


cp-violation in k, b and bs decays

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abstract

in this review we give an overview of cp-violation for k₀(\bar{k}₀), b₀^{q}(\bar{b}^q₀), q = d, s systems. direct cp-violation and mixing induced cp-violation are discussed.

1 introduction

symmetries have played an important role in particle physics. in quantum mechanics a symmetry is associated with a group of transformations under which a lagrangian remains invariant. symmetries limit the possible terms in a lagrangian and are associated with conservation laws. here we will be concerned with the role of discrete symmetries: space reflection (parity) p: \vec{x} \rightarrow -\vec{x}, time reversal t: t \rightarrow -t and charge conjugation c: particle \rightarrow antiparticle.

quantum electrodynamics (qed) and quantum chromodynamics (qcd) respect all these symmetries. also, all lorentz invariant local quantum field theories are cpt invariant. however, in weak interactions c and p are maximally violated separately but as we will see below, cp is conserved.

first indication of parity violation was revealed in the decay of a particle with spin parity j^p = 0^-, called k-meson into two modes k₀ \rightarrow \pi^+\pi^- (parity violating), and k₀ \rightarrow \pi^+\pi^-\pi^0 (parity conserving).

lee and yang in 1956, suggested that there is no experimental evidence for parity conservation in weak interaction. they suggested number of experiments to test the validity of space reflection invariance in weak decays. one way to test this is to measure the helicity of outgoing muon in the decay:

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]

the helicity of muon comes out to be negative, showing that parity conservation does not hold in this decay. in the rest frame of the pion, since \mu^+ comes out with negative helicity, the neutrino must also come out with negative helicity because of the spin conservation. thus confirming the fact that neutrino is left handed.

\[ \pi^+ \rightarrow \mu^+(-) + \nu_\mu \]

under charge conjugation,

\[ \pi^+ \xrightarrow{c} \pi^- \quad \mu^+ \xrightarrow{c} \mu^- \quad \nu_\mu \xrightarrow{c} \bar{\nu}_\mu \]

helicity \( h = \frac{\vec{p} \cdot \vec{q}}{|p|} \) under c and p transforms as,

1
\[ \mathcal{H} \overset{C}{\rightarrow} \mathcal{H}, \quad \mathcal{H} \overset{P}{\rightarrow} -\mathcal{H} \]

Invariance under \( C \) gives,

\[ \Gamma_{\pi^+\rightarrow\mu^+-\nu_\mu} = \Gamma_{\pi^-\rightarrow\mu^-+\bar{\nu}_\mu} \]

Experimentally,

\[ \Gamma_{\pi^+\rightarrow\mu^+-\nu_\mu} \gg \Gamma_{\pi^-\rightarrow\mu^-+\bar{\nu}_\mu} \]

showing that \( C \) is also violated in weak interactions. However, under \( CP \),

\[ \Gamma_{\pi^+\rightarrow\mu^+-\nu_\mu} \overset{CP}{\rightarrow} \Gamma_{\pi^-\rightarrow\mu^-+\bar{\nu}_\mu} \]

which is seen experimentally. Thus, \( CP \) conservation holds in weak interaction.

In the Standard Model, the fermions for each generation in their left handed chirality state belong to the representation,

\[ \left( \begin{array}{c} u_i \\ d_i \end{array} \right) : q(3, 2, 1/3) \]
\[ \bar{u}_i : (\bar{3}, 1, -4/3) \]
\[ \bar{d}_i : (3, 1, 2/3) \]

\[ \left( \begin{array}{c} \nu_{e^-} \\ e_i^+ \end{array} \right) : l(1, 2, -1/2) \]
\[ e_i^+ : (1, 1, 1) \]

of the electroweak unification group \( SU_C(3) \times SU_L(2) \times U_Y(1) \). Hence, the weak interaction Lagrangian for the charged current in the Standard Model is given by,

\[ \mathcal{L}_W = \bar{\psi}_i \gamma^\mu (1 - \gamma^5) \psi_j W^+_{\mu} + h.c. \]

where \( \psi_i \) is any of the left-handed doublet \( (i \) is the generation index). We note that the weak eigenstates \( d', s' \) and \( b' \) are not equal to the mass eigenstates \( d, s \) and \( b \). They are related to each other by a unitarity transformation,

\[ \left( \begin{array}{c} d' \\ s' \\ b' \end{array} \right) = V \left( \begin{array}{c} d \\ s \\ b \end{array} \right) \]  \( \text{(1)} \)

where \( V \) is called the \( CKM \) matrix.

\[ V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

\[ \simeq \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\ -\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\ A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1 \end{pmatrix} + O (\lambda^4) , \; \lambda = 0.22 \]  \( \text{(2)} \)

The unitarity of \( V, VV^\dagger = 1 \) gives,[Fig.1]
\[ V_{ud} V_{ub} + V_{cb} V_{cd} + V_{td} V_{tb} = 0 \]  

The second line in equation (2) expresses \( V \) in terms of Wolfenstein parametrization. Thus,

\[ \begin{align*}
V_{cb} &= A \lambda^2 \\
V_{ub} &= |V_{ub}| e^{-i \gamma} \\
V_{td} &= |V_{td}| e^{-i \beta}
\end{align*} \]

where,

\[ \tan \gamma = \frac{\eta}{\rho}, \quad \tan \beta = \frac{\bar{\eta}}{1 - \rho}, \quad \bar{\rho} = \rho (1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta (1 - \frac{\lambda^2}{2}). \]

In order to show that \( \mathcal{L}_W \) is \( CP \)-invariant, we first note that under \( C, P \) and \( CP \) operations the Dirac spinor \( \Psi \) transforms as follows:

\[ \begin{align*}
P \Psi (t, \vec{x}) P^{-1} &= \gamma^0 \Psi (t, -\vec{x}) \\
C \Psi (t, \vec{x}) C^{-1} &= -i \gamma^2 \gamma^0 \bar{\Psi}^T (t, \vec{x}) \\
T \Psi (t, \vec{x}) T^{-1} &= \gamma^1 \gamma^3 \Psi (-t, \vec{x})
\end{align*} \]

The effect of transformations \( C, P \) and \( CP \) on various quantities that appear in a gauge theory Lagrangian are given below:

| Transformation | Scalar | Pseudoscalar | Vector | Axial vector |
|---------------|--------|--------------|--------|-------------|
| \( P \)      | \bar{\Psi}_i \Psi_j | -i \bar{\Psi}_i \gamma_5 \Psi_j | \eta (\mu) \bar{\Psi}_i \gamma^\mu \Psi_j | -\eta (\mu) \bar{\Psi}_i \gamma^\mu \gamma^5 \Psi_j |
| \( C \)      | \bar{\Psi}_j \Psi_i | -i \bar{\Psi}_j \gamma_5 \Psi_i | -\bar{\Psi}_j \gamma^\mu \Psi_i | \bar{\Psi}_j \gamma^\mu \gamma^5 \Psi_i |
| \( CP \)     | \bar{\Psi}_j \Psi_i | -i \bar{\Psi}_j \gamma_5 \Psi_i | -\eta (\mu) \bar{\Psi}_j \gamma^\mu \Psi_i | -\eta (\mu) \bar{\Psi}_j \gamma^\mu \gamma^5 \Psi_i |

The vector bosons associated with the electroweak unification group \( SU_L (2) \times U (1) \) transform under \( CP \) as:

\[ \begin{align*}
W_\mu^\pm (\vec{x}, t) &\xrightarrow{CP} -\eta (\mu) W_\mu^\pm (-\vec{x}, t) \\
Z_\mu (\vec{x}, t) &\xrightarrow{CP} -\eta (\mu) Z_\mu (-\vec{x}, t) \\
A_\mu (\vec{x}, t) &\xrightarrow{CP} -\eta (\mu) A_\mu (-\vec{x}, t)
\end{align*} \]

where,

\[ \eta (\mu) = \begin{cases} +1, & \text{if } \mu = 0 \\ -1, & \text{if } \mu = 1, 2, 3 \end{cases} \]

The Lagrangian transforms as:

\[ \mathcal{L}_W \xrightarrow{CP} -\eta (\mu) \bar{\psi}_j \gamma^\mu (1 - \gamma^5) \psi_i W_\mu^+ + h.c. \]

Thus, the weak interaction Lagrangian in the Standard Model violates \( C \) and \( P \) but is \( CP \)-invariant.
It is instructive to discuss the restrictions imposed by CPT invariance. CPT invariance implies,

\[
\text{out} \langle f | \mathcal{L} | X \rangle = \text{out} \langle f | (\text{CPT})^{-1} \mathcal{L} \text{CPT} | X \rangle = \eta_T^* \eta_f \text{in} \langle f | (\text{CPT})\dagger \mathcal{L} (\text{CPT})^{-1}\dagger | X \rangle^* = \eta_T^* \eta_f \langle X | (\text{CPT})^{-1} \mathcal{L} (\text{CPT}) | f \rangle \text{in} = -\eta_T^* \eta_f \eta_{CP} \langle X | \mathcal{L} S_f | \bar{f} \rangle \text{out} = \eta_f \text{out} \langle \bar{f} | \mathcal{L}^\dagger | \bar{X} \rangle^* = \eta_f \exp(2i\delta_f) \text{out} \langle f | \mathcal{L} | X \rangle^* \tag{6}
\]

Hence, we get:

\[
\text{out} \langle \bar{f} | \mathcal{L} | \bar{X} \rangle = \eta_f \exp(2i\delta_f) \text{out} \langle f | \mathcal{L} | X \rangle^*
\]

\[
\bar{A}_f = \eta_f \exp(2i\delta_f) A_f \tag{7}
\]

In deriving the above result, we have put \( \tilde{f} = f \) where \( \tilde{f} \) means that momenta and spin are reversed. Since we are in the rest frame of \( X \), \( T \) will reverse only magnetic quantum number and we can drop \( \tilde{f} \). Further we have used,

\[
\text{CP} | X \rangle = - | \bar{X} \rangle \tag{8}
\]

\[
\text{CP} | f \rangle = \eta_f^{CP} | \tilde{f} \rangle \tag{9}
\]

\[
| f \rangle \text{in} = S_f | f \rangle \text{out} = \exp(2i\delta_f) | f \rangle \text{in} \tag{10}
\]

where \( \delta_f \) is the strong interaction phase shift. If CP-invariance holds, then,

\[
\text{out} \langle f | \mathcal{L} | X \rangle = \text{out} \langle \bar{f} | \mathcal{L} | \bar{X} \rangle \Rightarrow \bar{A}_f = A_f.
\]

Thus, the necessary condition for CP-violation is that the decay amplitude \( A \) should be complex. In view of our discussion above, under CP an operator \( O(\bar{x}, t) \) is replaced by,

\[
O(\bar{x}, t) \rightarrow O^\dagger (\bar{x}, t) \tag{11}
\]

The effective Lagrangian has the structure (\( L^\dagger = L \)),

\[
L = aO + a^* O^\dagger \tag{12}
\]

Hence, CP-violation requires \( a^* \neq a \). We now discuss the implication of CPT constraint with respect to CP violation of weak decays. The weak amplitude is complex; it contains the final state strong phase \( \delta_f \) and in addition it may also contain a weak phase \( \phi \). Taking out both these phases,

\[
A_f = \exp(i\phi) F_f = \exp(i\delta_f) \exp(i\phi) F_f | F_f | \nonumber
\]

CPT (Eq. (7)) gives,

\[
\bar{A}_f = \exp(2i\delta_f) \exp(-i\phi) \exp(-i\delta_f) | F_f | = \exp(-i\phi) F_f
\]
We conclude that the weak interaction Lagrangian in the Standard Model is \( CP \) invariant and since \( CP \) violation has been observed in hadronic sector (only in \( B, B_s \) and \( K \) decays) and not in leptonic sector, it is a consequence of mismatch between weak and mass eigenstates (i.e. the phases in \( CKM \) matrix) and/or the mismatch between \( CP \)-eigenstates,

\[
|X_{1,2}^0\rangle = \frac{1}{\sqrt{2}} \left[ |X^0\rangle \mp |\bar{X}^0\rangle \right]; \quad CP |X_{1,2}^0\rangle = \pm |X_{1,2}^0\rangle \quad (13)
\]

and the mass eigenstates i.e. \( CP \)-violation in the mass matrix. \( CP \)-violation due to mass mixing and in the decay amplitude has been experimentally observed in \( K^0 \) and \( B_d^0 \). For \( B_s \) decays, the \( CP \)-violation in the mass matrix is not expected in the Standard Model. In fact time dependent \( CP \)-violation asymmetry gives a clear way to observe direct \( CP \)-violation in \( B \) and \( B_s \) decays.

If \( CP \) is conserved,

\[
\langle X_2 | H | X_1 \rangle = \langle X_2 | (CP)^{-1} H (CP) | X_1 \rangle = -\langle X_2 | H | X_1 \rangle
\]

then,

\[
\langle X_2 | H | X_1 \rangle = 0.
\]

Thus \( |X_1\rangle \) and \( |X_2\rangle \) are also mass eigenstates. They form a complete set (in units \( \hbar = c = 1 \)),

\[
|\psi(t)\rangle = a(t) |X_1\rangle + b(t) |X_2\rangle
\]

\[
\frac{id|\psi(t)\rangle}{dt} = \begin{pmatrix} m_1 - \frac{i}{2} \Gamma_1 & 0 \\ 0 & m_2 - \frac{i}{2} \Gamma_2 \end{pmatrix} |\psi(t)\rangle. \quad (14)
\]

The solution is,

\[
a(t) = a(0) \exp \left(-im_1 t - \frac{1}{2} \Gamma_1 t\right)
\]

\[
b(t) = b(0) \exp \left(-im_2 t - \frac{1}{2} \Gamma_2 t\right)
\]

Suppose we start with the state \( |X^0\rangle \), i.e.,

\[
|\psi(0)\rangle = |X^0\rangle
\]
Then we get,
\[
\langle \psi(t) \rangle = \frac{1}{\sqrt{2}} \left[ \exp \left( -i m_1 t - \frac{1}{2} \Gamma_1 t \right) |X_1\rangle 
+ \exp \left( -i m_2 t - \frac{1}{2} \Gamma_2 t \right) |X_2\rangle \right]
\]
\[
= \frac{1}{\sqrt{2}} \left\{ \left[ \exp \left( -i m_1 t - \frac{1}{2} \Gamma_1 t \right) 
+ \exp \left( -i m_2 t - \frac{1}{2} \Gamma_2 t \right) \right] |X^0\rangle 
- \left[ \exp \left( -i m_1 t - \frac{1}{2} \Gamma_1 t \right) 
- \exp \left( -i m_2 t - \frac{1}{2} \Gamma_2 t \right) \right] |\bar{X}^0\rangle \right\}
\]
\]

(15)

However, in \(|X^0\rangle - |\bar{X}^0\rangle\) basis,
\[
\langle \psi(t) \rangle = a(t) |X^0\rangle + \bar{a}(t) |\bar{X}^0\rangle
\]
\[
\frac{d}{dt} \langle \psi(t) \rangle = M \langle \psi(t) \rangle
\]

the mass matrix \(M\) is not diagonal and is given by,
\[
M = m - \frac{i}{2} \Gamma = \left( \begin{array}{cc}
 m_{11} - \frac{i}{2} \Gamma_{11} & m_{12} - \frac{i}{2} \Gamma_{12} \\
 m_{21} - \frac{i}{2} \Gamma_{21} & m_{22} - \frac{i}{2} \Gamma_{22}
\end{array} \right)
\]

(16)

Hermiticity of matrices \(m_{\alpha\alpha'}\) and \(\Gamma_{\alpha\alpha'}\) gives \((\alpha = \alpha' = 1, 2)\),
\[
(m)_{\alpha\alpha'} = (m^\dagger)_{\alpha'\alpha}, \quad \Gamma_{\alpha\alpha'} = \Gamma^*_{\alpha'\alpha}
\]
\[
m_{21} = m^*_{12}, \quad \Gamma_{21} = \Gamma^*_{12}
\]

(17)

\(CPT\) invariance gives,
\[
\langle X^0 | M | X^0 \rangle = \langle \bar{X}^0 | M | \bar{X}^0 \rangle
\]
\[
m_{11} = m_{22}, \quad \Gamma_{11} = \Gamma_{22}
\]
\[
\langle \bar{X}^0 | M | X^0 \rangle = \langle X^0 | M | X^0 \rangle : \text{identity}
\]

(18)

Diagonalization of mass matrix \(M\) in eq. (16) gives,
\[
m_{11} - \frac{i}{2} \Gamma_{11} - pq = m_{1} - \frac{i}{2} \Gamma_{1}
\]
\[
m_{11} - \frac{i}{2} \Gamma_{11} + pq = m_{2} - \frac{i}{2} \Gamma_{2}
\]

(19)

where,
\[
p^2 = m_{12} - \frac{i}{2} \Gamma_{12}, \quad q^2 = m^*_{12} - \frac{i}{2} \Gamma^*_{12}
\]

(20)

The eigenstates are given by,
\[
|X_{1,2}\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left[ p|X^0\rangle \mp q|\bar{X}^0\rangle \right]
\]
2 $K^0 - \bar{K}^0$ Complex and $CP$–Violation in $K$-Decay

Consider the process,

\[ K^0 \rightarrow \pi^+ \pi^- \rightarrow \bar{K}^0, \quad |\Delta Y| = 2 \]

Thus, weak interaction can mix $K^0$ and $\bar{K}^0$,

\[ \langle K^0 | H | \bar{K}^0 \rangle \neq 0. \]

Off diagonal matrix elements are not zero. Thus, $K^0$ and $\bar{K}^0$ cannot be mass eigenstates. Select the phase:

\[ CP |K^0\rangle = -|\bar{K}^0\rangle. \]

Define,

\[ |K^0_1\rangle = \frac{1}{\sqrt{2}} \left[ |K^0\rangle - |\bar{K}^0\rangle \right] \]
\[ |K^0_2\rangle = \frac{1}{\sqrt{2}} \left[ |K^0\rangle + |\bar{K}^0\rangle \right] \]

Choose:

\[ CP |K^0_1\rangle = +|K^0_1\rangle \quad CP |K^0_2\rangle = -|K^0_2\rangle \]

where $K^0_1$ and $K^0_2$ are eigenstates of $CP$ with eigenvalues +1 and −1. Assuming $CP$ conservation,

\[ \langle \bar{K}^0 | M | K^0 \rangle = \langle K^0 | M | \bar{K}^0 \rangle \]

\[ m_{21} = m_{12} \quad \Gamma_{21} = \Gamma_{12} \]

where $m_{12}$ and $\Gamma_{12}$ are real. Thus,

\[ pq = m_{12} - \frac{i}{2} \Gamma_{12} \]
\[ m_1 = m_{11} - m_{12}, \quad \Gamma_1 = \Gamma_{11} - \Gamma_{12} \]
\[ m_2 = m_{11} + m_{12}, \quad \Gamma_2 = \Gamma_{11} + \Gamma_{12} \]
\[ \Delta m = m_2 - m_1 = 2m_{12}, \quad \Delta \Gamma = \Gamma_2 - \Gamma_1 = 2\Gamma_{12} \]

Since,

\[ CP \left( \pi^+ \pi^- \right) = (-1)^f (-1)^f = 1 \]

therefore, it is clear that,

\[ K^0_1 \rightarrow \pi^+ \pi^- \]

is allowed by $CP$ conservation.

However, experimentally it was found that long lived $K^0_2$ also decay to $\pi^+ \pi^-$ but with very small probability. Small $CP$ non conservation can be taken into account by defining,

\[ |K_S\rangle = |K^0_1\rangle + \varepsilon |K^0_2\rangle \]
\[ |K_L\rangle = |K^0_2\rangle + \varepsilon |K^0_1\rangle \]

(24)
where $\varepsilon$ is a small number. Thus $CP$ non conservation manifests itself by the ratio:

$$\eta_{+-} = \frac{A(K_L \to \pi^+ \pi^-)}{A(K_S \to \pi^+ \pi^-)} = \varepsilon$$

(25)

$$|\eta_{+-}| \simeq (2.286 \pm 0.017) \times 10^{-3}$$

Now $CP$ non conservation implies,

$$m_{12} \neq m_{12}^*, \quad \Gamma_{12} \neq \Gamma_{12}^*.$$  

(26)

Since $CP$ violation is a small effect, therefore,

$$\text{Im} m_{12} \ll \text{Re} m_{12} \quad \text{Im} \Gamma_{12} \ll \text{Re} \Gamma_{12}.$$  

(27)

Further, if $CP$- violation arises from mass matrix, then,

$$\Gamma_{12} = \Gamma_{12}^*.$$  

(28)

Thus, $CP$–violation can result by a small term $i \text{Im} m_{12}$ in the mass matrix given in Eq. [14],

$$M = \begin{pmatrix}
  m_1 - \frac{i}{2} \Gamma_1 & i \text{Im} m_{12} \\
  -i \text{Im} m_{12} & m_2 - \frac{i}{2} \Gamma_2
\end{pmatrix}.$$  

(29)

Diagonalization gives,

$$\varepsilon = \frac{i \text{Im} m_{12}}{(m_2 - m_1) - i (\Gamma_2 - \Gamma_1)/2}.$$  

(30)

Then from Eq. [23] up to first order, we get,

$$\Delta m = m_2 - m_1 \rightarrow m_{KL} - m_{KS}$$

$$= 2 \text{Re} m_{12}$$

$$\Delta \Gamma = \Gamma_2 - \Gamma_1 = \Gamma_L - \Gamma_S = 2 \Gamma_{12}$$

(31)

Eq. (15) is unchanged, replace,

$$m_1 \rightarrow m_S, \quad m_2 \rightarrow m_L$$

$$\Gamma_1 \rightarrow \Gamma_S, \quad \Gamma_2 \rightarrow \Gamma_L$$

Now,

$$\Delta m = m_L - m_S$$

$$\Delta \Gamma = \Gamma_L - \Gamma_S$$

$$\Gamma_S = \frac{\hbar}{\tau_S} = 7.367 \times 10^{-12} \text{ MeV},$$

$$\tau_S = (0.8935 \pm 0.0008) \times 10^{-10} \text{ s}$$

$$\Gamma_L = \frac{\hbar}{\tau_L} = 1.273 \times 10^{-14} \text{ MeV},$$

$$\tau_L = (5.17 \pm 0.04) \times 10^{-8} \text{ s}$$

$$\Delta \Gamma \simeq -\Gamma_S$$

$$m_L = m + \frac{1}{2} \Delta m$$

$$m_S = m - \frac{1}{2} \Delta m.$$  

(32)
Hence from Eq. (15),

\[
|\psi(t)\rangle \approx e^{-\frac{i}{2}mt} \left\{ e^{\frac{i}{2}t \Gamma_S} e^{\Delta \rho} + e^{-\frac{i}{2}\Delta \rho} \right\} |K^0\rangle
\]

(33)

Therefore, probability of finding $\bar{K}^0$ at time $t$ (recall that we started with $K^0$),

\[
P(K^0 \to \bar{K}^0, t) = \left| \langle \bar{K}^0 | \psi(t) \rangle \right|^2
\]

\[
= \frac{1}{4} \left( 1 + e^{-\Gamma_S t} - 2e^{-\frac{1}{2}\Gamma_S t} \cos(\Delta m t) \right)
\]

\[
= \frac{1}{4} \left( 1 + e^{-t/\tau_S} - 2e^{-\frac{1}{2}t/\tau_S} \cos(\Delta m t) \right)
\]

(34)

If kaons were stable ($\tau_S \to \infty$), then,

\[
P(K^0 \to \bar{K}^0, t) = \frac{1}{2} \left[ 1 - \cos(\Delta m t) \right]
\]

(35)

which shows that a state produced as pure $Y = 1$ state at $t = 0$ continuously oscillates between $Y = 1$ and $Y = -1$ state with frequency $\omega = \frac{\Delta m}{\hbar}$ and period of oscillation,

\[
\tau = \frac{2\pi}{(\Delta m/\hbar)}.
\]

(36)

Kaons, however, decay and their oscillations are damped.

By measuring the period of oscillation, $\Delta m$ can be determined.

\[
\Delta m = m_L - m_S = (3.489 \pm 0.008) \times 10^{-12} \text{ MeV}.
\]

(37)

Such a small number is measured as a consequence of superposition principle in quantum mechanics,

\[
\pi^- p \rightarrow K^0 \Lambda^0
\]

\[
\bar{K}^0 p \rightarrow \pi^+ \Lambda^0
\]

$\pi^+$ can only be produced by $\bar{K}^0$ in the final state. This would give a clear indication of oscillation.

Coming back to $CP$-violation,

\[
\varepsilon = i \frac{\text{Im} m_{12}}{\Delta m - i \Delta \Gamma/2}
\]

\[
\varepsilon = |\varepsilon| e^{i\phi_\varepsilon}
\]

(38)

\[
\tan \phi_\varepsilon = -2\frac{\Delta m}{\Delta \Gamma} = \frac{\Delta m}{\Gamma_S - \Gamma_L}
\]

\[
\approx \frac{2 \times 0.474 \Gamma_S}{0.998 \Gamma_S}
\]

\[
\Rightarrow \phi_\varepsilon = 43.59 \pm 0.05^0
\]

(39)

\[
|\varepsilon| = (2.229 \pm 0.012) \times 10^{-3}
\]

(40)

So far we have considered $CP$-violation due to mixing in the mass matrix. It is important to detect the $CP$-violation in the decay amplitude if any. This is done by looking for a difference
between $CP$-violation for the final $\pi^0\pi^0$ state and that for $\pi^+\pi^-$. Now due to Bose statistics, the two pions can be either in $I = 0$ or $I = 2$ states. Using Clebsch-Gordon (CG) coefficients,

$$A (K^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{3}} \left[ \sqrt{2} A_0 e^{i\delta_0} + A_2 e^{i\delta_2} \right]$$

$$A (K^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{3}} \left[ A_0 e^{i\delta_0} - \sqrt{2} A_2 e^{i\delta_2} \right]$$

(41)

Now $CPT$-invariance gives,

$$A (\bar{K}^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{3}} \left[ \sqrt{2} A_0^* e^{i\delta_0} + A_2^* e^{i\delta_2} \right]$$

$$A (\bar{K}^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{3}} \left[ A_0^* e^{i\delta_0} - \sqrt{2} A_2^* e^{i\delta_2} \right]$$

(42)

The dominant decay amplitude is $A_0$ due to $\Delta I = 1/2$ rule, $|A_2/A_0| \simeq 1/22$. Using the Wu–Yang phase convention, we can take $A_0$ to be real. Neglecting terms of order $\varepsilon \Re A_2/A_0$ and $\varepsilon \Im A_2/A_0$, we get,

$$\eta_{+-} \equiv |\eta_{+-}| e^{i\phi_{+-}} \simeq \varepsilon + \varepsilon'$$

$$\eta_{00} \equiv |\eta_{00}| e^{i\phi_{00}} \simeq \varepsilon - 2\varepsilon'$$

(43)

where,

$$\varepsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \Im \frac{A_2}{A_0}$$

(44)

Clearly $\varepsilon'$ measures the $CP$-violation in the decay amplitude, since $CP$-invariance implies $A_2$ to be real.

After 35 years of experiments at Fermilab and CERN, results have converged on a definitive non-zero result for $\varepsilon'$,

$$R = \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = \left| \frac{\varepsilon - 2\varepsilon'}{\varepsilon + \varepsilon'} \right|^2, \quad \varepsilon' \ll \varepsilon$$

$$\simeq \left| 1 - \frac{3\varepsilon'}{\varepsilon} \right|^2 \simeq 1 - 6 \Re (\varepsilon'/\varepsilon)$$

$$\Re (\varepsilon'/\varepsilon) = \frac{1 - R}{6} \approx 1.65 \pm 0.26 \times 10^{-3}.$$  

(45)

This is an evidence that although $\varepsilon'$ is a very small, but $CP$-violation does occur in the decay amplitude. Further we note from Eq. (44),

$$\phi_{\varepsilon'} = \delta_2 - \delta_0 + \frac{\pi}{2} \approx 42.3 \pm 1.5^0$$

where numerical value is based on an analysis of $\pi\pi$ scattering.
We now discuss the CP-asymmetry in leptonic decays of kaon.

\[
\frac{\Delta S}{\Delta Q} = 1 \\
K^+ \rightarrow \pi^0 + l^+ + \nu_l \\
K^0 \rightarrow \pi^- + l^+ + \nu_l = f \\
\bar{K}^0 \rightarrow \pi^+ + l^- + \bar{\nu}_l = f^* \text{ CPT} \\
\frac{\Delta S}{\Delta Q} = -1 \\
K^0 \rightarrow \pi^+ + l^- + \nu_l = g^* \\
\bar{K}^0 \rightarrow \pi^- + l^+ + \nu_l = g \text{ CPT}
\]

\[
A(K^0_L \rightarrow \pi^- + l^+ + \nu_l) = \frac{1}{\sqrt{2}} \left[ (1 + \epsilon)f + (1 - \epsilon)g \right] \\
A(K^0_L \rightarrow \pi^+ + l^- + \bar{\nu}_l) = \frac{1}{\sqrt{2}} \left[ (1 + \epsilon)g^* + (1 - \epsilon)f^* \right]
\]

The CP-asymmetry parameter \(\delta_l\):

\[
\delta_l = \frac{\Gamma(K^0_L \rightarrow \pi^- l^+ \nu_l) - \Gamma(K^0_L \rightarrow \pi^+ l^- \bar{\nu}_l)}{\Gamma(K^0_L \rightarrow \pi^+ l^- + \nu_l) + \Gamma(K^0_L \rightarrow \pi^- l^+ + \bar{\nu}_l)} = \frac{2 \text{Re}\epsilon[|f|^2 - |g|^2]}{|f|^2 + |g|^2 + (fg^* + f^*g) + O(\epsilon^2)}
\]

In the standard model \(\frac{\Delta S}{\Delta Q} = -1\) transitions are not allowed, thus \(g = 0\). Hence

\[
\delta_l \approx 2 \text{Re}\epsilon = (3.32 \pm 0.06) \times 10^{-3} \text{[Expt. value]}
\]

From Eq. (40), we get

\[
2 \text{Re}\epsilon = 2 |\epsilon| \cos \phi_\epsilon
\]

which gives on using experimental values for \(|\epsilon|\) and \(\phi_\epsilon\)

\[
2 \text{Re}\epsilon = (3.23 \pm 0.02) \times 10^{-3}
\]

in agreement with the experimental value for \(\delta_l\)

Finally we discuss CP-asymmetries for \(K \rightarrow 3\pi\) decays. The decays

\[
K^+ \rightarrow \pi^+ \pi^0 \pi^0, \pi^+ \pi^+ \pi^- \\
K^0 \rightarrow \pi^+ \pi^- \pi^0, \pi^0 \pi^0 \pi^0
\]

are parity conserving decays i.e. the parity of the final state is \(-1\). Now the C-parity of \(\pi^0\) and \((\pi^+ \pi^-)_l\) are given by

\[
C(\pi^0) = 1, \ C(\pi^+ \pi^-) = (-1)^l
\]

and G-parity of pion is \(-1\). Thus

\[
CP|\pi^0 \pi^0 \pi^0 > = -|\pi^0 \pi^0 \pi^0 > \\
CP|\pi^+ \pi^- \pi^0 > = (-1)^l|\pi^+ \pi^- \pi^0 >
\]
Hence CP-conservation implies
\[ K_2^0 \rightarrow \pi^0\pi^0\pi^0 \text{ allowed.} \]
\[ K_1^0 \rightarrow \pi^0\pi^0\pi^0 \text{ is forbidden.} \]
\[ K_1^0 \rightarrow \pi^+\pi^-\pi^0 \text{ allowed if } l_1 \text{ is odd.} \]
\[ K_2^0 \rightarrow \pi^+\pi^-\pi^0 \text{ allowed if } l_1 \text{ is even.} \]

Now G-parity of three pions $\pi^+\pi^-\pi^0$:
\[ G = C(-1)^I = (-1)^{l'} + I = -1 \]
Hence $l' = \text{even, } I(\text{odd}); I = 1, 3$
\[ l' = \text{odd, } I(\text{even}); I = 0, 2 \]

Only $l' = 0$ decays are favored as the decays for $l' > 0$ are highly suppressed due to centrifugal barrier. Hence $K_1^0 \rightarrow \pi^+\pi^-\pi^0$ is highly suppressed. Thus we have to take into account $I = 1, 3$ contributions as it requires $\Delta I = \frac{5}{2}$ transition.

Hence CP-asymmetries of $K^0 \rightarrow 3\pi$ decays are given by
\[ \eta_{000} = \frac{A(K_s \rightarrow \pi^0\pi^0\pi^0)}{A(K_L \rightarrow \pi^0\pi^0\pi^0)} = \frac{[i\text{Im}a_1 + \epsilon\text{Re}a_1]}{\text{Re}a_1 + i\text{Im}a_1} \approx \epsilon + i\frac{\text{Im}a_1}{\text{Re}a_1} \]
\[ \eta_{+0} = \frac{A(K_s \rightarrow \pi^+\pi^-\pi^0)}{A(K_L \rightarrow \pi^+\pi^-\pi^0)} \approx \epsilon + i\frac{\text{Im}a_1}{\text{Re}a_1} = \eta_{000} \]

3 $B^0 - \bar{B}^0$ Complex

For $B_q^0$ (q=d or s) we show below that both $m_{12}$ and $\Gamma_{12}$ have the same phase. Thus,
\[ m_{12} = |m_{12}| e^{-2i\phi_M} \]
\[ \Gamma_{12} = |\Gamma_{12}| e^{-2i\phi_M} \]

\[ |\Gamma_{12}| \ll |m_{12}| \]
\[ p^2 = e^{-2i\phi_M} [m_{12} - i |\Gamma_{12}|] \approx |m_{12}| e^{-2i\phi_M} \]
\[ q^2 = e^{2i\phi_M} [m_{12} - i |\Gamma_{12}|] \approx |m_{12}| e^{2i\phi_M} \]

\[ 2pq = 2|m_{12}| = (m_2 - m_1) - \frac{i}{2} (\Gamma_2 - \Gamma_1) \]
\[ \Rightarrow \Delta m_B = \frac{(m_2 - m_1)}{2} = |m_{12}| \]
\[ \Delta \Gamma = \Gamma_2 - \Gamma_1 = 0 \]

The above equations follow from the fact that,
\[ m_{12} - i\Gamma_{12} = \langle B_q^0 | H_{\text{eff}}^{\Delta B=2} | B_q^0 \rangle \]

$H_{\text{eff}}^{\Delta B=2}$ induces particle-antiparticle transition. For $\Delta m_{12}$, $H_{\text{eff}}^{\Delta B=2}$ arises from the box diagram as shown in Fig. 2, where the dominant contribution comes out from the $t$-quark. Thus,
\[ m_{12} \propto (V_{tb})^2 (V_{tb}^*)^2 m_t^2 \]
Now,
\[ \Gamma_{12} \propto \sum_f \langle \bar{B}_0^f | H_W | f \rangle \langle f | H_W | B_0^0 \rangle \]
where the sum is over all the final states which contribute to both \( B^0_q \) and \( \bar{B}_q^0 \) decays. Thus,
\[ \Gamma_{12} \propto (V_{cb}V^{*}_{cq} + V_{ub}V^{*}_{uq})^2 m_b^2 \propto (V_{tb})^2 (V^{*}_{tb})^2 m_b^2 \]
Hence we have the result that,
\[ \frac{\vert \Gamma_{12} \vert}{m_{12}} \sim \frac{m_b^2}{m_t^2} \]
Now \( B^0_d \to \bar{B}_d^0 \) transition:
\[ (V_{tb})^2 (V^{*}_{tb})^2 = A^2 \lambda^6 \left( (1 + \rho)^2 + \eta^2 \right) e^{2i\beta} \]
Hence,
\[ m_{12} = \vert m_{12} \vert e^{2i\beta}, \quad \Gamma_{12} = \vert \Gamma_{12} \vert e^{2i\beta}, \quad \phi_M = -\beta \]
On the other hand, \( B^0_s \to \bar{B}_s^0 \) transition:
\[ (V_{tb})^2 (V^{*}_{tb})^2 = |V_{ts}|^2 \approx A^2 \lambda^4 \]
\[ m_{12} = \vert m_{12} \vert, \quad \Gamma_{12} = \vert \Gamma_{12} \vert \]
\[ \phi_M = 0 \]
Also we have,
\[ \frac{\Delta m_{B_s}}{\Delta m_{B_d}} = \frac{\vert m_{12} \vert_s}{\vert m_{12} \vert_d} = \frac{1}{\lambda^2 \left[ (1 + \rho)^2 + \eta^2 \right]} \xi \]
where \( \xi \) is \( SU(3) \) breaking parameter.
Hence the mass eigenstates \( B^0_L \) and \( B^0_H \) can be written as:
\[ \begin{align*}
|B^0_L \rangle &= \frac{1}{\sqrt{2}} \left[ |B^0 \rangle - e^{2i\phi_M} |\bar{B}^0 \rangle \right] \quad CP = +1, \phi_M \to 0 \\
|B^0_H \rangle &= \frac{1}{\sqrt{2}} \left[ |B^0 \rangle + e^{2i\phi_M} |\bar{B}^0 \rangle \right] \quad CP = -1, \phi_M \to 0
\end{align*} \]
In this case, \( CP \) violation occurs due to phase factor \( e^{2i\phi_M} \) in the mass matrix.
Now one gets (from Eq. (15)), using Eqs.(49), (53) and (54),
\[ \begin{align*}
|B^0 (t) \rangle &= e^{-imt} e^{-i\frac{\Gamma}{2} t} \left\{ \cos \left( \frac{\Delta m}{2} t \right) |B^0 \rangle \\
&\quad -ie^{2i\phi_M} \sin \left( \frac{\Delta m}{2} t \right) |\bar{B}^0 \rangle \right\} \end{align*} \]
Similarly we get,

\[
|B^0(t)\rangle = -e^{-imt}e^{-\frac{\Gamma}{2}t}\left\{\cos\left(\frac{\Delta m}{2}t\right)|B^0\rangle -ie^{-2i\phi_M}\sin\left(\frac{\Delta m}{2}t\right)|B^0\rangle\right\}
\]

(56)

Suppose we start with \(B^0\) viz \(|B^0(0)\rangle = |B^0\rangle\), the probabilities of finding \(\bar{B}^0\) and \(B^0\) at time \(t\) is given by,

\[
P(B^0 \rightarrow \bar{B}^0, t) = |\langle \bar{B}^0 | B^0(t) \rangle|^2 = \frac{1}{2}e^{-\Gamma t} (1 - \cos(\Delta m) t)
\]

\[
P(B^0 \rightarrow B^0, t) = |\langle B^0 | B^0(t) \rangle|^2 = \frac{1}{2}e^{-\Gamma t} (1 + \cos(\Delta m) t)
\]

These are equations of a damped harmonic oscillator, the angular frequency of which is,

\[
\omega = \frac{\Delta m}{\hbar}
\]

We define the mixing parameter,

\[
r = \frac{\int_0^T |\langle \bar{B}^0 | B^0(t) \rangle|^2 dt}{\int_0^T |\langle B^0 | B^0(t) \rangle|^2 dt} = \frac{\chi}{1 - \chi} \xrightarrow{T \to \infty} \frac{(\Delta m/\Gamma)^2}{2 + (\Delta m/\Gamma)^2} = \frac{x^2}{2 + x^2}
\]

Experimentally, for \(B^0_d\) and \(B^0_s\),

\[
\Delta m_{B^0_d} = (0.507 \pm 0.005) \times 10^{-12}\text{hs}^{-1} = (3.337 \pm 0.033) \times 10^{-10}\text{MeV}
\]

\[
\Delta m_{B^0_s} = (17.77 \pm 0.10 \pm 0.007) \times 10^{-12}\text{hs}^{-1} = (1.17 \pm 0.01) \times 10^{-10}\text{MeV}
\]

\[
x_d = \left(\frac{\Delta m_{B^0_d}}{\Gamma_{B^0_d}}\right) = 0.77 \pm 0.008
\]

\[
x_s = \left(\frac{\Delta m_{B^0_s}}{\Gamma_{B^0_s}}\right) = 26.05 \pm 0.25
\]

From Eq. (55) and (56), the decay amplitudes for,

\[
B^0(t) \rightarrow f \quad A_f(t) = \langle f | H_w | B^0(t) \rangle
\]

\[
\bar{B}^0(t) \rightarrow \bar{f} \quad \bar{A}_f(t) = \langle \bar{f} | H_w | \bar{B}^0(t) \rangle
\]

(57)
are given by,

\[
A_f(t) = e^{-imt} e^{-i\frac{1}{2} \Gamma t} \left\{ \cos \left( \frac{\Delta m}{2} t \right) A_f - i e^{2i\phi_M} \sin \left( \frac{\Delta m}{2} t \right) \bar{A}_f \right\}
\]

(58)

\[
\bar{A}_f(t) = e^{-imt} e^{-i\frac{1}{2} \Gamma t} \left\{ \cos \left( \frac{\Delta m}{2} t \right) \bar{A}_f - i e^{-2i\phi_M} \sin \left( \frac{\Delta m}{2} t \right) A_f \right\}
\]

(59)

From Eqs. (58) and (59), we get for the decay rates,

\[
\Gamma_f(t) = e^{-\Gamma t} \left[ \frac{1}{2} \left( |A_f|^2 + |\bar{A}_f|^2 \right) + \frac{1}{2} \left( |A_f|^2 - |\bar{A}_f|^2 \right) \cos \Delta m t \right]
\]

(60)

\[
\bar{\Gamma}_f(t) = e^{-\Gamma t} \left[ \frac{1}{2} \left( |A_f|^2 + |\bar{A}_f|^2 \right) - \frac{1}{2} \left( |A_f|^2 - |\bar{A}_f|^2 \right) \cos \Delta m t \right]
\]

(61)

for \( \Gamma_f \) and \( \bar{\Gamma}_f \) change \( f \rightarrow \bar{f} \) and \( \bar{f} \rightarrow f \) in \( \Gamma_f \) and \( \bar{\Gamma}_f \) respectively.

As a simple application of the above equations, consider the semi-leptonic decays of \( B^0 \),

\[
B^0 \rightarrow l^+ \nu X^- : f \quad \text{for example } X^- = D^- \\
\bar{B}^0 \rightarrow l^- \bar{\nu} X^+ : \bar{f} \quad \text{for example } X^+ = D^+
\]

In the standard model, \( \bar{B}^0 \) decay into \( l^+ \nu X^- \) and \( B^0 \) decay into \( l^- \bar{\nu} X^+ \) is forbidden. Thus,

\[
\bar{A}_f = 0, \quad A_f = 0
\]

\[
\Gamma_f(t) = e^{-\Gamma t} \frac{1}{2} |A_f|^2 (1 + \cos \Delta m t)
\]

\[
\bar{\Gamma}_f(t) = e^{-\Gamma t} \frac{1}{2} |\bar{A}_f|^2 (1 - \cos \Delta m t), \quad : |\bar{A}_f| = |A_f|
\]

Hence,

\[
\delta = \frac{\int_0^\infty \Gamma_f(t) dt}{\int_0^\infty \bar{\Gamma}_f(t) dt} = \frac{x_d^2}{2 + x_d^2} = r_d
\]

Non zero value of \( \delta \) would indicate mixing. If, however, \( \bar{A}_f \neq 0 \) and \( A_f \neq 0 \) due to some exotic mechanism, then \( \delta \neq 0 \) even without mixing. Thus

\[
\frac{\Gamma(\mu^- X^+)}{\Gamma(\mu^+ X^-) + \Gamma(\mu^- X^+)} = \frac{r_d}{1 + r_d} = x_d
\]

\[
= 0.172 \pm 0.010 \quad \text{(Expt value)}
\]

which gives,

\[
x_d = 0.723 \pm 0.032
\]
4 CP-Violation in B-Decays

Case-I:

\[ |\bar{f}\rangle = CP |f\rangle = |f\rangle \]

For this case we get, from Eqs. (58) and (59),

\[ A_f(t) = \frac{\Gamma_f(t) - \bar{\Gamma}_f(t)}{\Gamma_f(t) + \bar{\Gamma}_f(t)} = \cos(\Delta mt) \left( |A_f|^2 - |\bar{A}_f|^2 \right) \]

\[ -i \sin(\Delta mt) \left( e^{2i\phi_M} A_f^\ast \bar{A}_f - e^{-2i\phi_M} A_f \bar{A}_f^\ast \right) / \left( |A_f|^2 + |\bar{A}_f|^2 \right) \]

\[ = \cos(\Delta mt) C_{\pi\pi} + \sin(\Delta mt) S_{\pi\pi} \]

where,

\[ C_{\pi\pi} = \frac{1 - |\bar{A}_f|^2 / |A_f|^2}{1 + |\bar{A}_f|^2 / |A_f|^2} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}, \quad \lambda = \frac{\bar{A}_f}{A_f} \]

This is the direct CP violation and,

\[ S_{\pi\pi} = \frac{2\text{Im} \left( e^{2i\phi_M} \lambda \right)}{1 + |\lambda|^2} \]

is the mixing induced CP-violation.

If the decay proceeds through a single diagram (for example tree graph), \( \bar{A}_f/A_f \) is given by (see Eqs. (15) and (16)),

\[ \lambda = \frac{\bar{A}_f}{A_f} = \frac{e^{i(\phi + \delta_f)}}{e^{i(-\phi + \delta_f)}} = e^{2i\phi} \]

where \( \phi \) is the weak phase in the decay amplitude. Hence from Eq. (62), we obtain,

\[ A_f(t) = \sin(\Delta mt) \sin(2\phi_M + 2\phi) \]

In particular for the decay,

\[ B^0 \to J/\psi K_s, \quad \phi = 0 \]

we obtain,

\[ A_{\psi K_s}(t) = \sin(2\phi_M) \sin(\Delta mt) = -\sin 2\beta \sin(\Delta mt) \]

and,

\[ A_{\psi K_s} = \frac{\int_0^\infty \left[ \Gamma_f(t) - \bar{\Gamma}_f(t) \right] dt}{\int_0^\infty \left[ \Gamma_f(t) + \bar{\Gamma}_f(t) \right] dt} \]

\[ = -\sin(2\beta) \frac{(\Delta m/\Gamma)}{1 + (\Delta m/\Gamma)^2} \]

Experiment : \( \left( \frac{\Delta m}{\Gamma} \right)_{B_d^0} = 0.776 \pm 0.008 \)

\( A_{\psi K_s} \) has been experimentally measured. It gives,

\[ \sin 2\beta = 0.678 \pm 0.025 \]
Corresponding to the decay $B^0 \rightarrow J/\psi K_s$, we have the decay $B_s^0 \rightarrow J/\psi \phi$. Thus for this decay

$$A_{J/\psi \phi}^{(t)} = -\sin 2\beta s \sin (\Delta m_{B^0} t)$$

$$A_{J/\psi \phi} = -\sin 2\beta s \frac{(\Delta m_{B^0}\Gamma s)}{1 + (\Delta m_{B^0}\Gamma s)^2}$$

In the standard model, $\beta s = 0$, $A_{J/\psi \phi} = 0$.

This is an example of $CP$-violation in the mass matrix. We now discuss the direct $CP$-violation. Direct $CP$-violation in $B$ decays involves the weak phase in the decay amplitude. The reason for this being that necessary condition for direct $CP$-violation is that decay amplitude should be complex as discussed in section 1. But this is not sufficient because in the limit of no final state interactions, the direct $CP$-violation in $B \rightarrow f$, $B \rightarrow \bar{f}$ decay vanishes. To illustrate this point, we discuss the decays $\bar{B}^0 \rightarrow \pi^+ \pi^-$. The main contribution to this decay is from tree graph (see Fig. 3); But this decay can also proceed via the penguin diagram (see Fig. 4).

The contribution of penguin diagram can be written as

$$P = V_{ub}V_{ud}^* f (u) + V_{cb}V_{cd}^* f (c) + V_{tb}V_{td}^* f (t)$$

(68)

where $f (u)$, $f (c)$ and $f (d)$ denote the contributions of $u$, $c$ and $t$ quarks in the loop. Now using the unitarity equation (3), we can rewrite Eq. (68) as,

$$P_c = V_{ub}V_{ud}^* (f (u) - f (t)) + V_{cb}V_{cd}^* (f (c) - f (t))$$

or

$$P_t = V_{ub}V_{ud}^* (f (u) - f (c)) + V_{tb}V_{td}^* (f (t) - f (c))$$

(69)

Due to loop integration $P$ is suppressed relative to $T$ but still its contribution is not negligible. The first part of Eq. (69) has the same CKM matrix elements as for the tree graph, so we can absorb it in the tree graph. Hence we can write (with $f = \pi^+ \pi^-)$,

$$\bar{A}_f = A \left( \bar{B}^0 \rightarrow \pi^+ \pi^- \right) = |T| e^{i(\gamma + \delta_T)} + |P| e^{i(\phi + \delta_P)}$$

(70)

where $\delta_T$ and $\delta_P$ are strong interaction phases which have been taken out so that $T$ and $P$ are real. $\phi$ is the weak phase in Penguin graph. $CP$-T invariance gives,

$$A_f \equiv A \left( B^0 \rightarrow \pi^+ \pi^- \right) = |T| e^{-i(\gamma - \delta_T)} + |P| e^{-i(\phi - \delta_P)}.$$ 

(71)

Hence direct $CP$-violation asymmetry is given by,

$$A_{CP} = \frac{-\Gamma (B^0 \rightarrow \pi^+ \pi^-) + \Gamma (\bar{B}^0 \rightarrow \pi^+ \pi^-)}{\Gamma (B^0 \rightarrow \pi^+ \pi^-) + \Gamma (\bar{B}^0 \rightarrow \pi^+ \pi^-)}$$

$$= \frac{1 - |\lambda|^2}{1 + |\lambda|^2}$$

$$= \frac{-2r \sin \delta \sin (\phi + \gamma)}{1 + 2r \cos \delta \cos (\gamma + \phi) + r^2}$$

(72)

where ($F_{\text{CKM}} = \text{CKM factor}$),

$$\delta = \delta_P - \delta_T \quad r \rightarrow F_{\text{CKM}} \frac{|P|}{|T|}$$

(73)
For the time dependent $CP$-asymmetry for $B^0 \rightarrow \pi^+ \pi^-$ decay we obtain from Eqs. (62) and (70),
\[
A(t) = C_{\pi\pi}(\cos \Delta m t) + S_{\pi\pi}(\sin \Delta m t),
\]
where the direct $CP$–violation parameter $C_{\pi\pi}$ and the mixing induced parameter $S_{\pi\pi}$ are given by,
\[
S_{\pi\pi} = \frac{2 \text{Im}[e^{2i\phi_m \lambda}]}{1 + |\lambda|^2} = -\frac{\sin(2\beta + 2\gamma) + 2r \cos \delta \sin(2\beta + \gamma - \phi) + r^2 \sin(2\beta - 2\phi)}{1 + 2r \cos \delta \cos(\gamma + \phi) + r^2}
\]
\[
C_{\pi\pi} = -A_{CP}
\]
As discussed above, we have two choices in selecting the Penguin contribution. For the first choice,
\[
\phi = \pi, \quad F_{\text{CKM}} = \frac{|V_{cb}| |V_{cd}|}{|V_{ub}| |V_{ud}|} = \frac{1}{(\bar{\rho}^2 + \bar{\eta}^2)^{1/2}}
\]
\[
r = \frac{1}{R_b |T|}
\]
For this case we have,
\[
C_{\pi\pi} = \frac{2r \sin \delta \sin \gamma}{1 + 2r \cos \delta \cos \gamma + r^2}
\]
\[
S_{\pi\pi} = -\frac{\sin(2\beta + 2\gamma) + 2r \cos \delta \sin(2\beta + \gamma) + r^2 \sin 2\beta}{1 + 2r \cos \delta \cos \gamma + r^2}
\]
\[
= \frac{\sin(2\alpha) + 2r \cos \delta \sin(\beta - \alpha) - r^2 \sin 2\beta}{1 + 2r \cos \delta \cos (\alpha + \beta) + r^2}
\]
For the second choice,
\[
\phi = \beta, \quad F_{\text{CKM}} = \frac{|V_{tb}| |V_{td}|}{|V_{ub}| |V_{ud}|} \approx \frac{\sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2}}{\sqrt{\bar{\rho}^2 + \eta^2}}
\]
\[
r = \frac{R_t |P_t|}{R_b |T|}
\]
So that in this case we get,
\[
C_{\pi\pi} = -A_{CP} = \frac{2r \sin \delta \sin \alpha}{1 - 2r \cos \delta \cos \alpha + r^2}
\]
\[
S_{\pi\pi} = \frac{\sin 2\alpha - 2r \cos \delta \sin \alpha}{1 - 2r \cos \delta \cos \alpha + r^2}
\]
For $B^+ \rightarrow \pi^+ \pi^-$, $B^0 \rightarrow \pi^0 \pi^0$, the decay amplitudes are given by,
\[
A_{00} = A(B^0 \rightarrow \pi^0 \pi^0) = \frac{1}{\sqrt{2}} T e^{i\delta_T} e^{i\gamma} \left[-r_c e^{i\delta_C T} + r e^{-i(\phi + \gamma - \delta + \delta_C T)}\right]
\]
\[
A_{+0} = A(B^+ \rightarrow \pi^+ \pi^0) = \frac{1}{\sqrt{2}} T e^{i\delta_T} e^{i\gamma} \left[1 + r_c e^{i\delta_C T}\right]
\]
\[
r_c = \frac{C}{T}, \quad \delta_{CT} = \delta_C - \delta_T
\]
Hence for $B^0 \rightarrow \pi^0 \pi^0$, the $CP$-asymmetries are given by

$$C_{\pi^0 \pi^0} = -A_{00}^{CP} = \frac{-2r/r_C \sin(\delta - \delta_{CT}) \sin(\gamma + \phi)}{1 + r^2/r_C^2 + 2r/r_C \cos(\delta - \delta_{CT}) \cos(\gamma + \phi)}$$

$$S_{\pi^0 \pi^0} = -\frac{\sin(2\beta + 2\gamma) - 2r/r_C \cos(\delta - \delta_{CT}) \sin(2\beta + \gamma - \phi) + r^2/r_C^2 \sin(2\beta - 2\phi)}{1 + r^2/r_C^22r/r_C \cos(\delta - \delta_{CT}) \cos(\gamma + \phi)}$$

For the case $\phi = \beta$, we get,

$$C_{\pi^0 \pi^0} = -A_{00}^{CP} = \frac{-2r/r_C \sin(\delta - \delta_{CT}) \sin(\alpha)}{1 + r^2/r_C^2 - 2r/r_C \cos(\delta - \delta_{CT}) \cos(\alpha)}$$

$$S_{\pi^0 \pi^0} = -\frac{\sin(2\alpha) - 2r/r_C \cos(\delta - \delta_{CT}) \sin(\alpha)}{1 + r^2/r_C^2 - 2r/r_C \cos(\delta - \delta_{CT}) \cos(\alpha)}$$

We end this section by considering the decays

$$\bar{B}^0 \rightarrow \phi K_s, \omega K_s$$

These decays satisfy the relations

$$\begin{bmatrix}
\frac{1}{\sqrt{2}} & \left( \frac{\langle \rho^0 \bar{K}^0 | H_W (s) | \bar{B}^0 \rangle - \langle \omega \bar{K}^0 | H_W (s) | \bar{B}^0 \rangle}{\langle \rho^0 \bar{K}^0 | H_W (s) | \bar{B}^0 \rangle} \right)
\langle \rho^0 | H_W (s) | \bar{B}^0 \rangle
\end{bmatrix}
\begin{bmatrix}
\frac{1}{2} (C - P + P_{EW}) - \frac{1}{2} (C + P + \frac{4}{3}P_{EW})
-(-P + \frac{2}{3}P_{EW})
\end{bmatrix} = 0$$

where $C, P$ and $P_{EW}$ are color suppressed, penguin and electroweak penguin amplitudes for these decay.

From the above equation, we obtain,

$$\frac{\langle \Gamma \rangle_{\omega K} + \langle \Gamma \rangle_{\rho K}}{\langle \Gamma \rangle_{\phi K}} \approx 1$$

$$\frac{S(\rho^0 K_s) + S(\omega K_s)}{2} \approx S(\phi K_s) = -\sin 2\beta$$

$$C(\rho^0 K_s) \approx -C(\omega K_s)$$

where we have neglected the terms of the order $r^2$. The parameter $r$ is defined below,

$$\langle \rho^0 \bar{K}^0 | H_W (s) | \bar{B}^0 \rangle = -|V_{cb}| |V_{cd}| |P| e^{i\delta_p} \left[ 1 - re^{i(\delta_C - \delta_p)} e^{-i\gamma} \right]$$

$$r = \frac{|C|}{|P|} \lambda^2 R_b$$

$$\frac{|C|}{|P|} \approx \frac{a_2}{a_4}$$

Assuming factorization for the electroweak penguin, we get from the above equation an interesting sum rule,

$$f_\rho F_1^{B-K}(m_\rho^2) - \frac{1}{3} f_\omega F_1^{B-K}(m_\omega^2) - \frac{2}{3} f_\phi F_1^{B-K}(m_\phi^2) = 0$$

Assuming $F_1(m_\rho^2) = F_1(m_\omega^2) = F_1(m_\phi^2) \approx F_1(1 \text{ GeV}^2)$, we get from the relation,

$$f_\rho - \frac{1}{3} f_\omega - \frac{2}{3} f_\phi = 0$$
which is reminiscent of current algebra and spectral function sum rules of 1960’s. The above sum rule is very well satisfied by the experimental values,

\[ f_\rho = (209 \pm 1) \text{ MeV}, \quad f_\omega = (187 \pm 3) \text{ MeV}, \quad f_\phi = (221 \pm 3) \text{ MeV}. \]

It is convenient to write, from Eqs. (60) and (61), the decay rates in the following form,

\[
\begin{align*}
\Gamma_f(t) - \Gamma_{\bar{f}}(t) + \Gamma_{\bar{f}}(t) - \Gamma_f(t)
&= e^{-\Gamma t} \left\{ \cos \Delta m t \left[ \left| A_f \right|^2 - \left| A_{\bar{f}} \right|^2 \right] + 2 \sin \Delta m t \left[ \text{Im} \left( e^{2i\phi_M} A_f^* \bar{A}_f \right) + \text{Im} \left( e^{2i\phi_M} A_{\bar{f}}^* \bar{A}_f \right) \right] \right\} \\
&= e^{-\Gamma t} \left\{ \cos \Delta m t \left[ \left| A_f \right|^2 + \left| A_{\bar{f}} \right|^2 \right] - \left( \left| A_f \right|^2 + \left| A_{\bar{f}} \right|^2 \right) \right\} + 2 \sin \Delta m t \left[ \text{Im} \left( e^{2i\phi_M} A_f^* \bar{A}_f \right) - \text{Im} \left( e^{2i\phi_M} A_{\bar{f}}^* \bar{A}_f \right) \right] \\
&= e^{-\Gamma t} \left\{ \cos \Delta m t \left[ \left| A_f \right|^2 + \left| A_{\bar{f}} \right|^2 \right] - \left( \left| A_f \right|^2 + \left| A_{\bar{f}} \right|^2 \right) \right\} + 2 \sin \Delta m t \left[ \text{Im} \left( e^{2i\phi_M} A_f^* \bar{A}_f \right) - \text{Im} \left( e^{2i\phi_M} A_{\bar{f}}^* \bar{A}_f \right) \right]
\end{align*}
\]

(73)

We now use the above equations to obtain some interesting results for the \( CP \) asymmetries for \( B \)-decays.

**Case-II:**

We first consider the case in which single weak amplitudes \( A_f \) and \( A'_{\bar{f}} \) with different weak phases describe the decays:

\[
\begin{align*}
A_f &= \langle f | L_W | B^0 \rangle = e^{i\phi} F_f \\
A'_{\bar{f}} &= \langle \bar{f} | L_W' | B^0 \rangle = e^{i\phi'} F'_{\bar{f}}
\end{align*}
\]

(75)

\( CPT \) gives,

\[
\begin{align*}
\bar{A}_f &= \langle \bar{f} | L_W | B^0 \rangle = e^{2i\delta_f} A_f^* \\
\bar{A}'_{f} &= \langle f | L_W' | B^0 \rangle = e^{2i\delta_{f'}} A_{f'}^*
\end{align*}
\]

(76)

Note \( \delta_f \) and \( \delta_{f'} \) are strong phases; \( \phi \) and \( \phi' \) are weak phases. The states \( | f > \) and \( | \bar{f} > \) are C-conjugate of each other such as states \( D^{(*)} - \pi^+ (D^{(*)} + \pi^-) \), \( D_s^{(*)} - K^+ (D_s^{(*)} + K^-) \), \( D^- \rho^+ (D^+ \rho^-) \).
Hence, we get from Eqs. (73), (74), (75) and (76),

\[
\mathcal{A}(t) \equiv \frac{[\Gamma_f(t) - \Gamma_f(t)] + [\Gamma_f(t) - \Gamma_f(t)]}{[\Gamma_f(t) + \Gamma_f(t)] + [\Gamma_f(t) + \Gamma_f(t)]} \\
= \frac{2|F_f|\frac{\Gamma_f'}{\Gamma_f}}{|F_f|^2 + |F_f'|^2} \sin \Delta mt \sin\left(2\phi_M - \phi - \phi'\right) \cos(\delta_f - \delta_f')
\]

(77)

\[
\mathcal{F}(t) \equiv \frac{[\Gamma_f(t) + \Gamma_f'] - [\Gamma_f(t) + \Gamma_f]}{[\Gamma_f(t) + \Gamma_f] + [\Gamma_f(t) + \Gamma_f]} \\
= \frac{|F_f|^2 - |F_f'|^2}{|F_f|^2 + |F_f'|^2} \cos \Delta mt \\
- \frac{2|F_f|\frac{\Gamma_f'}{\Gamma_f}}{|F_f|^2 + |F_f'|^2} \sin \Delta mt \cos\left(2\phi_M - \phi - \phi'\right) \sin(\delta_f - \delta_f')
\]

(78)

We now apply the above formula to $B \to \pi D$ and $B_s \to KD_s$ decays. For these decays,

\[\phi = 0, \quad \phi' = \gamma\]

\[\phi_M = \begin{cases} 
-\beta, & \text{for } B^0 \\
-\beta_s, & \text{for } B_s^0
\end{cases}\]

\[A_f = \langle D^- \pi^+ | \mathcal{L}_W | B^0 \rangle = F_f\]

\[A'_f = \langle D^+ \pi^- | \mathcal{L}_{W'} | B^0 \rangle = e^{i\gamma} F_f'\]

\[A_{f_s} = \langle K^+ D^- | \mathcal{L}_W | B_s^0 \rangle = F_{f_s}\]

\[A'_{f_s} = \langle K^- D^+ | \mathcal{L}_{W'} | B_s^0 \rangle = e^{i\gamma} F_{f_s}'\]

Note that the effective Lagrangians for decays ($q = d, s$) are given by,

\[\mathcal{L}_W = V_{cb} V^*_{uq} [\bar{q}\gamma^\mu (1 - \gamma_5) u] \left[\bar{c}\gamma_\mu (1 - \gamma_5) b\right] \quad (79a)\]

\[\mathcal{L}_{W'} = V_{ub} V^*_{cq} [\bar{q}\gamma^\mu (1 - \gamma_5) c] \left[\bar{u}\gamma_\mu (1 - \gamma_5) b\right] \quad (79b)\]

respectively. In the Wolfensteinn parametrization of CKM matrix,

\[
\frac{|V_{ub}| |V_{cq}|}{|V_{cb}| |V_{uq}|} = \lambda^2 \sqrt{\rho^2 + \eta^2},
\]

(80)

Define,

\[r = \lambda^2 R_b \frac{|F_f'|}{F_f} \left|\text{and } r_s = R_b \frac{|F_{f_s}'|}{F_{f_s}}\right|
\]

Thus, we get from Eqs. (77) and (78) for $B^0$ decays, (replacing $\frac{|F_f'|}{F_f}$ by $r$),
\[ \mathcal{A}(t) = -\frac{2r}{1 + r^2} \sin \Delta m_B t \sin (2\beta + \gamma) \cos \left( \delta_f - \delta_f' \right) \]

\[ \mathcal{F}(t) = \frac{1 - r^2}{1 + r^2} \cos \Delta m_B t - \frac{2r}{1 + r^2} \sin \Delta m_B t \cos (2\beta + \gamma) \sin \left( \delta_f - \delta_f' \right) \]  

(81)

For the decays,

\[ \bar{B}_s^0 (B_s^0) \rightarrow K^- D_s^+ (K^+ D_s^-) \]

\[ \bar{B}_s^0 (B_s^0) \rightarrow K^+ D_s^- (K^- D_s^+) \]

we get,

\[ \mathcal{A}_s(t) = -\frac{2r_s}{1 + r_s^2} \sin(\Delta m_{B_s} t) \sin (2\beta_s + \gamma) \cos \left( \delta_{f_s} - \delta_{f_s}' \right) \]

\[ \mathcal{F}_s(t) = \frac{1 - r_s^2}{1 + r_s^2} \cos \Delta m_{B_s} t - \frac{2r_s}{1 + r_s^2} \sin \Delta m_{B_s} t \cos (2\beta_s + \gamma) \sin \left( \delta_{f_s} - \delta_{f_s}' \right) \]  

(82)

We note that for time integrated CP-asymmetry,

\[ \mathcal{A}_s \equiv \frac{\int_0^\infty \left[ \Gamma_{f_s}(t) - \bar{\Gamma}_{f_s}(t) \right] dt}{\int_0^\infty \left[ \Gamma_{f_s}(t) + \bar{\Gamma}_{f_s}(t) \right] dt} = -\frac{2r}{1 + r^2} \sin (2\beta + \gamma) \frac{\Delta m_{B_s}/\Gamma_s}{1 + (\Delta m_{B_s}/\Gamma_s)^2} \cos (\delta_{f_s} - \delta_{f_s}') \]  

(83)

The CP–asymmetry \( \mathcal{A}_s(t) \) or \( \mathcal{A}_s \) involves two experimentally unknown parameters \( \sin (2\beta_s - \gamma) \) and \( \Delta m_{B_s} \). Both these parameters are of importance in order to test the unitarity of \( CKM \) matrix viz whether \( CKM \) matrix is the sole source of CP–violation in the processes in which CP–violation has been observed.

Within the case II, we discuss \( B \) decays into baryons and antibaryons.

So far we have discussed the CP-violation in kaon and \( \bar{B}_q - B_q^0 \) systems. There is thus a need to study CP-violation outside these systems.

The decays of \( B(\bar{B}) \) mesons to baryon-antibaryon pair \( N_1 \bar{N}_2 (\bar{N}_1 N_2) \) and subsequent decays of \( N_2, \bar{N}_2 \) or \( (N_1, \bar{N}_1) \) to a lighter hyperon (antihyperon) plus a meson provide a means to study CP-odd observables as for example in the process,

\[ e^- e^+ \rightarrow B, \bar{B} \rightarrow N_1 \bar{N}_2 \rightarrow N_1 \bar{N}_2 \pi, \quad \bar{N}_1 N_2 \rightarrow \bar{N}_1 N_2 \pi \]

The decay \( B \rightarrow N_1 \bar{N}_2 (f) \) is described by the matrix element,

\[ M_f = F_q e^{i\phi} \left[ \bar{u}(p_1)(A_f + \gamma_5 B_f)v(p_2) \right] \]  

(84)

where as \( B \rightarrow \bar{N}_1 N_2 (\bar{f}) \) is described by the matrix elements

\[ M'_f = F'_q e^{i\phi'} \left[ \bar{u}(p_2)(A'_f + \gamma_5 B'_f)v(p_1) \right] \]

where \( F_q \) is a constant containing CKM factor, \( \phi \) is the weak phase. The amplitude \( A_f \) and \( B_f \) are in general complex in the sense that they incorporate the final state phases \( \delta'_f \) and \( \delta''_f \) and
they may also contain weak phases $\phi_s$ and $\phi_p$. Note that $A_f$ is the parity violating amplitude ($p$-wave) whereas $B_f$ is parity conserving amplitude ($s$-wave). The CPT invariance gives the matrix elements for the decay $\bar{B} \to N_1 N_2(f)$:

$$\bar{M}_f = F_q e^{-i\phi} \left[ \bar{u}(p_2)(-A_f e^{2i\delta_f} + \gamma_5 B_f^* e^{2i\delta_f})v(p_1) \right]$$  \hspace{1cm} (85)

if the decays are described by a single matrix element $M_f$. If $\phi_s = 0 = \phi_p$ then CPT and CP invariance give the same predictions viz

$$\bar{\Gamma}_f = \Gamma_f, \quad \bar{\alpha}_f = -\alpha_f, \quad \bar{\beta}_f = -\beta_f, \quad \bar{\gamma}_f = \gamma_f$$  \hspace{1cm} (86)

The decay width for the mode $B \to N_1 \bar{N}_2(f)$ is given by,

$$\Gamma_f = \frac{m_1 m_2}{2\pi m_B^2} |p| |M_f|^2$$

$$= \frac{F_q^2}{2\pi m_B^2} |p| \left[ (p_1 \cdot p_2 - m_1 m_2) |A_f|^2 + (p_1 \cdot p_2 + m_1 m_2) |B_f|^2 \right]$$  \hspace{1cm} (87)

In order to take into account the polarization of $N_1$ and $\bar{N}_2$, we give the general expression for $|M_f|^2$,

$$|M_f|^2 = \frac{F_q^2}{16 m_1 m_2} Tr \left[ (\phi_1 + m_1)(1 + \gamma_5 \gamma \cdot s_1)(A_f + \gamma_5 B_f)(\phi_2 - m_2) \right. \left. \times (1 + \gamma_5 \gamma \cdot s_2)(A_f^* - \gamma_5 B_f^*) \right]$$  \hspace{1cm} (88)

where $s^\mu_1, s^\mu_2$ are polarization vectors of $N_1$ and $\bar{N}_2$ respectively ($p_1.s_1 = 0, \quad p_2.s_2 = 0, \quad s^2 = -1 = s^2$).

In the rest frame of $B$, we get,

$$|M_f|^2 = F_q^2 \frac{2E_1 E_2}{4 m_1 m_2} \left[ |a_s|^2 + |a_p|^2 \right]$$

$$\left\{ \frac{1 + \alpha_f \left( \frac{m_1}{E_1} n \cdot s_1 - \frac{m_2}{E_2} n \cdot s_2 \right)}{-\frac{m_1 m_2}{E_1 E_2} (n \cdot s_1)(n \cdot s_2)} \right\}$$

$$+ \beta_f n \cdot (s_1 \times s_2) + \gamma_f \left[ (n \cdot s_1)(n \cdot s_2) - s_1 \cdot s_2 \right]$$  \hspace{1cm} (89)

where,

$$a_s = \sqrt{\frac{p_1 \cdot p_2 + m_1 m_2}{2E_1 E_2}} B, \quad a_p = -\sqrt{\frac{p_1 \cdot p_2 - m_1 m_2}{2E_1 E_2}} A$$  \hspace{1cm} (90)

$$\alpha_f = \frac{2S_f P_f \cos(\delta^f - \delta^p)}{S^2_f + P^2_f}, \quad \beta_f = \frac{2S_f P_f \sin(\delta^f - \delta^p)}{S^2_f + P^2_f}$$

$$\gamma_f = \frac{S^2_f - P^2_f}{S^2_f + P^2_f}, \quad a_s = S_f e^{i\delta_f}, \quad a_p = P_f e^{i\delta_f}$$  \hspace{1cm} (91)

In the rest frame of $B$, due to spin conservation,

$$\left( \lambda_1 \equiv \frac{E_1}{m_1} n \cdot s_1 \right) = \left( \lambda_2 \equiv \frac{-E_2}{m_2} n \cdot s_2 \right) = \pm 1$$  \hspace{1cm} (92)
Thus, invariants multiplying $\beta_f$ and $\gamma_f$ vanish. Hence we have,

$$|M_f|^2 = \left(\frac{2E_1E_2}{m_1m_2}\right)F_q^2(S_f^2 + P_f^2)\left[(1 + \lambda_1\lambda_2) + \alpha_f(\lambda_1 + \lambda_2)\right]$$  \hspace{1cm} (93)

$$\Gamma_f = \Gamma_f^{++} + \Gamma_f^{--} = \frac{2E_1E_2}{2\pi m_B^2}\left|\bar{p}\right| F_q^2 \left[S_f^2 + P_f^2\right] = \bar{\Gamma}_f$$  \hspace{1cm} (94)

$$\Delta \Gamma_f = \frac{\Gamma_f^{++} - \Gamma_f^{--}}{\Gamma_f^{++} + \Gamma_f^{--}} = \alpha_f, \quad \Delta \bar{\Gamma}_f = \bar{\alpha}_f = -\alpha_f$$  \hspace{1cm} (95)

Eqs. \(94\) and \(95\) follow from $CP$ invariance. It will be of interest to test these equations.

Now $B^0_q$, $\bar{B}^0_q$ annihilate into baryon-antibaryon pair $N_1\bar{N}_2$ through $W$-exchange as depicted in Figs (5a) and (5b). $B^- \rightarrow N_1\bar{N}_2$ through annihilation diagram is shown in Fig (6). It is clear from Fig (5a) and (5b), that we have the same final state configuration for $B^0_q$, $\bar{B}^0_q \rightarrow N_1\bar{N}_2$. Thus, one would expect,

$$S_f' = S_f, \quad P_f' = P_f$$
$$\delta_s' = \delta_s, \quad \delta_p' = \delta_p$$  \hspace{1cm} (96)

Hence we have,

$$\Gamma_f' = \bar{\Gamma}_f' = r^2\Gamma_f; \quad r^2 = \frac{|F_q|^2}{|F_q|^2}$$  \hspace{1cm} (97)

$$\bar{\alpha}_f' = -\alpha_f' = \alpha_f = -\bar{\alpha}_f$$  \hspace{1cm} (98)

Above predictions can be tested in future experiments on baryon decay modes of $B$-mesons. In particular $\bar{\alpha}_f' = \alpha_f$ would give direct confirmation of Eqs. \(98\).

For the time dependent baryon decay modes of $B^0_q - \bar{B}^0_q$, we have: $(\phi = \gamma, \phi' = 0)$

$$\mathcal{A}(t) = \mathcal{A}^{++}(t) - \mathcal{A}^{--}(t) = \frac{2r \sin \Delta mt \sin(2\phi_M - \gamma)}{1 + r^2}$$  \hspace{1cm} (99)

$$\Delta \mathcal{A}(t) = \mathcal{A}^{++}(t) - \mathcal{A}^{--}(t) = 0$$  \hspace{1cm} (100)

$$\mathcal{F}(t) = \mathcal{F}^{++}(t) + \mathcal{F}^{--}(t) = \frac{1 - r^2}{1 + r^2} \cos \Delta mt$$  \hspace{1cm} (101)

$$\Delta \mathcal{F}(t) = \mathcal{F}^{++}(t) - \mathcal{F}^{--}(t) = \frac{1 - r^2}{2(1 + r^2)}(\alpha_f + \bar{\alpha}_f) \cos \Delta mt - \frac{4r \sin \Delta mt \sin(2\phi_M - \gamma)S_f P_f}{(1 + r^2)(S_f^2 + P_f^2)}$$  \hspace{1cm} (102)

where we have used Eqs. \(98\). For $B^0_d$, $r = -\lambda^2\sqrt{\rho^2 + \eta^2} \approx -0.02 \pm 0.006$ [4], $\phi_M = -\beta$; for $B^0_s$, $r = -\sqrt{\rho^2 + \eta^2} \approx -(0.40 \pm 0.13)$, $\phi_M = -\beta_s$.

Eq. \(99\) gives a means to determine the weak phase $2\beta + \gamma$ or $\gamma$ in the baryon decay modes of $B^0_d$ and $B^0_s$ respectively. Non-zero $\cos \Delta mt$ term in $\Delta \mathcal{F}(t)$ would give clear indication of $CP$ violation especially for baryon decay modes of $B^0_d$, for which $r^2 \leq 1$, so that $\frac{1 - r^2}{1 + r^2} \approx 1$. It may be noted that the time-dependent asymmetries arises because there are two independent amplitudes for the decays $B^0 \rightarrow N_1\bar{N}_2$, $\bar{N}_1N_2 : M_f, \ M'_f$.  

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The baryon decay modes of $B$-mesons not only provide a means to test prediction of $CP$ asymmetry viz $\alpha_f + \bar{\alpha}_{\bar{f}} = 0$ for charmed baryons (discussed above) but also to test the $CP$-asymmetry in hyperon (antihyperon) decays viz absence of $CP$-odd observables $\Delta \Gamma, \Delta \alpha, \Delta \beta$ discussed in [8]. Consider for example the decays,

$$B^0_q \to p\bar{\Lambda}_c^- \to p\bar{p}K^0(p\bar{\Lambda}_c^+ \to p\bar{p} \bar{\Lambda}_c^-\pi^- \to p\bar{p} \pi^+ \pi^-),$$

$$B^0_{\bar{q}} \to \bar{p}\Lambda_c^+ \to \bar{p}\bar{p}K^0(\bar{p}\Lambda_c^- \to \bar{p}\bar{p} \bar{\Lambda}_c^+\pi^- \to \bar{p}\bar{p} \pi^+ \pi^-)$$

By analyzing the final state $\bar{p}\bar{p}K^0, p\bar{p}K^0$, one may test $\alpha_f = -\bar{\alpha}_{\bar{f}}$ for the charmed hyperon. We note that for $\Lambda_c^+, c\tau = 59.9 \mu m$, whereas $c\tau = 7.8 cm$ for $\Lambda$–hyperon, so that the decays of $\Lambda_c^+$ and $\Lambda$ would not interfere with each other. By analysing the final state $\bar{p}\bar{p}\pi^-\pi^+ + \bar{p}\bar{p}\pi^-\pi^-$, one may check $CP$–violation for hyperon decays. One may also note that for $(B^0_d, \bar{B}^0_d)$ complex, the competing channels viz $B^0_d \to \bar{p}\Lambda_c^+, \bar{B}^0_d \to p\Lambda_c^-$ are doubly Cabibbo suppressed by $r^2 = \lambda^2 (\rho^2 + \eta^2)$ unlike $(B^0_s - \bar{B}^0_s)$ complex where the competing channels are suppressed by a factor of $(\rho^2 + \eta^2)$. Hence $B^0_d(\bar{B}^0_d)$ decays are more suitable for this type of analysis. Other decays of interest are,

$$B^- \to \bar{\Lambda}\bar{\Lambda}^- \to \bar{\Lambda}\bar{\Lambda}^- \to p\pi^- \bar{p}\pi^+ \pi^-$$

$$B^+ \to \bar{\Lambda}\bar{\Lambda}^+ \to \bar{\Lambda}\bar{\Lambda}^+ \to \bar{p}\pi^+ p\pi^- \pi^+$$

$$B^- \to \bar{p}\Lambda \to \bar{p}\bar{p}\pi^-$$

$$B^+ \to p\bar{\Lambda} \to p\bar{p}\pi^+$$

The non-leptonic hyperon (antihyperon) decays $N \to N'\pi (\bar{N} \to \bar{N}'\bar{\pi})$ are related to each other by $CPT$,

$$a_t(I) = \langle f^{out}_{H_W} | N \rangle = \eta_f e^{2i\delta_I} \langle f^{out}_{H_W} | \bar{N} \rangle = \eta_f e^{2i\delta_I} \bar{a}_t(I)$$

Hence,

$$\bar{a}_t(I) = \eta_f e^{2i\delta_I} a_t^*(I) = (-1)^{l+1} e^{i\delta_I} e^{-i\phi} |a_t|$$

where we have selected the phase $\eta_f = (-1)^{l+1}$. Here $I$ is the isospin of the final state and $\phi$ is the weak phase. Thus necessary condition for non-zero $CP$ odd observables is that the weak phase for each partial wave amplitude should be different. For instance for the decays $B^0(\bar{B}^0) \to p\bar{\Lambda}_c^- (\bar{p}\Lambda_c^+)$ we have,

$$\delta \Gamma = 0$$

$$\delta \alpha_f = -\tan (\delta_s - \delta_p) \tan (\phi_s - \phi_p)$$

$$\approx -\tan (\delta_s - \delta_p) \sin (\phi_s - \phi_p)$$

Case III:

Here $A_f \neq A_{\bar{f}}$.

$$A_f = \langle f | \mathcal{L}_W | B^0 \rangle = [e^{i\phi_1} F_1 f + e^{i\phi_2} F_2 f]$$

$$A_{\bar{f}} = \langle \bar{f} | \mathcal{L}_W | \bar{B}^0 \rangle = [e^{i\phi_1} F_1 \bar{f} + e^{i\phi_2} F_2 \bar{f}]$$

Examples:

$$B^0 \to \rho^- \pi^+(f) : A_f \quad B^0 \to \rho^+ \pi^-(\bar{f}) : A_{\bar{f}}$$
$B_s^0 \rightarrow K^*-K^+ \quad B_s^0 \rightarrow K^{*-}K^+$

CPT gives,

$$\bar{A}_{f,f} = \sum_i [e^{-i\phi_i} e^{2i\delta_{ij} f} F^*_i f]$$

Subtracting and adding Eqs. (74) and (73), we get,

$$\Gamma_f(t) - \bar{\Gamma}_f(t) = C_f \cos \Delta m t + S_f \sin \Delta m t$$

$$= (C - \Delta C) \cos \Delta m t + (S - \Delta S) \sin \Delta m t$$

$$\frac{\Gamma_f(t) - \bar{\Gamma}_f(t)}{\Gamma_f(t) + \bar{\Gamma}_f(t)} = C_f \cos \Delta m t + S_f \sin \Delta m t$$

$$= (C + \Delta C) \cos \Delta m t + (S + \Delta S) \sin \Delta m t$$

(103)

(104)

where

$$C_{f,f} = (C \pm \Delta C)$$

$$= \frac{|A_{f,f}|^2 - |\bar{A}_{f,f}|^2}{|A_{f,f}|^2 + |\bar{A}_{f,f}|^2}$$

$$= \frac{\Gamma_{f,f} - \bar{\Gamma}_{f,f}}{\Gamma_{f,f} + \bar{\Gamma}_{f,f}}$$

$$= \frac{R_{f,f}(1 - A_{CP}^{f,f}) - R_{f,f}(1 + A_{CP}^{f,f})}{\Gamma(1 \pm A_{CP})}$$

(105)

$$S_{f,f} = (S \pm \Delta S)$$

$$= \frac{2\text{Im}[e^{2i\phi_M} A_{f,f}^* \bar{A}_{f,f}]}{\Gamma_{f,f} + \bar{\Gamma}_{f,f}}$$

(106)

(107)

$$A_{CP}^{\bar{f}} = \frac{\bar{\Gamma}_{f} - \bar{\Gamma}_f}{\Gamma_f + \bar{\Gamma}_f}$$

$$A_{CP}^f = \frac{\bar{\Gamma}_f - \Gamma_f}{\Gamma_f + \bar{\Gamma}_f}$$

(108)

$$A_{CP} = \frac{(\Gamma_f + \bar{\Gamma}_f) - (\bar{\Gamma}_{f} + \Gamma_f)}{\Gamma_f + \bar{\Gamma}_f}$$

$$= \frac{R_f A_{CP}^f - R_f A_{CP}^{\bar{f}}}{\Gamma}$$

(109)

(110)

where

$$R_f = \frac{1}{2}(\Gamma_f + \bar{\Gamma}_f), \quad R_{\bar{f}} = \frac{1}{2}(\bar{\Gamma}_f + \Gamma_f)$$

$$\Gamma = R_f + R_{\bar{f}}$$

(111)
The following relations are also useful which can be easily derived from above equations:

\[
\frac{R_{f,f}}{R_f + R_f} = \frac{1}{2} \left[ (1 \pm \Delta C) \pm A_{CP} C \right]
\]
\[
\frac{R_f - R_f}{R_f + R_f} = |\Delta C + A_{CP} C|
\]
\[
\frac{R_f A_{CP}^f + R_f A_{CP}^f}{R_f + R_f} = [C + A_{CP} \Delta C]
\]

For these decays, the decay amplitudes can be written in terms of tree amplitude \(e^{i\phi_T} T_f\) and the penguin amplitude \(e^{i\phi_P} P_f\):

\[
A_f = e^{i\phi_T} e^{i\delta_f^T} |T_f| [1 + r_f e^{i(\phi_P - \phi_T)} e^{i\delta_f}]
\]
\[
A_f = e^{i\phi_T} e^{i\delta_f^T} |T_f| [1 + r_f e^{i(\phi_P - \phi_T)} e^{i\delta_f}]
\]

where \(r_{f,f} = \left| \frac{P_{f,f}}{T_{f,f}} \right|\), \(\delta_{f,f} = \delta_f^P - \delta_f^T\).

\[
\bar{A}_f = e^{-i\phi_T} e^{i\delta_f^T} |T_f| [1 + r_f e^{-i(\phi_P - \phi_T)} e^{i\delta_f}]
\]
\[
\bar{A}_f = e^{-i\phi_T} e^{i\delta_f^T} |T_f| [1 + r_f e^{-i(\phi_P - \phi_T)} e^{i\delta_f}]
\]

For \(B^0 \rightarrow \rho^- \pi^+\): \(\phi_T = \gamma, \phi_P = -\beta\)

For \(B^0 \rightarrow D^{*+} D^-\): \(A_f, \phi_T = 0, \phi_P = -\beta\)

Hence for \(B^0 \rightarrow \rho^- \pi^+, B^0 \rightarrow \rho^+ \pi^-\), we have

\[
A_f = |T_f| e^{-i\gamma} e^{i\delta_f^T} [1 - r_f e^{i(\alpha + \delta_f)}]
\]
\[
A_f = |T_f| e^{-i\gamma} e^{i\delta_f^T} [1 - r_f e^{i(\alpha + \delta_f)}]
\]

where

\[
r_{f,f} = \left| \frac{V_{ub}^* |V_{ud}| P_{f,f}}{|V_{ub}^* |V_{ud}| T_{f,f}} \right| = \frac{R_t |P_{f,f}|}{R_b |T_{f,f}|}
\]

and for \(B^0 \rightarrow D^{*-} D^+, B^0 \rightarrow D^{*+} D^-\), we have

\[
A_f = |T_f|^D e^{i\delta_f^D} [1 - r_f e^{i(-\beta + \delta_f^D)}]
\]
\[
A_f = |T_f|^D e^{i\delta_f^D} [1 - r_f e^{i(-\beta + \delta_f^D)}]
\]

where

\[
r_{f,f} = \frac{R_t |P_{f,f}|}{|T_{f,f}|}
\]

5 Final State Strong Phases

As we have seen the CP asymmetries in the hadronic decays of B and K mesons involve strong final state phases. Thus strong interactions in these decays play a crucial role. The short distance
strong interactions effects at the quark level are taken care of by perturbative QCD in terms of Wilson coefficients. The CKM matrix which connects the weak eigenstates will mass eigenstates is another aspect of strong interactions at quark level. In the case of semi leptonic decays, the long distance strong interaction effects manifest themselves in the form factors of final states after hadronization. Likewise the strong interaction final state phases are long distance effects. These phase shifts essentially arise in terms of S-matrix which changes an 'in' state into an 'out' state viz.

\[ |f\rangle_{\text{out}} = S|f\rangle_{\text{in}} = e^{2i\delta_f}|f\rangle_{\text{in}} \] (122)

In fact, the CPT invariance of weak interaction Lagrangian gives for the weak decay \( B(\bar{B}) \to f(\bar{f}) \)

\[ A_f \equiv \text{out} \langle \bar{f}|L_W|B \rangle = \eta_f e^{2i\delta_f} A^*_f \] (123)

It is difficult to reliably estimate the final state strong phase shifts. It involves the hadronic dynamics. However, using isospin, C-invariance of S-matrix and unitarity of S-matrix, we can relate these phases. In this regard, the decays \( B^0 \to f, \bar{f} \) described by two independent single amplitudes \( A_f \) and \( A'_f \) discussed in section 4 case (ii) and the decays described by the weak amplitudes \( A_f \neq A'_f \), described in section case (iii) are of interest.

The invariance of S-matrix viz. \( S_f = S_f \) would imply

\[ \delta_f = \delta'_f, \quad \delta_1f = \delta_1\bar{f}, \quad \delta_2f = \delta_2\bar{f} \]

In the above decays, \( b \) is converted into \( b \to c(u) + \bar{u} + d \). In particular, for the tree graph, the configuration is such that \( \bar{u} \) and \( d \) essentially go together into color singlet states will the third quark \( c(u) \) recoiling; there is a significant probability that system will hadronize as a two body final state. Thus at least for the tree amplitude \( \delta^T_f \) should be equal to \( \delta^T_{\bar{f}} \). To proceed further, we use the unitarity of S-matrix to relate the final state strong phases. The time reversal invariance gives

\[ F_f =_{\text{out}} \langle f|L_W|B \rangle =_{\text{in}} \langle f|L_W|B \rangle^* \] (124)

where \( L_W \) is the weak interaction Lagrangian without the CKM factor such as \( V^{*}_{ud}V_{ub} \). From Eq. (124), we have

\[ F_f^* =_{\text{out}} \langle f|S^\dagger L_W|B \rangle = \sum S^*_n f_n \] (125)

It is understood that the unitarity equation which follows from time reversal invaraince holds for each amplitude with the same weak phase. Above equation can be written in two equivalent forms:

1. Exclusive version of Unitarity

Writing

\[ S_{nf} = \delta_{nf} + iM_{nf} \] (126)

we get from Eq. (125),

\[ ImF_f = \sum M^*_{nf} f_n \] (127)
where $M_{nf}$ is the scattering amplitude for $f \rightarrow n$. In this version, the sum is over all allowed exclusive channels. This version is more suitable in a situation where a single exclusive channel is dominant one. To get the final result, one uses the dispersion relation.

2. Inclusive version of Unitarity

This version is more suitable for our analysis. For this case, we write Eq. (125) in the form

$$F^*_f - S^*_ff = \sum_{n \neq f} S^*_n F_n$$  \hspace{1cm} (128)

Parametrizing S-matrix as $S_{ff} \equiv S = \eta e^{2i\Delta}$, we get after taking the absolute square of both sides of Eq. (128)

$$|F_f|^2[(q + \eta^2) - 2\eta \cos 2(\delta_f - \Delta)] = \sum_{n,n' \neq f} F_n S^*_n F_{n'} S_{n'f}$$  \hspace{1cm} (129)

The above equation is an exact equation. In the random phase approximation, we can put

$$\sum_{n',n \neq f} F_n S^*_n F_{n'} S_{n'f} = \sum_{n \neq f} |F_n|^2 |S_{nf}|^2$$  \hspace{1cm} (130)

We note that in a single channel description:

$$(Flux)_{in} - (Flux)_{out} = 1 - |\eta e^{2i\Delta}|^2 = 1 - \eta^2 = \text{Absorption}$$

The absorption takes care of all the inelastic channels.

Similarly for the amplitude $F_f$, we have

$$F^*_f - S^*_ff = \sum_{\bar{n} \neq f} S^*_\bar{n} F_{\bar{n}}$$  \hspace{1cm} (131)

The C-invariance of S-matrix gives:

$$S_{fn} = \langle f | S | n \rangle = \langle f | C^{-1} CSC^{-1} C | n \rangle$$
$$= \langle \bar{f} | S | \bar{n} \rangle = S_{\bar{f} \bar{n}}$$  \hspace{1cm} (132)

Thus in particular C-invariance of S-matrix gives

$$S_{ff} = S_{ff} = \eta e^{2i\Delta}$$  \hspace{1cm} (133)

Hence from Eq. (129), using Eqs. (130–133), we get

$$\frac{1}{1 - \eta^2}[(q + \eta^2) - 2\eta \cos 2(\delta_f - \Delta) = \rho^2, \bar{\rho}^2$$  \hspace{1cm} (134)

where

$$\rho^2 = \frac{|F_n|^2}{|F_f|^2}, \quad \bar{\rho}^2 = \frac{|F_{\bar{n}}|^2}{|F_f|^2}$$  \hspace{1cm} (135)
It is convenient to write Eq. (134) in the form

\[ \sin^2(\delta_{f,j} - \Delta) = \frac{1 - \eta^2}{4\eta} \left[ \rho^2, \bar{\rho}^2 - \frac{1 - \eta}{1 + \eta} \right] \]

(136)

\[ 0 \leq (\delta_{f,j} - \Delta) \leq \theta \]

(137)

\[ -\theta \leq (\delta_{f,j} - \Delta) \leq 0 \]

(138)

where \( \theta = \sin^{-1}\sqrt{\frac{1-\eta}{2}} \).

The strong interaction parameters \( \Delta \) and \( \eta \) can be determined by strong interaction dynamics. Using \( SU(2) \), C-invariance of strong interactions and Regge pole phenomenology, the scattering amplitude \( M(s,t) \) for two particle final state can be calculated. (For details see ref. [12]). The s-wave scattering amplitude \( f \) for the decay modes \( \pi^+D^- (\pi^-D^+) \), \( K^+\pi^- \), \( \pi^+\pi^- \) which are s-wave decay modes of \( B^0 \) is given by

\[ f(s) = \frac{1}{16\pi s} \int_{-s}^{0} M(s') dt \]

where

\[ t \approx -\frac{1}{2} s (1 - \cos \theta) \]

Using the relation \( S = \eta e^{2i\Delta} = 1 + 2i f \), the phase shift \( \Delta \), the parameter \( \eta \) and the phase angle \( \theta \) can be determined. One gets (\( s = m_B^2 \))

\[ \pi^+D^- (\pi^-D^+) : \Delta \approx -7^\circ, \eta \approx 0.62, \rho_{\text{min}} \approx 0.23, \theta \approx 26^\circ \]

\[ K^+\pi^- \text{ or } K^0\pi^+ : \Delta \approx -9^\circ, \eta \approx 0.52, \rho_{\text{min}} \approx 0.31, \theta \approx 29^\circ \]

\[ \pi^+\pi^- : \Delta \approx -21^\circ, \eta \approx 0.48, \rho_{\text{min}} \approx 0.35, \theta \approx 31^\circ \]

Hence we get the following bounds

\[ \pi^+D^- (\pi^-D^+) : 0 \leq \delta_{f,j} - \Delta \leq 26^\circ \]

\[ K^+\pi^- \text{ or } K^0\pi^+ : 0 \leq \delta_{f,j} - \Delta \leq 29^\circ \]

\[ \pi^+\pi^- : 0 \leq \delta_{f,j} - \Delta \leq 31^\circ \]

For the tree amplitude, factorization implies \( \delta_f^T = 0 \). We can therefore take the point of view, the effective final state phase shift is given by \( \delta_f - \Delta \). We take the lower bounds for the tree amplitude so that final state effective phase shift \( \delta_f^T = 0 \). For the penguin we assume that the effective value of the final state phase shift \( \delta_f^P \) is near the upper bound. Thus for \( \pi^+D^- (\pi^-D^+) \), \( \delta_f^T = \delta_f^T \approx 0 \); for \( K^+\pi^- \), the phase shift \( \delta_{+-} = \delta_{+-}^P \approx 29^\circ \) where as for \( \pi^+\pi^- \), the phase shift \( \delta_{+-} = \delta_{+-}^P \approx 31^\circ \). These phase shifts are relevant for the Direct CP-asymmetries for \( B^0 \to K^+\pi^- \) and \( B^0 \to \pi^+\pi^- \) decays.

The decay \( B^0 \to K^+\pi^- \) is described by two amplitudes (For details see ref.[13])

\[ A(B^0 \to K^+\pi^-) = -\left[ P + e^{i\gamma_T} \right] = |P| \left[ 1 - r e^{i(\gamma + \delta_{+-})} \right] \]

\[ P = -|P| e^{-i\delta_P}, \quad T = |T| \]

\[ \delta_{+-} = \delta_P, \quad r = \frac{|T|}{|P|} \]

\[ A_{CP}(B^0 \to \pi^-K^+) = -\frac{2r \sin \gamma \sin \delta_{+-} R}{R} \]

\[ R = 1 - 2r \cos \gamma \cos \delta_{+-} + r^2 \]
Neglecting the terms of order $r^2$, 
\[ \tan \gamma \tan \delta_{+} - = \frac{-A_{CP} (B^0 \rightarrow \pi^- K^+)}{1 - R} \]

From the experimental values of $A_{CP} = (-0.097 \pm 0.012)$ and $R = 0.899 \pm 0.048$, with $\delta_{+} \approx 29^\circ$, we get $\gamma = (60 \pm 3)^\circ$. However for $\delta_{+} \approx 20^\circ$, (which corresponds to $\delta_f - \Delta \approx 20^\circ$; the value one gets for $\rho^2 = 0.65$), we get $\gamma = (69 \pm 3)^\circ$.

The phase shift $\delta_{+} \approx (20 \sim 29)^\circ$ for the $K^+ \pi^-$ is compatible with the experimental value of the direct CP-asymmetry for $B^0 \rightarrow K^+ \pi^-$ decay mode. For $\pi^+ \pi^-$, $\delta_{+} \sim 31^\circ$ is compatible with the value $(33 \pm 7 \pm 8 - 10)^\circ$ obtained by the authors of ref. [13]. In any case, our analysis shows that the upper limit for final state strong phase $\delta_f$ is around 30°. Finally, we note that the actual value of the effective final state phase shift ($\delta_f - \Delta$) depends on one free parameter $\rho$; the factorization implies $\delta_f^T = 0$ i.e. $(\delta_f - \Delta) = 0$ for the tree amplitude; for the penguin amplitude, $\delta_f^P$ depends on $\rho$; in any case it can not be greater than the upper bound.

6 CP Asymmetries and Strong Phases

Case II:

Now, we discuss the experimental tests to verify the equality (implied by C-invariance of S-matrix) of phase shifts $\delta_f$ and $\delta_f^P$ for the decays $B \rightarrow \pi D, \pi D^*, \rho D$ and $B_s \rightarrow K D_s, K D_s^*, K^* D_s$.

From Eqs. (81), we note that CP-asymmetries:
\[ -\frac{S_- + S_+}{2} = \frac{2r_D}{1 + r_D^2} \sin(2\beta + \gamma) \cos(\delta_f - \delta_f^P) \]
\[ -\frac{S_+ - S_-}{2} = \frac{2r_D}{1 + r_D^2} \cos(2\beta + \gamma) \sin(\delta_f - \delta_f^P) \]

involve the weak phase $2\beta + \gamma$ and strong phase $\delta_f - \delta_f^P$. These asymmetries are of interest because for 
\[ \delta_f = \delta_f^P, \frac{S_- + S_+}{2} = 0 \]
and 
\[ -\frac{S_- + S_+}{2} = \frac{2r_D}{1 + r_D^2} \sin(2\beta + \gamma) \]

Hence we can verify the equality of phases $\delta_f$ and $\delta_f^P$ and determine the weak phase $2\beta + \gamma$.

For $B_s^0$, replace $r_D \rightarrow r_s$, $\delta_f \rightarrow \delta_f$, $\delta_f^P = \delta_f^P$, and $\beta$ by $\beta_s$. In standard model $\beta_s = 0$.

The experimental results for the B decays are as follows discussed in section 4

\[ \begin{array}{ccc}
\frac{S_- + S_+}{2} & D^- \pi^+ & D^- \rho^+\\
0.046 \pm 0.023 & -0.037 \pm 0.012 & -0.024 \pm 0.031 \pm 0.009 \\
\frac{S_- + S_+}{2} & D^- \pi^+ & D^- \rho^+\\
0.022 \pm 0.021 & -0.006 \pm 0.016 & -0.098 \pm 0.055 \pm 0.018 \\
\end{array} \]
To determine the parameter \( r_D \) or \( r_s \), we assume factorization for the tree amplitude. Factorization gives for the decays \( B^0 \to D^+\pi^-, D^{*+}\pi^-, D^+\rho^-, D^+a_1^- \):

\[
|\tilde{F}_f| = |\tilde{T}_f| = G[f_{\pi}(m_B^2 - m_\pi^2)f_0^{B-D}(m_\pi^2), 2f_{\pi}m_B|\bar{p}|A_0^{B-D^*}(m_\pi^2), 2f_{\pi}m_B|\bar{p}|f_+^{B-D}(m_\pi^2), 2f_{\pi}m_B|\bar{p}|f_+^{B-D}(a_1^2)] (139)
\]

\[
|F'_f| = |T'_f| = G[f_D(m_B^2 - m_\pi^2)f_0^{B-\pi}(m_D^2), 2f_Dm_B|\bar{p}|A_0^{B-\rho}(m_D^2), 2f_Dm_B|\bar{p}|A_0^{B-a_1}(m_D^2)] (140)
\]

\[
G = \frac{G_F}{\sqrt{2}}|V_{ud}|_V|V_{tb}|_F, \quad G' = \frac{G_F}{\sqrt{2}}|V_{cd}|_V|V_{tb}|_F (141)
\]

\[
\Gamma(B^0 \to D^+\pi^-) = |V_{cb}|^2|f_0^{B-D}(m_\pi^2)|^2(2.281 \times 10^{-9})\text{MeV}
\]

\[
\Gamma(B^0 \to D^{*+}\pi^-) = |V_{cb}|^2|A_0^{B-D^*}(m_\pi^2)|^2(2.129 \times 10^{-9})\text{MeV}
\]

\[
\Gamma(B^0 \to D^+\rho^-) = |V_{cb}|^2|f_+^{B-D}(m_\rho^2)|^2(5.276 \times 10^{-9})\text{MeV}
\]

\[
\Gamma(B^0 \to D^+a_1^-) = |V_{cb}|^2|f_+^{B-D}(m_{a_1}^2)|^2(5.414 \times 10^{-9})\text{MeV}
\]

(142)

| Decay | Decay Width \((10^{-9} \text{ MeV} \times |V_{cb}|^2)\) | Form Factor | Form Factors \( h(w^*) \) |
|-------|--------------------------------|--------------|-----------------|
| \( B^0 \to D^+\pi^- \) | \((2.281)|f_0^{B-D}(m_\pi^2)|^2\) | 0.58 ± 0.05 | 0.51 ± 0.03 |
| \( B^0 \to D^{*+}\pi^- \) | \((2.129)|A_0^{B-D^*}(m_\pi^2)|^2\) | 0.61 ± 0.04 | 0.54 ± 0.03 |
| \( B^0 \to D^+\rho^- \) | \((5.276)|f_+^{B-D}(m_\rho^2)|^2\) | 0.65 ± 0.11 | 0.57 ± 0.10 |
| \( B^0 \to D^+a_1^- \) | \((5.414)|f_+^{B-D}(m_{a_1}^2)|^2\) | 0.57 ± 0.31 | 0.50 ± 0.27 |

Table 1: Form Factors

The decay widths for the above channels are given in the table 1 where we have used

\[
a_1^2|V_{ud}|^2 \approx 1, \quad f_\pi = 131\text{MeV}, \quad f_\rho = 209\text{MeV}, \quad f_{a_1} = 229\text{MeV}
\]

(143)

Using the experimental branching ratios and

\[
|V_{cb}| = (38.3 \pm 1.3) \times 10^{-3}
\]

we obtain the corresponding form factors given in Table 1.

\[
|f_0^{B-D}(m_\pi^2)| = 0.58 \pm 0.05 \\
|A_0^{B-D^*}(m_\pi^2)| = 0.61 \pm 0.04 \\
|f_+^{B-D}(m_\rho^2)| = 0.65 \pm 0.11 \\
|f_+^{B-D}(m_{a_1}^2)| = 0.57 \pm 0.31
\]

(144)

In terms of Isgur Wise variables:

\[
\omega = v \cdot v', \quad v^2 = v'^2 = 1, \quad t = q^2 = m_B^2 + m_{D^*}^2 - 2m_Bm_{D^*}\omega
\]

(145)
the form factors can be put in the following form

\[ f_{+}^{B-D}(t) = \frac{m_B + m_D}{2\sqrt{m_B m_D}} h_{+}(\omega), \quad f_{0}^{B-D}(t) = \frac{\sqrt{m_B m_D}}{m_B + m_D} (1 + \omega) h_{0}(\omega) \]

\[ A_{2}^{B-D^*}(t) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}} (1 + \omega) h_{A_{2}}(\omega), \quad A_{0}^{B-D^*}(t) = \frac{m_B + m_{D^*}}{2\sqrt{m_B m_{D^*}}} h_{A_{0}}(\omega) \]

\[ A_{1}^{B-D^*}(t) = \frac{\sqrt{m_B m_{D^*}}}{m_B + m_{D^*}} (1 + \omega) h_{A_{1}}(\omega) \]

(146)

Heavy Quark Effective Theory (HQET) gives:

\[ h_{+}(\omega) = h_{0}(\omega) = h_{A_0}(\omega) = h_{A_1}(\omega) = h_{A_2}(\omega) = \zeta(\omega) \]

where \( \zeta(\omega) \) is Isgur-Wise form factor, with normalization \( \zeta(1) = 1 \). For

\[ t = m_{\pi}^2, m_{\rho}^2, m_{a_1}^2 \]

\[ \omega^* = 1.589(1.504), 1.559, 1.508 \]

we get the form factors h’s given in Table 1.

In reference, the value quoted for \( h_{A_1}(\omega^*_{max}) \) is

\[ |h_{A_1}(\omega^*_{max})| = 0.52 \pm 0.03 \]

(147)

Since \( \omega^*_{max} = 1.504 \), the value for \( |h_{A_0}(\omega_{max})| \) obtained in Table 1 is in remarkable agreement with the value given in Eq. (147) that assumption for \( B^0 \to \pi D^*(\pi) \) decays is experimentally on solid footing and is in agreement with HQET.

From Eqs. (139) and (140), we obtain

\[ r_D = \lambda^2 R_b \left| \frac{T_f}{T_f} \right| \]

\[ = \lambda^2 R_b \left[ \frac{f_D(m_B^2 - m_{\pi}^2)}{f_{\pi}(m_B^2 - m_D^2)} f_{0}^{B-\pi}(m_D^2), \quad f_D^* f_{\pi}^{B-\pi}(m_{\pi}^2), \quad f_D A_{0}^{B-\rho}(m_{\pi}^2) \right] \]

(148)

where

\[ \frac{|V_{ub}|}{|V_{cd}|} \frac{|V_{cb}|}{|V_{ud}|} = \lambda^2 R_b \approx (0.227)^2(0.40) \approx 0.021 \]

(149)

To determine \( r_D \), we need information for the form factors \( f_{0}^{B-\pi}(m_D^2), f_{+}^{B-\pi}(m_D^2), A_{0}^{B-\rho}(m_D^2) \). For these form factors, we use the following values:

\[ A_{0}^{B-\rho}(0) = 0.30 \pm 0.03, A_{0}^{B-\rho}(m_{D^*}^2) = 0.38 \pm 0.04 \]

\[ f_{+}^{B-\pi}(0) = f_{0}^{B-\pi}(0) = 0.26 \pm 0.04, \quad f_{\pi}^{B-\pi}(m_{\pi}^2) = 0.32 \pm 0.05, \quad f_{0}^{B-D}(m_{D}^2) = 0.28 \pm 0.04 \]

Along with the remaining form factors given in Table, we obtain

\[ r_D = [0.018 \pm 0.002, \quad 0.017 \pm 0.003, \quad 0.012 \pm 0.002] \]

(150)

The above value for \( r_D^* \) gives

\[ -\left( \frac{S_{+} + S_{-}}{2} \right)_{D^*} = 2(0.017 \pm 0.003) \sin(2\beta + \gamma) \]

(151)
The experimental value of the CP asymmetry for $B^0 \rightarrow D^* \pi$ decay has the least error. Hence we obtain the following bounds

$$\sin(2\beta + \gamma) > 0.69$$

$$44^\circ \leq (2\beta + \gamma) \leq 90^\circ$$

or

$$90^\circ \leq (2\beta + \gamma) \leq 136^\circ$$

Selecting the second solution, and using $\beta \approx 43^\circ$, we get

$$\gamma = (70 \pm 23)^\circ$$

To end this section, we discuss the decays $\bar{B}^0_s \rightarrow D^+_s K^-, D^{*+}_s K^-$ for which no experimental data is available. However, using factorization, we get

$$\Gamma( \bar{B}^0_s \rightarrow D^+_s K^-) = (1.75 \times 10^{-10})|V_{cb} f_{D^+_s - D^{*+}_s} (m^2_{D^*_s})|^2 MeV$$

$$\Gamma( \bar{B}^0_s \rightarrow D^{*+}_s K^-) = (1.57 \times 10^{-10})|V_{cb} A_{D^{*+}_s - D^{*+}_s} (m^2_{D^*_s})|^2 MeV$$

SU(3) gives

$$|V_{cb} f_{D^+_s - D^{*+}_s} (m^2_{D^*_s})|^2 \approx |V_{cb} |f_{B^0_s - D^{*+}_s} (m^2_{D^*_s})|^2 = (0.50 \pm 0.04) \times 10^{-3}$$

$$|V_{cb} A_{D^{*+}_s - D^{*+}_s} (m^2_{D^*_s})|^2 \approx |V_{cb} |A_{B^0_s - D^{*+}_s} (m^2_{D^*_s})|^2 = (0.56 \pm 0.04) \times 10^{-3}$$

From the above equations, we get the following branching ratios

$$\frac{\Gamma( \bar{B}^0_s \rightarrow D^{(*)+}_s K^-)}{\Gamma_{B^0_s}} = (1.94 \pm 0.07) \times 10^{-4}[(1.96 \pm 0.07) \times 10^{-4}]$$

For $\bar{B}^0_s \rightarrow D^{*+}_s K^-$

$$r_s = R_b \left[ \frac{f_{D^+_s f_{B^0_s - D^{*+}_s} (m^2_{D^*_s})}}{f_{K} A_{B^0_s - D^{*+}_s} (m^2_{D^*_s})} \right]$$

Hence we get

$$-(\frac{S_+ + S_-}{2})_{D^{*+}_s K} = (0.41 \pm 0.08) \sin(2\beta_s + \gamma)$$

$$= (0.41 \pm 0.08) \sin \gamma$$

where we have used

$$R_b = 0.40, \quad \frac{f_{D^+_s}}{f_{K}} = 1.75 \pm 0.06, \quad \frac{f_{B^0_s - K^{(*)} (m^2_{D^*_s})}}{f_{K}} = 0.34 \pm 0.06$$

$$A_{B^0_s - D^{*+}_s} (m^2_{D^*_s}) = A_{B^0_s - D^{*+}_s} (0) = \frac{m_{B^0_s} + m_{D^{*+}_s}}{2\sqrt{m_{B^0_s}m_{D^{*+}_s}}} [\omega^*_{D^{*+}_s} = 1.453] = 0.52 \pm 0.03$$

$$= 0.58 \pm 0.03$$

Case III
We now confine ourselves to $B^0(\bar{B}^0) \to \rho^-\pi^+, \rho^+\pi^-(\rho^+\pi^-, \rho^-, \pi^+)$ decays only [13,14]. The experimental results for these decays are [6] as

$$\Gamma = R_f + R_f = (22.8 \pm 2.5) \times 10^{-6} \tag{163}$$

$$A_{CP}^f = -0.16 \pm 0.23, \quad A_{CP}^f = 0.08 \pm 0.12 \tag{164}$$

$$C = 0.01 \pm 0.14, \quad \Delta C = 0.37 \pm 0.08 \tag{165}$$

$$S = 0.01 \pm 0.09, \quad \Delta S = -0.05 \pm 0.10 \tag{166}$$

With the above values, it is hard to draw any reliable conclusion. Neglecting the term $A_{CP}C$ in Eqs. (112) and (113), we get

$$R_{\bar{f},f} = \frac{1}{2} \Gamma(1 \pm \Delta C) \tag{167}$$

$$R_{\bar{f}} - R_f = \Delta C \tag{168}$$

Using the above value for $\Delta C$, we obtain

$$R_{\bar{f}} = (15.6 \pm 1.7) \times 10^{-6} \tag{169}$$

$$R_f = (7.2 \pm 0.8) \times 10^{-6}$$

We analyze these decays by assuming factorization for the tree graphs[19]. This assumption gives

$$T_{\bar{f}} = \bar{T}_f \sim 2m_B f_\rho |\bar{p}| f_+(m^2_\rho) \tag{170}$$

$$T_f = \bar{T}_f \sim 2m_B f_\pi |\bar{p}| A_0(m^2_\pi) \tag{171}$$

Using $f_+(m^2_\rho) \approx 0.26 \pm 0.04$ and $A_0(m^2_\pi) \approx A_0(0) = 0.29 \pm 0.03$ and $|V_{ub}| = (3.5 \pm 0.6) \times 10^{-3}$, we get the following values for the tree amplitude contribution to the branching ratios

$$\Gamma^\text{tree}_f = (15.6 \pm 1.1) \times 10^{-6} \equiv |T_f|^2 \tag{172}$$

$$\Gamma^\text{tree}_f = (7.6 \pm 1.4) \times 10^{-6} \equiv |T_f|^2 \tag{173}$$

$$t = \frac{T_f}{T_{\bar{f}}} = \frac{f_\pi A_0(m^2_\pi)}{f_\rho f_+(m^2_\rho)} = 0.70 \pm 0.12 \tag{174}$$

Now

$$B_{\bar{f}} = \frac{R_{\bar{f}}}{|T_{\bar{f}}|^2} = 1 - 2r_f \cos \alpha \cos \delta_{\bar{f}} + r^2_{\bar{f}} \tag{175}$$

$$B_f = \frac{R_f}{|T_f|^2} = 1 - 2r_f \cos \alpha \cos \delta_f + r^2_f \tag{176}$$

Hence from Eqs. (169) and (173), we get

$$B_{\bar{f}} = 1.00 \pm 0.12$$

$$B_f = 0.95 \pm 0.11 \tag{177}$$
In order to take into account the contribution of penguin diagram, we introduce the angles \( \alpha'_{\text{eff}} \), defined as follows

\[
e^{i\alpha} A_{f,f} = |A_{f,f}| e^{-i\alpha'_{\text{eff}}}
\]
\[
e^{-i\beta} \bar{A}_{f,f} = |\bar{A}_{f,f}| e^{i\alpha'_{\text{eff}}}
\]

With this definition, we separate out tree and penguin contributions:

\[
e^{i\alpha} A_{f,f} - e^{-i\beta} \bar{A}_{f,f} = |A_{f,f}| e^{-i\alpha'_{\text{eff}}} - |\bar{A}_{f,f}| e^{i\alpha'_{\text{eff}}} = 2iT_{f,f} \sin \alpha
\]
\[
e^{i(\alpha + \beta)} A_{f,f} - e^{-i(\alpha + \beta)} \bar{A}_{f,f} = |A_{f,f}| e^{-i\alpha_{\text{eff}} - \alpha} = (2iT_{f,f} \sin \alpha) r_{f,f} e^{i\delta_{\text{eff}}}
\]

From Eq. (179), we get

\[
\left| \frac{T_{f,f}}{R_{f,f}} \right|^2 \sin^2 \alpha \equiv \frac{2\sin^2 \alpha}{B_{f,f}} = 1 - \sqrt{1 - A_{CP}^2 \cos 2\alpha'_{\text{eff}}} = 1 - \sqrt{1 - A_{CP}^2 \cos 2\alpha'_{\text{eff}}}
\]

\[
\sin 2\delta_{\text{eff}} = -A_{CP} \frac{\sin 2\alpha'_{\text{eff}}}{1 - \sqrt{1 - A_{CP}^2 \cos 2\alpha'_{\text{eff}}}}
\]

\[
\cos 2\delta_{\text{eff}} = \frac{1 - A_{CP}^2 \cos 2\alpha_{\text{eff}}}{1 - \sqrt{1 - A_{CP}^2 \cos 2\alpha'_{\text{eff}}}}
\]

From Eqs. (179) and (180), we get

\[
r_{f,f}^2 = \frac{1 - \sqrt{1 - A_{CP}^2 \cos 2\alpha'_{\text{eff}} - 2\alpha}}{1 - \sqrt{1 - A_{CP}^2 \cos 2\alpha'_{\text{eff}}}}
\]

\[
r_{f,f} \cos \delta_{\text{eff}} = \frac{\cos \alpha - \sqrt{1 - A_{CP}^2 \cos (2\alpha'_{\text{eff}} - \alpha)}}{1 - \sqrt{1 - A_{CP}^2 \cos 2\alpha'_{\text{eff}}}}
\]

\[
r_{f,f} \sin \delta_{\text{eff}} = \frac{-A_{CP} / \sin \alpha}{1 - \sqrt{1 - A_{CP}^2 \cos 2\alpha'_{\text{eff}}}}
\]

Now factorization implies \[23\]

\[
\delta_{\text{eff}}^T = 0 = \delta_{\text{eff}}^T
\]
Thus in the limit \( \delta_f^T \to 0 \), we get for Eq. (183)

\[
\cos 2\alpha_{eff}^f = -1, \quad \alpha_{eff}^f = 90^\circ
\]

\[
r_{f,f} \cos \delta_{f,f} = \cos \alpha
\]

\[
r_{f,f} \sin \delta_{f,f} = \frac{-A_{eff}^f}{\sqrt{1 - A_{eff}^f}}
\]

\[
r_{f,f}^2 = \frac{1 + \sqrt{1 - A_{eff}^f}}{1 - A_{eff}^f}
\]

\[
\approx \cos^2 \alpha + \frac{1}{4} A_{eff}^f \sin^2 \alpha
\]

The solution of Eq. (190) is graphically shown in Fig. 7 for \( \alpha \) in the range \( 80^\circ \leq \alpha < 103^\circ \) for \( r_{f,f} = 0.10, 0.15, 0.20, 0.25, 0.30 \). From the figure, the final state phases \( \delta_{f,f} \) for various values of \( r_{f,f} \) can be read for each value of \( \alpha \) in the above range. Few examples are given in Table 2

| \( \alpha \) | \( r_f \) | \( \delta_f \) | \( A_{CP}^f \approx -2r_f \sin \delta_f \sin \alpha \) |
|-----|-----|-----|-----|
| 80° | 0.20 | 29° | -0.19 |
|     | 0.25 | 46° | -0.36 |
| 82° | 0.15 | 22° | -0.11 |
|     | 0.20 | 46° | -0.28 |
| 85° | 0.10 | 29° | -0.10 |
|     | 0.15 | 54° | -0.24 |
| 86° | 0.10 | 46° | -0.14 |
|     | 0.15 | 62° | -0.26 |
| 88° | 0.10 | 70° | -0.19 |

Table 2:

For \( \alpha > 90^\circ \), change \( \alpha \to \pi - \alpha \), \( \delta_f \to \pi - \delta_f \). For example, for \( \alpha = 103^\circ \)

\[
r_f = 0.25, \quad \delta_f = 154^\circ, \quad A_{CP}^f \approx -0.22
\]

\[
r_f = 0.30, \quad \delta_f = 138^\circ, \quad A_{CP}^f \approx -0.40
\]

These examples have been selected keeping in view that final state phases \( \delta_{f,f} \) are not too large. For \( A_{CP}^f \), we have used Eq. (191) neglecting the second order term. An attractive option is \( A_{CP}^f = A_{CP}^\bar{f} \) for each value of \( \alpha \); although \( A_{CP}^f \neq A_{CP}^\bar{f} \) is also a possibility. \( A_{CP}^f = A_{CP}^\bar{f} \) implies \( r_f = r_{\bar{f}}, \delta_f = \delta_{\bar{f}} \).

Neglecting terms of order \( r_{f,\bar{f}}^2 \), we have

\[
A_{CP} \approx \frac{2 \sin \alpha (r_f \sin \delta_f - t^2 r_f \sin \delta_f)}{1 + t^2} = -\frac{A_{CP}^f - t^2 A_{CP}^\bar{f}}{1 + t^2}
\]

\[
C \approx -\frac{2t^2}{(1 + t)^2} (A_{CP}^f + A_{CP}^\bar{f})
\]

\[
\Delta C \approx \frac{1 - t^2}{1 + t^2} - \frac{4t^2 \cos \alpha}{(1 + t)^2} (r_f \cos \delta_f - r_{\bar{f}} \cos \delta_{\bar{f}})
\]
Now the second term in Eq. (196) vanishes and using the value of \( t \) given in Eq. (??), we get
\[
\Delta C \approx 0.34 \pm 0.06 \quad (197)
\]
Assuming \( A_{CP}^f = A_{CP}^\bar{f} \), we obtain
\[
\begin{align*}
A_{CP} &= -\frac{1 - t^2}{1 + t^2} A_{CP}^f \\
&= (0.34 \pm 0.06)(-A_{CP}^\bar{f}) \\
C &\approx -\frac{4t^2}{(1 + t^2)^2} A_{CP}^f \approx -(0.88 \pm 0.14) A_{CP}^f \quad (198)
\end{align*}
\]
Finally the CP asymmetries in the limit \( \delta_{f,\bar{f}}^T \to 0 \)
\[
\begin{align*}
S_f &= S + \Delta S = \frac{2\text{Im}[e^{2i\theta_M} A_f A_{\bar{f}}]}{\Gamma (1 - A_{CP})} \\
&= \sqrt{1 - C_f^2} \sin(2\alpha_{eff}^f + \delta)
\end{align*}
\]
\[
\begin{align*}
S_{\bar{f}} &= S - \Delta S = \frac{2\text{Im}[e^{2i\theta_M} A_f A_{\bar{f}}]}{\Gamma (1 - A_{CP})} \\
&= \sqrt{1 - C_f^2} \sin(2\alpha_{eff}^f - \delta)
\end{align*}
\]
\[
\begin{align*}
\Delta S &= \sqrt{1 - C_f^2} \cos \delta
\end{align*}
\]
The phase \( \delta \) is defined as
\[
\bar{A}_f = \frac{\bar{A}_f}{|A_f|} A_{\bar{f}} e^{i\delta} \quad (202)
\]
Hence we have
\[
\frac{S + \Delta S}{S - \Delta S} = -\sqrt{1 - C_f^2} \sqrt{1 - C_f^2}
\]

7 Conclusion

In weak interaction, both P and C are violated but CP is conserved by the weak interaction Lagrangian. Hence for \( X^0 - \bar{X}^0 \) complex (\( X^0 = K^0, B^0, B_s^0 \)): the mass matrix is not diagonal in \( |X^0\rangle \) and \( |\bar{X}^0\rangle \) basis. However, assuming \( CP \) conservation, the \( CP \) eigenstates \( |X_1^0\rangle \) and \( |X_2^0\rangle \) can be mass eigenstates and hence mass matrix is diagonal in this basis. The two sets of states are related to each other by superposition principle of quantum mechanics. This gives rise to quantum mechanical interference so that even if we start with a state \( |X^0\rangle \), the time evolution of this state can generate the state \( |X^0\rangle \). This is a source of mixing induced \( CP \) violation. However, both in \( K^0 - \bar{K}^0 \) and \( B^0 - \bar{B}^0 \) complex, the mass eigenstates \( |K^0_s\rangle \), \( |K^0_L\rangle \) and \( |B^0_L\rangle \), \( |B^0_R\rangle \) are not \( CP \) eigenstates. In the case of \( K^0 - \bar{K}^0 \) complex, there is a small admixture of wrong \( CP \) state characterized by a small parameter \( \epsilon \), which gives rise to the \( CP \) violating decay \( K^0_L \to \pi^+\pi^- \). This
was the first $CP$ violating decay observed experimentally. For $B^0 - \bar{B}^0$ complex, the mismatch between mass eigenstates and $CP$ eigenstates $|B^0_1\rangle$ and $|B^0_2\rangle$ is given by the phase factor $e^{2i\phi_M}$ where the phase factor is $\phi_M = -\beta$ in the standard model viz. one of the phases in the CKM matrix. For $B^0_s - \bar{B}^0_s$, there is no mismatch between $CP$ eigenstates $|B^0_{1s}\rangle$ and $|B^0_{2s}\rangle$ and the mass eigenstates. There is no extra phase available in CKM matrix, with three generations of quarks to accommodate more than two independent phases $\beta$ and $\gamma$; the unitarity of CKM matrix requires $\alpha + \beta + \gamma = \pi$.

The quantum mechanical interference gives rise to non-zero mass differences $\Delta m_K$, $\Delta m_B$ and $\Delta m_{B_s}$ between mass eigenstates. The mixing induced $CP$ violation involves these mass differences.

The $CPT$ invariance plays an important role in $CP$ violation in weak decays. $CPT$ invariance gives:

$$\tilde{A}_f = \eta_f e^{2i\delta_f} A^*_f, \quad A_f = e^{i\delta_f} e^{i\phi_f} |A_f|$$

where $A_f$ and $\tilde{A}_f$ are the amplitudes for the decays $X \to f$ and $\bar{X} \to \bar{f}$, the states $|f\rangle$ and $|\bar{f}\rangle$ being $CP$ conjugate of each other. For direct $CP$ violation, at least two amplitudes with different weak phases are required:

$$A_f = A_{1f} + A_{2f}$$

$CPT$ gives:

$$\tilde{A}_f = e^{2i\delta_f} A^*_{1f} + e^{2i\delta_{2f}} A^*_{2f}$$

$$A_{1f} = e^{i\delta_{1f}} e^{i\phi_{1f}} |A_{1f}|$$

where $(\delta_{1f}, \delta_{2f})$, $(\phi_{1f}, \phi_{2f})$ are strong final state phases and the weak phases respectively. Thus the direct $CP$ violation is given by

$$A_{CP} = \frac{\tilde{\Gamma}(X \to \bar{f}) - \Gamma(X \to f)}{\tilde{\Gamma}(X \to \bar{f}) + \Gamma(X \to f)}$$

where $\delta_f = \delta_{2f} - \delta_{1f}$, $\phi = \phi_2 - \phi_1$. Hence the necessary condition for non-zero direct $CP$ violation is $\delta_f \neq 0$ and $\phi \neq 0$.

In section 2, the $CP$ violation due to mismatch between $CP$ eigenstates $|K^0_1\rangle$, $|K^0_2\rangle$ and mass eigenstates $|K^0_{1s}\rangle$ and $|K^0_{2s}\rangle$ in terms of the parameter $\epsilon$ and direct $CP$ violation due to different weak phases between the decay amplitudes $A_0$ and $A_2$ are discussed.

**Section 4:**

**Case I**

The $CP$ violation for $B^0 \to f$ decay where $|\bar{f}\rangle = CP |f\rangle = |f\rangle$ are discussed. In particular for the decay $B^0 \to J/\psi K^0_S$ described by a single amplitude $A_f$, the $CP$ asymmetry is given by

$$A_{f/\psi K_S} = -\sin 2\beta \frac{\langle \Delta m_B / \Gamma \rangle}{1 + (\Delta m_B / \Gamma)}$$

It is a good illustration of $CP$ violation due to mismatch between mass and $CP$ eigenstates, involving the mixing parameter $\Delta m_B$. From the experimental values of $A_{f/\psi K_S}$ and $(\Delta m / \Gamma)_{B^0}$, the weak phase $2\beta$ is found to be $(43 \pm 3)\degree$. Corresponding to $B^0 \to J/\psi K^0_S$, we have $B^0_\Sigma \to J/\psi \phi$ and for this decay

$$A_{f/\psi} = -\sin 2\beta_s \frac{\langle \Delta m_{B^0_\Sigma} / \Gamma_S \rangle}{1 + (\Delta m_{B^0_\Sigma} / \Gamma_S)^2}$$

Any finite value of $A_{f/\psi}$ would imply $\beta_s \neq 0$ in contradiction with the standard model.
In this section for the case (i), both direct and mixing induced $CP$ violation viz. $A_{CP}$, $C_f$ and $S_f$ for $B^0 \to \pi^+\pi^-$ described by two amplitudes $T$ and $P_t$ given by tree and penguin diagrams is discussed. We find $C_{\pi\pi} = -A_{CP}(\pi\pi)$ and $S_{\pi\pi}$ is essentially given by

$$S_{\pi\pi} \approx (\sin 2\alpha + 2r \cos \delta \sin \alpha \cos 2\alpha), \quad r = \frac{R_t |P_t|}{R_b |T|}$$

$S_{\pi\pi} \neq 0$ even when final state phase $\delta = 0$.

**Case II**

We consider the decays described by two independent decay amplitudes $A_f$ and $A'_{\bar{f}}$ with different weak phases ($O$ and $\gamma$) where the final states $|f\rangle$ and $|\bar{f}\rangle$ are $C$ and $CP$ conjugate of each other such as the states $D^{(*)-}\pi^+$ ($D^{(*)+}\pi^-$), $D_s^{(*)-}K^+$ ($D_s^{(*)+}K^-$), $D^-\rho^+$ ($D^+\rho^-$).

It is argued in section 5, that $C$ and $CP$ invariance of hadronic interactions imply $\delta_f = \delta'_{\bar{f}}$.

As discussed in section 6, the equality of phases $\delta_f = \delta'_{\bar{f}}$ implies that time-dependent $CP$ asymmetries:

$$-\left(\frac{S_+ + S_-}{2}\right) = \frac{2r_{D^{(*)}}}{1 + 2r_{D^{(*)}}^2} \sin(2\beta + \gamma)$$

$$\frac{S_+ - S_-}{2} = 0$$

It is further shown that from the experimental value of $\frac{S_+ + S_-}{2}$ for $B^0 \to D^{*-}\pi^+$

$$\sin(2\beta + \gamma) > 0.69$$

$$44^\circ \leq 2\beta + \gamma \leq 90^\circ \quad \text{or} \quad 90^\circ \leq 2\beta + \gamma \leq 136^\circ$$

Selecting the second solution and using $2\beta \approx 43^\circ$, we get

$$\gamma = (70 \pm 23)^\circ$$

Using $SU(3)$, for the form factors for $B^0_s \to D^{*-}K^+$, we predict

$$-\left(\frac{S_+ + S_-}{2}\right) = (0.41 \pm 0.08) \sin(2\beta_s + \gamma)$$

In the standard model $\beta_s = 0$.

**Case III**

For the case (III) for which $A_f \neq A'_{\bar{f}}$ such as $B^0 \to \rho^+\pi^-$ : $A_f$ and $B^0 \to \rho^-\pi^+$ : $A_f$ where $A_{f,\bar{f}}$ are given by tree amplitude $e^{i\gamma}T_{f,\bar{f}}$ and penguin amplitude $e^{-i\beta}P_{f,\bar{f}}$ are discussed.

In section 6 case (iii), the factorization for the tree graph implies $\delta^T_{f,\bar{f}} \approx 0$. In the limit $\delta^T_{f,\bar{f}} \to 0$, it is shown that

$$r_{f,\bar{f}} \cos \delta_{f,\bar{f}} = \cos \alpha$$

$$r^2_{f,\bar{f}} \approx \cos^2 \alpha + A_{CP}^2 \sin^2 \alpha$$

Finally, in the limit $\delta^T_{f,\bar{f}} \to 0$, we get

$$\frac{S_f}{S_{\bar{f}}} = \frac{S + \Delta S}{S - \Delta S} = -\sqrt{\frac{1 - C^2_f}{1 - C^2_{\bar{f}}}}$$

To conclude:
1. No evidence that space-time symmetries are violated by fundamental laws of nature. The Translational and Rotational symmetries imply that space is homogeneous and isotropic.

Translational Symmetry $\Rightarrow$ Energy Momentum Conservation

Rotational Symmetry $\Rightarrow$ Angular Momentum Conservation

If we examine the light emitted by a distant object billions of light years away, we find that atoms have been following the same laws as they are here and now. (Translational Symmetry)

2. Discrete Symmetries are not universal; both C and P are violated in the weak interaction but respected by electromagnetic and strong interactions. There is no evidence for violation of time reversal invariance by any of the fundamental laws of nature.

3. Basic weak interaction Lagrangian is CP conserving. CP violation in weak interactions is a consequence of mismatch between mass eigenstates and CP eigenstates and or mismatch between weak and mass eigenstates at quark level. There is no evidence of CP violation in Lepton sector. There is no evidence that CP invariance is violated by any of the fundamentals laws of nature as implied by CPT invariance and T-invariance.

4. CP violation in weak decays is an example where basic laws are CP invariant but states at quark level contain CP violating phases.

5. The fundamental interaction governing atoms and molecules is the electromagnetic interaction which does not violate bilateral symmetry (left-right symmetry). In nature we find organic molecules in asymmetric form, i.e. left handed or right handed. This is another example where the basic laws governing these molecules are bilateristic symmetric but states are not. (Asymmetric initial conditions?)

6. **Baryon Asymmetry of the Universe: Baryogenesis:** No evidence for existence of antibaryons in the universe. $\eta = n_B/n_\gamma \sim 3 \times 10^{-10}$. The universe started with a complete matter antimatter symmetry in big bang picture. In subsequent evolution of the universe, a net baryon number is generated. This is possible provided the following conditions of Sakharov are satisfied

(a) There exists a baryon number violating interaction.

(b) There exist C and CP violation to induce the asymmetry between particle and antiparticle processes.

(c) Departure from thermal equilibrium of X-particles which mediate the baryon number violating interactions.

7. There seems to be no connection between CP violation required by baryogenesis and CP violation observed in weak decays.

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**Figure Captions:**

Figure 1 The Unitarity triangle

Figure 2 The Box Diagram

Figure 3 The Tree Diagram

Figure 4 The Penguin Diagram

Figure 5 (a) $W$-exchange diagram for $B^0_q \to N_1 \bar{N}_2 (M_f)$;
(b) $W$-exchange diagram for $B^0_q \to N_1 \bar{N}_2 (M_f')$

Figure 6 Annihilation diagram for $B^- \to N_1 \bar{N}_2$

Figure 7 Plot of equation $r_f \cos \delta_f = \cos \alpha$ for different values of $r$. For $80^\circ \leq \alpha \leq 103^\circ$. Where solid curve, dashed curve, dashed doted curve, dashed bouble doted and double dashed doted curve are corresponding to $r = 0.1$, $r = 0.15$, $r = 0.2$, $r = 0.25$ and $r = 0.3$ respectively.
This figure "Figure1-TheUnitaritytriangle.jpg" is available in "jpg" format from:

http://arxiv.org/ps/0907.3285v2
This figure "Figure2-TheBoxDiagram.jpg" is available in "jpg" format from:

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