Quantum Phase Transition of the Electron-Hole Liquid In the Coupled Quantum Wells

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**Abstract**

Many-component electron-hole plasma is considered in the Coupled Quantum Wells (CQW). It is found that the homogeneous state of the plasma is unstable if the carrier density is sufficiently small. The instability results in the breakdown into two coexisting phase - a low-density gas phase and a high-density electron-hole liquid. The homogeneous state of the electron-hole liquid is stable if the distance between the quantum wells $\ell$ is sufficiently small. However, as the distance $\ell$ increases and reaches a certain critical value $\ell_{cr}$, the plasmon spectrum of the electron-hole liquid becomes unstable. Hereupon, a quantum phase transition occurs, resulting in the appearance of the charge density waves of finite amplitude in both quantum wells. The strong mass renormalization and the strong $Z$-factor renormalization are found for the electron-hole liquid as the quantum phase transition occurs.
I. INTRODUCTION

For a long time the investigation of the 2D strongly correlated electron system attracts a great interest of both theorists and experimentalists (see e.g. Refs. [1–11]). The electron hole-plasma (EHP) in the coupled quantum wells (CQW), where the electrons are localized in one quantum well and the holes are localized in the other quantum well, occupies a special place among the low dimensional many-electron systems [12–33]. The interest in the CQW has greatly grown in the recent years due to the increasing ability of manufacturing the high quality quantum well structures in which electrons and holes are confined in the different spatial regions between which the tunneling can be made negligible [32]. The EHP in the CQW is a nonequilibrium one, but the electrons and the holes have a large lifetime due to the spatial separation [12]. Strong electron-hole correlations in such systems can result in the creation of excitons which are the bound electron-hole states. A possibility of the exciton Bose-Einstein condensation (BEC) as well as the superfluidity and the superconductivity in the COW are considered microscopically in Refs. [12, 13]. The gas-liquid transition, the features of the liquid exciton phase and the transition into the superfluid phase are studied as a function of the distance ℓ between the electron and the hole layers in the CQW in Ref.[14]. The strongly nonideal system of the excitons in the CQW considered as structureless bosons was considered in [24–26], the exciton correlation being taken into account in a semi-phenomenological way.

Below we propose a microscopical description of strongly-correlated multi-component electron-hole liquid (EHL) which is a nonideal multi-component plasma (EHP) in the CQW at zero temperature. The number of different kinds of the electrons and of the holes is assumed to be large. The electron-hole system in a many-valley semiconductors is a typical representative of the multi-component EHP [27]. As it was shown for the first time in Ref. [34], the multi-component EHP in bulk semiconductors possesses the unconventional Coulomb screening. Such remarkable feature is connected with occurrence of characteristic momentum $p_0$ and characteristic energy $\omega_0$ which far exceed the Fermi-momentum $p_F$ and the Fermi-energy $\varepsilon_F$, respectively. The parameters $p_0$ and $\omega_0$ determine the region of the plasmon spectrum which mainly responsible for the unconventional Coulomb screening in the multi-component EHP [34]. Such property of the multi-component EHP was employed for investigation of various features of the electron-hole liquid (EHL) [33, 36]. The features
inherent in the multi-component EHP are also relevant for the multi-component electron gas at the uniform positive background and the EHP and electronic gas with strong anisotropic electron spectrum in the quasi-one-dimensional and quasi-two-dimensional system \[34, 37-41\].

A possibility of a bulk phase transition of the EHP into EHL was considered in \[42\], followed by the numerous experimental and the theoretical investigations (see e.g. \[41, 43\]). This phase transition is a consequence of the instability of the neutral homogeneous EHP if it has a density smaller than certain critical value \(n_c\). The instability results in the appearance of drops of the EHL with the equilibrium density \(n_{eq} > n_c\). It is remarkable that, if the bulk EHP is a multi-component one, both \(n_c\) and \(n_{eq}\) are completely determined by the number of the component \(\nu\) \[34\].

The energy of the ground state and the chemical potential of the multi-component EHP in the CQW were calculated in Refs.\[44–46\] as a function of the electron density \(n\) (which is the same for the holes), the inter-plane distance \(\ell\), and the number of the components \(\nu \gg 1\). The critical concentration \(n_c = n_c(\ell, \nu)\) was found such that, for the concentration \(n < n_c\), a homogeneous in-plane charge distribution is unstable. Such instability resulted in the formation of the EHL with the equilibrium density \(n_{eq} = n_{eq}(\ell, \nu) > n_c\), \(n_{eq} \sim \nu^{3/2}\) if \(\ell \ll \nu^{-1} \ll 1\), and \(n_{eq} \sim \ell^{-3/2}\) if \(\ell \gg \nu^{-1}\) \[44\]. It is shown in Ref. \[44\] that, for the density \(n = n_{eq}\), the in-plane exciton radius is of the order of the average distance between the charge carriers within the quantum well. This fact does not evidence in favour of an existence of exciton as an structureless particle in the CQW. Instead, strong electron-hole correlations near the Fermi surface remain. These correlations, in turn, can result in the unconventional Coulomb screening (inherent in the multi-component EHL), and in the superconductivity induced due to the Coulomb interaction alone \[35, 36\].

In the present paper we investigate the features of the EHL in the CQW, whose existence is predicted in \[44, 46\]. Like these papers, the system of units is used in which the effective electron charge \(e^* = e/\sqrt{\kappa_0}\) (\(\kappa_0\) is the static permittivity), the bare electron mass \(m\) and the Planck constant \(\hbar\) are as follows \(e^* = m = \hbar = 1\). For such system of units, the effective Bohr radius, \(a_B = \hbar^2/me^* = 1\) which is taken as a length unit. For the sake of simplicity, we assume that the masses of electron and hole are equal. As is shown in \[45\], this assumption does not influence the result qualitatively but it simplifies the calculations significantly. According to Ref. \[46\], the plasmon spectrum of the EHL is stable for \(n = n_{eq}\).
if $\ell \ll 1$. In this case, as it is shown in the present paper, both the electron mass and the $Z$-factor for the Green function experience negligible renormalization induced by the Coulomb interaction. However, as the distance $\ell$ increases and reaches a certain critical value $\ell_{cr}$, the plasmon spectrum of the electron-hole liquid becomes unstable. Hereupon, a quantum phase transition occurs, resulting in the appearance of the charge density waves of finite amplitude in both quantum wells. The strong mass renormalization and the strong $Z$-factor renormalization are found for the electron-hole liquid as the quantum phase transition occurs. All the results obtained in Refs. [44–46] as well as in the present paper are based on the selection of the diagrams in the small parameters $1/\nu$. However, the results obtained seems to be qualitatively valid if the parameter $\nu$ is not too large. A relationship to the experiments available is considered.

II. GREEN FUNCTION IN THE MULTI-COMPARTMENT ELECTRON-HOLE PLASMA

The multi-component EHP in the CQW is described with the following Hamiltonian of the system $\hat{H} = \hat{H}_0 + \hat{V}_{\text{int}}$,

$$\hat{H}_0 = \sum_{\alpha\sigma\mathbf{k}} \frac{k^2}{2} a_{\alpha\sigma}^+ (\mathbf{k}) a_{\alpha\sigma} (\mathbf{k}),$$

$$\hat{V}_{\text{int}} = \frac{1}{2S} \sum_{\alpha\sigma\alpha'\sigma'} \mathbf{V}_{\alpha\alpha'} (\mathbf{q}) \times a_{\alpha\sigma}^+ (\mathbf{k}) a_{\alpha'\sigma'}^+ (\mathbf{k}') a_{\alpha'\sigma'} (\mathbf{k}' - \mathbf{q}) a_{\alpha\sigma} (\mathbf{k} + \mathbf{q}).$$

Here $\alpha = e$ stands for the electrons, while $\alpha = h$ stands for the holes; $\sigma = 1, \ldots, \nu$ labels the kind of the electron or the hole; $a_{\alpha\sigma}^+ (\mathbf{k})$ and $a_{\alpha\sigma} (\mathbf{k})$ are the electron or the hole creation and annihilation operators; $\mathbf{k}, \mathbf{k}', \mathbf{q}$ are the $2D$-momenta; $S$ is the area of the QWs. The Coulomb interaction $V_{\alpha\alpha'}$ is assumed to be independent of the kind of the particle, i.e. of the subscripts $\sigma$, and

$$V_{\alpha\alpha'} (\mathbf{q}) = \begin{cases} V_{ee} (q) = V_{hh} (q) = V = \frac{2\pi}{q}, & \alpha = \alpha'; \\ V_{eh} (q) = V' = -\frac{2\pi}{q} e^{-q\ell}, & \alpha \neq \alpha'. \end{cases}$$

A single-particle Green function $G_{\alpha\sigma} (K)$ depends neither on the subscript $\alpha$ nor on the subscript $\sigma$. Then,

$$G_{\alpha\sigma} (K) = \frac{1}{i\omega + \mu - k^2/2 - \Sigma (K)},$$

$$G (K) = \frac{1}{i\omega + \mu - k^2/2 - \Sigma (K)}.$$
where $\mu$ is a chemical potential, $\Sigma(K)$ is a self-energy part (SEP), $K = (i\omega, \mathbf{k})$, $\omega$ is the Matsubara frequency, $\mathbf{k}$ is the 2D-momentum. Like papers [44–46], the calculation of the Green function is based on the selection of the diagram in the small parameter $1/\nu \ll 1$. Let us represent the SEP as $\Sigma(K) = \Sigma_H + \Sigma^{(c)}(K)$, where $\Sigma_H = 2\pi n\ell$ is the $K$-independent Hartree contribution, and $\Sigma^{(c)}(K)$ involves both the exchange and the correlation contribution. Selecting the main sequence of the diagram in the parameter $1/\nu$ one obtains for the SEP $\Sigma^{(c)}(K)$ [44]

$$\Sigma^{(c)}(P) = -\int \frac{d\omega d^2k}{(2\pi)^3} U(K) G^{(0)}(\varepsilon+\omega, \mathbf{p}+\mathbf{k}).$$

(4)

Here the Green function is $G^{(0)}(K) = (i\omega + p_F^2/2 - k^2/2)^{-1}$, $p_F = 2\pi^{1/2}(n/\nu)^{1/2}$ is the Fermi momentum, $\varepsilon_F = 2\pi n/\nu$ is the Fermi energy, $n$ is the total concentration (the parameters $p_F, \varepsilon_F, n$ are the same for the electrons and the holes). The effective interaction reads

$$U(K) = \frac{V(k)}{1 - \chi(\ell) V(k) \Pi_0(K)},$$

(5)

where the polarization operator is given by

$$\Pi_0(K) = \nu \int \frac{d\omega_1 d^2k_1}{(2\pi)^3} G^{(0)}(K + K_1) G^{(0)}(K_1).$$

The function $\chi(\ell)$ is a monotonic, continuous and slowly varying one obeying the condition $\chi(\ell) = 2$ for $\ell \ll 1$ and $\chi(\ell) = 1$ for $\ell \gg 1$ [44].

We are interested in the $\Sigma^{(c)}(P)$ for the momenta and the frequencies which are close to the Fermi ones. On the, other hand, as shown in [44], the main contribution into integral (4) originates from the region around the $\omega \sim \omega_0 = n^{2/3}/2 \gg \varepsilon_F$ and $k \sim k_0 = n^{2/3} \gg p_F$. To calculate integral (4), one should take into account that for $\nu \gg 1$ the polarization operator $\Pi_0(K)$ can be substituted with its asymptotics for the momentum $k \gg p_F$ and the frequency $\omega \gg \varepsilon_F$ as

$$\Pi_0(K) = -nk^2/\left(\omega^2 + (k^2/2)^2\right).$$

(6)

Integral (4) is readily calculated by substitution $\mathbf{k} \rightarrow (\chi n)^{1/3} \mathbf{k}$, $\omega \rightarrow (\chi n)^{2/3} \omega$. Then, the calculation of the $\Sigma^{(c)}(P)$ for $p \ll k_0, \varepsilon \ll k_0^2$ results in the expression for the $\Sigma(P)$ in the form

$$\Sigma(P) = \Sigma(0, p_F) - i\varepsilon I_Z + \xi_p^{(0)} I_m, \quad \Sigma(0, p_F) = 2\pi n\ell - C (\chi n)^{1/3}, \quad \xi_p^{(0)} = (p^2 - p_F^2)/2$$

(7)

(8)
where \( I = C(\chi n)^{-1/3} \), \( I_m = C_m(\chi n)^{-1/3} \), \( \xi_p(0) = (p^2 - p_F^2)/2 \). The numerical calculation of the constants entering the \( \Sigma(P) \) gives \( C \approx 1.3, C_Z \approx 7.6 \), and \( C_m = 0.4 \).

The chemical potential \( \mu \) is determined via \( \Sigma(0, p_F) \) by the well-known relation

\[
\mu = \frac{p_F^2}{2} + \Sigma(0, p_F) = 2\pi n \left( \frac{1}{\nu + \ell} \right) - C(\chi n)^{1/3},
\]

and the Green function reads

\[
G(p) = G(i\varepsilon, p) = \frac{Z}{i\varepsilon - \xi^*_p}; \quad \xi^*_p = \frac{p^2 - p_F^2}{2m^*};
\]

\[
Z = \frac{1}{1 + I_z}; \quad \frac{1}{m^*} = \frac{1 + I_m}{1 + I_z} < 1.
\]

For the densities \( n < n_c = \left[ \frac{C\chi^{1/3}}{6\pi(\ell + 1/\nu)} \right]^{3/2} \), one has \( \partial\mu/\partial n < 0 \). This fact means an instability of the homogeneous EHP for sufficiently small densities. Then, chemical potential (9) determines the energy per particle

\[
E = \pi \frac{n}{\nu} + \pi n\ell - \frac{3}{4} C(\chi n)^{1/3}.
\]

This expression has a minimum for the density

\[
n_{eq} = \left[ \frac{C\chi^{1/3}}{4\pi(\ell + 1/\nu)} \right]^{3/2} > n_c.
\]

The minimum corresponds to the vanishing pressure. For this reason, the equilibrium state of the EHP at the density \( n = n_{eq} \) is the EHL.

Let us consider how the Coulomb interaction affects the effective mass \( m^* \) of the quasiparticle and the \( Z \)-factor of the renormalized Green function for the EHL, i.e. for the density \( n = n_{eq} \). Let \( \ell \ll 1 \). Then \( n_{eq} \gg 1 \). It follows from (11) that \( \Delta m = m^* - m \ll m \) and \( Z = 1 - \delta, \delta \ll 1 \). In the opposite case \( \ell \gg 1 \), one has \( n_{eq} \sim l^{-3/2} \approx 1 \). Then, according to (11), \( \Delta m = m^* - m \sim m \) and \( Z = 1 - \delta, \delta \sim 1 \) and the renormalization is significant. Thus, the renormalization induced by the Coulomb interaction is insignificant for \( \ell \ll 1 \) and is visible for \( \ell \gg 1 \).

III. VERTEX PART

To investigate the plasmon spectrum of the EHL and its stability, let us calculate the vertex part \( \Gamma_{\alpha\sigma,\alpha_1\sigma_1;\alpha'\sigma',\alpha'_1\sigma'_1} \) with two input fermion ends \( \alpha\sigma, \alpha_1\sigma_1 \) and two output fermion
ends $\alpha' \sigma', \alpha'_1 \sigma'_1$. In what follows, for brevity, we will use the notation $\Gamma_{\alpha \beta; \alpha' \beta'}$ instead of $\Gamma_{\alpha \sigma, \alpha_1 \sigma_1; \alpha' \sigma', \alpha'_1 \sigma'_1}$. Thus, we omit the subscripts $\sigma, \sigma_1 \sigma', \sigma'_1$. In particular, the notation $\alpha$, in fact, implies $\alpha \sigma$. This convention reflects the fact that the value of the vertex part does not depend on the value of the subscripts $\sigma, \sigma_1 \sigma', \sigma'_1$ at all. However, the omitted subscripts should be taken into account when the summation over such subscripts is necessary.

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FIG. 1: a)

The exact diagrammatic representation for the vertex function is given in Fig. 1a. In this figure the black circle with two input ends and two output ends represents the exact vertex part $\Gamma_{\alpha \beta; \alpha' \beta'}$; the black square with two input ends and two output ends represents the irreducible vertex part $\Gamma_{\alpha \beta; \alpha' \beta'}$. (Any diagram is called an irreducible one if it can not be cut across one interaction (dotted) line resulting in two uncoupled parts); the black triangular with one input end, one output end and one interaction end represents the irreducible vertex part $\Gamma_{\alpha \alpha' \beta \beta'}$, the wavy line denotes the effective Coulomb interaction $U_{\delta \delta'}$. In turn, the effective interaction $U_{\delta \delta'}$ is determined by the self-consistent diagrammatic equation in Fig. 1b, in which the dotted lines denote bare Coulomb interaction $U_{\delta \delta'}$, the inner lines with arrows denote the exact fermion Green functions. The diagrammatic equation in Fig. 1c is an exact relation between the irreducible vertex parts $\Gamma_{\alpha \beta; \alpha' \beta'}$ and $\Gamma_{\alpha \alpha' \beta \beta'}$. 

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So the analytic representation of the exact diagrammatic equation in Fig. 1a is given by

$$\Gamma_{\alpha\beta;\alpha';\beta'} = \Gamma_{\alpha\beta;\alpha';\beta'} + \sum_{\delta_1, \delta_2} \Gamma_{\alpha\beta;\delta_1} \cdot U_{\delta_1;\delta_2} \cdot \Gamma_{\beta\beta';\delta_2}^{(3)}$$

(14)

In Fig. 1b the self-consistent diagrammatic equation reads the effective interaction $U_{\delta_1;\delta_2}$, which enters Eq. (14). Thus,

$$U_{\delta_1;\delta_2} (K) = V_{\delta_1;\delta_2} (k) + \sum_{\rho, \eta} V_{\delta_1;\rho} (k) \Pi_{\rho;\eta} (K) U_{\eta;\delta_2}^{(14)} (K).$$

(15)

Here $\Pi_{\rho;\eta} (K)$ is the exact polarization operator which, according to Fig. 1b, reads

$$\Pi_{\rho;\eta} (K) = \Pi_{0}^{*} (K) \delta_{\rho} + \Pi_{\rho;\eta}^{(c)} (K).$$

(16)

In this equation, the polarization operator $\Pi_{0}^{*} (K)$ is determined as follows

$$\Pi_{0}^{*} (K) = \nu \int \frac{d\omega_1 d^2 k_1}{(2\pi)^3} G (K + K_1) G (K_1),$$

(17)

while the $\Pi_{\rho;\eta}^{(c)} (K)$ is determined via the vertex $\Gamma_{\alpha\beta;\alpha';\beta'}$ as is shown in Fig. 1b. The asymptotic expression for the $\Pi_{0}^{*} (K)$ is

$$\Pi_{0}^{*} (K) = -\nu \frac{Z^2}{2\pi} \frac{(kv_F)^2/2}{\omega^2 + (kv_F)^2/2}, \quad k \ll p_F$$

(18)

$$\Pi_{0}^{*} (K) = -n \frac{2Z^2 \xi_k^*}{\omega^2 + (\xi_k^*)^2}, \quad p_F \ll k \ll k_0 \sim n^{1/3}.$$  

(19)

The main diagrammatic sequence for the $\Gamma_{\alpha\beta;\alpha';\beta'}$ in the parameter $1/\nu \ll 1$ is shown in Fig. 2a. Let $\gamma_{\alpha\beta;\alpha';\beta'}$ (the light square) be irreducible bare vertex part which generates the main diagrammatic sequence for the vertex part $\Gamma_{\alpha\beta;\alpha';\beta'}$. One can show that the $\gamma_{\alpha\beta;\alpha';\beta'}$ is composed of two diagrams shown in Fig. 2b.

The simple reasoning reveals that $\gamma_{\alpha\beta;\alpha';\beta'}$ may be represented as follows: $\gamma_{\alpha\beta;\alpha';\beta'} = \gamma_{\alpha\beta;\alpha';\beta'}$. Then, e.g.,

$$\gamma_{ee} = \gamma_{hh} = \gamma; \quad \gamma (p_1, p_2, q) = -\int \frac{d^3 p}{(2\pi)^3} U (p) U (k - p) G (p_1 - p) [G (p_2 + p) + G (p_2 + k - p)]$$

(20)
FIG. 2: a) The main diagrammatic sequence for the $\Gamma_{\alpha\beta;\gamma\delta}$; b) the bare irreducible vertex function $\gamma_{\alpha\alpha'}$.

The effective interaction $U(K)$ in Eq. (20) is given by Eq. (15). An analysis of the integrand in Eq. (20) (which is similar to the analysis of the integrand in Eq. (4)) shows that the main contribution into integrals (20) originates from the region $k \sim k_0 \sim n^{1/3} \gg p_F$ and $\omega \sim \omega_0 \sim k_0^2 \gg \varepsilon_F$. For this reason, if the components of the external momenta $p_1, p_2, q$ are much smaller than $k_0, \omega_0$, then one can neglect $p_1, p_2, q$ in the integrand. Therefore, the vertex part $\gamma(p_1, p_2, q)$ does not depend on the $p_1, p_2, q$. After the simple transformation one obtains

$$\gamma = -\frac{1}{2n^2} \int \frac{d^2kd\omega}{(2\pi)^3} U(K) U(-K) (\Pi_0(K))^2. \quad (21)$$

To calculate integral (21), let us take into account that, for $\nu \gg 1$, Eq. (14) can be used for the polarization operator $\Pi_0(K)$ for large transfer momentum $k \gg p_F$. Then, the integral is readily calculated by substitution $k \rightarrow (\chi n)^{1/3} k$, $\omega \rightarrow (\chi n)^{2/3} \omega$. As a result, one obtains

$$\gamma = -C_\gamma \frac{1}{(\chi n)^{2/3}}, \quad C_\gamma \approx 0.4 \quad (22)$$

Similarly, for small external momenta one has

$$\gamma_{eh} = \gamma'; \quad \gamma' = -\frac{1}{2n^2} \int \frac{d^2kd\omega}{(2\pi)^3} U'(K) U'(-K) (\Pi_0(K))^2. \quad (23)$$

Here $U'(K)$ is the effective electron-hole interaction. As is mentioned above, integrals like (23) are determined by the region $k \sim k_0 \sim n^{1/3} \gg p_F$ and $\omega \sim \omega_0 \sim k_0^2 \gg \varepsilon_F$. For this region, the integrand is proportional to $U'(K) \sim V(k_0) \sim \exp(-k_0\ell)$. In what follows, we are interested in the densities $n \sim n_{eq}$ (see Eq. (13)). In this case for $\ell \ll 1$ the parameter $k_0\ell \ll 1$ and one has $U'(K) = -U(K)$. In the opposite case $\ell \gg 1$ one has $k_0\ell \sim \ell^{1/2} \gg 1$.
and the integrand in (23) vanishes. Thus, we have

\[ \gamma' = \gamma \text{ for } \ell \ll 1; \quad \gamma' = 0 \text{ for } \ell \gg 1 \]  

(24)

Since the bare vertex parts \( \gamma \) and \( \gamma' \) are constant, the irreducible vertex \( \Gamma_{\alpha\beta;\alpha';\beta'} \) depends only on the momentum transfer and, thus, \( \Gamma_{\alpha\beta;\alpha';\beta'} = \overline{\Gamma}_{\alpha\beta;\alpha';\beta'}(k, \omega) \). The main sequence of the diagram in the parameter \( 1/\nu \) for \( \overline{\Gamma}_{\alpha\beta;\alpha';\beta'} \) (see Fig. 2(a)) is easily sum for \( k \ll k_0 \) and \( \omega \ll \omega_0 \). Taking into account that the vertex part can be represented in the form \( \Gamma_{\alpha\beta;\alpha';\beta'} = \Gamma_{\alpha\beta;\alpha';\beta'}(k, \omega) \), we have

\[ \Gamma_{\alpha\beta;\alpha';\beta'}(k, \omega) = \Gamma_{\alpha\beta;\alpha';\beta'}(K) \delta_{\alpha\alpha'} \delta_{\beta\beta'}, \]  

(25)

These expressions are used to calculate the correlation part of the polarization operator \( \Pi_{\rho\eta}^{(c)}(K) \) (see Eq. (16)) and the vertex \( \Gamma_{\alpha,\alpha';\delta}^{(3)} \) (see. Fig. 1c). As a result, one obtains

\[ \Pi_{\rho\eta}^{(c)}(K) = \Pi_{\rho\eta}^{(c)}(K) \Gamma_{\rho\eta} \Pi_{\rho\eta}^{(c)}(K), \]  

(28)

\[ \Gamma_{\alpha,\alpha';\delta}^{(3)}(K) = \Gamma_{\alpha,\alpha';\delta}(K) = \delta_{\alpha\delta} + \Gamma_{\alpha,\delta}(K) \Pi_{\rho\eta}^{(c)}(K). \]  

(29)

Substituting (25), (28), (29) into Eq. (14) for the vertex \( \Gamma_{\alpha\beta;\alpha';\beta'} \), one has \( \Gamma_{\alpha\beta;\alpha';\beta'} = \Gamma_{\alpha\beta;\alpha';\beta'}(K) \delta_{\alpha\alpha'} \delta_{\beta\beta'} \), where

\[ \Gamma_{\alpha\beta;\alpha';\beta'} = \Gamma_{\alpha\beta;\alpha';\beta'}(K) = \frac{(V + \gamma) - [(V + \gamma)^2 - (V' + \gamma')^2] \Pi_{\rho\eta}^{(c)}}{[1 - (V + V' + \gamma + \gamma') \Pi_{\rho\eta}^{(c)}] [1 - (V - V' + \gamma - \gamma') \Pi_{\rho\eta}^{(c)}]}, \]  

(30)

\[ \Gamma_{\alpha\beta;\alpha';\beta'} = \Gamma_{\alpha\beta;\alpha';\beta'}(K) = \frac{V' + \gamma'}{[1 - (V + V' + \gamma + \gamma') \Pi_{\rho\eta}^{(c)}] [1 - (V - V' + \gamma - \gamma') \Pi_{\rho\eta}^{(c)}]}, \]  

(31)

IV. PLASMON SPECTRUM AND INSTABILITY

Let us investigate the plasmon spectrum of the EHP in the CQW which is determined by poles of vertex parts (30) and (31). First, let us consider the case \( l \ll 1 \). Then, it follows from (30), (31) that
\[ \Gamma = \frac{(V - \pi \ell)}{[1 - 2 (V - \pi \ell) \Pi_0]} + \frac{(\gamma + \pi \ell)}{[1 - 2 (\gamma + \pi \ell) \Pi_0]}. \]  

(32)

\[ \Gamma' = \frac{-(V - \pi \ell)}{[1 - 2 (V - \pi \ell) \Pi_0]} + \frac{(\gamma + \pi \ell)}{[1 - 2 (\gamma + \pi \ell) \Pi_0]}. \]  

(33)

Let us substitute (18) into (32) and (33) and replace the Matsubara frequency \( i\omega \) by the real frequency \( \omega \). The pole of the vertex parts \( \Gamma \) and \( \Gamma' \) is given by the second terms in Eq. (32) or Eq. (33). Then, the plasmon spectrum is determined by equation

\[ 1 + 2 (\gamma + \pi \ell) \frac{\nu}{2\pi} \frac{(kv_F)^2 / 2}{-\omega^2 + (kv_F)^2 / 2} = 0. \]  

(34)

The spectrum is stable if \( \omega \), which obey Eq. (34), is real. This takes place if

\[ n > \left[ \frac{C\gamma}{\chi^{2/3} (1/\nu + \ell)} \right]^{3/2}. \]  

(35)

Thus, if \( n > n_{cr} = \left[ \frac{C\chi^{1/3}}{6\pi(1/\nu + \ell)} \right]^{3/2} \), the plasmon spectrum is stable. In the opposite case, \( n < n_{cr} \), the pole takes place for imaginary \( \omega \). This denotes an instability of the plasmon spectrum. This instability just corresponds to the thermodynamic instability of the homogeneous EHP for the densities \( n < n_{cr} \) for which one has \( \partial\mu/\partial n < 0 \) [34]. Let us pay attention that for \( \ell \ll 1 \), the plasmon spectrum remains stable for the equilibrium EHL which has the density \( n_{eq} = \left[ \frac{C\rho^{1/3}}{4\pi(1/\nu + \ell)} \right]^{3/2} > n_c \).

Now let us investigate the plasmon spectrum for the case \( \ell \gg 1 \). Let us consider momenta and frequencies which obey the limitations \( (1/\ell) \ll k \ll k_0 \), \( \omega \ll \omega_0 \). In this case \( V'(k) \) vanishes. Also, according to (24), \( \gamma' = 0 \). Therefore, it follows from (31) that \( \Gamma' = 0 \). So in the case \( \ell \gg 1 \) Eq. (30) reads

\[ \Gamma (K) = \frac{V (k) + \gamma}{1 - \Pi^*_0 (K) (V (k) + \gamma)}. \]  

(36)

Let us substitute \( V, \Pi^*_0, \gamma \) by expressions Eqs. (2), (19), (22) for the momentums \( p_F \ll k \ll k_0 \) and change \( i\omega \) by \( \omega \) As a result, one obtains

\[ \Gamma (K) = \frac{\left( \frac{2\pi}{k} - \frac{C\gamma}{n^{2/3}} \right) (\omega^2 - (\xi_k^*)^2)}{\omega^2 - \xi_k^* (\xi_k^* + nZ \left( \frac{2\pi}{k} - \frac{C\gamma}{n^{2/3}} \right))}. \]  

(37)
A pole of the \( \Gamma (K) \) determines the plasmon spectrum and exists for the frequencies
\[
\omega^2_p (k) = \xi_k^* \left( \xi_k^* + nZ^2 \left( \frac{2\pi}{k} - \frac{C_\gamma}{n^{2/3}} \right) \right). 
\] (38)

Let us investigate a behavior of the plasmon spectrum for the EHL of a density \( n \sim n_{eq} \). One can easily see that, for small momentums \( k \ll k_0 \) which additionally belong to the interval \( n^{2/3} \lesssim k \lesssim n^{1/2} \), the plasmon frequency \( \omega_p (k) \) becomes imaginary. This means an instability of the homogeneous state of the EHL with respect to an appearance of the spatially inhomogeneous periodic in-plane charge distribution with a period characterized by the wave vector \( k \). Such a charge density fluctuation describes the charge density waves (CDW), which are in-phase for the electron and the hole layers. For the equilibrium EHL with \( n \sim n_{eq} \sim \ell^{-3/2} \), one has \( \frac{1}{\ell} \lesssim k \lesssim \frac{1}{\ell^{3/4}} \) and, thus, the period of the CDW obeys the condition \( \ell^{3/4} \lesssim D \lesssim \ell \).

V. CONCLUSION

Thus, for \( \ell \ll 1 \) the homogeneous state of the EHL with the density \( n_{eq} \sim \nu^{3/2} \) is stable. However, as the distance \( \ell \) increases, the plasmon spectrum becomes softer for finite momenta \( k \gtrsim \frac{1}{\ell} \). Then, for a certain \( \ell_{cr} \sim 1 \), there appears a momentum \( k = k_{cr} = 1/\ell_{cr} \) for which the plasmon frequency vanishes. As the distance \( \ell \) increases, the plasmon frequencies characterized by the wave-vector interval \( \frac{1}{\ell} \lesssim k \lesssim \frac{1}{\ell^{3/4}} \) become imaginary. As a result, the CDW appears. This feature of the plasmon spectrum implies that EHL in the CQW experiences a quantum phase transition in the parameter \( \ell \).

Note that some of the results obtained above are valid for multi-component electron gas at the positive background [1 4 6 37 40]. In particular, this concerns the effective mass renormalization, the \( Z \)-factor renormalization for the single-particle Green function, the dependence of the ground state energy and the chemical potential of the electrons. Also, the conclusion remains valid and leads to an instability of the ground state of the electron gas with respect to appearance of the CDW for sufficiently small density. However, a significant difference takes place: in contrast to EHL the electron gas at the positive background cannot find the equilibrium density to minimize the ground state energy since the electron density is settled by the positive background.

There are several experiments in which the EHL is seemed to be observed in CQW [47–12].
The result obtained in the present paper may be verified for the systems investigated in these papers.

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