Non-Topological Gauss-Bonnet type model of gravity with torsion

H. Niu∗
S. S. Chern Institute of Mathematics, Nankai University, Tianjin, 300-071, China

D. G. Pak†
S. S. Chern Institute of Mathematics, Nankai University, Tianjin, 300-071, China and
Center for Theoretical Physics, Seoul National University, Seoul 151-742, Korea

A non-topological Lorentz gauge model of gravity with torsion based on Gauss-Bonnet type Lagrangian is considered. The Lagrangian differs from the Lovelock term in four-dimensional space-time and has a number of interesting features. We demonstrate that the model admits a propagating torsion unlike the case of the topological Lovelock gravity. Due to additional symmetries of the proposed Gauss-Bonnet type Lagrangian the torsion has a reduced set of dynamical degrees of freedom corresponding to the spin two field, $U(1)$ gauge vector field and spin zero field. A remarkable feature is that the kinetic part of the Hamiltonian containing the spin two field is positively defined. We perform one-loop quantization of the model for a special case of constant Riemann curvature space-time background treating the torsion as a quantum field variable. We discuss a possible mechanism of emergent Einstein gravity as an effective theory which can be induced due to quantum dynamics of torsion.

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I. INTRODUCTION

Recently a modified Gauss-Bonnet gravity is a subject of intensive studies in constructing alternative cosmological models [1, 2]. The Gauss-Bonnet term appears also in low-energy effective action of superstring [3] which is a candidate for a consistent theory of quantum gravity unified with other fundamental interactions. On the other hand, the gauge approach to gravity based on gauging Lorentz and Poincare groups [4, 5, 6, 7, 8] can also lead to a consistent quantum theory of gravity in the framework of field theory formalism [9, 10, 11]. The extension of gravity models to the case of non-Riemannian space-time geometry reveals new possibilities towards construction of renormalizable quantum gravity with torsion [12, 13, 14, 15]. A Lorentz gauge model of gravity with Yang-Mills type Lagrangian including torsion has been developed further in [16] where it has been proposed that the Einstein gravity with a cosmological term can be induced as an effective theory due to quantum corrections of torsion. In that model the space-time metric is treated as a fixed classical field while the contortion (torsion) supposed to be a quantum field. Such a treatment of the metric is not satisfactory from the conceptual point of view since one has to assume the existence of the fixed classical space-time with a metric given a priori. In other words, we encounter the problem of space-time background dependence which is similar to the space-time dependence problem in superstrings. One possible way to resolve this problem is to generalize the Lorentz gauge model by extending the gauge group to Poincare one. In that case the gauge potential of Poincare group, the vielbein, becomes dynamical on equal footing with torsion. Another interesting possibility is to consider such a gravity model which assumes the existence of a pure topological phase with unfixed, arbitrary metric from the start.

In the present paper we consider a non-topological Gauss-Bonnet type gravity model with torsion and study its classical and quantum properties. We demonstrate that the model admits a propagating torsion, and this is strictly different from the Gauss-Bonnet gravity model in Lovelock form [17, 18]. The non-topological Gauss-Bonnet gravity represents an alternative theory to Yang-Mills type gauge model of gravity and it can provide the mechanism of emergent Einstein gravity via quantum dynamics of torsion as it was proposed recently in [16]. The main advantage of the present model is that in the absence of torsion the theory describes a pure topological phase of gravity with an arbitrary space-time metric which does not satisfy any equation of motion. The metric becomes dynamical only after inducing the Einstein-Hilbert term in the quantum effective action. It is remarkable that the torsion in our model possesses a reduced set of physical field components including a unique spin two field. So that the torsion can be interpreted as a
gravitational quantum counter-part to the metric treated as a classical field variable of the effective Einstein gravity. Besides, the spin two field component of torsion leads to a positively-definite kinetic term in the Hamiltonian, unlike the case of $R^2$ Lorentz gauge gravity models which have the well-known non-unitarity problem.

In section II, we describe the non-topological model of Gauss-Bonnet type gravity in the framework of Riemann-Cartan geometry. In section III, we study the canonical structure of the model in a special case of flat space-time metric and non-vanishing torsion which plays a role of the gauge potential in Lorentz gauge field theory. The quantization of the model in a constant curvature space-time background with quantum torsion field is considered in section IV. In the last section we discuss the possibility of generating the Einstein-Hilbert term and cosmological constant as a result of torsion radiative corrections.

II. THE MODEL

Let us start with the main outlines of Riemann-Cartan geometry. The basic geometric objects in approaches to formulation of gravity as a gauge theory of the Poincare group $[4,5,8]$ are the vielbein $e^m_a$ and the general Lorentz affine connection $A^c_{md}$. The infinitesimal Lorentz transformation of the vielbein $e^m_a$ is given by

$$\delta e^m_a = [\Lambda, e^m_a] = \Lambda^b_a e^m_b,$$  \hspace{1cm} (1)

where $\Lambda \equiv A^{cd}Q^{cd}$ is a Lie algebra valued gauge parameter, and $Q^{cd}$ is a generator of Lorentz Lie algebra. We assume that the vielbein is invertible and the signature of the flat metric $\eta_{ab}$ in the tangent space-time is Minkowskian, $\eta_{ab} = \text{diag}(+-++)$.

The covariant derivative with respect to Lorentz group transformation is defined in a standard manner

$$D_a = e^m_a (\partial_m + gA_m),$$  \hspace{1cm} (2)

where $A_m \equiv A_{mcd}Q^{cd}$ is a general affine connection taking values in the Lorentz Lie algebra, and $g$ is a new gravitational gauge coupling constant. For brevity of notation we will use a redefined connection which absorbs the coupling constant. The original Lorentz gauge transformation of the connection $A_m$ has the form

$$\delta A_m = -\partial_m A - [A_m, \Lambda].$$  \hspace{1cm} (3)

The affine connection $A_{mcd}$ can be rewritten as a sum of Levi-Civita spin connection $\varphi_{mc}^d(e)$ and contortion $K_{mc}^d$:

$$A_{mc}^d = \varphi_{mc}^d + K_{mc}^d,$$  \hspace{1cm} (4)

$$\varphi_{\mu a}^b(e) = \frac{1}{2} (e^{ab}\partial_\mu e_{\nu a} - e^{b\nu}_a \partial^\mu e_{\nu a} - e^{e\nu}_a \partial^\mu e_{\nu a} + \partial_\mu e_{\nu a} + e^{e\nu}_a \partial_\mu e_{\nu a} - \partial^\mu e_{\nu a}),$$  \hspace{1cm} (5)

The torsion and curvature tensors are defined in a standard way

$$[D_a, D_b] = T_{ab} = e^{e\nu}_a \partial_\nu e_{\mu b} + R_{ab},$$  \hspace{1cm} (6)

where $R_{ab} \equiv R_{abcd}Q^{cd}$. Under the decomposition $[4]$ the Riemann-Cartan curvature is splitted into two parts

$$R_{abcd} = \hat{R}_{abcd} + \tilde{R}_{abcd},$$  \hspace{1cm} (7)

$$\hat{R}_{abcd} = \hat{D}_a \varphi_{bc}^d + \varphi_{ac}^{\ e} \varphi_{be}^d - (a \leftrightarrow b),$$

$$\tilde{R}_{abcd} = \tilde{D}_a K_{bc}^d + K_{ac}^e K_{be}^d - (a \leftrightarrow b),$$

where the underlined indices stand for indices over which the covariantization has been performed.

With these preliminaries let us write down the well-known Lagrangian of Gauss-Bonnet gravity of Lovelock type in four-dimensional space-time $[17,18]$

$$\mathcal{L}_{\text{Lovelock}} = \beta_0 e_{abcd}R^{ab} \wedge R^{cd},$$  \hspace{1cm} (8)

where $\beta_0$ is a dimensionless constant and $R_{ab}$ is the Riemann-Cartan curvature two-form. Since the Lagrangian is represented by a closed differential form it does not produce equations of motion, so that the torsion has no propagating modes. We will consider a model of gravity with torsion based on the following Gauss-Bonnet type Lagrangian

$$\mathcal{L} = -\frac{1}{4} I_{GB} = -\frac{1}{4} (R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2).$$  \hspace{1cm} (9)

In a case of Riemannian space-time geometry, when torsion vanishes, the Lovelock term, $[8]$, reduces to the standard Gauss-Bonnet topological invariant which can be rewritten in its original form $I_{GB}$, $[9]$, in terms of Riemann curvature (up to a normalization factor). However, it is important to stress that in the presence of torsion the Gauss-Bonnet combination $I_{GB}$ is principally different from the Lovelock term $\mathcal{L}_{\text{Lovelock}}$. Namely, one can check that the Gauss-Bonnet Lagrangian in the form $[9]$ does not correspond to any topological invariant in Riemann-Cartan geometry since it can not be expressed as a total divergence. In the subsequent sections we will demonstrate that the contortion (torsion) in our model with Lagrangian $[4]$ reveals non-trivial dynamical properties and a number of interesting features.
III. GAUSS-BONNET TYPE GAUGE MODEL IN FLAT SPACE-TIME

The Lorentz gauge model described by Gauss-Bonnet type Lagrangian contains two sets of variables, the vielbein and the torsion. As we will see in the next section, the one-loop effective action with a constant curvature space-time background and quantum torsion possesses additional local symmetries which reduce the dynamical content of torsion. However, beyond one-loop approximation and with no assumption of constant curvature space-time the local symmetries may not survive. So that it is not obvious whether the number of dynamical torsion degrees of freedom remains the same in general. To understand the origin of the dynamical content of torsion we consider first the classical structure of Gauss-Bonnet gravity in the simplest case when the space-time metric is flat. In this limit the model represents a pure Lorentz gauge field theory with the quadratic Lagrangian \( \mathcal{L} \), and contortion \( K_{bcd} \) as a gauge potential. The Riemann-Cartan curvature field strength which can be written in a standard form (we keep for the Lorentz gauge field strength the same notation as for the Riemann-Cartan curvature)

\[ R_{abcd} = \partial_a K_{bcd} - \partial_b K_{acd} + K_{ac}^\varepsilon K_{bcd} - K_{bc}^\varepsilon K_{acd}. \tag{10} \]

Let us consider the canonical structure of the Lorentz gauge along the lines of canonical formalism of theories with constraints. The canonical momenta corresponding to the gauge potential \( K_{mcd} \) are defined as follows

\[ \pi_{mcd} = \frac{\partial \mathcal{L}}{\partial \dot{K}_{mcd}} = -R_{m0cd} + (\partial_a R_{mcd} + \delta_{mc} \delta R_{0c} + \frac{\gamma}{2} R\delta_{mcd} \delta_{bc} - (c \leftrightarrow d)), \tag{11} \]

where \( \dot{K}_{mcd} = \partial_b K_{mcd}. \) One can check that the canonical momenta \( \pi_{0cd} \) vanish identically and represent first-stage constraints

\[ \pi_{0cd} = 0. \tag{12} \]

There are other momenta, \( \pi_{\mu0\delta} \) (we use Greek letters for space indices), which do not contain terms with time derivatives of \( K_{mcd} \) and, due to this, they produce additional primary constraints

\[ \pi_{\mu0\delta} = -R_{0\mu0}\delta + R_{\mu\delta} + \delta_{\mu0} R_{0\delta} - \frac{1}{2} \delta_{\mu0} R. \tag{13} \]

In Lagrange formalism it follows that the Lagrange equations of motion for the field \( K_{\mu0\delta} \) are not dynamical but represent first order differential equations. These equations are solvable constraints in the theory which can be solved for \( K_{\mu0\delta} \) at least in principle. So that the components \( K_{\mu0\delta} \) do not represent dynamical degrees of freedom, and they can be excluded from the physical field spectrum.

The remaining canonical momenta \( \pi_{\mu\gamma\delta} \) have the following form

\[ \pi_{\mu\gamma\delta} = -R_{0\mu\gamma\delta} + \delta_{\mu\delta} R_{0\gamma} - \delta_{\mu\gamma} R_{0\delta}, \]

\[ \pi_{\mu0\delta} \equiv \pi_{\delta} = -R_{0\delta}. \tag{14} \]

These equations can be resolved to find the ”velocities” \( \dot{K}_{\mu\gamma\delta} \)

\[ \dot{K}_{\mu\gamma\delta} = -\pi_{\mu\gamma\delta} + \delta_{\mu\delta} \pi_{\gamma} + \frac{1}{2} \gamma R \delta_{\mu0\delta} + K_{\gamma0\delta} + K_{\gamma0\delta} \delta_{\mu0\delta}, \tag{15} \]

where \( \pi_{\gamma0\delta} = K_{\gamma0\delta} \). One can insert the “velocities” \( \dot{K}_{\mu\gamma\delta} \) into the initial Lagrangian

\[ \mathcal{L} = -\frac{1}{2} \pi_{\mu\gamma\delta}^2 + \pi_{\mu\gamma\delta} \delta_{\mu\delta} \frac{1}{2} \gamma R \delta_{\mu0\delta} + \hat{\mathcal{L}}(K), \]

\[ \hat{\mathcal{L}}(K) = -\frac{1}{4} R_{\mu0\delta}^2 - \frac{1}{4} R_{\nu0\delta\beta}^2 + \frac{1}{4} R_{\mu0\delta}^2 + \frac{1}{4} R_{\nu0\delta}^2, \tag{16} \]

where \( H^{*} = \pi_{mcd} \dot{K}^{mcd} - \mathcal{L}. \tag{17} \)

After inserting the functions \( \dot{K}_{\mu\gamma\delta} \), into the extended Hamiltonian \( H^{*} \) one obtains the Hamiltonian \( H^{(1)} \) with the partially resolved ”velocities” \( \dot{K}_{\mu\gamma\delta} \) and first-stage constraints

\[ H^{(1)} = \frac{1}{2} \pi_{\mu\gamma\delta} \pi_{\mu\gamma\delta} - \pi_{\mu\delta} \pi_{\mu\delta} - \pi_{\mu\gamma\delta} R_{0\mu\gamma\delta}, \]

\[ -\mathcal{L}(K) + \chi_{0cd} \Phi_{1cd}^{(1)} + 2 \lambda_{\mu0\delta} \Phi_{2\mu0\delta}^{(1)}, \]

\[ \Phi_{1cd}^{(1)} = \pi_{0cd}, \]

\[ \Phi_{2\mu0\delta}^{(1)} = \pi_{\mu0\delta} + \hat{R}_{0\mu0\delta} - \hat{R}_{\mu0\delta} + \frac{1}{2} \delta_{\mu0} (\hat{R} - 2 \hat{R}_{00}), \tag{18} \]

where \( \lambda_{0cd}, \chi_{0\mu\delta} \) are Lagrange multipliers. We can see that the first kinetic term in the Hamiltonian provides a positive contribution to the energy. There is no term
quadratic in momentum $\pi_{\mu0}\delta$ which could produce a negative energy contribution as in the case of Yang-Mills type $R^2$-gravity. This is a direct consequence of the specific structure of the Gauss-Bonnet combination. Still one has a negative contribution coming from the second term in $H^{(1)}$, so that the total Hamiltonian is not positively-defined. Notice, the Weyl type Lagrangian

$$\mathcal{L}_{\text{Weyl}} = \frac{1}{4} (R_{abcd} R^{abcd} - 2 R_{ab} R^{ab} + \frac{1}{3} R^2)$$  \hspace{1cm} (19)

leads to a Hamiltonian with a non-vanishing kinetic term $-\pi_{\mu0}\delta$, whereas the kinetic term $\pi_2^\alpha$ does not appear at all since the canonical momenta $\pi_\alpha$ vanish identically.

One can verify that the first-stage constraints commute to each other

$$\{\Phi^{(1)}_{1cd}, \Phi^{(1)}_{2ab}\} = 0.$$  \hspace{1cm} (20)

By direct calculating the Poisson brackets between the Hamiltonian $H^{(1)}$ and the first stage constraints $\Phi^{(1)}_{1cd}$ one can find the second-stage constraints

$$\{H^{(1)}, \Phi^{(1)}_{1\gamma\delta}\} = \Phi^{(2)}_{1\gamma\delta} = D^\gamma \pi_\delta + K^\gamma_{\alpha\delta}(\Phi^{(1)}_{2\alpha\delta} - \pi_{30\delta}),$$

$$\{H^{(1)}, \Phi^{(1)}_{10\delta}\} = \Phi^{(2)}_{10\delta} = \pi_{\mu\gamma} K_{\mu\delta} + \partial^\gamma (\Phi^{(1)}_{2\gamma\delta} - \pi_{30\delta}) - K_{\gamma\delta} (\Phi^{(1)}_{2\gamma\delta} - \pi_{30\delta}),$$  \hspace{1cm} (21)

where the covariantization is assumed on underlined indices. One can calculate the following Poisson bracket

$$\{\Phi^{(2)}_{1cd}, \Phi^{(1)}_{1ab}\} = 0.$$  \hspace{1cm} (22)

This implies that the Lagrange multiplier $\lambda_{abcd}$ can not be found as a solution of a new constraint. This is consistent with the fact of presence of the original Lorentz gauge symmetry due to which one can impose the Coulomb gauge condition $K_{0cd} = 0$.

The explicit expression for the second-stage constraint $\Phi^{(2)}_{2ab}$ turns out to be quite complicate. Because of this it is hard to find an explicit solution for the second Lagrange multiplier $\lambda_{a0\delta}$ as a solution of higher stage constraints. This obstacle reflects the high non-linearity of the Lagrange equation for $K_{a0\delta}$. Notice that, since $K_{\mu0\delta}$ satisfies first order differential equation, it can not be set to zero. That means there is no additional local symmetries in the full Lagrangian except for the original one given by $\delta_a K_{0cd}$. To analyze the number of local dynamical degrees of freedom in the theory it is enough to consider a free part of the Lagrangian $L_{\text{free}}$

$$L_{\text{free}} = -\frac{1}{2} (\partial_a K_{b0d})^2 + \frac{1}{2} (\partial^b K_{b0d})^2 + (\partial^a K_{b0d})^2$$

$$- 2 \partial^a K_{b0d} \partial^b K_{d0} + (\partial_a K_{b0d})^2 - (\partial^b K_{b0d})^2.$$  \hspace{1cm} (23)

One can verify that there are indeed only nine physical degrees of freedom corresponding to the field $K_{\mu\gamma\delta}$. To see that, notice that the free Lagrangian has additional two types of gauge symmetries under the following transformations

$$\delta K_{mc0} = \partial_c \tau_{cm} - \partial_m \tau_{cm},$$

$$\delta K_{mc0} = \partial_c \sigma_{cm} - \partial_m \sigma_{cm},$$  \hspace{1cm} (24)

where $\tau_{cd}, \sigma_{cd}$ are constrained gauge parameters satisfying the conditions

$$\sigma_{cd} = \sigma_{dc},$$

$$\tau_{cd} = -\tau_{dc}, \quad \partial^n \sigma_{cd} = 0,$$

$$\tau_{cd} = -\tau_{dc}. $$  \hspace{1cm} (26)

The constrained gauge parameter $\sigma_{cd}$ has six independent degrees of freedom whereas the parameter $\tau_{cd}$ has only three independent degrees of freedom. To count the independent degrees of $\tau_{cd}$ it is convenient to replace $\tau_{cd}$ with its dual counter-part

$$\tau_{cd} = -\frac{1}{2} \epsilon_{cdab} \tau^{*ab}.$$  \hspace{1cm} (27)

This replacement allows to express the dual gauge parameter $\tau^{*ab}$ in terms of a new vector $\psi_a$

$$\tau^{*ab} = \hat{D}_a \psi_b - \hat{D}_b \psi_a.$$  \hspace{1cm} (28)

The definition of $\psi_a$ implies a secondary gauge invariance

$$\delta_a \psi_a = \hat{D}_a \chi,$$  \hspace{1cm} (29)

which decreases the number of independent degrees of freedom up to three. So that, the total number of independent pure gauge degrees of freedom for the symmetries is nine. After subtracting six Lorentz gauge degrees we obtain finally nine physical degrees of freedom for $K_{b0d}$ in agreement with the results obtained in the canonical formalism.

In the next section we will consider the quantization of the Gauss-Bonnet gravity model in the presence of non-flat background metric corresponding to a constant curvature space-time. We will show that the number of additional local gauge symmetries will be decreased, however, the number of physical dynamical degrees of freedom remains the same.

IV. QUANTIZATION IN A CONSTANT CURVATURE SPACE-TIME BACKGROUND

One-loop effective action with a constant curvature space-time background and quantum torsion has been
calculated recently in the model with Yang-Mills type Lagrangian quadratic in Riemann-Cartan curvature \[16\]. In this section we consider perturbative quantization of the model with Gauss-Bonnet type Lagrangian, \(9\). The quantization procedure is similar to the covariant background quantization in supergravity \[20\]. We apply the quantization scheme based on functional integral. In background field formalism one starts with splitting the general gauge connection \(A_{mcd}\) into background (classical) and quantum parts

\[
A_{mcd} = A^{(cl)}_{mcd} + A^{(q)}_{mcd}. \tag{30}
\]

In this section we identify the classical field \(A^{(cl)}_{mcd}\) with the Levi-Civita connection \(\varphi_{mcd}(e)\) corresponding to the Riemannian space-time geometry and the quantum part \(A^{(q)}_{mcd}\) with contortion \(\kappa_{mcd}\) which represents quantum dynamical degrees of freedom.

Let us define two types of Lorentz gauge transformations consistent with the original gauge transformation \[3\] and splitting \[30\]:

(I) the classical, or background, gauge transformation

\[
\delta e^m_a = \Lambda^a_b e^m_b, \quad \delta \varphi_m = -\partial_\mu \Lambda - [\varphi, \Lambda], \quad \delta \kappa_m = -[\kappa, \Lambda], \tag{31}
\]

(II) the quantum gauge transformation

\[
\delta e^m_a = \delta \varphi_m(e) = 0, \quad \delta \kappa_m = -\hat{D}_m \Lambda - [\kappa, \Lambda], \tag{32}
\]

where \(\varphi_m \equiv \varphi_{mcd}Q^{cd}\), and the restricted covariant derivative \(\hat{D}_m\) is defined by means of the Levi-Civita connection only

\[
\hat{D}_m \Lambda = \partial_\mu \Lambda + [\varphi, \Lambda]. \tag{33}
\]

Notice that the restricted derivative \(\hat{D}_m\) is covariant under the classical Lorentz gauge transformation.

In one-loop approximation it is sufficient to keep only quadratic contortion terms in the Lagrangian. After integration by part and neglecting surface terms the quadratic Lagrangian can be reduced to the form

\[
L^{(2)} = -\frac{1}{4} I_{GB}(\hat{R}) + 2 \hat{D}_a \kappa_{bcd} (\hat{D}^a \kappa^{bcd} - \hat{D}^b \kappa^{acd})
- 4 (\hat{D}^a \kappa_{bcd})^2 + 8 \hat{D}_a \kappa_{bcd} \hat{D}^b \kappa^d - 4 (\hat{D}_a \kappa_{bd})^2
+ 4 (\hat{D}_b \kappa^d)^2 + 4 \hat{R}_{abc} K^{ace} \kappa^{b d} - 8 \hat{R}^{d c} (K^{e} \kappa_{bed} - K_{bed} \kappa^{e c})
- 2 \hat{R} (K^d_{a c d} + K_{b d}^e K^{c e})], \tag{34}
\]

where \(I_{GB}(\hat{R})\) is the topological Gauss-Bonnet density (up to an appropriate normalization factor).

To simplify the analysis of local dynamical degrees of freedom in the theory with curved space-time background (within the framework of perturbative quantization) we will use adiabatic approximation. So that the Riemann curvature is supposed to be covariant constant, i.e., \(\hat{D}_a \hat{R}_{bedc} = 0\).

An interesting feature of the quadratic Lagrangian \[34\] is the presence of additional local \(U(1)\) symmetry

\[
\delta_U(1) K_{bcd} = \frac{1}{3} (\eta_{bc} \hat{D}_d \lambda - \eta_{bd} \hat{D}_c \lambda), \quad \delta_U(1) \kappa_d = \hat{D}_d \lambda. \tag{35}
\]

This \(U(1)\) symmetry corresponds to the symmetry \[25\] in the flat space-time with the gauge parameter \(\lambda \equiv \sigma^e\). Notice, that a free part of the Yang-Mills type Lagrangian

\[
\mathcal{L}_{YM} = -\frac{1}{4} F_{abcd} F^{abcd} \tag{36}
\]

does not possess such a local \(U(1)\) symmetry.

It is convenient to decompose the contortion into irreducible parts

\[
K_{bcd} = Q_{bcd} + \frac{1}{3} (\eta_{bc} \hat{D}_d \lambda - \eta_{bd} \hat{D}_c \lambda) + \frac{1}{6} \epsilon_{bcde} S^e, \quad Q_{cd}^e = 0, \quad \epsilon_{abcd} Q_{bcd} = 0. \tag{37}
\]

For simplicity we choose the covariant constant background space-time as a Riemannian space-time of constant curvature

\[
\hat{R}_{abcd} = \frac{1}{12} \hat{R} (\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bc}). \tag{38}
\]

With this the Lagrangian \[34\] can be rewritten in the form

\[
L^{(2)} = -\frac{1}{4} I_{GB}(\hat{R}) - \frac{1}{2} (\hat{D}_a Q_{bcd})^2 + \frac{1}{2} \hat{D}^a Q^{bcd} \hat{D}_b Q_{acd}
+ (\hat{D}^a Q_{bcd})^2 - \frac{2}{3} \hat{D}^a Q_{bcd} \hat{D}^b K^d + \frac{1}{12} \hat{R} Q^{ace} Q_{ce b}
+ \frac{1}{9} (\hat{D}_b K_d - \hat{D}_d K_b)^2 + \frac{1}{12} (\hat{D}_a S^e)^2 - \frac{1}{36} \hat{R} S^2. \tag{39}
\]

An unexpected feature of the quadratic Lagrangian \[39\], is that it admits another local symmetry corresponding to the symmetry \[25\] in flat space-time limit. The symmetry is provided by the following transformations with a new constrained parameter \(\chi_{bc}\)

\[
\delta \chi_{Q_{bcd}} = \hat{D}_c \chi_{db} - \hat{D}_d \chi_{cb}, \quad \delta \chi_{K_{d}} = 0, \quad \delta \chi S_a = 0, \quad \chi_{bc} = \chi_{cb}, \quad \chi^c_c = 0, \quad \hat{D}_c \chi_{ced} = 0. \tag{40}
\]
The field $Q_{bcd}$ has sixteen field components in general. After subtracting six pure gauge degrees of freedom due to Lorentz gauge symmetry and five degrees due to $\chi$-symmetry one has exactly five physical degrees of freedom for the spin two field. The fact that we have only one physical spin two field is unexpected, and it does not occur in the gauge gravity model with Yang-Mills type Lagrangian $L_{YM}$, where the contortion $Q_{bcd}$ contains a pair of two spin fields, one of which produces a negative contribution to the Hamiltonian.

Notice, that the last term in the Lagrangian (39) prevents appearance of the local symmetry (24). This is not surprising, because the symmetries available in the case of flat space-time may not survive in curved space-time. Nevertheless, one can easily verify from the equations of motion for the vector field $S^a$ that only the temporal component $S^0$ is dynamical. The equations of motion for the space field components $S^\mu$ represent the first-order differential equations in respect to time derivative, so that they are constraints in the theory which can be solved. This implies that the field $S_a$ contains only one dynamical degree of freedom corresponding to spin zero field.

The fact that the field $S_a$ has only scalar dynamical degree of freedom completes the analogy between the metric tensor which has spin 2 and spin 0 irreducible components and the torsion fields $Q_{bcd}, S_a$ which have exactly six dynamical degrees of freedom corresponding to fields of spin two and zero. This can serve as an additional argument to our conjecture that the torsion represents a dynamic degree of freedom of quantum gravity and the classical metric tensor inherits its properties after the Einstein gravity emerges as an effective theory via quantum dynamics of torsion. We will discuss on this in more details in the last section. Notice, we have totally nine physical degrees of freedom for the fields $Q_{bcd}, K_a, S^a$ in agreement with the analysis presented in the previous section.

Having all local symmetries of the quadratic Lagrangian one can perform the formal quantization using the standard methods of quantum field theory. First we fix the gauge under the quantum type (II) gauge transformations, (22), which can be written for the torsion irreducible fields $Q_{bcd}, K_a, S_a$

$$\delta Q_{bcd} = \delta K_{bcd} - \frac{1}{3}(\eta_{bc}\delta K_d - \eta_{bd}\delta K_c) - \frac{1}{6}\epsilon_{bcdde}\delta S^e,$$

$$\delta K_d = -\hat{D}^c\Lambda_{cd},$$

$$\delta S^a = -\epsilon^{abcde}\hat{D}_b\Lambda_{cd},$$

where we keep only linear terms, that is enough in one-loop approximation. The simplest gauge fixing function we have chosen is the following

$$F_{1cd} = \hat{D}^bQ_{bcd}.\quad (42)$$

One has a simple transformation rule for the gauge function

$$\delta F_{1cd} = -\frac{2}{3}(\hat{D}\hat{D} + \hat{\Lambda})\Lambda_{cd}.\quad (43)$$

With this one can write down the corresponding gauge fixing term and Faddeev-Popov ghost Lagrangian

$$L^{(1)}_{gf} = -\frac{1}{2\xi_1}(\hat{D}^bQ_{bcd})^2,$$

$$L^{(1)}_{FP} = \bar{c}c(\hat{D}\hat{D} + \hat{R})c_{1cd},\quad (44)$$

where $\bar{c}, c$ are ghost fields. For simplicity we choose the gauge parameter $\xi_1 = 1$. Notice, that the gauge function $F_{1cd}$ is invariant under $U(1)$ and $\chi$-transformations, (45). To fix the gauge for the local $U(1)$ symmetry one has to introduce a second gauge fixing function which can be chosen as

$$F_2 = \hat{D}^bK_b.\quad (45)$$

The corresponding gauge fixing term and Faddeev-Popov Lagrangian have the following form

$$L^{(2)}_{gf} = -\frac{1}{\xi_2}(\hat{D}^bK_a)^2,$$

$$L^{(2)}_{FP} = \bar{c}_2\hat{D}\hat{D}c_2.\quad (46)$$

We choose a Feynman gauge ($\xi_2 = 1$) for simplicity. Finally, to fix the gauge for $\chi$-transformations one can choose the following gauge fixing function

$$F_{3bd} = \frac{1}{2}(\hat{D}^aQ_{bad} + \hat{D}^aQ_{dad}),$$

$$\eta^{bd}F_{3bd} = 0,$$

$$\hat{D}^bF_{3bd} = \frac{1}{2}\hat{D}^a\hat{D}^bQ_{bad} \simeq 0,\quad (47)$$

where the last equality takes place on the hypersurface $\hat{D}^bQ_{bcd} = 0$ in the configuration space of functions $\{Q_{bcd}\}$. One can easily find the corresponding gauge fixing and ghost terms

$$L^{(3)}_{gf} = -\frac{1}{2\xi_3}F_{3bd},$$

$$L^{(3)}_{FP} = \bar{\psi}_{cd}(\hat{D}\hat{D} - \frac{\hat{R}}{3})\psi_{cd}.\quad (48)$$

We set $\xi_3 = \frac{1}{2}$ which corresponds to symmetric gauge.

The final expression for a total one-loop effective Lagrangian is given by the sum of all gauge fixing and ghost terms
\[ \mathcal{L}_{ef}^{(2)} = -\frac{1}{4}I_{GB}(\hat{R}) - \frac{1}{2}(\hat{D}_a Q_{bcd})^2 + \frac{1}{2}(\hat{D}^\alpha Q_{bad})^2 - \frac{1}{2} \hat{D}^a Q_{bad} \hat{D}_c Q^{dcb} - \frac{2}{3} \hat{D}^a Q_{bad} \hat{D}^b K_d \]
\[ -\frac{1}{8} \hat{R} Q_{bca}^2 + \frac{\hat{R}}{6} Q^{bca} Q_{cda} + \frac{2}{9} (\hat{D}_a K_b)^2 + \frac{1}{18} \hat{R} K_d^2 - \frac{1}{36} F^{bca} (3 \hat{D}_a \hat{D}_b + \eta_{ab} \hat{R}) S_d + \sum_{i=1,2,3} \mathcal{L}_{F_i}. \quad (49) \]

The expression for the effective Lagrangian is ready for calculation of the one-loop effective action. The contributions of ghosts are given by scalar functional determinants which can be easily calculated in analytic form as in [16]. Calculation of the contribution produced by contortion is much more complicate due to the tensorial structure of the corresponding propagator. At least it can be estimated by using the high derivative expansion.

The propagator for the fields \( Q_{bcd}, K_d \) can be found straightforward. The principal part of such calculation is to find an inverse operator to \( G^{(pqr)}_{bcd} \) in the kinetic term

\[ \mathcal{L}^{kin}(Q) = \frac{1}{2} Q_{bcd} G^{(pqr)}_{bcd} Q_{pqr}. \quad (50) \]

We prove the existence of the propagator for \( Q_{bcd} \) by explicit calculating the inverse operator in flat space-time limit. In that limit one has

\[ G^{(pqr)}_{bcd} = \frac{1}{2} \delta^{[q}_{[c} \delta^{p]}_{[d]} \delta^{r]}_{[b]} \]

\[ K^{(pqr)}_{bcd} = \frac{1}{2} \delta^{[q}_{[c} \delta^{p]}_{[d]} \delta^{r]}_{[b]} \]

\[ B^{(pqr)}_{bcd} = \frac{1}{2} \delta^{[q}_{[c} \delta^{p]}_{[d]} \partial b \delta^{r]}_{[b]} \]

where, \( \square \equiv \partial^2 \), and we use the following rule for antisymmetrization over indices \([a, b] = 1/2 (ab - ba)\). The propagator \( G^{-1} \) can be defined as a right or left inverse operator to \( G \). We will choose a left operator, i.e., \( G^{-1} G = 1 \). The inverse operator reads

\[ G^{-1} = \frac{1}{\square} (1 + 4 K - 4 \hat{D} - 6 \hat{F}), \]

\[ F^{(pqr)}_{bcd} = \frac{1}{\square} \delta^{[q}_{[c} \partial d \delta^{p]}_{[b]} \partial b \delta^{r]}_{[b]} \]

The inverse operator to the kinetic operator \( G^{(S)}_{ab} = 3 \hat{D}_a \hat{D}_b + \eta_{ab} \hat{R} \) for the field \( S^a \) can be found for the non-flat space-time with a constant Riemann curvature in a complete form

\[ (G^{(S)})^{-1}_{ab} = \frac{1}{\hat{R}} \left( \delta_{ab} - \frac{1}{3} \hat{D}_a \right) \cdot \frac{1}{\hat{R} + \hat{D}_b}. \quad (53) \]

The inverse operator does not have additional poles, but it has a singularity at \( \hat{R} \rightarrow 0 \). This is related to the fact of appearance of the additional local symmetry [24] in flat space-time limit. Notice, that the curvature \( \hat{R} \) plays the role of a mass scale which makes worse the ultraviolet behavior of the propagator. Because of this the model looks non-renormalizable within the perturbative quantization scheme. One should notice that the scale \( \hat{R} \) appears to be a natural cut-off parameter related to the finite size of the Universe. So that, the standard perturbative technique of Feynman diagrams with unlimited internal momentum inside loops needs some improvement at least. Still it is possible that the model can be renormalizable non-perturbatively, since the original Lagrangian reveals high symmetry, and, there is no dimensional coupling constants in the theory.

V. DISCUSSION

In our previous paper [16] we have proposed a mechanism of dynamical generation of Einstein gravity and cosmological term via quantum corrections of torsion in the framework of Yang-Mills type Lorentz gauge model. The main ingredient of such a mechanism is the formation of torsion vacuum condensate which we assumed to be covariant constant

\[ \langle \tilde{R}_{abcd} \rangle = \frac{1}{2} M^2 (\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bc}), \quad (54) \]

where \( M^2 \) is a mass scale parameter characterizing the torsion condensate. The factor \( M^2 \) need to be positive since it corresponds to positive curvature space-time which can only be created during the vacuum transition from the trivial vacuum to the non-trivial one. Expanding the original classical Lagrangian \( \mathcal{L} \) [9], around the new vacuum by shifting \( R_{abcd} \rightarrow R_{abcd} + \langle \tilde{R}_{abcd} \rangle \) one obtains the following torsionless part of the Einstein-Hilbert effective action

\[ \mathcal{L}_{EH_{eff}} = -\frac{1}{4} I_{GB}(\hat{R}) - \frac{1}{2} \hat{R} M^2 - \frac{3}{2} M^4. \quad (55) \]

Notice, that the second and third terms in the effective action are completely identical to those in the effective action obtained from the Yang-Mills type Lagrangian [16].
One should emphasize that even though the vacuum averaged value $\langle R_{abcd} \rangle$ is specified by \cite{51}, the classical gravitational field $R_{abcd}$ is not constrained in general. The last term in the equation corresponds to a positive vacuum energy density which is supposed to be born during the vacuum transition. This is consistent with the positiveness of the scale parameter $M^2$ and with the fact that the torsion condensate \cite{51} corresponds to the gravitomagnetic part of the Riemann-Cartan curvature \cite{16}.

Notice that the torsion condensate \cite{51} is supposed to be covariant constant. Rigorously speaking, such a constant solution is unstable in general like the constant chromomagnetic field configuration in quantum chromodynamics which leads to vacuum instability \cite{21}. One can see that the ghost kinetic operator in (48) is not positively defined. This implies existence of tachyonic mode in the theory, so that the constant curvature solution corresponds to a false vacuum, and a true microscopic vacuum should be realized by some other non-trivial solution. In this respect the search of possible classical stable vacuum solutions in Einstein-Gauss-Bonnet gravity with torsion is of great importance \cite{22}.

In conclusion, we have considered a non-topological Gauss-Bonnet type model of gravity with dynamical torsion. The model has a number of advantages to compare with Yang-Mills type Lorentz gauge gravity. In the absence of torsion the model reduces to a pure topological Gauss-Bonnet gravity, i.e., one has a topological phase where the metric is not specified a priori. The metric obtains its dynamical content after dynamical symmetry breaking in the phase of effective Einstein gravity which is induced by quantum torsion corrections. Remarkably, the contortion in our model has only one physical spin two field which can be interpreted as a torsion quantum counter-part to the classical graviton. So that we don’t have to quantize the metric which can be treated as a classical object of the effective Einstein theory, whereas the torsion (its spin two field component) could be responsible for the quantum effects of gravitation. Another interesting result is that the Hamiltonian part corresponding to the spin two torsion component is positively defined unlike the case of Yang-Mills type Lorentz gauge theory. An obvious drawback of the model is still the presence of the vector mode $K_a$ which produces a negative contribution to the energy. One possible way to remove this difficulty is to extend the model by introducing supersymmetry which supposed to be unbroken at scale near the Planckian one. Possible applications of Lorentz gauge models of gravity with Lagrangians quadratic in Riemann-Cartan curvature in cosmology of early Universe will be considered elsewhere.

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\begin{thebibliography}{99}
\footnotesize
\bibitem{1} S. Nojiri, S.D. Odintsov, Phys. Lett. B631 (2005) 1; S. Nojiri, S.D. Odintsov and M. Sasaki, Phys. Rev. D71, 123509 (2005).
\bibitem{2} S. Nojiri, S.D. Odintsov, O.G. Gorbunova, J. Phys. A39 (2006) 6627; G. Cognola, E. Elizalde, S. Nojiri, S.D. Odintsov, S. Zerbini, Phys. Rev. D73, 084007 (2006); I.P. Neupane, B.M.N. Carter, J. Cosmol. Astropart. Phys. 0606 (2006) 004; B. Li, J.D. Barrow, D.F. Mota, Phys. Rev. D76, 044027 (2007).
\bibitem{3} D.G. Boulware and S. Deser, Phys. Rev. Lett 55, 2656 (1985).
\bibitem{4} R. Utiyama, Phys. Rev. 101, 1597 (1956);
\bibitem{5} T.W.B. Kibble, J. Math. Phys. 2, 212 (1961); D.W. Sciama, Rev. Mod. Phys. 36, 463 (1964); ibid., 1103.
\bibitem{6} K. Hayashi and T. Nakano, Progr. Theor. Phys. 38, 491 (1967); K. Hayashi and A. Bregman, Ann. Phys. (N. Y.) 75, 562 (1973); R. Utiyama and T. Fukuyama, Progr. Theor. Phys. 45, 612 (1971).
\bibitem{7} A.M. Brodskii, D. Ivanenko, and G.A. Sokolik, Sov. Phys. JETP 14, 930 (1974); D. Ivanenko, Procs. of Conf. on Theory of Gravitation, Gauthier-Villars, Paris, 1962, p.212, 215.
\bibitem{8} Y.M. Cho, Phys. Rev D14, 2521 (1976); Phys. Rev D14, 3341 (1976).
\bibitem{9} I. Antoniadis and E.T. Tomboulis, Phys. Rev. 33, 2756 (1986); D.A. Johnston, Nucl. Phys. B297, 721 (1988).
\bibitem{10} M. Carmelli, J. Math. Phys. 11, 2728 (1970); Nucl. Phys. B38, 621 (1972); Ann. Phys. (N. Y.) 113, 177 (1978); Group theory and General Relativity (International Series in Pure and Applied Physics, McGRAW-HILL Int. Book Co., 1977); E.A. Lord, Proc. Camb. Philos. Soc. 69, 423 (1971); Nuovo Cim. B11, 185 (1972).
\bibitem{11} M. Martellini and P. Sodano, Phys. Rev. D22, 1325 (1980).
\bibitem{12} D. Ivanenko and G. Sardanashvily, Phys. Rep. 94, 1 (1983).
\bibitem{13} F.W. Hehl, J.D. McCrea, E.W. Mielke, and Y. Ne’eman, Phys. Rep. 258, 1 (1995).
\bibitem{14} I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, Effective Action in Quantum Gravity (IOP, Bristol, 1992).
\bibitem{15} I.L. Shapiro, Phys. Repts., 357, 113 (2001).
\bibitem{16} D.G. Pak and S.W. Kim, Class. Quant. Grav. 25 (2008) 065011.
\bibitem{17} D. Lovelock, J. Math. Phys., 12, 498 (1971).
\bibitem{18} A. Mardones and J. Zanelli, Class. Quant. Grav. 8 (1991) 1545.
\end{thebibliography}
[19] D.M. Gitman, I.V. Tyutin, Quantization of fields with constraints (Springer-Verlag Berlin Heidelberg, 1990).
[20] B.M. Zupnik, D.G. Pak, Sov. J. Yadernaya Fizika, Vol.42, 710 (1985).
[21] G.K. Savvidy, Phys. Lett. B71, 133 (1977); N.K. Nielsen and P. Olesen, Nucl. Phys. B144, 376 (1978).
[22] F. Canfora, A. Giacomini and S. Willison, Phys. Rev. D76, 044021 (2007); F. Canfora, A. Giacomini and R. Troncoso, arXiv:0707.1056 [hep-th].