Adaptive Fuzzy Decentralized Control for Fractional-Order Nonlinear Large-Scale Systems with Unmodeled Dynamics

SHUAI SUI¹, (Member, IEEE), YONGLIANG ZHAN², JUNWEI JIN³, C. L. PHILIP CHEN⁴, (FELLOW, IEEE), AND SHAOCHENG TONG⁵, (Senior Member, IEEE)

¹The College of Science, Liaoning University of Technology, Jinzhou, 121001, China (e-mail: shuaisui2011@163.com)
²The College of Science, Liaoning University of Technology, Jinzhou, 121001, China (e-mail: zhan_yongliang@163.com)
³School of Artificial Intelligence and Big Data, Henan University of Technology, Zhengzhou, 450001, China (e-mail: jinjunwei24@163.com)
⁴School of Computer Science and Engineering, South China University of Technology, Guangzhou 510641, China (e-mail: philip.chen@ieee.org)
⁵The College of Science, Liaoning University of Technology, Jinzhou, 121001, China (e-mail: jztongsc@163.com)

This work was funded in part by the National Key Research and Development Program of China under number 2019YFA0706200 and 2019YFB1703600, in part by the National Natural Science Foundation of China (Nos. 62176111, 62173172, 62106068, 61751202, 61702195, 61751205, 61773188, U1813203, U1801262), the Doctoral Research Initiation of Foundation of Liaoning Province, China, under Grant 2021-BS-260, General project of Liaoning Provincial Department of Education, under Grant LJKZ0627. Corresponding author C. L. Philip Chen.

ABSTRACT This paper addresses the decentralized control issue for the fractional-order (FO) nonlinear large-scale strict-feedback systems with unmodeled dynamics. The unknown nonlinear functions are identified by fuzzy logic systems (FLSs) and the FO dynamic signals are introduced to dominate the unmodeled dynamics. Additionally, the fractional-order dynamic surface control (FODSC) design technique is introduced into the adaptive backstepping control algorithm to avoid the issue of “explosion of complexity”. Then, an adaptive fuzzy decentralized control scheme is developed via the FO Lyapunov stability criterion. It is proved that the controlled FO systems are stable and the tracking errors can converge to a small neighborhood of zero. The simulation example is provided to confirm the validity of the put forward control scheme.

INDEX TERMS Fractional-order system, large-scale systems, DSC, adaptive fuzzy decentralized control, backstepping, unmodeled dynamics.

I. INTRODUCTION

FRACTIONAL-ORDER nonlinear systems (FONSs) can be treated as the extension of integer-order nonlinear systems (IONSs), which is a research hotspot developed on the basis of fractional-order calculus theory. In practice, some real systems can be modeled by utilizing FONSs, for example, micro-electro-mechanical resonators [1], hyper-chaotic economic systems [2] and lithium batteries [3]. Recently, a growing number of scholars have begun to focus on the control issues of FONSs, and obtained some results [4]-[6]. A smooth adaptive backstepping control scheme was put forward in [4], which ensures the globally asymptotically stable of commensurate FONSs. The authors in [5] designed a backstepping-based adaptive tracking controller for the FONSs. Then, this approach was further studied in [6] for FONSs with states immeasurable. However, the above literatures only consider the case that the nonlinear functions in the controlled objects are known.

As we all know, FLSs and neural networks (NNs) have ability to approximate unknown nonlinear functions. Consequently, in references [7]-[16], they are utilized to identify unknown nonlinear functions in FONSs. The authors in [7]-[11] put forward several adaptive intelligent (fuzzy and NN) control strategies for FONSs with measurable or unmeasured states, respectively. Furthermore, the conventional backstepping control design technique adopted in the above-mentioned intelligent adaptive control strategies, which may lead to the issue of “explosion of complexity” since the repeated derivation of the virtual control functions. To solve this issue, several intelligent adaptive control strategies have been put forward in [12]-[13] by introducing fractional-order dynamic surface filters (FODSFs). Subsequently, to solve the issue of unmeasured states, the authors in [14]-[16] put forward some observer-based adaptive intelligent DSC strategies via
designing state observers. Nevertheless, the abovementioned control methods are limited to the single-input and single-output (SISO) FONSs.

In real life, such as traffic systems, network transportation systems, aerospace systems are often described by interconnected systems composed of many lower-dimensional subsystems. Because of the complex structure and high dimensions of interconnected systems, the decentralized control is often used. The decentralized control only relies on the local information of the subsystems, so compared with the central control, it has the characteristics of reducing the computational complexity and enhancing the robustness and reliability of the interactive operation fault [17]. Consequently, the intelligent decentralized control of nonlinear interconnected systems is favored by many scholars and has achieved a great progress in the past decades [18]-[22]. Recently, the adaptive decentralized controller and adaptive sliding-mode decentralized controller have been put forward by [23]-[24] for the FO nonlinear large-scale systems. Furthermore, the authors in [25]-[27] put forward intelligent adaptive backstepping decentralized control schemes for the uncertain nonwitched or switched FO nonlinear large-scale systems. Later, an observer-based intelligent adaptive decentralized DSC control strategy has been developed by the authors in [28]. However, the unmodeled dynamics are not considered in the above controlled objects.

The unmodeled dynamics refers to some dynamic characteristics lost when modeling the systems. It widely exists in practical systems and has strong uncertainty, which may cause the poor performance or even instability of the controlled systems. Because of this, this problem has also been paid attention by the majority of scholars. An output feedback tracking controller has been put forward in [29] to ensure the stability for the FO interconnected systems with unmodeled dynamics. Later, the authors in [10] put forward an intelligent adaptive method for SISO FONSs with unmodeled dynamics. Reference [10] gave the notion of Mittag-Leffler ISpS Lyapunov function for the FO interconnected systems with unmodeled dynamics. However, [10] is limited to SISO FONSs. It is necessary to design a control scheme for the FO nonlinear large-scale systems with unmodeled dynamics.

This paper considers the fuzzy adaptive decentralized control design issue for the FO nonlinear large-scale systems with unmodeled dynamics. The main features of this study are as below: i) For the issue of unmodeled dynamics, firstly, the systems are required to have Mittag-Leffler ISpS Lyapunov functions, and then some FO dynamic signals are introduced to solve this problem. ii) By applying FODSC technique, the put forward control strategy avoids the issue of “explosion of complexity” in the previous methods [7]-[11] and [24]-[25].

II. PROBLEM FORMULATIONS AND PRELIMINARIES

A. SYSTEM DESCRIPTIONS

The FO nonlinear large-scale systems are as follows:

\[
\begin{align*}
\mathcal{C}_0 D_t^\alpha z_i &= q_i(z_i, y_i, t) \\
\mathcal{C}_0 D_t^\alpha x_{i,j} &= f_{i,j}(X_{i,j}) + x_{i,j+1} + H_{i,j}(z_i, Y, t) \\
\mathcal{C}_0 D_t^\alpha x_{i,m_i} &= f_{i,m_i}(X_i) + u_i + H_{i,m_i}(z_i, Y, t) \\
y_i &= x_{i,1}
\end{align*}
\]

where \( \alpha \in (0,1) \), \( X_{i,j} = [x_{i,1}, x_{i,2}, \ldots x_{i,j}]^T \in \mathbb{R}^j \), \( X_{i,m_i} = X_i \), \( 1, 2, \cdots, M, j = 1, 2, \cdots, m_i \) are the system state vectors. \( u_i \) are the control inputs of the system and \( Y = [y_1, y_2, \cdots y_M]^T \in \mathbb{R}^M \) is the system output. \( f_{i,j}(.) \) are smooth unknown nonlinear functions. \( z_i \in \mathbb{R}^{m_i} \) are unmodeled dynamics and \( H_{i,j}(z_i, Y, t) \) are the dynamical disturbances. \( q_i(.) \) and \( H_{i,j}(.) \) are uncertain functions.

Remark 1: If the issue of unmodeled dynamics is ignored and there are no interconnected terms, the control objects (1) will become \( M \) independent SISO FONSs in strict-feedback form. For SISO FONSs in strict-feedback form have been extensively studied in [7]-[15]. However, this paper studies FO strict-feedback nonlinear large-scale systems, and considering the existence of unmodeled dynamic problem, it is more complex than the controlled systems in the previous references [7]-[15]. Therefore, the control design of this paper is more difficult.

Assumption 1 [12-13]: The given reference signals \( y_{i,d} \) are sufficiently smooth functions of \( t \) and \( y_{i,d} , \mathcal{C}_0 D_t^\alpha y_{i,d} \) and \( \mathcal{C}_0 D_t^\alpha (\mathcal{C}_0 D_t^\alpha y_{i,d}) \) are bounded. Additionally, it is assumed that there is a constant \( B_{i,0} > 0 \) such that \( y_{i,d}^2 + (\mathcal{C}_0 D_t^\alpha y_{i,d})^2 + (\mathcal{C}_0 D_t^\alpha (\mathcal{C}_0 D_t^\alpha y_{i,d}))^2 \leq B_{i,0} \).

Assumption 2 [10]: The system \( \mathcal{C}_0 D_t^\alpha z_i = q_i,1(z_i, y_i, t) \) has a Mittag-Leffler ISpS Lyapunov function \( V_{z_i}(z_i) \) such that

\[
\alpha_{i,1}(||z_i||) \leq V_{z_i}(z_i) \leq \alpha_{i,2}(||z_i||),
\]

\[
\mathcal{C}_0 D_t^\alpha V_{z_i}(z_i) \leq -c_i V_{z_i}(z_i) + \gamma_i ||y_i|| + d_i,
\]

where \( c_i > 0 \) and \( d_i > 0 \) are known constants and \( \alpha_{i,1}, \alpha_{i,2}, \gamma_i \) are \( k \)-\( \infty \) functions.

Assumption 3 [17, 19-20]: There exist a unknown constant \( q_{i,j}^* \geq 0 \) and known smooth functions \( \vartheta_{i,j,0} \geq 0 \) and \( \vartheta_{i,j,l} \geq 0 \) such that

\[
|H_{i,j}(z_i, Y, t)| \leq q_{i,j}^* \vartheta_{i,j,0} ||y_i|| + q_{i,j}^* \sum_{l=1}^{M} \vartheta_{i,j,l} ||y_l||
\]

where \( \vartheta_{i,j,0}(0) = 0 \).

Remark 2: Noting that Assumptions 1-3 are common assumptions and can be found in references [10, 12, 13, 17, 19, 20], respectively. The purpose of Assumption 2 is to explain the existence of Mittag-Leffler ISpS Lyapunov functions in the \( z_i \)- systems. The meaning of Assumption 3 is to deal with dynamical disturbances. However, the ways for addressing dynamical disturbances are not unique, such as

\[
|H_{i,j}(z_i, Y, t)| \leq \sum_{k=1}^{M} (q_{i,j,k}^1 ||z_i||^{k+1} + \sum_{l=1}^{M} q_{i,j,k}^l ||y_l||^{k+l})
\]
where $q_{i,j}^k$ and $q_{i,j}^k$ are unknown constants and $p = \{p_{i,j} | 1 \leq i \leq M, 1 \leq j \leq m_i\}$ is a known integer. In other words, Assumption 3 in the put forward control strategy can be replaced by other assumptions with the same function, and the control objectives can still be achieved after some modifications.

**Control objectives:** On the basis of FO Lyapunov stability criterion, an adaptive fuzzy decentralized controller is designed for systems (1), which makes all signals in the controlled systems are bounded as well as the tracking errors as small as possible.

**B. PRELIMINARIES**

**Definition 1 [30]:** The $\alpha$th Caputo fractional derivative can be defined as:

$$C^\alpha D_t^\alpha F(t) = \frac{1}{\Gamma(\alpha - \omega)} \int_0^t \frac{F(\zeta)(t - \zeta)^{\alpha - 1}}{(t - \zeta)\omega} d\tau$$

where $\omega 
\leq \alpha \leq \omega, \omega \in N^+, \Gamma(\cdot) = \int_0^{\infty} \tau^{\alpha - 1} e^{-\tau} d\tau$ is the Euler Gamma function with $\Gamma(1) = 1$.

**Definition 2 [30]:** The Mittag-Leffler function as:

$$E_{\alpha, \varphi}(\cdot) = \sum_{j=0}^{\infty} \frac{\gamma^j}{\Gamma(j\alpha + \varphi)}$$

where $\alpha, \varphi \in \mathbb{R}^+$ and $\gamma \in \mathbb{C}$, whose Laplace transform is as follows

$$L\{t^{\alpha-1}E_{\alpha, \varphi}(-\kappa t^\alpha)\} = \frac{s^{\alpha-\varphi}}{s^{\alpha} + \kappa}$$

where $\kappa \in \mathbb{R}$.

**Lemma 1 [30]:** Let $\alpha$ satisfy $\alpha \in (0, 2)$, $\beta \in \mathbb{R}$ and $\delta \in (\pi \alpha/2, \min\{\pi, \pi \alpha\})$, then one has

$$E_{\alpha, \beta}(\cdot) \leq \frac{\lambda}{1 + |\zeta|}$$

where $\lambda > 0, |\zeta| \geq 0$ and $\delta \leq |\arg(\zeta)| \leq \pi$.

**Lemma 2 [34]:** For any continuous function $f(x)$ with $f(0) = 0$, there exists a nonnegative smooth function $f$ with $\delta f/\delta x(0) = 0$ such that $f(x) \leq \hat{f}(x) + \varepsilon$.

**Lemma 3 [34]:** For a smooth function $g(x)$ with $g(0) = 0$, there exists a constant $\varepsilon > 0$, such that $|x| \leq \varepsilon g(x) + \varepsilon$.

**Remark 3:** Note the inequality $|x| \leq \varepsilon g(x) + \varepsilon$, where $g(x)$ represents a class of smooth functions with $g(0) = 0$. The selection of $g(x)$ is not unique, such as $x/4\varepsilon$ in [34] and $\tanh(x/\varepsilon)$ in [37]. Therefore, the inequality will hold by choosing an appropriate $g(x)$.

**Lemma 4 [10]:** If system (1) has a Mittag-Leffler ISPS Lyapunov function, then for any constants $c_i$ in $(0, c_i)$, any initial condition $z_{i,0} = z_i(t_0)$ and $r_{i,0} > 0$, for a function $\hat{\gamma}_i(y_i) \geq \gamma_i(y_i)$, there exists a finite $T^0 = T^0_i(c_i, r_i, z_i) \geq 0$, a nonnegative function $D_i(t_0, t)$ defined for all $t \geq t_0$ and a signal described by

$$C^\alpha D_t^\alpha r_i = -\bar{c}_i r_i + \hat{\gamma}_i(y_i) + \bar{d}_i, r_i(t_0) = r_i^0$$

such that $D_i(t_0, t)$ for all $t \geq t_0 + T^0$ and

$$V_{e_i} \leq r_i + D_i(t, t)$$

for all $t \geq t_0$.

A FLS in [31] is defined by

$$\hat{f}(x) = \frac{\sum_{i=1}^M \tilde{y}_i \prod_{i=1}^M \mu_{A_i}(x_i)}{\sum_{i=1}^M \prod_{i=1}^M \mu_{A_i}(x_i)}$$

In (9), $A_i$ and $G^i$ are fuzzy sets, $\mu_{A_i}(x_i)$ and $\mu_{G^i}(y_i)$ are their membership functions. $\tilde{y}_i = \max_{y \in R} R_i(y) = \gamma_i(y_i)$. Let

$$\psi_i = \prod_{i=1}^M \mu_{A_i}(x_i)$$

Denote $w^T = [\tilde{y}_1, \tilde{y}_2, \cdots \tilde{y}_M]^T$ and $\psi(x) = [\psi_1(x), \psi_2(x), \cdots \psi_M(x)]^T$, then FLS (9) is re-expressed by $\hat{f}(x) = w^T \psi(x)$.

**Lemma 5 [31]:** Suppose that $f(x)$ is a continuous function defined on a bounded compact set $\Omega$, then there exists a FLS $w^T \psi(x)$ such that

$$\sup_{x \in \Omega} |f(x) - w^T \psi(x)| \leq \delta$$

where $\delta > 0$ is a given constant.

Because $f_{i,j}(X_{i,j})$ in controlled FO systems (1) are unknown nonlinear functions, according to Lemma 5, it can be approximated by FLSs $f_{i,j}(X_{i,j}) \approx \psi_{i,j}(X_{i,j})$.

**Remark 4:** It is worth noting that, like references [9, 12, 35, 39], this paper select FLSs as the approximator to address unknown nonlinear functions in the controlled systems (1). However, there are some other nonlinear approximators, such as NNs [10, 11, 13, 38, 40] and neuro-fuzzy network system [15, 36], which can replace FLSs and achieve the same purpose.

**III. FUZZY ADAPTIVE DECENTRALIZED CONTROL DESIGN AND STABILITY ANALYSIS**

By adopting the backstepping-based control algorithm, the detailed design processes of the fuzzy adaptive decentralized controller will be given.

Consider the following coordinate transformations

$$\xi_{i,1} = x_{i,1} - y_{i,d}$$

$$\xi_{i,j} = x_{i,j} - \xi_{i,j-1}$$

$$v_{i,j} = \xi_{i,j} - \xi_{i,j-1}$$

$$i = 1, 2, \ldots, M, j = 2, \ldots, m_i$$

where $\xi_{i,1}$ are the tracking errors, $\xi_{i,j}$ are the dynamic surface errors, $\xi_{i,j}$ are the FODSF variables, $\xi_{i,j}$ are the FODSF output errors. $v_{i,j}$ and $v_{i,j}$ are the estimations of $w_{i,j}$ and $v_{i,j}$ respectively. $w_{i,j} = w_{i,j} - w_{i,j}$ and $v_{i,j} = v_{i,j} - v_{i,j}$ are the parameter errors. $v_{i,j}$ will be defined later.
Choose the Lyapunov function candidate as:

\[ V_i = \sum_{i=1}^{M} \left( \frac{1}{2} \rho_i \zeta_i^2 + \frac{1}{2} \bar{\rho}_i \zeta_i^2 + \frac{1}{2} \rho_i \zeta_i^2 + \frac{1}{2} \bar{\rho}_i \zeta_i^2 \right) \]  

(15)

where \( \rho_{i,1} > 0 \), \( \bar{\rho}_{i,1} > 0 \) and \( \rho_{i,2} \), \( \bar{\rho}_{i,2} > 0 \) are design constants, \( \nu_{i,1} = \max\{1, \rho_{i,1}, \bar{\rho}_{i,1}\} \).

From (14) and (15), and by utilizing the inequality \( C_0 D_0^p (x^T(t) x(t))/2 \leq x^T(t) C_0^T D_0^p x(t) \) in [32], the following inequality holds

\[ C_0 D_0^p V_i \leq \sum_{i=1}^{M} \left( \zeta_i (\zeta_i + \nu_{i,1} + \nu_{i,1} + \delta_{i,1} + \alpha_{i,1} + \bar{\alpha}_{i,1} + \bar{\alpha}_{i,1} + \bar{\alpha}_{i,1}) \right) \]  

(16)

where \( \nu_{i,1}, \nu_{i,1} \) are smooth nonnegative functions such that

\[ \sum_{i=1}^{M} \nu_{i,1} \geq 0 \]  

(17)

By the Young’s inequality, \( y_i = \zeta_i + \nu_{i,1} \). Assumption 3, (17) and Cauchy-Schwartz inequality, \( \sum_{i=1}^{2} a_i^2 \leq 2 \)

Design the virtual decentralized controller \( \varpi_i \), the adaptation laws \( C_0 D_0^p \varpi_i \) and \( C_0 D_0^p v_i \) as

\[ \varpi_i = -c_{i,1} \zeta_i - w_i^T \psi_i(\zeta_i) - \bar{\nu}_{i,1} \Phi_i - c_{i,1} \zeta_i + y_i + \zeta_i (\sum_{i=1}^{M} \vartheta_i,1,1(0)) \]  

(25)

\[ C_0 D_0^p v_i + \zeta_i \Phi_i - \vartheta_i,1,1 v_i \]  

(26)

where \( \tau_{i,1} \) and \( \bar{\nu}_{i,1} \) are design parameters.

Additionally, according to [10], \( \exists \lambda_i > 0 \) that can make the inequality \( \frac{1}{2} \left( \zeta_i^2 + \nu_{i,1}^2 \right) \leq \lambda_i \) hold. Then, by substituting (25)-(27) into (24), one can gain

\[ C_0 D_0^p V_i \leq \sum_{i=1}^{M} \left( \zeta_i \bar{\nu}_{i,1} \Phi_i - \zeta_i \Phi_i - \tau_{i,1} \varpi_i \right) \]  

(28)

Introduce a FODSF in [13] as

\[ \sigma_{i,10} C_0 D_0^p \zeta_i + \zeta_i = \varpi_i,1,1(0) = \varpi_i(0) \]  

(29)
where $\sigma_{i,j}$ is a constant.

By using (13) and (29), one has
\begin{equation}
\mathcal{C}_0 D_t^\alpha \psi_{i,j} = \mathcal{C}_0 D_t^\alpha \psi_{i,j} - \mathcal{C}_0 D_t^\alpha \psi_{i,j-1} = \zeta_{i,j+1} + \nu_{i,j} + \varpi_{i,j} + w_{i,j}^* \psi_{i,j}(X_{i,j}) + \delta_{i,j} + H(z_i, y_t) + \theta_{i,j}^2 \chi_{i,j+1} \end{equation}
where $\chi_{i,j+1}$ is a continuous function.

**Step i, j:** With the help of (1) and (13), one has
\begin{equation}
\mathcal{C}_0 D_t^\alpha \zeta_{i,j} = \mathcal{C}_0 D_t^\alpha x_{i,j} - \mathcal{C}_0 D_t^\alpha \varpi_{i,j} = \zeta_{i,j+1} + \nu_{i,j} + \varpi_{i,j} + w_{i,j}^* \psi_{i,j}(X_{i,j}) + \delta_{i,j} + H(z_i, Y_t) + \theta_{i,j}^2 \chi_{i,j+1} \end{equation}
The Lyapunov function candidate is chosen as:
\begin{equation}
V_j = V_j - 1 + \sum_{i=1}^{M} \left( \frac{1}{\eta_{i,j}} + \frac{1}{\eta_{i,j}} \right) \psi_{i,j}^2 + \frac{1}{\eta_{i,j}} \psi_{i,j}^2 \end{equation}
where $\rho_{i,j} > 0$ and $\theta_{i,j} > 0$ are design parameters. Similar to step i, 1, with the consideration of (31) and (32), and by utilizing inequality in [32], it follows that
\begin{equation}
\mathcal{C}_0 D_t^\alpha V_j \leq \mathcal{C}_0 D_t^\alpha V_j - 1 + \sum_{i=1}^{M} \left( \zeta_{i,j}(\zeta_{i,j+1} + \nu_{i,j}) + \varpi_{i,j} + w_{i,j}^* \psi_{i,j}(X_{i,j}) + \delta_{i,j} + H(z_i, Y_t) + \theta_{i,j}^2 \chi_{i,j+1} + \nu_{i,j} + \zeta_{i,j} \right)
\end{equation}
By the Young’s inequality, $y_i = \zeta_{i,j} + y_i^2$. Assumption 3, (17) and Cauchy-Schwartz inequality ($\sum_{i=1}^{2} a_{i}^{2k}$) the following results hold
\begin{equation}
\zeta_{i,j}(\nu_{i,j} + \delta_{i,j}) \leq \zeta_{i,j} + \frac{\delta_{i,j}^2}{2} + \frac{\nu_{i,j}^2}{2}
\end{equation}
where $\zeta_{i,j} = 2 \sum_{i=1}^{M} \eta_{i,j}, \delta_{i,j}$ and $d_{i,j}(t_0, t) = \frac{1}{\eta_{i,j}} \left( \frac{1}{\eta_{i,j}} \right) \psi_{i,j}^2 (2D_t^\alpha (t_0 + T))$.

Utilizing Lemma 2 gives
\begin{equation}
\psi_{i,j,0} \leq \psi_{i,j,0}(t_0) \leq 1
\end{equation}
where $\hat{\psi}_{i,j,0}(t) = \hat{\psi}_{i,j,0}(t_0) + 1$.
By using (13) and (45), one can obtain
\[ C^0 D^\alpha_i \nu_{i,j} = C^0 D^\alpha_i \kappa_{i,j} - C^0 D^\alpha_i \omega_{i,j} = -\frac{\partial \mathcal{H}}{\partial \nu_{i,j}} + G_{i,j}(\cdot) \]  
(46)

where \( G_{i,j}(\cdot) \) is constitutive function.

**Step i, m:** According to (1) and (13), one has
\[ C^0 D^\alpha_i \zeta_{i,m} = C^0 D^\alpha_i x_{i,m} - C^0 D^\alpha_i \kappa_{i,m} - 1 = w_{i,m}^T \psi_{i,m}(X_i) + \delta_{i,m} + u_i + H_{i,m}(\hat{z}_i, Y, t) - C^0 D^\alpha_i \kappa_{i,m} - 1 \]  
(47)
The whole Lyapunov function is chosen as
\[ V = V_{m-1} + \sum_{i=1}^{M} \left( \frac{1}{2} \dot{z}_{i,m}^2 + \frac{1}{\rho_{i,m}} \dot{w}_{i,m}^T \dot{w}_{i,m} \right) + H_{i,m}(\hat{z}_i, Y, t) - C^0 D^\alpha_i \kappa_{i,m} - 1 \]  
(48)

where \( \rho_{i,m} > 0 \) and \( \bar{\rho}_{i,m} > 0 \) are design parameters.

Similar to step i, 1, from (47) and (48), and by utilizing inequality in [32], one has
\[ C^0 D^\alpha_i V \leq C^0 D^\alpha_i V_{m-1} + \sum_{i=1}^{M} \left( \zeta_{i,m}(\dot{\delta}_{i,m} + u_i + \dot{w}_{i,m}^T \psi_{i,m}(X_i) + H_{i,m}(\hat{z}_i, Y, t) - C^0 D^\alpha_i \kappa_{i,m} - 1) \right. \]
\[ \left. - \frac{1}{\rho_{i,m}} \bar{\dot{w}}_{i,m}^T C^0 D^\alpha_i w_{i,m} + \frac{1}{\bar{\rho}_{i,m}} \dot{w}_{i,m}^T C^0 D^\alpha_i w_{i,m} \right) \]  
(49)

By the Young’s inequality, \( y_i = \zeta_{i,1} + y_{i,d} \), Assumption 3, (17) and Cauchy-Schwartz inequality \( (\sum a_i)^2 \leq 2^{2k} \sum_{i=1}^{2k} a_i^2 \), the following results hold
\[ \zeta_{i,m}(\dot{\delta}_{i,m} + u_i + \dot{w}_{i,m}^T \psi_{i,m}(X_i) + H_{i,m}(\hat{z}_i, Y, t) - C^0 D^\alpha_i \kappa_{i,m} - 1) \]
\[ \leq \zeta_{i,m} \left( q_{i,m}^T \dot{\vartheta}_{i,m,0}(\cdot) \right) \]  
(50)

\[ \zeta_{i,m}(\dot{\delta}_{i,m} + u_i + \dot{w}_{i,m}^T \psi_{i,m}(X_i) + H_{i,m}(\hat{z}_i, Y, t) - C^0 D^\alpha_i \kappa_{i,m} - 1) \]
\[ \leq \zeta_{i,m} \left( q_{i,m}^T \dot{\vartheta}_{i,m,0}(\cdot) \right) \]  
(51)

where \( \dot{\vartheta}_{i,m,0}(\cdot) \)
\[ q_{i,m}^T \dot{\vartheta}_{i,m,0}(\cdot) \leq \zeta_{i,m} \left( q_{i,m}^T \dot{\vartheta}_{i,m,0}(\cdot) \right) \]  
(52)

By applying Lemma 2, the following inequality holds
\[ \dot{\vartheta}_{i,m,0}(\cdot) \leq \hat{\vartheta}_{i,m,0}(\cdot) + 1 \]  
(53)
(57), the virtual decentralized controllers (25) and (41) and the FO parameter adaptation laws (26), (27), (42), (43), (58) and (59) are adopted, then whole control scheme can assure that all signals of the closed-loop system are bounded. Additionally, the tracking errors can be made as small as possible.

**Proof:** On the basis of Assumption 1 with a constant \( \pi_i > 0 \), the sets \( \Xi_{i,0} \) and \( \Xi_i \) are compact sets. Consequently \( \Xi_{i,0} \times \Xi_i \) is still a compact set. So \( \exists K_{i,j} > 0 \) such that \( |G_{i,j}| \leq K_{i,j} \).

By the Young's inequality, the following inequalities hold

\[
\nu_{i,h} G_{i,h} \leq \frac{\nu_{i,h}^2}{2} + \frac{K_{i,h}^2}{2}
\]

(62)

From (60)-(62), one can obtain

Proof:

\[
\nu_{i,h} G_{i,h} \leq \frac{\nu_{i,h}^2}{2} + \frac{K_{i,h}^2}{2}
\]

(62)

A repeated utilize of Lemma 1, we can obtain

\[
|E_{\alpha,1}(-\eta^\alpha)| \leq \frac{r}{1 + \eta^\alpha}
\]

(71)

Finally, (67) and the tracking error can be written as follows

\[
V(t) \leq V(0)\frac{r}{1 + \eta^\alpha} + \frac{\phi d}{\eta}, \quad t \geq 0
\]

(72)

where \( r \) is a positive constant. From (73), we further have

\[
|\xi_{i,1}| \leq \sqrt{2rV(0) + \frac{2\phi d}{\eta}}
\]

(73)

Based on (72), it is concluded that the controlled plant is stable. In view of (74), when \( t \) approaches infinity, we can get

\[
\lim_{t \to \infty} |\xi_{i,1}| \leq \sqrt{\frac{2\phi d}{\eta}}.
\]

Hence, it is concluded that all the signals of the closed-loop system are bounded and the tracking errors can achieve satisfactory performance. This completes the proof.

**IV. SIMULATION STUDY**

This section provides an example to illustrate the effectiveness of the put forward control algorithm.

**Example:** Consider the following FO nonlinear large-scale systems:

\[
\begin{align*}
C_0 \frac{D_t}{i} q_1 & = -z_1 + x_1 \times 0.5 \\
C_0 \frac{D_t}{i} q_{1,2} & = x_1 + x_2 + z_1 \sin(x_1) x_2 \\
C_0 \frac{D_t}{i} q_{2,1} & = \sin(x_3) x_2 + 1 + 1 \sin(x_1) x_2 \\
C_0 \frac{D_t}{i} q_{2,2} & = \sin(x_2) x_2 + 1 + 1 \sin(x_1) x_2 \\
C_0 \frac{D_t}{i} q_{2,3} & = \sin(x_2) x_2 + 1 + 1 \sin(x_1) x_2 \\
C_0 \frac{D_t}{i} y_1 & = x_1 + 1 \\
C_0 \frac{D_t}{i} y_2 & = \sin(x_2) x_2 + 1 + 1 \sin(x_1) x_2 \\
\end{align*}
\]

(74)

where \( \alpha = 0.98, f_{1,1}(X_{1,1}) = x_{1,1}, f_{1,2}(X_{1,2}) = \sin(x_{1}) x_{2}, f_{2,1}(X_{2,1}) = x_{2} x_{2}, f_{2,2}(X_{2,2}) = \frac{1}{2} x_{2} \sin(x_{2}), g_1(x_1, y_1, t) = -z_1 + x_1^2 + 1 + 0.5, g_2(z_2, y_2, t) = -z_2 + 0.5x_2^2 + 1 + 0.5, H_1(z_1, Y, t) = z_1 \sin(x_1) x_2, H_2(z_1, Y, t) = z_1 \sin(x_1) x_2, \) and \( H_2(z_1, Y, t) = z_1 \sin(x_1) x_2. \)

To make Assumption 2 hold for the \( z_1 \)-systems in (74), we can choose \( V_1(z_1) = z_1^2. \) Then, by the inequality \( C_0 \frac{D_t}{i} q_{i}^{(x)}(x(t)) \leq x^{(x)}(t) C_0 \frac{D_t}{i} q_{i}(x(t)) \) in [32] and Young's inequality, one can obtain

\[
C_0 \frac{D_t}{i} q_{i} V_1(z_1) \leq z_1(-z_1 + x_1^2 + 1 + 0.5) \leq -0.5z_1^2 + x_1^2 + 1 + 0.5
\]

(75)

\[
C_0 \frac{D_t}{i} q_{i} V_2(z_2) \leq z_2(-z_2 + 0.5x_2^2 + 1 + 0.5) \leq -0.5z_2^2 + x_2^2 + 1 + 0.5
\]

(76)
Taking $\alpha_{i,1}(\varepsilon_{i}) = 0.5\varepsilon_{i}^{2}$, $\alpha_{i,2}(|\varepsilon_{i}|) = 1.5\varepsilon_{i}^{2}$, $c_{i} = 0.5$, $\gamma_{1}(\varepsilon_{i}) = x_{4,1}^{1}$, $\gamma_{2}(\varepsilon_{i}) = \frac{1}{2}x_{2,1}^{2}$ and $d_{i} = \frac{1}{2}$. Then, Assumption 2 holds. By selecting $\tilde{c}_{i} = 0.2 \in (0, c_{i})$, two FO dynamic signals can be described as follows:

$$\frac{\partial}{\partial t}^{\sigma} r_{1} = -0.2r_{1} + x_{4,1}^{1} + \frac{1}{4} \quad (77)$$

$$\frac{\partial}{\partial t}^{\sigma} r_{2} = -0.2r_{2} + \frac{1}{4}x_{2,1}^{2} + \frac{1}{4} \quad (78)$$

Define five Gaussian-type membership functions as $\exp(-(x - l_{i})^{2}/2s_{i}^{2})$, $i = 1, \cdots, 5$, they are uniformly distributed on $[-2, 2]$. Their membership functions are shown in Fig. 1.

The desired reference signals are given as $y_{1,d} = \sin(t)$. The design parameters are chosen as $c_{1,1} = 10$, $c_{1,2} = 50$, $c_{2,1} = 20$, $c_{2,2} = 50$, $\rho_{i,j} = \bar{\rho}_{i,j} = 0.1$, $\rho_{i} = 1/13$, $\rho_{i} = 0.1$, $\tau_{i,j} = \bar{\tau}_{i,j} = 0.01$ and $\sigma_{i,1} = 0.2$. The initial conditions of $r_{i}(0) = 4$ and other variables are all zero.

Figs. 2-7 reveal the simulation results. Figs. 2 and 3 depict the trajectories of desired reference signal $y_{1,d}$ and the system output $y_{1}$. Fig. 4 shows the trajectories of states $x_{1,2}$. The trajectories in Fig. 5 reveals the tracking errors $\zeta_{1,1}$. The trajectories of $u_{2}$ are shown in Figs. 6 and 7.

V. CONCLUSIONS

This paper has investigated the tracking control problem for FO nonlinear large-scale systems with unmodeled dynamics. The FLSs and the FO dynamic signals have been adopted to cope with the unknown nonlinear continuous functions and unmodeled dynamics, respectively. By combining adaptive backstepping recursive design algorithm with the FO Lyapunov stability criterion, a stable fuzzy adaptive decentralized DSC strategy has been developed. The designed control scheme has the main features of ensuring the controlled system stable and also making the tracking errors to be smaller. The validity of the developed control approach has been confirmed via simulated example. For future work, the problem of unmeasurable state will be further considered,
such as references [14, 15, 41, 42] or the control design of fractional-order nonlinear multi-agent systems.

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SHUAI SUI received the B.S. degree and the M.S. degree in applied mathematics from Liaoning University of Technology, Jinhzhou, China, in 2011 and 2014, respectively. He is currently working toward the Ph.D. degree in computer science with the Faculty of Science and Technology, University of Macau, Macau, China. His current research interests include fuzzy control, adaptive control, and stochastic control.

YONGLIANG ZHAN received the B.S. degree in information and computing science from the Liaoning University of Technology, Jinhzhou, China, in 2019, where he is currently pursuing the M.E. degree in operational research and cybernetics.

His current research interests include fractional-order system, fuzzy control, adaptive control.

JUNWEI JIN received the B.S. degree from the Ningxia University, Ningxia, China in 2013 and the M.S. and Ph.D degree from the University of Macau, Macau, China in 2015 and 2019, respectively. Since May, 2019, he has been an assistant professor with the School of Artificial Intelligence and Big Data, Henan University of Technology. His current research interests include machine learning, computer vision and neural networks.

SHAOCHENG TONG received the B.S. degree in Mathematics from Jinhzhou Normal College, Jinhzhou, China, the M.S. degree in Fuzzy Mathematics from Dalian Marine University, Dalian, China, and the Ph.D. degree in Fuzzy Control from the Northeastern University, Shenyang, China, in 1982, 1988, and 1997, respectively. He is currently a Chair Professor with the Navigation College, Dalian Maritime University, Dalian, China. His current research interests include fuzzy control, adaptive control, neural networks.
C. L. PHILIP CHEN (S'88-M'88-SM'94-F'07) received the M. S. degree from the University of Michigan, Ann Arbor, MI, USA, and the Ph. D. degree from Purdue University, West Lafayette, IN, USA, all in electrical engineering, in 1985 and 1988, respectively. He is a Fellow of IEEE, American Association for the Advancement of Science (AAAS), International Association of Pattern Recognition (IAPR), Chinese Association of Automation (CAA), and Chinese Association of Automation (HKIE); a member of Academia Europaea (AE), European Academy of Sciences and Arts (EASA), and International Academy of Systems and Cybernetics Science (IASCYS). He is the editor-in-chief of the IEEE Transaction on Cybernetics, and an associate editor of several IEEE Transactions. He received IEEE Norbert Wiener Award in 2018 for his contribution in systems and cybernetics, and machine learnings. He is also a 2019 highly cited researcher in both Computer Science and Engineering by Clarivate Analytics. He is currently the Chair Professor and Dean of the College of Computer Science and Engineering, South China University of Technology. His current research interests include systems, cybernetics, and computational intelligence.