Gravitomagnetic effects

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Abstract

This paper contains a review of the theory and practice of gravitomagnetism, with particular attention to the different and numerous proposals which have been put forward to experimentally or observationally verify its effects. The basics of the gravitoelectromagnetic form of the Einstein equations is expounded. Then the Lense-Thirring and clock effects are described, reviewing the essentials of the theory. Space based and Earth based experiments are listed. Other effects, such as the coupling of gravitomagnetism with spin, are described and orders of magnitude are considered to give an idea of the feasibility of actual experiments.

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1 Introduction

The close analogy between Newton’s gravitation law and Coulomb’s law of electricity led many authors, in the past and also more recently, to investigate further similarities, such as the possibility that the motion of mass-charge could generate the analogous of a magnetic field. The magnetic field is produced by the motion of electric-charge, i.e. the electric current: the mass current would produce what is called "gravitomagnetic" field. Maxwell [1], in one of his fundamental works on electromagnetism, turned his attention to the possibility of
formulating the theory of gravitation in a form corresponding to the electromagnetic equations. However, he was puzzled by the problem of the energy of the gravitational field, i.e. the meaning and origin of the negative energy due to the mutual attraction of material bodies. In fact, according to him, the energy of a given field had to be "essentially positive", but this is not the case of the gravitational field. To balance this negative energy, a great amount of positive energy was required, in the form of energy of the space. But, since he was unable to understand how this could be, he did not proceed further along this line.

On the ground of Weber’s modification of the Coulomb law for the electrical charges, Holzmüller [2] first, then Tisserand [3], proposed to modify Newton’s law in a similar way, introducing, in the radial component of the force law a term depending on the relative velocity of the two attracting particles (as it is well described by North [4] and Whittaker [5]). In 1893 Oliver Heaviside [6] [7] investigated the analogy between gravitation and electromagnetism; in particular, he explained the propagation of energy in a gravitational field, in terms of a gravitoelectromagnetic Poynting vector, even though he (just as Maxwell did) considered the nature of gravitational energy a mystery. The formal analogy was then explored by Einstein [8] in the framework of General Relativity, and then by Thirring [9] who pointed out that the geodesic equation may be written in terms of a Lorentz force, acted by a gravitoelectric and gravitomagnetic field.

Any theory that combines Newtonian gravity together with Lorentz invariance in a consistent way, must include a gravitomagnetic field, which is generated by mass current. This is the case, of course, of General Relativity: it was shown by Lense and Thirring [10], that a rotating mass generates a gravitomagnetic field, which in turn causes a precession of planetary orbits. It is indeed interesting to notice that Lodge and Larmor, at the end of the nineteenth century discussed the effects of frame dragging on a non rotating interferometer [11], but within the framework of an aether-theoretic model This frame dragging corresponds in fact to the Lense-Thirring effect of General Relativity. However, at the beginning of the XX century, when Lense and Thirring published their famous papers, the effect named after them, which is indeed very small in the terrestrial environment, was far from being detectable, because of the technical difficulties and limitations of the time.

Contemporary improvements in technology have made possible to propose new ideas to reveal the Lense-Thirring precession by analyzing the data sets on the orbits of Earth satellites [12]. An actual experiment to observe the effects of the gravitomagnetic field of the Earth is about to fly; this is GP-B [13], which is described further on in this paper. Other experiments (like GP-C [14]) have been proposed to reveal the space-time structure, which is affected by gravitomagnetism, for example evidencing clock effects around a spinning massive object.

The main purpose of this paper is to review the various old and new tests of gravitomagnetism, which have been proposed in the past and may be viable
today, or in the next years. Considering the growing of technological skills, many experiments which were pure science-fiction ideas some decades ago, or nothing more than *gedanken experimente*, deserve today a detailed study. They would both represent new tests of General Relativity\(^1\) and give us the possibility of understanding how deep the analogy between the electrical and gravitational phenomena is.

In Section 2 we establish the theoretical apparatus, starting from Einstein’s equations in weak field approximation, to obtain the gravitoelectromagnetic equations. Then we present the tests of the rotation effects, starting from the Lense-Thirring effect which we analyze in details in Section 3. In Sections 4 and 5 we focus on the effects of rotation on clocks. The effect on signals propagation in both the terrestrial and the astrophysical environment are presented in Sections 6 and 7. The possibility of detecting the coupling of intrinsic Spin with the gravitomagnetic field is reviewed in Section 8. The raising of gravitomagnetic forces and their effects is presented in Section 9, while other gravitomagnetic effects are listed in Section 10.

Finally, in Section 11 we sketch the conclusions, stressing the possibility that a number of the effects presented in this paper could be revealed in the foreseeable future, taking into account both the improvement in technologies and the start of new promising research projects.

## 2 From Einstein field equations to Gravitoelectromagnetism

Einstein’s equations may be written in a very simple way, which leads straight to the analogy with Maxwell’s equations, if we consider the so called weak field approximation. We can use this kind of linear approximation if we deal with a source whose gravitational field is weak and, if the source is rotating, its rotation is not relativistic.

With the partial exception of the binary pulsar PSR 1913+16 (\[15\] sec. 5.6), the weak field is the normal condition for all the tests of General Relativity up to this moment.

We shall follow the standard treatment. So, let us start from the full non linear equations\(^2\)

\[
G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}
\]

We put

\[
g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}
\]

\(^1\)Of course, experiments could in principle discriminate between General Relativity and other theories of gravitation.

\(^2\)Notation: latin indices run from 1 to 3, while greek indices run from 0 to 3; the flat space time metric tensor is \(\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)\), in cartesian coordinates \(x^0 = ct, x^1 = x, x^2 = y, x^3 = z\); boldface letters refer to space vectors, like the position of a point \(\mathbf{x}\).
where $\eta_{\mu\nu}$ is the Minkowski metric tensor, and $|h_{\mu\nu}| \ll 1$ is a "small" deviation from it. Then we define

$$\overline{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h; \quad h = h^\alpha_\alpha$$

(3)

So if we expand the field equations (1) in powers of $\overline{h}_{\mu\nu}$ keeping only the linear terms, we obtain (sec. 18.1)

$$\Box \overline{h}_{\mu\nu} = 16\pi G c^4 T_{\mu\nu}$$

(4)

where we have also imposed the so called Lorentz gauge condition $\overline{h}_{\mu\alpha} \gamma^\alpha_\alpha = 0$. Equations (4) constitute the "linearized" Einstein’s field equations. The analogy with the corresponding Maxwell equations

$$\Box A^\nu = 4\pi j^\nu$$

(5)

is evident.

The solution of (4) may be written exactly in terms of retarded potentials:

$$\overline{h}_{\mu\nu} = -4\frac{G}{c^2} \int \frac{T_{\mu\nu}(t - |x - x'|/c, x')}{|x - x'|} d^3 x'$$

(6)

The role of the electromagnetic vector potential $A^\nu$ is played here by the tensor potential $\overline{h}_{\mu\nu}$, while the role of the four-current $j^\nu$ is played by the stress-energy tensor $T_{\mu\nu}$.

We look for solutions such that $|\overline{h}_{00}| \gg |\overline{h}_{ij}|, |\overline{h}_{0i}| \gg |\overline{h}_{ij}|$ (Mashhoon et. al [17]), and neglect the other terms, which are smaller.

The explicit expression for the tensor potential $\overline{h}_{\mu\nu}$ is then:

$$\overline{h}^{00} = \frac{4\Phi}{c^2}$$

(7)

$$\overline{h}^{0l} = -\frac{2A^l}{c^2}$$

(8)

Where $\Phi$ is the Newtonian or "gravitoelectric" potential

$$\Phi = -\frac{GM}{r}$$

(9)

while $A^l$ is the "gravitomagnetic" vector potential in terms of the total angular momentum of the system $S^l$.

$$A^l = \frac{G}{c} \frac{S^m n^k}{r^3} e^l_{nk}$$

(10)

$^3|h_{\mu\nu}| \ll 1$: as we shall see, in the solar system $|h_{\mu\nu}| \simeq |\Phi| \leq 10^{-6}$, where $\Phi$ is the newtonian potential. This makes the approximation consistent.

$^4$In this linear approximation, space time indices are lowered and raised using the flat-space time metric tensor $\eta_{\mu\nu}$; in particular space indices are raised and lowered by means of the euclidean tensor $\delta_{ij} = (1, 1, 1)$.

$^5$\varepsilon_{ijk}$ is the three-dimensional completely antisymmetric tensor of Levi-Civita.
It follows that $T_{00}^{c} = \rho$ is the "mass-charge" density. Hence the total mass $M$ of the system is

$$\int \rho d^3x = M$$

(11)

while $T_i^0 c = j^i$ represents the mass-current density, and the total angular momentum of the system is

$$S^i = 2 \int \varepsilon_{ijk} x' j^k c d^3x'$$

(12)

In terms of the potentials $\Phi, A$, the Lorentz gauge condition becomes

$$\frac{1}{c} \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \cdot A = 0$$

(13)

which, apart from a factor $1/2$, is the Lorentz condition of electromagnetism.

It is then straightforward to define the gravitoelectric and gravitomagnetic fields $E_G, B_G$:

$$E_G = -\nabla \Phi - \frac{1}{2c} \frac{\partial A}{\partial t}$$

(14)

$$B_G = \nabla \wedge A$$

(15)

Using equations (4), (13), (14), (15), and the definitions of mass density and current, we finally get the complete set of Maxwell’s equations for the so called gravitoelectromagnetic (GEM) fields:

$$\nabla \cdot E_G = -4\pi G \rho$$

(16)

$$\nabla \cdot B_G = 0$$

(17)

$$\nabla \wedge E_G = -\frac{1}{2c} \frac{\partial B_G}{\partial t}$$

(18)

$$\nabla \wedge B_G = \frac{1}{c} \frac{\partial E_G}{\partial t} - \frac{4\pi G}{c} j$$

(19)

$$E_G = -\nabla \Phi - \frac{1}{2c} \frac{\partial A}{\partial t}$$

(20)

$$B_G = \nabla \wedge A$$

(21)

$$\frac{1}{c} \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \cdot A = 0$$

(22)

Einstein’s field equations in this form correspond to a solution that describes the field around a rotating object in terms of gravitoelectric and gravitomagnetic potentials; the metric tensor can be read from the corresponding space-time invariant:

$$ds^2 = (1 + 2 \frac{\Phi}{c^2}) c^2 dt^2 + 4 dt (\mathbf{dr} \cdot \frac{A}{c}) - (1 - 2 \frac{\Phi}{c^2}) \delta_{ij} dx^i dx^j$$

(23)

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6In this equation and in the following ones, there is a factor $1/2$ which does not appear in standard electrodynamics: the effective gravitomagnetic charge is twice the gravitoelectric one; this is a remnant of the fact that the linear approximation of GR involves a spin-2 field, while "classical" electrodynamics involves a spin-1 field (see Wald [18], section 4.4)
Hence the gravitational field is understood in analogy with electromagnetism. For instance, the gravitomagnetic field of the Earth as well as of any other weakly gravitating and rotating mass may be written as a dipolar field:

\[ B_G = -\frac{4}{c} G \frac{3r(r \cdot S)}{2r^5} \]  

(24)

To complete the picture, a further analogy can be mentioned. In fact, using the present formalism, the geodesic equation for a particle in the field of a weakly gravitating, rotating object, can be cast in the form of an equation of motion under the action of a Lorentz Force. The geodesic equation is

\[ \frac{d^2 x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\beta} \frac{dx^\mu}{ds} \frac{dx^\beta}{ds} = 0 \]  

(25)

If we consider a particle in non-relativistic motion, we have \( \frac{dx^\alpha}{ds} \simeq 1 \), hence the velocity of the particle becomes \( \frac{v^i}{c} \simeq \frac{dx^i}{ds} \); at the same time we neglect terms in the form \( v^i v^h c^2 \). Limiting ourselves to static fields, where \( g_{\alpha\beta,0} = 0 \), it is easy to verify that the geodesic equation may be written as

\[ \frac{dv}{dt} = E_G + \frac{v}{c} \wedge B_G \]  

(26)

which shows that free fall, in the field of a massive rotating object, can be looked at as motion under the action of the Lorentz force produced by the GEM fields.

This is the basic background in which all tests of GEM take place. Even if we are more interested in the rotation effects, that is the gravitomagnetic effects, in the following sections we shall refer to gravitoelectromagnetic fields, which include also the gravitoelectric, or newtonian, part.

It is useful to exploit the analogy with electromagnetism, because, as we will see in a while, it simplifies the solutions of some problems. On the other hand, it is important to understand how far reaching this correspondence is.

## 3 The Lense-Thirring effect

The most famous gravitomagnetic effect is indeed the Lense and Thirring effect. Here we shall review both the effect per se, and the existing and proposed tests of it.

*The Mach Principle.* As we said in the Introduction, at the beginning of the XX century, Josef Lense and Hans Thirring [10], studied the effects of rotating masses within the relativistic theory of gravitation. Their starting intention was to incorporate the so called "Mach’s principle" [19] into the GR theory. Mach’s
belief was that the origin of inertia was in the global distribution of matter in the universe. A consequence of this assumption was that moving matter should somehow drag with itself nearby bodies. In particular rotating matter should induce rotation and specifically cause the precession of the axis of a gyroscope: a spinning body, which creates a gravitomagnetic field, somehow drags the free frame, which is a gyroscope.

Mach’s conjecture is fascinating, though, up to now, a definite and incontroversial incorporation of the principle into GR has not been attained. A more accurate analysis can be found in [17] and in the references therein.

The effect. The ‘gyroscopes’ that Lense and Thirring firstly took into account were planets and natural satellites acted upon by the rotating Sun, or Earth. The induced precession should show up in the orbital angular momentum of planets and satellites. The expected effect is indeed small, but, using artificial satellites with appropriately chosen orbits, it could appear as a precession of the orbital plane around the rotation axis of the Earth.

Let us start from the beginning. The metric tensor in the vicinity of a spinning mass in weak field approximation, may be read off from the space-time invariant (20 ch. 4)

\[
\begin{align*}
\text{ds}^2 &= \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \frac{4GMA}{cr} \sin^2 \theta d\phi dt \\
&= \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \left(1 + \frac{2GM}{c^2 r}\right) dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 + \frac{4GMA}{cr} \sin^2 \theta d\phi dt
\end{align*}
\]

Here \(a = S/Mc\), i.e. \(a\), apart from a \(c\) factor, represents the angular momentum of the source per unit mass; more precisely stated, it is the projection of the angular momentum three-vector on the direction of the rotation axis, divided by the mass and the speed of light.

To calculate the entity of the looked for precession rate, we have to solve the equations of motion of a test body in the gravitational field corresponding to the metric tensor (27): it is what Lense and Thirring did in their papers. It is however simpler and clearer to use the GEM equations ([20] ch.4).

The Lense-Thirring precession is analogous to the precession of the angular momentum of a charged particle, orbiting about a magnetic dipole; the orbital momentum of the particle divided by 2 and by \(c\) will then be the equivalent of the magnetic dipole moment of the particle. So, if

\[
\mathbf{B}_G = -\frac{G}{c} \frac{3r(r \cdot S) - Sr^2}{2r^5}
\]

is the gravitomagnetic field of the Earth (\(S\) is the angular momentum of the Earth) the torque on the angular momentum \(\mathbf{L}\) of the orbiting particle is

\[
\tau = \frac{\mathbf{L}}{2c} \wedge \mathbf{B}_G
\]

7Here the particle is assumed not to be itself a gyroscope, i.e. it is spinless.
The time derivative of the orbital angular momentum can then be written as

\[
\frac{d\mathbf{L}}{dt} = -G \frac{\mathbf{L} \wedge (3rr \cdot \mathbf{S} - \mathbf{S}r^2)}{c^2r^5}
\]  

(30)

From (30) we read the angular velocity of the precession \((\frac{d\mathbf{L}}{dt} = \mathbf{\Omega} \wedge \mathbf{L})\)

\[
\mathbf{\Omega} = G \frac{3rr \cdot \mathbf{S} - \mathbf{S}r^2}{c^2r^5}
\]  

(31)

If we take the average of \(\mathbf{\Omega}\) along the orbit, the effective angular velocity of precession is

\[
< \mathbf{\Omega} > = G \frac{3<rr \cdot \mathbf{S}> - <Sr^2>}{c^2r^5}
\]  

(32)

For an orbit with \(r \simeq R_{\text{Earth}}\) the magnitude of the precession rate is about 0.05 arcsec per year.

3.1 Observational verification

**Observational difficulties.** The gravitomagnetically induced orbital precession of artificial satellites is smaller than the effects caused by another kind of perturbation, such as the one generated by the quadrupole moment of the Earth. To detect it, however, many witty proposals have been made. For example Van Patten and Everitt proposed to use two polar satellites to eliminate the effect of the Earth’s quadrupole \[21\]. Ciufolini proposed to use two non-polar satellites, with opposite inclinations, for the same purpose \[12\].

**Direct evidences.** Analysing the existing laser ranging observations of the orbits of the satellites LAGEOS and LAGEOS II, Ciufolini et al. \[22\],\[23\] obtained the first direct measurement of the gravitomagnetic orbital perturbation due to the Earth’s rotation, i.e. the Lense-Thirring effect, within an accuracy of 20-30%.

**A further proposal.** Further improvements in the observation of the angular precession of the orbital plane could be attained with the LARES \[24\] proposed mission, which would be able to test the Lense-Thirring precession within an accuracy of 2-3% and would also be useful for a better understanding of fundamental gravitational phenomena such as testing the inverse square law, the equivalence principle, the PPN parameters etc. A new scenario for the LARES mission has been recently described by Iorio et al. \[25\].

**Indirect evidences.** Indirect evidences of the Lense-Thirring dragging of the inertial frames or, more generally, of gravitomagnetic influences were given by the periastron precession rate of the binary pulsar PSR 1913+16, and also by studying the laser ranging of the orbit of the Moon \[26\],\[27\],\[28\]. We consider these observations as indirect because the sought for precessions...
is in general combined with other effects some of which are not immediately disentangled from each other. Anyway the evidence for gravitomagnetic perturbations, considered as post newtonian forces between moving bodies, can nowadays be considered as clearly acquired, notwithstanding some contrary impressions still diffused a few years ago, as K. Nordtvedt points out \[29\].

**Lense - Thirring and QPO** The effects of the Lense - Thirring precession are of some interest in the study of quasi-periodic oscillations (QPO) in black holes \[30\],[31],[32].

### 3.2 Space based experiments

**Gravity Probe B.** Gravity Probe B (GP-B) is an experiment whose purpose is to detect the Lense-Thirring effect by measuring the precession rate of an orbiting gyroscope. It was proposed firstly by Schiff \[33\], and then developed by Fairbank, Everitt et al. \[13\],[34]. The actual launch of the satellite, which will carry four gyroscopes, is scheduled for the nearest future \[35\].

The precessional velocity of the spin of a gyroscope in a gravitational field is given by the general relation (\[15\] sec. 5.5)

\[
\Omega = -\frac{1}{2c^2} (v \wedge a) + G \frac{3rr \cdot S - S r^2}{c^5} + \frac{3}{2} G \frac{M r \wedge v}{r^3} \tag{33}
\]

The symbols \(v\) and \(a\) stand for the velocity and (non-gravitational) acceleration three-vectors. All terms in (33) can, in principle, be measured even on Earth. However, in the terrestrial environment, the first term, which is the Thomas precession, dominates. In free fall, i.e. for an orbiting gyroscope, there is no Thomas precession: only the second and third term remain, representing respectively the Lense-Thirring effect (depending on the angular momentum of the Earth) and the geodesic or de Sitter precession. The latter, which depends on parallel transport of a vector in curved space-time \[36\] has a magnitude about 100 times greater than the Lense-Thirring one. However for a gyroscope in a polar orbit, the angular velocity of the Lense-Thirring precession stays always in the revolution plane, while the angular velocity of the de Sitter precession is perpendicular to that plane. It is expected that GPB will measure both precessions.

Actually the mission will carry four identical gyroscopes. Each gyroscope is a fused quartz sphere coated with superconducting Niobium. The spheres float on a magnetic field and are set and maintained into rotation by a slight blow of Helium gas in an otherwise empty environment. The precession of the axis of each sphere will be revealed by a SQUID sensor. The expected value is 0.042 arcsec/year.

**Fundamental implications of GPB.** Recently Nayer and Reynolds \[37\] pointed out that Gravity Probe B could test some implications of the Randall-Sundrum

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8Whereas in the other examples the whole of the satellite’s orbit was to be considered as a gyroscope.
two-brane scenario: in fact this model predicts that Earth originated gravito-
magnetism should be some 16 orders of magnitude smaller than the one pre-
dicted by general relativity, hence GPB should see no gravitomagnetic effect at all. So, if a non-zero precession is actually measured, the peculiar two-brane scenario is somehow wrong. This is indeed an example of the persistency of a misconception according to which gravitomagnetism must still be verified. In fact, considering the other evidences for the existence of gravitomagnetic effects of the GR predictions size, the Randall-Sundrum two brane theory should already be considered as being disproved.

The HYPER Project  Thanks to the big improvement in the accuracy of experimental devices, it is possible to expect that in the future the effects of a rotating massive body on quantum particles can be detected (for further details on the quantum effects of rotation, see subsection 8). In particular, the available cooling techniques of gaseous atomic agglomerates makes it possible to prepare interfering atomic beams, which stay for a long time inside the interferometer and thus possess a long interaction time: this technology may prove to be useful to detect the Lense-Thirring effect too. A realization of this idea is attempted within the HYPER project [38], which is planned to put atomic interferometers in space with the aim of measuring the fine structure constant, the quantum gravity foam-structure of space time and the Lense-Thirring effect. This should be done using two atomic interferometers based on Mg, and two based on Cs atomic beams, placed in two orthogonal planes. The claimed resolution, for an integration time of 1000 s, should be \(10^{-14}\) rad/s. Contrary to GPB, where the cumulative effect over approximately one year is read out, in the case of HYPER the gravitomagnetic field would be measured locally, without integration over many days, but only in a few minutes. HYPER is planned to fly within the next 10 years.

3.3 Earth based experiments

Effects on a Pendulum. Braginsky et al. [39] proposed to detect the gravitomagnetic field of the Earth by studying its effect on the plane of swing of a Foucault pendulum, collocated at the South Pole. This experiment can be thought as an Earth based version of the GPB mission. The conceptual bases are very simple: the pendulum’s mass \(M\), swinging with velocity \(v\), is subjected to the gravitomagnetic field of the Earth at the pole \(\mathbf{B}_G = \frac{4\pi}{c} \frac{S}{m}\), which produces a force \(\mathbf{F}_{GM} = m\mathbf{\omega} \wedge \mathbf{B}_G\). This fact causes a precession of the principal axis of the pendulum, with respect to a fixed azimuth, with angular velocity

\[
\Omega_{GM} = \frac{B_G}{2c} = 0.281''/yr
\] (34)

or, in terms of the angle of precession for a duration of the experiment of \(\tau\):

\[
\Delta \phi_{GM} = \Omega_{GM} \times \tau = 0.046'' \frac{\tau}{60d}
\] (35)
As the authors point out, the main sources of errors are: frictional anisotropy, Pippard precession (due to variability of the angular momentum of the pendulum with respect to the rotation axis of the Earth, during the swing), frequency compensation, antiseismic isolation for the pendulum, atmospheric refraction and physical distortion for the telescope (used to point to a reference star, with respect to which the precession is evaluated), dynamic range for the readout system. Studies of these sources have to be done accurately, in order to determine the feasibility of the experiment, which could be a further verification of the gravitomagnetic effect.

**Lense-Thirring in the lab.** A ground-based test of the Lense-Thirring effect was proposed in 1988 by Cerдонio et al.\[40\]: they planned to compare the astrometric measurements of the terrestrial rotation (as performed by using the VLBI) with an inertial measurement of the angular velocity of the laboratory. The latter would be obtained using a new detector of local rotation, the so called Gyromagnetic Electron Gyroscope. This GEG would be made of a ferromagnetic rod rigidly inclosed in a superconducting shield, surrounded by a SQUID sensor; the two parts of the apparatus would be differently magnetized by rotation (Barnett and London effects) thus revealing the sought for rotation of the laboratory \[41\]. The comparison should be performed off-line, which is one of the advantages of their proposal. The experiment seems to be in the range of feasibility, even though the problem of isolation from seismic noise is serious (the possibility of using a spacecraft has been considered).

### 4 Clock Effect and Gravitomagnetism

**The effect.** A gravitomagnetic field affects the motion of standard clocks orbiting around a rotating mass and the proper time they measure. For example, the angular velocity of a rotating test body should be greater or lower depending on the direction of its angular momentum, with respect to the angular momentum of the source. Let us consider more closely the effect on the measure of time, starting from the Kerr metric \[42\], in Boyer-Lindquist coordinates \[43\]:

$$ds^2 = \left(1 - \frac{2\mu}{\rho^2}\right)c^2 dt^2 - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\theta^2 - \left(r^2 + a^2 + \frac{2\mu a^2 \sin^2 \theta}{\rho^2}\right) \sin^2 \theta d\phi^2 + \frac{4\mu a \sin^2 \theta}{\rho^2} d\phi c dt$$

where $\mu = \frac{GM}{c^2}$, $\rho^2 = r^2 + a^2 \cos^2 \theta$, and $\Delta = r^2 - 2\mu r + a^2$, $a = S/Mc$.

Let us start from a standard clock on a circular ($r = \cos t$) timelike geodesic orbit in the equatorial plane. From the geodesic equation

$$\frac{d^2x^\alpha}{ds^2} + \Gamma^\alpha_{\mu\beta} \frac{dx^\mu}{ds} \frac{dx^\beta}{ds} = 0$$

12
we obtain the only non trivial equation

\[(a^2 - \frac{r^3}{\mu})d\phi^2 - 2ad\phi cdt + c^2 dt^2 = 0 \] (38)

Depending on the direction of rotation of the clock, i.e. co-rotating or counter-rotating with respect to the rotation of the source, we have two different proper revolution periods measured by the clock after a closed orbit (2\pi change in the azimuthal \(\phi\) angle). In terms of the proper time of the clock one has:

\[\tau_\pm = T_0(1 - \frac{3\mu}{r} \pm \frac{a}{c}\omega_0)^{1/2} \] (39)

where \(T_0\) and \(\omega_0\) are the Keplerian period and frequency of the orbit, that is \(\omega_0 = \left(\frac{GM}{r^3}\right)^{1/2}\). So, the difference between the squares of the proper times is

\[\tau_+^2 - \tau_-^2 = 8\pi \frac{a}{c} T_0 \] (40)

This result is exact, i.e. approximation free; however, in most of the interesting cases, it suffices to use the lowest post-Newtonian order, so it turns out that

\[\tau_+ - \tau_- \simeq 4\pi \frac{a}{c} \] (41)

The result (41) is extremely appealing, because it is independent from both the radius of the orbit and the gravitational constant \(G\). This result was found also by Mitskevich and Pulido Garcia [45] and by one of us (A.T.) [46],[47],[48].

Using a geometric approach, based on the symmetry of space-time world lines of objects rotating on circular trajectories, it is showed that, for an axisymmetric metric tensor, the world lines are helices drawn on the flat bidimensional surface of a cylinder, so they can be easily studied from an euclidean point of view.

To give some numerical estimates of the magnitude of this effect, we may consider clocks orbiting around the Earth, and we obtain \(\tau_+ - \tau_- \simeq 10^{-7}\) s, which is well within the capability of the clocks available at present. In the next section, we shall analyze the difficulties which arise in preparing actual experiments to detect this effect.

We obtain a different result if we compare the proper times shown by two counter-orbiting clocks at the second conjunction event along equal radius trajectories in the equatorial plane [46]:

\[\tau_+ - \tau_- \simeq 8\pi \frac{\mu a}{c r} \] (42)

Formula (42) gives, in the case of the Earth, a time difference of \(\sim 10^{-16}\) s, which is much smaller than the time gap we obtained in (41). The proper time difference in clocks orbiting a rotating massive body, can be explained in terms of the gravitational Aharonov-Bohm effect [44],[49], where the phase of the radiation is influenced by the presence of a vector gravitomagnetic potential.
For an accurate analysis of the special classes of observers with regard to the gravitomagnetic clock effect see for example Bini et al. \cite{50,51} and references therein.

**Orbiting Clocks.** The use of orbiting clocks to detect the rotation effects in GR was proposed by Cohen et al. \cite{52} in 1988, when the accuracy and precision of atomic clocks was approaching the required sensitivity, and the possibility to place such clocks on satellites was in sight. They proposed to measure a synchronization gap between clocks in co-rotating and counter-rotating orbits, which they evaluated to be approximately $1.92 \times 10^{-17}$ s (see section \ref{section:5}).

**Gravity Probe C.** Rather than considering the synchronization gap, Gronwald et al. \cite{14} proposed to use clocks orbiting around the Earth to measure the gravitomagnetic effect expressed by formula \eqref{eq:41}, which is much larger than Cohen’s gap, and well within the measurement possibilities of modern atomic clocks \cite{53}. They studied a mission, called Gravity Probe C(lock), whose aim is to detect directly the gravitomagnetic field of the Earth. The name of this mission resembles closely to Gravity Probe B, and, as we shall see in a while, the fundamental nature of the effect they want to evidence is the same.

In practice the time difference \eqref{eq:41} can be obtained by comparison of the proper times shown by two clocks in co-rotating and counter-rotating orbits around the Earth, after one sidereal revolution. The relative gravitomagnetic variation of the orbital period with respect to the Keplerian period $T_0$ is

$$\frac{\tau_+ - \tau_-}{T_0} = \frac{2S}{Mc^2} \left(\frac{GM}{r^3}\right)^{1/2} \tag{43}$$

which gives a numerical estimate of about $4 \times 10^{-11}$ in the case of the Earth. Cohen and Mashhoon \cite{44} observed that formula \eqref{eq:43} is the same as the relative gravitomagnetic precession angle of Gravity Probe B gyroscopes, so it is expected that the difficulties of the two experiments are equivalent. It is not difficult to measure a time difference of $\simeq 10^{-7}$ s with modern technologies (future missions are planned to carry highly accurate clocks in space \cite{56}, and the GPS can be used to determine precisely the positions of the clocks), but as we are going to show, the difficulties of the experiments originate elsewhere.

First of all, the result \eqref{eq:41} is obtained considering two highly accurate and stable clocks, in two accurate and stable orbits: in fact the (gravitoelectric) post-newtonian corrections cancel exactly under the assumption that the orbits are identical, and only the gravitomagnetic correction remains.

Other error sources can be summarized in two groups:

\footnote{For complete reference, we mention that the main result of the Gravity Probe A mission was an accurate test of the gravitational redshift, by means of the sub-orbital flight of a rocket carrying a hydrogen maser clock \cite{54,55}.}

\footnote{The effect for more general orbits, with small inclination with respect to the equatorial plane is studied by Mashhoon et al. \cite{57} and the problem of closeness of geodesic orbits other than the equatorial ones is discussed. It is expected that the effect decreases with increasing inclination, and it becomes null for a polar orbit, because of symmetry.}

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i. errors due to the tracking of the actual orbits

ii. deviations from idealized orbits due to

- mass multipole moments of the Earth
- radiation pressure
- gravitational influence of the Moon, the Sun, and other planets
- other systematic errors (such as atmospheric disturbances)

It seems that errors due to the tracking of the orbits are those which require greater attention and careful study. In fact the present accuracy in determining the position of the clocks in space can be estimated (in terms of time difference) to be $\delta t \simeq 10^{-6}$ s [14], considering the available accuracy in angle and position measurements: it is one order of magnitude above the effect we want to measure. However it does not seem impossible to achieve the required precision; moreover the clock effect is cumulative so it can be made greater by performing many orbits, thus overcoming the statistical tracking errors too.

5 Correction to the Sagnac Effect

When a light source and a receiver are located on a turntable, which rotates with angular velocity $\omega$, the time for the round trip of the light rays along a closed path varies with the angular velocity itself. Moreover, the time will be different, with fixed $\omega$, if the light beam is co-rotating or counter-rotating. So, superimposing two oppositely rotating beams leads to a phase difference and to interference, or in the case of standing waves, to frequency shift and beats. This is the so called Sagnac effect [58], which can be explained in terms of Special Relativity, on the basis of the break of uniqueness of simultaneity in rotating systems [59].

The time lag between the two light beams, up to the first order in $v^2/c^2$ is

$$\delta \tau_S = 4\frac{A}{c^2}\omega$$

(44)

where $A$ is the area of the projection of the closed path followed by the beams around the platform, orthogonal to the rotation axis, and $\omega$ is the angular velocity of the observer.

Today the Sagnac effect has many applications, both for practical purposes and for fundamental physics; however the improvement of the accuracy in measurements introduces the need for precision corrections to the basic formula (44), such as the general relativistic ones. The latter were worked out by one of us (A.T) [60], considering the case of the Kerr field, in the post newtonian
approximation, and so they can be interpreted as gravitomagnetic corrections, originating from the off-diagonal term of the metric tensor. If the "rotating" platform is the Earth, and the circular path of the light rays is around the equator, then the "pure" Sagnac delay corresponding to formula (44) is

\[ \delta\tau_S = 4.12 \times 10^{-7} \text{ s} \] (45)

The correction depending on the pure mass term (Schwarzschild-like correction) is

\[ \delta\tau_M = 4\pi \mu \frac{R}{c^2} \Omega \simeq 2.84 \times 10^{-16} \text{ s} \] (46)

The first correction depending on the angular momentum of the Earth is

\[ \delta\tau_a = -8\pi \frac{a}{c} \frac{\mu}{R} \simeq -1.89 \times 10^{-16} \text{ s} \] (47)

where the radius, the mass, the angular momentum per unit mass and the angular velocity of the Earth were used; (47) in fact coincides with (42). Other corrections are worked out from the point of view of an observer on a geodetic orbit, but the numerical estimates are not much different or better than the ones we gave above. Anyway, it is reasonable that the fringe shift produced by the general relativistic corrections is in principle observable (with the time delays (46) and (47), using visible light, one obtains a \( \sim 10^{-2} \) fringe shift). Of course there are many technical difficulties in performing such experiments, both on Earth and in orbit, but they do not seem impossible to overcome in the next future.

// Satellites ring. An idea to verify the GR corrections to the Sagnac effect is to make use of a ring of orbiting satellites (such as those belonging to the GPS or to the future European Galileo system). A stationary ring configuration of satellites can be the way to force the light beams to run in a closed circuit around the Earth, both in co-rotating and counter-rotating direction. The time difference in the propagation times should reproduce the effect expressed by formula (42)-(47), once the much bigger classical Sagnac effect has been subtracted out. Of course here too the technical details need to be thoroughly worked out.

6 Effects of the angular momentum on Michelson-Morley experiments

Another manifestation of the effects of the angular momentum of the source of gravity in interferometric measurements can be studied considering the influence of the terrestrial rotation on the famous Michelson-Morley experiment. This is a well known experiment, which is often quoted in teaching the basic Special Relativity. The idea of the experiment is the following: let us imagine to place the interferometer on the equator, with one arm along the South-North direction and the other arm along the West-East direction. The light propagating
in the interferometer is subject to the influence of the gravitational field of the Earth, which changes the propagation time with respect to the flat space-time. Furthermore, the light propagating in the East-West arm should have a different time of propagation, depending on whether it propagates in co-rotating or counter-rotating sense.

Let us start from the null space-time element, in an axisymmetric field

\[ 0 = g_{tt} dt^2 + 2 g_{t\phi} dtd\phi + g_{rr} dr^2 + g_{\theta\theta} d\theta^2 + g_{\phi\phi} d\phi^2 \]  

We then suppose that the interferometer, placed near the equator, has short enough arms.

We can compute the difference in propagation times between the two arms of the interferometer, considering light world lines with \( r = \text{constant} \) only. This choice corresponds to limiting the study to light beams contained locally in a horizontal plane (this would actually require a wave guide locally shaped as a constant gravitational potential surface). It can be seen \([61]\) that the propagation time in the South-North-South arm is

\[ t_{SNS} = t_N + t_S = 2 \sqrt{-\frac{g_{\theta\theta}}{g_{tt} + 2 g_{t\phi} \Omega + g_{\phi\phi} \Omega^2}} \Phi \]  

while for the West-East-West arm we have

\[ t_{W EW} = 2 \sqrt{\frac{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}{g_{tt} + 2 g_{t\phi} \Omega + \Omega^2 g_{\phi\phi}}} \Phi \]  

where \( \Omega \) is the angular speed of the Earth and \( \Phi = l/R \). Here \( l \) is the (proper) length of the arm of the interferometer and \( R \) is the radius of the Earth. We assume in general that \( l \ll R \). Then, the difference in the time of flight along the two arms at the lowest order of a weak field approximation is

\[ \Delta t = t_{W EW} - t_{SNS} \simeq \frac{a^2 l}{R^2 c} \]  

The result \([51]\) is obtained in the presumption that a physical apparatus (bidimensional wave guide) obliges the light rays to move along constant radius paths; the order of magnitude estimate at the surface of the Earth for 1 m long interferometer arms is:

\[ \Delta t \sim 10^{-20} \text{ s} \]  

This effect is purely rotational and rather small but not entirely negligible. As we showed elsewhere \([61]\), it is possible to increase the value of \([52]\), using Fabry-Perot type interferometers, which permit multiple reflections. Then the time difference between the two paths would become \(10^{-17} - 10^{-16} \text{ s} \), which corresponds, for visible light, to \(10^{-2} - 10^{-1} \) fringe in the interference pattern. The obtained numeric value compares with the expected phase (and time) shifts in the gravitational wave interferometric detectors now under construction, as LIGO and VIRGO \([62],[63]\). There indeed a sensitivity is expected, in measuring
displacements, of the order of $10^{-16}$ m which corresponds to a time of flight difference 4 orders of magnitude lower than (52) and consequently a much higher sensitivity is required.

Of course the effect as such is a static one, producing a DC signal and it would be practically impossible to recognize its presence in the given static interference pattern. On the other hand the spectacular sensitivity of gravitational wave interferometric detectors is obtained at frequencies in the range $10^2 - 10^3$ Hz. To extract the information from the background and to profit of highly refined interferometric techniques we would have to modulate the signal. This result could be achieved, for instance, steadily rotating the whole interferometer in the horizontal plane. It is possible to design a configuration that, in principle, should allow for the measuring of this effect, even if a careful analysis of the technical details would of course be needed in order to proceed further.

7 Effects on signals propagation

Shapiro delay and time asymmetry. A well known effect on the propagation of light signals in a gravitational field is the time delay: for example, a light signal emitted from a source on the Earth toward another planet of the solar system, and hence reflected back undergoes a time delay during its trip (with respect to the propagation in flat space-time), due to the influence of the gravitational field of the Sun. Shapiro et al. [64] measured this time delay for radar-ecos from Mercury and Venus, using the radio-telescopes of Arecibo and Haystack. Anderson et al. [65] measured the time delay of the signals transmitted by Mariner 6 and 7 orbiting around the Sun. Finally Shapiro and Reasenberg obtained more accurate results using a Viking mission that posed a transponder on the surface of Mars: the theoretical prediction was verified within $\pm 0.1\%$ [66] [67].

Now, we can consider here the correction to the time of propagation due to the presence of a rotating source: the original Shapiro measurements accounted just for the presence of a massive source, described by the Schwarzschild solution, while we are going to work out the time delay in post newtonian approximation for a spinning source.

The metric tensor can be written in cartesian coordinates, so that we have:

$$ ds^2 = g_{tt}dt^2 + g_{xx}dx^2 + g_{yy}dy^2 + g_{zz}dz^2 + 2g_{xt}dxdt + 2g_{yt}dydt $$

supposing the angular momentum of the source is parallel to the $z$ axis. For a light ray propagating in the $xy$ plane we can consider a first approximation, in which it propagates in a straight line with $x = b = const$, from $y_1$ to $y_2$ ($y_1 < 0$, $y_2 > 0$). Then (53) becomes

$$ 0 = g_{tt}dt^2 + g_{yy}dy^2 + 2g_{yt}dydt $$

It is easy to see that we can write, approximately, the coordinate time of propagation from $y_1$ to $y_2$ in the form

$$ T(y_1, y_2) = T_0 + T_M + T_{Ma} + ... $$
where $T_M$, $T_{Ma}$, are the corrections to the Minkowski geometry propagation time $T_0$, and we neglect terms which are smaller. In particular, $T_M$ is the delay effect measured by Shapiro, while $T_{Ma}$ is the effect of the angular momentum we want to outline. We obtain

$$T_0 = \frac{y_2 - y_1}{c}$$

(55)

$$T_M = \frac{2GM}{c^3} \ln \frac{y_2 + \sqrt{b^2 + y_2^2}}{y_1 + \sqrt{b^2 + y_1^2}}$$

(56)

$$T_{Ma} = \frac{-2GMa}{c^3b} \left[ \frac{y_2}{\sqrt{b^2 + y_2^2}} - \frac{y_1}{\sqrt{b^2 + y_1^2}} \right]$$

(57)

To give some numerical estimates, let us consider a realistic situation in the solar system as, f.i., the propagation of signals from the Earth to Mercury; then $M = M_\odot$, $|y_1| = 1.5 \times 10^{11}$ m, $|y_2| = 6 \times 10^{10}$ m, $a_\odot \simeq 3 \times 10^3$ m, $b = 6.8 \times 10^8$ m, hence

$$T_0 = 700 \text{ s}$$

$$T_M = 1.1 \times 10^{-4} \text{ s}$$

$$T_{Ma} = 1.0 \times 10^{-10} \text{ s}$$

The first angular momentum correction is six orders of magnitude smaller than the main mass contribution, which has been measured in the past. It is interesting to investigate whether this correction could be measured in the context of a future space mission, maybe exploiting the asymmetry of this effect. In fact the sign of the correction changes depending on the fact that light propagates in the $x > 0$ or $x < 0$ region. A possible way to evidence this effect is proposed here, without entering into details, which will be given elsewhere.

Let us imagine light coming from far sources, which passes in the field of a rotating object, a situation which is common in astrophysics. Again, we are going to explain and evaluate the magnitude of this effect, without giving the details, that will be found elsewhere.

Let us consider the same metric tensor and coordinates of the previous section, and two symmetric light rays propagating in the $xy$ plane with constant $x$, that is $x^+ = b$ and $x^- = -b$. It is easy to show that, in the first post-newtonian order, the time difference between the two signals, when received, is

$$\delta \tau = -8 \frac{GMa}{bc^3}$$

(58)

If we consider the Sun, or respectively Jupiter, as sources of the gravitational field acting upon the light beams, and choose $b$ as small as possible, that is of the order of the radius of the source, we have the following numerical estimates:

$$\delta \tau_S = 1.6 \times 10^{-10} \text{ s}$$

(59)

$$\delta \tau_J = 4.2 \times 10^{-13} \text{ s}$$

(60)
The time difference can be evidenced by superimposing the two beams, and letting them interfere.

A good signal source could be a pulsar, because of its fine and stable timing. It seems that the Sun is not that good gravitational field source, because the light passing nearby it, i.e. in its corona, is subject to many perturbations that may hide the effect we want to see. We think that this idea deserves some further investigation, because it seems somehow promising.

Moreover, we want to point out that Ciufolini et al.[68],[69] have recently studied the problem of time delay due to spin, with particular attention to the possibility of detecting the effect of angular momentum by gravitational lensing.

In our calculation we have considered, for the sake of simplicity, rectilinear propagation of light rays. We know, since the very beginning of the GR tests, that this is not the case, because light is bent by the gravitational field. It is interesting to notice that the angular momentum of the source influences the bending of light too. The magnitude of the bending angle $\delta \phi$ turns out to be

\[ \delta \phi = \frac{4GM}{c^2b} \left( 1 - \frac{1}{c \cdot Mb} \right) \]  

(61)

where $b$ is the impact parameter, $S$ is the angular momentum of the source with mass $M$ and $n$ is a unit vector in the direction of the angular momentum of light about the center of the source-body. The terms outside the parenthesis in (61) is the pure Schwarzschild term, i.e. the gravitoelectric contribution. The relative correction to the pure mass term, in the case of the Sun, can be estimated in $10^{-6}$, which is very small. This shows that our toy model, which considers straight line propagation of light, is acceptable.

8 Gravitomagnetic coupling between Spin and Angular momentum

The GEM framework allows us to extend some well known effects of electrodynamics to the gravitational field. In fact, we have seen that the gravitomagnetic field of the Earth is analogous to a dipolar magnetic field. A spinning particle possesses a "gravitomagnetic dipole moment", i.e. a spin $\sigma$, which couples to the gravitomagnetic field of a rotating mass with an interaction energy analogous to the magnetic interaction $H = -m \cdot B$, between a magnetic dipole $m$ and the magnetic field $B$. The effects of this coupling on the deflection of polarized radiation have been studied in the past: it was found that the paths of right circularly and left circularly polarized photons, propagating in the gravitational field of a rotating object, split because of the coupling between the helicity and the angular momentum of the source [71][72], much like in a Stern-Gerlach experiment with polarized matter passing through an anisotropic magnetic field. However the deflection angle (in the case of photons of wavelength $\lambda$ propagating
around the Sun with distance of closest approach \( b \)

\[
\delta \phi \sim \lambda \frac{GS}{c^3 b^3}
\]  

(62)

is too small and so it was not detected in the first experiments done in the 70’s [73][74].

The effect should be observable for collapsed objects, thus being a way to measure their angular momentum. In the case of neutrinos, gravity-spin coupling should lead to a helicity flip, which should be important for neutron stars and supernovae [75]. Ahluwalia [76] studied the influence of angular momentum-spin coupling on quantum mechanical clocks, showing that they do not always redshift identically when moved from the field of a non rotating object to the field of a rotating source. The theoretical investigations on the role of intrinsic spin in gravitational interaction and the continuous improvement of precision measurement techniques have enlarged the horizon of possibly detectable effects.

Let us give orders of magnitude, following the treatment given by Mashhoon [77], where more references and a more detailed discussion can be found.

In analogy with the electrodynamic case, \( H = -\frac{\sigma}{c} \cdot \mathbf{B}_G = \sigma \cdot \Omega \) is the interaction Hamiltonian, where \( \Omega \) is the angular precession frequency given in eq (31) and \( \sigma \) is the spin of the particle under study. For an experiment performed on the surface of a rotating body, just like the Earth, in a laboratory at latitude \( \alpha \), it turns out that the interaction energy difference between particles of spin \( \sigma = \hbar \) polarized up and down (i.e. perpendicular to the surface) is

\[
E^+ - E^- = 4\hbar \frac{GS}{c^2 r^3} \sin \alpha
\]  

(63)

For the Earth and \( s = 1 \), we have \( \Delta E_E \simeq 2 \times 10^{-29} eV \); for Jupiter \( \Delta E_J \simeq 10^{-27} eV \), and the same for the Sun \( \Delta E_S \simeq 10^{-27} eV \). For a compact and fast rotating body, such as a neutron star, we could have \( \Delta E_{NS} \simeq 10^{-14} eV \).

Also light scattering around a rotating object is influenced by the polarization of the incident radiation, because a helicity-rotation coupling is present. It is expected that right circularly polarized (RCP) and left circularly polarized (LCP) waves be separated, in the gravitational deflection, by the small gravitomagnetic splitting angle (62). In addition to this effect, a rotation of the plane of polarization along the ray is expected. Furthermore, due to the different phase speed, the arrival times of positive and negative helicity radiation originating near a rotating object and propagating freely outward, are different [11]

\[
T^+ - T^- = -\frac{\lambda GS \cdot r}{c^3 \pi r^3}
\]  

(64)

where \( r \) is the distance between the emission point of the radiation and the center of the rotating source, whose angular momentum is \( S \); \( \lambda \) is the wavelength of the radiation. However, these effects are very small, but it may have

\footnote{Of course, we do not know the absolute value of the propagation times, because we do not know their origin; however it is possible to measure, in principle, their difference.}
astrophysical implications and they may become interesting in microlensing with polarized radiation.

The Hamiltonian $H = \sigma \cdot \Omega$ we introduced before, is position dependent, so there is a Stern-Gerlach force $F = -\nabla H$, which acts on the particle. This force causes a shift on the particle weight, which is different if the spin is polarized up or down. In fact, the effective weight for the particle is $W = mg - F \cdot \hat{r}$, hence it depends on the polarization:

$$W_{\pm} = mg \mp \frac{6s}{c^2r^4} \sin \alpha$$  \hspace{1cm} (65)

The ratio between the correction and the weight of a particle at the surface of the Earth is $\sim 10^{-29}$. Again the effect is too small to be measurable also in the next future.

9 Forces

Laboratory tests. In a paper published in 1977, Braginsky et al. proposed various laboratory based experiments, to test relativistic gravity. Among them, there were some tests of ”magnetic-type gravitational forces”, i.e. gravitomagnetic effects, which originate from the off-diagonal terms $g_{0i}$ of the metric tensor. They are characterized by the sensitivity to the direction of rotation of both the source and the detector.

One proposed experiment is the gravitational analogue of the Ampère experiment. The source is an axially symmetric body, and the detector is a small sphere, placed on the same axis as the main body. Both the source and the sphere rotate about the common axis. According to the fact that the rotation sense is the same or is opposite one has repulsion or attraction. The detector mass is the end of an arm of a torsion balance, which is used to measure the entity of the force. For reasonable values of the parameters, the order of magnitude of the force per unit mass ($F/m_d$) of the detector is

$$\frac{F}{m_d} \sim 10^{-20} \text{ m/s}^2$$  \hspace{1cm} (66)

The rotation is modulated at the eigenfrequency of the torque balance.

Another laboratory experiment, is a variant of the Davies experiment, which was proposed to measure the post-Newtonian ”dragging of the inertial frames” caused by the rotation of the Sun: two ideal light beams of infinitesimal wavelengths travel in a thin toroidal waveguide, around the rotating laboratory.

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12 It is interesting noticing that the force on the particle is mass-independent, hence the resulting acceleration is mass dependent, thus violating the universality of gravitational acceleration. Indeed, also the different deflection angle for RCP and LCP radiation is a violation of the principle of equivalence. However these are wave effects, and the universality of free fall is recovered in the JWKB limit.

13 Infinitesimal means that the wavelength is small compared to the dimensions of the waveguide, i.e. in the geometric optics limit.
source. If the wave guide is kept at rest with respect to a distant inertial frame, the standing wave pattern made by the two waves should move relative to the guide with the angular velocity $\Omega_D$, which represents the dragging of the inertial frames. For a reasonable value of the mass of the source, its size and its velocity of rotation, one has $\Omega_D \simeq 6 \times 10^{-21}\text{rad/s}$.

A time-changing gravitomagnetic field would produce a gravitational electromotive force, and this fact can be used to perform a Faraday-type experiment. The source of this field is a cylinder, set into uniform rotation, and moved up and down along its rotation axis. The detector consists in an axially symmetric sapphire crystal, which is mounted coaxially with respect to the source. The motion of the cylinder produces an induction gravitoelectric field, which drives oscillations in the detector at the same eigenfrequency of the source motion. The order of magnitude of the driving force per unit mass $m_d$ is (for reasonable values of the parameters)

$$\frac{F}{m_d} \simeq 10^{-20}\text{m/s}^2$$

or something less, according to the authors, since breaking effects may occur. It is possible, however, that in the future, a gravitational Faraday effect could be found in binary pulsars.

**Force on a Gyroscope.** In Gravitoelectromagnetism it turns out that the force acting on a gyroscope, in the field of a rotating object, can be calculated in analogy with the expression of the force acting on a magnetic dipole in a non-homogeneous magnetic field, and depends on the direction of rotation. If the gyro has angular momentum $L$, the force acting on it, is

$$F = \frac{1}{2c}(L \cdot \nabla)B_G$$

Out of the equatorial plane of the source a component of this force will be radial. The order of magnitude is given by

$$F \simeq \frac{G LS}{c^2 r^4}$$

If we consider a prototype gyroscope on the Earth, with reasonable mechanical parameters, the order of magnitude of the force we obtain is

$$F \simeq 10^{-17}\text{N}$$

This is a correction on the effective weight of the gyroscope, and, in principle, it could be detected. Indeed, we must remember that in 1989, an anomalous weight reduction on a rotating gyroscope was detected by Hayasaka and Takeuchi: a decrease of the gyroscope’s weight was detected only in its right rotation, while the weight

For instance $l = 1\text{ m}, m = 10\text{ Kg}, \omega = 10^3\text{ Hz}$, where $l$ is the suspension length, $m$ is the gyroscope’s mass, $\omega$ is its angular frequency.
remained unchanged in its left rotation, which ruled out the possibility of explaining it by means of our GEM picture. It must be mentioned, however, that afterwards different groups \[84\], \[85\], performing the same experiment, did not measure this mysterious reduction, which remained unexplained.

10 Other effects

We list here a number of other ideas which have been proposed to test gravitomagnetic effects.

10.1 In space

*Light propagation in the field of the Sun.* An approach similar to the one exploiting the asymmetry in the Shapiro delay, which considers the propagation of light signals in the gravitational field, can be found in the work of Davies \[86\]. Here the possibility of measuring the Angular Momentum of the Sun is studied, using the difference in elapsed times for radio signals travelling clockwise and counterclockwise around the Sun. These times are worked out for free (geodesic) electromagnetic waves, and, depending on the range of the solar rotation speed, the time difference is evaluated to be between \(1/6 \times 10^{-10}\) s and \(1/6 \times 10^{-8}\) s, which are not too different from the estimates we gave in (59).

*Tidal effects.* The possibility of performing an orbital test of gravitomagnetism, proposed by Braginski and Polnarev \[87\], was critically analyzed by Mashhoon and Theiss \[88\]: they studied the relativistic corrections to the Newtonian tidal accelerations, produced by a rotating system, and worked out the difficulties of measuring these effects in orbit around the Earth, showing that they are not different from those encountered by the Stanford’s Gravity Probe experiment.

Theiss \[89\] proposed another test of gravitomagnetism: he considered the possibility of measuring the effect of the terrestrial rotation on the tidal acceleration between test masses in a satellite. He showed that this effect should be unexpectedly large, and well within the possibility of detection by a satellite experiment proposed by Paik \[90\]. The aim of the mission was to measure, to very high accuracy, the gravitational (i.e. gravitoelectric) gradient of the Earth; the theoretical principles of detection of the GEM field by means of a superconducting gradiometer were worked out by Mashhoon et al. \[91\].

*Effects on the orbits.* De Felice \[92\] remarked that in the gravitational field of a spherical source General Relativity implies the presence of a radial thrust other than the centrifugal force, appearing when the angular velocity of a non-geodesic circular trajectory is changed. In order to maintain the same trajectory a correction would be needed; measuring this correction would be a means of detecting the GR effects. The required precision in the determination
of the parameters of an experiment in the Terrestrial environment would be $10^{-11}$ for the angular velocity and $10^{-13}$ in the value of the mass.

### 10.2 On Earth

**Effects on superconductors.** The effects of the gravitomagnetic field on pure superconductors were studied, by Li and Torr [93], with particular interest on the interplay between the magnetic and the gravitomagnetic field. The conclusion was that in a pure superconductor always exists a residual magnetic field produced by the gravitomagnetic field of the Earth. This residual magnetic field produces in turn a local perturbation of the gravitomagnetic external field.

**Effect on a spin $1/2$ system.** Another approach to the role of the gravitomagnetic field of the Earth on a quantum system with spin $1/2$ is found in [94], where the Rabi formula for two-level system transition is obtained, and the possibility of obtaining the quantum Zeno effect is investigated.

### 11 Conclusions and the future

As we have seen, in approximately 85 years of research a number of proposals have been put forward concerning the possibility to reveal the effects of gravitomagnetism. For a long time many of these ideas have kept an essentially speculative character, because in general the sought for effects are extremely weak within the solar system. Strong effects in the vicinity of neutron stars or black holes suffer of the fact that in systems allegedly composed of those objects a variety of other effects are present too and many parameters must be taken into account, so that it is not easy to draw uncontroversial conclusions.

Nowadays however the improvement in length and time measurement techniques, both in the lab and in space, have pushed the border of detectability somewhat below the size of part of the effects listed in this paper. Furthermore a number of space missions are under way or being planned, with different purposes, which could be fit for experiments on gravitomagnetism. In fact the cost of a dedicated mission is quite high, however in many cases it is possible to use the data collected from satellites sent for different purposes. This is in general the case for any spacecraft that carries transmitters, receivers and clocks. In the near future the European Galileo project will be realized. The satellites of those project, being designed for precision timing and positioning of the satellites themselves or of ground based stations, may easily be considered as ideal to reveal anisotropies in the propagation of electromagnetic signals in the terrestrial environment.

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