Abstract In this article we study an intermediate inflationary universe models using the Gauss–Bonnet brane. General conditions required for these models to be realizable are derived and discussed. We use recent astronomical observations to constraint the parameters appearing in the model.

1 Introduction

It is well known that one of the most exciting ideas of contemporary physics is to explain the origin of the observed structures in our universe. It is believed that inflation [1–6] can provide an elegant mechanism to explain the large-scale structure, as a result of quantum fluctuations in the early expanding universe, predicting that small density perturbations are likely to be generated in the very early universe with a nearly scale-free spectrum [7–11]. This prediction has been supported by early observational data, specifically in the detection of temperature fluctuations in the cosmic microwave background (CMB) by the COBE satellite [12]. The scheme of inflation [13–16] (see [17] for a review) is based on the idea that at early times there was a phase in which the universe evolved through accelerated expansion, during which the universe was dominated by a potential $V(\phi)$ of a scalar field $\phi$ (inflaton).

In the context of inflation we have the particular scenario of “intermediate inflation”, in which the scale factor evolves as $a(t) = \exp(At^f)$. Therefore, the expansion of the universe is slower than standard de Sitter inflation ($a(t) = \exp(\mathcal{H}t)$), but faster than power-law inflation ($a(t) = t^p; p > 1$). The intermediate inflationary model was introduced as an exact solution for a particular scalar field potential of the type $V(\phi) \propto \phi^{-4(f^{-1} - 1)}$, where $f$ is a free parameter [18–22]. Recently, a tachyon field in intermediate inflation was considered in [23], and a warm-intermediate inflationary universe model was studied in [24] (see also [25]).

The motivation to study intermediate inflationary model becomes from string/M-theory (for a review see [26–29]). This theory suggests that in order to have a ghost-free action high-order curvature invariant corrections to the Einstein–Hilbert action must be proportional to the Gauss–Bonnet (GB) term [30, 31]. GB terms arise naturally as the leading order of the expansion to the low-energy string effective action, where is the inverse string tension [32]. This kind of theory has been applied to possible resolution of the initial singularity problem [33], to the study of black-hole solutions [34–36], accelerated cosmological solutions [37], among others (see [38–44]). In particular, very recently, it has been found that for a dark-energy model the GB interaction in four dimensions with a dynamical dilatonic scalar field coupling leads to a solution of the form $a = \exp(At^f)$ [45], where the universe starts evolving with a decelerated exponential expansion. Here, the constant $A$ becomes given by $A = \frac{2}{\kappa^2}$ and $f = \frac{1}{2}$, with $\kappa^2 = 8\pi G$ and $n$ is a constant. Also, much attention has been focused on the Randall–Sundrum (RS) scenario, where our observable four-dimensional universe is modeled as a domain wall embedded in a higher-dimensional bulk space [46–48]. These kind of models can be obtained from superstring theory [49, 50]. For a comprehensible review on RS cosmology, see [51–53]. In this way, the idea that inflation, or specifically, intermediate inflation, comes from an effective theory at low dimension of a more fundamental string theory is in itself very appealing. Thus, in brane universe models the effective theories that emerge from string/M-theory lead to a Friedmann equation which is proportional to the square energy density.

When the five-dimensional Einstein–GB equations are projected on to the brane, a complicated Hubble equation is obtained [54–60]. Interestingly enough, this modified Friedmann equation reduces to a very simple equation...
$H^2 \propto \rho^q$ with $q = 1, 2, 2/3$ corresponding to General Relativity (GR), RS and GB regimes, respectively. This situation motivated the “patch cosmology” as a useful approach to study brane-world scenarios [61]. This scheme makes use of a non-standard Friedmann equation of the form $H^2 = \beta_4^2 \rho^{q}$. Despite all the shortcomings of this approximate treatment of extra-dimensional physics, it gives several important first-impact information. Recently, a closed inflationary universe in patch cosmology was considered in [62], and a tachyonic universes in patch cosmologies with $\Omega > 1$ was studied in [63].

The purpose of the present work is to study intermediate inflationary universe models, where the matter content is confined to a four-dimensional brane which is embedded in a five-dimensional bulk where a GB contribution is considered. We study these models using the approach of patch cosmology. On the other hand, a comprehensive study in the present work reveals that, intermediate inflation provides the possibility of density perturbation and gravitational wave spectra which differ from the usual inflationary prediction of a nearly flat spectrum with negligible gravitational waves. Furthermore, in the present model the tensor-to-scalar ratio $r$ is scale-dependent, and we have shown that a good fit to the WMAP5 observations.

The outline of the Letter is as follows. The next section we briefly review the cosmological equations in the GB brane world and present the patch cosmological equations for this model. In Sect. 3 presents a short review of the intermediate inflation in GB brane. In Sect. 4 the cosmological perturbations are investigated. Finally, in Sect. 5 we summarize our finding.

## 2 Cosmological equations in Gauss–Bonnet brane

We start with the five-dimensional bulk action for the GB brane world:

$$S = \frac{1}{2\kappa_5^2} \int_{\text{bulk}} d^5x \sqrt{-g_5} \left[ R - 2\Lambda_5 
+ \alpha \left( R_{\mu\nu\lambda\rho} R_{\mu\nu\lambda\rho} - 4 R_{\mu\nu} R_{\mu\nu} + R^2 \right) \right] 
+ \int_{\text{brane}} d^4x \sqrt{-g_4}(L_{\text{matter}} - \sigma).$$

(1)

where $\Lambda_5 = -3\mu^2 (2 - 4\alpha\mu^2)$ is the cosmological constant in five dimensions, with the $AdS_5$ energy scale $\mu$, $\alpha$ is the GB coupling constant, $\kappa_5 = 8\pi/m_5$ is the five-dimensional gravitational coupling constant and $\sigma$ is the brane tension. $L_{\text{matter}}$ is the matter Lagrangian for the inflaton field on the brane. We will consider the case that a perfect fluid matter source with density $\rho$ is confined to the brane.

A Friedmann–Robertson–Walker (FRW) brane in an $AdS_5$ bulk is a solution to the field and junction equations (see [54–60]). The modified Friedmann on the brane can be written as

$$H^2 = \frac{1}{4\alpha} \left[ (1 - 4\alpha\mu^2) \cosh \left( \frac{2\chi}{3} \right) - 1 \right],$$

(2)

$$\kappa_5^2 (\rho + \sigma) = \left[ \frac{2(1 - 4\alpha\mu^2)^3}{\alpha} \right]^{1/2} \sinh \chi,$$

(3)

where $\chi$ represents a dimensionless measure of the energy density $\rho$. In this work we will assume that the matter fields are restricted to a lower-dimensional hypersurface (brane) and that gravity exists throughout the space-time (brane and bulk) as a dynamical theory of geometry. Also, for 4D homogeneous and isotropic Friedmann cosmology, an extended version of Birkhoff’s theorem tells us that if the bulk space-time is AdS, it implies that the effect of the Weyl tensor (known as dark radiation) does not appear in the modified Friedmann equation. On the other hand, the brane Friedmann equation for the general, where the bulk space-time may be interpreted as a charged black hole was studied in [64–66].

The modified Friedmann equation (2), together with (3), shows that there is a characteristic Gauss–Bonnet energy scale [67]

$$m_{GB} = \left[ \frac{2(1 - 4\alpha\mu^2)^3}{\alpha\kappa_5^4} \right]^{1/8},$$

(4)

such that the GB high-energy regime ($\chi \gg 1$) occurs if $\rho + \sigma \gg m_{GB}^4$. Expanding (2) in $\chi$ and using (3), we find in the full theory three regimes for the dynamical history of the brane universe [54–60]:

$$\rho \gg m_{GB}^4 \quad \Rightarrow \quad H^2 \approx \left[ \frac{\kappa_5^2}{16\alpha} \rho \right]^{2/3} \quad \text{(GB)},$$

(5)

$$m_{GB} \gg \rho \gg \sigma \quad \Rightarrow \quad H^2 \approx \frac{\kappa_5^2}{6\sigma} \rho^2 \quad \text{(RS)},$$

(6)

$$\rho \ll \sigma \quad \Rightarrow \quad H^2 \approx \frac{\kappa_5^2}{3} \rho \quad \text{(GR)}.$$  

(7)

Clearly (5), (6) and (7) are much simpler than the full (2) and in a practical case one of the three energy regimes will be assumed. Therefore, patch cosmology can be useful to describe the universe in a region of time and energy in which [61]

$$H^2 = \beta_q^2 \rho^q,$$

(8)

where $H = \dot{a}/a$ is the Hubble parameter and $q$ is a patch parameter that describes a particular cosmological model under consideration. The choice $q = 1$ corresponds to standard general relativity with $\beta_1^2 = 8\pi/3m_p^2$, where $m_p$ is the four-dimensional Planck mass. If we take $q = 2$, we obtain the high-energy limit of the brane-world cosmology, in