Tests of Spacetime Symmetry with Particle Traps

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Abstract

Lorentz and CPT symmetry have been tested at high precision in numerous experiments. A general theoretical framework incorporating possible Lorentz and CPT violation in an extension of the standard model of particle physics has been developed. In this framework, analyses of several precision experiments have been performed to find unsuppressed symmetry-violating signals. This paper discusses features of the theory, presents results for trapped-particle systems, and reports bounds from recent experiments.

1 Introduction

Symmetry under the Lorentz and CPT transformations is a property of the standard model of particle physics. The possible violation of these symmetries has been investigated in the context of an underlying theory including also the gravitational interaction. Minuscule effects of Lorentz and CPT violation might then be detectable in high-precision experiments. The expected suppression of such effects would be the ratio of a low-energy scale to the Planck scale. These effects can be described by a general standard-model extension that allows CPT and Lorentz violation but retains the other conventional properties of quantum field theory, such as energy conservation, gauge invariance, and renormalizability.

Sensitivity to certain effects in the standard-model extension is known to exist in a variety of experiments. These include tests with muons, experiments with kaons and other neutral mesons, studies of the baryon asymmetry, measurements of cosmic birefringence, clock-comparison experiments, and investigations with spin-polarized solids. This paper will review investigations with low-energy trapped particles, focusing on tests with Penning-traps, and tests involving the spectroscopy of hydrogen and antihydrogen.

The extension of the SU(3)×SU(2)×U(1) standard model and quantum electrodynamics originates in the idea of spontaneous CPT and Lorentz breaking in an underlying context such as string theory. Violations of CPT and Lorentz symmetry are allowed in the theory as couplings that can be experimentally bounded, if not in fact measured. In this context, a particle with charge \( q \) and mass \( m \)
is described by a four-component spinor field \( \psi \) satisfying a Dirac equation with additional terms \([4, 21]\)

\[
(i \gamma^\mu D_\mu - m - a_\mu \gamma^\mu - b_\mu \gamma^5 \gamma^\mu - \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu} + ic_{\mu\nu} \gamma^\mu D^\nu + id_{\mu\nu} \gamma^5 \gamma^\mu D^\nu + ie_\mu D^\mu - f_\mu \gamma_5 D^\mu + \frac{1}{2} i g_{\mu\nu\lambda} \sigma^{\mu\nu} D^\lambda) \psi = 0 .
\] (1)

In this equation, \( A^\mu \) is the electromagnetic potential and \( iD_\mu \equiv i\partial_\mu - qA_\mu \). The symmetry violations are parametrized by a set of effective coupling constants \( a_\mu, b_\mu, c_{\mu\nu}, d_{\mu\nu}, e_\mu, f_\mu, g_{\mu\nu\lambda} \), and \( H_{\mu\nu} \); \( c_{\mu\nu} \) and \( d_{\mu\nu} \) are traceless, \( g_{\mu\nu\lambda} \) is antisymmetric in the first two indices, \( H_{\mu\nu} \) is antisymmetric, and all are real. The \( a_\mu, b_\mu, e_\mu, f_\mu, \) and \( g_{\mu\nu\lambda} \) terms break CPT, while those involving \( H_{\mu\nu}, c_{\mu\nu}, \) and \( d_{\mu\nu} \) preserve it. All of them are observer Lorentz covariant, but break particle Lorentz symmetry. Using a suitable field redefinition it is possible at first order to eliminate all the \( e_\mu \) and \( f_\mu \) terms and some of the \( g_{\mu\nu\lambda} \), so we set \( e_\mu, f_\mu, \) and \( g_{\mu\nu\lambda} \) equal to zero without any significant loss of generality \([4]\).

## 2 Symmetry Tests with Penning Traps

The Penning trap is a device that uses a uniform magnetic field and a quadrupole electric field to confine charged particles. The quantum behavior of the trapped particles can be studied with a high degree of precision and control. For example, it is possible to capture a single electron, positron, proton or antiproton and measure its motional frequencies over a period of several months. Two of these oscillation frequencies, which can be measured with precision better than a part in \( 10^8 \), are the cyclotron frequency \( \omega_c \) and the anomaly frequency \( \omega_a \). In the context of the standard-model extension, violations of Lorentz and CPT symmetry yield shifts of these frequencies. For an electron or positron, the leading-order shifts are

\[
\omega_c^e \approx \omega_c^+ \approx (1 - c_{00}^e - c_{11}^e - c_{22}^e) \omega_c ,
\]

\[
\omega_a^\mp \approx \omega_a \mp 2b_3^e + 2d_{30}^e m_e + 2H_{12}^e .
\]

In this notation, the superscript \( e^\pm \) refers to the positron or electron, and \( \omega_c^e, \omega_a^\pm \) represent the shifted frequencies. For other particles, for example protons and antiprotons, the expressions have appropriately modified superscripts.

### 2.1 Frequency-comparison tests

A category of Lorentz and CPT tests involves the comparison of frequencies that are equal in the conventional standard model of particle physics. Included among these are the cyclotron and anomaly frequencies of particles in Penning traps as compared with the corresponding antiparticle frequencies. In the standard-model extension, the electron-positron differences for the cyclotron and anomaly frequencies can be found from Eqs. (3) and (4):

\[
\Delta \omega_c^e \equiv \omega_c^e - \omega_c^e \approx 0 , \quad \Delta \omega_a^e \equiv \omega_a^e - \omega_a^e \approx -4b_3^e .
\] (4)
It follows that the dominant signal for CPT violation in Penning-trap experiments is a difference between the electron and positron anomaly frequencies. The $b_3$ coupling violates both Lorentz and CPT symmetry, so no leading-order contributions appear from CPT-preserving but Lorentz-breaking terms. Leading-order signals in cyclotron-frequency comparisons are suppressed in this context. A figure of merit for the test can be introduced as the ratio of a CPT-violating electron-positron energy-level difference and the basic energy scale \[ r_{\omega_a}^e \equiv \frac{|\mathcal{E}^{e-}_{n,s} - \mathcal{E}^{e+}_{n,-s}|}{\mathcal{E}^{e-}_{n,s}} . \] (5)

In this expression, $\mathcal{E}^{e-}_{n,s}$ and $\mathcal{E}^{e+}_{n,s}$ are energy eigenvalues of the full Penning-trap hamiltonians, with principal quantum numbers $n = 0, 1, 2, \ldots$ and spin $s = \pm 1$. For the nonrelativistic regime of relevance here, $\mathcal{E}^{e-}_{n,s}$ is essentially the rest mass $m_e$ and consequently Eq. (5) reduces to \[ r_{\omega_a}^e \approx \frac{|\Delta \omega_a^e|}{2m_e} \approx \frac{|2b_3^e|}{m_e} . \] (6)

One may estimate, for example, that if the anomaly frequencies were measured to an absolute precision of about 2 Hz, then a bound $r_{\omega_a}^e < 10^{-20}$ would be placed.

The Penning-trap group of Hans Dehmelt at the University of Washington in Seattle recently published a result based on this type of anomaly-frequency comparison [18]. A bound of \[ r_{\omega_a}^e < 1.2 \times 10^{-21} \] (7) was found from a reanalysis of earlier data for $g - 2$ experiments comparing single trapped electrons and single trapped positrons.

Previous CPT tests done with the Penning trap include comparisons of the gyromagnetic ratios of electrons and positrons. For example, one of the conventional figures of merit for CPT symmetry is \[ \left| \frac{g_+ - g_-}{g_{av}} \right| \lesssim 2 \times 10^{-12} . \] (8)

However, in the framework of the standard-model extension, CPT is broken without affecting the electron or positron gyromagnetic ratios. Thus, the theoretical value of the figure of merit in Eq. (8) would be zero even if CPT were broken, and this figure of merit is unsuitable in this theoretical context.

While it might appear from Eq. (4) that comparisons of cyclotron frequencies are insensitive to the CPT and Lorentz violations in the standard-model extension, this is in fact not entirely true. An experiment [17] by the group of Gerald Gabrielse at Harvard University compared the cyclotron frequencies of antiprotons and hydrogen ions and obtained a bound on a combination of Lorentz-violation couplings. This choice of ions was made to eliminate the difficulties of precisely reversing the electrode potentials when particles of opposite charge are loaded into
the trap [23]. Since the hydrogen ion $H^-$ and the antiproton both have negative charges, no electric-field reversal is necessary, and both particles can be simultaneously trapped. Established precision measurements of the electron mass and the $H^-$ binding energy can be used to estimate the theoretical value of the difference $\Delta \omega_{c}^{H^-} \equiv \omega_{c}^{H^-} - \omega_{c}^{\bar{p}}$ in conventional quantum theory. With these corrections for the two electrons in the $H^-$ ion, the experiment allows a comparison of the proton component of the $H^-$ ion with the antiproton.

In the context of the standard-model extension, this comparison of cyclotron frequencies is shifted at leading order by a combination of Lorentz-violating couplings [21]. A model-independent figure of merit

$$r_{\omega_c}^{H^-} \lesssim \left| \frac{\Delta \omega_{c}^{H^-}}{m_p} \right|$$

can be defined. One of the results of the Gabrielse experiment was the bound

$$r_{\omega_c}^{H^-} \lesssim 4 \times 10^{-26} .$$

Within the standard-model extension, this result limits a combination of Lorentz-violating, CPT-preserving couplings, including $c_{e00}^p$ and $c_{p00}^p$ which are not accessible in other similar experiments.

### 2.2 Sidereal-variation tests

The CPT- and Lorentz-violating couplings in the standard-model extension are constant vacuum expectation values of tensorial objects in a more fundamental theory. The physics of these couplings is approximately analogous to that of electrodynamics in macroscopic media [4]. Earthbound experiments sensitive to these minuscule couplings could seek to detect oscillations in experimental observables due to the rotation of the earth. These would be expected at various multiples of the earth’s sidereal frequency.

The conventional standard model predicts that the measured Penning-trap frequencies for an electron should remain constant provided the magnetic and electric fields remain constant. In the context of the standard-model extension, variations in the electron frequencies can be found from Eqs. (2) and (3) by noting that the indices in these expressions are given in the laboratory coordinate system that rotates against the fixed stars. A more complete discussion of tests of this type with the Penning trap is discussed in Ref. [21].

For a single electron in a Penning trap, the anomaly frequency $\omega_a^e$ is expected to have a variation with frequency equal to one sidereal day due to the index structure in Eq. (3). These indices are defined in terms of the magnetic field direction, which is fixed in the laboratory, but which rotates in the celestial equatorial coordinate system [12].
A model-independent figure of merit sensitive to the present effects may be defined in terms of the quantity [24]

\[ \Delta_{e/\omega_{e}} \equiv \frac{|E_{e/0+1} - E_{e/1-1}|}{E_{e/0-1}}, \tag{11} \]

which is essentially the ratio of the anomaly frequency to the rest mass of the electron. The amplitude of sidereal variations in this dimensionless quantity defines a figure of merit \( r_{e/\omega_{e}, \text{sidereal}} \) for this type of Lorentz-violating effect.

Data from an experiment confining a single electron in a Penning trap for several weeks have recently been reanalyzed by Mittleman of the Dehmelt trapping group. To search for sidereal variations, the data were partitioned into sidereal bins determined by the direction of the magnetic field. The bound obtained [19] is

\[ r_{e/\omega_{e}, \text{sidereal}} \leq 1.6 \times 10^{-21}. \tag{12} \]

In the present context, this constrains a combination of Lorentz-violating couplings, some of which also violate CPT.

3 Hydrogen and Antihydrogen

The hydrogen atom is one of the most studied systems in physics. Comparison of hydrogen (\( H \)) with antihydrogen (\( \bar{H} \)) requires the availability of antihydrogen atoms in quantities suitable for precision spectroscopy. As of October 2000, this is not a reality, although about a dozen events consistent with \( \bar{H} \) creation were reported in a 1995 experiment at CERN [25] and another dozen in a 1996 Fermilab experiment [26]. Current efforts by two experimental groups [27, 28] using the antiproton decelerator at CERN are underway to improve on these initial experiments and eventually create trapped antihydrogen for precision studies. Confinement would be within magnetic traps like the Ioffe-Pritchard trap [29].

An analysis of the spectra of \( H \) and \( \bar{H} \) in the context of the standard-model extension has been done for both free and trapped atoms [22].

3.1 Comparisons of hydrogen and antihydrogen

One of the spectral lines of importance is the two-photon 1S-2S transition because of its eventual expected measurement precision of a part in \( 10^{18} \). So far, relative precisions for this transition stand at a few parts in \( 10^{14} \) [30] for free hydrogen and a few parts in \( 10^{12} \) [31] for trapped hydrogen. The possible signals affecting the 1S-2S transition in free hydrogen in the context of the standard-model extension have been found to be suppressed by at least two powers of the fine-structure constant [22].

Turning to the analysis of trapped hydrogen and antihydrogen, it is useful to consider the case where the trap has a magnetic bias field \( B \) that splits the 1S and
2S levels into four hyperfine Zeeman levels, denoted in order of increasing energy by \(|a⟩_n, |b⟩_n, |c⟩_n, |d⟩_n\), with principal quantum number \(n = 1\) or \(2\), for both \(H\) and \(\overline{H}\). Only transitions involving the \(|c⟩\) and \(|d⟩\) are relevant because these are the two trapped states. For small values of the \(B\) field, transitions between the \(|d⟩_1\) and \(|d⟩_2\) states are field independent. So, by comparing the frequency \(ν^H_d\) for the 1S-2S transition \(|d⟩_1 \rightarrow |d⟩_2\) in \(H\) with the corresponding frequency \(ν^\overline{H}_d\) in \(\overline{H}\), effects due to magnetic-field instability and inhomogeneity would be minimized. However, the analysis again shows \(δν^H_d = δν^\overline{H}_d ≃ 0\) at leading order. There are no unsuppressed frequency shifts in this \(H\) transition or the corresponding \(\overline{H}\) transition.

An alternative would be consideration of the 1S-2S transition between the states \(|c⟩_1\) and \(|c⟩_2\) in \(H\) and \(\overline{H}\). In the present theoretical context, an unsuppressed frequency shift does indeed occur in this transition because the \(n\) dependence in the hyperfine splitting produces a spin-mixing difference between the 1S and 2S levels. The leading-order frequency shift is field dependent with a maximum at about \(B ≃ 0.01\) T. However, the strong field gradient at this value of \(B\) could severely limit the precision.

Another possibility for investigating CPT and Lorentz-violation in the context of the standard-model extension is to consider hyperfine transitions in the 1S ground state of \(H\). The analysis is done by considering the perturbative shifts in the energy levels of the relativistic \(H\) atom using the Dirac equation \([1]\). The CPT- and Lorentz-violating couplings give rise to field-dependent energy shifts of the \(|a⟩\) and \(|c⟩\) hyperfine levels and field-independent shifts of the \(|b⟩\) and \(|d⟩\) hyperfine levels in the 1S ground state of \(H\).

An interesting case is the \(|d⟩_1 \rightarrow |c⟩_1\) transition, also known as the \(F=1, Δm_F = ±1\) transition. While this transition can be measured at various frequencies, there is some advantage from a theoretical standpoint of selecting a magnetic field of about 0.65 T, since this minimizes suppression effects. At this field value, the leading-order difference in the frequencies \(ν^H_{c→d}\) and \(ν^\overline{H}_{c→d}\) is \(Δν_{c→d} ≃ ν^H_{c→d} − ν^\overline{H}_{c→d} ≃ −2b_3^c/π\). Within the context of the standard-model extension, this \(H-\overline{H}\) comparison isolates the CPT-violating coupling \(b_3\) for the proton and is therefore of interest as a clean CPT test. An appropriate model-independent figure of merit for any experimental comparison of this frequency in \(H\) and \(\overline{H}\) can be defined by \([22]\)

\[
 r_{rf,c→d}^H ≡ \frac{|(E^H_{1,d} - E^H_{1,c}) - (E^\overline{H}_{1,d} - E^\overline{H}_{1,c})|}{2π|Δν_{c→d}|/m_H},
\]

where the \(E\) denote relativistic energies for hydrogen and antihydrogen in the ground-state hyperfine level and where \(m_H\) is the atomic mass of \(H\). Assuming a frequency resolution of about 1 mHz would be possible with both species of particles, an upper bound of \(r_{rf,c→d}^H \lesssim 5 \times 10^{-27}\) can be estimated. The bound on the CPT- and Lorentz-violating coupling \(b_3\) would be \(|b_3^p| \lesssim 10^{-18}\) eV, an improvement of four orders of magnitude over bounds estimated for 1S-2S transitions.

Direct comparisons of frequencies in hydrogen with corresponding frequencies in antihydrogen are of course not possible until \(\overline{H}\) is readily available for spectroscopic
experiments.

3.2 Sidereal-variation tests in Hydrogen

The properties of the couplings in the standard-model extension mean that frequencies such as the ground-state hyperfine transition $\Delta \nu_{c \rightarrow d}$ in $H$ should have small variations due to the sidereal rotation of the earth. Thus interesting bounds can be placed on certain combinations of couplings using only hydrogen.

Such an experiment, searching for sidereal variations in the $F = 1$, $\Delta m_F = \pm 1$ transition of a $H$ maser has recently been completed at the Harvard-Smithsonian Center for Astrophysics [32]. The maser was run with a weak bias magnetic field of 0.6 mG, for which the corresponding Zeeman frequency is about 850 Hz. A double resonance technique was used to monitor this frequency, with a resolution of about 0.37 mHz. This bounds sidereal variations at the level of $1.5 \times 10^{-27}$ GeV. In the context of the standard-model extension, the parameters bounded here are a combination of electron and proton parameters,

$$|\tilde{b}_p^J + \tilde{b}_e^J| \leq 2 \pi \delta \nu_Z ,$$

where $\tilde{b}_J = b_J - m_e d_{0, J} - \frac{1}{2} \epsilon_{JKL} H_{KL}$ for both superscripts, and $\delta \nu_Z$ is the sidereal-frequency modulation of the Zeeman frequency [12].

A bound of $10^{-29}$ GeV has independently been placed on the electron parameter $\tilde{b}_e^J$ using a spin-polarized torsion pendulum [15]. Here, $J$ refers to spatial components in the nonrotating celestial coordinate system. This result was obtained from a reanalysis of data from the Eöt-Wash II experiment conducted at the University of Washington. Combining the hydrogen maser result mentioned above and this tight bound on the electron parameter, it can be inferred that the hydrogen maser experiment places the bound

$$\tilde{b}_p^J \leq 10^{-27} \text{GeV} .$$

4 Related tests

Clock-comparison tests have been used to study Lorentz symmetry and have resolutions of less than a $\mu$Hz, several orders of magnitude better than for the hydrogen-maser system. However, analysis of effects within the framework of the standard model extension is far more complex than for hydrogen, and has to rely on various nuclear models [12]. In comparison, the parameter combination $\tilde{b}_p^J$ bounded in the hydrogen-maser experiment is considerably cleaner than other comparable bounds from clock-comparison experiments, such as for the $^{199}$Hg/$^{133}$Cs system [12, 33].

An experiment with a dual-species $^{129}$Xe/$^3$He maser has recently placed a limit on a combination of CPT- and Lorentz-violating parameters within the standard-model extension [34]. With a resolution of about 45 nHz, the bound is

$$\tilde{b}_X^1 \equiv \sqrt{(\tilde{b}_X^p)^2 + (\tilde{b}_X^p)^2} = (4.0 \pm 3.3) \times 10^{-31} \text{GeV} ,$$

$$\tilde{b}_X^0 = (4.0 \pm 3.3) \times 10^{-31} \text{GeV} .$$
consistent with no Lorentz- and CPT-violating effects under reasonable statistical assumptions. This result, obtained in Walsworth’s laboratory at the Harvard-Smithsonian Center for Astrophysics, improves on the tightest previous limits \cite{12} for the CPT- and Lorentz-violating couplings of the neutron by a factor of more than six. Indications are that an improvement of about an order of magnitude will be possible with further refinements. In addition, a new experiment under development using a two-species $^{21}\text{Ne}/^{3}$He maser \cite{35} is expected to improve the resolution by a further order of magnitude.

Also of interest are recent bounds on Lorentz symmetry from neutrino-oscillation investigations \cite{36}.

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