A REAL OPTION APPROACH FOR INVESTMENT OPPORTUNITY VALUATION

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Abstract. In this paper, the valuation of an investment opportunity in a high-tech corporation using real option theory and modern capital budgeting is studied. Some key characteristics such as high-risk, multi-stage and technology life cycle of a high-tech project are considered in the proposed model. Since a real option is usually not tradable in the market, an actuarial approach is adopted in our study. We employ an irreversible regime-switching Markov chain to model the multi-stage and technology life cycle of the project in the high-tech industry. The valuation of captured real option can be formulated as the valuation of an American option with time-dependent strike price. For the purpose of practical implementation, a novel lattice-based method is developed to value the American option. Numerical examples are given to illustrate the proposed models and methods.

1. Introduction. In recent years, high-tech corporations have emerged as a leader in the world economy. An important and practical issue in high-tech corporations is how to evaluate the investment opportunity in such a high-risk industry. This issue is far from trivial due to some specific, or possibly, unique features of these projects including high-risk, multi-stage and technology life cycle, etc. Traditionally, the Discounted Cash Flow (DCF) analysis is a popular approach for valuation of an investment, project, company or asset. The method is based on a fundamental concept in economics, namely, the time value of money. Another approach, the Net Present Value (NPV), decides the value of an investment opportunity, project,
company, or asset by calculating the sum of present values from all future cash flows. Myers [19] questioned the use of the DCF to value an option by noting the uncertainty of future cash flows depending on whether the option is exercised or not. An investment may be described by an option since an investor is not committed to undertake it. Thus it was recommended in Myers [19] that an investment should be evaluated by modern option valuation technology rather than the DCF approach. Trigeorgis and Manson [28] pointed out that both the NPV and DCF approaches are based on the assumption that the estimated future cash flows can be obtained from the premise of future certainty. Consequently if uncertainty exists, both NPV and DCF cannot be applied.

In response to these criticisms, researchers have proposed the use of real option and decision analysis techniques for valuing risky projects in different industries. The real option approach assumes that the future is uncertain and management has the flexibility or the option to make different updated decisions as time passes, while traditional analysis such as the discounted cash flows approach may usually assume that all decisions are made in the beginning without the possibility to change. However, the actual business environment is fraught with uncertainties and risks. Decision makers have the flexibility to make appropriate changes in business investments and strategies when information is available and uncertainty becomes resolved. Traditional analyses neglect the flexibility of decision makers and this may significantly underestimate the value of an investment project. Decision makers can learn over time and revise their decisions when new information emerges. Therefore, in a dynamic decision-making process, certain strategic optionalities would appear in each particular project. These strategic optionalities may include the option to invest, abandon, contract, switch, choose, and so on. For example, Mun [18] shows that in the automobile manufacturing, General Motors applies real option approach to create switching options which allow the company to switch vendors when a certain raw material becomes too expensive. Once the type of the strategic option is identified for each project or at each stage of the project, the value of the option can be analyzed in a detailed manner and its implications for decision makings can be explored.

The real option approach has been used as a tool in different industries where strategic decision makings may be relevant. It seems that early applications of the real option approach for decision makings under uncertainty are in the oil and gas industries and the applications have later been extended into utilities, biotechnology, and now into telecommunications, high-tech, nuclear power investment, and real estates[3, 16, 22, 23, 24, 25, 39] and so on. Kellogg and Charnes [16] developed two methods, namely, decision-tree and binomial-lattice (which takes into account a growth option) to value a biotechnology company. Schwartz and Moon [22] proposed a model for valuing Internet companies using real option theory and modern capital budgeting techniques. They pointed out that the uncertainty about the key variables which determine the value of an Internet company plays a major role in the valuation. An extension of the model for the valuation of an Internet company has been proposed in [23]. In Smith and Nau [24] and Smith and McCardle [25], decision analysis and option pricing theory were integrated to evaluate risky projects and oil properties. Bollen [3] proposed an real option valuation framework that can incorporate a stochastic product life-cycle. A pentanomial lattice was then constructed [2] for the valuation of the real option. Bollen then concluded that ignoring the product may result in undervaluing the contraction option. In
Alexandra and Chen [1], the authors provided a general decision tree approach for determining the value of an investment opportunity and its optimal exercise path. West and Bengtsson [30] adopted a real option approach to analyze production design in global manufacturing with a view to valuing uncertainty in exchange rates. Ruhrmann et al. [21] presented a methodological approach to evaluate supplier development by formulating it as the valuation of real options, where uncertainty and flexibility attributed to changing corporate environment were taken into account in the decision making process on supplier development. Wu et al. [33] studied the joint pricing and quantity decisions in the presence of uncertain demand for different supply chain strategies. In a general context of decision making under uncertainty and its related risk management issues, one may refer to the survey paper by Wu and Olson [31] and some relevant papers in the special issues on enterprise risk management in operations [32]. Zhu [39] developed an evaluation model based on real option approach for the investment of nuclear power. Simulation techniques based on Least Squares Monte-Carlo was then employed to evaluate the value of nuclear power under economic uncertainty factors such as investment cost, electricity price, and nuclear accident.

The problem of evaluating high-tech investment and management has been discussed in the literature. Trigeorgis and Ioulianou [29] applied the real option theory to value a high-tech company and showed that compared with the terminal growth DCF assumption, their option-based portfolio Present Value of Growth Opportunities (PVGO) provides a better estimate of the firm grow prospects. In Brealey and Myers [4], it was found that a R&D investment in a high-tech corporation will bring in an option for the high-tech company. The option pricing theory can be employed to value it. Wu and Liou [35] developed a quantitative model for making investment decisions in Enterprise Resource Planning (ERP). They applied Dynamic Programming (DP) approach and incorporated two uncertain factors (revenue and cost) in their model. The results of their numerical examples show that the approach based on real option theory gives a more realistic valuation than the NPV approach. Recently, Wang et al. [34] applied a Fuzzy Multiple-Objective Linear Programming (FMOLP) method in decision models used by high-tech firms. The implementation results confirmed that their FMOLP model may potentially optimize outsourcing decisions in high-tech manufacturing firms. Tseng et al. [27] developed a performance evaluation model to measure the relative business performance of companies in high-tech manufacturing industry. Their method may provide some insights for high-tech manufacturing executives, collaborating companies and potential investors to analyze the company’s strengths and weaknesses. Colombo et al. [10] analyzed the effect of ownership structure on the performance of high-tech entrepreneurial firms. They found that the number of owner-managers has a positive effect on firm performance. Wang et al., [37] proposed an improved TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method to evaluate the provincial competitiveness of the Chinese high-tech industry. Unlike the traditional TOPSIS method, the improved TOPSIS method can improve the accuracy of the evaluation results by eliminating the linear correlation among indicators.

However, it seems that there is a relatively little work focusing on the valuation of high-tech investments taking into account of realistic features such as high-risk, multi-stage and technology life cycle in the high-tech industry. This partly motivates the development of an innovative and practically useful method for the captured
investment problem in high-tech companies. This paper considers an investment project in High-tech industry using the real option approach. The market price of the product and the sales in the future are uncertain. Decision makers are assumed to have the flexibility in making interim strategic decisions and the real option approach is adopted to analyze investment options and abandonment options by taking into account of the decision makers’ flexibility in making right decisions at right times. This paper contributes to the literature in at least two aspects. First, an irreversible regime-switching Markov chain is applied to model the multi-stage and cycle life of a project in the high-tech industry. Generally speaking, three stages are considered for a project in a high-tech company: R&D stage, Mature stage, and Decline stage. The model parameters may take very different values in different development stages. Second, instead of modeling only one source of uncertainty, two different sources of uncertainty are considered: the market price of the product \( p(t) \) and the sales \( q(t) \). The two variables are assumed to follow two independent geometric Brownian motions. This approach has been considered in [25] for valuing oil properties. In [24], an approach based on an utility function was adopted to value risky projects. In reality, it may be uneasy to elicit or quantify the risk preference of a decision maker. So this may represent a potential barrier for the practical implementation of the utility-based approach for valuing risky projects. Here, an approach based on a direct real-world valuation without recourse to an utility function is considered.

The rest of the paper is structured as follows. Section 2 describes the model and the real option valuation formulation. Section 3 presents a lattice-based method for the valuation of the real option relating to an investment opportunity in the high-tech industry. Numerical experiments and discussions are given in Sections 4 and 5, respectively. Finally, Section 6 summarizes the paper.

2. The model. In this section, we consider the situation that a decision maker (an individual or a corporation) has to invest or divest in a project of a high-tech company. The procedure in the valuation of an investment opportunity is developed in a discrete-time, finite-horizon framework. The state information of the decision maker at time \( t \) is described by the \( \sigma \)-field \( F_t \). The value of an investment opportunity is derived from the expected future cash flows of the high-tech company.

2.1. The market and the high-tech company. Owing to rapid advancement in technology and keen competition in global financial markets, industries such as biotechnology, computers, networks, and electronics have been populated by entrepreneurial high-tech firms [12, 27, 36]. The new product development capability is a key determinant for the success of a high-tech company. In order to compete and survive, high-tech firms are urged to innovate new products or services. For example, Apple, one of the world’s biggest technology companies, constantly innovates new products, such as iPhone 4, iPhone 5 and 6, through Research and Development (R&D) to stay competitive in the industry.

The uncertainty in sales at a particular site of a high-tech company cannot be hedged by tradeable securities in the financial market. Consequently, the uncertainty arising from investing in a risky project of a high-tech company cannot be completely hedged by trading in the market. This renders the market in the model considered here is incomplete. In this case, the subjective beliefs and/or preferences of the decision-marker may be relevant to valuing background uncertainties and risks.
The high-growth nature of the high-tech industry has attracted many investors despite its highly risky nature. It is found that the revenue rates, production rates and other indexes of a high-tech company appear to have different values in different periods. This observation implies the evolution of a high-tech company always follows a multi-stage way. Furthermore, according to Narayanan [20], the technology life-cycle may be seen as composed of four stages: Research and Development (R&D) stage, Ascent stage, Mature stage, and Decline stage.

To succeed in a competitive market and seize market share, high-tech companies have to make great efforts to develop new and better products. When a new product becomes popular, after a period of time, it will be digested by the consumers and at the same time, new or attractive products will be launched by competitors. It is therefore very difficult for a high-tech company to stay in the mature stage for a long time. In both Ascent and Mature stages, the project generates cash flows from the product sales. In these two stages, the amount of the profit for the high-tech company increases. Due to the relatively short duration of Mature stage and the similarities between Ascent stage and Mature stage, this paper combines the two stages (Ascent stage and Mature stage) together and denotes it as Mature stage. Thus, in our proposed model, there are only three stages for a high-tech company.

2.1.1. Stage 1: R&D stage. In this stage, the focus of the company is researching for a new product. This involves a substantial amount of development costs. It is not expected that the project generates any revenue during this stage. Furthermore, at the end of this stage the development may fail, and so the investment could be worthless. Lastly, the value of a high-tech company depends on the future outcome of the new product, and this makes the project highly risky.

2.1.2. Stage 2: Mature stage. The successful development of a new product in the R&D stage will result in productivity gains. In the Mature stage, the new product launches in the market and is accepted by the consumers gradually. Meanwhile, the product manufacturer will be a leader in the market until some other new products appear in the market. In this stage, the project generates cash flows from the product sales. The amount of the profit for the high-tech company is increasing, but can be highly volatile due to other factors such as macroeconomic conditions.

2.1.3. Stage 3: Decline stage. In this stage, there are some new and competitive products developed by other companies. Their products may replace or compete with the product developed from the project. The market demand and the sales of the product begin to decline. The high-tech company may generate decreasing cash flows because of the sales recession.

2.2. Irreversible regime-switching process for life cycle. In practice, the stages of life cycle of a firm may evolve randomly over time. A natural way to describe quantitatively the changes in the states of a life cycle over time is a Markov chain. Since jumps can only occur from an early stage to a later one, (i.e., irreversible), we describe the evolution of the three stages of the life cycle over time by a three-state irreversible Markov chain. The Markov chain is assumed to be observable here.

We assume that in the R&D stage of a new product, it will require a fixed time which is denoted as $t_{12}$. Actually, when a high-tech company intends to research and develop a new product, they will draw up a detailed research plan. For the sake of effectiveness of a research plan, the time for the R&D stage of a new product is
usually limited. At the end of the R&D stage, the development of the product is assumed to be successful with a given probability \( p \) and the failure probability is \( 1 - p \). The value of \( p \) can be obtained a priori by seeking opinions from experts (consultants and researchers) or the investor’s attitude towards the project. If the project is successfully developed at the deadline then the state of the model should jump from the R&D stage to the Mature stage at time \( t \). Otherwise, the research plan fails, there is no more future cash flow from the investment. The high-tech company would abandon the research of this product to avoid further loss of money. In this case, the valuation of this real option only depends on the information up to time \( t \).

Now we focus on the case in which there is new information after time \( t \). The company jumps into the Mature stage, and receives the cash flows generated by selling the products. It is possible that the company jumps to the Decline stage at any time since factors that cause a decline of selling may occur at any time. As time goes by, the product of the high-tech company is more likely to be replaced by other companies’ products. Both the market price and sales of the product decline. The high-tech company will finally get into the Decline stage. To simplify this transition process, the transition probability from the Mature stage to the Decline stage is assumed to follow an increasing function in time \( t \). This assumption can also be explained by the fact that as time goes by, the company will eventually jump to the Decline stage. If the company is in the Decline stage, then it will remain there until time \( T \).

2.3. Product price and sales. The product produced by the high-tech company is sold at the market price. Though the current sales and price are known given the current market information, the future sales and prices are uncertain. We borrow the idea of reservoir model in [25]. That is during the life cycle, both the market price \( p(t) \) and the sales \( q(t) \) of the product follow geometric Brownian motions. And the parameters are given, respectively, by \((\mu_p, \sigma_p)\) and \((\mu_q, \sigma_q)\) \((i = 2, 3, \ldots)\), representing the means and volatilities of the two processes in different stages. The dynamics of market price and sales are given, respectively, by

\[
\begin{align*}
    dp(t) &= \mu_p p(t) dt + \sigma_p p(t) dw_p(t), \quad (1) \\
    dq(t) &= \mu_q q(t) dt + \sigma_q q(t) dw_q(t). \quad (2)
\end{align*}
\]

Here \( w_p(t) \) and \( w_q(t) \) represent two independent standard Brownian motions. The initial values \( p(0) \) and \( q(0) \) are assumed to be known. And \( p(t) \) and \( q(t) \) may be linked due to the relationship between the demand and supply of products. However, to simplify our discussion, they are assumed to be independent.

The geometric Brownian motion model of spot prices and sales can be found in [25]. This paper considers the market price as a market uncertainty and the sales as a project-specific uncertainty which belongs to private uncertainties. This means that the market price \( p(t) \) somehow can be perfectly hedged by trading securities while the sales cannot be hedged. It is equivalent to setting the value of \( \mu_p \) as the risk-free interest rate in the formulation.

The instantaneous cash flow generated at time \( t \) is then given by

\[
c(t) = \gamma p(t) q(t) - c_o(t), \quad (3)
\]

where \( c_o(t) \) is the operating cost rate and \( 1 - \gamma \) is the proportion of taxes and various costs with respect to the revenues. The operating cost represents any cash outflows for business necessities, which may relate to the company’s facilities, personnel, and
A REAL OPTION APPROACH FOR INVESTMENT VALUATION

Since the objective of a high tech company changes in different stages, the value of \( c_0(t) \) is assumed to remain the same over one stage but be different in different stages. That is, \( c_0(t) \) depends on the Markov chain. Particularly, in the R&D stage, the high-tech company may spend much more money in employing highly skilled professionals and advanced facilities for innovating new products and thus \( c_0(t) \) in this stage is larger than in the other two stages.

2.4. Option valuation. We consider an option with maturity at time \( T \) and written on \( n \) underlying state variables. The evolution of the values of these state variables over time is described by \( n \) state processes \( X_1, X_2, \ldots, X_n \), for short, these processes are denoted as \( X = (X_1, X_2, \ldots, X_n) \). The terminal values of these processes are the only relevant information to determine the payoff of the option. Let \( \mathcal{F} := \{ F_t | t \in T \} \) be the filtration generated by the state process \( X \). The payoff of the option at time \( T \) is denoted by \( P(T, X(T)) \). The value of the option at time \( t (t \leq T) \) is denoted by \( V(t, X(t)) \) and we do not consider the situation where transaction costs are present. Boyle and Vorst [6] consider the pricing and hedging of options in the presence of transaction costs in a discrete-time binomial tree model. The incorporation of transaction costs in a multiple-asset situation and in a multivariate tree-based model may be quite complicated which may be worth an independent study.

If the market is complete and an European option is considered, i.e., it can be exercised only at the terminal time \( T \) then the option’s price at any time \( t < T \) is given by
\[
V(t, X(t)) = E^{Q}[e^{-r(T-t)}P(T, X(T)) | \mathcal{F}_t],
\]
where \( E^{Q} \) is the conditional expectation with respect to a risk-neutral probability and \( \mathcal{F}_t \) is the information up to time \( t \). The terminal value of this kind of option is the payoff at time \( T \), i.e., \( V(T, X(T)) = P(T, X(T)) \).

If the option is an American one, i.e., it can be exercised at any time before \( T \), then the price at any time \( t < T \) is given by
\[
V(t, X(t)) = \max_{\tau \in \mathcal{T}(t, T)} \left\{ E^{Q}[e^{-r(\tau-t)}P(\tau, X(\tau)) | \mathcal{F}_t]\right\},
\]
where \( \mathcal{T}(t, T) \) is the set of possible stopping times in \( [t, T] \) with respect to the information generated by the state process \( X \). The terminal value of an American option is the payoff at time \( T \), i.e., \( V(T, X(T)) = P(T, X(T)) \), if the American option has not yet been exercised. The above valuation paradigm can also be used to value capital budgeting projects and real options, see for instance, Smith and McCardle [25] and Alexander and Chen [1].

Due to the uncertainty in the sales, the investment risk of a high-tech project cannot be perfectly hedged. The subjective expectation or belief of a decision maker about the stream of cash flows to be realized from the investment would be relevant. Here to depict this situation, we adopt a subjective probability which may provide a representation of the belief of the decision maker to evaluate the stream of cash flows. We remark that in Huang et al. [14] that the demand process in a retail product is not tradeable, and therefore an actuarial valuation approach based on a subjective probability was adopted to value an option relating to the demand process.

2.4.1. Option to invest. In this subsection, we consider the situation that a decision maker has to choose whether to invest in the project or continue waiting at time
The decision opportunities for this decision maker are assumed at the discrete time points where \( h \) is the length of the discrete time step. The impact of different settings of \( h \) on real estate investment has been discussed in [1]. If the decision maker invests in the project at time \( t \), then the financial benefit to him is the sum of all future cash flows generated from time \( t + h \) to time \( T \):

\[
V_{T,h}(t, X(t)) = \max_{\tau \in T(t, T)} \left\{ E^P \left[ E^Q \left[ \sum_{s=t+h}^{T} e^{-r(s-t)}c(s, X(s)) \right] \right] \right\}
\]

where \( T(t, T) \) is the set of stopping times in \([t, T]\) with respect to the information generated by the state process \( X \). Here the conditional expectation, \( E^Q[-|\mathcal{F}_t^m] \), is taken with respect to the risk-neutral probability, given the market information up to and including time \( t \), the expectation \( E^P[-|\mathcal{F}_t] \) is taken with respect to the subjective probability, given current sales. Furthermore, according to [17], one can obtain the value of an American option and the optimal policy by comparing the continuation value with the wealth of the investor if the option is exercised at time \( t \). Then Eq. (7) can be rewritten as follows:

\[
\begin{align*}
V_{T,h}(t, X(t)) &= \max_{\tau \in T(h, T)} \left\{ E^P \left[ E^Q \left[ \sum_{s=t+h}^{T} e^{-r(s-t)}c(s, X(s)) - K_{in} |\mathcal{F}_t^m] |\mathcal{F}_t \right] \right] \right\}, \\
&= E^P\left[ E^Q\left[ e^{-rh} V_{T,h}(t + h, X(t + h)) |\mathcal{F}_t^m] |\mathcal{F}_t \right] \right].
\end{align*}
\]
At time $T$, the terminal value of the investment option is $V_{T,h}(T, X(T)) = 0$.

2.4.2. Option to abandon. In this subsection, suppose the decision maker has owned an investment at time 0 and he can decide to abandon the project at any time before $T$. If the decision maker decides to abandon the project at time $t$ then he/she can receive all the cash flows generated from the project before time $t$. Meanwhile, he/she should pay the corresponding cost $K_{di}$ for abandoning the project ($K_{di}$, similarly to $K$ in 2.4.1, is determined by the high-tech company, see for instance [25]). The abandonment cost ($K_{di}$) can be positive or negative. If the scrap value for an equipment at the site exceeds the other costs (for example, the cost of waste disposal) attributed to abandonment, a negative abandonment cost might incur. This divestment option is also an American option. Similarly, the value of the divestment option is obtained by taking the maximum between the payoff for exercising the option and the continuation value:

$$V_{a}^{T,h}(t, X(t)) = \max \left\{ \left( \sum_{s=h}^{t} e^{(t-s)} c(s, X(s)) - K_{di} \right), E^{Q}[e^{-r h} V_{a}^{T,h}(t + h, X(t + h))|F_{m}]|F_{t}] \right\}.$$  

(9)

In addition, we assume that the project must be abandoned at the end of the horizon and the value of the divestment option at time $T$ is given by

$$V_{a}^{T,h}(T, X(T)) = \sum_{s=h}^{T} e^{r(T-s)} c(s, X(s)) - K_{di}.$$  

3. Lattice construction. The lattice or tree approach to option pricing, such as the binomial model in Cox, Ross and Rubinstein (CRR) [9] and the trinomial model in Boyle [7, 8], is a popular method for pricing simple options. Indeed, market practitioners often use it as a numerical approach to approximate option prices. The approach we employ here is to construct a pentanomial lattice that represents the possible future paths of a regime-switching variable. The values of the cash flows can be computed for all the branches in the lattice. In the following, the construction of a pentanomial lattice is illustrated.

3.1. The recombined pentanomial lattice. In the CRR binomial tree model, in order to ensure that the rate of the stock price matches with the risk-free interest rate, the stock price is set to go up by a ratio $e^{\lambda \sqrt{h}}$ or go down by $e^{-\lambda \sqrt{h}}$ with certain specified probabilities. In the trinomial tree model, the stock price is allowed to remain unchanged, go up or go down by a ratio $e^{\lambda \sqrt{h}}$. Let $h$ be the size of time step and $r$ be the risk-free interest rate. Then the risk neutral probabilities $\pi_u$, $\pi_m$, $\pi_d$, corresponding to the situations that the stock price increases, remains the same and decreases, satisfy the following restrictions:

$$\begin{align*}
\pi_u e^{\lambda \sqrt{h}} + \pi_m + \pi_d e^{-\lambda \sqrt{h}} &= e^{rh}, \\
(\pi_u + \pi_d) \lambda^2 \sigma^2 h &= \sigma^2 h, \\
\pi_u + \pi_m + \pi_d &= 1,
\end{align*}$$  

(10)

where $\lambda$ should be greater than 1 so that the risk neutral probability measure exists.

There are two state variables in our model: the product’s market price $p(t)$ and the sales $q(t)$. Both of them follow a geometric Brownian motion as shown in Eq. (1) and Eq. (2). In order to approximate the evolution of $p(t) \cdot q(t)$ which appears in Eq. (6), we construct a lattice as a discretization of $p(t) \cdot q(t)$ by combining
two trinomial lattices of \( p(t) \) and \( q(t) \). There are three branches at each node of a trinomial lattice. If we just multiply the values of \( p(t) \) and \( q(t) \) to get the branches for \( p(t) \cdot q(t) \), then there are nine branches at each node of the lattice. Although the resulting lattice can represent both distributions of \( p(t) \) and \( q(t) \) accurately, the two trinomial lattices generally do not recombine effectively. One way to address this problem is to assume that the step sizes in both trinomial lattices are the same. In other words, suppose both the price and sales are allowed to remain unchanged, go up or go down by the same ratio \( e^{\sigma \sqrt{t}} \). There are five possible outcomes for the product \( p(t) \cdot q(t) \), which are \( e^{2\sigma \sqrt{t}}, e^{\sigma \sqrt{t}}, 1, e^{-\sigma \sqrt{t}} \) and \( e^{-2\sigma \sqrt{t}} \). It turns out that each node in the recombined lattice for \( p(t) \cdot q(t) \) has only five branches. The corresponding probability of each branch in the trinomial lattice are calculated in Section 3.2.

However, in our model, the means and the variances of the state variables \( p(t) \) and \( q(t) \) change in different regimes. To deal with this problem, Boyle and Tian [5], Kamrad and Ritchken [15] introduced more branches into the lattice so that extra information can be incorporated in the model. In this paper, we construct the tree by changing the probability measure if the regime state changes, instead of increasing the number of branches as in [2, 3]. Then the positions of the nodes cannot reflect their regime states since all of the regimes share the same lattice. Each of the nodes has many possible values which depend on its state. This method was introduced in Yuen and Yang [38].

### 3.2. Pentanomial lattice pricing model

In the previous subsection, in order to construct a simpler lattice, we propose to adjust the step sizes of the two trinomial lattices which correspond to \( p(t) \) and \( q(t) \). In this subsection, the details of the pentanomial lattice pricing model are discussed.

The up-jump ratio in each of the trinomial lattices is assumed to be \( e^{\sigma \sqrt{t}} \). In order to ensure that the ratio can be used in all regimes and the existence of the solutions of Eq. (11) and Eq. (12), we set

\[ \sigma > \max_{i=2,3}\{\sigma_{pi}, \sigma_{qi}\}. \]

As emphasised in Section 2.3, the product price is modeled as a source of market uncertainty. In regime \( i \) (\( i \in \{2,3\} \)), \( \pi_{pi}, \pi_{pi}^{u} \) and \( \pi_{pi}^{d} \) are the risk-neutral probabilities corresponding to the situations that the product price increases, remains the same and decreases, respectively. Thus for each \( i \in \{2,3\} \), we have

\[
\begin{align*}
\pi_{pi}^{u} e^{\sigma \sqrt{t}} + \pi_{pi}^{m} + \pi_{pi}^{d} e^{-\sigma \sqrt{t}} &= e^{r t h} \\
(\pi_{pi}^{u} + \pi_{pi}^{d}) \sigma^{2} h &= \sigma_{pi}^{2} h \\
\pi_{pi}^{u} + \pi_{pi}^{m} + \pi_{pi}^{d} &= 1.
\end{align*}
\]

On the other hand, \( \pi_{qi}^{u}, \pi_{qi}^{m} \) and \( \pi_{qi}^{d} \) are real-world probabilities corresponding to the situations that the sales increase, remains the same and decreases. The use of the real-world trinomial tree for the sales are in line with the decision analysis of [25]. For each \( i \in \{2,3\} \), we have

\[
\begin{align*}
\pi_{qi}^{u} e^{\sigma \sqrt{t}} + \pi_{qi}^{m} + \pi_{qi}^{d} e^{-\sigma \sqrt{t}} &= e^{\mu_{qi} t h} \\
(\pi_{qi}^{u} + \pi_{qi}^{d}) \sigma_{qi}^{2} h &= \sigma_{qi}^{2} h \\
\pi_{qi}^{u} + \pi_{qi}^{m} + \pi_{qi}^{d} &= 1.
\end{align*}
\]

A discretization of \( p(t) \cdot q(t) \) can be obtained by combining these two trinomial lattices. As a result, each of the nodes in the new lattice would have five branches.
Based on this pentanomial tree model, the calculation of the cash flows can be easy. The price of the real option can then be obtained by applying Eq. (8) and Eq. (9).

4. Numerical experiments and discussions. In this section, we apply the pentanomial lattice method to value investment opportunities for a project in a high-tech company. The sensitivity of option value to the parameters of the underlying regime-switching model is discussed. The behaviors of the real options in different types of high-tech companies are compared. For the purpose of comparison of the different high-tech companies, the time of the R&D stage is fixed as it is usually confirmed at the beginning of the project. Then four different cases are considered for the time jumping from the Mature stage to the Decline stage. In case 1, the jumping time is fixed. In other three cases, the jumping time will occur at any time over the time horizon, and the probabilities of a jump from the Mature stage to the Decline stage in the three cases are expressed in a constant, a concave and convex function of time $t$, respectively.

4.1. Application to a high-tech company. In this subsection, numerical experiments are presented to illustrate the practical implementation of our proposed models and methods. First, we assume that the life cycle time of a project in a high-tech company is 2 years, which is denoted as $T = 8$ (8 quarters or 24 months). During the life cycle, the time for the R&D stage is fixed and is assumed to be six months. This means the fixed time of the jumping from the R&D stage to the Mature stage is $t_{12} = 2$. After the R&D stage, the high-tech company will enter the Mature stage and the Decline stage in the remaining six time intervals.

The initial price and the initial amount of the product are set to be $p(0) = 2.5$ (dollars) and $q(0) = 1$, respectively. Other hypothetical values for the model parameters are given in Table 1. For the parameters like $r$ and $\gamma$ which depend on the market we refer to [25]. The values of other parameters decided by a high-tech company itself are set based on the features of a high-tech company, which are illustrated in Section 2.1. Actually, the real values of these parameters are decided seriously by the top management in a high-tech company since the setting of these values would affect the company’s profit.

The jumping time from the Mature stage to the Decline stage is denoted as $t_{23}$, and the probability distribution of $t_{23} = t$ $(t \geq t_{12})$ depends on the type of the high-tech company. Here we shall consider four different cases of probability distributions and they are presented in Table 2.

Case 1: The regime-switching time $t_{23}$ is fixed. Five sub-cases are presented in Table 3 by setting different combination of the time durations for the second and the third stages.

Case 2: The regime-switching time $t_{23}$ occurs at any time before the maturity $T$ with the same probability.

Case 3: The probability of the regime-switching time $t_{23}$ occurs at time $t$ before the maturity is a concave function of $t$. For example, this paper adopts $0.0853 \ln(4t)$ as the concave function in the following experiment. A rational regime-switching probability function of a high-tech company can be obtained by investing in the similar companies.

Case 4: The probability of the regime-switching time $t_{23}$ occurs at time $t$ before the maturity is a convex function of $t$. For example, this paper adopts $0.0516e^{0.4t}$ as the convex function in the following experiment. A rational
regime-switching probability function of a high-tech company can be obtained by investing in the similar companies.

**Table 1.** Parameters adopted in the model

| $T$ | $\sigma_p^2$ | $\sigma_q^2$ | $\mu_p^2$ | $\mu_q^2$ | $\sigma_p^2$ | $\sigma_q^2$ | $r$ | $p_{12}$ | $\gamma$ |
|-----|--------------|--------------|-----------|-----------|--------------|--------------|-----|---------|------|
| 8   | 0.38         | 0.3          | 0.2       | -0.2      | 0.4          | 0.3          | 0.0012 | 0.85    | 0.6   |

**Table 2.** Different cases for different types of high-tech companies

| Time $t_{12}$ | Case 1 | Case 2 | Case 3 | Case 4 |
|---------------|--------|--------|--------|--------|
| $t_{12} = 2$  | $t_{12} = 2$ | $t_{12} = 2$ | $t_{12} = 2$ |
| Time $t_{23}$ | Fixed time | Any time | Any time | Any time |
| $t_{23} = t$ | $t_{23} = a$ | $t_{23} = t$ | $t_{23} = t$ | $t_{23} = t$ |
| $t_{12} < t < T$ | $t_{12} < t < T$ | $t_{12} < t < T$ | $t_{12} < t < T$ |
| Probability of $t_{23} = t$ | Uniform rate | Concave function | Convex function |
| $t_{12} < t < T$ | $1/5$ | $0.0853 \ln(4t)$ | $0.0516e^{0.4t}$ |

Figure 1 shows the probability distributions of the jump time from the Mature stage to the Decline stage in Cases 3 and 4. In Case 3, the product produced by the entrant is easy to be replaced by other similar companies. With many counterparts in the market, competition for customers becomes fierce. Aggressive competition in the market results in a fast decrease in profit margins. Thus the company in Case 3 will have a relatively short Mature stage. In Case 4, the entrant is in a less competitive environment, and other companies cannot produce a similar product in a short time. If the research is successful, due to its breakthrough in techniques, the new product will monopolize the market for a while until a new product appears and replaces it. Figure 1 implies that in Case 4, the jump is more likely to occur when the time is close to $T$ and the expectation of the time staying in Mature stage will be greater than that in Case 3. In reality, the probability distribution of the jump time for a company depends on many factors, interested readers can refer to [11, 13, 26]. In the following, we shall include Case 3 and Case 4 in our numerical experiments.

Tables 3 and 5 show the amounts of the real option (invest & divest) values arising from the different cases. In these five subcases, the combinations of the durations for the Mature stage and the Decline stage are different. For example, (1,5) means the Mature stage takes 1 time step while the Decline stage takes up 5 time steps. Tables 4 and 6 show the cash amounts of the real option (invest & divest) values under different probability distributions of the jump time $t_{23}$. Tables 3 and 5 indicate that a longer Mature stage results in a higher option value. Recall that in the Mature stage, the product has already achieved acceptance in the market. The increase in growth rate of sales lead to an increase in profit margins. Hence if the Mature stage of the project is longer, the project is more worth to invest. From Tables 4 and 6, one can find that the option values in Case 4 are greater than those in Case 3. It is due to the fact that, in Case 4, the jump $t_{23}$ will occur at the time close to time $T$ with a higher probability, and the Mature stage in Case 4 is more likely to be longer than that of Case 3. These results also coincide with the findings presented in Tables 3 and 5.
4.2. Sensitivity analysis. In this subsection, we shall conduct numerical experiments to provide sensitivity analysis of our model. The method adopted here is to vary one parameter at one time, while other parameters are fixed to evaluate the impacts of the varying parameter on the real option valuation and the optimal times to exercise the two options.
Table 6. Divestment option values (dollars) in four cases with different probability distributions of the jump time $t_{23}$.

| Case of $t_{23}$ | Case 1 | Case 2 | Case 3 | Case 4 |
|------------------|--------|--------|--------|--------|
| Option Value     | 9.41   | 8.99   | 9.70   | 10.61  |

4.2.1. Impacts on the option valuation. First, we focus on the effects of the risk-free interest rate $r$ on the real option valuation. Since a risk-neutral probability is adopted in the valuation of the product price, the impact of the risk-free interest rate might not be limited to that on the discounted factor but also on the drift of the risk-neutral price process. It can be found in Figure 2 that the option value (invest & divest) decreases slowly as $r$ increases in each case. This may reflect that when $r$ increases, the discounted factor will be higher, and the change of the distribution probability of the product price cannot prevent the value of the project from becoming lower.

Figure 3 shows that the higher the proportion of taxes and various costs with respect to the revenues $(1 - \gamma)$, the lower the option value (invest & divest) will be. This is consistent with the intuition since an increase in the proportion of taxes and various costs certainly decreases cash flows from the new product sales and, thus, decreases the option value (invest & divest).

Figure 4 shows that the option value (invest & divest) in each case increases dramatically as $p_{12}$, the probability of successful development of a new product, increases. The reason is that an increase in $p_{12}$ increases the expected cash flows and, hence, increases the value of the option (invest & divest). Thus, the analysis of a company’s new product development capability is not only important for the company management but also represents an important piece of information for investors. This result suggests that when company management and investors calculate the value of the option (invest & divest), more attention should be paid on the estimation of the probability of a success in the new product development. Readers interested in company’s new product development capability can refer to [12]. Figure 5 shows that an increase in the mean of sales promotes the growth of the option value (invest & divest). This makes intuitive sense. An increase in the mean of sales enhances the expected production and the option value (invest & divest). An implication of this result is that the management of a high-tech company may pay attention to the production capacity and market demand of the new product. Then, they can design a rational mean of sales and attract more investments.

Figure 6 shows that when the investment (divestment) cost increases, the investment (divestment) option value drops first and then remains unchanged. This result implies that when the one-time cost in the Mature stage exceeds a certain level, it will not be optimal for the decision maker to exercise the option in the Mature stage. When the one-time cost in the Mature stage below the threshold, a higher one-time cost will result in a lower option value. This suggests that the management of the high-tech company may develop an appropriate one-time cost in the Mature stage to attract investors.

4.2.2. Impacts on the optimal times to exercise the two options. The real option approach developed in this paper can help decision makers to identify the projects which are undervalued. It also provides the information about optimal invest or devest time to decision makers. The numerical experiments showing the effect of
different parameters on the optimal invest and abandon time are also conducted in this paper. Four possible paths of \( p(t)q(t) \) in the pentanomial tree model are shown in Figure 7. For each path, the optimal times to exercise the options (invest & divest) are presented in Table 7 corresponding to different values of different parameters.

Table 7 implies that the changes of the interest rate \( r \) and the mean of sales in the Mature stage \( \mu_2^q \) may not affect the optimal times to exercise the options (invest & divest). While when the proportion of taxes and various costs with respect to the revenues \((1 - \gamma)\) increases, the results in Table 7 suggests the decision maker to exercise the investment (divestment) option later (earlier), and even give up the investment option. This is because the cash flow \( c(t, X(t)) \) would decrease as the value of \((1 - \gamma)\) increases. When \((1 - \gamma)\) goes up and is close to 1, it may be difficult to maintain a positive cash flow. Thus it may not be beneficial for the decision maker who has an investment option to invest in the project.
If the probability of successful development of a new product $p_{12}$ is very small, the decision maker is not suggested to exercise the option to invest in the R&D stage, but to invest in the Mature stage when the new product has been successfully developed. Then there is no need for the decision maker to bear the uncertainty resulting from the failure of development of new product. This can be seen from Table 7. That is when $p_{12}$ decreases from 0.84 to 0.04, the optimal time for exercising the option to invest changes from $t = 1$ to $t = 2$ for each path, which means the decision maker may wait for a longer time to invest in the project as the probability $p_{12}$ decreases. While for the one who has already invested in the project at time 0, the optimal time to exercise the option to divest for each path remains unchanged as $p_{12}$ changes.

It is also found in Table 7 that when the one-time investment cost $K_{1n}$ in the Mature stage goes up, the optimal time to invest in the project changes from $t = 2$ to $t = 1$ for each path.
Successful probability of the development of a new product
Option Value (dollars)
case1
case2
case3
case4
t = 1 for each path. This means that when $K_{in}$ exceeds a certain level, it is more preferable for the decision maker to invest in the R&D stage than in the Mature stage even though the development result of new product is unrevealed. When the one-time divestment cost $K_{di}$ in the Mature stage is $−2.5$, the optimal time to exercise the divestment option is $t = 5$ for each path in Table 7. This implies that when $K_{di}$ is negative and small enough, it may be optimal for the decision maker to abandon the project at the end of the Mature stage. While when the value of $K_{di}$ increases, the optimal exercise time may be postponed to the end of the decline stage $t = 8$.

5. Extensions. In this section, we consider some possible extensions to the model developed in the previous sections. In ever increasingly complex and uncertain market and business environments, there are many kinds of disruptive events such as

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{(a): Impacts of successful probability of development of a new product for investment option valuation, (b): Impacts of successful probability of development of a new product for divestment option valuation.}
\end{figure}
sharp or long economy declines, financial market bubbles and crashes, development of entirely new technologies or products by competitors etc. which are regularly observed on markets. The occurrences of disruptive events may affect the market price of the product, sales or production costs and eventually affect the cash flow of the project. The uncertainty from occurrences of disruptive events can be incorporated by introducing another stochastic multiplicative factor into the cash flow in Eq. (3), which means the proportion of taxes and various costs with respect to the revenues \(1 - \gamma\) also change overtime. The cash flows would then be given by

\[
\hat{c}(t) = \gamma(t)p(t)q(t) - c_o(t).
\]

The evolution of \(1 - \gamma(t)\) can be governed by a geometric Brownian motion. Traditionally, a Poisson process is employed to model the number of events and the random times that these events occur in a given time interval. A Poisson distribution with mean \(\lambda\) can be well approximated by the Normal distribution \(N(\lambda, \lambda)\)
for a large $\lambda$. And a large $\lambda$ in our model can be explained by the diversity of disruptive events. Hence, a geometric Brownian motion instead of a Poisson process is employed to describe the evolution of $1 - \gamma(t)$ over time. For the numerical implementation, the evolution of market price, sales and $1 - \gamma$ over time can be approximated by binomial trees. With the similar recombining techniques in Section 3, we can obtain a recombined lattice with four branches in each node by adjusting the step size in each of the binomial lattices. Then methods similar to that in Section 2 may be employed to value the investment opportunity with stochastic $1 - \gamma$.

Another possible source of uncertainty is from the costs of production. These costs of production may also change over time. This source of uncertainty can be modeled in the same way as disruptive events. Furthermore, the work presented here only considered the situation where the price process and the sales were driven by two independent Brownian motions. It is also interesting to further explore the situation where the price process and the sales are correlated using tree-based models.

Figure 6. (a): Impacts of one-time cost $K_{in}$ in mature stage for the investment option valuation, (b): Impacts of one-time Cost $K_{di}$ in Mature Stage for the divestment option valuation.
Figure 7. Four possible paths of $p(t)q(t)$ along the time in the pentanomial tree model.

Table 7. Impacts of the varying parameters on the optimal times to exercise the options. \("^{-}\) means that the option is not exercised at any time.

| Parameter | Invest Path 1 | Invest Path 2 | Invest Path 3 | Invest Path 4 | Divest Path 1 | Divest Path 2 | Divest Path 3 | Divest Path 4 |
|-----------|----------------|---------------|---------------|---------------|----------------|---------------|---------------|---------------|
| $r$       | $t = 1$        | $t = 1$       | $t = 1$       | $t = 1$       | $t = 8$        | $t = 8$       | $t = 8$       | $t = 8$       |
|           | $0.001$        | $0.0012$      | $0.0023$      | $0.0023$      | $0.0012$       | $0.0012$      | $0.0012$      | $0.0012$      |
| $1 - \gamma$ | $t = 1$      | $t = 1$       | $t = 1$       | $t = 1$       | $t = 8$        | $t = 8$       | $t = 8$       | $t = 8$       |
|           | $0.5$          | $0.5$         | $0.5$         | $0.5$         | $0.5$          | $0.5$         | $0.5$         | $0.5$         |
| $\mu_{12}$ | $t = 1$       | $t = 1$       | $t = 1$       | $t = 1$       | $t = 8$        | $t = 8$       | $t = 8$       | $t = 8$       |
|           | $0.04$         | $0.04$        | $0.04$        | $0.04$        | $0.04$         | $0.04$        | $0.04$        | $0.04$        |
| $\mu_{12}'$ | $t = 1$      | $t = 1$       | $t = 1$       | $t = 1$       | $t = 8$        | $t = 8$       | $t = 8$       | $t = 8$       |
|           | $0.05$         | $0.05$        | $0.05$        | $0.05$        | $0.05$         | $0.05$        | $0.05$        | $0.05$        |
| $\mu_{10}$ | $t = 1$       | $t = 1$       | $t = 1$       | $t = 1$       | $t = 8$        | $t = 8$       | $t = 8$       | $t = 8$       |
|           | $0.1$          | $0.1$         | $0.1$         | $0.1$         | $0.1$          | $0.1$         | $0.1$         | $0.1$         |
| $\mu_{d1}$ | $t = 1$       | $t = 1$       | $t = 1$       | $t = 1$       | $t = 8$        | $t = 8$       | $t = 8$       | $t = 8$       |
|           | $-0.5$         | $-0.5$        | $-0.5$        | $-0.5$        | $-0.5$         | $-0.5$        | $-0.5$        | $-0.5$        |

6. Conclusions. This paper presents a discrete time model for valuation of projects in the high-tech industry. The model takes into account the features of a project: high-risk and multi-stage. The evolution of state variables over time are governed by an irreversible regime-switching Markov chain which includes three stages: the R&D stage, the Mature stage and the Decline stage. Real option theory is then applied to value the project. Since a decision maker can invest in or divest from the project at any time before time $T$, the process of the valuation of the project can be formulated as an American option with time-dependent strike price. Given the non-tradeable nature of the investment opportunity in the high-tech industry, the standard Black-Scholes valuation approach cannot be applied. Here both real-world probability and risk-neutral probability are employed to value the real option as in [25]. Numerical examples are given to illustrate the implementation of the model. The impacts of some parameters, such as the risk-free interest rate, the mean of sales and the one time investment (divestment) cost, on the value of the
investment opportunity are also studied. The numerical results reveal that these parameters have material impacts on the values of the investment and divestment options. These results may also highlight the importance of parameter uncertainty in determining the value of the investment opportunity.

The model considered in this paper may pave the way to develop some modeling frameworks which integrate investment decisions in high-tech industry and financial sectors. For example, one possibly interesting direction for further research is to consider a general situation where the decision maker cannot only invest in the high-tech project, but also some financial securities such as a bond and an ordinary share. The object of the decision maker is to determine an optimal mix of investments in bond, share, and high-tech project so as to maximize an expected utility on terminal wealth. By utilizing the real option approach we employed here, the investment decision on the high-tech project can be formulated as the valuation of an American-style option. This American-style option will then be one of the investment opportunities in the optimal investment problem. This problem can be formulated as an optimal stopping-control problem which is rather uneasy to solve in a continuous-time modelling framework. However, using the numerical method based on trinomial tree considered in the present paper, one may develop a practical way to solve the problem numerically. To apply the trinomial tree approach, an additional trinomial tree lattice for the price movements of the ordinary share may be required and how to combine the additional trinomial tree with the two existing ones discussed in the present paper could be a non-trivial issue.

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