Fatigue damage evolution and finite element simulation of the samples under different notch morphology

Zhiqiang Wang, Xiaoguang Huang

College of Pipeline and Civil Engineering, China University of Petroleum (East China), Qingdao, Shandong, 266580, China

*E-mail: huangxg@upc.edu.cn

Abstract. Based on the theory of continuous damage mechanics and the principle of irreversible thermodynamics, the damage evolution model of low cycle fatigue is investigated. By programming the damage evolution model as UMAT subroutine in ABAQUS, the fatigue damage and crack initiation life of notch materials under specific loads are simulated, and the crack initiation location is determined. Furthermore, the effects of root radius, depth and opening angle on crack initiation life are investigated. The results show that the crack initiation occurs easily in the notch root where the damage is the most serious and the plastic strain accumulates fastest under cyclic loading. With the increase of notch depth and root radius, the crack initiation life decreases. For V-notched specimens, when the notch depth remains unchanged, the crack initiation life will decrease with the increase of the opening angle.

1. Introduction

Fatigue is a popular damage and failure mode of metal materials under alternating load. With the widespread application of metal materials in automobiles, machinery, aerospace, etc., fatigue failure has gradually become one of the main failure modes of such components. Generally, fatigue failure can be divided into three stages: crack initiation, crack propagation and failure fracture. For those materials without initial damage, the crack initiation life usually accounts for more than 80% of the total life. Hence, it is of great significance to study the crack initiation life of metal materials.

Damage mechanics is regarded as one of the most effective methods to study of fatigue damage evolution and predict fatigue life [1]-[5]. To meet the requirements of engineering design, fatigue components inevitably have different notch morphology. When under external loads, cracks often initiates at the structural notch on account of local stress concentration [6]. Therefore, it is extremely important to design the notch shape to improve the fatigue life of the notch members. Xie [7] used the modified Tanaka-Mura model to reveal the effect of gradient hardening thickness on the initiation location and lifetime of fatigue cracks. Jin [8] et al. found that the crack initiation lives of notched Mod.9cr-1Mo ferrite steel specimens were greatly dependent on the notch radius. Xing [9] et al. predicted the crack initiation life of notched plate specimens under high-low cycle fatigue load. However, there is little attention has been paid to the impact of notch morphology on crack initiation life. In this paper, the low-cycle fatigue damage evolution equation derived from the theory of damage mechanics was established and the influence of notch morphology on crack initiation life was analyzed.

2. Fatigue damage model

According to the classical damage theory, damage is usually defined as a phenomenon of deterioration of the internal properties of materials caused by the generation of micro-cracks and micro-cavities under
external loads. According to the definition of Kachanov[10] and Rabotnov[11], the damage variable for uniaxial samples can be expressed as follows:

$$D = 1 - \frac{S_D}{S}$$  \hspace{1cm} (1)

where $S_D$ and $S$ represent the effective bearing area of the damaged material, and the cross-sectional area of the material under no damage, respectively.

Based on the theory of continuous damage mechanics, the fatigue damage evolution equation can be described by dissipation potential. The damage dynamics equation can be written as

$$\dot{D} = -\frac{\partial \phi}{\partial Y}$$  \hspace{1cm} (2)

where $\phi$ and $Y$ denote dissipative potential function and strain energy. Also, the expression of dissipative potential $\phi$ is

$$\phi = \frac{Y^2}{2S_0} \frac{\dot{p}}{(1-D)^{\alpha_0}}$$  \hspace{1cm} (3)

where $S_0$ and $\alpha_0$ are material constants; $\dot{p}$ represents the plastic strain rate. The strain energy $Y$ is expressed as follows

$$Y = \frac{\sigma_{eq}^2 R_v}{2E(1-D)^2}$$  \hspace{1cm} (4)

where $\sigma_{eq}$ and $E$ denote equivalent stress and Young's modulus, respectively. $R_v$ is the parameter describing triaxial stress effect, and its expression is as follows

$$R_v = \frac{2}{3}(1+\nu)+3(1+2\nu)\left(\frac{\sigma_{II}}{\sigma_{eq}}\right)^2$$  \hspace{1cm} (5)

where $\nu$ is Poisson’s ratio, $\sigma_{II}$ represents the average stress and $R_v=1$ in the uniaxial state. By substituting equations (3)-(4) into equation (2), the fatigue damage evolution equation can be obtained

$$\dot{D} = \frac{\sigma_{eq}^2 \dot{p}}{2ES_0 (1-D)^{\alpha_0+2}} R_v$$  \hspace{1cm} (6)

Assuming that the stress-strain relationship of the material under cyclic loading can be characterized by the Ramberg-Osgood cyclic constitutive equation, so the plastic strain of each cycle is

$$\Delta \varepsilon_p = \left(\frac{\Delta \sigma}{K}\right)^\frac{1}{n}$$  \hspace{1cm} (7)

where $\Delta \varepsilon_p$ and $\Delta \sigma$ represent cyclic strain amplitude and cyclic stress amplitude respectively, $K$ is cyclic strength coefficient and $n$ denotes cyclic strain hardening index. By introducing the concept of damage, the plastic strain per cycle should be rewritten as

$$\Delta \varepsilon_p = \left[\frac{\Delta \sigma}{K(1-D)}\right]^\frac{1}{n}$$  \hspace{1cm} (8)

When the material is in a proportional loading or uniaxial stress, equation (8) can be expressed as

$$\frac{\Delta \sigma_{eq}}{1-D} = K \cdot \Delta \sigma^n$$  \hspace{1cm} (9)

Then, substitute equation (9) into equation (6) to obtain the low-cycle fatigue damage evolution equation

$$\dot{D} = \frac{K^2 \Delta \sigma^n R_v}{2ES_0 (1-D)^{\alpha_0}} \dot{p}$$  \hspace{1cm} (10)
Hence, the expression of fatigue damage of materials in a single cycle is as follows:

\[
\frac{dD}{dN} = \frac{K^2 \Delta P^{2n+1}}{2ES_v (1-D)\gamma (2n+1)} R_v = \frac{\Delta P^{\gamma+1}}{\Omega(\gamma+1)(1-D)\gamma n} R_v
\]

(11)

where \(\Omega\) and \(\gamma\) are material constants, \(\Omega = \frac{2ES_v}{K^2}\), \(\gamma = 2n\) [12].

In this paper, the crack initiation life of notched specimen is studied by coding UMAT subroutine of the fatigue damage evolution equation derived above. Since there is no identical engineering standard for critical crack length for crack initiation, micro-crack length up to 0.1-0.2mm is usually referred to as the size of crack initiation [14]. Crack initiation is considered when the failure element size reaches a critical crack initiation size.

3. Numerical damage simulation

3.1. Notch design of specimen

Figure 1. Geometry parameters of notch

Figure 2. FEM model

The present study concentrates on three main notch morphology, namely U-notched, C-notched, and V-notched. \(H\), \(\theta\), and \(R\) are the three main parameters describing the notch type, which respectively mean the notch depth, the opening angle, and the root radius, as shown in Figure 1.

3.2. Material parameters and finite element model

P92 steel plate specimens with length of 40mm, width of 20mm thickness of 5mm and a bilateral notch in the middle were selected to simulate the impact of notch on fatigue crack nucleation life. Material parameters are shown in Table 1 [13]. Taking the specimen with a semicircular notch of \(R=3\)mm (i.e. C-notched) as an example, a 1/8 model as shown in Figure 2 was established, and symmetric constraints were applied in the directions of model \(x\), \(y\), and \(z\). Also, the mesh type is C3D8R, and its size near the notch is 0.02mm to refine the mesh. Moreover, serrated cyclic loading with stress amplitude 131 MPa and frequency 0.5 Hz is applied to the end of the sample, and the stress ratio \(\xi=-1\).

Table 1. P92 steel material parameters.

|       | \(E\) (MPa) | \(v\)  | \(K\)  | \(n\)  | \(\Omega\) | \(\gamma\) | \(\alpha_0\) |
|-------|-------------|-------|--------|--------|------------|------------|------------|
|       | 125000      | 0.3   | 234    | 0.119  | 12.57      | 0.273      | 7.56       |

3.3. Numerical simulation results

Based on the finite element models of different notched specimens by using the parameter values in Figure 1, the relationship between the number of cycles and the damage of specimens under specific loads was simulated. Figure 3 shows the variation of damage with cycle number under the maximum load of 65.5MPa for materials with three semicircular notches with a radius of 3mm, 4mm, and 5mm respectively. The damage accumulation of three materials is slow at the primary stage, whereas, as the number of cycles increases, the damage accumulation also accelerates until the failure of the unit at the
critical damage stage. And the number of cycles required for failure of the element decreases with the increase of notch radius $R$. Figure 4 illustrates that the root of the notch has the greatest damage.

Besides, the distribution of damage distribution, equivalent stress and cyclic plastic strain in the $N$, $Z$ and $X$ directions of the semicircular notch specimen with $R=3\text{mm}$ as shown in Figure 5 were analyzed respectively. As can be seen from the results in Figures 5 and 6, the variation trend of the three variables is similar in all directions, and the maximum value occurs when the displacement is 0, i.e. the central position of the notch root. It indicates that the damage of the notch root element is the largest during the process of cyclic loading, and crack initiation will occur at the notch root first.

Furthermore, three types of U-notched, i.e. $H=2\text{mm}$, $H=3\text{mm}$, and $H=5\text{mm}$ were used to investigate the effect of notch depth $H$ on crack initiation life in this paper. When plotting the variation in the damage with the cycle number as shown in Figure 8, it can be clearly noted that the specimen with a notch depth of $H=2\text{mm}$ experienced almost no damage after 2000 cycles, and the notched specimen gets to critical damage value at about 1600 cycles when $H=2\text{mm}$, moreover, in the case of $H=5\text{mm}$, the element failures after 130 cycles. For the notch depth $H=3\text{mm}$, root radius $R=0$, and opening angles of $45^\circ$, $60^\circ$ and $90^\circ$ respectively, the research results of damage variation with cycle number under the same load are shown in Figure 8. Comparative analysis of the above plots shows that among the three mentioned notch morphology, the damage of the notch with an opening angle of $45^\circ$ shows the least variability, followed by the notch with an opening angle of $60^\circ$. The damage changed the most rapidly with the number of cycles at the opening angle of $90^\circ$. 
4. Conclusion

Based on the theory of continuous damage mechanics and the principle of irreversible thermodynamics, the damage evolution model of low cycle fatigue is investigated. By programming the damage evolution model as UMAT subroutine and coupling it to ABAQUS, the fatigue damage and crack initiation life of notch materials under specific loads are simulated, and the following conclusions are drawn:

a) The damage accumulates slowly at the initial stage, but the damage accumulates rapidly with the increase of the number of cycles until the cumulative damage reaches a critical value and crack initiation occurs.

b) The variation trend of notch damage, equivalent stress and cyclic plastic strain is similar along the direction of notch, and crack initiation is most easily induced at the notch root where the element damage is greatest.

c) With the increase of the radius of the notch root, the crack initiation life is longer. Also, when other parameters remain unchanged and the notch depth increases, the crack initiation life declines. Besides, for V-notched with a root radius of 0, the crack initiation life goes down as the opening angle greaten.

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