Influence of length on the noise delayed switching of long Josephson junctions

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The transient dynamics of long overlap Josephson junctions in the frame of the sine-Gordon model with a white noise source is investigated. The effect of noise delayed decay is observed for the case of overdamped sine-Gordon equation. It is shown that this noise induced effect, in the range of small noise intensities, vanishes for junctions lengths greater than several Josephson penetration length.

Keywords: long Josephson junctions; thermal fluctuations; noise delayed decay

I. INTRODUCTION

Josephson junctions are physical systems with nonlinear dynamics, which are interesting to investigate both from experimental and theoretical points of view. This is also in view of numerous applications of superconductive devices based on Josephson junctions, such as RSFQ devices, qubits, SQUIDs, etc. [Barone & Paternò, 1982; Likharev, 1986]. There are, in fact, a lot of open problems in Josephson junction’s dynamics, due to its nonlinear character. For some devices, as the RSFQ, minimization of the switching time is required for better performance [Pankratov & Spagnolo, 2004]. From this point of view, it is interesting to investigate the influence of thermal noise on the statistical properties of distributed Josephson junctions. It is also necessary to note that, currently, all Josephson junctions are manufactured with the use of optical and electron-beam lithography [Dorojevets, 2002; Makhlin et al., 2001], and can always be considered as distributed. Moreover macroscopic quantum phenomena in long Josephson junctions have attracted a lot of experimental and theoretical work recently [Weides et al., 2006; Mertens et al., 2006; Alﬁmov & Popkov, 2006; Kim et al., 2006; Fistul et al., 2003]. These junctions are characterized by one or more dimensions longer than the Josephson penetration length or depth [Barone & Paternò, 1982].

It was shown in different physical systems that thermal fluctuations can considerably increase the decay time of unstable states [Agudov & Malakhov, 1995; Malakhov & Pankratov, 1996; Agudov & Malakhov, 1999] and the lifetime of metastable states [Man tegna & Spagnolo, 1996; Agudov & Spagnolo, 2001; Spagnolo et al., 2004], producing a nonmonotonic behavior of these quantities as a function of the noise intensity. These are the effects of noise delayed decay (NDD) of unstable states and noise enhanced stability (NES) of metastable states. Both noise induced effects are due to the nonlinearity of the potential profile and are enhanced by the inverse probability current. The NDD effect, in particular, consists in the increase of the mean switching time (MST) of a Josephson junction due to the influence of noise. The main interest to analyze here long Josephson
junctons (LJJ) is the existence of noise delayed decay (NDD) effect for the restricted range of parameters, which is important for practical applications.

II. THE MODEL

In the frame of the resistive McCumber-Stewart model [Barone & Paternò, 1982; Likharev, 1986] the phase difference of the order parameter $\varphi(x,t)$ of a long Josephson junction of the overlap geometry (see Fig. 1) is described by the sine-Gordon equation

$$\beta \frac{\partial^2 \varphi}{\partial t^2} + \frac{\partial \varphi}{\partial t} - \frac{\partial^2 \varphi}{\partial x^2} = i \sin(\varphi) + i_f(x,t)$$

with the following boundary conditions

$$\frac{\partial \varphi(0,t)}{\partial x} = \frac{\partial \varphi(L,t)}{\partial x} = \Gamma.$$  

Here the time and the space are normalized to the inverse characteristic frequency $\omega_c^{-1}$ and the Josephson penetration length $\lambda_J$, respectively. The penetration length gives a measure of the distance in which dc Josephson currents are confined at the edges of the junction. $\beta = 1/\alpha^2$ is the McCumber-Stewart parameter, $\alpha = \omega_p/\omega_c$ is the damping, with $C$ the junction capacitance and $\omega_p = (2eI_c/\hbar C)^{1/2}$ the plasma frequency corresponding to the critical current $I_c$. The bias current density $i$ is normalized to the critical current density of the junction, $i_f(x,t)$ is the fluctuational current density, $\Gamma$ is the normalized magnetic field, $L = l/\lambda_J$ is the dimensionless length of the junction. Further, we will only consider the cases when $i > 1$ and $\Gamma = 0$, so in the potential profile there are no metastable states at all. Also let us consider only the case of homogeneous bias current distribution, when $i$ is constant along the junction. In the case where the fluctuations are treated as white Gaussian noise with zero mean, and the critical current density is fixed, its correlation function is

$$\langle i_f(x,t)i_f(x',t') \rangle = 2\gamma \delta(x-x')\delta(t-t'),$$

where

$$\gamma = I_T/(J_c \lambda_J)$$

is the dimensionless noise intensity, $J_c$ is the critical current density of the junction, $I_T = 2ekT/h$ is the thermal current, $e$ is the electron charge, $h$ the Planck constant, $k$ is the Boltzmann constant and $T$ is the temperature. It is possible to obtain the expressions (3) and (4) through the renormalization of the general formula for the noise intensity, derived for the fixed total critical current $I_c$ [Castellano et al., 1996; Pankratov, 2002].

Initially, the whole phase "string" $\varphi(x,0)$ is located at the position $\varphi_0$, so there are no vortices in the junction. The dynamics of such a phase "string" is similar to that of a real long string falling down on a tilted washboard potential addressing the general problem of the diffusion of an elastic string on a tilted periodic substrate [Cattuto & Marchesoni, 1997]. For our case $i > 1$, such initial phase is considered to be located in the inflection point of the potential profile $\varphi_0 = \pi/2$. It is possible to prepare such a state by a fast switching from the starting superconductive state, corresponding to a metastable state, to the resistive one, which becomes then the initial configuration of the system. Due to the unstable character of the initial position (see
number of realizations.

III. NOISE DELAYED SWITCHING

First, it is interesting to check the limiting transition of a long junction to a point one for small junction lengths $L \ll 1$. For a point junction, in the case of large damping $\beta \ll 1$, the MST was found analytically [Malakhov & Pankratov, 1996] for an arbitrary value of the noise intensity. That formula, however, was obtained for the case of constant critical current $I_c$, and the noise intensity in that case was: $\gamma_s = I_T/I_c$, where $I_c = J_c \lambda J L$. This means that we have to scale the noise intensity as $\gamma_s = \gamma/L$, with $\gamma$ given by Eq. (4). We get, therefore, the following closed expression for the MST in a long Josephson junction with constant critical current density

\[
\tau = \frac{L}{\gamma} \left\{ \int_{\varphi_0}^{\varphi_2} e^{-f(\varphi)} L/\gamma d\varphi \right\}.
\]

with \(f(\varphi) = \cos \varphi + i \varphi\).

In Eq. (5) \(\varphi_0\) is the coordinate of the initial delta-shaped distribution, and \(\varphi_{1,2}\) are the boundaries of the interval, delimiting the potential well or metastable state of the superconductive state, before the fast switching. To prevent misunderstanding with the use of such a renormalization procedure, one should consider the general expression for the noise intensity for fixed critical current [Castellano et al., 1996; Pankratov, 2002] and substitute the required bias current density in it. In Figs. 3, 4 the behavior of MST as a function of the dimensionless noise intensity $\gamma$ and the junction length $L$, respectively, is shown for two values of the bias current. The agreement between the theoretical results, obtained from Eq. (5), and the numerical simulations of Eq. (1), for a long junction, is very good not only in the limiting case $L \ll 1$, but even up to $L \sim 1$. In
Comparison between the theoretical results of Eq. (5) and the numerical simulations of Eq. (1), for two values of the bias current, namely $i = 1.2, 1.5$. Here the dimensionless junction length is $L = 0.1$.

Figs. 5, 6 the semilog plot of the MST versus the noise intensity $\gamma$, for different junction lengths, is shown. The NDD effect is present for junction lengths up to $L \gtrsim 5$, while for greater lengths it completely vanishes. This peculiarity is very important for RSFQ devices. This disappearance of NDD is due to the effective decrease of the noise intensity in comparison with the point junction.

For the same temperature level, the effective noise intensity $\gamma_s = \gamma/L$ for the long Josephson junction will be smaller by a factor $1/L$ in comparison with the point junction. So, for LJJ with rather large lengths ($L > 5$) the noise intensity will get out of the area of the NDD effect.

For example, from our simulations, we find that for great values of the noise intensity $\gamma \sim 100$, the NDD will exists even for LJJ with lengths $L \sim 10$, but such a range of $\gamma$ is not interesting from practical point of view. In other words, for large lengths of the distributed Josephson junctions, the random force $F_T \sim \sqrt{\gamma}$ becomes negli-
bly small in comparison with the deterministic force, caused by the potential profile
\[ u(\varphi) = 1 - \cos(\varphi) - i \varphi. \]

The standard deviation (SD) \( \sigma \) of the switching time is defined as
\[
\sigma = \sqrt{\langle t^2 \rangle - \langle t \rangle^2} = \int_0^\infty t^2 w(t) dt.
\]

For small noise intensities we can obtain an expression of \( \sigma(\varphi_0) \) for a long Josephson junction, by using the noise intensity renormalization, as before in Eq. (5), and the asymptotic expression derived for a point Josephson junction [Pankratov & Spagnolo, 2004]
\[
\sigma(\varphi_0) = \frac{1}{\omega_c} \sqrt{2\gamma/L} \left[ F(\varphi_0) + f_3(\varphi_0) \right] + \ldots (7)
\]
\[
F(\varphi_0) = f_1(\varphi_2) f_2(\varphi_0) - 2f_1(\varphi_2) f_2(\varphi_0) + f_1(\varphi_0) f_2(\varphi_0) + \frac{f_1(\varphi_0) - f_1(\varphi_2)}{\sqrt{1 - \sin(\varphi_0)^2}}.
\]
\[
f_1(x) = \frac{2}{\sqrt{4 - x^2}} \arctan \left( \frac{i \tan(x/2)}{\sqrt{4 - 1}} \right),
\]
\[
f_2(x) = \frac{1}{\sqrt{2} \sin(x)} \frac{1}{(i - \sin(x))^2},
\]
\[
f_3(\varphi_0) = \int_{\varphi_0}^{\varphi_2} \left[ \frac{\cos(x) f_1(x)}{(\sin(x))^3} - \frac{3}{2(\sin(x) - i)^3} \right] dx.
\]

In the following Fig. 7 the SD of the switching time versus the junction length \( L \) is shown, for two values of the bias current, namely \( i = 1.2, 1.5 \). The theoretical results of Eq. (7) show a quite good agreement with the numerical simulations of Eq. (1). Here the noise intensity is \( \gamma = 0.01 \).

In Fig. 8 the SD of the switching time versus the dimensionless noise intensity \( \gamma \) for four junction lengths, namely \( L = 0.1, 0.5, 2, 10 \). Here the bias current is \( i = 1.2 \).

In Fig. 9 the SD of the switching time versus the dimensionless noise intensity \( \gamma \) for the same junction lengths of Fig. 8. Here the bias current is \( i = 1.5 \).

The numerical simulations of the SD versus the noise intensity from Eq. (1), for two values of the bias current and for different junction lengths, are shown. All the behaviors of Figs. 7, 8, 9 are evaluated in a parameter range which is characteristic for RSFQ devices. The SD decreases with the length of the junction. This is because the dynamics controlling the switching event goes from a noise-induced regime, at very short junction lengths, to a deterministic regime, caused by the decrease of the effective noise intensity for very long junctions. The noisy regime is well visible in Figs. 8, 9 for the case \( L = 0.1 \), for which the \( \sigma \) increases with the noise in-
The deterministic regime, dominated by the potential profile, produces the asymptotic constant values of the SD (the curves for $L = 0.5, 2, 10$ in Figs. 8, 9). We note that for a higher bias current (Fig. ) the SD reaches lower asymptotic values, as we expect.

IV. CONCLUSIONS

In this work a study of the transient dynamics of long Josephson junctions, with bias currents greater than the critical current, is presented. We have found that, for the case of constant critical current density, the mean switching time has a maximum for small noise intensities and junction lengths. This is the noise delayed decay effect. This effect disappears for junctions with dimensionless lengths $L \gtrsim 5$, which can be important for the design of RSFQ devices based on Josephson junctions. The disappearance of NDD effect is explained by the decrease of the effective noise intensity with the length $L$ of the junction, by considering the scaling factor $1/L$. As a consequence the system gets out from the range of $\gamma$ values suitable to observe the noise delayed decay effect. Finally we observe for the MST and its SD, in the range of small noise intensities, a good agreement between the theoretical behaviors (Eqs. 5 and 7) and those obtained from numerical simulations of Eq. (1).

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