ON THE JOINT ESTIMATION OF PHASE NOISE AND TIME-VARYING CHANNELS FOR OFDM UNDER HIGH-MOBILITY CONDITIONS

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ABSTRACT

The combination of the effects of Doppler frequency shifts (due to mobility) and phase noise (due to the imperfections of oscillators operating at a high carrier frequency) poses serious challenges to Orthogonal Frequency Division Multiplexing (OFDM) wireless transmissions in terms of channel estimation and phase noise tracking performance and the associated pilot overhead required for that estimation and tracking. In this paper, we use separate sets of Basis Expansion Model (BEM) coefficients for modelling the time variation over intervals of several OFDM symbols of the channel paths and the phase noise process. Based on this model, an efficient solution approximating the maximum-likelihood joint estimation of these BEM coefficients is derived and shown to outperform state-of-the-art phase noise compensation methods.

Index Terms— OFDM, phase noise, mobility, BEM, PTRS, DMRS

1. INTRODUCTION

Beyond the fifth generation (5G) network’s physical layer will have to cope with a high degree of heterogeneity in terms of services and deployment scenarios e.g., situations of high mobility and transmission using the millimeter wave (mmWave) frequency bands. In case of perfect synchronization between the transmitter (Tx) and the receiver (Rx), Orthogonal Frequency Division Multiplexing (OFDM) offers high-quality wireless communications due to its robustness to fading channels, adequate spectral efficiency, and easy integration with the multiple-input and multiple-output (MIMO) technology. However, when working at a high carrier frequency, a time-varying phase difference between the Tx and Rx local oscillators results in the so-called Phase Noise (PN).

The PN is responsible for a multiplicative part which is common to all the OFDM symbol subcarriers i.e., Common Phase Error (CPE), and an additive part at the subcarrier level that introduces Inter-Carrier Interference (ICI) \cite{1, 2, 3}. As proposed in 3GPP standards \cite{4, 5}, Phase Noise Tracking Reference Signal (PTRS) pilots allows for CPE estimation and mitigation \cite{6, 7, 8, 9}. While CPE compensation results in performance improvement \cite{7}, ICI aware detection is needed to enable the use of high modulation and coding schemes (MCS) \cite{10}. Moreover, some of the upcoming 5G applications are expected to be deployed in mobility conditions, e.g. vehicular communications, where the fast channel variations also introduce ICI. An accurate channel state information (CSI) tracking these fast variations is needed under these conditions for ICI compensation and data recovery. The combination of PN and mobility can easily degrade the quality of the link unless sufficient pilot overhead is used to enable PN and CSI estimation. Under such conditions, a Basis Expansion Model (BEM) can typically model the channel dynamics. Since the order of the BEM model needed for most practical channel representation accuracy values is smaller than the number of subcarriers, lower pilot overhead is needed hence guaranteeing a higher spectral efficiency \cite{11, 12, 15, 16}. Since the Power Spectral Density (PSD) of PN decays rapidly beyond the loop bandwidth of the oscillator, the PN process can be sufficiently characterized by BEM consisting of a few lower-order spectral components, containing most of its energy. In \cite{13, 14}, methods for PN estimation based on such a BEM representation of the PN variations within an OFDM symbol are proposed while assuming the channel to be time-invariant within the OFDM symbols and perfectly known. A different BEM was used in \cite{7} for the same purpose, the problem of time-invariant channel estimation and data detection in the presence of PN was addressed in \cite{17} using a similar per-OFDM-symbol PN BEM and the joint estimation of the channel, the PN BEM and the data symbols vector. BEM-based estimation for time-varying channels in presence of PN was addressed in \cite{18}. However, a single BEM was used to model channel variations within OFDM symbols under the cumulative effect of Doppler frequency spread and PN.

Paper Contributions

- Most of the above-mentioned works address PN and CSI estimation as two distinct problems; and when they are addressed jointly, it is the cumulative effect of PN and Doppler that is modelled and estimated with the consequence of requiring large pilot overhead. By contrast, we propose a method to jointly tackle the time-varying channel and PN estimation that does not require such a large overhead.
- A low-complexity algorithm for approximating the
pilot-assisted Maximum Likelihood (ML) estimate for the above joint estimation problem is proposed.

• Using a BEM to model PN variations on the OFDM symbol level implicitly imposes the need for PN tracking pilots in every symbol. We instead solve the problem of estimating the coefficients of frame level PN and channel BEMs based on pilots e.g., 3GPP Demodulation Reference Signal (DMRS) [4] and PTRS [5], scattered throughout the frame.

• Numerical results obtained using simulations with realistic channel models, 5G frame structure, and channel coding demonstrate the benefits of the proposed approach with respect to CPE-based technique in terms of block error rate (BLER) for mmWave communications with user mobility.

2. SYSTEM MODEL

This section is dedicated to the baseband system model for an OFDM system experiencing PN and relative movement between the Tx and the Rx. Consider a Single Input Single Output (SISO) OFDM transmission frame consisting of $M$ consecutive OFDM symbols, each consisting of $K$ subcarriers with a total system bandwidth of $B_{\text{max}} = K\Delta f$ where $\Delta f = \frac{1}{T}$ is the subcarrier spacing and $T$ is the OFDM symbol duration. We designate the $K$-long vector of frequency domain samples of the $m$th OFDM symbol ($m \in \{0, \ldots, M - 1\}$) as $s_m$. For the Rx to be able to estimate the communication channel matrix, DMRS and PTRS pilots are inserted by the Tx within the symbols $\{s_m\}_{m=0,\ldots,M-1}$ following patterns defined by NR 5G specifications [4, 5]. The time domain samples of the $m$th OFDM symbol $x_m \in \mathbb{C}^{K \times 1}$ for $m = 0, \ldots, M - 1$ is formed by applying the Inverse Discrete Fourier Transform (IDFT) to the complex data symbols $s_k$ for $k = 0, \ldots, K - 1$, and by placing a copy of the last $K_{\text{cp}}$ samples ($K_{\text{cp}} < K$) in front of the symbol to form a cyclic prefix (CP) of length $K_{\text{cp}}$. This can be written as $x_m = A_{\text{cp}} F^H s_m$ where $A_{\text{cp}} \in \mathbb{R}^{(K+K_{\text{cp}}) \times K}$ is the CP insertion matrix and $F \in \mathbb{C}^{K \times K}$ is the DFT matrix. The discrete-time propagation channel from the Tx to the Rx is modelled as the sum of $P$ paths with delays $\{l_p\}_{p=1,\ldots,P}$ (indexed in increasing order). The time domain (TD) received samples in presence of RX side PN are given by

$$r_n = \sum_{p=1}^{P} p_n h_{p,n,x,n-l_p} + w_n, \quad (1)$$

where $p_n$ is the $n$-th PN sample, $h_{p,n}$ is the time-varying complex amplitude of the $p$-th path ($p \in \{1, \ldots, P\}$, $n \in 0, \ldots, N - 1$) with $N = M(K+K_{\text{cp}})$, and $w_n$ is the $\sigma^2$-variance additive white Gaussian noise (AWGN). Here, we assumed that PN is only present on the Rx side. This is done only for the sake of readability. All the proposed estimation algorithms are valid in the case where PN is also present at the Tx side. Each PN sample is given as $p_n = e^{j\theta_n}$, with $\theta_n$ defined by a first-order Autoregressive Model AR(1) with innovation variance $\sigma^2_{\text{PN}} = 4\pi B_{3\text{dB,PN}}\Delta t$, where $B_{3\text{dB,PN}}$ is the 3dB bandwidth of the PN [2]. In vector form, the PN is added by performing an element-wise multiplication between the received TD vector during the $m$-th OFDM symbol ($m = 0 \ldots M - 1$) and the PN vector $p_m$ associated with the same symbol and satisfying $[p_m]_n = p_{km+n} (n = 0 \ldots K + K_{\text{cp}} - 1, k \triangleq m(K+K_{\text{cp}}))$ to get

$$r_m = \text{diag} (p_m) B_{\text{cp}} H_m^{\text{TD}} A_{\text{cp}} F^H s_m + w_m, \quad (2)$$

where diag $(p_m)$ is a diagonal matrix with $p_m$ as its main diagonal, $w_m \sim \mathcal{C}N(0, \sigma^2 I_K)$, $B_{\text{cp}} \in \mathbb{R}^{K \times (K+K_{\text{cp}})}$ represents CP removal and $H_m^{\text{TD}} \in \mathbb{C}^{(K+K_{\text{cp}}) \times (K+K_{\text{cp}})}$ is the TD channel matrix defined in (3). Define $y_m \triangleq F r_m$ as the vector of received frequency domain (FD) samples of the $m$-th OFDM symbol. After some manipulations applied to (2),

$$y_m = F \sum_{p=1}^{P} \text{diag} (p_m) \text{diag} (h_{p,m}) T^l_p F^H s_m + n_m \quad (4)$$

where $T \in \mathbb{R}^{K \times K}$ is the permutation matrix defined in (3) and where $h_{p,m}$ is the $l_p$-th diagonal of matrix $B_{\text{cp}} H_m^{\text{TD}} A_{\text{cp}}$.

3. BASIS EXPANSION MODEL FOR TIME-VARYING CHANNEL AND PHASE NOISE ESTIMATION

In BEM approaches [11], $h_{p,m} \triangleq [h_{p,m}]_{n=0, \ldots, N-1}$ consisting of the channel samples associated with the $p$-th term in (1) is represented using $\alpha_{\text{ch},p} \in \mathbb{C}^{(Q_ch+1) \times 1}$ as

$$h_{p,m} = B_{\text{ch}} \alpha_{\text{ch},p}, \quad (5)$$

where the columns of $B_{\text{ch}} \in \mathbb{C}^{M(K+K_{\text{cp}}) \times (Q_ch+1)}$ are orthogonal basis sequences e.g., $[B_{\text{ch}}]_{n,q} = e^{j2\pi(q-Q_{ch})n/2}$ for exponential BEM. Thus, channel estimation boils down to estimating $P(Q_{ch}+1)$ unknowns $\alpha_{\text{ch}} \triangleq [\alpha_{\text{ch},1}^{T} \ldots \alpha_{\text{ch},P}^{T}]^{T}$ based on at least $P(Q_{ch}+1)$ transmitted pilot subcarriers.

3.1. Channel and PN Estimation with a Single BEM

In the presence of PN, a first approach is to use a single BEM to model the accumulative effect of mobility and PN on the channel. For that sake, we rewrite (4) as

$$y_m = F \sum_{p=1}^{P} T^l_p \text{diag} (h_{p,m} \odot p_m) F^H s_m + n_m$$

$$= F \sum_{p=1}^{P} T^l_p \text{diag} (F^H s_m)(p_m \odot h_{p,m}) + n_m \quad (6)$$

and we define $\alpha_{\text{ch},p,m} \in \mathbb{C}^{(Q_{ch},p,m+1) \times 1}$ as the BEM representation of the point-wise product vector $p_m \odot h_{p,m}$ associated with the basis matrix $B_{\text{ch},p,m} \in \mathbb{C}^{N \times (Q_{ch},p,m+1)}$. Let $K^o \leq MK$ designate the total number of DMRS and PTRS pilot subcarriers (including any necessary guard (null) subcarriers surrounding them) in the entire OFDM frame and define $y^o$ as the aggregate of the frame samples received at the positions of these subcarriers i.e., $y^o = A^o [y_0 \ldots y_{M-1}]^T$ where matrix $A^o \in \mathbb{R}^{K^o \times MK}$ has per row a single non-zero entry

1more precisely, per-sub-frame, using the 3GPP terminology
that is equal to one and which occupies the position of one of the pilots or guard subcarriers. Then

$$y^o = S(I_\alpha \otimes B_{ch,pn}) \alpha_{ch,pn} + \mathbf{n} = \mathbf{z} + \mathbf{n},$$

where $\otimes$ stands for the Kronecker product, $\mathbf{n}$ is the additive noise vector and $S \in \mathbb{C}^{MK \times M}$ is the sensing matrix linking the unknown vector $\alpha_{ch,pn}^T \triangleq [\alpha_{ch,pn,1}^T \ldots \alpha_{ch,pn,P}]^T$ to the measurements vector. Referring to (6), $S$ can be written as a block row of the following sub-matrices ($m = 0 \ldots M - 1$)

$$S_{(m-K+1):m} = A^\alpha F T^p \text{diag}(F^H s_m^o),$$

where $s_m^o$ equals $s_m$ at pilot subcarriers and zero elsewhere. Due to the i.i.d. Gaussian nature of $\mathbf{n}$, the log-likelihood function for estimating $\alpha_{ch,pn}$ from $y^o$ is $L(\alpha_{ch,pn}) = \|y^o - \mathbf{z}\|^2$ and the maximum likelihood estimate $\hat{\alpha}_{ch,pn} \triangleq \text{argmin}_{\alpha_{ch,pn}} L(\alpha_{ch,pn})$ is $(\dagger)$: Moore-Penrose inverse

$$\hat{\alpha}_{ch,pn} = (S(I_\alpha \otimes B_{ch,pn}))^\dagger y^o.$$  

The advantage of the single-BEM formulation is that the ML estimate as given by (9) is linear in $y^o$, thus guaranteeing low complexity. However, the pilot overhead becomes prohibitive, as validated in the simulations section, as soon as the $B_{ld,B,PN}$ is large enough. Indeed, the entries of the vector $h_p \otimes \mathbf{p}$ can be seen as the samples of a time-varying channel with increased Doppler spread. Modelling and estimating such a channel requires a BEM with a higher order ($Q_{ch,pn} > Q_{ch}$) and, consequently, a larger pilot overhead.

3.2. Channel and PN Estimation with Separate BEMs

We now model the PN vector $\mathbf{p}$ using a separate BEM as

$$\mathbf{p} = B_{pn} \alpha_{pn}$$

with $\alpha_{pn} \in \mathbb{C}^{Q_{pn} \times 1}$. Equation (4) can be rewritten as (10) where $A_m$ is a $K \times N$ selection matrix that is defined according to the $m$th OFDM symbol. Since (10) is no longer linear, an iterative approach is here investigated to estimate both the channel taps $h_p$ and the PN samples $\mathbf{p}$. In fact, the log-likelihood function $L(\alpha_{pn}, \alpha_{ch}) = \|y^o - \mathbf{z}(\alpha_{pn}, \alpha_{ch})\|^2$, with $\mathbf{z}(\alpha, \beta) \triangleq A^\alpha \times \left[z_0^T(s_{m}^o, \alpha, \beta) \ldots z_{M-1}^T(s_{M-1}^o, \alpha, \beta)\right]^T$ is quadratic in both the PN $\alpha_{pn}$ and the channel $\alpha_{ch}$.

One possible approach to search for $\text{argmin} L(\alpha_{pn}, \alpha_{ch})$ is alternating optimization [20]: during iteration $t \geq 1$ we derive $\alpha_{pn}^{(t)}$ that minimizes $L(\alpha_{pn}, \alpha_{ch}^{(t-1)})$ then $\alpha_{ch}^{(t)}$ that minimizes $L(\hat{\alpha}_{pn}^{(t)}, \alpha_{ch})$ until a stopping condition is satisfied. Note that $\alpha_{pn}^{(t)}$ can be found by using (9) with $B_{pn}$ replacing $B_{ch,pn}$ and with $S$ defined using (8) where its right-hand side (rhs) is $A^\alpha F \sum_{p=1}^P \text{diag}(A_m B_{ch,pn} \alpha_{ch,p}^{(t-1)})^T \text{diag}(F^H s_m^o)$. Then, $\hat{\alpha}_{ch}^{(t)}$ can similarly be computed using (9) with $B_{ch,pn}$ replacing $B_{ch,pn}$ and with $S$ defined using (8) with its rhs replaced by $A^\alpha F \sum_{p=1}^P \text{diag}(A_m B_{ch,pn} \alpha_{ch,p}^{(t-1)})^T \text{diag}(A_m B_{pn} \alpha_{pn}^{(t)})$. The pseudo-code of this procedure is reported in Algorithm 1. The conditions of [20, Theorems 2 and 3] are met, therefore Algorithm 1 provides a sequence of non-increasing log-likelihood values and an associated sequence of BEM coefficients that converge at least to a local solution to $\text{argmin} L(\alpha_{pn}, \alpha_{ch})$. Once the stopping condition is met, the estimated PN samples are used to compensate the PN in the received signal vector, while the frequency domain channel matrix (computed from the estimated $h_p$) is used for linear minimum mean square error detection of $\{s_m\}_{m=0\ldots M-1}$.

4. NUMERICAL RESULTS

This section provides numerical evaluations of the two approaches described above and compares them to the method of CPE estimation, widely studied in the literature [2] and

$$y_m \triangleq \sum_{p=1}^P \text{diag}(A_m B_{pn} \alpha_{pn}) \text{diag}(A_m B_{ch} \alpha_{ch,p})^T T^p F_m^H s_m + \mathbf{n}_m$$

$z_m(\mathbf{s}_m, \alpha_{pn}, \alpha_{ch})$. 

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
Algorithm 1: Channel and Phase Noise Estimation

Input: $Q_{ch}$, $Q_{pn}$, $t_{\text{max}}$, $\epsilon$, Output: $\{h_p^{(t)}\}_{p=1}^{P}$ and $p^{(t)}$

init $h_{p}^{(0)} \leftarrow 1 \quad \forall p = 1 \ldots P; \quad n = 0 \ldots N - 1$

for $t = 1, 2, \ldots$ do

$\hat{\alpha}_{\text{pn}}^{(t)} \leftarrow \arg \min L(\alpha_{\text{pn}}, \hat{\alpha}_{\text{ch}}^{(t-1)}), \quad p^{(t)} \leftarrow p(\hat{\alpha}_{\text{pn}}^{(t)})$ \hspace{1cm} (11)

$\hat{\alpha}_{\text{ch}}^{(t)} \leftarrow \arg \min L(\hat{\alpha}_{\text{pn}}^{(t)}, \alpha_{\text{ch}}), \quad h_{p}^{(t)} \leftarrow h_{p}(\hat{\alpha}_{\text{ch}}^{(t)})$ \hspace{1cm} (5)

end when $t = t_{\text{max}}$ or $\|\hat{\alpha}_{\text{ch}}^{(t)} - \alpha_{\text{ch}}^{(t-1)}\|^2 < \epsilon$

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Table 1. Simulation variables

| Parameter                              | Value       |
|----------------------------------------|-------------|
| (Modulation size, LDPC coding rate)    | (64,1/2)    |
| Channel model                          | 3G PTDLC100 |
| $B_{\text{max}}$, Transmission bandwidth | 8.640 MHz   |
| $f_c$, Carrier frequency               | 30 GHz      |
| $B_{3\text{dB}}$, 3dB bandwidth of the PN | 350 Hz     |
| BEM type                               | DPSS*       |

*Discrete Prolate Spheroidal Sequences (DPSS) as in [19]

The main simulation parameters are reported in Table 1. The performance metrics used for evaluation are BLER versus the Signal-to-Noise Ratio (SNR) and the MSE between the estimated and real PN samples. The CPE-based estimates are able to track the PN samples, guaranteeing a lower MSE. The performance of Algorithm 1 is compared with single-BEM PN-channel estimation (Sec. 3.1) and the CPE-based method for both low and high mobility. The comparison illustrates the advantage of the proposed solution over current PN compensation methods.

Proposed per-frame separate-BEMs approach yields the best BLER performance. Interestingly, the excessive requirement of the single-BEM approach in terms of pilot overhead makes its performance worse than the CPE-based method under realistic pilot patterns (10 DMRS symbols in every frame).

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5. CONCLUDING REMARKS

In this paper, we investigated the problem of the joint channel and PN estimation for an OFDM transmission with both Rx mobility and high carrier frequency. We proposed an approach where the first set of BEM coefficients models the time-variation over a frame of OFDM symbols of the channel paths, while the second one models the PN realization over the same interval. We provided an efficient approximate solution to the problem of ML joint estimation of these BEM coefficients. Numerical results showed the advantage of the proposed solution over current PN compensation methods.
6. REFERENCES

[1] J. P. Santacruz, S. Rommel, U. Johannsen, A. Jurado-Navas, and I. T. Monroy, “Analysis and compensation of phase noise in mm-wave OFDM ARoF systems for beyond 5G,” Journal of Lightwave Technology, vol. 39, no. 6, pp. 1602–1610, 2021.

[2] A. G. Armada, “Understanding the effects of phase noise in orthogonal frequency division multiplexing (OFDM),” IEEE transactions on broadcasting, vol. 47, no. 2, pp. 153–159, 2001.

[3] T. C. Schenk, R. W. Van der Hofstad, E. R. Fledderus, and P. F. Smulders, “Distribution of the ICI term in phase noise impaired OFDM systems,” IEEE Transactions on Wireless Communications, vol. 6, no. 4, pp. 1488–1500, 2007.

[4] 3GPP TS 38.211, “NR; Physical channels and modulation,” 2018.

[5] 3GPP R1-1702959, “PTRS design,” 2017.

[6] M. Chung, L. Liu, and O. Edfors, “Phase-noise compensation for OFDM systems exploiting coherence bandwidth: Modeling, algorithms, and analysis,” IEEE Transactions on Wireless Communications, vol. 21, no. 5, pp. 3040–3056, 2022.

[7] J.-C. Sibel, “Pilot-based phase noise tracking for uplink DFT-s-OFDM in 5G,” in 2018 25th International Conference on Telecommunications (ICT). IEEE, 2018, pp. 52–56.

[8] Y. Qi, M. Hunukumbure, H. Nam, H. Yoo, and S. Amuru, “On the phase tracking reference signal (PTRS) design for 5G new radio (NR),” in 2018 IEEE 88th Vehicular Technology Conference (VTC-Fall). IEEE, 2018, pp. 1–5.

[9] L. Zhang, J. Ma, Z. Wu, Y. Wang, C. Liu, and K. Habib, “Efficient cross-correlation algorithm for correction of common phase error employing preamble for orthogonal frequency division multiplexing (OFDM) receivers,” Wireless Personal Communications, vol. 118, no. 1, pp. 535–549, 2021.

[10] R. Krishnan, A. Graell i Amat, T. Eriksson, and G. Colavolpe, “Constellation optimization in the presence of strong phase noise,” IEEE Transactions on Communications, vol. 61, no. 12, pp. 5056–5066, 2013.

[11] S. Chen, G. Dai, and W. Rao, “A BEM for estimation of time-varying channels in OFDM,” in 2009 WRI International Conference on Communications and Mobile Computing, vol. 1. IEEE, 2009, pp. 256–259.

[12] N. M. Idrees, W. Haselmayr, D. Schellander, and A. Springer, “Time variant channel estimation using a modified complex exponential basis expansion model in LTE-OFDM systems,” in 21st Annual IEEE International Symposium on Personal, Indoor and Mobile Radio Communications. IEEE, 2010, pp. 603–607.

[13] P. Mathecken, T. Riihonen, S. Werner, and R. Wichman, “Constrained phase noise estimation in OFDM using scattered pilots without decision feedback,” IEEE Transactions on Signal Processing, vol. 65, no. 9, pp. 2348–2362, 2017.

[14] V. Syrjälä, T. Levanen, T. Ihalainen, and M. Valkama, “Pilot allocation and computationally efficient non-iterative estimation of phase noise in OFDM,” IEEE Wireless Communications Letters, vol. 8, no. 2, pp. 640–643, 2019.

[15] P. Mathecken, S. Werner, T. Riihonen, and R. Wichman, “Subspace-based phase noise estimation in OFDM receivers,” in 2015 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), 2015, pp. 3227–3231.

[16] P. Mathecken, T. Riihonen, S. Werner, and R. Wichman, “Constrained phase noise estimation in OFDM using scattered pilots without decision feedback,” IEEE Transactions on Signal Processing, vol. 65, no. 9, pp. 2348–2362, 2017.

[17] K. Shrivastav, R. Yadav, and K. Jain, “Joint MAP channel estimation and data detection for OFDM in presence of phase noise from free running and phase locked loop oscillator,” Digital Communications and Networks, vol. 7, no. 1, pp. 55–61, 2021. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S2352864820302686

[18] S. Zhang, D. Ren, W. Guo, and J. Su, “Pilot-assisted time-varying channel estimation for OFDM systems with phase noise,” in 2008 4th International Conference on Wireless Communications, Networking and Mobile Computing, 2008, pp. 1–4.

[19] X. Quan, F. Qi, X. Jing, S. Sun, H. Huang, and N. Chen, “Doubly selective channel estimation in ofdm system using optimized discrete prolate spheroidal sequences,” in The Proceedings of the Third International Conference on Communications, Signal Processing, and Systems, J. Mu, Q. Liang, W. Wang, B. Zhang, and Y. Pi, Eds. Cham: Springer International Publishing, 2015, pp. 365–372.

[20] J. Bezdek and R. Hathaway, “Convergence of alternating optimization,” Neural, Parallel & Scientific Computations, vol. 11, pp. 351–368, 12 2003.