Decay of long-lived quantum Hall induced currents in 2D electron systems

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Abstract. The decay of quasi-persistent circulating currents in the dissipationless quantum Hall regime has been observed. The currents induced by a time-varying magnetic field flow within a two-dimensional electron system (2DES) embedded in a GaAs–(Al,Ga)As heterojunction. The associated magnetic moment is measured using a highly sensitive magnetometer. The currents are observed to continue circulating for many hours after the magnetic field sweep is stopped indicating a very low sheet resistivity. Two distinct current decay regimes are observed, consisting of an initial exponential decay lasting a few tens of seconds followed by a much slower power-law decay. The presence of the fast initial decay, during which the current falls typically to half of its original value, indicates that the system is initially quite dissipative because the quantum Hall effect (QHE) has broken down due to the large induced current. As the current decays, the quasi-dissipationless QHE state recovers, resulting in the much slower decay, which the data suggest will persist for at least several days, much longer than has previously been suggested. The power-law form of the long decay suggests multiple relaxation paths for the system to return to equilibrium, each having a different characteristic time constant. This can equivalently be thought of as a resistivity which gradually falls with current.
1. Introduction

The quantum Hall effect (QHE) [1] occurs in semiconductor two-dimensional electron systems (2DESs) at low temperatures and high magnetic fields. When a magnetic field is applied to a 2DES the density of states (DOS) is modified from a constant value to a series of δ-functions separated by the cyclotron energy $\hbar \omega_c$. These quantized electron energy levels known as Landau levels are highly degenerate and in a real system are broadened by disorder. The degeneracy of a Landau level is proportional to the applied magnetic field, and the filling factor $\nu = n_e h/eB$ ($n_e$ is the electron number density of the 2DES) defines how many Landau levels are full at a given field. The degeneracy of the Landau levels leads to the formation of a mobility gap near integer $\nu$ where the DOS at the Fermi level is close to zero. As the magnetic field is swept through integer $\nu$, the Fermi energy lies in a mobility gap and $\rho_{xx}$ and $\sigma_{xx}$ both drop to a minimum. The QHE is characterized by a quantized Hall resistance ($\rho_{xx} = \hbar/ie^2$, where $i$ is an integer), with concurrent minima in the longitudinal resistance producing almost dissipationless electrical conduction ($\rho_{xx} \approx 0$) as the magnetic field is swept through integer $\nu$. It has previously been shown that large circulating currents are induced as the magnetic field is swept through integer $\nu$ [2, 3, 4, 5]. In an ideal 2DES at absolute zero $\rho_{xx} = 0$ and the induced circulating current should be persistent; however in a real disordered system or at nonzero temperature this is not the case. The decay of the induced circulating currents observed in a static magnetic field gives an insight into both the breakdown of the QHE at large currents [6], and into the relaxation processes by which the QHE system can return to equilibrium [3, 10].

The right-hand inset to figure 1 shows a schematic of the geometry of our experiment: a 2DES is subjected to a magnetic field and a circulating current is induced as the field is swept. The induced currents are thought to flow along the inner border between the outermost compressible strip at the edge of the 2DES, and the incompressible region within [7]. The currents are therefore isolated from the ohmic contacts at the edge of the 2DES, and are consequently not detected in conventional transport measurements, despite being typically at least an order of magnitude larger than the injected currents involved in such experiments [8, 9]. The circulating current induced by the sweeping field generates a magnetic moment perpendicular to the plane of the 2DES. There is a torque $\tau = m \times B = mB \sin \theta$ between the magnetic moment and the applied magnetic field. A measurement of $\tau$ therefore constitutes a measurement of the circulating current. Induced circulating currents were first observed by Eisenstein et al [2] using a torsion-balance magnetometer to measure the torque as a function of magnetic field. Similar experiments have been carried out to study the decay of these currents [3] and the breakdown of the QHE [10, 11].
Figure 1. Magnetometer output for sample 2 at 20 mK, the magnetic field was swept from 0 to 12 T at a rate of 30 mT s$^{-1}$. The left-hand inset is a schematic of the torsion-balance magnetometer. The right-hand inset is a schematic of the geometry of a magnetometry experiment. The normal to the 2DES is tilted by $\theta = 20\%$ with respect to the applied magnetic field.

Recently we have shown that laterally defined nanostructures such as quantum point contacts and quantum dots are sensitive to the presence of an induced circulating current [12]. Another method of detecting circulating currents uses single-electron transistors fabricated on top of the 2DES to detect the Hall electric field accompanying the circulating current via the Coulomb-blockade potential [4, 8].

Previous experiments [3, 10] have reported an exponential decay of current with time in the dissipationless QHE state, suggesting a single dominant decay path, causing the current to decay over a period of several hours. Here we report a much slower power-law dependence of the decay implying the presence of many different decay mechanisms, and consequently a resistivity that decreases with time (or equivalently, with current).

2. Experimental details

The torsion-balance magnetometry experiments were carried out on modulation-doped heterojunctions in a low-vibration dilution refrigerator as described by Matthews et al [13]. Two 2DESs were studied: sample 1: $n_e = 1.7 \times 10^{15}$ m$^{-2}$, $\mu = 200$ m$^2$ (Vs)$^{-1}$; sample 2: $n_e = 4.4 \times 10^{15}$ m$^{-2}$, $\mu = 50$ m$^2$ (Vs)$^{-1}$. The stated values were obtained in transport measurements at 4.2 K. The samples were uncontacted 10 $\times$ 6 mm rectangles.

The torsion-balance technique (left-hand inset to figure 1) uses a low-mass rotor, with Au evaporated on one side and suspended by a fine torsion fibre above two stator plates. The 2DES is mounted on the rotor with the normal to the 2DES at an angle of 20$\degree$ to the magnetic field.
The resulting torque causes a rotation of the rotor, which is measured using a capacitance bridge technique. Experiments were performed at magnetic fields up to 12 T and at temperatures in the range 20–800 mK. Figure 1 shows typical raw data: the torque caused by the magnetic moment (proportional to the circulating current) versus perpendicular magnetic field. Large induced currents are observed as the magnetic field is swept through the QHE conditions of integer Landau-level filling factor occupancy ($\nu = 1, 2, 3 \ldots$) (calculated from the perpendicular component of $B$ and the values of $n_e$ quoted above). Induced currents are observed at $\nu = 1–4$ for sample 1 and $\nu = 2–4, 6$ and 8 for sample 2. The current decay measurements reported here concentrate on Landau-level filling factors $\nu = 2$ and 4. The polarity of these induced currents reverses (not shown) when the sweep direction of the inducing magnetic field is reversed, confirming their identification as non-equilibrium features. The induced currents observed in figure 1 are quasi-persistent, meaning that the induced circulating current continues to flow around the sample for a considerable amount of time after the magnetic field sweep is halted. This behaviour is due to the small value of $\rho_{xx}$.

In order to measure the decay of the circulating current at a filling factor $\nu_0 ( = n_e h/eB_0)$, we induce a circulating current by sweeping the magnetic field from $B_0 + \Delta B$ to $B_0$ at a rate $dB/dt$. The magnetic field sweep is then stopped and the field maintained at a value $B_0$ for the duration of the experiment. Hereafter $\Delta B$ will be referred to as the ‘approach distance’ and $dB/dt$ as the ‘approach rate’.

The dilution refrigerator was run in single-shot mode during decay measurements to further eliminate mechanical noise, which would otherwise have occurred as the sorption pumps were cycled. The decays were monitored for up to 40 h, limited by temperature stability. As shown in figure 1 there is a background signal superimposed upon the signal from the circulating currents, which is caused by the background magnetization of the magnetometer and sample substrate. In order to quantify the decay rate of the induced currents, it is necessary to have an estimation of the zero level. Figure 2 shows the procedure for calculating the background signal and subtracting it from the decay data. Prior to each current-decay experiment the magnet is swept through integer filling factor. A second-order polynomial is then fitted to the data to either side of the peaks at

Figure 2. (a) Plot showing both polarities of induced current at $\nu = 2$ for sample 2 and a second-order polynomial fit to the background signal (dashed line). (b) Plot showing the same data after subtraction of the background signal.
Figure 3. Decay of induced current for sample 1 ($\nu = 2, 88 \pm 2\,\text{mK}$), inset, a logarithmic plot of the same data. The power-law decay has an exponent $n = -0.0742 \pm 0.0002$.

integer filling factor. The value of the polynomial fit at a given value of $\nu$ is equivalent to the magnitude of the background signal at that point. This value can then be subtracted from the raw data and the current decays plotted relative to their true zero value. The background level is determined again after the decay measurement, to check for variation in the background signal caused by drift in the room temperature electronics. Temperatures were monitored during the experiment and all the decays presented have temperature error bars indicated.

3. Results

Figure 3 shows the decay of an induced current at $\nu = 2$. The data display two distinct decay regimes. The initial exponential decay causes the current to fall to $\sim 70\%$ of its original value in the first 100 s. The subsequent power-law decay results in the current dropping from $\sim 70\%$ to $\sim 45\%$ in the ensuing 12 h. Having established that two distinct decay regimes occur, to obtain quantitative information about both regimes, we can separate the initial exponential decay from the subsequent power-law. To do this, we make use of the fact that the exponential component of the decay becomes negligible after a certain time, and fit the raw data in this regime (typically $t > 500\,\text{s}$) to a power-law $I = at^n$. To obtain the initial fast decay we then subtract the power-law dependence obtained, from the entire data set. The initial decay is then fitted to an exponential $I = I_0 \exp(-\lambda_{\text{init}}t)$. We have followed this procedure in obtaining the data for figures 4 and 5.

The decay shown in figure 3 has an approximately exponential decay (after power-law subtraction) for the first 100 seconds, with decay time constant $1/\lambda_{\text{init}} = 32 \pm 4\,\text{s}$. This relatively fast exponential decay is linked to the high-current breakdown of the QHE [10] and is only present when the initial circulating current is greater than the critical current required for QHE
Figure 4. Exponential decay time constant versus approach rate $dB/dt$ for sample 2 shown on a reverse semi-log plot for clarity ($\nu = 2$ at $150 \pm 2$ mK, $\Delta B = 0.1$ T) the error bars represent the uncertainty associated with the exponential fit. Inset is a semi-log plot of a typical fast decay with background signal and power-law removed (sample 2, $\nu = 2$ at 300 mK, $\Delta B = 0.058$ T, $dB/dt \approx 0.8$ mT s$^{-1}$). The decay has a time constant of $1/\lambda_{\text{init}} = 7.87 \pm 0.08$ s. The initial deviation from the exponential fit is a feature associated with the large inductance of the superconducting magnet.

breakdown. When this critical current is exceeded the system no longer displays its characteristic dissipationless conduction; instead it is quite dissipative. The QHE state then recovers as the circulating current falls.

The magnitude and time constant of the single-exponential decay depend both on the approach rate $dB/dt$ and the approach distance $\Delta B$. Figure 4 shows the dependence of the initial exponential time constant on the approach rate. The effect of increased approach rate is to increase the rate of decay. We also observe that the magnitude of the induced current is increased by a faster approach rate. It has been previously shown that the magnitude of the induced current increases with $dB/dt$ and $\Delta B$ up to the saturation magnetization at which high-current breakdown of the QHE occurs but cannot increase any further [14]. When in this regime, despite the circulating current not increasing beyond this critical value, the system can still be driven further into the high-current breakdown regime and the sample resistivity still increases.

Before the approach begins, there will already be an induced current present, having been induced by the sweep to the starting point $B = B_0 + \Delta B$. In the experiment of figure 4, this pre-existing induced current was of opposite polarity to the current induced by the approach to $B_0$. Consequently, it was in principle be possible to completely eliminate the initial exponential decay by an appropriate choice of $\Delta B$ and $dB/dt$ though we have not made use of this in the data presented.

The characteristics of the power-law decay are presented in figure 5. The filling-factor dependence displayed in figure 5(a) shows that as $B_0$ moves away from integer $\nu$ the magnitude of the initial induced current is reduced. The decay of the induced current also becomes faster.
Figure 5. (a) Induced current versus time for decays in the range $v = 2$–2.15, at 150 ± 8 mK. Note the decays are not offset. (b) Power-law exponent versus filling factor $v$ at 150 ± 8 mK. (c) Temperature dependence of the slow decay power-law exponent at $v = 2$ and 4. The uncertainties shown are due to slight temperature drifts over the several hours for which the decay was measured. All data shown for sample 2, $\Delta B = 0.058$ T, $dB/dt \approx 0.8$ mT s$^{-1}$.

indicated by the increasingly negative power-law exponent, as shown in figure 5(b). This contrasts with the initial exponential decay, which becomes slower away from integer $v$ because the smaller induced current drives the system less far into the high-current breakdown regime. Figure 5(c) shows the temperature dependence of the power-law decay exponent for sample 2 in the range 70–750 mK for $v = 2$ and 4. The data display an approximately linear dependence, with the exponent becoming increasingly negative with temperature signifying a faster decay. The decay of currents at $v = 4$ has a temperature dependence $\sim 3$ times faster than at $v = 2$.

In figure 5(b), points outside the range 1.875 $\leq v \leq 2.075$ have been excluded because as $v$ moves away from integer filling factor the induced current becomes smaller and closer to the noise level. There is also a tendency for the decays to become closer to exponential in nature both as $v$ moves away from integer filling factor, and as the temperature is raised. This deviation from a power-law behaviour probably accounts for the increasing scatter away from $v = \text{integer}$ and at elevated temperatures (especially for $v = 4$ where the induced current is much smaller as shown in figure 1).

The slow power-law decay is not affected by the choices of $\Delta B$ and $dB/dt$. 

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4. Discussion

In a sweeping magnetic field the circulating induced current results in a Hall voltage between the edges and the centre of the 2DES. The majority of the energy stored in the sample is stored capacitively in the Hall electric field between the circulating current at the edge of the sample and the centre of the 2DES [3]. For a non-ideal sample $\rho_{xx}$ and $\sigma_{xx}$ are nonzero allowing the charge circulating at the edge to return to the centre of the 2DES by scattering. In previous work [3] a simple semi-classical momentum scattering model was used to arrive at an estimate of $\rho_{xx}$. It was assumed that the decay time $\tau = 1/\lambda$ was the amount of time required on average for an electron to lose all of its tangential momentum so that it was free to move radially (reducing the tangential current) and thus helping the system return to equilibrium by discharging some of the capacitively stored energy in the sample. The following relationship between $\rho_{xx}$ and $\tau$ was obtained

$$\rho_{xx} = \rho_{xy}/\omega_c \tau.$$ 

The power-law decay can be pictured in two ways. The current may be assumed to flow in a single loop close to the edge of the sample. In this picture the power-law form of decay in the quasi-dissipationless QHE regime indicates the presence of multiple relaxation paths (i.e. multiple independent scattering events each with its own characteristic timescale), each of which contribute to $\rho_{xx}$. As the decay progresses and both the current and the Hall electric field fall, the faster relaxation routes vanish, leaving only slower relaxation paths. In the second, equivalent picture, one could envisage a disordered sample as containing many current loops of different shapes and sizes confined to wells in the disorder potential. In such a system each current loop has a characteristic time constant defined by the capacitance of the loop and the resistivity. The variation in decay rate between current loops would then explain the overall power-law decay rate observed. Faster decaying current loops will disappear first until a single current loop remains. In both these pictures, one would expect the power-law dependence to give way to a single exponential decay at very large times. There is no indication of this behaviour in our data after 45 000 s of decay, though if significantly longer measurements were made an observation of this exponential decay may be possible. In our previous study into the statistics of the noise accompanying breakdown of the QHE [15], the size of the noise jumps displayed a power-law dependence, suggesting multiple paths for QHE breakdown. It is interesting that we can now infer the same about the decay of circulating currents in the quasi-dissipationless QHE regime.

The power-law form of the relaxation can also be interpreted as a time-dependent or equivalently, current- or Hall-field-dependent $\rho_{xx}$. We can describe the power-law decay, $I = at^n$, as an exponential decay with a time-varying decay rate, $I = I_0 \exp(-\lambda(t) t)$. In this case

$$\lambda(t) = -\frac{n}{t} (\ln(t)).$$

In this description, the decay rate $\lambda(t)$ is proportional to $\rho_{xx}$. This implies that $\rho_{xx}$ decreases with time according to:

$$\rho_{xx} = -\frac{hm^*}{ve^3B} \left( \frac{n}{t} \ln(t) \right).$$

For the decay in sample 2 ($v = 2$, 150 mK, the top trace of figure 5(a)), this equation yields a value of $\rho_{xx} = 8.4 \times 10^{-13} \Omega/\square$ at $t = 100$ s falling to $\rho_{xx} = 4.3 \times 10^{-15} \Omega/\square$ after 45 000 s.
These values are consistent with those estimated in [3], in which a single exponential was fitted to data taken over a period of 800 s. The striking new feature of our studies is that the decays are sub-exponential, resulting in a $\rho_{xx}$ that decreases with time (approximately as $1/t$). This new behaviour has become apparent because we have been able to follow the decays for $\sim 60$ times longer than in previous experiments.

Chaubet and Geniet [16] calculated the transition probabilities associated with breakdown of the QHE. We note that their equation (25) could, for certain parameter values, result in decays similar to the power-law form we report. This is another suggestion of a link between the breakdown and quasi-dissipationless regimes.

The filling factor dependence shown in figure 5(b) indicates that the system is becoming more dissipative as the filling factor deviates from integer values. This demonstrates that $\rho_{xx}$ and $\sigma_{xx}$ do not remain constant across the entire quantum Hall plateaux.

Figure 5(a) showed that the exponential decay is present even at $\nu = 2.15$, which lies in the tail of the induced current. The current decays exponentially to zero in a few tens of seconds and there is no slow power-law decay. This can be attributed to intra-Landau level scattering [17] becoming the dominant relaxation mechanism away from integer $\nu$ [18]. The temperature dependence of the power-law decay shown in figure 5(c) suggests that electron-phonon interactions play an increasing role in the dissipation of the circulating electrons at integer $\nu$ as the temperature is increased. We also observe that the current at the start of the power-law decay $I_{0}$, which is equivalent to the critical current required for QHE breakdown [17] has a linear temperature dependence as previously reported [14].

Finally we note that a similar power-law form of decay is observed in the relaxation of magnetic moments in Ising spin glasses [19]. In the spin-glass phase a simple power-law decay is observed with a temperature-dependent exponent, while above the spin-glass transition a decay of the form $q(t) = ct^{-x} \exp(-\omega t^{-\beta})$ with temperature-dependent exponents $x(T)$ and $\beta(T)$ is observed. To distinguish between these different dependences in our experiment, improvements would have to be made to the temperature and mechanical stability. The comparison is intriguing nonetheless.

5. Summary

We have shown that the decay of induced currents in the QHE regime comprises an initial exponential decay caused by the high-current breakdown of the QHE followed by a much slower power-law decay once the quasi-dissipationless QHE state is restored. The power-law decay suggests the presence of multiple relaxation paths, each with its own characteristic time. The power-law form of decay also implies a $\rho_{xx}$ which is approximately proportional to $t^{-1}$ with calculated values decreasing by two orders of magnitude over the course of half a day. This has the important implication that significant residual induced currents are present for much longer than has previously been thought.

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