Cosmological parameters of $f(R)$ gravity with kinetic scalar curvature

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Abstract. We study modified $f(R)$ gravity theory with kinetic scalar curvature reducible to chiral cosmological model of a special type. New method of cosmological parameters calculation based on reduction of two-fields model to standard one with single scalar field is proposed. Parametric correspondence to observational data is shown for massive scalar field, power-law and intermediate inflation.

1. Introduction
The need of modification of Einstein gravity closely connected with discovery of the acceleration in the expansion of the Universe [1, 2]. After this discovery it became clear that GR could not explain this phenomena by natural way without introduction of additional fields (dark energy). Therefore there were studied modifications of gravity theory such as the Einstein-Gauss-Bonnet theory, scalar-tensor theory of gravity, $f(R)$ gravity and gravity theories with non-minimal coupling of a scalar field to the scalar curvature [3, 4].

Special attention should be paying to modifications of GR with higher-order corrections to the scalar curvature in the Einstein-Hilbert action. Such inclusion is dictated by quantum effects in the low-energy limit of string theory, superstrings, and supergravity, needed for the construction of a quantum theory of gravity [5].

Modified theories of gravity (MTG) should pass verifications on few levels. First of all, on cosmological level, the theory should admit inflationary stage as well as the later accelerated expansion of the Universe. For inflationary stage there need to have de Sitter and power-law solutions to be consistent with general description of the early inflationary and radiative-dominated stages. The following step is correspondence of theoretical predictions to observational data from cosmic missions WMAP, Planck and terrestrial ones BISEP2 and Keck Array CMB experiments. Generally speaking, the search of exact or approximated solutions and calculation of cosmological parameters are rather complicated and technically difficult tasks (see, for ex., [6, 7]). Therefore the reduction of MTG to GR with scalar fields (to chiral cosmological models [8, 9]) by virtue of Weyl conformal transformation is very useful and it simplifies the tasks of searching inflationary solutions and calculation of cosmological parameters.

General way of reducing the MTG with higher derivatives of the type
\[
S = \int d^4x \sqrt{-g} \left[ f(R, (\nabla R)^2, \Box R) \right]
\]
to Einstein gravity with two scalar fields was proposed in [10]. Cosmological applications and exact and approximate solutions were found in the work [11]. Also the simplified model with the action

\[ S = \int d^4x \sqrt{-g} \left[ f\left(R, (\nabla R)^2\right)\right], \]  

(2)

where

\[ f(R, (\nabla R)^2) = f_1(R) + X(R)R_{\mu\nu}R^{\mu\nu}, \]  

(3)

was investigated in [6, 7, 9].

In the present work we continue analysis of the model (2), in terms of chiral cosmological model [9], with the aim to present connection to observational data.

An important criterion for checking the correctness of cosmological inflation models is the comparison of the spectral characteristics of cosmological perturbations in these models and the observational data obtained from measurements of the CMB anisotropy [12]. Also, the possibilities of direct detection of relic gravitational waves by various methods [13-20] are considered, which will significantly reduce the number of possible evolution scenarios of the early universe. However, at the moment, the criterion for verification of the inflationary models is the upper limit on the ratio of the squares of the amplitudes of tensor and scalar cosmological perturbations (tensor-to-scalar ratio) \( r < 0.065 \) [12]. To show viable property of modified \( f(R) \) gravity with kinetic scalar curvature we proposed new method of cosmological parameters calculation based on the reduction of the initial chiral cosmological model to a single-filed cosmological one.

The paper is organized as follows: in section 2 we present the model equations. Section 3 is devoted to reduction of the chiral cosmological model, as the equivalent model of (2), to one-field model. In section 4 we demonstrate the algorithm of cosmological parameters calculation and confronting them with observational data using the example of massive scalar field model. In section 5 we study power-law inflation. Section 6 is devoted to study of cosmological parameters from ”Intermediate” inflation. Conclusion is presented in section 7.

2. The model equations

In the work [9] it was studied the model with the action

\[ S_1 = \int d^4x \sqrt{-g} f\left(R, (\nabla R)^2\right), \]  

(4)

where

\[ (\nabla R)^2 = g^{\mu\nu} \nabla_\mu R \nabla_\nu R, \quad f(R, (\nabla R)^2) = f_1(R) + X(R)\nabla_\mu R\nabla^\mu R. \]

Following by Naruko’s approach [10] the model (4) have been reduced to the Chiral Cosmological Model (CCM) [8] with the action

\[ S = \int \sqrt{-g} d^4x \left( \frac{R}{2} - \frac{1}{2} h_{AB}(\varphi) \partial_\mu \varphi^A \partial_\nu \varphi^B g^{\mu\nu} - V(\varphi) \right), \]  

(5)

considering in 2D metric of the target space

\[ ds^2 = d\chi^2 - e^{-\sqrt[3]{\chi}} X(\phi) d\phi^2, \quad \varphi^1 = \chi, \quad \varphi^2 = \phi \]  

(6)

and having the potential of interaction

\[ V(\chi, \phi) = \frac{1}{4} e^{-\sqrt[3]{\chi}} \left( \phi - e^{-\sqrt[3]{\chi}} f_1(\phi) \right). \]  

(7)
The CCM presentation of the model (5) give us possibility to apply the results of the work [8] to represent the equation of cosmological dynamics in the Friedmann-Robertson-Walker metric $ds^2 = -dt^2 + a(t)^2(dr^2 + r^2(d\theta^2 + \sin^2(\theta)d\phi^2))$ in the following way

$$3H^2 = \frac{1}{2} \dot{\chi}^2 - \frac{1}{2} e^{-\sqrt{\frac{2}{3}}\chi} X(\phi) \dot{\phi}^2 + \frac{1}{4} e^{-\sqrt{\frac{2}{3}}\chi} \left(-\phi + e^{-\sqrt{\frac{2}{3}}\chi} f_1(\phi)\right),$$

$$\dot{H} = -\frac{1}{2} \dot{\chi}^2 + \frac{1}{2} e^{-\sqrt{\frac{2}{3}}\chi} X(\phi) \dot{\phi}^2,$$

$$\ddot{\chi} + 3H \dot{\chi} - \frac{1}{2} \dot{\phi}^2 \sqrt{\frac{2}{3}} e^{-\sqrt{\frac{2}{3}}\chi} X(\phi) + \frac{1}{4} \sqrt{\frac{2}{3}} e^{-\sqrt{\frac{2}{3}}\chi} \left(\phi - 2e^{-\sqrt{\frac{2}{3}}\chi} f_1(\phi)\right) = 0,$$

$$X(\phi) \left(-3H \dot{\phi} - \dot{\phi} + \sqrt{\frac{2}{3}} \dot{\chi}\right) - \frac{1}{2} \ddot{\phi}^2 X'(\phi) - \frac{1}{4} + \frac{1}{4} e^{-\sqrt{\frac{2}{3}}\chi} f_1'(\phi) = 0.$$

In the work [9] there were presented examples of exact solutions of the model (5) -(7) and providing the model by additional material source.

To show a correspondence to observational data of the model (4) we develop the method of reduction of 2D CCM to a single field model.

### 3. Reduction to one-field model

To apply the standard method of cosmological parameters calculation (power spectrum, spectral indexes, tensor-to-scalar ratio) we reduce two scalar fields model to single field inflationary model assuming simple linear dependence between fields:

$$\phi(t) = k \chi(t), \quad k = \text{const.}$$

Evident relations between derivatives with respect to cosmic time

$$\dot{\phi}(t) = k \dot{\chi}(t), \quad \ddot{\phi}(t) = k \ddot{\chi}(t)$$

and derivatives wrt the field $\phi$

$$\frac{dX}{d\phi} = k^{-1} \frac{dX}{d\chi}, \quad \frac{df_1}{d\phi} = k^{-1} \frac{df_1}{d\chi}$$

there need to take into account in the action (5). The resulting action is

$$S = \int d^4x \sqrt{-g_E} \left(\frac{R_E}{2} - \frac{1}{2} \left[1 - k^2 e^{-\sqrt{\frac{2}{3}}\chi} X(\chi) \chi^2 - \frac{1}{4} f_1' e^{-\sqrt{\frac{2}{3}}\chi} - \frac{1}{4} k^2 e^{-\sqrt{\frac{2}{3}}\chi} \right] g_E^{\mu\nu} \chi_{,\mu} \chi_{,\nu} + \frac{1}{4} f_1(\chi) e^{-\sqrt{\frac{2}{3}}\chi} - \frac{1}{4} k^2 e^{-\sqrt{\frac{2}{3}}\chi} \right).$$

Then the equations of cosmological dynamics (8)-(11) are:

$$3H^2 = \frac{1}{2} \dot{\chi}^2 - \frac{1}{2} k^2 e^{-\sqrt{\frac{2}{3}}\chi} X(\chi) \chi^2 + \frac{1}{4} e^{-\sqrt{\frac{2}{3}}\chi} \left(-k \chi + e^{-\sqrt{\frac{2}{3}}\chi} f_1(\chi)\right),$$

$$\dot{H} = -\frac{1}{2} \dot{\chi}^2 \left(1 - k^2 e^{-\sqrt{\frac{2}{3}}\chi} X(\chi)\right).$$
\[ \ddot{\chi} + 3H \dot{\chi} - \frac{1}{2} k^2 \dot{\chi}^2 e^{-\sqrt{\frac{2}{3}} \chi} \left[ -\sqrt{\frac{2}{3}} \chi f(\chi) - \frac{d}{d\chi} \left( 2 \sqrt{\frac{2}{3}} f_1(\chi) \right) - \sqrt{\frac{2}{3}} k \chi + k \right] = 0. \]  

(18)

From equations (16) and (17) one can find, as in the case of standard GR cosmology, the consequence

\[ 3H^2 + \dot{H} = V(\chi, k\chi) = \frac{1}{4} e^{-\sqrt{\frac{2}{3}} \chi} \left( -k\chi + e^{-\sqrt{\frac{2}{3}} \chi} f_1(\chi) \right). \]  

(19)

To apply Ivanov-Salopek-Bond (ISB) method [21] we use (16)-(17) and freedom of choice the kinetic function \( X(\chi) \). Passing to the dependence of the Hubble function on the field \( \chi \) from (17) we have:

\[ H' = -\frac{1}{2} \dot{\chi} \left( 1 - k^2 e^{-\sqrt{\frac{2}{3}} \chi} X(\chi) \right). \]  

(20)

Then, the choice of the function \( X(\chi) \) can lead to an increase or decrease in the rate of the Hubble parameter change with a change in the field \( \chi \). We consider the case that allows us to use the ISB approach without significant modification. To this end, we select the function \( X(\chi) \) in the following form:

\[ X(\chi) = \pm e^{\sqrt{\frac{2}{3}} \chi}. \]  

(21)

This choice allows us to reduce the equation (19) to the following

\[ 3H^2 - \frac{2(H')^2}{1 \pm k^2} = \frac{1}{4} e^{-\sqrt{\frac{2}{3}} \chi} \left( -k\chi + e^{-\sqrt{\frac{2}{3}} \chi} f_1(\chi) \right). \]  

(22)

If we set \( k = 0 \) the ISB equation for standard Friedmann cosmology with the potential (19) is reproduced.

4. Algorithm of cosmological parameters calculation

Let us consider the algorithm of cosmological parameters (scalar spectral index \( n_s \), tensor-to-scalar ratio \( r \) and power spectrum \( P_S \) ) calculation using the example of exact solution for massive scalar filed [21]. The action for the single scalar field model is

\[ S_{sf} = \int d^4x \sqrt{-g} \left\{ R - \frac{1}{2} \phi, \phi g^{\mu \nu} - V(\phi) \right\}. \]  

(23)

We will use the set of exact solutions for scalar field model (23) which listed in the monograph [21].

The exact solution for massive scalar field defined by the potential

\[ V(\varphi) = \frac{m^2 \varphi^2}{2} - \frac{m^2}{3} \]  

(24)

and the Hubble parameter

\[ H(\varphi) = m \sqrt{\frac{1}{6} \varphi}. \]  

(25)

Scalar field evolution is

\[ \varphi(t) = -m \sqrt{\frac{2}{3} t} + \varphi_c = -m \sqrt{\frac{2}{3} (t - t_s)}, \]  

(26)
where \( t^* \) is time of the ending of Universe expansion, wherein \((t - t^*) < 0\). Note that the acceleration of the Universe is ending earlier. Considering inflationary stage of Universe evolution we assume that time is sufficient close to zero and, for the sake of simplicity, we assume \((t - t^*) = t < 0\).

Correspondence to the model (15) attains with the choice of the kinetic function \( X(\chi) \) from (21) and the function \( f_1(\phi) \) of the following form:

\[
f_1(\phi) = 4 \exp \left( \frac{k_2 \sqrt{2} \phi}{\sqrt{3(1 - k^2)}} \right) \left( \frac{m^2 \phi^2}{2} - \frac{m^2}{3} \right) + \exp \left( \frac{k \sqrt{2} \phi}{\sqrt{3(1 - k^2)}} \right) \frac{k^2 \phi}{\sqrt{1 - k^2}}. \tag{27}\]

Note the relations between fields are like follow:

\[
\begin{align*}
\phi &= k \chi; \\
\varphi &= \sqrt{1 - k^2} \chi; \\
\phi &= \frac{k \varphi}{\sqrt{1 - k^2}}.
\end{align*} \tag{28}\]

Reminding of the formal correspondence \( \phi = k \chi \) to scalar curvature \( R \) we note that \( f_1(R) \) is the part of standard representation containing the following terms \( A e^{c_1 R} R^2 + B e^{c_2 R/2} R - C e^{c_3 R} \), which appearing in the string theory [22, 23].

Now we are searching for the exact inflationary parameters, defined in [21]. First parameter one can find from the relation

\[
\epsilon = 2 \left( \frac{H'}{H} \right)^2 = - \frac{\dot{H}}{H^2}. \tag{29}\]

The result for the model (24)-(26) is

\[
\epsilon = \frac{3}{m^2 t^2}. \tag{30}\]

Next, we turn to Hubble parameter dependence on the number of e-foldings \( N(t) = - \int H(t) dt = \frac{m^2 t^2}{6} \). Using (29) in \( H = H(N) \) representation

\[
\epsilon = - \frac{dH}{dN} \frac{\dot{N}}{H^2} \tag{31}\]

and using dependencies on time we obtain \( \frac{dH}{dN} \) in the form:

\[
\frac{dH}{dN} = \frac{1}{i} \tag{32}\]

Taking into account the Hubble parameter dependence on time \( H(t) = \frac{m^2}{2} t \) we find the dependence \( t(N) = \frac{\sqrt{6N}}{m} \). Thus we found the dependence \( \epsilon = \epsilon(N) \) in the form \( \epsilon = \frac{1}{2N} \). If we take \( N = 60 \) then \( \epsilon = 0.0083 \). Calculation of the second exact inflationary parameter \( \delta = 2 \frac{H''}{H} = - \frac{\dot{H}}{2HH} \) gives us \( \delta = 0 \).

To define the scalar spectral parameter \( n_S \) we will use the refined formula [21, 24]

\[
n_S - 1 = 2 \left( \frac{\delta - 2\epsilon}{1 - \epsilon} \right). \tag{33}\]

As the result we obtain \( n_S \simeq 0.9664 \). Such a way we see that the result corresponds with good accuracy to the observational data of the Planck space mission.

Tensor spectral parameter for this model is defined from the relation

\[
n_T = - \frac{2\epsilon}{1 - \epsilon} \tag{34}\]
and equal to \( n_T = -0.0167 \).

Tensor-to-scalar relation we define by formula

\[
    r = 4s\epsilon,
\]

(35)

where \( s = 1 \) or \( s = 4 \) is the value of normalization of the tensor perturbations. If \( s = 1 \) we obtain \( r = 0.0332 < 0.1 \) and \( r = 0.0332 < 0.065 \) in the correspondence with observational data. If \( s = 4 \), \( r = 0.1328 \), this result does not correspond to observations.

To coordinate the model according to the power spectrum of scalar perturbations, we use the refined formula \[21, 24\]

\[
    P_S(k) = \frac{1}{2\epsilon} \left( \frac{H}{2\pi} \right)^2
\]

(36)

and the experimental value \( P_S(k) = 2.14 \times 10^{-9} \). As a result, we obtain that the mass of the scalar field \( \varphi \) is determined from the relation \( m^2 = 6.25 \times 10^{-11} \). From the comparison of the action (15), taking into account (21), we determine the relationship between the masses of the fields \( \varphi \) and \( \chi : m_\varphi = m_\chi/\sqrt{1 + k^2} \). That is, if it is necessary to coordinate with a specific particle mass, the mass \( m_\varphi \) can be decreased or increased. This fact emphasizes the difference between the single-field and multi-field models, that is, due to the presence of other fields, an effective change in mass for a single-field model is possible.

5. Power-law inflation

Let us consider the case of power-law inflation, the exact solution for which is represented as follows:

\[
    a(t) = a_s t^m, \quad H(t) = m/t,
\]

\[
    \varphi = \pm\sqrt{2}m \ln t + \varphi_s, \quad t(\varphi) = \exp\left( \frac{\varphi}{\pm\sqrt{2}m} \right),
\]

(37)

\[
    V(\varphi) = (m + 3m^2) \exp\left( -\sqrt{\frac{2}{m^2}} \varphi + V_0 \right).
\]

(38)

(39)

To make correlation with the model (15) we set the function \( f_1(\varphi) \) in the following form

\[
    f_1(\varphi) = \frac{k\varphi}{\sqrt{1 + k^2}} \exp\left( \sqrt{\frac{2}{3}} \frac{\varphi}{\sqrt{1 + k^2}} \right) - 4V_0 \exp\left( 2\sqrt{\frac{2}{3}} \frac{\varphi}{\sqrt{1 + k^2}} \right) - (m + 3m^2) \exp (k_m \varphi),
\]

(40)

where \( k_m = 2\sqrt{\frac{2/3}{1 + k^2}} - \sqrt{\frac{2\varphi}{m}} \).

The exact inflationary parameters for the power law inflation are

\[
    \epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{m},
\]

(41)

\[
    \delta = -\frac{\dot{H}}{2HH} = \frac{1}{m}.
\]

(42)

Suggesting the value \( m = 65 \), from the relation (33) we obtain

\[
    n_S - 1 = \frac{2\epsilon}{1 - \epsilon} = 0.968,
\]

(43)
The result is corresponded to observation data [12].

Next, following by general algorithm we define the dependence of e-folds number on time and inverse dependence

\[ N(t) = \int H(t) dt = m \ln \left( \frac{t}{C} \right), \quad t(N) = C \exp \left( \frac{N}{m} \right), \tag{44} \]

where \( C \) is an integration constant, taking in given case the sense of beginning of the inflation time \( C = t_i \).

Then using the relation (36), we obtain

\[ P_S(k) = m^2 \left( \frac{m^2}{2 \pi t_i} \right)^2 \exp \left( -\frac{2N}{m} \right) = 2.14 \times 10^{-9}. \tag{45} \]

Thus, in choosing system of units, from relation (45) for \( N = 60 \) we obtain \( t_i = 5 \times 10^5 \) what in terms of Planck time \( (t_{pl} = 5.4 \times 10^{-44} c) \) gives

\[ t_i = 5 \times 10^5 \times t_{pl}. \tag{46} \]

Also, from the equation (44) we determine the time of ending of the inflation stage

\[ t_e = 1.6 \times 10^6 \times t_{pl}. \tag{47} \]

Further we calculate tensor-to-scalar ratio

\[ r = 4s \epsilon = \frac{4s(n_S - 1)}{n_S - 3}, \quad r(s = 1) = 0.063 < 0.065, \quad r(s = 4) = 0.25 > 0.065. \tag{48} \]

Thus, for the normalisation \( s = 1 \) (unlike \( s = 4 \)) the amplitudes of primordial gravitational waves correspond to observation for the power law inflation.

6. Intermediate inflation

Intermediate inflationary models was introduced into consideration in the work [25] and recently there were investigation of cosmological parameters and confrontation them to observational data in [26]. We consider intermediate inflation in another presentation. Namely, we consider the case of an inflationary model based on the polynomial potential with the Hubble parameter \( H(\varphi) = A \varphi^m \) that is verified by observational data, as we will see, regardless of the choice of the parameter \( s \). The solution is

\[ V(\varphi) = 3A^2 \varphi^{2m} - 2A^2 m^2 \lambda \varphi^{2(m-1)}, \tag{49} \]

\[ \varphi(t) = [c_1 + 2Am(m-2)t]^{\frac{1}{2-m}}, \tag{50} \]

\[ H(t) = A [c_1 + 2Am(m-2)t]^{\frac{m}{2-m}}, \tag{51} \]

\[ a(t) = a_s \exp \left( -\frac{1}{4m} [c_1 + 2mA(m-2)t]^{\frac{2}{2-m}} \right), \tag{52} \]

where \( A, m \) and \( c_1 \) are an arbitrary constants.

To comply with these solutions, the function \( f_1(\varphi) \) is selected in the following form

\[ f_1(\varphi) = 4 \exp \left( \frac{2}{3} \sqrt{\frac{6}{1 + k^2}} \varphi \right) \left( 3A^2 \varphi^{2m} - 2A^2 m^2 \lambda \varphi^{2(m-1)} \right) + \exp \left( \frac{1}{3} \sqrt{\frac{6}{1 + k^2}} \varphi \right) \frac{k \varphi}{\sqrt{1 + k^2}}. \tag{53} \]
The inflationary parameters for this model are
\[ \epsilon = -\frac{\dot{H}}{H^2} = 2m^2 \left[ c_1 + 2Am(m-2)t \right] \frac{m}{m-2}, \]  
\[ \delta = -\frac{\dot{H}}{2HH} = 2m(m-1) \left[ c_1 + 2Am(m-2)t \right] \frac{m}{m-2}, \]  
with the following connections between them
\[ \delta = \left( \frac{m-1}{m} \right) \epsilon. \]  
\[ (54) \]
\[ (55) \]
\[ (56) \]

From the $e$-folds number
\[ N = -\frac{1}{4m} \left[ c_1 + 2mA(m-2)t \right] \frac{m}{m-2}, \]  
we obtain the following equation
\[ c_1 + 2mA(m-2)t = -(4mN)^{\frac{m}{m-2}}. \]  
\[ (57) \]
\[ (58) \]

From the inverse dependence $t = t(N)$ one has
\[ \mathcal{P}_S(N) = 16^{\frac{2m-1}{2-m}} \left( \frac{A}{4\pi m} \right)^2 \left[ 2^{-m+2}(mN)^{1-\frac{m}{m-2}} \right] \frac{2(m+1)}{2-m}, \quad \mathcal{P}_S(N = 60) = 2.14 \times 10^{-9}. \]  
\[ (59) \]

From this condition for $m = 1/3$ we obtain $A = 10^{-5}$.

The tensor-to-scalar ratio is
\[ r = \frac{4sm(n_s - 1)}{m(n_s - 3) - 2}. \]  
\[ (60) \]

For example, for $m = 1/3$ and $n_s = 0.968$ one has $r(s = 4) = 0.064 < 0.065$ and $r(s = 1) = 0.016 < 0.065$.

Thus, this type of inflationary model corresponds to observational constraint for both normalization of the amplitude of relict gravitational waves.

7. Conclusion
In this paper, we proposed the verification procedure based on observational data for cosmological inflation models founded on modified theories of gravity with a kinetic scalar curvature. The procedure means reduction of two-fields model to one-field version for standard Friedmann cosmology and calculation of cosmological parameters in it.

The original models with higher derivatives, through conformal transformations of the metric, were reduced to the chiral cosmological models with fixed target space metric and the potential of interaction [10, 21]. We use the CCM (5) - (7) with a special choice of the relation between scalar fields $\phi(t) = k \chi(t)$. This approach gives a correspondence with single-field models which can be performed on the basis of the expressions (19)–(22).

Therefore, on the basis of this representation, one can calculate the parameters of cosmological perturbations for the inflationary models based on the modified $f(R)$ gravity with a kinetic scalar curvature. We showed that inflationary model for massive scalar field can be matched to observation data and moreover the influence of the second field may drastically change the effective mass of the scalar particle. Also in the framework of this approach, a power-law inflation model was analyzed, for which compliance with observational data depends on the normalization of the tensor of relic gravitational waves $s = 1, 4$. Also, a model of intermediate inflation was considered, which is verified regardless of the choice of parameter $s$. 
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