Application of the Generalized Interpolation Material Point Method Coupled With the Finite Element Method for Rigid-Flexible Contact

Feng Chen*, Rong Chen*, Banghai Jiang

College of Arts and Science, National University of Defense Technology, Changsha, China

*Corresponding author e-mail: r.chen@nudt.edu.cn, *841744904@qq.com, 15252340@qq.com

Abstract. The paper shows the numerical simulation of the Generalized Interpolation Material Point Method (GIMP) coupled with the Finite Element Method (FEM) in rigid-flexible contact. The GIMP can overcome certain numerical noise existing in the original MPM and is used to simulate the great deformation of soft materials. A rigid material with a higher hardness can be described by the finite element method for limited deformation in rigid-flexible contact. First, the feasibility of GIMP was verified by the high-speed gelatin impact rigid wall. After that, this article shows the collision simulation of an iron bar and a gelatin bar. By comparing the material deformation, internal pressure distribution and the form of shock-wave obtained by GIMP with the calculation results of ls-dyna, it is shown that couple the GIMP and finite element method to deal with rigid-flexible contact is feasible.

1. Introduction
In the rigid-flexible contact problem, the rigid body tends to have a limited deformation, and the flexible body always has a large deformation. The Finite Element Method (FEM) has high computational efficiency when describing small deformation problems, but in the face of large deformation, it will encounter grid distortion or need to divide dense mesh, resulting in too small time step and long calculation time. Meshless methods such as the Material Point Method (MPM) developed by Sulsky et al. [1, 2] do not have mesh distortion. Instead, a regular background grid is used in each time step to describe the large deformation of the material. The Generalized Interpolation Material Point Method (GIMP) [3, 4] is an improved version of MPM, which can reduce the numerical instability of the MPM method to some extent. Therefore, the coupled GIMP and FEM to deal with the rigid-flex contact problem can play the respective advantages of these two methods.

2. The Generalized Interpolation Material Point Method

2.1. Governing equations
The Lagrangian movement description can be divided into two broad categories: the updated Lagrangian format and the total Lagrangian formulation. In the updated Lagrangian format, the governing equations are in the form:
\[ \rho(X,t)J(X,t) = \rho_0(X) \]  
(1)

\[ \sigma_{ij,j} + \rho b_i = \rho \ddot{u}_i \]  
(2)

\[ \rho \dot{W}^{int} = D_{ij} \sigma_{ij} \]  
(3)

The boundary conditions

\[
\left\{ \begin{array}{l}
\left( n_j \sigma_{ij} \right)_{\Gamma_t} = \overline{t}_i \\
u_i|_{\Gamma_u} = \overline{u}_i
\end{array} \right.
\]  
(4)

And initial conditions

\[
\left\{ \begin{array}{l}
\dot{u}(X,0) = \dot{u}_0(X) \\
u(X,0) = u(X)
\end{array} \right.
\]  
(5)

Where \( \rho \) represent the material density, \( X \) and \( J \) denote the Lagrangian coordinate and Jacobian determinant respectively, \( \sigma_{ij} \) is the Cauchy stress, \( w \) is the internal energy per unit volume, \( b_i \) is the body force per unit volume, \( u_j \) is the displacement, \( D_{ij} \) indicate the rate of deformation. \( \Gamma_u \) And \( \Gamma_t \) represent the displacement boundary and the prescribed traction boundary respectively.

Taking the virtual displacement \( \delta u_i \) as the test function the weak form of Eq. (2) can be obtained:

\[
\int_{\Omega} \rho \ddot{u}_i \delta u_i dV + \int_{\Omega} \sigma_{ij} \delta u_{ij} dV - \int_{\Omega} \rho b_i \delta u_i dV - \int_{\Gamma_t} \overline{t}_i \delta u_i dA = 0
\]  
(6)

2.2. MPM solution process

The object is dispersed into particles, and the density of the material point is approximated

\[ \rho(x) = \sum_{p=1}^{n_p} m_p \delta(x - x_p) \]  
(7)

Where \( m_p \) is the mass of the material particle \( p \), \( n_p \) denote the total number of particles and \( x_p \) represents the coordinate of particle \( p \), \( \delta \) is the Dirac delta function.

During the solution process, the material point and the background grid are completely fixed. In each time step, the material points move with the background mesh nodes. Therefor through the shape function \( N_p \) based on the background grid can achieve the mapping of information between the material point and background grid node.

The relation between particle displacement \( u_{ip} \) and nodal displacement \( u_{i0} \) are
\[ u_i = \sum_{j=1}^{n} N_{Ij} u_{ij} \]  

(8)

Substituting Eq. (7) and (8) into Eq. (6) and considering the arbitrariness of \( \delta u_d \) lead to

\[ \dot{p}_d = f_{int}^d + f_{ext}^d \]  

(9)

Where

\[ p_{id} = m_i v_{id} \]  

(10)

Is the grid nodal momentum, \( v_{id} \) is the grid nodal velocity, where

\[ m_i = \sum_{p=1}^{n_p} N_{ip} m_p \]  

(11)

Is the lumped grid nodal mass.

In the above equations, the internal grid nodal force and the external grid nodal force are

\[ f_{int}^d = -\sum_{p=1}^{n_p} N_{lp,j} \sigma_{ijp} \frac{m_p}{\rho_p} \]  

(12)

\[ f_{ext}^d = \sum_{p=1}^{n_p} N_{lp} \bar{t}_p h^{-1} \frac{m_p}{\rho_p} + \sum_{p=1}^{n_p} m_p N_{lp} \bar{b}_p \]  

(13)

\[ f_{id} = f_{ext}^d + f_{int}^d \]  

(14)

Where \( f_{id} \) is the grid nodal force, \( \sigma_{ijp} = \sigma_j(x_p), \ b_{ip} = b_i(x_p), \ \bar{t}_p = \bar{t}_i(x_p) \) is the force due to surface tractions, \( h \) denotes the supposed thickness of the layer of the boundary used to calculate the boundary integral in Eq. (5) [5].

2.3. **GIMP shape function**

The gradient of shape function of original MPM is discontinuous. The discontinuity results in reduced calculate accuracy or even complete distortion. Some newer formulations of MPM aim to reduce these algorithmic errors, such as Generalized Interpolation Material Point Method (GIMP) [3].

GIMP differs from MPM in the choice of shape function. In the original MPM, the density field was approximated as a collection of point masses using Dirac delta functions in Eq. (7). This equivalent to using single-point integration in calculation process. In the GIMP method, select the particle characteristic function as the trial function and take the background grid shape function as the test function.

For an axisymmetric coordinate system, assume that there are four material points in each cell, and the size of the material points remains constant throughout the calculation, using dimensionless coordinates

\[ \zeta = \left| (x_p - x_i) / L \right| \]  

(15)
Where $x_p$ and $x_I$ represent the coordinates of particle $p$ and grid node $I$ respectively, $L$ is the background grid size.

Then the shape function is [6]

$$
S_{lp}(\xi) = \begin{cases} 
\frac{7-16\xi^2}{8}, & \xi \leq 0.25 \\
1-\xi, & 0.25 < \xi \leq 0.75 \\
\frac{(5-4\xi)^2}{16}, & 0.75 < \xi \leq 1.25 \\
0, & \xi > 1.25
\end{cases}
$$

(16)

For three-dimensional problems

$$
S_{lp}(x) = S_{lp}(\xi)S_{lp}(\eta)S_{lp}(\zeta)
$$

(17)

3. Couple GIMP with FEM

3.1. Contact detection

Contact detection through the background grid. The physical quantity carried by the finite element node is mapped to the background grid using the same shape function as GIMP. If the two objects of $r$ and $s$ simultaneously contribute to the grid nodal momentum of $I$ and satisfy

$$
(v^r_{ii} - v^s_{ii})n^r_{ii} > 0
$$

(18)

Then we say object $r$ contacts with object $s$, where the out-surface normal vector of the object $b$ at the grid node $I$ can be calculated by the mass gradient

$$
n^b_{ii} = \sum_p m_p N_{lp,i}
$$

(19)

![Figure 1. Contact detection by background grid.](image)
3.2. Contact force
Suppose that object \(b\) contacts with no object firstly, and independently integrates momentum equation to obtain nodal test momentum and nodal test speed.

\[
\overline{p}^{b,k+1/2}_d = p^{b,k-1/2}_d + \Delta t^k f^{b,k}_d
\]

(20)

\[
\overline{v}^{b,k+1/2}_d = \overline{v}^{b,k-1/2}_d + \Delta t^k \frac{f^{b,k}_d}{m^{b,k}_i}
\]

(21)

At this time, if Eq. (20) is satisfied which indicates that two objects are in contact, the contact force needs to be applied, and the nodal momentum and nodal velocity are updated to

\[
p^{b,k+1/2}_d = \overline{p}^{b,k+1/2}_d + \Delta t^k f^{b,c,k}_d
\]

(22)

\[
v^{b,k+1/2}_d = \overline{v}^{b,k+1/2}_d + \Delta t^k \frac{f^{b,c,k}_d}{m^{b,k}_i}
\]

(23)

For adhesive contact, two objects satisfy the velocity continuous condition at the grid contact node \(I\)

\[
v^{r,k+1/2}_d - v^{s,k+1/2}_d = 0
\]

(24)

Substituting Eq. (23) into equation (24) to obtain contact force

\[
f^{r,c,k}_d = -f^{r,c,k}_d = \frac{m^{r,k}_i m^{r,k}_j}{(m^{r,k}_i + m^{r,k}_j)\Delta t^k} \left( \overline{v}^{r,k+1/2}_d - \overline{v}^{r,k+1/2}_d \right)
\]

(25)

3.3. Update position and velocity
Updating the position and velocity of material points by following equations

\[
x^{r,k+1}_l = x^{r,k}_l + \Delta t^{k+1/2} \sum_{i=1}^{\eta^g} p^{r,k+1/2}_d N^k_{l,p} / m^{r,k}_i
\]

(26)

\[
v^{r,k+1}_l = v^{r,k-1/2}_l + \Delta t^{k} \sum_{i=1}^{\eta^g} \left( f^{r,k}_d + f^{r,c,k}_d \right) N^k_{l,p} / m^{r,k}_i
\]

(27)

4. Numerical examples
4.1. High-speed gelatin impact rigid wall
Gelatin are commonly used in wound ballistics experiments to simulate biological tissue to study the damage effect of bullets or fragments on the human body. Gelatin is a typical soft material that is prone to great deformation and even crushing and liquefaction under certain pressure.
The simulation model bases on two-dimensional axisymmetric coordinate system to simulate a three-dimensional situation. The abscissa is the radial direction $R$ and the vertical coordinate is the height $z$. The gelatin cylinder block, with a radius of 5 cm and a height of 10 cm, impacts the rigid wall at a speed of -500 m/s. The gelatin can be modeled as an elastic-plastic material with polynomial equation of state [7]. The material parameters are shown in Table 1.

**Table 1. Values of material parameters for the gelatin.**

| $\rho_0$ (g/cm³) | $E$ (kPa) | $E_s$ (kPa) | $\sigma_0$ (kPa) | $C_0$ (GPa) | $C_1$ (GPa) | $C_2$ (GPa) | $C_3$ (GPa) |
|-----------------|---------|------------|----------------|-----------|---------|---------|---------|
| 1.03            | 850     | 10         | 220            | 0         | 2.38    | 7.14    | 11.9    |

Figure 1 shows the pressure distribution inside the cylinder, which is the results of the GIMP and the ls-dyna by FEM. The results of the two gelatin deformations are consistent with the internal pressure distribution. The deformation of the two results are in good agreement and showed a similar internal pressure distribution.

![Gelatin internal pressure distribution](image1)

**Figure 2.** Gelatin internal pressure distribution (25μs). The left is GIMP result and the right is ls-dyna result.

To investigate more specifically the propagation of shock waves generated by collisions, Figure 2 shows pressure-time history curves on the central axis near the 5μs, 15μs, 25μs, and 35μs moments, with shock waves propagating from bottom to top. It is observed that the pressure ranges and arrival times of the shock waves obtained by the two methods are basically identical. Since the GIMP method selects the pressure values of the material points near the axis of symmetry and ls-dyna selects the pressures of the finite elements near the axis of symmetry, there is a little difference and a certain error is introduced according to the size difference of material points and finite elements.
Figure 3. Pressure wave in the axial direction of the gelatin. The solid lines represent GIMP results and the dashed lines represent ls-dyna results.

4.2. Iron bar impact gelatin
The hardness of iron is much higher than that of gelatin. Therefore, the collision of an iron bar and gelatin is a typical rigid-flex contact problem. Set an iron bar with a radius of 1.5cm and a height of 3cm to impact the gelatin with a radius of 1cm and a height of 3cm placed on the rigid solid wall at a speed of -50m/s. The iron is described by the ideal elastic-plastic model with the Gruneisen equation of state, and the parameters are shown in Table 2 [8].

| $\rho_0$ (g/cm$^3$) | $G$ (GPa) | $\sigma_0$ (MPa) | $C_s$ (km/s) | $S$ | $\gamma_0$ |
|---------------------|-----------|-----------------|-------------|-----|----------|
| 7.83                | 83.1      | 455             | 4.955       | 0.454 | 1.81     |

Table 2. Values of material parameters for the iron bar.

Figure 3 shows the results of GIMP and ls-dyna calculations. The upper is iron and the lower is gelatin. The degree of deformation of gelatin is basically the same by the two methods, with a bulge at the top outside and a slight depression at the top center, indicating that the contact algorithm of the coupling GIMP and FEM is feasible.

Figure 4. Internal pressure distribution of iron bar impact gelatin (25μs). The left is GIMP result and the right is ls-dyna result.
Figure 4 shows the pressure-time history curves on the central axis of gelatin at 5μs and 15μs, with shock waves propagating from top to bottom. Observing this figure, the results obtained by the two methods are basically consistent. The curve obtained by ls-dyna is smoother, which is approximately equal to the average of the GIMP curve. The values obtained by the GIMP method fluctuate greatly between the material points. This may be due to the fact that the currently used GIMP does not completely eliminate the numerical noise generated by the material points crossing the grid.

![Graph showing pressure-time history curves](image)

**Figure 5.** Pressure wave in the axial direction of the gelatin. The solid lines represent GIMP results and the dashed lines represent ls-dyna results.

5. Conclusion

Compared to the finite element method, the meshless method is more suitable for dealing with large deformation problems. The advantage of the GIMP method is that it reduces the numerical noise caused by the discontinuity of the shape function gradient of original MPM. The accuracy of the GIMP method was verified by an example of a high-speed cylinder gelatin impacts the rigid wall. The feasibility of GIMP coupled FEM to treat rigid-flex contact was verified by the analysis of an iron bar impact the gelatin.

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