Structural design for a hybrid stone carving manipulator

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Abstract. With people's higher and higher requirements for environmental art, a large number of stone carving products are needed in the current urban construction. However, the traditional carving technology and equipment can not meet this demand. The development of robots to complete the processing of stone products, especially for the processing of special-shaped stones with complex shapes such as three-dimensional, multi-faceted and curved surfaces, has great social and economic benefits. In this paper, a stone carving manipulator is designed. In the processing of special-shaped curved stone, the structure of 3-TPS/TP hybrid manipulator is adopted, and its kinematic model and Jacobian matrix are established to realize the efficient automatic processing of special-shaped stone.

1. Introduction
Intelligent robot is one of the important technologies in a country's high-tech development level. It has broad application prospects. It is more and more widely used in current production and life. It is playing an increasingly important role instead of human beings \cite{1}.

Natural stone has been used as excellent building materials because of its elegant color, soft and beautiful, rich texture, wear and corrosion resistance, good mechanical properties and so on. Stone processing is a traditional industry with a long history. Compared with other industries, the degree of mechanization, automation and intelligence is not high enough \cite{2}. In terms of material cutting, glue filling, modeling, grinding and polishing, it basically depends on semi mechanization and semi automation, and there are still many parts completed manually.

It is an important research direction of stone processing equipment to apply intelligent robot technology to stone processing, make up for the shortcomings of traditional stone processing equipment, such as less freedom, inflexible movement and low precision, and realize the development of high efficiency, high quality and high precision of stone processing \cite{3}.

In this paper, a stone carving manipulator is designed to realize the carving of complex surface of special-shaped stone, processing and cutting of special-shaped column through series and parallel hybrid structure.

2. Configuration design

2.1. Movement form
The stone carving manipulator designed in this paper is a 5-DOF hybrid mechanism, which mainly includes series and parallel parts. The series part includes 2 degrees of freedom, which can move in
the x-axis and Y-axis directions; The parallel part includes three degrees of freedom, which can rotate around the x-axis, Y-axis and move along the z-axis. In addition, the tool can rotate around the z-axis direction.

2.2. Basic composition
The stone carving manipulator is mainly composed of beam, column, working head and bed, as shown in Figure 1.

Figure 1 Appearance of stone carving machine

Among them, the beam, column and bed form the series part of the stone carving manipulator. This bed structure is similar to the bed structure of the traditional stone carving machine, which can move in two directions: that is, along the length direction of the bed, the servo motor is used to drive the gear rack structure to move in the x-axis direction; Along the length direction of the beam, the servo motor is used to drive the ball screw structure to realize the movement in the y-axis direction.

The parallel part is the parallel working head part, which is mainly composed of a static platform, three driving chains, a constraint chain, a moving platform and a tool. Through the combination of different elongation of the three driving chains and the cooperation of the intermediate constraint chain, the engraving manipulator can rotate around the x-axis and Y-axis and move along the z-axis. The specific structure is shown in Figure 2.

The layout of the stone carving manipulator with this structure is very similar to that of the traditional stone carving machine. The change is to replace the working head of the ordinary carving machine with a parallel working head. The stone carving manipulator with this structure has simpler configuration, more flexible processing, more reliable work and higher accuracy, and can better realize the processing of complex three-dimensional special-shaped stones.
2.3. Structural design

The telescopic device is the key component of the driving chain of the parallel working head, which is mainly to achieve different elongation of the three driving chains, so that the parallel working head can rotate freely along the x-axis, Y-axis and z-axis, and then realize the engraving of complex curved surfaces of special-shaped stones, such as engraving patterns on marble columns. The specific structure of the expansion device is shown in Figure 3.

The specific movement process of the telescopic device is as follows: the servo motor 11 is connected with the ball screw 7 through the coupling 10, and the ball screw nut pair converts the rotary motion into linear motion, that is, the nut drives the keyway telescopic inner sleeve 5 to realize the linear motion in the vertical direction, so as to change the elongation of the driving chain, that is, change the posture of the changing platform, that is, change the posture of the tool, and realize the tool along the x-axis Rotate in the direction of Y axis and Z axis, cooperate with the series part to complete the three-dimensional curved surface carving of special-shaped stone.

In the telescopic device, the lead screw positioning adopts the positioning mode of "fixed at one end - supported at one end". The fixed end is the end connected with the servo motor, which is fixed by a diagonal contact ball bearing; The other end adopts an angular contact ball bearing, which is first
locked with a locking nut, and then interference matched with a sliding key sleeve outside the bearing,
so that the lower end of the lead screw can float and slide freely in the telescopic inner sleeve of the
keyway, so as to convert the rotary motion into linear motion, so as to achieve the change of the
extension of the driving chain.

3. Establishment of Jacobian matrix

3.1. Kinematic model of manipulator

Figure 4 shows the structure of the stone carving manipulator studied in this paper, and figure 5 shows
its kinematic model. The regular triangle $B_1B_2B_3$ represents the static platform, which $R_1$ is the radius
of its circumscribed circle, and the regular triangle $b_1b_2b_3$ represents the dynamic platform, which $R_2$
is the radius of its circumscribed circle. The dynamic and static platforms are composed of three driving
chains, namely $B_1C_1b_1, B_2C_2b_2, B_3C_3b_3$, which $B_i (i=1,2,3)$ represent the center of hook hinge, $b_i (i=1,2,3)$
represent the center of ball hinge and $C_i (i=1,2,3)$ represent the center of moving pair. The constraint
chain connects the static platform ($A_1$) and the dynamic platform ($A_2$), $A_1$ representing Hooke hinge
and $C_4$ representing moving pair.

Figure 4 Structure of parallel robot    Figure 5 Motion model of parallel robot

As shown in Figure 5, the static coordinate system $A_1$-XYZ is connected to the center of the static
platform, and the dynamic coordinate system $A_2$-xyz is connected to the center of the dynamic
platform. The Z axis passes through the center of the moving platform and is perpendicular to the
regular triangle $b_1b_2b_3$. The y-axis and Y-axis pass through the centers of the moving platform and the
static platform respectively, and coincide with the vertical line of the regular triangle. The motion of
the moving platform can be described by the rotation angle $\alpha, \beta$ around the X-axis, Y-axis and the
displacement $Z_P$ along the Z-axis.

3.2. Establishment of Jacobian matrix

3.2.1. Coordinate transformation equation

In the dynamic coordinate system, the coordinates of each hinge point can be described by the rotation
angle around the X-axis, Y-axis and the displacement in the Z-axis direction. The transformation
equation from dynamic coordinate system to static coordinate system is as follows:

$$(X_{hi}, Y_{hi}, Z_{hi}, I)^T = [T_{op}] (x_{hi}, y_{hi}, z_{hi}, I)^T$$

$$[T_{op}] = \begin{bmatrix} R & P \\ 0 & I \end{bmatrix}$$

$$R = \begin{bmatrix} \cos \beta & \sin \beta \sin \alpha & \sin \beta \cos \alpha \\ -\sin \beta & \cos \alpha & -\sin \alpha \\ \cos \beta & \sin \beta \sin \alpha & \cos \beta \cos \alpha \end{bmatrix}$$
The Jacobian matrix can be obtained from the coordinate transformation equation.

3.2.2. Jacobian matrix
It not only represents the velocity mapping relationship between the operation space and the joint space, but also represents the force transfer relationship between them \[4\]. It provides a convenient method for determining the static joint torque of the robot and the changes of velocity, acceleration and static force between different coordinate systems \[5\].

When the position of the point is known, the elongation length of the kinematic chain can be obtained through the inverse kinematics equation.

\[
L_i = \sqrt{(X_{b_i} - x_i \cos \beta - y_i \sin \alpha \sin \beta)^2 + (Y_{b_i} - y_i \cos \alpha)^2 + (z_{b_i} \cos \beta + y_i \cos \beta \sin \alpha + z_p)^2}
\]

(5)

\[
L_2 = \sqrt{(y_{b_1} \sin \beta \sin \alpha)^2 + (y_{b_2} - y_{b_1} \cos \alpha)^2 + (y_{b_2} \cos \beta \sin \alpha + z_p)^2}
\]

(6)

\[
L_3 = \sqrt{(X_{b_3} - x_i \cos \beta - y_i \sin \alpha \sin \beta)^2 + (Y_{b_3} - y_i \cos \alpha)^2 + (x_{b_3} \sin \beta + y_i \cos \beta \sin \alpha + z_p)^2}
\]

(7)

The relationship between the velocity of the moving chain reflected by the Jacobian matrix and the velocity of the center point of the moving platform is as follows:

\[
\nu_p = JT \nu_i
\]

(8)

\(\nu_p\) is the speed of the center point of the moving platform, \(\nu_i\) is the elongation speed of the constraint chain, \(J\) is the Jacobian matrix and \(T\) is the inverse Jacobian matrix, which represents the mapping relationship between the output speed of the end actuator and the input speed of the driving rod: \(T^{\times VD} = \nu_i (i=1,2,3)\). So:

\[
J = T^{-1} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}^{-1}
\]

(9)

The parameters in matrix \(T\) can be represented by \(L_1, L_2, L_3, R_1, R_2, \alpha, \beta\) and \(Z_P\).

3.3. Calculation example of Jacobian matrix
Assuming the tool length \(l_d=0.3m\), the angular velocity vector \(\omega=(\omega_x, \omega_y, \omega_z)^T\) of tool tip D can be obtained as follows:

\[
\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \frac{-0.94 \hat{y}}{\sqrt{0.09 - y^2}} \sqrt{\frac{0.09 - y^2}{0.09 - x^2 - y^2}} \pm \left( 0.09 - y^2 \right) \hat{x} + xy \hat{y} \sqrt{\frac{0.09 - y^2}{0.09 - x^2 - y^2}}
\]

(10)

Where: the position vector of tool tip point D in static coordinate system O-XYZ is: \( r_d = (x, y, z)^T \); The speed of point D can be: \( V_D = (\dot{x}, \dot{y}, \dot{z})^T \). If \(R_1=500mm\) and \(R_2=400mm\) are brought into equation (9) , and selected from the reachable poses of two random \(\alpha=-20^\circ, \beta=-20^\circ, Z_P=-500mm\) and \(\alpha=-10^\circ, \beta=15^\circ, Z_P=-500mm\), the Jacobian matrix value is calculated by MATLAB, as shown in Tab 1.
Tab 1 Example of Jacobian matrix

| name  | parameter     | $\alpha=-20^\circ, \beta=-20^\circ, Z_P=-500$mm | $\alpha=-10^\circ, \beta=15^\circ, Z_P=-500$mm |
|-------|---------------|-----------------------------------------------|-----------------------------------------------|
|       |               | \[ \begin{bmatrix} -0.43 & -0.36 & -0.45 \\ 1 & -1.68 & 1.35 \\ -1.14 & -0.17 & 1.93 \end{bmatrix} \] | \[ \begin{bmatrix} -0.38 & -0.35 & -0.34 \\ 0.88 & -1.77 & 1.09 \\ -1.32 & -0.09 & 1.51 \end{bmatrix} \] |

$\det(J) = 3.53 \quad 2.78$

### 4. Conclusion

This paper introduces the overall configuration of the hybrid stone carving manipulator, designs the structure of the key components, establishes its Jacobian matrix, and further completes the kinematics and dynamics analysis, strength design and verification, control system design and so on. By designing a reasonable stone carving manipulator, cooperating with the intelligent visual recognition system, supplemented by the tools required by different stones, the curve and surface processing of special-shaped stones can be realized.

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