Low-parametric equation for calculating the viscosity coefficient of nitrogen in liquid, gas and fluid states

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Abstract. The formulation for the viscosity coefficient of nitrogen is obtained. In the developed equation the dependence of the residua viscosity of various states of substance on the internal energy density is used. The new correlation represents the viscosity of nitrogen at temperatures from 70 K to 1000 K and pressures of up to 50 MPa within the limits of experimental uncertainties.

1. Introduction
The wide application of nitrogen in industry is conditioned by such characteristics as inertness and low temperature of transition to the liquid state, as well as the prevalence of gas. The use of nitrogen in industry is primarily associated with the oil and gas and metallurgical industries. The inert qualities of this gas can be used in fire-control units. In the chemical industry, nitrogen is used to produce nitrogen fertilizers and ammonia. Liquid nitrogen is also used as an effective refrigerant, and it is the main component of air.

The thermophysical properties of nitrogen are quite often measured with a high precision. The last equation of state for the thermodynamic properties of nitrogen was published by Span et al. [1]. It provides reasonable results up to extreme temperatures and pressures. At pressures less than 30 MPa from the triple point to the temperature of 523 K the uncertainty of measuring the density in the equation of state is 0.02%. The error at very high pressures (> 1 GPa) is 0.6% in terms of density [1].

A formula for calculating the viscosity of nitrogen with errors of ± 2% for its liquid and vapor states with the exception of the critical region is presented in [2]. The viscosity of a dilute gas is presented with an accuracy of ± 0.5% at temperatures of up to 1000 K. But at low temperatures of about 100K, as well as at high temperatures, there are differences between data from various sources on the viscosity of diluted gas; these differences exceed ± 1% and ± 0.5%, respectively. The new experimental values in [3, 4, 5, and 6] were used together with the most validated reference data to improve and obtain new correlations of zero density viscosity at low and high temperatures.

2. Equation for calculating viscosity
Different correlations and models are developed and allow us to describe reference and experimental data on viscosity in a wide range of parameters of state.

Using the modified equation [7, 8] to calculate the viscosity coefficient, the equation was obtained to describe the viscosity of nitrogen in a wide range of the state parameters. In this case, the dependence of the excess viscosity \( \Delta \eta (T, \rho) \) on the internal energy density is taken into account (figure 1).
\[ \eta_{\text{tot}}(T, P) = A \exp\left(\alpha \frac{x_0}{x_0 - x}\right) + B \left(\frac{T}{T_C}\right)^{0.25} \left(\frac{x}{x_0}\right)^{0.5} \exp(-\beta x / x_0) + \eta_m, \]  

(1)

Here \( \eta(T, P) \) is viscosity of the system in a given state, \( \mu \text{Pa s} \); \( T, P, V \) are temperature, pressure and specific volume of the system; \( \eta_{\text{en}}(T, \rho) = \eta_0(T) \Psi(h \rho) \) is Enskog viscosity of a hard sphere system, \( \eta_0(T) \) is viscosity of a dilute gas, \( \mu \text{Pa s} \); \( \Psi(h \rho) \) is density function; \( \rho = 1/V \) is substance density; \( x = \Delta U / V \) is internal-energy density; \( \Delta U = U_\text{a}(T) - U(P, T) \) is interaction energy; \( U(P, T) \) is inner energy of the system in a given state; \( U_\text{a}(T) \) is inner energy of an ideal gas; \( x_0 = H_0^0 / V_0 \), \( H_0^0 = U_\text{a}(0) - U(0, 0) \) is evaporation enthalpy at \( T \to 0 \) and \( P \to 0 \); \( V_0 = \lim_{T \to T_C, P \to 0} (V(T, P)) \) is specific volume of substance at absolute zero; \( T_C \) is temperature at the critical point, \( A, \alpha, B, \beta \) are individual adjustable substance constants.

Figure 1. Dependence of the excess viscosity \( \Delta \eta \) on the internal-energy density \( \Delta U/V \):

1-9 - experimental data of [3-5], [10-15], 10 - calculation by \( \Delta \eta = A \exp\left(\alpha \frac{x_0}{x_0 - x}\right) \).

The excess viscosity, where the “field” mechanism of momentum transfer (the first term in equation (1)) prevails, is found as the difference between viscosity of a substance in a given state and viscosity of a system of hard spheres according to Enskog formula \( \eta_{\text{en}} \). In the region of transition from a rarefied to a moderately dense gas, the contribution to the excess viscosity (the second term in equation (1)) is conditioned by the mixed mechanism of momentum transfer: the collisional (according to Enskog) and the field ones. The third summand in equation (1) contributes to the momentum transfer during particle “collisions”.

The equation (1) includes four individual adjustable coefficients that must be found from experimental or tabular viscosity data. In our calculations it was assumed that \( b = V_0 \). The physical parameters \( V_0 \) and \( H_0^0 \) are usually given in Tables of the thermodynamic properties of \( \text{N}_2 \). If there are no data for the enthalpy of evaporation, hypothetical volume of supercooled liquid, the number of adjustable parameters can increase to six: \( A, \alpha, B, \beta, V_0, \) and \( x_0 \).
The density and internal energy of nitrogen, included in the equation for determining the viscosity coefficient, were calculated using the developed unified low-parametric equation of state for calculating the thermodynamic characteristics of substances [9]. This equation includes 10 adjustable coefficients [9].

3. Analysis of obtained results

The comparison of experimental and tabulated data on nitrogen viscosity revealed that the largest uncertainties are in the region of high pressures and low temperatures, and the difference between these data at temperatures of about 100 K and below and pressures of up to 50 MPa can be as high as ±7%. Therefore, to develop equation (1), we used the most verified experimental [3-5], [10-15] and tabular data [2] in the considered range of temperatures and pressures.

To calculate the Enskog coefficient of viscosity of a hard sphere system $\eta_{en}$, we used the equation for the viscosity of a dilute gas from [6]. The calculations used the value of $H^0_0$ obtained in [1]. The coefficient $V_0$ was determined according to the Filippov - Timmermans rule $V_0/V_C = Z_C$, where $Z_C$ is the compressibility factor. The results of calculating nitrogen viscosity are shown in the Table and in figures 2–4. The Table gives the physical parameters of nitrogen (parameters of the characteristic points) and the values of $A$, $\alpha$, $B$, $\beta$, $H^0_0$, $V_0$, and $x_0$ coefficients, required for calculating the viscosity by equation (1), the ranges of description parameter, as well as the rms deviations of the calculated nitrogen viscosities from the tabulated values [2].

Table. Physical parameters of nitrogen; coefficients $A$, $\alpha$, $B$, $\beta$, $H^0_0$ and $V_0$ in equations (1) for calculating viscosity; ranges of the description parameters: by the temperature $\Delta T$, pressure $\Delta P$, mean-square $\sigma$ and weighted-mean AAD deviations of the calculated viscosity values from the tabulated values [2].

| Parameters | Value |
|------------|-------|
| $T_C$ (K)  | 126.192 |
| $P_C$ (MPa) | 3.3958 |
| $\rho_c$ (kgm$^{-3}$) | 313.3 |
| $A$ ($10^{-12}$ s) | 0.191003 |
| $\alpha$ | 0.700986 |
| $B$ | 8.01 |
| $\beta$ | 22.001 |
| $x_0$ (MPa) | 240.0 |
| $H^0_0$ (Jg$^{-1}$) | 229.995 |
| $V_0$ (cm$^3$g$^{-1}$) | 0.9237 |
| $\Delta T$ (K) | 70...1000 |
| $\Delta P$ (MPa) | 0.1...50 |
| $\sigma$, % | 2.101 |
| AAD, % | 1.0048 |

Figure 2 shows the calculated values of viscosity coefficient by the formula (1) as a function of the temperature and pressure on the isobars in comparison with the tabular data from [2]. Figures 3-4 shows deviations of the values of nitrogen viscosity calculated by (1) from the tabular [2] and experimental data [3-5], [10-15].
Figure 2. Comparison of nitrogen viscosities (line), calculated by formula (1) with tabular data of [2] (symbols) on isobars. 0.1 (1), 1(2), 4(3), 8(4), 10(5), 20(6), 50(7) MPa.

Figure 3. The deviations of the experimental data from calculated according to (1) viscosity values for nitrogen depending on the temperature on isobars 1-Clarke (1969); 2- Diller (1983); 3- Evers (2002); 4 - Interbeek (1969); 5 - Michels (1931); 6 - Vogel (2012); 7 - Golubev (1970); 8 - Seib (2006); 9 - Humberg (2018).

When comparing the calculated results according to the equation (1) and tabular data from [2], the largest deviations are 3 - 4. 5%. The deviations of experimental data from other literary sources are within the same limits. In the region of $T_C$ and $P_C$, as well as at high pressures and low temperatures, there is an increase in deviations between the tabulated [2] and calculated viscosity values (Figure 4). However, in this region, the experimental and reference data obtained by other authors diverge considerably.
Figure 4. The relative deviations of the tabular of [2] and experimental literature data from calculated according to (1) viscosity values for nitrogen depending on the temperature on isobars 1-Lemmon (2004); 2 - Seibt (2006); 3 - Vogel (2012); 4 - Golubev (1970); 5 - Diller (1983).

In addition, it should be emphasized that the uncertainties in calculating the viscosity coefficient by equation (1), consists of the error caused by the approximate nature of the unified equation of state for liquid and gas, used to calculate the density and internal energy, and the error in the approximate nature of equations (1).

4. Conclusions
A low-parametric unified equation for the viscosity of nitrogen containing 4 individual adjustable coefficients ($A, \alpha, B, \beta$) is derived. The range of validity of this equation for nitrogen is from 70 K to 1000 K at pressures of up to 50 MPa. The equation represents or predicts experimental viscosity data within their experimental uncertainties.

Acknowledgments
The research was carried out within the framework of the state assignment to the IT SB RAS 121031800219-2.

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