CFD Based Prediction of Discharge Coefficient of Sonic Nozzle with Surface Roughness

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Abstract. Due to its simplicity and accuracy, sonic nozzle is widely used in gas flow measurement, gas flow meter calibration standard, and flow control. The nozzle obtains mass flow rate by measuring temperature and pressure in the inlet during choked flow condition and calculate the flow rate using the one-dimensional isentropic flow equation multiplied by a discharge coefficient, which takes into account multiple non-isentropic effects, which causes the reduction in mass flow. Proper determination of discharge coefficient is crucial to ensure the accuracy of mass flow measurement by the nozzle. Available analytical solution for the prediction of discharge coefficient assumes that the nozzle wall is hydraulically smooth which causes disagreement with experimental results. In this paper, the discharge coefficient of sonic nozzle is determined using computational fluid dynamics method by taking into account the roughness of the wall. It is found that the result shows better agreement with the experiment data compared to the analytical result.

Keywords: sonic nozzle, discharge coefficient, boundary layer, mass flow, roughness, Computational Fluid Dynamics

Nomenclature

| Symbol | Definition |
|--------|------------|
| $A_t$  | Throat area, $m^2$ |
| $C^*$  | Critical flow function, dimensionless |
| $C_d$  | Discharge coefficient, dimensionless |
| $d$    | Throat diameter, $m$ |
| $m$    | Parameter defined by Geropp, dimensionless |
| $\dot{m}_{\text{actual}}$ | Actual mass flow rate ($kg \ s^{-1}$) |
| $\dot{m}_{\text{ideal}}$ | Isentropic one-dimensional mass flow rate ($kg \ s^{-1}$) |
| $p_0$  | Inlet stagnation pressure, $Pa$ |
| $R$    | Specific gas constant (dry air, $287 \ j \ kg^{-1} \ K^{-1}$) |
| $Re$   | Throat Reynolds number, dimensionless |
| $R_{\text{wall}}$ | Wall curvature radius, $m$ |
| $T_0$  | Inlet stagnation temperature, Kelvin |
| $\mu$  | Dynamic viscosity (dry air, 0.00001831 $Pa \ s$) |
| $\gamma$ | Specific heat ratio, dimensionless (dry air, 1.4) |
| $y^+$  | Dimensionless distance from wall |
1. Introduction

Unlike other types of flow measurement devices such as turbines or variable area flow meters, sonic nozzle requires no moving part, thus extending its life and reliability. Since it operates at choked flow condition, any pressure changes downstream of the nozzle may not affect mass flow through the nozzle. Therefore, it is a suitable choice for accurate gas flow measurement and flow control.

The nozzle operates at choked flow condition. Thus, when the inlet pressure and temperature is known (usually done by placing pressure and temperature sensor upstream of the nozzle), mass flow rate can then be calculated using isentropic one-dimensional flow equation multiplied by a discharge coefficient which serves as mass flow correction factor which takes into account non-isentropic and non-one dimensional effects. Accurate determination of the discharge coefficient is crucial in order to ensure accurate reading of mass flow.

Previous analytical discharge coefficient formulas derived by Hall [1] (which considers the effects of boundary layer in the nozzle wall, \( C_{d1} \)) and Geropp [2] (which considers multi-dimensional flow effects depending on geometry of nozzle, \( C_{d2} \)) assumed that the nozzle wall was hydraulically smooth [3], which can cause the formula to overestimate the experimental discharge coefficient.

In this study, Computational Fluid Dynamics is used to determine the discharge coefficient of sonic nozzle with surface roughness. The inlet pressure boundary condition is varied in accordance with the experiment data in order to observe the effect of Reynolds number to the discharge coefficient, and the back pressure is lowered sufficiently to ensure critical flow at the throat section and to avoid shock formation in divergent section of the nozzle [4].

2. Theoretical Discharge Coefficient

The dimensions of the nozzle and the inlet pipe in millimetre is shown in figure 1. Discharge coefficient \( C_d \) is the ratio of actual critical mass flow rate \( \dot{m}_{actual} \) to the ideal critical mass flow rate \( \dot{m}_{ideal} \)

\[
C_d = \frac{\dot{m}_{actual}}{\dot{m}_{ideal}}
\]

(1)

The critical mass flow rate \( \dot{m}_{ideal} \) for one-dimensional isentropic flow of an ideal gas can be calculated by [5]

\[
\dot{m}_{ideal} = \frac{A_t C^* p_0}{\sqrt{R T_0}}
\]

(2)

Where \( C^* \) (critical flow function) is given by

\[
C^* = \sqrt{\gamma \left( \frac{2}{\gamma+1} \right)^{\gamma+1}}
\]

(3)

The research of \( C_d \) is divided into three parts [3], namely: \( C_{d1} \) which considers mass flow reduction by viscous effects of boundary layers in the viscous region of the flow, \( C_{d2} \) which considers mass flow reduction in the inviscid region of the flow caused by the geometry of the nozzle, and \( C_{d3} \) (also called viral discharge coefficient) which considers the physical property of the gas, which is negligible when the gas is approximately ideal [4]. In this study, only \( C_{d1} \) and \( C_{d2} \) is considered. For Hall-Geropp nozzle, \( C_d \) can be expressed as

\[
C_d = C_{d1} C_{d2}
\]

(4)

Formulas for determining \( C_{d1} \) for laminar and turbulent flows in smooth wall nozzle are as follows [2, 4, 6]
\[ C_{d1} = \begin{cases} 1 - \frac{4}{\sqrt{Re.m}} \left( \frac{\gamma+1}{2} \right)^{\frac{1}{2(\gamma-1)}} \left( 3\sqrt{2} - 2\sqrt{3} + \frac{\gamma-1}{\sqrt{3}} \right), & \text{laminar flow} \vspace{0.5cm} \\ 1 - 0.00525 \left( \frac{d}{R_{\text{wall}}} \right)^{-0.4} \text{Re}^{-0.2}, & \text{turbulent Flow} \end{cases} \] (5)

Where

\[ Re = \frac{4\dot{m}_{\text{actual}}}{\pi d \mu} \] (6)

And

\[ m = \frac{2d}{R_{\text{wall}}} \left( \frac{\gamma+1}{2} \right)^{\frac{3\gamma-1}{\gamma-1}} \] (7)

While the formula for determining \( C_{d2} \) is as follows [1]

\[ C_{d2} = 1 - \frac{\gamma+1}{(2R_{\text{wall}}/d)^2} \left( \frac{1}{96} - \frac{8\gamma+21}{4608(2R_{\text{wall}}/d)^2} + \frac{754\gamma^2+1971\gamma+2007}{552960(2R_{\text{wall}}/d)^2} \right) \] (8)

Therefore, since the \( R_{\text{wall}}/d \) of the nozzle is 2.2, \( \gamma = 1.4 \), and it is assumed (due to the large Reynolds number regime in the experiment) that the boundary layer is turbulent, the theoretical discharge coefficient is given by

\[ C_d = (1 - 0.0071966 \text{Re}^{-0.2}) \times 0.998978 \] (9)

Figure 1 Inlet pipe and nozzle dimensions

3. CFD Simulation

3.1. Geometry and Meshing

3D model of the nozzle and inlet pipe is shown in figure 2. ICEM CFD software is used for meshing. Patch independent, quad dominant shell mesh is used as well as tetra/mixed volume meshing with octee mesh method. The cross section of the mesh is shown in Figure 3. The Prism mesh is used in the entire wall to ensure accurate simulation of boundary layer. The prism layers start in \( y^+ = 5 \), extending with a growth ratio of 1.2 for 17 layers as shown in figure 4.

Mesh quality values is used to assess the quality of the mesh and to ensure no negative volume exist in the mesh. Minimum element quality is 0.22 with maximum mesh quality of 1, averaging at 0.83 for a total of 9064479 elements. Therefore, the mesh is suitable for the simulation.
Figure 2 3D model

Figure 3 Cross section of the volume mesh

Figure 4 Prism layers
3.2. Solver Setting

CFX solver is used to solve Reynolds-Averaged steady state Navier-Stokes equation with shear stress transport turbulence model. It is assumed that the wall is adiabatic, and three different wall condition (smooth wall and rough wall with two different roughness) as well as inviscid flow condition is simulated. Wall roughness of both 1 micron and 2 micron is simulated to observe the effect of increased wall roughness to the discharge coefficient.

A total of five different inlet pressure is used in the simulation to observe the change of discharge coefficient with respect to change to Reynolds number. Aside from RMS residual target of 0.0001, conservation target of 0.001 is also used as convergence criterion to ensure no imbalances in the domain.

4. Results

All simulations satisfy the convergence criterion below the maximum number of iteration of 150. Figures 5, 6, 7 and 8 show the Mach number distribution for invsicid flow, viscous flow with smooth wall, viscous flow with wall roughness of 1 micron, and viscous flow with wall roughness of 2 micron, respectively. It can be seen that for viscous flows, both with smooth and rough walls, the viscosity effect cause reduction in Mach number near the walls of the nozzle, which forms the boundary layer.

Figure 9 shows the distribution of unit mass flow (product of flow density and velocity) at the wall of throat. As shown in the figure, there is a reduction of unit mass flow inside the boundary layer. This reduction in unit mass flow causes mass flow to be smaller compared to the ideal mass flow. As shown by the increasing wall distance in which the unit mass flow is equal to the freestream/core flow, the figure also shows that increasing wall roughness causes boundary layer thickness to increase as well, meaning that the boundary layer in the nozzle with rougher wall have larger displacement thickness, causing smaller effective throat radius compared to the smooth wall nozzle.

Figure 10 shows that this further reduction in effective throat diameter in nozzle with rough wall results in the decrease of discharge coefficient (thus in better agreement with the experiment result) compared to the nozzle with the smooth wall. For this simulation, a wall roughness of 2 micrometres causes the CFD result to better predict the experiment result by at least 43 percent compared to the smooth wall condition CFD.
Figure 6 Viscous flow Mach number distribution with smooth wall

Figure 7 Viscous flow Mach number distribution with 1 micron roughness wall
Figure 8 Viscous flow Mach number distribution with 2 micron roughness wall

Figure 9 Unit mass flow distribution inside boundary layer
Conclusion
The effect of surface roughness on discharge coefficient of sonic nozzle is analyzed using CFD. Five different inlet pressure is simulated to show the change in discharge coefficient with respect to Reynolds number, as well as two wall conditions, smooth and rough. Two different roughness values of 1 and 2 microns are simulated to observe the effect of increased roughness. The outlet pressure is set to be sufficiently small to avoid formation of normal shocks in the diffuser section of the nozzle which can cause difficulty in achieving convergence criterion. Prism mesh is used in wall region to ensure accurate capture of boundary layer. The CFX Solver is used to solve Reynolds-Averaged Navier-Stokes equation with shear stress turbulence modelling. All simulation reaches both RMS and conservation target of 0.0001 and 0.001, respectively, within 150 iteration steps.

The results are compared with existing analytical formula which assumes that the surface of nozzle is hydraulically smooth and confirmed by comparing the results to experimental data. As shown in the CFD result, viscosity effect causes unit mass flow defect inside the boundary which in turn causes reduction in mass flow in the nozzle. The boundary layer in the nozzle with the rougher wall is thicker than the nozzle with smooth wall, causing it to have more unit mass flow defect. As a result, the nozzle with the rougher wall has lower discharge coefficient over the entire simulated region of Reynolds number, and the nozzle with 2 micron wall roughness is found to be in better agreement with the experiment result.

References
1. I.M. Hall, Transonic flow in two-dimensional and axially-symmetric nozzles, Q. J.Mech. Appl. Math. (Pt. 4) (1962) 487–508.
2. D. Geropp, Laminare Grenzschichten in ebenen und Rotation-Symmetrischen Lavaldüsen, Deutsche Luft-und Raumfahrt Forschungsbericht, German, 1971, pp.71–90.
3. H. Ding et al, Approximate solution for discharge coeffecient of the sonic nozzle with surface roughness, Flow Measurement and Instrumentation 52 (2016) 227–232
4. H. Ding et al, Influence of divergent section on flow fields and discharge coefficient of ISO 9300 toroidal-throat sonic nozzle, Flow Measurement and Instrumentation 40 (2014)19-27
5. ISO 9300. Measurement of gas flow by means of critical flow venturi nozzles; 2005.
6. B.S. Stratford, The calculation of the discharge coefficient of profiled choked nozzles and optimum profile for absolute air flow measurement, J. R. Aeronaut. Soc. 68 (1964) 237–245.