Reexamining the Einstein-Podolsky-Rosen experiment, photon correlation and Bell’s inequality

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The purpose of this article is to show that the introduction of hidden variables to describe individual events is fully consistent with the statistical predictions of quantum theory. We illustrate the validity of this assertion by discussing two fundamental experiments on correlated photons which are believed to behave “violently non-classical”. Our considerations carry over to correlated pairs of neutral particles of spin one-half in a singlet state. Much in the spirit of Einstein’s conviction we come to the conclusion that the state vector of a system does not provide an exhaustive description of the individual physical system. We also briefly discuss an experiment on “quantum teleportation” and demonstrate that our completely local approach leads to a full understanding of the experiment indicating the absence of any teleportation phenomenon. We caution that the indiscriminate use of the term “Quantum Theory” tends to obscure distinct differences between the quantum mechanics of massive particles and the propagation of photons. It is emphasized that the properties of polarizers, beam splitters, halfwave plates etc. used in photon-correlation experiments are defined by the laws of classical optics. Hence, understanding the outcome of those experiments requires a well-founded interconnection between classical and quantum electrodynamics which we scrutinize for critical assumptions.

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I. INTRODUCTION

The numerous studies on entangled states and in particular on correlated photons during the past 20 years have repeatedly stirred up renewed interest in the seminal paper by Einstein, Podolsky and Rosen (commonly referred to as EPR) on the issue of whether or not physical reality can exhaustively be described within the framework of quantum mechanics. Their analysis is based on a meanwhile popular thought experiment that deals with a pair of correlated massive particles. In 1951 Bohm extended this thought experiment by considering two particles with spin in a singlet state. It is this form of the thought experiment that has been discussed in the literature ever since and will henceforth be referred to as EPRB-experiment.

Despite the plethora of articles and monographs that are fully or in part devoted to the EPRB-problem, there has been no successful attempt so far to explain the experiments within a local theory by introducing and averaging over a “hidden parameter” that represents an indispensable classical variable. We shall demonstrate that our naïve realistic local approach does not only yield the “correct” quantum mechanical result but that it applies as well to the fundamental experiment by Aspect et al. on correlated pairs of photons. Also in this case the result of our calculation is in complete agreement with the experiment.

The ERPB-experiment is commonly discussed in terms of pairs of particles that are emitted from a source in opposite directions having opposite transverse spin directions. The latter are thought to be measured by two identical instruments on either side of the source but sufficiently far away from it so that the respective particle passing through one of the measurement instrument cannot interact with the other particle running through the instrument in the opposite direction. Our analysis takes the naïve realist’s standpoint that the process of measurement on either side of the setup is strictly local in the sense that the measurement on one side has no effect on the measurement on the other side. Hence we deny the possibility of what Einstein termed spooky action at a distance. This implies that the removal of the measurement instrument on one side would not affect the measurement (in practice: the count rate) on the other side. But the original opposite orientation of the particle spins, dictated by the process of their generation, remains unaffected over any conceivable distance up to the entrance slits of the measurement instruments. This property is commonly referred to as “perfect correlation”. We question the validity of the standard assumption that the plane of polarization spanned by the two spins and the line of particle propagation is unknown and therefore indetermined in advance of measurement. Instead we identify the angle enclosed by the normal of this plane and some laboratory-fixed axis as a “hidden parameter” which attains the character of a random variable as one repeats the generation of pairs a sufficiently large number of times.

In Section VI we shall first discuss the case of “entangled” photons as the respective experiment has actually been carried out with considerable sophistication and great care. This applies as well to many similar experiments on correlated photons, one of which shall be the subject of Section VI. By contrast, experiments on correlated
pairs of massive particles and opposite transverse spin, which we discuss in Sections III and IV are mostly fictitious or less complete.

II. CORRELATED PAIRS OF PHOTONS

All the years of willful pondering have not brought me any closer to the answer to the question “what are light quanta”. Today every good-for-nothing believes he should know it, but he is mistaken...

Albert Einstein
(In a letter to M. Besso, 1951)

Since a photon (or “light quantum”) that has been emitted, for example, from an excited hydrogen atom delivers its energy completely to an absorber hydrogen atom, even when this atom is at an astronomical distance, we picture a photon as a point-like particle which is invisible. This will also prove to be consistent with its properties displayed in beam splitters and polarizers. As one knows, for example, from Schrödinger’s theory of the Doppler shift of atomic radiation the photon’s recoil is transferred to the emitter once it has been ejected, regardless whether or not it is absorbed some time later. Hence the assertion that a photon comes into existence only when it is observed (or “measured”) appears to have little justification.

In addition, there is a widespread belief that a photon cannot be associated with a certain polarization unless this property has been measured. We advance the opinion that this assertion is as implausible as unjustified. The indeterminacy of photon polarization before its disappearance of the incoming wave in favor of a secondary wave is the content of the fundamental Ewald-Oseen extinction theorem. (See, for example: M. Born and E. Wolf.) As soon as a wave train (associated with a photon) penetrates into an optical material it excites coherently its atoms and causes them to set up a secondary wave field. Thereby the wave train loses its energy. For simplicity we disregard in the following the slight departure from monochromacy in going from a plane wave to a wave packet and characterize the photon state of the interaction-free wave train by $|n_k\rangle$ where $\vec{k}$ denotes its wave vector. Its state at finite coupling to the atoms of the optical material may be described by

$$|\phi_\gamma(t)\rangle = c_1(t) |n_k^{(1)}\rangle + c_2(t) |n_k^{(2)}\rangle,$$

where

$$n_k^{(1)} = 1 \text{ and } n_k^{(2)} = 0 \text{ (for the vacuum state)}.$$

We assume a simple plausible time-dependence of the coefficients

$$c_1(t) = \frac{1}{\sqrt{2}} \left[ 1 - \tanh(2t/\tau) \right]^{\frac{1}{2}},$$
$$c_2(t) = \frac{1}{\sqrt{2}} \left[ 1 + \tanh(2t/\tau) \right]^{\frac{1}{2}},$$

which have the property

$$c_1^2(t) + c_2^2(t) = 1$$

and hence ensure the norm unity of $|\phi_\gamma\rangle$. The quantity $\tau$ denotes the absorption time. We reference the middle of the absorption time interval to $t = 0$. Since we have for simplicity substituted the wave train by a plane wave, the operator $\hat{E}$ of the electric field may be reduced to one term

$$\hat{E}(\vec{r}, t) = \sqrt{\frac{\hbar \omega_k}{2 \varepsilon_0 V}} \vec{e}_k \left[ \hat{a}_k e^{i(\vec{k} \cdot \vec{r} - \omega_k t)} + \hat{a}_k^{\dagger} e^{-i(\vec{k} \cdot \vec{r} - \omega_k t)} \right],$$

where $V$ is the normalization volume, $\omega_k$ the frequency, $\hat{a}_k, \hat{a}_k^{\dagger}$ denote the photon creation and annihilation operator, $\vec{e}_k$ is the unit vector of polarization, and $\varepsilon_0$ the vacuum permittivity. We have, further, introduced Planck’s constant $\hbar$ in the form $\hbar = \hbar/2\pi$. If one uses the commutation rules for $\hat{a}_k$ and $\hat{a}_k^{\dagger}$ it is straightforward to show that the expectation value of $\hat{E}$ can be cast as

$$\langle \phi_\gamma | \hat{E}(\vec{r}, t) | \phi_\gamma \rangle =$$
$$\sqrt{\frac{\hbar \omega_k}{2 \varepsilon_0 V}} \hat{a}_k(t) \cos[\vec{k} \cdot \vec{r} - \omega_k t];$$

where

$$\hat{a}_k(t) = 2c_1(t)c_2(t)\vec{e}_k = \frac{1}{\cosh(\frac{\tau}{\tau})} \vec{e}_k.$$
describes a bell-shape function of width $\tau$ centered at $t = 0$. Within this time span of the absorption process $\vec{E}(\vec{r}, t)$ oscillates like the electric field of an electromagnetic wave and therefore defines a plane of polarization in a completely classical way. It is the vector potential $\vec{A}(\vec{r}, t)$ connected with $\vec{E}(\vec{r}, t)$ through

$$\vec{E}(\vec{r}, t) = -\frac{\partial}{\partial t} \vec{A}(\vec{r}, t)$$

that goes into the Schrödinger equation of the atoms of the optical material and effects the build-up of secondary waves whose polarization is hence determined by the polarization of the incoming wave and not by observing it.

The article by Aspect et al. deals with a situation where pairs of visible photons, correlated in their linear polarization, are emitted at a rate of about $5 \times 10^7$ s$^{-1}$ from a $^{40}\text{Ca}$-source. They are monitored by two mirror symmetric instruments facing each other across the source. The setup is schematically shown in Fig.1 where the wave trains with which the two photons are associated are indicated together with their plane in which $\vec{E}(\vec{r}, t)$ oscillates. Just to simplify the ensuing discussions we shall henceforth refer to that plane as “the plane of polarization” although the latter is conventionally defined as the oscillation plane of the magnetic vector. The amplitude of the electric field vector is denoted by $E_\gamma$, the vector of light propagation by $\vec{c}_\gamma$. We define the direction of $E_\gamma$ to coincide with $\vec{c}_\gamma / c_\gamma \times \vec{n}_\gamma$, where $\vec{n}_\gamma$ denotes the normal of the plane of polarization. If the latter lies in the $x/y$-plane $\vec{n}_\gamma$ points along the $y$-direction. The vertical planes left and right of the source refer to the entrance faces of the polarizers, stations A and B, respectively, consisting of polarizing cubes that transmit light polarized along a direction $\vec{d}$ on the left-hand side and $\vec{b}$ on the right-hand side. Both cubes reflect the perpendicular polarization. Light in the direction of transmission and reflection is monitored by photomultipliers and coincidence counter electronics.

The energy density $u$ within each of the wave trains is given by

$$u = \varepsilon_0 E_\gamma^2.$$  

Hence, in the spirit of the above idea on photons, the quantity

$$\rho = \frac{u}{\hbar \omega_\gamma}$$  

has to be interpreted as the probability density of finding the photon in the respective wave train.

If the wave train contains only one photon $\rho$ yields unity on integration over the volume of the wave train. Hence, a detector of perfect quantum efficiency in line with the wave train’s propagation will definitely fire on its arrival if the cross section of the wave train would be the same size or smaller than the sensitive entrance face of the detector. If it has passed through a semi-reflecting/semi-transparent (50-50) beam splitter each of the two detectors at the end of the now occurring two beams will fire with only 50% probability, and there will be no coincidences of detector signals. If the wave train contains coherently two photons, the energy density $u$, and consequently $\rho$, will be larger by a factor of two so that $\rho_{\text{split}}$ in each of the beams behind the splitter integrates again to unity. However, this case deserves some more comment:

As follows from our considerations in connection with Eqs. 11 and 12 the two-photon state

$$|\phi_\gamma(t)\rangle = c_1(t) |n_k^{(1)}\rangle + c_2(t) |n_k^{(2)}\rangle$$

where $|n_k^{(1)}\rangle = 2$ can only yield an oscillating electric field $\vec{E}(\vec{r}, t)$ driving the build-up of the secondary field if $|n_k^{(2)}\rangle = 1$ because the operator $\vec{E}(\vec{r}, t)$ contains only first-order terms of $\hat{a}_k$ and $\hat{a}_k^\dagger$. Hence, the two-photon absorption $|n_k^{(1)}\rangle = 2 \rightarrow |n_k^{(2)}\rangle = 0$ happens in two consecutive steps giving rise to two secondary photons sequentially lined up in real-space. Since each of the two photons has a 50% chance to go to the transmission or reflection channel, there is a 25% chance that both photons go to the transmission or reflection channel. That means, there is a remaining 50% chance that one photon is transmitted and the other one is reflected. Although the probability currents in each of the beams integrates to one photon, there will only be a 50% chance for coincidence signals with the associated detectors.

In all what follows Eq. 13 and the implications just discussed will prove to be sufficient in analyzing the key experiments on photon correlation.

As for the experiment by Aspect et al. we only consider two photons that are emitted in a zero-recoil mode from an atom and hence travel in opposite directions. The photons are generated in a $^{40}\text{Ca}$-cascade transition: $4p^2 \rightarrow 4p\,4s \rightarrow 4s^2$. The first transition yields a photon of 5513 Å wavelength and keeps the dipole axis fixed for the second transition that yields a photon of 4227 Å wavelength. Because of the fixed dipole axis the two photons are both linearly polarized with a common plane of polarization which is conserved as they propagate.

When the left wave train penetrates into the cube (polarizer) of station A it divides up - according to Malus’ law - into a transmitted wave train with an amplitude $E_\gamma \cos(\theta - \varphi)$ of the primary electric field and into a reflected wave train with amplitude $E_\gamma \sin(\theta - \varphi)$. Here $\varphi$ denotes the angle that the normal of the polarization plane includes with the x-direction of the laboratory-fixed coordinate system whose z-axis coincides with the line of propagation of the two photons. The angle $\theta$ is correspondingly the angle that the direction $\vec{d}$ of the left polarizer includes with the x-direction. Its counterpart is the angle $\phi$ of the polarizer on the right-hand side. Here the respective wave train is decomposed into a transmitted wave train with electric field amplitude $E_\gamma \cos(\phi - \varphi)$
and a reflected wave train the electric field amplitude of which is $E_\gamma \sin(\phi - \varphi)$.

To characterize the associated energy densities we use superscripts A and B, respectively, for quantities referring to the left- and right-hand side (stations A and B) of the experimental setup. We, furthermore, use subscripts + and −, respectively, to denote the energy densities of the wave trains transmitted parallel or reflected perpendicular to $\vec{a}$ on the left-hand side, and likewise with respect to $\vec{b}$ on the right-hand side.

Hence we have according to Eq. (4)

$$u^A_+ = \varepsilon_0 E^2_\gamma \cos^2(\theta - \varphi) ; \quad u^A_- = \varepsilon_0 E^2_\gamma \sin^2(\theta - \varphi)$$

and correspondingly

$$u^B_+ = \varepsilon_0 E^2_\gamma \cos^2(\phi - \varphi) ; \quad u^B_- = \varepsilon_0 E^2_\gamma \sin^2(\phi - \varphi) ,$$

which yields for the probabilities of finding the respective photons in one of the channels (+ or −, respectively)

$$\hat{P}^A_+(\theta, \varphi) = \frac{u^A_+}{u^A_+ + u^A_-} = \cos^2(\theta - \varphi)$$

$$\hat{P}^A_-(\theta, \varphi) = \frac{u^A_-}{u^A_+ + u^A_-} = \sin^2(\theta - \varphi)$$

and

$$\hat{P}^B_+(\phi, \varphi) = \frac{u^B_+}{u^B_+ + u^B_-} = \cos^2(\phi - \varphi)$$

$$\hat{P}^B_-(\phi, \varphi) = \frac{u^B_-}{u^B_+ + u^B_-} = \sin^2(\phi - \varphi) .$$

The quantities $\hat{P}^{A/B}_+$ are proportional to the count rates in the associated channels of the experimental setup if one generates the photon pairs at a certain rate.

If the pairs would all be emitted with $E_\gamma$ lying in the same plane, but $\varphi$ would be unknown, one could determine this angle from the count rates by forming

$$\hat{P}^A_+(\theta, \varphi) - \hat{P}^A_-(\theta, \varphi) = \cos^2(\theta - \varphi) - \sin^2(\theta - \varphi) = \cos 2(\theta - \varphi)$$

This expression may be viewed as the degree of polarization of the incoming photon or just as its “polarization” with respect to $\vec{a}$. It equals $+1$ when $\varphi = \theta$ and $-1$ when $\varphi = \theta \pm \pi/2$. In the spin-resolved electron scattering at heavy atoms an analogous expression is used to define the polarization of an electron beam impinging on a Mott-detector where the difference in the left-right asymmetry of the pertinent differential cross section is used in place of $P_+ - P_-$. Incidentally, it is the Mott-detector that is actually used in true polarization experiments on massive particles as opposed to the fictional (completely inappropriate) Stern-Gerlach magnet commonly discussed in the context of EPRB-experiments.

As argued by Einstein in 1942, the result A of the measurement on photon 1 should not depend on the direction $\vec{b}$ of the polarizer at B acting on photon 2, and B should not depend on $\vec{a}$. Photon 2 is only correlated with photon 1 in the sense that its plane of polarization is the same as that of photon 1. That means in terms of the count rates at B:

$$\hat{P}^B_+(\phi, \varphi) - \hat{P}^B_-(\phi, \varphi) = \cos^2(\phi - \varphi) - \sin^2(\phi - \varphi) = \cos 2(\phi - \varphi)$$

To make sure that each count at A and B refers to the same pair, all four channels are checked by coincidence measurements.

The differences $\hat{P}^{A/B}_+ - \hat{P}^{A/B}_-$ may be interpreted as probabilities of “preferential detection” in the “+”-channel. They attain negative values if the photons are actually detected in the “−”-channel. Hence, the expression

$$\left( \hat{P}^A_+(\theta, \varphi) - \hat{P}^A_-(\theta, \varphi) \right) \left( \hat{P}^B_+(\phi, \varphi) - \hat{P}^B_-(\phi, \varphi) \right)$$

may be viewed as the probability of finding the photons at station A preferentially detected in the “+”-channel and those simultaneously monitored at B with the same preference. Because of Eqs. (8) and (9) this joint probability takes the form

$$\left( \hat{P}^A_+(\theta, \varphi) - \hat{P}^A_-(\theta, \varphi) \right) \left( \hat{P}^B_+(\phi, \varphi) - \hat{P}^B_-(\phi, \varphi) \right) = \cos 2(\theta - \varphi) \cos 2(\phi - \varphi) =$$
\[
\frac{1}{2} \cos^2 (\theta - \phi) + \frac{1}{2} \cos^2 (\theta + \phi - 2\varphi). \tag{10}
\]

Obviously, the first term on the right-hand side becomes \(\varphi\)-independent only if the polarizations of the right and left wave train lie in the same plane.

The pairs are emitted such that their planes of polarization are oriented at random. On performing a \(\varphi\)-average of Eq.\(\text{(10)}\) over the range \([-\frac{\pi}{2}, \frac{\pi}{2}]\) we obtain

\[
\left( \hat{P}_+^A(\theta, \varphi) - \hat{P}_-^A(\theta, \varphi) \right) \left( \hat{P}_+^B(\phi, \varphi) - \hat{P}_-^B(\phi, \varphi) \right) = \frac{1}{2} \cos 2(\theta - \phi). \tag{11}
\]

We may rewrite the expression on the left-hand side

\[
\left( \hat{P}_+^A(\theta, \varphi) - \hat{P}_-^A(\theta, \varphi) \right) \left( \hat{P}_+^B(\phi, \varphi) - \hat{P}_-^B(\phi, \varphi) \right) = P_{++}(\theta, \phi) + P_{--}(\theta, \phi) - P_{+-}(\theta, \phi) - P_{-+}(\theta, \phi) \tag{12}
\]

where

\[
P_{\pm\pm}(\theta, \phi) = \frac{P^A_{\pm}(\theta, \varphi)}{P^B_{\pm}(\phi, \varphi)}. \tag{13}
\]

From Eqs.\(\text{(10)}\) and \(\text{(11)}\) we have

\[
P_1 = \frac{P_{AB}}{P_{\pm\pm}} = \frac{1}{2} \tag{14}
\]

which states that the probability of finding, respectively, photon 1 at A or photon 2 at B in one of the two channels is equal to 0.5 on the average. We shall henceforth substitute \(\theta\) and \(\phi\) by the unit vectors \(\vec{a}\) and \(\vec{b}\) which these angles refer to.

The quantities \(P_{\pm\pm}(\vec{a}, \vec{b})\) represent joint probabilities. Hence, the conditional probability \(P^c_{++}(\vec{a}, \vec{b})\) of finding photon 1 at A in the "++"-channel if the companion photon 2 has been detected in the "+-"-channel at B, is given by

\[
P^c_{++}(\vec{a}, \vec{b}) = \frac{1}{P_1} P_{++}(\vec{a}, \vec{b}). \tag{15}
\]

The quantities \(P^c_{\pm\mp}(\vec{a}, \vec{b})\) for other sign combinations are defined analogously. Hence, if we set

\[
E = \frac{1}{P_1} \left( \hat{P}_+^A(\theta, \varphi) - \hat{P}_-^A(\theta, \varphi) \right) \left( \hat{P}_+^B(\phi, \varphi) - \hat{P}_-^B(\phi, \varphi) \right)
= P^c_{++}(\vec{a}, \vec{b}) + P^c_{--}(\vec{a}, \vec{b}) - P^c_{+-}(\vec{a}, \vec{b}) - P^c_{-+}(\vec{a}, \vec{b}) \tag{16}
\]

and use the above equations from \(\text{(11)}\) to \(\text{(15)}\), we obtain

\[
E = \cos 2(\theta - \phi) \equiv \cos 2(\vec{a}, \vec{b}). \tag{17}
\]

The quantity \(E\) constitutes the so-called correlation coefficient of the measurement on the two photons, and exactly this equation is fully confirmed by the experiment. By hindsight this may also be seen as justifying our assumption on the uniform distribution of the angle \(\varphi\).

As indicated in Fig.1 this angle describes the orientation of the plane of polarization in which the electric field of the wave trains oscillates. It hence represents, in the spirit of the EPR-article, an element of physical reality that allows one to predict with certainty how the wave trains impinging on their respective polarizers divide up into secondary wave trains with energy densities \(u_+\) and \(u_-\). Admittedly, these densities correlate only with the probabilities of transmission and reflection for an incoming single photon and thus represent typical elements of uncertainty. However, the actual choice made by the individual photon cannot be predicted by any of the existing theories.

It should also clearly be stated that quantum theory cannot make any definite prediction on the position of a photon within its associated wave train. The position along its line of propagation is only determined up to the length of the wave train. On the other hand, it appears to be out of the question that the distance of the two photons from the source remains exactly equal along this line of propagation and therefore represents another element of physical reality not described by quantum theory.

Furthermore, it is obvious from our treatment that a photon traversing the polarizer at A possesses now a polarization \(\hat{E}_\parallel \vec{a}\) that differs from that of the original (incoming) photon if \(\theta - \varphi \neq 0\). From the viewpoint of quantum theory, which is free from hidden parameters, a photon can have a definite polarization only after it has left the polarizer, which represents a rather implausible credo because the occurrence of this “definite polarization” can only be explained by assuming a classical functioning of the polarizer.

Quantum entanglement constitutes a particularly puzzling feature of describing two- or multi-photon correlation. This becomes even more apparent in the case of pairs that consist of identical massive particles with spin in an entangled singlet state. The latter implies equal probabilities of finding either particle with either spin orientation at station A and B, and only after the detector at A has measured a particle with “spin-up”, the detector at B yields definitely a coincidence signal that correlates with “spin-down” and vice versa. This amounts to an instantaneous \texttt{non-local} intercommunication between even very distant stations A and B. Hence, if one denies the reality of single events as described by our approach and adheres to the idea of quantum mechanical completeness defined by a state vector of the system, one is forced to ascribe the measurement a decisive influence on the system under study and to put up with \texttt{spooky action at a distance}.

As one follows the various steps in deriving Eq.\(\text{(11)}\) one recognizes that it describes basically a classical behavior of electromagnetic waves. This can be seen by considering a radio wave source that consists of a Hertzian oscillator of frequency \(\omega,\) whose dipole axis is intermittently turned at random such that the angle \(\varphi\) it includes with the \(x\)-axis becomes uniformly distributed over a unit circle in the \(x/y\)-plane. We assume that there are polarizers
positioned at A and B as depicted in Fig. 1. They may consist of a frame similar to a tennis racket that contains only one set of parallel (superconducting) strings with the second orthogonal set of strings missing. The plane of these strings is tilted by $45^\circ$ against the z-axis. The axis of these polarizer rackets includes angles $\theta$ and $\phi$, respectively, with the x-axis. A radio wave whose electric field vector $\vec{E}$ is parallel to the strings will be reflected by $90^\circ$, and it is transmitted if $\vec{E}$ is perpendicular to the strings. If $\vec{E}$ includes an angle $\phi$ with the x-axis, the relative intensities $I^{A/B}_{\pm\pm}$ for transmission and reflection are given by Eqs. (5) and (7). Hence, if one averages these intensities over a sufficiently long time and forms $I_{\pm\pm} = I^{A}_{\pm\pm}I^{B}_{\pm\pm}$ one obtains in complete analogy to Eq. (17)

$$E = 2[I_{++} + I_{--} - I_{+-} - I_{-+}] = \cos 2(\theta - \phi).$$

Here we have introduced a superscript “f” (for “fermion”) to indicate the reference to massive particles with half-integer spin.

The two equations yield a linear dependence of $E$ on $|\theta - \phi|$, viz.

$$E^{Bell} = P^{f}_{++} + P^{f}_{--} - P^{f}_{+-} - P^{f}_{-+} = -1 + 2\frac{|\theta - \phi|}{\pi}. \quad (20)$$

If one translates Bell’s considerations into the case of photons and applies them in particular to the experiment by Aspect et al., the corresponding linear dependence reads

$$E^{Bell} = 1 - 2\frac{2|\theta - \phi|}{\pi}, \quad (21)$$

which differs fundamentally from our result (17)

$$E = \cos 2(\theta - \phi)$$

although the two expressions agree for three particular values of $|\theta - \phi|$, viz. 0°, 45° and 90°.

As for the photon case his assumption is clearly inadmissible: a photon whose associated field vector $\vec{E}$ makes an angle $\theta - \phi$ with the polarizer at station A, has already a non-vanishing probability for going to the “+-”-channel as soon as $\theta - \phi$ differs from zero. This becomes obvious from tracing the origin of our results back to Eqs. (11) and (7).

Interestingly, Clauser and Horne discuss a model for the correlation of photon counts that bears some resemblance to our approach, but their ad hoc-assumption on the rates $P^{A}_{+}$ and $P^{B}_{-}$ such that $P^{A}_{+} - P^{B}_{-}$ agrees with the experiments lacks any physical foundation.

Bell’s highly recognized inequality (against which Aspect et al. checked their experiment) rests on the above assumption that leads to Eq. (21). Since the considered mechanism of sorting particles into two groups “+” and “-” is definitely unrealistic for photons, a test of hidden parameter theories that rest on Bell’s inequality is meaningless. In discussing the EPRB-experiment and summarizing the message of the above equations (18) and (19) in his inequality, Bell wanted to demonstrate that one cannot explain the “exact quantum mechanical result” without assuming action at a distance. A purely local mechanism sorting the members of particle pairs into “+” and “-” channels would definitely lead to a confirmation of his inequality. This is obviously not true for correlated photon pairs as we have shown in Section II where we derive the correct photon correlation factor (which violates Bell’s inequality) assuming a purely local mechanism of particle separation. One might argue that this only reflects the inadequacy of Bell’s line of reasoning for correlated photon pairs. However, as we shall demonstrate in the ensuing section, one can just as well explain the “exact quantum mechanical result” for the true EPRB-experiment by assuming again a purely local mechanism.
IV. LOCALITY VS. NON-LOCALITY

We start with the quantum mechanical expression for what is believed to be connected to the count rate in the four channels of an EPRB-setup where the counting is organized in complete analogy to the experiment by Aspect et al.:

\[ E = \langle \Psi_0 | \sigma_A \cdot \alpha \otimes \sigma_B \cdot \beta | \Psi_0 \rangle = -\cos(\alpha, \beta) \] (22)

which describes the statistical correlation of monitoring one of the two particles at station A with a spin component in line with \( \vec{a} \) and the counterpart of the two particles with a spin component in line with \( \vec{b} \). Here \( \Psi_0 \) stands for an entangled state (“Bell state”)

\[
\Psi_0 = \frac{1}{\sqrt{2}} \left( S_A(\vec{r}_1) e^{-i\vec{k} \cdot \vec{r}_1} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \otimes S_B(\vec{r}_2) e^{i\vec{k} \cdot \vec{r}_2} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} - S_A(\vec{r}_2) e^{-i\vec{k} \cdot \vec{r}_2} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \otimes S_B(\vec{r}_1) e^{i\vec{k} \cdot \vec{r}_1} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right). \] (23)

To retain the familiar notation for the Pauli spin matrices \( \sigma_A, \sigma_B \), we have rotated the coordinate system such that \( z \rightarrow x, \ y \rightarrow y \) and \( x \rightarrow z \). Hence \( \vec{k} \) denotes the particle momentum in the x-direction, and \( S_A(\vec{r}); S_B(\vec{r}) \) represent smooth, real-valued functions which vanish around the source and reach well into the Stern-Gerlach magnet at station A and B, respectively. They are sizeably different from zero only in a narrow cylinder around the x-axis and are essentially constant within this domain. Their squares integrate to unity. The unit vectors \( \vec{a} \) and \( \vec{b} \) are given by

\( \vec{a} = (0, -\sin \theta, \cos \theta); \quad \vec{b} = (0, -\sin \phi, \cos \phi) \).

Expression (22) follows simply from taking the expectation value of

\[ \sigma_A \cdot \vec{a} \otimes \sigma_B \cdot \vec{b} = -\sigma_y \otimes \sigma_y \sin \theta \sin \phi + \sigma_z \otimes \sigma_z \cos \theta \cos \phi - \sigma_y \otimes \sigma_z \sin \theta \cos \phi - \sigma_z \otimes \sigma_y \cos \theta \sin \phi. \]

In the spirit of our derivation that led to the set of Eqs. (11) to (13), \( E \) can alternatively be cast as

\[ E = \frac{1}{P_1} \left( \hat{P}_A(\theta, \varphi) - \hat{P}_A(\varphi, \theta) \right) \left( \hat{P}_B(\varphi, \theta) - \hat{P}_B(\theta, \varphi) \right) = P_{++} + P_{--} - P_{+-} - P_{-+} \] (24)

where \( P_{\pm \pm} = 2P_{\pm} \) and \( P_{\pm} \) is defined as before

\[ P_{\pm}(\vec{a}, \vec{b}) = \frac{1}{2} \left[ \hat{P}_A(\vec{a}, \vec{b}) \pm \hat{P}_B(\vec{a}, \vec{b}) \right] \] (25)

We are now in the position to discuss \( E \) as in the previous case of correlated photons. That means, we do not discuss pairs in entangled states but rather individual pairs, more precisely: a set of subsequently emitted pairs of particles, in which one of the particles is definitely moving toward A, the other one toward B. Correspondingly, we start again by considering the probability \( \hat{P}_A^\pm(\theta, \varphi) \) of finding the particle at A with spin up if it has entered the station in a state

\[ \psi_A(\vec{r}) = S_A(\vec{r}) e^{-i\vec{K} \cdot \vec{r}} \left[ \cos \varphi \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} - i \sin \varphi \left( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \right] \] (26)

whose spin encloses an angle \( \varphi \) with the z-axis. When it has been monitored in the “up”-channel its state is given by

\[ \psi_A^+(\vec{r}) = S_A(\vec{r}) e^{-i\vec{K} \cdot \vec{r}} \left[ \cos \varphi \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} - i \sin \varphi \left( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \right] \] (27)

From this we may determine the transition probability

\[ \left| \langle \psi_A^+(\vec{r}) | \psi_A(\vec{r}) \rangle \right|^2 = \left| \cos \varphi \cos \frac{\theta}{2} + \sin \varphi \sin \frac{\theta}{2} \right|^2, \]

that is

\[ \hat{P}_A^+(\theta, \varphi) = \cos^2 \left( \frac{\theta - \varphi}{2} \right). \] (28)

Likewise one obtains

\[ \hat{P}_A^-(\theta, \varphi) = \left( -i \cos \frac{\varphi}{2} \sin \frac{\theta}{2} + i \sin \frac{\varphi}{2} \cos \frac{\theta}{2} \right)^2 = \sin^2 \left( \frac{\theta - \varphi}{2} \right). \]

Hence

\[ \hat{P}_A^+(\theta, \varphi) - \hat{P}_A^-(\theta, \varphi) = \cos^2 \left( \frac{\theta - \varphi}{2} \right) - \sin^2 \left( \frac{\theta - \varphi}{2} \right) = \cos(\theta - \varphi). \]

The corresponding expression for station B reads

\[ \hat{P}_B^+(\theta, \varphi) - \hat{P}_B^-(\theta, \varphi) = \left[ \sin^2 \left( \frac{\phi - \varphi}{2} \right) - \cos^2 \left( \frac{\phi - \varphi}{2} \right) \right] = -\cos(\phi - \varphi). \]

If we insert this into Eq. (24) we obtain

\[ E = 2 \left( \hat{P}_A^+(\theta, \varphi) - \hat{P}_A^-(\theta, \varphi) \right) \left( \hat{P}_B^+(\phi, \varphi) - \hat{P}_B^-(\phi, \varphi) \right) = -\cos(\theta - \phi) \] (29)

which may alternatively be written

\[ E = -\vec{a} \cdot \vec{b} \] (30)

To make contact to Bell’s notation in his seminal paper2 we identify the angle \( \varphi \) in Eq. (26) with his single parameter \( \lambda \). Furthermore, we have to relate his functions \( A(\vec{a}, \lambda) \) and \( B(\vec{b}, \lambda) \) to our expressions \( \hat{P}_A^{\pm/B} \)

\[ A(\theta, \varphi) = A(\vec{a}, \lambda) = \sqrt{2} \left[ \hat{P}_A^+(\vec{a}, \lambda) - \hat{P}_A^-(\vec{a}, \lambda) \right] \]

and

\[ B(\phi, \varphi) = B(\vec{b}, \lambda) = \sqrt{2} \left[ \hat{P}_B^+(\vec{b}, \lambda) - \hat{P}_B^-(\vec{b}, \lambda) \right]. \]
The two spins and the line along which the two particles propagate span a plane whose normal encloses the angle $\varphi$ with the $z$-axis. We assume that $\varphi$ is uniformly distributed over the unit circle if the emission of pairs is repeated sufficiently often. Hence, we have for Bell’s probability distribution
\[ \rho(\lambda) = \frac{1}{\pi}. \]
Eq. (30) then takes the form
\[ E \equiv P_{\text{Bell}}(\vec{a}, \vec{b}) = \int_{-\pi/2}^{+\pi/2} \rho(\lambda) A(\vec{a}, \lambda) B(\vec{b}, \lambda) d\lambda = -\vec{a} \cdot \vec{b}, \]
where we have inserted our result (30) on the right-hand side. It is exactly this equality that is fundamentally questioned in Bell’s article: “It will be shown that this is not possible.”

He considers the property of $A(\vec{a}, \lambda)$ to be independent of the setting $\vec{b}$ at station B and $B(\vec{b}, \lambda)$ to be independent of $\vec{a}$ as defining the hypothesis of locality and restates the above assertion in a different context: “With these local forms, it is not possible to find functions $A$ and $B$ and a probability distribution $\rho$ which give the correlation $-\vec{a} \cdot \vec{b}$.

As we have already stated in Section III, he is led to this contradictory conclusion by his completely unfounded assumption on the $\varphi$-dependence of monitoring the particles at stations A and B.

To leave no shade of uncertainty, we emphasize again that our result Eq. (30) is based on the idea of what is commonly termed “local realism”. It is therefore exceedingly puzzling that even the most recent articles on this subject choose the Stern-Gerlach twin setup to illustrate the idea of “local realism” by explaining:

“Yet, for the two systems oriented in parallel (i.e. $\theta = \phi = 0$), once the, say, “up” detector of particle 1 has fired, we know with certainty that the “down” detector of particle 2 will register on the other side and vice versa.” (Zeilinger, 2)

At some other place of the article one reads:

”...neither [particle] has a well defined spin before it is measured.”

As we have stated at the outset, the latter assertion is without foundation and actually refuted by our derivation whose consistency with the experiment rests on the contrary assumption. As regards the first statement, it has to be recalled that the probability of detecting particle 1 in the “up”-channel is according to Eq. (29) $P^A_+ = \cos^2(\frac{1}{2} \varphi)$ and we have also for particle 2 in the “down”-channel $P^B_+ = \cos^2(\frac{1}{2} \varphi)$. Clearly, if $\varphi \neq 0$ and therefore $\cos^2(\frac{1}{2} \varphi) < 1$ particle 1 has a non-vanishing probability (yet smaller than unity) to be registered in the “up”-channel. But since we also have $P^B_- = \sin^2(\frac{1}{2} \varphi) < 1$, and consequently $P^B_+ > 0$ there is no guarantee that particle 2 will be detected in the “down”-channel.

The analogous experiment with photons is similarly commented in the literature and the conclusions are similarly besides the point.

V. QUANTUM TELEPORTATION

Experiments that are believed to demonstrate “quantum teleportation” have during the past decade gained considerable attention. We mention here only the ground breaking-work by Zeilinger and co-authors and refer the reader to the article by Greenberger et al. for a more detailed exposition of the world view of this school of thought. The experimental setup is schematically shown in Fig. 2 where a birefringent non-linear $\beta$-barium borate (BBO)-crystal acts as a parametric down-conversion source of photons pairs, each consisting of a so-called “signal” and an “idler”-photon. They are associated with the classical ordinary and extraordinary beam ("o-beam" and “e-beam”), the electric field vector of which oscillates, respectively, perpendicular to the principal plane or within it. That plane is spanned by the optic axis of the uniaxial crystal and the direction of the incoming UV-pulse (of 200 fs length and 390 nm wavelength) and is perpendicular to the drawing plane. The angle between these two directions is about 50°. Because it differs from zero and 90°, the directions of all signal photons lie on a cone with an axis in the principal plane. The directions of the idler photons span another cone whose axis lies also in the principal plane. The two cones intersect along two lines that enclose an angle of about 6°. They span a plane which coincides with the drawing plane in Fig. 2. Photons of pairs that are emitted along these lines, labeled (2) and (3), are in a particular way “entangled”. That means in our interpretation: their polarization is no longer uniquely determined by either lying in the principal plane or perpendicular to it. The electric field vector $\vec{E}_{\gamma_2}$ of photon 2 may now enclose any angle with the principal plane. But if one decomposes $\vec{E}_{\gamma_2}$ into components $\vec{E}_{\parallel_{\gamma_2}}$, $\vec{E}_{\perp_{\gamma_2}}$, parallel and perpendicular to the principal plane and performs the same decomposition on $\vec{E}_{\gamma_3}$ of photon 3, one finds $\vec{E}_{\parallel_{\gamma_2}} = \vec{E}_{\perp_{\gamma_2}}$ and $\vec{E}_{\parallel_{\gamma_3}} = \vec{E}_{\gamma_3}$. That means: if the normal of the polarization plane of photon 2 makes an angle $\varphi$ with the normal of the drawing plane, the normal of the corresponding plane of photon 3 makes an angle $\varphi + \frac{\pi}{4}$ with the normal of the drawing plane. (The latter is, incidentally, also common to all other beams shown in Fig. 2.) Hence, the polarizations of these two photons are always orthogonal. As in the previous sections, the angle $\varphi$ represents a random variable that attains a certain value at each emission act. The BBO-crystal does not completely absorb the incoming UV-pulse. The residual pulse leaving the crystal impinges on a small mirror (M) in front, is reflected and thereby forced to traverse the crystal another time in the opposite direction. On its way it generates another pair of “entangled” photons. The photon impinging on
Likewise we obtain continues its propagation after metallic reflection.

Thus, on integrating the densities $u$ and $V$ phase jump of $\pi$.

Hence, if the two photons possess orthogonal polarizations, that is when

$$\alpha = \frac{\pi}{2},$$

the probability $E^{(D1)}/h\omega_\gamma$ of finding one of the two photons at $D1$ becomes equal to the probability $E^{(D2)}/h\omega_\gamma$ at $D2$. That means: if there is a coincidence of the signals from the two detectors, the two photons must have had orthogonal polarizations. Since the polarization of photon 2 is orthogonal to that of photon 3, the latter must have the polarization of photon 1 after the polarizer.

All that is demonstrated by this experiment is that the D1/D2-coincidence electronics picks out of the $\varphi$-dependent set of pairs that consist of photons 2 and 3 just a photon 3 whose polarization is parallel to that of photon 1 after the polarizer. From our point of view there is nothing that would indicate a magic “quantum teleportation” of polarization from photon 1 to photon 3.

Since teleportation has gained considerable popularity in the recent past we want to illustrate the cogency of our conclusion by simplifying our line of argument in a thought-experiment using essentially the same setup:

We replace each pair of mutually orthogonal polarized photons at $D1$ becomes equal to the probability $E^{(D2)}/h\omega_\gamma$ at $D2$. That means: if there is a coincidence of the signals from the two detectors, the two photons must have had orthogonal polarizations. Since the polarization of photon 2 is orthogonal to that of photon 3, the latter must have the polarization of photon 1 after the polarizer.

All that is demonstrated by this experiment is that the D1/D2-coincidence electronics picks out of the $\varphi$-dependent set of pairs that consist of photons 2 and 3 just a photon 3 whose polarization is parallel to that of photon 1 after the polarizer. From our point of view there is nothing that would indicate a magic “quantum teleportation” of polarization from photon 1 to photon 3.

Since teleportation has gained considerable popularity in the recent past we want to illustrate the cogency of our conclusion by simplifying our line of argument in a thought-experiment using essentially the same setup:

We replace each pair of mutually orthogonal polarized photons, propagating along the beams (2) and (3), by a pair of “color-correlated” photons, which means, they are associated with two different frequencies that belong to two complementary colors adding up to white. The frequencies change statistically from pair to pair so as

\[ E^{(D1)} = u_1/2 \bar{E}^{(D1)} \]

\[ E^{(D2)} = u_2/2 \bar{E}^{(D2)} + \epsilon_0 |\bar{E}_{\gamma_1}| |\bar{E}_{\gamma_2}| \cos \phi \cos(\alpha + \pi) \]

\[ E^{(D1/D2)} = u_1/2 \bar{E}^{(D1)} + u_2/2 \bar{E}^{(D2)} + \epsilon_0 |\bar{E}_{\gamma_1}| |\bar{E}_{\gamma_2}| \cos \phi \cos(\alpha + \pi) \]

\[ E^{(D1/D2)} = u_1/2 \bar{E}^{(D1)} + u_2/2 \bar{E}^{(D2)} + \epsilon_0 |\bar{E}_{\gamma_1}| |\bar{E}_{\gamma_2}| \cos \phi \cos(\alpha + \pi) \]

\[ E^{(D1/D2)} = u_1/2 \bar{E}^{(D1)} + u_2/2 \bar{E}^{(D2)} + \epsilon_0 |\bar{E}_{\gamma_1}| |\bar{E}_{\gamma_2}| \cos \phi \cos(\alpha + \pi) \]

\[ E^{(D1/D2)} = u_1/2 \bar{E}^{(D1)} + u_2/2 \bar{E}^{(D2)} + \epsilon_0 |\bar{E}_{\gamma_1}| |\bar{E}_{\gamma_2}| \cos \phi \cos(\alpha + \pi) \]
to cover the full range of the visible spectrum. We substitute the polarizer “pol” by a filter that is set at some wavelength which may correspond to “yellow”, for example. The beam splitter is replaced by a detector that fires at white balance, that is, when the color of the photon arriving along path (2) is the white suplement of the yellow “passenger photon” (1). If this is the case, the photon travelling along path (3) must be yellow as well. Clearly there is no teleportation of color from photon (1) to (3), because the color of the latter is already set before photon (1) reaches the detector.

VI. MULTI-PHOTON ENTANGLEMENT

Spontaneous parametric down-convertion has so far proved to be the most effective source for polarization entangled photon pairs. The entanglement of more than two photons has been shown to be feasible by exposing a BBO-crystal to short pulses of ultraviolet light as in the experiments on “quantum teleportation” discussed in the previous section. Multi-photon entanglement is believed to yield the ultimate criterion for distinguishing local hidden variable theories from quantum mechanics. The purpose of this section is to again cast doubt on the validity of this belief. The setup of recent experiments dealing with the entanglement of 4 photons is as follows (s. for example the article by Weinfurter and associates):

Similar to the 2 forward-photons considered in the preceding section, the 4 photons are emitted into modes a and b defining two forward beams which coincide again with the two lines of intersection of the cones for signal and idler photons. Each beam is split up into two orthogonal beams by a non-polarizing beam splitter. Hence, after the two beam splitters one has 4 beams, all lying in the plane spanned by the original beams a and b which, together with the incident UV beam, form a y-shaped array. Each of the 4 beams enters into a polarizing beam splitter of the kind that was used by Aspect et al. Photons that are transmitted or reflected by these polarizing cubes are monitored by eight photon counters all of which are interconnected and checked by means of an eight-channel multi-coincidence logic. Each of the four cubes can be rotated around the incoming (and transmitted) beam. The respective rotation angles \( \phi_a, \phi_{a'}, \phi_b, \phi_{b'} \) are at reference zero when the reflected beams lie in the a/b-plane.

If one applies a reasoning similar to that of the previous section, “entanglement” of the 4 photons means:

The two photons of the “a”-beam have the same plane of polarization in common before they impinge on the non-polarizing beam splitter, and this applies also to the two photons of the “b”-beam. If the normal directions of these two planes of polarization enclose an angle of 90°, all four photons are correlated (“entangled”). By contrast, if this angle turns out to be a random variable, one is dealing with two uncorrelated pairs of correlated (“entangled”) photons.

Following the same line of reasoning as in Section II we obtain for the corresponding probabilities (\( \propto \) count rates)

\[
\hat{P}_+^a = \cos^2(\phi_a - \phi_a'); \quad \hat{P}_-^a = \sin^2(\phi_a - \phi_a');
\]

\[
\hat{P}_+^b = \cos^2(\phi_{a'} - \phi_a'); \quad \hat{P}_-^b = \sin^2(\phi_{a'} - \phi_a'),
\]

and analogous expressions for the \( b \)-beam where the expressions with \( \cos^2 \) and \( \sin^2 \) are just interchanged. If we were only dealing with the two photons of the “\( a' \)”-beam, the associated correlation factor would be given by

\[
E_a(\varphi_a) = 2 \left( \hat{P}_+^a + \hat{P}_-^a \right) \left( \hat{P}_+^{a'} + \hat{P}_-^{a'} \right) = \cos 2(\phi_a - \phi_{a'}) + \cos 2(\phi_a + \phi_{a'} - 2 \varphi_a).
\]

and likewise for the “\( b \)”-beam

\[
E_b(\varphi_b) = 2 \left( \hat{P}_+^b - \hat{B}_-^b \right) \left( \hat{P}_+^{b'} - \hat{B}_-^{b'} \right) = \cos 2(\phi_b - \phi_{b'}) + \cos 2(\phi_b + \phi_{b'} - 2 \varphi_b),
\]

where one has to observe that

\[
\varphi_b = \varphi_a + \frac{\pi}{2}.
\]

On the right-hand side of Eqs. (34) and (35) we have inserted a factor of 2 for reasons explained in connection with Eq. (15).

Since the count rates for all beams are statistically independent once they have passed the beam splitters and the polarizing cubes, we have for the total correlation factor

\[
E_{total}(\varphi) = E_a(\varphi_a) E_b(\varphi_b) =
\]

\[
[\cos 2(\phi_a - \phi_{a'}) + \cos 2(\phi_a + \phi_{a'} - 2 \varphi_b)] 
\times
\]

\[
[\cos 2(\phi_b - \phi_{b'}) + \cos 2(\phi_b + \phi_{b'} - 2 \varphi_b)].
\]

This can be recast as

\[
E_{total}(\varphi_a) =
\]

\[
\frac{1}{2} \left[ \cos 2(\phi_a - \phi_b - \phi_{a'} + \phi_{b'}) + \cos 2(\phi_a + \phi_b - \phi_{a'} - \phi_{b'}) 
+ \cos 2((\phi_a + \phi_{a'} - \phi_b - \phi_{b'}) - 2(\varphi_a - \varphi_b)) 
+ \cos 2((\phi_a + \phi_{a'} + \phi_b + \phi_{b'}) - 2(\varphi_a + \varphi_b)) \right].
\]

The first two terms on the right-hand side can be rewritten

\[
\frac{1}{2} \left[ \cos 2(\phi_a - \phi_b - \phi_{a'} + \phi_{b'}) + \cos 2(\phi_a + \phi_b - \phi_{a'} - \phi_{b'}) \right] =
\]

\[
\cos 2(\phi_a - \phi_{a'}) \cos 2(\phi_b - \phi_{b'}).
\]
If we now make use of Eq. (36) and average over \( \varphi_a \) we arrive at

\[
\mathcal{E}_{\text{total}} = \frac{1}{2} \cos 2(\phi_a + \phi_{a'} - \phi_b - \phi_{b'}) + \cos 2(\phi_a - \phi_{a'}) \cos 2(\phi_b - \phi_{b'}) .
\]  

(37)

The expression \( \cos 2(\phi_a + \phi_{a'} - \phi_b - \phi_{b'}) \) is termed “Greenberger-Horne-Zeilinger-(GHZ)-correlation function”. The factors of the product on the right-hand side have the EPRB-form (29) and thus refer to pairs of “intra-beam-correlated” photons. But the product appears only formally as part of the four-photon correlation. If the experiment also yields additional true “intra-beam-correlated” photons without inter-beam correlation that contribution would just appear with a different weight factor in front.

Except for such factors in front of the two terms on the right, our result is identical with that obtained by Weinfurter and collaborators. (It seems, however, that the argument of the cos-functions is erroneously by a factor of 2 too small in that article.) The result by Weinfurter and associates is based on a completely different reasoning and is thought to provide the ultimate proof that their experiment cannot be explained within a local theory. Obviously, that claim, which is also held by almost every researcher in this field, is unwarranted.

VII. CONCLUSIONS

Since quantum mechanics is manifestly non-local, it has become a widespread conviction that action at a distance constitutes only a feature that reflects this very fact. We have shown that this conclusion is without foundation. Effects of non-locality which are most clearly evidenced by two-slit experiments on massive particles, have nothing to do with action at a distance, but rather originate in an active role of the vacuum. This has become a widespread conviction that action at a distance constitutes only a feature that reflects this very fact. We have shown that this conclusion is without foundation. Effects of non-locality which are most clearly evidenced by two-slit experiments on massive particles, have nothing to do with action at a distance, but rather originate in an active role of the vacuum. Clearly, that claim, which is also held by almost every researcher in this field, is unwarranted.

The quantum character of the motion of massive particles is brought out by the modification of their classical propagation. In distinct contrast, the propagation of photons is completely controlled by the classical space/time behavior of the associated electromagnetic wave. This is most strikingly evidenced by the polarizers that are used in analyzing the photon correlation experiments. Clearly, these polarizers are designed by exclusively applying rules of classical optics. This applies as well to the other optical parts typical of the equipment, viz. mirrors, quarter- and halfwave plates, filters and phase shifters.
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