CP Violation in the Exclusive Decays

$B \rightarrow \pi e^+e^-$ and $B \rightarrow \rho e^+e^-$

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ABSTRACT

As a sequel to the calculation of the CP-violating asymmetry in the decay rates of $b \rightarrow d l^+l^-$ and $\bar{b} \rightarrow \bar{d} l^+l^-$, we address in this paper the asymmetry in exclusive channels $\bar{B} \rightarrow \pi e^+e^-$ and $\bar{B} \rightarrow \rho e^+e^-$, using form factors from two different models. In the invariant mass region $1 \text{ GeV} < \sqrt{s} < M_{J/\psi}$, the partial width asymmetry in the channel $\bar{B} \rightarrow \pi$ is $-6\% (-2\%)$, and that in the channel $\bar{B} \rightarrow \rho$, for one choice of form factors, is $-5\% (-2\%)$, assuming CKM parameters $\eta = 0.34$, $\rho = 0.3 (-0.3)$. We also calculate the forward-backward asymmetry $A_{FB}$ of the $e^-$ in the $e^+e^-$ centre-of-mass system, and find average values $\langle A_{FB} \rangle_{\bar{B} \rightarrow \pi} \equiv 0$, $\langle A_{FB} \rangle_{\bar{B} \rightarrow \rho} = -17\%$, to be compared with the inclusive result $\langle A_{FB} \rangle_{b \rightarrow d} = -9\%$. There is a CP-violating difference between $A_{FB}$ and the corresponding asymmetry in the antiparticle channel $\bar{A}_{FB}$. Formulae are given that are applicable to any FCNC channel $\bar{B} \rightarrow P_q(V_q) l^+l^-$, $q = s, d$, with $m_l \neq 0$, including lepton spin effects. An approximate procedure is used to incorporate the $\rho$, $\omega$, and $J/\psi$ resonances.

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I. MOTIVATION

We have recently calculated the $CP$-violating difference in the decay rates of the reactions $b \to d l^+ l^-$ and $\bar{b} \to \bar{d} l^+ l^-$, expected within the standard model [1]. The asymmetry in partial widths is directly proportional to $\text{Im} \left( V_{ub} V_{ud}^* \right) / \left( V_{tb} V_{td}^* \right)$, and is numerically equal to $-5\% (-2\%)$, assuming Cabibbo-Kobayashi-Maskawa (CKM) parameters $\eta = 0.34$, $\rho = 0.3 (-0.3)$.

In this paper we examine the exclusive channels $\bar{B} \to \pi e^+ e^-$ and $\bar{B} \to \rho e^+ e^-$. Although the branching ratios for individual channels are inevitably small, they probe different combinations of the Wilson coefficients $c_{\text{eff}}^7$, $c_{\text{eff}}^9$, $c_{10}$ appearing in the effective Hamiltonian, raising the possibility that the asymmetry might be substantially larger than in the inclusive reaction $b \to d l^+ l^-$. It may be noted that identification of the reaction $b \to d l^+ l^-$ in the presence of the much stronger decay $b \to s l^+ l^-$ will probably necessitate examination of the decay vertex, revealing the nature of the hadronic final state. In this paper we present results for the simplest exclusive channels $\bar{B} \to \pi e^+ e^-$ and $\bar{B} \to \rho e^+ e^-$. The formalism presented is general enough to be applied to any reaction induced by $b \to (s, d) l^+ l^-$ ($m_l \neq 0$), for any flavour-changing neutral current (FCNC) Hamiltonian characterized by

$$H_{\text{eff}} \sim G_\mu \bar{l} \gamma^\mu l + H_\mu \bar{l} \gamma^\mu \gamma^5 l,$$

where $G_\mu$ and $H_\mu$ are arbitrary combinations of the currents

$$\bar{f} \gamma_\mu(1 \pm \gamma^5)b, \quad \text{and} \quad \bar{f} \sigma_{\mu\nu} q^\nu(1 \pm \gamma^5)b, \quad f = s, d.$$ 

II. GENERAL FORMALISM

Assuming an effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{tq}^* \left( G_\mu \bar{l} \gamma^\mu l + H_\mu \bar{l} \gamma^\mu \gamma^5 l \right), \quad q = s, d,$$

and summing over vector meson polarizations, the differential cross section for the
exclusive decay $B \rightarrow P_q(V_q) l^+ l^-$ is given by the following formula\footnote{We use the convention $\epsilon_{0123} = +1$.}\footnote{\cite{footnote}}

$$
\frac{d\Gamma(B \rightarrow P_q(V_q) l^+ l^-)}{d\hat{s} \, d\hat{y} \, d\cos \theta} = \frac{G_F^2 M_B^9 \alpha^2}{2^{10} \pi^5} |V_{tb} V_{tq}|^2 \lambda^{1/2}(1, \hat{s}, \hat{M}_{P,V}^2) \sqrt{1 - \frac{4\hat{m}_t^2}{\hat{s}}} \delta(1 + \hat{s} - \hat{M}_{P,V}^2 - y) \sum_{s_+^-, s_-} \left\{ \begin{array}{l}
\frac{1}{2} \left[ (\hat{s} - 2\hat{m}_t^2) - 2(s_- \cdot s_+)\hat{m}_t^2 \right] \hat{W}^{L+R}_1 - \hat{m}_t(\hat{p}_+ \cdot s_- - \hat{p}_- \cdot s_+) \hat{W}^{L-R}_1 + 2\hat{m}_t^2 \hat{W}^+_1 \\
+ \left[ (\hat{p}_- \cdot v)(\hat{p}_+ \cdot v) + \frac{1}{2} \hat{m}_t^2(s_- \cdot s_+) - \hat{m}_t^2(s_- \cdot v)(s_+ \cdot v) - \frac{1}{4}(\hat{s} - 2\hat{m}_t^2) \right] \hat{W}^{L+R}_2 \\
+ \hat{m}_t \left[ (\hat{p}_- \cdot v)(s_+ \cdot v) - (\hat{p}_+ \cdot v)(s_- \cdot v) + \frac{1}{2}(\hat{p}_+ \cdot s_- - \hat{p}_- \cdot s_+) \right] \hat{W}^{L-R}_2 \\
+ \left[ (\hat{p}_- \cdot s_+)(\hat{p}_+ \cdot v)(s_- \cdot v) + (\hat{p}_+ \cdot s_-)(\hat{p}_- \cdot v)(s_+ \cdot v) - \frac{1}{2}(\hat{p}_- \cdot s_-)(\hat{p}_+ \cdot s_+) \right. \\
- (\hat{p}_- \cdot v)(\hat{p}_+ \cdot v)(s_- \cdot s_+) + \frac{1}{4}(\hat{s} - 2\hat{m}_t^2)[s_- \cdot s_+ - 2(s_- \cdot v)(s_+ \cdot v)] - \frac{1}{2}\hat{m}_t^2 \right] \hat{W}^+_2 \\
\left. - \frac{1}{2} i\epsilon_{\mu\nu\alpha\beta}[\hat{p}^\mu_\nu \hat{p}^\nu_\alpha[s_\beta(s_- \cdot v) + s_\beta(s_- \cdot v)] + v^\mu s^\nu s_\beta^\alpha[\hat{p}^\mu_\nu(\hat{p}_+ \cdot v) + \hat{p}^\beta_\nu(\hat{p}_- \cdot v)] \right] \hat{W}^-_2 \\
+ \hat{m}_t \left[ (\hat{p}_- \cdot s_+)(\hat{p}_+ \cdot v) + (\hat{p}_+ \cdot s_-)(\hat{p}_- \cdot v) - \frac{1}{2}\hat{s}(s_- + s_+) \cdot v \right] \hat{W}^{L+R}_3 \\
+ \frac{1}{2}[\hat{s}(\hat{p}_- - \hat{p}_+) \cdot v + 2\hat{m}_t^2[(\hat{p}_+ \cdot s_-)(s_+ \cdot v) - (\hat{p}_- \cdot s_+)(s_- \cdot v)] \right] \hat{W}^{L-R}_3 \\
+ \hat{m}_t \left[ (\hat{p}_- \cdot s_+)(\hat{p}_+ \cdot v) + (\hat{p}_+ \cdot s_-)(\hat{p}_- \cdot v) - \frac{1}{2}\hat{s}(s_- + s_+) \cdot v \right] \hat{W}^+_3 \\
- \hat{m}_t i\epsilon_{\mu\nu\alpha\beta}\hat{p}^\mu_\nu \hat{p}^\nu_\alpha(s_- + s_+)^\beta \hat{W}^-_3 \\
- \hat{m}_t^2 \left[ (\hat{p}_- \cdot s_+)(\hat{p}_- \cdot s_-) - \frac{1}{2}\hat{s}(1 + s_- \cdot s_+) \right] \left( \hat{W}^{L+R}_4 - \hat{W}^+_4 \right) \\
- \hat{m}_t^2 \left[ (\hat{p}_- \cdot s_+)(s_- \cdot v) + (\hat{p}_+ \cdot s_-)(s_+ \cdot v) - \frac{1}{2}\gamma v(1 + s_- \cdot s_+) \right] \left( \hat{W}^{L+R}_5 - \hat{W}^+_5 \right) \\
+ \hat{m}_t \left[ (\hat{p}_+ \cdot s_-)(\hat{p}_- \cdot v) - (\hat{p}_- \cdot s_+)(\hat{p}_+ \cdot v) + \frac{1}{2}\hat{s}(s_- - s_-) \cdot v \right] \hat{W}^{L-R}_5 \\
- \hat{m}_t^2 i\epsilon_{\mu\nu\alpha\beta}v^\mu s^\nu s_\alpha^\beta \hat{W}^-_5 \right\}. \quad (2.2)
$$

In Eq. (2.2) we introduced variables scaled by the $B$-meson mass, i.e.

$$
\hat{p}_i = \frac{p_i}{M_B}, \quad \hat{m}_t = \frac{m_t}{M_B}, \quad \hat{M}_{P,V} = \frac{M_{P,V}}{M_B}, \quad s \equiv q^2 = (\hat{p}_+ + \hat{p}_-)^2, \quad v^\mu \equiv \hat{p}_B^\mu, \quad (2.3)
$$

and the triangle function

$$
\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ac), \quad (2.4)
$$
so that
\[ \hat{p}_\pm \cdot v = \frac{1}{4} \left[ y \pm \lambda^{1/2}(1, \hat{s}, \hat{M}^2_{P,V}) \sqrt{1 - \frac{4\hat{m}^2_i}{\hat{s}}} \cos \theta \right], \quad y = 2v \cdot \hat{q}, \] (2.5)

\( \theta \) being the angle between \( l^- \) and the outgoing \( s \) or \( d \) quark in the \( l^+l^- \) centre-of-mass system. The four-vectors \( s_\pm \) and \( \hat{p}_\pm \) denote spins and momenta of the leptons respectively, and

\[ \hat{W}^{L\pm R} \equiv \hat{W}^L \pm \hat{W}^R, \] (2.6a)

\[ \hat{W}^{\pm} \equiv \hat{W}^L \pm \hat{W}^R. \] (2.6b)

The invariant form factors \( \hat{W}_i = \hat{W}_i(s, y) \) are defined via\(^2\)

\[ \hat{W}_{\mu \nu}^{LL} \equiv \frac{1}{M_B^2} \langle \bar{B}|(G - H)^\dagger_\mu |P(V)\rangle\langle P(V)|(G - H)_\nu |\bar{B}\rangle 
\]
\[ = -g_{\mu \nu} \hat{W}_1^{LL} + v_\mu v_\nu \hat{W}_2^{LL} - i\epsilon_{\mu \nu \alpha \beta} v^\alpha \hat{q}^\beta \hat{W}_3^{LL} + \hat{q}_\mu \hat{q}_\nu \hat{W}_4^{LL} + (\hat{q}_\mu v_\nu + \hat{q}_\nu v_\mu) \hat{W}_5^{LL}, \]
(2.7)

\[ \hat{W}_{\mu \nu}^{RR} = \hat{W}_{\mu \nu}^{LL}(H \rightarrow -H, \hat{W}_i^{LL} \rightarrow \hat{W}_i^{RR}), \] (2.8)

\[ \hat{W}_{\mu \nu}^{LR} \equiv \frac{1}{M_B^2} \langle \bar{B}|(G - H)^\dagger_\mu |P(V)\rangle\langle P(V)|(G + H)_\nu |\bar{B}\rangle 
\]
\[ = -g_{\mu \nu} \hat{W}_1^{LR} + v_\mu v_\nu \hat{W}_2^{LR} - i\epsilon_{\mu \nu \alpha \beta} v^\alpha \hat{q}^\beta \hat{W}_3^{LR} + \hat{q}_\mu \hat{q}_\nu \hat{W}_4^{LR} + (\hat{q}_\mu v_\nu + \hat{q}_\nu v_\mu) \hat{W}_5^{LR}, \]
(2.9)

\[ \hat{W}_{\mu \nu}^{RL} = \hat{W}_{\mu \nu}^{LR}(H \rightarrow -H, \hat{W}_i^{LR} \rightarrow \hat{W}_i^{RL}), \] (2.10)

with

\[ G_\mu = c_9^{\text{eff}} (\bar{f} \gamma_\mu P_L b) - 2 c_7^{\text{eff}} \bar{f} i\sigma_{\mu \nu} q^\nu \left( m_b P_R + m_f P_L \right) b, \] (2.11)

\[ H_\mu = c_{10} \bar{f} \gamma_\mu P_L b, \quad P_{L,R} = (1 \mp \gamma_5)/2, \quad f = s, d, \] (2.12)

and can be found in Appendices \[ \text{B} \] and \[ \text{C} \].

\(^2\)Note that these dimensionless quantities are different for \( \bar{B} \rightarrow P \) and \( \bar{B} \rightarrow V \) transitions.
Using the parameters listed in Appendix A and the analytic expressions for the Wilson coefficients, including next-to-leading order QCD corrections, given in Refs. [2–4], we obtain in leading logarithmic approximation

\[ c_7^{\text{eff}} = -0.315, \quad c_{10} = -4.642, \quad (2.13) \]

\[ c_1 = -0.249, \quad c_2 = 1.108, \quad c_3 = 1.112 \times 10^{-2}, \quad c_4 = -2.569 \times 10^{-2}, \]
\[ c_5 = 7.404 \times 10^{-3}, \quad c_6 = -3.144 \times 10^{-2}, \quad c_9 = 4.227, \quad (2.14) \]

and in next-to-leading approximation

\[ c_9^{\text{eff}} = c_9 + 0.124 \omega(\hat{s}) + g(\hat{m}_c, \hat{s}) (3c_1 + c_2 + 3c_3 + c_4 + 3c_5 + c_6) \]
\[ + \lambda_u [g(\hat{m}_c, \hat{s}) - g(\hat{m}_u, \hat{s})] (3c_1 + c_2) - \frac{1}{2} g(\hat{m}_q, \hat{s}) (c_3 + 3c_4) \]
\[ - \frac{1}{2} g(\hat{m}_b, \hat{s}) (4c_3 + 4c_4 + 3c_5 + c_6) + \frac{2}{9} (3c_3 + c_4 + 3c_5 + c_6), \quad (2.15) \]

with

\[ \lambda_u \equiv \frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}, \quad q = s, d, \quad (2.16) \]

and the one-loop function

\[ g(\hat{m}_i, \hat{s}) = -\frac{8}{9} \ln(m_i/m_b) + \frac{8}{27} + \frac{4}{9} y_i - \frac{2}{9} (2 + y_i) \sqrt{1 - y_i} \]
\[ \times \left\{ \Theta(1 - y_i) \left( \ln \left( \frac{1 + \sqrt{1 - y_i}}{1 - \sqrt{1 - y_i}} \right) - i\pi \right) + \Theta(y_i - 1) 2 \arctan \frac{1}{\sqrt{y_i - 1}} \right\}, \quad (2.17) \]

where \( y_i = 4\hat{m}_i^2/\hat{s} \). The function \( \omega(\hat{s}) \) in Eq. (2.13) represents the one-gluon correction to the matrix element of the operator \( O_9 = (e^2/16\pi^2)\bar{q}\gamma^\mu P_L b_a \bar{l} \gamma_\mu l \). In our discussion of exclusive channels, this correction may be regarded as a contribution to the form factors, and hence may be omitted (see also Ref. [4]).

As an alternative to the functions \( g(\hat{m}_u, \hat{s}) \) and \( g(\hat{m}_c, \hat{s}) \) describing the effects of \( u\bar{u} \) and \( c\bar{c} \) loops, we have also investigated an ansatz in which these functions are determined by the experimentally measured ratios \( R^{\rho,\omega}_{\text{had}}(\hat{s}) \) and \( R^{J/\psi}_{\text{had}}(\hat{s}) \), as described in detail in our previous paper [3]. In this way it is possible to incorporate the \( \rho, \omega \) and \( J/\psi, \psi' \) etc. resonances into the differential cross section in an approximate way, consistent with the idea of global duality. The numerical results for the average
\( CP \)-violating asymmetry \( \langle A_{CP} \rangle \) depend very little on which of these representations we choose for the function \( g(\hat{m}_{a,c}, \hat{s}) \).

If the spins of the leptons are not measured, we have

\[
\frac{d\Gamma}{ds \, dy \, d\cos \theta} = \frac{G_F^2 M_B^5 \alpha^2}{2^{10} \pi^5} |V_{tb} V_{tq}|^2 \lambda^{1/2}(1, \hat{s}, \hat{M}_{P,V}^2) \sqrt{1 - \frac{4\hat{m}_t^2}{\hat{s}}} \delta(1 + \hat{s} - \hat{M}_{P,V}^2 - y) \\
\times \left\{ 2\hat{s} \left( 1 + \frac{2\hat{m}_t^2}{\hat{s}} \right) \hat{W}_1^{L+R} + \left[ - \hat{s} + \frac{1}{4} \left[ y^2 - \lambda(1, \hat{s}, \hat{M}_{P,V}^2) \left( 1 - \frac{4\hat{m}_t^2}{\hat{s}} \right) \cos^2 \theta \right] \right] \hat{W}_2^{L+R} \\
- \hat{s} \lambda^{1/2}(1, \hat{s}, \hat{M}_{P,V}^2) \sqrt{1 - \frac{4\hat{m}_t^2}{\hat{s}}} \cos \theta \hat{W}_3^{L-R} \\
+ 2\hat{m}_t^2 \left[ 4(\hat{W}_1^+ - \hat{W}_1^{L+R}) + (\hat{W}_2^{L+R} - \hat{W}_2^+) + \hat{s}(\hat{W}_4^{L+R} + \hat{W}_4^+) + y(\hat{W}_5^{L+R} - \hat{W}_5^+) \right] \right\}.
\]

(2.18)

From this we may obtain the forward-backward asymmetry of \( l^- \) in the \( l^+l^- \) centre-of-mass system

\[
A_{FB}(\hat{s}) = \frac{\int_0^1 d\Gamma}{\int_{-1}^1 d\Gamma} \frac{d \cos \theta}{d\cos \theta} \frac{d \cos \theta}{d\cos \theta} = \frac{-3\lambda^{1/2}(1, \hat{s}, \hat{M}_{P,V}^2) \sqrt{1 - \frac{4\hat{m}_t^2}{\hat{s}}} \hat{W}_3^{L-R}}{\Sigma_{P,V}} \hat{W}_3^{L-R},
\]

(2.19)

where

\[
\Sigma_{P,V} = \left[ 12\hat{s}\hat{W}_1^{L+R} + \lambda(1, \hat{s}, \hat{M}_{P,V}^2)\hat{W}_2^{L+R} \right] \left( 1 + \frac{2\hat{m}_t^2}{\hat{s}} \right) + 12\hat{m}_t^2 \left[ 4(\hat{W}_1^+ - \hat{W}_1^{L+R}) \\
+ (\hat{W}_2^{L+R} - \hat{W}_2^+) + \hat{s}(\hat{W}_4^{L+R} - \hat{W}_4^+) + (1 + \hat{s} - \hat{M}_{P,V}^2)(\hat{W}_5^{L+R} - \hat{W}_5^+) \right].
\]

(2.20)

Integrating the differential cross section, Eq. (2.18), instead over \( y \) and \( \cos \theta \), we obtain the \( l^+l^- \) invariant mass spectrum

\[
\frac{d\Gamma}{ds} = \frac{G_F^2 M_B^5 \alpha^2}{3 \cdot 2^{10} \pi^5} |V_{tb} V_{tq}|^2 \lambda^{1/2}(1, \hat{s}, \hat{M}_{P,V}^2) \sqrt{1 - \frac{4\hat{m}_t^2}{\hat{s}}} \Sigma_{P,V}.
\]

(2.21)
Formulae analogous to those given above also apply to the inclusive reaction $\bar{B} \to X_q l^+ l^-$, which, at the level of the parton model, is simulated by the transition $b \to q l^+ l^-$ (see Appendix E for details).

III. PARTIAL WIDTH ASYMMETRY IN $\bar{B} \to \pi l^+ l^-$

A. Form factors

After these general remarks, we calculate the differential decay rate and $CP$-violating asymmetry in the reaction $\bar{B} \to \pi l^+ l^-$. The relevant matrix elements are parametrized using the invariant form factors introduced, e.g., by Colangelo et al. [6], i.e.

$$\langle \pi(p_\pi)|d\gamma_\mu P_{L,R} b|\bar{B}(p_B)\rangle = \frac{1}{2} \left\{ (2p_B - q)_\mu F_1(q^2) + \frac{M_B^2 - M_\pi^2}{q^2} q_\mu \left[ F_0(q^2) - F_1(q^2) \right] \right\} ,$$  \hspace{1cm} (3.1)

and

$$\langle \pi(p_\pi)|d\sigma_{\mu\nu} q^\nu P_{L,R} b|\bar{B}(p_B)\rangle = \frac{1}{2} \left[ (2p_B - q)_\mu q^2 - (M_B^2 - M_\pi^2) q_\mu \right] \frac{F_T(q^2)}{M_B + M_\pi} ;$$  \hspace{1cm} (3.2)

with $q = p_B - p_\pi$, and $P_{L,R} = (1 \mp \gamma_5)/2$.

B. Decay rate for $\bar{B} \to \pi l^+ l^-$

Using the general expression Eq. (2.18), along with the form factors $\hat{W}_i(\hat{s}, \hat{y})$, Eqs. (B3)–(B9), the triple differential decay rate becomes

$$\frac{d\Gamma(\bar{B} \to \pi l^+ l^-)}{d\hat{s} \; dy \, d \cos \theta} = \frac{G_F^2 M_B^5 \alpha^2}{2^{9/2} \pi^3} |V_{tb} V_{td}^*|^2 \lambda^{1/2}(1, \hat{s}, \hat{M}_\pi^2) \sqrt{1 - \frac{4\hat{m}_t^2}{\hat{s}}} \delta(1 + \hat{s} - \hat{M}_\pi^2 - \hat{y})$$

$$\times \left\{ \left[ |c_9^{\text{eff}} F_1(\hat{s}) - 2 c_7^{\text{eff}} \tilde{F}_T(\hat{s})|^2 + |c_{10} F_1(\hat{s})|^2 \right] \left( - \hat{s} + \frac{1}{4} [y^2 - \lambda(1, \hat{s}, \hat{M}_\pi^2) \cos^2 \theta] \right) + \hat{m}_t^2 |c_{10}|^2 \left[ |F_1(\hat{s})|^2 (2 - y + \hat{s}/2) + \frac{1}{2} A^2(\hat{s}) |\tilde{F}_1(\hat{s})|^2 \hat{s} \right. \right.$$  \hspace{1cm} (3.3)

$$\left. + A(\hat{s}) (y - \hat{s}) \text{Re} F_1^*(\hat{s}) \tilde{F}_1(\hat{s}) \right\}$$

$$+ \frac{\hat{m}_t^2}{\hat{s}} \lambda(1, \hat{s}, \hat{M}_\pi^2) \cos^2 \theta + 2 \hat{m}_t^2 |c_{10}|^2 \left[ |F_1(\hat{s})|^2 (2 - y + \hat{s}/2) + \frac{1}{2} A^2(\hat{s}) |\tilde{F}_1(\hat{s})|^2 \hat{s} \right.$$  \hspace{1cm} (3.3)
where
\[ \tilde{F}_T(\hat{s}) = \frac{F_T(\hat{s})}{1 + M_\pi}(\hat{m}_b + \hat{m}_d), \]  
\[ (3.4) \]
\[ \tilde{F}_1(\hat{s}) = F_0(\hat{s}) - F_1(\hat{s}), \]  
\[ (3.5) \]
\[ A(\hat{s}) = 1 - \hat{M}_\pi^2 \hat{s}. \]  
\[ (3.6) \]
Integration of the distribution in Eq. (3.3) over \( y \) and \( \cos \theta \) leads to the differential decay rate in the variable \( \sqrt{\hat{s}} \)
\[ d\Gamma(\bar{B} \to \pi^+ l^-) = \frac{G_F^2 M_B^5 \alpha^2}{3 \cdot 2^8 \pi^5} |V_{td}|^2 \frac{\sqrt{\hat{s}}}{\hat{s}} 4 \hat{m}_l^2 (1 - \hat{s}, \hat{M}_\pi^2) \hat{s} - 4 \hat{m}_l^2 \Sigma_\pi, \]  
\[ (3.7) \]
where we defined
\[ \Sigma_\pi = \left( |c_9^{\text{eff}} F_1(\hat{s}) - 2 c_7^{\text{eff}} \tilde{F}_T(\hat{s})|^2 + |c_{10} F_1(\hat{s})|^2 \right) \left( 1 + \frac{2 \hat{m}_l^2}{\hat{s}} \right) \lambda(1, \hat{s}, \hat{M}_\pi^2) \]
\[ + 12 \hat{m}_l^2 |c_{10}|^2 I(\hat{s}), \]  
\[ (3.8) \]
and
\[ I(\hat{s}) = |F_1(\hat{s})|^2(1 - \frac{\hat{s}}{2} + \hat{M}_\pi^2) + \frac{1}{2} A^2(\hat{s})|\tilde{F}_1(\hat{s})|^2\hat{s} + A(\hat{s})(1 - \hat{M}_\pi^2)\text{Re} F_1^*(\hat{s}) \tilde{F}_1(\hat{s}). \]  
\[ (3.9) \]
Eq. (3.7) agrees with Refs. [7, 8], when we set \( d \to s, M_\pi \to M_K \), and \( \hat{m}_d = 0 \) but \( \hat{m}_l \neq 0 \), and with Refs. [6, 9] in the case of \( \hat{m}_l = 0 \). The above form factors are related to those of Refs. [10, 11] through
\[ F_1(q^2) = f_+(q^2), \]  
\[ F_0(q^2) = f_+(q^2) + \frac{q^2}{M_B^2 - M_\pi^2} f_-(q^2), \]  
\[ F_T(q^2) = -(M_B + M_\pi)s(q^2). \]  
\[ (3.10a, b, c) \]
Using the Wolfenstein representation of the CKM matrix [12], we may write
\[ |V_{tb}V_{td}^*|^2 = A^2 \lambda^6[(1 - \rho)^2 + \eta^2] + O(\lambda^8) , \]  
\[ (3.11) \]
Table I: Branching ratio $\text{Br}(\bar{B} \to \pi e^+e^-)$ compared to $\text{Br}(\bar{B} \to X_d e^+e^-)$ for different values of $(\rho, \eta)$, excluding the region around the $J/\psi$ and $\psi'$ resonances ($\pm 20$ MeV). The labels “COL” and “MEL” denote the form factors of Refs. [6] and [11] respectively (see footnote 3).

| $(\rho, \eta)$ | MEL       | COL       |
|----------------|-----------|-----------|
| $(0.3, 0.34)$  | $1.5 \times 10^{-8}$ | $2.7 \times 10^{-7}$ |
|                | $0.9 \times 10^{-8}$   |           |
| $(-0.07, 0.34)$ | $3.1 \times 10^{-8}$ | $5.5 \times 10^{-7}$ |
|                | $1.9 \times 10^{-8}$   |           |
| $(-0.3, 0.34)$ | $4.4 \times 10^{-8}$ | $7.9 \times 10^{-7}$ |
|                | $2.7 \times 10^{-8}$   |           |

with four real parameters $\lambda \equiv \sin \theta_C$, $A$, $\rho$, and $\eta$, where $\eta$ is a measure of $CP$ violation in the standard model. Our results for the differential decay rate versus $\sqrt{\hat{s}}$ are shown in Fig. [1] for typical values of $(\rho, \eta)$, whereas the results for the branching ratio compared to the inclusive decay $\bar{B} \to X_d e^+e^-$ [1] are displayed in Table I.

C. $CP$-violating asymmetry

The $CP$-violating partial width asymmetry between $B$ and $\bar{B}$ decay is defined as follows

$$A_{CP}(\sqrt{\hat{s}}) = \frac{d\Gamma/d\sqrt{\hat{s}} - d\bar{\Gamma}/d\sqrt{\hat{s}}}{d\Gamma/d\sqrt{\hat{s}} + d\bar{\Gamma}/d\sqrt{\hat{s}}}$$

(3.12)

where

$$\frac{d\Gamma}{d\sqrt{\hat{s}}} = \frac{d\Gamma(\bar{B} \to \pi l^+l^-)}{d\sqrt{\hat{s}}}, \quad \frac{d\bar{\Gamma}}{d\sqrt{\hat{s}}} = \frac{d\Gamma(B \to \bar{\pi} l^+l^-)}{d\sqrt{\hat{s}}}$$

(3.13)

and we obtain

$$A_{CP}(\sqrt{\hat{s}}) = \frac{-2\text{Im} \lambda_{\pi} \Delta_{\pi}}{\Sigma_{\pi} + 2\text{Im} \lambda_{\pi} \Delta_{\pi}} \left(1 + \frac{2\hat{m}_{\pi}^2}{\hat{s}}\right) \lambda(1, \hat{s}, \hat{M}_{\pi}^2)$$

where $\Sigma_{\pi} = \lambda_{\pi} \Delta_{\pi}$ and $\lambda(1, \hat{s}, \hat{M}_{\pi}^2)$ is the first-order hadronic form factor.

Footnote 3: In our numerical calculations, we have used the form factors of Colangelo et al. [6], choosing the mass parameter in $F_1(q^2)$ and $F_T(q^2)$ to be $M_P = 5.3$ GeV. We also considered the alternative model of Melikhov and Nikitin [11], with $\bar{B} \to \pi$ and $\bar{B} \to \rho$ form factors specified in “Set 2”. 


\[ \approx -2\text{Im} \lambda_u \frac{\Delta_\pi}{\Sigma_\pi} \left( 1 + \frac{2\hat{m}_l^2}{s} \right) \lambda(1, \hat{s}, \hat{M}_\pi^2) \]

\[ = \frac{2\eta}{(1 - \rho)^2 + \eta^2} \frac{\Delta_\pi}{\Sigma_\pi} \left( 1 + \frac{2\hat{m}_l^2}{s} \right) \lambda(1, \hat{s}, \hat{M}_\pi^2) , \quad (3.14) \]

with

\[ \lambda_u \equiv \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} = \frac{\rho(1 - \rho) - \eta^2}{(1 - \rho)^2 + \eta^2} - i \frac{\eta}{(1 - \rho)^2 + \eta^2} + O(\lambda^2) , \quad (3.15) \]

\[ \Delta_\pi = \text{Im} \xi_1^* \xi_2 |F_1(\hat{s})|^2 - 2c_7^\text{eff} \text{Im} \xi_2 F_T^*(\hat{s}) F_1(\hat{s}) \frac{\hat{m}_b + \hat{m}_d}{1 + \hat{M}_\pi} , \quad (3.16) \]

\[ c_7^\text{eff} \equiv \xi_1 + \lambda_u \xi_2 , \quad (3.17) \]

and \( \Sigma_\pi \) defined in Eq. (3.8). In Table II we give the numerical values we have obtained for the average \( CP \)-violating asymmetry, along with the branching ratio, for a certain region of \( \sqrt{s} \), and show in Fig. \( \ref{fig:ACP} \) \( A_{CP} \) for the two form factor models previously mentioned, as a function of \( \sqrt{s} \). It should be noted that the asymmetry is essentially independent of the parametrization of form factors, as illustrated in Table II.

**Table II:** Branching ratio \( \text{Br} (\bar{B} \to \pi e^+ e^-) \) and average \( CP \)-violating asymmetry \( \langle A_{CP} \rangle \) for different values of \( (\rho, \eta) \) in the region \( 1 \text{ GeV} < \sqrt{s} < (M_{J/\psi} - 20 \text{ MeV}) \). The labels “COL” and “MEL” denote the form factors of Refs. \( \ref{ref:6} \) and \( \ref{ref:11} \) respectively (see footnote 3).

| \((\rho, \eta)\)   | \text{Br} (\bar{B} \to \pi e^+ e^-) | \langle A_{CP} \rangle |
|-------------------|-------------------------------------|---------------------|
| \((0.3, 0.34)\)   | MEL \( 0.8 \times 10^{-8} \) | \( -6.0 \times 10^{-2} \) |
|                   | COL \( 0.5 \times 10^{-8} \)  | \( -6.0 \times 10^{-2} \) |
| \((-0.07, 0.34)\) | MEL \( 1.6 \times 10^{-8} \)  | \( -3.1 \times 10^{-2} \) |
|                   | COL \( 1.1 \times 10^{-8} \)  | \( -3.1 \times 10^{-2} \) |
| \((-0.3, 0.34)\)  | MEL \( 2.2 \times 10^{-8} \)  | \( -2.2 \times 10^{-2} \) |
|                   | COL \( 1.5 \times 10^{-8} \)  | \( -2.2 \times 10^{-2} \) |
IV. PARTIAL WIDTH ASYMMETRY IN $\bar{B} \rightarrow \rho l^+l^-$

A. Form factors

The form factors for this process are defined as follows ($\epsilon_{0123} = +1$):

$$\langle \rho(p_\rho) | \bar{d}_\gamma \gamma_\mu P_L b | \bar{B}(p_B) \rangle = i \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_\rho^\alpha q^\beta \frac{V(q^2)}{M_B + M_\rho} - \frac{1}{2} \left\{ \epsilon^* (M_B + M_\rho) A_1(q^2) - (\epsilon^* \cdot q)(2p_B - q)_\mu \frac{A_2(q^2)}{M_B + M_\rho} - \frac{2M_\rho}{q^2} (\epsilon^* \cdot q) [A_3(q^2) - A_0(q^2)] q_\mu \right\} , \quad (4.1)$$

where $A_3$ can be written in terms of $A_1$ and $A_2$, i.e.

$$A_3(q^2) = \frac{M_B + M_\rho}{2M_\rho} A_1(q^2) - \frac{M_B - M_\rho}{2M_\rho} A_2(q^2) , \quad (4.2)$$

and

$$\langle \rho(p_\rho) | \bar{d}\gamma_\mu q^\nu P_{R,L} b | \bar{B}(p_B) \rangle = -2i \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_\rho^\alpha q^\beta T_1(q^2) \pm \left[ \epsilon^* (M_B^2 - M_\rho^2) - (\epsilon^* \cdot q)(2p_B - q)_\mu \left\{ T_2(q^2) \pm (\epsilon^* \cdot q) \left[ q_\mu - \frac{q^2}{M_B^2 - M_\rho^2} (2p_B - q)_\mu \right] T_3(q^2) \right\} , \quad (4.3)$$

$\epsilon^\mu$ being the $\rho$ polarization vector, and $q = p_B - p_\rho$.

B. Decay rate for $\bar{B} \rightarrow \rho l^+l^-$

Inserting the form factors $\hat{W}_i(\hat{s}, \hat{y})$, provided in Appendix C, into Eq. (2.18), we get

$$\frac{d\Gamma(\bar{B} \rightarrow \rho l^+l^-)}{d\hat{s} \, dy \, d \cos \theta} = \frac{G_F^2 M_B^5 \alpha^2}{210 \pi^5} |V_{tb} V_{td}^*|^2 \lambda^{1/2}(1, \hat{s}, \hat{M}_\rho^2) \sqrt{1 - \frac{4\hat{m}_\rho^2}{\hat{s}}} \delta(1 + \hat{s} - \hat{M}_\rho^2 - \hat{y}) \times \left\{ \mathcal{A}(\hat{s}, \hat{y}) + \mathcal{B}(\hat{s}, \hat{y}) \cos \theta + \mathcal{C}(\hat{s}, \hat{y}) \cos^2 \theta \right\} , \quad (4.4)$$

where the functions $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are defined as

$$\mathcal{A}(\hat{s}, \hat{y}) = \hat{s} \left( 1 + \frac{2\hat{m}_\rho^2}{\hat{s}} \right) \left[ (y^2 - 4\hat{s}) \alpha_1(\hat{s}) + (1 + \hat{M}_\rho)^2 \alpha_2(\hat{s}) \right] - \frac{1}{4} \left( y^2 - 4\hat{s} \right) \left\{ 2\hat{s} \alpha_1(\hat{s}) - \frac{(1 + \hat{M}_\rho)^2}{2\hat{M}_\rho^2} \alpha_2(\hat{s}) + \frac{2}{(1 + \hat{M}_\rho)^2} \left[ 1 - \frac{(2 - y)^2}{4\hat{M}_\rho^2} \right] \alpha_3(\hat{s}) - 2 \left[ 1 - \frac{(2 - y)}{2\hat{M}_\rho^2} \right] \text{Re} \alpha_4(\hat{s}) \right\}$$

$$+ 2\hat{m}_\rho^2 |c_{10}|^2 \left\{ -2(y^2 - 4\hat{s}) \frac{|V(\hat{s})|^2}{(1 + \hat{M}_\rho)^2} - 3(1 + \hat{M}_\rho)^2 |A_1(\hat{s})|^2 \right\}$$

$$+ \left[ 2(1 + \hat{M}_\rho) - \hat{s} \right] \left[ (2 - y)^2 - 4\hat{M}_\rho^2 \right] \frac{|A_2(\hat{s})|^2}{4\hat{M}_\rho^2 (1 + \hat{M}_\rho)^2}$$

$$\mathcal{B}(\hat{s}, \hat{y}) = \hat{s} \left( 1 + \frac{2\hat{m}_\rho^2}{\hat{s}} \right) \left[ (y^2 - 4\hat{s}) \alpha_1(\hat{s}) + (1 + \hat{M}_\rho)^2 \alpha_2(\hat{s}) \right] \left( 1 + \hat{M}_\rho \right) \left[ 1 - \frac{(2 - y)^2}{4\hat{M}_\rho^2} \right]$$

$$\mathcal{C}(\hat{s}, \hat{y}) = \hat{s} \left( 1 + \frac{2\hat{m}_\rho^2}{\hat{s}} \right) \left[ (y^2 - 4\hat{s}) \alpha_1(\hat{s}) + (1 + \hat{M}_\rho)^2 \alpha_2(\hat{s}) \right] \left[ 1 - \frac{(2 - y)^2}{4\hat{M}_\rho^2} \right]$$
with $f_i = y^2 - 4\hat{s}$ and $f_2 = (2 - y)^2 - 4\hat{M}_\rho^2$. The functions $\alpha_i(\hat{s})$, $i = 1, \ldots, 4$, are given in Appendix D.

The form factors defined in Eqs. (4.1)–(4.3) can be related to those of Ref. [11] via

$$V(q^2) = (M_B + M_\rho)g(q^2),$$

$$A_1(q^2) = \frac{f(q^2)}{M_B + M_\rho},$$

$$A_2(q^2) = -(M_B + M_\rho)a_+(q^2),$$

$$A_3(q^2) = \frac{f(q^2) + (M_B^2 - M_\rho^2)a_+(q^2)}{2M_\rho},$$

$$A_0(q^2) = \frac{q^2a_-(q^2) + f(q^2) + (M_B^2 - M_\rho^2)a_+(q^2)}{2M_\rho},$$

and

$$B(\hat{s}, y) = 4\hat{s}\lambda^{1/2}(1, \hat{s}, \hat{M}_\rho^2)\sqrt{1 - \frac{4\hat{M}_\rho^2}{\hat{s}}}\Re{c_{10}}\left\{A_1^*(\hat{s})V(\hat{s})\Re{c_{10}^{\text{eff}}} + \frac{2c_{10}^{\text{eff}}}{\hat{s}}\left[T_2^*(\hat{s})V(\hat{s})(1 - \hat{M}_\rho)(\hat{m}_b - \hat{m}_d) + A_1^*(\hat{s})T_1(\hat{s})(1 + \hat{M}_\rho)(\hat{m}_b + \hat{m}_d)\right]\right\},$$

$$C(\hat{s}, y) = -\frac{1}{4}\lambda(1, \hat{s}, \hat{M}_\rho^2)\left(1 - \frac{4\hat{m}_d^2}{\hat{s}}\right)\left\{-2\hat{s}\alpha_1(\hat{s}) + \frac{(1 + \hat{M}_\rho)^2}{2M_\rho^2}\alpha_2(\hat{s}) - \frac{2}{(1 + \hat{M}_\rho)^2}\left[1 - \frac{(2 - y)^2}{4M_\rho^2}\right]\alpha_3(\hat{s}) + 2\left[1 - \frac{(2 - y)^2}{2M_\rho^2}\right]\Re{\alpha_4}(\hat{s})\right\}.$$
\[ T_1(q^2) = -\frac{1}{2}g_+(q^2), \quad (4.9f) \]
\[ T_2(q^2) = -\frac{1}{2}\left[ g_+(q^2) + \frac{q^2 g_-(q^2)}{M_B^2 - M^2_\rho} \right], \quad (4.9g) \]
\[ T_3(q^2) = \frac{1}{2}\left[ g_-(q^2) - \frac{(M_B^2 - M^2_\rho)h(q^2)}{2} \right]. \quad (4.9h) \]

From Eq. (4.4) we obtain the differential decay rate in the variable \( \sqrt{s} \), by integrating over \( y \) and \( \cos \theta \)
\[ \frac{d\Gamma(\bar{B} \to \rho l^+ l^-)}{d\sqrt{s}} = \frac{G_F^2 M_B^5 \alpha^2}{3 \cdot 2^9 \pi^5} |V_{tb} V^*_{td}|^2 \lambda^{1/2}(1, \hat{s}, \hat{M}_\rho^2) \sqrt{\hat{s}} - 4\hat{m}_t^2 \Sigma_\rho, \quad (4.10) \]
with
\[ \Sigma_\rho = \left\{ 4\hat{s}\alpha_1(\hat{s}) + \frac{(1 + \hat{M}_\rho^2)^2}{\lambda(1, \hat{s}, \hat{M}_\rho^2)} \left[ 6\hat{s} + \frac{\lambda(1, \hat{s}, \hat{M}_\rho^2)}{2\hat{M}_\rho^2} \right] \alpha_2(\hat{s}) + \frac{\lambda(1, \hat{s}, \hat{M}_\rho^2)}{2\hat{M}_\rho^2(1 + \hat{M}_\rho^2)^2} \alpha_3(\hat{s}) - \frac{(1 - \hat{s} - \hat{M}_\rho^2)}{\hat{M}_\rho^2} \text{Re} \alpha_4(\hat{s}) \right\} \left( 1 + \frac{2\hat{m}_t^2}{\hat{s}} \right) \lambda(1, \hat{s}, \hat{M}_\rho^2) + 12\hat{m}_t^2|c_{10}|^2 \alpha_5(\hat{s}), \quad (4.11) \]
where \( \alpha_5(\hat{s}) \) can be found in Appendix [4]. The expression for the differential decay rate, Eq. (4.10), agrees with the result found by Geng and Kao [7] for \( \hat{m}_d = 0 \) but \( \hat{m}_t \neq 0 \), and with Greub et al. [9] in case of massless leptons, with the replacements \( d \to s \), and \( M_\rho \to M_{K^*} \) (see also Refs. [6, 13]). In Fig. [3], we plot the differential branching ratio as a function of \( \sqrt{s} \). Our results for the branching ratio for various values of \( \rho \) and \( \eta \) in the experimentally allowed domain, compared to the results for the inclusive decay \( \bar{B} \to X_d e^+ e^- \) [1], are summarized in Table [I].

**C. CP-violating asymmetry**

The \( CP \)-odd observable \( A_{CP} \), calculated in the channel \( \bar{B} \to \rho l^+ l^- \), is given by
\[ A_{CP}(\sqrt{s}) = \frac{-2\text{Im} \lambda_u \Delta_\rho}{\Sigma_\rho + 2\text{Im} \lambda_u \Delta_\rho} \left( 1 + \frac{2\hat{m}_t^2}{\hat{s}} \right) \lambda(1, \hat{s}, \hat{M}_\rho^2), \quad (4.12) \]
with \( \Sigma_\rho \) defined in Eq. (4.11), and
\[ \Delta_\rho = \text{Im} \xi^*_u \xi^*_d \left\{ 4\hat{s} \frac{|V(\hat{s})|^2}{(1 + \hat{M}_\rho^2)^2} + \frac{(1 + \hat{M}_\rho^2)^2}{\lambda(1, \hat{s}, \hat{M}_\rho^2)} \left[ 6\hat{s} + \frac{\lambda(1, \hat{s}, \hat{M}_\rho^2)}{2\hat{M}_\rho^2} \right] |A_1(\hat{s})|^2 \right. \]
\[ \left. + \frac{\lambda(1, \hat{s}, \hat{M}_\rho^2)}{2\hat{M}_\rho^2(1 + \hat{M}_\rho^2)^2} |A_2(\hat{s})|^2 - \frac{(1 - \hat{s} - \hat{M}_\rho^2)}{\hat{M}_\rho^2} \text{Re} A_1(\hat{s}) A_2^*(\hat{s}) \right\} \]
Table III: Branching ratio $\text{Br}(\bar{B} \to \rho e^+e^-)$ compared to $\text{Br}(\bar{B} \to X_d e^+e^-)$ for different values of $(\rho, \eta)$, excluding the region around the $J/\psi$ and $\psi'$ resonances ($\pm 20$ MeV). The labels “COL” and “MEL” denote the form factors of Refs. [6] and [11] respectively (see footnote 3).

| $(\rho, \eta)$ | $\text{Br}(\bar{B} \to \rho e^+e^-)$ | $\text{Br}(\bar{B} \to X_d e^+e^-)$ |
|----------------|-------------------------------------|-------------------------------------|
| $(0.3, 0.34)$  | MEL $2.4 \times 10^{-8}$            | COL $4.1 \times 10^{-8}$            |
|                |                                     | $2.7 \times 10^{-7}$               |
| $(-0.07, 0.34)$| MEL $5.0 \times 10^{-8}$            | COL $8.6 \times 10^{-8}$            |
|                |                                     | $5.5 \times 10^{-7}$               |
| $(-0.3, 0.34)$ | MEL $7.1 \times 10^{-8}$            | COL $1.2 \times 10^{-7}$            |
|                |                                     | $7.9 \times 10^{-7}$               |

As seen from Fig. 4 and Table IV, the result for the asymmetry $A_{CP}$ in $\bar{B} \to \rho e^+e^-$ differs considerably between the models of Refs. [6] and [11]. This difference can be traced to the very different behaviour of the form factor $T_3(q^2)$ in these models, especially the difference in sign (see Table 1 of [14]). The model of Stech [15] has a prediction for $T_3(q^2)$ qualitatively similar to that of [11]. It may also be noted that the inclusive asymmetry in the reaction $\bar{B} \to X_d e^+e^-$, calculated in [1], and reproduced in Fig. 5, has a resemblance to the exclusive asymmetry in $\bar{B} \to \rho e^+e^-$ shown in Fig. 4(b).
Table IV: Branching ratio \( Br(\bar{B} \to \rho e^+e^-) \) and average CP-violating asymmetry \( \langle A_{CP} \rangle \) for different values of \((\rho, \eta)\) in the region \( 1 \text{ GeV} < \sqrt{s} < (M_{J/\psi} - 20 \text{ MeV}) \). The labels “COL” and “MEL” denote the form factors of Refs. [6] and [11] respectively (see footnote 3).

| \((\rho, \eta)\) | \( Br(\bar{B} \to \rho e^+e^-) \) | \( \langle A_{CP} \rangle \) |
|------------------|----------------|------------------|
| \((0.3, 0.34)\) | MEL 0.9 \times 10^{-8} | -5.4 \times 10^{-2} |
| \(\approx 0\) | COL 1.3 \times 10^{-8} | |
| \((-0.07, 0.34)\) | MEL 1.7 \times 10^{-8} | -2.8 \times 10^{-2} |
| \(\approx 0\) | COL 2.6 \times 10^{-8} | |
| \((-0.3, 0.34)\) | MEL 2.4 \times 10^{-8} | -2.0 \times 10^{-2} |
| \(\approx 0\) | COL 3.7 \times 10^{-8} | |

V. CP VIOLATION AND FORWARD-BACKWARD ASYMMETRY

In the decay \( \bar{B} \to \pi l^+l^- \) the forward-backward asymmetry vanishes, since \( \hat{W}_3^{L-R} = 0 \) [cf. Eq. (2.19)], whereas for the \( \bar{B} \to \rho \) transition we find

\[
A_{FB}(\hat{s}) = 12\lambda^{1/2}(1, \hat{s}, \hat{M}_{\rho}^2) \sqrt{1 + \frac{4\hat{m}_l^2}{\hat{s}}} \frac{\text{Re} c_{10}}{\Sigma_{\rho}} \left\{ \hat{s}A_1^*(\hat{s})V(\hat{s})\text{Re} c_{10}^\text{eff} + 2c_{10}^\text{eff} \left[ T_2^*(\hat{s})V(\hat{s})(1 - \hat{M}_{\rho})(\hat{m}_b - \hat{m}_d) + A_1^*(\hat{s})T_1(\hat{s})(1 + \hat{M}_{\rho})(\hat{m}_b + \hat{m}_d) \right] \right\},
\]

(5.1)

with \( \Sigma_{\rho} \) defined in Eq. (4.11). Neglecting \( \hat{m}_l \) and \( \hat{m}_d \) in the above expression, we confirm the result of Ref. [9]. Our results for \( A_{FB} \) in the \( \bar{B} \to \rho \) channel are shown in Fig. 3, using the above-mentioned form factors. In addition, we plot in Fig. 4 the forward-backward asymmetry of the corresponding inclusive decay \( \bar{B} \to X_d e^+e^- \) by using the result derived in Ref. [16] for the \( b \to s \) analogue. Evaluation of the average forward-backward asymmetry in the exclusive and inclusive channels results in average values \( \langle A_{FB} \rangle_{\bar{B} \to \pi} = 0 \), \( \langle A_{FB} \rangle_{\bar{B} \to \rho} = -17\% \), and \( \langle A_{FB} \rangle_{b \to d} = -9\% \), where we have excluded the region around the \( J/\psi \) and \( \psi' \) resonances (±20 MeV). The numerical value for \( \langle A_{FB} \rangle_{\bar{B} \to \rho} \) differs very little between models [9] and [11].

Finally, we examine the CP-violating difference between \( A_{FB} \) and the corre-
sponding forward-backward asymmetry $A_{\text{FB}}$ in the antiparticle channel. The latter may be obtained by the replacement [see Eq. (2.13)]

$$c_9^{\text{eff}}(\lambda_u) \quad \longrightarrow \quad c_9^{\text{eff}} = c_9^{\text{eff}}(\lambda_u \rightarrow \lambda_u^*)$$

(5.2)

in Eqs. (4.11) and (5.1), which leads to

$$\delta_{\text{FB}} \equiv A_{\text{FB}} - A_{\text{FB}}^{\bar{c}}$$

$$= \frac{\eta}{[(1 - \rho)^2 + \eta^2]} \frac{24c_{10}^{\text{eff}}}{\Sigma_{\rho}(\Sigma_{\rho} + 4\text{Im} \lambda_u \Delta_{\rho})} \lambda^{1/2}(1, \hat{s}, \hat{\bar{M}}_{\rho}^2) \sqrt{1 - 4\hat{m}_t^2 \hat{s}}$$

$$\times \text{Re} \left\{ \hat{s} A_1^*(\hat{s}) V(\hat{s}) \Sigma_{\rho} \text{Im} \xi_2 - 2\Delta_{\rho} \left\{ \hat{s} A_1^*(\hat{s}) V(\hat{s}) \text{Re} c_9^{\text{eff}} \\
+ 2c_7^{\text{eff}} \left[ T_2^*(\hat{s}) V(\hat{s})(1 - \hat{M}_\rho)(\hat{m}_b - \hat{m}_d) + A_1^*(\hat{s}) T_1(\hat{s})(1 + \hat{M}_\rho)(\hat{m}_b + \hat{m}_d) \right] \right\} \right\},$$

(5.3)

where $\lambda_u, \xi_2, \Sigma_{\rho}$, and $\Delta_{\rho}$ are given in Eqs. (3.15), (3.17), (4.11), and (4.13) respectively. In Fig. 8 we plot the resulting difference in the forward-backward asymmetries for two sets of the Wolfenstein parameters $\rho$ and $\eta$, with $(\rho, \eta) = (-0.07, 0.2)$, and $(\rho, \eta) = (-0.07, 0.5)$.

VI. CONCLUSIONS

Flavour-changing neutral currents are a touchstone for weak interaction theories that make quantitative predictions for higher order effects (see, for example, Ref. [17]). In the case of the decays $b \rightarrow q \ell^+ \ell^-$ ($q = s, d$), the standard theory predicts a remarkable effective Hamiltonian, containing three coupling constants $c_7^{\text{eff}}, c_9^{\text{eff}}$ and $c_{10}^{\text{eff}}$ that are determined by the mass of the top quark. While the first of these is probed in the decay $b \rightarrow s \gamma$, the decays $b \rightarrow q \ell^+ \ell^-$ involve the magnitudes and relative signs of all three couplings. The reaction $b \rightarrow d \ell^+ \ell^-$ has the added piquancy of containing a large $CP$-violating phase, given by the argument of $V_{ub}V_{ud}^*/V_{tb}V_{td}^*$. In this paper, we have focussed on the $CP$-violating effects to be expected in the channels $\bar{B} \rightarrow \pi e^+e^-$ and $\bar{B} \rightarrow \rho e^+e^-$. Our results for the partial width asymmetry $A_{\text{CP}}$ are summarized in Tables II and IV, and in Figs. 2 and 4.
We have also calculated the forward-backward asymmetry $A_{FB}$ in the channel $\bar{B} \to \rho \ (\approx -17\%)$, and the $CP$-violating difference $\delta_{FB} \equiv A_{FB} - \bar{A}_{FB}$ between the meson and antimeson channels. This result is shown in Fig. 8. Together with our previous analysis of the inclusive decay $\bar{B} \to X_d l^+l^-$ [1], the present paper provides a complete profile of $CP$ violation in the sector $b \to d l^+l^-$ of the standard model. Our formalism is general enough to be applied to all reactions of the type $b \to q l^+l^- \ (q = s, d)$ induced by any effective Hamiltonian of the form (1.1)–(1.2).

Taking into account the typical branching ratio ($\sim 2 \times 10^{-8}$) and the typical $CP$-violating asymmetry ($\sim -5\%$), observation of $CP$ violation in the exclusive channels $\bar{B} \to \pi e^+e^-$ or $\bar{B} \to \rho e^+e^-$ will necessitate $\sim 10^{10}–10^{11}$ $B\bar{B}$ pairs, a challenging task that can only be contemplated at future hadron colliders. By the same token, an unexpectedly large asymmetry for these reactions would be a signal of new physics in the $b \to d e^+e^-$ sector, and a pointer to the existence of $CP$-violating sources outside the CKM matrix.

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APPENDIX A: INPUT PARAMETERS

\begin{align}
    m_b &= 4.8 \text{ GeV}, \quad m_c = 1.4 \text{ GeV}, \quad m_t = 176 \text{ GeV}, \quad m_u = m_d = 10 \text{ MeV} , \\
    M_B &= 5.27 \text{ GeV}, \quad M_\pi = 0.139 \text{ GeV}, \quad M_\rho = 0.768 \text{ GeV}, \quad M_\omega = 0.782 \text{ GeV} , \\
    M_W &= 80.2 \text{ GeV}, \quad \mu = m_b, \quad \sin^2 \theta_W = 0.23, \quad \Lambda_{QCD} = 225 \text{ MeV}, \quad \alpha = 1/129 , \\
    A &= 0.81, \quad \lambda = 0.2205, \quad \tau_B = 1.6 \times 10^{-12} \text{ s}, \quad m_e = 0.511 \text{ MeV} .
\end{align}

Further properties of the vector mesons can be found in Ref. [18].
APPENDIX B: $\bar{B} \to \pi^+ l^- \pi^-$ FORM FACTORS

Introducing the functions $\mathcal{D}_1$ and $\mathcal{D}_2$ via

$$\mathcal{D}_1 = c_9^{\text{eff}} F_1(\hat{s}) - 2 c_7^{\text{eff}} \tilde{F}_T(\hat{s}) - c_{10} F_1(\hat{s}) \; ,$$

and

$$\mathcal{D}_2 = c_9^{\text{eff}} \tilde{F}_1(\hat{s}) + 2 c_7^{\text{eff}} \tilde{F}_T(\hat{s}) - c_{10} \tilde{F}_1(\hat{s}) \; ,$$

where $F_1(\hat{s})$, $\tilde{F}_T(\hat{s})$, and $\tilde{F}_1(\hat{s})$ are defined through Eqs. (3.1), (3.2), (3.4), and (3.5), we obtain the following expressions for the form factors in $\bar{B} \to \pi^+ l^- l^-$

$$\hat{W}^{'LL}_1 = 0 \; ,$$

$$\hat{W}^{'LL}_2 = |\mathcal{D}_1|^2 \; ,$$

$$\hat{W}^{'LL}_3 = 0 \; ,$$

$$\hat{W}^{'LL}_4 = \frac{1}{4} \left[ |\mathcal{D}_1|^2 + A^2(\hat{s}) |\mathcal{D}_2|^2 - 2 A(\hat{s}) \text{Re}(\mathcal{D}_1^* \mathcal{D}_2) \right] \; ,$$

$$\hat{W}^{'LL}_5 = \frac{1}{2} \left[ A(\hat{s}) \text{Re}(\mathcal{D}_1^* \mathcal{D}_2) - |\mathcal{D}_1|^2 \right] \; ,$$

$$\hat{W}^{'RR}_i = \hat{W}^{'LL}_i \left( c_{10} \to -c_{10} \right) \; , \quad i = 1, \ldots, 5 \; ,$$

and, with the definition $\hat{W}^{'LL \pm R}_i \equiv \hat{W}^{'LL}_i \pm \hat{W}^{'RR}_i$, Eq. (2.6a), the relations

$$\hat{W}^{'+}_i = \hat{W}^{'L+R}_i \left( |c_{10}|^2 \to -|c_{10}|^2 \right) \; , \quad \hat{W}^{'-}_i = \hat{W}^{'L-R}_i \left( c_7^{\text{eff}} = 0, \text{Re} c_9^{\text{eff}} \to \text{Im} c_9^{\text{eff}} \right) \; .$$

The form factors $\hat{W}^{'LL}_i$ can be related to the ones found in Ref. [19] for the semileptonic decay $B \to D l \nu_l$, setting

$$c_7^{\text{eff}} = 0, \quad c_9^{\text{eff}} = -c_{10} = \frac{1}{2} \; ,$$

in Eqs. (B1) and (B2), and using Eqs. (3.5), (3.6), (3.10a), (3.10b).

APPENDIX C: $\bar{B} \to \rho^+ l^- \rho^-$ FORM FACTORS

In calculating the form factors $\hat{W}_i(\hat{s}, y)$ for the decay $B \to \rho^+ l^- l^-$ it is useful to introduce the notation

$$\mathcal{D}_3 = I_1(\hat{s}) - c_{10} \frac{V(\hat{s})}{1 + M_\rho^2} \; ,$$

(C1)
\[
\begin{align*}
\mathcal{D}_4 &= \frac{1}{2}(1 + \hat{M}_\rho) \left[ I_2(\hat{s}) - c_{10} A_1(\hat{s}) \right], \\
\mathcal{D}_5 &= -\frac{1}{2(1 + \hat{M}_\rho)} \left[ I_3(\hat{s}) - c_{10} A_2(\hat{s}) \right], \\
\mathcal{D}_6 &= -\frac{1}{\hat{s}} \left\{ I_4(\hat{s}) - c_{10} [A_3(\hat{s}) - A_0(\hat{s})]\hat{M}_\rho \right\},
\end{align*}
\]

with

\[
\begin{align*}
I_1(\hat{s}) &= c_9^{\text{eff}} \frac{V(\hat{s})}{1 + \hat{M}_\rho} + \frac{4c_7^{\text{eff}}}{\hat{s}} T_1(\hat{s})(\hat{m}_b + \hat{m}_d), \\
I_2(\hat{s}) &= c_9^{\text{eff}} A_1(\hat{s}) + \frac{4c_7^{\text{eff}}}{\hat{s}} (1 - \hat{M}_\rho) T_2(\hat{s})(\hat{m}_b - \hat{m}_d), \\
I_3(\hat{s}) &= c_9^{\text{eff}} A_2(\hat{s}) + \frac{4c_7^{\text{eff}}}{\hat{s}} \left[ (1 + \hat{M}_\rho) T_2(\hat{s}) + \frac{\hat{s}}{1 - \hat{M}_\rho} T_3(\hat{s}) \right] (\hat{m}_b - \hat{m}_d), \\
I_4(\hat{s}) &= c_9^{\text{eff}} [A_3(\hat{s}) - A_0(\hat{s})] \hat{M}_\rho - 2c_7^{\text{eff}} T_3(\hat{s})(\hat{m}_b - \hat{m}_d),
\end{align*}
\]

so that

\[
\begin{align*}
\hat{W}_1^{LL} &= \frac{1}{4} |\mathcal{D}_3|^2 (y^2 - 4\hat{s}) + |\mathcal{D}_4|^2, \\
\hat{W}_2^{LL} &= -|\mathcal{D}_3|^2 \hat{s} - 4|\mathcal{D}_5|^2 \left[ 1 - \frac{(2 - y)^2}{4M_\rho^2} \right] + \frac{|\mathcal{D}_4|^2}{M_\rho^2} - 4\text{Re} (\mathcal{D}_4 \mathcal{D}_5^\ast) \left( 1 - \frac{2 - y}{2M_\rho^2} \right), \\
\hat{W}_3^{LL} &= 2 \text{Re} (\mathcal{D}_4^\ast \mathcal{D}_4), \\
\hat{W}_4^{LL} &= -|\mathcal{D}_3|^2 - |\mathcal{D}_5|^2 - |\mathcal{D}_6|^2 - 2\text{Re} (\mathcal{D}_5 \mathcal{D}_6^\ast) \left[ 1 - \frac{(2 - y)^2}{4M_\rho^2} \right] + \frac{|\mathcal{D}_4|^2}{M_\rho^2} \\
&\quad + \text{Re} \mathcal{D}_4 (\mathcal{D}_5^\ast - \mathcal{D}_6^\ast) \frac{(2 - y)}{M_\rho^2}, \\
\hat{W}_5^{LL} &= \frac{1}{2} y |\mathcal{D}_3|^2 + 2 \left[ |\mathcal{D}_5|^2 - \text{Re} (\mathcal{D}_5 \mathcal{D}_6^\ast) \right] \left[ 1 - \frac{(2 - y)^2}{4M_\rho^2} \right] - \frac{|\mathcal{D}_4|^2}{M_\rho^2} \\
&\quad + \text{Re} \mathcal{D}_4 \left\{ \left[ 1 - \frac{3(2 - y)}{2M_\rho^2} \right] \mathcal{D}_5^\ast - \left[ 1 - \frac{2 - y}{2M_\rho^2} \right] \mathcal{D}_6^\ast \right\}.
\end{align*}
\]

As before, the remaining form factors \( \hat{W}_i^{RR} \), and \( \hat{W}_i^\pm \), \( i = 1, \ldots, 5 \), can be obtained by means of Eqs. (B8) and (B9). Using Eq. (B10), we reproduce the results derived by Boyd et al. [19] for the decay \( B \to D^* \nu \).
APPENDIX D: AUXILIARY FUNCTIONS

\[ \alpha_1(\hat{s}) = |I_1(\hat{s})|^2 + |c_{10} \frac{V(\hat{s})}{1 + \hat{M}_\rho}|^2, \]  
(D1)

\[ \alpha_2(\hat{s}) = |I_2(\hat{s})|^2 + |c_{10} A_1(\hat{s})|^2, \]  
(D2)

\[ \alpha_3(\hat{s}) = |I_3(\hat{s})|^2 + |c_{10} A_2(\hat{s})|^2, \]  
(D3)

\[ \alpha_4(\hat{s}) = I_2(\hat{s}) I_3(\hat{s}) + |c_{10}|^2 A_1(\hat{s}) A_2^*(\hat{s}), \]  
(D4)

\[ \alpha_5(\hat{s}) = -2 \left\{ \frac{|V(\hat{s})|^2}{(1 + \hat{M}_\rho)^2} \lambda(1, \hat{s}, \hat{M}_\rho^2) - 3(1 + \hat{M}_\rho)^2 |A_1(\hat{s})|^2 \right. 
\]
\[ + \frac{|A_2(\hat{s})|^2}{4 M^2(1 + \hat{M}_\rho)^2} \left[ 2(1 + \hat{M}_\rho^2) - \hat{s} \right] \lambda(1, \hat{s}, \hat{M}_\rho^2) + \frac{|A_3(\hat{s}) - A_0(\hat{s})|^2}{\hat{s}} \lambda(1, \hat{s}, \hat{M}_\rho^2) \]
\[ + \frac{1}{2 M^2} \left\{ \frac{2 \hat{M}_\rho}{\hat{s}} \text{Re} \left[ A_2(\hat{s})(1 - \hat{M}_\rho) - A_1(\hat{s})(1 + \hat{M}_\rho) \right] A_3^*(\hat{s}) - A_0^*(\hat{s}) \right\} \lambda(1, \hat{s}, \hat{M}_\rho^2), \]  
(D5)

where \( I_1, \ldots, I_3 \) have been given in the preceding Appendix, Eqs. (C5)–(C7).

APPENDIX E: \( \bar{B} \to X_{s,d} t^+ l^- \) FORM FACTORS

The differential decay rate for the inclusive reaction \( \bar{B} \to X_q t^+ l^- \), \( q = s \) or \( d \), may be written as

\[ \frac{d\Gamma(\bar{B} \to X_q t^+ l^-)}{d\hat{s} dy d\cos \theta} = \frac{G_F^2 m_b^5 \alpha^2}{210 \pi^6} |V_{tb} V_{tq}^*|^2 \lambda^{1/2}(1, \hat{s}, \hat{m}_q^2) \sqrt{1 - \frac{4 \hat{m}_q^2}{\hat{s}}} \sum_{s_+, s_-} \text{Im} \hat{T}(s_+, s_-; \hat{s}, y, \cos \theta), \]  
(E1)

where the expression for \( \hat{T} \) is given by the \{· · ·\} term in Eq. (2.2), with the replacement \( \hat{W}_i \to \hat{T}_i \), and the scaled variables

\[ \hat{p}_i = \frac{p_i}{m_b}, \quad \hat{m}_i = \frac{m_i}{m_b}, \quad \hat{s} \equiv \hat{q}^2 = (\hat{p}_+ + \hat{p}_-)^2, \quad \nu^\mu \equiv \hat{p}_b^\mu, \quad y = 2 \nu \cdot \hat{q}. \]  
(E2)
The form factors $\hat{T}_i, i = 1, \ldots, 5$, are defined through the time-ordered product

$$\hat{T}_{\mu\nu}^{LL} \equiv i \int d^4x \ e^{-iq\cdot x}\langle \overline{B}|T\left\{[G(x) - H(x)]_{\mu\nu}^\dagger, [G(0) - H(0)]_{\nu}\right\}|\overline{B}\rangle$$

$$= -g_{\mu\nu}\hat{T}_1^{LL} + v_\mu v_\nu \hat{T}_2^{LL} - i\epsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha\hat{q}^\beta\hat{T}_3^{LL} + \hat{q}_\mu \hat{q}_\nu \hat{T}_4^{LL} + (\hat{q}_\mu v_\nu + \hat{q}_\nu v_\mu)\hat{T}_5^{LL} ,$$

(E3)

$$\hat{T}_{\mu\nu}^{RR} = \hat{T}_{\mu\nu}^{LL}(H \rightarrow -H, \hat{T}_i^{LL} \rightarrow \hat{T}_i^{RR}) ,$$

(E4)

$$\hat{T}_{\mu\nu}^{LR} \equiv i \int d^4x \ e^{-iq\cdot x}\langle \overline{B}|T\left\{[G(x) - H(x)]_{\mu\nu}^\dagger, [G(0) + H(0)]_{\nu}\right\}|\overline{B}\rangle$$

$$= -g_{\mu\nu}\hat{T}_1^{LR} + v_\mu v_\nu \hat{T}_2^{LR} - i\epsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha\hat{q}^\beta\hat{T}_3^{LR} + \hat{q}_\mu \hat{q}_\nu \hat{T}_4^{LR} + (\hat{q}_\mu v_\nu + \hat{q}_\nu v_\mu)\hat{T}_5^{LR} ,$$

(E5)

$$\hat{T}_{\mu\nu}^{RL} = \hat{T}_{\mu\nu}^{LR}(H \rightarrow -H, \hat{T}_i^{LR} \rightarrow \hat{T}_i^{RL}) ,$$

(E6)

where $G$ and $H$ have already been defined in Eqs. (2.11) and (2.12) respectively.

Defining

$$\hat{T}_i^{L\pm R} \equiv \hat{T}_i^{LL} \pm \hat{T}_i^{RR} ,$$

(E7)

so that

$$\hat{T}_i^+ = \hat{T}_i^{L+R}(\left|c_{10}\right|^2 \rightarrow -\left|c_{10}\right|^2) , \quad \hat{T}_i^- = \hat{T}_i^{L-R}(c_7^{\text{eff}} = 0, \Re c_9^{\text{eff}} \rightarrow i\Im c_9^{\text{eff}}) ,$$

(E8)

we obtain the form factors of the parton model reaction $b \rightarrow q l^+ l^-$

$$\hat{T}_1^{L+R} = \frac{1}{(y - y_0 - i\epsilon)} \left\{ -\frac{4|c_7^{\text{eff}}|^2}{s^2} \left[ 6\hat{m}_q^2 \hat{s} + 2\hat{s} - y(1 + \hat{m}_q^2)(y - \hat{s}) \right] 
- 4\Re (c_7^{\text{eff}} c_9^{\text{eff}}) \left[ 2 - y(1 - \hat{m}_q^2)\frac{1}{\hat{s}} \right] + (|c_9^{\text{eff}}|^2 + |c_{10}|^2)(2 - y) \right\} ,$$

(E9)

$$\hat{T}_1^{L-R} = \frac{1}{(y - y_0 - i\epsilon)} \left\{ 4c_7^{\text{eff}} c_{10} \left[ 2 - y(1 - \hat{m}_q^2)\frac{1}{\hat{s}} \right] - 2\Re (c_9^{\text{eff}} c_{10})(2 - y) \right\} ,$$

(E10)

$$\hat{T}_2^{L+R} = \frac{1}{(y - y_0 - i\epsilon)} \left\{ -16|c_7^{\text{eff}}|^2 \left( 1 + \hat{m}_q^2 \right)\frac{1}{\hat{s}} + 4(|c_9^{\text{eff}}|^2 + |c_{10}|^2) \right\} ,$$

(E11)

\(^4\)It should be noted that the time-ordered product can be expanded in powers of $1/m_b$, using methods described in Ref. [20].
\[ \hat{T}_{2}^{L-R} = \frac{1}{(y-y_0 - i\epsilon)} \left\{ -8\text{Re} \left( c_5 \hat{c}_{10} \right) \right\}, \quad (E12) \]

\[ \hat{T}_{3}^{L+R} = \frac{1}{(y-y_0 - i\epsilon)} \left\{ \frac{8|c_7|^2}{s^2} (y-\hat{s})(1-\hat{m}_q^2) + 8\text{Re} \left( c_7 c_9 \hat{c}_{10} \right) \frac{1}{s} \right\}, \quad (E13) \]

\[ \hat{T}_{3}^{L-R} = \frac{1}{(y-y_0 - i\epsilon)} \left\{ -8c_7 c_9(1 + \hat{m}_q^2) \frac{1}{s} - 4\text{Re} \left( c_7 c_9 \hat{c}_{10} \right) \right\}, \quad (E14) \]

\[ \hat{T}_{4}^{L+R} = \frac{1}{(y-y_0 - i\epsilon)} \left\{ \frac{4|c_7|^2}{s^2} \left[ 2(1 + 3\hat{m}_q^2) + y(1 + \hat{m}_q^2) \right] - 8\text{Re} \left( c_7 c_9 \hat{c}_{10} \right) \frac{1}{s} \right\}, \quad (E15) \]

\[ \hat{T}_{5}^{L+R} = \frac{1}{(y-y_0 - i\epsilon)} \left\{ \frac{8|c_7|^2}{s^2} y(1 + \hat{m}_q^2) + 4\text{Re} \left( c_7 c_9 \hat{c}_{10} \right) \frac{1}{s} - 2(|c_9|^2 + |c_{10}|^2) \right\}, \quad (E16) \]

\[ \hat{T}_{5}^{L-R} = \frac{1}{(y-y_0 - i\epsilon)} \left\{ -4c_7 c_9(1 - \hat{m}_q^2) \frac{1}{s} + 4\text{Re} \left( c_7 c_9 \hat{c}_{10} \right) \right\}, \quad (E17) \]

with \( y_0 = 1 + \hat{s} - \hat{m}_q^2 \). The imaginary part \( \text{Im} \hat{T}_i(\hat{s}, y) \) is then obtained by the formal replacement

\[ \frac{1}{y-y_0 - i\epsilon} \longrightarrow \pi\delta(y-y_0). \quad (E18) \]
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FIGURE CAPTIONS

Figure 1 Differential branching ratio as a function of $\sqrt{\hat{s}}$, $\hat{s} \equiv q^2/M_B^2$, for the decay $B \to \pi e^+ e^-$ using the form factors of Ref. [6] (a) and Ref. [11] (b), including $\rho$, $\omega$, and $J/\psi$, $\psi'$ etc. resonances (solid curve), and choosing the Wolfenstein parameters to be $(\rho, \eta) = (-0.07, 0.34)$. The dashed line corresponds to the nonresonant invariant mass spectrum.

Figure 2 CP-violating partial width asymmetry in the decays $\bar{B} \to \pi e^+ e^-$ and $B \to \pi e^+ e^-$ as a function of $\sqrt{s}$ for $(\rho, \eta) = (-0.07, 0.34)$, including $\rho$, $\omega$, and $J/\psi$ resonances. Although we use form factors from two different models (Refs. [6] and [11]), the distributions are indistinguishable.

Figure 3 Differential branching ratio vs $\sqrt{\hat{s}}$ for the $B \to \rho e^+ e^-$ transition, using form factors of Colangelo et al. [6] (a) and Melikhov and Nikitin [11] (b) (see footnote 3). The dashed line represents the nonresonant invariant mass spectrum, whereas the solid line corresponds to the mass spectrum including the effects of $\rho$, $\omega$, and $J/\psi$ resonances. The Wolfenstein parameters are chosen to be $(\rho, \eta) = (-0.07, 0.34)$.

Figure 4 CP-violating partial width asymmetry in the exclusive channels $\bar{B} \to \rho e^+ e^-$ and $B \to \bar{\rho} e^+ e^-$ for $(\rho, \eta) = (-0.07, 0.34)$, using the two form factor models as in Fig. 3.

Figure 5 CP-violating partial width asymmetry in the inclusive decays $B \to X_d e^+ e^-$ and $B \to X_{\bar{d}} e^+ e^-$ for $(\rho, \eta) = (-0.07, 0.34)$.

Figure 6 Forward-backward asymmetry of $e^-$ in the $e^+ e^-$ centre-of-mass system in the decay $\bar{B} \to \rho e^+ e^-$ as a function of $\sqrt{s}$, including the effects of resonances (solid curve). Diagrams (a) and (b) correspond to two different form factor models, as described in the text. We also show, for comparison, the nonresonant distribution (dashed line).

Figure 7 Forward-backward asymmetry $A_{FB}$ vs $\sqrt{s}$ in the inclusive decay $\bar{B} \to X_d e^+ e^-$. The dashed line represents the nonresonant spectrum.
Figure 8 The $CP$-violating difference $\delta_{FB} \equiv A_{FB} - \bar{A}_{FB}$ for $(\rho, \eta) = (-0.07, 0.5)$ (solid line), and $(\rho, \eta) = (-0.07, 0.2)$ (dashed line), as a function of $\sqrt{s}$, neglecting the effects of resonances. Figs. (a) and (b) correspond to the form factors of Colangelo et al. [6] and Melikhov and Nikitin [11] respectively.
Fig. 1:
Fig. 2:
\[ \frac{d\text{Br}}{d\sqrt{s}} \]

(a) COL

\[ B \rightarrow \rho \]

(b) MEL

\[ B \rightarrow \rho \]

FIG. 3:
$A_{CP}$

(a) COL

$B \to \rho$

(b) MEL

$B \to \rho$

FIG. 4:
Fig. 5:
FIG. 6:
FIG. 8:

(a) COL

$B \rightarrow \rho$

(b) MEL

$B \rightarrow \rho$