A PID CONTROL METHOD BASED ON OPTIMAL CONTROL STRATEGY

HONG NIU*, ZHIJIANG FENG AND QUIN XIAO

College of Science, Liaoning Shihua University
Fushun Liaoning, 113001, China

YAJUN ZHANG

State Key Laboratory of Synthetical Automation for Process Industries
Northeastern University, Shenyang Liaoning 110819, China

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Abstract. A PID control method which combined optimal control strategy is proposed in this paper. The posterior unmodeled dynamics measurement data information are made full use to compensate the unknown nonlinearity of the system, and the unknown increment of the unmodeled dynamics is estimated. Then, a nonlinear PID controller with compensation of the posterior unmodeled dynamics measurement data and the estimation of the increment of the unmodeled dynamics is designed. Finally, through the numerical simulation, the effectiveness of the proposed method is verified.

1. Introduction. For a class of nonlinear system, the parameter identification algorithm and the intelligence estimation algorithm are combined to design a controller. Such as in [8, 9], adaptive fuzzy control method of stochastic nonlinear system and pure feedback adaptive control method are proposed. In [11, 10], an adaptive control method using fuzzy logic system to compensate the unknown nonlinear dynamical characteristics of the system is proposed. The nonlinear fuzzy sliding mode switching control method is proposed in [6, 7]. In [3, 4, 1, 14], switching control technologies based on neural networks and data driven method are proposed.

In [2], a new nonlinear control method is proposed. However, the method in [2] also directly conducts intelligent modeling, without making use of the historical data information. On this basis, a non-linear control method based on the measurable data of the pulp neutralization process and the historical data of the unmodeled dynamics is proposed in [15]. But, the known increment of the unmodeled dynamics is not used by the proposed method.

In this paper, the idea of paper [12] is adopted. The widely used PID control, unmodeled dynamics estimation technology are fully integrated. A non-linear PID control based on unmodeled dynamics compensation is proposed. The method

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* Corresponding author: Hong Niu.
makes full use of the input and output data information of the controlled plant, the unmodeled dynamics measurable historical data information and the non-linear compensation technology. By using ANFIS to estimate the unknown increment of the unmodeled dynamics, the compensator is designed. A better closed-loop control effect is achieved. Finally, the proposed control algorithm is verified by numerical simulation, the results show the effectiveness of the proposed algorithm.

2. Problem Description. A class of non-linear controlled plants can be described as

\[ y(k) = f [y(k - 1), \ldots, y(k - n_A), u(k - 1), \ldots, u(k - 1 - n_B)] \quad (1) \]

where \( u(k) \) and \( y(k) \) are the input and output, \( n_A \) and \( n_B \) are the orders of the model, and \( f(\cdot) \) is an unknown nonlinear function.

By using the idea of paper [5], the equation (1) can be transformed into a form as follows:

\[ y(k + 1) = -\mathcal{A} (z^{-1}) y(k) + B (z^{-1}) u(k) + v[x(k)] \quad (2) \]

where \( \mathcal{A} (z^{-1}) \) and \( B (z^{-1}) \) are respectively

\[
\mathcal{A} (z^{-1}) = -a_1 z^{-1} - a_2 z^{-2} - \cdots - a_{n_A} z^{-n_A}, \quad (3)
\]
\[
B (z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B}. \quad (4)
\]

\( v[x(k)] \) is unmodeled dynamics, \( x(k) \) is

\[ x(k) = [y(k), \ldots, y(k - n_A + 1), u(k), \ldots, u(k - n_B)]^T \quad (5) \]

The model (2) can be rewritten to the following form:

\[ A (z^{-1}) y(k + 1) = B (z^{-1}) u(k) + v[x(k)] \quad (6) \]

where \( A (z^{-1}) \) and \( B (z^{-1}) \) are

\[
A (z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{n_A} z^{-n_A} \quad (7)
\]
\[
B (z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{n_B} z^{-n_B} \quad (8)
\]

Considering \( v[x(k - 1)] \) can be obtained, the unknown unmodeled dynamics \( v[x(k)] \) can be expressed as the sum of \( v[x(k - 1)] \) plus the unknown incremental \( \Delta v(k) \) by using the historical data.

\[ v[x(k)] = v[x(k - 1)] + \Delta v(k) \quad (9) \]

Therefore, (6) can be further expressed as following

\[ A (z^{-1}) y(k + 1) = B (z^{-1}) u(k) + v[x(k - 1)] + \Delta v(k) \quad (10) \]

Control objective: A nonlinear PID controller is designed, which ensures that the input and output signals of the closed-loop system are bounded, making the output \( y(k) \) track the reference input \( w(k) \), and making tracking error less than or equal to the predetermined value \( \varepsilon (\varepsilon > 0) \), i.e.,

\[ \lim_{k \to \infty} |e(k)| = \lim_{\k \to \infty} |w(k) - y(k)| \leq \varepsilon \quad (11) \]

Condition 1:

\[ |v[x(k)]| \leq \varepsilon_1 \|x(k)\| + \varepsilon_2, \forall k \quad (12) \]

where \( \varepsilon_1 (0 \leq \varepsilon_1 < 1) \) and \( \varepsilon_2 > 0 \) are known constants.
3. Design of Nonlinear PID Controller.

3.1. Nonlinear PID Controller. A nonlinear PID controller with unmodeled dynamics compensation is proposed as follows.

\[
u(k) = u(k-1) + K_P[e(k) - e(k-1)] + K_Ie(k) + K_De(k) - 2e(k-1) + e(k-2) - \bar{K}(z^{-1}) \{ v[x(k-1)] + \Delta v(k) \}
\]  

where \( K_P, K_I \) and \( K_D \) are the proportional, integral and differential coefficients of the PID controller respectively, and \( \bar{K}(z^{-1}) \) is the polynomial of \( z^{-1} \). \( e(k) \) is the tracking error, defined as

\[
e(k) = w(k) - y(k)
\]  

where \( w(k) \) is the ideal output.

Using \( z^{-1} \), we can get

\[
[1 - z^{-1}]u(k) = [\bar{g}_0 + \bar{g}_1 z^{-1} + \bar{g}_2 z^{-2}]e(k) - \bar{K}(z^{-1}) \{ v[x(k-1)] + \Delta v(k) \}
\]  

where

\[
\bar{g}_0 = K_P + K_I + K_D
\]

\[
\bar{g}_1 = -K_P - 2K_D
\]

\[
\bar{g}_2 = K_D
\]

By substituting (14) into (15), it can be obtained that

\[
\bar{H}(z^{-1})u(k) = \bar{G}(z^{-1})w(k) - \bar{G}(z^{-1})y(k) - \bar{K}(z^{-1}) \{ v[x(k-1)] + \Delta v(k) \}
\]  

where \( \bar{H}(z^{-1}), \bar{G}(z^{-1}) \) and \( \bar{K}(z^{-1}) \) are polynomials of \( z^{-1} \). And \( \bar{H}(z^{-1}) \) and \( \bar{G}(z^{-1}) \) are

\[
\bar{H}(z^{-1}) = 1 - z^{-1}
\]

\[
\bar{G}(z^{-1}) = \bar{g}_0 + \bar{g}_1 z^{-1} + \bar{g}_2 z^{-2}
\]

The closed-loop system equation can be obtained by substituting (19) into (10).

\[
[\bar{H}(z^{-1})A(z^{-1}) + z^{-1}B(z^{-1})\bar{G}(z^{-1})]y(k+1) = B(z^{-1})\bar{G}(z^{-1})w(k) + [\bar{H}(z^{-1}) - B(z^{-1})\bar{K}(z^{-1})] \{ v[x(k-1)] + \Delta v(k) \}
\]  

From (22), it can be seen that by choosing \( \bar{K}(z^{-1}) \), the difference between \( \bar{H}(z^{-1}) \) and \( B(z^{-1})\bar{K}(z^{-1}) \) can be as small as possible.

In order to determine the parameters \( K_P, K_I, K_D \) and \( \bar{K}(z^{-1}) \), the following performance index is introduced.

\[
J = [P(z^{-1})y(k+1) - G(z^{-1})w(k) + Q(z^{-1})u(k) + K(z^{-1}) (v[x(k-1)] + \Delta v(k))]^2
\]  

where \( P(z^{-1}), G(z^{-1}), Q(z^{-1}) \) and \( K(z^{-1}) \) are polynomials of \( z^{-1} \).

In order to obtain the optimal control law that minimizes the performance index (23), the generalized output \( \phi(k+1) \) is defined as

\[
\phi(k+1) = P(z^{-1})y(k+1)
\]  

The generalized ideal output \( y^*(k+1) \) is defined as

\[
y^*(k+1) = G(z^{-1})w(k) - Q(z^{-1})u(k) - K(z^{-1}) (v[x(k-1)] + \Delta v(k))
\]  

The generalized output error \( e_g(k+1) \) is defined as

\[
e_g(k+1) = \phi(k+1) - y^*(k+1)
\]
The following Diophantine equation is introduced.

\[ P(z^{-1}) = F(z^{-1})A(z^{-1}) + z^{-1}G(z^{-1}) \]  
\[ (27) \]

The orders \( n_F \) and \( n_G \) of \( F(z^{-1}) \) and \( G(z^{-1}) \) are respectively

\[ n_F = 0 \]
\[ n_G = \max\{n_A - 1, n_P - 1\} \]  
\[ (28) \]
\[ (29) \]

From (28), it is can be obtained

\[ F = P(0) \]  
\[ (30) \]

\( G(z^{-1}) \) is chosen as follows:

\[ G(z^{-1}) = g_0 + g_1 z^{-1} + g_2 z^{-2} \]  
\[ (31) \]

According to (29), if

\[ n_G = \max\{n_A - 1, n_P - 1\} = n_A - 1, \]

the order \( n_A = 3 \) and \( P(z^{-1}) \) satisfies \( n_P \leq 2 \).

If \( n_G = \max\{n_A - 1, n_P - 1\} = n_P - 1, \)

Then, the order of \( P(z^{-1}) \) is \( n_P = 3 \), and the order of \( A(z^{-1}) \) is \( n_A \leq 2 \). The equations (10), (24) and (27) are available.

\[ P(z^{-1})y(k+1) = G(z^{-1})y(k) + H(z^{-1})u(k) + F(z^{-1})(v[x(k-1)] + \Delta v(k)) \]  
\[ (34) \]

and

\[ H(z^{-1}) = F(z^{-1})B(z^{-1}) \]  
\[ (35) \]

From the equations (24) and (34), we can obtain

\[ \phi(k+1) = G(z^{-1})y(k) + H(z^{-1})u(k) + F(z^{-1})(v[x(k-1)] + \Delta v(k)) \]  
\[ (36) \]

By substituting (36) into (23), a nonlinear PID controller can be obtained by minimizing \( J \).

\[ [H(z^{-1}) + Q(z^{-1})]u(k) = G(z^{-1})w(k) - G(z^{-1})y(k) \]

\[ - [K(z^{-1}) + F(z^{-1})](v[x(k-1)] + \Delta v(k)) \]  
\[ (37) \]

From (19) and (37), the corresponding relationship are as follows.

\[ \bar{H}(z^{-1}) = H(z^{-1}) + Q(z^{-1}) \]  
\[ (38) \]
\[ \bar{G}(z^{-1}) = G(z^{-1}) \]  
\[ (39) \]
\[ \bar{K}(z^{-1}) = K(z^{-1}) + F \]  
\[ (40) \]

Combining the formulas (20), (27), (38) and (39), the following formulas are established by offline selection of \( P(z^{-1}) \) and \( Q(z^{-1}) \).

\[ |P(z^{-1})B(z^{-1}) + Q(z^{-1})A(z^{-1})| \neq 0, \quad |z| > 1 \]  
\[ (41) \]

From the selected \( P(z^{-1}), \bar{G}(z^{-1}) \) and (16) - (18), the parameters of the PID controller can be obtained as follows.

\[ K_P = -(2g_2 + g_1) \]  
\[ (42) \]
\[ K_I = g_0 + g_1 + g_2 \]  
\[ (43) \]
\[ K_D = g_2 \]  
\[ (44) \]
In order to eliminate the influence of \(v[x(k-1)]\) and \(\Delta v(k)\) on the closed-loop system, by selecting \(Q(z^{-1})\), the selection \(K(z^{-1})\) should satisfy

\[
B(z^{-1})K(z^{-1}) = Q(z^{-1})
\] (45)

Therefore, \(\bar{K}(z^{-1})\) in (13) is

\[
\bar{K}(z^{-1}) = F + Q(z^{-1})/B(z^{-1})
\] (46)

3.2. Unmodeled Dynamics Incremental Estimation Based on ANFIS. Similar to [2], the unmodeled dynamics increment is estimated. When using the unmodeled dynamic estimation algorithm proposed, the following theorem can be obtained.

**Theorem 3.1.** Suppose \(v[x(k)]\) satisfy Condition1, and \(\varepsilon_1\) and \(\varepsilon_2\) are selected as small positive numbers. Moreover, according to the universal approximation theorem of the fuzzy system[13] for any \(\xi > 0\)

\[
|\bar{e}(k)| = |\Delta v(k) - \Delta \hat{v}(k)| \leq \xi
\] (47)

where \(\xi\) is an arbitrary small error limit specified in advance.

Therefore, the equation of the nonlinear PID controller with unmodeled dynamics increment estimation is

\[
[H(z^{-1}) + Q(z^{-1})]u(k) = G(z^{-1})w(k) - G(z^{-1})y(k) - [K(z^{-1}) + F(z^{-1})](v[x(k-1)] + \Delta \hat{v}(k))
\] (48)

3.3. Parameter Selection of PID Controller. When \(k \to \infty\) and \(v[x(\infty)]\) are constant, the steady-state value of \(K(z^{-1})\) is chosen as \(K(1)\) to satisfy the following requirement.

\[
Q(1) - B(1)K(1) = 0
\] (49)

From the above analysis, we can see that \(P(z^{-1})\) is the third-order polynomial of \(z^{-1}\). Therefore, let

\[
P(z^{-1}) = p_0 + p_1 z^{-1} + p_2 z^{-2} + p_3 z^{-3}
\] (50)

As can be seen from (35) and (38), it can be obtained:

\[
Q(z^{-1}) = \bar{H}(z^{-1}) - H(z^{-1}) = 1 - z^{-1} - FB(z^{-1})
\] (51)

Because \(F = P(0) = p_0\) and \(n_B = 1\), \(Q(z^{-1})\) is a first order polynomial or constant, that is, \(n_Q = 1\) or 0. Two cases are discussed below.

Let \(Q(z^{-1})\) is

\[
Q(z^{-1}) = q_0 + q_1 z^{-1}
\] (52)

As can be seen from (51) and (52), the choice of \(Q(z^{-1})\) is related to \(F = P(0) = p_0\), and

\[
\begin{cases}
q_0 = 1 - p_0 b_0 \\
q_1 = -1 - p_0 b_1
\end{cases}
\] (53)

The principle of choosing \(P(z^{-1})\) and \(Q(z^{-1})\) is to make the closed-loop characteristic polynomial \(T(z^{-1})\) stable, i.e.,

\[
P(z^{-1})B(z^{-1}) + Q(z^{-1})A(z^{-1}) = T(z^{-1})
\] (54)
According to the selected $P(z^{-1})$, the coefficients of $G(z^{-1})$ obtained from (27) are
\[ \begin{align*}
g_0 &= p_1 - p_0a_1 \\
g_1 &= p_2 - p_0a_2 \\
g_2 &= p_3
\end{align*} \tag{55} \]
Therefore, the equations for calculating the PID parameters are as follows.
\[ \begin{align*}
K_p &= -(2p_3 + p_2 - p_0a_2) \\
K_i &= p_1 - p_0a_1 + p_2 - p_0a_2 + p_3 \\
K_d &= p_3
\end{align*} \tag{56} \]
According to (45), the compensator $K(z^{-1})$ is
\[ K(z^{-1}) = \frac{Q(z^{-1})}{B(z^{-1})} = \frac{1 - p_0b_0 - (1 + p_0b_1)z^{-1}}{b_0 + b_1z^{-1}} \tag{57} \]

4. Performance Analysis.

**Lemma 4.1.** When the non-linear PID controller (48) is used to the system (10), the input-output equation of the closed-loop system is as follows
\[ \begin{align*}
\left[ \begin{array}{ccc}
\Gamma(z^{-1}) & 0 & y(k+1) \\
0 & \Gamma(z^{-1}) & u(k)
\end{array} \right] = \left[ \begin{array}{ccc}
B(z^{-1})G(z^{-1}) & A(z^{-1})G(z^{-1}) \\
A(z^{-1})K(z^{-1}) - P(z^{-1}) & 0
\end{array} \right] w(k) \\
+ \left[ \begin{array}{ccc}
Q(z^{-1}) - B(z^{-1})K(z^{-1}) \\
H(z^{-1}) + B(z^{-1})K(z^{-1})
\end{array} \right] \bar{e}(k) \\
+ \left[ \begin{array}{ccc}
Q(z^{-1}) - B(z^{-1})K(z^{-1}) \\
FA(z^{-1}) + A(z^{-1})K(z^{-1})
\end{array} \right] \Delta v(k)
\end{align*} \tag{58} \]
where $\Gamma(z^{-1}) = P(z^{-1})B(z^{-1}) + Q(z^{-1})A(z^{-1})$

**Proof.** From (34) and (37), it can be obtained
\[ P(z^{-1})y(k+1) = G(z^{-1})w(k) - Q(z^{-1})u(k) \]
\[ - K(z^{-1})v|x(k-1)| + F(z^{-1})\Delta v|x(k)| \tag{59} \]
From (10),(47) and (59), it can be obtained
\[ \begin{align*}
[P(z^{-1})B(z^{-1}) + Q(z^{-1})A(z^{-1})]y(k+1) \\
= B(z^{-1})G(z^{-1})w(k) + [H(z^{-1}) + B(z^{-1})K(z^{-1})]\bar{e}(k) \\
+ [Q(z^{-1}) - B(z^{-1})K(z^{-1})] \bar{e}(k) + \Delta v(k)
\end{align*} \tag{60} \]
\[ \begin{align*}
[P(z^{-1})B(z^{-1}) + Q(z^{-1})A(z^{-1})]u(k) \\
= A(z^{-1})G(z^{-1})w(k) + [FA(z^{-1}) + A(z^{-1})K(z^{-1})]\bar{e}(k) \\
- [A(z^{-1})K(z^{-1}) + P(z^{-1})] \Delta v(k)
\end{align*} \tag{61} \]
Combining (60) and (61), the equation (58) can be obtained. \hfill \square

**Theorem 4.2.** Assume that the system (10) satisfies the following conditions:
1. $v|x(k)|$ satisfies condition 1.
2. Select $P(z^{-1})$ and $Q(z^{-1})$ to satisfy
\[ P(z^{-1})B(z^{-1}) + Q(z^{-1})A(z^{-1}) \neq 0, \quad |z| > 1. \tag{62} \]
Choose $K(z^{-1})$ to satisfy
\[ Q(z^{-1}) = B(z^{-1})K(z^{-1}) \tag{63} \]
and

\[ Q(1) = B(1)K(1) \]  

(64)

Moreover, the tracking error \( e(k) \) of the closed-loop system satisfy

\[
\lim_{k \to \infty} |e(k)| = \lim_{k \to \infty} |w(k) - y(k)| \leq \varepsilon
\]

5. **Numerical Simulation.** The following discrete-time nonlinear systems are considered.

\[
y(k+1) = 2.6y(k) - 1.2y(k - 1) + u(k) + 0.65u(k - 1) + 1.25 \sin[u(k) + u(k - 1) + y(k) + y(k - 1)] - \frac{u(k) + u(k - 1) + y(k) + y(k - 1)}{1 + u(k)^2 + u(k - 1)^2 + y(k)^2 + y(k - 1)^2} + d(k)
\]

The origin is the equilibrium point of the system. \( d(k) = 0.01 \).

Obviously, the order of the system \( n_A = 2, n_B = 1 \). The parametric polynomial of the system is

\[ A(z^{-1}) = 1 - 2.6z^{-1} + 1.2z^{-2}, B(z^{-1}) = 1 + 0.65z^{-1}. \]

The unmodeled dynamics is

\[
v[x(k)] = 1.25 \sin[u(k) + u(k - 1) + y(k) + y(k - 1)] - \frac{u(k) + u(k - 1) + y(k) + y(k - 1)}{1 + u(k)^2 + u(k - 1)^2 + y(k)^2 + y(k - 1)^2} + d(k)
\]

where \( x(k) = [y(k), y(k - 1), u(k), u(k - 1)]^T \).

The weighting terms \( H(z^{-1}) = 1 + 0.975z^{-1}, G(z^{-1}) = 4.25 - 6.5625z^{-1} + 4.6856z^{-2}, Q(z^{-1}) = -0.5 - 0.65(z^{-1}), \) and \( K(z^{-1}) = -0.6485 \) are chosen offline.

The control objective is to make the output \( y(k) \) of the closed-loop system track the bounded reference input signal:

\[ w(k) = 2.0(\sin 2\pi k/50) \]

The simulation results are shown in Fig. 1-4.

Choose \( \varepsilon_1 = 0.001, \varepsilon_2 = 0.0001 \). In the unmodeled dynamic incremental estimation algorithm, the membership function of ANFIS is chosen as Gauss type, and each input is divided into three fuzzy subsets.

Fig. 1 is the output response of the closed-loop system using the proposed non-linear control algorithm. It can be seen from the figure that the tracking characteristic of the closed-loop system is improved by using the control algorithm in this paper.

Fig. 2 shows the output curve of the controller.

Fig. 3 shows the unmodeled dynamics and the curves of the estimated values obtained by the proposed estimation algorithm. As can be seen from the figure, the estimation algorithm is effective.

Fig. 4 is the estimation error. It can be seen from the figure that the estimation error increases with the increase of the output of the controller.
Figure 1. Performance of proposed PID control method (Output $y$, Reference Input $w$)

Figure 2. The controller $u$

Figure 3. The estimation of unmodelled dynamics
6. **Conclusion.** In this paper, a nonlinear PID control algorithm is proposed for a class of nonlinear systems. The proposed algorithm combines the optimal control theory with the conventional PID control algorithm, and the traditional control algorithm is thus be improved. The stability and convergence of the proposed control method are analyzed. Finally, through the numerical simulation, the simulation results verify the effectiveness of the proposed algorithm.

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E-mail address: niuhong@lnpu.edu.cn
E-mail address: 13889814533@163.com
E-mail address: yajunzhang@mail.neu.edu.cn
E-mail address: 704153933@qq.com