Symmetries of temporal correlators and the nature of hot QCD

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The temperature of the chiral restoration phase transition at \( \sim 130 \) MeV as well as the temperature of the center symmetry ("deconfinement") phase transition in a pure glue theory at \( \sim 300 \) MeV are two independent temperatures and their interplay determines a structure of different regimes of hot QCD. Given a chiral spin symmetry of the color charge and of the chromoelectric interaction we can conclude from observed symmetries of spatial and temporal correlators of \( N_F = 2 \) QCD with domain wall Dirac operator at physical quark masses that above the chiral symmetry restoration crossover around \( T_{pc} \) but below roughly \( 3T_{pc} \) there should exist an intermediate regime (the stringy fluid) of hot QCD that is characterized by approximate chiral spin symmetry and where degrees of freedom are chirally symmetric quarks bound into color singlet objects by the chromoelectric field. Above this intermediate regime the color charge and the chromoelectric field are Debye screened and one observes a transition to QGP with magnetic confinement.

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1. Introduction

Before the RHIC era there was a belief that at some critical temperature $T_c$ there should exist a deconfinement phase transition from the hadron gas to the quark gluon plasma (QGP) with free deconfined quarks and gluons. There is no dynamical breaking of chiral symmetry in a system of free quarks and consequently the spontaneously broken chiral symmetry below $T_c$ should be restored above $T_c$. So the critical temperature $T_c$ should be a common temperature of both deconfinement and of chiral restoration. At the RHIC era this view changed since both on the lattice and experimentally no phase transition was seen and instead there was a smooth but rather fast crossover observed on the lattice first in Wuppertal [1]. The quark condensate drops from its vacuum value at $T \sim 100$ MeV to practically zero at $T \sim 200$ MeV with a pseudocritical temperature around $T_{pc} \sim 155$ MeV. The Polyakov loop showed an inflection point only slightly above this temperature so after RHIC a new lore arised that in QCD there is a fast common deconfinement - chiral restoration crossover from hadron gas to QGP around the pseudocritical temperature $T_{pc} \sim 155$ MeV. This picture was also confirmed by the Bielefeld lattice group.

However, the inflection point of the nonrenormalized Polyakov loop was used to establish a "deconfinement temperature". If the renormalized Polyakov loop is studied, which is physical, see Fig. 1 [2], then one clearly sees that there is no hint of a deconfinement transition between 100 and 200 MeV, since the deconfinement transition should be accompanied by Polyakov loop evolution from 0 to 1. The renormalized Polyakov loop shows its evolution from small values to 1 in a broad temperature interval with inflection point around 300 MeV. This temperature coincides with the temperature of the center symmetry ("deconfinement") phase transition in a pure glue theory. Hence the evolution of the renormalized Polyakov loop suggests that in full QCD with light quarks a dramatic rearrangement in gluodynamics happens still around 300 MeV, which is however strongly smeared out by dynamical light quarks. This suggests that properties of hot QCD should be influenced by two independent temperatures, the chiral phase transition around $T_c \sim 130$ MeV [3] and by center symmetry phase transition at $T_d \sim 300$ MeV. There should be no unique pseudocritical temperature of combined chiral restoration and deconfinement crossover.

From the Polyakov loop correlators one can extract an effective potential between static sources (which is however dependent from an assumption whether such a potential is real or complex). Such a potential below and above $T_{pc}$ at physical and smaller than physical quark masses is shown in Fig. 2. [4]. We clearly see essentially the same potential at both temperatures. In both cases there is a flattening of the linear part of the potential that is related to the string breaking. At the same
time there is not yet Debye screening of the color charge. The Debye screening is by definition a screening of the negative Coulomb potential. Deconfinement could be associated with the Debye screening of the color charge [5]. Hence this potential tells that both below and above \( T_{pc} \) QCD is in the confining regime.

Another hint that in QCD there is no a simple fast crossover from hadron gas to QGP is suggested by experiments. The hadron gas is a gas. The QGP is also a gas with a large \( \eta/s \). Experimentalists tell us that at RHIC and LHC temperatures between \( \sim 150 \) and \( \sim 400 \) MeV they observe a liquid with a very small \( \eta/s \). Hence what is seen in experiments cannot be a QGP.

One obviously needs an objective information about degrees of freedom above \( T_{pc} \) to decide what physics and regimes take place there. The objective and model independent information can be delivered by symmetries that can be extracted from correlators on the lattice.

2. Symmetries of the color charge and chromoelectric interaction in QCD

The chromoelectric interaction is defined via the color charge (Lorentz-invariant)

\[
Q^a = \int d^3x \bar{\psi}(x) \frac{i}{2} \gamma^a \psi(x). \tag{1}
\]

The color charge has both \( SU(N_F)_L \times SU(N_F)_R \) and \( U(1)_A \) chiral symmetries which are also symmetries of the QCD Lagrangian. However it has an additional symmetry: the chiral spin \( SU(2)_{CS} \) symmetry [6, 7].

The \( SU(2)_{CS} \) chiral spin transformations are defined as follows:

\[
\psi \rightarrow \psi' = \exp\left(\frac{ie\Sigma}{2}\right) \psi \tag{2}
\]

with generators constructed from Euclidean gamma matrices \( \gamma_k; \ k = 1, 2, 3, 4 \)

\[
\Sigma = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}. \tag{3}
\]

The direct product of \( SU(2)_{CS} \) and of \( SU(N_F) \) can be embedded into \( SU(2N_F) \) group that contains as subgroups both \( SU(N_F)_L \times SU(N_F)_R \) and \( U(1)_A \) symmetries. The \( SU(2N_F) \) is also a symmetry of the color charge.

\[\text{This symmetry was reconstructed from a large degeneracy of hadrons observed on the lattice upon artificial truncation of the near zero modes of the Dirac operator at zero temperature [8, 9].}\]
In Minkowski space in a given reference frame the quark-gluon interaction part of the QCD Lagrangian can be split into temporal and spatial parts:

$$\bar{\psi} \gamma^\mu D_\mu \psi = \bar{\psi} \gamma^0 D_0 \psi + \bar{\psi} \gamma^i D_i \psi. \quad (4)$$

The covariant derivative $D_\mu$ includes interaction of the quark field $\psi$ with the gluon field $A^a_\mu$.

$$D_\mu \psi = (\partial_\mu - ig \frac{t^a}{2} A^a_\mu) \psi. \quad (5)$$

The temporal term contains interaction of the color-octet charge density

$$\bar{\psi}(x) \gamma^0 t^a \psi(x) = \psi(x)^\dagger t^a \psi(x) \quad (6)$$

with the chromoelectric part of the gluonic field. It is a singlet under the $SU(2)_{CS}$ and $SU(2N_F)$ transformations. The spatial part consists of a quark kinetic term and interaction with the chromomagnetic part of the gluonic field. It breaks $SU(2)_{CS}$ and $SU(2N_F)$. Hence interaction of quarks with the electric and magnetic components of the gluonic field can be distinguished by symmetry. Obviously one needs to fix a reference frame to make a discussion of electric and magnetic components sensible.

Emergence of approximate chiral spin and $SU(2N_F)$ symmetries in hadrons at zero temperature upon truncation of the near-zero modes of the Dirac operator observed in [8, 9] tells that while the magnetic interaction is located predominantly in the near-zero modes of the Dirac operator, a confining electric interaction is distributed among all modes of the Dirac operator. The quark condensate of the vacuum is associated only with the near-zero modes. Hence one can conclude from those results that confinement and chiral symmetry breaking in QCD are not directly related phenomena. Given this observation it was predicted that above $T_{pc}$, where the chiral symmetry is restored and the near-zero modes of the Dirac operator are suppressed by temperature effects, there should emerge the chiral spin and $SU(2N_F)$ symmetries and hence QCD should be still in the confining regime [10], for an overview see [11]. In order to understand physics and degrees of freedom in QCD above $T_{pc}$ one should study symmetry properties of correlators with respect to chiral spin and $SU(2N_F)$ transformations.

3. Emergence of the chiral spin and $SU(2N_F)$ symmetries in spatial correlators.

In this section we shortly overview the results on symmetries of spatial correlators obtained with JLQCD domain wall $N_F = 2$ configurations at physical quark masses in Refs. [12, 13] that have been reported at two previous lattice conferences.

In Fig. 3 we show a complete set of $J = 0,1$ correlators at different temperatures. One observes three distinct multiplets above the chiral symmetry restoration crossover and below roughly $T \sim 500$ MeV. The multiplet $E_1$ represents degenerate scalar and pseudoscalar correlators. This signals effective restoration of $U(1)_A$ symmetry that was previously observed in Ref. [14]. The approximately degenerate multiplets $E_2$ and $E_3$ arise from emerged approximate $SU(2)_{CS}$ and $SU(4)$ symmetries. These symmetries suggest that degrees of freedom in QCD above the chiral crossover but below $T \sim 500$ MeV are chirally symmetric quarks connected by the chromoelectric...
field into color singlet objects. This regime of QCD is referred to as a stringy fluid. At $T \sim 500$ MeV the chiral spin and $SU(4)$ symmetries disappear and only chiral symmetries remain. It suggests that the color charge and electric field get Debye screened and one observes a smooth transition to QGP with quasiquarks and quasigluons being the effective degrees of freedom.

4. Temporal correlators above $T_{pc}$

If above the chiral crossover the QCD action is approximately chiral spin and $SU(4)$ symmetric then it should also be observed in temporal correlators that are connected to observable spectral density.

The $t$-direction correlators in the hadron rest frame are
Transformations between interpolating vector operators, \(i = 1, 2, 3\). The left columns indicate the chiral representation for each operator. Red and blue arrows connect operators that transform into each other under \(SU(2)_L \times SU(2)_R\) and \(U(1)_A\), respectively. Green arrows connect operators that form triplets of \(SU(2)_{CS}\), \(k = 4\). The \(f_1\) and \(a_1\) operators are the \(SU(2)_{CS}\), \(k = 4\) – singlets. Purple arrows show the 15-plet of \(SU(4)\). The \(f_1\) operator is a \(SU(4)\)-singlet. From Ref. [7]

\[
C_T(t) = \sum_{x,y,z} <O_T(x,y,z,t)O_T(0,0)^\dagger>,
\]

where \(O_T = \bar{q} \gamma^\tau T \gamma q\) are all possible \(J = 0\) and \(J = 1\) local operators. The transformation properties of the \(J = 1\) operators with respect to \(U(1)_A\), \(SU(2)_L \times SU(2)_R\), \(SU(2)_{CS}\) and \(SU(4)\) are shown in Fig. 4 [7]. Emergence of these symmetries should be seen as a degeneracy of the corresponding correlators.

These correlators with the domain wall Dirac operator at physical quark masses with \(N_F = 2\) QCD (JLQCD configurations) normalized at \(n_t = 1\) calculated on \(48^3 \times 12\) lattices at 220 MeV [15] are shown in Fig. 5. Note that one needs a sufficiently large size \(N_t\) along the time direction to observe a real evolution of the temporal correlation functions. E.g. with \(N_t = 2\) by construction only chiral symmetries can be obtained since everything is fixed by the free quark fields at source and sink. Consequently at the moment we are limited only to the temperature \(T = 220\) MeV.

On the left side of Fig. 5 we show the correlators calculated with free, noninteracting quarks. Dynamics of free quarks are governed by the Dirac equation and only chiral symmetries exist. Indeed only degeneracies due to \(U(1)_A\) and \(SU(2)_L \times SU(2)_R\) symmetries are seen in meson correlators calculated for free quark gas. This pattern reflects correlators at very high temperatures since due to the asymptotic freedom at a very high \(T\) the quark-gluon interactions can be neglected.

On the right side of Fig. 5 we show the correlators calculated in full QCD at \(T = 220\) MeV. The pattern in full QCD is qualitatively different as compared to the free quark gas. In full QCD one clearly sees multiplets of all symmetries under discussion: \(U(1)_A\), \(SU(2)_L \times SU(2)_R\), \(SU(2)_{CS}\) and \(SU(4)\). This confirms results obtained with the spatial correlators.

5. Conclusions

Our results on symmetries of correlators of \(N_F = 2\) QCD at physical quarks masses allow to distinguish three different regimes of QCD at nonzero temperatures.
Figure 5: Temporal correlation functions for $48^3 \times 12$ lattices. The l.h.s. shows correlators calculated with free noninteracting quarks on the same lattice, and features a symmetry pattern expected from chiral symmetry. The r.h.s. presents full QCD data at a temperature of $T = 220$ MeV, which shows multiplets of all $U(1)_A$, $SU(2)_L \times SU(2)_R$, $SU(2)_{CS}$ and $SU(4)$ groups. From Ref. [15]

Figure 6: Three regimes of QCD. From Ref. [13].

At temperatures $0 - T_{pc}$ we have a hadron gas consisting of confined mesons with broken chiral symmetry. From the hadron gas there is a smooth chiral symmetry restoration crossover and one arrives at the stringy fluid that is characterized by chiral, $SU(2)_{CS}$ and $SU(4)$ symmetries. The electric confinement still persists and degrees of freedom are chirally symmetric quarks bound by the electric field into color singlet compounds. Around $T \sim 3T_{pc}$ the color charge gets Debye screened, the $SU(2)_{CS}$ and $SU(4)$ symmetries disappear (only chiral symmetries remain) and one observes a very smooth transition to QGP with quasiquarks and quasigluons being effective degrees of freedom.
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