Imaged substellar companions: not as eccentric as they appear? The effect of an unseen inner mass on derived orbits

Tim D. Pearce, Mark C. Wyatt and Grant M. Kennedy

Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge CB3 0HA, UK

Accepted 2013 October 24. Received 2013 October 15; in original form 2013 August 28

ABSTRACT

Increasing numbers of substellar companions are now being discovered via direct imaging. Orbital elements for some of these objects have been derived using star–companion astrometry, and several of these appear to have eccentricities significantly greater than zero. We show that stellar motion caused by an undetected inner body may result in the companion elements derived in such a way being incorrect, which could lead to an overestimation of the eccentricity. The magnitude of this effect is quantified in several regimes and we derive the maximum eccentricity error a third body could introduce in a general form, which may be easily applied to any imaged system. Criteria for identifying systems potentially susceptible to this scenario are presented, and we find that around half of the planets/companion brown dwarfs currently imaged could be liable to these errors when their orbital elements are derived. In particular, this effect could be relevant for systems within 100 pc with companions at >50 au, if they also harbour an unseen ∼10 M J object at >10 au. We use the Fomalhaut system as an example and show that a 10 per cent error could be induced on the planet’s eccentricity by an observationally allowed inner mass, which is similar in size to the current error from astrometry.

Key words: astrometry – planets and satellites: general – stars: individual: Fomalhaut.

1 INTRODUCTION

The past two decades have witnessed the birth of direct imaging as a technique to detect substellar companions, with the first discovery of an orbiting brown dwarf (Nakajima et al. 1995) and later a giant planet (Chauvin et al. 2004) via this method. Many more potential companions have since been imaged around other stars (www.exoplanet.eu; Schneider et al. 2011), with the method favouring large objects at wide separations from their hosts. In addition, the detection of orbital motion between imaging epochs has allowed constraints to be placed on some companion orbits (e.g. Soummer et al. 2011; Chauvin et al. 2012), with several appearing to have eccentricities significantly greater than zero (e.g. Neuhaus et al. 2010; Kalas et al. 2013).

Planet formation models generally favour the production of low-eccentricity companions, as any eccentricity excitations are quickly damped by the gas disc early on (Lissauer 1993). Gravitational instability is also thought to initially form protoplanets on low-eccentricity orbits (Boss 2011). Therefore, the existence of eccentric companions implies that some further process occurs beyond formation, which could be planet–planet scattering (Gladman 1993; Marzari & Weidenschilling 2002), 3+ body effects such as secular perturbations (Lee & Peale 2003), stellar flybys (Malmberg, Davies & Heggie 2011) or even planet mergers (Lin & Ida 1997) to name a few mechanisms. An accurate measure of eccentricity is very important for a dynamical understanding of these systems, and an overestimation of this quantity could result in an incorrect understanding of system evolution. A potential source of systematic overestimation of eccentricity in imaged systems is the subject of this paper.

Orbital elements of extrasolar companions detected via any method are generally derived in an astrocentric frame (relative to the star) assuming no other bodies in the system. However, if an undetected third mass were also present, then it would induce a stellar motion about the system barycentre, which could lead to the companion elements derived in this way being incorrect. This effect has already been examined for radial velocity (RV) detections; Rodigas & Hinz (2009) showed that 10–20 per cent of RV companions with eccentricities of 0.1–0.4 could actually be on circular orbits with an error introduced by an undetected outer companion, and Wittenmyer et al. (2013) identified several moderately eccentric single-planet systems that could be better fitted by two low-eccentricity planets. However, a similar effect has not been considered for wide-separation companions detected by imaging, where an additional mass could lie interior to this object and perturb the stellar motion.

Indeed, the existence of an unseen massive object interior to an imaged companion is often suggested if the latter is on a large eccentric orbit, as the inner mass may be required to scatter the observed object out to such a wide separation (Kalas et al. 2013).
In addition, long-term RV trends (e.g. Ségransan et al. 2011) and microlensed planets (Gaudi 2012) in some systems suggest that companions at \( \sim 10 \text{au} \) may be common, and these objects could have significant masses yet still remain unseen due to limitations in detection methods. High-contrast imaging at these separations is difficult and whilst RV surveys have excelled in locating short-period companions, detectable planets in Jovian-type orbits remain elusive. Furthermore, the precision of RV measurements is significantly reduced when applied to young stars due to stellar activity, yet it is in these systems that outer companions are easier to detect with imaging. This is highlighted by the case of \( \beta \) Pictoris, which shows that massive objects \((8M_J, \text{where } M_J \text{ is the mass of Jupiter})\) may exist around such stars yet evade RV detection (Lagrange et al. 2009, 2012).

If an unseen massive object existed in a system with a wide-separation imaged companion, then this companion could in fact orbit the star–inner object barycentre. The motion of the star about this barycentre would then cause the astrocentric elements of the imaged companion to vary with a period similar to that of the inner object (e.g. Morbidelli 2002), and hence its observationally derived orbital elements would be incorrect. In this work, we examine the effect of such a scenario on the derived eccentricity of the outermost object, which could be overestimated if an inner companion were present.

The layout of this paper is as follows. Sections 2 and 3 describe the theory work. In Section 2, we consider the case where the observed companion is on a circular barycentric orbit, in order to find the minimum mass of an unseen inner object required to make the outer body appear eccentric. We then generalize this to an eccentric outer companion in Section 3 to find the maximum error in eccentricity which could be induced by an inner object. We suggest criteria to identify systems potentially susceptible to these scenarios in Section 4, and Section 5 provides a step-by-step method which may be used to evaluate the magnitude of this effect for a given system. We apply this method to Fomalhaut and several other systems as examples. We remark on the detectability of an inner mass in Section 6, and discussion and conclusions are given in Sections 7 and 8.

2 OUTER OBJECT ON A CIRCULAR ORBIT

2.1 Negligible time between observations

First, we investigate how an object on a circular barycentric orbit may be given an apparent astrocentric eccentricity by an unseen inner companion. We assume that the outer object is small compared to the star, and is sufficiently distant that it undergoes two-body motion about the star–inner mass barycentre. We also make the initial assumption that the astrocentric position and velocity of the observed companion are both known at a single epoch (i.e. the time between observations required to derive the velocity is negligible compared to the inner object period), which will later be relaxed. Finally, the three-body system is assumed to be coplanar, though we will later show that any mutual inclination reduces the effect of an inner mass on the astrocentric elements of the outer body.

The set-up of this problem is shown in Fig. 1. The star and inner object form a circular binary, and the observed companion orbits the binary barycentre. At the moment of observation, the barycentric coordinate system is defined to be aligned with the binary separation vector, and the observed object has a true anomaly \( f \) in this frame. We also define an astrocentric coordinate system centred on the star, with the axes parallel to those in the barycentric frame. The astrocentric position of the observed companion \( r' \) is therefore given by \( r - r_s \), its barycentric position minus that of the star, and its astrocentric velocity \( v' \) is given by a similar expression. We will use primes to denote astrocentric parameters for the remainder of the paper, and the subscript \( i \) will be used to identify parameters associated with the inner object.

We can show that \( r' \) is given by

\[
r' = a \left( \begin{array}{c} \cos f \\ \sin f \end{array} \right) + \mu a_i \left( \begin{array}{c} 1 \\ 0 \end{array} \right),
\]

where \( a \) denotes the barycentric outer object semimajor axis, \( a_i \) is the binary separation and \( \mu = m_i/(m_s + m_i) \), where \( m_i \) and \( m_s \) are the masses of the inner object and star, respectively. Additionally, the velocity is

\[
v' = \sqrt{\frac{GM}{a} \left( \begin{array}{c} -\sin f \\ \cos f \end{array} \right)} + \mu \sqrt{\frac{GM}{a_i} \left( \begin{array}{c} 0 \\ 1 \end{array} \right)},
\]

where \( M = m_i + m_s \). To simplify the following, we introduce the parameter \( \alpha = a_i/a \) that, along with \( \mu \), contains all the information required to calculate the astrocentric coordinates. The fractional difference between the astrocentric radii, \( \delta r/r \equiv (r' - r)/r \), therefore has a maximum absolute value of \( \mu \alpha \). Similarly, \( \delta v/v = \mu / \sqrt{\alpha} \) at its maximum value. As we only consider unseen companions interior to the observed object, \( \alpha < 1 \) so the difference between the astrocentric and barycentric radii of the outer object is small whilst the velocity difference may be large. For example, a \( 0.01m_s \) object orbiting at \( \alpha = 0.1 \) would give the observed companion maximum \( \delta v/v \) and \( \delta r/r \) values of 0.03 and 0.001, respectively.

We convert the astrocentric Cartesian coordinates \( r' \) and \( v' \) into Keplerian orbital elements, and the resulting semimajor axis and eccentricity are shown as functions of a true anomaly \( f \) in Fig. 2.
The plotted functions are quite cumbersome, but to give the reader a feel for their behaviour we simplify them to the following first-order approximations. When $\alpha$ is small, $\delta v/v \gg \delta r/r$ and the behaviour of the osculating elements is completely dominated by the velocity shift. In this case, the semimajor axis and eccentricity reduce (to first order in $\mu$) to

$$a' - \alpha \approx \mu \left[ 1 + \frac{2}{\sqrt{\alpha}} \cos f \right]$$

(3)

and

$$e' \approx \mu \left[ 1 + \frac{4}{\sqrt{\alpha}} \cos f + \frac{1}{\alpha} \left( 1 + 3 \cos^2 f \right) \right]^{\frac{1}{2}},$$

(4)

which are roughly of order $\mu/\sqrt{\alpha}$, the same as the velocity shift. These equations provide a very good fit to the full functions, and hence the turning points in Fig. 2 may be estimated by substituting $f = 0, \pi/2$ and $\pi$ into the above. For example, if a companion on a 50 au circular orbit were observed about a solar-type star, an undetected 10 $M_J$ object at 1 au would cause the observed companion's apparent semimajor axis to vary between 43.5 and 57.5 au and its eccentricity to oscillate between 0.07 and 0.15 with a submaximum at 0.13.

Note that as $\alpha$ approaches unity, additional $\alpha$ terms caused by the radial shift $\delta r/r$ are no longer negligible in comparison to $1/\sqrt{\alpha}$, so equations (3) and (4) no longer hold. Regardless as we assume that the outer object undergoes two-body motion about the barycentre, the model is invalid in this regime due to three-body interactions. However, we do not consider such a scenario due to the nature of the problem; as $\alpha \rightarrow 1$, the mass of this object would have to be large to have any effect and should therefore be detectable. We do not consider $\alpha > 1$ for the same reason, and additionally the detected companion would be unlikely to orbit the barycentre in this case.

The maximum values of $\delta r$ and $\delta v$ occur when $f = 0$, i.e. all bodies are aligned, with the star farthest from the outer object. Here the stellar motion opposes the motion of the observed companion, and therefore $\delta v$ is maximized. We can differentiate the full equations for $a'$ and $e'$ and show that these elements are also maximum here. Therefore, by substituting $f = 0$ into the full equation for $e'$, we find an upper bound on $e'$ for each combination of $\mu$ and $\alpha$. This equation may be rearranged to find the minimum value of $\mu$ (as a function of $\alpha$) required to give the outer object an apparent astrocentric eccentricity $e'$. The resulting expression contains terms up to high orders in $\mu$; however, it may be approximated to better than 5 per cent accuracy by discarding terms greater than second order and multiplying by an empirical factor $F(e')$ to account for higher order terms. This yields the equation

$$\mu \gtrsim F(e') e' \left[ 1 + 4 \left( \frac{1}{\alpha} + \frac{1}{\sqrt{\alpha}} + \sqrt{\alpha} + a' \right) + 2a' \right]^{-\frac{1}{2}},$$

(5)

where $F(e') \equiv (1 + 0.3e')^{-\frac{1}{2}}$ is the empirically determined factor. Without this factor the above formula overestimates the minimum value of $\mu$ by $\sim 25$ per cent for high values of $e'$. The minimum value of $\mu$ is therefore only dependent on $\alpha$ and the observed astrocentric eccentricity, so is applicable to all systems. Fig. 3 shows this minimum mass as a function of $\alpha$; the contours were calculated using the full formalism rather than equation (5), but are well approximated by the latter. It is clear that the inner mass required to give the outer object a significant apparent eccentricity is generally large, typically in the giant planet-to-brown/dwarf red dwarf regime for a solar-type star.

As $\delta r/r$ is small, the approximation $\alpha \approx a'$ is generally very good, so this may be used to derive $a'$ from $\alpha$. Also as the minimum value of $e'$ is non-zero, we could progress in the same way as above to derive an upper limit on the inner object mass and thus bound $\mu$. 

Figure 2. Astrocentric semimajor axis and eccentricity of an outer object on a circular barycentric orbit, in the case where the time between observations is small. The turning points of both elements are denoted by subscripts, and may be well approximated by substituting $f = 0, \pi/2$ and $\pi$ into equations (3) and (4). Note that the astrocentric eccentricity is never zero, i.e. $e'_1 > 0$. The plots depend only on $\mu$ and $\alpha$, and are qualitatively the same for all parameters.

Figure 3. Minimum $\mu$ required to give an outer object on a circular orbit an apparent eccentricity $e'$, if the time between observations is much smaller than the inner object period (see Section 2.2 if this is not the case). Each line shows a different $e'$. Note the change in behaviour as $\alpha$ approaches unity as described in the text. This plot is independent of $m_o$ and $\alpha$. 

Downloaded from https://academic.oup.com/mnras/article-abstract/437/3/2686/1030509
by guest on 27 July 2018
in $\alpha$ space. However, this limit is not provided as the value is high (such an object would be identifiable using other methods, such as spectroscopy or imaging), so a better upper bound will be given by observational limits.

We have assumed that the three-body system is coplanar to derive the above bounds. If this condition is relaxed, we find that any mutual inclination reduces the difference between the two sets of outer object elements. This is to be expected; as noted above, the effect is maximized when the velocity shift $\delta v$ is greatest, i.e. when the direction of the stellar motion opposes that of the outer object. Mutual inclination reduces the stellar velocity component in the outer companion’s orbital plane, and hence lowers the velocity shift and thus its effect on the latter’s elements. Therefore, the value of $\mu$ derived using equation (5) will always be the minimum even if mutually inclined orbits are considered.

### 2.2 Non-negligible time between observations

Fig. 3 suggests that the minimum $\mu$ required to give a circular companion an apparent eccentricity may always be reduced by placing the inner object ever closer to the star. Unfortunately, there is a problem encountered in this regime, as the above assumes that the astrocentric coordinates of the outer object are known instantaneously. In reality, the velocity is derived by taking (at least) two images at two different epochs, and between these epochs the inner binary has also progressed about its orbit, as shown in Fig. 4. The effect of this motion will be significant if the time between observations is of the order of the inner binary period $T_i$, or greater, so is most important for inner objects on close orbits. It is these objects that were favoured by the previous results, because they suggested that even a small mass at this location could still have a significant effect on the apparent outer companion elements.

We now examine the case where two observations are made at times $t = 0$ and $\Delta t$. All bodies are again on coplanar circular orbits. We assume for simplicity that the time between observations is much smaller than the outer object period, which is valid as companions currently detectable by direct imaging generally exist far from their host star. This means that the motion of the outer object is approximately linear, with velocity $v' \equiv \Delta r'/\Delta t$. Without loss of generality, we can specify that at the time of the first observation, the system is in the same configuration as for the single-epoch case [i.e. the initial outer object position in the astrocentric frame is given by equation (1) with $f = f_i$]. At a time $\Delta t$ later, the observed companion will have a position

$$r_i = a \left( \cos(f_0 + \Delta f) \right) + \mu a \left( \cos(n_1 t) \right) \sin(n_1 t)$$

in the new astrocentric frame, where $\Delta f = \sqrt{GM/a^3} \Delta t$ and $n_i = 2\pi/T_i$. Therefore, we may estimate $v'$ as $(r_i - r_i')/\Delta t$, and using this and the companion’s position in one of the images we may proceed in deriving its astrocentric elements as before.

This time the elements are not only functions of $\mu$ and $a$ but also of $\Delta t$, and the resultant solutions are more complicated than in the previous case. However, to first order in $\mu$ (assuming $a$ is small) these elements may be well approximated as

$$a' = a \pm 2 a \zeta(\Delta t) \cos \left( f_0 - \frac{\Delta t}{T_i} \right)$$

and

$$e' \approx \mu \left[ 1 + 4 \sqrt{a} \zeta(\Delta t) \cos \left( f_0 - \frac{\Delta t}{T_i} \right) \right] + \frac{1}{a} \zeta^2(\Delta t) \left( 1 + 3 \cos^2 \left( f_0 - \frac{\Delta t}{T_i} \right) \right)^{1/2},$$

where

$$\zeta(\Delta t) \equiv \sin \left( \frac{\pi \Delta t}{T_i} \right).$$

Note that in the limit $\Delta t \to 0$, $\zeta(\Delta t) \to 1$ and the above expressions reduce to equations (3) and (4). The behaviour of these elements as a function of $\Delta t$ (using the full calculation rather than the first-order approximations above) is shown in Fig. 5.

There are several differences between this case and the $\Delta t/T_i \approx 0$ regime described earlier. First, the phases of $(a' - a)/a$ and $e'$ are now shifted in $f_0$ when compared to the single-epoch case, due to the changing object positions during the calculation. This manifests itself as the $f_0 - \pi \Delta t/T_i$ terms in the equations. Secondly, the multiple-epoch scenario reduces the amplitude of $(a' - a)/a$ and $e'$ when compared to the single-epoch case; Fig. 5 shows that the magnitude of these elements is a long-term decline as $\Delta t$ is increased. This can be explained by noting that the stellar motion, as well as the motion of the outer object, is effectively averaged by the use of multiple observation epochs. That is, as the observed astrocentric velocity is derived as $v' = \Delta r'/\Delta t$ where $r' = r - r_i$, the apparent velocity shift caused by the stellar motion is therefore $\Delta r_i/\Delta t$. For circular stellar motion, the velocity derived in this way will always be smaller than the true velocity, and so the effect of this averaging is to reduce $\delta v$ and therefore the amplitude of the outer object’s osculating elements. This manifests itself primarily as the $T_i/\Delta t$ term in equation (9), which causes the long-term $\sim 1/\Delta t$ declines in element amplitude visible in Fig. 5. Note that if $T_i/\Delta t > 1$, the inner binary makes at least one complete revolution between observations, and therefore the apparent stellar velocity as ‘seen’ by the outer object will be significantly reduced.

**Figure 4.** Triple system at a time $\Delta t$ after the first observation. Note that at both epochs the barycentric coordinate system is defined with respect to the inner binary position at $t = 0$. 

---

*Downloaded from https://academic.oup.com/mnras/article-abstract/437/3/2686/1030509 by guest on 27 July 2018*
α/Δ₁ is increased (not very significant in Fig. 2), the osculating elements oscillate as functions of \( f_0 \) (similar to Fig. 2), and the range over which they oscillate is denoted here by the shaded region. This plot is system specific and has been produced using \( μ = 0.001 \) and \( α = 0.008 \), but is qualitatively the same for all parameters. The quantities on the vertical axes are the same as in Fig. 2; note that the range tends to that of the simpler case as \( Δt \to 0 \).

Fig. 5 also shows that the elements undergo short-term oscillatory behaviour as a function of \( Δt/T_i \), and that the range over which they oscillate (and hence the dependence on the initial outer object true anomaly, \( f_0 \)) is zero when \( Δt/T_i \) is an integer. This is another effect of the apparently reduced stellar motion described above. First when \( Δt/T_i \) is an integer, the star has the same position at both observation epochs, and so its apparent velocity is zero. As the fractional radial shift \( δr/r \) caused by the unseen inner mass is negligible, in this case the outer object effectively ‘sees’ the star with no velocity and very little offset from the barycentre, and so the apparent astrocentric elements do not depend on the initial true anomaly of the outer companion. The only difference between the two sets of elements is therefore caused by this small barycentric offset and the use of the star’s mass to derive the astrocentric values, rather than the combined mass of the inner binary. Also when \( Δt/T_i \) is a half-integer (apart from when \( Δt/T_i = 1/2 \)), the binary is observed to have advanced by half an orbit, and so \( Δr_e \) and hence \( δv \) is maximum. This causes the submaxima in \( (\alpha' - α)/α \) and \( e' \) at these locations, although they may be slightly shifted due to the general \( ∼1/Δt \) decline. Note that for \( Δt/T_i < 1 \), the maxima lies at \( Δt/T_i = 0 \) rather than 1/2; this is because the binary has not yet made one complete revolution and so \( Δt/T_i \to 0 \) rather than a larger integer, and the apparent stellar velocity therefore tends to its true value. Finally, there is a very slight downward trend in \( (\alpha' - α)/α \) as \( Δt \) is increased (not very significant in Fig. 5 but pronounced in some cases) caused by the breakdown of the linear motion approximation.

The important thing to note for this case is that any non-zero \( Δt \) reduces the effect of an unseen inner mass when compared to the single-epoch scenario, and that this effect becomes significant if this object lies close to the star. Fig. 6 shows the minimum \( μ \) as a function of \( α \) required to give the outer an astrocentric eccentricity as before, only now for an example set of parameters with \( Δt \neq 0 \). The specific masses and turning points are system dependent. However, the plot is qualitatively the same for all parameters; at large \( α \), \( Δt/T_i \to 0 \) and the result tends to the simpler regime of Section 2.1. As \( α \) gets smaller the \( Δt \neq 0 \) case begins to dominate, and there is now a lower limit on \( μ \) to give a circular companion an apparent astrocentric eccentricity. Therefore, \( μ \) may not be ever reduced simply by moving the inner object closer to the star. Finally, the required mass increases sharply beyond this turning point, and subminima are also present due to \( T_i \) changing as a function of the inner binary semimajor axis.

Proceeding as for the simpler case, we may estimate the minimum unass me mass as a function of \( α \) required to give an observed circular companion an apparent astrocentric eccentricity \( e' \). By rearranging equation (8), we derive an approximate expression for this minimum mass that is valid when \( μ \) and \( α \) are small:

\[
μ \gtrsim e' \left( 1 + \frac{4}{\sqrt{α}} \zeta(Δt) + \frac{4}{α} \zeta^2(Δt) \right)^{-\frac{1}{2}}.
\]  

(10)
We may then differentiate this expression with respect to $\alpha$ and set it to zero to find the absolute minimum value of $\mu$. The first non-zero solution to the resulting equation occurs when

$$\tan(\Gamma) = \frac{3}{2} \Gamma,$$  \hspace{1cm} (11)

where $\Gamma \equiv \pi \Delta t / T_i$.

At this point as no information is known about the unseen inner mass, it makes sense to remove the dependence on $T_i$ from the above equations and replace it with $\tau$, the ratio of $\Delta t$ to the outer object period, which may be more intuitively estimated. Thus, $\Delta t / T_i = \tau / a^{3/2}$ and $\Gamma = \pi \tau / a^{3/2}$. Note that all of the system specific information (the star mass, $\Delta t$ and $a$) is contained within $\tau$. As a larger $\Gamma$ corresponds to a smaller value of $\alpha$, the smallest non-zero solution of equation (11) corresponds to the global minimum value of $\mu$. $\Gamma \approx 0.967$ at this point. Therefore, an inner mass will have the greatest effect on the astrocentric elements of the outer if

$$\frac{\Delta t}{T_i} \approx 0.31,$$  \hspace{1cm} (12)

i.e. the observational baseline is about a third of the inner object period. Substituting this into equation (10), we find that in order for an unseen inner mass to give a circular outer object an astrocentric eccentricity $e'$

$$\mu \gtrsim e' \left(1 + 2.30 \tau^{-1/3} + 1.32 \tau^{-2/3}\right)^{-\frac{1}{2}},$$  \hspace{1cm} (13)

and the location of this mass in order for $\mu$ to have the minimum possible value must be

$$\alpha \approx 2.19 \tau^{2/3}.$$  \hspace{1cm} (14)

The latter equation is independent of $e'$, and so the radius at which the inner object has the greatest effect is only dependent on $\alpha$ and $\tau$. The secondary minima in Fig. 6 correspond to higher $\Gamma$ solutions to equation (11), and the peaks correspond to a second set of $\alpha$-dependent roots to the differential of equation (10). Whilst the above equations are only approximate, they agree well with minimum masses and corresponding semimajor axis ratios calculated numerically without any simplifications.

Fig. 7 shows the absolute minimum $\mu$ as a function of $\tau$ and $e'$ calculated by a numerical grid search, which shows good agreement with equation (13) for $\tau \lesssim 10^{-2}$. Above this value, the two diverge as the linear motion approximation breaks down; assuming that the companion moves in a straight line between epochs will always introduce an error on the derived elements, and this error will increase as a greater fraction of the orbit is observed. Therefore, this effect is most apparent in the lower-right corner of the plot, where the difference between the outer object’s true velocity and that estimated linearly is sufficient to give the body an apparent eccentricity even in the absence of a third mass. This plot may be used to establish whether the apparent eccentricity of an observed companion could be entirely caused by the presence of an unseen inner mass.

The parameter $\tau$ is given by

$$\tau = \frac{\Delta t}{2\pi} \sqrt{\frac{G(m_\odot + m_\star)}{a^3}}$$  \hspace{1cm} (15)

and contains two unknowns, $m_\star$ and $a$. However, we again take advantage of $\delta r/r$ being small, and hence can make the approximation $r' \approx a$. Therefore, $\tau$ may be accurately estimated as

$$\tau \approx \Delta t \sqrt{m_\odot r^{5/3}},$$  \hspace{1cm} (16)

where $m_\odot$, $\Delta t$ and $r'$ are in units of solar masses, years and au, respectively. This approximation may be used in all of the above calculations. As an example suppose two observations of an object at 100 au from a solar-type star are made 1 yr apart, and orbital motion is detected between the epochs yielding an astrocentric eccentricity of 0.5. If the object is actually on a circular orbit, then $\tau = 10^{-3}$, and thus from Fig. 7 we see that $\min(\mu)$ is between 0.02 and 0.04 (the actual value is 0.035). Equation (14) shows that in order for $\mu$ to have this minimum value, the inner body must be located at 2.2 au.

3 OUTER OBJECT ON AN ELLIPTICAL ORBIT

We now generalize the above results to allow the outer object to have some eccentricity in the barycentric frame. As before an inner mass could potentially increase this eccentricity in the astrocentric frame. The apparent eccentricity may also now be decreased, i.e. an unseen mass could also make the companion appear less eccentric than it actually is. However, as circular orbits are generally favoured by planet formation models and highly eccentric companions point towards some disruptive dynamical event in the system’s history, we only focus on increasing the companion’s apparent eccentricity in this paper. The magnitude of this effect is expected to be roughly symmetrical, so an unseen mass could potentially increase or decrease the apparent eccentricity of an imaged companion by roughly the same amount. Therefore, the size of the potential eccentricity underestimation may also be estimated by the following method.

3.1 Negligible time between observations

We will proceed as before, by first analysing the $\Delta t = 0$ regime and then extending this to the multiple-epoch case. We again assume
the orbits to be coplanar, and the inner binary orbit is still circular. Equations (1) and (2) now become

\begin{equation}
    r' = \frac{a(1 - e^2)}{1 + e \cos f} \left( \begin{array}{c} \cos (\omega + f) \\ \sin (\omega + f) \end{array} \right) + \mu a_i \left( \begin{array}{c} 1 \\ 0 \end{array} \right)
\end{equation}

and

\begin{equation}
    \psi' = \sqrt{\frac{GM}{a(1 - e^2)}} \left( \begin{array}{c} -\sin(\omega + f) - e \sin \omega \\ \cos(\omega + f) + e \cos \omega \end{array} \right) + \mu \sqrt{\frac{GM}{a_i}} \left( \begin{array}{c} 0 \\ 1 \end{array} \right),
\end{equation}

where \(e\) is the barycentric eccentricity. As for the \(e = 0\) case, we use these equations to derive the outer object’s astrocentric elements. The system now has a 2D phase, given by the argument of periapsis \((\omega)\) and \(f\). The change in elements is maximized when the difference between the stellar motion and that of the outer body is greatest, which occurs when the outer object is at its pericentre. The maximum values of \(a'\) and \(e'\) therefore occur when \(\omega = f = 0\). Proceeding as before, we may again derive a lower limit on the unseen inner mass based on the observed companion’s astrocentric eccentricity, which now depends on its assumed barycentric eccentricity.

Fig. 8 shows the maximum fractional error in observed eccentricity, \(\Delta e'/e'\) \(\equiv (e' - e)/e'\), as a function of \(\mu\) and \(\alpha\) for different observed astrocentric eccentricities. Note that if \(\Delta e'/e' = 1\), then the outer object is on a circular orbit, and also that the errors plotted are positive (i.e. \(e' > e\)). The contours were again calculated using the full expression and we also derive a simplified analytical expression equivalent to equation (10), but it is cumbersome and so given in Appendix A. Fig. 8 is analogous to Fig. 3 as they are both independent of the star mass and semimajor axes.

The plot is qualitatively similar to Fig. 3, as \(\min(\mu)\) still follows a \(\sqrt{\alpha}\) dependence and turns over as other \(\alpha\) terms become non-negligible. We have neglected orbits for which the outer companion has pericentre interior to the orbit of the inner mass, as three-body dynamics would also be important in this region and the results would be incorrect. As for Fig. 3, this plot may be used to determine the maximum error on the derived eccentricity of a companion, given an observational upper limit on the inner mass as a function of its orbital radius.

### 3.2 Non-negligible time between observations

Once again, we consider the use of multiple observational epochs to derive the outer companion velocity. All analytics are now very inelegant and can be sensitive to simplifications, so there is little merit in reproducing them here. However, the resulting plots of \(\min(\mu)\) required to boost \(e\) up to \(e'\) as a function of \(\alpha\) are qualitatively the same as in Fig. 6, but shifted down slightly as \(\min(\mu)\) does not have to be as large. Fig. 6 is not shifted in \(\alpha\) by the introduction of non-zero \(e\), i.e. the global minimum value of \(\min(\mu)\) still occurs at the same ratio of semimajor axes as the \(e = 0\) case. This has been tested numerically across the entire parameter space. Therefore, equation (14) may still be used to locate the value of \(\alpha\) where the inner mass will have the greatest effect, although equation (13) no longer holds.

As the introduction of multiple observations again leads to an absolute minimum value of \(\mu\) required to give an outer object a given astrocentric eccentricity, we may produce a plot analogous to Fig. 7 that shows this minimum mass as a function of \(\tau\). This is presented in Fig. 9 for various astrocentric eccentricities, found using a numerical grid search. Note that the behaviour for \(\tau \gtrsim 10^{-2}\) is similar to that in Fig. 7 due to the breakdown of the linear motion approximation.

This plot may be used to establish whether the apparent eccentricity of an observed companion could be incorrect due to the effect of an unseen inner mass. However, there is one final problem; if the outer object may now have a barycentric eccentricity, we can no longer approximate the parameter \(\tau\) in the same way as before because the barycentric semimajor axis is unknown. However, we may constrain \(\tau\) to lie along a line in \(\Delta e'/e'\) space for the best case...
Figure 9. Minimum \( \mu_{\text{min}} \) required to induce a given fractional error in observed eccentricity, \( \Delta e/e' \), as a function of \( \tau \) for different astrocentric eccentricities. The values of \( \alpha \) corresponding to these minimum masses may still be found using equation (14). Note that the deviations from smooth contours on the right-hand side of the \( e' = 1.0 \) plot are numerical in nature, and should be ignored.
criteria a system must fulfil in order for this scenario to warrant consideration.

We require the system to harbour a directly imaged companion for which orbital motion has been detected, which is also much less massive than its parent star. We also require the parameter $\tau$ to be small in order for the inner body to have the greatest effect; Figs 7 and 9 show that $\tau$ must also be of the order of $10^{-2}$ or smaller if the primary effect on $e'$ is to be caused by an inner mass rather than a breakdown of the linear motion assumption. However, there are bounds on the minimum value of $\tau$ for a given system, set by observational limitations. First, $\Delta t$ may not take any arbitrary value; in reality we have a maximum observational baseline, $\max(\Delta t)$.

Substituting this into the equation for $\tau$, we arrive at an upper limit of

$$\tau < \max(\Delta t) \sqrt{\frac{m_2}{a^3}}. \quad (20)$$

where $\max(\Delta t)$, $m_2$, and $a$ are in units of years, solar masses, and au, respectively. There is also a lower limit on $\tau$, which arises because the difference in the angular separation of the companion between the two observational epochs must be large enough to be resolvable. If the orbit of this object is eccentric, then the largest change in angular position will occur if the orbit is face-on with the object at pericentre. If $\tau$ is small, we may approximate the change in true anomaly to be $\Delta f \approx 2\pi \tau \sqrt{(1 + e)/(1 - e)^2}$ at this point. Between the two epochs, the companion will move by an angular distance of approximately $\left| a(1 - e)/d \Delta f \right|$ at its pericentre as viewed from the Earth, where $d$ is the distance to the system. Therefore, the lower bound on $\tau$ is given by

$$\tau > \frac{3\sqrt{2}\theta_{\text{cen}}}{2\pi} \sqrt{\frac{1 - e}{1 + e} \frac{d}{a}}. \quad (21)$$

where $\theta_{\text{cen}}$ is the 1$\sigma$ half-width centroiding accuracy. The factor of $3\sqrt{2}$ comes from the requirement of a $3\sigma$ detection of orbital motion. If $\theta_{\text{cen}}$ is in radians, then $d$ and $a$ must be in the same units; alternatively if $\theta_{\text{cen}}$ is in arcseconds, then $d$ and $a$ are in parsecs and au, respectively.

The above equations show that for this scenario to be potentially important, the observed system must be nearby (small $d$) with a wide-separation companion, but not so wide that orbital motion is undetectable. Eliminating $\tau$ from the above equations, for orbital motion to be detected the semimajor axis must fulfil

$$a < m_2 \left( \frac{1 + e}{1 - e} \left( \frac{2\pi \max(\Delta t)}{3\sqrt{2}\theta_{\text{cen}} d} \right)^2 \right). \quad (22)$$

(again, $3\sqrt{2}$ comes from the requirement of a $3\sigma$ detection of orbital motion).

Additionally an absolute lower limit on $a$ is $d\theta_{\text{res}}/2$, where $\theta_{\text{res}}$ is the full-width instrument angular resolution. This arises from the observable star–companion separation, and is not $m_2$, or $e$ dependent. This is a lower bound because in general the detection of companions close to the star is contrast limited as opposed to resolution limited. Therefore, only very massive companions may be observed down to the ‘currently unresolvable’ limit, and lower mass objects must lie farther out to be detected. For example, whilst the resolution of the HST is 0.2 arcsec, its effective inner working angle is actually about 0.7 arcsec in the infrared (Krist 2006). The dependence of detectability on mass is not an issue for the outermost companions in this paper, because the analysis presented here is independent of this quantity provided that the imaged object is not massive enough to significantly perturb the inner binary. However, this dependence will affect our ability to detect an inner object, as
Effect of an unseen mass on derived orbits

Even a significant mass may be lost in the glare of the star if its semimajor axis is small enough (see Section 6).

Fig. 11 shows all of the above limits on $a$ as functions of $d$ for the $e = 0$ case, as well as the maximum and minimum values of $\tau$ from equations (20) and (21). We assume a maximum baseline of 10 yr and a resolution of 0.2 arcsec, that of the HST, for the above equations. The centroiding accuracy is taken to be 0.01 arcsec, which may be reached by current observations (e.g. Golimowski et al. 1998; Kasper et al. 2007; Neuhäuser et al. 2008, 2010). In fact, some observations have achieved even better accuracies; however, we will use 0.01 arcsec as a typical value because we feel it is a better representation of the current level of precision. We also plot the projected separations and distances for a selection of substellar companions detected by imaging (compiled from Reid et al. 2001; Wilson et al. 2001; Metchev & Hillenbrand 2004, 2006; Chauvin et al. 2005; Zuckerman & Song 2009; Tanner, Gelino & Law 2010; Rodriguez et al. 2012; Schneider et al. 2011). Companions with probable ($\geq 3\sigma$) and possible detections of orbital motion are highlighted, and we give details of these objects in Table 1. This should give the reader a feel for the region of parameter space occupied by these objects, in relation to that which may be important for the scenario described in this paper. Note that we have not plotted the astrocentric semimajor axes of the companions; we assume nothing about their orbits or orientations. However, the projected separations should provide order-of-magnitude approximations of $a$ sufficient for this plot.

As already stated, the effect of an inner mass on the derived elements of an outer companion will be most important if $\tau$ is small. This condition means that systems most susceptible to this effect would lie as high up Fig. 11 as possible (but below the upper limit for detectable orbital motion). Figs 7 and 9 suggest that $\tau \lesssim 10^{-2}$ for the inner body to significantly affect the derived elements of the outer body. It is clear that the area of parameter space where this scenario could be applicable is well populated by companions, so could be important for around half of currently imaged systems. Also note that the solid line scales as $(\max(\Delta t)/\theta_{\text{cen}})^2$, so as observational techniques improve and achieve longer time baselines, more objects could be discovered that would be susceptible to this scenario.

The next-generation Gemini Planet Imager (GPI; Macintosh et al. 2006) and the Spectro-Polarimetric High-contrast Exoplanet Research instrument (SPHERE; Dohlen et al. 2006) are expected to discover many companions within 100 au of young stars at 30–50 pc.
Whilst not populating the upper regions of Fig. 11, many of these objects could still be susceptible to eccentricity errors caused by unseen inner masses. Furthermore, as better centroiding precisions are achieved by these projects and others, orbital motion will be detectable using observations covering smaller fractions of companion orbits. This means that lower \( \tau \) values will be reached, and hence these objects would be more susceptible to eccentricity errors induced by unseen inner masses. Therefore, we conclude that the effect described in this paper could already be significant for many directly imaged systems and will be applicable to more imaged companions in the future.

### 5 HOW TO USE THIS PAPER

#### 5.1 Suggested method

In the above three sections, we described the set-up of the problem. We now suggest a step-by-step method to establish whether a derived astrocentric eccentricity is likely to be incorrect due to the presence of an inner object. We then apply this method to some example systems.

(i) Is the system suitable? It should have a known stellar mass and at least two images of the companion, between which orbital motion is observed. The user should also have estimates of the de-projected companion–star separation \( r' \) and astrocentric eccentricity \( e' \). If the inclination is unknown, then a lower bound on this eccentricity may be derived by varying the assumed line of sight position and velocity until a minimum is found. If the stellar rotation axis is known, then the inclination may be estimated by assuming that the star and companion are coplanar (e.g. Le Bouquin et al. 2009; Watson et al. 2011; Kennedy et al. 2013). The system should also lie above the horizontal \( \tau < 10^{-1} \) line in Fig. 11.

(ii) Could the companion be on a circular barycentric orbit? To establish this, estimate \( \tau \) using equation (16) and find the minimum inner mass required to give the observed \( e' \) using Fig. 7. Calculate the semimajor axis of this mass using equation (14); if an object with this mass and location cannot be ruled by observation, then it is possible for the imaged object to be on a circular orbit about the star–inner mass barycentre. If the inner mass is observationally excluded in this region but a larger mass may exist further out, use Fig. 3 or equation (5) to see if this object may lie farther from the star. Remember that these relations are not valid all the way up to \( \alpha = 1 \) due to three-body dynamics. Also note that the mass may exist closer in than the equation (14) value, but as the required mass rises steeply with decreasing distance (Fig. 6), this is a less useful consideration.

(iii) If the companion must be eccentric, what is the maximum error in this barycentric eccentricity that could be caused by an inner body? First, estimate \( \tau \) as a function of \( \Delta e/e' \) using equation (19), similar to that plotted in Fig. 10. Locate the graph in Fig. 9 that corresponds to the observed value of \( e' \), and overlay this \( \tau \) constraint on it. For each combination of \( \min(\mu) \) and \( \tau \) that lies in along this line, establish whether such an inner mass could exist at a distance given by equation (14), noting that \( \alpha \) will have to be estimated as \( \alpha/(1 - e) \). If it may, then use Fig. 9 to read off the maximum eccentricity error \( \Delta e/e' \) corresponding to this combination of \( \tau \) and \( \min(\mu) \). Again, if the inner object cannot exist in this region but a larger mass cannot be excluded further out, use Fig. 8 or the equation in Appendix A to calculate this minimum mass.

If an observer wishes to minimize the effect of this scenario on a system, the best technique would be to make multiple observations over a long baseline. This would increase \( \tau \) and thus reduce the average stellar velocity. The use of more than two observations would also reduce the effect of the third body, as the observed astrocentric elements would oscillate over time and so would vary depending on the pair of observations used to derive them. This could be used in some cases to exclude inner masses with periods shorter than the longest baseline.

#### 5.2 An example: the Fomalhaut system

Fig. 11 shows that the area of separation–distance parameter space where an unseen inner mass could affect the derived eccentricity...
of an observed object is well populated by companions. However, orbital motion has not been detected for many of these objects as the required observations are yet to be made. If additional measurements are taken in the near future, for example in an attempt to build up an exoplanet/brown dwarf eccentricity distribution, then the presence of unseen masses could induce significant errors on this distribution. However, until such measurements are made, the most susceptible planetary system is Fomalhaut, which we will use here as an example to demonstrate the above method.

The star has a mass of $1.92 M_\odot$, with a directly imaged planet (Fomalhaut-b) at 100 au in projection for which orbital motion has been observed over four epochs between 2004 and 2012 (Kalas et al. 2013). $\tau$ therefore lies between $10^{-4}$ and $10^{-2}$ from Fig. 11. The system also contains a narrow debris disc (Kalas, Graham & Clampin 2005); if the planet is assumed to lie in the plane of this disc, then its astrocentric position and velocity are $\sim 120$ au and $\sim 1$ au yr$^{-1}$, respectively, yielding an astrocentric eccentricity of about 0.8 (Kalas et al. 2013). Thus, this system is suitable for the method outlined in this work.

We first test the hypothesis that the planet’s orbit is actually circular in a barycentric frame and is aligned with the disc. Using equation (16) with $\Delta t = 7.6$ yr, we calculate $\tau$ to be 0.008. Fig. 7 shows that for Fomalhaut-b to be on a circular orbit with an inner mass giving it an astrocentric eccentricity of 0.8, the unseen inner companion must have $0.07 < \min(\mu) < 0.1$ for this value of $\tau$. Equation (14) shows that such a planet would exist at $\alpha = 0.09$, and therefore $a_1 = 11$ au. Such a planet is ruled out by photometric non-detections (Kenworthy et al. 2009, 2013), which place a model-dependent upper mass limit for $a_1 > 5$ au of $12-20 M_J (\mu \sim 0.006-0.01)$. This limit is an order of magnitude lower than required, so Fomalhaut-b cannot be on a circular orbit coplanar with the disc with its apparent eccentricity caused by an inner planet.

We now relax the condition that Fomalhaut-b must lie in the disc plane, to establish whether it is possible for the companion to have a circular orbit in any orientation. By varying the assumed line-of-sight position and velocity components of the companion, we derive a lower bound on its astrocentric eccentricity, which is 0.5–0.8 depending on the pair of observation epochs used. If we assume the orbital plane that gives $e' = 0.5$, then $a = 166$ au and $\Delta t = 1.7$. Therefore, $\tau = 0.001$, and Fig. 7 shows us that in order for Fomalhaut-b to be circular with this apparent eccentricity, $\min(\mu)$ must be between 0.02 and 0.04. This is still higher than the upper bound from observations, and so Fomalhaut-b has a barycentric eccentricity regardless of the chosen orbital plane.

Given that the orbit of Fomalhaut-b cannot be circular, we now wish to establish the maximum error in its barycentric eccentricity that could be caused by an observationally allowed unseen mass. We revert to the case where the planet and disc are coplanar. We first use equation (19) to constrain $\tau$ as a function of the barycentric eccentricity, as we may no longer estimate $\tau$ using equation (16) because we have no information about the true semimajor axis. Note that equation (19) again assumes the companion to be at its pericentre as this is the most favourable case for the scenario described in this paper. We overlay this $\tau$ constraint on the $e' = 0.8$ plot from Fig. 9, which is shown for the relevant parameters in Fig. 10. We know from the observational upper limits that $\mu \leq 0.006$, so the maximum value of $\Delta e/e'$ that this mass may induce occurs when the $\tau$ constraint intersects the $\mu = 0.006$ contour. This is at $\Delta e/e' = 0.088$, which corresponds to $e \geq 0.73$. Therefore, using equation (12) we see that a $12 M_J$ mass at $a_1 = 10$ au could introduce a 10 per cent error on Fomalhaut-b’s eccentricity.

Note that as we have four observations of Fomalhaut-b, we know that the linear velocity approximation is still good over at least eight years. Even though the inner planet/brown dwarf described above would have a period of about 20 yr, we cannot use b’s constant velocity to rule out such an object because the changing velocity of the star itself would be undetectable; the star would have moved by about $2\mu as = 0.1$ au (0.01 arcsec at 7.7 pc) over eight years, which would change the planet’s velocity by 1 per cent and would therefore be undetectable with the current precision. Furthermore, the uncertainties in $e'$ from the astrometry and assumptions about the orbital plane are large enough that the use of more than one pair of observations cannot rule out the scenario outlined above. We therefore conclude that, whilst Fomalhaut-b cannot be on a circular barycentric orbit, an unseen 12 $M_J$ companion at 10 au could result in an $\sim 10$ per cent overestimation of its astrocentric eccentricity, so it serves as a good example of a possible use of the above method.

### 5.3 Other example systems

In addition to Fomalhaut-b, we also applied the method to other applicable systems from Table 1. We excluded $\beta$ Pic-b as its $\tau$ value is too high, and HR7672-B due to RV constraints on the inner mass. We also excluded the 2M 0103(AB), HR 8799 and TWA 5 systems from analysis as they are known to host more than one companion, and the current method is therefore unsuitable. Note however that we do include HD 130948, in which the companion itself is a known binary, by treating the pair as a single object.

All of these companions have orbital motion which appears linear over the observational baseline, and their sky plane positions and velocities were therefore derived by fitting linear trends. However, unlike Fomalhaut, the majority of these systems do not have any additional information with which to predict the plane of the companion’s orbit. $\eta$ Tel does have a debris disc which is close to face-on, so if the companion and disc are aligned, then the former may be assumed to orbit in the sky plane (Smith et al. 2009). However, it is not clear that the two objects should necessarily be aligned, as the companion lies much farther from the star than the disc. Therefore, we derive astrocentric eccentricities for all companions in two extreme cases. In the first case, we assume that the orbit is constrained to the sky plane, and in the second case we assume that the orbit is orientated in such a way that the astrocentric eccentricity is minimized. Assuming that the companions are actually on circular orbits, we calculate the locations and masses of the lightest inner objects required to give the observed eccentricities. The results are shown in Table 2.

First, we consider the case where the orbits are assumed to lie in the sky plane. Four of these companions (GI 229-B, GJ 504-b, GQ Lupi-B and GSC 06214-00210-b), in spite of their high astrocentric eccentricities, could actually be on circular orbits with eccentricity errors introduced by observationally allowed inner masses. Furthermore, whilst the inner masses required for $\eta$ Tel-B and HD 130948-B, -C to be circular in the sky plane are observationally excluded, the maximum allowed inner masses could introduce eccentricity errors of 30 and 20 per cent, respectively. We found no upper mass limits in the literature for companions $\sim 5$ au from PZ Tel-A; however, the $\sim 200 M_J$ inner mass required to cause a large eccentricity error should be easily detectable. Therefore, this scenario may be quickly confirmed or excluded for this system.

In the second case, where the assumed line-of-sight position and velocity are varied, we see that many of the companions could actually be on circular orbits. However, three of the systems must have some astrocentric eccentricity regardless of the orbital plane.
Of these, Gl 229-B and GQ Lupi-B could be on circular orbits with their apparent eccentricity induced by unseen inner masses. Again we have no upper mass limits from the literature for companions close to PZ Tel-A; however, the 130 $M_J$ required could well be detectable with current instruments.

It is clear that the inner objects required for this scenario are typically tens of Jupiter masses, and many of them would inhabit the brown dwarf desert which could make their existence unlikely (Marcy & Butler 2000). However, it must be noted that brown dwarfs are occasionally observed in these locations (e.g. De Lee et al. 2013), and hence such objects may not be rejected purely due to this consideration.

### 6 DETECTABILITY OF THE UNSEEN MASS

Throughout this paper we have required a massive, unseen inner object to significantly affect the orbital elements of an outer companion. Such large masses may often be ruled out using observational constraints; however, we emphasize that this is by no means the case for all systems. We now summarize several detection methods which may provide upper limits on these masses.

First, there are limits from the images themselves. Whilst direct imaging is the best means to detect wide-separation companions, it is less suited to objects closer to the star owing to the huge contrast between the stellar flux and that of a smaller mass. The contrast ratio between a Jupiter mass planet and a solar-type star is $10^{-3}$ in the infrared (Traub & Oppenheimer 2010), and that of a brown dwarf to such a star is $10^{-7}$–$10^{-10}$ for L0 to T6 dwarfs, respectively (fig. 2.9, Bernat 2012). Whilst such contrast sensitivities are just beginning to be reached at wide angles from stars, companion detectability rapidly worsens closer in. For example, the Gemini Deep Planet Survey of young, nearby FGKM stars did achieve contrast sensitivities of $10^{-7}$ in some cases, but this value degraded sharply within 4 arcsec of the stars to around $10^{-5}$ at 1 arcsec (Lafrenière et al. 2007). Similarly, the International Deep Planet Survey of A and F stars, which also focused on young nearby systems, reached contrast ratios of $10^{-7}$ for some objects but only at separations greater than 6 arcsec (Vigan et al. 2012). At the distances of stars with wide-separation imaged companions (fig. 11), these angular scales correspond to tens or hundreds of au, which are much larger than the semimajor axes of masses typically required to introduce a significant error on an outer body’s eccentricity.

Projects such as SPHERE (Dohlen et al. 2006), GPI (Macintosh et al. 2006) and Project 1640 (Hinkley et al. 2011) should significantly increase the contrast sensitivity close to the star, and could rule out some of the inner companions required in this paper. However, even these instruments would struggle to identify objects of several to tens of Jupiter masses 5–10 au from stars at 50 pc (fig. 4, Beichman et al. 2010). A further problem lies in actually converting these contrast sensitivities into upper mass limits; this is not a problem in older systems, but is a challenge for objects orbiting young stars. This is because at early ages ($\lesssim 100$ Myr) hot and cold start models produce significantly different estimates of a companion’s luminosity (Spiegel & Burrows 2012). Young companions are also brighter (Marley et al. 2007) and hence more likely to be detected, which means that this problem is commonly encountered. All these considerations mean that for now direct imaging is not the best means to locate or exclude inner objects, although in many cases it provides the only mass constraint in the absence of any other detection method being applied.

The best upper limits on the masses of potential inner companions are likely to come from the RV technique, which is very good at detecting large objects orbiting close to the star. For a circular inner mass to be undetected to 3σ if RV data are available for at least half an orbit,\footnote{\[ \mu \sin i \lesssim 3 \times 10^{-4} \left( \frac{m_s}{1M_\odot} \right)^{1/2} \left( \frac{a_i}{10 \text{ au}} \right)^{1/2} \frac{K}{1 \text{ m s}^{-1}}, \]}

$$
\mu \sin i \lesssim 3 \times 10^{-4} \left( \frac{m_s}{1M_\odot} \right)^{1/2} \left( \frac{a_i}{10 \text{ au}} \right)^{1/2} \frac{K}{1 \text{ m s}^{-1}}
$$

where $i$ is the inclination ($i = 0$ being face-on) and $K$ is the 1σ RV sensitivity. $i$ is of the order of 1 m s$^{-1}$ for current techniques (Pepe et al. 2011). Whilst this method is therefore sensitive enough to rule out many unseen masses of the type described in this paper, it is not without its limitations. Most importantly, the detection limit in equation (23) becomes significantly degraded when the observational baseline is longer than the inner object’s orbital period. Indeed, even if the acceleration from the companion remains at a detectable level, the interpretation of such long-term RV trends remains unknown until one full orbital period has been sampled (e.g. Crepp et al. 2013). In addition, the star must be spectrally stable, which means RV is less effective at finding companions in young systems and particularly about A stars (e.g. Galland et al. 2006). Rotational broadening is also a problem for these stars. However, it is in these systems that imaging of outer companions is most successful, due to the decreasing companion luminosity with time (Baraffe et al. 2003). For this reason, RV and imaging surveys do not usually target the same stars, although there is some overlap between the two techniques. Finally, RV cannot detect companions if the system is face-on, which is the orientation favoured if the outer companion’s motion is to be determined via direct imaging. As a result of these caveats, many stars, including almost all of the

### Table 2. Locations and masses of the least massive inner objects required to give observed companions certain astrocentric eccentricities, if the companions are in fact on circular orbits. The eccentricities in the $e_{i=0}$ column have been derived with their orbits confined to the sky plane, and those in the $e_{\text{min}}$ column have been calculated by varying the assumed line-of-sight position and velocity until a minimum $e'$ was found. The ‘Allowed?’ column states whether this inner mass is observationally permitted. Note that $\eta$ Tel has a debris disc which lies roughly in the sky plane. We found no upper mass limits in the literature for companions $\sim 5$ au from PZ Tel-A.

| Companion            | $e_{i=0}$ | $\tau$ | $m_i$ (M$_J$) | $a_i$ (au) | Allowed? | $e'_{\text{min}}$ | $m_i$ (M$_J$) | $a_i$ (au) | Allowed? |
|----------------------|-----------|--------|---------------|------------|----------|------------------|---------------|------------|----------|
| Gl 229-B             | 0.9       | 0.003  | 47            | 1.8        | Yes      | 0.3             | 0.002         | 15         | 1.8      | Yes      |
| PZ Tel-B             | 1.0       | 0.04   | 210           | 5.5        | –         | 0.6             | 0.006         | 130        | 5.5      | –         |
| $\eta$ Tel-B         | 1.0       | 0.006  | 260           | 14         | No        | 0.0             | 0.006         | –          | –        | –        |
| GI 504-b             | 0.3       | 0.004  | 49            | 2.6        | Yes      | 0.0             | 0.004         | –          | –        | –        |
| GQ Lupi-B            | 0.9       | 0.002  | 50            | 3.7        | Yes      | 0.9             | 0.002         | 50         | 3.7      | Yes      |
| GSC 06214-00210-b    | 0.2       | 0.0004 | 5             | 3.7        | Yes      | 0.0             | 0.0003        | –          | –        | –        |
| HD 130948-B, -C      | 1.0       | 0.03   | 18            | 10         | No       | 0.0             | 0.030         | –          | –        | –        |
systems in Fig. 11, do not have RV data so large inner masses may not be ruled out. Indeed, the detection of an eccentric outer companion could provide motivation for an RV follow-up, to investigate whether an unseen inner mass is also present and responsible for this apparent high eccentricity.

Finally, stellar astrometry is also reaching the sensitivities required to detect companions, and this method is most sensitive to face-on orbits so could detect those missed by RV. However, this method also requires a baseline longer than the inner mass period. If such precision astrometry is available, the object may remain undetected to 3σ if

\[ \mu \gtrsim 2 \times 10^{-2} \left( \frac{a_1}{10 \text{ au}} \right)^{-1} \frac{d}{50 \text{ pc}} \frac{\theta_{\text{ast}}}{1 \text{ mas}}, \]  

(24)

where \( \theta_{\text{ast}} \) is the astrometric accuracy, which is currently of the order of 1 mas if many reference stars are available in the same field (e.g. Benedict et al. 2002; Sozzetti 2005). Due to the required baseline, it is still possible for Jupiter–brown dwarf mass objects to exist at \( \gtrsim 10 \text{ au} \) and remain undetected by precision astrometry.

The upcoming Gaia mission will bring about a significant improvement in astrometric precision, promising to reach sensitivities of 8 μas (Casertano et al. 2008). However, even Gaia will not be able to rule out many massive inner companions, due to the requirement that it observes the star for at least one full orbit of the inner body. Using the detection limits from figs 21 and 22 in Casertano et al. (2008) we see that whilst Jupiter mass planets could be detected to 3σ at 2–3 au from solar-type stars out to 200 pc, objects with significantly larger masses could still lie further out as the companion period increases beyond the 5 yr lifetime of the mission. In fact, brown dwarf mass objects could still exist undetected down to 10–20 au from 1 M⊙ stars at 10 pc, which means that Gaia will not be able to rule out many of the inner objects required for the scenario in this paper.

7 DISCUSSION

We have shown that the orbital elements of an imaged companion may be incorrectly derived due to the presence of an unseen inner mass. We demonstrated that a circular object would always appear eccentric if an inner mass were introduced, and showed that a non-negligible time between observations reduces the effect of this unseen mass on the companion’s orbital elements. We then provided a framework to identify the maximum eccentricity error an unseen mass could introduce as a function of readily derivable parameters, and also found the optimum location of such an object. We have demonstrated that many imaged companions could potentially be susceptible to this error, and showed that the eccentricity of Fomalhaut-b could have been overestimated by up to 10 per cent. Fomalhaut-b could have been overestimated by up to 10 per cent. Fomalhaut-b could have been overestimated by up to 10 per cent. Fomalhaut-b could have been overestimated by up to 10 per cent. Fomalhaut-b could have been overestimated by up to 10 per cent.

We have shown that the orbital elements of an imaged companion may be incorrectly derived due to the presence of an unseen inner mass. We demonstrated that a circular object would always appear eccentric if an inner mass were introduced, and showed that a non-negligible time between observations reduces the effect of this unseen mass on the companion’s orbital elements. We then provided a framework to identify the maximum eccentricity error an unseen mass could introduce as a function of readily derivable parameters, and also found the optimum location of such an object. We have demonstrated that many imaged companions could potentially be susceptible to this error, and showed that the eccentricity of Fomalhaut-b could have been overestimated by up to 10 per cent. Fomalhaut-b could have been overestimated by up to 10 per cent. Fomalhaut-b could have been overestimated by up to 10 per cent. Fomalhaut-b could have been overestimated by up to 10 per cent. Fomalhaut-b could have been overestimated by up to 10 per cent.

Effect of an unseen mass on derived orbits
optimum case, where both bodies are at their pericentres at the time of the observation, is very rare because the bodies do not spend much time around their pericentres. The odds of making an observation in this configuration are therefore very low. Hence, we conclude that, whilst the eccentricity error on an observed companion may be increased if the inner body is eccentric, in practice the error is almost always reduced in this regime. We have therefore not considered this any further.

8 CONCLUSIONS

We have shown that the use of direct imaging to derive the orbital elements of companion planets/brown dwarfs could lead to significant errors if an undetected inner mass is also present. The maximum effect of such a body on the derived eccentricity (and hence the semimajor axis) of the observed companion has been quantified for various cases, and we have also identified criteria to determine when this effect may be significant. We provide the reader with a step-by-step method to determine the maximum magnitude of this effect for any system, and apply it to several companions as examples. It appears that many of the currently imaged companions could be susceptible to this scenario when they have orbital motion detected.

ACKNOWLEDGEMENTS

We would like to thank Paul Kalas for discussing his 2013 paper before its publication, and our anonymous referee for their helpful comments and suggestions. TDP acknowledges the support of an STFC studentship, and MCW and GMK are grateful for support before its publication, and our anonymous referee for their helpful comments and suggestions.
APPENDIX A: MIN(μ) EQUATION FOR THE e ≠ 0 CASE

Here we present a simplified equation for the minimum value of μ required to give an eccentric outer object an astrocentric eccentricity e' if the time between observations is small. This is analogous to the e = 0 case given by equation (5), and shows good agreement with the lines in Fig. 8. As for the simpler case, the full version of this expression contains high orders of μ; however, terms arising from orders greater than two are now less dominant because the equation now includes a first-order term inside the square root. Therefore, the following formulae (up to second order in μ) are sufficiently accurate without an empirical scaling factor:

\[ A\mu^2 + B\mu + C \geq 0, \quad (A1) \]

where

\[ A \equiv \alpha^2 \frac{(1 + e)(3 + e)}{(1 - e)^2} + (2e + 1) \times \left( \frac{1 + e}{1 - e} + 4\sqrt{\alpha} \sqrt{1 - e} \frac{1 + e}{1 - e} + 4\sqrt{\alpha} \sqrt{1 - e^2} \right) \]

\[ + \frac{2}{\alpha} (2 + 3e)(1 - e) + (1 + e)^3, \]

\[ B \equiv 2e \left( 1 + e + \alpha \frac{1 + e}{1 - e} + \frac{2}{\sqrt{\alpha}} \sqrt{1 - e^2} \right), \]

\[ C \equiv e^2 - e'^2, \]

which may be solved for μ. Note that as e → 0, B → 0 and the solution tends to that of equation (5), and so in this case the empirical factor F(e') will again be required.

This paper has been typeset from a \TeX/LaTeX file prepared by the author.