Preparation of multi-party entanglement of individual photons
and atomic ensembles

Guo-Ping Guo*, Guang-Can Guo

Key Laboratory of Quantum Information, University of Science and Technology
of China, Chinese Academy of Science, Hefei, Anhui, P. P. China, 230026

Abstract

An experimental feasible scheme is proposed to generate Greenberger-Horne-Zeilinger (GHZ) type of maximal entanglement. Distinguishing from the previous schemes, this entanglement can be chosen between either atomic ensembles (stationary qubit) or individual photons (flying qubit), according to the difference applications we desire for it. The physical requirements of the scheme are moderate and well fit the present experimental technique.

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Entanglement of many parties is of fundamental interest to test quantum mechanics against local hidden theory [1,2]. Furthermore, it has many practical applications in various quantum information processing tasks such as quantum cryptography [3], computer [4], and teleportation [5]. It is also believed that with more subsystems entangled, quantum non-locality becomes more striking [1,6], and quantum entanglement is more useful in actual applications [7–9]. Great attention has been directed to get more and more subsystems entangled. In theory, numerous schemes have been proposed to generate multi-party entanglement with cavity [10,11], ion traps [12], spontaneous parametric down converter (SPDC) [13,14], and indistinguishable atoms in Bose-Einstein condensates [15]. In experiment, there are reports of demonstration of four-photon entanglement in SPDC [14] and four-particle

*Electronic address: harryguo@mail.ustc.edu.cn
entanglement in ion traps [16].

Based on the stimulated Raman emission of atomic ensemble and the associated collective enhancement effect, atomic ensemble has been shown useful in long distance quantum communication [17,18]. And there are also protocols to generate multi-party entanglement between atomic ensembles [19] or to entangle atomic ensembles with Stokes photon [20]. The inherent robust to realistic noise and imperfections of the collective states of the atomic ensembles enable them as a preferable choose for the stationary qubit of quantum computation and information. While the manipulation convenience with linear optics for the photon polarization states makes individual photon a suitable candidate for flying or transmitting qubit of the quantum computation and information.

Here we propose a scheme to generate GHZ type of maximal entanglement. Different from the previous multi-party entanglement preparation protocol with atomic ensembles, the entangled subsystems of the present scheme can be chosen as either atomic ensembles (stationary qubit) or individual photons (flying qubit), according to the difference application. However, due to the collective enhancement effect of the atomic ensemble Raman procession, the physical requirements of the present scheme are still moderate and well fit the current experimental technique.

As most works with atomic ensembles [17–20], the basic element of our scheme is also an ensemble of many identical atoms, whose experimental realization can be either a room-temperature atomic gas or a sample of cold trapped atoms. The relevant structure of the atom is shown in Fig. 1. From the three levels \( |g\rangle, |r\rangle, |l\rangle \), we can define two collective atomic operators \( s = (1/\sqrt{N^a}) \sum_{i=1}^{N^a} |g\rangle_i \langle s| \) with \( s = r, l \), where \( N^a \gg 1 \) is the total atom number. Initially the atoms are optically pumped to the ground state \( |g\rangle \), which is effectively a vacuum state \( |0\rangle \) of the operators \( r, l \). A basis of the polarization qubits can be defined from the states \( |R\rangle = r^\dagger |0\rangle \) and \( |L\rangle = l^\dagger |0\rangle \), which both have an experimentally demonstrated long coherence time [23–26]. In Raman processing, the atomic ensemble is transferred from the ground state \( |g\rangle \) to the excited state \( |e\rangle \) by a classical laser (the pump light) with Rabi frequency \( \Omega \). Shortly, this excited state will transit to the two metastable states \( |r\rangle \)
and $|l\rangle$ with equal probabilities. In these transitions $|e\rangle \rightarrow |r\rangle$ and $|e\rangle \rightarrow |l\rangle$, the atomic ensemble will respectively emit a Stokes photon, which is horizontally or vertically polarized. Due to the collective enhanced coherent interaction, these excitation modes $r$ and $l$ can be respectively transferred to optical excitation modes $h$ and $v$ with high precision. Then they can be detected by single-photon detectors, even for a free-space ensemble, which has been demonstrated in both in theory [27] and in experiments [24,25].

As pointed out in the paper [20], EPR entangled state $|\Psi^1\rangle = \left( r_a^\dagger h_p^\dagger l_a^\dagger v_p^\dagger \right) / \sqrt{2} |0_{ap}\rangle$ between atomic ensemble and Stokes photon can be prepared with a short off-resonant laser pulse in this atomic ensemble system. This preparation succeeds with a small probability $p$ for each Raman drive pulse, which can be controlled by adjusting light–atom interaction time $t_\Delta$ and pulse length [17,19]. Generally, the atomic ensemble and the Stokes photons can be totally written in the state

$$|\Psi_1\rangle = (I + p^\frac{1}{2}H + \sum_{j=2}^{\infty} \frac{(p^\frac{1}{2}H)^j}{j!}) |0_{ap}\rangle_1,$$

where $I$ is the identity operator, $H = (r_a^\dagger h_p^\dagger + l_a^\dagger v_p^\dagger) / \sqrt{2}$, and $|0_{ap}\rangle$ is the vacuum state of the whole system. Here $h^\dagger (v^\dagger)$ represents horizontal (vertical) mode creation operator of Stokes photon. For convenience we leave this state unnormalized. It is obvious that the probability to produce $m$ Stokes photons from an atomic ensemble decays exponentially with the number $m$. The preparation for this state has inherent resistance to noise and is well based on the current technology of laser manipulation [17,19]. Many applications, such as individual photons quantum memory, can be expected from this novel entanglement between atomic collective state (stationary qubit) and the individual photon polarization state (flying qubit) [18,20].

In order to generate multi-party entanglement, we can first prepare $n$ pair of entanglement between atomic ensembles and Stokes photons. A simply way is to illuminate $n$ atomic ensembles in turn with a pump classical light. Thus the whole system of the atomic ensembles and Stokes photons can be prepared in the state (which is un-normalized)
\[ |\Phi\rangle^{ap} = \prod_{i=1}^{n} |\Psi_i\rangle = \prod_{i=1}^{n} (I + \frac{p}{2} H + \sum_{j=2}^{\infty} \frac{(p/2 H)^j}{j!}) |0_{ap}\rangle_i, \]  

(2)

where the subscript \(i\) represents the \(i\)th atomic ensemble. In the expanding of this state, the terms involving \(n\) photons all have a coefficient of \(p^\frac{n}{2}\). With this state, we can thereby generate multi-particle entanglement of either \(n\) atomic ensembles or \(n\) photons.

In the following preparing procedure, we will employ a multi-photon GHZ states analyzer with linear optics as shown in Fig. 3. In this optical setup, the input photon in the mode \(h_{p_i}^\dagger\) (or \(v_{p_i}^\dagger\)) can be transferred into mode \((h_{D_i}^\dagger + v_{D_i}^\dagger)/\sqrt{2}\) (or \((h_{D_i}^\dagger - v_{D_i}^\dagger)/\sqrt{2}\)) , where we have assumed the notation 0 \(\equiv n\) for the detector’s subscript. Assume that there is only one photon in each input. When there are coincidence clicks between \(n\) detectors of this \(n\)-photon GHZ analyzer, and there are even (or odd) number of \(D_i\) among these \(n\) click detectors, the \(n\) photons are obviously measured in the state \(|M\rangle^\pm\) (or \(|M\rangle^-\)), where \(|M\rangle^\pm = (1/\sqrt{2})(\prod_{i=1}^{n} h_{p_i}^\dagger \pm \prod_{i=1}^{n} v_{p_i}^\dagger) |0\rangle^p\). Here \(|0\rangle^p\) represents the total vacuum state of the \(n\) photons.

Generally, this linear optics setup can distinguish the states \(|M\rangle^\pm\) from the other states of the \(n\)-party GHZ states. When \(n = 2\), it becomes a Bell states analyzer which can divide the four Bell states into three classes: \(|\Phi^+\rangle = \frac{1}{\sqrt{2}}(h_{p_1}^\dagger h_{p_2}^\dagger + v_{p_1}^\dagger v_{p_2}^\dagger) |0\rangle^p\), \(|\Phi^-\rangle = \frac{1}{\sqrt{2}}(h_{p_1}^\dagger h_{p_2}^\dagger - v_{p_1}^\dagger v_{p_2}^\dagger) |0\rangle^p\) and \(|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(h_{p_1}^\dagger v_{p_2}^\dagger \pm v_{p_1}^\dagger h_{p_2}^\dagger) |0\rangle^p\). Thereby this Bell states analyzer with linear optics can be straightforwardly applied in quantum information processing such as quantum teleportation. For the multi-photon case, much more applications can be expected for the \(n\)-photon GHZ states analyzer [21,22]. In the following, we will show how to generate multi-atomic-ensemble or multi-photon entanglement, with the above \(n\)-photon GHZ states analyzer.

As the \(n\) atomic ensembles and their emitting Stokes photons are totally in the state \(|\Phi\rangle^{ap}\), we respectively input those Stokes photons from the \(n\) atomic ensembles into the above \(n\)-photon GHZ states analyzer as Fig. 3. When there are coincidence clicks between \(n\) detectors of this \(n\)-photon GHZ analyzer and there are even (or odd) number of \(D_i\) among these \(n\) click detectors, the input Stokes photons are measured in the states \(|M\rangle^\pm\) or \(|M'\rangle\), where \(|M'\rangle\) represents the states that there are more than one photon in some optics input.
but still has the same coincidence clicks as the case that there is one and only one Stokes photon in each input. With this measurement, the residual $n$ atomic ensembles of the state $|\Phi\rangle^{ap}$ is projected into the states

$$\rho^a = |\Phi\rangle^a_\pm \langle \Phi | + \rho^a_{\text{vac}}.$$  \hspace{1cm} (3)

Here $|\Phi\rangle^a_\pm = (1/\sqrt{2})(\prod_{i=1}^n r_a^i \pm \prod_{i=1}^n l_a^i) |0\rangle^a$ is $n$-atomic-ensemble GHZ type maximal entangled state, which results from the case that the input Stokes photons are measured in the states $|M\rangle^\pm$. And $\rho^a_{\text{vac}}$ represents the $n$ atomic ensemble states when the Stokes photons are projected in the state $|\ M\rangle$. This is the case that some ensembles have more than one excitation and some others have no excitation. It is easy to see that the effect of the detector inefficiencies and loss of excitation can be also combined into the term $\rho^a_{\text{vac}}$. Note that for any practical application of the multi-party entanglement, the state preparation should be succeeded by a measurement of the polarization of the excitation on each ensemble [2,6–8]. There we only keep these results, for which excitation appears on each ensemble. Then the state $\rho^a$ can effectively yield $n$-atomic-ensemble GHZ type maximal entanglement as the states $|\Phi\rangle^a_\pm$ in any application.

As we can respectively transfer the atomic ensembles excitation modes $r$ and $l$ to optical excitation modes $h$ and $v$, the entanglement between $n$ atomic ensembles can be directly transferred to $n$ photons. Then we can get $n$-photon GHZ entanglement state straightly from the state $\rho^a$. Alternatively, we can measure the $n$ atomic ensembles of the state $|\Phi\rangle^{ap}$ in the basis $|N\rangle^\pm = (\prod_{i=1}^n r_a^i \pm \prod_{i=1}^n l_a^i)/\sqrt{2} |0\rangle^a$ to project the corresponding Stoke photons into GHZ entanglement state. This measurement can be done when the excitations of the $n$ atomic ensembles of the state $|\Phi\rangle^{ap}$ is transferred to $n$ optical excitations, and then measured with the same GHZ state analyzer. Similarly, those optical modes transferred from the atomic ensembles excitation modes can be measured in the state $|M\rangle^\pm$ with post-selection. This equals to measure the $n$ atomic ensembles in the basis $|N\rangle^\pm$. Thereby the remaining $n$ Stokes photons of the state $|\Phi\rangle^{ap}$ are projected in an entangled state, which can effectively yield $n$-photon GHZ type maximal entanglement as the state $|\Psi\rangle^p = \ldots$
\( (1/\sqrt{2})(\Pi_{i=1}^n h_{p_i}^\dagger + \Pi_{i=1}^n v_{p_i}^\dagger) |0\rangle^p = \langle N^\pm |\Phi\rangle^op \) in any application.

It has been shown that the inherent resilience to noise of the collective states of atomic ensemble can enable it as a well qualified candidate for stationary and register qubits of quantum information and computation [17,18]. On the other hand, the light is an ideal carrier of quantum information, and the individual photon polarization modes can be conveniently manipulated. According to the different applications, the present scheme can elegantly prepare either \( n \)-atomic-ensemble or \( n \)-photon entanglement in one experimental setup. Although post-selection is still needed as in most of multi-party entanglement generation schemes such as SPDC scheme [13,14], these states can yield effectively GHZ entanglement whenever they are put into applications.

We now give a brief discussion on the efficiency and the practical implementation of this proposal. As shown in the paper [17,19], we can control the probability of getting a Stokes photon \( p \sim 10^{-2} \) for a Raman driving pulse with a short light-atom interaction time \( t_\Delta \).

Then the \( n \) atomic ensembles has a probability of order of \( p^n \) to produce \( n \) Stokes photons.

When we only post-select the case that every ensemble has one excitation, the probability to get \( n \)-party entanglement is \( p^n/2^{n-1} \) which equals \( 10^{-6} \) in the case \( n = 3 \). Thus with a typical repetition frequency \( f_p = 10^7 Hz \) for the Raman pulses, we can get 10 pairs of three-party GHZ entangled states per second. The ensembles should be prepared into ground state before each round of Raman processing. It is also noted that the present entangling scheme suffers from the fast exponential degradation of the efficiency as most protocols with post-selection.

As the unknown phase differences between the Stokes photons are fixed by the optics setup, we can effortlessly balance them with some phase plates [17,19]. In the practical implementation, we can safely neglect the dark counts of the single-photon detectors in the coincidence detections [13]. The transferring of the atomic ensemble excitation mode to the optical mode can be high efficient. As long as the preparing procedure is accomplished in a time no longer than the coherence times of the atomic collective states \( T_{pre} \lesssim ms \), we can
also safely neglect the noise of the non-stationary phase drift induced by the pumping laser or by the optical channel.

In conclusion, we have proposed an experimental feasible scheme to prepare multi-party GHZ type entanglement between either atomic ensembles (stationary qubit) or individual photons (flying qubit) with post-selection in one experiment setup. Due to the collective enhancement effect of the atomic ensemble Raman procession, the physical requirements of this scheme are moderate and well fit for the current experimental technique.

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**Figure Captions:**

Figure 1: The relevant level structure with $|g\rangle$ the ground state, $|e\rangle$ the excited state, and $|r\rangle$, $|l\rangle$ the two metastable state for storing a qubit. The transition $|g\rangle \rightarrow |e\rangle$ is coupled by a classical laser (the pump light) with Rabi frequency $\Omega$, followed with two equal probability transitions $|e\rangle \rightarrow |r\rangle$ and $|e\rangle \rightarrow |l\rangle$, where right-handed and left-handed rotation forward-scattered Stokes photons are emitted respectively. For convenience, we assume off-resonant coupling with a large detuning $\Delta$.

Figure 2: Schematic drawing of the experimental setup for the generation of the entanglement state $|\Phi^p\rangle$. The frequency-selective filter separates the pump light from the Stokes photon. The ensembles are prepared in the ground state before each round of Raman procession.

Figure 3: Schematic of the experimental setup for the measurement $|M\rangle^\pm = (1/\sqrt{2})(\Pi_{i=1}^n h_{p_i}^\dagger \pm \Pi_{i=1}^n v_{p_i}^\dagger)|0\rangle^p$ of the $n$ Stokes photon in the state $|\Phi^p\rangle$. The polarizing beam splitters (PBS) reflect vertical photons and transmit horizontal photons. We can ad-
just the arrival time of the $n$ photons with the delay plates. The $\lambda/2$ plates are employed to rotate the polarization of the Stokes photon $i$ through $45^\circ$ to transfer the photon $h^{\dagger}$ mode into $(h^{\dagger} + v^{\dagger})/\sqrt{2}$ and $v^{\dagger}$ mode into $(h^{\dagger} - v^{\dagger})/\sqrt{2}$, where $\lambda$ is the wavelength of those photons. This optical setup transfers the mode $h^{\dagger}_{p_i}$ into $(h^{\dagger}_{D_{h}^{i}} + v^{\dagger}_{D_{v}^{i}})/\sqrt{2}$ and the mode $v^{\dagger}_{p_i}$ into $(h^{\dagger}_{D_{h}^{i - 1}} - v^{\dagger}_{D_{v}^{i - 1}})/\sqrt{2}$, where we have assumed the notation $0 \equiv n$ for the detector’s subscript. Then for the case that $n$ photon are detected and even (odd) number of detectors $D_{v}^{i}$ click, the $n$ photon are projected into state $|M\rangle^+$ (or $|M\rangle^-$).
Fig. 1. Guo

Fig. 2. Guo

Fig. 3. Guo