A numerical investigation of the heat transfer characteristics of water-based mango bark nanofluid flowing in a double-pipe heat exchanger

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1. Introduction

Heat exchange equipment is a device that aids the transfer of heat energy between two or more fluids that are at different temperatures. The fluids may be in direct contact or may be separated from mixing together. They are applied practically for a wide range of uses, in both heating and cooling processes. Although there have been numerous nanofluid-related studies, results and findings are still evolving and far from perfect. For example, an accurate correlation for the nanofluids’ thermophysical properties has eluded scientists up to now and this could be due to the fact that nanofluids vary in their percentage composition of solid particles, and as the percentage increases the physics governing the nanofluids changes. Therefore, it makes it impossible to use one model to accurately capture all its physical behaviour across all percentage compositions.

For new energy-efficient heat transfer equipment, it is imperative to have compact designs, but due to the inherently low thermal conductivity of conventional fluids, achieving compact designs has been difficult [3]. However, with the development of various nanofluids, the heat transfer properties of the base fluid are increased significantly, hence the prospect of achieving compact-size heat transfer devices have risen. Before the advent of nanofluids, micrometre or even millimetre-sized particles were used but the drawbacks included blocking of pipes, corrosion of conduits and high-pressure drop when applied in practice. Furthermore, they had rheological and stability issues wherein the processes. Although there have been numerous nanofluid-related studies, results and findings are still evolving and far from perfect. For example, an accurate correlation for the nanofluids’ thermophysical properties has eluded scientists up to now and this could be due to the fact that nanofluids vary in their percentage composition of solid particles, and as the percentage increases the physics governing the nanofluids changes. Therefore, it makes it impossible to use one model to accurately capture all its physical behaviour across all percentage compositions.

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particles settled very fast from the suspension, which also meant higher costs since the settled particles would have to be replaced each time. All these drawbacks made them unsuitable for use in practical applications.

Several existing articles [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] have been published which were dedicated to the study of nanofluids and the most relevant ones are presented in this section. Wen and Ding [16] studied the thermal characteristics of γ-Al2O3 nanoparticles dispersed in deionized water in the laminar flow regime under a constant wall heat flux condition with particular interest on the entrance region. It was found that the local heat transfer coefficient varied with regards to Reynolds number and volume fraction. Furthermore, it was found that there was a noticeable rise in the coefficient of heat transfer in the entrance region which led to a reduction in the thermal boundary layer thickness which reduced with axial distance. They suggested a possibility of smart entrance region to harness the particularly high performance at the entrance region. Nakhjavani et al. [17] produced silver nanoparticles via green synthesis method using green tea leaves. Their cost-effective method provided condition to control the average nanoparticle size. The particles produced were characterized employing x-ray diffraction, UV visualization, scanning electron microscope images, zeta potential measurements, digital light scattering and thermal conductivity measurements. From their results, they demonstrated that the produced samples of silver nanoparticles were pure in structure, closely identical in terms of morphology and showed longer stability when dispersed in deionized water. They also showed that small particles had higher thermal and antimicrobial performance. They also stated that, since green tea leaves were used for extracting the silver nanoparticles, the method was ecofriendly. The thermal conductivity of the dispersed silver nanoparticles in deionized water was found to be higher than that of deionized water. Sarafras et al. [18] experimentally measured and compared the thermal performance of deionized water and copper oxide water-based nanofluids at different operating conditions. Their results showed that if the applied heat flux was increased, the heat transfer coefficient increases for both of the test fluids at the distinguished heat transfer regions. Additionally, they found that by increasing the flow rate of fluid, heat transfer coefficient greatly increased at both regions. They showed that the fluid temperature at the inlet displaces the boundary between nucleate boiling and forced convection in that an increase in the inlet temperature, at lower heat fluxes leads to a dominance of nucleate mechanism. Sarafras et al. [19] performed an experimental investigation on how the fouling formation of nanoparticles affect the thermal performance of chevron type flat plate heat exchangers with CuO-water as the working fluid. In their study, a low frequency vibration was applied to mitigate the fouling formation. They showed that fouling thermal resistance was decreased by introducing vibration into the system as well as intensified overall thermal performance of the system. Nirkhah et al. [20] experimentally studied convective boiling heat transfer coefficient of spherical CuO (II) nanoparticles dispersed in water inside a vertical heat exchanger with the influence of different operating parameters such as mass and heat fluxes, concentration and subcooling temperature on forced convection and nucleate boiling heat transfer mechanism. In their study, they found that by increasing mass and heat fluxes, the heat transfer coefficient increased significantly for both heat transfer regions. Also, an increase in the nanoparticle weight concentration leads to an increase in the heat transfer coefficient in convective heat transfer and reduces the heat transfer coefficient in nucleate boiling resulting in the formation of nanoparticle deposition on the heat surface. Duangthongsuk and Wongwises [4] carried out an experimental study on heat transfer performance and pressure drop of TiO2-water nanofluid flowing in a vertical pipe in both the laminar and turbulent flow regimes. They found that the convective heat transfer increased with nanoparticle concentration in both laminar and turbulent flow regimes at a given particle size and Reynolds number. Additionally, they showed that at a given nanoparticle concentration and Reynolds number, the heat transfer coefficient was not sensitive to particle size. They further stated that the effect of particle size on the coefficient of heat transfer was due to the movement of the nanoparticles. Finally, they found a negligible effect of the nanoparticles on the pressure drop.

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Some numerical studies have been carried out on forced convection as well. Lemanowicz et al. [23] carried out a numerical simulation of nanofluid flow in a small diameter pipe and concluded that although one-phase simulation, as well as multiphase simulations, resulted in good agreement with the theoretical calculations and experimental results, they recommended using the multiphase model if possible. They stated that the multiphase approach allows one to investigate the behaviour of nanoparticles within the fluid and in some cases show more complete phenomena that would not be predicted by the single-phase model. Morteveji and Hejazian [24] carried out a numerical inspection based on the computational fluid dynamics (CFD) method, with a single-phase approach and suggested correlations to estimate the Nusselt number and friction factor based on dimensionless numbers and they observed that the modelling data were in good agreement with experimental data. Ahmed et al. [25] investigated the heat transfer enhancement in corrugated channels using SiO2-water nanofluid both experimentally and numerically and found that the average Nusselt number and heat transfer enhancement increases as the nanoparticles’ volume fraction increases at the expense of increased pressure drop. Ahmed et al. [26] numerically studied the heat transfer and nanofluid flow in a circular duct with vortex generator using the two-phase model and found that considering the nanofluid as two separate phases is more reasonable than assuming the nanofluid as a homogeneous single phase. Saberi et al. [27] employed both the single-phase and two-phase mixture models to study the laminar forced convective heat transfer of alumina-water and zirconia-water nanofluid through a vertical heated tube. Comparing their results with experiment showed that the mixture model gave a better result by 8% and 5% error for alumina-water and zirconia-water nanofluids respectively. The single-phase approach gave 13% and 8% error on prediction.

Bianco et al. [28] performed a numerical investigation on turbulent nanofluid flow in a circular tube subject to constant heat flux. The result showed that as opposed to the single-phase model, the multiphase model (mixture model) was found to be more accurate.

Akbari et al. [7] numerically investigated forced convection flow in a horizontal pipe. The results showed that the thermal prediction using the two-phase model was very sensitive to the particle concentration, while both the single-phase approach and the two-phase models predicted almost identical flow fields.

Regardless of the benefits of using these nanofluids, there are tangible health and environmental issues associated with their use. Hence the use of nanofluids is limited, due to the fact that most of the commonly available nanofluids are toxic, harmful, and dangerous to the humans and animals exposed to them either by inhalation, ingestion, and penetration or otherwise and they are also harmful to the environment, as reported by

He et al. [22] carried out an experimental study to investigate the heat and flow characteristics of TiO2-distilled water nanofluid flowing in a vertical pipe in both the laminar and turbulent flow regimes. They found that the convective heat transfer increased with nanoparticle concentration in both laminar and turbulent flow regimes at a given particle size and Reynolds number. Additionally, they showed that at a given nanoparticle concentration and Reynolds number, the heat transfer coefficient was not sensitive to particle size. They further stated that the effect of particle size on the coefficient of heat transfer was due to the movement of the nanoparticles. Finally, they found a negligible effect of the nanoparticles on the pressure drop.
researchers [30, 31, 32, 33, 34, 35, 36, 37, 38]. Tabet et al. [30] reported the adverse effects of industrial multiwalled carbon nanotubes on human pulmonary cells. Chang et al. [33] stated the health effects of exposure to nano-TiO\textsubscript{2}.

Additionally, environmental concerns due to use of non-biodegradable materials have been raised: researchers [32, 37] reported that there is amplified toxicological pollution on the environment due to the shape, size and chemical compositions of some of the nanotechnology products. They suggest that choosing less toxic materials will make huge positive impacts on the environment. Laboratory studies have shown that many nanoparticles, specifically those made of silver, copper, and zinc, have anti-microbial properties. While they may be useful for some medical applications, the introduction of such particles into the natural environment could pose a threat to beneficial microbial communities (bacteria, fungi, and archaea) such as those found in the soil [39]. Bio-nanoparticles which may be gotten from leaves, wood char and seeds could be environmentally friendly since humans are naturally exposed to these nanoparticles [40].

The experimental process of preparation and characterization of mango nanoparticle was thoroughly reported by Sharifpur et al. [40]. In their study, they prepared the nanofluid using a two-step method. In the first step, the nanoparticles were prepared by ball milling from mango bark and leaves that had been dried in sunlight. In the second step, the ultrasonic process was used to make the prepared nanoparticles suspend in deionized water to form the colloids; the required volume of nanoparticles was mixed with the necessary amount of water. This mixture was then subjected to an ultrasonic cavitation process for 1 h to prepare an even and stable fluid. They accessed the stability of the nanofluid and verified the stability with viscosity measurements at a constant temperature.

It is worth noting that the flow in the majority of practical applications in convective heat transfer is turbulent rather than laminar. This flow regime has a higher effectiveness of heat exchange than a laminar flow regime due to the mixing which happens on a microscopic scale with particle groups transported in a zig-zag path [1]. Hence this present study focuses on the turbulent flow condition.

The use of biomaterial nanoparticles in a base fluid under forced convection in a heat exchanger has not been addressed thoroughly in the literature. Thus, there is a gap in the literature that address the forced convection in a heat exchanger has not been addressed thoroughly in the literature. Further explanation can be found in Onyiriuka et al. [13]. The dimensional relationships of governing equations of the model as in Eqs. (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13), (14), (15), (16), and (17) are [42, 43, 44]:

Continuity,
\[ \nabla \cdot (\rho_n \vec{V}_n) = 0 \]  

Momentum,
\[ \nabla \cdot (\rho_n \vec{V}_n \vec{V}_n) = -\nabla P + \nabla \cdot (\mu_n \nabla \vec{V}_n) + \nabla \cdot \left( \sum_{k=1}^{n} \phi_k \mu_k \nabla \vec{V}_{a,k} \right) \]  

Energy,
\[ \nabla \cdot \left( \sum_{k=1}^{n} \phi_k \nabla H_k \right) = \nabla \cdot (k \nabla T) \]  

And volume fraction
\[ \nabla \cdot (\phi_k \rho_k \vec{V}_k) = -\nabla \cdot (\phi_k \rho_k \vec{V}_{a,k}) \]  

\[ \vec{V}_n = \frac{\sum_{i=1}^{n} \phi_i \rho_i \vec{v}_i}{\rho} \]  

\[ \rho = \sum_{i=1}^{n} \phi_i \rho_i \]  

\[ \mu = \sum_{i=1}^{n} \phi_i \mu_i \]  

\[ k = \sum_{i=1}^{n} \phi_i k_i \]

\( H_k \) is the sensible enthalpy for phases.

The drift velocity \((\vec{V}_{a,k})\) for the secondary phase is

\[ \vec{V}_{a,k} = \vec{v}_k - \vec{v}_n \]

The relative or slip velocity is defined as the velocity of the second phase \((p)\) relative to the velocity of the primary phase \((\bar{f})\):

\[ \vec{V}_{pf} = \vec{v}_p - \vec{v}_f \]

The drift velocity related to the relative velocity becomes:

\[ \vec{V}_{a,p} = \vec{V}_{pf} - \sum_{i=1}^{n} \vec{V}_p \phi_i \rho_i / \rho_n \]

Manninen et al. [45] and Naumann and Shiller [46] suggested the following equations for relative velocity \(\vec{V}_{pf}\) and the drag function \(f_{drag}\) as:

\[ \vec{V}_{pf} = \frac{\rho_p d_p^2}{18 \mu_{a,drag}} \frac{\rho_n - \rho_a}{\rho_p} \bar{a} \]  

\[ f_{drag} = \begin{cases} 1 + 0.15 Re_p^{0.687} & Re_p \leq 1000 \\ 0.0183 Re_p & Re_p \geq 1000 \end{cases} \]

Here the acceleration is determined by
\[ \bar{a} = \bar{g} - (\vec{V}_n \nabla) \vec{V}_n \]

\( d_p \) represents the diameter of the nanoparticles of the secondary phases while \( \bar{a} \) is the secondary phase particles’ acceleration.

The solids shear viscosity is defined as the addition of collisional and kinetic parts and the optional frictional part.

Syamlal et al. [47] provide the collisional part, which is a viscosity
influence due to collisions between particles taken from the kinetic theory of granular flow.

$$\mu_p,_{col} = \frac{4}{5} \phi_p \rho_p \delta_{p_{0 pp}} \left(1 + e_{pp} \right) \left( \Theta_p / \pi \right)^{1/2} \phi_p$$

(15)

while for the kinetic viscosity part, the Syamlal et al. [47] model is used for the calculations and is given as:

$$\mu_{p,kin} = \frac{\phi_p \rho_p \delta_{p_{0 pp}} \Theta_p}{6 \left(1 - e_{pp} \right)} \left[1 + \frac{5}{2} \left(1 + e_{pp} \right) \left(3e_{pp} - 1 \right) \phi_p \delta_{p_{0 pp}} \right]$$

(16)

and the bulk viscosity is the granular particles’ resistance to compression or expansion. The model is developed from the kinetic theory of granular flow based on Lui et al. [48].

$$\lambda_p = \frac{4}{5} \phi_p \rho_p \delta_{p_{0 pp}} \left(1 + e_{pp} \right) \left( \Theta_p / \pi \right)^{1/2}$$

(17)

wherein Eqs. (15),(16), and (17) $$\delta_{p_{0 pp}}$$ is the radial distribution function, $$\Theta_p$$ is the granular temperature and $$e_{pp}$$ is the restitution coefficient and $$\lambda_p$$ is the bulk viscosity.

### 2.2. Turbulence modelling

The realizable $$\kappa-\varepsilon$$ turbulent model was suggested by Shih et al. [49]. The equations for the turbulent kinetic ($$\kappa$$) and dissipation of turbulent kinetic energy ($$\varepsilon$$) used in the realizable $$\kappa-\varepsilon$$ turbulent model are defined in Eqs. (18), (19), (20), and (21) as:

$$\nabla(\rho \kappa v) = \nabla \left( \mu + \mu^t_{eff} \right) \nabla \kappa + G_k - \rho \varepsilon$$

(18)

$$\nabla(\rho \varepsilon v) = \nabla \left( \mu + \mu^t_{eff} \right) \nabla \varepsilon + \rho C_i S_i - \rho C_j \frac{\varepsilon^2}{\kappa + \sqrt{\varepsilon}}$$

(19)

Here,

$$C_i = \max \left[ 0.43, \frac{\kappa}{\varepsilon} \right]$$,

$$G_k = \mu_s S_k$$,

$$\kappa = \eta S_k$$, and

$$S_k = \sqrt{2} \rho_s S_k$$.

From Eq. (18), $$G_k$$ symbolizes the generation of turbulent kinetic energy due to the mean velocity gradients. Here, $$S_k$$ is the modulus of the mean rate of-strain tensor, $$\sigma_i$$ and $$\sigma$$ represents the effective Prandtl numbers for the turbulent kinetic energy and the rate of dissipation respectively.

Hence, $$\mu_t$$ is modeled as:

$$\mu_t = \frac{\rho \kappa^2}{\varepsilon} \left( A_3 + A_4 \frac{C_p}{C} \right)$$

(20)

Here, $$A_3$$ and $$A_4$$ are the model constants given as

$$A_3 = 4.04$$ and $$A_4 = 6 \cos \theta$$ respectively with

$$\phi = \frac{1}{3} \cos^{-1} \sqrt{6 \Omega_i}$$,

$$U' = \sqrt{2 \Sigma S_{i}} + \Omega_i + \Omega_v$$,

$$\Omega_i = \Omega_v - 3 F_i$$, and

$$W = \frac{\Sigma S_{i}}{S}$$

(21)

Here, $$\Omega_{i}$$ is the average rate of rotation tensor with the angular velocity $$\omega_{i}$$. In Eqs. (19) and (20), the model constants are $$C_i = 1.44$$, $$C_2 = 1.9$$, $$\sigma_i = 1.0$$ and $$\sigma = 1.2$$.

### 2.3. Heat transfer equations

The heat transfer rate of the heating fluid is given in Eq. (22) as [1]:

$$Q_{hw} = m_{hw} C_{p,w} (T_{in} - T_{out})$$

(22)

Here, $$Q_{hw}$$ represents the heat transfer rate of the hot water, $$m_{hw}$$ represents the mass flow rate of the hot water, while the heat transfer rate into the nanofluid is evaluated in Eq. (23) as [1]:

$$Q_{nf} = \eta \mu_{nf} C_{p,nf} (T_{in} - T_{out})_{nf}$$

(23)

Here, $$Q_{nf}$$ is the heat transfer rate of the nanofluid and $$\eta$$ is the mass flow rate of the nanofluid.

The average heat transfer rate between the hot water and the nanofluid is represented by $$Q_{avg}$$ and is given in Eq. (24) as [3]:

$$Q_{avg} = \frac{Q_{hw} + Q_{nf}}{2}$$

(24)

The heat transfer properties, $$N_{tu}$$ and $$h_{t}$$ of the nanofluids are estimated in Eqs. (25) and (26) [3]:

$$h_{t} = \frac{Q_{avg}}{T_{in} - T_{out}}$$

(25)

$$N_{tu} = h_{t} \frac{D}{k_{t}}$$

(26)

The logarithmic mean temperature difference (LMTD) is calculated in Eq. (27) [1]:

$$\text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln \left( \frac{\Delta T_1}{\Delta T_2} \right)}$$

(27)

where $$\Delta T_1$$ and $$\Delta T_2$$ represent the variation in temperature between the water and nanoparticle at the entrance and exit of the heat exchanger [1].

The heat exchanger effectiveness is calculated by Eq. (28) [1]:

$$\text{Effectiveness} = \frac{\text{Actual Heat Transfer Rate}}{\text{Maximum Possible Heat Transfer Rate}}$$

(28)

### 3. Methods

#### 3.1. The thermophysical properties of the nanofluid

Several models and correlations exist that can be used to calculate properties like density, heat capacity, thermal conductivity, and viscosity, as proposed and reported by researchers [7, 13, 28, 50, 51, 52, 53]. However, issues of estimating the and viscosity and thermal conductivity accurately still exist [13, 54]. Table 1 shows the properties of the base fluid and nanoparticles at 288 K [41, 55].

### 3.2. Nanofluid density and specific heat

The following equations are employed to calculate the density and heat capacity of the nanofluid in Eqs. (29) and (30) as:

$$\rho_{nf} = \left(1 - \phi \right) \rho_{w} + \phi \rho_{p}$$

(29)

$$\rho_{nf} C_{p,nf} = \left(1 - \phi \right) \left( C_{p,w} \rho_{w} \right) + \phi \left( C_{p,p} \rho_{p} \right)$$

(30)

### 3.3. Nanofluid dynamic viscosity

For the constant property assumption, the effective viscosity depends only on the volume fraction $$\phi$$ and is given according to [56] in Eq. (31) as:

| Physical properties of the base fluid and nanoparticles at 288 K [41, 55]. |
|---------------------------------------------------------------|
| **Properties** | **Water** | **Mango-bark** |
| $\text{Density} \ (\text{g} / \text{cm}^3)$ | 1.00 | 1.04 |
| $\text{Heat capacity} \ (\text{J} / \text{g} \cdot \text{K})$ | 1.4 | 1.4 |
| $\text{Thermal conductivity} \ (\text{W} / \text{m} \cdot \text{K})$ | 0.595 | 0.595 |
| $\mu \ (\text{Pascal} \cdot \text{seconds})$ | 0.001136 | 0.001136 |
\[ \mu_{nf} = \frac{\mu_w}{(1 - 2.5\phi)} \]  

(31)

3.4. Nanofluid thermal conductivity

For the constant properties assumption, the effective thermal conductivity is calculated by Eq. (32) [57]

\[ k_{nf} = k_{bf} \left[ 1 + 0.0193 + 0.00383 \left( \frac{\rho_w}{\rho_{bf}} \right) \sqrt{\frac{d_w}{d_{bf}}} \right] \]  

(32)

where \(d_{bf}\) is the effective size of carrier fluid molecule.

3.5. Geometric configuration and set-up

A two-dimensional axisymmetric geometry was considered for the double-pipe heat exchanger (Fig. 1a). The geometry was a counterflow horizontal double tube heat exchanger 1.5 m long, with nanofluid flowing inside the tube and the water flowing in the annulus. The inner tube was made from a smooth copper tube with an inner and outer diameter of 8.13 mm and 9.53 mm respectively. On the hand, the outer tube was made from a PVC tube with an internal diameter of 27.8 mm.

3.6. Grid sensitivity

For the purpose of justifying the correctness and the stability of the numerical results, a number of computations have been executed to determine the total number of grid points that will give accurate and acceptable results that are fit to define the flow and thermal field in the double-pipe geometry considered. The grid sensitivity investigation is done by changing the total number of grid spreading in both radial and axial directions. In a test case of water (the base fluid) with \(Re = 6 \times 10^3\), several combinations of grids were studied to justify that the numerical results do not change further with the change of grid distribution. This step is called the grid independence study or grid sensitivity analysis. The grid distributions studied and their metrics are Mesh 1 \(= 9 \times 1000\) (minimum orthogonal quality of 0.9 and maximum aspect ratio of 2.1), Mesh 2 \(= 15 \times 1500\) (minimum orthogonal quality of 0.8 and maximum aspect ratio of 4.0), Mesh 3 \(= 28 \times 3000\) (minimum orthogonal quality of 0.76 and maximum aspect ratio of 4.0), Mesh 4 \(= 38 \times 4000\) (minimum orthogonal quality of 0.89 and maximum aspect ratio of 5.1).

Fig. 2 depicts the variation of the heat transfer coefficient along the length of the double-pipe heat exchanger. It can be observed that there is only a 0.2% maximum deviation of Mesh 3 from Mesh 4. While the other two grid distribution (Mesh 1 and Mesh 2) deviates significantly. Furthermore, Fig. 3a is presented for \(Re = 5000\) to show the

Fig. 1. Domain (a) Geometrical configuration for the present study (b). Mesh for the present study.
variation of temperature along the inner radius at $x = 1.0$ m, Fig. 3b is presented again for $Re = 5000$ to show the variation of velocity along the inner radius at $x = 1.0$ m and Fig. 3c is presented for $Re = 5000$ to show the variation of turbulent kinetic energy profiles along the inner radius at a location $x = 1.0$ m. This is chosen since at that location both flow and thermal variables remain unchanged (fully developed turbulent flow). It can be seen that Mesh 3 and Mesh 4 generate the most reasonable results in the figures (Fig. 3) and that due to the number of elements in the various meshes, there is a variation in numerical solution and the differences found among Mesh 3 and Mesh 4 results are insignificant. Hence the grid distribution selected for the present computation is Mesh 3 as shown in Fig. 1b. It consisted of 28 elements in the radial direction and 3000 elements in the axial direction in order to save computation resources and avoid any inconsistencies in the numerical results. Additionally, the meshes were refined at locations close to the walls so that large variation of flow and thermal field behaviour near the wall are captured as shown in Fig. 1b. The aforementioned phenomenon is captured in the number of inflation layers of which its value in this study is 12.

4. Results

4.1. Validation of present numerical results

To ensure that we have solved the right equations we carried out this validation step for water and TiO$_2$ nanofluid.

4.1.1. Water

A plot for the variation of Nusselt number with Reynolds number for water is obtained from the present study and this plot is superimposed over the results from Gnielinski’s equation (Fig. 4). A maximum deviation of 5.4% and an average deviation of 4.1% was observed. This shows a very good agreement with Gnielinski’s correlation [58]. The variation can be attributed to near wall mesh distribution and temperature.

Fig. 2. Comparison of grid distributions for heat transfer coefficient along the length of the heat exchanger at $Re = 6100$.

Fig. 3. Grid sensitivity (a) Comparison of grid distributions for radial temperature at $x = 1.0$ m and $Re = 5000$. (b) Comparison of grid distributions for radial velocity at $x = 1.0$ m and $Re = 5000$. (c) Comparison of grid distributions for radial turbulent kinetic energy at $x = 1.0$ m and $Re = 5000$. 
nano-liquids of particle size of 21 nm for the TiO$_2$ nano-fluid. A maximum deviation of 5% can be observed which is within good accuracy and shows good agreement which results to $x = 0.1$ m from the inlet section. It can be deduced that the flow developed quite early. This is indeed a characteristic of turbulent flow and it is due to the intense mixing during the random fluctuations which overshadows the effects of momentum and heat diffusion. Also, the hydrodynamic and thermal entry lengths are about the same size not dependent of the Prandtl number.

Fig. 5 shows the comparison of the dimensionless radial velocity profiles for water along the tube radius at different axial locations for $Re = 8333$. The hydrodynamic entry length is given by $L_h = 1.359D Re^1/3$ which results to $x = 0.1$ m from the inlet section and it is dependent on the Reynolds number, which is the same for both water and nano-fluid, hence water and nano-fluid have the same $L_h$ (0.1 m). Furthermore, from the expression for $L_h$ it can be seen that the dependence of $L_h$ on Reynolds number is very small and the flow developed very quickly, like in the case of water in the tube. The reason for this is similar to the discussion on water in Fig. 6. In addition to that discussion, the thermophysical properties (specific heat, viscosity, and density) of the two fluids (water and nano-fluid) are however different. The effect due to Reynolds number being small (to the one-fourth power) makes these differences trivial.

Fig. 8 shows a plot comparing the dimensionless radial velocity profile of both water and nano-fluid at $x = 0.18$ m (sufficiently large to ensure fully developed flow) for $Re = 8333$. It can be seen that the nano-fluid radial velocity profile coincides with that of water. This is because it is independent of the volume fraction of the particles added. It must be noted here that the axial velocity profile for the nano-fluid increases with an increase in volume fraction for the same Reynolds number; this is because the Reynolds number depends on the density and viscosity of the fluid of which these two properties are directly proportional to the nanoparticle volume fraction. Hence for different nano-fluid volume fractions, the average velocities would be different to keep the Reynolds number steady. Furthermore, as nanoparticle concentration increases, the viscosity is enhanced at a substantially faster rate compared to the density.

Fig. 9 shows that the centerline velocity for water and nano-fluid coincides at the entrance region (counterflow). There is an overshoot of the centerline velocity by 0.02 m/s which settles quickly after 0.1 m (the entrance length). This agrees with the entry length of turbulent flow theory, which is due to intense mixing in the radial direction and eddy gradient at the wall and the accuracy is problem-specific.

4.1.2. TiO$_2$-water nano-fluid

We can satisfactorily say from the comparisons above that we have solved the right equations since our computational model is producing the correct results. Our computational model was then tested for nano-fluid. We tested TiO$_2$-water in the inner tube of the double-pipe heat exchanger for several Reynolds numbers. The comparison (Fig. 5) is done between this study and experimental data of [3] using their experimental set-up as boundary conditions: $V_b = 0.09334$ m/s, $t_i = 40 ^\circ$C, $t_w = 25 ^\circ$C with $\phi = 0.2\%$ and $d_p = 21$ nm for the TiO$_2$ nano-fluid. A maximum deviation of 5% can be observed which is within good accuracy and shows good agreement with experiments.

5. Results and discussion

Results presented henceforth for nano-fluid refers to water-based mango bark nano-fluids of particle size of $d_p = 100$ nm.

Fig. 6 show a comparison of dimensionless radial velocity profiles for water along the tube radius at different axial locations for $Re = 8333$. The hydrodynamic entry length is given by $L_h = 1.359D Re^{1/3}$, which results to $x = 0.1$ m from the inlet section. It can be deduced that the flow developed quite early. This is indeed a characteristic of turbulent flow and it is due to the intense mixing during the random fluctuations which overshadows the effects of momentum and heat diffusion. Also, the hydrodynamic and thermal entry lengths are about the same size not dependent of the Prandtl number.

Fig. 7 shows the comparison of the dimensionless radial velocity profiles for nano-fluid ($\phi = 0.2\%$) along the radius of the tube at different axial locations for $Re = 8333$. The hydrodynamic entry length is also given by $L_h = 1.359D Re^{1/3}$ which results to $x = 0.1$ m from the inlet section and it is dependent on the Reynolds number, which is the same for both water and nano-fluid, hence water and nano-fluid have the same $L_h$ (0.1 m). Furthermore, from the expression for $L_h$ it can be seen that the dependence of $L_h$ on Reynolds number is very small and the flow developed very quickly, like in the case of water in the tube. The reason for this is similar to the discussion on water in Fig. 6. In addition to that discussion, the thermophysical properties (specific heat, viscosity, and density) of the two fluids (water and nano-fluid) are however different. The effect due to Reynolds number being small (to the one-fourth power) makes these differences trivial.

Fig. 8 shows a plot comparing the dimensionless radial velocity profile of both water and nano-fluid at $x = 0.18$ m (sufficiently large to ensure fully developed flow) for $Re = 8333$. It can be seen that the nano-fluid radial velocity profile coincides with that of water. This is because it is independent of the volume fraction of the particles added. It must be noted here that the axial velocity profile for the nano-fluid increases with an increase in volume fraction for the same Reynolds number; this is because the Reynolds number depends on the density and viscosity of the fluid of which these two properties are directly proportional to the nanoparticle volume fraction. Hence for different nano-fluid volume fractions, the average velocities would be different to keep the Reynolds number steady. Furthermore, as nanoparticle concentration increases, the viscosity is enhanced at a substantially faster rate compared to the density.

Fig. 9 shows that the centerline velocity for water and nano-fluid coincides at the entrance region (counterflow). There is an overshoot of the centerline velocity by 0.02 m/s which settles quickly after 0.1 m (the entrance length). This agrees with the entry length of turbulent flow theory, which is due to intense mixing in the radial direction and eddy gradient at the wall and the accuracy is problem-specific.
motion. The velocity distribution remains unchanged in the fully developed region, as can be seen in the figure, and it depends on the geometry of the conduit and also the fluid and flow properties. In addition, the velocity which is along the centerline is called the maximum velocity and the deviation from it is called the velocity defect. According to the velocity defect law, the normalized velocity distribution in the core region of turbulent flow in a pipe is independent of the viscosity of the fluid but depends on the distance from the centreline. This is not surprising since the eddy motion is dominant in this region, and the effect of fluid viscosity is negligible. The characteristics of flow in the viscous sublayer are very important as they determine flow in the rest of the pipe.

The centerline temperature profile for water and nanofluid is presented in Fig. 10. Both profiles increase with length (counterflow) as they are heated by the hot fluid as they flow. The present set-up led to a temperature rise of 2.75 K at the centerline for both fluids and it is worth noting that the temperature distribution in the thermally fully developed region may change with $x$ in the direction of flow. This differs from the velocity distribution. The temperature profile can vary at various parts of the tube in the developed region, and it normally does. However, the dimensionless temperature distribution is constant in the thermally developed region when the temperature or heat transfer rate per unit area at the tube surface remains the same. The trend in Fig. 10 shows an almost linear increase in temperature of the cold fluid with length in the flow direction as it gains temperature from cooling the hot fluid and this trend is due to the fact that the heat flux is a linear function of the change in temperature and the length of the pipe while taking the thermal conductivity of the fluid into consideration.

The juxtaposition of the dimensionless temperature profiles for (a) $\varphi = 0\%$ (b) $\varphi = 0.2\%$ at several tube locations is presented in Fig. 11a and b respectively. It is evident from the figure that dimensionless temperature profiles for water and nanofluid do not change after $x = 0.1$ m, hence a thermally developed flow is observed after that point. However, due to wall effects, there are variations at the wall. Furthermore, we saw from Figs. 6 and 7 that both flow profiles are also hydrodynamically developed. We can safely infer fully developed flow from these two observations. Hence the heat transfer coefficient should remain constant in this region since velocity and normalized temperature distributions do
not change in the flow direction. It should also be noted here that since the Prandtl number increases with volume fraction, the resulting thermal length increases with volume fraction, hence the region is pushed back as volume fraction increases. Furthermore, the quickly thermally developed flow is due to the flow being turbulent, which is characterized by vigorous mixing during random fluctuations that obliterates the effect of momentum and heat diffusion; hence the hydrodynamic and thermal entry length are very short and about the same size.

From the plot of Nusselt number against dimensionless length for water and nano-fluid at $Re = 8333$ (Fig. 12), an almost constant Nusselt number in the fully developed region is observed for both water and nano-fluid as at the fully developed region (i.e. where the thermal and hydrodynamic boundary layers are fully developed) the thermal and flow behaviour no longer change. Additionally, it can be seen that the nano-fluid has a more effective convection than water. This is due to the particles added to the water which make heat flow more effective.

Fig. 13 shows the effect of volume fraction on the Nusselt number. It can be observed from the plot that the Nusselt number drops with an increase in volume fraction (for an increase of volume fraction by 1% there is an average drop of Nusselt number by 0.76) and also no dome shape was observed in the range of the volume fractions (0.2 – 6%) studied. The decrease of Nusselt number with volume fraction resulted from the fact that the thermal conductivity of the fluid increases with volume fraction and since the Nusselt number and thermal conductivity are inversely related, the Nusselt number decreases as thermal conductivity increases. Furthermore, the heat transfer coefficient increases with volume fraction since the thermal conductivity of the nano-fluid increases with volume fraction. It should be noted that the heat transfer coefficient is the rate of heat transfer between a solid surface (tube wall) and a fluid per unit temperature difference. So, the higher the heat transfer coefficient, the higher the heat rate: this is good for heat exchanger design, while the higher the $Nu$, the more potent the convection. The average Nusselt number of pure water, that is, when the volume fraction is zero was 43.5 which was much lower than that of the nano-fluid as can be seen in Fig. 13, where the lowest average Nusselt number was 68.8.

The Nusselt number can be seen not only to increase with Reynolds number in both fluids, as seen in Fig. 14 (for a $Re = 5000$, there was a 58% increase of Nusselt number, while for $Re = 13000$ there was a 45% increase), but at a lower Reynolds number the percentage increase of the Nusselt number is higher, while this increase decreases as the Reynolds number is increased. These observations are due to the fact that the Nusselt number is a strong function of the Reynolds number and bears a direct proportionality in variation: as the Reynolds number increases the Nusselt number also increases, and this relationship is an almost linear one. Also, as the Reynolds number increases, the effect of the nanoparticles is reduced, hence the decrease of the enhancement of heat transfer due to the nanoparticle addition. Furthermore, the Nusselt

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**Fig. 11.** Comparison of dimensionless temperature profiles for (a) $\phi = 0\%$ (b) $\phi = 0.2\%$.

**Fig. 12.** Nusselt number variation with dimensionless length for water and nano-fluid.

**Fig. 13.** Variation of Nusselt number with volume fraction.
number in the case of nanofluid was seen to be higher than that of the case of water; this was because the added particles which increased heat transfer performance and viscosity which resulted in more effective convection for nanofluid than for water.

The average heat transfer coefficient ratio, which is the ratio of the heat transfer coefficient of the nanofluids to the heat transfer coefficient of the base fluid, is plotted against the Reynolds number in Fig. 15. It is clearly seen that the ratio has its maximum value at the lowest Reynolds number. It is also clear from Fig. 15 that the nanofluid heat transfer coefficient is almost twice that of the base fluid. The decreasing trend with Reynolds number is due to the cancelling out of eddies due to particle random motion. This is similar to the trend discussed in Fig. 14. However, the lowest nanofluid average heat transfer coefficient is 1.3 times that of the base fluid.

Fig. 16 shows the plot of effectiveness with volume fraction. It is observed that the effectiveness increases with volume fraction, an overall increase of 0.019 of effectiveness over 0–6% volume fraction range, and this is because of the increase of thermal conductivity with volume fraction which leads to increased heat conduction with volume fraction.

However, caution must be stated here; just because a heat exchanger has high effectiveness and a certain flow condition does not mean it will have a higher heat transfer rate than some other low-effectiveness condition.

A plot of effectiveness with Reynolds number is shown in Fig. 17. Dome shape is observed for both fluids, showing that there exists a range of Reynolds numbers (7 000 – 10 000) where the effectiveness is least; this is because the effectiveness and the Reynolds numbers are related by a polynomial function. Also, nanofluid has a higher effectiveness value than water; this is because the heat transfer rate of nanofluid is higher than that of water due to an increase of thermal conductivity because of the added particles, hence higher heat exchanger effectiveness. High effectiveness values correspond to a small temperature difference between the hot and cold fluid, while a high heat transfer rate results from a large change in temperature. In a thermodynamic sense, higher effectiveness means a reduced value of the thermodynamic irreversibility and a smaller entropy generation. In other to get both high heat transfer and high effectiveness, the product of the surface area and overall heat transfer coefficient must increase. In addition, the heat transfer rate increases with nanoparticle volume fraction as opposed to the Nusselt number in Fig. 13. The effectiveness, therefore, represents the
thermodynamic performance of the heat exchanger.

The LMTD is plotted for various volume fractions in Fig. 18 and it is found to decrease with volume fraction, a similar trend with the Nusselt number and it is also due to the increase of thermal conductivity with volume fraction. It varied by a value of about 0.13 throughout the range of volume fractions studied.

Fig. 19 shows that the LMTD of the nanofluid is lower than water for all Reynolds numbers. This is again due to the increase of thermal conductivity with volume fraction as discussed in Fig. 18. However, the profile for water and nanofluid for the LMTD plot are similar in trends and vary over the range of Reynolds numbers studied; the range of LMTD varies from 11.71 to 12.21, which is a difference of 0.5, while the water has a higher LMTD of 0.03 than the nanofluid. We can see a difference in LMTD of the two fluids and a range of variation with Reynolds number.

Fig. 20 shows the average heat transfer coefficient against the Reynolds number with different nanofluid temperatures at a hot water flow rate and temperature of 0.14 m/s and 45 °C respectively. The effect of nanofluid temperature is seen at higher Reynolds numbers. Furthermore, the heat transfer coefficient is greater for a lower nanofluid temperature than for a higher temperature because an increase in nanofluid temperature leads to an increase in heat transfer rate, which leads to higher heat transfer coefficient [3]. However, in this study, it was found that at \( Re = 8000 \) where what exists on the left to point is exactly the opposite of what is to the right of the same point. This observed behavior is due to the polynomial relationship of the Reynolds number with the effectiveness of the heat exchanger and there exists an optimal Reynolds number in the design of the heat exchanger to exploit high thermal performance; this design point could be used to design high-performing heat exchangers.

Fig. 21 shows the average heat transfer coefficient as a function of Reynolds number with different hot water flow rates.

**Fig. 18.** LMTD variation with volume fraction.

**Fig. 19.** Variation of LMTD with Re for water and nanofluid.

**Fig. 20.** Average heat transfer coefficient as a function of Reynolds number with different nanofluid temperatures.

**Fig. 21.** Average heat transfer coefficient as a function of Reynolds number with different hot water flow rates.

**Fig. 22.** Average heat transfer coefficient as a function of Reynolds number with different hot water flow rates, at a hot water temperature of 40 °C and nanofluid temperature of 15 °C. It is clear that the average coefficient of heat transfer of the nanofluid increases directly proportional to the hot water flow rate until after a Re of 9 000 where the converse holds. This is also as a result of the polynomial relationship of Reynolds numbers and effectiveness of a heat exchanger as discussed in Fig. 20.

Fig. 22 shows the average heat transfer coefficient as a function of
Declarations

Author contribution statement

D. R. E. Ewim: Conceived and designed the experiments; Contributed reagents, materials, analysis tools or data; Wrote the paper.
E. J. Onyirika: Conceived and designed the experiments; Performed the experiments; Contributed reagents, materials, analysis tools or data; Wrote the paper.
A. O. Adelaja, O. O. Ighodaro: Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.
S. Bhattacharyya: Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Additional information

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