Enabling Long mmWave Aerial Backhaul Links via Fixed-Wing UAVs: Performance and Design

Mohammad Taghi Dabiri, Mazen Hasna, Senior Member, IEEE, Nizar Zorba, Senior Member, IEEE, Tamer Khattab, Senior Member, IEEE, and Khalid A. Qaraqe, Senior Member, IEEE

Abstract—We propose a fixed-wing unmanned aerial vehicles (UAV)-based millimeter wave (mmWave) backhaul architecture that is offered as a cost-effective and easy to deploy solution, to connect a disaster or remote area to the nearest core network. First, we fully characterize the single relay fixed-wing UAV-based communication system by taking into account the effects of realistic physical parameters, such as the UAV’s circular path, critical points of the flight path, heights and positions of obstacles, flight altitude, tracking error, the severity of UAV’s vibrations, the real 3D antenna pattern, mmWave atmospheric channel loss, temperature and air pressure. Second, we derive the distribution of the signal-to-noise ratio (SNR) metric, which is based on the sum of a series of Dirac delta functions. Using the SNR distribution, we derive analytical expressions for the outage probability and the ergodic capacity of the considered system as a function of all system parameters. To provide an acceptable quality of service for longer link lengths, we extend the analytical expressions to a multi-relay system. The accuracy of the analytical expressions are verified by Monte-Carlo simulations. Finally, by providing sufficient simulation results, we investigate the effects of key channel parameters such as antenna pattern gain and flight path on the performance of the considered system; and we carefully analyze the relationships between those parameters in order to maximize the average channel capacity.

Index Terms—Antenna pattern, backhaul links, positioning, mmWave communication, unmanned aerial vehicles (UAVs), fixed-wing UAVs.

I. INTRODUCTION

CLIMATE change has been the main cause for severe storms, flooding and hurricanes in the recent years. Over the past three decades, Europe has seen a sixty percent increase in extreme weather events [2]. Moreover, in the past three years, the average number of billion-dollar disasters in the United States was more than double the long-term average [3]. Parts of the world that have never experienced severe weather should be ready and plan for it now, while those who are more used to extreme natural events should be prepared for more [2]. One of the essential needs during and after a disaster event is providing a reliable connection link quickly to facilitate rescue operations, as well as to provide internet connectivity to the people escaping from the affected area [4]. Therefore, immediate and cost efficient high throughput solutions must be considered after natural disasters even better and more ubiquitous for 5G evolution and beyond.

Natural disasters such as earthquakes, hurricanes, tornadoes, floods, and other geologic processes can potentially cut or entirely destroy fiber infrastructure to the disaster area. Any disruption to the fragile fiber causes data disconnections that take days to find and repair. In addition, providing an alternative terrestrial wireless backhaul connectivity encounters serious challenges, including creating a line of sight (LoS) between the disaster area to the nearest core network, especially in forest and mountainous areas [5]. Due to their unique capabilities such as flexibility, maneuverability, and adaptive altitude adjustment, unmanned aerial vehicles (UAVs) acting as networked flying platforms (NFPs) can be considered as a promising solution to provide a temporary wireless backhaul connectivity while improving reliability of backhaul operations [6], [7], [8], [9]. More recently, millimeter wave (mmWave) backhauling has been proposed as a promising approach for aerial communications due to multiple reasons. First, unlike terrestrial mmWave communication links that suffer from blockage, the flying nature of UAVs offers a higher probability of LoS between communication nodes. In addition, the large available bandwidth at mmWave frequencies can provide high data rate in point-to-point aerial communication links, as needed for the backhaul communications. Moreover, to mitigate the negative effects of the high path-loss at the mmWave bands, the small wavelength enables the realization of a compact form of highly directive antenna arrays, which are suitable for flying platforms with limited payload. It is worth noting that, the cost of licensing mmWave spectrum is much lower than sub-6 GHz frequencies. Finally, the utilization of narrow beam high gain antennas at mmWave frequencies enhances the possibility of frequency reuse compared to sub-6 GHz frequencies.

A. Literature Review and Motivation

Recently, UAV-based mmWave backhaul links have been studied in few works [10], [11], [12], [13], [16], [17], [18], [19], [20], [21], [22], [23]. For instance, a 3D two-hop scheme is proposed in [10] and [11] wherein a user is connected.
to a base station (BS) by using a UAV-based backhaul link. In particular, the authors studied the performance of the considered network in both amplify-and-forward (AF) and decode-and-forward (DF) relaying protocols by considering realistic antenna radiation patterns for both BSs and UAVs based on practical models developed by 3GPP. A novel wireless backhaul link is introduced in [12] and [13] by installing reconfigurable intelligent surface (RIS) on high altitude UAVs to handle any sudden increase in traffic in an urban area. In [14] and [15], the authors investigated the use of aerial relay nodes to provide flexible and reconfigurable backhaul architecture by considering the effects of multipath propagation and dynamic link blockage in mmWave frequency bands. In [16], the achievable rate of the UAV-based mmWave wireless backhaul link is investigated and then, the authors analyzed the minimum cache hit probability to achieve a certain backhaul rate requirement. The UAV-BS location and bandwidth allocation problems is studied in [17] to maximize the throughput without exceeding the backhaul and access capacities. A novel spectrum management architecture for UAV-assisted mmWave networks is proposed in [18] to overcome the problem related to the spatio-temporal distribution of the wireless network traffic. In particular, with numerical results, the authors studied the performances of the proposed spectrum management for mmWave based backhaul in five different scenarios. The performance of UAV-based mmWave backhaul link is investigated in [19] and [20] where UAVs are equipped with linear and square array antennas. More recently, a fast algorithm for 3D optimal placement of rotary-wing UAVs is proposed in [21] to provide a long mmWave backhaul link. In [23], the authors studied a UAV-aided low latency mobile edge computing network with mmWave backhauling. However, the results of those studies are limited for rotary-wing UAVs.

Rotary-wing UAVs are used in cases where more maneuverability is required, for example, to provide internet services in crowded urban areas [24], [25]. To keep the rotary-wing UAVs stable in the air, its motors are required to individually speed up or slow down its propellers, which can be power consuming, mainly due to UAV inertia. Moreover, scaling the rotary-wing UAV up to a larger size faces major challenges because more energy is needed to change the speed of larger propellers. They also face restrictions on payload, altitude, and shorter flight times [26], [27]. Being able to fly for longer times, at higher altitudes, and with heavier payloads than rotary-wing UAVs are the greatest advantages of fixed wing UAVs [28]. All these characteristics make them suitable for remote or disaster area applications. Based on the results of [19] and [20], to design an aerial mmWave backhaul link based on a rotary-wing UAV, it is needed to find an optimal point in 3D space relative to the ground transmitter and receiver. However, fixed wing UAVs cannot hover or make sharp turns, and thus, the results of the aforementioned works are not directly applicable for fixed wing UAVs.

Finally, note that although fixed-wing UAV communications have been the subject of many works (see [29] and the references therein), most of them are to provide access links for users at lower frequencies, and to the best of our knowledge, a comprehensive study on the designing of the communication link based on fixed-wing UAVs seems necessary.

B. Contributions and Paper Structure

In this study, we consider a mmWave backhaul link based on fixed-wing UAVs, as shown in Fig. 1, that is offered as a cost effective and easy to deploy solution to connect a disaster or remote area to the nearest core network. Performance analysis of the considered fixed-wing UAV-based communication system are the main contributions of this work by taking into account important realistic parameters. Our detailed contributions are summarized as follows:

- We fully characterize the performance of single relay fixed-wing UAV-based communication systems by taking into account the effects of realistic physical parameters, such as the UAV’s circular path, critical points of the flight path, heights and position of obstacles, flight altitude, tracking error, severity of UAV’s vibrations, real 3D antenna pattern, mmWave atmospheric channel loss, temperature and air pressure.
- We derive the distribution of the signal-to-noise ratio (SNR), showing that the distribution of the end-to-end (E2E) SNR corresponds to the sum of a series of Dirac delta functions. Then, we derive analytical expressions for the outage probability and channel capacity of the considered system, as a function of the considered practical system parameters. The accuracy of the analytical expressions is verified with the results obtained from Monte-Carlo simulations.
- Within some scenarios, the use of single relay UAV will not be enough to guarantee the requested quality of service (QoS) for longer links. To provide an acceptable QoS for longer link length, we analyze the performance of the considered multi-relay system by providing several analytical expressions for outage probability and channel capacity under different scenarios.
- Finally, through extensive simulation results, we show the effects of key channel parameters such as antenna pattern gain and optimal flight path on the performance of the considered system, and we carefully study the relationships between those parameters in order to maximize the channel capacity.
Fig. 2. An illustration of the coordinated flight of the UAVs in a circular path so that the distance between the UAVs is constant and equal to $L_{uu}$ during the whole circular flight path.

The rest of this paper is organized as follows. We introduce the channel model of a fixed-wing UAV-based mmWave backhaul link in Section II. Analytical derivations along with the performance analysis of the considered system, in terms of the channel capacity and the outage probability are provided in Section III. Using the numerical and simulation results, we study the optimal parameters design of the considered system in section IV. Finally, conclusions and future road map are drawn in Section V.

II. THE SYSTEM MODEL

We consider a fixed-wing UAV acting as an NFP node in order to relay data from the nearest core network to the disaster or remote area, where $L_{ad}$ shows the distance between core network and remote area. First, we assume that the backhaul link is relayed to the remote area by one fixed-wing UAV. However, we will show that for longer values of $L_{ad}$, using only one UAV can not provide a desired QoS. Therefore, in the second part of this work, a relay system based on two or multiple fixed-wing UAVs is studied.

A. Single Relay System

We consider that a fixed-wing UAV rotates in a circular path with center $B_p$ and diameter $L_u$ as depicted in Fig. 1a because the circular motion approach with moderate speed could be preferable in terms of energy consumption [30]. Point $B_p$ shown in Fig. 1a is the closest and farthest point to the core network and remote area, respectively. On the other hand, $B_p$ is the farthest and closest point to the core network and remote area, respectively. Let $\psi_{s,min}$ and $\psi_{d,min}$ denote the minimum elevation angles of core network and remote area, respectively, $L_s$ represents the link length from core network to UAV (CU), $L_d$ denotes the link length between UAV and remote area, respectively, $A_s$ and $A_{us}$ as well as antennas $A_d$ and $A_{ud}$ try to adjust the direction of their antennas to each other. At first it may seem that by increasing the number of antenna elements, which leads to an increase in antenna gain, the system performance improves. However, in practical situations, increasing the antenna gain makes the system more sensitive to antenna misalignment. A change in the instantaneous speed and acceleration of the fixed-wing UAV, an error in the mechanical control system of UAV, mechanical noise, position estimation errors, tracking system error, air pressure, and wind speed can cause an alignment error between the antennas [31], [32], as graphically illustrated in Fig. 3a. Therefore, the alignment error is caused by the sum of a large number of independent errors, which

Table I

| Parameter | Description |
|-----------|-------------|
| $v \in \{ tx \}$ | This subscript is used to specify Tx and Rx antennas |
| $q \in \{ s,d \}$ | This subscript is used to specify $A_s$ and $A_d$ |
| $w \in \{ x,y \}$ | This subscript is used to specify $x$ and $y$ axes |
| $A_{us}$ | The NFP antenna directed toward $A_s$ |
| $A_{ud}$ | The NFP antenna directed toward $A_d$ |
| $A_s$ | Antenna of core network |
| $A_d$ | Antenna of remote area |
| $P_s$ | Transmitted power of $A_s$ |
| $P_d$ | Transmitted power of $A_d$ |
| $N_{auw}$ | Number of antenna elements of $A_{au}$ in $w_a$ axis |
| $N_{dw}$ | Number of antenna elements of $A_{du}$ in $w_d$ axis |
| $\theta_{aw}$ | Instantaneous misalignment of $A_{aw}$ in $w_a-z_a$ plane |
| $\theta_{dw}$ | Instantaneous misalignment of $A_{dw}$ in $w_d-z_a$ plane |
| $\mu_{aw}$ | Mean (offset) of RV $\theta_{aw}$ |
| $\mu_{dw}$ | Mean (offset) of RV $\theta_{dw}$ |
| $\sigma_{aw}$ | Variance of RV $\theta_{aw}$ |
| $\sigma_{dw}$ | Variance of RV $\theta_{dw}$ |
| $\lambda$ and $f_c$ | Wavelength and carrier frequency, respectively |
| $B_{p1}$ | The farthest and closest point to $A_s$ and $A_d$ |
| $B_{p2}$ | The farthest and closest point to $A_s$ and $A_d$ |
| $B_p$ | The center of UAV circular path |
| $\psi_{s,min}$ | Minimum elevation angle |
| $H_s$ | Height of NFP |
| $L_a$ | Link length of $A_{au}$ to $F_{pu}$ |
| $L_d$ | Horizontal distance between $A_s$ and $A_d$ |
| $L_{ad}$ | Horizontal distance between $A_{ad}$ and point $B_p$ |
| $L_{ud}$ | Diameter of the UAV circular flight path |
| $\theta_{R1}$ | Determines the UAV’s position in a circular path |

Multiple UAVs

$B_{p1}$ The center of first UAV circular path
$B_{p2}$ The center of second UAV circular path
$A_{au,i}$ The $i$th UAV antenna directed toward $A_{ad,i}$
$A_{ud,i}$ The $(i+1)$th UAV antenna directed toward $A_{ad,i}$
$N_e,i$ Number of antenna elements of $A_{au,i}$
$N_e,i$ Number of antenna elements of $A_{ud,i}$
$L_{aw}$ Distance between UAVs’ circular paths shown in Fig. 1b
$L_{au}$ Inter UAVs link length
$\theta_{aw,i}$ Instantaneous misalignment of $A_{aw,i}$ in $w_a-z_a$ plane
$\theta_{dw,i}$ Instantaneous misalignment of $A_{dw,i}$ in $w_d-z_a$ plane
$\mu_{aw,i}$ Mean (offset) of RV $\theta_{aw,i}$
$\mu_{dw,i}$ Mean (offset) of RV $\theta_{dw,i}$
$\sigma_{aw,i}$ Variance of RV $\theta_{aw,i}$
$\sigma_{dw,i}$ Variance of RV $\theta_{dw,i}$
$M$ Number of UAVs
$\tau_{au}$ Outage threshold
$\tau_{snr}$ SNR threshold

and $A_{ad}(N_{adx} \times N_{ady})$ denotes the array antenna of NFP directed toward the remote area, respectively. Antennas $A_s$ and $A_{us}$ as well as antennas $A_d$ and $A_{ud}$ try to adjust the direction of their antennas to each other. At first it may seem that by increasing the number of antenna elements, which leads to an increase in antenna gain, the system performance improves. However, in practical situations, increasing the antenna gain makes the system more sensitive to antenna misalignment. A change in the instantaneous speed and acceleration of the fixed-wing UAV, an error in the mechanical control system of UAV, mechanical noise, position estimation errors, tracking system error, air pressure, and wind speed can cause an alignment error between the antennas [31], [32], as graphically illustrated in Fig. 3a. Therefore, the alignment error is caused by the sum of a large number of independent errors, which

1 The minimum elevation angle is the minimum angle required to establish a LoS between the ground node and the nearest UAV.
is expected to have a normal distribution based on the central limit theorem. Let \( \theta_w \sim N(\mu_w, \sigma_w^2) \) be the instantaneous misalignment angle of \( A_q \) in \( w_q - z_q \) plane, where \( q \in \{s,d\} \) and \( w \in \{x,y\} \). Similarly, \( \theta_{quw} \sim N(\mu_{quw}, \sigma_{quw}^2) \) is assumed to be the instantaneous misalignment angle of \( A_uq \) in \( w_u - z_u \) plane.

### B. Multiple Relay System

For the longer values of \( L_{sd} \), we must increase the number of UAVs acting as relays to satisfy the required QoS along the entire circular flight path. The topology of a system with two UAVs is shown in Fig. 1b. For notational simplicity, all variables defined for a single-relay system are also valid for a multiple-relay system, except for few variables that are redefined below.

The mmWave signal first points to the first fixed-wing UAV, which is decoded and forwarded to the second UAV. Similarly, after receiving the signal, the second UAV relay decodes and forwards it to the remote area. Both fixed-wing UAVs rotate in a circular path with diameter \( L_{u1} \) as depicted in Figs. 1b and 2. The center of the circular path for the first and second UAVs are \( B^P_p \) and \( B^P_u \), respectively. Both UAVs fly at the same position, at the same speed and in the same direction in a circular path, so that the link length between the two UAVs remains constant along the entire circular flight path, as shown in Fig. 2. The considered link length between the two UAVs is \( L_{uu} = L_{u1} + L_{u2} \), where \( L_{u2} \) is the distance between two circular paths. Note that the parameter \( L_{u2} \) is a tunable parameter and has a significant effect on the performance of the considered system and thus, finding an optimal value for it is very important.

As shown in Fig. 3b, the two relay system consists of six mmWave array antennas. Four of the antennas are similar to the single relay system and the other two antennas are related to UAV-to-UAV (UU) link. We consider that \( z_u \) represents the propagation axis between the UAVs while axes \( x_u \) and \( y_u \) represent the array antenna plane perpendicular to the propagation axis. As shown in Fig. 3b, let \( A_{u1}(N_{u1} \times N_{u1}) \) denotes the first UAV antenna directed toward the second UAV which is characterized by \( N_{u1} \times N_{u1} \). Also, \( A_{uu+1}(N_{u1+1} \times N_{u1+1}) \) represents the second UAV antenna directed toward the first UAV which is characterized by \( N_{u1+1} \times N_{u1+1} \). Antennas \( A_{u1} \) and \( A_{uu+1} \) try to adjust the direction of their beams to each other, where \( i \in \{1, \ldots, M \} \) and \( M \) is the number of UAV relays. Let \( \theta_{uuw, i} \sim N(\mu_{uuw, i}, \sigma_{uuw, i}^2) \) be the instantaneous misalignment angle of \( A_uu \) in \( w_u - z_u \) plane, \( \theta_{uuw, i+1} \sim N(\mu_{uuw, i+1}, \sigma_{uuw, i+1}^2) \) be the instantaneous misalignment angle of \( A_{uu+1} \) in \( w_u - z_u \) plane.

For a multi relay system with \( M \) UAVs, we have \( M = 1 \) inter-UAV links. Depending on the type of DF relays used, as well as the symmetry of the inter-UAV links, the optimal parameter values of all links must be the same.

### C. Channel Propagation Loss

In normal atmospheric conditions, water vapor (H\(_2\)O) and oxygen (O\(_2\)) molecules are strongly absorptive of radio signals, especially at mmWave frequencies and higher. The resulting attenuation is in excess of the reduction in radiated signal power due to free-space loss. Channel loss (in dB) is usually expressed as

\[
h_{L,db}^o(f_c) = 20 \log \left( \frac{4\pi L}{\lambda} \right) + h_{L,db}^{aw}(f_c),
\]

where \( L \) is the link length (in m), \( \lambda \) is the wavelength (in m), \( f_c \) is mmWave frequency (in GHz), \( h_{L,db}^{aw}(f_c) = \frac{h_{L,db,km}^{aw}(f_c)/L}{1000} \) is the total attenuation due to oxygen and water (in dB), \( h_{L,db,km}^{aw}(f_c) = h_{L,db,km}^{o}(f_c) + h_{L,db,km}^{w}(f_c) \) is the attenuation per km due to oxygen and water (in dB/km). At 20\(^\circ\)C surface temperature and at sea level, approximate expressions for the attenuation constants of oxygen and water vapor (in dB/km) as defined by the International Telecommunications Union (ITU) are [33]:

\[
h_{L,db,km}^{o}(f_c) = 0.001 \times f_c^2 \\
\times \left\{ \begin{array}{ll}
6.09 & \text{for } f_c < 57 \\
4.81 & \text{for } 57 \leq f_c < 63 \\
4.13 & \text{for } 63 \leq f_c < 350 \\
0.19 & \text{for } \frac{f_c}{(f_c - 118.7)^2 + 2} \\
& \text{for } f_c < 350,
\end{array} \right.
\]

and

\[
h_{L,db,km}^{w}(f_c) = 0.0001 \times f_c^2 \rho_0 \left( \frac{3.6}{(f_c - 22.2)^2 + 8.5} + \frac{10.6}{(f_c - 38.3)^2} + 9 + \frac{8.9}{(f_c - 325.4)^2 + 26.3} \right),
\]

where \( \rho_0 = 7.5 \text{ g/m}^3 \) is the water vapor density at sea level, and \( h_{L,db,km}^{w}(f_c = 57) \) is the value of the first expression at \( f_c = 57 \) GHz. In general, the attenuation constants of oxygen and water vapor are functions of altitude, since they depend on factors such as temperature and pressure. These quantities are often assumed to vary exponentially with respect to height \( H \), as \( \rho(H) = \rho_0 \exp(-H/H_{\text{scale}}) \) where \( H_{\text{scale}} \) is known as the scale height, which is typically 1-2 km. From this, the specific attenuation as a function of height can be approximately modeled as

\[
h_{L,db,km}^{aw}(f_c, H) = h_{L,db,km}^{aw}(f_c) \exp(-H/H_{\text{scale}}).
\]

In our system model, both CU and UD links are slant. For a slant atmospheric path from height \( H_1 \) to height \( H_2 \) at
an angle $\psi$, the total atmospheric attenuation is obtained by integration from (4) as
\[
h^{\circ,w}_{\text{L, dBkkm}}(f_c) \approx \frac{h^{\circ,w,0}_{\text{L, dBkkm}}(f_c)}{\sin(\psi)} \left( e^{-H_1/H_{\text{scale}}} - e^{-H_2/H_{\text{scale}}} \right) H_s.
\]

(5)

D. 3D Antenna Pattern

Nowadays, advances in the fabrication of antenna array technology at mmWave bands allow the creation of large antenna arrays, with high antenna pattern gain in a cost effective and compact form, in order to compensate the negative effects of high propagation attenuation at mmWave bands. Antenna arrays at mmWave allow the creation of large 3D beamwidths, respectively, where the maximum directional gain of the antenna element, $G_{e,\text{max}}/10$ of each single antenna element is obtained as [34]

\[
\begin{align*}
G_{e,\text{max}} &= \max \left\{ -\left( G_{e,\text{MB},1} + G_{e,\text{MB},2} \right), F_m \right\}, \\
G_{e,\text{MB},1} &= -\min \left\{ -12 \left( \frac{\phi_{\text{MB}} - 90}{\psi_{\text{MB}}} \right)^2, G_{\text{SL}} \right\}, \\
G_{e,\text{MB},2} &= -\min \left\{ -12 \left( \frac{\theta_{\text{MB}}}{\psi_{\text{MB}}} \right)^2, F_m \right\}, \\
\theta_e &= \tan^{-1} \left( \frac{1+\sin^2(\phi_{\text{MB}})}{\sin(\psi_{\text{MB}})} \right),
\end{align*}
\]

(13)

where $\theta_{\text{MB}} = 65^\circ$ and $\phi_{\text{MB}} = 65^\circ$ are the vertical and horizontal 3D beamwidths, respectively. $G_{e,\text{max}}$ is the maximum directional gain of the antenna element, $F_m = 30$ dB is the front-back ratio, and $G_{\text{SL}} = 30$ dB is the side-lobe level limit.

If the amplitude excitation of the entire array is uniform, then the array factor $G_a(\theta_x, \theta_y)$ for a square array of $N \times N$ elements can be obtained as [35, eqs. (6.89) and (6.91)]

\[
G_a(\theta_x, \theta_y) = \frac{\sin \left( \frac{N(kd_x \sin(\theta_x) \cos(\phi) + \eta_x)}{2} \right) \sin \left( \frac{N(kd_y \sin(\theta_y) \sin(\phi) + \eta_y)}{2} \right)}{N \sin \left( \frac{N(kd_x \sin(\theta_x) \cos(\phi) + \eta_x)}{2} \right) \sin \left( \frac{N(kd_y \sin(\theta_y) \sin(\phi) + \eta_y)}{2} \right)},
\]

(8)

where $\eta_x$ and $\eta_y$ are progressive phase shifts between the elements along the $x$ and $y$ axes, respectively. For a fair comparison between antennas with different $N$, we assume that the total radiated power of antennas with different $N$ are the same. From this, we have

\[
G_0(N) = \left( \int_0^\pi \int_0^{2\pi} G'(\theta, \phi) \sin(\theta)d\theta d\phi \right)^{-1}.
\]

(9)

More details on the elements and array radiation pattern are provided in [34] and [35]. In addition, and without loss of generality, it is assumed that $\eta_x = \eta_y = 0$.

III. PERFORMANCE ANALYSIS

A. Single UAV Relay

For a given region with physical parameters such as air pressure, temperature, $\psi_{\text{min}}$, $\psi_{\text{d,min}}$, and $H_{\text{sd}}$, our aim is to adjust the tunable system parameters such as $N_{\text{az}}$, $N_{\text{ay}}$, $N_{\text{az}}$, $N_{\text{ay}}$, $N_{\text{daz}}$, $N_{\text{daz}}$, $H_{\text{amb}}$ and $L_{\text{sc}}$, to improve the system performance in terms of average capacity and outage probability. These two metrics are very important in the design of wireless communication systems. Our objective is to maximize the channel capacity with the outage probability as a constraint (it is less than a threshold, i.e., $P_{\text{out}} < P_{\text{out,th}}$, where $P_{\text{out,th}}$ is determined based on the requested QoS). Our optimization problem is formulated as:

\[
\begin{align*}
\max & \quad \bar{C}_{e,2e} \\
\text{s.t.} & \quad P_{\text{out}} < P_{\text{out,th}} \\
& \quad H_{u} > H_{u,\text{min}},
\end{align*}
\]

(10a)

where $\bar{C}_{e,2e}$ is the average channel capacity during the UAV flight time. Constraint (10c) is used to guarantee that the UAV is in the LoS of both core network and remote area throughout the entire flight path. Therefore, the minimum height of the UAV should be

\[
H_{u,\text{min}} = \max \left\{ \left( L_{\text{sc}} + \frac{L_{u,1}}{2} \right) \sin(\psi_{\text{min}}), \left( L_{\text{dc}} + \frac{L_{u,1}}{2} \right) \sin(\psi_{\text{d,min}}) \right\},
\]

(11)

to ensure it satisfies the LoS for both links. In (11), we have

\[
L_{\text{sc}} = \sqrt{L_{\text{sc},\min}^2 - H_{u}^2 + \frac{L_{u,1}^2}{4}}, \quad L_{\text{dc}} = \sqrt{L_{\text{dc},\min}^2 - H_{u}^2 + \frac{L_{u,1}^2}{4}},
\]

where $L_{u,\min}$ is the link length between core network and $B_{p1}$, while $L_{d,\min}$ is the link length between remote area and $B_{p2}$. We consider that the points $B_{p1}$, $B_{p1}$, and $B_{p2}$ are in $[x, y] = [0, 0], \left[ \frac{L_{u,1}}{2}, 0 \right]$, and $[-\frac{L_{u,1}}{2}, 0]$, respectively. Let $R_{1}$ indicates the path of a semicircle that starts from point $B_{p1}$ and reaches point $B_{p2}$. Therefore, each point on $R_{1}$ in the $[x - y]$ plane is specified as follows

\[
x_u = \frac{L_{u,1}}{2} \cos(\theta_{R1}), \quad y_u = \frac{L_{u,1}}{2} \sin(\theta_{R1}),
\]

(12)

where $0 < \theta_{R1} < \pi$. From this, the average channel capacity can be formulated as

\[
\bar{C}_{e,2e} = \frac{1}{\pi} \int_{\theta_{R1}=0}^\pi C_{e,2e}(\theta_{R1}) d\theta_{R1},
\]

(13)
where $C_{e2|\theta_{R1}}$ is the average end-to-end channel capacity conditioned on $\theta_{R1}$. For our system model, $C_{e2|\theta_{R1}}$ is a function of RVs $\theta_{xa}$, $\theta_{ya}$, $\theta_{ax}$, $\theta_{ay}$, $\theta_{dx}$, $\theta_{dy}$, $\theta_{adx}$, and $\theta_{ady}$, and is obtained as in (14), shown at the bottom of the page.

As in (16), shown at the bottom of the page, we have
\[
\theta_1 = \tan^{-1}\left(\sqrt{\tan^2(\theta_{dx}) + \tan^2(\theta_{dy})}\right) \quad \text{and} \quad \theta_q = \tan^{-1}\left(\sqrt{\tan^2(\theta_{ax}) + \tan^2(\theta_{ay})}\right).
\]
Moreover, $L_a$ and $L_d$ are functions of $H_u$, $L_{sc}$, and $\theta_{R1}$ as
\[
L_a = \sqrt{\left(L_{sc} - \frac{L_{al}}{2}\cos(\theta_{R1})\right)^2 + \frac{L_{al}^2}{4}\sin^2(\theta_{R1}) + H_a^2},
\]
\[
L_d = \sqrt{\left(L_{dc} + \frac{L_{dl}}{2}\cos(\theta_{R1})\right)^2 + \frac{L_{dl}^2}{4}\sin^2(\theta_{R1}) + H_d^2}.
\]

As can be seen, calculating the channel capacity from Eqs. (14)-(16) (15), shown at the bottom of the page), requires solving a 9-dimensional integral equation numerically, which is very time consuming. In order to analyze and design the system parameters optimally, it is necessary to provide more tractable and well-formed analytical expressions for performance metrics as a function of channel parameters. Therefore, in the following, we first present the distribution of the end-to-end SNR and then use it to calculate the performance metrics such as outage probability and channel capacity.

**Theorem 1:** The distribution of the end-to-end SNR conditioned on $\theta_{R1}$ is derived as
\[
\begin{align*}
J_{\gamma_1|\theta_{R1}}(\gamma_q|\theta_{R1}) &= \sum_{j_q=1}^{K_q} \sum_{j_a=1}^{K_a} T_q(j_q, N_{qz}) T_a(j_a, N_{az}) \\
& \times \delta\left(\gamma_q - \gamma_q'(j_q, j_a)\right),
\end{align*}
\]
where $\delta(\cdot)$ is Dirac delta function, $T_q(j_q, N_{qz})$ is derived as in (20), shown at the bottom of the next page, $\gamma_q'(j_q, j_a) = \gamma_q''(j_q, j_a) b_L(L_q(\theta_{R1}, L_{sc}, H_u))$, and
\[
\begin{align*}
\gamma_q''(j_q, j_a) &= \frac{10^{G_L/5} G_0(N_{qz}, N_{az}) G_0(N_{az}, N_{aqz}) P_{aq} N_{qz} N_{az}^2}{\sigma_n^2} \\
& \times \left(\frac{\sin\left(\frac{\theta_{ax}}{2}\right)}{\sin\left(\frac{\theta_{ay}}{2}\right)}\right)^2 \left(\frac{\sin\left(\frac{\theta_{dx}}{2}\right)}{\sin\left(\frac{\theta_{dy}}{2}\right)}\right)^2.
\end{align*}
\]

As can be seen, the closed-form expression provided in (18) is very simple and calculates the distribution of end-to-end SNR conditioned on $\theta_{R1}$ based on the sum of a series of Dirac delta functions. The Dirac delta function arises from

\[
\begin{align*}
C_{e2|\theta_{R1}} &= \frac{1}{16 \pi^2 \sigma_x \sigma_y \sigma_{ax} \sigma_{ay} \sigma_{dx} \sigma_{dy} \sigma_{adx} \sigma_{ady}} \iint_{\theta_{R1}} \frac{\exp\left(-\frac{(\theta_{ax} - \mu_{ax})^2}{2\sigma_{ax}^2}\right)}{2\pi \sigma_{ax}} \exp\left(-\frac{(\theta_{ay} - \mu_{ay})^2}{2\sigma_{ay}^2}\right) \exp\left(-\frac{(\theta_{dx} - \mu_{dx})^2}{2\sigma_{dx}^2}\right) \exp\left(-\frac{(\theta_{dy} - \mu_{dy})^2}{2\sigma_{dy}^2}\right) \\
& \times d\theta_{ax} d\theta_{ay} d\theta_{dx} d\theta_{dy} d\theta_{adx} d\theta_{ady} \left(\frac{1}{\theta_{R1}}\right)^{\pi/2} \left(\frac{1}{\theta_{R1}}\right)^{\pi/2} \left(\frac{1}{\theta_{R1}}\right)^{\pi/2} \left(\frac{1}{\theta_{R1}}\right)^{\pi/2} \left(\frac{1}{\theta_{R1}}\right)^{\pi/2} \\
& \times C_{e2|\theta_{R1}}' \min\left\{\log_2 \left(1 + \gamma_1(\theta_{ax}, \theta_{ay}, \theta_{dx}, \theta_{dy}, \theta_{adx}, \theta_{ady}|\theta_{R1})\right), \log_2 \left(1 + \gamma_2(\theta_{ax}, \theta_{ay}, \theta_{dx}, \theta_{dy}, \theta_{adx}, \theta_{ady}|\theta_{R1})\right)\right\}.
\end{align*}
\]
the approximation of the antenna pattern with $K_J$ and $K_u$ sectors. As the number of sectors increases, we expect the sectorized antenna pattern to approach the actual pattern. In the following, another approximate model for the end-to-end SNR distribution is presented with a lower computational complexity and the cost of lower accuracy.

**Proposition 1:** The distribution of the end-to-end SNR conditioned on $\theta_{R1}$ given in (18) can be simplified as

$$f_{\gamma_q|\theta_{R1}}(\gamma_q|\theta_{R1}) \simeq \sum_{j_u=1}^{K_u} \sum_{j_q=1}^{K_J} B_q(j_q, N_{qz}) \delta(\gamma_q - \gamma'_q(j_q, j_u)).$$

(21)

where $\sigma^2_{qz}$ is obtained in (48).

**Proof:** Please refer to Appendix C.

As can be seen, (21) has a lower computational load than (18), but, its accuracy is lower. In the following, the accuracy of the closed-form expressions with the results obtained from Monte-Carlo simulations is examined. However, the main feature of the provided analytical expression is that it is tractable, and will allow us to properly calculate the closed-form expressions for outage probability and channel capacity.

**Proposition 2:** Based on the channel distribution provided in Theorem 1, the average end-to-end channel capacity is obtained as

$$\bar{C}_{\text{e2e}} = \frac{1}{\pi} \int_{\theta_{R1}=0}^{\pi} \min \{ C_{uq|\theta_{R1}}, C_{du|\theta_{R1}} \} \, d\theta_{R1},$$

(23)

$$C_{uq|\theta_{R1}} = \sum_{j_u=1}^{K_u} \sum_{j_q=1}^{K_J} T_q(j_q, N_{qz}) T_{uq}(j_u, N_{uqz})$$

$$\times \log_2 \left( 1 + \gamma'_q(j_q, j_u) h_L(L_q(\theta_{R1}, L_{uc}, H_u)) \right).$$

(24)

Also, based on the channel distribution provided in Proposition 1, a simpler closed-form approximation for $C_{uq|\theta_{R1}}$ with lower computational load is obtained as

$$C_{uq|\theta_{R1}} \simeq \sum_{j_u=1}^{K_u} \sum_{j_q=1}^{K_J} B_q(j_q, N_{qz}) \log_2 \left( 1 + \gamma'_q(j_q, j_u) \right).$$

(25)

**Proof:** Please refer to Appendix D.

**Proposition 3:** Based on the channel distribution provided in Theorem 1, the end-to-end outage probability of the considered system conditioned on $\theta_{R1}$ is derived as

$$P_{\text{out}|\theta_{R1}} = P_{\text{out, su}|\theta_{R1}} + P_{\text{out, du}|\theta_{R1}} - P_{\text{out, su}|\theta_{R1}} P_{\text{out, du}|\theta_{R1}},$$

(26)

$$P_{\text{out, qu}|\theta_{R1}} = \sum_{j_u=1}^{K_u} \sum_{j_q=1}^{K_J} T_q(j_q, N_{qz}) T_{uq}(j_u, N_{uqz})$$

$$\times \mathbb{V} \left( \gamma_{th} - \gamma'_q(j_q, j_u) \right).$$

(27)

Also, based on the channel distribution provided in Proposition 1, a simpler closed-form approximation for $P_{\text{out, qu}|\theta_{R1}}$ with lower computational load is obtained as

$$P_{\text{out, qu}|\theta_{R1}} \simeq \sum_{j_u=1}^{K_u} \sum_{j_q=1}^{K_J} B_q(j_q, N_{qz}) \mathbb{V} \left( \gamma_{th} - \gamma'_q(j_q, j_u) \right).$$

(28)

**Proof:** Please refer to Appendix B.

The main important point about the analytical expressions presented in Propositions 1 and 3 is that, in addition to being tractable and very well formed, they are functions of all key channel parameters and we can easily analyze the effect of channel parameters on the outage probability and channel capacity of the considered system with a greater speed and lower computational load.

**B. Multiple Relay System**

Although the design of a multi-relay system is slightly more complicated than a single-relay system, the expressions obtained for a single-relay system can be easily extended to a multi-relay system as follows.
Proposition 4: The end-to-end channel capacity and outage probability of an $M$ relay system is derived respectively as

$$\mathcal{C}_{c2e} \simeq \frac{1}{\pi} \int_{\theta_{R1}=0}^{\pi} \min \{ \mathcal{C}_{uu}[\theta_{R1}], \mathcal{C}_{du}[\theta_{R1}], \mathcal{C}_{uu} \} \, d\theta_{R1},$$  (29)

and

$$\mathbb{P}_{\text{out}}[\theta_{R1}] = 1 - (1 - \mathbb{P}_{\text{out,uu}}[\theta_{R1}]) (1 - \mathbb{P}_{\text{out,du}}[\theta_{R1}]) \times (1 - \mathbb{P}_{\text{out,uu}}[\theta_{R1}])^{M-1},$$  (30)

where $\mathcal{C}_{uu}$ and $\mathbb{P}_{\text{out,uu}}[\theta_{R1}]$ are respectively obtained from Eqs. (23) and (28) by substituting parameters $N_{x1}$, $N_{y1}$, $N_{x2}$, $N_{y2}$, $\sigma_{w1}$, $\sigma_{w2}$, $\mu_{w1}$, $\mu_{w2}$, $L_{uu}$, $J_u$, and $h_L(L_{uu}, H_u)$ instead of $N_{x}$, $N_{y}$, $N_{xq}$, $N_{yq}$, $\sigma_{w}$, $\sigma_{q}$, $\mu_{w}$, $\mu_{q}$, $L_{q}$, $J_q$, and $h_L(L_q(\theta_{R1}), L_{uc}, H_u)$, respectively.

Proof: Based on the results of next remark and by following the method adopted in Appendices A, C, and B, the results can be proven.

Remark 1: The interior UAV link are symmetric, and the optimal values for the parameters of an interior UAV link can be used for the rest of the UAV links, and as a result, the design of a multi-relay system will be quite similar to a two-relay system.

Proposition 5: For the case where the antenna misalignment of the ground stations is small compared to the antenna misalignment of the UAV nodes, the channel capacity expression in (24) can be given asymptotically, at high SNR, in a simpler form as

$$\mathcal{C}_{c2e}(\theta_{R1}) = \min \{ \mathcal{C}_{uu}[\theta_{R1}], \mathcal{C}_{du}[\theta_{R1}], \mathcal{C}_{uu} \},$$  (31)

where

$$\mathcal{C}_{uu}[\theta_{R1}] \simeq \frac{1}{\ln(2)(\beta_{ux} - \beta_{uy})} \left[ \beta_{ux}^2 \left( \frac{\ln(S_{u0}(\theta_{R1}))}{\beta_{ux}} - 2 \right) - \beta_{uy}^2 \left( \frac{\ln(S_{u0}(\theta_{R1}))}{\beta_{uy}} - 2 \right) \right],$$

$$\mathcal{C}_{uu}[\theta_{R1}] \simeq \sum_{m=1}^{M} \frac{R_{uu}(m)}{\mathbb{R}_{uu}(m)} [\mathbb{R}_{uq}(m) \log_2 (S_{0}(\theta_{R1})) - 1]$$  (32)

and $S_{0}(\theta_{R1})$ is defined in (60), $\beta_{wu} = \frac{2\sigma_{wu}^2}{w_0^2(N_{wu})}$, $w \in \{x, y\}$, and $q \in \{s, d\}$, and

$$\beta_{wx} = \frac{2\sigma_{wx}^2}{w_0^2(N_{wx})}, \quad R_{wu}(m) = 1 + \frac{1-T_{wu}}{1+T_{wu}} \cos \left( \frac{\pi m}{M} \right),$$

$$\mathbb{R}_{wu}(m) = \frac{\mu_{wx}}{\mu_{wu}}, \quad \mathbb{R}_{uq}(m) = \frac{\mu_{wq}}{\mu_{wu}}, \quad T_q = \frac{\sigma_{wx}}{\sigma_{wu}}, \quad \sigma_{wx} = \max \{\sigma_{wx}, \sigma_{uwx}, \sigma_{wuq} \}, \quad \sigma_{uwq} = \min \{\sigma_{wuq}, \sigma_{wuq} \}.$$

Proof: Please refer to Appendix E.

According to (32), the following important point can be concluded. For the UU link, if $\ln(S_{u0}(\theta_{R1})) \gg 2\beta_{ux}$, then (32) is simplified as follows:

$$\mathcal{C}_{uu}[\theta_{R1}] \propto \log_2 (S_{u0}(\theta_{R1})).$$  (33)

Note that $S_{u0}(\theta_{R1})$ is a function of channel attenuation, antenna gain, and transmitted power. The condition $\ln(S_{u0}(\theta_{R1})) \gg 2\beta_{ux}$ is for the case where the antenna beamwidth is large (antenna gain is small) and $\sigma_{wuq}$ is very small. In this case, the effect of antenna misalignment errors can be ignored. However, in practical conditions, to compensate for the high attenuation in mmWave, the antenna gain is increased which leads to an increase in $\beta_{ux}$, and as a result, in practice, the approximation of (33) is not valid in most scenarios.

Proposition 6: For the case where the antenna misalignment of the ground stations is small compared to the antenna misalignment of the UAV nodes, the outage probability expression in (30) can be given asymptotically, at high SNR, in a simpler form as

$$\mathbb{P}_{\text{out}}[\theta_{R1}] \simeq \left( \sum_{m=1}^{M} \mathbb{R}_{uu}(m) \left( \frac{\gamma_{u}}{S_{0}(\theta_{R1})} \right) \mathbb{R}_{uu}(m) \right) \times \left( 1 - \frac{1}{\beta_{ux} - \beta_{uy}} \left( \sum_{m=1}^{M} \mathbb{R}_{uu}(m) \left( \frac{\gamma_{u}}{S_{0}(\theta_{R1})} \right) \mathbb{R}_{uu}(m) \right) \right) \times \left[ \beta_{ux} \left( \frac{\gamma_{u}}{S_{0}(\theta_{R1})} \right) \frac{1}{\beta_{ux}} - \beta_{uy} \left( \frac{\gamma_{u}}{S_{0}(\theta_{R1})} \right) \frac{1}{\beta_{uy}} \right]^{M-1}. $$  (34)

Proof: Please refer to Appendix F.

In many practical scenarios, the conditions of Propositions 6 and 7 are satisfied, because the ground stations are usually more stable than a UAV, and it does not face the weight and power limitations for using trackers with higher accuracy.

In the following, we will simplify the outage probability and channel capacity expressions for the case where the variances of the angular error along the $x$ and $y$ axes are almost the same, i.e., $\beta_{wu} \simeq \beta_{wuq} = \beta_{uy}$.

Proposition 7: For the case where $\sigma_{wuq} \simeq \sigma_{wuq}$, the outage probability and channel capacity are respectively simplified as

$$\mathbb{P}_{\text{out}}[\theta_{R1}] \simeq \left( \frac{\gamma_{u}}{S_{0}(\theta_{R1})} \right) \frac{\beta_{ux}}{\beta_{ux} - \beta_{uy}} \left( \frac{1}{\beta_{ux}} - \beta_{uy} \right)^{M-1}. $$  (35)

$$\mathcal{C}_{c2e}(\theta_{R1}) = \min \{ \mathcal{C}_{uu}[\theta_{R1}], \mathcal{C}_{du}[\theta_{R1}], \mathcal{C}_{uu} \},$$  (36)

where

$$\mathcal{C}_{uu}[\theta_{R1}] \simeq \frac{2\sigma_{wu}^2}{w_0^2(N_{wu})} \left[ \ln(S_{u0}(\theta_{R1})) - 4\beta_{ux} \right],$$

$$\mathcal{C}_{uu}[\theta_{R1}] \simeq \frac{2\sigma_{wu}^2}{w_0^2(N_{wu})} \left[ \ln(S_{u0}(\theta_{R1})) - 4\beta_{ux} \right].$$  (37)

Proof: Please refer to Appendix G.

When $2\beta_{ux} \gg \ln \left( \frac{\gamma_{u}}{S_{0}(\theta_{R1})} \right)$, (35) is simplified as

$$\mathbb{P}_{\text{out}}[\theta_{R1}] \simeq \left( \frac{\gamma_{u}}{S_{0}(\theta_{R1})} \right) \frac{\beta_{ux}}{\beta_{ux} - \beta_{uy}} \left( \frac{1}{\beta_{ux}} - \beta_{uy} \right)^{M-1}. $$  (38)

Under this condition, the diversity gain of the considered link is a function of $\frac{M-1}{\beta_{ux}} + \frac{1}{\beta_{ux}} + \frac{1}{\beta_{uy}}$. In other words, the diversity
TABLE II

| Parameters          | Values |
|---------------------|--------|
| $P_{tx}$            | 1 W    |
| $N_{qu}$            | 12-18  |
| $f_c$               | 70 GHz |
| $\rho_0$            | 7.5 g/m³ |
| $H_{ras}$           | 1.5 km |
| $\psi_{max}$        | 15°    |
| $d_{qu}$            | $\pi/2$ |
| $\sigma_{qu}$       | 0.3° & 0.5° |
| $\mu_{qu}$          | 0.2° & 0.3° |

gain is a function of the variance of the alignment error, the antenna pattern, and the number of relay UAVs.

IV. SIMULATIONS AND OPTIMAL SYSTEM DESIGN

By providing comprehensive simulations, the performance of the single-relay as well as the multi-relay systems is examined. The values of the parameters used in the simulations are listed in Table II. The Monte-Carlo simulations are used to show the accuracy of the provided analytical expressions. In the following, the single-relay system will be examined first, and then the multi-relay system will be studied for longer link lengths.

A. Single Relay Case

For single relay systems, one of the important parameters is the optimal position for point $B_p$, which determines the average position of the UAV in a circular motion. As discussed, the location of the point $B_p$ is adjusted in the sky with the parameter $L_{sc}$. Any change in the parameters $B_p$ and $L_{sc}$ affects the values of $L_s$ and $L_d$. In Fig. 4, the end-to-end outage probability and channel capacity are plotted versus $L_s$ for $\sigma_{qu} = 0.1^\circ$, $\sigma_{uq} = 1.4^\circ$, $\sigma_{uy} = 1.1^\circ$, $N_{qu}$ = 15, and $N_{qu} = 18$. As discussed in the previous section, the E2E performance depends on the performance of CU and UD links. Therefore, in Fig. 4, to get a better view, the performance of CU and UD links is also provided versus $L_s$. The results obtained from Fig. 4 can be expressed in the following two remarks.

Remark 2: For shorter links of $L_s$, the E2E performance can be well approximated with the performance of the UD link. However, for longer links of $L_s$, the E2E system performance is limited to the performance of the CU link.

Remark 3: The optimal value for $L_{sc}$ is very close to the length of $L_s$ for which the capacity of the CU link is equal to the capacity of the UD link.

To justify Remark 2, note that by increasing $L_s$, the performance of the CU link decreases and at the same time $L_d$ decreases and consequently the performance of the UD link improves. The accepted interval for $L_s$ and $L_d$ shown in Fig. 4a is to guarantee condition (10b). Based on (10b) and Remark 1, we can conclude the following remark.

Remark 4: In order to guarantee constraint (10b) along the circle flight path, it is necessary that $L_s < L_{s, max}$ and $L_d < L_{d, max}$ where $L_{s, max}$ and $L_{d, max}$ are obtained as

$$P_{out,tr} = \text{Prob}\{\gamma_s(\theta_{dx}, \theta_{dy}, \theta_{udx}, \theta_{udy})|L_{s, max} < \gamma_{th}\},$$

$$P_{out,tr} = \text{Prob}\{\gamma_d(\theta_{dx}, \theta_{dy}, \theta_{udx}, \theta_{udy})|L_{d, max} < \gamma_{th}\}.$$

Note that the use of Monte-Carlo simulations is very time consuming, especially for the lower values of outage probability. If Monte Carlo simulations are used to find the optimal values for tunable system parameters, it is necessary to independently run Monte-Carlo simulations in large numbers, and finally, select the optimal parameters from all the independent runs. As we see in the sequel, the performance of the considered system is highly dependent on the optimal values for the adjustable parameters such as the antenna patterns and the optimal position of the UAVs in 3D space. Therefore, for our system model, the search space to run Monte-Carlo simulation independently will be very large and thus, it will take a lot of time to optimally design the system parameters. For this aim, in this work, the closed-form expressions for outage probability as well as channel capacity were presented as functions of all key parameters of the considered system, which have a much shorter run time than Monte-Carlo simulations. In Figs. 4a and 4b, along with the simulation results, the two analytical expressions provided in Eqs. (24) and (28) for the outage probability as well as the analytical expressions provided in Eqs. (27) and (32) for the channel capacity are plotted. The results of Figs. 4a and 4b confirm the validity of the analytical expressions.

In Fig. 5, the outage probability of the single relay system is plotted versus both $N_{uq}$ and $N_{uy}$ for $\sigma_{ux} = 2^\circ$, $\sigma_{uy} = 0.5^\circ$, and two different values of $L_s = 10$ and 12 km. Since $\sigma_{ux} > \sigma_{uy}$, with increasing $N_{uy}$, the SNR in the receiver increases and thus, the performance of the system improves. Therefore, based on the results of Fig. 5 and for both $L_s = 10$ and 12 km, the optimal value for $N_{uy}$ is listed in Table II.

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
$N_{\text{uqy}} = N_{\text{u,max}}$ is 18. However, in the direction of the $x_q$ axis, although the SNR increases by increasing $N_{\text{uqy}}$, for larger values of $N_{\text{uqy}}$, the beamwidth decreases and the system becomes more sensitive to misalignment errors. Therefore, the outage probability increases. Moreover, based on the results of Fig. 5, we can conclude the following remark that decreases the search space and processing time during the optimal design of the considered system.

**Remark 5:** If $\sigma_{\text{uqy}} < \sigma_{\text{uqx}}$, then the optimal value for $N_{\text{uqy}}$ will be greater than the optimal value for $N_{\text{uqx}}$.

In order to obtain more information about the optimal selection of $N_{\text{uqx}}$, in Figs. 6a and 6b, the outage probability and the channel capacity of the single relay system is plotted for different values of $N_{\text{uqx}}$. From the results of Fig. 6, although the channel capacity increases with increasing $N_{\text{uqx}}$, the antenna beam bandwidth decreases for large $N_{\text{uqx}}$ and the system becomes more sensitive to alignment errors. Therefore, as we observe, the channel capacity is maximized for $N_{\text{uqx}} = 18$. However, for those values of $N_{\text{uqx}}$, we have $P_{\text{out}} > P_{\text{out,tr}}$ for all values of $L_s$ and therefore, the required QoS in condition (10b) is not guaranteed. It should be noted that the optimal value for $N_{\text{uqx}}$ cannot be determined from the results of Figs. 5 and 6. According to constraint (10b), in the entire flight path of the UAV, which is characterized by the parameter $-\pi < \theta_{R1} < \pi$, we should have $P_{\text{out}} > P_{\text{out,tr}}$.

Accordingly, in Fig. 7, the end-to-end performance of the single relay system is examined along the entire flight path. Since the circular flight path is symmetric with respect to $\theta_{R1}$, the outage probability and the channel capacity are provided for interval $-\frac{\pi}{2} < \theta_{R1} < \frac{\pi}{2}$ instead of interval $-\pi < \theta_{R1} < \pi$. Another important point is that the position of the circular flight path is controlled by the adjustable parameter $L_{sc}$.

2In practice, due to the weight and aerodynamic limitations of the UAV payload [36], a very large antenna can not be used and we have to consider a maximum for $N_{\text{uqy}}$. has a very important impact on the system performance. Based on (10), we seek to maximize the channel capacity while ensuring the constraints of (10), especially constraint (10b). In this work we consider $P_{\text{out,tr}} = 10^{-3}$. In Fig. 7b, in addition to the channel capacity, the average channel capacity over the entire circular flight path is also presented. Based on the results of Fig. 7, by increasing $N_{\text{uqx}}$, the average channel capacity increases. However, by increasing $N_{\text{uqx}}$, the performance of the considered system in terms of outage probability is not necessarily improved. Only those $N_{\text{uqx}}$ values that achieve outage probability lower than $10^{-3}$ for the whole interval $-\frac{\pi}{2} < \theta_{R1} < \frac{\pi}{2}$ are acceptable and are marked in black color in those figures. For the other $N_{\text{uqx}}$ values, which in part or for the whole circular route can not guarantee the constraint of (10b). For instance, for $L_{sc} = 10$ km, $N_{\text{uqx}} = 8$ only guarantees (10b), and therefore the maximum average channel...
capacity available for it is 5.3 bit/s/Hz. As a result, a very important point that can be deduced from the results of Fig. 7 is provided in the following remark.

Remark 6: To calculate the optimal UAV’s flight path as well as the optimal value for $L_{sc}$, it is better to first investigate the performance of the considered system versus $L_{s}$ since it gives a better view in terms of finding the acceptable interval for $L_{s}$ and $L_{d}$. In other words, by doing this we will find an acceptable range of $L_{sc}$ and thus, the search space to find the optimal values of system parameters will be significantly reduced. Then for each value of $L_{sc}$, we find the values of $N_{uu}$ that guarantee constraint (10b), and the largest of the obtained $N_{uu}$ results in the maximum achievable average channel capacity. This process will be repeated for all possible values of $L_{sc}$.

B. Multi-Relay Case

The use of a single relay system can ultimately guarantee a maximum length of $L_{ud}$. For example, for the parameters presented in Table II, the maximum possible length for $L_{ud}$ is 18.3 km, and for longer links, more UAVs should be used. In addition to the parameters considered for a single relay system, for a multi-relay system, it is necessary to find the optimal distance between the UAVs as well as the optimal antenna pattern used for communication links between the UAVs, which is studied in the following.

In Fig. 8, we evaluate the performance of a two-relay system in terms of both outage probability and channel capacity versus $\theta_{R1}$. Based on Eqs. (29) and (30), the performance of the system depends on the performance of the three CU, UU, and UD links. To get a better understanding, the performance of each link is also provided separately. As it turned out, for lower values of $L_{sc}$ obtained for the interval close to $\theta_{R1} = -90^\circ$, the system performance is limited to UD link. In Fig. 8a, we have specified this interval with the name of interval 1. Then, for the intermediate values of $\theta_{R1}$, the system performance is limited to the inter-UAV link or UU link marked with interval 2. For larger values of $\theta_{R1}$, the length of $L_{s}$ increases and thus, the system performance is limited to CU link determined by interval 3 in Fig. 8a. In addition, the accuracy of the analytical expressions has been confirmed using Monte-Carlo simulations. In Fig. 8, the label Analytical 1 refers to the analytical expression obtained based on the channel distribution function derived in (18), and the label Analytical 2 refers to the analytical expression obtained based on the channel distribution function provided in (22). As can be seen, Analytical 1 is more accurate. However, the results of Analytical 2 have less computational load than Analytical 1.

Note that the results of Fig. 8 are obtained for the values of $N_{tx1} = N_{tx2} = 10$, $L_{uu} = 6$ km, and $L_{sc} = 12$ km. However, those parameters are adjustable and by changing them, the performance of the considered system changes significantly. Accordingly, in Fig. 9, the outage probability of a two relay system for different values of $L_{sc}$ is plotted. The results of this figure clearly show the importance of finding an optimal value for $L_{sc}$. This indicates that for smaller values of $L_{sc}$, the system performance is limited to the UD link, and as $L_{sc}$ increases, the link length of $L_{d}$ becomes shorter, resulting in improved outage probability of the UD link. For intermediate values of $L_{sc}$, the outage probability on the circular flight path has less changes, indicating that the system performance is limited by the UAV link because the UAV link has a fixed link length along the circular flight path. For longer values of $L_{sc}$, it is observed that by increasing $\theta_{R1}$, the outage probability significantly increases. In this case, the system performance is limited to the CU link. In other words, it indicates that in half of the UAV’s circular path, the system performance has an acceptable outage probability, and in the other half, $L_{s}$ increases and the system performance is significantly reduced. Moreover, the results of Fig. 9a are provided for $N_{tx1} = N_{tx2} = 8$ and the results of Fig. 9b are for $N_{tx1} = N_{tx2} = 12$. It is observed that by changing $N_{tx1}$, the system performance as well as the optimal values for $L_{sc}$ also change. In addition, the results of Fig. 9 are obtained for a constant value $L_{uu}$. Similar to $L_{sc}$, we expect that by changing the values of $L_{uu}$, the outage probability of the considered two-relay system changes, significantly.

C. Optimal System Design

Finally, in order to optimally design the tunable parameters of the considered two-relay system, the method adopted in Tables III and IV can be used. For example, in Table III,
capacity is the highest average channel capacity. For example, according to constraint (10b), we calculate the average channel capacity on the critical points instead of the entire flight path. It is enough to be calculated on the entire flight path. Therefore, for $L_{uu} = 6$ km, the maximum achievable channel capacity is $C_{max} = 6.54$ bit/s/Hz that will be obtained for $L_{uu} = 6.5$ km and $N_{uu} = 12$. It does not guarantee constraint (10b) along the entire flight path. For $L_{uu} = 12$ km, the maximum achievable channel capacity is $C_{max} = 5.72$ bit/s/Hz that will be obtained for $L_{uu} = 6.5$ km and $N_{uu} = 12$. Note that the values of $L_{uu} = 6.5$ km and $N_{uu} = 12$ are the results should be repeated for the rest of the possible values of $L_{sc}$. Finally, we select the optimal value from the entire search space.

V. CONCLUSION

By taking into account the actual channel parameters such as the UAV vibrations, tracking error, real 3GPP antenna pattern, UAV’s height and flight path, and considering the effect of physical obstacles, the optimal design of a relay system based on fixed-wing UAV was investigated. In particular, we derived the distribution of SNR which is based on the sum of a series of Dirac delta functions. Then, we used the SNR distribution and derived the analytical expressions for the outage probability and the channel capacity of the considered system as a function of all real system parameters. After that, we extended the analytical expressions for a multi-relay system. The accuracy of analytical expressions was verified with the results obtained from Monte-Carlo simulations. Finally, by providing sufficient simulation results, we investigated the effects of key channel parameters such as antenna pattern gain and flight path on the performance of the considered system and we carefully studied the relationships between the link length and the minimum number of UAVs to establish the considered backhaul link. In addition, we investigated how to choose the optimal radius, center, and height of the fixed-wing UAVs in a circular flight path and the optimal choice of the antenna patterns in different environmental conditions in order to maximize average channel capacity.

APPENDIX A

SNR DISTRIBUTION

Due to the lower changes of the considered antenna’s gain pattern in the Roll direction, for the lower antenna misalignment (less than a few degrees), (16) can be approximated with good accuracy as follows:

$$\gamma_q(\theta_q, \theta_{uu}) \approx \gamma_q(\theta_{R1}, N_{qu}, N_{uuw}) \times \left( \frac{\sin \left( \frac{N_{qz} k d u q \sin(\theta_q)}{2} \right)}{N_{qz} \sin \left( \frac{k d u q \sin(\theta_{uu})}{2} \right)} \right)^2,$$

where

$$\gamma_q(\theta_{R1}, N_{qu}, N_{uuw}) = \frac{P_{uq}}{\sigma_n^2} \times 10^{\frac{G_{uuw}}{20} G_0(N_{qz}, N_{qu}) G_0(N_{uuw}, N_{uuw})},$$

and $\theta_q = \sqrt{\theta_{qz}^2 + \theta_{qu}^2}$, and $\theta_{uu} = \sqrt{\theta_{uuw}^2 + \theta_{uuq}^2}$. Since we have $\theta_{qz} \sim N(\mu_{qz}, \sigma_{qz}^2)$ and $\theta_{qu} \sim N(\mu_{qu}, \sigma_{qu}^2)$, the random variable $\theta_q$ follows the Beckmann distribution as [37]

$$f_{\theta_q}(\theta_q) = \frac{\theta_q}{2 \pi \sigma_{qz} \sigma_{qu}} \int_0^{2\pi} \exp \left( - \frac{(\theta_q \cos(\theta) - \mu_{qz})^2}{2 \sigma_{qz}^2} - \frac{(\theta_q \cos(\theta) - \mu_{qu})^2}{2 \sigma_{qu}^2} \right) d\theta.$$

Similarly, the distribution of RV $\theta_{uu}$ is obtained from (43) by substituting $\mu_{uuw}, \sigma_{uuw}, \sigma_{uuq}$, and $\sigma_{uuq}$ instead of $\mu_{qz}, \mu_{qu}$,
\[ \gamma \text{ and } \sigma_{q,2}, \text{ and } \sigma_{q,4}, \text{ respectively. Let us approximate (41) as} \]
\[
\gamma_q(\theta_q, \theta_{uq}|\theta_{RL}) \\
\simeq \frac{\gamma_{q1}(\theta_{RL}, N_{q,y}, N_{u,xy})}{N^2_{q,x}N^2_{u,xy}} \sum_{j_q=1}^{K_{J_q}} \sum_{j_u=1}^{K_{J_u}} \frac{\sin \left( \frac{2\pi j_u}{N_{u,xy}} \right)}{\sin \left( \frac{2\pi j_u}{N_{u,xy}} \right)^2} \frac{\sin \left( \frac{2\pi j_q}{N_{q,y}} \right)}{\sin \left( \frac{2\pi j_q}{N_{q,y}} \right)^2} \left( \sin \left( \frac{2\pi j_q}{N_{q,y}} \right) \right) \]
\[
\times \left[ \gamma(\theta_{uq} - \frac{2(j_u - 1)}{J_{N_{u,xy}}}) - \gamma(\theta_{uq} - \frac{2j_u}{J_{u,N_{u,xy}}}) \right] \\
\times \left[ \gamma(\theta_q - \frac{2(j_q - 1)}{J_{N_{q,y}}}) - \gamma(\theta_q - \frac{2j_q}{J_{q,N_{q,y}}}) \right], \tag{44} \]

where \( \gamma(x) = \begin{cases} 1 & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases} \) is the sign function, and the parameters \( J_q, J_u, \) and \( K \) are the natural numbers that for large values of \( J_q \) and \( J_u \), (44) tends to (41). Also, \( K = 1 \) refers to the main lobe of the antenna pattern and \( K > 1 \) refers to the number of side lobes. Using Eqs. (43), (44), and [38, (17)-20]), and after some derivations, the distribution of \( \gamma_q(\theta_q, \theta_{uq}) \) conditioned on \( \theta_{RL} \) is derived in (18).

**APPENDIX C**

**SNR DISTRIBUTION IN PROPOSITION 1**

The Beckmann distribution given in (43) can be approximated as [39]
\[
f_{\theta_q}(\theta_q) \simeq \frac{\theta_q}{\sigma^2_{q,c}} \exp \left( -\frac{\theta_q^2}{2\sigma^2_{q,c}} \right), \tag{47} \]
\[
\sigma^2_{q,c} = \left( \frac{3\mu^2_{q,x} \sigma_{q,x}^4 + 3\mu^2_{q,y} \sigma_{q,y}^4 + \sigma_{q,x}^6 + \sigma_{q,y}^6}{2} \right)^{\frac{1}{2}}. \tag{48} \]

Similarly, the distribution of RV \( \theta_{uq} \) is obtained from (47) by substituting \( \mu_{q,x}, \mu_{q,y}, \sigma_{u,x}, \) and \( \sigma_{u,y} \) instead of \( \mu_{q,x}, \mu_{q,y}, \sigma_{q,x}, \) and \( \sigma_{q,y} \), respectively. Using Eqs. (44), (48), and [40, eq. (3.321.4)], and after some derivations, the distribution of \( \gamma_q(\theta_q, \theta_{uq}) \) conditioned on \( \theta_{RL} \) is derived in (21).

**APPENDIX D**

**CHANNEL CAPACITY**

We approximate the average end-to-end channel capacity conditioned on \( \theta_{RL} \) provided in (14) as
\[
C_{\text{end}}|\theta_{RL} = \min \left\{ C_{\text{SU}}(\theta_{RL}), C_{\text{DU}}(\theta_{RL}) \right\}, \tag{49} \]
where \( C_{\text{SU}}(\theta_{RL}) \) and \( C_{\text{DU}}(\theta_{RL}) \) are the average channel capacities of CU and UD links conditioned on \( \theta_{RL} \), respectively. For our system model, \( C_{\text{UQ}}(\theta_{RL}) \) is a function of random variables (RVs) \( \theta_{q,x}, \theta_{q,y}, \theta_{u,x}, \) and \( \theta_{u,y} \) and can be obtained as
\[
C_{\text{UQ}}(\theta_{RL}) \\
= \frac{1}{4\pi^2 \sigma_{q,x} \sigma_{q,y} \sigma_{u,x} \sigma_{u,y}} \int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\pi/2} \log_2 \left( 1 + \gamma_q(\theta_{q,x}, \theta_{q,y}, \theta_{u,x}, \theta_{u,y}|\theta_{RL}) \exp \left( -\frac{(\theta_{q,x} - \mu_{q,x})^2}{2\sigma_{q,x}} \right) \right) \times \exp \left( -\frac{(\theta_{q,y} - \mu_{q,y})^2}{2\sigma_{q,y}} \right) \exp \left( -\frac{(\theta_{u,x} - \mu_{u,x})^2}{2\sigma_{u,x}} \right) \exp \left( -\frac{(\theta_{u,y} - \mu_{u,y})^2}{2\sigma_{u,y}} \right) d\theta_{q,x} d\theta_{q,y} d\theta_{u,x} d\theta_{u,y}. \tag{50} \]

where \( \gamma_q(\theta_{q,x}, \theta_{q,y}, \theta_{u,x}, \theta_{u,y}|\theta_{RL}) \) is obtained in (16). Although the expression given in (50) reduces the 8-dimensional integral to 4-dimensional, it still has a high computational time. Using Eqs. (18), (49), and (50) and performing a series of calculations, the end-to-end channel capacity is derived in (23). Also, using Eqs. (21), (49), and (50) and performing a series of calculations, another closed-form expression for \( C_{\text{UQ}}|\theta_{RL} \) with lower computational load is derived in (25).

**APPENDIX E**

A CD link consists of a CU link, \( (M-1) \) UU links, and a UD link.

1) **UU Link:** Following the results of (41), (16) is simplified as follows:
\[
\gamma_q(\theta_{u,x}, \theta_{u,y}, \theta_{u,x+1}, \theta_{u,y+1}|\theta_{RL}) \\
= S_{u0}(\theta_{RL}) \]

where

\[ S_{u0}(\theta_{R1}) = \frac{P_{L_q} h_L}{\sigma_n^2} G_0(N_{t,i}) G_0(N_{r,i+1}) G_{\text{max}}^2, \]  

(52)

\[ w_B(N_{t,i}) = \frac{1.061}{N_{t,i}}, \quad \text{and} \quad w_B(N_{r,i+1}) = \frac{1.061}{N_{r,i+1}}. \]  

Due to the symmetry, the optimal values for \( N_{t,i} \) and \( N_{r,i+1} \) are equal, that is, \( N_{t,i} = N_{r,i+1} = N_u \). For lower values of \( \theta_{u,u,t,i} \) and \( \theta_{u,u,r,i+1} \), we can approximate (51) as

\[ \gamma_{uw}(\theta_{u,u,t,i}, \theta_{u,u,r,i+1}, \theta_{u,u,r,i+1}) \theta_{R1} = S_{u0}(\theta_{R1}) \]

\[ \times \exp \left( -\frac{\theta_{u,u,t,i}^2 + \theta_{u,u,r,i+1}^2 + \theta_{u,u,r,i+1}^2}{w_B(N_u)} \right). \]

Let us define parameter \( S_2 \) as

\[ S_2 = \frac{\theta_{u,u,t,i}^2 + \theta_{u,u,r,i+1}^2}{s_{21}} + \frac{\theta_{u,u,r,i+1}^2}{s_{22}} \]  

(54)

Obtaining the distribution of \( S_2 \) is complicated. In most situations, the bias angle \( \mu_{uw,i} \) is negligible. Under this condition and due to the symmetry \( \sigma_{u,u,i} = \sigma_{u,u,r,i+1} = \sigma_{u,u} \) and \( \sigma_{uy,i} = \sigma_{uy,r,i+1} = \sigma_{uy} \), the RVs \( \theta_{u,u,t,i} \) and \( \theta_{u,u,r,i+1} \) have same distribution, and the RVs \( \theta_{u,u,i} \) and \( \theta_{u,u,r,i+1} \) have same distribution. Therefore, \( s_{21} = \sqrt{S_{21}} \) for \( i \in \{1, 2\} \) has a Rayleigh distribution as [42]

\[ f_{s_{21}}(s_{21}) = \frac{s_{21}}{\sigma_{u,u}^2} \exp \left( -\frac{s_{21}^2}{2\sigma_{u,u}^2} \right). \]  

(55)

Using Eqs. (55) and (54), and after some derivations, the PDF of \( S_2 \) is derived as

\[ f_{S_2}(S_2) = \frac{w_B(N_u)}{\sigma_{u,u}^2 - \sigma_{u,u}^2} \left[ e^{-\frac{w_B(N_u)}{\sigma_{u,u}^2} S_2} - e^{-\frac{w_B(N_u)}{\sigma_{u,u}^2} S_2} \right] \]  

(56)

Now, using Eqs. (53), (54), and (56), and after some derivations, the PDF of \( \gamma_{uw} \) is derived as

\[ f_{\gamma_{uw}}(\theta_{R1}) \gamma_{uw} \approx \frac{1}{2S_{u0}(\theta_{R1}) (\beta_{u,u} - \beta_{uy})} \times \left[ \frac{\gamma_{uw}}{S_{u0}(\theta_{R1})} \right]^{\frac{1}{\gamma_{uw} - 1}} - \left[ \frac{\gamma_{uw}}{S_{u0}(\theta_{R1})} \right]^{\frac{1}{\gamma_{uw} - 1}}, \]  

(57)

where \( 0 < \gamma_{uw} < S_{u0}(\theta_{R1}) \), \( \beta_{uw} = \frac{2\sigma_{uw}^2}{w_B(N_u)} \). Using (57), (14), from [40, eq. (2.02.5)], and after some derivations, the average channel capacity of UU link conditioned \( \theta_{R1} \) is derived in (32).

2) CU and UD Links: For CU and UD link, the CDF of \( \gamma_{uq}(\theta_{uq}, \theta_{uq}, \theta_{R1}) \) is derived in Appendix F. By taking the derivative of (65), the PDF of \( \gamma_{uq}(\theta_{uq}, \theta_{uq}, \theta_{R1}) \) is derived as

\[ f_{\gamma_{uq}}(\theta_{R1}) \gamma_{uq} = \frac{\gamma_{uq}(\theta_{R1})}{S_{uq}(\theta_{R1}) S_{uq}(\theta_{R1})} \]  

(58)

for \( 0 < \gamma_{uq} < S_{u0}(\theta_{R1}) \). Using (58) and [40, Eq. (2.728.1)], at high SNR, the channel capacity of (24) can be given asymptotically in (32).

APPENDIX F

In most practical situations, \( \sigma_{uq} \) is smaller than \( \sigma_{uq} \) and we can ignore the effect of \( \sigma_{uq} \) compared to that of \( \sigma_{uq} \) [31]. In this case, (16) is simplified as follows:

\[ \gamma_{uq}(\theta_{uq}, \theta_{uq}, \theta_{R1}) = S_{u0}(\theta_{R1}) \]

\[ \times \exp \left( -\frac{\theta_{uq}^2 + \theta_{uq}^2}{w_B(N_u)} \right). \]

(59)

where

\[ S_{u0}(\theta_{R1}) = \frac{P_{L_q} h_L (L_q(\theta_{R1} L_{x,c}, H_u))}{\sigma_n^2} G_0(N_u) G_0(N_u) G_{\text{max}}^2 \]  

(60)

and \( w_B(N_u) = \frac{1.061}{N_u} \). For lower values of \( \theta_{uq} \), we can approximate (59) as

\[ \gamma_{uq}(\theta_{uq}, \theta_{uq}, \theta_{R1}) \approx S_{u0}(\theta_{R1}) \exp \left( -\frac{\theta_{uq}^2 + \theta_{uq}^2}{w_B^2(N_u)} \right). \]

(61)

Let us define \( S_1 = \frac{\theta_{uq}^2 + \theta_{uq}^2}{w_B^2(N_u)} \) and \( \beta_{uq} = \frac{2\sigma_{uq}^2}{w_B^2(N_u)} \). Using [43, eqs. (3) and (5)], the distribution of \( S_1 \) is obtained as

\[ f_{S_1}(S_1) = \frac{1}{\beta_{uq} + \beta_{uq}} \exp \left( -\frac{\beta_{uq} + \beta_{uq}}{2\beta_{uq} \beta_{uq} S_1} \right) \times I_0 \left( \frac{\beta_{uq} + \beta_{uq}}{2\beta_{uq} \beta_{uq} S_1} \right), \]

(62)

where \( I_0(\cdot) \) is the modified Bessel function of the first kind with order zero. Based on Eq. (62) and using [44], the CDF of \( S_1 \) is derived as

\[ F_{S_1}(S_1) = Q \left( \frac{2S_1}{\beta_{uq} + \beta_{uq}}, \frac{2S_1}{\beta_{uq} + \beta_{uq}} \right) - Q \left( \frac{2S_1}{\beta_{uq} + \beta_{uq}}, \frac{2S_1}{\beta_{uq} + \beta_{uq}} \right), \]

(63)
where \( Q(a, b) \) is the Marcum Q-function, and
\[
\begin{aligned}
\mathcal{R}_1 &= \frac{\sqrt{1-T^2}}{2T} \sqrt{1 + \frac{1-T^2}{T^2}}, \\
\mathcal{R}_2 &= \mathcal{R}_1 \frac{1-T^2}{1+T^2}, \\
\mathcal{R}_3 &= \mathcal{T}_u = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}, \\
\sigma_{\text{max}} &= \max\{\sigma_{uqz}, \sigma_{uqy}\}, \\
\sigma_{\text{min}} &= \min\{\sigma_{uqz}, \sigma_{uqy}\}
\end{aligned}
\]
Using [45, eq. (5)], we can approximate (63) as
\[
F_{S_1}(S_1) \simeq 1 - \sum_{m=1}^{M} \mathcal{R}_3(m) \exp \left(-\frac{\mathcal{R}_4(m)}{\mathcal{R}_3(m)} S_1 \right)
\] (64)
where
\[
\begin{aligned}
\mathcal{R}_3(m) &= 1 +\frac{1-T^2}{1+T^2} \cos \left(\pi \frac{m-1}{M}\right), \\
\mathcal{R}_4(m) &= \frac{(1+T^2)^2}{2T^2} \mathcal{R}_3(m), \\
\mathcal{R}_5 &= \frac{2T}{M(1+T^2)}.
\end{aligned}
\]
Using Eqs. (59) and (64), the CDF of \( \gamma_q(\theta_{uqz}, \theta_{uqy}|\theta_R) \) is derived as
\[
F_{\gamma_q|\theta_R} \simeq \sum_{m=1}^{M} \mathcal{R}_6(q) \left( \frac{\gamma_q}{S_0(\theta_R)} \right) \mathcal{R}_7(q)
\] (65)
Also, using (57), the CDF of \( \gamma_{uu}(\theta_{uqz}, \theta_{uqy}|\theta_R) \) is obtained as
\[
F_{\gamma_{uu}|\theta_R} \simeq \frac{1}{(\beta_{uu} - \beta_{uy})} \times \left[ \beta_{ux} \left( \frac{\gamma_{uu}}{S_0(\theta_R)} \right)^{1/\beta_{uq}} - \beta_{uy} \left( \frac{\gamma_{uu}}{S_0(\theta_R)} \right)^{1/\beta_{uy}} \right].
\] (66)
Finally, using (65) and (66), at high SNR, the outage probability of (30) can be given asymptotically in (34).

**APPENDIX G**

1) **CU and UD Links:** When \( \beta_{uq} \simeq \beta_{uy} = \beta_{uu} \), (63) is simplified as [42]
\[
F_{S_1}(S_1) \simeq 1 - \exp \left(-\frac{S_1}{\beta_{uu}} \right).
\] (67)
Using Eqs. (59) and (67), the CDF of \( \gamma_q(\theta_{uqz}, \theta_{uqy}|\theta_R) \) is derived as
\[
F_{\gamma_q|\theta_R} \simeq \left( \frac{\gamma_q}{S_0(\theta_R)} \right)^{1/\beta_{uu}}
\] (68)
By taking the derivative of Eq. (68), the PDF of \( \gamma_q(\theta_{uqz}, \theta_{uqy}|\theta_R) \) is derived as
\[
f_{\gamma_q|\theta_R}(\gamma_q|\theta_R) \simeq \left( \frac{\gamma_q}{S_0(\theta_R)} \right)^{1/\beta_{uu}-1}
\] (69)
for \( 0 < \gamma_q < S_0(\theta_R) \). Using (58) and [40, Eq. (2.728.1)], at high SNR, the channel capacity of (32) can be given asymptotically in (37).

2) **UU Link:** By following the method of obtaining (66), when \( \beta_{ux} \simeq \beta_{uy} = \beta_{uu} \), the CDF of \( \gamma_{uu} \) is derived as
\[
F_{\gamma_{uu}|\theta_R}(\gamma_{uu}) = \left( \frac{\gamma_{uu}}{S_0(\theta_R)} \right)^{1/\beta_{uu}} \left[ 1 - \frac{1}{2\beta_{uu}} \ln \left( \frac{\gamma_{uu}}{S_0(\theta_R)} \right) \right].
\] (70)
Using (68) and (70), at high SNR, the outage probability in (34) can be simplified to (71). By taking the derivative of Eq. (70), the CDF of \( \gamma_{uu} \) is derived as
\[
f_{\gamma_{uu}|\theta_R}(\gamma_{uu}) = \frac{1}{2\beta_{uu}} \frac{S_0(\theta_R)}{\gamma_{uu}} \ln \left( \frac{S_0(\theta_R)}{\gamma_{uu}} \right) \times \gamma_{uu}^{-1}
\] (71)
Finally, using (71), applying a change of variable \( x = \ln(\gamma_{uu}) \), and after some derivations, the channel capacity of (24) can be given asymptotically in (37).

**REFERENCES**

[1] M. T. Dabiri, Mazen. O. Hasna, N. Zorba, and T. Khattab, “Long mmWave backhaul connectivity using fixed-wing UAVs,” in *Proc. IEEE Int. Conf. Commun. Workshops (ICC Workshops)*, May 2022, pp. 1035–1040.

[2] 5G Riders on the Storm. [Online]. Available: https://www.technews.com

[3] How 5G Can Improve Emergency Response Efforts During Extreme Weather. [Online]. Available: http://www.the5gexchange.com

[4] Z. Ullah, F. Al-Turjman, and L. Mostarda, “Cognition in UAV-aided 5G and beyond communications: A survey,” *IEEE Trans. Cognit. Commun. Netw.*, vol. 6, no. 3, pp. 872–891, Sep. 2020.

[5] B. Galkin, J. Kibida, and L. A. DaSilva, “Backhaul for low-altitude UAVs in urban environments,” in *Proc. IEEE Int. Conf. Commun. (ICC)*, Kansas City, MO, USA, May 2018, pp. 1–6.

[6] M. Alzenad, M. Z. Shakir, H. Yankomeroglu, and M. Alouini, “FSO-based vertical backhaul/fronthaul framework for 5G+ wireless networks,” *IEEE Commun. Mag.*, vol. 56, no. 1, pp. 218–224, Jan. 2018.

[7] W. Khawaja, I. Guvenc, D. W. Matolak, U. Fiebig, and N. Schreckenburger, “A survey of air-to-ground propagation channel modeling for unmanned aerial vehicles,” *IEEE Commun. Surveys Tuts.*, vol. 21, no. 3, pp. 2361–2391, 3rd Quart., 2019.

[8] Q. Wu and R. Zhang, “Common throughput maximization in UAV-enabled OFDMA systems with delay consideration,” *IEEE Trans. Commun.*, vol. 66, no. 12, pp. 6614–6627, Dec. 2018.

[9] M. Cui, G. Zhang, Q. Wu, and D. W. K. Ng, “Robust trajectory and transmit power design for secure UAV communications,” *IEEE Trans. Veh. Technol.*, vol. 67, no. 9, pp. 9042–9046, Sep. 2018.

[10] M. Banagar and H. S. Dhillon, “3D two-hop cellular networks with wireless backhauled UAVs: Modeling and fundamentals,” *IEEE Trans. Wireless Commun.*, vol. 21, no. 8, pp. 6417–6433, Aug. 2022.

[11] M. Banagar and H. S. Dhillon, “Fundamentals of 3D two-hop cellular networks analysis with wireless backhauled UAVs,” in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2021, pp. 1–6.

[12] H.-B. Jeon, S.-H. Park, J. Park, K. Huang, and C.-B. Chae, “An energy-efficient aerial backhaul system with reconfigurable intelligent surface,” *IEEE Trans. Wireless Commun.*, vol. 21, no. 8, pp. 6478–6494, Aug. 2022.

[13] H.-B. Jeon, S.-H. Park, J. Park, K. Huang, and C.-B. Chae, “RIS-assisted aerial backhaul system for UAV-Bs: An energy-efficiency perspective,” in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2021, pp. 1–6.

[14] M. Gapeyenko, V. Petrov, D. Moltchanov, S. Andreev, N. Himayat, and Y. Koucheryavy, “Flexible and reliable UAV-assisted backhaul operation in 5G mmWave cellular networks,” *IEEE J. Sel. Areas Commun.*, vol. 36, no. 11, pp. 2486–2496, Nov. 2018.

[15] N. Tafintsiev et al., “Aerial access and backhaul in mmWave BSS systems: Performance dynamics and optimization,” *IEEE Commun. Mag.*, vol. 58, no. 2, pp. 93–99, Feb. 2020.

[16] W. Wang, N. Cheng, Y. Liu, H. Zhou, X. Lin, and X. Shen, “Content delivery analysis in cellular networks with aerial caching and mmWave backhaul,” *IEEE Trans. Veh. Technol.*, vol. 70, no. 5, pp. 4809–4822, May 2021.

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.
Y. Yu, X. Bu, K. Yang, H. Yang, and Z. Han, “Backhaul-aware optimization of UAV base station location and bandwidth allocation for profit maximization,” IEEE Access, vol. 8, pp. 154573–154588, 2020.

Z. Feng, L. Ji, Q. Zhang, and W. Li, “Spectrum management for mmWave enabled UAV swarm networks: Challenges and opportunities,” IEEE Commun. Mag., vol. 57, no. 1, pp. 146–153, Jan. 2019.

M. T. Dabiri, H. Safi, S. Parsaeefard, and W. Saad, “Analytical channel models for millimeter wave UAV networks under hovering fluctuations,” IEEE Trans. Wireless Commun., vol. 19, no. 4, pp. 2868–2883, Apr. 2020.

M. T. Dabiri, M. Rezaee, V. Yazdanian, B. Maham, W. Saad, and C. S. Hong, “3D channel characterization and performance analysis of UAV-assisted millimeter wave links,” IEEE Trans. Wireless Commun., vol. 20, no. 1, pp. 110–125, Jan. 2021.

M. T. Dabiri, M. O. Hasna, T. Khattab, and K. Qaraqe, “A study of multihop mmW aerial backhaul links,” 2022, arXiv:2203.04387.

M. Y. Selim and A. E. Kamal, “Post-disaster 4G/5G network rehabilitation using drones: Solving battery and backhaul issues,” in Proc. IEEE Globecom Workshops (GC Wkshps), Dec. 2018, pp. 1–6.

Y. Yu, X. Bu, K. Yang, H. Yang, and Z. Han, “UAV-aided low latency mobile edge computing with mmWave backhaul,” in Proc. IEEE Int. Conf. Commun. (ICC), May 2019, pp. 1–7.

K. Meng, Q. Wu, S. Ma, W. Chen, and T. Q. S. Quek, “UAV trajectory and beamforming optimization for integrated periodic sensing and communication,” IEEE Wireless Commun. Lett., vol. 11, no. 6, pp. 1211–1215, Jun. 2022.

Q. Wu, Y. Zeng, and R. Zhang, “Joint trajectory and communication design for multi-UAV enabled wireless networks,” IEEE Trans. Wireless Commun., vol. 17, no. 3, pp. 2109–2121, Mar. 2018.

D. Yang, Q. Wu, Y. Zeng, and R. Zhang, “Energy tradeoff in ground-to-UAV communication via trajectory design,” IEEE Trans. Veh. Technol., vol. 67, no. 7, pp. 6721–6726, Jul. 2018.

G. Zhang, Q. Wu, M. Cui, and R. Zhang, “Securing UAV communications via joint trajectory and power control,” IEEE Trans. Wireless Commun., vol. 18, no. 2, pp. 1376–1389, Feb. 2019.

Y. Zeng, Q. Wu, and R. Zhang, “Accessing from the sky: A tutorial on UAV communications for 5G and beyond,” Proc. IEEE, vol. 107, no. 12, pp. 2327–2375, Dec. 2019.

G. Geraci et al., “What will be the future of UAV cellular communications be? A flight from 5G to 6G,” 2021, arXiv:2105.04842.

N. Babu, I. Donevski, A. Valcarce, F. Popovski, J. J. Nielsen, and C. B. Papadias, “Fairness-based energy-efficient 3-D path planning of a portable access point: A deep reinforcement learning approach,” IEEE Open J. Commun. Soc., vol. 3, pp. 1487–1500, 2022.

M. T. Dabiri, S. M. S. Sadough, and M. A. KHALIGHI, “Channel modeling and parameter optimization for hovering UAV-based free-space optical links,” IEEE J. Sel. Areas Commun., vol. 36, no. 9, pp. 2104–2113, Sep. 2018.

M. T. Dabiri et al., “UAV-assisted free space optical communication system with amplify-and-forward relaying,” IEEE Trans. Veh. Technol., vol. 70, no. 9, pp. 8926–8936, Sep. 2021.

J. P. Pena-Martin, J. M. Romero-Jerez, and F. J. Lopez-Martinez, “Generalized MGF of Beckmann fading with applications to wireless communications performance analysis,” IEEE Trans. Commun., vol. 65, no. 9, pp. 3933–3943, Sep. 2017.

B. Zhu, Z. Zeng, J. Cheng, and N. C. Beaulieu, “On the distribution function of the generalized Beckmann random variable and its applications in communications,” IEEE Trans. Commun., vol. 66, no. 5, pp. 2235–2250, May 2018.

R. Boluda-Ruiz, A. García-Zambrana, C. Castillo-Vázquez, and B. Castillo-Vázquez, “Novel approximation of misalignment fading modeled by Beckmann distribution on free-space optical links,” Opt. Exp., vol. 24, no. 20, pp. 22635–22649, Oct. 2016.

I. S. Gradsteyn and I. M. Ryzhik, Table of Integrals, Series, and Products, 7th ed. New York, NY, USA: Academic, 2007.

M. T. Dabiri, M. Hasna, N. Zorba, T. Khattab, and K. A. Qaraqe, “A general model for pointing error of high frequency directional antennas,” IEEE Open J. Commun. Soc., vol. 3, pp. 1978–1990, 2022.

A. Papoulis and S. U. Pillai, Probability, Random Variables, and Stochastic Processes. New York, NY, USA: McGraw-Hill, 2002.

M. Nakagami, “The m-distribution—A general formula of intensity distribution of rapid fading,” in Statistical Methods in Radio Wave Propagation. Amsterdam, The Netherlands: Elsevier, 1960, pp. 3–36.

2016. Hoytdistribution. Wolfram Research, Champaign, IL, USA. [Online]. Available: https://reference.wolfram.com/language/ref/HoytDistribution.html

G. N. Tavares, “Efficient computation of Hoyt cumulative distribution function,” Electron. Lett., vol. 46, no. 7, pp. 537–539, Apr. 2010.