High capacity spatial multimode quantum memories based on atomic ensembles

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We study spatial multimode quantum memories based on light storage in extended ensembles of Λ-type atoms. We show that such quantum light-matter interfaces allow for highly efficient storage of many spatial modes. In particular, forward operating memories possess excellent scaling with the important physical parameters: quadratic scaling with the Fresnel number and even cubic with the optical depth of the atomic ensemble. Thus, the simultaneous use of both the longitudinal and transverse shape of the stored spin wave modes constitutes a valuable and so far overlooked resource for multimode quantum memories.

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Introduction: Photons are ideal candidates for carrying quantum information, however, in order to process the information or store it for a later time, a quantum storage medium is needed. To achieve this, one needs to establish a controllable and efficient light-matter interface that will store light as a stationary excitation in the atomic medium while preserving quantum correlations. Quantum memories have already been demonstrated in a number of experiments based on atomic ensembles, see e.g. [1–6] as well as solid state systems, see e.g. [7–9]. Most of the realized memories support only a single mode, but in order to constitute a building block for a "quantum internet" [10] and for quantum computation they should preferably be capable of storing as many modes as possible [11, 12]. The protocols for multimode memories can be based on various degrees of freedom: spatial [13] or directional [14] modes as well as on controlled reversible inhomogeneous broadening (CRIB) [15, 16] and on atomic frequency combs (AFCs) [17], the latter of which has been realized experimentally [18].

In practice, experimentalists have access to limited physical resources and typically work with a given atomic ensemble where one wants to make full use of the capabilities that the available physical system has to offer. Limitations to the efficient storage of many modes can result, for instance, from the achievable density of cold atoms, imperfection of the optical pumping or the geometry of the atomic cloud. Typically in the experiments a single or a few independently acting transverse modes are employed.

Here, we study how to make full use of the additional resource given by the spatial extent of the atomic ensembles. We show that the simultaneous use of the longitudinal and transverse degrees of freedom allows for highly efficient storage of many spatial light modes resulting in capacities higher than previously expected. The number of modes one can store with high efficiency depends on the choice of the direction of retrieval relative to that of the storage process. We demonstrate that the forward operating memory, with the retrieved light traveling in the direction of the input signal, provides an excellent multimode memory resource. It has a remarkable scaling with the important physical parameters: the optical depth $d_0$ and the Fresnel number of the atomic ensemble $F$. Earlier estimations of the capacity of multimode memories predict a scaling with $\sqrt{d_0}$ for Raman memories as well as for protocols based on electromagnetically induced transparency (EIT) and a linear scaling for the CRIB protocols [19]. The dependence on the Fresnel number has only been roughly estimated in Refs. [12, 20] to be $\sim F^2$. Here, we show by direct calculation that the simultaneous use of the transverse and longitudinal shape of the stored spin wave mode leads to quantum memories with capacities that scale quadratically with the Fresnel number ($\sim F^2$) and even cubically with the optical depth of the atomic ensemble ($\sim d_0^3$). This is much better than expected from previous results. For comparison, we also study the backward operating spatial memory, with the retrieved light traveling in the opposite direction of the input light, and show that it can also serve as a high capacity multimode memory but with a slightly less promising scaling with the physical parameters.

Model: In order to analyze the capacity of the spatial multimode quantum memory, we use the three-dimensional theory for quantum memories based on Λ-type atomic ensembles presented in Ref. [21]. There, it was shown that the crucial physical parameters determining the quality of the quantum memory are the optical depth $d_0$ and the Fresnel number of the atomic ensemble $F$. We consider a cylindrically symmetric atomic ensemble with a Gaussian distribution in the radial direction $n(r) = n_0 \exp[-r^2/(2\sigma_r^2)]$ [see Fig. 1 (left)], where $n_0 = N_A/(2\pi L \sigma_r^2)$, $N_A$ is the number of atoms, $L$ is the length and $\sigma_r$ describes the width of the ensemble. The density along the longitudinal $z$ axis has been assumed constant for simplicity. The geometry of the ensemble is described by its Fresnel number $F = \sigma_r^2/\left(\lambda_0 L\right)$, where $\lambda_0$ is the wavelength of the light. The weak quantum field carries the quantum information to be stored into the atomic ensemble and couples states $|0\rangle$ and $|e\rangle$ with coupling strength $g$ [see Fig. 1 (right)]. States $|1\rangle$ and $|e\rangle$ are...
coupled by the strong classical control field, which sets the propagation direction of the retrieved light. $\Omega(t)$ is the Rabi frequency of the driving field and $\Delta$ denotes the detuning from the excited state $|e\rangle$, which spontaneously decays at a rate $\gamma$. The information is stored in a collective state of the ensemble represented by stationary spin wave excitations described by $S \sim \sum |0\rangle, |1\rangle$.

For simplicity, we solve the three-dimensional problem of the multimode quantum light-matter interface within the adiabatic approximation, where the excited state $|e\rangle$ is eliminated. The equations of motion within the paraxial approximation (in the co-moving frame $t' = t - z/c$) read [21]

$$\frac{d}{dz} \tilde{a}(\tilde{z}, \tilde{t}) = \left( -\frac{iK^2 \sigma^2}{4\pi F} - \frac{1}{2} \frac{d_0}{\frac{1}{2} + i\Delta} |\tilde{S}(\tilde{z}, \tilde{t})|^2 \right) \tilde{a}(\tilde{z}, \tilde{t})$$

$$- \frac{i\sqrt{d_0 \Omega(\tilde{t})}}{\frac{1}{2} + i\Delta} \tilde{S}(\tilde{z}, \tilde{t}),$$

$$\frac{d}{dt} \tilde{S}(\tilde{z}, \tilde{t}) = \frac{1}{2} |\tilde{a}(\tilde{t})|^2 \tilde{S}(\tilde{z}, \tilde{t}) - \frac{i\sqrt{d_0 \Omega(\tilde{t})}}{\frac{1}{2} + i\Delta} \overline{\tilde{a}}(\tilde{z}, \tilde{t}).$$

Here, we have introduced the dimensionless time $\tilde{t} = \gamma t'$, detuning $\tilde{\Delta} = \Delta/\gamma$, position $\tilde{z} = z/L$ and Rabi frequency $\tilde{\Omega}(\tilde{t}) = \Omega(\tilde{t})/\gamma$. The peak optical depth $d_0 = 4Lnn_0 |g|^2 / \gamma$ quantifies the absorption of the resonant light in the absence of the control field $\tilde{\Omega}(\tilde{t})$. We omit here the quantum noise since it is not needed for calculating the efficiency below [22]. The slowly varying light $\tilde{a}(\tilde{z}, \tilde{t})$ and spin wave $\tilde{S}(\tilde{z}, \tilde{t})$ operators have been expanded in a basis set of transverse mode functions $u_{mn}(\vec{r}_\perp)$ (see Ref. [21] for detailed derivations). Due to the sample symmetry as well as for numerical reasons, we chose a set of Bessel beams indexed by $n$ and the azimuthal quantum number $m$. The last terms in the above equations of motion describe the coherent interaction between light and matter, which is quantified by the optical depth, Rabi frequency, detuning and the coupling matrix $B_{mn,m'n'} = \iint d^2 \vec{r}_\perp u_{mn}(\vec{r}_\perp) u_{m'n'}(\vec{r}_\perp) n(\vec{r}_\perp) / n_0$.

Note, that we use here a compact vector and matrix notation where $a_{mn}(\tilde{z}, \tilde{t}) \rightarrow \tilde{a}(\tilde{z}, \tilde{t})$ and $B_{mn,m'n'} \rightarrow \tilde{B}$.

In order to solve the equations of motion (1), we perform the Laplace transform in time $\mathcal{L}\{g(t)\} = \int_0^\infty e^{-\omega t} g(t) dt$, which allows us to eliminate the differential equation for light. Here, we have assumed a constant driving field in space and time $\tilde{\Omega}$. In consequence, we can write the relation between the light modes and the spin wave (here for the forward operating memory) in the form of input-output beam splitter relations

$$\tilde{a}_{\text{out}}(\tilde{\omega}) = \int_0^1 d\tilde{z} \mathbb{K}[\tilde{\Omega}, \tilde{\omega}, \tilde{z}] \tilde{S}_0(\tilde{z}),$$

$$\tilde{S}_0(\tilde{z}) = \frac{1}{2\pi i} \int_{i\infty}^{\gamma} d\tilde{\omega} \tilde{K}^T[\tilde{\Omega}, \tilde{\omega}, 1 - \tilde{z}] \tilde{a}_{\text{in}}(\tilde{\omega}).$$

The transformation matrix $\mathbb{K}$ depends on frequency $\tilde{\omega}$, position $\tilde{z}$ and the physical parameters of the system $\tilde{\Omega}, \tilde{\Delta}, \tilde{F}$ and $d_0$. The analytical expressions for the matrix as well as beam splitter relations for the backward read-out are presented in detail in Ref. [21]. The efficiency of the quantum memory is the ratio between the number of outgoing and incoming light field excitations, which, assuming a normalized incoming light mode, can be written as

$$\eta = \int_0^\infty d\tilde{\omega} |\tilde{a}_{\text{out}}(\tilde{\omega})|^2 \sim \int d\nu \int d\nu' \tilde{a}_{\text{in}}(\nu) M[\nu, \nu'] \tilde{a}_{\text{in}}(\nu' \mathbb{K}^T).$$

Here, the kernel matrix $M$ gives full information about the relation between the input and output modes for a given set of physical parameters. After discretizing frequency and position, we diagonalize the large kernel matrix, which for the forward retrieval is $M[\nu, \nu'] \sim S[\Omega^*, \nu, 1 - \tilde{z}] K[\Omega, \tilde{\omega}, \tilde{z}] K[\Omega, \tilde{\omega}, \tilde{z}^*] \tilde{K}^T[\Omega^*, \nu', 1 - \tilde{z}^*]$. A set of characteristic efficiencies is thereby obtained as the eigenvalues of this matrix together with the corresponding set of incoming light modes $\tilde{a}_{\text{in}}(\nu)$. The optimal incoming light mode to store into the atomic ensemble corresponds to the eigenvector with the highest eigenvalue, which is the maximal efficiency of the memory. The remaining eigenvectors correspond to orthogonal modes that can be stored with lower efficiencies. We show in the following that, in general, there exist many light modes that give high efficiencies of the quantum memory.

The figure of merit for multimode quantum memories is the capacity, which can be defined in at least two different ways. Firstly, one can simply count the number of modes with an efficiency above a minimal value $\eta_{\min}$. Secondly, a more sophisticated measure can be obtained from the quantum capacity of a Gaussian channel with efficiency $\eta$, $Q(\eta) = \max\{0, \log_2 |\eta| - \log_2 |1 - \eta|\}$. $Q(\eta)$ is the average number of qubits that can be perfectly stored and retrieved from a particular mode with combined storage and retrieval efficiency $\eta$, provided many copies of such memory and optimal encoding, decoding and error correction of the stored information. The capacity of the memory is obtained by summing the capacities for all modes; only modes stored and retrieved with
a combined efficiency above $\eta = 0.5$ contribute due to the no-cloning theorem [24], so that $C = \sum_{\eta > 0.5} Q(\eta)$.

**Results:** In order to gain insight into the multimode character of the spatial quantum memory under consideration, we first calculate the capacity and the number of good orthogonal modes within subspaces with a fixed azimuthal quantum number $m$, see Fig. 2. All numerical results are obtained for resonant memories with $\Delta = 0$. Note, that we show only the non-negative values of the azimuthal quantum number $m$ since the modes for $m$ and $-m$ have the same efficiencies. The number of modes with high efficiency decreases with growing $|m|$ since light beams with $|m| > 0$ vanish to increasing degree towards the center of the atomic ensemble. Therefore, it is harder to focus the input light mode into the dense center of the atomic cloud, leading to a decreased effective optical depth $d_0$. One can see that for $d_0 = 100$ and $F = 1$, the forward operating memory yields higher capacities than the backward one. This can be explained by the fact that the driving light used for the backward retrieval reverses the longitudinal phase of the stored excitation but cannot properly reverse its transverse profile except for a spin wave with a uniform transverse phase. Thus in the case of any transverse phase gradient of the stored spin wave, the irreversible transverse phase leads to unwanted diffraction effects [21], which in consequence reduce the efficiencies and the number of modes that can be stored in the backward operating memory.

The total capacity of the forward operating quantum memory $C_f$ is presented in Fig. 3 (red closed circles and stars) as a function of the Fresnel number of the atomic ensemble $F$ for two values of the peak optical depth, $d_0 = 40$ and 100. In both cases, the total capacity $C_f$ reaches high values and grows quadratically with the Fresnel number, $C_f \sim F^2$. To investigate whether the scaling is independent of the two capacity measures mentioned here, we also plot the number of modes $N_f$ for $d_0 = 100$ for two values of the threshold efficiency $\eta_{\text{min}} = 0.5$ and 0.6. We see that the quadratic scaling with the Fresnel number $F$ is universal and either of these $N$ or $C$ may be used as the appropriate measure of the capacity.

The scaling with the Fresnel number depends, however, on the direction of the read-out. To compare the two operating modes of the memory, we also calculated the capacity of the backward memory (black open circles and stars). We find that in this case, the capacity only scales linearly with the Fresnel number but also achieves high values. Thus, even though the highest single mode efficiency for larger Fresnel numbers is achieved for backward operating memories [21], the number of good modes is larger for the forward memories in this $F$ regime.

The scaling of the capacity can be explained by considering the diffraction of light in the atomic medium. For the forward operating memory, the divergence angle of the incoming light beam is $\theta \sim \lambda_0/r_\perp$, where $r_\perp$ is the transverse waist of the stored stationary excitation. The mode can in this case be so focused that the maximum divergence angle becomes limited by the geometry of the ensemble: $\tan \theta_{\text{max}} \sim \sigma_\perp/L$. From this we obtain that the minimal achievable waist of the stored excitation is $r_{\perp,\text{min}} \sim \lambda_0 L/\sigma_\perp$. In consequence, the capacity of the memory, which is proportional to the ratio between the cross section area of the ensemble and the minimal waist squared, $C_f \sim \sigma_\perp^2/r_{\perp,\text{min}}^2 = F^2$, leading to the quadratic dependence on the Fresnel number. As mentioned above, in the case of backward operating memory,
the problem of irreversible transverse phase arises [21]. Therefore one cannot focus the light beam as strongly as in the case of the forward operating memory. Requiring the phase to be constant across the transverse profile leads to $\theta_{\text{max}} \sim r_{\perp}/L$ and in consequence to the capacity of the memory $C_b \sim \sigma_{\perp}^2 / r_{\perp,\text{min}}^2 = F$, showing the linear dependence on $F$.

Above, we have presented the multimode character of the transverse degrees of freedom in the quantum memory and its dependence on the Fresnel number of the atomic cloud. Now, we show that including simultaneously the transverse and longitudinal modes leads to much stronger scaling of the capacity with the optical depth $d_0$ than expected from considering these two degrees of freedom separately. We have optimized the full three-dimensional quantum memory and found that the capacity of the forward operating memory has a promising cubic scaling with the optical depth $C_f \sim d_0^3$, see Fig. 4, much better than previously expected. (The reason for using the number of modes $N_f$ with an efficiency of at least $\eta_{\text{min}} = 0.65$ in the figure was purely numerical. Large values of $F$ and $d_0$ make it hard to calculate all modes with $\eta > \eta_{\text{min}} = 0.5$ as required for the total capacity $C_f$.)

This remarkable cubic scaling of the capacity of the forward operating memory with the optical depth $d_0$ can be understood by means of the phase space volume of the incoming light mode $V_{\text{ps}} \sim k_{\perp,\text{max}}^2 r_{\text{max}}$, where $k_{\perp,\text{max}}$ is the maximal perpendicular wave number and $r_{\text{max}}$ is the maximal waist of the incoming light beam. If we consider light modes coming with different angles with respect to the ensemble axis $z$, the corresponding effective optical depth for a given angle $\theta$ can be expressed as $d_0(\theta) \sim d_0 \sigma_{\perp}/(L\theta)$ for $1 \gg \theta > \sigma_{\perp}/L$. Storage with a certain efficiency requires at least some critical optical depth $d_{\text{crit}}$, which leads to a maximal permitted divergence angle $\theta_{\text{max}} \sim \sigma_{\perp}d_0/(Ld_{\text{crit}})$. Therefore, the maximal perpendicular wave number is proportional to the optical depth, $k_{\perp,\text{max}} \sim \sigma_{\perp}d_0/(L\lambda_0d_{\text{crit}})$, which we have confirmed numerically. For the same reason, the maximal waist of the incoming light beam $r_{\text{max}} \sim \tan(\theta_{\text{max}})/L \sim \theta_{\text{max}}L \sim \sigma_{\perp}d/d_{\text{crit}}$ also scales linearly with the optical depth. In consequence, the number of available modes for each azimuthal quantum number $m$ is proportional to the phase space volume $V_{\text{ps}} \sim F d_0^2/d_{\text{crit}}^2$. On the other hand, the maximal azimuthal quantum number $m_{\text{max}} \sim \sigma_{\perp}k_{\perp,\text{max}} \sim Fd_0/d_{\text{crit}}$ scales linearly with the Fresnel number $F$ and cubically with the optical depth $d_0$. This is a much better scaling than the one achievable for other types of multimode memories with AFC or CRIB, where a linear scaling with the optical depth is obtained [19]. For comparison, we also provide the results for the backward operating memory, see Fig. 4. Here, the scaling is less promising $C_b \sim d_0^{3/2}$, but still high values of the capacity are achievable.

**Conclusion:** We have calculated the capacity of spatial quantum memories based on Λ-type atomic ensembles and thereby shown that they allow for storage of many light modes and exhibit a remarkable scaling with the important physical parameters. For memories operated in the forward direction, the capacity scales quadratically with the Fresnel number $F$ and cubically with the optical depth of the atomic ensemble $d_0$, which is much better than previously expected [19]. These results reveal that the transverse degrees of freedom combined with the longitudinal ones constitute a valuable resource for multimode quantum memories with excellent capacities. These results can be directly used in current experiments with extended ensembles of Λ-type atoms.

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