Fluxbranes Delaying Brane-Antibrane Annihilation

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Abstract

By intersecting the $RR$ charged $D_p - D_p$ pair ($p = 6, 4, 2, 0$) with the $RR F^7$-brane and by intersecting the $NSNS$ charged $F1 - F1$ and $NS5 - NS5$ pairs with the $NSNS F6$-branes, the possibility of stabilizing the brane-antibrane systems is considered. The behavior of the corresponding supergravity solutions indicates that the $RR F^7$-brane content of the solution plays the role of keeping the brane and the antibrane from annihilating each other completely since the two-brane configuration structure still persists in the vanishing inter-brane distance limit of the supergravity solution. In terms of the stringy description, we interpret this as representing that the $RR F^7$-brane “delays” the brane-antibrane annihilation process but only until this non-supersymmetric and hence unstable $F^7$-brane itself decays. Then next, the behavior of the supergravity solutions representing $F1 - F1$ and $NS5 - NS5$ again for vanishing inter-brane separation reveals that as they approach, these “$NS$”-charged brane and antibrane always collide and annihilate irrespective of the presence or the absence of the $NSNS F6$-brane. And we have essentially attributed this to the absence of (open) stringy description of the instability in the “$NS$”-charged case. This interpretation may provide a resolution to the contrasting features between the instability of “$R$”-charged brane-antibrane systems and that of “$NS$”-charged ones. Certainly, however, it poses another puzzle that in the “$NS$”-charged case, the quantum entity, that should take over the semi-classical instability as the inter-brane distance gets smaller, is missing. This is rather an embarrassing state of affair that needs to be treated with great care.

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1 Introduction

In the present work, we would like to address the issue of possible flux stabilization of the unstable, non-BPS $D_p - \bar{D}_p$ system. Thus it might be relevant to first remind why the $D_p - \bar{D}_p$ system is unstable to begin with by resorting to a simple argument that goes as follows. Consider a system consisting of a certain number $N$ of coincident $D_p$-branes separated by some distance from a system of $N$ coincident $\bar{D}_p$-branes, for simplicity, in flat $\mathbb{R}^{10}$. This system differs from the BPS system of $2N$ $D_p$-branes by the orientation reversal on the antibranes. In this system, the branes and the antibranes each break a different half of the original supersymmetry and the whole configuration is non-supersymmetric or non-BPS and hence is unstable. As a result, there is a combined gravitational and (RR) gauge attractive force between the branes and the antibranes at some large but finite separation leading to the semi-classical instability. At the separation of order the string scale, $\sim \sqrt{\alpha'} = l_s$, in particular, the open string connecting a $D_p$-brane to a $\bar{D}_p$-brane becomes tachyonic. What then would be the eventual fate or endpoint of this unstable $D_p - \bar{D}_p$-system? According to Sen [3], the endpoint could be the supersymmetric vacuum via the open string tachyon condensation. To be a little more concrete, in the $D_p - \bar{D}_p$ annihilation process, the open string tachyon behaves as a Higgs field and condenses to a minimum of its potential, breaking the worldvolume gauge symmetry to its diagonal subgroup,

$$U(N) \times U(N) \rightarrow U(N).$$

Now, if the outcome of this brane-antibranes annihilation were, as advocated by Sen [3], the supersymmetric vacuum, the residual gauge symmetry, i.e., the diagonal subgroup $U(N)$, should also disappear, presumably by the process suggested by Sen [3] or by Yi [4]. Regarding the conjectures on the possible endpoints of the unstable $D_p - \bar{D}_p$ systems, it is interesting to note that there are some suggestions on the obstructions against complete annihilation of the $D_p - \bar{D}_p$ system. One is the argument that due to the topological difference between the Chan-Paton bundle $E$ carried by the $D_p$-branes and $F$ carried by $\bar{D}_p$-branes, the endpoint could be a lower-dimensional D-brane instead. The other is the suggestion that endpoint could be a stable $D$-brane as a topological defect (soliton) arising in the worldvolume Higgs mechanism (i.e., gauge symmetry breaking) $U(N) \times U(N) \rightarrow U(N)$, namely the tachyon condensation, classified by the homotopy group

$$\Pi_{2n-1}(U(N)) = \mathbb{Z}, \quad \Pi_{2n}(U(N)) = 0,$$
with $N$ in the stable regime [2]. Having been convinced of the generic instability of the $D_p - \bar{D}_p$ system which is quantum (in terms of tachyon condensation) for very small brane-antibrane separation and is semi-classical in nature for large but finite separation, in the present work, we would like to discuss the possibility of stabilizing the brane-antibrane systems by intersecting the $RR$ charged $D_p - \bar{D}_p$ pair ($p = 6, 4, 2, 0$) with the $RR F_7$-brane [3, 4, 7] (which will be denoted henceforth by $(D_p - \bar{D}_p)||F7$) and intersecting $NSNS$ charged $F1 - F1$ and $NS5 - \bar{NS5}$ pairs with the $NSNS F6$-branes [7] (which similarly will be denoted by $(F1 - \bar{F}1)||F6$ and $(NS5 - \bar{NS5})||F6$ respectively). Since it is the $D6$ (or $\bar{D}6$) brane which has non-trivial coupling to the magnetic flux of $RR F7$-brane and the $NS5$ (or $\bar{NS5}$) brane that couples directly to that of the $NSNS F6$-brane, one might naturally expect that only the $D6 - \bar{D}6$ and $NS5 - \bar{NS5}$ systems, but no others, would be balanced, when the $RR F7$-brane and the $NSNS F6$-brane are intersected respectively, in an (unstable) equilibrium against the combined gravitational and gauge attractions. Within the context of analysis based on the explicit supergravity solutions, however, things turn out not to be so transparent. As we shall see in a moment, although the $RR F7$-brane fails to serve to stabilize the $D_p - \bar{D}_p$ pairs ($p = 4, 2, 0$) against the collapse for large but finite separation, when the branes and the antibranes are brought close enough together, the fluxbranes play the role of keeping them from merging and then annihilating each other completely. And the resolution to this apparent puzzle lies in the validity of the semi-classical supergravity description of the system. Namely, as the inter-brane separation gets smaller and smaller, say, towards the order of string scale $\sim \sqrt{\alpha'} = l_s$, the supergravity description becomes no longer reliable and the semi-classical instability should be replaced by the quantum, stringy instability expressed in terms of the open and closed string tachyon condensation. The conclusion we shall draw is that, if stated briefly, the $RR F7$-brane simply delays the annihilation process of the $D_p - \bar{D}_p$ systems only until the unstable $F7$-brane itself decays toward either a supersymmetric string vacuum or the nucleation of the $D6 - \bar{D}6$ pairs via the brany Schwinger process [3]. And here, we noticed the facts that firstly, the $F7$-brane breaks all the supersymmetries and hence should be unstable and decay [3, 8] and secondly, $D_{(p-1)} - \bar{D}_{(p-1)}$ pairs can generally be created from the $RR F_p$-brane background via the brane-version of Schwinger process. And eventually, the $RR$ fluxbrane can never eliminate the instability of the brane-antibrane system completely and hence the endpoint of these $(p|(D_p - \bar{D}_p), F7)$ systems ($p = 4, 2, 0$) would be either the supersymmetric vacuum or lower-dimensional branes arising as a result of topological obstruction argument given earlier. The $D6 - \bar{D}6$ system which stands out as a unique case, however, would be supported by the $F7$-brane against collapse. But again, this
would be true only within the time scale for the decay of $F7$-brane and once the $F7$-brane decays presumably leaving a supersymmetric closed string vacuum behind, even $D6$ and $\bar{D}6$ would collide and annihilate each other leaving yet another supersymmetric vacuum behind.

2  

**D — anti — D systems supported by RR fluxbrane**

In this section, we shall consider the intersection of non-BPS $D_{2p} - \bar{D}_{2p}$ systems with the magnetic $RR F7$-brane in IIA theory in order to study the role played by the $RR F7$-brane concerning the semi-classical and quantum (in terms of open string tachyon condensation) instability of the $D_{2p} - \bar{D}_{2p}$ systems. We first begin with the exact supergravity solutions representing the $D6 - \bar{D}6$ system and the intersection of this $D6 - \bar{D}6$ with a magnetic $RR$ flux 7-brane ($(D6 - \bar{D}6) || F7$ for short). The former possesses conical singularities which can be made to disappear by introducing the $RR$ magnetic field (i.e., the $RR F7$-brane) and properly tuning its strength in the latter. In order to demonstrate this, we need to along the way perform the M-theory uplift of the $D6 - \bar{D}6$ pair which leads to the Kaluza-Klein ($KK$) monopole/anti-monopole solution ($KK - dipole$ henceforth) first discussed in the literature by Sen [7] by embedding the Gibbons-Perry [8] $KK - dipole$ solution in $D = 11$ M-theory context. Indeed the $D6$-brane solution is unique among $D_{p}$-brane solutions in IIA/IIB theories in that it is a codimension 3 object and hence in many respects behaves like the familiar abelian magnetic monopole in $D = 4$. This, in turn, implies that the $D6 - \bar{D}6$ solution should exhibit essentially the same generic features as those of Bonnor’s magnetic dipole solution [9, 10] and its dilatonic generalizations [11, 10] in $D = 4$ studied extensively in the recent literature. As we shall see in a moment, these similarities allow us to envisage the generic nature of instabilities common in all unstable $D_{p} - \bar{D}_{p}$ systems in a simple and familiar manner. Then next, we consider the exact supergravity solutions representing the electrically $RR$-charged $D0 - \bar{D}0$ system and the intersection of this $D0 - \bar{D}0$ with a magnetic $RR$ flux 7-brane. This last system as well as $(D2 - \bar{D}2) || F7$ and $(D4 - \bar{D}4) || F7$ systems exhibit rather puzzling features and we attempt to provide resolutions to them later on.

2.1  

**$D6 - \bar{D}6$ pair supported by RR $F7$-brane**

(A)  

**$D6 - \bar{D}6$ pair in the absence of the magnetic field**

In string frame, the exact IIA supergravity solution representing the $D6 - \bar{D}6$ pair is
given by \[\text{(2)}\]

\[
d s_{10}^2 = H^{-1/2}[-\dot{t}^2 + \sum_{i=1}^{6} d x_i^2] + H^{1/2}[(\Delta + a^2 \sin^2 \theta) \left(\frac{d r^2}{\Delta} + d \theta^2\right) + \Delta \sin^2 \theta d \phi^2],
\]

\[
e^{2\phi} = H^{-3/2},
\]

\[
A_{[1]} = \left[\frac{2mra \sin^2 \theta}{\Delta + a^2 \sin^2 \theta}\right] d \phi, \quad F_{[2]} = dA_{[1]}
\]

where the harmonic function in 3-dimensional transverse space is given by

\[
H(r) = \frac{\Sigma}{\Delta + a^2 \sin^2 \theta}
\]

and \(\Sigma = r^2 - a^2 \cos^2 \theta, \quad \Delta = r^2 - 2mr - a^2\) with \(r\) being the radial coordinate in the transverse directions. The parameter \(a\) can be thought of as representing the separation between the brane and antibranes (we will elaborate on this shortly) and changing the sign of \(a\) amounts to reversing the orientation of the brane pair, so here we will choose, without loss of generality, \(a \geq 0\). \(m\) is the ADM mass of each brane and the ADM mass of the whole \(D6 - \bar{D}6\) system is \(M_{\text{ADM}} = 2m\) which should be obvious as it would be the sum of ADM mass of each brane when they are well separated. It is also noteworthy that, similarly to what happens in the Ernst solution \[\text{(13)}\] in \(D = 4\) Einstein-Maxwell theory describing a pair of oppositely-charged black holes accelerating away from each other (due to the Melvin magnetic universe content), this \(D6 - \bar{D}6\) solution in IIA theory is also static but \textit{axisymmetric} in these Boyer-Lindquist-type coordinates. As has been pointed out by Sen \[\text{(7)}\] in the M-theory \(KK\text{-dipole}\) solution case and by Emparan \[\text{(10)}\] in the case of generalized Bonnor’s solution, the IIA theory \(D6 - \bar{D}6\) solution given above represents the configuration in which a \(D6\)-brane and a \(\bar{D}6\)-brane are sitting on the endpoints of the dipole, i.e., \((r = r_+, \theta = 0)\) and \((r = r_+, \theta = \pi)\) respectively where \(r_+\) is the larger root of \(\Delta = 0\), namely \(r_+ = m + \sqrt{m^2 + a^2}\). Next, we turn to the conical singularity structure of this \(D6 - \bar{D}6\) solution. First observe that the rotational Killing field \(\psi_{\mu} = (\partial/\partial \phi)^{\mu}\) possesses vanishing norm, i.e., \(\psi_{\mu} \psi_{\mu} = g_{\alpha\beta} \psi^\alpha \psi^\beta = g_{\phi\phi} = 0\) at the locus of \(r = r_+\) as well as along the semi-infinite lines \(\theta = 0, \pi\). This implies that \(r = r_+\) can be thought of as a part of the symmetry axis of the solution. Namely unlike the other familiar axisymmetric solutions, for the case of the \(D6 - \bar{D}6\) solution under consideration, the endpoints of the two semi-axes \(\theta = 0\) and \(\theta = \pi\) do not come to join at a common point. Instead, the axis of symmetry is completed by the segment \(r = r_+\). And as \(\theta\) varies from 0 to \(\pi\), one moves along the segment from \((r = r_+, \theta = 0)\) where \(D6\) is situated to \((r = r_+, \theta = \pi)\) where \(\bar{D}6\) is placed. Then the natural question to be addressed is whether or not the conical singularities arise on different portions of the symmetry axis. This situation
is very reminiscent of the conical singularity structure in the generalized Bonnor’s dipole solution in Einstein-Maxwell-dilaton theory in $D = 4$ extensively studied by Emparan \cite{Emparan2003} recently. Thus below, we explore the nature of possible conical singularities in this $D_6 \pm \bar{D}_6$ solution in $D = 10$ type IIA theory following essentially the same avenue as that presented in the work of Emparan \cite{Emparan2003}. Namely, consider that ; if $C$ is the proper length of the circumference around the symmetry axis and $R$ is its proper radius, then the occurrence of a conical angle deficit (or excess) $\delta$ would manifest itself if $(dC/dR)|_{R\rightarrow 0} = 2\pi - \delta$. We now proceed to evaluate this conical deficit (or excess) assuming first that the azimuthal angle coordinate $\phi$ is identified with period $\Delta \phi$. The conical deficit along the axes $\theta = 0, \pi$ and along the segment $r = r_+$ are given respectively by

$$
\delta_{(0,\pi)} = 2\pi - \frac{\Delta \phi d\sqrt{g_{\phi\phi}}}{\sqrt{g_{\theta\theta}}} |_{\theta=0,\pi} = 2\pi - \Delta \phi, \tag{5}
$$

$$
\delta_{(r=r_+)} = 2\pi - \frac{\Delta \phi d\sqrt{g_{\phi\phi}}}{\sqrt{g_{rr}}} |_{r=r_+} = 2\pi - \left(1 + \frac{m^2}{a^2}\right)^{1/2} \Delta \phi
$$

where, of course, we used the $D_6 - \bar{D}_6$ metric solution given in eq.\!(3). From eq.\!(5), it is now evident that one cannot eliminate the conical singularities along the semi-axes $\theta = 0, \pi$ and along the segment $r = r_+$ at the same time. Indeed one has the options :

(i) One can remove the conical angle deficit along $\theta = 0, \pi$ by choosing $\Delta \phi = 2\pi$ at the expense of the conical angle excess along $r = r_+$ which amounts to the presence of a strut providing the internal pressure to counterbalance the combined gravitational and gauge attractions between $D_6$ and $\bar{D}_6$.

(ii) Alternatively, one can instead eliminate the conical singularity along $r = r_+$ by choosing $\Delta \phi = 2\pi(1 + m^2/a^2)^{-1/2}$ at the expense of the appearance of the conical angle deficit $\delta_{(0,\pi)} = 2\pi[1 - (a^2/(m^2 + a^2))^{1/2}]$ along $\theta = 0, \pi$ which implies the presence of cosmic strings providing the tension

$$
\tau = \frac{\delta_{(0,\pi)}}{8\pi} = \frac{1}{4} \left[1 - \left(\frac{a^2}{m^2 + a^2}\right)^{1/2}\right], \tag{6}
$$

that pulls $D_6$ and $\bar{D}_6$ at the endpoints apart.

Normally, one might wish to take the second option in which the pair of branes is suspended by open cosmic strings, namely $D_6$ and $\bar{D}_6$ are kept apart by the tensions generated by cosmic strings against the collapse due to the gravitational and gauge attractions. And the line $r = r_+, 0 < \theta < \pi$ joining $D_6$ and $\bar{D}_6$ is now completely non-singular. This recourse to cosmic strings to account for the conical singularities of the solution and to suspend the $D_6 - \bar{D}_6$ system in an equilibrium configuration, however, might appear as a rather ad
hoc prescription. Perhaps it would be more relevant to introduce an external magnetic field aligned with the axis joining the brane pair to counterbalance the combined gravitational and gauge attractions by pulling them apart. By properly tuning the strength of the magnetic field, the attractive inter-brane force along the axis would be rendered to vanish. Indeed this conical singularity structure of the $D6 - \bar{D}6$ system and its cure via the introduction of the external magnetic field of proper strength is reminiscent of Ernst’s prescription \([13]\) for the elimination of conical singularities of the charged $C$-metric and of Emparan’s treatment \([10]\) to remove the analogous conical singularities of the Bonnor’s magnetic dipole solution in Einstein-Maxwell and Einstein-Maxwell-dilaton theories and in the present work, we shall closely follow the formulation of Emparan \([10]\). Before doing so, however, we need to study the geometrical structure of the $D6 - \bar{D}6$ system in IIA theory given in eqs.(3) and (4) in some more detail.

(1) The meaning of parameter $a$ as the proper inter-brane distance

We now elaborate on our earlier comment that the parameter $a$ appearing in this supergravity solution can be regarded as indicating the proper separation between the brane and the antibrane. Notice first that for large $a$, the proper inter-brane distance increases as $\sim 2a$. Namely,

$$
l = \int_0^\pi d\theta \sqrt{g_{\theta\theta}}|_{r=r_+} = \int_0^\pi d\theta H^{1/4}(\Delta + a^2 \sin^2 \theta)^{1/2}|_{r=r_+} \\
\simeq \int_0^\pi d\theta a \sin \theta = 2a. \hspace{1cm} (7)$$

Meanwhile, as argued by Sen \([7]\), the proper inter-brane distance vanishes when $a \to 0$. In addition, that the limit $a \to 0$ actually amounts to the vanishing inter-brane distance can be made more transparent as follows. Recently, Brax, Mandal and Oz \([14]\) discussed the supergravity solution representing coincident $D_p - \bar{D}_p$ pairs in type II theories and studied its instability in terms of the condensation of tachyon arising in the spectrum of open strings stretched between $D_p$ and $\bar{D}_p$. Thus now, taking the $p = 6$ case for example, we first would like to establish the correspondence between our solution given above representing $D6 - \bar{D}6$ pair generally separated by an arbitrary distance and theirs. For specific but appropriate values of the parameters appearing in their solution, $(c_2 = 1(p > 3), c_1 = 0, r_0 = m/2)$ \([14]\) so as to represent a neutral, coincident $D6 - \bar{D}6$ pair, their solution is given in Einstein frame by

$$
d{s}_{E}^2 = \left[\frac{1 - r_0/\bar{r}}{1 + r_0/\bar{r}}\right]^{1/4} [-dt^2 + \sum_{i=1}^{6} dx_i^2] + \left[1 - \frac{r_0}{\bar{r}}\right]^{1/4} \left[1 + \frac{r_0}{\bar{r}}\right]^{15/4} [d\bar{r}^2 + \bar{r}^2(d\theta^2 + \sin^2 \theta d\phi^2)],$$

\[7\]
\( e^\phi = \left[ \frac{1 - r_0/\bar{r}}{1 + r_0/\bar{r}} \right]^{3/2}, \ A_{[1]} = 0 \)  

(8)

which, in string frame, using \( g^{E}_{\mu\nu} = e^{-\phi/2}g^{S}_{\mu\nu} \), becomes

\[
\begin{align*}
\text{ds}^2 &= \left[ 1 - r_0/\bar{r} \right] \left[ -dt^2 + \sum_{i=1}^{6} dx_i^2 + \left[ 1 - r_0/\bar{r} \right] \left[ d\bar{r}^2 + \bar{r}^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \right], \\
e^\phi &= \left[ 1 - r_0/\bar{r} \right]^{3/2}, \ A_{[1]} = 0.
\end{align*}
\]

(9)

Consider now, transforming from this \textit{isotropic} coordinate \( \bar{r} \) to the standard radial \( r \) coordinate

\[
\tilde{r} = \frac{1}{2} \left[ (r - 2r_0) + (r^2 - 4r_0r)^{1/2} \right] \quad \text{or inversely} \quad r = \tilde{r} \left( 1 + \frac{r_0}{\tilde{r}} \right)^2. \quad (10)
\]

Then their solution describing \((N = 1)D6\) and \((\tilde{N} = 1)\tilde{D}6\) now takes the form

\[
\begin{align*}
\text{ds}^2 &= \left( 1 - \frac{2m}{r} \right)^{1/2} \left[ -dt^2 + \sum_{i=1}^{6} dx_i^2 \right] + \left( 1 - \frac{2m}{r} \right)^{-1/2} \left[ d\bar{r}^2 + \bar{r}^2 \left( 1 - \frac{2m}{r} \right) (d\theta^2 + \sin^2 \theta d\phi^2) \right], \\
e^{2\phi} &= \left( 1 - \frac{2m}{r} \right)^{3/2}, \ A_{[1]} = 0.
\end{align*}
\]

(11)

Clearly, this solution indeed coincides with the \( a \rightarrow 0 \) (i.e., vanishing separation) limit of our more general \( D6 - \tilde{D}6 \) solution given in eq.(3). Actually, this aspect also has been pointed out in a recent literature [15]. And this confirms our earlier proposition that \( a \) indeed acts as a relevant parameter representing the proper inter-brane distance even for very small separation.

\( (2) \) \textit{The geometry near each pole of \( D6 - \tilde{D}6 \) pair}

Thus far, we have simply accepted that the supergravity solution given in eqs.(3),(4) represents the configuration of \( D6 - \tilde{D}6 \) pair. It would therefore be satisfying to demonstrate in a transparent manner that this is indeed the case. To this end, first note that the solution in eq.(3) clearly is asymptotically-flat as \( r \rightarrow \infty \) and in this asymptotic region, the \( RR \) tensor potential is indeed that of a “dipole”, i.e., \( A_{[1]} \rightarrow \frac{2ma}{r} sin^2 \theta d\phi = \frac{ma}{r}(1 - \cos \theta) d\phi \).

Also note that the axis of symmetry of the solution (i.e., the fixed point set of the isometry generated by the Killing field (\( \partial/\partial \phi \))) consists of the semi-infinite lines \( \theta = 0, \pi \) (running from \( r = r_+ \) to \( r = \infty \)) and the segment \( r = r_+ \) (running from \( \theta = 0 \) to \( \theta = \pi \)). And indeed at each of the poles, \((r = r_+, \theta = 0)\) and \((r = r_+, \theta = \pi)\), lies a (distorted) brane and (distorted) antibrane respectively. Thus in order to show this explicitly, we perform the change of coordinates from \((r, \theta)\) to \((\rho, \bar{\theta})\) given by the following transformation law [5, 10]

\[
r = r_+ + \frac{\rho}{2} (1 + \cos \bar{\theta}), \quad \sin^2 \theta = \frac{\rho}{\sqrt{m^2 + a^2}}(1 - \cos \bar{\theta}) \quad (12)
\]
where \( r_+ = m + \sqrt{m^2 + a^2} \) as given earlier. We start with the study of the metric near 
\((r = r_+, \theta = 0)\). Upon changing the coordinates as given by eq.(12) and then taking \( \rho \) to be 
much smaller than any other length scale involved so as to get near each pole, the \( D6 - \bar{D6} \) solution in eq.(3) becomes

\[
\begin{align*}
\text{ds}^2_{10} & \simeq g^{1/2}(\bar{\theta}) \left( \frac{\rho}{q} \right)^{1/2} \left[ -dt^2 + \sum_{i=1}^{6} dx_i^2 \right] + \left( \frac{q}{\rho} \right)^{1/2} \left[ g^{1/2}(\bar{\theta})(d\rho^2 + \rho^2 d\bar{\theta}^2) + g^{-1/2}(\bar{\theta}) \rho^2 \sin^2 \bar{\theta} d\phi^2 \right], \\
e^{2\phi} & \simeq \left( \frac{\rho}{q} \right)^{3/2} g^{3/2}(\bar{\theta}), \\
A_m^{[1]} & \simeq \frac{m \rho + a}{(m^2 + a^2)} g^{-1}(\bar{\theta})(1 - \cos \bar{\theta}) d\phi
\end{align*}
\]

(13)

where \( q = m r_+ / \sqrt{m^2 + a^2} \) is the RR charge of each \( D6 \)-brane and \( g(\bar{\theta}) = \cos^2(\bar{\theta}/2) + \left[ a^2/(m^2 + a^2) \right] \sin^2(\bar{\theta}/2) \). Namely in this small-\( \rho \) limit, the geometry of the solution reduces to that of the near-horizon limit of a \( D6 \)-brane. However, the horizon is no longer spherically-symmetric and is deformed due to the presence of the other brane, i.e., the \( \bar{D6} \)-brane located at the other pole. To elaborate on this point, it is noteworthy that the surface \( r = r_+ \) is still a horizon, but instead of being spherically-symmetric, it is elongated along the axis joining the poles in a prolate shape. Namely, the horizon turns out to be a \textit{prolate spheroid} with the distortion factor given by \( g(\bar{\theta}) \). And of course, it is further distorted by a conical defect at the poles. And similar analysis can be carried out near the other pole at which \( \bar{D6} \) is situated, i.e., near \((r = r_+, \theta = \pi)\). The limiting geometries above were valid for arbitrary values of \( "a" \), as long as we remain close to each pole. If instead we consider the limit of very large-\( a \), while keeping \((r - r_+) \) and \( a \sin^2 \theta \) finite, the supergravity solution in eq.(3) reduces, in this time, to

\[
\begin{align*}
\text{ds}^2_{10} & \simeq \left( 1 + \frac{q}{\rho} \right)^{-1/2} \left[ -dt^2 + \sum_{i=1}^{6} dx_i^2 \right] + \left( 1 + \frac{q}{\rho} \right)^{1/2} \left[ d\rho^2 + \rho^2 (d\bar{\theta}^2 + \sin^2 \bar{\theta} d\phi^2) \right], \\
e^{2\phi} & \simeq \left( 1 + \frac{q}{\rho} \right)^{-3/2}, \\
A_m^{[1]} & \simeq q(1 - \cos \bar{\theta}) d\phi
\end{align*}
\]

(14)

with \( q \to m \). Clearly, this can be recognized as representing the extremal \( D6 \)-brane solution with \( \rho^2 = \sum_{m=7}^{9} x_m^2 \). Indeed, this result was rather expected since, physically, taking the limit \( a \to \infty \) amounts to pushing one of the poles (say, \( \bar{D6} \)-brane) to a large distance and studying the geometry of the remaining pole (\( D6 \)-brane) which, as a consequence, should be spherically-symmetric. So we conclude that the solution given in eq.(3) indeed describes a \textit{dipole}, i.e., the \( D6 - \bar{D6} \) pair.
In order to introduce the external magnetic field with proper strength to counterbalance the combined gravitational and gauge attractions and hence to keep the $D6$ -- $\bar{D}6$ pair in an (unstable) equilibrium configuration, we now proceed to construct the supergravity solution representing $D6$ -- $\bar{D}6$ pair parallelly intersecting with a $RR$ $F7$-brane. This can be achieved by first uplifting the $D6$ -- $\bar{D}6$ solution in IIA theory to the $D=11$ $KK$ -- dipole solution in M-theory discussed by Sen [7] and then by performing a twisted $KK$-reduction on this M-theory $KK$ -- dipole. Thus consider carrying out the dimensional lift of the $D6$ -- $\bar{D}6$ solution given in eqs.(3) and (4) to $D=11$ via the standard $KK$-ansatz

$$ds_{11}^2 = e^{-\frac{2}{3}\phi}ds_{10}^2 + e^{\frac{4}{3}\phi}(dy + A_\mu dx^\mu)^2$$

(with $A_{[1]} = A_\mu d^\mu$ being the 1-form magnetic $RR$ potential in eq.(4)) which yields

$$ds_{11}^2 = [-dt^2 + \sum_{i=1}^{6} dx_i^2] + \sum \left[\frac{dr^2}{\Delta} + d\theta^2\right]$$ (16)

$$+ \frac{1}{\Sigma} [\Delta (dy - a \sin^2 \theta d\phi)^2 + \sin^2 \theta \{(r^2 - a^2) d\phi + ady\}^2].$$

This is the $KK$ monopole/anti-monopole solution in $D=11$ or the M-theory $KK$ -- dipole solution first given by Sen [7]. Similarly to the IIA theory $D6$ -- $\bar{D}6$ solution discussed above, this M-theory $KK$ -- dipole solution represents the configuration in which $KK$ monopole and anti-monopole are sitting on the endpoints of the dipole, i.e., $(r = r_+, \theta = 0)$ and $(r = r_-, \theta = \pi)$ respectively. Note that unlike the IIA theory $D6$ -- $\bar{D}6$ solution, this M-theory $KK$ -- dipole solution is free of conical singularities provided the azimuthal angle coordinate $\phi$ is periodically identified with the standard period of $2\pi$. Now to get back down to $D=10$, consider performing the non-trivial point identification $\phi \equiv (y + 2 \pi n_1 R, \ \phi + 2 \pi n_2 RB + 2 \pi n_2)$ (17)

(with $n_1, n_2 \in \mathbb{Z}$) on the M-theory $KK$ -- dipole solution in eq.(16), followed by the associated skew $KK$-reduction along the orbit of the Killing field

$$l = (\partial/\partial y) + B (\partial/\partial \phi)$$ (18)

where $B$ is a magnetic field parameter. And this amounts to introducing the adapted coordinate

$$\tilde{\phi} = \phi - By$$ (19)
which is constant along the orbits of \( l \) and possesses standard period of \( 2\pi \) and then proceeding with the standard KK-reduction along the orbit of \((\partial/\partial y)\). Thus we recast the metric solution, upon changing to this adapted coordinate, in the standard KK-ansatz

\[
\begin{align*}
    ds^2_{11} &= \left[-dt^2 + \sum_{i=1}^{6} dx_i^2\right] + \sum \left[\frac{dr^2}{\Delta} + d\theta^2\right] + \frac{\Delta + a^2 \sin^2 \theta}{\Sigma} dy^2 \\
    &+ \frac{2[\Delta a^2 - \Delta \alpha^2 \sin^2 \theta]}{\Sigma} dy (d\phi + Bdy) + \frac{\sin^2 \theta}{\Sigma} [(r^2 - a^2) + \Delta a^2 \sin^2 \theta] (d\tilde{\phi} + Bdy)^2
\end{align*}
\]

and then read off the 10-dimensional fields as

\[
\begin{align*}
    ds^2_{10} &= \Lambda^{1/2} \left[-dt^2 + \sum_{i=1}^{6} dx_i^2\right] + \sum \left[\frac{dr^2}{\Delta} + d\theta^2\right] + \Lambda^{-1/2} \Delta \sin^2 \theta d\tilde{\phi}^2, \\
    e^{\frac{4}{\Lambda} \phi} &= \Lambda, \\
    A_{[1]} &= \Lambda^{-1} \frac{\sin^2 \theta}{\Sigma} \left[B[(r^2 - a^2)^2 + \Delta a^2 \sin^2 \theta] + a[(r^2 - a^2) - \Delta]\right] d\tilde{\phi}, \\
    F_{[2]} &= (\partial_r A_\phi) dr \wedge d\tilde{\phi} + (\partial_\theta A_\phi) d\theta \wedge d\tilde{\phi}, \quad \text{where} \\
    \Lambda &= \frac{1}{\Sigma} \left\{[\Delta + a^2 \sin^2 \theta] + 2Ba \sin^2 \theta[(r^2 - a^2) - \Delta] + B^2 \sin^2 \theta[(r^2 - a^2)^2 + \Delta a^2 \sin^2 \theta]\right\}.
\end{align*}
\]

Note that this solution can be identified with a \( D6 - \bar{D}6 \) pair parallely intersecting with a magnetic \( RR\) \( F7 \)-brane since for \( B = 0 \), it reduces to the \( D6 - \bar{D}6 \) solution in eq.(3) while for \( m = 0 \) and \( a = 0 \), it reduces to a \( RR \) \( F7 \)-brane solution in IIA theory. To see this last point explicitly, we set \( m = 0 = a \) in eq.(21) to get

\[
\begin{align*}
    ds^2_{10} &= \Lambda^{1/2} \left[-dt^2 + \sum_{i=1}^{6} dx_i^2 + dr^2 + r^2 d\theta^2\right] + \Lambda^{-1/2} r^2 \sin^2 \theta d\tilde{\phi}^2, \\
    e^{2\phi} &= \Lambda^{3/2}, \\
    A_{[1]} &= A_\phi d\tilde{\phi} = \frac{Br^2 \sin^2 \theta}{1 + B^2 r^2 \sin^2 \theta}, \quad \text{where now} \\
    \Lambda &= (1 + B^2 r^2 \sin^2 \theta).
\end{align*}
\]

Clearly, this is a magnetic \( RR \) \( F7 \)-brane solution in type IIA theory. Also note that generally a \( D_{2p} \)-brane has a direct coupling to a \( RR \) \( F_{(2p+1)} \)-brane in IIA theory. Thus for the case at hand, the magnetic \( D6 \) and \( \bar{D}6 \)-brane content of the solution in eq.(21) couple directly to the magnetic \( RR \) 1-form potential of the \( F7 \)-brane content extracted in eq.(22) and as a result experience static Coulomb-type force that eventually keeps the \( D6 - \bar{D}6 \) pair apart against the gravitational and gauge attractions.

Lastly, we see if the conical singularities which were inevitably present in the \( D6 - \bar{D}6 \) seed
solution can now be eliminated by the introduction of this magnetic \( F^7 \)-brane content. To do so, notice that in this \((D6-\Bar{D6})||F7\) case, \( \psi^\mu \psi^\mu = g_{\phi\phi} = 0 \) has roots at the locus of \( r = r_+ \) as well as along the semi-infinite axes \( \theta = 0, \pi \). Thus we need to worry about the possible occurrence of conical singularities both along \( \theta = 0, \pi \) and at \( r = r_+ \) again. Assuming that the azimuthal angle coordinate \( \phi \) is identified with period \( \Delta \phi \), the conical deficit along the axes \( \theta = 0, \pi \) and along the segment \( r = r_+ \) are given respectively by

\[
\delta_{(0,\pi)} = 2\pi - \frac{\Delta \phi d}{\sqrt{g_{\phi\phi}}} |_{\theta=0,\pi} = 2\pi - \Delta \phi, \tag{23}
\]

\[
\delta_{(r=r_+)} = 2\pi - \frac{\Delta \phi d}{\sqrt{g_{\phi\phi}}} |_{r=r_+} = 2\pi - \left[ \frac{r_+ - m}{B(r_+^2 - a^2) + a} \right] \Delta \phi
\]

where, in this time, we used the \((D6-\Bar{D6})||F7\) metric solution given in eq.(21). Therefore, by choosing \( \Delta \phi = 2\pi \) and “tuning” the strength of the external magnetic field as

\[
B = \frac{(r_+ - m) - a}{(r_+^2 - a^2)} = \frac{\sqrt{m^2 + a^2} - a}{2mr_+} \tag{24}
\]

one now can remove all the conical singularities. As stated earlier, this removal of conical singularities by properly tuning the strength of the magnetic field amounts to suspending the \( D6 - \Bar{D6} \) pair in an (unstable) equilibrium configuration by introducing a force exerted by this magnetic field (i.e., the \( RR F^7 \)-brane) to counterbalance the combined gravitational and gauge attractive force. To see this in a qualitative manner \[\text{[1]}\], recall first that, when they are well separated, the distance between \( D6 \) and \( \Bar{D6} \) is given roughly by \( \sim 2a \) as shown in eq.(7) and in this large-\( a \) limit, the magnetic field strength given above in eq.(24) is \( B \approx m/4a^2 \). Next, since both the gravitational and \( RR \) gauge attractive forces between the branes would be given by \( m^2/(2a)^2 \) (where we used the fact that the \( RR \)-charge of a \( D6 \)-brane behaves like \( q \to m \) for large inter-brane separation as discussed earlier), the total attractive force goes like \( m^2/2a^2 \). Thus this combined attractive force would be counterbalanced by the repulsive force on the magnetic dipole of the \( D6 - \Bar{D6} \) pair, \( 2qB \approx 2m(m/4a^2) = m^2/2a^2 \) provided by the properly tuned magnetic field strength \( B \) of \( RR F^7 \)-brane give above in eq.(24).

### 2.2 \( D0 - \Bar{D0} \) pair supported by \( RR F^7 \)-brane

For the case of \( D0 - \Bar{D0} \) system, which is “electrically” \( RR \)-charged, it may seem irrelevant to attempt to intersect it with magnetic \( F^7 \)-brane to begin with. As we shall see later on in the appendix, however, the attempt to intersect it with electric \( RR \) fluxbrane via the twisted KK-reduction of \( W - \Bar{W} \) system in \( D = 11 \) supergravity fails. Thus the only remaining
option is to intersect it with magnetic $F7$-brane instead and see what effect this magnetic fluxbrane may have on the $D0\bar{D}0$ system regarding the stabilization particularly when the brane and the antibrane are brought close to each other. Thus we now start with the exact IIA supergravity solution representing the $D0\bar{D}0$ pair which is given, in string frame, by

$$ds_{10}^2 = H^{-1/2}[-dt^2] + H^{1/2}\left[\sum_{m=1}^6 dx_m^2 + (\Delta + a^2 \sin^2 \theta) \left(\frac{dr^2}{\Delta} + d\theta^2\right) + \Delta \sin^2 \theta d\phi^2\right],$$

$$e^{2\phi} = H^{3/2},$$

$$A_{[1]} = \left[\frac{2ma \cos \theta}{\Sigma}\right] dt, \quad F_{[2]} = dA_{[1]}$$

(25)

where again $H(r) = \Sigma/(\Delta + a^2 \sin^2 \theta)$. Then, as usual, by uplifting this $D0\bar{D}0$ solution in $D = 10$ IIA theory to $D = 11$, we can arrive at the $W\bar{W}$ (i.e., M-wave/anti-M-wave) solution given by

$$ds_{11}^2 = e^{-\frac{4}{3}\phi} ds_{10}^2 + e^{\frac{4}{3}\phi}(dy + A_{\mu} dx^\mu)^2$$

$$= -H^{-1}dt^2 + H\left(dy + \frac{2ma \cos \theta}{\Sigma} dt\right)^2$$

$$+ \left[\sum_{m=1}^6 dx_m^2 + (\Delta + a^2 \sin^2 \theta) \left(\frac{dr^2}{\Delta} + d\theta^2\right) + \Delta \sin^2 \theta d\phi^2\right].$$

(26)

Now, in order to construct the supergravity solution representing $D0\bar{D}0$ pair parallelly intersecting with a $RR F7$-brane by introducing the $RR F7$-brane content into the $D0\bar{D}0$ solution given in eq.(25), we, as usual, proceed to perform a twisted KK-reduction on this M-theory $W\bar{W}$ solution. Consider, therefore, performing the non-trivial point identification

$$(y, \phi) \equiv (y + 2\pi n_1 R, \phi + 2\pi n_1 RB + 2\pi n_2)$$

(27)

(with $n_1, n_2 \in \mathbb{Z}$) on the M-theory $W\bar{W}$ solution in eq.(26), followed by the associated skew KK-reduction along the orbit of the Killing field $l = (\partial/\partial y) + B(\partial/\partial \phi)$ where $B$ is again a magnetic field parameter. And this amounts to introducing the adapted coordinate $\tilde{\phi} = \phi - By$ which is constant along the orbits of $l$ and possesses standard period of $2\pi$ and then proceeding with the standard KK-reduction along the orbit of $(\partial/\partial y)$. The result is

$$ds_{10}^2 = \tilde{\Lambda}^{1/2}\left\{-H^{-1/2} dt^2 + H^{1/2}\left[\sum_{m=1}^6 dx_m^2 + (\Delta + a^2 \sin^2 \theta) \left(\frac{dr^2}{\Delta} + d\theta^2\right)\right]\right\}$$

$$+ \tilde{\Lambda}^{-1/2} H^{1/2} \Delta \sin^2 \theta \left\{B^2 \left(2ma \cos \theta\right)^2 dt^2 + \tilde{\phi}^2 - B \frac{4ma \cos \theta}{\Sigma} dt d\phi\right\},$$

(12)
\[ e^{\frac{4}{\sqrt{2}} \phi} = H \tilde{\Lambda}, \quad (28) \]
\[ A_{[1]} = \tilde{\Lambda}^{-1} \frac{2ma \cos \theta}{\Sigma} dt + \tilde{\Lambda}^{-1} H^{-1} B \Delta \sin^2 \theta d\tilde{\phi}, \]
\[ F_{[2]} = (\partial_r A_\Sigma) dr \wedge dt + (\partial_\theta A_\Sigma) d\theta \wedge dt + (\partial_r A_{\tilde{\phi}}) dr \wedge d\tilde{\phi} + (\partial_\theta A_{\tilde{\phi}}) d\theta \wedge d\tilde{\phi}, \quad \text{where} \]
\[ \tilde{\Lambda} = (1 + H^{-1} B^2 \Delta \sin^2 \theta). \]

This solution can be identified with a \( D0 - \bar{D}0 \) pair parallely intersecting with a magnetic \( RR F7 \)-brane since for \( B = 0 \), it reduces to the \( D0 - \bar{D}0 \) solution in eq.(25) while for \( m = 0 \) and \( a = 0 \), it reduces to a \( RR F7 \)-brane solution in IIA theory given in eq.(22). Again, we see if the conical singularities which were present in the \( D0 - \bar{D}0 \) seed solution can now be eliminated by the introduction of this magnetic \( F7 \)-brane content. To do so, notice that in this \( D0 - \bar{D}0 \) case, \( \psi^\mu \psi^\mu = g_{\tilde{\phi} \tilde{\phi}} = 0 \) still has roots at the locus of \( r = r_+ \) as well as along the semi-infinite axes \( \theta = 0, \pi \). Thus we need to worry about the possible occurrence of conical singularities both along \( \theta = 0, \pi \) and at \( r = r_+ \) again. Assuming that the azimuthal angle coordinate \( \tilde{\phi} \) is identified with period \( \Delta \tilde{\phi} \), the conical deficit along the axes \( \theta = 0, \pi \) and along the segment \( r = r_+ \) are given respectively by

\[ \delta_{(0, \pi)} = 2\pi - \left| \frac{\Delta \tilde{\phi} \sqrt{g_{\tilde{\phi} \tilde{\phi}}}}{\sqrt{g_{\tilde{\phi} \tilde{\phi}}}} \right|_{\theta=0, \pi} = 2\pi - \Delta \phi, \]
\[ \delta_{(r=r_+)} = 2\pi - \left| \frac{\Delta \tilde{\phi} \sqrt{g_{\tilde{\phi} \tilde{\phi}}}}{\sqrt{g_{rr}} dr} \right|_{r=r_+} = 2\pi - \left( 1 + \frac{m^2}{a^2} \right)^{1/2} \Delta \phi \]

where, of course, we used the \( D0 - \bar{D}0 \) case \( F7 \) metric solution given in eq.(28). To our dismay, but indeed as had been expected due to the reason stated earlier, the conical singularity structure essentially remains the same as that for the \( D0 - \bar{D}0 \) system, despite the introduction of the \( RR F7 \)-brane content into the system aiming at counterbalancing the combined gravitational and gauge attractions and hence keeping the system against the collapse. In other words, we still cannot remove the conical singularities along the axes, \( \theta = 0, \pi \) and along the segment \( r = r_+ \) at the same time. Certainly, this discouraging result demands physical explanation and indeed it can be attributed to the fact \( D0 \) (and \( \bar{D}0 \)) does not couple directly to the flux of \( RR F7 \)-brane and as a result experiences no Coulomb-type force from its presence. As we mentioned earlier, a \( D_p \)-brane couples only to the flux of a \( F_{(p+1)} \)-brane and this fact comes from the defining nature of the \( RR F_{(p+1)} \)-brane [6] according to which a \( F_{(p+1)} \)-brane is a \( (p + 2) \)-dimensional object in the \( (8 - p) \)-dimensional transverse space. And the core of this \( F_{(p+1)} \)-brane carries a \( (8 - p) \)-form magnetic \( RR \) field strength with flux piercing the transverse space. Therefore, only the \( D6 - \bar{D}6 \) system, which has non-trivial coupling to the \( F7 \)-brane, can be balanced in an (unstable) equilibrium against the com-
bined gravitational and gauge attractions. The others, such as \( D0 - \bar{D}0 \) we just discussed and \( D2 - \bar{D}2 \) and \( D4 - D4 \), the case of which can be examined in precisely the same manner, cannot be stabilized via the introduction of the magnetic \( F7 \)-brane simply because they do not have non-trivial coupling. For the cases of \( D0 - \bar{D}0 \) and \( D2 - \bar{D}2 \) systems, however, it may seem irrelevant to intersect them with magnetic \( F7 \)-brane to begin with since these configuration are electrically \( RR \) charged. But as will be demonstrated later on in the appendix, any attempt to intersect them with electric \( RR \) fluxbrane via the twisted KK-reduction from \( \mathrm{M-theory} \) \( W - \bar{W} \) and \( M2 - \bar{M}2 \) systems respectively in \( \mathrm{D=11} \) fails. Thus we are forced to intersect them with magnetic \( F7 \)-branes instead and see what effect this magnetic \( F7 \)-brane may have on \( D0 - \bar{D}0 \) or \( D2 - \bar{D}2 \) system concerning the stabilization particularly when they are brought closer and closer to each other. It turns out that there indeed is a non-trivial effect which is puzzling at first sight but admits convincing interpretation on second thought.

To discuss it in great detail, we first remind our earlier observation that the parameter “\( a \)” appearing in the supergravity solutions representing \( D2p - \bar{D}2p \) systems in \( \mathrm{IIA} \) theory can be thought of as representing the proper separation between the brane and the antibrane all the way to the zero distance. We now take the \( D0 - \bar{D}0 \) case which is under consideration and see what happens as the brane and the antibrane approach each other, namely as \( a \to 0 \).

First, in the absence of the magnetic \( RR \) \( F7 \)-brane content, the \( D0 - \bar{D}0 \) solution in the limit \( a \to 0 \) becomes

\[
\begin{align*}
    ds_{10}^2 &= \left( 1 - \frac{2m}{r} \right)^{1/2} [-dt^2] + \left( 1 - \frac{2m}{r} \right)^{-1/2} \left[ \sum_{m=1}^{6} dx_m^2 + dr^2 + r^2 \left( 1 - \frac{2m}{r} \right) (d\theta^2 + \sin^2 \theta d\phi^2) \right], \\
    e^{2\phi} &= \left( 1 - \frac{2m}{r} \right)^{-3/2}, \quad A_{[1]} = 0 \\
\end{align*}
\]

(30)

where we used \( \Sigma \to r^2 \), \( \Delta \to r^2(1 - 2m/r) \), and hence \( H \to (1 - 2m/r)^{-1} \) as \( a \to 0 \). In this limit, the opposite \( RR \) charges carried by \( D0 \) and \( \bar{D}0 \) annihilated each other since \( A_{[1]} = 0 \) and the solution now has the topology of \( R \times R^7 \times S^2 \). Particularly, the \( SO(3) \)-isometry in the transverse space implies that, as they approach, \( D0 \) and \( \bar{D}0 \) actually merge and as a result a curvature singularity develops at the center \( r = 0 \). On the other hand, looking at the solution representing the \( D0 - \bar{D}0 \) pair embedded in the magnetic \( F7 \)-brane, it does not appear to be possible to bring \( D0 \) and \( \bar{D}0 \) close enough to make them merge completely.

Thus to see this, consider the \( a \to 0 \) limit of the \( (D0 - \bar{D}0)||F7 \) solution

\[
\begin{align*}
    ds_{10}^2 &= \tilde{\Lambda}^{1/2} \left\{ \left( 1 - \frac{2m}{r} \right)^{1/2} [-dt^2] + \left( 1 - \frac{2m}{r} \right)^{-1/2} \left[ \sum_{m=1}^{6} dx_m^2 + dr^2 + r^2 \left( 1 - \frac{2m}{r} \right) d\theta^2 \right] \right\} \\
    &+ \tilde{\Lambda}^{-1/2} r^2 \left( 1 - \frac{2m}{r} \right)^{1/2} \sin^2 \theta d\phi^2, \\
\end{align*}
\]

(31)
\[e^{2\phi} = \tilde{\Lambda}^{3/2} \left(1 - \frac{2m}{r}\right)^{-3/2}, \quad A_{[1]} = \tilde{\Lambda}^{-1} \left(1 - \frac{2m}{r}\right)^2 Br^2 \sin^2 \theta d\tilde{\phi}\]

where now \(\tilde{\Lambda} = [1 + B^2 r^2(1 - 2m/r)^2 \sin^2 \theta]\) and \(A_{[1]}\) is the magnetic vector potential for the F7-brane. It is now obvious that for finite \(B \neq 0\), the portion of the transverse space fails to exhibit \(SO(3)\)-isometry. Instead, the solution now possesses axisymmetry, namely, the metric solution has explicit \(\theta\)-dependence coming from the factor \(\tilde{\Lambda}\) and this is the manifestation that even for very small separation, the two brane configuration structure still persists. In fact, the axisymmetry itself even in the limit \(a \to 0\) of the solution is no surprise as it has been expected to some extent since the \((D0 - \bar{D0})||F7\) solution involves the axisymmetric F7-brane content from the outset. Rather the point is that, in the \(a \to 0\) limit of the \((D0 - \bar{D0})||F7\) solution given above, one never knows whether this axisymmetry comes from the remaining F7-brane content after the complete merging of the \(D0 - \bar{D0}\) pair or from the surviving brane-antibrane configuration so long as one keeps the non-vanishing content of the branes, i.e., \(m \neq 0\). Thus generically, one should regard that even in the limit \(a \to 0\), the two brane structure may have a good chance to survive. Of course, if we turn off the F7-brane content, i.e., if we set \(B = 0\), then for \(a \to 0\), \(D0\) and \(\bar{D0}\) merge completely as they should. This observation indicates that although the magnetic F7-brane and \(D0\) (and \(\bar{D0}\)) do not directly couple and hence F7 fails to serve to stabilize the \(D0 - \bar{D0}\) system against the eventual collapse for finite separation, when \(D0\) and \(\bar{D0}\) are brought close enough together, the F7-brane turns out to play the role of keeping them from annihilating each other completely. And it is rather straightforward to see that the same is true for \(D2 - \bar{D2}\) and \(D4 - \bar{D4}\) systems as well (whose supergravity solutions are known \([12]\)). Within the context of the supergravity analysis, this picture is an apparent puzzle and demands some resolution. As was mentioned earlier in the introduction, one may naturally expect that the simplest endpoints of the semi-classical instability of the \(D_{2p} - \bar{D}_{2p}\) systems would be a supersymmetric vacuum. And since the introduction of \(RR\) F7-brane content cannot remove the semi-classical instability of the \(D_{2p} - \bar{D}_{2p}\) systems, one may still expect that they should eventually merge completely when they approach each other even in the presence of the fluxbrane. But rather to our surprise, this turned out not to be the case. Indeed, the possible answer to this puzzle may lie in the validity of the semi-classical supergravity description of the system. Namely, the supergravity solutions representing \((D_{2p} - \bar{D}_{2p})||F7\) systems cannot be trusted for stability analysis all the way down to \(a \to 0\) and obviously they invalidate as the separation between the branes approaches the string length scale, i.e., \(a \leq \sqrt{a'} = l_s\). Put differently, for very small separation of order \(a \sim \sqrt{a'} = l_s\),
the supergravity description of the system breaks down and we should in principle employ stringy analysis instead in terms of tachyonic mode arising for $a \leq l_s$ in the spectrum of open strings stretched between $D_p$ and $\bar{D}_p$. Firstly, as for the quantum instability associated with the non-BPS $D_p - \bar{D}_p$ system, it is by now widely accepted that for a nearly coincident $D_p - \bar{D}_p$ pair, the spectrum of open strings connecting $D_p$ and $\bar{D}_p$ develops a tachyonic mode and this open string tachyon “condenses” as $D_p$ and $\bar{D}_p$ annihilate each other to become a supersymmetric vacuum or evolve to produce a stable lower-dimensional brane. And it has been estimated by Callan and Maldacena [16] that the typical time scale for this $D_p - \bar{D}_p$ annihilation process is of order $1/\sqrt{g_s}$ where $g_s$ denotes the fundamental string coupling, $g_s = e^{\phi(\infty)}$. Secondly, as for the quantum instability associated with a RR fluxbrane, it has been conjectured and generally believed that it would presumably be linked to the closed string tachyonic mode. We now elaborate on this last point. As is well-known, one of the simplest ways to construct a RR $F7$-brane is via the “twisted” KK-reduction of the $D = 11$ Minkowski spacetime, which is a M-theory vacuum. As such, the $F7$-brane breaks all the supersymmetries and hence should be unstable and decay. Indeed, it has been known for some time [3, 8] that the Melvin-type fluxtube universe like the $F7$-brane actually decays at the rate given by $\Gamma \sim e^{-I}$, with “$I$” being the Euclidean instanton action and the instanton configuration related to this decay of the Melvin-type magnetic fluxtube universe is the Euclidean Kerr geometry in an arbitrary dimension. And it is generally expected that the endpoint of this RR $F7$-brane decay would be either a supersymmetric closed string vacuum or the nucleation of the $D6 - \bar{D}6$ pair via the brainy Schwinger process [3]. Particularly, it has been conjectured that the fluxbrane decay to a supersymmetric vacuum should be linked to the closed string tachyon condensation since it involves the decay of the spacetime itself [3, 17]. For the case at hand, we have both $D_p - \bar{D}_p$ pair and the RR $F7$-brane in the system and each is unstable for the reasons just stated. What is more, it is in many respects evident that the presence of the magnetic $F7$-brane, i.e., the external magnetic field changes the status of the quantum instability of the $D_p - \bar{D}_p$ system. Namely, due to the additional energy density introduced by the external magnetic field (i.e., the $F7$-brane), the total energy density of the $(D_{2p} - \bar{D}_{2p})||F7$ system now would be given by

$$E_{tot} = V(T) + 2M_D + \epsilon_{F7}$$

(32)

where again $V(T)$ and $M_D$ are the tachyon potential and the $D$-brane tension respectively and $\epsilon_{F7}$ denotes the contribution to the total energy density coming from $F7$-brane, i.e., the magnetic field energy density. Note that the tachyon potential $V(T)$ here would remain
unchanged from that in the absence of the magnetic RR $F_7$-brane as the $NS$-charged open strings stretched between $D_{2p}$ and $\bar{D}_{2p}$ have no direct coupling to the flux of RR $F_7$-brane. Now in this stringy description, as $D_{2p}$ and $\bar{D}_{2p}$ approach each other, the open string tachyon field $T$, having essentially the same potential as the one without $F_7$, may still condense, i.e., its mass squared may evolve from being negative around the false vacuum expectation value (vev) to being positive at the true vev, say, $T_0$ as it rolls down toward the (negative) minimum of the potential, $V(T_0)$. Even when the tachyon reaches the minimum $V(T_0)$ of its potential, however, the brane and the antibrane would not necessarily annihilate since the endpoint of this tachyon condensation is no longer a supersymmetric vacuum but instead it is a left-over $F_7$-brane for which the supersymmetry is completely broken, as can be deduced from the consideration, $E_{tot} = V(T_0) + 2M_{D} + \epsilon_{F_7} = \epsilon_{F_7}$ (where we used Sen’s conjecture $\Box$ in the absence of the $F_7$-brane content, $V(T_0) + 2M_{D} = 0$). Of course, there is another possibility in which the (negative) $V(T_0)$ cancels instead with the part or all of $\epsilon_{F_7}$ rather than it does exactly with $2M_{D}$. In this alternative situation, upon the tachyon condensation the left-over would be something that again breaks the supersymmetry completely. In other words, the brane-antibrane system would not be driven to the complete spontaneous annihilation as the endpoint of the tachyon condensation does not, in any case, enhance any supersymmetry of the system. As a result, as long as the $F_7$-brane is there, $D_{p}$ and $\bar{D}_{p}$ would not necessarily annihilate each other via the open string tachyon condensation and this quantum perspective is indeed consistent with the result of semi-classical supergravity analysis given earlier in which it has been demonstrated that $D_{2p} - \bar{D}_{2p}$ pairs may not necessarily merge and annihilate in the presence of the $F_7$-brane pairs even if they are brought close enough together. Besides, this argument to resolve the puzzling role played by the RR $F_7$-brane content in the $D_{2p} - \bar{D}_{2p}$ pairs ($p = 2, 1, 0$) holds true for the case of $D_6 - \bar{D}_6$ system as well although there, the RR $F_7$-brane has actually the direct coupling to the $D_6 - \bar{D}_6$ system and hence is able to provide the system with even a classical stability (i.e., an unstable equilibrium) generally for some finite separation between the branes. Namely, when $D_6$ and $\bar{D}_6$ are brought close enough to each other, the branes can be supported only until the $F_7$-brane itself disappears by decaying to a vacuum or to $D_6 - \bar{D}_6$ pairs via brany Schwinger process $\Box$. And this indicates that after all, the result of supergravity analysis given earlier was not totally wrong although it should not be naively trusted. $F_7$-brane, however, is itself unstable (as it breaks all the supersymmetries) and hence decays eventually. Therefore, the overall picture of the quantum instability of the $(D_{2p} - \bar{D}_{2p})||F_7$ systems can be stated as follows:
Within the time scale for the decay of magnetic $F7$-brane, $D_p - \bar{D}_p$ pair would be supported against collapse and the subsequent annihilation. Once $F7$-brane itself decays, $D_p$ and $\bar{D}_p$ would now annihilate each other presumably leaving supersymmetric vacuum behind as the tachyonic mode in the spectrum of the open strings stretched between $D_p$ and $\bar{D}_p$ condenses. Namely, the presence of the RR $F7$-brane just “delays” the annihilation process of $D_p - \bar{D}_p$ pair but can never eliminate the instability of the $D_p - \bar{D}_p$ pair completely!

Next, since it is the presence of the $F7$-brane which “delays” the annihilation of the $D_p - \bar{D}_p$ system, this decay mechanism might deserve closer examination. And it would be of particular interest to study what the effect of the presence of the $D_p - \bar{D}_p$ pair on the decay rate of the $F7$-brane really is. In order to estimate the $F7$-brane decay rate, all that is required is to find the instanton mediating the decay with the same asymptotics as those of the $F7$-brane since the two have to be matched in the asymptotic region. In the presence of the $F7$-brane alone and nothing else, it was rather straightforward to find the associated instanton configuration and that was, as mentioned, the higher-dimensional generalization of the Euclidean Kerr metric. And thus the evaluation of the corresponding Euclidean instanton action, $I(\text{instanton})$ was rather unambiguous, as well. For the case at hand, however, when both $F7$-brane and $D_p - \bar{D}_p$ pair are present, things get much more involved. Namely, since the presence of the $D_p - \bar{D}_p$ pair changes the asymptotics of the $F7$-brane geometry in a highly non-trivial fashion as we actually have seen earlier in eq.(21) or in eq.(28), it would be practically almost impossible to find the associated instanton configuration having this complicated asymptotics. And this, in turn, indicates that now the explicit evaluation of the corresponding Euclidean instanton action would not be available, either. Thus, one can only hope to determine whether the instanton action (decay rate) gets smaller (higher) or else it is the other way around. And the clue that would lead us to the right answer to this question is undoubtedly linked to whether or not the presence of the $D_p - \bar{D}_p$ pair would add more instability to the $F7$-brane accelerating its decay process. Unfortunately, any conclusive statement concerning this point appears to be beyond our reach for the moment, but our best guess is that presumably, the presence of the $D_p - \bar{D}_p$ pair would increase the instability of the $F7$-brane and hence elevate its decay rate. And this guess is based upon the fact that the $D_p - \bar{D}_p$ system is itself an unstable non-BPS configuration involving the open string tachyonic mode.
3. **NS–anti–NS systems supported by NSNS fluxbrane**?

Thus far we have considered the intersection of non-BPS $D_{2p} - \bar{D}_{2p}$ systems with the magnetic $RR F7$-brane in IIA theory in order to study the role played by the $RR F7$-brane concerning the semi-classical and quantum (in terms of open string tachyon condensation) instability of the $D_{2p} - \bar{D}_{2p}$ systems. Now the remaining unstable non-perturbative spectrum (or non-BPS solutions) of D=10, type IIA supergravity theory are $F_{1} - \bar{F_{1}}$ and $NS5 - \bar{NS5}$ systems. Since these are charged under the NSNS two-form tensor field $B_{[2]}$ (electrically for $F_{1} - \bar{F_{1}}$ and magnetically for $NS5 - \bar{NS5}$), now it would be natural to attempt to intersect them with the NSNS $F6$-brane and see if the NSNS $F6$-brane can play an analogous role regarding the semi-classical and quantum instability which is supposed to reside in these systems. At this point, it seems noteworthy that a (particularly special case of) NSNS $F6$-brane is related to the $RR F7$-brane via the chain of duality transformations, $U = TST$ with the $T$-duals acting on the same isometry direction [1]. As a result, it is tempting to expect that, for instance, the $(NS5 - \bar{NS5})||(NSNS F6)$ solution might as well be related to the $(D6 - \bar{D6})||(RR F7)$ solution via the same $U$-duality transformation just mentioned. As we shall see in a moment, however, this turns out not to be the case. Just as the $D6 - \bar{D6}$ system alone possesses direct coupling to the $RR F7$-brane but no other $D_{2p} - \bar{D}_{2p}$, only the $NS5 - \bar{NS5}$ but not $F_{1} - \bar{F_{1}}$ has direct coupling to the NSNS $F6$-brane. Thus one may naturally expect that the $NS5 - \bar{NS5}$ system would exclusively be counterbalanced against the combined gravitational and gauge attractions by the introduction of the NSNS $F6$-brane content into the system. As the fact that $(NS5 - \bar{NS5})||(NSNS F6)$ solution is not really related via the $U$-duality to the $(D6 - \bar{D6})||(RR F7)$ solution already signals, this naive expectation turns out not to hold either. Thus in the following, we shall discuss this rather puzzling issue in some detail and attempt to provide a relevant resolution.

### 3.1 NS5 – N\(\overline{S5}\) pair intersecting with NSNS F6-brane

We now start with the exact IIA supergravity solution representing the $NS5 - \bar{NS5}$ pair which is given, in string frame, by [12]

\[
\begin{align*}
\text{ds}^2_{10} &= -dt^2 + \sum_{i=1}^{5} dx_i^2 + H [dx_6^2 + (\Delta + a^2 \sin^2 \theta) \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \Delta \sin^2 \theta d\phi^2], \\
e^{2\phi} &= H,
\end{align*}
\]  

(33)
\[ B_{[2]} = \left[ \frac{2mra \sin^2 \theta}{\Delta + a^2 \sin^2 \theta} \right] dx_6 \wedge d\phi, \]
\[ H_{[3]} = dB_{[2]} = \frac{2mra \sin^2 \theta (r^2 + a^2 \cos^2 \theta)}{(\Delta + a^2 \sin^2 \theta)^2} dx_6 \wedge dr \wedge d\phi - \frac{4mra \sin \theta \cos \theta \Delta}{(\Delta + a^2 \sin^2 \theta)^2} dx_6 \wedge d\theta \wedge d\phi \]

where again, \( H(r) = \Sigma/(\Delta + a^2 \sin^2 \theta) \). Then, as usual, by using the M/IIA duality, we uplift this NS5 − NS5 solution in \( D = 10 \) IIA theory to \( D = 11 \), to get the \( M5 - \bar{M5} \) solution given by

\[ ds_{11}^2 = e^{-\frac{2}{3}\phi} ds_{10}^2 + e^{\frac{4}{3}\phi} (dy + A_\mu dx^\mu)^2 \]
\[ = H^{-1/3}[-dt^2 + \sum_{i=1}^5 dx_i^2] + H^{2/3}[dx_6^2 + dx_7^2 + (\Delta + a^2 \sin^2 \theta) \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \Delta \sin^2 \theta d\phi^2], \]
\[ F_{[4]}^{11} = \frac{2mra \sin^2 \theta (r^2 + a^2 \cos^2 \theta)}{(\Delta + a^2 \sin^2 \theta)^2} dx_6 \wedge dx_7 \wedge dr \wedge d\phi \]
\[ - \frac{4mra \sin \theta \cos \theta \Delta}{(\Delta + a^2 \sin^2 \theta)^2} dx_6 \wedge dx_7 \wedge d\theta \wedge d\phi. \]  

We first identify the coordinate on the M-theory circle as \( y = x_6 \). Then, consider taking the quotient of this \( (M5 - \bar{M5}) \) spacetime, namely identifying points along the orbit of the Killing field \( l = (\partial/\partial y) + B(\partial/\partial \phi) \), i.e.,

\[ (y, \phi) \equiv (y + 2\pi n_1 R, \phi + 2\pi n_1 RB + 2\pi n_2) \]

(with \( n_1, n_2 \in \mathbb{Z} \)). This amounts to introducing the “adapted” coordinate \( \tilde{\phi} = \phi - By \) which is constant along the orbits of \( l \) and possesses standard period of \( 2\pi \), i.e.,

\[ ds_{11}^2 = H^{-1/3}[-dt^2 + \sum_{i=1}^5 dx_i^2] \]
\[ + H^{2/3}[dx_7^2 + (\Delta + a^2 \sin^2 \theta) \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \Delta \sin^2 \theta (d\tilde{\phi} + Bdy)^2 + dy^2]. \]

Earlier, when constructing the RR F7-brane, we performed the usual KK-reduction along \((\partial/\partial y)\). But in this time, consider performing the the KK-compactification along the orbit of \((\partial/\partial x_7)\) instead, i.e.,

\[ ds_{11}^2 = e^{-\frac{4}{3}\phi} ds_{10}^2 + e^{\frac{4}{3}\phi} (dx_7 + A_\mu dx^\mu)^2 \]  

(37)

to get the 10-dimensional fields

\[ ds_{10}^2 = -dt^2 + \sum_{i=1}^5 dx_i^2 + H[(\Delta + a^2 \sin^2 \theta) \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \Delta \sin^2 \theta (d\tilde{\phi} + Bdy)^2 + dy^2], \]
\[ e^{\frac{4}{3}\phi} = H^{2/3}, \quad A_{[1]} = 0. \]  

(38)
In addition, from
\[ F_{[4]}^{11} = F_{[4]}^{IIA} + H_{[3]} \wedge dx_7, \]
we also get
\[ F_{[4]}^{IIA} = 0, \quad H_{[3]} = \frac{2mra \sin^2 \theta (r^2 + a^2 \cos^2 \theta)}{\Delta + a^2 \sin^2 \theta} dy \wedge dr \wedge d\tilde{\phi} - \frac{4mra \sin \theta \cos \theta \Delta}{\Delta + a^2 \sin^2 \theta} dy \wedge d\theta \wedge d\tilde{\phi} \]
which is the 3-form magnetic NSNS field strength sourced by the \( NS5 - \bar{NS5} \) pair, since the associated 2-form magnetic NSNS tensor potential is given by
\[ B_{[2]} = \left[ \frac{2mra \sin^2 \theta}{\Delta + a^2 \sin^2 \theta} \right] dy \wedge d\tilde{\phi}. \]
Thus this new solution can be identified with a \( NS5 - \bar{NS5} \) system intersecting with a \( NSNS \) F6-brane since for \( B = 0 \), it reduces to the usual \( NS5 - \bar{NS5} \) solution given earlier, while for \( m = 0 \) and \( a = 0 \), it reduces to (a special case of) \( NSNS \) F6-brane. To see this last point, set \( m = 0 \) and \( a = 0 \) in the solution given above to get
\[ ds_{10}^2 = -dt^2 + \sum_{i=1}^5 dx_i^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta (d\tilde{\phi} + B dx_9)^2 + dx_9^2, \quad e^{2\phi} = 1, \quad B_{[2]} = 0 \]
where we set \( y = x_9 \). Note first that this is indeed a special case of the more general \( NSNS \) F6-brane solution \[17\] of \( D = 10 \) type IIA theory. Certainly, this solution is locally-flat but it may have non-trivial topology. To see this, recall that this metric solution can be formally obtained from the flat metric via the shift \( \phi \rightarrow \phi + B x_9 \). As a result, a shift of \( x_9 \) by the period of the compactification circle \( 2\pi R \) induces rotation in the transverse plane by \( 2\pi RB \). Thus this metric becomes topologically non-trivial if \( BR \neq n \ (n \in \mathbb{Z}) \). It is interesting to note \[17\], however, that even for the topologically trivial case, if, particularly, \( BR = 2k + 1 \ (k = 0, \pm 1, \ldots) \), the superstring theory on this background is still non-trivial (i.e., not equivalent to that on the flat spacetime) since the spacetime fermions change its sign under \( 2\pi \) rotation in the transverse plane. And this means that the \( BR = 1 \) case (indeed all cases with \( BR = 2k + 1 \) are equivalent) represents a superstring with \textit{antiperiodic} fermionic boundary condition in \( x_9 \)-direction. And the magnetic \( NSNS \) fluxbrane content in the new solution above becomes noticeable if one carries out one more time of dimensional reduction of this solution in eq.(38) along \( (\partial / \partial x_9) \), i.e.,
\[ ds_{10}^2 = -dt^2 + \sum_{i=1}^5 dx_i^2 + H[(\Delta + a^2 \sin^2 \theta) \left( \frac{dr^2}{\Delta} + d\theta^2 \right)] + \Delta \sin^2 \theta (d\tilde{\phi} + B dx_9)^2 + dx_9^2 \]
\[ = ds_9^2 + e^{2\phi}(dx_9 + A_\alpha dx^\alpha)^2. \]
Then the resulting 9-dimensional fields are
\[
\begin{align*}
    ds_9^2 &= -dt^2 + \sum_{i=1}^{5} dx_i^2 + H[(\Delta + a^2 \sin^2 \theta) \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + f^{-1} \Delta \sin^2 \theta d\tilde{\phi}^2], \\
    e^{2\phi} &= Hf, \\
    A_{[1]} &= f^{-1} B \Delta \sin^2 \theta d\tilde{\phi}
\end{align*}
\]
with \( f = (1 + B^2 \Delta \sin^2 \theta) \). Evidently, the emergence of the KK gauge field \( A_{\tilde{\phi}} d\tilde{\phi} \) indicates that this 9-dimensional supergravity solution and hence its 10-dimensional ancestor given earlier indeed represent Melvin-type magnetic fluxbrane.

### 3.2 \( F1 - \bar{F}1 \) pair intersecting with \( NSNS \) F6-brane

Since \( F1 - \bar{F}1 \) system is “electrically” \( NSNS \) charged, it would seem natural to attempt to intersect it with an electric \( NSNS \) fluxbrane. Such an attempt, via the twisted KK-reduction of \( M2 - \bar{M}2 \) system, however, fails again since the electric fluxbrane constructed in this manner turns out to be trivial having a null structure just as what happens when one attempts to intersect a \( D0 - \bar{D}0 \) system with an electric \( RR \) fluxbrane that we discussed earlier. Thus we attempt to intersect the \( F1 - F1 \) system with the magnetic \( NSNS \) F6-brane instead and examine what effect this magnetic fluxbrane may have on the classical and quantum instability of the \( F1 - \bar{F}1 \) system.

We now start with the exact IIA supergravity solution representing the \( F1 - \bar{F}1 \) pair which is given, in string frame, by \[12\]
\[
\begin{align*}
    ds_{10}^2 &= H^{-1}[-dt^2 + dx_1^2] + \sum_{m=2}^6 dx_m^2 + (\Delta + a^2 \sin^2 \theta) \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \Delta \sin^2 \theta d\phi^2, \\
    e^{2\phi} &= H^{-1}, \\
    B_{[2]} &= -\left[ \frac{2ma \cos \theta}{\Sigma} \right] dt \wedge dx_1
\end{align*}
\]
where again, \( H(r) = \Sigma/(\Delta + a^2 \sin^2 \theta) \). Then by using the M/IIA duality, we uplift this \( F1 - \bar{F}1 \) solution in \( D = 10 \) IIA theory to \( D = 11 \), to get the \( M2 - \bar{M}2 \) solution given by
\[
\begin{align*}
    ds_{11}^2 &= H^{-2/3}[-dt^2 + \sum_{i=1}^{2} dx_i^2] + H^{1/3} \left[ \sum_{m=3}^{7} dx_m^2 + (\Delta + a^2 \sin^2 \theta) \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \Delta \sin^2 \theta d\phi^2 \right], \\
    A_{[3]} &= -\left[ \frac{2ma \cos \theta}{\Sigma} \right] dt \wedge dx_1 \wedge dx_2.
\end{align*}
\]
Killing field $l = (\partial/\partial y) + B(\partial/\partial \phi)$, i.e.,

$$(y, \phi) \equiv (y + 2\pi n_1 R, \phi + 2\pi n_1 RB + 2\pi n_2) \quad (47)$$

which amounts to introducing the “adapted” coordinate $\tilde{\phi} = \phi - By$ in terms of which the metric for the $M2 - \bar{M}2$ solution is rewritten as

$$ds_{11}^2 = H^{-2/3}[-dt^2 + \sum_{i=1}^{2} dx_i^2] + H^{1/3}[\sum_{m=3}^{6} dx_m^2 + (\Delta + a^2 \sin^2 \theta) \left(\frac{dr^2}{\Delta} + d\theta^2\right) + \Delta \sin^2 \theta (d\tilde{\phi} + Bdy)^2 + dy^2].$$

Consider now performing the the KK-compactification along the orbit of $(\partial/\partial x_9)$, i.e.,

$$ds_{11}^2 = e^{-\frac{2}{3}\phi} ds_{10}^2 + e^{\frac{4}{3}\phi} (dx_2 + A_\mu dx^\mu)^2 \quad (49)$$

to get the 10-dimensional fields

$$ds_{10}^2 = H^{-1}[-dt^2 + dx_1^2] + \sum_{m=3}^{6} dx_m^2 + (\Delta + a^2 \sin^2 \theta) \left(\frac{dr^2}{\Delta} + d\theta^2\right) + \Delta \sin^2 \theta (d\tilde{\phi} + Bdy)^2 + dy^2,$$

$$e^{\frac{4}{3}\phi} = H^{-2/3}, \quad A_{[1]} = 0. \quad (50)$$

In addition, from

$$A_{[3]}^{IIA} = A_{[3]}^{I1A} + H_{[2]} \wedge dx_2, \quad (51)$$

we also get

$$A_{[3]}^{I1A} = 0, \quad B_{[2]} = -\left[\frac{2ma \cos \theta}{\Sigma}\right] dt \wedge dx_1 \quad (52)$$

which precisely is the 2-form electric NSNS tensor potential sourced by a $F1 - \bar{F}1$ pair. Thus this new solution can be identified with a $F1 - \bar{F}1$ system intersecting with a NSNS $F6$-brane since for $B = 0$, it correctly reduces to the usual $F1 - \bar{F}1$ solution given earlier, while for $m = 0$ and $a = 0$, it reduces to (a special case of) NSNS $F6$-brane given in eq.(42). And next, the magnetic NSNS fluxbrane content in this new solution above becomes recognizable by reducing one more time down to 9-dimensions along $(\partial/\partial x_9)$, i.e.,

$$ds_{10}^2 = H^{-1}[-dt^2 + dx_1^2] + \sum_{m=2}^{5} dx_m^2 + (\Delta + a^2 \sin^2 \theta) \left(\frac{dr^2}{\Delta} + d\theta^2\right) + \Delta \sin^2 \theta (d\tilde{\phi} + Bdx_9)^2 + dx_9^2 = ds_{9}^2 + e^{2\phi}(dx_9 + A_\alpha dx^\alpha)^2. \quad (53)$$
The resulting 9-dimensional fields are

\[ ds^2_9 = H^{-1}[-dt^2 + dx_1^2] + \sum_{m=2}^5 dx_m^2 + (\Delta + a^2 \sin^2 \theta) \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + f^{-1} \Delta \sin^2 \theta d\tilde{\phi}^2, \]

\[ e^{2\phi} = f, \]

\[ A_{[1]} = f^{-1} B \Delta \sin^2 \theta d\tilde{\phi} \]

with \( f = (1 + B^2 \Delta \sin^2 \theta) \). Again, the emergence of the KK gauge field \( A_{\tilde{\phi}} d\tilde{\phi} \) indicates that this 9-dimensional supergravity solution and hence its 10-dimensional ancestor given above do represent Melvin-type magnetic fluxbrane.

It is rather obvious that for both \( NS5 - \bar{NS}5 \) and \( F1 - \bar{F}1 \) systems, the conical singularity structure remains the same although we have introduced into the system the \( NSNS \) F6-brane content again aiming at counterbalancing the gravitational and gauge attractions and hence keeping the brane-antibrane systems against collision and subsequent annihilation. Namely, we still cannot eliminate the conical singularities along the axes \( \theta = 0, \pi \) and along the segment \( r = r_+ \) at the same time. Moreover, there appears to be a rather unexpected point that needs to be clarified with care. Earlier, we discussed the intersection of the \( RR \) F7-brane with \( D_{2p} - \bar{D}_{2p} \) pairs in IIA-theory. There, we noticed that since generally a \( D_p \) brane couples directly to the flux of a \( F_{(p+1)} \)-brane, only the \( D6 - \bar{D}6 \) system, but not others, which has non-trivial coupling to the \( RR \) F7-brane, can be balanced in an unstable equilibrium against the combined gravitational (\( NSNS \)) and gauge (\( RR \)) attractions. Along this line of argument, for the case at hand, we may naturally expect that it would be the \( NS5 - \bar{NS}5 \) system, but obviously not \( F1 - \bar{F}1 \), which directly couples to the magnetic flux of the \( NSNS \) F6-brane and, as a result, can be balanced in an unstable equilibrium. This, however, turns out not to be the case. Namely, despite the introduction of the \( NSNS \) F6-brane content, the \( NS5 - \bar{NS}5 \) system, let alone the \( F1 - \bar{F}1 \) system, still preserve essentially the same conical singularity structure and hence exhibit the unaffected semi-classical instability. Indeed this puzzle has an immediate explanation and it is due to the fact that unlike the \( RR \) F7-brane content which carries the non-vanishing magnetic 2-form flux \( F_{[2]} \), the \( NSNS \) F6-brane content as has been constructed via the KK-reduction not along \( (\partial/\partial y) \) but instead along \( (\partial/\partial x_9) \) carries no non-trivial magnetic 3-form flux \( H_{[3]} \) proportional to the magnetic strength parameter \( B \). We already have witnessed this point in the expression for the pure \( NSNS \) F6-brane content that has been extracted by setting \( m = 0 \) and \( a = 0 \) in the \( (NS5 - \bar{NS}5)||F6 \) and \( (F1 - \bar{F}1)||F6 \) solutions given above. As a result, there is simply no magnetic \( NSNS \) flux for the \( NS5 - \bar{NS}5 \) pair to couple to and hence no repulsive force between the brane and the antibrane to counterbalance the
combined gravitational and gauge attractions. Unlike the \((D6 - \bar{D}6)||F7\) and \((D0 - \bar{D}0)||F7\) systems we discussed earlier, the point worthy of note in the present \((NS5 - \bar{NS5})||F6\) and \((F1 - \bar{F}1)||F6\) systems, however, lies in the fact that as the brane and the antibrane approach each other, i.e., as \(a \to 0\), they do merge and hence annihilate consistently with the fact that the introduction of the \(NSNS\) \(F6\)-brane content plays no role as far as in eliminating the semi-classical instability of these systems. To see this in an explicit manner, we take the \(F1 - \bar{F}1\) system, for example, and take the limit \(a \to 0\). First, in the absence of the \(NSNS\) \(F6\)-brane content,

\[
\begin{align*}
    ds^2_{10} &= \left(1 - \frac{2m}{r}\right) [-dt^2 + dx_1^2] + \sum_{m=2}^{6} dx_m^2 + dr^2 + r^2 \left(1 - \frac{2m}{r}\right) (d\theta^2 + \sin^2 \theta d\phi^2), \\
    e^{2\phi} &= \left(1 - \frac{2m}{r}\right), \quad B_{[2]} = 0
\end{align*}
\]

where we used \(\Sigma \to r^2\), \(\Delta \to r^2(1 - 2m/r)\), and hence \(H \to (1 - 2m/r)^{-1}\) as \(a \to 0\). In this limit, it appears that the opposite electric \(NSNS\) charges carried by \(F1\) and \(\bar{F}1\) annihilate each other since \(B_{[2]} = 0\) and the metric solution now has the topology of \(R \times R^7 \times S^2\). Particularly, the manifest \(SO(3)\)-isometry in the transverse space implies that, as they approach, \(F1\) and \(\bar{F}1\) actually merge and as a result a curvature singularity develops at the center \(r = 0\). On the other hand, consider the \(a \to 0\) limit of the \((F1 - \bar{F}1)||F6\) solution

\[
\begin{align*}
    ds^2_{10} &= \left(1 - \frac{2m}{r}\right) [-dt^2 + dx_1^2] + \sum_{m=3}^{6} dx_m^2 + dr^2 + r^2 \left(1 - \frac{2m}{r}\right) [d\theta^2 + \sin^2 \theta (d\tilde{\phi} + Bdy)^2] + dy^2, \\
    e^{2\phi} &= \left(1 - \frac{2m}{r}\right), \quad B_{[2]} = 0.
\end{align*}
\]

Evidently, even in the presence of non-zero magnetic field, i.e., \(B \neq 0\), the transverse space still exhibits \(SO(3)\)-isometry. The only effect of the non-zero \(NSNS\) magnetic field is to endow the transverse \((\tilde{\phi}, y)\) sector (where \(y = x_2\)) with non-trivial global topology and the local geometry of the transverse \((\theta, \tilde{\phi})\) sector is still that of \(S^2\). This indicates that since \(NSNS\) \(F6\)-brane and \(F1\) (and \(\bar{F}1\)) do not couple directly (since the first is magnetic whereas the second is electrically-charged under \(B_{[2]}\)), the \(F6\)-brane, playing no role in eliminating the semi-classical instability of the \(F1 - \bar{F}1\)-pair, simply cannot keep them from colliding and annihilating each other when \(F1\) and \(\bar{F}1\) are brought close enough together. And it should be clear that essentially the same is true for the case of \(NS5 - \bar{NS5}\) system. Namely as \(a \to 0\), \(NS5\) and \(\bar{NS5}\) do merge and annihilate each other despite the presence of the
NSNS F6-brane. Since this result is consistent with our naive expectation on the semi-classical endpoint of the “NS” brane-antibrane systems, we should feel comfortable with this conclusion. Nevertheless, one may be rather bewildered as this natural endpoint of the “NS” brane-antibrane systems turned out to be in sharp contrast with the puzzling picture of the semi-classical endpoint of the “R” brane-antibrane systems in the presence of the fluxbrane we discussed earlier. There, we observed that, despite its failure to serve to stabilize the $D_{2p} - \bar{D}_{2p}$ systems, the RR $F7$-brane played the role of keeping them from annihilating each other completely when $D_{2p}$ and $\bar{D}_{2p}$ are brought close enough together. And we attributed this apparent puzzle to the limitation of the semi-classical supergravity description which, for very small inter-brane separation of order the string scale, has to be replaced by the stringy description in which the $F7$-brane “delays” the “R” brane-antibrane annihilation process but only until the $F7$-brane itself decays.

Of course, there should be a resolution to this contrasting natures between the “NS”-charged case and “R”-charged case and it appears to be due to the fact that in the “NS”-charged case, there is simply no corresponding stringy description of the instability for very small separations. To be more precise, unlike in the “R”-charged case in which fundamental string $(F1)$ ending on $D_p$ (and $\bar{D}_p$) represented by the brane intersection rule, $(0|D_p, F1)$, develops, in its spectrum, a tachyonic mode which condenses as $D_p$ and $\bar{D}_p$ annihilate, in the “NS”-charged case, the fundamental string does not end on another fundamental string nor on NS5-brane, i.e., no intersection rules such as $(0|F1, F1)$ or $(0|F1, NS5)$ exists. The only possibilities known for the intersections among the NS-branes in IIA/IIB theories (deduced from T and S duality transformations) are $[19]$; $(1|F1, NS5)$, $(3|NS5, NS5)$. Thus there are simply no fundamental open strings connecting $F1 - \bar{F}1$ or $NS5 - \bar{NS5}$-pair and hence no associated tachyonic modes that replace the semi-classical instability of these NS brane-antibrane systems for very small separations to begin with. And on the side of the NSNS F6-brane (constructed via the KK-reduction as has been discussed earlier), it carries no magnetic 3-form flux $H_{[3]}$ to potentially shift the spectrum of open strings, if any. To summarize, in the “NS”-charged case, the stringy description of the instability is simply absent and only the semi-classical supergravity one exists and according to it, regardless of the presence or absence of the NSNS F6-brane, $F1 - \bar{F}1$ and $NS5 - \bar{NS5}$ systems are destined to collide and annihilate. And we only conjecture that the end points would be supersymmetric vacua.

Thus far, we have argued, in the “NS”-charged case, that the (open) stringy description is absent to represent the quantum instability in the $(NS5 - \bar{NS5})||F6$ and $(F1 - \bar{F}1)||F6$
systems. This interpretation may provide a resolution to the contrasting features between
the instability of “R”-charged brane-antibrane systems and that of ‘NS’-charged ones. Cer-
tainly, however, it poses another puzzle that in the “NS”-charged case, the quantum entity,
that should take over the semi-classical instability as the inter-brane distance gets smaller,
is missing. Although this is rather an embarrassing state of affair, there indeed appears
be an way out as long as the NSNS F6-brane content is present in the “NS”-charged
brane-antibrane systems. To get right to the point, in the “NS”-charged case the tachyonic
modes, that the closed string sector in the NSNS F6-brane background develops, appears
to be responsible for the quantum instabilities of the F1 − F1 and NS5 − N̄S5 pairs as well
as for that of the NSNS F6-brane itself. The rationale for this argument has its basis on
the work of Russo and Tseytlin [17, 18] in which they demonstrated in an explicit manner
that the closed string in the background of the NSNS F6-brane given in eq.(42) (which
is a special case of the more general species of the NSNS F6-brane) develops a tachyonic
mode provided the magnetic field strength parameter B is greater than the critical value,
B > B_{cr} = (R/2a') or equivalently if the radius of the M-theory circle is smaller than some
critical value, R ≤ R_{cr} = √(2a'). And this happens when no oscillator degrees are excited
and for zero KK-momentum mode but when there is a lowest winding mode along the M-
theory circle. And presumably such closed string tachyonic modes may survive even when
the “NS”-charged brane-antibrane pairs are present as well.
Thus to summarize, in the “R”-charged case, both the open string tachyonic mode living
in the D_{2p} − D_{2p} systems and the closed string tachyonic mode presumably associated with
the non-supersymmetric and hence unstable RR F7-brane are expected to contribute to the
decays of both the fluxbrane and the brane-antibrane systems. Meanwhile in the “NS”-
charged case, the closed string tachyonic mode alone known to arise due to the unstable
NSNS F6-brane with large magnetic field strength (as has been described above) appears
to be responsible for the quantum instabilities and hence the decay of both the fluxbrane
and the brane-antibrane systems presumably via some mechanism such as the condensa-
tion. This suggested resolution, however, is still not without limitation. Namely, one might
wonder what happens if one erases the NSNS F6-brane contents in the (NS5 − N̄S5)||F6
and (F1 − F̄1)||F6 systems. Even then, will this picture still holds true? That is, might
the closed strings living in the bulk play some role regarding the quantum instability of the
F1 − F̄1 and NS5 − N̄S5 pairs as well? At the present stage of the development of the
physics of unstable brane systems, this question cannot be answered in any definite fashion
yet but certainly needs to be considered in a serious manner.
4 Summary and discussions

In the present work, we raised and then resolved all the relevant puzzles concerning at least the semi-classical instabilities of the \(“R”\)-charged and \(“NS”\)-charged brane-antibrane systems in type IIA-theory. And in order to intersect \(D0 - \bar{D}0\) pair with the \(RR F7\)-brane and \(NS5 - \bar{NS}5\) and \(F1 - \bar{F}1\) pairs with the \(NSNS F6\)-brane, we had to, along the way, uplift these solutions to \(D = 11\) using the M/IIA duality. In this way, we have constructed \(W - \bar{W}\) (i.e., M-wave/anti-M-wave) in eq.(26), \(M5 - \bar{M}5\) in eq.(34) and \(M2 - \bar{M}2\) in eq.(46), respectively and to our knowledge, these supergravity solutions representing the M-theory brane-antibrane systems have not been discussed in the literature yet and hence make their first appearance in the present work. Next related to this, it is our next curiosity what the relevant avenue would be toward the study of instability of \(“R”\)-charged brane-antibrane systems in type IIB-theory such as \(D1 - \bar{D}1\) and \(D5 - \bar{D}5\) systems of which the explicit supergravity solutions are known \[12\] (it is rather curious that the \(D3 - \bar{D}3\) solution is not known \[12\] nor can be obtained via the \(T\)-dual transformations from the known \(D1 - \bar{D}1\) or \(D5 - \bar{D}5\) solution). Also it seems worthy of note that the results of the analysis presented in this work suggest that the semi-classical description, based on the supergravity solutions, for the instabilities of the \(“R”\)-charged brane-antibrane systems is indeed consistent with the stringy description in terms of Sen’s argument on the endpoint of the unstable branes. We now elaborate on this point. Firstly, in the absence of the magnetic \(RR F7\)-brane, the behavior of the supergravity solutions representing \(D_{2p} - \bar{D}_{2p}\) systems for \(a \to 0\) exhibits that as they approach each other, the brane and the antibrane actually merge and develop curvature singularity at the center, \(r = 0\). In the presence of the \(RR F7\)-brane, however, the behavior of the corresponding supergravity solutions for \(a \to 0\) indicates that the \(RR F7\)-brane content of the solution plays the role of keeping the brane and the antibrane from annihilating each other completely since the two-brane configuration structure still persists in the supergravity solution even for for very small separation. And in terms of the stringy description, we interpreted this as representing that the \(RR F7\)-brane “delays” the brane-antibrane annihilation process by introducing an additional energy density to the total energy density or equivalently by providing a non-supersymmetric background that survives all the way but only until this non-supersymmetric and hence unstable \(F7\)-brane itself decays. Obviously, this phenomenon of the delay of brane-antibrane annihilation by the \(RR F7\)-brane is a generic stringy effect depending crucially on the tachyonic mode in the string spectrum and the supersymmetry argument. Thus one would not expect it to have
any non-supersymmetric point particle field theory analog. Nevertheless, it is amusing to
realize that at least this effect is not counter-intuitive when compared with its counterpart
in ordinary point particle field theory. That is, consider the (electrically) charged particle-
antiparticle annihilation in the presence of a strong external electric field. Were it not for
the external field, nothing could stop the particle-antiparticle pairs from annihilating each
other. The external electric field, however, would relax its strength via the Schwinger process
of particle pair creation until it exhausts all of its energy. Due to this continuous creation
of particle-antiparticle pairs while the external field is alive, the over-all pair annihilation
process would slow down, namely, an effective delay of the pair annihilation would take
place. Once the external field vanishes by converting all of its energy into the particle pair
creations, then the usual pair annihilation in the free space will resume. Again, although this
example is not a relevant analog of our brany process, this comparison appears to indicate
that the brany phenomenon we discussed above does not look so unphysical after all.
Lastly, we have demonstrated that the behavior of the supergravity solutions representing
$F1-F1$ and $NS5-\bar{NS}5$ for $a \to 0$ reveals that as they approach, these “$NS$”-charged brane
and antibrane always collide and annihilate irrespective of the presence or the absence of the
$NSNS$ $F6$-brane. And we have essentially attributed this to the absence of (open) stringy
description of the instability in the “$NS$”-charged case, namely fundamental open string does
not end on another fundamental string nor on $NS5$-brane. We find that all these results
from the semi-classical analysis based on explicit supergravity solutions serve as indirect
evidences supporting Sen’s argument for the evolution of unstable $D_p-\bar{D}_p$ system according
to which as the separation between the pair becomes of order the string scale, the open
string connecting $D_p$ and $\bar{D}_p$ develops a tachyonic mode and the $D_p-\bar{D}_p$ pair annihilates to
a supersymmetric vacuum as the associated open string tachyon condenses, i.e., rolls down
to a minimum of its potential.
We now would like to add more words in this direction. As has been demonstrated in the
present work, from the semi-classical perspective based on relevant supergravity solutions,
the endpoint of unstable $D_p-\bar{D}_p$ system is represented by merging and subsequent “collapse”
of the brane and the antibrane. And according to the $a \to 0$ limit of the supergravity
solutions representing $D_p-\bar{D}_p$ systems, the outcome of this collapse turns out to be a
neutral black $p$-brane (since the opposite $RR$ charges are cancelled) having a “singular”
horizon at $r = 2m$ (with $m$ being the brane tension) as well as the curvature singularity at
the center $r = 0$. Meanwhile as has been suggested by Sen, from the stringy perspective
based on the open string field theory, the eventual fate of the non-BPS $D_p-\bar{D}_p$ system could
be a supersymmetric vacuum via the open string tachyon condensation. Namely, the brane and the antibrane merge and annihilate each other completely since firstly, the opposite $RR$ charges are cancelled and secondly, the total energy of the system, upon merging, may vanish

$$E_{\text{tot}} = V(T_0) + 2M_D = 0. \quad (57)$$

Thus according to this conjecture by Sen, the outcome of the brane-antibrane collision could be a complete annihilation into a supersymmetric vacuum. In the “$NS$”-charged case, however, the situation changes as we have seen in the text. There, in terms of the semi-classical description based on the exact supergravity solutions, the endpoint of unstable $F1 - \bar{F}1$ or $NS5 - \bar{NS}5$ system still appears to be merging and the “collapse”. And hence the outcome of this collapse is again the neutral black string or black 5-brane having singular horizon at $r = 2m$ as well as the curvature singularity at the center $r = 0$. However, since fundamental string does not end on another $F1$ nor on $NS5$ (namely no intersection rules such as $(0|F1, F1)$ or $(0|F1, NS5)$ exists), there is as a result, no stringy description available for the brane-antibrane annihilation in terms of open string tachyon condensation via Sen’s mechanism. This absence of the quantum mechanism for the outcome of $F1 - \bar{F}1$ or $NS5 - \bar{NS}5$ annihilation is indeed a very unnatural state of affair in light of the fact that $F1 - \bar{F}1$ and $NS5 - \bar{NS}5$ systems are just $U = ST$ duals to $D2 - \bar{D}2$ and to $D6 - \bar{D}6$ systems respectively. Certainly, therefore, a quantum, stringy description is in need for these $F1 - \bar{F}1$ and $NS5 - \bar{NS}5$ annihilations into (presumably) supersymmetric vacua.

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Appendix

A Intersecting $D0 - \bar{D}0$ with an electric $RR$ fluxbrane

As we mentioned earlier, since $D0 - \bar{D}0$ and $D2 - \bar{D}2$ systems are “electrically” $RR$ charged, it may be natural to attempt to intersect them with electric $RR$ fluxbranes. Attempt of this sort, however, via the twisted KK-reduction of $W - \bar{W}$ and $M2 - \bar{M}2$ systems respectively, fails since the electric $RR$ fluxbranes constructed in this way turns out to be essentially trivial. Thus in this appendix, by taking the $D0 - \bar{D}0$ case for example, we shall show in an
explicit manner that this is what actually happens.

Start again with the $W - \bar{W}$ solution in $D = 11$ M-theory given by eq.(26) in the text. We first write the transverse coordinates as $(y = x_1, x^m, r, \theta, \phi)$ ($m = 1, \ldots, 6$) with $y$ being identified with the coordinate on the M-theory circle. Next, we consider the twisted KK-reduction of this $D=11$ solution. However, since we are interested in generating an “electrically” $RR$-charged fluxbrane, we now choose to perform the following non-trivial point identification

$$(y, t) \equiv (y + 2\pi n_1 R, t + 2\pi n_1 R^2 E), \quad n_1 \in \mathbb{Z}$$

(with $E$ being the electric field parameter) followed by the associated skew KK-compactification along the orbit of the Killing field

$$l = (\partial/\partial y) + ER(\partial/\partial t).$$

In the twisted KK-reduction of this type, however, one should worry about the emergence of closed timelike curves. Therefore, we introduce the adapted coordinate

$$\tilde{t} = t - ERy$$

which is constant along the orbits of $l$ and in terms of which, the metric is free of closed timelike curves since now the adapted time coordinate $\tilde{t}$ has standard semi-infinite range $0 \leq \tilde{t} < \infty$. Finally, we proceed with the standard KK-reduction along the orbit of $(\partial/\partial y)$. The result is

$$
\begin{align*}
\dd s_{10}^2 &= \tilde{\Lambda}^{1/2} \left\{ -H^{-1/2}d\tilde{t}^2 + H^{1/2}\left[ \sum_{m=1}^{6} dx_m^2 + (\Delta + a^2 \sin^2 \theta) \left( \frac{dr^2}{\Delta} + d\theta^2 \right) + \Delta \sin^2 \theta d\phi^2 \right] \right\} \\
&\quad + \tilde{\Lambda}^{1/2}H^{3/2} \left\{ A_t^2 - \tilde{\Lambda}^{-1}[A_t(1 + A_t ER) - H^{-2}ER]^2 \right\} d\tilde{t}^2,
\end{align*}
$$

$$e^{\tilde{t}\phi} = H\tilde{\Lambda},$$

$$A_{[1]} = \tilde{\Lambda}^{-1}[A_t(1 + A_t ER) - H^{-2}ER] d\tilde{t},$$

$$F_{[2]} = (\partial_t A_t)dr \wedge d\tilde{t} + (\partial_\theta A_t) d\theta \wedge d\tilde{t}, \quad \text{where}$$

$$A_t = \frac{2ma\cos \theta}{\Sigma}, \quad \tilde{\Lambda} = [(1 + A_t ER)^2 - H^{-2}E^2 R^2].$$

We next consider the conical singularity structure of this new solution. To do so, notice that again in this case, $\psi^\mu \psi_\mu = g_{\tilde{t}\phi} = 0$ still has roots at the locus of $r = r_+$ as well as along the semi-infinite axes $\theta = 0, \pi$. Thus we need to worry about the possible occurrence of conical singularities both along $\theta = 0, \pi$ and at $r = r_+$ again. Assuming that the azimuthal angle
coordinate $\tilde{\phi}$ is identified with period $\Delta \tilde{\phi}$, the conical deficit along the axes $\theta = 0, \pi$ and along the segment $r = r_+$ are given respectively by

$$
\delta_{(0, \pi)} = 2\pi - \left. \frac{\Delta \tilde{\phi} d\sqrt{g_{\phi\phi}}}{\sqrt{g_{\theta\theta}} d\theta} \right|_{\theta=0, \pi} = 2\pi - \Delta \phi, \quad (62)
$$

$$
\delta_{(r=r_+)} = 2\pi - \left. \frac{\Delta \tilde{\phi} d\sqrt{g_{\phi\phi}}}{\sqrt{g_{rr}} dr} \right|_{r=r_+} = 2\pi - \left( 1 + \frac{m^2}{a^2} \right)^{1/2} \Delta \phi
$$

where, of course, we used the metric solution given in eq.(61). Note that the conical singularity structure remains essentially the same as those for the $D0 - \bar{D}0$ system despite the introduction into the system an electric $RR$ fluxbrane content. Indeed, this has been expected since the electric fluxbrane content we attempted to introduce via this twisted KK-reduction turns out to be a trivial one having null structure. In order to get the physical explanation for this failure, we examine the nature of the electric fluxbrane content. The pure electric fluxbrane content in this $(D0 - \bar{D}0)|(electric fluxbrane)$ solution given in eq.(61) can be extracted in an unambiguous manner simply by erasing the brane-antibrane content. That is, by setting $m = 0$ and $a = 0$, we get

$$
ds_{10}^2 = \tilde{\Lambda}^{1/2} [-dt^2 + \sum_{i=1}^{6} dx_i^2 + dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)] - \tilde{\Lambda}^{-1/2} E^2 R^2 dt^2
$$

$$
= \tilde{\Lambda}^{1/2} [-dt^2 + \sum_{i=1}^{6} dx_i^2 + dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)], \quad (63)
$$

$$
F_{[2]} = 0 \quad (vaccum)
$$

where $\tilde{\Lambda} = (1 - E^2 R^2)$ and in the last line, we redefined $t = (1 + \tilde{\Lambda}^{-1} E^2 R^2)^{1/2} \tilde{t}$. This is essentially a flat spacetime. And it means that although we attempted to introduce into the $D0 - \bar{D}0$ system an electric fluxbrane via the twisted KK-reduction by mixing the orbits of the Killing fields $(\partial/\partial y)$ and $(\partial/\partial \phi)$, it turns out that no non-trivial electric fluxbrane was generated. Precisely due to this null nature of the electric fluxbrane, the conical singularity structure remained unchanged as we have seen above. Lastly, it seems noteworthy that for the earlier twisted KK-reduction in which the orbits of the Killing fields $(\partial/\partial y)$ and $(\partial/\partial \phi)$ were mixed, the non-trivial magnetic fluxbrane was generated whereas for the present case when those of the Killing fields $(\partial/\partial y)$ and $(\partial/\partial \tilde{t})$ are mixed, trivial fluxbrane with null structure results. It is interesting to note that indeed, this is reminiscent of an well-known solution-generating technique in 4-dimensional Einstein-Maxwell theory [20] which can be stated as: the axial Killing vector $\psi^\mu = (\partial/\partial \tilde{\phi})^\mu$ in a vacuum spacetime generates a stationary, axisymmetric test electromagnetic field which asymptotically approaches a uniform mag-
netic field whereas the time-translational Killing vector $\xi^\mu = (\partial/\partial t)^\mu$ in a vacuum spacetime generates a stationary, axisymmetric electromagnetic field which vanishes asymptotically.

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