On the Mean Estimation using Stratified Double Median Ranked Set Sampling

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Abstract: Stratified Double Median Ranked Set Sampling (SDMRSS) method is suggested for estimating the population mean. The SDMRSS is compared with the Simple Random Sampling (SRS), Stratified Simple Random Sampling (SSRS) and Stratified Ranked Set Sampling (SRSS) methods. It is shown that SDMRSS estimator is an unbiased of the population mean and is more efficient than the SRS, SSRS and SRSS counterparts. Also, SDMRSS increase the efficiency of mean estimation for specific value of the sample size. The SDMRSS is applied on real data set.

Keywords: Ranked Set Sampling, Double Ranked Set Sampling, Median Ranked Set Sampling, Mean, Efficiency

Introduction

During the last years, the Ranked Set Sampling method which was proposed by McIntyre (1952) for estimating the population mean of pasture yields was developed and modified by many authors. Dell and Clutter (1972) showed that the mean of the RSS is an unbiased estimator of the population mean even if there are errors in ranking. Muttlak (1997) suggested using Median Ranked Set Sampling (MRSS) to estimate the population mean. Al-Saleh and Al-Kadiri (2000) introduced Double Ranked Set Sampling for estimating the population mean. Al-Saleh and Al-Omari (2002) suggested Multistage Ranked Set Sampling that increases the efficiency of estimating the population mean for specific value of the sample size. Jemain and Al-Omari (2006) suggested Multistage Median Ranked Set Sampling (MMRSS) to estimate a population mean. Syam et al. (2013) considered the problem of estimating the population mean using stratified Double Percentile Ranked Set Sample. For more details about RSS see for example Bouza (2002; Samawi, 1996; Ohyama et al., 2008; Al-Omari and Jaber, 2008; Jemain et al., 2007; Takahasi and Wakimoto, 1968).

In this study, stratified double median ranked set sampling method is suggested for estimating the population mean of symmetric and asymmetric distributions. The paper is organized as follows: In section 2, some of sampling methods considered in this study are presented. Estimation of the population mean is given in section 3. A simulation study is considered in section 4. A real life example using SDMRSS is discussed in section 5. Finally, conclusions are provided in section 6.

Sampling Methods

Stratified Simple Random Sampling

In stratified sampling, the population of N units is first divided into L subpopulations of sizes \(N_1, N_2, \ldots, N_L\) units, respectively. These subpopulations are non overlapping and together they comprise the whole population, i.e., \(N_1 + N_2 + \ldots + N_L = N\). The subpopulations are called strata and the proportional allocation is considered. When the strata have been determined, a sample is chosen from each stratum, the drawing is made in different strata. The sample sizes within the strata are denoted by \(n_1, n_2, \ldots, n_L\), respectively. If a simple random sample is taken from each stratum, the whole procedure is described as Stratified Simple Random Sampling (SSRS).

Ranked Set Sampling

The Ranked Set Sampling (RSS) is suggested by McIntyre (1952) can be described as follows: randomly select \(n\) samples of size \(n\) units each from the population of interest and then rank the units within each set with respect to the variable of interest. Select the smallest ranked unit from the first sample. The second smallest ranked unit from the second sample and the procedure is continued until the unit with the largest rank is selected for actual measurement from the \(n\)-th sample. Thus, a total of \(n\) measured units is obtained, one from each
ordered sample of size $n$ and this completed one cycle. The cycle may be repeated $m$ times if needed until $mn$ units have been measured.

**Median Ranked Set Sampling**

The MRSS procedure as proposed by Muttak (1997) depends on selecting $n$ random samples of size $n$ units from the population and the ranking of the units within each sample with respect to a variable of interest. If the sample size $n$ is odd, then from each sample select for the measurement the leftmost $\left(\frac{n+1}{2}\right)^{th}$ smallest ranked unit, i.e., the median of the sample. If the sample size $n$ is even, then select for the measurement from the first $\frac{n}{2}$ samples the leftmost $\frac{n}{2}^{th}$ smallest ranked unit of each set and from the second $\frac{n}{2}$ samples the rightmost $\frac{n}{2}+\frac{1}{2}^{th}$ smallest ranked unit. The cycle can be repeated $m$ times if needed to get a sample of size $mn$ units.

**Double Median Ranked Set Sampling**

The DMRSS (Samawi and Tawalbeh, 2002) is described as follows:

- Identify $n^2$ elements from the target population and divide these elements randomly into $n$ sets, each of size $n^2$ elements.
- If the sample size is even, chose from the first $\frac{n^2}{2}$ sets the $\frac{n}{2}^{th}$ smallest ranked unit of each set and from the second $\frac{n^2}{2}$ sets the $\frac{n}{2}+\frac{1}{2}^{th}$ smallest ranked unit. If the sample size is odd, select from all sets the $\frac{n+1}{2}^{th}$ smallest ranked unit. This step yields $n$ sets each of size $n$.
- Apply the MRSS procedure again on the sets obtained from Step (2) to obtain a DMRSS of size $n^2$.
- The cycle can be repeated $m$ times if needed to get a sample of size $mn$ units.

**Stratified Double Median Ranked Set Sampling**

Referring to section 2.1, if the double median ranked set sampling is used in each stratum, the whole method is described as Stratified Double Median Ranked Set Sampling (SDMRSS). To explain the method, the following example with two strata is considered, the first stratum with even sample size and the second stratum with odd sample size.

**Example 1**

Consider a population with two strata, $L = 2$ ($h = 1,2$) and in the first stratum there are 27 elements divided into 3 sets with 9 elements in each set and in the second stratum there are 64 elements divided into 4 sets with 16 elements in each set as the following.

Stratum (1): Assume that the 27 elements are:

$X^{(1)}_1, X^{(1)}_2, \ldots, X^{(1)}_9, X^{(1)}_{10}, \ldots, X^{(1)}_{18}, X^{(1)}_{19}, \ldots, X^{(1)}_{27}$.

After ranking the elements in each set, the following subsets are obtained:

- \[ \begin{bmatrix} X^{(1)}_{(11)} & X^{(1)}_{(12)} & X^{(1)}_{(13)} \\ X^{(1)}_{(21)} & X^{(1)}_{(22)} & X^{(1)}_{(23)} \\ X^{(1)}_{(31)} & X^{(1)}_{(32)} & X^{(1)}_{(33)} \end{bmatrix}, \begin{bmatrix} X^{(1)}_{(11)} & X^{(1)}_{(12)} & X^{(1)}_{(13)} \\ X^{(1)}_{(21)} & X^{(1)}_{(22)} & X^{(1)}_{(23)} \\ X^{(1)}_{(31)} & X^{(1)}_{(32)} & X^{(1)}_{(33)} \end{bmatrix} \]
- \[ \begin{bmatrix} X^{(1)}_{(11)} & X^{(1)}_{(12)} & X^{(1)}_{(13)} \\ X^{(1)}_{(21)} & X^{(1)}_{(22)} & X^{(1)}_{(23)} \\ X^{(1)}_{(31)} & X^{(1)}_{(32)} & X^{(1)}_{(33)} \end{bmatrix}. \]

Apply the MRSS method on each of the above sets to get three sets as the following:

Set (1): Assume that the 18 elements are:

$Y^{(1)}_{11}, Y^{(1)}_{12}, \ldots, Y^{(1)}_{19}, Y^{(1)}_{20}, \ldots, Y^{(1)}_{27}$.

Set (2): Assume that the 28 elements are:

$Y^{(2)}_{11}, Y^{(2)}_{12}, \ldots, Y^{(2)}_{20}, Y^{(2)}_{21}, \ldots, Y^{(2)}_{44}$.

Set (3): Assume that the 29 elements are:

$Y^{(3)}_{11}, Y^{(3)}_{12}, \ldots, Y^{(3)}_{20}, Y^{(3)}_{21}, \ldots, Y^{(3)}_{44}$.

After ranking the elements in each set, the following subsets are obtained:

- \[ \begin{bmatrix} Y^{(1)}_{(11)} & Y^{(1)}_{(12)} & Y^{(1)}_{(13)} & Y^{(1)}_{(14)} \\ Y^{(1)}_{(21)} & Y^{(1)}_{(22)} & Y^{(1)}_{(23)} & Y^{(1)}_{(24)} \\ Y^{(1)}_{(31)} & Y^{(1)}_{(32)} & Y^{(1)}_{(33)} & Y^{(1)}_{(34)} \end{bmatrix}, \begin{bmatrix} Y^{(2)}_{(11)} & Y^{(2)}_{(12)} & Y^{(2)}_{(13)} & Y^{(2)}_{(14)} \\ Y^{(2)}_{(21)} & Y^{(2)}_{(22)} & Y^{(2)}_{(23)} & Y^{(2)}_{(24)} \\ Y^{(2)}_{(31)} & Y^{(2)}_{(32)} & Y^{(2)}_{(33)} & Y^{(2)}_{(34)} \end{bmatrix} \]
- \[ \begin{bmatrix} Y^{(3)}_{(11)} & Y^{(3)}_{(12)} & Y^{(3)}_{(13)} & Y^{(3)}_{(14)} \\ Y^{(3)}_{(21)} & Y^{(3)}_{(22)} & Y^{(3)}_{(23)} & Y^{(3)}_{(24)} \end{bmatrix}. \]
Apply the MRSS on each of the 16 elements to get four sets as the following:

Set (1): \( Y_{(11)}^{(1)}, Y_{(12)}^{(1)}, Y_{(13)}^{(1)}, Y_{(14)}^{(1)} \)

Set (2): \( Y_{(21)}^{(2)}, Y_{(22)}^{(2)}, Y_{(23)}^{(2)}, Y_{(24)}^{(2)} \)

Set (3): \( Y_{(31)}^{(3)}, Y_{(32)}^{(3)}, Y_{(33)}^{(3)}, Y_{(34)}^{(3)} \)

Set (4): \( Y_{(41)}^{(4)}, Y_{(42)}^{(4)}, Y_{(43)}^{(4)}, Y_{(44)}^{(4)} \)

The elements of the double median ranked set sample in the second stratum are:

\[ X_{(21)}^{(2)}, X_{(22)}^{(2)}, X_{(23)}^{(2)}, X_{(24)}^{(2)} = X_{(11)}^{(1)} + X_{(21)}^{(2)} \]

Therefore, the SDMRSS units from the two strata are:

\[ X_{(11)}^{(1)}, X_{(12)}^{(1)}, X_{(13)}^{(1)}, X_{(21)}^{(2)}, X_{(22)}^{(2)}, X_{(23)}^{(2)}, X_{(14)}^{(1)}, X_{(24)}^{(2)} \]

### Estimation of the Population Mean

Assume that the variable of interest \( X \) has a density function \( f(x) \) and a cumulative distribution function \( F(x) \), with mean \( \mu \) and variance \( \sigma^2 \). Let \( X_1, X_2, \ldots, X_n \) be a SRS from \( f(x) \). Let \( X_{11}, X_{12}, \ldots, X_{16} \) be the orders of the \( i \)-th sample. Therefore, the measured units \( X_{i1}, X_{i2}, \ldots, X_{in} \) constitute the ranked set sample.

The notation for the DMRSS will be used by replacing the sets of ordered statistics \( X_{i1}, X_{i2}, \ldots, X_{in}, i = 1,2,\ldots,n \) which obtained from the sample \( X_{11}, X_{12}, \ldots, X_{16} \) with each of size \( n \). Let \( X_{(1,1)}, X_{(2,1)}, \ldots, X_{(n,1)} \) be the order statistics of the 1-th sample \( X_{(i,1)} \), \( X_{(2,1)}, \ldots, X_{(n,1)} \) constitute the ranked set sample.

The variance of the estimator \( \bar{X}_{SDMRSS} \) is given by:

\[
Var(\bar{X}_{SDMRSS}) = \frac{1}{n^2} \sum_{i=1}^{N} \sum_{h=1}^{n} \sigma^2_{h} \left( \frac{n-1}{n} \right) \left( \frac{n-2}{n} \right) \left( \frac{n-3}{n} \right) \left( \frac{n-4}{n} \right)
\]

The variance of \( \bar{X}_{SDMRSS} \) when \( n_h \) is odd is given by:

\[
Var(\bar{X}_{SDMRSS}) = \frac{1}{n^2} \sum_{i=1}^{N} \sum_{h=1}^{n} \sigma^2_{h} \left( \frac{n-1}{n} \right) \left( \frac{n-2}{n} \right) \left( \frac{n-3}{n} \right) \left( \frac{n-4}{n} \right)
\]

**Lemma 1.** \( \bar{X}_{SDMRSS} \) is an unbiased estimator of the mean of symmetric distribution.

**Proof.** Two cases are considered:

**First:** If the sample sizes in the strata \( n_h, h = 1,2,\ldots,L \) are even:

\[
\bar{X}_{SDMRSS} = \frac{1}{n} \sum_{h=1}^{L} \frac{1}{n_h} \left( \sum_{i=1}^{n_h} X^*_{h(i)} \right)
\]

\[
E(\bar{X}_{SDMRSS}) = \frac{1}{n} \sum_{h=1}^{L} \frac{1}{n_h} \left( \sum_{i=1}^{n_h} X_{h(i)} \right)
\]

\[
\bar{X}_{SDMRSS} = \frac{1}{n} \sum_{h=1}^{L} \frac{1}{n_h} \left( \sum_{i=1}^{n_h} X_{h(i)} \right)
\]

Since the distribution is symmetric about \( \mu \), then \( \mu^* = (\frac{1}{2}^* \mu^* + (\frac{1}{2}^* \mu^*) = 2\mu^* \). Therefore:

\[
E(\bar{X}_{SDMRSS}) = \frac{1}{n} \sum_{h=1}^{L} \frac{1}{n_h} \sum_{i=1}^{n_h} \mu^*_{h(i)}
\]
Based on Al-Saleh and Al-Kadiri (2000) we have

\[ \mu_h = \frac{1}{n_h} \sum_{i=1}^{n_h} \mu_i, \]

therefore:

\[ E(X_{SDMRSS}) = \sum_{h=1}^{L} W_h \cdot \mu_h = \mu. \]

Second: If the sample sizes in the strata \( n_h \), \( h = 1,2,\ldots, L \) are even, the variance of \( X_{SDMRSS} \) is:

\[ E\left( X_{SDMRSS}^2 \right) = E\left( \sum_{h=1}^{L} W_h \left( \sum_{i=1}^{n_h} X_i \right) \right) \]

\[ = \sum_{h=1}^{L} W_h \sum_{i=1}^{n_h} E(X_i) \]

\[ = \sum_{h=1}^{L} W_h \sum_{i=1}^{n_h} \mu_i = \mu \cdot E(X) = \mu. \]

Since the distribution is symmetric about the \( \mu \), then the mean = median, which implies \( \mu \left( \frac{n_h}{2} \right) = \mu \left( \frac{n_h}{2} \right) \).

Therefore:

\[ E\left( X_{SDMRSS}^2 \right) = \sum_{h=1}^{L} W_h \left( \sum_{i=1}^{n_h} \mu_i \right) = \sum_{h=1}^{L} W_h \cdot \mu_h = \mu. \]

**Lemma 2.** If the parent distribution is symmetric about \( \mu \), then \( Var(X_{SDMRSS}) < Var(X_{SRSS}) \).

**Proof.** Two cases are considered.

**First:** If the sample sizes in the strata \( n_h \), \( h = 1,2,\ldots, L \) are even. The variance of \( X_{SDMRSS} \) is:

\[ Var(X_{SDMRSS}) = \sum_{h=1}^{L} W_h \left( \sum_{i=1}^{n_h} \sigma^2_i \right) = \sum_{h=1}^{L} W_h \sigma^2_h. \]

Nevertheless, \( \sigma^2_h < \sigma^2_h \) for each stratum \( h = 1,2,\ldots, L \), this implies:

\[ Var(X_{SDMRSS}) < Var(X_{SRSS}) < Var(X_{SSRS}) < Var(X_{SRS}). \]

**Second:** The proof in case of odd sample sizes is similar.

### Simulation Study Based on SDMRSS

In this section, a simulation study is conducted to investigate the performance of SDMRSS for estimating the population mean. Symmetric and asymmetric distributions are considered for samples of sizes \( n = 9 \), \( 12, 14, 15, 18 \). Assume that the population is partitioned into two or three strata and proportional allocation is considered. The simulation is performed for the SRSS, SSRS, and SRS data sets from different distributions. The symmetric distributions are uniform and normal and the asymmetric distributions are exponential, gamma and Weibull. Using 100000 replications, estimates of the means, variances and Mean Squared Errors (MSE) are computed.

If the distribution is symmetric, then the efficiency of SDMRSS relative to \( T \) is defined by:

\[ eff\left( X_{SDMRSS}, \overline{X}_T \right) = \frac{Var\left( \overline{X}_T \right)}{Var\left( X_{SDMRSS} \right)}. \]

where, \( T = SRSS, SSRS, SRSS \).

But if the distribution is asymmetric, the efficiency is defined as:

\[ eff\left( X_{SDMRSS}, \overline{X}_T \right) = \frac{MSE\left( \overline{X}_T \right)}{MSE\left( X_{SDMRSS} \right)}. \]

The results are summarized in Table 1-5.

Based on the tables, we can conclude that gains in efficiency is attained using SDMRSS method as relative to other methods considered in this study to estimate the population mean of the variable of interest.

| Distribution  | SRSS | SSRS | SRS | SRSS | SSRS | SRS |
|---------------|------|------|-----|------|------|-----|
| Uniform (0,1) | 24.9532 | 29.0126 | 29.3264 | 38.1847 | 39.3721 | 40.8392 |
| Normal (0,1)  | 22.5374 | 24.3527 | 24.5821 | 34.6814 | 36.3885 | 35.6294 |
| Exponential (1) | 19.2431 | 17.6932 | 17.7357 | 23.3628 | 24.5624 | 22.9326 |
| Gamma (1,2)  | 17.4071 | 17.9437 | 17.8426 | 23.4775 | 24.2247 | 23.4635 |
| Weibull (1,2) | 17.3286 | 17.0184 | 16.9452 | 22.7541 | 22.8352 | 22.3715 |
Table 2. The efficiency of SDMRSS relative to SRSS, SSRS and SRS for \( n = 14 \) and \( n = 18 \) with two strata

|            | SRSS | SSRS | SRS | SRSS | SSRS | SRS |
|------------|------|------|-----|------|------|-----|
| Uniform (0,1) | 43.5647 | 47.8914 | 45.5718 | 60.8657 | 66.2043 | 63.2537 |
| Normal (0,1)  | 39.4729 | 41.7583 | 38.7629 | 45.3256 | 49.0021 | 49.3312 |
| Exponential (1) | 21.8473 | 23.1638 | 21.5547 | 22.6374 | 21.3546 | 21.9903 |
| Gamma (1,2)  | 23.2713 | 22.9246 | 22.3222 | 22.4617 | 22.3258 | 22.4366 |
| Weibull (1,2) | 21.2374 | 21.3621 | 21.2300 | 23.732 | 22.9557 | 29.8471 |

Table 3. The efficiency of SDMRSS relative to SRSS, SSRS and SRS for \( n = 15 \) and \( n = 18 \) with three strata

|            | SRSS | SSRS | SRS | SRSS | SSRS | SRS |
|------------|------|------|-----|------|------|-----|
| Uniform (0,1) | 50.5472 | 52.3169 | 51.7284 | 63.3704 | 67.32 | 74 |
| Normal (0,1)  | 35.643 | 38.3769 | 37.54 | 51.7528 | 52.5461 | 52.3271 |
| Exponential (1) | 24.0326 | 23.9002 | 23.2035 | 24.7361 | 25.3261 | 23.1043 |
| Gamma (1,2)  | 21.4375 | 22.2438 | 21.8793 | 25.0143 | 24.2637 | 23.9927 |
| Weibull (1,2) | 22.3258 | 22.1002 | 22.3726 | 24.8933 | 23.0143 | 23.8436 |

Table 4. The values of bias of SDMRSS for different distributions and different numbers of strata

| Sample size | No. of strata | Exp (1) | Gamma (1, 2) | Weibull (1, 2) |
|-------------|---------------|---------|--------------|---------------|
| \( n = 9 \), \( n_1 = 4 \), \( n_2 = 5 \) | 2 | 0.0565 | 0.1776 | 0.7564 |
| \( n = 12 \), \( n_1 = 5 \), \( n_2 = 7 \) | 2 | 0.0271 | 0.1989 | 0.3143 |
| \( n = 14 \), \( n_1 = 8 \), \( n_2 = 6 \) | 2 | 0.0168 | 0.1033 | 0.0276 |
| \( n = 18 \), \( n_1 = 10 \), \( n_2 = 8 \) | 2 | 0.0372 | 0.1101 | 0.0335 |
| \( n = 15 \), \( n_1 = 3 \), \( n_2 = 5 \), \( n_3 = 7 \) | 3 | 0.0106 | 0.0754 | 0.0021 |
| \( n = 18 \), \( n_1 = 4 \), \( n_2 = 6 \), \( n_3 = 8 \) | 3 | 0.0323 | 0.1011 | 0.0235 |

Table 5. The efficiency of SSRS, SRSS and SDMRSS relative to SRS for marks

| Sampling method | \( n = 7 \), \( n_1 = 4 \), \( n_2 = 3 \) | \( n = 12 \), \( n_1 = 4 \), \( n_2 = 7 \) | \( n = 14 \), \( n_1 = 8 \), \( n_2 = 6 \) |
|-----------------|---------------------------------|---------------------------------|---------------------------------|
| SRS Mean | 61.741 | 61.625 | 61.503 |
| Variance | 57.314 | 42.592 | 38.728 |
| SSRS Mean | 61.452 | 61.784 | 60.151 |
| Variance | 32.416 | 28.715 | 22.825 |
| Efficiency | 3.761 | 3.829 | 3.648 |
| SRSS Mean | 62.331 | 62.072 | 61.912 |
| Variance | 19.463 | 16.873 | 14.625 |
| Efficiency | 3.984 | 4.533 | 4.327 |
| SDMRSS Mean | 61.802 | 61.745 | 61.623 |
| Variance | 13.425 | 11.039 | 10.819 |
| Efficiency | 4.813 | 4.967 | 4.878 |

When the performances of the suggested SDMRSS estimators are compared, the efficiency of the suggested estimator is found to be more superior when the underlying distributions are symmetric as compared to asymmetric distributions.

The relative efficiency of SDMRSS estimator with respect to those estimators based on SRS, SSRS and SRSS are increasing as the sample size increases.

**Real Data Example using SDMRSS**

The marks of 787 students from scientific majors in Foundation Program at Qatar University during academic semester Fall 2011 are collected as a population to calculate its mean and variance. 100,000 samples using each of stratified double median ranked set sample, simple random sample, stratified ranked set sample and stratified simple random sample methods with sample size \( n = 7, 12, 14 \) using Mat lab 7 are generated. The mean and variance are obtained for each method and compared to evaluate the performance of SDMRSS to estimate the population mean for real data. Stratification is done according to the gender (males and females) and proportional allocation is considered.

Let \( x_{i1}, i = 1, 2, \ldots, 531 \) be the mark of \( i \)th female student in the population. Let \( x_{j2}, j = 1, 2, \ldots, 256 \) be the mark of \( j \)th male student in the population.
The mean $\mu$ and the variance $\sigma^2$ of the population are:

$$\mu = \frac{1}{787} \sum_{i=1}^{787} x_i = 61.98 \quad \text{and} \quad \sigma^2 = \frac{1}{787} \sum_{i=1}^{787} (x_i - \mu)^2 = 1060.86$$

The mean $\mu_1$ and the variance $\sigma_1^2$ of the female population are:

$$\mu_1 = \frac{1}{531} \sum_{i=1}^{531} x_{i_1} = 65.08 \quad \text{and} \quad \sigma_1^2 = \frac{1}{531} \sum_{i=1}^{531} (x_{i_1} - \mu)^2 = 1025.85$$

The mean $\mu_2$ and the variance $\sigma_2^2$ of the male population respectively are:

$$\mu_2 = \frac{1}{256} \sum_{j=1}^{256} x_{j_2} = 55.55 \quad \text{and} \quad \sigma_2^2 = \frac{1}{256} \sum_{j=1}^{256} (x_{j_2} - \mu)^2 = 1076.29$$

The skewness and median of the 787 students are -0.64 and 71.00. Skewness and median of 531 female students are -0.74 and 75.00. Skewness and median of 256 male students are -0.48 and 65.00.

Since the skewness for all students, female students and male students are -0.64, -0.74, -0.48, then the marks data are asymmetrically distributed, which means that SDMRSS estimator is biased and hence the mean squared errors of the estimators will be calculated. The efficiency of SSRS, SRSS and SDMRSS with respect to SRS are computed and summarized in Table 5.

From Table 5, the following are noticed:

- The values of estimated mean using SDMRSS are very close to the population mean
- The variance of SDMRSS estimator is less than the variances of SRS, SSRS and SRSS. Therefore, the efficiency values using SDMRSS are greater than those obtained using SRS, SSRS and SRSS
- Results in this real life example agrees with the theoretical results

**Conclusion**

In this study, new estimators of the population mean are suggested using SDMRSS. The performance of the estimators based on SDMRSS are compared with those using SRSS, SSRS and SRS for the same number of measured units. It is shown that SDMRSS estimators are unbiased of the population mean and are more efficient than their counterparts using SRSS, SSRS and SRS. Therefore, the SDMRSS is recommended for estimating the mean of symmetric and asymmetric distributions.

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**Author’s Contributions**

The work is the result of a full collaboration of the authors. However:

Mahmoud Ibrahim Syam: Carried out the simulations and application.

Amer Ibrahim Al-Omari: Participated in the theoretical part and discussion of this paper.

Kamarulzaman Ibrahim: Participated in reviewing the manuscript and giving ideas.

**Ethics**

This paper is original and has not published elsewhere.

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