In-medium $T$ matrix for neutron matter

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We calculate the equation of state of pure neutron matter, comparing the $G$-matrix calculation with the in-medium $T$-matrix result. At low densities, we obtain similar energies per nucleon, however some differences appear at higher densities. We use the self-consistent spectral functions from the $T$-matrix approach to calculate the $^{1}S_{0}$ superfluid gap including self-energy effects. We find a reduction of the superfluid gap by 30%.

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In the interior of neutron stars, neutron rich nuclear matter is present in a range of densities up to several times the normal nuclear density. Two properties of the neutron matter are important for the modeling of neutron stars. The equation of state, which serves as an input for the calculation of the mass of the star, and the superfluid properties of the dense matter, relevant for the cooling and the rotation of stars. Dense neutron matter cannot be studied directly in laboratory; therefore, it is important to have a reliable description of this many-body state from theoretical calculations.

Very advanced methods based on the Brueckner-Hartree-Fock (BHF) approximation were applied for the calculation of neutron matter properties [1]. Independently, approaches using the variational chain summation to tackle the nuclear many-body problem [2, 3] have been developed to a high level of accuracy. For symmetric nuclear matter, these calculations have shown the need to introduce a three-body force to reproduce the phenomenological saturation point. Relativistic corrections have also been estimated [3]. Variational and BHF methods allow to study the equation of state of neutron matter up to densities several times the normal nuclear matter density. Microscopic calculations of different pairing gaps in neutron matter are also available [2, 3, 4].

An alternative many-body scheme for the calculation of the properties of nuclear matter is the self-consistent in-medium $T$ matrix [2, 3, 4, 5, 6, 7].

\[ T = V + VGGT \]  

where the Green's functions $G^{-1} = \omega - p^2/2m - \Sigma$ in the ladder propagator are dressed by the self-consistent self-energy

\[ i\Sigma = Tr[T_A G]. \]  

The $T$-matrix approach is a conserving approximation [2] and gives consistent single-particle properties. The details of the equations and of the solution in the real time formalism can be found in [2, 3]. Since the self-energy has a non-zero imaginary part, we obtain in the $T$-matrix approximation a nontrivial single-particle spectral function.

At low densities, the $T$-matrix ladder diagrams are dominated by the particle-particle propagators and therefore should reduce to the $G$-matrix result (if the influence of the nucleon dressing is small at low densities). However, at higher densities the $\Phi$-derivable $T$-matrix approximation takes into account some diagrams of higher order in the hole-line expansion. It is then instructive to compare the results of the $T$-matrix calculation with the corresponding BHF calculation for a realistic interaction.

We perform self-consistent iterations of the $T$-matrix equations in pure neutron matter, using the separable Paris interaction [14, 15] and the same methods as in Ref. [4]. In Fig. 1 the resulting energy per nucleon is plotted as a function of the Fermi momentum. It is compared to the BHF calculation with the continuous choice of the single particle energy, using the same interaction and numerical grids. An excellent consistency of the two methods is found at low densities, as expected. This provides a powerful check of the methods used for the solution of integral equations, which for the $T$-matrix case involve refined numerical algorithms. At higher densities the neutron matter in the $T$-matrix calculation is less bound, leading to a considerably harder equation of state. It is known that the BHF results with the continuous choice are not modified considerably by 3 hole-lines corrections [14, 15, 6]. The $T$-matrix approximation is a different summation of some of the 3 and more hole-lines diagrams. We obtain the first quantitative estimate of the difference between the self-consistent $T$-matrix approximation and the BHF approach. Since the latter one is believed to be close to the converged result, we find an estimate of the accuracy of the $T$-matrix calculation.

Below $k_F = 1.3\text{fm}^{-1}$ the difference in the energy per particle from the two calculations is less than 2MeV. The discrepancy at higher densities could be attributed to the ring diagrams contribution. Also at higher densities meaningful results can be obtained only after inclusion of higher partial waves and three-body forces effects. In Fig. 1 results of modern variational calculations [3] and of a BHF calculation including contributions from 3 hole-

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FIG. 1: The energy per nucleon in pure neutron matter as a function of the Fermi momentum for the T-matrix (dotted line) and the BHF (solid line) calculations using the Paris potential. The dotted line represents the results of a variational chain summation [1] and the dashed-dotted line the results of a BHF calculation with 3 hole-lines contribution [17], both for a two-body Argonne V18 interaction.

lines diagrams [17] are also shown. Both methods are using a two-body Argonne V18 potential and give similar results. As shown in Ref. [14] the continuous BHF results are almost indistinguishable from the full BHF calculation with 3 hole-lines contributions included. Therefore, the difference between our BHF result (solid line in Fig. 1) and the variational or next order BHF results must be attributed to a different interaction used, and to the limited number of partial waves taken. The T-matrix energy per particle is larger than obtained in other approaches. Most importantly it is significantly larger than the continuous BHF calculation with the same interaction. It indicates that the T-matrix calculation does not give the correct energy per particle at higher densities.

For the analysis of superfluidity, we use the spectral function obtained in the T-matrix calculation. As noted above, the T-matrix approach gives reliable results for the single-particle properties in the medium. The effective mass in our calculation decreases from 1.07 to 1.02 nucleon masses with increasing density, which is only about 0.05 more than the corresponding BHF effective mass.

The superfluid gap equation with full spectral functions is

\[ \Delta(p) = \int \frac{d^3k}{(2\pi)^3} A(k, \omega) \frac{1 - 2f(E_k)}{2E_k} \Delta(k) \]  

where \( A(p, \omega) \) denotes the spectral function of the nucleon, including the diagonal self-energy \( \Sigma(p, \omega) \) (obtained in the T-matrix approximation) and the off-diagonal self-energy \( \Delta(p) \) (obtained from Eq. (3) itself): \( A(p, \omega) \) is the spectral function of the nucleon dressed with the diagonal self-energy only. Self-energy effects in the gap equation were studied in a number of papers [18, 19, 20, 21]. It was found that generally the superfluid energy gap is reduced by the in-medium dressing of nucleons. The mechanism of this reduction can be understood easily from the effective quasiparticle gap equation [18, 20]

\[ \hat{\Delta}(p) = Z_p \Delta(p) \]  

which is obtained as the quasiparticle limit of the gap equation with dressed propagators [18]. The superfluid energy gap is

\[ \hat{\Delta}(p) = Z_p \Delta(p) \]  

where

\[ Z_p = \left(1 - \frac{\partial \Sigma(p, \omega_p)}{\partial \omega}|_{\omega=\omega_p}\right)^{-1} \]  

and

\[ E_p = \sqrt{(\omega_p - \mu)^2 + \hat{\Delta}(p)^2} . \]

The usual BCS equation is obtained by putting \( Z_p = 1 \) in the above equations. In that case, the only influence of the medium comes through the modification of the dispersion relation \( \omega_p \).

Dispersive self-energy corrections to the gap equation are twofold. First, the superfluid energy gap is multiplied by the quasiparticle strength \( Z_{pF} < 1 \) [18], and second, the interaction strength in the gap equation is reduced by a factor \( Z_pZ_k \) [18]. In the case of symmetric nuclear matter at normal density the reduction of the superfluid gap due to self-energy effects is of about one order of magnitude [21].

In Fig. 3 we present the results of an analogous calculation for the \(^1S_0\) pairing in neutron matter for two interaction potentials [24, 25]. It is known that for the \(^1S_0\) gap different interaction potentials lead to similar superfluid gaps [4]. We recall that the spectral functions and the single-particle energies used in the gap equation are obtained from a separate self-consistent calculation.
in the normal phase. We find the $Z_p$ factor at the Fermi momentum $\approx 0.9$ for the range of densities studied. It is close to the one obtained from the BHF calculation without rearrangement terms [21]. The rearrangement terms in the BHF approach reduce this value further [20] (the $T$-matrix result includes the rearrangement contribution to the self-energy). We find a smaller $Z$ factor than the authors of reference [19]. The obtained values of $Z_p$ translate directly into the resulting superfluid gap, which we find in between the results of Refs. [3] and [20]. The average reduction factor is 0.7 in the region of the maximal gap. The effective gap equation (4) is an excellent approximation to the solution of the gap equation with full spectral functions (3). It is due to the fact that in this region of densities, the gap equation kernel is dominated by contributions close to the Fermi momentum, where the quasiparticle limit applies. We could not perform the self-consistent $T$-matrix calculation at densities below 0.016fm$^{-3}$; the expected disappearance of self-energy effects in the gap equation at low densities could not be explicitly demonstrated. This is in contrast to the results for the energy per particle where the BHF approach and the self-consistent $T$-matrix calculation merge at low density [3] (Fig 1).

In the gap closure region the ratio of the BCS gap to the gap obtained with dressed propagators is large. Such a strong reduction of the gap was found in symmetric nuclear matter at normal density [21], which is close to gap closure for the $^3S_1 - ^3D_1$ gap. Also the effective gap equation (4) and the gap equation with full spectral functions (3) give markedly different results.

In summary, we study pure neutron matter using the in-medium $T$-matrix approximation. We present a comparison of the energy per nucleon for neutron matter in the BHF and $T$-matrix approaches. We find similar binding energies at low densities, but a stiffer equation of state for the $T$-matrix calculation at higher densities. This is an explicit confirmation of the expected similarities and discrepancies between the two approaches.

We use self-consistent spectral functions in neutron matter to calculate the superfluid gap from the gap equation with dressed propagators. This allows us to test the accuracy of the effective gap equation (4), which incorporates to some extent self-energy effects. Using full spectral functions, we find, in the maximal gap region, a reduction of the gap of about 30% and a very good agreement with the effective gap equation. In the gap closure region, we find a stronger reduction. In this region the effective gap equation (4) is a less reliable estimate of self-energy effects. It should be remembered that also other in medium effects modify the superfluid gap in neutron matter [25, 26, 27, 28].

Acknowledgments

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