Abstract

After briefly remarking on alternatives for breaking the electroweak symmetry, I discuss the implication that recent precision experiments at LEP have for the symmetry breaking sector. The difficulties associated with generating fermion masses when the electroweak symmetry is broken dynamically are exposed and an alternative to the walking technicolor - extended technicolor scenario is suggested, based on models of substructure. Failures and lessons from trying to incorporate families and mixing in these latter schemes are also discussed.
Dynamical Electroweak Breaking: Issues and Challenges*

R .D. Peccei
Department of Physics, University of California, Los Angeles, CA 90024

*Invited talk at the “Salamfest”, held in honor of Abdus Salam at the International Centre for Theoretical Physics, Trieste, Italy, March 1993. To appear in the Proceedings of this conference.
It is a great pleasure and a singular honor for me to be talking at this “Salamfest”, particularly on a topic that is so closely connected to one of Salam’s central contributions to physics: the theory of the electroweak interactions. The model for the electroweak interactions put forth more than 25 years ago by Glashow, Salam and Weinberg [1], as more and more data was found to be in agreement with its predictions, has made a transition from model, to theory, to paradigm. LEP has provided the last chapter in this saga of success. The precise comparison of LEP data with the predictions of the Glashow, Salam and Weinberg theory provides overwhelming evidence that the electroweak interactions are indeed described by an $SU(2) \times U(1)$ gauge theory spontaneously broken to $U(1)_{em}$, with some custodial global symmetry guaranteeing that $\rho = 1$ [2]. Remarkably, however, even though the $SU(2) \times U(1)$ theory is well tested, the exact nature of the symmetry breaking mechanism is still essentially unknown. My remarks here will try to address this issue and some of its ramifications, particularly concerning dynamical symmetry breakdown.

As in any symmetry breakdown, the breakdown of the $SU(2) \times U(1)$ electroweak gauge symmetry to $U(1)_{em}$ is governed by an order parameter. What is uncertain, at the moment, is precisely what this order parameter is. Two alternative theoretical speculations exist concerning the nature of this order parameter. Either it is thought to be:

i) the vacuum expectation value (vev) of an elementary scalar field, $<\Phi>$, or it is assumed that

ii) it is related to a dynamical condensate of fermion - antifermion pairs, $<\bar{T}T>$, formed in a, as yet to be discovered, underlying theory.

Both the physics and philosophy behind these two alternatives are quite distinct. In particular, the first option is quite compatible with weak coupling, while the second option necessarily requires strong coupling.

The simplest way to break $SU(2) \times U(1) \to U(1)_{em}$ is to introduce a complex scalar doublet $\Phi$ into the electroweak theory, giving this field an asymmetric potential

$$V(\Phi) = \lambda(\Phi^\dagger \Phi - \frac{v^2}{2})^2,$$

(1)
which leads to the breakdown. The order parameter $\langle \Phi \rangle$ is then directly related to the scale $v \simeq (\sqrt{2}G_F)^{-1/2} \simeq 250 \text{ GeV}$ introduced in the potential and the breakdown occurs irrespective of the strength $\lambda$ of the potential. In supersymmetric versions of the Glashow, Salam and Weinberg model, where the presence of scalar fields is natural \[3\], the quartic scalar coefficients in fact are simply related to the $SU(2)$ and $U(1)$ couplings, so that in these cases these couplings are clearly weak \[1\]. Even without invoking supersymmetry, perturbative control of the theory argues for a weak effective coupling at the scale of the symmetry breakdown, $\lambda(v)$, so that the Landau pole in this coupling constant evolution is beyond the Planck mass, where surely new physics will enter \[5\].

Just as scalar vevs are naturally connected with weak coupling, having the $SU(2) \times U(1) \rightarrow U(1)_{em}$ order parameters be related to a fermion-antifermion condensate, $\langle \bar{T}T \rangle$, necessarily involves strong coupling. This is particularly clear if the interactions which gives rise to the fermion-antifermion condensates are those of a non Abelian gauge theory, as in technicolor \[6\].

The running coupling constant squared of such theories $\alpha_T(q^2)$, just like that of QCD, is weak at short distances but becomes strong as $q^2$ decreases. One can define a characteristic scale for these theories as the scale where $\alpha_T(q^2)$ becomes unity

$$\alpha_T(\Lambda_T^2) = 1 \ .$$

Condensate formation occurs precisely because the gauge interactions become strong. Thus the size of the order parameter $\langle \bar{T}T \rangle$ will be related to the scale where $\alpha_T(q^2)$ becomes strong, and one expects

$$\langle \bar{T}T \rangle \sim \Lambda_T^3 \ .$$  \(3\)

The physics of the symmetry breakdown of the electroweak interactions is most clearly manifested in the amplitudes for longitudinal gauge boson scattering. Thus, it is in $W_L W_L$ scattering where one may eventually see a distinction between the weak coupling and strong coupling alternatives discussed above. At very high energies $E >> M_W$, it is possible to establish a direct connection - through the, so called, equivalence theorem of Cornwall, Levin and Tiktopolous \[7\] - between the amplitudes of $W_L W_L$ scattering and

\footnote{In turn, weak coupling, necessitates having a light Higgs scalar in the spectrum, since $M_H^2 = 2\lambda(v) v^2$.}
those of their corresponding Goldstone bosons fields, \( w \).

Namely

\[
A_{W_LW_L} = A_{ww} + 0\left(\frac{M_W}{E}\right)
\]  

Clearly in the case where the order parameter is the elementary doublet vev \( \langle \Phi \rangle \), because the scalar self coupling \( \lambda \) is weak, the Goldstone boson scattering amplitudes will also be weak and one is led to expect weak interactions for the longitudinal gauge bosons. On the other hand, if the symmetry breakdown is dynamical, as a result of condensate formation, the Goldstone bosons interactions in the underlying theory are strong and thus one expects that there will be strong interactions among the \( W'_L \) s.

Even though the equivalence theorem is a high energy theorem, so that it cannot be strictly applied near threshold, there should not be much difference at low energy between the strongly coupled and weakly coupled symmetry breaking sectors. As is well known [9], the threshold behaviour of Goldstone boson interactions depends only on the metric of the coset space of the breakdown and not on any other details. Thus, independently of whether the Goldstone bosons \( w \) are strongly or weakly coupled, they have the same threshold interactions. Consequently, one expects that also the \( W'_L \) s should have the same interactions at low energy, independently of exactly how the \( SU(2) \times U(1) \) symmetry is broken.

The distinction between the two symmetry breaking mechanisms discussed, however, should become apparent at CM energies in the \( W_LW_L \) sub-system of order \( \hat{E}_{cm} \approx \sqrt{16\pi v^2} \approx 1.7 \text{ TeV} \) [10]. At these energies, in the strongly coupled case one expects resonance formation, but nothing particularly spectacular for the weakly coupled case. To get to these energies in the \( W_LW_L \) subsystem one needs the CM energies of the LHC and SSC, because of the energy degradation which occurs as one goes from protons, to quarks and, finally, to \( W' \) s. Many studies have been done to see if signals of strong \( W_LW_L \) scattering, such as resonance formation, could be visible at these future machines [11]. The difficulty, of course, is to dig out these signals from the background arising from ordinary two-\( W \) production. Although after cuts there are not many events left, in all cases examined signals of strong \( W_LW_L \) scattering are detectable at SSC energies, with these signals being somewhat more marginal for the LHC.

\(^2\)These are the fields which are absorbed in the Higgs mechanism [8] and serve to give the \( W' \) s mass.
Although the issue of what triggers the $SU(2) \times U(1)$ breaking will be eventually answered when the LHC and SSC become operational, it is interesting to ask whether one can tell anything already now about this question from the high precision electroweak data that has been gathered at LEP. LEP energies are quite “low” to be really directly sensitive to the symmetry breaking sector, but how the symmetry is broken can influence radiative corrections. In the elementary scalar case, as was pointed out first by Veltman [12], the dependence of the radiative corrections on the Higgs scalar mass, which typifies the symmetry breakdown, is given schematically by

$\text{rad. corr.} \sim \frac{\alpha}{\pi} \ell n \frac{M_H}{M_Z} + \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{M_H}{M_Z}\right)^2$.  

(5)

Thus, at leading order, one feels only logarithmic effects. Quadratic dependence on the Higgs mass is only felt at $0(\alpha^2)$ and is negligible if the Higgs scalar is light, as expected in this scenario.

The effects of a possible strong coupling symmetry breaking sector could be felt at LEP through modifications to the gauge propagators - the, so called, oblique corrections[13]. Since the typical scale where these effects should begin to be significant is of order $\sqrt{16\pi v^2} \gg M_Z$, it is sensible to expand the vacuum polarization tensors for the gauge fields in a power series in $q^2$, retaining just the first two terms [14]:

$$\Pi_{AB} (q^2) = \Pi_{AB} (0) + q^2 \Pi'_{AB} (0)$$  

(6)

Here the pair $\{AB\}$ spans the 4 possible gauge field configurations $\{AB = WW; ZZ; \gamma\gamma; \gamma Z\}$. Electromagnetic gauge invariance implies that $\Pi_{\gamma\gamma} (0) = \Pi_{\gamma Z} (0) = 0$, so that in toto one has 6 possible constants characterizing the vacuum polarization tensors in this approximation. Since one is assuming that the electroweak theory is $SU(2) \times U(1)$, three combinations of these constants can be fixed in terms of the three independent parameters which typify this theory, the two coupling constants $g_1$ and $g_2$ and $v$. $^3$ The other three combination of parameters can in principle be determined from experiment and then compared to what is expected by different schemes for symmetry breakdown.

$^3$More practically, instead of $g_1, g_2$ and $v$ one uses three other related parameters which are more accurately known experimentally: $\alpha, M_Z$ and $G_F$, the Fermi constant measured in $\mu$ decay.
A standard set of parameters, denoted by $S$, $T$ and $U$ by Peskin and Takeuchi \cite{15} and by other equivalent symbols by other authors, has emerged as being the most convenient to perform this analysis. It turns out that, of the three, only $S$ is moderately sensitive to the symmetry breaking sector, because $T$ and $U$ are dominated by other uncertainties.

Furthermore, if the symmetry breaking sector conserves an approximate vectorial $SU(2)$ symmetry - as it surely must, since experimentally the $\rho$ parameter is very near unity - one can write $S$ directly in terms of the vacuum polarization tensors for $SU(2)_V$ and $SU(2)_A$ currents, related to the $SU(2) \times U(1)$ currents, facilitating in this way the comparison with models of dynamical symmetry breaking. One finds \cite{14}

$$S = -4\pi \{\Pi'_{VV}(0) - \Pi'_{AA}(0)\} = \int_0^\infty \frac{ds}{s} [\nu(s) - a(s)] ,$$  \hspace{1cm} (7)

where the second line makes use of a spectral decomposition for the vector and axial vector current correlation functions.

One can extract what values of $S$ (and $T$) are allowed by comparing electroweak observables [like the hadronic width of the $Z$] to the theoretical predictions. To do so one needs some reference value of $m_t$ and $M_H$ to fix the expectations of the standard model, with elementary scalar field breaking \cite{5}.

That is, for each experimental observable one has formulas of the type \cite{15}

$$O_{\text{exp}} = O_{\text{sm}} + \alpha_0 S + \beta_0 T + \gamma_0 U$$  \hspace{1cm} (8)

with $\alpha_0$, $\beta_0$ and $\gamma_0$ computable coefficients [e.g. for $\Gamma_Z$, using as reference point $M_H = 1 \, \text{TeV}$, $m_t = 150 \, \text{GeV}$, one has $(\Gamma_Z)_{\text{exp}} = 2.487 \pm 0.010 \, \text{GeV}$; $(\Gamma_Z)_{\text{sm}} = 2.484 \, \text{GeV}$; $\alpha_0 = - 9.58 \times 10^{-3} \, \text{GeV}$; $\beta_0 = +2.615 \times 10^{-2} \, \text{GeV}$; $\gamma_0 = 0$]. A global fit of all precision electroweak data, performed by Peskin and Takeuchi \cite{15}, is shown in Fig. 1. As can be seen, the combined analysis of all data favors negative values of $S$ with

$$S \simeq -0 \, (1)$$  \hspace{1cm} (9)
This value can be compared to the expectations of various dynamical symmetry breaking models. The simplest of these models is technicolor [6]. Here one imagines that the condensates which break $SU(2) \times U(1)_{em}$ are those of an underlying $SU(N)_T$ theory, whose technifermions come in $n$ replications of left-handed doublets $(U_i D_i)_L$, $i = 1, 2, \ldots, n$ and right-handed singlets $U_{iR}, D_{iR}$ - much as ordinary quarks do. The condensates

$$<\bar{U}_i U_j> = <\bar{D}_i D_j> = \delta_{ij} \Lambda_T^3$$

(10)

then break $SU(2) \times U(1) \rightarrow U(1)_{em}$ precisely as $n$ replicas of scalar doublets $\Phi_i$ would. Because this theory is just a “scaled up” version of QCD, with the dynamical scale $\Lambda_T \sim v \sim 250$ GeV, one can estimate the parameter $S$ from QCD. By dominating the vector and axial vector spectral functions in QCD by the $\rho$ and $A_1$ poles one deduces that

$$S_{QCD} = \frac{6\pi f_\pi^2}{m_\rho^2}$$

(11)

and hence for the $SU(N)_T$ theory one has

$$S \simeq n \frac{N}{3} S_{QCD} \simeq n \frac{N}{3} \left( \frac{6\pi f_\pi^2}{m_\rho^2} \right) \simeq 0.08 nN$$

(12)

The vertical lines in Fig. 1 represent the expectations of such a simple technicolor model with $N = 4$ and $n = 1$ or 4 where, however, the QCD spectral integral was computed out without resorting to $\rho$ and $A_1$ dominance [15]. As can be seen from this figure, the theoretically “more realistic” $n = 4$ technicolor model is about $3\sigma$ away from the experimentally allowed region.

Although this result is not very encouraging for dynamical symmetry breaking theories, I do not believe it represents a death knell for these theories. First of all, one can argue, rather convincingly, that in more sophisticated theories with dynamical symmetry breaking of $SU(2) \times U(1)$ - the so called, walking technicolor theories [19], where $\alpha_T(q^2)$ runs slowly - that the integral over the spectral functions in $S$ should converge much more slowly,

---

6 These results agree quite well, however, with those obtained by assuming simple $\rho$ and $A_1$ dominance.
so that $S$ should be below the estimate given above [20]. Furthermore, if one worries about how fermions get mass in theories where $SU(2) \times U(1)$ is broken dynamically, it becomes quite clear that a naive scaling up of QCD will just not do! I turn to discuss this important point further.

Simple technicolor models do not address the issue of fermion masses unless one introduces some communication between ordinary fermions, $f$ - the quarks and leptons - and the fermions which condense, $T$. This is done ordinarily by introducing a second underlying theory - a, so called, extended technicolor theory (ETC) [21] - to connect the $f$ and $T$ fermions together. If these fermions sit in the same ETC representation and the ETC theory is a spontaneously broken gauge theory, broken at a large scale $\Lambda_{ETC}$, one generates in this way an effective four-fermion interaction

$$L_{ETC}^{eff} = \frac{1}{\Lambda_{ETC}^2} (\bar{T}_L T_R)(\bar{f}_L f_R).$$

(13)

This term, when the $T$ fermions condense $<\bar{T}_L T_R> \sim \Lambda_T^3$, gives the fermions $f$ a mass

$$m_f \sim \frac{<\bar{T}_R T_L>}{\Lambda_{ETC}^2} \sim \frac{\Lambda_T^3}{\Lambda_{ETC}^2}.$$  

(14)

Given that the technicolor scale $\Lambda_T \sim v$, the above formula has a generic difficulty. If one wants to generate with it the top quark mass, because top is so heavy, the scale $\Lambda_{ETC}$ cannot be too large. However, if $\Lambda_{ETC}$ is only in the $(1-10)$ TeV range, then one cannot avoid large flavor changing neutral current interactions (FCNC), which are not observed experimentally [22]. If $\Lambda_{ETC}$ is large, to avoid the FCNC problem, then one can never generate in this way large enough fermion masses.

Although the above conundrum is not the only problem of ETC theories, this problem can be ameliorated by walking technicolor models (WTC) [19], which have more realistic dynamics. Furthermore, WTC theories can also ameliorate some of the other endemic problems of ETC, like having too light pseudo Goldstone bosons. So, if one believes that the symmetry breakdown of $SU(2) \times U(1)$ to $U(1)_{em}$ is dynamical, it is much more sensible to focus on WTC models. Let me briefly indicate what is the underlying idea in these models. The identification of $\Lambda_T$, the scale of the condensate, with $v$, the scale related to the $W$ mass, is borrowed from QCD. In QCD, indeed, both the scale which measures the size of the quark condensate $<\bar{u}u>$ -
which triggers chiral symmetry breakdown - and that which measures the pion decay constant $f_\pi$ - associated with the coupling of the broken chiral currents to the Goldstone pion fields - are the same. That is

$$<\bar{u}u> \sim f^3_\pi \sim \Lambda^3_{QCD}$$

(15)

In WTC models, it turns out that the techniquark condensates $<\bar{T}T>$ have a scale which is much bigger than $v^3$. Hence, one can obtain rather large masses for sizable ETC scales $\Lambda_{ETC}$, avoiding the FCNC ↔ $m_t$ conundrum.

Both $<\bar{T}T>$ and $v$, the technipion decay constant, are related to the techniquark self energy $\Sigma(p)$, but probe different momentum scales in this function. Graphically, the relation of $<\bar{T}T>$ and $v^2$ to $\Sigma(p)$ can be easily inferred from Fig. 2. Retaining only the leading momentum behaviour - neglecting logarithmic terms - one has [23]

$$<\bar{T}T> \sim \int \frac{d^4p}{p^2} \Sigma(p) \sim \int dp^2 \Sigma(p) ,$$

while

$$v^2 \sim \int \frac{d^4p}{(p^2)^2} \Sigma^2(p) \sim \int \frac{dp^2}{p^2} \Sigma^2(p) .$$

(16)

For an asymptotically free theory the chirality breaking self energy $\Sigma(p)$ falls at large momentum as [24]

$$\Sigma(p) \sim \frac{\Sigma(0)^3}{p^2} ,$$

(17)

where again I have neglected logarithmic factors. Here $\Sigma(0)$ serves as the order parameter for the breakdown of the global chiral symmetries of the theory. Thus, one expects that it be of order of the dynamical scale of the technicolor theory: $\Sigma(0) \sim \Lambda_T$.

Because the integral over $\Sigma(p)$ which enters for $v^2$ is (essentially) convergent, one sees that $v^2$ probes the region of the techniquark self energy near $p^2 = 0$,

$$v \sim \Sigma(0) \sim \Lambda_T .$$

(18)

On the other hand, the condensate $<\bar{T}T>$ feels much more the large momentum structure of $\Sigma(p)$. In particular, the region of $p^2$ important for
fermion mass generation is $p^2 \sim \Lambda_{ETC}^2$ and one has

$$ m_f \sim \frac{\langle \bar{T}T \rangle}{\Lambda_{ETC}^2} \sim \frac{1}{\Lambda_{ETC}^2} \int_{\Lambda_{ETC}^2} \Sigma(p^2) \sim \Sigma(\Lambda_{ETC}) \quad . $$

(19)
The value of $\Sigma(\Lambda_{ETC})$ is not known a priori, without a dynamical calculation. If one assumes that at this scale the technifermion self energy already has achieved its asymptotic value $\Sigma(p) \sim \Sigma(0)^3/p^2$, then indeed one reproduces for $m_f$ the naive estimate of before:

$$m_f \sim \Sigma(\Lambda_{ETC}) \sim \frac{\Sigma(0)^3}{\Lambda_{ETC}^2} \sim \frac{\Lambda_f^3}{\Lambda_{ETC}^2}.$$ (20)

The assumption one makes in walking technicolor models is that the dynamics is such that at $\Lambda_{ETC}, \Sigma(\Lambda_{ETC})$ is still much above its ultimate asymptotic value. Thus one can have large masses for fermions even for large values of $\Lambda_{ETC}$, solving the FCNC $\leftrightarrow m_t$ conundrum.

To obtain a behaviour of the type

$$\Sigma(\Lambda_{ETC}) \gg \frac{\Sigma(0)^3}{\Lambda_{ETC}^2}$$ (21)

it is necessary that all physical quantities evolve slowly with momentum - hence the moniker walking for these theories. This can be achieved if the underlying theory is very nearly not asymptotically free or, perhaps, a fixed point theory [19]. Furthermore, if at $\Lambda_{ETC}$ the coupling constant of the walking technicolor theory is still rather strong, so that $\Sigma(p)$ does not take yet its perturbative asymptotic value, it is clear that one cannot simply decouple the ETC and the WTC dynamics from each other. The presence of the ETC interactions can therefore influence the self energy functions for different technifermions differently, which may provide a physical rationale for some of the observed hierarchies in the quark and lepton mass matrices [25].

Rather than pursue further ETC/WTC theories here, I would like to indicate an alternative possibility in which also the dynamics of fermion mass generation and that of $SU(2) \times U(1)$ breaking are naturally interlocked. This possibility is realized in composite technicolor models, where both the light fermions we see (the quarks and leptons) and the fermions which condense (the technifermions) are bound states of some fundamental preon theory. In these kinds of theories, the preon dynamics produces effective 4-fermion interactions involving technifermions and light fermions which are analogous to the, perhaps more familiar, ETC interactions. The formation of technifermion condensates, combined with the presence of these interactions, then allows mass to be generated for the light fermions.
In what follows I want to describe a toy attempt in this direction, developed in collaboration with S. Khlebnikov [26]. Our model produces one generation of quarks, say $t$ and $b$, as preon bound states \(^7\). These states are almost point-like, since their size is much smaller than their Compton’s wavelength

$$< r_q > \ll \frac{1}{m_q} . \quad (22)$$

However, one obtains a top-bottom mass hierarchy precisely because top is more extended than bottom,

$$\frac{m_t}{m_b} \approx \frac{< r_t >^2}{< r_b >^2} . \quad (23)$$

The root cause for quark mass generation in the model is the formation of certain $SU(2) \times U(1)$ breaking condensates of other fermionic preon bound states, which act precisely as techniquarks. Thus, in the model, the $W$ and $Z$ get mass dynamically by the same condensates which also generate quark masses. As it will become clear below, the model is rather uneconomical. Furthermore, the difficulties one encounters in trying to incorporate families in extensions of this model are also quite illustrative.

The model is based on a chiral gauged preon model. One can argue that certain of the global chiral symmetries in the model are preserved in the binding. As a result of these global chiral symmetries, there are a number of massless bound states $B$ in the spectrum. These states, however, acquire mass when one gauges some subset of the preons in a vector-like manner, as a result of condensate formation. After this further gauging, the original massless bound states $B$ split into three different set of states

$$B = \{q, T, M\} , \quad (24)$$

with $q$ being the quarks ($t, b$), $T$ being techniquarks and $M$ being megaquarks. The metacolor gauge interaction at the preon level leads to the formation of $< \bar{M} M >$ condensates which, in turn, serve to produce effective ETC interactions between the quarks and techniquarks. The technicolor gauge

\(^7\)Leptons are not included in the model, but there are no hypercharge anomalies since these are cancelled by the presence of the techniquark bound states in the spectrum.
interaction then, through the formation of $<TT>$ condensates, leads to the appearance of the $W$ and $Z$ masses and of mass terms for the quarks - these latter masses originating as a result of the effective ETC interactions.

In more detail, the preon dynamics of the model we considered is based on replicas of an $SU(6)$ preon theory with 10 Weyl preons in the fundamental representation, $F_a(a = 1, \cdots, 10)$, and one preon in the symmetric conjugate representation, $\bar{S}$. Such a theory is chiral, but has no gauge anomalies. It has a nominal chiral global symmetry

$$G = SU(10) \times U(1)_Q$$

(25)

where $U(1)_Q$ is the anomaly free combination of $F$ and $\bar{S}$ fermion number. However, one can argue dynamically \[27\] that a smaller symmetry than $G$ is preserved in the binding, namely

$$H = SU(6) \times SU(4) \times U(1)'_Q$$

(26)

Furthermore, one can identify the set of bound states $B$ which are massless by matching the $H$ anomalies at the preon level with those at the bound state level \[28\]. These states are readily seen to comprise two different kinds of states

$$B = \left\{ \begin{array}{l}
B_1 \sim (15; 1, -5/3) \\
B_2 \sim (6; 4, -5/6)
\end{array} \right.$$  

(27)

where, in the above, I have given the transformation of these states under $H$.

The bound states $B$ feel effective interactions among each other, as a result of their common underlying preonic structure\[8\]. There are $H$ invariant dimension 6 interactions, scaling as $\Lambda_c^{-2}$, with $\Lambda_c$ being the dynamical scale of the preon theory,

$$\mathcal{L}_9 = \frac{1}{\Lambda_c^2} (\bar{B} \gamma^\mu \lambda B)(\bar{B} \gamma_\mu \lambda B)$$

(28)

and dimension 9 interactions, scaling as $\Lambda_c^{-5}$

$$\mathcal{L}_9 = \frac{1}{\Lambda_c^5} B_1 B_2 \bar{B}_1 \bar{B}_2 \bar{B}_2 \bar{B}_2$$

(29)

\[8\] The states $B$ have the following schematic preonic structure: $B_{ab} \sim F_a^T \sigma_2 \sigma_\mu F_b \sigma^\mu \bar{S}$. 

13
These latter interactions are not so important in the one generation model under study, but they play an important role in multigeneration models, since they violate individual $B_1$ and $B_2$ number.

The actual model studied in [26] is based on 3 replicas of these $SU(6)$ preon models, with the individual models constructed so that among their $B$ states one generates, respectively, a $t_R$ state, a $b_R$ state and the $(\begin{array}{c} t \\ b \end{array})_L$ doublet of states. The first two $SU(6)$ models are identical in content with their 10 (right-handed) Weyl preons feeling different gauge interactions. One gauges an $SU(3)_c \times SU(3)_T \times SU(4)_M$ group of color, technicolor and metacolor interactions at the preon level, with the 10 $F_a$ preons transforming under this group as

$$F_a = \{(\bar{3}, 1, 1) \oplus (1, \bar{3}, 1) \oplus (1, 1, 4)\} \ . \quad (30)$$

The final $SU(6)$ preon theory has a doubled set of preons - 20 $F_a$ and 2 $\bar{S}$ - so as to be able to introduce at the preon level the electroweak $SU(2) \times U(1)$ interactions. These preons are (left-handed) Weyl states and are organized in doublets of $SU(2)$, except for the preons which feel the metacolor interactions, which are $SU(2)$ singlet states. Thus, in the last preon theory, the preons transform as

$$F_a = \{(3, 1, 1, 2) \oplus (1, \bar{3}, 1, 2) \oplus 2(1, 1, 4, 1)\}; \quad \bar{S} = (1, 1, 1, 2) \ . \quad (31)$$

The $B_1$ and $B_2$ massless bound states of each of these 3 $SU(6)$ preon theories can be classified immediately in terms of $SU(3)_c \times SU(3)_T \times SU(4)_M$. Since the antisymmetric combination of two $\bar{3}'s$ is a 3, one has

$$B_1 \sim \{3, 1, 1) \oplus (1, 3, 1) \oplus (\bar{3}, \bar{3}, 1)\}$$

and

$$B_2 \sim \{(3, 1, 4) \oplus (1, \bar{3}, 4)\} \quad (32)$$

Thus the $B_1$ states contains both quarks and techniquarks, while the $B_2$ states are the only massless bound states which feel metacolor.

Gauging the metacolor group will cause the formation of condensates of the $B_2$ states, produced by the right-handed $SU(6)$ preon theories, with those produced by the left-handed $SU(6)$ preon theory. These condensates tie the $\bar{S}$ preons to the $B_1$ and $B_2$ states.

$^9$All preons also have appropriate $U(1)$ quantum number [26].
left and right theories together, but preserve $SU(2) \times U(1)$. Specifically, one has two such condensates forming.\textsuperscript{[10]}

\[
< \bar{B}^{t_R}_2 B^{1L}_2 > \sim \Lambda_4^3 \quad ; \quad < \bar{B}^{b_R}_2 B^{2L}_2 > \sim \Lambda_4^3
\]  

(33)

These metacolor condensates, combined with the effective residual interactions (28) of the preon theory, give rise to a form of ETC interactions. How these ETC-like interactions arise is sketched schematically in Fig. 3. Because the individual $SU(6)$ preon theories have their own intrinsical dynamical scales, the resulting ETC interactions need not be the same for top and bottom. Indeed, as a result of the metacolor condensate formation one obtains

\[
\mathcal{L}_{\text{eff ETC}} = \frac{\Lambda_4^2}{\Lambda_L^2 \Lambda_{t_R}^2} (\bar{B}^L_1 \gamma_\mu \lambda B^L_1)(\bar{B}^t_R \gamma^\mu \lambda B^t_R) \\
+ \frac{\Lambda_4^2}{\Lambda_L^2 \Lambda_{b_R}^2} (\bar{B}^b_L \gamma_\mu \lambda B^b_L)(\bar{B}^b_R \gamma^\mu \lambda B^b_R)
\]  

(34)

\textsuperscript{10}The notation employed here is to append a superscript $t_R$, $b_R$ or $L$ to indicate to which preon theory the bound states belong. Note that there are 2 $B^L_2$ bound states formed, denoted by $B^{1L}_2$ and $B^{2L}_2$, respectively.
The $B_1$ states, recall, contain both quarks and techniquarks. Thus the above effective ETC interactions, once one turns on the technicolor interactions, will generate masses for the quarks. Although the technicolor condensates are expected to preserve a vectorial $SU(2)$ symmetry \cite{29}, the resulting top and bottom quark masses will be different since the effective ETC interactions for these states have different strength. If the scale of the technicolor condensate is $\Lambda_T$, the one obtains

$$m_t \sim \left( \frac{\Lambda^4 T^2}{\Lambda^2 L} \right) \frac{1}{\Lambda^2 t_R}; \quad m_b \sim \left( \frac{\Lambda^4 T^2}{\Lambda^2 L} \right) \frac{1}{\Lambda^2 b_R} \quad (35)$$

The masses of the top and bottom quarks will be small compared to the various preonic dynamical scales $\Lambda_L, \Lambda_{t_R}$ and $\Lambda_{b_R}$ - the compositeness scales - provided that these scales are large compared to the metacolor scale $\Lambda_4$ and the technicolor scale $\Lambda_T$. If this is so, these states will appear for all purposes as effectively elementary when probed with energies of $0(100 \text{ GeV})$. The particular hierarchy between $m_t$ and $m_b$ in the model is due to the difference in the dynamical scales of the two (right-handed) preon theories, $\Lambda_{t_R}$ and $\Lambda_{b_R}$. These scales typify the physical size of top and bottom

$$< r_t > \sim \frac{1}{\Lambda_{t_R}}; \quad < r_b > \sim \frac{1}{\Lambda_{b_R}} \quad . \quad (36)$$

Thus Eq. (35) contains the interrelation between mass and size alluded to earlier in Eq. (23).

It is easy to imagine a mechanical extension of this model, so as to introduce families of quarks \cite{26}. To get $N_f$ generations, all one has to do is to make $N_f$ copies of the $SU(6)^3$ preon model discussed above. In this extended model one can reproduce the hierarchy of the observed quark mass spectrum by assuming appropriate dynamical scales for all the various right-handed preon theories. That is,

$$m_f \sim \frac{1}{\Lambda^2 f_R} \sim < r_f >^2 \quad . \quad (37)$$

However, this naive extension of the toy one family model has many problems. These are both of a theoretical and a phenomenological nature. On the theoretical side, because this $(SU(6))^{3N_f}$ preon theory has so many preons, at
the preon level, all the vectorial interactions - color, technicolor and metacolor - are not asymptotically free. Thus, it is no longer clear whether one has control of the dynamics or that the assumed techniquark and metaquark condensates really occur. On the phenomenological side, perhaps even worse disasters occur. The extended theory \[26\] for \(N_f = 3\) has a natural \([U(1)_V]^3\) family symmetry which, if it is not broken somehow, prevents generating any quark mixing matrix at all. Thus, one has a quark mass hierarchy, but the Cabibbo, Kobayashi Maskawa matrix \(V_{(CKM)}\) is identically equal to unity:

\[
V_{CKM} = 1
\]  

(38)

It is, perhaps, useful to explain what prevents quark mixing in the model, as this is quite generic of models where generations are obtained by just blindly copying what happens in one family. For each generation a vectorial \(U(1)\) is preserved in the binding, under which all preons that carry color have charge +1, and all preons which carry technicolor carry charge −1. Naively, one would presume that this \([U(1)_V]^3\) symmetry is also preserved by all condensates. However, because of the chiral nature of the effective interactions experienced by the \(B_1\) and \(B_2\) bound states and because metacolor itself is probably strong at the compositeness scale, \[17\] it is not clear whether the Vafa - Witten theorem applies \[29\]. Thus, it is probable that the metacolor condensates do not respect the \([U(1)_V]^3\) family symmetry.

To get family mixing it is necessary that the metacolor condensates be not family diagonal

\[
< \bar{B}_{2i}^R B_{2j}^{1L} > \sim \Lambda_4^3 \Sigma_{ij}^1 ; \quad < \bar{B}_{2i}^R B_{2j}^{2L} > \sim \Lambda_4^3 \Sigma_{ij}^2
\]  

(39)

with

\[
\Sigma_{ij}^1 \neq \delta_{ij} ; \quad \Sigma_{ij}^2 \neq \delta_{ij}
\]  

(40)

Unfortunately, this assumption is not enough. Although these condensates appear to break \([U(1)_V]^3\) to a vectorial family symmetry \(U(1)_V\), the dynamics of the theory is such that there remains a discrete family symmetry, which ultimately prevents quark mixing. To understand this point, consider the analog to Eq. (34) which ensues after the formation of the nondiagonal

\[11\] Recall that metacolor is not asymptotically free in multigenerational models at the compositeness scale.
metacolor condensates given above. Although these interactions now connect $B_1$ bound states of different families, e.g.

$$\mathcal{L}_{\text{eff ETC}} = \frac{\Lambda_4^2}{\Lambda_{L_4}^2 \Lambda_{u_{Rj}}^2}(\bar{B}_{1i}\gamma^\mu \lambda B^{L}_{1i})(\bar{B}^{u\kappa}_{ij} \gamma^\mu \lambda B^{u\kappa}_{ij})$$

(41)

they still preserve the individual $B_{1j}$ - numbers since they always involve $\bar{B}_{1j} B_{1j}$ combinations.

To transmit the breaking of family number that occurs through the metacolor condensates, one needs to make use of the dimension 9 interactions of Eq. (29) which involve only $B_1$ fermions and $B_2$ antifermions (or vice versa). Using these interactions, and the metacolor condensates (39), then indeed one obtains effective interactions which break explicitly $[U(1)_V]^3$. Schematically, these interactions take the form

$$\mathcal{L}_{B_1 \text{ break}} = \frac{\Lambda_{16}^{16}}{\Lambda_{L_4}^{14} \Lambda_{u_{Rj}}^{6} \Lambda_{d_{Rk}}^{5}} B^{L}_{1i} B^{L}_{1i} B^{L}_{1i} B^{L}_{1i} B^{u\kappa}_{ij} B^{u\kappa}_{ij} B^{d\kappa}_{ik} B^{d\kappa}_{ik}$$

(42)

Unfortunately, although the above interaction breaks the $[U(1)_V]^3$ family symmetry, because of the structure of the theory there remains a $(Z_2)^3$ discrete family symmetry! This is a pity because otherwise, given the high powers of $\Lambda$ involved, the $B_1$ breaking Lagrangian would really act as a small dynamical perturbation to generate quark mixing.

Protective family symmetries or their remnants, like the $(Z_2)^3$ symmetry encountered above, are a generic feature of these simple replica models. These symmetries prevent quark mixing to occur, although one can generate arbitrary diagonal mass matrices. This problem may be solved in models where family structure occurs more naturally. Given the intriguing interrelation between the mass of the quarks and their extent, one can well imagine that in a more realistic model the largest of the diagonal elements in the mass matrices of quarks would involve a $c - t$ transition. In turn this leads one to speculate that, if there are FCNC in more realistic theories, these will be most strongly felt in the $c - t$ channel (and possibly in the $b - s$ channel) [26].

Attempts to generate fermion masses in models of dynamical symmetry breakdown of $SU(2) \times U(1)$ are salutary exercises. They serve to remind one that if one believes that the electroweak theory is broken dynamically, then
resolving the issue of fermion masses is absolutely crucial. Without a satisfactory solution to the fermion mass problem, it is difficult to give credence that $SU(2) \times U(1)$ is broken by some condensate of a yet to be discovered underlying theory. Continuing failures to satisfactorily resolve the fermion mass problem give further impetus to the idea that the electroweak theory of Glashow, Salam and Weinberg is broken down simply by a scalar vev. Before surrendering by default to this idea, however, it seems worthwhile to continue exploring some of these composite technicolor models, particularly since the dynamics of combined chiral and vectorial gauge theories is very rich and could still hide some interesting surprises. In doing so, one would be following the example of Salam, who was not afraid many times in his career to follow unfashionable paths, which eventually led to profound later insights. Whether the same will occur in this instance remains to be seen.

This work was supported in part by the Department of Energy under grant No. DE-FG03-91ER40662 TASK C

References

[1] S. L. Glashow, Nucl. Phys. 22 (1961) 579; A. Salam in Elementary Particle Theory ed. N. Svartholm (Almquist and Wiksell, Stockholm 1968); S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264

[2] See, for example, L. Rolandi, Proceedings of the XXVI International Conference on High Energy Physics, Dallas, Texas, August 1992, ed. J. R. Sanford, AIP Conference Proceeding No. 272 (AIP, New York, 1993)

[3] K. Wilson, as quoted in L. Susskind, Phys. Rev. D20 (1979) 2619; E. Gildener and S. Weinberg, Phys. Rev. D13 (1976) 3333; G ’t Hooft in Recent Developments in Gauge Theories, Cargese Lectures 1979, eds. G ’t Hooft et al (Plenum Press, New York, 1980).

[4] For a recent discussion, see for example, F. Zwirner, in Texas/Pascos ’92: Relativistic Astrophysics and Particle Cosmology, eds C. W. Akerlof and M. A. Srednicki; Ann. New York Acad. Sci. 688 (1993) 55

[5] L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B136 (1978) 115
[6] S. Weinberg, Phys. Rev. D13 (1976) 974; L. Susskind, Phys. Rev. D20 (1979) 2619

[7] J. M. Cornwall, D. M. Levin and G. Tiktopolous, Phys. Rev. D10 (1974) 1145; D11 (1975) 972 (E)

[8] P. W. Higgs, Phys. Lett 12 (1964) 132; Phys. Rev. Lett. 13 (1964) 508; F. Englert and R. Brout, Phys. Rev. Lett. 13 (1964) 321; G. S. Guralnik, C. Hagen and T. W. B. Kibble, Phys. Rev. Lett. 13 (1964) 585

[9] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177 (1969) 2239; C. Callan, S. Coleman, J. Wess and B. Zumino, Phys. Rev. 177 (1969) 2247

[10] B. W. Lee, C. Quigg and H. B. Thacker, Phys. Rev. D16 (1977) 1519

[11] For recent discussions, see for example, the contributions of M. S. Chanowitz and J. Bagger in the Proceedings of the XXVI International Conference on High Energy Physics, Dallas, Texas, August 1992, ed. J. R. Sanford, AIP Conference Proceedings No. 272 (AIP, New York, 1993)

[12] M. Veltman, Nucl. Phys. B123 (1977) 89; Acta Physica Polonica B12 (1981) 432

[13] B. W. Lynn, M. E. Peskin and R. G. Stuart in Physics at LEP, Vol. 1, eds J. Ellis and R. D. Peccei, CERN 86-02 (1986); D. Kennedy and B. W. Lynn, Nucl. Phys. B232 (1989) 1; M. Consoli, W. Hollik and F. Jegerlehner in Z Physics at LEP Vol. 1, eds G. Altarelli, R. Kleiss and C. Verzegnassi; CERN 89-08 (1989)

[14] M. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964; D. Kennedy and P. Langacker, Phys. Rev. Lett. 65 (1990) 2967; G. Altarelli and R. Barbieri, Phys. Lett. B253 (1991) 161

[15] M. Peskin and T. Takeuchi, Phys. Rev. D46 (1992) 381

[16] G. Altarelli, R. Barbieri and F. Caravagios, CERN preprint CERN TH 6770 93

20
[17] D. Kennedy and P. Langacker, Phys. Rev. D44 (1991) 1591

[18] G. Altarelli, R. Barbieri and S. Jadach, Nucl. Phys. B369 (1992) 3

[19] B. Holdom, Phys. Rev. D24 (1981) 1441; Y. Yamawaki, M. Bando and K. Matumoto, Phys. Rev. Lett. 56 (1986) 1335; T. Akiba and T. Yanagida, Phys. Lett. 169B (1986) 432; T. Appelquist, D. Karabali and L. Wijewardhana, Phys. Rev. Lett. 57 (1986) 982

[20] T. Appelquist and G. Triantaphyllou, Phys. Lett B278 (1992) 345; S. D. Hsu and R. Sundrum, Nucl. Phys. B. 391 (1993) 127

[21] S. Dimopoulos and L. Susskind, Nucl. Phys. B155 (1979) 237; E. Eichten and K. Lane, Phys. Lett. 90B (1980) 237

[22] S. Dimopoulos and J. Ellis, Nucl. Phys. B182 (1981) 505

[23] For a discussion, see for example, B. Holdom in Proceedings of the 1991 Nagoya Spring School on Dynamical Symmetry Breaking, ed. K. Yamawaki (World Scientific, Singapore 1992)

[24] K. D. Lane, Phys. Rev. D10 (1974) 2605; H. D. Politzer, Nucl. Phys. B117 (1976) 397

[25] For a discussion of this point and a semirealistic model, see for example, B. Holdom, Phys. Lett. B246 (1990) 169

[26] S. Khlebnikov and R. D. Peccei, Phys. Rev. D48 (1993) 361

[27] I. Bars and S. Yankielowicz, Phys. Lett. 101B (1981) 159

[28] G. ’t Hooft, Ref. [3]

[29] C. Vafa and E. Witten, Nucl. Phys. B234 (1984) 173
Figure 1: 68% C.L. and 90% C.L. curves in the $S$ and $T$ plane determined by the global fit to all data carried out by Peskin and Takeuchi [15]. Very similar results have also been obtained by [17], [18]. In the figure the predictions of the standard model and of an $SU(4)$ technicolor theory with $n = 1$ and $n = 4$ are shown, as a function of $m_t$. The crosses in the figure denote $m_t$ values which, starting from the bottom, increase by 20 GeV from $m_t = 90$ GeV.
Figure 2: Graphs relating $< \bar{T}T >$ (a) and $\nu^2$ (b) to the techniquark self energy $\Sigma(p)$.

Figure 3: Generation of effective ETC interactions, through metacolor condensation.