Hydrodynamic attractors, initial state energy and particle production in relativistic nuclear collisions

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G. Giacalone, AM, S. Schlichting, PRL (2019) [arXiv:1908.02866]
P. Hanus, AM, K. Reygers, PRC (2019) [arXiv:1908.02792]
Outline

- Motivation
- QCD equilibration in heavy ion collisions
- Entropy production from hydrodynamic attractor
- Connecting initial state energy and particle production
- Inferring entropy from experimental data
- Summary
Motivation
Revealing new phenomena of fundamental interactions

Heavy-ion collisions push nuclear matter in far-from-equilibrium state.

A new phase of matter is created — the Quark-Gluon Plasma (QGP).
Space-time picture of heavy ion collisions

- \( \tau \) ∼ \( \frac{1}{Qs} \)
- \( \tau_{\text{init}} \) ∼ \( \frac{1}{1\text{fm/c}} \)

Incoming nuclei: \( \text{Pb} \quad \text{Pb} \)

Particle escape
Fluid expansion
Pre-equilibrium
Energy deposition
Observed particles

\[ \langle \frac{dN_{\text{ch}}}{d\eta} \rangle \]
\[ \langle \frac{dE}{d\eta} \rangle \]
Space-time picture of heavy ion collisions

Initial Thermal \( \tau \sim \frac{1}{Q_s} \) s

\( \tau_{\text{init}} \sim 1 \text{fm/c} \)

PbPb Hydro

\( \tau = \text{const} \)

Particle escape

Fluid expansion

Pre-equilibrium

Energy deposition

Incoming nuclei

\begin{align*}
\langle \frac{dN_{\text{ch}}}{d\eta} \rangle \\
\langle \frac{dE}{d\eta} \rangle
\end{align*}
Space-time picture of heavy ion collisions

Initial

Thermal

$\tau \sim \frac{1}{Q_s}$

$\tau_{\text{init}} \sim 1\text{fm/c}$

PbPb

Hydro

Fluid expansion

Pre-equilibrium

Incoming nuclei

Particle escape

Energy deposition

$\langle \frac{dE}{d\eta} \rangle$

Observed particles

$\langle \frac{dN_{\text{ch}}}{d\eta} \rangle$

$\frac{dE}{d\eta}$
Space-time picture of heavy ion collisions

Initial
Thermal
$\tau \sim \frac{1}{Qs}$
$\tau_{\text{init}} \sim 1\text{fm/c}$

Pb-Pb Hydro

$t$ is constant

Particle escape
Fluid expansion
Pre-equilibrium
Incoming nuclei

Can we predict the number of produced particles?
Entropy production in central Au-Au collision at RHIC

Particle multiplicity is directly proportional to entropy at thermalization

\[
\langle \frac{dS}{dy} \rangle_{\tau_{\text{therm}}} \approx \frac{S}{N_{\text{ch}}} \left\langle \frac{dN_{\text{ch}}}{d\eta} \right\rangle
\]
Entropy production in central Au-Au collision at RHIC

Particle multiplicity is directly proportional to entropy at thermalization

Müller and Schäfer (2011)[1]
QCD equilibration in heavy ion collisions
Dynamics of $p \sim T$ quasi-particles is governed by Boltzmann equation.

$$\partial_\tau f_{g,q}(p) - \frac{p_z}{T} \partial_{p_z} f_{g,q}(p) = -C_{2\leftrightarrow2}[f] - C_{1\leftrightarrow2}[f]$$

1. 2 $\leftrightarrow$ 2 elastic scatterings: $gg \leftrightarrow gg$, $qq \leftrightarrow qq$, $qg \leftrightarrow qg$, $gg \leftrightarrow q\bar{q}$

$$|\mathcal{M}_{gg}^{gg}|^2 = \lambda^2 16 \frac{d_F C_F}{C_A^2} \left[ C_F \left( \frac{u}{t} + \frac{t}{u} \right) - C_A \left( \frac{t^2 + u^2}{s^2} \right) \right]$$

Hard Thermal Loop resumed propagators, screening mass $m_D \sim gT$

Contains the right physics for the “bottom-up” thermalization in QCD.

Baier, Mueller, Schiff, and Son (2001)[2]

see reviews by Teaney and Schlichting (2019) [3], Berges, Heller, AM and Venugopalan (in preparation)
Dynamics of $p \sim T$ quasi-particles is governed by Boltzmann equation.

$$\partial_\tau f_{g,q}(p) - \frac{p_z}{\tau} \partial_{p_z} f_{g,q}(p) = -C_{2\leftrightarrow2}[f] - C_{1\leftrightarrow2}[f]$$

1 $\leftrightarrow$ 2 medium induced collinear radiation: $g \leftrightarrow gg$, $q \leftrightarrow qg$, $g \leftrightarrow q\bar{q}$

$$|\mathcal{M}_{qq}|^2 = \frac{k' r^2 + p' r^2}{k^2 p'^2 p^2} \frac{F_q(k'; -p', p)}{p^3}$$

Resummed multiple scatterings with the medium (LPM suppression).

Contains the right physics for the "bottom-up" thermalization in QCD.

Baier, Mueller, Schiff, and Son (2001)[2]

see reviews by Teaney and Schlichting (2019) [3], Berges, Heller, AM and Venugopalan (in preparation)
First theoretically complete description of pre-equilibrium stages

- Initial state in heavy ion collisions is highly anisotropic.
- Fast isotropization is needed for fluid dynamical description.

Particle scatterings efficiently isotropize the system in QCD kinetic theory.

Longitudinal and transverse pressure anisotropies

\[ \frac{1}{3} \tau (\text{fm}) \]

Kurkela, AM, Paquet, Schlichting, Teaney, *PRL, PRC* (2018) [5, 6]

Kinetic theory smoothly connects the initial state and fluid descriptions.
First theoretically complete description of pre-equilibrium stages

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Kurkela, AM, Paquet, Schlichting, Teaney, PRL, PRC (2018) [5, 6]
Fermion production in QCD kinetic theory

- Initial state is dominated by gluonic fields.
- In chemical equilibrium $u, d, s$ quarks carry most of the total energy. Fermions are produced through fusion $gg \rightarrow q\bar{q}$ and splitting $g \rightarrow q\bar{q}$.

$\eta/s = 0.16$

d.o.f: 36 quark 16 gluon

Kurkela, AM, *PRL, PRD*, (2018) [7, 8]
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![Diagram showing energy density and time evolution in hydrodynamics and chemical equilibrium.](image)

Kurkela, AM, PRL, PRD, (2018) [7, 8]
Entropy production from hydrodynamic attractor
Boost-invariant equations of motion of 1D expansion at early times

Energy-momentum conservation $T^{\mu \nu} = \text{diag} (e, P_T, P_T, P_L)$

$$\partial_\tau e = -\frac{e + P_L}{\tau},$$

$\tau < R$

Need microscopic input: constitutive relation $P_L = P_L(e, \tau)$.

- Equilibrium: equation of state
  $$\frac{P_L}{e} \approx \frac{1}{3} \implies e \propto \tau^{-\frac{4}{3}}.$$

- Near-equilibrium: viscous constitutive equations
  $$\frac{P_L}{e} = \frac{1}{3} - \frac{16}{9} \frac{\eta/s}{\tau T} + \ldots.$$

$\eta/s$ —specific shear-viscosity.

Macroscopic evolution far from equilibrium?
Beyond gradient expansion: hydrodynamic attractor

Early collapse of different initial conditions — attractor

$$\tau \partial_\tau \log e = -\frac{P_L}{e} = -f(\tau T).$$

Heller and Spalinski (2015) [9]

Romatschke (2017) [10]
Emergence of effective macroscopic descriptions far-from-equilibrium

\[ \frac{P_L}{e} = f \left[ \tilde{w} = \frac{\tau T_{\text{eff}}}{4\pi \eta/s} \right], \text{ where } T_{\text{eff}} \propto e^{1/4}. \]

see reviews by
Florkowski, Heller and Spalinksi (2017)[11],
Romatschke and Romatschke (2017) [12]
Energy density evolution from equations of motion

\[ \tau \partial_\tau \log e = -\frac{P_L}{e} \quad \Rightarrow \quad e = \mathcal{E}(\tilde{w}) \times \frac{\left(e^{\tau^{4/3}}\right)_{\text{therm}}}{\tau^{4/3}}. \]

Universal early/late asymptotics

**Viscous hydro:**

\[ \mathcal{E}(\tilde{w} \gg 1) = 1 - \frac{2}{3\pi \tilde{w}} \]

**Free-streaming** \((e \sim \tau^{-1}) :\)

\[ \mathcal{E}(\tilde{w} \ll 1) = C_\infty^{-1} \tilde{w}^{4/9} \]
Entropy production from hydrodynamic attractor

- In equilibrium entropy per rapidity \( \frac{dS}{dy} = A_\perp (s\tau)_{\text{therm}} \) given by

\[
(s\tau)_{\text{therm}} \propto \left( e^{\frac{4}{3}} \right)^{\frac{3}{4}}_{\text{therm}}
\]

- Hydrodynamic attractor relates \( (e^{\frac{4}{3}})_{\text{therm}} \) to energy at early times

\[
(s\tau)_{\text{therm}} = \frac{4}{3} C^{3/4}_\infty \left( \frac{4\pi \eta}{s} \right)^{1/3} \left( \frac{\pi^2}{30} \nu_{\text{eff}} \right)^{1/3} (e\tau)^{2/3}_0.
\]

- Pocket formula for particle production from energy deposition

\[
\langle \frac{dN_{\text{ch}}}{d\eta} \rangle \approx A_\perp \frac{N_{\text{ch}}}{S} \frac{4}{3} C^{3/4}_\infty \left( \frac{4\pi \eta}{s} \right)^{1/3} \left( \frac{\pi^2}{30} \nu_{\text{eff}} \right)^{1/3} \left( \frac{1}{A_\perp} \langle \frac{dE_\perp}{d\eta} \rangle \right)_0^{2/3}
\]

All relevant-prefactors and powers included!

Giacalone, AM, Schlichting, *PRL* (2019) [13]
Connecting initial energy deposition to particle production
Energy deposition in high energy nucleus-nucleus collisions

Collisions of glasma sheets in color-glass condensate effective theory

Local saturation scale is proportional to nuclear thickness

\[ Q_s^2(x_\perp) \propto T(x_\perp). \]

Gluon liberation (up to $\log$–corrections)

- Gluon number  \( (n_\tau)_0(x_\perp) \propto T^<(x_\perp), \)
- Gluon energy \( (e_\tau)_0(x_\perp) \propto T^<(x_\perp)\sqrt{T^>(x_\perp)}. \)
Universal centrality dependence of particle multiplicity

Three predictors for particle multiplicity:

\[ \langle \frac{dN_{\text{ch}}}{d\eta} \rangle \propto \frac{dS_{\text{therm}}}{d\eta} , \frac{dN_{\text{gluons}}}{d\eta} , \langle \frac{dS_{\text{therm}}}{d\eta} \rangle . \]

- Equilibration
- No equilibration
- E-by-e fluctuations

![Graph showing centrality dependence](graph.png)

Centrality: \( \pi b^2 / \sigma_{\text{AA}} \)
Initial state energy density

Bjorken estimate of initial state energy density

\[ e_0 = \frac{1}{\tau_0 A_\perp} \frac{dE_{\perp}^{\text{initial}}}{dy} \approx \frac{1}{\tau_0 A_\perp} \frac{dE_{\perp}^{\text{final}}}{dy} . \]

Our pocket formula allows relating particle multiplicity to initial state energy

\[ e_0 \approx 270 \text{ GeV/fm}^3 \left( \frac{\tau_0}{0.1\text{fm}/c} \right)^{-1} \left( \frac{C_\infty}{0.87} \right)^{-9/8} \left( \frac{\eta/s}{2/4\pi} \right)^{-1/2} \]

\[ \left( \frac{A_\perp}{138\text{fm}^2} \right)^{-3/2} \left( \frac{dN_{\text{ch}}/d\eta}{1600} \right)^{3/2} \left( \frac{\nu_{\text{eff}}}{40} \right)^{-1/2} \left( \frac{S/N_{\text{ch}}}{7.5} \right)^{3/2} , \]

c.f. \( e \approx 0.3 \text{ GeV/fm}^3 \) near QCD cross-over.
Centrality dependence of initial state energy

Bands are variations of $C_\infty = [0.8-1.15]$, $\eta/s = [0.08-0.24]$
Inferring entropy from experimental data
Measuring entropy in heavy ion collisions

- Entropy formula for dilute gas of hadrons
  \[ S = \int \frac{d^3r d^3p}{(2\pi)^3} \left[ -f \ln f + (1 \pm f) \ln (1 \pm f) \right] \]

- Measured particle spectra constrain coordinate integral
  \[ \frac{dN}{d^3p} = \int d^3r f(r, p). \]

- Measured HBT radii give spatial spread of distribution
  \[ f(\vec{p}, \vec{r}) = \mathcal{F}(\vec{p}) \exp \left( -\frac{x_{\text{out}}^2}{2R_{\text{out}}^2} - \frac{x_{\text{side}}^2}{2R_{\text{side}}^2} - \frac{x_{\text{long}}^2}{2R_{\text{long}}^2} \right) \]

Data driven calculation of final \( dS/dy \) in the collision.  

Pal and Pratt (2003)
Entropy budget for 0–10% Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

Individual contributions of measured and inferred hadron species.

$\text{Hanus, AM and Reygers (2019) [14]}$

| particle | $(dS/dy)_{y=0}^{\text{one state}}$ | factor | $(dS/dy)_{y=0}^{\text{total}}$ |
|----------|---------------------------------|--------|--------------------------------|
| $\pi$    | 2182                            | 3      | 6546                           |
| $K$      | 605                             | 4      | 2420                           |
| $\eta$   | 399                             | 1      | 399                            |
| $\eta'$  | 66                              | 1      | 66                             |
| $p$      | 266                             | 2      | 532                            |
| $n$      | 266                             | 2      | 532                            |
| $\Lambda$ | 160                            | 2      | 320                            |
| $\Sigma$ | 58                              | 6      | 348                            |
| $\Xi$    | 39                              | 4      | 156                            |
| $\Omega$ | 8                               | 2      | 16                             |
| **total** |                                 |        | **11335**                      |

Entropy per measured charged hadron $S/N_{ch} = 6.7 \pm 0.8$. 
Space-time evolution of entropy and temperature

Pre-equilibrium and hydrodynamic simulations with KøMPøST and FluiduM

for KøMPøST see Kurkela, AM, Paquet, Schlichting and Teaney (2018) [5, 6]

for FluiduM, see Floerchinger, Grossi and Lion (2018) [15]

Hanus, AM and Reygers (2019) [14]
Initial state energy in central Pb-Pb collisions

\[ \langle dE/d\eta \rangle \text{ [GeV]} \]

\[ \eta/s = 0.08 \text{, } \eta/s = 0.16 \]

\[ \text{QGP } T > 156 \text{ MeV} \]
\[ \text{QGP+freeze-out} \]
\[ \text{KøMPøST} \]

Giacalone, AM, Schlichting, PRL (2019) [13]

Hanus, AM and Reygers (2019)[14]
Summary and Outlook

- Significant progress in understanding and modelling pre-equilibrium.
- Direct connection between initial and final states.
- Universal centrality dependence of particle multiplicity.
- Quantitative estimation of initial state energy.

Outlook:

- Corrections due to transverse expansion and incomplete equilibration.
- Equilibration phenomena in small collision systems

Giacalone, AM, Schlichting, in progress

Kurkela, AM, Törnvist, in progress
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[18] Aleksi Kurkela, Urs Achim Wiedemann, and Bin Wu. What attracts to attractors? 2019, 1907.08101.
Equilibration of perturbations

Non-linearities change the perturbation spectra

\[ s_{\text{therm}} \propto e_0^{2/3} \implies \frac{\delta s_{\text{therm}}}{s_{\text{therm}}} = \frac{2}{3} \frac{\delta e}{e_0}. \]

\( k = 0 \) perturbation evolution in kinetic theory:

\[ \frac{\delta e}{(e + T^{xx})} = \text{const.} \]

Keegan, Kurkela, AM and Teaney (2016) [16]
Equilibration of perturbations

Non-linearities change the perturbation spectra

\[ s_{\text{therm}} \propto e_0^{\frac{2}{3}} \implies \frac{\delta s_{\text{therm}}}{s_{\text{therm}}} = \frac{2}{3} \frac{\delta e}{e_0} = \frac{3}{4} \times \frac{8}{9} \frac{\delta e_0}{e_0}. \]

\( k = 0 \) perturbation evolution in kinetic theory: \( \frac{\delta e}{e + T^{xx}} = \text{const.} \)

Keegan, Kurkela, AM and Teaney (2016) [16]
Transverse pre-equilibrium evolution

KøMPøST— event-by-event kinetic pre-equilibrium for heavy ion collisions.

\[
\delta T^{\mu\nu}_x(\tau_{\text{hydro}}, x') = \int d^2 x' \ G^{\mu\nu}_{\alpha\beta}(x - x', \tau_{\text{hydro}}, \tau_{\text{EKT}}) \ \delta T^{\alpha\beta}_x(\tau_{\text{EKT}}, x').
\]

goes into hydro

linear response function

initial

https://github.com/KMPST/KoMPoST [17]

Kurkela, AM, Paquet, Schlichting and Teaney (2018)[5, 6]
Kinetic theory response functions

Invariant form of non-equilibrium response functions

\[ G^{\mu\nu}(\tau, \tau_0, |x - x_0|, e(\tau_0), \lambda) \Rightarrow G^{\mu\nu,\text{univ}}\left(\frac{\tau T_{\text{Id.}}}{\eta/s}, \frac{|x - x_0|}{(\tau - \tau_0)}\right) \]

All components of energy-momentum tensor generated by kinetic response

Kinetic response functions evolve from free-str.-like to hydrodynamic-like.

for details see Kurkela, AM, Paquet, Schlichting and Teaney (2018) [5]
Extraction of $R_{\text{out}} R_{\text{side}} R_{\text{long}}$ from data

Experimentally easier to measure one-dimensional Gaussian radius $R_{\text{inv}}$

We use phenomenological parametrizations to infer $R_{\text{out}} R_{\text{side}} R_{\text{long}}$

$R_{\text{inv}} (\text{fm}) = \left( R_{\text{out}} R_{\text{side}} R_{\text{long}} \right)^{1/3}$, $\alpha = 1.52$, $\beta = 0.51$

Data, Pb–Pb, 0–5%
Transverse pre-equilibrium evolution

Radial expansion in isotropization time approximation.

Opaque ($\hat{\gamma} \gg 1$) systems follow 1D attractor for $\tau \ll 2R$. 

Kurkela, Wiedemann and Wu (2019)[18]
Low $k$ expansion of kinetic response functions

$$\tilde{G}_{\text{energy}}(\tau, \tau_0, |k|) = \tilde{G}^0_s(\tau, \tau_0) \left( 1 - \frac{1}{2} |k|^2 (\tau - \tau_0)^2 \tilde{s}^{(2)}_s + \ldots \right),$$

$$\tilde{G}_{\text{energy}}^\text{mom.}(\tau, \tau_0, |k|) = \tilde{G}^0_s(\tau, \tau_0) \left( |k|(\tau - \tau_0) \tilde{s}^{(1)}_v + \ldots \right),$$

Taylor expansion coefficients of response functions to (initial):

Smooth evolution of low-wavelength response in kinetic pre-equilibrium.