On a small size floating dock structural analysis in oblique design waves by 3D-FEM approach

E Burlacu and L Domnisoru
"Dunarea de Jos" University of Galati, Department of Naval Architecture, Domneasca Street 111, 800201, Galati, Romania

E-mail: leonard.domnisoru@ugal.ro

Abstract. In this study we have enhanced the structural assessment analysis of a river-shipyard small size floating dock, with length overall of 60 m, for equivalent quasi-static oblique design wave conditions. The small size floating dock hull has two constructive layouts for the water ballast side tanks. The oblique design wave heading angle is in the range of 0 to 360 deg. The floating dock equilibrium in oblique equivalent design waves is achieved by own non-linear three parameters iterative program. The numerical analyses are done on full extended 3D-FEM structural models, from side to side and from stern to bow, of the small size floating dock, with specific boundary conditions for global and local strength computations. The modelling of the external oblique equivalent design wave pressure has requested to develop specialized user functions into FEM code. There are considered several loading scenarios of the small size floating dock, according to the shipbuilding classification societies. The structural assessment is based on three criteria: yielding stress, buckling and freeboard limit values. The numerical results of the structural assessment for the small size floating dock in oblique equivalent design waves made possible to establish the wave design height limits, according to the safety criteria restrictions.

1. Introduction
Although the floating docks should be operated in special sheltered conditions, with still water state (sw), in practice, according to the shipyard layout, also the wave conditions must be considered. Special cases are the wave conditions during the floating dock relocation operation [1].

In this study we have enhanced the strength analysis of a small size floating dock, with two constructive versions [2], from the single condition of head wave case μ=180 deg. [3], to the oblique EDW wave cases, with full range of the heading angle μ=0-360 deg. So, this study is able to deliver the polar diagrams of EDW oblique wave height limits by synthesize of the safety criteria, yielding stress, buckling and freeboard limits, according to the shipbuilding classification rules [4, 5].

Two 3D-FEM models of the small size dock are developed, full extended from side to side and all over the length. For the computation of the equilibrium conditions of the small size floating dock in oblique equivalent design waves, we have used an own numerical code [6], that includes a non-linear iterative algorithm for three equilibrium parameters sinkage, longitudinal and transversal trim, that defines the relative position between the dock base plane and the medium plane of the EDW oblique waves. Based on the own algorithm, the small size floating dock assessment in oblique waves is accomplished (figure 1).
2. Fundamentals of floating dock structural analysis in oblique design waves by 3D-FEM approach

For the numerical strength assessment in oblique design waves of the floating dock, we have used P_QSW [6] own code and Femap/NX Nastran [7] with user functions as in figure 1 logical flowchart.

(5) The 3D-FEM structural analysis
- application of the oblique equivalent design wave pressure on 3D-FEM model
- convergence criteria for three parameters
- boundary conditions for global and local strength on full extended 3D-FEM model;
- numerical strength analyses;
- used software: Femap/NX Nastran [7] and own developed user subroutines.

(6) The dock structure assessment
- yielding stress limit: \( \sigma_{\text{yield}} \leq \sigma_{\text{adm}} \)
- buckling strength criteria: \( B_{\text{min}} \geq B_{\text{adm}} \)
- freeboard criteria: \( F_{\text{min}} = H - z_{w,\text{max}} \geq f_s \)
- the polar diagrams of the equivalent oblique design wave height limit \( h_{\text{wlim}}(\mu) \) on heading angle range \( \mu = 0^{\circ} - 360^{\circ} \)
- floating dock operation limits assessment according to docking case

(4c) Oblique equivalent design wave EDW free surface \( z_w \)
\[
\begin{align*}
   z_w(x, y) &= d_m + x - x_f \cdot \theta + (y - y_f) \cdot \tan(\varphi) \pm \\
   &\pm \frac{h}{2} \cdot \frac{2\pi}{\lambda} (x \cos \mu + y \sin \mu) \\
   x &\in [0, L] ; \quad y \in [-B/2, +B/2] ; \quad \lambda = L \cos \mu \\
   &\pm \text{sagging and hogging wave conditions}
\end{align*}
\]

(4d) Equivalent beam model results in oblique EDW

Figure 1. The algorithm for floating dock structural analysis in oblique design waves by 3D-FEM approach.
The structural analysis of the floating dock in EDW oblique design waves, by 3D-FEM models with own developed algorithm, includes the following steps (figure 1):

- (1) **The floating dock data and operation cases parameters.** The input data for the dock structure, on board masses and offset lines have to be previous prepared [8]. The operation cases include the oblique equivalent design wave height range, \( h_w = 0 \div 2 \text{ m} \), corresponding to river-costal conditions, and the wave heading angle, \( \mu = 0 \div 90 \) (360) deg., \( \delta\mu = 15 \) deg., taking into account that the small size floating dock has double plane symmetry at centre line and amidships. For the selection of the maximum oblique EDW wave height at each loading case, the freeboard restrictions must be first taken into consideration.

- (2) **The 3D-CAD/FEM model.** In the case of oblique equivalent design waves EDW the pressure on the external hull shell is no longer symmetric on the sides as in the case of head waves [9]. So, for the numerical analyses the 3D model has to be extended not only over the whole dock length but also from side to side, increasing the necessary number of nodes and elements (table 2). According to the rules for FEM models [4, 5], the mesh average size is 200 mm, so that the dock structural models can be used for global and local strength assessment [10]. The FEM models are developed with quad and triangle shell elements, coupled membrane and Mindlin plate elements, close to a 3D stress and strain state [7, 10]. There are considered two constructive versions, with continuous CWT and non-continuous NWT ballast side tanks on the dock pontoon [2]. The on board masses are modelled as path distributed or lumped, including the water ballast and docked structure, so that the displacement \( \Delta \) corresponds to the analyzed loading case. The longitudinal and transversal position of the dock gravity centre remains unchanged for all the loading cases, \( x_G=L/2 \) and \( y_G=0 \), at the intersection of the amidships and centre plane, due to the double symmetry of the structure and on board mass distribution. For each constructive version, four specific loading cases are considered (table 2) [2], according to the floating dock rules [4, 5]. The mass diagram over the small size floating dock, for each loading case, is extracted from the 3D-FEM model using own developed subroutines implemented by command language of the FEM program [7].

- (3) **Setup of the data for the floating dock and oblique EDW equilibrium procedure.** For each loading case the equilibrium relative position between the dock hull and the oblique wave must be computed, by a non-linear iterative approach with three parameters [6], covering modules 4.a, 4.b, 4.c (figure 1). In the case of EDW equivalent design waves, a direct implementation in the FEM program of the iterative approach has been proven practically feasible only in the case of head waves (\( \mu=180 \) deg.), so that for the oblique waves (\( \mu=0\div360 \) deg.), we have developed and external source program P_QSW [6]. For this program as input data we have to import from the floating dock data the offset lines 3D model and the mass diagram from the 3D-FEM model.

- (4.a, 4.b, 4.c, 4.d) **Equilibrium procedure in oblique EDW.** For each floating dock constructive version and loading case (table 2), using the P_QSW [6] program, the operation conditions are cycled for oblique EDW wave height \( h_w \) and heading angle \( \mu \). For the 3D-FEM computation only the cases that satisfy the freeboard restrictions are selected (table 2). The non-linear iterative algorithm [6], for the dock and wave equilibrium computation, requires for each loading and operation conditions to carry out cycles on three parameters sinkage \( d_m \), longitudinal \( \theta \) and transversal \( \varphi \) trim, defining the relative position between the dock hull reference base plane and the medium plane of the quasi-static equivalent oblique wave. The convergence criteria (module 4.b) are formulated in terms of: floatability condition, the immerse volume \( V \) corresponds to the floating dock displacement \( \Delta=\rho V \); the longitudinal and transversal buoyancy centre position corresponds to the gravity centre position, \( x_G=x_B \) and \( y_G=y_B \). Based on the three equilibrium parameters, the oblique equivalent design wave EDW free surface \( z_e(x, y, \mu) \), \( x\in[0, L], y\in[-B/2, B/2] \) is computed (module 4.c), where \( L, B \) are the dock length and breadth, \( x_B=L/2, y_B=0 \) are the EDW oblique wave median plane centre position. The P_QSW [6] program can also deliver results in terms of floating dock equivalent beam model (module 4.d), as vertical and horizontal bending moments and shear forces, torsion moments, used for preliminary check out of global strength by statistical design rule values [4, 5], not included in this study.
-5) The 3D-FEM structural analysis. The external pressure from the oblique equivalent design wave \( p(x,y,z) \), \( x \in [0,L], y \in [-B/2,B/2], z \in [0,w] \), for each loading and operation cases, is applied on the floating dock hull external shell by own developed user functions, implemented in the FEM program Femap/NX Nastran [7]. Because the oblique wave pressure has an unsymmetrical distribution on the sides, at centre plane reference, special boundary conditions for the 3D-FEM models have to be considered [9, 10], in four nodes ND, one at fore peak and three at aft peak (table 1). So, the 3D-FEM models can be used for global and local strength assessment, where \( T_x, T_y, T_z \) and \( R_x, R_y, R_z \) are displacement and rotation degrees of freedom, in the Cartesian coordinated system. The numerical structural simulations involve: linear static analysis, under the assumption that the dock stress distribution on the external shell (module 5), for each loading and operation cases, is applied. Based on the three criteria, for each floating dock constructive version the polar diagrams of the oblique EDW wave height limit \( h_{\text{lim}}(\mu)_{\text{load}} \), over the whole wave heading angle range \( \mu = 0\text{-}360 \text{ deg.} \), are obtained. Comparison between the two constructive versions of the small size floating dock is made, in order to emphasize the differences for the structural safety in the case of oblique design wave conditions.

3. The main data of the small size floating dock models
For the small floating dock [2] we have analyzed two constructive versions, with continuous CWT (figure 3) and non-continuous NWT (figure 8) side ballast tanks. The 3D-FEM models are extended from side to side in order to be suitable for strength analysis in oblique EDW equivalent design waves, which requires special boundary conditions (table 1) and a dock-wave equilibrium algorithm (figure 1), so that the numerical model results full balanced.

Table 1. The boundary conditions setup on 3D-FEM models in oblique EDW waves, [9, 10].

| Position | Node | \( x \) | \( y \) | \( z \) | \( T_x \) | \( T_y \) | \( T_z \) | \( R_x \) | \( R_y \) | \( R_z \) |
|----------|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Fore     | ND_1 | 0      | 0      | 0      | \( \bullet \) | \( \bullet \) | \( \bullet \) | -      | -      | -      |
|          | ND_2 | 0      | 0      | 0      | -      | \( \bullet \) | -      | -      | -      | -      |
|          | ND_3 | 0      | B/2    | 0      | -      | -      | \( \bullet \) | -      | -      | -      |
|          | ND_4 | 0      | -B/2   | 0      | -      | -      | -      | \( \bullet \) | -      | -      |

(6) The dock structure assessment. The global and local strength assessment of small size floating dock is done by three criteria: yield stress admissible value \( \sigma_{\text{adm}} \), buckling admissible factor \( B_{\text{adm}} \) and freeboard safety value \( f_s \) (table 2), according to rules [4, 5]. Based on the three criteria, for each floating dock constructive version the polar diagrams of the oblique EDW wave height limit \( h_{\text{lim}}(\mu)_{\text{load}} \), over the whole wave heading angle range \( \mu = 0\text{-}360 \text{ deg.} \), are obtained. Comparison between the two constructive versions of the small size floating dock is made, in order to emphasize the differences for the structural safety in the case of oblique design wave conditions.

Table 2. The main data of the small size floating dock models, [2, 3].

| Length \( L \) (mm) | 60000 | Young module \( E \) (N/mm\(^2\)) | 2.1 \( \times \) \( 10^5 \) | Gravity centre \( x_G \) (mm) | 30000 | Breadth \( B \) (mm) | 20000 | Poisson ratio \( \nu \) | 0.3 | position \( y_G \) (mm) | 0 |
| Height \( H \) (mm) | 2000 | Material density \( \rho \) (kg/mm\(^3\)) | 7.8 \( \times \) \( 10^6 \) | Light, without | 1152 \( \times \) \( 10^3 \) |
| No. nodes \( N_D \) | 398995 | Yielding stress \( R_{YH} \) (N/mm\(^2\)) | 235 | docking mass | 960 \( \times \) \( 10^3 \) |
| No. elements \( N_E \) | 320771 | Stress \( \sigma_{\text{adm}} \) (N/mm\(^2\)) | 175 | Ship 1\( \times \)3 | 1980 \( \times \) \( 10^3 \) |
| No. elements \( N_E \) | 472830 | Buckling adm. \( B_{\text{adm}} \) | 1.5 | docking mass | 1788 \( \times \) \( 10^3 \) |
| No. elements \( N_E \) | 378210 | \( f_s \) (mm) | 300 | Water density \( \rho_w \) (kg/mm\(^3\)) | 1.0 \( \times \) \( 10^6 \) |
| Gravity acc. \( g \) (m/s\(^2\)) | 9.81 | Freeboard adm. \( f_s \) (mm) | 75 | Oblique eq. design wave \( \mu = 0\text{-}360 \text{ deg.} \) |
4. The strength analysis of the floating dock in oblique EDW waves, both constructive versions

A synthesis of the main 112 study cases, for global and local strength in oblique equivalent design waves, of the floating dock with four loading conditions (table 2), leads to the results from this section.

Figure 3 case CWT light \((h_w=1.93m)\) and figure 8 case NWT ship2 \((h_w=0.42m)\) present the equivalent von Mises stress in oblique EDW \(\mu=45\) deg (quarter-sea), by 3D-FEM full extended models of the small floating dock.

The equivalent von Mises stress distributions over the floating dock length \(x\in[0,L]\) are presented in figure 4 and figure 9 for the selected reference cases in oblique waves.

The buckling collapse modes of the two constructive floating dock versions, for the same oblique wave’s cases, are presented in figure 5 and figure 10.

The maximum values of von Mises stress and buckling factor, function to the heading angle \(\mu=0-180 \) deg, for CWT and NWT, light and ship2 cases, are presented in figures 6, 7, 11, 12.

Table 3 and table 4 present the maximum von Mises stress \(\sigma_{vonM,\text{max}}\), minimum buckling factor \(B_{\text{min}}\) and freeboard \(F_{\text{min}}\), at oblique EDW \(\mu=0-180\) deg., for all four loading cases of CWT and NWT dock.

**Figure 2.** Oblique EDW pressure, (a) hogging and (b) sagging, \(\mu=45\) deg., \(h_w=1.930m\), CWT dock.

**Figure 3.** CWT, light, \(\sigma_{vonM}\) (N/mm\(^2\)), \(\mu=45\)deg, \(h_w=1.93m\), (a) hogging and (b) sagging EDW.

**Figure 4.** CWT, light, \(\sigma_{vonM}\) (N/mm\(^2\)), \(\mu=45\)deg, \(h_w=1.93m\), (a) hogging and (b) sagging EDW.
Figure 5. CWT, light, $B_{\text{factor}}$, $\mu=45^\circ$, $h_w=1.93$ m, (a) hogging and (b) sagging EDW.

Figure 6. CWT, light, $\mu=0$–$180^\circ$, $h_w=1.93$ m, (a) stress $\sigma_{\text{vonM}}$ (N/mm$^2$) and (b) buckling $B_{\text{factor}}$.

Figure 7. CWT, ship2, $\mu=0$–$180^\circ$, $h_w=0.55$ m, (a) stress $\sigma_{\text{vonM}}$ (N/mm$^2$) and (b) buckling $B_{\text{factor}}$.

Figure 8. NWT, ship2, $\sigma_{\text{vonM}}$ (N/mm$^2$), $\mu=45^\circ$, $h_w=0.42$ m, (a) hogging and (b) sagging EDW.
Figure 9. NWT, ship2, $\sigma_{\text{vonM}}$ (N/mm$^2$), $\mu=45\text{deg}$, $h_w=0.42\text{m}$, (a) hogging and (b) sagging EDW.

Figure 10. NWT, ship2, $B_{\text{factor}}$, $\mu=45\text{deg}$, $h_w=0.42\text{m}$, (a) hogging and (b) sagging EDW.

Figure 11. NWT, ship2, $\mu=0\div180\text{deg}$, $h_w=0.42\text{m}$, (a) stress $\sigma_{\text{vonM}}$ (N/mm$^2$) and (b) buckling $B_{\text{factor}}$.

Figure 12. NWT light, $\mu=0\div180\text{deg}$, $h_{\text{limit}}$ from criteria: (a) stress $\sigma_{\text{vonM}}$ (N/mm$^2$) and (b) buckling $B_{\text{factor}}$. 
ck version, on all four loading cases, the only affecting the wave height limit, 1.93m at light case and 0.55m at ship restrictions result from the freeboard criteria (table 3, figures 3)

Combining the strength and freeboard criteria (table 4, polar diagrams of the oblique design wave height (eq. 13 and user functions in FEM program [7], for external EDW wave pressure application on the 3D-FEM full dock models (figures 1, 2).

For the continuous ballast tanks CWT floating dock version, on all four loading cases, the only restrictions result from the freeboard criteria (table 3, figures 3-7). The wave heading angle is not affecting the wave height limit, 1.93m at light case and 0.55m at ships 1, 2, 3 docking cases (figure 13).

5. Conclusions
Combining the strength and freeboard criteria (table 2), for the two versions of the floating dock, the polar diagrams of the oblique design wave height limit $h_{lim}(\mu)$ are presented in figure 13 and figure 14.

An enhanced procedure for the floating docks strength analysis has been developed, capable to take into account the local and global structural loads in oblique equivalent design waves, with unsymmetrical sides pressure, by an iterative non-linear approach with three parameters [6] and user functions in FEM program [7], for external EDW wave pressure application on the 3D-FEM full dock models (figures 1, 2).

| Case | $h_0$(m) | $d_0$(m) | $\theta$(rad) | $\phi$(rad) | $F_{w_0}$(m) | $\sigma_{\text{conM max}}$ (N/mm$^2$) | $B_{\text{min}}$ (-) |
|------|-----------|----------|-------------|-------------|-------------|---------------------------------|----------------|
| Light CWT | 1.930 | 0.960 | 0 | 0 | 0.075 (1.040) | hogg. | 56.95 59.96 59.94 56.20 54.46 42.92 35.30 | 23.99 |
| Ship1 CWT | 0.550 | 1.650 | 0 | 0 | 0.075 (0.350) | hogg. | 1.518 1.530 1.571 1.714 2.169 2.874 4.234 | 5.550 |
| Ship2 CWT | 0.550 | 1.650 | 0 | 0 | 0.075 (0.350) | hogg. | 47.76 47.79 47.79 47.74 47.54 46.80 46.26 | 46.68 |
| Ship3 CWT | 0.550 | 1.650 | 0 | 0 | 0.075 (0.350) | hogg. | 2.849 2.864 2.914 3.045 3.443 3.785 4.162 | 4.511 |

Table 3. The maximum stress $\sigma_{\text{conM max}}$, minimum buckling factor and freeboard, for CWT dock 3D model.

| Case | $h_0$(m) | $d_0$(m) | $\theta$(rad) | $\phi$(rad) | $F_{w_0}$(m) | $\sigma_{\text{conM max}}$ (N/mm$^2$) | $B_{\text{min}}$ (-) |
|------|-----------|----------|-------------|-------------|-------------|---------------------------------|----------------|
| Light NWT | 0.582 + 0.800 | 0.909 | 0 | 0 | 0.030 | hogg. | 0.582 0.587 0.615 0.696 1.041 1.800 1.800 | 0.030 |
| Ship1 NWT | 0.420 (0) | 1.490 | 0 | 0 | 0.300 (0.510) | hogg. | 73.58 73.46 73.42 73.44 73.33 55.33 53.73 | 41.10 |
| Ship2 NWT | 0.420 (0) | 1.490 | 0 | 0 | 0.300 (0.510) | hogg. | 3.68 2.39 3.21 3.21 54.17 89.65 54.84 | 2.833 |
| Ship3 NWT | 0.186 + 0.420 | 0.417 | 0 | 0 | 0.300 (0.510) | hogg. | 0.186 0.186 0.192 0.220 0.350 0.420 0.420 | 0.030 |

Table 4. The maximum stress $\sigma_{\text{conM max}}$, minimum buckling factor and freeboard, for NWT dock 3D model.

5. Conclusions
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An enhanced procedure for the floating docks strength analysis has been developed, capable to take into account the local and global structural loads in oblique equivalent design waves, with unsymmetrical sides pressure, by an iterative non-linear approach with three parameters [6] and user functions in FEM program [7], for external EDW wave pressure application on the 3D-FEM full dock models (figures 1, 2).

For the continuous ballast tanks CWT floating dock version, on all four loading cases, the only restrictions result from the freeboard criteria (table 3, figures 3-7). The wave heading angle is not affecting the wave height limit, 1.93m at light case and 0.55m at ships 1, 2, 3 docking cases (figure 13).

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For the continuous ballast tanks CWT floating dock version, on all four loading cases, the only restrictions result from the freeboard criteria (table 3, figures 3-7). The wave heading angle is not affecting the wave height limit, 1.93m at light case and 0.55m at ships 1, 2, 3 docking cases (figure 13).
3. For the non-continuous ballast tanks NWT floating dock version (table 4, figures 8-12), the yielding stress limit criterion involves no restrictions for all four loading cases. The buckling and freeboard criteria lead to the restrictions for wave height limit evaluation, with a significant influence of the heading angle values, $0.582 \div 1.800$ m at light case and $0.186 \div 0.420$ m at ship3 docking case. For ship1 and ship2 docking cases the restrictions are mainly from freeboard criteria, with wave height limit of 0.420 m (figure 14).

4. Further studies shall combine the analyses and limit criteria, by oblique quasi-static design waves [9], so that the overall operation safety of the floating dock is assessed.

![Global Strength 3D-FEM EDW h_{limit} CWT F.D.](image1)

![Global Strength 3D-FEM EDW h_{limit} NWT F.D.](image2)

**Figure 13.** CWT floating dock $h_{\text{limit}}(\mu)$ oblique EDW, 3D-FEM, polar diagram, $h_w=0\div2.0$ m, $\mu=0\div360$ deg.

**Figure 14.** NWT floating dock $h_{\text{limit}}(\mu)$ oblique EDW, 3D-FEM, polar diagram, $h_w=0\div2.0$ m, $\mu=0\div360$ deg.

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