Open inflation from non-singular instantons: Wrapping the universe with a membrane

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Abstract

The four-form field recently considered by Hawking and Turok couples naturally to a charged membrane, across which the effective cosmological constant has a discontinuity. We present instantons for the creation of an open inflationary universe surrounded by a membrane. They can also be used to describe the nucleation of a membrane on a pre-existing inflationary background. This process typically decreases the value of the effective cosmological constant and may lead to a novel scenario of eternal inflation. Moreover, by coupling the inflaton field to the membrane, the troublesome singularities which arise in the Hawking-Turok model can be eliminated.

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1 Introduction: The Hawking-Turok model

1.1 Wave function and gravitational instantons

As the recent developments in quantum cosmology have revived some long-standing disputes about the foundations of the field, it is appropriate for us to begin by stating our position on these questions as far as they relate to the work in this paper.

A gravitational instanton is a Euclidean solution of the Einstein equations with appropriate matter sources, and, in the case of cosmology, an (effective) cosmological constant. It can typically be applied in two ways. Within the wavefunction approach to quantum cosmology [1–3], it describes semiclassically the creation of the universe from nothing. The corresponding probability measure is obtained as the square of the amplitude of the wavefunction.

The amplitude of the Hartle-Hawking [1] wavefunction is given semiclassically by the exponential of minus the Euclidean instanton action. This wavefunction has the advantage that it is well-defined beyond minisuperspace, and mathematically simple; it merely paraphrases the rule that the likelihood of any state is proportional to the exponential of its total entropy. Precisely for this reason, however, it predicts that the inflaton field should start at the minimum of its potential, excluding any possibility of sufficient inflation. Even after the anthropic principle is applied, the predictions for $\Omega_0$ are unacceptable [4, 5].

This problem does not occur if Linde’s [2] or Vilenkin’s [3] wave functions are used. While they are not equivalent [6], they both effectively yield a probability measure of

$$P_L = P_V = \frac{1}{P_{HH}},$$

(1.1)
giving greatest weight to high values of the inflaton field. This typically leads to a long period of inflation yielding $\Omega_0 = 1$, which is consistent with recent estimates of $\Omega_{\text{matter}} + \Omega_\Lambda$.

Linde’s wavefunction was proposed as a tool to describe the instant of the creation of the universe, when the only relevant degree of freedom to be quantized was the scale factor. It cannot be generalized straightforwardly to include other degrees of freedom, since they would require the opposite Wick-rotation for stability [5, 7]. While Vilenkin’s wave function is perturbatively stable, it predicts that the universe is more likely to start with a pair
of black holes than without [8, 9]. Claims that Vilenkin’s proposal is “inapplicable” to this question [10] would seem to imply that his wave function is not sufficiently general to be of any fundamental interest.

Thus all three proposals for the wave function of the universe suffer from drawbacks; no single one of them answers all relevant questions satisfactorily. Yet they have proven powerful tools for investigating the initial conditions of the universe and other aspects of quantum gravity. Keeping in mind that none of them are rigorously derived from first principles, and that they may therefore provide answers to very different questions, it seems entirely appropriate to proceed with caution, and to apply each proposal only where it yields physically reasonable results. Independently of any probabilities assigned, there is little dispute that cosmological instantons correspond to the initial states of the universe that are semiclassically allowed.

The second application of gravitational instantons is far less controversial: the description of non-perturbative fluctuations on a pre-existing background [8, 11–14]. To obtain the semiclassical creation rate, $\Gamma$, for objects like black holes, domain walls, cosmic strings, or simply true vacua, one finds instantons that continue analytically to a Lorentzian universe containing the desired features. Its action is calculated, and then normalized by subtracting the action of the background instanton. (For cosmological instantons this subtraction is implicit in the construction of the saddlepoint path; see Ref. [15].) Then the creation rate is given by

$$\Gamma = \exp \left[ - (I_{\text{instanton}} - I_{\text{background}}) \right].$$

(1.2)

Note that in this formula, as in all equations below, we take $I$ to be the action of the full Euclidean solution, rather than the half-bounce that is actually used in the interpolating path; this will avoid many confusing factors of 2.

### 1.2 Action

The Hawking-Turok [16] (HT) model is Einstein gravity with a scalar field $\phi$ that has a generic effective potential of chaotic inflation, with a minimum at $\phi = 0$, and no other stationary points. They also include a four-form field,

$$F_{\mu\nu\rho\lambda} = \partial_{[\mu} A_{\nu\rho\lambda]},$$

(1.3)

which (unlike the inflaton) occurs naturally in 11D supergravity [17]. It has no classical dynamics in four dimensions, but can be used to cancel an
effective cosmological constant arising from the combined effects of other fields in the theory. The Euclidean action is

\[
I_{\text{ht}} = \int_M d^4x \sqrt{g} \left[ -\frac{R}{16\pi G} + \frac{1}{2} (\partial \phi)^2 + V(\phi) - \frac{1}{48} F_{\mu
u\rho\lambda} F^{\mu\nu\rho\lambda} \right] - \frac{1}{8\pi G} \int_{\partial M} d^3x \sqrt{h}K + \frac{1}{24} \int_{\partial M} d^3x \sqrt{h} F^{\mu\nu\rho\lambda} n_\mu A_{\nu\rho\lambda},
\]

(1.4)

where \( K \) is the trace of the second fundamental form, \( K_{ij} \) on any boundary with unit normal vector \( n_\mu \). The sign for the \( F^2 \) term differs from that used in Refs. [18–20], who use, in the 4D effective action, the same sign that occurs in the original 11D supergravity Lagrangian, so that the four-form field gives a positive cosmological constant. In the 11D theory, however, the four-form field makes a negative contribution towards the effective cosmological constant in the 4D subspace. In order to reproduce this from a 4D effective action, the sign of the \( F^2 \) term must be changed [21]. The first boundary term is the usual Gibbons-Hawking term [22]. The second boundary term must be included to obtain stationary action under variations that leave \( F \) fixed on the boundary. On-shell its value is negative twice the \( F^2 \) contribution in the volume term of the action.

1.3 Four-form field and effective cosmological constant

The four-form field equation, \( \nabla_\mu F^{\mu\nu\rho\lambda} = 0 \), has the solution,

\[
F^{\mu\nu\rho\lambda} = \frac{c}{i\sqrt{g}} \epsilon^{\mu\nu\rho\lambda},
\]

(1.5)

so that \( F^2 = -24c^2 \). For real \( c \) the four-form field will be real in the Lorentzian sector (as it should) and \( F^2 \) will be negative everywhere. Using this solution, it is easy to show [16, 20] that both the dynamics and the Euclidean action of the system can be reproduced from an action in which all terms related to \( F \) are dropped, and instead the effective potential is substituted as follows:

\[
V(\phi) \rightarrow U(\phi) = V(\phi) - \frac{1}{2} c^2.
\]

(1.6)

In order to explain the smallness of the observed effective cosmological constant, \( 8\pi G U_0 \), one can make an anthropic argument [13] to show that the
\( c^2 \) term will cancel the vacuum energy remaining after the end of inflation (\( \phi = 0 \)) almost exactly:

\[
c^2 = 2V_0, \tag{1.7}
\]

where \( V_0 = V(0) \). Since \( c \) is real, we must assume that \( V_0 > 0 \) for this cancellation to work. (Note that Brown and Teitelboim [18, 19] assume \( V_0 < 0 \) instead since they choose the opposite sign for the action of the four-form field.)

While the anthropic argument works well for fixing the effective cosmological constant, it predicts \( \Omega_0 = \Omega_{\text{matter}} + \Omega_\Lambda = 0.01 \) if the Hartle-Hawking no-boundary proposal [1] is used to determine the a priori probability distribution. At the end of this paper, we will sketch a new model of eternal inflation which avoids this problem and does not rely on the anthropic principle to explain regions of low effective cosmological constant.

### 1.4 Open Inflation

In an \( O(4) \)-invariant Euclidean spacetime with the metric

\[
ds^2 = d\sigma^2 + b^2(\sigma)(d\psi^2 + \sin^2 \psi d\Omega_2^2), \tag{1.8}
\]

the scalar field \( \phi \) and the three-sphere radius \( b \) obey the equations of motion

\[
\phi'' + \frac{3}{b} b' \phi' = U_{\phi'}, \quad b'' = -\frac{8\pi G}{3} b(\phi'^2 + U), \tag{1.9}
\]

where primes denote derivatives with respect to \( \sigma \).

With the regularity conditions \( b = 0, b' = 1, \) and \( \phi' = 0 \) at \( \sigma = 0 \) (the ‘North pole’), the standard distorted-four-sphere solution is obtained. Except in the neighborhood of the South pole, it is well approximated by an exact \( S^4 \),

\[
b(\sigma) = H^{-1}\sin H\sigma, \quad \phi(\sigma) = \phi_0, \tag{1.10}
\]

where \( H^2 = \frac{8\pi GU(\phi_0)}{3} \). Near the South pole, at \( \sigma = \sigma_f \), this approximation breaks down. The inflaton field diverges logarithmically, while the scale factor behaves like \( (\sigma_f - \sigma)^{1/3} \).

The solution can be cut in half along the line \( \psi = \pi/2 \), which removes half of each three-sphere. The resulting instanton has a boundary of vanishing second fundamental form. Across this surface, one can continue analytically to a Lorentzian spacetime [4, 23] with the time variable \( \tau \), given by \( \psi = \)

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The field $\phi$ will be independent of $\tau$. This metric describes an exponentially expanding shell of width $H^{-1}$, which is spatially nearly homogeneous except in a small region near $\sigma_f$ where it contains a timelike singularity.

Region II is only a part of a de Sitter-like universe. To obtain the remainder of the manifold, one can continue across the null hypersurface $\sigma = 0$ by taking $\tau = i\pi/2 + \chi$ and $\sigma = it$. This yields the metric

$$ds^2 = -dt^2 + a^2(t) \left( d\chi^2 + \sinh^2 \chi d\Omega_2^2 \right), \quad (1.12)$$
where \( a(t) = -ib[\sigma(t)] \approx H^{-1} \sinh Ht \). Its spacelike sections (defined by the hypersurfaces of constant inflaton field) are open. The part of the spacetime covered by these coordinates will be called region I. It looks from the inside like an infinite open universe, which inflates while the field \( \phi \) slowly rolls down to the minimum of its potential. When the vacuum energy ceases to dominate the evolution, the universe undergoes a transition to a radiation or matter-dominated open Friedman-Robertson-Walker model.

The real part of the Euclidean action of this solution comes entirely from the Euclidean sector, and is given by

\[
I_{\text{HT}} = -U(\phi_0) \int_M d^4x \sqrt{g} = -\frac{\pi}{GH^2}. \tag{1.13}
\]

1.5 Singularity problem

The singularity occurring at the South pole gives cause for concern. The divergence of the inflaton does not cause the volume term of the action to diverge, and the boundary term coming from the missing point is typically negligible. Nevertheless, singularities are not usually allowed in gravitational instantons [24]. In particular, the recent work of Vilenkin [24] would seem to imply that the singularity occurring in the HT model should be disallowed because a similar instanton would make flat space unstable.

Hawking and Turok [16] responded by arguing that, unlike in the flat-space case, the singularity would remain forever outside region I, the open bubble, where observations are made. It is clear from a causal diagram (Fig. 2), however, that the singularity is visible to observers in the open universe. This makes it necessary to search for non-singular solutions leading to open inflation.

Regular instantons have been constructed in Ref. [26] using solutions related to those of Coleman and De Luccia, and in Ref. [27] using axionic wormholes. Garriga argued in Ref. [28] that the singularity may disappear for compactified five-dimensional Kaluza-Klein instantons, at the expense of some fine-tuning. In Ref. [29] by the same author, the singularity is cut out using domain walls of negative energy density.

It seems to us, however, that allowing negative energy is just as problematic as allowing singularities. Therefore, we will consider only domain walls, or membranes, with strictly positive energy density. The divergence of the inflaton field is averted by coupling it to the domain wall. The advantage of this approach is that no particular features in the inflaton potential need
Figure 2: The causal structure of the HT solution. The thin dashed line is a typical spacelike section in the region I of open inflation. While the singularity never enters the open universe, this Penrose diagram shows that it can be seen by observers located there. The shaded region is the causal future of the singularity. It overlaps both with the open inflationary region (I) and with the post-inflationary open universe. While the lines corresponding to the singularity and the nucleation surface in this diagram may be distorted, reducing the region of overlap, the singularity will nevertheless be visible to all post-inflationary observers at sufficiently late times.

to be assumed. Like the original HT model, it works for generic models of chaotic inflation, but without the problem of a singularity.

Introducing a membrane into the model is no random fix. The membrane of supergravity is the natural source for the four-form field which is already in the theory. It ought to be included in any case. Such membranes are characterized by a mass and a charge. In Sec. 3 we construct a non-singular instanton containing an uncharged membranes (a domain wall). It is obtained as a regular portion of the HT instanton, glued to its own mirror image. This instanton describes the nucleation of a universe containing two regions of open inflation, separated by a domain wall. Alternatively,
it may be used to describe the spontaneous nucleation of a membrane in a pre-existing de Sitter background.

In Sec. 3 we consider charged membranes, across which the four-form field, and thus the effective cosmological constant, has a discontinuity. We show how to obtain an instanton containing a membrane by gluing portions of two different HT instantons together. Finally, we outline a new model of eternal inflation, driven by the four-form field, in which the membrane instanton plays a crucial role.

2 Domain wall wrapping an open universe

2.1 Domain walls

In cosmology, the term “domain wall” normally refers to two-dimensional topological defects which can form whenever there is a breaking of a discrete symmetry. Commonly, one thinks of the symmetry breaking in terms of some Higgs field, $\Phi$. If $\mathcal{M}_0$ denotes the vacuum manifold of $\Phi$ (i.e., the submanifold of the Higgs field configuration space on which the Higgs acquires a vacuum expectation value because it will minimize the potential energy $V(\Phi)$), then a necessary condition for a domain wall to exist is that $\pi_0(\mathcal{M}_0) \neq 0$. In other words, vacuum domain walls arise whenever the vacuum manifold is not connected. The simplest example of a potential which gives rise to vacuum domain walls is the classic ‘double well’, which is discussed in detail (along with many related things) in Ref. [30].

In general, domain walls are $(D - 2)$-dimensional defects (or extended objects) in $D$-dimensional spacetimes. As such, they are a common feature in the menagerie of objects which arise in the low-energy limit of string theory, as has been discussed in detail in Refs. [31] and [32]. In other words, in string theory a domain wall is just another ‘brane in the crowd’. In this case they will usually carry a charge. For $D = 4$, such membranes act as a source for the four-form field, and change its value. We will investigate the effects of charge in Sec. 3. It is instructive, however, to consider first an uncharged domain wall, which is characterized, in the thin wall approximation, only by its energy density, $\mu$.

Its Euclidean action is given by

$$I_{dw} = \mu \int d^3x \sqrt{h},$$

(2.1)
where the integral is over the domain wall world volume, and \( h = \det h_{ij} \) refers to the induced three-metric. The energy density of the domain wall is related to a discontinuity in the extrinsic curvature, \( K_{ij} \), via the Israel matching conditions:

1. A domain wall hypersurface is totally umbilic, i.e., the second fundamental form \( K_{ij} \) is proportional to the induced metric \( h_{ij} \) on each domain wall world sheet.

2. The discontinuity in the second fundamental form on each domain wall hypersurface is \( K_{ij}^+ - K_{ij}^- = 4\pi G \mu h_{ij} \),

where \( K_{ij}^- (K_{ij}^+) \) refers to the limiting value of the extrinsic curvature on the side of the wall to which the normal vector is (not) pointing. These conditions, together with the Gibbons-Hawking boundary terms evaluated on the wall, yield the on-shell action

\[
I_{DW} = -\frac{\mu}{2} \int d^3x \sqrt{h}; \tag{2.2}
\]

the volume integrals in the gravitational action, Eq. (1.4), are now understood to exclude the domain wall.

### 2.2 Cosmological domain wall instantons

In the instanton used by Hawking and Turok for the nucleation of an open inflationary universe, Eq. (1.10), the hypersurfaces of constant \( \sigma \) are trivially umbilic; we shall build our domain walls there. When the analytic continuation to a Lorentzian spacetime is performed, these surfaces remain forever just outside the region of open inflation, draping it but not intruding.

The idea is to cut the instanton along one of these surfaces and discard the singular part. The remaining part of the distorted four-sphere is then glued onto its mirror image. In the Lorentzian sector, this means we get rid of the part containing the singularity at \( \sigma_f \), but keep the part containing region I, the open universe; in fact we will obtain two such regions (see Fig. 3). This ‘cut, copy and paste’ procedure gives rise to a discontinuity in the second fundamental form, and thus to a domain wall. It also leads to a small discontinuity in the derivative of the inflaton field, \( \phi \); we will discuss in Sec. 2.3 how this can be attributed to an interaction of the field with the
The bubbles of open inflation are the forward light-cones.

Region 1

Domain wall (dark line) cut from each solution.

Identify.

Region II is outside the light cones.

Two ‘mirror-image’ Hawking-Turok solutions, identified along a $x =$constant domain wall hypersurface. The singularity is not shown, because the portion of region II containing the singularity has been thrown away.

Figure 3: Cut, copy and paste: How to construct a solution containing two regions of open inflation separated by a domain wall.

membrane. For now, we shall concentrate on the geometric aspects, using the approximation of constant $\phi$ throughout the instanton.

We thus take the portion of the four-sphere, Eq. (1.10), corresponding to $0 \leq \sigma < \sigma_m$ and match it to its mirror image. The extrinsic curvature at $\sigma = \sigma_m$ will be given by

$$K^\pm_{ij} = \pm \frac{b'(\sigma_m)}{b(\sigma_m)} h_{ij},$$

(2.3)

where the prime denotes differentiation relative to $\sigma$. According to the Israel conditions, this gives rise to a domain wall of energy density $\mu$:

$$K^+_{ij} - K^-_{ij} = 4\pi G \mu h_{ij},$$

(2.4)
whence
\[ \mu = \frac{b'(\sigma_m)}{2\pi G b(\sigma_m)}. \]  

(2.5)

We are assuming, of course, that \( \mu \) is a fixed parameter in the theory. Then the above equation tells us where to cut:

\[ \sigma_m = H^{-1} \arccot 2\pi G \mu H^{-1}. \]  

(2.6)

In particular, for positive energy domain walls, we find that \( b' > 0 \). In other words, one must cut before the surface of maximum expansion, \( \sigma < \sigma_{\text{max}} \), and discard the larger part of the instanton. Gluing the remaining \( \sigma < \sigma_m \) sector of the \( S^4 \) instanton to its mirror image, we thus obtain an instanton shaped like a lens, or perhaps a hamburger, with the domain wall playing the role of the beef (see Fig. 4).

Figure 4: The hamburger instanton is obtained by gluing a portion of the \( S^4 \) instanton to its mirror image.

The corresponding Lorentzian solution is obtained by the same Coleman-De Luccia analytic continuations that were employed for the HT instanton,
Eq. (1.10). The fact that a part of the four-sphere is missing means that the Lorentzian shell surrounding the open universe ('region II') will be correspondingly thinner than the usual $H^{-1}$. The domain wall resides in the center of this shell, at constant $\sigma$.

The Euclidean action can easily be calculated from the on-shell actions for the $S^4$ instanton, Eq. (1.13), and the domain wall, Eq. (2.2):

$$I_{HT/D} = -U(\phi_0) \int_M d^4x \sqrt{g} - \frac{\mu}{2} \int d^3x \sqrt{h}.$$  (2.7)

Adjusting for the smaller four-volume that arises from restricting to $\sigma < \sigma_m$ on each half of the hamburger, and noting that the Euclidean world volume of the domain wall is a three-sphere of radius $b(\sigma_m)$, one obtains

$$I_{HT/D} = -\frac{\pi}{GH^2} (1 - \cos H \sigma_m).$$  (2.8)

As we discussed in the introduction, the hamburger instanton can be applied in two ways. It may be interpreted to describe the semiclassical creation from nothing of two open universes surrounded and separated by a membrane. Alternatively, one can use it to describe the spontaneous nucleation of a domain wall in a pre-existing de Sitter-like universe. To obtain the semiclassical creation rate, one must first subtract the background action, Eq. (1.13). This yields

$$\Gamma = e^{-(I_{HT/D} - I_{HT})} = \exp \left(-\frac{\pi}{GH^2} \cos H \sigma_m \right).$$  (2.9)

### 2.3 Using domain walls to avoid the singularity

The domain wall we have introduced effectively cuts the distorted-four-sphere instanton, discards the larger (singular) part, and pastes the smaller part to its mirror image to form the hamburger instanton. The two parts are not completely symmetric, however, since the inflaton field, $\phi$, grows slowly with Euclidean time, and continues to grow across the domain wall. It would therefore diverge near the South pole in the same way that it does in the HT case, leading to a mild singularity.

In the presence of a domain wall, however, it is easy to prevent this behavior. All it takes is a small coupling of the domain wall to the field:

$$\mu = \mu(\phi).$$  (2.10)
Garriga [29] used the coupling $\mu = \mu_0 - \alpha e^{\kappa \phi}$ in a different context, but for our purposes there is no need to assume any specific form. With the $\phi$-dependence of $\mu$, the inflaton field moves in the effective potential

$$U_\mu(\phi) = U(\phi) + \delta(\sigma - \sigma_m)\mu(\phi). \quad (2.11)$$

This leads to a discontinuity of $\phi'$ across the domain wall:

$$\phi'_- - \phi'_+ = \mu_\phi. \quad (2.12)$$

The instanton will be non-singular if it is made completely symmetric, i.e. if $\phi'_- = -\phi'_+$. Thus we obtain the regularity condition

$$\mu_\phi[\phi(\sigma_m)] = -2\phi'(\sigma_m). \quad (2.13)$$

This does, of course, require fine-tuning of the coupling. We can turn the argument around, however, by noting that $\mu_\phi$ generically depends on the value of $\phi$ at the domain wall, which is approximately $\phi_0$. Requiring regularity for a given coupling thus fixes the initial value of the inflaton field. Therefore, the coupling can be used to dial $\phi_0$, and thus the value of $\Omega_0$.

3 Charged membrane wrapping an open universe

3.1 Fundamental Membranes

A feature of the $p$-branes of supergravity theories is that they are naturally associated with $(p + 2)$ forms, which may be regarded as the field strengths for $(p + 1)$ potentials. The potential couples electrically to the worldvolume of the $p$-brane. Thus, by Eq. (1.3), the four-form field $F_{\mu \nu \rho \lambda}$ is related to the three-form potential $A_{\nu \rho \lambda}$, which couples to the worldvolume of a membrane, i.e. a two-dimensional extended object. In four spacetime dimensions, one would regard a two-dimensional extended object as a ‘domain wall’. Put the other way around, one can couple any domain wall to the four-form in this way. Such a coupling was considered in a cosmological context in [18, 19], where it was shown that the effective cosmological constant can be decreased through the nucleation of membranes which couple to the four-form.

In string theory there are in general three distinct classes of $p$-branes, which are distinguished according to how the tension of the brane varies
with the string coupling constant. Let $g_s$ denote the string coupling constant, which, crudely speaking, measures the size of the eleventh dimension in M-theory. Then the three classes, consisting of fundamental branes, Dirichlet branes (D-branes), and solitonic branes, may be defined as follows [33]. For fundamental $p$-branes, the tension $T_p$ does not depend at all on the string coupling. For D($p$)-branes, the tension $T_p$ varies inversely as the string coupling ($T_p \sim 1/g_s$). Finally, for solitonic branes the tension varies inversely as the square of the coupling ($T_p \sim 1/g_s^2$).

The vacuum expectation value of the dilaton field, $g_s$, may not be constant in general. If the effective energy density of a domain wall depends on a field which is spacetime dependent, one may not use the Israel conditions, since these conditions are derived subject to the assumption that the energy density of the wall is constant. Thus, in order to describe D8-branes (in ten dimensions) which are domain walls in backgrounds with a varying dilaton, for example, one needs to generalize the Israel conditions in some way. The correct modification of the Israel conditions will be presented in a future paper [34].

Here, however, we are going to assume that the domain walls couple as fundamental branes to the four-form. This assumption, which was implicitly made in [18, 19], is important, because it means that we do not have to worry about any dilatonic dependence for the energy density of the domain walls, and so we are justified in using the Israel conditions to construct domain wall hypersurfaces.

With this in mind, we follow the constructions in Ref. [18, 19] and couple our domain walls to the four-form of [16]. In Ref. [16], the four-form was introduced in an attempt to ‘solve’ the cosmological constant problem. Dirac quantization for the charges supported by these branes [33] implies that the effective cosmological constant generated in this way will jump by an integral multiple of a discrete unit of charge, as one moves from one side of the membrane to the other side. This does not necessarily imply a fine-tuning problem for the four-form of HT [16], simply because their four-form is not induced by a membrane. This is analogous to the situation in ordinary electromagnetism, where it only makes sense to talk about charge quantization once electric sources have been introduced. In the absence of elementary particles, electric flux can take any value.
3.2 Action and Solutions

The Euclidean action of a charged membrane is given by

\[ I_{dw} = \mu \int d^3x \sqrt{h} + \frac{e}{6} \int d^3x A_{\nu\rho\lambda} \epsilon^{\nu\rho\lambda} \]  

(3.1)

The solution for the four-form field will still be given by Eq. (1.5), except that the new \( \delta \)-function term in its equation of motion leads to a discontinuity of the field strength across the domain wall:

\[ F^{\mu\nu\rho\lambda} = \frac{c_\pm}{i\sqrt{g}} \epsilon^{\mu\nu\rho\lambda}, \]  

(3.2)

with

\[ c_- = c_+ - e. \]  

(3.3)

As before, the super- or subscript ‘+’ refers to the region before the membrane \((\sigma < \sigma_m)\), and ‘−’ to the region beyond. The instanton will consist of sections of two distorted four-spheres of different radii, matched across the membrane:

\[ b(\sigma) = H_+^{-1} \sin H_+ \sigma \quad (\sigma < \sigma_m), \]  

(3.4)

\[ b(\sigma) = H_-^{-1} \sin H_- (\sigma_f - \sigma) \quad (\sigma > \sigma_m), \]  

(3.5)

where

\[ H_\pm^2 = 8\pi G U_\pm(\phi_0)/3, \]  

(3.6)

and

\[ U_\pm(\phi) = V(\phi) - \frac{1}{2} c_\pm^2. \]  

(3.7)

We use the approximation \( \phi(\sigma) = \phi_0 \). It is straightforward to generalize the method used in Sec. 2.3 to remove the singularity in the charged case; we will therefore not worry about it here. Also, the analytic continuation to a Lorentzian spacetime is exactly as for the HT instanton without domain walls, or with uncharged domain walls (see Sec. 1.4).

The matching analysis is more interesting now. As the solution is regular, there will be two open universes, emerging from light cones at \( \sigma = 0 \) and \( \sigma = \sigma_f \). They will differ in the value of the cosmological constant after inflation ends, \( U_0 = V_0 - \frac{1}{2} c_+^2 \). In order for the universe to live long enough, and for galaxies to form, it is necessary that \( U_0 \approx 0 \) in one of the two open regions. We may choose this region to be on the ‘+’ side. As we restrict to compact instantons, we shall not consider Anti-de Sitter spacetimes. Therefore we will
take \( e > 0 \), so that the ‘−’ bubble will retain a higher residual cosmological constant. Then the effective cosmological constant in the ‘+’ portion of the instanton will be lower, and the radius of curvature larger, than in the ‘−’ portion. From the positivity of the membrane energy, it then follows that the ‘+’ portion will always be less than half of an \( S^4 \). The ‘−’ portion, however, can be either less (\( \eta = 1 \)) or more (\( \eta = -1 \)) than half.

With \( c_+ \) fixed by anthropic arguments, the values of \( H_\pm \) depend on \( \phi_0 \) and \( e \) according to Eqs. (3.3) and (3.7). The energy density, \( \mu \), and the requirement that the scale factor be continuous across the membrane fix \( \sigma_m \) and \( \sigma_f \). It was convenient in the previous section to eliminate \( \mu \) in favor of \( \sigma_m \); similarly, it will simplify expressions now if we eliminate in favor of \( b_m = b(\sigma_m) \), which is related to \( \mu \) by

\[
\mu = \frac{\sqrt{1 - H_+^2 b_m^2} + \eta \sqrt{1 - H_-^2 b_m^2}}{4\pi G b_m}.
\]

(3.8)

Thus we shall specify instantons by \((H_+, H_-, b_m, \eta)\) instead of the equivalent \((\phi_0, e, \mu)\).

On-shell the charge term vanishes and the instanton action is given by

\[
I_{HT/M} = -\frac{\pi}{2 G H^2} \left( 1 - \eta \sqrt{1 - H_-^2 b_m^2} \right) - \frac{\pi}{2 G H^2} \left( 1 - \sqrt{1 - H_+^2 b_m^2} \right).
\]

(3.9)

This is the relevant action if we consider the creation from nothing of a universe containing two bubbles of open inflation with different cosmological constants, separated by a charged membrane.

If we are interested in the spontaneous nucleation of a charged membrane on a background de Sitter-like universe, we must subtract the action of the background instanton to get the pair creation rate. There are two possible backgrounds: the four-spheres of radius \( H_\pm^{-1} \). Using the ‘+’ background corresponds to the nucleation of a membrane inside of which the cosmological constant is larger. By Eqs. (1.2) and (1.13), the creation rate is exp\((-\Delta I_{HT/M}^\pm)\), with

\[
\Delta I_{HT/M}^\pm = \frac{\pi}{2 G H^2} \left( 1 + \sqrt{1 - H_\pm^2 b_m^2} \right) - \frac{\pi}{2 G H^2} \left( 1 - \eta \sqrt{1 - H_\pm^2 b_m^2} \right).
\]

(3.10)

Starting from the ‘−’ background, on the other hand, means that the membrane nucleation decreases the cosmological constant. This occurs at a rate...
of \( \exp(-\Delta I_{HT/M}) \), with

\[
\Delta I_{HT/M} = \frac{\pi}{2GH^2} \left( 1 + \eta \sqrt{1 - H^2 b_m^2} \right) - \frac{\pi}{2GH^2} \left( 1 - \sqrt{1 - H^2 b_m^2} \right). \tag{3.11}
\]

Starting from a given background, a little algebra shows that either type of nucleation is suppressed, and that the increase of the effective cosmological constant is more suppressed than the decrease:

\[
\Delta I_{HT/M}^+ > \Delta I_{HT/M}^- > 0. \tag{3.12}
\]

Thus, the cosmological constant is, on average, driven down by membrane creation. For an appropriate range of parameters, it can relax to a value within experimental bounds, and will not decrease below zero \([18, 19]\). The problems with this kind of mechanism are two-fold. First, why should the unit of membrane charge be so small as to be able to tune the cosmological constant to an accuracy of \(10^{-122}\)? It is simply unclear where such a scale should come from. However, this problem is still less serious than the original cosmological constant problem; once the scale is explained, dynamic fine-tuning takes place automatically.

As was pointed out in Refs. \([18, 19]\), however, the main problem is the slowness of the process. By the time the final jump of the effective cosmological constant occurs, the inflaton field will be in the minimum of its potential, and the open universe contained in the membrane will be empty. There is one way to circumvent this problem, which we will turn to next. (We should also point out that Linde \([5]\) is proposing a non-dynamical fix for the cosmological constant in the wavefunction framework. In this scenario, the anthropic principle selects an acceptable value for the cosmological constant just like in Ref. \([16]\), but the problem of low \(\Omega_0\) is avoided.)

### 3.3 Eternal inflation driven by the four-form

Let us assume that the universe is in a period of inflation, with \(\phi\) far from its minimum and slowly rolling down; also assume that \(U_0 = V_0 - \frac{1}{2} c^2\) has not been fine-tuned and is many orders of magnitude larger than current constraints allow. The cosmological constant would need to be corrected by membrane creation at least 60 e-foldings before inflation ends in order to obtain a realistic value of \(\Omega_0\). Given the exponential suppression of membrane nucleation events, they are extremely unlikely to take place even during the
entire classical roll-down of the inflaton field. The need to complete the
dynamic tuning, rather than just achieve some random decrease of the cos-
 cosmological constant, exacerbates this problem.

Linde has pointed out \cite{35} that for generic models of chaotic inflation,
there is a critical value of the effective cosmological constant beyond which
random quantum fluctuations of the inflaton field dominate over the classical
decrease. As the number of horizon volumes grows exponentially, the effective
cosmological constant will increase roughly in half of the new domains, so
that inflation continues forever in some regions.

If the cosmological constant arises purely from the inflaton potential (i.e.
if $V_0 = c = 0$), the regime of eternal inflation lies beyond a critical value of
$\phi$, for which de Sitter space is hot enough to support the strong quantum
fluctuations. The exit from the eternal phase occurs locally if $\phi$ jumps below
the critical value. This leads a region of spacetime into the regime of classical
slow-roll, and allows inflation to end there. The entire observable universe
would be contained in such a region.

In models with a four-form field, there is a different way to support eternal
inflation: If $U_0$ is sufficiently large, the temperature of the spacetime,
$T \sim H/2\pi$, will support a stochastic quantum evolution of the inflaton field for
all values of $\phi$, even $\phi = 0$. In this eternally inflating universe, all values of $\phi$
will be realized in different regions, from $\phi = 0$ to $\phi = \phi_{Pl}$. Then the number
of horizon volumes is unbounded, and the suppression of membrane creation
is no longer an obstacle. In fact, the exit from this type of eternal inflation
occurs not though a jump of $\phi$, but instead by membrane nucleation pushing
$U_0$ below the critical value.

Because of the stochastic distribution of the values of $\phi$, more than 60
e-foldings of inflation will occur inside most membranes. For sufficiently high
values of $\phi$, the transition may simply be one from $F^2$-driven eternal inflation
to ‘ordinary’ eternal inflation. For lower values of $\phi$, the classical decrease
immediately begins. This makes no difference; the only thing that matters
is that the exit sets $U_0 \approx 0$.

Most membranes that are spontaneously created will fail to do so, because
they will have the ‘wrong’ charge. They may trigger a local exit from eternal
inflation, but without leading to an acceptable value of the present cosmo-
logical constant. We do not worry about these membranes; they merely add
to the vast vacuum-dominated regions abundant in eternal inflation. However,a small proportion of membranes will have just the right charge (some
appropriate multiple of the unit charge), such that the cosmological constant
is neutralized, or nearly enough neutralized, by a single nucleation event. In an eternally inflating universe, any small but semiclassically non-vanishing probability is large enough. There will be regions in which inflation ends and galaxies can form, typically with \( \Omega_{\text{matter}} + \Omega_\Lambda = 1 \), although smaller values of \( \Omega_0 \) will also occur. That we live in such a region is no more surprising than the fact that we do not live in intergalactic space.

Such models may still suffer from some of standard problems that come with combining a fundamental eleven-dimensional theory with inflationary theory: the inflaton potential must be inserted ad hoc, and there is no convincing argument why the unit membrane charge, and the background value of \( F^2 \), should not both be of Planckian order. But it has several strong advantages over other proposals. First, like in all models of eternal inflation, any initial conditions are completely obliterated, allowing us to avoid the minefield of wavefunction proposals. Second, as in Refs. \[\text{[18, 19]}\], the effective cosmological constant is neutralized dynamically, with no need to resort to anthropic arguments. Third, it predicts a total value of \( \Omega_0 = 1 \), which is consistent with recent supernova observations.

4 Summary

We have shown that it is possible to couple domain walls to the four-form field recently resuscitated by Hawking and Turok to ‘solve’ the cosmological constant problem, and that one may regard such membranes as sources for the effective cosmological constant. For generic inflaton potentials, we constructed non-singular instantons that give rise to a Lorentzian universe containing two regions of open inflation, separated by a charged or uncharged membrane.

Such instantons may be interpreted to describe the creation of singularity-free, open inflationary universes from nothing. They also mediate the semiclassical production of membranes on a pre-existing inflating background; on average, this process drives down the effective cosmological constant. We proposed a model of eternal inflation in which regions with acceptable values of \( \Omega_0 \) and \( \Omega_\Lambda \) are produced dynamically.

The membranes we considered wrap the open universes, hovering just outside the light-cones that bound the regions of open inflation. Parts of the inflationary and postinflationary open universe are in their causal future. Fluctuations generated by perturbing these domain walls travel into the bub-
ble of open inflation. It would be interesting to understand whether they can have observable effects.

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