Research Article

Optimization Analysis of Automotive Asymmetric Magnetic Pole Permanent Magnet Motor by Taguchi Method

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In order to improve the air-gap flux density of the permanent magnet synchronous motor and reduce the cogging torque, a novel structure with asymmetric magnetic poles for automobile was proposed. Based on the characteristics of the parallel magnetic circuit, the magnetic flux path diagram is established. And the equivalent magnetic circuit model is established by the equivalent magnetic circuit method. The Taguchi method is used to be a multiobjective optimization algorithm. The total harmonic distortion of the air-gap flux density is the first optimization goal. The second and third optimization goals are the cogging torque and the average of output torque, respectively. And the torque ripple is a constraint condition. The optimized parameter combination is obtained by the Taguchi method. Finite element simulation analysis and prototype test are carried out for the optimized motor structure. The results show that the total harmonic distortion of air-gap flux density is reduced by 36.7% comparing with the initial structure. The cogging torque is reduced by 26.0%. And the average output torque is increased by 4.8%.

1. Introduction

Electric automobile has been widely concerned around the world because of the superiority of energy saving and environmental protection. As one of the core technologies of electric automobiles, the driving motor system has become the key component. High power density, high efficiency, and wide speed range have become current research directions. At present, driving motors of electric automotive include induction motor, direct current motor, and permanent magnet synchronous motor (PMSM). PMSM has become the most promising drive motor due to its advantages of high power density, high efficiency, and simple structure [1, 2].

According to the relative position of the permanent magnet and the rotor core, PMSM can be divided into surface mount type and interior type. The interior type can be divided into three types: radial type, tangential type, and hybrid type. Their magnetization directions of the permanent magnets are different. At present, the interior PMSM is used widely in industrial engineering. Many automotive companies such as Daimler, Toyota, and BYD have widely applied interior PMSM to their electric automobile [3, 4].

However, the air-gap flux density waveform of PMSM is similar to a square wave. It has many higher harmonics. And high cogging torque is not conducive to the output performance [5].

In view of above disadvantages, domestic and foreign scholars carried out studies on the influence of output performance and different magnetic pole shapes. It is determined that the air-gap flux density and the cogging torque are greatly affected by the shape of the magnetic poles. Appropriate magnetic pole shape can improve the air-gap flux density and reduce cogging torque [6, 7]. Toyota adopted an interior PMSM structure of V-type and tangential type on Prius hybrid automobile. Oak Ridge National Laboratory proposed a U-type motor structure. The air-gap flux density is improved, and the cogging torque is reduced. But the amount of permanent magnets is increased [8]. Biasing the magnetic poles and changing the pole arc coefficient can also improve the motor performance [9–11]. In [12], the outer edge of the magnet of the surface mount PMSM was cut to improve air-gap magnetic field. But it makes the manufactural process more difficult. And it can cause sudden changes in magnetic field. For optimization methods, domestic scholars proposed
2. Motor Structure and Magnetic Circuit Analysis

2.1. Motor Structure. This paper proposes PMSM structure asymmetric magnetic poles. Its three-dimensional structure is shown in Figure 1. It is an interior-type PMSM with 8 poles and 12 slots. Its winding is concentrated winding. Each of its magnetic poles is a single-layer arrangement. The shape of magnetic poles is similar with "λ." Each magnetic pole is combined with two pieces of rare earth permanent magnet materials. The thickness and length of them are different. The relevant structure parameters are shown in Table 1.

2.2. Motor Effective Flux Analysis. This paper takes a traditional V-type structure as comparing structure. Each pole effective magnetic flux of two motor structures is analyzed. Figure 2 shows the comparison diagrams of the flux paths. It can be seen that the effective flux of the traditional V-type structure is a single path. It is provided by two adjacent permanent magnets. The effective flux of asymmetric magnetic pole structure consists of two paths in parallel. A path is provided by a single long permanent magnet. And another path is provided by two adjacent permanent magnets.

Schematic diagrams of flux paths have been shown above. The equivalent magnetic circuit models of two motor structures are established, respectively, by the equivalent magnetic circuit method. And they are shown in Figure 3.

In Figure 3, \( F_1 \) and \( F_2 \) are the magnetomotive force. They are, respectively, provided by the long permanent magnet and the short permanent magnet. \( F_d \) is the direct axis component of the armature reaction. \( G_{ml} \) and \( G_{mS} \) are the internal equivalent permeance of the long permanent magnet and the short permanent magnet, respectively. \( 2G_{ms2} \) is the permeance between the long permanent magnet and the rotor core. \( 2G_{ms2} \) is the permeance between the short permanent magnet and the rotor core. \( 2G_{rs} \) is the leakage permeance of long permanent magnet. And \( 2G_{ls} \) is the leakage permeance of short permanent magnet. \( G_T \) is the air-gap permeance. \( G_I \) is the permeance of the stator teeth. \( G_{r2} \) is the permeance of the rotor core between long permanent magnet and the air gap. \( G_{r1} \) is the permeance of the rotor core between the short permanent magnet and the air gap. \( G_{rp} \) is the permeance of the rotor core between the long and short permanent magnets. \( \Phi_m \) is the total magnetic flux in a magnetic circuit. It is provided by the long and short permanent magnet. \( \Phi_{m1} \) is the effective magnetic flux. \( \Phi_{r1} \) and \( \Phi_{r2} \) are the leakage flux of long permanent magnet and short permanent magnet, respectively.

In Figure 3(b), \( \Phi_{m1} \) is an extra magnetic flux in the equivalent magnetic circuit model. And it only appears in an asymmetric magnetic pole structure. \( G_{r2} \) and \( G_{r1} \) are connected in parallel. \( G' \) is used to represent the total permeance of \( G_{r2} \), \( G_{rp} \), \( 2G_{ms2} \), \( G_{ns} \), and \( 2G_{ls} \).

According to the superposition principle, \( G' \) meets the following conditions:

\[
\frac{1}{G'} = \frac{1}{G_{r2}} + \frac{1}{G_{rp}} + \frac{(2/G_{ls}) \times ((1/G_{ms}) + (2/G_{ms2}))}{(2/G_{ls}) + (1/G_{ms}) + (2/G_{ms2})}. \quad (1)
\]
$G_e$ is used to represent the total permeance of $G_{r1}$ and $G'$.
It meets the following conditions:

$$\frac{1}{G_e} < \min \left\{ \frac{1}{G_{r1}}, \frac{1}{G'} \right\}.$$  (2)

Above all, the structure with asymmetric magnetic poles can reduce the total permeance in the magnetic circuit. Its total effective magnetic flux is more than a traditional V-type structure if the magnetomotive force is the same.

3. Motor Structure Optimization

In order to obtain a better geometric motor shape, this paper makes some improvements on the rotor structure. The shape of rotor edge is changed. The auxiliary slots are used inside the rotor. The length of the air gap becomes nonuniform. The parameters are optimized by an appropriate optimization method. And the performance of the rated operating point is selected for research and analysis.

3.1. Optimization Ideas and Optimization Methods

The method of changing the air-gap length is used in this paper. The air-gap length becomes nonuniform. And the effective magnetic flux direction is adjusted. The specific measures are as follows. The auxiliary slots are used inside the rotor. It can enhance the magnetization ability and reduce the rotational inertia. The distance between auxiliary slots and a permanent magnet is $d$. The outer edge of the rotor is changed to be eccentric. The eccentric distance is $h$. Straight line is used in the junction of the eccentric circle. Its length is $l$. It can eliminate sharp corners on the outer edge, as shown in Figure 4.

The above improvement method can improve the air-gap flux density waveform. The schematic diagram of the expected waveform can be represented in Figure 5. It is more sinusoidal than the initial structure.

This paper uses the Taguchi method to optimize the parameters. The Taguchi method was first proposed by Dr. Genichi Taguchi who is a Japanese quality control expert. It is a local optimization method based on orthogonal experiments. It can quickly determine the combination of multiob-

![Figure 3: The equivalent magnetic circuit models: (a) traditional V-type; (b) asymmetrical magnetic poles.](image)

![Figure 4: Rotor structure optimization.](image)

![Figure 5: Schematic diagrams of the air-gap magnetic density waveform: (a) initial structure and waveform; (b) improved structure and expected waveform.](image)

| Level | A (mm) | B (mm) | C (mm) |
|-------|--------|--------|--------|
| 1     | 2      | 6      | None   |
| 2     | 4      | 6.5    | 4      |
| 3     | 6      | 4      | 4.5    |
| 4     | 8      | \     | \      |
| 5     | 10     | \     | \      |
| 6     | 12     | \     | \      |
Objective optimization with the least number of experiments. The basic design process is as follows:

1. Determine optimization goals according to requirements
2. Determine optimization variables and corresponding level values
3. Establish an orthogonal experimental matrix

Table 3: Nonstandard orthogonal experimental matrix.

| Experiment number | A  | B  | C  |
|-------------------|----|----|----|
| 1                 | 1  | 1  | 1  |
| 2                 | 2  | 1  | 2  |
| 3                 | 3  | 1  | 3  |
| 4                 | 4  | 1  | 2  |
| 5                 | 5  | 1  | 3  |
| 6                 | 6  | 1  | 1  |
| 7                 | 1  | 2  | 3  |
| 8                 | 2  | 2  | 1  |
| 9                 | 3  | 2  | 2  |
| 10                | 4  | 2  | 1  |
| 11                | 5  | 2  | 2  |
| 12                | 6  | 2  | 3  |
| 13                | 1  | 3  | 2  |
| 14                | 2  | 3  | 3  |
| 15                | 3  | 3  | 1  |
| 16                | 4  | 3  | 3  |
| 17                | 5  | 3  | 1  |
| 18                | 6  | 3  | 2  |

Table 4: Orthogonal experimental results.

| Experiment number | THD (%) | $T_{cog}$ (mN•m) | $T_{avg}$ (N•m) |
|-------------------|---------|------------------|-----------------|
| 1                 | 40.35   | 298.86           | 13.32           |
| 2                 | 37.93   | 294.42           | 13.57           |
| 3                 | 37.25   | 286.83           | 13.58           |
| 4                 | 33.91   | 277.12           | 13.60           |
| 5                 | 35.86   | 280.57           | 13.59           |
| 6                 | 38.68   | 284.29           | 13.46           |
| 7                 | 40.23   | 297.56           | 13.32           |
| 8                 | 36.95   | 286.89           | 13.54           |
| 9                 | 37.06   | 281.55           | 13.57           |
| 10                | 31.01   | 274.46           | 13.78           |
| 11                | 35.67   | 277.92           | 13.65           |
| 12                | 37.47   | 282.65           | 13.61           |
| 13                | 40.62   | 364.42           | 13.45           |
| 14                | 38.27   | 324.96           | 13.56           |
| 15                | 37.72   | 314.61           | 13.56           |
| 16                | 34.95   | 297.53           | 13.71           |
| 17                | 36.82   | 304.35           | 13.69           |
| 18                | 37.73   | 302.57           | 13.61           |

Table 5: The overall average value of experimental results.

|               | THD (%) | $T_{cog}$ (mN•m) | $T_{avg}$ (N•m) |
|---------------|---------|------------------|-----------------|
| Average value | 37.734  | 302.570          | 13.606          |

Table 6: The average value of THD.

| Level | A     | B     | C     |
|-------|-------|-------|-------|
| 1     | 40.40 | 36.33 | 36.92 |
| 2     | 37.72 | 36.40 | 36.15 |
| 3     | 37.34 | 37.68 | 36.34 |
| 4     | 33.29 | \    | \    |
| 5     | 36.11 | \    | \    |
| 6     | 37.96 | \    | \    |

Table 7: The average value of $T_{cog}$.

| Level | A     | B     | C     |
|-------|-------|-------|-------|
| 1     | 320.28| 287.02| 294.02|
| 2     | 302.09| 283.51| 295.67|
| 3     | 294.33| 318.07| 293.91|
| 4     | 283.04| \    | \    |
| 5     | 287.61| \    | \    |
| 6     | 289.84| \    | \    |

Table 8: The average value of $T_{avg}$.

| Level | A     | B     | C     |
|-------|-------|-------|-------|
| 1     | 13.36 | 13.52 | 13.56 |
| 2     | 13.56 | 13.60 | 13.57 |
| 3     | 13.57 | 13.68 | 13.65 |
| 4     | 13.70 | \    | \    |
| 5     | 13.64 | \    | \    |
| 6     | 13.56 | \    | \    |

(4) Use the finite element method to solve the orthogonal experiment matrix
(5) Analyze the influence of each optimization variable and determine the optimal parameters
(6) Verify the optimization results by finite element analysis

3.2 Determination of Optimization Goals and Optimization Variables. In this paper, the Taguchi method is used to optimize the air-gap flux density waveform, the cogging torque, and the output torque. And it must ensure that the torque ripple does not increase. Therefore, the total harmonic distortion of the air-gap flux density (THD) is the first optimization goal. The cogging torque ($T_{cog}$) is the second optimization goal. The average of output torque ($T_{avg}$) is the third optimization goal. And the torque ripple is a constraint condition. It requires that THD and $T_{cog}$ reduces.
and $T_{\text{avg}}$ increases as much as possible. And the torque ripple does not increase.

In addition, $h$ is chosen as the optimization variable. $l$ is chosen as the optimization variable. $d$ is chosen as the optimization variable. The initial values of the three optimization variables are $(2, 6.5, 4)$. On this basis, appropriate values are selected as the level values. And optimization parameters and level values are shown in Table 2. Especially, "none" means no auxiliary slot.

### 3.3. Orthogonal Experimental Designs and Solution.

The optimization goals, optimization variables, and corresponding level values have been determined above. The number of level values is not exactly the same. The traditional standard orthogonal experiment matrix cannot be established. According to principles of the Taguchi method experiment designing, the nonstandard orthogonal experimental matrix is established as shown in Table 3.

The rated operating point performance of each experiment can be calculated by using the two-dimensional finite element analysis method. The orthogonal experimental results are shown in Table 4. The calculation formula of THD is shown in formula (3). And the calculation of $T_{\text{avg}}$ is shown in formula (5). $T_{\text{cog}}$ can be obtained by the simulation analysis.

\[
\text{THD} = \sqrt{\sum_{i=2}^{n} x_i^2} \times 100\% , \tag{3}
\]

\[
T_{\text{avg}} = \frac{(T_{\text{max}} + T_{\text{min}})}{2} , \tag{4}
\]

where $x_1$ is the effective value of the fundamental wave, $x_i$ is the effective value of higher harmonics, and $T_{\text{max}}$ and $T_{\text{min}}$...
are the maximum and minimum output torque, respectively. And they can be obtained by the simulation analysis.

3.4. Results Processing and Analysis. In the previous section, the results of each experiment were obtained by the two-dimensional finite element method. The overall average value of the experimental results can be calculated by formula (5). And the results are shown in Table 5.

\[ m = \frac{1}{18} \sum_{i=1}^{18} m_i, \]  

(5)

The average value of each optimization variable can be calculated, respectively, at a certain level value. To give an example, the average value can be obtained by the following formula when the optimization variable \( A \) is selected as the level 1.

\[ m_{A1}(\text{THD}) = \frac{1}{3} (\text{THD}(1) + \text{THD}(7) + \text{THD}(13)). \]  

(6)

Similarly, the average values of \( \text{THD}, \ T_{\text{cog}}, \) and \( \ T_{\text{avg}} \) can be obtained with reference to the calculation method of the above formula. And the results of \( \text{THD}, \ T_{\text{cog}}, \) and \( \ T_{\text{avg}} \) are shown in Table 6, Table 7, and Table 8, respectively.

If a single goal is optimized, the optimization combination can be selected in Table 6, Table 7, and Table 8. To optimize \( \text{THD}, \) a combination of \( A4, B1, \) and \( C2 \) can be selected. To optimize \( T_{\text{cog}}, \) a combination of \( A4, B2, \) and \( C3 \) can be selected. And a combination of \( A4, B3, \) and \( C3 \) can be selected to optimize \( T_{\text{avg}}. \)
It is necessary to analyze the influence of each optimization variable when multiobjective optimization is carried out. Based on the previous calculation, the proportions can be calculated by formula (7).

\[
SS = \frac{1}{x} \sum_{i=1}^{x} (m_x - \bar{m})^2,
\]

where \( m_x \) is the value of each optimization goal under a certain level; \( \bar{m} \) is the average value of each optimization goal; \( x \) is the number of levels, and SS is the variance of the optimization variable under a certain performance. And the calculation results are shown in Table 9.

In the above, the experimental results of the Taguchi method optimization are analyzed. And the proportions of the influence of each variable on the optimization goal are obtained. It can be seen that optimization variable \( A \) has a great influence on the three optimization goals. THD and \( T_{cog} \) are the smallest when level 4 is selected. And \( T_{avg} \) is the largest. Optimization variable \( B \) has a great influence on \( T_{cog} \). Therefore, optimization variable \( B \) chooses level 2. And optimization variable \( C \) has a little influence on each optimization goal and chooses level 3. In other words, the eccentric distance is 8 mm. The length of the straight line is 6.5 mm. And the distance between auxiliary slots and a permanent magnet is 4.5 mm.

4. Simulation Analysis and Experimental Verification

4.1. Finite Element Simulation Analyses. According to the above optimization scheme, the motor structure after optimization is obtained and designed. The structure is shown in Figure 6. And the static magnetic field analysis is shown in Figure 7.

The optimized motor structure is analyzed by finite element simulation. The air-gap flux density waveform and harmonic analysis are compared and analyzed in Figure 8. The waveform after optimization meets an excepted waveform.

Coggging torque comparison is shown in Figure 9. And output torque comparison is shown in Figure 10. With the two-dimensional finite element method simulation analysis, it can be calculated that THD is 28.9%. The \( T_{cog} \) is 284.3 mN\cdotm. And the \( T_{avg} \) is 13.7 N\cdotm.

It can be seen from Figure 10 that the \( T_{avg} \) is improved. And the range of the output torque ripple is reduced. The torque ripple rate (\( T_{ripple} \)) represents the torque ripple of the output torque. Its calculation is as follows:
\[ T_{\text{ripple}} = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{avg}}} \times 100\%. \] (8)

It can be calculated that \( T_{\text{ripple}} \) before optimization is 41.1% by the above formula. And \( T_{\text{ripple}} \) after optimization is 24.7%. It decreases by 16.4%.

The performance comparison before and after optimization is shown in Figure 11. Compared with before optimization, THD is reduced by 36.7%. \( T_{\text{cog}} \) is reduced by 26.0%. And \( T_{\text{avg}} \) is increased by 4.8%.

4.2. Prototype Manufacture and Experiment. In order to verify the effectiveness of the simulation analysis, the rotor structure after optimization is manufactured. It is shown in Figure 12. The prototype is assembled and placed on a cogging torque test platform for experiment. The cogging torque test platform is shown in Figure 13.

The cogging torque experimental curve is shown in Figure 14. The maximum value is 280.8 mN\( \cdot \)m. It is similar to the simulation value. Maximum friction torque is 346.8 mN\( \cdot \)m.

In order to obtain the maximum torque and efficiency of asymmetrical magnetic pole PMSM, it is necessary to test the characteristic curve of the motor. The prototype is placed on the dynamometer test platform for a full load test. It is shown in Figure 15. And the characteristic curve of the motor is obtained as shown in Figure 16.

In Figure 16, the maximum speed of the motor is 5000 revolutions per minute. The maximum output torque is 62 N\( \cdot \)m. And maximum efficiency exceeds 90%.

5. Conclusions

This paper proposes a novel asymmetric magnetic pole PMSM for automobiles. The equivalent magnetic circuit model is established. Many improvements have been made to the rotor structure. The Taguchi method is used to obtain the optimal solution of each optimization variable. Combined with finite element analysis and prototype experiment, the effectiveness of the structure and optimization method is verified. In addition, some conclusions are drawn as follows:

1. The novel asymmetric magnetic pole permanent magnet synchronous motor has parallel effective magnetic flux. Compared with the traditional V-type structure, the total permeance is reduced. And the effective magnetic flux content is increased.
2. The Taguchi method is used to optimize the motor structure. A nonstandard orthogonal experimental matrix is established. The original 54 sets of experiments are replaced by 18 combinations of experiments. The number of experiments is reduced obviously. And the finite element method verifies the effectiveness of the method.
3. The eccentric distance of the rotor has the greatest influence on the total harmonic distortion of the air-gap flux density and the average output torque.
4. Compared with the performance before optimization, the total harmonic distortion of the air-gap flux density is reduced by 36.7%. The cogging torque is reduced by 26.0%. The average output torque is increased by 4.8%. And the output torque ripple is reduced by 16.4%.

Data Availability

All data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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