Fuzzy pid control of electromechanical actuator system

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Abstract. In order to meet the needs of modern high-performance aircraft, the fuzzy PID control strategy was used to design the electromechanical actuator control system. This control strategy combines the advantages of fuzzy control and PID control, and has better adaptability to time-varying complex systems. The electromechanical actuator control system adopted the three-loop structure, and current loop, speed loop and position loop were built on the platform of MATLAB / Simulink. Considering that the discrete universe fuzzy PID control algorithm has good real-time performance with small online computation, the fuzzy PID controller in discrete universe was designed in the position loop and the system was simulated. The simulation results show that the dynamic and steady performance of discrete universe fuzzy PID control is better than that of traditional PID control, and it is more suitable for electromechanical actuator system.

1. Introduction

As the terminal actuator, the electromechanical actuator can convert the command signal into the rudder angle quickly and accurately [1], which directly determines the actual performance of the flight control system.

The traditional PID controller has fixed parameters and is easy to implement. It is widely used in the electromechanical actuator control system [2]. However, the electromechanical actuator system is a multivariable, nonlinear and time-varying complex system. Traditional PID control can not meet the requirements of high precision. Sliding mode control has the advantages of wide application and fast response. However, chattering is difficult to be eliminated in the process of operation, and measures must be taken to reduce it to a tolerable range [3-5]. Internal model control (IMC) has less on-line adjustment parameters and good tracking performance, but it is limited for complex systems with uncertainties and nonlinearity [6]. Active disturbance rejection control (ADRC) can suppress the external disturbance in the process of system operation [7], but it is difficult to adjust so many parameters.

Fuzzy PID control is based on fuzzy theory [8]. According to the actual response, the optimal adjustment of PID parameters is realized automatically by fuzzy reasoning. Fuzzy PID control has better adaptability to time-varying complex system [9,10], and is widely used.
in servo systems [11-14].

2. Model of electromechanical actuator system

2.1. Mathematical Model OF Electromechanical Actuator

In this paper, brushless DC motor was used as the servo motor of the electromechanical actuator studied. It is assumed that the armature conductor of brushless DC motor is continuously and evenly distributed on the armature surface, without considering the hysteresis loss and eddy current loss. The armature reaction ignored, the three-phase winding is symmetrically distributed [15].

The equivalent circuit of brushless DC motor can be illustrated as figure 1.

![Equivalent circuit of brushless DC motor.](image)

The motion equation of the electromechanical actuator is expressed as follows [16,17].

**Phase voltage balance equation:**

\[
\begin{bmatrix}
  \frac{du_a}{dt} \\
  \frac{du_b}{dt} \\
  \frac{du_c}{dt}
\end{bmatrix} =
\begin{bmatrix}
  R_a & 0 & 0 \\
  0 & R_b & 0 \\
  0 & 0 & R_c
\end{bmatrix}
\begin{bmatrix}
  i_a \\
  i_b \\
  i_c
\end{bmatrix}
+ \begin{bmatrix}
  L - M & 0 & 0 \\
  0 & L - M & 0 \\
  0 & 0 & L - M
\end{bmatrix}
\begin{bmatrix}
  \frac{di_a}{dt} \\
  \frac{di_b}{dt} \\
  \frac{di_c}{dt}
\end{bmatrix}
+ \begin{bmatrix}
  e_a \\
  e_b \\
  e_c
\end{bmatrix}
\] (1)

Where, \( u_a, u_b, u_c \) is respectively the phase voltage (V), \( R_a, R_b, R_c \) respectively represents the resistance of three-phase stator windings (Ω), \( L, M \) respectively represents self-inductance and mutual inductance (H). \( i_a, i_b, i_c \) is the phase current \( A \), \( e_a, e_b, e_c \) is the phase back EMF voltage \( V \).

**Electromagnetic torque equation:**

\[
T_e = \left( e_a i_a + e_b i_b + e_c i_c \right) / \omega
\] (1.2)

Where, \( T_e \) is the electromagnetic torque generated by armature current (N m), \( \omega \) is the motor speed (rad/s).

**Torque balance equation:**

\[
T_e - T_L = J \frac{d\omega}{dt} + B_i \omega
\] (1.3)

Where, \( T_L \) is the load torque converted to motor shaft (N m), \( J \) is the total moment of inertia.
converted to motor shaft \((kg \cdot m^2)\), \(B_v\) is the equivalent viscous friction coefficient of the system \((N \cdot m / (rad \cdot s^{-1}))\).

### 2.2. Position Servo System Model of Electromechanical Actuator

The position servo control system of electromechanical actuator adopted three loop control strategy, which built current loop, speed loop and position loop in turn. The position controller adopted PID controller integrated with intelligent algorithm, and the current loop and speed loop adopted PI controller to guarantee the steady-state accuracy and dynamic tracking performance of the system. The model of position servo control system is shown in figure 2.

![Figure 2. Servo control system model of electromechanical actuator.](image)

### 3. Design of fuzzy pid system

Fuzzy PID control algorithm combines the characteristics of fuzzy control algorithm and PID control algorithm, which has several advantages such as simple structure and strong adaptability. As is shown in figure 3, the structure of fuzzy PID controller is composed of fuzzy controller and PID regulator. The fuzzy controller takes the error \(e\) as the input and the output are the correction values of PID parameters, that is, \(\Delta K_p, \Delta K_i, \Delta K_d\). In the figure, \(k_e, k_v\) are the quantitative factors that transform the actual universe into the fuzzy universe. \(K_{up}, K_{ui}, K_{ud}\) are the proportion factors that transform the fuzzy universe into the actual universe.
The fuzzy relationship between the three PID parameters and the pair of $e$ and $ec$ should be found out. Based on this condition, the fuzzy control principle would be utilized to modify the PID parameters online. The expression of fuzzy tuning PID controller is as follows.

$$
\begin{align*}
\Delta K_p &= K_{p0} + \Delta K_p \\
\Delta K_i &= K_{i0} + \Delta K_i \\
\Delta K_d &= K_{d0} + \Delta K_d
\end{align*}
$$

(2.1)

Where, $K_{p0}, K_{i0}, K_{d0}$ are the optimal PID parameters adjusted by traditional experience method. The fuzzy PID control algorithm was applied to the position loop of the electromechanical actuator servo system to track the position signal.

### 3.1. Input Variables and Output Variables

For the fuzzy PID controller in the position loop of electromechanical actuator servo system, the input variables were set as the system error between the given rudder angle and the actual rudder angle and the system error change rate. Meanwhile, the three correction values of PID controller parameters were taken as the output variables.

### 3.2. Membership Function and Fuzzy Rules

The fuzzy subset of input and output language values was set as \{NB,NM,NS,ZO,PS,PM,PB\}. The membership functions were selected as Z-shaped membership function (corresponding to NB), triangular membership functions and S-shape membership function (corresponding to PB).

The fuzzy control rule table was established with the consideration of the dynamic response and steady-state error requirements of the system. Table 1 is the fuzzy rule table corresponding to $\Delta K_p$.

| $e$ | $ec$ |
|-----|------|
| NB  | NB   |
| NM  | NM   |
| NS  | NS   |
| ZO  | ZO   |
| PS  | PS   |
| PM  | PM   |
| PB  | PB   |
| NB  | PB   |
| NM  | PB   |

Table 1. The fuzzy rule table corresponding to $\Delta K_p$. 
3.3. Design of Fuzzy PID Control System in Discrete Universe

When the universe is discrete, the number of quantized inputs is limited. According to the different combinations of input conditions, the corresponding control variables can be calculated off-line to form a control table. This control table can be called directly in the actual control with small amount of online calculation. This kind of off-line calculation and on-line look-up table fuzzy control method is easy to meet the requirements of real-time control [18].

Figure 4. Structure diagram of discrete universe fuzzy PID controller

\( k_e, k_{ec} \) were the quantification factors. Suppose that the actual variation ranges of language variables \( e, de / dt \) were \([-a, a]\) and \([-b, b]\) respectively and the fuzzy domains were \(\{-n_j, \cdots, -1, 0, 1, \cdots, n_i\} \quad (i = 1, 2)\), then the quantification factors can be as follows.

\[
k_e = n_1 / a
\]

\[
k_{ec} = n_2 / b
\]

\( K_{up}, K_{ai}, K_{ud} \) were the proportion factors. Suppose that the actual change ranges of \( K_{up}, K_{ai}, K_{ud} \) were respectively \([-c, c],[-d, d],[-f, f]\) and the fuzzy domains were \(\{-n_j, \cdots, -1,0,1, \cdots, n_i\} \quad (i = 3,4,5)\), then the proportion factors can be as follows.
The fuzzy universe of system error $e$ was taken as $[-3, 3]$. The fuzzy universe of error change rate $e_{c}$ was $[-3, 3]$. In addition, the fuzzy universe of output variable $\Delta K_{p}$ was $[-0.3, 0.3]$, the fuzzy universe of output variable $\Delta K_{i}$ was $[-0.06, 0.06]$, and the fuzzy universe of output variable $\Delta K_{d}$ was $[-3, 3]$.

Let the known inputs be $X_{0}$ and $Y_{0}$, and the corresponding fuzzy sets were $A^{'1}$ and $B^{'1}$. Let the outputs be $Z_{10}, Z_{20}, Z_{30}$, and the corresponding fuzzy sets were $C_{1}^{'}C_{2}^{'}, C_{3}^{'}$. Taking $C_{1}^{'}$ as an example, let the fuzzy control rules be $R_{i}^{'}(i = 1, 2, \cdots, 49)$.

$R_{1}^{'}$: if $x$ is NB and $y$ is NB, then $z_{1}^{'}$ is PB. 
also $R_{2}^{'}$: if $x$ is NB and $y$ is NM, then $z_{1}^{'}$ is PB.

also $R_{49}^{'}$: if $x$ is PB and $y$ is PB, then $z_{1}^{'}$ is NB.

Fuzzy set is described by its membership functions. The fuzzy operation adopts single point fuzzy set, that is

$$
\mu_{A}^{'}(x) = \begin{cases} 1 & (x = x_{0}) \\ 0 & (x \neq x_{0}) \end{cases}
$$

(2.5)

$$
\mu_{B}^{'}(y) = \begin{cases} 1 & (y = y_{0}) \\ 0 & (y \neq y_{0}) \end{cases}
$$

(2.6)

Take $C_{1}^{'}$ for example. According to the theory of fuzzy logic, if the operation of ‘and’ adopted intersection method, the operation of ‘also’ also used union method, the operation of ‘synthesis’ employed maximum-minimum method, and fuzzy implication utilized minimum operation, then

$$
C_{1}^{'} = (A^{'} \times B^{'}) \circ R = (A^{'} \times B^{'}) \circ \bigcup_{i=1}^{49} R_{i}^{'} = \bigcup_{i=1}^{49} (A_{i}^{'} \times B_{i}^{'}) \circ \left[ (A_{i} \times B_{i}) \rightarrow C_{1i}^{'} \right] 
$$

$$
= \bigcup_{i=1}^{49} \left[ A_{i}^{'} \rightarrow C_{1i}^{'} \right] \cap \left[ B^{' \prime} \circ (B_{i} \rightarrow C_{1i}) \right] = \bigcup_{i=1}^{49} C_{1i}^{'} \cap C_{1i}^{'} = \bigcup_{i=1}^{49} C_{1i}^{'}
$$

The maximum membership degree method and the centroid method can be used to defuzzify $C_{1}^{'}$. The platform of System Test (Simulink Test) and fuzzy logic toolbox in MATLAB can be employed to realize the fuzzy reasoning test in discrete universe, obtaining the fuzzy control table. According to the fuzzy control rules in Table.1, the following table was the fuzzy control table in discrete universe corresponding to $Z_{10}$. 

\[
K_{up} = \frac{c}{n_3} \\
K_{ui} = \frac{d}{n_4} \\
K_{ud} = \frac{f^{'}}{n_5}
\]
Table 2. The fuzzy rule table in discrete universe corresponding to $Z_{10}$.

| $x_0$ | $y'_0$ |
|-------|--------|
| -3 | 0.27 | 0.27 | 0.20 | 0.20 | 0.10 | 0.00 | 0.00 |
| -2 | 0.27 | 0.27 | 0.20 | 0.10 | 0.10 | 0.00 | -0.10 |
| -1 | 0.20 | 0.20 | 0.20 | 0.10 | 0.00 | -0.10 | -0.10 |
| 0  | 0.20 | 0.20 | 0.10 | 0.00 | -0.10 | -0.20 | -0.20 |
| 1  | 0.10 | 0.10 | 0.00 | -0.10 | -0.10 | -0.20 | -0.20 |
| 2  | 0.10 | 0.00 | -0.10 | -0.20 | -0.20 | -0.27 | -0.27 |
| 3  | 0.00 | 0.00 | -0.20 | -0.20 | -0.20 | -0.20 | -0.27 |

4. Simulation experiment and analysis

According to the structure of the servo control system and the above control laws, the position control of the electromechanical actuator was simulated in MATLAB environment. The control effects of discrete universe fuzzy PID control strategy and classical PID control strategy on position signal tracking of servo system were to be compared with each other, and the superiority of the control strategy designed in this paper would be verified.

The specific parameters of the electromechanical actuator system are as follows. Equivalent moment of inertia $J = 4 \times 10^{-5} \text{kg} \cdot \text{m}^2$, stator phase inductance $L_s = 0.02825 \text{mH}$, stator phase resistance $R_s = 0.143 \Omega$, torque coefficient $K_T = 0.0263N \cdot \text{m} / \text{A}$, back EMF coefficient $K_e = 2.7541 \times 10^{-3} \text{V} / \text{rpm}$. The selected motor is a certain type of brushless DC motor of MAXON company, whose rated voltage is $24 \text{V}$ and rated torque is $311 \text{mN} \cdot \text{m}$.

4.1. System Indexes and Simulation Condition

The indexes of servo control system of electromechanical actuator were as follows.

1) Maximum output angle $10^\circ$;
2) Maximum response frequency $10 \text{ Hz}$;
3) Maximum load torque $2.4N \cdot \text{m} \left(10^5\right)$.

The sinusoidal signal and step signal taken as the given position signal respectively, the simulation was carried out under load. The load was flexible, and the load torque was proportional to the rudder deflection angle.

4.2. Simulation results of maximum rudder deflection response

The input signal frequency was $1 \text{ Hz}$ and the amplitude was $10$. Under the condition of load, the traditional PID control law and fuzzy PID control law were respectively applied to the position loop of the servo system of the electromechanical actuator. After that, the dynamic and static performance of the system was recorded.
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Figure 5. Comparison graph of maximum rudder deflection response under load.

Table 3. Simulation data of maximum rudder deflection response

|                         | Traditional PID | Fuzzy PID |
|-------------------------|-----------------|-----------|
| Maximum amplitude error (°) | 0.350           | 0.0900    |
| Maximum phase error (°)   | 12.6            | 5.40      |

The input signal was sinusoidal signal, whose frequency was 1 Hz and amplitude was 10. According to figure 5 and table 3, the maximum phase error and maximum amplitude error of the system under traditional PID control exceeded those under Fuzzy PID control. The maximum amplitude error of the system under Fuzzy PID control was 74.29% lower than that in traditional PID control system. The maximum phase error of the system under Fuzzy PID control law was reduced by 57.14% compared with the traditional PID control system.

4.3. Simulation Results of Step Response Characteristics

Under the condition of load, the step signal with amplitude of 50% of the maximum deflection angle was input, that is, the step signal with amplitude of 5. The traditional PID control law and fuzzy PID control law were respectively applied to the position loop of the servo system of the electromechanical actuator. Then, the dynamic and static performance of the system was recorded. The simulation results were shown in figure 6 and table 4.
Figure 6. Comparison graph of system step response under load.

Table 4. Simulation data of step response under load.

|                      | Traditional PID | Fuzzy PID |
|----------------------|-----------------|-----------|
| Rise time/ms         | 57.4            | 28.8      |
| Settling time/ms     | 110.2           | 58.1      |
| Steady-state error/(°)| 0.0340          | 0.0210    |

It can be seen from figure 6 and table 4 that the rise time of the system controlled by fuzzy PID controller was about 49.83% shorter than that of classical PID control under step response with load. Under the step response with load, the regulating time of fuzzy PID controller was 47.28% shorter than that of classical PID control. The steady-state error of the system under Fuzzy PID control was reduced by 38.24% compared with the classical PID control.

4.4. Simulation Results of Frequency Response Characteristics

Under the condition of load, the input signal amplitude was 1/10 of the maximum deflection angle, that is, the frequency was 10Hz and the amplitude was 1. The traditional PID control law and the fuzzy PID control law were applied to the position loop of the servo system respectively, and the maximum amplitude error and maximum phase error of the system under the stable state were recorded. The simulation results were shown in figure 7 and table 5.
According to figure 7 and table 5, the maximum phase error and maximum amplitude error of the system under traditional PID control surpassed those under Fuzzy PID control. The maximum amplitude error of the system under Fuzzy PID control was 57.14% lower than that in traditional PID control system. The maximum phase error of the system under Fuzzy PID control law was reduced by 88.89% compared with the traditional PID control system.

5. Conclusion

Based on the model of electromechanical actuator position servo system, the traditional position loop PID control algorithm was improved by using fuzzy PID control algorithm. The fact that the discrete universe fuzzy PID control algorithm has small online computation and good real-time performance taken into consideration, the traditional PID controller and the discrete universe fuzzy PID controller were respectively applied to the position loop of an aircraft electromechanical actuator system, and the step response characteristics and frequency response characteristics under load were compared. The simulation results showed that, compared with the traditional PID controller, the fuzzy PID controller has better dynamic response characteristics and higher steady-state accuracy for time-varying system, which obviously improved the control effect of the system.

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