Implications of the Electroweak Precision Data:

a 1996 Update

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Abstract

The most reliable prediction for the Higgs-boson mass, $M_H$, is obtained in a fit to the leptonic $Z$-resonance observables $\Gamma_l$ and $s^2_W$ combined with the $W$-boson mass, $M_W$, and top-quark mass, $m_t$, measurements. The corresponding bounds on $M_H$ are independent of potential uncertainties related to $R_b$, $R_c$, and $\alpha_s(M_Z^2)$, and they are not significantly further improved by including also the experimental information on the inclusive (hadronic and total) $Z$-boson decays. At the $1\sigma$ level, we obtain $M_H \lesssim 360$ GeV using $s^2_W$(LEP + SLD) and $M_H \lesssim 540$ GeV using $s^2_W$(LEP). Our analysis in terms of effective parameters confirms previous conclusions with increased accuracy. In the mass parameter, $\Delta x$, and the mixing parameter, $\varepsilon$, pure fermion loops are sufficient, while for the coupling parameter, $\Delta y$, ($M_H$-insensitive) bosonic contributions are essential for consistency with experiment, thus providing indirect empirical evidence for the non-Abelian structure of the theory.

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Employing the most recent 1996 electroweak precision data \[1\], in the present note we update the previous analyses \[2, 3, 4\] with respect to

(i) their implications on constraining the mass \(M_H\) of the Higgs scalar within the Standard Model (SM), in comparison with previous results \[5, 6\],

(ii) their implications in terms of the effective parameters \(\Delta x, \Delta y, \varepsilon, \text{and } \Delta y_b\), which quantify deviations from custodial SU(2) and SU(2)×U(1) symmetry within an effective Lagrangian \[3, 4, 7, 8\] for electroweak interactions at the Z-boson resonance and are closely related to the \(\varepsilon_i\) parameters of Ref. \[9\].

Bounds on the Higgs-boson mass

As discussed in a recent analysis \[3\], the 1995 electroweak precision data \[10\] lead to the following 1σ bounds for the mass of the Higgs boson:

\[
M_H = 148^{+263}_{-103} \text{ GeV} \quad \text{using} \quad s_W^2(\text{LEP+SLD})|_{95} = 0.23143 \pm 0.00028,
\]

\[
M_H = 343^{+523}_{-219} \text{ GeV} \quad \text{using} \quad s_W^2(\text{LEP})|_{95} = 0.23186 \pm 0.00034. \quad (1)
\]

These values and the corresponding upper bounds of \(M_H < 400 \text{ GeV}\) and \(M_H < 900 \text{ GeV}\) at the 1σ level are considerably higher than some of the results which appeared in the literature (compare e.g. Ref. \[6\]). The difference between (1) and results obtained by some other authors is essentially due to the exclusion of the 1995 value of \(R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})\) in the four-parameter \((m_t, M_H, \alpha(M_Z^2), \alpha_s(M_Z^2))\) fits which leads to the results \[1\]. In fact, it was argued in Ref. \[3\] that the exclusion of the experimental value for \(R_b\) from the set of data used to deduce the Higgs-boson mass was necessary in view of the upward shift of \(R_b^{\text{exp}}|_{95} = 0.2219 \pm 0.0017\) by more than three standard deviations with respect to the SM prediction. Indeed, inclusion of the 1995 value of \(R_b^{\text{exp}}\) in the set of data being fitted,

(i) within the SM effectively amounts to imposing a value of the top-quark mass, \(m_t\), which lies substantially below the result of the direct measurements at the Tevatron. This is a consequence of the enhancement of the 1995 value of \(R_b^{\text{exp}}\) in conjunction with the increase of the \((M_H\text{-insensitive})\) theoretical prediction for \(R_b\) with decreasing \(m_t\). It is precisely this preference of a low value of \(m_t\) which in turn implies a low value for \(M_H\) due to the well-known \((m_t, M_H)\) correlation in the other observables entering the fit. Since the low value of \(m_t\) is at variance with the direct Tevatron measurement, also the low value of \(M_H\) resulting from this fit was rejected.

(ii) allowing for a non-standard \(Z \rightarrow b\bar{b}\) vertex parametrized by an adjustable constant, yields a value of \(m_t\) consistent with the Tevatron measurement. At the same time, however, the stringent upper bounds on \(M_H\) resulting from including \(R_b^{\text{exp}}\) in the SM

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\[1\] Not including the direct Tevatron measurement of \(m_t\) in the set of data used in the fit, using \(s_W^2(\text{LEP+SLD})\), we obtained \[3\] \(m_t = 153 \pm 11 \text{ GeV}\) compared with \(m_t^{\text{exp}}|_{95} = 180 \pm 12 \text{ GeV}\) \[11\] from the Tevatron measurement of 1995. In the same fit, we obtained \(M_H = 35^{+50}_{-18} \text{ GeV}\).
fits are lost. The results for $M_H$ in this non-standard fit approximately coincide with the results (1), obtained by excluding $R_b^{\text{exp}}$ from the data set in the fit based on the unmodified SM.

Accordingly, in Ref. [2] it was concluded that the values (1) were indeed the only reliable ones obtainable from the 1995 data.

The most pronounced change in the 1996 data [1] relative to the 1995 data [10] occurred in $R_b$ and $R_c$. $R_b$ is now given by $R_b^{\text{exp}} = 0.2178 \pm 0.0011$ being hardly two standard deviations above the SM prediction for $R_b$ for $m_t = 175 \text{ GeV}$, while $R_c^{\text{exp}}$ is now in perfect agreement with the SM. We recall that owing to its relatively large error $R_c^{\text{exp}}$ only marginally influences the fits within the SM. We expect that the above-mentioned downward shift of $m_t$, occurring as a consequence of including $R_b^{\text{exp}}$ in the set of data being fitted, will be less drastic in the fit based on the 1996 value of $R_b^{\text{exp}}$. In addition to the change in $R_b^{\text{exp}}$, the Tevatron result for the top-quark mass has changed from the 1995 value of $m_t^{\text{exp}} = 180 \pm 12 \text{ GeV}$ [11] to the 1996 value of $m_t^{\text{exp}} = 175 \pm 6 \text{ GeV}$ [12]. As a consequence of both, the lower value of $R_b^{\text{exp}}$ and the more precise value of $m_t^{\text{exp}}$, we expect a very much reduced sensitivity of the results for $M_H$ on whether $R_b^{\text{exp}}$ is included in or excluded from the fit. Since the experimental values of other observables have not changed very much, we expect that the result for $M_H$ to be obtained from the full set of 1996 data will be close to the values (1).

In what follows, we present a detailed analysis, in order to investigate in how far the above expectation is actually reflected in the results obtained from the 1996 data.

The results of a four-parameter ($m_t, M_H, \alpha(M_Z^2), \alpha_s(M_Z^2)$) fit to the 1996 data, which are taken from Ref. [1] and explicitly listed in Tab. 1 are shown in the $(M_H, \Delta \chi^2)$ plots of Fig. 1 and in Tab. 2. In this fit, $M_H$ and $\alpha_s(M_Z^2)$ are treated as free fit parameters, while for $m_t$ and $\alpha(M_Z^2)$ the experimental information (central value and error) is taken into account. The reduced error in $m_t^{\text{exp}} = 175 \pm 6 \text{ GeV}$, in comparison with the results in Ref. [2], leads to a steeper $\Delta \chi^2$ distribution in Fig. 1 and a corresponding reduction of the errors on $M_H$ in Tab. 2. We observe that the results for $M_H$, as expected, are indeed relatively independent of whether the value for $R_b^{\text{exp}}$ is or is not taken into account in the fits. In Fig. 1 and Tab. 2 we also present the results obtained from fits to the “leptonic sector” (see Tab. 1), i.e. to the restricted set of data consisting of $s_{W}^2$, $M_W$, $\Gamma_1$ accompanied by $m_t^{\text{exp}}$ and $\alpha(M_Z^2)$. It is noteworthy that the results on $M_H$ from this restricted set of data are as accurate as the results from the full data sample, without being plagued, however, by potential uncertainties related to $R_b^{\text{exp}}$ and $\alpha_s(M_Z^2)$. In fact, the quality of the fits in terms of $\chi^2_{\text{min}}/\text{d.o.f.}$ according to Tab. 2 in the case of the leptonic sector is better than in the case of the full set of data.

The central values of $M_H$ deduced from the full set of 1996 data approximately resemble the values (1) from the 1995 set with $R_b^{\text{exp}}$ excluded. Indeed, in the case of $s_{W}^2$ (LEP+SLD), the value of $M_H = 158^{+148}_{-84} \text{ GeV}$ resulting from the full 1996 set of data is close to the value (1) of $M_H = 148^{+263}_{-103} \text{ GeV}$ [2] obtained upon excluding $R_b^{\text{exp}}$ from the 1995 set of data, while both values in turn differ appreciably from the value of $M_H = 81^{+134}_{-52} \text{ GeV}$ [2] as obtained from the full set of 1995 data.

2 Fits of the Higgs-boson mass to the 1996 set of data were also performed in Refs. [1] [7]. The results corresponding to the ones of Tab. 3, as far as available, are in good agreement with ours.
Turning to the dependence of the central value and the upper bound on $M_H$ on the input for $s_w^2$, we note that according to Tab. 1 the values of $s_w^2$(LEP + SLD) and $s_w^2$(LEP) are only one standard deviation apart from each other. Replacing $s_w^2$(LEP + SLD) by the higher value of $s_w^2$(LEP) nevertheless leads to an appreciable increase in the central value for $M_H$ by $\sim 100$ GeV, while the upper $1\sigma$ limits on $M_H$ increase by $\sim 200$ GeV. We finally note that $s_w^2$(SLD), which is approximately 3$\sigma$ below $s_w^2$(LEP), when taken by itself in conjunction with all other data implies $M_H = 14^{+25}_{-10}$ GeV, which seriously violates the lower bound of 65 GeV from the direct Higgs-boson search at LEP. Obviously the rather unclear empirical information on $s_w^2$, despite the high precision of the measurements, represents one of the main sources of uncertainty in $M_H$ fits at present.

The delicate interplay of the experimental results for $s_w^2$, $R_b$ and $m_t$ in constraining $M_H$ is visualized in the two-parameter ($m_t, M_H$) fits shown in Fig. 2. The (moderately) enhanced value of $R_b^{\text{exp}}$ still has the tendency of pulling down the fit values for $m_t$ to lower values than the ones obtained by the Tevatron measurement. Addition of the fairly precise value of $m_t^{\text{exp}} = 175 \pm 6$ GeV practically removes the dependence on $R_b^{\text{exp}}$ in accordance with the results for $M_H$ already displayed in Fig. 1 and Tab. 2.

The dependence of $m_t$ and $M_H$ on the central value as well as the error of the input parameter $\alpha(M_Z^2)$ has been taken into account by including it as a fit parameter in the four-parameter fits, which lead to the results in Fig. 1 and Tab. 4. As a deviation in $\alpha(M_Z^2)$ by, e.g., the amount of one standard deviation is by no means excluded, it is instructive to study the effect of varying the (fixed) input value of $\alpha(M_Z^2)$ by one standard deviation, as displayed in the second row of Fig. 2. We note that the fit results and the corresponding variations with different $\alpha(M_Z^2)$ in the case of the leptonic sector, which are not shown in Fig. 2, are very close to the ones obtained for the full set of data without $R_b^{\text{exp}}$, as shown in Fig. 1 (first column, second row). The fit results for $\alpha_s(M_Z^2)$, which has been treated as free fit parameter in the four-parameter fits, are consistent with $\alpha_s(M_Z^2) = 0.123 \pm 0.006$ resulting from an event-shape analysis at LEP [14]. Nevertheless, it is instructive to inspect also the dependence of the fit results for $m_t$ and $M_H$ on a fixed input for $\alpha_s(M_Z^2)$. The effect of varying $\alpha_s(M_Z^2)$ is illustrated in the third row of Fig. 2.

The last row of Fig. 2 shows the dependence of the 1$\sigma$ contours for the effective weak mixing angle, $s_w^2$. The level of sensitivity reached by the data for $s_w^2$ becomes apparent by noting that the shift in $M_H$ resulting from replacing $s_w^2$(LEP + SLD) by $s_w^2$(LEP) is of roughly the same magnitude as the shift due to the change in $\alpha(M_Z^2)$ by one standard deviation shown in the second row of Fig. 2.

The above discussion based on Tab. 2 and Figs. 1 and 2 may be summarized by concluding that the $R_b^{\text{exp}}$ effect of the 1995 data of effectively pulling down the fit value for $m_t$, and consequently of $M_H$, is still present in the 1996 data, even though the magnitude of the effect is strongly reduced. Nevertheless, one notices that the central values of $M_H$ obtained from the leptonic sector and based on the full set of data upon excluding $R_b^{\text{exp}}$ are very close together, while differing by $\sim 30$ GeV from the result from the full set of data. Taking into account the fact that the 1$\sigma$ errors of $M_H$ obtained in all three cases
are approximately the same, while $\chi^2_{\text{min}}/\text{d.o.f.}$ is best in the fit to the leptonic set of data, we prefer to quote this result of

$$ M_H = 190^{+174}_{-102} \text{ GeV}, \quad \text{using} \quad s^2_w(\text{LEP + SLD}) = 0.23165 \pm 0.00024, $$

$$ M_H = 296^{+243}_{-143} \text{ GeV}, \quad \text{using} \quad s^2_w(\text{LEP}) = 0.23200 \pm 0.00027, \quad (2) $$

as our final one from the 1996 electroweak data. We stress again that (2) has the advantage of being independent of $R_{\text{exp}}$ and $\alpha_s(M_Z^2)$. In comparison with the result (1) based on the 1995 set of data, the upper 1$\sigma$ bounds (2) on $M_H$ based on the 1996 set of data are strongly reduced to $M_H \lesssim 360$ GeV and $M_H \lesssim 540$ GeV, for $s^2_w(\text{LEP+SLD})$ and $s^2_w(\text{LEP})$, respectively.

| leptonic sector | hadronic sector |
|-----------------|-----------------|
| $\Gamma_1 = 83.91 \pm 0.11$ MeV | $R = 20.778 \pm 0.029$ |
| $s^2_w(\text{LEP}) = 0.23200 \pm 0.00027$ | $\sigma_h = 41.508 \pm 0.056$ |
| $s^2_w(\text{SLD}) = 0.23061 \pm 0.00047$ | $R_b = 0.2179 \pm 0.0012$ |
| $s^2_w(\text{LEP + SLD}) = 0.23165 \pm 0.00024$ | $R_c = 0.1715 \pm 0.0056$ |
| $M_W = 80.356 \pm 0.125$ GeV | $\Gamma_T = 2494.6 \pm 2.7$ MeV |

| input parameters | correlation matrices |
|------------------|---------------------|
| $M_Z = 91.1863 \pm 0.0020$ GeV | $\sigma_h$ |
| $G_\mu = 1.16639(2) \cdot 10^{-5}$ GeV$^{-2}$ | $R$ |
| $\alpha(M_Z^2)^{-1} = 128.89 \pm 0.09$ | $\Gamma_T$ |
| $\alpha_s(M_Z^2) = 0.123 \pm 0.006$ | $R_b$ |
| $m_b = 4.7$ GeV | $R_c$ |
| $m_t = 175 \pm 6$ GeV | $\sigma_h$ |

Table 1: The precision data used in the fits, consisting of the LEP data [1], the SLD value [13] for $s^2_w$, and the world average [14] for $M_W$. The partial widths $\Gamma_1$, $\Gamma_h$, $\Gamma_b$, and $\Gamma_c$ are obtained from the observables $R = \Gamma_h/\Gamma_1$, $\sigma_h = (12\pi\Gamma_1\Gamma_h)/(M_Z^2\Gamma_T^2)$, $R_b = \Gamma_b/\Gamma_h$, $R_c = \Gamma_c/\Gamma_h$, and $\Gamma_T$ using the given correlation matrices. The data in the upper left-hand column will be referred to as “leptonic sector” in the fits. Inclusion of the data in the upper right-hand column will be referred to as fitting “all data”. If not stated otherwise, the theoretical predictions are based on the input parameters given in the lower left-hand column of the table, where $\alpha(M_Z^2)$ is taken from Ref. [15], $\alpha_s(M_Z^2)$ results from the event-shape analysis [16] at LEP, and $m_t$ represents the direct Tevatron measurement [12].
Figure 1: $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ is plotted against $M_H$ for the $(m_t, M_H, \alpha(M_Z^2), \alpha_s(M_Z^2))$ fit to various sets of physical observables. For a chosen input for $s^2_W$, as indicated, we show the result of a fit to

(i) the full set of 1996 data, $s^2_W$, $M_W$, $\Gamma_T$, $\sigma_t$, $R$, $R_b$, $R_c$, together with $m_t^{\text{exp}}$, $\alpha(M_Z^2)$,

(ii) the 1996 set of (i) upon exclusion of $R_b$,

(iii) the 1996 "leptonic sector" of $s^2_W$, $M_W$, $\Gamma_L$, together with $m_t^{\text{exp}}$, $\alpha(M_Z^2)$. 
Table 2: The results of the \((m_t, M_H, \alpha(M_Z^2), \alpha_s(M_Z^2))\) fits to various sets of experimental data, as indicated. In detail, the rows (i) to (iv) are based on:

(i) the full 1996 set of \(s_w^2, M_W, \Gamma_T, \sigma_h, R, R_b, R_c\), together with \(m_t^{exp}, \alpha(M_Z^2)\),
(ii) the set as in (i) but excluding \(R_b\),
(iii) the 1996 “leptonic sector” of \(s_w^2, M_W, \Gamma_1\), together with \(m_t^{exp}, \alpha(M_Z^2)\),
(iv) the 1995 set as in (i), but excluding \(R_b\),

where for \(s_w^2\) the values of \(s_w^2(LEP + SLD)\) and \(s_w^2(LEP)\) are used in the fits, as indicated. The values for \(\alpha(M_Z^2)\) obtained in the fits reproduce the input given in Tab. \(\text{I}\) and accordingly are suppressed in the present table. For the leptonic sector \(\alpha_s(M_Z^2) = 0.123\) is used as fixed input (needed only for two-loop corrections), whereas \(\alpha_s(M_Z^2)\) is treated as unconstrained fit parameter in the other fits.

Figure 2: The results of the two-parameter \((m_t, M_H)\) fits within the SM are displayed in the \((m_t, M_H)\) plane. The different columns refer to the sets of experimental data used in the corresponding fits,

(i) “all data \(\setminus R_b\)”: \(s_w^2(LEP + SLD), M_W, \Gamma_T, \sigma_h, R, R_c\),
(ii) “all data”: \(R_b\) is added to set (i),
(iii) “all data + \(m_t^{exp}\)”: \(R_b, m_t^{exp}\) are added to the set (i).

The second and third row shows the shift resulting from changing \(\alpha(M_Z^2)^{-1}\) and \(\alpha_s(M_Z^2)\), respectively, by one standard deviation in the SM prediction. The fourth row shows the effect of replacing \(s_w^2(LEP + SLD)\) by \(s_w^2(LEP)\) and \(s_w^2(SLD)\) in the fits. Note that the 1\(\sigma\) boundaries given in the first row are repeated identically in each row, in order to facilitate comparison with other boundaries. The value of \(\chi^2_{\text{min}}/\text{d.o.f.}\) given in the plots refers to the central values of \(\alpha(M_Z^2)^{-1}\) and \(\alpha_s(M_Z^2)\). In all plots the empirical value of \(m_t^{exp} = 175 \pm 6\) GeV is also indicated.
Figure 2:
Analysis in terms of effective parameters

We turn to the second part of the present work and investigate electroweak interactions from the point of view of the effective Lagrangian developed in Refs. [3, 4, 7, 8]. This approach allows for a detailed assessment of which elements of the electroweak theory are quantitatively tested by the precision experiments at the Z-boson resonance. In particular, the question in how far the non-Abelian structure of the theory enters the predictions (and in how far it is tested) and questions on the empirical evidence for the Higgs mechanism may be answered by this approach.

In addition to the canonical input parameters—the Fermi coupling $G_\mu$, the effective electromagnetic coupling at the Z-boson mass scale, $\alpha(M_Z^2)$, and the Z-boson mass, $M_Z$—for the interactions of the vector bosons with charged leptons the effective Lagrangian contains three parameters, $\Delta x$, $\Delta y$, $\varepsilon$. The parameters $\Delta x$, $\Delta y$, $\varepsilon$ are related to SU(2) breaking in the vector-boson masses, in their couplings to charged leptons, and in the mixing among the neutral vector bosons, respectively. For vanishing values of $\Delta x$, $\Delta y$, $\varepsilon$, in the leptonic sector the Lagrangian coincides with the one of the SM in the unitary gauge, and its (tree-level) predictions coincide with the $\alpha(M_Z^2)$-Born approximation of the SM. In order to extend the Lagrangian to the interactions of neutrinos and quarks, additional parameters, related to the corresponding couplings, have been introduced in Ref. [4]. They are given by $\Delta y_\nu$ for the neutrino, $\Delta y_b$ for the bottom quark, and $\Delta y_h$ for the remaining light quarks.

In our analysis we restrict ourselves to a four-parameter ($\Delta x$, $\Delta y$, $\varepsilon$, $\Delta y_b$) fit, assuming that $\Delta y_\nu$ and $\Delta y_h$ are well represented by vertex corrections in the SM. We note in passing that the corrections entering $\Delta y_\nu$ and $\Delta y_b$ only depend on vector-boson–fermion interactions and do not involve the non-Abelian structure of the theory (see Ref. [4]). The results of our fit to the data of Tab. 1 are shown in Tab. 3 and Fig. 3.

The comparison in Tab. 3 with the results of the analysis in Ref. [4], which have been based on the 1994 data, shows a significant decrease of the experimental errors to roughly two thirds of their 1994 value. The most drastic shift in the central values of the parameters occurred in $\Delta y_b$, due to the shift of the experimental value for $R_b$, as discussed in the preceding section. The 1σ ellipse in the ($\Delta y_b$, $\varepsilon$) plane of Fig. 3 now includes the value of $m_t = 160$ GeV, while with the 1994 value it only reached values of $m_t \sim 100$ GeV.

Otherwise, the present analysis at an improved level of accuracy confirms conclusions drawn previously [3, 4]:

(i) The effective parameters deviate significantly from zero, the value which corresponds to the $\alpha(M_Z^2)$-Born approximation of the SM. Genuinely electroweak corrections are definitely required within the SU(2)×U(1) theory to obtain consistency with the experimental results.

3When comparing the present results with the ones of Ref. [4], one should notice that in Ref. [4] the value of $\alpha_s(M_Z^2) = 0.118 \pm 0.007$ was used and that the ellipses shown in the plots for the effective parameters were scaled by a factor 1.4 (corresponding to a confidence level of 61% but not 83%, as erroneously indicated in Ref. [4]).

4Prior to the start of LEP experiments it was stressed [19] that the difference between the dominant fermion-loop and the full one-loop corrections sets the scale for genuine electroweak precision tests. This precision was first reached in 1994 [3] but not yet in 1993 [8, 20].
(ii) In the case of the “mass parameter” $\Delta x$ and the “mixing parameter” $\varepsilon$, the pure contributions of the fermion loops to the $W^\pm$, $Z$, and $\gamma$-boson propagators, i.e. neglecting all contributions involving loops containing vector bosons,

$$\Delta x \simeq \Delta x_{\text{ferm}}, \quad \varepsilon \simeq \varepsilon_{\text{ferm}},$$

lead to consistency with the experimental data. Note that $\Delta x_{\text{ferm}}$ contains a dominating $m_t^2$ and a log $m_t$ term besides a small constant contribution, while $\varepsilon_{\text{ferm}}$ is dominated by a large constant term and contains an additional log $m_t$ contribution. The (logarithmic) Higgs-boson mass dependence is entirely contained in the bosonic contributions, $\Delta x_{\text{bos}}$ and $\varepsilon_{\text{bos}}$, to $\Delta x$ and $\varepsilon$. In view of the results of the $M_H$ fits discussed above, it is not surprising that these bosonic contributions are not well resolved by the projection of the ellipsoid into the $(\Delta x, \varepsilon)$ plane shown in Fig. 3. The shift to somewhat higher values of $M_H$, if $s_W^2(\text{LEP + SLD})$ is replaced by $s_W^2(\text{LEP})$, or if $\alpha(M_Z^2)^{-1}$ is replaced by $\alpha(M_Z^2)^{-1} + \delta \alpha(M_Z^2)^{-1}$, is nevertheless clearly visible in Fig. 3. Note that both the uncertainty in the experimental value of $s_W^2$ as well as the one in $\alpha(M_Z^2)$ mainly affect the mixing parameter $\varepsilon$.

(iii) In the “coupling parameter” $\Delta y$ a contribution beyond fermion loops,

$$\Delta y = \Delta y_{\text{ferm}} + \Delta y_{\text{bos}},$$

is definitely required for consistency with the data. As seen in Tab. 3, the large negative fermion-loop contribution in the SM is overcompensated by a positive bosonic part which is practically independent of $M_H$. As pointed out in Ref. [21], the large fermionic and bosonic contributions to $\Delta y$ may be traced back to the use of the low-energy parameter $G_\mu$ as input for the theoretical predictions. Indeed, the main fermionic and bosonic contributions to $\Delta y$ are identical to the loop contributions $\Delta y^{\text{SC}}$, connecting the low-energy charged-current coupling, $g_{\text{W}^\pm}(0) \propto G_\mu M_W^2$, with the high-energy charged-current coupling, $g_{\text{W}^\pm}(M_W^2) \propto G_\mu M_W^2/(1 + \Delta y^{\text{SC}})$, appearing in the leptonic width of the W boson. For details, we refer to Ref. [21]. Even though the theoretical value of $\Delta y$ is extremely insensitive to $M_H$ and may even be derived in the framework of a massive vector-boson theory [22], the agreement of the theoretical prediction with experiment constitutes significant positive empirical evidence for the non-Abelian sector of the SM.

We finally note that the isolation of the experimentally resolved dominant, $M_H$-insensitive bosonic corrections (in $\Delta y$) from the small, $M_H$-dependent ones (in $\Delta x$ and $\varepsilon$) is a specific feature of our choice of parameters, naturally implied by examining SU(2) breaking in an effective Lagrangian. This separation is not present in the widely used $\varepsilon_i$ parameters [9] related to ours via [4]

$$\varepsilon_1 = \Delta x - \Delta y + 0.2 \times 10^{-3}, \quad \varepsilon_2 = -\Delta y + 0.1 \times 10^{-3},$$
$$\varepsilon_3 = -\varepsilon + 0.2 \times 10^{-3}, \quad \varepsilon_b = -\Delta y_b/2 - 0.1 \times 10^{-3}.$$  

In $\varepsilon_1$, the $M_H$-dependent contribution to the mass parameter, $\Delta x$, appears in linear combination with the $M_H$-insensitive bosonic correction contained in the coupling parameter $\Delta y$. 

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Table 3: The results for the four parameters $\Delta x$, $\Delta y$, $\varepsilon$, $\Delta y_b$.
(a) obtained by a fit to the experimental data, as specified (the 1994 results are taken from Ref. [4]). The first error is statistical ($1\sigma$), the second due to the deviation by replacing $\alpha(M_Z^2)^{-1} \rightarrow \alpha(M_Z^2)^{-1} \pm \delta \alpha(M_Z^2)$, and the third due to $\alpha_s(M_Z^2)^{-1} \rightarrow \alpha_s(M_Z^2)^{-1} \pm \delta \alpha_s(M_Z^2)$. (b) as predicted within the SM for $m_t = 175$ GeV and $M_H = 300$ GeV. The “full” predictions, which include the complete one-loop and leading two-loop corrections (see Ref. [4]), are compared to the $O(\alpha)$ fermion-loop contributions. The first upper/lower deviations correspond to $\pm 6$ GeV in $m_t$, the second (if given) to the variations of $M_H$ from 60 GeV (lower) to 1 TeV (upper).

| Tab. 3a | '96 data $s_w^2$(LEP + SLD) | '94 data $s_w^2$(LEP) |
|---------|----------------------------|------------------|
| $\Delta x/10^{-3}$ | $11.8 \pm 3.2 \pm 0.2 \mp 0.1$ | $12.6 \pm 3.2 \pm 0.2 \mp 0.0$ |
| $\Delta y/10^{-3}$ | $8.4 \pm 3.3 \pm 0.2 \pm 0.4$ | $8.9 \pm 3.3 \pm 0.2 \pm 0.4$ |
| $\varepsilon/10^{-3}$ | $-3.8 \pm 1.2 \pm 0.5 \pm 0.3$ | $-4.9 \pm 1.2 \pm 0.5 \pm 0.3$ |
| $\Delta y_b/10^{-3}$ | $6.7 \pm 3.8 \pm 0.0 \pm 4.4$ | $6.1 \pm 3.8 \pm 0.0 \pm 4.4$ |

| Tab. 3b | theory | full | ferm. 1-loop |
|---------|--------|-------|--------------|
| $\Delta x/10^{-3}$ | $11.8^{+0.7}_{-0.7}^{1.4} \pm 1.1$ | $11.7^{+0.8}_{-0.8}$ |
| $\Delta y/10^{-3}$ | $7.3^{+0.1}_{-0.1}^{1.0} \pm 0.2$ | $-6.4^{+0.1}_{-0.1}$ |
| $\varepsilon/10^{-3}$ | $-5.2^{+0.6}_{-0.0}^{1.0} \pm 1.3$ | $-5.8^{+0.0}_{-0.0}$ |
| $\Delta y_b/10^{-3}$ | $11.8^{+1.1}_{-1.0}^{1.1} \pm 0.1$ | 0 |
Figure 3: The projections of the 1σ ellipsoid of the electroweak parameters $\Delta x$, $\Delta y$, $\varepsilon$, $\Delta y_b$ obtained from the 1996 set of data in comparison with the SM predictions. Both the results obtained from using $\bar{s}_W^2$(LEP) and $\bar{s}_W^2$(LEP + SLD) as experimental input are shown. The full SM predictions correspond to Higgs-boson masses of 100 GeV (dotted with diamonds), 300 GeV (long-dashed dotted) and 1 TeV (short-dashed dotted) parametrized by the top-quark mass ranging from 120 GeV to 220 GeV in steps of 20 GeV. The pure fermion-loop prediction is also shown (short-dashed curve with squares) for the same values of $m_t$. The arrows indicate the shifts of the centres of the ellipses upon changing $\alpha(M_Z^2)^{-1}$ to $\alpha(M_Z^2)^{-1} + \delta\alpha(M_Z^2)^{-1}$ and $\alpha_s(M_Z^2)$ to $\alpha_s(M_Z^2) + \delta\alpha_s(M_Z^2)$. 
Conclusions

With the 1996 data, the agreement between experiment and the predictions of the electroweak Standard Model (SM) has become even more impressive. From the analysis in terms of the parameters in an effective Lagrangian we learned that the non-Abelian structure of the theory entering the bosonic loops is quantitatively supported by the empirical data. The upper 1σ bounds on the Higgs-boson mass, $M_H$, have improved to 360 GeV and 540 GeV based on $s^2_W$ (LEP + SLD) and $s^2_W$ (LEP), respectively. Changing $\alpha(M^2_Z)$ by one standard deviation leads to changes in the upper bounds on $M_H$ which are similar to the ones induced by the difference between $s^2_W$ (LEP + SLD) and $s^2_W$ (LEP). At the 95% C.L. we obtain $M_H \lesssim 550$ GeV and $M_H \lesssim 800$ GeV, respectively. We stress that these bounds already follow from the reduced set of data of $s^2_W$, $M_W$, $\Gamma_l$, $m^\text{exp}_t$, and $\alpha(M^2_Z)$, thus avoiding potential uncertainties related to $R_b$, $R_c$, and $\alpha_s(M^2_Z)$. Moreover, the bounds on $M_H$ are not significantly further improved by including also the experimental information on the inclusive (hadronic and total) Z-boson decays. The fact that the bounds on $M_H$ lie in the perturbative regime of the SM may be interpreted as supporting the concept of the Higgs mechanism, even though the ultimate answer to the question of its realization in nature cannot be given as long as experimental evidence for the existence of the Higgs boson is missing.

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