Black holes in the varying speed of light theory

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We consider the effect of the Varying Speed of Light theory on non-rotating black holes. We show that in any varying-\(c\) theory, the Schwarzschild solution is neither static nor stationary. For a no-charged black hole, the singularity in the Schwarzschild horizon cannot be removed by coordinate transformation. Hence, no matter can enter the horizon, and the interior part of the black hole is separated from the rest of the Universe. If \(\dot{c} < 0\), then the size of the Schwarzschild radius increases with time. The higher value of the speed of light in the very early Universe may have caused a large reduction in the probability of the creation of the primordial black holes and their population. The same analogy is also considered for the charged black holes.

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1. INTRODUCTION

Although the Standard Big Bang (SBB) model of the Universe provides a successful model and does not conflict with astronomical observations, several preassumptions in this model, which were introduced as initial values rather than derived aspects, can be considered as its weakness. According to the SBB model the flatness, horizon, primordial seeds of galaxies, magnetic monopole, and cosmological constant (\(\Lambda\)) problems cannot be derived from the theory [1].

Efforts to derive the above problems as results of the theory have mainly introduced the inflationary scenario of the Universe [2, 3, 4, 5]. In this scenario, immediately after the big bang, the Universe experienced a superluminal expansion due to potentials that convert gravitational attraction into repulsion. During the last two decades of twentieth century, the various models of inflation were improved [6] but there are still some open questions that must be answered by these models. For instance, no successful microscopic foundation for inflation has been presented. At this time, no models of inflation are fully satisfactory [7]; hence, one can feel free to search for other theories.

Varying Speed of Light (VSL) models have therefore been introduced as alternatives to the inflationary models [7, 8, 9, 10, 11, 12, 13]. They solve the problems of SBB model and are also in agreement with the theories that allow the fine structure constant, \(\alpha\), to vary, for instance, theories that try to bind high-energy physics and standard cosmology with the idea of dimensional reduction. Since high-energy theories use higher dimensional spaces and low-energy physics is four-dimensional, then there must be a dimension reduction mechanism to lower dimensions. Such a mechanism usually lets one or more of the constants have time or space dependencies [14, 15]. Observations also confirm the time variation of \(\alpha\) [16]. However, any variation in \(\alpha = e^2/\hbar c\) can be explained as a variation in \(c\), \(\hbar\) and (or) \(e\) [17]. The theory in which \(e\) has a time dependency was proposed by Beckenstein [18]. In this theory, the vacuum, as a dielectric medium, screens the electric charge. Although VSL theory can be transformed into a varying-\(e\) theory by a suitable choice of transformation, the dual of minimal VSL theory is not a minimal varying-\(e\) theory itself [17]. Also, new observations of some supernovae show that the universe is accelerating [19, 20, 21], which is in agreement with VSL theory.

Different models of the VSL have been introduced, but none of them can be regarded as an exact mechanism for the dynamics of \(c\) as yet [9, 11, 22, 23, 24]. For instance, in one model, Albrecht and Magueijo [8] postulated that light traveled much faster in the early Universe and, due to a phase transition at a critical time \(t_c\), the velocity of light fell suddenly and changed to its current value. In a model proposed by Barrow [9], \(c\) is a smooth power-law function of time and reaches from a very large value to today’s value. In another model [22, 23, 24], a phase of spontaneous breaking of a local Lorentz-invariance generates a large increase in the speed of light. Reformulation of this model leads to a bimetric model. That is, there are two separate metrics, one is associated with gravity and one with electrodynamics, which are related to each other with a vector or gradient of a scalar field.

There has been some efforts, in the literature, to see different consequences of the VSL models, and in this article we intend, by a few plausible assumptions, to examine their consequence(s) for black holes.

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2. THE FIELD EQUATION

As a first consequence of the VSL theory, a dynamical field should appear somehow in gravitational theory, hence the exact form of the Einstein equation may not be valid anymore. Actually, when \( c(x^a) \) varies, an obvious problem appears in the heart of general relativity, the Einstein equation itself, i.e.

\[
G_{ab} = \frac{8\pi G}{c^4} T_{ab}. \tag{1}
\]

For the Bianchi identity implies that the covariant divergence of the left-hand side of (1) must be zero, hence one gets

\[
\nabla_a \left( \frac{8\pi G}{c^4} T^{ab} \right) = 0. \tag{2}
\]

In the classical view, when \( c \) is fixed, (2) is in agreement with energy-momentum conservation, but when \( c \) varies, it is not valid anymore. To solve this problem, the following two main ideas [13] are suggested:

1. Adding other term(s) to \( T^{ab} \), but it is important to note that even when one wants to derive a vacuum solution, the right-hand side of (1) may not be zero.

2. By neglecting the energy-momentum conservation, any change in \( c \) can act as a source of matter creation. If it is so, any experiment that leads to violation of this conservation can be used to estimate the changing ratio of \( c \). In this case, for a vacuum solution, the right-hand side is zero.

However, there are also some works that handle the dynamics of the VSL models, for example, see ref. [22].

3. THE SCHWARZSCHILD METRIC

As there is not a universally accepted theory for dynamics of the VSL models, which includes a field for varying \( c \), one may find it useful to analyze the consequences of these models even in an elementary, simple way. For instance, we take an advantage of the correspondence principle or the principle of minimal coupling as an indication for further procedure in this work. As the same advantage has almost been taken in considering a way to amend and (or) generalize the Friedmann equations, which were written for the Einstein equation in a preferred frame with constant \( c \), to propose a varying-\( c \) model in ref. [25].

The Schwarzschild metric is a spherically symmetric vacuum solution to the Einstein equation that is asymptotically flat, i.e.,

\[
ds^2 = \left( 1 - \frac{2m}{r} \right) c^2 dt^2 - \left( 1 - \frac{2m}{r} \right)^{-1} dr^2 - r^2 d\Omega^2, \tag{3}
\]

with \( m \) as a constant of integral. The Birkhoff theorem implies that this solution is also static. Using the weak-field limit, which arises purely from assuming geodesic motion and the Newtonian limit, leads to an interpretation of the constant \( m \), as a geometrized mass, to be equal to \( GM/c^2 \) of a point mass \( M \) situated at the origin, i.e., the Schwarzschild metric becomes

\[
ds^2 = \left( 1 - \frac{2GM}{c^2 r} \right) c^2 dt^2 - \left( 1 - \frac{2GM}{c^2 r} \right)^{-1} dr^2 - r^2 d\Omega^2. \tag{4}
\]

It is well-known that the Schwarzschild metric is not only a vacuum solution to the Einstein equation, but it is a particular exact vacuum solution with fixed \( G \) to the Brans-Dicke theory of gravity as well [26].

Now, if either \( G \) or \( c \) is dependent on space or time, then an explicit calculation of (4) shows that some of the Einstein tensor components will no longer be zero and hence such a line element obviously does not correspond to a vacuum solution of the standard Einstein equation.

Before we proceed further, we should clear up the following assumption as well. In a metric one usually writes \( dx^0 = c \, dt \), the component of \( g_{00} \) is the coefficient of \( dx^0 \) and not the coefficient of \( dt \) (except in the especial case of \( c = 1 \) [30]). However, if there would be a \( c(x^a) \) in a component of \( g_{ab} \), its derivative(s) should appear in the Einstein tensor. Hence, in the case of vacuum, i.e., \( T_{ab} = 0 \), the dynamic equation should be \( G_{ab} \neq 0 \) in the VSL theory, as expected in the first suggestion of the previous section. This means that the Birkhoff theorem is no longer applicable here.
Therefore, if one looks for a spherically symmetric gravitational field solution for a vacuum case in the VSL theory, it should be a plausible assumption that one would expect, somehow, to have a generalization of the Schwarzschild metric. One may justify it better with the correspondence principle. Hence, with the aid of the principle of minimal coupling, we assume that the simplest generalization of this metric in the case of the VSL theory would be as follows

\[ ds^2 = \left( 1 - \frac{2GM}{c(t)^2 r} \right) c(t)^2 dt^2 - \left( 1 - \frac{2GM}{c(t)^2 r} \right)^{-1} dr^2 - r^2 d\Omega^2 . \]

(5)

Obviously for constant \( c \), (5) goes to the standard Schwarzschild metric; and with the metric (5) one will not get the standard Einstein vacuum equation as expected.

Perhaps, it could be more accurate if one assumes \( c = c(t, r) \). However, in a preferred frame theory, as in the most of the work appearing in the literature to date regarding the VSL models, when cosmological solutions are considered, i.e., homogeneous and isotropic space-time, one can assume \( c \) as a function of time only.

The metric (5) is no longer static or even stationary, this is a penalty that one must pay for this generalization. We should emphasize that one should have expected such a result, for a time dependence for \( c \) of these black holes could seriously tend to zero. This means that the probability of creation of primordial black holes to decrease. This means that the population of primordial black holes is much lower than expected, hence, the probability of any possible observation of these black holes could seriously tend to zero.

Like the classical Schwarzschild solution, this line element in addition to an essential singularity in the center, has a singularity at

\[ r_s = \frac{2GM}{c(t)^2} , \]

namely, at the Schwarzschild radius.

A question arises: is the singularity at (6) in the VSL theory removable? To answer this question, one may calculate the Riemann tensor scalar invariant as usual, but with the assumption of varying \( c \). The resultant is

\[
R_{abcd}R^{abcd} = 12 \frac{(\frac{\dot{c}}{c})^2}{r^4} - 8 \frac{(\frac{\dot{c}}{c})^2 \dot{c}(t)}{r^2 c(t)^3 (1 - \frac{\dot{r}}{r})^2} - 16\frac{(\frac{\dot{c}}{c})^2 \dot{c}(t)^2 (1 + \frac{\dot{r}}{r})}{r^2 c(t)^4 (1 - \frac{\dot{r}}{r})^3} \\
+ 4 \frac{(\frac{\dot{c}}{c})^2 \dot{c}(t)^2}{c(t)^6 (1 - \frac{\dot{r}}{r})^4} - 32 \frac{(\frac{\dot{c}}{c})^2 \dot{c}(t)^2 \dot{c}(t)}{c(t)^7 (1 - \frac{\dot{r}}{r})^5} \\
+ 64 \frac{(\frac{\dot{c}}{c})^2 \dot{c}(t)^4}{c(t)^8 (1 - \frac{\dot{r}}{r})^6} .
\]

(7)

It is clear, this scalar at \( r = r_s \), besides \( r = 0 \), tends to infinity. It means that the Schwarzschild radius, as \( c \) varies, is also an essential singularity. Hence, no coordinate transformation can be found to let the matter enter the horizon[31], i.e. nothing can pass the horizon. The interior part of a black hole is separated from its exterior, i.e., the whole manifold. This situation remains the same until the speed of light reaches a constant value during an epoch of time. Consider the Schwarzschild radius as an essential singularity in the VSL models can also be found in the literature[24], however, in another context.

The mass \( M \) in (6) is not only the interior mass of a black hole, but any outer matter that is attracted towards a black hole (especially in a spherically symmetric configuration) should be included in \( M \). Hence by attracting \( M \) more matter, the mass \( M \), and, respectively the Schwarzschild radius, which is the only parameter describing a black hole, increases. Although this is not due to the VSL theory, it guaranties the dynamic evolution of the Schwarzschild radius in the VSL theory.

To see the effect of the VSL theory on the Schwarzschild radius, from (6), one can write

\[
\frac{\dot{r}}{r} = -2 \frac{\dot{c}(t)}{c(t)} ,
\]

(8)

for a black hole with fixed mass \( M \). Regarding (8), when \( c \) is decreasing, i.e., \( \dot{c}(t) < 0 \), the Schwarzschild radius increases with time. In the case of \( \dot{c}(t) > 0 \), the Schwarzschild radius decreases.

The above effects are removed when the speed of light takes a fixed value. The Lorentz-invariance is then restored and matter can pass the Schwarzschild horizon of the black hole.

In the early Universe, when the speed of light was much higher than now (at least \( 10^{30} \) higher in order to solve the SBB problems[8]), (6) shows that the Schwarzschild radius associated with a particular mass \( M \), was much smaller, therefore it should have caused the probability of creation of primordial black holes to decrease. This means that the population of primordial black holes is much lower than expected, hence, the probability of any possible observation of these black holes could seriously tend to zero.
4. THE CHARGED BLACK HOLES

In this section, we extend the above procedure for the charged black holes in the case of the VSL theory. Hence, regarding the same analogy for the Reissner-Nordstrøm metric, one may assume the following solution

\[
ds^2 = \left(1 - \frac{2GM}{c(t)^2r} + \frac{\varepsilon^2}{r^2}\right) c(t)^2 dt^2 - \left(1 - \frac{2GM}{c(t)^2r} + \frac{\varepsilon^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2,
\]

where \(\varepsilon\) is interpreted as the net (geometric) electric charge of the black hole. Therefore, the Riemann tensor scalar invariant will be

\[
R_{abcd}R^{abcd} = 12 \left(\frac{\varepsilon^2}{r^2} - \frac{\varepsilon^2}{r^2}\right)^2 + \frac{1}{2} \left(\frac{\varepsilon^2}{r^2}\right)^2 - 8 \frac{(\varepsilon^2/c(t)^2 - 3\varepsilon^2/c(t)^2)}{r^2c(t)^3(1 - \frac{\varepsilon^2}{r^2} + \frac{\varepsilon^2}{r^2})^2} \right.
\]

\[
+ 16 \frac{(\varepsilon^2/c(t)^2)(1 + \varepsilon^2/c(t)^2 - 6\varepsilon^2/c(t)^2) + (\varepsilon^2)}{r^2c(t)^3(1 - \frac{\varepsilon^2}{r^2} + \frac{\varepsilon^2}{r^2})^3} \right.
\]

\[
+ 4 \frac{(\varepsilon^2/c(t)^2)^2}{c(t)^6(1 - \frac{\varepsilon^2}{r^2} + \frac{\varepsilon^2}{r^2})^4} - 32 \frac{(\varepsilon^2/c(t)^2)^2(\varepsilon^2/c(t)^2)(1 + \varepsilon^2)}{c(t)^7(1 - \frac{\varepsilon^2}{r^2} + \frac{\varepsilon^2}{r^2})^5} \right.
\]

\[
+ 64 \frac{(\varepsilon^2/c(t)^2)^4(1 + \varepsilon^2)^2}{c(t)^8(1 - \frac{\varepsilon^2}{r^2} + \frac{\varepsilon^2}{r^2})^6}.
\]

where \(\varepsilon\) is assumed to be a constant. Again, as is obvious, the above scalar in the case of \(\varepsilon^2 \leq (GM/c^2) \equiv m^2\), at \(r = r_\pm\), where

\[
r_\pm = m \pm (m^2 - \varepsilon^2)^{\frac{1}{2}}, \tag{11}
\]

also tends to infinity, hence it leads to essential singularities there. However, in the case of \(\varepsilon^2 > m^2\), there is only the usual essential singularity at \(r = 0\).

It is well worth noting that by assuming \(\varepsilon = const.\), one has forced the usual electric charge to vary, so we have

\[
\varepsilon = \frac{1}{c(t)^2} \left(\frac{G}{4\pi\epsilon_0}\right)^{\frac{1}{2}} q, \tag{12}
\]

where \(\epsilon_0 = \frac{1}{4\pi\epsilon_c}\). Now, by a plausible assumption that \(\mu_0\) is a constant, the electric charge should vary as \(q \propto \frac{1}{\varepsilon}\), to keep \(\varepsilon\) as a fixed value. Any other assumption about the dependency or constancy of the electric charge results in more complicated calculations. For example, in the case of \(q = const.\), an analogous metric would be

\[
ds^2 = \left(1 - \frac{2GM}{c(t)^2r} + \frac{\mu_0\tilde{G}}{4\pi} \right) c(t)^2 dt^2 - \left(1 - \frac{2GM}{c(t)^2r} + \frac{\mu_0\tilde{G}}{4\pi}\right)^{-1} dr^2 - r^2 d\Omega^2. \tag{13}
\]

5. CONCLUSIONS AND REMARKS

We have used a vacuum case for the VSL theory, i.e., \(T_{ab} = 0\) where \(G_{ab} \neq 0\). With this assumption we have achieved the following.

- In spite of time-dependence, the generalized Schwarzschild metric remains spherically symmetric. Hence, with \(c(t)\), and not \(c(t, r)\), the metric can only radiate with monopole symmetry.
- The Schwarzschild radius of a (non charged) black hole is not a removable singularity. No matter can pass through the horizon.
- The Schwarzschild radius is dynamic even though no matter is attracted towards the black hole. It increases when \(c\) is decreasing and vice versa.
- If, in the very early epoch of the Universe, the value of \(c\) was much more larger than its current value, then the probability of creation of the primordial black holes, hence their population, and consequently the probability of observing them is greatly reduced.
When one considers a charged black hole with a fixed $\varepsilon$, essential singularities, besides $r = 0$, arise only if $\varepsilon^2 \leq m^2$ at $r = r_{\pm}$.

It should be emphasized that if $c$ attains a fixed value, then all the above effects are switched off and the classical properties of the black holes are restored. Actually, this is the result of the way we embarked on the generalization of the Schwarzschild metric. However, one should be aware that simply writing in a time variation in the Schwarzschild solution may not lead to an exact behavior, for example, in the Brans-Dicke theory of gravitation, when $G$ is time dependent, other solutions arise including vacuum Friedmann Universes that are not black holes. Although, once again, as there is not yet a concrete dynamical theory for the VSL models, we aim to consider their consequence(s) through a self-consistent procedure in this work. Nevertheless, when varying-$c$ creates extra terms in the Einstein equation, it should alter the previous dynamics, as, usually a change in the dynamics is mainly achieved by altering the Lagrangian of the system. For example, one knows that adding higher derivative correction terms to a Lagrangian does not only mean that they will perturb the original theory, but their presence, as unconstrained terms even with small coefficients, make the new theory completely different from the original one [28]. However, having an extended horizon for black holes looks somehow consistent with a change of speed of light.

Finally, while we were revising this manuscript, we came across ref. [29] in which a similar conclusion for horizon growth is deduced.

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[30] Also, no difficulty would arise even if one assumes $x^{2'} = c_1 t$, for it should be noted that one can write $dx^0 = c_1' dt$, for example, when $c$ is a polynomial function of $t$ only.
[31] Note that $r = r_e$ is still the radius of hypersurfaces of $r$ equal to a constant that is the event horizon, i.e., $g^{ab} \frac{\partial r}{\partial x^a} \frac{\partial r}{\partial x^b} = 0 \Rightarrow r = r_e$.
[32] We use the word “attracting” here instead of “absorbing” in the classical models, to clear up any possible misunderstandings of the essential singularity properties of the Schwarzschild radius derived above.