ANGULAR SEPARATIONS OF LENSED QUASAR IMAGES

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ABSTRACT

We have analyzed the observed image separations of the gravitationally lensed images of quasars (QSOs) for a possible correlation with the source redshift. Contrary to the previously noted anticorrelation based on a smaller data set, no correlation is found for the currently available data. We have calculated the average image separations of the lensed QSOs as a function of source redshifts for isothermal spheres with cores in a flat universe, taking into account the amplification bias caused by lensing. The shape of the distribution of the average image separation as a function of redshift is very robust and is insensitive to most model parameters. Observations are found to be roughly consistent with the theoretical results for models that assume the lens distribution to be (1) a Schechter luminosity function that, however, cannot produce images with large separation and (2) mass condensations in a cold dark matter universe, as given by the Press-Schechter theory if an upper limit of $1 \times 10^{13} - 7 \times 10^{13} M_\odot$ is assumed on the mass of the condensations.

Subject heading: gravitational lensing

1. INTRODUCTION

The phenomenon of gravitational lensing is extremely useful for understanding the large-scale structure of the universe. Studying the observed properties of the images in individual lenses can provide knowledge of the mass distribution and mass-to-light ratio in these lenses (Grogin & Narayan 1996; Tyson, Kochanski, & Dell’Antonio 1998). A statistical analysis of the observed lens systems, on the other hand, can help us to restrict the values of the cosmological parameters (Falco, Kochanek, & Munoz 1998; Link & Pierce 1998). Based on the frequency of gravitational lensing, upper limits have been placed on the value of the cosmological constant (Fukugita et al. 1992; Kochanek 1996). It recently has been pointed out by Park & Gott (1997, hereafter PG) that the image separations of the observed lenses are strongly negatively correlated with source redshifts, $z_s$, the anticorrelation being much stronger than that predicted by standard cosmologies. PG considered several possible ways to strengthen the anticorrelation. A steeper mass profile, merger of galaxies, and an increase in their mass by in-fall strengthen the anticorrelation. A steeper mass profile, merger of galaxies, and an increase in their mass by in-fall strengthen the anticorrelation.

Williams (1997, hereafter LLRW) demonstrated that the theoretical upper limit to the image separations shows a strong anticorrelation with the source redshift, consistent with the observations, provided (1) the lensing galaxies have logarithmic surface mass densities that gradually change with radius, (2) there is a dispersion in the lensing properties of galaxies like the central surface mass density or velocity dispersion, and (3) the characteristic length scale of dark matter halos of galaxies scales as $L_s$, with $a \approx 0.4$.

More data on lensed quasars (QSOs) have become available since 1997, and the number of lensed QSOs is now roughly twice the number in the PG sample. The PG sample also contained some doubtful lenses. The new data set has to be examined afresh for the presence or absence of the correlation between image separation and source redshift. The main aim of the present paper is to analyze the currently available data to look for a possible correlation. We then want to compare the observed distribution of average image separation as a function of source redshift with the results of a detailed calculation of statistical lensing of galaxies, with a view of obtaining constraints on the values of various parameters entering the calculations. The plan of the paper is as follows: In § 2 we present the analysis of the data using various statistical tests to quantify the presence of any correlation. In § 3 calculations for lensing statistics are presented, in particular of the average image separation as a function of source redshift, for a flat universe, realistic lens distributions (taking into account the amplification bias), and different cosmological models. In §§ 4 and 5 we present the results and conclusions, respectively.

2. IS THERE A CORRELATION?

The anticorrelation noted by PG is, as observed by them, mainly due to six low-redshift ($z_s \leq 2.15$), large-separation ($\Delta \theta > 4''$) images, namely, 0240–343, 0957+561, 1120+019, 1429−008, 1634+267, and 2345+007. Out of these, only one (0957+561) was a confirmed lens. Since then, Kochanek, Falco, & Munoz (1999) have argued against the lensed nature of most of the wide-separation QSO pairs (hereafter WSQPs). Studying the optical and radio properties of these pairs, they concluded that all of the WSQPs with $\Delta \theta$ between $3''$ and $10''$ are binary quasars rather than gravitationally lensed images with a one-sided 2 $\sigma$ (1 $\sigma$) upper limit of 22% (8%) on the lens fraction of these QSO pairs. They explained the high incidence of occurrence of binary QSOs (which is 2 orders of magnitude higher than that given by the quasar-quasar correlation function) as being due to the enhanced quasar correlation function in the merger of galaxies. No lensing galaxy could be found in front of these WSQPs so if they are lensed images, the lensing mass, although of the magnitude of a cluster, has to be completely invisible. Peng et al. (1999) have searched for a lensing galaxy in front of Q1634+267 at the lens optical wavelength using the near-infrared camera and multiobject spectrometer (NICMOS) and showed that the lens has to...
have $M/L \geq 690 h_{65}^{-1} (1200 h_{65}^{-1})$ for $\Omega = 0.1 (1.0)$ and $H_0 = 65 h_{65} \text{ km s}^{-1} \text{ Mpc}^{-1}$. They therefore suggest that the double “images” may be binary QSOs rather than multiple images of a single QSO. Very similar spectra of the two images, however, defy this conclusion. Peng et al. (1999) have compared the spectral similarities of 14 pairs of QSOs with separations between 3" and 10" that are having very similar redshifts (with the velocity differences between pairs being $\leq 500 \text{ km s}^{-1}$) with the spectral similarities of randomly chosen QSO pairs from the Large Bright Quasar Survey. They conclude that there is a $\leq 3\%$ probability that a randomly drawn sample of 14 QSO pairs would show spectra as similar as those of the observed pairs. So unless a viable theory to explain the similarities of the spectra of QSOs in merging galaxies can be developed, one cannot discard the lens hypothesis for the WSQPs.

New data on lensed QSOs have become available since 1997. Forty-nine confirmed or likely lenses have been compiled by the CfA/Arizona Space Telescope Lens Survey (CASTLES), out of which the source redshift is available for 39 QSOs. These include 15 lenses from the PG sample. For reasons stated above, we may combine these with the five WSQPs from the PG sample (0240, 1120, 1429, 1634, and 2345) to get an extended sample of 44 lenses. We have considered four separate samples for our analysis, namely, (1) PG20: the sample used by PG of 20 QSOs; (2) PG15: the PG sample without the five WSQPs; (3) CAST39: the CASTLES sample of 39 lenses; and (4) EXT44: the extended sample containing 39 CASTLES lenses and five WSQPs. The angular separation versus source redshift for these samples is plotted in Figures 1a–1d, respectively. These samples were searched for the presence of correlation by performing a Spearman rank correlation test. A Kolmogorov-Smirnov (KS) test also was performed to determine if the image separations for sources with redshifts smaller than 2.5 and sources with redshifts greater than 2.5 were taken from the same distribution. We also obtained the best straight-line fits to the unbinned data of the four samples, which are plotted in the figure. The results are shown in Table 1. It is clear that PG20 shows highly significant anticorrelation between $\Delta \theta$ and $z_s$. The anticorrelation weakens but persists at about a 1.8 $\sigma$ level after the removal of the five WSQPs (PG15). CASTLES data, on the other hand, do not show any correlation. Adding the five WSQPs to CASTLES data (EXT44) gives rise to anti-correlation, but it is very weak and is statistically insignificant. A KS test also shows that for the EXT44 sample, the probability is quite large that the lens separations for sources with redshift smaller and larger than 2.5 are taken from the same distribution. We thus conclude that the

\[\text{FIG. 1.—Image separation vs. source redshift for four samples: (a) PG data of 20 QSOs, (b) PG data without the five WSQPs, (c) CASTLES data of 39 QSOs, (d) CASTLES data with the five WSQPs. Triangles represent the WSQPs. The best-fit straight line for each data set is plotted.}\]
strong anticorrelation noted by PG was mainly due to the inclusion of five questionable QSOs that formed a large fraction of about 25% of their sample. The present data set of 39 confirmed lenses does not show any correlation by itself and even after the inclusion of the five questionable lenses, which now form only 11% of the sample.

3. DETAILS OF CALCULATIONS

As seen above, no correlation exists between the image separations and the source redshifts, and it seems possible that results of the standard theoretical models may be consistent with the observations without the need of any drastic assumptions. It is possible that we may be able to constrain some of the parameters entering the calculations. In this section we present the details of our calculation of lensing statistics, which differ from previous calculations only in the way the amplification bias has been taken into account.

We have assumed the mass distribution of the lenses to be that of iso-thermal spheres with cores, as found to be consistent with the observations without the need of any drastic assumptions. It is possible that we may be able to constrain some of the parameters entering the calculations. In this section we present the details of our calculation of lensing statistics, which differ from previous calculations only in the way the amplification bias has been taken into account.

We have assumed the mass distribution of the lenses to be that of iso-thermal spheres with cores, as found to be required by Maoz & Rix (1993) in order to explain the lensing events in the Hubble Space Telescope snapshot survey. We have not taken into account the effects of ellipticity in the lensing galaxies. Since the ellipticity mainly affects the relative numbers of double and quadruple lenses (Keeton, Kochanek, & Saljak 1997), it will not affect our results. For the lens distribution as a function of mass and redshift, we consider the Schechter luminosity function of galaxies and the mass condensations in a cold dark matter (CDM) universe obtained using the Press-Schechter theory (hereafter referred to as the PS distribution). The former is given by

\[ \Phi(L) dL \propto \left( \frac{L}{L^*} \right)^{\alpha} e^{-L/L^*} dL, \]  

with \( \Phi(L) \) being the number of galaxies with luminosity \( L \) per unit of comoving volume. Here \( \alpha = -1.1 \) (Marzke et al. 1998). We have assumed that the comoving number density of galaxies is independent of redshift. Thus, the normalization constant in the luminosity function does not affect the calculation presented below. The circular velocity dependence of the luminosity is taken to be \( L/L^* = (\sigma/\sigma_*)^4 \) with \( \sigma_* = 225 \text{ km s}^{-1} \) for elliptical galaxies (Faber & Jackson 1976; de Vaucouleurs & Olson 1982; Kochanek 1994) and \( L/L^* = (\sigma/\sigma_*)^{2.6} \) with \( \sigma_* = 144 \text{ km s}^{-1} \) for spiral galaxies (Tully & Fisher 1977; Fukugita & Turner 1991), which are assumed to be 70% of all the galaxies (Postman & Geller 1984). The PS distribution is given by

\[ n(v_c, z) dv_c = -\frac{3(1.67)^3}{(2\pi)^{3/2} v_c^{3/2}} \frac{\delta_z H_0^2 (1 + z)^{5/2}}{\Delta(r_0)} \times \frac{d\ln \Delta}{d\ln v_c} \exp \left[ -\frac{\delta^2 (1 + z)^2}{2\Delta^2 (r_0)} \right] dv_c. \]  

Here \( n(v_c, z) \) is the number density of mass condensations with circular velocity \( v_c \) at redshift \( z \), \( \delta_z = 1.68 \), and the functional form of \( \Delta(r_0) \) for the CDM power spectrum of density perturbation is

\[ \Delta(r_0) = 16.3b^{-1}(1 - 0.3909r_0^{0.1} + 0.4814r_0^{0.2})^{-10}, \]  

where \( b \) is the bias parameter. The mass and circular velocity of a halo are related to the comoving radius \( r_0 \) and redshift \( z \) by

\[ M = \frac{4\pi}{3} \rho_0 r_0^3, \quad v_c = 1.67(1 + z)^{1/2} H_0 r_0, \]  

with \( \rho_0 \) being the mean density of the universe. The density fluctuation amplitudes are taken from \( N \)-body simulation work (Narayan & White 1988; Mo, Miralda-Escude, & Rees 1993).

We consider several flat-world models with different values of the cosmological constant. Following Fukugita et al. (1992), we use the angular diameter distances between the lens and the observer \((D_{\odot}L)\), between the source and the observer \((D_{\odot}S)\), and between the lens and the source \((D_{LS})\) and take the critical impact parameter for SIS lens to be

\[ a_{cr} = 4\pi \left( \frac{\sigma_{||}}{c} \right)^2 \frac{D_{OL}D_{LS}}{D_{OS}}, \]  

with \( \sigma_{||} \) being the one-component velocity dispersion for the lens equal to \( v_c/\sqrt{2} \). The angular diameter distance formulae for various types of world models are taken from Fukugita et al. (1992). The lensing cross section for ISC is given by (Hinshaw & Krauss 1987), \( \sigma = \pi l_0^2 \), \( l_0 \) being the maximum impact parameter for lensing, given by

\[ l_0 = [(a_{cr}^2 + 5a_{cr} r_c - 0.5r_c^2) - 0.5r_c^{1/2}(r_c + 4a_{cr})^{3/2}]^{1/2}, \]  

with \( r_c \) being the core radius of the lens mass distribution. The amplification of an image is obtained from \( \lambda = (b/l)(db/dl) \). Here \( b/l \) is the impact parameter and \( b \) the image position in the lens plane obtained by solving the lens equation

\[ b^3 + 2bl^2 + b(l^2 + 2a_{cr} r_c - a_c^2) + 2la_{cr} r_c = 0. \]  

The average image separation for a given value, \( z_s \), is given by

\[ \langle \Delta \theta \rangle = \int_{z_s}^{z_f} \int_{v_c}^{v_s} \int_{l}^{l_s} n_l(z_s, z_l, v_s, v_c, M_{lim}^{(l)}) \Delta \theta(z_s, z_l, v_s, v_c, l) \, dl \, dv_c \, dv_s \, dz_l. \]  

Here \( \Delta \theta(z_s, z_l, v_s, v_c, l) \) is the separation between the two brighter images produced by a lens at \( z_l \) with circular velocity \( v_c \) and impact parameter \( l \), and \( n_l(z_s, z_l, v_s, v_c, M_{lim}^{(l)}) \) is the number of observable sources (with \( m_{obs} < M_{lim}^{(l)} \)) at the redshift \( z_s \), which will be lensed by galaxies at a redshift \( z_l \) with circular velocity \( v_c \) and impact parameter \( l \).
velocity $v_c$ and impact parameter $l$, such that both of the brighter images will have a luminosity higher than that for $M_{B}^{\lim}$. This can be obtained by multiplying the QSO luminosity function by the optical depth for lensing and integrating over the observable magnitude interval. This can be written as

$$n_d(z_s, z_l, M_{B}^{\lim}) = \int_{M_{B} = -\infty}^{M_{B}^{\lim} + 2.5 \log A} dM_B \phi(M_B) \frac{\partial^3 \tau(z_s, z_l, v_c, l)}{\partial z_l \partial v_c \partial l}.$$  

Here $M_{B}^{\lim}$ is the absolute magnitude of a source at $z_s$ corresponding to the limiting apparent magnitude $m_{B}^{\lim}$ of the survey, assumed here to be 18. The results are quite insensitive to the value of $m_{B}^{\lim}$. The amplification $A(z_s, z_l, v_c, l)$ being that of the weaker of the two bright images, and $\phi(M_B)$ is the QSO luminosity function, which is taken from Wallington & Narayan (1993). The variable $\frac{\partial^3 \tau(z_s, z_l, v_c, l)}{\partial z_l \partial v_c \partial l}$ is the optical depth for lensing of a source at $z_s$ by lenses with velocity dispersion $v_c$ at redshift $z_l$ with impact parameter $l$. This is given by

$$\frac{\partial^3 \tau(z_s, z_l, v_c, l)}{\partial z_l \partial v_c \partial l} = n_d(v_c, z_l) 2\pi c \frac{dt}{dz_l},$$

with $n_d(v_c, z_l)$ being the number density of the lenses with circular velocity $v_c$ at $z_l$ and $2\pi c dt$ being the cross section for lensing for impact parameters between $l$ and $dl$.

The unnormalized probability of lensing for a given image separation $\Delta \theta$ can be computed from

$$\frac{\partial p(z_s, \Delta \theta)}{\partial \Delta \theta} = \int_{0}^{z_s} dz_l \int_{0}^{l_0} dl m_d(z_s, z_l, M_{B}^{\lim}, l, \Delta \theta),$$

where

$$m_d(z_s, z_l, M_{B}^{\lim}, l, \Delta \theta) = \int_{M_{B} = -\infty}^{M_{B}^{\lim} + 2.5 \log A} dM_B \phi(M_B) \frac{\partial^3 \tau(z_s, z_l, l, \Delta \theta)}{\partial z_l \partial \Delta \theta \partial l}.$$  

is the observable number of QSOs (obtained as above by multiplying the QSO luminosity function and the optical depth of lensing and integrating over the observable magnitude range) lensed by lenses at redshift $z_l$ with impact parameter $l$ producing image separation $\Delta \theta$, the amplification $A(z_s, z_l, l, \Delta \theta)$ being that of the weaker of the two bright images with image separation $\Delta \theta$, and

$$\frac{\partial^3 \tau(z_s, z_l, l, \Delta \theta)}{\partial z_l \partial \Delta \theta \partial l} = n_d(v_c, z_l) 2\pi c \frac{dt}{d\Delta \theta} \frac{c dt}{dz_l},$$

with $v_c$ being the circular velocity of lenses at $z_l$, which will yield the image separation $\Delta \theta$ for impact parameter $l$. For this calculation, we have ignored the dependence of image separation on the impact parameter, which is very weak (Hinshaw & Krauss 1987), and have used the value of image separation by the ISCs at the zero-impact parameter. We have verified that this assumption does not lead to errors larger than 1%. This is given by

$$\Delta \theta = 2 \frac{a_{cr}}{D_{DL} \left(1.0 - 2 \frac{r_c}{a_{cr}}\right)^{1/2}}.$$  

The value of $a_{cr}$ for a given $\Delta \theta$ obtained from the above equation is used to obtain the necessary value of $v_c$ as

$$v_c = c \left(\frac{a_{cr} D_{OS}}{2\pi D_{OL} D_{LS}}\right)^{1/2}.$$  

4. RESULTS AND DISCUSSION

We have plotted in Figure 2a the unnormalized differential probability $\partial p(z_s, \Delta \theta)/\partial \Delta \theta$ as a function of image separation for the two lens distribution functions for several redshifts. We have used a value of 0.2 kpc for the core radius for these calculations. This is the upper limit obtained by Wallington & Narayan (1993) from the observed absence of central images for the lensed QSOs. Models of individual lenses imply somewhat lower core radii (Kochanek 1995). An upper limit of 1.4 kpc has been obtained from the results of N-body simulations of the gravitational collapse of density peaks by Dubinski & Carlberg (1991). The slope of the lens distribution (as a function of luminosity or circular velocity) for the Schechter luminosity function is steeper than that for the PS distribution. As a result, the probability distribution is broader for the PS distribution compared to that for the Schechter distribution. The slope of the probability distribution for each lens distribution is, however, almost independent of the source redshift for $\Delta \theta > 0.4$, which indicates that the average value of $\Delta \theta$ may not be very sensitive to $z_s$.

Figures 2b and 2c show the effect of varying the values of various parameters on the differential probability distribution. Figure 2b shows that increasing the core radius suppresses the probability for small values of separations (curves 2, 3, 4, and 5). Again, however, the slope of the distribution is independent of the value of the core radius for $\Delta \theta > 0.3$ for $r_c < 0.2$ kpc. The image separation increases with the increase in core radius as the probability for small separations is suppressed (Hinshaw & Krauss 1987). However, the effect of including the amplification bias (which increases the probability for small values of $\Delta \theta$, as can be seen by comparing curves 3 and 6) cancels the effect of the increase in the core radius, as has been noted by Hinshaw & Krauss (1987). This results in the probability distribution being almost independent of $r_c$ for $\Delta \theta > 0.3$ for the assumed range of $r_c$ values. Higher values of the core radius change the probability distribution more significantly for larger values of separations. The effect of increasing the bias parameter is to reduce the probability for large values of $\Delta \theta$, as seen by comparing curves 1 and 3. In Figure 2c we have plotted the probability distribution for three cosmological models, including the cosmic accordance model proposed by Ostriker & Steinhardt (1995). For PS distribution, the peak shifts to lower $\Delta \theta$ values with an increase in $\Lambda$.
cutoff on the mass of the condensations. The probability distribution for a cutoff of 600 km s$^{-1}$ is shown in Figure 2a for comparison. As is expected, the absence of large masses reduces the probability for large separations drastically. Porciani & Madau (2000) have shown that the probability of lensing is consistent with observations if one assumes that the condensations with mass $\geq 3.5 \times 10^{13} M_\odot$ have nonsingular mass distribution. However, as explained above, for $r_c \leq 0.2$ kpc, the effect of the finite core radius is partly compensated for by the amplification bias for $\Delta \theta > 0^\prime.3$, and it is not possible to suppress the probability for separations greater than $4^\prime$. A cutoff on the mass of condensation therefore is needed to reduce the probability for these separations.

Figure 3a shows that the $\langle \Delta \theta \rangle$ values are almost independent of the source redshift. The decrease of $\langle \Delta \theta \rangle$ with $z_s$ is somewhat more pronounced for the assumption of PS distribution for small values ($\leq 2$) of $z_s$. The decrease is larger when the distance formulae for empty beams are used compared to that for the case of a filled beam (curves 1 and 2). The values of $\langle \Delta \theta \rangle$ are sensitive to the values of the upper limit on the circular velocity of the condensations used in the calculation. An increase in the upper limit from 750 to 1000 km s$^{-1}$ increases the values of $\langle \Delta \theta \rangle$ by a factor of $\geq 1.5$ (curves 2 and 4). Change in the bias parameter (curves 4 and 6) again changes the absolute values but does not alter the slope of the distribution. As expected from Figure 2b, the values of $\langle \Delta \theta \rangle$ are almost independent of the core radius (curves 4 and 5). We thus see that the observed image separation as a function of redshift (as seen from the best-fit line) is in reasonable agreement (in view of the uncertainties of the best fit given in Table 1) with the theo-
retical results for an upper limit on circular velocity of about 600–750 km s\(^{-1}\) for \(b = 1\) for PS distribution. This value of course is not absolute since higher values will be needed for higher bias parameters. Use of a Schechter luminosity function, however, produces considerably lower values of \(\langle \Delta \theta \rangle\).

In Figure 3b we have shown the results for different assumptions about the change in the core radius with redshift and with circular velocities of the condensations as suggested by LLRW. Assuming \(r_c \propto v_c^{0.5}, 1.0, 2.0\) (curves 4, 5, and 6) does not change the results significantly. Similarly, the assumption of a redshift dependence of \(r_c \propto (1 + z)^{-2}\) (curves 1 and 2) also changes the results only by a small amount. This is expected in view of the very weak dependence of \(\langle \Delta \theta \rangle\) on core radius. Thus, the observed absence of correlation is consistent with the results for a flat universe even if the core radius varies with luminosity contrary to the suggestion of LLRW. In Figure 3c we have plotted the results for different cosmologies. Values of \(\langle \Delta \theta \rangle\) decrease with an increase in \(\Lambda\).
The results presented above show that the average image separation is not very sensitive to (1) the values of core radii, (2) different assumptions about the dependences of core radii on luminosity or redshift, and (3) the values of the cosmological constant between 0 and 0.65. The values are sensitive to the upper limit used for the circular velocity for the PS distribution. For $b = 1$, core radii between 0 and 0.2 kpc, and a cosmological constant between 0 and 0.65, an upper limit of about 600–750 km s$^{-1}$ on the circular velocities of the dark matter halos for the PS distribution is roughly consistent with the observations of image separations. It may be noted that the observed values of image separations do show considerable scatter at any given source redshift. A large scatter is expected from the flat probability distributions as a function of $\Delta \theta$ (Fig. 2). The differential probability of lensing as a function of angular separation cannot be directly compared with the observations because of the small number of observed lenses at any given redshift. However, if we take all the observed lenses together, irrespective of their redshifts, then we see that six of 49 lenses in the full CASTLES data and 11 of 54 in the extended data including the WQPs have image separation between 3" and 8". This requires the ratio of probabilities for separations between 3" and 8" to that for separations between 0.3" and 3" to be 0.14 and 0.26, respectively, for the two samples. We have calculated this probability ratio to be 0.62, 0.35, and 0.15 for upper limits of 750, 600, and 500 km s$^{-1}$, respectively, for the PS distribution for $b = 1$, $\Omega = 1$, and $r_c = 0.2$ kpc. The ratio is 0.04 for the Schechter function.

Keeton, Christlein, & Zabludo† (2000) have considered the detailed galactic luminosity function dependent on type and environment (Bromley 1998a, 1998b). They obtained a good match between their calculated probability and the image separation distribution for 49 QSO lenses in the CASTLES data. We have calculated the probability distribution and the average image separation using the luminosity function used by them. The results are shown in Figures 2c and 3c. The probability peaks at a higher value of image separation, and as a result the values of $\langle \Delta \theta \rangle$ are higher and can be considered to be in agreement with the CAST39 sample. However, we note that even for this luminosity function it is not possible to get values of image separations larger than 6". The ratio of probabilities for separations between 3" and 8" to that for separations between 0.3" and 3" is 0.12.

5. Conclusions

We have analyzed the observed image separations of lensed QSOs for a possible correlation with the source redshift. A correlation was noted earlier for the then available data by PG. The present data of 39 confirmed or likely lensed QSOs, even when combined with five wide-separation QSO pairs that are doubtful lenses, do not show any statistically significant correlation. LLRW had shown that if the core radii scale with a luminosity of $L^{0.4}$, then an anticorrelation is expected theoretically because of the presence of dispersion in the lensing properties of galaxies. We have calculated the average image separations of the lensed QSOs as a function of source redshifts for isothermal spheres with cores in a flat universe, taking into account the amplification bias caused by lensing. We do not find the strong anticorrelation stipulated by LLRW. In fact, the shape of the distribution of average image separation as a function of source redshift is very robust and is insensitive to the change in parameters. As a result, we are unable to obtain meaningful constraints on any of the parameters. The assumption of the Schechter luminosity function is unable to produce separations larger than 6". The use of PS distribution, on the other hand, yields a large number of wide-separation lenses even for a nonzero core radius and necessitates the assumption of a cutoff on the mass of the condensations or large values of core radii for condensations with large masses. It may be noted that the PS distribution also has been found to be remarkably successful in explaining the observed distributions of the QSO absorption lines (Das & Khare 1999).

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