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Abstract. Exhaled droplets of an infected person can spread diseases with the expiratory flows in indoor environments. This work focuses on how the thermal stratification affects the evaporation and dispersion of exhaled droplets by using a theoretical integral model. Pure water droplets with different diameters (10-240μm) are exhaled and disperse with different RHs (0, 30%, 50% and 60%). Results show that for medium droplets (50-170μm), the thermal stratification can obviously weaken the evaporation of droplets due to less heat and mass transfer between droplets and ambient air. The droplets were similarly influenced when the ambient RH increases from 0 to 50%, particularly, the 50μm droplets showed an obvious fluctuation trend with the jet flow at the lock-up height in the stratified environment. When RH increased to 60%, a possible condensation phenomenon occurred on droplets, increasing the suspending time of droplets in the air. This theoretical model will be useful to control infectious diseases in rooms, especially when the thermal stratification exists.

1. Introduction
Respiratory droplets or droplet nuclei in the indoor environment have been recognized as carriers of potential pathogens as they were first identified by Wells [1]. Both the evaporation of exhaled droplets and dispersion of droplet nuclei are key processes for disease transmission. Studies show that the fate of expiratory droplets via respiratory activities, or more specifically, how far the droplets with different sizes travel, is highly dominated by the exhaled flow and subsequently by the ambient airflow [2,3]. As to the indoor environment with vertical temperature gradient such as in displacement ventilation (DV), previous studies demonstrate that at certain conditions human exhaled pollutants would be trapped or locked at the breathing height due to the thermal stratification, which was referred to the lock-up height [4-6]. Persons with the breathing height of the lock-up layer will be considered at high risk [4]. However, the influence of the lock-up phenomenon of the exhaled airflow on the droplet evaporation and dispersion has not been well understood.

This paper complements the previous studies on the lock-up effect on the dispersion of exhaled droplets with a jet integral model, in which the item of buoyancy flux is revised by a buoyant acceleration. The governing equations of evaporation and movement of a single droplet are combined with the integral model. The dispersion of the droplets or droplet nuclei is iterated with time by dating the results obtained from a group of ordinary differential equations (ODEs).

2. Mathematical models
2.1 Jet integral model in thermally stratified environments
As proposed by Zhou et al. [5], the respiratory process is treated as a continuous exhalation without inhalation and the exhaled flow is simplified as a horizontal buoyant jet here. In our preliminary study,
an integral model has been proved to be capable of capturing the physics of the exhaled jet flow
 dispersion, such as half-width, velocity, tracer concentration and trajectory, emphatically in indoor
 environments with a thermal stratification [6]. For a single droplet, it is also necessary to get the local
temperature distribution of the exhaled airflow.

The ambient has a stable density distribution \( \rho_a(y) \), determined by the vertical temperature
distribution \( T_a(y) \) in the thermally-stratified indoor environment. The distribution functions \( f(r, \phi) \) on
the cross-sectional plane is specified by Gaussian profile [6]. The integral quantities in the turbulent jet
zone about an element \( ds \) on the central trajectory satisfy the following principles of conservation [6]:

\[
\frac{d}{ds}\left[ \pi b^2 (u_c + 2 u_c \cos \theta) \right] = E
\]

(1)

\[
\frac{d}{ds}\left[ \frac{1}{2} \pi b^2 (u_c + 2 u_c \cos \theta)^2 \cos \theta \right] = E u_a + F_D \sin \theta
\]

(2)

\[
\frac{d}{ds}\left[ \frac{1}{2} \pi b^2 (u_c + 2 u_c \cos \theta)^2 \sin \theta \right] = \pi b^2 (\frac{\rho_a - \rho_0}{\rho_0}) g - F_D \cos \theta
\]

(3)

\[
\frac{d}{ds}\left[ \pi b^2 (u_c \frac{\lambda^2}{\lambda + 1} + \lambda^2 u_c \cos \theta) \right] (\frac{\rho_a - \rho_0}{\rho_0}) = \pi b^2 (u_c + 2 u_c \cos \theta) \frac{\rho_0}{\rho_0} \frac{\partial \theta}{\partial y} \sin \theta
\]

(4)

\[
\frac{d}{ds}\left[ \pi b^2 (u_c \frac{\lambda^2}{\lambda + 1} + \lambda^2 u_c \cos \theta) \right] c_c = 0
\]

(5)

\[
\frac{d}{ds}\left[ \pi b^2 (u_c \frac{\lambda^2}{\lambda + 1} + \lambda^2 u_c \cos \theta) \right] T_c = - \pi b^2 (u_c + 2 u_c \cos \theta) \frac{d \theta}{dy} \sin \theta
\]

(6)

where \( b \) is half width, \( E \) entrainment rate, \( F_D \) ambient drag force and \( \lambda \) dispersion rati.

2.2 Movement of a single droplet

The movement equation and the displacement of droplet can be given by

\[
\frac{d u_p}{dt} = g \left( u \frac{\rho_d}{\rho_p} \right) \frac{3 C_d}{4 \rho_p} \frac{u_p - u_p^*}{|u_p - u_p^*|}
\]

(7)

\[
\frac{d v_p}{dt} = u_p - u_p^*
\]

(8)

where \( C_d \) is the drag coefficient.

2.3 Evaporation of a single droplet

Droplet evaporation is a combination of heat and mass transfer between droplets and their surroundings:

\[
- \frac{d m_p}{dt} = I_d = \frac{2 \pi d_p M_V D_e}{RT_e} \ln \left( \frac{p_{pd}}{p_{pv}} \right)
\]

(9)

\[
m_p \rho_p \frac{dT_p}{dt} = \pi d_p^2 K_g \frac{T_e - T_p}{T_p} \frac{Nu - I_d L_V}{L_V}
\]

(10)

where \( I_d \) is evaporation rate, \( M_V \) molecular weight of vapor, \( D_e \) diffusion coefficient, \( K_g \) thermal
conductivity, \( L_V \) latent heat of vaporization, \( C \) correction factor, \( Sh \) Sherwood number and \( Nu \) Nusselt
number. With the ODEs (1)-(10), a fourth-order Runge-Kutta algorithm [7] is employed to solve the
governing equations for the evaporation process and trajectory of a droplet after being expelled with the
exhaled jet flow.

3. Results and discussion

3.1 Model validation

Experimental works of Smolik et al. [8] and Ranz and Marshall [9] were employed here to validate our
model. From Fig.1, it can be seen that the prediction agrees well with the experimental results of
motionless and freely falling droplets, which suggests that the integral model is valid to reveal
evaporation and dispersion of the exhaled droplets in indoor environments.
3.2 Effects of lock-up phenomenon on droplet dispersion

The evaporation time of a droplet of certain diameter could be derived, as well as the falling time, which was defined as the time a droplet took to fall 2m away. In the numerical calculations, the diameter of person’s mouth was set at 0.02m, a stable expiratory temperature at 33°C and the air temperature at 20°C. The dispersion in a uniform environment is included here to make a comparison and thus highlight the characteristics in stratified environments.

Fig. 2 gives how the exhaled droplet diameter decreases with time when the air is dry (RH of 0%). It can be seen that small droplets evaporated faster and large droplets deposited faster. A 60μm droplet evaporated completely at less than 2s and a 220μm droplet fell onto the ground at about 4s. In thermally stratified environment, the evaporation time is longer than that in uniform environment. For a 160μm droplet, it can completely evaporate in the air before reaching the ground in uniform environment, while it deposited to ground before total evaporation in stratified environment.

![Figure 2. Variations of droplet diameter with time: (a) in thermally uniform environment; (b) in thermally stratified environment](image)

The evaporation-falling time curve of different sizes of droplets is plotted in Fig. 3. It can be seen that for droplets of less than 100μm, there is no difference in the evaporation time in uniform and stratified, so is the falling time for droplets of larger than 200μm. However, for droplets of 100-200μm, the evaporation time shows longer in stratified environment than that in uniform environment, whereas the deposition time becomes shorter. The intersection of evaporation time and falling time on curves means the largest droplet that could totally evaporate in the air after release from the breathing height of 2m. In stratified environment, the point moves to the left, which means the thermal stratification works as a barricade for evaporation and accelerates the deposition for medium droplets. Therefore, parts of the exhaled droplets can be deposited by changing the indoor airflow in order to decrease the number of suspended droplets with the expiratory jet flow.
3.3 Effects of RH on droplet dispersion in stratified environment

Fig. 4 and Fig. 5 show the influence of the ambient humidity in stratified environment. It can be seen from Fig. 4, high RH can obviously postpone the evaporation process of some droplets, since air with a higher RH has a lower potential in absorbing the water vapor. For both 10 and 50\(\mu\)m droplets, they can totally evaporate before they deposit to the ground, all with evaporation times less than 10s. For 200\(\mu\)m droplets, they can quickly fall onto the ground within 10s. However, for 100\(\mu\)m droplets, it is found that with low RHs, droplets can totally evaporate before reaching the ground. As RH rises to 50\%, droplets can only partially evaporate and drop down to the floor instead of moving along with airflows.

The temperature of the droplet changes depending on both the surrounding relative humidity and air temperature. Fig. 5 shows the comparison of the evaporation-falling time of droplets. In this simulation, the ambient air temperature in uniform environment is held constant 293.15K, while in stratified environment, the temperature gradient is \(dT/dy=2^\circ\text{C}/\text{m}\). Results show that higher RH contributed to longer evaporation times and shorter falling times, which is reflected the left-moving turning points of curves that represent the largest droplet that could totally evaporate in the air after release from the breathing height. Compared with the uniform environment, it seems that the thermal layer weakens the mass transfer for medium droplets less than 90\(\mu\)m, equivalent to a higher RH. For the droplets larger than 170\(\mu\)m droplets, the thermal stratification accelerates their deposition.

With RH increasing to 60\%, a local hump is observed in the curve of diameter variations with time, seeing Fig. 6(a). This phenomenon is induced by a possible condensation of water vapor in ambient air on droplets. The exhaled air from a person is at 100% RH and a higher temperature \((T_{p0}=306\text{K})\), along
with lower ambient air temperature \((T_{\infty}=293.15K)\), so it is easy to result in condensation upon the traveling droplet. It is noted from Fig. 6(b) that in a few seconds, droplet temperature rapidly declines to a low value that is close to ambient air temperature. The horizontal velocity has the same trends in different RHs, seeing Fig. 6(c). With the current model, RH does not change the velocity field of the ambient air, and so it cannot affect the droplet horizontal velocity directly. However, a hump phenomenon is also seen in Fig. 6(d) when RH increases to 60%, the possible reason for which is momentum conservation. When the condensation occurs, the mass of the droplet increases so the vertical velocity decreases. Redrow et al. [10] explored the evaporation of airborne sputum droplets in uniform environment and also found a condensation phenomenon occurred at 80%, higher than our 60%, which is reasonable because the impact of the temperature gradient in stratified environment is equivalent to a higher RH according to the above results.

![Figure 6](image)

**Figure 6.** Condensation upon droplets at high RH:
(a) droplet diameter with time; (b) droplet temperature with time;
(c) droplet horizontal velocity with time; (d) droplet vertical velocity with time

### 4. Conclusions

This study analyzed the evaporation and motion of exhaled droplets in thermally stratified environment and thermally uniform environment using a jet integral model. Compared with thermally uniform environment, it was found that small droplets and larger droplets can evaporate and deposited quickly in the air, respectively, which were hardly influenced by the vertical thermal stratification and ambient RH. However, for medium droplets, it was revealed that the vertical temperature gradient in thermally stratified environment works as a barricade for evaporation and accelerates the deposition, and deeper stratification causes it more obvious. The evaporation and falling of droplets were similarly influenced when the ambient RH increases from 0 to 50%, during which the 50μm droplets showed an obvious fluctuation trend with the jet flow at the lock-up height in the stratified environment. When RH increased to 60%, a possible condensation of ambient air occurred on droplets, which increased the suspending time of droplets in the air, as well as the inhalation risk by the susceptible.

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