Thermospin Hall effect generated by thermal influence and thermoelectric effect

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Abstract

In this paper, we present the theoretical predication of a thermospin Hall effect, in which a transverse spin current can be generated in semiconductors in the presence of spin-orbit coupling by a frequency-dependent longitudinal temperature gradient. The thermospin Hall effect has a number of qualitative distinctions in comparison with the spin Hall effect driven by an electric field. Because of the thermoelectric effect, there is no net charge current but there is a heat flow from the hot side to the cold side. We perform the theoretical calculation of dynamical thermospin Hall conductivity in a two-dimensional Rashba spintronic system. It has been shown that the direct interband optical transition dominates the ordering and manipulation of spin in the generation of a transverse intrinsic spin current. In view of the role of the thermoelectric effect, the contributions to the thermospin Hall effect are classified as that originating from a direct contribution of thermal electronic diffusion and that from the compensatory electron flow in balance with the thermal diffusion. In physical terms, we explain the phenomenon as the spin-orbit coupling exerting force on electronic orbital motions, which are driven by the thermoelectric properties, and manipulating the spin-orientation-dependent motions. For a finite system, the analysis yields evidence that the spin accumulation around the edges of a plate determines the magnetization. In equilibrium, a field created by a magnetization gradient emerges in the direction perpendicular to the temperature gradient. The experimental observation of the thermospin Hall effect is proposed by measuring the longitudinal temperature difference with the injection of a transverse spin current and by analyzing the Hall angle. In addition, in order to achieve pure spin accumulation in the spin Hall effect, an extension of the thermospin Hall effect for exciting electron-hole pairs in semiconductors is proposed.

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Rapid developments in technology and manufacturing are providing us with favorable circumstances for fabricating high-speed and low-powered electronic devices that use the spin properties of electrons. This will alter the current situation that microchips use only the charge properties of electrons. In order to efficiently utilize the advantageous functions of spin-based recording and information processing, one of the crucial issues is to accomplish pure spin injection virtually. In fact, besides its potential applications in spintronic devices, the electrical generation and manipulation of spin flux in semiconductors also has its own fundamental physics worthy of being studied. Among such studies, the spin Hall effect in narrow-band semiconductors has been a focal point of research in last few years. As a quantum mechanical degree of freedom attached to electrons, the spin can couple with the orbital motion of an electron through an internal electric field. A simple image of this coupling may be obtained as follows: The orbital motion of an electron in an internal electric field creates a magnetic field in its vicinity. On account of the electron spin, an electron has an intrinsic magnetic moment. The interaction between the electron spin magnetic moment and the magnetic field created by its orbital motion is regarded as the spin-orbit (SO) coupling. The SO interaction energy leads to a spin-splitting in the energy spectrum of moving electrons, even in the absence of any magnetic field. For a two-dimensional electron gas (2DEG) in a heterostructure, such as InAs- and In$_{1-x}$Ga$_x$As quantum wells, the structural inversion asymmetry of the confining potential due to the presence of the heterojunction results in the spin splitting of the conduction band in momentum (k) space. The formation of spin-splitting bands due to Rashba SO coupling has been measured in a number of materials, e.g., in heterostructures based on InAs or HgTe. Experimentally, Rashba SO coupling can be changed externally, e.g., by applying additional back-gate voltage to the structure. The change in the strength of SO coupling induces a modulation of electronic band structures, which is equivalent to the manipulation of electron spins. Hence, the spin-orientation-dependent motion can eventually be controlled by regulating the voltage gate without a magnetic field. Similar to a magnetic field exerting Lorentz force transversely on the moving charge, the SO coupling produces a transverse ”force” on the moving spin. Under the spin ”force,” electrons with opposite spin orientations drift off their initial direction of motion and tend to separate spatially in opposite directions. If the motion of electrons is driven by an external electric field, a result is that a spin current is generated in the direction perpendicular to the electric field, without an accompanying transverse charge.
current. Differing from the phenomenon that an extrinsic mechanism causes the spatial separation of electrons with different spin orientations, the phenomenon that is merely due to the SO coupling in solids is subsequently referred to as the intrinsic spin Hall effect\textsuperscript{5,6}. Recently, related studies have been extended to address the quantum spin Hall effect by employing the existence of bulk gap and gapless edge states in a time-reversal invariant system with SO coupling\textsuperscript{15,16}.

It is common knowledge that an electric current can be generated not only by an applied electric field but also by a temperature gradient in solids\textsuperscript{17}. This gives rise to a great number of interesting thermoelectric phenomena. A particularly interesting transport property is the thermoelectric (Seebeck) effect, i.e., the temperature gradient produces an electric potential gradient which can drive an electric current. Due to the extreme sensitivity of 2DEG to changes in electronic structure at the Fermi energy\textsuperscript{18}, the thermoelectric powers of 2DEG in Al\textsubscript{x}Ga\textsubscript{1-x}As/GaAs heterojunctions at low temperatures have attracted much attention\textsuperscript{19}. Studies show that the thermal features of semiconductor materials can play an important role in transport. In practice, a sensitive probe of the transport mechanism has included some proposals based on the use of thermoelectric power. One can, therefore, expect that this mechanism should play a role in semiconductor spintronic materials and will provide a challenging opportunity in developing new thermospintronic devices, whose thermoelectric properties can be controlled by SO coupling.

Here we introduce a new ”member” of the spin Hall effect ”family” - the thermospin Hall effect. The present study shows that a transverse spin current can be generated in a 2DEG in the presence of SO coupling by a longitudinal thermal gradient. Correspondingly, thermospin Hall conductivity is defined as the ratio between the transverse spin current and the longitudinal thermal gradient. We schematically illustrate these phenomena in Figure 1(a). A heuristic picture of the thermospin Hall effect is the combination of the thermal diffusion of electrons, the thermoelectric effect, and the spin-orientation-dependent side drift driven by SO coupling. When there exists the time-dependent temperature gradient $\nabla T$ (parallel to the $\hat{x}$ direction) in 2DEG, an electric potential distribution is built up by the Seebeck effect. The spatial distribution of the electric potential induces a backflow of electrons, which tends to balance the charge current driven by $\nabla_x T$. Meanwhile, the time-dependent electric potential stimulates electronic transitions between spin-splitting bands. As a result, no net average electric current flows in the $\hat{x}$ direction. Since moving electrons have not
only charge but also spin, the SO coupling in the system exerts a spin force on moving spins in the $\hat{y}$ direction, i.e., perpendicular to the temperature gradient. The force depends on both the spin orientation and the direction of electron orbital motion. Electrons with opposite spins are forced to move oppositely in the $\hat{y}$ direction. Hence, while not accompanied by net charge current, a pure spin current, associated with electronic transitions between spin-splitting bands, is generated. To differentiate from the spin Hall effect driven by an electric field, we regard this phenomenon as the thermospin Hall effect. Here the thermoelectric effect and the transition between spin-splitting states play important roles.

Regarding this phenomenologically, the spin current can, in general, be expressed in the form
\[ \mathbf{J}^{(s)} = \mathbb{L}^{(se)} \cdot \mathbf{E} + \mathbb{L}^{(sq)} \cdot (-\nabla T/T), \]
where $\mathbb{L}^{(se)}$ and $\mathbb{L}^{(sq)}$ are the spin conductivity tensor and the thermospin conductivity tensor, respectively. The induced electric field is determined by the balancing of the longitudinal electric current, i.e., $E_x = S \nabla_x T$ with Seebeck coefficient $S$. In the linear response, the corresponding spin current in the $\hat{y}$ direction can be written as
\[ J_y^{(s)} = \sigma_{TH}^{SH} \nabla_x T, \]
where $\sigma_{TH}^{SH} = \left( TSL_{yz}^{(se)} - L_{yz}^{(sq)} \right) / T$ is the thermospin Hall conductivity.

To formulate thermospin Hall conductivity, it is necessary to calculate the nonequilibrium carrier current density influenced by electric potential and temperature gradients. The crucial point in studying the spin thermal transport is to clarify the influence of temperature in the microscopic theoretical description. There is a long history of microscopic study in thermoelectric transport phenomena. Since temperature is a statistical property of the system, there is no Hamiltonian to describe the thermal gradient. Therefore, although the linear response to an external electric field leads unambiguously to the Kubo formula for the electric spin Hall conductivity tensor, its extension to the calculation of thermospin Hall conductivity is not straightforward. Fortunately, in 1964, Luttinger provided microscopic proof that thermoelectric transport coefficients are given by the corresponding current-current correlation function$^{20}$. In the framework of a so-called ”mechanical” derivation, an inhomogeneous gravitational field is introduced to produce energy flow and temperature fluctuation$^{19–21}$. Theoretically, an energy density $H (\mathbf{r})$ behaves as if it had a mass density $H (\mathbf{r}) / c^2$, which interacts with the gravitational field $- (1/c^2) \psi (\mathbf{r}, t)$. Varying $\psi$ will cause an energy current to flow. Further, a varying energy density gives rise to a temperature gradient. The macroscopic currents arising in a nonequilibrium system are proportional to the driving forces $E - (T/e) \nabla (\mu/T)$ and $T \nabla (1/T) - \nabla \psi$, where $- (T/e) \nabla (\mu/T)$ ($\mu$ is chemical potential and $e$ is electron charge) and $T \nabla (1/T)$ are statistical forces, while $\mathbf{E}$ and $- (1/c^2) \nabla \psi$ are
the electric field and gradient of the gravitational field, respectively. Due to the induced temperature gradient, a compensating energy current flowing in the opposite direction brings the system into equilibrium. In this way, the variation of the added energy fluctuation is balanced by a temperature distribution. This leads to the identification \( \nabla \psi = \nabla T/T \). Einstein’s relationship is tenable — it relates the response to electrical field \( \mathbf{E} \) and gravitational field gradient \( \nabla \psi \) to the observed concentration gradient \( (T/e) \nabla (\mu/T) \) and temperature gradient \( T \nabla (1/T) \), respectively. In equilibrium, zero current conditions lead to the relations \( \mathbf{E} = (T/e) \nabla (\mu/T) \) and \( T \nabla (1/T) = \nabla \psi \). The transport coefficients response to \( T \nabla (1/T) \) equals that to \( \nabla \psi \) and the same is true for the coefficients of \( \mathbf{E} \) and \( (T/e) \nabla (\mu/T) \). Hence, it is only necessary to consider the system response to dynamical forces for calculations of transport coefficients. This theoretical description is then valid for spin-dependent transport. The linear response theory enables us in practice to perform analytic derivations of spin Hall conductivity in the case of non-uniform temperature \( T \). In the presence of SO coupling, the spin-orientation dependence has been found to be involved in the interaction between energy density and the gravitational field. Generalizing Einstein relations to the spin-dependent response theory, the spin-dependent thermal coefficients can be obtained on the analogy of spin-dependent (charge and spin) conductivities.

We now present a theoretical calculation of the spin Hall current generated by thermal influence and the thermoelectric effect. Here we consider a two-dimensional Rashba spintronic system in the presence of a time-dependent temperature gradient\(^{21-23} \), i.e., studying the dynamical response to a temperature gradient alternating with nonzero frequency. This yields frequency-dependent thermopower and dynamical thermospin Hall conductivity. The Hamiltonian in the presence of infinitesimal time-dependent electric field \( \mathbf{E}(\mathbf{r}, t) = -\nabla \varphi(\mathbf{r}, t) \) and gradient of the gravitational field \( \nabla \psi(\mathbf{r}, t) \) is given by \( H = H_0 + \frac{1}{c} \mathbf{J}^{(e)} \cdot \mathbf{A}(\mathbf{r}, t) + \frac{1}{e} \mathbf{J}^{(q)} \cdot \mathbf{N}(\mathbf{r}, t) \), where vector potentials \( \mathbf{A}(\mathbf{r}, t) = \frac{c}{i\omega} \mathbf{E}(\mathbf{r}) e^{-i\omega t+0^+t} \) and \( \mathbf{N}(\mathbf{r}, t) = \frac{e}{i\omega} \nabla \psi(\mathbf{r}) e^{-i\omega t+0^+t} \), which are adiabatically switched on from the infinitely remote past \( t = -\infty \), interact with electrical and energy currents, respectively. \( H_0 \) is the Rashba Hamiltonian and can be written in a second quantized form, \[ \sum_{k, s} E_{k, s}^{(0)} \hat{a}_{k, s}^\dagger \hat{a}_{k, s}, \] where \( \hat{a}_{k, s} \) (\( \hat{a}^\dagger_{k, s} \)) is the annihilation (creation) operator for an electron with momentum \( \hbar \mathbf{k} \) and band \( s \). \( s = \pm \) denotes two spin-splitting dispersion branches’ energy \( E_{k, s}^{(0)} = \hbar^2 k^2 / 2m^* + s\lambda k \) with Rashba SO coefficient \( \lambda \), the effective electron mass \( m^* \) and the in-plane momentum \( \hbar \mathbf{k} = \hbar \sqrt{k_x^2 + k_y^2} \). Corresponding electric current operator \( \mathbf{J}^{(e)} \) and
heat current operator $\mathbf{J}^{(q)}$ take the form

$$\mathbf{J}^{(e)} = e \sum_{k,s} \mathbf{K}_{k,s}^{(s)} a_{k,s}^+ a_{k,s} - ie \sum_{k,s} \mathbf{K}_{k,s}^{(s)} a_{k,s}^+ a_{k,-s}$$

and

$$\mathbf{J}^{(q)} = \sum_{k,s} \left( E_{0,k,s}^{(s)} - \mu \right) \mathbf{K}_{k,s}^{(s)} a_{k,s}^+ a_{k,s} - i \sum_{k,s} \left( \frac{\hbar^2 k^2}{2m^*} - \mu \right) \mathbf{K}_{k,s}^{(s)} a_{k,s}^+ a_{k,-s}$$

with $\mathbf{K}_{k,s}^{(s)} = (\hbar/m^*) \mathbf{k} [1 + s (\lambda m^*/\hbar^2)]$ and $\mathbf{K}_{k,s}^{(s)} = s (\lambda/\hbar) [(\mathbf{k} \times \hat{z}) / k]$. From the expressions of currents it can be seen that, responding to the frequency-dependent electric field and temperature gradient, the contributions to currents are composed of two parts: the intraband current and the interband current. The latter comes mainly from the anomalous velocity term related to the transition between spin-splitting subbands. To pinpoint the specific details of subband transitions that are responsible for the generation of the spin Hall current, we keep the discussion within “clean” limits, specifically that the system has no disorder, impurity, or electron-phonon interaction$^{24,25}$. In evaluating spin states with spin-splitting subbands, the equation of motion in the linear response to $\mathbf{A}$ and $\mathbf{N}$ leads to

$$\langle a_{k,s}^+ a_{k,s} \rangle^{(1)} = 0$$

and

$$\langle a_{k,s}^+ a_{k,-s} \rangle^{(1)} = -i s \frac{\lambda}{\hbar k} F_{k,s}^{s,-s} (\omega) \left[ \mathbf{k} \times \left( \frac{e}{c} A + \frac{\hbar^2 k^2}{2m^*} \mathbf{N} \right) \right] \cdot \hat{z},$$

where $F_{k,s}^{s,-s} (\omega) = (f_{k,s} - f_{k,-s}) / (\hbar \omega' + E_{0,k,s}^{(0)} - E_{0,k,-s}^{(0)})$ with $\omega' = \omega + i0_+$ and Dirac-Fermi distribution function $f_{k,s}$. These mean that only the interband transitions cause dynamical currents while intraband terms do not contribute to conduction if the system is free of any disorders. After a straightforward calculation, the frequency-dependent currents in the linear response are found as $\mathbf{J}^{(n)} (\omega) = \mathbb{L}^{(ne)} (\omega) \cdot \mathbf{E} + \mathbb{L}^{(nq)} (\omega) \cdot (-\nabla T / T)$, where $n = 1$ denotes charge ($e$) and $n = 3$ denotes heat ($q$). The corresponding transport coefficients, the order of $\lambda^2$, can be cast in the form $\mathbb{L} (\omega) \propto \sum_{k,s} g_{k,s}^{(n)} (\omega)$ with $g_{k,s}^{(n)} (\omega) = k^n F_{k,s}^{s,-s} (\omega) \mathbb{I}$ and two dimensional unit tensor $\mathbb{I} = \hat{x} \hat{x} + \hat{y} \hat{y}$. For $E_y = \nabla_y T = 0$ and $\nabla_x T \neq 0$, $j^{(e)} = 0$ is a result of the complete cancellation in opposite electric currents owing to electrons flowing between hot and cold sides in the system. As a consequence of the thermoelectric effect, a longitudinal electric field $E_x$ is induced by the temperature gradient, i.e., $E_x = S \nabla_x T$, where $S = L^{(eq)} / (TL^{(ee)})$ is the Seebeck coefficient.

Accompanying the electron motion in a Rashba SO coupling system, a spin force, oriented perpendicular to the temperature gradient, acts on its spin. Although the net electric current
vanishes, the number of electrons moving in opposite directions is not of an equal amount. The excess of moving electrons will contribute to generating the spin Hall current. The drift processes of electrons are opposite with respect to their spin orientations being up or down. In that the opposite spins move in opposite directions, this sets up a transverse spin current in the system. The spin current operator in the second quantized form is given by
\[ J_z^{(s)} = \frac{\hbar^2}{2m^*} \sum_{k,s} \mathbf{k} \alpha^\dagger_{k,s} \mathbf{a}_{k,-s}. \] It is quite evident that the spin current is generated not by the displacement from the electron distribution function, but by the contribution from transitions between spin-split bands. The calculation shows that the only nonzero component of the spin current is in the \( \hat{y} \) direction, i.e., perpendicular to the temperature gradient. It is noted that the contributions to the spin current are naturally classified into a direct thermal spin Hall current, caused by a temperature gradient
\[ -\left( L^{(sq)} / T \right) \nabla_x T, \]
and a thermoelectric spin Hall current, by thermoelectric effect
\[ SL^{(se)} \nabla_x T, \]
where \( L^{(sq)} \) and \( L^{(se)} \) relate to \( s \mathbb{G}_{k,s}^{(2)} (\omega) \) and \( s \mathbb{G}_{k,s}^{(4)} (\omega) \), respectively, and are in the order of \( \lambda \). The total spin Hall current can be written, in terms of the thermospin Hall conductivity
\[ \sigma_{TE}^{SH} = \frac{(TSL^{(se)} - L^{(sq)})}{T}, \] as \( J_y^{(s)} = \sigma_{TE}^{SH} \nabla_x T. \)

Figure 2 shows the temperature dependence of thermospin Hall conductivity against the strength of SO coupling. Here a finite frequency is chosen and the thermospin Hall conductivity is frequency dependent. It is readily seen that the thermospin Hall conductivity \( \sigma_{TH}^{SH} (\omega) \) is non-vanished when the frequency \( \omega \) is in the region
\[ 2\lambda k_{F,+} \leq \hbar \omega \leq 2\lambda k_{F,-}, \]
where
\[ k_s = \sqrt{2m^* \lambda^2 / \hbar^2 \sqrt{4\mu + 2m^* \lambda^2 / \hbar^2} - 2m^* \lambda^2 / \hbar^2}. \]
The interband transitions mainly appear between energies
\[ \hbar \omega_+ = 2\lambda k_{F,+} \] and \( \hbar \omega_- = 2\lambda k_{F,-}, \) which correspond respectively to the minimum and maximum photon energy required to induce optical transitions between the initial \( s = -1 \) and final \( s = +1 \) spin-split bands. The width of the resonant window is almost independent of temperature because the change of temperature is merely broadening the range of electron distribution at the Fermi energy. However, increasing temperature shifts the resonant window toward the high-frequency regime. In the inset of Figure 2, we show the shifting of the resonant frequency window with respect to altering the strength of the SO coupling. As seen in Figure 2, the magnitude of thermospin Hall conductivity depends on the temperature and displays as hyperbolic in nature. It exhibits the behavior \( 1/T \) at low temperature if the frequency is within the range \( (\omega_+, \omega_-) \). When the temperature increases, the thermospin Hall conductivity decreases rapidly, while at high temperature, it tends slowly to a frequency-dependent low finite value. In contrast, if the frequency is out-
side the range \((\omega_+ \omega_-)\), the thermospin Hall conductivity tends to zero at low temperature. With increasing temperature, the thermospin Hall conductivity increases first and achieves a maximum value at a certain temperature. With further increasing temperature, the thermospin Hall conductivity will decrease and tend to a frequency-dependent low finite value. These findings indicate that the measurement of thermospin Hall current can be controlled by modulating electrical gating and by adjusting the frequency of the temperature gradient. The observation of a relatively strong magnitude in the frequency domain hinges on the appropriate temperature.

If the 2DEG is confined as a plate with a constant gate voltage applied in the \(\hat{z}\) direction (controlled strength of SO coupling), as illustrated in Figure 1(a), we attach its two ends to heat baths with high temperature \(T_H\) on the left and low temperature \(T_L\) on the right (in the \(\hat{x}\) direction). The heat current, associated with moving electrons, flows between the left and the right ends. The SO coupling manifests as a spin force on the spin of electrons in the direction perpendicular to the heat flow. As a result, the formation of spin-orientation-dependent electronic motion leads to spins with opposite signs preferentially deviating in opposite directions. An excess of up spins will be accumulated on one side of the plate and an excess of down spins on the opposite side. The spin alignment of electrons (ferromagnetic profiles) on the edge determines the magnetization, i.e., the magnetization \(M^\uparrow\) is distributed in the region near one edge and \(M^\downarrow\) near another edge. The magnetizations in the \(+\hat{z}\) and \(−\hat{z}\) directions produce stray fields in the \(+\hat{y}\) and \(−\hat{y}\) directions, respectively. The field exerts the force on the incoming spin-polarized flow through the interaction \(\mu_B j_y \nabla_y M_z\) and stops any further spin exchange. Equilibrium is created due to the balancing of the spin force of SO coupling, the electric force due to charge accumulation, and the force owing to the gradient of magnetization. The spin accumulation profiles persist after the spin currents have vanished. Then, in the \(\hat{y}\) direction, a potential distribution \(V_{SH}\) is generated on the upper half-plane and the voltage \(−V_{SH}\) on the lower half-plane, as illustrated in Figure 1(a). SO coupling is equivalent to the effective SU(2) gauge potential, transverse to the temperature gradient in the Pauli Hamiltonian. As such, similar to the Nernst effect in a conductor plate, a potential distribution built up from stray magnetization can emerge in the \(\hat{y}\) direction under SO coupling (instead of a magnetic field in the \(\hat{z}\) direction in the Nernst effect) and a temperature bias in the \(\hat{x}\) direction.

The predication of an effective potential caused by a magnetization gradient opens up
possibilities for measuring the thermospin Hall signal. One such possibility is that, to confirm the thermospin Hall effect experimentally, the characteristics of light reflected in magnetizations owing to spin alignment can be sought. Recently, several groups reported optical detections of spin accumulation with opposite signs at the sample edges in current-biased nonmagnetic semiconductors. In these experiments, Rashba-type spin splitting can be identified via optical experiments instead of magnetotransport measurements. Optical experiments have also been used to study the generation of a pure spin current. Another possibility that emerges is the spin accumulation induced by a thermoelectric effect, providing us a reciprocal way to confirm the thermospin Hall effect. In fact, the predication of the thermospin Hall effect offers us a new pathway to obtain spin information by means of the detection of longitudinal temperature differences when a spin current is transversely injected. As a transverse spin current is injected in a nonmagnetic semiconductor, the effect is essentially that electrons with opposite spins flow in opposing transverse directions. The SO interaction in the system leads electrons to move toward the same longitudinal side of the sample. This results in a charge accumulation and induces a longitudinal temperature gradient by means of the thermoelectric effect. In fact, similar electrical measurements, based on the idea of the inverse spin Hall effect, have been achieved with a charge accumulation. In addition, due to the opening up of new channels for thermospin transition accompanied by heat-radiating absorption and excitation, some unique thermal-optical properties can be observed in thermospin Hall systems. Different from experiments where the induced magnetic moment is non-destructively detected in a non-contacting way using a magnetometer, the thermospin Hall effect provides a promising method for detecting inhomogeneity of spin accumulation in semiconductor materials, employing a thermistor probe and a thermogalvanometer.

It becomes of interest to us to decide whether the thermal effect in heterogeneous semiconductors might be of use in practical spintronics. Spin-splitting states are asymmetric in the momentum distribution of electrons in Rashba spintronic systems and can usually be controlled experimentally by tuning external parameters. Therefore, the topic of the thermospin properties of Rashba spintronic systems, such as the thermoelectric-driven spin current proposed in this study, is very rich in terms of basic physics and device applications. Studying the spin-dependent Hall angle allows us to address the problem of the validity of spin-orientation-dependent edge states in the spin Hall effect. The Hall angle Θ is defined
by \( \tan \Theta = E_y/E_x \), where \( E_y \) and \( E_x \) are the transverse and longitudinal potential gradients, corresponding respectively to completely different mechanisms: the former stems from the induced spin accumulation due to the SO coupling, and the latter, in essence, originates from the thermoelectric effect. The Hall angle in equilibrium is a reflection of the thermospin and thermoelectric effects of a given system. Because the spin experiences a spin force \( (\propto j^{(s)} \times \hat{z}) \), we note that the spin diffusion process is rotated away from the \( \hat{x} \) direction by a Hall angle \( \Theta \). Under the interaction between the spin-polarized flow and the gradient field of magnetization, a backflow current \( J_x^{(e)} = \mu_B L^{(es)} \nabla_y M_z \) is generated with the aid of an anomalous Hall/converse spin Hall effect. It is convenient to regard this as an anomalous electric Hall effect induced by the magnetization gradient. This backflow electric current is rotated away from the \( -\hat{x} \) direction by the Hall angle \( \Theta \). It is worthwhile to point out that the generation of a Hall-like backflow electric current originates entirely from the interaction \( \mu_B j_y^s \nabla_y M_z \) breaking the time-reversal symmetry\(^34\). In equilibrium, \( J_x^{(e)} = J_y^{(s)} = 0 \) yields for the longitudinal (\( E_x \)) and transverse (\( E_y \)) potential gradients, \( E_x = \left( L^{(eq)}/T L^{(se)} \right) \nabla_x T \) and \( E_y = -\left( \hbar/e \right) \left[ \left( L^{(ce)} L^{(se)} - L^{(ee)} L^{(sq)} \right)/T L^{(se)} L^{(eq)} \right] \nabla_x T \), respectively. Therefore, we obtain \( \tan \Theta = \left( \hbar/e \right) \left[ \left( L^{(ce)} L^{(se)} - L^{(eq)} L^{(se)} \right)/L^{(se)} L^{(eq)} \right] \). Figure 3 presents an estimation of the Hall angle in the thermospin Hall effect. The most prominent features of frequency dependence of the Hall angle are sharp divergences around \( \hbar \omega = 2\lambda k_F,\pm \) (Figure 3(b)). The absolute value decreases rapidly as \( \omega \) deviates from \( 2\lambda k_F,\pm \). The distance between divergences is temperature dependent (Figure 3(a)). The reason is that both spin transport and the magnetization on the edges are dominated by those electrons which are thermally created in relatively wider energy bands. The sign of the Hall angle depends on whether the frequency is inside or outside the frequency range \( \Delta \omega = (\omega_-, \omega_+) \). In addition, the transverse potential gradient decreases as temperature increases. The reason is that, in principle, the spin alignment decreases as the contribution of spin splitting is suppressed by thermal energy with the increase of temperature. In practice, the spin diffusion length also slowly decreases at high temperature.

Finally, we should point out that a charge accumulation accompanies the electronic spin Hall effect, due to an excess of electrons with the same spin orientation gathering around edges. Such a charge population might produce an influence in optical measurements. To eliminate the influence caused by the charge accumulation, it is essential to achieve pure spin accumulation in the spin Hall effect. This can be realized by the thermophotovoltaic
generation of electron-hole pairs in semiconductor systems. This is similar to the optical schemes for generating spin current\textsuperscript{29,30}, which are also valid for the thermospin Hall effect. For situations in which the Fermi level lies between the conduction and valence bands, the absorption of "photon" energy excites an electron in the valence band to move into the conduction band. As a result, an electron-hole pair is created. The charge currents due to electrons and holes under the temperature gradient are opposite and equal in magnitude. In this case, there is no net electric current. Correspondingly, the thermopower is negative for electrons and positive for holes\textsuperscript{35}. As it has been described for Rashba systems, the spins of both moving electrons and holes exert spin force. In identifying the pair formed by an electron and hole with opposite spins, it can be seen that the forces exerted on them are opposite. Such spin asymmetry of an electron-hole pair results in the pair breaking and the electron and hole separating spatially under the spin force. In narrow-gap insulators, the electron-hole interaction is very weak, due to sufficient screening. In these systems, excitonic effects do not dominate the spectrum and one-electron spectrum approximation is applicable. Therefore, electrons and holes with the same spin orientation will deviate in the same direction. As illustrated in Figure 1(b), the motions of electrons and holes from the central excitation spot upward for up spins and downward for down spins results in the spin current. In this way, the opposing charge currents due to electrons and holes are converted into the intrinsic spin current. There also exist two kinds of contributions in the thermospin Hall effect: one is the direct contribution from spin asymmetry in the electron-hole pair \(- (L^{(eq)}/T)\) and another is from the thermoelectric effect \(SL^{(se)}\). For a plate, there is only net magnetization but no net charge in edges because the populations of electrons and holes are equal and have the same spin orientations. No charge distribution in the \(\hat{y}\) direction means no electric field is generated in the \(\hat{y}\) direction either. Hence, no Joule heating is caused. This characteristic is particularly suitable, not only for a dissipationless experimental measurement, but also for device applications at low power loss. We emphasize that the survival of thermospin Hall phenomena in semiconductors strongly depends upon the spin-splitting band structure, which will be influenced in principle by the phonon spectrum\textsuperscript{36} and the peculiarities of the scattering mechanisms\textsuperscript{37}. The electron-phonon interaction in the thermospin Hall effect are interesting in practice because the thermoelectric effect would be drastically changed by the electron-phonon interaction, ranging from the weak-coupling limit\textsuperscript{38} to the strong-coupling regime\textsuperscript{39}. This may give further insight both for the nature of
the thermospin Hall effect and for spintronic applications.

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**Figures**
FIG. 1: Schematic illustration of a traverse spin-orientation-dependent motion in SO coupling systems generated by longitudinal temperature influence: (a) electron gas and (b) the scheme proposed for excitation of electron-hole pairs. The left and right ends of two-dimensional systems are contacted to the periodically modulated hot and cold baths, respectively. The heat current, associated with moving electrons, flows between the left and right ends. The corresponding charge current is balanced by the thermoelectric effect. The spin-up (spin-down) carriers are shown moving in the +\( \hat{y} \) (−\( \hat{y} \)) direction, yielding no transverse net charge current. As a result, a spin current, without accompanying net charge current, is generated in the direction perpendicular to the temperature gradient. The magnetizations in opposite out-of-plane orientations are formed in the opposite edges if the systems are plates.
FIG. 2: The thermospin Hall conductivity versus the strength of SO coupling and temperature in the presence of a periodically modulated temperature gradient. Here the frequency has been taken 0.2 in the unit of Fermi energy. The inset shows the frequency-dependent thermospin Hall conductivity with five different strengths of SO coupling.
FIG. 3: Dependence with frequency of the Hall angle. The upper panel (a) displays variances of the Hall angle for temperatures ranging roughly from low (solid line) up to room temperature (dash line), for a fixed Rashba coupling. The lower panel (b) shows variances of the Hall angle with three different strengths of SO coupling.