Typical Scales in the Spatial Distribution of QSOs

Zugan Deng\textsuperscript{2}, Xiaoyang Xia\textsuperscript{3}, Li-Zhi Fang\textsuperscript{1}

\textsuperscript{1} Physics Department and Steward Observatory, University of Arizona, Tucson, Arizona 85721

\textsuperscript{2} Graduate School, Chinese Academy of Sciences, Beijing, P.R.China

\textsuperscript{3} Physics Department, Tianjing Normal College, Tianjing, P.R.China

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Abstract

We present results of searching for the possible typical scales in the spatial distribution of QSOs. Our method is based on the second derivative of the two-point correlation function. This statistic is sensitive to the scale of the maximum in the spectrum $P(k)$ of the density perturbation in the universe. This maximum or bend scale can be detected as the wavelengths of the periodic component in the second derivative of the integral correlation function. For various QSO samples compiled from surveys of pencil-beam and bright QSOs, a typical scale of about $93 \pm 10\ h^{-1}\ Mpc$ for $q_0 = 0.5$ has been detected. This typical scale is in good agreement with that found in the spatial distributions of galaxies, clusters of galaxies, and CIV absorption systems of QSOs if $q_0$ is taken to be $\sim 0.2$. Therefore, it is likely a common or universal scale in the large scale structure traced by these objects. This result is consistent with the assumption that the typical scale comes from a characteristic scale in the spectrum of the density perturbation in the universe.

Subjects headings; cosmology - QSO: clustering
1. Introduction

According to the standard scenario of structure formation in the universe, the initial perturbation produced by quantum fluctuation of scalar fields during the inflationary era is scale-invariant. The power spectrum of the initial perturbation is assumed to be \( P(k) \propto k^n \), where \( k \) is the wavenumber of the perturbation, and the spectral index \( n \sim 1 \). Therefore, no typical scales exist in the very early universe. Subsequent evolution of the universe leads to a deviation of the density perturbation spectrum from a scale-invariant one. Typical scales emerge from the distribution of cosmic matter. For instance, in a linear regime, the density spectrum can be approximated as the following form (Peacock 1991; Mo et al. 1993)

\[
P(k) = \frac{k}{1 + (k\lambda/2\pi)^{2.4}}
\]

where \( \lambda \) is a typical scale, on which \( P(k) \) is the maximum, i.e. \( P(k) \) bends from \( \propto k \) to \( k^{-1.4} \) at \( k \sim 2\pi/\lambda \). The clustering of galaxies and clusters of galaxies showed that the bend scale \( \lambda \) should be larger than about 100 h\(^{-1}\)Mpc. On the other hand, the anisotropy of cosmic background radiation indicates that \( \lambda < 1000 \text{ h}^{-1}\text{Mpc} \). Therefore, in the standard model, the linear evolution of density perturbation brings out at least one typical scale in the range between 100 and 1000 h\(^{-1}\)Mpc in the spectrum.

The possible existence of typical scales in non-standard scenario of the structure formation has also been proposed. For a modified inflation model (Starobinsky, 1992), it has been found that a typical scale of about 100 h\(^{-1}\)Mpc in the cluster-cluster correlation is crucial in determining the peculiarities of the inflation and the nature of the dark matter (Kotok et al. 1993).

Observations have indeed discovered structures in the distribution of galaxies or clusters with scales as large as about 100 h\(^{-1}\)Mpc, including the great void (Kirshner et al. 1981), filaments and sheets (Haynes and Giovannelli, 1986), the Great Wall (de Lapparent et al. 1988), the Great Attractor (Dressler et al. 1987) and the 128 Mpc ‘periodicity’ of pencil beam sample (Broadhurst et al. 1990). However, these observed scales cannot be identified as the bend scale in the density spectrum.

In the last two years, systematic approaches to the typical scales in the spatial distribution of galaxies and clusters have been done by several groups. Buryak, et al. (1991, 1992) developed a method to probe typical scale from one-dimensional samples. Einasto and Gramann (1993) investigated the possible observational phenomena in the distribution of clusters and galaxies related to the bend scale in \( P(k) \). An extensive search for the typical scales has been made by Mo, et al. (1992a, b). Using the method of the second derivative of the integral two point correlation function, they have detected typical scales in the distribution of galaxies and clusters, especially, a scale of 130 h\(^{-1}\)Mpc commonly exists in samples including the deep pencil-beam
survey (Broadhurst, et al. 1990), deep redshift surveys of Abell clusters (Huchra, et al. 1990) and QDOT survey of IRAS galaxies (Rowan-Robinson, et al. 1990).

In this paper we extend this investigation to QSOs, i.e. searching for the possible typical scales in the spatial distribution of QSOs. Our motivations are twofold. First, the distributions of low-redshift objects like galaxies showed that the bend in the density spectrum may occur at wavelengths $\lambda \sim 150 \ h^{-1}\text{Mpc}$ (Peacock 1991; Vogeley, et al. 1992; Vogeley & West 1992; Mo, et al. 1992a,b; Jing & Valdarnini 1993). However, the wavelengths involved are already comparable to the sizes of the samples used, and the fair-sample assumption may then be questionable. This problem should be less severe for QSOs because we can have QSO samples with size much greater than $150 \ h^{-1}\text{Mpc}$. Secondly, if the typical scales detected in the structures of galaxy and cluster do come from the characteristic scales like the bend in the spectrum of perturbation, it should be measurable in the QSO distribution as well, unless QSOs trace substantially different large-scale structures than galaxies and clusters do. Therefore, it is important to see if the distribution of QSOs is consistent with the assumption that the typical scales found in galaxies and clusters are ‘universal’.

It is interesting to note that the scale $100 \ h^{-1}\text{Mpc}$ has already been mentioned in early studies of QSO clustering. About a decade ago, using the nearest neighbor analysis, Chu and Zhu (1983) showed that the distribution of QSOs listed in the sample Bolton and Savage (1979) deviates from the Monte Carlo samples on the scale of about $100 \ h^{-1}\text{Mpc}$. Some authors also suggested the existence of isolated groups with comoving scales of about $100 \ h^{-1}\text{Mpc}$ (e.g., Crampton, et al. 1989; Clowes & Campusano 1991). But these results do not provide a convincing argument for the scale considered. We will use more rigorous statistic to detect the typical scales in the samples of Boyle, et al. (1990, 1991) and Foltz, et al. (1987, 1989).

Our plan is to give a brief description of the method in section 2, the results of typical scale analysis of QSO samples in section 3, a comparison of the typical scales of QSOs with that of galaxies in section 4, and a conclusion in section 5.

2. Method of Detecting Typical Scales

Statistics based on the amplitude of the two point correlation function $\xi(r)$ is the most popular method in the study of large scale structure. This method is, however, not adequate for detecting typical scales. The amplitude and the correlation length $r_0$ in the two-point correlation function $\xi(r) = (r/r_0)^{-1.8}$ do not relate to the bend scale in a simple way. For a given density spectrum $P(k)$, the two point correlation function can be calculated by

$$\xi(r) = \frac{B}{r} \int_0^\infty \sin krP(k)kdk \hspace{1cm} (2)$$

where B is a constant. Generally, the bend in the spectrum $P(k)$ only leads to a slight drop in the amplitude of the correlation function on the scale of bending. This
means, only the amplitudes of $\xi(r)$ at $r \sim \lambda$ are useful to probe $\lambda$. However, on such large scales, the absolute value of the amplitudes of the two-point correlation function has a large statistical error due to the uncertainty in the mean density of objects considered. Structures on larger scales with density contrast less than the uncertainty of the mean density will be masked by the noise of the two-point correlation function. This problem is especially severe for QSO samples because the mean density of QSOs is redshift-dependent. Even with a homogeneous sample, it is still difficult to calculate the evolution of the mean volume density, because the deceleration parameter $q_0$ is poorly determined.

The method developed by Mo, et al. (1992a,b) and Einasto and Gramann (1993) is based on the second derivative of the two-point correlation function. We will introduce this method by a slightly different way in order to demonstrate its advantage in searching for the typical scale, especially the bend scale in $P(k)$. Let us consider the behavior of $\xi(r)$ when $r$ is large. Eq.(2) shows that, for a spectrum with a maximum like that in eq.(1), the dominate term of $\xi(r)$ when $r \geq \lambda$ should be a periodic function of $r$ with wavelength equal to about $\lambda$. For instance, if one takes an approximate form of the spectrum (1) as follows: $P(k) = k$ for $k < 2\pi/\lambda$ and $P(k) = k^{-1.4}$ for $k > 2\pi/\lambda$, the dominant term of $\xi(r)$ at large $r$ will be $r^{-m} \cos(2\pi/\lambda r)$, where $m \sim 2$. The second derivative of $\log \xi(r)$ is then proportional to $\cos(2\pi r/\lambda)$. Therefore, the bend scale $\lambda$ can be detected by the wavelengths of the periodic components in the second derivative $d^2 \log \xi(r)/dr^2$. Of course, such periodic components will also be masked by the noise given by the uncertainty of the mean density. However, it is well known from statistics that, for a given noise masked data set, identifying periodic components is easier than determining the absolute value of the amplitudes of the correlation function. Considering the sizes of QSO samples usually are much greater than the wavelengths of the periodic components involved, the statistic of detecting periodic component in a QSO sample would be more effective than that of determining the amplitude.

In actual work, the usual two-point correlation function $\xi(r)$ is replaced by the function $\Xi(r) = 1 + \bar{\xi}(r)$, where $\bar{\xi}(r)$ is the integral two-point correlation function defined by

$$\bar{\xi}(r) = \frac{3}{r^3} \int_0^r \xi(x)x^2 dx$$

When $r \geq \lambda$, $\Xi(r)$ has about the same behavior as $\xi(r)$. Therefore, we can also use the statistic of $d^2 \log \Xi(r)/dr^2$ to detect the typical scales. In measuring clustering of high redshift objects on large scales, the statistic based on the integral two point correlation function $\bar{\xi}(r)$ is sometimes more advantageous than $\xi(r)$. The reasons are as follows.

First, for determining the two-point correlation function $\xi(r)$, one needs a choice of bin size of the separation of QSO pair. The binning may lead to false periodic
components in the $\xi(r)$ with wavelengths equal to the harmonics of the bin scale, and then a misidentification of the typical scales. Moreover, the total number and number density in available samples of QSOs are very low, and the binning will cause a large fluctuation in $\xi(r)$ if the bin scale is chosen too small. These puzzles can be avoided by using the statistic $\bar{\xi}(r)$ because, according to the definition eq.(3), it does not bin.

Second, from eqs. (2) and (3), $\bar{\xi}(r)$ can be related to the density spectrum by

$$\bar{\xi}(r) = \frac{3B}{r^3} \int_0^\infty [\sin kr - kr \cos kr] \frac{P(k) dk}{k}$$

(4)

where the window function $[\sin kr - kr \cos kr]/k$ dies off faster at large $k$ than that for $\xi(r)$ [eq.(2)]. Therefore, the statistical result will be less severely affected by clustering on small scales (Mo, et al. 1993).

When the boundary effect is negligible, $\Xi(r)$ is given by

$$\Xi(r) = \frac{N_{dd}(r) \times N_r}{N_{dr} \times N}$$

(5)

where $N_{dd}(r)$ is the number of QSO pairs with separation less than $r$, $N_{dr}(r)$ is the mean number of object pairs between observed and random samples, $N$ and $N_r$ are the total numbers of objects in real and random samples, respectively.

Eq.(5) shows that $\Xi(r)$ is given by an un-normalized integrated pair counts of QSOs, the result does not sensitively depend on the mean number density of QSOs. Therefore, the uncertainty in the mean density is avoided from the beginning. As a consequence, this method is not sensitive to the selection function used for generating the random samples as well. In this paper, we will fit one-dimensional samples by a cubic polynomial, and three-dimensional samples by a linear function in the redshift range of each sample.

In calculating the derivative of $\Xi(r)$, we will meet differences like $N_{dd}(r + \Delta r) - N_{dd}(r)$. Obviously, this difference will be dominated by noise when $\Delta r$ is less than the mean distance $D$ of nearest neighbor of QSOs in the sample. This will be the main source of the error in the derivative of the correlation function when $\Delta r \leq D$. In order to suppress the influence of this noise in small wavelengths, we smooth $\Xi(r)$ by convolution integral $\Xi(r) = \int \Xi(r') S(r - r') dr'$, where the smoothing function $S(r)$ is equal to 1 when $|r - r'| < L$, and 0 otherwise, and taking the smooth scale $L$ to be equal to or less than $D$. Fluctuations with wavelengths less than the scale $L$ will totally be suppressed in the function $\Xi(r)$ by the smoothing, while all inhomogeneities with scales comparable to or larger than the scale $L$ will not be affected by the smoothing. Our algorithm is to use this smoothed function $\bar{\Xi}(r)$ to calculate the second derivative $\Delta \theta(r) \equiv \frac{d^2 \log \bar{\Xi}(r)}{dr^2}$.

The statistical significance of the peaks in the second derivative $\Delta \theta(r)$ can be measured by the standard deviation $\sigma$ which is estimated by Monte Carlo samples.
generated under the same selection conditions as the real samples. Usually we take 100 random samples to calculate the standard deviation. Comparing the curve $\Delta \theta(r)$ of the real samples with that of random samples, we can infer the statistical significance of peaks appearing in the $\Delta \theta(r)$ of real samples. Estimating the significance in this way has the advantage that the edge effects are automatically avoided.

The periodic components in $\Delta \theta(r)$ can be detected by power spectrum analysis (PSA). The wavelengths of these periodic components are the typical scales. The statistical significance of the existence of periodic components in the second derivative, $\Delta \theta''(r)$, can be estimated by the usual way of power spectrum analysis.

This method has been used to analyze 1- and 3-dimension samples of optical and IRAS galaxies or clusters of galaxies, and a common scale of $130 \pm 10 \ h^{-1}\text{Mpc}$ was detected (Mo, et al. 1992a,b). The samples used for analysis include the deep pencil-beam surveys (Broadhurst et al. 1990), deep redshift surveys of Abell clusters (Huchra et al. 1990) and QDOT survey of IRAS galaxies (Rowan-Robinson et al. 1990). Therefore, the scale of $130 \ h^{-1}\text{Mpc}$ might be a candidate for the bend scale $\lambda$ in the initial density spectrum $P(k)$. Since the structures with scales as large as about 100 $h^{-1}\text{Mpc}$ in the present universe should still remain in the linear evolutionary stage, the typical scale found in the distribution of galaxies and clusters should probably also be measurable in the distributions of high redshift objects. Therefore, one should expect the existence of a 100 $h^{-1}\text{Mpc}$ typical scales in QSO distribution if QSOs trace the high peaks in the density field as galaxies and clusters of galaxies do.

3. Statistical Results

3.1 Pencil-beam samples

The one-dimensional samples of QSOs used in our analysis are formed from Boyle et al. 1990 (BFSP) and 1991 (BJS). BFSP contains about 420 QSOs identified in a complete ($B \leq 21$), ultraviolet excess (UVX) survey, which covers 34 pencil beam fields, each has about 0.35 square degrees. These pencil-beam fields are scattered over eight $5^\circ \times 5^\circ$ UK Schmidt fields. BJS includes 61 QSOs identified in a complete to $B \leq 22$ survey done by multicolored technique in three pencil-beam fields at high galactic latitudes.

Redshift distribution of QSOs listed in BSFP and BJS are plotted in Figure 1a and b, respectively. As well known, the UVX and multicolored technique are likely to provide QSO candidates with high completeness when redshift $z \leq 2.2$ (Véron 1983). On the other hand, the imposed stellar morphological criterion may cause the incompleteness at redshift less than about 0.6. Figure 1 shows that most QSOs in the BFSP and BJS are in the redshift range of 0.6 to 2.2 and the number of QSOs dramatically decreases outside this interval. Therefore, the samples consisting of QSOs with redshifts from 0.6 to 2.2 in BSFP and BJS should be largely complete and unbiased. We adopt, respectively, QSOs with $0.6 < z < 2.2$ in 1) BSFP and 2)
BJS as two parent samples in our statistic.

Because each field covers only about $0.35 \, \text{deg}^2$, the size of their cross section is about $20 \, h^{-1}\text{Mpc}$ at $z \sim 1$. One can consider these samples as one-dimensional if we focus on the structures with scales much larger than $20 \, h^{-1}\text{Mpc}$. Each one-dimensional sample can be seen as a representation of the three-dimensional distribution in a given direction. Obviously, some features shown in these samples are direction-dependent.

In order to reduce the influence of the local features on the statistics, we use combined sample, which consists of a number of the pencil-beam samples in different directions. For such combined samples, the individual features of the pencil-beam fields should be less important.

We made four subsamples called QN, QS, QSGP and QBJS, which consist of QSOs in the fields of northern sky, southern sky (excepted those in the Schmidt field along southern galactic pole direction), southern galactic pole and that given by BJS survey, respectively. Table 1 shows the numbers of pencil-beam fields $N_p$, numbers of QSOs $N_q$ for each subsample. It should be pointed out that a Schmidt field is about $5^\circ \times 5^\circ$, two pencil-beam fields in the same Schmidt plate have a mean separation of about $2^\circ.5$ which corresponds to an across scale of about $90 \, h^{-1}\text{Mpc}$ at $z \sim 1$. Thus, in studying the structures with scales larger than $100 \, h^{-1}\text{Mpc}$, the pencil-beam samples in the same Schmidt plate should not be considered as totally independent samples, i.e. different pencil-beams may imprinted by a same structure on scale larger than the separation of the pencil-beams.

Figures 2a-d present the results of the second derivative, $\Delta \theta(r)$, for each subsample. The bold lines show the $\Delta \theta(r)$ for the real samples, and the light lines are the standard deviations $\pm 1\sigma$ given by random samples. Since the mean distance of nearest neighbor QSOs in these samples is equal to or larger than about $50 \, h^{-1}\text{Mpc}$, the smoothing scale $L$ is taken to be $40 \, h^{-1}\text{Mpc}$. The sharp dips appearing in Figure 2 on scales less than about $40 \, h^{-1}\text{Mpc}$ are caused by that, for all random samples, $N_{dr}(r)$ is zero on small scales. It can be clearly seen from Figure 2 that, for all subsamples of QSOs, the curves of $\Delta \theta(r)$ show periodically distributed peaks (and valleys). Figure 3a and b show the same calculation as Figure 2 but the smoothing scale $L$ is taken to be $20 \, h^{-1}\text{Mpc}$. As expected, more fluctuations with short wavelengths appeared in Figure 3, while the fluctuations on scales larger than $40 \, h^{-1}\text{Mpc}$ are the same as the case of $L = 40 \, h^{-1}\text{Mpc}$.

The statistical significances of each peak (and valley) in $\Delta \theta(r)$ are marginally higher than $\pm \sigma$, mostly in the range of $1 - 2.5 \, \sigma$. This result is the same as that given by previous studies. Up to now, almost all QSO structures detected with scales greater than $10 \, h^{-1}\text{Mpc}$ are in the significance level of $2 - 3 \, \sigma$ (Iovino & Shaver, 1988; Bahcall & Chokshi 1991; Boyle & Mo 1993). However, our method is not only based on the individual peak in $\Delta \theta(r)$, but in the regular or periodic distribution of these peaks (and valleys). Figure 4 and 5 plot, respectively, the power spectrum
of the $\Delta \theta(r)$ for sample QS and QBJS with $L = 40$ and $20 \, h^{-1}\text{Mpc}$. One can find from these figures that 1) all power spectrum show a peak around $\lambda \sim 90 \, h^{-1}\text{Mpc}$ with confidence no less than 99% (for $L=40 \, h^{-1}\text{Mpc}$); 2) if we use the width of the peak as a measure of the uncertainty of the wavelength, the mean wavelength for one-dimensional samples is $90 \pm 8 \, h^{-1}\text{Mpc}$; 3) for each subsample the spectrum of $L = 20$ and $40 \, h^{-1}\text{Mpc}$ have the same shape, therefore these statistical results are independent of the smoothing length.

3.2 Three dimensional samples

The three-dimensional sample used in this paper is compiled from the LBQS survey (Foltz et al., 1987, 1989; Hewett et al., 1991; Chaffee et al., 1991; Morris et al., 1991). The LBQS survey presented more than 1000 QSOs with $m_j \leq 18.5$. The total area of these fields is about 800 square degrees. It is one of the largest and uniformly selected QSO sample up to date. The redshift interval of this sample is between 0.2 and 3.3. An artificial cut-off has been made at redshift 0.2, because too many stars mixed into the candidates below this redshift. As the redshifts of QSOs becomes higher than 3.3, the Ly$\alpha$ line will move out of the $j$ band. The sample can be considered to be complete in the magnitude interval $16.0 < m_j < 18.7$. Figure 6a plots the redshift distributions of QSOs in samples LBQS.

The first sample compiled from this survey is called LBQS, which contains of all LBQS QSOs with redshift in the range from 1.0 to 2.2. The limitation of $1.0 < z < 2.2$ comes from the following consideration. a) Each plate used in LBQS survey covers an area of about $6^{\circ} \times 6^{\circ}$, which spans $\sim 200 \, h^{-1}\text{Mpc}$ and higher when $z > 1$ and $q_0 = 0.5$. Therefore, if we are interesting in probing structures with scales equal to or larger than $100 \, h^{-1}\text{Mpc}$, only the sub-samples with $z > 1$ can be treated as a three dimension one. b) The number of the LBQS QSOs drops rapidly when $z > 2.2$. The mean distance of nearest neighbor QSOs at $z > 2.2$ is equal to or larger than $\sim 100 \, h^{-1}\text{Mpc}$. Therefore, the data at $z > 2.2$ are no longer suitable for probing structures with scales of about $100 \, h^{-1}\text{Mpc}$.

To study the possible influence of the foreground objects on the typical scales, such as that given by gravitational lensing effect, we compiled a sample called LBQS-V, which consist of all LBQS QSOs with $1.0 < z < 2.2$ excepting those in field of the nearest supercluster Virgo (Figure 6b). The redshift distributions of both LBQS and LBQS-V are quite smooth. Samples LBQS and LBQS-V are listed in Table 2.

Figures 7a and b plotted the result of $\Delta \theta(r)$ for the LBQS and LBQS-V, respectively. The ranges of $\pm 1\sigma$ given by random samples are shown by the light curves. The ratio of signal to noise in 3-dimensional samples appears to be higher than that of the pencil beam samples. The significance of the peaks (and valleys) now is about $3\sigma$. As in the 1-dimensional sample, the significant peaks distributed regularly or periodically. Figures 8 and 9, respectively, the power spectrum of samples LBQS and LBQS-V with $L = 40$ and $20 \, h^{-1}\text{Mpc}$. The mean wavelength is $\sim 95 \pm 9 \, h^{-1}\text{Mpc}$.\noindent
This is the same as that of pencil beam samples.

The value of $\lambda \sim 90 - 100 \ h^{-1}\text{Mpc}$ found here is in agreement with those obtained from the amplitudes of the correlation function (Mo & Fang 1993). Using a best fit of the integral correlation function to the power spectrum $P(k)$ [eq.(1)], it found $\lambda \sim 100 - 200 \ h^{-1}\text{Mpc}$. Therefore, the scale of $\sim 100 \ h^{-1}\text{Mpc}$ seems to be universal for the various QSO samples considered.

4. Difference of Typical Scales between QSOs and Galaxies

The typical scale found in QSO distribution shows a difference from that of galaxies and clusters of galaxies (Mo, et al. 1992a), which was found to be $130 \pm 10 \ h^{-1}\text{Mpc}$ by the same method. It is generally believed that the structures with scales larger than about $50 \ h^{-1}\text{Mpc}$ should still remain in linear evolution regime. If the typical scale comes from the bend scale in the initial density spectrum, the comoving typical scale of QSOs should be the same as that of galaxies and clusters of galaxies. Therefore, it is necessary to study the possible origin of the observed difference between the typical scales of QSOs, and galaxies and clusters.

If the typical scale is assumed to be local, i.e. being only a feature of nearby galaxies, one should not expect that the same typical scale shows up in the spatial distribution of QSOs. However, it has been found that the typical scale of $130 \ h^{-1}\text{Mpc}$ exist in almost all samples of galaxies and clusters of galaxies (Mo, et al, 1992a, b). On the other hand, the fact that no difference has been found between the results of LBQS and LBQS-V (Figure 6a and b) indicates that the $95 \ h^{-1}\text{Mpc}$ typical scale of QSOs may not be affected (at least in current error bar) by gravitation lensing of a local structures like the Virgo clusters. Therefore, one cannot explain the difference of the typical scale between galaxies and QSOs as a local effect.

If galaxies and QSOs trace different aspects of the density fields in the universe, we should not expect that galaxies and QSOs have the same typical scales. Considering the bias mechanism for galaxies is probably no longer useful for QSO formation, one may reasonably assume that the QSO-traced structures are different from that traced by galaxy. Indeed, the number density of QSOs is much less than galaxies. Therefore, in terms of bias model, the biasing threshold of QSO should be higher than galaxies. Bower, et al. (1993) recently proposed a bias model describing cooperative formation of galaxies, in which the threshold of galaxy formation is scale-dependent, it is lower in a domain with higher mean density, and higher in the area with lower mean density. This is equal to replacing $P(k)$ by $P(k)B(k)$, where the bias function $B(k)$ is decreasing with $k$ increasing. Obviously, the bend scale of “spectrum” $P(k)B(k)$ will be greater than that of $P(k)$.

However, this explanation encounters difficult if we consider the following facts. First, QSO clustering satisfies the same power law correlation function as galaxies and clusters (Shanks et al. 1988; Fang et al. 1985; Shaver 1988; Chu & Zhu 1989; Crampton, Cowley & Hartwick 1989; Boyle 1991). Second, QSO clustering is the same
as small groups of galaxies or poor clusters (Bahcall & Chokshi 1991). It has been known for a decade that low redshift QSOs are preferentially located in poor clusters or groups of galaxies. This was found by the QSO-galaxy covariance function (Yee & Green 1987), CIV-associated absorption in high redshift radio-loud QSOs (Flotz et al 1988), clustering analyses of the QSO distribution and galaxy environments around of QSOs (Ellingson et al 1991a). It has also been shown that the velocity dispersion of galaxies around QSOs is $\sim 400 \text{ km s}^{-1}$ (Ellingson et al 1991b). This means, QSOs trace the same density field as poor clusters do. Therefore, the formation and radiation of QSOs may not provide an effective mechanism that leads to the difference of typical scales between QSOs and clusters of galaxies.

Now we turn to the explanation based on $q_0$-dependence of the typical scale. In the previous sections, all scales are calculated under the assumption that the universe is of Einstein-de Sitter, and thus the deceleration parameter $q_0$ is taken to be 0.5. As it has been pointed out by Shank et al (1987) and Mo, et al. (1992a, b), the typical scale is crucially dependent on $q_0$ for high redshift objects like QSOs. Figures 10 is the power spectrum of the $\Delta \theta(r)$ for QBJS when $q_0 = 0.2, 0.4$ and 0.7, respectively. From the peaks in Figure 10 one can see a systematic increase of the wavelength with the decrease of $q_0$. This relationship is plotted in Figure 11. It shows that when $q_0 = 0.2$, the typical scale of QSOs is $125 \pm 11 \text{ h}^{-1}\text{Mpc}$. For other samples, we found about the same result. Figure 12 and 13 showed that all power spectrum of samples QN, QSGP, LBQS and LBQS-V have a peak at $130 \pm 15 \text{ h}^{-1}\text{Mpc}$.

Since galaxies and clusters have low redshifts, their typical scales do not depend on $q_0$. Therefore, the typical scale of QSOs is in good agreement with that of galaxies and clusters (Mo, et al. 1992a, b) when $q_0 \sim 0.2$. In other words, if one assumes that the comoving value of the typical scale can be used as a “standard cosmological rod” in the linear regime of an expanding universe, the universe should be of $q_0 \sim 0.5$. Considering various uncertainties in the typical scales of galaxies, clusters, and QSOs, it would be better to say that $q_0$ should be less than 0.5 at 95% confidence.

It is interested to point out that the conclusion of a low total mass density universe ($q_0 \sim 0.1$) has also been proposed by several independent researchers (Park, et al. 1992; Bahcall and Cen, 1992; Vogeley, et al., 1992). In the paper by Mo, et al (1992b), the $q_0$-dependence of the typical scales of the CIV absorption systems and the Ly$\alpha$ forests of high redshift QSOs has been studied. Using their result, one can find that the value of $q_0$, at which the typical scale of CIV system is the same as that of galaxies and cluster, is $q_0 \sim 0.2 - 0.3$. Therefore, when $q_0 = 0.2$, the $130 \text{ h}^{-1}\text{Mpc}$ typical scale is likely universal among the samples of galaxies, clusters, QSOs and the CIV absorption systems of QSOs. However, for this $q_0$, the typical scale of Ly$\alpha$ forest is different from $130 \text{ h}^{-1}\text{Mpc}$. This is understandable because the CIV systems probably originated from absorption of clouds associated with galaxies (Waymann, et al. 1979; Young et al, 1982), and Ly$\alpha$ comes from clouds which are unable to form.
QSOs and galaxies. In a word, it would be reasonable to say that the $130\, h^{-1}\text{Mpc}$ typical scale is universal in the distributions of galaxies, clusters and quasars.

5. Conclusion and Discussion

The possible typical scales in the distribution of QSOs have been detected by means of the second derivative of integrated correlation function. A typical scale of about $93 \pm 10\, h^{-1}\text{Mpc}$ when $q_0 = 0.5$ have been detected with considerable confidence in available 1- and 3-dimension samples of QSOs. This typical scale is probably “universal” for various subsamples of QSOs. If $q_0$ is taken to be 0.2, the QSO typical scale becomes the same as that of galaxies and clusters of galaxies. One can then have the following conclusions.

1. The existence of a $\sim 100\, h^{-1}\text{Mpc}$ typical scale in the spatial distribution of QSOs further strengthens the picture that QSOs probably trace the same larger scale density field of the universe as galaxies and clusters of galaxies do.

2. The redshifts range and sizes of QSO samples used here are totally different from that of galaxies and clusters of galaxies. The agreement of the QSO typical scale with the result of galaxies and clusters suggests that the detected typical scale is most likely universal in the large scale structure. This is consistent with the assumption that the typical scale is due to a characteristic scale in the initial perturbation spectrum, such as the bend scale in the density spectrum at the linear regime.

3. If the typical scale comes from the initial spectrum of the density perturbation, one can use the comoving values of the typical scale as a cosmic standard length when this scale remains in linear evolution. Accordingly, if we require that the comoving value of typical scale of QSOs is the same as that of galaxies, clusters and CIV absorption systems, the typical scale measurement favors an open universe, i.e. $q_0 < 0.5$.

4. The bend scale is model-dependent. For instance, CDM model should have a lower bending scale than that of a hybrid model. The formation time of structures with scale as large as the bend scale is also model-dependent. Therefore, the fact that the structures of bend scale exist at the high redshift seen for QSOs distribution may help discriminate among various cosmological models.

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### Table 1
Data of 1-dimensional samples

| Sample  | $N_p$ | $N_q$ | Notes                                      |
|---------|-------|-------|--------------------------------------------|
| QN      | 13    | 141   | northern sky (BSFP)                        |
| QS      | 14    | 154   | southern sky excepting southern galactic pole (BSFP) |
| QSGP    | 7     | 94    | southern galactic pole (BSFP)              |
| QBJS    | 3     | 52    | (BJS)                                      |

### Table 2
Data of 3-dimensional samples

| Sample | $N_p$ | $N_q$ | Notes                                                  |
|--------|-------|-------|--------------------------------------------------------|
| LBQS   | 18    | 510   | Foltz et al, 1987, 1989; Hewett et al. 1991 and Chaffee et al., 1991, Morris et al. 1991 |
| LBQS-V | 14    | 399   | excepting Virgo fields                                 |
|        |       |       | (references are the same as LBQS)                     |
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Figure captions:

**Figure 1** Redshift histograms of QSOs listed in pencil beam surveys. a. BFSP given by Boyle et al. (1990), and b. BJS given by Boyle et al. (1991).

**Figure 2** Curves of $\Delta \theta(r)$ of samples a) QN; b) QBJS; c) QS; and d) QSGP. Light lines show the curves of $\pm \sigma$ given by a average of 100 random samples. $q_0$ is taken to be 0.5, and smoothing scale $L = 40 \, h^{-1}\text{Mpc}$.

**Figure 3** Curves of $\Delta \theta(r)$ of samples a) QN and b) QS. These curves are obtained in the same way as Figure 2, but taking the smoothing scale $L = 20 \, h^{-1}\text{Mpc}$.

**Figure 4** Power spectrum of $\Delta \theta(r)$ for sample QS. The smoothing length $L$ is taken to be 40 and 20 $h^{-1}\text{Mpc}$, and $q_0 = 0.5$. For peaks with $P \geq 7.6$, the confidence of the existence of a periodic component is $\geq 99\%$.

**Figure 5** Power spectrum of $\Delta \theta(r)$ for sample QBJS.

**Figure 6** Redshift distributions of QSOs listed in the LBQS survey (Foltz et al. 1988, 1989): a. LBQS, consisting of all LBQS QSOs; b. LBQS-V, consisting of all LBQS QSOs, but excepting those in the area of the Virgo cluster.

**Figure 7** Curves of $\Delta \theta(r)$ of three-dimensional samples, a. LBQS; b. LBQS-V, respectively. $q_0$ is taken to be 0.5.

**Figure 8** Power spectrum of $\Delta \theta(r)$ for sample LBQS.

**Figure 9** Power spectrum of $\Delta \theta(r)$ for sample LBQS-V.

**Figure 10** Power spectrum of $\Delta \theta(r)$ for sample QBJS. The deceleration parameter is taken to be $q_0 = 0.2, 0.4, 0.7$, respectively.

**Figure 11** QSO’s Typical scale in sample QBJS as a function of $q_0$.

**Figure 12** Power spectrum of $\Delta \theta(r)$ for samples QN and QSGP when $q_0 = 0.2$ and $L = 40 \, h^{-1}\text{Mpc}$.

**Figure 13** Power spectrum of $\Delta \theta(r)$ for samples LBQS and LBQS-V when $q_0 = 0.2$ and $L = 40 \, h^{-1}\text{Mpc}$.