AdS/CFT Correspondence and Supersymmetry

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Abstract

We use the group-theoretic interpretation of the AdS/CFT correspondence which we proposed earlier in order to lift intertwining operators acting between boundary conformal representations to intertwining operators acting between bulk conformal representations. Further, we present the classification of the positive energy (lowest weight) unitary irreducible representations of the $D = 6$ superconformal algebras $osp(8^*/2N)$.

1 Introduction

Recently there was renewed interest in (super)conformal field theories in arbitrary dimensions. This happened after the remarkable proposal in [1].

1Lectures at the 1st Summer School in Modern Mathematical Physics, Sokobanja, Yugoslavia, 13-25.8.2001; to appear in the Proceedings of the School.

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according to which the large $N$ limit of a conformally invariant theory in $d$ dimensions is governed by supergravity (and string theory) on $d+1$-dimensional $AdS$ space (often called $AdS_{d+1}$) times a compact manifold. Actually the possible relation of field theory on $AdS_{d+1}$ to field theory on $M_d$ has been a subject of long interest, cf., e.g., [2, 3, 4], and also [3] for discussions motivated by recent developments, and additional references. The proposal of [4] was elaborated in [5] and [6] where was proposed a precise correspondence between conformal field theory observables and those of supergravity: correlation functions in conformal field theory are given by the dependence of the supergravity action on the asymptotic behavior at infinity. More explicitly, a conformal field $O$ corresponds to an AdS field $\phi$ when there exists a conformal invariant coupling $\int \phi_0 O$ where $\phi_0$ is the value of $\phi$ at the boundary of $AdS_{d+1}$ (17). Furthermore, the dimension $\Delta$ of the operator $O$ is given by the mass of the particle described by $\phi$ in supergravity [7]. After these initial papers there was an explosion of related research of which interest to us are two aspects: 1) calculation of conformal correlators from AdS (super)gravity and various questions of holography, cf., e.g., [8-78]; 2) matching of supergravity and superstring spectra with superconformal theories, cf., e.g., [79-140].

For the first aspect of the AdS/CFT correspondence one of the main features furnishing it is that the boundary $M_d$ of $AdS_{d+1}$ is in fact a copy of $d$-dimensional Minkowski space (with a cone added at infinity); the symmetry group $SO(d, 2)$ of $AdS_{d+1}$ acts on $M_d$ as the conformal group. The fact that $SO(d, 2)$ acts on $AdS_{d+1}$ as a group of ordinary symmetries and on $M_d$ as a group of conformal symmetries means that there are two ways to get a physical theory with $SO(d, 2)$ symmetry: in a relativistic field theory (with or without gravity) on $AdS_{d+1}$, or in a conformal field theory on $M_d$.

In an earlier paper [19] we gave a group-theoretic interpretation of the above correspondence. In fact such an interpretation is partially present in [138] for the $d = 3$ Euclidean version of the AdS/CFT correspondence in the context of the construction of discrete series representations of the group $SO(4, 1)$ involving symmetric traceless tensors of arbitrary nonzero spin. In short the essence of our interpretation is that the above correspondence is a relation of representation equivalence between the representations describing the fields $\phi$, $\phi_0$ and $O$. There are actually two kinds of equivalences. The first kind is new (besides the example from [138] mentioned above) and was proved in [19] - it is between the representations describing the bulk fields and the boundary fields. The second kind is well known - it is the
equivalence between boundary conformal representations which are related by restricted Weyl reflections, the representations here being the coupled fields $\phi_0$ and $\mathcal{O}$. Our interpretation means that the operators relating these fields are intertwining operators between (partially) equivalent representations. Operators giving the first kind of equivalence for special cases were actually given in, e.g., [7, 8, 9, 12, 16] - in [13] they were constructed in a more general setting from the requirement that they are intertwining operators. The operators giving the second kind of equivalence are provided by the standard conformal two-point functions. Using both equivalences we have found that the bulk field has naturally two boundary fields, namely, the coupled fields $\phi_0$ and $\mathcal{O}$, the limits being governed by the corresponding conjugated conformal weights $d - \Delta$ and $\Delta$. Thus, from the point of view of the bulk-to-boundary correspondence the coupled fields $\phi_0$ and $\mathcal{O}$ are generically on an equal footing. [The appearance of two boundary fields was used later in [22] in a slightly different context, namely, that the theory with the same classical AdS Lagrangian can be interpreted in terms of two different CFT's with the conjugated dimensions. This is possible only for sufficiently negative AdS-mass-squared, so that both dimensions would not be lower than the unitarity bound.]

In the present paper we review also the results of [29] in order to lift intertwining operators acting between boundary conformal representations to intertwining operators acting between bulk conformal representations.

For the second aspect of the AdS/CFT correspondence one of the most important tasks is the classification of the UIRs of the corresponding superalgebras. Particularly important are those for $D \leq 6$ since in these cases the relevant superconformal algebras satisfy [3] the Haag-Lopuszanski-Sohnius theorem [40]. Until recently such classification was known only for the $D = 4$ superconformal algebras $su(2,2/N)$ [41] (for $N = 1$), [42] [43] [44] [45]. Recently, the classification for $D = 3, 5, 6$ was given [46] but the results were conjectural and there was not enough detail in order to check these conjectures. In view of the interesting applications [110, 120, 132] of $D = 6$ unitary irreps to the analysis of OPEs and 1/2 BPS operators we decided to reexamine the list of UIRs of the $D = 6$ superconformal algebras $osp(8^*/2N)$ in detail [43]. We confirm all but one of the conjectures of [46] and thus, we give the final list of UIRs for $D = 6$. Our main tool is the explicit construction of the norms. This, on the one hand, enables us to prove the unitarity list, and, on the other hand, enables us to give explicitly the states of the irreps. We give a brief summary of [43] in
2 Conformal field theory representations

As in [19] we consider the Euclidean version of the AdS/CFT correspondence. For definiteness we use the following defining relation of the Sitter group $G$:

$$G = \{g \in GL(d+2, \mathbb{R}) \mid g^\dagger \eta g = \eta = \text{diag}(-1, \ldots, -1, 1), \quad \det g = 1, \quad g_{d+2,d+2} \geq 1\}$$  \hspace{1cm} (1)

Thus, $G = SO_e(d+1,1)$, i.e., it is the identity component of $O(d+1,1)$, ($g^\dagger$ is the transposed of $g$). Note that for $d$ even some expressions are simpler if we work with the extended de Sitter group:

$$G' = \{g \in GL(d+2, \mathbb{R}) \mid g^\dagger \eta g = \eta, \quad g_{d+2,d+2} \geq 1\}$$  \hspace{1cm} (2)

which includes reflections of the first $d+1$ axes. The representations of $G$ used in conformal field theory are called (in the representation theory of semisimple Lie groups) generalized principal series representations (cf. [147]). In [138, 148, 149] they were called elementary representations (ERs). They are obtained by induction from the subgroup $P = MAN$, where $M = SO(d)$ is the Euclidean Lorentz group, $A$ is the one-dimensional dilatation group, $N$ is the group of special conformal transformations (isomorphic to $\mathbb{R}^d$), $P$ is called a parabolic subgroup of $G$. The induction is from unitary irreps of $M = SO(d)$, from arbitrary (non-unitary) characters of $A$, and trivially from $N$. There are several realizations of these representations. We give now the so-called noncompact picture of the ERs - it is the one actually used in physics.

The representation space of these induced representations consists of smooth functions on $\mathbb{R}^d$ with values in the corresponding finite-dimensional representation space of $M$, i.e.:

$$C_\chi = \{f \in C^\infty(\mathbb{R}^d, V_\mu)\}$$  \hspace{1cm} (3)

where $\chi = [\mu, \Delta]$, $\Delta$ is the conformal weight, $\mu$ is a unitary irrep of $M$, $V_\mu$ is the finite-dimensional representation space of $\mu$. In addition, these functions have special asymptotic expansion as $x \to \infty$. The leading term
of this expansion is $f(x) \sim \frac{1}{(x^2)^{\Delta}} f_0$, (for more details we refer to [138, 148, 149]). The representation $T^\chi$ acts in $C^\chi$ by:

$$(T^\chi(g)f)(x) = |a|^{-\Delta} \cdot D^\mu(m) f(x')$$

(4)

where the nonglobal Bruhat decomposition $g = \tilde{n} m a n$ is used:

$$g^{-1}\tilde{n} = \tilde{n} m a^{-1} n^{-1}, \quad g \in G, \tilde{n}, \tilde{n}' \in \tilde{N}, m \in M, a \in A, n \in N$$

(5)

where $\tilde{N}$ is the abelian group of Euclidean translations (isomorphic to $\mathbb{R}^d$), $D^\mu(m)$ is the representation matrix of $\mu$ in $V^\mu$.

Note that the representation data given by $\chi = [\mu, \Delta]$ fixes also the value of the Casimir operators $C_i$ in the ER $C^\chi$, independently of the latter reducibility. For later use we write:

$$C_i f(x) = \lambda_i(\mu, \Delta) f(x), \quad i = 1, \ldots, \text{rank } G = [\frac{d}{2}] + 1,$$

(6)

Next, we would like to recall the general expression of the conformal two-point function $G^\chi(x_1 - x_2)$ (for special cases cf. [150, 151, 152, 153], for the general formula with special stress on the role of the conformal inversion, cf. [154], also [138]):

$$G^\chi(x) = \frac{\gamma^\chi}{(x_2)^\Delta} D^\mu(r(x))$$

(7)

$$r(x) = \begin{pmatrix} \tilde{r}(x) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in M, \quad \tilde{r}(x) = \frac{1}{x^2} x_i x_j - \delta_{ij}$$

where $\gamma^\chi$ is an arbitrary constant for the moment. (Note that for $d$ even $r(x) \in O(d)$, so we work with $G'$, cf. [2].)

Finally, we note the intertwining property of $G^\chi(x)$. Namely, let $\tilde{\chi}$ be the representation conjugated to $\chi$ by a restricted Weyl reflection, i.e., by the nontrivial element of the restricted Weyl group $W(G, A)$ [138]. Then we have:

$$\tilde{\chi} \doteq [\tilde{\mu}, d - \Delta], \quad \text{for } \chi = [\mu, \Delta],$$

(8)

where $\tilde{\mu}$ is the mirror image representation of $\mu$. (For $d$ odd $\tilde{\mu} \cong \mu$, while for $d$ even $\tilde{\mu}$ may be obtained from $\mu$ by exchanging the representation
labels of the two distinguished Dynkin nodes of \( SO(d) \).) Then there is the following intertwining operator \[154, 138\]:

\[
G \chi : C \tilde{\chi} \rightarrow C \chi, \quad T^\chi(g) \circ G \chi = G \chi \circ T ^\tilde{\chi}(g), \quad \forall g,
\]

\[
(G \chi f)(x_1) = \int G \chi(x_1 - x_2) f(x_2) \, dx_2, \quad dx \equiv d^d x,
\]

the last line uses our symbolic notation for partial equivalence between \( \chi \) and \( \tilde{\chi} \). Note that because of this equivalence the values of all Casimirs coincide:

\[
\lambda_i(\bar{\mu}, d - \Delta) = \lambda_i(\mu, \Delta), \quad \forall i.
\]

\section{Representations on the bulk space}

In the previous section we discussed representations on \( \mathbb{R}^d \cong \tilde{N} \) induced from the parabolic subgroup \( MAN \) which is natural since the abelian subgroup \( \tilde{N} \) is locally isomorphic to the factor space \( G/MAN \) (via the Bruhat decomposition). Similarly, it is natural to discuss representations on the bulk space \( S \cong \tilde{N}A \) which are induced from the maximal compact subgroup \( K = SO(d + 1) \) since the solvable group \( \tilde{N}A \) is isomorphic to the factor space \( G/K \) via the global \textit{Iwasawa} decomposition \( G = \tilde{N}AK \) (cf. the details in [19]). Namely, we consider the representation spaces:

\[
\hat{\mathcal{C}}_\tau = \{ \phi \in C^\infty(\mathbb{R}^d \times \mathbb{R}_{>0}, U_\tau) \}
\]

where \( \tau \) is an arbitrary unitary irrep of \( K \), \( U_\tau \) is the finite-dimensional representation space of \( \tau \), with representation action:

\[
(\hat{T}_\tau^r(g)\phi)(x, |a|) = \hat{D}_\tau^r(k)\phi(x', |a'|)
\]

where the Iwasawa decomposition is used:

\[
g^{-1}\tilde{n}_x a = \tilde{n}_x' a' k^{-1}, \quad g \in G, \ k \in K, \ \tilde{n}_x, \tilde{n}_x' \in \tilde{N}, \ a, a' \in A
\]

\[3\text{Note that in [19] the bulk space } S \cong \tilde{N}A \cong SO(d + 1, 1)/SO(d + 1) \text{ was called de Sitter space, though in the literature the latter name is used for the space } SO(d + 1, 1)/SO(d, 1). \text{ The latter space was used recently for extensive study of the so-called } dS/CFT \text{ correspondence, cf., e.g., [155] [156] [157] [158].]
and \( \hat{D}^\tau(k) \) is the representation matrix of \( \tau \) in \( U_\tau \). However, unlike the ERs, these representations are reducible, and to single out an irrep equivalent, say, a subrepresentation of an ER, one has to look for solutions of the eigenvalue problem related to the Casimir operators.

In the actual implementation of (13) and (14) we use the following parametrization of \( k \):

\[
k = \begin{pmatrix}
k_{ij} & k_{i,d+1} & 0 \\
k_{d+1,j} & k_{d+1,d+1} & 0 \\
0 & 0 & 1
\end{pmatrix} \in K, \quad (k_{\alpha\beta}) \in SO(d+1).
\]

Further we shall need also the unique decomposition:

\[
k = m(k)k_f, \quad m(k) = \begin{pmatrix}
\tilde{m}(k) & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \in M, \quad k_f = \begin{pmatrix}
\tilde{k}_f & 0 \\
0 & 1
\end{pmatrix} \in K
\]

representing the decomposition of \( K \) into its subgroup \( M \) and the coset \( K/M : K \cong M K/M \). Explicitly, we have (for \( k_{d+1,d+1} \neq -1 \)):

\[
\tilde{m}(k) = \left(k_{ij} - \frac{1}{1 + k_{d+1,d+1}} k_{i,d+1} k_{d+1,j}\right)
\]

\[
\tilde{k}_f = \left(\delta_{ij} - \frac{2}{1 + x_i x_j} x_i x_j - \frac{2}{1 - x_i^2} x_i^2 \frac{1 - x_i^2}{1 + x_i^2}\right) \tilde{x}_x, \quad x \in \mathbb{R}^d,
\]

\[x_i = k_{d+1,i}/(1 + k_{d+1,d+1}).\]

Note that \( \tilde{x}_x = \left(\tilde{k}_x \quad 0\right) \) appeared in (1.30a) of [138].

Further, we would like to extract from \( \hat{C}_\tau \) a representation that may be equivalent to \( C_\chi, \chi = [\mu, \Delta] \). The first condition for this is that the \( M \)-representation \( \mu \) is contained in the restriction of the \( K \)-representation \( \tau \) to \( M \). Another condition is that the two representations would have the same Casimir values \( \lambda_i(\mu, \Delta) \). Having in mind the degeneracy of Casimir values for partially equivalent representations (e.g., (11)) we add also the appropriate asymptotic condition. Furthermore, from now on we shall suppose that \( \Delta \) is real. Thus, we shall use the representations:

\[
\hat{C}_\chi^\tau = \{ \phi \in \hat{C}_\tau : C_i \phi(x, |a|) = \lambda_i(\mu, \Delta) \phi(x, |a|), \quad \forall i, \quad \mu \in \tau|_M, \\
\phi(x, |a|) \sim |a|^\Delta \varphi(x) \text{ for } |a| \to 0 \}
\]
Unlike their action on the ERs $C_\chi$ the Casimirs $C_i$ here are differential operators and the elements of $\hat{C}_\chi^{\tau}$ are solutions of the equations in (18).

In [13] it was shown that generically the functions in (18) have also a second limit with $\Delta \rightarrow d - \Delta$:

$$\tilde{\varphi}(x) = \lim_{|a| \rightarrow 0} |a|^{\Delta - d} \phi(x, |a|)$$  \hspace{1cm} (19)

For generic representations this establishes the following important relation:

$$\hat{C}_\chi^{\tau} = \hat{C}_{\tilde{\chi}}^{\tau}, \quad \chi = [\mu, \Delta], \quad \tilde{\chi} = [\tilde{\mu}, d - \Delta].$$  \hspace{1cm} (20)

For non-generic representations the second asymptotic expansion of $\phi$ contains logarithms (cf., e.g., (7.45) of [13] and [10, 15]), and then the representations $\chi$ and $\tilde{\chi}$ are only partially equivalent:

$$\hat{C}_\chi^{\tau} \simeq \hat{C}_{\tilde{\chi}}^{\tau}.$$  \hspace{1cm} (21)

4 Intertwining relations between conformal and bulk representations

This Section contains the main result of [19], explicating the relations between CFT and bulk representations as intertwining relations. We first give the intertwining operator from the bulk to the CFT realization. The operator is mapping a function on the bulk space to its boundary value and was used in a restricted sense (explained below) in many papers, starting from [7].

**Theorem:** Let us define the operator:

$$L_\chi^{\tau} : \hat{C}_\chi^{\tau} \rightarrow C_\chi,$$  \hspace{1cm} (22)

with the following action:

$$\left( L_\chi^{\tau} \phi \right)(x) = \lim_{|a| \rightarrow 0} |a|^{-\Delta} \Pi_\mu^{\tau} \phi(x, |a|)$$  \hspace{1cm} (23)

where $\Pi_\mu^{\tau}$ is the standard projection operator from the $K$-representation space $U_\tau$ to the $M$-representation space $V_\mu$, which acts in the following way on the $K$-representation matrices:

$$\Pi_\mu^{\tau} \hat{D}(k) = D^\mu(m(k)) \Pi_\mu^{\tau} \hat{D}(k)$$  \hspace{1cm} (24)
where we have used (16). Then \( L^\tau_\chi \) is an intertwining operator, i.e.:
\[
L^\tau_\chi \circ \hat{T}^\tau(g) = T^\chi(g) \circ L^\tau_\chi, \quad \forall g \in G .
\] (25)

In addition, in (23) the operator \( \Pi^\tau_\mu \) acts in the following truncated way:
\[
\Pi^\tau_\mu \hat{D}^\tau(k) = D^\mu(m(k)) \Pi^\tau_\mu
\] (26)
The proof is given in [19].

Next we consider the operator inverse to \( L^\tau_\chi \) which would restore a function on the bulk space from its boundary value, as discussed in [7-18]. Again what was new in [19] was that the operator was defined as intertwining operator between exactly defined spaces in a more general setting. Moreover, the operator was constructed just from the condition that it is an intertwining integral operator. Namely, we started with the operator:
\[
\tilde{L}^\tau_\chi : C_\chi \rightarrow \hat{C}^\tau_\chi ,
\] (27)
using the following Ansatz:
\[
(\tilde{L}^\tau_\chi f)(x,|a|) = \int K^\tau_\chi(x,|a|;x') f(x') \, dx'
\] (28)
where \( K^\tau_\chi(x,|a|;x') \) is a linear operator acting from the space \( V_\mu \) to the space \( U^\tau_\chi \), and supposed that \( \tilde{L}^\tau_\chi \) is an intertwining operator, i.e.:
\[
\hat{T}^\tau(g) \circ \tilde{L}^\tau_\chi = \tilde{L}^\tau_\chi \circ T^\chi(g) , \quad \forall g \in G .
\] (29)

From this we obtained (cf. the details in [19]) that \( K^\tau_\chi \) is fixed up to an overall multiplicative constant \( N^\tau_\chi \) and explicitly is:
\[
K^\tau_\chi(x,|a|;x') = K^\tau_\chi(x-x',|a|)
\]
\[
K^\tau_\chi(x,|a|) = N^\tau_\chi \left( \frac{|a|}{x^2 + |a|^2} \right)^{d-\Delta} \hat{D}^\tau(k_{x-\frac{a}{|a|}}) \Pi^\mu_\tau
\] (30)
where \( \Pi^\mu_\tau \) is the canonical embedding operator from \( V_\mu \) to \( U^\tau_\chi \), such that \( \Pi^\mu_\tau \circ \Pi^\mu_\tau = 1_\mu \), and \( k_x \) is given in (17).

The above operator exists for arbitrary representations \( \tau \) of \( K = SO(d+1) \) which contain the representation \( \mu \) of \( M = SO(d) \). We use the standard
The $SO(p)$ representation parametrization: $[\ell_1, \ldots, \ell_p]$, ($\hat{p} \equiv \lceil \frac{p}{2} \rceil$), where all $\ell_j$ are simultaneously integer or half-integer, all are positive except for $p$ even when $\ell_1$ can also be negative, and they are ordered: $|\ell_1| \leq \ell_2 \leq \ldots \leq \ell_{\hat{p}}$. The condition that $\tau = [\ell_1', \ldots, \ell_{\hat{d}}']$, ($\hat{d} \equiv \lceil \frac{d+1}{2} \rceil$), contains $\mu = [\ell_1, \ldots, \ell_{\hat{d}}]$, ($\tilde{\ell} \equiv \lceil \frac{d}{2} \rceil$), explicitly is:

$$
|\ell_1'| \leq \ell_1 \leq \ldots \leq \ell_{\hat{d}} \leq \ell_1', \quad \text{d odd, } \hat{d} = \tilde{d} + 1
$$
$$
-l_1' \leq \ell_1 \leq \ell_1' \leq \ldots \leq \ell_{\hat{d}} \leq \ell_1', \quad \text{d even, } \hat{d} = \tilde{d}
$$

(31)

If one is primarily concerned with the ERs $\chi = [\mu, \Delta]$ it is convenient to chose a 'minimal' representation $\tau(\mu)$ of $K = SO(d+1)$ containing $\mu$. This depends on the parity of $d$. Thus, for $\mu$ as above, when $d$ is odd we would choose:

$$
\tau(\mu) = [\ell_1, \ell_1, \ldots, \ell_{\hat{d}}] \quad \text{or} \quad \tilde{\tau}(\mu) = [-\ell_1, \ell_1, \ldots, \ell_{\hat{d}}], \quad \mu \cong \tilde{\mu}, \quad (32)
$$

while for even $d$ we would choose:

$$
\tau(\mu) = ||\ell_1|, \ell_2, \ldots, \ell_{\hat{d}}| = \tau(\tilde{\mu}) \cong \tilde{\tau}(\mu) = \tilde{\tau}(\tilde{\mu}), \quad \tilde{\mu} = [-\ell_1, \ell_2, \ldots, \ell_{\hat{d}}].
$$

(33)

Thus, in the odd $d$ case for each $\mu$ we would choose between two $K$-irreps which are mirror images of one another, while in the even $d$ case to each two mirror-image irreps of $M$ we choose one and the same irrep of $K$.

The explicit examples which appeared in the literature are actually in the cases in which $\tau = \tau(\mu)$, e.g., $[7, 8, 9, 12, 16]$, though there is no such interpretation as we have here.

5 Intertwining operators on the bulk space

In this section we review [29]. We show how to lift intertwining operators acting between boundary conformal representations to intertwining operators acting between bulk conformal representations. Of course, for generic representations there is nothing to do, since the pairs of equivalent boundary representations $\chi$ and $\tilde{\chi}$ are equivalent to the same bulk representation. However, for nongeneric boundary representations when $\chi$ and $\tilde{\chi}$ are partially equivalent but not equivalent, the situation is much more interesting. In this case besides the pair $\chi_0 \equiv \chi$ and $\tilde{\chi}_0 \equiv \tilde{\chi}$ there exist $\hat{d}$ more such
pairs $\chi_i$ and $\tilde{\chi}_i$ ($i = 1, ..., \tilde{d}$) so that these $2\tilde{d} + 2$ ERs are partially equivalent between themselves, (for the explicit parametrization of these ERs we refer to [138, 148] for early partial cases, and [159] for the general case). These partial equivalences are realized by $2\tilde{d}$ intertwining differential operators. The latter come in pairs, i.e., if there exists an intertwining differential operator $D$ acting from $C_{\chi_j}$ to $C_{\chi_{j'}}$ ($j, j' = 0, 1, ..., \tilde{d}; j \neq j'$), then there exists an intertwining differential operator $D'$ acting from $C_{\tilde{\chi}_j}$ to $C_{\tilde{\chi}_{j'}}$.

Now we shall use these operators and the operators bulk↔boundary operators of the previous section to build operators acting between bulk representations. For notational simplicity we write $\chi, \chi'$ instead of $\chi_j, \chi_{j'}$. We start with the operator $D$ acting from $C_\chi$ to $C_{\chi'}$, where $\chi' = [\mu', \Delta']$, and lift it to an operator $\tilde{D}$ acting from $\hat{C}_\tau \chi$ to $\hat{C}_\tau \chi'$, where $\tau'$ is a UIR of $K$ containing $\mu'$. Explicitly, we have:

$$\tilde{D} : \hat{C}_\tau \chi \to \hat{C}_\tau \chi', \quad \tilde{D} = \tilde{L}_{\chi'} \circ D \circ L_{\chi}$$

(34)

Analogously, we have for the operator $D'$ acting from $\hat{C}_{\chi'}$ to $\hat{C}_{\tilde{\chi}'}$:

$$\tilde{D}' : \hat{C}_{\tilde{\chi}'} \to \hat{C}_{\tilde{\chi}}', \quad \tilde{D}' = \tilde{L}_{\tilde{\chi}} \circ D' \circ L_{\chi}$$

(35)

The explicit parametrization and expressions of all possible operators $\tilde{D}, \tilde{D}'$ will be given elsewhere.

Finally, we notice that all operators that we have used may be found on
the following diagram:

\[
\begin{array}{ccc}
\hat{C}_\tilde{\chi} & \simeq & \hat{C}_\chi \\
L^\tau_\tilde{\chi} & \leftrightarrow & \bar{L}^\tau_\tilde{\chi} \\
\tilde{L}^\tau_\chi & \leftrightarrow & L^\tau_\chi \\
C_{\tilde{\chi}} & \downarrow & \bar{G}_{\tilde{\chi}} & \leftarrow & \bar{G}_\chi & \rightarrow & C_{\chi} \\
\mathcal{D}' \uparrow & \downarrow & \mathcal{D} \\
C_{\tilde{\chi}'} & \downarrow & \bar{G}_{\tilde{\chi}'} & \leftarrow & \bar{G}_{\chi'} & \rightarrow & C_{\chi'} \\
L^\tau_{\tilde{\chi}'} & \leftrightarrow & \bar{L}^\tau_{\tilde{\chi}'} \\
\tilde{L}^\tau_{\chi'} & \leftrightarrow & L^\tau_{\chi'} \\
\hat{C}^\tau_{\tilde{\chi}'} & \simeq & \hat{C}^\tau_{\chi'}
\end{array}
\]

(36)

6 Positive energy UIRs of D=6 conformal supersymmetry

In this section we review [135]. The superconformal algebras in \( D = 6 \) are \( \mathcal{G} = \text{osp}(8^* / 2N) \) (real forms of \( \text{osp}(8/2N) \simeq D(4, N) \), [160]). We label their physically relevant representations by the signature:

\[
\chi = [d; n_1, n_2, n_3; a_1, \ldots, a_N]
\]

(37)

where \( d \) is the conformal weight, \( n_1, n_2, n_3 \) are non-negative integers which are Dynkin labels of the finite-dimensional irreps of the \( D = 6 \) Lorentz
algebra $so(5, 1)$, and $a_1, ..., a_N$ are non-negative integers which are Dynkin labels of the finite-dimensional irreps of the internal (or $R$) symmetry algebra $usp(2N)$. The even subalgebra of $osp(8^*/2N)$ is the algebra $so^*(8) \oplus usp(2N)$, and $so^*(8) \cong so(6, 2)$ is the $D = 6$ conformal algebra.

In [135] we gave a constructive proof for the UIRs of $osp(8^*/2N)$ following the methods used for the $D = 4$ superconformal algebras $su(2, 2/N)$, cf. [143, 144, 145]. The main tool is an adaptation of the Shapovalov form on the Verma modules $V^x$ over the complexification $G_C = osp(8/2N)$ of $G$. The UIRs are realized as irreducible factor-modules of the Verma modules $V^x$. (The reducibility conditions of $V^x$ are derived according to [161].) The main result is:

**Theorem:** All positive energy unitary irreducible representations of the conformal superalgebra $osp(8^*/2N)$ characterized by the signature $\chi$ in (37) are obtained for real $d$ and are given in the following list:

$$
d \geq d_{11}^- = \frac{1}{2}(3n_1 + 2n_2 + n_3) + 2r_1 + 6, \quad \text{no restrictions on } n_j \quad (38)
d = d_{21}^- = \frac{1}{2}(n_3 + 2n_2) + 2r_1 + 4, \quad n_1 = 0 \quad (39)
d = d_{31}^- = \frac{1}{2}n_3 + 2r_1 + 2, \quad n_1 = n_2 = 0 \quad (40)
d = d_{41}^- = 2r_1, \quad n_1 = n_2 = n_3 = 0 \quad (41)
$$

where $d_{j1}^-$ are the four distinguished reducibility points of the Verma modules:

$$
d_{j1}^- \doteq \frac{1}{2}(3n_1 + 2n_2 + n_3) - 2 \sum_{s=1}^{j-1} n_j + 2r_1 + 8 - 2j \quad (42)
$$

**Remark:** For $N = 1, 2$ the Theorem was conjectured by Minwalla [146], except that he conjectured unitarity also for the open interval $(d_{31}^-, d_{21}^-)$ with conditions on $n_j$ as in (40). We should note that this conjecture could be reproduced neither by methods of conformal field theory [110], nor by the oscillator method [162] (cf. [146]), and thus was in doubt. To compare with the notations of [146] one should use the following substitutions: $n_1 = h_2 - h_3$, $n_2 = h_1 - h_2$, $n_3 = h_2 + h_3$, $r_1 = k$, and $h_j$ are all integer or all half-integer. The fact that $n_j \geq 0$ for $j = 1, 2, 3$ translates into: $h_1 \geq h_2 \geq |h_3|$, i.e., the parameters $h_j$ are of the type often used for representations of $so(2N)$ (though usually for $N \geq 4$). Note also that the statement of the Theorem is arranged in [146] according to the possible values of $n_i$ first and then the possible values of $d$. To compare with the notation of [110] we use the substitution $(n_1, n_2, n_3) \rightarrow (J_3, J_2, J_1)$. Some UIRs at the four exceptional
points \( d_{ij} \) were constructed in \([4]\) by the oscillator method (some of these were identified with Cartan-type signatures like \((37)\) in, e.g., \([140], [126]\)).

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