Theoretical investigation of optical phenomenon from nanometric antireflex layers

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Summary. Within the paper were studied the physical phenomenon which characterize the passing of light through a dielectric environment, a case which can be applied to eyeglass lenses with a nanometric antireflex layer. Were determined the optical characteristics required or such nanometric layers. Was studied practically the increase of light transmission, respectively the reduction of reflection for this new manufacturing technology for eyeglass lenses and the nanometric antireflex layers. The study was completed with photometric measurements and measurements for the resistance of the antireflex coatings compared for several manufacturing technologies.

1. Introduction

The purpose of the nanometric antireflex layers is to reduce reflection, and at the same time increase transmission. Usually, the treatment must be applied on both sides of the material to be coated. Thus, in this context, we define the nanometric antireflex layer as the one contributing to the increase of transmittance and, in the ideal case, to its maximization.

An optical coating is composed from a succession of thin layers (the optical thicknesses are comparable with the wavelength of the incident light) from different optical materials.

We call a material layer as being nanometric when the effects of the interference can be detected in reflected or transmitted light. The detection is due to the presence of coherent fringes corresponding to the constructive interference. If the effect of the interference can not be observed, it means the layer is too thick, case described as being incoherent. Thus, we aim to optimize the thickness of the layer, by using the theoretical modeling, in order to produce the destructive interference of the reflected waves and the constructive interference of the transmitted waves.

Considering the theme regarding the antireflex layers applied on a certain substrate is required a theoretical approach of the phenomenon of light passing through the separation surface between two environments.

There are two methods to approach the problem: Elastic optics method (Fresnel and Huygens) and electromagnetic optics method (Maxwell). Some phenomena can be explained by the theory of the particle behavior in an elastic medium, and some phenomena can be explained by the theory of the behavior of a transverse wave.
Thus, phenomena like: The Compton effect, the photoelectric effect (photodiode), image intensifiers, correction of optical systems, etc., are explained by the particle behavior. And phenomena like: Reflection, refraction, diffusion, diffraction, interference, etc., are explained by the wave behavior.

Further on we will approach the issue by employing the second method.

2. The electromagnetic optics method

In order to study the phenomenon of light passing through a medium, is considered that the light has an electromagnetic nature and that the matter is a system of electrical charges. The light is part of the radiation domain of the electromagnetic spectrum (EM). The spectrum represents the domain of frequencies, wavelengths and energies for various types of radiations.

The photon is considered the smallest particle (quanta) of the electromagnetic field. The quantity of energy emitted/absorbed is not a continuous value, but comes in quanta (discreet variable).

In theory is considered that the photon has a dual wave-particle behavior. The experimental results attest not only the particle properties of the photons but also the validity of the principles of energy and momentum conservation in any process, including for the individual interactions between photons and elementary particles.

Any photon has an energy:

\[ E = h \times 3.58 \times 10^{-15}[J] \]  
where \( h = 6.6 \times 10^{-34}[Js] \) Planck's constant

\( E \) – kinetic energy; \( f \) – frequency

We know that for any radiation we have the formula:

\[ \nu = f \times \lambda \] (2)

The speed of the emitted radiation could be determined experimentally, but we know that in vacuum any radiation has \( \nu = c \) (the speed limit which can be reached by a body is \( c = 299 792 458 \text{ m/s} \) regardless of the environment in which it moves).

\[ \rightarrow \lambda = c / f \] (3)

Thus, a radiation can be characterized in a vacuum by \( \lambda \) or by \( f \). But, when the radiation passes through various media, the frequency is conserved and the speed is reduced. As a consequence, two radiations vary by their frequency (regardless of the medium).

According to Maxwell's theory for a complete approach are introduced the electrical and magnetic properties of the material defined by the well known notion of the refractive index.

The absolute refractive index (or plainly the refractive index) represents the ratio \( n \) between the speed of light in a vacuum \( c \) and the speed of light in a medium \( \nu \).

\[ n = \frac{c}{\nu} \] (4)

If we have two different successive media we can then write:

\[ n_1 = \frac{c_1}{\nu_1} ; \quad n_2 = \frac{c_2}{\nu_2} \] (5)

The ratios show how many times the absolute speed was reduced in that medium.

The relative refractive index represents the ratio. \( n_{21} \) between \( n_2 \) and \( n_1 \).

\[ n_{21} = \frac{n_2}{n_1} \] (6)

The ratios show how many times the speed was reduced/increased in medium \( n_2 \) compared to \( n_1 \). In many media, the intensity of the light is quickly attenuated as it enters the material.

If we write the refractive index \( N \) as a complex number:

\[ N = n - ik \] (7)
Where \( n \) is the absolute refractive index, \( k \) is the extinction coefficient, and we introduce that in the Maxwell equations then the physical phenomenon corresponds with the mathematical description.

Both \( n \) and \( k \) depend on the wave frequency. If \( k > 0 \), then the light is absorbed, and if \( k = 0 \), then the light crosses the respective medium without losses.

### 2.1. Light wave equation. Polarization state

The vibrations and quanta propagation are treated as sinusoidal movements of the type:

\[
y = A \times \sin \left( \frac{2\pi t}{T} \right) = A \times \sin(\omega t)
\]

Where \( A \) is the amplitude of the wave, \( t \) is the time of an oscillating movement, and \( 1/T \) represents the number of movements per second (frequency).

The equation can also be written as a function of the wavelength \( \lambda \) and the wave number \( k \), which depend on the speed \( v \) and frequency \( f \) of the wave.

\[
y(x, t) = A \cos \left( \omega t - kx \right) = A \cos \left( \frac{2\pi f}{v} (vt - x) \right)
\]

\[
k = \frac{2\pi f}{v} = \frac{\omega}{v}
\]

In the specialty literature \( y(x, t) \) is called the wave elongation, and the argument of the cosine function is called the wave phase.

We consider: The incidence of the light on a plane separation surface between two dielectric media with different dielectric constants. The electromagnetic wave (EM) has the angle \( i \) with the incidence normal line according to Figure 1.

In the general case the light is natural, meaning the orientation of the cross created by the electric vector and the magnetic vector changes from moment to moment. The EM wave does not need a medium for propagation; the electric field (E) generates a magnetic field (H) and vice-versa, meaning they generate each other. Still, at any time, each of these vectors can be decomposed in two components, one parallel (s) with the incidence plane and the other perpendicular (p) to this plane.

\[
E = E_s + E_p \Leftrightarrow A = A_s + A_p \text{ (} A = \text{amplitudinea vectorului electric incident)}
\]

The analysis is reduced to the observing of two particular cases, which appear as a result of the Brewster incidence. As a result of the incidence with a Brewster angle, the reflection of the natural light is totally polarized in the incidence plane, and the refraction of the natural light is totally polarized in the transmission plane. As such:

- The electric vector oscillates in the incidence plane (and the magnetic one is normal in this plane). The phenomenon is called \( p \) polarization.

\[
A = A_s + 0
\]

\[
R || = \frac{tg(i - t)}{tg(i + t)} \times A ||
\]
\[
T_{\parallel} = \frac{2 \sin t \cos i}{\sin(i + t) \cos(i - t)} = A_{\parallel}
\]  

- The electric vector oscillates perpendicularly to the incidence plane (and the magnetic one is in this plane). The phenomenon is called \textit{s polarization}.

\[
E = 0 + E_p
\]
\[
R = \frac{\sin(i - t)}{\sin(i + t)} \times A
\]
\[
T = \frac{\sin\times\cos i}{\sin(i + t)} \times A
\]

For each case is calculated the wave phase:

\[
tg(\delta_p/2) = \frac{(\sin^2 i - n_{z1}^2)^{1/2}}{\cos i}
\]
\[
tg(\delta_s/2) = \frac{(\sin^2 i - n_{z1}^2)^{1/2}}{n_{z1}\cos i}
\]

And the phase difference between the two is:

\[
tg(\delta/2) = \frac{\cos \times (\sin^2 i - n_{z1}^2)^{1/2}}{\cos i}
\]

\subsection*{2.2. Mathematical formalities in the approach of the thin antireflex layers. Thickness of the layers}

The optical properties of each layer are described completely by the complex refractive index and the geometrical thickness \(d\). The mathematical formula used for the determination of the spectral characteristics of the covering is the matrix one, which characterizes each layer by a matrix. [1]

\[
\begin{bmatrix} E_0 \\ H_0 \end{bmatrix} = \prod_{r=1}^{n} \begin{bmatrix} \cos \delta_j & \frac{i\sin \delta_j}{n_j} \\ \frac{in_j \sin \delta_j}{\cos \delta_j} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1/n_m \end{bmatrix}
\]

\[
\delta_j = \frac{2\pi n_j d \cos i}{\lambda}
\]

where \(\delta_j\) is the phase thickness of layer \(j\), also indicating the phase displacement of the wave; \(d\) is the geometrical thickness of layer \(j\); \(n_j\) is the refractive index of the film crossed by the wave; \(n_m\) is the refractive index of the substrate. The angle \(i\) is obtained from Snell's law, more precisely from the law of refraction.

From the relation of the phase difference (5) can be determined the optimal optical thickness for obtaining the antireflex effect. It can be observed that the solution is an integer number of quarters/halves of waves.

\[
\delta = m \left(\frac{\pi}{4}\right), m = 0,1,2,3,...
\]

\textbf{Case I:} \(m\) is even, thus \(\cos \delta = \pm 1\) and \(\sin \delta = 0\), with a thickness of the layer equal with a multiple of wave half lengths. The layers of this type are often referred to as being absent because they yield the unit matrix for certain wavelengths and thus, when multiplying between the matrices of each layer, they provide no contribution.

\textbf{Case II:} \(m\) is odd, thus \(\sin \delta = \pm 1\) and \(\cos \delta = 0\), with a thickness of the layer equal with a multiple of a wave quarter length. Thus, the layers with a thickness of a quarter wave are the ones for which a satisfactory antireflec effect is obtained.

In a medium devoid of absorption, the calculation of the reflectance is made by separating the irradiations/intensities of the two waves, transmitted and reflected. The irradiation or light intensity is defined as the flow of energy carried by a wave on the surface unit.
2.3. Fresnel coefficients

The Fresnel coefficients are defined in the case of normal incidence, respectively the coefficients corresponding to the amplitude for reflection and transmission.

The reflection coefficient is the ratio between the reflected amplitude \( R \), and the incident one \( A \), of the electrical field. The refractive index of the incidence medium is marked as \( n_0 \), and the one for the layer as \( n_1 \):

\[
\rho = \frac{R}{A} = \frac{n_0 - n_1}{n_0 + n_1}
\]

(24)

From the formula for the reflection coefficient we observe that \( \rho \) becomes negative in the case when the light is reflected at the separation surface between a less dense medium with a denser medium (higher refractive index) from an optical perspective. This represents the change of the phase with \( \pi \) for the reflected wave ion the first medium.

The transmission coefficient is the ratio between the transmitted and the incident amplitude.

\[
\sigma = \frac{T}{A} = \frac{2n_0}{(n_0 + n_1)^2}
\]

(25)

We define the factors for reflection \( R \), and respectively transmission \( T \).

The reflectance/reflection factor is equal with the ratio between the reflected intensity and the incident one, and the transmittance/transmission factor is equal with the ratio between the transmitted intensity and the incident one.

\[
R = \rho^2 = \left(\frac{n_0 - n_1}{n_0 + n_1}\right)^2
\]

(26)

\[
T = \frac{n_1}{n_0} \sigma^2 = \frac{2n_0}{(n_0 + n_1)^2}
\]

(27)

For the case when there is no absorption in the translucent medium, the sum of the reflected and transmitted energies is equal with the value of the incident energy.

\[
\rho^2 + \sigma^2 = 1 ; \quad T + R = 1
\]

(28)

The analysis of the antireflex layers for the oblique incidence is recommended only in the cases of the coatings perfected for a narrow spectrum of wavelengths, case in which we separate the light wave in two components with linear polarization.

The transmission is independent on the angle of the incident light. For the theoretical handling can be employed the instrument of the transmission potential for a layer, defined as the ratio between the irradiation leaving the layer and the incident one. It is a measure of the theoretical maximum transmission capacity, ideal for a non absorbent medium.

\[
\psi = \frac{I_{\text{out}}}{I_{\text{in}}} = \frac{T}{(1 - R)}
\]

(29)

Thus, for a maximum transmission, the reflection is zero, which is equivalent with adding an antireflex layer.

3. Conclusion. Summary

As such, the reflection for a layer can be calculated with the formula:

\[
R = \frac{n_1^2(n_0 - n_m)^2 - (n_0^2 - n_1^2)(n_1^2 - n_m^2)\sin\left(\frac{2\pi d n_1}{\lambda}\right)}{n_1^2(n_0 - n_m)^2 + (n_0^2 - n_1^2)(n_1^2 - n_m^2)\sin\left(\frac{2\pi d n_1}{\lambda}\right)}
\]

(30)

Where \( n_1, n_0, n_m \) are the refractive indexes of the layer, incidence medium and substrate; \( d \) is the geometrical thickness of the thin layer. [6]

The thickness and refractive index of the layer satisfy the following principles in order to have a zero reflection for a certain wavelength:
\[\frac{2\pi n_1 d}{\lambda} = \frac{(2m + 1)\pi}{2}, \quad m = 0, 1, 2, 3, \ldots \quad (31)\]

\[n_1 = \sqrt{n_0 \cdot n_m} \quad (32)\]

For a certain AR layer, the transmittance will be maximum for the values:
\[4n_1 d, \ 4n_1 d/3, \ 4n_1 d/5, \ \ldots, \ 4n_1 d/(2m + 1). \quad (33)\]

The wavelength corresponding to the maximum transmittance will obey the rule:
\[\lambda_1 = 3\lambda_2 = 5\lambda_3 = \ldots = (2m + 1)\lambda_{m+1} \quad (34)\]

Thus, the transmittance for the coating with a monolayer of metallic oxides has the theoretical values calculated in Table 1.

**Table 1.** Refractive indexes and maximum transmittance for coatings with MgF2, TiO2, SiO2 on substrate OMB-99, with refractive index 1.5326 calculated according to formula (9) for the reflection coefficient and for T = 1 - 2R (there are two reflections, not considering the absorption)

|                | Refractive index | Reflection (%) | Transmittance (%) |
|----------------|-----------------|----------------|-------------------|
| MgF2           | 1.3777          | 1.135          | 97.73             |
| TiO2           | 2.6142          | 40.150         | 19.700            |
| SiO2           | 1.4585          | 2.640          | 94.721            |

For thicknesses different to \(\lambda/4\) we require complex calculations, facilitated by specialty software programs which employ graphical analysis methods.

The theoretical study includes graphs regarding the spectral reflection for the normal and oblique incidence depending on the refractive indexes for the layers and medium, etc.

![Figure 2-A. Incident angle of reflection for the substrate OMB-99 with no coating](image-url)
The reflection (Figure 2-A) and transmission (Figure 2-B) depending on the incidence angle for the lens designed in the first part of this paper. The graph is for the substrate OMB-99(special optical monomer) with no coating, the polarization cases s and p. Can be observed the value of 56° for the Brewster angle from the reflection graph.

Figure 2-B. Incident angle of transmission for the substrate OMB-99 with no coating

Figure 3-A. Incident angle of reflection for the substrate OMB-99 with coating on both faces of the lens

Figure 3-B. Incident angle of transmission for the substrate OMB-99 with coating on both faces of the lens
The reflection (Figure 3-A) and transmission (Figure 3-B) depending on the incidence angle for the lens designed in the first part of this paper. The graph is for the substrate OMB-99 with coating on both faces of the lens with MgF₂ (0.25) + SiO₂ (0.5) + TiO₂ (0.25), the polarization cases s and p.

Can be observed the increase of the reflection intensity, respectively the decrease of the transmission intensity at the same time with the incidence angle.

4. References

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