Bolt Load Calculation for Hybrid Joint Considering Friction and Nonlinear Stiffness under Combined Thermal and Mechanical Loads

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Abstract. An improved analytical method for calculating the bolt load distribution in single-lap or double-lap multi-bolt composite-metal joint structure considering material nonlinear behaviour and thermal stress is proposed in this paper. The analytical method is based on the stiffness method and the composite material mechanics. The bolt load ratios calculation results of a three-bolt joint structures by the improved method is compared with the results from finite element (FE) method and the relative error less than 7%. The great agreement verifies the accuracy of the analytical method. Besides, this improved has more considering factors and higher accuracy than traditional stiffness method and less calculating cost than FE method.

Keywords: Composite-metal mechanical joint; Thermal stress; Bolt load calculation;

1. Introduction
In the structural design of aircraft, composite-metal mechanical joint is widely used in the connection structure between outer wing and central wing since its high reliability, load-carrying capacity and convenience to disassemble. However, bolt load distribution and the location of the stress concentration bolt, which are closely related to the failure behaviour, in the composite–metal joint structure are difficult to assign especially under combined thermal-mechanical loads. There are mainly three ways to calculate the load distribution of joints structure: analytical method[1], finite element (FE) method[2] and experiment tests[3]. FE method can simulate nearly all effects in the joint structure with more computational costs[4]. Experimental tests has poor economy and difficult to accurately measure the bolt load[5]. The analytical method has few calculation costs but the calculation accuracy is unsatisfactory owing to less factors considering[6]. The analytical method based on the spring method is extended to investigate the effects of clearance [7] and hole location errors [8]. An improved stiffness method considering the thermal stress, friction effects and nonlinear behaviour is developed in this paper for accuracy calculation of bolt load in the joint structures.

2. The improved stiffness method
A single-lap five-bolt joint connection structure shows in Fig.1 is discussed to investigate the improved analytical method of bolt load distribution based on stiffness method. The numbers of bolts in the
drawing from left to right are 1#-5#. The upper plate A has the same material as all the titanium alloy bolts, and the lower connecting plate B is made of IM7/8552 laminate.

**Figure 1.** The schematic diagram of single-lap five-bolt joint connection structure.

**Friction force.** In the initial stage of loading, the static friction between the plates is the dominant factor of load rise. According to the classical Coulomb law, the plates begin to slide relative to each other only when the external load is greater than the static friction force $F_0$, as illustrated in Eq. 1:

$$P \geq \mu F_p = F_0,$$

where $P$ is external load, $\mu$ is the static friction coefficient between laminates, $F_p$ is the tightening force, $F_0$ is the static friction force. After the connection plates begins to slide relative, the deformation of connected plate caused by friction can be calculated by sliding friction force and flexibility:

$$u^f = C^f \cdot F^f,$$

where $u^f$ is the deformation of connected plate around the bolt, $C^f$ is the shear flexibility of connected plate around the bolt, $F^f$ is the friction force around the bolt. $C^f$ can be calculated as Eq. 3:

$$C^f = t / (A^b \cdot G_{XZ}),$$

where $t$ is thickness of laminate plate, $A^b$ is the contact area between the bolt and laminate plate, which is nearly same as the contact area between gasket or nut (without gasket) and laminate. $G_{XZ}$ is out-plan shear module.

Thus, according to the equilibrium condition of force, there is a general equilibrium equation between bolt load, friction force and external load, as shown in Eq. 4:

$$F_1 + F_2 + F_3 + F_1^f + F_2^f + F_3^f = P,$$

where, $F_n$ is the bolt load of bolt NO.$n$, $F_n^f$ is the friction force around bolt NO.$n$.

The load transferred between two adjacent nails of plate A is as follows:

$$\begin{aligned}
F_{12}^A &= P - F_1 - F_1^f \\
F_{23}^A &= P - F_1 - F_2 - F_1^f - F_2^f \\
&\vdots \\
F_{n-1,n}^A &= P - (F_1 + \cdots + F_{n-1}) - (F_1^f + \cdots + F_{n-1}^f)
\end{aligned}$$

The load transferred between two adjacent nails of plate B is as follows:

$$\begin{aligned}
F_{12}^B &= F_1 + F_1^f \\
F_{23}^B &= (F_1 + F_2) + (F_1^f + F_2^f) \\
&\vdots \\
F_{n-1,n}^B &= (F_1 + \cdots + F_{n-1}) + (F_1^f + \cdots + F_{n-1}^f)
\end{aligned}$$

According to Eq. 1 to Eq. 6, the load of each bolt, in the upper and lower connecting plates between adjacent bolts and the external load on the whole connecting structure are shown in Fig. 2.
Figure 2. Schematic diagram of loads on the bolt and connecting plates.

Thermal environment. When the temperature changes, the materials with different properties in the composite-metal mechanical joint structure will produce different thermal deformation.

The distance ion both plate A and plate B between two adjacent nails is equal:

\[ L_A = L_B. \]  
(7)

Assuming the deformation of plate A is \( \Delta n_{A,n-1} \), the deformation of plate B is \( \Delta n_{B,n-1} \), shear deformation of the bolt NO.n-1 is \( \Delta n_{n-1} \), shear deformation of the bolt NO.n is \( \Delta n_n \). The deformation compatibility equation is shown in Eq. 8:

\[ L_A + \Delta n_{A,n-1} + \Delta n = L_B + \Delta n_{B,n-1} + \Delta n_{n-1}. \]  
(8)

By substituting Eq. 7 into Eq. 8, the compatibility conditions of multi row single row nails can be obtained as follows:

\[ \Delta n_{A,n-1} + \Delta n = \Delta n_{B,n-1} + \Delta n_{n-1}. \]  
(9)

The deformation of bolt and laminate between two adjacent bolts under load is shown in Eq. 10:

\[
\begin{align*}
\Delta n_{n-1} &= F_n \cdot C_n^p, \\
\Delta n_{A,n-1} &= F_{n-1,n} \cdot C_{n-1,n}^A + \alpha_A \cdot \Delta T \cdot L_A - C_n^f \cdot F_n^f, \\
\Delta n_{B,n-1} &= F_{n-1,n} \cdot C_{n-1,n}^B + \alpha_B \cdot \Delta T \cdot L_B - C_{n-1}^f \cdot F_{n-1}^f
\end{align*}
\]  
(10)

where, \( F_n \) is the bolt load of bolt NO.n; \( C_n^p \) is the shear flexibility of bolt NO. n; \( \Delta n \) is the displacement of bolt NO.n; \( F_{n-1,n}^A \) and \( F_{n-1,n}^B \) are the axial load between bolts NO.n and NO.n-1 in plates A and B, respectively. \( C_{n-1,n}^A \) and \( C_{n-1,n}^B \) are axial tension flexibility of plates A and B between bolts NO.n and NO.n-1, respectively. \( \Delta n_{A,n-1} \) and \( \Delta n_{B,n-1} \) are deformation of plates A and B between bolts NO.n and NO.n-1, respectively. \( \alpha_A \) and \( \alpha_B \) are coefficient of thermal expansion of plates A and B, respectively.

The deformation diagram between two adjacent bolts is shown in Fig. 3.

Figure 3. The deformation diagram between two adjacent bolts

Nonlinear deformation of single bolt. The nonlinear deformation properties of single bolt was captured by a tension test of single bolt joint specimen. The stress and strain can be calculate as in Eq. 11:

\[ \sigma = P / (D \cdot t), \quad \varepsilon = \alpha \cdot U / D. \]  
(11)

where \( \sigma \) and \( \varepsilon \) are stress and strain of the joint structure, \( U \) is the displacement deformation of the joint area, \( D \) is the diameter of bolt, \( t \) is the thickness of laminate plate, \( \alpha \) is a constant depends on the
connection form, \(a=1\) for single-lap, \(a=2\) for double-lap. There are four stages in the deformation process as shown in Fig. 4.

![Figure 4. The four stages of stress-strain curve in single bolt test](image)

The connection flexibility of bolt in linear deformation was calculated as shown in Eq. 12:

\[
C_b = \frac{\delta}{P} = \frac{(t_A + t_B)^2}{E_b D^3} + 3.72 \left( \frac{1}{t_A E_x} + \frac{1}{t_B E_x} \right),
\]

where \(C_b\) is the flexibility of bolt, \(\delta\) is shear deformation, \(t_A\) and \(t_B\) are thickness of plates A and B, respectively, \(D\) is bolt diameter, \(E_b\) is young’s module, 3.72 is the coefficient about the bolt. The Ramberg-Osgood equation was used to express the nonlinear damage process before the failure point:

\[
\varepsilon = \frac{\sigma}{E} + \left( \frac{\sigma}{K} \right)^{\frac{1}{n}},
\]

where \(E\) is equivalent module in the linear deformation process, \(K\) is secant module, \(n\) is nonlinear parameter, \(K\) and \(n\) were obtained by least squares regression method, the equivalent flexibility can be deduced as shown in Eq. 14.

\[
C_n = C_b + \frac{1}{n^{1/K}} \left( \frac{p}{D - t_K} \right)^{\frac{1}{n - 1}} = C_b + f(K, n).
\]

The equivalent flexibility equals to the sum of the flexibility in linear deformation and the nonlinear damage term, which is the function \(f(K, n)\) in Eq. 14.

In order to simulate the nonlinear behavior of composite laminate, a progressive loading was adopted in the calculation. The incremental relation is calculated as Eq. 15:

\[
\Delta e^m = C_n^m \Delta N^m,
\]

where \(\Delta e^m\) is incremental strain of composite laminate in \(m\)th increment, \(\Delta N^m\) is the external load. \(C_n^m\) is the compliance matrix, which deduced through the classical laminated plate theory of composite mechanics.

The flexibility of plates A and B between two adjacent bolts is shown in Eq. 16:

\[
C_{n-1,n}^A = \frac{t_A^{n-1,n}}{E_x W_{n-1,n}^A t_A^{n-1,n}}, \quad C_{n-1,n}^B = \frac{L_B^{n-1,n}}{E_x W_{n-1,n}^B t_B^{n-1,n}},
\]

where \(E_x^A\) and \(E_x^B\) are tension modules of plates A and B, respectively, \(W_{n-1,n}^A\) and \(W_{n-1,n}^B\) are the weight of plates A and B, \(t_A^{n-1,n}\) and \(t_B^{n-1,n}\) are thickness of plates A and B, \(L_A^{n-1,n}\) and \(L_B^{n-1,n}\) are bolt pitch between bolts NO.\(n\) and NO.\(n-1\).
Based on the above equilibrium conditions, compatibility conditions and load displacement relationship, a set of equations with bolt load as unknown parameters can be formed, as shown in Eq. 17:

\[
\begin{align*}
F_1 C_1^b & = (P - F_1) C_{12}^A - F_1 C_{12}^B + \alpha_A \cdot \Delta T \cdot L_A - \alpha_B \cdot \Delta T \cdot L_B \\
F_2 C_2^b & = (P - F_2) C_{12}^A - F_2 C_{12}^B + \alpha_A \cdot \Delta T \cdot L_A - \alpha_B \cdot \Delta T \cdot L_B \\
\vdots & \\
F_{n-1} C_{n-1}^b & = (P - \sum_{j=1}^{n-1} F_j) C_{n-1,n}^A - (\sum_{j=1}^{n-1} F_j) C_{n-1,n}^B + \alpha_A \cdot \Delta T \cdot L_A - \alpha_B \cdot \Delta T \cdot L_B \\
F_n & = P - \sum_{j=1}^{n} F_j
\end{align*}
\] (17)

Eq. 18 can be simplified as matrix form in Eq. 17:

\[
CF = D,
\] (18)

where \(C\) is the matrix related to the flexibility of connecting plates A and B and bolts, \(F\) is the vector composed of unknown bolt loads, and \(D\) is the deformation of connection plates A and B due to external load. Then the matrix form of the final calculation equations of pin load is as follows:

\[
F = C^{-1}D.
\] (19)

### 3. Finite element method

A FE model of the joint structure in Fig.1 is established in ABAQUS with 15544 elements. Fig. 5 shows the bolt connection area. For the purpose of simulating the nonlinear deformation of fastener, the plastic constitutive of titanium alloy is defined in the FEM model and geometric nonlinearly is considering in the model. The FE model is meshed with first-order continuum rectangular shape element (C3D8I). The surface-to-surface contact constraints is set in the contact area between the bolt and the connecting plate and the plate between the upper and lower connecting plates. A totally external load of 100KN applied on the right end surface of the upper plate which is the composite laminate and a fixed boundary condition is defined on the left end.

![Figure 5. The FE model of five-bolt-joint](image)

### 4. Results and discussion

Fig. 6 comparing the bolt load distribution ratios calculated by the improved analytical method and FE method. The largest relative errors between analytical method and FE method at #1 bolt is 6.96% because all factors were considered in the FE model, such as the contact between the bolt and the plate and the 3D effect of the joint members. As the error is acceptable, the calculation result is dependable and the improved analytical method is reliable.
Figure 6. Comparison of bolt load ratios of different method at various temperature

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