Idea and Theory of Particle Access

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Abstract: Aiming at some problems existing in the current quality of service (QoS) mechanism of large-scale networks (i.e. poor scalability, coarse granularity for provided service levels, poor fairness between different service levels, and improving delay performance at the expense of sacrificing some resource utilization), the paper puts forward the idea and theory of particle access. In the proposed particle access mechanism, the network first granulates the information flow (that is, the information flow is subdivided into information particles, each of which is given its corresponding attributes), and allocates access resources to the information particle group which is composed of all the information particles to be transmitted, so as to ensure that the occupied bandwidth resources is minimized on the premise of meeting the delay requirements of each information particle. Moreover, in the paper, the concepts of both information particle and information particle group are defined; The relationships between the average access bandwidth requirement, the peak average access bandwidth requirement, and the minimum reachable access bandwidth of an information particle group are analyzed; The influences of time attribute and attribute of bearing capacity of an information particle group on the minimum reachable access bandwidth are analyzed; An effective method for the calculation of the minimum reachable access bandwidth of an information particle group is given, and a particle access algorithm based on dynamically adjusting the minimum reachable access bandwidth is proposed; Furthermore, several properties, such as the upper and the lower bounds of the minimum reachable access bandwidth, the property of linear superpositions of multiple information particle groups, the property of linear splitting of an information particle group, and a closed form of expression
of the minimum reachable access bandwidth and so on, of the minimum reachable access bandwidth(s) of information particle group(s) are proved; Finally, the joint optimization of the mechanism of network wave and the mechanism of particle access is studied. For a given information particle group transmitted over a given primary path, a method to obtain the minimum reachable transmission bandwidth, which can guarantee the end-to-end delays of all the information particles contained in the information particle group, is proposed. The research of the paper pave a new way for further improving QoS mechanisms of large-scale networks, and lay the corresponding theoretical foundation.

*Keywords*: Network Resource Allocation, QoS, Particle Access, Information Particle, Information Particle Group, Delay Guarantee, Minimum Reachable Access Bandwidth, Superposition of Multiple Information Particle Groups, Splitting of an Information Particle Group
I. Introduction

As we all know, compared with “circuit switching” technology, “packet switching” technology can significantly improve the utilization of network resources, so it has become one of the key technologies supporting today’s Internet. In packet switching systems, because the network resources are fully shared among various traffic flows, the problem of QoS guarantee becomes more challenging [1]. The basic starting point of QoS mechanism is to ensure various performance requirements of different traffic flows under the limited network resources. Therefore, two mechanisms of QoS guarantee, namely “integrated services” and “differentiated services” are proposed.

The core idea of integrated services [2] is to reserve a certain amount of resources in all the forwarding nodes belonging to the end-to-end path connecting the source node and the destination node of a traffic flow to meet the resource requirement of the traffic flow (this kind of QoS mechanism of providing resource reservation service for traffic flows is also called as “flow based” mechanism). In essence, integrated services can be regarded as an imitation of circuit switching mechanism over packet switching mechanism, so good QoS guarantee can be achieved for traffic flows by using the mechanism of integrated services. However, once this mechanism is applied in large-scale communication networks (such as the Internet), one will inevitably face the need to reserve resources for hundreds of traffic flows on each router at the same time, and maintain the corresponding statuses of all these traffic flows, which poses a great challenge to the design and the implementation of a network router! In short, the main problem faced by the mechanism of integrated services is its poor scalability.

The mechanism of differentiated services [3] is proposed to solve the problem of poor scalability faced by the mechanism of integrated services. The basic idea is to only divide packets of traffic flows in the network into several categories, and provide differentiated services for different types of packets at the intermediate forwarding nodes (this kind of QoS mechanism of providing differentiated services for different types of packets is also called as “class based” mechanism). Although this mechanism solves the problem of poor scalability faced by integrated services to a certain extent, there are still some noteworthy problems existing in the mechanism of differentiated services.

First, let us consider the possible problems of differentiated services in the case that packets of
traffic flows are only divided into fewer types. For example, among the proposed mechanisms for
differentiated services, a simple mechanism that can provide differentiated services is proposed, that
is, the so-called “expedited forwarding” mechanism [4]. In the expedited forwarding mechanism,
packets of traffic flows are only divided into two types, namely the “regular” type and the “expedited”
type. Relative to the “regular” packets, the “expedited” packets have absolute priority for the
utilization of network resources. The problem is that even for packets, belonging to the same
“expedited” type or “regular” type, significantly different performance requirements still exist. In
this way, it is somewhat “rough” to only divide the packets into two categories, so it is difficult to
finely guarantee the QoS requirement of each traffic flow. In short, by using the mechanism of
differentiated services, if the packets of traffic flows are only divided into fewer types, it is
difficult for the network to provide refined QoS guarantee.

Secondly, let us think over the possible problems of the mechanism of differentiated services
when packets of traffic flows are divided into many more types. For example, in differentiated
services, a relatively complex mechanism, the so-called “assured forwarding” mechanism have been
proposed [5]. In short, in the assured forwarding mechanism, packets are first divided into four
different priorities, and then three levels of packet dropping are further defined. This is equivalent
to finally dividing the packets of traffic flows into twelve service levels and providing differentiated
services for different levels accordingly. For a mechanism just like the assured forwarding, since it
supports more service levels, it can provide more refined differentiated services. However, facing
with a large number of service types, how to ensure the fairness of resource allocation among all
these service levels becomes more and more difficult to be dealt with. That is to say, with the
increase of the number of service levels, the fairness relationships between different service levels
that need to be properly handled with in the resource allocation strategy become more and more
complicated (theoretically, assured forwarding needs to handle \( \sum_{i=1}^{11} i=66 \) pairs of fairness
relationships between all the service levels!), which makes it a very challenging problem to
reconcile the fairness between various service levels. In short, by adopting the mechanism of
differentiated services, if packets of traffic flows are divided into many more types, it is more
difficult to ensure fairness between different service levels.

The fairness problems of the mechanism of differentiated services pointed out above will
further bring about other problems. On the one hand, with the emergence of new types of network services, it is likely to be found out that the available service levels cannot provide satisfactory QoS guarantee for the new appeared services. In order to solve this problem, it is natural that one thinks of designing new service levels for new types of services and adds them to the existing class based QoS mechanism. Therefore, one needs to consider not only how to design the corresponding resource allocation strategy for the new service levels according to their specific performance requirements, but also how to make the newly defined service levels coexist “peacefully” with the existing service levels in the shared QoS guarantee mechanism. Obviously, with the increasing number of service levels defined, it will become more and more difficult to design an extra new service level which can coexist “peacefully” with the existing service levels. Once again, it can be seen that the problem of scalability arises. On the other hand, if the network allows an application to set its service level on its own will, the application vendor is very likely to intentionally label the packets generated by its developed application to some service level with high priority, which inevitably leads to the abuse of some service levels that are easier to acquire network resources, and further amplifies the harm caused by the existing unfairness between service levels, and even causes the final collapse of the network!

In recent years, people pay more and more attention to the research on how to solve the problem of delay guarantee in wireless access networks [6]. Among many solutions, some of them improve QoS for delay sensitive traffic by improving the total system throughput, scheduling strategies, or queuing strategies, and so on. However, these strategies cannot provide deterministic delay guarantee. Although some access strategies can provide deterministic delay guarantee, it is achieved at the expense of sacrificing some network access resources. For example, in some access strategies based on resource reservation, the network reserves a certain amount of resources for some expected packets of delay sensitive traffic flows in advance, so as to significantly improve the performance of these traffic flows. However, the arrival characteristics of some delay sensitive traffic have great uncertainty, which leads to the phenomenon that some pre reserved resources are wasted (that is, the expected packets do not arrive within the time period corresponding to the reserved resources).

In short, in order to guarantee the QoS for delay sensitive traffic, available solutions often sacrifice the utilization of network resources in exchange for the improvement of delay performance.
Aiming at the problems existing in the mechanism of QoS guarantee in large-scale networks, the paper proposes the idea and theory of particle access. The major contributions are listed as follows:

1) The basic idea of particle access is proposed, and the concepts of both information particle and information particle group are given;

2) The relationships between the average access bandwidth requirement, the peak average access bandwidth requirement, and the minimum reachable access bandwidth of an information particle group are analyzed;

3) The influences of time attribute and attribute of bearing capacity of an information particle group on the minimum reachable access bandwidth are analyzed;

4) An effective method for the calculation of the minimum reachable access bandwidth of an information particle group is proposed, and a particle access algorithm based on dynamically adjusting the minimum reachable access bandwidth is given;

5) The upper bound and the lower bound of the minimum reachable access bandwidth of an information particle group are given, respectively;

6) The property of linear superpositions of multiple information particle groups under specific constraints is proved. The property of linear splitting of an information particle group is also proved;

7) A closed form expression of representing the minimum reachable access bandwidth of an information particle subgroup as the product of its vector of switching functions and its vector of average access bandwidth requirements is given;

8) For a given information particle group transmitted over a given primary path, a method to obtain the minimum reachable transmission bandwidth, which can guarantee the end-to-end delays of all the information particles contained in the information particle group, is proposed.

The paper is organized into nine parts: in Section I, the problems existing in the current QoS mechanisms are pointed out; In Section II, the basic idea of the paper is proposed, that is, the idea of particle access is introduced into the access mechanism of networks; In Section III, the basic concept of information particle is defined; In Section IV, the concept of information particle group is further defined; In Section V, basic properties of the minimum reachable access bandwidth of an information particle group are proved; In Section VI, the intrinsic mechanism of the influences of
time attribute and attribute of bearing capacity of an information particle group on the minimum reachable access bandwidth is analyzed; In Section VII, based on the basic framework of the theory built in the paper, an effective method for the calculation of the minimum reachable access bandwidth of an information particle group is given, and a particle access algorithm based on dynamically adjusting the minimum reachable access bandwidth is proposed. Furthermore, several properties of the minimum reachable access bandwidth(s) of information particle group(s) are proved; In Section VIII, the joint optimization method of the mechanism of network wave and the mechanism of particle access is proposed. Finally, conclusions are drawn in Section IX.
II. Basic idea

This section describes the basic idea of particle access. We organize the descriptions of this section into answers to several key problems, so as to clearly outline the basic idea.

1) How to use the concept of information particle to realize a simple and universal modeling for network traffic flows?

The four main performance measurements of the QoS guarantee for end-to-end network traffic flows are [1]: bandwidth, delay, jitter, and packet loss rate. If the bandwidth and the performance of end-to-end delay in the network are acceptable, the delay jitter and the packet loss rate can be improved by using some caching technologies residing on the receiver side and some packet retransmission technologies, respectively. Thus, among the four performance measurements, bandwidth and delay are more important. Furthermore, if the number of information bits $\ell$ born by a packet and its tolerable end-to-end delay $L$ are given, one can easily estimate that the bandwidth demand of the packet is $B = \ell/L$. Therefore, considering that there is a simple conversion relationship between bandwidth and delay, and the delay is often the basis for estimating the required bandwidth, it is evident that among the four QoS measurements mentioned above, delay is the most critical one. This is why we firmly grasp the measurement of delay in the proposed traffic flow model which serves as the basis of the idea of particle access.

In order to build a mechanism which can effectively support the end-to-end QoS guarantee in large-scale networks, it is necessary to set up a simple and general model for the end-to-end traffic flows, based on which an efficient and flexible QoS mechanism can be built. In section I, we have analyzed the problems faced by flow based and class based QoS mechanisms. To solve these problems, it is believed that the new traffic model should not dwell on the level of each traffic flow or on the level of each traffic class, but should further focus on the more basic and fine-grained level, that is, the level of each packet, which makes the descriptions of the characteristics of end-to-end traffic flows are more fundamental. Therefore, the following assumptions are made:

a) An end-to-end traffic flow can be divided into several information blocks with a certain size (i.e. bearing capacity $\ell$ of information, unit: bit);

b) Each information block corresponds to a value of end-to-end delay tolerance (i.e. the deadline $L$ of the information block, unit: second);
c) For an information block with a bearing capacity of $\ell$ and a deadline of $L$, if the network can successfully transmit all the information bits it bears to the receiving end before $L$, the transmission of the information block is regarded as valid.

In this paper, we define the information block with the attributes of bearing capacity and deadline as an information particle (to highlight that it is the cornerstone for constructing the QoS mechanism), and accordingly, an end-to-end traffic flow is modeled as an information particle flow composed of information particles (see sections III and IV for relevant details).

2) How to achieve efficient processing of information particles?

In the mechanism of particle access, network routers need to deal with the access and transmission of information particle groups composed of a large number of information particles. Specifically, it is necessary for routers to decide how much access bandwidth is allocated to the information particle groups and how to realize the orderly and efficient transmission of information particles within a given bandwidth. Generally speaking, there are three possible methods to be adopted:

a) Determining the required bandwidth and transmission time instant for each information particle according to the attributes of each of them;

b) According to the attributes of each information particle, some statistical characteristics of the corresponding information particle group are evaluated, and a rough estimation of the bandwidth required by the whole information particle group is obtained. Of course, in addition to evaluating the total bandwidth required by the information particle group, it is also necessary to further schedule the transmission of each information particle contained in the information particle group;

c) According to the attributes of each information particle, the bandwidth required by the corresponding information particle group is accurately evaluated, and based on which, the transmission of each information particle contained in the information particle group is further scheduled.

It is easy to be found that in method “a”, because a single information particle is processed one by one, it is bound to bring about huge processing overhead. Therefore, method “a” is the least desirable one of the three methods. As for method “b”, it has less processing overhead and can bring
about better scalability, but the statistical evaluation method adopted leads to large deviation in the estimation of required bandwidth. If the estimation of bandwidth is too large relative to the real one, it will cause the unnecessary waste of access resources. On the other hand, if the estimated access bandwidth is too small relative to the real one, it will affect the effective transmissions of some information particles, and eventually affect the QoS of the end-to-end traffic flow. For method “c”, if one can find a simple and accurate method to evaluate the required bandwidth and a simple and effective transmission strategy for information particles, this method is undoubtedly the most ideal one. Because we have found out a simple and accurate bandwidth evaluation method and a simple strategy which can realize efficient transmission of information particles under a given access bandwidth, method “c” is naturally adopted to realize efficient particle access in the paper (see sections V, VI and VII for relevant details).

3) How to allocate access bandwidth for an information particle group? How to achieve efficient transmission of an information particle group?

For the former problem, our idea is to find out what is the minimum access bandwidth (defined as “minimum reachable access bandwidth of an information particle group” in the paper) that can make the transmission of all the information particles valid under the condition that the corresponding information particle group is given (that is, when the basic attributes of the information particles constituting the information particle group are known). For the latter problem, the basic idea of the paper is to find out a simple and effective transmission strategy that can achieve the minimum reachable access bandwidth. It can be seen that only by solving these two key problems at the same time can one say that an efficient method to transmit information particle group has been found.

In sections V and VI, we analyze the relationships between the average access bandwidth requirement, the peak average access bandwidth requirement, and the minimum reachable access bandwidth of an information particle group, and point out that at least there exist a transmission strategy of EDF (early deadline first), which can achieve efficient transmissions of all information particles under the constraint of the minimum reachable access bandwidth. Furthermore, in Theorem VII.1 proved in section VII, a simple method of obtaining the minimum reachable access bandwidth of an information particle group is proposed.
4) How to dynamically adjust the access bandwidth of an information particle group?

With the passage of time and the advancement of the transmission procedure of each information particle constituting an information particle group, the key attributes of the information particles contained in the information particle group (such as the attribute of bearing capacity and the deadline attribute) show some time variability, which leads to the time variability of the minimum reachable access bandwidth required by the information particle group. Therefore, we define the time point when the minimum reachable access bandwidth changes as the inflection point of the required access bandwidth of the information particle group. In Section VII, we systematically analyze the properties of the inflection point of the required access bandwidth, and put forward a method to determine the inflection point and the corresponding newly changed minimum reachable access bandwidth at that point. Furthermore, based on the theoretical results proved in the paper, in Section VII, a particle access algorithm based on dynamically adjusting the minimum reachable access bandwidth is proposed. The algorithm dynamically adjusts the minimum reachable access bandwidth of the information particle group at inflection point of the required access bandwidth, so as to ensures that the access efficiency is maintained to be one during the whole procedure of the access and transmission of the information particle group (that is, the optimal utilization of the allocated bandwidth can be achieved).

5) What are the advantages of particle access?

After comparing the problems pointed out in Section I with the basic idea of particle access described in this section, it is not difficult for us to summarize the advantages of particle access as follows:

a) Due to the efficient computation of the access bandwidth and the exerted transmission strategy of an information particle group, particle access has good scalability;

b) Since the service quality of an information flow is guaranteed at the level of each information particle, particle access can achieve more fine-grained QoS guarantee;

c) Since the idea of providing differentiated services at the level of each traffic flow or at the level of each traffic class is abandoned, particle access can completely solve the intrinsic problems of fairness existing between different traffic flows or traffic classes;
d) Since it has the ability of dynamically adjusting the minimum reachable access bandwidth, the mechanism of particle access can dynamically maximize the utilization of bandwidth resources for an information particle group while still meeting the delay requirement of each information particle.
III. Definition of an information particle

In this section, the concept of an information particle, the basic attributes of an information particle (including the bearing capacity, the initial time instant and the deadline, etc.), the effective survival time interval, the effective survival time span, the instantaneous transmission rate of an information particle, the transmission strategy exerted to an information particle, the validity of a transmission strategy, the average access bandwidth requirement of an information particle, and the arrival status of an information particle, etc. are defined.

Definition III.1: An information particle

An information particle is the information block to be transmitted with specific basic attributes (including the bearing capacity, the initial time instant and the deadline, etc.), which is identified by its sequence number \( i \in \mathbb{Z}^+ \).

Definition III.2: Time instant \( t \in \mathbb{R}^+ \) (unit: second) on the time axis

In the paper, time instant \( t=0 \), refers to the current time instant of the network. Time instant \( t>0 \), refers to the future time instant of the network. Time instant \( t<0 \), refers to the past time instant of the network. Unless otherwise specified, this article only considers the case of \( t \geq 0 \).

Definition III.3: Bearing capacity \( \ell_i \in \mathbb{R}^+ \) of an information particle (unit: bit)

The amount of information contained in the information bits born by the information particle \( i \) to be transmitted at the current time \( t \). With the passage of time, the bearing capacity \( \ell_i \) may decrease (because some information bits of the information particle \( i \) have been transmitted). Unless otherwise specified, this article only considers the case of \( 0 < \ell_i < +\infty \).

Note III.1:

For the convenience of subsequent processing, the bearing capacity of an information particle is treated as a positive real number in this article.
Definition III.4: Initial time instant \( t_{b,i} (\in \mathbb{R}) \) of an information particle (unit: second)

Relative to the current time \( t \), the earliest time when the access network can start the transmission of the information particle \( i \). Unless otherwise specified, this article only considers the case of \( t_{b,i} = 0 \).

Definition III.5: Deadline of an information particle \( t_{e,i} (\in \mathbb{R}^+) \) (unit: second)

Relative to the current time \( t \), the latest time when the access network can transmit the information particles \( i \). Unless otherwise specified, this article only considers the case of \( t_{e,i} > t_{b,i} \).

Note III.2:

In the following text, only valid information particles are considered, that is, an information particle \( i \) that meets the following two conditions at the same time is considered to be valid:

1) The bearing capacity of it should satisfy \( 0 < \ell_j < +\infty \).

2) The deadline of it must be greater than its initial time instant, i.e. \( t_{e,i} > t_{b,i} \).

Definition III.6: Effective survival time interval \( D_{e,i}^{\text{me}} \) of an information particle

The effective survival time interval of an information particle is defined as the closed time interval \( D_{e,i}^{\text{me}} \equiv [t_{b,i}, t_{e,i}] \), that is, it is meaningful to transmit and process the information particle \( i \) within \( D_{e,i}^{\text{me}} \).

Definition III.7: Effective survival time span of an information particle \( L_{\text{e,i}}^{\text{me}} (\in \mathbb{R}^+) \) (unit: second)

As for the information particle \( i \), it is defined as the time length covered by its effective survival time interval \( D_{e,i}^{\text{me}} \), i.e. \( L_{\text{e,i}}^{\text{me}} \equiv t_{e,i} - t_{b,i} \).

Definition III.8: Instantaneous transmission rate \( r_i(t) (r_i(t) \in \mathbb{R}, 0 \leq r_i(t) < +\infty) \) of an
information particle $i$ at time $t$ (unit: bits per second)

It is defined as the instantaneous information transmission rate of the information particle $i$ at a specific time instant $t$.

Definition III.9: Transmission strategy $S_i$ exerted to an information particle $i$

It is defined as the distribution function $r_i(t)$ of the instantaneous transmission rate within the effective survival time interval $D_i^{\text{int}}$ of an information particle $i$, i.e. $S_i \triangleq r_i(t) (t \in D_i^{\text{int}})$.

Definition III.10: Validity of the transmission strategy $S_i$ for an information particle $i$

If all the information bits carried by an information particle $i$ can be successfully transmitted out at a limited instantaneous transmission rate within the effective survival time interval $D_i^{\text{int}}$ of the information particle $i$ by using the transmission strategy $S_i$, it is said that the transmission strategy $S_i$ is valid for the information particle $i$. Specifically, as for a valid $S_i$, $\int_{t \in D_i^{\text{int}}} r_i(t)\,dt = \ell_i$ and $r_i(t) < +\infty (t \in D_i^{\text{int}})$ should be satisfied.

Definition III.11: The average access bandwidth requirement of an information particle $B_{\text{av},i} (\in \mathbb{R}^+)$ (unit: bits per second)

For an information particle $i$, $B_{\text{av},i} \triangleq \ell_i / \mathcal{L}_i^{\text{int}}$ is define as the average access bandwidth requirement of $i$ in the access network.

Definition III.12: Arrival status of an information particle

For an information particle $i$, if its initial time instant is $t_{i,0}=0$, its arrival status is regarded as “arrived”. Unless otherwise specified, this paper only considers the case that the information particle is “arrived”.

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IV. Definition of an information particle group

In this section, we define the concept of an information particle group, the incremental sorting operation performed on an information particle group, the bearing capacity of an information particle group, the effective survival time interval of an information particle group, the effective survival time span, the transmission strategy exerted to an information particle group, the validity of a transmission strategy exerted to an information particle group, the average access bandwidth requirement of an information particle group, the peak average access bandwidth requirement, the principal subgroup of an information particle group, the reachable access bandwidth of an information particle group, the arrival status of an information particle group, the access efficiency of an information particle group under a given access bandwidth, the inflection point of the required access bandwidth, the largest principal subgroup of an information particle group, a switching function of an information particle group, the vector of switching functions of an information particle subgroup, and the vector of average access bandwidth requirements of an information particle subgroup and so on.

Definition IV.1: An information particle group

A nonempty set containing $N \geq 1$ valid information particles, denoted as $Q$.

Definition IV.2: Incremental sorting operation $Op_{inc}(Q)$ performed on an information particle group $Q$

Given the information particle group $Q$, and let it be composed of $N \geq 1$ information particles. If the information particles are sorted from small to large according to their deadlines, and the information particle ranked first are given the sequence number “1” (its deadline is $t_{e,1}$), the information particle ranked second are given the sequence number “2” (its deadline is $t_{e,2} \geq t_{e,1}$),..., and the information particle ranked $N$ are given the sequence number “$N$” (its deadline is $t_{e,N} \geq t_{e,N-1}$). This sorting operation for the information particles contained in $Q$ is called incremental sorting operation, which is denoted as $Op_{inc}(Q)$. 
Definition IV.3: Bearing capacity $\ell_{Q}$ (\(\in \mathbb{R}^{+}\)) of an information particle group (unit: bit)

For an information particle group $Q$, the sum of the bearing capacities of all the information particles contained therein, i.e. $\ell_{Q} \triangleq \sum_{i \in Q} \ell_{i}$. Unless otherwise specified, this article only considers the case of $0 < \ell_{Q} < +\infty$.

Definition IV.4: Effective survival time interval $D_{Q}^{sw}$ of an information particle group

For an information particle group $Q$, the union of the effective survival time intervals of all the information particles contained therein, i.e. $D_{Q}^{sw} \triangleq \bigcup_{i \in Q} D_{i}^{sw}$.

Definition IV.5: Effective survival time span $L_{Q}^{sw}$ (\(\in \mathbb{R}^{+}\)) of an information particle group (unit: second)

For an information particle group $Q$, the total time length covered by its effective survival time interval $D_{Q}^{sw}$, which is denoted as $L_{Q}^{sw}$ (\(\in \mathbb{R}^{+}\)).

Definition IV.6: Transmission strategy $S_{Q}$ exerted to an information particle group $Q$

It is defined as the set of transmission strategies $S_{i}$ corresponding to all the information particles $i$ ($i \in Q$) contained in an information particle group $Q$, i.e. $S_{Q} \triangleq \{S_{i} | i \in Q\}$ (Note: for the convenience of subsequent analyses, for the time instant $t$ that exceeds the effective survival time interval $D_{i}^{sw}$ of the information particle $i$ ($i \in Q$) and belongs to the effective survival time interval $D_{Q}^{sw}$ of the information particle group $Q$, it is defined that the corresponding instantaneous transmission rate at the time instant $t$ is $r(t) \triangleq 0$).

Definition IV.7: The validity of a transmission strategy $S_{Q}$ exerted to an information particle group $Q$

If all the transmission strategies $S_{i}$ ($i \in Q$) contained in $S_{Q}$ are valid (see Definition III.10),
the transmission strategy $S_Q$ is said to be valid for the information particle group $Q$.

**Definition IV.8:** The average access bandwidth requirement $B_{av,Q} (\in \mathbb{R}^+)$ of an information particle group $Q$ (unit: bits per second)

For an information particle group $Q$, define $B_{av,Q} \triangleq \ell_Q / L_Q$ as its average access bandwidth requirement.

**Definition IV.9:** The peak average access bandwidth requirement $B_{pk,Q} (\in \mathbb{R}^+)$ of an information particle group $Q$ (unit: bits per second)

The maximum value of the average access bandwidth requirements of all the nonempty subsets (including $Q$ itself) contained in the information particle group $Q$. (Note: there may be more than one information particle subgroup achieving the peak average access bandwidth requirement)

**Definition IV.10:** A principal subgroup of an information particle group $Q$

If the average access bandwidth requirement of an information particle subgroup $Q' (Q' \subseteq Q)$ is $B_{av,Q'} = B_{pk,Q}$, then $Q'$ is a principal information particle subgroup of $Q$. (Note: there may be more than one information particle subgroup in $Q$)

**Definition IV.11:** The reachable access bandwidth $B_{re,Q} (\in \mathbb{R}^+, B_{re,Q} < +\infty)$ of an information particle group $Q$ (unit: bits per second)

Within the effective survival time interval $D_Q^e$ of $Q$, if there exist at least one valid transmission strategy $S_Q$ for $Q$, so that the total instantaneous transmission rate satisfies $\sum_{t=0}^{\infty} r_i(t) \leq B_{re,Q} (0 < B_{re,Q} < +\infty, t \in D_Q^e)$, it is said that the access bandwidth $B_{re,Q}$ is reachable for $Q$.

**Definition IV.12:** The minimum reachable access bandwidth $B_{re,Q}^{\min} (\in \mathbb{R}^+)$ of an information particle group $Q$ (unit: bits per second)
The minimum value of all the reachable access bandwidths of an information particle group $Q$ in its effective survival time interval $D_Q^{sw}$.

**Definition IV.13: The arrival status of an information particle group**

For an information particle group $Q$, if the arrival statuses of all the information particles contained in it are “arrived”, the arrival status of $Q$ is defined to be “arrived”. Unless otherwise specified, this paper only considers the case that the information particle group is “arrived”.

**Definition IV.14: The access efficiency of an information particle group $Q$ under a given access bandwidth and within its effective survival time interval**

For an information particle group $Q$, given its effective survival time interval $D_Q^{sw}$ and the access bandwidth $B_{r',o} (\in \mathbb{R}^+)$ (Note: this access bandwidth should be reachable), the corresponding access efficiency is defined as: $\eta_{Q,r',o} = \frac{t_0}{\int \left(B_{r',o} \cdot L_{oQ} \right) dt}$. Combined with **Definition IV.8**, it can be obtained that $\eta_{Q,r',o}$ can also be expressed as: $\eta_{Q,r',o} = \frac{B_{oQ}}{B_{r',o}}$.

**Definition IV.15: The inflection point of the required access bandwidth of an information particle group $Q$**

Given an information particle group $Q$, an incremental sorting operation $Op_{inc}(Q)$ is performed on it. If the information particle with the sorting sequence number of $N_{r,o} (\geq 1)$ is the one with the largest sequence number among all the information particles contained in all the principal information particle subgroups in $Q$, then $N_{r,o}$ is defined as the inflection point of the required access bandwidth of $Q$. (Note: the physical meaning of $N_{r,o}$ will be clarified in Section VII)

**Definition IV.16: The largest principal subgroup of an information particle group $Q$**

Given an information particle group $Q$, an incremental sorting operation $Op_{inc}(Q)$ is performed on it. Define the principal information particle subgroup containing the information
particle with the sequence number of $N_{r, Q}$ as the largest principal subgroup of $Q$, which is denoted as $Q^r_{cr}$.

**Definition IV.17:** An information particle subgroup $Q_i$ of the information particle group $Q$

Given an information particle group $Q$ composed of $N$ ($\geq 1$) information particles, and an incremental sorting operation $Op_{inc}(Q)$ is performed on it. $Q_i$ ($Q_i \subseteq Q$) is defined as the information particle subgroup composed of all the information particles whose sorting sequence numbers are not greater than $i$ ($1 \leq i \leq N$).

**Definition IV.18:** An information particle group with a sequence number $Q^i$ denotes an information particle group with a sequence number $i$.

**Definition IV.19:** A switching function of an information particle group $Q$ with a sequence number $i$

Given an information particle group $Q$ composed of $N$ ($N \geq 2$) information particles, and an incremental sorting operation $Op_{inc}(Q)$ is performed on it. A switching function $\delta_{i, Q}$ of an information particle group $Q$ with a sequence number $i$ ($2 \leq i \leq N - 1$) is defined as:

$$
\delta_{i, Q} = \begin{cases} 
1 & B^{\min}_{iv, Q_i} \geq B^{\min}_{iv, Q_i} \\
0 & B^{\min}_{iv, Q_i} < B^{\min}_{iv, Q_i}
\end{cases}
$$

**(IV.1)**

**Note IV.1:**

According to **Definition IV.17** and **Definition IV.19**, we know that $\delta_{i_1, Q_i_1} = \delta_{i_2, Q_i_2} = \cdots = \delta_{i_{N-1}, Q_i_{N-1}} = \delta_{i, Q}$, where $Q_{i_1} \subseteq Q_{i_2} \subseteq \cdots \subseteq Q_{i_{N-1}} \subseteq Q_N = Q$.

**Definition IV.20:** The vector of switching functions of an information particle subgroup $Q_e$ ($Q_e \subseteq Q$)

Given an information particle group $Q$ composed of $N$ ($N \geq 2$) information particles, and an incremental sorting operation $Op_{inc}(Q)$ is performed on it. Let $Q_e$ ($Q_e \subseteq Q$) denote the
information particle subgroup composed of all the information particles whose sorting sequence numbers are not greater than \( n \) \((2 \leq n \leq N)\). The vector \( \delta_{Q_n} \) of switching functions of \( Q_n \) is defined as a \( n \times 1 \) column vector. That is,

\[
\delta_{Q_n} \triangleq \begin{cases} 
\left( \delta_{n-1,Q_n}, 1 - \delta_{n-1,Q_n} \right)_{1\times n} & (n = 2) \\
\prod_{j=1}^{n-1} \delta_{j,Q_n}, (1 - \delta_{n-1,Q_n}) \prod_{j=1}^{n-1} \delta_{j,Q_n}, \ldots, (1 - \delta_{n-2,Q_n}) \prod_{j=1}^{n-2} \delta_{j,Q_n}, \ldots, (1 - \delta_{n-1,Q_n}) \prod_{j=1}^{n-1} \delta_{j,Q_n} \right)_{1\times n} & (2 < n \leq N) 
\end{cases}
\]

\[(\text{IV.2})\]

**Definition IV.21:** The vector of average access bandwidth requirements of an information particle subgroup \( Q_n (Q_n \subseteq Q) \)

Given an information particle group \( Q \) composed of \( N \) \((N \geq 2)\) information particles, and an incremental sorting operation \( Op_{inc}(Q) \) is performed on it. Let \( Q_n (Q_n \subseteq Q) \) denote the information particle subgroup composed of all the information particles whose sorting sequence numbers are not greater than \( n \) \((2 \leq n \leq N)\). The vector \( B_{av,Q_n} \) of average access bandwidth requirements of \( Q_n \) is defined as a \( n \times 1 \) column vector. That is,

\[
B_{av,Q_n} \triangleq \left( B_{av,Q_2}, B_{av,Q_3}, \ldots, B_{av,Q_{n-1}}, B_{av,Q_n} \right)_{1\times n}
\]

\[(\text{IV.3})\]
V. The relationships between the average access bandwidth requirement, the peak average access bandwidth requirement, and the minimum reachable access bandwidth of an information particle group

In this section, the relationship between the reachable access bandwidth of an information particle group and the average access bandwidth requirements of its subgroups is proved (Lemma V.1.1); It is proved that the access efficiency of an information particle group is not greater than one under a given reachable access bandwidth and within its effective survival time interval (Corollary V.1.1.1); The relationship between the minimum reachable access bandwidth of an information particle group and the average access bandwidth requirements of its subgroups is proved (Corollary V.1.1.2); The relationship between the minimum reachable access bandwidth of an information particle group and its peak average access bandwidth requirement is proved (Corollary V.1.1.3); The necessary and sufficient conditions for an access bandwidth to be reachable are given (Lemma V.1.2); The reachability of the peak average access bandwidth requirement is proved (Corollary V.1.2.1), and finally the important conclusion which states that the minimum reachable access bandwidth of an information particle group is equal to its peak average access bandwidth requirement is proved (Theorem V.1).

**Lemma V.1.1:** Given an information particle group $Q$, let any of its reachable access bandwidth be $B_{r,v}$. If the average access bandwidth requirement of any subset $Q'$ (including $Q$ itself) contained in $Q$ is $B_{a,v}$, there is $B_{r,v} \geq B_{a,v}$.

**Proof:**

According to the definition of the reachable access bandwidth $B_{r,v}$ of an information particle group $Q$ (see Definition IV.11), the following unequal relationship holds

$$B_{r,v} \geq \sum_{i \in Q} r_i(t) \geq \sum_{i \in Q'} r_i(t), \quad t \in D_{Q}'$$

(V.1)

Integrate over the above inequality within the effective survival time interval $D_{Q}'$, it can be obtained that

$$\int_{t \in D_{Q}'} B_{r,v} dt \geq \int_{t \in D_{Q}'} (\sum_{i \in Q} r_i(t)) dt$$

(V.2)
According to the definition of the bearing capacity $\ell_{Q'}$ of an information particle subgroup $Q'$ (see Definition IV.3), there is

$$\int_{r \in Q'} \left( \sum_{i \in Q'} r_i(t) \right) dt = \ell_{Q'} \tag{V.3}$$

Substitute the above relationship into equation (2), we get

$$R_{r,v,Q} \geq \frac{\ell_{Q'}}{L_{d,Q}} = R_{r,v,Q'} \tag{V.4}$$

\[\square\]

**Corollary V.1.1.1:** The access efficiency $\eta_{Q,v,r,Q}$ of an information particle group $Q$ under a given reachable access bandwidth $B_{r,v,Q}$ and within its effective survival time interval $D_{Q}^{out}$ is not greater than one, i.e. $\eta_{Q,v,r,Q} \leq 1$.

**Proof:**

Let the information particle subgroup $Q' = Q$, we can have $R_{r,v,Q} \geq R_{r,v,Q}$ from Lemma V.1.1.

Furthermore, in combination with Definition IV.14, there is $\eta_{Q,v,r,Q} = \frac{R_{r,v,Q}}{B_{r,v,Q}} \leq 1$. \[\square\]

**Corollary V.1.1.2:** Given an information particle group $Q$, let its minimum reachable access bandwidth be $B_{r,v,Q}^{min}$. If the average access bandwidth requirement of any subset $Q'$ in $Q$ (including $Q$ itself) is $B_{r,v,Q'}$, there is $B_{r,v,Q}^{min} \geq B_{r,v,Q'}$.

**Proof:**

It can be directly proved by the definition of the minimum reachable access bandwidth $B_{r,v,Q}^{min}$ of an information particle group $Q$ (see Definition IV.12) and the conclusion given in Lemma V.1.1. \[\square\]

**Corollary V.1.1.3:** Given an information particle group $Q$, let its minimum reachable access bandwidth be $B_{r,v,Q}^{min}$ and its peak average access bandwidth requirement is $B_{r,v,Q}^{pk}$,
there is $B_{r,0}^\text{min} \geq B_{w,0}^f$.

Proof:

It is known that $B_{r,0}^\text{min} \geq B_{r,0}'$ holds for any subset $Q'$, and $B_{w,0}^f \triangleq \max\{B_{w,0}\}$, $Q' \subseteq Q$ can be obtained immediately from Definition IV.9, so $B_{r,0}^\text{min} \geq B_{w,0}^f$ holds from Corollary V.1.1.2. ■

Lemma V.1.2: Given an information particle group $Q$ (composed of $N \geq 1$ information particles). An incremental sorting operation $Q_{\text{ Inc}}$ is performed on $Q$ (see Definition IV.2). Let $Q_i$ ($Q_i \subseteq Q$) be the information particle subgroup composed of all the information particles in $Q$ whose sorting sequence numbers are not greater than $i$ ($1 \leq i \leq N$). The bearing capacity and the effective survival time span of $Q_i$ are $\ell_{Q_i}$ and $L_{t_{lim}}$, respectively. The sufficient and necessary conditions for an access bandwidth $B$ to be reachable for $Q$ is that $\frac{\ell_{Q_i}}{B} \leq L_{t_{lim}}$ holds for all $Q_i$ ($Q_i \subseteq Q$, $1 \leq i \leq N$).

Proof:

First, we prove that if the access bandwidth $B$ is reachable, then $\frac{\ell_{Q_i}}{B} \leq L_{t_{lim}}$ is true for all $Q_i$ ($Q_i \subseteq Q$, $1 \leq i \leq N$). According to the definition of the reachable access bandwidth of an information particle group $Q$ (see Definition IV.11), there is a valid transmission strategy $S_Q$ for $Q$, so that the corresponding distribution functions $r_j(t)$ ($j \in Q$) of the instantaneous transmission rates satisfy the following relationship

\[
\int_{t_{lim}} B dt \geq \int_{t_{lim}} \left(\sum_{j=1}^{N} r_j(t)\right) dt \\
\geq \int_{t_{lim}} \left(\sum_{j=1}^{i} r_j(t)\right) dt \\
= \ell_{Q_i} \quad (1 \leq i \leq N)
\]

From the above equation, it can be obtained that

\[
B \cdot L_{t_{lim}} \geq \ell_{Q_i} \quad (1 \leq i \leq N)
\]

Therefore, the following equation holds for all the information particle subgroups $Q_i$

\[
\frac{\ell_{Q_i}}{B} \leq L_{t_{lim}}
\]
Secondly, we prove that if \( \frac{\ell_{Q_i}}{B} \leq L_{\ell_{Q_i}} \) is true for all \( Q_i \), \( 1 \leq i \leq N \), then the access bandwidth \( B \) is reachable. If \( \frac{\ell_{Q_i}}{B} \leq L_{\ell_{Q_i}} \) is true for all \( Q \), it means that for all the constituent information particles \( i \) \( (1 \leq i \leq N) \) in \( Q \), if EDF (early deadline first) transmission strategy is adopted and each particle in \( Q \) is transmitted one by one at the instantaneous rate \( B \), the transmission of each constituent information particle can be completed before its deadline, i.e. \( t_{i,j} \) \( (1 \leq i \leq N) \). In other words, under the bandwidth constraint \( B \), it is found that at least EDF transmission strategy is valid for the transmission of the information particle group \( Q \). Therefore, \( B \) is a reachable access bandwidth. ■

**Discussion V.1:**

By slightly deforming the conclusion of Lemma V.1.2, it can be seen that the sufficient and necessary conditions for an access bandwidth \( B \) to be reachable for \( Q \) is that \( \frac{\ell_{Q_i}}{L_{\ell_{Q_i}}} \leq B \) holds for all information particle subgroups \( Q_i \), \( 1 \leq i \leq N \). According to the definition of the average access bandwidth requirement of an information particle group (Definition IV.8), it can be known that Lemma V.1.2 actually points out that the sufficient and necessary conditions for an access bandwidth \( B \) to be reachable is that \( B_{av,Q} = \frac{\ell_{Q}}{L_{\ell_{Q}}} \leq B \) holds for all the information particle subgroups \( Q \), \( 1 \leq i \leq N \). This conclusion can be used to verify whether a given access bandwidth can effectively bear an information particle group (i.e. meets the requirements for bearing capacities and deadlines of all information particles in it).

**Corollary V.1.2.1:** Given an information particle group \( Q \), the access bandwidth \( B_{av,Q} \) is reachable.

**Proof:**

Given an information particle group \( Q \), consider that it is composed of \( N \) \( (\geq 1) \) information particles. Let \( Q \), \( (Q \subseteq Q) \) indicate the information particle subgroup composed of all the information particles in the information particle group \( Q \) whose sorting sequence number is not
greater than \( i (1 \leq i \leq N) \) after the operation \( O_{p_{inc}}(Q) \). According to the definition of the peak average access bandwidth requirement \( B_{av,Q}^{pk} \) of an information particle group \( Q \) (see Definition IV.9), there is

\[
B_{av,Q}^{pk} \geq B_{av,Q} = \frac{Q_i}{L_{\gamma_i}} \quad (1 \leq i \leq N)
\]  \hspace{1cm} (V.8)

Therefore, the following formula holds for all \( Q \),

\[
\frac{Q_i}{B_{av,Q}^{pk}} \leq L_{\gamma_i}
\]  \hspace{1cm} (V.9)

According to Lemma V.1.2, \( B_{av,Q}^{pk} \) is reachable. ■

**Theorem V.1:** Given an information particle group \( Q \), its minimum reachable access bandwidth and its peak average access bandwidth requirement are \( B_{re,Q}^{\min} \) and \( B_{av,Q}^{pk} \) respectively, then there is \( B_{re,Q}^{\min} = B_{av,Q}^{pk} \).

**Proof:**

From Corollary V.1.1.3, we can know \( B_{re,Q}^{\min} \geq B_{av,Q}^{pk} \). And from Corollary V.1.2.1, it is known that for a given information particle group \( Q \), the access bandwidth \( B_{av,Q}^{pk} \) is reachable for \( Q \).

Therefore, it can be concluded that \( B_{re,Q}^{\min} = B_{av,Q}^{pk} \). ■

**Discussion V.2:**

1) The above theorem provides the basic method to determine the minimum reachable access bandwidth of an information particle group, that is, the minimum reachable access bandwidth can be obtained by determining the peak average access bandwidth requirement of the information particle group.

2) According to the definition of the peak average access bandwidth requirement of an information particle group (Definition IV.9), it is known that it will not vary with the reassignment of the sequence numbers of information particles contained in the corresponding information particle group (such as performing incremental sorting operation \( O_{p_{inc}}(Q) \), etc.).
Since the minimum reachable access bandwidth of an information particle group is equal to its peak average access bandwidth requirement, the minimum reachable access bandwidth will not vary with the reassignment of the sequence numbers of information particles contained in the information particle group.

**Corollary V.1.1:** All access bandwidth constraints that are not less than the minimum reachable access bandwidth of an information particle group are reachable

Given an information particle group \( Q \), its minimum reachable access bandwidth is \( B_{\text{re},Q}^{\text{min}} \), then the access bandwidth constraint \( B \geq B_{\text{re},Q}^{\text{min}} \) is reachable.

**Proof:**

According to the definition of the minimum reachable access bandwidth of an information particle group (Definition IV.12), it can be seen that the access bandwidth constraint \( B = B_{\text{re},Q}^{\text{min}} \) is reachable. Furthermore, according to Lemma V.1.2, since access bandwidth \( B_{\text{re},Q}^{\text{min}} \) is reachable, all the information particle subgroups \( Q_i (Q_i \subseteq Q, 1 \leq i \leq N) \) of \( Q \) should satisfy the relationship

\[
B_{\text{re},Q_i} = \frac{f_{Q_i}}{L_{Q_i}} \leq B_{\text{re},Q}^{\text{min}}.
\]

For access bandwidth constraint \( B > B_{\text{re},Q}^{\text{min}} \), all the information particle subgroups \( Q_i (Q_i \subseteq Q, 1 \leq i \leq N) \) satisfy \( B_{\text{re},Q_i} = \frac{f_{Q_i}}{L_{Q_i}} \leq B_{\text{re},Q}^{\text{min}} < B \). Therefore, once again, according to Lemma V.1.2, the access bandwidth constraint \( B > B_{\text{re},Q}^{\text{min}} \) is reachable. ■
VI. The influences of the time attribute and the attribute of bearing capacity of an information particle group on the minimum reachable access bandwidth

In this section, it is proved that with the increase of the deadline of one information particle in a given information particle group, the average access bandwidth requirement (Theorem VI.1) and the minimum reachable access bandwidth (Corollary VI.1.1) of the information particle group will not increase (or even decrease); It is proved that (Theorem VI.2) after adding a new information particle into a given information particle group, the minimum reachable access bandwidth of the information particle group will not decrease (or even increase); It is proved that with the decrease of the bearing capacity of an information particle in a given information particle group, the average access bandwidth requirement (Theorem VI.4) and the minimum reachable access bandwidth (Corollary VI.4.1) of the information particle group will not increase (or even decrease); It is further proved that for a given information particle group that meets a certain bandwidth constraint, it can still meet the bandwidth constraint by adjusting the deadline attribute (Theorem VI.3) and/or the bearing capacity attribute (Theorem VI.5) of a newly arrived information particle.

Theorem VI.1: Given an information particle group \( Q \) composed of \( N (\geq 1) \) information particles. Select any of an information particle \( i \ (i \in Q) \) in \( Q \) and increase its deadline from \( t_{e_i} \) to \( t'_{e_i} \) (i.e. \( t'_{e_i} \geq t_{e_i} \)). If the average access bandwidth requirement of \( Q \) varies accordingly from \( B_{av,Q} \) to \( B'_{av,Q} \) with the variation of the deadline of \( i \), there is

\[
B_{av,Q} \geq B'_{av,Q}.
\]

Proof:

Suppose that as the deadline of the information particle \( i \) increases from \( t_{e_i} \) to \( t'_{e_i} \), the effective survival time span of the information particle group \( Q \) varies from \( L_{\delta_i} \) to \( L'_{\delta_i} \). It can be seen from the definition of the average access bandwidth requirement (see Definition IV.8)

\[
B_{av,Q} = \frac{\ell_Q}{L_{\delta_i}} \quad \text{(VI.1)}
\]

\[
B'_{av,Q} = \frac{\ell_Q}{L'_{\delta_i}} \quad \text{(VI.2)}
\]
With the deadline of the information particle \( i \) \((i \in Q)\) increasing from \( t_{e,i} \) to \( t'_{e,i} \), there is

\[
L'_{e_Q} \geq L_{e_Q}
\]

(VI.3)

Based on the above three equations, we have \( B_{av,Q} \geq B'_{av,Q} \). ■

**Corollary VI.1.1**: Given an information particle group \( Q \) composed of \( N (\geq 1) \) information particles. Select any of an information particle \( i \) \((i \in Q)\) in \( Q \) and increase its deadline from \( t_{e,i} \) to \( t'_{e,i} \) (i.e. \( t'_{e,i} \geq t_{e,i} \)). If the minimum reachable access bandwidth of \( Q \) varies accordingly from \( B_{re,Q}^{\min} \) to \( B_{re,Q}^{\min'} \) with the variation of the deadline of \( i \), there is \( B_{re,Q}^{\min} \geq B_{re,Q}^{\min'} \).

**Proof:**

As the deadline of the information particle \( i \) increases from \( t_{e,i} \) to \( t'_{e,i} \), the peak average access bandwidth requirement of \( Q \) changes from \( B_{av,Q}^{pk} \) to \( B_{av,Q}^{pk'} \). The conclusion in **Theorem VI.1** is also applicable to any of a nonempty subset \( Q' \) (including \( Q \) itself) of \( Q \), that is, as the deadline of the information particle \( i \) increases from \( t_{e,i} \) to \( t'_{e,i} \), the average access bandwidth requirement of \( Q' \) will not increase. Therefore, combined with the definition of the peak average access bandwidth requirement (see **Definition IV.9**), there is

\[
B_{av,Q}^{pk'} \leq B_{av,Q}^{pk}
\]

(VI.4)

According to **Theorem V.1**, we have

\[
B_{re,Q}^{\min} = B_{re,Q}^{pk}
\]

(VI.5)

\[
B_{re,Q}^{\min'} = B_{re,Q}^{pk'}
\]

(VI.6)

Based on the above three equations, we have \( B_{re,Q}^{\min} \geq B_{re,Q}^{\min'} \). ■

**Theorem VI.2**: Given an information particle group \( Q \) composed of \( N (\geq 1) \) information particles. Let its minimum reachable access bandwidth be \( B_{re,Q}^{\min} \). A new information
particle \( i (i \notin Q) \) is added into \( Q \). Therefore, a new information particle group 

\[ Q' = i \bigcup Q \] 

is formed. Assume that the minimum reachable access bandwidth of \( Q' \) is 

\[ B_{\text{re},Q'}^{\min}, \text{there is } B_{\text{re},Q'}^{\min} \geq B_{\text{re},Q}^{\min}. \]

Proof:

Based on the definition of the peak average access bandwidth requirement (see Definition IV.9), there is

\[ B_{\text{pk},Q}^{i} \geq B_{\text{av},Q}^{i} \]  \hspace{1cm} (VI.7)

According to Theorem V.1, we have

\[ B_{\text{re},Q}^{\min} = B_{\text{pk},Q}^{i} \]  \hspace{1cm} (VI.8)

\[ B_{\text{re},Q'}^{\min} = B_{\text{pk},Q'}^{i} \]  \hspace{1cm} (VI.9)

Based on the above three equations, we have \( B_{\text{re},Q'}^{\min} \geq B_{\text{re},Q}^{\min} \). ■

**Theorem VI.3:** Given an information particle group \( Q \) composed of \( N \) (\( \geq 1 \)) information particles. Let its minimum reachable access bandwidth be \( B_{\text{re},Q}^{\min} \). A new information particle \( i (i \notin Q) \) is added into \( Q \), and a new information particle group \( Q' (Q' = i \bigcup Q) \) is formed. The minimum reachable access bandwidth of \( Q' \) is \( B_{\text{re},Q'}^{\min} \), and its bearing capacity is \( f_{Q'} < +\infty \). It is assumed that the adjustment of the minimum reachable access bandwidth \( B_{\text{re},Q'}^{\min} \) of \( Q' \) can be achieved by only changing the deadline \( t_{i,e} \) of a newly added information particle \( i \). And an access bandwidth constraint \( B (\in \mathbb{R}^+) \) which satisfies the relationship of \( B > B_{\text{re},Q}^{\min} \) is given. It can be concluded that there always exists a value of \( t_{i,e} (\in \mathbb{R}^+) \) which makes the corresponding \( B_{\text{re},Q'}^{\min} \) satisfy the relationship of

\[ B_{\text{re},Q'}^{\min} \leq B. \]

Proof:

Set the deadline \( t_{i,e} \) to a positive real number \( t' (\in \mathbb{R}^+) \) with \( t' > t_{i,e} \). Accordingly, the
corresponding minimum reachable access bandwidth of $Q'$ is denoted as $B_{\min, r, Q'}$. Therefore, there are two cases to be considered, namely

**Case 1:** $B_{\min, r, Q'} \leq B$

In this case, by setting $t_{e,i}$ to $t'$, the proposition can be satisfied.

**Case 2:** $B_{\min, r, Q'} > B$

From the definition of the peak average access bandwidth requirement (see **Definition IV.9** and the assumption that the bearing capacity $\ell_{Q'} < +\infty$ (see **Definition IV.3**), it can be seen that

$$\lim_{t_{e,i} \to +\infty} B_{av, Q'}^p = B_{av, Q}^p$$  \hfill (VI.10)

Furthermore, based on **Theorem V.1**, we have

$$B_{\min, r, Q'} \to +\infty \Rightarrow B_{av, Q'} = B_{av, Q} = B_{av, Q}^\min$$  \hfill (VI.11)

Namely

$$B_{\min, r, Q'} \to +\infty \Rightarrow B_{\min, r, Q'} < B$$  \hfill (VI.12)

According to the definition of limit, there must be a positive real number $\delta$ ($> t'$) which makes $B_{\min, r, Q', > \delta} \leq B$ hold when $t_{e,i} > \delta$. Therefore, the proposition in this case holds.

Since the above two cases must be one of them, the proposition is proved. ■

---

**Theorem VI.4:** Given an information particle group $Q$ composed of $N$ ($\geq 1$) information particles. Select any of an information particle $i$ ($i \in Q$) in $Q$ and decrease its bearing capacity from $\ell_i$ to $\ell'_i$ (that is $0 < \ell'_i \leq \ell_i$). As the bearing capacity of $i$ varies, the average access bandwidth requirement of $Q$ changes from $B_{av, Q}$ to $B_{av, Q}'$. It can be concluded that $B_{av, Q} \geq B_{av, Q}'$.

**Proof:**

As the bearing capacity of the information particle $i$ decreases from $\ell_i$ to $\ell'_i$, the bearing capacity of the information particle group $Q$ varies from $\ell_Q$ to $\ell'_Q$. It can be seen from the definition of the average access bandwidth requirement (see **Definition IV.8**).
\[ B_{av,Q} = \frac{\ell_Q}{L_{2Q}} \] \hspace{1cm} (VI.13)

\[ B'_{av,Q} = \frac{\ell'_Q}{L_{2Q}} \] \hspace{1cm} (VI.14)

Since the bearing capacity of the information particle \( i (i \in Q) \) decreases from \( \ell_i \) to \( \ell'_i \), we have

\[ \ell'_Q \leq \ell_Q \] \hspace{1cm} (VI.15)

Based on the above three equations, it can be concluded that

\[ B_{av,Q} \geq B'_{av,Q}. \] ■

**Corollary VI.4.1:** Given an information particle group \( Q \) composed of \( N (\geq 1) \) information particles. Select any of an information particle \( i (i \in Q) \) in \( Q \) and decrease its bearing capacity from \( \ell_i \) to \( \ell'_i \) (that is \( 0 < \ell'_i \leq \ell_i \)). As the bearing capacity of \( i \) varies, the minimum reachable access bandwidth of \( Q \) changes from \( B_{re,Q}^{min} \) to \( B_{re,Q}'^{min} \). It can be concluded that \( B_{re,Q}^{min} \geq B_{re,Q}'^{min} \).

**Proof:**

As the bearing capacity of information particle \( i \) decreases from \( \ell_i \) to \( \ell'_i \), the peak average access bandwidth requirement of the information particle group \( Q \) changes from \( B_{av,Q}^{pk} \) to \( B_{av,Q}'^{pk} \). Based on Theorem VI.4 and the definition of the peak average access bandwidth requirement (see Definition IV.9), there is

\[ B_{av,Q}^{pk} \leq B_{av,Q}^{pk}' \] \hspace{1cm} (VI.16)

According to Theorem V.1, we have

\[ B_{re,Q}^{min} = B_{av,Q}^{pk} \] \hspace{1cm} (VI.17)

\[ B_{re,Q}'^{min} = B_{av,Q}'^{pk} \] \hspace{1cm} (VI.18)

Based on the above three equations, it can be concluded that \( B_{re,Q}^{min} \geq B_{re,Q}'^{min} \). ■

**Theorem VI.5:** Given an information particle group \( Q \) composed of \( N (\geq 1) \) information
particles. Let its minimum reachable access bandwidth be $B_{rQ}^{\min}$. A new information particle $i$ ($i \notin Q$) is added into $Q$, and a new information particle group $Q'$ ($Q' = i \cup Q$) is formed. The minimum reachable access bandwidth of $Q'$ is $B_{rQ'}^{\min}$. It is assumed that the adjustment of the minimum reachable access bandwidth $B_{rQ'}^{\min}$ of $Q'$ can be achieved by only changing the bearing capacity $\ell_i$ of a newly added information particle $i$. And an access bandwidth constraint $B (\in \mathbb{R}^+)$ which satisfies the relationship of $B > B_{rQ}^{\min}$ is given. It can be concluded that there always exists a value of $\ell_i (\in \mathbb{R}^+)$ which makes the corresponding $B_{rQ'}^{\min}$ satisfy the relationship of $B_{rQ'}^{\min} \leq B$.

Proof:

Set the bearing capacity $\ell_i$ to a positive real number $\ell' (\in \mathbb{R}^+)$. Accordingly, the corresponding minimum reachable access bandwidth of $Q'$ is denoted as $B_{rQ'\ell}^{\min}$. Therefore, there are two cases to be considered, namely

**Case 1:** $B_{rQ'\ell}^{\min} \leq B$

In this case, by setting $\ell_i$ to $\ell'$, the proposition can be satisfied.

**Case 2:** $B_{rQ'\ell}^{\min} > B$

From the definition of the peak average access bandwidth requirement (see Definition IV.9), it can be seen that

$$\lim_{\ell_i \to \ell'} B_{rQ'}^{pk} = B_{rQ'}^{\min}$$  \hspace{1cm} (VI.19)

Furthermore, based on Theorem V.1, we have

$$B_{rQ'\ell}^{\min} = \lim_{\ell_i \to \ell'} B_{rQ'}^{pk} = B_{rQ'}^{\min} = B_{rQ}^{\min}$$  \hspace{1cm} (VI.20)

Namely

$$B_{rQ'\ell}^{\min} = B_{rQ}^{\min} < B$$  \hspace{1cm} (VI.21)

According to the definition of limit, there must be a positive real number $\delta (< \ell')$ which makes $B_{rQ'\ell_i < \delta} \leq B$ hold when $\ell_i < \delta$. Therefore, the proposition in this case holds.
Since the above two cases must be one of them, the proposition is proved. ■
VII. A particle access algorithm based on dynamically adjusting the minimum reachable access bandwidth

In this section, a more concise expression of the minimum reachable access bandwidth of an information particle group is proved (Theorem VII.1); In Corollary VII.1.1, it is proved that when the access bandwidth of a given information particle group is set to be equal to its minimum reachable access bandwidth, the access efficiency of any of its principal information particle subgroup reaches one (that is, for any of its principal information particle subgroup, the allocated access bandwidth can be fully utilized). The problem of each principal information particle subgroup being composed of which information particles is studied (Corollary VII.1.2), and the inclusion and included relationships among several principal information particle subgroups are proved (Corollary VII.1.3). The necessary and sufficient conditions for several information particles forming or not forming a principal information particle subgroup are proved (Corollary VII.1.4). The fact that the minimum reachable access bandwidth will decrease at the inflection point of the access bandwidth requirement of an information particle group is proved (Corollary VII.1.5). Based on the theoretical basis built in this section, a particle access algorithm based on dynamically adjusting the minimum reachable access bandwidth is proposed (Algorithm VII.1). The upper bound (Corollary VII.1.6) and the lower bound (Corollary VII.1.7) of the minimum reachable access bandwidth of an information particle group are given, respectively. The property of linear superpositions of multiple information particle groups under specific constraints is proved (Corollary VII.1.8). The property of linear splitting of an information particle group is also proved (Corollary VII.1.9). A closed form expression of representing the minimum reachable access bandwidth of an information particle subgroup as the product of its vector of switching functions and its vector of average access bandwidth requirements is given (Corollary VII.1.10). And it is proved that the vector of switching functions has and only has one component with its value being equal to one (Corollary VII.1.11).

A. The theorem of the minimum reachable access bandwidth of an information particle group

| Theorem VII.1: Given an information particle group $Q$ composed of $N \geq 1$ information particles, and an incremental sorting operation $Op_{inc}(Q)$ is performed on it. | 35 / 86 |
Let $B_{av,Q}$ indicate the average access bandwidth requirement of the information particle subgroup $Q_i (Q_i \subseteq Q)$ composed of all the information particles whose sorting sequence numbers are not greater than $i (1 \leq i \leq N)$, then there is $B_{av,Q} = \max_{i=1,...,N} \{B_{av,Q_i}\}$.

**Proof:**

Consider the information particle with sequence number of $N$ after the operation of $Op_{inc}(Q)$. For the case of $N=1$, since the proposition conclusion obviously holds, its proof is omitted. Next, let us consider the case of $N \geq 2$:

The set $Q_{(N,j),k} (0 \leq j \leq N-1, k = 1, 2, ...)$ is defined as the information particle subgroup composed of one information particle with sequence number of $N$ and $j$ information particles whose sequence numbers are smaller than $N$ (Note: different information particle subgroup is distinguished by the corresponding subscript of $k$), i.e. $Q_{(N,j),k} \subseteq Q$. In particular, when $j=0$, $Q_{(N,0),k}$ represents an information particle subgroup containing only the information particle with sequence number of $N$; When $j=N-1$, $Q_{(N,N-1),1}$ represents a subgroup of information particles composed of all the information particles whose sequence numbers are not greater than $N$, i.e. $Q_{(N,N-1),1} = Q_N$ (in fact $Q_N = Q$). According to the definition of the average access bandwidth requirement (see Definition IV.8), it can be obtained that

$$B_{av,Q_{(N,j),k}} = \frac{\ell_{Q_{(N,j),k}}}{L_{D_{av,Q_{(N,j),k}}}} = \frac{\sum_{i \in Q_{(N,j),k}} \ell_i}{L_{Q_{(N,j),k}}},$$

(VII.1)

In the information particle group $Q$, it can be obtained from the definition of the effective survival time span of an information particle group (see Definition IV.5)

$$L_{D_{av,Q}} = L_{D_{av,Q}}$$

(VII.2)

Combing the above two equations, we have
That is, as to the information particle \( N \), the maximum value of \( B_{av,\Omega_{N,1}} \) is \( B_{av,Q} \) (and it is obvious that \( B_{av,\Omega_{N,1}} = B_{av,Q} \)). Furthermore, according to equation (VII.3) and the definition of the peak average access bandwidth requirement of an information particle group \( Q \) (see Definition IV.9), the following equation can be obtained

\[
B_{av,Q}^{\text{pk}} = \max \{ B_{av,\Omega_{N,1}}^{\text{pk}}, B_{av,Q} \}
\tag{VII.4}
\]

By iteratively using the above conclusion, we can get

\[
B_{av,Q}^{\text{pk}} = \max \{ B_{av,\Omega_{N,1}}^{\text{pk}}, B_{av,Q} \} = \max \{ \max \{ B_{av,\Omega_{N,2}}^{\text{pk}}, B_{av,Q} \}, B_{av,Q} \} = \max \{ B_{av,\Omega_{N,2}}^{\text{pk}}, B_{av,Q} \} = \max \{ \max \{ B_{av,\Omega_{N,3}}^{\text{pk}}, B_{av,Q} \}, B_{av,Q} \} = \max \{ B_{av,\Omega_{N,3}}^{\text{pk}}, B_{av,Q} \} = \ldots = \max \{ B_{av,Q}, \ldots, B_{av,Q} \}
\tag{VII.5}
\]

According to Theorem V.1, we have

\[
B_{av,Q}^{\text{min}} = B_{av,Q}^{\text{pk}} = \max \{ B_{av,Q} \}
\tag{VII.6}
\]

Discussion VII.1:

The above proposition provides a simple way of calculating the minimum reachable access bandwidth of an information particle group.

B. Some properties of information particle subgroups

**Corollary VII.1.1:** Given an information particle group \( Q \) composed of \( N \) (\( \geq 1 \)) information particles. Let its access bandwidth be equal to its minimum reachable access bandwidth \( B_{av,Q}^{\text{min}} \). The access efficiency of any of its principal information particle subgroup
Proof:

According to the definition of the access efficiency (see Definition IV.14), the definition of a principal information particle subgroup (see Definition IV.10) and Theorem V.1, we can get

\[
\eta_{Q''_r | \min Q} = \frac{B_{m,Q''_r}}{B_{\min \tau,Q}} = \frac{B_{m,Q}}{B_{\min \tau,Q}} = 1. \quad \blacksquare
\]

Discussion VII.2:

Access efficiency \( \eta_{Q''_r | \min Q} = 1 \), which means that when the access bandwidth of an information particle group \( Q \) is limited to its minimum reachable access bandwidth \( B_{\min \tau,Q} (=B_{m,Q}) \), within the effective survival time interval \( D_{Q''_r} \) covered by the principal information particle subgroup \( Q''_r \), the access resources with bandwidth \( B_{\min \tau,Q} \) are completely occupied by the transmission of \( Q''_r \), that is, the bandwidth resources are fully utilized.

Corollary VII.1.2: Given an information particle group \( Q \) composed of \( N (\geq 1) \) information particles, and an incremental sorting operation \( Q_{\text{inc}}(Q) \) is performed on it. If the maximum sequence number of all the information particles contained in a principal information particle subgroup \( Q''_r \) (\( Q''_r \subseteq Q \)) is \( k (k \geq 1) \), \( Q''_r \) must contain all the information particles in \( Q \) whose sequence numbers are greater than or equal to 1 and less than or equal to \( k (k \geq 1) \).

Proof:

For the case of \( k=1 \), the proposition is obviously true. Proof omitted.

Consider the case of \( k > 1 \). Reduction to absurdity is adopted. Assuming that there exist an information particle \( j (1 \leq j < k, \ell_j > 0) \) in \( Q \) which does not belong to the principal information particle subgroup \( Q''_r \), \( j \) and \( Q''_r \) can then be combined to form another information particle subgroup \( Q''_r = Q'' \cup j \). According to the definition of the average access bandwidth requirement (see Definition IV.8), we have
\[ \begin{align*}
B_{w,Q''} &= \frac{\sum_{i \in Q''} \ell_i}{L_{D_{w,Q''}^0}} = \frac{\sum_{i \in Q''} \ell_i}{L_{D_{w,Q''}^0}} \\
&= \frac{\ell_j + \sum_{i \in Q''} \ell_i}{L_{D_{w,Q''}^0}} + B_{w,Q''} > B_{w,Q''}
\end{align*} \quad \text{(VII.7)}
\]

The relationship \( L_{D_{w,Q''}^0} = L_{D_{w,Q''}^0} \) is used in the above equation. This is because that the sequence number of the information particle \( j \) is smaller than that of the information particle \( k \), so the effective survival time interval \( D_{Q''}^{w0} \) of the information particle subgroup \( Q'' \) and the effective survival time interval \( D_{Q''}^{w0} \) of \( Q'' \) are completely overlapped. The above equation is in contradiction with the premise (i.e. the information particle subgroup \( Q'' \) is a principal information particle subgroup). ■

**Discussion VII.3:**

The above proposition reveals that there exists continuity among the sequence numbers of information particles constituting a principal information particle subgroup.

**Corollary VII.1.3:** Given an information particle group \( Q \) composed of \( N (\geq 1) \) information particles, and an incremental sorting operation \( Op_{inc}(Q) \) is performed on it. Let the corresponding maximum sequence numbers of information particles contained in principal information particle subgroups \( Q'_{cr1} (Q'_{cr1} \subseteq Q) \) and \( Q'_{cr2} (Q'_{cr2} \subseteq Q) \) are \( k_1 (k_1 \geq 1) \) and \( k_2 (k_2 \geq 1) \) respectively, and \( k_1 < k_2 \). It can be concluded that \( Q'_{cr1} \) is a true subset of \( Q'_{cr2} \), i.e. \( Q'_{cr1} \subset Q'_{cr2} \).

**Proof:**

According to **Corollary VII.1.2**, \( Q'_{cr1} \) contains all the information particles with sequence numbers greater than or equal to 1 and less than or equal to \( k_1 \) in \( Q \), and \( Q'_{cr2} \) contains all the information particles with sequence numbers greater than or equal to 1 and less than or equal to \( k_2 (k_2 > k_1) \) in \( Q \), so it is evident that \( Q'_{cr2} \) contains \( Q'_{cr1} \). ■

**Discussion VII.4:**
The above proposition suggests that for an information particle group \( Q \) containing finite number of elements, there must exist a largest principal information particle subgroup (defined in Definition IV.16) containing all the other principal information particle subgroup.

**Corollary VII.1.4:** Given an information particle group \( Q \) composed of \( N \geq 1 \) information particles, and an incremental sorting operation \( Op_{inc}(Q) \) is performed on it. Let \( Q_k (Q_k = \{ j \mid 1 \leq j \leq k, j \in Q \}) \) be the information particle subgroup composed of all the information particles in \( Q \) whose sequence number is not greater than \( k \) \((k \geq 1)\), and the effective survival time span of \( Q_k \) is \( L_{Q_k}^{\text{av}} \). It can be concluded that

1. The sufficient and necessary condition for \( Q_k \) being a principal information particle subgroup of \( Q \) is:
   \[
   L_{Q_k}^{\text{av}} = \left( \sum_{i=1}^{k} \ell_i \right) / B_{av,Q} \cdot \left( \sum_{i=1}^{k} \frac{1}{b_i} \right)
   \]
2. The sufficient and necessary condition for \( Q_k \) being not a principal information particle subgroup of \( Q \) is:
   \[
   L_{Q_k}^{\text{av}} > \left( \sum_{i=1}^{k} \ell_i \right) / B_{av,Q} \cdot \left( \sum_{i=1}^{k} \frac{1}{b_i} \right)
   \]

**Proof:**

First, let us prove the conclusion (1). If \( Q_k \) is a principal information particle subgroup of \( Q \), according to the definition of a principal information particle subgroup (see Definition IV.10), the average access bandwidth requirement of \( Q_k \) is \( B_{av,Q_k} = B^{av}_{av,Q} \), i.e.

\[
B_{av,Q_k} = \left( \sum_{i=1}^{k} \ell_i \right) / L_{Q_k}^{\text{av}} = B^{av}_{av,Q}.
\]

Therefore, there is \( L_{Q_k}^{\text{av}} = \left( \sum_{i=1}^{k} \ell_i \right) / B^{av}_{av,Q} \); Furthermore, if \( L_{Q_k}^{\text{av}} = \left( \sum_{i=1}^{k} \ell_i \right) / B^{av}_{av,Q} \) holds, there is \( B_{av,Q_k} = \left( \sum_{i=1}^{k} \frac{1}{b_i} \right) / B^{av}_{av,Q} \). Therefore, according to the definition of a principal information particle subgroup (see Definition IV.10), \( Q_k \) is a principal information particle subgroup of \( Q \). Therefore, conclusion (1) is proved.

Next, let us prove the conclusion (2). If \( Q_k \) is not a principal information particle subgroup of \( Q \), according to the definition of a principal information particle subgroup (see Definition IV.10), the average access bandwidth requirement of \( Q_k \) is \( B_{av,Q_k} < B^{av}_{av,Q} \), i.e.

\[
B_{av,Q_k} = \left( \sum_{i=1}^{k} \ell_i \right) / L_{Q_k}^{\text{av}} < B^{av}_{av,Q}.
\]

Therefore, there is \( L_{Q_k}^{\text{av}} > \left( \sum_{i=1}^{k} \ell_i \right) / B^{av}_{av,Q} \); Furthermore, if \( L_{Q_k}^{\text{av}} > \left( \sum_{i=1}^{k} \ell_i \right) / B^{av}_{av,Q} \) holds, there is \( B_{av,Q_k} = \left( \sum_{i=1}^{k} \frac{1}{b_i} \right) / L_{Q_k}^{\text{av}} < B^{av}_{av,Q} \). Therefore, according to the definition of a principal information particle
subgroup (see Definition IV.10). $Q_k$ is not a principal information particle subgroup of $Q$.

Therefore, conclusion (2) is proved. ■

Discussion VII.5:

1) On one hand, based on the above proposition, if the information particles contained in an information particle group $Q$ are transmitted according to the EDF transmission strategy with the access bandwidth being set to be $B_{av,Q}$, and moreover, if the completion time of the transmission of the information particle with sequence number of $k (k \geq 1)$ is 
\[
\left( \sum_{i=1}^{k} \lambda_i \right) \frac{1}{B_{av,Q}} < L_{DQk}^a,
\]
which indicates that the completion time of the transmission of the information particle $k$ is ahead of its deadline $t_{e,k}$, it can be concluded that the information particle subgroup $Q_k (Q_k = \{ j | 1 \leq j \leq k, j \in Q \})$ must not be a principal information particle subgroup of $Q$.

2) On the other hand, based on the above proposition, if the completion time of the transmission of the information particle with sequence number of $k (k \geq 1)$ is 
\[
\left( \sum_{i=1}^{k} \lambda_i \right) \frac{1}{B_{av,Q}} = L_{DQk}^a,
\]
which indicates that the completion time of the transmission of the information particle $k$ is just the same as its deadline $t_{e,k}$, it can be concluded that the information particle subgroup $Q_k (Q_k = \{ j | 1 \leq j \leq k, j \in Q \})$ must be a principal information particle subgroup of $Q$.

**Corollary VII.1.5:** Given an information particle group $Q$ composed of $N (\geq 1)$ information particles, and an incremental sorting operation $Op_{Inc}(Q)$ is performed on it. Let $Q_k (Q_k = \{ j | 1 \leq j \leq k, j \in Q \})$ be the information particle subgroup composed of all the information particles in $Q$ whose sequence number is not greater than $k (k \geq 1)$, and the effective survival time span of $Q_k$ is $L_{DQk}^a$. If the inflection point of the access bandwidth requirement of $Q$ is $N_{cr,Q} (\geq 1)$, and $N_{cr,Q} < N$. Let
\[
\alpha = \max_{k = N_{cr,Q} + 1}^{N} \left\{ \sum_{i=N_{cr,Q}+1}^{k} \lambda_i \left( \frac{1}{L_{DQk}^a} - \frac{1}{L_{DQk}^a} \right) \right\},
\]
then $\alpha < B_{av,Q}^a$ holds.

**Proof:**
Assuming that when \( k = j (N_{crQ} + 1 \leq j \leq N) \), the value of \( \sum_{i=N_{crQ}+1}^{k} \ell_i \left( L_{D_i^{avQ}} - L_{D_i^{avQ}} \right) \) reaches the maximum, i.e

\[
\alpha = \frac{\sum_{i=N_{crQ}+1}^{k} \ell_i}{L_{D_i^{avQ}} - L_{D_i^{avQ}}} \quad \text{(VII.8)}
\]

According to the definition of a principal information particle subgroup of \( Q \) (refer to Definition IV.16), it is known that the information particle subgroup \( Q_{N_{crQ}} \) \( (Q_{N_{crQ}} = \{ i | 1 \leq i \leq N_{crQ}, i \in Q \}) \) is the largest principal information particle subgroup of \( Q \), so the following equation holds

\[
B_{avQ}^{pk} = B_{avQ} = \frac{\sum_{i=1}^{N_{crQ}} \ell_i}{L_{D_i^{avQ}} - L_{D_i^{avQ}}} \quad \text{(VII.9)}
\]

Moreover, let us define an auxiliary variable \( \sigma \) as

\[
\sigma \triangleq (\alpha - B_{avQ}) (L_{D_i^{avQ}} - L_{D_i^{avQ}}) \quad \text{(VII.10)}
\]

By combining Equation VII.8 and Equation VII.10, it can be obtained that

\[
B_{avQ}^{pk} = \frac{\sum_{i=N_{crQ}+1}^{k} \ell_i - \sigma}{L_{D_i^{avQ}} - L_{D_i^{avQ}}} \quad \text{(VII.11)}
\]

According to Equation VII.9 and Equation VII.11, there is

\[
B_{avQ}^{pk} = \frac{\sum_{i=1}^{N_{crQ}} \ell_i}{L_{D_i^{avQ}} - L_{D_i^{avQ}}} = \frac{\sum_{i=N_{crQ}+1}^{k} \ell_i - \sigma}{L_{D_i^{avQ}} - L_{D_i^{avQ}}} = \frac{\sum_{i=1}^{N_{crQ}} \ell_i + \sum_{i=N_{crQ}+1}^{k} \ell_i - \sigma}{L_{D_i^{avQ}} - L_{D_i^{avQ}}} = \frac{\sum_{i=1}^{N_{crQ}} \ell_i}{L_{D_i^{avQ}}} - \frac{\sigma}{L_{D_i^{avQ}}} \quad \text{(VII.12)}
\]

Based on the definition of \( N_{crQ} \) (refer to Definition IV.15), we have \( B_{avQ} < B_{avQ}^{pk} \), which is further substituted into Equation VII.12, it can be obtained that

\[
\sigma = (B_{avQ} - B_{avQ}^{pk}) \cdot L_{D_i^{avQ}} < 0 \quad \text{(VII.13)}
\]

From Equation VII.10, Equation VII.13 and Theorem V.1, we have the conclusion that
Based on Corollary 1, 2 and 3 of Theorem VII.1, and Definitions IV.15 and IV.16, it can be seen that when the access bandwidth of an information particle group $Q$ is limited to its minimum reachable access bandwidth $B_{min, Q}$ and EDF transmission strategy is adopted, it is within the effective survival time interval $D^{int}_{Q', \alpha}$ covered by its largest principal information particle subgroup $Q'_{\alpha}$ (its corresponding effective survival time span is denoted as $L_{Q', \alpha}$) that the access resources with bandwidth of $B_{min, Q}$ are completely occupied by the transmission of information carried by $N_{cr, Q}$ information particles belonging to $Q'_{\alpha}$. The completion time of the transmission of the information particle with sequence number $N_{cr, Q}$ is $\left(\sum_{i=1}^{N_{cr, Q}} \ell_i \right)/B_{min, Q} = L_{Q', \alpha}$, which indicates that the completion time of the transmission of the information particle $N_{cr, Q}$ is exactly the same as its deadline $t_{cr, N_{cr, Q}}$. Therefore, at this time instant, it can be observed that under a given access bandwidth $B_{min, Q}$ and within its effective survival time interval $D^{int}_{Q', \alpha}$, the access efficiency of the largest principal information particle subgroup $Q'_{\alpha}$ is $\eta_{Q', \alpha} = 1$.

2) Here, the physical meaning of $\alpha \triangleq \max_{i-N_{cr, Q}+1} \left\{ \frac{1}{B_{min, Q}} \sum_{i=N_{cr, Q}+1}^{L_{Q'} - L_{Q', \alpha}} \ell_i \right\}$ proposed in the above proposition is further explained and discussed. When considering the information particle group $Q$ in the preceding article, we have not clearly pointed out that the determination of the values of its attributes is closely related to the specific time point. In fact, we do refer to specific time points to determine the exact attribute values of an information particle group and those of each information particles contained in it.

Without losing generality, first let us consider the time instant at $t = 0$. At that time, we
observed that the information particle group $Q(t = 0)$ (Note: here we explicitly marked the corresponding time for the information particle group) is composed of $N$ valid information particles. By performing the incremental sorting operation $Op_{inc}(Q(t = 0))$, sorting the $N$ information particles contained therein and giving each of these information particles a corresponding sequence number, the information particle group $Q(t = 0)$ can be specifically determined as a set composed of $N$ information particles, i.e. $Q(t = 0) = \{ i | 1 \leq i \leq N \}$.

Relative to the current time instant $t = 0$, the bearing capacity of the information particle $i$ in $Q(t = 0)$ is $\lambda_i (>0)$, and its deadline is $e_i (>0)$. How about the situation within the time interval $[0, t_{e,N_{e,Q(t=0)}}]$? According to Theorem VII.1, the minimum reachable access bandwidth corresponding to $Q(t = 0)$ can be obtained, i.e. $B_{rec,Q(t=0)}^{min} = B_{rec,Q(t=0)}^{EDF} = \max \{ B_{av,Q(t=0)} \}$. Based on Definitions IV.15 and IV.16, the inflection point $N_{cr,Q(t=0)}$ of the access bandwidth requirement and the corresponding largest principal information particle subgroup $Q^*_{cr}(t = 0) = \{ i | 1 \leq i \leq N_{cr,Q(t=0)}, i \in Q(t = 0) \}$ of $Q(t = 0)$ can also be obtained. Relative to the current time instant $t = 0$, let the deadline of the information particle with sequence number of $N_{cr,Q(t=0)}$ be $t_{e,N_{cr,Q(t=0)}}$. Based on Corollary 1, 2, and 3 of Theorem VII.1, it can be seen that within the time interval $[0, t_{e,N_{cr,Q(t=0)}}]$, if the access bandwidth of the whole information particle group is limited to $B_{rec,Q(t=0)}^{min}$ ($= B_{rec,Q(t=0)}^{EDF}$) and the EDF transmission strategy is adopted, the access resources with a bandwidth of $B_{rec,Q(t=0)}^{min}$ ($= B_{rec,Q(t=0)}^{EDF}$) will be completely occupied by the transmission of the information carried by the information particles in $Q(t = 0)$ whose sequence numbers are not greater than $N_{cr,Q(t=0)}$, and all the information carried by these information particles will be transmitted out before time $t_{e,N_{cr,Q(t=0)}}$. In other words, within the time interval $[0, t_{e,N_{cr,Q(t=0)}}]$, given the access bandwidth and the adopted EDF transmission strategy, we can fully know the transmission procedure of the information particle group.

From time $t = t_{e,N_{cr,Q(t=0)}}$, the initial information particle group $Q(t = 0)$ contains only the
remaining \( N - N_{v,e(t=0)} \) information particles, and at this time instant it becomes the information particle group \( Q(t = t_{e,N_{v,e(0)}}) \). Then, what are the minimum reachable access bandwidth and the peak average access bandwidth requirement of the information particle group \( Q(t = t_{e,N_{v,e(0)}}) \) relative to the time \( t = t_{e,N_{v,e(0)}} \)? Obviously, the conclusion in Theorem VII.1 can still be applied to the information particle group \( Q(t = t_{e,N_{v,e(0)}}) \). Based on Theorem VII.1, it is not difficult to be found that

\[
\alpha \doteq \max_{k=N_{v,e(0)}+1}^{N} \left\{ \sum_{i=N_{v,e(0)}+1}^{k} \left( L_{Q_{k-1}}^\omega - L_{Q_{k-2}}^\omega \right) \right\}
\]

is the minimum reachable access bandwidth of the information particle group \( Q(t = t_{e,N_{v,e(0)}}) \). Therefore, the conclusion of Corollary VII.1.5 is that the minimum reachable access bandwidth \( B_{v,e(Q(t=0))}^{\text{min}} \) corresponding to the information particle group \( Q(t = 0) \) must be greater than the minimum reachable access bandwidth \( B_{v,e(Q(t=t_{e,N_{v,e(0)}}))}^{\text{min}} \) corresponding to the information particle group \( Q(t = t_{e,N_{v,e(0)}}) \). Therefore, time \( t = t_{e,N_{v,e(0)}} \) is precisely the inflection point where the minimum reachable access bandwidth of the information particle group changes, which is why we define the sequence number of the information particle with sequence number \( N_{v,e(Q(t=0))} \) in \( Q(t = 0) \) as the inflection point of its access bandwidth requirement. Of course, in the above discussion, in order to explain that the properties of an information particle group vary with time, we explicitly indicate the corresponding times. For the sake of brevity, in the other part of this paper, we still omit the annotation of the corresponding time without causing ambiguity.

3) The conclusion of Corollary VII.1.5 shows that when EDF transmission strategy is adopted, the required minimum reachable access bandwidth may decrease as information particles are transmitted out one by one. In order to ensure the delay requirements of each information particles and fully utilize the network access bandwidth as much as possible, it is necessary to design an information particle group access algorithm which can dynamically adjust the minimum reachable access bandwidth (see Algorithm VII.1).

C. Dynamic access algorithm of an information particle group

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Algorithm VII.1: Particle access algorithm based on dynamically adjusting the minimum reachable access bandwidth

1: Update time $t$ to the current time;
2: Initialize the set $Q$ ($Q$ is the set of information particles bearing the information to be transmitted (i.e., information particle group)) to include only the information particles to be transmitted at the current time;
3: WHILE ($Q \neq \emptyset$) DO
4: Refresh the attributes (i.e., bearing capacity and deadline) of all the information particles contained in $Q$ relative to the current time $t$;
5: Perform incremental sorting operation $Op_{inc}(Q)$ on $Q$;
6: Based on Theorem VII.1, the minimum reachable access bandwidth of the current information particle group $Q$ is calculated and updated as $B_{re}^{\text{min}}$;
7: Based on Definition IV.10 and Definition IV.15, calculate and update the inflection point of the access bandwidth requirement of the current information particle group $Q$ as $N_{cr,Q}$;
8: Take $B_{re,Q}^{\text{min}}$ as the access bandwidth, the first $N_{cr,Q}$ information particles in $Q$ are transmitted successively according to the EDF transmission strategy (once the information particles are transmitted, they will be deleted from the set $Q$);
9: Update time as $t \leftarrow t + t_{r,N_{cr,Q}}$;
10: END WHILE

Note VII.1:

If new information particles arrive during the procedure of the transmission, the current transmission operation should be interrupted immediately and the algorithm is executed again from step 1.

D. Some properties of the minimum reachable access bandwidths of one or more information particle groups
Corollary VII.1.6: The upper bound of the minimum reachable access bandwidth $B_{\text{re},Q}^\text{min}$ of an information particle group

Given an information particle group $Q$ containing $N$ ($\geq 1$) information particles, let its minimum reachable access bandwidth be $B_{\text{re},Q}^\text{min}$. The average access bandwidth requirement of the information particle $i$ ($1 \leq i \leq N$) contained in $Q$ is $B_{av,i}^i \triangleq \ell_i / L_{\varnothing}$ (Definition III.11), then there is $B_{\text{re},Q}^\text{min} \leq \sum_{i=1}^{N} B_{av,i}$ (when $L_{D_{\varnothing}} = L_{D_{\varnothing}}^2 = \ldots = L_{D_{\varnothing}}^N$, the equation holds).

Proof:

It is proved by mathematical induction.

As for the case of $N = 1$, the information particle group $Q$ contains only one information particle, that is, the information particle $i$ ($i = 1$). According to the definition of the peak average access bandwidth requirement of an information particle group (Definition IV.9), we have $B_{av,Q}^i = B_{av,i}^i$ ($i = 1$). Furthermore, according to Theorem V.1, we have $B_{\text{re},Q}^\text{min} = B_{av,Q}^i$. Based on the above, it can be obtained that $B_{\text{re},Q}^\text{min} = B_{av,i}^i$ ($i = 1$). Therefore, when $N = 1$, the proposition holds (i.e. the equation holds).

As for the case of $N = n$ ($n > 1$), it is assumed that the proposition holds (i.e. $B_{\text{re},Q}^\text{min} \leq \sum_{i=1}^{n} B_{av,i}$ holds). In the following, let us consider the case of $N = n + 1$. Without loss of generality, firstly an incremental sorting operation $Op_{inc}(Q)$ is performed on $Q$ (Note: performing an incremental sorting operation will not change the minimum reachable access bandwidth of the corresponding information particle group). The sorted information particle group $Q$ still contains exactly the same $n + 1$ information particles, just the sequence numbers of each information particle in it may differ before and after being sorted. Let $B_{\text{re},Q_{n+1}}^\text{min}$, $B_{av,Q_{n+1}}^\text{pk}$, and $B_{av,Q_{n+1}}^\text{av}$ represent the minimum reachable access bandwidth, the peak average access bandwidth requirement, and the average access bandwidth requirement of an information particle subgroup $Q_{n+1}$ (Definition IV.17) of $Q$, respectively. According to Equation VII.4 and Theorem V.1, we have

$$\max \{B_{av,Q_{n+1}}^\text{pk} \}$,

which can be expressed as
\[ B_{rev,Q_{last}}^{\text{min}} = \delta_{n,Q} B_{rev,Q_{last}}^{\text{min}} + (1 - \delta_{n,Q}) B_{av,Q_{last}} \]  

(VII.15)

where \( \delta_{n,Q} \) is the switching function (Definition IV.19) of \( Q \) with a sequence number \( n \).

Below, we will discuss the cases of \( \delta_{n,Q} = 1 \) and \( \delta_{n,Q} = 0 \), respectively. When \( \delta_{n,Q} = 1 \) (i.e. \( B_{rev,Q_{last}}^{\text{min}} \geq B_{av,Q_{last}} \)), due to the assumption that \( B_{rev,Q_{last}}^{\text{min}} \leq \sum_{i=1}^{n} B_{av,i} \) holds, there is

\[ B_{rev,Q_{last}}^{\text{min}} = B_{rev,Q_{last}}^{\text{min}} \leq \sum_{i=1}^{n} B_{av,i} < \sum_{i=1}^{n+1} B_{av,i} \]  

(VII.16)

It shows that when \( \delta_{n,Q} = 1 \), the proposition holds. As for the case of \( \delta_{n,Q} = 0 \) (i.e. \( B_{rev,Q_{last}}^{\text{min}} < B_{av,Q_{last}} \)), we have

\[ B_{rev,Q_{last}}^{\text{min}} = B_{rev,Q_{last}}^{\text{min}} = \frac{\ell_{Q_{last}}}{L_{Q_{last}}} \]

\[ = \sum_{i=1}^{n} \frac{\ell_{i}}{L_{D_{i}}} = \sum_{i=1}^{n} \frac{\ell_{i}^\prime}{L_{D_{i}}} + B_{av,n+i} \]  

(VII.17)

When \( L_{\ell_{i}} = L_{\ell_{i}^\prime} = \ldots = L_{\ell_{n+i}} = L_{D_{i}} \), the equation in the above Equation VII.17 holds. This indicates that for the case of \( \delta_{n,Q} = 0 \), the proposition also holds.

Based on the above, it can be obtained that the proposition holds for the case of \( N = n + 1 \).

Discussion VII.7:

According to the above Corollary VII.1.6, when the deadlines of all the \( N \) (\( \geq 1 \)) information particles contained in an information particle group \( Q \) are equal, the minimum reachable access bandwidth can be quickly calculated out by using equation \( B_{rev,Q}^{\text{min}} = \sum_{i=1}^{n} B_{av,i} \).
the information particle \( i (1 \leq i \leq N) \) contained in \( Q \) is \( B_{av,i} \triangleq \ell_i / L_{D_{av}} \) (Definition III.11), then there is \( B_{\text{min}} \geq \sum_{i=1}^{N} B_{av,i} / N \).

**Proof:**

It is proved by mathematical induction.

As for the case of \( N = 2 \), the information particle group \( Q \) contains only two information particles. Without loss of generality, firstly an incremental sorting operation \( Op_{im} (Q) \) is performed on \( Q \) (Note: performing an incremental sorting operation will not change the minimum reachable access bandwidth of the corresponding information particle group). According to Equation VII.15, we have

\[
B_{\text{min}} = \delta_{1,0} B_{\text{min}} + (1 - \delta_{1,0}) B_{av,0},
\]

(VII.18)

Where \( \delta_{1,0} \) is the switching function (Definition IV.19) of \( Q \) with a sequence number 1. Below, we will discuss the cases of \( \delta_{1,0} = 1 \) and \( \delta_{1,0} = 0 \), respectively. When \( \delta_{1,0} = 1 \) (i.e. \( B_{\text{min}} \geq B_{av,0} \)), there is

\[
B_{\text{min}} = B_{av,1} \geq B_{av,0} > B_{av,2}
\]

(VII.19)

And then, there is

\[
2B_{\text{min}} = 2B_{\text{min}} = 2B_{av,1} = B_{av,1} + B_{av,1} > B_{av,1} + B_{av,2}
\]

(VII.20)

Therefore, it can be obtained that \( B_{\text{min}} > \frac{B_{av,1} + B_{av,2}}{2} \) holds. When \( \delta_{1,0} = 0 \) (i.e. \( B_{\text{min}} < B_{av,0} \)), there is

\[
B_{\text{min}} = B_{av,1} < B_{av,0}
\]

(VII.21)

And then, there is
Therefore, it can be obtained that \( B_{\text{re},Q_i}^{\min} > \frac{B_{\text{re},1} + B_{\text{re},2}}{2} \) also holds in this case. Based on the above, it can be obtained that \( B_{\text{re},Q}^{\min} > \frac{\sum_{i=1}^{n} B_{\text{re},j}}{n} \) holds for the case of \( N = 2 \).

As for the case of \( N = n \ (n > 2) \), it is assumed that the proposition holds (i.e. \( B_{\text{re},Q}^{\min} > \frac{\sum_{i=1}^{n} B_{\text{re},j}}{n} \) holds). In the following, let us consider the case of \( N = n + 1 \). Without loss of generality, firstly an incremental sorting operation \( \text{Op}_{\text{Inc}}(Q) \) is performed on \( Q \) (Note: performing an incremental sorting operation will not change the minimum reachable access bandwidth of the corresponding information particle group). The sorted information particle group \( Q \) still contains exactly the same \( n + 1 \) information particles, just the sequence numbers of each information particle in it may differ before and after being sorted. According to Equation VII.15, we have

\[
B_{\text{re},Q_{n+1}}^{\min} = \delta_{n,Q} B_{\text{re},Q_n}^{\min} + (1 - \delta_{n,Q}) B_{\text{re},Q_{n+1}},
\]

where \( \delta_{n,Q} \) is the switching function (Definition IV.19) of \( Q \) with a sequence number \( n \).

Below, we will discuss the cases of \( \delta_{n,Q} = 1 \) and \( \delta_{n,Q} = 0 \), respectively. When \( \delta_{n,Q} = 1 \) (i.e. \( B_{\text{re},Q_n}^{\min} = B_{\text{re},Q_{n+1}} \)), there is

\[
(n + 1) B_{\text{re},Q_{n+1}}^{\min} = (n + 1) B_{\text{re},Q_n}^{\min} = n B_{\text{re},Q_n}^{\min} + B_{\text{re},Q_n},
\]

\[
> \sum_{i=1}^{n} B_{\text{re},j} + B_{\text{re},Q_n}^{\min} > \sum_{i=1}^{n} B_{\text{re},j} + B_{\text{re},Q_{n+1}}.
\]

\[
= \sum_{i=1}^{n} B_{\text{re},j} + \frac{\ell_{Q_n}}{L_{Q_n}^{\min}} - \sum_{i=1}^{n} B_{\text{re},j} + \frac{\sum_{i=1}^{n+1} \ell_{i}}{L_{Q_{n+1}}^{\min}}
\]

\[
> \sum_{i=1}^{n+1} B_{\text{re},j}.
\]

It shows that when \( \delta_{n,Q} = 1 \), \( B_{\text{re},Q_{n+1}}^{\min} > \frac{\sum_{i=1}^{n+1} B_{\text{re},j}}{n + 1} \) holds. As for the case of \( \delta_{n,Q} = 0 \) (i.e.
\[ B_{r_i Q_i}^{\text{min}} < B_{av_i Q_i} \), we have
\]
\[
(n+1)B_{r_i Q_i}^{\text{min}} = (n+1)B_{av_i Q_i} < nB_{av_i Q_i} + B_{av_{i+1}} > nB_{r_i Q_i} + B_{av_{i+1}} > \sum_{i=1}^{n} B_{av_i} + B_{av_{n+1}} = \sum_{i=1}^{n} B_{av_i} \]

(VII.24)

It shows that when \( \delta_{n,Q} = 0 \), \( B_{r_i Q_i}^{\text{min}} > \frac{\sum_{i=1}^{n} B_{av_i}}{n+1} \) also holds.

Based on the above, it can be obtained that the proposition holds for the case of \( N = n + 1 \).■

Discussion VII.8:

According to the above Corollary VII.1.7, for an information particle group \( Q \) containing \( N \) (\( \geq 2 \)) information particles, an access bandwidth constraint of less than or equal to \( \sum_{i=1}^{N} B_{av_i} \) must not be reachable. Therefore, this conclusion can be used to quickly determine the non-reachability of a given access bandwidth constraint.

Corollary VII.1.8: Linear superpositions of multiple information particle groups under specific constraints

Given \( M + 1 \) (\( M \geq 1 \)) information particle groups \( Q' \) (\( 1 \leq i \leq M + 1 \)), each of which contains \( N \) (\( \geq 1 \)) information particles and has been sorted by performing an incremental sorting operation \( Op_{\ell_i}(Q') \) (\( 1 \leq i \leq M + 1 \)). The deadlines of information particles in \( Q' \) (\( 1 \leq i \leq M + 1 \)) with their sequence numbers being \( j \) (\( 1 \leq j \leq N \)) are all equal to \( t_{r_i,j} \).\n
And, the bearing capacities of information particles in \( Q' \) (\( 1 \leq i \leq M + 1 \)) with their sequence numbers being \( j \) (\( 1 \leq j \leq N \)) are \( \ell_{j,Q'} \) (\( 1 \leq i \leq M + 1 \)), respectively, and satisfy the relationship

\[ \ell_{j,Q'} = \sum_{i=1}^{M} \alpha_i \ell_{j,Q'} \] (\( \alpha_i > 0 \)). \( B_{r_i Q_i}^{\text{min}} \) (\( 1 \leq i \leq M + 1; 1 \leq j \leq N \)) is the minimum reachable access bandwidth of the information particle subgroup \( Q' \) of \( Q' \) (\( 1 \leq i \leq M + 1 \)) (according to
Definition IV.17, $Q'$ is the information particle subgroup composed of all the information particles whose sorting sequence numbers are not greater than $j$. The sequence of switching functions of the information particle group $Q'$ ($1 \leq i \leq M+1$) is denoted as $\delta_{n,Q'} (1 \leq i \leq M+1; 1 \leq n \leq N-1)$ (Definition IV.19), and it is assumed that the relationship $\delta_{n,Q'} = \delta_{n,Q''} = \ldots = \delta_{n,Q^{(M)}} (1 \leq n \leq N-1)$ holds. Then, it can be concluded that the following relationships must hold.

$$B_{\min_{re,Q'}} = \sum_{i=1}^{M} \alpha_i B_{\min_{re,Q'}}^i (1 \leq j \leq N)$$  \hspace{1cm} (VII.25)

and

$$\delta_{n,Q'} = \delta_{n,Q''} = \ldots = \delta_{n,Q^{(M)}} (1 \leq n \leq N-1)$$  \hspace{1cm} (VII.26)

Proof:

It is proved by mathematical induction.

Firstly, let us consider the case of $j = 1$. According to Theorem V.1, we have

$$B_{\min_{re,Q'}} = B_{\min_{Q'}} = B_{\max_{Q'}}$$

$$= \frac{f_{Q'}}{t_{r,1}} = \frac{f_{Q^{(M)}}}{t_{r,1}} = \sum_{i=1}^{M} \alpha_i f_{i,Q'}$$

$$= \sum_{i=1}^{M} \alpha_i B_{\min_{re,Q'}}^i$$

(VII.27)

Therefore, as for the case of $j = 1$, it can be obtained that $B_{\min_{re,Q'}} = \sum_{i=1}^{M} \alpha_i B_{\min_{re,Q'}}^i$ holds.

As for the case of $j = n (1 < n < N)$, it is assumed that the proposition holds (i.e. $B_{\min_{re,Q'}} = \sum_{i=1}^{M} \alpha_i B_{\min_{re,Q'}}^i$ holds). In the following, let us consider the case of $j = n+1 (1 < n < N)$.

According to Equation VII.15, we have

$$B_{\min_{re,Q'}} = \delta_{n,Q'} B_{\min_{re,Q'}}^i + (1 - \delta_{n,Q'}) B_{\max_{re,Q'}}^i$$  \hspace{1cm} (VII.28)

According to the question setting of $\delta_{n,Q'} = \delta_{n,Q''} = \ldots = \delta_{n,Q^{(M)}}$, we can discuss the cases of

$\delta_{n,Q'} = \delta_{n,Q''} = \ldots = \delta_{n,Q^{(M)}} = 1$ and $\delta_{n,Q'} = \delta_{n,Q''} = \ldots = \delta_{n,Q^{(M)}} = 0$, respectively. When

$\delta_{n,Q'} = \delta_{n,Q''} = \ldots = \delta_{n,Q^{(M)}} = 1$ (i.e. $B_{\min_{re,Q'}} \geq B_{\max_{re,Q'}} (1 \leq i \leq M)$), there is
\[
\sum_{i=1}^{M} \alpha_i B_{\nu_i Q_i^*} \geq \sum_{i=1}^{M} \alpha_i B_{\nu_i Q_i^*}
\]  
(VII.29)

where, according to the above assumption, there is
\[
B_{\nu_i Q_i^*}^\min = \sum_{i=1}^{M} \alpha_i B_{\nu_i Q_i^*}
\]  
(VII.30)

And then, there is
\[
\sum_{i=1}^{M} \alpha_i B_{\nu_i Q_i^*} = \sum_{i=1}^{M} \left( \frac{\ell_{Q_i^*}}{t_{\nu_i Q_i^*}} \right) = \sum_{i=1}^{M} \left( \frac{\alpha_i \sum_{j=1}^{i} \ell_{Q_i^*}}{t_{\nu_i Q_i^*}} \right)
\]
\[
= \frac{\sum_{j=1}^{i} \sum_{i=1}^{M} \alpha_i \ell_{Q_i^*}}{t_{\nu_i Q_i^*}} = \frac{\sum_{j=i+1}^{\infty} \ell_{Q_i^*}}{t_{\nu_i Q_i^*}}
\]  
(VII.31)

Substituting Equation VII.30 and Equation VII.31 into Equation VII.29 yields
\[
B_{\nu_i Q_i^*}^\min \geq B_{\nu_i Q_i^*}
\]  
(VII.32)

Considering the definition of a switching function \( \delta_{\nu_i Q_i^*} \) (Definition IV.19), that is
\[
\delta_{\nu_i Q_i^*} = \begin{cases} 
1 & B_{\nu_i Q_i^*}^\min \geq B_{\nu_i Q_i^*} \\
0 & B_{\nu_i Q_i^*}^\min < B_{\nu_i Q_i^*}
\end{cases}
\]  
(VII.33)

Therefore, according to the above two equations, it can be obtained that \( \delta_{\nu_i Q_i^*} = 1 \), i.e.
\[
\delta_{\nu_i Q_i^*} = \delta_{\nu_i Q_i^*} = \ldots = \delta_{\nu_i Q_i^*} = \delta_{\nu_i Q_i^*}
\]  
holds. By combining Equation VII.28 and Equation VII.30, it can be obtained that
\[
B_{\nu_i Q_i^*}^\min = B_{\nu_i Q_i^*} = \sum_{i=1}^{M} \alpha_i B_{\nu_i Q_i^*}
\]  
(VII.34)

As for the case of \( \delta_{\nu_i Q_i^*} = \delta_{\nu_i Q_i^*} = \ldots = \delta_{\nu_i Q_i^*} = 0 \) (i.e. \( B_{\nu_i Q_i^*}^\min < B_{\nu_i Q_i^*} \) (1 \( \leq i \leq M \)), there is
\[
\sum_{i=1}^{M} \alpha_i B_{\nu_i Q_i^*} < \sum_{i=1}^{M} \alpha_i B_{\nu_i Q_i^*}
\]  
(VII.35)

Substituting Equation VII.30 and Equation VII.31 into the above equation, there is
\[
B_{\nu_i Q_i^*}^\min < B_{\nu_i Q_i^*}
\]  
(VII.36)

Considering the definition of a switching function \( \delta_{\nu_i Q_i^*} \), it can be concluded that \( \delta_{\nu_i Q_i^*} = 0 \), i.e.
\[
\delta_{\nu_i Q_i^*} = \delta_{\nu_i Q_i^*} = \ldots = \delta_{\nu_i Q_i^*} = \delta_{\nu_i Q_i^*}
\]  
holds. By combining Equation VII.28 and Equation VII.31, there is
Based on the above analyses of the case of $\delta_{n,Q} = \delta_{n,Q'} = ... = \delta_{n,Q''} = 1$ (Equation VII.34) and that of $\delta_{n,Q} = \delta_{n,Q'} = ... = \delta_{n,Q''} = 0$ (Equation VII.37), it can be concluded that the following relationship holds

$$B_{\min}^{\text{re,Q}_{n,Q'}} = B_{av,Q_{n,Q'}} = \sum_{i=1}^{M} a_i B_{av,Q_{n,i}}$$ (VII.37)

The above equation can be further written as follows

$$B_{\min}^{\text{re,Q}_{n,Q'}} = \begin{cases} \sum_{i=1}^{M} a_i B_{\min,Q_{n,i}}, & (\delta_{n,Q} = \delta_{n,Q'} = ... = \delta_{n,Q''} = \delta_{n,Q'''} = 1) \\ \sum_{i=1}^{M} a_i B_{av,Q_{n,i}}, & (\delta_{n,Q} = \delta_{n,Q'} = ... = \delta_{n,Q''} = \delta_{n,Q'''} = 0) \end{cases}$$ (VII.38)

Based on the above, it can be obtained that $B_{\min}^{\text{re,Q}_{n,Q'}} = \sum_{i=1}^{M} a_i B_{\min,Q_{n,i}}$ holds for the case of $j = n + 1$. ■

Discussion VII.9:

In the above Corollary VII.1.8, the information particle group $Q^{M+1}$ can be regarded as a kind of linear superposition of $M$ ($M \geq 1$) information particle groups $Q' (1 \leq i \leq M)$ with similar structures (i.e. containing the same number of information particles, information particles contained having the same distribution of deadlines, and each information particle group having the same sequence of switching functions, etc.). Corollary VII.1.8 further reveals that there exist a simple linear relationship between the minimum reachable access bandwidth of information particle group $Q^{M+1}$ and the minimum reachable access bandwidths of information particle groups $Q' (1 \leq i \leq M)$. Furthermore, it can be observed that the linear superposition of information particle groups with similar structures does not result in further compression of the demand for bandwidth resources.

Corollary VII.1.9: Linear splitting of an information particle group

Given $M + 1$ ($M \geq 1$) information particle groups $Q' (1 \leq i \leq M + 1)$, each of which contains $N$ ($\geq 1$) information particles and has been sorted by performing an incremental
sorting operation \( Op_{inc}(Q') \) \((1 \leq i \leq M + 1)\). The deadlines of information particles in \( Q' \) \((1 \leq i \leq M + 1)\) with their sequence numbers being \( j \) \((1 \leq j \leq N)\) are all equal to \( t_{e,i} \). And, the bearing capacities of information particles in \( Q' \) \((1 \leq i \leq M + 1)\) with their sequence numbers being \( j \) \((1 \leq j \leq N)\) are \( \ell_{j,Q(i)} \) \((1 \leq i \leq M + 1)\), respectively, and satisfy the relationship

\[
\ell_{j,Q(i)} = \alpha_{i}\ell_{j,Q^{0,i}} \quad (\alpha_{i} > 0) \quad B_{min}^{\text{re},Q^{0,i}} \quad (1 \leq i \leq M + 1; 1 \leq j \leq N)
\]

is the minimum reachable access bandwidth of the information particle subgroup \( Q' \) of \( Q' \) \((1 \leq i \leq M + 1)\). The sequence of switching functions of the information particle group \( Q' \) \((1 \leq i \leq M + 1)\) is denoted as \( \delta_{n,Q'} \) \((1 \leq i \leq M + 1; 1 \leq n \leq N - 1)\) \((\text{Definition IV.19})\). Then, it can be concluded that the following relationships must hold.

\[
B_{\text{min}}^{\text{re},Q(i)} = \alpha_{i} B_{\text{re},Q^{0,i}} \quad (1 \leq i \leq M, 1 \leq j \leq N) \quad (\text{VII.40})
\]

and

\[
\delta_{n,Q'} = \delta_{n,Q'} = \ldots = \delta_{n,Q'} = \delta_{n,Q'} (1 \leq n \leq N - 1) \quad (\text{VII.41})
\]

**Proof:**

It is proved by mathematical induction.

Firstly, let us consider the case of \( j = 1 \). According to \textbf{Theorem V.1}, we have

\[
B_{\text{min}}^{\text{re},Q(i)} = \frac{r_{j,Q'}}{t_{e,j}} = \frac{r_{j,Q^{0,i}}}{t_{e,1}} = \alpha_{i} B_{\text{re},Q^{0,i}} = \alpha_{i} B_{\text{re},Q^{0,i}} \quad (1 \leq i \leq M) \quad (\text{VII.42})
\]

Therefore, as for the case of \( j = 1 \), it can be obtained that \( B_{\text{min}}^{\text{re},Q(i)} = \alpha_{i} B_{\text{re},Q^{0,i}} \quad (1 \leq i \leq M) \) holds.

As for the case of \( j = n \) \((1 < n < N)\), it is assumed that the proposition holds (i.e. \( B_{\text{min}}^{\text{re},Q(i)} = \alpha_{i} B_{\text{re},Q^{0,i}} \quad (1 \leq i \leq M) \) holds). In the following, let us consider the case of \( j = n + 1 \) \((1 < n < N)\). We can discuss the cases of \( \delta_{n,Q^{0,i}} = 1 \) and \( \delta_{n,Q^{0,i}} = 0 \), respectively. When \( \delta_{n,Q^{0,i}} = 1 \), there is

\[
B_{\text{min}}^{\text{re},Q^{0,i}} \geq B_{\text{re},Q^{0,i}} \quad (\text{VII.43})
\]
where, according to the above assumption, there is

\[
B_{\text{re} Q_{i+1}}^\text{min} = \frac{B_{\text{re} Q_i}}{\alpha_i} \quad (1 \leq i \leq M)
\]  

(VII.44)

and

\[
B_{\text{av} Q_{i+1}} = \frac{\sum_{j=1}^{\ell} \overrightarrow{f_j Q_i}}{t_{x, n_i}} = \frac{\sum_{j=1}^{\ell} \overrightarrow{f_j Q_{i+1}}}{t_{x, n_i+1}} \quad (1 \leq i \leq M)
\]  

(VII.45)

Substituting the above two equations into Equation VII.43 yields

\[
B_{\text{re} Q_i}^\text{min} \geq B_{\text{av} Q_i} \quad (1 \leq i \leq M)
\]  

(VII.46)

Combining with the definition of a switching function \( \delta_{\text{n} Q_i} \) (Definition IV.19), it can be obtained that \( \delta_{\text{n} Q_i} = 1 \) \((1 \leq i \leq M)\), i.e. \( \delta_{\text{n} Q_i} = \delta_{\text{n} Q_{i+1}} = \ldots = \delta_{\text{n} Q_{i+\ell}} = \delta_{\text{n} Q_{i+2\ell}} \) holds. According to Equation VII.15, we have

\[
B_{\text{re} Q_i}^\text{min} = \delta_{\text{n} Q_i} B_{\text{re} Q_i} + (1 - \delta_{\text{n} Q_i}) B_{\text{av} Q_i}
\]  

(VII.47)

Since \( \delta_{\text{n} Q_i} = 1 \) \((1 \leq i \leq M)\), it can be obtained that

\[
B_{\text{re} Q_i}^\text{min} = B_{\text{re} Q_i} = \alpha_i B_{\text{re} Q_i}^\text{min} = \alpha_i B_{\text{re} Q_i}^\text{min} \quad (1 \leq i \leq M)
\]  

(VII.48)

For the case of \( \delta_{\text{n} Q_i+\ell} = 0 \), there is

\[
B_{\text{re} Q_i}^\text{min} < B_{\text{av} Q_i}
\]  

(VII.49)

Similarly, it can be concluded that

\[
B_{\text{re} Q_i}^\text{min} < B_{\text{av} Q_i} \quad (1 \leq i \leq M)
\]  

(VII.50)

Combining with the definition of a switching function \( \delta_{\text{n} Q_i} \), it can be obtained that \( \delta_{\text{n} Q_i} = 0 \) \((1 \leq i \leq M)\), i.e. \( \delta_{\text{n} Q_i} = \delta_{\text{n} Q_{i+1}} = \ldots = \delta_{\text{n} Q_{i+\ell}} = \delta_{\text{n} Q_{i+2\ell}} \) also holds in this case. Similarly, from Equation VII.15 and Equation VII.45, it can be obtained that

\[
B_{\text{re} Q_i}^\text{min} = B_{\text{av} Q_i} = \alpha_i B_{\text{re} Q_i}^\text{min} = \alpha_i B_{\text{re} Q_i}^\text{min} \quad (1 \leq i \leq M)
\]  

(VII.51)

Based on the above analyses (Equation VII.48 and Equation VII.51), it can be concluded
that both $B\min_{\alpha_i \epsilon_i} = \alpha_i B\min_{\alpha_i \epsilon_i}$, $(1 \leq i \leq M)$ and $\delta_{\alpha_i \epsilon_i} = \delta_{\alpha_i \epsilon_i}$ hold for the case of $j = n + 1$. ■

**Discussion VII.10:**

In the above Corollary VII.1.9, if $\sum_{i=1}^{M} \alpha_i = 1$ is set, it can be regarded as linearly splitting the information particle group $Q^{M+i}$ into $M$ ($M \geq 1$) information particle groups $Q_i^{1} (1 \leq i \leq M)$ with similar structures (i.e. containing the same number of information particles, information particles contained having the same distribution of deadlines, and each information particle group having the same sequence of switching functions, etc.), and at the same time maintaining the total bandwidth requirement unchanged, i.e. $B\min_{\alpha_i \epsilon_i} = \sum_{i=1}^{M} \delta_{\alpha_i \epsilon_i}$. This provides a feasible approach for dividing an information particle group with a high demand of bandwidth resources into several information particle groups each with lower demands for bandwidth resources, and then by simultaneously using multiple channels each with smaller bandwidths to bear the splitted information particle group.

**Corollary VII.1.10:** A closed form expression of the minimum reachable access bandwidth of an information particle subgroup

Given an information particle group $Q$, which contains $N$ ($\geq 2$) information particles and has been sorted by performing an incremental sorting operation $Op_{inc}(Q)$. The vector of switching functions (Definition IV.20) and the vector of average access bandwidth requirements (Definition IV.21) of an information particle subgroup $Q_n$, $Q_{\leq n}$, $2 \leq n \leq N$) are $\delta_{Q_n}$ and $B_{av, Q_n}$, respectively. Then, it can be concluded that the following relationship must hold.

$$B\min_{\alpha_i \epsilon_i} = \delta_{Q_n} \cdot B_{av, Q_n} \quad (VII.52)$$

**Proof:**

As for the case of $n = 2$, according to Equation VII.15, Definition IV.20, and Definition IV.21, we have
Therefore, it can be seen that for the case of $n = 2$, $B_{\alpha_{r},Q_{0}}^{\min} = \delta_{\alpha_{r},Q_{0}}^{\tau} \cdot B_{\alpha_{r};Q_{0}}$ holds.

For the case of $n > 2$, mathematical induction is used. Firstly, let us prove the case of $n = 3$.

According to Equation VII.15, there is

\[
B_{\alpha_{r},Q_{0}}^{\min} = \delta_{2,Q_{0}} B_{\alpha_{r},Q_{0}}^{\min} + (1 - \delta_{2,Q_{0}}) B_{\alpha_{r};Q_{0}} = \delta_{2,Q_{0}} \left( \delta_{1,Q_{0}} B_{\alpha_{r},Q_{0}}^{\min} + (1 - \delta_{1,Q_{0}}) B_{\alpha_{r};Q_{0}} \right) + (1 - \delta_{2,Q_{0}}) B_{\alpha_{r};Q_{0}}
\]

\[
= \delta_{1,Q_{0}} \delta_{2,Q_{0}} B_{\alpha_{r};Q_{0}} + (1 - \delta_{1,Q_{0}}) \delta_{2,Q_{0}} B_{\alpha_{r};Q_{0}} + (1 - \delta_{2,Q_{0}}) B_{\alpha_{r};Q_{0}}
\]

\[
= \delta_{1,Q_{0}} \delta_{2,Q_{0}} B_{\alpha_{r};Q_{0}} + (1 - \delta_{1,Q_{0}}) \delta_{2,Q_{0}} B_{\alpha_{r};Q_{0}} + (1 - \delta_{2,Q_{0}}) B_{\alpha_{r};Q_{0}}
\]

\[
= (\delta_{1,Q_{0}} \delta_{2,Q_{0}}, (1 - \delta_{1,Q_{0}}) \delta_{2,Q_{0}}, 1 - \delta_{2,Q_{0}}) \begin{pmatrix} B_{\alpha_{r};Q_{0}} \\ B_{\alpha_{r};Q_{0}} \\ B_{\alpha_{r};Q_{0}} \end{pmatrix}
\]

\[
= \delta_{\alpha_{r},Q_{0}}^{\tau} \cdot B_{\alpha_{r};Q_{0}}
\]

Therefore, it can be seen that for the case of $n = 3$, $B_{\alpha_{r},Q_{0}}^{\min} = \delta_{\alpha_{r},Q_{0}}^{\tau} \cdot B_{\alpha_{r};Q_{0}}$ holds.

Assuming that when $n = k$ ($k > 3$), $B_{\alpha_{r};Q_{0}}^{\min} = \delta_{\alpha_{r},Q_{0}}^{\tau} \cdot B_{\alpha_{r};Q_{0}}$ holds. Let us consider the case of $n = k + 1$. According to Equation VII.15, we have

\[
B_{\alpha_{r},Q_{0},t}^{\min} = \delta_{k,Q_{0}} B_{\alpha_{r},Q_{0},t}^{\min} + (1 - \delta_{k,Q_{0}}) B_{\alpha_{r};Q_{0},t} = \delta_{k,Q_{0}} \cdot \delta_{\alpha_{r},Q_{0}}^{\tau} \cdot B_{\alpha_{r};Q_{0}} + (1 - \delta_{k,Q_{0}}) B_{\alpha_{r};Q_{0},t}
\]

\[
= \delta_{k,Q_{0}} \cdot \delta_{\alpha_{r},Q_{0}}^{\tau} \cdot B_{\alpha_{r};Q_{0}} + (1 - \delta_{k,Q_{0}}) B_{\alpha_{r};Q_{0},t}
\]

\[
= \delta_{k,Q_{0}} \cdot \delta_{\alpha_{r},Q_{0}}^{\tau} \cdot B_{\alpha_{r};Q_{0}} + (1 - \delta_{k,Q_{0}}) B_{\alpha_{r};Q_{0},t}
\]

\[
= \delta_{k,Q_{0}} \cdot \delta_{\alpha_{r},Q_{0}}^{\tau} \cdot B_{\alpha_{r};Q_{0}} + (1 - \delta_{k,Q_{0}}) B_{\alpha_{r};Q_{0},t}
\]

\[
= \delta_{k,Q_{0}} \cdot \delta_{\alpha_{r},Q_{0}}^{\tau} \cdot B_{\alpha_{r};Q_{0}} + (1 - \delta_{k,Q_{0}}) B_{\alpha_{r};Q_{0},t}
\]
\[
\begin{align*}
&= \left(\prod_{j=1}^{k} \delta_{j, Q_{n+1}}^{,} \right) \\
&\quad \times \left(1 - \delta_{k, Q_{n+1}} \right) \prod_{j=2}^{k} \delta_{j, Q_{n+1}}^{,} \cdots \left(1 - \delta_{2, Q_{n+1}} \right) \cdots \left(1 - \delta_{1, Q_{n+1}} \right) \\
&\quad \times \left(1 - \delta_{k, Q_{n+1}}^{,} \right) \delta_{k, Q_{n+1}}^{,} \delta_{k, Q_{n+1}}^{,} \delta_{k, Q_{n+1}}^{,} \delta_{k, Q_{n+1}}^{,} \\
&= \delta_{Q_{n+1}}^{,} B_{r\rightarrow Q_{n+1}}^{,} (VII.55)
\end{align*}
\]

Therefore, it can be seen that for the case of \( n = k + 1 \), \( B_{r\rightarrow Q_{n+1}}^{\min} = \delta_{Q_{n+1}}^{,} \cdot B_{r\rightarrow Q_{n+1}}^{,} \) also holds.

Based on the above, it can be concluded that \( B_{r\rightarrow Q_{n+1}}^{\min} = \delta_{Q_{n+1}}^{,} \cdot B_{r\rightarrow Q_{n+1}}^{,} \) holds for the case of \( n > 2 \). ■

Discussion VII.11:

The above Corollary VII.1.10 reveals that the key factors of determining the minimum reachable access bandwidth of an information particle group are its vector of average access bandwidth requirements and its vector of switching functions. It is believed that this provides a useful guidance for further in-depth research on computations such as superpositions and splittings of information particle groups in the future.

**Corollary VII.1.11**: The vector of switching functions of an information particle subgroup has and only has one component with its value being equal to one

Given an information particle group \( Q \), which contains \( N \) (\( \geq 2 \)) information particles and has been sorted by performing an incremental sorting operation \( Op_{Inc} (Q) \). The vector of switching functions (Definition IV.20) of an information particle subgroup \( Q_{n} (Q_{n} \subseteq Q, 2 \leq n \leq N) \) is \( \delta_{Q_{n}} \). Then, it can be concluded that among \( n \) components of \( \delta_{Q_{n}} \), there is one and only one component with its value being equal to one.

**Proof:**

As for the case of \( n = 2 \), according to Definition IV.20, we have

\[
\delta_{Q_{2}}^{,} = \left(\delta_{1, Q_{2}}^{,}, 1 - \delta_{1, Q_{2}}^{,} \right) (VII.56)
\]

According to Definition IV.19, the switching function \( \delta_{1, Q_{2}}^{,} \) can only be taken as 0 or 1. It can be obtained that
\[
\delta_{\theta}^r = \begin{cases} 
(1,0) & (\delta_{\theta,i} = 1) \\
(0,1) & (\delta_{\theta,i} = 0)
\end{cases}
\] (VII.57)

Therefore, it can be seen that the proposition holds in the case of \( n = 2 \).

Next, let us consider the case of \( n > 2 \). In this case, according to Definition IV.20, the vector \( \delta_{\theta,i} \) can be generally expressed as follows:

\[
\delta_{\theta,i} = \left( \delta_{\theta,1}, \delta_{\theta,2}, \ldots, \delta_{\theta,(i-1)}, \delta_{\theta,i}, \delta_{\theta,(i+1)}, \ldots, \delta_{\theta,n} \right)^T
\]

(VII.58)

where \( \delta_{\theta,1} \) \( \equiv \prod_{j=1}^{n-1} \delta_{\theta,j}, \) \( \delta_{\theta,i} \equiv (1-\delta_{\theta,i-1}) \prod_{j=i}^{n-1} \delta_{\theta,j} \) \( (2 \leq i \leq n-1), \) \( \delta_{\theta,n} \equiv 1-\delta_{\theta,n-1}. \)

Firstly, let us prove that the \( n \) components of \( \delta_{\theta,i} \) cannot all be zero. Here, reduction to absurdity is adopted. Assuming that all the \( n \) components of \( \delta_{\theta,i} \) are 0, \( \delta_{\theta,n-1} = 1 \) can be obtained from the premise that \( \delta_{\theta,n} = 0. \) Then, from \( \delta_{\theta,n-1} = 1 \) and \( \delta_{\theta,n-1} = 0, \delta_{\theta,n-2} = 1 \) can be derived. By repeating this reasoning process, we can obtain that \( \delta_{\theta,j} = 1 (1 \leq j \leq n-1), \) from which \( \delta_{\theta,1} \equiv \prod_{j=1}^{n-1} \delta_{\theta,j} = 1 \) can be derived. This contradicts the assumption that all the \( n \) components of \( \delta_{\theta,i} \) are 0. Therefore, it is impossible for all the \( n \) components of \( \delta_{\theta,i} \) being equal to 0.

Next, we will prove that only one term among \( n \) components of \( \delta_{\theta,i} \) can be equal to 1. Considering the fact that the \( n \) components of \( \delta_{\theta,i} \) cannot all be 0, there must be at least one component that is equal to 1. Below, we will consider three different cases one by one. Firstly, assuming that \( \delta_{\theta,1} = 1, \delta_{\theta,j} = 1 (1 \leq j \leq n-1) \) is obtained, and then \( \delta_{\theta,i} = 0 (2 \leq i \leq n) \) can be easily obtained (i.e. all the other \( n-1 \) components are 0). Secondly, assuming that \( \delta_{\theta,n} = 1, \) then \( \delta_{\theta,n-1} = 0 \) can be obtained, and \( \delta_{\theta,i} = 0 (1 \leq i \leq n-1) \) must hold (i.e. all the other \( n-1 \) components are 0). Finally, assuming that \( \delta_{\theta,2} = 1 (2 \leq i \leq n-1), \) we have \( \delta_{\theta,i} = 0 (2 \leq i \leq n-1), \) and it can be easily obtained that \( \delta_{\theta,i} = 0 (1 \leq j \leq i-1). \) On the other hand, since \( \delta_{\theta,1} = 1, \) we can also obtain that \( \delta_{\theta,j} = 1 (i \leq j \leq n-1). \) And then, \( \delta_{\theta,j} = 0 (i+1 \leq j \leq n) \) can be obtained.

Therefore, in this case, it can be concluded that all the other \( n-1 \) components must be 0.
VIII. The joint optimization with the mechanism of network wave

Aiming at the disorder problem of the utilization of network resources commonly existing in multi-hop transmission networks, in [7], we propose the idea and the corresponding supporting theory, i.e. theory of network wave, by constructing volatility information transmission mechanism between the sending nodes and their corresponding receiving nodes of a pair of paths (composed of two primary paths), so as to improve the orderliness of the utilization of network resources. However, in this work, the problems of dividing information blocks into smaller granularity, i.e. information packets, and how to guarantee the delay requirements of all the information packets in end-to-end transmissions are not considered.

On the other hand, in the above proposed theory of particle access, the problem of how to guarantee the end-to-end delay of an information particle group on a multi-hop path is not considered. In view of this, in this section, the joint optimization of the mechanism of network wave and the mechanism of particle access is studied. For a given information particle group transmitted over a given primary path, a method to obtain the minimum reachable transmission bandwidth, which can guarantee the end-to-end delays of all the information particles contained in the information particle group, is proposed. It is shown that the combination of the mechanism of network wave and the mechanism of particle access is an effective way to solve the problem of end-to-end multi-hop transmission.

A. Descriptions of the problems

At the time instant of \( t = 0 \), consider a primary path \( P \) composed of \( N+1 \) nodes (see Definition III.1 in [7]), starting from node 1 (the source node) in \( P \), an information particle group \( Q \) (see Definition IV.1) containing \( M (\geq 1) \) information particles is relayed through node 2, node 3,..., node \( i,..., \) node \( N (\geq 1) \), and finally arrives at node \( N+1 \) (the destination node).

**Problem VIII.1:** As shown in the sub-figure A of Fig. VIII.1, assuming that the starting time instant of the initial time beat is \( t_{\beta_0} (\geq 0) \) (that is, the initial time instant when node 1 is activated). With \( T_{\beta_p} (T_{\beta_p} \geq T'_{\beta_p}) \) (\( T_{\beta_p} \) and \( T'_{\beta_p} \) are the reachable period and the intrinsic period of \( P \), respectively. See Definition VI.2 and Definition VI.4 in [7]) as its period, the primary path \( P \) executes the
Algorithm VII.1 (i.e., the algorithm for the transmission of information blocks based on the reachable period of a primary path) proposed in [7]. During the execution of Algorithm VII.1, the duration of each time beat is set to be $T_B \ (> 0)$, and the transmission bandwidth of each relaying
node is \( B \) (bits/s). After performing the incremental sorting operation on the information particle group \( Q \) (see Definition IV.2), the bearing capacity of the information particle \( i \ (1 \leq i \leq M) \) in \( Q \) and the deadline for its departure from node \( N \ (\geq 1) \) are \( f_i \) and \( t_{i,j}^{(N)} \ (\geq t_{bs} + N \cdot T_b) \), respectively (Note: the first half of the superscript of \( t_{i,j}^{(N)} \) represents the sequence number of the node which the information particle leaving from, and the second half represents the corresponding sequence number of the problem being considered).

Then, what is the minimum reachable transmission bandwidth \( B_{P,Q}^{min,1} \) (bits/s) of each relaying node that can ensure that all the \( M \) information particles contained in \( Q \) reach the destination node \( N+1 \) before their corresponding deadlines \( t_{i,j}^{(N)} \) (that is, the final time instant of each information particle leaving from the node \( N \) is no later than its deadline) (To be noted that the second half of the superscript of \( B_{P,Q}^{min,1} \) indicates its corresponding sequence number of the problem being considered)?

**Note VIII.1:**

1) In order to make full use of bandwidth resources, the transmission of an information particle at each relaying node is allowed to span multiple time beats. Therefore, the moment when an information particle leaves a specific node is indeed the moment when the last bit of the information particle “leaves” the node;

2) For simplicity, it is assumed that the current active node completes the transmission of all the information particles (all or part of them) waiting to be transmitted at the end of each time beat when the node being considered is active;

3) For all the information particles, from \( t = 0 \) to the time when they finally reach the destination node of the primary path, they need to experience at least \( t_{bs} + N \cdot T_b \) time delays. Therefore, only the cases with deadlines \( t_{i,j}^{(N)} \geq t_{bs} + N \cdot T_b \) are considered here (that is, it is believed that the cases with deadlines \( t_{i,j}^{(N)} < t_{bs} + N \cdot T_b \) cannot be realized);

4) The system defined in Problem VIII.1 is hereinafter referred to as “System VIII.1”.
Problem VIII.2: As shown in the sub-figure B of Fig. VIII.1, assuming that the starting time instant of the initial time beat is \( t_{b_0} (\geq 0) \). With \( T_{s_p} \) \((T_{s_p} \geq T_{s_p}^e)\) as its period, the primary path \( P \) executes the Algorithm VII.1 proposed in [7]. During the execution of Algorithm VII.1, the duration of each time beat is set to be \( T_p (\geq 0) \), and the transmission bandwidth of each relaying node is \( B \) (bits/s). Let us only consider the node 1 of \( P \). After performing the incremental sorting operation on the information particle group \( Q \), the bearing capacity of the information particle \( i \) \((1 \leq i \leq M)\) in \( Q \) is \( \ell_i \), and the deadline for its departure from node 1 is

\[
\ell^{(1, 2)}_{e,i} \triangleq \ell^{(N, 1)}_{e,i} - (N-1) \cdot T_p
\]  

(VIII.1)

where \( \ell^{(N, 1)}_{e,i} \) is given in Problem VIII.1.

Then, what is the minimum reachable transmission bandwidth \( B_{P, 1}^{\min} \) (bits/s) of node 1 that can ensure that all the \( M \) information particles contained in \( Q \) leave from node 1 before their corresponding deadlines \( \ell^{(1, 2)}_{e,i} \)?

Note VIII.2:
1) The system defined in Problem VIII.2 is hereinafter referred to as “System VIII.2”;
2) System VIII.2 is mapped from System VIII.1.

Problem VIII.3: As shown in the sub-figure C of Fig. VIII.1, assuming that the starting time instant of the initial time beat is 0 (that is, the left shift operation is performed on the time axis relative to System VIII.2). With \( T_{s_p} \) \((T_{s_p} \geq T_{s_p}^e)\) as its period, the primary path \( P \) executes the Algorithm VII.1 proposed in [7]. During the execution of Algorithm VII.1, the duration of each time beat is set to be \( T_p (\geq 0) \), and the transmission bandwidth of each relaying node is \( B \) (bits/s). Let us only consider the node 1 of \( P \). After performing the incremental sorting operation on the information particle group \( Q \), the bearing capacity of the information particle \( i \) \((1 \leq i \leq M)\) in \( Q \) is \( \ell_i \), and the deadline for its departure from node 1 is

\[
\ell^{(1, 3)}_{e,i} \triangleq \ell^{(1, 2)}_{e,i} - t_{b_0}
\]  

(VIII.2)

where both \( \ell^{(1, 2)}_{e,i} \) and \( t_{b_0} \) are given in Problem VIII.2.
Then, what is the minimum reachable transmission bandwidth $B_{P,Q}^{\text{min},3}$ (bits/s) of node 1 that can ensure that all the $M$ information particles contained in $Q$ leave from node 1 before their corresponding deadlines $t_{i,j}^{(1,3)}$?

**Note VIII.3:**
1) The system defined in Problem VIII.3 is hereinafter referred to as “System VIII.3”;
2) System VIII.3 is mapped from System VIII.2.

**Problem VIII.4:** As shown in the sub-figure D of Fig. VIII.1, assuming that the starting time instant of the initial time beat is 0. With $T_{S_p}$ ($T_{S_p} \geq T_{S_p}$) as its period, the primary path $P$ executes the Algorithm VII.1 proposed in [7]. During the execution of Algorithm VII.1, the duration of each time beat is set to be $T_B$ ($>0$), and the transmission bandwidth of each relaying node is $B$ (bits/s). Let us only consider the node 1 of $P$. After performing the incremental sorting operation on the information particle group $Q$, the bearing capacity of the information particle $i$ ($1 \leq i \leq M$) in $Q$ is $\ell_i$, and the deadline for its departure from node 1 is

$$t_{i,j}^{(1,4)} = T_B \left(1 + T_{S_p} \left[ \frac{t_{i,j}^{(1,3)} - T_B}{T_B \cdot T_{S_p}} \right] \right) \quad (\text{VIII.3})$$

where $t_{i,j}^{(1,3)}$ is given in Problem VIII.3.

Then, what is the minimum reachable transmission bandwidth $B_{P,Q}^{\text{min},4}$ (bits/s) of node 1 that can ensure that all the $M$ information particles contained in $Q$ leave from node 1 before their corresponding deadlines $t_{i,j}^{(1,4)}$?

**Note VIII.4:**
1) The system defined in Problem VIII.4 is hereinafter referred to as “System VIII.4”;
2) System VIII.4 is mapped from System VIII.3.

**Problem VIII.5:** As shown in the sub-figure E of Fig. VIII.1, assuming that the starting time instant of the initial time beat is 0. Let us only consider the node 1 of $P$. Let node 1 remain active at all
times (that is, it is always in the sending state), and it only outputs at the end of each time beat. After performing the incremental sorting operation on the information particle group \( Q \), the bearing capacity of the information particle \( i \) \((1 \leq i \leq M)\) in \( Q \) is \( \ell_{i} \), and the deadline for its departure from node 1 is 

\[
\ell_{i}^{(1,5)} = \ell_{i}^{(1,4)} - T_{b} \cdot (T_{s} - 1),
\]

(VIII.4)

where \( \ell_{i}^{(1,4)} \) is given in \textbf{Problem VIII.4}.

Then, what is the minimum reachable transmission bandwidth \( B_{P,Q}^{\min,5} \) (bits/s) of node 1 that can ensure that all the \( M \) information particles contained in \( Q \) leave from node 1 before their corresponding deadlines \( \ell_{i}^{(1,5)} \)?

\textbf{Note VIII.5:}

1) The system defined in \textbf{Problem VIII.5} is hereinafter referred to as “\textit{System VIII.5}”;
2) \textit{System VIII.5} is mapped from \textbf{System VIII.4}.

\textbf{B. Relationships between key parameters of each system}

In this subsection, we will prove that the minimum reachable transmission bandwidths corresponding to the five systems defined above are equal to each other. In this way, we can naturally transform the problem of solving \textbf{Problem VIII.1} into the problem of solving \textbf{Problem VIII.5} (and the method of solving \textbf{Problem VIII.5} has already been given in this paper). Furthermore, we will also prove that by setting the period of the primary path as its intrinsic period, it will be beneficial to minimize the end-to-end transmission bandwidth.

\textbf{Lemma VIII.1.1:} \( B_{P,Q}^{\min,1} = B_{P,Q}^{\min,2} \).

\textbf{Proof:}

First, let us prove that \( B_{P,Q}^{\min,1} \) of \textbf{System VIII.1} is also reachable for \textbf{System VIII.2}.

Considering that \( B_{P,Q}^{\min,1} \) is reachable for \textbf{System VIII.1}, for all the information particles contained
in information particle group $Q$, there is at least one set of corresponding time instants $t_i^{(N, 3)}$ $(1 \leq i \leq M)$ of leaving node $N$ in System VIII.1 satisfying the following relationships:

$$t_i^{(N, 3)} \leq t_{ij}^{(N, 1)}$$  \hspace{1cm} (VIII.5)

Let $t_i^{(1, 3)}$ $(1 \leq i \leq M)$ indicate the time instant when an information particle $i$ in System VIII.1 leaves node 1. We have

$$t_i^{(1, 3)} = t_i^{(N, 3)} - (N - 1) \cdot T_b$$  \hspace{1cm} (VIII.6)

Combing the above two equations, it can be obtained that

$$t_i^{(1, 3)} = t_i^{(N, 3)} - (N - 1) \cdot T_b \leq t_{ij}^{(N, 1)} - (N - 1) \cdot T_b = t_{ij}^{(N, 2)}$$  \hspace{1cm} (VIII.7)

Let $t_i^{(1, 2)}$ $(1 \leq i \leq M)$ indicate the time instant when an information particle $i$ in System VIII.2 leaves node 1. According to the relations of $t_i^{(1, 2)} = t_i^{(1, 1)}$ $(1 \leq i \leq M)$, time instants $t_i^{(1, 3)}$ are mapped to System VIII.2. Based on the above equation, we can get

$$t_i^{(1, 2)} \leq t_i^{(1, 2)}$$  \hspace{1cm} (VIII.8)

This shows that when the transmission bandwidth of node 1 in System VIII.2 is $B_{P, Q}^{min, 1}$, for all the information particles contained in the information particle group $Q$, there is at least one set of time instants $t_i^{(1, 2)}$ $(1 \leq i \leq M)$ of leaving node 1 satisfying the relationships of $t_i^{(1, 2)} \leq t_i^{(1, 2)}$. Therefore, the transmission bandwidth $B_{P, Q}^{min, 1}$ is reachable for System VIII.2.

Next, let us prove that $B_{P, Q}^{min, 2}$ of System VIII.2 is also reachable for System VIII.1. Considering that $B_{P, Q}^{min, 2}$ is reachable for System VIII.2, for all the information particles contained in information particle group $Q$, there is at least one set of corresponding time instants $t_i^{(1, 2)}$ $(1 \leq i \leq M)$ of leaving node 1 in System VIII.2 satisfying the following relationships:

$$t_i^{(1, 2)} \leq t_i^{(1, 2)}$$  \hspace{1cm} (VIII.9)

According to the relations of $t_i^{(1, 1)} = t_i^{(1, 2)}$ $(1 \leq i \leq M)$, time instants $t_i^{(1, 2)}$ are mapped to System VIII.1. Based on the above equation, we have

$$t_i^{(N, 3)} = t_i^{(1, 1)} + (N - 1) \cdot T_b \leq t_i^{(1, 2)} + (N - 1) \cdot T_b = t_i^{(N, 3)}$$  \hspace{1cm} (VIII.10)
This shows that when the transmission bandwidths of all the relaying nodes in System VIII.1 are set to be $B_{P,Q}^{\text{min},2}$, for all the information particles contained in the information particle group $Q$, there is at least one set of time instants $t_i^{(N,1)}$ ($1 \leq i \leq M$) of leaving node $N$ satisfying the relationships of $t_i^{(N,1)} \leq t_i^{(N,3)}$. Therefore, the transmission bandwidth $B_{P,Q}^{\text{min},2}$ is reachable for System VIII.1.

To sum up, considering that $B_{P,Q}^{\text{min},1}$ and $B_{P,Q}^{\text{min},2}$ are the minimum reachable transmission bandwidths of System VIII.1 and System VIII.2, respectively, it can be obtained that $B_{P,Q}^{\text{min},1} = B_{P,Q}^{\text{min},2}$ must hold.

### Lemma VIII.1.2: $B_{P,Q}^{\text{min},2} = B_{P,Q}^{\text{min},3}$

**Proof:**

First, let us prove that $B_{P,Q}^{\text{min},2}$ of System VIII.2 is also reachable for System VIII.3. Considering that $B_{P,Q}^{\text{min},2}$ is reachable for System VIII.2, for all the information particles contained in information particle group $Q$, there is at least one set of corresponding time instants $t_i^{(1,2)}$ ($1 \leq i \leq M$) of leaving node 1 in System VIII.2 satisfying the following relationships:

$$t_i^{(1,2)} \leq t_i^{(1,3)}$$  \hspace{1cm}  (VIII.11)

According to the relations of $t_i^{(1,3)} = t_i^{(1,2)} - t_{d,i}$ ($1 \leq i \leq M$), time instants $t_i^{(1,2)}$ are mapped to System VIII.3. Based on the above equation, we have

$$t_i^{(1,3)} = t_i^{(1,2)} - t_{d,i} \leq t_i^{(1,2)} - t_{d,i} \leq t_i^{(1,3)}$$  \hspace{1cm}  (VIII.12)

This shows that when the transmission bandwidth of node 1 in System VIII.3 is set to be $B_{P,Q}^{\text{min},2}$, for all the information particles contained in the information particle group $Q$, there is at least one set of time instants $t_i^{(1,3)}$ ($1 \leq i \leq M$) of leaving node 1 satisfying the relationships of $t_i^{(1,3)} \leq t_i^{(1,3)}$. Therefore, the transmission bandwidth $B_{P,Q}^{\text{min},2}$ is reachable for System VIII.3.

Next, let us prove that $B_{P,Q}^{\text{min},3}$ of System VIII.3 is also reachable for System VIII.2.
Considering that \( B_{P,Q}^{\text{min}, 3} \) is reachable for System VIII.3, for all the information particles contained in information particle group \( Q \), there is at least one set of corresponding time instants \( t_{i}^{(1,3)} \) (1 \( \leq \) \( i \) \( \leq \) \( M \)) of leaving node 1 in System VIII.3 satisfying the following relationships:

\[ t_{i}^{(1,3)} \leq t_{i}^{(3,3)} \]  

(VIII.13)

According to the relations of \( t_{i}^{(1,2)} = t_{i}^{(1,3)} + t_{gs} \) (1 \( \leq \) \( i \) \( \leq \) \( M \)) , time instants \( t_{i}^{(1,3)} \) are mapped to System VIII.2. Based on the above equation, we have

\[ t_{i}^{(1,2)} = t_{i}^{(1,3)} + t_{gs} \leq t_{i}^{(1,3)} + t_{gs} = t_{i}^{(1,2)} \]  

(VIII.14)

This shows that when the transmission bandwidth of node 1 in System VIII.2 is set to be \( B_{P,Q}^{\text{min}, 3} \), for all the information particles contained in the information particle group \( Q \), there is at least one set of time instants \( t_{i}^{(1,3)} \) (1 \( \leq \) \( i \) \( \leq \) \( M \)) of leaving node 1 satisfying the relationships of \( t_{i}^{(1,2)} \leq t_{i}^{(1,3)} \).

Therefore, the transmission bandwidth \( B_{P,Q}^{\text{min}, 3} \) is reachable for System VIII.2.

To sum up, considering that \( B_{P,Q}^{\text{min}, 2} \) and \( B_{P,Q}^{\text{min}, 3} \) are the minimum reachable transmission bandwidths of System VIII.2 and System VIII.3, respectively, it can be obtained that \( B_{P,Q}^{\text{min}, 2} = B_{P,Q}^{\text{min}, 3} \) must hold.

---

Lemma VIII.1.3: \( B_{P,Q}^{\text{min}, 3} = B_{P,Q}^{\text{min}, 4} \).

Proof:

First, let us prove that \( B_{P,Q}^{\text{min}, 3} \) of System VIII.3 is also reachable for System VIII.4.

Considering that \( B_{P,Q}^{\text{min}, 3} \) is reachable for System VIII.3, for all the information particles contained in information particle group \( Q \), there is at least one set of corresponding time instants \( t_{i}^{(1,3)} \) (1 \( \leq \) \( i \) \( \leq \) \( M \)) of leaving node 1 in System VIII.3 satisfying the relationships of \( t_{i}^{(1,3)} \leq t_{i}^{(3,3)} \). It can be seen from the second point of Note VIII.1 that for System VIII.3, the time instants when information particles leave node 1 can only be \( T_{g} \cdot (1 + k \cdot T_{s}) \) (\( k \geq 0, k \in \mathbb{Z} \)). Therefore, the relationships of \( t_{i}^{(1,3)} \leq t_{i}^{(3,3)} \) (1 \( \leq \) \( i \) \( \leq \) \( M \)) imply that the relationships of
\[ t^{(1,3)}_i \leq T_b \left( 1 + T_{Sp} \frac{t^{(1,3)}_j - T_b}{T_b \cdot T_{Sp}} \right) \] hold. Hence, it can be obtained that
\[ t^{(1,3)}_i \leq T_b \left( 1 + T_{Sp} \frac{t^{(1,3)}_j - T_b}{T_b \cdot T_{Sp}} \right) = t^{(1,4)}_{ij}. \]
According to the relations of \( t^{(1,4)}_i = t^{(1,3)}_i \), time instants
\( t^{(1,3)}_i \) are mapped to System VIII.4. Since \( t^{(1,3)}_i \leq t^{(1,4)}_{ij} \), we have \( t^{(1,4)}_i \leq t^{(1,4)}_{ij} \) \( (1 \leq i \leq M) \). This shows that when the transmission bandwidth of node 1 in System VIII.4 is set to be \( B_{P, Q}^{min, 3} \), for all the information particles contained in the information particle group \( Q \), there is at least one set of time instants \( t^{(1,4)}_i \) \( (1 \leq i \leq M) \) of leaving node 1 satisfying the relationships of
\[ t^{(1,4)}_i \leq t^{(1,4)}_{ij} \leq t^{(1,3)}_i \]. Therefore, the transmission bandwidth \( B_{P, Q}^{min, 3} \) is reachable for System VIII.4.

Next, let us prove that \( B_{P, Q}^{min, 4} \) of System VIII.4 is also reachable for System VIII.3. Considering that \( B_{P, Q}^{min, 4} \) is reachable for System VIII.4, for all the information particles contained in information particle group \( Q \), there is at least one set of corresponding time instants
\( t^{(1,4)}_i \) \( (1 \leq i \leq M) \) of leaving node 1 in System VIII.4 satisfying the relationships of
\[ t^{(1,4)}_i \leq t^{(1,4)}_{ij} \leq t^{(1,3)}_i \]. Since
\[ t^{(1,4)}_i = T_b \left( 1 + T_{Sp} \frac{t^{(1,3)}_j - T_b}{T_b \cdot T_{Sp}} \right) \leq T_b \left( 1 + T_{Sp} \frac{t^{(1,3)}_j - T_b}{T_b \cdot T_{Sp}} \right) = t^{(1,3)}_i, \]
we have \( t^{(1,4)}_i \leq t^{(1,4)}_{ij} \leq t^{(1,3)}_i \). According to the relations of \( t^{(1,3)}_i = t^{(1,4)}_{ij} \), time instants \( t^{(1,4)}_i \) are mapped to System VIII.3. Since \( t^{(1,4)}_i \leq t^{(1,3)}_i \), we have \( t^{(1,3)}_i \leq t^{(1,3)}_{ij} \) \( (1 \leq i \leq M) \). This shows that when the transmission bandwidth of node 1 in System VIII.3 is set to be \( B_{P, Q}^{min, 4} \), for all the information particles contained in the information particle group \( Q \), there is at least one set of time instants \( t^{(1,3)}_i \) \( (1 \leq i \leq M) \) of leaving node 1 satisfying the relationships of
\[ t^{(1,3)}_i \leq t^{(1,3)}_{ij} \leq t^{(1,4)}_i \] \( (1 \leq i \leq M) \). Therefore, the transmission bandwidth \( B_{P, Q}^{min, 4} \) is reachable for System VIII.3.

To sum up, considering that \( B_{P, Q}^{min, 3} \) and \( B_{P, Q}^{min, 4} \) are the minimum reachable transmission bandwidths of System VIII.3 and System VIII.4, respectively, it can be obtained that
\[ B_{P, Q}^{min, 3} = B_{P, Q}^{min, 4} \] must hold. \[ \blacksquare \]
Lemma VIII.1.4: $B_{P,Q}^{\text{min}, 4} = B_{P,Q}^{\text{min}, 5}$.

Proof:

First, let us prove that $B_{P,Q}^{\text{min}, 4}$ of System VIII.4 is also reachable for System VIII.5.

Considering that $B_{P,Q}^{\text{min}, 4}$ is reachable for System VIII.4, for all the information particles contained in information particle group $Q$, there is at least one set of corresponding time instants $t^{(1,4)}_i (1 \leq i \leq M)$ of leaving node 1 in System VIII.4 satisfying the relationships:

$t^{(1,4)}_i \leq t^{(1,4)}_{e,i} (1 \leq i \leq M)$. Since both $\left\lfloor \frac{t^{(1,4)}_i - T_B}{T_B \cdot T_{S_y}} \right\rfloor$ and $\left\lfloor \frac{t^{(1,4)}_{e,i} - T_B}{T_B \cdot T_{S_y}} \right\rfloor$ are integers, we have

$$\left\lfloor \frac{t^{(1,4)}_i - T_B}{T_B \cdot T_{S_y}} \right\rfloor = \frac{t^{(1,4)}_i - T_B}{T_B \cdot T_{S_y}}$$ \hspace{1cm} (VIII.15)

$$\left\lfloor \frac{t^{(1,4)}_{e,i} - T_B}{T_B \cdot T_{S_y}} \right\rfloor = \frac{t^{(1,4)}_{e,i} - T_B}{T_B \cdot T_{S_y}}$$ \hspace{1cm} (VIII.16)

Mapping $t^{(1,4)}_i$ and $t^{(1,4)}_{e,i}$ to System VIII.5, it can be obtained that

$$t^{(1,5)}_i = t^{(1,4)}_i - T_B \cdot (T_{S_y} - 1) \cdot \left\lfloor \frac{t^{(1,4)}_i - T_B}{T_B \cdot T_{S_y}} \right\rfloor = t^{(1,4)}_i - T_B \cdot (T_{S_y} - 1) \cdot \left( \frac{t^{(1,4)}_i - T_B}{T_B \cdot T_{S_y}} - 1 \right)$$ \hspace{1cm} (VIII.17)

$$= \frac{t^{(1,4)}_i - T_B \cdot (T_{S_y} - 1)}{T_{S_y}}$$

and

$$t^{(1,5)}_{e,i} = t^{(1,4)}_{e,i} - T_B \cdot (T_{S_y} - 1) \cdot \left\lfloor \frac{t^{(1,4)}_{e,i} - T_B}{T_B \cdot T_{S_y}} \right\rfloor = t^{(1,4)}_{e,i} - T_B \cdot (T_{S_y} - 1) \cdot \left( \frac{t^{(1,4)}_{e,i} - T_B}{T_B \cdot T_{S_y}} - 1 \right)$$ \hspace{1cm} (VIII.18)

$$= \frac{t^{(1,4)}_{e,i} - T_B \cdot (T_{S_y} - 1)}{T_{S_y}}$$

Since $t^{(1,4)}_i \leq t^{(1,4)}_{e,i} (1 \leq i \leq M)$, and by further combing the above two equations, we have

$t^{(1,5)}_i \leq t^{(1,5)}_{e,i} (1 \leq i \leq M)$. This shows that when the transmission bandwidth of node 1 in System VIII.5 is set to be $B_{P,Q}^{\text{min}, 4}$, for all the information particles contained in the information particle
group $Q$, there is at least one set of time instants $t^{(1, 5)}_i$ ($1 \leq i \leq M$) of leaving node 1 satisfying the relationships of $t^{(1, 5)}_i \leq t^{(1, 5)}_{e,i}$ ($1 \leq i \leq M$). Therefore, the transmission bandwidth $B^\text{min}_{P, Q}^{4}$ is reachable for System VIII.5.

Next, let us prove that $B^\text{min}_{P, Q}^{5}$ of System VIII.5 is also reachable for System VIII.4. Considering that $B^\text{min}_{P, Q}^{5}$ is reachable for System VIII.5, for all the information particles contained in information particle group $Q$, there is at least one set of corresponding time instants $t^{(1, 5)}_i$ ($1 \leq i \leq M$) of leaving node 1 in System VIII.5 satisfying the relationships: $t^{(1, 5)}_i \leq t^{(1, 5)}_{e,i}$ ($1 \leq i \leq M$). Mapping $t^{(1, 5)}_i$ and $t^{(1, 5)}_{e,i}$ to System VIII.4, we have

$$t^{(1, 4)}_i = T_{s_y} \cdot \left( t^{(1, 5)}_i - \frac{T_{s_y}(T_{s_y} - 1)}{T_{s_y}} \right)$$  \hspace{1cm} (VIII.19)$$

and

$$t^{(1, 4)}_{e,i} = T_{s_y} \cdot \left( t^{(1, 5)}_{e,i} - \frac{T_{s_y}(T_{s_y} - 1)}{T_{s_y}} \right)$$  \hspace{1cm} (VIII.20)$$

Since $t^{(1, 5)}_i \leq t^{(1, 5)}_{e,i}$ ($1 \leq i \leq M$), by further combining the above two equations, it can be obtained that $t^{(1, 4)}_i \leq t^{(1, 4)}_{e,i}$ ($1 \leq i \leq M$). This shows that when the transmission bandwidth of node 1 in System VIII.4 is set to be $B^\text{min}_{P, Q}^{5}$, for all the information particles contained in the information particle group $Q$, there is at least one set of time instants $t^{(1, 4)}_i$ ($1 \leq i \leq M$) of leaving node 1 satisfying the relationships of $t^{(1, 4)}_i \leq t^{(1, 4)}_{e,i}$ ($1 \leq i \leq M$). Therefore, the transmission bandwidth $B^\text{min}_{P, Q}^{5}$ is reachable for System VIII.4.

To sum up, considering that $B^\text{min}_{P, Q}^{4}$ and $B^\text{min}_{P, Q}^{5}$ are the minimum reachable transmission bandwidths of System VIII.4 and System VIII.5, respectively, it can be obtained that $B^\text{min}_{P, Q}^{4} = B^\text{min}_{P, Q}^{5}$ must hold.

\begin{center}
\begin{tabular}{|c|}
\hline
\textbf{Theorem VIII.1}: $B^\text{min}_{P, Q}^{1} = B^\text{min}_{P, Q}^{2} = B^\text{min}_{P, Q}^{3} = B^\text{min}_{P, Q}^{4} = B^\text{min}_{P, Q}^{5}$. \\
\hline
\end{tabular}
\end{center}

Proof:
According to Lemma VIII.1.1, Lemma VIII.1.2, Lemma VIII.1.3 and Lemma VIII.1.4, the proposition can be instantly proved.

Theorem VIII.2: In System VIII.1, given \( N \) (the number of relaying nodes in a primary path \( P \)), \( t_{th} \) (the starting time instant of the initial time beat) and \( T_b \) (the duration of each time beat). Let the information particle group transmitted over the primary path \( P \) be \( Q \), and after the incremental sorting operation is performed on \( Q \), the deadline for the information particle \( i \) (\( 1 \leq i \leq M \)) to leave the node \( N \) (\( \geq 1 \)) is \( t_{(i-1)}^{(N)} \) (\( 1 \leq i \leq M \)). With \( T_{s_p} \) \((T_{s_p} \geq T_{s_p}'\) \( T_{s_p} \) and \( T_{s_p}' \) are the reachable period and the intrinsic period of \( P \), respectively) as its period, the primary path \( P \) executes the Algorithm VII.1 proposed in [7]. Let \( B_{p,Q}^{\min,1} \) denote the minimum reachable transmission bandwidth corresponding to the adopted period of \( T_{s_p} \). The relation of \( B_{p,Q}^{\min,1} \geq B_{p,Q}^{\min,1} \) holds.

Proof:

Based on Equation (VIII.1) and Equation (VIII.2), we have

\[
T_{s_p}^{(i,3)} = T_{s_p}^{(i,3)} - (N-1) \cdot T_b - t_{th} \tag{VIII.21}
\]

By combing Equation (VIII.3), it can be obtained that

\[
T_{s_p}^{(i,4)} \leq T_b \cdot \left(1 + T_{s_p} \cdot \left[\frac{T_{s_p}^{(i,3)}}{T_b \cdot T_{s_p}'} - \frac{T_{s_p}}{T_b \cdot T_{s_p}'}\right]\right) \tag{VIII.22}
\]

Furthermore, combing Equation (VIII.18) and Equation (VIII.22), and since \( T_{s_p} \geq T_{s_p}' \), we can get

\[
T_{s_p}^{(i,5)}(T_{s_p}) = \frac{T_{s_p}^{(i,4)}}{T_{s_p}} + \frac{T_{s_p} \cdot (T_{s_p} - 1)}{T_{s_p}} \tag{VIII.23}
\]

\[
= T_{s_p}^{(i,4)} - \frac{T_{s_p} \cdot (T_{s_p} - 1)}{T_{s_p}} \tag{VIII.23}
\]

\[
= T_{s_p} \cdot \left[\frac{T_{s_p}^{(i,3)}}{T_b \cdot T_{s_p}'} - \frac{T_{s_p}}{T_b \cdot T_{s_p}'}\right] + 1 \tag{VIII.23}
\]

\[
\leq T_{s_p} \cdot \left[\frac{T_{s_p}^{(i,3)}}{T_b \cdot T_{s_p}'} - \frac{T_{s_p}}{T_b \cdot T_{s_p}'}\right] + 1 \tag{VIII.23}
\]

\[
= T_{s_p}^{(i,5)}(T_{s_p}') \tag{VIII.23}
\]
where $t_{i,j}^{(1)}(T_{s_i})$ and $t_{i,j}^{(1)}(T_{s_i}^*)$ represent the deadlines for the information particle $i$ ($1 \leq i \leq M$) leaving the node 1 with $T_{s_i}$ and $T_{s_i}^*$ being the corresponding transmission period in System VIII.5, respectively. It can be obtained from Corollary VI.1.1

$$B_{{P,Q}^5}^{\min}(T_{s_i}) = B_{{P,Q}^5}^{\min}(T_{s_i}) \geq B_{{P,Q}^5}^{\min}(T_{s_i}^*) = B_{{P,Q}^5}^{\min}(T_{s_i}^*)$$  \hspace{1cm} (VIII.24)

where $B_{{P,Q}^5}^{\min}(T_{s_i})$ and $B_{{P,Q}^5}^{\min}(T_{s_i}^*)$ represent the minimum reachable access bandwidths in System VIII.5 with the transmission period being $T_{s_i}$ and $T_{s_i}^*$, respectively. Furthermore, according to Theorem VIII.1, we have

$$B_{{P,Q}^1}^{\min}(T_{s_i}) = B_{{P,Q}^5}^{\min}(T_{s_i}) \geq B_{{P,Q}^5}^{\min}(T_{s_i}) = B_{{P,Q}^5}^{\min}(T_{s_i}^*)$$  \hspace{1cm} (VIII.25)

Hence, it can be concluded that the relation of $B_{{P,Q}^1}^{\min}(T_{s_i}) \geq B_{{P,Q}^5}^{\min}(T_{s_i}^*)$ holds.

Note VIII.6:

1) This theorem shows that by setting the transmission period of the primary path as its intrinsic period $T_{s_i}^*$, the minimum reachable transmission bandwidth of System VIII.1 will be optimized (that is, among all the minimum reachable transmission bandwidths corresponding to various configurable transmission periods, the minimum reachable transmission bandwidth corresponding to the transmission period of $T_{s_i}^*$ is the smallest one), which is the significance of joint optimization of the mechanism of network wave and the mechanism of particle access lying in!

C. The joint optimization algorithm of the mechanism of network wave and the mechanism of particle access

In this subsection, we first describe the algorithm steps (that is, the following Algorithm VIII.1) for calculating the minimum reachable transmission bandwidth of System VIII.1. Next, we describe the algorithm steps (that is, the following Algorithm VIII.2) of allocating each information particle of an information particle group into each successive active time beats of node 1.

Algorithm VIII.1: algorithm steps for calculating the minimum reachable transmission bandwidth
bandwidth of System VIII.1

1: BEGIN

2: Map the deadline $t_{e,i}^{(N-1)}$ of the information particle $i \ (1 \leq i \leq M)$ in System VIII.1 to the deadline $t_{e,i}^{(N-2)} = t_{e,i}^{(N-1)} - (N-1) \cdot T_B$ of the information particle $i$ in System VIII.2;

3: Map the deadline $t_{e,i}^{(1,2)}$ of the information particle $i \ (1 \leq i \leq M)$ in System VIII.2 to the deadline $t_{e,i}^{(1,3)} = t_{e,i}^{(1,2)} - T_{bb}$ of the information particle $i$ in System VIII.3;

4: //According to Theorem VIII.2, in order to optimize the transmission bandwidth, the

5: //transmission period of the primary path is set as its intrinsic period $P_{ST}$.

6: Map the deadline $t_{e,i}^{(1,3)}$ of the information particle $i \ (1 \leq i \leq M)$ in System VIII.3 to the deadline $t_{e,i}^{(1,4)} = T_B \left(1 + T_{Sp} \cdot \frac{t_{e,i}^{(1,3)} - T_B}{T_B \cdot T_{Sp}}\right)$ of the information particle $i$ in System VIII.4;

7: Map the deadline $t_{e,i}^{(1,4)}$ of the information particle $i \ (1 \leq i \leq M)$ in System VIII.4 to the deadline $t_{e,i}^{(1,5)} = t_{e,i}^{(1,4)} - T_B \cdot (T_{Sp} - 1) \left(\frac{t_{e,i}^{(1,4)} - T_B}{T_B \cdot T_{Sp}}\right)$ of the information particle $i$ in System VIII.5;

8: For the information particle group in System VIII.5, apply Theorem VII.1 given in this paper to calculate the minimum reachable access bandwidth $B_{min}^{\alpha Q}$ of the information particle group;

9: Get the minimum reachable transmission bandwidth as $B_{P,Q}^{min,5} = B_{P,Q}^{min,\alpha Q}$;

10: According to Theorem VIII.1, the minimum reachable transmission bandwidth of System VIII.1 is $B_{P,Q}^{min,1} = B_{P,Q}^{min,5}$, that is, when executing Algorithm VII.1 proposed in [7], the transmission bandwidths from node 1 to node $N$ are configured as $B_{P,Q}^{min,1}$.

11: END

Note VIII.7:
1) The information particle group has been incrementally sorted before the algorithm is executed;
2) In each mapping step, the bearing capacity \( \ell_i \) \((1 \leq i \leq M)\) of each information particle remains unchanged.

Algorithm VIII.2: the algorithm steps to calculate the time beats for the primary path (that is, for node 1) transmitting the information particle \( i \) \((1 \leq i \leq M)\) and the corresponding allocations of transmission resources

1: BEGIN
2: Initialize the time pointer as \( t_p = 0 \);
3: Initialize the sequence number of the information particle being processed as \( i = 1 \);
4: WHILE \((i \leq M)\) DO
5: // The processing procedure follows the increasing of the sequence number of
6: // the information particle being processed until all the information particles in the
7: // information particle group are processed
8: The sequence number of the starting time beat of the transmission of information particle \( i \) at node 1 is \( n_\alpha = \left\lceil \frac{t_p}{T_B} \right\rceil \);
9: The sequence number of the ending time beat of the transmission of information particle \( i \) at node 1 is \( n_\beta = \left\lfloor \frac{t_p + \ell_i / B_p^{\min_i}}{T_B} \right\rfloor \);
10: IF \( (n_\alpha = n_\beta) \) THEN
11: // In this case, the information particle \( i \) will be transmitted out within one time beat of node 1
12: The amount of information transmitted by the information particle \( i \) in the time beat \( n_\alpha \) is \( \ell_i \) (bits);
13: ELSE
14: // In this case (that is, \( n_\alpha < n_\beta \)), the information particle \( i \) will be transmitted out
15: // within multiple time beats of node 1

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17: As for the information particle $i$, the amount of information transmitted within the time beat $n_a$ is $B_{P,Q}^{\min,1} \left( \frac{t_p}{T_B} \cdot T_B - t_p \right)$ (bits);

18: As for the information particle $i$, the amount of information transmitted within the time beat $n_B$ is $B_{P,Q}^{\min,1} \left( t_p + \frac{\ell_i}{B_{P,Q}^{\min,1}} \cdot (n_B - 1) \cdot T_B \right)$ (bits);

19: IF ($n_a < n_B - 1$) THEN

20: // In this case, the transmission of information particle $i$ at node 1 will span more than two time beats

21: //more than two time beats

22: As for the information particle $i$, the amount of information transmitted between the time beat $n_a$ and $n_B$ (excluding the time beat $n_a$ and $n_B$) is $B_{P,Q}^{\min,1} \cdot T_B \cdot (n_B - n_a - 1)$ (bits);

23: END IF

24: END IF

25: Refresh the time pointer as $t_p := t_p + \frac{\ell_i}{B_{P,Q}^{\min,1}}$;

26: Refresh the sequence number of the information particle being processed as $i := i + 1$;

27: END WHILE

28: END

Fig. VIII.2 the relationships between some key time points in Algorithm VIII.2 (The sequence number of a time beat shown in the figure indicates the counter of the time beats when node 1 is active)
Note VIII.8:

1) For the better understanding of the above algorithm, please refer to Fig. VIII.2;

2) The information particle group has been incrementally sorted before the algorithm is executed;

3) The “sequence number of a time beat” in the algorithm is serve as the counter of the time beats when node 1 is active. For example, if the sequence number is \( n \), it corresponds to the time beat when node 1 is active for the \( n \) th time;

4) The description of the algorithm only focuses on how the amount of information carried by the information particles is distributed among the time beats corresponding to node 1. As for the transmission of these information particles in other relaying nodes, the Algorithm VII.1 in [7] is strictly followed and executed accordingly;

5) The above algorithm does not specify how the information bits arranged in a specific time beat should be transmitted in the corresponding time beat. In fact, any transmission method that can ensure that the information bits arranged in a specific time beat are transmitted before the end of the time beat is considered feasible.
IX. Conclusions

Aiming at some problems existing in the current QoS guarantee mechanism in large-scale networks (i.e. poor scalability, coarse granularity for provided service levels, poor fairness between different service levels, and improving delay performance at the expense of sacrificing some resource utilization), this paper puts forward the basic idea and theory of particle access. In the proposed particle access mechanism, the network allocates access resources to the information particle group which is composed of all the information particles to be transmitted, so as to ensure that the occupied bandwidth resources is minimized on the premise of meeting the delay requirements of each information particle. Specifically, in the paper, the concepts of both information particle and information particle group are defined; The relationships between the average access bandwidth requirement, the peak average access bandwidth requirement, and the minimum reachable access bandwidth of an information particle group are analyzed; The influences of time attribute and attribute of bearing capacity of an information particle group on the minimum reachable access bandwidth are analyzed; An effective method for the calculation of the minimum reachable access bandwidth of an information particle group is given, and a particle access algorithm based on dynamically adjusting the minimum reachable access bandwidth is proposed; Furthermore, several properties of the minimum reachable access bandwidth(s) of information particle group(s) are proved; Finally, the joint optimization of the mechanism of network wave and the mechanism of particle access is studied. For a given information particle group transmitted over a given primary path, a method to obtain the minimum reachable transmission bandwidth, which can guarantee the end-to-end delays of all the information particles contained in the information particle group, is proposed.

The research results of this paper provide a new idea for further improving the QoS mechanism of large-scale networks, and lay the corresponding theoretical foundation.

In the following, some important conclusions are summarized:

1) In Theorem V.1, it is pointed out that the minimum reachable access bandwidth of an information particle group is equal to its peak average access bandwidth requirement;

2) In Theorem VI.3 and Theorem VI.5, it is proved that for a given information particle group which meets a certain bandwidth constraint, it can still meet the bandwidth constraint by adjusting the deadline attribute and/or the bearing capacity attribute of a newly arrived
3) In Theorem VII.1, a more concise and easy to be handled expression of the minimum reachable access bandwidth of an information particle group is given;

4) In Corollary VII.1.5, the fact that the minimum reachable access bandwidth decreases at the inflection point of the access bandwidth requirement of an information particle group is proved;

5) In Algorithm VII.1, a particle access algorithm based on dynamically adjusting the minimum reachable access bandwidth is proposed;

6) The upper bound (Corollary VII.1.6) and the lower bound (Corollary VII.1.7) of the minimum reachable access bandwidth of an information particle group are given, respectively;

7) The property of linear superpositions of multiple information particle groups under specific constraints is proved (Corollary VII.1.8). The property of linear splitting of an information particle group is also proved (Corollary VII.1.9);

8) A closed form expression of representing the minimum reachable access bandwidth of an information particle subgroup as the product of its vector of switching functions and its vector of average access bandwidth requirements is given (Corollary VII.1.10). And it is proved that the vector of switching functions has and only has one component with its value being equal to one (Corollary VII.1.11);

9) In Algorithm VIII.1 and Algorithm VIII.2, for a given information particle group transmitted over a given primary path, a method to obtain the minimum reachable transmission bandwidth, which can guarantee the end-to-end delays of all the information particles contained in the information particle group, is proposed.
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Appendix A: Symbol List

| Symbol | Description |
|--------|-------------|
| $\ell_i$ | The bearing capacity of an information particle (refer to Definition III.3) |
| $t_{b,i}$ | The initial time instant of an information particle (refer to Definition III.4) |
| $t_{e,i}$ | The deadline of an information particle (refer to Definition III.5) |
| $D_i^{mi}$ | The effective survival time interval of an information particle (refer to Definition III.6) |
| $L_i^{e}$ | The effective survival time span of an information particle (refer to Definition III.7) |
| $r_i(t)$ | The instantaneous transmission rate of an information particle (refer to Definition III.8) |
| $S_i$ | The transmission strategy exerted to an information particle $i$ (refer to Definition III.9) |
| $B_{av,i}$ | The average access bandwidth requirement of an information particle (refer to Definition III.11) |
| $Q$ | An information particle group (refer to Definition IV.1) |
| $Op_{inc}(Q)$ | The incremental sorting operation performed on an information particle group $Q$ (refer to Definition IV.2) |
| $\ell_Q$ | The bearing capacity of an information particle group (refer to Definition IV.3) |
| $D_Q^{mi}$ | The effective survival time interval of an information particle group (refer to Definition IV.4) |
| $L_Q^{e}$ | The effective survival time span of an information particle group (refer to Definition IV.5) |
| $S_Q$ | The transmission strategy exerted to an information particle group $Q$ (refer to Definition IV.6) |
| $B_{av,Q}$ | The average access bandwidth requirement of an information particle group $Q$ (refer to Definition IV.8) |
| $B_{av,Q}^{pk}$ | The peak average access bandwidth requirement of an information particle group $Q$ |
| **Symbol** | **Description** |
|------------|----------------|
| $B_{\text{re},Q}$ | The reachable access bandwidth of an information particle group $Q$ (refer to Definition IV.11) |
| $B_{\text{re},Q}^\text{min}$ | The minimum reachable access bandwidth of an information particle group $Q$ (refer to Definition IV.12) |
| $\eta_{Q\theta_n,Q}$ | The access efficiency of an information particle group $Q$ under a given access bandwidth and within its effective survival time interval (refer to Definition IV.14) |
| $N_{\text{cr},Q}$ | The inflection point of the required access bandwidth of an information particle group $Q$ (refer to Definition IV.15) |
| $Q_{cr}^*$ | The largest principal subgroup of an information particle group $Q$ (refer to Definition IV.16) |
| $Q_i$ | After an incremental sorting operation $\text{Op}_\text{inc}(Q)$ is performed, $Q$ ($Q \subseteq Q$) is the information particle subgroup composed of all the information particles in $Q$ whose sorting sequence numbers are not greater than $i$ (refer to Definition IV.17) |
| $Q^i$ | An information particle group with sequence number $i$ (refer to Definition IV.18) |
| $\delta_{i,Q}$ | Switching function of an information particle group $Q$ with sequence number $i$ (refer to Definition IV.19) |
| $\delta_{Q_n}$ | Vector of switching functions of an information particle subgroup $Q_n$ ($Q_n \subseteq Q$) (refer to Definition IV.20) |
| $B_{\text{re},Q_n}$ | Vector of average access bandwidth requirements of an information particle subgroup $Q_n$ ($Q_n \subseteq Q$) (refer to Definition IV.21) |
| $I_{\text{glob}}$ | The starting time instant of the initial time beat (refer to Problem VIII.1) |
| $T_B$ | The duration of a time beat (refer to Problem VIII.1) |
| $t_{\text{de}}^{(N,N)}$ | The deadline of an information particle leaving from node $N$ in System VIII.1 (refer to Problem VIII.1) |
| $B_{\text{min},1,P,Q}$ | The minimum reachable transmission bandwidth of each relaying node in |
| Equation | Description |
|----------|-------------|
| $t_{ex_i}^{(1,2)}$ | The deadline of an information particle $i$ leaving from node 1 in System VIII.2 (refer to Problem VIII.2) |
| $B_{min,2}^{P,Q}$ | The minimum reachable transmission bandwidth of node 1 in System VIII.2 (refer to Problem VIII.2) |
| $t_{ex_i}^{(1,3)}$ | The deadline of an information particle $i$ leaving from node 1 in System VIII.3 (refer to Problem VIII.3) |
| $B_{min,3}^{P,Q}$ | The minimum reachable transmission bandwidth of node 1 in System VIII.3 (refer to Problem VIII.3) |
| $t_{ex_i}^{(1,4)}$ | The deadline of an information particle $i$ leaving from node 1 in System VIII.4 (refer to Problem VIII.4) |
| $B_{min,4}^{P,Q}$ | The minimum reachable transmission bandwidth of node 1 in System VIII.4 (refer to Problem VIII.4) |
| $t_{ex_i}^{(1,5)}$ | The deadline of an information particle $i$ leaving from node 1 in System VIII.5 (refer to Problem VIII.5) |
| $B_{min,5}^{P,Q}$ | The minimum reachable transmission bandwidth of node 1 in System VIII.5 (refer to Problem VIII.5) |
