The research of discrete and continuous models of impact devices by numerical methods

A M Slidenko and V M Slidenko

1Voronezh State Technical University, 14 Moskovsky Avenue, 394006, Voronezh, Russia
2National Technical University of Ukraine "Igor Sikorsky Kyiv Politechnic Institute", Kyiv, Ukraine

E-mail: alexandr.slidenko@yandex.ru, viktorslidenko@gmail.com

Abstract. The article gives the research results of discrete and continuous models of an impact device by numerical methods. The discrete model makes it possible to evaluate the process of impulsive force of the striker with the tool taking into account the aftereffect of the resistance of the working medium. For the rod model, the initial boundary conditions are formulated, which take into account the impulsive force with the determination of the impact force by the Hertz method. The characteristics of nonlinear rigid and dissipative constraints depend on the difference in the displacements of the contact rod ends. The interaction of the tool with the working medium, in particular with the rock, is modeled by similar constraints. These processes are described by initial-boundary value problems with wave equations, which are different in initial and boundary conditions. Mixed difference schemes are used to solve problems. For the rods of variable cross-section, the voltage distribution along their length is determined. The results obtained are compared with the solutions by the Fourier method of model problems.

1. Introduction
The work of some authors is devoted to the study of impact devices with structural elements of the rod type. In papers [1-6], initial boundary value problems are formulated for rods of constant cross-section and their interaction, which is carried out through elastic constraints. In this formulation, the solution of problems is found by the d'Alembert and Fourier methods. In papers [3, 7], various formulations of problems for rod systems with impulse loads are given. In papers [8, 9], a difference method was applied with an impulse load on the end of the rod, which was modeled by determining the initial velocity for the rod sections located in the "initial load" zone. Ideally, the size of the zone $\varepsilon \rightarrow 0$, the value $\varepsilon$ is taken in the calculations where the limiting solution is obtained by the Fourier method [10].

Two-mass discrete model makes it possible to take into account the aftereffect of the resistance of the "working medium" with acceptable accuracy. The use of the rod model revealed the presence of high frequency oscillations. These oscillations lead to fluctuations in the speed sign of the tool end contact with the working medium, as a result the determination of the transition point to the lower part of the resistance characteristic (switching points) is problematic.
The aim of this work is to analyze discrete and continuous models concerning the characteristics of impulse force of the striker with the tool, taking into account the non-linear resistance of the working medium. The impact force is determined similarly to the Hertz model \[1, 3\], and the resistance of the working medium is taken as a piecewise linear dependence with a floating switching point (the beginning of working medium unloading). Two schemes of impulse loading for rods of variable cross-section and the application of mixed difference schemes for solving initial-boundary value problems are considered. In the first case, the return from the working medium of the impulse to the tool and its transmission to the striker is simulated. In the second case, the striker contacts the tool and transmits an impulse to it if the migration of the striker end exceeds the migration of the tool end. Otherwise, the connection between the striker and the tool is significantly weakened and the interaction practically ceases. This scheme takes into account the resistance of the working medium with the aftereffect \[7\]. For comparison of solutions and correction of difference schemes, the solution of the initial-boundary value problem for the rods of constant cross section, obtained by the Fourier method in \[10\], is used.

2. Discrete and continuous mathematical models

The structural diagram of the elements of the hydroimpulsive system (GIS) – impact-type devices \[7\], and the design diagram in the form of a discrete model are shown in figure 1.

Controlling the acceleration of the striker (1) to contact with the tool (2) is implemented by a hydraulic system supplying fluid to line (6) and draining fluid through line (7). Zones A and B are used for constructive formation of damping parameters.

A discrete two-mass model of an impulse impact device is shown in figure 1 (b). The initial-value problem for the two-mass model was considered in the form

\[
m_1 \frac{d^2 x_1}{dt^2} = -p(x_1, x_2) + G_0 \left( \frac{dx_2}{dt} - \frac{dx_1}{dt} \right),
\]

\[
m_2 \frac{d^2 x_2}{dt^2} = p(x_1, x_2) + G_0 \left( \frac{dx_1}{dt} - \frac{dx_2}{dt} \right) - R(x_2, W_1, W_2), \quad t \in [0, T],
\]

\[
\frac{dx_1}{dt}(0) = W_1, \quad \frac{dx_2}{dt}(0) = 0,
\]

\[
x_1(0) = 0, \quad x_2(0) = 0.
\]
Here \( x_1, x_2 \) – the displacements of discrete elements, \( m_1, m_2 \) – the reduced masses of the striker and the tool, \( P_u(x_1, x_2) \) – the force of impact interaction, \( R(x_2, W_2, X_2) \) – the resistance of the working medium, \( W_1 \) – the pre-impact velocity of the striker, \( W_2 \) – the velocity of the tool, \( G_0 \) – the dissipative resistance, \( t \) – the time.

In order to apply numerical methods, the system of second-order differential equations (1) - (2) is reduced to a system of first-order equations. New variables are given:

\[
\begin{align*}
y_0 &= x_1, & y_1 &= \frac{dx_1}{dt} = \frac{dy_0}{dt}, & y_2 &= x_2, & y_3 &= \frac{dx_2}{dt} = \frac{dy_2}{dt}.
\end{align*}
\]

Taking into account the agreed notations, we arrive at a system of four ordinary differential equations of the first order

\[
\begin{align*}
dy_0 &= y_1, & m_1 \frac{dy_1}{dt} &= -P_u(y_0, y_2) + G_0(y_3 - y_1), \\
m_2 \frac{dy_3}{dt} &= P_u(y_0, y_2) + G_0(y_1 - y_3) - R(y_2, W_2, X_2), & \frac{dy_2}{dt} &= y_3.
\end{align*}
\]

The initial conditions take the form

\[
\begin{align*}
y_0(0) &= 0, & y_1(0) &= 0, \\
y_2(0) &= W_1, & y_3(0) &= 0.
\end{align*}
\]

Let us consider a continuous rod-type model, an approximation of the impact force and the resistance force of the working medium.

Different design load schemes are shown in figures 2, 3. Figure 2 (a) shows a model for taking into account the action of the working medium reaction, in particular of the rock, on the striker (2) and the body of the impact device through the tool (1). In the calculations, the variable cross-section areas were determined by parametric formulas in order to ensure an equal volume of the rods. The form of the dependences of the rod cross-section areas on the length is shown in figure 2 (b).

![Figure 2](image_url)

**Figure 2.** (a) Design load schemes of the device: A – interaction zone of the striker (2) with the tool (1); C – Interaction zone of the tool with the "working medium"; B – interaction zone of the striker with the body of the device; (b) Dependences of the cross-section area of the rods on the length: (1) – cylinder; (2) – a cylinder with a conic part; (3) – wavy profile.

The scheme in figure 3 (a) shows the process of impact device interaction (1) with the tool (2) and the latter with the working medium. In accordance with the Hertz model, the interaction of the rods upon impact is determined by the contact zone. The assessment of interaction forces:
Let us write down the vibration equations for the rods position; the change of characteristics of the properties of the rod material, is taken in the range of \( K_0 = 2 \cdot 10^{10} - 6 \cdot 10^{10} \) N/m\(^a\). In the Hertz model \( \alpha = 3/2 \). The resistance aftereffect of the working medium is represented by the parametric dependence

\[
R(V, W, X_2) = \begin{cases} 
   c_1 V, & \text{if } (0 \leq V \leq X_2) \land (W \geq 0), \\
   c_2 V + (c_2 - c_1) X_2, & \text{if } (c_2^{-1}(c_2 - c_1) X_2 \leq V \leq X_2) \land (W < 0), \\
   0, & \text{if } (0 \leq V < c_2^{-1}(c_2 - c_1) X_2) \land W < 0, \\
   c_1 V, & \text{if } V < 0.
\end{cases}
\]

(10)

In formula (10), \( V \) – the displacement, \( W \) – the speed of the Shank end, \( c_1, c_2, c_3 \) – the characteristics of the working medium properties, \( X_2 \) – the coordinate of the tripping point at the change of the speed sign of the contact with the working medium of tool end (figure 3 (b)).

**Figure 3.** (a) Design load schemes of the device: A – the contact interaction zone of the striker (1) with the tool (2); B – interaction zone of the tool with the working medium; (b) The resistance dependence of the working medium on the displacement and direction of the movement.

Let us insymbol: \( U(t, x) \) – displacement of the section \( x \) of the first rod (1) from the equilibrium position; \( V(t, y) \) – displacement of the section \( y \) of the second rod (2) from the equilibrium position. Let us write down the vibration equations for the rods

\[
\frac{\partial^2 U(t, x)}{\partial t^2} = a_i^2 \left( \frac{1}{S_i(x)} \frac{dS_i(x)}{dx} \frac{\partial U(t, x)}{\partial x} + \frac{\partial^2 U(t, x)}{\partial x^2} \right), 0 \leq x \leq L_i,
\]

(11)

\[
\frac{\partial^2 V(t, y)}{\partial t^2} = a_i^2 \left( \frac{1}{S_i(y)} \frac{dS_i(y)}{dy} \frac{\partial V(t, y)}{\partial y} + \frac{\partial^2 V(t, y)}{\partial y^2} \right), 0 \leq y \leq L_2, t \in [0, T].
\]

(12)

Here \( a_i = \sqrt{\frac{E_i}{\rho_i}} \) – the propagation speed of an elastic wave in the rod material, \( E_i \) – elastic modulus, \( \rho_i \) – the density of the rod material, \( S_i(x) \) – the cross-section area of the \( i \)-rod, \( i=1,2 \).
Let us consider the initial conditions for the loading according to the scheme shown in figure 2 (a). When \( t=0 \) the left end of the tool is loaded by the pulse \( P \). This means that a small part of the tool \( \varepsilon \) has an initial speed, which is determined by the formula

\[
W_i = P \left( \rho \int_0^\varepsilon S_1(x)dx \right)^{-1}.
\] (13)

Initial conditions have the form

\[
U(0, x) = \begin{cases} W_1, & 0 \leq x \leq \varepsilon, \\ 0, & x > \varepsilon. \end{cases}, \quad V(0, y) = 0, \quad \frac{\partial V}{\partial t}(0, y) = 0.
\] (14)

The boundary condition for the left end of the first rod reflects the absence of a load:

\[
S_1(0)E_1 \frac{\partial U}{\partial x}(t, 0) = 0.
\] (15)

The resistance of the elastic and dissipative elements is taken into account at the right end of the first rod.

\[
S_1(L)E_1 \frac{\partial U}{\partial x}(t, L) = -K_0(U(t, L) - V(t, 0))^2 + G_0 \left( \frac{\partial V}{\partial t}(t, 0) - \frac{\partial U}{\partial t}(t, L) \right).
\] (16)

For the first scheme, the load was taken \( k = 1 \), for the second scheme, it was \( k = 1.5 \).

Let us consider the initial and boundary conditions for the load shown in figure 3 (a).

The first striker-rod moves with a speed \( W_1 \), and the second tool-rod moves with a speed \( W_2 \leq W_1 \). After contact of the rods, their joint movement occurs. Under the condition \( U(t, L) \geq V(t, 0) \), the resistance coefficients of the elastic and dissipative elements imitate the impact process, and under the condition \( U(t, L) < V(t, 0) \), these coefficients are significantly less and imitate some technological connection. Thus, the boundary condition (16) is preserved, but now the coefficients \( K_0 \) and \( G_0 \) are the functions from the difference between the displacements of the contact ends of the striker and the tool:

\[
K_0 = \begin{cases} K_{01}, & \text{if } U(t, L) \geq V(t, 0), \\ K_{02}, & \text{if } U(t, L) < V(t, 0). \end{cases}, \quad G_0 = \begin{cases} G_{01}, & \text{if } U(t, L) \geq V(t, 0), \\ G_{02}, & \text{if } U(t, L) < V(t, 0). \end{cases}
\] (17)

The equality of forces is assumed at the contact ends.

\[
S_2(0)E_2 \frac{\partial V}{\partial y}(t, 0) = S_1(L)E_1 \frac{\partial U}{\partial x}(t, L).
\] (18)

The conditions at the right end of the second rod simulate the process of interaction of the rod-tool with the working medium, taking into account the aftereffect:

\[
S_2(L_2)E_2 \frac{\partial V}{\partial x}(t, L_2) = -K_1V(t, L_2) - G_1 \frac{\partial V}{\partial t}(t, L_2) - R \left( V(t, L_2) \frac{\partial V}{\partial t}(t, L_2), X_2 \right).
\] (19)

3. Numerical methods

The initial problem (1) - (4) is solved by the Runge-Kutta method. The switching point of the resistance of the working medium is carried out by changing the speed sign of the second discrete element according to an explicit scheme. The \( X_2 \) coordinate at each time step is determined by an
iterative method. The transition to a system of four first-order differential equations (5) - (8) is carried out to implement the numerical method.

The solution to the initial-boundary value problem (11) - (19) is found by the finite difference method. Let us consider this method in detail.

Let us introduce a grid in each of the coordinate systems $Ox$, $Oy$ and the notation:

$$h_1 = \frac{L_1}{N_1}, \quad x_0 = 0, \quad x_i = x_{i-1} + h_1, \quad i = 1, 2, ..., N_1; \quad h_2 = \frac{L_2}{N_2}, \quad y_0 = 0, \quad y_j = y_{j-1} + h_2, \quad j = 1, 2, ..., N_2,$$

$$\tau = \frac{T}{M}, \quad t_0 = 0, \quad t_n = t_{n-1} + \tau, \quad n = 1, 2, ..., M;$$

$$\Delta S_1(x_i) = \frac{S_1(x_{i+1}) - S_1(x_{i-1})}{2h_1}, \quad \Delta S_2(y_j) = \frac{S_2(y_{j+1}) - S_2(y_{j-1})}{2h_2},$$

$$\Delta_2 U^n_i = \frac{U^n_{i+1} - 2U^n_i + U^n_{i-1}}{h_1^2}, \quad \Delta_2 V^n_j = \frac{V^n_{j+1} - 2V^n_j + V^n_{j-1}}{h_2^2}, \quad \Delta U^n_i = \frac{U^n_{i+1} - U^n_{i-1}}{2h_1}, \quad \Delta V^n_j = \frac{V^n_{j+1} - V^n_{j-1}}{2h_2}.$$

Let us write the difference equations (mixed difference schemes) [11]

$$\frac{U^{n+1}_i - 2U^n_i + U^{n-1}_i}{\tau^2} = \sigma_1 a_1^2 \left[ \frac{\Delta S_1(x_i)}{S_1(x_i)} \Delta U^n_{i-1} + \Delta_2 U^n_{i} \right] + (\zeta - 1) \sigma_1 a_1^2 \left[ \frac{\Delta S_1(x_i)}{S_1(x_i)} \Delta U^n_{i+1} + \Delta_2 U^n_{i} \right], \quad i = 1, 2, ..., N_1 - 1;$$

$$\frac{V^{n+1}_j - 2V^n_j + V^{n-1}_j}{\tau^2} = \sigma_2 a_2^2 \left[ \frac{\Delta S_2(y_j)}{S_2(y_j)} \Delta V^n_{j-1} + \Delta_2 V^n_{j} \right] + (\zeta - 1) \sigma_2 a_2^2 \left[ \frac{\Delta S_2(y_j)}{S_2(y_j)} \Delta V^n_{j+1} + \Delta_2 V^n_{j} \right], \quad j = 1, 2, ..., N_2 - 1.$$

The parameter $\zeta$ determines the type of the difference scheme. The weighting coefficients $\sigma_1$ and $\sigma_2$ are selected based on the analysis results of solutions to model problems to ensure the stability of the scheme and acceptable accuracy. The approximation of the initial conditions is taken in the form:

$$U^n_i = 0, \quad V^n_j = 0, \quad \frac{U^n_i - U^n_0}{\tau} = 0, \quad \frac{V^n_j - V^n_j}{\tau} = \begin{cases} W_i, & 0 \leq x_i \leq \varepsilon, \\ 0, & x_i > \varepsilon, \end{cases}, \quad i = 1, ..., N_1 - 1, \quad j = 1, ..., N_2 - 1.$$

The approximation of the boundary conditions at the ends of the first rod:

$$S_1(0) E_1 \frac{U^{n+1}_1 - U^{n+1}_0}{h_1} = 0,$$

$$S_1(L_1) E_1 \left( U^{n+1}_{N_1} - U^{n+1}_{N_1-1} \right) h_1 = -K_0 \left( U^{n+1}_{N_1} - V^{n+1}_{N_1} \right) h_1 + G_0 \left( V^{n+1}_0 - V^{n+1}_0 - \left( U^{n+1}_{N_1} - U^{n+1}_{N_1} \right) \right),$$

$$S_2(0) E_2 \frac{V^{n+1}_1 - V^{n+1}_0}{h_2} = S_1(L_1) E_1 \frac{U^{n+1}_1 - U^{n+1}_{N_1-1}}{h_1}.$$

The approximation of the boundary conditions at the right end of the second rod:
Let us consider the sweep method for both systems and find the relations between the unknown parameters. We seek a solution to system (27) in the form

\[ U_{i-1}^{n+1} = \alpha_i U_i^n + \beta_{i-1}, \quad i = 1, 2, ..., N_i. \]  

Let us write the formulas for determining the sweep method coefficients

\[ \alpha_i = \frac{A_i}{B_i - C_i \alpha_{i-1}}, \quad \beta_i = \frac{C_i \beta_{i-1} + D_i}{B_i - C_i \alpha_{i-1}}. \]  

3.1 Solution algorithm for the difference problem

Systems of equations (20) and (21) will be reduced to the standard form:

\[ A_i U_{i+1}^{n+1} - B_i U_i^{n+1} + C_i U_{i-1}^{n+1} = -D_i, \]

\[ a_j V_{j+1}^{n+1} - b_j V_j^{n+1} + c_j V_{j-1}^{n+1} = -d_j. \]  

From boundary condition (25) and recurrence formula (29) when \( i = 1 \), the system of equations follows

\[ U_0^{n+1} = U_1^{n+1}, \quad U_0^{n+1} = \alpha_0 U_1^{n+1} + \beta_0. \]

From this system we get \( \alpha_0 = 1, \beta_0 = 0 \). Using formulas (30) we can calculate \( \alpha_i \) and \( \beta_i \), \( i = 1, 2, ..., N_i - 1 \). To implement backward path of the elimination, it is necessary to calculate \( U_{N_i}^{n+1} \) taking into account conditions (25) and (26). We seek a solution to system (28) in the form

\[ V_{j+1}^{n+1} = \delta_j V_j^{n+1} + \gamma_j, \quad j = 0, 1, 2, ..., N_2 - 1. \]  

From the system of equations (24), (25), (29) when \( i = N_i \), and (33) when \( j = 0 \), we find a solution that depends on the coefficients of the backward elimination for system (31) \( \delta_0 \) and \( \gamma_0 \):

\[ U_{N_i}^{n+1} = \left( \frac{G_2 \tau + \gamma_0 + \beta_{N_i-1} q \left( \frac{\tau}{G_0} - \frac{\delta_0}{p(1-\delta_0)} \right)}{\alpha_{N_i-1} q \left( \frac{\delta_0}{p(1-\delta_0)} - \frac{\tau}{G_0} \right) + \left( \frac{1 - \delta_0}{p(1-\delta_0)} + \frac{\tau}{G_0} \right)} \right)^{-1} \]

\[ U_{N_i}^{n+1} = \alpha_{N_i-1} U_{N_i} + \beta_{N_i-1}, \]

where \( p = \frac{S_2(0)E_2}{h_2^2}, \quad q = \frac{S_1(L_1)E_1}{h_1}, \quad G_2 = -K_0 \left( U_{N_i}^n - V_0^* \right), \quad G_0 = \frac{G_0}{\tau} \left( U_{N_i}^n - V_0^* \right), \]

\[ K_0 = \begin{cases} 
G_{01}, & \text{if } U_{N_i}^n \geq V_0^*, \\
G_{02}, & \text{if } U_{N_i}^n < V_0^*. 
\end{cases} \]  

We get the dependencies:

\[ V_{i+1}^{n+1} = \frac{1 + \frac{\tau}{G_0}}{\frac{1}{p} + \frac{\tau}{G_0}} U_{N_i}^{n+1} + \left( 1 + \frac{q}{p} + \frac{\tau}{G_0} \right) U_{N_i}^{n+1} - \frac{G_2 \tau}{G_0} U_{N_i}^{n+1}, \quad V_0^{n+1} = V_1^{n+1} - q \left( U_{N_i}^{n+1} - U_{N_i-1}^{n+1} \right) p^{-1}. \]  

\[ (26) \]
To determine the coefficients $\delta_0$ and $\gamma_0$, it is necessary to take into account the boundary conditions (26) at the right end of the second rod and the assumed solution to the second system of equations (28):

$$S_2(L_2)E_2\left(V_{N_2}^{n+1} - V_{N_2-1}^{n+1}\right)h_2^{-1} = -K_1V_{N_2}^{n+1} - G_1\left(V_{N_2}^{n+1} - V_{N_2}^n\right)h_2^{-1} - R\left(V_{N_2}^n, \left(V_{N_2}^n - V_{N_2}^{n+1}\right)h_2^{-1}, X_2\right),$$

$$V_{N_2-1}^{n+1} = \delta_{N_2-1}V_{N_2}^{n+1} + \gamma_{N_2-1}.$$  \hspace{1cm} (35)

From the system (35) it follows

$$\delta_{N_2-1} = \left(E_2 S_2(L_2) + K_1 h_2 + \frac{G_1 h_2}{\tau}\right)\left(E_2 S_2(L_2)\right)^{-1},$$

$$\gamma_{N_2-1} = -\frac{G_1 h_2}{\tau E_2 S_2(L_2)} V_{N_2}^n - R\left(V_{N_2}^n, \left(V_{N_2}^n - V_{N_2}^{n+1}\right)h_2^{-1}, X_2\right)\frac{h_2}{E_2 S_2(L_2)}.\hspace{1cm} (36)\hspace{1cm} (37)

From the system (28, 31), the formulas are correct for the sweep coefficients

$$\delta_{j-1} = (b_j\delta_j - a_j\delta_j c_j)^{-1}, \hspace{0.5cm} \gamma_{j-1} = (\gamma_j(b_j - c_j\delta_{j-1} - d_j\gamma_j)^{-1}, \hspace{0.5cm} j = N_2 - 1, N_2 - 2, ..., 2, 1. \hspace{1cm} (38)$$

Model problems are used to select the parameters for the implementation of the difference scheme, the analytical solution of model problems was obtained by the Fourier method [7, 8, 10]. The algorithm is implemented in the Mathcad system.

The general scheme of studying models by numerical methods is shown in figure 4.

**Figure 4.** The general scheme of studying models by numerical methods.

### 4. Results of computational experiments

Figure 5 shows graphs of changes in the resistance force of the working medium over time (figure 5 (a)) and depending on the movement of the second discrete element (figure 5 (b)). Options of continuous switching and discontinuous switching are considered. These options are associated with
the inaccuracy in determining the \( X_2 \) coordinate of the switching point according to the speed sign of the second discrete element.

\[ R, N \]

\[ W_2 > 0 \]

\[ W_2 < 0 \]

\[ V, m \]

\[ T \]

**Figure 5.** Change in the resistance force of the working medium with three switching options:
1 – no aftereffect; 2 – continuous switching; 3 – discontinuous switching:
(a) – time dependence; (b) – dependence on the displacement of the second discrete element

Basic parameters: \( K_0 = 2 \cdot 10^{10} \text{ N/m}^{3/2}; \) \( m_1 = 50 \text{ kg}; \) \( m_2 = 100 \text{ kg}; \) \( W_1 = 10 \text{ m/s}; \) \( G_0 = 6 \cdot 10^3 \text{ Ns/m}. \)

Oscillations of discrete elements in time are shown in figure 6. Figure 7 (a) shows the influence of the switching inaccuracy of the resistance force according to the speed sign of the second discrete element. Figure 7 (b) demonstrates the period \( T \) of the impact element interaction.

\[ U, V, m \]

\[ T \]

**Figure 6.** (a) Oscillations of discrete elements with different options of switching the resistance of the working medium: 1, 2, 3 – the first element; 1', 2', 3' – is the second element;
(b) The initial section of the collision: 1 – the first element; 2 – second element

Parameters: \( K_0 = 2 \cdot 10^{10} \text{ N/m}^{3/2}; \) \( m_1 = 50 \text{ kg}; \) \( m_2 = 100 \text{ kg}; \) \( W_1 = 10 \text{ m/s}; \) \( G_0 = 6 \cdot 10^3 \text{ Ns/m}. \)

Let us consider the results of solving difference problems.

The study of model problems was carried out to get consistent results, the solution of which was found by the Fourier method. This made it possible to choose rational parameters of the difference scheme. In the first scheme, the impact is made on the left end of the first rod and is simulated in a known way: the initial velocity is attributed to a small part of the rod. The oscillations obtained by the difference method are compared with the Fourier solution of the model problem for the first rod in the presence of only rigid resistance at the right end [8]. Problem solving graphs are shown in figure 7.
Figure 7. (a) Oscillations of the contact ends of the rods (two-layer scheme) (1) – tool; (2) – device body; (3) – Fourier solution of the model problem; (b) Voltage distribution along the length of the rod; 1 – two-layer scheme; 2 – three-layer scheme. Parameters: $P=550$ Ns; $L_1=0.96$ m; $L_2=1$ m; $K_0=600000$ N/m; $S_1=0.014$ m$^2$; $S_2=0.29$ m$^2$.

The impact process and the development of high-frequency oscillations of the cross-sections of the striker and the tool are modeled when calculating the second load scheme.

For a continuous model, the use of the iterative method for determining the $X_2$ coordinate of the switching point (Fig. 3 (b)) is problematic due to high-frequency oscillations of the speed sign of the contact end of the tool. In this regard, mode selection should be done according to the parameters of low-frequency oscillations of the equivalent two-mass model.

Figure 8, 9 show solutions of initial-boundary value problems with non-linear stiffness of the intermediate element and the presence of a dissipative element. Figure 8, b shows the dependence of the displacements of the contact ends of the tool and striker (1,2), the tool with the working medium (3). The occurrence of high-frequency oscillations (800-900 Hz) is shown.

The period of contact interaction of the rods can be estimated by the value $T = (6 \times 10^{-4}) \, \text{s}$, (figure 8, b).

Figure 9 shows the voltage distribution in the sections of the striker and the tool with a tapered shape (figure 9 a, b) and cylindrical (figure 9, c, d).

When the condition is $U_{Ni} \geq V_0$ the contact ends of the striker and the tool are compressed. If this condition is violated, the contact is stopped.
The voltage distribution along the length of the striker and the tool depends on the characteristics of rigidity, pre-impact velocity and geometric parameters of the striker and tool. The presented algorithm makes it possible to evaluate these dependencies and this is important for the design of impact devices.

**Conclusion**

Let us list the main research results.

1. The proposed algorithm for studying the model of impulse interaction of the striker and the tool in the presence of non-linear rigid and dissipative constraints, taking into account the resistance aftereffect of the working medium, can work as a calculation method for designing impact devices.

2. A parametric method is proposed for determining the force characteristic of the resistance of the working medium by using the iterative calculation of the coordinates of the switching point (the point where the tool starts unloading). The iterative method for a discrete element provides continuous switching due to the monotonicity of the speed sign on a short time interval.

3. The application of the approximation method of the non-linear in the boundary condition of contact according to an explicit scheme using the method of sequential sweeping made it possible to calculate the first stage of the impulse interaction of the contact ends of the striker and the tool. The estimated duration of the contact interaction of the striker and the tool was 400 ... 900 μs.

4. The dependence option of the stiffness and dissipation coefficients on the difference in the displacements of the contact ends of the striker and the tool of variable cross-sections is tested. The developed program makes it possible to make calculations for various dependences of the cross-sectional areas of the striker and the tool (cylinder, cone, wavy profile). The efficiency of application of two-layer and three-layer difference schemes for an approximate solution of the initial-boundary value problem describing the impulse interaction of the striker and the tool with variable cross sections is shown.
References

[1] Alimov O D, Manzhosov V K and Eremyants V E 1985 A stroke. The propagation of wave deformations in shock systems (Moscow: Nauka) p 358

[2] Ivanov A P 1997 System Dynamics with Mechanical Collisions (Moskow International Education Program) p 336

[3] Manzhosov V K and Novikov D A 2015 Modeling of transient and limit motion cycles of vibro-shock systems with discontinuous characteristics: monograph (Ulyanovsk: Ulstu) p 236

[4] Zhukov I A and Molchanov V V 2014 Rational designing two-stage anvil blocks of impact mechanisms Advanced Materials Research. V 1040 pp 699-702

[5] Zhukov I A, Dvornikov L T and Nikitenko S M 2016 About creation of machines for destruction of rock with formation of apertures of various cross-section IOP Conference Series: Materials Science and Engineering V 124 1 012171

[6] Zhukov I A, Repin A A and Timofeev E G 2018 Automated calculation and analysis of impacts generated in mining machine by anvil blocks of complex geometry IOP Conf. Series: Earth and Environmental Science 134 012071 doi :10.1088/1755-1315/134/1/012071

[7] Slidenko V M and Slidenko A M 2017 Mathematical Modeling of Shok-wave Processes of Hydropulse System of Mining Machines: monograf (Kyiv: KPI alter Igor Sikorski Publishing house “Polytechnic”) p 220

[8] Slidenko A M and Slidenko V M 2019 Numerical Resarch Methodofan Impact Device Model Journal of Physics: Conference Series 1203 012086.

[9] Slidenko A.M. and Slidenko V. M. 2020 Models of hysteresis oscillation damping at pulse loadings Journal of Physics: Conf. Series 1479 012098

[10] Aramanovich I G and Levin V I 1969 Equations of Mathematikal Physics (Moskow: Science) p 288

[11] Samarskii A A 2001 The Theory of Difference Schemes (New York – Basel: Marcel Dekker, Inc) p 761