 3.032 | 3.041 | $2^3P_2$ | 2.906 |
| $3^3P_2$ | 3.438 | 3.431 | $3^3P_2$ | 3.295 |

Table 3: $D$ and $D_s$ spectra. †Average value of 2290 and 2308 MeV by Belle. All in GeV.
Figure 2: The \((N, M^2)\)-trajectories for the \(c\bar{s}\) states. The predicted masses are displayed numerically in Table 3.

be indeed the lightest non-singlet scalar meson.

BaBar recently reported the discovery of a new \(D_{sJ}(2860)\) state with a mass of 2856.6 ± 1.5 ± 5.0 MeV and width of 48 ± 7 ± 10 MeV\[13\]. BaBar observed this state only in the \(D^0K^+\) or \(D^+K_S^0\) system and found no evidence in the \(D^{*0}K^+\) and \(D^{*+}K_S^0\) channels, therefore the possible \(J^P\) of the \(D_{sJ}(2860)\) is 0\(^+\), 1\(^-\), 2\(^+\), 3\(^-\), \cdots\).

Based on our prediction as shown in Table 3, the \(D_{sJ}(2860)\) mass is about 213 MeV higher than the 2\(^3\)S\(_1\) \(c\bar{s}\) mass (2653 MeV), and 82 MeV higher than the 1\(^3\)D\(_1\) \(c\bar{s}\) mass (2775 MeV). The decay pattern and total width of the \(D_{sJ}(2860)\) for various quantum number assignments has been investigated by Zhang et al.\[42\] in a \(^3P_0\) decay model. If the \(D_{sJ}(2860)\) is the 2\(^3\)S\(_1\) \(c\bar{s}\) state, its total width is about 90 MeV, and the dominant decay modes are \(DK^*\) with \(\Gamma(DK^*) = 24\) MeV and \(D^*K\) with \(\Gamma(D^*K) = 64\) MeV, and the \(DK\) mode is only 12 KeV\[42\]. If the \(D_{sJ}(2860)\) is the 1\(^3\)D\(_1\) \(c\bar{s}\) state, its total width is about 132 MeV, and \(\Gamma(DK) : \Gamma(D_3\eta) : \Gamma(D^*K) : \Gamma(D_3^*\eta) : \Gamma(DK^*) \approx 42 : 12 : 7 : 1 : 4\)[42]. Both the predicted mass and width for the 2\(^3\)S\(_1\) or 1\(^3\)D\(_1\) \(c\bar{s}\) are significantly larger than the experimental data of the \(D_{sJ}(2860)\), making the assignment of the \(D_{sJ}(2860)\) as the 2\(^3\)S\(_1\) or 1\(^3\)D\(_1\) \(c\bar{s}\) unfavorable.

However, the \(D_{sJ}(2860)\) mass is close to our predicted 2\(^3\)P\(_0\) \(c\bar{s}\) mass (2839 MeV) and 1\(^3\)D\(_3\) \(c\bar{s}\)
mass (2815 MeV). If the $D_{sJ}(2860)$ is the $2^3P_0$ $c\bar{s}$ state, its total width becomes around 54 MeV, consistent with the measured width of the $D_{sJ}(2860)$, 48 ± 7 ± 10 MeV, within errors, the dominant decay modes are $DK$ with $\Gamma(DK) = 37$ MeV and $D_s\eta$ with $\Gamma(D_s\eta) = 16$ MeV[42], and the decay modes $D^*K$, $D_s^*\eta$ and $DK^*$ are forbidden. The suggestion that the $D_{sJ}(2860)$ can be identified as the $2^3P_0$ $c\bar{s}$ has been given by [12, 43]. If the $D_{sJ}(2860)$ is the $1^3D_3$ $c\bar{s}$ state, its total width is about 37 MeV, also compatible with 48 ± 7 ± 10 MeV within errors, the dominant decay modes are $DK$ with $\Gamma(DK) = 22$ MeV and $D^*K$ with $\Gamma(D^*K) = 13$ MeV[42]. The assignment of the $D_{sJ}(2860)$ as the $3^−$ $c\bar{s}$ has been proposed by [44]. At present, only the decay $D_{sJ}(2860) → DK$ is observed experimentally, which is not enough to distinguish the above two possible assignments, therefore, the assignment of the $D_{sJ}(2860)$ as the $2^3P_0$ or $1^3D_3$ $c\bar{s}$ seems favorable by the available experimental information.

It should be noted that for the $2^3P_0$ $c\bar{s}$ state, the decay modes $D^*K$, $D_s^*\eta$ and $DK^*$ are forbidden, and the $1^3D_3$ $c\bar{s}$ state has a large $D^*K$ decay width and a small $D_s\eta$ decay width ($\sim 1.2$ MeV[42]), therefore, the further experimental search of the $D_{sJ}(2860)$ in the $DK^*$, $D^*K$, $D_s^*\eta$ and $D_s\eta$ decay modes would be certainly desirable for distinguishing the above two possible assignments.
Our predicted mass of the $2^3S_1 c\bar{s}$ state is around 2653 MeV, in excellent agreement with the prediction $2658 \pm 15$ MeV obtained by Chang from Bethe-Salpeter equation\cite{45}, and 48 MeV lighter than the $D_{sJ}(2690)/D_{sJ}(2700)$ mass$^5$. If the $D_{sJ}(2690)/D_{sJ}(2700)$ is the $2^3S_1 c\bar{s}$ state, its total width is about 103 MeV in a $^3P_0$ decay model\cite{12}, consistent with the measured width of $112 \pm 7 \pm 36$ or $115 \pm 20^{+36}_{-32}$ MeV, and the dominant decay modes are $DK$ with a width of 22 MeV and $D^*K$ with a width of 78 MeV\cite{12}. Also, the $D_{sJ}(2690)/D_{sJ}(2700)$ mass is about 74 MeV lighter than our predicted $1^3D_1 c\bar{s}$ mass (2775 MeV), if it is the $1^3D_1 c\bar{s}$ state, the calculations performed by Ref.\cite{42} within a $^3P_0$ model show its total width is 73 MeV, also roughly consistent with the experimental data, and the dominant decay modes are $DK$ with a width of about 49 MeV and $D_s\eta$ with a width of about 13 MeV. Therefore, the $D_{sJ}(2690)/D_{sJ}(2700)$ as either a $2^3S_1$ or $1^3D_1 c\bar{s}$ seems consistent with experimental data\cite{42}. Considering that the $D_{sJ}(2690)/D_{sJ}(2700)$ mass is about 48 MeV higher than $M_{c\bar{s}(2^3S_1)}$, while 74 MeV lower than $M_{c\bar{s}(1^3D_1)}$, we tend to suggest the $D_{sJ}(2690)/D_{sJ}(2700)$ is most likely a mixture of the $D_s(2^3S_1)$ and $D_s(1^3D_1)$. It has been found that\cite{12} in the presence of the $2^3S_1 c\bar{s}$ mixing with $1^3D_1 c\bar{s}$ (with the mixing angle of approximately $-0.5$ radians), the predicted total width of the $D_{sJ}(2690)/D_{sJ}(2700)$ (with mass set to 2688 MeV) by a $^3P_0$ model becomes about 110 MeV, in good agreement with the data.

5 Summary and conclusion

Masses of the $D$ and $D_s$ states lying on the $1^3S_1$-like trajectories are estimated in Regge phenomenology. The predicted masses agree well with the recent results by some modified quark models. It therefore appears likely that Regge phenomenology is capable of describing the $D$ and $D_s$ masses with reasonable accuracy.

Our predictions show that masses of the recent observed charmed states such as the $D_{s0}(2317)$, $D_{sJ}(2860)$ and $D_{sJ}(2690)/D_{sJ}(2700)$ can be reasonably reproduced in the simple quark-antiquark picture. Based on our analysis, we suggest the $D_{sJ}(2317)$ can be identified as the conventional

$^5$The average value of the BaBar mass and Belle mass is 2701 MeV
$1^3P_0$ $c\bar{s}$ state, the assignment of the $D_{sJ}(2860)$ as the $D_s(2^3P_0)$ or $D_s(1^3D_3)$ seems favorable, and the $D_{sJ}(2690)/D_{sJ}(2700)$ is most likely a mixture of the $D_s(2^3S_1)$ and $D_s(1^3D_1)$.

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