Driven degenerate three-level cascade laser

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We analyze a degenerate three-level cascade laser coupled to an external coherent light via one of the coupler mirrors and vacuum reservoir in the other, employing the stochastic differential equation associated with the normal ordering. We study the squeezing properties and also calculate the mean photon number of the cavity radiation. It turns out that the generated light exhibits up to 98.3% squeezing under certain conditions pertaining to the initial preparation of the superposition and the amplitude of the driving radiation. Moreover, the mean photon number is found to be large where there is a better squeezing. Hence it is believed that the system under consideration can generate an intense squeezed light.

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I. INTRODUCTION

Interaction of three-level atoms with a radiation has attracted a great deal of interest in recent years \[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\]. It is believed that an atomic coherence is found to be responsible for various important quantum features of the emitted light. In general, the atomic coherence can be induced in a three-level atom by coupling the levels between which a direct transition is dipole forbidden by an external radiation \[1, 2, 3, 4, 5, 6\] or by preparing the atom initially in a coherent superposition of these two levels \[7, 8, 9\]. It is found that the cavity radiation exhibits squeezing under certain conditions for both cases \[4, 9, 10, 11\].

In a cascade three-level atom the top, intermediate, and bottom levels are conveniently denoted by \(|a\rangle\), \(|b\rangle\), and \(|c\rangle\) in which a direct transition between levels \(|a\rangle\) and \(|c\rangle\) is dipole forbidden. When the three-level cascade atom decays from \(|a\rangle\) to \(|c\rangle\) via the level \(|b\rangle\) two photons are generated. If the two photons have identical frequency, then the three-level atom is referred to as a degenerate. Hence we define a degenerate three-level laser as a quantum optical system in which degenerate three-level atoms in a cascade configuration and initially prepared in a coherent superposition of the top and bottom levels are injected at a constant rate into a cavity. These atoms are removed from the cavity after some time. We hence realize that, a degenerate three-level laser is a two photon device in which squeezing properties are expected to occur due to the correlation between these two photons \[2, 4\].

We consider a degenerate three-level cascade laser coupled to a vacuum reservoir via a single-port mirror and the bottom level of the atoms on the other hand is coupled to the top level by an external resonant coherent light as shown in Fig. 1. Some authors have already studied such a scheme in which the atomic coherence is induced by an external radiation and when initially the atoms are prepared in the top level \[4, 10\] and bottom level \[6\]. They found that the three-level laser in these cases resemble the parametric oscillator for a strong radiation. Moreover, recently Saavedra et al. \[1\] studied the \(\lambda\) three-level laser when the atoms are initially prepared in a coherent superposition and the forbidden transition is induced by driving with strong external radiation. They found that there is lasing without population inversion with the favorable noise reduction occurs for equal population of the two levels and when the initial coherence is maximum. Therefore, we consider the case when the atoms are initially prepared to be in an arbitrary atomic superposition and in addition driven on resonance externally by a coherent radiation. We restrict our analysis to the regime of lasing without population inversion.

In this communication, we study the squeezing properties of the cavity radiation and we also calculate the mean photon number using the stochastic differential equation associated with the normal ordering. We prefer to employ the classical stochastic relations to the corresponding quantum operator for obvious reason that it is mathematically easier to deal with. In particular, we calculate the quadrature variance and the mean photon number for cases when the atoms are initially prepared to be in the bottom level and when they are having equal probability of being in the top and bottom levels.

\[
\begin{align*}
\Omega & \quad \omega_a & \quad \hat{a} \\
\omega_a & \quad |a\rangle & \quad \omega_a & \quad |b\rangle & \quad \omega_a \hat{a} \\
& & \quad |c\rangle
\end{align*}
\]

FIG. 1: Schematic representation of a coherently driven degenerate three-level atom in a cascade configuration. The transitions between \(|a\rangle - |b\rangle\) and \(|b\rangle - |c\rangle\) at frequency \(\omega_a\) each are taken to be resonant with the cavity. The transition \(|b\rangle - |c\rangle\) is dipole forbidden and can be induced by driving the atom externally with...
resonant radiation of frequency $2\omega_a$.

II. MASTER EQUATION

The interaction of a degenerate three-level atom with a single-mode light can be described in the rotating-wave approximation and in the interaction picture by the Hamiltonian of the form

$$\hat{H}_{AR} = ig \left[ a \langle a | b \rangle + | b \rangle \langle c | - \langle b | a \rangle + | c \rangle \langle b | \right] \hat{a}^\dagger,$$

where $g$ is the coupling constant, which is taken to be the same for both transitions, and $\hat{a}$ is the annihilation operator for the cavity mode. On the other hand, the three-level cascade atom for which its bottom level is coupled to the top level by a resonant coherent light can be expressed in the rotating-wave approximation and in the interaction picture by the Hamiltonian of the form

$$\hat{H}_C = i\frac{\Omega}{2} \left[ |c\rangle \langle a| - |a\rangle \langle c| \right],$$

where $\Omega$ is a real-positive constant proportional to the probability amplitudes for the atom to be in the top and bottom levels, respectively. This corresponds to the fact that the three-level atom is initially prepared to be in a coherent superposition of the top and bottom levels. Hence the initial density operator for the atom described by the quantum state (4) would be

$$\hat{\rho}_A(0) = \rho_{aa}^{(0)} |a\rangle \langle a| + \rho_{ac}^{(0)} |a\rangle \langle c| + \rho_{ac}^{(0)} |c\rangle \langle a| + \rho_{cc}^{(0)} |c\rangle \langle c|,$$

where

$$\rho_{\alpha\beta}^{(0)} = C_\alpha^*(0)C_\beta(0),$$

with $\alpha, \beta = a, b, c$.

Next we seek to determine the time evolution of the density operator. In this regard, we first assume that the three-level atoms initially prepared in a coherent superposition of the top and bottom levels are injected into a cavity at constant rate $r_\alpha$ and removed after some time $\tau$, which is long enough for the atoms to decay spontaneously to levels other than the middle or the lower. For convenience, the atomic spontaneous decay rate $\gamma$ is taken to be the same for the two upper levels. In the good cavity limit, $\gamma \gg \kappa$, where $\kappa$ is the cavity damping constant, the cavity mode variables change slowly compared with the atomic variables. Hence the atomic variables will reach steady state in relatively short time. In this case, the time derivative of such variables can be set to zero, while keeping the remaining terms at time $t$. This procedure is usually referred to as the adiabatic approximation scheme. Moreover, since the coupling constant is believed to be small, we confine ourselves to a linear analysis that amounts to dropping the higher order terms in $g$.

Now applying the linear and adiabatic approximation schemes, it can be established in the good cavity limit that the time evolution of the density operator for the cavity mode, driven by a coherent light on resonance, and coupled to a vacuum reservoir, takes the form

$$\frac{d\hat{\rho}(t)}{dt} = \frac{AC}{2B} \left[ 2\hat{a}^\dagger \hat{\rho} \hat{a} - \hat{a} \hat{a}^\dagger \hat{\rho} + \hat{\rho} \hat{a}^\dagger \hat{a} \right] + \frac{AD}{2B} \left[ \hat{a} \hat{a}^\dagger \hat{\rho} - \hat{a}^\dagger \hat{a} \hat{\rho}^* + \hat{\rho} \hat{a}^\dagger \hat{a} \right] + \frac{AE}{2B} \left[ \hat{a}^\dagger \hat{\rho} - \hat{a} \hat{\rho}^* + \hat{\rho} \hat{a}^\dagger \hat{a} \right] + \frac{AF}{2B} \left[ \hat{a}^\dagger \hat{\rho} - \hat{a} \hat{\rho}^* + \hat{\rho} \hat{a}^\dagger \hat{a} \right],$$

where

$$A = \frac{2r_a g^2}{\gamma^2}, \quad B = \left( 1 + \frac{\Omega^2}{\gamma^2} \right) \left( 1 + \frac{\Omega^2}{4\gamma^2} \right),$$

$$C = \rho_{aa}^{(0)} \left( 1 + \frac{\Omega^2}{4\gamma^2} \right) - \rho_{ac}^{(0)} \frac{3\Omega}{2\gamma} + \rho_{cc}^{(0)} \frac{3\Omega^2}{4\gamma^2},$$

$$D = \rho_{aa}^{(0)} \frac{3\Omega^2}{4\gamma^2} + \rho_{ac}^{(0)} \frac{3\Omega}{2\gamma} + \rho_{cc}^{(0)} \left( 1 + \frac{\Omega^2}{4\gamma^2} \right),$$

$$E = -\rho_{aa}^{(0)} \frac{\Omega}{2\gamma} \left( 1 - \frac{\Omega^2}{2\gamma^2} \right) - \rho_{ac}^{(0)} \left( 1 - \frac{\Omega^2}{2\gamma^2} \right) + \rho_{cc}^{(0)} \frac{\Omega}{2\gamma} \left( 1 + \frac{\Omega^2}{4\gamma^2} \right),$$

$$F = -\rho_{aa}^{(0)} \frac{\Omega}{2\gamma} \left( 1 + \frac{\Omega^2}{4\gamma^2} \right) - \rho_{ac}^{(0)} \left( 1 - \frac{\Omega^2}{2\gamma^2} \right) + \rho_{cc}^{(0)} \frac{\Omega}{2\gamma} \left( 1 - \frac{\Omega^2}{2\gamma^2} \right).$$
On the other hand, the time evolution of the density operator for a single-mode cavity radiation coupled to a vacuum reservoir via a single-port mirror is found using the standard method \[^{13}\] to be

\[
\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}_S(t), \hat{\rho}(t)] + \frac{\kappa}{2}[2\hat{a}\hat{\rho}\hat{a} - \hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{\rho}\hat{a}^\dagger\hat{a}],
\]

where \(\kappa\) is the cavity damping constant. With the aid of Eqs. (7) and (13), the master equation describing the cavity radiation of the driven degenerate three-level cascade laser coupled to a vacuum reservoir turns out to be

\[
\frac{d\hat{\rho}(t)}{dt} = \frac{AC}{2B}[2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}^\dagger\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{\rho}\hat{a}^\dagger\hat{a}] + \frac{1}{2} \left( \frac{AD}{B} + \kappa \right)[2\hat{a}\hat{\rho}\hat{a} - \hat{a}^\dagger\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{\rho}\hat{a}^\dagger\hat{a}] + \frac{AF}{2B}[\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}^\dagger\hat{\rho}\hat{a}^\dagger + \hat{\rho}\hat{a}] + \frac{AF}{2B}[\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}^\dagger\hat{\rho}\hat{a}^\dagger + \hat{\rho}\hat{a}].
\]

\[\text{(15)}\]

### III. STOCHASTIC DIFFERENTIAL EQUATION

We now proceed to drive the pertinent stochastic differential equations associated with the normal ordering. To this end, making use of Eq. (15) and the fact that

\[
\frac{d}{dt}(\hat{a}(t)) = Tr\left(\frac{d\rho}{dt}\hat{a}\right),
\]

one can readily see that

\[
\frac{d}{dt}(\hat{a}(t)) = -\frac{\mu}{2}\hat{a}(t) + \beta\hat{a}^\dagger(t),
\]

\[\text{(17)}\]

\[
\frac{d}{dt}(\hat{a}^\dagger(t)\hat{a}(t)) = -\mu\hat{a}^\dagger(t)\hat{a}(t) + \beta[\hat{a}^\dagger(t)^2 + \hat{a}^\dagger(t)^2] + \frac{AC}{B},
\]

\[\text{(19)}\]

where

\[
\mu = \frac{A}{B}(D - C) + \kappa,
\]

\[\text{(20)}\]

\[
\beta = \frac{A}{2B}(E - F).
\]

\[\text{(21)}\]

We notice that the operators in Eqs. (17), (18), and (19) are in the normal order. Hence we can express these equations in terms of the c-number variables associated with the normal ordering as

\[
\frac{d}{dt}\langle\alpha(t)\rangle = -\frac{\mu}{2}\langle\alpha(t)\rangle + \beta\langle\alpha^*(t)\rangle,
\]

\[\text{(22)}\]

\[
\frac{d}{dt}\langle\alpha^2(t)\rangle = -\mu\langle\alpha^2(t)\rangle + 2\beta\langle\alpha(t)\alpha(t)\rangle - \frac{AF}{B},
\]

\[\text{(23)}\]

\[
\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle = -\mu\langle\alpha^*(t)\alpha(t)\rangle + \beta\left[\langle\alpha^2(t)\rangle + \langle\alpha^2(t)\rangle \right] + \frac{AC}{B}.
\]

\[\text{(24)}\]

On the basis of Eqs. (22), it is possible to write

\[
\frac{d}{dt}\alpha(t) = -\frac{\mu}{2}\alpha(t) + \beta\alpha^*(t) + E(t),
\]

\[\text{(25)}\]

where \(E(t)\) is the corresponding noise force the properties of which remain to be determined.

The expectation value of Eq. (25) has the same form as (22) provided that the noise force has a zero mean,

\[
\langle E(t)\rangle = 0.
\]

\[\text{(26)}\]

One can also verify applying Eqs. (22), (23), (24), and (25), along with the fact that the noise force at time \(t\) does not correlate with the cavity mode variables at the earlier times that

\[
\langle E(t')E(t)\rangle = -\frac{AF}{B}\delta(t - t'),
\]

\[\text{(27)}\]

\[
\langle E(t)E^*(t')\rangle = \frac{AC}{B}\delta(t - t').
\]

\[\text{(28)}\]

We notice that Eqs. (26), (27), and (28) represent the mean and correlation properties of the noise force.

Furthermore, it proves to be more convenient to introduce new variables defined as

\[
\alpha_\pm(t) = \alpha^*(t) \pm \alpha(t),
\]

\[\text{(29)}\]

so that one can easily see with the aid of Eq. (25) and its complex conjugate that

\[
\frac{d}{dt}\alpha_\pm(t) = -\frac{\lambda_\mp}{2}\alpha_\pm(t) + E_\mp(t) \pm E(t),
\]

\[\text{(30)}\]

where

\[
\lambda_\mp = \mu \mp 2\beta.
\]

\[\text{(31)}\]

The formal integration of Eq. (30) results in

\[
\alpha(t) = a_+(t)\alpha(0) + a_-(t)\alpha^*(0) + F_-(t) + F_+(t),
\]

\[\text{(32)}\]

in which

\[
a_\pm(t) = \frac{1}{2}\left(e^{-\frac{\lambda_\mp t}{2}} \pm e^{-\frac{\lambda_\pm t}{2}}\right).
\]

\[\text{(33)}\]
Thus taking Eqs. (27) and (28) into consideration, we express the initial atomic coherence of the top and bottom levels as

\[
\rho_{ac}^{(0)} = |\rho_{ac}^{(0)}| e^{i\theta},
\]

where \(\theta\) is the phase factor. It can also be easily checked that

\[
|\rho_{ac}^{(0)}| = \sqrt{\rho_{aa}^{(0)} \rho_{cc}^{(0)}}.
\]

Moreover, upon introducing a new parameter defined by

\[
\rho_{aa}^{(0)} = \frac{1 - \eta}{2},
\]

\[
\rho_{cc}^{(0)} = \frac{1 + \eta}{2},
\]

\[
\rho_{ac}^{(0)} = \frac{\sqrt{1 - \eta^2}}{2} e^{i\theta}.
\]

Therefore, on the basis of Eqs. (39), (40), (41), and (45), Eq. (46) finally takes, for \(\theta = 0\), the form

\[
\langle \alpha_{\pm}^2(t) \rangle = \frac{A \left( \frac{\Omega}{\gamma} \left( 1 - 3\eta + \frac{\Omega^2}{\gamma^2} \right) + \sqrt{1 - \eta^2 \left( 1 - \frac{\Omega^2}{2\gamma^2} \right)} \right)}{\chi_{\pm}}
\]

\[
 \pm \frac{A \left( 1 - \eta + \frac{\Omega^2}{\gamma^2} \left( 2 + \eta \right) - \sqrt{1 - \eta^2 \frac{\Omega^2}{2\gamma^2}} \right)}{\chi_{\pm}}
\]

which reduces at steady state to

\[
\langle \alpha_{\pm}^2(t) \rangle_{ss} = \frac{A \left( \frac{\Omega}{\gamma} \left( 1 - 3\eta + \frac{\Omega^2}{\gamma^2} \right) + \sqrt{1 - \eta^2 \left( 1 - \frac{\Omega^2}{2\gamma^2} \right)} \right)}{\chi_{\pm}}
\]

\[
 \pm \frac{A \left( 1 - \eta + \frac{\Omega^2}{\gamma^2} \left( 2 + \eta \right) - \sqrt{1 - \eta^2 \frac{\Omega^2}{2\gamma^2}} \right)}{\chi_{\pm}}
\]

in which

\[
\chi_{\pm} = \kappa \left( 1 + \frac{\Omega^2}{\gamma^2} \right) \left( 1 + \frac{\Omega^2}{4\gamma^2} \right) + A \left( \frac{1 - \Omega^2}{2\gamma^2} \right) \eta \nonumber
\]

\[
+ \sqrt{1 - \eta^2 \frac{3\Omega^2}{2\gamma^2}} + \frac{\Omega}{2\gamma} \left( 1 + \frac{\Omega^2}{\gamma^2} \right).
\]

Hence the variance of the quadrature operators (39) at steady state turn out to be

\[
\Delta a_{\pm}^2 = 1 \pm \frac{A \left( \frac{\Omega}{\gamma} \left( 1 - 3\eta + \frac{\Omega^2}{\gamma^2} \right) + \sqrt{1 - \eta^2 \left( 1 - \frac{\Omega^2}{2\gamma^2} \right)} \right)}{\chi_{\pm}}
\]

\[
 \pm \frac{A \left( 1 - \eta + \frac{\Omega^2}{\gamma^2} \left( 2 + \eta \right) - \sqrt{1 - \eta^2 \frac{\Omega^2}{2\gamma^2}} \right)}{\chi_{\pm}}
\]

We see from Fig. (2) that the light produced by a degenerate three-level cascade laser externally driven on resonance by a coherent radiation exhibits squeezing for
certain values of $\Omega/\gamma$ and $\eta$ for a given linear gain coefficient. It is found for $\kappa = 0.2$ and $A = 0.33$ that the squeezing occurs for all values of $\Omega/\gamma$ when $\eta < 0.5$. It is also possible to notice that the more there are atoms initially in the upper level, the better would be the resulting squeezing.

In order to study the dependence of the squeezing on the amplitude of the driving radiation and initially injected atomic coherence closely we consider various cases of interest. In this respect, it is not difficult to check for $\Omega = 0$ that

$$
\Delta a^2_{\pm} = \frac{\kappa + A(1 \pm \sqrt{1 - \eta^2})}{A\eta + \kappa}. 
$$

(51)

The same result has been obtained for instance by Feseha [11].

We found that the light generated when all atoms are initially in the lower level exhibits squeezing for $\Omega > 3.5\gamma$. In this case a significant squeezing is obtained in the vicinity of a particular amplitude of the driving radiation for each value of the linear gain coefficient. It is also found that a squeezing of nearly 35% occurs at $\Omega = 10.1\gamma$ for $A = 0.99$. In addition, it is not difficult to see from Fig. (4) that the degree of squeezing decreases with the amplitude of the driving radiation for larger values of $\Omega/\gamma$. Though the squeezing increases with the linear gain coefficient in this case, we cannot use arbitrary values of $A$, since the steady state consideration fails to be applied for $A > 0.99$, for $\eta = 1$ and $\Omega = 3.5\gamma$.

Furthermore, when the atoms are initially prepared with equal probability to be in the top and bottom levels, substantial degree of squeezing is found for small values of $\eta$. This indicates that the more atoms are injected into the cavity at a time the more the degree of the squeezing of the cavity radiation would be. In particular, a maximum of 98% squeezing occurs at $\eta = 0.02$ for $A = 1000$. The correlated emission initiated by the initial atomic coherence is responsible for the reduction of the fluctuations of the noise in one of the quadrature components below the classical limit.

Moreover, if initially all atoms are in the lower level, $\eta = 1$, Eq. (50) takes the form

$$
\Delta a^2_{\pm} = 1 \mp \frac{A}{\chi_{\pm}} \left[ \frac{\Omega}{\gamma} \left( 1 + \frac{\Omega^2}{4\gamma^2} \right) + A \left( 1 - \frac{\Omega^2}{2\gamma^2} \right) \mp \frac{\Omega}{2\gamma} \left( 1 + \frac{\Omega^2}{\gamma^2} \right) \right]. 
$$

(52)

(53)

With

$$
\chi_{\pm} = \kappa \left( 1 + \frac{\Omega^2}{\gamma^2} \right) \left( 1 + \frac{\Omega^2}{4\gamma^2} \right) + A \left[ 1 - \frac{\Omega^2}{2\gamma^2} \right] \mp \frac{\Omega}{2\gamma} \left( 1 + \frac{\Omega^2}{\gamma^2} \right). 
$$

We clearly see from Fig. (3) that the degree of squeezing increases with the linear gain coefficient and a substantial degree of squeezing is found for small values of $\eta$.
η = 0, we get

\[
\Delta a^2_\pm(t) = 1 \pm A \left[ \frac{\Omega}{2\gamma} \left( \frac{\Omega^2}{4\gamma^2} - 2 - \frac{\Omega}{2} \right) + \frac{1}{\lambda''_\pm} \right] + A \left[ 1 + \frac{\Omega^2}{4\gamma^2} - \frac{3\Omega^4}{2\gamma^2} \right],
\]

in which

\[
\lambda''_\pm = \kappa \left( 1 + \frac{\Omega^2}{\gamma^2} \right) \left( 1 + \frac{\Omega^2}{4\gamma^2} \right) + A \left[ \frac{3\Omega}{2\gamma} + \frac{\Omega}{2\gamma} \left( 1 + \frac{\Omega^2}{\gamma^2} \right) \right].
\]

We see from Fig. (5) that the light produced by the degenerate three-level laser, when the atoms are initially prepared with equal probability of being in the top and bottom levels, exhibits substantial degree of squeezing for small values of \(\Omega/\gamma\). It is found that a maximum squeezing of 98.3% occurs at \(\Omega = 0.012\gamma\) for \(A = 1000\). As we have seen before the degree of squeezing decreases with the amplitude of the classical radiation for larger values of \(\Omega/\gamma\), but it increases with the linear gain coefficient throughout.

On the basis of the definition of the parameter \(\eta\) (Eq. (44)) we notice that for \(\eta = 0\), \(\rho_{aa}^{(0)} = \rho_{cc}^{(0)} = \rho_{ac}^{(0)} = 1/2\), which corresponds to a maximum initial atomic coherence. But, for \(\eta = 1\), \(\rho_{aa}^{(0)} = \rho_{cc}^{(0)} = 0\) and \(\rho_{ac}^{(0)} = 1\), which is related to the absence of injected atomic coherence at the beginning. It is not difficult to see from Eq. (51) that there is no squeezing property when the atoms are initially prepared with maximum or minimum atomic coherence, if they are not driven externally (\(\Omega = 0\)). However, as shown in Fig. (3) the maximum squeezing occurs when the atoms are prepared with initial coherence very close to the maximum possible value in this case. We also observe from Figs. (4) and (5) that the external coherent radiation initiates the correlation between the photons which leads to squeezing when \(\eta = 0\) or \(\eta = 1\). When there is no injected atomic coherence a squeezing close to 50% is obtained near particular amplitude of the external radiation. This result agrees with the prediction of H. Xiong et al. [6] that three-level laser in which the atoms are initially prepared in the bottom level and externally driven by strong radiation resembles parametric oscillator.

In addition, we realize upon comparing the results shown in Figs. (3), (4), and (5) that a better squeezing can be obtained when the atoms are initially prepared with a maximum atomic coherence and also driven externally with a coherent radiation of relatively small amplitude. Likewise, the maximum noise reduction when the atoms are injected into a resonant cavity with maximum atomic coherence and driven with classical radiation for a three-level laser is obtained [1]. Though the external radiation induces the coherence which is believed to be the cause of squeezing, we observe that pumping the atoms with stronger radiation than required destroys squeezing. In connection to what we have seen before, we note that the degree of squeezing would be maximum for certain values of the initial atomic coherence that depends on the rate at which the atoms are injected into the cavity.

We hence conclude that the cavity radiation exhibits significant squeezing for certain values of the amplitude of the driving radiation and initial preparation of the superposition, where the degree of squeezing increases with the linear gain coefficient.

\section{V. MEAN PHOTON NUMBER}

The mean photon number of the cavity radiation

\[
\bar{n} = \langle a^\dagger(t)a(t) \rangle
\]

can be expressed, with the aid of Eq. (29), in the form

\[
\bar{n} = \frac{\langle a^2_+(t) \rangle - \langle a^2_-(t) \rangle}{4}.
\]

On account of Eq. (51), we then find at steady state

\[
\bar{n} = -\frac{A \left[ \frac{\Omega^2}{2\gamma^2}(1 - 3\eta) + \frac{\Omega^2}{2\gamma^2} - \left( 1 - \eta + \frac{\Omega^2}{2\gamma^2}(2 + \eta) \right) \right]}{4\lambda_-} + \frac{A \sqrt{1 - \eta^2} \left( \frac{\Omega^2}{2\gamma^2} - 1 - \frac{3\Omega^4}{2\gamma^2} \right)}{4\lambda_-} + \frac{A \left[ \frac{\Omega^2}{2\gamma^2}(1 - 3\eta) + \frac{\Omega^2}{2\gamma^2} + \left( 1 - \eta + \frac{\Omega^2}{2\gamma^2}(2 + \eta) \right) \right]}{4\lambda_+} - \frac{A \sqrt{1 - \eta^2} \left( \frac{\Omega^2}{2\gamma^2} - 1 + \frac{3\Omega^4}{2\gamma^2} \right)}{4\lambda_+}.
\]
One can clearly see from Fig. (6) that no light is produced when initially all atoms are in the bottom level and if there is no external driving radiation. We also notice that as a result of the external driving it is possible to generate an intense light from the laser even when the atoms are initially in the lower level. This demonstrates the mechanism of lasing without population inversion.

Now we seek to consider various cases of interest. For $\Omega = 0$, Eq. (58) reduces
$$\bar{n} = \frac{A(1 - \eta)}{2(A\eta + \kappa)},$$
which is the same as the result obtained for instance by Fessaha [10] in the absence of the driving radiation. We see from Eq. (59) that the mean photon number would be zero when there is no driving light and all atoms are initially in the bottom level, and one gets the most intense light when all atoms are initially in the upper level as expected. In addition, for $\eta = 1$, Eq. (58) takes the form
$$\bar{n} = -\frac{A}{4\chi''} \left[ -\frac{\Omega^2}{\gamma} + \frac{\Omega^3}{2\gamma^2} + \frac{3\Omega^2}{2\gamma^3} \right]$$
$$+ \frac{A}{4\chi''} \left[ -\frac{\Omega^2}{\gamma} + \frac{\Omega^3}{2\gamma^2} + \frac{3\Omega^2}{2\gamma^3} \right].$$

As indicated in Fig. (7), the intensity of the light subsequently decreases if we keep on increasing the strength of the driving light.

On the other hand, for $\eta = 0$, we readily get from Eq. (58) that
$$\bar{n} = -\frac{A}{4\chi''} \left[ \frac{\Omega^2}{2\gamma} \left( 1 + \frac{\Omega^2}{\gamma} + \frac{\Omega}{\gamma} \right) - \frac{3\Omega^2}{2\gamma^2} + \frac{\Omega^2}{\gamma^2} \right]$$
$$+ \frac{A}{4\chi''} \left[ \frac{\Omega^2}{2\gamma} \left( 1 + \frac{\Omega^2}{\gamma} + \frac{\Omega}{\gamma} \right) - 2 - \left( \frac{\Omega}{\gamma} + \frac{\Omega^2}{\gamma^2} \right) \right].$$

We notice that the intensity of the produced light decreases with the strength of the external coherent light if the atoms are initially prepared to have equal probability of being in the top and bottom levels for $\Omega < \gamma$. We clearly see from Figs. (7) and (8) that the mean photon number increases with the linear gain coefficient.

VI. CONCLUSION

We present a detailed analysis of the squeezing properties of the light produced by the degenerate three-level cascade laser coupled to a vacuum reservoir via one of the coupler mirrors and an external resonant coherent radiation in the other. We found that the cavity radiation exhibits up to 98.3% squeezing under certain conditions pertaining to the initial preparation of the superposition and strength of the coherent radiation.
Driving the atoms with an external coherent radiation affects both the degree of squeezing and intensity of the generated light. We found that when the atoms are driven externally with a strong radiation the resulting squeezing and intensity of the cavity radiation are considerably reduced. Thus we cannot see the practical advantageous of this mechanism in this respect. However, we also found that an intense radiation with a substantial degree of squeezing can be generated specially when the atoms are initially prepared with equal probability of being in the bottom and top levels where there is no squeezing in the absence of the driving radiation. In addition, it is possible to get a squeezed light when the atoms are initially in the bottom level by this mechanism where there is no radiation at all in the absence of driving. We hence realize that driving mechanism can be considered as an option for producing a squeezed light when it is difficult to prepare the atoms in an arbitrary initial superposition. On the other hand, though it appears reasonable to expect enhancement of squeezing when we externally drive atoms with an arbitrary initial superposition, we are unable to confirm this for all possible cases from our analysis. However, driving the atoms with external radiation is found to significantly improve the squeezing, if the atoms are prepared initially with maximum atomic coherence.

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