Multi-boson effects and the normalization of the two-pion correlation function

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I. INTRODUCTION

Two-particle Bose-Einstein (BE) interferometry (also known as Hanbury Brown-Twiss (HBT) intensity interferometry) as a method for obtaining information on the space-time geometry and dynamics of high energy collisions has recently received intensive theoretical and experimental attention. Detailed investigations revealed that high-quality two-particle correlation data constrain not only the geometric size of the particle-emitting source but also its dynamical state at particle freeze-out [1–3].

Two different definitions of two-pion correlation function are employed in the literature [1–14]. The first starts from the measured invariant i-pion inclusive distribution

\[ N_i(p_1, p_2, \ldots, p_i) = E_{p_1} \cdots E_{p_i} \frac{1}{\sigma} \left. \frac{d^3\sigma}{dp_1 dp_2 \cdots dp_i} \right|_{p_1 + \cdots + p_i = E}, \]  

which is normalized via

\[ \int \frac{d^3p_1}{E_{p_1}} \cdots \frac{d^3p_i}{E_{p_i}} N_i(p_1, \ldots, p_i) = \langle n(n-1) \cdots (n-i+1) \rangle \]  

(2)

to the i\textsuperscript{th} order factorial moment of the pion multiplicity distribution, and defines the two-particle correlator as

\[ C^I(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1)N_1(p_2)}. \]  

(3)

The second definition instead employs the normalized i-pion production probability

\[ P_i(p_1, \ldots, p_i) = \frac{N_i(p_1, \ldots, p_i)}{\langle n(n-1) \cdots (n-i+1) \rangle} \]  

(4)

and defines

\[ C^{II}(p_1, p_2) = \frac{P_2(p_1, p_2)}{P_1(p_1)P_1(p_2)}. \]  

(5)

It follows that

\[ C^{II}(p_1, p_2) = \frac{(n)^2}{(n(n-1))} C^I(p_1, p_2). \]  

(6)

Recently, Miśkowiec and Voloshin [14] argued that the first definition is preferable because it is based directly on measured quantities and it is consistent with the often used theoretical expression

\[ C(q, K) = 1 + \frac{1}{\int d^4 x S(x, K) e^{i q x}} \frac{1}{\int d^4 y S(y, p_2)}. \]  

(7)

Here \( K = (p_1 + p_2)/2, q = p_1 - p_2, q^0 = E_{p_1} - E_{p_2}, \) and \( S(x, K) \) is the emission function of the source. In this paper we will stress that the first definition has the additional advantage that it provides information not only about \textit{the shape of the correlator}, but also through its normalization about \textit{the pion multiplicity distribution} which is lost in the second definition. We will show that it can be exploited to search for multi-pion symmetrization effects and may thus be a useful ingredient in the HBT analysis of 2-pion correlation functions.

II. A SIMPLE EXAMPLE

To illustrate the importance of the normalization of the 2-pion correlator let us start with a simple example in which we consider the following multi-pion states:

\[ |\phi\rangle_m = A_m \exp([-\hat{B}^\dagger]^m)|0\rangle, \]  

\[ \hat{B}^\dagger = \int d^3p j(p) a^\dagger(p). \]  

(8)

(9)

The states (8) are normalizable for \( m \leq 2; \) the normalization \( A_m \) ensures \( m|\langle \phi|\phi \rangle_m = 1. \) For \( m = 1, |\phi\rangle_1 \) is a standard coherent state (8) with \( A_1 = \exp(-n_0/2) \) and \( n_0 = \int d^3p |j(p)|^2. \) Then the pion multiplicity distribution is of Poisson form,

\[ P(n) = \frac{n_0^n}{n!} \exp(-n_0), \]  

(10)

and the two-pion correlators (8) and (9) are given by

\[ C^I(p_1, p_2) = C^{II}(p_1, p_2) = 1. \]  

(11)

For \( m = 2 \) we have \( A_2 = (1 - 4n_0^2)^{\frac{1}{2}}, \) and the pion multiplicity distribution is given by
\[ P(n) = \begin{cases} 0 & , \ n = 2k + 1 \text{ odd}, \\ (1 - 4n_0^2)^{\frac{3}{2}} \frac{(2k)!}{(k!)^2 n_0^{2k}} & , \ n = 2k \text{ even}. \end{cases} \]  

(12)

Calculating the correlation functions we find

\[ C^I(p_1, p_2) = 2 + \frac{1}{4n_0^2}, \quad C^{II}(p_1, p_2) = 1. \]  

(13)

One observes that now \( C^I \) is different from \( C^{II} \) due to the fact that the pion multiplicity distribution is no longer of Poisson form. However, although \(|\phi_2\rangle\) is clearly not a coherent state, \( C^{II} \) is again equal to 1. The use of the second definition \( \langle \rangle \) thus does not allow to distinguish between the states \(|\phi_1\rangle\) and \(|\phi_2\rangle\), whereas the first definition \( \langle \rangle \) clearly does. One may, of course, argue that the only difference between \( C^I \) and \( C^{II} \) is the normalization factor which can be obtained independently by measuring the pion multiplicity distribution. Our point is that important information on the pion multiplicity distribution can also be extracted directly from the properly normalized correlation function, and that this opportunity should not be given away by working with probabilities rather than directly with the measured cross sections.

### III. Multi-Boson Effects on the Correlator and Its Normalization

We will now consider a more physical model and show again that the use of the second definition leads to a loss of interesting information about the source. It is well known that in relativistic heavy-ion collisions the pion multiplicity is so large that it may be necessary to take multi-pion BE correlations into account. In the following we will use a specific class of density matrices for multi-pion systems to study multi-pion BE correlation effects on the two-pion correlation function, thereby generalizing the conclusions of Ref. \( \langle \rangle \). For this class of ensembles it was shown in \( \langle \rangle \) that, after including multi-pion correlation effects, the two-pion and single-pion inclusive distributions can, in the notation of \( \langle \rangle \), \( \langle \rangle \), \( \langle \rangle \), \( \langle \rangle \), be written in the following simple form \( \langle \rangle \):

\[
N_1(p) = E_p H(p, p), \\
N_2(p_1, p_2) = E_{p_1} E_{p_2} \left[ H(p_1, p_1) H(p_2, p_1) \right. \\
\left. + H(p_1, p_2) H(p_2, p_1) \right], \\
H(p_1, p_2) = \sum_{i=1}^{\infty} G_i(p_1, p_2).
\]

(14)\n
(15)\n
(16)

The \( G_i(p, q) \) are defined as

\[
G_i(p, q) = \int \rho(p, p_1) d^3 p_1 \rho(p_1, p_2) \cdots d^3 p_{i-1} \rho(p_{i-1}, q).
\]

(17)

where \( \rho(p, p) \) is a Fourier transform of the source emission function \( g(x, K) \):

\[
\rho(p_i, p_j) = \int d^3 x g(x, K_{ij}) e^{i q_i \cdot x}.
\]

(18)

Here \( K_{ij} = (p_i + p_j)/2 \) and \( q_{ij} = p_i - p_j \). Inserting the expressions \( (\langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \) into Eq. \( (\langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \) one obtains

\[
C^I(p_1, p_2) = 1 + \frac{H(p_1, p_2) H(p_2, p_1)}{H(p_1, p_1) H(p_2, p_2)}. \]

(19)

This correlator goes to 1 as \( q \to \infty \) and to 2 as \( q \to 0 \). (Final state interactions are neglected here.) Thus even dramatic multi-boson effects as discussed below do not affect the interpretation of the correlator \( C^I \) — although they change the multiplicity distribution towards Bose-Einstein form they do not lead to genuine phase coherence.

Explicit expressions for the pion multiplicity distribution and its first two moments \( \langle n \rangle \), \( \langle n(n - 1) \rangle \) for the model studied here can be found in \( \langle \rangle \). Since \( H(p_1, p_2) = H^*(p_2, p_1) \), the second term in Eq. \( (\langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \) is always positive, ensuring that \( C - 1 \) is positive definite. The normalization conditions \( \langle \rangle \) therefore imply that for the class of systems studied here and in \( \langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \) one has always \( \langle n(n - 1) \rangle > \langle n \rangle^2 \) (see Fig. 3 below). Obviously, Eq. \( (\langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \) can therefore not apply to systems with multiplicity distributions \( P(n) \) which give \( \langle n(n - 1) \rangle < \langle n \rangle^2 \) (e.g., for systems with fixed event multiplicity \( \langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \)).

The structure of \( (\langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \) permits to introduce, in analogy to \( (\langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \), a modified source distribution \( S(x, K) \) via

\[
H(p_1, p_2) = \int d^4 x S(x, K) e^{i q \cdot x}
\]

(20)

such that the correlator \( (\langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \) can be written in the form \( \langle \rangle \). \( S(x, K) \) is related to the original source distribution \( g(x, K) \) via Eqs. \( (\langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \). \( S(x, K) \) includes all higher order multiparticle BE symmetrization effects. When interpreting measured single particle spectra and two-particle correlations one must keep in mind that the extracted information on the source corresponds to the effective source distribution \( S(x, p) \) rather than to the emission function \( g(x, p) \). The following example shows that these two functions can differ considerably; but we will also see that an important clue as to how much they differ will be provided by the normalization of the correlator.

As shown in \( \langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \) the recursion relations for the functions \( G_i \) in \( (\langle \rangle \langle \rangle \langle \rangle \langle \rangle \langle \rangle \) can be solved analytically for the class of model ensembles studied here if the following source distribution \( g(x, p) \) is assumed:

\[
g(r, t, p) = n_0 \left( \frac{1}{2 \pi R^2} \right)^{3/2} \exp \left( -\frac{r^2}{2 R^2} \right) \\
\times \left( \frac{1}{2 \pi \Delta^2} \right)^{3/2} \exp \left( -\frac{p^2}{2 \Delta^2} \right) \delta(t).
\]

(21)
\(g(r, t, p)\) is the Wigner density of the source in the absence of multi-pion symmetrization effects. It is obtained in [14,13] by folding the Wigner densities of individual Gaussian wavepackets with a classical phase-space distribution \(\rho_{\text{class}}\) for their centers (Eq. (19) in [20]; see also [17]). The parameters \(R, \Delta\) in (21) are thus combinations of the wavepacket width \(\sigma\) with the spatial and momentum space widths \(R_{\text{class}}\) and \(\Delta_{\text{class}}\) of \(\rho_{\text{class}}\) (see Eqs. (20,21) in [28]). While the width parameters \(R_{\text{class}}, \Delta_{\text{class}}\) of the classical distribution \(\rho_{\text{class}}\) are unconstrained, the widths \(R, \Delta\) of the Wigner density \(g(r, t, p)\) which result from the folding procedure always satisfy the quantum mechanical uncertainty relation \(R\Delta \geq \hbar/2\).

The input multiplicity distribution is taken to be Poissonian as in [10]; its mean value \(n_0\) can be interpreted as the mean pion multiplicity in the absence of Bose-Einstein correlations [16-17]. By inspection of Eqs. (16)-(18) we compute

\[
N_1(p) = E_p H(p, p) = E_p \int d^4 x S(x, p) \tag{22}
\]

![Fig. 1](image1.png)

**FIG. 1.** Multi-pion correlation effects on the single-pion spectrum. The dash-dotted line corresponds to the input distribution \(\int d^4 x g(x, p)\) with \(R = 3\) fm and \(\Delta = 140\) MeV. The other lines correspond to the measured distribution for various values of the average pion multiplicity \(\langle n \rangle\) per event. Also given are the corresponding average phase space densities \(d\) (see text).

as well as the normalized single-pion probability distribution in momentum space

\[
P_1^{(n)}(p) = \frac{E_p}{\langle n \rangle} \int d^4 x S(x, p) = \frac{E_p}{\langle n \rangle} \sum_{i=1}^{\infty} G_i(p, p). \tag{23}
\]

The mean pion multiplicity \(\langle n \rangle\) is given by

\[
\langle n \rangle = \int d^3 p d^4 x S(x, p) = \sum_{i=1}^{\infty} \int d^3 p G_i(p, p). \tag{24}
\]

For the model [21] \(P_1^{(n)}(p)\) is a function of \(p = |p|\) only. It is shown in Fig. 1 for different observed average pion multiplicities \(\langle n \rangle\). Next to the value \(\langle n \rangle\) we also give the average pion phase-space density of the system,

\[
d = \frac{\langle n \rangle}{(2R\Delta)^3}. \tag{25}
\]

One sees that as \(d\) increases the pions concentrate in momentum space at low momenta. This reflects their bosonic nature: pions like to be in the same state.

The instantaneous nature of the (effective) emission functions \(g(x, p)\) and \(S(x, p)\) (see [21]) allows for inversion of the Fourier transform (23): writing \(S(x, p) = S(r, p)\delta(t)\) we have

\[
S(r, K) = \int \frac{d^3 q}{(2\pi)^3} H(K + \frac{q}{2}, K - \frac{q}{2}) e^{iq \cdot r}. \tag{26}
\]

We define the normalized source distribution in coordinate space \(P^{(n)}(r)\) via

\[
P^{(n)}(r) = \frac{\int d^3 K S(r, K)}{\int d^3 K d^3 r S(r, K)} = \frac{1}{\langle n \rangle} \int \frac{d^3 K d^3 q}{(2\pi)^3} H(K + \frac{q}{2}, K - \frac{q}{2}) e^{iq \cdot r}. \tag{27}
\]

![Fig. 2](image2.png)

**FIG. 2.** Multi-pion correlation effects on the spatial pion distribution. The dash-dotted line corresponds to the input momentum distribution \(\int dt d^3 p g(x, p)\) with parameters \(R = 3\) fm and \(\Delta = 140\) MeV. The other lines correspond to the effective spatial distribution which would be extracted from HBT measurements, for various values of the average pion multiplicity \(\langle n \rangle\) per event. Also given are the corresponding average phase space densities \(d\) (see text).

Due to the spherical symmetry of (21) it is a function of \(|r|\) only. The function \(H\) in (26) is known analytically [16-19] to be a simple Gaussian in \(q\), rendering the Fourier transform trivial. The resulting \(P^{(n)}(r)\) is shown in Fig. 2 for different pion phase-space densities. One sees that with increasing phase-space density the multi-pion BE correlations also lead to a concentration...
of the pions in coordinate space. The fact that multi-pion BE correlations lead to a reduction of the HBT radius has been observed previously \[13,17,25\]. The radius extracted from HBT interferometry reflects the typical length scale of $P^{(n)}(r)$; it is always smaller than the input geometric radius $R$ of the source and depends on the mean pion multiplicity per event.

**IV. THE HIGH DENSITY LIMIT**

Taking the limit of a highly condensed Bose gas, $d = (2R\Delta)^3 \rightarrow \infty \[30\]$, the multiplicity distribution and 1- and 2-particle spectra can be determined analytically \[3\]:

$$P(n) = \frac{\langle n \rangle^n}{(\langle n \rangle + 1)^{n+1}}, \quad (28)$$

$$N_1(p) = E_p \frac{(n)}{(2\pi\Delta^2)\hbar^2} \exp \left( -\frac{p^2}{2\Delta^2} \right), \quad (29)$$

$$N_2(p_1, p_2) = 2N_1(p_1)N_1(p_2), \quad (30)$$

$$\Delta^2 \approx \frac{\Delta}{2R} \leq \Delta^2. \quad (31)$$

In this limit the correlation functions are

$$C^I(p_1, p_2) = 2, \quad C^{II}(p_1, p_2) = 1. \quad (32)$$

Multi-pion BE correlations change the original Poisson distribution into the Bose-Einstein multiplicity distribution \[28\]. Correspondingly, $\langle n(n-1) \rangle$ changes from 1 to 2.

In \[3\] this change exactly compensates the fact that the correlator $C^I$ no longer decays as a function of $q=p_1-p_2$, and from the resulting $C^{II} \equiv 1$ one might thus be misled to conclude (incorrectly) that the source exhibits phase coherence.

In Fig. 3 we show the ratio $\langle n \rangle^2/\langle n(n-1) \rangle$ as a function of the average pion phase-space density. In the lower diagram we plot it as a function of the ratio of input parameters $n_0/(2R\Delta)^3$, in the upper diagram we use as a measure of the phase-space density the analogous ratio formed with the measured average multiplicity $\langle n \rangle$. For low phase-space densities and large systems $(2R\Delta \gg 1)$

$$\langle n(n-1) \rangle = \langle n \rangle^2 = n_0^2. \quad (33)$$

For large phase-space densities $d \gg 1$, the ratio $\langle n \rangle^2/\langle n(n-1) \rangle$ decreases, eventually approaching for $d \rightarrow \infty$ the value 0.5 which reflects Bose-Einstein statistics. The critical phase-space density for the transition from Poisson statistics with $\langle n(n-1) \rangle = \langle n \rangle^2$ to Bose-Einstein statistics with $\langle n(n-1) \rangle = 2\langle n \rangle^2$ depends on the total phase-space volume $(2R\Delta)^3$, but for $2R\Delta \gg 1$ it occurs near $d \approx 0.3$. For $d \gg 1$, multi-boson symmetrization effects become dominant. The Bose condensation limit is reached at a finite critical value for the mean input multiplicity $n_0$:

$$n_0 \rightarrow n_c = (R\Delta + \frac{1}{2})^3 \geq 1. \quad (34)$$

As $n_0$ approaches the critical value (which for large systems $2R\Delta \gg 1$ corresponds to $n_c/(2R\Delta)^3 \approx \frac{1}{2}$), the observed mean multiplicity $\langle n \rangle$ and phase-space density $d$ (as well as the total energy) go to infinity \[16\]. In this sense the limit $n_0 \rightarrow n_c$ here is analogous to the limit $\mu_\pi \rightarrow m_\pi$ in a thermalized pion gas of infinite volume in the grand canonical formalism.
V. NORMALIZED CORRELATION FUNCTIONS FROM EXPERIMENT

A direct experimental determination of the phase-space density \( d = \langle n \rangle / (2\pi R)^3 \) in high energy collisions is not easy. The sources created in such collisions feature strong collective expansion [21], and therefore only small fractions of the total collision region (so-called regions of homogeneity) contribute effectively to the two-particle correlator \( \tilde{N} \). This means that in the parametrization \( \tilde{N} \) we should use \( R^2 = R_{\text{hom}}^2 + 1/(4\Delta^2) \) where \( R_{\text{hom}} \) is the (pair momentum dependent) homogeneity radius which, in the absence of strong multi-pion effects, is equal to the HBT radius parameter \( R_{\text{HBT}} \). Without more detailed model studies it is then, however, unclear what fraction of the total observed multiplicity \( \langle n \rangle \) comes from a single such homogeneity region.

On the other hand Fig. 3 suggests that, within our model class of event ensembles, the ratio \( \frac{\langle n \rangle^2}{\langle n(n-1) \rangle} \) is a useful indicator for the average phase-space density \( d \) in the source and thereby also for the expected multi-pion symmetrization effects on the 1- and 2-particle spectra which need to be taken into account in an extraction of the source size from HBT measurements.

In the experiment one usually fits the two-particle correlator with the functional form

\[
C_{\exp}(p_1, p_2) = C_{\exp}(q, K) = N' \left( 1 + \lambda f(q, K) \right)^2, \tag{35}
\]

where the function \( f \) vanishes as \( q \to \infty \). Obviously, \( N' \) depends on the chosen definition of the correlation function: For the definition \( \tilde{N} \) the normalization is always \( N' = 1 \) (see Eq. (13)), while for the definition \( \tilde{N} \) it is \( N' \tilde{N} = \frac{\langle n \rangle^2}{\langle n(n-1) \rangle} \) (see Eq. (8)). But in both cases \( N' \) is well-defined and thus should not be treated as a free fit parameter. Therefore we now discuss shortly an algorithm for the experimental construction of the two-particle correlator which is guaranteed [32] to give the correct value for \( N', \) without relying on an actual measurement of the multiplicity distribution.

We write the single-pion inclusive distribution as

\[
N_1(p) = \frac{E_p}{N_{\text{ev}}} \sum_{i=1}^{N_{\text{ev}}} \nu_i(p). \tag{36}
\]

\( N_{\text{ev}} \) is the total number of collision events, and \( \nu_i(p) \) is the number of pions with momentum \( p \) in collision \( i \). The two-particle distribution can be expressed as

\[
N_2(p_1, p_2) = \frac{E_{p_1} E_{p_2}}{N_{\text{ev}}} \sum_{i=1}^{N_{\text{ev}}} \tilde{\nu}_{i,1}(p_1, p_2), \tag{37}
\]

where \( \tilde{\nu}_{i,1}(p_1, p_2) \) is the number of pion pairs with momenta \( (p_1, p_2) \) in collision event \( i \), and the double index \( i \) indicates that both particles are from the same event.

These definitions satisfy the normalization conditions \( \tilde{N} \).

While \( N_2(p_1, p_2) \) is constructed by selecting pion pairs from the same events, the denominator \( N_1(p_1)N_1(p_2) \) can be generated by combining pion pairs from different events \( \tilde{N}_2 \)[33][1][1][2][4]. The proper prescription is

\[
N_1(p_1)N_1(p_2) = \frac{E_{p_1} E_{p_2}}{N_{\text{ev}}(N_{\text{ev}} - 1)} \sum_{i,j=1, i \neq j}^{N_{\text{ev}}} \tilde{\nu}_{i,j}(p_1, p_2) \tag{38}
\]

where \( \tilde{\nu}_{i,j}(p_1, p_2) = \nu_i(p_1)\nu_j(p_2) \). One easily checks that

\[
\int \frac{d^3p_1}{E_{p_1}} \frac{d^3p_2}{E_{p_2}} N_1(p_1)N_1(p_2) = \langle n \rangle^2. \tag{39}
\]

The ratio of (37) and (38) thus gives the properly normalized correlator \( C' \). The above equations are true for unbiased events. Trigger biases and limited experimental acceptances can induce residual correlations in the event-mixed “background” [33] which must be corrected for separately (see [8] for an extensive discussion).

For large \( N_{\text{ev}} \), the evaluation of (38) is very time consuming; it also leads to a statistically unnecessarily accurate result for the denominator in (3). In practice one can live with fewer event pairs for event mixing, by replacing in (38) the number \( N_{\text{ev}} \) by a much smaller number \( N'_{\text{ev}} \). As long as \( N'_{\text{ev}} - 1 > N_{\text{ev}} \) one can still ensure that the contribution of the denominator to the statistical error of the final correlation function is negligible [32].

The correlator \( C'' \) differs from \( C' \) only by the different normalization. It can be constructed by taking the ratio of the following two expressions [8]:

\[
P_2(p_1, p_2) = \frac{E_{p_1} E_{p_2}}{N'_{\text{pairs}}} \sum_{i=1}^{N'_{\text{ev}}} \tilde{\nu}_{i,1}(p_1, p_2), \tag{40}
\]

\[
P_1(p_1)P_1(p_2) = \frac{E_{p_1} E_{p_2}}{N'_{\text{pairs}}} \sum_{i,j=1, i \neq j}^{N'_{\text{ev}}} \tilde{\nu}_{i,j}(p_1, p_2). \tag{41}
\]

\( N'_{\text{pairs}} \) and \( N''_{\text{pairs}} \) are the total numbers of “correlated” and “uncorrelated” pion pairs, respectively:

\[
N'_{\text{pairs}} = N_{\text{ev}} \cdot \langle n(n-1) \rangle, \tag{42}
\]

\[
N''_{\text{pairs}} = N_{\text{ev}}(N_{\text{ev}} - 1) \cdot \langle n \rangle^2. \tag{43}
\]

VI. CONCLUSIONS

We have shown that in principle the normalization of the two-particle Bose-Einstein correlation function contains valuable information on the multiplicity distribution of the event ensemble. Both theoretically and experimentally the absolute normalization of the correlation function should thus be controlled as well as possible. We presented a variant of a previously suggested
experimental algorithm [14] for the construction of the correlator which guarantees correctly normalized correlators. Within a specific model class of event ensembles which recently received extensive theoretical attention we showed that in systems with large pion phase-space densities multi-pion symmetrization effects can lead to interesting measurable effects on the normalization of the correlator. We suggest a careful study of this normalization as an alternate method for searching for strong multi-pion symmetrization effects in high-multiplicity hadronic and heavy-ion collisions.

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[29] The expressions for $G_{2}(p,q)$ in Refs. [16,17] and [18,19] look at first sight quite different, but they become identical after suitable rearrangements [18].
[30] Since $2R_{\Delta} \geq 1$, the limit indicated in the text can only be achieved by letting $(n) \rightarrow \infty$ or $n_{0} \rightarrow n_{e}$ (see Eq. (34)).
[31] Combining this result with Eqs. (12) and (13) gives a correlator of the form $C^{II}(q,K) = \mathcal{N}(1 + |f(q,K)|^{2})$ with $\mathcal{N} = 1/[1 + (2R_{\Delta})^{-3}] \approx 1 - (2R_{\Delta})^{-3}$. A similar result was also obtained in Refs. [23] in the limit of small phase-space densities, although with quite different assumptions about the event ensemble. In particular, in Refs. [23] the events were assumed to have a fixed pion multiplicity $n$. For this reason the factor $\mathcal{N}$ in Refs. [23] cannot be associated with the pion multiplicity distribution $P(n)$; still, the leading dependence of the normalization of the correlator on multipion effects enters through the same factor involving the phase-space volume $(2R_{\Delta})^{3}$ of the source.
[32] This algorithm agrees with the recent suggestion by Miśkowiec and Voloshin [14] up to a minor detail: these authors suggested to use for the denominator in the correlation function mixed pairs constructed from $N_{\text{mix}}(N_{\text{mix}}-1)$ event pairs, $N_{\text{mix}}$ being chosen such that $N_{\text{mix}}(N_{\text{mix}}-1) \equiv N_{\text{ev}}$ where $N_{\text{ev}}$ is the number of events from which the correlated pairs for the numerator are extracted. With this choice the normalizing prefactors in the numerator and denominator of the correlation cancel automatically. Our algorithm uses a larger number of event pairs for event mixing and rescales the denominator accordingly; this ensures that the statistical error of the correlation function is dominated by the “signal” in the numerator, and it also agrees with current experimental practice (D. Miśkowiec, private communication).
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