The Dirichlet Super-Three-Brane in Ten-Dimensional Type IIB Supergravity

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Abstract

We give the full supersymmetric and \(\kappa\)-symmetric action for the Dirichlet three-brane, including its coupling to background superfields of ten-dimensional type IIB supergravity.
1. Introduction

The recent incorporation of Dirichlet $p$-branes in string theory has led to major advances in our understanding of the non-perturbative aspects of the theory [1,2,3,4,5]. This includes issues like the rôles played by solitonic states in non-perturbative string theories and results concerning string duality [6,7,8] as well as fundamental explanations of results known from semi-classical gravity [9].

An important aspect is the fascinating rôle played by standard supergravity theories in uncovering deep connections between different perturbative sectors of the full non-perturbative theory [10,11,12]. It is now clear that eleven-dimensional supergravity, which previously seemed completely disconnected from string theory, plays a rather central, unifying rôle in the full theory [11], e.g. when discussing a higher-dimensional origin of $p$-brane solutions [13] and various kinds of duality symmetries appearing in lower-dimensional ($D \leq 10$) theories [11,14-21 and references therein] (we do not aim at making the reference list complete).

In the same spirit as one obtains supergravity theories from strings in ten dimensions and below, it was argued some time ago that one should seek to explain the existence of eleven-dimensional supergravity in terms of the enigmatic quantized supermembrane [22], or, as suggested more recently, in terms of the even less understood M-theory [11,15,16,23,24], of which the membrane might be a low-energy manifestation. That such a connection might exist is indicated by the fact that $p=2$ (and $p=5$) branes [25,26] occur as solutions to the $D=11$ supergravity field equations and by the relation between $\kappa$-symmetry and the constraints on the $D=11$ supergeometry that implies the field equations.

However, some interesting chiral string theories and their $p$-brane solutions fall outside this scheme, as emphasized in [12], and instead hint at the existence of new supergravity-like theories in yet higher dimensions [27], F-theory [28] (or Y-theory [12]) in twelve dimensions or fundamentally new theories in thirteen [29,30,31] dimensions or above, often with non-standard signatures [28,30,31]. The search for these exotic theories would probably benefit from a more detailed understanding of the less exotic objects known to exist in eleven dimensions and below. One particularly interesting piece of new information would be the construction of $\kappa$-symmetric covariant actions which involve vector or antisymmetric tensor supermultiplets in a fundamental way.

Higher-dimensional extended objects of various kinds have turned out to be extremely useful in many of the recent discussions of these matters (see e.g. refs. [5,10]), in spite of the fact that they are in many cases rather poorly understood. For example, in ten and eleven dimensions full knowledge of the covariant super- and $\kappa$-symmetric action including background couplings has been obtained only for the string ($p=1$) [32,33] in $D=10$, $N=1$ supergravity [34] and type IIB supergravity [35], for the membrane ($p=2$) [22] in $D=11$ [36,37] and for the 5-brane in $D=10$ [38]. Although a more complete picture [39,40] of possible world-volume supermultiplets [41] is now emerging, none of the actions based on vector or antisymmetric tensor supermultiplets have yet been constructed. A case that falls in between is the membrane in type IIA supergravity which has Ramond–Ramond (RR) charges and hence is a D-brane with $p=2$. We are here referring to the fact that the action for this type IIA membrane can be obtained by dualizing [39, 42,43,44] the
eleventh coordinate for the membrane in $D=11$ to a vector, thereby generating an action containing an abelian world-volume vector field. Using the formulation of Bergshoeff et al. [22], the action derived in this way by Townsend [43] is of Brink–DiVecchia–Howe–Tucker (BDHT, a.k.a. Polyakov) type with a non-propagating world-volume metric [45,46]. Elimination of this auxiliary metric by means of its field equation leads to a Dirac–Born–Infeld (DBI) action. That such an action must appear is known from open string calculations in the case of constant field strength for the Born–Infeld vector field, at least for the bosonic part of the action [2,47]. In fact, a fully covariant target space supersymmetric and world-volume $\kappa$-symmetric DBI action has not yet been presented in the literature.

It is the purpose of this paper to derive such an action for the case of the Dirichlet 3-brane in type IIB string theory. In this case the answer can not be deduced by dualizing any previously known action. Instead, our derivation starts from the bosonic DBI action and knowledge about the structure of the necessary Wess–Zumino terms obtained from the superspace formulation of type IIB supergravity. As is the case for the membrane propagating in the background of eleven-dimensional supergravity, the action for the D3-brane in type IIB superspace is only $\kappa$-symmetric provided the background fields satisfy the proper on-shell constraints (see ref. [22]). Similar results in the case of type IIB were obtained some years ago by Grisaru et al. [33], who coupled the type IIB supergravity to the string in the NS-NS sector. Note however that in this case one does not probe the constraints of the field strengths coming from the RR sector. In the case of the D3-brane considered here on the other hand, all constraints necessary to derive the field equations of type IIB supergravity [35] are involved in establishing the $\kappa$-invariance of the action.

The plan of the paper is as follows. In section 2 we explain our notation and conventions, and present the generalization of the bosonic action to a $\kappa$-symmetric action in flat type IIB superspace. Section 3 is devoted to the construction of the couplings to the on-shell type IIB background superfields, which as mentioned above involves all the field strengths of the theory, i.e., both the ones that originate in the NS-NS sector and the ones from the RR sector. For the bosonic part of the action the exact relation between the BDHT version and the DBI version is known from the work of ref. [48]. A summary of our results including a discussion of the $\kappa$-symmetric BDHT-type extension of ref. [48] and some final comments are collected in section 4.

2. The $\kappa$-symmetric $p=3$ action in flat IIB superspace

We will start the construction by considering the bosonic DBI action

$$\mathcal{L}_{\text{DBI}} = -\sqrt{-\det(g_{ij} + F_{ij})}, \quad (2.1)$$

where $g_{ij}$ is the pullback to the four-dimensional world-volume of the flat ten-dimensional target space metric $\eta_{ab}$ and $F_{ij}$ is the field strength of the world-volume vector field appearing in all D-brane actions generated from open strings. (The standard factor of $\frac{\alpha'}{2\pi}$ in front of $F_{ij}$ is set
equal to one in this paper.) The pullback is written in terms of $\partial_i X^a$ where $X^a$ are coordinates in the flat ten-dimensional bosonic target space. As a first step we now generalize the corresponding world-volume form $dX^a$ to a globally supersymmetric form by introducing the type IIB fermionic coordinates $\theta^a$. Since we will throughout this paper use real sixteen-component Majorana–Weyl spinors and the corresponding $\gamma$-matrices we must remember that type IIB spinors have an extra two-dimensional $SO(2)$ index and that indices $\alpha, \beta, \ldots$ are thus to be viewed as composite indices representing the tensor product of a Majorana–Weyl index and an $SO(2)$ index. The $\gamma$-matrices are extended in a similar fashion and all these quantities will as a consequence remain real. The $\gamma$-matrices anticommute to $2\eta_{ab}$ (with a suppressed $\delta_{\alpha \beta}$) which we choose to be a mostly positive flat metric.

The supersymmetric world-volume variables (forms) for the case where there are no background fields are

$$d\theta^a,$$

$$\Pi^a = dX^a + id\bar{\theta}\gamma^a \theta,$$

$$\mathcal{F} = dA - idX_a \wedge (d\bar{\theta}\gamma^a K \theta) + \frac{1}{2} (d\bar{\theta}\gamma_a \theta) \wedge (d\bar{\theta}\gamma^a K \theta).$$

The expression for $\mathcal{F}$ contains the matrix $K$, which is a real $2 \times 2$-matrix with $K^2 = 1$. It acts on the $SO(2)$ indices mentioned above and together with $I$ and $J$ ($I^2 = -1$, $J^2 = K^2 = 1$, $IJ = K$ and $I, J, K$ anticommuting) they behave as gamma-matrices (generators) for $SL(2; \mathbb{R}) \cong Spin(1, 2)$, or, equivalently, as the imaginary split quaternionic units. A convenient basis (although unnecessary for any calculations) is

$$I = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad K = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

from which one possible correspondence with a complex spinor formalism may be read off: $I\psi \leftrightarrow -i\bar{\psi}$, $J\psi \leftrightarrow i\bar{\psi}^*$, $K\psi \leftrightarrow \psi^*$. Concerning the conventions for ordinary forms, we have here adopted the ones that are standard in superspace differential geometry, namely $\mathcal{F} = \frac{1}{2} d\xi^i \wedge d\xi^j \mathcal{F}_{ij}$ (note the order of the indices). As will be discussed in the next section, these globally supersymmetric extensions arise naturally when taking the flat superspace limit of the D3-brane in a general background. The reason we present the flat case first is that it clarifies how the $\kappa$-symmetry works without any need for knowledge about the rather complicated superspace formulation of type IIB supergravity.

The rigid supersymmetry transformations under which the above forms are invariant are

$$\delta_\epsilon \theta^a = \epsilon^a,$$

$$\delta_\epsilon X^a = -i(\bar{\theta}\gamma^a \epsilon),$$

$$\delta_\epsilon A = idX_a (\bar{\theta}\gamma^a K \epsilon) - \frac{1}{6} (d\bar{\theta}\gamma_a \theta) (\bar{\theta}\gamma^a K \epsilon) - \frac{1}{6} (d\bar{\theta}\gamma_a K \theta) (\bar{\theta}\gamma^a \epsilon),$$

where $\epsilon$ is a constant target space IIB spinor and a world-volume scalar. It is a priori clear that $A$
must transform under supersymmetry, since it contains two of the eight physical bosonic degrees of freedom matching the fermionic ones. We will comment further on this issue in section 3. To prove the invariance of $\mathcal{F}$ one must make use of the “cyclic Fierz identity”

$$(\tilde{A} \gamma_a B)(\tilde{C} \gamma^a D) = -\frac{1}{2}(\tilde{A} \gamma_a e_I D)(\tilde{B} \gamma^a e_I C) - \frac{1}{2}(\tilde{A} \gamma_a e_I C)(\tilde{B} \gamma^a e_I D),$$

(2.5)

where $e_I = 1, I, J, K$ and summation over the index $I$ is understood. Note that $d\theta$ anticommutes with $\theta$ but commutes with another $d\theta$. By writing $A = \alpha \psi$, with $\alpha$ an odd parameter, the above identity, which is valid for bosonic spinors $A, B, C, D$, can be used to derive Fierz identities for any kind of spinors. Equation (2.5) is the only Fierz identity needed throughout this paper. By using it twice, one finds, e.g.,

$$(\gamma_a)(\alpha\beta(\gamma^a)_{\gamma}\delta) = -(\gamma_a J)(\alpha\beta(\gamma^a J)_{\gamma}\delta) = -(\gamma_a K)(\alpha\beta(\gamma^a K)_{\gamma}\delta),$$

(2.6)

identities which are used repeatedly below in calculations concerning $\kappa$-symmetry and Bianchi identities.

Turning to the $\kappa$-symmetry, we know from previous experience that the $\kappa$-transformations of $\theta^a$ and $X^a$ differ from the $\varepsilon$-transformations above only by a sign in the transformation for $X^a$ (this fact will find its natural explanation in section 3 — a $\kappa$-transformation is rather half a covariant derivative, and commutes with supersymmetry). The $\kappa$-transformation of $A$ is completely fixed once one demands that $\kappa$-transformations are supersymmetric, i.e., that $\delta_{\varepsilon}$ and $\delta_{\kappa}$ commute. To determine the overall constant, one makes the inspired guess that $\delta_{\kappa} g_{ij}$ and $\delta_{\kappa} \mathcal{F}_{ij}$ should end up having a similar structure. Using the transformations

$$\delta_{\varepsilon} \theta^a = \kappa^a,$$
$$\delta_{\kappa} X^a = i(\bar{\theta} \gamma^a \kappa),$$
$$\delta_{\kappa} A = -idX_a(\bar{\theta} \gamma^a K \kappa) + \frac{1}{2}(d\bar{\theta} \gamma_a \theta)(\bar{\theta} \gamma^a K \kappa) + \frac{1}{2}(d\bar{\theta} \gamma_a K \theta)(\bar{\theta} \gamma^a \kappa),$$

(2.7)

we find

$$\delta_{\kappa} \Pi_i^2 = 2i(d\bar{\theta} \gamma^a \kappa),$$
$$\delta_{\kappa} \mathcal{F} = -2i\Pi_a \wedge (d\bar{\theta} \gamma^a K \kappa).$$

(2.8)

In order for the bosonic and fermionic fields in the world-volume action to have equal numbers of degrees of freedom we must as usual project out half of the components of the local space-time spinor parameter $\kappa$. This is accomplished by imposing $\kappa = \frac{1}{2} (1 + \Gamma) \kappa$, where

$$\Gamma = \frac{\varepsilon_{ijkl}}{\sqrt{-\det(g + \mathcal{F})}} \left( \frac{1}{24} \gamma_{ijkl} I - \frac{1}{4} \mathcal{F}_{ij} \gamma_{kl} I + \frac{1}{8} \mathcal{F}_{ij} \mathcal{F}_{kl} I \right)$$

(2.9)
satisfies $\Gamma^2 = 1$. Here, $\gamma_{i_1 \ldots i_n}$ is the pullback to the world-volume of the antisymmetrized (weight one) product of $n$ $D=10$ $\gamma$-matrices, and thus $\gamma_i \gamma_j = g_{ij}$. In proving that the matrix $\Gamma$ squares to one, it is convenient to rewrite the DBI lagrangian using

$$\det(g_{ij} + \mathcal{F}_{ij}) = \det(g_{ij})(1 - \frac{1}{2} \text{tr}(g^{-1} \mathcal{F})^2) + \det(\mathcal{F}_{ij}), \quad (2.10)$$

and make use of identities like

$$\det \mathcal{F} = \left( \frac{1}{8} \varepsilon^{ijkl} \mathcal{F}_{ij} \mathcal{F}_{kl} \right)^2,$$

$$(g^{-1} \mathcal{F})^4 = \frac{1}{2} (g^{-1} \mathcal{F})^2 \text{tr}(g^{-1} \mathcal{F})^2 - \det(g^{-1} \mathcal{F}), \quad (2.11)$$

valid for antisymmetric matrices $\mathcal{F}_{ij}$. Given an antisymmetric tensor, the form of $\Gamma$ is unique up to the choice for the matrix $J$, which in turn depends on the choice of the matrix $K$ in the transformations. A perhaps trivial remark is that just by trying to construct a projection matrix for $p=3$, one is forced to introduce a pair of Majorana–Weyl spinors of equal chirality. The question of chirality is related to the sign of the square of $\varepsilon^{i_1 \ldots i_{p+1}} \gamma_{i_1 \ldots i_{p+1}}$, which is $(-1)^{\frac{1}{2}p(p+1)+1}$. Already this observation tells much about the structure of supersymmetric branes of diverse dimensionalities. The construction of $\Gamma$ is of course a key ingredient in establishing $\kappa$-symmetry, and therefore in the entire construction of the supersymmetric brane action.

After a somewhat tedious calculation one arrives at the following result for the $\kappa$-variation of the DBI lagrangian:

$$\delta_{\kappa} \left( -\sqrt{-\det(g + \mathcal{F})} \right) = \varepsilon^{ijkl} \left( -\frac{1}{8} (\partial_l \partial_j \gamma_{i k l} I\kappa) + i \mathcal{F}_{ij} (\partial_k \partial_l \gamma_{i l} J\kappa) \right). \quad (2.12)$$

The computations are simplified somewhat if one instead of inserting the projected parameter $\frac{1}{2}(1+\Gamma)\kappa$ just uses $\Gamma\kappa$. Note that the integral of the right hand side can be nicely written as the integral of a world-volume four-form,

$$\delta_{\kappa} I_{\text{DBI}} = \int \left( 2i(\partial \gamma_{(3)} I\kappa) - 2i \mathcal{F}(\partial \gamma_{(1)} J\kappa) \right), \quad (2.13)$$

where $\gamma_{(n)} = \frac{1}{n!} d\xi^{i_n} \wedge \ldots \wedge d\xi^{i_1} \gamma_{i_1} \ldots \gamma_{i_n}$. The result that the $\kappa$-variation of the DBI action gives the integral of a differential form is of course a necessary condition for the existence of an invariant action built out of just a DBI term and a Wess–Zumino (WZ) term.

In order to find the WZ part of the action, it is in principle possible to make a general ansatz and determine the exact expression by demanding cancellation with (2.13). However, a better alternative would be to first consider its general superspace structure discussed in refs. [49,50], and make use of the already solved type IIB superspace Bianchi identities [35] to obtain the exact expression. The flat limit is then expected to produce the same answer as the more direct approach.
alluded to above. Since this will be carried out in detail in section 3, where we will discuss the coupling of the D3-brane to a general type IIB supergravity background, we end this section by simply giving the expression for the WZ action whose variation will cancel the one from the DBI term:

$$I_{WZ} = \int e^{\tilde{F}} \mathcal{C} = \int (C_4 + \tilde{F} \wedge C_2) \right) ,$$

where

$$C_{ab} = 0 , \quad C_{a\beta} = i(\gamma_a J_\theta)_{\beta} \quad C_{\alpha\beta} = (\gamma^a \theta)_{\alpha}(\gamma_a J_\theta)_{\beta} ,$$

and

$$C_{abcd} = 0 , \quad C_{abc\delta} = -i(\gamma_{abc} J_\theta)_{\delta} \quad C_{ab\gamma\delta} = -(\gamma^c \theta)_{\gamma} (\gamma_{abc} J_\theta)_{\delta} - 2(\gamma_a J_\theta)_{\gamma}(\gamma_b K_\theta)_{\delta} , \quad C_{a\beta\gamma\delta} = \gamma^a \theta_{\alpha}(\gamma^b \theta)_{\beta} (\gamma_c \theta)_{\gamma}(\gamma_{abc} J_\theta)_{\delta} - 3(\gamma^a \theta)_{\alpha}(\gamma_a J_\theta)_{\beta}(\gamma^b \theta)_{\gamma}(\gamma_b K_\theta)_{\delta} .$$

where (anti-)symmetrization is understood. In the following section, the various $\theta$-extended quantities introduced here will receive a proper explanation in terms of the flat limit of the superspace formulation of the target space type IIB supergravity theory.

When one considers an ordinary $p$-brane, only the first term in equation (2.14) is present, $I_{WZ}^{(p)} = \int C_{(p+1)}$, and the crucial $\gamma$-matrix identity, yielding the “brane scan”[51], is

$$(\gamma^{a_1 \cdots a_{p-1} a})_{\alpha\beta}(\gamma_a)_{\gamma\delta} = 0 .$$

The corresponding identity here, which can be seen as responsible for the existence of the D3-brane, and which is also crucial for the Bianchi identities of the following section, is

$$(\gamma^{abc})_{(\alpha\beta}(\gamma_c)_{\gamma\delta)} - 2(\gamma^a J_{(\alpha\beta}(\gamma^b K)_{\gamma\delta)} = 0 .$$

A proper classification of similar identities in different dimensionalities should provide a “D-brane scan” [39].

3. Coupling to a General IIB Background

As is well known (see e.g. ref. [35]), the field theory of ten-dimensional type IIB supergravity contains in the bosonic sector the metric $g_{mn}$, a complex scalar $\lambda = C_{(0)} + ie^{-\phi}$, a complex two-form $a_{(2)} = C_{(2)} + iB_{(2)}$ and a self-dual real four-form $C_{(4)}$, and in the fermionic sector the complex spinor
\[ \Lambda_\alpha \text{ and the Rarita–Schwinger field } \psi_m^\alpha. \] Bosonic fields originating in the RR sector of the string are here denoted as \( C_{(n)} \). The indices \( m, n, \ldots \) are ten-dimensional world indices which will be combined with \( \mu, \nu, \ldots \) to ten-dimensional super-world indices \( M, N, \ldots \). The superspace vielbein is then defined by

\[ E^A = dZ^M E_M^A, \quad (3.1) \]

where \( dZ^M \) are the superspace differentials \((dX^m, d\theta^\mu)\). Our superspace differential geometry conventions are those of Wess and Bagger \([52]\).

Since the action for a Dp-brane in general involves a DBI term with couplings only to NS-NS fields while the RR couplings appear exclusively in the WZ term, it is not possible to work with the above complex combinations of NS-NS and RR fields. It turns out to be necessary to keep also the spinors real, as we did in the previous section. It will even turn out that the natural choice of field strength does not correspond to the real and imaginary part of a complex field. As a consequence, some formulae below may look somewhat unfamiliar to a reader used to the previous superspace treatments of type IIB supergravity \([35]\). Real Majorana–Weyl spinors are here written in terms of composite indices \( \alpha, \beta, \ldots \) in tangent space or \( \mu, \nu, \ldots \) in curved superspace.

In superspace all ordinary component fields are introduced as first components of their corresponding superfields. From the gauge fields one then constructs the super-field strengths and derives the super-Bianchi identities. In order to reduce the enormous field content of these superfields down to the on-shell content one introduces constraints on some of the components of the super-field strengths. When these constraints are inserted in the Bianchi identities, the latter cease to be identities, and if the constraints are properly chosen the equations so obtained are just the supergravity equations of motion. For the membrane in eleven-dimensional supergravity it turns out that it is exactly these on-shell constraints (modulo different choices of the conventional ones) that must be imposed in order for the membrane action to be \( \kappa \)-symmetric \([22]\). We will now show that the same phenomenon is at work here.

The action is \( I = I_{\text{DBI}} + I_{\text{WZ}} \), where

\[ I_{\text{DBI}} = - \int d^4 \xi \sqrt{-\det(\hat{g} + e^{-\Phi})} \quad (3.2) \]

and

\[ I_{\text{WZ}} = \int e^{\Phi} \hat{C}. \quad (3.3) \]

This expression formally agrees with refs. \([53,50,44,49]\), with all bosonic fields replaced by the corresponding superfields. Here \( \hat{g} \) refers to the pullback of the ten-dimensional flat metric \( \eta_{ab} \), i.e.,

\[ \hat{g}_{ij} = E_i^a E_j^b \eta_{ab}, \quad (3.4) \]

which in an arbitrary background involves the vielbein through \( E_i^A = (\partial_i Z^M) E_M^A \). The use of hats indicates coupling to a general type IIB superspace background. For instance, in the NS-NS
sector the superfield $B_{MN}$ enters through $\hat{\mathcal{F}} = \frac{1}{2} d\xi^i \wedge d\xi^j (F_{ij} - \hat{B}_{ij})$ where $\hat{B}_{ij} = E_j^A E_i^B B_{AB}$. In the WZ term the pullback of the RR potential $\hat{C}$ is defined in a similar fashion. Note that we have adopted the definition of $\hat{C}$ introduced in ref. [50] where this RR potential is considered as a sum of forms of different degrees. In the case of type IIB they are all of even degree. In fact, such a sum was already discussed by Cederwall et al. [54] in the investigation of $p$-brane algebras and their general extensions.

The advantage of equation (3.2) over the form $\mathcal{L}_{\text{DBI}} = - e^{-\phi} (-\det(\hat{g}_s + \hat{\mathcal{F}}))^\frac{1}{2}$, $\hat{g} = e^{-\frac{\phi}{2}} \hat{g}_s$, is that the metric $\hat{g}$ is the SL(2; $\mathbb{R}$)-invariant Einstein metric, whereas $\hat{g}_s$ is the string metric and transforms under SL(2; $\mathbb{R}$). For $\hat{g}$ (and all associated superfields) we can use the conventional supergravity constraints [35], while the torsion corresponding to the vielbein of $\hat{g}_s$ necessarily contains explicit dilaton factors. It is a property peculiar to $p=3$, and essential for its duality properties, that the naked Einstein metric occurs in the action [55, 44, 56].

When written in this way, the D3-brane action is manifestly invariant under type IIB superspace reparametrizations. One might wonder how this is reconciled with the coupling to open strings where the boundary conditions break $N = 2$ to $N = 1$. The answer is identical to the situation for the gauge invariance of an antisymmetric tensor field coupling to the bulk of an open string. Naïvely, the gauge invariance is destroyed by the presence of a boundary, but when the boundary is a D-brane it is restored by the coupling of the endpoints to the vector potential $A$ on the world-volume. The gauge transformation $\delta B = d\lambda$ is accompanied by $\delta A = \lambda$, so that the combination $\hat{\mathcal{F}} = F - \hat{B}_{(2)}$ is invariant. For $N = 2$ supersymmetry, an exactly parallel mechanism is at work, and this is the concrete interpretation of the third equation in (2.4), which is exactly what is necessary to keep $\hat{\mathcal{F}}$ invariant in the flat case [11, 43]. In fact, once the extra terms in the modified supersymmetric field strength are understood as the pullback of the superfield $B$, the just mentioned gauge transformation can be used to move the transformations freely between $A$ and $B$, and in particular to keep $A$ inert under supersymmetry and $\kappa$-symmetry.

We now proceed to the issue of $\kappa$-symmetry. Contrary to the situation in section 2, we now have full knowledge of the WZ term from the outset. This gives us the possibility to perform the calculations in a slightly different manner. By first varying the WZ term with parameter $\Gamma \kappa$ one finds that the calculations simplify quite a bit. The reason for this is that one directly obtains the factor $\mathcal{L}_{\text{DBI}}^{-1}$, which also follows from the variation of the DBI term if written in the form

$$
\delta_{\kappa} \mathcal{L}_{\text{DBI}} = -\frac{1}{12} \varepsilon^{ijkl} \varepsilon^{i'j'k'l'} \left( \hat{g}_{ii'} + e^{-\frac{\phi}{2}} \hat{\mathcal{F}}_{ii'} \right) \left( \hat{g}_{jj'} + e^{-\frac{\phi}{2}} \hat{\mathcal{F}}_{jj'} \right) \times \left( \hat{g}_{kk'} + e^{-\frac{\phi}{2}} \hat{\mathcal{F}}_{kk'} \right) \left( \kappa \hat{g}_{ll'} + e^{-\frac{\phi}{2}} \hat{\mathcal{F}}_{ll'} \right) - \frac{1}{2} \delta_{\phi} \hat{g} - \frac{1}{2} \delta_{\phi} \hat{\mathcal{F}}_{ll'} .
$$

Consequently, there will be no need for any of the matrix identities essential for proving $\Gamma^2 = 1$, and we find that the comparison of the terms coming from the variation of the DBI and WZ parts, respectively, is rather straightforward.

In order to derive the variations to be inserted in equation (3.5) we need the variations of the...
superspace coordinates. These can be written in the following form [22]:

\[(\delta_{\kappa}Z^{M})E_{M}^{a} = 0 \ , \quad (\delta_{\kappa}Z^{M})E_{M}^{a} = \kappa^{a} \ . \quad (3.6)\]

We then find that

\[\delta_{\kappa}\hat{g}_{ij} = 2E_{(i}^{\ a}E_{j)}^{B}\kappa^{a}T_{\alpha\beta} , \quad \delta_{\kappa}\hat{\varphi}_{ij} = -E_{i}^{B}E_{i}^{A}\kappa^{a}H_{\alpha\beta} , \quad \delta_{\kappa}\phi = \kappa^{a}\partial_{a}\phi \ . \quad (3.7)\]

To be explicit, the transformation of \(\hat{B}\) induced by (3.6) is

\[\delta_{\kappa}\hat{B} = \mathcal{L}_{\kappa}\hat{B} \equiv (d_{i} + i_{d})\hat{B} = di_{\kappa}\hat{B} + i_{\kappa}\hat{H} , \quad \text{where the first term is cancelled in} \ \delta_{\kappa}\hat{F} \ \text{by the gauge transformation} \ \delta A = i_{\kappa}\hat{B} \ (\text{see the discussion above}), \ \text{leaving} \ \delta_{\kappa}\hat{F} = -i_{\kappa}\hat{H} . \]

At this point one needs to solve the superspace Bianchi identities [35] to verify that the for us relevant on-shell constraints (those with dimension 0 and \(\frac{1}{2}\)) are

\[T_{\alpha\beta}^{c} = 2i(\gamma_{c}^{\alpha}\beta) , \quad T_{ab}^{c} = 0 , \quad T_{\alpha\beta}^{\gamma} = -(J)_{(\alpha}^{\gamma}(J\Lambda)_{\beta)} + (K)_{(\alpha}^{\gamma}(K\Lambda)_{\beta)} \]
\[+ \frac{1}{2}(\gamma_{a}J)_{\alpha\beta}(\gamma_{a}K)_{\gamma\beta} - \frac{1}{2}(\gamma_{a}K)_{\alpha\beta}(\gamma_{a}J)_{\gamma\beta} , \quad H_{\alpha\beta\gamma} = 2i\hat{\varphi}(\gamma_{c}K)_{\alpha\beta} , \quad H_{abc} = -e^{\hat{\varphi}}(\gamma_{bc}K\Lambda)_{\alpha} , \]
\[\partial_{a}\phi = 2\Lambda\alpha . \quad (3.8)\]

(We will demonstrate in a little while that this configuration is the correct one.) Thus, we can write the variation in the following form

\[\delta_{\kappa}\hat{g}_{ij} = 4i(\hat{E}_{(i}^{\gamma}\kappa)_{j)} , \quad \delta_{\kappa}\hat{\varphi}_{ij} = -4ie^{\hat{\varphi}}(\hat{E}_{(i}^{\gamma}\kappa)J_{j)} + e^{\hat{\varphi}}(\hat{\Lambda}\gamma_{ij}K\kappa) , \quad \delta_{\kappa}\phi = 2\kappa\Lambda . \quad (3.9)\]

Modulo boundary terms, the variation of the WZ term gives

\[\delta_{\kappa}I_{\text{WZ}} = \int \left( (i_{\kappa}\hat{R})_{(1)} + \hat{\varphi} \wedge (i_{\kappa}\hat{R})_{(2)} + \frac{1}{2} \hat{\varphi} \wedge \hat{\varphi} \wedge (i_{\kappa}\hat{R})_{(0)} \right) , \quad (3.10)\]

where the hatted field strengths \(\hat{R}\) are pullbacks of \((i_{\kappa}R)_{(n)} = \frac{1}{n!}E_{\alpha_{1}A_{1}}^{A_{n}} \wedge \ldots \wedge E_{\alpha_{n}}^{A_{1}}R_{\alpha_{1}A_{2}...A_{n}} . \)

Thus we find that only super-field strength components with at least one spinor index are relevant.
The formal sum of the curvature forms can be written \( R = e^B d(e^{-B} C) = dC - H \wedge C \), i.e.,

\[
R(5) = dC(4) - H \wedge C(2) ,
R(3) = dC(2) - H \wedge C(0) ,
R(1) = dC(0) ,
\]

so the Bianchi identities read \( d(e^{-B} R) = 0 \), i.e.,

\[
\begin{align*}
dR(5) + H \wedge R(3) &= 0 , \\
dR(3) + H \wedge R(1) &= 0 , \\
dR(1) &= 0 .
\end{align*}
\]

In the real basis employed here, the superspace constraints that enter into the variation of the WZ term read (the exact relation to the constraints given by Howe and West in ref. \[35\] will soon be established)

\[
\begin{align*}
R_{abc\alpha \beta} &= 2i(\gamma_{abc} I)_{\alpha \beta} , & R_{abcd\alpha} &= 0 , \\
R_{a\beta \gamma} &= -2ie^{-\phi/2}(\gamma_a J)_{\beta \gamma} , & R_{ab\gamma} &= -e^{-\phi/2}(\gamma_{ab} J \Lambda)_{\gamma} , \\
R_{\alpha} &= 2e^{-\phi} (I \Lambda_{\alpha}) .
\end{align*}
\]

All curvatures with more than two spinor indices have dimension lower than zero, and must vanish. We now insert these constraints in \( \delta \kappa L_{\text{WZ}} \) and at the same time substitute \( \kappa \) by \( \Gamma \kappa \), where the projection matrix in a general background is

\[
\Gamma = \frac{\varepsilon^{ijkl}}{\sqrt{-\det(g + e^{-\phi/2} F)}} \left( \frac{1}{24} \gamma_{ijkl} I - \frac{1}{4} e^{-\phi/2} \hat{F}_{ij} \gamma_{kl} J + \frac{1}{8} e^{-\phi/2} \hat{F}_{ij} \hat{F}_{kl} I \right) .
\]

Note that the denominator is just minus the DBI lagrangian. It is then just a matter of writing out the various terms and verifying that they cancel the ones coming from the variation of the DBI term, thus proving the \( \kappa \)-symmetry of the action \( I = I_{\text{DBI}} + I_{\text{WZ}} \). Since the constraints that go into this calculation can not be relaxed while keeping the system \( \kappa \)-invariant *, we have also shown the equivalence between demanding \( \kappa \)-invariance and imposing the field equations for a D3-brane propagating in a type IIB supergravity background.

Here we make the observation that the projection matrix in (3.14) can be expressed in a more elegant way as

\[
\Gamma \, d^4 \xi = \frac{1}{Z_{\text{DBI}}} \exp \left( e^{-\phi/2} \hat{F} \right) \gamma I ,
\]

where

\[
\gamma \equiv \bigoplus_{i \in \mathbb{N}} \gamma_{(2i)} (-K)^i = \mathbb{I} - \gamma_{(2)} K + \gamma_{(4)} ,
\]

* Although the constraints given in equations (3.8) and (3.13) depend to a certain extent on the choice of conventions, this choice is physically irrelevant.
and it is understood that only terms of form degree four are to be included in the expansion. Considering that the WZ term takes the universal form (3.3) and that this form bears a close resemblance to (3.15), we believe that the latter expression may be relevant for a general formulation of supersymmetric D-branes.

To make this picture complete, we would like to establish contact with the type IIB supergravity formulation of ref. [35]. The torsion components at dimension 0 and $\frac{1}{2}$ are stated in equation (3.8) and the Bianchi identities for the torsion are straightforward to check at this level. The constrained components of the SL(2; $\mathbb{R}$)-invariant three-form field strengths are

$$
\begin{bmatrix}
\tilde{H} \\
\tilde{R}
\end{bmatrix}_{\alpha\beta\gamma} = 
\begin{bmatrix}
2i(\gamma_c K)_{\alpha\beta} \\
-2i(\gamma_c J)_{\alpha\beta}
\end{bmatrix},
\begin{bmatrix}
\tilde{H} \\
\tilde{R}
\end{bmatrix}_{\alpha\beta\gamma} = 
\begin{bmatrix}
-(\gamma_{bc} K)_{\alpha} \\
-(\gamma_{bc} J)_{\alpha}
\end{bmatrix},
$$

(3.17)

and they fulfill the Bianchi identities

$$
D \begin{bmatrix}
\tilde{H} \\
\tilde{R}
\end{bmatrix} + M \begin{bmatrix}
\tilde{H} \\
\tilde{R}
\end{bmatrix} = 0 ,
$$

(3.18)

where $M$ is the connection formed from the scalar field $2 \times 2$ matrix $\mathcal{Y}$ belonging to the group manifold of SL(2; $\mathbb{R}$):

$$
M = \mathcal{Y}^{-1} d\mathcal{Y} = \begin{bmatrix}
P & P' + Q \\
P' - Q & -P
\end{bmatrix} .
$$

(3.19)

The physical scalars belong to the coset SL(2; $\mathbb{R}$)/U(1), and the covariant derivative includes the U(1) part, $Q$, of the connection. The SL(2; $\mathbb{R}$)-invariant three-form field strengths carry U(1) charge two, so the dimension zero components have charge zero. It is consistent with the Maurer–Cartan equation for $M$,

$$
0 = dP + 2Q \wedge P' = DP ,
0 = dP' - 2Q \wedge P = DP' ,
0 = dQ + 2P \wedge P' ,
$$

(3.20)

to make the gauge choice $Q = -P'$, thus making the connection lower-triangular, and this allows one to solve the Maurer–Cartan equations for $P$ and $P'$ in terms of the physical scalars as $P = \frac{1}{2} d\phi$, $P' = \frac{1}{2} e^\phi dC_0$. We have $P'_\alpha = (J\Lambda)_\alpha$. After rescaling $\tilde{H}$ with $e^{\phi} / 2$ and $\tilde{R}$ with $e^{-\phi} / 2$ one arrives exactly at the configuration of equations (3.8) and (3.13) and the Bianchi identities (3.12). When checking the Bianchi identities $dH = 0$, $dR_{(3)} + H \wedge R_{(1)} = 0$ (which are absolutely necessary, considering the way the potentials enter the action above) explicitly, we need to observe that these field strengths carry U(1) charge zero, so the dimension zero components are of charge $-2$. It is a convincing test of the procedure carried out above that it actually dictates the constraints and Bianchi identities for the fields of type IIB supergravity.

Having established the $\kappa$-symmetry of the D3-brane action in an on-shell type IIB supergravity background, it is instructive to return to the case of flat superspace. As mentioned in section 2, the flat limit should reproduce all the $\theta$-extensions introduced in the beginning of that section.
From the only non-vanishing component of the flat torsion, $T_{\alpha\beta} = 2i(\gamma^c)_{\alpha\beta}$, and the definition of the superspace torsion we conclude that one choice of flat vielbeins is

$$E^a_m = \delta^a_m, \quad E^a_\alpha = 0, \quad E^\alpha_\mu = \delta^\alpha_\mu.$$  

(3.21)

This immediately leads to the identification of the one-form $\Pi^a$ used in section 2 with the flat limit of

$$(dZ^M)E^a_M = dX^m \delta^a_m + i\theta \gamma^a \theta = \Pi^a.$$  

(3.22)

In the case of antisymmetric tensor fields we would like to solve the flat constraints on the field strengths for the corresponding flat potential. The definition of the flat NS-NS three-form $H_{ABC}$ in terms of the flat torsion is

$$H_{ABC} = 3\partial\{A_{BC}\} + 3T_{(AB}B_{D)C}.$$  

(3.23)

Here $\{\ldots\}$ refers to graded symmetrizations. The only non-zero component of $H_{ABC}$ in the flat limit is $H_{a\beta\gamma} = 2i(\gamma_aK)_{\beta\gamma}$, which is reproduced using the potential

$$B_{ab} = 0, \quad B_{a\beta} = -i(\gamma_aK)_{\beta}, \quad B_{\alpha\beta} = -(\gamma^a\theta)(\alpha(\gamma_aK)_{\beta}).$$  

(3.24)

Thus, we find that $F_{ij} = F_{ij} - E_j^B E_i^A B_{AB}$ gives exactly the $\theta$-extended $F$ used in section 2. The other field strengths work in a similar way.

4. Conclusions and comments

In this paper we have so far shown that the sum of the DBI and the WZ terms that represents the coupling of the D3-brane to a background four-form potential of type IIB supergravity can be made both $\kappa$-symmetric and supersymmetric in target space in a unique and rather straightforward way. In ref. [48] the bosonic part of the DBI action for the D3-brane was shown to be equivalent on-shell to the BDHT-type action

$$\mathcal{L} = \sqrt{-\gamma} \left\{ -\frac{1}{2} \text{tr}(\gamma^{-1}g) + \sqrt{\left(1 + \frac{1}{4}\text{tr}(\gamma^{-1}\hat{F})^2\right)^2 + \Delta(\gamma^{-1}\hat{F})} \right\}$$  

(4.1)

by using the field equation for the world-volume metric $\gamma$ to algebraically eliminate it. $\Delta$ is given by $\Delta(X) = \det X - \frac{1}{16}(\text{tr}X^2)^2$. This kind of bosonic action was discussed recently for several important cases by Townsend [43], and by Tseytlin [44] who demonstrated that together with the WZ term
the bosonic action exhibits certain duality properties. Furthermore, the square root in the above action can be removed by introducing also a non-propagating scalar on the world-volume:

$$\mathcal{L} = \sqrt{-\gamma} \left\{ -\frac{1}{2} \text{tr}(\gamma^{-1} g) + \varphi \left[ (1 + \frac{1}{4} \text{tr}(\gamma^{-1} \tilde{F})^2)^2 + \Delta(\gamma^{-1} \tilde{F}) \right] + \varphi^{-1} \right\}. \quad (4.2)$$

The $\kappa$-symmetrization of this action follows the same pattern as for the DBI action. In fact, one can easily convince oneself that the solution presented here for the transformation rules work equally well for the BDHT form of the action. It is interesting to note in this context that using the so-called 1.5 order formalism when establishing the invariance of the action, it does not matter whether the matrix $\Gamma$ in the projection matrix contains the super-DBI action in the denominator or the superversion of the BDHT-type action derived in ref. [48]; in any case, the denominator will not be as simple as $\sqrt{-\gamma}$, and the analysis of ref. [48] makes it quite clear that the cases $p = 1, 2$ are special in this respect.

Our results may thus be summarized as follows: the action

$$I_{D3} = \int \sqrt{-\gamma} \left\{ -\frac{1}{2} \text{tr}(\gamma^{-1} \tilde{g}) + \varphi \left[ (1 + \frac{1}{4} \text{tr}(\gamma^{-1} e^{-\frac{2}{\kappa} \tilde{\mathcal{H}}})^2)^2 + \Delta(\gamma^{-1} e^{-\frac{2}{\kappa} \tilde{\mathcal{H}}}) \right] + \varphi^{-1} \right\} + \int e^{\varphi} \tilde{C} \quad (4.3)$$

and all other forms of it obtained by using algebraic world-volume field equations are invariant under the $\kappa$-transformations of section 3, provided the background fields satisfy the IIB supergravity field equations.

We have not explicitly discussed the properties of our D3-brane action under the $\text{SL}(2; \mathbb{Z})$ S-duality group (which is the quantum remainder of the $\text{SL}(2; \mathbb{R})$ above) of the type IIB string theory. Knowing how the self-duality works in the bosonic case [44,56], it should be fairly easy to repeat those calculations with all background fields replaced by the corresponding superfields.

It would also be interesting to study the system under consideration in terms of world-volume instead of space-time supersymmetry. The field content matches that of $N = 4$ super-Maxwell theory (or super-Yang–Mills in the non-abelian case): a gauge potential, four spinors (we envisage that gauge fixing the $\kappa$- and reparametrization symmetries will turn the world-volume scalars $\theta$ into world-volume spinors, as happens for the fermions in the Green–Schwarz superstring), and six scalars (the transverse components of $X$). The DBI action belongs to a very restricted class of non-linear generalizations of the Maxwell theory that upon supersymmetrization does not give rise to propagating auxiliary fields [57], and it is most likely that the maximally extended world-volume theory, should it be quantized, is a finite quantum field theory. It is not obvious that there will be reason to perform such a quantization in a string theory context — being D-branes, these objects do not seem to play the rôle of truly fundamental constituents in any “string theory” vacuum. At the present level of understanding, the actions treated in this paper play a rôle as low-energy effective actions, whose “classical” variation gives the necessary correction to the open
string \( \beta \)-function. We should not be surprised, however, if a more profound understanding of non-perturbative “string theory” would involve something like quantized D-branes.

While this paper deals exclusively with 3-branes, it is clear that the mechanisms at work for other type IIA or IIB Dirichlet branes will be quite analogous, and once the ice is broken the formulation of the corresponding actions is a matter of algebra. We also believe, although this is less obvious, that the additional insights gained here may be used in a broader context, namely for \( p \)-brane actions with antisymmetric tensor supermultiplets \([43]\). Interesting cases that fall into this category are the five-brane in eleven dimensions \([40, 49]\) and perhaps also \( p \)-branes living in target spaces of dimension twelve and higher \([10, 28, 29, 30, 43]\). The generalization of the DBI action to non-abelian vector fields \([58]\) is another obvious issue. Unfortunately, in spite of the fact that there are many possible non-abelian generalizations of the BDHT-type actions given above, not much can be achieved until one understands the technicalities of how to eliminate the world-volume metric from the action. We hope to come back to these questions in the future.

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