Eccentric domination decomposition of graphs

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Abstract
A decomposition \((G_1, G_2, \ldots, G_n)\) of \(G\) is said to be an eccentric domination decomposition (EDD) if i) \(E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)\) ii) Each \(G_i\) is connected iii) \(\gamma_{ed}(G_i) = i, i = 1, 2, \ldots, n\). If a graph \(G\) has EDD, we say that \(G\) admits eccentric domination decomposition.

Keywords
Decomposition, Domination, Eccentric domination decomposition.

AMS Subject Classification
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1. Introduction

In this article, all the terminologies from the graph theory are used in the case of Frank Harary [3]. A simple undirected graph without loops or multiple edges are considered here. The theory of domination is one of the fastest growing fields of graph theory, Which has been investigated by S.T. Hedetniemi [4]. A set \(D \subseteq V(G)\) of vertices in a graph \(G\) is a dominating set if every vertex \(v\) in \(V - D\) is adjacent to a vertex in \(D\). The Minimum cardinality of a dominating set of \(G\) is called the domination number of \(G\) and is denoted by \(\gamma(G)\).

A set \(D \subseteq V\) is an eccentric dominating set if \(D\) is a dominating set of \(G\) and for every \(v \in V - D\), there exists at least one eccentric point of \(v\) in \(D\).

If \(D\) is an eccentric dominating set, then every superset \(D' \supseteq D\) is also an eccentric dominating set. But \(D'' \subseteq D\) is not necessarily an eccentric dominating set.

An eccentric dominating set \(D\) is a minimal eccentric dominating set if no proper subset \(D'' \subseteq D\) is an eccentric dominating set. The minimum cardinality of an eccentric dominating set \(\gamma_{ed}(G)\) is known as minimum eccentric dominating set. This concept was introduced by T.N. Janakiraman, M. Bhanumathi and S. Muthaiam [5].

The decomposition of graphs is another important field of graph theory. Several authors studied various types of decompositions by imposing conditions on \(G_i\) in the decomposition. Let \(G = (V, E)\) be a simple connected graph with \(p\) vertices and \(q\) edges. If \(G_1, G_2, \ldots, G_n\) are connected edge disjoint subgraphs of \(G\) with \(E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)\) then \((G_1, G_2, \ldots, G_n)\) is said to be a Decomposition of \(G\). Motivated by the concepts of Ascending Domination Decomposition [7] and Continuous Monotonic Decomposition [2] we introduce a new concept Eccentric Domination Decomposition of a graphs.

2. Eccentric Domination Decomposition \(\{EDD\}\)

Definition 2.1. A decomposition \((G_1, G_2, \ldots, G_n)\) of \(G\) is said to be an Eccentric Domination Decomposition if
i) \(E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)\)
ii) Each \(G_i\) is connected
iii) \(\gamma_{ed}(G_i) = i, i = 1, 2, \ldots, n\).

If a graph \(G\) has EDD, we say that \(G\) admits Eccentric Domination Decomposition.

Theorem 2.2. \(K_{1,n}\) admits Eccentric domination decomposition.

Proof. Let \(G = K_{1,n}\). Let \(G_1\) be a subgraph obtained from \(K_{1,n}\) by taking the edge \(uu_1\). Then \(\gamma_{ed}(G_1) = 1\). We also see
that $G_2 = K_{1,n} - G_1$ and $\gamma_{ed}(G_2) = 2$. Hence $\psi = \{G_1, G_2\}$ is an EDD for $K_{1,n}$.

Theorem 2.3. Complete bipartite graph $k_{m,n}$ admits Eccentric Domination Decomposition.

Proof. Let $V = X \cup Y$ be a bipartition of $k_{m,n}$ with $|X| = m$ and $|Y| = n$. Let $X = \{x_1, x_2, \ldots, x_m\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$. Let $G_1$ be a subgraph obtained from $k_{m,n}$ by taking the edge $x_1y_1$. Then $\gamma_{ed}(G_1) = 1$. We also see that $G_2 = k_{m,n} - G_1$ and $\gamma_{ed}(G_2) = 2$. Hence $\psi = \{G_1, G_2\}$ is an EDD for $k_{m,n}$.

Theorem 2.4. $SL_m$ has an EDD $\psi = \{G_1, G_2, \ldots, G_n\}$ if and only if $SL_m$ has $\frac{n^2 - n + 2}{2}$ vertices.

Proof. Slanting ladder $SL_m$ obtained from two path $u_1, u_2, \ldots, u_m$ and $v_1, v_2, \ldots, v_m$ by joining each $u_i$ with $v_{i+1}$, $1 \leq i \leq m - 1$. To prove $SL_m$ has an EDD.

Suppose $SL_m$ has $\frac{n^2 - n + 2}{2}$ vertices.

Let $G_1 = \{u_1, u_2\}$

$G_2 = \{u_1, v_1, v_2\}$

$G_3 = \{u_2, u_3, u_4, v_2, v_3, v_4\}$

Case i) In the above construction of $G_1, G_2, \ldots, G_n$ if we add the vertices $1, 2, \ldots, n$ in $SL_m$ then there will be remaining 1 to $n$ vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of $G_1$. Therefore the resulting decomposition does not admit EDD. Hence $\psi = \{G_1, G_2, \ldots, G_n\}$ is an $SL_m$.

Conversely suppose $SL_m$ has an EDD.

To prove that $SL_m$ has $\frac{n^2 - n + 2}{2}$ vertices.

Suppose $SL_m$ has no $\frac{n^2 - n + 2}{2}$ vertices.

The following are the two possibilities.

Case i) In the above construction of $G_1, G_2, \ldots, G_n$ if we add the vertices $1, 2, \ldots, n$ in $SL_m$ then there will be remaining 1 to $n$ vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of $G_1$. Therefore the resulting decomposition does not admit EDD. Hence $\psi = \{G_1, G_2, \ldots, G_n\}$ is an $SL_m$.

Conversely suppose $SL_m$ has an EDD.

Let $G_1 = \{u_1, v_1\}$

$G_2 = \{u_1, u_2, v_1, v_2\}$

$G_3 = \{u_2, u_3, u_4, v_2, v_3, v_4\}$

To prove $SL_m$ has an EDD.

Suppose $SL_m$ has $\frac{n^2 - 6n + 8}{2}$ vertices. Clearly $\gamma_{ed}(G_1) = i, i = 1, 2, \ldots, n$. We observe that the minimum eccentric dominating set of $G_n$ has $n$ vertices $TL_m$ has $\frac{2n^2 - 6n + 8}{2}$ vertices. Clearly $\gamma_{ed}(G_i) = i, i = 1, 2, \ldots, n$. and hence $\psi = \{G_1, G_2, \ldots, G_n\}$ is an $TL_m$.

Conversely suppose $TL_m$ has an EDD.

To prove that $TL_m$ has $\frac{2n^2 - 6n + 10}{2}$ vertices.

The following are the two possibilities.

Case i) In the above construction of $G_1, G_2, \ldots, G_n$ if we add the vertices $1, 2, \ldots, n$ in $TL_m$ then there will be remaining 1 to $n$ vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of $G_i$. Therefore the resulting decomposition does not admit EDD. Hence $\psi = \{G_1, G_2, \ldots, G_n\}$ is an $TL_m$.

Conversely suppose $TL_m$ has an EDD.

To prove that $TL_m$ has $\frac{2n^2 - 6n + 10}{2}$ vertices.

Suppose $TL_m$ has no $\frac{2n^2 - 6n + 10}{2}$ vertices.

The following are the two possibilities.

Case i) In the above construction of $G_1, G_2, \ldots, G_n$ if we add the vertices $1, 2, \ldots, n$ in $TL_m$ then there will be remaining 1 to $n$ vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of $G_i$. Therefore the resulting decomposition does not admit EDD. Hence $\psi = \{G_1, G_2, \ldots, G_n\}$ is an $TL_m$.

Conversely suppose $TL_m$ has an EDD.

To prove that $TL_m$ has $\frac{2n^2 - 6n + 10}{2}$ vertices.

Suppose $TL_m$ has no $\frac{2n^2 - 6n + 10}{2}$ vertices.
move the vertices $1, 2, \ldots, n$ in $T L_m$ then there will be remaining 1 to $n-1$ vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of $G_i$. Therefore the resulting decomposition does not admit EDD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption. □

**Theorem 2.6.** $P_p \odot K_1$ has an EDD $\psi = \{G_1, G_2, \ldots, G_n\}$ if and only if $P_p \odot K_1$ has $\frac{n^2 - n + 2}{2}$ vertices.

**Proof.** Let $P_p = \{u_1, u_2, \ldots, u_p\}$ be a path. If we attach the vertices $u'_1, u'_2, \ldots, u'_p$ to $u_1, u_2, \ldots, u_p$ respectively then we get $P_p \odot K_1$.

To prove $P_p \odot K_1$ has an EDD.

Suppose $P_p \odot K_1$ has $\frac{n^2 - n + 2}{2}$ vertices. Let $G_1 = \{u_1, u'_1\}$ $G_2 = \{u_1, u_2, u'_2\}$ $G_3 = \{u_2, u_3, u_4, u'_3, u'_4\}$ .... $G_n = \{u_p, u_{p+1}, \ldots, u_p, u'_{p+1}, \ldots, u'_p\}$ clearly $\gamma_{ed}(G_i) = i$, $i = 1, 2, \ldots, n$. We observe that the minimum eccentric dominating set of $G_n$ has $n$ vertices $P_p \odot K_1$ has $\frac{n^2 - n + 2}{2}$ vertices. Clearly $\gamma_{ed}(G_i) = i$, $i = 1, 2, \ldots, n$ and hence $\psi = \{G_1, G_2, \ldots, G_n\}$ is an $P_p \odot K_1$.

Conversely suppose $P_p \odot K_1$ has an EDD.

To prove that $P_p \odot K_1$ has $\frac{n^2 - n + 2}{2}$ vertices.

Suppose $P_p \odot K_1$ has no $\frac{n^2 - n + 2}{2}$ vertices.
The following are the two possibilities.

Case i) In the above construction of $G_1, G_2, \ldots, G_n$ if we add the vertices $1, 2, \ldots, n$ in $P_p \odot K_1$ then there will be remaining 1 to $n$ vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of $G_i$. Therefore the resulting decomposition does not admit EDD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption.

Case ii) In the above construction of $G_1, G_2, \ldots, G_n$ if we remove the vertices $1, 2, \ldots, n$ in $P_p \odot K_1$ then there will be remaining 1 to $n-1$ vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of $G_i$. Therefore the resulting decomposition does not admit EDD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption. □

**3. Conclusion**

From this paper, we get a knowledge of the eccentric domination decomposition of graphs.

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