Time-dependent Casimir-Polder forces
and partially dressed states

R. Passante

Istituto di BioFisica - Sezione di Palermo, Consiglio Nazionale delle Ricerche,
Via Ugo La Malfa 153, I-90146 Palermo, Italy

F. Persico

INFN and Dipartimento di Scienze Fisiche ed Astronomiche,
Universitá degli Studi di Palermo, Via Archirafi 36, I-90123 Palermo, Italy

(Dated: December 20, 2018)

Abstract

A time-dependent Casimir-Polder force is shown to arise during the time evolution of a partially
dressed two-level atom. The partially dressed atom is obtained by a rapid change of an atomic
parameter such as its transition frequency, due to the action of some external agent. The elec-
tromagnetic field fluctuations around the atom, averaged over the solid angle for simplicity, are
calculated as a function of time, and it is shown that the interaction energy with a second atom
yields a dynamical Casimir-Polder potential between the two atoms.

PACS numbers: 12.20.Ds

*Electronic address: passante@iaif.pa.cnr.it
Time-dependent phenomena in quantum electrodynamics have recently been the object of a renewed interest, particularly in connection with the effects which involve time-dependent quantum fluctuations of the vacuum and lead to time-dependent Casimir forces (see e.g. \[1\]). On the other hand, Casimir-Polder intermolecular interactions are long-range interactions between neutral atoms or molecules, originating from their common interaction with the (vacuum) quantum radiation field \[2, 3\]. Like the Casimir forces, they are also a manifestation of the quantum nature of the electromagnetic field. In the case of two isotropic ground-state atoms, the Casimir-Polder potential behaves as \(r^{-6}\) in the so-called “near zone” and as \(r^{-7}\) in the “far zone”, \(r\) being the intermolecular separation, assumed larger than the overlap region of the electronic wavefunctions of the atoms. The length scale separating the near and far zones is a typical (average) atomic transition wavelength from the ground state \[1\]. The near-zone part of the potential is the usual van der Waals potential for neutral atoms or polarizable bodies, and involves only the electrostatic dipole-dipole interaction between the fluctuating atomic dipole moments. The far zone part of the potential involves also the interaction with the transverse radiation field, and in particular it stems from the exchange of pairs of virtual photons between the atoms.

Many physical models for the Casimir-Polder forces have been proposed, in order to gain a qualitative understanding of the origin of these intermolecular forces. Some of the models proposed trace the physical origin of the Casimir-Polder potential to the vacuum fluctuations and some others to the radiation reaction field (for a review see \[4\] and references therein; three- and many-body Casimir-Polder forces have also been considered in the literature\[5, 6, 7\]). The Casimir-Polder potential between ground-state neutral atoms has also been related to the presence of a virtual photon cloud around each ground-state atom, both for the two- \[8\] and many-body cases \[9\]. In fact, the virtual photons dressing an atom generate space-dependent fluctuations of the electric and of the magnetic field around the atom, and their interaction with other atoms yields the Casimir-Polder potential \[4\]. Thus Casimir-Polder forces probe the field fluctuations generated by an atom in vacuo. In other words, the Casimir-Polder potential between two atoms directly yields the electromagnetic field fluctuations generated by the virtual photon cloud dressing a ground-state neutral atom. More precisely, it is the average force which is observed, because Casimir-Polder forces are fluctuating forces \[10\].

The situation becomes more interesting if one of the atoms is in a dynamic situation,
because a change in time of the electromagnetic field fluctuations is expected in this case, leading to time-dependent Casimir-Polder forces. This was investigated in the case of the dressing of an initially bare atom and in the complementary case of the undressing of a fully dressed atom using a simple effective Hamiltonian for the atom-field interaction [11]. In the first case, it was found that: i) at time \( t \), the electric and magnetic energy densities of the field fluctuations around the dressing atom are the same as those of the fully dressed source for \( r < ct \), while for \( r > ct \) they are the same as in the absence of the atom, as required by the causality principle; ii) at \( r = ct \), a singularity of the energy density is present, which propagates with velocity \( c \); iii) the interaction energy of these field fluctuations with the second “test” atom yields a time-dependent potential [11]. This potential is the analogue of the Casimir-Polder potential in a dynamical situation. In the second case complementary results were obtained. A bare field source however cannot be easily produced in the laboratory. Thus the evolution from a bare to a dressed state, which was considered in [11], is an extreme situation which cannot be considered realistic because the atom-field interaction can never be switched off. In addition, the evolution from bare to dressed states significantly involves high-frequency modes of the field, which are known to generate the ultraviolet divergences of quantum field theory. For these reasons, we have more recently considered a more sophisticated model where the evolution of an initially partially dressed atom is involved [12]. Partially dressed states of an atom are dressed states with an incomplete virtual photon cloud [13]. This model is free from the unrealistic assumption of an initially bare atom, and in [12] it has been shown that the high-frequency modes of the radiation field, which would eventually lead to the ultraviolet divergences, do not play a significant role in the evolution of the system.

In this letter we consider the time-dependent energy density of the electromagnetic field fluctuations around a partially dressed atom, and calculate explicitly the dynamics of the electromagnetic field fluctuations around it, averaging over the solid angle for simplicity. We also consider their interaction energy with a second “test” atom and the resulting time-dependent Casimir-Polder force. We use the same model of partially dressed atom as in [12]; the reader is referred to this paper for more detail on the model used.

We consider a two-level atom interacting with the electromagnetic radiation field, in the Coulomb gauge and within dipole approximation. The Hamiltonian describing this system
is ($\hbar = 1$)

$$H' = \omega_0 S_z + \sum_{kj} \omega_k a^\dagger_{kj} a_{kj} + \sum_{kj} \epsilon_{kj} \left( a_{kj} - a^\dagger_{kj} \right) (S_+ - S_-) \tag{1}$$

$S$ are the pseudospin atomic operators and the coupling constant $\epsilon_{kj}$, in the multipolar coupling scheme is [4]

$$\epsilon_{kj} = -\sqrt{\frac{2\pi \omega_k}{V}} \hat{e}_{kj} \cdot \mathbf{d} \tag{2}$$

where $\mathbf{d}$ is the atomic dipole moment.

We assume that up to time $t = 0$ the atom is in its dressed ground state. The dressed ground state, at second-order in the electric charge, is

$$|g\rangle = \left( 1 - \frac{1}{2} \sum_{kj} \frac{\epsilon_{kj}^2}{(\omega_0 + \omega_k)^2} \right) |\downarrow 0_{kj}\rangle + \sum_{kj} \frac{\epsilon_{kj}}{\omega_0 + \omega_k} |\uparrow k_{kj}\rangle$$

$$- \sum_{kjk'j'} \frac{\epsilon_{kj}\epsilon_{k'j'}}{(\omega_0 + \omega_k)(\omega_0 + \omega_{k'})} |\downarrow j\rangle |k\rangle |k'\rangle |j'\rangle \tag{3}$$

where $\downarrow (\uparrow)$ denotes the atomic ground (excited) state, and $kj$ indicates photon states.

We now assume that at $t = 0$ the atom is subjected to an abrupt change $\Delta \omega_0$ in its transition frequency (for example, by the action of an external electric field; see [12] for more detail). Abrupt changes in the parameters of atom-photon Hamiltonian have been considered previously in connection with radiative processes of different kinds (see e.g. [14]). In our case, the atomic transition frequency is $\omega_0$ for $t < 0$ and $\omega_0 + \Delta \omega_0$ for $t > 0$. A consequence of this change is that the state (3), which is an eigenstate of $H'$ for $t < 0$, is not an eigenstate of the total Hamiltonian for $t > 0$, because the Hamiltonian has changed. Thus, the state $|g\rangle$ must evolve for $t > 0$, and in this way we have generated a partially dressed state. The Hamiltonian of the system for $t > 0$ is then

$$H = H' + \Delta \omega_0 S_z \tag{4}$$

where $H'$ is the Hamiltonian for $t < 0$, given by (1).

The state of the system at $t > 0$ has been obtained in [12] at the second order in $\epsilon_{kj}$ and first order in $\Delta \omega_0$. Using this state, we can evaluate at $t > 0$ the change of the electric and magnetic energy density around the atom with respect to that at times $t < 0$. After lengthy calculations, we obtain ($t > 0$)

$$\delta \mathcal{E}_E(r, t) = \frac{1}{8\pi} \langle E^2(r, t) \rangle - \frac{1}{8\pi} \langle E^2(r, t < 0) \rangle$$
\[
\begin{align*}
= -\Delta \omega_0 \frac{c^2}{(2\pi)^3} \int dkdk'k'^2k'^2 \left\{ \left( d^2 j_0(kr)j_0(k'r) + \frac{1}{k^2} j_0(kr)(d\cdot\nabla^r)j_0(k'r) \right) \\
+ \frac{1}{k^2} j_0(kr)(d\cdot\nabla^r)j_0(kr) + \frac{1}{k^2k'^2} d_j d_j (\nabla^r \nabla^r j_0(kr) \left( \nabla^r j_0(k'r) \right) \right) \right\} \\
x \times F(\omega_k, \omega_{k'}, t) \right\} + cc
\end{align*}
\] (5)

for the electric energy density, where \( \nabla^r \) is the gradient operator with respect to the coordinate \( r \), and

\[
\begin{align*}
\delta \mathcal{E}_M(r, t) &= \frac{1}{8\pi} (B^2(r, t)) - \frac{1}{8\pi} (B^2(r, t < 0)) \\
&= -\Delta \omega_0 \frac{c^2}{(2\pi)^3} \int dkdk'k'^2k'^2 \left\{ \left( d^2 \nabla^r j_0(kr) \cdot \nabla^r j_0(k'r) \right) \\
- (d\cdot\nabla^r j_0(kr))(d\cdot\nabla^r j_0(k'r)) G(\omega_k, \omega_{k'}, t) \right\} + cc
\end{align*}
\] (6)

for the magnetic energy density. The functions \( F(\omega_k, \omega_{k'}, t) \) and \( G(\omega_k, \omega_{k'}, t) \) assume a simple form in the far zone \( r >> c/\omega_0 \), where we can assume \( \omega_k << \omega_0 \) [13]. In this case, we have

\[
F(\omega_k, \omega_{k'}, t) = G(\omega_k, \omega_{k'}, t) = \frac{1}{\omega_0^2(\omega_k + \omega_{k'})} \left( 1 - e^{-i(\omega_k + \omega_{k'})t} \right)
\] (7)

In order to simplify the calculation of (5-6), we perform an average of the energy densities over a sphere of radius \( r \) centered on the atom. The integrals on \( k, k' \) can be performed explicitly, and the final result is (valid only in the far zone)

\[
\begin{align*}
\delta \mathcal{E}_E(r, t) &= \frac{1}{4\pi} \int d\Omega \delta \mathcal{E}_E(r, t) \\
&= -\Delta \omega_0 \frac{cd^2}{24\pi^2\omega_0^2} \left( \frac{13}{2^{2/7}} (1 - \Theta(r - ct)) + \frac{13}{2r^6} \delta(r - ct) - \frac{5}{2r^5} \delta'(r - ct) \\
+ \frac{1}{3r^4} \delta''(r - ct) + \frac{1}{30r^2} \delta^{(iv)}(r - ct) \right)
\end{align*}
\] (8)

\[
\begin{align*}
\delta \mathcal{E}_M(r, t) &= \frac{1}{4\pi} \int d\Omega \delta \mathcal{E}_M(r, t) \\
&= \Delta \omega_0 \frac{cd^2}{24\pi^2\omega_0^2} \left( \frac{133}{32r^7} (1 - \Theta(r - ct)) + \frac{133}{2r^6} \delta(r - ct) - \frac{29}{r^5} \delta'(r - ct) \\
+ \frac{41}{6r^4} \delta''(r - ct) - \frac{5}{6r^3} \delta'''(r - ct) + \frac{1}{30r^2} \delta^{(iv)}(r - ct) \right)
\end{align*}
\] (9)

These expressions have a structure similar to that obtained for the dressing of the bare source [11]. We note that the most diverging terms, i.e. those proportional to the fourth derivative of the delta function, cancel in the total (electric plus magnetic) energy density,
coherently with the known fact the the time evolution of the field energy in this model is less affected by the high-frequency field modes compared to the bare atom case [12]. Moreover, they are proportional to the atom’s electric static polarizability $\alpha \sim d^2/\omega_0$. The further proportionality to $\Delta \omega_0/\omega_0$, which is a quantity much smaller than one, indicates that all single-atom effects related to the time-dependent field fluctuations given by (8) and (9) are quite small, although in a system of many atoms the effects may become measurable.

The presence of the $\Theta$ function in (8,9) ensures causality in the propagation of the energy density. A singularity of the electric and magnetic energy density is present at $r = ct$. When a second atom, described by its static electric polarizability $\alpha$, assumed isotropic for simplicity, is placed at a distance $r$ from the first atom, the resulting Casimir-Polder interaction potential between the two atoms can be obtained as

$$V(r, t) = -\frac{1}{2} \alpha \langle E^2(r, t) \rangle$$

where $\alpha$ is the static polarizability of the second atom [16, 17]. A similar interaction energy involving the magnetic field fluctuations exists, proportional to the magnetic polarizability of the second atom. The potential energy $V(r, t)$ in equation (10) yields a time-dependent Casimir-Polder potential between the two atoms; contrarily to previous calculations, it has been calculated in a situation which does not involve unrealistic bare states, because it results from the evolution of a partially dressed atom. This result is the counterpart of the dynamical Casimir effect; however, a noteworthy difference is that in the present case matter has been treated dynamically, whereas the usual description of the dynamical Casimir effect includes the presence of matter only through the boundary condition on the field operators [18]; only very recently, corrections to the results based on external boundary conditions have been introduced in the calculation of the Casimir effect for macroscopic bodies for a 1D cavity with a moving mirror [19]. As mentioned in the introduction, the main point of this paper is to show that a time-dependent Casimir-Polder counterpart of the time-dependent Casimir force exists in QED and does not depend on unrealistic assumptions such as an initially bare atom. A reliable estimate of the intensity of this force requires a more detailed model for the partially dressed atom, but it is evident that the dynamical Casimir-Polder interaction investigated in this paper within an isolated pair of atoms, although conceptually important, is a very tiny effect. In fact, the time-dependent force, compared to the usual time-independent Casimir-Polder force, which has been directly observed [20], contains an
extra factor $\Delta \omega_0/\omega_0 << 1$. We hope to discuss this point in a future publication.

In conclusion, we have considered a model of a partially dressed field source in the framework of quantum electrodynamics. This model consists of a dressed two-level atom, interacting with the electromagnetic radiation field, which is suddenly subjected to a rapid change of its transition frequency due to an external agent. The dressed ground state of the atom becomes a partially dressed state after this change. The time evolution of the energy density of its dressing photon cloud has been considered, and it has been shown that this process yields a time-dependent Casimir-Polder force on a second neutral atom placed at a distance \( r \). We have shown that this new effect is in principle detectable, since the treatment we have given is free of the unrealistic assumption of an initial bare atom.

This work was supported by the European Union under contract No. HPHA-CT-2001-40002 and in part by the bilateral Italian-Japanese project 15C1 on Quantum Information and Computation of the Italian Ministry for Foreign Affairs. Partial support by Ministero dell’Università e della Ricerca Scientifica e Tecnologica and by Comitato Regionale di Ricerche Nucleari e di Struttura della Materia is also acknowledged.

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