Exclusive heavy meson pair production by $\gamma \gamma$ collision in heavy quark effective theory

S. Y. Choi\textsuperscript{1} and H. S. Song

Center for Theoretical Physics and Department of Physics, Seoul National University, Seoul 151-742, Korea

ABSTRACT

The spin and flavor symmetries of the heavy quark effective theory are employed to discuss the exclusive cross sections for pair production of heavy mesons in photon-photon collision. The ratios of the exclusive cross sections for heavy mesons are obtained explicitly and the validity of results is discussed.

\textsuperscript{1}E-mail : SYCHOI@KRSNUCC1.bitnet
1 Introduction

Physical processes of heavy mesons involving heavy quarks, such as the $c$ and $b$ quarks, have recently attracted both theoretical and experimental interests.

The experimental data have improved considerably and also proposals [1] for detailed study of such processes have been made. On the other hand, new theoretical ideas developed recently have led to the formulation of an effective heavy quark theory [2, 3, 4, 5] with which the analysis of physical processes is greatly simplified due to additional symmetries arising in the heavy quark limit where the heavy quark mass is very large.

One of the symmetries is a heavy flavor symmetry, namely an SU($N$) for $N$ heavy flavors, under which the heavy quarks are rotated into one another. The second is called the spin symmetry that arises due to the decoupling of the spin degrees of freedom in the heavy quark limit. And one of the important properties of the heavy quark effective theory is that the heavy quark velocity is difficult to change as long as nonperturbative aspects of QCD with a typical scale $\Lambda_{\text{QCD}}$ are concerned but it can be only modified by perturbative processes such as an electroweak interaction.

These symmetries and the so-called velocity superselection rule have been very successful in their applications to weak decays of heavy hadrons [3, 7] and exclusive heavy hadron production in $e^+e^-$ annihilation [8, 9, 10]. While the additional symmetries are broken by terms of the order $\Lambda_{\text{QCD}}/m_H$, where $m_H$ is the mass of the heavy meson involved in the particular process and by QCD radiative corrections of the order $\alpha_S(m_H)$, these corrections may be treated systematically and have been calculated in some cases.

In this paper, we use symmetries of the heavy quark effective theory to discuss pair production of heavy mesons in $\gamma\gamma$ collision.

The cross section for production of a pair of heavy mesons in two-photon reactions is a function of the dimensionful variables $s = (k_1 + k_2)^2$, $m_H^2$, $\Lambda_{\text{QCD}}^2$ and $Q^2$, which is a generic notation for the absolute value of momentum transfer due to $t$- and $u$-channel heavy quark exchanges not involved in $e^+e^-$ collisions. We consider the kinematic regime where $\sqrt{s}$ is reasonably larger than $m_H$, and both $m_H^2$ and $Q^2$ are much larger than $\Lambda_{\text{QCD}}^2$. Then the pair production of heavy mesons in the $\gamma\gamma$ (as well as $e^+e^-$) collisions may be visualized as a heavy quark pair production followed by the fragmentation of the heavy quark into a heavy meson involving the light QCD degrees of freedom. The approximate visualization may get more concrete with an
appropriate angular cutoff to the production distribution of heavy mesons.

The scattering amplitudes of large-angle exclusive processes might be computed, in the leading order, from Born diagrams shown in figs. (a-h). The solid line is for the heavy quark and the dashed line for the light degree of freedom in the heavy meson. However, for the heavy meson containing a heavy quark, the scattering amplitudes of figs. (c-h) are strongly suppressed, since the diagrams require the exchange of a hard gluon to produce the final state, while those of figs. (a) and (b) do not. Therefore, in the heavy quark limit, only the two diagrams (a) and (b) contribute to the production of a pair of heavy mesons.

Basically, the two photon system is distinctly different from the $e^+e^-$ system in the following aspects. First, the former has positive charge conjugation, while the latter has negative one. Thus the C conservation of both QED and QCD guarantees a separate and complementary investigation of final meson states by $\gamma\gamma$ and $e^+e^-$ collisions. On the other hand, virtual photons are also available at $e^+e^-$ or $pp(\bar{p})$ colliders so that spin-one final states can be explored. Finally, the on and off shell nature and the polarization of each of the (virtual or real) incident photons can be continuously varied, allowing detailed tests of the theory. The last two aspects are not considered and only on-shell photons are taken into account in the present work.

The paper is organized as follows. In section 2, we determine heavy meson wave functions through symmetries of the heavy quark effective theory and apply them to obtain the exclusive cross sections for the production of heavy mesons in $\gamma\gamma$ collision and their ratios. In section 3, our results are discussed and some conclusions are made.

## 2 Cross sections and ratios

We employ the heavy quark effective theory to determine the wave functions of heavy mesons formed by a non-tractable fragmentation mechanism from heavy quarks and to apply them to the production mechanism of heavy mesons, especially, in the $\gamma\gamma$ collision.

In general, a heavy meson containing a single heavy quark is mostly viewed as a freely propagating point-like color source (a heavy quark), dressed by strongly interacting light degrees of freedom (so-called “brown mucks”) bearing appropriate values of color, baryon number, angular momentum and
parity to make up the observed physical state. The heavy meson is described by the velocity vector \( v^\mu \) \((v^2 = 1, v^0 > 0)\) and the residual momentum vector \( k^\mu \), which form the total four momentum \( p^\mu (= m_H v^\mu + k^\mu) \). The velocity \( v^\mu \) is prohibited from changing by the velocity superselection rule and \( k^\mu \) is of the order \( \Lambda_{\text{QCD}} \). Moreover heavy quark states and heavy anti-quark states are completely decoupled because of an unsurmountable mass gap \( 2m_H \) and thus, in each velocity sector of the theory, there are two separate SU(2) spin symmetries to enable us to connect different spin states of heavy mesons and those of heavy anti-mesons, respectively.

These spin symmetries can be used to determine wave functions of heavy mesons and their antimesons explicitly \([13, 14]\) and to derive relations between the matrix elements of any Dirac structure \( \Gamma \) which is to be sandwiched between heavy quark fields. The Dirac structure \( \Gamma \) denoting the production of a pair of heavy quarks through \( t- \) and \( u- \) channel heavy quark exchanges in \( \gamma \gamma \) collision is given by

\[
\Gamma = \frac{m_H (1 + \gamma_5)}{v_1 \cdot k_1} \psi_1^{\dagger} + \psi_2^{\dagger} \frac{m_H (1 + \gamma_5)}{v_1 \cdot k_2} \psi_1, \tag{1}
\]

where \( \epsilon_1 \) and \( \epsilon_2 \) are wave vectors of the incident photons with four momentum vectors \( k_1 \) and \( k_2 \), respectively, and \( v_1(v_2) \) is the four velocity of the heavy meson (antimeson). The first term in eq. (1) corresponds to the \( t \)-channel exchange and the second to the \( u \)-channel exchange.

A multiplet of heavy mesons, transformed into one another by the spin symmetry, should have the same mass \( m_H \) and thus their genuine mass differences at most of the order \( \Lambda_{\text{QCD}} \) are neglected in the heavy quark limit.

The matrix wave functions of heavy mesons containing light degrees of freedom of zero orbital angular momentum as well as complying with Lorentz invariance and parity are as follows:

\[
\begin{align*}
M_v &= \frac{1 + \gamma_5}{2} : \text{a pseudoscalar meson}, \\
M^*_v &= -\frac{1 + \gamma_5}{2} : \text{a vector meson}, \\
\tilde{M}_v &= \frac{1 - \gamma_5}{2} : \text{a pseudoscalar antimeson}, \\
\tilde{M}^*_v &= -\frac{1 - \gamma_5}{2} : \text{a vector antimeson}. \tag{2}
\end{align*}
\]
The complex conjugates $\bar{M}_v$ and $\bar{M}_v^*$ are defined as

$$\bar{M}_v = \gamma^0 M_v^+ \gamma^0, \quad \bar{M}_v^* = \gamma^0 M_v^{*+} \gamma^0,$$

(3)

respectively. Then the matrix element of two heavy mesons may be expressed by just one form-factor $[8, 9]$ as

$$< M_{v_1} \bar{M}_{v_2} | \bar{h} \Gamma \bar{h} | 0 > = m_H \zeta(v_1 \cdot v_2) \text{tr}(\bar{M}_{v_1} \Gamma \bar{M}_{v_2}),$$

(4)

where $h (\bar{h})$ is the heavy quark (anti-quark) field, respectively, and $\zeta(v_1 \cdot v_2)$ is the analogue of the dimensionless Isgur-Wise function for the time-like momentum transfer $v_1 \cdot v_2$, which is in the center of mass frame

$$v_1 \cdot v_2 = \frac{s}{2m_H^2} - 1.$$  

(5)

Note that only one Isgur-Wise form factor is needed for heavy mesons. Therefore, the ratios of the exclusive production of the $M \bar{M}, M^* \bar{M}(M \bar{M}^*)$ and $M^* \bar{M}^*$ meson pairs in the $\gamma \gamma$ collision will be independent of the unknown coefficient function $\zeta(v_1 \cdot v_2)$.

For simplicity, let us introduce a variable $z$ as

$$z = \beta \cos \theta,$$

(6)

where $\theta$ is the scattering angle and $\beta = \sqrt{1 - 4m_H^2/s}$ in the center of mass frame. The general form of differential cross section for $\gamma \gamma \rightarrow M \bar{M}$, where $M$ is a heavy meson, is given by

$$\frac{d\sigma}{dz}(\gamma \gamma \rightarrow M \bar{M}) = \frac{\pi \alpha^2}{8} N_C q_H^4 |\zeta(v_1 \cdot v_2)|^2 \frac{1}{s} \text{tr}(\bar{M}_{v_1} \Gamma \bar{M}_{v_2}) |^2,$$

(7)

where $q_H$ is the charge of a heavy quark and $N_C$ is the color factor which is 3 for SU(3)$_C$. Taking an average over initial photon polarizations and a sum over final vector meson polarizations, we find for the differential cross sections

$$\frac{d\sigma}{dz}(\gamma \gamma \rightarrow M \bar{M}) = \sigma_0 \times \frac{f(\beta, z)}{(1 - z^2)^2},$$

$$\frac{d\sigma}{dz}(\gamma \gamma \rightarrow M^* \bar{M}(M \bar{M}^*)) = \sigma_0 \times \frac{s}{8m_H^2} \frac{2z^2 - 3z^4 + f(\beta, z)}{(1 - z^2)^2},$$

$$\frac{d\sigma}{dz}(\gamma \gamma \rightarrow M^* \bar{M}^*) = \sigma_0 \times \frac{s}{2m_H^2} \frac{(\beta^2 - z^4) + 3f(\beta, z)}{(1 - z^2)^2},$$

(8)
where
\[ \sigma_0 = \pi \alpha^2 N C q_H^4 \frac{\zeta(v_1 \cdot v_2)^2}{s}, \]
\[ f(\beta, z) = \beta^4 + (z^2 - \beta^2)^2. \] (9)

The ratios of differential cross sections are \( \beta^2 : \frac{s}{8m_H^2} \beta^2 : \frac{s}{4m_H^2} + \frac{3}{2} \beta^2 \) at \( \theta = \frac{\pi}{2} \)
and \( \beta^2 : 1 : 2 + 3 \beta^2 \) at the forward or backward direction \( (\theta = 0 \text{ or } \pi) \),
respectively. The production of two vector mesons dominates in both cases.

Integrating the differential cross sections over \( z \) we get the ratios
\[ \sigma(\gamma\gamma \to M \tilde{M}) : \sigma(\gamma\gamma \to M^* \tilde{M}) : \sigma(\gamma\gamma \to M^* \tilde{M}^*) = h_1(\beta) : \frac{s}{8m_H^2} h_2(\beta) : 3h_1(\beta) + \frac{s}{4m_H^4} h_3(\beta), \] (10)

where
\[ h_1(\beta) = \frac{\beta}{1 - \beta^2} (2\beta^4 - 4\beta^2 + 3) + (\beta^4 + \beta^2 - \frac{3}{2}) \log \frac{1 + \beta}{1 - \beta}, \]
\[ h_2(\beta) = \frac{(1 + \beta^2)^2}{2} \left[ \log \frac{1 + \beta}{1 - \beta} - \frac{2\beta}{1 + \beta^2} \right], \]
\[ h_3(\beta) = -6\beta + (\beta^2 + 3) \log \frac{1 + \beta}{1 - \beta}. \] (11)

Near the threshold \( \sqrt{s} = 2m_H \), the \( \beta \) value is small enough to be used as an expansion parameter. As \( \beta \to 0 \), the pseudoscalar meson pair production is strongly suppressed and the ratios of other total cross sections come close to 1 : 6 (See fig. 4).

3 Discussion

In this paper we have obtained the ratios (10) of the exclusive cross section in the limit where effects of the order of \( \Lambda_{QCD}/m_H \) and \( \alpha_S(m_H) \) may be neglected and momentum transfers are large compared to \( \Lambda_{QCD}^2 \). We find that near the threshold \( \sqrt{s} = 2m_H \) the process \( \gamma\gamma \to M \tilde{M} \) is highly suppressed and the ratios of other total cross sections become 1 : 6. In other words, the vector meson production is dominant and any pseudoscalar meson should be accompanied by a vector meson as its production companion in the \( \gamma\gamma \)
collision. The result is quite different from that in the $e^+e^-$ case where the ratios are $1:2:7$ at threshold.

From eq. (8), we can show that the differential cross sections near threshold are

$$\frac{d\sigma}{dz}(\gamma\gamma \rightarrow M^*\bar{M}) \propto \cos^2\theta, \quad \frac{d\sigma}{dz}(\gamma\gamma \rightarrow M^*\bar{M}^*) \propto 2. \quad (12)$$

The angular dependence implies:

- The final pseudoscalar and vector meson system has a $P$-state wave function $Y_{10}(\cos \theta)$.
- The final vector meson pair system has an isotropic distribution, i.e. no angular momentum.

Naturally the ratios are expected to be $1:6$ near threshold, which is consistent with the explicit calculation.

Our result has a finite region of validity. It is quite likely that, at the low end around the threshold, important resonance effects arise and influence the ratios. Thus our prediction would work only in the energy region at least a few GeV higher than $\sqrt{s} = 2m_H$.

The upper limit of validity is complicated. We should consider QCD radiative corrections as well as effects of higher order in $\Lambda_{QCD}/m_H$. The latter corrections [8, 14] can be systematically treated. However, the former are difficult to treat, because of non-perturbative QCD contributions. Besides, the angular distributions are forwardly and backwardly peaked and the cross sections become too small at high energies to verify any theoretical prediction.

Consequently, the appropriate choice of the region of validity is highly required. In the $\gamma\gamma$ production of heavy mesons the appropriate region of validity is the one where the CM energy $\sqrt{s}$ is a couple of GeV higher than $2m_H$ but not so very high for data analyses, and the $t$- or $u$-channel momentum transfer $Q^2$ should be much larger than $\Lambda^2_{QCD}$ with a view to excluding non-perturbative QCD effects in the heavy quark pair production by two photon before hadronization.

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Figure captions

Figure 1: Feynman diagrams contributing to heavy meson pair production in γγ collision to lowest order in αS at the QCD parton level. The solid line is for the heavy quark and the dashed line for the light quark.

Figure 2: Ratios of total cross sections $\sigma(\gamma \gamma \rightarrow M \tilde{M})/\sigma(\gamma \gamma \rightarrow M^* \tilde{M})$ and $\sigma(\gamma \gamma \rightarrow M^* \tilde{M}^*)/\sigma(\gamma \gamma \rightarrow M^* \tilde{M})$ versus the heavy meson velocity $\beta$. 