Reliability Assessment on Stability of Slopes Reinforced with Anti-sliding Piles in Spatially Variable Soils

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Abstract. A reliability-based analysis method considering spatial variability of soil properties was proposed for stability analysis of pile-reinforced slope system in this paper. A total of 71 three dimensional (3D) finite element simulation results were used to develop a Polynomial Regression (PR) model relating the safety factor (FOS) of the slope system to the key input parameters. The PR model was proved to be reasonably accurate with a high coefficient of determination. It was subsequently implemented into a modified spreadsheet involving First-order reliability method (FORM) and spatial variability factors to perform spatial variability and reliability analyses. Based on the results, some useful conclusions are arrived with regard to the effects of spatial variability factors. In addition, a useful guideline for geotechnical engineers to choose suitable pile locations in engineering practices is provided.

1. Introduction

Reinforcement of unstable slopes has been a vital geotechnical issue that needs to be emphasized to guarantee the slope stability, personal safety and property safety. The use of anti-sliding piles to prevent slope failure has been widely adopted due to its effectiveness and convenience of construction[1].

The slope with the same safety factor may exhibit different risk level due to the variability of the slope properties. Numerous studies have been conducted in recent years to investigate the slope stability based on reliability analyses considering the uncertainties and spatial variability of the soil properties[2-4].

The objective of this study is to present a procedure to evaluate the slope stability reinforced with anti-sliding piles from a probabilistic perspective and the spatial variability of soil properties is taken into account. To achieve this goal, a Polynomial Regression (PR) model evaluating the safety factor (FOS) of the pile-reinforced slope system was firstly established through plenty of numerical computations using the 3D finite element software PLAXIS. Then a spreadsheet was developed which combines the use of the PR model, the First-order reliability method (FORM), and the spatial variability factors to conduct spatial variability reliability analysis for the pile-reinforced slope.
2. Numerical analyses

2.1 Slope geometry and boundary conditions
In order to obtain the PR model for relating the FOS of the slope system to input key parameters, many 3D finite element numerical calculations were performed by means of PLAXIS. Because of symmetry, only a representative region including two piles and surrounding soil was considered as schematically plotted in figure 1(a). The geometry and dimensions of the numerical model are shown in figure 1(b) and (c).

![Figure 1. Schematic illustration of soil model](image)

In terms of the boundary conditions, the nodes on the bottom of the mesh are fixed in both the -x- and -y- directions and the nodes along the four vertical boundaries are constrained from moving in the direction normal to the planes.

2.2 Material properties
The soil adopted the linear elastic perfectly plastic Mohr Coulomb failure criterion. The input key parameters that used to predict the FOS via PR model are listed in table 1.

| Categories          | Number | Parameter                      | Range of parameter |
|---------------------|--------|--------------------------------|--------------------|
| Soil parameters     | 1      | Cohesion (c, KPa)              | 5, 15, 25, 35, 45, 55, 65 |
|                     | 2      | Internal friction angle (φ, degrees) | 15, 20, 25, 30, 35 |
|                     | 3      | Soil unit weight (γ, kN/m³)     | 16, 17, 18, 19, 20 |
| Pile parameters     | 4      | Pile length (H, m)             | 3, 6, 9, 12, 15, 18 |
|                     | 5      | Dimensionless pile spacing (S/D) | 2, 3, 4, 5         |
|                     | 6      | Dimensionless piles location (ξx=Lx/L) | 0.3, 0.4, 0.5, 0.6, 0.7 |
| Geometry            | 7      | Slope inclination angle (α, degrees) | 40, 45, 50       |

3. Polynomial Regression (PR) model
Seven key input parameters were taken into consideration to develop the PR model. A total of 71 assumed cases were analyzed in this study. Moreover, the safety factor of the slope would increase with the increasing of the pile length until the pile length exceeds a certain length. The particular length is defined as "Critical Length". A semi-empirical formulation that estimating the Critical Length has also been proposed.

Based on the FE analyses results, the PR models for estimating FOS and Critical Length of the slope system were presented and take the following forms:

\[
\begin{align*}
\text{FOS} &= a + a_1 \alpha + a_2 (S / D) + a_3 (L_x / L) + a_4 H + a_5 \gamma + a_6 c + a_7 \varphi + a_8 \alpha^2 \\
&\quad + a_9 (S / D)^2 + a_{10} (L_x / L)^2 + a_{11} H^2 + a_{12} \gamma^2 + a_{13} c^2 + a_{14} \varphi^2 + a_{15} \alpha \varphi \\
\text{CL} &= b_1 + b_2 \alpha + b_3 (L_x / L) + b_4 \gamma + b_5 c + b_6 \varphi + b_7 \alpha^2 + b_8 (L_x / L)^2 \\
&\quad + b_9 \gamma^2 + b_{10} \varphi^2 + b_{11} \alpha \varphi
\end{align*}
\]

The accuracy of the estimation model can be evaluated by coefficient of determination (R²), the
greater the deterministic coefficient, the more accurate the estimation model. FOS and Critical Length obtained by estimating model versus those obtained via FE calculations are plotted in figure 2. The coefficients of Equation (1) and (2) are listed in table 2 and 3, respectively. The two equations are reasonably accurate with a high coefficient of deterioration ($R^2$) of 0.9781 and 0.9539, respectively.

![Figure 2](image)

**Table 2** Response surface coefficients for FOS

| $a_0$  | 2.8142 | $a_4$  | 0.0372 | $a_8$  | 0     | $a_{12}$  | 0.0066 |
|-------|--------|-------|--------|-------|-------|-----------|--------|
| $a_1$ | -0.0152| $a_5$ | -0.2963| $a_9$ | 0.0018| $a_{13}$  | -0.0003|
| $a_2$ | -0.0432| $a_6$ | 0.0459 | $a_{10}$ | -2.9148| $a_{14}$  | -0.0002|
| $a_3$ | 2.7615 | $a_7$ | 0.0462 | $a_{11}$ | -0.0002| $a_{15}$  | 0.0005 |

**Table 3** Response surface coefficients for Critical Length

| $b_0$  | 33.3055 | $b_1$  | -0.5959 | $b_2$  | 5.9648 | $b_3$  | -0.0009 |
|-------|---------|-------|---------|-------|--------|-------|---------|
| $b_4$ | -0.545  | $b_5$ | 0.0044  | $b_6$ | 12.8129| $b_7$ | 0.0002  |
| $b_8$ | 0.0059  | $b_9$ | 0.005   |

The calculation of FOS should combine Equation (1) and Equation (2). The Critical Length is calculated by Equation (2) first and compared with input pile length, if it is greater than input pile length, the input pile length is taken into Equation (1) to evaluate the FOS. Otherwise, the Critical Length is taken into Equation (1) to evaluate the FOS.

It is worth noting that the two estimation models (Equation (1) and Equation (2)) are based on regression analysis of a limited number of FE parametric studies, therefore, the use of the two equations requires that the actual slope conditions fall within the range of parameters listed in table 1.

4. Reliability analysis

4.1 The variance reduction factor

A dimensionless variance reduction function $\Gamma^2$ was proposed by Vanmarcke[5], which is a function of the scale of fluctuation $\theta$ and characteristic length $L$. The variance reduction function $\Gamma^2$ is adopted in reliability analysis to take into consideration the spatial variability and it has been successfully applied in many studies[6-7]. The variance reduction factor is given by:

$$
\Gamma^2 = \frac{1}{2} \left( \frac{\theta}{L} \right)^2 \left[ 2L \frac{\partial}{\theta} - 1 + \exp \left( -\frac{2L}{\theta} \right) \right] 
$$

in which $L$ = characteristic length; and $\theta$ = scale of fluctuation. The positive square root of the
The variance reduction factor is used in this paper and it is regarded as the standard deviation reduction factor ($\Gamma$).

The $\theta = 2, 5, 10, 20, 40, 60, 80, 100$ m and $L = 7, 10, 13, 16$ m are adopted respectively in this study. It worth noting that the $\theta$ values are commonly used in spatial variability analysis and the characteristic length $L$ is closed to the slope height.

### 4.2 The developed spreadsheet for spatial variability analysis

The spreadsheet set up for spatial variability analysis is shown in figure 3. This spreadsheet is modified from the original version of Goh et al.[8]. The spatial factors are introduced into the spreadsheet, which is listed in Cells G10:G12. For brevity, the use procedure of the spreadsheet is omitted here and Goh et al.[8] can be referred to for detail use.

**Figure 3. Modified spreadsheet for spatial variability analysis**

### 4.3 Consideration of spatial variability of soil properties

In this section, the parameters listed in table 4 are adopted. The coefficient of variation of cohesion and friction angle are assumed to be 0.3 and 0.1, respectively. The influence of $\theta$ and $L$ on reliability index $\beta$ and failure probability $P_f$ are plotted in figure 4 (a) and (b), respectively. The case without considering spatial variability is also shown in figure 4 (a) and is represented by red dashed line. There is a sharp decrease of $\beta$ with the increase of $\theta$ at the beginning, and finally the $\beta$ considering spatial variability converges to 1.4323 that without considering spatial variability. This indicates that the reliability index could be too conservative without considering spatial variability of soil properties.

**Table 4 Parameters for reliability assessment considering spatial variability**

| Parameters                        | Mean value | Value of COV |
|-----------------------------------|------------|--------------|
| Random variables                  |            |              |
| Cohesion ($c$, kPa)               | 7          | 0.3          |
| Internal friction angle ($\phi$, degrees) | 25         | 0.1          |
| Deterministic parameters          |            |              |
| Soil unit weight ($\gamma$, kN/m$^3$) | 18         | -            |
| Pile length ($H$, m)              | 9          | -            |
| Slope inclination angle ($\alpha$, degrees) | 45         | -            |
| Dimensionless pile spacing ($S/D$) | 3          | -            |
| Dimensionless piles location ($\xi_x=L_x/L$) | 0.5         | -            |
Figure 4 Influence of spatial variability factors of soil properties on (a) reliability index $\beta$, and (b) failure probability $P_f$.

4.4 Influence of pile location

Figure 5 (a) and (b) show the effects of dimensionless pile location on reliability index $\beta$, considering the spatial variability of soil properties. The coefficient of variation of cohesion and friction angle are assumed to be 0.3 and 0.1, respectively. It is obviously that reliability index $\beta$ is significantly influenced by pile location since $\beta$ tends to increase with $\xi$ and then decrease after reaching the middle location of the slope. Piles at middle location of the slope would provide higher reliability index than piles at other location, which is in good agreement with the result of Li and Liang [9]. It can also be drawn that lower $\theta$ and greater $L$ will result in higher reliability index, same as illustration above.

Figure 5 influence of pile location on reliability index (a) $\theta = 2$ and 5 m, and (b) $\theta = 10$ and 20 m.

5. Summary and conclusions

A reliability-based approach was proposed in this paper to evaluate the stability of pile-reinforced slope system from a probabilistic perspective. Based on the results of spatial variability and reliability analyses, major conclusions can be drawn as following:

1) The PR model proposed in this study is reasonably accurate to predict the FOS of pile-reinforced slope system.

2) The spatial variability factors influence reliability index significantly, the reliability index could be too conservative if the spatial variability of soil properties is neglected; The lower scale of fluctuation $\theta$ and greater characteristic length $L$ can result in higher reliability index.

3) The location of anti-sliding piles is a vital factor that affects the stability of the slope system. For homogeneous soil slope, the piles at the middle location of the slope may provide the highest reliability index for the given diameter and pile spacing. This will provide useful guideline for geotechnical engineers to choose suitable pile locations in engineering practices.

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