Model of the polarization extinction ratio change due to multiple reflection of laser radiation from the faces of the terbium-gallium garnet crystal in Faraday rotator

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Abstract. In this work we investigated the phenomena in the Faraday rotator, that can affect the polarization state of transmitting laser radiation. We considered two factors that can cause deterioration of radiation extinction ratio and, therefore, isolation properties of Faraday cell: multiple reflections of radiation from the faces of the TGG crystal and imperfection of the laser beam input relative to the central axis of the crystal. We were able to calculate the impact of these factors on polarization extinction ratio of laser radiation transmitting through the crystal.

1. Introduction

Nowadays, much attention is paid to the development of laser sources with different wavelengths based on the effect of nonlinear frequency conversion of the polarized radiation. The efficiency of this conversion depends on the extinction ratio and the direction of polarization, which is controlled by the terbium-gallium garnet (TGG) magneto-optical crystal, placed in a magnetic field. TGG crystals are widely used for optical isolators, rotators and modulators, based on Faraday effect. When linearly polarized optical radiation passes through this crystal in a magnetic field, the polarization plane rotates by an angle proportional to the magnitude of the magnetic field and the crystal length. However, after passing the TGG crystal radiation can partially depolarize, what can lead to the malfunction of the optical element. In the literature, the mechanisms of radiation depolarization due to the effect of photoelasticity and temperature dependence of the Verdet constant under conditions of inhomogeneous heating of the crystal by laser radiation were described in detail [1]. This paper for the first time presents an analysis of the extinction ratio change of the polarization of radiation passing through a TGG crystal, taking into account the effect of multiple reflections from the crystal faces.

2. Mathematical model of change of polarization extinction of laser radiation in TGG crystal

It is known that the Faraday effect is nonreciprocal: when the radiation passes through it twice, it rotates at a double angle. Thus, multiple reflections from the crystal faces lead to the presence of components with different polarization states what can cause depolarization of the laser radiation. Therefore, when using magneto-optical crystals without antireflective (AR) coating or with weak AR coating, this effect can lead to significant deterioration of polarization extinction ratio.
2.1. The effect of multiple reflections of radiation in a magneto-optical element in the case of a small reflection coefficient from the crystal facets.

First, we have studied the effect of multiple reflections in the presence of high-transmission AR coating that provides a reflection coefficient as low as 0.1%. Modelling was performed for linearly polarized radiation with an initial extinction ratio of 40 dB. In the case of low reflectance from crystal facets, we can consider only one additional component – radiation that has experienced two reflections inside the crystal and, therefore, has passed through it three times. Due to the non-reciprocity of the Faraday effect, the corresponding vectors of the second pair of field components \((E_{s2}, E_{p2})\) will be rotated by a triple angle of Faraday rotation \((3\theta)\) relative to the initial position. Consequently, this field vectors will be rotated by a double angle of Faraday rotation \((2\theta)\) relative to the pair of field vectors \((E_{s1}, E_{p1})\) of radiation that passed the crystal without additional reflections as shown on Figure 1. The relations between different field components are the following:

\[
10 \log_{10} \left( \frac{E_{s1}}{E_{p1}} \right)^2 = 40; \quad \left| \frac{E_{s1}}{E_{s2}} \right| = \left| \frac{E_{p1}}{E_{p2}} \right| = 0.001; \quad (1)
\]

The values of the maximum and minimum polarization components in terms of radiation power:

\[
P_{\min} = \left| -E_{s1} \sin \alpha + iE_{p2} \cos \alpha + e^{i\varphi} (E_{s2} \sin(2\theta - \alpha) + iE_{p2} \cos(2\theta - \alpha)) \right|^2 \quad (2)
\]

\[
P_{\max} = \left| E_{s1} \cos \beta + iE_{p2} \sin \beta + e^{i\varphi} (E_{s2} \cos(2\theta - \beta) - iE_{p2} \sin(2\theta - \beta)) \right|^2 \quad (3)
\]

where \(i\) - imaginary unit, \(\varphi\) – optical phase delay between two pairs of radiation components.

In the model, we have taken the value of the Faraday rotation angle \(\theta\) equal to 45° in accordance with the value used in the traditional Faraday isolator. Unequal \(\alpha\) and \(\beta\) angles represents the case when the experimenter adjust polarimeter to the minimum and maximum of radiation power, independently. But, taking this angles equal better represents the effect of depolarization in real Faraday isolators, since directions to the minimum and maximum are assumed to be perpendicular.

To determine the \(\alpha\) and \(\beta\) values, we used the maximum search function in Wolfram Mathematica for equation:

\[
Ext = 10 \log_{10} \left( \frac{P_{\max}}{P_{\min}} \right) \quad (4)
\]

where \(P_{\min}, P_{\max}\) are determined from (2,3) provided that \(\alpha = \beta\), and using relations (1). It was turned out, that in the case of high transmittance AR coating the values of these angles will not exceed 3.5°. Thus, in this case the direction to the minimum and maximum polarization components of the output radiation will practically coincide with the direction of the first pair of field vectors \((E_{s1}, E_{p1})\).
The calculated from (1-3) dependence of the extinction ratio of output radiation on the optical phase delay $\varphi$ between two radiation components is shown on the Figure 2.

![Figure 2. The dependence of the polarization extinction ratio of laser radiation after the TGG crystal on the phase delay between components](image1)

![Figure 3. The dependence of the polarization extinction ratio of laser radiation after the TGG crystal from the phase delay in the absence of antireflective coating](image2)

As can be seen, the extinction ratio change even in the presence of AR coating varies up to 0.85 dB depending on phase delay between two radiation components.

2.2. The effect of multiple reflections of radiation in a magneto-optical element in the case of a high reflection coefficient.

Let us consider another extreme case - the change of extinction ratio in the absence of AR coating, i.e. taking into account Fresnel reflection coefficients from crystal facets (about 11% for TGG crystal). In this approximation, it will be insufficient to consider only one re-reflected component. For better accuracy, we decided to include two more components, which were reflected 4 and 6 times. Therefore, the expressions (2,3) will have the following form:

$$P_{\text{min}} = | -E_{S1} \sin \alpha + iE_{P1} \cos \alpha + e^{i\varphi} (E_{S2} \sin(2\theta - \alpha) + iE_{P2} \cos(2\theta - \alpha)) + e^{i2\varphi} (E_{S2} \sin(4\theta - \alpha) + iE_{P2} \cos(4\theta - \alpha)) + e^{i3\varphi} (E_{S2} \sin(6\theta - \alpha) + iE_{P2} \cos(6\theta - \alpha)) |$$  \hspace{1cm} (5)

$$P_{\text{max}} = | E_{S1} \cos \beta + iE_{P1} \sin \beta + e^{i\varphi} (E_{S2} \cos(2\theta - \alpha) - iE_{P2} \sin(2\theta - \alpha)) + e^{i2\varphi} (E_{S2} \cos(4\theta - \alpha) - iE_{P2} \sin(4\theta - \alpha)) + e^{i3\varphi} (E_{S2} \cos(6\theta - \alpha) - iE_{P2} \sin(6\theta - \alpha)) |$$  \hspace{1cm} (6)

At the same time, we will look for the maximum of the expression (4) where $P_{\text{min}}$, $P_{\text{max}}$ are determined from (5,6) provided that $\alpha = \beta$, and different field components relations are the following:

$$10\log_{10} \left( \frac{E_{S1}}{E_{P1}} \right)^2 = 40; \quad \left| \frac{E_{S1}}{E_{S2}} \right| = \left| \frac{E_{P1}}{E_{P2}} \right| = 0.1;$$  \hspace{1cm} (7)

The calculated from (4-7) dependence of the extinction ratio of output radiation on the phase delay $\varphi$ in the case of absence of AR-coating is shown on the Figure 3. As you can see, the polarization extinction ratio in such a magneto-optical element can both significantly improve and decrease up to 20 dB depending on the phase delay $\varphi$. Proposed model allows determination of laser radiation depolarization for any other facet reflection values. For example, for AR coating with 1% reflection coefficient maximum decrease of polarization extinction ratio can reach 6 dB. However, it should be noted that this model does not take into account the processes of absorption and scattering of light during its propagation through the crystal.
2.3. The effect of the nonideality of the laser beam alignment

In addition, we have calculated how the extinction ratio can be influenced by the fact that the output beam can have a fraction of the radiation that has not passed through the crystal due to imperfect laser beam alignment. So, we considered an ideal Gaussian beam which center was shifted relative to the center of the magneto-optical crystal. Part of radiation passing outside the crystal will not rotate its polarization plane, what result in decrease in the polarization extinction ratio of the output beam. Thus, the relative position of the vectors is shown in Figure 4, where pairs of vectors \((E^\text{ex}_S, E^\text{in}_P)\) and \((E^\text{ex}_P, E^\text{in}_S)\) spatially separated and parameter \(\nu = \frac{P^\text{in}_S}{P^\text{ex}_S + P^\text{in}_S} = \frac{P^\text{in}_P}{P^\text{ex}_P + P^\text{in}_P}\).

To calculate the fraction of the rotated radiation, we integrated the Gaussian power distribution in the beam along the input face of the crystal. The diameter of crystal in the model was 4 mm, the full width at half maximum of the laser beam was 1.5 mm. Then we wrote an expression (8) describing the power dependence on the rotation angle of the output polarimeter \(\phi\), for which we found the minimum and maximum values that will be orthogonal.

\[
P_{\text{rot}}(\phi) = \cos^2(\theta - \phi) + \frac{\nu}{1 - \nu} \cos^2 \phi + \frac{P^\text{ex}_P}{P^\text{ex}_S} \sin^2(\theta - \phi) + \frac{P^\text{in}_S}{P^\text{in}_P} \frac{\nu}{1 - \nu} \sin^2 \phi
\]

(8)

Having this expression, we can obtain the dependence of the extinction ratio on the offset of the Gaussian beam from the center of the crystal, as shown on Figure 5.

![Figure 4. The mutual location of the vectors in the case of imperfection of the plant](image)

![Figure 5. Extinction ratio dependence on laser beam offset \(y_0\) from the centre of the crystal](image)

3. Conclusion

We have demonstrated that polarization extinction ratio of radiation passing through a magneto-optical element can significantly change due to the existence of reflections from crystal faces. We have calculated the limits of such changes for crystal with and without AR coating. It was found that it is important to take into account the phase difference between the different components of transmitted laser radiation. The influence of the laser beam offset from the center of the magneto-optical crystal on the radiation depolarization value was also estimated.

The work was carried out within the framework of the state task.

References

[1] E A Khazanov, “Thermooptics of magnetoactive media: Faraday isolators for high average power lasers”, Physics ± Uspekhi 59 (9), pp, 886 - 909 (2016).