pQCD Calculations of Heavy Quark and $J/\psi$ Production

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Abstract. We review the present status of theoretical predictions for both closed ($J/\psi$) and open heavy quark production in high energy collisions, and their comparisons to experimental data.

1. Introduction

The aim of this talk is to review the theoretical predictions of perturbative QCD (pQCD) for the production of either closed ($J/\psi$) or open heavy quark states in high energy collisions, and to compare them to the available experimental measurements. While the focus of Quark Matter is of course on heavy ion collisions, we shall however only describe predictions for the far simpler case of hadron collisions, aiming at establishing benchmarks for more complex environments. A further restriction is to consider only collinear factorization, since it is in this framework that calculations are most developed and, by often being carried out beyond leading order, they allow for an estimate of their own theoretical uncertainty.

2. $J/\psi$ production

The literature on heavy quarkonium production is extremely vast and it is impossible to fully cover it in a short talk. Hence the decision to describe only a specific approach, and to select a few processes able to highlight both its strengths and its weaknesses (for more comprehensive reviews see [1–3]).

In heavy quarkonium physics one immediately recognizes three energy scales, the heavy quark mass $m$, its momentum $mv$ and its kinetic energy $mv^2$, which allow to separate the problem into different parts. In particular, the relatively large scale of the heavy quark mass allows one to identify a short distance, perturbative component, controlled by the value of the strong coupling: $\alpha_s(m_c) \simeq 0.35$ and $\alpha_s(m_b) \simeq 0.2$ respectively. Next, considerations involving a Coulomb-like potential for the strong interaction, $V(r) \sim -\alpha_s(1/r)/r$, and the virial theorem lead to the relation $v \simeq \alpha_s(mv)$. 
This equation can be solved to yield $v_c \sim 0.6$ and $v_b \sim 0.3$, pointing therefore to the existence of a second ‘small’ parameter.

The two small parameters $\alpha_s$ and $v$ suggest a double expansion in their powers. The theoretical tool to exploit this situation in a theoretically consistent manner was proposed by Bodwin, Braaten and Lepage in 1994 in the form of an effective theory, Non-Relativistic QCD (NRQCD) [4]. This approach establishes the following factorisation theorem:

$$\sigma[H] = \sum_n \sigma_n(\mu) \langle O_n^H(\mu) \rangle,$$

i.e. the cross section for the production of the quarkonium state $H$ is given by a sum of products of a short distance cross section, calculable in perturbative QCD (pQCD), and a long-distance NRQCD matrix element. The first term contains the prediction for the production of a heavy quark-antiquark pair in a given spin, angular momentum and colour (both singlet and octet) state $n$. The second factor is a non-perturbative contribution accounting for its hadronisation into the heavy quarkonium state: it cannot be calculated, but NRQCD scaling rules, in terms of powers of $v$, allow for a controlled truncation of the series. Finally, $\mu$ is a factorisation scale.

This approach (often mistakenly called the ‘Colour Octet Model’) extends the Colour Singlet Model (CSM) in a systematic way, in that no divergences should be left in higher order calculations, and the series can be meaningfully truncated according to the scaling rules.

The first phenomenological success of the NRQCD approach was the explanation of the very large $J/\psi$ and $\psi'$ production at the Tevatron in $p\bar{p}$ collisions [5]: adding colour-octet contributions and fitting their non-perturbative NRQCD matrix elements to data, their values turned out to be roughly one-hundredth of the corresponding colour singlet ones (fixed either by decay rates or by potential models), and therefore in agreement with the scaling rule $\langle O_8^{J/\psi} \rangle / \langle O_1^{J/\psi} \rangle \sim v^4/2N_c \simeq 0.02$, $N_c = 3$ being the number of colours.

The first solid theoretical success can instead probably be considered the full next-to-leading order calculation [10] of all the $ij \to$ quarkonium processes, $i$ and $j$ being a light quark or a gluon. It was known since the late Seventies that some of these processes develop an uncanceled singularity in the CSM. In the NRQCD approach this singularity is seen to be correctly canceled by higher order corrections to the corresponding colour octet matrix element. One interesting recent development along these lines is the discovery [11] that the definition of the NRQCD matrix elements must be updated for this property to continue to hold to next-to-next-to-leading order.

Beyond these first successes, the NRQCD approach has had mixed results. One of the first problems to surface was the contribution of colour octet channels to $J/\psi$ photoproduction: with the normalisation fixed by the Tevatron fits, the theoretical predictions seemed to overshoot [6] the HERA data near the elastic region $z = 1$. In fact, it was soon argued that in this region large higher order contributions are likely to be present, and a meaningful comparison is only possible after a proper all-order
resummation. Once such a resummation was performed, the prediction was indeed found to be in much better agreement [7–9].

A second instance of a possible discrepancy between NRQCD predictions and experimental data was found in the exclusive process $e^+e^- \rightarrow J/\psi + \eta_c$. Leading order NRQCD predictions put its cross section around 4–5 fb, while experimental measurements performed by Belle [12] and BaBar [13] put it around 20 fb. Also in this case it was however soon realised that higher order contributions can be important. A recent paper (see [14] and references therein) puts together a number of such contributions, and arrives at a prediction of $17.5 \pm 5.7$ fb, in good agreement with the measurements.

The message from these two cases is therefore that when NRQCD predictions are to leading order the theoretical uncertainties from missing higher orders can be huge, partly as a consequence – especially for charm – of the relatively large size of the expansion parameters $\alpha_s(m_c)$ and $v_c$. It is therefore advisable, when judging how NRQCD fares in comparisons to experimental measurements, not to give too much weight to discrepancies with leading order predictions, and suspend the judgment at least until more reliable calculations are available.

The third and final kind of comparison between experimental data and NRQCD predictions which we address here revolves around the polarisation of $J/\psi$'s produced at the Tevatron. The naive NRQCD result is that, since most of these particles are predicted to come from gluon fragmentation and the gluon is nearly real at large transverse momentum, the $J/\psi$ should retain its transverse polarisation. In fact, more detailed calculations predict no net polarisation at low $p_T$, and then a steady increase towards transverse polarisation as $p_T$ increases. Unfortunately, the data do not seem to support this picture. Both Tevatron Run I data and preliminary Run II data are rather compatible with little or no polarisation even at largish $p_T$ (10 – 20 GeV), in contradiction with the prediction (see e.g. [3] for more details and references).

There is, at the moment, no clear understanding of what might cause this discrepancy. If the “blame” is to lie on the theoretical side, the NRQCD velocity scaling rules, which dictate the relative importance of the higher orders, might be to blame. Once more, however, one should remember that all the calculations are leading order ones, and more accurate predictions might once again change this picture.

3. Open heavy quark production

While theoretical predictions for production of closed heavy quark systems like $J/\psi$ pose evidently an extra level of challenge, it is obvious that they can be at best only as good as the predictions for open heavy quarks. In this section we shall therefore explore this issue. The produced system being simpler, the calculations can be more refined and accurate. On the other hand, the higher precision leads to the need for extra precautions when studying effects like non-perturbative fragmentation which, if the overall accuracy were lower, could be treated much more naively or even almost forgotten altogether.
Modern heavy quark phenomenology tries to provide theoretical descriptions for observables as close as possible to those which are really measured by experiments, within realistic phase space acceptances. This means that the typical process which is calculated looks like the following one:

\[ pp^{pQCD} \rightarrow Q^{NP fragm.} \rightarrow H_Q^{decay} \rightarrow e, \]  

where \( Q \) is the heavy quark, \( H_Q \) a heavy hadron, and \( e \) represents a generic final state observable, for instance a lepton. While the decay can be a complicated process, whose detailed description – typically via Monte Carlo techniques – is needed to faithfully reproduce the experimental measurement, it is nevertheless the first part of the reaction, up to the production of the heavy hadron \( H_Q \), which is controlled by QCD and which requires most of our attention. This process actually contains two different phases, which are not unambiguously separable: the first is the description in perturbative QCD of the production of the heavy quark \( Q \) in the hard collision; the second is its non-perturbative fragmentation into the heavy hadron. They both need to be carefully studied – and properly matched – in order to achieve a good theoretical precision.

The first building block of heavy quark production calculations is the factorisation theorem established by Collins, Soper and Sterman [15], which allows the cross section for production of a heavy quark to be written, up to higher twist corrections suppressed either by the heavy quark mass or by the (larger) transverse momentum, as a convolution of a calculable short distance perturbative term and of the parton distribution functions for light partons only. Next-to-leading order (NLO) calculations based on this theorem were successively performed [16–18], and constitute today the backbone of all phenomenological predictions.

While total heavy quark cross sections are a genuine perturbative prediction, and are usually reliably given by fixed order NLO calculations, for differential distributions – closer to what is really measured – one must also account for additional effects. On one hand, potentially large perturbative logarithms need to be resummed to all orders [19]. On the other, one needs to consider the momentum degradation related to the hadronisation of the quarks into the heavy hadrons. This non-perturbative contribution cannot be calculated, but theoretical arguments show that it is relatively small for heavy quarks: roughly speaking, momentum degradation is predicted to scale like \( O(\Lambda/m) \), \( \Lambda \) being an hadronic scale. Hence, once this contribution is properly defined, extracted from highly accurate \( e^+e^- \) data, and consistently and correctly used in hadronic collisions, it will not degrade the accuracy provided by the perturbative calculations, even if its numerical effect will be non-negligible. One framework which implements all this is the so-called FONLL [20], and many of the comparisons which will be shown in the following are based on it.

An essential component of a reliable and modern phenomenological prediction is a sensible estimate of its theoretical uncertainty. Such uncertainty cannot of course be rigorously established in terms of standard deviations. Rather, it is usually estimated with a mix of good practice and common sense. The perturbative calculations contain
artificial ingredients, like the factorisation and renormalisation scale, whose effect on the final result when varied allows one to estimate its uncertainty. Properly used, and coupled with experience, this tool can usually provide a reasonably good estimate, provided of course that the calculation itself is at least minimally reliable (we can’t expect an unreliable calculation to provide a reliable estimate of its own uncertainty...). One important aspect to keep in mind is that the theoretical uncertainty bands which usually result should not be regarded as gaussian errors: their ‘central curve’ does not necessarily represent the ‘best’ prediction. In fact higher order calculations, where available, tend to disprove this identification. It is therefore wiser to regard

Figure 1. Some theory vs.data comparisons for a variety of exclusive measurements of charm and bottom cross section in different experiments. Plots from [21–24], see original references therein.
Figure 2. Non-photonic electron cross sections from PHENIX and STAR, compared to the FONLL theoretical prediction of [28]

an uncertainty band more like an almost flat probability distribution for the ‘correct’ prediction, with the probability of it falling within the band depending on the way it has been obtained, but possibly (again from experience) around 80-90% for a typical NLO calculation.

Figure 1 shows some recent comparisons between experimental results and theoretical predictions (in the form of uncertainty bands) for production of charm and bottom quarks at various colliders (see [25] for a recent review of fixed target data). One can see a generally good agreement within both experimental and theoretical errors. While the data sometimes tend to lie on the upper edge of the theoretical band, they are nevertheless compatible, and moreover a next-to-next-to-leading calculation is likely to increase yet a little more the theoretical cross sections, hence improving the agreement.

Keeping in mind these comparisons, we consider now the production of charm and bottom in $pp$ collisions at RHIC, as measured via the transverse momentum distribution of non-photonic electrons. Both STAR [26] and PHENIX [27] have performed this measurement, and FONLL predictions – with theoretical simulation of the whole process down to the decay into electrons – are available from Ref. [28]. We can see from Fig. 2 that while the comparison with PHENIX data looks qualitatively similar to those previously considered, the STAR cross section seems instead to be more significantly larger than the theoretical prediction, albeit with large uncertainties.

4. Conclusions

NRQCD seems to be a theoretically sound approach to quarkonium production, and has enjoyed a number of phenomenological successes. Many of the predictions calculated in this framework are however only leading order ones, and therefore still subject to large uncertainties. More detailed calculations can certainly help constrain the theory more. They might also help better understanding the issue of $J/\psi$ polarisation in $p\bar{p}$ collisions, presently the biggest problem for this approach.
As far as calculations for open heavy quark production are concerned, they seem to do a good job in reproducing most of the experimental measurements for realistic observables within proper phase space acceptance regions. One might therefore conclude that even for charm, whose low mass and larger non-perturbative hadronisation corrections might cast doubts on the predictive power of the theory, pQCD actually manages to deliver good results. In the light of a fairly large number of theory–experiment comparisons with qualitatively similar behaviour, the recent measurement by STAR sticks out as somewhat different. More investigations will be necessary to figure out what is going on.

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