Time and Entropy from Semi-classical Tunneling of the Cosmological Scale Function*

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Abstract

Two major apparently unrelated problems, that of the origin of time in the universe associated with quantum gravity and of the entropy in de Sitter cosmological models, are found to have their origin in a single physical phenomenon: the semi-classical tunneling through a classically forbidden region of the cosmological scale factor. In this region there is a mixing of the states of quantum matter and those of the semi-classical gravity which produces a thermal mixture of the matter states and hence an "entropy;" this same mixing effect brings about the conversion of a parametric time variable into a physical intrinsic time.

1 Introduction

Black holes and certain cosmological models have been under extensive and intensive study in recent years in the context of Bekenstein-Hawking type

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thermal states, i.e. thermal states associated with an event horizon. These studies involve general relativity (at the borderline with its quantum character), quantum field theory and statistical mechanics. There remain many questions about these systems of a very fundamental character and though most of the effects involved are not readily observable, they nonetheless represent a laboratory for a great number of fundamental questions in each of the separate disciplines as well as to the connections between them. What is the source of the thermal state in these cases? Are they really thermal states: are these configurations which look like thermal states really pure quantum states or do they represent a density matrix? If they do how does one reconcile an evolution from a pure state to a density matrix? Is the level of semi-classical gravity sufficient to explain the effects? For the cosmological case there is the additional question of starting from a quantum gravity state of the wave function of the universe for which there is no evidently no time dependence and proceeding to the time evolution of the universe; this is called in the literature the problem of time. There is, of course, here as well as in general the problem of time direction. The number of papers dealing with both the subject of entropy source for black holes and related problems as well as the efforts devoted to an answer to the "problem of time" are numerous and varied and to extensive to be included in the present work.[1]

We wish to present here an approach which ties these problems to a particular model of the universe and is carried out in a special approximation. Thus we take an extreme position of particularity. This has the disadvantage that it requires special conditions to hold, but they do happen to be close to what corresponds to the real universe. The second advantage is that even if it is not the correct explanation, the fact that one can put the results explicitly will add significantly to the understanding of more general and models worked out less completely. We will argue that the justification of using semi-classical gravity is that it is necessary for time to be meaningful in this context, i.e. for an eigenstate for the whole universe. The time seen in the universe, the cosmic time may thus be explained without it necessarily being the answer the question of time in quantum gravity in general. That question may be quite independent of the subject of the time evolution in our observable universe.

We are able to give an answer to a considerable group of the above questions in the context of a single physical idea, the semi-classical tunneling of the cosmological scale function out of a classical forbidden domain. In this domain quantum matter states are mixed with those of semi-classical gravity.
yielding a thermal mixture of the matter states and a further thermal contribution of the gravity associated with this thermal mixing of the matter. Gravity in itself remains adequately described by the single homogeneous function, the multiplicity comes from the matter states. On the same basis gravity becomes semi-classical via the mixing of highly excited levels of the matter states. The mixing of matter states produces an effect that the parametric time represents a reciprocal temperature of the thermal mixture of the matter in the forbidden regime and behaves like a physical time in the Lorenzian region when the scale function penetrates the barrier. The time evolution follows the semi-classical orbits of the gravity and there is a foliation of the modified spacelike slices which reappear in the semi-classical description.

Brief statement of results: The various problems are solved on the basis of a semi-classical tunneling occurring on small scales of the universe in going from a classically forbidden region to a classically permitted region of the cosmological scale function denoted $a(t)$. We are dealing with quantum matter in the form of conformally coupled scalar bosons, with mass and self-interaction. Gravity is taken to be that associated with a homogeneous isotropic space-time (FRW metric) with a positive curvature ($k=1$) and a positive cosmological constant $\Lambda(\Lambda_{eff}) > 0$. Though the sign of the cosmological constant is a necessary feature of our theory, the sign of the curvature is one of convenience, as is the conformal coupling of the bosons. There then exists a tunneling region for $a$ at a scale of the order of magnitude of the de Sitter event horizon. Our action has the form

$$I = \frac{1}{16\pi} \int d^4x \sqrt{|g|} [R - 2\Lambda] + \int d^4x \sqrt{|g|} \left[ \sum_{\mu=0}^{3} \frac{R}{6} \phi^2 - m^2 \phi^2 + \lambda \phi^4 \right]. \quad (1)$$

The metric, using conformal time is:

$$ds^2 = a^2(\eta)[d\eta^2 - d\chi^2 - \sin^2 \chi(d\theta^2 + \sin^2 \theta d\phi^2)]. \quad (2)$$

We study solutions of the Wheeler DeWitt equation in the above minisuperspace model. Let us consider this as a realistic model of the universe. In past work\cite{2} and \cite{3} we have taken $\Lambda$ as a phenomenological constant.

\footnote{Units: $\hbar = c = G = 1$}
Here we wish to consider a $\Lambda_{eff}$ which has its origin in a special solution of a classical condensate for the $\phi$ which can occur for a Higgs mass ($m^2 < 0$). A phase transition which removes the condensate removes the cosmological constant. The entropy grows as a result of this transition. This work is in progress.

We make a conformal transformation for the $\phi$ fields.

$\phi = \phi/a \quad g_{\mu\nu} = a^2 \gamma_{\mu\nu}$, \hspace{1cm} (3)

$\gamma_{\mu\nu}$ corresponds to the static Einstein metric. Using the FRW solution we can write our action in the form:

$$I = \frac{3}{16\pi} \int d^4x \sqrt{|\gamma|} [a^2 - a^2 + \frac{\Lambda}{3} a^4] + \int d^4x \sqrt{|\gamma|} \frac{1}{2}[\gamma^{\mu\nu} \phi,_{\mu} \phi,_{\nu} - \phi^2 - \mu^2 a^2 \phi^2 + \lambda \phi^4].$$ \hspace{1cm} (4)

We follow Banks’ analysis of the first order Born-Oppenheimer approximation, regarding the semi-classical gravity as the heavy degree of freedom and the quantum bosonic matter as the light degree of freedom, giving an effective Schroedinger equation in terms of the parametric time variable obtained from solving the equations of motion of the semi-classical gravity. This time is shown to be associated with a foliation of modified spacelike hypersurfaces. It is essential to note that the modified Schroedinger equation obtained here is dependent on the mixing of the semi-classical gravity with the quantum matter degrees of freedom!

One universe, considered as a unique one which represents a closed system, being in an eigenstate has no time regardless of whether the eigenvalue is zero, i.e. this holds quite separately of the arguments about the constraint conditions. Notice also that even if the eigenvalue is zero, this does not imply that the operator is zero, there may be many states corresponding to this eigenvalue. Such state can only acquire a time by some of the states going semi-classical and this can only occur when some sum over highly excited states of the degree of freedom as to become classical, or as here when other states are mixed with this degree of freedom. It is unphysical for the state of the universe at time scales a few orders of magnitude greater than Planck time to treat the matter as well as gravity semi-classically unless there is some condensate of the matter. The only feasible time variable for this situation is an intrinsic time variable of the kind that we obtain.
We have approached the problem in three different complementary ways. Each provides insight and understanding of aspects of the problem less obvious in the other approaches. We have made a statistical mechanical evaluation of the entropy\(^2\), thus showing that what we obtain is a real statistical state. We have analyzed the problem in terms of the wave function of the universe approach by two different ways, Born-Oppenheimer analysis\(^3\) of the wave function of the universe and the evaluation by a functional integral approach. The Born-Oppenheimer approach with the inclusion of the leading higher order term leads to the understanding that the time variable which is the solution of the classical equations of motion is also a time variable for the Schroedinger equation for the matter. This holds, however, only if the matter wave function is not an eigenstate of the matter Hamiltonian. Thus the conversion of the parametric time to the time of the Schroedinger equation requires a mixing of matter states, here by the intermediary of mixture with the semi-classical gravity. This feature is not at all obvious in the functional integral approach, and, in fact, has been overlooked by many people using the semi-classical gravity as a source for a time variable. We evaluate the probability weight of being at a particular scale function \(a\); this quantity we denote \(W(a)\). This is presumed to describe the state of the universe at cosmic scales some orders of magnitude greater than Planck lengths. We evaluate a quantity which is formally a density matrix. That quantity may be equivalent to the scalar product of the wave function of the universe with itself if certain fluctuations are small enough. Similarly there is another step in the procedure which involves interchange of the order of integration between a time variable fixing the constraint and an integral over matter configurations. In terms of the quantity calculated we find a well defined intrinsic time variable and a thermal admixture of the matter the above calculated quantity being the entropy when \(a\) has a value greater than or equal to the outer turning point of the cosmological scale function.

In section 2 we will review some of the special properties of gravitational statistical thermodynamics as a background for our specific discussion. This aspect of the problem is the most carelessly treated feature in the literature of this subject. In Section 3 we consider the question of an appropriate energy and Hamiltonian for our model and then write down an expression for the corresponding entropy. Section 4 introduces the Wheeler-DeWitt equation and the constraints; we introduce the concept of the semi-classical, intrinsic foliation to replace the extrinsic one which has been eliminated by the diffeomorphism invariance. Section 5 deals with a higher order Born-
Oppenheimer approximation in which a parametric time is to be identified as physical time; this parametric time also has a possible imaginary domain where it will subsequently associated with a reciprocal temperature. The functional integral evaluation of $W(a)$ is presented in section 6, where the contribution from the tunneling region is identified with the exponential of the entropy. This quantity is analyzed in terms of the relation between density matrix and a pure state. Conditions are discussed under which this could be in effect equivalent to a pure state; fluctuations of the wave function at the turning points of the scale function must be sufficiently small. The last section concludes with a summary and a discussion of further work and various problems about the above work.

2 Gravitational statistical thermodynamics

Is there any thermodynamic equilibrium for gravitating systems? The answer, we suggest, is a qualified yes. Namely, although there is no absolute maximum of entropy valid for gravitational systems since its Hamiltonian and action are indefinite quantities, there can be a local maximum of the entropy which is sufficiently long lived and to which the corrections are sufficiently small to be a useful description and calculational device. Energy is always problematic quantity in general relativity. We would like to suggest that systems and geometries which lack some conserved energy-like quantity cannot be expected to have any thermodynamic equilibrium configuration. Such a conserved ”energy” itself implies some special time and related temperature variable. In the next section we will review a procedure to define such a conserved energy for metrics that correspond to cases with a conformal time-like killing vector. There must be some kind of generalization of the conserved energy being related to time translation invariance.

While many fundamental questions are asked of the quantum mechanical and general relativistic behavior of these systems, the treatment of statistical mechanics is done very uncritically. Gravitating systems cannot strictly be divided into subsystems in the sense commonly used in thermodynamics to define canonical or grand canonical ensembles. If one is sufficiently far away from a phase transition such division is not problematic. However, in terms of phase transitions the open systems are not independent thermodynamic systems with fixed intensive parameters given by a reservoir (e.g. temperature). It is very common that people doing the statistical mechanics of
black holes or cosmology begin with an expression for the free energy as a representation of thermal behavior. There is no justification in taking the partition function as the representative function of thermodynamic equilibrium for these systems. This is commonly done by invoking conventional statistical mechanics—separation of system into large and small subsystem with the large subsystem being regarded as a heat bath and the small one as the systems which has a canonical ensemble distribution. This separation is fundamentally incorrect and the stability of the small subsystem is, in general, not that of the positive heat capacity. For example, York’s correction of Hawking’s canonical ensemble treatment of the stability of a black hole in a radiation cavity is evidently the correct way to evaluate the canonical ensemble where a temperature is fixed at the boundary surface at finite radius. This result appears to be independent of the universe external to the box, but is it really independent? It is unclear under what circumstances the York calculation could be realized physically. An exception may be the case of a black hole produced spontaneously by a density fluctuation. In such case the particle horizon limits the region which could be effected by the black hole and there the temperature is indeed fixed at the surface of the particle horizon, producing a finite region of space inside of which the black hole could be found in a stable canonical ensemble. Hawking’s result is correct for the case where the black hole and its surrounding cavity are alone in the universe with the temperature fixed at infinity.

Even in standard systems, it is the microcanonical ensemble which is fundamental and is based on conserved macroscopic quantities, especially the energy. Here the quasi-ergodic hypothesis serves as the fundamental assumption and there has been established some reasonable justification of this principle. Other ensembles are derivative quantities and their validation is based on the applicability of the thermodynamic limit for conventional systems. Due to the long range attractive forces, this is not a valid principle for gravity and so one must proceed with caution in applying any conventional wisdom to gravitational systems. This caution is singularly lacking in many of the works written in this field. Just one example will be given at this point. If the starting point is the microcanonical ensemble one can derive the free energy, in a saddlepoint evaluation of the energy constraint. If one begins from the canonical ensemble one can evaluate the entropy from a saddle point evaluation of the partition function. The common equation relating the two is that which is valid in the thermodynamic limit, namely the Legendre transformation. However, the two ensembles being inequivalent
for gravitational systems, the fluctuations differ and the two different ways of evaluating the entropy are not equivalent!

3 Energy and entropy of the cosmological model

Horwitz and Katz [7] found a conserved energy for FRW cosmological metrics using a reference metric scheme developed earlier by Katz, Lynden-Bell and Israel [8]. The key to this approach to defining a conserved energy is to introduce a reference metric which has a static time-like Killing vector. If the physical metric can be mapped smoothly onto the reference metric without singularities, then the conserved current defined relative to the reference metric can be shown to lead to a conserved energy which corresponds to the physical energy for all cases where an energy is known. Katz et al. [8] dealt with the case of asymptotically flat space-times. The conserved energy is a function of the metric only. For the cosmological case, at least for a closed universe, this constant is shown to be equal to zero. A covariant Hamiltonian was derived which was shown to satisfy the following identity:

\[ H = E + \int \sqrt{|g|} (T^\mu_\nu - G^\mu_\nu) \xi^\nu_{conf} d\Sigma_\mu = D, \]  

where the symbol D represents the dilatation generalized to include gravity as in the work of [2]; the energy being zero, the Hamiltonian can be identified with the dilatation. The Hamiltonian is derived from the action

\[ I = \frac{1}{16\pi} \int (R - 2\Lambda) \sqrt{|g|} d^4x + \int d^4x [\partial_\alpha (k^\alpha \sqrt{|g|})] + I_{\text{matter}}. \] 

This vector \( k^\alpha \), which depends on both metrics in the action produces the same effect as the York-Hawking surface term to eliminate the second derivatives except that here the reference space appears explicitly. The formalism used is to express all quantities in terms of the reference metric like the approach developed by Rosen [9] for a different purpose. The modified Christoffel symbol \( \Delta_{\nu\lambda} \) is defined (the bar referring to the reference metric)

\[ \Delta^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda} - \bar{\Gamma}^\alpha_{\mu\nu} \]  

and then

\[ k^\alpha = g^{\alpha\beta} \Delta^\nu_{\nu\beta} - g^{\mu\nu} \Delta^\alpha_{\mu\nu}. \]
If we now quantize semi-classically; our identity remains, with the conserved energy depending only on the metric remaining zero and:

\[
\hat{D}_{ren} = \hat{H}_{ren}, 
\]

and the renormalized Hamiltonian remains equal to the conserved energy \( E=0 \) in the sense that

\[
\hat{H}\,|\Psi\rangle = E\,|\Psi\rangle = 0. 
\]

We can define the entropy by an assumed quasi-ergodic theorem, that the entropy is the log of the sum over states satisfying the constraint that \( D_{ren} = 0 \). Thus we have

\[
expS = Tr(\delta(\hat{H} - E)) = Tr(\delta(\hat{D}))
\]

for our case, where we have restored the \( E \) which may differ from zero for example, for nonclosed universes; our entropy expression in terms of \( D \) remains valid.

Horwitz and Weil\cite{2} evaluated the cosmological entropy this way some years ago establishing a first principles calculation of the entropy. The result found was a generalization of the Gibbons-Hawking de Sitter entropy which was found to be thermodynamically unstable. The stable entropy is associated with a state where the universe has a finite density of thermal bosons. This increases the entropy by a modest amount. Furthermore establishing the principle that the quantity being calculated is a thermodynamic entropy and not something that just formally looks like an entropy.

4 Wheeler-DeWitt Equation

Writing the Wheeler-DeWitt equation for our minisuperspace model

\[
(\hat{H}_G + \hat{H}_M)\Psi(a,\phi) = 0, 
\]

The Hamiltonian density\footnote{Properly for the Hamiltonian we should replace the time derivatives by the corresponding momentum variables; that connection is sufficiently obvious that we have chosen to retain this form for conciseness.} of the gravity is

\[
h_G = -\frac{3}{16\pi}(\dot{a}^2 + V(a)) 
\]
where the gravitational potential is
\[ V(a) = [a^2 - \frac{\Lambda}{3}a^4]. \quad (14) \]

The matter Hamiltonian density is here
\[ h_M = \frac{1}{2}(\dot{\phi}^2 + (\nabla \phi)^2) + \mu^2 a^2 \phi^2 - \Lambda \phi^4). \quad (15) \]

The Wheeler-DeWitt approach begins by making a 3+1 split of the spacetime and considering the Hamiltonian defined on an arbitrary spacelike hypersurface. Setting up a foliation of the space-like hypersurfaces, can be considered to define the time variable. In a minisuperspace model a preferred foliation might be associated with a time-like Killing vector. The Hamiltonian in arbitrary coordinates can be expressed in terms of a lapse function associated with a normal to the spacelike hypersurface and a shift vector associated to transformations on the hypersurface. In general using considerations of symmetry one can make arbitrary change of the lapse function and arbitrary coordinate transformations on the space-like hypersurface. These invariance properties lead to 4 constraint equations denoted respectively as the Hamiltonian and momentum constraints. For general relativity the Hamiltonian can be written completely in terms of the constraints and the lapse function and shift 3-vector. We can quantize by usual canonical methods, finding the momenta and replacing the classical momenta by the derivatives with respect to canonical coordinates:
\[ p_a \sim \dot{a} \rightarrow \frac{\delta}{\delta a} \]
\[ p_\phi \sim \dot{\phi} \rightarrow \frac{\delta}{\delta \phi}, \]

One commonly follows the Dirac method and associates the constraint with a zero eigenvalue of the constraint operators. The constraint equations and the wave function of the universe are functions of the induced metric \( h_{ij} \) on the spacelike hypersurface and the matter coordinates. We proceed with the Dirac approach where we transfer the constraint condition to an eigenvalue problem on constraining our problem to be the zero eigenvalue of the Hamiltonian and momentum constraint operators acting on the wave function. The
Hamiltonian, consisting of a gravity and a matter part when acting the wave function are familiarly known as the Wheeler-DeWitt equation:

\[ [\hat{H}_G + \hat{H}_M] \Psi(h_{i,j}, \phi^\alpha) = 0 \]  

(16)

where the \( \phi^\alpha \) are here our boson matter fields. Corresponding equations exists for the momentum constraints.

What happens to space-like hypersurfaces when we apply the constraint condition? One might consider two possibilities: either the space-like hypersurfaces remain distinct and the quantization which is carried out on some arbitrary one to is retained with transfer operator to any other surface absent. Alternatively we might consider that the wave function satisfying the constraint equation includes contributions from all the hypersurfaces. It is easy in the functional integral formalism below of (23) to see that the latter is the case. The result for the fixed energy case is a superposition of the multitude of spacelike hypersurfaces. We will return to this explicitly below after we have presented the functional integral evaluation below. This is manifestly a superposition of the amplitudes summed over the original leaves. Choosing a common saddlepoint of \( a(t) \) and \( t \) in the functional integral, we precisely get a path through the modified hypersurface evaluated at \( a \)'s determined by the classical equations. The interchange of orders of integration when justified as in our case, carries us over from a many-fingered time situation which has not such simple interpretation to a well defined time variable and a well defined path once we have specified initial value conditions and assuming that there is a unique choice, or a well-determined basis to choose one of several possible choices.

5 Born-Oppenheimer analysis

The essential approximation of Born-Oppenheimer is to assume that the wave function can be written as a product wave function of matter and gravity parts. The zeroth approximation has the matter wave function depending on the gravity part only parametrically. We shall be interested in including the next order approximation.

\[ \Psi(a, \phi) = \chi(a, \phi)\psi(a). \]  

(17)

Then the product wave functions are determined by the 1st order Born-Oppenheimer equations:
\[
\frac{\langle \chi | \hat{h} | \Psi \rangle}{\langle \chi | \chi \rangle} = (\dot{a})^2 + V(a) + \langle \hat{h}_M \rangle = 0.
\tag{18}
\]

where
\[
\langle \hat{h}_M \rangle = \frac{\langle \chi | \hat{h}_M | \chi \rangle}{\langle \chi | \chi \rangle}.
\tag{19}
\]

The second equation is then found to be
\[
\frac{\langle \psi | \hat{H} | \Psi \rangle}{\langle \psi | \psi \rangle} = [\hat{H}_M - \langle \hat{H}_M \rangle] \chi(a, \phi) - \frac{\partial (\ln \psi)}{\partial a} \frac{\partial \chi}{\partial a} = 0.
\tag{20}
\]

Since we are dealing with semi-classical gravity then we have
\[
\frac{\partial (\ln \psi)}{\partial a} = i \dot{a} \quad \frac{\partial (\ln \psi)}{\partial a} \frac{\partial}{\partial t(a)} = \frac{\partial}{\partial t(a)}
\tag{21}
\]

where the quantity \( t(a) \) is determined from the solution of (18) and is to be identified as a parametric time variable. It can be either real or imaginary. We choose to identify it with the physical time of the universe on the basis of (20)-(21) being in effect a Schroedinger equation for the matter wave function. Notice, however, one feature of the result, namely that there is no Schroedinger equation if the \( \chi \)'s are eigenfunctions of the matter wave function. The last term of (20) is by its very nature a mixing term; in this case it mixes semi-classical degrees of freedom with matter degrees of freedom. Consequently our pure wave function appears to have a superposition of all matter eigenstates. As we shall see below this mixture is produced in the tunneling region and the imaginary time of that region will appear as a reciprocal temperature. The mixture of the matter states will be a thermal admixture. Solving (18) for \( t(a) \) we find the equation
\[
t(a) = 2 \int_{a_{ref}}^a \frac{da}{\sqrt{-\frac{16\pi}{3} \langle \hat{h}_M \rangle - a^2 + \frac{4}{3} a^4}} = 2 \int \frac{da}{\sqrt{A(a)}}
\tag{22}
\]

Then for \( A(a) > 0 \), \( t(a) \) is real, while for \( A(a) < 0 \), \( t(a) \) is imaginary; \( A = 0 \) gives the turning points.

Functional integral evaluation

The Born-Oppenheimer approach was important in terms of emphasizing certain physical features of the approximation used. However, functional integral evaluation is preferrable for other aspects of the problem.
which has the desired properties of yielding a semi-classical thermal universe evolving with the time given by the Born-Oppenheimer Schroedinger equation has the form of a density matrix which we will evaluate by functional integral methods. This density matrix is potentially equivalent to the square of the Wheeler-DeWitt wave function under conditions which we will describe, but not verify in the present paper. Thus we consider the weight function for the universe to be at a scale $a$:

$$W(a) = \int D[\phi(\vec{x})] \int \frac{d\tau}{2\pi} W_{KB} \int D[a] \int D[\phi(x)] e^{-iI(\{a\},\{\phi(\vec{x})\})}.$$  

(23)

The energy constraint is fixed by integration over a “time-like” coordinate $\tau$. The semi-classical path of integration of $a$ proceeds from the value of the argument of $W(a)$ to a turning point and back. The choice of the reference turning point which corresponds to a choice of boundary conditions and sets a direction to time is to choose the lower turning point in the tunneling region denoted $a_-$. However, to carry out our evaluation of this integral we proceed to one further approximation. We interchange the order of the $\phi$ integral with that of the $\tau$ integral. Evaluating the time integral by saddlepoint method, we would otherwise have our time saddlepoint depend on $\phi(\vec{x})$. Interchanging the orders allows us to convert the result from a many fingered time solution to a single time. The justification for this interchange has been discussed at some length by Brout and Venturi[12]. The essential point is that the many degrees of freedom of the matter must produce sufficiently small fluctuations to justify this step. We shall assume that conditions which Brout and Venturi have found for its validity hold here. Since both the $a$ and the $\phi$ are periodic functions of $\tau$, if we carry out the following integral within the tunneling region. Then

$$\int_{\phi(0,\vec{x})=\phi(\tau,\vec{x})=\phi(\vec{x})} D[\phi] e^{-I_M(\{\phi\},a)} = e^{-\tau F}.$$  

(24)

This is the functional integral over the boson degrees of freedom for an $a$ inside the tunneling region giving us a partition function with the inverse temperature equal to the return path in time from $a$ to $a_-$. Thus the above expression for an $a$ in the tunneling region, with the $\phi$ integral carried out
takes the form

\[ W(a) = \int_{\tau_0-i\infty}^{\tau_0+i\infty} \frac{d\tau}{2\pi i} \int_{a(0)=a(\tau)=a} D[a]e^{-I(a)-\tau F}. \] (25)

The common saddle point of the two remaining integrals corresponds to the energy condition found above for the Born-Oppenheimer equation, with the addition that the expectation value of the energy is found to be precisely the thermal expectation value

\[ \langle \hat{h}_M \rangle = \frac{\int D[\phi]e^{\phi}[-I_M]\hat{h}_M}{\int D[\phi]e^{\phi}[-I_M]} = \frac{\text{const}}{\tau^4}. \] (26)

The energy condition thus is that found by the Born-Oppenheimer treatment above

\[ \dot{a}^2 + a^2 - \frac{\Lambda}{3}a^4 = -\frac{16\pi}{3} \langle \hat{h}_M \rangle \equiv -\epsilon(\tau). \] (27)

Integrating this equation inside the tunneling domain, we can solve the equation by seeking for the self consistent solution of

\[ \tau(a) = 2 \int_{a_-}^{a} \frac{da}{\sqrt{-\epsilon(\tau) + a^2 - \frac{\Lambda}{3}a^4}}. \] (28)

When \( a = a_+ \) we will identify this as the reciprocal temperature of the physical domain. That is

\[ \beta = 2 \int_{a_-}^{a_+} \frac{da}{\sqrt{-\epsilon(\beta) + a^2 - \frac{\Lambda}{3}a^4}}. \] (29)

Furthermore we can then identify the quantity \( W(a_+) \) with the physical entropy. Thus we obtain the equation

\[ W(a_+) = e^{\exp[S]}. \] (30)

For \( a > a_+ \), \( W(a) \) is independent of \( a \), but maintains the weight function of the statistical distribution.

If we were to continue the path to \( a \)'s on the decreasing stage of the universe, since our paths all begin at some \( a \) go out to infinity retrace the path back to \( a_- \), we will find an entropy which again increases when we penetrate the tunneling region on recollapse of the universe. Thus we find
that the universe is “born”, beginning its temporal expansion in a thermal inflationary state with a finite temperature as measure in conformal time. This asymptotically de Sitter state has necessarily a finite density of thermal bosons -like the stable black hole case of radiation in a radiation cavity-. We have elsewhere shown that the Gibbons-Hawking\cite{[13]} thermal de Sitter solution is unstable.\cite{[11]} The crucial point of our argument is that though the formal procedure of Hawking and coworkers found the suggestive result in which the black hole and de Sitter cosmology appear thermal, their approach misses the fact that the presence of thermal matter is not only a reflection of the black hole emission, but is fundamental to the explanation and the source of the entropy of the problem. The attempts to show that the entropy is either an entanglement entropy or an effect near the horizon are at least in that respect in accord with our point of view. Thus the recent results of finding due to these terms a black hole entropy differing from $A/4$ are precisely in the spirit of our results for the cosmological case. But as to the source of the entropy we do not agree with these other points of view. At least for the cosmology case we have shown explicitly the source of the entropy to be a mixing of matter states with the semi-classical gravity, with no connection to particle states, no connection to a separation of states inside and outside the horizon.

6 Concluding remarks

We have demonstrated how both time in cosmology and the thermal state of a cosmological asymptotically de Sitter universe arise from semi-classical tunneling of gravity in the presence of quantum matter. Both of these quantities have their origin in the mixing of semi-classical gravity and matter in the tunneling region. The same mixing effect provides the multiplicity of high energy states necessary to yield the classical behavior of the gravity. We accept the point of view that the problem is solved in terms of the specifics of “our universe”, i.e. that we require a positive cosmological constant in the early universe to make it begin to evolve. We have presented solutions only for a uniform, isotropic universe. Whether our results are limited to that case is not clear, but I think it would appear that we at least require the universe to begin with conformal time-like symmetry. As we have stated above we claim that as long as a single universe is the case in point and that we are dealing with the wave function of the total universe, hence a closed
system time can only appear as intrinsic quantity and only when gravity is semi-classical.

Left unresolved in the present work is the question to what extent the thermal inflationary universe evolving with an intrinsic time given by the modified Schroeding equation is or is not in an essentially pure state. This requires a detailed analysis of the $\phi(\vec{x})$ integrals at both turning points. A determination that the integrals are sharply peaked at one value would establish the equivalence. At the lower turning point this is necessary to identify the density matrix with the square of the wave function. At the upper turning point this is necessary to establish that the integration over the $\phi(\vec{x})$ does not convert the square of the wavefunction into a density matrix. The argument of order interchange as a separate issue is evidently necessary for the result to be valid. The question of coherence for our results thus cannot even be addressed until the above verification is accomplished.

Two major generalizations are necessary before these results can reach their full impact. In the first place we need to consider in detail a more general model of the matter which can both provide us with the switching cosmological constant and the phase transition between inflation and Friedmann cosmology. We have work in this direction in progress. Secondly we must discover how to extend the approach to black holes. There may be some differences in even important details but the basic physical source of the entropy would be expected to be the same.

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