Hysteresis in a quantized superfluid ‘atomtronic’ circuit

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Atomtronics is an emerging interdisciplinary field that seeks to develop new functional methods by creating devices and circuits where ultracold atoms, often superfluids, have a role analogous to that of electrons in electronics. Hysteresis is widely used in electronic circuits—it is routinely observed in superconducting circuits and is essential in radio-frequency superconducting quantum interference devices. Furthermore, it is as fundamental to superfluidity (and superconductivity) as quantized persistent currents, critical velocity and Josephson effects. Nevertheless, despite multiple theoretical predictions, hysteresis has not been previously observed in any superfluid, atomic-gas Bose–Einstein condensate. Here we directly detect hysteresis between quantized circulation states in an atomtronic circuit formed from a ring of superfluid Bose–Einstein condensate obstructed by a rotating weak link (a region of low atomic density). This contrasts with previous experiments on superfluid liquid helium where hysteresis was observed directly in systems in which the quantization of flow could not be observed, and indirectly in systems that showed quantized flow. Our techniques allow us to tune the size of the hysteresis loop and to consider the fundamental excitations that accompany hysteresis. The results suggest that the relevant excitations involved in hysteresis are vortices, and that dissipation has an important role in the dynamics. Controlled hysteresis in atomtronic circuits may prove to be a crucial feature for the development of practical devices, just as it has in electronic circuits such as memories, digital noise filters (for example Schmitt triggers) and magnetometers (for example superconducting quantum interference devices).

Hysteresis is a general feature of systems where the energy has two (or more) local minima separated by an energy barrier. A schematic of this type of energy landscape is shown in Fig. 1a. A canonical example of hysteresis is the Landau theory of ferromagnetism, where the order parameter, \( \kappa \), is the magnetization, and the energy, \( E(\kappa) \), has two minima (stable states) corresponding to the magnetization being aligned parallel or, respectively, antiparallel to the applied magnetic field. For a Bose–Einstein condensate (BEC) in a ring-shaped trap, these minima represent stable flow states of the system, and their energies depend on the applied rotation rate of the trap, \( \Omega \) (here this rotation is created using a rotating repulsive perturbation). With no interatomic interactions, there is only one minimum in the energy landscape of the BEC. With the addition of interactions, an energy barrier can appear, creating two (or more) stable flow states. This barrier stabilizes the flow, making the BEC a superfluid.

The energy of the barrier is not generally known for superfluid systems; depending on the parameters of the system, it could be related to the energy required to create elementary excitations such as phonons, solitons or vortices. However, the stable states are well known. Rotation of a superfluid in a ring is characterized by a quantized rotation frequency, \( n\Omega_0 \), where \( n \) is the winding number, \( \Omega_0 = \hbar/mR^2 \) is the rotational quantum, \( \hbar \) is Planck’s constant divided by \( 2\pi \), \( m \) is the mass of an atom and \( R \) is the mean radius of the trap. The energy of the superfluid in the frame that rotates with the trap depends on the relative velocity between the superfluid and the trap, and the energy is proportional to \((n - \Omega/\Omega_0)^2\).

Any ring-shaped superfluid necessarily exhibits both hysteresis and a critical rotation rate, \( \Omega_c^+ \) (or, equivalently, a critical velocity), because all these effects fundamentally arise from the energy barrier that creates superfluidity. To understand this, we plot the energy of the stable states and the energy barrier as a function of \( \Omega \) (Fig. 1b, c). Figure 1b shows the ‘swallowtail’ energy structure characteristic of the superfluid. If the system begins in \( n = 0 \), the flow is stable until \( \Omega = \Omega_0^+ \), where the energies of the \( n = 0 \) state and the barrier are equal. At this point, the \( n = 0 \) state is no longer stable and a transition occurs to \( n = 1 \), which has lower energy. If \( \Omega \) is now decreased, this state is stable until \( \Omega < \Omega_0^- \), where the flow changes. Thus, a typical hysteresis loop is traced, as shown in the lower panel in Fig. 1b. We note that, although \( \Omega_0^+ \) and \( \Omega_0^- \) are the same relative to the superfluid flow, they are generally different in the lab frame and thus appear as hysteresis. Furthermore, in the hysteretic case, they are different from a more general definition of critical rotation (or velocity) that involves the onset of phase transitions in the system.

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Figure 1. Origin of hysteresis. a. Schematic energy landscape of a hysteric system. As a function of an order parameter, \( \kappa \), the energy can have local minima (squares, circles), which represent stable states, separated by a local maximum (stars), which forms an energy barrier, \( E_b \). This landscape is shown for five values of the applied field, \( F \) (for superfluidity, \( F \rightarrow \Omega \), the rotation rate of the trap). b. Plotted as a function of \( \Omega \) for a superfluid, the energy of the minima (solid) and maximum (dashed) form a swallowtail (top), which exhibits hysteresis (bottom). c. This swallowtail structure is periodic in \( \Omega_0 \) states above \( E_2 \) are unstable.
of dissipation or the creation of excitations. At $\Omega^\pm_0$, the hysteretic system may create excitations or experience dissipation, but both cease after the transition is made. Measurement of a hysteresis loop, in addition to measuring $\Omega^\pm_1$, shows an important feature of the underlying energy landscape: the system has at least two stable states. Bi-stability of a moving BEC has been demonstrated independently of quantized states or critical velocities\textsuperscript{25,26}. Finally, unlike that for ferromagnetism, this energy structure is periodic in $\Omega$, with period $\Omega_0$ (Fig. 1c). Similar periodic swallowtail energy structures are predicted for superfluids trapped in a lattice\textsuperscript{27}.

Our superfluid system is a BEC of \textsuperscript{23}Na atoms in a ring-shaped optical dipole trap (Fig. 2a). To induce flow, we use a blue-detuned laser to create a rotating repulsive potential, depleting the density in a small portion of the ring and thereby creating a weak link\textsuperscript{27}. The intensity of the laser sets the height of this potential, $U$. Without this weak link, superfluid flow in the ring should be quite stable\textsuperscript{24}, with $\Omega^\pm_1 \gg \Omega_0$. Changing $U$ will change the critical angular velocities, $\Omega^\pm_0$, and the size of the hysteresis loop. Rotating the weak link in the azimuthal direction at angular frequency $\Omega$ can drive transitions, or phase slips, between the quantized circulation states\textsuperscript{9} (Fig. 2b).

To observe hysteresis in these phase slips, we use a two-step experimental sequence (Fig. 2c). After condensing the atoms into the ring trap, the BEC is prepared in either the $n = 0$ or the $n = 1$ circulation state by either not rotating the weak link or by rotating it at $\Omega_1 = 1.1$ Hz. The fidelity with which this procedure generates the expected initial state is $\approx 97\%$. We then rotate the weak link at various angular velocities, $\Omega_2$, in the range $0.3$ to $1.2$ Hz, for an additional 2 s. In step 1, $U$ is ramped to $U_1 \approx 1.1\mu_0$, where $\mu_0$ is the global chemical potential. In step 2, $U$ is ramped to a chosen $U_2$. The transitions $n = 0 \rightarrow 1$ and $n = 1 \rightarrow 0$ occur at different values of $\Omega_2$ and form hysteresis loops (Fig. 3a–f). Each plot shows the measured hysteresis loop for a specific $U_2$ value. As $U_2$ is increased, both $\Omega^+_0$ and $\Omega^-_0$ approach $\Omega_0/2$; that is, the hysteresis loop becomes smaller. The observed transitions are not sharp, unlike those in Fig. 1b. The dominant broadening mechanism is probably shot-to-shot atom number fluctuations, but the non-zero temperature ($\sim 100$ nK) may also contribute (Supplementary Information).

**Figure 2** | Experimental set-up and procedure. a, Schematic and in situ images of our trap, which is formed by crossing a ring-shaped dipole trap for radial confinement and a sheet trap for vertical confinement. b, Schematic and in situ images of a ring rotated by a repulsive weak link. c, Two-step experimental sequence: the height, $U$, of the repulsive potential and the angular rotation rate, $\Omega$, as a function of time. Step 1 sets the initial winding number using $\Omega_1$ (either 0 or 1.1 Hz) and $U_1$ ($\approx 1.1\mu_0$), step 2 probes the hysteresis with $\Omega_2$ and $U_2$ (see text).

**Figure 3** | Hysteresis data. a–f, Hysteresis loops with sigmoid fits. The red up-triangles and blue down-triangles show the winding number $n$ averaged over $\sim 20$ shots when starting with $n = 0$ and, respectively, $n = 1$. All error bars show the 68% confidence interval. The fits determine $\Omega^+_0$ and $\Omega^-_0/2$ (vertical grey lines; Methods) and their uncertainties. g, Hysteresis loop size versus $U_2$. The green circles show the experimental data. The magenta line and band are respectively the prediction and uncertainty of an effective one-dimensional hydrodynamic model\textsuperscript{28}. The open and filled cyan diamonds and their uncertainties are the results of our GPE simulation with $A = 0$ and, respectively, $A = 0.01$.

Figure 3g shows the measured size of the hysteresis loop, $(\Omega^+_0 - \Omega^-_0)/\Omega_0$, as a function of the strength of the weak link; the size of the loop monotonically decreases with increasing $U_2/\mu_0$ until it reaches a value consistent with zero near $U_2/\mu_0 \approx 0.75$. To predict the size of the hysteresis loop, we used two models. First, we used an effective one-dimensional model that computes the fluid velocity in the rotating frame as a function of $\Omega$. We assume that $\Omega^\pm$ will occur when this velocity reaches the local speed of sound\textsuperscript{29}. We also simulated our system with the three-dimensional, time-dependent Gross–Pitaevskii equation (GPE). These two approaches predict hysteresis and are consistent, suggesting that both theories predict that $\Omega^\pm_0$ is determined by the sound speed. Despite occurring at the sound speed, the observed excitations in the GPE simulation are vortex–antivortex pairs. Perhaps most strikingly, there is a large discrepancy between our models and experiment (Fig. 3g).

One property of the system that our models fail to include is dissipation. As another approach, we added dissipation to the GPE phenomenologically\textsuperscript{28}, by modifying it as follows:

$$i\hbar \frac{\partial \psi}{\partial t} = \left(1 - iA\right) \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(x,y,z,t) + gN|\psi|^2 - \mu \right] \psi$$

Here $\psi$ is the BEC wavefunction, $g$ is the interaction strength, $V$ is the externally applied potential (trap and weak link), $N$ is the atom number, $\mu$ is the chemical potential of the initial stationary state and $A$ is the dissipation parameter. With $A = 0.01$, a reasonable value for our experiment, the hysteresis loop size decreases as shown in Fig. 3g but not significantly by comparison with the discrepancy with experiment. Increasing the dissipation parameter does not improve the agreement (Supplementary Information). However, it is clear that dissipation is important. In fact, dissipation is essential and implicitly assumed in the energy landscape picture described in Fig. 1: dissipation allows the system to relax to the minima of the landscape; without dissipation, the system cannot change its energy.

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To gain insight into the role of dissipation, we consider a toy model where the relevant excitations are vortex–antivortex pairs, and derive the associated energy landscape. If an antivortex and a vortex were to be nucleated at the inner and, respectively, outer edges (Fig. 4, left inset), move to the centre and annihilate, then the winding number would change by one unit. The energy of a vortex–antivortex pair in a perfectly-hard-walled ring trap in the presence of a vortex field has been derived using the method of images\(^{26}\). In the limit that the width of the annulus, \(d\), is much less than \(R\), this energy reduces to

$$E = \pi \rho d R s^2 + 2\pi \hbar p_{\text{ISO}} \rho \hbar^2 \frac{m}{2} + \pi \rho d R s^2 \ln \left[ \frac{d}{d_s} \sin \left( \frac{\pi s}{d} \right) \right]$$

where \(s\) is the separation between the vortices, \(\nu\) is the velocity of the superfluid, \(\rho\) is the effective two-dimensional mass density, and \(\xi = \left( \frac{\hbar^2}{2m g(N)} \right)\) is the healing length of the condensate and is therefore the core size. This equation applies to a system with a uniform annulus width, \(d\), and uniform velocity, \(\nu\). To apply this model to our system, we take \(d\) to be the effective width of the annulus in the weak-link region and we take \(\nu = \nu_{\text{m}}\) the maximum velocity in the weak link.

For \(\nu_{\text{m}} = 0\), equation (1) has a maximum at \(s = d/2\) and diverges negatively at \(d = 0\). Such divergence is unphysical, because the non-zero radii of the vortices prevent them from coming arbitrarily close to each other or the wall. We assume that the distance of closest approach to the walls is \(C\) and that that between vortices is \(2C\), where \(C\) is of order unity. Thus, \(s\) ranges from \(s_{\text{min}} = 2C\) to \(s_{\text{max}} = d - 2C\). We assume that vortices annihilate at \(s_{\text{min}}\) and enter the annulus at \(s_{\text{max}}\).

We plot the energy landscape described above in Fig. 4 (right inset) for several different \(\nu_{\text{m}}\) values and constant \(d\). The two stable states, at \(s_{\text{min}}\) and \(s_{\text{max}}\), represent a winding number difference of one. This implies that for a phase slip to occur the vortex pair must nucleate at either \(s_{\text{max}}\) or \(s_{\text{min}}\) and move to the opposite extreme. This happens when the energy barrier disappears, that is, when \(\frac{dE}{ds}|_{s=s_{\text{min}}} = \frac{dE}{ds}|_{s=s_{\text{max}}} = 0\). This defines the critical velocity, \(\nu_{\text{m}} = \nu_{\text{c}}\), as

$$\nu_{\text{c}} = \pm \frac{\pi \hbar}{md d_s} \cot \left( \frac{2C}{d} \right) \text{ (2)}$$

where the plus and minus signs respectively refer to starting at \(s_{\text{min}}\) and \(s_{\text{max}}\).

To compare this model to our experiment, we computed the critical velocity in the weak link from the transitions in the hysteresis loops.

The critical velocity is not a simple function of \(U/\mu_0\) and \(Q_2\); rather, the requirements of quantized winding number and, in a frame co-rotating with the weak link, continuity of flow, require a self-consistent solution for the flow velocity around the entire ring (Methods). Figure 4 shows the result of this calculation. In direct contrast to the local speed of sound in the weak link, which decreases as \(\sqrt{1 - U/\mu_0}\), we find that the critical velocity increases. The observed critical velocity is well fitted to equation (2) with a single value of \(2C/\xi\). This value implies a distance of closest approach \(2C = 0.6d\). Over the range of interest, the value of \(C\) ranges from 1.5 to 0.7, agreeing with the assumption that it is of order unity (\(C\) is calculated using the best estimates of \(\xi\) and \(d\), both of which vary with \(U\)). The fact that the data can be fitted using this crude model suggests that vortices are the relevant excitations and that equation (1) (or something that captures similar physics) gives a good prediction of the energy landscape.

Our hysteretic system has the essential features of the radio-frequency superconducting quantum interference device (SQUID), just as SQUIDs detect magnetic fields, our analogous system can detect rotations. Hysteresis has an important role in radio-frequency SQUIDs, where it is used as a readout mechanism. In our system, hysteresis will also be important, allowing for greater accuracy by cancelling systematic effects. The hysteresis loops are centred on \(\pm Q_2/2\) for different directions of rotation; therefore, we can measure the asymmetry in the measurements of \(\pm Q_2/2\) to extract an unknown bias rotation. Such measurements may cancel out effects such as asymmetries in the ring potential.

We have measured hysteresis in a dilute atomic-gas BEC, a phenomenon that is as fundamental to superfluidity as the existence of persistent currents and critical velocities. Our studies suggest that the elementary excitations involved in hysteresis are vortices and that dissipation has an important role in the dynamics. We suspect that more sophisticated models that include dissipation will yield better agreement. Finally, beyond being an atomtronic rotation sensor, it is possible that in the hysteretic regime this device could act as classical memory or a digital noise filter in future atomtronic circuits.

**METHODS SUMMARY**

The ring-shaped BEC, which contains approximately \(4 \times 10^5\) \(^{23}\text{Na}\) atoms, is created from a cloud of laser-cooled atoms by evaporation, first in a magnetic trap and then in a ring-shaped optical dipole trap. The optical dipole trap is shaped roughly like a flattened torus (Methods), with measured harmonic trap frequencies of \(472(4)\) Hz in the vertical direction and \(188(3)\) Hz in the radial direction. (All uncertainties are the uncorrelated combination of 1\(\sigma\) statistical and systematic uncertainties unless stated otherwise.) The mean radius of the trap is \(19.5(4)\) \(\mu\)m. The weak link is created by a blue-detuned laser beam (Methods). Time-of-flight expansion of the condensate allows us to determine the winding number by measuring the size of the central hole that appears in the cloud.

**Online Content** Any additional Methods, Extended Data display items and Source Data are available in the online version of the paper; references unique to these sections appear only in the online paper.

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Author Information Reprints and permissions information is available at www.nature.com/reprints. The authors declare no competing financial interests. Readers are welcome to comment on the online version of the paper. Correspondence and requests for materials should be addressed to G.K.C. (gretchen.campbell@nist.gov).
METHODS

Optical dipole traps. Our optical dipole trap is formed by the combination of two laser beams. A blue-detuned ($\lambda = 532$ nm) laser beam passes through a ring-shaped intensity mask, and the shadow is imaged onto the atoms, forming a repulsive, ring-shaped potential. This trap combines with an attractive confining potential in the vertical direction, generated by a red-detuned ($\lambda = 1,064$ nm) laser beam shaped like a sheet. If the imaging resolution were perfect, the trap would be hard-walled in the vertical and radial directions to be 3.2 and 8.1 in Figs 3 and 4; the relative uncertainties between the points are smaller.

We calibrated the weak link by observing the atomic density depletion caused by the weak-link potential (Supplementary Information). The dominant uncertainty in $U_1/\mu_0$ is in the common calibration and is reflected in the horizontal error bars in Figs 3 and 4; the relative uncertainties between the points are smaller.

BEC parameters. Approximately $4 \times 10^6$ atoms comprise the BEC after evaporation first in a magnetic time-orbiting potential trap and subsequently in the optical traps described above. We estimate the global chemical potential, $\mu_0$, to be $\mu_0/\hbar = 2 \pi \times 1.7$ kHz and the corresponding Thomas–Fermi full-widths in the vertical and radial directions to be 3.2 and 8.1 $\mu$m, respectively. Given this mean radius, we expect that $\Omega_0 = 1.19(4)$ Hz, in rough agreement with the measured value of $\Omega_0 = 1.05(5)$ Hz (by assuming the hysteresis loops are centred on $\Omega_0/2$).

Measurement of the winding number. To measure the final rotational state after rotating, the BEC is released from the trap and imaged after a 10-ms time of flight. As the BEC expands, rotation will cause a hole to appear in the centre. As with the trap described above, we estimate the global chemical potential, $\mu_0$, to be $\mu_0/\hbar = 2 \pi \times 1.7$ kHz and the corresponding Thomas–Fermi full-widths in the vertical and radial directions to be 3.2 and 8.1 $\mu$m, respectively. Given this mean radius, we expect that $\Omega_0 = 1.19(4)$ Hz, in rough agreement with the measured value of $\Omega_0 = 1.05(5)$ Hz (by assuming the hysteresis loops are centred on $\Omega_0/2$).

Estimating the uncertainty in the average winding number. Given that the outcome of any given experiment is either $n = 0$ or $n = 1$, traditional methods of estimating the uncertainty in the mean value (for example Gaussian statistics) are not applicable. The uncertainty in this average can be estimated by the cumulative beta distribution, which is appropriate for experiments that yield binary results.

Fitting the hysteretic transitions and determining $\Omega_0^2$ and its uncertainty. We use a sigmoid of the form $1/\cosh^{-1}(\Omega - \Omega_0)/\Delta \Omega_0 + 1$ to fit the data as in Fig. 3, where $\Omega_0$ and $\Delta \Omega_0$ are the fit parameters. Although this fit well describes the data, the relationship between $\Omega_0^2$ and the fit parameters depends on the mechanisms for the broadening of the transition. For example, consider a model where thermal fluctuations drive the system over the energy barrier. This would occur when the energy barrier becomes of the order of $k_B T$, where $k_B$ is the Boltzmann constant and $T$ is the temperature. The dynamics of this process is random and would lead to phase slips at lower values of $\Omega_0$ for the $0 \rightarrow 1$ transition (and higher values for $1 \rightarrow 0$ transitions). In principle, this effect would cause a broadening of the transition region, and the zero-temperature $\Omega_0^2$ would then correspond to the value of $\Omega_2$ where probability for a transition equals unity. However, a different mechanism could be responsible for the broadening. In particular, atom number fluctuations can change $U_1/\mu_0$ and, therefore, $\Omega_0^2$, from shot to shot. On average, this leads to a broadening. On the basis of the experimentally observed change in $\Omega_0^2$ versus the strength of the weak link, and our atom number shot-to-shot fluctuations of $-16\%$ (which represent the peak-to-peak fluctuations for 95% of the data), we expect the transitions to be approximately $0.12$ Hz wide, compared with the average of $0.18$ Hz. Because atom number fluctuations explain most of the width, we take $\Omega_0^2 = \Omega_0$, and, to account for the possibility of finite-temperature or other unknown broadening effects, take the 1$\sigma$ uncertainty to be $(3/2)\Omega_0$.

Extracting the critical velocity. Extracting the critical velocity in our system is non-trivial because the flow must satisfy the requirements of quantized winding number and continuity of flow in the frame rotating with the weak link. (Continuity of flow does not occur in any other frame.) One counterintuitive result of these requirements is that moving the weak link will impart some angular momentum to the superfluid as viewed from the fixed, laboratory frame, even in the $n = 0$ state.

To extract the critical velocity given these constraints, we work in the rotating frame. The velocity, $v_n$, of atoms in the rotating frame is related to the rotation rate of the weak link by

$$v_n = \frac{m}{\hbar} \int_0^{2\pi} v_0(\theta) R d\theta + 2\pi \frac{\Omega}{\Omega_0} = 2\pi n$$

where $\theta$ is the azimuthal angle. This equation is an expression of the Bohr–Sommerfeld quantization condition. The first term represents the phase accumulated by the atoms after integrating once around the ring, and the second term represents the Sagnac phase that appears as a result of transforming into the rotating frame. In the rotating frame, the atom velocity, $v_0(\theta)$, and the mass density, $\rho(\theta)$, satisfy a continuity equation, $\rho(\theta) v_0(\theta) = \rho(\theta) c(\theta)$, where $c_1$ and $c_2$ are any two azimuthal angles. Given $U_1/\mu_0$, we determine the equivalent one-dimensional density, $\rho(\theta)$, by integrating over the radial and vertical directions of our cloud using the Thomas–Fermi approximation. For a given rotation rate, $\Omega$, and density, $\rho(\theta)$, equation (3) and the continuity equation uniquely determine $v_n(\theta)$ and, in particular, the velocity in the weak link, $v_n = \max[v_n(\theta)]$. The critical velocity is then taken to be the value of $v_n$ when the weak link is rotated at the critical rotation rate $\Omega_c$.

When $\Omega_c^2 - \Omega_0^2 \rightarrow 0$, this method of extracting the critical velocity is unreliable and thus we neglect the point near $U = 0.8\mu_0$ (Supplementary Information). Going slightly farther into the regime where $U > \mu_0$ results in the BEC becoming simply connected.

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