Surface/State correspondence and $ TT $ deformation

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The surface/state correspondence suggests that the bulk co-dimensional two surface could be dual to the quantum state in the holographic conformal field theory (CFT). Inspired by the cutoff-AdS/$ TT $-deformed-CFT correspondence, we propose that the quantum states of two-dimensional $ TT $-deformed holographic CFT are dual to some particular surfaces in the AdS$_3$ gravity. In particular, the time slice of the cut-off surface is dual to the ground state of the $ TT $-deformed CFT. We examine our proposal by studying the entanglement entropy and quantum information metric. We find that the complexity of the ground state in the deformed theory is consistent with the one of a particular cMERA and the holographic complexity via CV or CA prescription.

I. INTRODUCTION

The study of the black hole thermodynamics inspired G. ’t Hooft and L. Susskind [1, 2] to propose holography as the guiding principle of quantum gravity. One concrete realization of the holographic principle is the AdS/CFT correspondence, which states that quantum gravity in anti-de Sitter (AdS) spacetime could be dual to a conformal field theory (CFT) living on the asymptotical AdS boundary [3]. Among various issues in AdS/CFT, the emergence of bulk geometry is outstanding. One promising proposal is the surface/state correspondence [4, 5], which claims that any co-dimension two convex surface $ \Sigma $ corresponds to a quantum state described by a density matrix $ \rho (\Sigma) $ in the dual Hilbert space. The proposal is based on the recent development of holographic entanglement entropy [6, 7] and interesting connection between the AdS/CFT and the tensor network [8, 9]. The physical picture of the surface/state correspondence is intuitive and has inspired many new discoveries, such as the holographic entanglement of purification [10]. However, the concrete form of the correspondence is still far from clear.

In this paper, we study the surface/state correspondence in the context of newly discovered cutoff-AdS/$ TT $-deformed-CFT (cAdS/dCFT) correspondence [11]. The study of $ TT $ deformed was pioneered by Smirnov and Zamolodchikov [12, 13]. They studied the deformation triggered by the composite operator $ T_{zz} T_{zz} - T_{z \bar{z}}^2 $ in two-dimensional (2D) quantum field theory, and found that an integrable quantum filed theory is still integrable after the $ TT $ deformation. More interestingly, it was conjectured that the $ TT $-deformed holographic CFT$_2$ in the large central charge limit could be dual to the cutoff AdS$_3$ gravity provided that the sign of the deformation parameter is chosen suitably [11]. For some related works, see [14–18]. As the boundary moves into the bulk, one gets a series of codimension-one hypersurfaces. These hypersurfaces are exactly where the $ TT $-deformed CFTs live.

Their codimension-two time slices are where the quantum states of the $ TT $-deformed CFTs dwell. This fact inspires us to propose that the surface/state correspondence can be realized by the cAdS/dCFT correspondence. These co-dimension two-time-slices in AdS$_3$ correspond to the vacuum states of the $ TT $-deformed CFTs, while those in the BTZ background correspond to the mixed states at finite temperature. We will examine this proposal quantitatively by studying the entanglement entropy and complexity in both field theory and bulk gravity.

Entanglement entropy plays an important role in modern physics, especially in quantum information theory and quantum gravity. The milestone development is the holographic entanglement entropy. The authors in [6, 7] proposed that the entanglement entropy in holographic CFT can be calculated holographically by the area of the bulk minimum surface anchoring on the boundary entanglement surface. The appearance of minimal surface, which is called the RT-surface, encouraged people to conjecture that the spacetime may emerge from the quantum entanglement [19]. Another piece of evidence supporting this the conjecture comes from the analog between tensor network [20] and the RT surface [8]. The connection between tensor network and holographic entanglement surface led to the works on the spacetime reconstruction from tensor network [21], which further led to the surface/state correspondence [4, 5]. Thus it is very interesting and necessary to examine our proposal from the viewpoint of entanglement entropy.

Though entanglement plays a crucial role in studying the dual gravity, it is not enough [22], especially when one tries to understand the interior of the black hole. The complexity of a quantum state was thus introduced as a dual probe to investigate the growth rate of the Einstein-Rosen bridge [22, 23]. There have been two conjectures about the holographic complexity: the CV conjecture [24] and CA conjecture [25]. These two conjectures share some similarities but differ in many ways [26, 27]. More importantly, how to define a complexity in quantum field theory is still under intense investigations [28, 30]. Since we are studying the surface/state correspondence which was inspired from the tensor network, we will adopt the definition from [29, 33].
Our study could be seen from another point of view. In [34], the quantum information metric in a CFT with respect to a small marginal deformation has been studied. Here we study the quantum information metric with respect to an irrelevant deformation.

II. SURFACE/STATE CORRESPONDENCE, $T\bar{T}$ DEFORMATION AND ENTANGLEMENT

A. Surface/State correspondence

Let’s briefly review the surface/state correspondence [4-5]. It claims that any codimension two convex surface $\Sigma$ corresponds to a quantum state described by a density matrix $\rho(\Sigma)$ in the dual Hilbert space. When this surface is closed and topologically trivial, the state is given by a pure state $|\Phi(\Sigma)\rangle$, as shown in Figure 1. Especially, if $\Sigma$ is a time slice of AdS boundary, $|\Phi(\Sigma)\rangle$ is simply the ground state of the dual CFT.

![Figure 1: A codimension two surface $\Sigma$ corresponds to a quantum state $|\Phi(\Sigma)\rangle$.](image1)

A subregion $A$ of $\Sigma$ has its own dual state $\rho_A$, and we can define the entanglement entropy of $A$ as $S_A^{\Sigma} = -\text{tr}_A \rho_A \log \rho_A$. It was proposed [4-5] that $S_A^{\Sigma}$ satisfies the generalized Ryu-Takayanagi formula: $S_A^{\Sigma} = \frac{\text{Area}(\gamma^*)}{4G_N}$, where $\gamma^*$ is a minimal surface in the bulk whose boundary is given by $\partial A$, as shown in Figure 2.

![Figure 2: Generalized Ryu-Takayanagi formula.](image2)

B. $T\bar{T}$-deformed CFT

When a 2-dimensional quantum field theory is deformed by the $T\bar{T}$ operator, we obtain a family of theories $T^{(\mu)}$. Moving infinitesimally from $T^{(\mu)}$ to $T^{(\mu+\delta\mu)}$, the action changes as

$$S^{(\mu)} \rightarrow S^{(\mu+\delta\mu)} = S^{(\mu)} + \delta\mu \int d^2 x O^{(\mu)},$$

$$O^{(\mu)} = T^{(\mu)} \bar{T}^{(\mu)} - Q^{(\mu)^2},$$

where $T^{(\mu)} = -2\pi T_{zz}^{(\mu)}$, $\bar{T}^{(\mu)} = -2\pi \bar{T}_{zz}^{(\mu)}$, $Q^{(\mu)} = 2\pi T_{zz}^{(\mu)}$ are the energy-momentum tensor of theory $T^{(\mu)}$. The deformation parameter $\mu$ has dimension $(\text{Length})^2$ and this deformation is irrelevant. The original theory sits at $\mu = 0$. If the quantum field theory is a holographic CFT, the cAdS/dCFT correspondence claims that the deformed theory $T^{(\mu)}$ is dual to a gravitational theory living in a finite region in AdS$_3$ with a radial cutoff at $r = r_c$, where the metric can be written as

$$ds^2 = \frac{L^2}{r^2} dr^2 + \frac{r^2}{L^2} g_{\alpha\beta} dx^\alpha dx^\beta, \quad (r \leq r_c).$$

Here $L$ is the AdS radius. Within our convention, $\mu$ and $r_c$ are related by

$$\mu = \frac{6L^4}{\pi c r_c^3}. \quad (4)$$

The theory $T^{(\mu)}$ lives on the new boundary at $r = r_c$ which is a codimension one surface. The time slice $S^{r_c}$ of the new boundary is a codimension two surface on which the quantum states of $T^{(\mu)}$ dwell, as shown in Figure 3.

![Figure 3: Quantum state $|\Omega^{(\mu)}\rangle$ of $T^{(\mu)}$ lives on $S^{r_c}$.](image3)

C. Entanglement entropy

The similarity between Figure 1 and Figure 3 inspires our proposal: cAdS/dCFT is one kind of realization of the surface/state correspondence. The state corresponding to the surface $S^{r_c}$ is the quantum state of theory $T^{(\mu)}$. Especially, when the $T\bar{T}$ deformation is performed on the CFT ground state, the surface $S^{r_c}$ will correspond to the ground state $|\Omega^{(\mu)}\rangle$ of $T^{(\mu)}$, which could be expressed as

$$\langle \varphi | \Omega^{(\mu)}\rangle = \frac{\int_{\delta(t=0)=\varphi} D\phi \, e^{-\int_0^t dt \int dx L^{(\mu)}}}{(\int D\phi \, e^{-\int_0^t dt \int dx L^{(\mu)}})^{1/2}}. \quad (5)$$

Here $L^{(\mu)}$ is the Euclidean Lagrangian density of theory $T^{(\mu)}$, $t$ and $x$ are the Euclidean coordinates. The boundary condition should satisfy $\phi(t = 0) = \varphi$. Note that through $T\bar{T}$
deformation, we can only obtain the corresponding states of surfaces like $S^r$ which comes from the uniformly moving inward of the original boundary. The corresponding states of more general codimension two surfaces are still unknown.

In our study we do not need to require the existence of a BTZ black hole in the bulk. This means that we focus on the ground state of $TT$-deformed holographic CFT at zero temperature. Without the black hole, the metric $\Lambda$ is invariant under the scaling $r \rightarrow r/\lambda$, $x \rightarrow x/\lambda$, and the cut-off theory is invariant under the rescaling $r_c \rightarrow r_c/\lambda$, $t \rightarrow \lambda t$, $x \rightarrow \lambda x$. Since $\mu \sim r^{-2}$, we see that $(\mu, t, x)$ is equivalent to $(\lambda^2 \mu, \lambda t, \lambda x)$. We may certainly include an BTZ black hole in the bulk. In this case the time-slice $S^{r_c}$ corresponds to a mixed state describing the $TT$-deformed CFT at a finite temperature.

In [38] it was shown that the RT formula still holds for cAdS/dCFT to the leading order of the deformation parameter. Subsequently the study has been generalized to multiple intervals [39] at finite temperature. More works on the entanglement entropy showed that the RT formula still works in cAdS/dCFT [40][41]. Now the holographic entanglement entropy is captured by the length of the geodesic ending on the cut-off surface. This is exactly the generalized RT formula in the surface/state correspondence [5].

It is illuminating to compare the cut-off surfaces with the layers in cMERA. The coarse graining in cMERA is reflected in the quantum states in different layers. On the other hand, the cut-off surfaces corresponding to different $TT$ deformations play similar role, as shown in the holographic RG flow argument[11].

III. SURFACE/STATE CORRESPONDENCE, $TT$ DEFORMATION AND COMPLEXITY

Complexity captures the information beyond entanglement. In this section, we study the complexity of the ground state of $TT$-deformed CFT and compare it with the complexity of the cMERA which has been studied in [29][33]. The holographic complexity of the ground state is studied as well.

We follow the definition of complexity for field theory from [29][33]. For two quantum states $|\Phi(\xi)\rangle$ and $|\Phi(\xi+d\xi)\rangle$ where $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ are the parameters of the state space, the (squared) Hilbert-Schmidt distance is defined by

$$d_{HS}^2(\xi) \equiv 1 - \langle\Phi(\xi + d\xi)|\Phi(\xi)\rangle^2 = g_{ij}d\xi_id\xi_j,$$  (6)

in which $g_{ij}$ is the Fisher information metric. For two points $|\Phi(0)\rangle$ and $|\Phi(\lambda_{max})\rangle$ in the state space, one can find a curve $|\Phi(\lambda)\rangle$ connecting them which has the minimum length under the Fisher information metric. Supposing the reference state $|\Psi\rangle = |\Phi(0)\rangle$ and the target state $|\Upsilon\rangle = |\Phi(\lambda_{max})\rangle$, then the complexity between $|\Upsilon\rangle$ and $|\Psi\rangle$ is defined by

$$C_{(|\Psi\rangle, |\Upsilon\rangle)} = \int_0^{\lambda_{max}} d_{HS}(\lambda) = \int_0^{\lambda_{max}} \sqrt{g_{\lambda\lambda}} d\lambda.$$  (7)

Note that (7) is invariant under the reparametrization of $\lambda$.

A. The complexity of cMERA

The definition (7) seems good but difficult to apply due to the very high dimensions of the state space. Fortunately, the situation is simplified for cMERA. The layers in cMERA can be labeled by one parameter $u$ and the corresponding quantum states by $|\Phi(u)\rangle$. We suppose that the curve composed by $|\Phi(u)\rangle$ is exactly the minimum curve for computing the complexity and the metric along this curve is just the Fisher information metric, even though more choices exist. According to [33], one can write the line element as

$$d_{HS}(u) \equiv 1 - |\langle\Phi(u + du)|\Phi(u)\rangle|^2 = N(u)g_{uu}du^2.$$  (8)

Here

$$N(u) = \text{Vol} \cdot \int_{|k| \leq \Lambda e^u} d^dk,$$  (9)

is the volume of the effective phase space at $u$-th layer, in which $d$ is the spacial dimension, $\text{Vol}$ is the volume of $R^d$, $k$ is the momentum, $\Lambda = 1/e$ is the ultraviolet cutoff and $\Lambda e^u$ is the momentum cutoff for the $u$-th layer. Using $N(u)$, one can define a series of complexity of cMERA [29][33] by

$$C(n) = \int N(u)\frac{d}{du} \sqrt{g_{uu}} du.$$  (10)

Note that $C^{(2)}$ is just the complexity defined by Fisher information metric. For the ground state of 1+1 dimensional massless free scalar, its cMERA complexity has been worked out as

$$C^{(1)}_{cMERA} = \frac{\text{Vol} \cdot \Lambda}{2}, \quad C^{(2)}_{cMERA} = \sqrt{\text{Vol} \cdot \Lambda},$$  (11)

where $\text{Vol}$ is the length of $R^1$ and $\Lambda$ is the momentum cutoff of cMERA. $\Lambda$ can also be understood as the momentum cutoff at different layer $u$ in the same cMERA. So we have

$$C^{(1)}_{|\Phi(u)\rangle} = \frac{\text{Vol} \cdot \Lambda e^u}{2}, \quad C^{(2)}_{|\Phi(u)\rangle} = \sqrt{\text{Vol} \cdot \Lambda e^u},$$  (12)

where $\Lambda e^u$ is the momentum cutoff at $u$-th layer.

B. The complexity of the ground state of $TT$-deformed CFT

In the case of the $TT$-deformed CFT, the Hilbert-Schmidt distance between the ground states of theories $T^{(n)}$ and $T^{(\mu + du)}$ is

$$d_{HS}^2(\mu) \equiv 1 - |\langle\Omega^{(\mu + du)}|\Omega^{(\mu)}\rangle|^2 = N(\mu)g_{\mu\mu}d\mu^2,$$  (13)

where $N(\mu)$ is similar to $N(u)$,

$$N(\mu) = \text{Vol}/\sqrt{\mu}$$  (14)

with $1/\sqrt{\mu}$ being the momentum cutoff of theory $T^{(\mu)}$ and $\text{Vol}$ being the length of $R^1$. Then the complexity of the ground state of $TT$-deformed 2D CFT could be defined as

$$C^{(n)}(\mu) = \int N(\mu)\frac{d}{d\mu} \sqrt{g_{\mu\mu}} d\mu.$$  (15)
Once the metric $g_{\mu\nu}$ is obtained from (15), one can get the complexity of $|\Omega(\mu)|$ from above formula.

The distance (13) can be calculated by using the method in [32]. We consider the $T\bar{T}$-deformed CFT living on the plane. Then $|\langle \Omega(\mu+\mu_{\epsilon})|\Omega(\mu)\rangle|^2$ can be expressed by the path integral

$$
\langle \Omega(\mu+\mu_{\epsilon})|\Omega(\mu)\rangle = \frac{\int D\phi \exp[-\int dx d\delta\mathcal{L}(\phi) + \int_0^\infty dt d\delta\mathcal{L}(\phi + \mu_{\epsilon})]}{\sqrt{Z(\mu+\mu_{\epsilon})Z(\mu)}}
$$

where $Z(\mu)$ is the partition function of theory $\mathcal{T}(\mu)$. According to the definition of $T\bar{T}$ deformation, we have

$$
\mathcal{L}(\mu+\mu_{\epsilon}) - \mathcal{L}(\mu) = \delta\mathcal{L} = d\mu \cdot O(\mu)(t,x).
$$

Introducing an UV cutoff $\epsilon$, the ground state becomes

$$
\tilde{\Omega}(\mu+\mu_{\epsilon}) = \frac{\langle \exp[-\int dx d\delta\mathcal{L}] \rangle}{\langle \exp[-\int dx d\delta\mathcal{L}] \rangle^{1/2}},
$$

where $H(\mu)$ is the Hamiltonian of the theory $\mathcal{T}(\mu)$. The cutoff $\epsilon$ depends on $\mu$ implicitly $\epsilon \sim \sqrt{\mu}$. Then we get

$$
\langle \tilde{\Omega}(\mu+\mu_{\epsilon})|\Omega(\mu)\rangle = \frac{\langle \exp[-\int dx d\delta\mathcal{L}] \rangle}{\langle \exp[-\int dx d\delta\mathcal{L}] \rangle^{1/2}},
$$

where $\langle \ldots \rangle$ denotes the expectation value in the ground state $|\Omega(\mu)\rangle$. Expanding the above formula to the second order of $d\mu$, one gets

$$
1 - |\langle \tilde{\Omega}(\mu+\mu_{\epsilon})|\Omega(\mu)\rangle|^2 = d\mu^2 \int_\epsilon^\infty dt \int_{-\infty}^\infty dx dx' \langle O(\mu)(t,x)O(\mu)(t',x') \rangle.
$$

Here the time reversal symmetry $\langle \delta\mathcal{L}(t,x)\delta\mathcal{L}(-t',x') \rangle = \langle \delta\mathcal{L}(t,x)\delta\mathcal{L}(t',x') \rangle$ was used.

When $\mu = 0$, one has $O(0) = T\bar{T}$ where $T,\bar{T}$ are the energy-momentum tensor of the original CFT. So we have

$$
\langle O(0)(t,x)O(0)(t',x') \rangle = \frac{e^2}{A((t-t')^2 + (x-x')^2)^2}.
$$

The general expression of $\langle O(\mu)(t,x)O(\mu)(t',x') \rangle$ is unknown. However, its behavior can be obtained by dimensional analysis. Since $(\mu, t, x)$ is equivalent to $(\lambda^2 \mu, \lambda t, \lambda x)$, we find that the correlation function takes the form

$$
\langle O(\mu)(t,x)O(\mu)(t',x') \rangle = \frac{e^2[1 + \sum_{i=1}^\infty a_i ((t-t')^2 + (x-x')^2)^i]}{4((t-t')^2 + (x-x')^2)^2},
$$

where the coefficients $a_i$ are constants that cannot be determined now.

Plugging (22) into (20), we get

$$
1 - |\langle \tilde{\Omega}(\mu+\mu_{\epsilon})|\Omega(\mu)\rangle|^2 = N(\mu) g_{\mu\nu}d\mu^2 = N \cdot \text{Vol} \cdot \mu^{-5/2}d\mu^2.
$$

Here $\epsilon \sim \sqrt{\mu}$ was used. $N$ is a dimensionless constant and Vol is the length of $R^1$. Since $N(\mu) = \text{Vol}/\sqrt{\mu}$, we get

$$
g_{\mu\nu} = \frac{N}{\mu^2}.
$$

Plugging these into the definition of complexity (15), we get

$$
C^{(\mu)}(\mu) = \int N(\mu) \sqrt{g_{\mu\nu}} d\mu \propto \left( \frac{\text{Vol}}{\sqrt{\mu}} \right)^{\frac{3}{2}}.
$$

Now we compare the above result with the complexity of cMERA (11). Since $1/\sqrt{\mu}$ is the momentum cutoff of theory $\mathcal{T}(\mu)$, the results are consistent given $\Lambda \leftrightarrow 1/\sqrt{\mu}$. Different ground state $|\Omega(\mu)\rangle$ in $T\bar{T}$ deformed CFT corresponds to $|\Phi(u)\rangle$ at different layer $u$ in cMERA. This supports our proposal that cAdS/dCFT is a realization of surface/state correspondence. More concretely, the corresponding quantum state of surface $\mathcal{S}^c$ is the ground state $|\Omega(\mu)\rangle$ in theory $\mathcal{T}(\mu)$.

### C. Holographic complexity

There are two conjectures about the holographic complexity: CV conjecture and CA conjecture. The CV conjecture claims that the complexity of a quantum state on the boundary time slice $\Xi$ is proportional to the maximum volume of the spacelike hypersurface which meets the boundary on $\Xi$:

$$
C_V(\Xi) = \max_{\Xi = \partial B} \left[ \frac{\mathcal{V}(B)}{G_N L} \right] .
$$

Here $B$ is a hypersurface which meets the boundary on $\Xi$ and $\mathcal{V}(B)$ is its volume. The CA conjecture claims that the complexity of a quantum state on the boundary time slice $\Xi$ is

$$
C_A(\Xi) = \frac{I_{\text{WdW}}}{\pi \hbar} .
$$

Here $I_{\text{WdW}}$ is the onshell action in the WdW patch which is the closure of all spacelike surfaces with boundary $\Xi$. The WdW patch for the Poincare patch of $AdS_3$ we considered here is shown in Figure [4].

#### 1. The CV conjecture

The $T\bar{T}$ deformed CFT living on a 2-dimensional plane is dual to the Poincare patch of $AdS_3$ with boundary at finite cutoff:

$$
ds^2 = -\frac{r^2}{L^2} dt^2 + \frac{L^2}{r^2} dr^2 + \frac{r^2}{L^2} dx^2 , \quad (r \leq r_c).
$$

The deformed theory $\mathcal{T}(\mu)$ lives on the new boundary $r = r_c$ satisfying (4). Its Penrose diagram is shown in Figure [4].

We are only interested in the complexity of the ground state $|\Omega(\mu)\rangle$ in the theory $\mathcal{T}(\mu)$. Since $|\Omega(\mu)\rangle$ is just the quantum state at time $t = 0$ in theory $\mathcal{T}(\mu)$, the boundary $\Xi$ is at ($t = 0$).
Figure 4: The WdW patch for the Poincare patch of AdS. We introduced a regularized surface $r = \epsilon$.

Due to the symmetry, $B$ is the hypersurface with $t = 0$. Its volume is

$$V(B) = \int dx \int_0^r dr \sqrt{\frac{L^2 + r^2}{L^2}} = \text{Vol} \cdot r_c,$$

where Vol is the length of $R^3$. So the complexity is

$$C_V(\Omega^{(\mu)}) = \frac{\text{Vol} \cdot r_c}{G_N L} \propto \frac{\text{Vol}}{\sqrt{\mu}},$$

where the relation (4) was used. Comparing with the result from field theory, we find that

$$C_V(\Omega^{(\mu)}) \sim C^{(1)}(\mu).$$

This implies that $C^{(1)}$ is a more suitable definition of complexity for field theory.

### 2. The CA conjecture

To investigate the CA conjecture, it is convenient to introduce the tortoise coordinate $r^*(r) = \int \frac{L^2}{r^2} dr = -\frac{L^2}{r}$. We construct the Eddington-Finkelstein outgoing and in-falling coordinates as $u = t - r^*(r)$, $v = t + r^*(r)$ respectively. Then the metric of the Poincare patch of AdS$_3$ can be written as

$$ds^2 = -f(r)du^2 - 2dudr + \frac{r^2}{L^2} dv^2,$$

$$= -f(r)dv^2 + 2dudr + \frac{r^2}{L^2} dx^2,$$

where $f(r) = r^2/L^2$. The past null boundary of the WdW patch for time slice $(t = 0, r = r_c)$ is $u = u_c = \frac{L^2}{r_c}$. The future null boundary is $v = v_c = -\frac{L^2}{r_c}$.

The calculation of CA conjecture is straightforward and the final result is

$$C_A(\Omega^{(\mu)}) = \frac{\text{Vol} \cdot r_c}{8\pi^2 h G_N L} \log \frac{\tilde{L}^2}{L^2}.$$  \hspace{1cm} (34)

Here $\tilde{L}$ is a constant introduced to remove the ambiguity associated with the normalization of the tangent vectors of the null boundaries. Comparing with the result from CV conjecture, we find that they are consistent up to a constant factor $\frac{1}{8\pi^2} \log \frac{\tilde{L}^2}{L^2}$. We should require $\tilde{L} > L$ to ensure they have the similar behavior. Comparing the result of CA conjecture with that from field theory, we find

$$C_A(\Omega^{(\mu)}) \sim C^{(1)}(\mu).$$  \hspace{1cm} (35)

Thus for the ground state of $\mathcal{T}\mathcal{T}$ deformed CFT, the complexity defined by $C^{(1)}$ is consistent with the corresponding holographic CA and CV conjectures.

### IV. CONCLUSIONS AND DISCUSSIONS

The surface/state correspondence is an interesting proposal to study the emergence of spacetime. In this letter, we propose that cAdS/cCFT correspondence is a realization of surface/state correspondence. More concretely, when the surface is a time slice of the moved boundary, the corresponding state is just the ground state of the $\mathcal{T}\mathcal{T}$-deformed CFT. This proposal is novel. The study of the entanglement entropy of the $\mathcal{T}\mathcal{T}$-deformed CFT gives us the first evidence for our proposal. We further tested our proposal from the perspective of the complexity. The complexity of the ground state of $\mathcal{T}\mathcal{T}$-deformed CFT is consistent with that of the cMERA for the ground state of a massless free scalar, and also with the holographic complexity.

In this work, we only studied the complexity of the ground state of the $\mathcal{T}\mathcal{T}$ deformed CFT. To test our proposal about defining the complexity via $\mathcal{T}\mathcal{T}$ deformation, it would be interesting to generalize our study to thermofield double states in the future. Since the thermofield double state is dual to the eternal BTZ black hole, we can check whether the complexity of the thermofield double state obtained by our method is consistent with the holographic complexity of the eternal BTZ black hole.

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