A Diquark Chiral Effective Theory and Exotic Baryons

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Abstract

A chiral effective theory of quarks and diquarks is formulated and applied to exotic tetraquarks and pentaquarks. The effective theory is similar to the chiral quark effective theory with the addition of diquark degrees of freedom and couplings to quarks. Chiral symmetry through generalized Goldberger-Treiman relations, fixes the mass splitting between the even and odd parity diquarks to be about 600 MeV. We provide an estimate of the parameters of the effective theory from both the random instanton model and known data on the low lying scalar nonet. We assess the static properties of the exotic baryons, such as their masses, magnetic moments and decay widths. We show that the small decay widths of the newly reported exotics are largely due to a large tunneling suppression of a quark between a pair of diquarks. We find \( \Gamma (\Theta^+ \rightarrow K^+ n) \approx 2.5 \sim 7.0 \text{ MeV} \) and \( \Gamma (\Xi^{--} \rightarrow \Xi^- \pi^-) \approx 1.7 \sim 4.8 \text{ MeV} \).

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1 Introduction

A number of recent experiments [1, 2, 3] have reported the occurrence of narrow baryonic excitations with exotic quantum numbers. Among the members of an anti-decuplet, an isospin singlet $\Theta^+(1540)$ and an isospin quadruplet $\Xi_{3/2}(1860)$ were discovered. While several experiments are set to improve on the current ones, only the masses and upper bounds on the widths of the exotics were measured. The reported hadronic width of the cascade was smaller than the experimental resolution of 18 MeV of the NA49 experiment, which is rather remarkable. The parity of these exotics is expected to be positive.

Two decades ago the SU(3) version of the Skyrme model predicted a low lying antidecuplet and a 27-plet with exotic quantum numbers, in particular an antidecuplet with an isosinglet $\Theta^+$ carrying spin-parity assignment $1/2^+$ and strangeness $+1$ with a mass of 1530 MeV and a width less than 15 MeV [4], a remarkable prediction. This notwithstanding, the occurrence of exotics in QCD calls for a quark model description.

The observed exotic baryons carry quantum numbers of at least five quarks. In this spirit, quark-diquark models were recently suggested [5, 6, 7] to account for the occurrence of low lying multiquark states with exotic quantum numbers. Scalar and tensor diquarks are strongly bound by color-exchange or instanton interactions in the color $\mathbf{3}$ channel, providing a natural way for the multiquark states to organize. The scalars are in the antitriplet of flavor while the tensors are in the sextet. A pair of P-wave scalars bind to an antistrange quark to form a low-lying positive parity exotic in the $\mathbf{10}$ of flavor [5]. A pair of S-wave scalar and tensor bind to an antistrange quark to form a low lying positive parity exotic in the $\mathbf{27}$ of flavor [6]. These exotic arrangements are falsifiable by experiments [8] or by lattice calculations [9].

In this paper we formulate an effective chiral theory of quarks and diquarks with strong correlations in the color antitriplet and flavor antitriplet (scalar) and sextet (tensor). Since the typical diquark mass is of order $400 - 600$ MeV [11] it sits mid-way between the confinement scale with $\Lambda_{QCD} \approx 200$ MeV and the chiral symmetry breaking scale $\Lambda_{CSB} \approx 4\pi f_\pi$. The chiral effective theory we are seeking is analogous to the chiral quark effective theory formulated by Georgi and Manohar [12], with the addition of diquark degrees of freedom and their couplings to quarks.
The Chiral Effective Lagrangian

In this section we will discuss some chiral aspects of the constituent diquarks and derive a chiral Lagrangian for their interactions with the pseudoscalar nonet of mesons.

2.1 Diquarks

The color antitriplet diquarks are described by local fields. The scalar diquark is a flavor antitriplet defined as

\[
\varphi_{S\alpha}^i (x) = \lim_{y \to x} \frac{|y - x|}{\kappa_S^S} \epsilon^{ijk} \epsilon_{\alpha \beta \gamma} \bar{\psi}_{cj}^\gamma (x) \gamma_5 \psi^\beta_k (y),
\]

and the tensor diquark is a flavor sextet defined as

\[
\varphi_{T\alpha ij}^{mn} (x) = \lim_{y \to x} \frac{|y - x|}{\kappa_T^T} \epsilon_{\alpha \beta \gamma} \bar{\psi}_{c\{i}^\gamma (x) \sigma_{mn} \psi_{j\}}^\beta (y).\]

\(\kappa_{S,T}\) are mass scales for the diquark fields, \(\gamma_{S,T}\) are anomalous dimensions of the diquark correlators. The Greek indices denote colors, while \(i, j, k = 1, 2, 3\) denote flavors and \(\sigma_{mn} = i[\gamma_m, \gamma_n]/2\) is the covariantized spin matrix. The tensor diquark is antisymmetric in spin and symmetric in flavor (the laces in \(\mathbb{F}\)). The 3 time components in the tensor are parity odd, while the 3 space components are parity even. \(\psi_c \equiv C \bar{\psi}^T\) is a charge conjugated field of a quark field, \(\psi\). The charge conjugation matrix is given as \(C = i \gamma_2 \gamma_0\). Being a strong correlator, the diquark field has a decay width and a form factor, which will be characterized by its couplings to quarks. The couplings may have momentum dependence. Note that a chiral transformation turns (1) into a pseudoscalar of arbitrary flavor. The locality of the fields and the Pauli principle restricts the pseudoscalar to be again an antitriplet in flavor.

Thus, under a chiral transformation (1) mixes with the pseudoscalar diquark

\[
\varphi_{P\alpha}^i (x) = \lim_{y \to x} \frac{|y - x|}{\kappa_S^S} \epsilon^{ijk} \epsilon_{\alpha \beta \gamma} \bar{\psi}_{cj}^\gamma (x) \psi^\gamma_k (y).
\]

The same arguments apply to the tensor which yields mixing between its even and odd parity content.

2.2 Scalars

The chiral effective Lagrangians for the scalar diquarks require the pseudoscalars as well. For that we introduce the left/right combinations of linear chiral diquark fields
\[ \phi_{R,L} = \frac{1}{2} (\phi_S \pm i \phi_P) \] (4)

which transform as (3, 1) (R) and (1, 3) (L) under rigid SU(3)_R \times SU(3)_L. Let \( \Sigma = \sigma + i \Pi \) be the linear representation of the scalar plus pseudoscalar nonet that transforms as (3, 3).

The chiral symmetric part of the Lagrangian for scalar-pseudoscalar diquarks to lowest order reads

\[
\mathcal{L}_{3\times3} = |D_\mu \phi_R|^2 - \Delta^2 |\phi_R|^2 + \frac{g_\pi}{2} \phi_R^\dagger \Sigma \phi_L - \frac{i}{4f_\pi^2} (g_A - 1) \left( D_\mu \phi_L^\dagger (\Sigma^\dagger \partial^\mu \Sigma) \phi_L + \text{h.c.} \right) + \left( L \leftrightarrow R, \Sigma \leftrightarrow \Sigma^\dagger \right). \] (5)

In the vacuum chiral symmetry is spontaneously broken with \( \Sigma \) developing a v.e.v. Specifically, \( \Sigma = \xi f_\pi \) with the non-linear unitary chiral fields \( \xi = e^{i \pi / 2 f_\pi} \), where \( \pi = \pi_a T_a \) and \( T_a \) are SU(3) generators in the adjoint representation with a normalization \( \text{tr} (T_a T_b) = 1/2 \delta_{ab} \).

The non-linear diquark fields follow from (4) through

\[ \varphi_L = \xi \phi_L \quad \varphi_R = \xi^\dagger \phi_R. \] (6)

Inserting (6) into (5) yields the lowest order chiral Lagrangian

\[
\mathcal{L} = |(D_\mu + i R_\mu) \varphi_R|^2 + \frac{1}{2} g_\pi f_\pi \varphi_R^\dagger \varphi_L - \Delta^2 \varphi_R^\dagger \varphi_R + (L \leftrightarrow R) + \frac{1}{2} (g_A - 1) \left[ (D_\mu + i R_\mu) \varphi_R \right]^\dagger A^\mu \varphi_R + \text{h.c.} + (L \leftrightarrow R) - \left( g_S \varphi_{Si}^\alpha \epsilon^{ijk} \epsilon_{\alpha\beta\gamma} \psi_{cj}^\gamma \gamma_5 \psi_k^\gamma + g_P \varphi_{Pi}^\alpha \epsilon^{ijk} \epsilon_{\alpha\beta\gamma} \psi_{cj}^\beta \psi_k^\gamma + \text{h.c.} \right) + \mathcal{L}_{\text{int}}(\varphi_S, \varphi_P, \psi, \bar{\psi}) + \mathcal{L}_\chi^{\text{Q}}. \] (7)

All terms retained are chirally invariant for \( g_S = g_P \). The Yukawa couplings \( g_S \) and \( g_P \) induce mass splitting between different flavors. The higher-order interactions of diquarks and quarks are denoted by \( \mathcal{L}_{\text{int}} \), which may contain the (chirally) covariant derivatives and explicit mass breaking terms.\footnote{Because of chiral symmetry, the pions couple to quarks and diquarks with a derivative coupling. Hence, the pion contribution to the scalar mass, and the magnetic moments of pentaquarks will be suppressed by \( 1/\Lambda_{\text{CSB}} \), compared to that of Yukawa coupling of diquarks, as discussed in section 3.}

The usual interactions of quarks and Nambu-Goldstone bosons are
contained in $\mathcal{L}_{\chi Q}$ (chiral quark effective theory). The relevant degrees of freedom of the diquark chiral effective theory are constituent quarks, diquarks, gluons, and pions. When the diquarks are absent or infinitely heavy, the effective Lagrangian should reduce to the chiral quark effective theory of Georgi and Manohar. The power-counting rule should be same and the diquark field scales like $f_\pi$.

Since the diquark transforms like a color antitriplet, the $SU(3)_c$ covariant derivative is given as

$$D_\mu \varphi = \partial_\mu \varphi + ig_s A_\mu^a T^a \varphi. \quad (8)$$

Chiral symmetry is enforced through

$$i R_\mu = (V_\mu - iA_\mu), \quad i L_\mu = (V_\mu + iA_\mu), \quad (9)$$

with

$$V_\mu = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right), \quad A_\mu = \frac{i}{2} \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right). \quad (10)$$

The chiral Lagrangian (7) mixes scalar and pseudoscalar diquarks through the axial vector current. The pseudoscalar diquarks are heavy and broad (above the two quark continuum). Indeed, the chiral invariant masses in (7) yield the canonical contributions

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} M_S^2 |\varphi_S|^2 - \frac{1}{2} M_P^2 |\varphi_P|^2 \quad (11)$$

with

$$M_S^2 = \Delta^2 - \frac{g_\pi f_\pi}{2}, \quad M_P^2 = \Delta^2 + \frac{g_\pi f_\pi}{2}. \quad (12)$$

The spontaneous breaking of chiral symmetry pushes the scalar down (light) and the pseudoscalar up (heavy). The splitting in the chiral limit is given by

$$M_P^2 - M_S^2 = g_\pi f_\pi, \quad (13)$$

which is a generalized Goldberger-Treiman relation for diquarks. The dimensional parameter $g_\pi$ relates to the pion-diquark coupling as is evident in the linear representation. Indeed,
\( g_{\pi D} = g_\pi/(M_P + M_S) \) is just the pion-diquark coupling following from (5) through a non-relativistic reduction \([10]\). Thus \( M_P - M_S = g_{\pi D} f_\pi \). The pion-diquark coupling is about twice the pion-quark coupling \( g_\pi Q \), and about two-third the pion-nucleon coupling \( g_{\pi N} \). Thus

\[
M_P - M_S = g_{\pi D} f_\pi \approx \frac{2}{3} g_{\pi N} f_\pi \approx \frac{2}{3} M_N ,
\]

where the standard Goldberger-Treiman relation in the chiral limit was used. In the chiral limit, the pseudoscalar diquark is about 600 MeV heavier than the scalar diquark and decouples. The scalar mass is not fixed by chiral symmetry and will be determined empirically below.

2.3 Tensors

The chiral effective Lagrangian for tensor diquarks can also be constructed in a similar way. For that we separate the covariant tensor field into its “electric” and “magnetic” spin components

\[
\phi_E^k = i \phi_T^0 \quad \phi_T^k = \frac{1}{2} \epsilon^{krs} \phi_T^r \phi_T^s
\]

with the color and flavor indices unchanged. The chiral components of the tensor field follows in the form

\[
\phi_R^k = \frac{1}{2} \left( \phi_M^k - i \phi_E^k \right) \quad \phi_L^k = \frac{1}{2} \left( \phi_M^k + i \phi_E^k \right) .
\]

Under rigid \( \Lambda_{R,L} \) chiral transformations, \((13)\) transform as \( \Lambda_{R,L} \phi_{R,L} \Lambda_{R,L}^T \). The lowest order chiral Lagrangian involving the scalar-pseudoscalar nonet field \( \Sigma = \sigma + i \Pi \) with the tensor diquarks in the linear representation reads

\[
\mathcal{L}_{3\times 3} = |D_\mu \phi_R|^2 - \Delta_T^2 |\phi_R|^2 + \frac{g_{T\pi}}{2} \phi_R^\dagger \Sigma \phi_L \Sigma^T \\
- \frac{i}{4 f_\pi^2} (g_A - 1) \left( D_\mu \phi_L^\dagger (\Sigma^\dagger \partial^\mu \Sigma) \phi_L + \text{h.c.} \right) \\
+ \left( L \leftrightarrow R \right) , \Sigma \leftrightarrow \Sigma^T ,
\]

which is analogous to \((5)\) except for the pion-tensor pseudoscalar coupling term (third term).
In the non-linear representation we introduce

\[ \varphi_R = \xi^\dagger \phi_R \xi^* \quad \varphi_L = \xi \phi_L \xi^T, \]  

(18)
in terms of which the chiral effective Lagrangian with pseudoscalar mesons and tensor diquarks now read

\[ \mathcal{L} = \left| \left( D_\mu + i R_\mu . + . i R^T_\mu \right) \varphi_R \right|^2 + \frac{1}{2} g_{T\pi} f^2 \pi \varphi_R^\dagger \varphi_L + \Delta^2 \varphi_R^\dagger \varphi_R + (L \leftrightarrow R) \]

\[ + \frac{1}{2} (g_A - 1) \left\{ \left( D_\mu + i R_\mu . + . i R^T_\mu \right) \varphi_R \right\}^\dagger A^\mu \varphi_R + \text{h.c.} \]  

(19)

\[ - \left( g_{T\pi} \varphi_{T\alpha \beta \gamma} \bar{\psi}_c \left\{ i \sigma^{\alpha \beta} \psi \right\}_j + \text{h.c.} \right). \]

The dots stand for the position of \( \varphi \) inside the bracket. The Yukawa coupling \( g_T \) is the same for even/odd parity tensors in the chiral limit. \((19)\) should be added to \((7)\) as the most general chiral effective lagrangian involving both scalars and tensors in leading order. The mass terms for the electric and magnetic tensors are

\[ M^2_M = \Delta^2 - g_{T\pi} f^2 \pi \]

\[ M^2_E = \Delta^2 + g_{T\pi} f^2 \pi, \]

leading to a generalized Goldberger-Treiman relation

\[ M^2_E - M^2_M = g_{T\pi} f^2 \pi. \]

(20)

(21)

Again \( g_{T\pi} f_\pi / (M_M + M_E) \) plays the role of the pion-tensor coupling as is clear from \((17)\). Assuming the latter to be of the order of \( 2/3 \) the pion nucleon coupling, we conclude that the splitting between the electric (parity-odd) and magnetic (parity-even) diquarks is comparable to the splitting between the pseudoscalar and scalar diquark and of the order of \( 600 \) MeV. The magnetic tensor diquark with positive parity is lighter than the electric tensor diquark with negative parity. The magnetic tensor diquark was used in a recent analysis of the pentaquark \([3]\).

In what follows, we will focus on the chiral effective Lagrangian involving only the scalar diquarks. The inclusion of the tensors in our analysis is straightforward.
Figure 1: Scalar coupling in the effective theory. j is the diquark current.

3 The static properties of exotic baryons

In this section we will provide both a theoretical and an empirical determination of the scalar diquark parameters in (7) by using: i. results from the random instanton model to the QCD vacuum; ii. known data on the nonet of scalar mesons. Both extractions will be shown to be consistent.

3.1 Random Instanton Model

Some of the parameters in (7) and (19) have been measured in the random instanton model of the QCD vacuum at a cutoff of the order of the inverse instanton size of 3/fm [11]. For completeness we quote the (2 flavor) scalar and tensor parameters in the table below where the masses are in MeV and the couplings are in GeV² [11]

| M_S  | M_T  | G_S   | G_T   |
|------|------|-------|-------|
| 420±30| 570±20| 0.22±0.01| 0.13±0.00 |

(22)

The couplings G’s are defined on shell, e.g. [11]

\[ \langle 0|\varphi_{S\alpha}(0)|\varphi_{S\beta}(K)\rangle = \frac{G_S}{\sqrt{3}} \delta^{ij} \delta_{\alpha\beta} \],

(23)

with \( K^2 = M_S^2 \). \( \varphi_{S\alpha}(0) \) is calculable in our effective Lagrangian approach. Indeed, in the flavor symmetric limit we have (See Fig. 1)

\[ \langle 0|\varphi_{S\alpha}(0)|\varphi_{S\beta}(K)\rangle = 32 g_S J(K) \delta_{\alpha\beta} \delta^{ij} \],

(24)

with

\[ J(K) = \int \frac{d^4q}{(2\pi)^4} \frac{m^2 - K_+ \cdot K_-}{(K_+^2 + m^2)(K_-^2 + m^2)} \],

(25)
after rotation to Euclidean momenta ($K^2 = -M_S^2$). Here $m$ is the flavor symmetric constituent quark mass, and $K_\pm = (K/2 \pm q)$. Since (25) diverges it requires regularization. For a comparison to the random instanton model it is perhaps physical to use a covariant cutoff of the order of the instanton size used in [11], i.e. $\Lambda \approx 3/\text{fm}$. As a result, the measured on-shell $G_S$ in the random instanton model translates to our off-shell $g_S$ as $g_S \approx 2.85 (G_S/\Lambda^2)$ or $g_S^2 \approx 3.03$ at a cutoff scale of the order of 0.6 GeV. Dimensional regularization leads to a consistent result for a comparable renormalization scale. Indeed, in the minimal subtraction scheme (25) reads

$$J_{E}^{MS}(K) = \left(\frac{M_S}{4\pi}\right)^2 \left[ 1 + \frac{3m^2}{4M_S^2} + \frac{m^2}{2M_S^2} \ln \left(\frac{m^2}{\mu^2}\right) \right].$$

(26)

For $\mu \approx m \approx M_S$ we get $g_S \approx 1.63 (G_S/M_S^2)$ or $g_S^2 \approx 2.05$. The current results for $g_S$ as extracted from the random instanton model are consistent with the empirical estimates we now discuss.

### 3.2 Scalar Nonet

The same effective parameters can also assessed from experiment. Indeed, the scalar constituent mass of the diquark can be related to the measured scalar meson masses whereby the latter are bound diquark-anti-diquark scalars in our effective theory. From (7) it follows that the scalar diquark mass reads (see Fig. 2)

$$M_{jk}^2 = M_S^2 + \frac{4g_S^2}{\pi^2} \left[ (m_j m_k - m_j^2 - m_k^2) + \frac{m_j^3 \ln \left(\frac{m_j^2}{\mu^2}\right) + m_k^3 \ln \left(\frac{m_k^2}{\mu^2}\right)}{m_j + m_k} \right],$$

(27)

where $\mu$ is the renormalization point and $m_j$ are the constituent quark mass of the $j$-th flavor ($j \neq k$). The imaginary part of Fig. 2 vanishes below threshold. While it may broaden the diquarks off-shell, that is in the bound state configuration of the scalar meson, we expect the broadening to be small and ignore it.
Assuming the light nonet of scalars to be composed of a bound diquark and antidiquark [3, 4], we arrive at the mass formula

\[
M(a_0) = M(f_0) = M_{us} + M_{ds} - B \\
M(\kappa) = M_{us} + M_{ud} - B \\
M(\sigma) = 2M_{ud} - B ,
\]  

(28)

where \( B \) is the binding energy of the diquark-anti-diquark pair [15]. The mass formula (28) fixes the scalar diquark masses, coupling and their binding energy in the tetraquark configuration. Thus

\[
2M(\kappa) = \frac{1}{2} [M(a_0) + M(f_0)] + M(\sigma),
\]  

(29)

which works very well with the experimental values, \( M(a_0) = M(f_0) = 980 \text{ MeV} \), \( M(\kappa) = 800 \text{ MeV} \), and \( M(\sigma) = 500 \sim 600 \text{ MeV} \). Taking the renormalization point \( \mu = 200 \text{ MeV} \) and \( 350 \text{ MeV} \leq M_S \leq 480 \text{ MeV} \), we get in units of GeV for the masses and the binding energy

\[
\begin{array}{|c|c|c|c|c|}
\hline
M_S & M_{us}, M_{ds} & M_{ud} & B & g_S^2 \\
\hline
0.35 & 0.56 & 0.38 & 0.15 & 2.64 \\
0.40 & 0.61 & 0.43 & 0.25 & 2.92 \\
0.42 & 0.63 & 0.45 & 0.28 & 3.03 \\
0.45 & 0.66 & 0.48 & 0.34 & 3.19 \\
0.48 & 0.69 & 0.51 & 0.40 & 3.36 \\
\hline
\end{array}
\]  

(30)

At the renormalization point \( \mu = 200 \text{ MeV} \) and the scalar mass \( M_S = 420 \pm 30 \text{ MeV} \), obtained in the random instanton model, the results for the scalar coupling are in good agreement with the results from the random instanton model. We note that the scalar couplings for different \( M_S \) are small giving rise to corrections of order \( 4g_S^2/\pi^2 \) or less than 10%. The smallness of the higher-order corrections indicates that indeed the diquark picture captures the correct physics of QCD around 400 \sim 600 \text{ MeV}. The diquark anti-diquark bound state should be treated relativistically, since the binding energy is comparable to the rest mass energy.

In the diquark effective theory, we can also calculate the mass difference in the anti-decuplet. In particular

\[
M(\Xi^{--}_{3/2}) - M(\Theta^+) = (2M_{ds} + m_u) - (2M_{ud} + m_s) = 320 \sim 180 \text{ MeV},
\]  

(31)

which works reasonably well with the newly reported splitting from NA49 [10]. In reaching (31) we have used the fact that the remaining one-gluon exchange between the constituent diquarks and quarks is flavor blind (perturbative).
3.3 Magnetic Moments

Now, we consider the magnetic moments of exotic baryons, which are important quantum numbers in photo-production of exotic baryons. First, we consider $\Theta^+$, whose quark content is $(ud)^2\bar{s}$. Being a scalar, the diquark does not carry any magnetic moment. So, in leading order the magnetic moment of $\Theta^+$ is equal to that of the anti-strange quark plus that of the orbiting P-wave diquarks,

$$\vec{\mu}_m(\Theta^+) = \vec{\mu}_s + \vec{\mu}_L + \delta\vec{\mu}_m,$$

where $\mu_s = 0.75$ n.m. is the magnetic moment of the anti-strange quark and $\mu_L$ is the magnetic moment of the orbiting diquarks, and $\delta\mu_m$ is a correction due to the quantum fluctuation of diquarks. Fermi statistics and the range of the effective interaction force the pair of $(ud)$ diquarks to bind in a P-wave, resulting in positive-parity pentaquarks. Thus, for $400 \text{ MeV} \leq M_S \leq 450 \text{ MeV}$

$$\delta\mu_m = \mu_u \frac{g_S^2}{16\pi^2} \frac{m_u^2}{M_{ds}^2 - m_u^2} \left[ 1 - \frac{M_{ds}^2}{M_{ds}^2 - m_u^2} \ln \left( \frac{M_{ds}^2}{m_u^2} \right) \right] + (u \leftrightarrow d).$$

Since $g_S^2$ is of order one, the one-loop correction due to the diquarks to the magnetic moment of the anti-strange quark is quite small, $\delta\mu_m \sim 5 \times 10^{-3}$ n.m. We find $\mu_m(\Theta^+) = 0.71$ n.m. for $M_S = 400$ MeV for $J^P = \frac{1}{2}^+$, if we take $m_u \simeq m_d \simeq 360$ MeV and $\mu_u = 1.98$ n.m., $\mu_d = -1.10$ n.m., while $\mu_m(\Theta^+) = 0.56$ n.m. for $M_S = 450$ MeV. For $J^P = \frac{3}{2}^+$, we get $\mu_m(\Theta^+) = 2.21 \sim 2.06$ n.m. with $400 \text{ MeV} \leq M_S \leq 450 \text{ MeV}$. The values we have obtained are in general larger than the values obtained in the chiral quark-soliton model or those obtained in the QCD sum-rule estimate. Note that the leading correction is less than a per cent and thus the calculations are quite reliable.
The present approach shows how to assess the magnetic moments of the full octet and antidecuplet made of scalar diquarks. The same approach can be used to assess the magnetic moments of the exotics in the 27-plet for the case of a scalar and a tensor pair as discussed above. Also with the chiral effective Lagrangians (7) and (19) we may estimate the masses of the bound exotics and their decay widths using a 3-body bound state formulation. These issues and others will be reported elsewhere in a longer analysis. Instead we proceed to show why generically the newly observed exotics carry small widths in the diquark description.

4 Small Decay Widths

Since the scalar and tensor diquark masses are smaller than the constituents ($M_{jk} < m_j + m_k$ for $M_{S,T} < 650$MeV), they are stable against decay near mass shell. In a bound exotic such as $\Theta^+$ and $\Xi^-$, the diquarks orbit in a P-wave. They are held together with the antistrange quark via colored Coulomb and confining forces. In such a configuration, the diquarks are nearby and tunneling of one of the quarks between the two diquarks may take place.

Indeed, consider the decay process $\Theta^+ \to K^+ n$ as depicted in Fig. 4. Here a d quark tunnels from a diquark $ud$ to the other diquark to form a nucleon $udd$ and an off-shell $u$ quark, which is annihilated by the anti-strange quark. (If $u$ were to tunnel, the decay is to $K^0 p$ with a comparable decay width.) The decay width is therefore given as

$$\Gamma = \lim_{v \to 0} \sigma(\bar{s} + \phi_{ud} + \phi_{ud} \to K^+ + n) v |\psi(0)|^2,$$

where $v$ is the velocity of $\bar{s}$ in the rest frame of the target diquark and $\psi$ is the 1S wave function of the quark-diquark inside the pentaquark. The differential cross section for the annihilation process is then (see Fig. 4)

$$d\sigma = \frac{(2\pi)^4 |\mathcal{M}|^2}{4\sqrt{(p_1 \cdot p_2)^2 - m_s^2 M_{ud}^2}} 4 e^{-2S_0} d\Phi(p_1 + p_2; k_1, k_2),$$

with the tunneling probability $e^{-2S_0}$ and the phase space

$$d\Phi(p_1 + p_2; k_1, k_2) = \delta^4(p_1 + p_2 - k_1 - k_2) \frac{d^3k_1}{(2\pi)^3 2E_3} \frac{d^3k_2}{(2\pi)^3 2E_4}. \quad (37)$$

Assuming that the annihilation amplitude is factorizable (QCD corrections are small), we find

$$\mathcal{M} = \frac{g_A g}{\sqrt{2} f_K} \bar{s}_s(p_1) \gamma_5 \frac{1}{k_1 - \not{p}_1 - m_u} \gamma_5 v_d(k_2), \quad (38)$$
where $v_{s,d}$ are the wave functions of $\bar{s}$ and $d_c$, respectively. Integrating over the phase space and taking $v \to 0$, we find the decay width

$$\Gamma_{\Theta^+} \simeq 5.0 \, e^{-2S_0} \, \frac{g_s^2 g_\rho^2}{8\pi f_K} \, |\psi(0)|^2.$$  

(39)

Using the WKB approximation, we have for the tunneling amplitude

$$e^{-S_0} = \langle n | T e^i \int d^4 x \, E_{\text{int}} | d, \phi_{ud} \rangle \approx e^{-\Delta E r_0},$$  

(40)

where $\Delta E = (m_u + m_d) - M_{ud}$ is the diquark binding energy and $r_0$ is the average distance between two diquarks in the pentaquark (See Fig. 5). Of course, the pair in a P-wave senses centrifugation but this is cutoff at short relative distances by Pauli blocking which provides a repulsive core, making the height in Fig. 5 likely higher. So our tunnelling estimate below will be on the lower side. An estimate of the average distance between two diquarks in $\Theta^+$, follows from

$$M_{\Theta^+} = 2M_{ud} + m_\bar{s} + \frac{2}{M_{ud} r_0^2},$$  

(41)

where the third contribution is the rotational energy of diquarks in a P-wave. Using the empirical mass $M_{\Theta^+} = 1540$ MeV we find $r_0 = (150 \text{ MeV})^{-1}$ and $\Delta E = 270$ MeV. The P-wave repulsion estimate is a bit on the lower side as discussed in [6] for a pair of scalar diquarks. This notwithstanding, the WKB estimate yields a tunnelling amplitude of order $e^{-1.8} \simeq 0.17$.

The $1S$ wave function of the quark-diquark at the origin can be written as

$$\psi(0) = \frac{2}{a_0^{3/2}} \frac{1}{\sqrt{4\pi}}.$$  

(42)

where $a_0$ is the Bohr radius of the quark-diquark bound state. Assuming they are non-relativistic, we get by the dimensional analysis $a_0 \simeq (2\overline{m} B)^{-1/2}$, where $\overline{m} = 250$ MeV is the reduced mass and $B$ is the binding energy of the quark-diquark bound state. Taking
Figure 5: Tunnelling of a quark from one diquark to another.

$B = 100 \sim 200$ MeV, comparable to the pentaquark binding energy, $g^2 = 3.03$ and $g_A = 0.75$ from the quark model, we find

$$\Gamma_{\Theta^+} \simeq 2.5 \sim 7.0 \text{ MeV.} \quad (43)$$

The tunnelling process reduces the decay width by a factor of 50 to 100, compared to normal hadronic decays which do not have any tunnelling process [19]. We claim that the unusual narrowness of the exotic baryons is naturally explained in the diquark picture. For the $\Xi_{^-3/2}^{-}$ isospin quadruplet, we find the height of the potential barrier, $\Delta E_1 \simeq 270$ MeV with an average separation of P-wave strange-diquarks $r_1 \simeq (270 \text{ MeV})^{-1}$. Then, we find the decay width for $\Xi_{^-3/2}^{-} \to \Xi^{-} \pi^{-}$ to be for $B = 100 \sim 200$ MeV

$$\Gamma_{\Xi_{^-3/2}^{-}} \simeq 0.51 \ e^{-2\Delta E_1 r_1} \frac{g^2 g_A^2}{8\pi f^2} |\psi_u(0)|^2 \simeq 1.7 \sim 4.8 \text{ MeV}, \quad (44)$$

where $\psi_u$ is the wave function of the anti-up quark and one of the diquarks in $\Xi_{^-3/2}^{-}$. The relative (partial) decay width of $\Xi_{^-3/2}^{-}$ and $\Theta^+$, already observed at CERN SPS and at LEPS SPring-8, respectively, is

$$\frac{\Gamma_{\Xi_{^-3/2}^{-}}}{\Gamma_{\Theta^+}} \simeq 0.1 \left( \frac{f_K |\psi_u(0)|}{f_\pi |\psi_u(0)|} \right)^2 e^{2(\Delta E r_0 - \Delta E_1 r_1)} \simeq 0.7. \quad (45)$$

5 Conclusions

We have formulated a chiral quark-diquark effective theory to analyze the properties of the newly discovered multiquark exotics. The chiral effective theory involves the low lying scalar and tensor diquarks suggested by the random instanton model of the QCD vacuum, and
its parameters are well constrained by the observed scalar mesons treated as tetraquarks or bound pairs of diquark-anti-diquark. Chiral symmetry alone shows that the even parity scalar and tensor diquarks are about 600 MeV lighter than their odd parity chiral partners. This maybe the expected splitting between even-odd parity multiquark exotics with diquark content.

The diquark chiral effective theory captures correctly the physics of QCD between the confinement scale and the chiral symmetry breaking scale. It is similar to the chiral quark effective theory suggested by Georgi and Manohar with the addition of diquark degrees of freedom and their couplings to quarks and other particles like pions and gluons.

We have suggested that the smallness of the strong decay widths of the newly reported exotics is naturally explained as a quark tunnelling from one pair of diquark to the other. This phenomenon does not occur in non-exotic strong baryon decays. Our WKB estimate shows that the strong decay rate in exotics is about two orders of magnitude suppressed in comparison to non-exotics. We find the decay width of $\Theta^+ \to K^+ n$ to be $\Gamma_{\Theta^+} = 2.5 \sim 7.0$ MeV and the decay width of $\Xi^{-} \to \pi^- \Xi^-$, $\Gamma_{\Xi^{-}} = 1.7 \sim 4.8$ MeV, unusually narrow as seen by several experiments.

The chiral diquark effective theory proposed here, maybe used to refine the magnetic moment calculations in the exotic antidecuplet (scalar-scalar) or 27-plet (scalar-tensor) channels. It can be used to evaluate the axial charge couplings, and exotic colored Coulomb bound states made of pairs of diquarks for tetraquark and pentaquark states. These issues and others will be addressed elsewhere.
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Note Added

After submitting our paper we became aware of the newly reported charmed pentaquark by the H1 collaboration in hep-ex/0403017. In our chiral diquark description with diquark doublers, the observed charmed pentaquark maybe interpreted as the $1/2^-$ parity partner of the expected $1/2^+$. Specifically, $\Theta^+_c$ at 1.5 GeV assumed $1/2^+$ with scalar-scalar diquarks, would have a chiral partner $1/2^-$ at 2.1 GeV with scalar-pseudoscalar diquarks. Similarly, its charm counterpart $\Theta^0_c$ with $1/2^+$ at 2.5 GeV would have a chiral partner $1/2^-$ at 3.1 GeV. This identification is consistent with a recent soliton calculation using heavy-spin and chiral symmetry in hep-ph/0403184.

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