Combined effect of Zeeman splitting and spin-orbit interaction on the Josephson current in a S-2DEG-S structure

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We analyze new spin effects in current-carrying state of superconductor-2D electron gas-superconductor (S-2DEG-S) device with spin-polarized nuclei in 2DEG region. The hyperfine interaction of 2D electrons with nuclear spins, described by the effective magnetic field $B$, produces Zeeman splitting of Andreev levels without orbital effects, that leads to the interference pattern of supercurrent oscillations over $B$. The spin-orbit effects in 2DEG cause strongly anisotropic dependence of the Josephson current on the direction of $B$, which may be used as a probe for the spin-orbit interaction intensity. Under certain conditions, the system reveals the properties of $\pi$-junction.

PACS numbers: 74.80.Fp, 31.30.Gs, 71.70.Ej, 73.20.Dx.

The spin-orbit (SO) and hyperfine (HF) interactions in GaAs heterojunctions and similar 2D quantum Hall systems attract permanent theoretical and experimental attention. The hyperfine field of the nuclear spin subsystem acting upon the spins of charge carriers may reach $10^4$ G. At low temperatures, the nuclear spin relaxation time can be macroscopically long, so the nonequilibrium spin population in heterojunctions, once created, is conserved during hundreds of seconds. The Zeeman splitting combined with a strong spin-orbit coupling in GaAs/AlGaAs 2DEG gives rise to a novel class of coherent phenomena, e.g., the spontaneous Aharonov-Bohm effect.

![FIG. 1. A model of the superconductor - 2DEG - superconductor device based on GaAs/AlGaAs heterojunction. The normal $\textbf{n}$ is directed towards the Al-doped layer.](image)

In this paper we discuss mesoscopic spin-orbit effects in Josephson current flowing across the S-2DEG-S structure (Fig. 1) with polarized nuclei in 2DEG region. The transfer of the Josephson current through the normal conducting layer is provided by the Andreev reflection of electrons and holes at the NS-interfaces, which convert normal electron excitations into Cooper pairs in the superconducting banks. In a pure system with length $d$ smaller than the electron scattering length, the interference between coherent electron states and retro-reflected hole states produces the set of spin-degenerate Andreev energy levels $E_\lambda(\Phi)$, which depend on the quantum numbers $\lambda$ and on the difference $\Phi$ of the order parameter phases in superconducting electrodes. In short structures, $d < \xi_0$ ($\xi_0$ is the coherence length in the superconductor), the Josephson current can be presented as the sum of currents transferred by individual Andreev bound states (see, e.g., Ref. 5 and references therein),

$$ I(\Phi) = -\frac{2e}{\hbar} \sum_\lambda \frac{\partial E_\lambda(\Phi)}{\partial \Phi} \tanh \frac{E_\lambda(\Phi)}{2T}, \quad (1) $$

and must be sensitive to the HF and SO interaction which eliminates spin degeneracy of the Andreev levels.

The contact HF interaction in a semiconductor is described by the Hamiltonian

$$ \hat{H}_{hf} = (8\pi/3)\mu_B\gamma_h \sum_i \mathbf{I}_i \mathbf{\sigma} \delta(\mathbf{r} - \mathbf{R}_i). \quad (2) $$

Here $\mu_B$ is the Bohr magneton, $\gamma_h$ is the nuclear magneton, and $\mathbf{I}_i$, $\mathbf{\sigma}$, $\mathbf{R}_i$, $\mathbf{r}$ are nuclear and charge carrier spins and positions, respectively. It follows from Eq. (2) that if the nuclear spins are polarized, $\sum_i \mathbf{I}_i \neq 0$, the charge carrier spins feel the effective HF field $B$ which may cause spin splitting in 2DEG of the order of one tenth of the Fermi energy $E_F$. The influence of the Zeeman splitting solely on a supercurrent was studied first in Ref. 5 for the SFS junction (F denotes ferromagnetic metal). It was shown that the spin splitting suppresses the critical current and produces its oscillations over the intrinsic magnetic field localized within the F-layer.

The SO interaction of a charge carrier with the interface potential in GaAs/AlGaAs heterojunctions is modeled by the Bychkov-Rashba term

$$ \hat{H}_{so} = (\alpha/\hbar) [\mathbf{\sigma} \mathbf{p}] \mathbf{n}, \quad (3) $$

where $\alpha = 0.6 \times 10^{-9}$ eV cm for holes and $\alpha = 0.25 \times 10^{-9}$ eV cm for electrons, $\mathbf{p}$, $\mathbf{n}$ are the charge
carrier spin and momentum, \( n \) is the normal to the interface directed towards the Al-doped layer.

The configuration proposed in Fig. 1 has the following characteristic features:

i) the nuclear field \( B \) is localized outside the superconductors and does not influence the pairing mechanism;

ii) it affects only the electron spins and does not modify the space motion of charge carriers, whereas usual magnetic field causes strong orbital effects and transforms Andreev levels into Landau bands. This produces oscillations of the electron momentum. The spinor function \( u(x) \) obeys Eq. (11) with a reduced Hamiltonian, in which the quasi-classical approximation for the kinetic energy and SO operators is used,

\[
\gamma(h,w) = \arcsin \left[ \frac{\sum_{k=\pm 1} (1+kn_kn_{-k})}{2} \sin^2 \left( \frac{A_+ + kA_-}{2} \right) \right]^{1/2}
\]

which has the standard structure of the equation for Andreev levels in current-carrying SNS junction. An additional term \( \sigma \gamma \), where \( \sigma = \pm 1 \) indicates the spin direction, describes Zeeman splitting of the Andreev levels renormalized by the SO interaction. In terms of the BdG wave mechanics, the spin effects change phase relations between the wave functions of the incident and retro-reflected quasiparticles. This produces oscillations of the Andreev levels with the change of the interaction parameters \( h, w \) which enter the oscillating phase shift \( \sigma \gamma \) in Eq. (14). As a result, the Josephson current in Eq. (1) also reveals oscillations with \( s = \pm 1 \) indicating two signs of \( z \)-component of the electron momentum. The spinor function \( u(x) \) obeys Eq. (11) with a reduced Hamiltonian, in which the quasi-classical approximation for the kinetic energy and SO operators is used,

\[
\gamma(h,w) = \arcsin \left[ \frac{\sum_{k=\pm 1} (1+kn_kn_{-k})}{2} \sin^2 \left( \frac{A_+ + kA_-}{2} \right) \right]^{1/2}
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\]
the SO interaction \((\mathbf{w} = 0)\), \(E_n(\Phi)\) depends only on the modulus of \(\mathbf{h}\):

\[
\gamma(\mathbf{h}, 0) = \arcsin [\sin (hd/hv)]. \tag{17}
\]

It is helpful to consider this anisotropy for a single electron state with fixed direction of the SO vector \(\mathbf{w}\). As follows from Eqs. (14)-(16), the energy of Andreev level depends only on the angle between \(\mathbf{h}\) and \(\mathbf{w}\) and their moduli, being insensitive to the rotation of \(\mathbf{h}\) around \(\mathbf{w}\). At \(\mathbf{h} \parallel \mathbf{w}\), the effects of SO interaction vanish, as in Eq. (17). These conclusions can be extended to the angle dependence of the Josephson current in Eq. (1) in a narrow 2DEG channel which holds a single electron mode \((\rho = p_F, p_y = 0)\). In this extreme case, the vectors \(\mathbf{w}\) of all electrons are directed along the \(y\)-axis and create a fixed reference frame for the Zeeman vector \(\mathbf{h}\).

At arbitrary length of the 2DEG region, Eq. (14) can be solved only numerically. Below we consider an analytically solvable case of 2DEG channel much shorter than the coherence length \(\xi_0\), when the left-hand side of Eq. (14) is negligibly small,

\[
E_n(\Phi) = s\Delta \cos (\Phi/2 + \gamma), \quad (s, \sigma) = \pm 1. \tag{18}
\]

At \(\gamma = 0\), Eq. (18) describes two spin-degenerate Andreev levels in a superconducting constriction \(\mathbf{H}_c\) which traverse across the whole energy gap with the change of \(\Phi\) and intersect each other at \(\Phi = \pi\). The spin effects split each level into two spin-dependent terms and, in addition, expand them into four energy bands which transfer the Josephson current

\[
I(\Phi) = \frac{e\Delta}{\hbar} \int_{-p_F}^{p_F} \frac{dp_y}{2\pi \hbar} \sum_{\sigma = \pm 1} \tanh \left[ \frac{\Delta}{2T} \cos \left( \frac{\Phi}{2} + \gamma \right) \right] \times \sin \left( \frac{\Phi}{2} + \gamma \right). \tag{19}
\]

At \(h = 0\), Eq. (19) is reduced to a 2D analogue of the current-phase relationship \(I(\Phi)\) for pure constriction.

The set of curves \(I(\Phi)\) calculated numerically at \(T = 0\) for various directions and magnitudes of Zeeman field combined with SO interaction is presented in Fig. 2b-d in dimensionless variables

\[
\mathbf{H} = (hv_F/d)\mathbf{h}, \quad \mathbf{W} = (hv_F/d)\mathbf{w}. \tag{20}
\]

In comparison with those plotted for \(W = 0\) in Fig. 2a.

The common features of these dependencies are drastic variations of the shape of \(I(\Phi)\) at \(H \sim 1\), and the rapid change of sign of the derivative \(dI/d\Phi\) at \(\Phi = \pi\) in small field \(H\). It is interesting that the curves at \(W = 0\) and at \(W = 1\), \(\mathbf{H} \parallel \mathbf{y}\) (Fig. 2a,b) are similar each to other, as well as the curves for \(W = 1\), \(\mathbf{H} \parallel \mathbf{x}\) and \(W = 1\), \(\mathbf{H} \parallel \mathbf{z}\) (Fig. 2c,d). This reflects the results of our analysis of the anisotropy of Andreev levels in 1D case which appears to be qualitatively applicable for 2D system: the SO effects are relatively small at \(\mathbf{H} \parallel \mathbf{y}\) and approximately isotropic under rotation of Zeeman field around the \(y\)-axis.

In Fig. 3 we present oscillations of the critical current \(I_c(H, T) = I_c(0, 0)\) on the dimensionless magnetic field \(H\), at \(W = 1\), \(\mathbf{H} \parallel \mathbf{x}, \mathbf{y}, \mathbf{z}\) (solid curves) and \(W = 0\), \(\mathbf{H}\) directed arbitrarily (dashed curves). Upper pairs of curves were calculated for \(T = 0\), lower pairs for \(T = 0.9T_c\).

The positive sign of \(dI_c/d\Phi\) at \(\Phi = \pi\), which occurs at \(H \neq 0\) (Fig. 2), means that this state can be stable.
and may produce persistent current in the ground state of a superconducting loop with high enough inductance (junction, [21]). On the other hand, the negative sign of \( dI/d\Phi(0) < 0 \), which occurs within the certain field range at \( W = 0 \) or at \( W \neq 0, H \parallel y \), signifies instability of usual ground state with \( \Phi = 0 \) (note that the SO interaction stabilizes this state at \( H \perp y \), at least at \( T = 0 \)). The results of a numerical analysis of stability of states \( \Phi = 0, \pi \) for various directions and magnitudes of the Zeeman field and intensities of the SO interaction are shown in Fig. 4.

In summary, we have shown that the Josephson current in a mesoscopic S-2DEG-S structure is highly sensitive to the combined action of the Zeeman field and spin-orbit interactions. In particular, the critical current reveals oscillations and anisotropy with respect to the Zeeman field \( B \), and the regions of stability at \( \Phi = \pi \) (like in junctions) emerge. We assumed hyperfine interaction of electrons with polarized nuclei as the source of electron spin polarization, though similar effects should be observed in external magnetic field lying in the 2DEG-plane (to avoid orbital effects). In order to access the regime of strong interaction \( (H \sim 1, W \sim 1) \) in short 2DEG bridge \((d < \xi_0)\) considered here, the interaction energies \( h, w \) of 2DEG should exceed \( \Delta \). Since the HF and SO interaction magnitudes in GaAs/AlGaAs heterojunctions reach at most 1 K in temperature scale, the banks of short S-2DEG-S structure should be preferably fabricated from a superconductor with low \( T_c \leq 1 \) K. This restriction can be significantly softened in long \((d \gg \xi_0)\) S-2DEG-S junctions where the interaction energies should be comparable with the small distance between Andreev levels: \( h, w \sim \hbar v_F/d \ll \Delta \).

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