A Minimal Model for Neutral Naturalness and pseudo-Nambu-Goldstone Dark Matter

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ABSTRACT: We outline a scenario where both the Higgs and a complex scalar dark matter candidate arise as the pseudo-Nambu-Goldstone bosons of breaking a global $SO(7)$ symmetry to $SO(6)$. The novelty of our construction is that the symmetry partners of the Standard Model top-quark are charged under a hidden color group and not the usual $SU(3)_c$. Consequently, the scale of spontaneous symmetry breaking and the masses of the top partners can be significantly lower than those with colored top partners. Taking these scales to be lower at once makes the model more natural and also reduces the induced non-derivative coupling between the Higgs and the dark matter. Indeed, natural realizations of this construction describe simple thermal WIMP dark matter which is stable under a global $U(1)_D$ symmetry. We show how the Large Hadron Collider along with current and next generation dark matter experiments will explore the most natural manifestations of this framework.

KEYWORDS: Beyond the Standard Model, Neutral Naturalness, Dark Matter
1 Introduction

The Standard Model (SM) of particle physics has great agreement with experiment, however it cannot be the complete theory of nature. One of the most pressing theoretical problems within the SM is the hierarchy between the weak and Planck scales. Both composite Higgs models and constructions which protect the Higgs mass through a new symmetry predict new particles or states with masses at or below the TeV scale.

Beyond this theoretical puzzle, there is overwhelming experimental evidence for dark matter (DM) which also points to new particles and interactions beyond the SM. While there is a vast and varied spectrum of possible DM candidates, weakly interacting massive particles (WIMPs) are perhaps the most theoretically compelling. This is especially the case when viewed through the lens of the hierarchy problem. Then the DM can naturally obtain a weak scale mass and couplings, providing the observed DM density through thermal freeze-out.

However, both symmetry based explanations of Higgs naturalness and thermal WIMPs have become increasingly constrained by experiment. Searches at the Large Hadron Collider (LHC) have pushed the limits on the colored symmetry partners of SM quarks to the TeV scale. At the same time a host of direct detection experiments are driving the limits on WIMP DM cross sections toward the so-called neutrino floor. With the severity of these
constraints many new and interesting ideas for both Higgs naturalness and DM have been explored.

Years before the Higgs was discovered [1, 2], it was pointed out that if the Higgs mass parameter is insensitive to high scales because of a new symmetry, the symmetry partners of the SM quarks do not need to carry SM color [3–7]. Since the discovery of colored symmetry partners to SM quarks has not followed the discovery of the Higgs, more realizations of color neutral naturalness have been explored [8–17]. Connecting DM to neutral naturalness began with the Dark Top [6], and has flourished in the context of twin Higgs models [18–28].

In twin Higgs models, the Higgs is a pseudo-Nambu-Goldstone boson (pNGB) of a global $SO(8)$ symmetry breaking to $SO(7)$. The variety in DM candidates typically comes not from the symmetry breaking structure, but by making particular assumptions about the particle content in the twin sector. Other neutral naturalness pNGB constructions [7, 11, 12, 16, 17] employ smaller symmetry groups, but this move toward minimality makes it more difficult to accommodate simple DM candidates.

However, as demonstrated in [29, 30], the six pNGBs that spring from $SO(7)/SO(6)$ can be associated with the Higgs doublet (respecting the custodial symmetry) along with a complex scalar DM stabilized by a global $U(1)_D$.  The mass of the DM and its couplings to the Higgs are determined by the symmetry breaking structure and the low energy fields that transform under the symmetry. This necessarily includes the top quark for the model to address the hierarchy problem. As a consequence, the collider bounds on colored top partners lead to couplings between the Higgs and the DM that are near or beyond the experimental limits [29, 32].

In the following section we construct a neutral natural version of the $SO(7)/SO(6)$ symmetry breaking pattern. As expected, the quark symmetry partners are charged under a hidden color group $SU(3)_c$ rather than SM color. This mean they can be much lighter, allowing for additional freedom in the Higgs non-derivative coupling to the DM. These SM color neutral top partners are electroweak charged and break the DM shift symmetry, generating the DM potential. Thus, more natural top partner masses can lead to Higgs portal direct detection signals that may not be fully explored until the next generation of dark matter experiments. However, we do find that nearly all natural parameter choices lie above the neutrino floor.

In addition, the new fields related to the top quark exhibit quirky [33] dynamics. These less studied particles can be discovered at the LHC, providing a complementary probe of the model. In Sec. 3 we outline the most promising collider searches, including both prompt and displaced signals. We find that the LHC already bounds the quirks with $U(1)_D$ charges. Because these particles determine the coupling between the Higgs and the dark matter, these collider bounds immediately inform the sensitivity of dark matter experiments to the pNGB WIMPs. We also calculate the corrections to the electroweak precision tests (EWPT) due to the presence of the new electroweak charged states.

In Sec. 4 we discuss the DM phenomenology, showing which parameter values lead to the correct thermal relic density and elucidate how direct and indirect searches probe the model. We find collider searches and direct detection experiments provide complementary probes.

\footnote{A coset like $SO(6)/SO(5)$ leads to five pNGBs which comprise the Higgs doublet and a real scalar field which can be a dark matter candidate [31], however the stability of DM requires an additional dark party.}
both delving into the natural parameter space along different directions in parameter space. While current limits allow versions of this framework with $\sim 10\%$ tuning, next generation searches should be able to discover the quirks or DM, often in multipole channels, down to $\sim 1\%$ tuning. Following our conclusions, in Sec. 5, we include two appendices to provide details relating to the work.

2 Neutral Naturalness from $SO(7)/SO(6)$

In this section, we describe a neutral naturalness model which includes the Higgs doublet and a complex scalar DM candidate as pNGBs. This model is related to that of the Refs. [29, 30], but crucially has color neutral top partners. The global symmetry structure is $SO(7) \times SU(7)$, where $SU(7) \supset SU(3)_c \times SU(3)_c \times U(1)_X$ includes the SM color group as well as a hidden sector color denoted $SU(3)_c$. The additional $U(1)_X$ ensures SM fields have their measured hypercharges. At some scale $f$ the global $SO(7)$ symmetry is broken to $SO(6) \supset SO(4)_C \times SO(2)_D \simeq SU(2)_L \times SU(2)_R \times U(1)_D$. Here the $SO(4)_C \simeq SU(2)_L \times SU(2)_R$ is the familiar custodial symmetry with $SU(2)_L$ being the usual SM weak gauge group and $SO(2)_D = U(1)_D$ is the global symmetry that stabilizes the DM. This construction also breaks the DM’s shift symmetry in a new way. In particular, through color neutral vector-like quarks in addition to the color neutral top partners. As we see below, the DM mass and its non-derivative couplings are proportional to the masses of these color neutral vector-like quarks.

2.1 The Gauge Sector

We begin with the interactions amongst the NGBs and the gauge fields. The NGB fields can be parameterized nonlinearly as

$$\Sigma = e^{-i \Pi / f} \Sigma_0, \quad \text{with} \quad \Pi = \sqrt{2} \pi_a T_a,$$

(2.1)

where $\Sigma_0 = (0, 0, 0, 0, 0, f)^T$ and $T_a$ are the broken generators of $SO(7)$ with $a = 1, \cdots , 6$, see Appendix A for details. We immediately find

$$\Sigma = \frac{f}{|\pi|} \sin \left( \frac{|\pi|}{f} \right) \left( \pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, |\pi| \cot \left( \frac{|\pi|}{f} \right) \right)^T,$$

(2.2)

where $|\pi| \equiv \sqrt{\pi_a T_a}$. We can then write the leading order NGB Lagrangian as

$$\mathcal{L}_{\text{NGB}} = \frac{1}{2} (D_\mu \Sigma)^T D^\mu \Sigma,$$

(2.3)

where the covariant derivative is given by

$$D_\mu = \partial_\mu - ig W_\mu^a T_a - ig' B_\mu (T_3^A + X).$$

(2.4)

Note that the electric charge of fields is defined by $Q = T_3^A + X$, or the hypercharge is defined as $Y = T_3^A + X$.

The first four NGBs are related to the usual Higgs doublet $H = (h^+, h^0)^T$ by

$$(\pi_1, \pi_2, \pi_3, \pi_4) = \left( -\frac{h^+ - h^{*+}}{\sqrt{2}}, \frac{h^+ + h^{*+}}{\sqrt{2}}, \frac{h^0 - h^{0*}}{\sqrt{2}}, \frac{h^0 + h^{0*}}{\sqrt{2}} \right).$$

(2.5)
In unitary gauge when $h^+ = 0$ and $h^0 = \bar{h}/\sqrt{2}$ we have

\[(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6) = \left(0, 0, 0, \bar{h}, \sqrt{2} \text{Im}\chi, \sqrt{2} \text{Re}\chi\right),\]  

(2.6)

where we have defined $\chi = (\pi_6 + i\pi_5)/\sqrt{2}$ as a complex scalar which is our DM candidate.

It is convenient to make the field redefinition [34],

\[\sin \left(\frac{|\pi|}{f}\right) \frac{\pi^a}{|\pi|} \rightarrow \frac{\pi^a}{f}.\]

(2.7)

We can then write NGB field as

\[\Sigma = \left(0, 0, 0, \bar{h}, \sqrt{2} \text{Im}\chi, \sqrt{2} \text{Re}\chi, \sqrt{f^2 - h^2 - 2|\chi|^2}\right),\]

(2.8)

in unitary gauge. The NGB Lagrangian has the leading order terms

\[\mathcal{L}_{\text{NGB}} = \frac{1}{2} (\partial_\mu h)^2 + \frac{1}{2} |\partial_\mu \chi|^2 + \frac{1}{2} \left(\frac{\bar{h} \partial_\mu \bar{h} + \chi^* \partial_\mu \chi + \chi \partial_\mu \chi^*}{f^2 - h^2 - 2|\chi|^2}\right)^2 + \frac{\bar{h}^2}{4} \left[g^2 W^+ W^- + \frac{1}{2} (gW^3 - g'B_\mu)^2\right].\]

(2.9)

When $\bar{h}$ gets a vacuum expectation value (VEV) of $v \approx 246$ GeV we write

\[\bar{h} = v + c_v h, \quad \text{with} \quad c_v \equiv \sqrt{1 - \frac{v^2}{f^2}},\]

(2.10)

to ensure that $h$ is canonically normalized. We also define $s_v \equiv v/f$.

2.2 The Quark Sector

The quark fields include particles charged under both SM color and the hidden color group. In terms of $SO(7)$ and $SU(6)$ representations we have the left- and right-handed quarks as $Q_L = (7, 6)$ and $T_R = (1, 6)$. These can be split up schematically in terms of fields in the $3$ of their respective color groups

\[Q_L = (Q_L, \hat{Q}_L), \quad T_R = (t_R, \hat{T}_R),\]

(2.11)

where we have put a hat on fields charged under the hidden color group. More explicitly we write out the low lying left-handed fields as

\[
\begin{align*}
\sqrt{2} Q_L &= \begin{pmatrix}
SU(3)_C \vline \quad SU(3)_{\hat{C}}
\end{pmatrix} \\
&= \begin{pmatrix}
ib_L & \hat{ib}_L \\
b_L & \hat{b}_L \\
it_L & \hat{i}_L \\
\bar{t}_L & \bar{\hat{t}}_L \\
0 & iX_L - iY_L \\
0 & X_L + \hat{Y}_L \\
0 & \sqrt{2} T_L
\end{pmatrix},
\end{align*}
\]

(2.12)
Figure 1: These Feynman diagrams show cancelation of quadratic divergences due to $SU(3)_c$ top quark and $SU(3)_{\hat{c}}$ top quark for the SM-like Higgs $H$ (upper row) and the complex scalar $\chi$ (lower row).

where $q_L = (t_L, b_L)^T$ is the usual $SU(2)_L$ quark doublet. This is similar in spirit to Refs. [11, 12, 29] which use incomplete quark multiplets. One can imagine the other fields lifted out of the low energy spectrum by vector-like masses, or as in extra dimensional models [5, 7] that the boundary conditions of the bulk fields are such that the zero modes vanish. In order to obtain the correct hypercharge for $t_L$, $t_R$, and $b_L$, both $Q_L$ and $T_R$ have a $U(1)_X$ charge of 2/3. The Yukawa coupling term $\bar{Q}_L \Sigma T_R$ then implies that the NGBs have zero $X$ charge, which in particular implies that $\chi$ has no SM gauge charges.

The top sector couplings follow from

\[ L \supset \lambda_{\ell} \bar{q}_L \Sigma T_R + \text{h.c.} \]

\[ = \lambda_{\ell} \left[ - \left( \bar{q}_L t_R + \bar{q}_L \hat{T}_R \right) H + \chi^* \bar{X}_L \hat{T}_R + \chi \bar{Y}_L \hat{T}_R + f \left( 1 - \frac{|H|^2 + |\chi|^2}{f^2} + \ldots \right) \bar{T}_L \hat{T}_R \right] + \text{h.c.}, \]

(2.13)

where $q_L = (t_L, b_L)^T$ and $\hat{q}_L = (\hat{t}_L, \hat{b}_L)^T$ are $SU(2)_L$ doublets and we have restored the eaten NGBs for the moment into the Higgs doublet $H$, and defined $\hat{H} = i\sigma^2 H^*$. From these interactions we obtain the one-loop diagrams in Fig. 1 relevant to the mass corrections for $H$ and $\chi$. The leading contributions from the top quark are doubled by $\hat{q}_L$ interaction, but this combination is exactly cancelled by $\hat{T}_L$. Like in [16], the contributions from fields carrying SM color and those carrying hidden color are not equal. Note that the DM shift symmetry is broken by the SM color neutral top partner $\hat{T}$ and $U(1)_D$ charged fermions $\hat{X}_L, \hat{Y}_L$.

The hidden color fields can be lifted through vector-like mass terms with new heavy states. We can write down the mass terms

\[ L_{\text{vec mass}} = -m_Q \bar{q}_L \hat{q}_R - m_X \bar{X}_L \hat{X}_R - m_Y \bar{Y}_L \hat{Y}_R + \text{h.c.}, \]

(2.14)

where $\hat{q}_R = (\hat{t}_R, \hat{b}_R)^T$ is an $SU(2)_L$ doublet. We take these to be free parameters as we calculate the scalar potential.
2.3 The Scalar Potential

We are interested in the obtaining the potentials for both the Higgs and the DM. This is obtained from the Coleman-Weinberg (CW) potential [35]

\[ V_{\text{CW}} = -\frac{N_c}{8\pi^2} \Lambda_{\text{UV}}^2 \text{Tr} M^2 - \frac{N_c}{16\pi^2} \text{Tr} \left[ \mathcal{M}^4 \left( \ln \frac{\Lambda_{\text{UV}}^2}{\Lambda_{\text{UV}}} - \frac{1}{2} \right) \right], \]

(2.15)

where \(\mathcal{M}^2\) is the Dirac fermion mass squared matrices, with masses as functions of \(\tilde{h}\) and \(\chi\). We note first that there is no quadratic sensitivity to to the cut off because \(\text{Tr} \mathcal{M}^2\) is independent of the scalar fields. However, we do find logarithmic sensitivity because

\[ \text{Tr} \mathcal{M}^4 = \frac{\lambda_t^4}{2} \tilde{h}^4 + h^2 \lambda_t^2 (m_Q^2 - \lambda_t^2 f^2) + 2\lambda_t^2 |\chi|^2 (m_X^2 + m_Y^2), \]

(2.16)

where we have dropped field independent terms.

Any remaining terms in the scalar potential, such as quartic mixing of \(\tilde{h}\) and \(\chi\) or a \(|\chi|^4\) term, are independent of \(\Lambda_{\text{UV}}\) and so are robustly determined by the low energy physics. Clearly, in order for electroweak symmetry to break we need the Higgs mass parameter to be negative, so we require \(f\lambda_t > m_Q\). From Eq. (2.10) we see that Higgs couplings to SM fields will be reduced by \(c_w\). As in other pNGB Higgs construction, this implies that \(f\) exceeds \(v\) by a factor of a few. As in [16] we find there must be a cancellation between independent terms \((m_Q\) and \(\lambda_t f)\) to obtain the correct Higgs mass. This motivates defining

\[ \lambda_t^2 f^2 \equiv m_Q^2 (1 + \delta_m). \]

(2.17)

For simplicity, in this work we take the vector-like masses of the DM sector to be equal

\[ m_X = m_Y \equiv m_V. \]

(2.18)

This mass scale is related to \(m_Q\) by the ratio \(r_Q = m_V^2 / m_Q^2\). In this limit we find one of the dark fermion mass eigenstates is exactly \(m_V\), while the others are determined by a cubic equation. We then find the scalar potential, which has the general form of

\[ V(h, \chi) = \frac{1}{2} \mu_h^2 \tilde{h}^2 + \frac{\lambda_h}{4} \tilde{h}^4 + \mu_X^2 |\chi|^2 + \lambda_X |\chi|^4 + \lambda_{h\chi} \tilde{h}^2 |\chi|^2. \]

(2.19)

The potential parameters are calculated from the CW potential in Eq. (2.15). We find

\[ \mu_h^2 = \frac{3\lambda_t^2 f^2}{8\pi^2} \left[ m_Q^2 \ln \frac{\Lambda_{\text{UV}}^2}{m_Q^2} - \lambda_t^2 f^2 \ln \frac{\Lambda_{\text{UV}}^2}{\lambda_t^2 f^2} + \frac{\lambda_t^2 f^2 m_Q^2}{m_Q^2 - \lambda_t^2 f^2} \ln \frac{\lambda_t^2 f^2}{m_Q^2} \right], \]

(2.20)

\[ \lambda_h = \frac{3\lambda_t^4}{16\pi^2} \left[ \ln \frac{\Lambda_{\text{UV}}^2}{\lambda_t^2 f^2} \right] - \frac{1}{2} \frac{(3m_Q^2 - \lambda_t^2 f^2)^2}{(m_Q^2 - \lambda_t^2 f^2)^2} - m_Q^4 \frac{3m_Q^2 - \lambda_t^2 f^2}{(m_Q^2 - \lambda_t^2 f^2)^3} \ln \frac{\lambda_t^2 f^2}{m_Q^2}, \]

(2.21)

\[ \mu_X^2 = \frac{3\lambda_t^2 m_V^2}{4\pi^2} \left[ \ln \frac{\Lambda_{\text{UV}}^2}{\lambda_t^2 f^2} + \frac{m_V^2}{m_V^2 - \lambda_t^2 f^2} \ln \frac{\lambda_t^2 f^2}{m_V^2} \right], \]

(2.22)

\[ \lambda_X = -\frac{3\lambda_t^4 m_V^4}{4\pi^2 (m_V^2 - \lambda_t^2 f^2)^2} \left[ 2 + \frac{m_V^2 + \lambda_t^2 f^2}{m_V^2 - \lambda_t^2 f^2} \ln \frac{\lambda_t^2 f^2}{m_V^2} \right], \]

(2.23)

\[ \lambda_{h\chi} = -\frac{3\lambda_t^4 m_V^2}{8\pi^2} \left[ \frac{2m_Q^2 - \lambda_t^2 f^2}{(m_Q^2 - \lambda_t^2 f^2)(m_V^2 - \lambda_t^2 f^2)} + \frac{m_Q^2 (2m_Q^2 - m_V^2)}{(m_Q^2 - \lambda_t^2 f^2)^2 (m_Q^2 - m_V^2)} \ln \frac{\lambda_t^2 f^2}{m_V^2} \right]. \]
We find \( \delta \) means \( \delta \mu \) to the correct Higgs mass. In the limit of small Twin Higgs \([36]\) and since we know \( \langle \chi \rangle = 0 \) and \( \langle \mathcal{F} \rangle = v \). With \( \mu_\mathcal{F}^2 < 0 \) and \( \mu_\mathcal{F}^2 > 0 \), this is the deepest vacuum as long as \( \lambda_h \chi < \lambda^2 \chi \). However, when \( \lambda_h \chi > \lambda^2 \chi \), the vacuum with \( \langle \chi \rangle \neq 0 \) becomes a saddle point rather than a minimum, so the deepest stable vacuum still has \( \langle \chi \rangle = 0 \). In this case we find

\[
\mu_h^2 = -\lambda_h v^2, \quad m_h^2 = 2e_v \lambda_h v^2, \quad m_\chi^2 = \mu_\chi^2 + \lambda_\chi v^2.
\]  

Since we know \( v \simeq 246 \) GeV and the Higgs mass \( m_h \simeq 125 \) GeV, therefore \( \lambda_h \simeq 0.13 \) and \( \mu_h \simeq 89 \) GeV. The constraints on Higgs couplings (see Sec. 3) imply that \( f \gtrsim 3v \), which means \( \delta_m \ll 1 \). It then makes sense to expand the potential terms to leading order in \( \delta_m \).

We find

\[
\mu_h^2 = -\frac{3\lambda_h^2}{8\pi^2} m_Q^2 + \mathcal{O}(\delta_m), \quad \lambda_h = \frac{3\lambda_h^4}{16\pi^2} \left[ \frac{2}{3} + \ln \frac{\Lambda_{\text{UV}}^4}{m_Q^2 \lambda_h^2} + \mathcal{O}(\delta_m) \right],
\]

\[
\mu_h^2 = \frac{3\lambda_h^2}{4\pi^2} \left[ \ln \frac{\Lambda_{\text{UV}}^4}{m_Q^2} + \frac{r_Q}{1-r_Q} \ln r_Q + \mathcal{O}(\delta_m) \right],
\]

\[
\lambda_h = \frac{3\lambda_h^4}{4\pi^2 (1-r_Q)^2} \left[ \frac{1+r_Q}{1-r_Q} \ln \frac{1}{r_Q} - 2 + \mathcal{O}(\delta_m) \right],
\]

\[
\lambda_\chi = \frac{3\lambda_\chi^4}{8\pi^2 (1-r_Q)^2} \left[ \frac{3-r_Q}{2} - \frac{r_Q(2-r_Q)}{1-r_Q} \ln \frac{1}{r_Q} + \mathcal{O}(\delta_m) \right].
\]

Here we have taken \( \ln \frac{\Lambda_{\text{UV}}^4}{m_Q^2} \) to be order one, as expected for a cutoff of a few TeV.

The Higgs potential has logarithmic dependence on \( \Lambda_{\text{UV}} \). This is similar to both the Twin Higgs \([36]\) and \( \text{SO}(6)/\text{SO}(5) \) constructions \([11]\) where sizable UV contributions lead to the correct Higgs mass. In the limit of small \( \delta_m \) and taking \( \lambda_h^4 \) = 0.936 we find \( \mu_h^2 \approx -(146)^2 \) GeV\(^2\) and \( \lambda_h \approx 0.13 \) for \( m_Q = 800 \) GeV and \( \Lambda_{\text{UV}} = 3 \) TeV. These are similar to the SM listed above, so that a suitable UV completion, perhaps composite or holographic, can easily accommodate the measured Higgs mass.

At the same time the quartic couplings that involve \( \chi \) are determined completely by the low-energy theory. Thus, we can make robust predictions about the DM without knowledge of the UV completion. In Fig. 2 we see that these quartics are order \( 10^{-2} \) over a wide span of \( r_Q \). This gives the value of the DM self-interactions as well as its coupling strength to the Higgs. The value of \( r_Q \) is constrained by collider production of the hidden color fermions which is taken up in the following section.

### 2.3.1 Tuning

The Higgs potential obtained above also allows us to determine tuning of the Higgs mass parameter. We use the formula

\[
\Delta = \frac{2\delta \mu^2}{\mu_h^2} \left( \frac{m_Q}{m_h^2} \right)^{-1},
\]

where \( \delta \mu^2 \) is the leading one-loop correction to the Higgs mass parameter

\[
\delta \mu^2 = \frac{N_c}{8\pi^2} \lambda_h^2 (m_Q^2 - \lambda_\chi^2 f^2) \ln \frac{\Lambda_{\text{UV}}^4}{m_Q^2} = -\frac{3\lambda_h^4 \delta_m}{8\pi^2} m_Q^2 \ln \frac{\Lambda_{\text{UV}}^4}{m_Q^2}.
\]
Figure 2: The quartic couplings involving the DM field $\chi$ as a function of $r_Q = m_{V}^2/m_{Q}^2$. We have neglected all terms of order $\delta_m$.

Clearly, this tuning depends sensitively on $\delta_m$, and is greatly reduced when $\delta_m \ll 1$.

It is useful to connect $\delta_m$ to $v/f$. This is done by simply minimizing the part of the Higgs potential that depends on $\ln \Lambda_{UV}$. This leads to the relation

$$\frac{v^2}{f^2} = 1 - \frac{m_Q^2}{\lambda_t^2 f^2} = \frac{\delta_m}{1 + \delta_m},$$

similar to what was found in [16]. We rewrite this as

$$\delta_m = \frac{v^2/f^2}{1 - v^2/f^2},$$

(2.33)

to see that $\delta_m$ roughly tracks the tuning required to misalign the vacuum, as it should, for it is by choosing $\delta_m$ small that we obtain the correct Higgs mass. This makes clear that taking $\delta_m$ small is not an additional tuning, but the only tuning required to realize the correct Higgs potential. For instance, when $f/v = 3$ (10) we find $\delta_m \approx 0.125 (0.01)$ which corresponds to $\sim 10\% \ (1\%)$ tuning.

3 Collider phenomenology

The collider signals of this model arise from the Higgs sector and the production and decay of the hidden color quirks. To determine both these effects we need the physical mass states of the hidden sector fermions. The relevant mass matrix $\mathcal{M}_F$ is

$$- \left( i_L, \hat{T}_L \right) \mathcal{M}_F \left( \hat{t}_R \right) = - \left( i_L, \hat{T}_L \right) \begin{pmatrix} m_Q & m_t \\ 0 & -c_v \lambda_t f \end{pmatrix} \left( \hat{t}_R \right),$$

(3.1)

As noted in the previous section to obtain the correct Higgs mass without introducing additional fine-tuning, we require,

$$m_Q = \frac{\lambda_t f}{\sqrt{1 + \delta_m}} = c_v \lambda_t f,$$

(3.2)
where $\delta_m$ is given in Eq. (2.33). In the following, we fix the vector-like mass for the quirk doublet $m_Q$ to the this value. Note that we can use this relation to define $f/v$ in terms of $m_Q$:

$$f/v = \sqrt{1 + \frac{m_Q^2}{2m_t^2}}.$$  \hspace{1cm} (3.3)

The physical masses are obtained by diagonalizing the fermion mass matrix by the transformation $R(\theta_L)^T M_F R(\theta_R)$, where the rotation matrices are

$$R(\theta_i) = \begin{pmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{pmatrix}.$$  \hspace{1cm} (3.4)

The mass eigenvalues are given by

$$m^2_{\pm} = m_Q^2 \left(1 + \frac{m_t^2}{2m_Q^2} \pm \frac{m_t}{m_Q} \sqrt{1 + \frac{m_t^2}{4m_Q^2}}\right),$$  \hspace{1cm} (3.5)

and the mixing angles are

$$\sin 2\theta_L = -\sin 2\theta_R = \frac{1}{\sqrt{1 + \frac{m_t^2}{4m_Q^2}}}.$$  \hspace{1cm} (3.6)

In other words, $\theta_L = -\theta_R \equiv \theta$. The unmixed states are described by Dirac fermions $\hat{T}_\pm$ with masses $m_{\pm}$, their couplings to SM fields are given in Appendix B.

### 3.1 Scalar Sector

Like other pNGB Higgs models we find the tree level couplings of the Higgs to SM states are reduced. In our case they are reduced by $c_v$, which follows immediately from Eq. (2.10). This leads to the usual bound of $f \gtrsim 3v$ from the LHC measurement of Higgs couplings to gauge bosons. It may also lead to interesting signals at the HL-LHC and future colliders. At the same time, the existence of new fermionic states with electric charge that couple to the Higgs amplifies its coupling to photons. As in the quirky little Higgs model [7], this pushes the rate of $h \rightarrow \gamma\gamma$ closer to the SM prediction [37].

Explicitly, we find the Higgs width into diphotons is approximately

$$\Gamma(h \rightarrow \gamma\gamma) = \frac{\alpha^2 m_h^3}{256\pi^3 v^2} \left| c_v \left[ A_V \left(\frac{4m_V^2}{m_h^2}\right) + \frac{4}{3} A_F \left(\frac{4m^2}{m^2_h}\right) \right] + \frac{m_t \sin \theta}{m_+} \left(c_v \cos \theta - \sqrt{2}s_v \sin \theta\right) \frac{4}{3} A_F \left(\frac{4m_+^2}{m_h^2}\right) + \frac{m_t \cos \theta}{m_-} \left(c_v \sin \theta + \sqrt{2}c_v \cos \theta\right) \frac{4}{3} A_F \left(\frac{4m_-^2}{m_h^2}\right) \right|^2.$$  \hspace{1cm} (3.7)

In Fig. 3 we see how the production of a given Higgs to SM final state rate changes relative to the SM prediction as a function of $m_Q$. The blue curve shows the usual result for tree level Higgs coupling deviations, while the dashed orange curve denotes the decay into two photons. We see that the latter is slightly increased relative to the other rates. However, the deviation is small enough that it would likely require a future lepton collider to measure...
Figure 3: Ratio of the production of a the Higgs boson and subsequent prompt decay into SM final states as a function of $m_Q$. The $h \rightarrow \gamma\gamma$ line is given by the dashed orange curve, while all other prompt SM final states fall along the solid blue curve.

it [38–40]. Current Higgs coupling measurements require this ratio be no less than 0.8, and the HL-LHC is expected to reach a precision corresponding to about 0.9 [41]. We see that these already begin to probe $v/f$, but do not reach beyond about 10% tuning.

The Higgs also develops a loop level coupling to the gluons of the hidden QCD. Similar to coupling to the photon, we find the Higgs coupling to hidden gluons takes the form

$$c_{\hat{g}} = \frac{m_t}{m_+} \left( c_v \cos \theta - \sqrt{2} s_v \sin \theta \right) \left( 3 \frac{4}{4} A_F \left( \frac{4m_{+}^2}{m_{h}^2} \right) \right)$$

This leads to the Higgs width into hidden gluons

$$\Gamma(h \rightarrow \hat{g}\hat{g}) = \frac{\hat{\alpha}_s m_h^5}{288\pi^3 v^2 |c_{\hat{g}}|^2} \left( 6 \pi v m_{h}^2 - m_0^2 \right)^2 \Gamma_{SM}(h(m_0) \rightarrow X_{SM}X_{SM})$$

where $\hat{\alpha}_s = \tilde{g}_s^2/(4\pi)$ is the hidden sector strong coupling parameter, $\tilde{G}_{\mu\nu}$ is the hidden gluon field strength, and

$$\Gamma(h \rightarrow \hat{g}\hat{g}) = \frac{\hat{\alpha}_s m_h^5}{288\pi^3 v^2 |c_{\hat{g}}|^2} \left( 6 \pi v m_{h}^2 - m_0^2 \right)^2 \Gamma_{SM}(h(m_0) \rightarrow X_{SM}X_{SM})$$

which may contribute to a detectable Higgs width at future lepton colliders.

Since the states charged with hidden color carry $U(1)_X$ charge, they are electrically charged under the SM. Bounds from LEP imply that such states cannot be lighter than about 100 GeV. Consequently, the lightest hadrons of the hidden confining group are the glueballs. The lightest glueball state is a $0^{++}$ and has a small mixing with the Higgs. This allows the glueballs to decay with long lifetime to SM states. From [42] we find the glueball partial width into SM states to be

$$\Gamma (0^{++} \rightarrow X_{SM}X_{SM}) = |c_{\hat{g}}|^2 \left[ \frac{\hat{\alpha}_s}{6\pi v m_{h}^2 - m_0^2} \right] \Gamma_{SM}(h(m_0) \rightarrow X_{SM}X_{SM})$$
while this scale has its drawbacks [45] in the pure gauge limit there are not many physical scales to choose from.
Figure 5: The allowed region in the $S$–$T$ plane leaving the $U$ parameter free [57]. The colored points (orange, green, blue, and red) indicate the values of $S$ and $T$ for $f/v = 3, 4, 5, 6$ with UV cutoff scale $2, 3, 4, 5$ TeV.

### 3.2 Electroweak Precision Test

Extensions of the SM are constrained by precision electroweak measurements. The constraints can be expressed in terms of the oblique parameters $S$, $T$, and $U$ [54, 55]. The contributions to $U$ are typically small, so we only compute the contribution to $S$ and $T$. These contributions arise from the new electroweak charged fermions inducing important radiative corrections to gauge boson propagators. In addition, the modified coupling of the Higgs boson to gauge bosons leads to an infrared log divergence [56]. We find the leading contributions to be

\[
S \approx \frac{2N_c m_t^2}{15\pi m_Q^2} + \frac{1}{12\pi} \frac{v^2}{f^2} \ln \left( \frac{\Lambda_{UV}^2}{m_h^2} \right),
\]

\[
T \approx \frac{13N_c m_t^4}{120\pi m_Z^2 m_Q^2 \sin^2 2\theta_W} - \frac{3}{16\pi \cos^2 \theta_W} \frac{v^2}{f^2} \ln \left( \frac{\Lambda_{UV}^2}{m_h^2} \right),
\]

where $\Lambda_{UV}$ is UV cutoff scale, $\theta_W$ is the usual weak mixing angle, and the factor of $N_c$ comes from the number of dark QCD color.

As expected, the contributions from vector-like fermions, $\hat{X}$ and $\hat{Y}$, cancel as well as the power law UV divergences. These contributions are compared to the experimental fits and found to lie within the 68% and 95% allowed regions as provided by the Gfitter collaboration [57]. In Fig. 5, we plot $S$ and $T$ with $U$ free, for the input parameters $m_h = 125$ GeV and $m_t = 172.5$ GeV. The colored points in the figure correspond to values of $f/v = 3, 4, 5, 6$ (orange, green, blue, and red) and $\Lambda_{UV}$, 2, 3, 4, 5 TeV. With increasing the value of $f$, the value of $S$ and $T$ approach the SM value.
3.3 Quirky Signals

The new fermions (\(\hat{T}_\pm, \hat{X}, \text{ and } \hat{Y}\)) can be produced at colliders through Drell-Yan due to their hypercharge of 2/3. We parameterize the couplings of any fermion \(f\) to the \(Z\) boson and the photon by

\[
\mathcal{L} \supset \frac{g}{2c_W}Z_\mu \gamma^\mu(v_f - a_f \gamma^5)f + eQ_f A_\mu \gamma^\mu f,
\]

where \(c_W\) and \(s_W\) is the cosine and sine of the weak mixing angle while \(g\) and \(e = g s_W\) are the weak and electric couplings, respectively. As an example, SM fermions have \(v_f = T_3 - 2Q_s\) and \(a_f = T_3\). We then find the partonic cross section for \(qq \rightarrow Z, \gamma \rightarrow ff\) to be

\[
\sigma(q\bar{q} \rightarrow ff)(\tilde{s}) = \frac{\pi \alpha_Z^2 N_c}{12 N_c \tilde{s}(1 - m_Z^2/\tilde{s})} \sqrt{1 - \frac{4m_f^2}{\tilde{s}}} \times \left\{ (1 + \frac{2m_f^2}{\tilde{s}}) \left[ |v_q v_f + 4s_W c_W Q_q Q_f \left(1 - \frac{m_f^2}{\tilde{s}}\right)|^2 + |a_q v_f|^2 \right] \right. \\
\left. + \left(1 - \frac{4m_f^2}{\tilde{s}}\right) \left[ |v_q a_f|^2 + |a_q a_f|^2 \right] \right\},
\]

where \(\alpha_Z \equiv g^2/(4\pi c_W^2)\). In Fig. 6 we see the fermion cross sections at a 14 TeV proton-proton collider. We used MSTW2008 PDFs [58] and a factorization scale of \(\sqrt{\tilde{s}}/2\).

All the fermions charged under the hidden color group have masses much above 100 GeV due to LEP bounds on charged particles. The hidden confining scale is of the order of a few GeV, so we expect them to exhibit quirky [33] dynamics, which can give a variety of new signals at colliders [59–64]. After production they are connected by a string of hidden color flux which, because there are no light hidden color states, is stable against fragmentation. The quirky pair behaves as though connected by a string with tension \(\sigma \sim 3.6\tilde{\Lambda}_{QCD}^2\) [65], see also [66].
Much of the subsequent dynamics can be treated semi-classically. Since these quirks carry electric charge the oscillating particles radiate soft-photons, quickly shedding energy until they reach their ground state \[67, 68\]. Annihilation is strongly suppressed in states with nonzero orbital angular momentum, so in nearly every case the quirks do not annihilate until they reach the ground state. Since the quirks are accelerated by the string tension, we can estimate their acceleration as \( a = \sigma / m_f \). Then, using the Larmor formula we can estimate the radiated power as

\[
P = \frac{8\pi\alpha}{3} a^2 = \frac{8\pi\alpha\sigma^2}{3m_f^2},
\]

where \( \alpha = e^2 / (4\pi) \). The time it takes the quirky bound state to drop to its ground state is given by \( K / P \), where \( K \) is the kinetic energy of the quirks. Taking \( K \sim m_f \) we can then estimate the de-excitation time \( T_d \) as

\[
T_d \sim \frac{3m_f^3}{8\pi\alpha \left( \frac{3.6\Lambda_{\text{QCD}}^2}{\text{GeV}} \right)^2} \sim 4 \times 10^{-19} \text{ sec} \left( \frac{m_f}{800 \text{ GeV}} \right)^3 \left( \frac{6 \text{ GeV}}{\Lambda_{\text{QCD}}} \right)^4.
\]

Clearly, the de-excitation is very fast, leading to prompt annihilation.

Depending on the masses of the hidden \( \hat{b} \) quark, the \( \hat{T}_\pm \) could \( \beta \)-decay by emitting a \( W \). When the mass splitting is small the de-excitation is faster and the states typically annihilate. However, if the splitting is large it is most likely that both top-like states transition to bottom-like states. These would then de-excite and annihilate in the same way, though there would be additional \( W \)s in the final state.

If the \( \hat{b} \) quarks are not too heavy, then \( \hat{T}_\pm \hat{b} \) combinations can be produced through the \( W \) boson. If these states are similar in mass so that \( \beta \)-decay is slow then the bound states can lead to visible signals, like \( W\gamma \) resonances, with appreciable rates. This is because the electric charge of the state prevents its decay into hidden gluons. However, larger splittings allow the heavier state to decay to the lighter promptly, diluting these signals significantly.

Because the quirks are fermions there are four \( s \)-wave states, one singlet and three triplet. Following [14] we assume that each of these states is populated equally by production, so we take the total width \( \Gamma_{\text{tot}} \) of the bound state to be

\[
\Gamma_{\text{tot}} = \Gamma_s + 3\Gamma_t,
\]

where \( \Gamma_s \) and \( \Gamma_t \) are the widths of the singlet and triplet states respectively.

For the \( \hat{T}_\pm \hat{b} \) states which carry weak isospin the dominant quirkonium decays are to \( WW \) with a branching fraction of about 75\%. This comes from the chiral enhancement in this decay. This signal has been searched for at the LHC by both ATLAS [69, 70] and CMS [71, 72]. The next largest fractions are into \( Zh \), at the 10\% level, which can be compared to ATLAS [73] and CMS [74, 75] searches. All other visible final states are suppressed well below the percent level, see Fig. 7. Of these, the most likely LHC signal is a new scalar resonance decaying to \( WW \), though this does depend on the \( b \)-quirk mass. As shown in Fig. 8, current searches are not yet sensitive to these signals. Here we assume a production of the \( \hat{T}_- \hat{T}_- \) directly, and through production of the \( \hat{T}_+ \) state which then decays to a soft \( Z \) and \( \hat{T}_- \). While the LHC is not yet sensitive to these signals, the high luminosity run (dashed red line) will probe the most natural regions of parameter space [76].
Figure 7: Branching fraction for the s-wave quirkonium composed of the $T_-$ (left) and $\tilde{X}$ or $\tilde{Y}$ (right) quirks. The $T_-$ quirks dominant decays are into weak gauge bosons $W^+W^-$, and we assume $\delta_m = 0.1$. The dominant branching fraction for the $\tilde{X}$ and $\tilde{Y}$ is into hidden gluons.

Figure 8: Left: Comparison of LHC 13 reach in $W^+W^-$ resonance searches for a $\tilde{T}_-\tilde{T}_-$ bound state (black) to the theoretical 14 TeV prediction (blue) and expected sensitivity of the high luminosity run (dashed red). Right: Comparison of LHC 13 reach in dilepton resonance searches for $\tilde{X}\tilde{X}$ and $\tilde{Y}\tilde{Y}$ bound states (black) to the theoretical 13 TeV prediction (blue). Contributions from $\tilde{T}$ states included for $m_Q = 800$ (1000) GeV in red dashed (purple dash-dotted) curve for DM masses greater than 100 GeV.

The $\tilde{X}$ and $\tilde{Y}$ particles only couple to visible states through hypercharge, hence there is no rate into $Zh$ and the rate into $WW$ vanishes when the $Z$ mass can be neglected. The largest coupling is to hidden gluons, so this dominates the branching fractions. These gluons shower and hadronize into hidden QCD glueballs, some fraction which may have displaced decays at the LHC [77]. However, they can also annihilate into $\bar{f}f$ and EW gauge bosons through their hypercharge coupling, see Fig. 7. Of these, dilepton and diphoton channels have the greatest discovery potential because the signal is so clean, which has motivated searches at both ATLAS [78, 79] and CMS [80, 81]. In the right panel of Fig. 8 we compare the reach of the ATLAS search [79] to the theoretical prediction. We see that quirks below about 550 GeV are in tension with current collider bounds. Seeing that the predicted cross section is near the experimental limit, it is likely that by the end of the LHC run 3, with 300 fb$^{-1}$, any quirks of this type below a TeV will be discovered. Further LHC runs can probe even larger $m_V$, but we note that taking this mass larger does not affect the naturalness of the Higgs mass. It does, however, indicate that the DM is heavier, see Eq. (2.27).

When $m_V > m_\tau + m_\chi$ the $\tilde{X}, \tilde{Y}$ quirks will quickly decay, $\tilde{V} \rightarrow \tilde{T}_- + \chi$. In this case the powerful dilepton resonance search will not apply. Instead, the production cross section
for $T_-$ bound states must include this, in general small, additional mode. A similar story holds if $m_\perp > m_V + m_\chi$, where now the $\hat{T}_- \chi$ quirk decays promptly to an $\hat{X}$ or $\hat{Y}$ and a DM scalar. Then, the dilepton bounds would apply to the $\hat{T}$ production. For lighter $m_Q$ this can strengthen the bound on $m_V$. The red dashed and purple dash-dotted lines on the dilepton bound in Fig. 8 correspond to taking $m_Q = 800$ GeV and $m_Q = 1000$ GeV, respectively, and the DM mass of 100 GeV. By taking the DM heavier these lines would cut off earlier, at $m_V = m_\perp - m_\chi$. In summary, standard collider searches for prompt visible objects do constrain $m_\hat{\chi} > 550$ GeV, but the other parameters of the model are less restricted. However, both the displaced searches related to the hidden sector glueballs and dilepton and diboson resonance searches can provide evidence for the hidden QCD sector at the LHC. As we shall see in the next section, this parametric freedom can lead to viable DM, and complementary search strategies from DM experiments.

4 Dark matter phenomenology

In this section we detail the phenomenology of the DM candidate $\chi$, the complex scalar charge under the global symmetry $U(1)_D$. As mentioned above, this global symmetry stabilizes the DM. All the SM fields and the quirky top partners $\hat{T}_\pm$ are $U(1)_D$ neutral, whereas the quirky fermions $\hat{X}$ and $\hat{Y}$ are charged. The $U(1)_D$ global symmetry is exact, so we can associate a discrete dark $Z_2$ parity under which,

$$\chi \xrightarrow{Z_2} -\chi, \quad \hat{X} \xrightarrow{Z_2} -\hat{X}, \quad \hat{Y} \xrightarrow{Z_2} -\hat{Y}, \quad (4.1)$$

but more generally we simply consider particles in this sector as carrying a global dark charge, which prevents their decay. Since the quirky states $\hat{X}$ and $\hat{Y}$ have the fractional SM electric charge $2/3$ they cannot be the DM. However, the SM neutral complex scalar $\chi$ is our DM candidate as long as it is the lightest $U(1)_D$ charged particle.

To determine the success of this scalar as explaining the observed DM in the universe, in what follows we calculate the relic abundance and DM-nucleon cross section for the direct detection in our model. We then consider the dark matter annihilation for the indirect detection and impose the collider constraints on our parameter space. We find that much of the natural parameter space of this model has not yet been conclusively probed by experiment, but is expected to be covered next several years.

4.1 Relic abundance

The thermal relic density of the scalar $\chi$ is obtained using the standard freeze-out analysis. Figures 9 and 10 show the relevant Feynman diagrams for the DM annihilation and semi-annihilation/conversion, respectively. The Boltzmann equation for the DM annihilation and semi-annihilation/conversion processes is

$$\frac{d\bar{n}_\chi}{dt} = -3Hn_\chi - \langle \sigma_{\chi\phi\phi}' n_{\chi\phi\phi} \rangle \left( n_\chi^2 - \bar{n}_\chi^2 \right) - \langle \sigma_{\chi\phi\phi}' n_{\chi\phi\phi}^T \bar{n}_{\hat{V}} \rangle \left( n_\chi n_{\hat{V}} - \bar{n}_\chi \bar{n}_{\hat{V}} \right) - \langle \sigma_{\chi\phi\phi}' n_{\hat{V}} \bar{n}_{\hat{V}} \rangle \left( n_\chi n_{\hat{V}} - \bar{n}_\chi \bar{n}_{\hat{V}} \right)$$
where $\phi(\phi')$ are the SM fields: $h, t, W, Z, \gamma, \cdots$. Also, $H$ is the Hubble parameter and $n_{i}$ is the number density of species $i$, whereas the $\tilde{n}_{i}$ is its thermal equilibrium value. The quantity $\langle \sigma_{ijkl} v_{\text{Møl}} \rangle \equiv \langle \sigma_{ij} (ij \rightarrow kl) v_{\text{Møl}} \rangle$ is the thermal averaged cross-section of the initial states $ij$ to final states $kl$ with $v_{\text{Møl}}$ being the Møller velocity. The last term in the first line of Eq. (4.2) describes the dynamics of the standard DM annihilation to the SM final states as shown in Fig. 9. The second and third lines describe the semi-annihilation and conversion processes shown in Fig. 10.

The dominant DM annihilation channels are to the SM, i.e. $\chi \chi \to WW, hh, ZZ, t\bar{t}, b\bar{b}$, while the semi-annihilation and conversion processes are only relevant if the masses the quirk states $(\bar{V}, \bar{T}_{\pm})$ are similar to $m_{\chi}$. When the quirk masses are much larger than the DM, their thermal distributions are Boltzmann suppressed, making semi-annihilation or conversion processes very rare as compared to the standard annihilation processes. The relevant Feynman rules to calculate the DM annihilation or semi-annihilation processes are given in Appendix B. The DM relic abundance is computed using the public code micrOMEGAs [82].

Before discussing these results we emphasis some of the features of this model.

- The top partners are SM color neutral, therefore the symmetry breaking scale $f$ may be at or below a TeV. This leads to significant improvements in the fine-tuning while simultaneously allowing a larger window for the pNGB DM masses in comparison to colored top partner models [29, 31, 32].

- The DM annihilations to SM are dominated by $s$-channel Higgs exchange. The amplitude for such processes is,

$$M_{\chi \chi \phi \phi'} \propto \left( \frac{s}{f^2} - 2\lambda_{h\chi} \right) v,$$  

(4.3)
where \( s = 4m_X^2 \). The \( s \) dependent term originates from the derivative coupling \( \partial_\mu h \partial_\mu (\chi^* \chi) \), while the \( \lambda_{hX} \) term is a loop induced explicit breaking of the \( \chi \) shift symmetry, see Eq. (2.19).

- When the standard DM annihilation processes dominate (which we see below is typically the case), the DM relic abundance can be estimated as,
  \[
  \Omega \chi h^2 \approx 0.12 \left( \frac{2.2 \times 10^{-26} \text{cm}^3 \text{s}^{-1}}{\langle \sigma(\chi\chi^* \rightarrow \text{SM}) v_{\text{Mol}} \rangle} \right), \tag{4.4}
  \]
  where 0.12 is the observed DM relic abundance by the PLANCK satellite [83].

- The thermal averaged annihilation cross section to SM fields via \( s \)-channel Higgs exchange is proportional to
  \[
  \langle \sigma(\chi\chi^* \rightarrow \text{SM}) v_{\text{Mol}} \rangle \propto \frac{1}{m_X^2} \left( \frac{4m_X^2}{f^2} - 2\lambda_{hX} c_v^2 \right)^2, \tag{4.5}
  \]
  which implies that in the limit \( \lambda_{hX} \rightarrow 0 \), i.e. no explicit shift symmetry breaking, the cross section is proportional to \( m_X^2 / f^2 \). Hence, for a given \( m_X \) the relic abundance, \( \Omega \chi h^2 \), scales as \( f^4 \).

- For \( m_X^2 / f^2 \ll 1 \), \( \chi \) annihilation proceeds through the portal coupling \( \lambda_{hX} \). When \( m_X^2 \sim \lambda_{hX} f^2 / 2 \) the annihilation cross-section drops due to cancellation between the \( s \)-channel process, enhancing the relic abundance. For \( m_X^2 \gg \lambda_{hX} f^2 / 2 \) the DM relic abundance falls like \( 1/m_X^2 \) for fixed \( f \).

In Fig. 11, we show the relic abundance \( \Omega \chi h^2 \) for two benchmark values of \( \lambda_{hX} = 0.005 \) and 0.025 as a function of \( m_X \) with fixed \( f/v = 4, 6, 8, 10 \). Notice that for masses below 50 GeV the DM tends to be overproduced. This is because the thermal averaged cross-section in this region is directly proportional to the portal coupling \( \lambda_{hX} \), which direct detection constrains to be relatively small (see below). On the other hand, for \( m_X \sim m_h / 2 \) the relic abundance drops sharply due to the resonant enhancement of the Higgs portal cross-section. For DM masses \( m_X^2 \sim \lambda_{hX} f^2 / 2 \) there is cancelation in the cross-section as a result the relic abundance enhances which produces the peaks in Fig. 11. For larger DM masses the cross section is proportional to \( m_X^2 / f^4 \) and the relic density drops as DM mass increases.

For the case \( \lambda_{hX} = 0.005 \) (left-panel), the relic density curves terminate when the DM becomes heavier than the quirk states \( \hat{X}, \hat{Y} \). These states are bound by the dark color force into quirky bound states, which then efficiently annihilate due to their electric charge, making them an unsuitable thermal DM candidate. There is also a sharp drop in the relic density at the end of each curve, which is due to an \( s \)-channel resonant enhancement of semi-annihilation processes, i.e. \( \chi \hat{V} \rightarrow \hat{T}_\pm \rightarrow \hat{T}_\pm \text{SM} \), as shown in Fig. 10. The semi-annihilation processes are only significant when \( m_X \approx m_V \approx m_\pm / 2 \) and in most of the parameter space are inefficient as compared to the standard annihilation processes. Since the portal coupling \( \lambda_{hX} \) is proportional to \( r_Q = m_V^2 / m_Q^2 \) it can be reduced for relatively light vector-like quirks \( \hat{V} \). However, collider searches at the LEP and LHC put a lower bound these vector-like quirks, see Sec. 3.3.

We see that the smallest mass that produces the correct DM thermal relic is near the Higgs resonance region, above \( \sim 50 \text{ GeV} \). This is fairly independent of \( f/v \) and \( \lambda_{hX} \).
However, the largest DM masses which leads to correct relic abundance does depend on $f/v$ and $\lambda_{h\chi}$. Since naturalness prefers a smaller $f/v$ and $\lambda_{h\chi}$ is constrained by direct detection (see below), we find that restricting $f/v \leq 10$ puts an upper bound of $m_{\chi} \lesssim 1$ TeV for obtaining the correct relic.

### 4.2 Direct detection

The WIMP DM scenario is being thoroughly tested by direct detection experiments. We here highlight the main features of our pNGB DM construction where direct detection null results are explained naturally.

At tree-level the DM-nucleon interaction is only mediated by $t$-channel Higgs exchange. As discussed above, the DM-Higgs interaction has two sources: (i) the derivative coupling $\sim (\partial^\mu h) (\partial^\nu (\chi^* \chi))/f^2$, and (ii) the portal coupling $\sim \lambda_{h\chi} h\chi^*\chi$. The strength of the derivative interaction in a $t$-channel process is suppressed by the DM momentum transferred, $t/f^2 \sim (100$ MeV$)^2/f^2 \ll 1$. For all practical purposes we can neglect such interactions. Hence the only relevant interaction for direct detection is the portal coupling $\lambda_{h\chi}$.

In this case, the spin-independent DM-nucleon scattering cross-section $\sigma_{\chi N}^{SI}$ can be approximated as (see e.g. [29, 31]),

$$\sigma_{\chi N}^{SI} \simeq \frac{f_N^2 m_N^4}{\pi m_h^4 \frac{m_h^2}{m_{\chi}}} \approx 2.5 \times 10^{-46} \left( \frac{\lambda_{h\chi}}{0.025} \right)^2 \left( \frac{300 \text{ GeV}}{m_{\chi}} \right)^2,$$

where $m_N$ is the nucleon mass and $f_N \approx 0.3$ encapsulates the Higgs-nucleon coupling. The current bound on the spin-independent DM-nucleon cross-section for mass range $\sim [50, 1000]$ GeV is by XENON1T with one ton-year of exposure time [84]. For instance, the upper limit on the spin-independent DM-nucleon cross-section for DM mass 300 GeV is

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\[^3\]There are 1-loop processes involving the quirk states and the electroweak bosons which contribute to the DM-nucleon scattering. These processes are suppressed compared to tree-level, so we neglect them.
Figure 12: The DM-nucleon cross section as a function of DM mass $m_\chi$ with all points producing the observed relic abundance $\Omega_{\text{obs}} h^2 = 0.12 \pm 0.0012$. The gray points above gray curve are excluded by current XENON1T bounds, whereas the colored points (corresponding to particular $f/v$ values) are allowed.

$\sim 2.5 \times 10^{-46}$ at 90% C.L. From Eq. (4.6) it is clear that the $\sigma_{SI}^{\chi N}$ is directly proportional to the square of the portal coupling $\lambda_{h\chi}^2$ and inversely proportional to the square of DM mass $m_\chi^2$. Hence to satisfy the direct detection constraints we either need to reduce the portal coupling $\lambda_{h\chi}$ or increase the DM mass.

One feature of this minimal model is that $\lambda_{h\chi}$ is determined by a small number of low-energy parameters: the vector-like masses of the quirks, $m_V$ and $m_Q$. However, as noted above in Eq. (3.2), the top partners quirk mass $m_Q = c_v \lambda_t f$ is fixed in terms of $f$ to obtain the correct Higgs mass. Hence, the free parameters are $m_\chi$, $f$, and $r_Q \equiv m_V^2 / m_Q^2$. As discussed above, one can specify $f$ by requiring the correct DM relic abundance and $r_Q$ can be exchanged with $\lambda_{h\chi}$, which is constrained by direct detection.

In Fig. 12 we show the spin-independent DM-nucleon cross section $\sigma_{SI}^{\chi N}$ as a function of DM mass $m_\chi$. We have performed a random scan of the parameter space for $f/v \in [3, 10]$ and $m_V \in [m_\chi, 4\pi f]$. The lower value of the $f/v = 3$ choice is enforced by the SM Higgs coupling measurement and electroweak measurements data, while the upper value of $f/v = 10$ limits the tuning to $\sim 1\%$. The lower value of $m_V$ makes sure that $\chi$ is the lightest state charged under $U(1)_D$. All the points shown in the plot correspond to the correct relic abundance $\Omega_\chi h^2 = \Omega_{\text{obs}} h^2 \pm 5\sigma$, where $\Omega_{\text{obs}} h^2 = 0.12 \pm 0.0012$ is the observed DM relic density as measured by the Planck satellite [83]. The gray (pentagon) points above the gray line are excluded by the XENON1T [84]. All the colored points (color barcoded with $f/v$) are allowed by the current XENON1T constraint. The dashed gray line indicates the expected XENONnT bound [84] which covers much of the more natural parameter space. However, there are points allowed below this bound above the so-called neutrino-floor (red dotted), which could be discovered by next generation detectors, e.g. LZ [85] and DARWIN [86].
Figure 13: The parameter space with all the points producing correct relic abundance in the $m_\chi$ vs $\langle \sigma v \rangle_{WW}$. The colored (gray) points allowed (excluded) by the XENON1T1y direct detection experiment. The allowed points are color coded for different $f/v$ values.

4.3 **Indirect detection**

We now turn to indirect detection. There are a variety of experiments searching for DM annihilations in the Milky Way galaxy and nearby dwarf galaxies, which are assumed to be dominated by DM. The typical signals of DM annihilation to the SM particles leads to gamma-rays, gamma-lines, and an excess of secondary products like antipositrons and antiprotons in cosmic-rays (CR). In particular, the experimental data can be used to put upper bounds on the various annihilation channels, including $WW, ZZ, hh, t\bar{t}, b\bar{b}, \tau^+\tau^-, \cdots$.

In our model the DM dominantly annihilates into $WW, hh, ZZ, t\bar{t}$ final states. We calculate the present day DM thermal averaged annihilation to the SM particles at zero velocity by using micrOMEGAs [82]. We find that the DM thermal annihilation cross-section is $\langle \sigma v \rangle \approx 2.2 \times 10^{-26} \text{ cm}^3/\text{s}$ for parameter values that produce the correct relic abundance. The fraction of annihilation cross-section to $W^+W^-$ is $\sim 45\%$ and $hh/ZZ \sim 25\%$ for $m_\chi \gtrsim m_h$. Whereas the branching fraction is dominantly into $W^+W^-$ for $m_\chi \in [m_W, m_h]$.

In Fig. 13 we show the DM annihilation cross section to $W^+W^-$, $\langle \sigma v \rangle_{WW}$, in units of $[10^{-26} \text{ cm}^3/\text{s}]$ as function of $m_\chi$. All the data points in this figure produce correct DM relic abundance and satisfy the XENON1T direct detection constraint. Because these points have $m_\chi > m_W$, the most dominant annihilation channels are the $WW, ZZ, hh$. In the following we summarize the most sensitive indirect detection probes in the mass range of interest.

**Gamma-rays:** The most robust indirect detection bounds are due to **Fermi-LAT** [87] and **Fermi-LAT+DES** [88] with six years of data from 15 and 45 DM dominated dwarf spheroidal galaxies (dSphs), respectively. Theses constraints are considered robust because the uncertainties associated with propagation of gamma rays are relatively small. The **Fermi-LAT** results [87] provide upper-limits on the DM thermal annihilation cross section into several
SM final states including $WW, b\bar{b}, \tau^+\tau^-$, whereas, the updated analysis Fermi-LAT+DES \cite{88} only includes the $b\bar{b}$ and $\tau^+\tau^-$ channels. These bounds do not constraint any of the parameter space allowed by the direct detection. However, Fermi-LAT has provided expected 95% C.L. upper-limits for the DM thermal annihilation into $b\bar{b}$ and $\tau^+\tau^-$ channels with 15 years of data and 60 dSPhs \cite{89}. One can interpolate the projected upper-limit from the $\langle \sigma v \rangle_{b\bar{b}}$ to $\langle \sigma v \rangle_{WW}$ by a simple rescaling $\langle \sigma v \rangle_{WW} \simeq 1.33 \langle \sigma v \rangle_{b\bar{b}}$ in the DM mass range of our interest. In Fig. 13 we show the projected 95% C.L. sensitivity on $\langle \sigma v \rangle_{WW}$ by Fermi-LAT with 15 years and 60 dSPhs by the solid (gray) curve. This sensitivity sets a lower-limit on the DM mass $m_\chi \gtrsim 150$ GeV.

Cosmic-rays: The flux of antipositrons and antiprotons in the cosmic-rays (CR) provides another indirect probe of DM annihilation in the Galaxy. In particular recent precise AMS-02 CR antiproton flux data \cite{90} has led to strong constraints on the DM thermal annihilation. In Refs. \cite{91, 92} the AMS-02 antiproton flux data was used to put stringent constraints on DM with masses in range $[150, 1000]$ GeV. The AMS-02 95% C.L. exclusion constraint on $\langle \sigma v \rangle_{WW}$ as obtained by CCK \cite{91} is shown in Fig. 13 as dash-dotted (blue) curve. This constraint excludes most of the data points between DM masses $m_\chi \in [225, 375]$ GeV. However, these constraint has large systematic uncertainties, mainly due to CR propagation and diffusion parameters \cite{91}. The updated analysis by (CHKK) \cite{93} reveals a weaker constraint in the $W^+W^-$ channel, which is also given by a dash-dotted (blue) curve. Even though the updated AMS-02 analysis does not constrain our model, future AMS CR antiproton data are likely to. Another future CR experiment is the Cherenkov Telescope Array (CTA) which is expected to be sensitive to large DM masses \cite{94}. In Fig. 13 we show the projected sensitivity of CTA for DM annihilation to $W^+W^-$ with Galactic Diffuse Emission (GDE) Gamma model of Ref. \cite{95}, as a dashed (red) curve, for two assumptions of systematic error. The most optimistic implies that CTA will probe DM masses above $\sim 300$ GeV, though this is quickly weakened when systematic errors are included.

5 Conclusion

We have outlined a framework in which the Higgs and a scalar DM candidate arise pNGBs of a broken global symmetry. Because the symmetry partners of the top quark do not carry SM color, the induced scalar potential between the Higgs and the DM, which is UV insensitive, allows for improved fine-tuning and simultaneously explains null results for WIMP DM searches. The quantitative success of this framework is summarized by Fig. 14 in the $m_\chi$ vs $\lambda_{h\chi}$ plane with the color of scanned points corresponding to values of $f/v \in [3, 10]$. This corresponds to fine-tuning in the model of about 10% to 1%, respectively.

The phenomenology can be specified by the DM mass $m_\chi$, the global symmetry breaking scale $f$, and the vector-like mass $m_V$ of the quirky fermions, which is the source of breaking the $\chi$ shift symmetry. As shown in Sec. 2.3 we can trade $m_V$ for $\lambda_{h\chi}$. Hence, the three free parameters of the model are $m_\chi, f/v$, and $\lambda_{h\chi}$.

The points in Fig. 14 scan in $m_\chi \sim [50, 1000]$ GeV and $\lambda_{h\chi} \sim [0.2, 0.0005]$ while required to produce the correct relic abundance $\Omega_\chi h^2 = 0.12 \pm 5(0.012)$. The gray (pentagon) points are excluded at 90% C.L. by the direct detection experiment XENON1T with one year exposure time \cite{84}. Future direct detection XENONnT 90% C.L. reach is overlaid as the dash-dotted (black) curve, which would cover much of the allowed parameter space. Next
Figure 14: The parameter space with all the points producing correct relic abundance in the $m_\chi$ vs $\lambda_{h\chi}$. The colored (gray) points allowed (excluded) by the XENON1T1y direct detection experiment. The allowed points are color coded for different $f/\nu$ values.

generation experiments that will descend toward the neutrino floor will fully explore this framework.

The next most stringent constraint is due to the LHC bound on the vector-like mass $m_V \gtrsim 550$ GeV of the quirky fermions $\hat{X}, \hat{Y}$ as shown in Fig. 8. This limit from the ATLAS collaboration search for dilepton resonances with 139 fb$^{-1}$ data is due to the annihilation of quirks $\hat{V}\hat{V}$ to $\ell^+\ell^-$. We show the bound in Fig. 14 as red (hexagon) points. Since the portal coupling $\lambda_{h\chi}$ is proportional to $m_V^2$, the lower-bound on $m_V$ translates to a DM mass and $f/\nu$ dependent lower-bound on $\lambda_{h\chi}$. We have also shown dashed (red) contours of $m_V = 1$ TeV to 10 TeV which shows how future LHC runs may be able to discover quirks in much of the natural parameter space. The complementarity between collider and direct detection could lead to both discovery and confirmation of this construction in the coming years, or its exclusion.

In Fig. 14 we also show how indirect detection gamma-rays 95% C.L. constraints from the Fermi-LAT 15 years with 60 dSphs as blue (star) points. This puts a lower limit on the DM mass $m_\chi \gtrsim 150$ GeV. We have not shown in this plot the indirect detection constraints from the cosmic-rays experiments AMS-02 because of their large systematic uncertainties. However, in the future such uncertainties may be reduced, allowing experiments like AMS and CTA to provide another complementary probe, and hopefully discovery, of this model.

In summary, this framework of WIMP dark addresses the hierarchy problem without colored symmetry partners, and consequently is only tuned at the 10% level while agreeing with all experimental bounds. However, existing experiments will soon be able to discover or exclude these more natural realizations of the model. After the searches of the HL-LHC run and next generation direct detection experiments models with fine tuning at or better than 1% may be thoroughly probed.
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A \textit{SO}(7) Generators

In this appendix we collect all the relevant details. The \textit{SO}(7) generators in the fundamental representation can be written as,

\begin{align}
T^{a_{L,R}}_{ij} &= -\frac{i}{2} \left[ \frac{1}{2} \epsilon^{abc} \left( \delta_i^b \delta_j^c - \delta_i^c \delta_j^b \right) \right] \pm \left( \delta_a^4 \delta_j^i - \delta_a^i \delta_j^4 \right), \quad a_{L,R} = 1, 2, 3, \\
T_{ij}^{ab} &= -\frac{i}{\sqrt{2}} \left( \delta_i^a \delta_j^b - \delta_i^b \delta_j^a \right), \quad b = 5, 6; \quad a = 1, \ldots, b - 1, \\
T_{ij}^{\hat{a}} &= -\frac{i}{\sqrt{2}} \left( \delta_i^\hat{a} \delta_j^7 - \delta_j^\hat{a} \delta_i^7 \right), \quad \hat{a} = 1, \ldots, 6,
\end{align}

where \(i, j = 1, \ldots, 7\). We have chosen the normalization \(\text{Tr} \left[ T^a T^b \right] = \delta^{ab} \). The unbroken generators \(T^{a_{L,R}}_{ij}, T_{ij}^{ab}\) correspond to the \textit{SO}(6), whereas the broken generators \(T_{ij}^{\hat{a}}\) correspond to the \textit{SO}(7)/\textit{SO}(6) coset. Note that \(T^{a_{L,R}}_{ij}\) correspond to the custodial \textit{SO}(4)_{C} \cong \text{SU}(2)_{L} \times \text{SU}(2)_{R }\) subgroup of \textit{SO}(6).

B Feynman rules and Quirk Processes

In this appendix we record formulae for quirk production and decay widths. The relevant Feynman rules are given in Table 1. The decays are typically similar to the results [96, 97], using the methods outlined in [98, 99]. The couplings of the \(Z\) to fermions are taken to be

\begin{equation}
\frac{g}{2c_W} \gamma^\mu (v_i - a_i \gamma_5),
\end{equation}

where \(c_W \equiv \cos \theta_W\). For convenience we define the following

\begin{equation}
R_i = \frac{m_i^2}{M^2}, \quad \beta_{ij} = \sqrt{1 - 2(R_i + R_j) + (R_i - R_j)^2},
\end{equation}

where \(M\) is the mass of the relevant bound state. The number of colors in the quirk confining group is \(N_c\).

We calculate the cross section \(pp \to Z, \gamma \to \bar{f}f\) from the quark \(q\) initiated partonic cross section \(\sigma\) into a quirk \(Q\) pair by

\begin{equation}
\sigma(pp \to QQ)(s) = \sum_q \int_{4m_q^2}^s d\tau L_{\bar{q}\bar{q}}(\bar{q}\bar{q} \to \bar{Q}Q)(\bar{\tau} s),
\end{equation}

where

\begin{equation}
L_{\bar{q}\bar{q}}(\tau) = \int_{\tau}^1 \frac{dx}{x} \left[ f_q(x) f_{\bar{q}} \left( \frac{\tau}{x} \right) + f_{\bar{q}} \left( \frac{\tau}{x} \right) f_q(x) \right],
\end{equation}

\vspace{-2cm}

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is denoted \( Z \) and not exactly known, so we simply give each decay width in units of the unknown factor on the radial wavefunction.

| Decay Width | Formula |
|-------------|---------|
| \( (p_1 + p_2)^2 - 2 \frac{c^2 \lambda_{h\chi}}{f} \) | \( \frac{2c_v c_0}{f^2} (p_1 + p_2)^2 - p_1 p_2 - 3 \frac{c^2 \lambda_{h\chi}}{f} \) |
| \( (p_1 + p_2)^2 - 2 \frac{c^2 \lambda_{h\chi}}{f} \) | \( \frac{i 2c_v c_0}{f^2} (p_1 + p_2)^2 - p_1 p_2 - 3 \frac{c^2 \lambda_{h\chi}}{f} \) |
| \( (p_1 + p_2)^2 - 2 \frac{c^2 \lambda_{h\chi}}{f} \) | \( \frac{i 2c_v m_f^2}{v}, \quad V = W^+, Z \) |

Table 1: Some of the most relevant Feynman rules of our model are listed in this table, see the text for the corresponding notation.

Because the quirk states decays from all \( \ell > 0 \) states are strongly suppressed \([33]\) we only consider decays of the singlet \( ^1S_0 \) and triplet \( ^3S_1 \) states. Each of these decay widths depends on the radial wavefunction \( R(0) \) of the quirk bound state. This factor is nonperturbative and not exactly known, so we simply give each decay width in units of the unknown factor \( |R(0)|^2 \).

The neutral states are composed of fermionic quirks \( Q \) with mass \( m_Q \). In this case the \( Z \) couplings are labeled \( v_Q \) and \( a_Q \), and the electric charge is denoted \( Q_Q \). The mass is denoted \( m_Q \) and we take the meson mass to be \( M \), which for heavy constituents is approximately \( 2m_Q \).

We begin with decays to fermion pairs. These fermions have \( Z \) couplings \( v_f \) and \( a_f \) as well as electric charge \( Q_f \). They also come in \( N_c \) colors. The decays to \( \bar{f} f \) are,

\[
\Gamma( ^1S_0 \to \bar{f} f) = \frac{2N_c N_c a_W^2 a_0^2 a_0^2 m_f^2 m_Q^2}{c_W^4 m_z^4 m_f^4 m_Q^2} \beta_{ff},
\]

\[ (B.5) \]

\[ ^4 \text{This introduces a relative factor of two compared to the } Z \text{ couplings used by } [96, 97]. \]
\[ \Gamma(3S_1 \rightarrow f \bar{f}) = \frac{N_c N_v \alpha_W^2}{12 M^2} \beta_{ff} \left[ (1 + 2 R_f) \left( 4 s_W^2 Q f Q f + \frac{v_Q v_f}{c_W^2 (1 - R_Z)} \right)^2 + \frac{v_Q^2 a_f^2 \beta_{ff}}{c_W^2 (1 - R_Z)^2} \right] , \]  

(B.6)

where \( \alpha_W \equiv g^2/(4\pi) \). Next, we turn to decays into \( Z \gamma \),

\[ \Gamma(3S_0 \rightarrow Z \gamma) = \frac{8 N_c \alpha_W \alpha Q^2 v_Q^2}{c_W^2 M^2} (1 - R_Z) , \]  

(B.7)

\[ \Gamma(3S_1 \rightarrow Z \gamma) = \frac{8 N_c \alpha_W \alpha Q^2 a_f^2 m_Q^2}{3 c_W^2 m_Z^2 M^2} (1 - R_Z^2) . \]  

(B.8)

The decays to \( ZZ \) \footnote{Note the erratum of [96] in reference to \( \Gamma(3S_0 \rightarrow ZZ) \) and \( \Gamma(3S_1 \rightarrow Zh) \). In addition, the \( \Gamma(3S_0 \rightarrow f \bar{f}) \) depends on the axial coupling of the \( 3S_0 \) constituents to the \( Z \), as clarified in [97].},

\[ \Gamma(1S_0 \rightarrow ZZ) = \frac{N_c \alpha_W^2 (v_Q^2 + a_Q^2)^2}{4 M^2 c_W^2 (1 - 2 R_Z)^2} \beta_{ZZ}^3 \]  

(B.9)

\[ \Gamma(3S_1 \rightarrow ZZ) = \frac{N_c \alpha_W^2 v_Q^2 a_Q^2}{3 c_W^2 M^2 R_Z (1 - 2 R_Z^2)} \beta_{ZZ}^5 . \]  

(B.10)

Next, to \( Zh \),

\[ \Gamma(1S_0 \rightarrow Zh) = \frac{N_c \alpha_W^2 a_Q^2 M^2}{16 m_Z^2 c_W^2} \beta_{Zh}^3 \]  

(B.11)

\[ \Gamma(3S_1 \rightarrow Zh) = \frac{N_c \alpha_W^2 v_Q^2 a_Q^2 \beta_{Zh}}{12 c_W^2 M^2 m_Z^2} \left\{ 2 + \frac{1}{4 R_Z} \left( 1 + R_Z - R_h \right)^2 \right\} \left[ \frac{2 m_Q R_Z}{1 - R_Z} - \frac{v \lambda_Q (1 + R_Z - R_W)}{1 - R_Z - R_h} \right]^2 \]

\[ + \frac{R_Z}{2} \left( 1 - R_Z - R_h \right) \left[ \frac{2 m_Q R_Z}{1 - R_Z} - \frac{v \lambda_Q (1 + R_Z - R_W)}{1 - R_Z - R_h} \right] + \frac{\beta_{Zh}^4 v_Q^2 a_Q^2}{4 R_Z (1 - R_Z - R_h)^2} \right\} , \]  

(B.12)

where \( \lambda_Q \) is the Yukawa coupling of the quirks to the Higgs. Finally, to \( h \gamma \),

\[ \Gamma(1S_0 \rightarrow h \gamma) = 0 , \]  

(B.13)

\[ \Gamma(3S_1 \rightarrow h \gamma) = \frac{N_c \alpha_Q^2 \lambda_Q (1 - R_h)}{3 \pi M^2} . \]  

(B.14)

One might expect decays to scalar pairs like \( hh \) and, in the case of the \( \hat{X} \) and \( \hat{Y} \) quirks, \( \chi \chi^* \). However, \( CP \) and angular momentum conservation forbid such decays from the s-wave states, though higher angular momentum states do allow these decays.

We now turn to decays into \( W^+W^- \). We label the \( SU(2)_L \) partner of \( Q \) by \( q \), with mass \( m_q \) etc. The \( W \) couplings \( v_W \) and \( a_W \) are defined by the interaction

\[ \frac{g}{2 \sqrt{2}} \gamma^\mu (v_W - a_W \gamma^5) . \]  

(B.15)

We note that this decay depends upon the electric charge of particle that makes up the bound state in a nontrivial way. This is due to the diagrams related to the \( t \)- or \( u \)-channel exchange of the \( SU(2)_L \), partner of the particle making up the bound state.
by a quirk with positive charge involve a different diagram than those with negative charge. None of these subtleties affect the singlet case, but we do distinguish the triplet cases as $S_1^{(\pm)}$, where the superscript denotes whether the quirk has positive or negative electric charge. The decays to $W^+W^-$ are

\[ \Gamma(1S_0 \rightarrow W^+W^-) = \frac{N_\varepsilon \alpha_W}{8 M^2 (1 + 4 R_q - 4 R_W)^2} \beta^3_{W^+W^-}, \]  
\[ \Gamma(3S_1^{(\pm)} \rightarrow W^+W^-) = \frac{N_\varepsilon \alpha_W^3}{192 R_W^3 M^2} \left\{ \frac{16 v_W^2 a_W^2 S}{(1 + 4 R_q - 4 R_W)^2} - 6 R_W \left[ \frac{v_W^2 (m_Q - m_q) + a_W^2 (m_Q + m_q)}{1 + 4 R_q - 4 R_W} \right]^2 \right. 
+ (1 + 10 R_W) \left[ 4 Q Q s^2 W + \frac{2 v_Q}{1 - R_Z} \mp \frac{m_q}{m_Q} \frac{v_W^2 - a_W^2}{1 + 4 R_q - 4 R_W} \right]^2 
+ 2 R_W (5 + 6 R_W) \left[ 4 Q Q s^2 W + \frac{2 v_Q}{1 - R_Z} \mp \frac{v_W^2 + a_W^2}{1 + 4 R_q - 4 R_W} \right]^2 \right\}. \]  

We also record the decays involving hidden gluons. These are taken from [100].

\[ \Gamma(1S_0 \rightarrow \hat{g} \hat{g}) = \frac{N_\varepsilon^2 - 1}{N_\varepsilon M^2} \alpha_s^2, \]  
\[ \Gamma(3S_1 \rightarrow \hat{g} \hat{g} \hat{g}) = \frac{(N_\varepsilon^2 - 1)(N_\varepsilon^2 - 4)(\pi^2 - 9)}{9 \pi N_\varepsilon^2 M^2} \hat{\alpha}_s^3, \]  
\[ \Gamma(3S_1 \rightarrow \gamma \hat{g} \hat{g}) = \frac{4 Q Q^2 (N_\varepsilon^2 - 1)(\pi^2 - 9)}{3 \pi M^2 N_\varepsilon} \alpha \hat{\alpha}_s^2, \]

where we have denoted the hidden sector strong coupling by $\hat{\alpha}_s$. Finally, the singlet state can also decay to photons

\[ \Gamma(1S_0 \rightarrow \gamma \gamma) = \frac{4 N_\varepsilon Q^4}{M^2} \alpha_s^2. \]

**References**

[1] ATLAS Collaboration, G. Aad et al., “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” *Phys. Lett.* **B716** (2012) 1–29, [arXiv:1207.7214].

[2] CMS Collaboration, S. Chatrchyan et al., “Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC,” *Phys. Lett.* **B716** (2012) 30–61, [arXiv:1207.7235].

[3] Z. Chacko, H.-S. Goh, and R. Harnik, “The Twin Higgs: Natural electroweak breaking from mirror symmetry,” *Phys. Rev. Lett.* **96** (2006) 231802, [hep-ph/0506256].

[4] R. Barbieri, T. Gregoire, and L. J. Hall, “Mirror world at the large hadron collider,” [hep-ph/0509242].

[5] G. Burdman, Z. Chacko, H.-S. Goh, and R. Harnik, “Folded supersymmetry and the LEP paradox,” *JHEP* **02** (2007) 009, [hep-ph/0609152].

[6] D. Poland and J. Thaler, “The Dark Top,” *JHEP* **11** (2008) 083, [arXiv:0808.1290].

[7] H. Cai, H.-C. Cheng, and J. Terning, “A Quirky Little Higgs Model,” *JHEP* **05** (2009) 045, [arXiv:0812.0843].

[8] N. Craig, S. Knapen, and P. Longhi, “Neutral Naturalness from Orbifold Higgs Models,” *Phys. Rev. Lett.* **114** no. 6, (2015) 061803, [arXiv:1410.6808].
[9] N. Craig, S. Knapen, and P. Longhi, “The Orbifold Higgs,” JHEP 03 (2015) 106, [arXiv:1411.7393].
[10] B. Batell and M. McCullough, “Neutrino Masses from Neutral Top Partners,” Phys. Rev. D92 no. 7, (2015) 073018, [arXiv:1504.04016].
[11] J. Serra and R. Torre, “Neutral naturalness from the brother-Higgs model,” Phys. Rev. D97 no. 3, (2018) 035017, [arXiv:1709.05399].
[12] C. Csáki, T. Ma, and J. Shu, “Trigonometric Parity for Composite Higgs Models,” Phys. Rev. Lett. 121 no. 23, (2018) 231801, [arXiv:1709.08636].
[13] C. Csáki, T. Ma, and J. Shu, “Trigonometric Parity for Composite Higgs Models,” Phys. Rev. Lett. 121 no. 23, (2018) 231801, [arXiv:1709.08636].
[14] T. Cohen, N. Craig, G. F. Giudice, and M. Mccullough, “The Hyperbolic Higgs,” JHEP 05 (2018) 091, [arXiv:1803.03647].
[15] H.-C. Cheng, L. Li, E. Salvioni, and C. B. Verhaaren, “Singlet Scalar Top Partners from Accidental Supersymmetry,” JHEP 05 (2018) 057, [arXiv:1803.03651].
[16] J. Serra, S. Stelzl, R. Torre, and A. Weiler, “Hypercharged Naturalness,” JCAP 1510 no. 10, (2015) 054, [arXiv:1505.07113].
[17] I. Garcia Garcia, R. Lasenby, and J. March-Russell, “Twin Higgs Asymmetric Dark Matter,” Phys. Rev. D92 no. 5, (2015) 055034, [arXiv:1505.07109].
[18] M. Farina, “Asymmetric Twin Dark Matter,” JCAP 1511 no. 11, (2015) 017, [arXiv:1506.03520].
[19] M. Freytsis, S. Knapen, D. J. Robinson, and Y. Tsai, “Gamma-rays from Dark Showers with Twin Higgs Models,” JHEP 05 (2016) 018, [arXiv:1601.07556].
[20] M. Farina, A. Monteux, and C. S. Shin, “Twin mechanism for baryon and dark matter asymmetries,” Phys. Rev. D94 no. 3, (2016) 035017, [arXiv:1604.08211].
[21] Y. Hochberg, E. Kuflik, and H. Murayama, “Twin Higgs model with strongly interacting massive particle dark matter,” Phys. Rev. D99 no. 1, (2019) 015005, [arXiv:1805.09345].
[22] H.-C. Cheng, L. Li, and R. Zheng, “Cosscattering/Coannihilation Dark Matter in a Fraternal Twin Higgs Model,” JHEP 09 (2018) 098, [arXiv:1805.12139].
[23] J. Terning, C. B. Verhaaren, and K. Zora, “Composite Twin Dark Matter,” Phys. Rev. D99 no. 9, (2019) 095020, [arXiv:1902.08211].
[24] S. Koren and R. McGehee, “Freeze-Twin Dark Matter,” JCAP 1811 no. 11, (2018) 050, [arXiv:1809.09106].
[25] M. Badziak, G. Grilli Di Cortona, and K. Harigaya, “Natural Twin Neutralino Dark Matter,” JHEP 12 (2019) 023, [arXiv:1904.02560].
[33] J. Kang and M. A. Luty, “Macroscopic Strings and ‘Quirks’ at Colliders,” *JHEP* 11 (2009) 065, [arXiv:0805.4642].

[34] B. Gripaios, A. Pomarol, F. Riva, and J. Serra, “Beyond the Minimal Composite Higgs Model,” *JHEP* 04 (2009) 070, [arXiv:0902.1483].

[35] S. R. Coleman and E. J. Weinberg, “Radiative Corrections as the Origin of Spontaneous Symmetry Breaking,” *Phys. Rev.* D7 (1973) 1888–1910.

[36] R. Contino, D. Greco, R. Mahbubani, R. Rattazzi, and R. Torre, “Precision Tests and Fine Tuning in Twin Higgs Models,” *Phys. Rev.* D96 no. 9, (2017) 095036, [arXiv:1702.00797].

[37] G. Burdman, Z. Chacko, R. Harnik, L. de Lima, and C. B. Verhaaren, “Colorless Top Partners, a 125 GeV Higgs, and the Limits on Naturalness,” *Phys. Rev.* D91 no. 5, (2015) 055007, [arXiv:1411.3310].

[38] K. Fujii *et al.*, “Physics Case for the 250 GeV Stage of the International Linear Collider,” arXiv:1710.07621.

[39] R. Franceschini *et al.*, “The CLIC Potential for New Physics,” arXiv:1812.02093.

[40] FCC Collaboration, A. Abada *et al.*, “FCC Physics Opportunities,” *Eur. Phys. J.* C79 no. 6, (2019) 474.

[41] M. Cepeda *et al.*, “Report from Working Group 2,” *CERN Yellow Rep. Monogr.* 7 (2019) 221–584, [arXiv:1902.00134].

[42] J. E. Juknevich, “Pure-glue hidden valleys through the Higgs portal,” *JHEP* 08 (2010) 121, [arXiv:0911.5616].

[43] Y. Chen *et al.*, “Glueball spectrum and matrix elements on anisotropic lattices,” *Phys. Rev.* D73 (2006) 014516, [hep-lat/0510074].

[44] D. Curtin and C. B. Verhaaren, “Discovering Uncolored Naturalness in Exotic Higgs Decays,” *JHEP* 12 (2015) 072, [arXiv:1506.06141].

[45] T. DeGrand and E. T. Neil, “Repurposing lattice QCD results for composite phenomenology,” *Phys. Rev.* D101 no. 3, (2020) 034504, [arXiv:1910.08561].

[46] D. Curtin *et al.*, “Long-Lived Particles at the Energy Frontier: The MATHUSLA Physics Case,” *Rept. Prog. Phys.* 82 no. 11, (2019) 116201, [arXiv:1806.07396].

[47] N. Craig, A. Katz, M. Strassler, and R. Sundrum, “Naturalness in the Dark at the LHC,” *JHEP* 07 (2015) 105, [arXiv:1501.05310].

[48] D. Buttazzo, F. Sala, and A. Tesi, “Singlet-like Higgs bosons at present and future colliders,” *JHEP* 11 (2015) 158, [arXiv:1505.05488].

[49] A. Ahmed, “Heavy Higgs of the Twin Higgs Models,” *JHEP* 02 (2018) 048, [arXiv:1711.03107].

[50] Z. Chacko, C. Kilic, S. Najjari, and C. B. Verhaaren, “Testing the Scalar Sector of the Twin Higgs Model at Colliders,” *Phys. Rev.* D97 no. 5, (2018) 055031, [arXiv:1711.05300].

[51] C. Kilic, S. Najjari, and C. B. Verhaaren, “Discovering the Twin Higgs Boson with Displaced Decays,” *Phys. Rev.* D99 no. 7, (2019) 075029, [arXiv:1812.08173].

[52] S. Alipour-Fard, N. Craig, S. Gori, S. Koren, and D. Redigolo, “The second Higgs at the lifetime frontier,” arXiv:1812.09315.

[53] A. Ahmed, B. M. Dillon, and S. Najjari, “Dilaton portal in strongly interacting twin Higgs models,” *JHEP* 02 (2020) 124, [arXiv:1911.05085].

[54] M. E. Peskin and T. Takeuchi, “A New constraint on a strongly interacting Higgs sector,” *Phys. Rev. Lett.* 65 (1990) 964–967.

[55] M. E. Peskin and T. Takeuchi, “Estimation of oblique electroweak corrections,” *Phys. Rev.* D46 (1992) 381–409.

[56] R. Contino, “The Higgs as a Composite Nambu-Goldstone Boson,” in *Physics of the large and the small, TASI 09, proceedings of the Theoretical Advanced Study Institute in*
[57] J. Haller, A. Hoecker, R. Kogler, K. Mönig, T. Peiffer, and J. Stelzer, “Update of the global electroweak fit and constraints on two-Higgs-doublet models,” *Eur. Phys. J.* C78 no. 8, (2018) 675, [arXiv:1803.01853].

[58] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, “Parton distributions for the LHC,” *Eur. Phys. J.* C63 (2009) 189–285, [arXiv:0901.0002].

[59] R. Harnik, G. D. Kribs, and A. Martin, “Quirks at the Tevatron and Beyond,” *Phys. Rev.* D84 (2011) 035029, [arXiv:1106.2569].

[60] M. Farina and M. Low, “Constraining Quirky Tracks with Conventional Searches,” *Phys. Rev.* Lett. 119 no. 11, (2017) 111801, [arXiv:1708.02243].

[61] S. Knapen, H. K. Lou, M. Papucci, and J. Setford, “Tracking down Quirks at the Large Hadron Collider,” *Phys. Rev.* D96 no. 11, (2017) 115015, [arXiv:1708.02243].

[62] J. A. Evans and M. A. Luty, “Stopping Quirks at the LHC,” JHEP 06 (2019) 090, [arXiv:1811.08903].

[63] J. Li, T. Li, J. Pei, and W. Zhang, “Uncovering quirk signal via energy loss inside tracker,” arXiv:1911.02223.

[64] J. Li, T. Li, J. Pei, and W. Zhang, “The quirk trajectory,” arXiv:2002.07503.

[65] B. Lucini, M. Teper, and U. Wenger, “Glueballs and k-strings in SU(N) gauge theories: Calculations with improved operators,” *JHEP* 06 (2004) 012, [hep-lat/0404008].

[66] M. Teper, “Large N and confining flux tubes as strings - a view from the lattice,” *Acta Phys. Polon.* B40 (2009) 3249–3320, [arXiv:0912.3339].

[67] G. Burdman, Z. Chacko, H.-S. Goh, R. Harnik, and C. A. Krenke, “The Quirky Collider Signals of Folded Supersymmetry,” *Phys. Rev.* D78 (2008) 075028, [arXiv:0805.4667].

[68] R. Harnik and T. Wizansky, “Signals of New Physics in the Underlying Event,” *Phys. Rev.* D80 (2009) 075015, [arXiv:0810.3948].

[69] ATLAS Collaboration, M. Aaboud et al., “Search for heavy resonances decaying into WW in the $e\nu\mu\nu$ final state in $pp$ collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector,” *Eur. Phys. J.* C78 no. 1, (2018) 24, [arXiv:1710.01123].

[70] ATLAS Collaboration, M. Aaboud et al., “Search for WW/WZ resonance production in $\ell\nu qq$ final states in $pp$ collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector,” JHEP 03 (2018) 042, [arXiv:1710.07235].

[71] CMS Collaboration, A. M. Sirunyan et al., “Search for massive resonances decaying into WW, WZ or ZZ bosons in proton-proton collisions at $\sqrt{s} = 13$ TeV,” JHEP 03 (2017) 162, [arXiv:1612.09159].

[72] CMS Collaboration, A. M. Sirunyan et al., “Search for heavy resonances decaying into a $W$ or $Z$ boson and a Higgs boson in dijet final states at $\sqrt{s} = 13$ TeV,” *Phys. Rev.* D97 no. 7, (2018) 072006, [arXiv:1708.05379].

[73] ATLAS Collaboration, M. Aaboud et al., “Search for heavy resonances decaying into a $W$ or $Z$ boson and a Higgs boson in final states with leptons and $b$-jets in 36 fb$^{-1}$ of $\sqrt{s} = 13$ TeV $pp$ collisions with the ATLAS detector,” JHEP 03 (2018) 174, [arXiv:1712.06518]. [Erratum: JHEP11,051(2018)].

[74] CMS Collaboration, V. Khachatryan et al., “Search for heavy resonances decaying into a vector boson and a Higgs boson in final states with charged leptons, neutrinos, and $b$ quarks,” *Phys. Lett.* B768 (2017) 137–162, [arXiv:1610.08066].

[75] CMS Collaboration, A. M. Sirunyan et al., “Search for heavy resonances decaying into two Higgs bosons or into a Higgs boson and a $W$ or $Z$ boson in proton-proton collisions at 13 TeV,” JHEP 01 (2019) 051, [arXiv:1808.01365].
[76] **ATLAS Collaboration** Collaboration, “HL-LHC prospects for diboson resonance searches and electroweak vector boson scattering in the WW/WZ → ℓνqq final state,” Tech. Rep. ATL-PHYS-PUB-2018-022, CERN, Geneva, Oct, 2018. https://cds.cern.ch/record/2645269.

[77] Z. Chacko, D. Curtin, and C. B. Verhaaren, “A Quirky Probe of Neutral Naturalness,” *Phys. Rev. D94* no. 1, (2016) 011504, [arXiv:1512.05782].

[78] **ATLAS Collaboration**, M. Aaboud et al., “Search for new phenomena in high-mass diphoton final states using 37 fb−1 of proton–proton collisions collected at √s = 13 TeV with the ATLAS detector,” *Phys. Lett. B775* (2017) 105–125, [arXiv:1707.04147].

[79] **ATLAS Collaboration**, G. Aad et al., “Search for high-mass dilepton resonances using 139 fb−1 of pp collision data collected at √s = 13 TeV with the ATLAS detector,” *Phys. Lett. B796* (2019) 68–87, [arXiv:1903.06248].

[80] **CMS Collaboration**, A. M. Sirunyan et al., “Search for physics beyond the standard model in high-mass diphoton events from proton-proton collisions at √s = 13 TeV,” *Phys. Rev. D98* no. 9, (2018) 092001, [arXiv:1809.00327].

[81] **CMS Collaboration**, C. Collaboration, “Search for a narrow resonance in high-mass dilepton final states in proton-proton collisions using 140 fb−1 of data at √s = 13 TeV,”.

[82] G. Bélanger, F. Boudjema, A. Goudelis, A. Pukhov, and B. Zaldivar, “micrOMEGAs5.0: Freeze-in,” *Comput. Phys. Commun.* 231 (2018) 173–186, [arXiv:1801.03509].

[83] **Planck Collaboration**, N. Aghanim et al., “Planck 2018 results. VI. Cosmological parameters,” [arXiv:1807.06209].

[84] **XENON Collaboration**, E. Aprile et al., “Dark Matter Search Results from a One Ton-Year Exposure of XENON1T,” *Phys. Rev. Lett. 121* no. 11, (2018) 111302, [arXiv:1805.12562].

[85] **Fermi-LAT Collaboration**, M. Ackermann et al., “Searching for Dark Matter Annihilation from Milky Way Dwarf Spheroidal Galaxies with Six Years of Fermi Large Area Telescope Data,” *Phys. Rev. Lett. 115* no. 23, (2015) 231301, [arXiv:1503.02641].

[86] **Fermi-LAT, DES Collaboration**, A. Albert et al., “Searching for Dark Matter Annihilation in Recently Discovered Milky Way Satellites with Fermi-LAT,” *Astrophys. J. 834* no. 2, (2017) 110, [arXiv:1611.03184].

[87] **Fermi-LAT Collaboration**, E. Charles et al., “Sensitivity Projections for Dark Matter Searches with the Fermi Large Area Telescope,” *Phys. Rept. 636* (2016) 1–46, [arXiv:1605.02016].

[88] **AMS Collaboration**, M. Aguilar et al., “Antiproton Flux, Antiproton-to-Proton Flux Ratio, and Properties of Elementary Particle Fluxes in Primary Cosmic Rays Measured with the Alpha Magnetic Spectrometer on the International Space Station,” *Phys. Rev. Lett. 117* no. 9, (2016) 091103.

[89] A. Cuoco, M. Krämer, and M. Korsmeier, “Novel Dark Matter Constraints from Antiprotons in Light of AMS-02,” *Phys. Rev. Lett. 118* no. 19, (2017) 191102, [arXiv:1610.03071].

[90] M.-Y. Cui, Q. Yuan, Y.-L. S. Tsai, and Y.-Z. Fan, “Possible dark matter annihilation signal in the AMS-02 antiproton data,” *Phys. Rev. Lett. 118* no. 19, (2017) 191101, [arXiv:1610.03840].

[91] A. Cuoco, J. Heisig, M. Korsmeier, and M. Krämer, “Constraining heavy dark matter with cosmic-ray antiprotons,” *JCAP 1804* (2018) 004, [arXiv:1711.05274].
[94] T. Bringmann, C. Eckner, A. Sokolenko, L. Yang, and G. Zaharias, “Probing the sensitivity of the Cherenkov Telescope Array to Dark Matter in the Galactic Center,” TeV Particle Astrophysics 2018, Berlin, Germany.

[95] D. Gaggero, D. Grasso, A. Marinelli, M. Taoso, and A. Urbano, “Diffuse cosmic rays shining in the Galactic center: A novel interpretation of H.E.S.S. and Fermi-LAT γ-ray data,” Phys. Rev. Lett. 119 no. 3, (2017) 031101, [arXiv:1702.01124].

[96] V. D. Barger, E. W. N. Glover, K. Hikasa, W.-Y. Keung, M. G. Olsson, C. J. Suchyta, III, and X. R. Tata, “Superheavy Quarkonium Production and Decays: A New Higgs Signal,” Phys. Rev. D35 (1987) 3366. [Erratum: Phys. Rev.D38,1632(1988)].

[97] R. Fok and G. D. Kribs, “Chiral Quirkonium Decays,” Phys. Rev. D84 (2011) 035001, [arXiv:1106.3101].

[98] J. H. Kuhn, J. Kaplan, and E. G. O. Sañiani, “Electromagnetic Annihilation of e+ e- Into Quarkonium States with Even Charge Conjugation,” Nucl. Phys. B157 (1979) 125–144.

[99] B. Guberina, J. H. Kuhn, R. D. Peccei, and R. Ruckl, “Rare Decays of the Z0,” Nucl. Phys. B174 (1980) 317–334.

[100] K. Cheung, W.-Y. Keung, and T.-C. Yuan, “Phenomenology of squarkonium,” Nucl. Phys. B811 (2009) 274–287, [arXiv:0810.1524].