Gravitational Coupling and
Dynamical Reduction of The Cosmological Constant

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Abstract

We introduce a dynamical model to reduce a large cosmological constant to a sufficiently small value. The basic ingredient in this model is a distinction which has been made between the two unit systems used in cosmology and particle physics. We have used a conformal invariant gravitational model to define a particular conformal frame in terms of large scale properties of the universe. It is then argued that the contributions of mass scales in particle physics to the vacuum energy density should be considered in a different conformal frame. In this manner, a decaying mechanism is presented in which the conformal factor appears as a dynamical field and plays a key role to relax a large effective cosmological constant. Moreover, we argue that this model also provides a possible explanation for the coincidence problem.

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1 Introduction

There are now strong observational evidences that the expansion of the universe is accelerating. These observations are based on type Ia supernova [1] and Cosmic Microwave Background Radiation [2]. The standard explanation invokes an unknown component, usually referred to as dark energy. It contributes to energy density of the universe with \( \Omega_d = 0.7 \) where \( \Omega_d \) is the corresponding density parameter, see e.g., [3] and references therein. A candidate for dark energy which seems to be both natural and consistent with observations is the cosmological constant.
constant \[3\] \[4\] \[5\]. However, in order to avoid theoretical problems \[4\], other scenarios have been investigated. Among these scenarios, there are quintessence \[6\], tachyons \[7\], phantom \[8\], quintom \[9\], modified gravity \[10\] and so forth.

There are two important problems that are related to the cosmological constant. The first problem, usually known as the fine tuning problem, is the large discrepancy between observations and theoretical predictions on its value \[4\]. The second problem concerns with the coincidence between the observed vacuum energy density and the current matter density. While these two energy components evolve differently as the universe expands, their contributions to total energy density of the universe in the present epoch are the same order of magnitude. In the present work our main focus will be on the first problem and some comments will be offered concerning the second one.

There have been many attempts trying to resolve the fine tuning problem \[4\]. Most of them are based on the belief that the cosmological constant may not have such an extremely small value at all times and there should exist a dynamical mechanism working during evolution of the universe which provides a cancelation of the vacuum energy density at late times \[11\]. As noted in \[12\], such a mechanism should have two important characteristics. Firstly, since any mass scale in particle physics contributes to vacuum energy density much larger than the observational bound, the mechanism should not be sensitive to a particular type of contribution and should work equally well for every mass scale introduced by elementary particle physics. Secondly, it should work whenever these contributions are considered at cosmological level since the discrepancy manifests when one compares them with relevant cosmological observations. This latter strongly suggests that construction of a mechanism for relaxing these contributions should somehow take into account the two unit systems usually used in cosmology and particle physics. In fact, the small upper limit is obtained in a unit system which is defined in terms of large scale cosmological parameters. On the other hand, the theoretical predictions are based on a natural unit system which is suggested by quantum physics. These two unit systems are usually related by a conversion factor which is independent of space and time. In other terms they are related by a global unit transformation. The point we wish to make here is that the large discrepancy between observations and theoretical predictions on the vacuum energy density arises when one prejudices that the two unit systems should be indistinguishable up to a constant conversion factor in all spacetime points, or related by a global unit transformation. Such a global transformation clearly carries no dynamical implications and the use of a particular unit system is actually a matter of convenience. It means that one may arbitrarily use the unit system suggested by quantum physics to describe the evolution of the universe or the cosmological unit system to describe dynamical properties of an elementary particle.

In the present work we would like to consider a theoretical scheme in which an explicit recognition is given to the distinguished characteristics of these two unit systems. In such a theoretical scheme one should no longer accept the triviality one usually assigns to a unit transformation. In this respect, we shall consider local unit transformations (conformal transformations \[13\]) relating different standard of units (conformal frames) via general spacetime dependent conversion (conformal) factors. In this language observations and theoretical predictions on the vacuum energy density are actually carried out in two different conformal frames. We emphasize that local unit transformations give a dynamical meaning to changes of unit systems and can be consequently taken as a basis for constructing a dynamical mechanism which works
due to cosmic expansion. Along this line of investigation, we shall show that the conformal factor which relates the two unit systems plays the role of a dynamical field which can eventually reduce the effective cosmological constant to a small value consistent with observations. We also show that this dynamical reduction of the vacuum energy density can also lead to a possible alleviation of the coincidence problem. Throughout this paper we work in units in which $\hbar = c = 1$ and the sign conventions are those of MTW [14].

2 The Model

We begin with considering the general form of vacuum sector of a scalar tensor theory

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left\{ F(\phi) R + Z(\phi) g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + V(\phi) \right\}$$

(1)

Here $g$ is the determinant of $g_{\mu\nu}$ and $R$ is the scalar curvature. The functions $F(\phi)$, $Z(\phi)$ and $V(\phi)$ are arbitrary functions of the real scalar field $\phi$. Different parameterizations have been used in the literature. However, we would like to consider the case that the action (1) remains invariant under conformal transformations

$$\bar{g}_{\mu\nu} = e^{-2\sigma} g_{\mu\nu}$$

(2)

$$\bar{\phi} = e^{\sigma} \phi$$

(3)

where $\sigma$ is a smooth dimensionless spacetime function. The conformal transformation (2) implies that all spacetime intervals transform according to

$$\bar{ds} = e^{-\sigma} ds$$

(4)

while coordinates are fixed. This can be interpreted as changes of standards of length and time or transformation of unit systems [13]. In this view under a conformal transformation all dimensional quantities are transformed according to their dimensions. Therefore if one assigns a mass $\mu$ to the scalar field $\phi$, it should be transformed as

$$\bar{\mu} = e^{\sigma} \mu$$

(5)

With this fact in mind, the parametrization $F(\phi) = \frac{1}{6} \phi^2$, $Z(\phi) = 1$ and $V(\phi) = -\frac{1}{3} \mu^2 \phi^2$ reduces the action (1) to

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left\{ \frac{1}{6} R \phi^2 + g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{3} \mu^2 \phi^2 \right\}$$

(6)

This action with $\mu = 0$ is the field theoretic version of the so-called Hoyle-Narlikar theory [15]. It is also shown [16] that $f(R)$ theories of gravity [10] in the Palatini formalism can be cast in the form of (6) with an appropriate potential. 

\*\*One may consider a self-interacting scalar field by adding the term $\lambda \phi^4$ to the potential $V(\phi)$. This would not change the conformal symmetry of the action (6).
The action (6) is invariant under transformations (2), (3) and (5). Since Noether’s theorem relates a basic symmetry to a conservation law, two different conservation laws can be attributed to the gravitational system (6). Firstly, general covariance requires that (6) remains unchanged under general coordinate transformations while the system of units is held fixed. This is related to the Bianchi identities or conservation of the stress tensor of any matter system which couples with (6). Secondly, conformal invariance leads to a relation between the trace of the stress tensor and $\mu$ [17]. In the case that $\mu = 0$, only traceless matter systems can be coupled with (6).

The conformal invariance of (6) is broken if one assigns a particular constant value to $\mu$. In this way one characterizes the unit system in which a particular measurement is carried out. In a cosmological context, the most suggestive choice for $\mu$ is $\mu \sim H_0$ with $H_0^{-1}$ being the present Hubble radius of the universe. In the corresponding conformal frame, which is referred from now on as the cosmological frame, a constant configuration can be assigned to the scalar field. It is given by $\phi^{-2} \sim G$ with $G$ being the gravitational constant [17]. The action (6) then reduces to

$$\frac{1}{\kappa} \int d^4x \sqrt{-g} (R - 2\mu^2) \quad (7)$$

where $\kappa$ gives the gravitational constant. This corresponds to the usual Einstein-Hilbert action with a small but nonzero cosmological constant induced due to finite size of the universe. However, the cosmological constant receives strong contributions from various mass scales introduced by elementary particle physics. To incorporate these contributions to (7) it should be noted that they belong to a conformal frame which has properties entirely different from those used to define the cosmological frame. To define this atomic conformal frame or unit system, one considers local characteristics of a typical elementary particle and neglects the large scale properties of the universe [12] §. If the metric tensor of this conformal frame is denoted by $\bar{g}_{\mu\nu}$, it is then related to $g_{\mu\nu}$ by (2). We therefore write the action (7) in the form

$$S = \frac{1}{\kappa} \int d^4x \sqrt{-g} (R - 2\mu^2) - \int d^4x \sqrt{-\bar{g}} L(\bar{g}_{\mu\nu}, \bar{\psi}) \quad (8)$$

where $L(\bar{g}_{\mu\nu}, \bar{\psi})$ is the Lagrangian density of some matter field $\bar{\psi}$ in the atomic conformal frame. As an illustration we take $L(\bar{g}_{\mu\nu}, \bar{\psi})$ to be Lagrangian density of a real massive scalar field

$$L(\bar{g}_{\mu\nu}, \bar{\psi}) = \bar{g}^{\mu\nu} \nabla_\mu \bar{\psi} \nabla_\nu \bar{\psi} + \bar{m}^2 \bar{\psi}^2 \quad (9)$$

The quantity $\bar{m}$ corresponds to the mass of the scalar field in the atomic unit system. In terms of the background variables ($g_{\mu\nu}$, $\psi$), (9) can be written as

$$L(\bar{g}_{\mu\nu}, \bar{\psi}) = e^{4\sigma} \left\{ g^{\mu\nu} \nabla_\mu \psi \nabla_\nu \psi + \psi^2 g^{\mu\nu} \nabla_\mu \sigma \nabla_\nu \sigma + 2\psi g^{\mu\nu} \nabla_\mu \psi \nabla_\nu \sigma + \bar{m}^2 e^{-2\sigma} \psi^2 \right\} \quad (10)$$

If we suppose that the scalar field $\psi$ has a constant average value in the cosmological frame we can set $\psi=\text{constant}$. In this case (10) reduces to

$$L(\bar{g}_{\mu\nu}, \bar{\psi}) = \psi^2 e^{4\sigma} \left\{ g^{\mu\nu} \nabla_\mu \sigma \nabla_\nu \sigma + \bar{m}^2 e^{-2\sigma} \right\} \quad (11)$$

\[\text{For arguments concerning the physical status of Einstein and Jordan frames see, for example, [18].}\]

\[\text{§In fact, the basic idea that a viable cosmological model should contain a dynamical distinction between the unit systems used in cosmology and particle physics is not new. See, for example, [19].}\]
Combining (8) and (11) leads to
\[ S = \int d^4x \sqrt{-g} \{ R - 2\mu^2 - \kappa \alpha (g^{\mu\nu} \nabla_\mu \sigma \nabla_\nu \sigma + \bar{m}^2 e^{-2\sigma}) \} \] (12)

where \( \psi^2 = \alpha \). Note that \( \sigma \) appears as a dynamical field which allows us to investigate the evolution of \( \bar{m} \) as the universe evolves. We now follow the consequences of the action (12) by writing the field equations
\[ G_{\mu\nu} + \mu^2 g_{\mu\nu} = \kappa T^\sigma_{\mu\nu} \] (13)
\[ \Box \sigma + \bar{m}^2 e^{-2\sigma} = 0 \] (14)

where
\[ T^\sigma_{\mu\nu} = \alpha (\nabla_\mu \sigma \nabla_\nu \sigma - \frac{1}{2} g_{\mu\nu} \nabla_\gamma \sigma \nabla_\gamma \sigma - \frac{1}{2} \bar{m}^2 e^{-2\sigma} g_{\mu\nu}) \] (15)

In these equations the exponential coefficient for \( \bar{m}^2 \) emphasizes the dynamical distinction between the two unit systems mentioned above. In an expanding universe this distinction is expected to increase since cosmological scales enlarge as the universe expands and, as suggested by (4), the conformal factor \( e^\sigma \) must grow with time. This automatically provides us with a dynamical reduction of \( \bar{m} \) in the cosmological frame. That this intuitive picture is actually consistent with the field equations is illustrated in the following:

Applying (13) and (14) to the spatially flat Friedmann-Robertson-Walker metric, yields
\[ 3H^2 - \mu^2 = \kappa \rho_\sigma \] (16)
\[ 2\dot{H} + 3H^2 - \mu^2 = -\kappa p_\sigma \] (17)
\[ \ddot{\sigma} + 3H \dot{\sigma} - \bar{m}^2 e^{-2\sigma} = 0 \] (18)

where
\[ \rho_\sigma = \frac{1}{2} \alpha (\dot{\sigma}^2 + \bar{m}^2 e^{-2\sigma}) \] (19)
\[ p_\sigma = \frac{1}{2} \alpha (\dot{\sigma}^2 - \bar{m}^2 e^{-2\sigma}) \] (20)

Here \( H = \dot{a}/a \) is the Hubble parameter and overdot indicates differentiation with respect to the coordinate time \( t \). Due to homogeneity and isotropy, the field \( \sigma \) is taken to be only a function of time. Assuming that the universe follows a power law expansion, namely that \( H \sim t^{-1} \), equation (18) gives the solution
\[ e^\sigma = \sigma_0 t \] (21)

with \( \sigma_0 \sim \bar{m} \). In equation (16), one then obtains \( \Lambda_{\text{eff}} = \mu^2 - \bar{m}^2 e^{-2\sigma} \sim t^{-2} \) which is consistent with the observational bound.

The other problem attributed to the cosmological constant is the coincidence between matter and vacuum energy densities in total energy density of the universe. In standard \( \Lambda \text{CDM} \) model energy density of matter dilutes as \( \rho_m \sim a^{-3} \) while vacuum energy density is a constant. Thus one should explain that in the history of the universe why we live in an epoch in which these two energy densities are of the same order of magnitude. One possible explanation is that the vacuum energy density is not actually a constant and evolves during expansion of the
universe. If the evolution of the vacuum density is the same as that of the matter density we then arrive at a possible solution of the problem. Let us now examine this property in the model presented here. To do this we should add a cosmic matter system to the action (12),

\[ S = \int d^4x \sqrt{-g} \left\{ (R - 2\mu^2) + \kappa [L_m(g_{\mu\nu}) - \alpha (g^{\mu\nu} \nabla_\mu \sigma \nabla_\nu \sigma + \bar{m}^2 e^{-2\sigma})] \right\} \]  

(22)

where \( L_m \) denotes lagrangian density corresponding to the cosmic matter. Note that this cosmological matter couples with the metric \( g_{\mu\nu} \) describing the cosmological frame. The equation (13) then generalizes to

\[ G_{\mu\nu} + \mu^2 g_{\mu\nu} = \kappa (T^m_{\mu\nu} + T^\sigma_{\mu\nu}) \]  

(23)

where \( T^m_{\mu\nu} \) is the stress tensor corresponding to the cosmological matter system. Applying the Bianchi identities and using the relation (15), one can easily check that the stress tensors \( T^\sigma_{\mu\nu} \) and \( T^m_{\mu\nu} \) are separately conserved

\[ \nabla^\mu T^\sigma_{\mu\nu} = \nabla^\mu T^m_{\mu\nu} = 0 \]  

(24)

We take the cosmic matter to be a dust system (perfect fluid with zero pressure) with energy density \( \rho_m \). In this case the equation (16) takes the form

\[ 3H^2 - \mu^2 = \kappa (\rho_m + \rho_\sigma) \]  

(25)

while (17) and (18) remain unchanged. We then define the ratio \( r = \frac{\rho_m}{\rho_\sigma} \) and use the equations (17) and (25) to obtain

\[ r = -1 - \frac{2\dot{H}}{\kappa \rho_\sigma} \frac{p_\sigma}{\rho_\sigma} \]  

(26)

This relation implies that \( r \) remains constant during expansion of the universe if \( p_\sigma \) and \( \rho_\sigma \) evolves as \( \dot{H} \). If the universe follows a power law expansion then \( H \sim t^{-1} \). In this case, matter and vacuum densities have the same time evolution if \( p_\sigma \sim \rho_\sigma \sim t^{-2} \). Inspection of (19) and (20) reveals that this condition is actually consistent with the decaying law of \( \bar{m} \) which is represented by (21).

As the last point we remark that different couplings of matter systems with gravity, as indicated in (22), may seem to be in conflict with equivalence principle or universality of free fall. However, it should be noted that this principle is supported by very precise experiments which have been carried out in the present epoch and it is not clear that it does hold during evolution of the universe. Noting the relation (21), one infers that the dynamical field \( \sigma \) which makes the gravitational coupling of cosmic matter and atomic matter to be different during evolution of the universe takes at present time a constant value \( \bar{m} e^{-\sigma} \sim H_0^{-1} \). This implies the same coupling of the two types of matter systems in the present epoch if \( \alpha \sim 1 \).

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\(^{*}\)One may note that the model presented here shares some likeness with the quintessence models. However, quintessence models need appropriate potential functions for a scalar field which are usually constructed by fine tuning. Moreover, there is no a precise physical meaning which can be attributed to the scalar field.
3 Conclusion

We argued that the large discrepancy between observations and theoretical estimations on vacuum energy density may be attributed to an interrelation between the unit systems by which these quantities are usually measured. We have proposed a dynamical model in which these two unit systems are related by a local unit transformation. The basic ingredient in our model is the gravitational coupling of different contributions to vacuum energy density coming from elementary particle physics. These contributions belong to a conformal frame that is dynamically distinct from the cosmological frame in which the cosmological observations are carried out. Mathematically, it means that these contributions couple with a metric $\bar{g}_{\mu\nu}$ which is conformally related to the metric defined in the cosmological frame $g_{\mu\nu}$ by a spacetime dependent conformal factor. We would like to underline two important features of such a gravitational coupling. Firstly, different mass scales introduced by particle physics have variable contributions to vacuum energy density in the cosmological frame. Secondly, these variations are controlled by the conformal factor. Note that this conformal factor is characterized by $\sigma$ field which automatically finds a kinetic term in the action (12) when we write a particular field theory, denoted here by $L(\bar{g}_{\mu\nu}, \bar{\psi})$, in terms of the background variables. From a physical point of view, one expects that $\sigma$ be an increasing function of time since due to expansion of the universe the cosmological scales should enlarge with respect to the atomic scales. This then provides us with a dynamical reduction of the cosmological constant which works due to cosmic expansion. As the last point, we remark that decaying of the cosmological constant with expansion of the universe provides a possible explanation for the coincidence problem. Alleviating the problem needs the same time evolution for both matter and vacuum energy densities. We have shown that this behavior is actually consistent with our field equations.
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