Rapidity equilibration and longitudinal expansion at RHIC

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Abstract: The evolution of net-proton rapidity spectra with $\sqrt{s_{NN}}$ in heavy relativistic systems is proposed as an indicator for local equilibration and longitudinal expansion. In a Relativistic Diffusion Model, bell-shaped distributions in central collisions at AGS energies and double-humped nonequilibrium spectra at SPS show pronounced longitudinal collective expansion when compared to the available data. The broad midrapidity valley recently discovered at RHIC in central Au + Au collisions at $\sqrt{s_{NN}}= 200$ GeV indicates rapid local equilibration which is most likely due to deconfinement, and fast longitudinal expansion of the locally equilibrated subsystem. A prediction is made for Au + Au at $\sqrt{s_{NN}}= 62.4$ GeV.

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Net-baryon rapidity distributions have proven to be sensitive indicators for local equilibration and deconfinement in relativistic heavy-ion collisions [1, 2]. Whereas statistical model analyses of multiplicity ratios of produced particles [3] appear to be consistent with the assumption that the system reaches equilibrium and can therefore be described by a temperature and a chemical potential, it is clear that one has to consider the distribution functions of the relevant observables in order to determine whether the system has indeed reached, or gone through, thermal equilibrium.

Distribution functions of produced particles in transverse momentum or energy have been shown in many analyses to be close to thermal equilibrium, if one takes into account the collective transverse expansion of the system [4]. Here, the expansion velocities rise as the energy increases from AGS via SPS to RHIC when they are estimated based on thermal distribution functions.

In longitudinal direction, however, it is more difficult to determine the degree of equilibration, and the collective expansion velocity. It is the main purpose of this note to provide a schematic macroscopic approach that allows to obtain both quantities. There have been earlier attempts to extract longitudinal collective velocities at AGS and SPS energies by assuming that the rapidity distributions are thermal, and that the remaining large difference to the data is then due to expansion [5]. However, the net-baryon rapidity distributions at AGS, SPS and RHIC energies are clearly nonequilibrium distributions even in central collisions [2], and also the longitudinal distributions of produced particles are not fully thermalized [16].

To account for the nonequilibrium behavior of the system, the evolution of net-proton rapidity spectra with incident energy \( \sqrt{s_{NN}} = 4.9 \) to 200 GeV is studied analytically in a Relativistic Diffusion Model (RDM) [2], and compared to AGS [6], SPS [7] and RHIC [8] data. In addition to the nonequilibrium-statistical evolution as described in the RDM, collective longitudinal expansion is considered.

The model [2,9,10,11] is based on a generalized Fokker-Planck equation (FPE) for the distribution function \( R(y,t) \) in rapidity space, \( y = 0.5 \cdot \ln((E+p)/(E-p)) \),

\[
\frac{\partial}{\partial t}[R(y,t)] = -\frac{\partial}{\partial y}[J(y)[R(y,t)]] + D_y(t)\frac{\partial^2}{\partial y^2}[R(y,t)]^{2-q}.
\]

(1)

with the nonextensivity-parameter \( q \) [12], and the rapidity diffusion coefficient \( D_y \) that
contains the microscopic physics, and accounts for the broadening of the distribution functions through interactions and particle creations.

The FPE can be solved analytically in the linear model case q=1, with constant $D_y$, a linear drift function

$$J(y) = (y_{eq} - y)/\tau_y,$$

and the rapidity relaxation time $\tau_y$. This is the so-called Uhlenbeck-Ornstein process for the relativistic invariant rapidity \[2,11\]. The equilibrium value is $y_{eq} = 0$ in the center-of-mass for symmetric systems, whereas $y_{eq}$ is calculated from the given masses and momenta for asymmetric systems.

Using $\delta-$function initial conditions at the beam rapidities for net baryons, analytical solutions of Eq.(1) are obtained for various values of $\tau_{int}/\tau_y$ (Fig.1 bottom as an example for Pb + Pb at SPS). Here the interaction time $\tau_{int}$ is the time from nuclear contact to freeze-out, or the integration time of Eq.(1). It determines how close to equilibrium the system can come, and it is obtained from dynamical models or from parametrizations of two-particle correlation measurements. In central Au + Au collisions at 200 A GeV, this yields $\tau_{int} \simeq 10 fm/c$ \[13\]. For known interaction times, the rapidity relaxation times and diffusion coefficients can then be obtained from the data. Otherwise, $\tau_{int}/\tau_y$ and the rapidity width coefficient

$$\Gamma_y = \left[ 8 \cdot \ln(2) \cdot D_y \cdot \tau_y \right]^{1/2}$$

(3)
can be determined from the peak positions and width of the data, respectively, through the use of the equations for the mean values and the variances \[2\] that are derived from Eq.(1).

The transport coefficients $D_y$ and $\tau_y$ are, however, macroscopically related to each other through a dissipation-fluctuation theorem with the equilibrium temperature $T$. This relation has been derived in \[14\] from the requirement that the stationary solution of Eq.(1) is equated with a Gaussian approximation to the thermal equilibrium distribution in $y$-space. At fixed incident energy, this weak-coupling rapidity diffusion coefficient turns out to be proportional \[14\] to the equilibrium temperature $T$ as in the analysis of Brownian motion (Einstein relation)

$$D_y \propto \frac{T}{\tau_y}.$$
When compared to heavy-ion collision data at energies above the SIS region of 1-2 GeV, rapidity spectra that are calculated using this relation are consistently narrower [14] than the data because collective processes (in particular, collective longitudinal expansion) are not included in the weak-coupling dissipation-fluctuation theorem. Hence, the width coefficients $\Gamma_y$ as obtained from Eqs.(3,4) are replaced by effective values $\Gamma_y^{eff}$. Depending on the system and the incident energy, they are typically factors of 2-5 larger than the calculated values.

In this work, I assume that the whole discrepancy $\Gamma_y^{eff} - \Gamma_y$ is due to longitudinal expansion. I obtain the mean collective expansion velocity in $\pm z$-direction for baryons of rest mass $m_0$ and relativistic mass $m$ from the relativistic velocity

$$v = \sqrt{1 - (m_0/m)^2}.$$  \hspace{1cm} (5)

For $f$ degrees of freedom ($f=1$ in rapidity space) the effective mass is

$$m = m_0 + \frac{f}{2} \cdot (T_{eff} - T).$$ \hspace{1cm} (6)

In this expression, the mean energy content of the expansion is written as $E_{coll} = f/2 \cdot (T_{eff} - T)$ with an effective temperature $T_{eff}$ and the equilibrium temperature $T$ in analogy to the classical expression for the mean energy. Hence, the expansion enhances the particle mass $m_0$ to its relativistic value $m$.

The thermal equilibrium temperature $T$ should be associated with kinetic freeze-out. Typical values of $T$ at RHIC energies are 110 MeV. This is significantly below the chemical freeze-out temperature of about 170 MeV [3] that is obtained from fits of hadron abundances. In the weak-coupling case without collective effects, $T_{eff}=T$ such that $v_{coll}=0$, and the distribution functions remain too narrow as compared to the data. With expansion, the diffusion coefficient is enlarged to its effective value, and since it is proportional to the temperature, $T_{eff}$ is obtained from $T$ using the same enhancement factor.

With Eqs.(5,6), the collective velocity becomes

$$v_{coll}^{||} = \left[1 - \left(\frac{m_0}{m_0 + \frac{f}{2} \cdot (T_{eff} - T)}\right)^2\right]^{1/2}$$ \hspace{1cm} (7)

with the limiting cases $v_{coll}^{||} = 1$ for $T_{eff} >> T$, and $v_{coll}^{||} = 0$ for $T_{eff} = T$.  

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Comparing the solutions of Eq. (1) to Au + Au central collision (5 per cent of the cross section) data at AGS-energies $\sqrt{s_{NN}} = 4.9$ GeV [6], it turns out that due to the relatively long interaction time $\tau_{int}$ and hence, the large ratio $\tau_{int}/\tau_y \simeq 1.08$ (Table I), the system is in rapidity space very close to thermal equilibrium, with longitudinal collective expansion at $v_{coll}^{||} = 0.49$ (Fig.1, upper frame). The bell-shaped experimental distribution is in good agreement with the solution of Eq. (1). The distributions remain bell-shaped also at lower energies [6].

This situation changes at the higher SPS energy of $\sqrt{s_{NN}} = 17.3$ GeV. Here net-proton Pb + Pb rapidity spectra corrected for hyperon feeddown [7] show two pronounced peaks in central collisions, which arise from the penetration of the incident baryons through the system. The gradual slow-down and broadening is described using Eq. (1) as a hadronic diffusion process with subsequent collective expansion, $v_{coll}^{||} = 0.75$ (Table I). The associated nonequilibrium solutions with expansion for various values of $\tau_{int}/\tau_y$ are shown in the lower frame of Fig.1. The system clearly does not reach the dash-dotted equilibrium solution. Hence, both nonequilibrium properties, and collective expansion are required to interpret the broad rapidity spectra seen at the SPS.

Within the current framework, no indication for deconfinement of the incident baryons or other unusual processes can be deduced from the net-proton rapidity data at AGS and SPS energies, because the crucial midrapidity region is here too small.

This is, however, different at RHIC energies $\sqrt{s_{NN}} = 200$ GeV. The RDM nonequilibrium solution exhibits pronounced penetration peaks with collective longitudinal expansion $v_{coll}^{||} = 0.93$, but it fails to reproduce the BRAHMS net-proton data [8] in the broad midrapidity valley (solid curve in Fig.2, bottom): The diffusion of the incident baryons due to soft scatterings is not strong enough to explain the net baryon density in the central rapidity region. The individual nonequilibrium solutions $R_1$ and $R_2$ are Gaussians, and if they fit the data points near $y=2$, they necessarily grossly underpredict the midrapidity yield because it is in their tails.

This central region can only be reached if a fraction of the system undergoes a fast transition to local thermal equilibrium, dashed curve in Fig.2, bottom. With collective expansion of this locally equilibrated subsystem of 22 net protons ($v_{coll}^{||} = 0.93$), the flat midrapidity BRAHMS data are well reproduced in an incoherent superposition of
nonequilibrium and equilibrium solutions of Eq.\[1\]

\[
\frac{dN(y, t = \tau_{\text{int}})}{dy} = N_1 R_1(y, \tau_{\text{int}}) + N_2 R_2(y, \tau_{\text{int}}) + N_{eq} R_{eq}^{\text{loc}}(y).
\]  

(8)

The fast transition of a subsystem of \(N_{eq} \simeq 55\) baryons, or 22 protons to local thermal equilibrium suggests that the associated participant partons have reached local equilibrium through the sudden enhancement in the number of degrees of freedom that accompanies deconfinement. Microscopically, the large gap to midrapidity is thus bridged through hard scatterings of partons with subsequent thermal equilibration, rather than diffusion of nucleons. The fact that even hard partons can participate significantly in equilibration processes is evidenced by the high-\(p_T\) suppression found in Au + Au at RHIC.

The local equilibrium distribution \(R_{eq}^{\text{loc}}\) with expansion is a solution of Eq.\[1\] for time to infinity with an enlarged (effective) diffusion coefficient that is related to the effective equilibrium temperature. The nonequilibrium solutions \(R_1, R_2\) of Eq.\[1\] are calculated with the same value of the diffusion coefficient. The underlying picture is that expansion affects nonequilibrium and equilibrium distributions in a similar fashion.

In a schematic calculation for the lower RHIC energy of \(\sqrt{s_{NN}} = 62.4\) GeV, I have used the rapidity diffusion coefficient from 200 GeV to obtain the result in the upper frame of Fig.2. Two pronounced penetration peaks can be seen, together with a narrow midrapidity valley. The corresponding data have been taken, and are presently being analyzed by the BRAHMS collaboration [15]. Once they are available, an adjustment of \(\Gamma_y^{\text{eff}}\) and \(N_{eq}^{\text{loc}}\) may be required. One may expect another correction because the total net proton number could change, although the baryon number is conserved. Note, however, that this change was found to be negligible at SPS energies.

The RDM-results can be further refined if the linear drift function is replaced by

\[
J(y) = -\alpha \cdot m_{\perp} \sinh(y)
\]

(9)

with the transverse mass \(m_{\perp} = \sqrt{m^2 + p_{\perp}^2}\), because this yields exactly the Boltzmann distribution as the stationary solution of Eq.\[1\] for \(q=1\). This entails, however, to solve Eq.\[1\] numerically. A numerical solution is also required if one tries to account for multiparticle effects through nonextensive statistics [12] with an explicit nonlinearity in the FPE, \(1 < q < 1.5\). In this case, an approximate result may be obtained from a linear superposition...
of power-law solutions of Eq.\[10\]. The local equilibrium fraction is

\[ R_{\text{loc,eq}}^{\text{q}} \propto [1 - (1 - q) \frac{m_{\perp}}{T} \cosh(y)]^{1/q} \]  

with \( +\infty \int_{-\infty}^{+\infty} R_{\text{loc,eq}}^{\text{q}} dy = 1 \). For \( \sqrt{s_{NN}} = 200 \text{ GeV} \), \( m_{\perp} = 1.1 \text{ GeV} \), \( T = 110 \text{ MeV} \), \( q = 1.4 \) and \( N_{\text{eq}} = 22 \) protons, Eq.\[10\] yields the dotted curve in Fig.2 (bottom). It lies in between the Boltzmann-Gibbs local equilibrium results without and with expansion. Hence, it simulates to some extent the collective effects.

Whereas local equilibration in rapidity space occurs only for a small fraction of the participant baryons in central collisions at RHIC energies, the opposite is true for produced hadrons. This can be inferred from recent applications of the RDM with three sources (located at the beam rapidities, and at midrapidity) and \( \delta \)-function initial conditions \[16\]. About 78% of the produced hadrons in central Au + Au collisions at 200 A GeV are found to be locally equilibrated in pseudorapidity space, whereas the corresponding number is 17% in central d + Au.

To conclude, I have interpreted recent results for central collisions of heavy systems at AGS, SPS and RHIC energies in a Relativistic Diffusion Model (RDM) for multiparticle interactions based on the interplay of nonequilibrium and local equilibrium (”thermal”) solutions. In the linear version of the model, analytical results for the rapidity distribution of net protons in central collisions have been obtained and compared to data. The enhancement of the diffusion in rapidity space as opposed to the expectation from the weak-coupling dissipation-fluctuation theorem has been interpreted as collective expansion, and longitudinal expansion velocities have been determined from a comparison between RDM-results and data based on a relativistic expression for the collective velocity. A prediction for net-proton rapidity distributions at \( \sqrt{s_{NN}} = 62.4 \text{ GeV} \) yields a smaller midrapidity valley than at 200 GeV which will soon be compared with forthcoming data.
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TABLE I. Parameters for heavy relativistic systems at AGS, SPS and RHIC-energies.
The beam rapidity is expressed in the c.m. system. The fit parameter $\tau_{int}/\tau_y$ determines how fast the net-baryon system equilibrates in rapidity space. The second free parameter is the effective rapidity width coefficient $\Gamma^y_{eff}$. It includes the effect of expansion. The deduced longitudinal expansion velocity is $v_{coll}^{||}$. At 62.4 GeV, $\Gamma^y_{eff}$ will need adjustment(*) to forthcoming data.

| Lab   | System     | $\sqrt{s_{NN}}$ (GeV) | $y_{b.c.m.}$ | $\tau_{int}/\tau_y$ | $\Gamma^y_{eff}$ | $v_{coll}^{||}$ |
|-------|------------|------------------------|--------------|----------------------|------------------|----------------|
| AGS   | Au + Au    | 4.9                    | 1.60         | 1.08                 | 1.45             | 0.49           |
| SPS   | Pb + Pb    | 17.3                   | 2.91         | 0.81                 | 2.43             | 0.75           |
| RHIC  | Au + Au    | 62.4                   | 4.20         | 0.34                 | 2.94*            | 0.86           |
| RHIC  | Au + Au    | 200                    | 5.36         | 0.26                 | 4.29             | 0.93           |
Figure captions

**FIG. 1:** Net-proton rapidity spectra in the Relativistic Diffusion Model (RDM), solid curves. Central Au + Au at AGS energies $\sqrt{s_{NN}} = 4.9$ GeV (data from [6]) is close to a longitudinally expanding equilibrium distribution (top). Central Pb + Pb at SPS energies $\sqrt{s_{NN}} = 17.3$ GeV (data from [7]) remains relatively far from thermal equilibrium and therefore, double-humped with two penetration peaks (middle). Dashed curves are thermal equilibrium distributions without collective expansion, dash-dotted curves with expansion. The bottom panel shows analytical Pb + Pb RDM-solutions for $\tau_{int}/\tau_y = 0.08, 0.1, 0.2, 0.5, 0.8, 1.0, 1.1, 1.3$, all with longitudinal collective expansion, Table I. The transition from bell-shaped (AGS) to double-humped (SPS) is clearly shown in the RDM.

**FIG. 2:** Net-proton rapidity spectra in the RDM (solid curves) for central collisions of Au + Au at RHIC energies $\sqrt{s_{NN}} = 62.4$ GeV (top), and 200 GeV (middle, central 5% of the cross section) compared to BRAHMS data [8] at the higher energy. The net proton content is 158. The high midrapidity yield is not attainable through soft scatterings in the Relativistic Diffusion Model. Instead, this region containing $\approx 22$ protons at 200 GeV is populated through hard scatterings with ensuing local equilibration (dashed curve), and collective expansion (dashed areas). See Table I for parameters. The dotted curve is the nonlinear local equilibrium solution of the FPE for $q=1.4$, cf. text.
