Resurrecting the Power-law, Intermediate, and Logamediate Inflations in the DBI Scenario with Constant Sound Speed

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Received 2017 August 4; revised 2017 November 28; accepted 2017 December 19; published 2018 February 5

Abstract

We investigate the power-law, intermediate, and logamediate inflationary models in the framework of DBI non-canonical scalar field with constant sound speed. In the DBI setting, we first represent the power spectrum of both scalar density and tensor gravitational perturbations. Then, we derive different inflationary observables including the scalar spectral index \(n_s\), the running of the scalar spectral index \(\text{d}n_s/\text{d} \ln k\), and the tensor-to-scalar ratio \(r\). We show that the 95% CL constraint of the Planck 2015 \(T + E\) data on the non-Gaussianity parameter \(f_{\text{DBI}}\) leads to the sound speed bound \(c_s \geq 0.087\) in the DBI inflation. Moreover, our results imply that, although the predictions of the power-law, intermediate, and logamediate inflation in the standard canonical framework \((c_s = 1)\) are not consistent with the Planck 2015 data, in the DBI scenario with constant sound speed \(c_s < 1\), the result of the \(r - n_s\) diagram for these models can lie inside the 68% CL region favored by Planck 2015 TT,TE,EE+lowP data. We also specify the parameter space of the power-law, intermediate, and logamediate inflation for which our models are compatible with the 68% or 95% CL regions of the Planck 2015 TT,TE,EE+lowP data. Using the allowed ranges of the parameter space of the intermediate and logamediate inflationary models, we estimate the running of the scalar spectral index and find that it is compatible with the 95% CL constraint from the Planck 2015 TT,TE,EE+lowP data.

Key words: early universe – inflation

1. Introduction

Inflation is a central part of modern cosmology. In this scenario, it is assumed that a fast accelerated expansion has taken place in a short period of time before the radiation-dominated era (Guth 1981; Albrecht & Steinhardt 1982; Linde 1982, 1983). Consideration of this accelerated expansion can successfully resolve the well-known problems of classical cosmology (Guth 1981; Albrecht & Steinhardt 1982; Linde 1982, 1983). Besides, the perturbations created during inflation can be regarded as the origin of the large-scale structure formation as well as the anisotropies of the cosmic microwave background (CMB) radiation (Mukhanov & Chibisov 1981; Hawking 1982; Guth & Pi 1982; Starobinsky 1982). Indeed, the general prediction of the inflationary scenario that the primordial perturbations should be adiabatic and nearly scale-invariant is well in agreement with the recent observational data provided by the Planck satellite (Ade et al. 2016a, 2016b).

Despite its achievements, the inflationary paradigm suffers from fundamental ambiguities. In particular, so far we do not know what the nature of the scalar field driving inflation (inflaton) is, what shape the inflaton potential possesses, at which energy scale the inflationary era of our universe occurred, and what gravitational theory is valid at that era (Baumann 2009). In addition, the conventional models of inflation suffer from the eta problem that challenges the basic principles of them (Copeland et al. 1994). There exist no convincing solutions to these problems in the context of the standard inflationary framework based on a single canonical scalar field in the Einstein gravity (Baumann 2009; Martin et al. 2014). However, so far remarkable attempts have been made to solve these drawbacks in the alternative models to the standard canonical inflation. One important class of these alternative models is the Dirac–Born–Infeld (DBI) inflation, which has robust motivations from string theory (Alishahiha et al. 2004; Silverstein & Tong 2004). In the DBI inflation, the role of the inflaton is played by the radial coordinate of a D3-brane moving in a warped region (throat) of a compactification space (Alishahiha et al. 2004; Silverstein & Tong 2004). From the physical standpoint, this may be interpreted as the impact of the higher dimensions on our four-dimensional spacetime, which leads to the appearance of the DBI Lagrangian in the effective action through the dimensional reduction. Hence, the DBI framework proposes a convincing candidate for the inflaton field. Furthermore, by investigating the dynamics of the inflaton on the throat of the compactification space, some suggestions have been presented for the inflaton potential that make it possible for us to circumvent the eta problem (Kachru et al. 2003; Firouzjahi & Tye 2005; Shandera & Tye 2006; Baumann et al. 2007, 2008, 2009; Krause & Pajer 2008; Easson & Gregory 2009).

From the theoretical point of view, the DBI inflation can be included in the class of the non-canonical inflationary models in which the kinetic term in the action is different from the canonical one (Armendariz-Picón et al. 1999; Garriga & Mukhanov 1999). The outstanding feature of the non-canonical inflationary models is the fact that in these models, the propagation speed of the density perturbations, the so-called “sound speed,” can be different from the light speed. Due to this property, the value of the tensor-to-scalar ratio is generally reduced in this class of models relative to the standard canonical inflationary setting (Unnikrishnan et al. 2012), which supplies a better consistency with the Planck 2015 observational data (Ade et al. 2016a). In addition, because of this property, the non-canonical inflationary models are capable of providing rather large values for the primordial
non-Gaussianity (Alishahiha et al. 2004; Chen et al. 2007; Li et al. 2008; Tolley & Wyman 2010), and this feature may be used to discriminate between the canonical and non-canonical inflationary models by increasingly precise measurements in the future.

Inflation with the DBI scalar field has been extensively studied in the literature (Kachru et al. 2003; Alishahiha et al. 2004; Silverstein & Tong 2004; Chen 2005a, 2005b, 2005c; Firouzjahi & Tye 2005; Shandera & Tye 2006; Baumann et al. 2007, 2008, 2009; Chen et al. 2007; Peiris et al. 2007; Spalinski 2007, 2008; Gauthier & Akhoury 2008; Krause & Pajer 2008; Li et al. 2008; Bessada et al. 2009; Easson & Gregory 2009; Tzirakis & Kinney 2009; Tolley & Wyman 2010; Cai et al. 2011; van de Bruck et al. 2011; Miranda et al. 2012; Tsujikawa et al. 2013; Nazavari et al. 2016; S. Rasouli et al. 2017, in preparation). The basic concepts of this model have been introduced in Silverstein & Tong (2004), where the authors have analyzed the motion of a rolling scalar field expressly in the strong coupling regime of the field theory and have developed their study to cosmological contexts based on the minimal coupling of this kind of field theory to four-dimensional gravity. In Alishahiha et al. (2004), the authors have studied the perturbations in the DBI inflation to the nonlinear order and have shown that the primordial non-Gaussianity parameter has a direct relation with the Lorentz factor (or, equivalently, with the sound speed) of the model, and consequently this model is capable of providing large values for the primordial non-Gaussianity. Further studies done later verified these conclusions (Chen et al. 2007; Li et al. 2008; Tolley & Wyman 2010). In Kachru et al. (2003), Firouzjahi & Tye (2005), Shandera & Tye (2006), Baumann et al. (2007, 2008, 2009), Krause & Pajer (2008), and Easson & Gregory (2009), some attempts have been made to reconstruct the inflationary potentials in the DBI setting to ameliorate the eta problem in describing the dynamics of the early universe. The Hamilton–Jacobi formalism in the context of DBI scalar field theories was introduced in Silverstein & Tong (2004), and then was discussed with more details in Spalinski (2007). Different DBI inflationary models have been investigated in Gauthier & Akhoury (2008), Peiris et al. (2007), Bessada et al. (2009), Tzirakis & Kinney (2009), Miranda et al. (2012), Tsujikawa et al. (2013), and Nazavari et al. (2016), and validity of their predictions has been appraised with the CMB data. In van de Bruck et al. (2011), the DBI inflation has developed in the context of a scalar-tensor theory of gravity. Finally, in Cai et al. (2011) and S. Rasouli et al. (2017, in preparation), the DBI inflation has been studied in the warm inflationary scenario in which during inflation the inflaton energy density is transformed to the radiation energy density constantly, so that the universe can transit to the radiation-dominated era without resorting to any additional reheating processes.

In the present work, we study the power-law (Lucchin & Matarrese 1985; Halliwell 1987; Yokoyama & Maeda 1988; Liddle 1989), intermediate (Barrow 1990; Barrow & Liddle 1993; Barrow et al. 2006; Barrow & Nunes 2007), and logamediate (Barrow & Nunes 2007) scale factors in the DBI inflation with constant sound speed. These scale factors have special importance in cosmological contexts because they appear as a class of possible indefinite solutions from imposing weak general conditions on cosmological models. In Barrow (1996), it has been discussed that we can find eight conceivable asymptotic cosmological solutions, of which three give rise to non-inflationary expansions. Three others result in the power-law \((a(t) \propto t^q\) where \(q > 1\), de Sitter \((a(t) \propto \exp(\mathcal{H}t)\) where \(\mathcal{H}\) is constant), and intermediate \((a(t) \propto \exp(At^\lambda)\) where \(A > 0\) and \(0 < \lambda < 1\) inflationary expansions. The remaining two inflationary expansions have asymptotic behaviors in logarithmic form. Although these scale factors have remarkable cosmological significance, they are not consistent with the Planck 2015 observational results (Ade et al. 2016a) in the standard canonical inflationary scenario \((\epsilon = 1)\), as has been explicitly shown in Rezaazadeh et al. (2015, 2016, 2017a). In the present paper, we aim to refine the power-law, intermediate, and logamediate inflationary models in light of the current observational results by considering them in the DBI scalar field setting. Motivated by Spalinski (2008) and Tsujikawa et al. (2013), we assume in our work that the sound speed is constant during inflation. We examine the viability of each scale factor in light of the Planck 2015 results (Ade et al. 2016a) and also present some observational constraints on them.

The outline of this paper is as follows. In Section 2, we review briefly the basic equations governing the background cosmology in the DBI inflation. In addition, we also represent the power spectra of the primordial scalar and tensor perturbations. Then, in Sections 3–5, we respectively investigate the power-law, intermediate, and logamediate DBI inflationary models with constant sound speed. Finally, in Section 6, we summarize our conclusions.

2. DBI Inflation

Within the framework of Einstein’s gravity, the action of non-canonical scalar field models is given by (Armenazar-Picon et al. 1999; Garriga & Mukhanov 1999)

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R + \mathcal{L}(X, \phi) \right],
\]

where \(g\) and \(R\) are the determinant of the background metric and the Ricci scalar, respectively. Throughout this paper, we work in units where the Planck reduced mass is taken equal to unity, \(M_p = (8\pi G)^{-1/2} = 1\), for the sake of simplicity. In the present work, we concentrate on the DBI non-canonical scalar field with the Lagrangian density (Alishahiha et al. 2004; Silverstein & Tong 2004)

\[
\mathcal{L}(X, \phi) \equiv f^{-1}(\phi) \left[ 1 - \sqrt{1 - 2f(\phi)}X \right] - V(\phi),
\]

where \(f(\phi)\) is the warp factor, while \(X \equiv -g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}/2\), and \(V(\phi)\) are the canonical kinetic term and potential of the DBI scalar field \(\phi\), respectively.

For the DBI Lagrangian (2), the energy density and pressure of the non-canonical scalar field \(\phi\) are given by

\[
\rho_\phi \equiv 2X \mathcal{L}_X - \mathcal{L} = \frac{\gamma - 1}{f(\phi)} + V(\phi),
\]

\[
p_\phi \equiv \mathcal{L} = \frac{\gamma - 1}{\gamma f(\phi)} - V(\phi),
\]

where the subscript, “\(X\)” indicates the partial derivative with respect to \(X\). Moreover,

\[
\gamma \equiv \frac{1}{\sqrt{1 - 2f(\phi)}X}
\]
is the parameter determining the relativistic limit of brane motion in a warped background. Note that the parameter $\gamma$ is defined in analogy of the Lorentz factor appearing in relativistic particle dynamics.

The sound speed is generally defined by

$$c_s^2 \equiv \frac{\rho_{\phi,X}}{\rho_{\phi}},$$

(6)

which specifies the propagation speed of the inflaton fluctuations $\delta \phi$ relative to the homogeneous background. In order for the sound speed to be physical, it should be real and subluminal, $0 < c_s^2 \leq 1$ (Franche et al. 2010). Applying Equations (3)–(5), the sound speed in DBI inflation takes the form

$$c_s = \sqrt{1 - 2f(\phi)X} \frac{1}{\gamma}.$$  

(7)

For the homogeneous and isotropic background spacetime, we consider the spatially flat Friedmann–Robertson–Walker (FRW) metric. Therefore, varying the action (1) with respect to the metric yields the Friedmann equations in the DBI inflation as follows:

$$H^2 = \frac{1}{3\Omega_\phi}$$

(8)

$$\dot{H} = -\frac{1}{2}(\rho_{\phi} + p_{\phi}).$$

(9)

Here, $H = \dot{a}/a$ is the Hubble parameter, and $a$ is the scale factor $a$ of the universe. The dot denotes the derivative with respect to the cosmic time $t$. Considering the FRW metric, the canonical kinetic term is simplified to $X = \dot{\phi}^2/2$.

Taking the variation of action (1) with respect to $\phi$ gives the equation of motion of the scalar field as

$$\ddot{\phi} + \frac{3H}{\gamma^2} \dot{\phi} + \frac{V'(\phi)}{\gamma^3} + \frac{f'(\phi)(\gamma + 2)(\gamma - 1)}{2f(\phi)\gamma(\gamma + 1)}\dot{\phi}^2 = 0,$$

(10)

where the prime denotes the derivative with respect to $\phi$. For the case of $\gamma = 1$, this equation recovers the well-known equation of motion in the standard model of inflation. Note that Equation (10) can also be obtained from the continuity equation

$$\dot{\rho}_\phi + 3H(\rho_{\phi} + p_{\phi}) = 0,$$

(11)

where $\rho_{\phi}$ and $p_{\phi}$ are given by Equations (3) and (4), respectively.

Substituting Equations (3) and (4) into (9), and using Equation (5), one can easily obtain

$$\dot{\phi} = -\frac{2}{\gamma}H'(\phi),$$

(12)

which is the same as that obtained for the DBI inflationary model in the Hamilton–Jacobi formalism (Spalinski 2007).

In the study of inflation, it is convenient to express the extent of the universe expansion in terms of the $e$-fold number

$$N \equiv \ln \frac{a_e}{a}.$$  

(13)

The above definition is equivalent to

$$dN = -Hdt.$$  

(14)

In the DBI inflationary framework, it is useful to define the Hubble slow-roll parameters as follows:

$$\varepsilon_1 \equiv -\frac{\dot{H}}{H^2},$$

(15)

$$\varepsilon_{i+1} \equiv \frac{\varepsilon_i}{H\dot{\varepsilon}_i},\quad (i \geq 1).$$

(16)

Furthermore, it is convenient to define sound slow-roll parameters in the following forms:

$$\xi_{s1} \equiv \frac{\dot{c}_s}{Hc_s},$$

(17)

$$\xi_{s(i+1)} \equiv \frac{\xi_{si}}{Hc_{si}},\quad (i \geq 1).$$

(18)

During inflation, the Hubble and sound slow-roll parameters are usually much less than unity, and considering these conditions is known as the slow-roll approximation.

It is straightforward to show that using Equations (3), (7), (12), and (15), the Friedmann Equation (8) can be written as

$$\frac{V(\phi)}{3H^2} + \frac{2}{3} \frac{\varepsilon_1}{1 + c_s} = 1.$$  

(19)

Since in the slow-roll regime, $\varepsilon_1 \ll 1$, we therefore can ignore the second term on the left-hand side of the above equation versus the first one. Consequently, in the DBI inflation, the first Friedmann equation in the slow-roll approximation takes the form (Chen et al. 2007; Miranda et al. 2012)

$$H^2 \simeq \frac{1}{3}\frac{V(\phi)},$$

(20)

which is same as that obtained in the standard canonical inflation. In addition, in the slow-roll approximation, the first term on the left-hand side of Equation (10) can be neglected in front of the next terms, and therefore using Equations (5) and (7), we find (Miranda et al. 2012)

$$3H\ddot{\phi} + c_s V' \simeq 0.$$  

(21)

In this paper, our main goal is to compare the results of some DBI inflationary models with the Planck 2015 CMB data (Ade et al. 2016a). Therefore, it is useful to mention how the Planck Collaboration (Ade et al. 2016a) applies the CMB anisotropy data to constrain the inflationary observables. The Planck Collaboration (Ade et al. 2016a) connects the CMB angular power spectra to the scalar and tensor primordial power spectra via the transfer functions $\Delta_{A,B}(k)$ and $\Delta_{A,B}(k)$ as follows:

$$C_l^{A,B,T} = \int_0^\infty \frac{dk}{k} \Delta_{A,B}(k)\Delta_{A,B}(k)P_T(k),$$

(22)

$$C_l^{A,B,E,B} = \int_0^\infty \frac{dk}{k} \Delta_{A,B}(k)\Delta_{A,B}(k)P_T(k),$$

(23)

where $A, B = T, E, B$ refer to temperature mode $T$ and polarization modes $E$ and $B$. The transfer functions $\Delta_{A,B}(k)$ and $\Delta_{A,B}(k)$ specify the evolution of the perturbations after their re-entry into the Hubble horizon. These functions generally have to be computed numerically using the Boltzmann codes such as CMBFAST (Seljak & Zaldarriaga 1996) or CAMB.
(Lewis et al. 2000), which depend on the parameters of the background cosmological model. The Planck Collaboration (Ade et al. 2016a) assumes that the background dynamics of the universe after inflation is followed by the standard cosmological model ΛCDM. In addition, the Planck team (Ade et al. 2016a) parameterizes phenomenologically the primordial power spectra of the scalar and tensor perturbations, respectively, as follows:

$$\mathcal{P}_s(k) = \mathcal{P}(k_s)$$

$$\times \left( \frac{k}{k_s} \right)^{n_s - 1 + \frac{1}{2} d n_s / d \ln k}$$

$$\times \left( \frac{k}{k_s} \right)^{n_s - 1 + \frac{1}{2} d n_s / d \ln k}$$

where $n_s (n_s)$, $d n_s / d \ln k$ ($d n_s / d \ln k$), and $d^2 n_s / d (\ln k)^2$ are the scalar (tensor) spectral index, the running of the scalar (tensor) spectral index, and the running of the running of the scalar spectral index, respectively. The ratio of the tensor power spectrum to the scalar one is denoted by $r$, and that is a crucial inflationary observable to discriminate between inflationary models. All of these observables are evaluated at the pivot scale denoted by $k_s$. Note that definitions (24) and (25) are completely independent of the inflationary model, and consequently the Planck 2015 results (Ade et al. 2016a) are applicable for each model in which the subsequent evolution of the universe after inflation is described by the ΛCDM model. Throughout this paper, we focus on the Planck 2015 data set fitted on the base of the model ΛCDM + r + $d n_s / d \ln k$. In this model, it is supposed that a quasi-de Sitter expansion has happened during inflation. Note that a quasi-de Sitter expansion generally leads to almost scale-invariant power spectra for the primordial perturbations, which is in good agreement with the recent observational data. In our work, we also assume that the perturbations generated during the slow-roll inflation are adiabatic, so that we can neglect the entropy generation during this period. We further assume after inflation, the dynamics of the background cosmology is followed by the ΛCDM model. In the model ΛCDM + r + $d n_s / d \ln k$, only the amplitude of the tensor perturbations $\mathcal{P}_t(k_s)$ is taken into account, and the scale dependence of these perturbations, which is expressed by $n_s$, is supposed to be negligible. But regarding the scalar perturbations, not only their scale dependence, which is measured by $n_s$, is included, but also the scale dependence of the quantity $n_s$, which is given by $d n_s / d \ln k$, is taken into account. Here, we use all of assumptions taken by the Planck Collaboration (Ade et al. 2016a) and apply their observational constraints on inflation to examine the viability of our DBI inflationary model.

Now, we turn to obtaining the equations of the inflationary observables in the DBI inflationary scenario. For this purpose, we first note that the power spectrum of the scalar perturbations in this model is given by (Garriga & Mukhanov 1999)

$$\mathcal{P}_s = \frac{H^2}{8 \pi^2 c_s \varepsilon_1} \bigg|_{c_s k = aH}.$$  

This expression should be evaluated at the time of sound horizon exit at which $c_s k = aH$. The recent value reported by the Planck Collaboration for the scalar perturbation amplitude at the horizon crossing of the pivot scale $k_s = 0.05$ Mpc$^{-1}$ is $\ln[10^{10} \mathcal{P}(k_s)] = 3.094 \pm 0.034$ (Planck 2015 TT,TE,EE + lowP data; Ade et al. 2016a). In order to specify the scale dependence of the scalar perturbations, it is conventional to define the scalar spectral index as

$$n_s - 1 \equiv \frac{d \ln \mathcal{P}_s}{d \ln k}.$$  

The reported value by the Planck 2015 collaboration for the scalar spectral index is $n_s = 0.9644 \pm 0.0049$ (68% CL, Planck 2015 TT,TE,EE+lowP; Ade et al. 2016a). In the slow-roll regime, the Hubble parameter $H$ and the sound speed $c_s$ vary much more slowly than the scale factor $a$ of the universe (Garriga & Mukhanov 1999), and hence the relation $c_s k = aH$ leads to

$$d \ln k \approx H dt = -dN.$$  

Applying Equations (15)–(17) and (26)–(28), we can obtain a relation for the scalar spectral index in the non-canonical inflationary framework as

$$n_s = 1 - 2 \varepsilon_1 - \varepsilon_2 - \varepsilon_3.$$  

Furthermore, with the help of Equations (15)–(18), and (29), we can obtain the running of the scalar spectral index as

$$\frac{d n_s}{d \ln k} = -2 \varepsilon_1 \varepsilon_2 - \varepsilon_2 \varepsilon_3 - \varepsilon_3 \varepsilon_1.$$  

The CMB constraint on this observable is $d n_s / d \ln k = -0.0085 \pm 0.0076$ (68% CL, Planck 2015 TT, TE, EE+lowP; Ade et al. 2016a).

Now, we concentrate on the tensor perturbations in the setting of DBI non-canonical inflation. The tensor power spectrum of this scenario is given by (Garriga & Mukhanov 1999)

$$\mathcal{P}_t = \frac{2H^2}{\pi^2} \bigg|_{c_t k = aH}.$$  

It should be noted that, contrary to the scalar perturbations that freeze when they leave the sound horizon ($c_t k = aH$), the tensor perturbations freeze when they cross the Hubble horizon ($k = aH$) outward of it. However, the difference between the sound horizon exit and the Hubble horizon exit is negligible to the lowest order in the slow-roll parameters (Garriga & Mukhanov 1999; Unnikrishnan et al. 2012). One should note that the tensor power spectrum (31) in the non-canonical inflationary setting is the same as that obtained in the standard model of canonical inflation because the tensor modes (or gravitational waves) depend on the gravitational part of the action (1) that is identical in both of these models that are based on Einstein’s gravity.

The scale dependence of the tensor perturbations is measured by the tensor spectral index defined as

$$n_t \equiv \frac{d \ln \mathcal{P}_t}{d \ln k}.$$  

No precise measurement for the tensor spectral index $n_t$ currently exists, so we have to wait for future observations (Simard et al. 2015). Using Equations (15), (28), and (31), the
tensor spectral index (32) reads

\[ n_t = -2\epsilon_t. \] (33)

As mentioned above, the tensor-to-scalar ratio is defined as

\[ r \equiv \frac{P_T}{P_s}. \] (34)

The Planck 2015 data have provided the upper limit on tensor-to-scalar ratio as \( r < 0.149 \) (95% CL, Planck 2015 TT,TE,EE+lowP; Ade et al. 2016a), which is a central criterion for discriminating between inflationary models. For the DBI inflation, using Equations (26) and (31), it is straightforward to show that the tensor-to-scalar ratio (34) takes the form

\[ r = 16c_s\epsilon_t, \] (35)

where we have neglected the difference between the sound horizon exit and the Hubble horizon exit in the slow-roll approximation (Garriga & Mukhanov 1999; Unnikrishnan et al. 2012).

Combining Equations (33) and (35), it is easy to get

\[ r = -8c_s n_t, \] (36)

which is known as the consistency relation of the DBI inflation. This differs from the standard consistency relation \( r = -8n_t \) in the canonical inflation. However, as was explained previously, the Planck 2015 constraints on inflation that we will consider in the present paper are applicable for all slow-roll inflationary models, independent of their consistency relations. Of course, if one considers the Planck bounds on \( r \) and applies the consistency relation of the model, one can find some constraints on \( n_t \) that are obviously model-dependent.

One other important observable is the primordial non-Gaussianity that can be generated during inflation. It can be used as a powerful criterion to discriminate between different inflationary scenarios (Babich et al. 2004). There are different types of non-Gaussianity parameters containing the equilateral, squeezed, and orthogonal shapes that arise respectively in models with higher-derivative interactions resulting in non-trivial speeds of sound (Alishahiha et al. 2004; Chen et al. 2007; Li et al. 2008; Tolley & Wyman 2010), multi-fields inflation (Bartolo et al. 2002; Sasaki 2008), and models with non-standard initial states (Chen et al. 2007; Holman & Tolley 2008). In our case, we deal with a single field inflation, and so the signal of primordial non-Gaussianity is peaked at the equilateral triangle configuration.

For the DBI models of inflation, the Planck Collaboration (Ade et al. 2016b), following Alishahiha et al. (2004), considers phenomenologically the bispectrum of the primordial perturbations as

\[
B_{\Phi}^{\text{DBI}}(k_1, k_2, k_3) = \frac{6A_{NL}^2}{(k_1k_2k_3)^3} \frac{(-3/7)}{(k_1 + k_2 + k_3)^2} \times \left\{ \sum_i k_i^5 + \sum_{i<j} [2k_i^2k_j - 3k_i^3k_j^2] + \sum_{i \neq j} [k_i^3k_jk_l - 4k_i^2k_j^2k_l] \right\},
\] (37)

which is close to the equilateral shape. Here, the subscript \( \Phi \) refers to the Bardeen gauge-invariant gravitational potential (Bardeen 1980), which its power spectrum \( P_{\Phi}(k) = A/k^{4-n_t} \) is normalized to \( A^2 \). There also exist sub-leading order terms in the above expression that lead to extra non-separable shapes, but these are expected to be much smaller without special fine-tuning. The DBI models of inflation predict the non-linearity parameter as (Alishahiha et al. 2004; Chen et al. 2007)

\[
f_{\text{NL}}^{\text{DBI}} = -\frac{35}{108} \left( \frac{1}{c_s^2} - 1 \right). \] (38)

The Planck Collaboration (Ade et al. 2016b) has constrained the non-separable shape given by Equation (37) and found \( f_{\text{NL}}^{\text{DBI}} = 2.6 \pm 61.6 \) from temperature data and also \( f_{\text{NL}}^{\text{DBI}} = 15.6 \pm 37.3 \) from both temperature and polarization data at 68% CL. Apart from this, they have applied their 95% CL constraints on Equation (38) and reported the following lower bounds on the sound speed of the DBI inflation as

\[ c_s \geq 0.069 \quad (95\% \text{ CL}, \, T \, \text{ only}), \] (39)

\[ c_s \geq 0.087 \quad (95\% \text{ CL}, \, T + E), \] (40)

where we will consider the lower bound (40) in the next sections. In what follows, we investigate the power-law, intermediate, and logaminate inflations driven by the DBI scalar field with constant speed of sound and examine the viability of these models in light of the Planck 2015 observational results.

### 3. Power-law DBI Inflation

The first case that we study is the power-law scale factor

\[ a(t) = a_0 t^q, \] (41)

where \( q > 1 \) is a constant parameter. This scale factor has already been investigated in the framework of DBI inflation with constant sound speed in Tsujikawa et al. (2013), where the authors have derived the warp factor and inflationary potential corresponding to this model. They also have presented the bound on the sound speed of the model deduced from the Planck 2013 constraints on the primordial non-Gaussianity. Here, we proceed to examine this model in greater detail, and in particular we are interested in displaying the explicit prediction of this model in the \( r - n_t \) plane, which is a crucial test to check the admissibility of each inflationary model in light of the observational data. Also, we specify the parameter space of the power-law DBI inflation for which our model is compatible with the 68% or 95% CL regions of the Planck 2015 TT,TE,EE+lowP data.

Considering scale factor (41) and using Equation (12) for the constant \( \gamma \), one can find

\[
\dot{\Phi} = \sqrt{2c_s q} \frac{\Phi}{t}. \] (42)

The above differential equation can be easily solved to give

\[
\Phi = \sqrt{2c_s q} \ln t, \] (43)

where the integration constant has been put to zero without loss of generality. With the help of Equations (41) and (43), it is a trivial task to show that the Hubble parameter \( H \) is expressed in
terms of the DBI scalar field $\phi$ as

$$H = q \exp \left( -\frac{\phi}{\sqrt{2c_fq}} \right)$$  \hspace{1cm} (44)$$

For the constant $c_f$, using Equations (7), (42), and (43) and $X = \phi^2/2$, the warp factor acquires the form

$$f(\phi) = \frac{(1 - c_f^2)}{2c_fq} e^{\sqrt{\frac{2}{c_f}} \phi}.$$  \hspace{1cm} (45)$$

In the slow-roll approximation, we can use Equation (44) in the Friedmann Equation (20), and find the inflationary potential as

$$V(\phi) = 3q^2 e^{-\sqrt{\frac{2}{c_f}} \phi}.$$  \hspace{1cm} (46)$$

This results reflects the fact that in our DBI framework with constant sound speed, the power-law inflation arises from an exponential potential, and this is in agreement with the result found in Tsujikawa et al. (2013). Also, in the case of $c_s = 1$, the above equation indicates that the power-law inflation is driven by the exponential potential $V(\phi) \propto e^{-\sqrt{\frac{2}{c_f}} \phi}$, which is the well-known result that we expect in the standard canonical inflationary setting (Lucchin & Matarrese 1985; Halliwell 1987; Yokoyama & Maeda 1988; Liddle 1989).

Considering the power-law scale factor (41), the first Hubble slow-roll parameter (15) becomes

$$\varepsilon_1 = \frac{1}{q},$$  \hspace{1cm} (47)$$

whereas the other Hubble slow-roll parameters vanish. From the above equation it is evident that $\varepsilon_1$ is constant during the inflationary era, and hence it cannot reach unity at the end of inflation, which means that in this model, inflation cannot terminate by slow-roll violation (i.e., $\varepsilon_1 = 1$; Martin et al. 2014; Zhang & Zhu 2014; Rezazadeh et al. 2015, 2016, 2017a, 2017b).

Substituting Equation (47) into (26), the scalar power spectrum is obtained as

$$P_s = \frac{q^3}{8\pi^2c_f^2t^2}.$$  \hspace{1cm} (48)$$

For the power-law inflation (41), solving the differential Equation (14) gives $t$ in terms of the e-fold number $N$ as

$$t = C e^{-\frac{N}{2}},$$  \hspace{1cm} (49)$$

where $C$ is the constant of integration that can be determined by applying the condition $N_e = 0$ at the end time of inflation $t = t_e$, which is a direct consequence of definition (13) for the number of e-foldings. Thus, we set $C = t_e$ and consequently

$$t = t_e e^{-\frac{N}{2}}.$$  \hspace{1cm} (50)$$

By inserting Equation (50) into (48), the power spectrum of scalar density perturbations is given in terms of the e-fold number $N$ as

$$P_s = \frac{q^3}{8\pi^2c_f^2t_e^2} e^{2N}.$$  \hspace{1cm} (51)$$

Fixing the amplitude of the scalar power spectrum in the above equation at the horizon crossing, one can determine the end time of inflation $t_e$ in terms of the other parameters of the model. Using Equation (47) in (29) and keeping in mind that $\varepsilon_2 = \varepsilon_{21} = 0$, the scalar spectral index is obtained as

$$n_s = 1 - \frac{2}{q}.$$  \hspace{1cm} (52)$$

As we show, the result (52) does not depend on the e-fold number $N$, and hence it leads to a vanishing running of the scalar spectral index,

$$\frac{dn_s}{d\ln k} = 0,$$  \hspace{1cm} (53)$$

satisfying the 95% CL constraint deduced from Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a).

Using Equations (35) and (47), the tensor-to-scalar ratio will be

$$r = \frac{16c_s^2}{q}.$$  \hspace{1cm} (54)$$

We see that $r$ like $n_s$ is independent of $N$, and therefore we can combine Equations (52) and (54) to reach

$$r = 8c_s(1 - n_s),$$  \hspace{1cm} (55)$$

implying a linear relation between $r$ and $n_s$. The existence of a linear relation between these two observables for the power-law inflation also appears in other inflationary scenarios, such as the standard canonical inflation (Tsujikawa et al. 2013), the tachyon inflation (Rezazadeh et al. 2017b), and the Brans-Dicke inflation (Tahmasebzadeh et al. 2016). Note that we can only combine the equations for $n_s$ and $r$ to eliminate the constant parameters, and this is not allowed for the dynamical variables like the cosmic time $t$, the scalar field $\phi$, or the e-fold number $N$ (Rezazadeh et al. 2017b). Otherwise, one may incorrectly avoid the essential requirement that the inflationary observables must be evaluated at the time of horizon exit (Rezazadeh et al. 2017b).

Now, we use Equations (52) and (54) to plot the $r - n_s$ diagram of our DBI power-law inflation model as demonstrated in Figure 1. In this figure, the prediction of the model is shown by the black lines for different values of sound speed $c_s$ while the parameter $q$ is taken to be varying in the range $q > 1$. It should be noted that the selected values for $c_s$ satisfy the sound speed bound $c_s \geq 0.087$ obtained in the previous section due to the 95% CL constraint of the Planck 2015 data on the non-Gaussianity parameter $f_{NL}$. Also, in this figure, the marginalized joint 68% and 95% CL regions of the Planck data (Ade et al. 2016a) have been specified. From the figure, it is clear that for the case $c_s = 1$ corresponding to the power-law inflation in the standard canonical scalar field framework, the result of model is completely ruled out by the Planck 2015 TT, TE,EE+lowP data (Ade et al. 2016a). But taking $0.083 \leq c_s \leq 0.29$ and $0.29 \leq c_s \leq 0.63$, the predictions of power-law inflation in DBI non-canonical scalar field model are compatible with the joint 68% and 95% CL regions of the Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a), respectively.

In Table 1, we further have determined the intervals of $q$ that our model with some specified values of $c_s$ can be in agreement with Planck 2015 TT,TE,EE+lowP data. In order to specify the allowed ranges of the parameter $q$ in Table 1, we locate the prediction of the model in the $r - n_s$ plane for different values of $q$ and find the values of $q$ for which the result of the model
lies on the boundaries of the 68% or 95% CL regions of the CMB data.

It is useful to determine the parameter space of $q$ and $c_s$ for consistency of our model with the Planck 2015 constraints. This parameter space is represented in Figure 2, where the darker and lighter blue areas point to the regions for which the $-r_{ns}$ prediction of model verifies, respectively, the 68% and 95% CL constraints of the Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a) to draw Figure 2, we have provided a computational code to calculate $n_s$ and $r$ by using of Equations (52) and (54) for given values of $q$ and $c_s$, and then we check whether or not the prediction of the model can lie inside the 68% or 95% CL regions of the observational data.

In order to compare the results of our model with the observational data more quantitatively, we evaluate the inflationary observables, including $n_s$, $r$, and $f_{NL}^\text{DBI}$, with the help of Equations (52), (54), and (38), respectively. These observables are estimated for different values of $c_s$ and $q$, and the results are presented in Table 2. Using the obtained values for $n_s$ and $r$, we check the consistency of the model in the $r-n_s$ plane in comparison with Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a) and summarize our results in the last column of Table 2. It should be noted that the quantitative results of Table 2 verify the qualitative findings of Figure 2. The values of $f_{NL}^\text{DBI}$ presented in the fifth column of Table 2 are compatible with the lower bound $c_s \geq 0.087$ imposed by the

### Table 1

| $c_s$ | $q$ (68% CL) | $q$ (95% CL) |
|-------|--------------|--------------|
| 0.1   | [47, 83]     | [41, 111]    |
| 0.2   | [53, 78]     | [44, 109]    |
| 0.3   | ...          | [48, 104]    |
| 0.4   | ...          | [52, 101]    |
| 0.5   | ...          | [58, 97]     |
| 0.6   | ...          | [67, 83]     |
| $c_s \geq 0.63$ | ... | ... |

### Table 2

| $c_s$ | $q$ | $n_s$ | $r$ | $f_{NL}^\text{DBI}$ | $r-n_s$ Consistency |
|-------|-----|-------|-----|---------------------|---------------------|
| 0.1   | 65  | 0.9692| 0.0246| -32.083            | 68% CL              |
| 0.2   | 65.5| 0.9695| 0.0489| -7.778             | 68% CL              |
| 0.3   | 76  | 0.9737| 0.0632| -3.277             | 95% CL              |
| 0.4   | 76.5| 0.9739| 0.0837| -1.701             | 95% CL              |
| 0.5   | 77.5| 0.9742| 0.1032| -0.972             | 95% CL              |
| 0.6   | 75  | 0.9733| 0.1280| -0.576             | 95% CL              |
| 0.7   | 80  | 0.9750| 0.1400| -0.337             | ...                 |
4. Intermediate DBI Inflation

The next model that we investigate in the DBI scenario is the intermediate scale factor (Barrow 1990; Barrow & Liddle 1993; Barrow et al. 2006; Barrow & Nunes 2007)

\[ a(t) = a_0 \exp(\lambda t), \]  

where \( a_0 > 0, A > 0, \) and \( 0 < \lambda < 1 \) are constant parameters. Note that the intermediate inflation driven by a DBI scalar field has already been investigated in Nazavari et al. (2016), but here we follow a quite different approach. In Nazavari et al. (2016), the authors have chosen the AdS warp factor \( f(\phi) = \lambda/\phi^4 \) in their investigation, while our work is based on the assumption of constant sound speed, which enables us to determine the warp factor as well as the inflationary potential associated with the intermediate scale factor \( (56) \).

With the intermediate scale factor \( (56) \), the Hubble parameter becomes

\[ H = \frac{A\lambda}{t^{1-\lambda}}. \]  

(57)

For the constant sound speed, inserting Equation \( (57) \) into \( (12) \) yields

\[ \dot{\phi} = \sqrt{\frac{2c_sA\lambda(1-\lambda)}{t^{2-\lambda}}} \phi. \]  

(58)

Integrating the above equation gives

\[ \phi = 2\sqrt{\frac{2c_sA(1-\lambda)}{\lambda}} t^{\sqrt{\frac{2}{\lambda}}}. \]  

(59)

By replacing \( t \) from Equation \( (59) \) into \( (57) \), we reach

\[ H = \lambda^2 \left( \frac{A}{\lambda} \right)^{\frac{1}{\lambda}} [8c_s(1-\lambda)]^{\frac{1-\lambda}{2\lambda}} \phi^{\frac{2(1-\lambda)}{\lambda}}. \]  

(60)

With the help of Equations \( (7) \), \( (58) \), and \( (59) \), and also using \( X = \phi^{\sqrt{\frac{2}{\lambda}}} \), the background warp factor can be found as

\[ f(\phi) = 4^{\frac{1-\lambda}{\lambda}}(1-c_s^2) \left[ \frac{\lambda-1}{c_sA(1-\lambda)} \right]^\frac{1}{\lambda} \phi^{\frac{2(1-\lambda)}{\lambda}}. \]  

(61)

In addition, inserting Equation \( (60) \) into the first Friedmann Equation \( (20) \), the brane potential for this model in the slow-roll approximation reads

\[ V(\phi) = 3\lambda^2 \left( \frac{A}{\lambda} \right) [8c_s(1-\lambda)]^2 \phi^{\frac{4(1-\lambda)}{\lambda}}. \]  

(62)

This equation implies that in our DBI setting, like the canonical framework (Barrow 1990; Barrow & Liddle 1993; Barrow et al. 2006; Barrow & Nunes 2007), the intermediate inflation \( (56) \) stems from the inverse power-law potential \( V(\phi) \propto \phi^{4(1-\lambda)/\lambda} \).

Considering the Hubble parameter \( (57) \), the Hubble slow-roll parameters \( (15) \) and \( (16) \) read

\[ \varepsilon_1 = \frac{(1-\lambda)}{A\lambda} t^{-\lambda}, \]  

(63)

Using Equations \( (57) \) and \( (63) \) in \( (26) \), we obtain the power spectrum of primordial density perturbations as

\[ P_\text{s} = \frac{(A\lambda)^3}{8\pi^2c_s(1-\lambda)} t^{3\lambda-2}. \]  

(65)

By integrating the differential equation \( dv = -Hdt \) in which \( H \) is now given by Equation \( (57) \), one can express the cosmic time \( t \) in terms of the \( e \)-folds number \( N \) for the intermediate inflation as

\[ t = \left( t^\lambda - \frac{N}{A} \right)^{\frac{1}{\lambda}}. \]  

(66)

where \( t_e \) refers to the end time of inflation. Now, by replacing \( t \) from Equation \( (66) \) into \( (65) \), the power spectrum of scalar perturbations acquires the form

\[ P_\text{s} = \frac{(A\lambda)^3}{8\pi^2c_s(1-\lambda)} \left( t^\lambda - \frac{N}{A} \right)^{\frac{3\lambda-2}{\lambda}}. \]  

(67)

Also, using Equations \( (63) \), \( (64) \), and \( (66) \) in \( (29) \), the scalar spectral index reads

\[ n_s = 1 - \frac{2 - 3\lambda}{A\lambda \left( t^\lambda - \frac{N}{A} \right)}. \]  

(68)

where we have set \( \varepsilon_1 = 0 \) because the sound speed is constant in our model. It should be noted that the parameter \( \lambda \) should be in the range of \( 0 < \lambda < 2/3 \) because for \( 2/3 < \lambda < 1 \) we obtain a blue-tilted spectrum \( (n_s > 1) \) that is completely ruled out by the Planck 2015 data (Ade et al. 2016a). In addition, for \( \lambda = 2/3 \), we obtain the scale-invariant Harrison-Zel’dovich spectrum \( (n_s = 0) \), which is also at odds with the Planck 2015 observations (Ade et al. 2016a).

By applying Equations \( (63) \), \( (64) \), and \( (66) \) in \( (30) \), and keeping in mind that \( \varepsilon_1 = \varepsilon_2 = 0 \), we can further calculate the running of the scalar spectral index for our model as

\[ \frac{dn_s}{d\ln k} = \frac{2 - 3\lambda}{A^2 \lambda \left( t^\lambda - \frac{N}{A} \right)}. \]  

(69)

In order to find the relation of the tensor-to-scalar ratio in our model, we use Equations \( (63) \) and \( (66) \) in \( (35) \), and we will have

\[ r = \frac{16c_s(1-\lambda)}{A\lambda \left( t^\lambda - \frac{N}{A} \right)}. \]  

(70)

Now, we are interested in determining the parameter \( t_e \) in terms of the other parameters of the model. For this purpose, we evaluate the scalar power spectrum \( (67) \) at the epoch of horizon crossing with the \( e \)-fold number \( N_s \). Solving the resulting equation for \( t_e \) yields

\[ t_e = \left[ \frac{1}{\ln \left( 10^{10}P_{s0} \right) - P_{s0}} \right]^{\frac{1}{\lambda}} + N_s. \]  

(71)

where \( P_{s0} \left|_{N=N_s} \right. \) is a constant number given by \( \ln \left[ 10^{10}P_{s0} \right] = 3.094 \pm 0.034 \) according to the 68% CL.
constrain from Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a). Substituting Equation (71) into Equations (68), (69), and (70), we find, respectively,

\[ n_s = 1 - (2 - 3\lambda) \left( \frac{8\pi^2c_s(1 - \lambda)}{(A\lambda)^2} P_s^\prime \right)^{\frac{1}{2\lambda}}, \quad (72) \]

\[ \frac{dn_s}{d\ln k} = \frac{2 - 3\lambda}{A^2\lambda} \left( \frac{8\pi^2c_s(1 - \lambda)}{(A\lambda)^3} P_s^\prime \right)^{\frac{1}{2\lambda}}, \quad (73) \]

\[ r = 16c_s(1 - \lambda) \left( \frac{8\pi^2c_s(1 - \lambda)}{(A\lambda)^2} P_s^\prime \right)^{\frac{1}{2\lambda}}. \quad (74) \]

The above observables are completely independent of the dynamical quantities such as the cosmic time \( t \), the inflaton field \( \phi \), and the e-fold number \( N \). As a result, we can easily combine Equations (72) and (74) to find

\[ r = \frac{16c_s(1 - \lambda)}{2 - 3\lambda} (1 - n_s), \quad (75) \]

which demonstrates a linear relation between \( r \) and \( n_s \) in our DBI intermediate model. With \( c_s = 1 \), this is in agreement with the results found in Barrow et al. (2006) and Barrow & Nunes (2007) for the intermediate inflation in the standard canonical setup. It is interesting that for \( \lambda \to 0 \), the relation (75) reduces to the linear relation (55) obtained in the previous section for the DBI power-law inflation.

We use Equations (72) and (74) to plot the \( r - n_s \) diagram for the intermediate inflation (56) driven by the DBI scalar field (2). This diagram is illustrated in Figure 3 for some typical values of \( c_s \) and \( \lambda \) with varying \( A \) in the range of \( A > 0 \).

![Figure 3](image.png)

Figure 3. Same as Figure 1, but for the intermediate inflation (56) in the DBI setting (2) for some typical values of \( c_s \) and \( \lambda \) with varying \( A \) in the range of \( A > 0 \).

### Table 3

| \( \lambda \) | \( A (68\% \text{ CL}) \) | \( \frac{dn_s}{d\ln k} \) | \( A (95\% \text{ CL}) \) | \( \frac{dn_s}{d\ln k} \) |
|---|---|---|---|---|
| 0.1 | [945, 150] | [3.507 \times 10^{-5}, 1.040 \times 10^{-4}] | [84, 195] | [1.892 \times 10^{-3}, 1.372 \times 10^{-4}] |
| 0.2 | [9, 14] | [8.415 \times 10^{-5}, 2.548 \times 10^{-4}] | [8, 7.17] | [4.832 \times 10^{-3}, 3.276 \times 10^{-4}] |
| 0.3 | [1.32, 1.75] | [1.658 \times 10^{-4}, 4.622 \times 10^{-4}] | [1.215, 2.06] | [9.162 \times 10^{-3}, 6.248 \times 10^{-4}] |
| 0.4 | [0.209, 0.254] | [3.090 \times 10^{-4}, 8.193 \times 10^{-4}] | [0.196, 0.287] | [1.678 \times 10^{-3}, 1.130 \times 10^{-3}] |
| 0.5 | [0.0368, 0.0405] | [6.712 \times 10^{-4}, 1.445 \times 10^{-3}] | [0.0351, 0.0439] | [3.522 \times 10^{-4}, 2.109 \times 10^{-3}] |
| 0.6 | ... | ... | ... | ... |
| \( \lambda \geq 0.627 \) | ... | ... | ... | ... |

Note. Furthermore, the estimated values for the running of the scalar spectral index \( d n_s / d \ln k \) are presented in the table that satisfy the 95% CL constraint of the Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a).
Equations (72) and (74) we determine the values of $A$ for which the results of the model in the $r - n_s$ plane stand on the boundary of the 68% or 95% CL regions of the observational data. In addition, in Table 3, we have estimated the running of the scalar spectral index $d n_s / d \ln k$ for our model by the use of Equation (73). The estimated values for this observable fulfill the 95% CL constraint of the Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a).

In Figure 4, we have specified the parameter space of $A$ and $\lambda$ for which the $r - n_s$ result of our model with some specified values of $c_s$ is compatible with the 68% or 95% CL regions of the Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a).
order to get this diagram, we have employed a numerical code similar to the one used to provide Figure 2. Here, our code computes \( n_s \) and \( r \) by using of Equations (72) and (74), respectively, and then projects the values of \( A \) and \( \lambda \), which are compatible with the 68% or 95% CL regions of the observational data in a two-dimensional counter-plot as shown in Figure 4.

It is useful to compare the predictions of the DBI intermediate model with the CMB data in a more quantitative manner. In Table 4, we have tabulated the inflationary observables for \( \lambda = 0.1 \) and some typical values of \( c_s \) and \( A \).

![Figure 5](image)

**Table 4**

| \( c_s \) | \( A \) | \( n_s \) | \( \frac{d_n}{dn} \) | \( r \) | \( r^{\text{DBI}}/r^{\text{NL}} \) | \( r - n_s \) | consistency |
|---|---|---|---|---|---|---|---|
| 0.1 | 100 | 0.9607 | \( 9.106 \times 10^{-5} \) | 0.0333 | -32.083 | 68% CL |
| 0.2 | 120 | 0.9699 | \( 6.433 \times 10^{-5} \) | 0.0560 | -7.778 | 68% CL |
| 0.3 | 120 | 0.9661 | \( 6.748 \times 10^{-5} \) | 0.0861 | -3.277 | 95% CL |
| 0.4 | 150 | 0.9735 | \( 4.129 \times 10^{-5} \) | 0.0898 | -1.701 | 95% CL |
| 0.5 | 150 | 0.9732 | \( 4.239 \times 10^{-5} \) | 0.1137 | -0.972 | 95% CL |
| 0.6 | 152 | 0.9733 | \( 4.198 \times 10^{-5} \) | 0.1358 | -0.576 | 95% CL |
| 0.7 | 152 | 0.9726 | \( 4.410 \times 10^{-5} \) | 0.1623 | -0.337 | ... |

For the scale factor (76), the Hubble parameter reads

\[
H = \frac{A\lambda(\ln t)^{\lambda - 1}}{t}. \tag{77}
\]

Furthermore, by using Equations (15) and (16), it is simple to show that the first three slow-roll Hubble parameter are given as follows:

\[
e_1 = \frac{(\ln t - \lambda + 1)}{A\lambda(\ln t)^\lambda}, \tag{78}
\]

\[
e_2 = -\frac{(\lambda - 1)(\ln t - \lambda)}{A\lambda(\ln t - \lambda + 1)(\ln t)^\lambda}, \tag{79}
\]

\[
e_3 = \frac{\lambda((\ln t)^2 - (2\lambda^2 - \lambda + 1)\ln t + (\lambda - 1)\lambda^2)}{A\lambda(\ln t)^{\lambda + 1}(\ln t - \lambda)(\ln t - \lambda + 1)}. \tag{80}
\]

It should be noted that unlike the power-law and intermediate DBI inflationary models, here because of the complexity of the calculations, we cannot find the analytical functions for the brane potential \( V(\phi) \) and the background warp factor \( f(\phi) \) corresponding to the logamediate DBI inflation.

5. Logamediate DBI Inflation

The last model that we study within the framework of the DBI scalar field is the logamediate inflation characterized by the scale factor (Barrow & Nunes 2007)

\[
a(t) = a_0 \exp\{A(\ln t)^{\lambda}\}. \tag{76}
\]

where \( a_0 > 0, A > 0, \) and \( \lambda \geq 1 \) are constant parameters. In the case of \( \lambda = 1 \), the logamediate scale factor (76) is converted to the power-law one \( a(t) = a_0 t^q \), where \( q = A \).
where the constant of integration has been set equal to zero without loss of generality. The solution of the above equation for $t$ is

$$t = \exp \left[ \left( \frac{\lambda + 1}{8 \epsilon_c A \lambda} \right)^{\frac{1}{\lambda - 1}} \phi \right].$$

(84)

Substituting the above solution into Equation (77), the Hubble parameter is obtained as

$$H = H_0 \phi^{\alpha/2} \exp \left[ -B \phi^{\beta} \right],$$

(85)

where we have defined

$$H_0 \equiv A \lambda B^{\lambda - 1},$$

(86)

$$B \equiv \left( \frac{\lambda + 1}{8 \epsilon_c A \lambda} \right)^{\frac{1}{\lambda - 1}},$$

(87)

$$\alpha \equiv \frac{4(\lambda - 1)}{\lambda + 1},$$

(88)

$$\beta \equiv \frac{2}{\lambda + 1}.$$  

(89)

For the constant sound speed, from Equation (7) and using $X = \phi^2/2$, we can derive the warp factor of this model as

$$f(\phi) = f_0 \phi^\alpha \exp \left[ 2B \phi^\beta \right],$$

(90)

where

$$f_0 \equiv \frac{1 - c_s^2}{2 \epsilon_c H_0}.$$  

(91)

In addition to the late time limit, if we assume the slow-roll conditions, then using Equation (85) in (20) one can obtain the inflationary potential as

$$V(\phi) = V_0 \phi^\alpha \exp \left[ -2B \phi^\beta \right],$$

(92)

where the parameter $V_0$ is defined as

$$V_0 \equiv 3H_0^2 = 3A(\lambda B^{\lambda - 1})^2.$$  

(93)

It can be readily checked that in the case of $\epsilon_s = 1$, the DBI potential (92) reduces to the one obtained for the logamediate inflation in the standard canonical scenario (Barrow & Parsons 1995; Barrow & Nunes 2007; Rezaazadeh et al. 2017a).

In what follows, we proceed to estimate the inflationary observables for the present model. Using Equations (77) and (78), the power spectrum of scalar density perturbations (26) yields

$$P_\delta = \frac{(A \lambda)^3 (\ln t)^{\lambda - 2}}{8 \pi^2 c_s t^2 (\ln t - \lambda + 1)}.$$  

(94)

Also, by replacing Equations (78)–(80) and $\epsilon_{s1} = \epsilon_{s2} = 0$ into Equations (29), (30), and (35), we obtain

$$n_s = \frac{A \lambda (\ln t)^4}{(\ln t - \lambda + 1)^2} \times \frac{A \lambda (\ln t)^4 - A \lambda (\ln t - 1)(\ln t)^4}{2(\ln t)^2 + 5(\lambda - 2) \ln t - 3\lambda^2 + 5\lambda - 2},$$  

(95)

$$\frac{dn_s}{d \ln k} = \left( \lambda - 1 \right) \left( A \lambda (\ln t)^4 (\ln t - \lambda + 1) \right)^{-2} \times \left[ 2(\ln t)^3 - (7\lambda - 4)(\ln t)^2 \right. \\

\left. + (8\lambda^2 - 9\lambda + 3)(\ln t - 3\lambda^2 - 5\lambda + 2) \right].$$  

(96)

$$r = \frac{16c_s^2 (\ln t - \lambda + 1)}{A \lambda (\ln t)^3}.$$  

(97)

Notice that for the case of $\lambda = 1$, the above equations reduce to Equations (52), (53), and (54) obtained in Section 3 for the DBI power-law inflation. It makes sense because in this case the logamediate scale factor (76) changes into the power-law form $a(t) \propto t^{q}$ in which $q = A$.

Solving the differential Equation (14) with the Hubble parameter (77), one can find

$$t = \exp \left[ \left( \ln t_c \lambda - \frac{N_s}{A} \right)^{\frac{1}{\lambda}} \right],$$  

(98)

where the integration constant has been determined by considering the end of inflation condition, i.e., $N_c \equiv N(t = t_c) = 0$. In order to find the parameter $t_c$ in terms of the other model parameters, we fix the amplitude of scalar perturbations $P_\delta$ at the horizon exit $e$-fold number $N_*$, according to the Planck observational results. To this aim, we first obtain $P_\delta$ by replacing Equation (98) into (94) as

$$P_\delta = \frac{(A \lambda)^3 (\ln t_c)^{\lambda - 2}}{8 \pi^2 c_s \left[ (\ln t_c)^{\lambda} - \frac{N_*}{A} \right]^{1/\lambda} - \lambda + 1} \times \exp \left[ -2 \left( \ln t_c \lambda - \frac{N_*}{A} \right)^{1/\lambda} \right].$$  

(99)
We recall that the quantity $\mathcal{P}_{\tau^*}$ is a constant number constrained as $\ln[10^{10}\mathcal{P}_{\tau^*}] = 3.094 \pm 0.034$ due to Planck 2015 TT, TE, EE+lowP data at 68% CL (Ade et al. 2016a). We apply this constraint to obtain the parameter $\tau_e$ in terms of the other model parameters. It should be noted that Equation (99) cannot be solved analytically, and we have to employ a numerical method for this purpose. In the next step, we replace $\tau_e$ from Equation (99) into (98) to find the time of horizon crossing $t_w$ for given parameters $n, A, \lambda$, and $N^*_w$. Surprisingly, our numerical computations imply that $t_w$ is completely independent of $N^*_w$. Such a situation also appears in the study of logamediate inflation in $f(T)$-gravity (Rezazadeh et al. 2017a).

Here, we repeat the deduction of Rezazadeh et al. (2017a) to justify the mentioned surprising consequence. Following

Figure 6. Same as Figure 4, but for the logamediate inflation (76) in the DBI scenario (2) with different values of the constant sound speed $c_s$. 

We recall that the quantity $\mathcal{P}_{\tau^*}$ is a constant number constrained as $\ln[10^{10}\mathcal{P}_{\tau^*}] = 3.094 \pm 0.034$ due to Planck 2015 TT, TE, EE+lowP data at 68% CL (Ade et al. 2016a). We apply this constraint to obtain the parameter $\tau_e$ in terms of the other model parameters. It should be noted that Equation (99) cannot be solved analytically, and we have to employ a numerical method for this purpose. In the next step, we replace $\tau_e$ from
In Figure 6, the parameter space of $A$ and $\lambda$ are specified for some typical values of $c_s$, in which the result of the $r-n_s$ diagram is compatible with the 68% or 95% CL joint regions of the Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a). To plot this diagram, we have utilized the same numerical method as that used in Figure 4 corresponding to the intermediate DBI inflation.

At the end of this section, we present some numerical values for the inflationary observables in the logamediate DBI inflation. These values are listed in Table 6 for $\lambda = 2$ and some typical values of $c_s$ and $A$. For each case we have examined the $r-n_s$ consistency and concluded that our results are in agreement with those illustrated in Figure 6.

6. Conclusions

Here, we studied the power-law, intermediate, and logamediate inflations driven by the DBI non-canonical scalar field. The DBI inflation is inspired from string theory, and in this model the role of inflaton is performed by the radial coordinate of a D3-brane moving on the throat of the compactification space. We first investigated the basic equations governing the cosmological background evolution in the DBI inflationary scenario. In particular, we showed that the slow-roll first Friedmann equation in the DBI setting is identical to that of the standard canonical scenario. Subsequently, with the help of scalar and tensor power spectra in the setup of DBI inflation, we obtained the inflationary observables including the scalar spectral index $n_s$, the running of the scalar spectral index $d_n_s/d\ln k$, and the tensor-to-scalar ratio $r$. We showed that the 95% CL constraint from temperature and polarization data of the Planck 2015 collaboration on the non-Gaussianity parameter leads to the lower bound $c_s \geq 0.087$ on the sound speed of the DBI inflation.

In addition, we assumed the sound speed to be constant during inflation and examined viability of the power-law, intermediate, and logamediate DBI inflationary models in light of the Planck 2015 observational data. The first scale factor that we investigated was the power-law scale factor $a(t) \propto t^q$ where $q > 1$. We showed that in the slow-roll approximation, this scale factor arises from the ascending exponential warp factor $f(\phi) \propto \exp \left( \frac{1}{q} \phi \right)$ and the descending exponential potential $V(\phi) \propto \exp \left( - \frac{1}{q} \phi \right)$. We further showed that the intermediate scale factor $a(t) \propto \exp (A t^{\lambda})$ where $A > 0$ and $0 < \lambda < 1$ is driven by the power-law warp factor $f(\phi) \propto \phi^{2(2-\lambda)/\lambda}$ and the inverse power-law potential $V(\phi) \propto \phi^{4(1-\lambda)/\lambda}$. In the study of logamediate scale factor $a(t) \propto \exp [A (\ln t)^{\lambda}]$ where $A > 0$ and $\lambda > 1$, we derived the warp factor $f(\phi) \propto \phi^{-\alpha/2} \exp [2B \phi^{\beta}]$ and the inflationary potential $V(\phi) \propto \phi^{\alpha} \exp [-2B \phi^{\beta}]$ where $\alpha$, $\beta$, and $B$ are positive constants. Note that in the case of $c_s = 1$, the slow-roll results of the DBI inflationary potentials corresponding to the aforementioned scale factors are the same as those obtained in the standard canonical framework.

Our study implies that, although the power-law, intermediate, and logamediate inflationary models in the standard canonical framework are completely ruled out by the Planck 2015 results, in the DBI setting with constant sound speed they can be compatible with the Planck observations. We showed that the predictions of these DBI models in the $r-n_s$ plane can lie inside the 68% CL joint region of the Planck 2015 TT,TE, EE+lowP data (Ade et al. 2016a), if we take the sound speed $c_s$...
to be sufficiently less than the light speed ($c = 1$). For the power-law, intermediate, and logamediate DBI models, we specified their parameter space for which the $r - n_s$ predictions of these models are compatible with 68% or 95% CL marginalized joint regions of the Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a). In our work, we further estimated the running of the scalar spectral index $d n_s / d \ln k$ for the allowed ranges of the parameter space of the intermediate and logamediate DBI inflationary models and concluded that the estimated values satisfy the 95% CL constraint of the Planck 2015 TT,TE,EE+lowP data (Ade et al. 2016a).

The authors thank the referee for valuable comments. The work of A. Abdolmaleki has been supported financially by Research Institute for Astronomy and Astrophysics of Maragha (RIAAM) under research project No. 1/5237-116.

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