Algorithms for increasing the reliability of primary measurement information

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Annotation:
The work analyzes the process of predicting (extrapolating) the state of an object; if we use the results of previous measurements, simulate the equation of the dynamics of the process at the time of the subsequent measurement, applying the approximation of the measured process by a given class of function, selected based on the physical properties of the process under study.

Keywords:
Measured value, converter, filtration, linear function, emergency limit, averaging, control, error, coefficient, linearization, reliability, technological equipment

Introduction. Control over the work of industrial production involves the collection of the necessary information about technological processes from sensors, its processing and transfer to operational personnel [1,2]. When solving control problems, a number of sequentially performed operations are carried out to process the measured information [3]: analytical calibration of the sensor; extra- and interpolation of discretely measured values; filtering from interference; integration and averaging of measured values; introduction of corrections for temperature, pressure and consumption of material and energy flow; plausibility check of measured values, etc.

The algorithms of the information and measurement subsystem of the APCS are designed to collect information from analog sensors, number-pulse discrete and time-coded signals with a given polling period, from sensors of discrete and number-pulse initiative signals, to process the entered information, to connect the operator technologist with the system and entering operational messages on the progress of the technological process.

The information subsystem performs standard processing of the measured parameters [4]: scaling and linearization of measured values; plausibility check; introduction of pressure and temperature corrections for process flow rates; exponential smoothing (filtration); control of measured values for compliance with specified regulatory and emergency limits; averaging measurement results.

The outputs of the primary measuring converters are sequentially interrogated by the commutator, and the measured signals are fed further to the analog-to-digital converter (ADC). The frequency of uniform sampling of one signal in a stationary random process is determined by the upper boundary frequency of the spectrum of the useful signal, the permissible measurement error, the root-mean-square error of the measuring channel, and the variance of the random process itself.

The transformation of the measured value into the machine code of the control complex (UVK) occurs in accordance with the expression: 

\[ Y = f_1\{f_2[f_1(X)]\}, \]  

(1)
Where \( X \) - measured value; \( Y \) - machine code; \( f_1 \) - linear function of converting ADC code into machine code; \( f_2 \) - linear function of converting input signals into ADC code; \( f_3 \) - dependence between the measured value and the electrical signal of the primary measuring transducer.

The operation of filtering the electrical signal coming from the primary measuring transducer makes it possible to determine the random component of the measurement error. To do this, you need to know the frequency characteristics of the signal and interference.

For filtering, an easy-to-implement exponential smoothing algorithm is often used:

\[
Y_n = \alpha X_n + (1 - \alpha)Y_{n-1},
\]

where \( X_n \) - measured linearized parameter value; \( Y_{n-1} \) - the smoothed value in the previous step; \( \alpha \) - smoothing factor (filter constant).

At the first step \( Y_1 \) at \( n = 1 \) it is set equal to \( X_1 \). The regular component of the measurement error in some cases can be compensated for using the algorithms for correcting the sensor readings.

Correction factor \( K_1 \) with a deviation of temperature \( T \) and pressure \( P \) from the calculated \( T_0 \) and \( P_0 \) is determined by the following expressions

\[
K_1 = \frac{P + 0.1}{T + 273} K ; \quad K = \frac{T_0}{P_0},
\]

where \( T_0 \) - design gas temperature (°K); \( P_0 \) - design pressure, MPa (MPa).

Corrected calculation is calculated by the formula

\[
Y = K_1 Y_L,
\]

where \( Y_L \) - measured linearized flow rate.

Changing the characteristics of sensors or measuring channels leads to the appearance of measurement errors that cannot be established by the described methods of primary information processing. In the process control system, when solving control problems, the results of measuring many parameters are used simultaneously. Therefore, the probability of obtaining an incorrect result in the event of failure of individual measuring channels increases sharply. In this regard, the control of the reliability of the initial measurement information is one of the most important functions of the process control system [5].

In the simplest case, information reliability control is carried out along the technologically possible boundaries of the measured value change. Violation of these limits usually indicates a malfunction of the sensor or measurement channel:

\[
X_{\text{BRI}} \leq X_i \leq X_{\text{BRI}},
\]

where \( X_{\text{BRI}} \) - the lower limit of the possible change of the variable \( X \); \( X_{\text{BRI}} \) - the upper limit of the possible change of the variable \( X \); \( X_i \) - current value of variable \( X \) at \( i \)-th polling cycle.

If a violation is detected, a message is sent to the operator's control panel. Further processing of this parameter is carried out using its last valid value.

Control over the course of the technological process is carried out along the lower regulatory limit (NWG), upper regulatory limit (VRG), lower emergency limit (NAG), upper emergency limit (VAG), the permissible rate of change of parameters (SKI) within:

\[
X_{\text{RAG}} \leq X_i \leq X_{\text{BAG}},
\]

where \( X_{\text{RAG}} \) - lower emergency limit of the measured process variable; \( X_{\text{BAG}} \) - upper emergency limit of the measured process variable \( X \);

\[
X_{\text{HFF}} \leq X_i \leq X_{\text{BFF}},
\]
where is $X_{HPF}$ - lower regulatory limit of the measured process variable $X$; $X_{BPF}$ - upper regulatory limit of the measured process variable $X$;

$$\Delta X = |X_i - X_{i-1}| \leq \delta X_{\text{max}},$$

(8)

where is $\delta X_{\text{max}}$ - the largest allowable increment of the variable during the polling cycle.

To analyze the operation of technological equipment, as well as the technological stage as a whole, it is necessary to determine the average values of the costs used in the production of raw materials, reagents, energy, etc. The averaging of the measured values of the parameters is carried out according to the formula

$$Y_G = \frac{1}{n} X_i + \left(1 - \frac{1}{n}\right) Y_{G-1},$$

(9)

where is $X_i$ - $i$-th measured linearized parameter value; $n$ - the number of parameter polls at a given averaging interval.

In the practice of creating an automated process control system, there is a need for scaling and linearization of measured values.

For sensors with a linear character of the scale, the $X$ values are calculated by the formula

$$X = B + (A - B)N/L;$$

(10)

for square-law sensors

$$X = \sqrt{(A^2 - B^2)N/L + B^2},$$

(11)

where is $X$ - parameter value; $A$ - true value of the end of the scale of the sensor; $B$ - true value of the beginning of the scale of the sensor; $N$ - ADC code; $L = 1000$ - the maximum value of the ADC code.

Для эффективного управления процессом оперативному персоналу it is necessary to have information about the current values of the monitored parameters, indirectly measured values, technical and economic indicators of the operation of units and production sites [6].

Traditional models of information processing in order to increase its reliability are based on the assumption that the mathematical model of the research object is low, both at the stage of obtaining information and at the forecasting horizon. Obviously, a more preferable option is when the information processing algorithm is able to automatically recognize the models and adapt to them.

Taking into account the "aging" of information and the dynamics of changes in the model, the problem of statistical synthesis of optimal algorithms for processing measurement information can be formulated as follows:

- based on theoretical considerations or accumulated experience, an analytical expression is selected for the information processing algorithm, which depends on unknown parameters;
- restrictions on unknown coefficients are set;
- the form of the optimality criterion for the method of processing the measurement results is selected or determined;
- the problem of finding the conditional extremum of the optimality criterion J is solved under the given constraints by varying the values of unknown coefficients;
- the method and algorithm of information processing is determined.

Let us consider the formulated problem in relation to the cases of geometric and arithmetic smoothing.

Suppose we have a simple random sample of independent and equally accurate measurements of the same quantity $X = \{X_1, X_2, ..., X_n\}$, the variances of which are the same and equal to $\sigma^2$. It is required to find an estimate satisfying the property of unbiasedness based on the results of primary measurements. We are looking for the estimate $\overline{X}$ in the form of a linear combination of measurements:
\[ \bar{X} = \sum_{i=1}^{n} C_i X_i , \]  

where is \( C_i \) - some constant factors to be determined.

The solution to the problem gives the following expressions for the variance of the estimate:

\[ D(\hat{x}) = \delta^2 \left( \frac{1-\alpha}{1-\alpha^2} \right)^2 \left( 1-\alpha^{2n} \right) \left( \frac{1-\alpha}{1+\alpha} \right) \left( 1-\alpha^n \right) \]  

\[ \tilde{D}_{g.c.} = \frac{D(\hat{x})}{\delta^2} = \frac{1}{1+\alpha} \frac{1+\alpha^n}{1-\alpha^n} , \]  

where is \( \alpha \) - smoothing constant.

As you can see,

\[ \min \frac{D(\hat{x})}{\delta} = \frac{1}{n} \]  

at \( \alpha = 1 \).

The geometric smoothing method, which is a finite-dimensional analogue of exponential smoothing, is used for sampling a limited volume, and the unbiasedness condition is satisfied for any sample size.

The choice of the smoothing constant value (or the number of measurements \( M \) in the moving average) assumes a compromise between the speed of response to model changes (increase and decrease in \( M \)), as well as the quality of filtering random noise. Since the range of variation of the smoothing constant is very large (usually from 0.01 to 0.3, and the sample size in the moving average is from 6 to 200), the task is to find a choice method in each specific case.

If we assume that the “age” of the current measurement is zero, the previous measurement is one, and that in the future the “age” of previous measurements is increased by one, then the average “age” in the moving average is equal to the average of the “ages” of all individual measurements taken with weights, equal for these individual measurements:

\[ \rho_{c.c.} = \frac{1}{M} (0 + 1 + 2 + \ldots + M - 1) = \frac{M(M-1)}{2} \cdot \frac{1}{M} = \frac{M-1}{2} . \]  

With geometric smoothing, the average “age” of measurements is determined as

\[ \rho_{g.c.} = \left( \sum (n-k) \alpha^{n-k} \right) \frac{1-\alpha}{1-\alpha^n} \frac{1-\alpha^{n-1}}{1-\alpha^n} - (n-1) \frac{\alpha^n}{1-\alpha^n} ; \]  

\[ \lim \rho_{g.c.} = \frac{n-1}{2} ; \]  

\[ \lim \rho_{g.c.} = \frac{1}{1-\alpha} , \alpha \rightarrow \infty \]  

Let us formulate a general formulation of the problem of predicting the characteristics of a stochastic process in terms of the statistical theory of optimal estimation \([7,8]\).

Let there be a production process, the current state of which at time \( i \) can be described by an \( n \)-dimensional random vector \( x_i \). Observation of the process at time \( j \) can be represented as an \( n \)-dimensional vector \( z_j \), statistically related to the state vector. Let the sequence of observations \( z_j (j=1,k) \) be defined. It is required to determine the estimate of the state vector for some given \( s \)-th moment of time, if \( x_0 \) is inaccessible to direct observation.
Since the state vector \( x_i \) and the observation vector \( z_i \) are statistically related, the trajectory \( x \) can be estimated from the observations \( z \). Of the possible estimates, it is necessary to choose the one that has the highest accuracy, that is, it sets the estimated trajectory closest to the trajectory \( x \).

We denote the estimate obtained on the basis of observations through \( \hat{x}_i \)

\[
\hat{x}_i = \varphi(z_j), \quad (j = 1, k).
\]

(20)

Consider the dynamic relation:

\[
x_{n+1} = Fx_n + GU_n,
\]

where is \( F \) and \( G \)- characterize, respectively, the dynamic properties of the process and the constraints on the input signal; \( U_n \) is an independent Gaussian random sequence with zero mathematical expectation and covariance matrix \( G \).

Observations are determined by the ratio:

\[
z_n = Hx_n + V_n.
\]

(22)

where is \( H \) - characterizes the constraints imposed on observations of \( x_n \); \( V_n \) is a sequence similar to \( V_n \) with the covariance matrix \( R \).

The initial conditions are \( x(0) \) and

\[
M[x(0)] = x, \quad \text{cov}[x(0), x(0)] = P_0.
\]

(1)

The F, G and H values can change over time.

The task of getting an estimate \( \hat{x}(0 \leq i \leq n) \) staged as follows [42]. Measurements given \( z_i \), \( z_2, \ldots, z_n \). Estimates required \( \hat{x}_{i/n}(0 \leq i \leq n) \) according to measurement data that minimize the criterion:

\[
J = \frac{1}{2} \left[ (x_{0/n} - x)^T (x_{0/n} - x) + \sum_{i=0}^{n-1} \frac{1}{2} \left( (z_{i+1} - Hx_{i+1/n})^T H^{-1} (z_{i+1} - Hx_{i+1/n}) \right) \right] \times \left( z_{i+1} - Hx_{i+1/n} \right) + \left[ Q^{1/2} (U_{i/n})^T Q^{-1} (U_{i/n}) \right],
\]

(23)

with restrictions

\[
\hat{x}_{i/n} = Fx_{i/n} + GU_{i/n},
\]

(24)

Here it is necessary to know the optimal (minimizing criterion \( J \)) estimate \( \hat{x}_{i/n} \), those. the first thing to do is to solve the filtration problem. The optimal solution can be obtained using the following ratios:

\[
x_{n/n} = Fx_{(n-1)/(n-1)} + P_{n/n} H^T R^{-1} \left( z_n + HP_{n/(n-1)} \right),
\]

\[
P_{n/n} = P_{n/(n-1)} H^T \left( HP_{n/(n-1)} H^T + R \right)^{-1} HP_{n/(n-1)},
\]

(25)

\[
F_{n/(n-1)} = FP_{n/(n-1)} F^T + GQG^T.
\]

Let the predicted process be specified in the form of a multidimensional discrete system, the transition of which from the \( i \) th state to the \((i + 1)\) th state is described by the multidimensional difference equation:

\[
x_{i+1} = A_i x_i + \epsilon_i,
\]

(26)

where \( A_i \) is known dimension transition matrix \((nxn)\); \( x_i \) is a column vector of phase coordinates at the \( i \)-th time of dimension \((nx1)\); is a column vector of additive noise of dimension \((nx1)\).

At the moment of time \( i \), the measurement \( z_i \) is carried out, which is related by the vector equation to the phase state vector:

\[
z_i = D_i x_i + \sigma_i
\]

(27)

where \( z_i \) is a column-vector of dimensions \((mx1)\); \( D_i \) is a matrix of known dimension coefficients \((mxn)\); is a column vector of additive noise of dimension \((nx1)\).
We will assume that random processes and are independent with mean values equal to zero: and symmetric, non-negative definite covariance matrices: \( M(\sigma_0^T) = Q \), \( Mm[\sigma^T] = R \).

The equation allows one to obtain extrapolated estimates of the state vector at the \((i + 1)\) -th moment, if the previous estimates are known

\[ \hat{x}_{(i+1)} = A_{(i+1)} \hat{x}_i, \]

The extrapolated estimate errors are characterized by the following covariance matrix:

\[ K_{(i+1)} = A_{(i+1)} K_i A_{(i+1)}^T + Q. \]

The choice of the linear form of the optimal estimate is determined by the following main factors [9]. First, in most technical problems, the linear operator is easier to implement using linear devices. Second, for a normal process, when the A-optimality criterion is satisfied, the optimal linear operator is generally a linear operator, i.e. it is impossible to improve the values of the criterion by passing from a linear operator to any other non-linear operator.

In accordance with the above, the new estimate is written in the form:

\[ \hat{x}^+_i = \hat{x}_i + F(z_i - D\hat{x}_i), \]

where \( F_i \) - dimension gain matrix (nxm).

We represent expression (26) in the form

\[ \hat{x}^+_i - x_i = \hat{x}_i - x_i + F[-D_i(\hat{x}_i - x_i) + \sigma_i], \]

where \( \hat{x}_i \) - the true value of the phase state vector of the process under study.

Then the error of the new estimate of the phase state will be equal to:

\[ \varepsilon^+_i = \varepsilon_i + F[\sigma_i - \sigma_i, \varepsilon_i]. \]

Let us define the covariance matrix of the error of the new state estimate of the controlled process. By definition, we have:

\[ k^+_i = M(\varepsilon^+_i, \varepsilon^+_i)^T. \]

Performing the operation of determining the mathematical expectation and using the matrix inversion lemma [10], one can show that the system describing the optimal filter has the form:

\[ \hat{x}^+_i = \hat{x}_i + F(z_i - D\hat{x}_i), \]

\[ F_i = k_i D^T (k_i + D_k D^T)^{-1}, \]

\[ k^+_i = k_i - k_i D^T (k_i + D_k D^T)^{-1} - D_k. \]

This filter is a Kalman type filter [11] and provides estimates of the parameters of the predicted process by the minimum of the generalized variance of errors (the minimum of the trace of the covariance matrix \( k^+_i \)).

The system characterizing the optimal predictive filter is as follows:

\[ \hat{x}_{(i+1)} = A_{(i+1)} \hat{x}_{(i+1)}, \]

\[ k_{(i+1)} = A_{(i+1)} k^+_i A_{(i+1)}^T + \sum_{n=1}^{i} A_{(n+1),(i+1)} Q_{(n+1),(n+1)} A_{(i+1),j}^T. \]

The predictive filter (35) does not impose any restrictions on the form of the interference distribution laws and. It is enough only that the first two initial moments are given.

Thus, in the case of a linear relationship between measurements and the phase vector of the state, normal distribution laws of measurement errors and estimates of the state of the process, the procedures found by the criterion of the maximum information are equivalent.

The process of predicting (extrapolating) the state of an object can be carried out using the results of previous measurements, i.e. simulate the equation of the dynamics of the process at the time of each subsequent measurement, applying the approximation of the measured process by a given class.
of function, selected based on the physical properties of the process under study. In this case, the simplest form of approximation of the equation of dynamics of a nonlinear process is its representation in the form of a Taylor series for the known quantization interval

\[ x_{(t+i)} = x_i + x_i \Delta t + \frac{1}{2!} x_i \Delta t^2 + \ldots, \]

where \( x_i = x(t_i) \) - the value of the measured parameter at the moment \( T = t_i \).

Since the values of the derivatives are not known exactly, then to determine we will use the previous measurements at the moment, ... etc. Carrying out the linear approximation operation on the quantization interval and keeping the three terms of the expansion in the Taylor series, after some transformations, we can come to an expression of the form:

\[ x_{(t+i)} = A_i x_i + A_{(i-1)} x_{(i-1)} + A_{(i-2)} x_{(i-2)} + \gamma, \]

where \( \gamma \) is the column vector of the approximation error, consisting of the term of the error due to a finite number of expansion terms and the term due to the inaccuracy of calculating the derivatives; \( A_{(i-j)} \) - constant coefficient matrix.

The last expression (37) is a second-order predictor and allows one to predict the state of the process at the \( t \) moment of time.

Without loss of generality, we now assume that the numerical characteristics of the approximation error satisfy the conditions:

\[ M(\gamma) = 0; \quad M(\gamma, \gamma^T) = Q. \]

The generalized form of the quasi-optimal filter, when the formulation of forecasting problems under conditions of a priori uncertainty of the process model presupposes finding estimates of the parameters of the object with an arbitrary specified lead interval \( s \geq 1 \) using a predictor of any order, has the form:

\[
\begin{align*}
\hat{x}_{(t+s)} &= \left( \prod_{j=1}^{s} A_{(t+j)} \right) \hat{x}_{(t)} - B_{(t+s)} \gamma^T, \\
x_{(t+s)} &= Q_{(t+s)} \hat{x}_{(t+s)} + \sum_{j=1}^{s} B_{(t+j)} \gamma \hat{x}_{(t+j)} \\
\hat{x}^*_i &= \hat{x}_i + F_i (Z_i - D_i \hat{x}_i), \\
k^*_i &= k_i - k_i D_i^T (R_i + D_i k_i D_i^T)^{-1} D_i k_i.
\end{align*}
\]

To obtain the predicted state of the process, it is necessary to introduce into consideration a new vector of variables \( x \) of increased dimension and find the value \( \hat{x}_{(t+s)} \). From the resulting vector it is possible to select the desired vector of the state of the process \( Y_{(t+s)} \).

**Conclusion.** Summarizing, it can be noted that we have shown: in the case of a priori given linear mathematical models of technological processes and systems, an optimal adaptive filter of the Kalman type is effective from the point of view of predictive estimates. The problem of synthesis of a quasi-optimal predictive filter is solved in the case of a priori uncertainty in the knowledge of the mathematical model of an object, which allows predicting its state for each subsequent quantization interval.

The problem of statistical synthesis of optimal algorithms for processing measurement information, taking into account the "aging" of data and the dynamics of changes in the model of the measuring transducer, is formulated as applied to the cases of geometric and arithmetic smoothing, with a priori specified linear mathematical models of technological processes and control and management systems that are effective in terms of predictive estimates. The problem of synthesizing a quasi-optimal
predictive filter is solved in the case of a priori uncertainty in the knowledge of a mathematical model that allows predicting the state of an object for each subsequent quantization interval.

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