A Fast and Accurate Method for Computing the Microwave Heating of Moving Objects

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Abstract: In this paper, we show a fast and accurate numerical method for simulating the microwave heating of moving objects, which is still a challenge because of its complicated mathematical model simultaneously coupling electromagnetic field, thermal field, and temperature-dependent moving objects. By contrast with most discrete methods whose dielectric parameters of the heated samples are updated only when they move to a new position or even turn a circle, in our simulations a real-time procedure is added to renew the parameters during the whole heating process. Furthermore, to avoid the mesh-mismatch induced by remeshing the moving objects, we move the cavity instead of samples. To verify the efficiency and accuracy, we compared our method with the arbitrary Lagrangian–Eulerian method, one of the most accurate methods for computing this process until now. For the same computation model, our method helps in decreasing the computing time by about 90% with almost the same accuracy. Moreover, the influence of the rotational speed on the microwave heating is systematically investigated by using this method. The results show the widely used speed in domestic microwave ovens, 5 rpm, is indeed a good choice for improving the temperature uniformity with high energy efficiency.

Keywords: moving objects; microwave oven; numerical simulation; updating dielectric properties

1. Introduction

Microwave heating has been widely used in many fields in the past few decades, including food processing, chemical engineering, medical health, and even graphene production [1–4]. Compared with traditional heating methods, it has numerous advantages like selective energy absorption, high-speed startup, high energy efficiency, and volumetric heating [5–7]. However, uneven heating, mainly caused by the mixed modes of the microwave cavity, is still one of the major drawbacks, which hinders the widespread applications of microwave heating in various fields [8]. For a domestic microwave oven, adding a turntable is one of the most effective methods to improve the uniformity of temperature distribution during the microwave heating process [9,10]. In this case, the heated sample goes through different locations with different intensities of electromagnetic fields by rotating with the turntable, so that each part of the sample absorbs roughly the same amount of microwave energy. However, as one of the most effective and economical methods for designing and optimizing microwave heating, numerical simulation of this process is quite difficult because its mathematical model simultaneously couples electromagnetic field, thermal field, and temperature-dependent moving objects [9].
To simulate this kind of microwave heating, Geedipalli et al. proposed dividing a continuous rotation cycle into 24 steps for the first time, i.e., the turntable rotates 15° per step. The electromagnetic field distribution of each step was calculated and stored as the heating source for the following heat transfer analysis [11]. However, in their computation the dielectric properties of the sample were considered as temperature-independent. Actually, they usually vary with temperature during the heating process, which may further affect the absorption of electromagnetic energy. As a result, the temperature field may change dramatically since the applied microwave power mostly reaches thousands of watts especially in industrial applications [12].

To solve this problem, Pitchai et al. added a procedure to renew the dielectric properties after the temperature distribution was calculated for each time step [13,14]. However, the heated objects should be remeshed after moving to another location and the mesh elements are normally different from those of the previous step. Therefore, the temperature of the new mesh node cannot exactly be inherited from that of the original mesh node, which creates calculation errors for each remeshing. For a long heating process, the errors will gradually be accumulated and the accuracy of the calculation will become worse and worse. To obtain a better mesh-match during the rotation, a definite sliding interface between the moving and fixed parts was introduced by Liu et al. for ensuring that the nodes were connected during rotating [9]. However, this method is only suitable for heated objects with simple shapes such as cuboids and cylinders, because the setup of the sliding interface will be very complicated for an object with irregular shapes.

To simplify the temperature iteration in the heated samples between the different rotational positions, Chen et al. rotated the microwave cavity instead of the rotating object. At the same time, the dielectric properties of the heated samples were renewed when the turntable rotates to a new position (typical) or rotates a circle (simplified) (i.e., updating at the beginning of each rotational step (every 0.83 s) and at the beginning of each cycle (every 10 s).) [15]. However, their untimely updating strategy may lead to serious calculation errors especially for the temperature-sensitive samples heated by a high-power microwave because the dielectric properties can change significantly even in each time step [12]. Therefore, in order to obtain more accurate simulation results, the dielectric properties of the heated sample should be updated more frequently at each position, rather than assumed to be constant. Moreover, this method is also too time-consuming because too many physical fields are involved. The computation times of heating frozen potatoes for 6 minutes with these two approaches are 550 hours and 96 hours, respectively.

Recently, Zhou et al. proposed a continuous algorithm by using the arbitrary Lagrangian–Eulerian (ALE) method [16], in which the computational mesh moves in a prescribed way and deforms according to the variation of a free surface for ensuring the continuous rotation of the heated sample and the turntable. Moreover, the dielectric properties of samples are updated in real-time just like in actual microwave heating. This simulation model has been validated by their experiments. However, it took about 6.24 hours to simulate a small rotating sample (40 mm × 50 mm × 10 mm) being heated for a short time (12 s). Obviously, for larger samples being heated for longer time, the mesh will increase dramatically and the computation time will be intolerable.

In this paper, a fast and accurate numerical method is shown to simulate microwave heating that simultaneously couples electromagnetic field, thermal field, and temperature-dependent moving objects. During our whole simulation, the oven cavity is rotated instead of the potato, naturally avoiding the temperature inheritance errors provoked by the mesh-mismatch inside the moving objects. More importantly, by contrast with most discrete methods which update the dielectric parameters of the heated samples only when they move to a new position or even turn a circle, in our simulations the electromagnetic field and the heat transfer are two-way coupled and solved each time, like these parameters being updated in a real-time way. To evaluate the efficiency and accuracy of our method, a 3-D fine element method (FEM)-based microwave oven model with rotating turntables is developed. Compared with the ALE method, one of the most accurate methods for computing the microwave heating of moving objects till now, our method takes only about 11% of the calculation time with
almost the same accuracy for the same computation model. Moreover, using our method the effects of discrete angles under different rotation speeds are also studied and this showed that the slower rotational speed is, the smaller the rotation angle is needed to ensure the accuracy of the simulation. Finally, we also systematically investigate the influence of the rotational speed of the turntable on the microwave heating of two different shaped samples (i.e., cuboid and cylinder). The results show that a widely used speed in domestic microwave ovens, 5 rpm, is indeed a good choice for improving the temperature uniformity with high energy efficiency. Our method may be helpful in optimizing other parameters of microwave heating, especially with heat-sensitive materials, fast and accurately.

2. Mathematical Model

2.1. Electromagnetic Field and Its Boundary Conditions

The electric field \( E \) with an angular frequency \( \omega \) distributed in a microwave oven including the food can be obtained by solving Maxwell’s waveform equation,

\[
\nabla \times \mu^{-1} (\nabla \times E) - k_0^2 \left( \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0} \right) E = 0, \tag{1}
\]

where \( \varepsilon_0 \) is the vacuum permittivity and \( k_0 \) is the wavenumber of free space; \( \varepsilon_r \), \( \mu_r \), and \( \sigma \) are the relative permittivity, the relative permeability and the electrical conductivity of the food, respectively. Boundary conditions, specified on the walls of the microwave oven, are assumed to be perfect electrical conductors. Therefore, the tangential component of the electric field should be zero at the interface between the air and the walls of the microwave oven cavity, i.e.,

\[
E_{\text{tangential}} = 0. \tag{2}
\]

2.2. Heat Transfer and Its Boundary Conditions

For microwave heating, the electromagnetic power \( Q_e(t) \) absorbed by the heated sample is taken as the heat source, which can be given by:

\[
Q_e(t) = \frac{1}{2} \omega \varepsilon_0 \varepsilon'' |E|^2, \tag{3}
\]

where \( \varepsilon'' \) is the imaginary part of the complex relative permittivity of the heated food. \( |E| \) is the magnitude of the electric field. Then, the temperature distribution of the sample can be obtained by solving the heat transfer equation expressed as:

\[
\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q_e(t), \tag{4}
\]

where \( T \) is the temperature; \( \rho \), \( C_p \), and \( k \) are the density, the heat capacity, and the heat transfer coefficient of the heated sample, respectively.

Besides the heat transfer inside the heated sample, the energy exchange between the sample and its surrounding air is also included in the proposed model by adding the equation of air convection,

\[
-k \nabla T = h_c (T - T_a), \tag{5}
\]

where \( h_c \) and \( T_a \) are the convection heat transfer coefficient and the air temperature, respectively.

Additionally, the energy exchange between the rotating turntable and its surroundings (i.e., the heated food and air) is not taken into account. In other words, the thermal boundaries between the rotating turntable and its surroundings are considered as insulation,

\[
-n \cdot q = 0, \tag{6}
\]
where $n$ is the normal unit vector of the heated sample surface, $q$ is the heat flux.

2.3. Simulation Strategy

Simulating microwave heating coupled electromagnetic and thermal fields of moving objects is usually very complicated and extremely time-consuming. To address this problem, we show an efficient and flexible method where each continuous rotation circle is divided into many discrete points, i.e., the moving objects change their positions many times. At each position, the moving objects keep stationary for a period, which is further divided into smaller electromagnetic time steps. During each electromagnetic time step, the dissipated power density inside the heated sample is calculated by solving Equations (1)–(3) in the frequency domain with an iterative linear system solver, Flexible Generalized Minimum RESidual (FGMRES). Then, the temperature distribution field in the time domain is obtained by taking the power as the heat source to solve Equations (4)–(6) with a parallel sparse direct solver (PARDISO). With the calculated temperature, the dielectric properties of the sample are updated at the end of each electromagnetic time steps. The calculated temperature and updated dielectric properties are taken as the initial values in the calculation of the following time step. Since the calculated electromagnetic time steps are usually less than 0.01 s, the dielectric properties can be considered as real-time changed just like the actual microwave heating. After the calculation at one position is done, we instantaneously rotate the remains, instead of the heated sample, to the next position and reconduct the above calculations until the desired heating time is reached. The flowchart of the proposed method is outlined in Figure 1.

![Flowchart of the proposed method](image)

Figure 1. Overall flowchart of the proposed method.

2.4. Multiphysics Simulation of a 3-D Model

For validating our method, we take a 3-D model like the article of Zhou et al. [16], which includes four parts: a cavity, a glass turntable, a rectangular waveguide, and a potato sample (50 mm × 60 mm × 15 mm), as shown in Figure 2. The initial temperatures of the air, rotating turntable, and the moving objects inside the oven are all 20 °C. The potato, whose dielectric properties are considered as temperature-dependent, is centered at the glass turntable. The waveguide filled with air works in the TE$_{10}$ mode, providing a power of 300 W and operating at 2.45 GHz. The input parameters of the geometric model are given in Table 1 [9,16–19].
Moreover, we also operate memory by the ALE method \cite{16}, which is shown to be effective for reducing memory requirements in 3-D simulations. Therefore, a mesh convergence study for 3-D electric field calculation is performed by the proposed method and the ALE method to determine the reasonable amount of mesh elements. Figure 3 shows the normalized power absorption (NPA) as a function of the total number of elements for the proposed method and the arbitrary Lagrangian–Eulerian (ALE) method. It is crucial to choose an appropriate number of mesh elements for numerical simulation because this affects not only the calculation accuracy but also the calculation time. Therefore, a mesh convergence study for 3-D electric field calculation is performed by the proposed method and the ALE method to determine the reasonable amount of mesh elements. Figure 3 shows the normalized power absorption (NPA) as a function of the total number of elements in the mesh. It can be clearly seen that the simulation results are mesh-independent after the number of elements reaches 100,000. Accordingly, the total elements consisting of 138,438 and 126,480 quadratic tetrahedrons are used in the proposed method and the ALE method, respectively.

![Diagram](image_url)

**Figure 2.** Schematic of the microwave oven system used for modeling (all sizes in millimeters).

| Property                        | Domains | Value                                      | Source |
|---------------------------------|---------|--------------------------------------------|--------|
| Relative permittivity           | Air     | 1                                          | \cite{18} |
|                                 | Potato  | \((-6.4 \times 10^{-3}T^2 + 0.2T + 56.8)\) | \cite{9} |
|                                 | Glass   | 4                                          | \cite{16} |
| Conductivity (S/m)              | Air     | 0                                          | \cite{18} |
|                                 | Potato  | 0                                          | \cite{18} |
|                                 | Glass   | 0                                          | \cite{16} |
| Thermal conductivity (W/m-K)    | Potato  | \(5.998 \times 10^7\)                     | \cite{17} |
| Density (kg/m^3)                | Potato  | 1050                                       | \cite{17} |
| Specific heat capacity (J/kg-K) | Potato  | 3630                                       | \cite{17} |
| Heat transfer coefficient       | Potato  | 10                                         | \cite{19} |

**Figure 3.** Normalized power absorption (NPA) as a function of the total number of elements for the proposed method and the arbitrary Lagrangian–Eulerian (ALE) method.
As a discrete simulation, the continuous rotation of the turntable and the heated food should be divided into many discrete steps and the discrete angle plays an important role in obtaining the temperature distribution with good accuracy. To study the effects of rotational angle on simulation, the average temperatures of the potato at the 24th second under different speeds and different rotation angles are calculated, as shown in Figure 4. This shows that when the rotational speed is faster than 5 rpm, a rotation angle of 30° per time step is sufficient to obtain accurate results. However, when the speed is lower than 5 rpm, a smaller angle should be selected to ensure the accuracy of the simulation results. Therefore, 30° rotational degrees per step is suggested to obtain reasonable results by the proposed method in the next study, i.e., the oven is assumed to be at 12 discrete positions for each rotation cycle.

![Figure 4. Average temperature of potato slice at different rotation angles per time step under different speeds.](image)

In order to evaluate the accuracy and efficiency of our method, we simulate the same 3-D model and compare our results with those obtained by the ALE method [16], which has been proved to be more efficient and accurate than the conventional discrete method for simulating the microwave heating of moving objects and verified by experiments. Moreover, we also study the effects of rotational speeds on the microwave heating process. In addition, the effects of food deformation, moisture migration, and phase change are ignored because the heating time is very short. All the simulations are performed by COMSOL Multiphysics in the workstation with an operating memory of 96-GB RAM, running on an Intel(R) Xeon(R) CPU X5680 3.33GHz.

3. Results and Discussion

The rotational speed of the glass turntable is set to 6 rpm and the total heating time is 10 s, that is, the turntable only rotates one turn in this simulation. As previously discussed, the rotation angle of each calculation step is taken as 30°. Correspondingly, the rotating parts of the model will stay at each location for 0.834 s until the next step of calculation. As shown in Figure 5, the electric field at 0 s, 0.834 s, 1.667 s, and 2.5 s calculated by our method are compared with those obtained by the ALE method. These heating times correspond to the rotating parts rotating 0°, 30°, 60°, and 90°, respectively. It is clear that the electric field distribution calculated by our method agrees very well with that calculated by the ALE method.
3.1. Temperature Distribution inside the Heated Potato

The scattering parameters of the waveguide port are also plotted at the end of each second during the entire heating process. As shown in Figure 6, the two curves fit very well and the average relative error of the scattering parameters is only about 0.53% [12]. This further confirms that the proposed method can calculate the electric field distribution inside the microwave oven with good predictability.

Figure 5. Multislices of the electric field of the potato inside of a microwave oven computed by the ALE method and the proposed method after heating 0 s, 0.834 s, 1.667 s, and 2.5 s (unit: V/m).

Figure 6. $|S_{11}|$ computed by the proposed method and the ALE method.

Figure 7 displays the outline of the potato and the temperature distributions on the upper surface of the potato at the 10th second. It clearly shows that the temperature distributions are obviously non-uniform and their shapes are very similar. For example, both of them have a “hot ring” at the center of the potato and the four corners are relatively cool. For further quantitative verification, Figure 8 gives the temperature profile of a potato sample as a function of time at five specific locations defined in Figure 9. As shown in Figure 8a to Figure 8e, the temperatures calculated by the two methods are matched very well, too. Meanwhile, the average temperature curves obtained by these two methods are also in good consistency as plotted in Figure 8f. The qualitative and quantitative analyses of the temperature field prove that the proposed method can be applied to calculate the temperature field of the food with good predictability.
Figure 7. Outline of potato and temperature distributions on the upper surface of the potato obtained by the ALE method and the proposed method at the 10th second (top view, unit: °C).

Figure 8. Comparison of the time-dependent temperature calculated by the proposed method and the ALE method at the five specific points.

Figure 9. The specific locations of the five points (7.5 mm from the top surface).

To further evaluate the reliability and efficiency of our method, the standard deviation (SD) of a point P is defined by [16]:

$$ SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (T_i - \bar{T}_i)^2}, $$

(7)
where $T_i$ is the temperature of point $P$ at a specific time step $i$ and $\overline{T}_i$ is the average temperature of the whole potato, and $n$ is the total time steps of the calculation. The $SD$ values of the points P1 to P5 and the total computing time are listed in Table 2. It can be clearly noticed that the $SD$s calculated by the proposed method are very close to those obtained by the ALE method. It is worth mentioning that compared with the ALE method, the proposed method provides almost the same accuracy only using $5555/50117$ or $11\%$ of the computing time. This may possibly be attributed to the movement of the turntable. To guarantee the reliability of the deformed mesh and the continuous rotating in the ALE method, the turntable can only move a small angle and at each step the EM fields and the heat transfer analysis should be solved, which greatly increases the computational time because many steps are needed to simulate rotating a circle. Moreover, the meshes in the air domain of the ALE method are deformed as the turntable moves, causing a decrease in the mesh element quality (MEQ). To solve this problem, the meshes in the air domain will be remeshed when MEQ < 0.2, which also requires a lot of calculation time. This confirms that the proposed method is much more efficient than the ALE method with desirable accuracy.

### Table 2. Total computation time and standard deviation (SD) in different calculations at five locations.

| Property             | ALE Method | Proposed Method |
|----------------------|------------|-----------------|
| $SD$ in P1           | 9.28       | 9.37            |
| $SD$ in P2           | 4.89       | 5.29            |
| $SD$ in P3           | 6.26       | 6.49            |
| $SD$ in P4           | 1.58       | 1.80            |
| $SD$ in P5           | 6.22       | 6.60            |
| Average $SD$ for all points | 6.19       | 6.45            |
| Total computation time, s | 50,117     | 5555            |

3.2. The Influence of Rotational Speed on the Microwave Heating

The speed of the rotating turntable always acts as an important factor in optimizing the domestic microwave oven, which was infrequently investigated by previous moving-mesh methods because this was too time-consuming [16]. Here, we use the same model verified above to study the influence of the rotational speed on the microwave heating. To mimic a real microwave oven, the input power of the waveguide is added to 1 KW and three rotational speeds, i.e., 10 rpm, 5 rpm, and 2.5 rpm, are calculated and compared. To ensure more accurate simulation results, a finer mesh with 163,965 quadratic tetrahedral elements and a smaller rotation angle of 15° are selected for the computation, i.e., the rotation parts will stay at each location for 0.25 s, 0.5 s, and 1 s, respectively. The microwave heating lasts 24 s. Additionally, for comparing the heating uniformity we also compute for the potato remaining stationary during the whole microwave heating.

Firstly, the time-dependent scattering parameter $S_{11}$ of the rectangular waveguide, which reflects the effective output microwave power of the microwave oven, is quantitatively given in Figure 10. It is clear that all scattering parameters of the microwave ovens with rotating potatoes fluctuate with the time and the time interval of the ups and downs are almost a half period of the rotation of the turntable, suggesting that the effective output power is sensitive to the location of the heated potato. Moreover, with the increment of heating time, the fluctuant $S_{11}$ gradually increases and the amplitude of fluctuation decreases. This can be explained based on the temperature-dependent dielectric properties of the potato. As shown in Table 1, both real and imaginary parts of the dielectric constant of the potato decrease with the increment of temperature from 20 to 100 °C, indicating that its microwave absorption also decreases. Therefore, when the heating time adds, the temperature of the potato increases and the absorbed microwave power decreases.
are the temperature field distribution on the top surface of the potato after heating 24 s at different rotational speeds as shown in Figure 11. It can be seen that all temperature distributions have similar patterns, whereas the dark areas are more obvious for the rotational speeds of 2.5 rpm. This means that a slow rotational speed is not good for improving the heating uniformity.

![Figure 10](image1)

**Figure 10.** $|S^2|$ computed by the proposed method at different speeds.

Secondly, we compare the temperature field distribution on the top surface of the potato after heating 24 s at different rotational speeds as shown in Figure 11. It can be seen that all temperature distributions have similar patterns, whereas the dark areas are more obvious for the rotational speeds of 2.5 rpm. This means that a slow rotational speed is not good for improving the heating uniformity.

![Figure 11](image2)

**Figure 11.** Simulated temperature distributions on the top surface of the cubic potato after microwave heating for 24 s with different rotation speeds (top view, unit: °C).

To further qualitatively compare the effects of speed on the temperature inside the potato, the maximum and minimum temperatures are also plotted. As shown in Figure 12a, for low rotational speeds, the turntable rotates faster, the minimum temperature of the potato is higher. When the turntable rotates faster than 5 rpm, the minimum temperature of the potato is almost no longer affected by the speed. On the other hand, the rotational speed is an effective way to lower the maximum temperature of the potato as presented in Figure 12b. It is clear that the maximum temperature roughly decreases with the increment of rotational speed. However, for the speed of 5 rpm and 2.5 rpm, the influence of the speed on the maximum temperature is not obvious. Combining Figure 12a,b, it can be concluded that increasing the rotational speed is an effective way to decrease the difference between the maximum and minimum temperatures of the potato and improve the heating uniformity. Additionally, the maximum temperature of the potato with a stationary turntable is higher than all those with the rotating ones, and the minimum temperature is far lower, which leads to severe heating unevenness.
her two speeds (8 1 2 5 T, suggesting the r 5 1 2 1 5 2 1 1 r 5 r eds (top view, (1 (2 2 = = 4 1 = r 8 1 r 2 (4 2 4 = 1 1 r (T 8 (T ° are -2 = 76x158 × 779 Appl. Sci. speeds, the minimum, maximum, and average temperatures of the cylindrical potato heated with these middle and “cold spot” in the center. Moreover, to quantitatively compare the e 3, i.e., 10 rpm, 5 rpm, and 2.5 rpm, on microwave heating of a cylindrical potato (10 mm (height) × 30 mm (radius)) centered in the turntable with the same microwave cavity. The total heating time is also set to 24 s and the mesh (height) × 30 mm (radius) centered in the turntable (5 rpm absorbs the most energy, agreeing well with the average temperature presented in Figure 13. As we know, 1− |S_{11}|^2 is proportional to the effective output microwave power of the waveguide. According to the data of $S_{11}$ the microwave energy absorbed by the potato can be estimated by an integral of $1− |S_{11}|^2$ over time. The calculated integrals show that the potato rotating at the speed of 5 rpm absorbs the most energy, agreeing well with the average temperature presented in Figure 13.

Finally, to reconfirm the most suitable speed obtained above, we also study the effects of these three speeds, i.e., 10 rpm, 5 rpm, and 2.5 rpm, on microwave heating of a cylindrical potato (10 mm (height) × 30 mm (radius)) centered in the turntable with the same microwave cavity. The total heating time is also set to 24 s and the mesh with 161,619 quadratic tetrahedral elements and the rotational angle of 15° are selected for the computation. Figure 14 compares the temperature field distribution on the top surface of the cylindrical potato after heating 24 s at different rotational speeds. The temperature distribution patterns of cylindrical potato are also similar under these three speeds with “hot ring” on the middle and “cold spot” in the center. Moreover, to quantitatively compare the effects of rotational speeds, the minimum, maximum, and average temperatures of the cylindrical potato heated with these
three different speeds are also given in Figure 15. It clearly shows that the average temperature at the speed 5 rpm is slightly higher than that at the other two speeds, which reconfirm that 5 rpm is indeed a good choice for improving the temperature uniformity with high energy efficiency.

**Figure 14.** Outline of potato and simulated temperature distributions on the top surface of the cylindrical meshed potato after microwave heating for 24 s with different rotation speeds (top view, unit: °C).

**Figure 15.** The minimum (a), maximum (b) and average (c) temperatures of the cylindrical shaped mashed potato heated with different speeds.

### 4. Conclusions

In this paper, we show a fast, flexible and accurate numerical method to simulate temperature-dependent moving objects coupling the electromagnetic field and thermal field. To avoid the temperature inheritance errors caused by mesh-mismatch, the cavity is rotated in the opposite of the actual rotation direction. More importantly, the dielectric properties of the potato are updated instantaneously with respect to the changing of temperature field during the whole heating simulation.

To demonstrate the advantage of our method, firstly, a 3-D model with moving objects is presented and the simulated results are compared with those obtained by the ALE method, which is one of the most accurate methods and has been verified experimentally. Our proposed method is more efficient with an 89% decrease of the computation time with almost the same predictability.

Secondly, using our method, the effects of discrete angles under different rotation speeds are also studied. The results show that the slower rotational speed is, the smaller the effective rotation angle is to ensure the accuracy of the simulation results.

Finally, we also systematically investigate the influence of the rotational speed of the turntable on the microwave heating of two different shaped samples (i.e., cuboid and cylinder), which were infrequently studied by previous moving-mesh methods because this was too time-consuming. The results show that a widely used speed in domestic microwave ovens, 5 rpm, is indeed a good choice for improving the temperature uniformity with high energy efficiency. Our method may be helpful for optimizing microwave heating with heat-sensitive materials.
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