The Diagnostics of Optical and Thermal Characteristics of Semitransparent Media at Modeling Phase Influence of Oscillating Convection and Thermoradiation

Fedor A. Zagumennov¹, Vladimir G. Merzlikin²,³ and Andrey V. Bystrov²

¹Department of Instrument Production Techniques, Bauman Moscow State Technical Un, 2-ya Baumanskaya str., 5/1, Moscow, 105005, Russia
²Department of Industrial Economics, Plekhanov Russian Un of Economics, Streymanny, 36, Moscow 117997, Russia
³Department of Technology and Equipment of Mechanical Engineering, Moscow Polytechnic Un, Bol’shaya Semenovskaya, 38, Moscow 107023, Russia

E-mail: sgw32@yandex.ru

Abstract. The paper suggests diagnostics method of non-stationary combined regime heating of semitransparent materials base on an evaluation of the interconnected complex of optical parameters (extinction – \( b_{rad} \), scattering - \( \sigma \) and absorption \( \kappa \) indices) and thermophysical (thermal diffusivity \( \alpha_T \) and conductivity \( \kappa_T \)) ones using simulation the general temperature field \( \vartheta(\omega,\omega_T,\Delta\varphi_{\text{conv}}) = \vartheta_{\text{rad}}(\omega,\omega_T,b_{rad},\Delta\varphi_{\text{rad}}) + \vartheta_{\text{conv}}(\omega,\omega_T,b_{conv},\Delta\varphi_{\text{conv}}) \) including the additive terms due to the convective and radiation fluxes with a phase shift \( \Delta\varphi_{\text{conv}} \) within the external oscillating (frequency \( \omega \)) heat load. A one-dimensional problem of finding the temperature for a semitransparent material in the absence of aggregative transitions, neglecting long wavelength re-emission was considered in the analytic representation. The classical solution (of the homogeneous heat equation) and the particular one (of the inhomogeneous one with an internal thermal radiant source) defined summable terms with independent phases \( \Delta\varphi_{\text{conv}} \), \( \Delta\varphi_{\text{rad}} \) and different indices respectively for attenuation \( b_{\text{conv}} = \sqrt{\omega/2\alpha_T} \) (conductive heatwave) and for diffraction extinction \( b_{\text{rad}} = \sqrt{\kappa_T^2 + \omega^2} \) (radiant heatwave). In-phase production of additive term \( \vartheta_{\text{rad}}(\omega=0,\omega_T) \) simultaneously with external convection impact on the exposed surface allowed for the first time to establish interconnected complex \( b_{\text{rad}} = \sqrt{\omega \cdot \text{ctg}(\Delta\varphi_{\text{rad}})/\alpha_T} \). The obtained formulas for specific temperature distributions allow you to use them to estimate values of optical and thermal parameters of a semitransparent material from experimentally recorded phase jumps of the temperature field during the action of alternate impact of initial phased convective and radiation components of the external heat flux. The results of the study can be used both for experimental diagnostics and to select the heat regimes for combined radial and convective processing of semitransparent ceramics, composites, and biomaterials.

1. Introduction

Traditionally in thermal physics it is important to analyze the heat regimes of materials under the influence of external oscillating convective and thermoradiation heat fluxes with different phase shift
Studies on the calculated and experimental estimates of the temperature fields for technical materials and natural environments are known [2-5].

But the interpretation of numerical and experimental estimates significantly complicated when predicting the impact of phased components of convective and radiant heating on formation of general temperature regime with predominance of conductive heating or alternative volumetric radiant overheating for semitransparent materials.

Therefore, obtaining an analytical solution is an urgent task allowing to evaluate an interconnected complex of optical and thermophysical characteristics of semitransparent material will depend on experimentally recorded temperature field characteristics, produced by the action of an external nonstationary convective-thermodynamic heat flux, changing according to a harmonic law with equal frequencies \( \omega \) of its components and simulated phase shift [6-8].

2. Mathematical model of combined convective heating and volumetric radiant overheating

The ambient temperature \( \theta_A(t) \) and probe radiation flux \( q_{rad}(t) \) with phase shift \( \Delta \), are set taking into account the time averages of the constants \( \theta_A \), \( q_{rad0} \) and amplitudes \( \theta_A, q_{rad1} \) of harmonic oscillations:

\[
\theta_A(t) = \theta_A + \theta_{A1} \cdot \cos(\omega t),
\]

\[
q_{rad}(t) = q_{rad0} + q_{rad1} \cdot \cos(\omega t - \Delta).\]

We are guided by the theory of radiative-conductive subsurface heating of semitransparent material with thickness \( 0 < z < H \) under the influence of radiant component overheating together with convective surface heating in one-dimensional approximation. Therefore the calculated temperature is determined not only by thermophysical parameters but also by optical properties.

The inhomogeneous heat equation should include the thermal source function \( F(z, t, \omega) \) which is caused by penetrating radiation:

\[
\frac{\partial \theta}{\partial t} = a_r \cdot \frac{\partial^2 \theta}{\partial z^2} + F_{rad} \cdot \frac{\partial \theta}{\partial z} + F_{rad}(z, \omega) = \frac{q_{rad}(t, \omega, \Delta)}{1 - A} \cdot b_{rad} \cdot \exp\left(-2b_{rad} \cdot z - A \cdot b_{rad}^2 \cdot (z + H)\right),
\]

where \( c \) – specific heat capacity, \( \rho \) – density, \( K_r \) – thermal conductivity, \( a_r = K_r / c \rho \cdot \) temperature diffusivity coefficient.

Boundary conditions on the exposed surface are determined by the influence of convection fluxes and long wavelength nonpenetrating radiation in the absence of aggregative transitions:

\[
z = 0, \quad -K_r \frac{\partial \theta(0, t)}{\partial z} = \alpha_{turb} \cdot [\theta_A \cdot \cos(\omega t) - \theta(0, t)] + e_{\text{eff}} \cdot \left[\left[\theta_A \cdot \cos(\omega t)\right]^4 - \left[\theta(0, t)\right]^4\right],
\]

where \( \alpha_{turb} \) – coefficient of turbulent heat exchange, \( e_{\text{eff}} \) – Stephan-Boltzmann coefficient, \( e_{\text{eff}} \) - effective emissivity coefficient.

Our examined samples of semitransparent material should be considered as scattering and weakly absorbing mediums and ensure the necessary reflection and transmission for fallen radiation. The methodology of calculative-theoretical estimations of the optical parameters is well known [4, 5], including version developed earlier by authors [6] which is used in this work.

The penetration of radiation is defined by volumetric coefficients of reflection, transmission and absorption, depending on scattering and absorption indexes of the given optically inhomogeneous environment.

Obtained numerical values of extinction \( b_{rad} \) and scattering \( \sigma \) indexes for example the ceramic layer are determined by experimental values of albedo \( A \) and absorption index \( \kappa \). The given calculations were based on a functional relationship of these optical parameters [6]:

\[
b_{rad} = \sqrt{\kappa^2 + p \cdot \kappa \cdot \sigma}, \quad A = \frac{b_{rad} - \kappa}{b_{rad} + \kappa},
\]
where $b_{rad}$ - extinction index is considered as attenuation coefficient, $A$ – albedo is reflection coefficient of semi-infinite layer with spherical scattering indicatrix ($p = 1$) in two-flux approximation of solution for radiative transfer equation.

For known samples of porous ceramics (based on silica, zirconia aluminum) $\kappa \sim 0.01-1$ m$^{-1}$ and $\sigma \sim 1-3000$ m$^{-1}$, albedo changes in interval $A \sim 30-99\%$ with $b_{rad} < 10-30$ m$^{-1}$. It allows us to offer simple optical models for materials for solving engineering problems of complex heat transfer.

Thus it is possible to calculate the absorbed radiant energy function (2) within UV, visible and near IR spectral ranges for most artificial and natural radiant heat sources.

Considered physical and mathematical model of the radiant heat exchange is caused by technological conditions for convective (e.g. flame machining) or radiant (e.g. laser one) processing for materials. Intensive heat fluxes can exceed tens of MW/m$^2$. At temperatures of aggregative transitions up to $\sim 2000-3000$K for the refractory ceramics (type $Al_2O_3$, $SiO_2$, $ZrO_2$) the generated fluxes of self - IR radiation from expose surface may not exceed 1 MW/m$^2$.

Therefore, to obtain an analytical solution in the boundary conditions (3) authors will be neglected longwave re-emission on the external border and this thermal problem becomes linear:

$$z = 0, \quad -K_r \frac{\partial \theta(0,t)}{\partial z} = \alpha_{rad} r_1 \cdot \{[\theta(\omega t) - \theta(0,t)]; \quad z \to \infty, \quad \theta(z,t) \to 0.$$ (5)

because of the accepted linear mathematical model for solving the problem total temperature field in semitransparent material can be considered as the result of independent additive contributions calculated temperature distributions $\theta_{c+r}(z,\omega t, \theta_1, q_{rad}, \Delta \phi_{conv}, b_{conv}, \Delta \phi_{rad})$ and $\theta_{rad}(z,\omega t, \Delta \phi_{rad}, b_{rad}, \Delta \phi_{rad})$, respectively (See Figure 1):

**Figure 1.** Block scheme of external (1) oscillating heat load on the semitransparent material (2) by components of convective surface heating $\theta_d(t)$ and subsurface radiant overheating $q_{rad}(t)$ (with a phase shift $\Delta \phi_{conv}$), which form and model the general temperature field $\theta(z,t)$ with additive terms $\theta_{c+r}(z,t)$ and $\theta_{rad}(z,t)$ with appropriate phases $\Delta \phi_{conv}$ ($b_{conv}$, $b_{rad}$) and $\Delta \phi_{rad}$ ($b_{rad}$) with exponential conductive attenuation (Line 3, with index $b_{rad}$) and radiative extinction (Line 4, with index $b_{conv}$) for their heat waves

I - surface convective heating (1a) causing a heat wave with phase $\Delta \phi_{conv}$, and index attenuation $b_{conv} = \sqrt{\omega / 2\kappa}$ at a conductive mechanism of heat propagation inside the material, but corrected by volumetric overheating, produced by penetrating radiation (1b);

II - volumetric overheating by penetrating thermal radiation (1b) with phase $\Delta \phi_{rad}$ and extinction index $b_{rad}$ (4) with conductive-radiate heat transfer.

Besides, for the constant components of the external temperature $\theta_d(t)$ (1a) and the incident radiation flux $q_{rad0}$ (1b) the trivial exact solution is known [1], which will not be considered in our analysis of generated oscillating temperature fields.
Then for a total analytical solution for formed temperature fields we can limit ourselves to the influence only variable radiant \( q_{\text{rad}} \cdot \cos(\omega t - \Delta \phi_{\text{rad}}) \) and convective \( \theta_{\text{A1}} \cdot \cos(\omega t) \) components of harmonic oscillations of an external heat load.

The formulated problem (1-5) admits an analytical solution.

Since the heat conductivity equation (2) is inhomogeneous with the desired temperature field \( \theta(z,t) \), then it is possible to find separate solutions in the form of independent components:

1 - conductive component of homogeneous equation with oscillating convective heating at the boundary (5) and

2 - particular solution for the inhomogeneous equation (3) with thermoradiation subsurface overheating.

Then, taking into account the initial temperature, the general solution for (1-5) is [8]:

\[
\theta(z,\omega t,\theta_{\text{in}},q_{\text{rad}},\Delta \phi_{\text{conv}}) = \theta_{\text{in}}(z)+\Delta \phi_{\text{conv}}(z,\omega t,\theta_{\text{in}},q_{\text{rad}},\Delta \phi_{\text{rad}}) = \\
= \sqrt{c_1^2 + c_2^2} \cdot \cos\left(\omega t - \frac{\omega}{2a_r} \cdot z - \Delta \phi_{\text{rad}}(z,\omega t)\right) \cdot e^{-b_{\text{rad}} \cdot z},
\]

\[
\Delta \phi_{\text{rad}} = \text{arcsin} \left(\frac{c_2}{c_1^2 + c_2^2}\right), \quad \Delta \phi_{\text{conv}} = -\text{arcsin} \left(\frac{\omega}{\sqrt{\omega^2 + a_r^2}}\right).
\]

\[
c_1 = \frac{K_r \sqrt{\frac{\omega}{2a_r} + \alpha_{\text{turb}}}}{\Delta}, \quad c_2 = \frac{K_r \sqrt{\frac{\omega}{2a_r}} \cdot \Phi_1 + K_r \frac{\omega}{2a_r} \cdot \Phi_2}{\Delta}.
\]

\[
\Phi_1 = \alpha_{\text{turb}} \cdot \theta_{\text{in}} - \left(K_r b_{\text{rad}} + \alpha_{\text{turb}}\right) \frac{q_{\text{rad}} \cdot b_{\text{rad}} \cdot (1 - A)}{c \cdot \rho} \cdot d_1, \quad \Phi_2 = \frac{q_{\text{rad}} \cdot b_{\text{rad}} \cdot (1 - A)}{c \cdot \rho} \cdot \left(K_r b_{\text{rad}} + \alpha_{\text{turb}}\right) \cdot d_2,
\]

\[
\Delta = \left(K_r \sqrt{\frac{\omega}{2a_r} + \alpha_{\text{turb}}} \right)^2 + K_r \cdot \frac{\omega}{2a_r}.
\]

On the surface of the medium, the temperature is defined by the formula:

\[
\theta(z = 0,t) = \sqrt{c_1^2 + c_2^2} \cdot \cos\left(\omega t - \Delta \phi_{\text{rad}}(z,\omega t)\right) - \frac{q_{\text{rad}} \cdot b_{\text{rad}} \cdot (1 - A)}{c \cdot \rho} \cdot \cos\left(\omega t - \Delta \phi_{\text{rad}}(z,\omega t)\right) \cdot \\
\sqrt{\omega^2 + a_r^2} \cdot b_{\text{rad}}^2.
\]

Note also the case of maximum heating when the oscillations of the external temperature (1a) and the thermoradiation term in (7) can occur in the phase \( \Delta \phi_{\text{rad}} = 0 \) (See formula 6).

This is done if the phase \( \phi_{\text{conv}} \) in the incident radiant flux (1b), the frequency \( \omega \) and the optical-thermophysical parameters of the medium are related by interconnected specific complex (Figure 2):

\[
b_{\text{rad}} = \sqrt{k^2 + \kappa \sigma} = \sqrt{\omega \cdot c tg (\Delta \phi_{\text{conv}}) / a_r}.
\]
Figure 2. The dependence of semitransparent material albedo $A$ vs the phase shift $\Delta \phi_{\text{conv}}$ (between convective and radiant components of the external oscillating heat flux with period $T=3600$ s) during maximum heating by surface convective impact (1a) with in-phase radiant overheating $\theta_{\text{rad}}(z,t,\omega,\Delta\phi_{\text{rad}})$ (6) with $\Delta \phi_{\text{rad}}=0$ for samples:

- porous ceramics (Lines #1a, #1b with a thermal conductivity $K_{T1}=0.1$ W/m·K and thermal diffusivity $\alpha_{T}=2.2\cdot10^{-7}$ m$^2$/s coef).
  Line # 1a for $\kappa_{11}=1$ m$^{-1}$, $\sigma_{11}=450$ m$^{-1}$. Line # 1b for $\kappa_{12}=2$ m$^{-1}$, $\sigma_{11}=450$ m$^{-1}$.
- monolithic ceramics (Lines #2a, #2b with a thermal conductivity $K_{T2}=2$ W/m·K and thermal diffusivity $\alpha_{T}=4.4\cdot10^{-6}$ m$^2$/s coef).
  Line # 2a for $\kappa_{21}=1$ m$^{-1}$, $\sigma_{21}=45$ m$^{-1}$. Line #2b for $\kappa_{22}=2$ m$^{-1}$, $\sigma_{21}=45$ m$^{-1}$.

Model parameters: $\theta_{41}=\pm500$ K, $\theta_{40}=900$ K; $\alpha_{\text{turb}}=100$ W/m$^2$·K; $q_{\text{rad}1}=\pm10$ and $q_{\text{rad}2}=10$ kW/m$^2$

3. Conclusion

The obtained formulas for general solution $\theta(z,t)$ (6) with additive terms $\theta_{c+r}(z,t), \theta_{\text{rad}}(z,t)$ allow you to use them to estimate values of optical and thermal parameters of a semitransparent material based on interconnected specific complex (8) from experimentally recorded phase jumps (7) of the temperature of the exposed surface at the alternating energy effect of initial phased convective and radiation components of the external heat fluxes.

The results of the study can be used both for experimental diagnostics and to select the heat regimes for combined radial and convective processing of semitransparent ceramics, composites, and biomaterials.

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