Analysis of Bayesian method in queuing system customer service locket Semarang Tawang railway station

Sugito, Mustafid, A Prahatama, B Warsito, A A Salsabela

Department of Statistics, Diponegoro University, Semarang, Indonesia 50275

Corresponding author: aurumuyunaas@gmail.com

Abstract. Jamming is one of the serious problems in Indonesia caused by the increase of vehicles. The government has been made solution for this situation for example was public transportation. The train is one of the suitable public transportation because of the ticket price was cheap. Tawang Railway Station Semarang was the biggest railway station in Semarang. In the specific day such long holiday or celebration day, many people have chosen train to take them. This make a queuing situation on the counter of station especially on customer service locket which provide information about the station and more. Queue theory models provide the random of arrival and service time. The Bayesian theory suits to handle the problem of queuing that has been working for several times to repair the prior information with new information of sample and also can make a combination of the distribution after assumption steady state. Based on the analysis of the queue models for customer service are (G/G/c):(GD/∞/∞) from the posterior distribution with combination from prior distribution and likelihood sample. The prior distribution used in this research is Poisson. The likelihood sample used Uniform Discrete and Negative Binomial distribution. The posterior distribution is a combination of Uniform Discrete and Beta distribution. Queue models can be used to count the size of the system performance. Based on the calculations and analysis with RGui, it can be concluded that the queuing system of customer service locket have been good because its steady state and busy probability is higher than jobless probability.

1. Introduction

The volume of vehicles is increasing over time, and road facilities that cannot accommodate the number of vehicles crossing the road cause a traffic jam. One way to reduce congestion is by using public transportation. Train is alternative transportation that is often used by people to travel, especially with long routes. The increase in passenger volume that usually occurs on major holidays or holidays will create a situation of waiting in long lines at the ticket counter service and cause a queuing phenomenon.

The queuing process itself is a process associated with the arrival of customers, waiting in line if it cannot be served, being served, and finally leaving the facility [7]. In previous research [10] has discussed the right model for describing queues and also the performance measure of the queuing system at the customer service section. The Bayesian method exists a method that improves previous research if known new information regarding the sample from the study [8]. The Bayesian method is suitable for use as an analysis of the queuing system apart from being able to easily combine prior information, which becomes substantial in a queuing system that has been running for some time, the Bayesian method can handle common problems such as limitations in queuing parameter space and combining distributions which is possible after the steady state conditions are met [1].
2. Literature review

2.1 Customer Service Locket
The customer service section is the section that provides information services to customers who come to Tawang Station regarding all matters concerning train departures, train schedules, rail routes and other matters relating to the operation of Tawang Station. This counter also serves refunds or refunds and changes the schedule with certain conditions.

2.2 Steady State Size
For example, \( \lambda \) is the average number of customer arrivals to the service per unit time, \( \mu \) is the average customer who has been served per unit of time, and \( c \) is the number of facilities where the service is (server), then \( \rho \) is defined as the ratio between the average customers come (\( \lambda \)) with the average customer served per unit time (\( \mu \)) [9] or it can be written as follows:

\[
\rho = \frac{\lambda}{c} \mu
\]  

2.3 Kolmogorov Smirnov Test
1. Determining Hypotheses
   \( H_0 \): The distribution of the sample follows the distribution specified
   \( H_1 \): The distribution of the sample does not follow a defined distribution
2. Determining the Level of Significance
   The level of significance used \( \alpha = 5\% \)
3. Test Statistics
   \[
   D=\max \left( \max \left( |S(x_i) - F_0(x_i)|, (|S(x_{i-1}) - F_0(x_i)|) \right) \right) \quad (2)
   \]
   with:
   \( S(x_i) \): the cumulative probability function calculated from the sample data
   \( F_0(x_i) \): hypothesized distribution function (cumulative probability function)
   \( r \): the number of different x-value
4. Test Criteria
   Reject \( H_0 \) if it is at the \( \alpha \) significance level if the D value \( \geq \) the \( D_{\text{table}} \) value (1- \( \alpha \)) or if the sig value < \( \alpha \) value. \( D_{\text{table}} (\alpha) \) is the critical value obtained from the Kolmogorov-Smirnov table [4].

2.4 Poisson Distribution
A discrete random variable X has a Poisson distribution with \( 0 < \lambda \), if it has the following pdf form [2]:

\[
f(x) = \frac{e^{-\lambda x} \lambda^x}{x!}, \quad x = 0,1,\ldots \quad (3)
\]

2.5 Negative Binomial Distribution
A discrete random variable X has a negative binomial distribution with \( 0 < \lambda < 1 \) and q = 1-\( \lambda \), if it has the following pdf form [2]:

\[
f(x) = \left( \frac{x - 1}{n - 1} \right) \lambda^n (1 - \lambda)^{x-n}, \quad x = 1,2,\ldots \quad (4)
\]

2.6 Uniform Discrete Distribution
A discrete random variable X has a discrete uniform distribution in integers 1,2,\ldots, N if it has the following pdf form [2]:

\[
f(x) = \frac{1}{N}, \quad x = 1,2,\ldots,N \quad (5)
\]
2.7 Beta Distribution
A continuous random variable X has a beta distribution with 0 <a and 0 <b in the following pdf format [2]:
\[
f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}, 0 < x < 1
\]  
\(6\)

Queueing Model (G/G/c): (GD/∞/∞)
The formula for finding performance measures in the model (G / G / c) : (GD / ∞ / ∞) is as follows [5]:
\[
L_q = \left( 1 - \frac{\rho}{c(1-\rho)^2} \right) \frac{P_0}{\lambda^2 v(t) + v(t') \lambda^2}
\]  
\(7\)

with:
\[v(t) = \frac{1}{(\mu^2)^2}\]
\[v(t') = \frac{1}{(\lambda^2)^2}\]
\[L_s = L_q + r\]  
\(8\)
\[W_q = \frac{L_q}{\lambda}\]  
\(9\)
\[W_s = W_q + \frac{1}{\mu}\]  
\(10\)

2.8 Bayesian Description
It is known that \(x' = (x_1, \ldots, x_n)\) is a vector of n observations with the probability distribution \(p(x | \lambda)\) is the conditional probability y if it is known that the value of the k parameter \(\lambda\) is \(\lambda' = (\lambda_1, \ldots, \lambda_k)\). It is known that \(\lambda\) has its own probability distribution, \(p(\lambda)\) [3], with
\[
p(x | \lambda)p(\lambda) = p(\lambda | x)p(x) = p(x, \lambda)
\]  
\(11\)
so it can be write:
\[
p(\lambda | x) = \frac{p(x | \lambda)p(\lambda)}{p(x)}
\]  
\(12\)
or,
\[
p(\lambda | x) = cp(x | \lambda)p(\lambda)
\]  
\(13\)

2.9 Prior Distribution
Prior is divided into 2 groups based on their Likelihood function [3]:

1. Relating to the distribution form of the pattern identification results
   a. Prior conjugate, refers to the model analysis reference, especially in the formation of its likelihood function.
   b. Prior non-conjugate, prior to the model, does not consider the pattern forming its likelihood function.

2. Regarding the determination of each parameter in the prior distribution pattern.
   a. Informative prior refers to giving the parameters of the prior distribution that has been chosen whether or not the conjugate prior distribution is.
   b. Prior is non-informative, if the prior distribution selection is not based on existing data or prior distribution which does not contain information about the \(\lambda\) parameter.

One form of non-informative prior approach is to use Jeffrey's method. This method states that the prior distribution \(f(\lambda)\) is the square root of Fisher's information expressed in
\[
f(\lambda) = \left[ I(\lambda) \right]^{1/2}
\]  
\(14\)

\(I(\lambda)\) is the expected value of Fisher's information
\[
I(\lambda) = -E_\lambda \left[ \frac{\partial^2 logf(x;\lambda)}{\partial \lambda^2} \right]
\]  
\(15\)
if \( \lambda = (\lambda_1, ..., \lambda_p) \) is a vector, used
\[
f(\lambda) = [\text{det}I(\lambda)]^{1/2}
\]
(16)

1(\lambda) is the Fisher information matrix (p x p) with index (i, j), then
\[
I(\lambda) = -E_{\lambda} \left[ \frac{\partial^2}{\partial \lambda_i \partial \lambda_j} \log f(x; \theta) \right] \text{ dengan } i = 1,2,...,p; j = 1,2,...,p
\]
(17)

2.10 Likelihood Function
The likelihood function is the joint density function of n random variables \( X_1, X_2, ..., X_n \) and is expressed in the form \( f(x_1, x_2, ..., x_n; \lambda) \). If \( X_1, X_2, ..., X_n \) represent a random sample of \( f(x; \theta) \) [2], then
\[
L(\lambda) = f(x_1; \lambda) f(x_2; \lambda) ... f(x_n; \lambda)
\]
\[
L(\lambda) = \prod_{i=1}^{n} f(x_i; \lambda)
\]
(18)

2.11 Posterior Distribution
The posterior distribution is an amalgamation of the sample distribution and prior distribution [3]. If the data \( x, p(x | \lambda) \) in the equation \( p(\lambda | x) = cp(x | \lambda) p(\lambda) \) can be viewed as a function not of \( x \) but of \( \lambda \), then it can be written that the likelihood function of \( \lambda \) is conditionally \( y \) can be written as \( L(\lambda | x) \).
The bayesian equation can be written as follows
\[
p(\lambda | x) = L(x | \lambda) p(\lambda)
\]
(19)
in other words, the Bayes theorem shows that the probability distribution for the posterior \( \lambda \) in the \( x \) data is proportional to the distribution for the prior \( \lambda \) of the data and the conditional likelihood \( \lambda \) with \( x \).

posterior \( \propto \) likelihood \( x \) prior

3. Research methods
The data used in this study are primary data taken from the results of direct observations made for 7 days and prior research data, namely Yustiti's (2014) research. The steps of analysis are as follows:

a. Researchers conducted research at Tawang Station Semarang counters to obtain data on the number of customers who came and the number of customers served per 60 minutes in customer service locket Semarang Tawang Railway Station.
b. The data obtained must meet the steady state conditions (\( \rho = \lambda / (\mu c) <1 \)).
c. Researchers are looking for the right distribution for the arrival of customers and customers who are served with easy fit software.
d. Researchers perform a fit test of the Kolmogorov-Smirnov distribution for the appropriate distribution.
e. The researcher determines the likelihood of the sample distribution.
f. Researchers entered the prior distribution in Yustiti's (2014) study.
g. Researchers determine the posterior distribution by entering the prior distribution values from previous studies. The distribution of the newly drawn sample is the likelihood value and the posterior probability is calculated (posterior \( \propto \) likelihood \( x \) prior)
h. Researchers determine the appropriate queuing model. Judging from the distribution of arrivals, distribution of services, the number of server, queuing discipline used is first out (FCFS), capacity in the system and the source of the call.
i. The researcher determines the system performance, namely the estimated number of customers in the system (Ls), the estimated number of customers in the queue (Lq), the waiting time in the system (Ws), and the waiting time in queue (Wq) with the syntax RGui.
j. Researchers made conclusions about the counter services at Tawang Station Semarang.
4. Results

4.1 Steady State Size

Table 1. Steady State Size

| Locket         | Lambda  | Miuo | C   | ρ   |
|----------------|---------|------|-----|-----|
| Customer Service | 14.08163 | 14.08163 | 2   | 0.5 |

The value of ρ less than 1 means that this state meets the steady state conditions.

4.2 Distribution Fit Test

The distribution suitability test is used to determine whether the data on the arrival rate and service rate at the customer service counter follow a certain distribution.

Figure 1 (a) uniform discrete for arrival time (b) negative binomial for arrival time (c) uniform discrete for server time (d) negative binomial for server time
Table 2. Kolmogorov Smirnov Result

|                      | D Count | D Table | P Value | Alfa  | Decision | Conclusion                          |
|----------------------|---------|---------|---------|-------|----------|-------------------------------------|
| Discrete Uniform of  | 0,12889 | 0,19401 | 0,3587  | 0,05  | H_0 accepted | Data distributed Uniform Discrete   |
| arrival              |         |         |         |       |          |                                    |
| Discrete uniform of  | 0,12245 | 0,19401 | 0,42089 | 0,05  | H_0 accepted | Data distributed Uniform Discrete   |
| service              |         |         |         |       |          |                                    |
| Negative Binomial of | 0,17543 | 0,19401 | 0,08647 | 0,05  | H_0 accepted | Data distributed Negative Binomial  |
| arrival              |         |         |         |       |          |                                    |
| Negative Binomial of | 0,10869 | 0,19401 | 0,57151 | 0,05  | H_0 accepted | Data distributed Negative Binomial  |
| service              |         |         |         |       |          |                                    |

4.3 Bayesian Prior

a. Poisson Distribution as prior

Using the Jeffrey method, we find the prior distribution of the Poisson distribution as follows:

\[
f(x; \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}
\]

\[
\log f(x; \lambda) = \log \left( \frac{e^{-\lambda} \lambda^x}{x!} \right)
\]

\[
= x \log \lambda - \log x! - \lambda
\]

\[
\frac{\partial \log f(x; \lambda)}{\partial \lambda} = x - \lambda
\]

\[
\frac{\partial^2 \log f(x; \lambda)}{\partial \lambda^2} = -x
\]

\[
I(\lambda) = -E \left[ -\frac{x}{\lambda^2} \right]
\]

\[
= \frac{1}{\lambda} E(x)
\]

\[
= \frac{1}{\lambda^2} (\lambda)
\]

\[
= \frac{1}{\lambda}
\]

\[
f(\lambda) = \frac{1}{\lambda}
\]

\[
= \frac{1}{\sqrt{\lambda}}
\]

So Prior distribution of \(f(x; \lambda) = \frac{1}{\sqrt{\lambda}}\)

Likelihood

a. Likelihood Negative Binomial Distribution

\[
Likelihood = \prod \left( \frac{x - 1}{n - 1} \lambda^x (1 - \lambda)^{x-n} \right)
\]

\[
= \frac{x}{\lambda} \prod \left( \frac{1}{n - 1} \lambda^x (1 - \lambda)^{x-n} \right)
\]

b. Likelihood Uniform Diskrit Distribution, DU(\lambda), \lambda = 1,2,...

\[
Likelihood = \prod \left( \frac{1}{\lambda} \right)
\]
Posterior

a. Prior Poisson Distribution and Likelihood Negative Binomial

\[ \text{Likelihood from data sample distributed Poisson}(\lambda) \]
\[ \text{Posterior } \propto \text{Prior } \times \text{Likelihood} \]

\[ \text{Posterior} = \frac{1}{\lambda^n} \left( \prod \left( \frac{x - 1}{n - 1} \right) \lambda^n (1 - \lambda) \Sigma x - n \right) \]
\[ = \prod \left( \frac{(n + x - 2)!}{(x - 1)! (x - n)!} \right) \lambda^n - 1/2 (1 - \lambda) \Sigma (x - n) \]
\[ \sim \text{Beta}(\Sigma n + 1/2 \Sigma (x - n) + 1) \]

b. Prior Poisson Distribution and Likelihood Discrete Uniform

\[ \text{Likelihood from data sample distributed Discrete Uniform}(\lambda) \]

\[ \text{Posterior} = \frac{1}{\sqrt{\lambda}} \left( \frac{1}{\lambda n} \right) \]
\[ = \frac{1}{\lambda^{1/2 + n}} \]
\[ \sim \text{Discrete Uniform}\left(\lambda^{1/2} \Sigma n\right) \]

4.4 Queueing Model

Table 3. Model of the Queuing

| Arrival distribution | Service distribution | Model |
|----------------------|----------------------|-------|
| Discrete Uniform     | Discrete Uniform     | (G/G/2):(GD/∞/∞) |
| Discrete Uniform     | Negative Binomial    | (G/G/2):(GD/∞/∞) |
| Negative Binomial    | Discrete Uniform     | (G/G/2):(GD/∞/∞) |
| Negative Binomial    | Negative Binomial    | (G/G/2):(GD/∞/∞) |

4.5 Size of the Performance System

The performance measure of the queuing system from the queue model of each counter is obtained from the Gui R output as follows:
Figure 2 size of the system performance

Table 4. Size of system performance

| Loket                | Lq       | Ls     | Wq     | Ws     |
|----------------------|----------|--------|--------|--------|
| Customer Service     | 0.001681 | 1.001681 | 0.00012 | 0.07113 |

5. Conclusion

Based on research that has been carried out at Tawang Station Semarang, the following conclusions are obtained: Possible queuing models for customer service counters (UNIFORM DISKRIT / UNIFORM DISKRIT / 2) : (GD / ∞ / ∞), (UNIFORM DISKRIT / BETA / 2) : (GD / ∞ / ∞), (BETA / UNIFORM DISKRIT / 2) : (GD / ∞ / ∞), and (BETA / BETA / 2) : (GD / ∞ / ∞). Based on the value of the system performance measure obtained from the output of Gui R for each counter, it is found that the average number of customers estimated in the system is 1.00168 customers or if it is rounded up to 2 customers. The average value of the number of customers estimated in the queue was 0.00168; customer or if rounded to 1 customer in line. The time required in the system is 4.26780 minutes and the time required to queue is 0.00720 minutes. The probability of busy service is 33.33%, with the probability of being unemployed 66.67%.

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