Hierarchi problem

Sergey G Rubin
National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe highway 31, Moscow, 115409, Russia
E-mail: sergeirubin@list.ru

Abstract. The way to solve the hierarchy problem based on multidimensional gravity is discussed. Various metrics of deformed extra space are produced at the Planck scale. It is shown that the Higgs vacuum value depends on a metric of extra space and hence their different numerical values are realized in various universes. An interval of the Higgs vacuum values is proved include zero value. Our universe belongs to a set of universes the vacuum values of which are close to zero.

1. Introduction
Main problem discussed in this paper concerns the smallness of electroweak scale or more definitely the vacuum average of the Higgs field comparing to the Planck scale.

In terms of the multidimensional gravity vacuum state \( v \) of the Higgs field depends on a metric \( G \) of an extra space, \( v = v(G) \). As was shown in [1], [2], [3] the metric continuously varies in wide range. Each specific metric is realized in a specific universe. Our Universe is associated to those extra metric which gives extremely small ratio \( v/M_{Pl} \). Prompt realization of this idea is described below. This approach assumes existence of large number of various universes, see also [4].

2. Multidimensional Lagrangian
Let us consider a space \( M = M_4 \times V_n \) with metric

\[
d s^2 = G_{AB} dZ^A dZ^B = g_{\mu\nu}(x) dx^\mu dx^n + G_{ab}(y) dy^a dy^b \tag{1}
\]

Here \( M, M' \) are the manifolds with metrics \( g_{\mu\nu}(x) \) and \( G_{ab}(x,y) \) respectively. \( x \) and \( y \) are the coordinates of the subspaces \( M \) and \( M' \). We will refer to 4-dim space \( M \) and \( n \)-dim compact space \( M' \) as a main space and an extra space respectively. Here the metric has the signature \( (+ - - - ...) \), the Greek indices \( \mu, \nu = 0, 1, 2, 3 \) refer to 4-dimensional coordinates). Latin indices run over \( a, b, ... = 4, 5, ... \). Consider the Higgs field action acting in 6-dim manifold

\[
S = \frac{m^{D-2}_D}{2} \int d^D Z \sqrt{|G|} \left[ f(R) + L_H \right]; \tag{2}
\]

\[
f(R) = u_1(R - R_0)^2 + u_2 \tag{3}
\]

\( L_H \) is a Lagrangian of the proto-Higgs field

\[
L_H = \{ \partial_A H^+ G^{AB} \partial_B H - U(H^+H) \} \tag{4}
\]

\[
U = b_2 (H^+ H) + b_4 (H^+ H^2). \tag{5}
\]
Equations of motion are

\[ \nabla_{+}^{2}H = b_{2}H + 2b_{4} (H^{+}H) H; \quad h.c. \]  

(5)

Classical equations for the metric of extra space have the following form

\[ R_{ab}f' - \frac{1}{2} f(R) G_{ab} \nabla^{2} - \nabla_{a} \nabla_{b} fR + G_{ab} \nabla^{2} f' = \frac{1}{m_{D}^{2}} T_{ab}, \]  

(6)

+ additional conditions.

Here the first term proportional to \( R_{4} \) is omitted due to inequality \( R_{4} \ll R_{n} \) assumed throughout the paper. The choice of metric \( G_{\theta \theta} = -r(\theta)^{2}; \ G_{\phi \phi} = r(\theta)^{2} \sin^{2}(\theta) \) leads to the Ricci scalar expressed in terms of the radius \( r(\theta) \).

As a result explicit form of equation (6) to be solved numerically is

\[ \partial_{\theta}^{2}R + \cot \theta \partial_{\theta}R = -\frac{1}{2} r(\theta)^{2} \left( R_{0}^{2} - R^{2} \right) + w_{2}/u_{1}. \]  

(7)

As the additional conditions let us fix the metric at the point \( \theta = \pi: r(\pi) = r_{\pi}; r'(\pi) = 0; R(\pi) = R_{\pi}; R'(\pi) = 0 \). From here on we will use the units \( m_{D} = 1 \). Evidently, there is a set of solutions to system (6) depending on additional conditions. Maximally symmetrical extra spaces which are used in great majority of literature represent a small subset of this set.

Let us seek for the solution in the form

\[ H(Z) = H(x, y) = \chi(x)W(y) \]  

(8)

where the function \( \chi(x) \) relates to the Higgs field. The proto-Higgs Lagrangian density acquires the form

\[ L_{\chi} = \{ W(y)^{2} \partial_{\mu} \chi(x)^{+} g^{\mu \nu} \partial_{\nu} \chi(x) + \chi(x)^{+} \chi(x) G^{yy}(\partial_{y} W)^{2} \]  

\[-b_{2} W(y)^{2} \chi(x)^{+} \chi(x) - b_{4} W(y)^{4} [\chi(x)^{+} \chi(x)]^{2} \]  

(9)

after substitution (8) into (4).

Equation (5) is transformed into equation

\[ \nabla_{x}^{2} W(y) + b_{2} W(y) = -2b_{4} \chi^{+}(x) \chi(x) W(y)^{3} \simeq \]  

\[-2b_{4} \langle \chi^{+}(x) \chi(x) \rangle W(y)^{3}. \]  

(10)

with additional conditions

\[ W(\pi) = W_{\pi}; \quad W'(\pi) = 0. \]  

(11)

Dimensionless average \( \langle \chi^{2} \rangle \) is supposed to be of order unity. Integrating out extra-coordinates \( y \) in action (2) with Lagrangian (9) we obtain the known Higgs action

\[ S_{H} = \int d^{4}x \sqrt{g(x)} \{ \partial_{\mu} h(x)^{+} g^{\mu \nu} \partial_{\nu} h(x) + \lambda_{eff} v_{eff}^{2} h(x)^{+} h(x) - \lambda_{eff} [h(x)^{+} h(x)]^{2} \} \]  

(12)

where

\[ \lambda_{eff} \equiv \frac{4B_{4}}{K^{2}}, \quad v_{eff}^{2} \equiv -\frac{B_{2}}{K \lambda_{eff}}. \]  

(13)
and

\[ K = \frac{m_D^{D-2}}{2} \int d^n y \sqrt{|G(y)|} W(y)^2 \]

\[ B_2 = \frac{m_D^{D-2}}{2} \int d^n y \sqrt{|G(y)|} \left[ b_2 W(y)^2 - G^{yy} (\partial_y W)^2 \right] \]

\[ B_4 = \frac{m_D^{D-2}}{2} b_4 \int d^n y \sqrt{|G(y)|} W(y)^4. \]  

(14)

The Higgs field \( h \) is connected to the proto-Higgs field \( \chi \) as \( h(x) \equiv \sqrt{K} \chi \). After the Higgs discovery its self coupling is known and is expressed in terms of the Higgs particle mass \( \lambda_{\text{eff}} = m_{\text{Higgs}}^2 / (2 v_{\text{Higgs}}) = 125^2 / (2 \cdot 246^2) = 0.13 \). This value determines parameter \( b_4 \) according to expressions (13), (14). The main aim of the paper is to find conditions for inequality

\[ v_{\text{eff}}^2 \equiv \frac{1}{\lambda_{\text{eff}}} |B_2| / K \ll m_D^2 \]  

(15)

see (13), representing the Hierarchy problem. As was discussed above it is enough to prove that values \( v_{\text{eff}} \) is settled in an extremely tight interval including zero value. The left hand side of inequality (15) depends on random additional conditions caused by space-time foam.

Dependence of the Higgs vacuum average \( v \) on the boundary value \( W_\pi \) was obtained numerically. The curve intersects zero at point \( W_\pi = 0.3 \). It means that universes with the vacuum average be arbitrary close to zero do exist. We are in one of such universe.

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References

[1] Rubin S 2015 EPJC 75 333 (arXive:gr-gc/1503.05011)
[2] Gani V, Dmitriev A and Rubin S 2015 Int. J. Mod. Phys. D 24 1545001 (arXive:gr-gc/1411.4828)
[3] Rubin S 2015 Gravitation & Cosmology 21 143 (arXive:gr-gc/1403.2062)
[4] Freivogel B and Susskind L 2004 Phys.Rev. D 70 126007 (arXive:hep-th/0408133)