NONPARAMETRIC RECONSTRUCTION OF ABELL 2218 FROM COMBINED WEAK AND STRONG LENSING

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ABSTRACT

We apply a new nonparametric technique to reconstruct, with uncertainties, the projected mass distribution of the inner region of Abell 2218 using combined strong and weak lensing constraints from multiple-image systems and arclets with known redshifts. The reconstructed mass map broadly resembles previous, less detailed parametric models, but when examined in detail it shows several substructures not necessarily associated with light but strongly required by the lensing data. In particular, the highest mass peak is offset by \( \sim 30 \, h_{\text{50}}^{-1} \) kpc from the main light peak, and projected mass-to-light ratio in the directions of different cluster galaxies varies by at least a factor of 10. On comparison with mass estimates from models of the X-ray emitting gas, we find that the X-ray models underpredict the enclosed mass profile by at least a factor of 2.5; the discrepancy gets worse if we assume that mass traces light to the extent allowed by the lensing constraints.

Key words: dark matter — galaxies: clusters: individual (Abell 2218) — gravitational lensing — X-rays

1. INTRODUCTION

Galaxy clusters are thought to be the largest and most recently assembled gravitationally bound entities in the universe, and thus an accurate determination of their masses is of utmost importance for a full understanding of the formation and evolution of cosmic structures and for mapping of dark matter in large scales and constraining cosmological parameters. Traditional methods for inferring mass distributions in clusters are (1) dynamical methods, in which the observed line-of-sight velocity distribution of the luminous cluster galaxies is used in conjunction with the virial theorem, and (2) X-ray methods, in which the investigation of the X-ray emission of the intracluster hot gas is used to trace the cluster potential. Both these traditional methods depend on restrictive assumptions about the geometrical and dynamical states of clusters. A more sophisticated method, which proved to be very reliable and independent of any prior assumptions, is gravitational lensing.

Theoretically, the above three methods have to yield the same cluster masses if the clusters are dynamically relaxed, but recent literature shows a lot of discussion and disagreement of cluster masses derived from observed velocity dispersions, observations of X-ray emitting gas, and gravitational lensing (Miralda-Escude & Babul 1995; Wu & Fang 1997). Early studies based upon a few selected clusters claimed a mass estimate discrepancy of at least \( \sim 2-3 \). This discrepancy can well be attributed to projection effects, nonthermal pressure, or the fact that the specific cluster is still in the formation era. Evidently, the problem is relevant to the precise determination of mass distribution and to the dynamical evolution of those clusters. For clusters that are still in an ongoing merging phase, i.e., not in hydrostatic equilibrium, the hot gas does not follow the gravitational potential of the cluster, and therefore the X-ray cluster mass is uncertain and should be different from the gravitational lensing-derived mass and/or even the virial mass.

Within gravitational lensing, there are two distinct methods for inferring masses from lensing observations: (1) the parametric model-fitting method for the strong lensing region (giant arcs and multiple images) and (2) statistical distortion method for the weak lensing regions (weakly distorted single arclets). The parametric model fitting starts from the innermost regions of galaxy cluster, where lensing information is richest and extends outward. It requires fitting parametrized profiles to cluster-sized galaxies (Kneib et al. 1996, hereafter KESCS96). It works efficiently with one or two multiple-image systems but becomes difficult to implement with several multiple-image systems spanning outward, and hence the fitted model becomes nonunique. On the other hand, the statistical distortion method starts from regions where the distortions are very tiny and weak, i.e., outer regions of galaxy clusters, and goes inward. It requires averaging statistically the ellipsities of background galaxies over patches of sky, typically \( \sim 20' \) in size, to trace the cluster mass distribution from its shear field (Kaiser & Squires 1993, hereafter KS93; Kaiser 1995; Schneider & Seitz 1995). Methods based on the KS93 algorithm suffer from a global invariance transformation known as the “mass-sheet degeneracy” and also cannot be used to probe the inner regions of rich clusters because the lensing properties change very rapidly over that sampling scale.

In intermediate regions, with no occurrence of multiple images but having highly distorted arclets, neither of the above methods can be used. However, several attempts to reconstruct the mass distribution in this region were made (Kaiser 1995; Schneider & Seitz 1995; Seitz & Schneider 1996), but their formulation of the problem made it nonlin-
ear and still suffers from the so-called mass-sheet degeneracy and boundary effects.

Obviously, the problem of mapping the cluster mass distribution from gravitational lensing requires an independent nonparametric method that can be used simultaneously in regions with varying lensing strength. The required method should disentangle the systematic effects that plagued the existing methods encoded in forms of nonuniqueness.

In this paper we present a new and general reconstruction technique that combines regions with varying lensing strength and overcomes the drawbacks of the existing methods. The technique is basically an extension of the nonparametric cluster inversion described by AbdelSalam et al. (1998, hereafter Paper I) for the strong lensing regime to incorporate the extra information encoded in the observations of single/weakly distorted arclets. The extension results in a method that combines strong and weak lensing data in a mass reconstruction and overcomes the mass-sheet degeneracy. In general, the method is nonparametric and similar to the weak lensing (or statistical distortions) method of KS93, but our formulation of the problem is manifestly linear in all the regimes, and hence, the problem becomes simpler and can thereby easily include the strong lensing regime.

We apply the technique to the spectacular cluster lens A2218 to reconstruct its projected mass distribution, with uncertainties, from combined strong and weak lensing. This cluster has previously been studied through parametric lens modeling of strong lensing and arcs by Kneib et al. (1995) and Kneib et al. (1996); KESCS96, nonparametric inversion from statistical distortions by Squires et al. (1996), and parametric modeling of statistical distortions by Smail et al. (1997).

2. RECONSTRUCTION METHOD

The method we will follow here is a free-form or nonparametric one that reconstructs a pixelated mass distribution.

It is basically the same as in Paper I, except that (1) in this work we implement constraints from weak, as well as strong, lensing whereas the earlier work implemented only strong lensing and described the extension to weak lensing, and (2) this paper uses Gaussian pixels whereas Paper I used square pixels—the difference is tiny for the results of the following section. Since Paper I already has a full description, we will only summarize the technique here.

2.1. Pixelization of the Mass Distribution

The lens plane is divided into \( N \times N \) pixels with interpixel distance \( a \). The \( mn \)-th pixel is a Gaussian tent with dispersion \( a/2 \) and peak height \( \kappa_{mn} \). That is to say, if the \( mn \)-th pixel is centered at \( \theta_{mn} \), it has a mass profile of

\[
\kappa_{mn} \exp \left( \frac{-2(\theta - \theta_{mn})^2}{a^2} \right).
\] (1)

Hereafter we refer to \( a \) as the pixel size. We measure \( \kappa_{mn} \) in units of the critical density \( \Sigma_{\text{crit}} \) for sources at infinity. Thus the total mass is

\[
M_{\text{total}} = \frac{a^2 \pi}{2} \Sigma_{\text{crit}} \sum_{mn} \kappa_{mn}.
\] (2)

The appropriately scaled arrival time of a light ray from a background source at an unlensed angular position \( \beta \) via a point \( \theta \) in the pixelated lens plane to the observer is

\[
\tau(\theta) = \frac{1}{2} (\theta - \beta)^2 - \frac{D_{ds}}{D_s} \sum_{mn} \kappa_{mn} \psi_{mn}(\theta),
\] (3)

where

\[
\psi_{mn}(\theta) = \frac{1}{\pi} \int \exp \left( \frac{-2(\theta' - \theta_{mn})^2}{a^2} \right) \ln |\theta - \theta'| d^2 \theta'.
\] (4)

Thus, \( \kappa_{mn} \psi_{mn}(\theta) \) represents the contribution of the \( mn \)-th pixel to the total gravitational potential of the lens. Its derivatives with respect to \( \theta \) represent contributions to the bending angle and amplification components.

See the Appendix for explicit expressions for the function \( \psi_{mn} \) and its derivatives.

2.2. Lensing Observables and Constraint Equations

The basic approach of our technique is that the lens equation \([\nabla \cdot \tau(\theta) = 0]\) and shape parameters or distortions of gravitationally lensed images \([\nabla^2 \tau(\theta) = 0]\), at all values of \( \theta \) corresponding to the observed image locations, are considered rigid linear constraints on the mass distribution.

Writing the lens equation at an observed image location \( \theta_i \)

\[
\beta = \theta_i - \frac{D_{ds}}{D_s} \sum_{mn} \kappa_{mn} \nabla_{\theta} \psi_{mn}(\theta_i),
\] (5)

each image supplies us with a two-component constraint equation. But we have to solve for the unknown source position \( \beta \). Thus the number of constraints on the mass distribution from multiple images is \( 2(n_{\text{sources}} - 1) \). These strong lensing constraints are linear equality constraints.

The constraints from the observed shape parameters of distorted images (arclets) can be obtained from the Hessian of the arrival-time function. If an arclet at \( \theta_i \) is observed to be stretched along the direction \( \theta_i' \) by at least a factor of \( \epsilon \) compared with the perpendicular direction \( \theta_i' \), then by considering the \( \theta_i' \) and \( \theta_i' \) components of the inverse amplification matrix we have

\[
\epsilon \left| \frac{\partial^2}{\partial \theta_i' \partial \theta_i'} \tau(\theta_i) \right| \leq \left| \frac{\partial^2}{\partial \theta_i' \partial \theta_i'} \tau(\theta_i) \right|.
\] (6)

If we can independently infer the image parity, we can remove the absolute value signs in equation (6), which will then supply us with a linear constraint equation. But we have to solve for the unknown source position \( \beta \). Thus the number of constraints on the mass distribution from multiple images is \( 2(n_{\text{sources}} - 1) \). These strong lensing constraints are linear equality constraints.

While constraints in the form of equation (6) can be used for arclets in multiply imaged systems (and we do so), they are most useful for singly imaged arclets (weak lensing). For statistical distortions the form of the constraint stays exactly the same. If the shear field is known accurately enough, the inequality can become an equality and can be supplemented by the additional constraint

\[
\frac{\partial^2}{\partial \theta_i' \partial \theta_i'} \tau(\theta_i) = 0.
\] (7)

In our work the weak lensing constraints are expressed in implicit form (eq. [6]), whereas in previous work (e.g., KS93; Kaiser 1995; Seitz & Schneider 1996) it is usual to express these in explicit form. The advantage of the implicit
by themselves are insufficient to constrain the lens uniquely. The fact that the number of pixels leaves a vast family of mass distributions, all perfectly constrained with the lensing observations. It is now necessary to add more information based on some criteria for physical plausibility.

As discussed in Paper I, it is particularly interesting to consider mass distributions that minimize

$$\sum_{mn} \left[ (\sum_{mn'} \kappa_{mn'}) L_{mn} - \kappa_{mn} \right]^2 + \sigma^4 \sum_{mn} (\nabla^2 \kappa_{mn})^2,$$

where $L_{mn}$ represents the light profile normalized to unit total luminosity. In equation (10), the first term tends to minimize the $M/L$ variations, while the second term smooths the mass map on scales of $\leq \sigma$. Such minimization may be considered as regularization with respect to the light distribution and the smoothing scale $\sigma$ and is readily implemented via standard numerical algorithms, such as the NAG routine E04NFF. It serves two purposes: (1) as a test whether light is indeed a fair or biased tracer of mass, and (2) a basis for Monte Carlo simulations to estimate uncertainties in the mass map. We go into details in § 3, but for now we emphasize strongly that our technique considers the light distribution as a secondary information subservient to the rigid constraints from the observed lensing data.

3. Lensed Observables in Abell 2218

Abell 2218 is an exceptionally rich lensing cluster at redshift $z = 0.175$, which hosts seven multiple-image systems and over 100 arclets. In the present paper, we will use all the secure information that lensing provides in this first application of combined strong and weak lensing. Our analysis of A2218 is based on the archival Hubble Space Telescope (HST) Wide Field Planetary Camera 2 (WFPC2) images, and we take the redshifts of the resolved images from Ebbels et al. (1998).

Throughout this paper we use $\Omega = 1; \Lambda = 0; H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$, which corresponds to an angular size of $1'' = 3.84$ kpc at the redshift of Abell 2218.

To refer to individual clusters and background galaxies, we will follow the three-digit numbering scheme of Le Borgne et al. (1992); but for objects referred to often we adopt simpler names below.

3.1. Multiply Lensed Features

Of the multiply imaged background galaxies, we use three that have secure spectroscopic redshifts and one that has a photometric redshift. Positions and redshifts are listed in Table 1 and plotted in Figure 2.

The "a" system ($z = 2.515$)—The spectacular arc system No. 384 clearly shows an internally symmetric pattern of

![Diagram](image)

**Table 1**

| ID   | $z$  | Image | $x$ (arcsec) | $y$ (arcsec) |
|------|------|-------|--------------|--------------|
| 384  | 2.515| a1    | 14.5         | 19.1         |
| 328  | 2.515| a2    | 17.0         | 15.4         |
| 328  | 0.702| b1    | -14.7        | 16.1         |
| 389  | 0.702| b2    | -17.8        | -15.4        |
| 289  | 1.034| c1    | -61.7        | 7.2          |
|      | 1.034| c2    | -61.5        | 0.6          |
| 730  | 1.1  | d1    | -75.2        | -1.4         |
| 1.1  | 1.1  | d2    | -74.6        | -6.0         |
| 1.1  | 1.1  | d3    | -73.6        | -9.0         |
unresolved knots that characterizes the system as a fold arc comprising two merging images (which we will call a1 and a2) with opposite parities. It is identified as an image of a background star-forming galaxy with the visible knots representing the H\textsc{ii} regions.

The "b" system ($z = 0.702$).—The identification of the multiple images in this system is somewhat complicated. The red arc No. 359 was initially identified by Pello et al. (1992) as an image of a background spheroidal galaxy. It was later interpreted by Kneib et al. (1995) using a ground-based image as two merging images forming a fold arc with No. 328 as a counterimage. However, the improved resolution of HST revealed that such a configuration for arc No. 359 cannot be true since a faint extension (KESCS96) connecting it to No. 337 is now revealed, thus strongly suggesting that it is also a counterimage. On the other hand, Nos. 337 and 389 yield a similar color to No. 359, thus indicating that No. 389 is another counterimage of No. 359. Since such a system is complicated and no simple model can explain it, we tried various input configurations. The only plausible solutions can be obtained when Nos. 328 and 389 (which we call b1 and b2) are considered as two images from the same source, and No. 359 (which we call "b3?") is an arclet arising from another source, possibly a different component of the same lensed galaxy.

The "c" system ($z = 1.034$).—The blue arc No. 289 has a large amount of internal structure in the southern part, which is highly magnified but appears to be slightly distorted and thus characterized as singly imaged while the tail visible on the north is highly elongated with tiny breaks and definitely multiple imaged. Nevertheless, this complex configuration can be explained by a background source lying near a cusp caustic but with a high portion of it being outside the cusp and observed as the southern end of the arc, while the portion within the caustic is observed as the northern end. We consider the northern part as two merging segments (and call these c1 and c2), and consider the southern part (which we call "c3?") as a singly imaged arclet.

### TABLE 2

| ID    | $z$     | $x$    | $y$    | $\theta$ | $\epsilon$ |
|-------|---------|--------|--------|----------|------------|
| 145   | 0.628   | -2.5   | 103.9  | 26.57    | 2.5        |
| 159   | 0.564   | 5.1    | 105.4  | 80.54    | 2.0        |
| 158   | 0.723   | -41.8  | 62.9   | 19.65    | 2.2        |
| 242   | 0.635   | 30.3   | 88.8   | 117.9    | 5.0        |
| 231   | 0.563   | -9.7   | 57.8   | 29.48    | 4.0        |
| 381   | 0.521   | 35.9   | 34.6   | 157.4    | 2.5        |
| 317   | 0.474   | -7.3   | 27.5   | 167.7    | 1.5        |
| 306   | 0.450   | -97.5  | -44.8  | 135.0    | 1.8        |
| 464   | 0.476   | 10.6   | -23.3  | 45.0     | 6.0        |
| 467   | 0.475   | 17.0   | -19.7  | 48.01    | 4.0        |
| 297   | 0.450   | -21.7  | 23.2   | 65.22    | 5.0        |
| 456   | 0.538   | 20.6   | -11.6  | 49.76    | 3.5        |
| 238   | 0.635   | -44.4  | 20.1   | 49.40    | 4.0        |
| 273   | 0.800   | -41.4  | 16.6   | 68.75    | 2.5        |
| 205   | 0.693   | -72.1  | 16.3   | 90.00    | 1.2        |
| 431   | 0.675   | 39.9   | 21.9   | 110.0    | 6.0        |
| 444   | 1.030   | 24.6   | -2.8   | 75.96    | 7.0        |
| 359   | 0.702   | -20.5  | -3.7   | 98.75    | 5.0        |
| 289   | 1.034   | -63.7  | -8.9   | 77.20    | 6.0        |

Notes.—The positions $x$ and $y$ are in arcseconds, and the position angle $\theta$ is in degrees, while $\epsilon$ is a lower bound on the elongation. The final two arclets, b3? and c3?, are probably parts of the corresponding multiple-image systems in Table 1, but we have not used these as multiple-image constraints. We have taken all the arclets as minima of the arrival time except for b3?, which is evidently a saddle.

The "d" system ($z = 1.1 \pm 0.3$).—This is a faint thin arc, known as No. 730, with a number of bright knots visible. Three elongated components (which we call d1, d2, and d3) can be easily distinguished along the arc, which characterizes it as a cusp arc.

### 3.2. Secure Single Distorted Arclets

Ebbels et al. (1998) have spectroscopically measured the redshifts of a number of arclets with different degrees of
accuracy in the HST WFPC2 area. We use all the arclets for which they have quality 1 redshifts, and we use arclets with quality 2 redshifts in regions where quality 1 redshifts are not available. Table 2 gives details of the 18 arclets used. The last two arclets constitute parts of multiple-image systems.

3.3. Luminosity Distribution
The apparent magnitudes in the R band of the foreground galaxies are taken from Le Borgne et al. (1992). We considered only those with apparent magnitudes ≤ 20, and we find that there are 33 of them enclosed within the HST image. The pixelated light distribution $L_{mn}$ is obtained by replacing each of the 33 galaxies by a Gaussian light profile of dispersion 10". Figure 3 shows a contour plot of this smoothed luminosity distribution.

4. RECONSTRUCTED MASS DISTRIBUTIONS AND UNCERTAINTIES
Our optimal mass map of A2218 is shown in Figure 4. It uses a pixel size of 2.29 and regularizes with respect to the light distribution with smoothing scale $\sigma = 6\arcsec$, while strictly obeying all the strong and weak lensing constraints. (We found $\sigma = 6\arcsec$ to be the smallest value that eliminated obvious artifacts in the mass map. We use this value throughout this paper except where otherwise noted.) Our mass map is broadly similar to the previous parametric models (K95; KEKCS96), which show that the cluster is clearly bimodal, with one clump being less massive and with detailed substructures implied by the lensing data.

It is of interest to examine also other reconstructed mass maps obtained using different criteria. For example, one can reconstruct a mass map by assuming zero-light distribution for the cluster (i.e., setting $L_{mn} = 0$ in eq. [10]); this is shown in Figure 5 and is roughly the minimal matter distribution needed to reproduce the data. Or one can reconstruct a mass map with a much higher smoothing parameter; Figure 6 shows a mass map using $\sigma = 19\arcsec$. It is reassuring to see that all the main features are still present in these cases.

The spikes visible in all our mass maps are due to the fact that we are using local constraints from sparsely sampled data, i.e., arclets. The peaks of the spikes are very robust between different reconstructions, but the wings are very variable. As a result the total mass shows large variation, although the allowed variation is quite well bounded above and below. Statistical distortion maps going to larger radii would, we expect, constrain the total mass much better and hence reduce this problem. We leave this extension for future work, hopefully with several HST pointings.

Our reconstruction technique naturally lends itself toward calculating error estimates on the pixelated mass.

Fig. 4.—Mass distribution of Abell 2218, reconstructed using strong and weak lensing constraints and regularizing with respect to the light distribution and a smoothing scale of 6\arcsec. The density is in units of the critical density for sources at infinity, $\Sigma_{\text{crit}} = 3.1 \times 10^{14} h_0^{-2.5} M_{\odot} \text{arcsec}^{-2}$; contours are in steps of 0.1. Figures to follow in this paper always use the same units, and they use the same contour steps unless otherwise noted. The total mass in the field is $2.75 \times 10^{14} h_0^{-2.5} M_{\odot}$.
distribution. We use a Monte Carlo procedure, randomizing the positions of the cluster galaxies or by rotating the entire light distribution by various angles and constructing an ensemble of new mass maps (see Fig. 7). The fractional dispersion

\[ \Delta \kappa_{mn} = \left( \frac{\langle \kappa_{mn}^2 \rangle}{\langle \kappa_{mn} \rangle^2} - 1 \right)^{1/2} \]  

(11)

over this ensemble is a measure of the pixel-by-pixel fractional uncertainty in the mass map. This is shown in Figure 8 as a contour plot of the computed \( \Delta \kappa_{mn} \). Regions in the lens plane enclosed by \( \Delta \kappa_{mn} \leq 0.2 \) are very well constrained. Clearly, the best constrained regions have a high number density of images. In Figure 9, we quantify this by plotting \( \Delta \kappa_{mn} \) binned over circles of radius 10\(^\prime\) versus the number of images enclosed. As expected, the fractional uncertainty decreases with the number of images.

5. MASS-LIGHT DISCREPANCIES

While our mass reconstruction broadly resembles previous parametric models, we find significant differences that contradict a basic assumption of the parametric modeling method, that of modeling the cluster as a smooth mass distribution plus small clumps associated with bright galaxies. Two of these differences are worth discussing in detail.

First, we find that the highest mass peak is significantly offset from the brightest light peak, the center of the cD galaxy No. 391. The offset is highly significant given the uncertainties. To test this further, we did a mass reconstruction in a smaller field around this main peak but with a finer pixel size of 1\(^\prime\). We find (see Fig. 10) that the offset is still persistent and is about 10\(^\prime\) toward the second peak of the cluster. It is interesting that the direction of the offset between the mass and light peaks detected from our modeling is similar to that found by Markevitch (1997) between the temperature and light peak.

Second, we find that the secondary mass peak is not centered on the bright galaxy No. 244. Again, we did a mass reconstruction on a smaller field using 1\(^\prime\) pixels, this time near No. 244; this is shown in Figure 11. The mass distribution in this region, has two peaks in the directions of two galaxies at least 2.5 mag fainter in \( R \) than No. 244, while No. 244 itself is in a valley between these two peaks. This is also highly significant given the uncertainties and is seen both in the main reconstruction and in the “zoom in.” If the three mass concentrations are indeed associated with the identified galaxies, the 2.5 mag difference implies that the associated \( M/L \) ratios vary by more than a factor of 10.

These new results we reported are robust to the pixel size, shape, and light distribution of the cluster.

6. COMPARISON WITH X-RAY MODELS

The X-ray emission by clusters is usually modeled assuming that the underlying potential of the cluster is spherically symmetric and that the hot gas is in hydrostatic
equilibrium. It is important to test these assumptions through comparison with lensing, which does not depend on these assumptions. Miralda-Escudé & Babul (1995) argued that the strong lensing data could not be reconciled with an equilibrium spherical model for the X-ray emitting at the observed temperature—to agree with lensing the gas temperature would have to be much higher, or equivalently, lensing requires at least twice as much mass as the gas.

Fig. 6.—Mass map using much more smoothing: $\sigma = 19''$

Fig. 7.—Contour map of the projected mass distribution. This corresponds to Fig. 4 and is plotted here for comparison with Fig. 8.

Fig. 8.—Fractional uncertainty of the mass distribution. The crosses mark the position of the two dominant cD galaxies in A2218.
model. They suggested that the gas might be partly supported by turbulent motion or by magnetic fields or might be multiphased. Another possibility is that the cluster is still undergoing a merging phase, and hence the gas would not be expected to trace the gravitational potential.

In Figure 12 we plot the enclosed projected mass from a model for the hot gas by Allen (1998) and enclosed mass from our reconstructions. The lowest enclosed mass from lensing we obtained for the “zero-light” reconstruction is shown in Figure 5; the highest is for our optimal reconstruction (Fig. 4), which tends to minimize $M/L$ variation and thus extrapolates mass into regions with current lensing data. Other models correspond to various other regularizations. We see from Figure 12 that the mass discrepancy is a factor of 2.5 for even the unrealistic-looking model of Figure 5.

7. ANALYSIS OF INDIVIDUAL MULTIPLE-IMAGE SYSTEMS

We have examined the arrival-time surfaces, critical curves, and caustics for all the multiple-image systems and
some of the arclets. Arrival-time contours are a good way of verifying that a mass distribution reproduces the given image position properties. In this section we discuss two of the multiple-image systems in detail and report any predicted counterimages.

7.1. The a1-a2 System

The arrival-time surface (see Fig. 13) reproduces the image positions of the two segments a1 and a2 comprising the fold arc. Moreover, the arrival-time surface predicts a third image 20 above the dominant mass clump, very close to the observed arclet No. 468, also predicted by K96 as a counterimage for the same arc. The mirror symmetry seen across the arc a1-a2 implies that the critical curve is passing through them. Exploring the critical curves at the redshift (z = 2.515) of the system confirms that a1-a2 is indeed a fold arc resulting from a background source lying close to a beak-to-beak caustic. The critical curves and caustics implied by our mass map are shown in Figure 14.

7.2. The b1-b2 System

The arrival-time surface (see in addition to Fig. 15), reproducing the positions of b1 and b2 as required, also predicts an image near the observed position of b3 but not exactly coinciding with it, which suggests that b3 is an image of a different component of the same lens galaxy. Figure 16 shows the critical curve caustics at the redshift (z = 0.702) of this system. The predicted position of the source is in a region where the caustics are very convoluted and this would result in a complex image configuration. As an auxiliary test, we inspect the arrival-time surface for the arclet b3 itself (see Fig. 17). The position of the arclet b3 is, of course, exactly reproduced, and moreover, four extra images are predicted. The position of two of the predicted images matches fairly with those of the b1 and b2. This result strongly supports the scenario that the images b1, b2, and No. 337 are counterimages of the arclet b3.

7.3. Statistical Magnification Map

Since most work on cluster lensing at present is based on statistical distortion of images, it is interesting to examine the statistical magnification map. We calculate the statistical magnification as follows: We first divide the field into square regions 10 or 20 in size. Within each square we compute the magnification matrix at 25 random points, using our optimal mass reconstruction and redshifts ran-

![Fig. 14.—Critical curves and caustics at the redshift of the a1-a2 system (z = 2.515). The top panel shows the critical curves with the two circles marking the positions of the two merging images. The bottom panel is the corresponding caustics with the circle marking the predicted source position.](image)

![Fig. 15.—Arrival-time surface for the b1-b2 system at z = 0.702. Circles mark the observed positions of b1 and b2.](image)
Fig. 16.—Critical curves (top) and caustics (bottom) at redshift $z = 0.702$. The circles in the top panel mark positions of observed images, while that in the bottom panel marks the predicted position of the source.

domly chosen from those of the multiply imaged systems; of these we discard any points corresponding to axis ratios greater than 5 and average the rest. This averaged matrix we call the statistical magnification. Such a procedure roughly mimics the observational procedure of averaging ellipticities while discarding obvious arclets, though of course it is not the same, because for real data the absolute magnification is not usually available.

Figure 18 shows the statistical magnification we obtained by averaging over $10^\circ$ and $20^\circ$ squares. It illustrates a cautionary fact: in the strong lensing region the statistical magnification is dominated by noise. The reason is that a critical curve might be crossed, and this can change the shear from $\sim 1$ to $\infty$ and back to $\sim 1$ again over a $10^\circ$ scale. Discarding arclets with high-axis ratios does not cure this problem—Figure 18 (left) shows two places with statistical magnification corresponding to axis ratios greater than 5 even though individual location with such high-axis ratios have been excluded; this is because two low-axis ratio magnification matrices on opposite sides of a critical curve can result in a high-axis ratio one if averaged.

We conclude that statistical shear must not be used in the strong lensing region of clusters; instead, constraints from

Fig. 17.—Arrival-time surface for b3? at $z = 0.702$. The circle marks the observed position of b3?.

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We conclude that statistical shear must not be used in the strong lensing region of clusters; instead, constraints from
8. CONCLUSIONS

The present paper is, to our knowledge, the first to combine constraints from strong and weak lensing in cluster mass reconstruction. The method we describe recovers a pixelated mass distribution that strictly obeys constraints from lensing observations and uses the light distribution of the cluster as subordinate information that may be overridden by lensing constraints. We explore the projected (dark) matter distribution of Abell 2218 with an unprecedented level of detail. Our mass map shows that the primary mass peak is offset from the light peak and that projected mass-to-light variations of galaxy-sized components are severely inconsistent with the galaxy M/L scaling deduced from dynamical arguments. In general, these imply that mass does not follow light on a range of length scales in A2218. We also confirm and elaborate a previous result—that current X-ray mass models significantly underestimate the mass of a cluster that is able to reproduce the observed lensing. Our results conclude that mass estimates from lensing are at least 2.5 times that from X-ray models. This suggests that in at least some clusters intracluster hot gas does not trace the gravitational potential.

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APPENDIX

INTEGRALS OVER GAUSSIAN PIXELS

Integrating equation (4) yields the coefficient of the $mn$th pixel's contribution to the potential:

$$\psi_{mn}(\theta_x, \theta_y) = \frac{a^2}{4} \left[ \ln(u) - \text{Ei}(-u) + \gamma_E \right], \quad u \equiv \frac{2\theta^2}{a^2}, \quad (A1)$$

where Ei denotes the exponential integral and $\gamma_E$ is Euler's constant. Thus the first derivatives of the coefficients of the deflection potential, i.e., the components of the deflection angle, are

$$\partial_x \psi_{mn}(\theta_x, \theta_y) = \frac{\theta_x}{u} \left[ 1 - \exp(-u) \right], \quad \partial_y \psi_{mn}(\theta_x, \theta_y) = \frac{\theta_y}{u} \left[ 1 - \exp(-u) \right]. \quad (A2)$$

The second derivatives of the potential are

$$\partial_{xx} \psi_{mn}(\theta_x, \theta_y) = \frac{1}{u} \left( 1 - \frac{2\theta_x^2}{\theta^2} \right) \left[ 1 - \exp(-u) \right] + \frac{2\theta_x^2}{\theta^2} \exp(-u),$$

$$\partial_{yy} \psi_{mn}(\theta_x, \theta_y) = \frac{1}{u} \left( 1 - \frac{2\theta_y^2}{\theta^2} \right) \left[ 1 - \exp(-u) \right] + \frac{2\theta_y^2}{\theta^2} \exp(-u),$$

$$\partial_{xy} \psi_{mn}(\theta_x, \theta_y) = \frac{2\theta_x \theta_y}{\theta^2} \left\{ \exp(-u) - \frac{1}{u} \left[ 1 - \exp(-u) \right] \right\}. \quad (A3)$$
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