Multiparticle entanglement is of important resources for quantum communication and quantum computation. The goal of this paper is to characterize general multipartite entangled states according to shallow quantum circuits. It is first proved that any genuinely multipartite entanglement on finite-dimensional spaces can be generated by using 2-layer shallow quantum circuit consisting of two biseparable quantum channels, which has the smallest nontrivial circuit depth in the shallow quantum circuit model. Further, a semi-device-independent entanglement model depending on the local connection ability in the second layer of quantum circuits is proposed. This implies a complete hierarchy of distinguishing genuinely multipartite entangled states. It shows a completely different multipartite nonlocality from the quantum network entanglement. These results show new insights for the multipartite entanglement, quantum network, and measurement-based quantum computation.

1. Introduction

Entanglement is an important quantum property of two or more systems in quantum mechanics associated with Schrödinger evolution equations. A bipartite entanglement is defined as it cannot be decomposed into an ensemble of separable states. This allows verifying any bipartite entanglement beyond all separable states using entanglement witness. Another device-independent method is inspired by Einstein–Podolsky–Rosen (EPR) steering or Bell inequality which can witness stronger quantum nonlocalities from only the statistics of local measurements on an entanglement.

Although the fully separable model is easily extended for multi-particle systems, it is useless for verifying the genuinely multipartite nonlocality. Instead, some stronger separable models are constructed for special goals. One is the so-called biseparable model which is defined to distinguish the genuinely multipartite entanglement from all the biseparable states. This allows us to witness the genuine multipartite nonlocality beyond the fully separable model even for quantum networks. Another model is using Greenberger–Horne–Zeilinger (GHZ)-paradox in the all-versus-nothing test manner. If the local tensor decomposition is considered, the high-dimensional model or quantum network entanglement may rule out any network separable state consisting of small entanglements that are shared by partial parties. Different from the biseparable model, this model provides a device-independent verification of unknown entanglement devices. Another is from the particle-losing model for characterizing the entanglement robustness against losing partial systems. This can imply a different hierarchy of well-known entanglements including GHZ state and Dicke states going beyond other models. All of these entanglement models can only justify special multipartite systems. A natural problem is to explore proper model for general systems.

Bravyi et al. investigated 2D hidden linear function problem in terms of constant-depth quantum circuit using special quantum gates. This is further extended to other circuits over special gates. These results intrigue new ideas to explore multipartite entanglement with shallow quantum circuits. Especially, each bipartite separable state can be generated by using one layer of quantum separable channel, as Figure 1a. This suggests a novel model for generating multipartite entanglement by using different layers of biseparable completely-positive trace-preserving (BCPTP) channels from fully separable states, as shown in Figure 1b. A nature problem is what is the relationship among the entangled states, circuit depth, and quantum channels.

The goal of this work is to characterize multipartite entanglement based on shallow quantum circuits of bipartite quantum channels. We first prove any multipartite entanglement can be generated by using a 2-layer shallow quantum circuit consisting of two biseparable completely-positive trace-preserving channels. If the second layer consists of a convex combination of local fully separable channels with one joint channel, the present model further implies a complete hierarchy for characterizing any multipartite entanglement according to the joint ability in its generation circuits. This second layer can be further regarded as an adversarial model in cryptographic applications.
Figure 1. Schematic shallow quantum circuit for quantum state generations. a) A bipartite separable system with a 1-layer quantum circuit. $E(\cdot)$ is a bipartite separable channel defined by $E(\cdot) = \sum_i K_i \otimes S_i$, where Kraus operators $K_i$ and $S_i$ satisfy $\sum_i K_i^\dagger \otimes S_i^\dagger S_i = 1$ with the identity operator 1. b) An $n$-partite entanglement with $k$-layer quantum circuits. Each $E_i(\cdot)$ is a bipartite separable quantum channel. c) A tripartite entanglement with 2-layer quantum circuits. One is from a biseparable quantum channel and the other is from a local joint model in adversarial scenarios. Here, two parties in the second layer who share particles $B$ and $C$ may perform local joint operations defined by a biseparable quantum channel $E_{A|BC}(\cdot)$ over the bipartition $A$ and $\{B, C\}$.

2. Result

2.1. Genuinely Multipartite Entanglement Generated with Shallow Quantum Circuits

A general isolated $d$-dimensional quantum system is represented by a normalized vector $|\phi\rangle$ in Hilbert space $\mathcal{H}_d$. Instead, an open system is described by probabilistically mixing an ensemble of pure states $\{|\phi_i\rangle\}$, that is, $\rho = \sum_i p_i |\phi_i\rangle \langle \phi_i|$, where $\{p_i\}$ is a probability distribution. An $n$-particle pure state $|\Phi\rangle$ on Hilbert space $\otimes_{i=1}^n \mathcal{H}_i$ is biseparable\cite{10} if it can be represented by $|\Phi\rangle = |\phi_i\rangle |\psi_j\rangle_\mathcal{I}$ with two pure states $|\phi_i\rangle$ and $|\psi_j\rangle$, where $I$ and $\mathcal{I}$ are bipartition of $\{A_1, \ldots, A_n\}$. Note that $|\Phi\rangle$ can be generated from a fully separable state $|0\rangle^\otimes n$ with a 1-layer shallow circuit of biseparable quantum channel defined by $E(\cdot) := U_j \otimes V_j$, where $U_j$ and $V_j$ are $n$-partite entanglement models in terms of shallow quantum circuits. Especially, define a biseparable completely positive trace-preserving (BCPTP) channel\cite{10,19} on Hilbert space $\mathcal{H}_i \otimes \mathcal{H}_j$ as

$$E_i(\rho) = \sum_j (K_j \otimes S_j) \rho (K_j^\dagger \otimes S_j^\dagger)$$

where $K_i, S_i$ are respective Kraus operators on Hilbert space $\mathcal{H}_i$ and $\mathcal{H}_j$ and satisfy $\sum_i K_i^\dagger \otimes S_i^\dagger S_i = 1$ with the identity operator 1, $\mathcal{H}_i = \otimes_{A_1,G_i} \mathcal{H}_i$, and $\mathcal{H}_j = \otimes_{A_j,G_j} \mathcal{H}_j$. For a given separable state $\rho_{bs} = \sum_i p_i |\phi_i\rangle \langle \phi_i|$ over a given bipartition $I$ and $\mathcal{I}$, it is straightforward to show from Equation (2) that there is a BCPTP channel $E_i$ with $K_i : |0\rangle \mapsto \rho_{bs}$ and $S_i : |0\rangle \mapsto \phi_i$ such that

$$\rho_{bs} = E_i(|0\rangle \langle 0|^\otimes n)$$

This means that any biseparable state\cite{10} can be generated by a probabilistically convex combination of BCPTP channels, that is,

$$\rho_{bs} = \sum_i q_i E_i(|0\rangle \langle 0|^\otimes n)$$

where the summation is over any proper subset of $\{A_1, \ldots, A_n\}$. Thus BCPTP channel provides an equivalent representation of the biseparable model\cite{10}. Our goal in what follows is to explore the multipartite entanglement in terms of its shallow generation circuits consisting of BCPTP channels.

Note that one layer of BCPTP channel can only build a biseparable state\cite{10}. This implies that the nontrivial example should be at least two layers. Especially, for any genuinely $n$-partite entanglement on finite-dimensional Hilbert spaces $\otimes_{i=1}^n \mathcal{H}_i$,\cite{10} assume that the Schmidt decomposition with respect to the bipartition $I = \{A_i\}$ and $\mathcal{I} = \{A_2, \ldots, A_n\}$ is given by

$$|\Phi\rangle = \sum_{i=1}^d \sqrt{\lambda_i} |\phi_i\rangle_i |\psi_i\rangle_\mathcal{I}$$

where $\lambda_i$’s are Schmidt coefficients satisfying $\sum_i \lambda_i = 1$. $\{|\phi_i\rangle\}$ are orthogonal states of $A_i$, and $\{|\psi_i\rangle\}$ are orthogonal states of all the systems in $\mathcal{I}$. There is a unitary transformation $U$ on Hilbert space $\mathcal{H}_i$ satisfying (Note S1, Supporting Information)

$$U : |\psi_i\rangle_\mathcal{I} \mapsto |\phi_i\rangle_i |\psi_i\rangle_\mathcal{I}, \ i = 1, \ldots, d$$

where $\{|\psi_i\rangle\}$ are orthogonal states of all the systems in $\mathcal{I} = \{A_2, \ldots, A_n\}$. Thus the state $|\Phi\rangle$ is generated by a 2-layer shallow quantum circuit consisting of two BCPTP channels. By considering the probabilistic mixture of BCPTP channels, it implies a general result for multipartite entanglement on finite-dimensional Hilbert spaces.

Theorem 1. For $n$-partite state $\rho$ on finite-dimensional Hilbert spaces, it can be generated by a 2-layer quantum circuit given by

$$\rho = E_2 \circ E_1(|0\rangle \langle 0|^\otimes n)$$

$= \sum_{i,j} p_i q_j E_i E_j(|0\rangle \langle 0|^\otimes n)$

where $E_i$ and $E_j$ are respective probabilistic mixtures of BCPTP channels $E_i$ and $E_j$, and $\{p_i\}$ and $\{q_j\}$ are probability distributions.

Theorem 1 holds for any pure or mixed state on finite-dimensional Hilbert space. Equation (8) implies a universal circuit with two depths for building any state from a fully product state, as shown in Figure 2. This means any entanglement can be
generated by a 2-layer shallow circuit consisting of two BCPTP channels. Note that the 2-layer quantum circuit is the smallest nontrivial shallow circuits. Theorem 1 implies the strong generation ability of small shallow circuits. This raises an immediate problem whether or not similar results hold in infinite-dimensional Hilbert spaces.

The present shallow circuit consisting of BCPTP channels is stronger than the standard quantum circuit model with an unfix circuit depth using two-particle joint operations.[28,29] The second layer circuit of Theorem 1 means that the BCPTP channel may activate multipartite entanglement from biseparable states. This kind of entanglement swapping is the core of quantum networks.[30,31]

2.2. k-Connection Genuinely Entanglement Generated with 2-Layer Shallow Quantum Circuits

In Theorem 1, a 2-layer circuit model may be too strong both in theory and applications. The main reason is that both biseparable quantum channels allow any bipartition decomposition. Instead, we consider a one-side biseparable channel in the second layer while the first layer is to prepare a biseparable state. Here, one local joint operation $E_i$ may be performed on a local set $I \subseteq \{A_1, \ldots, A_n\}$ while separable operations are performed on each particle in the complement set $\overline{I}$. The joint channel $E_i$ can be regarded as semi-device-independent scenarios in secure applications such as quantum secret sharing, as shown in Figure 1c, where partial adversaries who own systems in $I$ may cooperate to recover other parties’ information by performing joint operations while legal others are remotely distributed and then not allowed to perform joint operations. Denote $e_i$ as the number of particles in $I$. Define a k-connection BCPTP channel $E^{(k)}_{\rho}$ on Hilbert space $H_I \otimes H_{\overline{I}}$ with $e_i \leq k$ as

$$E^{(k)}_{\rho}(\rho) = \sum_i (K_i \otimes (\otimes_{j \in (I)} S_{ij})) \rho ((K_i \otimes (\otimes_{j \in (I)} S_{ij})))$$

where $K_i$ and $S_{ij}$ are respective Kraus operators on Hilbert space $H_I$ and $H_{\overline{A}_i}$, and satisfy $\sum_i (K_i \otimes (\otimes_{j \in (I)} S_{ij}) (\otimes_{j \in (I)} S_{ij})) = 1$. The present $k$-connection BCPTP channel is of state-dependent. Our goal here is to explore genuinely multipartite entanglement in terms of its generation circuits with the quantum channel in Equation (9) in the second layer.

### Definition 1

An n-partite state is $k$-connection genuinely entanglement ($k$-CGE) if it is not a $k$-connection biseparable state given by

$$\rho^{(k)}_{\rho} = \sum_{\ell : e_i \leq k} p_{\ell} E^{(\ell)}_{\rho} (|0\rangle \langle 0|^{\otimes n})$$

where $E^{(\ell)}_{\rho}$ are $\ell$-connection BCPTP channels in terms of the bipartition $I$ and $\overline{I}$, $\{p_{\ell}\}$ is a probability distribution, and $E^{(\ell)}_{\rho}$ is a convex combination of BCPTP channels.

Similar to the proof of Theorem 1, the Schmidt decomposition of a given n-particle pure state $|\Phi\rangle$ on $d^n$-dimensional Hilbert space $\mathbb{H}_n^\otimes$ is given by

$$|\Phi\rangle = \sum_{i=1}^N \sqrt{\lambda_i} |\phi_i\rangle |\psi_i\rangle$$

where $\lambda_i$’s are Schmidt coefficients satisfying $\sum_i \lambda_i = 1$, $\{|\phi_i\rangle\}$ are orthogonal states of all the systems in $I$, and $\{|\psi_i\rangle\}$ are orthogonal states of all the systems in $\overline{I}$. From Equation (11) we have $N \leq \min\{d^\ell, d^n/d^\ell\}$. This implies that $N \leq d^n/d^\ell \leq d^n/(2\ell)$ for any integer $\ell$ with $\ell \geq n/2 + 1$. There is a unitary transformation $U$ on Hilbert space $H_I$ satisfying

$$U : |\psi_i\rangle_i \mapsto |0\rangle_\lambda |\psi_i\rangle_i, i = 1, \ldots, N$$

where $A_j \in \overline{I}$ and $\{|\psi_i\rangle\}$ are orthogonal states of the particles in $J := \overline{I} - \{A_j\}$. So, the state $|\Phi\rangle$ is an $\ell$-connection biseparable. This implies a general result for generating multipartite entanglement with different connection abilities as follows.

**Theorem 2.** For an $n$-partite state $\rho$ on finite-dimensional Hilbert space, it can be generated by a 2-layer shallow quantum circuit as

$$\rho = \sum_{\ell : e_i \leq k} p_{\ell} E^{(\ell)}_{\rho} (|0\rangle \langle 0|^{\otimes n})$$

where $E^{(\ell)}_{\rho}$ is a convex combination of BCPTP channels $E_{\ell}$ and $E^{(\ell)}_{\rho}$ is a convex combination of $k$-connection BCPTP channels $E^{(\ell)}_{\rho}$ defined in Equation (9) with $k \geq n/2 + 1$.

Theorem 2 rules out the possibility of $k$-CGE for large integer $k$. The situation is different for small $k$. For special case of $n = 2$, Definition 1 is reduced to the standard separable model of bipartite systems.[1,2] For each $n \geq 3$, from Definition 1 any biseparable state[10] is 1-connection biseparable state. In general, the connection ability $k$ is state-dependent.

### 2.3. A Complete Hierarchy of Genuinely Multipartite Entanglement

Note that any $k$-connection biseparable state is an $s$-connection biseparable state for any $s \geq k$. This implies a complete hierarchy for all the multipartite entangled states, as shown in Figure 3. The largest set contains 1-CGEs, that is, the genuinely multipartite entanglement in the biseparable model.[10] Instead, the smallest set consists of the strongest multipartite entanglement, that
is, $k_{\text{max}}$-CGE with $k_{\text{max}} = \lfloor \frac{d}{2} \rfloor$. Since each subset of $k$-CGE is not empty (see examples in what follows), the new classification is strict from Theorems 1 and 2, that is, each entanglement belongs to the only set $k$-CGE while it is not in $k + 1$-CGE for some $k$.

For any $n$-particle pure state $|\Phi\rangle$ on a finite-dimensional Hilbert space $\mathcal{H}_n$, from Equation (11) the orthogonality of $\{|\phi_i\rangle\}_i$ allows for constructing a unitary transformation (Equation (12)) if and only if its Schmidt number satisfies $N \leq d^{k-1}$ with $\ell^*_i = k$. This implies a directive way to find the connection-ability $k$ for a given pure state using its Schmidt numbers of reduced density matrices (Note S2, Supporting Information).

**Theorem 3.** An $n$-particle pure state on a $d^n$-dimensional Hilbert space $\mathcal{H}_n$ is $k$-CGE if and only if the Schmidt number of the reduced density matrix of any $k$ particles is larger than $d^{k-1}$.

From Definition 1 any genuinely multipartite entanglement in the biseparable model$^{[10,32]}$ is 1-CGE. Moreover, the present $k$-CGE is stronger than the robust entanglement with the robustness-depth $k$ since the particle-losing channel$^{[24]}$ is local CPTP channel. From Theorem 3 it generally requires to evaluate Schmidt numbers of almost all the reduced density matrices of $s$-particle with $s \leq 2^n$. This yields to a NP hard problem for general entanglement because of exponential number $(O(d^2/n))$ of reduced states. Instead, it is easy for special states.

**2.4. Examples**

**Example 1.** An $n$-particle Greenberger–Horne–Zeilinger (GHZ) state$^{[12]}$ is given by

$$|\text{GHZ}\rangle = \sum_{i=1}^{d} a_i |i\cdots i\rangle_{A_1\cdots A_n}$$

(15)

where $a_i$s satisfy $\sum_{i=1}^{d} a_i^2 = 1$. It is easy to verify the state in Equation (15) is 1-CGE from its permutational symmetry.

**Example 2.** Consider an $n$-particle $W$-type state$^{[31]}$

$$|W\rangle_{A_1\cdots A_n} = \sum_{i=1}^{d} a_i |1\rangle_i + a_{n+1} |1 \cdots 1\rangle$$

(16)

where $|1\rangle_i$ denotes the $i$-excitation defined by $|1\rangle_i = |0\rangle^{\otimes i-1}|1\rangle|0\rangle^{\otimes n-i}$. From Theorem 3 it is easy to prove $|W\rangle$ is a 2-CGE for $a_i \neq 0$, $i = 1, \ldots, n + 1$. This can be extended for general $n$-qubit Dicke states with $s$ excitations$^{[14]}$ given by

$$|D_{s,n}\rangle = \frac{1}{\sqrt{L_s}} \sum_{i_1\cdots i_s} |i_1\cdots i_s\rangle_{A_1\cdots A_s}$$

(17)

where $L_s$ is the normalization constant. It is a $k$-CGE with $k = \lfloor \log_2(s + 1) \rfloor + 1$ (Note S3, Supporting Information). This is beyond previous models$^{[10,32]}$ which do not distinguish GHZ state and $W$ state. Moreover, the state $|D_{s,n}\rangle$ is equivalent to $|D_{(d-1)^{s+n-1}}\rangle$ under local unitary operations. This implies the strongest nonlocality of Dicke state $|D_{s,n}\rangle$ with $s = \lfloor (d - 1)^n / 2 \rfloor$.

**Example 3.** Another example is entangled quantum network which may show different nonlocalities beyond single entanglement$^{[6,7,35]}$. As resource states of measurement-based quantum computation$^{[36]}$ the so-called cluster states$^{[36]}$ may be generated by generalized Einstein–Podolsky–Rosen (EPR) states$^{[14]}$ as shown in Figure 4. This kind of entanglement is 1-CGE (Note S4, Supporting Information). Similar result can be extended for generalized graph states$^{[37]}$ consisting of generalized EPR$^{[31]}$ and GHZ states$^{[12]}$. Instead, some quantum networks may show different connection abilities. One example is the $n$-particle completely-connected network $\mathcal{N}_c$, where each pair shares one bipartite entanglement $|\phi_i\rangle$. Recent result shows the joint state of any $k$-partite subnetwork in $\mathcal{N}_c$ is entangled for $k \geq 2$.$^{[10,28]}$ Hence, from Theorems 2 and 3 the joint state of $\mathcal{N}_c$ is a $k_{\text{max}}$-CGE with $k_{\text{max}} = \lfloor \frac{1}{2} \rfloor$. This means that $\mathcal{N}_c$ shows stronger nonlocality than GHZ state (Equation (15)) and $W$ state (Equation (16)) for any $n > 4$ in the present model. This is different from the robust-entanglement model$^{[24]}$, where both the $W$ state and $\mathcal{N}_c$ has the same robust-depth. Remarkably, it is converse to the recent result$^{[19–21]}$ which proves GHZ and $W$ states have stronger entanglement beyond quantum networks. This shows a surprising feature of the genuinely multipartite nonlocality beyond bipartite scenarios, that is, it is of model-dependent.
Algorithm 1 Verifying any n-partite quantum network $\mathcal{N}_q$ consisting of bipartite entanglement

**Input:** Finite-size network $\mathcal{N}_q$ 

**Output:** $k$, satisfying that $\mathcal{N}_q$ is at most $k$-connection entanglement

1. Find the connectedness degree $r_i$ for every party $A_i$ with $i = 1, \ldots, n$.
2. Rearrange all parties with decreasing order into $A_1, \ldots, A_n$ for simplicity.
3. Find $J$ such that $r_j = \min(r_i)$ with $j \in J$. Let $m = |J|$.
4. For $s = 1 : m$
   
   (a) Let $A_i \cap J = \emptyset$ for $i \in J$. For the parties $A_i$ with $i \notin J$, they share the most bipartite entangled states with parties in $\overline{J}$, compared with other parties in $\overline{J}$ and $A_i \in J$.
   
   (b) Evaluate $s_{\text{out}}$ and $f_i$
   
   (c) If $s_{\text{out}} + 2 f_i \geq s_{\text{out}}$, set $s = s + 1$
   
   (d) Output $k \leq \min(s_i, \ldots, s_n)$

For general quantum networks, it is generally difficult to find the largest $k$ such that the total state of $\mathcal{N}_q$ is $k$-CGE. Here, we provide a polynomial-time algorithm (Algorithm 1) for estimating the upper bound of $k$ for featuring its connection ability. This is inspired by Lemmas 1 and 2 (Note S5, Supporting Information).

For each party $A_i$ with the minimal connectedness degree $r_i$, from Lemma 1 (Note S5, Supporting Information) if $s_{\text{out}} + 2 f_i \geq s_{\text{out}}$ for some $t$ there is a CPTP mapping to disentangle all the particles shared by $A_i$. Hence, the total state of $\mathcal{N}_q$ is $t$-connection biseparable. The time complexity of the step (i) is at most $O(n)$.

The time complexity of step (iii) is at most $O(n)$. For a given party $A_i$ with $j \in J$, the time complexity of the step (b) is at most $O(n^2)$. Hence, the total time complexity is at most $O(n^3)$.

Some examples are shown in Figure 4. For the chain quantum network in Figure 4a, the party $A_1$ shares one bipartite entanglement (red line) with other parties out of $A$, and shares one bipartite entanglement (green line) with $A_2$. From Lemma 1 the chain quantum network in Figure 4a is 1-connection biseparable, where the party $A_1$ can be disentangled with other parties out of $A$ by using joint operation of $A_1$ and $A_2$. Moreover, it is genuinely multipartite entanglement $^{[10,21]}$ that is, any local operation cannot disentangle one party. Thus the chain quantum network in Figure 4a is 1-CGE. Similar result holds for the star quantum network in Figure 4b and cyclic quantum network in Figure 4c. For a completely connected quantum network in Figure 4d, the party $A_1$ shares two bipartite entangled states (red lines) with others out of $A$. There are three bipartite entangled states (green lines) shared by parties in $A$. From Lemma 1, it is 3-connection biseparable while any two parties cannot jointly disentangle one party. Hence, this quantum network is 2-CGE, where the party $A_1$ can be disentangled with others out of $A$ by using joint operation of the parties $A_1, A_2, A_3, A_4$. In general, we can prove an $n$-partite completely connected network is $k_{\text{max}}$-CGE with $k_{\text{max}} = \lfloor \frac{n+1}{2} \rfloor$. Similarly, from Lemma 2 the planar quantum network in Figure 4e is 1-CGE while the cubic quantum network is 2-CGE.

**2.5. Robustness of k-CGE**

From Equation (10) all the $k$-connection biseparable states constitute a convex set $S_k$. This allows to verify a general entangle-
explore the intrinsic nonlocality or the most reasonable model for multipartite systems.

We investigated general genuinely-generation multipartite entanglement with the help of shallow circuits. We proposed a two-layer shallow circuit model to characterize all the multipartite states in terms of biseparable completely positive trace-preserving channels. We further defined a multi-layer circuit model with one-side local-connection. The one-side local joint operation has provided a general standard for characterizing the connecting ability of general multipartite entanglement. We obtained a simple hierarchy of multipartite entanglement in terms of the connection ability. The new entanglement witness is used to verify the entanglement robustness. These results should be interesting in multipartite entanglement theory, quantum communication, and quantum computation.

Supporting Information
Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest
The authors declare no conflict of interest.

Author Contributions
M.X.L. and S.-M.F. conceived the idea. M.X.L. wrote the majority of the paper and S.M.F. reviewed this main results.

Data Availability Statement
Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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biseparable quantum channel, genuinely multipartite entanglement, measurement-based quantum computation, semi-device-independent, quantum networks, shallow quantum circuits

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