A new attempt for radiation hydrodynamics simulation with anisotropic and non-equilibrium distribution

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Abstract. We present a computational method for solving time-dependent radiation moment equations with an Eddington tensor which represents an anisotropic field of radiation. For estimating the Eddington tensor, we propose a conical-ray method covering the whole solid angle with which the steady-state radiative transfer equation is solved. Computed radiation flux shows an excellent agreement with an analytical solution using the proposed conical-ray method even if a light source is located at a distant place from the computed point.

1. Introduction

A flow dynamics in high-temperature gases or plasmas is often influenced by energy transport of radiation emitted from flowfield itself. When we analyze such a flowfield numerically, it is necessary to couple radiative transfer equation with hydrodynamic equations in a certain form. If the density of the flowing matter is low enough, i.e., for optically thin case, we can conduct a formulation with a steady-state assumption of radiation field since electromagnetic waves spread more quickly than the characteristic speed of the flowfield. On the other hand, a diffusion approximation can be applied for solving the radiative transfer equation because of an isotropic radiation field for optically thick case in which the matter density is relatively high. However, for a flowfield rapidly changing with space and time such as laser ablation plasma, supernova explosion (neutrino transfer), and so on, radiative transfer property depends on the dynamics of the field. In such a case, a phenomenon cannot be described appropriately with one of two above-mentioned limits, but a method of connecting two limits smoothly is needed.

For moment equations of radiative transfer, an Eddington tensor defined by radiation energy density and stress tensor is responsible for an anisotropic property of radiation field. In the present paper, a new computational technique which takes the anisotropic radiation suitable for radiation hydrodynamics simulation into consideration by nonlocally determining the Eddington tensor is examined as following the works of Stone et al. [1] and Hayes & Norman [2]. We introduce a conical-ray method for estimating a precise Eddington tensor even with a point source which is difficult to be resolved with the conventional ray method. Test problems have been performed under a simple condition, and the proposed method is validated as compared with the conventional one.
2. Numerical modeling

2.1. Estimation of Eddington tensor with ray-tracing technique

The value of the Eddington tensor highly depends on optical thickness of the matter considered as mentioned in the previous section. In the optically thick case, since the radiation is easy to be absorbed, and influence of it, therefore, cannot reach a distant place, the Eddington tensor can be described as a function of locally obtained physical quantities. In this case, the transport process of radiation becomes in diffusion, then it is approximated by

\[ f_\nu = \frac{1}{3} I \]

with this limit, where \( I \) is the unit tensor. However, when not optically thick, or when optical thickness is nonuniform, it is necessary to estimate the Eddington tensor from nonlocal physical quantities because the radiation field receives the influence of the matter in a distant place. In this paper, in order to take such a situation into consideration, a steady-state transfer equation is solved only for estimating the Eddington tensor including the anisotropic information of radiation, and then the time-dependent moment equations are solved with it. This procedure was first proposed by Stone et al. [1].

In the previous our work [3], radiation rays were set to be at the solid angles corresponding to the Gauss points of the discrete ordinate (so-called \( S_N \)) method [4]. If we obtain a specific radiation intensity \( I_{\nu n} \) by a ray-tracing manner with a \( n \)-th ray, the Eddington tensor can be approximated by the following discrete integration,

\[
f_\nu = \frac{\sum_{n} w_n I_{\nu n} \Omega_n \Omega}{\sum_{n} w_n I_{\nu n}},
\]

where \( P_\nu, E_\nu, \) and \( \Omega \) are the radiation stress tensor, the radiation energy density, and the unit vector of propagation direction, respectively. Subscripts \( \nu \) and \( n \) denotes the frequency and the ray index. The variable \( w_n \) is a weight of the \( n \)-th Gauss point for the numerical integration.

If the rays are discretely arranged in a solid angle as treated in the conventional work, it is impossible to take in the information on the cell which does not exist along the path of the rays and may become important in the optically thin media. Although increasing a number of rays simply improves this fault, a huge number of rays and, of course, a tremendous computational cost are required for touching all the cells. Then, in order to include the information other than the ray passing cells, we consider a conical ray (see Fig. 1) which has a certain margin around the original ray and do the ray-tracing with the physical quantities volume-averaged over the cells within this margin. Other procedures such as the original ray setting and the numerical integration are performed in the same manner as the previous study.

Figure 2 shows a sample of the arrangement of conical rays in a 1/8 sphere. The original rays are set to the angles corresponding to the \( S_8 \) method. Three types of conical ray can be considered in this case. There may be many candidates of the method to determine the solid angles (i.e., the radii \( r_a, r_b, \) and \( r_c \) in Fig. 2) covered by each cone. In this study, however, we set them to let the following value \( I \) be minimum: \( I = \sum_{i}^{n_{pairs}} (r_i + r_j - L_i)^2 \) for \( r_i + r_j - L_i > 0 \), where \( j \) expresses an adjoining cone and \( L_i \) is an interval with the adjacent cone. The sum is performed for all the types of pair connecting with the red lines in Fig. 2.

2.2. Numerical procedure for solving radiation moment equations

The definition position of each physical quantity follows the manner of Ref. [2] with staggered grids. Therefore, \( E_\nu \) and the diagonal element of \( P_\nu \) are the cell-centered quantities, the radiation energy flux \( F_\nu \) is defined at the cell interface, and the non-diagonal element of \( P_\nu \) is defined at the cell edge. Each element of the Eddington tensor is similarly defined as \( P_{\nu_\nu} \).
With the definition given in this way, the 0th- and 1st-order moment equations can be written in the following forms with a partially implicit method,

\[
E^{n+1}_{\nu_{i,j}} = E^n_{\nu_{i,j}} + \Delta t \left[ 4\pi \eta^n_{\nu_{i,j}} - c\chi^n_{\nu_{i,j}} E^{n+1}_{\nu_{i,j}} + (\nabla \cdot F^{n+1})_{i,j} \right],
\]

(2)

\[
F^{n+1}_{\nu_{i,j}} = \frac{F^n_{\nu_{i,j}}}{1 + c\chi^n_{\nu_{i,j}} \Delta t} - \frac{c^2 \Delta t}{1 + c\chi^n_{\nu_{i,j}} \Delta t} \nabla \cdot \left( f^n_{\nu} E^{n+1}_{\nu} \right)_{i,j},
\]

(3)

where \( \eta_{\nu} \) and \( \chi_{\nu} \) are the emissivity and the opacity, respectively. These forms are also almost same as proposed by Hayes & Norman [2]. Note that our formulation is developed for two-dimensional simulation while this paper only presents the one-dimensional results. We solve the system of equations for \( E^{n+1}_{\nu_{i,j}} \) using a family of conjugate gradient method.

3. Test problem
A simple problem for validating our proposed method is presented. A constant heat flux \( F_0 \) from surface of a small sphere is given as a boundary condition for a spherically symmetric simulation. Propagation of the radiation to a medium with a uniform opacity was calculated without fluid motion. For further simplicity, it is supposed that the opacity does not have dependency for frequency, and the emissivity from the medium is set to be zero. Considering an optically thin situation in which the solution is more sensitive for anisotropic radiation, we performed simulations with opacity \( \chi = 0.2 \) over the region where the optical depth is comparable to or larger than unity. In Figs. 3 and 4, the horizontal axis shows the optical depth from the surface of the central sphere. Red lines denote plots of radiation energy flux \( F_r \) at each \( \Delta t = 7.5(\chi/c) \).

Figure 3 shows a computational result with the conventional ray method. One can see the reasonable agreement with an analytical solution by first two plots. This feature indicates the advantage using the Eddington tensor estimated by ray-tracing method while the \( P_1 \) method (almost diffusion approximation) must deviate from the analytical solution even at this early stage [3]. However, after the next plot, the obtained solution suddenly depart from the analytical one. Here, a blue line in the figure represents the diagonal element of the calculated Eddington tensor. Note there is a discontinuity at optical depth \( \sim 0.7 \). This discontinuity actually results from that the all arranged rays miss the central light source. This is an unavoidable
shortcoming of the conventional ray method. After missing the light source, the computed flux is always overestimated, and the behavior of the solution is similar to the $P_1$ method because the Eddington tensor in that regime is close to the isotropic one ($1/3$).

Figure 4 shows the result using the proposed conical-ray method. With the conical-ray method, the radiation flux shows an excellent agreement with the analytical solution even if the light source is located at a distant place from the computed point. Moreover, the computed Eddington tensor smoothly approaches to unity which is the exact value in the case of free-streaming. However, the small deviation is found at the vicinity of the light source. This discrepancy comes from a relatively coarse resolution rather than the conventional method due to the averaging over the overlapping cones. An inaccurate solution near the light source may lead to an issue for radiation hydrodynamics simulations. Further investigation is needed to overcome this unwelcome feature.

4. Conclusion
We have developed a new method for solving the radiation transfer equation aiming to be coupled with a radiative flowfield. Conical rays are introduced in order to estimate the Eddington tensor accurately by solving the steady-state transfer equation even with a distant light source. The computed results show that the proposed method reasonably reproduces an analytical radiation energy flux with a light source in an optically thin situation while the conventional ray method cannot at the distant place from the source. Since the present conical-ray method is slightly expensive for coupling computation with a flowfield, we should explore an effective computation within a feasible cost, e.g., utilizing the GPU technology.

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