ICE LINES IN CIRCUMBINARY PROTOPLANETARY DISKS

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ABSTRACT

I examine the position of the ice line in circumbinary disks heated by steady mass accretion and stellar irradiation and compare with the critical semimajor axis, interior to which planetary orbits are unstable. There is a critical binary separation, dependent on the binary parameters and disk properties, for which binaries with separations larger than this critical value have ice lines that lie interior to the boundary of stability. For an equal-mass binary comprised of 1 M⊙ components, this critical separation is \( \approx 1.04 \) AU, and scales weakly with mass accretion rate and Rosseland mean opacity (\( \propto [M_{\text{K}}]^{2/9} \)). Assuming a steady mass accretion rate of \( M \sim 10^{-8} M_{\odot} \text{ yr}^{-1} \) and a Rosseland mean opacity of \( \kappa \sim 1 \text{ cm}^2 \text{ g}^{-1} \), I show that \( \geq 80\% \) of all binary systems with component masses \( M_1 \leq 2.0 M_{\odot} \) have ice lines that lie interior to the critical semimajor axis. This suggests that rocky planets should not form in these systems, a prediction which can be tested by looking for planets around binaries with separations larger than the critical separation with \textit{Kepler} (difficult) and with microlensing.

Key words: binaries – general – planets and satellites: formation – protoplanetary disks – stars: pre-main sequence

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1. INTRODUCTION

The “ice line,” or “snow line,” is the point in protoplanetary disks at (and beyond) which the temperature is low enough that water ice can condense from the nebular gas (Hayashi 1981). Stellar irradiation, mass accretion, and general disk properties (e.g., opacity, surface mass density distribution) determine the position of the ice line. The position of the ice line is weakly dependent on the disk environment; stellar irradiation, mass accretion, and general disk properties determine the position. These properties vary in time as the disk evolves, and thus the position of the ice line is also time-dependent (Davis 2005; Garaud & Lin 2007). The temperature threshold below which ice can form is \( \sim 145–170 \) K, depending on the partial pressure of the nebular water vapor (Podolak & Zücker 2004; Lecar et al. 2006). The ice line is important in the context of planetary formation because it separates the inner region of rocky planet formation from the outer region of gaseous and icy planet formation (Ida & Lin 2005; Kennedy & Kenyon 2008).

Gas giant formation by core accretion occurs when protoplanet core masses are sufficient to attract gas from the nebula (Mizuno 1980; Pollack et al. 1996). Bodenheimer & Pollack (1986) show that the timescale for gas giant formation is set by the core mass (see also Ikoma et al. 2000). Thus, if the core mass is too small, it will not be able to accrete enough gas to form a gas giant before the gas is removed. Past the ice line, condensation of water ices increases the disk surface density, allowing the formation of gas giants by core accretion within the 1–10 Myr lifetime of disk gas (Zuckerman et al. 1995; Pascucci et al. 2006). It is therefore important to know where the snow line lies in protodisks, as it holds much information about where gas giants are formed and how their orbits evolve in time.

Only recently have circumbinary planets been discovered. There are currently six known circumbinary planets in five different systems found by \textit{Kepler} (Doyle et al. 2011; Welsh et al. 2012; Orosz et al. 2012a, 2012b). These unambiguous discoveries have motivated a growing body of literature regarding planetary formation in circumbinary disks. In this Letter, I study ice-line positions in circumbinary disks and investigate the parameters that determine their locations. In Section 2, I describe a simple circumbinary disk model, and in Section 3, I calculate ice-line positions and describe results. Conclusions are presented in Section 4.

2. MODEL

2.1. Disk Model and Stellar Parameters

Similar to the treatment of Lecar et al. (2006) in the case of a single star, I consider a circumbinary disk heated by steady mass accretion as well as stellar irradiation from the binary. I adopt a circumbinary disk with a flared geometry, as described in Chiang & Goldreich (1997). The surface mass density distribution is \( \Sigma = \Sigma_0 (a/\text{AU})^{-3/2} \), with \( \Sigma_0 = 10^3 \text{ g cm}^{-2} \) (Weidenschilling 1977).

Several simulations have shown that the inner edge of circumbinary disks should be truncated between 1.8\( a_{\text{bin}} \) and 2.6\( a_{\text{bin}} \), where \( a_{\text{bin}} \) is the binary semimajor axis, due to tidal torques exerted by the binary (Artymowicz et al. 1991; Artymowicz & Lubow 1994; Günther & Kley 2002; Pierens & Nelson 2007; MacFadyen & Milosavljević 2008). There has been general agreement with these predictions from observations of directly imaged circumbinary disks, most notably that found around GG Tauri by Dutrey et al. (1994; see also Beust & Dutrey 2005; Duvert et al. 1998; Ireland & Kraus 2008). Furthermore, Holman & Wiegert (1999) have shown that there is a critical semimajor axis for planets orbiting a binary. Within this critical semimajor axis, the planets are too strongly affected by the binary and their orbits are unstable. Holman & Wiegert (1999) provide the relation

\[
a_c = [(1.60 \pm 0.04) + (5.10 \pm 0.05)e] + \mu + (-2.22 \pm 0.11)e^2 + (4.12 \pm 0.09)\mu
\]

\[
+ (-4.27 \pm 0.17)e\mu + (-5.09 \pm 0.11)\mu^2 + (4.61 \pm 0.36)e^2\mu^2\]

where \( a_c \) is the critical semimajor axis, \( \mu = m_2 / (m_1 + m_2) \) is the mass ratio of the binary, and \( e \) is the eccentricity of the binary.
To generalize this to the case of a binary, I replace \( M \) term usually associated with Equation (2), as it is a very small correction for this study due to the truncated inner edges of circumbinary disks. I adopt a constant value of \( M = 10^{-8} \, M_\odot \, \text{yr}^{-1} \) for the mass accretion rate.\(^1\)

The temperature due to accretional flux diffusing in an optically thick medium from the midplane to the surface of the disk is

\[
T_{\text{acc}}^4 = \frac{3}{4} \left( \tau_R + \frac{2}{3} \right) T_{\text{eff,acc}}^4,
\]

where \( \tau_R \) is the Rosseland optical depth:

\[
\tau_R = \int_0^\infty \kappa_R \rho(r, z) \, dz,
\]

which I approximate as \( \tau_R \approx \frac{\kappa_R \Sigma(r)}{2} \), where \( \kappa_R \) is the Rosseland mean opacity. I adopt the opacity value of \( \kappa_R \approx 1 \, \text{cm}^2 \, \text{g}^{-1} \) from D’Alessio et al. (2001), using their dust model with a maximum grain size of \( a_{\text{max}} = 1 \) mm and grain size distribution power-law slope of \( p = 3.5 \). Jang-Condell & Sasselov (2004) showed that the dependence of this opacity on the temperature structure of a disk is small between 100 and 300 K, so I assume that \( \kappa_R \) is constant throughout the regions of the disk I examine.

\(^1\) Hartmann et al. (1998) found a median accretion rate for T Tauri stars of \( \sim 10^{-8} \, M_\odot \, \text{yr}^{-1} \), with a spread as large as one order of magnitude, in the Taurus and Chamaeleon I molecular cloud complexes. I discuss the sensitivity of my results to this choice in Section 3.

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**Figure 1.** Illustration showing the various geometric configurations of the binary, each leading to a different position of the ice line. The sign of the eccentricity terms depends on whether the stars are at apastron (+) or periastron (−). The star labeled as “1” is the primary and the star labeled as “2” is the secondary. Configurations B and C are geometrically similar, just with the primary and secondary switched.
and the temperature of the interior of the disk is $T_{\text{int}, B} = (F_B / 2 \sigma)^{1/4}$. For configuration C, similar in geometry to B, shown in Figure 1, I assume the secondary fully eclipses the primary, yielding an internal disk temperature of $T_{\text{int}, C} = (F_C / 2 \sigma)^{1/4}$, where $F_C$ is given by Equation (7) but with $R_{1,1} \rightarrow R_{a,2}$, $T_{a,1} \rightarrow T_{a,2}$, $a_1 \rightarrow a_2$, and $a_{b,1} \rightarrow a_{b,2}$. Note that in the case of equal-mass binaries, configurations B and C are degenerate. The configuration that corresponds to the most heating by irradiation depends on the binary mass ratio and eccentricity.

Combining heating due to accretion and irradiation, the midplane temperature of this circumbinary disk model is given by

$$T_{\text{mid}} = T_{\text{acc}}^4 + T_{\text{irr}}^4.$$  

### 3. RESULTS

I find the position of the ice line by determining the point in the circumbinary disk where Equation (8) is equal to 160 K. The precise temperature of ice condensation will be in the range 145–170 K (Podolak & Zucker 2004), but the position of the ice line will not change significantly within this range, so I adopt an intermediate value of 160 K.

Figure 2 shows the maximum ice-line position relative to the critical semimajor axis, $a_c$, as a function of the binary separation for both equal-mass binaries (top panel) and unequal-mass binaries (bottom panel) on circular orbits. The unequal-mass systems are constructed from a 0.5 $M_\odot$ star as a secondary. For the $\mu \sim 0.2$, $\mu = 0.25, \mu = 0.33$, and $\mu = 0.42$, the primary systems have a mass of 2 $M_\odot$, 1.5 $M_\odot$, 1 $M_\odot$, and 0.7 $M_\odot$, respectively.

In all systems I consider that have component masses $\lesssim 2.0 M_\odot$, heating by accretion dominates the determination of the ice-line position. Assuming that for separations from the ice line inward there is only accretional heating, Equation (8) can be approximated as $T_{\text{mid}} = T_{\text{acc}}$. Combining this with Equations (2) and (3), along with the fact that $\tau_R \gg 2/3$ in this regime,

$$a_{\text{ice}}^{\text{acc}} \approx 2.13 \left( \frac{M_{\text{bin}}}{M_\odot} \right)^{2/9} \left( \frac{k_R}{1 \text{ cm}^2 \text{ g}^{-1}} \right)^{2/9} \left( \frac{M}{10^{-8} M_\odot \text{ yr}^{-1}} \right)^{2/9} \left( \frac{T_{\text{ice}}}{160 \text{ K}} \right)^{-8/9},$$  

which shows the position of the ice line for a given total binary mass $M_{\text{tot}}$ is independent of the binary separation. Normalizing this ice-line position to the critical semimajor axis, which is proportional to the binary separation for a given mass ratio and eccentricity, it is clear that $a_{\text{ice}} / a_c \propto a_{\text{bin}}^{-1}$. This proportionality holds for any binary eccentricity and mass ratio, provided that no component of the binary has a mass $\lesssim 2.0 M_\odot$, so that accretion is the dominant heating mechanism out to the position of the ice line.

The most important information to be gleaned from Figure 2 is that in each case there is a critical binary separation for which $a_{\text{ice}} / a_c = 1$, which I define as $a_{\text{crit}}^{\text{acc}} = a_{\text{bin}}(a_{\text{ice}} / a_c = 1)$. For binary separations equal to and larger than $a_{\text{crit}}^{\text{acc}}$, the ice line is always interior to the inner disk edge. Analytically, employing Equation (9) and the expression for the critical semimajor axis as a function of binary separation for equal-mass binaries on circular orbits ($a_c \approx 2.4 a_{\text{bin}}$),

$$a_{\text{ice}}^{\text{acc}} \approx 0.89 \left( \frac{\text{AU}}{a_{\text{bin}}} \right) \left( \frac{M_{\text{bin}}}{M_\odot} \right)^{2/9} \left( \frac{k_R}{1 \text{ cm}^2 \text{ g}^{-1}} \right)^{2/9} \left( \frac{M}{10^{-8} M_\odot \text{ yr}^{-1}} \right)^{2/9} \left( \frac{T_{\text{ice}}}{160 \text{ K}} \right)^{-8/9},$$  

which is good to within a few percent for all the equal-mass systems shown in Figure 2 with component masses $\lesssim 2.0 M_\odot$. Setting $a_{\text{ice}}^{\text{acc}} / a_c = 1$ yields an expression for $a_{\text{crit}}^{\text{acc}}$. The equal-mass binaries with component masses $\gtrsim 2.5 M_\odot$ heat their disks primarily by stellar irradiation. Assuming $T_{\text{mid}} = T_{\text{irr}} = T_{\text{ice}}$ for equal-mass binaries on circular orbits in configuration A shown in Figure 1 (which provides the maximum amount of heating by irradiation) yields no closed-form solution for $a_{\text{crit}}^{\text{acc}} / a_c$. However, an analytical expression for $a_{\text{crit}}^{\text{acc}}$ resulting from heating by stellar irradiation can be obtained:

$$a_{\text{crit}}^{\text{acc}} \approx 0.198 \left( \frac{\text{AU}}{4000 \text{ K}} \right)^{8/3} \left( \frac{R_{\ast}}{4 \text{ R}_\odot} \right) \left( \frac{T_{\text{ice}}}{160 \text{ K}} \right)^{-7/3}. $$  

If the binaries are allowed to have eccentric orbits, the critical semimajor axis, $a_c$, grows via Equation (1). All unequal-mass binaries considered and the equal-mass binaries with component masses $\lesssim 1.0 M_\odot$ remain in the accretion-dominated regime for all eccentricities. However, for equal-mass binaries with component masses $\gtrsim 1.0 M_\odot$, irradiation dominates at
distances near the ice line for binary separations equal to the critical value, \( a_{\text{bin}}^{\text{crit}} \). In both the accretion- and irradiation-dominated regimes, since \( \Delta a_{\text{bin}} \) always increases, \( a_{\text{bin}}^{\text{crit}} \) decreases with eccentricity. Figure 3 shows the change in \( a_{\text{bin}}^{\text{crit}} \) as a function of binary eccentricity for systems of both equal- and unequal-mass binaries.

For the systems in the accretion-dominated regime, the change in \( a_{\text{bin}}^{\text{crit}} \) as a function of binary eccentricity can be expressed analytically by combining Equations (9) and (1):

\[
\Delta a_{\text{bin}}^{\text{crit}} \approx \frac{1}{\Phi(e, \mu)} - \frac{1}{\Phi(0, \mu)} \times 2.13 \text{ AU} \left( \frac{M_{\text{tot}}}{M_\odot} \right)^{2/9} \times \left( \frac{\kappa_{\text{g}}}{1 \text{ cm}^2 \text{ g}^{-1}} \right)^{2/9} \left( \frac{M}{10^{-8} M_\odot \text{ yr}^{-1}} \right)^{2/9} \times \left( \frac{T_{\text{ice}}}{160 \text{ K}} \right)^{-8/9},
\]

where \( \Phi(e, \mu) \) is the coefficient for the critical semimajor axis, given by the expression in the brackets of Equation (1). For the accretion-dominated systems in Figure 3, I plot these analytic curves. I find no simple, analytic form for \( \Delta a_{\text{bin}}^{\text{crit}} \) in the irradiation-dominated regime. The main point from Figure 3 is that binary eccentricity serves to decrease \( a_{\text{bin}}^{\text{crit}} \) in all cases.

I also examine how the maximum ice line changes in time, as the binary components contract to the main sequence, for equal-mass binaries on circular orbits. Using the stellar evolution models of Siess et al. (2000), I calculated \( a_{\text{bin}}^{\text{crit}} \) for each system as a function of time, as the binary components approach the main sequence. For equal-mass binaries consisting of stars less massive than \( \sim 2 M_\odot \), the temperature and luminosity do not increase enough to dominate disk heating over steady mass accretion until they reach ZAMS. However, even at ZAMS, the ice line is not increased much from the initial position at formation time. Furthermore, by the time stars less massive than \( \sim 2 M_\odot \) reach the ZAMS, their disks will have dissipated and any planet formation should have already occurred. Conversely, equal-mass binaries consisting of stars more massive than \( \sim 2 M_\odot \) will indeed have their ice-line locations significantly increased as they contract toward the main sequence. The effective temperature of such massive stars increases rapidly as they approach the main sequence, and stellar irradiation quickly becomes the dominant heating mechanism for their disks. Since \( t_{\text{ZAMS}} \) for massive stars is much less than disk lifetimes, any planet formation around massive star binaries will necessarily continue after they reach the main sequence. For the purposes of this study, I thus restrict my conclusions to binaries consisting of low- to intermediate-mass components, \( M_\ast \lesssim 2 M_\odot \).

It is also worth noting that the mass accretion rate among T Tauri stars declines as the stars age (Hartmann et al. 1998). However, since \( a_{\text{acc}}^{\text{bin}} \propto M^{2/9} \), a declining mass accretion rate will serve to move the ice line inward. Lecar et al. (2006) pointed out that the disk lifetime scales as \( 1/M \) and that the lifetime of the minimum-mass solar nebula disk for a constant \( M = 10^{-8} M_\odot \text{ yr}^{-1} = 2 \times 10^6 \text{ yr} \) or roughly the lifetime of the nebular gas. Accretion rates higher than \( 10^{-8} M_\odot \text{ yr}^{-1} \) would further constrain the (already difficult to explain) timescales for giant planet formation. Nonetheless, the scaling of the ice line in the accretion-dominated regime with the mass accretion rate is weak and thus variations of an order of magnitude would not significantly change the results of this study.

It is interesting to note that there is a maximum binary separation (i.e., \( a_{\text{bin}}^{\text{crit}} \)) for which systems with separations larger than this maximum value have ice lines that lie interior to the critical semimajor axis. Circumbinary disks around systems with \( a_{\text{bin}} \gtrsim a_{\text{bin}}^{\text{crit}} \) will therefore have condensed water ice throughout the entire disk, meaning that rocky planets should not form in such systems. Using the properties of the observed binary population, I calculate the fraction of these binary systems with \( a_{\text{bin}} \gtrsim a_{\text{bin}}^{\text{crit}} \).

RagHAVan et al. (2010) determine the orbital period distribution of solar-type stars with stellar companions to be consistent with a Gaussian, with a mean of \( \log P = 5.03 \) and \( \sigma_{\log P} = 2.28 \), where \( P \) is in days. I assume that this period distribution found by RagHAVan et al. (2010) is the same for all equal-mass binaries with component masses in the range I consider, 0.5 \( M_\odot \lesssim M_\ast \lesssim 2.0 M_\odot \). I then compute the periods corresponding to \( a_{\text{bin}}^{\text{crit}} \) for equal-mass binaries with component masses in this range and integrate over a Gaussian distribution with the above parameters to estimate the fraction of binaries with ice lines interior to their inner disk edge.

For equal-mass binaries with total masses \( M_{\text{tot}} \lesssim 4.0 M_\odot \), the percentage of binary systems that have ice lines interior to the critical semimajor axis, at \( t_{\text{ZAMS}}/10 \) and later, ranges from 81% \( (M_{\text{tot}} = 4 M_\odot) \) to 84% \( (M_{\text{tot}} = 1 M_\odot) \). These fractions represent lower limits because I conservatively choose the “maximum”
ice line in each system. Since the value of $a_{\text{crit}}$ I find for the lowest mass ratio system I consider ($\mu \sim 0.2$) is approximately the same as that of the equal-mass binary with component masses of $2.0 M_{\odot}$ and corresponds to the largest value of $a_{\text{crit}}$ for all unequal-mass systems examined, the same fraction of 80% is also a conservative lower limit for the fraction of unequal-mass binaries with ice lines interior to the boundary of stability.

4. CONCLUSION

I have shown that ice lines lie interior to the critical semimajor axis for $\geq 80\%$ of all equal and unequal-mass binaries with components less massive than $M_* \lesssim 2 M_{\odot}$. This suggests that rocky planets should not form in these systems; only gas giant and icy planets. At the very least, there should be systematically larger planets around these systems relative to planets orbiting binaries with separations less than the critical value.

These results can be tested by finding more circumbinary planets, specifically planets around binaries with separations larger than the critical separation. This is possible, albeit difficult, with Kepler, because there are fewer eclipsing binaries at larger separations and planetary transit probabilities decrease with orbital period. Microlensing can also detect circumbinary planets at a range of binary separations and seems to be the most promising method for testing the results of this study (Han 2008).

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