Composite Models on a safe road to the Planck scale

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Abstract. We present the first serious attempt to define a model of composite pseudo-Nambu-Goldstone Higgs with partial compositeness for all standard fermions that is valid up to the Planck scale. The main ingredient is the presence of a large multiplicity of fermions in the microscopic gauge-fermion description, which allows us to use large-$N_f$ techniques to show the presence of an interactive UV fixed point for the gauge couplings (UV safety). We also present results for the Dark Matter relic density and direct detection in the example model that is UV-completed.

1. Introduction

The possibility that the electroweak symmetry breaking in the Standard Model (SM) is dynamically generated by a confining and condensing strong interaction is still a valid alternative to the Higgs mechanism in the SM. The latter is described in terms of an elementary scalar field, with all the drawbacks associated to it (hierarchy, non-explanation of the negative mass squared, etc.). In dynamical models, the scalar sector is replaced by a new strong sector: this idea is in fact as old as the SM itself [1, 2]. In the original proposal, heavily based on Quantum Chromodynamics (QCD), no light scalar arose in the spectrum, thus this possibility is ruled out by the recent discovery of a 125 GeV Higgs-like boson. On the other hand, a light Higgs boson can still be present arising as a pseudo-Nambu-Goldstone boson (pNGB) of a broad global symmetry breaking pattern [3]. This mechanism, first proposed in the early 80’s, has received a revival with the discovery of the holographic principle [4] that links a strongly interacting conformal field theory to extra dimensions. Thus, the Higgs arises as a gauge field in the holographic picture [5], in analogy to models of Gauge-Higgs Unification [6].

Following the holographic approach, most of the recent work has been focusing on effective field theory approaches [7, 8], where little attention is given to the microscopic origin of the confining dynamics. The UV completion of the model is thus assumed to be either a conformal theory of unknown origin, or extra dimensions. An alternative, inspired by QCD, is to describe an underlying microscopic model in terms of gauge and fermion degrees of freedom. By doing so, one first realises that non-minimal cosets in terms of number of pNGBs are the norm. For instance, the minimal allowed coset would be $SU(4)/Sp(4)$ [9], which contains 5 pNGBs organised as a Higgs doublet (4 degrees of freedom) plus a gauge singlet. All the allowed cosets fall in 3 classes: $SU(N)/Sp(N)$, with $N$ even, if the fermions are in a pseudo-real irreducible
representation (irrep) of the confining gauge force, \(SU(N)/SO(N)\) if they are in a real irrep, and finally \(SU(N) \times SU(N)/SU(N)\) if they are in a complex irrep. This observation strongly limits the possible cosets to study.

Another ingredient that came into play, and is important in model building, is the concept of fermion partial compositeness \([10]\). In this paradigm, the SM fermions acquire their mass by coupling linearly to some fermionic operators in the composite sector. This needs to be compared to the older idea to couple them in bi-linear operators, and it was introduced to give a generic solution to the problem of potentially dangerous flavour changing neutral currents. In the holographic approach partial compositeness materialises in the fact that the SM fermions are the lightest states of a bulk propagating fermion, thus their localisation in the extra dimension determines the degree of compositeness \([11, 12]\). In underlying gauge-fermion theories, this mechanism requires the presence of four-fermion couplings between one SM fermion and fermions in the strong sector, which have the proper quantum numbers. This calls for the issue of assigning QCD quantum numbers to the strong sector in order to allow couplings to the quarks.

In these proceedings we will focus on models that feature an underlying gauge-fermion description, as described above. In particular, we will focus on two aspects: the possibility to UV complete such models in order to give them a microscopic description valid up to the Planck scale, and the possibility to feature a Dark Matter candidate among the additional pNGBs. As we will see, both features will heavily rely on the underlying gauge-fermion description.

About the first point, attempts to build “UV completions” were put forward in Refs \([13, 14, 15]\): however, they are not genuine UV completions but simple underlying descriptions. The main reason is that no attempt is given to explaining the origin of the couplings responsible for top partial compositeness, thus the presence of four-fermion interactions imposes a cut-off to the theory that is very close to the one of the effective description in terms of pNGBs. Nevertheless, such descriptions have the benefit of lattice studies that can shed some light on the low-energy properties of the models \([16, 17, 18, 19]\). One interesting idea emerged in Ref. \([13]\): top partial compositeness can be accommodated by adding a second species of fermions, which transform under a different irrep than those condensing in the Higgs sector. In this way, QCD interactions are sequestered to the new sector and do not interfere with the electroweak symmetry breaking. \(^1\)

Here we will go beyond this approach and try to genuinely define the theory above the validity of the effective pNGB description: our approach will be based on large \(N_f\) resummation techniques, see Ref. \([21, 22, 23]\). In fact, extending partial compositeness to all fermions of the SM necessarily implies the presence of a large number of fermions in the second sector. This naturally drives the theory towards a UV interacting fixed point, which renders the model valid up to arbitrary scales.

The second point is the presence of a Dark Matter candidate. As already mentioned, underlying gauge-fermion descriptions naturally predict the presence of non-minimal cosets, which include additional pNGBs. In the minimal model, \(SU(4)/Sp(4)\), the additional singlet has been the first Dark Matter candidate of this kind \([24]\), however it decays via topological anomalies. The most minimal model that features a Dark matter candidate is the minimal coset with complex irreps, based on \(SU(4) \times SU(4)/SU(4)\) \([25]\). The properties of the Dark matter candidate in the case of bilinear top mass terms have been studied in Ref. \([26]\). Other examples include \(SU(6)/Sp(6)\) \([27]\) and \(SU(6)/SO(6)\) \([28]\). For completeness, works considering cosets that do not have a simple underlying gauge-fermion description have also been considered in Refs \([29, 30, 31, 32]\).

\(^1\) The only exception is a QCD-like model in Ref. \([20]\).
Table 1. Minimal cosets with a pNGB Higgs doublet arising from an underlying gauge-fermion theory. The fourth column shows the $SU(2)_L$ irrep, with the hypercharge as subscript. The last three columns show some properties of the explicit models, with the nomenclature M1-M12 from Ref. [33], and MV being the model from Ref. [20].

| psi irrep | coset | pNGBs | EW charges | models | $\psi_{HC}$ | Lattice results |
|-----------|-------|-------|------------|--------|-------------|----------------|
| pseudo-real | $SU(4)/Sp(4)$ | 5 | $2_{\pm 1/2} \oplus 1_0$ | M8-M9 | $Sp(4)$, $SO(11)$ | $Sp(4)$ |
| real | $SU(5)/SO(5)$ | 14 | $2_{\pm 1/2} \oplus 1_0 \oplus 3_{\pm 1} \oplus 3_0$ | M1-M7 | $SU(4)$, $Sp(4)$, $SO(7)$, $SO(9)$, $SO(10)$ | $SU(4)$, $Sp(4)$ |
| complex | $SU(4) \times SU(4)/SU(4)$ | 15 | $2 \times 2_{\pm 1/2} \oplus 3_0 \oplus 1_{\pm 1} \oplus 2 \times 1_0$ | M10-M12 | $SO(10)$, $SU(4)$, $SU(5)$ | $SU(10)$, $SU(3)$ |

2. Running safely to the Planck scale

As already mentioned, our analysis is based on the low energy models listed in Ref. [13] (see also Ref. [33]). These models were designed to give underlying fermion-gauge descriptions to the interactions that give rise to the composite pNGB Higgs and top partial compositeness. The QCD interactions are sequestered to a second sector of underlying fermions, $\chi$, that transform under a different irrep under the confining Hypercolour (HC) than the underlying fermions, $\psi$, generating the Higgs. A complete, but brief, list of models is presented in table 1. We also included the model of Ref. [20] where $\psi$ and $\chi$ transform under the same irrep. The top partners, i.e. the operators that couple to the top fields, arise as bound states of the form $\psi\psi\chi$ or $\psi\chi\chi$, depending on the specific model.

One limitation of these models is the fact that they can accommodate only for the generation of the top mass. The reason is that many more underlying fermions need to be added in order to include operators that can couple to the bottom, and also to leptons and the other two generations. Thus, the model will feature too many fermionic degrees of freedom and asymptotic freedom is lost. In other words, the theory will not confine at low energies. In our work in Ref. [34] we turned this drawback into a benefit: in fact, adding a fermion $\chi$ for each SM fermion (i.e., allowing partial compositeness for all SM fermions) increases dramatically the fermionic degrees of freedom to the point that we can employ large $N_f$ techniques to resum their effect on the running of the gauge couplings. It has been observed in the literature that the resummed beta function allows for a zero in the UV, thus there is a strong hint of the presence of a UV interacting fixed point [35]. In turn this implies that our models also feature an interacting UV fixed point, thus they can be trusted up to arbitrarily high scales.

While this picture sounds simple, it is not that easy to realise it in practice. We will present here an explicit example with model M10, motivated by the fact that it has a pNGB Dark Matter candidate. The model is based on the HC group $SO(10)_{HC}$, and the underlying fermions charged under $SU(10)_{HC}$ are listed in table 2. The underlying fermions that characterise M10 (i.e., the low energy part of it) are the ones with $\approx 0$ mass. They will make sure that the theory confines and generates a pNGB effective theory below the scale $\Lambda_{HC} \approx 4\pi f$, where $f$ is the decay constant of the composite Higgs. As a reference, we can keep in mind the following scales: $f \approx 1$ TeV (in order to escape bounds from electroweak precision tests) and $\Lambda_{HC} \approx 10$ TeV.

At the scale $\Lambda_{HC}$, we introduce two more $\chi$'s, with the quantum numbers that allow for the generation of partial compositeness for the bottom quark and tau lepton (thus completing the third generation). Their presence pushes the theory inside an IR conformal window [36, 37, 38], preferably at strong coupling. The role of this walking region [39] is to generate a large enough
**Table 2.** UV-completed model M10 - all fermions are Dirac spinors.

|   | $SO(10)_{HC}$ | $SU(3)_c$ | $SU(2)_L$ | $U(1)_Y$ | mass        |
|---|--------------|-----------|-----------|----------|------------|
| $\psi_L$ | 16          | 1         | 2         | 0        | $\sim 0$   |
| $\psi_{u,d}^{R}$ | 16          | 1         | 1         | $\pm1/2$ | $\sim 0$   |
| $\chi_u^3$ | 10          | 3         | 1         | 2/3      |            |
| $\chi_u^3$ | 10          | 3         | 1         | $-1/3$   | $\sim \Lambda_{HC}$ |
| $\chi^{1,2}_u$ | 3          | 1         | 2         |          |            |
| $\chi^{1,2}_d$ | 10         | 3         | 1         | $-1/3$   | $\Lambda_{Fl} \gg \Lambda_{HC}$ |
| $\chi^{1,2}_l$ | 1          | 1         | 1         | $-1$     |            |

Figure 1. Plot of the renormalisation group running of the gauge couplings, $\alpha_i$, above the electroweak scale. The continuous lines correspond to one-loop running, while the dashed lines show the large-$N_f$ resummation above $\Lambda_{Fl} = 10^{8.5}$ GeV. The upper panel contains the running of $\alpha_{10}$ at strong coupling. Note that $\alpha_2$ also runs towards the UV fixed point $\alpha_2 = 0.86$ (not shown in the plot).

The split between the scale where the partial compositeness couplings are generated and the scale of compositeness. At a much higher scale, $\Lambda_{Fl}$, we introduced two more copies of the $\chi$’s, in order to generate partial compositeness for all generations. Now, the number of fermions is such that large-$N_f$ resummation can be employed for all gauge couplings. The running, without large-$N_f$, is shown in figure 1 by the solid lines. The green one corresponds to the confining $SO(10)_{HC}$, with the strong coupling walking represented as a sketch because a perturbative calculation in that regime is clearly not trustworthy. Lattice studies will therefore be necessary to study this region and the validity and duration of the walking regime [16, 17, 18, 19]. It is clear that all the gauge couplings start growing towards the UV once the third-generation partial compositeness is introduced. This growth will be tamed by large-$N_f$ resummation.
2.1. Large-$N_f$ results

Contrary to calculations already present in the literature \[40, 41, 42, 43\], where $N_f$ refers to the multiplicity of a single fermion type, in our case we have fermions that have different quantum numbers under the 4 gauge groups of the model. Thus, we need to define a different effective $N_f$ for each group and resum them all at the same order.

For each gauge group, therefore, we define a normalised coupling \[35\] that takes into account the multiplicity of fermions $f$ charged under it:

$$K_i \equiv N_i T_i \frac{\alpha_i}{\pi} = \frac{\alpha_i}{\pi} \sum_l n_l T_l(r_l); \quad (1)$$

with $i = 1, 2, 3, 10$ labelling the 4 gauge groups. We fix the overall normalisation with the fundamental irreps, i.e. $T_1 = T_{10} = 1$ and $T_2 = T_3 = 1/2$. Moreover, for $U(1)$ it suffices to replace in the formula $T(r_f) \rightarrow Y_2 f$. The multiplicities $N_i$, that vary with each group, are all considered formally to be of order "$N_f$", thus we can resum them at the same time and at the same order. For the UV-complete model M10, we find the following values:

$$N_1 = 93, \quad N_2 = 22, \quad N_3 = 66, \quad N_{10} = 25. \quad (2)$$

The resummed evolution equations, keeping all terms in the expansion, in general read

$$\frac{\partial \ln K_i}{\partial \ln \mu} \equiv \beta_i(K_i) = 2K_i \left[ 1 + \sum_n \frac{1}{N_f^n} B^{(n)}_i \right], \quad (3)$$

with the first-order terms equal to

$$\frac{B^{(1)}_i}{N_f} = \frac{1}{N_i} \left[ c_{i,i} H_1(K_i) + \sum_{\substack{j \neq i}} c_{i,j} F_1(K_j) \right], \quad \text{for } i = 2, 3, 10; \quad (4)$$

and

$$\frac{B^{(1)}_1}{N_f} = \frac{1}{N_1} \left[ c_{1,1} F_1(K_1) + \sum_{\substack{j \neq 1}} c_{1,j} F_1(K_j) \right]. \quad (5)$$

The coefficients $c_{i,j}$ can be computed numerically in terms of the quantum numbers of the fermions. For M10, this yields

$$c_{1,1} = \frac{1927}{3348}, \quad c_{1,2} = \frac{3}{44}, \quad c_{1,3} = \frac{211}{99}, \quad c_{1,10} = 15; \quad \text{c}_{2,1} = \frac{1}{186}, \quad c_{2,2} = \frac{3}{2}, \quad c_{2,3} = \frac{9}{11}, \quad c_{2,10} = \frac{5}{54}; \quad c_{3,1} = \frac{1116}{211}, \quad c_{3,2} = \frac{9}{44}, \quad c_{3,3} = \frac{3}{8}, \quad c_{3,10} = \frac{5}{9}; \quad c_{10,1} = \frac{3}{31}, \quad c_{10,2} = \frac{3}{11}, \quad c_{10,3} = \frac{8}{11}, \quad c_{10,10} = \frac{5}{2}. \quad (6)$$

The functions $H_1$ and $F_1$ are generated by the resummation of fermion bubbles on the gauge propagators of the two-loop diagrams, and their explicit expression can be found in Ref. [41]. They have a special property: they feature a pole above which the resummation does not converge, and the pole drives the function towards negative values. This implies that near the pole the beta function at leading order will vanish. The poles, therefore, act as an effective barrier that does not allow the gauge couplings to grow beyond such a point in the UV. This
property is at the origin of the postulated UV fixed point. For the two functions relevant here, the poles stand at:

\[ H_1(K^*) \to -\infty \text{ for } K^* = 3, \quad F_1(K^*) \to -\infty \text{ for } K^* = \frac{15}{2}. \]  

These barriers will thus drive our model to a complete UV fixed point only if the value of the couplings at the threshold \( \Lambda_{\text{Fl}} \) are below the pole: the evolution towards the UV will stop at that value where the beta function vanishes and the theory approaches a fixed point. Numerically, in M10 this implies:

\[ \alpha_1(\Lambda_{\text{Fl}}) < 0.25, \quad \alpha_2(\Lambda_{\text{Fl}}) < 0.86, \quad \alpha_3(\Lambda_{\text{Fl}}) < 0.28, \quad \alpha_{10}(\Lambda_{\text{Fl}}) < 0.38. \]  

Satisfying the above constraints provides an upper bound on \( \Lambda_{\text{Fl}} \) as all the gauge couplings (except \( \alpha_{10} \)) increase towards the UV above \( \Lambda_{\text{HC}} \). We should also remind the reader that \( \Lambda_{\text{Fl}} \) cannot be too low, otherwise we risk generating dangerous flavour-changing neutral currents when the partial compositeness four-fermion interactions are generated. Thus, we will generically require \( \Lambda > 10^5 \text{ TeV} \) for generic flavour violating effects.

We show in figure 1 the one-loop running of the 4 gauge couplings above the electroweak scale, assuming \( \Lambda_{\text{HC}} = 10 \text{ TeV} \) for the model in table 2. In this case, the coupling that first crosses the upper limit in Eq. (8) is the QCD one \( \alpha_3 \), shown in red. The thin horizontal line, representing the bound, is crossed at \( 10^9 \text{ GeV} \), which is therefore the upper limit on the value of \( \Lambda_{\text{Fl}} \). We thus add the \( \chi \)'s for the light generations at \( \Lambda_{\text{Fl}} = 10^8 \text{ GeV} \) and plot in dashed lines the running after the large-\( N_f \) resummation. We can see that all the couplings run to the UV fixed point (for \( SU(2) \) it is not shown in the plot because of the larger value). The behaviour of \( \alpha_3 \) after \( U(1) \) saturates the asymptotic value is a numerical artefact, nevertheless it shows the impact of the \( U(1) \) running on the other gauge couplings as a large \( F_1(\alpha_1) \) will affect all the \( \beta \)-functions in Eq. (5). This example shows that a completely UV-safe composite Higgs and Dark Matter model with partial compositeness is indeed feasible.

3. Dark Matter phenomenology

The low energy description of this model contains electroweakly charged pNGBs generated by the condensation of the \( \psi \) underlying fermions. Out of the cosets shown in table 1, only the \( SU(4)^2/SU(4) \) one enjoys a potentially stable pNGB Dark Matter candidate. The parity, which is a combination of charge conjugation and an \( SU(4) \) rotation, has been studied in Ref. [25].

While the phenomenology of the model with bi-linear mass operators has been studied in Ref. [26], here we will focus on the case where the top mass is generated via partial compositeness. We will thus focus on two scenarios: one based on a holographic model and one based on the UV-complete model discussed in the previous section.

In both cases, the pNGBs that are odd under the dark parity transform as a second Higgs doublet, a real triplet, a charged and a neutral singlet. In general, the lightest state will be a mixture of the three neutral components. We will here discuss the general strategy we employed in the calculation, and show the explicit results in the following sections.

For the top partial compositeness, after appropriately choosing the representation of the fermionic operator, we write down the linear mixing of the right-handed top and of the left-handed doublet. This leaves two free parameters, i.e. the two pre-Yukawas, which are partly fixed by the value of the top mass. At one loop level, this mixing will also generate a potential for the pNGBs, including the Higgs. Here we use the standard technique of computing the loops and imposing maximal symmetry [44] to guarantee the finiteness of the contribution. Note that there is an alternative way, namely computing a basis of operators generated by the same
pre-Yukawa, treated as spurions [45]. This second technique would be more appropriate if the
top partners, i.e. the massive resonances associated with the fermionic operators, were heavy
and/or strongly coupled. In fact, the loop calculation can only be trusted in a regime of light
and weakly coupled composite fermions [46], which is not always the case in strongly interacting
dynamics. It should also be noticed that the two techniques do not necessarily lead to the same
results, as it was shown in a specific case in Ref. [47].

In this work, we also considered the contribution of the gauge loops and of bare masses for
the technifermions, see Ref. [25]. In particular, the masses play an important role in determining
the nature of the lightest stable pNGB. There are two possible masses that can be added: one
for the left-handed doublet $\psi_L$ and one for the right-handed doublet $\psi_R$ (see table 2). These
two masses explicitly break $SU(4) \rightarrow SU(2)_L \times SU(2)_R$ (if $m_{\psi_L} = \pm m_{\psi_R}$, then a global $SU(4)$
is preserved). We can define, therefore, an asymmetry between the two as

$$\delta = \frac{m_{\psi_L} - m_{\psi_R}}{m_{\psi_L} + m_{\psi_R}}.$$  

We observe that, in general, the lightest neutral state is dominantly the $SU(2)_L$ real triplet for
$\delta < 0$, and the singlet for $\delta > 0$, while an equal mixture occurs for $\delta \sim 0$. Interestingly this
pattern does not depend crucially on the top mass mechanism.

To partly fix the free parameters of the model, we first impose that the electroweak scale is
correctly generated at the minimum of the potential and then impose the correct value of the
Higgs and top masses. This fixes only 3 of the free parameters of the model. We then scan over
the remaining ones to check for points that satisfy the correct relic abundance.

3.1. Results for a holographic model

The first case we analyse corresponds to a model inspired by holography. In fact, in order to
preserve the dark parity, we need to choose an appropriate representation for the top partners
under the global $SU(4) \times SU(4)$ symmetry. The simplest choice is a representation that is real
under charge conjugation, i.e. $(4, \bar{4})$. With this choice, the dark parity maps this representation
on itself as it acts as a transpose on the $SU(4)^2$ matrix. As we will see in the next section, this
choice is not possible in the UV complete models studied in the previous section.

![Figure 2](image-url)

**Figure 2.** Scan of the re-weighted Spin-Independent cross section compared to current and
future Direct Detection constraints [48, 49, 50] for $\delta = 0.5$ (left) and $\delta = -0.5$ (right). The
colour encodes the value of the relic density, where the green line indicates the points saturating
the Planck value. [Plots from the first version of Ref. [34]]
The results of the scan are shown in figure 2, where we show the Spin-Independent cross section re-weighted to the correct value of the relic density versus Direct Detection exclusions. The colour of the points encodes the value of the relic density, with the green lines marking points that saturate the Planck bound. Points that tend to the blue hues are under-dense, thus not excluded. We see that in the case of the dominantly singlet ($\delta > 0$), the relic density is saturated by a relatively low mass, with $M_{DM} \approx 250$ GeV. This is due to the typically low annihilation cross section, which is dominated by $\eta\eta \rightarrow \gamma\gamma, ZZ, W^+W^-$ channels. Direct Detection just touches the allowed region, which will however be completely excluded by the next generation experiments. Note that Direct Detection is only due to a trilinear coupling of the scalar Dark Matter candidate to the Higgs.

For $\delta < 0$, the Dark Matter candidate is dominantly a triplet. While a low mass region starting at the same value is present, with annihilation again dominantly in $W^+W^-$, for points where the $\eta\eta \rightarrow t\bar{t}$ channel is dominant larger masses up to 1.8 TeV are needed. However, Direct Detection (XENON1T) already excludes masses up to 1.5 TeV, thus only the high mass end is left available.

Note that the value of $f$ is not fixed here, as it is correlated to the value of the Dark Matter mass. In the scan we have $f = v/\sin \theta$, with $\theta$ varying within the range $[0.003, 0.3]$. The preferred values of the masses fall in the range that is typically preferred by electroweak precision bounds too.

![Figure 3. Same as figure 2, but for the UV complete model M10. [Plots from the revised version of Ref. [34]]](image)

### 3.2. Results for the safe model

We now turn our attention to the UV-complete model M10 (in fact these results also apply to M11). In this case, the representation of the top partners cannot be chosen to be $(4, \bar{4})$ due to the quantum numbers of the underlying fermions. In fact, to make a baryon $\psi\psi\chi$, we need to combine $16 \times 16 \times 10$, while the vacuum is characterised by $16 \times \bar{16}$. It turns out, therefore, that the allowed representations are $(4, \bar{4}) \oplus (4, \bar{4})$ or $(6, 1) \oplus (1, 6)$. These representations are clearly not mapped onto themselves by the dark parity if taken individually, thus one needs to make sure that a symmetric coupling to the charge-conjugate representations are also present. \(^2\)

While this seems contrived, it is actually a natural choice in terms of the underlying model.

\(^2\) The dark parity would map $(4, 4) \leftrightarrow (\bar{4}, \bar{4})$ and $(6, 1) \leftrightarrow (1, 6)$. 
The easiest way to see this is to write the four-fermion couplings in terms of Weyl spinors: we define by a super-script \( l \) and \( r \) the left- and right-handed components of the Dirac spinors from table 2. In the case of the \((6,1) \oplus (1,6)\) representation, for instance, the four-fermion interactions for the right-handed top \( t^r \) would thus read

\[
y_1 \left( \frac{(r^C \chi_3^l) (\psi_u^l \psi_d^l)}{\Lambda^2} \right) + y_2 \left( \frac{(r^C \chi_3^l) (\psi_u^r \psi_d^r)}{\Lambda^2} \right).
\]

(10)

The dark parity would be preserved if \( y_1 = y_2 \). Note that the only difference between the two operators is the chirality of the \( \psi \)-spinsors, thus a symmetric scenario can be easily achieved if the operator is generated by a scalar mediator that couples to both chiralities of the spinors.

The results of the scan for the choice \((4,4) \oplus (\bar{4},\bar{4})\) is shown in figure 3. Similar to the plot in the previous section, we show points with different hues indicating the relic abundance, with the one tending to blue being under-abundant. The points in dark green saturate the correct value. Once again, we observe that for dominantly singlet Dark Matter candidate, i.e. \( \delta > 0 \), the points have a light mass, close to \( M_{DM} \approx 250 \text{ GeV} \). However, contrary to the previous case, the SI cross section can be much smaller in this case, thus evading current and future experiments.

For the dominantly triplet case, \( \delta < 0 \), a similar mass range as before is accessed, between 250 GeV and 2 TeV. However, we see that now the large mass points lie close to the exclusion limit and will be probed by future Direct Detection experiments, while the low mass points allow for very small and undetectable Spin Independent cross sections.

4. Conclusions and Outlook

Models where the electroweak symmetry breaking is dynamically generated and the Higgs emerges as a composite pNGB are still a valid alternative to the elementary Higgs nature of the SM. In these proceedings we have addressed the issue of UV completing this scenario, which is currently dominantly studied at the effective model level. We presented the first serious attempt to define a theory all the way up to the Planck scale.

Our main construction is based on the observation that, in order to generate masses via partial compositeness to all the SM fermions, a microscopic fermion-gauge theory needs to contain a large number of fermions charged under the SM gauge symmetries as well as the confining HC interactions. This in turns allows us to use large-\( N_f \) techniques. This resummation hints at the presence of an interacting UV fixed point in the gauge running. As such, the theory can be extrapolated up to arbitrarily high energies. A key constraint on the model building is the fact that none of the gauge couplings should be larger than the fixed value once the large multiplicity of fermions enters in the game. Thus, there is an upper limit on the value of the mass of such fermions. We showed in an explicit example based on a confining \( SO(10)_{HC} \) interaction that this is indeed allowed. We showed that the new fermions can be added at a scale close to \( 10^9 \text{ GeV} \), above which all gauge couplings run to a safe point. In this work we did not address the origin of the four-fermion interactions responsible for the partial compositeness. However, we showed that they can be easily generated via scalar interactions. As long as the scalar mediator are added close to the highest scale in the model, i.e. the threshold for the many fermions, the theory remains natural (i.e., no unjustified hierarchies are present). The next step in validating this model would be to check that the Yukawa couplings between the scalar mediators and the underlying fermions also run to a UV fixed point. This will be the scope of our future explorations.

We have also addressed the phenomenology of the Dark Matter candidate in scenarios where the dominant coupling is the top partial compositeness. We find two interesting regions: one where the mass of the Dark Matter is \( \approx 250 \text{ GeV} \), and one where it can extend up to 2 TeV. Interestingly, next generation Direct Detection experiments can probe most of the parameter
space of the model. However, in particular at low mass, there are regions of small Spin-Independent cross section on nuclei. However, the LHC may have a chance to cover this region of the parameter space. We leave this exploration for a future study.

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