Reflective thinking in solving an algebra problem: a case study of field independent-prospective teacher

Agustan S\textsuperscript{1,2}, Dwi Juniati\textsuperscript{3}, Tatag Yuli Eko Siswono\textsuperscript{3}
\textsuperscript{1} Faculty of Teacher Training and Education, Universitas Muhammadiyah Makassar, Indonesia
\textsuperscript{2} Doctoral Program Student of Mathematics Education, Indonesia
\textsuperscript{3} Department of Mathematics, Universitas Negeri Surabaya, Indonesia
E-mail: agustan@unismuh.ac.id, juniati@yahoo.com, tatagsiswono@unesa.ac.id

Abstract. Nowadays, reflective thinking is one of the important things which become a concern in learning mathematics, especially in solving a mathematical problem. The purpose of this paper is to describe how the student used reflective thinking when solved an algebra problem. The subject of this research is one female student who has field independent cognitive style. This research is a descriptive exploratory study with data analysis using qualitative approach to describe in depth reflective thinking of prospective teacher in solving an algebra problem. Four main categories are used to analyse the reflective thinking in solving an algebra problem: (1) formulation and synthesis of experience, (2) orderliness of experience, (3) evaluating the experience and (4) testing the selected solution based on the experience. The results showed that the subject described the problem by using another word and the subject also found the difficulties in making mathematical modelling. The subject analysed two concepts used in solving problem. For instance, geometry related to point and line while algebra is related to algebra arithmetic operation. The subject stated that solution must have four aspect to get effective solution, specifically the ability to (a) understand the meaning of every words; (b) make mathematical modelling; (c) calculate mathematically; (d) interpret solution obtained logically. To test the internal consistency or error in solution, the subject checked and looked back related procedures and operations used. Moreover, the subject tried to resolve the problem in a different way to compare the answers which had been obtained before. The findings supported the assertion that reflective thinking provides an opportunity for the students in improving their weakness in mathematical problem solving. It can make a grow accuracy and concentration in solving a mathematical problem. Consequently, the students will get the right and logic answer by reflective thinking.

1. Introduction
The competence in solving mathematical problems is one of the competencies which must be owned by a student in learning mathematics [1]. To solve a mathematical problem, it is required a high level thinking skills which it can encourage students's curiosity in order that there is a desire to solve mathematical problems encountered. One of the thinking skills which can be applied in solving mathematical problem is reflective thinking [2] because reflective thinking is a type of high-level thinking which can encourage students's curiosity [3]. This is in line with demands of curriculum 2013 in Indonesia which is found key words on main and basic competencies as science behavioral includes curiosity, objective, fair, thorough, meticulous, diligent, careful, responsible, open, critical, creative, innovative and caring environment. Moreover the skills required to face the 21st century challenges, namely cognitive skills, intrapersonal skills, and
interpersonal skills. It confirms that reflective thinking is a future competence that should be
taught to students to adapt changes and respond demands of the 21st century. In addition,
Goodell [4] and Ville [5] stated that one of goals of the educational personnel and institutions
creates a prospective teacher who is responsible and capable reflective thinking. King and
Kitchener stated that, by reflective thinking, student can solve more complex problems because
student’s sense will be directed and solution of the problem being solved tend to be correct
and appropriate [6]. In addition, Ambrose [7], Gelter [8] and Koszalka [6] stated that reflective
thinking gives opportunities for students to justify misconceptions by helping students to think
about what they did and why they did it. This indicates that the reflective thinking occurs
after mathematical problem solving conducted with the aim checking the error of the concepts
used and try to justify these misconceptions, so that it can develop students’ skills in using
mathematical concepts [9]. Every student has a unique character so as to learn, master, process
information, solve problems, and to teach the material well, in this case mathematics, someone
will do it in also different way [10]. Every student has cognitive style. Differences in student’s
cognitive styles affect students’ skills in reflective thinking and solving problems. Cognitive
styles correlate with intellectual and perceptual behaviour. Intellectual refers to a person’s
ability to think, while perceptually refers to a person’s ability to view or interpret anything.
Another concept of cognitive style developed by Witkin, et.al [19] based on continuous global
analytical. Based on this concept, cognitive styles are divided into field-independent (FI) and
field-dependent (FD). Individuals who are field-independent (FI) prefer to separate the parts of a
number of patterns and analysis patterns based on their components. Another hand, individuals
who are field-dependent (FD) tend to see a pattern as a whole, not separate it into parts.

Based on the explanation above, reflective thinking is one of the important tools that help
students to improve their curiosity. It gives also opportunities for students in justifying their
mistakes or misconceptions when they have solved mathematical problem and try to improve
their weakness. As a consequence, in this research paper, the researchers will describe how
the subject uses her reflective thinking after solving algebra problem who has field independent
cognitive style. It is interesting to investigate subject’s reflective thinking to give us description
and insight how reflective thinking can be applied in solving mathematical problem to get the
best answer logically. As a result, we can choose the best way to teach the students in providing
a chance to look for their mistakes in solving mathematical problem and to justify them so that
their answers tends to be true.

2. Literature Review
Currently reflective thinking has became the most prominent issues in the literature, in particular
on professional education of teachers [11], [12]. Reflective thinking is active, persistent, and
careful consideration of any belief or suppose from of knowledge in the light of the grounds
that support it and the conclusion to which it tends [13]. Furthermore, Dewey [14] suggested
that there are six phases in reflective thinking, namely: (a) an experience; (b) spontaneous
interpretation of the experience; (c) naming the problem or question that arise out of the
experience; (d) generating possible explanations for the problem or question posed; (e) ramifying
the explanation into full-blown hypotheses. Meanwhile, according to Lee [15], there are five
phases reflective thinking, namely: (a) problem context; (b) Problem definition; (c) Seeking
possible solution; (d) experimentation; (e) evaluation; (f) acceptance/rejection. Rodgers
[16] argues that there are four phases in the process of reflective thinking as follows: (a)
presence to experience; (b) description of experience; (c) analysis of experience; (d) intelligent
action/experimentation. Zehavi and Mann [17] stated that the reflective thinking for the level of
meta-cognitive referred to four categories: electoral techniques, monitoring process of solutions,
insight or ingenuity, and conceptualization which connects the concept and meaning. To gain an
overview of the process of reflective thinking in solving mathematical problems, previously it is
need to be synchronized several components of reflective thinking processes that exist, namely the components of the process of reflective thinking by John Dewey [13] abbreviated PBRD; by Lee [15] abbreviated PBRL; by Rodgers [16] abbreviated PBRR, by Zehavi and Mann [17] abbreviated PBRZM. From the components of reflective thinking processes served by each figures who examined reflective thinking process, there are similarities indicators. It is obtained results of the construction of thinking reflective process contained in the following table I which consists of four stages. These four stages are: (1) Formulation and synthesis of the experience; (2) Orderliness of experience; (3) Evaluation of experience; and (4) Testing the selected solution based on the experience.

| Table 1: Construction of Reflective Thinking Process |
|-----------------------------------------------|
| Dewey (PBDR) | Lee (PBRL) | Rodgers (PBRR) | Zehavi & Mann (PBRZM) | Reflective Thinking |
| Formulation and synthesis of the experience | Formulate a problem, reflecting on the experience | Formulate a problem, reflecting on the experience | Formulate a problem, reflecting on the experience | Reflective Thinking |
| Ordering the procedure | Problem definition, refining | Problem definition, refining | Problem definition, refining | Reflective thinking |
| Developing possible explanations for the experience | Analysis of the experience | Analysis of the experience | Analysis of the experience | Reflective thinking |
| Testing the explanations | Testing the explanations, refining | Testing the explanations, refining | Testing the explanations, refining | Reflective thinking |
| Reflecting on the selected solution | Reflecting on the selected solution | Reflecting on the selected solution | Reflecting on the selected solution | Reflective thinking |

Everyone has a different character (unique) so as to learn, master, process information, solve problems, and to teach the material well, someone will do it in a different way also [10]. Every student has a characteristic which consistent when organizing and processing the information obtained. This characteristic is known as cognitive style. Tennant [18] and Witkin et.al [19] stated that cognitive style is a characteristic of thinking process which is consistent and reflected on students. Furthermore, Witkin et.al [19] had developed a concept of cognitive style which is based on continuous global analyst. Based on this concept, cognitive styles are divided into field-independent and field-dependent. Field independent are individuals who are more analytical, they can choose the stimulus based on the situation, so that their perception is only a small part affected when there is a change in the situation. They prefer to separate the parts of a number of patterns and analyze patterns based on their components. While, Field dependent are individuals who tend to get difficulties in differentiating stimulus through situations that owned so that their perception is easily influenced by the manipulation of the surroundings. They tend to see a pattern as a whole, it is not separate it into parts. It can be concluded that the individuals who are Field Independent (FI) have a tendency to respond to stimulus by using their own perceptions, more analytical, and they analyze patterns based its components. Meanwhile, individuals who are Field Dependent (FD) have a tendency to respond a stimulus by using the environment as a basic requirement in perception, and they tend to see a pattern as a whole (global), not separate parts.

3. Method
This paper aims to explore and investigate student’s reflective thinking process in solving algebra problem. As a consequence, the researchers adopted exploration approach to analyze the data by using qualitative approach. [20]. To get this aims, the participant was given a task of solving algebra problem and then the participant was interviewed. The participant is a student in Department of Mathematics Education, Universitas Muhammadiyah Makassar, Indonesia. To determine the participant, it will be used Group Embedded Figure Test (GEFT) which consists of 25 items. The test is divided into three sessions. The first session consists of 7 items as session of practice, the second session consists of nine items and the third session consists of nine items. The second and the third session serve as session assessed. The criteria used in the selection of
the participant using the criteria according to Kepner and Neimark [21], participant who can give right answer 0-9 classed as Field Dependent (FD) and 10-18 classed as Field Independent (FI). Furthermore, it was determined 1 participant who is female and has Field Independent (FI) cognitive styles. In addition, the participant was also considered able to verbally communicate well.

4. Result
Based on the construction of reflective thinking process of the subject in solving an algebra problem that is divided into four stages, the researchers investigated and found out what happened on every stage related to reflective thinking in solving algebra problem which had been done by the participant. The participant had been interviewed deeply on matters concerning the four stages and linkages that may affect it. The results of the interview toward the participant on data collection illustrating the process of reflective thinking in solving algebra problem by a prospective teacher who has field independent cognitive style.

The first stage, formulation and synthesis of the experience, the participant described problem by using his own words and explained the problem clearly related to the main point of the problem. The participant also found the concepts which were related to the problem given. Furthermore, when the subject identified the concept or subject matter which is related to the problem, the participant mentioned not only the concept that ever the participant used in solving a problem before, but also the participant explained that the concept which will be used in solving algebra problem. Moreover, the participant found the difficulties to change from the problem to mathematical modelling.

From the transcript, it can be explained that the participant described the problem by using another word however participant’s explanation still has the same idea from the problem. For more clearly, the following is the excerpt of interview excerpt based on this stage.

Researcher: What did you understand the problem?
Participant: I understood that Sukri is a trader. The distance his home and his shop is 20 km. There are 60 mangoes must bring to the shop. ... Taufan always eats one mango per kilometre. We are demanded to find out a strategy in bringing mangoes as much as possible. This problem related to the concept of algebra arithmetic operation (addition, subtraction, division and multiplication), linear equations, linear inequalities and geometry. When I wanted to solve this, I got difficulties to make a mathematical modelling ....

The second stage, orderliness of experience, the participant described a strategy in solving algebra problem given. The participant gave an explanation related to steps in choosing an operation well and the difficulties when applying strategies. The participant also investigated two concepts used in solving problem and explained impact development of his strategy to solve other problems. For more clearly, the following is the excerpt of interview from participant.

Researcher: Based on your experience, please tell me about your strategies to solve this problem!
Participant: In my opinion, this problem related to a concept of algebra and geometry. For algebra, ... For geometry, ... I tried to solve this by drawing a sketch. I found difficulties to apply my strategies (drawing). I was less conscientious and less concentration. It made my answer almost wrong. If I met other problem, I will apply this strategies and develop it. As a consequence, it can help me to solve it easier. However, I used much time.

From the transcript, it can be explained that participant analyzes specifically two concepts used in solving problem. For instance, geometry is related to point and line and algebra is related to algebra arithmetic operation. The participant made drawing as a way to find answer or solution from the problem presented. The difficulties which were earned by the participant applying her strategies are a lack of concentration and not careful making her answer almost wrong. Moreover, the participant explained that she tended to use the same way to solve other problems. The participant also adapted her strategies in order to develop her solution to get the
right answer. Furthermore, development of strategies was based on problem context where it
gave effect not only positive impact but also a negative impact. The positive impact facilitated
to take the right answer and a negative impact took much time in solving problem.

The third stage, **evaluation of experience**, the participant explains how to determine
criteria of a good and proper solution. The participant find also strengths and weaknesses
from subject’s solutions and stated how to improve the weaknesses which are owned or carried
when solving problems. For more clearly, the following is an excerpt of the interview from the
participant.

Researcher: *Please, define a criteria of a good settlement*  
Participant: *Firstly, the criteria of a good settlement must be obvious and done step by step. The second, accuracy in understanding problem. There are four aspects to get an effective solution namely the ability to understand meaning of words, making mathematical modelling, performing mathematical calculations, interpreting the results obtained. I used many ways to solve this so that I believed my answer tended to be true. I sometimes made mistakes when I didn’t have enough time. It can become that I were not careful and I didn’t concentrate. However, I will exercise myself to solve other problem. In order that, it made me habitual to solve problem.*

From the transcript, it can be concluded that participant can give the explanation about
the criteria of solution, namely (a) coherent; (b) accuracy in solving problem. Furthermore,
participant makes opinion that solution must have four aspect to get effective answer, specifically
(a) skill to understand meaning every words; (b) skill to make mathematical modelling; (c) skill
to calculate mathematically; (d) skill to interpret solution obtained logically. The participant
stated that less conscientious and less concentration become weaknesses in solving problem
and to revise and improve the weakness then it is need a habit to solve other problem. As a
consequence, someone will not be rigid when facing problems.

The last stage, **testing the selected solution based on the experience**, the participant
explained how to make a decision in using operation and how to test internal consistency or
mistake in operation or procedure. Clearly, the following is an excerpt of interview from the
participant.

Researcher: *How would you define that procedure which you used had been appropriate with the principle of mathematical operation?*  
Participant: *In my opinion, my procedure is correct. I noted the information known, asked, then how is completion and to make me sure with my answer. I tried to solve it in several times and in many ways. ... I also checked the operation which I used and interpreted my answer logically by linking with the context of the problem.*

From the transcript, it can be concluded that the participant illustrated that it was needed
decision to choose a way or procedure which can help to find out the right answer. The procedure
used was suitable with principle and the rule of mathematical operation by taking down all
informations. Furthermore, to test the internal consistency or error in operation or in solution,
the participant checked and looked back related procedures and operations which had been used.
Moreover, the participant tried to resolve the problem in a different way to compare the answers
which had been obtained before.

Based on explanation above, it can be described that the participant states that the first
way, Taufan eats one mango per kilometre so that there is no mango arrives in the shop. The
second way, it is still the same case, however, when Taufan arrives in the shop, the last mango
is not ate. The last way, the participant solved by making stopover in the street which is passed
by Taufan. If participant divides 2 parts the street, she will get 1 stopover and 10 mangoes
can be in the shop. If participant divided 4 parts, she will get 3 stopovers and 15 mangoes. If
participant divided 5 parts, she will get 4 stopovers and 16 mangoes. The last, if participant
divides 10 parts the street, participant will get 9 stopovers and 16 mangoes.

Therefore, the participant made conclusion that 16 mangoes is the right answer. The first
reason is that there are two the same answers which describe consistency of participant’s answer. The second one is the participant interpreted her answers and connected to the context of problem.

5. Conclusion
Based on the result, it can be concluded that reflective thinking can be investigated after someone did a mathematical problem solving. Reflective thinking provides opportunities for the students to improve their weakness. When they get some difficulties and find some mistakes or some misconceptions related to their answer, they will correct it and try to interpret it to obtain logical answer. Reflective thinking also can make a grow accuracy and concentration when the students solve a mathematical problem. Consequently, the students will get the right and logic answer by reflective thinking. Above all, this research paper provides insight into how promote reflective thinking that could be trained to prospective teacher that impact when prospective teacher become a real teacher in society.

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