Abstract. The Tevatron collider at Fermilab provides a very rich environment for the study of $b$-hadrons. One of the most important analyses within the $B$ physics program of the CDF experiment is $B_s^0$ mixing. Since the time this school was held, several improvements in the $B_s^0$ mixing analysis have made possible the measurement of the $B_s^0$ oscillation frequency, result that has been presented at the FPCP 2006 Conference. We thank the organizers of the Corfu School for allowing us documenting here these latest results.

1. Introduction
The Tevatron collider at Fermilab, operating at $\sqrt{s} = 1.96$ TeV, has a huge $b$ production rate which is about three orders of magnitude higher than the production rate at $e^+e^-$ colliders running on the $\Upsilon(4S)$ resonance. Among the produced $b$-hadrons, there are heavy and excited states which are currently mainly accessible at the Tevatron, such as $B_s^0$, $B_c$, $\Lambda_b$ and $B^{**}$. The aim of the $B$ physics program of the CDF experiment is to provide constraints to the CKM matrix by taking advantage of the unique features of a hadron collider.

2. The CDF II detector
The CDF II detector is described in detail elsewhere [1], and consists of a spectrometer immersed in a 1.4 T magnetic field, surrounded by electromagnetic and hadronic calorimeters and muon detectors. The important features needed for this measurement include the 15 layer inner silicon strip detector (SVX) for precision vertex determination; the 96 layer outer drift chamber (COT) used for both precision tracking and $dE/dx$ particle identification; time-of-flight counters outside the COT for low momentum charged kaon identification; and calorimeters and muon drift chambers for lepton identification.

Charm and beauty hadrons are selected using a three-level trigger system that exploits the kinematics of heavy flavor production and decay and the long lifetimes of $D$ and $B$ mesons. At level one, charged particle track pairs are identified using COT data, with particle $p_T > 2$ GeV/c. At level two, the SVX data is matched to the COT tracks to identify secondary vertices from heavy flavor decays. This trigger capability is crucial for the measurement because it allows a 1000-fold rejection of backgrounds while retaining a high purity sample of heavy flavor decays. Both hadronic and semileptonic samples are obtained from this trigger selection. The third level trigger performs offline track reconstruction and lepton identification.
3. $B_s^0$ mixing in the Standard Model

In the Standard Model, the $B_s^0$ meson exists in two $CP$-conjugate states, $|B_s^0\rangle = |\bar{b}s\rangle$ and $|\bar{B}_s^0\rangle = |bs\rangle$. The two mass eigenstates of the $B_s^0$ meson, $B^H_s$ and $B^L_s$ (Heavy and Light), are not $CP$-eigenstates, but are mixtures of the two $CP$-conjugate quark states:

$$|B^H_s\rangle = p|B^0_s\rangle - q|\bar{B}^0_s\rangle,$$

$$|B^L_s\rangle = p|B^0_s\rangle + q|\bar{B}^0_s\rangle,$$

with $p^2 + q^2 = 1$. The mass difference between the $B^H_s$ and $B^L_s$ eigenstates is defined as

$$\Delta m \equiv m_H - m_L,$$

$$\Gamma = \frac{\Gamma_H + \Gamma_L}{2},$$

where $m_{H,L}$ and $\Gamma_{H,L}$ denote the mass and decay width of $B^{H,L}_s$. The probability $P$ for a $B^0_s$ meson produced at time $t = 0$ to decay as $B^0_s$ at proper time $t > 0$ is given by

$$P \approx \frac{\Gamma}{2} e^{-\Gamma t} [1 - \cos(\Delta m_s t)],$$

neglecting effects from $CP$ violation as well as a possible lifetime difference between the heavy and light $B^0_s$ mass eigenstates.

4. Analysis overview

Particle-antiparticle oscillations have been observed and well established in the $B^0_s$ system. The mass difference $\Delta m_d$ is measured to be $\Delta m_d = (0.505 \pm 0.005) \text{ ps}^{-1}$ [3]. However, observing the oscillation signal in the $B^0_s$ system has been challenging so far. The 95% confidence level (C.L.) lower limit for the $B^0_s$ mass difference is $\Delta m_s > 14.5 \text{ ps}^{-1}$ [3].

We use signals of fully reconstructed $B^0_s$ hadronic decays \(^1\) to $D^-_s \pi^+(\pi^+\pi^-)$ and $B^0_s$ leptonic decays to $D^-_s \ell^+\nu$, where $\ell = e, \mu$. The proper decay time of each signal candidate is calculated from the measured distance between the production and decay positions, the $B^0_s$ meson momentum, and the $B^0_s$ mass. The hadronic and semileptonic decay modes are complementary. Due to the larger branching ratio, the semileptonic decays provide higher statistics and stringent limits on smaller values of $\Delta m_s$. However, the semileptonic decays have poorer decay time resolution due to the missing neutrino and the unseen photon in $D^-_s \rightarrow D^-_s \gamma$. The fully reconstructed hadronic decays provide much better time resolution and therefore better sensitivity for rapid oscillations. In addition to good time resolution, the measurement of $\Delta m_s$ requires efficient and clear separation of mixed and unmixed events. The charges of the decay products unambiguously determine the flavor ($B^0_s$ or $\bar{B}^0_s$) at the time of decay. However, the flavor at the time of production can only be inferred from charge correlations with other particles in the event. Since $b$-hadrons are produced in pairs, we use charged particles in the second $b$-jet to define “opposite side” flavor tags based on lepton identification or overall jet charge. We use a complementary technique, “same-side tagging”, that exploits the charge correlation between kaons and the $B^0_s$ flavor in the fragmentation of the $b$-jet. The same-side tagging technique is critical for the measurement because it increases the effective statistics on the final flavor-tagged samples by a factor of four. In practice, the production flavor will be correctly tagged with a probability $P_{\text{tag}}$, which is smaller than one, but larger than one half (which corresponds to a random tag). The dilution is $D \equiv 2P_{\text{tag}} - 1$.

\(^1\) Throughout this document references to a specific charge state imply the charge-conjugate state as well.
5. Signal reconstruction

To reconstruct \(B_0^s\) candidates, we first select \(D_s^-\) candidates. We use \(D_s^- \rightarrow \phi \pi^-, D_s^- \rightarrow K^*(892)^0 K^-,\) and \(D_s^- \rightarrow \pi^+\pi^-\pi^-,\) with \(\phi \rightarrow K^+K^-\) and \(K^*(892)^0 \rightarrow K^-\pi^+\); we require that \(\phi\) and \(K^*(892)^0\) candidates be consistent with the known mass and width [3] of these two resonances. For the \(K^*(892)^0 K^-\) final state, we remove candidates that are consistent with the decay \(D_s^- \rightarrow K^+\pi^-\pi^-\). No particle identification is used to select these combinations. The \(D_s^-\) candidates are combined with one or three additional charged particles to form \(D_s^- \ell^+\nu, D_s^- \pi^+,\) or \(D_s^- \pi^+\pi^+\pi^-\) candidates. Leptons are identified using the calorimeter and muon detector response and the COT \(dE/dx\) information. The \(D_s^-\) and other \(B_0^s\) decay products are constrained to originate from a common vertex in three dimensions, and the \(B_0^s\) and \(D_s^-\) decay vertices are required to be significantly displaced from the \(p\bar{p}\) collision vertex. Figure 1 shows the final mass distribution for \(B_0^s \rightarrow D_s^- \pi^+\) candidates (left) and invariant \(D_s^-\) mass distribution for all combined \(B_0^s \rightarrow D_s^- \pi^+\) candidates (right).

6. Decay-time reconstruction

The transverse decay length \(L_{xy}(B)\) is defined as the displacement in the transverse plane from the primary event vertex to the reconstructed \(B\) decay point. The \(B\) meson proper decay time is given by

\[
ct(B) = L_{xy}(B) \frac{M_B}{p_T(B)},
\]

where \(M_B\) is the \(B\) mass [3]. For the semileptonic \(B\) decays we must substitute the \(B\) decay by the \(\ell D\) system. Since the \(B\) meson is not fully reconstructed, the pseudo proper decay time \(ct^*\) of the reconstructed \(B\) meson is computed as

\[
ct \equiv ct^* k,
\]
The $k$-factor corrects between the reconstructed $p_T(\ell D), L_{xy}(\ell D)$ and the unknown $p_T(B), L_{xy}(B)$ in the data. The $k$-factor distribution is obtained from a MC simulation of the signal semileptonic decays, taking into account the sample composition. Different $k$-factor distributions are used, as a function of the lepton-$D$ mass. As shown in Figure 2, the $k$-factor distribution changes considerably with the lepton-$D$ mass, making events with large lepton-$D$ mass more valuable.

**Figure 2.** $k$-factor distribution for several $m(\ell D)$ mass regions for $B_s^0 \to D^- s \ell^+ \nu, D_s^- \to \phi \pi^-$ decays.

Due to some cuts on variables related to the reconstructed proper decay time distribution, the proper decay time does not follow a pure exponential (modulo resolution and $k$-factor effects), but it is biased. This bias is corrected for using Monte Carlo simulation.

### 6.1. Decay-time resolution

The determination of the $ct$ resolution is a necessary piece of input for derivation of a proper result on $\Delta m_s$. The most precise determination comes from lifetime measurements in the exclusively reconstructed modes involving $J/\psi$ decays. The $ct$ resolution is part of the fitting procedure, and it is determined with the prompt events, in which another track from the primary vertex is combined with the $J/\psi$ candidate. Depending on the exact reconstruction details, the measured $ct$ uncertainty must be scaled by factors between 1.1 and 1.4, which implies that we underestimate the true $ct$ resolution. For analyses which rely on samples obtained with sculpting on the $ct$ distribution, the prompt component is highly reduced, making the determination of the $ct$ resolution problematic. We use unbiased prompt $D$ candidates to measure $ct/\sigma_{ct}$, and then apply a correction for each event, depending on the topology and on several kinematical quantities.
Table 1. Oscillation frequency $\Delta m_d$ measured at CDF.

| sample       | $\Delta m_d$ [ps$^{-1}$]                      |
|--------------|-----------------------------------------------|
| semileptonic | $0.509 \pm 0.010$ (stat.) $\pm 0.016$ (syst.) |
| hadronic     | $0.536 \pm 0.028$ (stat.) $\pm 0.006$ (syst.) |

Table 2. Opposite-side flavor taggers performance.

| tagger      | $\epsilon D^2$ [%] |
|-------------|---------------------|
| muon        | $0.55 \pm 0.05$     |
| electron    | $0.30 \pm 0.03$     |
| jet charge  | $0.70 \pm 0.06$     |
| combined    | $1.55 \pm 0.08$     |

7. Opposite-side flavor tagging

One of the ingredients for measuring flavor oscillations is identifying whether a $B$ meson was produced as a $B$, which contains a $b$ antiquark, or as a $\bar{B}$, which contains a $b$ quark. We refer to this production flavor identification as $b$-flavor tagging. The methods of $b$-flavor tagging are classified into opposite-side and same-side $b$-flavor taggers. Opposite-side taggers exploit the fact that $b$ quarks are mostly produced in $b \bar{b}$ pairs. Same-side flavor tags are based on the charge of particles produced in association with the $b$-hadron. The performance of the $b$ flavor tags is quantified by their efficiency $\epsilon$ and dilution $D$.

In 20% of the cases, the opposite-side $b$ decays leptonically either into an electron or a muon ($b \to \ell^- X$). The charge of the lepton is correlated to the charge of the decaying $B$ meson. Depending on the type of the $B$ meson, there is a certain probability of oscillation between production and decay (0% for $B^+$, 17.5% for $B^0_d$ and 50% for $B^0_s$). Therefore this tagging algorithm already contains an intrinsic dilution.

The average charge of an opposite-side $b$-jet is correlated to the charge of the opposite-side $b$ quark and can thus be used to determine the opposite-side $b$-flavor. The main challenge of this jet charge tagger (JQT) is to select the $b$-jet. Information of a displaced vertex or displaced tracks in the jet helps to identify $b$-jets. This tagging algorithm has high tagging efficiency, but the dilution is relatively low.

8. Flavor taggers calibration

For setting a limit on $\Delta m_s$ knowledge of the flavor taggers performance is crucial. For the opposite-side flavor taggers it is measured in kinematically similar $B^0_d$ and $B^+$ samples, whereas the same-side flavor tagger performance has to be obtained from Monte Carlo. The $\Delta m$ analysis involves complex fits with many parameters which combine several $B$ flavors and several decay modes, various different taggers, and deals with complex templates for mass and lifetime fits for various sources of background. Therefore, the measurement of $\Delta m_d$ is, in addition to the calibration of the opposite-side taggers, a very important test of the fitter framework. The $\Delta m_d$ measurements reproduced in table 1 are compatible with the PDG average value. The performance of the different opposite-side flavor taggers is summarized in table 2.
9. Same-Side flavor tagging
During the fragmentation and the formation of the $B_0^s$ meson there is a left over $\bar{s}$ quark which is likely to form a $K^+$. Hence, if there is a nearby charged particle, which is additionally identified as a kaon, it is quite likely that it is the leading fragmentation track and its charge is then correlated to the flavor of the $B_0^s$ meson. While the performance of an opposite-side tagger does not depend on the flavor of the $B$ on the signal side, the same-side tagger performance depends on the signal fragmentation process. Therefore the opposite-side performance can be measured in $B_0^s$ and $B^+$ samples, and can then be used for setting a limit on the $B_0^s$ mixing frequency. But when using a same-side tagger for a limit on $\Delta m_s$, one must rely on Monte Carlo simulation.

CDF has performed extensive data and Monte Carlo comparisons of several quantities related to the tagging, and determined the tagging candidate by selecting the most likely kaon track. The particle identification likelihood combines information of the $dE/dx$ and from the Time-of-Flight system. A comparison between data and PYTHIA Monte Carlo for the average dilution obtained by using that variable is shown in Figure 3. Finally, the overall tagging performance as computed from Monte Carlo is

$$\epsilon D^2(B_0^s \rightarrow D_s^- (\phi \pi^-) \pi^+) = 4.0^{+0.8}_{-1.2}\%.$$  

Figure 3. Comparison between data and PYTHIA Monte Carlo for the average dilution obtained by selecting the most likely kaon track as the tag.

10. Amplitude Scan
The likelihood term describing the tagged proper decay time of a neutral $B$ meson is modified by including an additional parameter multiplying the cosine, the so-called amplitude $A$,

$$\mathcal{L} \propto 1 \pm A D \cos(\Delta m t).$$  

The parameter $A$ is left free in the fit while $D$ is known and fixed in the scan. This method [4] involves performing one such $A$-fit for each value of the parameter $\Delta m$, which is fixed at each step; in the case of infinite statistics, optimal resolution and perfect tagger parameterization and calibration, one would expect $A$ to be unity for the true oscillation frequency. In practice, the output of the procedure is accordingly a list of fitted values ($A$, $\sigma_A$) for each $\Delta m$ hypothesis. A particular $\Delta m$ hypothesis is excluded to a 95% confidence level in case the following relation is observed: $A + 1.645 \sigma_A < 1$. The sensitivity of a mixing measurement is defined as the $\Delta m$ value for which $1.645 \sigma_A = 1$. 
The scan shown in Figure 4 is obtained when the method is applied to the hadronic $B^0_d$ samples of the CDF experiment, using the combined opposite-side tagging algorithms [5]. The expected compatibility of the measured amplitude with unity in the vicinity of the true frequency, $\Delta m_d = 0.5 \text{ ps}^{-1}$, is verified.

![Figure 4](image)

**Figure 4.** Amplitude scan for $\Delta m_d$ in $B^0_d \rightarrow J/\psi K^*(892)^0$ and $B^0_d \rightarrow D^- K^+$ decay modes. The scan is compatible with unit around the result of the actual $\Delta m_d$ fit.

11. Results
The result of the combined amplitude scan on $\sim 1 \text{ fb}^{-1}$ is shown in Figure 5. The sensitivity for the combination of all hadronic and semileptonic decay modes is $25.8 \text{ ps}^{-1}$, and the 95% C.L. lower limit is $\Delta m_s > 16.7 \text{ ps}^{-1}$.

![Figure 5](image)

**Figure 5.** $B^0_s$ oscillation amplitude as a function of the oscillation frequency $\Delta m_s$. The dotted curve is the value of $1.645 \sigma_A$ as a function of $\Delta m_s$, indicating a sensitivity of $\Delta m_s = 25.8 \text{ ps}^{-1}$.

The 95% confidence level limit is significantly lower than the expected limit because the amplitude shows a value consistent with unit near $\Delta m_s = 17.3 \text{ ps}^{-1}$. To assess the significance of this deviation, CDF looks at the ratio of the likelihood function at $A = 0$ and $A = 1$, as
shown in Figure 6. The maximum likelihood ratio is at $\Delta m_s = 17.3$ ps$^{-1}$ and has a value of $\log(L^1/L^0) = 6.75$. The probability that random tags background could fluctuate to mimic such a signature is 0.2%. Under the hypothesis that this is a signal for $B_s^0 - \bar{B}_s^0$ oscillations, CDF measures

$$\Delta m_s = 17.31^{+0.33}_{-0.18} \text{ (stat.)} \pm 0.07 \text{ (syst.)} \text{ ps}^{-1}.$$ 

The systematic error of this measurement is completely dominated by the $ct$ scale uncertainty, which is of the order of 0.4%.

![Figure 6. Combined log-likelihood ratio as a function of $\Delta m_s$.](image)

**Acknowledgments**

J. Piedra is supported by the EU funding under the RTN contract: HPRN-CT-2002-00292, Probe for New Physics.

**References**

[1] R. Blair et al, The CDF II detector: Technical Design Report, FERMILAB-PUB-96-390-E.
[2] The CDF Collaboration, Measurement of the $B_s^0 - \bar{B}_s^0$ oscillation frequency, [http://www-cdf.fnal.gov/physics/new/bottom/bottom.html](http://www-cdf.fnal.gov/physics/new/bottom/bottom.html).
[3] S. Eidelman et al. [Particle Data Group], [http://pdg.web.cern.ch/pdg](http://pdg.web.cern.ch/pdg).
[4] H.G. Moser, A. Roussrie, Mathematical methods for $B^0 \bar{B}^0$ oscillation analyses, NIM A384 (1997), 491-505.
[5] J. Piedra, Determination of $\Delta m_s$ and absolute calibration of flavor taggers for the $\Delta m_s$ analysis, FERMILAB-THESIS-2005-27.