Anisotropic Metamaterial Optical Fibers

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Abstract: Internal physical structure can drastically modify the properties of waveguides: photonic crystal fibers are able to confine light inside a hollow air core by Bragg scattering from a periodic array of holes, while metamaterial loaded waveguides for microwaves can support propagation at frequencies well below cutoff. Anisotropic metamaterials assembled into cylindrically symmetric geometries constitute light-guiding structures that support new kinds of exotic modes. A microtube of anodized nanoporous alumina, with nanopores radially emanating from the inner wall to the outer surface, is a manifestation of such an anisotropic metamaterial optical fiber. The nanopores, when filled with a plasmonic metal such as silver or gold, greatly increase the electromagnetic anisotropy. The modal solutions in anisotropic circular waveguides can be uncommon Bessel functions with imaginary orders.

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Optical fibers form the backbone of optical communications systems worldwide. Their analogues in the microwave regime are hollow or coaxial metallic waveguides, which are indispensable in applications requiring field confinement, low losses, and high power-handling capability. The introduction of physical structure in the form of a periodic array of microscopic air holes running along the fiber axis in the photonic crystal fiber has made it possible to confine light even within a central hollow air core. The photonic crystal fiber depends on Bragg scattering and photonic bandgap effects for confining light. Birefringence of the modes in these fibers usually results from a two-fold asymmetry, and even small birefringence created by applied strain provides for many sensor applications. However, these fibers usually have no structure or structural anisotropy perpendicular to the fiber axis.

Metamaterials are structured composite materials that can be designed for specific electromagnetic responses across the spectrum, not readily available in nature. The structural units of the metamaterials are subwavelength in size, which makes metamaterials amenable to description as effective media. The geometry and materials of the structural units can be chosen to yield a wide variety of desired responses and dispersions for the metamaterial. Structural anisotropy of the units results in anisotropic effective medium parameters like the dielectric permittivity or magnetic permeability. This anisotropy can be extremely large compared to natural anisotropic responses in crystals.

Interesting phenomena have been shown in microwave rectangular and circular waveguides loaded with metamaterials, including propagation at frequencies below the fundamental mode of the unfilled waveguide. Similarly, the unprecedented control over the metamaterial properties as well as the enormous flexibility of applications possible with a fiber makes it attractive to combine them into a common platform. The immense difficulty of practically assembling small nanostructured metamaterial units inside the micrometer sized fibers has essentially discouraged even the discussion of such systems for optical frequencies and only some cylindrically layered tubes or with coaxially oriented nanowires that are otherwise uniform along the axis, have been theoretically discussed as metamaterial optical fibers. We show here that a fiber or circular waveguide containing an anisotropic metamaterial embodies a very unique system that can support highly uncommon modes described by Bessel and Neumann functions of imaginary orders. A nanoporous alumina (Al₂O₃) microtube with nanopores that emanate radially from an inner surface to a nanoporous surface provides a physical manifestation of such an anisotropic metamaterial fiber. Well known anodization techniques have been adapted to form such fibers. The possibility of filling up the pores with plasmonic metals like gold and silver makes possible a fiber made of a hyperbolic metamaterial with large resonant plasmonic interactions. The nanopores can be filled up by various materials by various methods and makes this system very attractive for sensor applications.

We start by considering a cylindrically symmetric waveguide formed of a material with relative dielectric permittivity

\[
\overline{\varepsilon} = \varepsilon_0 \begin{pmatrix} \varepsilon_r & 0 & 0 \\ 0 & \varepsilon_\phi & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}.
\]  

That is, the waveguide material has different dielectric responses along the axial, radial and azimuthal directions and the material is spatially homogeneous. For simplicity, we will deal with the coaxial case in which the concentric inner \((r = R_1)\) and outer \((r = R_2)\) surfaces of the waveguide are perfect electric conductors (PEC). In this case, the transverse electric (TE)
modes with $E_z = 0$ and the transverse magnetic (TM) modes with $H_z = 0$ decouple and the Maxwell's equations become separable. We obtain solutions within the anisotropic medium for the TE modes as

$$H_z = [AJ_\nu(k_r r) + BY_\nu(k_r r)] \exp[i(m\phi + \beta z)]$$  \hspace{1cm} (2)

where $J_\nu$ and $Y_\nu$ are the Bessel and Neumann functions of order $\nu$, $\beta$ is the propagation constant, and $m$ is a non-zero integer. To simplify, if one considers only dielectric anisotropy with $\mu = 1$, and $\varepsilon_z = \varepsilon_{\phi}$, we have $k_r^2 = \varepsilon_{\phi} \omega^2 / c^2 - \beta^2$ and $\nu^2 = \{(\varepsilon_{\phi} \omega^2 / c^2 - \beta^2) / (\varepsilon_{\phi} \omega^2 / c^2 - \beta^2)\} m^2$. Analogous expressions are obtained for the TM modes as

$$E_z = [AJ_\nu(k_r r) + BY_\nu(k_r r)] \exp[i(m\phi + \beta z)]$$  \hspace{1cm} (3)

where with $\mu = 1$, and $\varepsilon_z = \varepsilon_{\phi}$, we have $k_r^2 = \varepsilon_{\phi} \omega^2 / c^2 - \beta^2 \varepsilon_{\phi} / \varepsilon_z$ and $\nu^2 = \{(\varepsilon_{\phi} \omega^2 / c^2 - \beta^2 \varepsilon_{\phi} / \varepsilon_z) / (\varepsilon_{\phi} \omega^2 / c^2 - \beta^2)\} m^2$. The coefficients $A$ and $B$ are related by setting the tangential component of the TM mode, $E_z = 0$, at the PEC boundaries, resulting in

$$J_\nu (k_r R_1) Y_\nu (k_r R_2) - J_\nu (k_r R_2) Y_\nu (k_r R_1) = 0,$$  \hspace{1cm} (4)

which determines $\beta$ and the dispersion of the mode.

Computed solutions for some modes in anisotropic fibers are shown in Fig. 1 where the solutions for a positive index waveguide are also shown for comparison. The nature of the modes critically depends on $\nu$ and $k_r$. When $k_r$ is imaginary, the modal solutions assume the form of Bessel and Neumann functions of the second kind. The anisotropic nature of the waveguide allows $\nu$ to be fractional and even imaginary ($\nu^2 < 0$), which is not possible in isotropic systems. Physical applications of Bessel functions with imaginary orders have been rarely reported [9]. For real $k_r$, Bessel (TM) modes with imaginary orders result for $\varepsilon_z \omega^2 / c^2 < \beta^2 < \varepsilon_{\phi} \omega^2 / c^2$. These modes are very interesting in that the higher-order (large $m$) modes are localized near the central region of the waveguide (see Fig. 1). In contrast, the large $m$ whispering gallery modes are concentrated at the edges of a waveguide filled with an isotropic material. Further,
Fig. 2. Scanning electron micrographs of the nanoporous alumina microtube. Note the presence of the radially oriented non-branching nanopores. The nanoporous outer surface and the impermeable barrier oxide layer at the inner tubular surface are shown in the insets. The brittle alumina microtube cracks when cleaved for the SEM imaging.

these imaginary Bessel modes undergo large oscillations near the center and need not converge at \( r = 0 \) ([9]) – a reflection of the geometric singularity at the axis, which cannot be attained in a physical system. The PEC boundary condition applied at \( r = R_1 \) prevents any issues with the mathematical singularities at the origin. The number of oscillations near \( r = 0 \) increases with the order \( \nu \) of the mode. Even for modes with real fractional orders that result for anisotropic fibers, there is a comparatively larger confinement of the higher-order \( m \) modes (Fig. 1). Another important aspect is the case of TM modes with imaginary orders that have no cutoff frequency when \( \varepsilon_r < 0 \) and \( \varepsilon_r > 0 \) as then \( \beta^2 = \varepsilon_r \omega^2/c^2 + \varepsilon_r/|\varepsilon_z|k_z^2 \) that are distinct from the quasistatic TEM mode. Whereas the anisotropy described above may be realized in microwave waveguides using thin-wire metamaterials [12] [14], the task of practically fabricating a metamaterial fiber for optical/NIR frequencies with nanoscale features is daunting. We realized, however, that a wire mesh metamaterial in the cylindrical geometry could be obtained by anodization / electrodeposition methods. Starting with a high purity (99.999%) aluminum wire that was cleaned and electropolished, a two-step anodization [8] was carried out at 40V in 0.3 M oxalic acid to obtain a nanoporous alumina layer of several micrometers to several tens of micrometers thickness. The nanoporous alumina consists of an inner impermeable alumina layer near the center from which nanopores radially emanate and terminate on a nanoporous surface formed by the nanopores as shown in Fig. 2. The nanopores may now filled with a plasmonic metal like silver, gold or nickel by electrodeposition techniques [13]. This results in an extremely large anisotropy in the waveguide. The aluminum wire at the center may be retained or etched away using CuCl2. If retained, it effectively forms a PEC at the inner surface (\( R_1 \)), while a simple thick coating of aluminum on the outer nanoporous surface will provide a PEC there (\( R_2 \)). This yields the exact physical manifestation of the coaxial metamaterial fiber discussed above. The thickness of the porous alumina layer, the inner core diameter and interpore separation and pore sizes are flexibly changed by choice of the thickness of the initial aluminum wire, acid environment and anodization voltage and the times for electropolishing, anodization and etching. We have obtained upto 5 cm long microtubes with alumina thicknesses ranging from 2 to 80 \( \mu \)m, inner core diameters from 3 to 40 \( \mu \)m and outer diameters of upto 100 \( \mu \)m.
An effective anisotropic dielectric tensor for the nanostructured medium can be obtained approximately from a simple Maxwell-Garnet homogenization [15] applied in the curvilinear geometry. Considering electric fields parallel to the nanopores or plasmonic nanorods, we obtain an effective permittivity component as

\[ \varepsilon_r = \varepsilon_i f + (1 - f) \varepsilon_h, \]

where \( \varepsilon_h \) is the dielectric permittivity of host medium alumina, \( \varepsilon_i \) is the dielectric permittivity of the inclusion metal nanowire or nanopore void and \( f \) is the areal fill fraction of the pore or nanorod on the constant \( r \) surface. Similarly, considering the electric fields perpendicular to the nanopores or nanorods, the other effective permittivity components are obtained as

\[ \varepsilon_\phi = \varepsilon_z = \frac{(1 + f) \varepsilon_i \varepsilon_h + (1 - f) \varepsilon_h^2}{(1 - f) \varepsilon_i + (1 + f) \varepsilon_h}. \]

In this picture, only the areal fill fraction \( f \) of the pore/nanorod determines the effective medium properties and the anisotropy results in \( \varepsilon_r \neq \varepsilon_\phi \) and \( \varepsilon_z \simeq \varepsilon_\phi \). The nanopore cross-section approximately reduces linearly with the radial distance in alumina microtubes anodized at constant voltage, and the areal fill fraction changes with the radial distance as

\[ f = \frac{(2\pi q^2 r)}{(\sqrt{3}d^2 R)}, \]

where \( q \) is the pore diameter and \( d \) is the pore separation at some given radius \( R \). As a consequence, these fibers are actually spatially inhomogeneous (See Fig. 3), which will affect the nature of the modes. Typically, nanowires made of plasmonic materials that have Re(\( \varepsilon \)) < 0 results in an extreme effective anisotropy with \( \varepsilon_r < 0 \) and \( \varepsilon_\phi > 0 \). While anisotropy itself makes it possible to have modes with imaginary orders, the indefinite permittivity with \( \varepsilon_r < \varepsilon_\phi \) (as \( \varepsilon_r < 0 \) and \( \varepsilon_\phi > 0 \)) that results when plasmonic nanorods fill the nanopores makes the conditions for modes with imaginary orders immediately accessible.

The guidance of light through a bent section of a nanoporous anisotropic fiber and a aluminium core is shown in Fig. 3. The scattering from inhomogeneities and other defects is strong, but the evidence for light confinement and guidance is clear. Although the nanoporous alumina material used here might not possess optimal optical properties, the nanostructure may be converted into glasses through chemical processes or sol-gel methods similar to those in Ref. [8]. Hence, low-loss anisotropic optical fibers can be obtained. It is also important to realize that the generic nature of these modal solutions are not limited by the application of PEC boundaries. Having dielectric boundaries, for example by having a hollow core or no aluminum coating on the outer surface, causes the waveguide to be inhomogeneous and support hybrid (the so-called HE and EH) modes. Fibers with hollow cores are important for various issues of confinement of light as in hollow core photonic crystal fibers. The special case in which the core is hollow but the PEC boundary condition at \( r = R_2 \) is maintained is, in fact, the anisotropic metamaterial-lined circular waveguide. When the metamaterial liner exhibits certain properties, e.g. a permittivity tensor containing near-zero and negative elements, these waveguides permit propagation well below their natural cutoff frequencies [5]. This has important applications in the miniaturization of microwave waveguide systems requiring access to the interior volume such as in fluid heating, electron-beam propagation, and traveling-wave magnetic-resonance imaging. As open-ended waveguide probe antennas, these structures may enable subwavelength spatial resolutions with high efficiency in near-field antenna characterization [14].

We will point out some of the astonishing range of possibilities that arise with anisotropic metamaterial optical fibers. The Bessel modes with imaginary orders have enormous implications for coupling to the near-field modes of small sources as they have fast varying fields along both the radial and azimuthal directions localized near the center of the fiber. The subwavelength image resolution possible with near-field imaging by a hyperlens is also due to this effect [11]. Usually, one is able to couple light to the whispering gallery modes only though the
cladding due to the localization of these modes at the edges. The Bessel modes with imaginary orders can enable a butt-coupling to the high order $m$ modes to couple to the near-field modes of a radiating source. Further, these Bessel modes with imaginary orders occupy the entire cross section of the fibre and have implications for the amplification of these modes in the presence of gain media without spatial hole burning effects. The presence of plasmonic nanorods inside medium results in large local electromagnetic fields that are so important for nonlinear interactions. The guidance of fields within the fiber with large nonlinear interactions has immense implications for effects like super continuum generation due to dispersion of ultra-short pulses.

The enormous surface areas available within the nanopores for adsorbing molecules coupled with the fiber geometry and resonant modes makes this system highly suitable for sensor applications. In case of hollow core fibers, liquids can be flown through the central microtube and the enhanced fields of the higher order $m$ modes localized near the center could be a very useful for detecting molecules in the liquids.

We have demonstrated here a new paradigm of metamaterial fibers and circular waveguides. These anisotropic metamaterial fibers support exotic modes described by Bessel and Neumann functions of fractional and imaginary orders, a rare physical application of these mathematical objects. Many of the properties presented here are generic to circular waveguides with anisotropic metamaterial fillings and have wide ranging applications across the electromagnetic spectrum from the rf to optical frequencies. Nanoporous alumina microtubes with or without a central hollow core are presented as a physical manifestation of such an anisotropic metamaterial optical fiber.

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