Electrical spin driving by $g$-matrix modulation in spin-orbit qubits

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In a semiconductor spin qubit with sizable spin-orbit coupling, coherent spin rotations can be driven by a resonant gate-voltage modulation. Recently, we have exploited this opportunity in the experimental demonstration of a hole spin qubit in a silicon device. Here we investigate the underlying physical mechanisms by measuring the full angular dependence of the Rabi frequency as well as the gate-voltage dependence and anisotropy of the hole $g$-factors. We show that a $g$-matrix formalism can simultaneously capture and discriminate the contributions of two mechanisms so far independently discussed in the literature: one associated with the modulation of the $g$-factors, and measurable by Zeeman energy spectroscopy, the other not. Our approach has a general validity and can be applied to the analysis of other types of spin-orbit qubits.

The spin-orbit (SO) interaction is a relativistic effect coupling the motional and spin degrees of freedom of a particle. In semiconductors, a sufficiently strong SO coupling can allow for electric-dipole spin resonance (EDSR) [1], namely the coherent rotation on an electron spin driven by a radio-frequency electric-field modulation. This opportunity can be exploited for the control of semiconductor spin qubits, where quantum information is encoded in the spin state of a confined electron (or hole). In this context, electrically-driven spin manipulation has practical advantages. As opposed to conventional magnetic-field-mediated electron spin resonance [2–4], it can result in faster manipulation [5,6] and higher qubit fidelity [7]. SO coupling also offers an intrinsic pathway to EDSR, with no need for dedicated control tools such as micromagnets generating local magnetic-field gradients [8,9].

There exist two distinct mechanisms for EDSR ultimately related to SO coupling. The first one was originally observed in a GaAs/AlGaAs heterostructure and was named $g$-tensor magnetic resonance ($g$-TMR) [10]. This mechanism requires anisotropic and spatially varying electron [10–13] (or hole [14,15]) $g$-factors. In essence, an alternating electric-field induces spatial oscillations of the electronic wave function translating into time-dependent $g$-factors and a non-collinear modulation of the Zeeman vector, i.e. the product of the $g$-tensor and the applied static magnetic field. The second mechanism was experimentally observed in a variety of III-V semiconductor quantum dots [16–19]. It is not associated with $g$-factor modulations. Instead, it can be understood as originating from an effective time-dependent magnetic field proportional to the alternating electric-field, to the static magnetic field, and to the inverse spin-orbit length [20,21].

In general, the two mechanisms are expected to coexist in the same device, since they share a common SO origin. Here we investigate this coexistence in a hole spin qubit confined to a silicon quantum dot. In this device, Rabi oscillations of the spin-1/2 hole state are electrically driven by a resonant microwave modulation applied to a gate voltage. By correlating the anisotropy of the Rabi frequency with the angle and gate-voltage dependence of the Zeeman splitting, we succeed in discriminating the two contributions to EDSR. In a linear approximation, the electrically driven hole spin resonance can be modeled in terms of the gate-induced modulation of a $g$-matrix $\hat{g}$. Only part of this modulation can be related to the gate voltage dependence and anisotropy of the Zeeman splitting, which is in principle accessible to static measurements. This part is responsible for a generalized version of the $g$-TMR mechanism, which we shall name Zeeman Modulation Resonance (ZMR). The remaining part of the $g$-matrix modulation, intrinsically inaccessible to Zeeman splitting measurements, can only be extracted through time-domain measurements of the Rabi frequency as a function of the magnetic field orientation. This component corresponds to the mechanism discussed in Ref. [22], which does not lead to gate-induced $g$-factor variations. For this reason, we shall refer to it as iso-Zeeman EDSR (IZR).

The device, shown in Fig. 1a, is a metal-oxide-semiconductor (MOS) trigate silicon nanowire field-effect transistor (FET) [23]. The source (S) and drain (D) are degenerately boron-doped reservoirs of holes, whereas the channel, oriented along [110], is undoped. Two top gates in series (G1, G2) tune a double quantum dot at the base temperature $T = 15 \text{ mK}$ of a dilution cryostat. At finite drain bias $V_d$, the transport of holes through the double dot results in pairs of triangles of DC current $I_d$ in the gate voltage diagram $V_{G1}-V_{G2}$. We work in the region of Fig. 1a, where Pauli Spin Blockade (PSB) [24] is revealed by current rectification at the base of the triangles and by magneto-transport measurements [25,27]; $(1,1)$ and $(0,2)$ denote the parity-equivalent excess charges of the double dot, though each dot contains between 10 and 30...
holography, as in Ref. [4].
In Fig.[1], we operate the device as a spin-orbit qubit. The gates are biased in PSB (yellow star in Fig.[1]), while a continuous microwave signal of frequency $f$ is applied to gate 2 in a magnetic field $B = (B, 0, 0)$. The microwave excitation drives hole spin transitions in both quantum dots, thereby lifting PSB. The current increases when the photon energy $h f$ matches the Zeeman splitting $\Delta E = |g^*| \mu_B B$ between two spin states, $h$ being Planck’s constant, $\mu_B$ Bohr’s magneton and $g^*$ the effective hole $g$-factor of the resonant dot for this orientation of $B$. From Fig.[1] we obtain $|g_L^*| = 1.96 \pm 0.02$ and $|g_R^*| = 2.02 \pm 0.02$ in the left and right dot, respectively. In Fig.[1] we report Rabi oscillations of the spin in dot R. The spin is initialized by PSB at the yellow star in Fig.[1]. The coherent manipulation is accomplished by a microwave burst of duration $\tau$ applied in the Coulomb blockade regime at the bias point $V_0$ marked by a blue dot in Fig.[1]. Spin readout relies again on PSB. The cycle initialization/manipulation/readout is repeated continuously and results in an oscillating current $I_d$ as a function of $\tau$, signature of the coherent spin rotations [6][10].
We next investigate the anisotropy of the effective hole $g$-factor in dot R at bias point $V_0$. For a given magnetic field, we measure $I_d$ as a function of $f$ for a fixed burst time while applying the cycle described above. The current trace shows a peak at the resonance frequency $h f = |g^*| \mu_B B$ (inset of Fig.[2]). Such a measurement is repeated 400 times in order to get the resonance frequency distribution shown in Fig.[2]. Finally $g^*(V_0, B)$ is extracted from the peak of the Gaussian distribution. The iso-surfaces of $\Delta E^2 = g^{*2} \mu_B^2 B^2$ being ellipsoids (28 and later discussion), six values of $g^*$ along different magnetic field directions completely characterize the anisotropy of the $g$-factor. The latter can be mapped onto:

$$|g^*| = \sqrt{(g^*_1 B_1)^2 + (g^*_2 B_2)^2 + (g^*_3 B_3)^2} / |B|, \quad (1)$$

where $B_1, B_2, B_3$ are the components of $B$ along the prin-
principal magnetic axes $X, Y, Z$, and $g_1^*, g_2^*, g_3^*$ are the corresponding effective $g$-factors. We find $|g_1^*| \approx 2.08$, $|g_2^*| \approx 2.48$ and $|g_3^*| \approx 1.62$. Fig. 2 shows the ellipsoidal iso-surfaces $\Delta E^2$ in the measurement frame $\{x, y, z\}$. Figs. 2a and 2b compare the full angular dependence of $g^*$ expected from Eq. (1) to the experimental values as a function of the elevation angle $\theta$ and azimuthal angle $\varphi$ of $B$ in the measurement frame. Any pixel of Fig. 2a is a distinct spin resonance experiment. The magnetic axis $Y$ associated to $g_2^*$ is almost aligned with the $y$ (nanowire) axis of the measurement frame. The other two orthogonal magnetic axes do not match crystallographic axes. We have measured a similar in-plane $g^* \sim 2 - 2.6$ (depending on gate voltage) and out-of-plane $g^* \sim 1.5$ in other $p$-type nanowire FETs [15]. $g^*$ is significant in all directions, suggesting a non-purely heavy flavor of the confined holes [27, 29].

The $g$-factors can also be tuned by the electric field. An example is the orange distribution of Fig. 2c, measured at the manipulation point $V_1$ (orange dot in Fig. 1b). Similar changes for other orientations of $B$ result in a small (yet sizeable) rotation of the principal magnetic axes.

In addition to the Zeeman response, we characterize the dependence of the qubit Rabi frequency on the magnetic field orientation. Fig. 3a displays the Rabi frequency for 291 directions of $B$. For each pixel, $|B|$ is adjusted so that the spin resonance sticks to 9 GHz in order to drive the spin at the same microwave power. The Rabi frequency ranges from 3 MHz to 40 MHz; it is maximal for $B \parallel z$, while is minimal along the $x$ direction.

We now aim to describe the electric and magnetic tunability of $g^*$ (Figs. 2b, 2c) and the anisotropy of the Rabi map (Fig. 3b) in an unified picture. Our approach consists in a generalization of the $g$-TMR formalism of Ref. [10]. The qubit is modeled as a Kramers doublet $|\uparrow\rangle$ and $|\downarrow\rangle$ in a magnetic field $B$. The effective two-level Hamiltonian is:

$$H = \frac{1}{2} \mu_B \sigma \cdot \hat{g} \cdot B,$$

(2)

where $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of Pauli matrices and $\hat{g}$ is a $3 \times 3$ matrix with 9 independent components. In principle any linear-in-$B$ Hamiltonian can be mapped onto Eq. (2) up to an irrelevant energy shift. The Zeeman splitting between the two eigenstates of $H$ is $\Delta E = \langle g^* \rangle \mu_B B$, with $|g^*| \approx |\hat{g} \cdot b|$ and $b = B/B$. The symmetric Zeeman tensor $\hat{G} \equiv \langle \hat{g} \cdot \hat{g} \rangle$ defined by

$$(\Delta E)^2 = \mu_B^2 |\hat{g} \cdot B|^2 = \mu_B^2 (\hat{B} \cdot \hat{G} \cdot B)$$

(3)

can be reconstructed from $\Delta E$ for six orientations of $B$. $\hat{G}$ completely characterizes the variations of $|g^*|$ given by Eq. (1) and shown in Fig. 2 and Fig. 2b. The eigenvectors of $\hat{G}$ are the principal magnetic axes $X, Y, Z$, while the corresponding eigenvalues $g_1^{*2}, g_2^{*2}$ and $g_3^{*2}$ are the squares of the principal $g$-factors.

We next assume that $\hat{g}$ is gate-voltage dependent, and write $\hat{g}(V_G) \approx \hat{g}(V_0) + (V_G - V_0) \hat{g}^\prime(V_0)$, where $\hat{g}^\prime = \partial \hat{g} / \partial V_G$. Upon application of a microwave burst $V_G(t) = V_0 + V_{ac}\sin(2\pi ft)$ resonant with the transition between the two states of the doublet ($hf = \Delta E$), the Rabi frequency $f_R = |f_R|$ reads [14]:

$$f_R = \frac{\mu_B B V_{ac}}{2|g^*|} \left[ \hat{g}(V_0) \cdot b \right] \times \left[ \hat{g}^\prime(V_0) \cdot b \right],$$

(4)

with $|g^*| = |\hat{g}(V_0) \cdot b|$ (see Supplementary Material [30]). Importantly, the anisotropy of $g^*$ (characterized by $\hat{G}$) and its electrical tunability (described by $\hat{G}^\prime = \partial \hat{G} / \partial V_G$) do not provide enough information to predict the Rabi frequencies with Eq. (4). The matrix elements of $\hat{g}(V_0)$ depend on the choice of axes for the magnetic field and of a basis set for the Kramers doublet [Eq. (2)] - at variance with the elements of $\hat{G}$, which depend only on the choice of a frame of the magnetic field [Eq. (3)]. As a consequence, $\hat{g}$ is defined by the experimental $\hat{G}$ up to a rotation of the Kramers basis. We note that $\hat{g}$ can always be diagonalized in the magnetic axes frame $\{X, Y, Z\}$ ($\hat{g} = \hat{g}_{\text{diag}}(g_1^{*2}, g_2^{*2}, g_3^{*2})$) by an appropriate (but implicit) choice of Kramers basis (Refs. [22, 51] and [30]); yet it is practically impossible to exploit a common basis for the reconstruction of $\hat{g}(V_0)$ and $\hat{g}^\prime(V_0)$ from $\hat{G}(V_0)$ and $\hat{G}^\prime(V_0)$.

As an implication, we emphasize that a gate-voltage modulation of $\hat{g}$ can give rise to a finite Rabi frequency [Eq. (4)] but no variations in the Zeeman splitting, i.e. $\hat{G}^\prime = 0$. Indeed, the electrical tunability of the Zeeman tensor $\hat{G}$ is related to the derivative of the $\hat{g}$ matrix by

$$\hat{G}^\prime = \langle \hat{g} \cdot \hat{g}^\prime + \hat{g}^\prime \cdot \hat{g} \rangle,$$

(5)

so that $\hat{G}$ is zero if $\langle \hat{g} \cdot \hat{g}^\prime \rangle$ is an anti-symmetric matrix.

Such a mechanism for EDSR will be named “Iso-Zeeman
Resonance” (IZR) in order to underline that \( \hat{g} \), but not \( \hat{G} \), depends on \( V_G \). A notable example is the set-up of Ref. [22], where a harmonic quantum dot in a static magnetic field is moved around its equilibrium position by an homogeneous alternating electric field, and EDSR is mediated by intrinsic SO coupling. In these conditions, \( \hat{G}' = 0 \) because the electric field does not change the shape of the confinement potential and, therefore, the Zeeman splitting; however, \( \hat{g}' \neq 0 \) since the vector potential breaks translational symmetry. The \( g \)-matrix formulation of the theory of Ref. [22] for harmonic and arbitrary confinement potentials is discussed in [30].

On the other hand, the electrical variations \( \hat{G}' \) of the Zeeman tensor result in a non-zero \( \hat{g}' \) [Eq. (1)] driving spin rotations according to Eq. (4). We refer to this mechanism as “Zeeman Modulation Resonance” (ZMR). Extreme cases of ZMR are Refs. [10, 14], where the magnetic axes do not even depend on gate voltages, but are set by the symmetry of the system. Therefore, spin oscillations result only from modulations of the principal \( g \)-factors \( g'_i \), as in the conventional \( g \)-TMR scenario.

Both ZMR and IZR are ultimately due to the SO interaction, but they manifest in different ways. ZMR is driven by modulations of the shape of the confinement potential by the electric field, which result in variations of the Zeeman splitting. IZR can be interpreted as the effect of the alternating motion of the dot as a whole (as, e.g., in Ref. [22]), but is invisible in the Zeeman splitting.

Our formalism allows to discriminate IZR and ZMR within Fig. 3a. The electrical modulations of \( \hat{g} \) due to ZMR and IZR can be characterized by two matrices, \( \hat{g}'_{\text{ZMR}} \) and \( \hat{g}'_{\text{IZR}} \), such that \( \hat{g}'_{\text{ZMR}} + \hat{g}'_{\text{IZR}} \equiv \hat{g}' \); \( \hat{f}_R \) splits accordingly into \( \hat{f}_{\text{ZMR}} + \hat{f}_{\text{IZR}} \), where \( \hat{f}_{\text{ZMR}} \) and \( \hat{f}_{\text{IZR}} \) are given by Eq. (4) using \( \hat{g}'_{\text{ZMR}} \) and \( \hat{g}'_{\text{IZR}} \) as input, respectively. \( \hat{g}'_{\text{ZMR}} \) and \( \hat{g}'_{\text{IZR}} \) are defined from the decomposition of \( \hat{g} \cdot \hat{g}' \) into a symmetric and an anti-symmetric matrix. As expected, \( \hat{g}'_{\text{ZMR}} = \hat{g}'^{-1} \cdot \hat{G}'/2 \) is fully determined by the dependence of the symmetric Zeeman tensor on gate voltage, while \( \hat{g}'_{\text{IZR}} \) is totally independent on \( \hat{G}' \) (30) for details).

We compute \( \hat{G}' \) from the experimental tensors \( \hat{G}(V_0) \) and \( \hat{G}(V_1) \) (with \( V_1 - V_0 = 0.25 \text{ mV} \approx V_{\text{ac}} \)) quoted in Fig. 2. In the magnetic axes frame \( \{X, Y, Z\} \) at \( V_{G2} = V_0 \), we find:

\[
\hat{G}'(V_0) = \begin{bmatrix}
-17.9 & 21.1 & 7.2 \\
21.2 & 17.1 & -19.8 \\
7.2 & -19.8 & 9.1
\end{bmatrix} \cdot V^{-1}.
\]

We can finally fit the missing \( \hat{g}'_{\text{IZR}} \) contribution onto the Rabi frequency map of Fig. 3a. This yields:

\[
\hat{g}'(V_0) = \begin{bmatrix}
-4.3 & -2.4 & -3.2 \\
10.5 & 3.4 & -28.0 \\
8.6 & 30.5 & 2.8
\end{bmatrix} \cdot V^{-1}
\]

in the same magnetic axes and in the (yet unknown)

Kramers basis where \( \hat{g}(V_0) \) is diagonal.

Both the Kramers basis and the principal magnetic axes rotate when varying \( V_G \); since both \( \hat{G}'(V_0) \) and \( \hat{g}'(V_0) \) are non-diagonal, which demonstrates concomitant IZR and ZMR. The Rabi frequencies calculated from \( \hat{g}_d \) and \( \hat{g}' \) via Eq. (4) are plotted in Fig. 3b. We ascribe the small discrepancies to the experimental uncertainty on \( \hat{G}'(V_0) \). Finally, we address the contributions of IZR and ZMR to the Rabi frequency. Fig. 4 shows the maps of \( \hat{f}_{\text{IZR}} \) and \( f_{\text{ZMR}} \) obtained from Eq. (4) using \( \hat{g}'_{\text{IZR}} \) and \( \hat{g}'_{\text{ZMR}} \) as inputs, respectively. \( f_{\text{ZMR}} \) (Fig. 4b) is about three times smaller than the maximal \( f_{\text{IZR}} \) (Fig. 4a), but still non-negligible. The IZR map of Fig. 4 is consistent with an in-plane spin-orbit field \( B_{\text{SO}} \parallel \hat{x} \) (as \( f_{\text{IZR}} \) is minimal along that direction). Since the holes are mostly confined along \( z \) by the structure and gates field, this implies that the microwave signal on gate 2 essentially drives motion of the hole along the nanowire axis \( y \). As a matter of fact, dot R is controlled by gates 1 and 2 on Fig. 4b, and is likely shifted under the spacer between the gates around this bias point. A sizable ZMR arises on top of IZR due to the changes of the complex confinement potential with gate voltage. Note that the total Rabi frequency \( f_R \) also depends on the relative orientation of \( f_{\text{IZR}} \) and \( f_{\text{ZMR}} \) (30). To conclude, coherent spin rotations in presence of SO coupling are achieved by the composition of two effects, one (ZMR) led by modulations of the shape of the potential, the other (IZR) resulting from the dot motion in the spin-orbit field. ZMR can be characterized by the gate voltage dependence of the Zeeman tensor, which is uniquely defined by the measurement of the Zeeman splitting for six orientations of the magnetic field. The missing IZR contribution can be extracted from the anisotropy of the Rabi frequency. Both IZR and ZMR can be described in an unified picture by a generalized \( g \)-matrix and by its derivative with respect to gate voltage, which completely characterize the effects of SO coupling (in the linear regime). Measuring and understanding the interplay between ZMR and IZR shall help to optimize
SUPPLEMENTAL MATERIAL

In this supplementary material we address (i) the details of the $g$-matrix formalism for the Rabi frequency, (ii) the reformulation in the $g$-matrix formalism of a known example [22] of spin-orbit mediated spin resonance at finite $B$, (iii) the separation between IZR and ZMR contributions and (iv) the experimental procedure to extract IZR and ZMR matrices. We also make some concluding remarks on the $g$-matrix formalism.

Model

In this Section, we introduce the $g$-matrix and the symmetric Zeeman tensor, and discuss their relations; then we derive the formula for the Rabi frequency in the $g$-matrix formalism.

The $g$-matrix

The Hamiltonian of a Kramers doublet $\{|\uparrow\rangle, |\downarrow\rangle\}$ in a homogeneous magnetic field $B$ can be written:

$$H = \frac{1}{2} \mu_B \vec{\sigma} \cdot \hat{g} \cdot B,$$

where $\mu_B$ is Bohr’s magneton, $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is the vector of Pauli matrices, $\hat{g}$ is the $g$-matrix (a real $3 \times 3$ matrix), and $\cdot$ is the matrix product. Any linear-in-$B$ two-level Hamiltonian can in principle be mapped onto Eq. (S1) up to an irrelevant energy shift. The 9 elements of the $g$-matrix are independent unless symmetries reduce the number of degrees of freedom. In order to get further insights into the significance of the $g$-matrix, we may factor $\hat{g} = \hat{U} \cdot \hat{g}_d \cdot \hat{V}$, where $\hat{g}_d = \text{diag}(g_1, g_2, g_3)$ is diagonal and $\hat{U}, \hat{V}$ are unitary matrices with determinant +1 (singular value decomposition):

$$H = \frac{1}{2} \mu_B (\vec{\sigma} \cdot \hat{g}_d \cdot (\vec{V} \cdot B)).$$

The columns of $\vec{V}$ define three direct, orthonormal magnetic axes $X, Y$ and $Z$. $\vec{\sigma}' = (\hat{U} \cdot \vec{\sigma})$ sets three new spin matrices $(\sigma'_1, \sigma'_2, \sigma'_3)$, or, equivalently, three new orthogonal quantization axes for the pseudo-spin of the Kramers doublet. Therefore, there must exist an unitary transform $R$ in the $\{|\uparrow\rangle, |\downarrow\rangle\}$ subspace such that $R^\dagger \sigma'_i R = \sigma_i$ for all $i$’s [32]. The columns of $R$ define a new basis $\{|\uparrow\rangle_Z, |\downarrow\rangle_Z\}$ for the two levels system in which:

$$H = \frac{1}{2} \mu_B (g_1 B_1 \sigma_1 + g_2 B_2 \sigma_2 + g_3 B_3 \sigma_3),$$

where $B_1, B_2$ and $B_3$ are the components of $B$ along the magnetic axes $X, Y$ and $Z$. Hence, the $g$-matrix can be made diagonal with an appropriate choice of real space axes for the magnetic field and basis set for the two levels system [28, 31]. The states $|\uparrow\rangle_Z$ and $|\downarrow\rangle_Z$ can be identified as the up and down pseudo-spin states along $Z$ as they are the eigenstates of $H$ for magnetic fields $B \parallel Z$. 

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The (symmetric) Zeeman tensor

Rewriting Eq. [S1] as

\[ H = \frac{1}{2} \mu_B |\mathbf{g} \cdot \mathbf{B}| \sigma_u, \]  

(S4)

where \( \sigma_u = t^u \cdot \sigma \) and \( u = \mathbf{g} \cdot \mathbf{B}/|\mathbf{g} \cdot \mathbf{B}| \), the Zeeman splitting \( \Delta E \) between the eigenstates of \( H \) reads:

\[ \Delta E = \mu_B |\mathbf{g} \cdot \mathbf{B}|. \]  

(S5)

This can be conveniently cast in the form:

\[ \Delta E^2 = \mu_B^2 (t^\mathbf{B} \cdot t^\mathbf{g} \cdot \mathbf{B}) = \mu_B^2 (\mathbf{B} \cdot \mathbf{G} \cdot \mathbf{B}), \]  

(S6)

where \( \mathbf{G} = t^\mathbf{g} \cdot \mathbf{g} \) is the symmetric Zeeman tensor. From a practical point of view, \( \mathbf{G} \) can be constructed from the measurement of \( \Delta E^2 \) for six orientations of the magnetic field. Note that \( \mathbf{G} \) only depends on the choice of a frame for the magnetic field. On the contrary \( \mathbf{g} \) depends on the choice of a frame for the magnetic field and on a choice of basis set \( \{|\uparrow\rangle, |\downarrow\rangle\} \) for the Kramers doublet. Any rotation \( R \) of the \( \{|\uparrow\rangle, |\downarrow\rangle\} \) basis set results in a corresponding rotation \( \mathbf{g}_R = U(R) \cdot \mathbf{g} \) of the g-matrix, which leaves the Zeeman tensor \( \mathbf{G}_R = t^{\mathbf{g}_R} \cdot \mathbf{g}_R = t^\mathbf{g} \cdot \mathbf{g} = \mathbf{G} \) invariant (as expected, since the Zeeman splittings must not depend on the choice of the \( \{|\uparrow\rangle, |\downarrow\rangle\} \) basis set). It follows from Eq. [S3] that the eigenvalues of \( \mathbf{G} \) are \( g_1^2, g_2^2, \) and \( g_3^2 \) while the eigenvectors of \( \mathbf{G} \) are the magnetic axes \( \mathbf{X}, \mathbf{Y} \) and \( \mathbf{Z} \). The characterization of the Zeeman splittings therefore brings the principal \( g \)-factors and associated magnetic axes, but leaves \( |\uparrow\rangle \mathbf{Z} \) and \( |\downarrow\rangle \mathbf{Z} \) unspecified.

Although not a limitation for many purposes, this may hinder the measurement of \( \mathbf{g}' \), the derivative of \( \mathbf{g} \) with respect to the gate voltage \( V \). Indeed, if \( |\uparrow\rangle \mathbf{Z} \) and \( |\downarrow\rangle \mathbf{Z} \) depend on \( V \), then \( \mathbf{g}(V) \) is diagonal in a different (yet implicit) basis set at each gate voltage. It is then not possible to reconstruct \( \mathbf{g}' \) from the measurement of the Zeeman tensor at different gate voltages.

The Rabi frequency in the g-matrix formalism

We now derive the formula for the Rabi frequency when the g-matrix is dependent on a single control parameter, in this case a gate voltage \( V \). When this gate voltage is varied around \( V = V_0 \), the Hamiltonian can be written:

\[ H(V) = \frac{1}{2} \mu_B t^\mathbf{g} \cdot \mathbf{g}(V) \cdot \mathbf{B} \]

\[ \simeq \frac{1}{2} \mu_B t^\mathbf{g} \cdot \mathbf{g}(V_0) + \mathbf{g}(V_0) \delta V \cdot \mathbf{B}, \]  

(S7)

where \( \mathbf{g}' \) is the derivative of \( \mathbf{g} \) with respect to \( V \) and \( \delta V = V - V_0 \). Let us introduce the Larmor vector \( \mathbf{h}\Omega = \mu_B \mathbf{g}(V_0) \cdot \mathbf{B}/2 \) and its gate-voltage derivative \( \mathbf{h}\Omega' = \mu_B \mathbf{g}'(V_0) \cdot \mathbf{B}/2 \). Then,

\[ H(V) = \mathbf{h}\Omega = \mathbf{h}\Omega + \mathbf{h}\Omega' \delta V |\sigma_\omega + \mathbf{h}\Omega' \delta V |\sigma^{\omega'}, \]  

(S8)

with \( \omega = \Omega/|\Omega| \) and \( \omega' = \Omega'/|\Omega'| \). Splitting \( \Omega' = \Omega'_{\parallel} + \Omega'_{\perp} \) into components parallel and perpendicular to \( \Omega \),

\[ H(V) = \mathbf{h}\Omega + \mathbf{h}\Omega_{\parallel} \delta V |\sigma_\omega + \mathbf{h}\Omega_{\perp} |\delta V |\sigma^{\omega'}_. \]  

(S9)

\( \Omega' \) characterizes gate-driven modulations of the Larmor (spin precession) frequency, while \( \Omega'_{\perp} \) characterizes spin rotations. For a radio-frequency (RF) \( \delta V = V_{ac} \sin(|\Omega|/t) \) resonant with the transition between the eigenstates of \( H(V_0) \), the Rabi frequency \( f_R \) reads [10, 13]:

\[ \hbar f_R = \mathbf{h}\Omega_{\perp} V_{ac} \]

\[ = \hbar \omega \times \Omega_{\perp} V_{ac} \]

\[ = \frac{\mu_B V_{ac}}{2 |g^*|} |\mathbf{g}(V_0) \cdot \mathbf{b} \times \mathbf{g}'(V_0) \cdot \mathbf{b}|, \]  

(S10)
where \( \mathbf{b} = \mathbf{B}/B \) is the unit vector along the magnetic field and \(|g^*| = |\hat{g}(V_0) \cdot \mathbf{b}| \) is the effective \( g \)-factor along that direction. This may be conveniently written \( f_R = |\mathbf{f}_R| \), with:

\[
h_fR = \frac{\mu_B BV_{ac}}{2|g^*|} [\hat{g}(V_0) \cdot \mathbf{b}] \times [\hat{g}'(V_0) \cdot \mathbf{b}].
\] (S11)

Note that the Larmor frequency \(|\Omega|/(2\pi)\) and the Rabi frequency \(|\mathbf{f}_R|\) do not depend on the choice of the basis set for the Kramers doublet. The orientation of \( \Omega \) and \( \mathbf{f}_R \), however, does (any change of \(|\uparrow\rangle, |\downarrow\rangle\) basis set results in a rotation of \( \Omega \) and \( \mathbf{f}_R \) [22]. \( \Omega \) and \( \mathbf{f}_R \) actually define the axis of precession and the axis of rotation of the pseudo-spin in the Bloch sphere defined by the \(|\uparrow\rangle, |\downarrow\rangle\) states and the magnetic field frame.

**Example: Electric Dipole Spin Resonance in a finite magnetic field**

Any mechanism giving rise to Rabi oscillations with frequency proportional to \( B \) and \( V_{ac} \) shall be captured by Eq. (S10). This excludes spin rotations at zero-field [33] or showing significant non-linearities [34].

As an example, we can cast the spin-orbit mechanism of Ref. [22] in the \( g \)-matrix formalism. We consider a quantum dot in the effective mass approximation, with strong confinement along \( z \), harmonic confinement \( V(x, y) = m\omega_0^2(x^2 + y^2)/2 \) in the \((xy)\) plane, and in-plane Rashba plus Dresselhaus spin-orbit coupling [22]. We assume an isotropic \( g \)-factor \( g_0 \). An electric field \( \mathbf{E} = E_0(e_x x + e_y y) \sin(\omega t) \) is applied in the \((xy)\) plane in order to drive Rabi oscillations between the \(|\downarrow\rangle \) and \(|\uparrow\rangle\) states \((e_x^2 + e_y^2 = 1 \), the spin being quantized along \( z \)). Then, according to Ref.[22]

\[
h_fR = 2g_0\mu_B |\mathbf{B} \times \Omega_0|,
\] (S12)

at resonance, where:

\[
\Omega_0 = -\frac{eE_0}{m\omega_0^2} \left( \frac{e_y}{\lambda_-} + \frac{e_x}{\lambda_+}, 0 \right),
\] (S13)

and \( \lambda_{\pm} = h/[m(\beta \pm \alpha)] \), \((\alpha, \beta)\) being the Rashba and Dresselhaus spin-orbit constants. From the effective Hamiltonian of the quantum dot in the static limit \((\omega \to 0)\), [22]

\[
H_{\text{eff}} = \frac{1}{2} g_0\mu_B \mathbf{B} \cdot \mathbf{\sigma} + g_0\mu_B (\mathbf{B} \times \Omega_0) \cdot \mathbf{\sigma} ,
\] (S14)

the matrices \( \hat{g} \) and \( \hat{g}' \) can be readily identified (the derivative being taken with respect to \( E_0 \) instead of \( V \)):

\[
\hat{g} = g_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\] (S15a)

\[
\hat{g}' = 2g_0 \begin{pmatrix} 0 & 0 & -\Theta_{0,y} \\ 0 & 0 & +\Theta_{0,x} \\ +\Theta_{0,y} & -\Theta_{0,x} & 0 \end{pmatrix},
\] (S15b)

with \( \Theta_0 = \Omega_0/E_0 \). Insertion of Eqs. [S15] into Eqs. [S10] yields back Eq. [S12], showing that the present \( g \)-matrix formalism indeed captures the EDSR mechanism of Ref.[22]

**Discussion: IZR vs ZMR**

In the above set-up the Rabi oscillations are mediated by spin-orbit coupling when the quantum dot is moved around its equilibrium position by the alternating electric field. This example highlights the distinction between the IZR and ZMR contributions introduced in the main text.

Indeed, in a harmonic potential, the alternating electric field moves the center of the dot around but does not change the harmonic shape of the confinement potential. Therefore, the Zeeman splittings are expected to be independent on the position of the dot. As a matter of fact,

\[
\hat{G}' = 'g \cdot \hat{g}' + 'g' \cdot \hat{g}
\] (S16)
is zero because $\dot{q}'$ is anti-symmetric. While gauge invariance imposes that $\hat{G}$ does not change upon translation of the dot in an uniform magnetic field, $\dot{g}$ does depend on the position of the dot because the magnetic vector potential breaks translational symmetry.

The set-up of Ref. 22 hence gives rise to pure “IZR” (i.e., modulations of the $g$-matrix without modulations of the Zeeman splittings). In an arbitrary potential/non-uniform alternating electric field, the motion of the dot will, in general, come along with changes of the shape of the confinement potential resulting in modulations of the Zeeman splittings (extra ZMR contributions to $\dot{q}'$). These modulations of the Zeeman splittings share, however, the same microscopic origin as IZR, namely spin-orbit coupling, and shall therefore be treated on the same footing.

As discussed in the main text, we can always split $\dot{t}\hat{g} \cdot \dot{g}' = \hat{S} + \hat{A}$, where $\hat{S}$ is a symmetric and $\hat{A}$ is an anti-symmetric matrix. Then, Eq. (S16) sets $\hat{S} = \hat{G}'/2$. We can next introduce the ZMR matrix
\[
\dot{g}_{ZMR} = \dot{t} \hat{g}^{-1} \cdot \hat{G}'/2
\] (S17)

and the IZR matrix
\[
\dot{g}_{IZR} = \dot{t} \hat{g}^{-1} \cdot \hat{A},
\] (S18)

and split Eq. (S11) as $f_R = f_{ZMR} + f_{IZR}$, where $f_{ZMR}$ and $f_{IZR}$ are the contributions of $\dot{g}_{ZMR}'$ and $\dot{g}_{IZR}'$ to the Rabi frequency. The ZMR matrix captures the modulations of the Zeeman tensor, while the IZR matrix captures the pure rotations of $\{|\uparrow\rangle_Z, |\downarrow\rangle_Z\}$ that do not give rise to modulations of the Zeeman splittings. If the Rabi oscillations are mediated by spin-orbit coupling, ZMR can be interpreted as the contribution from the changes of the shape of the potential, and IZR as the contribution from the motion of the dot in the electric field.

In most situations IZR coexists with ZMR. IZR is expected to dominate over ZMR whenever the motion of the dot does not come along with significant variations of the shape of the potential, as outlined in Ref. 22. On the opposite, ZMR prevails in highly symmetric situations and/or for specific orientations of the magnetic field where the gate potential only modulates the $g_i$'s (as in Ref. 14). If the gate potential also controls the magnetic axes $\{X, Y, Z\}$, ZMR is typically accompanied by IZR.

**Extraction of IZR and ZMR contributions**

As discussed in section 1, the $\dot{g}'$ matrix can not usually be reconstructed from the measurement of the Zeeman splittings since the IZR contribution does not give rise to modulations of the Zeeman tensor. Nonetheless, $\dot{g}_{IZR}'$ can be extracted from the Rabi frequency map. We detail the procedure below.

First, the symmetric Zeeman tensor $\hat{G} = \dot{t} \hat{g} \cdot \dot{g}'$ is constructed from the measurement of the Zeeman splittings along 6 independent directions. The eigenvalues of $\hat{G}$ are the square of the principal $g$-factors $g_1^2$, $g_2^2$, and $g_3^2$, while the eigenvectors of $\hat{G}$ are the principal magnetic axes $X$, $Y$, and $Z$. In the magnetic axes frame, there exists a (yet implicit) basis set $\{|\uparrow\rangle_Z, |\downarrow\rangle_Z\}$ for the Kramers doublet such that:
\[
\hat{g} \equiv \hat{g}_d = \begin{pmatrix}
g_1 & 0 & 0 \\
0 & g_2 & 0 \\
0 & 0 & g_3
\end{pmatrix}.
\] (S19)

Note that there might be an ambiguity on the sign of the $g_i$'s (all assumed with the same sign here).

The matrix $\hat{G}'$ is extracted from the measurement of $\hat{G}$ for two nearby gate voltages $V = V_0$ and $V = V_0 + \delta V$. In the principal magnetic axes and $\{|\uparrow\rangle_Z, |\downarrow\rangle_Z\}$ basis set at $V = V_0$, the ZMR matrix is then $\dot{g}_{ZMR}' = \hat{g}_d^{-1} \cdot \hat{G}'/2$.

The three independent elements of $\hat{A}$ are fitted to the dependence of the Rabi frequency on the magnetic field orientation using Eq. (S10) (in principle, a measurement of the Rabi frequency along three directions shall be sufficient; here we made a least square fit on the whole map). The amplitude $V_{ac}$ of the RF field needs to be included in the fit if the attenuation of the RF lines is not exactly known. Note that the Rabi frequency is measured at constant Zeeman splitting (resonance frequency $|\Omega|/(2\pi) = 9$ GHz) instead of constant magnetic field in order to ensure that the attenuation does not vary from one field orientation to another.

This procedure yields in the present case $g_1(V_0) = 2.08$ along $X = (0.82, 0.19, -0.53)$, $g_2(V_0) = 2.48$ along $Y =
FIG. S1. (a) Map of $|f_R| - |f_{IZR}| - |f_{ZMR}|$ as a function of the orientation of the magnetic field (see main text for a definition of $\theta$ and $\phi$). (b) Map of $\cos(\theta_Z)$ as a function of the orientation of the magnetic field, where $\theta_Z$ is the angle between $f_{IZR}$ and $f_{ZMR}$. These two maps are plotted at constant Zeeman splitting $|g^*\mu_B B/h| = 9$ GHz as in the main text.

$$(-0.22, 0.98, 0.01), \ g_3(V_0) = 1.62 \ \text{along} \ \mathbf{Z} = (0.52, 0.11, 0.84),$$

$$\hat{g}'_{ZMR}(V_0) = \begin{pmatrix} -4.31 & 5.07 & 1.73 \\ 4.26 & 3.45 & -4.01 \\ 2.22 & -6.12 & 2.82 \end{pmatrix} \ \text{V}^{-1}, \quad (S20)$$

and

$$\hat{g}'_{IZR}(V_0) = \begin{pmatrix} 0.0 & -7.45 & -4.97 \\ 6.26 & 0.0 & -23.99 \\ 6.38 & 36.60 & 0.0 \end{pmatrix} \ \text{V}^{-1}. \quad (S21)$$

We estimate $V_{ac} = 0.41$ mV, close to the value expected for the RF setup. The fact that $G'$ is not diagonal in this low-symmetry device shows that the principal magnetic axes (as well as, presumably, the basis set $\{|\uparrow\rangle_Z, |\downarrow\rangle_Z\}$) rotate with the gate voltage. The calculated map of Rabi frequencies $f_R$ is plotted on Fig. 3b of the main text, while the IZR contribution $|f_{IZR}|$ and the ZMR contribution $|f_{ZMR}|$ are plotted in Fig. 4a and Fig. 4b, respectively.

The ZMR map of Fig. 4b shows a complex dependence on the magnetic field orientation that reflects the complex structure of the potential in the device. As for the IZR diagram of Fig. 4a, we point out that, although the model of Ref. 22 can not be applied directly, $|f_{IZR}|$ reproduces one of the most salient feature of Eq. (S12): two well-defined minima along $\mathbf{x} = [1\bar{1}0]$ suggest that the effective spin-orbit field is perpendicular to the nanowire. Since the static electric field in the dot is dominated by the vertical ($\parallel z$) component, this implies that the RF electric field on gate 2 essentially drives motion of the hole in dot R along the nanowire axis $y$. This is consistent with the fact that dot R is controlled by both gates 1 and 2 on Fig. 1b of the main text, and is, therefore, likely located under the spacer between gate 1 and gate 2 at this bias point. This calls for a detailed modeling of the potential and spin-orbit interactions in these devices, however beyond the scope of this paper.

We emphasize that, in general, $f_R < |f_{IZR}| + |f_{ZMR}|$, because $f_{IZR}$ and $f_{ZMR}$ are not aligned (IZR and ZMR drive rotations around different axes). In order to highlight this correction, we have plotted in Fig. S1 the difference $f_R - |f_{IZR}| - |f_{ZMR}|$ along with $\cos(\theta_Z)$, where $\theta_Z$ is the angle between $f_{IZR}$ and $f_{ZMR}$. There is a large sector around $\theta = 90^\circ$, $\varphi = 135^\circ$ where IZR and ZMR tend to cancel each other. The design of the device might be optimized in order to align $f_{IZR}$ and $f_{ZMR}$ as best as possible.
As discussed earlier, Eq. (S10) holds in any device where the Rabi frequencies at finite magnetic field are linear in $B$ and $V_{ac}$. It can therefore be used as a “compact model” for the control of the device. The analysis of the orientational dependence of $f_R$, $f_{IZR}$ and $f_{ZMR}$ combined with appropriate modeling may provide valuable information about the confinement potential and spin-orbit interactions in the qubits.

As far as modeling is concerned, the $g$-matrix formalism is a very efficient way to compute the map of Rabi frequencies. Indeed, the $g$-matrix at a given gate voltage $V = V_0$ can be calculated from the eigenstates at zero magnetic field using simple perturbation theory and its derivative can be obtained from finite differences between gate voltages $V = V_0$ and $V = V_0 + \delta V$ (note that in a numerical calculation the basis set $\{|\uparrow\rangle, |\downarrow\rangle\}$ can be set explicitly). The complete map of Rabi frequencies can hence be constructed from two electronic structure calculations at zero magnetic field but different gate voltages, without the need for an explicit sampling over magnetic field orientations.

Concluding remarks
$R$ can be cast in the form

$$R = \begin{pmatrix} \alpha e^{i\theta} & -\beta^* \\ \beta e^{i\theta} & \alpha^* \end{pmatrix}$$

with $|\alpha|^2 + |\beta|^2 = 1$. In the basis set $\{|\uparrow\rangle', |\downarrow\rangle'\} = R\{|\uparrow\rangle, |\downarrow\rangle\}$, the Hamiltonian reads $H' = \mu_B \sigma' \cdot \hat{g} \cdot B / 2$, with $\sigma' = R^\dagger \sigma R$. Yet $\sigma' = \hat{U} \cdot \sigma$, with:

$$\hat{U} = \begin{pmatrix} \text{Re}[(\alpha^2 - \beta^2)e^{i\theta}] & \text{Im}[(\alpha^2 - \beta^2)e^{i\theta}] & 2\text{Re}(\alpha^* \beta) \\ -\text{Im}[(\alpha^2 + \beta^2)e^{i\theta}] & \text{Re}[(\alpha^2 + \beta^2)e^{i\theta}] & 2\text{Im}(\alpha^* \beta) \\ -2\text{Re}(\alpha\beta e^{i\theta}) & -2\text{Im}(\alpha\beta e^{i\theta}) & |\alpha|^2 - |\beta|^2 \end{pmatrix}.$$ 

Hence $H' = \mu_B \sigma' \cdot \hat{g}' \cdot B / 2$, with $\hat{g}' = \hat{U} \cdot \hat{g}$. The $\hat{U}$ matrix is unitary with determinant +1. Therefore, any rotation of the $\{|\uparrow\rangle, |\downarrow\rangle\}$ basis set results in a corresponding rotation of the $g$-matrix. Conversely, any unitary $3 \times 3$ matrix $U$ with determinant +1 can be mapped onto Eq. (), and associated with a unitary transform $R$ in the $\{|\uparrow\rangle, |\downarrow\rangle\}$ subspace.

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