Proximity Effect and Charging in Mesoscopic Normal Metal - Superconductor Junction Systems

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Abstract

Charging effects in mesoscopic junctions suppress the tunneling of electrons. In normal metal - superconductor systems they also suppress the proximity effect, thereby revealing the nature of the microscopic processes and of the ground state of the system. The effect can be made visible since the charging and proximity effect and hence the transport properties of the system can be modulated by gate voltages. If the superconducting electrode is isolated, electron number parity effect lead to an even-odd asymmetry in the proximity effect.

74.50 +r, 73.40 gk
Single electron charging effects influence the transport through small capacitance junction systems [1]. If part of the system is superconducting new phenomena have been observed. They include parity effects in systems containing superconducting islands [2,3], which arise since a single unpaired electron in a superconducting island raises the energy by the gap energy $\Delta$ [4]. This makes the correlated tunneling of two electrons by the process of an Andreev reflection an important channel for charge transfer [5]. Charging effects also influence the supercurrent through SSS or SNS structures with a mesoscopic normal island in the middle [6,7].

When a superconductor and a normal metal are put in contact, Cooper pairs leak to the normal region and a superconducting order parameter is induced in the metal. Since charging effects inhibit tunneling they reduce the proximity effect. Both can be modulated by gate voltages, which opens the way to modulate supercurrents. This creates the possibility to detect the effect and interesting perspectives for future applications [8].

The proximity effect had been studied in macroscopic junction systems by Aslamazov, Larkin and Ovchinnikov (ALO) [9]. We show how charging effects in mesoscopic systems modify their results. We first study the system shown schematically in the upper inset of Fig. 1: a small normal island separated from a bulk superconductor (with gap $\Delta$) by a low capacitance tunnel junction. The island is coupled capacitively to a gate voltage. In the second part of this article we consider the situation where the island is superconducting and study the proximity effect in the normal electrode and the modification of its transport properties. In this case electron number parity effects can be observed in the proximity effect.

The system is described by the Hamiltonian

$$H = H_0 + H_t + H_{ch}$$

where $H_0$ refers to the electrons in the normal island and the superconductor, $H_t$ describes tunneling, and $H_{ch}$ the capacitive Coulomb interaction. In the presence of the gate voltage source the electrostatic charging energy is

$$E_{ch}(q,Q_g) = \frac{q e - Q_g}{2C}.$$ 

Here $q$ is the number of excess electrons on the island, $C$ its total capacitance, i.e. the sum of the junction capacitance and that to the gate voltage. The gate voltage is responsible for the offset charge $Q_g = q_g e = C_g V_g$.

The essence of the proximity effect is a nonvanishing pair amplitude $\langle \psi_N \psi_N \rangle$ induced in the normal metal by the coupling to the superconductor. The size of the induced energy gap $\Delta_{ind} = \lambda \langle \psi_N \psi_N \rangle$ depends further on the effective pairing interaction $\lambda$ in the metal. To lowest order in the tunneling matrix element $|T|^2$, the pair amplitude is represented by the diagram shown in the lower inset of Fig. 1. It has been analyzed by (ALO) [9]. In order to evaluate the influence of the charging energy (which is a nonperturbative effect) we formulate the problem in an effective action approach, generalizing the work of Ref. [10].

The partition function of the system can be recast as a path integral over a phase variable which is related to the voltage drop at the junction interface $\phi(\tau) = (2e/h) \int_0^\tau d\tau' V(\tau')$. Since the total number of charges on the island is an integer, values of the phase which differ by $4\pi$ have to be identified. Accordingly, the partition function includes a summation over winding numbers $\phi(\beta) = \phi(0) + 4\pi n$ [11]. In the second arrangement with a superconducting island parity effects at low temperatures suppress odd number charge states. A restriction to even number states corresponds to the boundary conditions $\phi(\beta) = \phi(0) + 2\pi n$. The charging energy, including the effect of the gate voltage, is described by the action

$$S_{ch} = \int_0^\beta \left( \frac{q e - Q_g}{2C} \right) d\tau' V(\tau').$$
The effect of tunneling, in an expansion in the tunneling matrix element, is described by the action

\[
S_{ch} = \int_{0}^{\beta} d\tau \left[ \frac{C}{2} \left( \frac{\hbar}{2e} \frac{\partial \phi}{\partial \tau} \right)^2 + iQ_{g} \frac{\hbar}{2e} \frac{\partial \phi}{\partial \tau} \right]
\]  

(1)

The function\( G \) is short-ranged in space and the tunneling matrix elements are localized to the junction interface \( r = \rho \). We can write the pair amplitude as

\[
\langle \psi_{N}\psi_{N} \rangle_{r,\tau;\tau_{0}} = \frac{G}{G_{0}} \frac{V}{N_{N}(0)S} \int d\tau \int d\tau' \int d^{2}\rho K(r_{0},\rho,\tau,\tau';\tau_{0})F_{S}(\rho,\rho,\tau - \tau';\tau_{0})h(\tau,\tau';\tau_{0})
\]  

(3)

Here \( G_{S,N}(\tau) \) and \( F_{S,N}(\tau) \) are quasiclassical diagonal and off-diagonal Greens functions of the superconducting and normal electrodes, respectively. The first contribution describes quasiparticle tunneling. At low temperatures in an ideal system it is suppressed below the gap voltage, but it sets in when the gain in charging energy exceeds the energy gap for quasiparticle excitations. The second term (written here for later comparison) vanishes if one of the electrodes is normal, \( F_{N} = 0 \). If both electrodes are superconducting it describes Cooper pair tunneling in the general form (incl. the so-called “quasiparticle - Cooper pair interference” effect). The two electrons tunnel in a correlated way at times \( \tau \) and \( \tau' \) within the time scale \( \Delta^{-1} \). Higher order terms proportional to \(| T |^{4} \) describe further correlated two-particle tunneling. This includes Andreev reflection which becomes an important channel for charge transfer, when the gap \( \Delta \) exceeds the scale of the charging energy \( E_{C} = e^{2}/2C \).

The pair amplitude \( \langle \psi_{N}\psi_{N} \rangle \) in the normal metal can be calculated by adding a source term to the Hamiltonian \( \hat{H}_{N} = \int_{0}^{\beta} d\tau \int d^{3}\lambda \psi_{N}(r,\tau)\hat{N}_{N}(r,\tau)\psi_{N}(r,\tau) \) and taking the functional derivative \( \langle \psi_{N}\psi_{N} \rangle_{r,\tau_{0}} = -\delta \ln(Z)/\delta \lambda(r,\tau_{0})|_{\lambda=0} \). This produces a term similar to the second one in (4), generalized to the situation where the magnitude of the order parameter depends on space and time. Noting that the Gorkov Greens function \( F_{S} \) in the superconductor is short-ranged in space and the tunneling matrix elements are localized to the junction interface \( (r = \rho) \) we can write the pair amplitude as

\[
\langle \psi_{N}\psi_{N} \rangle_{r_{0},\tau_{0}} = \frac{G}{G_{0}} \frac{V}{N_{N}(0)S} \int d\tau \int d\tau' \int d^{2}\rho K(r_{0},\rho,\tau,\tau';\tau_{0})F_{S}(\rho,\rho,\tau - \tau';\tau_{0})h(\tau,\tau';\tau_{0})
\]  

(3)

The function \( p(r,\rho,t) \) satisfies a diffusion equation with the boundary condition that the probability current normal to the surface of the island vanishes \([12]\).

In the classical case the function \( h = 1 \), and the ALO result is recovered. The quantum fluctuations due to charging are taken into account by the phase correlator \( h(\tau,\tau';\tau_{0}) = \langle \exp\{i(\phi(\tau) + \phi(\tau') - 2\phi(\tau_{0}))/2\}\rangle_{S_{ch}} \). The expectation value has to be calculated using the charging action \([1]\) including a summation over winding numbers (note that the weight for
The function \( h(\tau, \tau'; \tau_0) \) is the amplitude for forward single electron transfers at times \( \tau \) and \( \tau' \) (similar to the term involving \( \cos \left[ \frac{\phi(\tau) + \phi(\tau')}{2} \right] \) in \( S_t \) ) and a 2e transfer back at time \( \tau_0 \). The correlator, evaluated in the charge representation, is

\[
\langle \exp \left[ i \phi(\tau) + \phi(\tau') - i \phi(\tau_0) \right] \rangle_{S_{ch}} = \exp \left( E_C |\tau' - \tau| \right) \langle \exp \left[ 2E_C [1 + (q - q_g)] (\tau + \tau' - 2\tau_0) \right] \rangle_q
\]

for \( \tau < \tau' < \tau_0 \) and similar expressions for other relations between \( \tau, \tau' \) and \( \tau_0 \). The average \( \langle \rangle_q \) is taken by summing over discrete charge states \( q = 0, \pm 1, \pm 2, ... \) with weight \( \exp \left[ -\beta E_C (q - q_g)^2 \right] \) properly normalized.

The charging effects are most important in the case of a very small island \( hD/d^2 \gg \max(E_C, \Delta) \) (here \( d \) is the linear dimension of the island). In this case the propagator \( p(\mathbf{r}_1, \mathbf{r}_2, \xi - \xi') \sim V^{-2} \delta(\xi - \xi') \) is essentially a constant; substituting this expression in eq. (3) we obtain

\[
\langle \psi_N \psi_N \rangle = \frac{GV}{G_0 \beta^3} \sum_{\omega_\nu, \omega_\eta > 0} \frac{\Delta}{\omega_\mu + \omega_\eta + \omega_\eta} h(\omega_\nu - \omega_\mu, \omega_\mu - \omega_\eta) e^{i\chi}.
\]

In larger islands \( p(\mathbf{r}_1, \mathbf{r}_2, t) \) describes the decay of the pair amplitude. In the absence of inelastic and pairbreaking effects, the decay length is given by the thermal diffusion length \( \sqrt{\hbar D/T} \). Pair breaking effects can be easily incorporated. They influence the Cooperon propagator, and thus proximity effect and decay length.

The modification of the pair amplitude as compared to the classical case depends on temperature and applied gate voltage. The temperature dependence is shown in Fig. 1. For \( E_C/\Delta = 0 \) (upper curve) the pair amplitude diverges logarithmically for \( T \to 0 \). This is the result of ALO \[9\]. For finite values of the charging energy the proximity effect is suppressed; also the divergence at \( T = 0 \) is removed. If the pairing interaction \( \lambda \) is finite, eq. \[3\] acquires an additional factor \( (1 + \lambda \omega_\mu T \sum_{\omega_\nu > 0} \frac{1}{\omega_\nu})^{-1} \). For repulsive \( \lambda \) this leads to an even more pronounced suppression of the induced pair amplitude at very low temperatures and a reentrant behavior.

The charging effects and the proximity effect can be modulated by gate voltages as shown in Fig. 2. The periodicity of the modulation is 1e, reflecting the effect of single electron tunneling. Even though the rate of single electron tunneling may be low for low voltages, it is still sufficient at low sweeping rates to reset the state of the system in a setup where the island is a normal metal.

In the second arrangement, where a small superconducting island is coupled to a film of normal metal the modulation is even more pronounced. In this case a single excitation created in the superconductor does not find a partner to recombine. Hence the energy of odd electron number states lies higher by \( \Delta \) than that of the corresponding normal island. As a result a single electron tunnels at low temperatures only if the gain in charging energy exceeds the cost in excitation energy. The excitation energy can be regained when another electron enters the island \[4,13\]. This leads to the 2-e periodic long-short cyclic dependence observed in the properties of SN electron boxes \[8\] or transistors \[2\] and studied theoretically in Refs. \[6,7\]. As long as the temperature is low we can describe the new
situation by summing in (5) only over even charges \( q = 0, \pm 2, \ldots \) in the range where even electron number states have the lower energy, \( E_{\text{ch}}(\text{even}, Q_g) < E_{\text{ch}}(\text{even} \pm 1, Q_g) + \Delta \), or only over odd charges \( q = \pm 1, \pm 3, \ldots \) in the range where odd states win. The pronounced dependence of the induced pair amplitude as a function of the gate voltage is shown in Fig. 3. At low temperatures the pair amplitude diverges logarithmically at the points of degeneracy \( E_{\text{ch}}(\text{even}, Q_g) = E_{\text{ch}}(\text{even} \pm 1, Q_g) + \Delta \). Only at these points the low temperature divergence of the ALO result is not removed. In either case the divergence is an artifact of the perturbation theory. It is regularized by higher order tunneling processes, which give rise to a lifetime broadening [14].

In the case where \( \Delta > E_C \) quasiparticle tunneling remains suppressed for all values of the gate voltage. In this case the process which transfers charges and resets the system is Andreev reflection [5,15]. A 2e-periodic picture emerges with peaks in the induced pair amplitude at odd integer values of \( q_g \) (see Fig. 3).

The modulation of charging effects helps to make the proximity effect visible. As an application we consider the situation where a superconducting electrode is placed on top of a normal film. If the film is thin enough a uniform pair amplitude will be induced below the electrode. A current through this normal film depends on the induced superconductivity \( \Delta \). If the electrodes have a low capacitance and are coupled to the voltage source, the current through the normal film can be modulated.

In summary we have presented an effective action description of charging effects in normal metal-superconductor tunnel junctions and shown that the Coulomb interaction in the island suppresses the proximity effect. We recover the classical results of ALO. The charging is accounted for by an extra phase correlation function modifying the classical expressions. It removes the low temperature divergences obtained in the classical limit. Several extensions can be included, e.g. relaxation processes due to the flow of Ohmic currents and the effect of a gap in the normal island induced by the proximity effect. Both regularize the divergence of the classical result.

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FIGURES

FIG. 1. Temperature dependence of the induced pair amplitude (in units of $Ge^2/hV$) at $Q_g = 0$ for $E_C/\Delta = 0$ (upper curve), $E_C/\Delta = 0.23$ (middle curve), and $E_C/\Delta = 0.45$ (lower curve). Upper inset: One of the setups studied. A small normal island (N) is coupled to a superconductor (S) by a tunnel junction. The gate voltage $V_g$ applied via a capacitor $C_g$ is used to modulate the charging effects. Lower inset: The induced pair amplitude in the normal metal (N) to second order in the tunneling in diagrammatic language. F is the Gorkov Greens function in the superconductor (S), G is the ordinary Greens function.

FIG. 2. Dependence of the proximity effect on the induced charge $Q_g = C_g V_g$ for $T/T_C = 0.05$, $E_C/\Delta = 0.23$ (upper curve) $E_C/\Delta = 0.45$ (middle curve) and $E_C/\Delta = 0.7$ (lower curve). The curves are periodic in $Q_g$ with period $1e$.

FIG. 3. Dependence of the pair amplitude on $Q_g$ in the case where the island is superconducting and parity effects reduce the regime of the odd electron number state for $T = 0$ and (a) $E_C/\Delta = 0.45 < 1$, for curve (b) $E_C/\Delta = 1.8 > 1$ (no single electron transitions). The curves have now period $2e$. 
Fig. 1, lower inset
Fig. 1, upper inset