Meson Masses in Nuclear Matter

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Abstract

Mass shifts $\Delta m$ of particles in nuclear matter relative to their vacuum values are considered. A general formula relating $\Delta m(E)$ ($E$ is the particle energy) to the real part of the forward particle-nucleon scattering amplitude $Re f(E)$ is presented and its applicability domain is formulated. The $\rho$-meson mass shift in nuclear matter is calculated at $2 \lesssim E_\rho \lesssim 7$ GeV for transversally and longitudinally polarized $\rho$-mesons with the results: $\Delta m_\rho^T \sim 50$ MeV and $\Delta m_\rho^L \sim 10$ MeV at normal nuclear density.

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The problem of how the properties of mesons and baryons change in nuclear matter in comparison to their free values has attracted a lot of attention recently. Among these properties the first of interest are mass shifts of particles in nuclear matter. This interest is related to the fact that it was possible to calculate by QCD sum rules and on the lattice almost all masses of low lying mesons and baryons, and a hope appears to extend these calculations to the case of particles is embedded in nuclear medium. On the other hand, the values of particle masses can be measured experimentally - at least some of them - and some data started to appear. In this aspect experiments on heavy ion collisions, in which the dependence of particle masses on nuclear density can be found, are very promising.

In early theoretical investigations of this problem \cite{1,2} one or another model of strong
interaction of particles in nuclear matter was used. In the pioneering work by Drukarev and Levin the use of QCD sum rules for the calculation of nucleon mass shift in nuclear matter was suggested. Later this method was applied also to calculation of meson masses (for recent reviews see [4, 5]). Among the latter the most interesting is the case of light vector mesons and, especially, of $\rho$. The reason is that a $\rho$-meson produced inside the nucleus decays also there and can be observed by its partial decay into $e^+e^-$. So, the characteristics of $\rho$-meson inside the nucleus can be directly measured.

The masses of vector mesons in nuclear matter were calculated in [2, 7–13]. (In Ref. [9] a universal ratio of particle masses in nuclear matter to their vacuum values was suggested.) However, the results obtained by different methods do not coincide. Moreover, there is no agreement as to whether the $\rho$ mass decreases or increases in nuclear medium in comparison with its value in vacuum: in Refs. [2, 7, 12] $\Delta m = (m_{\rho})_{\text{nucl}} - m_{\rho} > 0$, while in Refs. [9, 10, 13] $\Delta m < 0$. Since the interaction of $\rho$-meson with nucleons in medium is energy dependent, one may expect that the mass shift is also energy dependent. This problem was not considered in the investigations mentioned above: only the case of $\rho$-meson at rest was considered. But $\rho$-mesons at rest are not good objects from experimental points of view. In experiments on nuclei $\rho$-mesons as a rule are produced with energies of order of 1 GeV or more. Finally, for a moving $\rho$-meson the interaction with matter of the meson polarized transversally or longitudinally is different. So, one may expect that the mass shifts in nuclear matter of transversally and longitudinally polarized $\rho$-mesons are also different. For all of these reasons a new consideration of this problem is desirable.

We start with general considerations applicable to any particle imbedded in nuclear matter. Let us accept the standard assumption in the treatment of the problem in view [4, 6]: the interaction of the particle with a nucleon in matter is not affected by other nucleons, i.e. the nuclear matter can be considered as an inhomogeneous macroscopic medium. This immediately restricts the particle wave length: $\lambda = k^{-1} \ll d$, where $d$ is the mean internucleon distance. Numerically this means that the particle momentum $k$ must be larger than a few hundred MeV. Since we assume that the particle is created inside the nucleus, we must require that its formation length $l_{\text{form}}$ is less than the nucleus radius $R$

$$l_{\text{form}} \sim \frac{E}{m m_{\text{char}}},$$

(1)
where $E$ and $m$ are the particle energy and mass, $m_{\text{char}} \sim m_{\rho}$ is the characteristic strong interaction scale. Eq. (1) implies an upper limit on the particle energy, $E_\rho < 15$ GeV for middle weight nuclei. An additional restriction on the upper value of the particle momentum $k$ arises from the requirement that for the observation of the mass shift the particle must mainly decay inside the nucleus, $k/\Gamma m < R$. This gives $k_\rho < 6$ GeV, $k_\omega < 300$ MeV, $k_\phi < 200$ MeV for $\rho$, $\omega$, and $\phi$, correspondingly. Comparison of lower and upper limits for the particle momenta shows that for $\omega$ and $\phi$ the assumption of independent scattering on individual nucleons in the nucleus and the possibility of experimental observation are in contradiction. So, we are left only with $\rho$.

In consideration of the particle mass shifts in nuclear matter, or, equivalently, of the mean effective potential acting on the particle in matter, we use the general method suggested long ago for treatment of propagation of fast neutrons in nuclei [14] (see also [15]). The main idea is that for $\lambda \ll d \ll R$ the effect of medium on the particle propagation can be described by attenuation and refraction indexes. Attenuation of particles moving in the direction of $z$-axis at a distance $z$ is equal to $\exp(-\rho \sigma z)$, where $\rho = A/V$ is the nuclear density, $A$ is the atomic number, $V$ is the nucleus volume, and $\sigma$ is the total cross section of the interaction of the particle with nucleons. (Strictly speaking $\rho \sigma = (Z \sigma_p + N \sigma_n)/V$.) Using the optical theorem

$$k \sigma = 4\pi \text{Im} f(E),$$

where $f(E)$ is the forward scattering amplitude, we can write that the modulus of the particle wave function in matter is proportional to

$$|\psi| \sim \exp \left[ -\rho \frac{2\pi z}{k} \text{Im} f(E) \right]$$

This formula is evidently generalized to the wave function itself

$$\psi \sim \exp \left[ i\rho \frac{2\pi z}{k} f(E) \right]$$

Eq. (4) is correct if $|f| \ll d = (V/A)^{1/3}$: only in this case the scattering on each nucleon can be considered as independent and interference effects can be neglected [15]. $\text{Re} f(E)$ is related to the refraction index of matter for particle propagation [14]. We want to decribe
the propagation of a particle through nuclear matter introducing an effective mass \( m_{\text{eff}} = m + \Delta m \). This means that (leaving absorption aside)

\[
\psi \sim e^{ikz}, \quad k = \sqrt{E^2 - m_{\text{eff}}^2} \approx k - \frac{m_{\text{eff}}}{k} \Delta m
\]  

(5)

By comparing Eqs. (4) and (5) we get

\[
\Delta m(E) = -2\pi \frac{\rho}{m} \text{Re} f(E)
\]  

(6)

The expression in Eq. (6) for \( \Delta m \) has the meaning of an effective potential acting on the particle in medium [14,15]. For the correction to the particle width we have in a similar way

\[
\Delta \Gamma(E) = \frac{\rho}{m} k \sigma(E)
\]  

(7)

All the above statements are general and can be applied to any particle in nuclear matter. Let us now turn to the most interesting case, the \( \rho \)-meson.

In order to find \( \rho N \) forward scattering amplitude we use the vector dominance model (VDM) and the relation which follows from VDM (see, e.g. [16])

\[
f_{\gamma N} = 4\pi\alpha \left( \frac{1}{g_{\rho}^2} f_{\rho N} + \frac{1}{g_{\omega}^2} f_{\omega N} + \frac{1}{g_{\phi}^2} f_{\phi N} \right)
\]  

(8)

The last term in the r.h.s. of Eq. (8) can be safely neglected: as follows from \( \phi \)-photoproduction data, it is small. Basing on the quark model, assume \( f_{\omega N} \approx f_{\rho N} \). Since \( g_{\omega}^2/g_{\rho}^2 \approx 8 \), the contribution of \( \omega \) to the r.h.s. of Eq. (8) is also small. Therefore, according to Eq. (8) \( \text{Re} f_{\rho N}(E) \) is expressed through \( \text{Re} f_{\gamma N}(E) \). The latter can be found from the photoproduction data through the dispersion relation with one subtraction,

\[
\text{Re} f_{\gamma N}(E) = f_{\gamma N}(0) + \frac{E^2}{(2\pi)^2 P} \int_{E_{th}}^{\infty} dE' \frac{\sigma_{\gamma N}(E')}{E'^2 - E^2},
\]  

(9)

where \( P \) denotes principle value, \( \sigma_{\gamma N}(E) \) is the total photoproduction cross section, \( E_{th} = \mu + \mu^2/2m_N \), \( \mu \) and \( m_N \) are the pion and nucleon masses, and \( f_{\gamma N}(0) \) is given by the Thompson formula, \( f_{\gamma p}(0) = -\alpha/m_p, f_{\gamma n} = 0 \).

The VDM relation Eq. (8) holds only for the amplitude of transversally polarized vector meson \( f_{\rho N}^T \), since \( f_{\gamma N} \) is the scattering amplitude of the real transversally polarized photon. In Eq. (8) the \( \rho \)-meson energy \( E_\rho \) is related to the photon energy by the requirement that
the masses of hadronic states produced in $\rho N$ and $\gamma N$ scattering should be equal, $E_\rho = E_\gamma - m_\rho^2/2m_N$.

It is known that VDM works well starting from $\gamma$ energies about 2 GeV, where one may expect the VDM accuracy of about 30% and better at higher energies (see, e.g. [16]). At these energies the nucleon Fermi motion can be neglected. In calculation of $\text{Re}f_{\gamma N}(E)$ according to Eq. (9) we used the PDG data [17] on photoproduction on deuteron. For the high-energy tail the Donnachie–Landshoff fitting formula [18] for $\sigma_{\gamma p}$ was used, and it was assumed that $\sigma_{\gamma D}/\sigma_{\gamma p} = \text{const}$ starting from $E_\gamma = 20$ GeV. The results for $\text{Re}f^T_{\rho N}$ and $\Delta m^T_\rho$ at normal nuclear density $\rho = (4\pi r_0^3/3)^{-1}$, $r_0 = 1.25$ fm, are shown in Fig. 1 as functions of $E_\rho$. The mass shift in the energy region, where our consideration is valid, $2 \, \text{GeV} < E_\rho < 7$ GeV, is positive ($\rho$ mass increases in nuclear matter) and is of order of 50 MeV. However, the condition $|\text{Re}f| < d \sim 2$ fm is not well fulfilled. Probably the main effect of interference of different nucleons is screening and the true values of $\Delta m_\rho$ are a bit smaller than our results.

Now consider the longitudinal $\rho$-meson. In this case, unlike the transverse $\rho$, it is impossible to relate the forward scattering amplitude of $\rho$ to that of the real photon, but it is still possible to have such a relation for the virtual photon. We assume that VDM holds for virtual photons if the photon virtualities are not large, less or of order of $m_\rho^2$. For the transverse scattering amplitude the generalization of Eq. (8) to the virtual photon is

$$f^T_{\gamma N}(E_\gamma, q^2) = 4\pi\alpha \sum_{V=\rho,\omega,\phi} \frac{m_V^4}{(q^2 - m_V^2)^2} \frac{1}{g_V^2} f^T_{VN}(E_V)$$

For the longitudinal scattering amplitude the generalization of VDM has the form

$$f^L_{\gamma N}(E_\gamma, q^2) = 4\pi\alpha \sum_{V=\rho,\omega,\phi} \frac{|q^2|m_V^2}{(q^2 - m_V^2)^2} \frac{1}{g_V^2} f^L_{VN}(E_V)$$

Eqs. (10) and (11) can be proved in models incorporating direct $\gamma N$ interaction. The denominators in these equations correspond to the assumption that at $Q^2 = -q^2 \lesssim m_V^2$ the dominant intermediate states in the $\gamma$-channel are vector mesons and the contributions of higher states can be neglected. The factor $q^2$ in the numerator of Eq. (11) is a kinematical factor that evidently follows from the requirement of vanishing $f^L_{\gamma N}$ at $q^2 = 0$. The absolute value $|q^2|$ arises, since $\text{Im}f^L_{\gamma N}$ is positive at $q^2 < 0$ as well as at $q^2 > 0$. This corresponds to
the fact that while for transverse photon (or any transverse or longitudinal vector meson) the polarization vector squared is \( e^2 = -1 \), for longitudinal virtual photon we put \( e^2 = 1 \) in order to get a positive cross section (see [16]). The relation between \( E_\rho \) and \( E_\gamma \) is now
\[
E_\rho = E_\gamma - (m_\rho^2 + Q^2) / 2m_N.
\]

Of course, the accuracy in determination of \( \text{Re}f_{\rho N}(E) \) basing on the data for the virtual photon scattering amplitude will be worse than in the case of real photon, but for the longitudinal \( \rho \)-meson even such information will be valuable. \( \text{Re}f_{\gamma N}^{T,L}(E, Q^2) \) can be found from the data on deep inelastic scattering in the same way as was done for the real photon.

The dispersion relation takes the form
\[
\text{Re}f_{\gamma N}^{(T,L)}(E, Q^2) = f_{\gamma N}^{(T,L)}(0, Q^2) - \frac{\alpha}{m_N} P \int_0^1 dx' \frac{1 + 4m_N^2x'^2/Q^2}{x'^2 - x^2} F_2(x', Q^2) \frac{(1, R)}{1 + R}
\]

(12)

where \( x = Q^2 / 2\nu, \nu = m_N E, F_2(x, Q^2) \) is the nucleon structure function, and \( R = \sigma_L / \sigma_T \) is the ratio of longitudinal to transverse photon cross sections.

Consider first the case of transverse photon and check whether starting from the deep inelastic scattering data we can get the values of \( \text{Re}f_{\rho N}^T(E) \) close to those we have already found from photoproduction. We choose \( Q^2 = 0.5 \text{ GeV}^2 \) and take \( F_2^p(x, 0.5 \text{ GeV}^2) \) from the data compilation done by Ji and Unrau [19]. The ratio \( F_2^n / F_2^p \) was taken from [20] for \( x < 0.2 \). For \( x > 0.2 \), where the data at small \( Q^2 \) are absent, we assume \( F_2^n / F_2^p = 0.75 \). The information about \( R \) at small \( Q^2 \) is scarce. Basing on the data from Refs. [21, 24] we assume \( R_p = R_n = 0.3 \). We also assume that at \( Q^2 = 0.5 \text{ GeV}^2 \) the subtraction term in Eq. (12) is given by the one-nucleon intermediate state, as it takes place in the Thompson formula. The one-nucleon intermediate state contributes also to the integral in Eq. (12). Its total contribution to Eq. (12) is
\[
\text{Re}f_{\gamma N}^T(\nu, Q^2)_{\text{one-nucl}} = -\frac{\alpha}{m_N} \left[ F_E^2(Q^2) + \frac{1}{4}Q^4G_M^2(Q^2)\frac{1}{\nu^2 - Q^4/4} \right],
\]

(13)

where \( F_E \) and \( G_M \) are the nucleon electric Pauli and magnetic Sachs formfactors. The results of our calculation show that the shape of the curve for \( \text{Re}f_{\rho N}^T(E_\rho) \) obtained from the data at \( Q^2 = 0.5 \text{ GeV}^2 \) is similar to the curve \( \text{Re}f_{\rho N}^T(E_\rho) \) in Fig. 1, but the absolute values are 30–40% smaller. Since the factor \( (Q^2 + m_\rho^2)^2 / m_\rho^4 \approx 3.4 \) connecting the values of \( f_{\gamma N}^T(E_\gamma, Q^2) \) and \( f_{\rho N}^T(E_\rho) \) is rather large, this fact can be considered as an indication that the accuracy of VDM for the problem considered is of order 30–40%.
The calculation of $\text{Re} f_{7N}(E, Q^2)$ is similar. The only difference appears in the subtraction term in Eq. (12). In [22] it was proved that $f_{7N}(0, Q^2)$ at small $Q^2$ is given by the one-nucleon intermediate state and it was argued that its contribution dominates up to $Q^2 = 0.5 \text{ GeV}^2$. The contribution of one-nucleon intermediate state to $f_{7N}(\nu, Q^2)$ is

$$\text{Re} f_{7N}(\nu, Q^2)_{\text{one-nucl}} = -\alpha m_N Q^2 \left[ \frac{1}{4m_N^2} F_M^2(Q^2) + \frac{1}{\nu^2 - Q^4/4} G_E^2(Q^2) \right],$$

(14)

where $F_M$ and $G_E$ are the nucleon magnetic Pauli and electric Sachs formfactors.

The results of calculation of $\text{Re} f_{\rho N}(E_{\rho})$ and $\Delta m_{\rho}^L(E_{\rho})$ are plotted in Fig. 1. As is seen from Fig. 1 in the energy range $E_{\rho} = 2 - 7 \text{ GeV}$ $\Delta m_{\rho}^L$ is essentially smaller than $\Delta m_{\rho}^T$. Although the uncertainty in the determination of $\Delta m_{\rho}^L$ is rather large, we believe that this qualitative conclusion will be intact in a true theory. Since at rest $\Delta m_{\rho}^T = \Delta m_{\rho}^L$, one should expect a strong energy dependence of $\Delta m_{\rho}^T$ and/or $\Delta m_{\rho}^L$ in the domain $m_{\rho} < E_{\rho} < 2 \text{ GeV}$. This is not surprising in the framework of our approach, since there are resonances in this domain and strong variations of $\text{Re} f_{\rho N}(E_{\rho})$ and $\Delta m_{\rho}(E_{\rho})$ are very likely. The main sources of uncertainty in our approach are the assumption of independent scattering on nucleons in the nucleus (Fermi gas approximation) and the use of VDM, especially for the virtual photon. We estimate the uncertainty as $\sim 30 - 50\%$ for $\Delta m_{\rho}^T$ and as a factor of $\sim 2$ for $\Delta m_{\rho}^L$. The broadening of the $\rho$ width calculated according to Eq. (7) is large: $\Delta \Gamma_{\rho}^T \approx 300 \text{ MeV}$, $\Delta \Gamma_{\rho}^L \approx 100 \text{ MeV}$ at $E_{\rho} = 3 \text{ GeV}$ and normal nuclear density.

A few remarks are in order comparing our consideration with the previous ones. Strictly speaking no direct comparison can be done, since all previous calculations refer to the mass shift of $\rho$-meson at rest, while the applicability domain of our results is $E_{\rho} > 2 \text{ GeV}$. As was mentioned above, one may expect a strong energy dependence of $\Delta m_{\rho}(E)$ in the interval $m_{\rho} < E_{\rho} < 2 \text{ GeV}$. (Even the sign difference in $\Delta m_{\rho}$ obtained here and in Refs. [3–6,9–13] cannot be considered as a contradiction, since $\text{Re} f_{\rho N}(E)$ may change sign going through resonances, as it indeed happens with $\text{Re} f_{7N}(E)$.) But we would like to emphasize one important point. The basic physical content of our approach is the statement that the meson mass shift in nuclear matter is determined by the meson-nucleon interaction and scattering proceeding at rather large distances, not much less than internucleon distances. The main point of Refs. [3, 6, 13] was the assumption that the mass shifts are determined by small distances and that the QCD sum rule method developed for the calculation of small
distance contributions can be applied to this problem. Since our basic formula is general and contains no assumptions (apart from the Fermi gas approximation, which is a common point in all approaches) the values of \( f_{\rho N} \sim 1 \text{ fm} \) obtained above clearly demonstrate that large distances are indeed of importance in this problem. In the calculations of Refs. [6,10,12,13] the operator product expansion (OPE) for the virtual photon - nucleon forward scattering amplitude was used and a few terms in OPE were kept. As is well known the OPE in this case is a light-cone expansion, and the expansion parameter along the light-cone is \( 1/x = 2\nu/Q^2 \). For the \( \rho \)-meson at rest \( \nu \sim m_N m_{\rho}, \ Q^2 \sim m_{\rho}^2 \), and \( 1/x \sim 2m_N/m_{\rho} \approx 2.5 \). Therefore, there are no reasons to keep only a few terms in this expansion, as was done in [6,10,12,13]. This fact, of course, is the manifestation of the physical statement made above about importance of large distances in the problem discussed.

Finally, we would like to mention that a similar treatment of in-medium pions using the data on \( \pi N \) forward scattering amplitudes extracted from the phase analysis in Ref. [23] shows a strong energy dependence of the pion mass shift for \( 400 \text{ MeV} < E_\pi < 1500 \text{ MeV} \): \( \Delta m_\pi = 30 - 70 \text{ MeV} \) for normal nuclear density.

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FIGURES

FIG. 1. Energy dependence of $-\text{Re}f_{\rho N}^T$ and $-\text{Re}f_{\rho N}^L$ (upper and lower solid curves, left scale), and of $\Delta m_{\rho}^T$ and $\Delta m_{\rho}^L$ (upper and lower dashed curves, right scale) at normal nuclear density.