An LES study of turbulent flow over in-line tube-banks and comparison with experimental measurements

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Abstract Turbulent flow across an in-line array of tube-banks with transverse and longitudinal pitch $P_T/D = 2.67$, and $P_L/D = 2.31$, has been simulated successfully by Large Eddy Simulation (LES) based on the dynamic Smagorinsky subgrid scale model (SGS), in which a wall-layer model is used to reduce the computational cost. The flow structures across the tube-banks were examined through the normalized Q criterion. The surface pressure characteristics from the middle cylinder within each column of cylinders are found to agree well with the existing experimental data, as did also the values of drag and lift coefficients. These results indicate that cylinders from the second column experience the minimum drag force and maximum lift force fluctuation. Spectral analyses were performed for velocity signals sampled behind each middle cylinder axis, which show that the dominant vortex shedding frequency does not vary across the tube-banks. On this basis, we also examined the shear layer instability. Finally, we report auto-correlation functions for streamwise and cross velocity fluctuations as a function of the spanwise length.

Keywords LES, cylinder, tube-bank, surface pressure characteristics, vortex shedding, shear-layer instability
1 Introduction

Turbulent flow over tube-banks has been traditionally modelled using the Reynolds-Averaged Navier-Stokes (RANS) equations with different turbulence models (see [1, 2, 3, 4, 5]). The flow across tube-banks is very unsteady, characterized by strong vortex shedding and bluff-body wakes. Rodi[6] has demonstrated the difficulty or even impossibility of accurately simulating the flow phenomena using the RANS methodology with the standard $k-\epsilon$ turbulence model. The astonishingly rapid development of Large-Eddy Simulation (LES) (see [7, 8, 9]) has shown the potential to more accurately simulate simple flow phenomena since LES resolves the large-scale unsteady motion directly and requires only modelling of the universal small-scale turbulence structures.

It is still a challenging task for numerical simulations to obtain an accurate prediction of unsteady flow separation at high Reynolds number across a single cylinder, not to mention flow across tube-banks since the flow exhibits strong unsteadiness and complex vortex structures. Whether the dynamic boundary layer around a cylinder is accurately resolved with appropriate numerical techniques and resolution (see [10, 11, 12, 13]) is crucial to predicting the unstable region where turbulence is generated, the instability of shear layer ([14]) and the physics of the wake ([15, 16]). Hence the conventional LES of turbulent flows across a single cylinder and tube-banks is an extremely expensive endeavour at high Reynolds number flow ([11, 17]).

In recent years, LES has been used to simulate turbulent flows across in-line and staggered tube-banks (see [18, 19, 20, 21, 22]) and demonstrated its feasibility and effectiveness. Barsamian and Hassan[18] carried out a two-dimensional LES calculation of flow over tube bundle arrays using two subgrid scale models and studied the power spectra and bound spectra of drag and lift forces. Later extension in three-dimensional LES by Hassan and Barsamian[19] was used to study velocity profile, power spectra density (PSD) of velocities and forces, auto-correlation functions of streamwise and transverse velocities in a flow past a tube bundle at Reynolds number of 21700 based on the free stream velocity and cylinder diameter. Rollet-Miet et al.[2] performed LES based on a Finite Element Method for a turbulent, incompressible flow around a staggered array of tubes and compared the results with the measurements from Simonin and Barcouda[23]. Beale and Spalding[1] performed an LES of transient flow in a relatively low Reynolds number regime of $Re \in [30, 3000]$ based on the gap velocity and cylinder diameter. Both in-line square and staggered-
square tube-banks were studied in their work which accounted for pressure drop, lift, drag and heat transfer. Liang and Papadakis [21] employed an unstructured grid Finite-Volume Method (FVM) based LES to study the vortex shedding characteristics inside a staggered tube bundle.

The simulation of turbulent flow over tube-banks can be simplified to model merely a single circular cylinder, provided that the cylinders are packed so closely that wake vorticity does not turn up. In this case, the computational domain is reduced to a single periodic circular cylinder with four cylinder quarters around it. Hence, periodic boundary conditions are assumed in the streamwise and cross-flow direction. Benhamadouche and Laurence [4] carried out a comprehensively comparative study of turbulent flow across a single periodic cylinder in a tube bundle with LES, coarse LES and URANS. In their study, LES with a wall function modelling method gives the best results when compared with Simonin and Barcouda [23]'s experimental data and DNS results from Moulinec et al. [24]. Moulinec et al. [25] carried out the diagonal Cartesian method (DCM) based DNS to study turbulent flow past an “element cell” in a tube-banks with four sets of grids. The Reynolds number was equal to 6000 based on the bulk velocity and the circular diameter. They compared the results on the mean velocity and r.m.s values from the finest cell with the datum measured by Simonin and Barcouda [23] and numerical results calculated by Rollet-Miet et al. [2], who has shown the feasibility of an “element cell” as an LES computational domain. Following the work of Moulinec et al. [25], Moulinec et al. [26] further investigated the wake turbulence between a “wide element” consisting of 16 circular cylinders using a three-dimensional DNS for $Re \in [50, 6000]$ based on the bulk velocity.

In the present study, in contrast to previous research work (see [2, 4, 21, 24, 26]), a full scale turbulent flow across an in-line tube-banks was computed with a three-dimensional LES. The numerical technique was based on the Finite-Volume Method (FVM) using wall-layer modelling on unstructured grids with a collocated arrangement for all the unknown flow variables. Particular attention was given to the investigation of detailed statistics around the circular cylinder in the middle cylinders of each column, which were compared with the available experimental data of Shim [27], Hill et al. [28] and Shim et al. [29].

The rest of the this paper is structured as follows. The computational methodology and geometry are presented first. Then, a detailed comparison and discussion of mean and r.m.s surface
pressure distribution on the middle cylinders from each column is given. In addition to that, the corresponding drag and lift force, frequency analysis of velocity signals and auto-correlations of streamwise and cross-wise velocities in the spanwise direction, which complement the existing experimental measurements, are reported. Finally, conclusions are drawn.

2 Computational methodology

2.1 Formulation of a dynamic Smagorinsky model

The governing equations for LES are obtained by spatially filtering the Navier-Stokes equations. In this process, the eddies that are smaller than the filter size used in the simulations are filtered out. Hence, the resulting filtered equations govern the dynamics of large eddies in turbulent flows. A spatially filtered variable that is denoted by an overbar is defined using a convolution product (see [30])

\[ \overline{\phi}(x, t) = \int_D \phi(y, t) G(x, y) dy \]  

where \( D \) denotes the computational domain, and \( G \) the filter function that determines the scale of the resolved eddies.

In the current study, the finite-volume discretization employed itself provides the filtering operation as

\[ \overline{\phi}(x, t) = \frac{1}{V} \int_D \phi(y, t) dy, \quad y \in V \]  

where \( V \) denotes the volume of a computational cell. Hence, the implied filter function, \( G(x, y) \) in eq. 2, is a top-hat filter given by

\[ G(x, y) = \begin{cases} 
\frac{1}{V} & \text{for } |x - y| \in V \\
0 & \text{otherwise} 
\end{cases} \]  

Filtering the continuity and Navier-Stokes equations, the governing equations for resolved scales in LES are obtained

\[ \frac{\partial \overline{m_i}}{\partial x_i} = 0 \]
An LES study and in-line tube-banks

\[
\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nabla \cdot \left( \nu \nabla \bar{u}_i \right) - \frac{\partial \tau_{ij}}{\partial x_j} \tag{5}
\]

where \( \tau_{ij} \) denotes the subgrid scale (SGS herefrom) stress tensor defined by

\[
\tau_{ij} = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j \tag{6}
\]

The filtered equations are unclosed since the SGS stress tensor \( \tau_{ij} \) is unknown. The SGS stress tensor can be modelled based on an isotropic eddy-viscosity model as:

\[
\tau_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij} = -2 \nu_t \bar{S}_{ij} \tag{7}
\]

where \( \nu_t \) denotes the SGS eddy viscosity, and \( \bar{S}_{ij} \) is the resolved rate of strain tensor given by

\[
\bar{S}_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \tag{8}
\]

where \( \nu_t \) is computed in terms of the Smagorinsky \([31]\) type eddy-viscosity model using

\[
\nu_t = C_{\nu} \overline{\Delta}^2 |\bar{S}| \tag{9}
\]

where \( C_{\nu} \) denotes the Smagorinsky coefficient, \( |\bar{S}| \) the modulus of rate of strain tensor for the resolved scales,

\[
|\bar{S}| = \sqrt{2 \bar{S}_{ij} \bar{S}_{ij}} \tag{10}
\]

and \( \overline{\Delta} \) denotes the grid filter length obtained from

\[
\overline{\Delta} = V^{1/3} \tag{11}
\]

Consequently, the SGS stress tensor is computated as following

\[
\tau_{ij} - \frac{1}{3} \delta_{ij} \tau_{ij} = -2 C_{\nu} \overline{\Delta}^2 |\bar{S}| \bar{S}_{ij} \tag{12}
\]

This model claims to be simple and efficient. It needs merely a constant \textit{in priori} value for \( C_{\nu} \). Nevertheless, work from \([32, 33, 34]\) has shown different values of \( C_{\nu} \) for distinct flows. Hence, the
major drawback of the model used in LES is that there is an inherent inability to represent a wide range of turbulent flows with a single value of the model coefficient $C_v$. Given that the turbulent flow over tube-banks in the present study is fully three-dimensional, the standard Smagorinsky SGS model is not used here to compute the coefficient $C_v$.

Germano et al. [35] proposed a new procedure to dynamically compute the model coefficient $C_v$ based on the information obtained from the resolved large scales of motion. The new procedure employs another coarser filter $\tilde{\Delta}$ (test filter) whose width is greater than that of the default grid filter. Applying the test filter to the filtered Navier-Stokes equations, one obtains the following equations

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \tilde{u}_i \tilde{u}_j}{\partial x_j} = - \frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \tilde{u}_i}{\partial x_j} \right) - \frac{\partial T_{ij}}{\partial x_j} \tag{13}$$

where the tilde denotes the test-filtered quantities. $T_{ij}$ represents the subgrid scale stress tensor from the resolved large scales of motion and is given by

$$T_{ij} = \tilde{u}_i \tilde{u}_j - \tilde{u}_i \tilde{u}_j \tag{14}$$

The quantities given in (6) and (14) are related by the Germano identity:

$$\mathcal{L}_{ij} = T_{ij} - \tilde{\tau}_{ij} \tag{15}$$

which represents the resolved turbulent stress tensor from the SGS tensor between the test and grid filters, $T_{ij}$ and $\tau_{ij}$. Applying the same Smagorinsky model to $T_{ij}$ and $\tau_{ij}$, the anisotropic parts of $\mathcal{L}_{ij}$ can be written as

$$\mathcal{L}_{ij} - \frac{1}{3} \mathcal{L}_{kk} \delta_{ij} = -2CM_{ij} \tag{16}$$

where

$$M_{ij} = \tilde{\Delta}^2 |\tilde{\Sigma}| \tilde{S}_{ij} - \tilde{\Delta}^2 |\tilde{\Sigma}| \tilde{S}_{ij} \tag{17}$$

One hence obtains the value of $C$ from (17) that is solved on the test filter level and then apply it to Eq. (12). The model value of $C$ is obtained via a least squares approach proposed by Lilly [36], since Eq. (17) is an overdetermined system of equations for the unknown variable $C$. Lilly [36]
An LES study and ... in-line tube-banks

defined a criterion for minimizing the square of the error as

\[ E = (L_{ij} - \frac{\delta_{ij}}{3}L_{kk} - 2CM_{ij})^2 \]  

(18)

In order to obtain a local value, varying in time and space in a fairly wide range, for the model constant \( C \), one takes \( \frac{\partial E}{\partial C} \) and sets it zero to get

\[ C = \frac{1}{2} \frac{L_{ij}M_{ij}}{M_{ij}M_{ij}} \]  

(19)

A negative \( C \) represents the transfer of flow energy from the subgrid-scale eddies to the resolved eddies, which is known as back-scatter and regarded as a desirable attribute of the dynamic model.

2.2 The Werner and Wengle wall layer model

The Large Eddy Simulation (LES) of turbulent flow over tube-banks is hampered by expensive computational cost incurred when the dynamic and thin near-wall layer is fully resolved. To obviate the computational cost associated with calculating the wall shear stress from the laminar stress-strain relationship that requires the first cell to be put within the range of \( y^+ \approx 1 \), Werner et al. [37] proposed a simple power-law to replace the law of the wall, in which the velocity profile on a solid wall is given as following,

\[ u^+ = \begin{cases} 
 y^+ & \text{for } y^+ \leq 11.81 \\
 A(y^+)^B & \text{for } y^+ > 11.81 
\end{cases} \]  

(20)

where \( A = 8.3 \) and \( B = 1/7 \). An analytical integration of Eq. (21) results in the following relations for the wall shear stress

\[ |\tau_w| = \begin{cases} 
 \frac{2\mu|u_p|}{\Delta y} & \text{for } y^+ \leq 11.81 \\
 \rho \left[ \frac{1-B}{2} A^{1+B} \left( \frac{\mu}{\rho \Delta y} \right)^{1+B} + \frac{1+B}{A} \left( \frac{\mu}{\rho \Delta y} |u_p| \right) \right]^{\frac{2}{1+B}} & \text{for } y^+ > 11.81 
\end{cases} \]  

(21)

where \( u_p \) is velocity component parallel to the wall and given by:

\[ |u_p| = \frac{\mu}{2\rho \Delta y} A^{\frac{2}{1+B}} \]  

(22)
2.3 Flow configuration of in-line tube-banks

The flow configuration is shown in figure 1 and the coordinate system depicted in figure 2. Flow is from left to right and normal to the cylinder axis. The computational domain is of size $L_x \times L_y \times L_z = 28D \times 16D \times 2D$, where $D$ denotes the cylinder diameter. This configuration is based on the second test case considered in Shim[27] which measures surface pressure distributions and fluctuating lift forces and was performed in a suction-type wind tunnel. It consists of four-column in-line tube bundles with transverse pitch-to-diameter ratio ($P_T/D$) $S_T$ of 2.67 and longitudinal pitch-to-diameter ratio ($P_L/D$) $S_L$ of 2.31, respectively. The Reynolds number $Re_o$ based on the free stream velocity $U_o$ and the cylinder diameter $D$ equals to 9600, and $Re_g$ based on the gap streamwise velocity between two cylinders is equal to 15200.

The Navier-Stokes solver used in this work uses a cell-centered, collocated grid arrangement finite-volume (FV) discretization method. All spatial terms in the momentum equations are discretized by the bounded central differencing scheme, which not only boasts the advantage of low numerical diffusion of central-differencing scheme but also eliminates unphysical oscillations in the solution fields. The spatial discretization scheme is based on a multi-dimensional, least squares cell-based gradient reconstruction scheme to guarantee a second-order spatial accuracy. In order to prevent unphysical checker-board pressure field, this study employs a procedure similar to that proposed by Rieh and Chow[38]. The Gear’s implicit, three-level second-order accurate scheme is employed for temporal discretization. A generalized fractional-step method is employed for the overall time-advancement.

The computational grid is evident in figure 3. The total number of grid elements used for the present simulation is 2730240. The mesh has an embedded region of fine mesh designed for each cylinder in order to enhance the mesh resolution near the cylinder without incurring too large an increase in the total number of mesh elements. 96 grid points hence are allocated along the cylinder surface. The grid spacing on the cylinder in the radial, circumferential, and spanwise direction are $\Delta r/D = 1.4 \times 10^{-2}$, $\Delta \theta/D = 3.27 \times 10^{-2}$, $\Delta z/D = 5.0 \times 10^{-2}$, respectively. The first cell adjacent
An LES study and ... in-line tube-banks

to the cylinder is within the range $\Delta y^+ < 11.8$ in wall units$^1$ that satisfies the requirements of the Wener-Wengle wall-layer model for LES. Prior to the present simulation, with the standard Smagorinsky subgrid scale model, a coarser grid simulation were carried out to determine the resolution.

With fully developed turbulent flow, periodic boundary conditions are justified to use along the normal ($y$) and spanwise ($z$) direction. For the inlet boundary condition, a simple uniform velocity profile is assumed and the turbulent intensity set to zero. Hence, the turbulence fluctuations at the inlet was not accounted for temporally and spatially. Nevertheless, a length $5D$ before the first column bank is used to allow the development of turbulence. At the exit boundary, the solution variables from the adjacent interior cells are extrapolated to satisfy the mass conservation.

The simulation is advanced with a non-dimensional time step $\Delta tU_o/D \approx 2 \times 10^{-3}$ that yields maximum Courant-Friedrichs-Lewy (CFL) number of 0.5. For results presented here, the first-order statistics are collected by integrating the governing equations over an interval of $30D/U_o$, and all the statistics are averaged over the 40 sampling points along the spanwise direction.

3 Results and discussions

To provide an overview of the development of turbulent flow across the four-column in-line tube-banks, wake vortices visualized using the Q criterion (see [39]) are presented first. Then, time-resolved pressure distributions provide quantitative information on surface pressure fluctuations, which are compared with experimental measurements ([27]). Following this, the time histories of coefficient of drag $C_D$ and lift are given. The development of vortex shedding behind the cylinder in the middle column are investigated via examining the corresponding energy spectrum in the wake. The coherence of vortex shedding along the length of the middle cylinder is studied through computing the auto-correlation function of each velocity fluctuation component. In the present work, the turbulent flow across tube-banks has been considered to have reached the statistically stationary state after a simulation time of $T = 200D/U_o$. All the statistics presented here are

$^1$ The superscript $+$ denotes a non-dimensional quantity scaled using the wall variables, e.g. $y^+ = yu_\tau/\nu$, where $\nu$ is the kinematic viscosity and $u_\tau = \sqrt{\tau_w/\rho}$ is the wall friction velocity based on the wall shear stress $\tau_w$, and which is a velocity scale representative of velocities close to a solid boundary.
computed after this transient stage. Further, the statistics are averaged in the periodic spanwise direction.

3.1 Instantaneous flow field

The contours of turbulent kinetic energy (TKE) at a given time across the four-in-line tube banks are presented in figure 4 using a normalized $Q$-criterion $= 8 \times 10^{-2}$. The $Q$-criterion, proposed by Hunt et al. [39], is defined as the second invariant of velocity gradient tensor $\nabla u$ for incompressible flows by the following expression

$$Q = \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} = \frac{1}{2} \omega_i^2 - e_{ij}^2$$

(23)

where $e_{ij}^2$ and $\omega_i$ denotes the symmetric and antisymmetric parts of $\nabla u$, respectively.

The instantaneous flow field shows the salient feature of the wake dynamics where a wide, yet different range of scales behind every column of cylinders can be observed. As far as the first column cylinders are concerned, the flow shows no unexpected properties, but a few points are worth noting for comparison with flow patterns behind other cylinders.

Firstly, the boundary layer on each individual cylinders of the first column remains laminar up to the separation point, and it undergoes transition to turbulence in the separated shear layer. Whilst the boundary layer separation on the cylinders from the downstream columns is much delayed and so that the wake is much narrow, resulting in a much smaller coefficient of drag. This principally results from the inflow conditions for the downstream cylinders. Figure 5 shows a close-up of the vortex motion around the cylinders across the middle plane, again shown contours of TKE in terms of the same normalized-$Q$ criterion. It is evident that the turbulence level is quite high at the front side of the downstream cylinders.

Secondly, figure 4 illustrates different flow pattern of vortex travelling downstream each column of cylinders. Large coherent structures are visible in the wake of first column of cylinders. Nevertheless, the classical von Karman vortex streets fail to arise because the second cylinder column lies within the range of the recirculation region of flow behind the first column and hence suppresses the vortex street formation in the wake. Another effect of the downstream cylinder is to
increase the wake instabilities further. Large flow structures are lost and broken into small eddies, producing ultimately a fully developed grid turbulence after the final cylinder column.

3.2 Surface pressure characteristics

Figure 6a presents time-averaged surface pressure distributions against $\theta$ from the front stagnation point for the middle circular cylinder, taken from the first column to the fourth column, respectively. The results of Shim[27] are shown for comparison.

The surfaces pressures are presented in terms of the coefficient of pressure

$$C_p = \frac{\langle p \rangle_T - p_{ref}}{q_{ref}}$$

where $\langle p \rangle_T$ denotes an ensemble average across the spanwise direction for all the sampling points on the cylinder surface over the sampling time interval $T$, though the vortex shedding does not necessarily occur in phase over the whole spanwise direction. The time-averaged boundary layers on either side of each circular cylinder are assumed to be symmetrical. $q_{ref}$ is the dynamic pressure in terms of free stream velocity $u_o$ and fluid density $\rho$, which is given by

$$q_{ref} = \frac{1}{2} \rho u_o^2$$

To make $C_p$ equal to unit at the front stagnation point for every cylinder, the corresponding static pressure $p_{ref}$ is calculated according to equation 24 first, $C_p$ is hence determined around the cylinder surface. This procedure was also used in the work of Shim[27] for calculating $C_p$. Hence, in view of the transverse pitch ratio $S_T = 2.67$ and in terms of the continuity equation $u_g/u_o = S_T/(S_T - d)$, one obtains the corresponding converting factor for the related quantities.

Very good agreements for the time-averaged surface pressure distribution around the four cylinders are observed between the LES calculations and the experimental measurements of Shim[27] among the four figures of Figure 6. Other quantities, for example, the r.m.s pressure distribution and vortex shedding frequency are also very comparable. They shall be shown in later figures in this paper. For the cylinder from the first column, note that the LES data in figure 6a contain a kink near $\theta = 85^\circ$, which indicates the presence of the laminar boundary layer separating from
the upper and lower surface of the cylinder. This transition region from the experimental data of Shim [27] is not as readily perceived as in the LES computation, in that the measurements were taken in 10-degree increments from the forward stagnation point to the opposite side of the cylinder. For the discernible wiggle from the present calculations in the range of $\theta \in [80, 120]$, the likely reason is entrainment of shear layer fluid on to the cylinder surface owing to the interference from the close arrangement of cylinders.

To the best of the authors' knowledge, there is so far no information available on the pressure distribution around the surfaces of cylinders in a tube bank from LES. It is of interest, thus, to show mean pressure distribution around the surface of the downstream cylinders in terms of the equation 24 and to further compare the results measured by Shim [27]. As far as the positive values of base $C_p$ obtained from the downstream cylinders are concerned, it also results from the definition of $C_p$ in this work. It can be observed that the results from the two distinct approaches are very comparable across the figure 6b, c, d. Because of the wake from the first column of cylinders which impinges upon the downstream second column of cylinders, a rise of mean pressure value is to be expected within the windward side. In particular, as can be observed from the figure 6a and 6b, they display distinct shapes for mean pressure distribution. The rise of mean pressure distribution is clearly discernible within the range of $\theta \in [0^\circ, 40^\circ]$ in figure 6b. It is interesting to note that the two peaks lie nearly to the same position around $\theta = 40^\circ$ in Figure 6b. Moreover, it is worthwhile noting that the difference from the pressure of front stagnation point and the base pressure is reduced significantly compared to the corresponding cylinder from the first column. This is attributed to the turbulence level of approaching flow since it is located within the wake. In contrast to figure 6b, the rise is not observed for $C_p$ from the third and forth column in figure 6c, d. This can be explained as the wake from downstream cylinders is much narrow and more mixed than the one behind the first column.

The r.m.s value of pressure distribution around the surfaces of the four cylinders are shown in the four figures 9b, b, c, d along with the Shim [27]'s data. First to note is that the pressure fluctuates more than 50% for the downstream cylinders. This indicates that instantaneous surface pressure different from the time-averaged value significantly and further demonstrates that the URANS methodology is not suitable for the present work. Figure 9b exhibits relatively high and
An LES study and ... in-line tube-banks

| Case               | Maximum of $C'_p$ | $C'_p(90^\circ)$ |
|--------------------|-------------------|------------------|
| Present LES ($Re_p = 15270$) |                   |                  |
| C1                 | 0.236(110°)       | 0.159            |
| C2                 | 0.584(40.4°)      | 0.425            |
| C3                 | 0.640(40.4°)      | 0.441            |
| C4                 | 0.544(36.7°)      | 0.377            |
| Experiments [27]   |                   |                  |
| C1                 | 0.457(110°)       | 0.438            |
| C2                 | 0.641(40°)        | 0.539            |
| C3                 | 0.658(40°)        | 0.592            |
| C4                 | 0.658(40°)        | 0.582            |
| Experiments [40]   |                   |                  |
| Single cylinder $Re = 10k$ | 0.292 | 0.282 |

Table 1: Comparison of results for r.m.s pressure distribution $C'_p$

uniform values of fluctuating pressure distributions around the first cylinder from the findings of Shim [27]. One must suspect this according to the work of Norberg [40] at a comparable Reynolds number that $C'_p$ exhibits a very low level at the frontal stagnation line ($\theta = 0^\circ$). Second, it can be observed that the general trend is in reasonably good agreement with the measurements of [27] except for the first column cylinder. Moreover, the position of first peak as shown in the figure 9b corresponds to the same angle in the figure 6a that indicates the tripping of laminar boundary layer separation. The second peak after the shoulder of the cylinder results from the reattachment of boundary layer separation on the surface.

Table 1 shows a comparison of r.m.s value and maximum value of pressure fluctuations from the present LES computation with the experimental values of Shim [27] and Norberg [40]. It can be observed that the values from the present calculations match very well with the measurements, especially for the angular position within the windward side at which the maximum r.m.s value of fluctuating pressure occurs. In addition, one interesting point is that the width of wakes from the second, third and fourth column cylinder is very close. The maximums on downstream cylinders are caused by the impingement of shedding-vortex from the upstream cylinders. The low r.m.s values of pressure fluctuation compared with experimentally measured ones on the leeward side result from the relatively weak wake predicted by the present LES with wall-layer modelling. It is also worthwhile emphasizing that the calculated results at $\theta = 90^\circ$ are significantly higher that the value at a comparable $Re = 10k$ compiled in Norberg [40] for a single circular cylinder.
Finally, judging from the shape of mean pressure distribution around the surface in the two figures, it stands to reason that the present calculation is capable of accurately predicting the pattern or dynamics of flow across tube-banks.

3.3 Drag and lift coefficients

To further validate the present study with experiments, table 2 summarizes the flow parameters concerning $C_D$ and $C'_L$ along with experimental measurements. The coefficient of mean drag per unit span is defined by:

$$C_D = \frac{F_D}{ld\rho u_o^2/2}$$

(26)

where $l$ denotes the spanwise length of the cylinder; $F_D$ denotes the form drag force caused by the surface pressure distribution through ignoring the viscous drag force, which is obtained by an integration of mean pressure distribution around the cylinder. Thus, $C_D$ is given by

$$C_D = \int_{0^\circ}^{180^\circ} \bar{C}_P \cos(\theta) d\theta.$$  

(27)

It is evident that the results of $C_D$ predicted by the present LES study agree favorably well with the experimental measurements (27) except under-predict $C_D$ for the second column cylinder; the magnitudes of $C'_L$ obtained from this work shows reasonable agreement with experimentally measured values except for the first column cylinder due to the reason discussed before. The results for $C_D$ and $C'_L$ are also interpreted in terms of with the free stream velocity $u_o$, which is based on the conversion factor discussed in section 3.2 and clearly a significant variable concerning $C_D$ and $C'_L$ as shown in table 2. Through interpreting this way, the drag experienced by the first column cylinder is increased considerably. Similar observations apply to the rest of downstream cylinders. But the increment for the first column cylinder is distinct from the remaining ones. Comparing the value $C_D = 1.941$ in terms of $Re_\beta = 15270$ with $C_D = 1.185$ (11) for a comparable Reynolds number, it can be observed that $C_D$ for the first column cylinder predicted in this LES study is considerably higher that the value for a unconfined single smooth circular cylinder. This can be explained that the distinct discrepancy interpreted through $u_o$ is undoubtedly a consequence
An LES study and ... in-line tube-banks

Based on $u_g$

| Case       | $C_D$ Based on $u_g$ | $C_L'$ | $C_D$ Based on $u_o$ | $C_L'$ |
|------------|---------------------|--------|----------------------|--------|
| Present LES| C1 0.767 0.228       | 1.941  | 0.579                |        |
|            | C2 0.404 0.655       | 1.022  | 1.656                |        |
|            | C3 0.454 0.650       | 1.146  | 1.645                |        |
|            | C4 0.464 0.507       | 1.174  | 1.284                |        |
| Experiments| Estimated           | 2.022  | 0.127-0.202          |        |
|            | C2 0.324 0.55-0.65   | 0.820  | 1.391-1.645          |        |
|            | C3 0.465 0.60-0.70   | 1.176  | 1.518-1.771          |        |
|            | C4 0.476 0.52-0.60   | 1.204  | 1.316-1.518          |        |

Table 2: Comparison of results for $C_D$ and $C_L'$

of a higher pressure coefficient which results from higher separation velocities in confined flow situations ([42]) as shown in figure 8.

In the light of the foregoing discussion it becomes evident that the $C_D$ for the downstream cylinders would be much higher than an unconfined circular cylinder as well. Nevertheless, compared with the standard value $C_D = 1.185$ ([1]), table 2 shows comparable values for $C_D$. Thus, it seems reasonable that the transverse pitch ratio $S_T = P_T/D = 2.67$ does not give rise to the same effect on the drag for the downstream cylinder as for the first column. However, in the present study, the approaching stream for different column cylinders is of different turbulence level which brings about appreciable effects on the vortex shedding and drag force. For example, the free stream for the first column cylinder is assumed uniform. Whilst the downstream cylinders encounter significantly high turbulence level because they lie in the wake. Hence, the blockage ration and the turbulence level of approaching stream, two conflicting factors, result in a drag coefficient $C_D$ for the downstream cylinder which is not significant different from an unconfined circular smooth cylinder. This is demonstrated in figure 8 which shows $C_D$ for cylinder C2, C3, C4 is significant lower than the value of C1.

In selecting extra experimental data, the focus is given to those measured on a single circular smooth cylinder in free stream flow or in confined flows with a comparable blockage ratio to the present configuration. The data for $C_L'$ from Norberg [40] are determined by the following...
correlation

\[ C'_L = 0.52 - 0.06 \times [\log(Re/1600)]^{-2.6} \quad (5.4 \times 10^3 < Re < 2.2 \times 10^5) \] (28)

which covers the upper bound of sub-critical Reynolds number range. \( C'_L \) does not display much variation when the Reynolds number remains below the critical value. Richter and Naudascher\(^4\)’s data, which are extrapolated from their experimental observations performed at a smooth circular cylinder in a wind-tunnel with a blockage ratio of 1/4, are included for further comparisons. It can be observed that, when interpreting \( C'_L \) in terms of \( u_o \), the values of \( C'_L \) for downstream cylinders are significantly higher than that of an unconfined circular cylinder and do not fall in the scatter region of a confined circular cylinder (\(^2\)). This indicates that the vortex shedding from downstream cylinders may be augmented further by the feature of approaching wake turbulence from upstream cylinders. Consequently, it yields considerably higher values of \( C'_L \) on downstream cylinders as presented in figure 9 in which \( C'_L \) is interpreted with the free stream velocity \( u_o \).

Moreover, two interesting points can be derived from figure 8 and 9. First, as far as the fluctuating magnitudes of the two variables are concerned, it can be observed that the first column displays smaller values than downstream cylinders. The second observation is that the maximum fluctuation of \( C'_L \) is achieved on the second or the third column cylinder. A similar phenomenon was reported by Liang and Papadakis\(^2\).

3.4 Shear-layer instability and vortex shedding

Figure 10 presents close-up views of an instantaneous velocity vector map in the middle plane of the flow domain around the four cylinders C1, C2, C3, C4. In accordance with the results of previous researchers (\(^1\)), in the sub-critical regimes, the separating shear layers become turbulent. In figure 10a, it can be observed that small-scale vortexes are being formed in the shear layers behind C1. Nevertheless, such small vortexes appear not to be formed behind the downstream cylinders as shown in figure 10b. This can be explained that the approaching stream for the downstream cylinders, the wake of upstream cylinders, is of particularly high turbulence level, so that there is no transition that is closely connected with the vortex formation in the shear layer.
Figure 11 presents a statistically significant sample of time histories of velocity fluctuations at a point \( (x/D = 0.55, y/D = 0.65) \) with respect to the center of the cylinder that lies in the near wake. The power spectrum density is obtained by an ensemble average across the 40 sampling stations in the homogeneous spanwise direction. The fairly sharp peaks, the Strouhal frequency \( f_{St} \), characterize the predominant vortex shedding. Moreover, it can be observed that there is another peak \( (f_{sl}) \) that represents the frequency of shear-layer vortices and is significantly higher and is of a relatively broader band than \( f_{St} \). For this broadband feature of shear layer vortices, Dong et al.\[43\] ascribed this complex phenomena to a few factors, e.g. the Karman vortex formation, the varying momentum thickness and the oscillation of the separation line. However, the value for \( f_{sl} \) predicted for the first column cylinder fails to match the well-known \( Re^{0.67} \) law for an unconfined circular cylinder \( \[44\] \). This is consistent with the observations from Brun et al.\[45\] that indicate there is no universal Reynolds number dependence of \( f_{sl}/f_{St} \) for two cylinders placed side by side.

The time histories and corresponding power spectrum densities for the three downstream cylinders are presented in figure 12, figure 13 and figure 14. The fundamental frequency of vortex shedding is well pronounced for the three cylinders. From Gerrard\[46\] and Gerrard\[47\], the fundamental shedding frequency behaviors like a mean rather than a fluctuating quantity because the strengths of the vortexes depend most strongly on the mean rate of shedding of vorticity, which is governed by the mean behavior of the separated shear layer. Consequently, it is reasonable to expect that the fundamental shedding frequency will show little variations for downstream cylinders in the sub-critical range of Reynolds number. Nonetheless, there is no signature for the shear layer vortexes observed. This may result from the feature of significant inhomogeneity in the crosswise direction associated with the wake from the upstream cylinders.

It is worthwhile noting that the dominant frequency of vortex shedding predicted in the present study is evaluated in terms of the mean velocity across the gap \( u_g \). It is of interest to compare the predicted value with the experimental observations, especially with the universal Strouhal number \( St^* \) proposed by Roshko\[48\] that is defined \( fd^*/u^* \) in terms of the wake width between the rows of vortexes \( d^* \) and the wake velocity \( u^* \) obtained from the free-streamline theory. These are summarized in the table 3 along with an extrapolated value from the measurements for a confined circular cylinder by Richter and Naudascher \[42\]. It can be observed that Shim\[27\]’s
measurement as well as the present predicted value fall well within the range 95% of the universal Strouhal number for the sub-critical range of Reynolds number.

3.5 Correlation length for vortex shedding

To examine the spatial structure of vortex shedding behind the cylinders, figure 15 presents the auto-correlation functions for the streamwise and crosswise velocity components for the 40-sampling stations \((x/D = 0.55, y/D = 0.65)\) with respect to the axis of the cylinder the across the homogeneous spanwise direction. It is defined as

\[
R_{ij}(x; z, t) = \frac{u_i'(x; t)u_j'(x + z; t)}{u_i'^2(x; t)}
\]  

(29)

From figure 15a it can be observed for the first column cylinder C1 that \(R_{u'u'}\) and \(R_{v'v'}\) are decreasing monotonically to zero within the range of \(L/D = 1\). This implies that the spanwise length of the biggest eddy from vortex shedding approximately equals to the diameter of the cylinder. This feature has been demonstrated for an unconfined single circular cylinder by previous researchers. Nonetheless, the fact that \(R_{u'u'}\) and \(R_{v'v'}\) do not tend asymptotically towards zero at large separation distance is concerned with the periodic boundary condition employed for the homogeneous spanwise direction in the present LES study.

Nevertheless, from figure 15b,c,d it can be observed that the downstream cylinders C1, C2 and C3 display distinct behaviors with respect to the auto-correlation as a function of the spanwise length for the streamwise and crosswise velocity fluctuations. First, within the length of \(L/D = 1\), \(R_{u'u'}\) and \(R_{v'v'}\) do not decease to zero. Secondly, \(R_{v'v'}\) wiggles across the middle part of the
An LES study and ... in-line tube-banks

cylinder $L/D \in [0.5, 1.5]$. For both of the discrepancies from the first column cylinder C1, it may result from the mixing of the shedding vortexes from different column cylinders, hence it gives rise to complex eddy patterns of the wake.

4 Concluding remarks

Turbulent flow across in-line tube-banks with transverse and longitudinal pitch $P_T/D = 2.67$ and $P_L/D = 2.31$, respectively, has been studied successfully by Large Eddy Simulation (LES) based on the dynamic Smagorinsky subgrid scale model (SGS) with a wall-layer model. Flow structures across the tube-banks based on the normalized Q criterion is presented. The middle cylinder from each column is chosen to present results and compared with experiments. The surface pressure characteristics observed in Shim's experiment are well reproduced irrespective of some discrepancies that can be attributed to the difficulty in numerically mimicking the inflow condition of the experiment. Quite satisfying agreement is observed between the simulation and experimental observations for the drag and lift coefficients, which indicates the second column cylinder experiences the minimum drag force and maximum lift force fluctuation. A frequency analysis for velocity signals at the position with respect to each cylinder axis ($x/D = 0.55, y/D = 0.65$) is presented and compared with experimental as well as theoretical work. These results show that the dominant vortex shedding frequency does not show variations across the tube bank. Nevertheless, the instability frequency of shear layer is not observed for the downstream cylinders. As far as the first column cylinder is concerned, the shear layer instability observed does not show agreement with the universal value for an unconfined single circular cylinder; however, this supports the recent experimental measurements by Brun et al. Finally, auto-correlation functions for streamwise and cross velocity fluctuations as a function of the spanwise length are investigated. They indicate the turbulent eddy behind downstream cylinders are of more complex structure than the first column cylinder as result of the mixing shedding vortexes from different column cylinders.
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Fig. 1: Configuration of the four-column in-line tube banks
Fig. 2: Configuration of the four-row in-line tube bank. The $x$– axis indicates the freestream flow direction; $y$– and $z$–axis respectively indicate the transverse and spanwise direction.
Fig. 3: Computational domain and mesh in the $x - y$ plane.
Fig. 4: The filtered flow structure development across the four-in-line tube banks, iso-surface of the second invariant of velocity gradient tensor, colored by the resolved turbulent kinetic energy (TKE)
Fig. 5: Vortex motion around cylinders at the middle plane cut
Fig. 6: Averaged mean $C_p$ as a function of angle from the front stagnation point, (a) $C_1$, (b) $C_2$, (c) $C_3$, (d) $C_4$.
Fig. 7: Surface fluctuating pressure distribution as a function of angle from the front stagnation point over 30 non-dimensional time units, (a) $C_1$, (b) $C_2$, (c) $C_3$, (d) $C_4$. 
Fig. 8: Time history of $\bar{C}_D$, (a) $C_1, C_2$ (b) $C_3, C_4$
Fig. 9: Time history of $C'_L$, (a) C1, C2 (b) C3, C4
Fig. 10: Instantaneous velocity vector map in the middle plane of flow domain (a) C1, (b) C2, (c) C3, (d) C4
Fig. 11: Time histories of velocity signal fluctuations behind cylinder C1 and the corresponding power spectrum density.
Fig. 12: Time histories of velocity signal fluctuations behind cylinder C2 and the corresponding power spectrum density.
Fig. 13: Time histories of velocity signal fluctuations behind cylinder C3 and the corresponding power spectrum density.
Fig. 14: Time histories of velocity signal fluctuations behind cylinder C4 and the corresponding power spectrum density.

Time histories of velocity signal fluctuations behind cylinder C4 and the corresponding power spectrum density.
Fig. 15: Auto-correlation of streamwise and crosswise velocity fluctuations as a function of spanwise length (a) \( C_1 \), (b) \( C_2 \), (c) \( C_3 \), (d) \( C_4 \).