Discriminating among interpretations for $X(2900)$ states

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[T.B. & E. Swanson, 2008.12838]
[T.B. & E. Swanson, 2009.05352]
Experimental properties

Two $X(2900)$ states in $B^+ \rightarrow D^+ X, X \rightarrow D^- K^+$

Their minimal flavour content is $ud\bar{s}\bar{c}$

$X_0(2900)$ \quad 2.866 \pm 0.007 \pm 0.002 \text{ GeV} \quad 0^+$

$X_1(2900)$ \quad 2.904 \pm 0.005 \pm 0.001 \text{ GeV} \quad 1^-$
Experimental properties

Two $X(2900)$ states in $B^+ \rightarrow D^+ X$, $X \rightarrow D^- K^+$

Their minimal flavour content is $ud\bar{s}\bar{c}$

$X_0(2900)$ \hspace{1cm} $2.866 \pm 0.007 \pm 0.002$ GeV \hspace{1cm} $0^+$

$X_1(2900)$ \hspace{1cm} $2.904 \pm 0.005 \pm 0.001$ GeV \hspace{1cm} $1^-$

$\bar{D}^* K^*$ \hspace{1cm} $2.902$ GeV
Experimental properties

Two $X(2900)$ states in $B^+ \rightarrow D^+ X, X \rightarrow D^- K^+$

Their minimal flavour content is $ud\bar{s}\bar{c}$

| State      | Mass (GeV)         | Parity  |
|------------|--------------------|---------|
| $X_0(2900)$| $2.866 \pm 0.007 \pm 0.002$ | $0^+$   |
| $X_1(2900)$| $2.904 \pm 0.005 \pm 0.001$ | $1^-$   |
| $\bar{D}^* K^*$ | $2.902$ GeV        | $0^+, 1^+, 2^+$ (in S-wave) |
Experimental properties

Two $X(2900)$ states in $B^+ \rightarrow D^+ X, X \rightarrow D^- K^+$

Their minimal flavour content is $ud\bar{s}\bar{c}$

$X_0(2900) \quad 2.866 \pm 0.007 \pm 0.002 \text{ GeV} \quad 0^+$
$X_1(2900) \quad 2.904 \pm 0.005 \pm 0.001 \text{ GeV} \quad 1^-$

$\bar{D}^* K^* \quad 2.902 \text{ GeV} \quad 0^+, 1^+, 2^+ \text{ (in S-wave)}$
$\bar{D}_1(2420)K \quad 2.917 \text{ GeV}$
Experimental properties

Two $X(2900)$ states in $B^+ \rightarrow D^+ X$, $X \rightarrow D^- K^+$

Their minimal flavour content is $u d \bar{s} \bar{c}$

$\begin{array}{lll}
X_0(2900) & 2.866 \pm 0.007 \pm 0.002 \text{ GeV} & 0^+ \\
X_1(2900) & 2.904 \pm 0.005 \pm 0.001 \text{ GeV} & 1^- \\
\bar{D}^* K^* & 2.902 \text{ GeV} & 0^+, 1^+, 2^+ \text{ (in S-wave)} \\
\bar{D}_1(2420) K & 2.917 \text{ GeV} & 1^- \text{ (in S-wave)} \\
\end{array}$
Models
Tetraquark

Tetraquark interpretations:

▶ [Karliner & Rosner, 2008.05993]
▶ [He, Wang & Zhu, 2008.07145]
▶ [Zhang, 2008.07295]

But

▶ Mass inconsistent with variational quark model [Lu, Chen, Dong, 2008.07340]
▶ Analogy with bound lattice $ud\bar{b}\bar{b}$ is questionable
▶ If $X(2900)$ are orbital/radial excitations, where are ground states?
▶ No evidence for bound $ud\bar{s}\bar{c}$ in lattice [Hudspith et al 2006.14294], quark model [Zouzou et al 1986], QCD sum rules [Agaev et al 1907.04017]
▶ $1^-$ state awkward (P-wave)
Many models for $0^+ X_0(2900)$:

- **isoscalar** $\bar{D}^* K^*$ with vector hidden gauge [Molina et al 1005.0355, 2008.11171]
- **isoscalar** $\bar{D}^* K^*$ with effective Lagrangian [Liu et al 2008.07389]
- **isovector** $\bar{D}^* K^*$ with effective Lagrangian [He and Chen 2008.07782]
- **isoscalar** $\bar{D}^* K^*$ with heavy quark symmetry [Hu et al 2008.06894]
- **isoscalar** $\bar{D}^* K^*$ with molecular and diquark d.o.f. [Chen et al 2008.07516, Xue et al 2008.09516]

The $1^- X_1(2900)$ is more difficult:

- **virtual state from** $\bar{D}_1(2420) K$ [He and Chen 2008.07782]
Triangle [Liu et al 2008.07190]
Triangle [Liu et al 2008.07190]

Channels with thresholds near 2900 MeV
Triangle [Liu et al 2008.07190]

(a) $B^+ \rightarrow \chi_{c1} \rightarrow D^+ \rightarrow D^{*-} \rightarrow D^- \rightarrow K^+$

(b) $B^+ \rightarrow D_{sJ}^+ \rightarrow D_1^{0} \rightarrow K^+ \rightarrow K^0 \rightarrow D^-$

colour-suppressed!
Triangle [Liu et al 2008.07190]

 colour-suppressed!
Triangle [Liu et al 2008.07190]

colour-suppressed!
Triangle with FSIs [T.B. & Swanson 2008.12838]
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Triangle with FSIs [T.B. & Swanson 2008.12838]

[Diagram showing a triangle with vertices labeled B^+, D_s^{(*)}, K^{(*)}, and K. Arrows and lines indicate interactions and driving channels.]

driving channels with interactions
This is an example fit. Given parametric freedom, can't really distinguish triangle scenario (weak FSIs) from resonance scenario (strong FSIs)
Discriminating among models

[T.B. & E. Swanson, 2009.05352]
\[ \bar{b} \rightarrow \bar{c}(c\bar{s}) \] Cabibbo-favoured

colour-favoured

Other diagrams gives wrong flavours and/or are colour suppressed
\[ n\bar{n} = \frac{(u\bar{u} + d\bar{d})}{\sqrt{2}} \]
Note the absence of
\[ B^+ \rightarrow D^+ D^- K^+ \]
flavour only, e.g. \( \bar{D}^* K^0 \) or \( D^0 K^0 \)
\[
\begin{align*}
\bar{D}^0 & \quad X & \quad D^- \\
K^0 & \quad K^+ & (\bar{D}^0) \\
D^+ & \quad \quad (K^0)
\end{align*}
\begin{align*}
\bar{D}^0 & \quad X^+ & \quad \bar{D}^0 \\
K^+ & \quad K^+ & \quad D^0
\end{align*}
\]
\[ \begin{align*}
D^0 & \quad \bar{D}^0 \\
X & \quad D^- \\
K^- & \quad (\bar{D}^0) \\
K^+ & \quad (K^-)
\end{align*} \]
3 $B^+$ modes...

$B^+ \rightarrow D^+ X$, $X \rightarrow D^- K^+$,
$B^+ \rightarrow D^+ X$, $X \rightarrow \bar{D}^0 K^0$,
$B^+ \rightarrow D^0 X^+$, $X^+ \rightarrow \bar{D}^0 K^+$.
3 $B^+$ modes...
$B^+ \rightarrow D^+ X$, $X \rightarrow D^- K^+$,
$B^+ \rightarrow D^+ X$, $X \rightarrow \bar{D}^0 K^0$,
$B^+ \rightarrow D^0 X^+$, $X^+ \rightarrow \bar{D}^0 K^+$. 
... and another 3 for $B^0$
All six modes related by isospin properties of operator $O$, which follows from nature of $X$. 
Results for $X_1(2900)$ and its charged partners
(For $X_0(2900)$ scale by 5.6 / 30.6)

| $B(B \to D \bar{D}K)$ | $B^+ \to D^+ X$, $X \to D^- K^+$ | $B^0 \to D^0 X$, $X \to D^- K^0$ | $B^+ \to D^+ X$, $X \to \bar{D}^0 K^+$ | $B^0 \to D^0 X$, $X \to D^- K^+$ | $B^+ \to D^0 X^+$, $X^+ \to \bar{D}^0 K^+$ | $B^0 \to D^+ X^-$, $X^- \to D^- K^0$ |
|------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| $B(B \to D \bar{D}K)$  | 2.2 ± 0.7                         | 2.7 ± 1.1                         | 15.5 ± 2.1                       | 10.7 ± 1.1                       | 14.5 ± 3.3                       | 7.5 ± 1.7                        |

$f(B \to DX, X \to \bar{D}K)$

| Triangle, QE        | 30.6 | 23.2 | 0 | 0 | 4.6 | 8.3 |
| Triangle, OPE       | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT       | 30.6 | 23.2 | $1.1 \left(1 - \frac{C_0}{C_1}\right)^2$ | $1.5 \left(1 - \frac{C_0}{C_1}\right)^2$ | $1.2 \left(1 + \frac{C_0}{C_1}\right)^2$ | $2.1 \left(1 + \frac{C_0}{C_1}\right)^2$ |
| Resonance, $I = 0$  | 30.6 | 23.2 | 4.3 | 5.8 | 0 | 0 |
| Resonance, $I = 1$  | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, $I$ mixed| 30.6 | 23.2 | $4.3 \tan^2 \left(\theta + \frac{\pi}{4}\right)$ | $5.8 \tan^2 \left(\theta - \frac{\pi}{4}\right)$ |

$\Delta f/f$

| $\Delta f/f$ | 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |
Results for $X_1(2900)$ and its charged partners
(For $X_0(2900)$ scale by 5.6 / 30.6 )

| $B(B \to D \bar{D}K)$ | $B(B \to D \bar{D}K)$ | $B(B \to D \bar{D}K)$ | $B(B \to D \bar{D}K)$ | $B(B \to D \bar{D}K)$ | $B(B \to D \bar{D}K)$ |
|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| 2.2 ± 0.7            | 2.7 ± 1.1            | 15.5 ± 2.1           | 10.7 ± 1.1           | 14.5 ± 3.3           | 7.5 ± 1.7            |

$f(B \to DX, X \to \bar{D}K)$

| Method                  | Triangle, QE | Triangle, OPE | Triangle, EFT | Resonance, $I = 0$ | Resonance, $I = 1$ | Resonance, $I$ mixed | $\Delta f/f$ |
|------------------------|--------------|--------------|--------------|-------------------|-------------------|---------------------|-------------|
|                        | 30.6         | 30.6         | 30.6         | 30.6              | 30.6              | 30.6                | 0.1         |
|                        | 23.2         | 23.2         | 23.2         | 23.2              | 23.2              | 23.2                | 0.53        |
|                        | 0            | 1.1          | 1.1          | 4.3               | 4.3               | 4.3                 | 0.36        |
|                        |              |              |              | 0                 | 5.8               | 5.8                 | 0.35        |
|                        |              |              |              |                   |                   |                     | 0.41        |
|                        |              |              |              |                   |                   |                     | 0.41        |
|                        |              |              |              |                   |                   |                     |             |
Results for $X_1(2900)$ and its charged partners
(For $X_0(2900)$ scale by 5.6 / 30.6)

| $B(\rightarrow D\bar{D}K)$ | $2.2 \pm 0.7$ | $2.7 \pm 1.1$ | $15.5 \pm 2.1$ | $10.7 \pm 1.1$ | $14.5 \pm 3.3$ | $7.5 \pm 1.7$ |
|---------------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $f(\rightarrow DX, X \rightarrow \bar{D}K)$ |                     |                |                |                |                |                |
| Triangle, QE            | 30.6            | 23.2           | 0              | 0              | 4.6            | 8.3            |
| Triangle, OPE           | 30.6            | 23.2           | 1.1            | 1.5            | 1.2            | 2.1            |
| Triangle, EFT           | 30.6            | 23.2           | $1.1 \left(1 - \frac{C_0}{C_1}\right)^2$ | $1.5 \left(1 - \frac{C_0}{C_1}\right)^2$ | $1.2 \left(1 + \frac{C_0}{C_1}\right)^2$ | $2.1 \left(1 + \frac{C_0}{C_1}\right)^2$ |
| Resonance, $I = 0$      | 30.6            | 23.2           | 4.3            | 5.8            | 0              | 0              |
| Resonance, $I = 1$      | 30.6            | 23.2           | 4.3            | 5.8            | 18.6           | 33.4           |
| Resonance, $I$ mixed    | 30.6            | 23.2           | $4.3 \tan^2 \left(\theta + \frac{\pi}{4}\right)$ | $5.8 \tan^2 \left(\theta - \frac{\pi}{4}\right)$ |  |
| $\Delta f/f$            | 0.1             | 0.53           | 0.36           | 0.35           | 0.41           | 0.41           |
A triangle diagram with quark-exchange (QE), one-pion exchange (OPE) or effective field theory (EFT) interactions

| $B(B \to D\bar{D}K)$ | $B^+ \to D^+X$, $X \to D^-K^+$ | $B^0 \to D^0X$, $X \to \bar{D}^0K^0$ | $B^+ \to D^+X$, $X \to \bar{D}^0K^0$ | $B^0 \to D^0X$, $X \to D^-K^+$ | $B^+ \to D^0X^+$, $X^+ \to \bar{D}^0K^+$ | $B^0 \to D^+X^-$, $X^- \to D^-K^0$ |
|-----------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| $B(B \to D\bar{D}K)$ | $2.2 \pm 0.7$                     | $2.7 \pm 1.1$                     | $15.5 \pm 2.1$                   | $10.7 \pm 1.1$                   | $14.5 \pm 3.3$                   | $7.5 \pm 1.7$                    |

| $f(B \to DX, X \to \bar{D}K)$ |
|--------------------------------|
| Triangle, QE                  | 30.6                              | 23.2                              | 0                                | 0                                | 4.6                              | 8.3                              |
| Triangle, OPE                 | 30.6                              | 23.2                              | 1.1                              | 1.5                              | 1.2                              | 2.1                              |
| Triangle, EFT                 | 30.6                              | 23.2                              | $1.1 \left(1 - \frac{C_0}{C_1}\right)^2$ | $1.5 \left(1 - \frac{C_0}{C_1}\right)^2$ | $1.2 \left(1 + \frac{C_0}{C_1}\right)^2$ | $2.1 \left(1 + \frac{C_0}{C_1}\right)^2$ |
| Resonance, $I = 0$            | 30.6                              | 23.2                              | 4.3                              | 5.8                              | 0                                | 0                                |
| Resonance, $I = 1$            | 30.6                              | 23.2                              | 4.3                              | 5.8                              | 18.6                             | 33.4                             |
| Resonance, $I$ mixed          | 30.6                              | 23.2                              | $4.3 \tan^2 \left(\theta + \frac{\pi}{4}\right)$ | $5.8 \tan^2 \left(\theta - \frac{\pi}{4}\right)$ |

| $\Delta f/f$                 | 0.1                               | 0.53                              | 0.36                             | 0.35                             | 0.41                             | 0.41                             |
| $B(B \to D\bar{D}K)$ | $B^+ \to D^+X$, $X \to D^-K^+$ | $B^0 \to D^0X$, $X \to \bar{D}^0K^0$ | $B^+ \to D^+X$, $X \to \bar{D}^0K^0$ | $B^0 \to D^0X$, $X \to D^-K^+$ | $B^+ \to D^0X^+$, $X^+ \to \bar{D}^0K^+$ | $B^0 \to D^+X^-$, $X^- \to D^-K^0$ |
|---------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\mathcal{B}$      | 2.2 ± 0.7        | 2.7 ± 1.1        | 15.5 ± 2.1      | 10.7 ± 1.1      | 14.5 ± 3.3      | 7.5 ± 1.7       |

$f(B \to DX, X \to \bar{D}K)$

|                   | Triangle, QE    | Triangle, OPE   | Triangle, EFT  | Resonance, $I = 0$ | Resonance, $I = 1$ | Resonance, $I$ mixed |
|-------------------|-----------------|-----------------|-----------------|--------------------|--------------------|----------------------|
| $f$               | 23.2            | 1.1             | 1.1             | 0                  | 4.3                | 4.3 tan² $(\theta + \frac{\pi}{4})$ |
| Resonance, $I = 0$| 30.6            | 0               | 5.8             | 0                  | 18.6               | 5.8 tan² $(\theta - \frac{\pi}{4})$ |
| Resonance, $I = 1$| 30.6            | 23.2            | 4.3             | 5.8                | 33.4               | 0                    |
| Resonance, $I$ mixed | 30.6          | 23.2            | 4.3             | 5.8                | 33.4               | 0                    |

$\Delta f/f$

|                   | 0.1             | 0.53            | 0.36            | 0.35               | 0.41               | 0.41               |

Resonance, either molecular or tetraquark
Mixed isospin case relevant for molecule
\[ f(B \rightarrow DX, X \rightarrow \bar{D}K) = \frac{\mathcal{B}(B \rightarrow DX, X \rightarrow \bar{D}K)}{\mathcal{B}(B \rightarrow D\bar{D}K)} \]

|                  | \( B^+ \rightarrow D^+X, \)
|------------------|----------------------------------|
|                  | \( X \rightarrow D^-K^+ \)       | \( B^0 \rightarrow D^0X, \)
|                  |                                   | \( X \rightarrow \bar{D}^0K^0 \) |
|                  | \( B^+ \rightarrow D^+X, \)
|                  |                                   | \( X \rightarrow D^-K^+ \)       |
|                  | \( B^0 \rightarrow D^0X, \)
|                  |                                   | \( X \rightarrow \bar{D}^0K^0 \) |
|                  | \( B^+ \rightarrow D^0X^+, \)
|                  |                                   | \( X^+ \rightarrow \bar{D}^0K^+ \) |
|                  | \( B^0 \rightarrow D^+X^-, \)    | \( X^- \rightarrow D^-K^0 \)     |
| \( \mathcal{B}(B \rightarrow D\bar{D}K) \) | 2.2 ± 0.7                         | 2.7 ± 1.1                         | 15.5 ± 2.1                      | 10.7 ± 1.1                      | 14.5 ± 3.3                      | 7.5 ± 1.7                       |

\[ f(B \rightarrow DX, X \rightarrow \bar{D}K) \]

|                  | Triangle, QE                         | Triangle, OPE                        | Triangle, EFT                         | Resonance, \( I = 0 \)                     | Resonance, \( I = 1 \)                     | Resonance, \( I \) mixed                     |
|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
|                  | 30.6                            | 23.2                            | 0                              | 0                              | 4.6                             | 8.3                             |
|                  | 30.6                            | 23.2                            | 1.1                            | 1.5                            | 1.2                             | 2.1                             |
|                  | 30.6                            | 23.2                            | 1.1 \left(1 - \frac{C_0}{C_1}\right)^2 | 1.5 \left(1 - \frac{C_0}{C_1}\right)^2 | 1.2 \left(1 + \frac{C_0}{C_1}\right)^2 | 2.1 \left(1 + \frac{C_0}{C_1}\right)^2 |
|                  | 30.6                            | 23.2                            | 4.3                            | 5.8                            | 0                               | 0                               |
|                  | 30.6                            | 23.2                            | 4.3                            | 5.8                            | 18.6                            | 33.4                            |
|                  | 30.6                            | 23.2                            | 4.3 tan^2 \left(\theta + \frac{\pi}{4}\right) | 5.8 tan^2 \left(\theta - \frac{\pi}{4}\right) |

\[ \Delta f/f \]

|                  | 0.1                             | 0.53                            | 0.36                           | 0.35                           | 0.41                           | 0.41                            |
\[
\begin{align*}
f(B \to DX, X \to \bar{D}K) &= \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)} \\
\end{align*}
\]

- Relations among matrix elements and small correction (B lifetime)

| \( B \to D\bar{D}K \) | 2.2 \( \pm \) 0.7 | 2.7 \( \pm \) 1.1 | 15.5 \( \pm \) 2.1 | 10.7 \( \pm \) 1.1 | 14.5 \( \pm \) 3.3 | 7.5 \( \pm \) 1.7 |
|----------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|

\[
f(B \to DX, X \to \bar{D}K)
\]

| Triangle, QE | 30.6 | 23.2 | 0 | 0 | 4.6 | 8.3 |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Triangle, OPE | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT | 30.6 | 23.2 | \(1 - \frac{C_0}{C_1}\)^2 | 1.5 \(1 - \frac{C_0}{C_1}\)^2 | 1.2 \(1 + \frac{C_0}{C_1}\)^2 | 2.1 \(1 + \frac{C_0}{C_1}\)^2 |
| Resonance, \( I = 0 \) | 30.6 | 23.2 | 4.3 | 5.8 | 0 | 0 |
| Resonance, \( I = 1 \) | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, \( I \) mixed | 30.6 | 23.2 | 4.3 \(\tan^2 (\theta + \frac{\pi}{4})\) | 5.8 \(\tan^2 (\theta - \frac{\pi}{4})\) |

| \( \Delta f/f \) | 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |
\[ f(B \rightarrow DX, X \rightarrow \bar{D}K) = \frac{\mathcal{B}(B \rightarrow DX, X \rightarrow \bar{D}K)}{\mathcal{B}(B \rightarrow D\bar{D}K)} \]

| \( \mathcal{B}(B \rightarrow D\bar{D}K) \) | 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |
|----------------------------------|-----------|-----------|------------|------------|------------|----------|

\[ f(B \rightarrow DX, X \rightarrow \bar{D}K) \]

| Triangle, QE | 30.6 | 23.2 | 0 | 0 | 4.6 | 8.3 |
| Triangle, OPE | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT | 30.6 | 23.2 | 1.1 \(1 - \frac{C_0}{C_1} \)^2 | 1.5 \(1 - \frac{C_0}{C_1} \)^2 | 1.2 \(1 + \frac{C_0}{C_1} \)^2 | 2.1 \(1 + \frac{C_0}{C_1} \)^2 |
| Resonance, \( I = 0 \) | 30.6 | 23.2 | 4.3 | 5.8 | 0 | 0 |
| Resonance, \( I = 1 \) | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, \( I \) mixed | 30.6 | 23.2 | 4.3 \(\tan^2 (\theta + \frac{\pi}{4}) \) | 5.8 \(\tan^2 (\theta - \frac{\pi}{4}) \) |

\( \Delta f/f \)

| 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |
\[ f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)} \quad \text{enhancement if 3-body small} \]

| \( B \to D\bar{D}K \) | \( B^+ \to D^+X, \\ X \to D^-K^+ \) | \( B^0 \to D^0X, \\ X \to \bar{D}^0K^0 \) | \( B^+ \to D^+X, \\ X \to \bar{D}^0K^0 \) | \( B^0 \to D^0X, \\ X \to D^-K^+ \) | \( B^+ \to D^0X^+, \\ X^+ \to \bar{D}^0K^+ \) | \( B^0 \to D^+X^-, \\ X^- \to D^-K^0 \) |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| \( \mathcal{B}(B \to D\bar{D}K) \) | 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |

| \( f(B \to DX, X \to \bar{D}K) \) |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Triangle, QE | 30.6 | 23.2 | 0 | 0 | 4.6 | 8.3 |
| Triangle, OPE | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT | 30.6 | 23.2 | \( 1.1 \left(1 - \frac{C_0}{C_1}\right)^2 \) | \( 1.5 \left(1 - \frac{C_0}{C_1}\right)^2 \) | \( 1.2 \left(1 + \frac{C_0}{C_1}\right)^2 \) | \( 2.1 \left(1 + \frac{C_0}{C_1}\right)^2 \) |
| Resonance, \( I = 0 \) | 30.6 | 23.2 | 4.3 | 5.8 | 0 | 0 |
| Resonance, \( I = 1 \) | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, \( I \) mixed | 30.6 | 23.2 | \( 4.3 \tan^2 (\theta + \frac{\pi}{4}) \) | \( 5.8 \tan^2 (\theta - \frac{\pi}{4}) \) |

| \( \Delta f/f \) | 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |
\[
f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)}
\]

| \(B \to D^+X, X \to D^-K^+\) | \(B^0 \to D^0X, X \to \bar{D}^0K^0\) | \(B^+ \to D^+X, X \to \bar{D}^0K^0\) | \(B^0 \to D^0X, X \to D^-K^+\) | \(B^+ \to D^0X^+, X^+ \to \bar{D}^0K^+\) | \(B^0 \to D^+X^-, X^- \to D^-K^0\) |
|----------------|----------------|----------------|----------------|----------------|----------------|
| \(\mathcal{B}(B \to D\bar{D}K)\) | 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |

\[
f(B \to DX, X \to \bar{D}K)
\]

| Triangle, QE | 30.6 | 23.2 | 0 | 0 | 4.6 | 8.3 |
| Triangle, OPE | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT | 30.6 | 23.2 | \(1.1 \left(1 - \frac{C_0}{C_1}\right)^2\) | \(1.5 \left(1 - \frac{C_0}{C_1}\right)^2\) | \(1.2 \left(1 + \frac{C_0}{C_1}\right)^2\) | \(2.1 \left(1 + \frac{C_0}{C_1}\right)^2\) |
| Resonance, \(I = 0\) | 30.6 | 23.2 | 4.3 | 5.8 | 0 | 0 |
| Resonance, \(I = 1\) | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, \(I\) mixed | 30.6 | 23.2 | \(4.3 \tan^2(\theta + \frac{\pi}{4})\) | \(5.8 \tan^2(\theta - \frac{\pi}{4})\) |

\[
\Delta f/f
\]

| 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |

**fractional uncertainty**
\[ f(B \rightarrow DX, X \rightarrow \bar{D}K) = \frac{\mathcal{B}(B \rightarrow DX, X \rightarrow \bar{D}K)}{\mathcal{B}(B \rightarrow D\bar{D}K)} \]

| \( \mathcal{B}(B \rightarrow D\bar{D}K) \) | \( B^+ \rightarrow D^+X, \quad X \rightarrow D^-K^+ \) | \( B^0 \rightarrow D^0X, \quad X \rightarrow D^0K^0 \) | \( B^+ \rightarrow D^+X, \quad X \rightarrow \bar{D}^0K^0 \) | \( B^0 \rightarrow D^0X, \quad X \rightarrow D^-K^+ \) | \( B^+ \rightarrow D^0X^+, \quad X^+ \rightarrow \bar{D}^0K^+ \) | \( B^0 \rightarrow D^+X^-, \quad X^- \rightarrow D^-K^0 \) |
|---|---|---|---|---|---|---|
| 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |

| \( f(B \rightarrow DX, X \rightarrow \bar{D}K) \) |
|---|
| Triangle, QE | 30.6 | 23.2 | 0 | 0 | 4.6 | 8.3 |
| Triangle, OPE | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT | 30.6 | 23.2 | \( 1.1 \left( 1 - \frac{C_0}{C_1} \right)^2 \) | \( 1.5 \left( 1 - \frac{C_0}{C_1} \right)^2 \) | \( 1.2 \left( 1 + \frac{C_0}{C_1} \right)^2 \) | \( 2.1 \left( 1 + \frac{C_0}{C_1} \right)^2 \) |
| Resonance, \( I = 0 \) | 30.6 | 23.2 | 4.3 | 5.8 | 0 | 0 |
| Resonance, \( I = 1 \) | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, \( I \) mixed | 30.6 | 23.2 | \( 4.3 \tan^2 \left( \theta + \frac{\pi}{4} \right) \) | \( 5.8 \tan^2 \left( \theta - \frac{\pi}{4} \right) \) |

| \( \Delta f/f \) | 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |

existing channel is largest
\[ f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)} \]

| Mode | \( B^+ \to D^+X, X \to D^-K^+ \) | \( B^0 \to D^0X, X \to \bar{D}^0K^0 \) | \( B^+ \to D^+X, X \to \bar{D}^0K^0 \) | \( B^0 \to D^0X, X \to D^-K^+ \) | \( B^+ \to D^0X^+, X^+ \to \bar{D}^0K^+ \) | \( B^0 \to D^+X^-, X^- \to D^-K^0 \) |
|------|------------------|------------------|------------------|------------------|------------------|------------------|
| \( \mathcal{B}(B \to D\bar{D}K) \) | 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |

\[ f(B \to DX, X \to \bar{D}K) \]

| Model | \( \text{Triangle, QE} \) | \( \text{Triangle, OPE} \) | \( \text{Triangle, EFT} \) | \( \text{Resonance, } I = 0 \) | \( \text{Resonance, } I = 1 \) | \( \text{Resonance, } I \text{ mixed} \) |
|------|------------------|------------------|------------------|------------------|------------------|------------------|
| \( 30.6 \) | 23.2 | 0 | 1.1 \( (1 - \frac{c_0}{c_1})^2 \) | 1.5 \( (1 - \frac{c_0}{c_1})^2 \) | 1.2 \( (1 + \frac{c_0}{c_1})^2 \) | 2.1 \( (1 + \frac{c_0}{c_1})^2 \) |
| \( 30.6 \) | 23.2 | 1.1 | 1.5 \( (1 - \frac{c_0}{c_1})^2 \) | 1.2 \( (1 + \frac{c_0}{c_1})^2 \) | 2.1 \( (1 + \frac{c_0}{c_1})^2 \) |
| \( 30.6 \) | 23.2 | 4.3 | 5.8 | 0 | 0 |
| \( 30.6 \) | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| \( 30.6 \) | 23.2 | 4.3 \( \tan^2 \left( \theta + \frac{\pi}{4} \right) \) | 5.8 \( \tan^2 \left( \theta - \frac{\pi}{4} \right) \) |

\[ \Delta f/f \]

| \( 0.1 \) | \( 0.53 \) | \( 0.36 \) | \( 0.35 \) | \( 0.41 \) | \( 0.41 \) |

Neutral mode is comparable general prediction, same for all models.
\[ f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)} \]

| \( B^+ \to D^+X, \) \( X \to D^-K^+ \) | \( B^0 \to D^0X, \) \( X \to \bar{D}^0K^0 \) | \( B^+ \to D^+X, \) \( X \to \bar{D}^0K^0 \) | \( B^0 \to D^0X, \) \( X \to D^-K^+ \) | \( B^+ \to D^0X^+, \) \( X^+ \to \bar{D}^0K^+ \) | \( B^0 \to D^+X^-, \) \( X^- \to D^-K^0 \) |
|-----------------|-----------------|------------------|-----------------|-----------------|-----------------|
| \( \mathcal{B}(B \to D\bar{D}K) \) | 2.2 \( \pm \) 0.7 | 2.7 \( \pm \) 1.1 | 15.5 \( \pm \) 2.1 | 10.7 \( \pm \) 1.1 | 14.5 \( \pm \) 3.3 | 7.5 \( \pm \) 1.7 |

\[ f(B \to DX, X \to \bar{D}K) \]

| Triangle, QE | 30.6 | 23.2 | 0 | 0 | 4.6 | 8.3 |
| Triangle, OPE | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT | 30.6 | 23.2 | \( 1.1 \left( 1 - \frac{C_0}{C_1} \right)^2 \) | \( 1.5 \left( 1 - \frac{C_0}{C_1} \right)^2 \) | \( 1.2 \left( 1 + \frac{C_0}{C_1} \right)^2 \) | \( 2.1 \left( 1 + \frac{C_0}{C_1} \right)^2 \) |
| Resonance, \( I = 0 \) | 30.6 | 23.2 | 4.3 | 5.8 | 0 | 0 |
| Resonance, \( I = 1 \) | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, \( I \) mixed | 30.6 | 23.2 | \( 4.3 \tan^2 (\theta + \frac{\pi}{4}) \) | \( 5.8 \tan^2 (\theta - \frac{\pi}{4}) \) |

\( \Delta f/f \)

| | | | 0.36 | 0.35 | 0.41 | 0.41 |

remaining predictions discriminate among models
uncertainties are large but predictions still discriminate
\[ f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)} \]

| $B^+ \to D^+X, \ X \to D^-K^+$ | $B^0 \to D^0X, \ X \to D^0K^0$ | $B^+ \to D^+X, \ X \to D^-K^+$ | $B^0 \to D^0X, \ X \to D^-K^+$ | $B^+ \to D^0X^+, \ X^+ \to \bar{D}^0K^+$ | $B^0 \to D^+X^-, \ X^- \to D^-K^0$ |
|---|---|---|---|---|---|
| $\mathcal{B}(B \to D\bar{D}K)$ | 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |

| $f(B \to DX, X \to \bar{D}K)$ |
|---|
| Triangle, QE | 30.6 | 23.2 | 0 | 0 | 4.6 | 8.3 |
| Triangle, OPE | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT | 30.6 | 23.2 | $1.1 \left(1 - \frac{C_0}{C_1}\right)^2$ | $1.5 \left(1 - \frac{C_0}{C_1}\right)^2$ | $1.2 \left(1 + \frac{C_0}{C_1}\right)^2$ | $2.1 \left(1 + \frac{C_0}{C_1}\right)^2$ |
| Resonance, $I = 0$ | 30.6 | 23.2 | 4.3 | 5.8 | 0 | 0 |
| Resonance, $I = 1$ | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, $I$ mixed | 30.6 | 23.2 | $4.3 \tan^2\left(\theta + \frac{\pi}{4}\right)$ | $5.8 \tan^2\left(\theta - \frac{\pi}{4}\right)$ | |

| $\Delta f/f$ | 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |
\[ f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)} \]

| Production Mode | \(B^+ \to D^+X, \ X \to D^-K^+\) | \(B^0 \to D^0X, \ X \to \bar{D}^0K^0\) | \(B^+ \to D^+X, \ X \to \bar{D}^0K^0\) | \(B^0 \to D^0X, \ X \to D^-K^+\) | \(B^+ \to D^0X^+, \ X^+ \to \bar{D}^0K^+\) | \(B^0 \to D^+X^-, \ X^- \to D^-K^0\) |
|-----------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| \(\mathcal{B}(B \to D\bar{D}K)\) | 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |

\[
f(B \to DX, X \to \bar{D}K)\]

| Contribution | Triangle, QE | Triangle, OPE | Triangle, EFT | Resonance, \(I = 0\) | Resonance, \(I = 1\) | Resonance, \(I\) mixed |
|--------------|-------------|-------------|-------------|----------------|----------------|----------------|
| Value        | 30.6        | 30.6        | 30.6        | 30.6           | 30.6           | 30.6           |
| Value        | 23.2        | 23.2        | 23.2        | 23.2           | 23.2           | 23.2           |
| Value        | 0           | 1.1         | \(1.1 \left( 1 - \frac{C_0}{C_1} \right)^2\) | \(1.5 \left( 1 - \frac{C_0}{C_1} \right)^2\) |
| Value        | 0           | 1.5         | \(1.2 \left( 1 + \frac{C_0}{C_1} \right)^2\) | \(2.1 \left( 1 + \frac{C_0}{C_1} \right)^2\) |
| Value        | 4.6         | 1.2         | 2.1         | 4.3 tan^2 \(\theta + \frac{\pi}{4}\) |
| Value        | 8.3         | 2.1         | 33.4        | 5.8 tan^2 \(\theta - \frac{\pi}{4}\) |

| \(\Delta f/f\) | 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |

**same production mode**

**different final state**
\[ f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)} \]

| \( B \to DX, X \to \bar{D}K \) | \( B^+ \to D^+X, X \to D^-K^+ \) | \( B^0 \to D^0X, X \to \bar{D}^0K^0 \) | \( B^+ \to D^+X, X \to \bar{D}^0K^0 \) | \( B^0 \to D^0X, X \to D^-K^+ \) | \( B^+ \to D^0X^+, X^+ \to \bar{D}^0K^+ \) | \( B^0 \to D^+X^-, X^- \to D^-K^0 \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \mathcal{B}(B \to D\bar{D}K) \) | 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |

\[ f(B \to DX, X \to \bar{D}K) \]

- Triangle, QE: 30.6, 23.2, 0, 4.6, 8.3
- Triangle, OPE: 30.6, 23.2, 1.1, 1.5, 1.2, 2.1
- Triangle, EFT: 30.6, 23.2, \( 1.1 \left( 1 - \frac{C_0}{C_1} \right)^2 \), \( 1.5 \left( 1 - \frac{C_0}{C_1} \right)^2 \), \( 1.2 \left( 1 + \frac{C_0}{C_1} \right)^2 \), \( 2.1 \left( 1 + \frac{C_0}{C_1} \right)^2 \)
- Resonance, \( I = 0 \): 30.6, 23.2, 4.3, 5.8, 0, 0
- Resonance, \( I = 1 \): 30.6, 23.2, 4.3, 5.8, 18.6, 33.4
- Resonance, \( I \) mixed: 30.6, 23.2, \( 4.3 \tan^2 \left( \theta + \frac{\pi}{4} \right) \), \( 5.8 \tan^2 \left( \theta - \frac{\pi}{4} \right) \)

\( \Delta f/f \)

- 0.1, 0.53, 0.36, 0.35, 0.41, 0.41

**Selection Rule**: The selection rule is indicated where the entries are highlighted.

**Same Production Mode, Different Final State**: This is indicated by the boxed entries in the table.
\[
f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)}
\]

|                          | $B^+ \to D^+X, X \to D^-K^+$ | $B^0 \to D^0X, X \to \bar{D}^0K^0$ | $B^+ \to D^+X, X \to \bar{D}^0K^0$ | $B^0 \to D^0X, X \to D^-K^+$ | $B^+ \to D^0X^+, X^+ \to \bar{D}^0K^+$ | $B^0 \to D^+X^-, X^- \to D^-K^0$ |
|--------------------------|-------------------------------|---------------------------------|---------------------------------|-------------------------------|---------------------------------|-------------------------------|
| $\mathcal{B}(B \to D\bar{D}K)$ | $2.2 \pm 0.7$                 | $2.7 \pm 1.1$                  | $15.5 \pm 2.1$                  | $10.7 \pm 1.1$               | $14.5 \pm 3.3$                 | $7.5 \pm 1.7$                |

|                          | $B^+ \to D^+X, X \to D^-K^+$ | $B^0 \to D^0X, X \to \bar{D}^0K^0$ | $B^+ \to D^+X, X \to \bar{D}^0K^0$ | $B^0 \to D^0X, X \to D^-K^+$ | $B^+ \to D^0X^+, X^+ \to \bar{D}^0K^+$ | $B^0 \to D^+X^-, X^- \to D^-K^0$ |
|--------------------------|-------------------------------|---------------------------------|---------------------------------|-------------------------------|---------------------------------|-------------------------------|
| $f(B \to DX, X \to \bar{D}K)$ |                               |                                 |                                 |                               |                                 |                               |
| Triangle, QE             | 30.6                          | 23.2                           | 0                               | 0                             | 4.6                             | 8.3                            |
| Triangle, OPE            | 30.6                          | 23.2                           | 1.1                            | 1.5                           | 1.2                             | 2.1                            |
| Triangle, EFT            | 30.6                          | 23.2                           | $1.1 \left(1 - \frac{C_0}{C_1}\right)^2$ | $1.5 \left(1 - \frac{C_0}{C_1}\right)^2$ | $1.2 \left(1 + \frac{C_0}{C_1}\right)^2$ | $2.1 \left(1 + \frac{C_0}{C_1}\right)^2$ |
| Resonance, $I = 0$       | 30.6                          | 23.2                           | 4.3                            | 5.8                           | 0                              | 0                              |
| Resonance, $I = 1$       | 30.6                          | 23.2                           | 4.3                            | 5.8                           | 18.6                           | 33.4                           |
| Resonance, $I$ mixed     | 30.6                          | 23.2                           | $4.3 \tan^2 \left(\theta + \frac{\pi}{4}\right)$ | $5.8 \tan^2 \left(\theta - \frac{\pi}{4}\right)$ |                                 |                               |
| $\Delta f/f$             | 0.1                           | 0.53                           | 0.36                           | 0.35                          | 0.41                           | 0.41                           |
\[ f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)} \]

| $B^+ \to D^+X$, $X \to D^-K^+$ | $B^0 \to D^0X$, $X \to \bar{D}^0K^0$ | $B^+ \to D^+X$, $X \to \bar{D}^0K^0$ | $B^0 \to D^0X$, $X \to D^-K^+$ | $B^+ \to D^0X^+$, $X^+ \to \bar{D}^0K^+$ | $B^0 \to D^+X^-$, $X^- \to D^-K^0$ |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\mathcal{B}(B \to D\bar{D}K)$ | 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |

Constraint on contact terms

- Triangle, QE
- Triangle, OPE
- Triangle, EFT
- Resonance, $I = 0$
- Resonance, $I = 1$
- Resonance, $I$ mixed

\[ \Delta f/f \]

same production mode
different final state
\[ f(B \rightarrow DX, X \rightarrow \bar{D}K) = \frac{\mathcal{B}(B \rightarrow DX, X \rightarrow \bar{D}K)}{\mathcal{B}(B \rightarrow D\bar{D}K)} \]

| \(\mathcal{B}(B \rightarrow D\bar{D}K)\) | \(B^+ \rightarrow D^+X, X \rightarrow D^-K^+\) | \(B^0 \rightarrow D^0X, X \rightarrow \bar{D}^0K^0\) | \(B^+ \rightarrow D^+X, X \rightarrow \bar{D}^0K^0\) | \(B^0 \rightarrow D^0X, X \rightarrow D^-K^+\) | \(B^+ \rightarrow D^0X^+, X^+ \rightarrow \bar{D}^0K^+\) | \(B^0 \rightarrow D^+X^-, X^- \rightarrow D^-K^0\) |
|---|---|---|---|---|---|---|
| 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |

\[ f(B \rightarrow DX, X \rightarrow \bar{D}K) \]

| Triangle, QE | 30.6 | 23.2 | 0 | 0 | 4.6 | 8.3 |
| Triangle, OPE | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT | 30.6 | 23.2 | \(1.1 \left(1 - \frac{C_0}{C_1}\right)^2\) | \(1.5 \left(1 - \frac{C_0}{C_1}\right)^2\) | \(1.2 \left(1 + \frac{C_0}{C_1}\right)^2\) | \(2.1 \left(1 + \frac{C_0}{C_1}\right)^2\) |
| Resonance, \(I = 0\) | 30.6 | 23.2 | 4.3 | 5.8 | 0 | 0 |
| Resonance, \(I = 1\) | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, \(I \text{ mixed}\) | 30.6 | 23.2 | \(4.3 \tan^2\left(\theta + \frac{\pi}{4}\right)\) | 5.8 \(\tan^2\left(\theta - \frac{\pi}{4}\right)\) |

| \(\Delta f/f\) | 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |

*constraint on isospin mixing angle*

*same production mode, different final state*
\[ f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)} \]

|                                | \( B^+ \to D^+X, X \to D^-K^+ \) | \( B^0 \to D^0X, X \to \bar{D}^0K^0 \) | \( B^+ \to D^+X, X \to \bar{D}^0K^0 \) | \( B^0 \to D^0X, X \to D^-K^+ \) | \( B^+ \to D^0X^+, X^+ \to \bar{D}^0K^+ \) | \( B^0 \to D^+X^-, X^- \to D^-K^0 \) |
|--------------------------------|----------------------------------|------------------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| \( \mathcal{B}(B \to D\bar{D}K) \) | 2.2 \( \pm 0.7 \)                | 2.7 \( \pm 1.1 \)                      | 15.5 \( \pm 2.1 \)              | 10.7 \( \pm 1.1 \)              | 14.5 \( \pm 3.3 \)              | 7.5 \( \pm 1.7 \)               |
| \( f(B \to DX, X \to \bar{D}K) \) |                                  |                                          |                                  |                                  |                                  |                                  |
| Triangle, QE                  | 30.6                             | 23.2                                     | 0                                | 0                                | 4.6                              | 8.3                              |
| Triangle, OPE                  | 30.6                             | 23.2                                     | 1.1                              | 1.5                              | 1.2                              | 2.1                              |
| Triangle, EFT                  | 30.6                             | 23.2                                     | 1.1 \( (1 - \frac{C_0}{C_1})^2 \) | 1.5 \( (1 - \frac{C_0}{C_1})^2 \) | 1.2 \( (1 + \frac{C_0}{C_1})^2 \) | 2.1 \( (1 + \frac{C_0}{C_1})^2 \) |
| Resonance, \( I = 0 \)         | 30.6                             | 23.2                                     | 4.3                              | 5.8                              | 0                                | 0                                |
| Resonance, \( I = 1 \)         | 30.6                             | 23.2                                     | 4.3                              | 5.8                              | 18.6                             | 33.4                             |
| Resonance, \( I \) mixed       | 30.6                             | 23.2                                     | 4.3 \( \tan^2 (\theta + \frac{\pi}{4}) \) | 5.8 \( \tan^2 (\theta - \frac{\pi}{4}) \) |                                  |                                  |
| \( \Delta f/f \)               | 0.1                              | 0.53                                     | 0.36                             | 0.35                             | 0.41                             | 0.41                             |

Similar patterns for neutral X in neutral B decays.
\[ f(B \rightarrow DX, X \rightarrow \bar{D}K) = \frac{\mathcal{B}(B \rightarrow DX, X \rightarrow \bar{D}K)}{\mathcal{B}(B \rightarrow D\bar{D}K)} \]

| \( B^+ \rightarrow D^+X, \) \( X \rightarrow D^-K^+ \) | \( B^0 \rightarrow D^0X, \) \( X \rightarrow \bar{D}^0K^0 \) | \( B^+ \rightarrow D^+X, \) \( X \rightarrow \bar{D}^0K^0 \) | \( B^0 \rightarrow D^0X, \) \( X \rightarrow D^-K^+ \) | \( B^+ \rightarrow D^0X^+, \) \( X^+ \rightarrow \bar{D}^0K^+ \) | \( B^0 \rightarrow D^+X^-, \) \( X^- \rightarrow D^-K^0 \) |
|---|---|---|---|---|---|
| \( \mathcal{B}(B \rightarrow D\bar{D}K) \) | 2.2 ± 0.7 | 2.7 ± 1.1 | 15.5 ± 2.1 | 10.7 ± 1.1 | 14.5 ± 3.3 | 7.5 ± 1.7 |

\[ f(B \rightarrow DX, X \rightarrow \bar{D}K) \]

| Triangle, QE | 30.6 | 23.2 | 0 | 0 | 4.6 | 8.3 |
| Triangle, OPE | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT | 30.6 | 23.2 | 1.1 \( \left( 1 - \frac{C_0}{C_1} \right)^2 \) | 1.5 \( \left( 1 - \frac{C_0}{C_1} \right)^2 \) | 1.2 \( \left( 1 + \frac{C_0}{C_1} \right)^2 \) | 2.1 \( \left( 1 + \frac{C_0}{C_1} \right)^2 \) |
| Resonance, \( I = 0 \) | 30.6 | 23.2 | 4.3 | 5.8 | 0 | 0 |
| Resonance, \( I = 1 \) | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, \( I \) mixed | 30.6 | 23.2 | 4.3 tan² \( \theta + \frac{\pi}{4} \) | 5.8 tan² \( \theta - \frac{\pi}{4} \) | |

\( \Delta f/f \)

| 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |

similar patterns for neutral X in neutral B decays
\[ f(B \to DX, X \to \bar{D}K) = \frac{\mathcal{B}(B \to DX, X \to \bar{D}K)}{\mathcal{B}(B \to D\bar{D}K)} \]

| \( B^+ \to D^+X, \ X \to D^-K^+ \) | \( B^0 \to D^0X, \ X \to \bar{D}^0 K^0 \) | \( B^+ \to D^+X, \ X \to \bar{D}^0 K^0 \) | \( B^0 \to D^0X, \ X \to D^-K^+ \) | \( B^+ \to D^0 X^+, \ X^+ \to \bar{D}^0 K^+ \) | \( B^0 \to D^+ X^-, \ X^- \to D^- K^0 \) |
|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| \( \mathcal{B}(B \to D\bar{D}K) \) | 2.2 ± 0.7                        | 2.7 ± 1.1                        | 15.5 ± 2.1                       | 10.7 ± 1.1                       | 14.5 ± 3.3                       | 7.5 ± 1.7                        |

\[ f(B \to DX, X \to \bar{D}K) \]

- **Triangle, QE**: 30.6
- **Triangle, OPE**: 30.6
- **Triangle, EFT**: 30.6
- **Resonance, \( I = 0 \)**: 30.6
- **Resonance, \( I = 1 \)**: 30.6
- **Resonance, \( I \) mixed**: 30.6

\( \Delta f/f \) | 0.1 | 0.53 | 0.36 | 0.35 | 0.41 | 0.41 |
\[ f(B \to D^+ X, X \to \bar{D} K) = \frac{\mathcal{B}(B \to D^+ X, X \to \bar{D} K)}{\mathcal{B}(B \to D^- K)} \]

discriminate among models

|                      | \( B^+ \to D^+ X, X \to D^- K^+ \) | \( B^0 \to D^0 X, X \to \bar{D}^0 K^0 \) | \( B^+ \to D^+ X, X \to \bar{D}^0 K^0 \) | \( B^0 \to D^0 X, X \to D^- K^+ \) | \( B^+ \to D^0 X^+, X^+ \to \bar{D}^0 K^+ \) | \( B^0 \to D^+ X^-, X^- \to D^- K^0 \) |
|----------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| \( \mathcal{B}(B \to D \bar{D} K) \) | 2.2 ± 0.7                         | 2.7 ± 1.1                         | 15.5 ± 2.1                        | 10.7 ± 1.1                        | 14.5 ± 3.3                        | 7.5 ± 1.7                         |

\[ f(B \to D^+ X, X \to \bar{D} K) \]

- Triangle, QE: 
  - 30.6
- Triangle, OPE: 
  - 30.6
- Triangle, EFT: 
  - 30.6
- Resonance, \( I = 0 \): 
  - 30.6
- Resonance, \( I = 1 \): 
  - 30.6
- Resonance, \( I \) mixed: 
  - 30.6

\[ \Delta f/f \]

- 0.1
- 0.53
- 0.36
- 0.35
- 0.41
- 0.41

charged partners

absent only in \( I=0 \) resonance scenario
\[
f(B \rightarrow DX, X \rightarrow \bar{D}K) = \frac{\mathcal{B}(B \rightarrow DX, X \rightarrow \bar{D}K)}{\mathcal{B}(B \rightarrow D\bar{D}K)}
\]

| \( B^+ \rightarrow D^+X, \) \( X \rightarrow D^-K^+ \) | \( B^0 \rightarrow D^0X, \) \( X \rightarrow \bar{D}^0K^0 \) | \( B^+ \rightarrow D^+X, \) \( X \rightarrow \bar{D}^0K^0 \) | \( B^0 \rightarrow D^0X, \) \( X \rightarrow D^-K^+ \) | \( B^+ \rightarrow D^0X^+ , \) \( X^+ \rightarrow \bar{D}^0K^+ \) | \( B^0 \rightarrow D^+X^-, \) \( X^- \rightarrow D^-K^0 \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \mathcal{B}(B \rightarrow D\bar{D}K) \) | 2.2 ± 0.7        | 2.7 ± 1.1        | 15.5 ± 2.1       | 10.7 ± 1.1       | 14.5 ± 3.3       | 7.5 ± 1.7       |

\[
f(B \rightarrow DX, X \rightarrow \bar{D}K)
\]

| Triangle, QE  | 30.6 | 23.2 | 0   | 0   | 4.6 | 8.3 |
| Triangle, OPE | 30.6 | 23.2 | 1.1 | 1.5 | 1.2 | 2.1 |
| Triangle, EFT | 30.6 | 23.2 | 1.1 \( \left(1 - \frac{C_0}{C_1}\right)^2 \) | 1.5 \( \left(1 - \frac{C_0}{C_1}\right)^2 \) | 1.2 \( \left(1 + \frac{C_0}{C_1}\right)^2 \) | 2.1 \( \left(1 + \frac{C_0}{C_1}\right)^2 \) |
| Resonance, \( I = 0 \) | 30.6 | 23.2 | 4.3 | 5.8 | 0   | 0   |
| Resonance, \( I = 1 \) | 30.6 | 23.2 | 4.3 | 5.8 | 18.6 | 33.4 |
| Resonance, \( I \) mixed | 30.6 | 23.2 | 4.3 \( \tan^2 \left(\theta + \frac{\pi}{4}\right) \) | 5.8 \( \tan^2 \left(\theta - \frac{\pi}{4}\right) \) | 0.41 | 0.41 |

\[
\Delta f/f = \begin{array}{c}
0.1 \\
0.53 \\
0.36 \\
0.35 \\
0.41 \\
0.41 \\
\end{array}
\]

enormous fit fractions in \( I=1 \) resonance scenario
Resonance scenarios only:

\[ \mathcal{B}(B \to DX, X \to \bar{D}K) = \mathcal{B}(B \to DX) \mathcal{B}(X \to \bar{D}K). \]
Resonance scenarios only:

\[ \mathcal{B}(B \to DX, X \to \bar{D}K) = \mathcal{B}(B \to DX) \mathcal{B}(X \to \bar{D}K). \]
Resonance scenarios only:

\[ B(B \to DX, X \to \bar{D}K) = B(B \to DX)B(X \to \bar{D}K). \]

Note: \( B(X \to \bar{D}K) \ll 1 \) is natural!
so \( B(B \to DX) \) could be very large.
Conclusions
Triangle and resonance scenarios fit experimental data in amplitude model.
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Assuming
- dominance of colour-favoured transitions, and
- isospin
we get relations among fit fractions
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Triangle and resonance scenarios fit experimental data in amplitude model.

Assuming
- dominance of colour-favoured transitions, and
- isospin
we get relations among fit fractions.

Six possible modes
- some new modes have very large fit fraction
- pattern can discriminate among models.

(absent only in I=0 resonance scenario)