Tensor gauge boson production in high-energy collisions

S. Konitopoulos, R. Fazio\(^{(a)}\) and G. Savvidy

Institute of Nuclear Physics, Demokritos National Research Center - Agia Paraskevi, GR-15310 Athens, Greece, EU

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Abstract – We considered spin-two gauge boson production in the fermion pair annihilation process and calculated the polarized cross-sections for each set of helicity orientations of initial and final particles. Adding up all sixteen amplitudes and averaging over the initial particle spins, we obtained the unpolarized cross-section. The angular dependence of these cross-sections is compared with the similar annihilation cross-sections in QED with two photons in the final state, with two gluons in QCD and W-pair in the electroweak theory.

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An infinite tower of massive particles of high spin naturally appears in the spectrum of different string field theories. It is generally expected that in the tensionless limit or, what is equivalent, at high energy and fixed-angle scattering the string spectrum becomes effectively massless and it is of great importance to find out the corresponding Lagrangian and its genuine symmetries [1–6]. In the open-string theory with Chan-Paton charges these massless states can combine into the infinite tower of non-Abelian tensor gauge fields [7] and one could guess that the corresponding Lagrangian quantum field theory should be described by some extension of the Yang-Mills theory.

A possible extension of the non-Abelian theory to non-Abelian tensor gauge fields and the construction of a gauge-invariant Lagrangian was made recently in [8–10]. Recall that non-Abelian gauge fields are defined as rank- \((s+1)\) tensor gauge fields \(A^a_{\mu_1\ldots\mu_s}\) (see footnote \(^1\)) and that one can construct infinite series of forms \(L_s\) \((s = 1, 2, 3, \ldots)\) which are invariant with respect to the extended gauge transformations \([8–10]\). These forms are quadratic in the field strength tensors \(G^a_{\mu\nu,\lambda\ldots\lambda_s}\). The resulting gauge-invariant Lagrangian \(\mathcal{L}\) defines cubic and quartic self-interactions of charged gauge quanta carrying a spin larger than one \([8–10]\). The gauge-invariant Lagrangian describing dynamical tensor gauge bosons of all ranks has the form \([8–10]\):

\[
\mathcal{L} = \mathcal{L}_{YM} + g_2(L_2 + L_2') + \ldots,
\]

where \(\mathcal{L}_{YM}\) is the Yang-Mills Lagrangian. For the lower-rank tensor gauge fields the Lagrangian has the following form \([8–10]\):

\[
L_1 = -\frac{1}{4}G^a_{\mu\nu}\mathcal{G}^a_{\mu\nu},
\]

\[
L_2 = -\frac{1}{4}G^a_{\mu\nu,\lambda}\mathcal{G}^a_{\mu\nu,\lambda} - \frac{1}{4}G^a_{\mu\nu}\mathcal{G}^a_{\mu\nu,\lambda\lambda},
\]

\[
L_2' = \frac{1}{4}G^a_{\mu\nu,\lambda}\mathcal{G}^a_{\mu\nu,\lambda} + \frac{1}{4}G^a_{\mu\nu,\lambda}\mathcal{G}^a_{\mu\nu,\lambda\lambda} + \frac{1}{2}G^a_{\mu\nu}\mathcal{G}^a_{\mu\nu,\lambda\lambda},
\]

where the generalized field strength tensors are

\[
G^a_{\mu\nu,\lambda} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu,
\]

\[
G^a_{\mu\nu,\lambda} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} (A^b_\mu A^c_\nu + A^b_\nu A^c_\mu),
\]

\[
G^a_{\mu\nu,\lambda\rho} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} (A^b_\mu A^c_\nu - A^b_\nu A^c_\mu) + g f^{abc} (A^b_\mu A^c_\nu A^d_\rho + A^b_\nu A^c_\mu A^d_\rho - A^b_\nu A^c_\mu A^d_\rho + A^b_\mu A^c_\nu A^d_\rho),
\]

The definition of the Lagrangian forms \(L_s\) and \(L_s'\) for higher-rank fields can be found in the previous publications \([8–10]\). The fermions are defined as Rarita-Schwinger spinor-tensor fields \(\psi^a_{\lambda_1\ldots\lambda_s}\) with mixed transformation properties of Dirac four-component wave function and are totally symmetric tensors of rank \(s\) over the indices \(\lambda_1\ldots\lambda_s\) (the index \(a\) denotes the Dirac index and will be suppressed in the remaining part of the article). All

\(^{(a)}\)On a leave of absence from: Departamento de Física, Universidad Nacional de Colombia - Bogotá, Colombia.

\(^1\)The tensor gauge fields \(A^a_{\mu_1\ldots\lambda_s}(x), s = 0, 1, 2, \ldots\) are totally symmetric with respect to the indices \(\lambda_1\ldots\lambda_s\). A priori the tensor fields have no symmetries with respect to the first index \(\mu\). In particular we have \(A^a_{\mu\lambda} \neq A^a_{\lambda\mu}\) and \(A^a_{\mu\lambda} = A^a_{\mu\rho\lambda} \neq A^a_{\lambda\mu\rho}\). The adjoint group index \(a = 1, \ldots, N^2 - 1\) in the case of the SU\((N)\) gauge group.
fields of the \( \{ \psi \} \) family are isotopic multiplets \( \psi^{i}_{\lambda \ldots \lambda_{s}}(x) \) belonging to the same representation \( \sigma_{i}^{\lambda} \) of the compact Lie group \( G \) (the index \( i \) denotes the isotopic index). The invariant Lagrangian is the sum \( [9] \)

\[
L_F = L_D + f_{3/2} L_{3/2} + \ldots ,
\]

where \( L_D \) is the Dirac Lagrangian:

\[
L_D = \bar{\psi} i \gamma^{\mu} (\delta_{ij} i \partial_{\mu} + g \sigma_{ij} A_{\mu}^{a}) \psi^{j} = \bar{\psi} (i \partial + g A) \psi,
\]

and for the spin–vector field \( \psi^{a}_{\mu} \) together with the additional rank-2 spin-tensor \( \psi_{\mu \lambda} \), the invariant Lagrangian has the form \( [9] \)

\[
L_{3/2} = \bar{\psi} \lambda \gamma_{\mu} (i \partial_{\mu} + g A_{\mu}) \psi_{\lambda} + \frac{1}{2} \bar{\psi} \lambda \gamma_{\mu} (i \partial_{\mu} + g A_{\mu}) \psi_{\lambda} + \frac{1}{2} \bar{\psi} \lambda \gamma_{\mu} (i \partial_{\mu} + g A_{\mu}) \psi_{\lambda} + g \bar{\psi} \lambda \gamma_{\mu} A_{\mu} A_{\lambda} \psi_{\lambda} + \frac{1}{2} g \bar{\psi} \lambda \gamma_{\mu} A_{\mu} A_{\lambda} \psi_{\lambda}.
\]

The above expressions define interacting gauge field theory with infinitely many gauge fields. Not much is known about physical properties of such gauge field theories and in the present paper we shall focus our attention on the lower-rank tensor gauge field \( A_{\mu}^{a} \lambda \), which describes in this theory charged gauge bosons of spin two. We are interested in studying the first nontrivial interaction processes which are predicted by this tensor gauge field theory. In particular we shall consider a possible production of charged spin-two gauge bosons \([8–10]\).

Our intention in this article is to calculate the leading-order differential cross-section of spin-two gauge boson production in the fermion pair annihilation process \( f + \bar{f} \rightarrow T + T \) and to analyze the angular dependence of the polarized cross-sections for each set of helicity orientations of initial and final particles \([18]\) and shall compare them with the corresponding cross-sections for photons and gluons in QED and QCD, as well as with the \( W \)-pair production in the electroweak theory.

The annihilation process is illustrated in fig. 1. Working in the center-of-mass frame, we make the following assignments: \( p_{-} = (E_{-}, \vec{p}_{-}) \), \( p_{+} = (E_{+}, \vec{p}_{+}) \), \( k_{1} = (\omega_{1}, \vec{k}_{1}) \), \( k_{2} = (\omega_{2}, \vec{k}_{2}) \), where \( p_{\pm} \) are momenta of the fermions \( f \bar{f} \) and \( k_{1,2} \) momenta of the tensor gauge bosons \( TT \). All particles are massless \( p_{-}^{2} = p_{+}^{2} = k_{1}^{2} = k_{2}^{2} = 0 \). In the center-of-mass frame the momenta satisfy the relations \( \vec{p}_{+} = -\vec{p}_{-} \), \( \vec{k}_{2} = -\vec{k}_{1} \) and \( E_{-} = E_{+} = \omega_{1} = \omega_{2} = E \). The invariant variables of the process are

\[
s = 2(k_{1} \cdot k_{2}), \quad t = -\frac{s}{2} (1 - \cos \theta), \quad u = -\frac{s}{2} (1 + \cos \theta),
\]

where \( s = (2E)^{2} \) and \( \theta \) is the scattering angle.

The Feynman rules for the Lagrangian \((1), (4)\) can be derived from the functional integral over the fermion fields \( \bar{\psi}, \psi_{\mu}, \psi_{\mu \nu}, \ldots \) and over the gauge boson fields \( A_{\mu}^{a}, A_{\mu \nu}^{a}, \ldots \) \([8–10]\). The Dirac indices are not shown, the indices of the symmetry group \( G \) are \( i, j = 1, \ldots, d(r) \), where \( d(r) \) is the dimension of the representation \( r \) and \( a = 1, \ldots, d(G) \), where \( d(G) \) is the number of generators of the group \( G \).

In the momentum space the interaction vertex of vector gauge boson V with two tensor gauge bosons T—the VTT vertex—has the form\( ^{2}[9,10]\)

\[
V_{\alpha \alpha'} \beta \gamma \gamma' (k, p, q) = -g F_{\alpha \alpha' \beta \gamma \gamma'}(k, p, q),
\]

where

\[
F_{\alpha \alpha' \beta \gamma \gamma'}(k, p, q) = [\eta_{\alpha \beta}(p - k), \gamma_{\gamma}, \eta_{\alpha' \gamma'}(k - q)]
\]

(8)

The Lorentz indices \( \alpha, \beta \) and momentum \( k \) belong to the first tensor gauge boson, the \( \gamma, \gamma' \) and momentum \( q \) belong to the second tensor gauge boson, and the Lorentz index \( \beta \) and momentum \( p \) belong to the vector gauge boson. The vertex is shown in fig. 2. Vector gauge bosons are conventionally drawn as thin wavy lines, tensor gauge bosons are thick wavy lines.

It is convenient to write the differential cross-section in the center-of-mass frame with the tensor boson produced into the solid angle \( d\Omega \) as

\[
d\sigma = \frac{1}{2s} |M|^{2} \frac{1}{32\pi^{2}} d\Omega,
\]

where the final-state density is \( d\Phi = \frac{1}{32\pi^{2}} d\Omega \).

\( ^{2} \) See formulas \((62), (65) \) and \((66)\) in \([10]\).
orientations of initial and final particles together with the unpolarized cross-section. Using the explicit form of the vertex operator $P^{\mu \rho \nu \beta}$ (7), (8) and the orthogonality properties of the tensor gauge boson wave functions

\[
k_1^+ \epsilon_{\mu \alpha} (k_1) = k_1^\alpha \epsilon_{\mu \alpha} (k_1) = 0,
k_2^\alpha \epsilon_{\mu \alpha} (k_2) = k_2^\beta \epsilon_{\mu \beta} (k_2) = 0,
\]

(11)

where the last relations follow from the fact that $k_1 \parallel k_2$ in the process of fig. 1, we shall get

\[
\mathcal{M}^{\mu \alpha \rho \beta} \epsilon^*_{\mu \alpha} (k_1) \epsilon^*_{\rho \beta} (k_2) =

\]

\[
(ig)^2 \bar{\psi}(p_+) \left\{ \Gamma^{\mu \rho \nu \beta} \right\} \frac{1}{4} \frac{g^{\alpha \beta}}{p_- - k_2} t^\beta \gamma^\nu + \gamma^\rho t^\beta \frac{1}{4} g^{\alpha \beta} \gamma^\mu

+i f^{abc} t^c \gamma^\rho (k_2 - k_1)^\mu

\frac{1}{k_2^2} \left\{ g^{\nu \beta} g^{\alpha \sigma} - \frac{1}{2} g^{\alpha \beta} g^{\nu \sigma} \right\} u(p_-) \epsilon^*_{\mu \alpha} (k_1) \epsilon^*_{\rho \beta} (k_2),
\]

(12)

As the next step we shall calculate the above matrix element in the helicity basis for initial fermions and final tensor gauge bosons. This calculation of polarized cross-sections is very similar to the one in QED [13]. The right- and left-handed spinors wave functions are

\[
u^R (p_-) = \sqrt{2E} (0, 0, 0, 1), \quad \nu^L (p_-) = (0, 0, -1, 0)
\]

(13)

and the tensor gauge boson wave functions for circular polarizations along the $k_1$-direction are

\[
e_R^{\mu \alpha} (k_1) = \frac{i}{2} (0, \cos \theta, i, - \sin \theta) \otimes (0, \cos \theta, i, - \sin \theta),
e_L^{\mu \alpha} (k_1) = \frac{i}{2} (0, - \cos \theta, i, \sin \theta) \otimes (0, - \cos \theta, i, \sin \theta).
\]

(14)

It is easy to check that the wave functions (14) are orthonormal, $e_R^{\mu \alpha} (k_1) e_L^{\nu \beta} (k_1) = 0$, $e_R^{\mu \alpha} (k_1) e_R^{\nu \beta} (k_1) = 1$, $e_L^{\mu \alpha} (k_1) e_L^{\nu \beta} (k_1) = 1$, and fulfill eqs. (11). The helicity states for the second gauge boson are $e_R^{\nu \beta} (k_2) = \epsilon_R^{\nu \beta} (k_2)$, $e_L^{\nu \beta} (k_2) = \epsilon_L^{\nu \beta} (k_2)$, where $k_1^\alpha = (E, E \sin \theta, 0, E \cos \theta)$ and $k_2^\alpha = (E, -E \sin \theta, 0, -E \cos \theta)$. Now we can calculate all sixteen matrix elements between states of definite helicities. Let us start with $f_R f_L \rightarrow T_R T_R$. The scattering amplitude (12) for these particular helicities $\mathcal{M}^{\mu \alpha \nu \beta} \epsilon^R_{\mu \alpha} (k_1) \epsilon^R_{\nu \beta} (k_2)$ contains three terms. By plugging explicit expressions for the helicity wave functions (13), (14) into the matrix element (12), we can find the first term

\[
(ig)^2 \bar{\psi}^R (p_+) \left\{ \Gamma^{\mu \rho \nu \beta} \right\} \frac{1}{4} \frac{g^{\alpha \beta}}{p_- - k_2} t^\beta \gamma^\nu + \gamma^\rho t^\beta \frac{1}{4} g^{\alpha \beta} \gamma^\mu u(R_+) e^R_{\mu \alpha} (k_1) e^R_{\nu \beta} (k_2) =

\]

\[
\frac{(ig)^2}{4} t^\rho t^\beta \sin \theta,
\]

then the second one

\[
(ig)^2 \bar{\psi}^L (p_+) \left\{ \Gamma^{\mu \rho \nu \beta} \right\} \frac{1}{4} \frac{g^{\alpha \beta}}{p_- - k_1} t^\beta \gamma^\nu + \gamma^\rho t^\beta \frac{1}{4} g^{\alpha \beta} \gamma^\mu u(R_-) e^L_{\mu \alpha} (k_1) e^L_{\nu \beta} (k_2) =

\]

\[
- \frac{(ig)^2}{4} t^\rho t^\beta \sin \theta.
\]
and finally the third one

\[ \langle ig \rangle^{2}_{0} P_{+} \left\{ i f^{abc} t_{c} \left( \frac{k_{2} - k_{1}}{k_{1}^{2}} \right) \left( g^{\mu \nu} g^{\alpha \beta} - \frac{1}{2} g^{\mu \beta} g^{\alpha \nu} \right) \right\} \]

\[ \times u^{R}(p_{-}) e^{R}_{\mu}(k_{1}) e^{R}_{\nu}(k_{2}) = -i \frac{\langle ig \rangle^{2}}{2} f^{abc} t_{c} \sin \theta, \]

so that all together we will give

\[ M_{RL}^{\alpha \beta} e^{R}_{\nu}(k_{1}) e^{R}_{\nu}(k_{2}) = \frac{\langle ig \rangle^{2}}{4} \left( \left( f^{a} t^{b} - 2i f^{abc} t_{c} \right) \sin \theta = -i \frac{\langle ig \rangle^{2}}{2} f^{abc} t_{c} \sin \theta. \right) \]  

(15)

To compute the cross-section, we must square the matrix element (15) and then average over the symmetries of the initial fermions and sum over the symmetries of the final tensor gauge bosons. This gives

\[ \sum |M|_{RL \rightarrow RR}^{2} = \frac{g^{4}}{16 d(r)} \text{tr}(f^{abc} t_{c} f^{bd} t_{d}) \sin^{2} \theta = \frac{g^{4}}{16 d(r)} C_{2}(r) C_{2}(G) \sin^{2} \theta, \]

(16)

where the invariant operator \( C_{2} \) is defined by the equation \( t^{a} t^{a} = C_{2} 1 \). Similarly, using the helicity functions (13) and (14), we can calculate the amplitude \( f_{f_{L} \rightarrow T_{L}} \). This gives

\[ \sum |M|_{RL \rightarrow LL}^{2} = \frac{g^{4}}{16 d(r)} \text{tr}(f^{abc} t_{c} f^{bd} t_{d}) \sin^{2} \theta = \frac{g^{4}}{16 d(r)} C_{2}(r) C_{2}(G) \sin^{2} \theta. \]  

(17)

The amplitude \( f_{f_{L} \rightarrow T_{L}} \) vanishes because the factor common to all the three pieces of this amplitude \(-g^{\nu \rho} e^{R}_{\nu}(k_{1}) e^{R}_{\nu}(k_{2})=\) is equal to zero. Thus only four amplitudes out of sixteen are nonzero: \( f_{f_{L} \rightarrow T_{R}}, f_{f_{L} \rightarrow T_{L}}, f_{L}, f_{R} \rightarrow T_{R}, f_{L}, f_{R} \rightarrow T_{L}. \) From this analysis it follows that the total spin angular momentum of the final state is one unit less than that of the initial state, therefore one unit of spin angular momentum is converted to the orbital angular momentum and the final state is a \( P \)-wave.

We can calculate now the leading-order polarized cross-sections for the tensor gauge boson production in the annihilation process. Plugging matrix elements (16) into our general cross-section formula in the center-of-mass frame (9) yields

\[ d \sigma_{f_{f_{L} \rightarrow T_{R} T_{R}}} = \frac{\alpha^{2}}{s} \frac{C_{2}(r) C_{2}(G)}{64 d(r)} \sin^{2} \theta \text{d} \Omega, \]

(18)

where \( \alpha = \frac{g^{2}}{4 \pi}. \) For the rest of the helicities we shall get: \( d \sigma_{f_{f_{L} \rightarrow T_{R} T_{L}}} = d \sigma_{f_{f_{L} \rightarrow T_{L} T_{R}}} = d \sigma_{f_{f_{L} \rightarrow T_{L} T_{L}}} = d \sigma_{f_{L} f_{R} \rightarrow T_{L} T_{L}}, \) where for the \( SU(N) \) group we have \( C_{2}(r) C_{2}(G) = \frac{(N^{2} - 1)}{128 N}. \) Adding up all sixteen amplitudes and dividing by four in order to average over the initial particle spins, we shall get the unpolarized cross-section which simply coincides with (18).

This cross-section should be compared with the analogous annihilation cross-sections in QED and QCD. Indeed, let us compare this result with the electron-positron annihilation into two transversal photons. The \( e^{+} e^{-} \rightarrow \gamma \gamma \) annihilation cross-section [14] in the high-energy limit is

\[ d \sigma_{\gamma \gamma} = \frac{\alpha^{2}}{s} \frac{1 + \cos^{2} \theta}{\sin^{2} \theta} \text{d} \Omega, \]

(19)

except for very small angles of order \( m_{e} / E. \) The cross-section has a minimum at \( \theta = \pi / 2 \) and then increases for small angles [15]. The quark pair annihilation cross-section into two transversal gluons \( q \bar{q} \rightarrow gg \) in the leading order of the strong coupling \( \alpha_{s} \) is

\[ d \sigma_{gg} = \frac{\alpha^{2}}{s} \frac{C_{2}(r) C_{2}(r)}{4 d(r)} \left[ 1 + \cos^{2} \theta \right] \text{d} \Omega \]

(20)

and also has a minimum at \( \theta = \pi / 2 \) and increases for small scattering angles [16]. The production cross-section of spin-two gauge bosons (18) shows a dramatically different behaviour \(-\sin^{2} \theta—\) with its maximum at \( \theta = \pi / 2 \) and decreases for small angles. It is also instructive to compare this result with the angular dependence of the \( W \)-pair production in the electroweak theory. The high-energy production of longitudinal gauge bosons \( W_{0}^{\pm} \) is [17]

\[ d \sigma_{e^{+} e^{-} \rightarrow W_{0}^{+} W_{0}^{-}} = \frac{\alpha^{2}}{s} \left[ 1 + 4 \sin^{2} \theta \right] \text{d}\Omega, \]

(21)

where \( \cos \theta = \frac{m_{W}}{m_{Z}} \) and it is similar to the spin-two transversal gauge boson production (18).

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