Determination of weak phases $\phi_2$ and $\phi_3$ from $B \to \pi \pi, K \pi$ in the pQCD method

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(November 7, 2018)

I. INTRODUCTION

One of the most exciting aspect of present high energy physics is the exploration of CP violation in B-meson decays, allowing us to overconstrain both sides and three weak phases $\phi_1 (=\beta)$, $\phi_2 (=\alpha)$ and $\phi_3 (=\gamma)$ of the unitarity triangle of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] and to check the possibility of New Physics. The “gold-plated” mode $B_d \to J/\psi K_s$ [2], which allows us to determine $\phi_1$ without any hadron uncertainty, recently measured by BaBar and Belle collaborations [3]: $\phi_2 = (25.5 \pm 4.0)^\circ$. There are many other interesting channels with which we may achieve this goal by determining $\phi_2$ and $\phi_3$ [4].

In this letter, we focus on the $B \to \pi^+ \pi^-$ and $K \pi$ processes, providing promising strategies for determining the weak phases of $\phi_2$ and $\phi_3$, by using the perturbative QCD method.

The perturbative QCD method (pQCD) has a predictive power demonstrated successfully in exclusive 2 body B-meson decays, specially in charmless B-meson decay processes [5]. By introducing parton transverse momenta $k_\perp$, we can generate naturally the Sudakov suppression effect due to the resummation of large double logarithms $\exp\left(-\frac{2k_\perp^2}{m^2}\right)$, which suppress the long-distance contributions in the small $k_\perp$ region and give a sizable average $< k_\perp^2 > \sim \Lambda M_B$. This can resolve the end point singularity problem and allow the applicability of pQCD to exclusive B-meson decays. We found that almost all of the contributions to the matrix element come from the integration region where $\alpha_s/\pi < 0.3$ and the perturbative treatment can be justified.

In the pQCD approach, we can predict the contribution of non-factorizable term and annihilation diagram on the same basis as the factorizable one. A folklore for annihilation contributions is that they are negligible compared to W-emission diagrams due to helicity suppression. However the operators $O_{b,6}$ with helicity structure $(S-P)(S+P)$ are not suppressed and give dominant imaginary values, which is the main source of strong phase in the pQCD approach. Therefore we have a large direct CP violation in $B \to \pi^\pm \pi^\mp, K^\mp \pi^\mp$, since large strong phase comes from the factorized annihilation diagram, which can distinguish pQCD from other models [4].

II. EXTRACTION OF $\phi_2(=\alpha)$ FROM $B \to \pi^+ \pi^-$

Even though isospin analysis of $B \to \pi \pi$ can provide a clean way to determine $\phi_2$, it might be difficult in practice because of the small branching ratio of $B^0 \to \pi^0 \pi^0$. In reality to determine $\phi_2$, we can use the time-dependent rate of $B^0(t) \to \pi^+ \pi^-$ including sizable penguin contributions. The amplitude can be written by using the c-convention:

$$A(B^0 \to \pi^+ \pi^-) = V_{ub}^* V_{ud} A_u + V_{cb}^* V_{cd} A_c + V_{tb}^* V_{td} A_t,$$

$$= V_{ub}^* V_{ud} (A_u - A_t) + V_{cb}^* V_{cd} (A_c - A_t),$$

$$= -(|T_c| e^{i\delta_c} e^{-i\phi_3} + |P_c| e^{i\phi_3})$$

Penguin term carries a different weak phase than the dominant tree amplitude, which leads to generalized form of the time-dependent asymmetry:

$$A(t) = \frac{\Gamma(\bar{B}^0(t) \to \pi^+ \pi^-) - \Gamma(B^0(t) \to \pi^+ \pi^-)}{\Gamma(B^0(t) \to \pi^+ \pi^-) + \Gamma(\bar{B}^0(t) \to \pi^+ \pi^-)}$$

$$= S_{\pi\pi} \sin(\Delta m t) - C_{\pi\pi} \cos(\Delta m t)$$

where

$$C_{\pi\pi} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2}, \quad S_{\pi\pi} = \frac{2Im(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2}$$

satisfies the relation $C_{\pi\pi}^2 + S_{\pi\pi}^2 \leq 1$. Here

$$\lambda_{\pi\pi} = |\lambda_{\pi\pi}| e^{2i(\phi_2 + \Delta \phi_2)} = e^{2i\phi_2} \frac{1 + R e^{i\phi_3} e^{i\phi_3}}{1 + R e^{i\phi_3} e^{-i\phi_3}}$$

PACS numbers: 13.25.Hw, 13.25.Ft
with $R_c = |P_c/T_c|$ and the strong phase difference between penguin and tree amplitudes $\delta = \delta_p - \delta_T$. The time-dependent asymmetry measurement provides two equations for $C_{\pi\pi}$ and $S_{\pi\pi}$ for three unknown variables $R_c$, $\delta$ and $\phi_2$.

When we define $R_{\pi\pi} = \frac{\overline{Br}(B^0 \rightarrow \pi^+\pi^-)}{Br(B^0 \rightarrow \pi^+\pi^-)}|_{\text{tree}}$, where $\overline{Br}$ stands for a branching ratio averaged over $B^0$ and $B^0$, the explicit expression for $S_{\pi\pi}$ and $C_{\pi\pi}$ are given by:

$$R_{\pi\pi} = 1 - 2R_c \cos \phi_1 \cos \phi_2 + R_c^2,$$

(6)

$$R_{\pi\pi} S_{\pi\pi} = \sin 2\phi_2 + 2R_c \sin(\phi_1 - \phi_2) \cos \delta - R_c^2 \sin 2\phi_1,$$

(7)

$$R_{\pi\pi} C_{\pi\pi} = 2R_c \sin(\phi_1 + \phi_2) \sin \delta.$$

(8)

If we know $R_c$ and $\delta$, then $\phi_2$ can be determined from the experimental data on $C_{\pi\pi}$ versus $S_{\pi\pi}$.

Since pQCD provides $R_c = \frac{0.23 \pm 0.07}{0.05}$ and $-41^\circ < \delta < -32^\circ$, the allowed range of $\phi_2$ at present stage is determined as $55^\circ < \phi_2 < 100^\circ$ as shown in Fig. 1. Since we have a relatively large strong phase in pQCD, in contrast to the QCD-factorization ($\delta \sim 0^\circ$), we predict large direct CP violation effect of $A_{cp}(B^0 \rightarrow \pi^+\pi^-) = (23 \pm 7)\%$ which will be tested by more precise experimental measurement in near future. In numerical analysis, since the data by Belle collaboration [8] is located outside allowed physical regions, we considered only the recent BaBar measurement [9] with 90% C.L. interval taking into account the systematical errors:

- $S_{\pi\pi} = 0.02 \pm 0.34 \pm 0.05$ [-0.54, +0.58]
- $C_{\pi\pi} = -0.30 \pm 0.25 \pm 0.04$ [-0.72, +0.12].

The central point of BaBar data corresponds to $\phi_2 = 78^\circ$ in the pQCD method.

![Fig. 1. Plot of $C_{\pi\pi}$ versus $S_{\pi\pi}$ for various values of $\phi_2$ with $\phi_1 = 25.5^\circ$, $0.18 < R_c < 0.30$ and $-41^\circ < \delta < -32^\circ$ in the pQCD method. Here we consider the allowed experimental ranges of BaBar measurement within 90% C.L. Dark areas are allowed regions by pQCD for different $\phi_2$ values.](image1)

![Fig. 2. Plot of $\Delta \phi_2$ versus $\phi_2$ with $\phi_1 = 25.5^\circ$, $0.18 < R_c < 0.30$ and $-41^\circ < \delta < -32^\circ$ in the pQCD method.](image2)

**III. EXTRACTION OF $\phi_3(=\gamma)$ FROM $B^0 \rightarrow K^+\pi^-$ AND $B^+ \rightarrow K^0\pi^+$ PROCESSES**

By using tree-penguin interference in $B^0 \rightarrow K^+\pi^-(\sim T' + P')$ versus $B^+ \rightarrow K^0\pi^+(\sim P')$, CP-averaged $B \rightarrow K\pi$ branching fraction may lead to non-trivial constraints on the $\phi_3$ angle [10]. In order to determine $\phi_3$, we need one more useful information on CP-violating rate differences [11]. Let’s introduce the following observables:

$$R_K = \frac{\overline{Br}(B^0 \rightarrow K^+\pi^-) \tau_+}{Br(B^+ \rightarrow K^0\pi^+) \tau_0} = 1 - 2r_K \cos \delta \cos \phi_3 + r_K^2 \sin^2 \phi_3,$$

(9)

$$A_0 = \frac{\Gamma(B^0 \rightarrow K^-\pi^+) - \Gamma(B^0 \rightarrow K^+\pi^-)}{\Gamma(B^- \rightarrow K^0\pi^-) + \Gamma(B^+ \rightarrow K^0\pi^+)} = A_{cp}(B^0 \rightarrow K^+\pi^-) R_K = -2r_K \sin \phi_3 \sin \delta.$$

(10)

where $r_K = |T'/T|$ is the ratio of tree to penguin amplitudes in $B \rightarrow K\pi$ decay and $\delta = (\delta_T' - \delta_T)$ is the strong phase difference between tree and penguin amplitudes. After eliminate $\sin \delta$ in Eq.(8)-(9), we have

$$R_K = 1 + r_K^2 \pm \sqrt{(4r_K^2 \cos^2 \phi_3 - A_{cp}^2 \cot^2 \phi_3)}.$$  

(11)

Here we obtain $r_K = 0.201 \pm 0.037$ from the pQCD analysis [11] and $A_0 = -0.110 \pm 0.065$ by combining recent
BaBar measurement on CP asymmetry of $B^0 \to K^+\pi^-$: $A_{cp}(B^0 \to K^+\pi^-) = -10.2 \pm 5.0 \pm 1.6\%$ [9] with present world averaged value of $R_K = 1.10 \pm 0.15$ [12].

IV. CONCLUSION

We discussed two methods to determine the weak phases $\phi_2$ and $\phi_3$ within the pQCD approach through
1) Time-dependent asymmetries in $B^0 \to \pi^+\pi^-$, 2) $B \to K\pi$ processes via penguin-tree interference. We can already obtain interesting bounds on $\phi_2$ and $\phi_3$ from present experimental measurements. Our predictions within the pQCD method is in good agreement with present experimental measurements in charmless B-decays. Specially our pQCD method predicted a large direct CP asymmetry in $B^0 \to \pi^+\pi^-(23 \pm 7\%)$ decay, which will be a crucial touch stone in order to distinguish our approach from others in future precise measurements. More detail works on other methods in $B \to \pi\pi, K\pi$ [14] and $D^{(*)}\pi$ processes [15] will appear elsewhere.

ACKNOWLEDGMENTS

It is a great pleasure to thank A.I. Sanda, E. Paschos, H.-n. Li and other members of PQCD working group for fruitful collaborations and joyful discussions. I would like to thank S.J. Brodsky, H.Y. Cheng and M. Kobayashi for their hospitality and encouragement. This work was supported in part by Science Council of R.O.C. under Grant No. NSC-90-2811-M-002 and in part by Grant-in Aid for Scientific Research from Ministry of Education, Science and Culture of Japan.

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