A Versatile Numerical Method for the Multi-Fluid Plasma Model in Partially- and Fully-Ionized Plasmas

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Abstract. We present an innovative numerical method that solves for the multi-fluid plasma equations, including the transport, frictional, and chemical reactions terms, coupled to full Maxwell's equations. The numerical method features a scheme for the electromagnetic field with a proper scaling for the numerical dissipation, a scheme that solves flows at all speeds regimes (from subsonic to supersonic), and implicit time integration to tackle the stiffness of the system. Verification of the numerical scheme is also presented in a wide variety of plasma conditions.

1. Introduction
The dynamic interaction between magnetic fields and plasmas underlies a great variety of fascinating phenomena that occur in the universe. Most of the numerical simulations of magnetized plasmas have focused on solving the Magnetohydrodynamic (MHD) equations (e.g., [1]). However, astrophysical and laboratory plasmas are rich in non-ideal processes that are not contained in the ideal MHD approximation. These effects include the collisional resistivity, the anisotropy caused by the gyration of ions and electrons around the magnetic fields lines, the decoupling between the motion of the plasma species, the presence of neutral atoms or molecules, turbulent transport, etc [2].

The most complete and universal description of plasma is contained in the kinetic equation that, together with Maxwell’s equations, describes the evolution of the distribution function in the phase space for each plasma species. Based on the kinetic equation, Particle-in-Cell (PIC) (e.g.,[3, 4, 5]) are mostly used for fully-ionized low-collisional plasmas such as the ones in space weather applications [6] and nuclear fusion plasmas (e.g., [7]). Alternatively, the direct solution to the Vlasov equation ([8] and references therein) in a six-dimensional grid is proposed. However, the numerical solution of these approaches is computationally very expensive and unaffordable to simulate full-domain high-density plasma phenomena.
The MHD approach is able to simulate full-scale dynamics although it only deals with large-scale low-frequency phenomena and therefore is not able to represent some of the important phenomena. Different modifications of the MHD equations are proposed in order to tackle the non-ideal processes. The Hall-MHD approach extends the induction equation of the MHD equations in order to tackle finite ion gyroradius effects [1]. For partially-ionized plasma, the effect of the neutrals is tackled with an ambipolar diffusion (Pedersen dissipation) term in the induction equation [9].

Other methods that combine the kinetic and the fluid approach are proposed to capture the non-ideal dynamics. Hybrid codes use a hydrodynamic model for the electrons while the ions are represented with a PIC model (e.g., [10]). Alternatively, the two-way coupling of MHD simulations with PIC is recently proposed. In this approach, a limited part of the domain evolves with particle methods whereas the remaining is represented with a MHD approach [11, 12, 13]. In Fig. 1, we summarize the different plasma models as function of the collisionality and the magnetization of the plasma.

In this article, we propose a multi-fluid solver as an alternative to kinetic and MHD models. The multi-fluid plasma description is obtained by taking moments are of the Boltzmann equation for each species. Therefore, mass, momentum, and energy conservation laws are solved for each species within the plasma. The different species are hence treated as separate fluids that can interact among each other through collisions and reactions, while the charged species interact with the electromagnetic field.

Recently, different numerical methods that solve for the multi-fluid equations coupled to the Maxwell’s equations have been proposed. The ANTHEM code [15, 16] first carried out simulations of the fully-ionized ideal two-fluid (electron+ion) model. More recently, a high-order discontinuous Galerkin method have been proposed for the ideal two-fluid model by Loverich & Shumlak [17], Loverich et al. [18], Srinivasan & Shumlak [19], and Sousa & Shumlak [20]. Baboolal [21], Baboolal & Bharuthram [22], and Kumar & Mishra [23] propose finite difference methods for the ideal two-fluid (electron+ion) model. Alternatively, finite volume discretizations are developed by Shumlak & Loverich [24], and Hakim et al. [25]. For partially-ionized plasmas, Meier [26] proposes a two-fluid model that considers neutrals and ionized particles as separate fluids, interacting by means of chemical reactions, collisional momentum and energy exchanges. Numerical solutions in chromospheric conditions are obtained by Leake et al. [27, 28], Murphy & Lukin [29], Maneva et al. [30], and Alvarez-Laguna et al. [31].

In the present paper, we present a novel finite volume method that is based on upwind schemes for discretizing the multi-fluid equations coupled to the full Maxwell’s equations. Compared to previous multi-fluid finite volume schemes, the present scheme rescales the numerical dissipation...
of the numerical flux of the Maxwell’s equations. By doing this, the numerical flux takes into account the effect of the source terms, responsible for the coupling between the fluid and the electromagnetic dynamics. Similarly, the numerical scheme uses an AUSM$^+$-up (Advection Upstream Splitting Method) for the fluid equations, which provides an accurate solution in all velocity regimes from subsonic to supersonic. Second-order accuracy is obtained by total variation diminishing (TVD) reconstruction with a Venkatakrishnan limiter for near-smooth flows. Finally, the last main difference to previous schemes is the fully-implicit time discretization that provides numerical stability for large CFL numbers. These novelties result in a robust scheme that is able to reproduce different regimes, from ideal MHD to multi-fluid equations both in the collisional and collisionless regime.

2. The multi-fluid plasma equations
The general multi-fluid plasma equations consider mass, momentum and total energy balance equations for the plasma species $\alpha$, as follows,

$$\frac{\partial \rho_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \vec{u}_\alpha) = \dot{\rho}_\alpha,$$

$$\frac{\partial \rho_\alpha \vec{u}_\alpha}{\partial t} + \nabla \cdot (\rho_\alpha \vec{u}_\alpha \vec{u}_\alpha + p_\alpha \vec{I}) = \nabla \cdot \tilde{\pi}_\alpha + \vec{F}_\alpha + \sum_{\beta \neq \alpha} \vec{R}_{\alpha \beta}^{\text{elastic}} + \vec{R}_{\alpha}^{\text{react}},$$

$$\frac{\partial}{\partial t} \left[ \mathcal{U}_\alpha + \rho_\alpha \frac{u^2_\alpha}{2} \right] + \nabla \cdot \left[ \left( \mathcal{U}_\alpha + \rho_\alpha \frac{u^2_\alpha}{2} \right) \vec{u}_\alpha \right] = \nabla \cdot (\vec{u}_\alpha \cdot \tilde{\pi}_\alpha - \vec{q}_\alpha - p_\alpha \vec{u}_\alpha) + \rho_\alpha \vec{F}_\alpha \cdot \vec{u}_\alpha + \sum_{\beta \neq \alpha} \dot{\mathcal{Q}}_{\alpha \beta}^{\text{elastic}} + \dot{\mathcal{Q}}_{\alpha}^{\text{react}}.$$

Here, $\rho_\alpha$ is the mass density, $\vec{u}_\alpha$ is the macroscopic velocity, $p_\alpha$ is the scalar pressure, $\tilde{\pi}_\alpha$ is the stress tensor, $\vec{q}_\alpha$ is the heat flux, $\mathcal{U}_\alpha$ is the internal energy, $\vec{F}_\alpha$ is the external force, $\vec{R}_{\alpha \beta}^{\text{elastic}}$ and $\dot{\mathcal{Q}}_{\alpha \beta}^{\text{elastic}}$ are the exchanges of momentum and energy due to elastic binary collisions, and $\dot{\rho}_\alpha$ and $\dot{\mathcal{Q}}_{\alpha}^{\text{react}}$ are the mass and energy production due to chemical reactions.

The external force in presence of electromagnetic fields reads $\vec{F}_\alpha = m_\alpha e_\alpha \left( \vec{E}_\alpha + \vec{u}_\alpha \times \vec{B} \right)$. The electric and magnetic fields $\vec{E}$ and $\vec{B}$ are determined by Maxwell’s equations as

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times \vec{E} = 0,$$

$$\frac{\partial \vec{E}}{\partial t} - c^2 \nabla \times \vec{B} = -\frac{j}{\epsilon_0},$$

$$\nabla \cdot \vec{B} = 0,$$

$$\nabla \cdot \vec{E} = \frac{\rho_c}{\epsilon_0},$$

where $c$ is the speed of light, $\epsilon_0$ is the permittivity of free space, the total electric current density $j = \sum_n n_a e_\alpha \vec{u}_\alpha$, and the total charge density $\rho_c = \sum_n e_\alpha n_\alpha$. 

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3. Multi-dimensional finite volume method on unstructured meshes
We summarize the finite volume method described in Alvarez-Laguna et al. [33, 34]. We propose a discretization that is based on the finite volume method for the purely hyperbolic equations Maxwell’s equations [35] coupled to the general multi-fluid equations (1)-(3).

We apply the spatial discretization:

\[
\frac{d\mathbf{U}_i}{dt}|_{\Omega_i} + \sum_{j \in \mathcal{D}_i} \mathbf{H}_{ij}|_{\partial \Omega_{ij}} + \sum_{j \in \mathcal{D}_i} \mathbf{G}_{ij}|_{\partial \Omega_{ij}} = \sum_{j \in \mathcal{D}_i} \mathbf{S}_j|_{\Omega_i},
\]

where the cell average \( \mathbf{U}_i \) is the array of unknowns, \( \mathbf{H}_{ij} \) and \( \mathbf{G}_{ij} \) are respectively the numerical inviscid and diffusive fluxes at the interface between cells \( i \) and \( j \), and \( \mathbf{S}_i \) is the cell-average source term (external forces, reaction and collisional terms). \( |\Omega_{ij}| \) is the cell volume and \( |\partial \Omega_{ij}| \) is the face area. In the following sections, we will present the numerical schemes that are used in the present work to approximate the inviscid numerical flux. These are a modified-CIR (Courant, Isaacson, and Rees) for Maxwell’s equations and a generalization of the AUSM\(^+\)-up (Advection Upstream Splitting Method) for multi-fluid plasma equations.

3.1. Modified-CIR scheme for Maxwell’s equations
Numerical dissipation is necessary to ensure stable solutions without spurious numerical oscillations [36]. However, an excessive numerical dissipation can hinder the accuracy of the method.

The CIR scheme [37] is an upwind discretization that reads:

\[
\mathbf{H}_{ij} = \mathbf{A}_n^+ \mathbf{U}_L + \mathbf{A}_n^- \mathbf{U}_R = \frac{\mathbf{H}(\mathbf{U}_L) + \mathbf{H}(\mathbf{U}_R)}{2} - \frac{1}{2} |\mathbf{A}_n| (\mathbf{U}_R - \mathbf{U}_L),
\]

where \( \mathbf{U}_{L/R} = (B_x, B_y, B_z, E_x, E_y, E_z, \Psi, \Phi)_{L/R} \) are the variables on the left and right of the face, \( \mathbf{A}_n^\pm = \Lambda_n^\pm \Lambda_n^{-1} \) and \( \Lambda_n \) is the diagonal matrix that contains either the positive or negative eigenvalues of \( \mathbf{A} \) respectively. In this case, the eigenvalues are \( \lambda = \pm \epsilon, \pm \epsilon, \pm \gamma \epsilon, \pm \chi \epsilon \). The matrix \( |\mathbf{A}_n| = \Lambda |\Lambda|^{-1} \) is the numerical dissipation, where \( |\Lambda| = \frac{\Lambda_+ - \Lambda_-}{2} \). Expression of the matrices \( \mathbf{A}_n^\pm \) can be found in Munz et al. [35].

In the multi-fluid plasma model, Maxwell’s equations are coupled to the multi-fluid equations through the source terms. The CIR scheme is based on the eigenvalue structure of the numerical flux and does not take into account the scales of the fluid-electromagnetic coupled problem.

The numerical dissipation of the original CIR scheme reads:

\[
|\mathbf{A}_n| = \begin{bmatrix}
|\mathbf{A}_n|^B & 0 & 0 \\
0 & |\mathbf{A}_n|^E & 0 \\
0 & 0 & |\mathbf{A}_n|^{HDC}
\end{bmatrix},
\]

where

\[
|\mathbf{A}_n|^B = c \mathbf{D}^B, \quad |\mathbf{A}_n|^E = c \mathbf{D}^E, \quad \text{and} \quad |\mathbf{A}_n|^{HDC} = c \mathbf{D}^{HDC}.
\]

The matrices \( \mathbf{D}^B, \mathbf{D}^E, \) and \( \mathbf{D}^{HDC} \), defined in Alvarez-Laguna et al. [34], are function of the unit normal vector to the face \( \vec{n} \) and the divergence cleaning constants \( \gamma \) and \( \chi \).

The equivalent equation including the numerical dissipation, as shown by Alvarez-Laguna et
as noted by MacCormack \[38, 39\] and Alvarez-Laguna et al. \[33, 34\], this numerical dissipation is not balanced with the flux terms when the equations are coupled to the dynamics of a conducting fluid. The reason for that is that the CIR scheme is built upon the Riemann solution to a purely electromagnetic problem. However, the exact solution for the Riemann problem that couples the multi-fluid equations to Maxwell’s equations is not known. Owing to that reason, we propose to rescale the numerical dissipation in order to account for the scales of the fluid-electromagnetics coupled problem.

The propagation of waves depends on the multi-fluid model in consideration. Two different cases are studied in this article: 1) the two-fluid (ion+neutral) model for partially ionized plasmas that considers electrons and ions coupled and assumes charge neutrality [27], and 2) the ideal two-fluid (electron+ion) model [24].

We note that when charge neutrality is assumed and the electric field follows resistive Ohm’s law, the behaviour of the electromagnetic waves is similar to the MHD model. In Alvarez-Laguna et al. [33], we propose a similar scaling to MacCormack’s [38]. This results in the following numerical dissipation matrices:

\[
|\mathbf{A}_n|^B = \mathbf{D}^B, \quad |\mathbf{A}_n|^E = c^2 \mathbf{D}^E, \quad \text{and} \quad |\mathbf{A}_n|^HDC = \begin{bmatrix} 1 & 0 \\ 0 & c^2 \end{bmatrix} \mathbf{D}^{HDC}. \tag{13}
\]

Alternatively, the ideal two-fluid plasma model contains high frequency dispersive waves, such as the plasma waves. As noted by Goedbloed & Poedts [40], the symmetry between the electric field and the magnetic field is lost in the two-fluid model. While the magnetic field waves are in the MHD regime (i.e., Alfvén and magnetosonic waves), the electric field fluctuates in much higher frequency, i.e., the plasma frequency. Owing to that reason, Alvarez-Laguna et al. [34] propose the following rescaling of the numerical dissipation:

\[
|\mathbf{A}_n|^B = \hat{r}_D^p v_A \mathbf{D}^B, \quad |\mathbf{A}_n|^E = c \mathbf{D}^E, \quad \text{and} \quad |\mathbf{A}_n|^HDC = c \mathbf{D}^{HDC}. \tag{14}
\]

where \(\hat{r}_D = \max(r_D/l_0, \Delta x/l_0)\), \(r_D\) is a characteristic value of the Debye length in the background plasma, and \(l_0\) is a characteristic macroscopic length.

Note that the rescaling of the numerical dissipation that is given in Eqs. (13) and (14) maintains the linearity of Maxwell’s equations. This choice reduces the number of algebraic operations of the algorithm and improves the numerical convergence of the linear system in the implicit solver. Additionally, the use of implicit time discretization that allows for long time scales helps to damp the high-frequency waves (e.g., pure electromagnetic waves) that does not need to be resolved. More complex methods, such as computing the roots of the dispersion relation of the coupled fluid-Maxwell system in each interface, as proposed by Amano [41], is a more sophisticated alternative to our choice.
3.2. AUSM$^+$-up for all speeds extended to multi-fluid equations

The AUSM method splits the numerical inviscid flux into advective and pressure flux terms, as follows:

$$
\mathbf{H}_s(\mathbf{U}) = \vec{F}_s^{(c)} \cdot \vec{n} = \begin{pmatrix}
\rho_s \vec{u}_s \cdot \vec{n} \\
\rho_s \vec{u}_s \vec{u}_s \cdot \vec{n} + p\vec{n} \\
\rho_s H_s \vec{u}_s \cdot \vec{n}
\end{pmatrix} = \begin{pmatrix}
\rho_s \\
\rho_s \vec{u}_s \\
\rho_s H_s
\end{pmatrix} + p_s \begin{pmatrix}
0 \\
\vec{n} \\
0
\end{pmatrix} = M_s a_s \begin{pmatrix}
\rho_s \\
\rho_s \vec{u}_s \\
\rho_s H_s
\end{pmatrix} + p_s \begin{pmatrix}
0 \\
\vec{n} \\
0
\end{pmatrix},
$$

(15)

where $M_s = \vec{u}_s \cdot \vec{n}/a_s$. The AUSM scheme discretize the numerical flux for species $s$ at the interface $|\partial \Omega_{ij}|$ as:

$$
\mathbf{H}_{ij,s} = \mathbf{H}_{ij,s}^{(A)} + \mathbf{H}_{ij,s}^{(P)},
$$

(16)

where $\mathbf{H}_{ij,s}^{(A)}$ and $\mathbf{H}_{ij,s}^{(P)}$ are the upwinded advective and pressure fluxes, respectively. The former reads:

$$
\mathbf{H}_{ij,s}^{(A)} = M_{1/2,s} a_{1/2,s} \begin{pmatrix}
\rho_s \\
\rho_s \vec{u}_s \\
\rho_s H_s
\end{pmatrix}
\begin{cases}
L, & \text{if } M_{1/2,s} > 0 \\
R, & \text{otherwise}
\end{cases}
$$

and

$$
\mathbf{H}_{ij,s}^{(P)} = p_{1/2,s} \begin{pmatrix}
0 \\
\vec{n}_{ij} \\
0
\end{pmatrix}.
$$

(17)

The expressions for $M_{1/2,s}$, $a_{1/2,s}$, and $p_{1/2,s}$ are given in Alvarez-Laguna et al. [33]. The advantage of AUSM is its ability to achieve shock resolution, monotonicity, positivity, and entropy-satisfying properties. Additionally, it is designed to improve accuracy for low Mach number regime. This property is important in the multi-fluid model as the electron speed of sound is much greater than this of ions.

3.3. Total variation diminishing (TVD) reconstruction and implicit time stepping

Second-order accuracy is obtained with a weighted linear least square reconstruction [42] with Venkatakrishnan’s limiter for near-smooth flow regions [32]. We observe that the choice of the limiter has a strong impact in the ideal two-fluid (ion+electron) model. Owing to the presence of dispersive waves, the use of a limiter with a correction for near-smooth regions reduces the numerical dissipation.

The time discretization uses a fully-implicit three-point backward Euler scheme, as explained in Alvarez-Laguna et al. [33].

3.4. COOLFluiD framework

The numerical method has been implemented into COOLFluiD [43], a modern world-class high performance computing (HPC) platform that allows a proper efficiency on massively-parallel architectures up to several tens of thousands of cores (see Fig. 2). The multi-fluid/Maxwell code presented in this article has been tested on a number of supercomputer facilities. These include the ThinKing cluster of the Flemish Supercomputer Centre (VSC), NASA’s Pleiades supercomputer, the XC40 Hazel Hen system at HLRS, the Blue Gene/Q JUQUEEN at JSC,
Figure 2: Strong scaling on NASA Pleiades for 0.5 billion-cells 3D grid. In this case, as the size of the mesh is kept constant during the test, the scaling of the algorithm is limited by communication.

and VKI’s local clusters. Additionally, in a work that is not included in this article, the algorithm has been ported to graphics processing units (GPUs).

The multi-fluid/Maxwell numerical method and validation is explained in detail in Alvarez-Laguna et al. [33, 34]. Additionally, it is used to study the propagation of waves in the chromosphere in Maneva et al. [30] and the impact of radiation on chromospheric magnetic reconnection [31].

4. Numerical results

4.1. Accuracy analysis

As seen in Section 3.1, an excessive numerical dissipation can damage the accuracy of the numerical method. We perform an accuracy analysis in order to demonstrate that we achieve second-order accuracy in space with the TVD scheme explained above.

The scheme in the charge-neutrality limit is benchmarked in the simulation of a MHD isodensity vortex that is advected at 45° to the horizontal direction [44, 45, 46]. In Fig. 3a, the $L_2$ norm of the error is shown for the electric field (solid black), the magnetic field (dashed black), and the velocity field (red) in different meshes. We observe that second-order is achieved with the proposed scheme when we compare the previous curves with the slope corresponding to the second-order accuracy that is shown in blue.

Alternatively, we assess the accuracy of the scheme for the ideal two-fluid (electron+ion) model where charge neutrality is not assumed. We study a two-fluid circularly polarized wave, as described in Alvarez-Laguna et al. [34]. The results of this study are shown in Fig. 3b. In this figure, the $L_2$ norm of the error is shown for the electric field (solid black), the magnetic field (dashed black), the velocity field of electrons (red), and ions (blue) in different meshes that are compared to the second-order slope. Second-order accuracy is also achieved with the proposed rescaling of the electromagnetic field numerical dissipation without the charge-neutrality assumption. In Alvarez-Laguna et al. [34] it is shown that second-order accuracy is not achieved when the numerical dissipation is not rescaled.
4.2. **MHD shocks resolution**

We test the divergence cleaning method and the ability to capture shocks in two ideal-MHD cases, namely, the Orszag-Tang vortex [47] and the MHD rotor [48]. In Fig. 4, we show the evolution of the thermal pressure in the Orszag-Tang vortex. The comparison with a constrained-transport MHD code shows that the shocks are resolved by the AUSM method. Similarly, in Fig. 5, the result for the magnetic pressure of the MHD rotor shows that the proposed method is able to properly capture the MHD shocks while fulfilling the divergence constraint.

4.3. **Stiff source terms associated to elastic collisions and chemical reactions**

The proposed method, as it uses fully-implicit time integration, is able to handle very stiff source terms. The chemical reactions and elastic collisions terms introduce a wide range of time scales which may result in very restricted time steps for explicit discretizations. In order to test a model that considers stiff source terms, we solve for the reactive and collisional partially-ionized two-fluid model that is proposed by Leake et al. [27]. This model includes chemical reactions, collisional momentum and energy exchange, transport fluxes, and anisotropic heat conduction.

In Alvarez-Laguna et al. [31], we study magnetic reconnection under chromospheric conditions for different ionization levels. In Fig. 6, we show the magnetic field lines and electric current for two cases with initial ionization level of $\Psi_0 = 0.5\%$ (top) and $\Psi_0 = 50\%$ (bottom). Collisional and chemical reaction terms are computed with the local plasma parameters. The reconnection in the chromosphere strongly depends on the role played by ambipolar diffusion that is produced by the presence of neutrals. As seen in Fig. 6, the weakly ionized case ($\Psi_0 = 0.5\%$) presents a very sharp current sheet as compared to the more ionized case ($\Psi_0 = 50\%$). The collisions between neutrals and ions is responsible for the ambipolar diffusion which narrows the current sheet and enhances reconnection. In Fig. 7, we show the reconnection rate for a series of simulation for different ionization levels between $\Psi_0 = 0.5 - 50\%$. The reconnection rate is larger for low ionization cases.

In order to correctly capture the separation between the motions of ions and neutrals, as well as the chemical non-equilibrium in chromospheric magnetic reconnection, multi-fluid models are needed.
Figure 4: Top: evolution of thermal pressure of the Orszag-Tang vortex at $t = 3$ on a mesh of $200 \times 200$ elements. Bottom: comparison between the proposed method and a constrained transport solution [49] over a line at $y = 0.625\pi$ at $t = 3$.

Figure 5: Magnetic pressure and magnetic field lines at $t = 0.2962$ of the MHD rotor on a mesh of $400 \times 400$.

Figure 6: Current and magnetic field lines for a magnetic reconnection in chromospheric conditions. Two initial ionization levels are studied $\Psi_0 = 0.5\%$ (top) and $\Psi_0 = 50\%$.

Figure 7: Reconnection rate under chromospheric conditions for different ionization levels.
4.4. Ideal two-fluid (electron+ion) plasma model
The ideal two-fluid plasma model considers ions and electrons as different fluids. From the numerical point of view, this system of equations is difficult to solve as the dynamics and scales of ions and electrons are very different because of their mass disparity. Additionally, even though the plasma remains quasi-neutral in lengths that are larger than the Debye length, small separation of the charges produces plasma waves that are very high frequency.

We simulate the Geospace Environmental Modeling (GEM) magnetic reconnection challenge [50] with a two-fluid model considering ions and electrons. The mass ratio is artificially reduced to 25 and the speed of light to $10v_A$ where $v_A$ is the Alfvén speed. In Fig. 8, we show the momentum and temperature of electrons and ions. We observe the decoupling of the motion of ions and electrons that is responsible for the Hall effect. Additionally, in the temperature field of ions we note the formation of shocks as well as shock-shock interactions. Alternatively, the electrons do not present shocks as their speed of sound is much larger than this of ions.

The results presented in Fig. 8 qualitatively compare well to those of Fig. 14 of Hakim et al. [25]. A quantitative comparison of the reconnection rate that is obtained with the present numerical method is presented in Alvarez-Laguna et al. [34].

5. Conclusions
An innovative numerical method that solves for the multi-fluid plasma equations, including the transport, frictional, and chemical reactions terms, coupled to Maxwell’s equations has been proposed. The numerical method features implicit time integration to tackle the stiffness of the system, a scheme that solves flows at all speeds regimes (from subsonic to supersonic), and a
scheme for the electromagnetic field with a proper scaling for the numerical dissipation.

The numerical tool has been extensively verified and proves its versatility to represent a wide range of plasma regimes — from low-subsonic to supersonic, from reactive partially-ionized to fully-ionized, and from collisionally dominated to collisionless and magnetically dominated. The reusable implementation of the code allows for selecting the number of species and the closure models of choice. Hence, the same code can be used to simulate ideal and resistive MHD, reactive and collisional ion-neutral, and ion-electron models.

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