Generic features of the phase transition in cold and dense quark matter

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We investigate the phase transition in cold and dense quark matter in an intuitive way that shares common features of the effective model approaches. We first express the quasi-particle contribution to the thermodynamic potential with the dynamical mass $M$ and then discuss how we can understand the possible first-order phase transition with and without the vector interaction from the saturation curve on the plane of the energy per particle and the density. We next extend our analysis including inhomogeneity and discuss the relation between the order of the phase transition and the saturation curve. We emphasize that the saturation curve is useful to infer qualitative nature of the phase transition even without knowing the explicit solution of the ground state.

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I. INTRODUCTION

The quest for the phase diagram of strongly interacting matter out of quarks and gluons (i.e. matter described by Quantum Chromodynamics – QCD) is one of the most challenging problems in modern theoretical and experimental physics. There are many speculations on the QCD phase diagram from theory such as the color-superconducting phase [1], the quarkyonic state [2] and the triple-point-like structure [3], the QCD critical point [4], and so on (see Refs. [5, 6] for comprehensive reviews). Available experimental information [7] is, however, too limited to constrain uncertainties on those speculative possibilities (see Ref. [8] for an attempt and also Refs. [9, 10] for physical interpretations). Among others, the QCD critical point search is vigorously ongoing in the present and future experimental facilities as well as in the first-principle calculation of the lattice-QCD simulation.

The QCD critical point would be, if discovered, a landmark for our understanding on QCD matter. In an infinite-volume system at equilibrium, the fluctuations are expected to show critical behavior and thus the criticality would serve as experimental signatures [11]. There are many theoretical proposals and experimental data taken from the beam-energy scan program at Relativistic Heavy Ion Collider (RHIC) in the Brookhaven National Laboratory. It is an urgent question to make clear whether the QCD critical point exists and, if any, where it is located.

Because of the notorious sign problem with finite quark chemical potential $\mu_q$, the importance sampling breaks down in finite-density simulations. Although theoretical attempts are making steady progresses, (temporal) finite-volume effects are not easily treatable [12], and it is still difficult to extract any reliable conclusion even on a qualitative level. Then, under these circumstances, there are three major passages toward the QCD phase diagram studies (except for recent developments in the functional method [13]).

1. One can discuss the critical phenomena assuming the QCD critical point. This is a common strategy of theory in general. Since the critical properties are universal, one can make model-independent predictions. The virtue of this approach is the generality, but it does not give any clue about the concrete structure of the QCD phase diagram.

2. One can utilize the effective model description with a reasonable choice of the model parameters [14]. The location of the critical point is sensitive to model details. Not that all model results are model dependent, but the nature of the phase transition at high density strongly depends on a part of the model setup, as we will elucidate later.

3. One can make a conjecture on the phase structure based on generic properties of QCD such as symmetries [15, 16] and the degrees of freedom in a particular limit [2]. Because the argument lacks for concrete dynamics unlike the model study, one should check individually which scenario is favorable in reality. Nevertheless, such a conjecture from physics deliberation provides us with a useful guideline for model analysis.

The aim of the present work is to establish a path from 2 to 3 in the above classification. This is a route rather opposite to conventional approaches. Instead of choosing a particular model description, we shall extract the essential ingredients common in most model studies and try to unveil the underlying physics mechanism in a way free from model artifacts. In particular, by looking at the saturation curve, i.e. the energy per particle as a function of density, we can clearly see the nature of the liquid-gas phase transition, which also enables us to understand why the vector interaction would disfavor the first-order phase transition. It is a straightforward extension to include inhomogeneity as the chiral spiral for simplicity, and we can then find that the phase structure still has rich contents, which is again understandable from the saturation curve.

II. FIRST-ORDER PHASE TRANSITION AT ZERO TEMPERATURE

Let us start our analysis utilizing the same setup as Ref. [17]. We treat cold and dense quark matter in
a quasi-particle description. This means that we assume a Fermi liquid of quark matter, which should be valid for bulk thermodynamic quantities as long as $T$ is small enough and the Landau damping is a minor effect. Strictly speaking, our strategy would work in a density region between two onsets: one for quark deconfinement and the other for color superconductivity. It is very hard to quantify deconfinement and a phenomenological study of the equation of state [18] implies that quark-hadron crossover may start around the baryon density $\rho_B \sim 2\rho_0$ with the normal nuclear density $\rho_0 \simeq 0.17$ fm$^{-3}$. The phase structure involving color superconductivity is more ambiguous and severely dependent on the models around $\rho_B \sim 5\rho_0$ [19]. Therefore we should restrict the validity of our treatment within a range $2\rho_0 \lesssim \rho_B \lesssim 5\rho_0$. This is, however, a rather conservative estimate and should be loosened at a higher temperature where quarks would be more liberated.

In this way the thermodynamic potential from quasiparticles, $\Omega_{\text{matter}}$, is expressed as a function of the effective mass $M$ in a form of

$$\Omega_{\text{matter}}[M]/V = -\int_0^{\mu_q} d\mu' \rho(\mu') - 4N_cN_fT\int \frac{d^3p}{(2\pi)^3} \ln(1 + e^{-\omega_p/T}), \quad (1)$$

where $\rho(\mu)$ is the quark number density defined by $\rho(\mu) = 2N_cN_f\int \frac{d^3p}{(2\pi)^3} n_F(\omega_p - \mu) - n_F(\omega_p + \mu)$ with the Fermi-Dirac distribution function, $n_F(\omega_p) = (e^{\omega_p/T} + 1)^{-1}$, and the quasi-particle energy, $\omega_p = \sqrt{p^2 + M^2}$. It is important to note that this $\mu_q$-dependent matter part is common in any quark models such as the (P)NJL and the (P)QM models [14]. Then, the model uncertainty is unavoidable in the vacuum part.

In a quasi-particle picture of quarks the vacuum part could be expressed as $\Omega_0[M]/V = -2N_cN_f\int \Lambda \frac{d^3p}{(2\pi)^3} \omega_p + U[M]$ with a potential term. If we postulate that $U[M] = (M - m)^3/(4g_4)$, then $\Omega_0[M] + \Omega_{\text{matter}}[M]$ exactly amounts to the thermodynamic potential in the NJL model with the bare mass $m$ [20]. To implement the $U(1)$ anomaly in the three-flavor case, we may add a term $-g_4(M - m)^3$ in $U[M]$. From now on, we shall adopt a more general form of $\Omega_0[M]$ inspired by the Ginzburg-Landau expansion, i.e.

$$\Omega_0[M]/V = a(M^2 - M^2)^2 - bM - cM^3. \quad (2)$$

Although the thermodynamic potential in hand is extremely simple, this setup sufficiently grasps the generic features of the phase transition in cold and dense quark matter. One may wonder if this polynomial form would miss a logarithmic singularity as discussed in Ref. [21]. There are two reasons why this is not a serious problem to our analysis: First of all, such a logarithmic singularity is related to the infrared singularity of massless fermion loops. As we will see later, we are more interested in the massive case than the chiral limit and the effect of the logarithmic singularity is only minor then. Second, this logarithmic term has no effect for the first-order phase transition at large $\mu_q$ and $T = 0$ because the phase transition typically exists around $M \sim M_0$ (see Fig. 6), which is far from the singularity near $M = 0$.

To enter the regime at higher temperature, one should consider the meson fluctuations that may give rise to T-dependent coefficients in Eq. (2). Therefore, strictly speaking, our analysis is valid only in the region with $\mu_q \gg T$. In what follows we consider only the $c = 0$ case, for we are interested in the mechanism in favor of the first-order phase transition and $c \neq 0$ would trivially stabilize the first-order transition.

Figure 1 shows the typical behavior of the potential. As discussed in Ref. [17] the matter part $\Omega_{\text{matter}}$ always has a minimum at $M = 0$ because the baryon density is the largest when quasi-particles are massless. Let us consider the condition for the first-order phase transition in the case of $T = 0$ in which Eq. (1) simplifies as: $\Omega_{\text{matter}}[V] = -N_cN_f/(12\pi^2)(p_F\mu^3 - \frac{2}{3}M^2p_F\mu + \frac{2}{3}M^4\ln((\mu + p_F)/(\mu - p_F)))\theta(\mu - M)$ with $p_F = \sqrt{\mu^2 - M^2}$. In Ref. [17] the upper bound for the curvature $a$ was estimated under a reasonable but limited situation, $\mu_q \simeq M_0$. We can relax this numerically only to find that a first-order phase transition can remain in the chiral limit ($b = 0$) unless we choose unphysical parameters so that a phase transition takes place at $\mu_q \gg M_0$. Then $\Omega_{\text{matter}}$ stretches far beyond $M \sim M_0$ and the phase transition is no longer of first order.

This simple analysis tells us that the first-order phase transition at $T = 0$ can occur since $\Omega_{\text{matter}}$ is proportional to $\theta(\mu - M)$ and $\Omega$ does not have to contain a $M^6$ term, while $\Omega$ is sometimes assumed to take a form of $c_2M^2 + c_4M^4 + c_6M^6$ at $T \neq 0$. Thus, the present
formalism based on the quasi-particle approximation is more appropriate for the investigations of cold and dense quark matter.

Furthermore, we must add a term $\propto \rho^2$ in $\Omega_{\text{matter}}$, which stems from the vector-channel interaction $(\bar{\psi} \gamma_\mu \psi)^2$ that is chiral symmetric [22], i.e.

$$\Omega_{\text{vec}}[M]/V = g_v \rho^2,$$  \tag{3}

which can be evaluated with $\rho$ numerically which is obtained as $\rho = \frac{N_c N_f}{8\pi^2} (\mu^2 - M^2)^{1/2} \theta(\mu - M)$ at $T = 0$. We should note that in the mean-field NJL model with the vector interaction, usually, the vector interaction would shift the chemical potential, which pushes the energy up by $\sim 2g_v \rho^2$, and the condensation energy is negative, $-g_v \rho^2$, leading to $\sim 2g_v \rho^2 - g_v \rho^2 = g_v \rho^2$ in total. Here we simply postulate this in a form of Eq. (3).

For a deeper insight, Fig. 2 is quite instructive. This figure shows the location of two degenerate minima in the potential (i.e. the dynamical mass) when $\mu_\nu$ takes a value at the first-order phase transition. For example, in the chiral limit, the dynamical quark mass jumps from $M \approx M_0$ to $M = 0$. The jump is naturally reduced at larger $b$ (larger quark mass) and eventually only crossover remains beyond the bend of the curves in Fig. 2. One can notice that the curve substantially shrinks with positive $g_v$ which disfavors the first-order phase transition.

It is interesting to see that the vector interaction has only a minor impact for $b = 0$. This is because the minimum at $M = 0$ is intact as long as chiral symmetry is exact at $b = 0$ and $\rho$ and thus the vector interaction is still very small at $M = M_0$. This observation is, however, not completely free from the model choice. If the phase transition is located at $\mu_\nu > M_0$ with some other choice of parameters, the potential minimum around $M = M_0$ is also influenced substantially by the density effect and thus the first-order phase transition could be diminished by the vector interaction. This part of uncertainty is not relevant, for we are interested in the physical world with finite quark mass after all.

Guided by Fig. 2 we shall specifically look at the following three cases: (1) $b = g_v = 0$ (first-order), (2) $b = 0.08M_0^3$ and $g_v = 0$ (weak first-order), and (3) $b = 0.08M_0^3$ and $g_v = 0.12/M_0^3$ (crossover).

For later convenience we shall plot the energy per particle $\varepsilon/\rho_B$ at $T = 0$ in Fig. 3, where $\varepsilon = \Omega/V + \mu_B \rho_B - \Omega_0/V$ is the internal energy density measured from the hadronic vacuum with $M \sim M_0$ (before a finite density appears), and $\rho_B = \rho/N_c$ is the baryon number density. If the curve has a minimum as a function of $\rho_B$, i.e. $d(\varepsilon/\rho_B)/d\rho_B = \mu_B/\rho_B - \varepsilon/\rho_B^2 = 0$, the pressure difference becomes zero, which indicates a first-order phase transition of the general liquid-gas type (see Ref. [23] for a review and also Ref. [24] for experimental studies). Therefore, whenever $\varepsilon/\rho_B$ has a minimum as a function of $\rho_B$, the $T = 0$ system must have a first-order phase transition in the same way as the (symmetric) nuclear matter phase transition at $\mu_B = M_N - B$ with $M_N \approx 939$ MeV being the nucleon mass and $B \approx 16$ MeV the nuclear binding energy. At the second-order transition, the energy curve should be flat at the point of inflection. This kind of analysis on quark matter is well known in the context of quark droplets [25] but less applied in the phase diagram research. What is necessary for the existence of the critical point (first-order phase transition) is a convex structure of the curve (saturation property), which is a general statement that does not rely on any model nor Ansatz.

Because this point of the liquid-gas transition is so important, let us recall here how an intermediate density between $\rho = 0$ and the saturation density $\rho = \rho_0$ can be realized in this case. If the energy per particle has a min-

![FIG. 2](image1)

![FIG. 3](image2)
that the vector interaction \( \rho \) as a spatial average over bubbles with the core with \( \rho \sim \rho_0 \) in the empty vacuum. Though the surface energy effect is not considered in the above schematic figure where a simple nucleon-gas picture is depicted, the actual bubble shapes in a nuclear liquid depend on the surface term \( a_S \), etc.

minimum as schematically shown in the upper panel of Fig. 4 it would be energetically preferable to form bubbles with the core with \( \rho \sim \rho_0 \) rather than a homogeneous distribution of dilute \( \rho \). If we consider the surface energy, the density gradient (Weizsäcker) term, and the charge neutrality, bubbles should take optimal shapes such as the nuclear pasta (spaghetti, lasagna, etc) [26]. Such a state of matter is nothing but a mixed phase associated with the first-order phase transition, and importantly, this argument already implies the existence of an inhomogeneous ground state near the liquid-gas transition. In other words, if a mixed phase is characterized by a typical wave number \( q \), how can we strictly distinguish such a phase from an inhomogeneous ground state? One may think that in the case of quark matter the inhomogeneity pattern has been considered repeatedly in various contexts such as the pion condensation in nuclear matter [28], large-\( N_c \) QCD [29], the Overhauser instability [30], the quarkyonic spiral with confining force [31], and so on. The dispersion relation (4) should be plugged into \( \Omega_{\text{matter}}/V \) in Eq. (1). Unlike the normal dispersion relation, we see that a large part of the mass effect can be absorbed by \( q \sim M \), with which \( \rho \) is no longer suppressed even at large \( M \). This is the reason why a first-order phase transition can occur from the homogeneous hadronic phase to the chiral spiral where \( M \) is substantially large. Also, we should point out that the Ginzburg-Landau analysis in Ref. [32] to conclude that the chiral spiral is less favored might be inadequate; the largest energy gain in \( \Omega_{\text{matter}}/V \) comes from the region with large \( M \) where the Ginzburg-Landau expansion should not work.

The physical mechanism to lower the total energy is the Overhauser effect as argued in Ref. [30]. In the ordinary Overhauser instability the momenta of the spin-up component are shifted up by \( p_F \) and those of the spin-down component are shifted down by \( p_F \), so that a gap opens where two energy dispersion relations cross. In (1+1)-dimensional NJL model the situation is completely analogous [27]: a choice of \( q = 2\mu_1 \) eliminates the \( \mu_1 \) dependence and the energy gain originates from the fact that \( \rho \) is completely insensitive to \( M \) and thus \( \rho \) is never suppressed by \( M \) in contrast to the homogeneous solution. In (3+1)-dimensional case, on the other hand, not only \( p_z \) but also \( p_\perp \) share the Fermi momentum, and so the optimal \( q \) is not 2\( \mu_1 \) but rather \( q \sim M \) which will be confirmed by numerical calculations later.

Thus, \( \Omega_{\text{matter}} \) always tends to favor the chiral spiral with \( q \sim M \), while it is \( \Omega_0 \) that would hinder the growth of \( q \). In the leading order the vacuum part has an expansion in terms of \( q \) as

\[
\Omega_0[M,q]/V = \Omega_0[M,q = 0]/V + (\alpha M^2 + \beta b)q^2, \tag{5}
\]

where the first term with \( \alpha > 0 \) is a “kinetic” term against spatial modulation. This term should be van-
$$\frac{\alpha}{M_0} = 0.25$$

Figure 5. Typical phase diagrams with the chiral spiral. The solid curve in the lower-$\mu_q$ side represents the homogeneous chiral phase transition of first order with $b = g_\nu = 0$ with which the chiral spiral region surrounded by the first-order phase boundaries is attached. For $b = 0.08M_0^2$ and $g_\nu = 0.12/M_0^2$ the homogeneous first-order transition and thus the QCD critical point no longer appear, but the inhomogeneous region is enlarged as shown in the higher-$\mu_q$ side with a first-order boundary (solid curve) terminating at $P$ followed by a second-order boundary (dashed curve).

ishing at either $M = 0$ or $q = 0$, so the expansion should start with $M^2q^2$. One can estimate $\alpha$ using a chiral model, but one should be careful to not pick an unphysical term $\sim \Lambda^2q^2$ up from gauge-variant regularization. The latter term $\propto \beta$ comes from a phase of the current mass term associated with the basis change from $\psi$ to $\psi'$. Quantitative details may depend on $\alpha$ and $\beta$, but qualitative features as we discuss below do not rely on any specific choice of them.

Figure 5 shows typical behavior of the phase boundaries on the $\mu_q^{-1}T$ plane with zero and non-zero $b$ and $g_\nu$. For demonstration we chose $\alpha = 0.25$ and $\beta = 0.25/M_0$. Then in the lower-$\mu_q$ side of Fig. 5 we see that there is an island structure of the chiral spiral surrounded by the first-order boundaries. The solid curve extending to smaller $\mu_q$ represents a first-order phase transition associated with the homogeneous condensate only. It should be mentioned that the first-order phase transition at $b = 0$ in the high-$T$ and small-$\mu_q$ region, which is not of our present interest, might have been artificially strengthened due to the lack of the logarithmic singularity in Eq. (2). The first-order boundary of inhomogeneity at smaller $\mu_q$ stays very close to this curve. This is because the effective potential becomes very shallow near the first-order phase transition in the homogeneous case as clearly recognized in the total potential presented in Fig. 1. The secondary first-order boundary at larger $\mu_q$ is much weaker because $M$ and thus $q$ are small there. (Note that, in the chiral limit, $q$ may not decrease but only increase in a narrow region of $\mu_q$ as shown in Ref. [30]. This tendency near the first-order phase transition is partially seen also in the massive case in Fig. 7.) The corresponding saturation curve of $\epsilon/\rho_B$ is shown by a long-dashed curve with the label “CS” in Fig. 3, from which a minimum at lower energy is apparent. We note that the inhomogeneity island in the vicinity of the first-order phase transition is consistent with our intuitive discussions of the mix phase formation below Fig. 4.

With the vector interaction included, the so-called QCD critical point is easily washed out [17, 32, 33]. Interestingly, however, as shown in the higher-$\mu_q$ side of Fig. 5 and especially at $P$ in this figure, there is a chance that the critical point (strictly speaking, tri-critical point) is revived driven by the inhomogeneous condensate. The question is then how robust this observation is. In fact it has been reported that the soliton solution [34] is more stable than the chiral spiral and also it exhibits a second-order phase transition rather than a first-order one [32].

Let us then consider when the second-order phase transition is possible in view of the saturation curve in Fig. 3. To have a second-order phase transition from the hadronic phase (with homogeneous $M \sim M_0$) to a general inhomogeneous state, there must be an energy curve that is tangent to the hadronic branch (solid curves from $\rho_B = 0$) and going below it. The curves do not have to be flat because there is a small energy difference before and after a finite density appears, which is further enhanced by $1/\rho_B^2$ in the slope of the saturation curve. To avoid a first-order transition, moreover, the energy curve should be monotonically increasing with increasing $\rho_B$.

Such a situation is not allowed, for example, in the far bottom curves (at $b = g_\nu = 0$) in Fig. 3. In this case with the saturation energy lower than that at $\rho_B = 0$, we can conclude that only a first-order phase transition is possible however complicated and optimized modulations we introduce. The situation is different with finite $b$ and/or $g_\nu$. It is clear on a qualitative level that a larger $g_\nu$ would ease better inhomogeneous states to develop, for the dashed chiral-spiral curve could be then easily extended down to $\rho_B = 0$ monotonically. This means that the phase transition between the homogeneous and inhomogeneous states could be of second order. Therefore, unfortunately, the existence of the critical point $P$ is again not a robust conclusion especially with the vector interaction.

From a plain physical interpretation, it would be the most natural to have continuous phase transitions that border the inhomogeneous island. Such an intuition is based on the picture of the liquid-gas phase transition. In fact, if the boundary is a first-order phase transition, there will appear a density regime that can be described only as a mixed state. It is the role of the chiral condensate in quark matter that makes a difference from the situation in nuclear matter. The density modulation inherent in a mixed state can be mimicked by the modulation in the chiral condensate, which would lead to an inhomogeneous ground state of quark matter with lower energy. This is exactly what happens with the soliton solution.
and in Refs. \cite{32,34}. Indeed, at the onset of solitonic inhomogeneity, localized domain-walls start appearing, which approaches sinusoidal patterns at larger \( \mu_q \). The density profile has peaks arising from the kinks and this situation is reminiscent of a mixed state picture as schematically depicted in the bottom of Fig. 4. It would interesting to figure out the saturation curve corresponding to the solitonic solution. This is beyond our current scope, but it presumably goes below the chiral-spiral curves and is smoothly merged with the hadronic branch at smaller \( \mu_B \).

Finally let us take a closer look at the solution with \( b = 0.08M_0^4 \) and \( g_v = 0.12/M_0^4 \). Figures 6 and 7 show the behavior of the constituent mass \( M \) and the wave-number \( q \), respectively, as functions of \( \mu_q \) and \( T \). At a glance one may notice that \( q \sim M \) holds in the chiral-spiral region as we discussed. The structure of the chiral-spiral island is quite characteristic. In view of Fig. 7, one might say that the island is surrounded by a steep “cliff” at smaller \( \mu_q \) and a gentle “beach” at larger \( \mu_q \) \cite{39}.

Such a structure of the island should be quite robust because the energy gain is mainly attributed to \( q \sim M \). Hence, the cliff stands with a large energy gain at smaller \( \mu_q \) where \( M \) is still large, and the inhomogeneous state gradually becomes indistinguishable from the homogeneous state as \( M \) gets smaller at larger \( \mu_q \).

\section{IV. SUMMARY}

We have developed a picture of the first-order phase transition of quark matter based on the saturation curve and the liquid-gas phase transition. From this picture we discuss the relation between the order of the phase transition and the behavior of the saturation curve. We demonstrated this using a simple Ansatz of the chiral spiral, but the argument itself is not limited to such a special choice. As a matter of fact, because the chiral spiral can be mapped to the conventional pion condensation \cite{28} that is killed by the spin-isospin interaction, it may be likely that the chiral spiral should be suppressed by the axial-vector interactions \( \sim (\bar{\psi}\gamma_5\gamma_\mu\psi)^2 \) or \( \sim (\bar{\psi}\gamma_5\tau_\mu\tau_\nu\psi)^2 \), and eventually superseded by others such as the soliton-like modulation and more generally multiple-wave superpositions. Even in this case the saturation curve would provide us with valuable information on the nature of the phase transition.

We can think of several directions as future extensions. It may be interesting to seek for some connections between our saturation considerations and the Ginzburg-Landau analyses as in Ref. \cite{36}. Also, the interplay between the chiral spiral and the external magnetic field would deserve further investigations \cite{37}. We are actually working in this direction to clarify the phase structure with three axes, \( \mu_q \), \( T \), and \( B \), including the spatially inhomogeneous state \cite{38}.

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[40] This is a concrete manifestation of the “Happy Island” conjectured by Larry McLerran [35].