Brane-Antibrane Systems Interaction under Tachyon Condensation

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The interaction between a parallel brane-antibrane and brane-antibrane is investigated by regarding the brane-antibrane pair as a kink or anti-kink type tachyon condensed state. As the kink-type tachyon condensed state is known as a non-BPS brane we expand the Lagrangian of tachyon effective field theory to the quadratic order in the off-diagonal fluctuation and then use the zeta-function regularization and Schwinger perturbative formula to evaluate the interaction within a kink-kink or a kink-antikink. The results show that while the kink and kink has repulsive force the kink and anti-kink has attractive force and may annihilate by each others. We therefore evaluate the free energy at finite temperature and determine the critical temperature above which the stable state of kink-antikink system may be found.

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1 Introduction

Sen’s conjectures on tachyon condensation [1] have drawn attention to various non-BPS D-brane configurations in string theory [2-6]. According to the conjectures the potential height of the tachyon potential exactly cancels the tension of the original unstable D-brane and, at the stable true vacuum, the original D-brane disappears and closed string theory is realized. Also, the kink-type tachyon condensations correspond to lower dimensional D-branes and organize “Descent relations” between stable and unstable D-branes [2]. These conjectures have been examined from various viewpoints in string/M theory, such as boundary string field theories (BSFT) [7]. While the simplified field theory model of tachyon dynamics [3,4] is a powerful tool in describing various aspects of these dynamics in a simpler context.

The generalization to the brane-antibrane system has also been investigated [5,6]. It is seen that, for a Dp-¯Dp pair system the kink-like configuration of the tachyon leads to a non-BPS D(p-1) brane [5]. In a recent paper [6], Hashimoto and Sakai used a tachyon effective field theory of a non-BPS brane to construct a classical solution representing a parallel brane-antibrane. From the solution the energy for the brane and antibrane with distance $x$ is proportional to $x^n e^{-x^2/4}$. The exponential factor in here denotes the appearance of an excitation of a string connecting the two branes.

In this paper we use the tachyon effective field theory of a non-BPS brane [3,4,8] to investigate the interaction between a parallel brane-antibrane and brane-antibrane. We regard one brane-antibrane pair as a kink and another pair as a kink or an anti-kink configuration. As the kink or anti-kink in here is corresponding to the tachyon condensed state it could be described by the non-BPS brane. We therefore expand the corresponding Lagrangian to the quadratic order in the off-diagonal fluctuation to evaluate the interaction within a parallel kink-kink or a parallel kink-antikink. Using the zeta-function regularization and Schwinger perturbative formula [9-11] we show that the force between the kink and kink is repulsive. However, the force between the kink and anti-kink is attractive and thus may annihilate by each others. We also evaluate the free energy at finite temperature and determine the critical temperature above which the stable state of kink-antikink system may be found.

Note that the free energy for the brane-antibrane system at finite temperature had been
evaluated in [12] in which the D-Դ system is simply regarded as a free-gas thermal system with classical tachyon field mass + tachyon classical field potential. However, our treatments are to consider the interaction between a parallel brane-antibrane and brane-antibrane which is different from them.

2 Tachyon Effective Field Theory

Lagrangian of the two derivative truncation of boundary string field theory is given by [3,4,8]

\[ L_{D_p} = -\tau_p \left( \frac{1}{2} \partial_\mu T \partial_\mu T + 1 \right) e^{-T^2/4}. \]  \hspace{1cm} (2.1)

where \( \tau_p \) is the tension of the unstable \( D_p \) brane. The unstable vacuum at \( T = 0 \) corresponds to the original \( D_p \) brane, and its energy density is exactly equal to the \( D_p \) brane tension. On the other hand, the stable vacuum at \( T = \pm \infty \) is thought of as the ”closed string vacuum” with vanishing energy. It is seen that there is another stable one soliton solution with \( T = \sqrt{2}x \) which represents a kink solution with center at \( x = 0 \). This kink solution has tension \( \tau_{p-1} = 2\sqrt{2\pi} \tau_p \) which is reasonably close to the actual value of \( \tau_{p-1} = \sqrt{2\pi} \tau_p \). It was also shown in [3] that the fluctuation modes about this soliton have integer mass squared level spacing. Thus, this model nicely conforms with the Sen conjectures [1].

2.1 Lagrangian of Brane-Antibrane System

It is known that the kink-type tachyon condensation of a brane-antibrane pair becomes a unstable D-branes [2]. In order to describe kink-kink or kink-antikink system the tachyon field needs to be generalized to a 2 by 2 matrix [4,8]. In this paper we use the following Lagrangian of the tachyon effective field

\[ L = -\tau_p Tr \left[ \frac{1}{2} \partial_\mu \bar{T} e^{-\bar{T}T/8} \partial_\mu T e^{-\bar{T}T/8} + e^{-\bar{T}T/4} \right]. \]  \hspace{1cm} (2.2)

This Lagrangian has been derived from the string theory by Kutasov, Marino and Moore [4]. Minahan [4] had also used this Lagrangian to investigate the property of stretched strings.
in tachyon condensation models. Note that in the kink-antikink system the tachyon field become complex-valued and the tachyon potential in the above equation shall take the form $Tr(e^{-TT/4})$, as expected. The tachyon field can be expressed as

$$T = \left( \begin{array}{cc} \sqrt{2}(x - x_0) & T_+ \\ T_- & -\sqrt{2}(x + x_0) \end{array} \right)$$  \hspace{1cm} (2.3a)$$

in which $T_+ = \bar{T}_- [4,8]$. The diagonal parts represent a kink (i.e., a non-BPS brane after the tachyon condensation of a brane-antibrane pair) with the center located at $x_0$ and an antikink with center located at $-x_0$. The off-diagonal complex tachyon fields are $T_+$ and $T_-$ which represent the lowest energy excitation of the strings stretched between a non-BPS brane (i.e., a kink) and non-BPS antibrane (i.e., an anti-kink). In a same way the tachyon field corresponding a kink with the center located at $x_0$ and a kink with center located at $-x_0$ is

$$T = \left( \begin{array}{cc} \sqrt{2}(x - x_0) & T_+ \\ T_- & \sqrt{2}(x + x_0) \end{array} \right)$$  \hspace{1cm} (2.3b)$$

To calculate the Lagrangian we first use the Pauli matrix $\sigma_i$ to express the tachyon field (2.3a) as

$$T = -\sqrt{2}x_01 + \sqrt{2}x\sigma_z + \frac{1}{2}(T_+ + T_-)\sigma_x + \frac{i}{2}(T_+ - T_-)\sigma_y.$$  \hspace{1cm} (2.4)$$

Then, using the formula

$$e^{x\sigma_x+y\sigma_y+z\sigma_z} = \cosh(\sqrt{x^2+y^2+z^2}) + \frac{x\sigma_x + y\sigma_y + z\sigma_z}{\sqrt{x^2+y^2+z^2}}\sinh(\sqrt{x^2+y^2+z^2}),$$  \hspace{1cm} (2.5)$$

the Lagrangian expanding to the second order in the off-diagonal tachyon field becomes

$$L_{(K-K)} \approx - \left[ 4K(x, x_0) + K(x, x_0) \partial_\mu T_+ \partial_\mu T_- + U(x, x_0) T_+T_- \right].$$  \hspace{1cm} (2.6a)$$
in which and hereafter we let $\tau_p = 1$. We also define

$$K(x, x_0) \equiv e^{-(x^2 + x_0^2)/2} \cosh(xx_0).$$  \hspace{1cm} (2.7a)$$
$$U(x, x_0) \equiv e^{-(x^2 + x_0^2)/2} \left( \frac{x_0}{x} \sinh(xx_0) - \cosh(xx_0) \right).$$  \hspace{1cm} (2.7b)$$

After the integration the first term in the (2.6) can be expressed as

$$\int_{-\infty}^{\infty} dx [e^{-(x^2 + x_0^2)/2} \cosh(xx_0)] = \frac{1}{2} \int_{-\infty}^{\infty} dx [e^{-(x-x_0)^2/2} + e^{-(x+x_0)^2/2}] = \sqrt{2\pi}. \hspace{1cm} (2.8)$$

Thus, the first term clearly represent the energy density of the independent kink and antikink, which is irrelevant to our investigation and is neglected hereafter.

To find the interaction between kink and antikink we shall integrate out the off-diagonal tachyon field in (2.6). However, in the above truncated Lagrangian the coefficients before the tachyon kinetic term and mass term, i.e. $K(x, x_0)$ and $U(x, x_0)$, are coordinate dependent and we have to adopt some approximations to proceed.

Our prescription is to redefined a new real field $\Phi_i$ by

$$T_\pm = \sqrt{K(x, x_0)} (\Phi_1 \pm i\Phi_2).$$ \hspace{1cm} (2.9)$$

Then we have the following relation

$$\int dx \ K(x, x_0) \partial_\mu T_+ \partial_\mu T_- = - \int dx \left[ \Phi_1 \partial^2 \Phi_1 + \Phi_2 \partial^2 \Phi_2 + M_k(x, x_0) \left( \Phi_1^2 + \Phi_2^2 \right) \right], \hspace{1cm} (2.10a)$$

$$\int dx \ U(x, x_0) T_+ T_- = \int dx \ M_u(x, x_0) \left( \Phi_1^2 + \Phi_2^2 \right), \hspace{1cm} (2.10b)$$

in which

$$M_k(x, x_0) \equiv - \frac{1}{4} K(x, x_0)^{-2} \partial_\mu K(x, x_0) \partial_\mu K(x, x_0) + \frac{1}{2} K(x, x_0)^{-1} \partial_\mu \partial_\nu K(x, x_0)$$

$$= \frac{1}{4} (2 - x^2) - \frac{1}{2} (1 - x^2) x_0^2 + \frac{1}{12} (3 - 2x^2) x_0^4 + O(x_0^6), \hspace{1cm} (2.11a)$$

$$M_u(x, x_0) \equiv \frac{U(x, x_0)}{K(x, x_0)} = -1 + x_0^2 - \frac{1}{3} x_0^4 + O(x_0^6). \hspace{1cm} (2.11b)$$
This means that our approximation is to consider the kink antikink at short distance, i.e. $x_0 < 1$ (note that we have set $\tau_p = 1$). The associated Hamiltonian operator with respect to the fields $\Phi_i$ can be expanded as

\begin{align}
H &= H_0 + V_1 + O(x_0^4), \\
H_0 &\equiv \partial_\mu \partial_\mu + \left(\frac{3}{2} - \frac{x^2}{4}\right), \\
V_1 &\equiv -\frac{1}{2}(1 + x^2)x_0^2, \\
\end{align}

(2.12)

(2.13a)

(2.13b)

It is fortunate that the operator $H_0$ has eigenfunction of parabolic cylinder function $D_n(x)$ [13], i.e.

\begin{align}
\frac{d^2}{dx^2}D_n(x) + (n + 1 + \frac{1}{2} - \frac{x^2}{4})D_n(x) &= 0, \\
D_n(x) &= 2^{-\frac{n}{2}}e^{-\frac{x^2}{4}}(2\pi)^{-\frac{1}{4}}(\sqrt{n!})^{-1}H_n\left(\frac{x}{\sqrt{2}}\right),
\end{align}

(2.14)

(2.15)

in which $H_n$ is the Hermite function. Note that the above definition renders the parabolic cylinder function $D_n(x)$ been normalized.

For the tachyon field (2.3b) the Lagrangian expanding to the second order in the off-diagonal tachyon field becomes

\begin{align}
L_{(K-K)} \approx -[4K(x, x_0) + K(x, x_0) \partial_\mu T_+ \partial_\mu T_-].
\end{align}

(2.16)

and the associated Hamiltonian operator become

\begin{align}
H &= H_0 + V_1 + O(x_0^4), \\
H_0 &\equiv \partial_\mu \partial_\mu + \left(\frac{1}{2} - \frac{x^2}{4}\right), \\
V_1 &\equiv -\frac{1}{2}(1 + x^2)x_0^2,
\end{align}

(2.17)

(2.18a)

(2.18b)
The existence of the exact solution of operator $H_0$ allows us, as in the conventional quantum field [11], to use the zeta-function regularization method to evaluate the renormalized effective action with a help of the Schwinger perturbative formula [10]. Using the effective action we can then obtain the interaction and free energy in the kink-antikink systems. The method is briefly described in below.

### 2.2 Effective Action and Schwinger Perturbation Method

First, for a field $\Phi$ with action $S[\Phi]$ the effective action $W$ defined by

$$e^{iW} = \int D\Phi e^{iS[\Phi]},$$

(2.19)

can be evaluated by the $\zeta$-function regularization [9-11]

$$W = -i\frac{1}{2} \ln[\text{Det}(H)] = -i\frac{1}{2} [\zeta'(0) + \zeta(0)\ln(\mu^2)],$$

(2.20)

We will take parameter $\mu = 1$ in this paper. The $\zeta$ function can be evaluated from the relation

$$\zeta_H(\nu) = [\Gamma(\nu)]^{-1} \int dx dt \int_0^\infty ids (is)^{\nu-1} <x,t|e^{-isH}|x,t>,$$

(2.21)

where the operator $H$ is defined in (2.12) ol (2.16). Next, as the quantities $V_1$ is small we can use the following expansion [10]

$$\text{Tr} e^{-isH} = \text{Tr}[e^{-isH_0} - is e^{-isH_0} V_1],$$

(2.22)

to expand the $\zeta$ function by

$$\zeta_H(\nu) = \zeta(\nu)_0 + \zeta(\nu)_1 + \cdots,$$

(2.23)

where $\zeta_i(\nu)$ are defined in below.

$$\zeta_0(\nu) = [\Gamma(\nu)]^{-1} \int dx dt \int_0^\infty ids (is)^{\nu-1} <x,t|e^{-isH_0}|x,t>,$$

(2.24a)
\[
\zeta_{V_1}(\nu) = [\Gamma(\nu)]^{-1} \int dx dt \int_0^\infty ids \left( (-is)^{\nu-1}(-is) < x, t \mid e^{-isH_0}V_1 \mid x, t > \right),
\]

(2.24b)

In the next we will use the above relations to evaluate the \( \zeta \) function and then obtain the energy and free energy in the kink-antikink systems.

### 3 Calculations of Zeta-Function and Effective Action

The \( \zeta_0(\nu) \) does not depend on \( x_0 \) and is irrelevant to our discussions. We can neglect it. The other zeta functions are calculated in below for the zero temperature and finite temperature.

#### 3.1 Zero Temperature : Kink-Antikink

From the definition we have the relation

\[
\zeta_{V_1}(\nu) = [\Gamma(\nu)]^{-1} \int dx dt \int_0^\infty ids \left( (-is)^{\nu-1}(-is) < x, t \mid e^{-isH_0} \left( -\frac{1}{2}(1 + x^2)x_0^2 \right) \mid x, t > \right)
\]

\[
= [\Gamma(\nu)]^{-1} \int dx dt \int_0^\infty dss \sum_n \int \frac{p_0}{2\pi} \left( (-s)^{n-p_0^2} \left( \int dx \left( \frac{1}{2}(1 + x^2)x_0^2 \right) D_n(x)^2 \right) \right)
\]

\[
= \nu \sum_n \left[ \int \frac{p_0}{2\pi} \frac{1}{(p_0^2 + n)^{\nu+1}} \right] \left[ (n+1)x_0^2 \right]
\]

\[
= \nu x_0^2 \frac{\Gamma(\frac{1}{2})\Gamma(\nu + \frac{1}{2})}{2\pi \Gamma(\nu + 1)} \sum_n \frac{1+n}{n^{\nu+\frac{1}{2}}} = \nu x_0^2 \frac{\zeta(-\frac{1}{2}) + \zeta(\frac{1}{2})}{2}.
\]

(3.1a)

To obtain the above result we have inserted the complete set \( |p_0, n > < p_0, n | \) before the operator \( H_0 \) and replace \( s \) by \( -is \). We have also transferred to the Euclidean space by \( p_0 \rightarrow ip_0 \). Note that repeatedly using the recursion of the Hermite function \( H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x) \) we can expressed \( x^2H_n \) as a linear combination of \( H_m \) and then from the orthogonality of the Hermite function we have a relation
\[
\int dx x^2 D_n(x) D_n(x) = \sqrt{(n+1)(n+2)} \delta_{n+2,m} + (2n+1) \delta_{n,m} + \sqrt{(m+1)(m+2)} \delta_{n-2,m}.
\]

This formula has been used to obtain the result (3.1a).

Now, substituting (3.1a) into (2.7) we can find that the bounded configuration of kink-antikink has the energy

\[
E_{K-K}(x_0) = -\frac{1}{2} x_0^2 \left[ \zeta\left(-\frac{1}{2}\right) + \zeta\left(\frac{1}{2}\right) \right] \approx 0.83 x_0^2. \tag{3.2a}
\]

This result shows an attractive linear force between a kink and an antikink with short distance. The contribution from the next order of \(x_0^4\) could be evaluated in the same way [11]. While the first order have revealed the desired physical property we shall neglect it.

### 3.2 Zero Temperature : Kink-Kink

In the same way, for the kink-kink system we have the relation

\[
\zeta_{V_1}(\nu) = [\Gamma(\nu)]^{-1} \int dx dt \int_0^\infty ds (is)^{\nu-1} (-is)^{-1} e^{-isH_0} \left( -\frac{1}{2} (1 - x^2) x_0^2 \right) | x_0^2 | x >
\]

\[
= \nu x_0^2 \frac{\Gamma(\frac{1}{2}) \Gamma(\nu + \frac{1}{2})}{2\pi \Gamma(\nu + 1)} \sum \frac{-n}{n^{\nu+\frac{1}{2}}} = \nu x_0^2 \frac{2}{\pi} \left[ - \zeta\left(\frac{1}{2}\right) \right]. \tag{3.1b}
\]

Thus the substituting the above equation (3.1b) into (2.7) we can find that the bounded configuration of kink-kink has the energy

\[
E_{K-K}(x_0) = -\frac{1}{2} x_0^2 \left[ - \zeta\left(\frac{1}{2}\right) \right] \approx -0.104 x_0^2. \tag{3.2b}
\]

This result shows a repulsive force between a kink and a kink as desired.
3.3 Finite Temperature

To evaluate the zeta function for the kink-antikink at finite temperature we shall replace

\[ p_0 \rightarrow i \frac{2\pi}{\beta} p_0, \]  

\[ \int dp_0 \rightarrow i \frac{2\pi}{\beta} \sum_{p_0}, \]  

in which \( p_0 \) is an integral and temperature \( T = \beta^{-1} \). Under these replacements the free energy \( F \) is defined by [9]

\[ \beta F = -\frac{1}{2} \zeta'(0). \]  

Now following the prescription in the zero-temperature case we have the relation

\[ \zeta_V(\nu) = [\Gamma(\nu)]^{-1} \int dx dt \int_0^\infty ds (s)^{\nu-1}(-s) <x, t | e^{-sH_0} \left( -\frac{1}{2} (1 + x^2)x_0^4 \right) | x, t > \]

\[ = \frac{\nu}{\beta} x_0^2 \sum_{n=1} \sum_{p_0} \frac{1 + n}{\left( \frac{2\pi p_0}{\beta} \right)^2 + n}^{\nu+1}. \]  

(3.6)

To proceed let is first rewrite the above equation as

\[ \frac{\nu}{\beta} x_0^2 \left( \frac{2\pi}{\beta} \right)^{-2\nu-2} \left[ 2 \sum_{n=1,\beta=1} \frac{1 + n}{p_0^2 + \left( \frac{\beta}{2\pi} \right)^2 n}^{\nu+1} + \ldots \right] \]

\[ = \frac{\nu}{\beta} x_0^2 \left( \frac{2\pi}{\beta} \right)^{-2\nu} \left[ 2 \sum_{\nu=1} \int dx \frac{1 + x \left( \frac{2\pi}{\beta} \right)^2}{p_0^2 + x}^{\nu+1} + \ldots \right] \]

\[ = \frac{\nu}{\beta} x_0^2 \left( \frac{2\pi}{\beta} \right)^{-2\nu} \left[ 4 \sum_{\nu=1} \int dy \frac{1 + y^2 \left( \frac{2\pi}{\beta} \right)^2}{p_0^2 + y^2}^{\nu+1} + \ldots \right] \]

\[ = \frac{2}{\beta} x_0^2 \left[ \left( \frac{2\pi}{\beta} \right)^{-2\nu} \zeta(2\nu) - \left( \frac{2\pi}{\beta} \right)^{-2\nu+2} \zeta(2\nu - 2) + \ldots \right] \]

\[ = \frac{2}{\beta} x_0^2 \left[ \ln(kT) + 2\zeta(3) (kT)^2 + \ldots \right] \]  

(3.7)
in which the ... represent terms which are smaller compared to the leading term at high
temperature, or those are $O(\nu^2)$ which do not contribute to the effective action. To obtain
the above result we have used a simple relation

$$\int dy \frac{y^\beta}{(y^2 + M^2)^\alpha} = \frac{\Gamma(\frac{1+\beta}{2})\Gamma(\alpha-\frac{1+\beta}{2})}{(M^2)^{\alpha-\frac{1+\beta}{2}}\Gamma(\alpha)}$$

(3.8)

The reflation formula [14]

$$\pi^{-\frac{z}{2}} \Gamma\left(\frac{z}{2}\right) \zeta(z) = \pi^{-\frac{1-z}{2}} \Gamma\left(\frac{1-z}{2}\right) \zeta(1-z),$$

(3.9)

has been used to regularize the divergence in the summation over $p_0$. (i.e., $\nu \zeta(1+\nu) =
1 + \gamma \nu + ...$, in which $\gamma$ is the Euler constant)

Substituting (3.7) into (3.5) we see that the free energy becomes local maximum at $x_0$ at
high temperature, in contrast to the zero-temperature case in which the potential of kink-
antikink system is minimum at $x_0 = 0$. This indicates that the kink-antikink system may
become stable at high temperature.

Our next work is to find the transition temperature at which the zero-temperature phase
(which is stable $x_0 = 0$) is transferred to the high-temperature phase (which is stable at
$x_0 \neq 0$). However, as the stable distance at high temperature is very large, it is beyond
the small $x_0$ approximation used in this paper. Therefore let us turn to investigate the
low-temperature case.

At low temperature $\frac{2\pi}{\beta} \ll 0$, we can use the Euler-Maclauriu formula

$$\sum_{k} f(k) = \int dx f(x) - \frac{1}{12} f'(0) - \frac{1}{720} f''''(0), \quad if \ f^{(n)}(\infty) = 0,$$

(3.10)

to take the summation over $p_0$ in (3.6). The integration term becomes just that at zero term
which have been presented at (3.1), and the $f'(0)$ terms is easy to calculate. From (3.6) and
(3.1) the final result is

$$\zeta_{V_1}(\nu) = \frac{\nu}{\beta} x_0^2 \sum_{n=1} \sum_{p_0} \frac{1+n}{\left(\frac{2\pi p_0}{\beta}\right)^2 + n}^{\nu+1}.$$
\[ \sum_n \left[ \int \frac{p_0}{2\pi (p_0^2 + n)^{\nu+1}} \right] \left[ (n + 1)x_0^2 \right] + \frac{\nu}{6\beta} x_0^2 \left( \frac{2\pi}{\beta} \right)^2 \left[ \zeta(\nu + 2) + \zeta(\nu + 1) \right] \]

\[ = \nu x_0^2 \left( \frac{1}{2} \left[ \zeta\left(\frac{1}{2}\right) + \zeta\left(\frac{1}{2}\right) \right] + (2\pi)^2 (kT)^3 \left[ \frac{1+\gamma}{6} + \frac{1}{6} \zeta(2) \right] \right) \]

\[ \approx \nu x_0^2 \left( -1.33 + 0.617(2\pi)^2 (kT)^3 \right) . \] (3.11)

The transition temperature thus is at

\[ kT_c \approx \left( \frac{1}{2\pi^2} \right)^{1/3} . \] (3.12)

These complete our investigations. We thus conclude that at the above critical temperature the kink-antikink system becomes stable.

4 Conclusion

In this paper we investigate the interactions within a parallel kink-kink and within a parallel kink-antikink. We regard the pair of brane-antibrane as a configuration of kink or antikink which is a non-BPS brane and then use the tachyon effective field theory of non-BPS brane \[3,4,8\] to investigate the interaction in this brane-antibrane systems. We expand the Lagrangian of tachyon effective field theory to the quadratic order in the off-diagonal fluctuation and use the zeta-function regularization and Schwinger perturbative formula \[9-11\] to find the to evaluate the interaction within a kink-kink or a kink-antikink. The results show that while the kink and kink has repulsive force the kink and anti-kink has attractive force and may be annihilated by each others. Especially, we also evaluate the free energy at finite temperature and determine the critical temperature above which the stable state of kink-antikink system may be found.

Finally let us make two remarks:

(1) Although the kinks or antikinks on type II brane-antibrane pairs correspond to unstable type II D-branes, which are uncharged, we can distinguish between the kink and the
antikink (the notation used in this paper) in the following way. Since the brane has not yet been annihilated by antibrane in a $D - \bar{D}$ pairs, there shall be a finite distance (which is assumed to be larger than the small distance $x_0$ used in eq.(2.11)) between the brane and antibrane. Also, as brane has positive charge while antibrane has negative charge the brane-antibrane pairs can be, for example, regarded as an electric dipole system. Now, for the brane-antibrane pairs along the x-axial we may call $(D - \bar{D})$ pairs a kink while $(\bar{D} - D)$ pairs an antikink. If a kink, i.e. a $(D_a - \bar{D}_a)$ pairs is nearly overlapped on the another kink, i.e. a $(D_b - \bar{D}_b)$ pairs, then the repulsive force between $D_a$ and $D_b$, and repulsive force between $\bar{D}_a$ and $\bar{D}_b$ will lead to a repulsive force between kink and kink. On the other hand, if a kink, i.e. a $(D_a - \bar{D}_a)$ pairs, is nearly overlapped on the another antikink, i.e. a $(\bar{D}_b - D_b)$ pairs, then the attractive force between $D_a$ and $\bar{D}_b$, and attractive force between $\bar{D}_a$ and $D_b$ will lead to an attractive force between kink and antikink.

(2) Our treatment in this paper has used the Minahan-Zwiebach model [3,4] to analyze the kink-antikink system. This model is a simplified field theory model of tachyon dynamics and is a powerful tool in describing various aspects of these dynamics in a simpler context. It is known to share common properties with string theory. However, the small distance expansions in eq.(2.11a) and eq.(2.11b) is just a result of derivative truncation of BSFT [7]. It will be interesting to investigate the problem by using of an effective action (from string field theory) to describe a short distance computation and to see the validity of our procedure of integrating out the off diagonal terms of the tachyon, which seemingly still remain light in the short distance regime. The problem with large $x_0$ is surely physical interesting and is also remained to be investigated.
1. A. Sen, “Tachyon Condensation on the Brane Antibrane System” , JHEP 9808 (1998) 012 , hep-th/9805170 ; “Descent Relations Among Bosonic D-branes”, Int. J. Mod. Phys. A14 (1999) 4061, hep-th/9902105 ; “Non-BPS States and Branes in String Theory”, hep-th/9904207; “Universality of the Tachyon Potential”, JHEP 9912 (1999) 027, hep-th/9911116.

2. S. Moriyama and S. Nakamura, “Descent Relation of Tachyon Condensation from Boundary String Field Theory” ; Phys. Lett. B506 (2001) 161, hep-th/0009246 ; K. Hashimoto and S. Nagaoka, “Realization of Brane Descent Relations in Effective Theories” , Phys. Rev D66 (2002) 02060011, hep-th/0202079.

3. B. Zwiebach, “A Solvable Toy Model for Tachyon Condensation in String Field Theory” , JHEP 0009 2000 028 , hep-th/0008227 ; J. A. Minahan and B. Zwiebach, “Field Theory Models for Tachyon and Gauge Field String Dynamics”, JHEP 0009 2000 029 , hep-th/0008231 . J. A. Minahan and B. Zwiebach, “Effective Tachyon Dynamics in Superstring Theory” , JHEP 0103 2001 038 , hep-th/0009246 . J. A. Minahan and B. Zwiebach, “Gauge Fields and Fermions in Tachyon Effective Field Theories” , JHEP 0102 2001 034 , hep-th/0011226.

4. D. Kutasov, M. Marino and G. Moore, “Remarks on Tachyon Condensation in Superstring Field Theory”, hep-th/0010108; Minahan, “Stretched Strings in Tachyon Condensation Models” , JHEP 0205 2002 024 , hep-th/0203108.

5. P. Kraus and F. Larsen, “Boundary String Field Theory of the D̄D System” , PR D63 2001 106004 , hep-th/0012198 ; T. Takayanagi, S. Terashima and T. Uesugi, “Brane-Antibrane Action from Boundary String Field Theory”, JHEP 0103 2001 019, hep-th/0012210.

6. K. Hashimoto and N. Sakai, “Brane - Antibrane as a Defect of Tachyon Condensation ”, hep-th/0209232.

7. E. Witten, “On Background Independent Open String Field Theory” , Phys. Rev. D46 (1992) 5467, hep-th/9208027; “Some Computations in Background Independent
8. A. A. Gerasimov and S. L. Shatashvili, “On non-abelian structures in field theory of open strings,” JHEP 0106, 066 (2001) [arXiv:hep-th/0105245]; E. T. Akhmedov, “Non-Abelian structures in BSFT and RR couplings”, hep-th/0110002. V. Pestun, “On non-Abelian low energy effective action for D-branes”, JHEP 0111, 017 (2001), hep-th/0110092.

9. S. Hawking, Commun. Math. Phys. 55, 133 (1976)

10. J. Schwinger, Phys. Rev. 82, 664 (1951); L. Parker, ” Aspects of quantum field theory in curved spacetime: effective action and energy-momentum tensor” in ”Recent Developments in Gravitation”, ed. S. Deser and M. Levey (New York: Plenum, 1977).

11. W. H. Huang, One-loop Effective Action on Rotational Spacetimes: ζ - Function Regularization and Swinger Perturbative Expansion, Ann. Phys. 254, 69 (1997), hep-th/0211079; W. H. Huang, ”Semiclassical gravitation and quantization for the Bianchi type I universe with large anisotropy ”, Phys. Rev. D 58 (1998) 084007, hep-th/0209092; W. H. Huang, Conformal Transformation of Renormalized Effective in Curved Spacetimes, Phys. Rev. D 51, 579 (1995).

12. U. H. Danielsson, A.Guijosa, and M. Kruczenski, ” Brane-Antibrane Systems at Finite Temperature and the Entropy of Black Branes”, JHEP 0109 (2001) 011, hep-th/00106201; M. Majumdar and A. Davis, ”Cosmological Creation of D-branes and anti-D-branes” , JHEP 0203 (2002) 056 , hep-th/0202148.
13. I. S. Gradshteyn and I. M. Ryzhik, ”Table of Integrals, Series and Products.” Academic Press. New York 1980.

14. J. Ambjorn and S. Wolfram, Ann. Phys. 147 (1983) 1; E. Elizalde, S. d. Osdinsov, A. Romeo, A. A. Bytseko, and S. Zerbini, ”Zeta Regularization Techniques with Applications”, World Scientific, 1994.