Dynamic susceptibility and dynamic correlations in spin ice

M. I. Ryzhkin¹, I. A. Ryzhkin¹ and S. T. Bramwell²

¹ Institute of Solid State Physics RAS - 2 Academician Ossipyan Str., Chernogolovka, Moscow District 142432, Russia
² London Centre for Nanotechnology and Department of Physics and Astronomy, University College of London 17–19 Gordon Street, London WC1H0AJ, UK

received 19 September 2013; accepted in final form 6 November 2013
published online 2 December 2013

PACS 75.10.Bk – Classical spin models
PACS 75.40.Gb – Dynamic properties (dynamic susceptibility, spin waves, spin diffusion, dynamic scaling, etc.)
PACS 28.20.Cz – Neutron scattering

Abstract – Here we calculate the dynamic susceptibility and dynamic correlation function in spin ice using the model of emergent magnetic monopoles. Calculations are based on a method originally suggested for the description of dynamic processes in water ice (non-equilibrium thermodynamics approach). We show that for $T \to 0$ the dynamic correlation function reproduces the transverse dipole correlations (static correlation function) characteristic of spin ice in its ground state and explains in what sense spin ice is non-ergodic. At non-zero temperatures the dynamic correlation function includes an additional longitudinal component which decreases as the temperature decreases. Both terms (transverse and longitudinal) exhibit identical Debye-like dependences on frequency but with different relaxation times: the magnetic Coulomb interaction of monopoles reduces the longitudinal relaxation time with respect to the transverse one. We calculate the dielectric function analogue for the magnetic monopole gas and discuss how the non-equilibrium thermodynamics approach exposes corrections to the Debye-Hückel theory of magnetic monopoles and the concept of "entropic charge".

Copyright © EPLA, 2013

Introduction. – Spin ice is the name given to compounds such as Ho₂Ti₂O₇, Dy₂Ti₂O₇ which demonstrate unusual magnetic correlations [1]. The magnetic ions Ho³⁺ and Dy³⁺ sit at the vertices of regular tetrahedra linked into a three-dimensional pyrochlore lattice. Due to strong anisotropy the atomic magnetic moments (spins) of the magnetic ions can be directed only along local anisotropic axes connecting the centers of nearest tetrahedra. The ground state is characterized by the ice rule: two spins of each tetrahedron are directed toward its center, and two other spins, away from its center (see fig. 1). This rule leads to a ground state degeneracy that diverges exponentially with the number of spins [2,3] and to implicit topological ordering. To illustrate this it is useful to present a ground state configuration as a set of strongly entangled strings (one of them is shown in fig. 1). Each such string is a line drawn through lattice bonds with magnetic ions along spins, and the ice rule ensures that strings can either be closed or they end at the sample boundary. Spins are strongly correlated along strings; therefore long strings contribute to long-range correlations, but short and closed strings do not contribute to these correlations. The final result depends on the relative proportion of long and short strings. From the string picture it becomes obvious that there might be long-range correlations between spins in spin ice.

Those correlations between spins can be characterized by an equilibrium (or static) correlation function for the local magnetization. As was shown in [4–6] by averaging over all ground-state configurations this function has a striking dipole-like form:

$$S_{\alpha\beta}(q) = \langle M_{\alpha}(q)M_{\beta}(-q) \rangle \propto \left( \delta_{\alpha\beta} - \frac{q_{\alpha}q_{\beta}}{q^2} \right).$$  \hspace{1cm} (1)

From the same string picture, with single spin flip dynamics, it is obvious that any ground state configuration is frozen: one cannot change it without the breaking some of strings or without violating the ice rule. The flipping of a spin in the ground state breaks the ice rule in two neighboring tetrahedra: one tetrahedron has spins directed three-in and one-out, while its neighbor has one-in and three-out. These tetrahedra can be considered as positive and negative emergent magnetic monopoles respectively [7,8]. By means of further spin flips the
magnetic monopoles can move through the lattice and thereby change the spin configuration (see fig. 2). The magnetic monopole model affords an economical way of describing the dynamic processes in spin ice, as one substitutes the strongly correlated system of spins by one of dilute and weakly interacting magnetic monopoles.

The dynamic susceptibility and dynamic correlation functions of a magnetic system are basic properties used to express the linear response of the system to diverse experimental probes, most importantly neutron scattering and magnetometry. The purpose of our paper is to show how these basic properties of spin ice at non-zero temperatures can be obtained in the framework of the magnetic monopoles approach. We also discuss their relationship with static analogs obtained for the ground state and with results of other works.

**Basic equations.** — To calculate the dynamic susceptibility we find the magnetization of spin ice in a time dependent and inhomogeneous magnetic field using Jacard’s method originally suggested for electric relaxation in common water ice [9]. Its detailed description, corresponding to the notations of the present paper, can be found in [10]. In this method monopole fluxes \( j_{\pm} \) and concentrations \( n_{\pm} \), magnetization \( M \) and magnetic field \( (H, h) \) (external and generated by magnetic monopoles, respectively) are defined by the following set of equations:

\[
\begin{align*}
    j_{\pm} + D_{\pm} \nabla n_{\pm} &= \frac{\sigma_{\pm}}{Q_{\pm}} \left[ Q_{\pm} (H + h) - \eta_{\pm} \Phi \Omega \right], \\
    \partial \Omega / \partial t &= \sum_{\pm} \eta_{\pm} j_{\pm}, \\
    \partial n_{\pm} / \partial t &= -\nabla \cdot j_{\pm}, \\
    \nabla \cdot h &= 4\pi \sum_{\pm} Q_{\pm} \delta n_{\pm}.
\end{align*}
\]

In this set eqs. (2) define linear response fluxes of magnetic monopoles due to applied disturbances. Here \( \sigma_{\pm}, D_{\pm}, Q_{\pm} = \pm Q \) are specific conductivities, diffusion coefficients, and magnetic charges of positive and negative magnetic monopoles respectively, quantities \( \eta_{\pm} = \pm 1 \) describe signs and type of magnetic ordering due to monopole fluxes. Quasiparticle fluxes are induced by the external magnetic field \( H \) and by the magnetic field \( h \) generated by the magnetic monopoles.

It makes sense to clarify the origin of the field \( h \) in more detail. Initially we have a system of spins with short-range exchange and long-range dipole interaction. In the ground state the dipole interaction can be included into exchange interaction due to a self-screening effect [11]. In an excited state with a low concentration of magnetic monopoles, the dipole interaction leads to the long-range magnetic Coulomb interaction between magnetic monopoles. The simplest way to see that is the analogy between water ice and spin ice based on the resemblance of Hamiltonians for the both problems [12,13], which means that monopole defects in spin ice interact in a similar way to ionic defects in water ice. The Coulomb interaction between magnetic monopoles —and hence the dipole interaction between spins— is taken into account here by a self-consistent magnetic field \( h \), and eventually will result to the difference between longitudinal and transverse relaxation times (see eqs. (11)–(14) below).

The configuration vector \( \Omega \) and concentration gradients also contribute to the fluxes. A physical sense of the configuration vector is very simple, it is proportional to the magnetization \( M = Q \Omega \). As discussed further below, eqs. (2) are close analogs to ones for fluxes of electrons and holes in semiconductors with the exception of the entropic term proportional to the configuration vector \( \Omega \). A parameter \( \Phi \) was calculated in [14]:

\[
\Phi = \frac{8 \alpha k_B T}{\sqrt{3}}.
\]

In this set eqs. (2) define linear response fluxes of magnetic monopoles due to applied disturbances. Here
It can be also considered as a characteristic of entropic interaction [15].

Equation (3) defines a modification of the spin ordering by magnetic monopole fluxes (see the spins along dotted lines in fig. 1,2), eqs. (4) are continuity equations, and eq. (5) is the magnetic analog of Poisson’s equation. Specific conductivities and diffusion coefficients are related by the expressions \( \sigma_{\pm} = Q_{\pm} D_{\pm} n_{0\pm}/k_{B} T = Q_{\pm} n_{0\pm} \), where \( n_{0\pm} = n_{0} \) and \( \mu_{\pm} \) are the concentrations and mobilities of monopoles respectively. Note that eqs. (4) are approximate in that they exclude chemical kinetic terms that describe the creation and annihilation of magnetic monopoles, as well as bound pair formation.

In Fourier presentation eqs. (2)–(5) become linear algebraic ones, and they can be written in the forms (further we use the notation for variables in Fourier presentation):

\[
j_{\pm} + D_{\pm} i q \delta n_{\pm} = \pm \frac{\sigma_{\pm}}{Q_{\pm}} \left[ Q \left( H + h \right) - \Phi \Omega \right],
\]

\[
i \omega \Omega = Q \left( j_{+} - j_{-} \right),
\]

\[
\omega \delta n_{\pm} = q \cdot j_{\pm},
\]

\[
i q \cdot h = 4 \pi Q \left( \delta n_{+} - \delta n_{-} \right).
\]

**Results.** – Eliminating first \( \delta n_{\pm} \), then \( j_{\pm} \), we find the components of magnetization \( M_{\alpha} \) and magnetic field \( h_{\alpha} \). Skipping simple calculations we give final expressions for the case of equal diffusion coefficients \( D = D_{+} = D_{-} \):

\[
M_{\alpha} = \left[ \frac{Q^{2}/\Phi}{1 - i \omega \tau} \left( \delta_{\alpha\beta} - 2 q_{\alpha} q_{\beta} / q^{2} \right) \right] H_{\beta},
\]

\[
h_{\alpha} = - \frac{4 \pi Q^{2}/\Phi}{\tau / \tau' - i \omega \tau' + \tau D q^{2} / q^{2}} q_{\alpha} q_{\beta} H_{\beta},
\]

where transverse and longitudinal relaxation times are defined by the following equations, respectively:

\[
\frac{1}{\tau} = \frac{2 \Phi D_{0}}{k_{B} T}, \quad \frac{1}{\tau'} = \frac{2(4 \pi Q^{2} + \Phi) D_{0}}{k_{B} T}.
\]

From eq. (11) we get the following expression for the magnetic susceptibility that relates external field and generated magnetizations:

\[
\chi_{\alpha\beta}(q, \omega) = \frac{Q^{2} / \Phi}{1 - i \omega \tau} \left( \delta_{\alpha\beta} - 2 q_{\alpha} q_{\beta} / q^{2} \right)
+ \frac{Q^{2} / \Phi}{\tau / \tau' - i \omega \tau' + \tau D q^{2} / q^{2}} q_{\alpha} q_{\beta}.
\]

We may also calculate the susceptibility that relates the total internal field to the magnetisation,

\[
M_{\alpha} = \tilde{\chi}_{\alpha\beta}(q, \omega) \tilde{H}_{\beta} = \tilde{\chi}_{\alpha\beta}(q, \omega) (H_{\beta} + h_{\beta}).
\]

Using eqs. (11), (12) it easy to get for \( \tilde{\chi}_{\alpha\beta}(q, \omega) \) the following expression:

\[
\tilde{\chi}_{\alpha\beta}(q, \omega) = \frac{Q^{2} / \Phi}{1 - i \omega \tau} \left( \delta_{\alpha\beta} - 2 q_{\alpha} q_{\beta} / q^{2} \right)
+ \frac{Q^{2} / \Phi}{1 - i \omega \tau' + \tau D q^{2} / q^{2}} q_{\alpha} q_{\beta}.
\]

In the limit \( \omega = 0, q \to 0 \), both the longitudinal and transverse \( \chi_{\alpha\beta}(q, \omega) \) tend to the thermodynamic isothermal susceptibility \( \chi_{\epsilon} = M / H_{\text{internal}} \), that can be obtained from the (sample shape-independent) internal energy \( U = U(S, M) \) (here \( H_{\text{internal}} = H + h \), \( S = \text{entropy} \)). In the same limit \( \chi_{\alpha\beta}(q, \omega) \) tends to different values for longitudinal and transverse “apparent” susceptibilities, \( \chi_{\epsilon} = M / H_{\text{external}} \) (here \( H_{\text{external}} = H \)). These susceptibilities derive from the function \( U' = U + U_{\text{mag}} \), where \( U_{\text{mag}} \) is the magnetostatic energy, that depends on sample shape. In electrical systems, analogues of \( \tilde{\chi}, \chi \) are variously referred to as the “proper” or “screened” response function, and the “improper” or “unscreened” response function respectively, but these labels do not imply that one is more relevant to experiment than the other. In the present case the discontinuous behaviour of \( \chi \) in the limit \( \omega = 0, q \to 0 \) reflects the genuine shape-dependent properties of \( \chi_{\epsilon} \) that arise from truncation of the long-range dipole interaction at the sample boundaries. In the following we refer to \( \tilde{\chi} \) as the “generalised” susceptibility and \( \chi \) as the “apparent” susceptibility. The generalised susceptibility eq. (16) can be got from the apparent susceptibility eq. (14) by means of equalization of relaxation times \( \tau' \to \tau \). Hence we consider the apparent susceptibility \( \chi_{\alpha\beta} \), and respective results for the generalised susceptibility can be got by means of the simple substitution \( \tau' \to \tau \).

Using eq. (14) and the fluctuation-dissipation theorem in its classical form

\[
\text{Im} \chi_{\alpha\beta}(q, \omega) = \frac{\omega}{2 k_{B} T} S_{\alpha\beta}(q, \omega),
\]

we get the dynamical correlation function \( S_{\alpha\beta}(q, \omega) \). Omitting simple computations we get

\[
S_{\alpha\beta}(q, \omega) = 4 Q^{2} D n_{0} \left[ \frac{1}{1/\tau^{2} + \omega^{2}} \left( \delta_{\alpha\beta} - 2 q_{\alpha} q_{\beta} / q^{2} \right) \right] + \frac{1}{(1/\tau' + D q^{2})^{2} + \omega^{2}} q_{\alpha} q_{\beta}.
\]

The respective formula for the generalised correlation function has the form

\[
\tilde{S}_{\alpha\beta}(q, \omega) = 4 Q^{2} D n_{0} \left[ \frac{1}{1/\tau^{2} + \omega^{2}} \left( \delta_{\alpha\beta} - 2 q_{\alpha} q_{\beta} / q^{2} \right) \right] + \frac{1}{(1/\tau + D q^{2})^{2} + \omega^{2}} q_{\alpha} q_{\beta}.
\]

Once again, transverse and longitudinal correlation functions \( \tilde{S}_{\alpha} \) become equal in the limit \( q \to 0, \omega = 0 \), while the correlation functions \( S_{\alpha} \) reach different values in this limit, giving rise to a discontinuity at \( q = 0 \).
Analysis of the results. – Now let us discuss the physical sense of the obtained results and compare them with results of other papers.

First, we discuss the fundamental question: does the model of emergent monopoles agree with Maxwell’s equations? The answer is yes. Indeed, the applied field is solenoidal $q_o H_o = 0$. Then we have to write an induction of magnetic field in the form:

$$B_o = H_o + 4\pi M_o + h_o = H_o + \frac{Q^2/\Phi}{1 - i\omega \tau} \left( \delta_{\alpha\beta} - \frac{q_o q_{\beta}}{q^2} \right) H_{\beta}. \quad (20)$$

Therefore the magnetic induction is also solenoidal, $q_o B_o = 0$, in agreement with Maxwell’s equation \( \nabla \cdot \mathbf{B} = 0 \) (there are no genuine magnetic monopoles). This condition is satisfied due to the exact cancellation of longitudinal terms from eqs. (11), (12) in eq. (20). Therefore the accounting of the field $\mathbf{h}$ into eqs. (2) is fundamentally important: without this field one comes to misleading conclusion that genuine magnetic monopoles exist. The exact cancellation of longitudinal terms from $M_o$ and $h_o$ is always maintained: for different diffusion coefficients and at all values of temperature. Also note that the term $h_o$ in eq. (20) may seem unusual in comparison with common definitions of magnetic induction. Why must we include the supplementary term $h_o$? In fact this term arises from nearly free monopoles, and in some sense it is similar to the contribution of free electrons to an electrical induction in semiconductors [16]. For example, consider the properties of eqs. (7)–(10) with magnetic properties substituted for electric ones: \( \{\mathbf{H}, \mathbf{h}, \mathbf{M}\} \rightarrow \{\mathbf{E}, \mathbf{e}, \mathbf{P}\} \) i.e. the electrical applied field, screening field and polarisation, respectively. Suppressing the third term on the right in eq. (7) then gives equations characteristic of an electrical conductor. Including it gives spin ice both “conducting” and “dielectric” properties.

Second, we note that the magnetization in eq. (11) includes both transverse and longitudinal terms. Both terms have Debye dependence on frequency, but with different relaxation times. The first (transverse) term takes a transverse relaxation time $\tau$, the second (longitudinal) one takes a longitudinal time $\tau'$, see eqs. (13). Using eq. (6) one can estimate a ratio of times as $\tau / \tau' \approx 1 + 8.3 / T \gg 1$ for low temperature $T < 2$K. The distinction between these two timescales exists in the response functions $\chi, \bar{S}$ but not in $\chi, \bar{S}$, suggesting that their difference will be manifest only in experimental observations where sample shape-dependence plays a role (see above).

Third, for time dependent correlation functions we get the following expressions:

$$S_{\alpha\beta}(\mathbf{q}, t) = \frac{\sqrt{3} Q^2}{8a} \left[ \left( \delta_{\alpha\beta} - \frac{q_o q_{\beta}}{q^2} \right) e^{-t/\tau} \right.$$ \begin{equation}
+ \left. \frac{e^{-\left(1/\tau' + Dq^2/\tau'\right)t}}{\tau / \tau' + \tau Dq^2/q^2} \frac{q_o q_{\beta}}{q^2}, \right] \quad (21)$$

In the limit $T \rightarrow 0$ for all $t > 0$ we come to the equation

$$S_{\alpha\beta}(\mathbf{q}, t) = S_{\alpha\beta}(\mathbf{q}, t) = \frac{\sqrt{3} Q^2}{8a} \left( \delta_{\alpha\beta} - \frac{q_o q_{\beta}}{q^2} \right), \quad (22)$$

which coincides with the static correlation function [4–6]. For coincidence with [4] one should take parameter $\lambda = 1$. The independence on time of (23) means that at zero temperature there are not any relaxation processes. But if we first tend $t \rightarrow \infty$ we come to the different result:

$$S_{\alpha\beta}(\mathbf{q}, t) = S_{\alpha\beta}(\mathbf{q}, t) = 0. \quad (24)$$

That is, there are relaxation processes (correlation functions tend to zero as time tends to infinity). This contrast emphasises that spin ice becomes non-ergodic only at finite observation time.

Finally we compare the formulae (21), (22) with results from the works [17–19]. Equation (22) looks exactly like the result of [17–19] for Heisenberg’s model (note that spin ice considered here is described by Ising’s model) if we equalize:

$$8\Gamma \lambda T \rightarrow \tau^{-1}, \quad 8\Gamma J a^2 \rightarrow D, \quad \xi^2 \rightarrow D \tau. \quad (25)$$

But eq. (21) has a slightly different form. There are two characteristic lengths in eq. (21): the diffusion length $\xi_{\text{diff}} = \xi = (\tau D)^{1/2}$ and the Debye screening length $\xi_D = (4\pi Q^2 / \Phi + 1)^{1/2} / \tau D$. This difference is a consequence of neglecting the Coulomb interaction between monopoles (that is by neglecting the field $h_o$) in [17–19]. In fact a magnetic Coulomb interaction leads to the inequality $\xi_{\text{diff}} \gg \xi_D$ for $T < 2$K.

The longitudinal susceptibility was considered by one of us in a recent article [17]. Our present result for the longitudinal susceptibility coincides with that of [20] in the limit $T \rightarrow 0$ and differs from it in the general case. But for generalised susceptibility both the results exactly coincide.

A static ($\omega = 0$) tensor correlation function for dipolar spin ice that agrees with our $S_{\alpha\beta}(\mathbf{q}, 0)$ eq. (18) was presented in a very recent publication [21]. There the longitudinal-transverse discontinuity in $S_{\alpha\beta}(\mathbf{q})$ at $q = 0$ was recommended as a diagnostic in neutron scattering measurements. However the available experimental evidence on spin ice [22,23] suggests that neutron scattering measures $\bar{S}$ rather than $\bar{S}$ in the $q \rightarrow 0$ limit, suggesting that the discontinuity may not be observable. More detailed studies of this question would be welcome.

Discussion. – We now explore how our results for dynamical susceptibilities and correlation functions are related to other properties of the monopole model and to experiments.
Strictly, our model should only apply at low temperature $T < 1\, \text{K}$, as most of our starting equations hold only when the concentration of monopoles is much lower than the concentration of regular tetrahedra without violations of ice rules. However, numerical evidence shows that the monopole picture applies up to at least $10\, \text{K}$ [24] and experiments [25] find consistency with the nonlinear thermodynamic approach in this regime. It would therefore seem that even in the dense monopole regime ($T > 2\, \text{K}$), monopoles may be treated as weakly interacting quasi-particles with densities and transport coefficients normalised by charge screening, as in the analogous case of strong electrolytes.

The canonical model of charge screening in electrolytes is Debye-Hückel theory, which has been applied to the monopole model in [15]. In contrast to our non-equilibrium thermodynamics method, this is an equilibrium statistical mechanics approach, based on linearisation of the Poisson-Boltzmann equation. The central concept in Debye-Hückel theory is the screened Coulomb potential $\sim e^{-\xi_D q/r}$ which may be obtained from the static longitudinal dielectric function.

To explore the relationship with Debye-Hückel theory it is therefore interesting to use our results to calculate a dielectric function analogue. We start by defining a longitudinal apparent susceptibility from eq. (14) for $\omega = 0$:

$$\chi^L(q) = \frac{e^{Q^2/\Phi}}{1 + 4\pi Q^2/\Phi + \tau D q^2} = \frac{\chi_T \xi^{-2}}{\xi_D^2 + q^2}.$$  \hspace{1cm} (26)

where $\chi_T \equiv \chi(q = 0, \omega = 0)$ is the isothermal bulk susceptibility. Next we consider the response of a system to an applied longitudinal field, as might arise in practice from the static longitudinal dielectric function.

The screened magnetic Coulomb potential per unit charge is given by the Fourier transform ($FT$):

$$\phi(r) = FT \left[ \frac{1}{q^2 e^{\xi^L(q)}} \right] = \frac{2}{\pi} \int_0^\infty \frac{\sin(qr)}{q r e^{\xi}(q)} \, dq.$$  \hspace{1cm} (28)

The result is

$$\phi(r) = e^{-r/\xi_D} + 1/4\pi \chi_T r(1 + 1/4\pi \chi_T).$$  \hspace{1cm} (29)

When $\xi_D$ is large, this becomes the Coulomb form, $1/r$. When $\chi_T$ is large, it becomes the Debye-Hückel form, but at long distances there is a crossover to a modified Coulomb form $A/r$ with reduced amplitude $A = 1/(1 + 4\pi \chi_T)$. A fixed test monopole, imagined as the terminus of an infinitely long string, is not completely screened by the monopole gas [26]. By integrating Poisson’s equation over all space, it is easily shown that the positive charge induced by a negative test charge at the origin is less than one whole unit of charge $Q$ by the factor $4\pi \chi_T A = 1/(1 + 1/4\pi \chi_T)$. Note that eq. (29) applies for large $\xi_D$ only: for small $\xi_D$ the cutoff given by the lattice constant must be accounted for, but this is beyond the scope of the theory discussed here. In analysis of the experimental specific heat of Dy$_2$Ti$_2$O$_7$ [15] the cutoff dominates at temperatures above $\approx 1\, \text{K}$.

It is also interesting to discuss the relation of the present theory to the concept of entropic charge [15]. If we represent the entropic free energy $W_{\text{ent}} = 4k_B T a\Omega^2/\sqrt{3}$ (see [14] and eq. (6)) by an effective magnetostatic energy $W_{\text{mag}} = 2\pi M^2$, then we arrive at the entropic charge $q_{\text{ent}} = \sqrt{2k_B T a/\pi \sqrt{3}}$. However, it is clear that the entropic charge derived in this way does not have any local meaning: the separation of two monopoles may either increase or decrease the configuration vector, leading to an attractive or repulsive interaction respectively. It seems that the configuration vector gives a more complete account of entropic forces than does entropic charge, at least in the long-wavelength limit.

The correlation function $\tilde{S}(q,0)$ eq. (18) derived here is qualitatively similar to that observed in experiment on Ho$_2$Ti$_2$O$_7$ [27] and we have calculated the long-range $B$-field that contributes to the local response sensed by $\mu$SR and NMR. A full description of these experiments would further need to take into account the local magnetic structure. In the future it would be interesting to find other experimental probes that directly isolate the coarse grained fields $H$, $M$ and $B$, to which the present theory should be most directly applicable. In conclusion, the non-equilibrium thermodynamic approach has the advantages of thermodynamic consistency, physical transparency, and ease of generalization, but the inherent drawbacks of any thermodynamic quasiparticle description: lack of accounting for local details and a range of application that is limited by the accuracy of the quasiparticle picture. We note a recent alternative approach to static correlations [21] that is useful in connecting with cases where our approach in its present form breaks down, for example at higher temperature or on diluted spin lattices [28]. Further developments of our approach could include accounting for the chemical kinetic terms in eq. (4), which are relevant to the Wien effect and bound (Bjerrum) pair formation [29]. Also, we have given the results for the case of equal diffusion coefficients, but it is not difficult to generalise to the case where $D_+ \neq D_-$, and in this case there is an additional reason for generation of space magnetic charge: the difference of magnetic monopole conductivities leads to the formation of magnetic space charge arising as a result of their movement. The present approach could also be adapted to describe the coupling of magnetic currents with other thermodynamic forces such as thermal gradients [30] and electric fields [31], and should be generally useful for predicting consequences of the monopole model that can be tested against experiment.
REFERENCES

[1] Harris M. J., Bramwell S. T., McMorrow D. F. et al., Phys. Rev. Lett., 79 (1997) 2554.
[2] Pauling L., J. Am. Chem. Soc., 57 (1935) 2680.
[3] Ramirez A. P., Hayashi A., Cava R. J. et al., Nature, 399 (1999) 333.
[4] Youngblood R., Axe J. D. and McCoy B. M., Phys. Rev. B, 21 (1980) 5212.
[5] Isakov S. V., Gregor K., Moessner R. et al., Phys. Rev. Lett., 93 (2004) 167204.
[6] Henley C. L., Phys. Rev. B, 71 (2005) 014424.
[7] Ramirez A. P., Hayashi A., Cava R. J. et al., Nature, 399 (1999) 333.
[8] Youngblood R., Axe J. D. and McCoy B. M., Phys. Rev. B, 21 (1980) 5212.
[9] Isakov S. V., Gregor K., Moessner R. et al., Phys. Rev. Lett., 93 (2004) 167204.
[10] Henley C. L., Phys. Rev. B, 71 (2005) 014424.
[11] Ramirez A. P., Hayashi A., Cava R. J. et al., Nature, 399 (1999) 333.
[12] Youngblood R., Axe J. D. and McCoy B. M., Phys. Rev. B, 21 (1980) 5212.
[13] Isakov S. V., Gregor K., Moessner R. et al., Phys. Rev. Lett., 93 (2004) 167204.
[14] Henley C. L., Phys. Rev. B, 71 (2005) 014424.
[15] Ramirez A. P., Hayashi A., Cava R. J. et al., Nature, 399 (1999) 333.
[16] Youngblood R., Axe J. D. and McCoy B. M., Phys. Rev. B, 21 (1980) 5212.