LETTER

The SVD beamformer with diverging waves: a proof-of-concept for fast aberration correction

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Abstract

The rise of ultrafast ultrasound imaging—with plane or diverging waves—paved the way to new applications of ultrasound in biomedical applications. However, propagation through complex layers (typically fat, muscle, and bone) hinder considerably the image quality, especially because of sound speed heterogeneities. In difficult-to-image patients, in the case of the hepatic steatosis for instance, a good image and a reliable sound speed quantification are crucial to provide a powerful non-invasive diagnosis tool. In this work, we proposed to adapt the singular value decomposition (SVD) beamformer method for diverging waves and thus present a novel aberration correction approach for widely used curved arrays. We probed its efficiency experimentally both in vitro and in vivo. Besides the proposed matrix formalism, we explored the physical meaning of the SVD of ultrafast data. Finally, we demonstrated the ability of the technique to improve the image quality and offer new perspectives particularly in quantitative liver imaging.

1. Introduction

In the last 20 years, ultrafast ultrasound imaging (Smith et al 1991, Sandrin et al 1999, Tanter et al 2002b, Tanter and Fink 2014, Jensen et al 2016) based on received parallel beamforming (Bruneel et al 1977, Shattuck et al 1984, von Ramm et al 1991) has been widely broadening the application range of echography. Using a small set of single transmissions to image the whole medium with the full transducer array, it offered a frame rate capability typically up to 10 000 images per second. Either plane or diverging waves have been used, and the transmission rate was largely dictated by the compromise between frame rate and image quality (Tabei et al 2003, Montaldo et al 2009). Ultrafast ultrasound images still suffer from potential aberrations due to sound speed heterogeneities and complex tissue layers. They affect both the image quality and any ensuing quantitative assessment such as speed-of-sound or Doppler flow. For instance, liver ultrasound is made challenging by successive muscle and fat layers that may strongly distort the wave front (O’donnell and Flax 1988, Pinton et al 2011). Research efforts towards non-invasive quantification of liver steatosis, Non Alcoholic SteatoHepatitis (NASH) diagnosis, and steatosis staging with ultrasound imaging (Palmeri et al 2008, Dasarathy et al 2009, Sasso et al 2010, Deffieux et al 2015, Imbault et al 2017, Jakovljevic et al 2018, Dioguardi Burgio et al 2019, Nguyen et al 2019) are numerous using either dispersion of shear waves, ultrasonic backscattering attenuation or acoustic sound speed. Speed-of-sound was recently proposed to be a potential quantitative biomarker of the disease (Imbault et al 2018, Kumagai et al 2019). Nevertheless, to provide significantly improved results, the aberration correction issue affecting the estimation of the acoustic sound speed parameter remains highly challenging. In liver, as in abdominal organs, prostate or heart, diverging waves are generally preferred (Lockwood et al 1998, Cikes et al 2014, Papadacci et al 2016) since they overcome the limited field of view accessible with plane waves. Besides, they suffer less from
aberration effects (Rau et al 2019). For each diverging wave, a virtual source is placed behind the transducer and different backscattered echoes from transmissions from successive virtual sources are coherently compounded to form the final ultrafast image.

In an attempt to propose a solution to the longstanding problem of real time aberration correction in medical ultrasound, we recently introduced a mathematical framework and beamforming technique based on the singular value decomposition (SVD) of ultrafast data acquired with plane waves, the so-called SVD beamformer (Bendjador et al 2020). Many interesting approaches for aberration correction have been developed (Dahl 2017, Chau et al 2019, Lambert et al 2020a, 2020b) and most are designed to exploit the spatial coherence of the backscattered speckle noise (Mallart and Fink 1991, Aubry and Derode 2009, Osmanski et al 2012, Shin et al 2018). The SVD beamformer is such an approach and has the strong advantage to be straightforward to implement, and to allow computations of correction in real time. Here, we extend the SVD beamforming approach to diverging waves which provides, with a single mathematical operation performed on the ultrafast raw data, both the corrected image and the estimation of the complex aberration law. We build the ultrafast compound matrix in the basis defined by the positions of the virtual sources and investigate its first singular vectors.

After an introduction of the matrix formalism for SVD beamforming with diverging waves, we provide experimental validation of its in vitro and in vivo efficiency. SVD beamforming yields a simple and real-time approach for aberration correction implemented on curved arrays for difficult-to-image patients which could pave the way to novel and precise sound speed quantification techniques in liver.

2. SVD Beamforming for diverging transmissions

2.1. Theory

Scattering and propagation of ultrasound in a given medium is fully characterized by its Green function $h_{0i}(t)$, relating each pixel $p$ of the image to each element $i$ of the ultrasonic transducer (Prada and Fink 1994). In Fourier space, the Green’s functions form the propagation operator $H(\omega)$, and convolution operations conveniently become products. In the following section, the variables will thus be defined in the Fourier domain as functions of the pulsation $\omega$. The formalism described here being valid at each frequency, we will assume $H = H(\omega) = H(\omega_0)$, where $\omega_0$ is the central frequency of the transmitted signal. This monochromatic assumption will be used in the equations below for a sake of clarity.

Let $E(\omega)$ be the $[N_e, 1]$-emission vector in the canonical space, at each frequency, where $N_e$ is the number of elements in the ultrasound array. $E(\omega)$ contains, for each element, the Fourier transform of the transmitted time-signal at a frequency $\omega$. Assuming a lossless propagation, the signal $S$ received on the array can then be expressed as a matrix product:

$$S = H\Gamma^*HE$$

Equation (1) described the propagation at emission ($^tH$) and at reception (H) through the scattering medium, modeled by the matrix $\Gamma$.

The ultrasound image formation requires to compensate for both emission and reception propagation in the so-called beamforming process. Based on the propagation model $H_0$, this consists in estimating:

$$I = {^tH_0^*H^*}\Gamma^*HE$$

Where $^*$ stands for the complex conjugate.

Conventional B-mode ultrasound relies on the use of successive focused emissions $E_i$. Each emission $E_i$ creates the $i$-th line of the image and their successive accumulation results in the conventional B-mode image. Because $H_0$ is the ideal propagation model from the pixels to the array, $H_0^*$ contains, in each column the Green’s function phase conjugate for the pixel $p$. This corresponds to a transmit vector focusing on pixel $p$ that we can denote $E_i = H_0^*p$. In the end, the beamformed image $I_m$ is given by:

$$I_m = \text{diag}(H_0^*H^*H_0)$$

Bendjador et al 2020 proposed to describe the coherent plane wave compounding by introducing a change-of-basis matrix $P$ between the canonical space and the space defined by the plane waves at different transmit angles in equation (3). Here, the matrix formalism proposed in equation (2) can be adapted to diverging emissions using a change-of-basis matrix $P$ between canonical space and the space defined by the transmitting sources positions. $N_e$ and $N_s$ being respectively the number of elements in the array and the number of transmitting virtual sources, $P$ is a $[N_s, N_e]$- matrix. The final image, obtained by diverging wave compounding, is described similarly with a filter potential $PP^{*}$ when there are less sources than transducer elements:
\[ I_m = \text{diag}(H^a_i \ H^t \ H^p \ P^a H^s_i) \] (4)

We propose to introduce the Ultrafast Compound Matrix \( R \) (Bendjador et al. 2020) in the basis of diverging emissions. It contains the \( N_s \) beamformed images for each virtual source. Thus, \( R \) is a \([N, N_s]\)-matrix where \( N \) is the number of pixels in the image. It can be written:

\[ R = (H^a_i \ H^t \ H^p) \circ (H^s_i \ P^a) \] (5)

From equation (5), the SVD Beamformer approach, developed in (Bendjador et al. 2020) for plane wave transmits, can be transposed to diverging wave transmits. Therefore, we introduce the sources coherence matrix measuring the coherence between ultrasound images acquired with different virtual sources positions, and defined by:

\[ C_S = R^*R \] (6)

The best image achievable, corrected from any aberration, is the one that maximizes the module of coherence between sources. Considering first an aberrating phase and amplitude screen, the aberration can be expressed as a diagonal matrix in the canonical space. In typical medical ultrasound configurations, and when \( P^{-1} \) is defined, we can assume that the spatial frequency variations are sufficiently low to approximate the aberration in the emitting sources \( S \) remaining diagonal. Although the phase screen approximation is a simplistic approximation of the aberrations, we will see that more realistic and complex aberrating layers can be described by a set of aberrating phase and amplitude screens for different regions of the medium. We thus define \( A \) to be the aberration correction matrix in the diverging transmits space. Under this assumption, the corrected image is given by RA since the aberrations have been compensated. The covariance, between different transmitting sources, of the ideal image is maximal. Thus, correcting the aberrations consists in maximizing the covariance of the image. This solution ascertains \( X = \text{diag}(A) \). It can be expressed as a Rayleigh quotient maximization:

\[ J(X) = \frac{\text{tr}(RX^*RX)}{X^*X} = \frac{X^*(RX)X}{X} \] (7)

Interestingly, the Rayleigh quotient is maximized by the first eigen vector of \((RX^*R)\). This matrix being Hermitian, its eigen vectors are also singular vectors of \( R \). So, the first singular vector of \( R \) maximizes the Rayleigh quotient and consequently, the covariance of beamformed data. This leads us to write the SVD of \( R \):

\[ R = \sum_{i=1}^{N_s} \lambda_i V_i(x, y) U_i^* \] (8)

Where \( S \) designates the emission space, constituted by the diverging sources.

The first spatial singular vector \( V_i \), having the highest singular value, represents the best linear fit of the lines of \( R \). Each of them contains the \( N_s \) complex amplitudes of a given pixel seen by the different diverging waves. Thus, the first singular vector maximizes the coherence between the images of the pixel acquired with different sources. The first singular vector \( U_1^* \) contains the aberration, in phase and amplitude, in the diverging emissions space. So \( U_1 \) gives as demonstrated above the aberration correction. \( V_i \) is then the corrected image where all pixels behave as point reflectors independently of the virtual source position. Physically, it means that for the first singular vector, all sources see the same image of the medium, as they would in absence of any aberration.

### 2.2. Methods

To assess the SVD beamforming efficiency with diverging transmissions, ultrasound acquisitions were programmed on a 256-channel research scanner (Verasonics Research Systems). We used a 192-element curve probe, at a central frequency of 3.9 MHz, pitch size of 0.35 mm and radius of curvature of 5.7cm (GE C1-6D probe). Diverging waves were transmitted by placing virtual sources behind the transducer at a height close to curvature center, and at homogeneously distributed abscissas covering the transducer width. A set of \( N_s = 101 \) diverging waves, was transmitted from different sources positions at a Pulse Repetition Frequency (PRF) = 10 kHz.

Acquisitions were performed in vitro on two commercial ultrasound phantoms (ATS551 - 1450 m s\(^{-1}\) and CIRS054GS—1540 m s\(^{-1}\)). We also tested the approach on in vivo liver data, on patients from the QUID -NASH research project which aims at evaluating non-invasive tools in type 2 diabetic patients with NASH. The ultrasound emission sequences were calibrated to meet the FDA Track 3 Recommendations, and the clinical trial was approved by the French National Agency of Health and Medicine under the protocol 2018-A00311-54.

Images were acquired during fasting, with patients in supine position, using an intercostal approach on the right liver lobe during a 3-second neutral breath apnea.

For each transmitting source, the image was individually beamformed, leading to \( N_s \) independent images of the medium. Their coherent summation would consist in the classical coherent compounding method with diverging waves (Lockwood et al. 1998, Hazard and Lockwood 1999, Jensen et al. 2006, Papadacci et al. 2016). The
ultrafast compound matrix $R$ is built with all complex (IQ) images from the $N_S$ transmissions before compounding. $R$ is reshaped in a 2D Casorati $[N, N_S]$-matrix and SVD is performed. It separates the image-space variations from the transmission-space variations (Bendjador et al 2020). As shown in figure 1, the filtering of the first singular vector provides both the corrected image ($V_1$) and the aberration law ($U_1^T$). For this illustration purpose, a simulated aberration was introduced in post-processing and is fully recovered by the SVD beamforming process.

As for SVD beamforming with plane waves, the process may be performed successively on individual regions of interest called ‘patches’ to take the limited isoplanatism angle of the aberration into account (Tanter et al 1998). Indeed, the spatial extension of the aberration in practical configurations hinders the phase and amplitude screen approximation when the SVD beamforming is applied to a very large region. By subdividing the full imaging into smaller patches, within which isoplanatism approximation remains valid, SVD beamforming allows to correct highly complex aberrations. The typical patch size is not universal and mostly depends on the nature of the aberrator. It decreases with the increase of spatial frequencies in the aberration law.

3. Results

First, we investigated the image correction ability of the SVD beamforming with diverging waves by considering the spatial singular vector $V(x, z)$ containing the image corrected from aberrations. We probed its efficiency on an in vitro phantom containing anechoic cysts (ATS551, figure 2(a)), using a default sound speed assumption of 1540 m s$^{-1}$ and introducing, the same virtual aberration as in figure 1. SVD beamforming provided a B-mode image of significantly higher quality by simultaneously correcting the default sound speed estimation and the aberration law. This was quantified using the contrast ratio $CR$ defined by:

$$CR = \frac{\mu_{\text{in}}}{\mu_{\text{out}}}$$

Where $\mu_{\text{in}}$ and $\mu_{\text{out}}$ respectively define the mean B-mode image intensity inside and outside an anechoic cyst. The contrast improvement was obtained by computing the curve differences from ultrafast compound to SVD beamformer (blue - orange). We found improvements by 5.3 dB in vitro and by 4.3 dB and 3.7 dB in patients liver (figures 2(b), (c)). Qualitatively, one can also see an improvement in the in vivo structures definition which is a promising result of the SVD beamforming approach with diverging waves. One may note that patient 1 has a much thinner fat layer above the liver than patient 2. That makes the patient 1 ‘easier-to-image’ with ultrasound, and apparently easier to correct with our method too. Patient 2 is indeed a case further from our thin screen hypothesis made in the theoretical demonstration.

The ultrasonic imaging sequence was based on many sources ($N_S = 101$) in order to reach optimal image quality and detailed description of the aberration. We previously demonstrated that $N_S$ could be significantly
reduced to satisfy ultrafast frame rates while still achieving significant correction (Sasso et al. 2010). In figure 2, contrasts plots, we quantified the influence of the number of independent virtual sources on the aberration correction by applying our approach down to 5-source acquisitions. Both for in vitro and in vivo experiment, the contrast rise due to SVD beamforming remain notable, respectively of 4.7 dB, 2.0 dB and 1.6 dB. Thus, SVD beamformer improvements stand - even if the true aberration is not fully depicted by the limited number of diverging waves.

Last, we investigated how the phase and amplitude aberration of the first singular vector could allow us to quantify the sound speed in homogeneous medium, or at least inform on the accuracy of the beamforming sound speed (as first discussed in Bendjador et al. 2020). Indeed, when taking a sufficiently large patch, the SVD beamformer does not correct local aberrations anymore and rather gives a global deviation due to a mismatch with the speed of sound in the medium. When the sound speed chosen for beamforming is lower than the actual sound speed, the phase aberration is a convex hyperbole (and concave when the chosen sound speed is higher). The optimal sound speed is determined by the flattest curve in phase aberration. This means that all images provided by individual virtual sources are in phase. The optimal average sound speed corresponds to the mean sound speed integrated along the path from the transducer array to the center of the patch.

Figure 3 shows the application of this problem in the CIRS speckle phantom of reference sound speed 1540 m s \(^{-1}\). For a sake of clarity, only five sound speeds are represented even though an increment of 5 m s \(^{-1}\) was used for the range of tested beamforming speeds from 1480 m s \(^{-1}\) to 1600 m s \(^{-1}\).

The emission delays remain constant when the beamforming speed of sound changes. Thus, we had to compute new sources positions for each of the tested sound speed values. This can be performed by solving an over-constrained system to find the best source position correction which might not be uniquely defined. Here, we assumed that the best solution was the weighted average of all solutions, for a given transmit. After SVD Beamforming, the first singular vector yields the phase aberration law. In such a homogeneous medium, we obtained phase laws of different curvatures. The flattest curve was reached with the beamforming sound speed of 1540 m s \(^{-1}\) ±/− 5 m s \(^{-1}\), which corresponds to the expected value; the variance being given by the chosen span for beamforming sound speeds. SVD beamforming thus allows to get a first estimate of the global sound speed of a medium, for a more accurate initial image formation.

4. Discussion

In this article, we proposed an adaptation of SVD beamforming for diverging wave transmission. After setting up the matrix formalism, we demonstrated experimentally that it allows real-time image correction that could be
The wave number should be considered. The wave number would then be described as a complex value.

Rigorously, to account for the attenuation and dispersion when solving the wave equation, the imaginary part of the retrieval can provide a useful in ultrafast ultrasound imaging of difficult-to-image patients. We also presented how the phase aberration retrieval can provide a first integrated sound speed estimation of the whole medium.

In the theoretical part of this Letter, we described the ultrasound propagation as a simple lossless model. Rigorously, to account for the attenuation and dispersion when solving the wave equation, the imaginary part of the wave number should be considered. The wave number would then be described as a complex value $k(\omega) = \frac{2\pi c}{\lambda} + i\alpha(\omega).$ This breaks the monochromatic assumption as well as the reciprocity of the wave equation. The phase aberration would then be different at each frequency. Attenuation is somehow considered by this approach since SVD beamforming optimizes coherence, also in amplitude, between sources. In the case of a constant acoustic loss homogeneously distributed in space, the absorption effects during propagation do not strongly modify the spatial distribution of time shifts and amplitudes of the wavefront (Tabei et al. 2003). In such configurations, neglecting these acoustic losses and correcting only phase shifts was found to be sufficient in a first approximation (Tanter et al. 2000, Tanter et al. 2002a). Ultimately, our approach provides a first order efficient correction under the lossless and monochromatic assumptions, and more extensive work on theoretical models could consider both attenuation and dispersion in the image formation problem.

Then, we recalled the physical meaning for the SVD of ultrafast diverging wave data and took benefit from it to achieve real-time estimation of aberration corrections. Indeed, the mathematical singular value decomposition intrinsically allows to separate spatial variables and transmission basis related variables. The first singular vectors give on the one hand the corrected image and on the other hand the aberration amplitude and phase. Interestingly, this operation allows almost real-time aberration correction since computation times of SVD are already of the order of 100 ms. The experimental validation was achieved by performing acquisitions using diverging waves transmitted with a curved array well suited for clinical liver imaging. The approach could be easily adapted for phased array geometries for instance for cardiac imaging or transcranial imaging.

In strongly aberrating media, such as the skull for transcranial imaging, the aberration correction may be jointly implemented in the receive mode. To do so, the aberration law obtained in transmission by SVD Beamforming can be transposed to the canonical basis and used to correct the reception time delays to further improve the overall aberration correction. This additional step would enhance the correction effect of our approach but also increase the computation times.

For this proof-of-concept demonstration, we first used a large number of sources (Nt = 101). We then showed that even if the aberration complexity might not be fully described by a limited amount of transmits, SVD beamforming still allows aberration correction. The improvement was confirmed by a contrast increase both in vivo and in vitro. The ability to lower the number of sources is indeed a key towards ultrafast Doppler imaging where a trade-off has to be found between image quality and frame rate. Thus the efficiency of SVD beamformer with lower transmits already offers a promising tool for adaptive Doppler imaging.

We also showed a first example of SVD beamforming application to sound speed estimation. Here, our goal is to minimize the phase of the first singular vector, which is the key towards aberration correction. On a speckle phantom, by beamforming at different sound speeds and comparing the aberration laws, we demonstrated that we obtained hyperbolic aberration laws for the wrong sound speeds, and that the flattest law corresponded to the actual sound speed of the medium. This first estimation remains only valid within an homogeneous region, in the absence of any aberration. Though, one can circumvent this limit by applying SVD beamformer on isoplanatic patches. In each of them, the sound speed is then supposed to be homogeneous and one would be able to differentiate the tissues within an in vivo ultrasound image. One drawback of this approach is that it requires to re-beamform at each sound speed, and thus loses the real-time interesting aspect of SVD beamforming. Further developments are currently studied to solve the inverse problem between the local aberration laws in the virtual sources basis and the sound speed distribution.
Finally, the potential of SVD beamforming, shown for plane wave transmits, is transposable to diverging waves transmits and aberration correction with a straightforward method. This will become particularly interesting when studied in vivo on difficult-to-image patients. The first application of SVD beamforming for sound speed estimation also paves the way to new approaches for the long-standing and challenging issue of local speed-of-sound imaging.

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