On EM-Driven Size Reduction of Antenna Structures With Explicit Constraint Handling

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ABSTRACT Simulation-driven miniaturization of antenna components is a challenging task mainly due to the presence of expensive constraints, evaluation of which involves full-wave electromagnetic (EM) analysis. The recommended approach is implicit constraint handling using penalty functions, which, however, requires a meticulous selection of penalty coefficients, instrumental in ensuring optimization process reliability. This paper proposes a novel size reduction technique with explicit handling of design constraints. Our approach employs trust-region gradient-based procedure as an underlying optimization engine, and adjusts the search radius based on assessing the quality of representing constraints by auxiliary linear expansion models (versus their exact evaluation through EM analysis), rather than based on the quality of objective rendition. This unconventional utilization of trust region framework leads to a procedure that is demonstrably superior over implicit methods, as indicated by comprehensive numerical studies involving three broadband antennas, and benchmarking against state-of-the-art techniques. The two most attractive features of our methodology are simplicity, and no need to tune the algorithm for a specific problem at hand.

INDEX TERMS Compact antennas, constrained optimization, gradient-based search, simulation-driven design.

I. INTRODUCTION

Preserving small size is an important consideration in the design of contemporary antenna systems. It has become even more essential with the emergence of new application areas, for which compact dimensions is a prerequisite (internet of things [1], wearable [2] and implantable devices [3], mobile communications [4] including 5G [5], medical imaging [6]). Structural miniaturization can be achieved by the appropriate selection of antenna architecture [7], or with the aid of topological modifications, e.g., stubs [8], defected ground structures [9], shorting pins [10], etc. Notwithstanding, achieving the smallest possible size while fulfilling requirements imposed on electrical and field performance figures, requires meticulous and simultaneous tuning of all antenna parameters. Rigorous numerical optimization is instrumental in this process [11]. At the same time, for the sake of accuracy, but also the lack of alternatives (e.g., parameterized equivalent network representations), dimension tuning of the majority of antenna structures has to be carried out using full-wave electromagnetic (EM) simulation models. This entails considerable computational expenses.

High cost of EM-driven antenna optimization is a difficulty that may be alleviated to a certain extent using methods such as adjoint sensitivity [12], sparse Jacobian updates [13] (in the context of local tuning), dedicated solvers [14], or surrogate-assisted methods [15], involving both data-driven [16], and physics-based models [17], as well as machine learning methodologies [18], often combined with sequential sampling routines [19]. Yet, perhaps the biggest challenge in optimization-based size reduction of antenna structures is handling of design constraints. The major issue is that the constraints related to antenna performance figures, e.g., maximum allowed in-band reflection or axial ratio levels, acceptance levels for bandwidth or side lobes, etc., are expensive ones, i.e., require EM analysis to be evaluated. Controlling these is numerically demanding. A possible workaround is...
a penalty function approach, where appropriately quantified constraint violations contribute to the primary objective [20]. This allows for reformulating the size reduction problem into an unconstrained task; however, the reliability of the optimization process becomes highly dependent on the setup of penalty coefficients [21]. Other approaches reported in the literature include feasible region boundary search [22], in which the size-reduction oriented objective function is altered depending on the current constraint violation level to increase penalization of the latter, as well as a relaxation procedure [23], where size-reduction-oriented steps are interleaved with impedance matching improvement ones, to facilitate identification of the compact design that still satisfies the electrical performance figures of interest. These algorithms; however, are relatively complex to implement and not straightforward to generalize for arbitrary number of constraints.

This paper proposes a novel approach to EM-driven size reduction of antenna structures, which is based on explicit constraint handling. The presented algorithm employs the trust-region (TR) gradient-based procedure with numerical derivatives. The search radius is updated in consecutive iterations based on the accuracy of representing the constraints through their linear approximation models, which enables a precise control thereof without using any external parameters that need to be set up (such as penalty coefficients in the penalty function approach). This is in contrast to conventional TR frameworks, where the search radius adjustment is guided by the accuracy of primary objective rendition. As demonstrated using three broadband antenna structures, our methodology permits extremely accurate handling of design constraints without the necessity of tuning the algorithm to a specific antenna structure at hand. At the same time, it allows better miniaturization ratios than penalty function techniques.

II. ANTENNA SIZE REDUCTION WITH EXPLICIT CONSTRAINTS

This section introduces the optimization-based size reduction procedure considered in the paper. The emphasis is put on explicit handling of design constraints within the trust-region framework.

A. EM-DRIVEN SIZE REDUCTION. PROBLEM FORMULATION

Let $x = [x_1 \ldots x_n]^T$ denote the number of independent designable (typically, geometry) parameters of the antenna structure of interest. The size reduction task can be formulated as

$$x^* = \arg \min_x \ U(x)$$

where the objective function $U(x) = A(x)$, i.e., the antenna size at the design $x$ (e.g., the footprint area for planar structures). The problem (1) is subject to the inequality constraints $g_k(x) \leq 0$, $k = 1, \ldots, n_g$, and equality constraints $h_l(x) = 0$, $k = 1, \ldots, n_h$. In practice, the constraints are usually of inequality type and related to electrical and field properties, e.g., $|S_1(x, f)| \leq -10 \text{ dB}$ for the frequencies $f$ within the antenna operating band $F$, axial ratio $AR(x, f) \leq 3 \text{ dB}$ for $f \in F$, gain variation $\Delta G(x, f) \leq 2 \text{ dB}$ for $f \in F$, or realized gain $G(x, f_0) \geq 8 \text{ dB}$ at the center frequency $f_0$. In the case of multi-band antennas, $F$ might be a set-theory union of disjoint frequency intervals.

B. EXPLICIT HANDLING OF EXPENSIVE CONSTRAINTS. THE CONCEPT

Performance-related constraints are expensive to evaluate as their values are extracted from EM-simulated antenna characteristics. From numerical standpoint, this is a major challenge as conventional optimization routines also require constraint gradients. The constraints themselves may not even be differentiable (typically being formulated in a minimax form), and contain a certain level of numerical noise inherent to most EM solvers, stemming from adaptive meshing methods, termination criteria of the simulation process, etc.

The common workaround is implicit handling using a penalty function approach [21], where appropriately scaled constraint violations complement the main objective function to be minimized (here, antenna size). The bottleneck of this approach is the necessity of careful tuning of penalty coefficients, which often entails repetitive optimization runs.

In this paper, we introduce a mechanism for explicit treatment of expensive constraints, adopting the trust-region (TR) framework [24]. The TR algorithm is an iterative procedure for solving (1), which produces approximations $x^{(i)}$, $i = 0, 1, \ldots$, to $x^*$. Subsequent vectors $x^{(i)}$ are obtained through local optimization of a local (usually, linear, e.g., first-order Taylor, or quadratic) models $U^{(i)}_L$ of the objective function $U$, established at the current iteration point $x^{(i)}$, as

$$x^{(i+1)} = \arg \min_{x} \ U^{(i)}_L(x) \quad \text{subject to } g_k(x) \leq 0, k = 1, \ldots, n_g$$

(2)

subject to $g_k(x) \leq 0, k = 1, \ldots, n_g$ (we only assume inequality constraints to simplify the notation). The size $d^{(i)}$ of the search region is adaptively adjusted using the standard TR rules [24].

In the case of size reduction, the objective function is cheap to evaluate (computed directly from the parameter vector $x$). The challenge is to maintain feasibility of solutions given high cost of evaluating the constraints. To achieve this, we employ linear expansion models of the constraints and control the search region size $d^{(i)}$ based on the reliability of this representation during consecutive iterations of the optimization algorithm.

Let $r(x)$ stand for aggregated vector of EM-simulated antenna responses (reflection, axial ratio, gain, etc.). Consider a linear model $L^{(i)}_0(x)$ of $r(x)$ in the vicinity of $x^{(i)}$

$$L^{(i)}_0(x) = r(x^{(i)}) + J(x^{(i)}) \cdot (x - x^{(i)})$$

(3)

Here, $J(x^{(i)})$ is a sensitivity (Jacobian) matrix of $r$ at $x^{(i)}$, estimated using finite differentiation. The approximation model of the $k$th constraint $g_k(r(x))$ (here, explicit dependence of constraints on antenna responses is emphasized),
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**FIGURE 1.** Constraint prediction using linear model $g_{L,k}$ (4). The top picture shows design relocation from $x^{(i)}$ to $x^{(i+1)}$ upon solving sub-problem (2). The design is relocated to minimize antenna size while satisfying the constraint (as predicted by $g_{L,k}$). The bottom-left picture illustrates a situation when linear model prediction is satisfactory. The bottom-right picture shows a case when the prediction is poor and the new design is, in fact, infeasible according to the true constraint value evaluated through EM analysis. This will lead to a reduction of the TR size in the next iteration (cf. (7), (8)).

is denoted as

$$g_{L,k}(x) = g_k(L^{(i)}(x)) \quad (4)$$

The new iteration point $x^{(i)}$ is obtained by solving (2) subject to $g_{L,k}(x) \leq 0$, $k = 1, \ldots, n_g$.

The critical part of the procedure is to appropriately update the TR size $d^{(i)}$. Lower values improve the alignment of $g_{L,k}(x)$ with $g_k(r(x))$ within the search region, whereas higher values permit larger steps in the parameter space. A decision upon adjusting $d^{(i)}$ should be based on the accuracy of accounting for solution feasibility by $g_{L,k}(x)$ as well as feasibility status itself. The overall adjustment scheme is arranged as follows:

$$d^{(i+1)} = \begin{cases} 2d^{(i)} & \text{if } \rho \geq 0.75 \\ \frac{d^{(i)}}{2} & \text{if } 0.25 \leq \rho < 0.75 \\ \frac{d^{(i)}}{3} & \text{if } \rho < 0.25 \end{cases} \quad (5)$$

**FIGURE 2.** Flow diagram of the proposed size reduction algorithm with explicit constraint handling.

**TABLE 1.** Verification antennas.

| Antenna | Substrate | Designable Parameters [mm] | Other Parameters [mm] |
|---------|-----------|---------------------------|-----------------------|
| I       | RF-35     | $x = [L_g, g, a, L_z, w_1, o]^T$ | $w_0 = 2a + a$, $w_y = 1.7$ |
| II      | RF-35     | $x = [L_z, dR, R_{rel}, dL, dw]$ | $w_0 = 1.7$ |
| III     | FR4       | $x = [L_g, L_o, L_z, dL, dW, dW \ a b]^T$ | $W_z = 3.0$ |

where $\rho$ is the ‘constraint’ gain ratio, accounting for the actual versus predicted change of constraint violations. The multiplication factors and thresholds in (5) correspond to the standard TR setup (cf. [24]).

The gain ratio $\rho$ represents the worst-case scenario across all considered constraints, i.e., we have

$$\rho = \min\{\rho_1, \ldots, \rho_{n_g}\} \quad (6)$$
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The strategy for computing individual gain ratios $\rho_k$ for constraints $g_k(x)$, $k = 1, \ldots, n_g$, will be discussed in the next section.

### C. GAIN RATIO CALCULATION

Let $g_k^{(i)}$ be the acceptance threshold for the violation of the $k$th constraint at iteration $i$. The following rules are established for calculating $\rho_k$:

- **Rule 1**: If $g_k(r(x^{(i)})) > G_k^{(i)}$, i.e., large constraint violation at the beginning of iteration has been observed, then
  \[
  \rho_k = \frac{g_k(r(x^{(i)}) - g_k(r(x^{(i+1)}))}{g_{L,k}(x^{(i+1)}) - g_{L,k}(x^{(i)})} \tag{7}
  \]

- **Rule 2**: If $g_k(x^{(i)}) \leq G_k^{(i)}$, i.e., acceptable constraint violation has been observed, then
  \[
  \rho_k = \frac{1}{2} \left[ 1 + \text{sgn} \left( g_k(r(x^{(i)})) - g_k(r(x^{(i+1)})) \right) \right] \tag{8}
  \]

- **Rule 3**: Having $\rho_k$ computed as in (7) or (8), it is updated as follows: if $\rho_k < 0$ and $g_k(r(x^{(i+1)})) \leq G_k^{(i)}$, set $\rho_k = 0.5$.

The purpose of these rules is to penalize poor prediction capability of the linear model $g_{L,k}$ for large constraint violations (Rule 1), or the lack of feasibility status improvement for small values of constraint violation (Rule 2). The same two rules are used to reward good prediction capability, and design relocation towards feasible region. Furthermore, Rule 3 overwrites the previous rules if the actual constraint violation at the new iteration point $x^{(i+1)}$ is small (i.e., lower than $G_k^{(i)}$). This is necessary to avoid erratic behavior when the design is close to the feasible region boundary (i.e., constraint violations are close to zero and may assume either positive or negative sign). Figure 1 shows examples of good and poor constraint prediction of the constraint $g_k$ by the linear model $g_{L,k}$.

Next, we address the acceptance thresholds $G_k^{(i)}$. In order to provide more leeway for infeasible solutions at the early stages of the optimization process (which facilitates the allocation of small-size designs), the thresholds are linked to the convergence status of the algorithm, and the maximum allowed constraint violation $G_k^{\max}$ (user-defined). Let $\varepsilon$ be the termination threshold of the algorithm, assuming the termination condition: $||x^{(i+1)} - x^{(i)}|| < \varepsilon$ or $d^{(i)} < \varepsilon$. The

**TABLE 2. Optimization results.**

| Antenna | Optimization approach | Antenna footprint $A$ [mm$^2$]$^1$ | Std$(A)^2$ | Constraint violation $D$ [dB]$^3$ | Std$(D)$ [dB]$^4$ |
|---------|------------------------|------------------------------------|------------|----------------------------------|-----------------|
| I       | $\beta = 10^2$        | 305.4                              | 49.7       | 6.7                              | 1.7             |
|         | $\beta = 10^3$        | 318.1                              | 42.6       | 1.2                              | 0.4             |
|         | $\beta = 10^4$        | 317.7                              | 42.3       | 0.4                              | 0.7             |
|         | $\beta = 10^5$        | 318.8                              | 43.3       | 0.05                             | 0.2             |
|         | $\beta = 10^6$        | 320.9                              | 45.8       | 0.06                             | 0.3             |
| II      | Penalty function approach | 314.1                              | 42.3       | 0.3                              | 0.2             |
|         | Adaptive penalty factors | 323.0                              | 42.5       | 0.03                             | 0.08            |
|         | This work (explicit constraint handling) | 328.4                          | 41.6       | 0.04                             | 0.08            |
| III     | Penalty function approach | 113.7                              | 9.07       | 8.4                              | 0.53            |
|         | $\beta = 10^2$        | 250.4                              | 24.0       | 1.2                              | 0.5             |
|         | $\beta = 10^3$        | 318.6                              | 60.0       | 0.14                             | 0.1             |
|         | $\beta = 10^4$        | 331.6                              | 63.4       | 0.10                             | 0.14            |
|         | $\beta = 10^5$        | 367.6                              | 51.9       | 0.05                             | 0.11            |
|         | Adaptive penalty factors | 281.6                              | 37.1       | 0.23                             | 0.15            |
|         | This work (explicit constraint handling) | 284.7                          | 41.6       | 0.04                             | 0.08            |
|         | $\beta = 10^2$        | 56.1                                | 3.8        | 8.6                              | 0.60            |
|         | $\beta = 10^3$        | 212.8                              | 14.3       | 1.0                              | 0.40            |
|         | $\beta = 10^4$        | 255.0                              | 25.1       | 0.15                             | 0.10            |
|         | $\beta = 10^5$        | 280.1                              | 47.4       | 0.05                             | 0.07            |
|         | $\beta = 10^6$        | 285.8                              | 29.6       | 0.00                             | 0.01            |
|         | Adaptive penalty factors | 215.6                              | 3.6        | 0.25                             | 0.14            |
|         | This work (explicit constraint handling) | 224.4                          | 6.7        | 0.10                             | 0.21            |

$^1$Optimized antenna footprint averaged over ten algorithm runs.
$^2$Standard deviation of the optimized antenna footprint averaged over ten algorithm runs.
$^3$Average constraint violation $D = 3.1$ GHz $\leq 10.6$ GHz: $\text{max}(S(f)) + 10$.
$^4$Standard deviation of the constraint violation $D$, averaged over ten algorithm runs.
convergence status at the iteration $i$ can be defined as

$$C^{(i)} = \max \left\{ 1, \min \left\{ \frac{||x^{(i+1)} - x^{(i)}||, d^{(i)}}{\varepsilon} \right\} \right\}$$

(9)

Using this, we define the acceptance threshold for the next iteration as

$$G_{k}^{(i+1)} = G_{k, \text{max}} \min \left\{ 1, \alpha C^{(i)} \right\}$$

(10)

As the convergence factor $C^{(i)}$ assumes large values at the early stages of the optimization process, the threshold $G_{k}^{(i+1)}$ is also larger (yet, it cannot exceed $G_{k, \text{max}}$ to permit excessive violations). Upon convergence, it is decreased to $\alpha G_{k, \text{max}}$, where $\alpha$ is a small positive number, e.g., 0.1 (not critical for the algorithm operation).

D. OPTIMIZATION ALGORITHM

The operation of the size reduction algorithm with explicit constraint handling has been summarized in the form of the flow diagram in Fig. 2. It should be emphasized that the only control parameters of the procedure, apart from the termination threshold, are the maximum allowed constraint violations $G_{k, \text{max}}$ and the scaling factor $\alpha$, neither of which is critical.

The new design $x^{(i+1)}$ is accepted if the gain ratio $\rho$ is positive. This, according to the rules of Section II, $C$, occurs if the design is feasible or constraint violation has been sufficiently improved (cf. (7)).

III. DEMONSTRATION CASE STUDIES

This section discusses validation of the optimization procedure introduced in Section II, involving three compact broadband antennas. The results are compared to those obtained using a penalty function approach with different values of penalty coefficients, as well as adaptive penalty factors [22].

A. VERIFICATION CASE STUDIES

The verification antenna structures have been shown in Fig. 3, whereas Table 1 provides information concerning the substrates and geometry parameters. In all cases, the design goal...
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C. DISCUSSION

The results presented in Table 2 indicate that not only the presented methodology allows for a precise control of the design constraints (here, maximum in-band reflection level), but also ensures improved miniaturization rates as compared to the penalty function approach (for comparable average constraint violation levels). Explicit approach is also competitive to adaptive penalty function technique in terms of handling constraint violations (which results in slightly larger average antenna sizes). Furthermore, the performance of the proposed algorithm is consistent across the verification antenna set. No need to tune the algorithm is another important advantage, which is in contrast to the benchmark, where the values of penalty coefficients have a major effect on the optimization process performance; also, their optimum values are problem dependent.

IV. CONCLUSION

This paper proposed a novel approach to EM-driven size reduction of antenna components with explicit handling of design constraints, and their predicted violations incorporated into the adjustment of the search radius of the underlying trust-region framework. Comprehensive verification experiments demonstrate that our methodology allows for a precise control of electrical performance figures while ensuring superior miniaturization rates, as compared to a penalty function approach. At the same time, it is comparable to adaptive penalty coefficient approach while offering better accuracy in maintaining the required constraint levels. Additional advantages of the technique include simple implementation, and the lack of control parameters that would have to be adjusted to tailor the algorithm to a specific antenna structure.

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