Metric transformation of the surface of a tight punch for forming sheet shells of double curvature

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Abstract. For the first time, the problem of parametric reduction of the circumscribing surface of the shell of double curvature to the main axes and to the planes of symmetry defined by Gaussian coordinates is considered in detail. The solution of this problem is essential for ensuring the conditions of symmetrical shaping of the double-curvature circumflex shells by tight fit. The tight punch is presented in the form of a technological electronic layout, containing metric information of the surface of double curvature and storing information of the geometric alignment of the skin that extends to the outer circumferential surface of the unit. In this case, the circumferential surface of the shell of double curvature remains invariant with respect to the coordinate transformation.

1. Introduction

The aerodynamic shape of the aircraft is described by two basic concepts – it is the outline and contour of the surface. The outline refers to the surface of the airframe, streamlined by the air flow in flight, and the contour – the line of intersection of the outline with the coordinate planes of the aircraft XYZ (figure 1).

![Figure 1. Diagram of the coordinate planes.](image-url)
The outline of the aircraft is formed by the external skins. This is an external structural shell element that serves to give the outline of the airframe a certain (for example, streamlined) shape and is involved in the perception of the aerodynamic load. The outer skins used in aircraft are made of a sufficiently strong and rigid sheet material; they can resist normal and tangential forces and work for all types of loads. Currently, when designing and calculating the shells, the skin is taken into account as a load-bearing power element of the aircraft structure.

Most of the skins have the external shapes of the outline-forming surfaces corresponding to the theoretical contours of the aircraft, set by the aerodynamic model of the airframe surface [1]. The fuselage of the aircraft is designed entirely as a large circumscribing object by means of contours of parallel cross-sections passing through a series of points of the longitudinal X-axis, as in figure 1. After determining the cross-sections, you can also build longitudinal curves that connect them into a single three-dimensional shape. This is a way to define curves in a space in which a Cartesian coordinate system is introduced and consists in projecting these curves onto the plane of symmetry (x, y) and onto the construction plane (x, z) of the plane. Traditionally, these contour curves were drawn in full size at points using mechanical splines. The process is called the plaz method of constructing surfaces [2].

The specificity of the design and the complexity of the external forms of the aircraft surfaces does not allow us to coordinate the geometric parameters of the airframe assembly units, skins, technological equipment and other mating parts and to link them in the aggregate-assembly production with the help of traditional machine-building drawings. The only room, the size of which allowed to do this, is a structural plaz, which was drawn in projections on the plane (x, y) and (x, z) at a scale of 1:1 contours of units, assemblies and individual skins. The constructive plaz is used for making templates. Templates as information carriers are copies of contours that transfer the specified shapes and sizes to the sheaths and their volumetric equipment.

A feature of modern aggregate and assembly production is the increasing complexity of contours and increasing requirements for their quality, including the accuracy of reproducing shapes and sizes [3]. In turn, the reproduction of the shapes and sizes of aggregates, assemblies and sheaths is characterized by the specificity of the methods and means used. There are three groups of size matching methods:

- methods based on the use of only rigid media of shapes and sizes (templates);
- methods based, on the one hand, on the use of analytical descriptions of surfaces associated with aerodynamic contours, and on the other hand, on the use of structural plazas and rigid carriers of shapes and sizes (templates) for linking structural elements that are not related to the aerodynamic contours of the product;
- methods based on the plazeless production of the circumference equipment on the basis of analytical (numerical) setting of all geometric parameters of the skin and other structural elements, components and assemblies of the product, including the use of the holographic principle of linking information.

For the construction of surfaces began to use a computers, because serial testing required, as a rule, the use of more accurate analytical and graphical methods of linking. The geometric linking of structural elements remained the same complex multi-factor task, and it was performed first on the basis of the plaz-template method using only rigid carriers of shapes and sizes, and then using the calculation-plaz method using analytical methods used in drawing plaz lines using drawing machines [4].

At present, automated systems have appeared based on the development of the apparatus for mathematical modeling of objects and production processes using modern computer technology and CNC equipment. These are CAD/CAM systems, on the basis of which new methods have been developed that allow describing the vast majority of surface variants based on the parametric representation of curves and surfaces. New methods use analytical curves as systems of lines lying on the projected surface, but not the surface itself, which was a breakthrough in the design and construction of surfaces of complex shapes [5].
Parametric representation of curves and surfaces has advantages. Since we have to calculate tangents, normals, curvatures, etc., we need a parametrization for which the differentiation operation is easily performed. Parametric equations have certain advantages. For example, to obtain a graphic image of a curve or surface, the coordinates of their points corresponding to certain parameter values are calculated sequentially. The parametric method of defining curves frees you from being bound to any particular coordinate system. In this case, the shape of the curve or surface does not depend on the choice of the coordinate system. Therefore, the development of the parametric method can be considered natural, but without a computer, its use is impossible.

With the advent of computers, more universal methods were developed, moving away from traditional graphical methods. Modern operators, who have replaced the plaz surface specialists, should use various systems for determining surfaces. Systems of this type were made fully automatic, so that the only input information was the coordinates of the points through which the projected surface should pass. The principle of such systems is based on the method of two-dimensional interpolation. Thus, the spread of analytical methods for setting contours, automation of calculations and recording programs for CNC equipment contributed to the development of an independent method for the formation of shapes and sizes of mating structural elements, i.e., the method of plazeless linking of structural elements of an aircraft airframe.

The plazeless linking of the dimensions is carried out by means of an aerodynamic model of the water-forming surface obtained by calculation. The formation of interconnected working contours of technological equipment is ensured by its precise production on CNC machines. When using a plazeless coupling, the accuracy of the coupling of the mated structural elements is directly dependent on the accuracy of their manufacture. Since the external contours of the aircraft are set analytically, the use of plazeless linking means extends to the elements of the technological equipment, including the tight punches associated with the theoretical contours of the products. However, a complete three-dimensional description of the product is the basis for a plazeless production preparation [3].

Over the past decade, information technology has come a long way in its development. The emergence of «heavy» integrated CAD/CAM/CAE systems allows aviation engineers to perform an electronic description of the product with an accuracy of 1-3 microns, using the principle of three-dimensional parameterization [7]. The essence of parameterization is that the part is performed in a computer assembly as an object not with specifically specified dimensions, but with interrelated variables, when one of them changes, the entire part is rebuilt. The use of the parameterized model principle allows you to modify a huge number of assemblies of the production object within a few hours and is limited only by the capabilities of computer technology.

Such capabilities are inherent in the electronic description of the product – an electronic layout (EL) and the process of modeling the aircraft begins, as with traditional methods, with the creation of a geometric model of the surface, which was used to build cross-section contours when splitting plazas and developing programs for manufacturing outline equipment on CNC equipment. Geometric models of the surface were presented both in the form of a rigid immutable model and in a parametric form. Depending on the functions performed, the description of the surface was presented in the form of a frame model, or a surface model, or in the form of a solid.

We can assure you that the production technologies of aviation equipment, even with the use of modern digital technologies, remain the same, despite the fact that the number and complexity of manufacturing plazs and outline equipment during the spatial linking procedure on the EL has sharply decreased. Tight punches can be made not according to control casts from the surface standard (the basic standard), but directly on CNC machines without drawing the plaz and without making templates. For individual sections of the surface of the tight punch, its contours are equidistant with respect to the external theoretical outline and represent a real breakdown of the lines of frames, stringers and other power elements of the airframe.

However, due to the need to solve new geometric problems related to the shaping of a flat sheet by fitting it into a shell part, we need a different position in space of the surface of the tight punch, the shape of which will correspond to the first and second fundamental quadratic forms that define the
surface up to its position in space. The meaning of the first and second quadratic forms is that they allow us to construct a surface under certain conditions [6]. Then, as the coordinate lines on the surface, it is better to take a network of lines of curvature.

In the context of differential geometry, these lines of curvature are contours of normal surface sections that have a geodesic origin according to the Gauss metric [7]. They are consistent with the internal geometry of the surface, the metric properties of which remain unchanged when it bends. This is the great importance of metric concepts, and some surface properties related to the structure of a differential quadratic form for a curve element on a surface can be investigated independently of space. Such properties of surfaces are called internal, and geometry based on the study of differential quadratic forms is called internal geometry.

Then, following Gauss, we define a surface in three-dimensional Euclidean space in a parametric way by setting three equations \( x^i = x^i(q^1, q^2) \), where \( i = 1, 2, 3 \). The parameters \( q^1, q^2 \) define a grid of curves on the surface, called curvilinear, or Gaussian coordinates. This has become the main tool for measuring surface curvature. Such tools made it possible to determine two types of surface curvature, known today as the Gaussian curvature \( K \) and the mean curvature \( H \). Therefore, the geometry of the shell does not coincide with the geometry of the skin for the assembly of the aircraft unit, since these geometries were tied to different categories of curves and surfaces: contour-contour-forming and Gaussian-parametric. Switching from one category to another and back again is not difficult when using modern digital technologies.

There is still no regulatory and technical documentation (RTD) in production, which determines the information content and status of EL in the technological documentation. Therefore, the production units still work with drawings, plazas and templates, and the EL practically «hang» and do not participate in the pre-production processes. Even when developing control programs for the manufacture of parts, their geometry is verified on the basis of plaz information, including tight punches. The main problem is that in order to ensure interchangeability, the volumetric contour-forming equipment, including the tight punches, are directly connected to the theoretical contours of the aerodynamic contours of the aircraft units.

However, there are already methods that allow you to describe any contours on a computer and build any parametric lines, including the conjugate orthogonal grid of surface curvatures. Therefore, we propose to create a technological electronic layout (TEL) that contains the geometric information that is necessary to solve a specific technological problem of obtaining the details of the cladding by shaping the skin. This primarily concerns the tight punch, the geometry of which will differ from the original electronic layout of the EL. Then the design of the technological process will be carried out using application systems, and the initial information will be transmitted in the form of TEL and drawings and supported by the information environment of pre-production.

2. Geometric elaboration of a tight punch

The process of forming the sheet material with a tight fit remains the main one in the production of shell parts of skins. The created metal forming equipment (tight presses) is unique, specialized and must be program-controlled. Much attention is paid to the problems of automating this process. As it was found, the surfaces of the tight punches are directly related to the theoretical contours of the aerodynamic contours of the aircraft units. This leads to the fact that the longitudinal line of curvature of the surface of the tight punch does not coincide with the direction of the load action, or rather with the direction of the forming contour of the shell. This circumstance violates the symmetry of the tightness forming, which leads to a violation of monotony and to the manifestation of instability of plastic deformation in the form of folds and rupture of the sheet billet [8].

The proportionality of the tensile loads and the monotonicity of the deformation of the sheet billet in the process of forming the skin under the conditions of a flat stress state will be preserved when its main directions coincide with the directions of the lines of curvature. They are consistent with the internal geometry of the surface, the metric properties of which remain unchanged when it bends. The study of internal geometry can be performed without considering its surface in space, including in the spatial
A rectangular coordinate system formed by the coordinate planes of the aircraft. Figure 2 shows the original tight punch, on the surface of which the shell part of the skin is located according to the contours of the outline.

![Figure 2](image)

**Figure 2.** The original tight punch, on the surface of which the shell part of the skin is located according to the contours of the outline.

Figure 3 shows a converted tight punch, on the surface of which the shell part of the skin is also located according to the contours of the outline, but the upper ridge of the surface points is occupied by the forming contour of the longitudinal curvature of this shell. This is clearly seen when combining two tight punches on the outer surface (figure 4). The outer surface is one, because it is invariant with respect to its representation in different coordinate systems. In the spatial rectangular coordinate system formed by the coordinate planes of the aircraft, a tight punch is shown, the surface of which corresponds to the contours of the outline. In a curved coordinate system formed by two lines of curvature on a surface with a pole point as the origin, a tight punch is shown after our geometric study.

![Figure 3](image)

**Figure 3.** A converted tight punch, on the surface of which the shell part of the skin is also located, where the upper ridge of the surface points occupied the forming contour of the longitudinal curvature of this shell.
Figure 4. The combination of the original and converted tight punches on the outer surface, on which the shell part of the skin is located.

Figure 5 shows the results of the geometric study with the indication of the values of the geometric parameters in normal cross-sections, the upper points of which along the forming contour correspond to the coordinates of the points of descent of the sheet blank from the tight punch at any stage of the step-by-step wrapping.

Figure 5. The results of the geometric study with the indication of the geometric parameters f and R in the normal cross sections of the tight punch.
The procedure for the geometric study of a tight punch is associated with the transition from the outer circumferential surfaces to the surface of double curvature, tied to the grid of the main lines of curvature. Means of parametric determination of surfaces in terms of Gaussian coordinates with the help of Gaussian parameters \( u \) and \( v \) were introduced, and the installation of the tight punch should be made so that the position of the shaping contour of the surface is aligned with the first vertical plane \( F_1 \). The second vertical plane \( F_2 \), orthogonal to the first, contains the position of the contour of the central cross-section that intersects the shaping contour at the "pole" (point \( O \)) of the surface of the shell of double curvature. As a result of this binding, both the orthogonal planes \( F_1 \) and \( F_2 \) pass into the vertical planes of symmetry of the tight press, which determine the position of the normal \( Z_0 \) in the "pole" of the surface of the shell of double curvature and the directions of the lines of curvature \( X_0 \) and \( Y_0 \) (figure 6).

![Figure 6](image)

**Figure 6.** a) the surface of the shell of double curvature; b) the forming longitudinal contour (2L); c) the central transverse contour (\( f_0 \)).

The installation of the base of the tight punch on the press table should ensure a constant position of the forming contour in the vertical plane of the press symmetry. This is the first time we have made such a transition. As a result, we have gained great advantages in automating the design and control of the processes of forming skin-covering shells of any surface, as well as complex geometric shapes. It became possible to distinguish on such a surface the position of the forming contour of the tight punch, which will be combined with the longitudinal vertical plane of the press, including the pole of the surface and passing through the middle of the central sponge.

For the contour of the central cross-section of the surface, it is enough to simply set the location of the axial symmetry line for the classical surface of the second order touching at the pole, as part of the toroidal surface (figure 7). As a result, it is possible to determine the properties of the local shape at the pole of the surface of the tight punch. This approach can also be used for other shapes, by selecting a classical second-order surface that touches it at the pole, which greatly facilitates the analytical approach to modeling, which adequately reflects the real process of shaping the skin.
For the central cross-section, you can set a cylindrical coordinate system: $r$, $\beta$ and the «imaginary» axis of axial symmetry $c$-$c$ of this contour. Then we will refer to the main geometric parameters of the double curvature shell (see figure 6):

- the radii of curvature of the contours of the surface sections by the vertical planes of symmetry $F_1$ and $F_2$ at the point $O$: $R_1$ (longitudinal) and $R_2$ (transverse);
- length of the contour of the longitudinal section of the surface $2L$ by the vertical plane of symmetry $F_1$ (forming contour);
- the length of the contour of the edge section of the surface $2l_k$ with a plane parallel to $F_1$;
- deflection of the contour of the central cross-section $f_0$ in the vertical plane of symmetry of the surface $F_2$;
- longitudinal $2\alpha_k$ and transverse $2\beta_k$ angles of the contours of the cross-sections of the surface by the vertical planes of symmetry $F_1$ and $F_2$ of the tight punch;
- the value of $a$, equal to $(R_1 - R_2)$ in figure 4
- shell width $2B$.

The contours of the cross sections of the shell surface with the vertical planes of symmetry $F_1$ and $F_2$ are plane curves that intersect at the point $O$ with the radii $R_1$ and $R_2$. The values that are the inverse of the radii:

$$k_1 = \frac{1}{R_1}; k_2 = \frac{1}{R_2}$$  (1)

are the normal curvatures of these lines on the surface that intersect at the point $O$ and have the properties of extremality: one of them is the maximum, and the other is the minimum. In this case, the product of the curvature values determines the Gaussian curvature $K$ at the point $O$. The directions of the tangents to these lines of the shell surface at the point $O$ determine the main directions $x_0$ and $y_0$, which lie simultaneously in the tangent plane and in the planes of symmetry $F_1$ and $F_2$.

Then the geometric shape of the shell of the double curvature will be locally characterized at the point $O$ by the value of the Gaussian curvature. The Gaussian curvature as a metric parameter characterizes the deformation of the shell of a given geometric shape in the «pole». The Gaussian curvature of a surface is considered an invariant of the curvature tensor of plane lines passing through the point $O$ and retains its value when bending. This deformation is called isometric bending, in which the lengths of the lines drawn on the surface of the shell and the angles between them are preserved. Only the geometric shape of the double-curvature shell changes, but its surface remains isometrically similar to the surface before bending.

In the coordinate system of the main directions $x_0$ and $y_0$ lying in the tangent plane, the surface of the shell in the vicinity of the point $O$ can be represented as a function:
\[ z_0 = \frac{1}{2} \cdot (k_1 \cdot x_0^2 + k_2 \cdot y_0^2) \] (2)

The explicit form of specifying the surface in this form is used to find the characteristic metric properties of the shell. Just as some circle is close to the point of the curve in infinite proximity, so some classical quadratic surface of the second order will be close to the surface of the shell in the vicinity of the point \( O \). It can include, in particular, a part of the external toroidal surface.

The considered method of parametric representation of the surface in terms of coordinates with the help of Gaussian parameters \( u \) and \( v \), will allow to ensure the stress-strain state of the workpiece during the fitting in the main axes of stresses and deformations, but under the condition that the direction of the fitting is aligned with the vertical plane of symmetry \( F_1 \) of the surface of the fitting punch. To do this, it is sufficient to ensure that the plane in which the central clamping sponge of the tight press lies is perpendicular to the tangent at the point of convergence of the forming contour of the tight punch and the constant position of this contour in the vertical plane of symmetry \( F_1 \).

3. Geometric provision of symmetrical fit

Modeling of the processes of forming the skin is based on the geometric model of the shell, shown in figure 5. It should provide the calculation at any stage of the step loading when the sheet billet is stretched, the coordinates of the point of its descent from the punch, the angle of inclination at the point of descent, the length of the blank laid on the punch and the trajectory of the point near the press clamp relative to the forming contour of the punch, etc. The step process in the control programs is based on the increments of either the angle of descent of the workpiece from the punch, or the length of the workpiece laid on the punch. At the same time, the technological features of the process and the design models of shaping are such that the fitting can only be performed along strictly convex punches in the longitudinal direction.

Essentially, the surface of the shell is given by a grid of plane curves lying on the secant normal planes. A set of such lines is a discrete frame of the surface. The planes of symmetry \( F_1 \) and \( F_2 \), shown in figure 6, are used as the main planes on which the cross-section contours are projected. The gaps between the frame lines are filled graphically by projections using two-dimensional interpolation methods. Thus, mathematical modeling of the geometric shape of the shell is reduced to obtaining an approximating curve for a given discrete set of points that meets the requirements of continuity and smoothness.

In addition to the requirements for the approximating curve, it is necessary to take into account two features: first, the source data may contain gross errors (when measuring coordinates, operator errors when entering coordinate values into computer memory, etc.); second, the source data inevitably contains sampling errors. Since classical methods of curve approximation, such as Lagrange polynomial interpolation, root-mean-square interpolation, cubic equation interpolation in the form of Beziers or B-splines, may not always meet the above requirements, it becomes necessary to develop a special apparatus for geometric modeling of the class of problems under consideration.

In computer-aided design, the development of methods for determining the smoothness of surfaces is of great interest. Currently, the best mathematical methods for determining the smoothness of a surface use orthogonal meshes of minimum and maximum curvature or Gaussian curvature \( K \). For the parametric representation of surfaces, the Gaussian parametric coordinates \( u \) and \( v \) are used and the differential geometric parameters are determined at the intersection point of two lines defining the neighborhood of the synthesized surface. A surface is defined by sets of flat lines written in parametric form:

\[ x = x(t); \]
\[ y = y(t); \]
\[ z = z(t); \] (3)

where \( t \) – the parameter, when changing from -1 to 1, the point runs along the line from beginning to end, have continuous derivatives of the 2nd order (figure 8).
The coordinates of the nodes in the network are calculated directly from (3) for known values of the parameter \( t \), and the derivatives from (3) in the form of tangents:

\[
\dot{r}_u = (x_t, y_t, z_t) \quad \text{and} \quad \dot{r}_v = (\hat{x}_t, \hat{y}_t, \hat{z}_t)
\]  

(4)

The vector \( m \) is the unit vector of the normal to the surface and is calculated from the values of the tangents known from (4) in the form:

\[
m = \frac{r_u \times r_v}{|r_u \times r_v|},
\]  

(5)

Figure 8. Parametric representation of the surface using the Gaussian parameters \( u \) and \( v \).

where the notation of the form \( |r_u \times r_v| \) is the norm of the vector.

Having the nodal parameters (4) and (5), it is already possible to build efficient algorithms for reducing the surfaces of double curvature to the lines of curvature. In general, it is necessary to have a surface equation of two variables:

\[
x = x(u, v);
\]

\[
y = y(u, v);
\]

\[
z = z(u, v),
\]  

(6)

in the neighborhood of each grid node and find the second derivatives for a more accurate description of the surface geometry based on the application of mixed derivatives:

\[
\ddot{r}_{uv} = (\ddot{x}_{uv}, \ddot{y}_{uv}, \ddot{z}_{uv})
\]  

(7)

However, since these equations do not exist, we can estimate (7) by the values of the first and second derivatives of a pair of lines intersecting at a given node. The basis for this estimate is the assumption that in the vicinity of the node, the lines of the discrete frame differ slightly from the corresponding normal sections of the surface to be constructed.

Take any of the tangents to the line \( u \) or \( v \), for example \( \dot{r}_u \), then we can write:

\[
\dot{r}_u \cdot m = 0
\]  

(8)

Differentiating this ratio by the length of the arc \( S \) of the selected curve, we get:

\[
\ddot{r}_u \cdot m + \dot{r}_u \cdot \dot{m} = 0
\]  

(9)

Given that \( \ddot{r}_u = N_u \) is the normal to the curve at a given point and, by substituting in the coordinate directions, we get:
\[
m_{u} = -\frac{N_{u}m}{r_{u}}; \quad m_{v} = -\frac{N_{v}m}{r_{v}};
\]

From the expression of the coefficients of the second differential quadratic Gauss form, we write:

\[
\hat{r}_{uv} \cdot m = -\frac{r_{u}m_{u} + r_{v}m_{v}}{2}
\]

Substituting (10) in (11), multiplying on the right by \(m\) and taking into account that \(m^{2}=1\), we get:

\[
\hat{r}_{uv} = \frac{1}{2} \left[ N_{u} \frac{r_{u}r_{v}}{(r_{u})^{2}} + N_{v} \frac{r_{u}r_{v}}{(r_{v})^{2}} \right]
\]

It can be seen from (12) that the mixed derivative is a weighted average of the normals to the coordinate lines at their intersection. The weight coefficients are complexes of tangent vectors. Many orthogonal lines can be drawn through any nodal point of the surface, only two of them are the main curvatures that satisfy the condition \(\hat{r}_{uv} = 0\). In practice, when constructing a discrete framework, one tends to define one of the families of surface lines. As a rule, this is a set of cross-sections. The second family is defined kinematically as the family of equidistant lines.

It is known that the kinematic method of forming a water-forming surface consists in determining a one-parameter family of curves forming the desired surface. If we are dealing with a family of straight generators, for example, one-line surfaces, then one of the curvatures, in our case \(k_{2}\) is 0, then the Gaussian curvature \(K\) at all points is also zero and the surface is unfolding, i.e. it can be unfolded into a plane.

For the surface of the shell of a biconvex shape \((K>0)\), according to the condition \(K = |k_{1} \cdot k_{2}| = \max\), you can set the position of the generator passing through the point \(O\) and the radii of curvature at this point \(R_{10}\) and \(R_{20}\) in the main directions of the surface \(x_{0}\) and \(y_{0}\).

Here, the generating, for example, \(v\)-curve moves along some guiding \(u\)-curve, and the surface is given by the equation:

\[
r(u, v) = r(v) \cdot [T(u)],
\]

where \(r(v)\) – parametric equation of the generatrix \((0 \leq v \leq 1)\);

\(T(u)\) – means the transformation of the generatrix \(r(v)\) and determines its shape in the process of moving and changing along the guide.

In our case, the shape of the generatrix is determined by the magnitude and direction of the deflection of the cross-section of the shell surface (see figure 6). If we take the length of the arc or the angle of the arc measured from the center point as the parameter \(t\), then we have:

\[
|\hat{r}(\alpha)| = 1; \quad |\hat{r}(\alpha)| = k_{1};
\]

\[
|\hat{r}(\beta)| = 1; \quad |\hat{r}(\beta)| = k_{2};
\]

This consideration of the surface does not contradict, but, on the contrary, facilitates the search for the directions of the main axes \(x_{0}\) and \(y_{0}\). It was noted above that the combination of curvatures of any node point of a discrete frame:

\[
K = k_{\min} \cdot k_{\max}
\]

characterizes the local shape of the surface, and at point \(O\) the shape of the contiguous quadratic surface as a whole. Thus, we get a smooth convex surface. In the reasoning, we used a vector parametric representation that is compatible with the use of spatial transformations of three-dimensional coordinates.

Thus, in order to study the circumferential surfaces of a tight punch, taking into account the properties of the classical quadratic shell in contact, the geometric data during processing must go through the following stages: construction of a discrete frame; setting geometric parameters in the cell nodes; transformation of the discrete frame into a model of a continuous surface; representation of the surface in the main axes; setting the curvature contours with second-order curves.
4. Summary
To determine the directions of the main axes \( x_0 \) and \( y_0 \) at the point \( O \), we considered the surface of the shell as a whole, when it is defined as a one-parameter family of curves of transverse contours located at their upper point along the shaping contour in normal planes. Moreover, in the vicinity of the point \( O \), the surface of the shell is close to the quadratic surface (2), which belongs to the surface of the toroid type.

For the practical implementation of the tightness scheme under conditions of symmetry, it is necessary, first of all, to solve the problem associated with the representation of the shell surface in the main axes. The solution of the problem is achieved by using a combination of the following techniques when forming a biconvex shell.

Based on the 3D model of the shell surface, the vertex (point \( O \)), the main axes \( x_0 \) and \( y_0 \), and the coordinates of the points of the forming contour and the central cross-section located in the central planes of symmetry \( F_1 \) and \( F_2 \) of the tight press are determined.

The symmetry of the working surface can be created by «building up» the surface on the computer due to «smooth crosslinking» in the direction of the \( x_0 \) and \( y_0 \) axes. Then the forming contour will be located in the plane of symmetry \( F_1 \) and will be aligned with one of the lines of curvature passing through the point \( O \).

Such a setting of the forming contour of the tight punch provides, at any stage of step loading, the calculation of the coordinates of the vanishing points of the workpiece from the punch, the angle of inclination at the vanishing points, the length of the blank laid on the punch and the trajectory of the point near the press clamp relative to the forming contour.

The step-by-step process should be constructed for the surface reduced to the main axes and planes in the control programs for forming tight shells under the conditions of symmetry and their combination with the main axes of anisotropy of the sheet billet and the coordinate planes of the tight press.

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