Phases as Hidden Variables of Quantum Mechanics

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Quantum mechanical wave functions have phases. These phases either initial or acquired during time evolution usually do not enter the final expressions for observable physical quantities. Nevertheless in many cases the observable physical quantities implicitly depend on the phases. Hence we may regard the phases as a sort of hidden variables of Quantum Mechanics. Neglecting the phase role makes inexplicable the peculiar quantum effects such as particle interference, Einstein-Podolsky-Rosen correlation and many others. To the contrary the adequate inclusion of phases into consideration reduces QM puzzles and mysteries to simple and obvious triviality.

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I. INTRODUCTION

Various proofs exist that hidden variables are incompatible with the principles and rules of Quantum Mechanics (see, e.g. [1–3]). However there are quantities that may be regarded as a sort of hidden variables of Quantum Mechanics. These quantities are the phases of wave functions. As a rule these phases do not enter the final QM expressions for the observed physical quantities though frequently they implicitly determine their values or even the very existence of QM effects. Neglecting the phase role makes inexplicable the peculiar quantum effects such as particle interference, Einstein-Podolsky-Rosen correlation and many others. To the contrary the adequate inclusion of phases into consideration reduces QM puzzles and mysteries to simple and obvious triviality.

II. BRA, KET AND WAVECLE

Dirac in his notorious book [4] introduced the bra and ket functions (or vectors in Hilbert space) and the corresponding expression (bracket) for the observable physical quantity. Let us consider an observable physical quantity $c$ for the simple case of a quantum particle in some space volume $V$. In the usual notations and in Dirac’s notation we have:

$$\hat{c} = \langle \psi | C | \psi \rangle \equiv \int_{V} d\mathbf{r} d\mathbf{r}' \psi^\dagger(\mathbf{r}, t) C(\mathbf{r}|\mathbf{r'}) \psi(\mathbf{r}', t)$$  \hspace{1cm} (1)

Three quantities stand in this expression: the bra-wave function (or bra-vector) $\langle \psi | \equiv \psi^\dagger(\mathbf{r}, t)$ then the operator of physical observable $\hat{c}$ with the kernel $C(\mathbf{r}|\mathbf{r'})$ and the ket-wave function (ket-vector) $| \psi \rangle \equiv \psi(\mathbf{r'}, t)$.

The bra and ket functions are under permanent action of Hamiltonian $H$ and satisfy the equations of motion (Schrödinger equations). We have for the ket-function

$$(\partial_t + iH)\psi(x, t) = \psi(x, 0)\delta(t)$$  \hspace{1cm} (2)
(for brevity we take $\hbar = 1$ and do not write the vector signs for space coordinates $\mathbf{r} \equiv x$). The solution of the equation (2) in the resolvent form or as the multiplicative integral is given by

$$\psi(x, t) = \frac{1}{\partial_t + iH} \psi(x, 0) \delta(t) = \Pi_0^t (1 - iH dt) \psi(x, 0)$$

(3)

For the bra-function we have analogously

$$\psi^\dagger(x, t) = \frac{1}{\partial_t - iH} \psi^\dagger(x, 0) \delta(t) = \Pi_0^t (1 + iH dt) \psi^\dagger(x, 0)$$

(4)

The multiplicative integrals at right in (3) and (4) show how the Hamiltonian acts at the wave function changing it in every moment of time.

For time-independent $H$ the solutions simplify:

$$\psi(x, t) = \exp(-iHt) \psi(x, 0), \quad \psi^\dagger(x, t) = \exp(iHt) \psi^\dagger(x, 0)$$

(5)

For the eigenfunctions of Hamiltonian $H \psi_p = \epsilon_p \psi_p$ which we take as the set the normalized and orthogonal functions

$$\langle p|k \rangle = 0, \quad \langle k|p \rangle = 0, \quad \langle p|p \rangle = 1, \quad \langle k|k \rangle = 1$$

the time dependence reduces to the phase change $\psi \equiv \psi_p$:

$$\psi_p^\dagger (x, t) = e^{+i\epsilon_p t - i\varphi_p} \psi_p^\dagger(x), \quad \psi_p (x, t) = e^{-i\epsilon_p t + i\varphi_p} \psi_p (x)$$

(6)

Here $\varphi_p$ is the initial phase.

Now let us decompose the initial wave function $\psi(x, 0) \equiv \psi_0$ into a series of Hamiltonian eigenfunctions. Using (6) we get for the bra-function $\psi^\dagger(x, t) \equiv \psi^\dagger(t)$ and ket-function $\psi(x, t) \equiv \psi(t)$ the expressions:

$$\psi(t) = \sum_p a_p e^{-i\epsilon_p t + i\varphi_p} \psi_p, \quad \psi^\dagger(t) = \sum_p a_p^* e^{+i\epsilon_p t - i\varphi_p} \psi_p^\dagger$$

(7)

The equations of motion for bra and ket are linear so the sum of solutions is also a solutions. The expressions for physical observable quantities are bilinear on bra and ket. This peculiarity of quantum mechanics leads to the existence of two different contributions to the value of a physical quantity. Indeed, substituting (7) into (1) with $C \equiv U(x)$ we obtain two parts of the observable $U$. The first part contains only diagonal matrix elements $\langle p|U|p \rangle$ of the operator of observable $U$ and does not depend on time and initial phases:

$$U = \sum_p \langle p|U|p \rangle |a_p|^2 = \sum_p \langle p|U|p \rangle F_p$$

(8)

Here $F_p$ is the occupation number (or distribution function) for the quantum state $p$. The sum of $F_p$ over $p$ is equal to the total number $N$ of quantum particles: $N = \sum_p F_p$. For one particle $F_p$ describes the probability for a particle to be in the state $p$.

Another part of $U$ contains the non-diagonal matrix elements $\langle p|U|k \rangle$ and depends on time and initial phases. We denote it as $\Delta U(t)$:

$$\Delta U(t) = \sum_{p \neq k} \langle p|U|k \rangle a_p^* a_k e^{i(\epsilon_p - \epsilon_k)t - i(\varphi_p - \varphi_k)}$$

(9)

Note that this contribution vanishes after averaging over time or initial phases.
The total observable $U(t)$ is given by the sum of (8) and (9):

$$U(t) = \overline{U} + \Delta U(t), \quad \overline{\Delta U(t)} = 0 \quad (10)$$

We see that the physical quantity $U(t)$ is the sum of the constant background $\overline{U}$ and the alternating fluctuations $\Delta U(t)$ over this background with zero mean value. Note that phases explicitly enter only into the fluctuation part of observable data (9). Implicitly the phases are contained also in the background part (8) since the probability $F_p$ there is formed by the bra and ket with the same phases. One may say that $F_p$ is the mean number of bra and ket with the same phases.

Since the operator $U$ can be arbitrary the state of a quantum object is determined by the bra and ket taken in one moment of time. It is reasonable to put just such bra+ket pairs (of functions or objects behind them) into correspondence to the potentially observable quantum particles. As it is known they are not waves and not corpuscles revealing nevertheless the wave and corpuscle properties. Following Eddington let us call such objects by wavicles. The bra and ket have phases and the wavicles also have phases which are equal to the bra and ket phase differences. The expressions (8-10) show that there are zero phase wavicles formed by bra and ket with the same phase. These wavicles contribute to the constant background of physical quantities looking like classical particles and revealing themselves as corpuscles. The wavicles with phases either initial or acquired during time evolution until the observation moment reveal themselves as quantum fluctuations. Their contributions into observable quantities depend on phases and vanish after phase averaging. Just these wavicles with phases are responsible for the wave properties of quantum particles.

III. DIAGRAMS

Now let us depict all written above as quantum diagrams. The diagrams are much more transparent than letter formulae. We use the diagrams where each diagram picture has one to one correspondence to the letter formulae and one can easily write the analytical expression for a diagram picture.

The quantum diagram pictures for the wave functions (bra and ket), the observable quantities, the occupation numbers and the action of a perturbation potential are presented in the Figure 1.

Because of simplicity of these diagrams and their direct connection with analytical formulae here no rigorous derivation is necessary. (For more details see [5–7,11]).
The diagram (F1.1) at the left represents the bra and ket by two lines with time arrows which correspond $\psi^i(t)$ and $\psi(t)$. The time goes from bottom $t = 0$ to the top time $t$. At $t = 0$ we have $\psi^i(0)$ for the bra and $\psi(0)$ for the ket. The line intervals between these points show time evolution and correspond the resolvents $1/(\partial_t - iH)$ for the bra and $1/(\partial_t + iH)$ for the ket. Going from top to bottom of the diagram we get:

\[ \frac{1}{\partial_t + iH} = \frac{1}{\partial_t} + \frac{1}{\partial_t}(-iH)\frac{1}{\partial_t} + \frac{1}{\partial_t}(-iH)\frac{1}{\partial_t}(-iH)\frac{1}{\partial_t} + \ldots \]

(11)

and the analogous expression for the bra with the substitution $(-iH) \rightarrow (+iH)$. The symbols $1/\partial_t$ correspond to time integrations over intervals where nothing occur while symbols $H$ describe the momentary Hamiltonian action under which the bra or ket states abruptly change.

The resolvents can be understood as the infinite series of events when the Hamiltonian $H$ acts during the time interval $(0\delta t)$ by zero, one, two, three and more times. For the ket we have:

\[ \frac{1}{\partial_t + iH + iV} = \frac{1}{\partial_t + iH} + \frac{1}{\partial_t + iH}(-iV)\frac{1}{\partial_t + iH} + \ldots \]

(12)

Here events correspond only to the perturbation action while the Hamiltonian action is taken into account as the part of normal evolution. The points on the lines describes the perturbation actions. Note that after the perturbation point the bra remains bra and the ket remains ket. The $k \rightarrow p$ transition under the action of $V$ gives the usual expression of the perturbation theory. For the ket we have

\[ \langle p | \frac{1}{\partial_t + iH}(-iV)\frac{1}{\partial_t} + i\epsilon_p | k \rangle = \frac{1}{\partial_t + i\epsilon_p}(-i\langle p|V|k \rangle)\frac{1}{\partial_t + i\epsilon_k} \]

(13)

and the similar expression for the bra. Note that in written formulae the future is at left and the past is at right, so the ordinary left to right writing order of symbols is against the diagram time direction.

The diagram (F1.2) shows the time evolution of the wavicle for the quantum state $k$ occupied with the probability $F_k$. The wavicle is composed by the pair of bra+ket lines with the same quantum indices and initial phases. Again going from top to bottom of the diagram we get:

\[ F_k(t) = \frac{1}{\partial_t - i\epsilon_k + i\epsilon_k}F_k(0)\delta(t) = F_k(0)\frac{1}{\partial_t}\delta(t) = F_k(0)\Theta(t) \]

(14)

Here $\Theta(t) = (1/\partial_t)\delta(t)$ is the step-function of Heaviside, $\Theta(t) = 0$ for $t < 0$ and $\Theta(t) = 1$ for $t \geq 0$. The bra and ket of the wavicle have phases which are time-dependent. But the wavicle phase remains zero because of the phase equality $e^{i\epsilon_k t - i\varphi_k}e^{-i\epsilon_k t + i\varphi_k} = 1$.

The initial state occupation number $F_k(0)$ is represented by the horizontal bar at $t = 0$. Such symbols and the time ordering of events (points) on the diagram lines are the main differences between our diagrams and the widely used Feynman diagrams (see, e.g. 8–10).

The diagram (F1.3) shows the mean contribution to the physical quantity $\mathcal{U}$ from the wavicles of the occupied state $k$. The value is given by the matrix element $\langle k | U | k \rangle$ which corresponds to the top point where the bra and ket lines enter. This point describes the
"classical device" of Bohr needed for a measurement. Measurements really are complicated physical processes with many stages. All concrete details of such processes and the necessary averaging of final data are implicitly included in the values of corresponding matrix elements. Bohr’s "classicality" of a device simply means that two undetectable elements of Quantum World (i.e. bra and ket) unite in the device to become detectable in our Classical World.

The contribution to the fluctuation part $\Delta U$ of the observable $U$ is shown in the diagram (F1.4). This part is given by the bra and ket with different quantum indices and initial phases as it is shown in (9). Reading the diagram we get ($t \geq t'$):

$$
\langle k|U|p \rangle \frac{1}{\partial_t - i\epsilon_k + i\epsilon_p} a_k^\dagger a_p e^{-i(\phi_k - \phi_p)} \delta(t - t') = \langle k|U|p \rangle a_k^\dagger a_p e^{i(\epsilon_k - \epsilon_p)(t-t')-i(\phi_k - \phi_p)}
$$

(15)

Let us note that the measurement points in the diagrams (F1.3) and (F1.4) differ essentially from the perturbation points in the diagram (F1.5). Perturbations points in bra and ket lines do not change their properties, i.e. the bra remains bra and ket remains ket. To the contrary the measurement points where the bra and ket line unites signify the end of normal (unitary) evolution prescribed by the motion equations.

The bra and ket lines with different indices can be obtained by a perturbation action or by the exchange of bra or ket between the wavicles of occupied states.

IV. QUANTUM EXCHANGE

If we take at time $t$ two wavicles of the occupied states $k$ and $p$ we will get two bra and two ket with two random phases $\phi_k(t) = \epsilon_k t - \varphi_k$ and $\phi_p(t) = \epsilon_p t - \varphi_p$. (The phases $\phi(t)$ enter in phase multipliers as $e^{i\phi(t)}$ for the bra and $e^{-i\phi(t)}$ for the ket.) From these four independent objects of Quantum World we can form four wavicles potentially observable in our Classical World. Two of them have zero phase $\beta = 0$ while two other have equal and sign-opposite phases $\alpha(t)$ and $-\alpha(t)$:

$$
\beta \equiv \phi_p(t) - \phi_p(t) = \phi_k(t) - \phi_k(t) = 0, \quad \alpha(t) \equiv \phi_p(t) - \phi_k(t) = -[\phi_k(t) - \phi_p(t)]
$$

(16)

The phase $\alpha$ is always random even if $\epsilon_k = \epsilon_p$ and all contributions of such wavicles into observable quantities are random and vanish after phase averaging.

Now let us take two independent detectors and find the observable quantities $A$ and $B$ related to the wavicles of $\beta$ and $\alpha$ types. Since for the $\alpha$-type wavicles $\overline{A} = 0$ and $\overline{B} = 0$ we consider the correlated values $AB$ or $BA$. These values taken in one time moment are depicted in the Figure 2.

![FIG. 2: Exchange correlation](image-url)
There are also analogous two diagrams with substitutions $A \rightarrow B$ and $B \rightarrow A$ which we do not depict for brevity. The diagrams correspond to the jointly averaged product of the measurement values in both detectors.

The left diagram shows the contribution of two wavicles with zero phases. We see that no links between them exist and their contribution is simply the product of two independent values. One wavicle hits one detector while other wavicle hits another detector or vice versa. In this case either separate or joint averaging of the detector data give the same result. Reading this diagrams we have (together with analogous one) the uncorrelated detector contributions:

$$ (AB)_{\text{uncor}} = \langle p|A|p\rangle\langle k|B|k\rangle + \langle k|A|p\rangle\langle p|B|k\rangle \quad (17) $$

The right diagram shows the contribution of two wavicles with the same but sign opposite phases. The detector data of such wavicles contain the multipliers $\exp(i\alpha)$ and $\exp(-i\alpha)$ and therefore vanish after separate phase averaging in each detector. However, after the joint averaging the multipliers cancel each other and the joint result becomes phase independent. The corresponding contribution is given by

$$ (AB)_{\text{cor}} = \pm\left[\langle p|A|k\rangle\langle k|B|p\rangle + \langle k|A|p\rangle\langle p|B|k\rangle\right] F_p F_k \quad (18) $$

The total average $AB$ is the sum of the uncorrelated and correlated parts:

$$ AB = \left[\langle p|A|p\rangle\langle k|B|k\rangle \pm \langle p|A|k\rangle\langle k|B|p\rangle\right] F_p F_k + (A \Leftrightarrow B) \quad (19) $$

The sign of correlation contribution is negative for fermions and positive for bosons.

We see from (19) and the diagrams in the Figure 2 how the exchange of the bra or ket in the zero-phase wavicles produces two wavicles with the same and sign-opposite phases. These wavicles become phase correlated and any physical observable quantities from these wavicles also become correlated. They are just the so-called entangled quantum particles which are so popular in the literature. The graphic transformation of the left diagram in the Figure 2 into the right one inevitably leads to the line intersection or Schrödinger Verschränkung (i.e. entanglement). The intersection results in the sign difference between bosons and fermions. (In the Figure 2 the correlation diagram is depicted without line intersection to make more transparent the origin of phase correlation).

Let us emphasize that phase correlated (entangled) wavicles have no magic properties and any action on one of them has no influence on another. The phase correlation is the real "common cause at the past" for EPR-correlation, Hunbery Brown-Twiss effect and many other experimentally observable correlation phenomena.

Note that such phase correlated wavicles are always present in any many-particle quantum systems. For example in many electron systems they are responsible for the appearance of exchange Coulomb energy. Two coherent charge fluctuations there have enough time to interact by the quick Coulomb potential and give the corresponding contribution to the energy. Ignoring the phase role in this effect leads to the usual explanation in QM manuals by obscure words: "It is a quantum effect with no classical analogy".

In the same way the EPR-correlation on macroscopic distances gave rise to the multitude of similar "explanations" like nonlocality, retrocausality, multitude of worlds or other non-physical fantasies. No such fantasies are needed for the simple physical picture of quantum exchange correlation. (For the details see [11,13,14]).

The quantum exchange between two wavicles in the same state leads to the difference in the behavior of bosons and fermions. If we put $p = k$ in the formula (19) we get $AB \equiv 0$ for fermions since the exchange contribution cancels the product of independent $A$ and $B$ contributions. In this way the Pauli prohibition for two fermions to be in the same states realizes.

For the bosons the exchange contribution doubles the uncorrelated contribution $AB \rightarrow 2AB$. For $A = B = 1$ we get the well-known formula for the occupancy number fluctuations.
of independent bosons and fermions:

$$\delta F_p \delta F_k \equiv F_p F_k - F_p F_k = F_p (1 \pm F_p) \delta_{pk}$$  \hspace{1cm} (20)$$

Here $F_p \delta_{pk}$ is the Poisson autocorrelation term.

V. PHASE AND PROBABILITY

We see above that the phase-independent wavicles (or $\beta$-type wavicles) form the permanent background of physical quantities. The phase-dependent wavicles (or $\alpha$-type wavicles) create fluctuations over this background with mean zero values. Now let us consider the probabilities of these physical processes. It is convenient to take as a simple example the mean value of space-position operator $U(x) = \delta(x - R)$ or the potential equal to zero except the close vicinity of space point $R$. In this point a wavicle appears in our observed Classical World when its bra and ket meet each other. For the quantum states $k$ with the wave function $\psi_k$ the wavicle background contribution (see the diagram F1.3 or the left diagrams of Fig.2) is given by:

$$U(R) = |\psi_k(R)|^2 F_k \hspace{1cm} \overline{U} = \int_V U(R) dR = F_k$$  \hspace{1cm} (21)$$

These expressions have clear physical meaning. It gives the probability to find a wavicle of a given state $k$ in a given space point $R$. We see also that if the quantum state is occupied $F_k \neq 0$ its bra and ket will meet certainly somewhere in the system. For $F_k = 1$ we come to the well-known Born rule to treat $|\psi(R)|^2$ as the probability for a quantum particle to be in the space point $R$. For many quantum states and $N$ quantum particles that can occupy them we have obviously

$$U(R) = \sum_k |\psi_k(R)|^2 F_k \hspace{1cm} \overline{U} = \sum_k F_k = N$$  \hspace{1cm} (22)$$

Thus we see that zero-phase wavicles look like classical particles thus demonstrating corpuscular properties. However, there is a principal difference between classical particles (i.e. material points) and wavicles. The point-particles in our Classical World always exist as really observable entities whereas the wavicles appear there only after measurements (i.e. after the encounter of their bra and ket constituents). Before such encounters the wavicles are only potentially observable.

Now let us consider the phase-dependent wavicles ($\alpha$-type wavicles) that are responsible for peculiar quantum effects. First of all note that we should have at least two quantum states for their appearance (see the diagram F1.4). At least two occupied states are needed for quantum exchange (see the diagrams of Fig.2).

The contribution of $(kp)$-wavicles (i.e. $k$-bra and $p$-ket) is given by the non-diagonal matrix element $\langle k|U|p \rangle = \psi_k^\dagger(R) \psi_p(R)$ together with the corresponding phase multipliers (15) and the mean distribution value as $\sqrt{F_k F_p}$. The $(pk)$-wavicles give the complex-conjugated contribution.

Thus for the phase-dependent $\alpha$-type wavicles the total contribution to the observable quantity $\Delta U(R, t)$ is given by:

$$\Delta U(R, t) = \sum_{k \neq p} \psi_k^\dagger(R) \psi_p(R) e^{i \alpha_{pk}(t)} \sqrt{F_k F_p} + c.c$$  \hspace{1cm} (23)$$
We see here the sum of contributions of various bra+ket pairs (wavicles with various phases). Their phases are random quantities (see (15)). Because of symmetry between bra and ket $\Delta U(R,t)$ is a real quantity and can be rewritten as

$$\Delta U(R,t) = \sum_{k \neq p} |\psi_k(R)\psi_p(R)| \sqrt{F_k F_p} e^{i\Phi_k(R,t)} + c.c \quad (24)$$

The total phase $\Phi_{kp}(R,t)$ is the sum of $\alpha_{kp}(t)$ and the phase $\gamma_{kp}(R)$ of the product $\psi_k(R)\psi_p(R) = |\psi_k(R)\psi_p(R)| e^{i\gamma_{kp}(R)}$. Note that the phase $\gamma_{kp}$ is nonzero even for real space wave functions because of their orthogonality. For two real orthogonal functions there should be a number of space points where their product changes its sign. In these points the phase $\gamma$ change by $\pm \pi$. Thus $\Delta U(R,t)$ is the sum of complex quantities $pe^{i\Phi}$ with $p > 0$ and the phase $\pm \Phi$. Note that being integrated over the system volume $\Delta U(R,t)$ vanishes as well as it vanishes after phase averaging. The negative parts of $\Delta U(R,t)$ require the introduction of negative or even complex numbers in order to treat them in a probabilistic way.

Let us divide $\rho \cos \Phi$ into two parts:

$$\rho \cos \Phi = \rho[\cos^2(\Phi/2) - \sin^2(\Phi/2)] \equiv (+\rho)P + (-\rho)Q, \quad P + Q = 1 \quad (25)$$

One part becomes the probability of positive result $\rho$ while other part becomes the probability of negative result $-\rho$. The total probability remains unity as it should be. For random phase we have mean zero probabilities:

$$\cos \Phi = \cos^2(\Phi/2) - \sin^2(\Phi/2) = 1/2 - 1/2 = 0 \quad (26)$$

Now let us consider the case where phase-dependent wavicles appear under perturbation action from initial zero-phase wavicles. As an example take the electron of the hydrogen atom of a stationary orbit at time $t=0$ (see the diagram F1.3). Then it emits or absorbs a photon and passes to another stationary orbit. To get the final orbit from the initial orbit two transitions are necessary (bra-bra) and (ket-ket) (see the diagrams F1.5). Since the bra and ket are independent objects these transitions occur randomly at different time moments $t_1$ and $t_2$. Thus we get the (kk)-wavicle before $t_1$ and the (pp)-wavicle after $t_2$. The intermediate (pk) or (kp) wavicles with phases $\pm \Phi(t)$ exist during random time interval $\Delta t = t_2 - t_1$. They describe Bohr "quantum jumps" between stationary orbits. These "jumps" were the object of fierce disputes between Bohr and Schrödinger in the heroic time of QM. Unfortunately both used only wave-function language (ket language) and naturally came to nothing. In the bra+ket language the inevitability of fast "jumps" between prolonged stationary orbits becomes evident.

Now consider the action of soft potentials that unable to cause transitions between quantum states but can change wave function phases. Taking a constant perturbation potential $V$ which commutes with Hamiltonian $H$ we get according to (5):

$$e^{-iHt-iVt}\psi(x,0) = e^{-iVt}\psi(x,t), \quad e^{iHt+iVt}\psi^\dagger(x,0) = e^{iVt}\psi^\dagger(x,t) \quad (27)$$

Neglecting switch on/off perturbations this (adiabatic) potential during its action $\Delta t$ create additional phases $\varphi = \pm V\Delta t$ for the bra and ket. Despite such perturbations the wavicle phase remains the same when the perturbation acts in the same way on its bra and ket. However, if the actions on the bra and ket are different the wavicle will acquire the additional phase. In the usual interference experiments a flow of zero-phase wavicles go through two slits or two channels. Then they are detected as screen marks or detector clicks. Thus there are two ways for a wavicle of the flow. It can go by one or another way thus revealing its "corpuscular nature" and creating the simple sum of two background pictures. Alternatively it can go through both ways (its bra goes one way while its ket another way or vice versa). In
this case the wavicle acquires a phase. Then it participates in the constructive or distractive interference according to its phase (see (25)). Note that interference only redistributes background picture and does not change its intensity. The total number of events (marks or clicks) remains the same.

The result of the two-way interference is shown in the Fig.3. The contributions of two left diagrams describe the corpuscular wavicle conduct when it passes one of possible two ways. Two right diagrams show how the wavicle reveals its wave property by passing both ways.

![Two-way interference](image)

FIG. 3: Two-way interference

We see now that the great mystery of QM (i.e. quantum particle self-interference) will become an obvious triviality when we interpret the mysterious wavicle as a bra+ket pair. The necessary but puzzling wavicle passage through two slits also becomes natural. The use of the bra+ket language helps to make more transparent other interference phenomena [15].

VI. PHASE AND SECOND QUANTIZATION

The second quantization is the usual and highly popular instrument of QM mathematics. But there are no phases in the forms that we use above. Of course, rightly used, the method can adequately treat all quantum effects where phases play a role. However the explicit absence of phases frequently impedes the proper interpretation of mathematical expressions and understanding of the physics of described phenomena. For instance, it is difficult to see the "common cause of the past" for the quantum exchange correlation (see the diagrams of Fig.2 and the expression (19)). Also it is not easy to treat superconduction phenomena without introduction of "anomalous averaging" which violates the formal rules of second quantization.

Though phases are absent in the expressions of observable quantities they are present in the time-dependent amplitudes of second quantization operators. The creation and annihilation operators (better to call them the bra and ket operators) are given by

\[ a^\dagger(t) = e^{iHt}a^\dagger e^{-iHt} = e^{i\omega t}a^\dagger, \quad a(t) = e^{iHt}ae^{-iHt} = e^{-i\omega t}a \]  

where \( \omega \equiv \epsilon_k \) is the energy of a given state and the Hamiltonian has the form

\[ H = \sum_k \epsilon_k a_k^\dagger(t)a_k(t) = \sum_k \epsilon_k a_k^\dagger a_k \]
The initial phases in the state occupancy can be introduced according to convention \( \varphi = \omega t_0 \) where \( t_0 \) is the time moment when the bra and ket meet to form a wavicle.

In the second quantization mathematics the lines on the diagrams (see the figures above) correspond to the average commutators of bra and ket operators \( \langle \Psi | [\hat{a}(t_1)\hat{a}^{\dagger}(t_2)]_\mp |\Psi \rangle \) while wavicles correspond to their averaged one-time correlators \( \langle \Psi | \hat{a}^{\dagger}(t)\hat{a}(t) |\Psi \rangle \).

Note that two-time correlators \( \langle \Psi | \hat{a}^{\dagger}(t_2)\hat{a}(t_1) |\Psi \rangle \) or \( \langle \Psi | \hat{a}^{\dagger}(t_1)\hat{a}(t_2) |\Psi \rangle \) represent separate bra or ket at time \( t_2 > t_1 \) with occupancy numbers \( F \) at time \( t_1 \). They have rapidly oscillating phase multipliers \( e^{\pm i\omega(t_2-t_1)} \) and do not describe observable physical quantities. Wavicles with phases can have two different phase multipliers \( e^{i\omega t} \) and \( e^{-i\omega t} \) and oscillate with frequency differences \( \Delta \Omega = \omega - \omega' \) which can be small. In principle they are observable and describe time-dependent fluctuations. With implicit phase inclusion they are represented by two-particle two-time correlators of the form \( \langle \Psi | \hat{a}^{\dagger}(t_1)\hat{a}_1(t_2)\hat{a}^{\dagger}_2(t_2)\hat{a}_2(t_1) |\Psi \rangle \). Such correlators correspond to mean observable quantities (see, e.g. \( \text{[20]} \)).

VII. THERMAL BATH AND QUANTUM BATH

In the previous sections we did not distinguish the occupancy numbers and distribution functions. In quantum mechanics one uses the microscopic occupation numbers which for fermions are equal to 0 or 1 and several units for bosons. In kinetics, however, instead of them it is more convenient to use so-called "coarse-grained" distribution functions which are averaged for many adjacent quantum states or for many trials. Such functions can have arbitrary state occupation values. Among them the most important ones are the equilibrium distribution functions. They are the Fermi-Dirac distribution for fermions and the Bose-Einstein distribution for bosons as well as the Boltzman distribution for classical particles.

The equilibrium distributions do not require concrete physical mechanisms for realizations and follow from the general thermodynamic principle of maximal entropy. To justify the application of this principle one should postulate the existence of random interactions of very small intensity between the system under consideration and its surrounding. Such interactions of various nature constitute the thermal bath (or thermostat) which ensures the equilibrium of the system and the relaxation to it after various perturbations.

The thermostat acts on the distribution functions which are formed by zero-phase wavicles (i.e. the bra+ket pairs with equal frequencies and initial phases). These wavicles represent Bohr’s stationary orbits which in many respect look as classical particles. The wavicles formed by bra and ket with largely different frequencies oscillate rapidly and serve as Bohr’s quantum jumps of brief duration between the stationary orbits. The jumps occur under actions of random potentials which should be sufficiently hard to initiate transitions between the orbits. Such potentials are the parts of thermostat.

The experience shows that bra and ket of zero-phase wavicles prefer to keep themselves together thus looking at sufficiently large space and time scales as single objects (e.g. photons in light beams). The interference shows that a wavicle can change (lose or acquire phases) under the action of soft (adiabatic) potentials \( \text{[27]} \) when their bra or ket suffer different potential actions. If the potential changes equally the bra and ket phases the wavicle phase will remain zero. Thus the association of bra and ket of zero-phase wavicles (e.g. their going by the same path) favors the conservation of zero-phase wavicles while the dissociation reduces their number. Since this number is invariant in average \( \text{[22]} \) the processes of losing or acquiring phases by zero-phase wavicles are similar to the establishment of equilibrium by thermostat actions.

Taking such observations into account we come to the conclusion that phase-independent wavicles find themselves in a sort of equilibrium as compare with phase-dependent wavicles. The always existing random adiabatic potentials may act as a mechanism supporting zero-
phase equilibrium for wavicles. By the analogy with thermostat (Thermal Bath) we may call these potentials by "Quantum Bath" or "quantostat".

VIII. PHASE ORIGIN

The bra and ket space functions can be identical and real. Then the difference between bra and ket with the same indices in this case are the rotation multipliers $e^{\pm i\omega t}$. In the diagrams (see Fig.1) the bra-multiplier $e^{+i\omega t}$ corresponds to the down-line arrow while the ket-multiplier $e^{-i\omega t}$ corresponds to the up-line arrow. The arrows means only the sign of rotation and not the evolution along or against time direction. The evolution always is going along the time according to the causality condition. It becomes obvious if we include damping into the frequency (real points in diagram lines).

The time evolution of bra and ket $e^{\pm i\omega t}$ reminds the complex solution of the harmonic oscillator equation. The $\cos \omega t$ and $\sin \omega t$ are also solutions and they are real. But they do not satisfy the general causality condition that the variation of a quantity at the time $(t + 0)$ is determined by its value at the time $(t - 0)$. Two exponential solutions satisfy this condition so their use as two amplitudes in this sense is preferable. For an oscillator the classical bra and ket amplitudes $(a^\dagger$ and $a)$ as well as their quantum analogues include the coordinate and momentum parts. Therefore they describe simultaneously the position and the velocity. The imaginary unit permits to unite these complimentary physical quantities as a single entity at the same time retaining their separate existence. The phase $\phi(t) = \omega t$ reflects the ratio between these parts in the amplitudes and implicitly also in their product:

$$1 = e^{i\omega t}e^{-i\omega t} = (\cos \omega t + i \sin \omega t)(\cos \omega t - i \sin \omega t) = \cos^2 \omega t + \sin^2 \omega t \equiv P + K = 1$$  \hspace{1cm} (30)

The potential energy and position correspond to the real part of the solutions while the kinetic energy and velocity correspond to their imaginary part. At a given moment of time two relative parts of the total energy exist as the potential energy $P = \cos^2 \omega t$ and the kinetic energy $K = \sin^2 \omega t$. One can also consider $P$ and $K$ as the probabilities to find the energy in its potential or kinetic forms. If we accept the indivisibility of a single quantum then it will reveal itself in two incompatible events with the probabilities $P + K = 1$. For $N \gg 1$ quanta they form two distinct parts as the classical potential and kinetic energies with equal mean values.

Now divide the time interval $(0T)$ by a number of parts $\Delta t_j$. Then ignoring the state variations we can compare the unobserved quantum evolution with the corresponding potentially observed evolution as an equality:

$$1 = \Pi_0^T e^{i\omega \Delta t_j} e^{-i\omega \Delta t_j} = \Pi_0^T (\cos^2 \omega \Delta t_j + \sin^2 \omega \Delta t_j) = \Pi_0^T (p_j + k_j) = 1$$  \hspace{1cm} (31)

At the left we see the bra and ket unobservable quantum evolution. At the right it transforms by possible observations into a series of incompatible events with probabilities $p_j + k_j = 1$ for complimentary quantities. One can include also a possible fast quantum jumps between stationary orbits with different frequencies $\omega \rightarrow \omega_j$. This way the phases implicitly govern the results of observations.

For non real space wave functions (i.e. for plain waves) one should add the space parts of phases to the time and initial parts of bra and ket phases.

IX. CONCLUSION

The use of the bra+ket language for the interpretation of QM formulae permits to get simple and natural answers on a number of questions which "one cannot ask" in the usual
The bra and ket phases and the resulting wavicle phase are the necessary elements of the physical picture though as a rule they do not appear in the final expressions for the observed physical quantities.

The quanta of energy that are the basic elements of Quantum World may be imagined as localized entities with permanent oscillations between their complimentary (potential and kinetic) components. The oscillations usually are too fast and because of this the separate bra or ket are unobservable. But their combinations in the form of wavicles have much less oscillation frequencies and emerge in our Classical World as observed quantum particles which actually become classical objects. The implicit phases of bra and ket components of wavicles cause the peculiar "non-intuitive" properties of wavicles.

Let us emphasize that any adequate interpretation of QM mathematics is impossible without the natural physical picture of bra+ket=wavicle.

One can compare the practice of getting reasonable explanations of quantum phenomena by the incomplete ket-language as an attempt to march using only one leg.

As a result it inevitably leads to various non-physical fantasies grossly contradicting all established principles of Physics inherited from the past or to the capitulation "it is impossible to understand Quantum Mechanics".

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1. J. Neumann "Mathematical Foundation of Quantum Mechanics", Princeton University, (1955)
2. J.S. Bell "Introduction to the hidden variables question", In: "Foundation of Quantum Mechanics", New York, (1971)
3. F.J. Bellifante "A survey of Hidden Variable Theories", Pergamon Press. (1973)
4. P.A.M. Dirac "The Principles of Quantum Mechanics", Oxford, Clarendon Press, (1958)
5. S.V.Gantsevich, V.L.Gurevich, M.I.Muradov, D.A.Parshin, Phys.Rev.(B), 32(19), 14006, (1995)
6. R.Katilius, S.V.Gantsevich, V.D.Kagan, M.I.Muradov, Fluct Noise Let. 9(4), 373, (2010)
7. R. Katilius, S.V. Gantsevich, Fluct. Noise Let., 12, N4, 1350023 (2013)
8. R. Feynman, Rev. Mod. Phys., 29, 205, (1957)
9. J.D. Bjorken, S.D. Drell "Relativistic Quantum Mechanics", McGraw Book Comp., (1964).
10. S. Raims, "Many-electron theory", North-Holland Pub.Comp., Amsterdam-London, (1972)
11. S.V. Gantsevich "EPR paradox in quantum diagrams", arXiv, 1701.00448
12. S.V. Gantsevich "Quantum mechanics and common sense", arXiv, 1609.08427
13. S.V. Gantsevich and V.L. Gurevich, Teor.Phys. 2(2), 63 (2017)
14. S.V. Gantsevich and V.L. Gurevich, Phys.Solid State 60, 1 (2018)
15. S.V. Gantsevich and V.L. Gurevich, Phys.Solid State, 61(11), 2104, (2019)