Didactical design research of quadratic function based on learning obstacle and learning trajectory

R M Ruli¹, S Prabawanto² and E Mulyana²

¹ Departemen Pendidikan Matematika, Sekolah Pascasarjana, Universitas Pendidikan Indonesia, Jl. Dr. Setia Budhi No. 229, Bandung 40154, Indonesia
² Departemen Pendidikan Matematika, Universitas Pendidikan Indonesia, Jl. Dr. Setia Budhi No. 229, Bandung 40154, Indonesia

*redomartila@student.upi.edu

Abstract. Students understanding on the topic of quadratic equation and quadratic function is separated each other, whereas both are interconnected. This research aimed to formulate a didactic design, which will help junior high students understand the linkage between both topics. By employing a didactical design research (DDR), this research involved the process of repersonalization and recontextualization of quadratic function to explore the learning trajectories and learning obstacles within such a process. This design aimed to represent the context of the assigned problem, and was analyzed through the following steps: 1) the explorative prediction of students’ responses related to the assigned problem; 2) collaborative learning to lead the students in understanding the linkage of both concepts: quadratic equation and quadratic function. Assigned by a contextual problem related to quadratic function, the students could widen their understanding about quadratic equation, but simultaneously encountered difficulties in relating the topic with quadratic function. In this case, collaborative learning helped them in doing so.

1. Introduction

The process of learning mathematics is strongly influenced by how students view the mathematics itself. Therefore, teachers should direct how students view math to create conducive learning environment. Because of basically in the learning process, mathematics is often identified with negative connotation by students. Most of the students make math as a subject that is strongly avoided and even feared by students. Therefore, in the process of learning mathematics, the teachers should be able to create a harmonious, friendly, and fun class atmosphere so that the students have an interest and willingness to follow the learning process. Therefore, the teacher should try to create a condition where the students can be active and creative in following the learning activities, such as giving the students the opportunity to think creatively and to talk and write something, to give the exercises that require responsibility and to provide the challenging work to do. This will make students feel more confident and create a learning atmosphere that can support the establishment of good relationships between teachers, students and mathematics subjects in the classroom.

The process of learning mathematics basically related to three things, namely: teachers, students, and materials[1]. These three things must go hand in hand for the success of a learning process. In addition to these three things, the success in learning is also determined from how the teacher designing the
learning process. A good teaching and learning process should be preceded by planning or designing a good learning [2]. Instructional design is a big responsibility for designing teaching and learning process [3]. In designing the learning process of mathematics, teachers must involve a complex set of processes, confusing and unique. Started from conducting curriculum analysis to determine what topics and lesson themes will be delivered [4]. In this case, the teacher will make a learning trajectory about the material presented to the students. Learning trajectory helps teachers understand students’ level of knowledge as the key to presenting what students need, it is intended that teachers can understand the material to be delivered [5]. By understanding the learning trajectory of a material that will be taught, the teacher can create a learning design that can help students to understand the subject matter better.

In order to achieve these objectives, in the preparation of a lesson plan, the teachers must be able to perform repersonalization and recontextualization in advance to examine the concept of mathematics more deeply seen from the relevance of the concepts and the context. Repersonalization is a process of mathematization as mathematicians do, if a concept is related to the concept before and after it. While recontextualization is an attempt to rethink the context used in the learning process. Thus, before doing the learning process, a teacher needs to examine the mathematical concepts that will be taught in more depth and see the relationship of interconnection between the concept and context. Various experiences gained from the process of repersonalization and recontextualization become valuable materials for teachers when teachers try to overcome the difficulties experienced by students and sometimes the difficulty is exactly the same process he experienced during the repersonalization and recontextualization [1].

Basically, mathematic is a discipline that consists of several topics. Mathematical topics given to students in elementary to secondary education include numbers and operations, algebra, geometry, measurement and data analysis, and measurement (National Council of Teachers of Mathematics, 2000). The topics that are always become a problem for the students are the topics related to symbols and notations. Students often have difficulty in describing problems in the form of mathematics or in the form of symbols and expressing ideas that have not been fulfilled as a whole [6].

One of the topics where students often experience obstacles in learning is a function, especially a quadratic function. When drawn backward, the topic of quadratic function is essentially a merging of algebra and geometry topics. In this case, the two topics are the topics that become obstacles for students to understand math. For algebraic topics, students have difficulty in solving problems related to variables and equations. The number of upper secondary students who have problems in learning algebra as they do not understand the meaning of the equation [7]. If students have not been able to interpret an equation in algebra, then students will have difficulty in solving a problem related to algebra. While on the topic of geometry, students are basically easier to understand because students have been doing basic concepts of geometry in everyday life. However, students have difficulties when asked to find a solution of a problem of school geometry. So, to be able to understand the concept of quadratic functions, teachers should be able to design learning that is able to build students' understanding of the topic of algebra and geometry.

There are some barriers commonly experienced by students in teaching the concept of functions, especially the squared function. Istiqomah based on his research found some of the epistemological obstacles of functional topics, namely: (1) epistemological barriers related to the existing concept of definition of function, (2) epistemological constraints related to the context of variation of information available on the question, (3) the epistemological barriers associated with the students’ ability to communicate the existing information into functional notation, and (4) the epistemological barriers associated with functional concepts with other mathematical concepts especially in the concept of numbers, equations and algebraic operations [8].

Kotsopoulos found that middle-class students showed difficulties when performing squared factorization [9]. The difficulty arises when students were asked to repeat the fact of multiplication. He stated that the students were confused when given non-routine questions as they usually do, for example: $x^2 + 3x + 1 = x + 4$. Rahmawati, Suparta & Suweken found some obstacles that arise when
students were exposed to open issues on the topic of quadratic functions, namely: 1) students do not understand the purpose of the problem given; 2) students felt confused in using “formula”, this is because the students were accustomed to work by using formulas; 3) students tended to memorize a problem solving; 4) students were not used to using elaboration; 5) some students solved the problems that were given in groups, not independent [10].

The same obstacles were also found by Ellis & Grinstead on the topic of quadratic functions, among others: 1) the relationship between algebra, how to solve problems with tables and also graphical representations; 2) graphic display as whole object; 3) the struggle to correctly interpret the role of the parameters; 4) the tendency to make a false generalization of the linear function. They found the students’ difficulties in connecting algebra with graphical representation in which two-thirds of the students they interviewed explained the purpose of parameter \(a\) in \(y = ax^2 + bx + c\) as a slope of a quadratic function. This is not true since the slope is not part of the quadratic function, but is part of the linear function [11].

Learning barriers experienced by these students impacted on the influence of the level of mathematical ability of the students. One of the efforts in improving the students’ mathematical abilities is by learning that accommodates students to construct their own knowledge. The National Council of Teachers of Mathematics (2000) stated that students must study mathematics with understanding, actively building new knowledge based on prior experience and knowledge. It is the task of a teacher to help and accommodate the student in building his or her own understanding of a material, especially on the subject matter. The basic knowledge that students must possess in understanding the functional material is to understand how to interpret the signs used for algebra, the variables, and to solve algebraic expressions and equations and to relate the understanding of algebra into the understanding of geometry, and vice versa.

To realize the learning process as expected, the instructional materials that are designed must provide the support for the learning process. The didactic design that is designed based on the Learning Obstacle experienced and the Learning Trajectory that students pass through in studying certain materials is expected to facilitate students to better understand the material. The task of the math teacher becomes broader than before, during and after the learning takes place is to create a didactical situation [12]. so, the learning process occurs within the students. The main role of a teacher is to create a didactic situation so that the learning process occurs within the students. The didactic situation is designed and assembled in such a way as to be a good didactic design [1].

2. Method
This research focuses on studying the learning obstacle and quadratic functional trajectory learning and designing didactic design based on learning obstacle and learning trajectory so that the didactic design is expected to improve and develop the learning process in a better direction and can overcome the learning obstacle experienced by students. The subjects of the research were divided into two groups. The first group of the research subjects, in which to know the difficulty of learning, was the students of grade X senior high school which have studied quadratic function materials. The second group of the research subjects, in which the research was on the use of early didactic design of the concept of quadratic functions, was the students of grade IX junior high school. The main instrument in this study was the researchers themselves, where the researchers’ functions were in determining the research factors, selecting the information as a source of data, determining the quality of data, analyzing and making conclusions from the data obtained. However, the additional instruments were used to obtain the information needed. The types of instruments used in this study were test instruments and non-test instruments. The test instrument was used to test learning obstacle and test the didactic design that has been created. For non-test instrument used were interview, observation, and documentation.
3. Result and discussion

3.1. The result of analysis on learning obstacle test and student interviews
Based on the result of learning obstacle test to 23 students of grade X and the result of interview with 3 students, the learning barrier are as follows:

3.2. Didactic learning obstacles
The students experienced didactic obstacles in studying the problems of quadratic equations and quadratic functions identified from interviews with 3 students. In the problems given during the trial of learning obstacle, the students had difficulties in understanding and representing the problems given. As can be seen in the problem number 1 at figure 1, the students were not familiar with the problems given, the limitations of students in solving the problem at number 1 is due to the students’ knowledge of the given material is limited to procedural knowledge.

![Figure 1. Students answer in the problem number 1.](image1)

This can be seen at figure 2 when the students were asked to solve the problems in number 3 that already in the form of equations, the number of students who completed and answered correctly were more than the problem number 1. This is likely because the students were familiar with problems that already form equations.

![Figure 2. Students answer in the problem number 3.](image2)
As for the problem number 2 that can be seen at figure 3, based on the results of learning obstacle test and student interviews, found that students had difficulties in solving the problems given was the topic that only briefly taught in the class, so the students had not really understand how to represent the function into the form of quadratic function graph.

![Figure 3. Students answer in the problem number 2.](image)

3.3. Epistemological Learning obstacle
The students experienced epistemological barriers when creating a representation of a problem either in the form of equations or in graphical form. This is showed from the results of experimental learning obstacle and also the results of student interviews. For the number 1 problem, it appears that the students did not have epistemological barriers, because of the 23 students who follow the trial of learning obstacle as much as 6 students already can make representations into the image form of the problems given, but in solving the problem the students were more likely to enumerate how length and width of the given problem by extent. Then as many as 6 students also have been able to solve the problems given by making representations into the form of quadratic equations. But for problem number 2, the majority of students had epistemological barriers in representing the given problem into graphic form. Most of the students had difficulty in determining the x and y coordinate points. While on problem number 3, found no epistemological obstacles of students in solving the problems given, because as many as 16 students have been able to solve the problems given, but only almost entirely answered by way of factoring. This indicates that the students seem to have a concept image that solving a quadratic equation can only be done by one way.

4. Conclusion
Based on the results of the analysis learning obstacle, the next step that will researchers do to overcome of students learning obstacle is to do the analysis of learning trajectory on the topic of the quadratic function. In addition, another way that can help reduce students’ learning obstacles regarding quadratic function lesson is to create a learning design in accordance with their characteristics and materials’ characteristics that will be taught, so that their understanding of quadratic function is not gained partially.
References
[1] Suryadi D 2010 Metapedadidaktik dan Didactical Design Research (DDR): Sintesis Hasil Pemikiran Berdasarkan Lesson Study dalam Teori, Paradigma, Prinsip, dan Pendekatan Pembelajaran MIPA dalam Konteks Indonesia (Bandung: FPMIPA UPI)
[2] Kunandar 2014 Penilaian Autentik: Penilaian Hasil Belajar Peserta Didik Berdasarkan Kurikulum 2013 (Jakarta: PT Raja Grafindo Persada)
[3] Isman A 2011 Instructional Design in Education: New Model. The Turkish Online Journal of Educational Technology 10 (1)
[4] Suratno T 2016 Didaktik and Didactical Design Research in D Suryadi, E Mulyana, T Suratno, D A K Dewi and S Y Maudy (Eds) Monograf Didactical Design Research (Bandung: Rizki Press)
[5] Clements D H and Sarama J 2009 Learning and Teaching Early Math: The Learning Trajectories Approach (New York: Routledge)
[6] Junaidah 2015 Meningkatkan Kemampuan Pemahaman, Komunikasi dan Disposisi Matematis Siswa SMP Melalui Pendekatan Kontekstual (Bandung: Tesis PPs UPI)
[7] Kieran C 1979 Children’s Operational Thinking Within the Context of Bracketing and The Order of Operations. In D Tall (Ed) Proceeding of The Third International Conference for The Psychology of Mathematics Education (Conventry England: Mathematics Education research centre, Warwick University)
[8] Istiqomah D N 2015 Learning Obstacles terkait Kemampuan Problem Solving pada Konsep Fungsi Matematika SMP Seminar Nasional Matematika dan Pendidikan Matematika UNY 2015
[9] Kotsopoulos D 2007 Unraveling student challenges with quadratics: A Cognitive approach Australian Mathematics Teacher
[10] Rahmawati N L T, Suparta I N and Suwaken G 2016 Pembelajaran Dengan Visual Scaffolding Untuk Mengembangkan Kemampuan Siswa dalam Menyelesaikan Masalah Terbuka Materi Fungsi Kuadrat Prosiding Seminar Nasional MIPA 2016
[11] Ellis A B and Grinstead P 2008 Hidden lessons: How a focus on slope-like properties of quadratic functions encouraged unexpected generalizations. The Journal of Mathematical Behavior
[12] Brousseau G 1997 Theory of Didactical Situations in Mathematics (New York: Kluwer Academic Publisher)