Higgs boson production at $e^+e^-$ colliders: a model independent approach.

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Abstract

We consider non-Standard Model physics effects using an effective lagrangian parameterization. We determine the operators whose effects are most significant and extract the sensitivity to the scale of new physics generated by the existing data. We then consider processes containing the Higgs particle in $e^+e^-$ colliders as a probe for new physics effects, and demonstrate their usefulness in this area.
The possibility of determining new physics effects by precision measurements has been pursued for a long time in an effort to provide insights into the interactions that lie beyond the Standard Model. Many of these efforts were carried out for a specific model of non-Standard Model physics [1].

Recently it has become evident that these studies should be complemented with a model independent approach where all possible non-standard effects are parameterized by means of an effective lagrangian [2, 4]. This formalism is model and process independent and thus provides an unprejudiced analysis of the data. Such an approach will be pursued in this paper. We will parameterize all non-standard effects using the coefficients of a set of effective operators (which respect the symmetries of the Standard Model). These operators are chosen so that there are no a-priori reasons to suppose that the said coefficients are suppressed. In this respect the present analysis differs from others appearing in the literature [3] which concentrate on operators related to the vector-boson self interactions. In using a manifestly gauge invariant parametrization we diverge from those studies aimed at elucidating the rigidity of the Standard Model to violations of its symmetries (whose drawbacks have been emphasized previously [4]).

The effective lagrangian approach requires a choice of the low energy particle content. In this paper we will assume that the Standard Model correctly describes all such excitations (including the Higgs particle) [1]. Thus we imagine that there is a scale Λ, independent of the Fermi scale, at which the new physics becomes apparent. Since the Standard Model is renormalizable and the new physics is assumed to be heavy due to a large dimensional parameter ∼ Λ, the decoupling theorem [5] is applicable and requires that all new physics effects be suppressed by inverse powers of Λ. All such effects are expressed in terms of a series of local gauge invariant operators of canonical dimension > 4; the catalogue of such operators up to dimension 6 is given in Refs. [6] (there are no dimension 5 operators respecting the global and local symmetries of the Standard Model).

For the situation we are considering it is natural to assume that the underlying theory is weakly coupled (else radiative corrections will drive the Higgs mass to Λ unless the low energy particle content is modified to effect cancelations, we will not pursue this possibility). Thus the relevant property

1Other approaches can be followed, assuming, for example, an extended scalar sector or the complete absence of light physical scalars.
of a given dimension 6 operator is whether it can be generated at tree level by the underlying physics. The coefficient of such operators are expected to be $O(1)$; in contrast, the coefficients of loop-generated operators will contain a suppression factor $\sim 1/16\pi^2$. The determination of those operators which are tree level generated is given in Ref. [7].

The strategy which we follow in this paper is to develop the effects of the tree-level-generated operators containing leptons and scalars in various processes. We will consider the constraints implied by current high-precision data and predict the sensitivity to new effects at LEP2 and a proposed version of the NLC.

The operators which we will consider are

$$
\begin{align*}
\mathcal{O}_\phi &= \frac{1}{3} (\phi^\dagger \phi)^3 \\
\mathcal{O}_{\partial\phi} &= \frac{i}{2} \partial_\mu (\phi^\dagger \phi) \partial^\mu (\phi^\dagger \phi) \\
\mathcal{O}^{(1)}_\phi &= \left(\phi^\dagger \phi\right) \left[ (D_\mu \phi)^\dagger D^\mu \phi \right] \\
\mathcal{O}^{(3)}_\phi &= \left(\phi^\dagger D_\mu \phi\right) \left[ (D_\mu \phi)^\dagger \phi \right] \\
\mathcal{O}_{\phi\ell} &= i \left(\phi^\dagger D_\mu \phi\right) \left(\bar{\ell} \gamma^\mu \ell \right) \\
\mathcal{O}^{(3)}_{\phi\ell} &= i \left(\phi^\dagger \tau^I D_\mu \phi\right) \left(\bar{\ell} \gamma^\mu \tau^I \ell \right)
\end{align*}
$$

Where we have omitted those that contain quark fields. The complete list of tree level generated operators can be found in Ref. [7]. Note that the modifications to the $WWZ$ and $WW\gamma$ vertices are not in this list; this implies, as has been repeatedly emphasized, that LEP2 will not be sensitive to these effects. The NLC will have enough sensitivity to probe these anomalous couplings, still its sensitivity to non-Standard Model processes generated by (1) will be significantly larger. The effects of the above operators present the widest windows into physics beyond the Standard Model.

Given the above list the lagrangian which we will use in the following calculations is

$$
\mathcal{L} = \mathcal{L}^{(SM)} + \frac{1}{\Lambda^2} \sum_i \left\{ \alpha_i \mathcal{O}_i + \text{h.c.} \right\}
$$

The above operators modify the couplings of the leptons to the $Z$ and to the $W$ gauge bosons; they also modify the $\rho$ parameter, the Fermi constant and the normalization of the Higgs field. Only this last effect is not probed in the existing data.

Footnote:

2If there is a large number of loop graphs this suppression factor can be reduced but, simultaneously, the masses not protected by a symmetry will, in general, be driven to the scale $\Lambda$. 

3
The electron vector and axial couplings to the \( Z \), \( g_V(e) \) and \( g_A(e) \), and the neutrino coupling to the \( Z \), \( g_\nu \), receive the contributions [8]

\[
|\delta g_V(e)| = \frac{v^2}{2\Lambda^2} |\alpha^{(1)} + \alpha^{(3)}| \lesssim 0.0021,
\]

\[
|\delta g_A(e)| = \frac{v^2}{2\Lambda^2} |\alpha^{(1)} + \alpha^{(3)} - \alpha_{\phi e}| \lesssim 0.00064,
\]

\[
|\delta g_\nu| = \frac{v^2}{2\Lambda^2} |\alpha^{(1)} - \alpha^{(3)}| \lesssim 0.0018;
\]

where the \((1\sigma)\) experimental constraints are also indicated. At the \(3\sigma\) level these bounds then correspond to the constraints

\[
\Lambda_{\text{TeV}} \gtrsim \frac{2.5}{\sqrt{|\alpha^{(1)}|}} \quad \frac{2.5}{\sqrt{|\alpha^{(3)}|}} \quad \frac{2.7}{\sqrt{|\alpha_{\phi e}|}},
\]

where \(\Lambda_{\text{TeV}}\) is the scale of new physics in TeV units.

Similarly the contributions to the \( \rho \) parameter arise from \( O^{(3)}_\phi \), explicitly

\[
|\delta T| = \frac{4\pi}{s_w^2} |\alpha^{(3)}| \frac{v^2}{\Lambda^2} \lesssim 0.4
\]

This bound [9] implies \(\Lambda_{\text{TeV}} \gtrsim 1.7/\sqrt{|\alpha^{(3)}|}\) (at \(3\sigma\)).

The Fermi constant receives contributions from \( O^{(3)}_{\phi e}, O^{(1)}_{\phi} \) and form the four fermion operator \((\bar{\ell}\gamma^\mu \tau^I \ell)^2/2.\) We will use \( G_F \), the fine structure constant and the \( Z \) mass as our input parameters, then the effects of the \( O \) on \( G_F \) and \( m_Z \) are observed in deviations of the \( W \) mass form its Standard Model prediction. None of the high precision measurements constrain \(\alpha^{(1)}_{\phi} \) since, without direct observation of the Higgs, the tree-level effects of this operator are absorbed in the wave function renormalization of the scalar doublet.

The excellence of the above constraints has been used to claim that the possibilities of observing new physics effects at LEP2 are greatly diminished (if not absent), having been preempted by LEP1 [10]. While this is true for all effects relating to the coupling of the fermions to the gauge bosons, it is not so for the couplings of the Higgs to the other fields. It is precisely on these effects that we will concentrate upon in the following. It will be
assumed that the Higgs will be detected and studied and we will use the high precision measurements expected from LEP2 to constrain physics beyond the Standard Model by observation of reactions where the Higgs is produced.

We will concentrate on two reactions, namely

\[ e^+ e^- \rightarrow Z H \quad e^+ e^- \rightarrow H \bar{\nu} \nu \]  \hspace{1cm} (6)

At LEP2 the second reaction is dominated by the Bjorken process (followed by \( Z \rightarrow \bar{\nu} \nu \)), at higher energies \( W \) fusion will dominate (at least within the \( R_\xi \) gauges); we will include both sets of graphs.

Using the expression (2), with \( O_i \) defined in (1), we calculate the relevant amplitude for \( e^+ e^- \rightarrow Z H \); the Feynman graphs are given in figure 1. The Feynman rules are extracted as usual from (2), the resulting amplitude squared is

\[ |A|^2 = \frac{1}{4} \left( |a_L|^2 + |a_R|^2 \right) \left[ s + \frac{(t - m_H^2)(u - m_H^2)}{m_Z^2} \right] \]  \hspace{1cm} (7)

where

\[ a_L = \frac{2g}{v c_w} \frac{m_Z^2}{s - m_Z^2} \left[ \frac{2s_w^2 - 1}{2} \left( 1 + \delta_Z \right) + \delta \epsilon_L \frac{s}{m_Z^2} \right] \]

\[ a_R = \frac{2g}{v c_w} \frac{m_Z^2}{s - m_Z^2} \left[ s_w^2 \left( 1 + \delta_Z \right) + \delta \epsilon_R \frac{s}{m_Z^2} \right] \]  \hspace{1cm} (8)

and

\[ \delta_Z = \frac{v^2}{2\Lambda^2} \left( \alpha^{(1)}_\phi + \alpha^{(3)}_\phi \right) - 2\alpha_\phi \delta \]

\[ \delta \epsilon_L = -\frac{v^2}{2\Lambda^2} \left( \alpha^{(1)}_\phi \delta \ell + \alpha^{(3)}_\phi \right) \]

\[ \delta \epsilon_R = -\frac{v^2}{2\Lambda^2} \alpha_{\phi e} \]  \hspace{1cm} (9)

where \( s, t \) and \( u \) stand for the usual Mandelstam variables. As mentioned above, we will use the Fermi constant \( G_F \), the \( Z \) mass \( m_Z \) and the fine structure constant as input parameters; all other quantities are to be expressed in
terms of these numbers. We used $c_w = g/\sqrt{g^2 + g'^2}$ and $s_w = g'/\sqrt{g^2 + g'^2}$; the vacuum expectation value $v$ denotes the minimum of the scalar potential and receives a modification due to the presence of $O_\phi$. The mass $m_z$ denotes the physical mass of the $Z$ particle.

![Feynman graphs](image)

Fig. 1 Feynman graphs contributing to the process $e^+e^- \rightarrow ZH$.

Similarly we evaluate the amplitude for the process $e^+e^- \rightarrow H\bar{\nu}\nu$. The Feynman graphs are given in figure 2. The resulting amplitude squared is

$$|A|^2 = 8|X_v|^2 (p_2' \cdot p_1) (p_2 \cdot p_1') + 2|X_s|^2 (p_2' \cdot p_1') (p_2 \cdot p_1)$$

(10)

where

$$X_v = \frac{g^2}{v} \frac{m_w^2}{(q_1^2 - m_w^2)(q_2^2 - m_w^2)} \left[ 1 + \delta_W + \frac{q_1^2 + q_2^2}{m_w^2} \delta g_L \right]$$

$$+ \frac{g^2 - g'^2}{2v} \frac{m_z^2}{(k_1^2 - m_z^2)(k_2 - m_z^2)} \left[ 1 + \delta_Z + \frac{2\delta_{\nu_L} k_2^2 - (2/c_w) \delta \epsilon_L k_1^2}{m_z^2} \right]$$

$$X_s = \frac{2g'^2}{v} \frac{m_z^2}{(k_1^2 - m_z^2)(k_2 - m_z^2)} \left[ 1 + \delta_Z + \frac{2\delta_{\nu_L} k_2^2 + (1/s_w) \delta \epsilon_L k_1^2}{m_z^2} \right]$$

(11)

In the above

$$\delta g_L = \frac{v^2}{\Lambda^2} \alpha_{\phi\ell}^{(3)}$$

$$\delta_{\nu_L} = \frac{v^2}{2\Lambda^2} \left( \alpha_{\phi\ell}^{(3)} - \alpha_{\phi\ell}^{(1)} \right)$$

$$\delta_W = -\frac{v^2}{2\Lambda^2} \left( 2\alpha_{\phi\phi} - \alpha_{\phi}^{(1)} + \alpha_{\phi}^{(3)} \right);$$

(12)
the other quantities were defined previously. The momentum assignments are indicated in figure 2; $p_{1,2}$ are incoming while $p'_{1,2}$ are outgoing.

![Feynman graphs](image)

Fig. 2 Feynman graphs contributing to the process $e^+e^- \rightarrow H\bar{\nu}\nu$.

Note that the corrections to the Standard Model expressions are of two types. There are small modifications to the coefficients and, more importantly, there are terms which grow with the momenta and which appear to violate unitarity for sufficiently large $s$. This is, of course, only a signal that the approximations used break down for sufficiently large energy; this problem is never encountered since (9) is only valid for $s \ll \Lambda^2$. Nonetheless the presence of these terms generates the most significant deviations from the Standard Model. This effect is reminiscent of the delayed unitarity effects [11] which, though not dramatic, do provide non-trivial sensitivity into new physics effects.

The results for the total cross sections for the two processes are given in figure 3. In presenting these results we chose values for $\alpha_{\phi_1}^{(1,3)}$, $\alpha_{\phi_2}$ and $\alpha_{\phi_3}$ which saturate the $(3\sigma)$ bounds derived from (4,5); the signs were chosen to maximize the effects. The rest of the $\alpha_i$ were chosen equal to one. We also restricted the range of $s$ to the point where the deviations from the Standard Model generated by the dimension 6 operators are $\sim 100\%$, at which point the approximations used break down.
Fig. 3 Total cross sections for the process $e^+e^- \rightarrow H\bar{\nu}\nu$ and $e^+e^- \rightarrow ZH$. The values for the constants $\alpha_i$ and other restrictions are specified in the text. The solid lines correspond to the Standard Model predictions whereas the dotted ones represent the effective lagrangian results.

In evaluating the cross section we have expressed the weak-mixing angle, the vacuum expectation value, etc. in terms of the above-mentioned input parameters. The expressions are cumbersome and will not be given explicitly here; they can be extracted from the results presented in Ref. [6].

It is clear from the plots in figure 3 that the new physics contributions to the $e^+e^- \rightarrow H\bar{\nu}\nu$ process are unobservable. In contrast the deviations from the Standard Model for the reaction $e^+e^- \rightarrow ZH$ can be quite significant. To illustrate the implications of this result we calculated the statistical
significance $N_{SD}$ of the deviations from the Standard Model for the Bjorken process. This quantity is defined by

$$N_{SD} = \frac{|\sigma - \sigma_{SM}| L}{\sqrt{\sigma L}}$$

(13)

where $\sigma$ and $\sigma_{SM}$ denote the total and Standard Model cross sections respectively, and $L$ denotes the luminosity. The results are presented in figure 4.

![Graph showing statistical significance of new physics effects for the process $e^+e^- \rightarrow ZH$.](image)

**Fig. 4** Statistical significance of the new physics effects for the process $e^+e^- \rightarrow ZH$. The non-monotonic behavior of the curves is due to our having chosen the $\alpha_i$ which saturate the existing bounds (at the 3$\sigma$ level) obtained in (4,5), see the text for details.

As above, we have chosen the $\alpha_i$ which saturate the bounds (4,5) at the 3$\sigma$ level. The results in figure 4 then give the maximum possible value of $N_{SD}$ to be observed at LEP2 which is also consistent with the existing measurements. As can be seen from this figure, the sensitivity of LEP2 reaches several TeV.
when this process is considered. The situation is further improved for a Next Linear Collider of 500GeV CM energy and 20 fb\(^{-1}\) luminosity.

Finally we compute the sensitivity to the scale of new physics \(\Lambda\) as a function of the energy of the \(e^+e^-\) collider used to probe this type of new physics. We present the various discovery limit contours in figure 5.

\[\Lambda(\text{TeV})\]

\[\sqrt{s}\ (\text{TeV})\]

\[N_{SB}=0.5\]

\[N_{SB}=3.0\]

\[N_{SB}=5.0\]

Fig. 5 Sensitivity to the scale of new physics \(\Lambda\) for a given CM energy \(\sqrt{s}\) using the reaction \(e^+e^- \rightarrow ZH\). Denoting by \(L\) the luminosity, the curves correspond to \(m_H = 70\text{GeV}\), \(L = 0.5\text{ fb}^{-1}\): solid line; \(m_H = 150\text{GeV}\), \(L = 20\text{ fb}^{-1}\): dashed line; \(m_H = 500\text{GeV}\), \(L = 20\text{ fb}^{-1}\): dotted line. We chose \(\alpha_i = 1\) (for which the current 3\(\sigma\) bounds imply \(\Lambda \gtrsim 2.7\text{TeV}\)).

The above results demonstrate a significant sensitivity of near-future accelerators to new physics effects. The approach presented here, applicable to the case where there is a light Higgs excitation, is based on the segregation of those new-physics effects that can occur at tree level. In case the careful examination of this process (and others similar to it) provides no hint of
deviations from the Standard Model it will be necessary to conclude that the corresponding operators are suppressed in the underlying theory. This is a non-trivial statement that will eliminate several types of interactions. For example, the absence of a deviations in the $eeZ$ vertex significantly constrains the mixings and masses of a new neutral vector boson and also the mixings of the electron with new heavy leptons. Since the general form of the interactions that give rise to the operators (1) is known [7], even the absence of deviations from the Standard Model can be translated into useful information: the various contributions to the effective operators must either be suppressed or cancelations must be present.

It is interesting to provide some models which can generate the operators studied in this paper. One can consider, for example the addition to the Standard Model of a vector-like fermion $\Psi^I$ which is an $SU(2)_L$ triplet and a $U(1)_Y$ singlet; in this case $\alpha^{(1,3)}_{\ell\ell}$ will be non-zero and proportional to the $\bar{\ell}\tau^I\Psi^I\bar{\phi}$ coupling constant. The same operators are generated by a heavy $Z'$ exchange; in this case $\Lambda$ corresponds to the vacuum expectation value in the new $U(1)$ and the Standard Model scalar doublet and leptons are assumed to have non-zero quantum numbers in the new $U(1)$ group. We also note that the MSSM [12] does not generate the operators under consideration (unless R-parity violations are included). The observation of strong deviations from the Standard Model in the above processes would then argue against this model.

It is worth noticing that at high energy colliders the $H\bar{\nu}\nu$ production channel, having much bigger cross section than the $ZH$ one, can be used to measure $H$ properties (e.g. $m_H$) while the $ZH$ should be utilized to observe deviations from the Standard Model prediction. Therefore the both processes considered here would be needed and are in fact complementary: $H\bar{\nu}\nu$ having very small corrections, but providing large production rate and $ZH$ with it’s substantial sensitivity to non-standard physics.

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