Testing vector-tensor gravity with current cosmological observations

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Abstract.

A certain vector-tensor theory of gravitation (VT) has been recently applied to cosmology (Phys. Rev. D, 89, 2014, 044035). It leads to encouraging results. The zero order energy density of the vector field accounts for the cosmological constant. It has been recently proved that the VT vector field cannot play the role of the electromagnetic field. The evolution of the scalar perturbations is different in VT and general relativity. Tensor fluctuations evolve in the same way in both theories. Here, the VT evolution equations of the scalar modes are appropriately written, and the initial conditions at high redshift – for numerical integration– are given. The codes COSMOMC and CAMB are modified for applications to VT cosmology and, then, by using the new version of COSMOMC, statistical methods (Markov chains) allow us to fit theoretical VT predictions with current cosmological data obtained in various experiments. Previous encouraging fits were based on WMAP7 data, SN Ia observations, and a minimal model involving seven cosmological parameters. Here a similar preliminary study based on PLANCK data and updated SN Ia observations is presented.

1. Introduction and Generalities

In any vector-tensor theory, there are two fields, the metric $g_{\mu\nu}$, and a four-vector $A^\mu$. Various theories of this type have been studied (see [1], [2]). From these fields, we may define: The antisymmetric tensor $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, where the symbol $\nabla$ stands for a covariant derivative, and quantities $R$ and $g$, which are the scalar curvature and the determinant of the matrix $g_{\mu\nu}$ formed by the covariant components of the metric, respectively.

Here, we are concerned with a cosmological model, which is based on a certain vector-tensor theory of gravitation. This theory is hereafter referred to as VT (see [3]). The basic VT equations may be derived from the following action [4]:

$$I = \int \left[ \frac{R}{16\pi G} + (\frac{\gamma}{2} - \varepsilon)F_{\mu\nu}F^{\mu\nu} + \gamma (\nabla_\mu A^\mu)^2 - \rho (1 + \varepsilon) \right] \sqrt{-g} \, d^4x ,$$

(1)

where the dimensionless parameters $\gamma$ and $\varepsilon$ must satisfy the inequality $2\varepsilon > \gamma$, to avoid ghosts, and the relation $\gamma > 0$ to have a positive cosmological energy density for the vector field $A^\mu$. Quantity $\rho$ is the conserved energy density of an isentropic perfect fluid, $\varepsilon$ is its internal energy density [see [5] and [6] for details], and $G$ is the constant of gravitation.
The VT basic equations may be found in [4]. Here, these equations are directly written in the cosmological case. We write the zero order (background universe) equations, plus the first order corrections due to scalar perturbations of \( g_{\mu\nu}, A^\mu \), and the energy momentum tensor of the cosmological fluid \( T^\mu_\nu \). Vector perturbations do not contribute at first order, and there are no new tensor perturbations associate to \( A^\mu \), which implies that the tensor modes evolve as in general relativity (GR).

Hereafter, our signature is \((-+,+,-,+)\), Greek (Latin) indices run from 0 to 3 (1 to 3), units are chosen in such a way that the speed of light is \( c = 1 \), the present value of the scale factor is \( a = 1 \) (flat universe), the coordinate and conformal times are \( t \) and \( \tau \), respectively, and finally, whatever quantity \( S \) may be, \( S_B \) stands for its background value and \( \dot{S} \) is its derivative with respect to the conformal time.

The background universe is assumed to be flat, homogeneous, and isotropic. It contains matter and radiation. In this flat background, the metric has the Robertson-Walker form, and the covariant components of the VT vector field are \( [A_0 B(\tau), 0, 0, 0] \). Then, at zero order (no perturbations), we get

\[
3 \frac{\dot{a}^2}{a^2} = 8\pi G a^2 (\rho_B + \rho_B^A) \tag{2}
\]

\[
-2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = 8\pi G a^2 (P_B + P_B^A) \tag{3}
\]

\[
D_B \equiv (\nabla \cdot A)_B = -\frac{1}{a^2} [A_0 B + 2\frac{\dot{a}}{a} A_0 B] = \text{constant} , \tag{4}
\]

where \( \rho_B \) and \( P_B \) are the background energy density and pressure of the cosmological fluid (baryons, dark matter, massless neutrinos and radiation), \( \nabla \cdot A \equiv \Delta_\nu A^\nu \), and \( \rho_B^A \) \( (P_B^A) \) is the energy density (pressure) of the field \( A^\mu \). These quantities may be easily calculated from the \( A^\mu \) energy momentum tensor [4] to get

\[
\rho_B^A = -P_B^A = \gamma D_B^2 = \text{constant} , \tag{5}
\]

which proves that \( \rho_B^A \) plays the role of dark energy with the vacuum equation of state \( W = P_B^A/\rho_B^A = -1 \) [3]. The resulting cosmology involves all the free parameters and equations of the standard \( \Lambda \)CDM model, plus Eq. (4), which may be numerically solved for appropriate initial conditions (see below) to get function \( A_{0B}(\tau) \). Parameters \( \gamma \) and \( \epsilon \) and the sign of \( A_{0B} \) kept arbitrary at zero order (background) and also at first order (see [4]).

Let us now include scalar perturbations in VT [4]. It is done as in GR. The metric and the energy momentum tensor of the cosmological fluid are perturbed as in [7]. All the physical quantities are expanded in terms of the scalar harmonics \( Q^{(0)} \), which are plane waves in the flat case; e.g., the photon energy density is \( \rho_{\gamma} = \rho_{\gamma B} [1 + \delta_{\gamma}(k, \tau) Q^{(0)}] \), \( k \) being the comoving wavenumber and \( \delta_{\gamma}(k, \tau) \) the first order relative perturbation associated to \( \rho_{\gamma} \) in momentum space, where the perturbed cosmological equations are treated. The metric expansion involves the functions \( h(k, \tau) \) and \( \eta(k, \tau) \). See [7, 4] to get a full list of the perturbation functions in GR.

In VT, a new function is associated to the expansion of the field \( A^\mu \). It has been justified [3, 4] that the most appropriate scalar mode, \( D^{(0)} \), associated to \( A^\mu \) may be defined as follows:

\[
D = \nabla \cdot A = D_B^{(0)}[1 + D^{(0)}(k, \tau) Q^{(0)}] \tag{6}
\]

In momentum space, the \( A^\mu \) field equations reduce to the following second order differential equation:

\[
\ddot{D}^{(0)} + 2\frac{\dot{a}}{a} \dot{D}^{(0)} + k^2 D^{(0)} = 0 . \tag{7}
\]
Since only the scalar mode \( D^{(0)} \) is involved in the last equation, this mode is not coupled to any other scalar mode. Let us now write the equations for \( h \) and \( \eta \) since most of them are different from the corresponding equations in standard cosmology based on GR. These equations are

\[
k^2 \eta - \frac{1}{2} \frac{\dot{a}}{a} \dot{h} = 4\pi G \left[ -a^2 \rho B \delta - 2\gamma D_B (a^2 D_B D^{(0)} + A_{0B} \dot{D}^{(0)}) \right]
\]

\[
k^2 \dot{\eta} = 4\pi G [a^2 (\rho_B + P_B) \theta + 2\gamma k^2 A_{0B} D_B D^{(0)}]
\]

\[
\ddot{h} + 2 \frac{\dot{a}}{a} \dot{h} - 2k^2 \eta = -24\pi G [a^2 P_B \pi_L - 2\gamma D_B (a^2 D_B D^{(0)} - A_{0B} \dot{D}^{(0)})]
\]

\[
\ddot{\eta} + 6 \dot{\eta} + 2 \frac{\dot{a}}{a} (\dot{h} + 6 \dot{\eta}) - 2k^2 \eta = -24\pi G a^2 (\rho_B + P_B) \sigma .
\]

The terms involving \( \gamma \) are characteristic of VT; so, for \( \gamma = 0 \), these equations become identical to those of GR. The remaining equations have the same form in both GR and VT (see [7, 4]).

The complete system of equations must be solved for appropriate initial values of \( D^{(0)}, \dot{D}^{(0)} \), and any other scalar perturbation.

As it is usual, the initial conditions are calculated at redshift \( z = 10^8 \), in the radiation dominated era. At this redshift, all the cosmological scales were outside the horizon \( (k\tau << 1) \), the background functions \( a \) and \( A_{0B} \) may be assumed to be proportional to powers of \( \tau \), and any scalar mode \( Y \)–including \( D^{(0)} \)– may be expanded in the form

\[
Y = \sum_{n,m} \beta_{nm} \kappa^n \tau^m .
\]

The smallness of \( k\tau \) for cosmological scales, the existence of growing and decaying terms in Eq. (12), and additional considerations allow us to calculate the \( n \) and \( m \) values being relevant for each mode. Length but straightforward calculations leads to the VT initial conditions. For the background, we get

\[
\tau_{in} = \left( \frac{\dot{a}}{a} \right)^{-1} \text{ in}, \quad (A_{0B})_{in} = -\frac{D_B}{5(1 + z_{in})^2} \left( \frac{\dot{a}}{a} \right)_{in} .
\]

The initial conditions for the scalar modes are identical to those of GR (see [7, 4]), except for the following modes:

\[
D^{(0)} = C_1 \kappa^n , \quad \dot{D}^{(0)} = 0 ,
\]

where \( C_1 \) is an arbitrary constant, and

\[
h = C_2 (k\tau)^2 + C_4 (k\tau)^4 , \quad \theta = \theta_0 = -\frac{1}{18} C_2 k^4 \tau^3 - \frac{1}{30} C_4 k^6 \tau^5 ,
\]

\[
\theta_\nu = -\frac{23 + 4R_\nu}{18(15 + 4R_\nu)} C_2 k^4 \tau^3 - \frac{1}{30} C_4 k^6 \tau^5 ,
\]

where constant \( C_2 \) is another arbitrary parameter, and \( C_4 = \left[ 8\pi GC_1 \rho_\nu / 3(1 + z_{in})^2 \right] (\dot{a}/a)_{in}^2 \). The initial conditions only involve the arbitrary parameters \( C_1 \) and \( C_2 \), which are fully independent of the parameters \( \gamma \) and \( \varepsilon \) involved in the action (1). The evolution of the background universe and its linear perturbations do not allow us to calculate \( \gamma \) and \( \varepsilon \). Here, these parameters are kept undetermined.

For \( C_1 \neq 0 \), the same evolution equations and initial conditions as in standard GR cosmology are recovered and, then, \( C_2 \) is the unique normalization constant. For \( C_1 = 0 \), the VT scalar modes deviate from those of the standard ΛCDM model. Deviations depend on \( |C_1| \) (see [4]).
2. Numerical results and discussion

In the framework of GR, there are available codes, e.g., CAMB, to calculate the predictions of a given cosmological model (angular CMB power spectra, energy density power spectrum, and so on), and there are other codes, as COSMOMC, which use statistical techniques to fit model predictions and current observational data (CMB PLANCK data, updated data from SN Ia observations, and data from other experiments and galaxy surveys). By using these codes, the cosmological parameters may be either estimated or bounded.

Once evolution equations and initial conditions have been derived for both the background universe and the scalar modes of VT (see section 1), the codes CAMB and COSMOMC may be modified for VT cosmological applications. Modifications must be well planned since the original versions of these codes have been designed to work in standard GR cosmology. Such versions involve various conditions leading to accurate and fast calculations, which are not appropriate for VT applications. We have carefully modified recent versions of CAMB and COSMOMC to work with PLANCK data, and with current data about galaxy surveys, Ia supernovae, and so on. The resulting codes are hereafter denoted CAMB-VT and COSMOMC-VT.

Numerical calculations (performed with CAMB-VT) prove that quantities $T_1 = \ddot{a}h + h\dot{a}$, $T_2 = \dot{h}$ and $T_3 = \dot{\eta}$, and other quantities involved in the evolution equations of the CMB photon distribution function (see [7]) evolve in the same way, until small redshifts, in both GR and VT. For redshifts $z \leq 5$ and spatial scales smaller than a few hundreds of megaparsec, some of these VT quantities show a significant separation with respect to those of GR. See, e.g., Fig. (1), where quantity $T_1$ is represented in terms of $z$ for various $k$-values. At redshifts smaller than $\sim 2$, an oscillatory evolution of $T_1$ arises in VT, whereas no oscillations appear in GR. For too large spatial scales there are no differences between VT and GR. Deviations between both theories are responsible for the differences between the CMB angular power spectra of VT and GR. Very small integration steps are necessary to numerically estimate some integrals, at least, inside the oscillatory intervals associated to small enough scales and low redshifts. CAMB and COSMOMC codes have been modified to consider all the significant scales and also to integrate up to zero redshift.

![Figure 1](image_url)

**Figure 1.** Quantity $T_1 = \ddot{a}h + h\dot{a}$ in terms of the redshift $z$ for various scales. The comoving wavenumbers in units of $Mpc^{-1}$ are shown in the top of each panel.
By using CAMB-VT, the CMB angular power spectra corresponding to various values of constant $C_1$ have been calculated. They are shown in Fig. 2 together with the $C_\ell$ values observed by PLANCK (top panel) and WMAP7 (bottom panel). Since the two $C_\ell$ samples are rather different just in the region where the VT and GR predicted spectra separate, namely, for $\ell$ values smaller than \(\sim 100\), the statistical fit –obtained with COSMOMC-VT– could be rather different for PLANCK (here) and WMAP ([4]) data; in particular, for the $C_1$ parameter.

In order to verify these suspicions, we have run the code COSMOMC-VT to fit observations and predictions by using seven parameters: $C_1$, plus the six usual parameters of standard cosmology used to fit WMAP and PLANCK data. These parameters are $\Omega_b h^2$, $\Omega_{DM} h^2$, $\tau$, $n_s$, $\log(10^{10} A_s)$, and $\theta$, where $\Omega_b$ and $\Omega_{DM}$ are the density parameters of baryons and dark matter, respectively, $h$ is the reduced Hubble constant, $\tau$ is the reionization optical depth, $n_s$ is the spectral index of the power spectrum of scalar modes, and $A_s$ is the normalization constant of the same spectrum whose form is $P(k) = A_s k^{n_s}$; finally, the parameter $\theta$ is defined by the relation $\theta \times 10^{-2} = d_A(z_*)/r_s(z_*)$, where $d_A(z_*)$ is the angular diameter distance at decoupling redshift $z_*$, and $r_s(z_*)$ is the sound horizon at the same redshift. Moreover, all the calculations have been performed under the following basic assumptions: the background is flat, perturbations are adiabatic, the lensing effect is considered, there are no massive neutrinos, the equation of state of the dark energy is $P = W \rho$ with $W = -1$, vector and tensor modes are negligible, the mean CMB temperature is $T_{CMB} = 2.726$, the effective number of relativistic species is 3.046, and the total number of effectively massless degrees of freedom is $g_\ast = 10.75$. Only PLANCK and SN Ia data are considered to look for statistically admissible values of the cosmological parameters.

The COSMOMC-VT runs –described above– have given the mean likelihoods displayed in Fig. (3), as well as the best fits presented in Table 1, where we see that the six common parameters of the best GR and VT fits are very similar, and also that the $C_1$ value of the best VT fit is close to $3 \times 10^{-9}$. Similar results were obtained in [4] from the WMAP7 data. In each panel of Fig. (3), there is a red spot on the left hand side. The best fit is inside this spot, but there are other elongated spots where the mean likelihood is smaller (color) than in the main
Figure 3. Each panel corresponds to a pair of parameters. In all cases, one of these parameters is $C_1$. The mean likelihood function is displayed by using a red green yellow blue color bar. The greatest (smallest) values are red (dark blue).

Table 1. Best fits: values of the basic cosmological parameters

| THEORY | $C_1 \times 10^{-8}$ | $\Omega_b h^2$ | $\Omega_{DM} h^2$ | $\tau$ | $n_s$ | $\log[10^{10} A_s]$ | $\theta$ |
|--------|----------------------|----------------|------------------|-------|------|-------------------|-------|
| GR     | 0.0                  | 0.0221         | 0.119            | 0.0927| 0.963| 3.103             | 1.041 |
| VT     | 0.2903               | 0.0220         | 0.119            | 0.0909| 0.963| 3.091             | 1.041 |

red spot. More research is necessary to interpret these spots. They could be due to a poor convergence of COSMOMC-VT, but they could also have a real statistical meaning. In this second case, the enlargement of the data set by considering PLANCK and SN Ia observations plus cosmological data from other experiments might lead to interesting conclusions.

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