Numerical analysis on rigidity of cylindrical honeycomb cores under radial loads

Sachiko Ishida\textsuperscript{1*}, Nur Asyikin binti Ahmad\textsuperscript{2}, and Koki Oka\textsuperscript{1}

\textsuperscript{1}Department of Mechanical Engineering, Meiji University
\textsuperscript{2}Ichikoh Industries, Ltd.
\textsuperscript{*}sishida@meiji.ac.jp

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Abstract. Although the rigidity of flat honeycomb cores has been previously studied, the rigidity of honeycomb cores with a unique shape has not been extensively investigated. This study aims to determine the rigidity of cylindrical honeycomb cores under static radial loads via linear finite element analysis. Two numerical cylindrical honeycomb models are demonstrated: one satisfies the radial core height alignment condition with distorted core walls; the other cancels the distortion of core walls, but its core height is not completely aligned with the radial direction. The geometrical differences between the models are small. However, our analyses not only reveal clear differences in rigidity but also demonstrate that the differences in rigidity depended on the constraint conditions applied on the models. These results imply that cylindrical honeycomb cores can be applied in the mechanical components that undergo radial loads, such as vehicle tires, robotic rovers, bearings, and liquid containers.

Keywords: Structure design, Mechanical design, Core structures, Composite materials, Metamaterials, Origami

1. Introduction

Honeycomb cores are widely used to construct industrial structures such as airplanes [1] and railway vehicles [2] owing to their advantage of being lightweight with rigid characteristics. As these structures have complex curved surfaces, the cores are required to follow their shape. Generally, curved honeycomb cores are manufactured by cutting flat honeycomb cores to the desired shape. As for the origami-inspired manufacturing method, Saito et al. [3, 4] developed a new method of fabricating a 3D wing shape with curved honeycomb cores by folding a flat sheet. Both these methods can widen the shape designability range of honeycomb cores. However, from a mechanical perspective, the possibility of arranging the core height direction...
has not been discussed, and the cores are all aligned in parallel regardless of the direction of the load to be applied. As mentioned by Gibson et al. [5] in their theoretical and experimental study, the out-of-plane rigidity of flat honeycomb cores (i.e., rigidity in the core height direction) is significantly larger than the in-plane rigidity (i.e., rigidity in the orthogonal directions); the freely curved honeycomb cores given by cutting flat honeycomb cores thus retain high rigidity only in the unique direction, i.e., the core height direction. For designing the core directions, Maleczek’s method [6] for axisymmetric honeycomb cores is simple and realistic, wherein flat honeycomb cores are rotationally expanded and bent to form a closed cylindrical shape. However, the thus-formed honeycomb cores are warped outward because they are anticlastic structures oppositely curved in two orthogonal directions. Moreover, the core walls are distorted owing to the bending forces.

Based on the above-mentioned studies, Ishida [7] developed geometrical designs for cylindrical honeycomb cores that were exactly straight and not warped. Ishida mathematically revealed that designs satisfying both the conditions of radial core height alignment and flat core walls were not possible. Put differently, the original designs (hereinafter referred to as radiation model) satisfied the radial core height alignment condition, but the core walls were distorted; in contrast, the revised designs (hereinafter referred to as revised model) cancelled the distortion of core walls, but the core height was not exactly aligned with the radial direction. Although the geometrical differences between these two models are small, the influence of geometry on the mechanical properties are unclear. Therefore, this study focuses on revealing the mechanical behaviors of both cylindrical models under static radial loads via linear finite element analysis.

2. Numerical models and conditions

2.1. Radiation model and revised model

Figure 1(a) shows the construction of conventional flat honeycomb cores by the corrugation method [8]. Flat sheets are folded into corrugated shapes, and the adjacent corrugated sheets are layered and glued to construct hexagonal hollows. The Cartesian coordinates \( (x, y, z) \) are defined as shown in Fig. 1 (a), where \( x \), \( y \), and \( z \) represent the direction of the lay sheets, core width, and core height, respectively. To consider a cylindrical surface, the Cartesian coordinates are transformed to cylindrical coordinates \( (x, \theta, r) \), where \( x \) and \( r \) denote the directions of lay sheets and core height, respectively, and \( \theta \) is the circumferential direction (Fig. 1 (b)). Thus, cylindrical honeycomb cores can be fabricated following a fabrication method similar to that of conventional flat honeycomb cores [7].

The radiation model shown in Fig. 2 is a numerical model of the cylindrical honeycomb cores, wherein the directions of all corrugated fold lines are aligned with radial direction \( r \). The model was designed to support radial loads, such as inner pressure, because flat
honeycomb cores are highly rigid along the core height direction according to the theoretical consideration on flat honeycomb cores [5]. However, the four core walls (of the six) in a hexagon of the radiation model got twisted (Fig. 2 (c)), as the fold lines were arranged in a position twisted from each other. In contrast, the revised model shown in Fig. 3 is another numerical model of the cylindrical honeycomb cores designed to keep the core walls flat; however, the direction of the corrugated fold lines was not exactly radial.

Figure 1: Construction of (a) flat honeycomb cores and (b) cylindrical honeycomb cores by the corrugation method

Figure 2: Geometry of radiation model: (a) overview, (b) top view, and (c) schematic of enlarged core

Figure 3: Geometry of revised model: (a) overview, (b) top view, and (c) schematic of enlarged core
In this study, the two numerical models are numerically analyzed without any face sheets, which are generally used for practical application to sandwich them in order to focus on the effects of core configuration on rigidity. The dimensions of both models are identical, as shown in Table 1. Although the geometry of the core walls was slightly different (i.e., flat or twisted), the difference in volume or weight of the models was negligible. The construction process shown in Fig. 1 indicates that the thickness of the practical honeycomb cores is not uniform; however, the core thickness of the numerical models was regarded as uniform for simplicity.

2.2 Numerical conditions

Aluminum alloy, which is a versatile material used for honeycomb cores in the market, was utilized for the numerical models. The linear material properties of typical aluminum alloy shown in Table 2 were adopted because elastic deformation and rigidity under static load are discussed herein, and the material was modeled as an isotropic material. For discretization, quadrilateral and partially triangular shell elements with mesh size of 0.83 mm were adopted for both numerical models. Four constraint conditions described in Fig. 4 were tested to represent possible practical usages. Detailed description is below:

(a) The outer core edges in line were fixed, in which a working condition such as a rolling tire under air pressure was assumed.

(b) The core walls on one end of the cylindrical honeycomb cores were fixed, in which a working condition such as a liquid container under inner pressure without a lid was assumed.

(c) The core walls on both ends were fixed, in which a liquid container with a lid was assumed.

(d) A single core edge in the middle was fixed, as a case the least constraint possible.

Finally, a static radial load of 100 N was uniformly applied on the inner edges outward. The value of 100 N was selected for simplicity and did not have practical intents. Because this study was performed via linear static analysis using alloys with linear material properties, the deformation or stress induced was considered to be proportional to the force applied. In addition, any initial stresses that could occur on practical honeycomb cores through the manufacturing process were not included in the computation.
Table 2: Material properties

| Material              | Aluminum alloy |
|-----------------------|----------------|
| Density, $\rho$ [kg/m$^3$] | 2770           |
| Young’s modulus, $E$ [GPa]     | 71             |
| Poisson’s ratio, $\nu$         | 0.33           |

Figure 4: Constraint conditions: (a) outer core edges in line are fixed, (b) core walls on one end are fixed, (c) core walls on both ends are fixed, and (d) single core edge in the middle is fixed

3. Numerical results

3.1 Displacements

Figure 5 shows the deformed distances under the radial load of 100 N, and the deformed shapes are displayed in real scale. Under the constraint condition (a), the fixed cores of both numerical models never displaced (blue), whereas the cores located at the opposite side of the fixed cores displaced outward as the cylinders got inflated (Figs. 5 (a-1) and 5 (a-2)); specifically, the radiation model was warped outward. The maximum displacement was observed at the end of the models. Under the constraint condition (b), the cores were deformed outward depending on the distance from the fixed cores, and the maximum displacements were observed at the opposite end of the fixed cores (Figs. 5 (b-1) and 5 (b-2)). On the contrary, under the constraint condition (c), the displacement was sufficiently small on both models, but the maximum displacements were observed in the middle of both models. Finally, under the constraint condition (d), the radiation model was warped outward depending on the distance from the fixed cores, whereas the revised model was relatively straight. The maximum displacements were observed at the ends of the cylinders on both models (Figs. 5 (d-1) and 5 (d-2)).
3.2 Von Mises stress

Figure 6 shows the von Mises stress under the radial load of 100 N. Under any constraint condition of (a)–(d), the stress was concentrated at the intersections of the core walls (i.e., corners of the hexagons), because they acted as supporting positions for deformation of the core walls. Under the constraint condition (a), the maximum von Mises stress, which was caused by tensile deformation of the core walls because of inflation-like displacements of the cylinders, was observed at the intersections near the fixed hexagonal edges on both models (Figs. 6 (a-1) and 6 (a-2)). Under the constraint condition (b), the maximum von Mises stress was observed at the intersections near the opposite end to the fixed end, where the hexagonal shapes were largely distorted (Figs. 6 (b-1) and 6 (b-2)). Under the constraint condition (c), the maximum von Mises stress was observed at the intersections near both the fixed ends on both models owing to tensile deformation of the core walls (Figs. 6 (c-1) and 6 (c-2)). However, compressive deformation was locally observed on the core walls in the middle of the revised model because they supported the radial load under the condition that the model was sufficiently fixed and the displacement was constrained. Finally, under the constraint condition (d), the maximum von Mises stress was observed at the intersections near both the ends, where the hexagonal shapes were largely distorted (Figs. 6 (d-1) and 6 (d-2)).

Depending on the constraint conditions, the maximum von Mises stress was over yield stress of a typical aluminum alloy, indicating that the honeycomb cores would collapse in practical
applications. Here again, the elastic relationship of stress induced to the unique force applied is focused, and plastic deformation or failure modes are not discussed.

![Figure 6](image)

**Discussion**

### 4.1 Rigidity along radial direction

Figure 7 shows the radial displacements at the position where the maximum displacement was observed under the constraint conditions (a)–(d). Under the constraint condition (a), the radiation model displaced by a larger degree depending on the distance from the fixed cores, whereas the revised model retained the original semi-circular shape at the positions close to the fixed cores. The ratio of the maximum displacements of the revised model to the radiation model was $\frac{14.4 \text{ mm}}{16.0 \text{ mm}} = 0.900$. The absolute values of the maximum displacement numerically obtained in this study are parameter-dependent, such that different designs of the numerical models return different values. However, the values proportionally change on both the models with similar dimensions, as long as linear computation is performed. Thus, the rigidity of the revised model was higher by 10% than that of the radiation model, along the radial direction. Under the constraint condition (b), the displaced shapes remained circular on both the models, but the radiation model exhibited larger displacement than the revised model at all positions. As the ratio of the maximum displacements of the revised model to the radiation...
model was 10.9 mm/13.0 mm = 0.838, the rigidity of the revised model was approximately 16% higher than that of the radiation model along the radial direction. In contrast, under the constraint condition (c), no difference in displacement between the models was visible against the original circular shape even at the maximum displacement position shown in Fig. 7 (c). However, the absolute values of the maximum displacements were 7.38 μm and 8.16 μm for the radiation model and the revised model, respectively; thus, the ratio was 8.16 μm/7.38 μm = 1.11. This resulted in the rigidity of the revised model being approximately 11% lower than that of the radiation model in the radial direction, when the models were sufficiently fixed even though the amount of displacement was trivial. Finally, under the constraint condition (d), the radiation model was inflated uniformly, whereas the revised model was not displaced at the side of the fixed cores because the entire shape was still straight. As the ratio of the maximum displacements was 12.0 mm/14.0 mm = 0.857, the rigidity of the revised model was approximately 14% higher than that of the radiation model along the radial direction.

Under the constraint conditions (a), (b), and (d), the models were not sufficiently fixed and were globally displaced outward at the positions apart from the fixed cores. In these cases, the rigidity of the revised model was higher than that of the radiation model. This result indicates that global displacements under radial load can be reduced by the following important factors: using a model with flat core walls (i.e., revised model) and sacrificing the core height alignment in the radial direction (i.e., radiation model). In contrast, under the constraint condition (c), the models were sufficiently fixed at both the ends, and global displacements were negligible. In this case, the model with radial core height alignment is appropriate from a high-rigidity perspective.

Figure 7: Radial displacement of radiation model (blue) and revised model (red) against their original shapes (black) at maximum displacement positions: (a)–(d) correspond to the constraint conditions in Fig. 4 (a)–(d)
4.2 Rigidity in circumferential direction

In this section, the rigidity along the circumferential direction $\theta$ under a radial load is considered. Because the numerical models of cylindrical honeycomb cores were displaced similar to an inflated balloon, the inflation mechanism of a cylindrical membrane [9] is adopted.

When a uniform pressure $p$ is applied along the radial direction onto the inner surface of a cylindrical membrane of diameter $D$ and height $h$ (Fig. 9), the tensile force along the circumferential direction $T_c$ is given by solving the equilibrium equation as follows:

$$2T_c = \frac{1}{2} phD \int_0^\pi \sin \theta \, d\theta$$

$$= phD$$

$$\therefore T_c = \frac{1}{2} phD \tag{1}$$

where $D$ is given by $D = (D_o + D_0)/2 = D_o - c$ under the assumption that the core height $c$ is comparatively small against the diameter $D$. In our study, a radial force $F$ of 100 N was applied on the inner surface of the cylinder; the pressure $p$ is given by

$$p = \frac{F}{\pi Dh} \tag{2}$$

Thus, the tensile stress $\sigma_\theta$ caused by the tensile force along the circumferential direction is obtained by

$$\sigma_\theta = \frac{T_c}{hc} = \frac{pD}{2c} = \frac{F}{2\pi ch} \tag{3}$$

Subsequently, the strain in the circumferential direction $\varepsilon_\theta$ is given by the ratio of change in circumference $\Delta l$ to the original circumference, as follows:

$$\varepsilon_\theta = \frac{\Delta l}{\pi D} \tag{4}$$

In this study, the change in circumference $\Delta l$ was calculated from planar displacements at the positions where the maximum displacements were observed (Fig. 7). Thus, the rigidity or the Young’s modulus of the numerical models along the circumferential direction is given by

$$E_\theta = \frac{\sigma_\theta}{\varepsilon_\theta} = \frac{FD}{2ch\Delta l} \tag{5}$$
Figure 10 shows the Young’s moduli of both models under the constraint conditions (a)–(d) obtained by substituting the geometrical dimensions of the models provided in Table 1 in Eq. (5). Under the constraint conditions (a), (b), and (d), the ratios of the Young’s moduli of the revised model to the radiation model were 0.223 MPa/0.130 MPa = 1.72, 0.146 MPa/0.0967 MPa = 1.51, and 0.182 MPa/0.106 MPa = 0.72, respectively. This resulted in the rigidity of the revised model being approximately 72%, 51%, and 72% higher than that of the radiation model, respectively, in the circumferential direction. In contrast, under the constraint condition (c), the ratio of the Young’s modulus was given as 127 MPa/133 MPa = 0.95; thus, the rigidity of the revised model was approximately 5% lower than that of the radiation model in the circumferential direction. It is reasonable that these trends are similar to those of rigidity in the radial direction.

Figure 10: Young’s modulus along the circumferential direction of radiation model (blue) and revised model (red): (a)–(d) correspond to the constraint conditions in Fig. 4 (a)–(d)

4.3 Von Mises stress

Figure 11 shows the maximum von Mises stress on the models under the constraint conditions (a)–(d). Under the constraint conditions (a), (b), and (d), the ratios of the maximum von Mises stress of the revised model to the radiation model were 593 MPa/720 MPa = 0.823, 502 MPa/618 MPa = 0.812, and 455 MPa/577 MPa = 0.788, respectively. This resulted in the stresses on the revised model being approximately 18%, 17%, and 21% lower than those on the radiation model, respectively; therefore, the revised model had less risk of failure. In contrast, under the constraint condition (c), the ratio of the maximum von Mises stress was...
10.2 MPa/8.73 MPa = 1.17; the stress on the revised model was approximately 17% higher than that on the radiation model. The values of the maximum von Mises stress were considerably smaller than those under other constraint conditions; however, it is remarkable that the rank order of the models was opposite in a case where the models were sufficiently fixed.

In the cases established herein, the maximum stress was induced by tensile deformation of the core walls due to global inflation-like displacements and not by the local compressive deformation of the core walls. It is thus incontrovertible that the model with less displacements was less stressed than the other model.

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Figure 11: Maximum von Mises stress on radiation model (blue) and revised model (red); (a)–(d) correspond to the constraint conditions shown in Fig. 4 (a)–(d)

5. Conclusions

In this study, two numerical models of cylindrical honeycomb cores were designed, and their radial and circumferential rigidities under radial load were analyzed. As the radial displacements of the radiation model, which had radial core height alignment, were larger than those of the revised model under equivalent conditions, the revised model was suitable for less displacement or high rigidity. However, only when the models were firmly fixed and global displacements were restricted, the radiation model had the advantage of high rigidity in terms of usage. Regarding the von Mises stress, the model with less displacements was less stressed than the other model, because the maximum stress was induced by tensile deformation of the core walls due to global inflation-like displacements.

Mechanical components such as vehicle tires, robotic rovers, bearings, and liquid containers, which undergo radial loads, are possible practical applications of the cylindrical honeycomb cores. However, structural rigidity under sharing, bending, and any possible dynamic loads that can be applied requires detailed research. Further investigation can be conducted using numerical and experimental approaches.
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