1. Introduction

Free-space optical (FSO) links are supposed to operate through atmosphere which contains fog, smoke, dust, rain, smog and charged particles. The terrestrial FSO link provide a viable last mile solution for high speed connectivity without the need of digging necessary to lay the conventional fiber [1]. Atmospheric particles like fog, snow, etc attenuate the transmitted signal propagating through the line-of-sight FSO links. Among all the different attenuating factors, fog is the most serious deterrent [2, 3]. Fog can be characterized by liquid water content (LWC), optical visibility, drop size distribution and temperature [4]. The scattering, absorption and extinction of laser beam propagating through the atmosphere are associated with fog droplets sizes, their effective radii [3] and the microphysical properties of fog, specifically liquid water content [3, 5].

The LWC is the measure of total mass of water per unit volume of the droplets in air. The LWC is expressed in g/m³. LWC is used to characterize the different types of fog/clouds. In the absence of precipitation higher amount of liquid water content decreases visibility and indicates dense fog [5, 6].

Fog is an accumulation of water droplets or ice crystal fines accompanied by hygroscopic, water-saturated fine particles that reduce visibility. Under dense fog conditions the visibility decreases below 100 m and cause severe signal attenuation for FSO links [2]. For light fog the visibility remains in the range of 500–1000 m relatively less deterrent for FSO links.

For wide spread acceptability of FSO links there is need to investigate thoroughly different weather impairment on FSO links [7]. The prediction of optical attenuation in lower atmospheric visibility ranges due to water hazes, fogs, and clouds has been thoroughly investigated and researched [8].

To the best of the authors’ knowledge this is 1st attempt to find a distribution model for LWC for terrestrial FSO links. Fog drop size distribution is widely modelled as a modified gamma distribution [9, 10]. The current work found Gamma distribution as the best fit distribution for both. The results provide the statistical behaviour of LWC and visibility, which is of high importance for Optical Wireless systems and networks.

**Keywords:** Free-space Optics (FSO), Optical Wireless Communication, Liquid Water Content (LWC), Visibility.
of high importance for optical wireless communication and networks and will provide a thorough understanding of visibility and LWC and their variations.

2. Experimental Setup

The measurement campaign was carried out at the Department of Frequency Engineering, Czech Metrology Institute (CMI) Prague, Czech Republic from 08. 01. 2009 till 31.05.2009. Prague is located at latitude 50° 05' 12" N longitude 14° 24' 59" E and at altitude of 191 m. Prague has continental weather. Prague has the mean air temperature 10.4° C from 1971–2000 [11]. The FSO systems were installed 26 meters above the ground level. The link margin of the two FSO systems allows the measurement of specific attenuation up to 180 dB/km for 830 nm wavelength systems and 130 dB/km for 1550 nm system. Optical calibration was performed before deploying the FSO devices. A received power is obtained from the calibrated Received Signal Strength Indicator (RSSI) signal of the FSO link. Meteorological conditions are identified by means of a color video camera and an automatic weather observation system located near the FSO receivers. The system uses Vaisala sensors for the measurement of temperature, humidity, air pressure, velocity and direction of the wind. The VAISALA PWD 11 equipment measures the atmospheric visibility (5% definition) values in the range from 50 m up to 2000 m using forward scattered light in the angle of 45°. The PVM-100 device is used to measure liquid water content LWC (g/m$^3$) and integrated particle surface area PSA (cm$^2$/m$^3$) of fog.

The meteorological data is synchronized in time with the hydrometer attenuation measurement. The received FSO signal levels and the meteorological data are recorded synchronously on a PC’s hard disk. In this article we analyzed 5 months real time measurement data of reduced visibility and LWC due to fog for terrestrial FSO links. In the measurement campaign, we sampled data at a rate of one sample per minute. The other instruments were calibrated in a way to measure the specific quantity at the last second of the every minute in parallel with the optical attenuations. We selected data set for analysis where the visibility was less than 1 km. Figs 1 and 5 shows the reduced data set of LWC and Visibility. It is important to note that the time axis of Figs 1 and 5 is not continuous.

3. Results and Analysis

The department of Frequency Engineering at Czech Metrology Institute is conducting measurement campaign at path links of 100 m and 853 m using the wavelengths of 1550 nm, 830 nm and 850 nm along with visibility, LWC and integrated particle surface area (PSA). A 5 months measured data of visibility and LWC were collected for detailed analysis. In the measurement campaign we sampled data at a rate of one sample per minute. The other instruments were calibrated in a way to measure the specific quantity at the last second of the every minute in parallel with the optical attenuations. In metrology theory, it is admitted that fog exists when visibility is decreased to less to less than 1 km. Therefore we selected the data set for analysis where the visibility was less than 1 km. Fig. 1 shows the reduced data set. It is also important to remark that the time axis in the figure is not continuous. The detailed statistical analysis to find the best suited distribution model for visibility and LWC data is provided in subsequent sections.

A. Visibility

Visibility is defined as the distance to an object at which the image contrast drops to a certain percentage of the original contrast of the object, equivalent to a certain transmission threshold $T_H$ over the atmospheric path. Two different definitions for threshold exist, 2% and 5%. The 5% transmission threshold is more commonly used at airports to denote the “runway visibility range” RVR [12, 13]. Another meteorological definition of visibility is the actual distance at which a person can discern an ideal dark object against the horizon sky [14]. The visibility range is technically measured at the center of sensitivity for the human eye (where the sensitivity of the human eye is maximum), at 550 nm with a spectral band width of typically 250 nm. Different visibility based empirical models have been proposed to calculate optical attenuation from visibility, Kim [15], Pierce [16], Kruse [17] and Al-Naboulsi [18]. Here we presented the statistical characterization of 5 months measured visibility data. The analysis of measured visibility is provided in Fig. 1.

The horizontal axis of Fig. 1 shows the minutes of the days and the vertical axis shows the visibility measured in meters. It is clear from Fig. 1 that there occurred certain dense fog events when the visibility is less than 100 m. The descriptive statistics of the visibility is provided in Table 1.

It is important to mention that the minimum value of measured visibility is 80 m (from Table 1) while the maximum value is 991 m. And also the skewness of the data is positive which shows that the right tail of the distribution will be longer as compared to the left tail.
After having explored the descriptive statistics we proceed further to find the best fit distribution for visibility measured data. We compared all the continuous distribution on the visibility data by analysing the probability density function (PDF) and Cumulative distribution function (CDF). We performed comparison among all the distribution by comparing Quantile-Quantile-Plot (QQ plot). QQ plot is used for comparing two probability distributions by plotting their quantiles against each other. Here we show the results of two best fitted distribution for visibility data. The PDF and CDF of the measured visibility data is provided in Figs 2 and 3 respectively.

Figs. 2 and 3 show that the selected distribution models can be used for statistical characterization of the behavior of measured visibility data. We performed the goodness of fit by observing their

Descriptive statistics of the measured visibility

| Serial No | Statistical Quantity | Values |
|-----------|----------------------|-------|
| 1         | Sample Size          | 2073  |
| 2         | Range                | 911   |
| 3         | Mean                 | 343.74|
| 4         | Variance             | 37688.0|
| 5         | Std. Deviation       | 194.13|
| 6         | Std. Error           | 4.2639|
| 7         | Skewness             | 1.1366|
| 8         | Min                  | 80    |
| 9         | 25% (Q1)             | 197.5 |
| 10        | 50% (Median)         | 291   |
| 11        | 75% (Q3)             | 443   |
| 12        | 90%                  | 637.2 |
| 13        | 95%                  | 752.2 |
| 14        | Max                  | 991   |

QQ plot of measured visibility to find the best fit model between the selected two models. The QQ plot of measured visibility is provided in Fig. 4. In Fig. 4 it is clear that Gamma distribution is performing well as compared to Lognormal distribution for measured visibility. The QQ plot for Gamma distribution is following the normal line as compared to that of Lognormal distribution.

The probability density function for Gamma distribution is provided in equation (1).

\[ f(x) = \frac{\kappa (x - y)^{\kappa - 1}}{\beta^\kappa \Gamma(\kappa)} - \exp\left(-\frac{(x - y)\beta}{\kappa}\right) \]  

(1)

Where \( \kappa \) is a continuous shape parameter \( (\kappa > 0) \), continuous shape parameter \( (\alpha > 0) \), \( \beta \) is scale parameter \( (\beta > 0) \) and \( y \) is location parameter. We computed the optimum parameters for Gamma distribution for measured visibility which are provided in Table 2.
We concluded that Gamma distribution (4 parameters Gamma Distribution model) is the best suited model for measured visibility in the absence of precipitation. The proposed distribution model can be used under fog conditions.

B. Liquid water content

LWC is the measurement of mass of water in fog/cloud in a specified amount of dry air. LWC can be expressed as g/m³. LWC is different for different types of fog and clouds. The classification of clouds and fog is highly related to the amount of LWC and its origin. The combination of LWC and its origin allows to readily predict the types of conditions that will be, most likely, in the vicinity of the FSO links [19]. Fog that has very low densities contains very small amount of water and so eventually results in lower values of LWC i.e., about 0.05 g/m³ for a moderate fog (visibility range around 300 m). Much higher values of LWC (around 0.5 g/m³) result in the formation of thick or dense fog (visibility range of about 50 m) [10]. Fog characterized by several physical parameters like particle size distribution, temperature, humidity and LWC has been extensively modeled by drop size distribution and visibility range.

The analysis of the measured LWC is provided in Fig. 5.

![Fig. 5 Analysis of measured LWC](image)

The horizontal axis of Fig. 5 shows the minutes of the days and the vertical axis shows the LWC measured in g/m³. The descriptive statistics of the measured LWC is provided in Table 3.

| Serial No | Statistical Quantity | Values |
|-----------|----------------------|--------|
| 1         | Sample Size          | 2073   |
| 2         | Range                | 0.399  |
| 3         | Mean                 | 0.06422|
| 4         | Variance             | 0.00281|
| 5         | Std. Deviation       | 0.05304|
| 6         | Std. Error           | 0.00116|
| 7         | Skewness             | 1.4467 |
| 8         | Min                  | 0.001  |
| 9         | 25% (Q1)             | 0.021  |
| 10        | 50% (Median)         | 0.053  |
| 11        | 75% (Q3)             | 0.093  |
| 12        | 90%                  | 0.128  |
| 13        | 95%                  | 0.17   |
| 14        | Max                  | 0.4    |

It is obvious from Table 3 that 95 % of the time the LWC remains below 0.17 g/m³.

We applied curve fitting techniques to find the best fit distribution model for measured LWC. We compared all the continuous distribution on the LWC data by analysing their PDF and CDF. We performed comparison among all the distribution by comparing the QQ plot. The PDF, CDF of the measured LWC data is provided in Figs. 6 and 7 respectively. Fig. 6 shows that both selected distribution models can be used for statistical characterization of LWC under fog conditions. But the results of QQ plot suggest that Gamma distribution is a better suited model for LWC than compared Beta distribution.
We performed the goodness of it by observing the QQ plot of the measured LWC. The QQ plot is provided in Fig. 8. It is obvious from Fig. 8 that Gamma distribution model is performing well as compared to Beta distribution. QQ plot for Gamma distribution is converging but QQ plot for Beta distribution is deviating for higher values of LWC. The distribution function for three parameters Gamma distribution is provided in equation (2)

$$f(x) = \frac{(x - y)^{\alpha - 1}}{\beta^\alpha \Gamma(\alpha)} \exp\left(-\frac{(x - y)/\beta}{\beta}\right)$$

Where $\alpha$ is a shape parameter ($\alpha > 0$), $\beta$ is scale parameter ($\beta > 0$) and $y$ is location parameter. We computed the optimum parameters for Gamma distribution for measured visibility which are provided in Table 2.

We concluded that Gamma distribution (3 parameters Gamma Distribution) is the best suited model for measured LWC in the absence of precipitation. The proposed distribution model can be used under fog conditions ranging from dense fog to light fog and in the absence of precipitation.

4. Conclusions

A detailed statistical analysis of 5 months real time measured data of visibility and LWC for terrestrial FSO links has been presented to find the best fit distribution model for visibility and LWC. Our findings show that in reduced visibility, due to fog, 4 parameters Gamma distribution best fit the measured data of visibility and 3 parameters Gamma distribution is the best fit distribution model for LWC in the absence of precipitation. Both proposed model can be used under fog conditions ranging from dense fog to light fog and in the absence of precipitation.

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