Observation of pairs of atoms at opposite momenta in an equilibrium interacting Bose gas

Antoine Tenart, Gaëtan Hercé, Jan-Philipp Bureik, Alexandre Dareau and David Clément

Quantum fluctuations play a central role in the properties of quantum matter. In non-interacting ensembles, they manifest as fluctuations of non-commuting observables, quantified by Heisenberg inequalities. In the presence of interactions, additional quantum fluctuations appear, from which many-body correlations and entanglement arise. Weak interactions are predicted to deplete Bose–Einstein condensates by the formation of correlated pairs of bosons with opposite momenta. Here we report the observation of these atom pairs in the depletion of an equilibrium interacting Bose gas. Our measurements of atom–atom correlations, both at opposite and close-by momenta, allow us to characterize the equilibrium many-body state. We also show that the atom pairs share the properties of two-mode squeezed states, including relative number squeezing. Our results illustrate how interacting systems acquire non-trivial quantum correlations as a result of the interplay between quantum fluctuations and interactions.

Interaction-induced quantum correlations and entanglement are essential features of many-body physics that arise from interactions. Understanding these properties poses challenges for both experimentalists and theorists. In experiments, various methods can now probe the correlations between individual particles in interacting systems. Therefore, quantitative tests of many-body theories become accessible in regimes where theoretical predictions exist. A conceptually simple example, which we study here, is that of weakly interacting bosons, described by the celebrated Bogoliubov theory.

In a seminal paper, N. Bogoliubov introduced a theoretical description of weakly interacting bosons, motivated by the observation of the superfluidity of liquid helium. In that context, an emblematic prediction of this theory is the linear spectrum of excitations as fluctuations of non-commuting observables, quantified by Heisenberg inequalities. A second central result is a microscopic description of the many-body ground state as a macroscopic occupation of the lowest-energy state of the trap, namely, a Bose–Einstein condensate (BEC), and a fraction of bosons outside the BEC—namely, quantum depletion—originating from interaction-induced quantum fluctuations. Quantum depletion is expected to extend over a large momentum range and features correlations between opposite momenta \(k/−k\). Measurements of momentum density \(\rho(k)\) have confirmed the presence of these large momentum components and Bogoliubov’s prediction of the quantum depleted fraction. However, ground-state correlations at opposite momenta have not yet been observed.

An intuitive picture of the origin of \(k/−k\) correlations can be derived from Bogoliubov’s theory that assumes weak interactions and only considers processes involving two atoms in the BEC whose momenta are close to zero. To ensure momentum conservation after the interaction, the two bosons leaving the BEC (to populate the quantum depletion) have opposite momenta (that is, \(k/−k\)), and they form a momentum-correlated pair. Although apparently simple, this mechanism should be contrasted with nonlinear processes generating similar momentum-correlated pairs in out-of-equilibrium configurations, where nonlinearities efficiently drive resonant processes with both momentum and energy conservation and for which (semi-)classical pictures hold. Out-of-equilibrium examples range from parametric downconversion (PDC) in quantum optics and the dissociation of diatomic molecules in atomic physics to elastic collisions in high-energy physics and at ultralow temperatures. In contrast, individualizing the creation of a single \(k/−k\) pair in the equilibrium many-body ground state would violate (kinetic) energy conservation, indicating a failure of classical pictures in the case of quantum depletion. This highlights the collective nature of these two-body interactions, delocalized over the BEC wave function, as a result of quantum fluctuations.

These interaction-induced quantum fluctuations create a coherent superposition of \(k/−k\) pairs that play a central role in the equilibrium properties of interacting bosons. Though their origin differs from that of PDC photon pairs, the Hamiltonian terms of pair creation are analogous and lead to similar paired states. Therefore, we expect the correlations between momenta \(k\) and \(-k\) to be analogous to those of two-mode squeezed states. Such correlations illustrate the non-trivial nature of the Bogoliubov many-body ground state as two-mode squeezed states constitute a primary resource of entanglement with continuous variables. Observing these fascinating properties requires probing interacting bosons with a single-particle resolution in momentum space. To this end, our quantum-gas experiment combines a long time of flight (TOF) to access momentum space, as well as a three-dimensional (3D) detection method of individual metastable helium-4 (‘He) atoms.

The experiment starts with the production of weakly interacting ‘He BECs of a few thousand atoms in a 3D optical lattice. The lattice amplitude is fixed at \(V = 7.75E_r\), where \(E_r = \hbar^2 / (2m d^2)\) is the recoil energy, \(\hbar\) is the Planck constant, \(m\) is the atomic mass, \(d = 775\) nm is the lattice spacing and \(k_r = 2\pi / d\) is the associated momentum scale. The shallow lattice allows us to enhance interactions and increase quantum depletion (from 0.2% to 5.0%), although it remains in the validity range of the Bogoliubov description. The lattice is then abruptly switched off and the gas is allowed to expand during a long TOF of 296 ms. In a TOF experiment from a lattice, interactions are effectively switched off abruptly and the in-trap many-body wave function is projected onto the momentum basis. Therefore, the measured 3D atom coordinates yield the in-trap momentum of each atom using a ballistic relation. In the reconstructed atom distributions, we can post-select atoms belonging to a specific
volume $\Omega_g$ and compute atom–atom correlations over $\Omega_r$. We use this to exclude atoms from the BEC and only study the depletion (Fig. 1b). Finally, statistical averages are obtained from recording about 2,000 atom distributions (Methods). To identify pairs, we use the following integrated atom–atom correlations:

$$g^{(2)}_A(\delta k) = \frac{\int d^3\mathbf{k} \langle a(\mathbf{k})a(\mathbf{k}+\delta \mathbf{k})a(\mathbf{k})a(\mathbf{k}+\delta \mathbf{k}) \rangle}{\int d^3\mathbf{k} \rho(\mathbf{k})\rho(\delta \mathbf{k}-\mathbf{k})},$$

(1)

where $\rho(\mathbf{k}) = \langle a^\dagger(\mathbf{k})a(\mathbf{k}) \rangle$. With this definition, a peak located at $\delta \mathbf{k} = 0$ signals pairs of atoms at opposite momenta.

In Fig. 1c, we present one-dimensional (1D) cuts of the pair correlations $g^{(2)}_A(\delta k)$ measured in the depletion of lattice BECs, and observe a peak located at $\delta \mathbf{k} = 0$. For these data, we find, on average, about 100 atoms and 0.5 atom pairs per shot in $\Omega_g$ (Supplementary Information). A crucial experimental parameter for obtaining this signal is the detection efficiency, which we have recently increased to 0.53(2) (Methods). The observation of atom pairs with opposite momenta in the depletion of equilibrium interacting BECs is a central result of this work. Identifying their origin, however, requires accounting for the effect of temperature.

In our experiment, temperature $T$ should increase the thermal population of depletion without contributing to the $k' - k$ correlations. This is because we probe large momenta corresponding to single-particle excitations of the Bogoliubov spectrum (see below). Therefore, when the temperature increases, the number of pairs becomes a negligible fraction of the total depletion, making their detection nearly impossible. This suggests that the $k' - k$ peak rapidly vanishes with temperature, a sensitivity limiting its range of observation but also providing us with a means to confirm its origin. Indeed, an essential aspect of our experiment is the ability to produce BECs in the low-temperature regime, namely, $k_b T \ll \mu$, where thermal depletion ($\sim 10\%$; Fig. 1) is not much greater than quantum depletion ($\sim 5\%$; Fig. 1). Here $k_b$ is the Boltzmann constant and $\mu$ is the chemical potential. This low-temperature regime, namely, $k_b T / \mu \approx 0.3$ (Fig. 1), is accessible in the lattice because it enhances interactions.

To study temperature sensitivity of the $k' - k$ peak, we slightly increase the gas temperature to maintain a large BEC (Methods), and repeat the correlation measurement. The two datasets (non-heated and heated) are shown in Fig. 2. The increase in temperature translates into an increase in density $\rho(\mathbf{k})$, visible in the log-scale plot shown in Fig. 2b. No $k' - k$ peak is visible in the heated BEC, confirming that a finite temperature does not contribute to $k' - k$ correlations (Fig. 2a). Our description is further validated by the observation of a $k' - k$ peak of intermediate amplitude at intermediate temperature (Supplementary Information).

It is also illuminating to contrast the temperature sensitivity of $k' - k$ correlations with that of local correlations at $k' \approx k$. These local correlations reflect bosonic bunching and are quantified by

$$g^{(2)}_N(\delta k) = \frac{\int d^3\mathbf{k} \langle a(\mathbf{k})a(\mathbf{k}+\delta \mathbf{k})a(\mathbf{k})a(\mathbf{k}+\delta \mathbf{k}) \rangle}{\int d^3\mathbf{k} \rho(\mathbf{k})\rho(\delta \mathbf{k}+\mathbf{k})},$$

(2)

where a peak located at $\delta \mathbf{k} = 0$ signals bunching. In Fig. 2c, we plot $g^{(2)}_N(\delta k)$ for both low-temperature and heated datasets. We find...
similar bunching in both cases. The contrasted behaviour of $k^-k$ and $k/k$ correlations with temperature highlights their different nature: the $k^-k$ correlations reflect quantum correlations present at $T=0$ induced by the atom pairs; the $k/k$ correlations correspond to bosonic bunching revealing thermal (chaotic) statistics, independent of $T$. Having identified the origin of the $k^-k$ correlations, we proceed with a quantitative characterization to assess their role in the properties of the equilibrium interacting Bose gas.

Our microscopic picture of correlations builds on the Bogoliubov approximation, according to which the Hamiltonian of weakly interacting bosons is diagonal in the quasi-particle basis. Therefore, all the quantum states have Gaussian statistics and Wick’s theorem applies. Two-body correlations can thus be decomposed into two parts: $\langle a^\dagger(k)a^\dagger(k')a(k)a(k') \rangle = G_{\Delta}^{(2)}(k,k') + G_{N}^{(2)}(k,k')$, where $G_{\Delta}^{(2)}(k,k') = |\langle a^\dagger(k)a(k') \rangle|^2$ are anomalous correlations and $G_{N}^{(2)}(k,k') = \rho(k')\rho(k) + |\langle a^\dagger(k)a(k') \rangle|^2$ are normal correlations. We base our analysis on the fact that the contributions of anomalous and normal correlations are well separated in momentum space. Indeed, anomalous correlations are present only for $k' \sim -k$ (ref. 29) and reflected in the momentum-integrated function $g_{\Delta}^{(2)}$, whereas normal correlations are present only for $k' \sim k$, manifesting in $g_{N}^{(2)}$. Furthermore, although quantum depletion exhibits both anomalous and normal correlations, thermal excitations contribute only to $g_{N}^{(2)}$ here, because only thermal excitations with a strong phononic character, that is, at momenta $|k| \ll 1$ ($\xi$ is the BEC healing length), exhibit anomalous correlations. In the probed range of momenta ($0.85 \lesssim |k| \lesssim 1.15$), this thermal contribution is negligible. To compare Bogoliubov’s prediction to the experiment, we fit the bell-shaped correlation peaks with a Gaussian function, from which we extract the amplitudes ($g_{\Delta}^{(2)}(0)$ and $g_{N}^{(2)}(0)$) and r.m.s. widths ($\sigma_\Delta$ and $\sigma_N$) of anomalous and normal correlations, respectively.

The width of the correlation peaks is inversely proportional to the in-trap size of the associated component: $\sigma_\Delta$ is set by the size of the thermal component, whereas $\sigma_N$ is set by the extension of quantum depletion, limited to that of the BEC by definition. Because the thermal component exceeds the BEC in size, one expects $\sigma_\Delta \geq \sigma_N$, which is confirmed in all the datasets shown in Figs. 2a and 3a.

Furthermore, we study how $\sigma_\Delta$ varies with the BEC size $L_{\text{BEC}}$ by producing low-temperature BECs with varying atom numbers $N$ ($N \approx 2.5(5) \times 10^{13}$ to $N \approx 10.0(1) \times 10^{13}$) and by exploiting the fact that the diffraction peaks in $\rho(k)$ have a root mean square (r.m.s.) width $\sigma_{\text{BECC}}$ set by $L_{\text{BEC}}$. As explained in Supplementary Information and shown in Fig. 3, our measurement is in quantitative agreement with the Bogoliubov prediction $\sigma_\Delta = \sigma_{\text{BECC}}$ (ref. 29) when we account for the effect of small shot-to-shot centre-of-mass fluctuations.

We now discuss the peak amplitudes that reveal the nature of correlations. As mentioned above, $g_{\Delta}^{(2)}(0)$ signals bosonic bunching. Theoretically, it is maximally contrasted, that is, $g_{\Delta}^{(2)}(0) = 2$, and unaffected by temperature, because any equilibrium state of the Bogoliubov theory exhibits thermal (chaotic) statistics for $k' \sim k$. Although this is expected for thermal excitations, it is less obvious for quantum depletion—a pure state. In that case, this property results from the destruction of quantum coherences in the measurement of local correlations, similar to two-mode squeezed states when only one mode is probed29. An analysis of the data shown in Fig. 2c yields $g_{\Delta}^{(2)}(0) = 2.05(6)$, confirming that the local statistics are thermal (Supplementary Information). More importantly, the anomalous amplitude $g_{\Delta}^{(2)}(0)$ is expected to increase with the inverse of density $\rho(k)$. This is suggested by the Bogoliubov picture of quantum depletion based on $k^-k$ pairs: similar to two-mode squeezed states, the particle number in one mode $k$ (at $T=0$) is equal to the number of pairs, leading to $g_{\Delta}^{(2)}(0) \sim 1/\rho(k)$. To verify this prediction for the measured integrated correlations, we vary the average density $\rho_{\Omega_f} = \int \rho(k) dk$ by analysing the datasets with different total atom numbers (Fig. 3b). Again, there is a pronounced difference between the normal and anomalous correlations, and we find that $g_{\Delta}^{(2)}(0) \sim 1$ is indeed inversely proportional to $\rho_{\Omega_f}$. Furthermore, the anomalous amplitude reaches values that largely exceed the bunching amplitude. Observing $g_{\Delta}^{(2)}(0) \gg g_{\Delta}^{(2)}(0)$ constitutes a violation of the Cauchy–Schwartz inequality for
Fig. 3 | Peak widths and amplitudes of the observed atom–atom correlations. a, Peak r.m.s. width of normal correlation \( \sigma_N \) (blue squares) and anomalous correlation \( \sigma_A \) (green circles) as a function of the BEC atom number \( N_{BEC} \). The green shaded area represents the expected value from the measured BEC width \( \sigma_{BEC} \) (Supplementary Information). b, Peak amplitudes of normal correlation \( g^{(2)}_N(0) - 1 \) (blue squares) and anomalous correlation \( g^{(2)}_A(0) - 1 \) (green circles) as a function of inverse density \( 1/\rho_{BEC} \). Although the amplitude of bunching is perfectly contrasted and constant, namely, \( g^{(2)}_N(0) \simeq 2 \), the amplitude of the \( k' - k \) peak linearly increases with \( 1/\rho_{BEC} \). The vertical error bars correspond to standard deviation of the mean over the three directions of momentum space. The horizontal error bars correspond to one standard deviation. The blue dashed line indicates a correlation amplitude \( g^{(2)}_N(0) = 2 \).

Classical fluctuating quantities. It also suggests sub-Poissonian number differences between modes at opposite momenta. A direct calculation on the experimental data confirms this assertion (Fig. 4). We compute the normalized variance of the atom number difference, \( \xi^2 = \langle (N_k - N_{k'})^2 \rangle / \langle N_k \rangle \) (Supplementary Information) and measure \( \xi^2_{k' - k} = 0.992(4) < \xi^2_{k'k' - k} = 1.004(4) \) in the low-temperature data (Fig. 4). This number squeezing is small in comparison to that found with discrete (spin) variables, being limited by the (uncorrelated) thermal population and finite detection efficiency. Finally, our observation of \( g^{(2)}_N(0) > g^{(2)}_A(0) \) fulfills the Bush–Parentani criterion for certifying entanglement in the continuous variable \( k \), under the reasonable assumption that \( \langle \sigma'(k)\sigma(-k) \rangle = 0 \). However, an experimental confirmation of this assertion is necessary to demonstrate entanglement.

In summary, this work reports the observation of \( k' - k \) pairs in the depletion of an interacting Bose gas, and the characterization of the associated quantum correlations and number squeezing. It also demonstrates that a TOF experiment combined with single-atom detection is a sensitive and quantitative probe of in-trap quantum fluctuations in interacting systems, which can be used to characterize non-Gaussian many-body quantum states in the future. A fermionic analogue of this experiment using \(^3\text{He}^*\) to study the pairing of momenta would also be of great interest.

Online content
Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of
data and code availability are available at https://doi.org/10.1038/s41567-021-01381-2.

Received: 7 April 2021; Accepted: 10 September 2021; Published online: 4 November 2021

References
1. Heisenberg, W. Über den anschaulichen inhalt der quantentheoretischen kinematik und mechanik. Z. Phys. 43, 172–198 (1927).
2. Amico, L., Fazio, R., Osterloh, A. & Vedral, V. Entanglement in many-body systems. Rev. Mod. Phys. 80, 517–576 (2008).
3. Bogoliubov, N. On the theory of superfluidity. J. Phys. 11, 23 (1947).
4. Lee, T. D., Huang, K. & Yang, C. N. Eigenvalues and eigenfunctions of a Bose system of hard spheres and its low-temperature properties. Phys. Rev. 106, 1135–1145 (1957).
5. Esteve, J., Gross, C., Weller, A., Giovanazzi, S. & Oberthaler, M. Squeezing the adiabatic preparation of ultracold lattice bosons in the vicinity of the Mott transition. Phys. Rev. Lett. 126, 045301 (2021).
6. Cayla, H. et al. Hanbury Brown and Twiss bunching of phonons and of the quantum depletion in an interacting Bose gas. Phys. Rev. Lett. 125, 165301 (2020).
7. Loudon, R. & Knight, P. L. Squeezed light. J. Mod. Optic. 34, 709–759 (1987).
8. Braunstein, S. L. & Van Loock, P. Quantum information with continuous variables. Rev. Mod. Phys. 77, 513–577 (2005).
9. Orzel, C., Tuchman, A. K., Fenselau, M. L., Yasuda, M. & Kasevich, M. A. Squeezed states in a Bose–Einstein condensate. Science 291, 2386–2389 (2001).
10. Esteve, J., Gross, C., Weller, A., Giovanazzi, S. & Oberthaler, M. Squeezing and entanglement in a Bose–Einstein condensate. Nature 455, 1216–1219 (2008).
11. Bucker, R. et al. Twin–atom beams. Nat. Phys. 7, 608–611 (2011).
12. Miller, A., Pines, D. & Nozières, P. Elementary excitations in liquid helium. Phys. Rev. D 127, 1452–1464 (1962).
13. Ozeri, R., Katz, N., Steinhauer, J. & Davidson, N. Colloquium: bulk Bogoliubov excitations in a Bose–Einstein condensate. Rev. Mod. Phys. 77, 187–205 (2005).
14. Fontaine, Q. et al. Observation of the Bogoliubov dispersion in a fluid of light. Phys. Rev. Lett. 121, 183604 (2018).
15. Stepnov, P. et al. Dispersion relation of the collective excitations in a resonantly driven polariton fluid. Nat. Commun. 10, 3869 (2019).
16. Griffin, A., Snake, D. W. & Stringari, S. (eds) Bose–Einstein Condensation (Cambridge Univ. Press, 1995).
17. Xu, K. et al. Observation of strong quantum depletion in a gaseous Bose–Einstein condensate. Phys. Rev. Lett. 99, 180405 (2006).
18. Lopes, R. et al. Quantum depletion of a homogeneous Bose–Einstein condensate. Phys. Rev. Lett. 119, 190404 (2017).
19. Burnham, D. C. & Weinberg, D. L. Observation of simultaneity in parametric production of optical photon pairs. Phys. Rev. Lett. 25, 84 (1970).
20. Greiner, M., Regal, C. A., Stewart, J. T. & Jin, D. S. Probing pair–correlated fermionic atoms through correlations in atom shot noise. Phys. Rev. Lett. 94, 110401 (2005).
21. Aransin, G. First observation of correlations between high transverse momentum charged particles in events from the CERN proton-antiproton collider. Phys. Lett. B 118, 173–177 (1982).
22. Perrin, A. et al. Observation of atom pairs in spontaneous four–wave mixing of two colliding Bose–Einstein condensates. Phys. Rev. Lett. 99, 150405 (2007).
23. Cayla, H. et al. Single–atom–resolved probing of lattice gases in momentum space. Phys. Rev. A 97, 061609 (2018).
24. Tenart, A. et al. Two–body collisions in the time–of–flight dynamics of lattice Bose superfluids. Phys. Rev. Research 2, 013017 (2020).
25. Walls, D. F. & Milburn, G. J. Quantum Optics (Springer-Verlag, 2008).
26. Butera, S., Clermont, D. & Carusotto, I. Position– and momentum–space two–body correlations in a weakly interacting trapped condensate. Phys. Rev. A 103, 013302 (2021).
27. Pezzè, L., Smerzi, A., Oberthaler, M. K., Schmied, R. & Treutlein, P. Quantum metrology with nonclassical states of atomic ensembles. Rev. Mod. Phys. 90, 035005 (2018).
28. Busch, X. & Parentani, R. Quantum entanglement in analogue Hawking radiation: when is the final state nonseparable? Phys. Rev. D 89, 105024 (2014).
29. Bergschneider, A. et al. Experimental characterization of two–particle entanglement through position and momentum correlations. Nat. Phys. 15, 640–644 (2019).
30. Schweigler, T. et al. Experimental characterization of a quantum many–body system via higher–order correlations. Nature 545, 323–326 (2017).

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.
© The Author(s), under exclusive licence to Springer Nature Limited 2021
Methods

Computation of atom–atom correlations. The correlation functions \( g^{(1)}_k(\delta k) \) and \( g^{(2)}_k(\delta k) \) are numerically computed from the measured atom distributions with a similar two-step procedure. The first step computes the correlations over the momentum-space volume \( \Omega \), in each atom distribution, recorded from a single experimental run, and then averaging the correlations over the set of experimental runs performed under identical conditions. This step computes the numerator in equations (1) and (2), namely, \( \int_{\Omega} \langle a^\dagger(\vec{k})a(\vec{k} \pm \delta \vec{k})a(\vec{k})a(\vec{k} \pm \delta \vec{k}) \rangle d\vec{k} \). The second step aims at evaluating the denominator \( \int_{\Omega} \rho(\vec{k})\rho(\vec{k} \pm \delta \vec{k})d\vec{k} \) in equations (1) and (2) and involves merging all the atom distributions used in the first step and only then computing the correlations. Atom–atom correlations present in a single atom distribution become negligible over the \( \sim 2,000 \) distributions merged together; therefore, correlations in the second step are computed over an ensemble of statistically independent atoms.

The computed functions \( g^{(1)}_k(\delta k) \) and \( g^{(2)}_k(\delta k) \) are 3D. When plotting a 1D cut through \( g^{(1)}_k(\delta k) \) or \( g^{(2)}_k(\delta k) \), a small transverse integration \( \Delta k_\perp \) is used to improve the signal-to-noise ratio at the expense of reducing the amplitude of the correlation peaks. The amplitudes \( g^{(1)}_k(0) \) and \( g^{(2)}_k(0) \) are obtained from studying the effect of transverse integration (Supplementary Information).

Detection efficiency of the helium detector. To avoid any disturbance induced by stray magnetic fields over the long free fall (~45 cm), the \( ^{3}He^* \) atoms trapped in the \( m_f = +1 \) sub-state are transferred to the non-magnetic sub-state \( m_f = 0 \) after the gas has been released from the optical trap. Any atom that would remain in the \( +1 \) state after this step would then be removed by applying a strong magnetic gradient. Here we use a two-photon Raman transition (detuned by 800 MHz from the \( 2S_\perp \rightarrow 2P_\perp \) transition) to transfer all the atoms in the \( m_f = 0 \) state; therefore, the detection efficiency per atom is only limited by that of the \( ^{3}He^* \) detector. To optimize the latter, we used funnel-type microchannel plates (Hamamatsu F9142-01 MOD6) whose open-to-air ratio is close to unity (~0.95). We have calibrated the total detection efficiency per atom using absorption images as a reference for the atom number and have measured it to be 0.53(2).

Heating procedure. Equilibrium-lattice BECs in the low-temperature regime are produced by ramping up the lattice amplitude at a rate of 0.3E_r per millisecond, and holding the atoms at the final amplitude of \( V = 7.75E_r \) for 5 ms. At this final lattice amplitude, the ratio of the on-site interaction energy \( U \) to the tunnelling energy \( V \) is \( U/V = 5 \). We have recently shown that this procedure produces equilibrium states of the Bose–Hubbard Hamiltonian with low-entropy \( S/N = 0.8(1) k_B \), where \( S/N \) is the entropy per atom. From holding the atoms at the final amplitude for a longer duration, the gas is continuously heated over time (attributed to imperfections such as spontaneous emission or mechanical vibrations). For the heated data (Fig. 2), we hold the atoms for 500 ms at \( V = 7.75E_r \) that is, for hundreds of tunnelling times (225 \( \times h\)/\( f \)).

Data availability

All data shown in this paper are available from the corresponding author upon reasonable request.

Acknowledgements

We thank A. Aspect, A. Browaeys, I. Carusotto, H. Cayla and T. Roscilde for their valuable comments on the manuscript, and acknowledge fruitful discussions with the members of the Quantum Gas group at Institut d’Optique. We acknowledge financial support from the LabEx PALM (grant number ANR-10-LABX-0039), the Région Ile-de-France in the framework of the DIM SIReTiq, the ‘Fondation d’entreprise X’core pour la Recherche’ and the Agence Nationale pour la Recherche (grant number ANR-17-CE30-0020-01). D.C. acknowledges support from the Institut Universitaire de France.

Author contributions

A.T. and G.H. carried out the experiments. All the authors contributed to the data analysis, progression of the project and writing of the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

Supplementary information The online version contains supplementary material available at https://doi.org/10.1038/s41567-021-01381-2.

Correspondence and requests for materials should be addressed to David Clément.

Peer review information Nature Physics thanks the anonymous reviewers for their contribution to the peer review of this work.

Reprints and permissions information is available at www.nature.com/reprints.