Stress-strain state in soil massif when interacting with incompressible barrette and raft

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Abstract. The statement and solution of problems on the interaction of incompressible barrette with the soil massif as part of a pile-raft foundation are considered, taking into account the barrette steps, diameters, the embedded length, as well as nonlinear soil behaviors by analytical and numerical methods using Plaxis-3d. It is shown that these parameters have a significant effect on the stress-strain state (SSS) of the soil massif interacting with the barrette-raft system. By taking the approach above engineers entirely manage to gain formulas so that to determine the SSS in incompressible barrette, as well as to assess the settlement of this system. For the sake of achieving the solution to problem above, the behaviors of the barrette-raft system, the soil massif are assumed to be elastic.

1. Introduction

When a barrette-raft system interacts with the surrounding soil, a complex and heterogeneous SSS arises, in which the geometrical parameters such as barrette steps, embedded length, diameters exercise substantial influence. With an increase in the number of barrettes, the quantitative assessment of the SSS of the barrette-raft-soil massif becomes significantly sophisticated. The SSS assessment is necessary to figure out the principle of distribution of the load between surfaces and the heel of the barrette, as a result, the settlement of the barrette-raft-soil massif and the bearing capacity of the barrette-raft system absolutely can be estimated accurately. Therefore, the authors present an analytical solution to the problems above and conduct comparison with the results achieved from plaxis-3d in order to estimate the efficiency of the solution.

2. Statement problem

Considering a barrette-raft system with given soil properties, as well as dimension and boundary conditions of the soil massif, which is presented in the figure 1, we assume that the load $p$ applied to the raft is distributed to the surrounding and underlaying soil in terms of compressive stress, shear stress and punching stress respectively. Hence the following problems are necessitated to be clarified:

- The equation for distribution of the compressive stress under the raft with depth;
- The equation for distribution of the shear stress on surfaces of the barrette’s body.
- The equation for determination of the punching stress under the heel of the barrette
- The equation for estimating the punching settlement of the heel and the entire settlement of the barrette-raft-soil massif.
3. Program research

\[ \sigma_N \omega + \sigma_s (1 - \omega) = p \]  

(1)

where \( \sigma_N \) is the compressive stress at the head of the barrette, \( \sigma_s \) is the stress in the surrounding soil under the raft plate, \( \omega \) is a non-dimensional parameter and calculated by the ratio \( \frac{ab}{AB} \).

The barrette settlement with the load \( p \) develops due to the shear deformation of the surrounding soil within the affected zone around the barrette with the range from \( 2a \times 2b \) to \( 2A \times 2B \) along the barrette’s body and the vertical sides of the soil massif. Thereby, the increment of the settlement at the depth \( z \) can be calculated by the following dependency:

\[ dS(r, z) = -\gamma(r, z)dr \]  

(2)

in which

\[ \gamma(r, z) = \tau(r, z) / G \]  

(3)

\[ \tau_a(r, z) = \frac{(B - r)}{(B - b)} , \quad \tau_b(r, z) = \frac{(A - r)}{(A - a)} \]  

(4)

Putting the attaining the equation of the shear stresses into expression (1), then we achieve the distribution of the settlement on the lateral surfaces corresponding to the edges \( 2a \) and \( 2b \) of the incompressible barrette \( S_{\text{side } a} \) and \( S_{\text{side } b} \), respectively.

\[ S_{\text{side } a}(z) = -\frac{\tau_a(z)}{G} \int_0^z \frac{(B - r)^2}{(B - b)^2}dr = \tau_a(z) \frac{B - b}{3G} + C_1 \]  

(5)

\[ S_{\text{side } b}(z) = -\frac{\tau_b(z)}{G} \int_{A-a}^z \frac{(A - r)^2}{(A - a)^2}dr = \tau_b(z) \frac{A - a}{3G} + C_2 \]  

(6)

Considering the boundary condition of the soil massif and the calculation scheme, it is obvious that when \( r \) is taken equal to \( B \) and \( A \), respectively, \( S_{\text{side } a}(r = B) \) and \( S_{\text{side } b}(r = A) \) turn out to be \( C_1 \) and \( C_2 \).
\( C_2 \), respectively. It is confirmed that the value of the settlement on edges 2a and 2b in the soil massif and the value of settlement due to the compressive stress \( \sigma_i \) under the raft plate are the same i.e.

\[
S_{\text{side } a}(r=l) = C_1 = S_{\text{side } a}(r=b) = C_2 = \sigma_m z + \sigma_m (L-l)
\]

(7)

where \( m \) and \( m_0 \) are the values of the coefficients of volume compressibility of the soil massif in the range of the barrette’s body and the underlying soil, respectively.

Substituting the received equation (7) to expressions (5) and (6) we finally get:

\[
S_{\text{side } a}(z) = \frac{-\tau_a(z)}{G} \frac{8}{\pi} \frac{(B-r)^2}{(B-b)^2} r = \tau_a(z) \frac{B-b}{3G} + \sigma_m z + \sigma_m (L-l)
\]

(8)

\[
S_{\text{side } b}(z) = \frac{-\tau_b(z)}{G} \frac{4}{\pi} \frac{(A-r)^2}{(A-a)^2} r = \tau_b(z) \frac{A-a}{3G} + \sigma_m z + \sigma_m (L-l).
\]

(9)

The settlement of the barrette’s heel is procured from the known dependency of the settlement of a rectangular rigid stamp taking into account the coefficient \( K_{\text{r}} < 1 \) on the load \( p \), then we get:

\[
S_R = \frac{\sigma_R (1-v_p) a K_i w}{G_0} = \sigma_R K
\]

(10)

\( G_0 \) and \( v_p \) are deformation parameters of the underlying soil layer, while \( w \) is the coefficient taking into account the shape of the stamp (rectangle, square).

\[
K = \frac{(1-v_p) a K_i w}{G_0}
\]

(11)

\( K_i \) is a non- dimensional coefficient which denotes the effect of the barrette’s shape and the embedded depth on the settlement of the underlying soil. We can refer to N M Doroshkevich, V V Znamensky, and V I Kudinov’s article [9] for \( K_i \).

\[
K_i = \frac{1}{(1-v_0^2) w} \left\{ \frac{(1+v_0)(3-4v_0)}{4\pi(1-v_0)} \left[ \ln \left[ \left( n^2 + 1 \right)^{1/2} + n \right] \right] + \frac{(1+v_0)(8v_0^2-12v_0+5)}{4\pi(1-v_0)} \left[ \ln \left[ \frac{m^2 n^2 + m^2 + 16}{m^2 n^2 + m^2 + 16} \right] + m \right] + \frac{2m(1+v_0)}{\pi(1-v_0)(m^2 n^2 + m^2 + 16)} \left[ \left( m^2 + 16 \right)^{1/2} + \left( m^2 n^2 + 16 \right)^{1/2} \right] + \frac{2(1+v_0)2v_0-1}{\pi m(1-v_0)} \left[ \arcsin \left[ 4n \left( m^2 + 16 \right)^{1/2} \right] + \arcsin \left[ 4n \left( m^2 n^2 + 16 \right)^{1/2} \right] - \frac{\pi}{2} \right] \right\}
\]

(12)

in which \( n = \frac{b}{a} \); \( m = \frac{2a}{l} \).

Determining the calculated value \( \sigma_R \) in the linear statement, the degree of its approximation to the limit state should be checked (\( \sigma_R < \sigma_R’ \)). This test can be taken place by the formula of L. Prandtl [11]:

\[
\sigma_R = (\gamma d + c_0 \cot \varphi_b) \frac{1 + \sin \varphi_b}{1 - \sin \varphi_b} \exp(\pi \cot \varphi_b) - c_0 \cot \varphi_b
\]

(13)

\( d \) is the depth at the level of barrette’s heel (m);
\( \gamma \) is the unit weight of soil from the ground surface to depth \( d \) (kN/m³);
\(c_0\) and \(\varphi_0\) are the friction angle and the cohesion of the underlying soil layer (kPa) and (radian).

Considering the interaction of an incompressible barrette with the soil mass with the origin of coordinates \(z = 0\) at the barrette’s heel level, by considering the equilibrium conditions for the element \(dz\) (referred to calculation scheme in the figure 1), it is affirmed that the increment of the stress \(d\sigma_z(z)\) is taken equal to the increment of the shear stress on the lateral surfaces between the barrette and the surrounding soil.

\[
4ab \, d\sigma_z(z) = (4r_a a + 4r_b b) \, dz
\]

Therefore, it is reformed as follows

\[
\frac{d\sigma_z(z)}{dz} = \frac{\tau_a a + \tau_b b}{ab}
\]

Basing on the equilibrium condition of the settlement on the barrette’s side surfaces \(S_{\text{side } a}(z) = S_{\text{side } b}(z)\), we achieve the dependency of the shear stress \(\tau_a(z)\) on \(\tau_b(z)\)

\[
\tau_b(z) = \tau_a(z) \frac{B-b}{A-a}
\]

The value \(\tau_b(z)\) in expression (15) is replaced by the received from (16), thereby we have:

\[
\frac{d\sigma_z(z)}{dz} = \tau_a(z) \frac{(A-a)a + (B-b)b}{(A-a)ab} = \tau_a(z)I
\]

where \(I = \frac{(A-a)a + (B-b)b}{(A-a)ab}\)

The principle of distribution of the shear stress on the surfaces of the barrette depending on the depth \(z\) (m) complies the exponential function proposed by Sidorov V.V. [16] as follow:

\[
\tau_a(z) = \tau_a(0) \exp(-\alpha z)
\]

where \(\tau_a(0)\) is the maximum value of the shear stress at the depth \(z = 0\) corresponding to the calculation scheme; coefficient \(\alpha\) is given in the range \([\frac{3}{l}; \frac{5}{l}]\); \(l\) (m) is embedded length of the barrette.

Conducting integration of differential equation (17) with respect to \(z\) and taking into account expression (19) we attain the expression indicating the value of compressive stress in the body of incompressible barrette at any depth \(z\) (m) within the embedded length \(l\) (m)

\[
\sigma_a(z) = \tau_a(0)I_1 e^{-\alpha z} \, dz = \tau_a(0)I_1 \frac{e^{-\alpha z}}{\alpha} + C_3
\]

when \(z = 0\), \(\sigma_a(0) = \tau_a(0) \frac{I_1}{\alpha} + C_3 = \sigma_R\), \(C_3 = \sigma_R + \tau_a(0) \frac{I_1}{\alpha}\)

Replacing \(C_3\) in expression (20) by the received value from (21), \(\sigma_b(z)\) conspicuously turns out to be as follows:

\[
\sigma_b(z) = \tau_a(0)I_1 \frac{e^{-\alpha z}}{\alpha} + \tau_a(0) \frac{I_1}{\alpha} + \sigma_R
\]

From the equilibrium condition between the settlement of the surrounding soil at the level \(z = l\), taking into account the incompressible property of barrette, we evidently attain the expressions:

\[
S_{\text{side } a}(l) = S_R
\]

\[
S_R = \sigma_R K = \tau_a(l) \frac{B-b}{3G} + \sigma_m l + \sigma_m (L-l)
\]
Putting \( l \) into equation (19) and then superseding \( \tau_s(l) \) in (24) by the received expression, we obviously get:

\[
\sigma_K = \tau_s(0)e^{-\frac{a-l}{m}}B - b + \sigma_s m_l + \sigma_s m_0 (L - l)
\]

(25)

where \( m_l \) and \( m_0 \) are the values of the coefficients of volume compressibility of the surrounding soil in the range of length \( l \) and the underlying soil, respectively. \( S_K = \sigma_K \) is the settlement gained owing to the punching of the barrette’s heel into the underlying soil layer.

In the similar way of analyzing above, the expression of compressive stress \( \sigma_N \) at the head of the barrette appears as follows:

\[
\sigma_N = \sigma_k(l) = \tau_s(0)I_1 \frac{e^{-\frac{a-l}{m}}}{\alpha} + \tau_s(0)I_1 \frac{l}{\alpha} + \sigma_K
\]

(26)

Considering the equality of the settlement of the barrette and the underlying soil layer at the level \( z=0 \), we decidedly procure the following expressions:

\[
S_{\text{side}}(0) = S_R
\]

(27)

\[
\sigma_K = \tau_s(0)e^{-\frac{a-0}{m}}B - b + \sigma_s m_0 (L - l)
\]

(28)

\[
\sigma_K = \tau_s(0)B - b + \sigma_s m_0 (L - l)
\]

(29)

Juxtaposing expression (29) with (25) above, we definitely get a new equation:

\[
\tau_s(0)e^{-\frac{a-l}{m}}B - b + \sigma_s m_l + \sigma_s m_0 (L - l) = \tau_s(0)B - b + \sigma_s m_0 (L - l)
\]

(30)

\[
\tau_s(0) = \sigma_s \frac{3G}{B - b} \frac{m_1}{1 - e^{-\frac{a-l}{m}}}
\]

(31)

Substituting the received expression from (31) for \( \tau_s(0) \) in (29) and (26), respectively, we attain:

\[
\sigma_K = \sigma_s \frac{m_1}{1 - e^{-\frac{a-l}{m}}} + \sigma_s m_0 (L - l)
\]

(32)

\[
\sigma_N = \sigma_k(l) = \sigma_k \frac{3G I_1}{B - b} \frac{m_1}{1 - e^{-\frac{a-l}{m}}} \frac{e^{-\frac{a-l}{m}}}{\alpha} + \sigma_s \frac{3G}{B - b} \frac{m_1}{1 - e^{-\frac{a-l}{m}}} \frac{l}{\alpha} + \sigma_K
\]

(33)

and finally

\[
\sigma_s = \sigma_K \frac{K(1 - e^{-\frac{a-l}{m}})}{m_1 + m_0 (L - l)(1 - e^{-\frac{a-l}{m}})}
\]

(34)

\[
\sigma_N = \sigma_s \frac{3G I_1 m_1}{B - b} \frac{1}{\alpha} + \sigma_K
\]

(35)

Continuing transformation by replacing \( \sigma_s \) in (35) from (34), the dependency of \( \sigma_N \) on \( \sigma_s \) turns out to be as follows:

\[
\sigma_N = \sigma_K \left[ \frac{K(1 - e^{-\frac{a-l}{m}})}{m_1 + m_0 (L - l)(1 - e^{-\frac{a-l}{m}})} \frac{3G I_1 m_1}{B - b} \frac{1}{\alpha} + 1 \right]
\]

(36)

In order to determine the punching stress \( \sigma_K \) underneath the barrette’s heel, we put \( \sigma_N \) and \( \sigma_s \) from expressions (36) and (34), respectively, into (1)

\[
p = \sigma_K \left[ \frac{K(1 - e^{-\frac{a-l}{m}})}{m_1 + m_0 (L - l)(1 - e^{-\frac{a-l}{m}})} \frac{3G I_1 m_1}{B - b} \frac{1}{\alpha} + 1 \right] \omega + \sigma_K \frac{K(1 - e^{-\frac{a-l}{m}})(1 - \omega)}{m_1 + m_0 (L - l)(1 - e^{-\frac{a-l}{m}})}
\]

(37)
\[ \sigma_R = \frac{p}{I_2} \]  

in which \( I_2 = \frac{K(1-e^{-a \alpha})}{m_1 + m_b(L-l)(1-e^{-a \alpha})} \left[ \frac{3G}{B-b} + \frac{\omega + 1 - \omega}{\alpha} \right] + \omega \) \tag{39}

Placing \( \sigma_R \) from (38) in (34) and then putting the received expression into (31), we conspicuously achieve the final expressions for calculating \( \sigma_z \) and \( \tau_z(0) \):

\[ \sigma_z = \frac{p}{I_2} \frac{K(1-e^{-a \alpha})}{m_1 + m_b(L-l)(1-e^{-a \alpha})} \]

\[ \tau_z(0) = \frac{p}{I_2} \frac{3G K m_l}{(B-b) m_1 + m_b(L-l)(1-e^{-a \alpha})} \alpha \]  \tag{41}

The entire settlement of the barrette-raft-soil massif is formed of two terms

\[ S_{\text{soil massif}} = S_R + S_{\text{underlying soil layer}} \]  \tag{42}

\[ S_{\text{soil massif}} = \frac{\sigma_R (1 - v_0) a K w}{G_0} + p m_0 (L-l) \]  \tag{43}

The dependency of compressive stress \( \sigma_z(z) \) in the body of the barrette on the depth \( z \) (m) can be shown as follows:

\[ \sigma_z(z) = \tau_z(0) I_1 \frac{e^{-a \alpha}}{\alpha} + \tau_z(0) \frac{I_1}{\alpha} + \sigma_R = \frac{p}{I_2} \frac{3G K m_l}{m_1 + m_b(L-l)(1-e^{-a \alpha})} \alpha I_1 \frac{1-e^{-a \alpha}}{1-e^{-a \alpha}} + \frac{p}{I_2} \]  \tag{44}

The dependency of shear stress \( \tau_z(z) \) on the surfaces of the barrette on the depth \( z \) (m) within the embedded length \( l \) (m):

\[ \tau_z(z) = \tau_z(0) \exp(-a \alpha z) = \frac{p}{I_2} \frac{3G K m_l}{m_1 + m_b(L-l)(1-e^{-a \alpha})} e^{-a \alpha z} \]  \tag{45}

The distribution of the compressive stress \( \sigma_y(z) \) under the raft plate is written as follows:

\[ \sigma_y(z) = \frac{p}{1-\omega} - \sigma_y(0) \frac{\omega}{1-\omega} \]  \tag{46}

Considering example \( p = 500 \text{ kN}; a = 0.75 \text{ m}; b = 1.5 \text{ m}; A = 3.75 \text{ m}; B = 4.5 \text{ m}; l = 30 \text{ m}; L = 40 \text{ m}; a = b = 45^\circ; K_1 = 0.722; G_1 = 50000 \text{ kPa}; G_0 = 30000 \text{ kPa}; w = 1.22; v_1 = 0.35; v_0 = 0.3; m_k = 4.615 \times 10^{-3} \text{ kPa}^{-1}; m_b = 9.905 \times 10^{-6} \text{ kPa}^{-1}; \gamma_l = 19 \text{ kN/m}^3; \gamma_0 = 21.1 \text{ kN/m}^3; \varphi_0 = 19^\circ; c_0 = 60 \text{ kPa}; \alpha = 0.1. \)

Then we get: \( \sigma_R = 2843.13 \text{ kPa}; \sigma_y = 4141 \text{ kPa}; \sigma_N = 7111.57 \text{ kPa}; \sigma_s = 27.74 \text{ kPa}; S_R = 0.044 \text{ m}; \tau_z(0) = \tau_y(0) = 206 \text{ kPa}; S_{\text{soil cell}} = 0.091 \text{ m} \)

The result achieved by the PLAXIS 3D in linear formulation with the uniformly distributed load \( p = 500 \text{ kPa} \) is represented as follows.
Figure 2. Iso-fields of the entire settlement of the barrette-raft-soil massif.

Figure 3. Iso-fields of the vertical stress of the incompressible barrette.

Figure 4. Shear stress on a lateral surface of the incompressible barrette.
The average values of the stress under the barrette’ heel, at the head of the barrette, the settlement of the soil massif and the maximum shear stress procured by the numerical method are performed as follows:

\[ \sigma_{R}^{\text{plaxis}} = 2700.2 \text{kPa}; \quad \sigma_{N}^{\text{plaxis}} = 7265 \text{kPa}; \quad S_{\text{soil cell}}^{\text{plaxis}} = 0.096 \text{ m} \quad \tau_{a}^{\text{plaxis}}(0) = 195.6 \text{ kPa}. \]

The difference of the values shown above between the analytical method and numerical method varies in the range from 2% to 5.3%.

The dependencies curve of \( \sigma_{b} \), \( \sigma_{s} \), \( \tau_{a} \), and \( \tau_{b} \) on \( z \) (m) corresponding to the analytical solution.

**Figure. 5.** Vertical stress \( \sigma_{b} \) in the body of incompressible barrette on depth \( z \) (m).

**Figure. 6.** Vertical compressive stress \( \sigma_{s} \) in surrounding-soil massif on depth \( z \) (m).

**Figure. 7.** Shear stress \( \tau_{a} \) and \( \tau_{b} \) on the lateral surfaces 2a and 2b of the incompressible barrette on depth \( z \) (m).

4. Conclusions:

- The geometric parameters of the barrette-raft-soil massif, as well as the mechanical and physical properties of the surrounding, and the underlying soil layer exercise a substantial influence in the stress-strain-state (SSS) of them.
- It is figured out that the distributed load transferred from the raft to the lateral surfaces and the heel is proportional to their geometric and mechanical parameters and this proportion is variable
with depth. The proportion of the stress at the barrette heel and the barrette head accounts for 37%.

- The juxtaposition of the result attained by using analytical and numerical methods is clearly represented (the divergence is about 6%), and the solution to the problems is able to be suggested when carrying out a calculation of the entire settlement as well as bearing capacity of the barrette.

- In order to adjust the proportion mentioned above and mobilize dramatically the bearing capacity soil layer under the barrette heel, the dimensional parameters of the soil massif and barrette need to be chosen rationally.

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