Fading Improves Connectivity in Vehicular Ad-hoc Networks
Samar Elaraby, Student Member, IEEE and Sherif M. Abueleinin

Abstract—Connectivity analysis is a crucial metric for network performance in vehicular ad-hoc networks (VANETs). Although VANET connectivity has been intensively studied and investigated under channel models without fading for their simplicity, the connectivity probability provided by these studies does not mimic the real-world scenarios that suffer from channel impairments. However, the connectivity probability was too challenging to be caught by a closed formula due to the emerging complexity associated with the randomness in a fading channel. In this paper, we precisely estimate the connectivity probability using a proposed graph-based algorithm. The results prove that Rayleigh-fading channels reinforce the connectivity compared to no-fading models at the same level of transmitting power and vehicle densities. Using simulations and analysis, we thoroughly investigate these outcomes and their causes providing closed formulas for single-link connectivity and an upper bound for vehicle connectivity under fading conditions. We also exploit the numeric results of the proposed algorithm to deduce an upper bound for the VANET connectivity probability under Rayleigh-fading channels.

Index Terms—Channel Fading, Rayleigh Fading, Highway VANETs, VANET Connectivity, Graph Theory.

I. INTRODUCTION

Intelligent transportation systems no longer belong to the future due to the recent advances in their key enablers, especially vehicle communications. The latter can provide a various number of applications ranging from safety and traffic alert dissemination to dynamic routing planning to entertainment, gaming and content sharing [1]–[3]. Furthermore, vehicle communication was envisioned as a technology accelerator for autonomous vehicles to hit the road [4]. Internet of vehicles (IoV) is another emerging technology that mainly relies on vehicular ad-hoc networks (VANETs) [5], [6]. Vehicle communications have been immensely developed pushing towards the release of new emerging standards [7]–[9]. Among the efforts, connectivity analysis has been well investigated in the literature as a crucial metric for network performance.

In VANETs, vehicles are supposed to communicate with each other in one- or multi-hop routes without the need to an infrastructure network. Therefore, VANETs are considered as mobile ad-hoc networks (MANETs) having their nodes arranged in predetermined one-dimensional (1D) roads. The connectivity analysis is, accordingly, simplified and influenced by traffic headway and vehicle mobility alongside the communication environment. Traffic headway is defined as the distance from the front bumper of one vehicle to the front bumper of its successive. While the exponential distribution is widely accepted to describe free-flow traffic headways, other theoretical and empirical models were suggested for better headway modeling under other traffic conditions [10]–[14].

Communication channel modeling has also a significant impact on connectivity analysis. In the literature, several studies relied on the fixed communication range model [13], [15]–[23], also known as the unit disc model [24]–[27], which neglects the effects of the channel. In such models, the communication range of any vehicle, within which the received signal-to-noise ratio (SNR) is kept higher than a certain threshold value, is restricted to a predefined circle with the vehicle in its center. Even though this assumption simplifies the computational complexity of connectivity analysis, it is understood to only convey the network average behavior [27]. Therefore, the unit disc model has been severely criticized in the literature [25], [27]. For example, [26] proved that the average node degree, which is the average number of vehicles connected to one vehicle, of the unit disc is nearly double its counterpart in log-normal fading models at higher vehicle densities.

A. Related Work

Different studies in the literature have considered connectivity analysis under channel impairments [23]–[32]. More specifically, [28] analyzes the effect of lognormal shadowing and Rayleigh fading on connectivity by finding an expression for the probability that two nodes are connected if the distance between them is less than the communication range. References [29]–[32] followed similar methodology, but considering additional fading models. Their method is accurate in describing the connectivity between any two successive vehicles, but it becomes challenging when analyzing network connectivity. The channel impairments can cause one node to be unreachable by its nearest neighbor while, at the same time, reachable by a node that is further away. This fact falsifies the connectivity analysis delivered in [31], where the network connectivity probability is defined as the probability that each two consecutive nodes are connected.

It is widely agreed that channel impairments, especially fading, would negatively impact the connectivity of VANETs as it does for regular communications. In addition to the deteriorated node degree in [26], the authors of [33] claimed that fading channels require higher transmission power or vehicle density to achieve the same level of connectivity as of the unit disc model. It is even explicitly stated in [34] that Rayleigh-fading channels have negative impacts on connectivity of ad-hoc networks, and there would be no improvement
in connectivity except by means of diversity. Moreover, the connectivity probability of VANETs deduced in [31] shows deteriorated connectivity in fading channels compared to the unit disc channel.

On the contrary, while the authors of [35] were investigating the impact of shadowing on connectivity, they argued that the higher the fading variance is, the higher the connectivity probability of MANETs becomes. Besides, [36] claims that fading increases the probability of long links yielding to improved connectivity in ad-hoc networks. A similar remark was observed under certain conditions in [37], [38] in static wireless ad-hoc networks (VANETs) that have their nodes uniformly distributed in a 2D space. To the best of our knowledge, the reported findings of the improved network connectivity in fading channels were only restricted to MANETs and static VANETs under certain assumptions.

These findings should be thoroughly assessed before they are generalized to VANETs due to the problem restrictions. Not only do vehicles constitute 1D queues, but also VANETs employ different headway distributions, and their vehicle densities, considering free-flow traffic, are much lower than node densities in other ad-hoc networks. In this paper, we tackle this issue by proposing a graph-based algorithm that can precisely capture the vehicular network behavior. Using analysis and simulations, we negate the claims of [26], [31], [33] and prove the improved connectivity in VANETs under Rayleigh-fading conditions associated with free-flow traffic.

B. Paper Organization

First, we define the connectivity problem in VANETs in Section II. Then, the VANET model is presented in Section III along with VANET connectivity in the unit disc model. Graph-based simulations are discussed in Section IV, while Section V provides a detailed analysis of the single-link and vehicle connectivity in a Rayleigh-fading channel. The network connectivity is then delivered in Section VI before the conclusions are presented in Section VII.

II. Instantaneous Connectivity in VANETs

The definition of network connectivity can vary according to applications if considered from a physical-layer perspective. The one considered in this paper copes with the extreme case, where all nodes should be connected altogether to form one whole mesh at any given time. In other words, each pair of nodes can exchange message packets through a single- or multi-hop route at any given time. This definition does not tolerate even partitioned networks, in which nodes are gathered in groups each of which has its nodes connected to each other but completely disconnected from the other groups. In broadcasting applications, for example, the delivery of broadcast messages to all the nodes on a certain road segment is then sustained with considerable delays. The connectivity analysis, therefore, assesses the probability that the network is connected on the physical layer. Although the connectivity would change with vehicle mobility over time, this paper, like [13], [16], [18]–[22], considers the instantaneous connectivity, which limits the analysis to a snapshot where all vehicles are considered fixed. The vehicle mobility and speed still influence the intervehicle spacing and vehicle density. Consequently, the instantaneous connectivity can estimate the minimum average vehicle density that maintains a certain level of connectivity.

The network connectivity is also influenced by communication channels between nodes. In an environment that is free of channel impairments, the signals are attenuated over the space according to the free-space propagation model. The wave propagations form circles of fixed received power around the transmitter, which is connected to the receivers located inside a circle of a certain threshold. When a multipath propagation is considered, one signal can encounter different paths between the transceivers. These copies of the signal that arrive with different time delays sum up constructively or destructively, and their random behavior is captured by statistical models. In this paper, we consider the Rayleigh-fading model which is valid when line-of-sight (LoS) propagation is absent. In this case, the circles of fixed received power render to irregular random contours around the transmitter that vary rapidly over time as expected in a fast-fading environment.

Fading has its negative implications on direct wireless links between any two certain nodes. Ad-hoc networks, however, can substitute direct links with multi-hop routes, and their connectivity definition gets beyond the connectivity between a single pair of nodes. We then consider different definitions of connectivity in VANETs:

Single-link connectivity is the probability that two nodes are connected with a direct link. Each vehicle that maintains a single link with another vehicle is called its linked neighbor. Two-side vehicle isolation is the probability that a vehicle has no linked neighbors at all. One-side vehicle isolation. Each vehicle has neighbors both in front of and behind it (see Fig. 5a), which are known as forward and backward neighbors, respectively. One-side vehicle isolation is then defined as the probability that a vehicle has no linked neighbors in a predefined side. Vehicle connectivity is the probability that a vehicle is not isolated. Thus it represents the complement event of vehicle isolation, and accordingly has one- and two-side definitions. Network connectivity, or simply connectivity, is the probability that all the nodes form one unpartitioned cluster.

In the following sections, we compare those definitions in both the unit disc and Rayleigh-fading models in order to better comprehend the effects of fading on connectivity.

III. VANET Model and Connectivity at Fixed Communication Range

In this paper, the considered VANET model is of a multi-lane segment of a highway with a length of \( L \). Due to vehicle sparseness on a highway, the vehicles, with a vehicle density \( \rho \), are free to choose their own speed. Thus, their movements are independent of each other and this behavior is captured in the free-flow traffic model [11].

A. Free-Flow Traffic Model

According to traffic theory, any sensor placed along a highway observes the arrival of vehicles as a Poisson process, as
long as the vehicles follow free-flow traffic. Consequently, the intervehicle spacings between every two consecutive vehicles were proved to be i.i.d. random variables with exponential distributions [39]. Let the intervehicle spacing between vehicle \(V_i\) and \(V_{i+1}\), where \(i = 1, 2, \ldots, N-1\), be a random variable \(X_i\) and its probability density function (PDF) be

\[
f_{X_i}(x) = \rho e^{-\rho x}, \quad x \geq 0
\]

Since the lane separation is extremely small compared to the intervehicle spacing along the road, the former is commonly neglected under the free-flow traffic conditions.

### B. Single-Link Connectivity at a Fixed Communication Range

The connectivity of our interest is the instantaneous connectivity captured as a snapshot of the network at any given moment. The instantaneous single-link connectivity between any two vehicles depends on the transmitting power, vehicle density, and communication channel between them [35]. We assume that the power transmitted from all vehicles is identical, which leads to omitting it from the analysis. Following the free-space propagation model, the received SNR can be defined as

\[
\gamma = \frac{\beta P_T}{d^\alpha} P_{\text{noise}}
\]

where \(P_T\) is the transmitted power, \(\beta\) is a constant associated with the path loss model, \(\alpha\) is the PLE, and \(P_{\text{noise}}\) is the noise power. At a certain PLE, if we consider a circle around the transmitter, the received power, and the SNR accordingly, will remain constant along this circle. The fixed communication range \(r\) can then be determined from Eq. (2) as the distance \(d\) that maintains a certain level of received SNR \(\Psi\).

While we have defined single-link connectivity in Section II to include the connectivity between any two nodes, we restrict it to the first successive node only in the unit disc model. Since a vehicle that is connected to its second nearest neighbor definitely has a link to its first successive neighbor, the connectivity to far neighbors contains redundant information. Therefore, direct links to farther neighbors are omitted in the probabilistic analysis. Any two consecutive vehicles are connected if each vehicle is located within the communication range of the other. Thus, the single-link connectivity becomes

\[
P_{SL}^{(1)} = P(X_i \leq r) = F_{X_i}(r) = 1 - e^{-\rho r}
\]

where the superscript of \(P_{SL}^{(1)}\) refers to the first successive neighbor restriction, and \(F_{X_i}(r)\) is the cumulative distribution function (CDF) of the intervehicle spacing [22].

### C. VANET Connectivity

The network connectivity, accordingly, is the probability that each vehicle is connected to its first successive neighbor. For a network of \(N\) vehicles, the network connectivity is conditioned by the existence of \(N-1\) links, each of which connects two different adjacent vehicles. Hence, the network probability can be evaluated as discussed in [22] by

\[
P_{UD} = \prod_{i=1}^{N-1} P(X_i \leq r) = (1 - e^{-\rho r})^{N-1}
\]

Since \(r\) is assumed to be fixed, the VANET connectivity depends on both vehicle density and the total number of vehicles. It can be inferred that the higher the average vehicle density, the higher the connectivity probability we can achieve.

### IV. Graph Theory Approach for Connectivity

A vehicular network can be represented as a graph, where its vertices are the vehicles and its edges are the connections between the vehicles if exist. Graphs are classified into weighted and unweighted graphs. For weighted graphs, each edge is associated with a weight representing a predetermined distance between the two vertices. In contrast, unweighted graphs suspend the edge weights and assign every edge to a weight of one. Graphs can also be categorized as directed and undirected graphs. In directed graphs, an edge between two nodes indicates that the connection is valid from one node to the other and not the other way around. In VANETs, the corresponding graph is assumed to be unweighted. The communication channel between any two vehicles is considered identical in both ways under the unit disc and Rayleigh-fading channels. Therefore, the undirected graphs are found to best represent the considered scenarios.

Graphs can be represented mathematically by two different types of matrices. First is the adjacency matrix that represents the graph structure, i.e., its vertices and edges. The adjacency matrix \(A\) is a matrix whose element \(A_{ij}\) is one if there exists an edge between the nodes \(v_i\) and \(v_j\) and zero otherwise. Second is the Laplacian matrix, which is deduced from \(A\) to provide a vivid representation of the graph connectivity. The Laplacian matrix \(L\) contains both of edge information in its off-diagonal elements and node degrees in its diagonal. Moreover, the Laplacian matrix differentiates between partitioned and connected graphs. The number of partitions in one graph can be evaluated using the eigenvalues of the Laplacian matrix. The number of zero eigenvalues is itself the number of the partitions in the graph represented by that Laplacian matrix [40]. Since all the eigenvalues of a real Laplacian matrix are positive, the second smallest eigenvalue, called algebraic connectivity, determines the graph connectivity [41].

Based on the graph algebraic connectivity, we numerically predict the connectivity probability of highway VANETs. A pseudocode of the procedure is presented in Algorithm 1 under the unit disc model. First, a weighted matrix \(W\) is generated to maintain the exponentially-distributed intervehicle spacings. Then, the unweighted adjacency matrix \(A\) is acquired by comparing each element of the weighted matrix to the fixed communication range \(r\). Next, the second-smallest eigenvalue is determined from the Laplacian matrix \(L\). If the second-smallest eigenvalue is not zero, the graph is declared connected. Finally, the connectivity probability is evaluated over a massive graph ensemble.

Simulations were held to examine the proposed algorithm, and the results are illustrated in Fig. 1. In the simulations, we considered a multi-lane road segment of length 10Km and repeated the process at different average vehicle densities and fixed communication ranges. The simulation results were found identical to the analytic curves generated from Eq. (4).
Algorithm 1 A pseudocode of the proposed simulations

1: Set the value of the communication range r
2: Set a matrix $W$ as a zero matrix of size $N \times N$
3: Set $counter = 0$
4: for every iteration do
5:    Set a vector $y$ with $N - 1$ random numbers that are exponentially distributed with a mean of $\frac{1}{\lambda}$
6:    Calculate the upper triangular part of $W$ as $W_{ij} := \sum_{m=i}^{j-1} y_m \forall j > i$
7:    Calculate the lower triangular part of $W$ as $W_{ji} := W_{ij}$
8:    $A := W \leq r$
9:    Set the diagonal degree matrix $D$ with its diagonal elements $d_{ii} := \sum_{j=1}^{N} A_{ij}$
10:   $L := D - A$
11:   Set $\lambda_2$ to the second-smallest value of $\text{eign}(L)$
12:   if $\lambda_2 \neq 0$ then
13:       $counter := counter + 1$
14:   Calculate $P_c := counter / \#\text{iterations}$

Fig. 1: Connectivity probability under a unit disc communication channel, with $L = 10$Km.

V. CONNECTIVITY IN RAYLEIGH-FADING CHANNELS

The wave propagation through real-world communication channels suffers from multipath fading. This, in turn, affects the communication transmissions, as the receiving terminals receive a number of delayed versions of the transmitted signals. Under the free-flow traffic, where the vehicles are widely separated, the LoS propagation is almost absent, and therefore the PDF of received signal amplitudes follows a Rayleigh distribution [42]. In this section, we thoroughly study connectivity over a Rayleigh-fading channel.

A. Communication Range Distribution

While the PDF of received signal amplitudes is Rayleigh distributed, the PDF of the received power, and in turn the SNR, follows an exponential distribution [43]. Thus, the PDF of the received SNR $\gamma$ at a certain distance $d$ is

$$f_\gamma(z) = \frac{1}{\bar{\gamma}} e^{-\frac{z}{\bar{\gamma}}} = \frac{d^\alpha P_{\text{noise}}}{\beta P_T} e^{-\frac{d^\alpha P_{\text{noise}}}{\beta P_T}}$$

where $\bar{\gamma} = \frac{\beta P_T}{d^\alpha P_{\text{noise}}}$ is the average SNR [28] (cf. Eq. (2)).

In a Rayleigh-fading channel, the communication range becomes the contour that maintains an SNR threshold $\Psi$ (see Fig. 3b), which varies rapidly over time and space. Since the received SNR at a certain distance is a random variable over the Rayleigh-fading channel, the communication range has acquired a random behavior as well. In order to detect its probability distribution, let the communication range $R$ be a random variable with a CDF of

$$F_R(x) = 1 - P(\gamma(x) > \Psi) = 1 - e^{-\frac{x^\alpha P_{\text{noise}}}{\beta P_T}}, x \geq 0$$

Note that the CDF considers the probability that the communication range does not exceed a certain distance $x$. That reflects that the received SNR at that distance can no longer maintain the threshold value $\Psi$.

From the resultant CDF, it can be inferred that the communication range follows a Weibull distribution under a Rayleigh-fading channel with a PDF of

$$f_R(x) = \alpha \left(\frac{1}{\lambda}\right)^\alpha x^{\alpha-1} e^{-\left(\frac{x}{\lambda}\right)^\alpha}$$

where $\lambda^\alpha = \frac{\Psi P_{\text{noise}}}{\beta P_T}$, and $\lambda$ is the scale parameter of the Weibull distribution. The average communication range, then, can be deduced as

$$E[R] = \alpha \left(\frac{1}{\lambda}\right)^\alpha \int_0^{\infty} x^{\alpha} e^{-\frac{x^\alpha}{\lambda^\alpha}} dx = \lambda \Gamma\left(1 + \frac{1}{\alpha}\right)$$

where $\Gamma(.)$ is the gamma function [44]. The average communication range represents a circle around the transmitter that averages the randomness of the communication range and its irregular varying contours. According to the unit disc model, this circle with a fixed communication range is believed to average the VANET behavior in a Rayleigh-fading channel since it averages the communication range.

B. Single-Link Connectivity

Over a Rayleigh-fading channel, the vehicles can reach farther neighbors, regardless of their connectivity to their closer neighbors. The communication channels to various neighbors are independent of each other, and only depend on the propagation environment between each two vehicles. Therefore, the single-link connectivity should consider any neighbor in the surroundings as first described in Section III. In this case, the intervehicle distance between any two vehicles in the network should be determined first.

The distance between any pair of vehicles follows a different probabilistic distribution from the one used if they become successive neighbors, i.e., there is no other vehicle between them. In order to distinguish between the intervehicle spacing, which is the distance between two successive vehicles, and the distance between any two vehicles, we refer to the latter as intervehicle distance. Since the intervehicle spacing is exponentially distributed, the intervehicle distance between any vehicle and its $m^{th}$ neighbor is the sum of $m$ independent exponentially-distributed intervehicle spacings (see Fig. 3a).

Let the intervehicle distance to the $m^{th}$ neighbor be $Z_m$, and
then $Z_m \triangleq \sum_{i=1}^{m} X_i$, where $X_i$ with the exponential PDF described in Eq. (1). Thus, the PDF of $Z_m$ would be

$$f_{Z_m}(x) = f_{X_1}(x) \ast f_{X_2}(x) \cdots \ast f_{X_m}(x)$$

(9)

where the operator $\ast$ is the linear convolution. The solution of the $n$-fold convolution of exponential distributions follows an Erlang distribution with a PDF of

$$f_{Z_m}(x; m) = \frac{\rho^m}{(m-1)!} x^{m-1} e^{-\rho x}, \quad x \geq 0$$

(10)

where the operator $(\cdot)!$ is the factorial $[45]$. In a Rayleigh-fading channel, the single-link connectivity probability can be reasonably viewed as the probability that the intervehicle distance is within the vehicle communication range. Those two variables constitute two independent random variables. Hence, the single-link connectivity probability can be deduced as follows

$$P_{SL|Ray}(Z_m \leq R; m) = \int_0^{\infty} \int_x^{\infty} f_{Z_m,R}(x, r; m) dr dx = \int_0^{\infty} (1 - F_R(x)) f_{Z_m}(x; m) dx$$

(11)

where $f_{Z_m,R}(x, r)$ is the joint probability function. By substituting Eqs. (9) and (10) into Eq. (11), a general form of the single-link connectivity probability in a Rayleigh-fading channel is acquired

$$P_{SL|Ray} = \frac{\rho^m}{(m-1)!} \int_0^{\infty} x^{m-1} e^{-\rho x - (\frac{x}{\bar{r}})^m} dx$$

(12)

which should be computed numerically. A closed formula of Eq. (12), however, can be derived when the PLE is set to 2;

$$P_{SL|Ray} = \frac{\rho^m}{(m-1)!} \left( \frac{\rho \lambda^2}{2} \right)^m e^{2x^2} \sum_{k=0}^{m-1} \frac{(m-1)}{k} \left(-2 \rho \lambda\right)^k \Gamma \left( \frac{k+1}{2}, \frac{\rho \lambda^2}{4} \right)$$

(13)

where the $\Gamma(m, x)$ is the upper incomplete gamma function. For the sake of comparison, we derive a general form for single-link connectivity in a unit disc scenario, where any neighbor is considered, as follows

$$P_{SL|UD} = P(Z_m \leq r; m) = \frac{\rho^m}{(m-1)!} \int_0^{r} x^{m-1} e^{-\rho x} dx = 1 - e^{-\rho r} \sum_{k=0}^{m-1} \frac{(\rho r)^k}{k!}$$

(14)

It is clearly obvious that the single-link connectivity to the closest neighbor, i.e., $m = 1$, tends to Eq. (3).

The single-link connectivity of a Rayleigh-fading channel is represented as solid lines in Fig. 2. As depicted, the single-link connectivity to the closest neighbors falls behind that of the unit disc. Moreover, the Rayleigh-fading channel could not even sustain the same level of connectivity probability acquired by a unit disc with a communication range that is either three fourths or even half as high as it is in the Rayleigh channel. Surprisingly, that behavior was completely reversed for the farthest neighbors. Not only does the Rayleigh single-link connectivity surpass the connectivity of a unit disc with the same or a higher communication range, but also it reaches new neighbors that were not involved before in the coverage of fixed ranges. This remark matches the results reported in [35]–[38] for MANETs and static WANETs.

Graph-based simulations were also implemented to validate the theoretical results. An algorithm, identical to Algorithm 1, was used to perform these simulations. The required information about the single-link connectivity was, however, gathered from the off-diagonal elements of the adjacency matrix $A$ in Step 8, and there was no need to proceed to the following steps. Besides, the communication range in Step 1 was chosen to be a Weibull-distributed random variable to mimic the Rayleigh fading channel. The simulation results, then, matched theories as illustrated in Fig. 2.

C. Vehicle Connectivity

We define vehicle isolation according to the sufficient conditions required by a vehicle to eliminate the network connectivity. Thereafter, we can evaluate how a single vehicle affects the network connectivity in the two different channel models.
Accordingly, we get two different types of vehicle isolation, each of which matches one of the considered channel models. In a unit disc, a blocked connectivity to one side of a vehicle is sufficient and necessary to drop the connectivity. By a one-side vehicle connectivity, we only consider the connections in a predefined direction, either the forward direction to the vehicles in front of any considered vehicle or the backward direction. In Fig. 3a, if the forward direction is considered, the third vehicle from the right is disconnected from its forward neighbors, leading to preventing the network connectivity. The same result can be concluded when the backward direction is considered; the second vehicle from the right is, then, the isolated node that blocks the network connectivity. Thus, the vehicle connectivity in the unit disc channels can be narrowed to be one-sided.

In contrast, the one-side isolation is not sufficient to halt the connectivity in Rayleigh-fading environments. As in Fig. 3b, although the coverage of the black vehicle does not contain any forward neighbors, its connectivity is maintained by its link to its first backward neighbor. Consequently, the two-side isolation is sufficient but not necessary for burdening the connectivity, and then it is the convenient candidate for describing the vehicle connectivity under the Rayleigh-fading assumptions. However, the one-side vehicle connectivity of Rayleigh channels is also evaluated as an auxiliary step and for comparisons with the unit disc model.

The probability that a vehicle is connected from one side can be expressed as

\[ P_1 = P \left( \bigcup_{m=1}^{M} (Z_m \leq R; m) \right) = 1 - P \left( \bigcap_{m=1}^{M} (Z_m > R; m) \right) \]

where \( M \) should be the whole number of nodes located on one side of a vehicle. However, this number can be reduced to span only the neighbors within a certain area around the vehicle in order to relax the computations.

With a Rayleigh-fading channel, the single-link connectivity to different neighbors can be reasonably viewed as uncorrelated but dependent events. As the connectivity is controlled by the communication channel to different neighbors, which can be assumed uncorrelated in a Rayleigh channel if the received antennas are at least spaced by a wavelength apart. Besides, it also depends on the distance to different nonconsecutive neighbors \( Z_m \), which can be assumed uncorrelated but dependent as the distance has to increase with the neighbor order. Consequently, the vehicle connectivity should be expressed in terms of conditional probabilities. For simplicity, we assume that the single-link connectivity to different neighbors are independent of each other in order to derive an upper bound for vehicle connectivity. The latter helps out deducing an upper bound for connectivity probability in Section VI. Thus, the vehicle connectivity from one side becomes

\[ P_{1|Ray} = 1 - \prod_{m=1}^{M} \left( 1 - \frac{\rho^m}{(m-1)!} \int_0^\infty x^{m-1} e^{-\rho x} \frac{1}{x} dx \right) \]

Since the vehicle connectivity on one side is independent of the connectivity of the other side, the vehicle connectivity in a Rayleigh-fading channel can be evaluated as

\[ P_{V|Ray} = \left( 1 - \prod_{m=1}^{M} \left( 1 - \frac{\rho^m}{(m-1)!} \int_0^\infty x^{m-1} e^{-\rho x} \frac{1}{x} dx \right) \right)^2 \]

On the other hand, the vehicle connectivity of a unit disc is much simpler as the connections to remote neighbors do not represent unique situations. Therefore, the vehicle connectivity in Eq. (15) tends to be the single-link connectivity probability to the first neighbor, which can be declared as

\[ P_{V|UD} = P(Z_m \leq r; m = 1) = 1 - e^{-\rho r} \]

Graph-based simulations were executed using the proposed algorithm in Section IV. The vehicle connectivity is related...
to and can be evaluated from the degree matrix D in Step 9 of Algorithm 1. A comparison between vehicle connectivity in both Rayleigh-fading and unit disc channels based on the simulation results is represented in Fig. 4. Even though the Rayleigh-fading channel deteriorates the connectivity to near vehicles, the graph simulations show that any vehicle would maintain the same one-side connectivity of the unit disc scenario. That becomes anticipated after the results of Fig. 2. Meanwhile, the (two-side) vehicle isolation under a Rayleigh assumption was proven to be lower than the sufficient (one-side) vehicle isolation of the unit disc, leading to better vehicle connectivity in the Rayleigh-fading scenarios. That implies that the Rayleigh-fading channels would probably experience an improved network connectivity. Eventually, the vehicle connectivity probability in Eqs. 16 and 17 were proved to serve as upper bounds for actual vehicle connectivity.

VI. GRAPH-BASED PROOF OF IMPROVED CONNECTIVITY

With improved vehicle connectivity under a Rayleigh-fading channel, it is crucial to analyze the VANET connectivity to detect the fading implications on it. The definition of connectivity in unit disc channels cannot be extended to fading channels. As in the unit disc, it is sufficient to check whether each vehicle is connected to its first neighbor, the property that is not sufficient in a Rayleigh channel. The connectivity can be retained by many other network topologies. Furthermore, provided that each vehicle is not isolated, the whole network may be partitioned. That leaves defining a closed formula for the connectivity probability nearly infeasible. Therefore, we rely on graph theory to numerically predict the connectivity probability in a Rayleigh-fading channel with Monte-Carlo simulations. Using these simulations, we also provide an upper bound for connectivity probability.

A. Graph-based Simulations

For the sake of comparison, the average communication range in Rayleigh-fading channel was set identical to the fixed one of the unit disc. This ensures that the two cases employ the same SNR threshold to assess the connectivity. The other parameters including the transmit power, road segment length, average vehicle density, and intervehicle distribution were kept the same. Based on the proposed algorithm, graph-based simulations were executed for $10^4$ iterations, and the results are shown in Fig. 5. The simulations embrace the improved connectivity of Rayleigh-fading channels over the unit disc. For different communication ranges, Rayleigh channels achieve higher connectivity probability $P_c$ at different vehicle densities. All connectivity curves tend to reach the unity probability faster in the Rayleigh-fading channel, providing higher connectivity probabilities at lower vehicle densities.

These advantages were accomplished because of the reinforced connections to extended number of vehicles. At the absence of LoS, albeit the reflected versions of the transmitted signals may destruct each other obstructing the connectivity to near neighbors, they have the chance to sum up constructively for wider ranges compared to the unit disc. This, in turn, leads to connecting more separated partitions of the network and raising the connectivity probability. It can also be viewed as a sort of diversity since each node has several neighbors to receive from. If the links to some of them experience deep fade, the others are likely to mitigate it and provide connectivity. Although other types of diversity are manmade, the diversity provided here is granted by the cooperation manner of ad-hoc networks.

B. Connectivity Upper Bound Formula

While deducing a closed form for the connectivity probability of a Rayleigh-fading channel is not straightforward, an upper bound may be more convenient. Any connected graph intuitively has all of its nodes connected to one neighbor at least. However, if all nodes of one network has at least one linked neighbor, the network may be either connected or partitioned. The Laplacian matrix can distinguish between the two cases easily, but it is a nontrivial task from a statistical perspective. Since the assured vehicle connectivity all over the vehicles of the network is necessary but not sufficient for the VANET connectivity, the joint probability that all vehicles are connected can be, then, thought of as an upper bound for the VANET connectivity.

According to Eqs. 16 and 17, vehicle connectivity depends on the number of surrounding vehicles $M$ considered. Choosing $M$ as low as 1 will omit the Rayleigh constructive impact on connectivity, the problem faced the connectivity analysis of 31. Thus, $M$ should be chosen optimally to tightly cover the true connectivity probability. Furthermore, the two-side vehicle connectivity provides a sparser VANET topology as it considers the probability of a vehicle being connected to at least one neighbor out of $2M$ (i.e., $M$ of each side). Consequently, the problem of finding the upper bound of $P_c$ can rely on the one-side vehicle connectivity

$$P_c^{UB} = \min_M (P_{1Ray})^{N-1} \quad s.t. \quad P_c^{UB} \geq P_c \quad (19)$$

Note that the one-side vehicle connectivity, evaluated in Eq. 16, was also an upper bound as shown in Fig. 4. With a maximum deviation of around 0.1 at a vehicle density of 0.003
vehicles/m, this deviation would diminish exponentially with the total number of vehicles according to Eq. (19).

Using simulations, with the results in Fig. 6, the minimum number of vehicles $M$ that should be included in connectivity calculations was inferred to be 4. In other words, the probability that every vehicle of a VANET is connected to at least one neighbor of its closest 4 forward neighbors provides a tight upper bound for the VANET connectivity probability. However, considering only 3 neighbors can provide tighter bounds at lower vehicle densities, and then a combination of the two values can be used to better express the connectivity.

VII. CONCLUSIONS

Rayleigh-fading channels were proved to have an improved connectivity compared to unit disc channels. In the absence of LoS propagation, the scattered signals have more chances to add up constructively over wider ranges, providing connectivity to a larger number of neighbors. However, the scattering phenomenon deteriorates the single-link connectivity probability to the closest neighbors. Graph-based simulations were the most convenient method to capture this behavior of Rayleigh channels. Due to the lack of necessary and sufficient conditions for the network connectivity, it is nearly infeasible to describe this behavior with closed formulas. Instead, and with the help of the proposed graph-based simulations, a mathematical upper bound for the connectivity was derived to be the probability that every vehicle is connected to one of its first four successors.

REFERENCES

[1] S. Ucar, S. C. Ergen, and O. Ozkasap, “Multihop-cluster-based IEEE 802.11p and LTE hybrid architecture for VANET safety message dissemination,” IEEE Transactions on Vehicular Technology, vol. 65, no. 4, pp. 2621–2636, 2016.
[2] T. S. Darwish, K. Abu Bakar, and K. Haseeb, “Reliable intersection-based traffic aware routing protocol for urban areas vehicular ad hoc networks,” IEEE Intelligent Transportation Systems Magazine, vol. 10, no. 1, pp. 60–73, 2018.
[3] Y. Toor, P. Mühlthaler, A. Laouiti, and A. De La Fortelle, “Vehicle ad hoc networks: Applications and related technical issues,” IEEE Communications Surveys and Tutorials, vol. 10, no. 3, pp. 74–88, 2008.
[4] Y. Maalej, S. Sorour, A. Abdel-Rahim, and M. Guizani, “Vanets meet autonomous vehicles: Multimodal surrounding recognition using manifold alignment,” IEEE Access, vol. 6, pp. 29026–29040, 2018.
[5] F. Yang, S. Wang, J. Li, Z. Liu, and Q. Sun, “An overview of internet of vehicles,” China Communications, vol. 11, no. 10, pp. 1–15, 2014.
[6] O. Kaiwartya, A. H. Abdullah, and X. Liu, “Internet of vehicles: Motivation, layered architecture, network model, challenges, and future aspects,” IEEE Access, vol. 4, pp. 5356–5373, 2016.
[7] IEEE P802.11p/D6.01, “Part 11: Wireless lan medium access control (MAC) and physical layer (PHY) specifications-amendment 7: Wireless access in vehicular environments,” Apr. 2009.
[8] IEEE Std, “IEEE Standard for Wireless Access in Vehicular Environments (WAVE) Multi-Channel Operation,” Sep. 2010.
[9] IEEE, “Family of standards for wireless access in vehicular environments (WAVE)—IEEE 1609 series. IEEE, 2013.
[10] M. Krbálek and P. Šeba, “The statistical properties of the city transport in Cuernavaca (Mexico) and random matrix ensembles,” Journal of Physics A: Mathematical and General, vol. 33, no. 26, pp. 229–234, 2000.
[11] A. Y. Abul-Magd, “Modeling highway-traffic highway distributions using superstatistics,” Physical Review E - Statistical, Nonlinear, and Soft Matter Physics, vol. 76, no. 5, pp. 14–17, 2007.
[12] R. Nagel, “The effect of vehicular distance distributions and mobility on VANET communications,” in IEEE Intelligent Vehicles Symposium, Proceedings, pp. 1190–1194, 2010.
[13] L. Cheng and S. Panichpapiboon, “Effects of intervehicle spacing distributions on connectivity of VANET: A case study from measured highway traffic,” IEEE Communications Magazine, vol. 50, no. 10, pp. 90–97, 2012.
[14] L. Li and X. M. Chen, “Vehicle headway modeling and its inferences in macroscopic/microscopic traffic flow theory: A survey,” Transportation Research Part C: Emerging Technologies, vol. 76, pp. 170–188, 2017.
[15] H. X. Yang and M. Tang, “Adaptive routing strategy on networks of mobile nodes,” Physica A: Statistical Mechanics and its Applications, vol. 402, pp. 1–7, 2014.
[16] Z. Khan, P. Fan, and S. Fang, “On the connectivity of vehicular ad hoc network under various mobility scenarios,” IEEE Access, vol. 5, pp. 22559–22565, 2017.
[17] C. Chen, L. Liu, T. Qiu, K. Yang, F. Gong, and H. Song, “ASGR: An artificial spider-web-based geographic routing in heterogeneous vehicular networks,” IEEE Transactions on Intelligent Transportation Systems, pp. 1–17, 2018.
[18] Y. Meng, Y. Dong, X. Liu, and Y. Zhao, “An interference-aware resource allocation scheme for connectivity improvement in vehicular networks,” IEEE Access, vol. 6, pp. 51319–51328, 2018.
[19] S. M. Abuelenin and A. Y. Abul-Magd, “Effect of minimum headway distance on connectivity of VANETs,” AEU - International Journal of Electronics and Communications, vol. 69, no. 5, pp. 867–871, 2015.
[20] S. M. Abuelenin and A. Y. Abul-Magd, “Corrigendum to ‘Effect of minimum headway distance on connectivity of VANETs’ [AEU – Int. J. Electron. Commun. 69(5) (2015) 867–871],” AEU - International Journal of Electronics and Communications, vol. 83, p. 566, 2018.
[21] S. M. Abuelenin and A. Y. Abul-Magd, “Empirical study of traffic velocity distribution and its effect on VANETs’ connectivity,” in International Conference on Connected Vehicles and Expo (ICCVE), pp. 391–395, 2014.
[22] S. Panichpapiboon and W. Pattara-Aitkom, “Connectivity requirements for self-organizing traffic information systems,” IEEE Transactions on Vehicular Technology, vol. 57, no. 6, pp. 3333–3340, 2008.
[23] S. C. Ng, W. Zhang, Y. Zhang, Y. Yang, and G. Mao, “Analysis of access and connectivity probabilities in vehicular relay networks,” IEEE Journal on Selected Areas in Communications, vol. 29, no. 1, pp. 140–150, 2011.
[24] N. Akhtar, S. C. Ergen, and O. Ozkasap, “Vehicle mobility and communication channel models for realistic and efficient highway VANET simulation,” IEEE Transactions on Vehicular Technology, vol. 64, pp. 248–262, Jan. 2015.
[25] X. Jin, W. Su, and W. Yan, “Quantitative analysis of the VANET connectivity: Theory and application,” in IEEE Vehicular Technology Conference, pp. 1–5, 2011.
[26] N. Akhtar, O. Ozkasap, and S. C. Ergen, “VANET topology characteristics under realistic mobility and channel models,” in IEEE Wireless Communications and Networking Conference, WCNC, pp. 1774–1779, 2011.
[27] D. Naboulsi and M. Fiore, “Characterizing the instantaneous connectivity of large-scale urban vehicular networks,” IEEE Transactions on Mobile Computing, vol. 16, no. 5, pp. 1272–1286, 2017.
[28] D. Miorandi and E. Altman, “Connectivity in one-dimensional ad hoc networks: A queueing theoretical approach,” Wireless Networks, vol. 12, no. 5, pp. 573–587, 2006.

[29] S. Ukkusuri and L. Du, “Geometric connectivity of vehicular ad hoc networks: Analytical characterization,” Transportation Research Part C: Emerging Technologies, vol. 16, no. 5, pp. 615–634, 2008.

[30] N. P. Chandrasekharan and B. Ancharav, “Connectivity analysis of one-dimensional vehicular ad hoc networks in fading channels,” Eurasip Journal on Wireless Communications and Networking, vol. 2012, pp. 1–16, 2012.

[31] A. Babu and V. M. Ajeer, “Analytical model for connectivity of vehicular ad hoc networks in the presence of channel randomness,” International Journal of Communication Systems, vol. 26, pp. 927–946, Jul. 2013.

[32] C. H. Mar and W. K. G. Seah, “An analysis of connectivity in a MANET of autonomous cooperative mobile agents under the Rayleigh fading channel,” in IEEE 61st Vehicular Technology Conference, vol. 4, pp. 2606–2610, May 2005.

[33] D. Miorandi, E. Altman, and G. Alfano, “The impact of channel randomness on coverage and connectivity of ad hoc and sensor networks,” IEEE Transactions on Wireless Communications, vol. 7, no. 3, pp. 1062–1072, 2008.

[34] C. Bettstetter and C. Hartmann, “Connectivity of wireless multihop networks in a shadow fading environment,” Wireless Networks, vol. 11, no. 5, pp. 571–579, 2005.

[35] R. Hekmat and P. Van Mieghem, “Connectivity in wireless ad-hoc networks with a log-normal radio model,” Mobile Networks and Applications, vol. 11, no. 3, pp. 351–360, 2006.

[36] X. Zhou, S. Durrani, and H. M. Jones, “Connectivity of ad hoc networks: Is fading good or bad?” in 2nd International Conference on Signal Processing and Communication Systems, 2008.

[37] O. Georgiou, C. P. Dettmann, and J. P. Coon, “Network connectivity: Stochastic vs. deterministic wireless channels,” in IEEE International Conference on Communications, pp. 77–82, 2014.

[38] S. Yousefi, E. Altman, R. El-Azouzi, and M. Fathy, “Analytical model for connectivity in vehicular ad hoc networks,” IEEE Transactions on Vehicular Technology, vol. 57, pp. 3341–3356, Nov. 2008.

[39] Sherif M. Abuelenin received the B.Sc. degree in electronics and communications from Suez Canal University, Egypt in 1999, the M.Sc. degree in Electrical Engineering from Tuskegee University in 2002, and the Ph.D. degree from Auburn University in 2005. He joined Tuskegee University as an assistant professor of Electrical Engineering in 2005. From 2007 to 2011 he served as an assistant professor in the faculty of Engineering Sciences, Sinai University, Egypt. In 2011, he joined the faculty of engineering, Port-Said University, where he currently is an Associate Professor of Electrical engineering. His research interests include signal processing for communications, vehicular communications, and fuzzy logic systems.

[40] L. Stanković and E. Sejdić, eds., Vertex-frequency analysis of graph signals. Signals and Communication Technology, Springer International Publishing, 1st ed., 2019.

[41] M. Fiedler, “Algebraic connectivity of graphs,” Czechoslovak Mathematical Journal, vol. 23, no. 2, pp. 298–305, 1973.

[42] G. Acosta-Marum and M. A. Ingram, “Six time- and frequency- selective empirical channel models for vehicular wireless LANs,” IEEE Vehicular Technology Magazine, vol. 2, pp. 4–11, Dec. 2007.

[43] A. Goldsmith, Wireless communications. Cambridge University Press, 2005.

[44] I. S Gradsteyn and I. M Ryzhik, Table of integrals, series, and products. Academic Press, 7th ed., 2007.

[45] M. Akkouchi, “On the convolution of exponential distributions,” Journal of the Chungcheong Mathematical Society, vol. 21, no. 4, pp. 501–510, 2008.

Samar Elaraby received her M.Sc. and B.Sc. degrees in Electronics and Communication Engineering from Port Said University, Egypt, in 2017 and 2011 respectively. Since 2012, she has been a research assistant with the Faculty of Engineering, Port Said University, Egypt. Her current research interests focus on theories and applications of signal processing and machine learning.