Two-flavor color superconductivity in the Fierz-transformed Nambu–Jona-Lasinio model

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The color superconducting phase of two-flavor quark matter is studied under a Fierz-transformed Lagrangian. In the Fierz-transformed Lagrangian both the quark-antiquark and quark-quark channel are included. Two parameters \(\alpha\) and \(\beta\) are introduced to weight the Fierz-transformed quark-antiquark Lagrangian and quark-quark Lagrangian, respectively. The couplings of different interaction channel are thus fixed by the two coefficients other than five free parameters. The interplay between chiral symmetry restoration and the formation of color superconducting phases are discussed. It is found that the increase of \(\alpha\) leads the chiral phase transition from first order to crossover. The calculated specific heat shows jump at critical temperature which confirms the phase transition. The increase of \(\beta\) leads the pseudo-critical chemical potential reduce. With large \(\beta\) the superconductors gap \(\Delta\) can be larger than the typical value and the well-known BCS relation between critical temperature and gap is well fitted.

I. INTRODUCTION

Investigation of deconfined quark matter is a hot issue in the study of the strong interaction. The quark-gluon plasma (QGP) which can be reproduced by relativistic heavy ion collisions (RHICs) experiment \textsuperscript{[1, 2]} may exists as gluons deconfine in the adjoint representation at high temperature. At low temperature and less baryon density, hadrons and mesons are confined that all states are color singlets. The phase transition from hadronic matter to the QGP is a broad crossover at zero baryon chemical potential. Although the transition property at high densities is still debated, but it is widely expected to be first-order \textsuperscript{[3, 4]} and evidences in study of the EOS of massive compact stars show that quark-matter cores may exist in the stars and it could be a first order phase transition \textsuperscript{[3, 4]}.

The compact star mass and radius heavily depended on the quark core mass and radius. But the regime of low temperatures and moderate densities is not accessible by perturbative QCD up to now, and the QCD critical point is uncertain. There are so many uncertainties in connecting the hadron phase and quark phases. In the schematic phase diagram, there is a possible phase transition between the nuclear superfluid and color superconductors \textsuperscript{[5, 9]} at sufficiently large densities and low temperatures. Many kinds of pairing patterns and condensate exist, such as meson, diquark and four-quark condensate \textsuperscript{[10, 14]}. In-depth research on hadron and quark color super-conductance and super-fluidity, which provide useful information on the phase transition \textsuperscript{[15, 16]} and viscous and diffusive effects near the QCD critical point \textsuperscript{[17, 20]}, could have potential applications to heavy ion collisions and astrophysics of neutron stars \textsuperscript{[21, 25]}. For the quark matter, quark condensate plays a key role in quark mass formation and phase transitions. With the spontaneous chiral symmetry breaking the quark-antiquark condensates provide the main contribution of the effective mass. In analogy with Cooper pairs of electrons with opposite momentum and opposite spin, cold deconfined quarks could become paired to form color superconductor with diquark condensate. To study color superconductor it can be started by applying the color current interaction \(-g(\bar{\psi}\gamma_{\mu}\lambda_{a}\psi)^{2}\) directly, with a free coupling \(g\) which is the common practice. After taking the mean field approximation, only the scalar current \(-g(\bar{\psi}\gamma_{0}\lambda_{a}\psi)^{2}\) remains. To incorporate into more interaction channel it always resorts to the Fierz transformation of the original colore current interaction \textsuperscript{[10, 20, 27]}. On the opposite side, the standard NJL Lagrangian contains only scalar and pseudo-scalar interactions but without the colore current interaction. However, with the Fierz transformation we can produce color current interaction and the vector interaction with which the chemical potential needs to be revised. The Fierz transformation is a mathematically equal description of the original Lagrangian which means the original Lagrangian and its Fierz transformation should be all contained in the complete Lagrangian with appropriate weight. Besides the color current interaction, with the rearrangement of the position of quark and antiquark, the Fierz transformed Lagrangian contains an additional scalar term which will modify the contribution of the scalar four-point interaction in the standard Lagrangian. The introduce of weighting parameters of the original Lagrangian and its Fierz transformations is not equal to redefine the couplings of variant interaction channel. In our previous paper, the introduction of weighting parameter seems mathematically equal to a redefinition of the coupling of vector channel interaction \textsuperscript{[28, 31]}. In Ref. \textsuperscript{[30]}, the modification to the effective chemical potential is written as \(\mu_{r} = \mu - \alpha G' / |N_{c}(1 - \alpha) + \alpha|\) where \(\alpha\) is the weighing parameter, \(G'\) is determined by the low energy experiment data and \(n\) is the vector condensate. If the original Lagrangian has a vector channel interaction \(\bar{\psi}\gamma_{\mu}\lambda_{a}\psi\) and \(\bar{\psi}\gamma_{\mu}\lambda_{a}\psi\) but \(\bar{\psi}\gamma_{\mu}\lambda_{a}\psi\) or \(\bar{\psi}\gamma_{\mu}\lambda_{a}\psi\) then the weight parameter would be \(\alpha\) to redefine the coupling of \(\bar{\psi}\gamma_{\mu}\lambda_{a}\psi\).
in the form of \(-G_V(q\gamma_0q)^2\), the effective chemical potential is \(\mu_r = \mu - 2G_V n\). As compared the expressions of \(\mu_r\) and \(\mu_r'\), it is easy to see that the adjusting of weighting parameters \(\alpha\) is equal to adjust the vector coupling \(G_V\). But this kind of equivalence is a result that we have neglected the color interaction terms \(G(q\gamma_0\lambda_0q)^2\) in the Fierz-transformed Lagrangian for a color neutral system. Also we will show latter that the Fierz transformation gives a fixed ratio of couplings for some interaction channels which is different from any other works.

In this paper, we study the quark-antiquark and diquark condensate from a Lagrangian that contains standard NJL Lagrangian and its Fierz transformation with quark-quark and quark-antiquark interactions to investigate their competition and mutual influence. To keep a global antisymmetry of the wave function, the diquark condensates can involve two colored quarks with spin-0 or with spin-1 in a single color. The estimated gap of the vector channel is much smaller than the scalar channel \([32]\). In this paper we focus on the two-flavor superconductor phase is to consider color current interaction with introduction of color current coupling or color superconductor phase is to consider color current coupling or with spin-1 for a single color. The estimated gap of the diquark condensates can involve two colored quarks with spin-0 or with spin-1.

II. MODEL

The standard NJL Lagrangian with four point interaction is

\[
\mathcal{L} = \bar{\psi}(i\beta - m)\psi + g \left\{ (\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5\vec{	au}\psi)^2 \right\}.
\] (1)

Under the mean field approximation, only the scalar channel remains which governs the chiral restoration. When considering the contribution of the vector channel, the usual method is to introduce a vector coupling constant. Similarly, the conventional way to study the color superconductor phase is to consider color current interaction with introduction of color current coupling or the color electric and magnetic coupling constants. But all these terms exist in the Fierz transformation of the standard NJL Lagrangian. Taking into account the anticommutation rules for fermions, the four Fermi interaction

\[
\mathcal{L}_{\text{int}} = g_I(q\bar{q}q)^2 = g_I\Gamma_{ij}\Gamma_{kl}\bar{q}_i\bar{q}_j\bar{q}_k\bar{q}_l
\] (2)

change to

\[
\mathcal{L}_{\text{ex}} = -g_I\Gamma_{il}\Gamma_{kj}\bar{q}_i\bar{q}_j\bar{q}_k\bar{q}_l
\] (3)

and

\[
\mathcal{L}_{\text{qq}} = g_I\Gamma_{ij}\Gamma_{kl}\bar{q}_i\bar{q}_k\bar{q}_j\bar{q}_l.
\] (4)

The three interactions, \(\mathcal{L}_{\text{int}}, \mathcal{L}_{\text{ex}}\) and \(\mathcal{L}_{\text{qq}}\), are mathematically equal, but apparently the contributions of different interaction channels vary under the mean field approximation.

In order to make the Lagrangian under the mean field approximation as close as possible to the original interaction, we take the following Lagrangian

\[
\mathcal{L} = \bar{\psi}(i\beta - m)\psi + (1 - \alpha - \beta)\mathcal{L}_{\text{int}} + \alpha\mathcal{L}_{\text{ex}} + \beta\mathcal{L}_{\text{qq}}. 
\] (5)

In this way, the vector channel and color current interactions are naturally included. Although the interactions are still included arbitrary numbers, the meanings are different. The \(\alpha\) and \(\beta\) weight the contribution from different interactions and set up constraints on the vector channel and diquark channel. Different from models that staring from the color current interactions, the coupling ratio \(G_S/H_S\) of quark-antiquark and diquark is no longer a constant \([16]\).

A. The Fierz transformations

The Fierz transformation is under spinor, flavor and color spaces. The exchange diagrams lead to \(\mathcal{L}_{\text{ex}}\) for the \(\bar{q}q\) interactions and for the scalar and pseudo-scalar channel we have operators of the interactions

\[
[s]_{\alpha\beta',\alpha'\beta} = \frac{1}{4} \left[ s_p - p + v - a + \frac{t}{2} \right]_{\alpha\beta',\alpha'\beta'},
\]

\[
[p]_{\alpha\beta',\alpha'\beta} = \frac{1}{4} \left[ -s_p + p + v - a - \frac{t}{2} \right]_{\alpha\beta',\alpha'\beta'},
\] (6)

where

\[
s_{\alpha\beta,\alpha'\beta'} = 1_{\alpha\beta}, l_{\alpha'\beta'},
\]

\[
p_{\alpha\beta,\alpha'\beta'} = (\gamma_\alpha)_{\alpha\beta}, (\gamma_\beta)_{\alpha'\beta'}.
\]

\[
u_{\alpha\beta,\alpha'\beta'} = (\gamma_\mu)_{\alpha\beta}, (\gamma_\nu)_{\alpha'\beta'}.
\]

\[
a_{\alpha\beta,\alpha'\beta'} = (\gamma_\mu\gamma_5)_{\alpha\beta}, (\gamma_\nu\gamma_5)_{\alpha'\beta'}.
\]

\[
t_{\alpha\beta,\alpha'\beta'} = (\sigma^{\mu\nu})_{\alpha\beta}, (\sigma^{\mu\nu})_{\alpha'\beta'}.
\]

In the quark-antiquark channel, the transformation matrix in the \(U(N)\) symmetry is

\[
\begin{pmatrix}
\frac{1}{N-1} & \frac{1}{N} & \frac{1}{N} \\
\frac{1}{N} & \frac{1}{N-1} & \frac{1}{N} \\
\frac{1}{N} & \frac{1}{N} & \frac{1}{N-1}
\end{pmatrix}
\] (8)

with \(a = 1, 2, \ldots, N^2 - 1\). So in the flavor space with \(N = 2\)

\[
1 \cdot 1 \rightarrow \frac{1}{2} 1 \cdot 1 + \frac{1}{2} \tau \cdot \tau,
\] (9)

\[
\tau \cdot \tau \rightarrow \frac{1}{2} 1 \cdot 1 - \frac{1}{2} \tau \cdot \tau.
\] (10)

Here all the subscripts are neglected for simplification. And in the color space with \(N = 3\)

\[
\lambda_0 \cdot \lambda_0 \rightarrow \frac{1}{3} \lambda_0 \cdot \lambda_0 + \frac{1}{2} \lambda \cdot \lambda.
\] (11)
Here, only transformation of the color single was considered and more details can refer to the appendix of [16]. A full transformation under spinor, flavor and color spaces results in
\[
\frac{1}{4}(s - p + v - a + \frac{1}{2}t) \otimes (\frac{1}{2}1 \cdot 1 + \frac{1}{2}\gamma \cdot \tau) \otimes (\frac{1}{2}\lambda_0 \cdot \lambda_0 + \frac{1}{2}\lambda \cdot \lambda) \frac{1}{2}[-s + p + v - a - \frac{1}{2}t] \otimes (\frac{1}{2}1 \cdot 1 - \frac{1}{2}\gamma \cdot \tau) \otimes (\frac{1}{2}\lambda_0 \cdot \lambda_0 + \frac{1}{2}\lambda \cdot \lambda).
\]
(12)

In the quark-quark channel, the transformation matrix in the \( U(N) \) symmetry is
\[
\begin{bmatrix}
(1)_{ij}(1)_{kl} \\
(\tau_a)_{ij}(\tau_a)_{kl}
\end{bmatrix} = \left( \frac{\Delta}{N} \right)^2 \left( \frac{\Delta}{N} \right)^{k+l} \left( \begin{array}{cc}
(\tau_S)_{ik}(\tau_S)_{lj} \\
(\tau_A)_{ik}(\tau_A)_{lj}
\end{array} \right).
\]
(13)

Here, the subscripts \( S \) and \( A \) to show that the generators are symmetry or anti-symmetry under transposition. The total transformation to quark-quark channel is
\[
\frac{1}{4}[s - p + v - a - \frac{1}{2}t] \otimes (\frac{1}{2}1 \cdot 1 + \frac{1}{2}\gamma \cdot \tau) \\
\otimes (\frac{1}{2}\lambda_S \cdot \lambda_S + \frac{1}{2}\lambda_A \cdot \lambda_A) + \frac{1}{2}[s - p + v] \\
\otimes (\frac{1}{2}1 \cdot 1 - \frac{1}{2}\gamma \cdot \tau) \otimes (\frac{1}{2}\lambda_S \cdot \lambda_S + \frac{1}{2}\lambda_A \cdot \lambda_A).
\]
(14)

The most important diquark channel are the \( q \bar{q} \gamma_5 C r_2 \lambda_A q \) and \( q \gamma_5 r_2 \lambda_A q \). But in Eq. [14], the colored vector channel is cancelled out. Then we have only the colored scalar channel which read as
\[
\frac{1}{2} \lambda \cdot \lambda.
\]
(15)

Under the mean field approximation, only the scalar, vector and diquark channel remain which are listed here with
\[
\mathcal{L}_{nt} = g(q \bar{q})^2,
\]
(16)
\[
\mathcal{L}_{ex} = \frac{g}{12}((q \bar{q})^2 - 2g \bar{q} q g) + \frac{g}{8}((q \bar{q} g)^2 - 2g \bar{q} q g),
\]
(17)
\[
\mathcal{L}_{qq} = \frac{g}{8}((q \gamma_5 C r_2 \lambda_A q \bar{q} \gamma_5 C r_2 \lambda_A q).
\]
(18)

Considering the weighting parameters in [18], the effective couplings for different interaction channels are
\[
G_4^{(0)} = (1 - \alpha - \beta)g, \quad G_8^{(0)} = \frac{\alpha}{8}g,
\]
(19)
\[
G_8^{(8)} = \frac{-\alpha}{6}g, \quad G_9^{(8)} = \frac{-\alpha}{4}g, \quad H = \frac{\beta}{8}g.
\]
(20)

The presence of \( \delta \) brakes the color-\( SU(3) \), and only the red and green quarks participate in the condensate. The quark condensates of red and blue quarks are expected to be different. Introducing the colored quark mass and color chemical potential
\[
M_r = M_0 + \frac{1}{\sqrt{3}}M_8, \quad M_b = M_0 - \frac{2}{\sqrt{3}}M_8,
\]
(27)
\[
\phi_8 = \frac{2}{\sqrt{3}}(\phi_r - \phi_b), \quad \mu_8 = \frac{2}{\sqrt{3}}(\mu_r - \mu_b),
\]
(28)

we have
\[
M_r = m - \frac{2}{3}(6G_0^{(0)} + 2G_8^{(8)})\phi_r - \frac{2}{3}(3G_8^{(0)} - 2G_8^{(8)})\phi_b,
\]
(29)
\[
M_b = m - \frac{2}{3}(6G_0^{(0)} - 4G_8^{(8)})\phi_r - \frac{2}{3}(3G_8^{(0)} - 4G_8^{(8)})\phi_b.
\]
The thermodynamic potential by evaluating the trace and performing the Matsubara sum at nonzero temperature is given by

\[
\Omega(T, \mu) = -4 \int \frac{d^3p}{(2\pi)^3} \left\{ 2 \left( \frac{\omega_- + \omega_+}{2} + T \ln \left( 1 + e^{-\omega_-/T} \right) + T \ln \left( 1 + e^{-\omega_+/T} \right) + E_{p,b} + T \ln \left( 1 + e^{-E_{p,b}/T} \right) \right) \right\} + V + \text{const.}
\]

Here, \( E_{p,b} = \sqrt{p^2 + M_b^2} \) and the dispersion laws for colored quarks read as

\[
\omega_\pm = \sqrt{\left( \sqrt{p^2 + M_F^2} \pm \mu \right)^2 + |\Delta|^2}, \tag{31}
\]

which produces gap of \( 2\Delta \) for the coupled quark in the particle-hole excitation spectrum.

The stationary points of the thermodynamic potential give the expectation values of condensates which are

\[
\phi_r = -4 \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{M_r s - \tilde{\mu}_r t}{2\omega_-} \tanh \left( \frac{\omega_-}{2T} \right) + \frac{M_r s + \tilde{\mu}_r t}{2\omega_+} \tanh \left( \frac{\omega_+}{2T} \right) \right\}, \tag{32}
\]

\[
\phi_b = -4 \int \frac{d^3p}{(2\pi)^3} \frac{M_b}{E_{p,b}} \left( \frac{1 - n_{p,b}(T, \tilde{\mu}_b)}{2\omega_-} \tanh \left( \frac{\omega_-}{2T} \right) - n_{\bar{p},b}(T, \tilde{\mu}_b) \right) \tag{33}
\]

\[
n_r = 4 \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{\tilde{\mu}_r (s - \tilde{p}^2)}{2\omega_-} \tanh \left( \frac{\omega_-}{2T} \right) + \frac{\tilde{\mu}_r (s + \tilde{p}^2) + M_r t}{2\omega_+} \tanh \left( \frac{\omega_+}{2T} \right) \right\}, \tag{34}
\]

\[
n_b = 4 \int \frac{d^3p}{(2\pi)^3} \left( n_{p,b}(T, \tilde{\mu}_b) - n_{\bar{p},b}(T, \tilde{\mu}_b) \right) \tag{35}
\]

\[
\delta = -8 \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{\Delta s - \Delta \nu t}{2\omega_-} \tanh \left( \frac{\omega_-}{2T} \right) + \frac{\Delta s - \Delta \nu t}{2\omega_+} \tanh \left( \frac{\omega_+}{2T} \right) \right\}. \tag{36}
\]

The blue quark is unpaired, and here \( n_{p,b} \) and \( n_{\bar{p},b} \) are the usual Fermi occupation functions for the blue quarks and antiquarks. With the parameters given before, these self-consistent equations give the quark mass about 267 MeV at zero temperature and zero chemical potential.

### III. RESULTS

#### A. The chiral phase transition

With the given parameters under three-momentum cut-off regularization, the chiral phase transition is first order under low temperature and crossover under high temperature. The quark mass as function of chemical potential and temperature is presented in Fig. 1 with equal weight for the three parts of interaction.

![FIG. 1: Contour plot of quark mass as function of chemical potential and temperature of equal weights when \( \alpha = \beta = 1/3 \).](image)

The first order transition will turn to crossover as the the vector channel is more involved with large \( \alpha \), as shown in the upper panel of Fig. 2. More discussions on the effect of the vector channel can be found in previous works [28–30].

The first order phase transition still remains as we increase the weight of diquark interaction channel. As \( \beta \) increases the pseudo-critical chemical potential decreases. In the lower panel of Fig. 2, the obvious change occurs when \( \beta \) is larger than 0.8. When \( \alpha = 0 \) and \( \beta = 0.5 \) the ratio of coupling strength of the scalar quark-antiquark channel and diquark channel is \( G_s^{(0)} : H = 8 : 1 \) and thus the diquark channel heavily suppressed. So to manifest the colored diquark effect on the phase transition, the weight \( \beta \) must be larger than that.

#### B. Mass and quark number difference

The Fierz-transformed Lagrangian \( L_{\text{ex}} \) has included both the scalar channel and vector channel, the increase of \( \alpha \) will also increase the scalar interaction. We can study the competition between scalar channel and vector channel by fixing \( \beta \). In order to reflect the competition between vector channel and colored diquark interaction channel more clearly, we fix the weight of direct interaction term \( L_{\text{int}} \) and study the results under different \( \alpha \) and \( \beta \).
The 2SC model does not guarantee that the color neutrality is fulfilled. The red and blue quark have different effective masses and number densities as the gap appears to be nonzero.

At low chemical potential the mass and quark number density are equal for red and blue quark but increase rapidly when the chemical potential is larger than some specific values, as shown in Fig. 3. The mass difference is less than 1 MeV. Since the chiral symmetry is partly restored at high chemical potential, the mass difference drops to almost zero. The difference of number density increases as $\beta$ increases. We can also see in Fig. 3 that the difference of number density decreases at high chemical potential for $\beta = 0.8$. It may indicates that the rate of formation of quark pairs should be reduced at high chemical potential. At more large chemical potential the model may become sensitive to the cut-off and finally it reaches the limits of the model.

C. Gap and specific heat

A direct manifestation of color super conduct of super fluid is the appearance of a non-negligible gap in the dispersion law. The gap $\Delta$ varies with temperature and chemical potential. The calculated results are presented in Fig. 4 and 5. The gap increases with chemical potential and $\beta$, and only for relative large $\beta$ the interactions give rise to a non-negligible gap. Since the Lagrangian is dominated by the Fierz-transformed diquark part, the gap $\Delta$ can be larger than 100 MeV. Critical chemical potential exists in the Fig. 4 and before critical point the gap $\Delta$ is zero.

The diquark could be de-paired as temperature increases just the same as in the BSC theory for electrons.
The diquark condensate vanishes and the gap decreases to zero as temperature increases. There is a well-known BCS relation that

\[ T_c \simeq 0.57 \Delta_0, \]  

where \( \Delta_0 \) is the gap at zero temperature and \( T_c \) is the temperature where gap begin to be zero [34]. We plot the \( \Delta/\Delta_0 \sim T \) relation calculated at \( \mu = 400 \) MeV in Fig. [3]. At various parameter set, the critical temperature = \( T_c \) is located between 0.5 and 0.6.

![Fig. 5: Diquark condensate as function of temperature at \( \mu = 400 \) MeV.](image)

![Fig. 6: Normalized condensates as function of temperature. \( \Delta_0 \) is the gap at zero temperature.](image)

Furthermore, we calculate the specific heat as function of temperature. At fixed chemical potential, the specific heat is given by

\[ c_v = -T \frac{\partial^2 \Omega}{\partial T^2}. \]  

We show the results in Fig. [4] The specific heat increases from zero at low temperature. When the temperature continues to increase the specific heat has a jump-down at large chemical potential and \( \beta \). The jump-down occurs at the critical temperature. At fixed chemical potential, the critical temperature increases with \( \beta \).

![Fig. 7: Specific heat as function of temperature at fixed chemical potential.](image)

### IV. SUMMARY

Without performing mean field approximation, the original Lagrangian and its Fierz transformation are numerically identical. The Fierz-transformed Lagrangian contains a variety of interaction channel. But at the mean field level, only some specific terms remain. We take the idea that all these equal interaction should be included in one combined Lagrangian with different weights.

In this paper, starting from the standard NJL Lagrangian with only scalar and pseudo scalar interaction other than from the color current interactions, we have firstly composed Lagrangian with its Fierz transformation. The five coupling constants are replaced by two weighting parameters. The Fierz-transformed Lagrangian contains both quark-antiquark interaction included the vector channel interaction and quark-quark interaction. In the quark-quark interaction we care about the scalar and pseudo-scalar channel, but only scalar channel remain after the fierz transformation.

The competition of the two transformed Lagrangian are studied. We have calculated the chiral phase transition. It shows that the vector channel and diquark channel interaction have opposite effect on the chiral transition as chemical potential increase. The vector channel interaction postpones the transition while the diquark interaction advances the chiral phase transition as the chemical potential increases.

We have also calculated the colored quark mass and number density difference, the superconductor gap and the specific heat. The mass difference of colored quark can be negligible. But critical chemical potential exits where the quark mass and number density difference begin to increase from zero and the gap \( \Delta \) also increases.
We have calculated the gap as function of temperature as specific chemical potential for different weight parameters. It is found that although the critical temperature and gaps are varied but the normalized gap as function of temperature are almost coincides. The ratios of critical temperature and the gap at zero temperature are located between 0.5 and 0.6 which are in consistent with the BCS relation with the ratio of 0.57.

In the end we confirm the phase transition to superconductor phase by the calculation of specific heat.

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