Topological phases and pumps in the Su–Schrieffer–Heeger model periodically modulated in time

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Abstract

By the Floquet theory, we transform the Su–Schrieffer–Heeger model with the periodically modulated nearest-neighbor (NN) and next-nearest-neighbor (NNN) interactions into an effective 2D model, which holds the total Chern number of $\pm 1$ modulated by the parameter $\theta$. Under a staggered electric potential, the topological phase diagrams of the effective 2D model are reshaped and similar to the well-known Haldane model. While under a staggered Zeeman field, the topological phase diagram has the same shape as the former case, but with different Chern numbers, such as the spin and valley Chern numbers. With the combination of the staggered Zeeman field and the electric field, the effective 2D model holds even richer topological phases. At last, we find some types of topological pump, which can generate the time-averaged current without any bias voltage. The current depends on their different Chern numbers. In other words, we can modulate the parameters to obtain various Chern numbers to control the topological pump.

Keywords: topological, floquet, SSH model

(Some figures may appear in colour only in the online journal)
(NNN) hoppings were modulated by some phase parameters [17]. In contrast to these works, we periodically modulate the \( h_N \) and NNN hoppings in time with the combination of the staggered Zeeman field and staggered electric potential [25] to acquire abundant topological phases characterized by the spin Chern number, the valley Chern number and the spin-valley Chern number [26–28]. By the Floquet theory, we can obtain richer topological phases than Li et al.’s work with the effect of the external fields. In addition, some topological pump will also happen in our system. The type of the pump depends on its topological phase. The pump mechanism results from the NN and NNN hoppings periodically modulated in time.

This paper is organized as follows. In section 2, we introduce 1D tight-binding model with the NN and NNN hoppings modulated in time. In section 3, we transform the 1D model into an effective 2D model with the use of the Floquet theory and give some discussions on the Floquet theory. In section 4, we give the main results for the topological phases and topological pump effect of this effective 2D model.

2. Models and methods

2.1. 1D SSH model

We consider a 1D dimerized lattice with one unit cell containing A and B sites, shown in figure 1. Here the NN and NNN hoppings are periodically modulated in time, and the corresponding bonds have opposite phases [29]. We also apply both perpendicular staggered Zeeman field and electric potential to this lattice. Thus the tight-binding Hamiltonian of this system consists of the SSH chain (\( H_{\text{SSH}} \)) and the external field (\( H_{\text{ex}} \))

\[
H = H_{\text{SSH}} + H_{\text{ex}}, 
\]

where

\[
H_{\text{SSH}} = \sum_m | t_1 c_m^A c_{m+1}^C + t_2 c_m^A c_{m+1}^C + t_3 c_m^B c_{m+1}^C | + \sum_N \left[ t_B c_m^A c_{m+1}^{A, +} + t_C c_m^B c_{m+1}^{B, +} \right] + \text{H.C.},
\]

\[
H_{\text{ex}} = \sum_{\alpha=1}^2 \mu_\alpha \epsilon c_m^\alpha c_m^\alpha + \sum_{i=1}^N \delta c_i^a c_i^a,
\]

In equation (1), \( c_m^A, c_m^B, c_{m+1}^C \) are the fermion creation (annihilation) operators on the sublattice A and B in the Nth unit cell. In the first term, \( t_1 = t \left(1 + 2 \delta \cos \omega t \right) \) and \( t_2 = t \left(1 - 2 \delta \cos \omega t \right) \) with \( \delta \) being the driving amplitude. This term represents the NN hopping with opposite phases. The second term represents the NNN hopping, \( t_3 = 2 \theta \cos (\omega t + \theta) \). In these expressions, \( h \) is the driving amplitude and \( \theta \) is a phase parameter which can vary from 0 to \( 2\pi \) continuously. The external fields (\( H_{\text{ex}} \)) includes a perpendicular staggered Zeeman field \( M \) and an electrical staggered potential \( \Delta \). The latter is induced by the site-dependent electric field, where \( \mu_\alpha = +1 \) (1) for the A (B) site. In addition, \( \sigma^z \) is the Pauli matrix of z component.

This model periodically modulated in time can be treated by the Floquet theory introduced in references [21, 22]. If a Hamiltonian has the time-periodic property [30, 31]: \( H(t) = H(t + T) \), we can apply the Floquet formalism to get the so-called Floquet states [32] \( \varphi_\alpha = e^{-i \omega \epsilon} | \theta_\alpha (t) \rangle \), where \( | \theta_\alpha (t) \rangle = | \theta_\alpha (t + T) \rangle \) and \( \omega \) are the Floquet modes and \( \epsilon_\alpha \) are the associated quasi-energies. One gets this Floquet operator \( HF = H(t) - i \delta t \) by inserting these solutions into the time-dependent Schrodinger equation, thus \( HF | \theta_\alpha (t) \rangle = \epsilon_\alpha | \theta_\alpha (t) \rangle \). By using the Fourier transformation, we have

\[
| \theta_\alpha (t) \rangle = \sum_m e^{i m \omega t} | \varphi_m^{\alpha} \rangle \quad \text{and} \quad H_N = \frac{1}{T} \int_0^T HF e^{-i \omega t} dt.
\]

One can get

\[
\sum_m H_N^{cm} | \varphi_m^{\alpha} \rangle = \epsilon_\alpha | \varphi_m^{\alpha} \rangle,
\]

where

\[
H_N^{cm} = \frac{1}{T} \int_0^T H(t) e^{i (m-n) \omega t} dt - m \omega \delta_{nm}, \quad \text{(2)}
\]

which is the Floquet Hamiltonian expanded in the Hilbert space [33].

Then we can use the Floquet Hamiltonian to rewrite the Hamiltonian \( H \) as

\[
H_N^{SSH} = \sum_N \delta_{nm} c_m^A c_{m+1}^C + \sum_N \delta_{nm} c_m^B c_{m+1}^C + \text{H.C.}
\]

\[
+ \sum_N \delta_{nm} (c_m^A c_{m+1}^C \delta_{nm+1} + c_m^B c_{m+1}^C \delta_{nm+1}) + \text{H.C.}
\]

\[
+ \sum_N \delta_{nm} (c_m^A c_{m+1}^C + c_m^B c_{m+1}^C) + \text{H.C.}, \quad \text{(3)}
\]

where \( N \) is the number of unit cells and the Floquet modes are represented by the indices \( n \) and \( m \).

2.2. 2D Effective model

Followed from the work of Gomez-Leon and Platero [21], one can treat the Floquet modes \( n \) as a second parameter and transform it into the momentum \( k_f \) in an extra dimension (f direction). Thus the Hamiltonian in equation (2) can be regarded as an effective 2D tight-binding model. But the last term in equation (2) breaks the translational symmetry in this effective 2D model. Actually we can treat the frequency \( \omega \) as small enough in order to be satisfied with the translational symmetry. Thus the last term in the Hamiltonian (2) can be regarded as an effective electric field \( E_{\text{eff}} = \omega \) [21, 22]. By this
approximation, we may follow the Bloch theorem and define
the effective wavevector $k_f = \frac{2\pi}{m_0}n = \frac{2\pi}{\theta}$, where $n (n = \frac{2\pi}{\theta})$ is
the photon number. We obtain an effective momentum space
with $k_f$ ranged in $[-\pi, \pi]$.

By using the Fourier transformation and the periodic boundary condition, we can easily rewrite the Hamiltonian $H$ as

$$H_{SSH} = \sum_k \rho_k^* h(k) \rho_k,$$

where $\rho_k = (a_k, b_k)$ and $h(k) = \begin{bmatrix} B_1 & B_2 \\ B_1 & B_4 \end{bmatrix}$ with

$$B_1 = 4h \cos k_x \cos (k_f - \theta),$$
$$B_2 = t + 2\delta \cos k_f + i (\cos k_x - i \sin k_x) - 2\delta \cos k_f (\cos k_x - i \sin k_x),$$
$$B_3 = t + 2\delta \cos k_f + i (\cos k_x + i \sin k_x) - 2\delta \cos k_f (\cos k_x + i \sin k_x),$$
$$B_4 = 4h \cos(k_x) \cos(k_f + \theta).$$

Alternatively, $h(k)$ can be expressed in the form

$$h(k) = h_I + d(k) \cdot \sigma,$$

where $I$ is the identity matrix and $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices. The coefficients of Pauli matrices are defined as

$$h_I = 2h \cos(k_x) (\cos(k_f - \theta) + \cos(k_f + \theta)), d_x = t + 2\delta \cos k_f + t \cos(k_x) - 2\delta \cos k_f \cos(k_x),$$
$$d_y = -2\delta \cos k_f \sin(k_x) + t \sin(k_x),$$
$$d_z = 2h \cos(k_x) (\cos(k_f - \theta) - \cos(k_f + \theta)).$$

The coefficient $d_x$ can be extended in the following form which includes the Hamiltonian $H_{ex}$

$$d_x = 2h \cos(k_x) (\cos(k_f - \theta) - \cos(k_f + \theta)) + \Delta + sM,$$

where $s = 1 (-1)$ for spin-up (spin-down). By diagonalizing the Hamiltonian, one gets the eigenvalues $E(k) = h_I \pm \sqrt{\gamma}$ where $\gamma = d_x^2 + d_y^2 + d_z^2$. By setting $d_x^2 + d_y^2 = 0$, we find the Hamiltonian has two different extremum (Dirac) points; $K = (\pi, \frac{\pi}{2})$ and $K' = (\pi, \frac{-\pi}{2})$ in $k_x - k_f$ space. The Hamiltonian with the NN and NNN hoppings in the $x$ direction, as well as with the coupling of different photon numbers (figure 2(a)), have been translated in $k_x - k_f$ space. And the band structure with NN and NNN hoppings in figure 2(b) shares the same Dirac points with the band structure with only NN hopping in figure 2(c). But the effect of NNN hopping makes the contribution to the Dirac mass used to modulate topological phases. We will discuss it in the next section. If
$d_1$ is zero, the effective 2D model is gapless; otherwise the system is gapped. Near these points, we can use the Dirac equation to describe the effective low-energy physics.

2.3. Topological invariance quantities

For the effective momentum space $\{k_x, k_f\}$, where $k_f$ can be regarded as the second parameter in addition to the momentum in the real space ($k_x$). So the Chern number can be calculated by integrating the Berry curvature over the effective 2D Brillouin zone,

$$C = \frac{1}{2\pi} \int dk dk_f \left( \partial_{k_f} A_{k_x} - \partial_{k_x} A_{k_f} \right),$$

(7)

where $A_{k_x} = i \langle \varphi_+ (k) | \partial_{k_x} \varphi_+ (k) \rangle$ and $A_{k_f} = i \langle \varphi_+ (k) | \partial_{k_f} \varphi_+ (k) \rangle$ are the Berry connection defined in 2D effective space, with $\varphi_+ (k)$ obtained from the Hamiltonian in equation (5). As a topological invariant, the Chern number can be used to characterize the topological property of the 2D system. In this work, we can regard the 1D system as the effective 2D system so that the Chern number is also used to characterize topological property of 1D system [34, 35].

Expanded near the two Dirac points, the Hamiltonian in equation (5) can be expressed as

$$H (\tau) = 4t \delta \tau k_x \sigma_1 - t k_x \sigma_y + m_1^\tau \sigma_z,$$

(8)

where

$$m_1^\tau = 4h^* \sin \theta + \Delta + sM,$$

and $\tau = 1 (-1)$ for $K (K')$. We can transform equation (8) into the standard form,

$$H (\tau) = \sum_{ij} A_{ij} (\tau) k_i \sigma_j + m_1^\tau \sigma_z,$$

(9)

where the summation is over $x$ and $y$ dimensions. According to equation (9), one can get the resulting matrix $A$ as

$$A (\tau) = \begin{pmatrix} \frac{e^2}{2h} & 4\delta \tau \\ -1 & 0 \end{pmatrix}.$$

(10)

It is clear that the two Dirac points ($K$ and $K'$) have opposite values for Det ($A$). Here we analytically derive the Chern number of the lower filled band into a simple formula. As we know [22], the Hall conductance may be written as

$$\sigma_{xy} = \frac{e^2}{2h} \text{sgn} (m_1^\tau) \text{sgn} (\text{Det} (A)),$$

(11)

or the four independent Chern numbers are expressed as

$$C_1^\tau = \frac{1}{2} \text{sgn} (m_1^\tau) \text{sgn} (\text{Det} (A))$$

(12)

due to that the Hall conductance is quantized in units of $\frac{e^2}{2h}$. With the help of equation (10), we can rewrite equation (12) as

\[\text{Figure 3.} \hspace{1cm} \text{(a) Total Chern number as a function of parameter } \theta, \hspace{0.5cm} \text{(b)–(d): energy band structures in the 2D effective model with (b) } \theta = -0.5\pi, \hspace{0.5cm} \text{(c) } \theta = 0, \hspace{0.5cm} \text{(d) } \theta = 0.5\pi. \hspace{1cm} \text{We set } t = 1 \text{ eV}, \delta = 1, \ h = 0.2t. \]
with $\tau = 1 \ (-1)$ for $K (K')$ and $s = +1 \ (-1)$ for spin-up (spin-down). The formula $m'_s = 4h\tau \sin \theta + \Delta + sM$ means that the different Dirac point raises up different Dirac gap with the same external field.

In order to keep the numerical calculation of the Chern number (equation (7)) correct, the Fermi level should be kept in the bulk gap, or the Chern number is ill defined: the lower number (equation (7)) correct, the Fermi level should be kept in equation (6). However, one can choose suitable $h_I$ to make sure that the two bands can be separated by a gap. Thus one can use the simple formula to clearly see how the topological property of the effective 2D system changes.

Before introducing the topological phase of the effective 2D system, we define four independent Chern numbers $C_i$ introduced before: the total Chern number $C$, the spin Chern number $C_s$, the valley Chern number $C_v$, and the spin-valley Chern number $C_{sv}$ [26],

\[
C = C^K_+ + C^{K'}_+ + C^K_- + C^{K'}_-,
\]
\[
C_s = \frac{1}{2} \left( C^K_+ + C^{K'}_+ - C^K_- - C^{K'}_- \right),
\]
\[
C_v = C^K_+ - C^K_- + C^{K'}_+ - C^{K'}_-,
\]
\[
C_{sv} = \frac{1}{2} \left( C^K_+ - C^{K'}_+ - C^K_- + C^{K'}_- \right).
\]  

From equation (15), one gets that the boundary of the topological phase transition is defined by the condition of the two Dirac mass $\pm 4h \sin \theta = 0$ and these phase transition points are $\theta = 0, \pm \pi$. With this condition, one can plot the phase diagram associated with $\theta$ for the effective 2D system in figure 3. The total Chern number $C$ is just modulated by the parameter $\theta$, where $C = -1$ is in the region of $\theta = (-\pi, 0)$ and $C = 1$ in the region of $\theta = (0, \pi)$. From figures 3(b)–(d), the gap closing and reopening means the topological phase transition. It is interesting that the 1D model with the NN and NNN hoppings periodically modulated in time exhibits a topological phase transition with the parameter $\theta$.

Next, we give detailed discussions on the topological phases with the external fields on this effective 2D model.

1. Perpendicular staggered electric potential

In this case, the staggered electric potential $\Delta$ is applied on the effective 2D model, and the Dirac mass is modified by $m'_s = \Delta + \tau 4h \sin \theta$. The condition to determine the phase boundary is that the Dirac mass $m'_s = 0$ or $\frac{\Delta}{4h} = \pm \sin \theta$. With this condition, one can plot the phase diagram in figure 4(a). In contrast to the phase diagram in figure 3(a), there is no new topological phase, but reshaped structure of the phase diagram. And the effective 2D model under the staggered potential shares a similar structure of the phase diagram of 2D Haldane model [18].

As well know [36], the Haldane model can hardly be realized in ordinary condensed matter due to the especially staggered magnetic flux. Here we give an easy way to realize the Haldane model.

2. Perpendicular staggered Zeeman field

If this Zeeman field is taken into account in the effective 2D model, the Dirac mass is $m'_s = sM + \tau 4h \sin \theta$. The condition of the phase boundary is that the Dirac mass equal $\theta$.
to zero $m^+_x = 0$, or $\frac{M}{4\hbar} = \pm \sin \theta$ with $s = +1$ and $-1$ for spin-up or spin-down. With this condition, the phase diagram is plotted in figure 4(b), which has the same structure as figure 4(a). But the topological phase in figure 4(b) is completely different from figure 4(a). The spin and valley Chern numbers emerge under the staggered Zeeman field. Previous works [9, 11] about the effective 2D model just showed the total Chern number but not the other Chern numbers. Here we find that this effective 2D model may be utilized to obtain other Chern numbers with the staggered Zeeman field.

(3) Perpendicular staggered Zeeman field and electric potential

If both of the Zeeman filed and electric potential are taken into account for the effective 2D model, the Dirac mass is modified by $m^+_x = sM + r\hbar^2 \sin \theta + \Delta$. The condition of the phase boundary is $m^+_x = 0$ or $\frac{M}{4\hbar} = \pm \sin \theta - \frac{\Delta}{4\hbar}$ for spin-up, and $\frac{M}{4\hbar} = \pm \sin \theta + \frac{\Delta}{4\hbar}$ for spin-down. With this condition, one can get abundant topological phase diagrams as in figures 5(a) and (b). The topological phases depend on three parameters $\frac{M}{4\hbar}$, $\frac{\Delta}{4\hbar}$ and $\theta$. We set the region $\frac{M}{4\hbar} > 1$ for figure 5(a) and the region $0 < \frac{\Delta}{4\hbar} < 1$ for figure 5(b). In addition, if we set $\frac{\Delta}{4\hbar} = 0$, the phase diagram is reduced to figure 4(b). In contrast to the figure 5(a), the new topological phase appears in the overlap region in figure 5(b). In one word, the effective 2D model can be utilized to realize the four Chern numbers with the help of both staggered potential and Zeeman field.

Here we emphasize that these topological properties are very robust in disorders. We add the on-site disorders (within the range of $[-0.1 \text{ eV}, 0.1 \text{ eV}]$) to the eight-site supercell SSH system. Then we numerically calculate the Berry curvature and Chern numbers for these disordered systems with the methods presented in our previous work [37]. We have verified that all the Chern numbers are unchanged in the phase diagrams of figures 3 and 4.

3.2. Floquet topological pump

As we mentioned, the last term in equation (3) can be proved to be an effective electric field $E_{\text{eff}} = \omega$ in the additional $f$ direction [21, 22]. Following [22], one can rewrite the unidirectional current of the 1D model in the $n$th photon-number state as

$$j_x (n) = \sigma_{xy} \omega = \frac{e^2}{h} C \omega,$$

where $\sigma_{xy} = \frac{e^2}{h} C$ and $C$ can be replaced by $C_x, C_y$ and $C_{xy}$ [38, 39] as well. So the sum of all the states of the 1D model can be written as

$$j_x (t) = \sum_n j_x (n) e^{-i n \omega t}.$$

This formula is the time-dependent one-dimensional current in the $x$ direction. We take the time-averaged current of $j_x (t)$ over the period $T$ as

$$\bar{j}_x = \frac{1}{T} \int_0^T j_x (t) dt = \frac{1}{T} \int_0^T \sum_n j_x (n) e^{-i n \omega t} dt = j_x (0) = \frac{e^2}{h} C \omega.$$

The $\bar{j}_x$ above may also become the spin current $\bar{j}_{sx}$, the valley current $\bar{j}_{vx}$ and the spin-valley current $\bar{j}_{sxvx}$ when the total Chern number $C$ is replaced by $C_x, C_y$ and $C_{xy}$ in equation (16). The time-averaged current $\bar{j}_x$ ($\bar{j}_{sx}$, $\bar{j}_{vx}$ and $\bar{j}_{sxvx}$) is dependent on the Chern number $C (C_x, C_y$ and $C_{xy})$, which means the effective 2D model with different topological state has different types of the time-averaged current. In other words, this model can generate a unidirectional current with an effective electric field, which is also the ratchet effect [22].
Here we generalized this ratchet effect into the spin and valley degree of freedom. Since they are related to the topological property, we call these time-averaged current $j_x$ without bias voltage as the topological pump.

Now, we use the topological phase in figure 4 to discuss these topological pumps in detail. In figure 4(a), the total Chern number adjusted by the parameters is $\pm 1$ or 0 in certain range of $\theta$ and $\Delta_4 h$, which is equivalent to that the topological pump has opposite direction (or zero) in $x$ axis. It is interesting that the parameter $\theta$ and $\Delta_4 h$ can control the time-averaged charge current $\overline{j}_x$ depicted in figure 6(a). In figure 4(b), the topological phases are dependent on the parameters $M_4 h$ and $\theta$. And the parameter $M_4 h$ can motivate the new topological phases with $C_s$ and $C_v$. In others word, the topological pump is for the spin/valley current in the $x$ direction depicted in figure 6(b). In figure 5, the total Chern number ($C$), the spin Chern number ($C_s$), the valley Chern number ($C_v$) and the spin-valley Chern number ($C_{sv}$) all have the corresponding charge, spin, valley and spin-valley time-averaged currents in the $x$ direction, which are not shown here.

3.3. Experimental realization

The 1D SSH model can be easily realized in many systems, such as plasmonics, photonics and circuit QED [14–16], which is rationally used as our model. In a single degenerate optical cavity, the arbitrary long-range hopping periodically modulated in time can be realized [40, 41]. In one word, the 1D SSH model periodically modulated in time is experimentally implemented. The time-averaged current is proportional to the Chern number and an effective electric field $\omega$ which should be small enough to make sure the condition of translational symmetry in our 2D model. Actually we can take $\omega$ large enough in the above conditions to make the time-averaged current obviously observed. And the antiferromagnetic exchanged field can be implemented, in which the 1D SSH model is coupled to an antiferromagnetic insulator with large mismatching [42]. And with the perpendicular electric field, the staggered electric field can be easily obtained due to the mismatched sublattice $A$ and $B$. With the reasonable parameter $\theta$, nonzero value of the staggered antiferromagnetic exchanged and electric field can obtain the time-averaged current. In addition, the light-induced nonequilibrium Floquet phase has been realized by applying coherent phono modes [43, 44]. With a periodic electric field, the ratchet, equal to the pump, can be also realized in graphene breaking the spatial symmetry [45].

4. Summary

In summary, we utilize the effective 2D model and the Floquet theorem to study the topological phases of the SSH lattice with NN and NNN ac driven perturbations. In the presence of staggered potential and Zeeman field, we find this effective 2D model have four Chern numbers, which is different from previous works with just the total Chern number. What is interesting is that there exists nonzero time-averaged current. This topological pump is dependent on its topological phase. The mixing of two external fields in the effective 2D model generates very rich nontrivial phases, which also correspond to the rich types of topological pumps.

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