The birth of strange stars:
kinetics, hydrodynamics and phenomenology
of supernovae and GRBs
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Abstract
We present a short review of strange quark matter in supernovae and
related explosions, with particular attention to the issue of the propagation
of the combustion in the dense stellar environment. We discuss the insta-
bilities affecting the flame and present some new results of application to
the turbulent regime. The transition to the distributed regime and further
deflagration-to-detonation mechanism are addressed. Finally we show that
magnetic fields may be important for this problem, because they modify the
flame through the dispersion relations which characterize the instabilities. A
tentative classification of explosive phenomena according to the value of the
average local magnetic field affecting the burning and the type of stellar sys-
tem in which this conversion is taking place is presented. As a general result,
we conclude that “short” conversion timescales are always favored, since the
burning falls in either the turbulent Rayleigh-Taylor (or even the distribu-
ted) regime, or perhaps in the detonation one. In both cases the velocity is several
orders of magnitude larger than \( v_{\text{lam}} \), and therefore the latter is irrelevant
in practice for this problem. Interesting perspectives for the study of this
problem are still open and important issues need to be addressed.

1 Introduction

Intensive work in the 60s and 70s definitely established the concept of elemen-
tary constituents of nucleons (quarks and gluons). At increasing center-of-
mass energy in experimental searches of the elementary components (partons)
of protons and other hadrons revealed new physics in need of a theoretical
framework to be developed. The theory of “new” strong interactions (as
opposed to the “old” nuclear physics) was constructed in parallel, first fo-
cused on classification schemes (or, as is called today, flavor physics) and
later on finding a theory to describe the dynamics. The development and
success of gauge theories in the '70s eventually leaded to a non-abelian version based on the $SU(3)_c$ symmetry group \[1\] as a “natural” candidate for a theory of strong interactions. The fundamental quantum number carried by the elementary constituents (quarks) was named ”color”, and consequently the dynamics involving quarks and gauge fields (gluons) become known as Quantum Chromodynamics (QCD for short).

It was considered by many somewhat puzzling that repeated efforts to find these entities as free particles (asymptotic states) failed. Subsequent work elaborated on a striking feature of the theory: that the interactions themselves preclude the appearance of the quarks and gluons outside ordinary hadrons, they remain confined inside them at low energies. Another property was soon demonstrated to hold when momentum transfer scales $Q$ became large enough. This is the so-called asymptotic freedom, and states that the colored particles behave as if they were free in the limit $Q \to 0$. Actually, there is an energy (or momentum) scale above which color quantum number is not confined any more, but how large the momentum transfer should be (or in other words, which is energy, as measured by the temperature or density of the ensemble allowing the deconfinement) is still a matter of debate. These developments mean that the early universe passed through a deconfinement $\to$ confinement phase transition along its cooling, although less certainty holds for the densities of the ”natural” laboratories (neutron stars) in which compression would deconfine hadronic matter. The earliest calculations \[2\] using reasonable models for both the confined and deconfined phases imprinted on successive researchers the uncertain conclusion that quarks and gluons (forming a state known as the quark-gluon plasma, or QGP) should appear at densities above a large threshold, say, $10 \times \rho_0$; with $\rho_0$ the nuclear saturation density.

From the starting of these calculations it has proved very difficult to reliably determine the transition points, and also the nature of the transition itself (at least when full numerical calculations were out of sight \[3\]). Most of the times the conclusions had to be extracted from simultaneous extrapolations of both a quark model, expected to be valid for $\rho \to \infty$, and an hadronic model valid around $\rho_0$ but uncertain much above it. Since there is no certainty in either one, the final result is always subject to reasonable doubts. The “induction” of a definite order of the transition because of the adopted functional forms of the thermodynamical quantities of both sides. Nevertheless these serious and honest attempts have proliferated until today, given that the transition is still elusive (the extensively studied finite
temperature case still has some small uncertainty in the value of $T_c$ and a quite consensual assessment of the order, see [1] for details). Recent analysis [5] of hadronic flows have added a lot of excitement to these topics, since it appears that the QGP was indeed produced in heavy ion collisions, but the asymptotic form is not reached, rather behaving as a glass-like system. Needless to say, this kind of studies attract a lot of attention and offers a concrete form to glimpse the deconfined state of hadronic matter, yet to be characterized and understood.

2 Stable strange quark matter?

While the study of the quark-gluon plasma occupied many studies in connection with the early Universe and compact stars, a much radical proposal emerged in the 80s about it, which may be described as follows: it is true that the asymptotic freedom property guarantees that quarks and gluons will be the ground state of QCD at high densities/temperatures, but it says nothing about the ground state at lower densities or temperatures. The everyday experience strongly suggests that ordinary hadrons confine the quarks/gluons and thus constitute the "true" (in the sense of $\rho \to 0$ and $T \to 0$) ground state of hadronic matter. The emerging strange matter hypothesis came precisely to challenge this "common sense" statement: it says that the true ground state of hadronic matter is a particular form of the QGP, differing from the ordinary matter by the presence of a key quantum number (strangeness). This is counterintuitive to many people, but a careful look at the physical arguments shows no inconsistency whatsoever, at least in principle.

An argument for the SQM being the true ground state can be made as follows: as is well-known the quantity that determines which phase is preferred is the Gibbs free energy per particle $G/n$ as a function of the pressure (we impose $T = 0$ hereafter as appropriate for highly degenerate hadronic matter, it is easy to see that the term $-TS$ in the free energy disfavors SQM at high temperatures). As $P$ is increased starting from the neighborhood of the nuclear matter saturation point $\rho_0$ the asymptotic freedom guarantees that there has to be a switch from nuclear matter (N) to elementary hadronic constituents, that is, the lighter quarks $u$ and $d$. The point at which this is supposed to happen will be labelled as $P_c$. Thus, the doubts stated above about the appearance of the QGP inside neutron stars may be now restated as whether the pressure at the center is larger or smaller than $P_c$. 

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However, this is where the concept of strangeness plays an important role. Strangeness is the flavor quantum number carried by $\Lambda$s and other heavy hadrons. At the elementary level, it is carried by a different quark $s$, with current mass in the ballpark of $\sim 100\, MeV$, that is, light enough to be present at a few times the nuclear saturation density. While creating strangeness in hadrons costs energy (because strange hadrons are heavier than non-strange ones; for instance, the $\Lambda$s are heavier than the neutrons and so on); this trend is reversed inside the QGP. The reason is simply the Pauli exclusion principle: a new Fermi sea in the liquid (the one of the $s$ quark) allows a rearrangement of the energy, and sharing it lowers the energy per particle. However, the gain is not precisely known, but it is not impossible to imagine lowering the free energy per particle to a value that would be lower than the mass of the neutron $m_n$ even when $P \rightarrow 0$. If realized, this would preclude the (strange) QGP to decay into ordinary hadrons because this would cost energy and the SQM would have been created. Put it simply, the compression would liberate the elementary components that quickly create their own way of surviving. We stress that all these are bulk (i.e. large number) concepts, and it is central to the SQM hypothesis to reach a strangeness per baryon of the order one (and exactly one if the strange quark had no mass to deplete its relative abundance). This is not possible in a few-body system like a nucleus, because each weak decay creating a strangeness unit contributes roughly with a factor $G^2_{\text{Fermi}}$ to the amplitude, and thus the simultaneous decays are strongly suppressed; this is why it has been very difficult to produce even doubly strange nuclei, let alone higher multiplicity ones. However, once quarks roam free in the QGP they can easily decay by $u+d \rightarrow u+s$ because there is plenty of phase space for the products until equilibrium is reached. These bulk estimates have been always one way or another behind the idea of SQM.

As it stands, the SQM hypothesis is very bold. It conjectures that every hadron we see around us is in a metastable state, and if conditions for creation of a large net strangeness were met, the matter would not make back ordinary hadrons (technically it is said that SQM constitutes a non-topological soliton stabilized against decays by a conserved charge, the baryon number, see [6] for a thorough discussion of this case and related ones). The general idea of reaching extreme conditions and stabilizing the QGP is already apparent in the paper of Bodmer [7], later reintroduced and refined in references [8, 9, 10] and colorfully discussed in the paper of Witten [11], which was fundamental to give a big boost to SQM research.
SQM as a theoretical construction is interesting, but finding it in nature would be infinitely more. Key questions of SQM such as whether it does exist or not, and whether it has been ever produced in the Universe are still unsolved. On the other hand, we begin for the first time to have the possibility of falsifying these basic questions mainly thanks to the new generation of space telescopes (HST, Chandra, XMM) and neutrino observatories (SNO, Kamioka, Icecube), to name just a few. These instruments may be used to look for exotic states in compact stars and their birth events.

Many applications of SQM in astrophysics were foreseen during the first decade after its official birth [11] and early infancy [12]. Since astrophysical insight has shown to be essential in the determination of fundamental questions related to SQM, we shall focus briefly in a very definite (and important) astrophysical problem, trying to give an assessment and pointing on the uncertainties and possible directions that may be explored in the near future. We thus restrict our discussion to SQM in compact stars, and more specifically, on how a seed of SQM may grow and propagate throughout a just-born neutron star. This has been a popular choice for an energy source in GRBs and core-collapse supernovae, therefore it is important to establish its basic features with confidence to build on them.

3 SQM in protoneutron stars: effects in core-collapse supernovae

As a “natural” environment in which SQM might form, core collapse supernovae has received reasonable attention [13, 14, 15, 16]. Despite of more than three decades of theoretical research and hard numerical modelling, the processes that cause the explosion of massive stars are still not understood ([17]). If, as the more recent and detailed numerical simulations suggest, the neutrino-driven mechanism works on special conditions only, the current paradigm for explaining massive star explosions would have to be deeply revised. “Conventional” physics has now turned attention to the role of rotation inside the progenitor and magnetic fields [18, 19], possibly relating this problem to the GRB one [21]. Although it is still too early for making definitive conclusions, investigations including the possible transition to deconfined QCD phases may be relevant to this problem. The first studies of SQM in supernovae ([20, 13, 22, 23, 16]) showed that this hypothetic subnu-
clear energy source is more than adequate to contribute to the explosion, and that some observed characteristics in the neutrino emission of SN1987A may be naturally explained within this scenario ([24, 25, 20]) (a second peak in the neutrino emission is naturally predicted in these models, and such signal has been tentatively associated to the late neutrinos from SN1987A detected by Kamiokande, which have to be otherwise interpreted as a statistical gap within the current paradigm).

From a wide perspective, supernovae are perhaps the only astrophysical events in which we could have the possibility of making a “multiwavelength” detection (neutrinos, various electromagnetic wavelengths, gravitational waves) of the process of SQM formation. However, these calculations are still in the infancy, and just bold expectations have been formulated. Some specific simulations [26] have addressed (negatively) a few questions posed in GRB models. In addition, a firmer detailed observational background would be needed, which imperatively needs the occurrence of a number of supernova explosions in the neighborhood of our galaxy, and thus is out of any human control (in turn the instrumentation must be improved greatly).

Second, although the general picture of SQM formation in supernovae has been qualitatively constructed, no systematic calculations have been made. There are also many unresolved questions related to strong interactions at high densities, which introduce an uncomfortable degree of uncertainty in all conclusions. We shall attempt below to describe the basics of the SQM propagation problem, a subject that has been addressed in the literature over the years from the kinetic/energetics point of view [28, 29, 30, 31, 32], but has a high degree of complexity from the coupling to hydrodynamics, much in the same way thermonuclear supernovae do. We will be guided by the work done in the latter problem, even though most of our discussion is new (i.e. unpublished) for the specific problem of SQM propagation. We shall later attempt to sketch the effects of the magnetic fields for the propagation, which leads to a tentative classification of the different phenomenological events.

4 SQM combustion dynamics: early stages

As discussed and agreed in the literature, a seed of SQM must become active or form following the standard bounce onto the former iron core. We shall not address this problem of the seed here, just assuming that by some of the proposed mechanisms [33, 20] the seed of SQM is present within $\sim$ seconds
after the bounce (if the quick appearance is bypassed, a late conversion could ensue \[34\] but without effects in supernovae). The neutron-SQM interface must then propagate outwards powered by the energy release of converted neutrons, much in the same way as a laboratory combustion.

It seems reasonable to assume the combustion to begin as a \textit{laminar} deflagration, in which the diffusion of $s$ quarks set the scale for the flame length $l_{th}$. This has been actually the subject of early calculations \[27, 28, 29, 30\], in which a plane front approximation was used to obtain the laminar velocity $u_{lam}$ as a function of temperature, density and other relevant quantities. The result $u_{lam} \leq 10^4 \text{cm s}^{-1}$ suggested that a just-born NS would convert to a SS in $\sim 100 \text{s}$ or so. From the combustion theory point of view this is equivalent to decouple completely the kinetics of the burning from the hydrodynamics of the flow in the star. Nevertheless, the reasonable convergence of several approaches to the calculation of $u_{lam}$ gives some confidence that the result is reliable within the approximations.

In a situation as such (a combustion starting around the center and propagating outwards), it has been known for many years \[35, 36\] that small perturbations are unstable for all wavelengths at a linear level. In fact Horvath and Benvenuto \[37\] calculated the perturbation growth for this specific problem with the resulting condition

$$j^4 < 4\sigma g \rho_1 \rho_2^2 \frac{1}{(\rho_2 - \rho_1)}$$

where $j$ is the mass flux onto the flame, $\sigma$ the surface tension, $g$ the local gravitational acceleration and $\rho_1, \rho_2$ the densities of the “fuel” (neutron matter) and “ashes” (SQM) respectively. As it stands, this is impossible to satisfy for \textit{any} deflagration (in particular, the laminar), because by its very definition $j^2 = (P_2 - P_1)\rho_2 \rho_1 / (\rho_2 - \rho_1)$, and thus a deflagration which must obey $P_2 < P_1$ and $\rho_2 < \rho_1$ making the r.h.s a negative number. This way, the flame wrinkles in a timescale $\leq$ the dynamical timescale $\tau_{dyn} \sim 10^{-3} \text{s}$ (as appropriate in a protoneutron star). Thus, the strong statement made by Landau and Darrieus is confirmed at the linear level.

Numerical calculations of the Landau-Darrieus instability beyond the linear level \[38\] show the formation of \textit{cusps}, leading to quadratic and higher-order terms in the dispersion relation and stabilizing the flame \[39\]. The flame acquires a cellular shape and accelerates, since the contact area between the fuel and ashes increases. The stationary, scale-invariant amplitude of this cusps leads to an acceleration of the flame, with velocity described in
this regime as

$$u_{cell} = u_{lam} \left(1 + 0.4(1 - \mu)^2\right)$$  \hspace{1cm} (2)$$

with $\mu = \rho_2/\rho_1$. The flame velocity is higher than in the laminar regime by a modest amount for all reasonable compression ratios $\mu$. An alternative cellular flame model has been developed by Blinnikov and Sasorov [40]. They observe that the wrinkled flame can be described with a fractal model, which in the 2-D case yields

$$u_{cell} \simeq u_{lam} \left(\frac{l}{l_{crit}}\right)^{D_{cell}}$$  \hspace{1cm} (3)$$

with $D_{cell}$ the fractal dimension of the surface and $l_{crit}$ a suitable minimum length. A calculation of the latter quantity finally yields

$$u_{cell} \simeq u_{lam} \left(\frac{l_{max}}{l_{crit}}\right)^{0.6(1-\mu)^2}$$  \hspace{1cm} (4)$$

where we have imposed the radial distance $l_{max}$ as the maximum scale for which this theory is valid. Arguments related to the propagation of L-D unstable flames suggest that $l_{crit}$ may be identified with the Markstein length [41], or at least $\sim 100 l_{th}$. Eq. (4) leads to the same conclusion as before: there is a modest acceleration of the flame and stabilization at the small scales.

In Ref. [37], the extreme assumption that velocity of the flame can not become supersonic, we obtained a (small) maximum length for this regime to hold. This should be rather interpreted as the scale beyond which the above L-D description breaks down definitely, due to combined additional physical effects that we now address.

Since the gravitational pull is always being exerted onto the flame, one could have anticipated that the cell structure can not be scale-invariant indefinitely, and in fact disruption of the bubbles does occur [42]. A turbulent cascade dominates the burning above certain length, which can be estimated from the point when the velocity of turbulent fluctuations $u'(l)$ becomes equal to $u_{cell}$. This defines the so-called Gibson scale $l_{gib}$ [43]. Imposing a Kolmogorov spectrum (it is now established that this is more accurate than the so-called Bolgiano-Obukhov spectrum for 3D, whereas the latter is relevant for 2D models) $u'(l) \propto l^{1/3}$, it can be shown that the scaling of $l_{gib}$ is
where the fluctuations have to be normalized to the largest scales $L$ encountered in the system. Given that the turbulence itself can not become supersonic (the speed of sound is already $\sim c$ in the problem), and using the former value $u_{cell} \geq 10^4 \text{cm s}^{-1}$, we obtain for $l_{gib}$ the value of $\sim 10^{-4} \text{cm}$ as an extreme upper limit, and decreasing with time. This is, however, initially much larger than $l_{th}$ in the diffusive regime, and allows a classification of the burning into the flamelet regime: while the flame propagation is still determined by diffusion, the total burning is in turn controlled by turbulence in a turbulent region called the flame brush. In the flamelet regime, for all scales $\gg l_{gib}$, the turbulent velocity $u_{turb}$ and front width $l_{turb}$ are determined by the Kolmogorov spectrum at the larger scales. The important point to stress here is that the turbulent eddy turnover controls the transport and fuel consumption, quite unlikely a pure laminar regime [44, 45]. Diffusion processes do not play the dominant role once the flamelet regime is achieved, quickly after the start of the combustion. We note that if $l_{gib}$ decreases below the value of $l_{th}$ one can no longer talk of a laminar regime and the burning is likely described by the distributed regime, in which turbulent eddies disrupt the flame and dominate the burning on macroscopic and microscopic scales. We shall assume that the flamelet regime exists and proceed to describe the large-scale physics, keeping in mind the possibility of being bypassed in favor of the distributed regime.

5 SQM combustion dynamics: turbulent large-scale regime

While at the small scales L-D instability affects the flame eventually leading to the flame brush in the way described above, on still larger scales, buoyancy of hot burned fuel (SQM) dominates the dynamics of the process as a consequence of the Rayleigh-Taylor instability. In fact, one obtains essentially the R-T results by letting $u_{lam} \to 0$ in the L-D analysis. The classical solution of this problem [46] is well-known and indicates that in the linear regime the perturbations grow exponentially with a time scale $\tau_{RT} = (4\pi l/g)^{1/2}(\rho/\Delta \rho)^{1/2}$, with $\Delta \rho = \rho_1 - \rho_2$. After the modes attain amplitudes similar to the originally unperturbed, the merging/fragmentation of the bubbles and the shear
Kelvin-Helmholtz instability between bubble surfaces give rise to a turbulent mixing layer. In models with a single bubble scale, the velocity is

\[ u_{RT} \simeq \frac{1}{2} \sqrt{At \times l \times g_{eff}} \simeq \frac{1}{2} \sqrt{\frac{1}{2} (1 - \mu) \times l \times g} \]  

with \( At \) the Atwood number and \( l \) the radius of the tube in the experiment [47]. In astrophysical problems a single-scale expression is seldom enough to describe the intrinsic multi-scale system, and a 1-D model containing most of the relevant physics, the so-called Sharp-Wheeler model [48] is widely used to calculate evolution of the flames. In this picture the combustion front advances into the cold unburned fuel with a speed of

\[ u_{SW} \sim \frac{1}{20} (1 - \mu) gt \]  

It is clear that the bubble radius evolves linearly with the distance to the center. The Sharp-Wheeler speed eq.(7) can be identified with the effective speed of the burning provided the latter is completed inside the R-T mixing zone.

Fractal models have also been employed as an alternative description for the R-T regime [49, 50], with a generic prediction that can be summarized as

\[ u_{R-T} = u_{lam} \left( \frac{L}{l_{min}} \right)^{n/2} \]  

where \( n = 2(D - 2) \) relates the index to the fractal index \( D \), \( L \) is the scale at which the turbulent velocities equal the R-T instability velocity, and \( l_{min} \) is the smallest scale that can still deform the flame front (bounded from below by the Gibson scale defined above).

For both the Sharp-Wheeler model eq.(7) and the fractal model of eq.(8) the velocity increase is very large respect to the “kinetic” laminar models for the same problem. This is quite analogous to the carbon burning regime in type I supernova models, in which all the hydrodynamical aspects are being considered together with the reaction kinetics. It is important to remark that in all the cases the flames can be still defined quite properly, and that energetics determined by Hugoniot curves are still valid, as they should.

As performed in Type I SN studies, we plot in Fig. 1 the relevant velocities for the burning flame as a function of the scale. From a simple inspection
of this figure, it is clear that the $n \rightarrow SQM$ combustion should accelerate substantially when evolving at relatively low radii ($certainly \ll 1km$, still deep in the stellar interior. Further evolution of the flame will be discussed below, ending with some of the expected consequences and phenomenological features.

6 SQM combustion dynamics: distributed regime and the transition to the detonations branch (DDT)

The evolution of the flames described below is now quite well established and substantiated by several numerical simulations. One may wonder about the final outcome of the burning process when the flame is well within the R-T stage. The possibility that turbulence disrupts the flame on microscopic scales, which would not be well-defined any more, can be adopted as a rough intuitive description of the distributed regime. In the latter mixed regions of fuel and ashes burn in regions that have a distribution of temperatures interact strongly with the turbulence. Alternatively, the combustion may reach the edge of the star without reaching the distributed regime.

More rigourously, the distributed regime can be characterized by the inequality $l_{gib} < l_{th}$. It is not clear whether this condition is achieved in the $n \rightarrow SQM$ conversion. As suggested above, it may be achieved directly in the early stages. However, and in spite that $l_{gib}$ decreases along the propagation, $u'$ is clearly bounded from above by $c$. Therefore, $l_{gib}$ may be short for the distributed regime to be reached if it is not reached in the early stages, and this is a point that needs a detailed investigation.

One may nevertheless entertain the possibility of a distributed regime in the problem because it is one of the expected pathways to the detonation branch of the combustions. In these scenarios a detonation (self-propagated combustion mediated by a shock) can start, for example, by means of the Zel’dovich gradient mechanism [51]. For this to occur, a macroscopic region of the mixed fuel/ashes should be able to burn “at once” (i.e. allowing a supersonic phase velocity), which requires a very shallow temperature gradient $\nabla T$. It is not known how large the critical macroscopic region should actually be, detailed calculations show that its value for the WD carbon burning problem is $L_c \sim 10^4 cm$, and it is likely much smaller in our problem. Estimations
of the size of the distributed flames yield essentially \( l_{\text{dist}} = \alpha l_{\text{th}} K a \), where \( \alpha \) is a pure number and \( K a \) is the Karlovitz number, used in turbulence studies as a measure of the quotient of diffusive to eddy turnover time. Physically, if \( l_{\text{dist}} \) can stretch to reach the \( L_c \) value, the system would satisfy at least a necessary condition for a transition to detonation (since the deflagrations come first, this is call in the literature as Deflagration-to-Detonation Transition, or DDT, [52]). This condition can be combined with the expression \( l_{\text{gib}} = l_{\text{th}} / K a^2 \) to yield the relation

\[
\alpha^{1/3} K a \geq \left( \frac{L_c}{l_{\text{gib}}} \right)^{1/3}
\]

converted into a bound on \( \alpha \) when we observe that the distributed regime starts at \( K a > 1 \). \( L_c \) values larger than \( \sim 10 \text{ cm} \) would not allow the burning to become a detonation (DDT) by the Zel’dovich gradient mechanism. Thus, a necessary, but not sufficient, condition for the DDT can be established whenever \( L_c \leq 10 \text{ cm} \).

Another condition for DDT within the gradient mechanism is related to the hierarchy of time scales of mixing, burning and dynamical. Contrary to the WD explosion problem, we have already seen that \( \tau_{\text{dyn}} \) is always much longer than \( \tau_{\text{burn}} \) (identified with the weak interaction time scale \( \tau_W \sim 10^{-8} \text{ s} \)) to create the strangeness). Therefore we have

\[
\tau_{\text{mix}} \leq \tau_W \ll \tau_{\text{dyn}}
\]

which yields, after substituting

\[
\frac{L_c}{u'(L_c)} \leq \tau_W
\]

using for the turbulent velocity fluctuations the estimate \( u'(L) = (1/2) \sqrt{g_{\text{eff}} L} \) [53], we obtain an upper bound on \( L_c \)

\[
L_c \leq 10^{-5}(1 - \mu) \text{ cm}
\]

This is a small length over which to mix fluids, and would make the former condition on the Karlovitz number (eq. 9) irrelevant, likely leading to a DDT phenomenon immediately. Other mechanisms for DDT do exist, but is too difficult to discuss them in connection with our problem at this stage.
From all the above discussion we believe it is clear that the examination of the laminar diffusive regime is just a part of the whole very complex problem. The full evolution of the burning $n \rightarrow SQM$ can be accurately described by using the so-called reactive Euler equations

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{u}) = 0$$

(13)

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla.(\rho \mathbf{u} \mathbf{u}) + \nabla P = 0$$

(14)

$$\frac{\partial E}{\partial t} + \nabla.((E + P) \mathbf{u}) = 0$$

(15)

$$\frac{\partial X_i}{\partial t} + (\mathbf{u} \cdot \nabla)X_i = R(T, \rho, X_i)$$

(16)

with $X_i$ are the fraction of each quarks and the reaction rates $R(T, \rho, X_i)$ have to be calculated at finite temperature for the dense environment (see, i.e. [16]). Due to enormously disparate length scales, ranging from $\sim$ few fm to perhaps $\sim$ km, a model of the flame can facilitate the calculations, otherwise it is known that resolving the full structure demands a huge computational investment [54].

7 Role of $n \rightarrow SQM$ conversion in supernovae

In the original proposal [20, 13] of a fast combustion mode in supernovae, a newtonian calculation was employed to estimate the dynamical quantities, in particular the energy that could be transferred to the outer layer of a stalled shock in a massive star. We have seen that a complex but quick sequence of phenomena affects the flame, even if initially starts as a slow laminar combustion. If energy can not be directly transferred to the outer layers, SQM formation may still be important because of the production of neutrinos by appropriate reactions in the deconfined phase. The binding energy of the strange star has to be released as well [55], much in the same way as the binding energy of the neutron star in the standard picture. Although new fresh neutrinos could in principle produce a late revival of the stalled shock wave, other features than the total released energy are essential such as spectral features of the neutrino emission, and more importantly (if the
transition happens to be somewhat delayed) the exact time of its occurrence, since if it occurs too late there will be no way to explode the star by the shock reheating mechanism at all.

While it is still not clear whether the detonation mode is feasible, since it requires fast transport of heat to sustain the front and a working DDT mechanism (if it is not initiated “directly”), assuming the latter case, and since the conversion is not expected to be exothermic all the way down to zero pressure it is unavoidable that a detonation will become a standard shock wave beyond some radius (assuming the MIT Bag model for SQM this radius is the one for which $E - 3P = 4B$). This shock wave will propagate outwards and the question is whether or not it will be able to transfer its energy and complete the work unfinished by the unsuccessful prompt shock wave. In turn, a more moderate turbulent combustion (subsonic but still very fast) may be the final outcome instead of a detonation, and its propagation would mix the material on macroscopic scales due to the action of Landau-Darrieus and Rayleigh-Taylor instabilities. Its role in the reenergization of the stalled shock, possibly by neutrino transfer, has not been calculated as yet.

A better understanding of the previous sequence of combustion processes will also give information about the timescale of the conversion of the star, which is closely related to the different observational signals. These calculations also constitute an important task for the near future.

8 Delayed conversions, compact star structure and gamma-ray bursts

Up to now we have considered the hydrodynamics of the reactive flows with the assumption of its occurrence well inside the first seconds after the prompt shock bounce. If the just-born protoneutron stars do not collapse to black holes due to accretion in the early stages \cite{56}, and within the SQM hypothesis, then pure strange stars, made up entirely of strange quark matter from the center to the surface, may be the compact remnants of supernovae. But even in the case of absolute stability, if the transition is not triggered during the supernova explosion, all observed “normal” neutron stars would be in a metastable state, which is quite difficult to imagine because of ISM contamination arguments \cite{33, 57, 58} and the mismatch $\tau_{\text{conv}} \ll \tau_{\text{star}}$ between the timescale in which favorable conditions for conversion occur $\tau_{\text{conv}}$.  

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and the lifetime of the star $\tau_{\text{star}}$. According to recent calculations the deconfinement transition is more likely to occur by heating and compression during the Kelvin-Helmholtz phase of proto-neutron star (PSN) evolution (see, for instance, [59]). If it did not happen there, once the PNS has cooled to temperatures below $\sim 1\text{MeV}$, only accretion from a companion star or strangelet contamination would allow the transition (and many barriers may preclude its occurrence), even in the case where it is energetically favored. Thus, the existence of strange stars is determined not only by fundamental questions concerning the true ground state of dense matter but also by the exact physical conditions in the specific astrophysical environments together with the plausibility of the conversion mechanisms in these situations.

The SQM conversion has been repeatedly associated with gamma-ray bursts. Many works in the past have explored the idea that the conversion of NM into SQM in NSs may be an energy source for GRBs ([60, 61, 62, 63, 65, 66, 67]). These models mostly address spherically symmetric conversions of the whole NS rendering isotropic gamma emission. Accumulating observational evidence suggests that at least “long” GRBs are strongly asymmetric, jet-like outflows, a feature that needs some crucial ingredient in the SQM physics formation-propagation to proceed. The “short” burst subclass was not obviously asymmetric prior to HETE2 and SWIFT data, but now evidence has mounted for a substantial asymmetry (but not extreme) in them. The association of Type Ib/c with a few GRBs has reinforced the investigation of underlying explosion mechanisms, and the absence of a temporal break in most of the light curves (interpreted in terms of a collimated jet effect) is a puzzling feature that might be related to the total energy budget in a yet unclear resolution.

A new potentially important feature recently recognized in this class of models is that if a conversion to SQM actually begins near the center of an NS, the presence of a moderate magnetic field $B$ ($\sim 10^{13}$ G) will originate a prompt asymmetric gamma emission, which may be observed as a short, beamed GRB after the recovery of a fraction of the neutrino energy via $\nu\bar{\nu} \rightarrow e^+e^- \rightarrow \gamma\gamma$ [68]. The basic physical effect is again related to the instabilities described in the former sections: the influence of the magnetic field expected to be present in NS interiors quenches the growth of the hydrodynamic instabilities in the equatorial direction of the star (parallel to the magnetic field) while it allows them to grow in the polar one. As a result, the flame will propagate much faster in the polar direction, and this will result in a strong (transitory) asymmetry in the geometry of the just formed
core of hot SQM, which will resemble a cylinder orientated in the direction of the magnetic poles of the NS. While it lasts, this geometrical asymmetry gives rise to a bipolar emission of the thermal neutrino-antineutrino pairs produced in the process of SQM formation. This is because almost all the thermal neutrinos generated in the process of SQM formation will be emitted in a free streaming regime through the polar cap surface, and not in other directions due to the opacity of the matter surrounding the cylinder. The neutrino-antineutrino pairs annihilate into electron-positron pairs just above the polar caps of the NS, giving rise to a relativistic fireball, thus providing a suitable form of energy transport and conversion to gamma-emission that may be associated to short gamma-ray bursts. A unifying scheme in which SQM appearance produces spherical ejection phenomena to highly asymmetric gamma beaming, as a more or less continuous function of the magnetic field $B$ and the astrophysical system under examination may be possible, and is tentatively sketched in Table 1.

**Table 1.** Tentative classification of explosive events due to SQM in several stellar systems

| Mag. field (G) | Type II SN | LMXB-HMXB* | AIC(?)† |
|---------------|------------|-------------|---------|
| 0 < $B$ < $10^{12}$ | "normal" SN | spherical, weak short GRB | UV-X flash |
| $B$ $\sim$ $10^{13}$ | bipolar SN | bipolar, strong short GRB | bipolar UV-X flash |
| $B$ $\geq$ $10^{14}$ | ? | jet-like, weak short GRB | jet-like UV-X flash |
| $B$ $\gg$ $10^{15−16}$ | – | -no SQM formation- | – |

* only if $NM \rightarrow SQM$ conversion is sometimes suppressed when a NS is formed.
† upper limit to the rate $\sim 10^{-4}yr^{-1}galaxy^{-1}$ needs to be revised if SQM burning occurs modifying nucleosynthetic yields.

We are still very far from a thorough understanding of magnetic field effects, and a reliable simulation is even more challenging than simulating the $B = 0$ reactive Euler equations (13-16). However, we believe that it is fair to say that magnetic fields are relevant for the physics of the conversion, even at moderate values. In summary, we may be witnessing an ultimate subnuclear energy source in action, powering SN-GRBs if SQM exists.
9 Conclusions

We have presented a discussion of the main features of hypothetic $n \rightarrow SQM$ conversions inside neutron stars. We have shown that even if the initial state of the process could be a laminar deflagration, the hitherto ignored hydrodynamic instabilities (Landau-Darrieus and Rayleigh-Taylor) quickly take over and determine the propagation through the vast majority of the star, in a regime of turbulent deflagration [37, 68]. Models which ignore hydrodynamics altogether or just concentrate on the energy conditions to determine the combustion mode miss completely this important features. In particular, the association of long timescales (up to $10^3 - 10^4$ s) of GRBs based on the identification of a laminar deflagration as the relevant timescale in the process is not tenable [71]. Other proposed models differ in their kinetic aspects, for example, models in which energy is obtained by pairing quarks [72, 73, 74, 75] typically operate on strong interaction timescales, and thus may be thought as an isocoric burning, i.e. much faster than the described instability scenario. Still other energy transfer mechanisms are possible [76], and certainly the issue of neutrino transport from the reaction zone ahead has never been addressed in detail [77], although there is plenty of energy carried by them.

It is still possible that all these regimes could be bypassed in favor of a “prompt” detonation mode started at the very central region, for example, by the sudden conversion of a macroscopic small region, further sending a shock wave with $\sim$ half of the initial overpressure [69]. Propagation of such a combustion mode is in principle possible [20, 70], but more detailed studies have yet to be performed on this problem by coupling properly the energy transport to the structure of the flame front. Models treating the conversion much in the same way as a plain phase transition are even more remotely relevant to the actual physics.

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Figure 1: Scales in the SQM burning problem.
At a given instant the regimes dominating the burning are shown as a function of the lengthscale. Below $\sim 100l_{th}$ the laminar flame ensues. Cells appear above that scale and produce a weakly-dependent velocity (as described by fractal models, for instance). Above $l_{gib}$ cellular stabilization fails and above a transition scale the buoyancy ultimate dominates the burning $u_{RT} \propto l^{1/2}$. It should be kept in mind that the distributed regime may be directly reached, disrupting the flame that no longer follows the regimes of Fig.1
