Open Strings in PP-Wave Background from Defect Conformal Field Theory

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Abstract

We consider open strings ending on a D5-brane in the pp-wave background, which is realized in the Penrose limit of $AdS_5 \times S^5$ with an $AdS_4 \times S^2$ brane. A complete list of gauge invariant operators in the defect conformal field theory is constructed which is dual to the open string states.
1 Introduction

AdS/CFT correspondence [1, 2, 3] (for a review, see [4]) has lead to deep understandings of string theory and field theory. However, until recently, much of the progress in this direction has been limited to supergravity approximations due to the difficulty when one has Ramond-Ramond background. Recently, it has been shown that string theory can be fully solved in the pp-wave background even in the presence of RR flux [5, 6] in the light-cone Green-Schwarz formalism. Shortly after this development, Berenstein, et. al. [7] have put forward an exciting proposal that tests AdS/CFT correspondence beyond the supergravity approximation. More specifically, they have related closed string states in the pp-wave background with operators of the dual $\mathcal{N} = 4$ SYM with large R-charge $J \sim \sqrt{N}$ and finite $\Delta - J$. Many interesting papers have subsequently followed [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

In this article, we extend the results of [7] to the case of open strings ending on a D5-brane in the pp-wave background. We consider a large number of D3-branes and a single D5-brane in the near-horizon limit. The resulting system is $AdS_5 \times S^5$ with the D5-brane spanning an $AdS_4 \times S^2$. Recently extending the idea of [25, 26], De Wolfe, et. al. [27] have proposed that its dual field theory is a defect conformal field theory in which the usual $\mathcal{N} = 4$ bulk SYM theory is coupled to a 3-dimensional conformal defect. This defect field theory has been further studied by [28] in which they demonstrate quantum conformal invariance for the non-abelian case. By taking the Penrose limit [29, 30] of this setup, one obtains a D5-brane in the pp-wave background. We construct a complete list of gauge invariant operators in the defect conformal field theory which is dual to the open string states ending on the D5-brane. Interestingly, boundary conditions of open stings on the D5-brane are encoded in the way symmetry is broken by the defect and in specific form of defect couplings in the dual field theory.

This paper is organized as follows. In section 2, we give a brief review of the D-brane setup and the field content of the defect conformal field theory. In section 3, we discuss the Penrose limit of this background and obtain the open string spectrum. In the last section, we propose a list of gauge invariant operators dual to the open string states.
While this manuscript was being prepared for publication, article [31] containing some overlap with our results was posted on the web.

2 Review of defect conformal field theory

In this section, we briefly review the D3-D5 brane setup of [25, 26] and the field content of its dual defect conformal field theory discussed in [27]. The interested reader can find further details in the aforementioned papers. We start with the coordinate system in which the world-volume of a stack of $N$ D3-branes span the directions $(x^0, x^1, x^2, x^9)$ and a single D5-brane spans the directions $(x^0, x^1, x^2, x^3, x^4, x^5)$. The D-branes sit at the origin of their transverse coordinates. The setup is summarized in the following table:

|   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D3| x | x | x |   |   |   |   |   |   | x |
| D5| x | x | x | x | x |   |   |   |   |   |

The presence of the D5-brane breaks 16 spacetime supersymmetries to 8 supersymmetries and reduces the global symmetry group $SO(6)$ to $SO(3) \times SO(3)$, where each $SO(3)$ acts on the 345 and 678 coordinates respectively. In AdS/CFT correspondence, one is interested in taking the near horizon limit where the string coupling $g \to 0$ and $N \to \infty$ with the product $gN$ fixed. In this limit, we have the D5-brane spanning an $AdS_4 \times S^2$ subspace of $AdS_5 \times S^5$. The dual conformal field theory of type IIB string theory in this background is $\mathcal{N} = 4$ SYM theory [1] that lives on the boundary of $AdS_5$ parameterized by $(x^0, x^1, x^2, x^9)$. The D5-brane introduces a codimension one conformal defect on this boundary located at $x^9 = 0$. An analogous model can be considered for the $AdS_3 \times S^3$ case, where an $AdS_2$ brane introduces a one-dimensional defect in the dual CFT [32]. Such a reasoning has been used by [33, 34] to construct boundary states for the $AdS_2$ branes.

It has been argued by DeWolfe, et. al. [27] that type IIB string theory in $AdS_5 \times S^5$ with a $AdS_4 \times S^2$ brane is dual to a defect conformal field theory wherein a subset of fields
of $d = 4, \mathcal{N} = 4$ SYM couples to a $d = 3, \mathcal{N} = 4$ $SU(N)$ fundamental hypermultiplet on the defect preserving $SO(3, 2)$ conformal invariance and 8 supercharges. Let us summarize the field content of the defect conformal field theory relevant for our purposes. Denote the $SU(2)$ acting on the 345 directions as $SU(2)_H$ and the one acting on the 678 directions as $SU(2)_V$. Then we have the usual bulk $d = 4, \mathcal{N} = 4$ vector multiplet which decomposes into a $d = 3, \mathcal{N} = 4$ vector multiplet and a $d = 3, \mathcal{N} = 4$ adjoint hypermultiplet. The bosonic components of the vector multiplet are $A_\mu (\mu = 0, 1, 2), X^6, X^7, X^8$, with the scalars transforming as a 3 of $SU(2)_V$, while those of hypermultiplet are $A_9, X^3, X^4, X^5$, with the scalars as a 3 of $SU(2)_H$. The derivatives of $X^3, X^4, X^5$ along the 9-direction, which is normal to the defect, are also a part of the vector multiplet. The four adjoint $d = 4$ Majorana spinors of $\mathcal{N} = 4$ SYM transform as a $(2, 2)$ of $SU(2)_V \times SU(2)_H$, which is denoted as $\lambda^{im}$. Under the dimensional reduction to $d = 3$, they decompose into pairs of 2-component $d = 3$ Majorana spinors, $\lambda_1^{im}$ and $\lambda_2^{im}$, where the former is in the vector multiplet and the latter in the hyper multiplet. We also have a $d = 3, \mathcal{N} = 4$ $SU(N)$ fundamental hypermultiplet on the defect. It consists of complex scalars $q^m$ transforming as a 2 of $SU(2)_H$ and $d = 3$ Dirac spinors $\Psi^i$ transforming as a 2 of $SU(2)_V$. They are coupled canonically to 3-dimensional gauge fields $A_\mu$. Hence supersymmetry will induce couplings to the rest of the bulk vector multiplet as well via defect F-term, while the bulk hypermultiplet does not couple to the defect hypermultiplet at tree level. This fact will play a crucial role in reproducing the open string spectrum in section 4. The field content of interest is summarized in the following table.

| Field           | Spin | $SU(2)_H$ | $SU(2)_V$ | $SU(N)$ | $\Delta$ |
|-----------------|------|-----------|-----------|---------|----------|
| $X^3, X^4, X^5$ | 0    | 1         | 0         | adjoint | 1        |
| $X^6, X^7, X^8$ | 0    | 0         | 1         | adjoint | 1        |
| $\lambda^{im}$  | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | adjoint | $\frac{3}{2}$ |
| $q^m$           | 0    | $\frac{1}{2}$ | 0         | $\mathcal{N}$ | $\frac{1}{2}$ |
| $\bar{q}^m$     | 0    | $\frac{1}{2}$ | 0         | $\bar{\mathcal{N}}$ | $\frac{1}{2}$ |
| $\Psi^i$        | $\frac{1}{2}$ | 0         | $\frac{1}{2}$ | $\mathcal{N}$ | 1        |
| $\bar{\Psi}^i$  | $\frac{1}{2}$ | 0         | $\frac{1}{2}$ | $\bar{\mathcal{N}}$ | 1        |
Field theory action takes the form

$$S = S_4 + S_3,$$  \hspace{1cm} (1)

where $S_4$ is the usual $d = 4$, $\mathcal{N} = 4$ SYM part and $S_3$ is the $d = 3$ defect CFT action with defect couplings to $d = 4$, $\mathcal{N} = 4$ SYM fields. They are derived in [27] using the preserved $d = 3$, $\mathcal{N} = 4$ supersymmetry and the global symmetries. The authors of [27] convincingly argue that the chiral primary operators in the defect CFT are

$$\bar{q}^m \bar{\sigma}^{(ln}X^{j_l}_H \ldots X^{j_l}_H)q^n,$$  \hspace{1cm} (2)

where we define a shifted Pauli matrices $\bar{\sigma}^I$ ($I = 3, 4, 5$) as $\sigma^{I-2}$ and (...) denotes traceless symmetrization. These operators will turn out to be the important building blocks for open strings ending on the D5-brane in section 4.

### 3 Open strings in pp-waves

Let us now consider the Penrose limit of near-horizon geometry of D3-D5 brane setup described in the previous section. It is convenient to introduce global coordinates on $AdS_5 \times S^5$ in taking the Penrose limit. The metric takes the form

$$ds^2 = R^2 \left[ -dt^2 \cosh^2 \rho + d\rho^2 + \sinh^2 \rho d\Omega_3^2 + d\psi^2 \cos^2 \varphi + d\varphi^2 + \sin^2 \varphi d\Omega_3^2 \right],$$  \hspace{1cm} (3)

where $R^4 = 4\pi g\alpha'^2 N$. We introduce light-cone coordinates $\tilde{x}^\pm = (t \pm \psi)/2$ and take the Penrose limit ($R \to \infty$ with $g$ fixed) after rescaling coordinates as follows

$$\tilde{x}^+ = x^+, \quad \tilde{x}^- = \frac{x^-}{R^2}, \quad \rho = \frac{r}{R}, \quad \theta = \frac{y}{R}. \hspace{1cm} (4)$$

After rescaling $x^\pm$ to introduce a mass scale, $\mu$, the metric and the Ramond-Ramond form takes the form

$$ds^2 = -4dx^+dx^- - \mu^2 \bar{z}^2 dx^+d\bar{z} + d\bar{z}^2,$$  \hspace{1cm} (5)

$$F_{+1234} = F_{+5678} = \mu.$$  \hspace{1cm} (6)
where $\vec{z} = (z^1, \ldots, z^8)$. The SO(2) generator, $J = -i\partial_\psi$, rotates the 34 plane in the original D3-D5 setup. One finds that
\begin{align}
2p^- &= -p_+ = i\partial_{z^+} = i\partial_{\chi^+} = i(\partial_t + \partial_\psi) = \Delta - J, \\
2p^+ &= -p_- = i\partial_{z^-} = \frac{i}{R^2}(\partial_t - \partial_\psi) = \frac{\Delta + J}{R^2}.
\end{align}
Therefore, the Penrose limit corresponds to restricting to operators with large $J \sim \sqrt{N}$ and finite $\Delta - J$. Notice that we are in the large 't Hooft coupling regime since we keep $g$ fixed.

In the Penrose limit, the string action reduces to the following form in the light-cone gauge
\begin{equation}
S = \frac{1}{2\pi\alpha'} \int d\tau \int_0^{\pi\alpha' p^+} d\sigma \left[ \frac{1}{2} \dot{z}^2 - \frac{1}{2} \dot{\psi}^2 - \frac{1}{2} \mu^2 z^2 + i \left( \frac{1}{2} S_1 \partial_+ S_1 + \frac{1}{2} S_2 \partial_- S_2 - \mu S_1 \Gamma^{1234} S_2 \right) \right],
\end{equation}
where $S_i$ are positive chirality $SO(8)$ spinors. One can readily see that taking the light-cone gauge leads to Neumann boundary conditions for $x^+, x^-$ in the open-string sector since
\begin{equation}
\partial_\sigma x^- = \frac{\partial_\tau z^i \partial_\sigma z^i}{p^+}.
\end{equation}

We identify $(z^5, z^6, z^7, z^8)$ directions with the original $(x^5, x^6, x^7, x^8)$ directions and $z^4$ with the orthogonal direction to D5 brane in $AdS_5$. We label the coordinates in the Penrose limit such that the boundary conditions for the D5-brane are given as

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| N | N | N | N | N | D | N | D | D |

where N and D denote Neumann and Dirichlet boundary conditions respectively. For $S_i$, the appropriate boundary condition is [35]
\begin{equation}
S_2 = \Gamma^{1235} S_1.
\end{equation}
The boundary condition for the fermions effectively reduces the degree of freedom by half. Taking the Penrose limit and taking the light-cone gauge break the symmetry group $SO(3, 2) \times$

\footnote{We have chosen the null geodesic in the Penrose limit to lie on the D5-brane because in the light-cone gauge, Neumann conditions are automatically imposed on $x^\pm$.}
$SU(2)_H \times SU(2)_V$ to $SO(3) \times SU(2)_V$. The full open string spectrum on a D5-brane has recently been computed by [23]. The mode expansions for the bosonic part are

$$z_{NN}^I(\tau, \sigma) = \cos(\mu \tau) z_0^I + \frac{1}{\mu} \sin(\mu \tau) p_0^I + i \sum_{n=1}^{\infty} \frac{1}{\sqrt{\omega_n}} e^{-i\omega_n \tau} \cos \left( \frac{n \sigma}{\alpha' p^+} \right) a_n^I + c.c., \quad (12)$$

$$z_{DD}^I(\tau, \sigma) = i \sum_{n=1}^{\infty} \frac{1}{\sqrt{\omega_n}} e^{-i\omega_n \tau} \sin \left( \frac{n \sigma}{\alpha' p^+} \right) a_n^I + c.c., \quad (13)$$

where we have defined

$$\omega_n = \sqrt{\mu^2 + \frac{n^2}{4(\alpha' p^+)^2}}. \quad (14)$$

Important difference between the Neumann and Dirichlet expansions is that the Dirichlet expansion does not have a zero mode. This gives rise to 4 massive bosonic oscillators. Similarly, eight zero modes coming from fermions form 4 massive fermionic oscillators and their contribution to the zero point energy exactly cancel the contribution from the bosonic zero modes. Due to the mass term, fermionic creation and annihilation operators have $+1/2$ and $-1/2$ eigenvalues with respect to $\Gamma^{45}$ respectively, and both transform separately as doublets of $SU(2)_V$. Hence, the light cone vacuum should be a singlet of $SU(2)_V$ for the fermionic zero modes, thus correctly reproducing D5-brane SYM multiplet.

The light cone Hamiltonian is given as

$$2p^- = -p_+ = H_{lc} = \sum_{n=0}^{\infty} N_n \sqrt{\mu^2 + \frac{n^2}{4(\alpha' p^+)^2}}, \quad (15)$$

where $N_n$ denotes the total occupation number of that mode for both bosonic and fermionic oscillators. Rewriting the Hamiltonian in variables better suited for $AdS_5 \times S^5$, one notes that a typical string excitation contributes to $\Delta - J = 2p^-$ with frequency

$$(\Delta - J)_n = \sqrt{1 + \frac{\pi g N n^2}{J^2}}. \quad (16)$$

This point has been clarified in [20].

This point is to be contrasted with [31] where all creation operators have the same quantum number of the symmetry under consideration.
4 Open strings from defect conformal field theory

In this section, we construct a list of gauge invariant operators in the defect CFT dual to states in the open string Hilbert space. Recall that \( J \) is the generator of rotation on the \( X^3 - X^4 \) plane. Define

\[
Z \equiv \frac{1}{\sqrt{2}} (X^3 + iX^4), \quad \sigma^Z \equiv \frac{1}{\sqrt{2}} (\bar{\sigma}^3 + i\bar{\sigma}^4) = \frac{1}{\sqrt{2}} (\sigma^1 + i\sigma^2),
\]

Both the operators \( Z \) and \( \bar{q}^m \sigma^Z_{mn} q^n \) have \( \Delta = J = 1 \). The fact that \( Z \) belongs to the bulk hypermultiplet will be important later. We propose that the light-cone vacuum corresponds to

\[
|0,p^+\rangle_{l.c.} \leftrightarrow \frac{1}{N^{J/2}} \sum_{l=0}^{J} 1_{N^{J/2+1}}^{\sigma^Z_{mn}} \bar{q}^m \bar{q}^Z \cdots \bar{q}^{Z_{l-1}} q^n.
\]

As mentioned above, this is a chiral primary operator with \( \Delta = J \) found in [27]. Because it is a chiral primary, \( \Delta - J = 0 \) in the strong 't Hooft coupling limit. This property agrees with the fact that the light-cone vacuum has zero energy. Furthermore, it does not transform under \( SU(2)_V \) as one expects from the light-cone vacuum.

For excited states, as in the closed string case [7], we insert proper operators with \( \Delta - J = 1 \) without phases for zero modes and with appropriate phases for nonzero modes. Here we consider Neumann and Dirichlet directions separately since there are several crucial differences.

For the zero mode excitations along the Neumann directions, we have the following correspondence:

\[
a_{0}^{1}|0,p^+\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \frac{1}{N^{J/2+1}} \sigma^Z_{mn} \bar{q}^m \bar{q} Z^l (D_0 Z)^{J-l} q^n, \quad (19)
\]

\[
a_{0}^{2}|0,p^+\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \frac{1}{N^{J/2+1}} \sigma^Z_{mn} \bar{q}^m \bar{q} Z^l (D_1 Z)^{J-l} q^n. \quad (20)
\]

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\( ^4 \)To be rigorous, the directions \( x^0, x^1, x^2, x^9 \) are related to the original coordinates by a conformal transformation after wick rotation as in the radial quantization[7]. This transformation leaves the 9 direction orthogonal to the defect.
\[ a_0^3|0, p^+\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \frac{1}{N_{j/2+1}} \sigma^Z_{mn} \bar{q}^m Z^l (D_2 Z) Z^{j-l} q^n, \]  
\[ a_0^5|0, p^+\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \frac{1}{N_{j/2+1}} \sigma^Z_{mn} \bar{q}^m Z^l X^5 Z^{j-l} q^n. \]

The above open string states are associated with preserved symmetries of the D5 brane. They are massive however since the symmetries do not commute with the light cone Hamiltonian. Hence, these operators are obtained from the vacuum operator (18) by acting corresponding preserved symmetries in the defect conformal field theory[7, 20]. For example, the fourth operator (22) is obtained by acting a \( SU(2)_H \) rotation on the vacuum operator. The rotation also acts on the boundary fields \( \bar{q}^m \) and \( q^n \) giving rise to additional terms such as
\[ \tilde{\sigma}^5_{mn} \bar{q}^m Z^{j+1} q^n. \]

For notational simplicity, we have suppressed this term in the above list. Likewise, the other three operators have additional boundary contributions. In the weak ’t Hooft coupling regime, these operators have \( \Delta - J = 1 \). Since they are in the same multiplet as the chiral primary operator (18), their dimensions are also protected by supersymmetry.

For nonzero mode excitations along the Neumann directions, we insert operators with cosine phases\(^5\)
\[ a^n_1|0, p^+\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \frac{\sqrt{2} \cos \left( \frac{\pi n l}{J} \right)}{N_{j/2+1}} \sigma^Z_{mn} \bar{q}^m Z^l (D_0 Z) Z^{j-l} q^n, \]
\[ a^n_2|0, p^+\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \frac{\sqrt{2} \cos \left( \frac{\pi n l}{J} \right)}{N_{j/2+1}} \sigma^Z_{mn} \bar{q}^m Z^l (D_1 Z) Z^{j-l} q^n, \]
\[ a^n_3|0, p^+\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \frac{\sqrt{2} \cos \left( \frac{\pi n l}{J} \right)}{N_{j/2+1}} \sigma^Z_{mn} \bar{q}^m Z^l (D_2 Z) Z^{j-l} q^n. \]

\(^5\)This point is also noticed in [31].

\(^6\)In principle, we should assign phases including the boundary contributions. Again, for simplicity, we suppress them since it does not affect following conclusions.
\[a^{i_5}_n|0, p^+\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \sqrt{2} \cos\left(\frac{\pi nl}{J}\right) \sigma_{m}^{\star} q^{m} z^l X^5 Z^{J-l} q^{n}. \]  

The factor of $\sqrt{2}$ is necessary for correct normalization of the free Feynman diagram in the two-point function. Notice that unlike the closed string case, the operators with single insertions are not trivially zero which reflects the fact that there is no level matching condition for open strings. In addition, the sign of $n$ has no significance, which corresponds to the fact that there is only one set of oscillators instead of both the left and right moving sectors.

We can compute the anomalous dimension of these operators following the closed string case discussed in the appendix of [7]. The only difference from the closed string case is that the exponential phase has been replaced by the cosine phase. For example, let $O$ be the fourth operator (22) above. The contribution from $\frac{1}{2\pi g} \int d^4 x 2Tr[X^5 ZX^5 \bar{Z}]$ in the bulk action gives

\[
\langle O(x)O^*(0) \rangle = \frac{\mathcal{N}}{|x|^{2\Delta}} \left[ 1 + \frac{1}{J} \sum_{l=0}^{J-1} N(2\pi g) 8 \cos \left(\frac{\pi nl}{J}\right) \cos \left(\frac{\pi n(l + 1)}{J}\right) \frac{1}{4\pi^2} \log |x| \Lambda \right] 
\]

\[
= \frac{\mathcal{N}}{|x|^{2\Delta}} \left[ 1 + \frac{1}{J} \sum_{l=0}^{J-1} N(2\pi g) 4 \left\{ \cos \left(\frac{\pi n(2l + 1)}{J}\right) + \cos \left(\frac{\pi n}{J}\right) \right\} \frac{1}{4\pi^2} \log |x| \Lambda \right] 
\]

\[
= \frac{\mathcal{N}}{|x|^{2\Delta}} \left[ 1 + N(2\pi g) 4 \cos \left(\frac{\pi n}{J}\right) \frac{1}{4\pi^2} \log |x| \Lambda \right], \tag{28}
\]

where $\mathcal{N}$ is a normalization factor and $\Lambda$ is the UV cutoff scale. As argued in [7], contributions from other Feynman diagrams cancel this contribution when $n=0$. Therefore, the full contribution can be taken into account by simply replacing $\cos \left(\frac{\pi n}{J}\right)$ with $\cos \left(\frac{\pi n}{J}\right) - 1$.

Finally, we have to the leading order

\[
\langle O(x)O^*(0) \rangle = \frac{\mathcal{N}}{|x|^{2\Delta}} \left[ 1 - \frac{\pi g N n^2}{J^2} \log |x| \Lambda \right]. \tag{29}
\]

Therefore, one gets

\[
(\Delta - J)_n = 1 + \frac{\pi g N n^2}{2 J^2} = 1 + \frac{n^2}{8(\alpha'p^+)^2}. \tag{30}
\]

This is exactly the first order expansion of light-cone energy of corresponding string states.
Now consider the directions with Dirichlet boundary conditions. As mentioned earlier, the associated mode expansions do not have zero modes. For nonzero mode excitations, we insert appropriate operators with *sine* phases as follows

\[
a^{\dagger}_{4} n|0, p^{+}\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \sqrt{2} \sin\left(\frac{\pi n l}{J}\right) \sigma_{mn}^{Z} Z^l (D_{9}Z) Z^{J-l} q^{n},
\]

(31)

\[
a^{\dagger}_{6} n|0, p^{+}\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \sqrt{2} \sin\left(\frac{\pi n l}{J}\right) \sigma_{mn}^{Z} Z^l X^{6} Z^{J-l} q^{n},
\]

(32)

\[
a^{\dagger}_{7} n|0, p^{+}\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \sqrt{2} \sin\left(\frac{\pi n l}{J}\right) \sigma_{mn}^{Z} Z^l X^{7} Z^{J-l} q^{n},
\]

(33)

\[
a^{\dagger}_{8} n|0, p^{+}\rangle_{l.c.} \leftrightarrow \frac{1}{\sqrt{J}} \sum_{l=0}^{J} \sqrt{2} \sin\left(\frac{\pi n l}{J}\right) \sigma_{mn}^{Z} Z^l X^{8} Z^{J-l} q^{n}.
\]

(34)

Notice that the sine phases naturally kill the zero modes when \( n = 0 \). We should ask what is the fate of the operators with insertions along the Dirichlet directions without phase. These operators are obtained by acting on the vacuum operator (18) with symmetries broken by the defect\(^7\). Therefore, their dimensions are not generally protected. In fact, the operators \( X^{6}, X^{7}, X^{8} \) are in the bulk vector multiplet and couple to the defect hyper multiplet via defect F-term. Similarly, the normal derivative \( D_{9}Z \) couples to the defect hyper multiplet despite the fact that \( Z \) itself is in the bulk hyper multiplet[27]. This boundary interaction gives rise to large anomalous dimensions of order \( N/J \sim J \) when one inserts operators *without phases*. Hence such operators will disappear in the strong 't Hooft coupling regime as implied by the open string spectrum. Nevertheless, once we include the sine phase, boundary interactions are suppressed by a factor of \( \sin^{2}\left(\frac{\pi n}{J}\right) \sim 1/J^{2} \), and they can be ignored to the leading order in \( 1/J \). Therefore, the only contribution to anomalous dimensions comes from the bulk interaction. The computation is essentially the same as above, and the result agrees with the open string spectrum.

For fermionic excitations, we insert \( J = 1/2 \) components of \( \lambda^{im} \), which is just \( \lambda^{11} \).\(^8\) As in

\(^7\)As a result, they do not act on \( q \) and \( \bar{q} \) unlike the case for Neumann directions.

\(^8\)We take \( m \) to be the quantum number of \( J \), which is a generator of Cartan subalgebra of \( SU(2)_{H} \).
the bosonic sector, the number of zero modes is half of that of non-zero modes. Hence, we need a similar mechanism to remove possible gauge theory operators corresponding to the 4 unphysical zero modes. The symmetry breaking pattern and the form of defect couplings in the defect CFT again allow one to do this consistently. Recall that the operators $\lambda_1^{i_1}$ and $\lambda_2^{i_1}$ are in the vector and hyper multiplets respectively. Only $\lambda_1^{i_1}$ couples to the defect hypermultiplet while $\lambda_2^{i_1}$ can be obtained from $Z$ by acting preserved supersymmetries. Therefore, we associate sine and cosine phases with $\lambda_1^{i_1}$ and $\lambda_2^{i_1}$ respectively. As in the bosonic sector, this assignment reproduces the open string spectrum in the fermionic sector.

5 Conclusion

In this article, we have considered a Penrose limit of type IIB string theory on $AdS_5 \times S^5$ with a D5-brane spanning an $AdS_4 \times S^2$ whose dual field theory is $\mathcal{N} = 4$ SYM coupled to a 3-dimensional conformal defect. The Penrose limit gives rise to a D5-brane in the pp-wave background. The limit corresponds to looking at a subsector of operators in the dual field theory with large $J \sim \sqrt{N}$ and finite $\Delta - J$ in the large 't Hooft coupling regime. We have studied perturbative open string spectrum on this brane and constructed a complete list of gauge invariant operators dual to the open string states from the defect conformal field theory. The peculiar feature of defect couplings, symmetry breaking pattern in the dual field theory, and sine-cosine phases are essential to reproduce the proper boundary conditions for the open strings.

One can also consider several M D5-branes. Then the defect hypermultiplet gets an additional $U(M)$ index with $q^m$ and $\bar{q}^n$ transforming as $M$ and $\bar{M}$ of $U(M)$ respectively. This naturally induces Chan-Paton factors at the ends of open strings as expected.

It would be interesting to construct defect conformal field theories arising from other supersymmetric brane intersections and study their Penrose limits. Then we expect to find specific defect couplings and symmetry breaking patterns which reflect the boundary conditions of the D-branes in this limit.

\footnote{They also transform $q$ and $\bar{q}$ into $\Psi$ and $\bar{\Psi}$. Therefore, when we insert $\lambda_2^{i_1}$, we have additional boundary terms with $q$ or $\bar{q}$ replaced by $\Psi$ or $\bar{\Psi}$.}
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