Continuous variable methods in relativistic quantum information: characterization of quantum and classical correlations of scalar field modes in noninertial frames

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Abstract
We review a recently introduced unified approach to the analytical quantification of correlations in Gaussian states of bosonic scalar fields by means of Rényi-2 entropy. This allows us to obtain handy formulae for classical, quantum, total correlations, as well as bipartite and multipartite entanglement. We apply our techniques to the study of correlations between two modes of a scalar field as described by observers in different states of motion. When one or both observers are in uniform acceleration, the quantum and classical correlations are degraded differently by the Unruh effect, depending on which mode is detected. Residual quantum correlations, in the form of quantum discord without entanglement, may survive in the limit of an infinitely accelerated observer Rob, provided they are revealed in a measurement performed by the inertial Alice.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Relativistic quantum information (RQI) is a blooming area of research devoted to the study of quantum information concepts and processes under relativistic conditions [1–4]. Traditional streams of investigation in the domain of RQI have included the characterization of entropy and entanglement between modes of a quantum field as perceived by observers in different states of motion [1, 5–12], the production of entangled particles in curved spacetimes and models of the expanding universe [13–15], the investigation and applications of spacelike and timelike entanglement extracted from the quantum vacuum [16, 17], the modification and generation
of entanglement by moving cavities [18–21] and the analysis of quantum communication
protocols such as teleportation and key distribution in noninertial reference frames
[22–24]. The main theoretical ingredients for RQI ventures are a marriage of quantum field
theory on one hand [25] and the formalism of quantum information theory [26] on the other.
For fermionic (Grassman) fields, described by field operators \( \hat{c} \) subject to anticommutation
relations \( \{ \hat{c}, \hat{c}^\dagger \} = 1 \), one can investigate state properties and mode correlations by employing
the quantum information techniques usually adopted for states of multi-qubit systems, where
by ’qubit’ we mean a two-level quantum system [26]. For bosonic scalar fields, described by
field operators \( \hat{b} \) satisfying canonical commutation relations \( [\hat{b}, \hat{b}^\dagger] = 1 \), each mode lives in
an infinite-dimensional Hilbert space and represents a so-called continuous variable system.
Techniques from continuous variable quantum information [27–31] are thus potentially very
useful for RQI investigations involving scalar fields.

In this paper, we shall present a collection of relevant methods and measures to quantify
state properties and correlations in modes of a free scalar field. We shall focus on Gaussian
states and transformations [29], as they arise naturally in a number of contexts in RQI [8, 24, 25,
32–35] (see [21] for a recent overview), and they enjoy tractable mathematical expressions.
As Gaussian states are the states of any physical system in the harmonic approximation
[36, 37], or so-called small oscillations limit, they lend themselves as first-choice testbeds for
novel theoretical investigations; therefore it is no surprise that their role in RQI ventures has
become so prominent. Moreover, some transformations, such as those associated to the change
of coordinates between Minkowski and Rindler observers in flat spacetimes, are naturally
associated—with no approximation—to Gaussian operations [25]. In fact, the Unruh effect on
scalar fields [38], and the closely related Hawking effect in the presence of a black hole [39],
can be formally described in terms of the action of a Gaussian amplification channel [23, 40].
Gaussian states are furthermore particularly easy to prepare and control in a range of setups
including primarily quantum optics, atomic ensembles, trapped ions, optomechanics, as well
as hybrid interfaced networks thereof [30]. This could make them candidates of choice for
the implementation of explorative experiments to, at least, simulate relativistic phenomena in
the quantum optical setting, e.g., in the spirit of [41] (keeping in mind the warnings advanced
in [42]).

It is however appropriate to stress that the above mentioned liaison between relativity
and quantum information holds, to date, to a formal equivalence at mathematical level. We
stress that the main purpose of our work is in fact to provide mathematical tools that might
be useful for ongoing and future research in RQI: in this respect, our work aims to deploy
solid theoretical methods and results rather than to develop actually feasible experimental
proposals. The reader must be aware that concerns about the ultimate physical meaningfulness
of certain RQI findings have been advanced. For instance, global relativistic quantum field
modes cannot be measured by ideal measurements, i.e. measurements that map eigenstates of
an observable into themselves [43]. Still, analysing how basic quantum field theory predictions
affect fundamental quantum correlations between global field modes may be instructive to
better grasp the basics of the mechanisms involved. Surely, translating the results obtained
in this scenario to carry out real experiments is not straightforward at all, and more refined
approaches should thus be adopted. Here we limit ourselves to mention some recent proposals
regarding localized projective measurements [44] and particle detector models [45], deferring
the discussion to the concluding section for additional remarks.

Let us briefly recall some previous works related to our analysis. A comprehensive
characterization of the degradation and redistribution of entanglement between modes of
a bosonic scalar field was developed in [8] by means of Gaussian quantum information
techniques. In that paper, entanglement and total correlations in the state of two field
modes—described as a two-mode squeezed (Gaussian) entangled state from a fully inertial perspective—were found to degrade if one or both observers undergo uniform acceleration (see also [32]). In the case of one inertial (Alice) and one noninertial observer (Rob, living in Rindler region \( \text{I} \)), the lost entanglement was interpreted as redistributed genuine tripartite entanglement among Alice, Rob and an observer (known in the literature as anti-Rob) living in the causally disconnected Rindler region \( \text{II} \). A similar analysis for fermionic fields was reported in [7].

Entanglement [46] is, however, not the only form of quantum correlation. A finer description of quantumness versus classicality of correlations in bipartite quantum states has been recently put forward [47, 48]. Measures such as the one-way classical correlation and the quantum discord have been now computed [49–52] and measured experimentally [53] for Gaussian states, and provide a deeper insight into the nature of correlations compared to the entanglement/separability dichotomy. In rough terms, classical correlations correspond to how much, at most, the ignorance that one observer (say Alice) has about the marginal state of her subsystem, is reduced when the other observer (say Rob) performs a measurement on his subsystem [48]. This has to be maximized over all possible measurements on Rob’s side. Complementarily, the genuinely quantum correlations, as captured by the quantum discord [47], are those destroyed in the above described process of a marginal measurement on one subsystem only. Including the optimization over measurements, this corresponds to how much, at least, a marginal measurement disturbs the state of a composite system, which is a distinctively quantum feature. In this sense general quantum correlations are always revealed by means of marginal measurement processes [54]. Formal definitions of these quantities will be provided later; the interested reader can refer e.g. to a recent review [55] for further details. It is immediately clear that the above concepts for classical and quantum correlations have, unlike entanglement, an intrinsically non-symmetric nature. If we swap over the roles of the two observers, quite different results can be obtained. In particular, it is possible that quantumness of correlations can be revealed, or detected, by measurements on one subsystem, but not on the other. This is precisely what will be found to happen in the state of two scalar field modes in the limit of infinite acceleration of Rob: quantum discord is destroyed (like entanglement)—and classical correlations unaffected—if Rob is the measuring party, while enduring quantum correlations remain detectable if the inertial observer Alice is in charge of the measurement (see also [56]).

The paper contents and structure are as follows. In section 2 we review the formalism of continuous variable Gaussian states and their informational properties [29]; we adopt a recently introduced unified approach to the study of Gaussian correlations (including entanglement, classical, quantum and total correlations) by means of Rényi-2 entropy [52]. In section 3 we recall the basics of the Unruh effect for scalar fields, and we apply the introduced techniques to characterize how various forms of correlations are affected by acceleration of one or both observers detecting two field modes which are in an entangled Gaussian state from a fully inertial perspective. In section 4 we draw our concluding remarks and outline relevant perspectives.

2. Gaussian states, operations, information and correlation measures

2.1. Continuous variable systems

A continuous variable system of \( N \) canonical bosonic modes is described by a Hilbert space \( \mathcal{H} = \bigotimes_{k=1}^{N} \mathcal{H}_k \) resulting from the tensor product structure of infinite-dimensional Fock spaces
\( \mathcal{H}_k \), each of them associated to a single mode [27–29]. For instance, one can think of a noninteracting quantized scalar field (such as the electromagnetic field), whose Hamiltonian

\[
\hat{H} = \sum_{k=1}^{N} \hbar \omega_k \left( \hat{b}_k^\dagger \hat{b}_k + \frac{1}{2} \right),
\]

describes a system of an arbitrary number \( N \) of harmonic oscillators of different frequencies, the modes of the field. Here \( \hat{b}_k \) and \( \hat{b}_k^\dagger \) are the annihilation and creation operators of an excitation in mode \( k \) (with frequency \( \omega_k \)), which satisfy the bosonic commutation relations

\[
[\hat{b}_k, \hat{b}_{k'}^\dagger] = \delta_{kk'}, \quad [\hat{b}_k, \hat{b}_{k'}] = [\hat{b}_k^\dagger, \hat{b}_{k'}^\dagger] = 0.
\]

From now on we shall assume for convenience natural units with \( \hbar = c = 1 \). The corresponding quadrature phase operators ('position' and 'momentum') for each mode are defined as

\[
\hat{q}_k = \left( \hat{b}_k + \hat{b}_k^\dagger \right) \sqrt{2}, \quad \hat{p}_k = \frac{\left( \hat{b}_k - \hat{b}_k^\dagger \right)}{i \sqrt{2}}.
\]

We can group together the canonical operators in the vector

\[
\hat{R} = (\hat{q}_1, \hat{p}_1, \ldots, \hat{q}_N, \hat{p}_N)^T \in \mathbb{R}^{2N},
\]

which enables us to write in compact form the bosonic commutation relations between the quadrature phase operators,

\[
[\hat{R}_k, \hat{R}_l] = i \Omega_{kl},
\]

where \( \Omega \) is the \( N \)-mode symplectic form

\[
\Omega = \bigoplus_{k=1}^{N} \omega, \quad \omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.
\]

The space \( \mathcal{H}_k \) is spanned by the Fock basis \( \{|n\rangle_k \} \) of eigenstates of the number operator \( \hat{n}_k = \hat{b}_k^\dagger \hat{b}_k \), representing the Hamiltonian of the noninteracting mode via equation (1). The Hamiltonian of each mode is bounded from below, thus ensuring the stability of the system. For each mode \( k \) there exists a different vacuum state \( |0\rangle_k \in \mathcal{H}_k \) such that \( \hat{b}_k |0\rangle_k = 0 \). The vacuum state of the global Hilbert space will be denoted by \( |0\rangle = \bigotimes_k |0\rangle_k \).

The states of a continuous variable system are the set of positive trace-class operators \( \{\rho\} \) on the Hilbert space \( \mathcal{H} = \bigotimes_{k=1}^{N} \mathcal{H}_k \). Alternatively, for continuous variable systems, any state can be conveniently described by the so-called Wigner quasi-probability distribution, obtained as the Wigner–Weyl transform from \( \rho \) [57], and defined as

\[
W_{\rho}(\xi) = \frac{1}{\pi^N} \int_{\mathbb{R}^{2N}} \chi_{\rho}(\kappa) e^{i \kappa^T \Omega \xi} d^{2N} \kappa,
\]

where \( \xi \) and \( \kappa \) belong to the real \( 2N \)-dimensional space \( \Gamma = (\mathbb{R}^{2N}, \Omega) \), which is called phase space in analogy with classical Hamiltonian dynamics, and \( \chi_{\rho} \) is the characteristic function of \( \rho \),

\[
\chi_{\rho}(\kappa) = \text{tr} [\rho \hat{D}(\kappa)],
\]

with

\[
\hat{D}(\kappa) = e^{i \kappa^T \Omega \xi}
\]

being the Weyl displacement operator.
2.2. Gaussian states

The set of Gaussian states is, by definition, the set of states of a continuous variable system whose characteristic function and Wigner phase-space distribution are positive-everywhere, Gaussian-shaped functions. Gaussian states, such as coherent, squeezed and thermal states, are thus completely specified by the first and second statistical moments of the phase quadrature operators. As the first moments can be adjusted by marginal displacements, which do not affect any informational property of the considered states, we shall assume them to be zero, \( \langle \hat{R} \rangle = 0 \) in all the considered states without loss of generality. The important object encoding all the relevant properties of a Gaussian state \( \rho \) is therefore the covariance matrix (CM) \( \sigma \) of the second moments, whose elements are given by

\[
\sigma_{j,k} = \text{tr}[\rho [\hat{R}_j, \hat{R}_k]].
\]

(9)

We can then write the Wigner distribution (equation (6)) of a generic \( N \)-mode undisplaced Gaussian state in the compact form

\[
W_\rho (\xi) = \frac{1}{\pi^N \sqrt{\det \sigma}} \exp(-\xi^T \sigma^{-1} \xi).
\]

(10)

One can see that in the phase-space picture, the tensor product structure is replaced by a direct sum structure, so that the \( N \)-mode phase space is \( \bigoplus_k \Gamma_1 = \bigoplus_k \mathbb{R}^2 \), where \( \Gamma_k = (\mathbb{R}^2, \omega) \) is the marginal phase space associated with mode \( k \). Similarly, the CM for product states of the form \( \otimes_k \rho_k \) will be the direct sum \( \oplus_k \sigma_k \) of individual covariance matrices for each subsystem.

In particular, the global vacuum state \( |0\rangle \) of a \( N \)-mode scalar field is a Gaussian state with CM \( \sigma_0 = \bigoplus_{k=1}^N \mathbb{I} \) where \( \mathbb{I} \) denotes here the \( 2 \times 2 \) identity matrix. If we partition our system into two subsystems \( A \) and \( B \) \( [\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B] \), each grouping \( N_A \) and \( N_B \) modes, respectively (with \( N_A + N_B = N \)), the CM of a \( N \)-mode bipartite Gaussian state \( \rho_{AB} \) with respect to such a splitting can be written in the block form

\[
\sigma_{AB} = \begin{pmatrix} \sigma_A & \mathbb{S}_{AB} \\ \mathbb{S}_{AB}^T & \sigma_B \end{pmatrix}.
\]

(11)

We refer the reader to [27, 29, 31, 37] for further details on the structural and formal description of Gaussian quantum states in phase space.

2.3. Gaussian operations

Gaussian unitaries. An important role in the theoretical and experimental manipulation of Gaussian states is played by unitary operations \( \hat{U} \) which preserve the Gaussian character of the states on which they act. They are generated by Hamiltonian terms which are at most quadratic in the field operators. By the metaplectic representation, any such unitary operation at the Hilbert space level corresponds, in phase space, to a symplectic transformation, that is, a linear transformation \( S \) which preserves the symplectic form \( \Omega : S^T \Omega S = \Omega \). Symplectic transformations on a \( 2N \)-dimensional phase space form the real symplectic group \( Sp(2N, \mathbb{R}) \).

Such transformations act linearly on first moments and by congruence on covariance matrices, \( \sigma \mapsto S \sigma S^T \). Ideal beam splitters, phase shifters and squeezers are all described by some kind of symplectic transformation (see e.g. [58]). For instance, the two-mode squeezing operator

\[
\hat{U}_{i,j}(r) = \exp \left[ r (\hat{b}_i \hat{b}_j^\dagger - \hat{b}_j \hat{b}_i) \right]
\]

(12)

corresponds to the symplectic transformation

\[
S_{i,j}(r) = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\sinh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}.
\]

(13)

where the matrix is understood to act on the pair of modes \( i \) and \( j \).
**Gaussian measurements.** In quantum mechanics, two main types of measurement processes are usually considered [26]. The first one is constituted by projective (von Neumann) measurements, which are defined by a set of Hermitian positive operators \( \{ \Pi_i \} \) such that \( \sum_i \Pi_i = 1 \) and \( \Pi_i \Pi_j = b_{ij} \Pi_i \). A projective measurement maps a state \( \rho \) into a state \( \rho_i = \frac{\Pi_i \rho \Pi_i}{\text{tr}[\Pi_i \rho \Pi_i]} \) with probability \( p_i = \text{tr}[\Pi_i \rho \Pi_i] \). If we focus on a local projective measurement on the subsystem \( B \) of a bipartite state \( \rho_{AB} \), say \( \Pi_i = \mathbb{1}_A \otimes \Pi_i \), the subsystem \( A \) is then mapped into the conditional state \( \rho_A|\Pi_i = \text{tr}_B[\Pi_i \rho_{AB}] \Pi_i \). The second type of quantum measurements are known as POVM (positive operator-valued measure) measurements and amount to a more general class compared to projective measurements. They are defined again in terms of a set of Hermitian positive operators \( \{ \Pi_i \} \) such that \( \sum_i \Pi_i = \mathbb{1} \), but they need not be orthogonal in this case. In the following, by ‘measurement’ we will refer in general to a POVM.

In the continuous variable case, the measurement operations mapping Gaussian states into Gaussian states are called Gaussian measurements. They can be realized experimentally by appending ancillae initialized in Gaussian states, implementing Gaussian unitary (symplectic) operations on the system and ancillary modes, and then measuring quadrature operators, which can be achieved e.g. by means of balanced homodyne detection in the optics framework [51]. Given a bipartite Gaussian state \( \rho_{AB} \), any such measurement on, say, the \( N_B \)-mode subsystem \( B = \{ B_1, \ldots, B_{N_B} \} \), is described by a POVM of the form [59]

\[
\Pi_B(\eta) = \pi^{-N_B} \int \Pi_B(\eta) d^{2N_B} \eta = 1 \quad \text{and} \quad \Lambda_B^{-1} = \text{the density matrix of a (generally mixed) } N_B \text{-mode Gaussian state with CM } \Gamma_B \text{ which denotes the so-called seed of the measurement.}
\]

The conditional state \( \rho_{A|\Pi_B(\eta)} \) after the measurement \( \Pi_B(\eta) \) has been performed on \( B \) has a CM \( \hat{\sigma}_A^\Pi \) independent of the outcome \( \eta \) and given by the Schur complement [60]

\[
\hat{\sigma}_A^\Pi = \sigma_A - \mathcal{s}_{AB}(\sigma_B + \Gamma_B^\Pi)^{-1} \mathcal{s}^T_{AB},
\]

where the original bipartite CM \( \sigma_{AB} \) of the \( N \)-mode state \( \rho_{AB} \) has been written in block form as in equation (11).

### 2.4. Gaussian information measures in terms of Rényi-2 entropy

An extensive account of informational and entanglement properties of Gaussian states, using various well-established measures, can be found for instance in [27, 29, 61, 62]. Here we follow a novel approach introduced in [52], to which the reader is referred for further details and rigorous proofs.

Rényi-\( \alpha \) entropies [63] constitute a powerful family of additive entropies, which provide a generalized spectrum of measures of (lack of) information in a quantum state \( \rho \). They find widespread application in quantum information theory (see [52] and references therein), while their role in holographic theories is attracting a certain interest from the gravity community as well [64]. They are defined as

\[
S_\alpha(\rho) = \frac{1}{1 - \alpha} \ln \text{tr}(\rho^\alpha),
\]

and reduce to the conventional von Neumann entropy in the limit \( \alpha \to 1 \). The case \( \alpha = 2 \) is especially simple, \( S_2(\rho) = -\ln \text{tr}(\rho^2) \).
For arbitrary Gaussian states, the Rényi entropy of order 2 satisfies the strong subadditivity inequality \[52\]; this allows us to define relevant \textit{bona fide} Gaussian measures of information and correlation quantities, encompassing entanglement and more general quantum and classical purity, or, equivalently, lack of information, i.e. ignorance) will thus be the Rényi-2 entropy, \[ S_2(\rho) = \frac{1}{2} \ln(\det(\rho)), \] (18)
which is 0 on pure states (\(\det(\sigma) = 1\)) and grows unboundedly with increasing mixedness of the state. This measure is directly related to the phase-space Shannon entropy of the Wigner distribution \(W_\rho\) of the state \(\rho\) \[\text{(10)}, \text{defined as } H(W_\rho(\xi)) = -\int W_\rho(\xi) \ln[W_\rho(\xi)] d^2\xi \ [65].\]
Indeed, one has \(H(W_\rho(\xi)) = S_2(\rho) + N(1 + \ln \pi) \ [52].\)

**Total correlations.** For a bipartite Gaussian state \(\rho_{AB}\) with CM as in equation \(11\), the total correlations between subsystems \(A\) and \(B\) can be quantified by the Rényi-2 mutual information \(I_2\), defined as \[52\]
\[ I_2(\sigma_{AB}) = S_2(\sigma_A) + S_2(\sigma_B) - S_2(\sigma_{AB}), \]
(19)
which measures the phase-space distinguishability between the Wigner function of \(\rho_{AB}\) and the Wigner function associated to the product of the marginals \(\rho_A \otimes \rho_B\), which is, by definition, a state in which the subsystems \(A\) and \(B\) are completely uncorrelated.

**Entanglement.** In the previous paragraph, we talked about total correlations, but that is not the end of the story. In general, we can discriminate between classical and quantum correlations. A bipartite pure state \(|\psi_{AB}\rangle\) is quantum-correlated, i.e. is ‘entangled’, if and only if it cannot be factorized as \(|\psi_{AB}\rangle = |\phi_A\rangle \otimes |\chi_B\rangle\). On the other hand, a mixed state \(\rho_{AB}\) is entangled if and only if it cannot be written as \(\rho_{AB} = \sum_i p_i \rho_A \otimes \rho_B\), that is, a convex combinations of product states, where \(\{p_i\}\) are probabilities and \(\sum_i p_i = 1\). Unentangled states are called ‘separable’.

The reader can refer to \[46\] for an extensive review on entanglement. In particular, one can quantify the amount of entanglement in a state by building specific measures. For Gaussian states, any measure of entanglement will be a function of the elements of the CM only \[29\].

A measure of bipartite entanglement \(E_2\) for Gaussian states based on Rényi-2 entropy can be defined as follows \[52\]. Given a Gaussian state \(\rho_{AB}\) with CM \(\sigma_{AB}\), we have
\[ E_2(\sigma_{AB}) = \inf_{\gamma_{AB} : 0 < \gamma_{AB} \leq \sigma_{AB}, \det \gamma_{AB} = 1} \frac{1}{2} \ln(\det(\gamma_A)), \] (20)
where the minimization is over pure \(N\)-mode Gaussian states with CM \(\gamma_{AB}\) smaller than \(\sigma_{AB}\).

For a pure Gaussian state \(\rho_{AB} = |\psi_{AB}\rangle \langle \psi_{AB}|\) with CM \(\sigma_{AB}^{\text{pure}}\), the minimum is saturated by \(\gamma_{AB} = \sigma_{AB}^{\text{pure}}\), so that the measure of equation \(20\) reduces to the pure-state Rényi-2 entropy of entanglement,
\[ E_2(\sigma_{AB}^{\text{pure}}) = S_2(\sigma_A) = \frac{1}{2} \ln(\det(\sigma_A)), \] (21)
where \(\sigma_A\) is the reduced CM of subsystem \(A\). For a generally mixed state, equation \(20\) amounts to taking the Gaussian convex roof of the pure-state Rényi-2 entropy of entanglement, according to the formalism of \[62\]. Closed formulae for \(E_2\) can be obtained for special classes of two-mode Gaussian states \[52\]. The Rényi-2 entanglement is additive and monotonically nonincreasing under Gaussian local operations and classical communication.
Classical correlations. For pure states, entanglement is the only kind of quantum correlations. A pure separable state is essentially classical, and the subsystems are not correlated at all. On the other hand, for mixed states, one can identify a finer distinction between classical and quantum correlations, such that even most separable states display a definite quantum character [47, 48].

Conceptually, one-way classical correlations are those extractable by local measurements; they can be defined in terms of how much the ignorance about the state of a subsystem, say A, is reduced when the most informative local measurement is performed on subsystem B [48]. The quantum correlations (known as ‘discord’) are, complementarily, those destroyed by local measurement processes, and correspond to the change in total correlations between the two subsystems, following the action of a minimally disturbing local measurement on one subsystem only [47]. For Gaussian states, Rényi-2 entropy can be adopted once more to measure ignorance and correlations [52].

To begin with, we can introduce a Gaussian Rényi-2 measure of one-way classical correlations [48–50, 52]. We define

$$J_2^{A|B} = \sup_{\mathcal{M}_B} \frac{1}{2} \ln \left( \frac{\det \sigma_A}{\det \tilde{\sigma}_A} \right);$$

$$J_2^{B|A} = \sup_{\mathcal{M}_A} \frac{1}{2} \ln \left( \frac{\det \sigma_B}{\det \tilde{\sigma}_B} \right);$$

where the one-way classical correlations $J_2^{A|B}$, with Gaussian measurements on A, have been defined accordingly by swapping the roles of the two subsystems, $A \leftrightarrow B$. Note that, for the same state $\rho_{AB}$, $J_2^{A|B} \neq J_2^{B|A}$ in general: the classical correlations depend on which subsystem is measured (see footnote 1).

Quantum correlations. We can now define a Gaussian measure of quantumness of correlations based on Rényi-2 entropy. Following the landmark study by Ollivier and Zurek [47], and the recent investigations of Gaussian quantum discord [49, 50, 52], we define the Rényi-2 discord as the difference between mutual information (19) and classical correlations (22),

$$D_2^{A|B} = I_2^{A:B} - J_2^{A|B};$$

$$D_2^{B|A} = I_2^{A:B} - J_2^{B|A};$$

The discord is clearly a nonsymmetric quantity as well (see footnote 1). It captures general quantum correlations even in the absence of entanglement [47, 55].

Let us remark that we have defined classical and quantum correlations by restricting the optimization over Gaussian measurements only. This means that, potentially allowing for

1 Notice the directional notation ‘A|B’ to indicate ‘A given B’, i.e. to specify that we are looking at the change in the informational content of A following a minimally disturbing marginal measurement on B. For entanglement and total correlations there is no direction as those quantities are symmetric, so the notation ‘A : B’ is adopted instead.

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more general non-Gaussian measurements, one could obtain higher classical correlations and lower quantum ones. However, some numerical and partial analytical evidence support the conclusion that, for two-mode Gaussian states, Gaussian measurements are optimal for the calculation of general one-way classical and quantum discord [50, 66]. Certainly, restricting to the practically relevant Gaussian measurements makes the problem dramatically more tractable, as one can obtain closed analytical expressions for equations (22) and (23) for the case of \( A \) and \( B \) being single modes, that is, \( \rho_{AB} \) being a general two-mode Gaussian state [50, 52]. We make explicit use of these formulæ to derive the results of section 3. Further, restricting to Gaussian measurements also corresponds closely to the reality of what is implementable in laboratory with present day technology [53].

Finally, let us observe that

\[
\frac{I_2(\sigma_{A,B}^{\text{pure}})}{2} = J_2(\sigma_{A,B}^{\text{pure}}) = J_2(\sigma_{B,A}^{\text{pure}}) = D_2(\sigma_{A,B}^{\text{pure}}) = D_2(\sigma_{B,A}^{\text{pure}}) = E_2(\sigma_{A,B}^{\text{pure}}) = S_2(\sigma_A) = S_2(\sigma_B).
\]

for pure bipartite Gaussian states \( \rho_{AB} \) of an arbitrary number of modes. That is, general quantum correlations reduce to entanglement, and an equal amount of classical correlations is contained as well in pure states.

3. Unruh effect and correlations of scalar field modes in noninertial frames

3.1. Rudiments of the Unruh effect

There are excellent references in the literature about the Unruh effect [38], see e.g. [67] for a recent review; the physics of it will be most likely covered in detail elsewhere in this special issue. We shall briefly recall the phenomenon for the purpose of setting up our notation.

It is well known that different quantization procedures for observers in different states of motion, i.e. inertial and noninertial observers, of a quantum field in a flat spacetime may introduce not only non-trivial effects on particle generation, but also on the behaviour of the correlations between field modes. The setting we wish to investigate is the following. We consider a \((1+1)\)-dimensional Minkowski spacetime with coordinates \((t, z)\), which we can adopt as proper coordinates for an inertial observer Alice moving in the Minkowski plane. In such a context, the proper coordinates of an observer Rob moving with uniform proper acceleration \(a\) are the Rindler coordinates \((\tau, \zeta)\). Two different sets of Rindler coordinates are needed for covering region \(I, II\) of the Minkowski spacetime (see figure 1), and are given by

\[
\begin{align*}
I: & \quad at = e^{\alpha \zeta} \sinh(\alpha \tau), \quad az = e^{\alpha \zeta} \cosh(\alpha \tau), \\
II: & \quad at = -e^{\alpha \zeta} \sinh(\alpha \tau), \quad az = -e^{\alpha \zeta} \cosh(\alpha \tau).
\end{align*}
\]

These sets of coordinates define two Rindler regions (respectively \(I\) and \(II\)) that are causally disconnected from each other.

Now, let us consider a free quantum scalar field; its quantization in the Minkowski coordinates is not equivalent to the one in the Rindler ones, since the solutions of the Klein–Gordon equation in the two coordinate systems are different. In particular, a Minkowski vacuum state of a field mode described by an inertial observer Alice is expressed in Rindler coordinates as a two-mode squeezed state:

\[
|0_M\rangle = \hat{U}_{1,II}(r)|n\rangle_I|n\rangle_{II} = \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r |n\rangle_I|n\rangle_{II}.
\]
where $\hat{U}_{I,II}(r)$ is exactly the two-mode squeezing operator of equation (12), that encodes the particle pair production between the two Rindler wedges. Here the dimensionless ‘acceleration parameter’ $r$ is proportional to the Unruh temperature $T$:

$$\cosh^{-2} r = 1 - e^{-\frac{k_B T}{\hbar \omega}}, \quad T = \frac{\hbar a}{2 \pi k_B}, \quad \tag{28}$$

with $k_B$ being the Boltzmann constant, and $\omega$ being the frequency of the mode.

Adopting the Heisenberg picture, we have that the Rindler field mode operators $\hat{b}_I, \hat{b}_II$ are connected to the Minkowski ones $\hat{b}_M$ via a Bogoliubov transformation [25],

$$\hat{b}_M = \cosh r \hat{b}_I - \sinh r \hat{b}_II. \quad \tag{29}$$

A noninertial observer Rob with uniform acceleration $a$ is confined to Rindler region $I$ and has no access to the opposite region. Thus, the equilibrium state from Rob’s viewpoint, in the Schrödinger picture, is obtained by tracing over the modes in the causally disconnected region $II$,

$$\rho_I = \text{tr}_II \left\{ \hat{U}_{I,II}(r)(\langle 0M \rangle_II \otimes \langle 0 | 0 \rangle_M)I \hat{U}^\dagger_{I,II}(r) \right\} \quad \text{tr}_II \left\{ \hat{U}_{I,II}(r)(\langle 0M \rangle_II \otimes \langle 0 | 0 \rangle_M)I \hat{U}^\dagger_{I,II}(r) \right\}$$

$$= \frac{1}{\cosh^2 r} \sum_{n=0}^{\infty} \tanh^{2n} r |n\rangle_II \langle n |. \quad \tag{30}$$

One can then see that the Minkowski vacuum is described, by a uniformly accelerated observer Rob, as a particle-populated thermal state with temperature $T$ given by equation (28).

This phenomenon, called Unruh effect [38], has a well known formal analogue in quantum optics [57]: an input signal beam $I$ in the state $|0M\rangle$ interacts with an idler vacuum mode $II$ (ancilla) via a two-mode squeezing transformation $\hat{U}_{I,II}(r)$ (realized by parametric down-conversion) with squeezing $r$; tracing over the output idler mode, the output signal is left precisely in the mixed thermal state $\rho_I$ of equation (30). Overall the non-unitary transformation from input to output, or from inertial to noninertial frame, corresponds to the action of a bosonic amplification channel [23, 40].
One can question how a different state (other than the vacuum) of a scalar field mode, described as $|\psi\rangle$ in Minkowski coordinates, is perceived by a noninertial observer Rob confined to Rindler region $I$. In seminal RQI investigations [6–8], it was implicitly assumed that a Minkowski mode with a sharp frequency transforms into a single frequency Rindler mode too. This assumption has been proven incorrect [11]: Minkowski modes prepared in states other than the vacuum, e.g. single-particle states, are effectively described as oscillatory, non-peaked broadband wavepackets from a Rindler perspective. However, a valid ‘single-mode approximation’ can be still employed if one considers in general a class of Unruh modes [38, 25] of the massless scalar field, rather than Minkowski modes. Such modes are purely positive-frequency combinations of standard plane waves in Minkowski coordinates, but enjoy a special property: they are mapped into single frequency modes in Rindler coordinates. The interested reader can refer e.g. to [11, 9] for further details. Unruh modes form a complete basis of solutions of the field equations that can span any physical state, and are very apt to make calculations, therefore qualifying as suitable candidates for our exploratory investigation. Although they have been shown to suffer some pathologies (delocalization and oscillatory behaviour near the acceleration horizon) that might hinder their physical realization [11, 24], one can in principle design plausible models of non-point-like detectors which couple effectively to a single Unruh mode [68].

Having clarified these important issues of physical nature, let us recall the mathematical results. Let $|\psi\rangle_U$ denote the state of a Unruh mode of the field from an inertial perspective, characterized by the creation operator

$$\hat{b}^+_I|0_M\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} \frac{\tanh n \sqrt{n+1}}{n} |\Phi_n\rangle,$$

with $|\Phi_n\rangle = q_L|n\rangle|n+1\rangle_r + q_R|n+1\rangle|n\rangle_r$. (31)

where $|q_R|^2 + |q_L|^2 = 1$ and $|\Phi_n\rangle$ is the state of a single Rindler mode of frequency $\omega$, see [11] for details. If we fix $q_R = 1, q_L = 0$, then the formal analogy with the bosonic amplification channel still holds for any inertial state $|\psi\rangle_U$ of the Unruh mode, and a formula akin to equation (30) can be still used to determine the state $\rho_I$ of the field as described in Rindler coordinates by the noninertial observer Rob [25, 11, 40]. One has, namely

$$\rho_I = \text{tr}_I\left\{\hat{U}_{II}(r)|\langle\psi\rangle_U\rangle_I \otimes |0\rangle\langle 0|_I|\hat{U}^+_I(r)\right\},$$

(32)

where the Rindler modes $I$ and $II$ have a definite frequency $\omega$. If $|\psi\rangle$ is the Unruh vacuum, which coincides with the Minkowski vacuum $|0_M\rangle$, then equation (32) reduces to equation (30). In general, $\rho_I$ will be some other mixed state from a noninertial perspective. We remark that one might choose different values of $q_{R,L}$ in equation (31), which could result in interesting phenomena such as enhancement rather than degradation of quantum correlations from noninertial perspectives [12]; we leave those settings for further analysis, focusing here on the case $q_R = 1$.

The crucial observation to make is that the two-mode squeezing transformation $\hat{U}_{II}(r)$ is a Gaussian operation, and the amplification channel is a Gaussian channel, i.e. they preserve the Gaussianity of the input states. Therefore, if the inertial state $|\psi\rangle$ is chosen to be Gaussian in equation (32), the transformed states in Rindler coordinates remain Gaussian as well, and the methods from the previous section can be readily employed to characterize how informational properties are perceived in different reference frames [8]. This holds for general Bogoliubov transformations [21].
3.2. The setting

In this paper we focus on a massless scalar field whose state, as seen from an inertial Minkowski frame, involves all modes in the vacuum, but for two Unruh modes $A$ and $R$ which are initialized in a pure, entangled (Gaussian) two-mode squeezed state with squeezing $s$ [8, 32], characterized by a CM of the form

$$\sigma_{AR}^{(M)}(s) = S_{A,R}(s)\mathbb{I}_{AR}S_{A,R}^T(s) = \begin{pmatrix}
\cosh(2s) & 0 & \sinh(2s) & 0 \\
0 & \cosh(2s) & 0 & -\sinh(2s) \\
\sinh(2s) & 0 & \cosh(2s) & 0 \\
0 & -\sinh(2s) & 0 & \cosh(2s)
\end{pmatrix},$$

where $\mathbb{I}_{AR}$ is the CM of the two-mode vacuum $|0_M\rangle_2 \otimes |0_M\rangle_2$, and $S_{i,j}$ defined in equation (13) is the phase-space (symplectic) representation of the two-mode squeezing operation of equation (12). Note that, compared with the previous subsection, we are considering here two populated Unruh modes (rather than a single one) which are already correlated from an inertial perspective.

Let us then first analyse the correlations between the two modes $A$ and $R$ in Minkowski coordinates. Let Alice be the observer associated to the description of the mode $A$, and let Rob be the observer who describes mode $R$. In our analysis, we are avoiding the issues associated to the practical detection of those modes: the reader can consider the terms ‘observers’ and ‘coordinates’ as synonyms for all practical purposes. From a fully inertial perspective (i.e. if both observers are inertial), the correlations in the state $\sigma_{AR}^{(M)}(s)$ are given by equation (24), that is, entanglement, quantum and classical correlations are all equal to

$$C_2(\sigma_{AR}^{(M)}(s)) = \ln[\cosh(2s)]$$

(where we have introduced the common symbol $C_2$ for ‘correlations’), while total correlations are clearly $T_2 = 2C_2$. All the correlations increase unboundedly with increasing initial squeezing $s$. For $s \rightarrow \infty$ the two modes of the field asymptotically tend to realize, from an inertial perspective, the Einstein–Podolski–Rosen perfectly correlated state.

Now, we present the core of our work. We shall consider two settings. In the first one, say (a), the mode $A$ is still expressed in Minkowski coordinates, i.e. Alice is an inertial observer, while $R$ is now described by Rindler coordinates, i.e. Rob undergoes uniform acceleration characterized by an acceleration parameter $r$. In the second picture, say (b), Alice and Rob are both subjected to uniform acceleration characterized by acceleration parameters $w$ and $r$, respectively. The world lines for the two settings are depicted schematically in figure 1. Entanglement redistribution phenomena under these prescriptions have been studied for instance in [8, 34] for scalar fields.

For setting (a), the complete description of the problem involves three modes, mode $A$ described by the inertial Alice, mode $R$ by the noninertial Rob in Rindler region $I$, and mode $\bar{R}$ by a noninertial observer anti-Rob confined to Rindler region $II$ (the prefix ‘anti’ is just used for labelling observers in region $II$). This is because the mode $R$ is mapped to two sets of Rindler coordinates, respectively for region $I$ and $II$. Consistently, setting (b) involves additionally a fourth mode, $\bar{A}$, as we now have a noninertial observer anti-Alice confined to Rindler region $II$ as well.

Let us now analyse the description of the two settings in the Gaussian phase-space formalism. Recalling that we can work at the CM level, we have already seen from equation (32) that the change from Minkowski to Rindler coordinates corresponds to a two-mode squeezing operation for each single Unruh mode, i.e. $|\psi\rangle_U = \hat{U}_{II}(r)(|\psi\rangle_I \otimes |0\rangle_{II})$, where $\hat{U}_{II}(r)$ is associated to the symplectic transformation $S_{I,II}(r)$.

In the first setting, therefore, since the observer Rob is accelerating uniformly, the original two-mode entangled states described by Alice and Rob in Minkowski coordinates becomes
‘distributed’ among three observers, i.e. we need three systems of coordinates to describe it, specifically associated with Alice, Rob for the region \(I\) and anti-Rob for region \(II\). The Gaussian state of the complete system has CM given by \([8]\)

\[
\sigma^{(a)}_{A\bar{A}R}(s, r) = [I_A \oplus S_{R,\bar{R}}(r)][\sigma^{(M)}_{AR}(s) \oplus I_{\bar{R}}][I_A \oplus S_{R,\bar{R}}(r)]^T,
\]

where \(S_{R,\bar{R}}(r)\) is the squeezing operation correspondent to the change of coordinates due to the noninertiality of Rob, and we have used the fact that the CM of a vacuum state is the identity matrix.

Similarly, a change of coordinates for mode \(A\) as well implies a further two-mode squeezing operation \(S_{A,\bar{A}}(w)\) where Alice is now a noninertial observer confined in region \(I\) and anti-Alice in region \(II\). Therefore, the CM for the complete state in the second setting reads

\[
\sigma^{(b)}_{A\bar{A}R}(s, w, r) = [S_{A,\bar{A}}(w) \oplus S_{R,\bar{R}}(r)][\sigma^{(M)}_{A\bar{A}}(s) \oplus I_{\bar{R}}][S_{A,\bar{A}}(w) \oplus S_{R,\bar{R}}(r)]^T.
\]

Note that if we set \(w = 0\), then mode \(\bar{A}\) decouples and setting (b) reduces to setting (a).

### 3.3. Correlations in noninertial frames

**Correlations between physical observers.** First of all, we are interested in the correlations between the two field modes \(A\) and \(R\) as described by the observers Alice and Rob in the two settings described above. To obtain \(\sigma^{(a,b)}_{AR}\), we need to trace equations (35) and (36) over the inaccessible degrees of freedom associated with modes in Rindler region \(II\). The latter modes appear to have acquired correlations with the inaccessible degrees of freedom associated with modes in Rindler region \(I\) and the case of setting (a) can be retrieved by choosing \(r\) of \(A\) and \(w\) of \(R\). Therefore, the physical state of modes \(A\) and \(R\) should be detected by Alice and Rob as more mixed and less correlated, intuitively, with increasing acceleration of one or both observers. The correlations in the reduced states \(\sigma^{(a,b)}_{AR}\) will be compared with the ones available from a fully inertial perspective, \(C_2(\sigma^{(M)}_{AR})\), given by equation (34). A comparative summary of our results is illustrated in figure 2.

We find *prima facie* that the total correlations (equation (19)) are degraded as functions of \(r\) and \(w\) as expected. For the general setting (b), we get

\[
I_2(\sigma_{AR}^{(b)}) = \ln \left[ \frac{(\cosh^2(r) \cosh(2s) + \sinh^2(r))(\cosh(2s) \cosh^2(w) + \sinh^2(w))}{\cosh(2r) \cosh^2(s) \cosh(2w) - \sinh^2(s)} \right],
\]

and the case of setting (a) can be retrieved by choosing \(w = 0\). The total correlations are never completely destroyed under the Unruh effect in the considered settings. If Alice stays inertial (setting (a)), we get

\[
\lim_{r \to \infty} \frac{I_2(\sigma_{AR}^{(a)})}{I_2(\sigma_{AR}^{(M)})} = \frac{1}{2},
\]

while if Alice is in high uniform acceleration as well (setting (b)), we get

\[
\frac{1}{4} \leq \lim_{r, w \to \infty} \frac{I_2(\sigma_{AR}^{(b)})}{I_2(\sigma_{AR}^{(M)})} \leq \frac{1}{2}.
\]

It is interesting to evaluate how the total correlations decompose into classical and genuinely quantum components. We find that the classical correlations revealed in a marginal measurement are unaffected by the state of motion of the observer who is performing the measurement, but carry a signature of the state of motion of the other observer. Specifically, in setting (a), if the noninertial observer Rob implements a Gaussian measurement on mode \(R\)
and we compute the ensuing classical correlations (equation (22)) between the modes $R$ and $A$, we find
\[
\mathcal{J}_2(\sigma_{A|R}^{(a)}) = C_2(\sigma_{A|R}^{(M)}) = \ln[\cosh(2s)],
\] (40)
independently of Rob’s acceleration parameter $r$. In other words, if Alice stays inertial, she does not experience any lack of information on the state of mode $A$ depending on whether Rob detects $R$ in an inertial or a noninertial frame. We can then conclude, in this specific sense, that such one-way classical correlations are unaffected by the Unruh effect. If we swap the roles over and let Alice be the one who implements the marginal measurement, however, we find instead that the classical correlations do depend on Rob’s acceleration parameter $r$, yet they do not depend on Alice’s acceleration parameter $w$: they are then the same in both settings (a) and (b) and given by
\[
\mathcal{J}_2(\sigma_{A|R}^{(b)}) = \ln[\text{sech}(2r)(\cosh^2(r) \cosh(2s) + \sinh^2(r))].
\] (41)
For high acceleration of Rob ($r \gg 0$) and large inertial correlations ($s \gg 0$), we have
\[
\lim_{r,s \to \infty} \mathcal{J}_2(\sigma_{A|R}^{(b)}) = C_2(\sigma_{A|R}^{(M)}) - \ln 2,
\] (42)
meaning that Rob experiences a lack of no more than one bit (which in our units amounts to $\ln 2$) of classical correlations even if Alice stays inertial, as predicted in [8].

Figure 2. Correlations between field modes $A$ and $R$ as described by Alice and Rob in the following settings. (a) Alice is inertial, and Rob undergoes uniform acceleration with acceleration parameter $r$. (b) Alice and Rob both undergo uniform acceleration, with parameters $w = 2r$ and $r$, respectively. From a fully inertial perspective, the state of modes $A$ and $R$ is a two-mode squeezed state with squeezing parameter $s$ (we choose $s = \text{arccosh}(e)/2$ in the plots, so that the inertial quantum and classical correlations amount to 1). In the top row, classical and quantum correlations (quantified respectively by the one-way measure $\mathcal{J}_2$ and the discord $\mathcal{D}_2$) are revealed through marginal measurements on $R$. In the bottom row, they are revealed through marginal measurements on $A$. Entanglement $\mathcal{E}_2$ and total correlations $\mathcal{I}_2$ are instead symmetric quantities. Notice how the classical correlations in the bottom row are the same for both settings. Notice also how entanglement vanishes at a finite $r$ for setting (b), while discord can only vanish in the infinite acceleration limit. Discord revealed through measurements on $A$ does not vanish in this limit when Alice stays inertial.
By combining the analysis of mutual information and classical correlations we can draw conclusions about the Unruh effect on general quantum correlations as quantified by the discord (equation (23)). We find that in setting (a), the discord revealed through measurements on \( A \) converges to a finite value in the limit \( r \to \infty \), given by

\[
\lim_{r \to \infty} D_2(\sigma_{AR}^{[a]}(r)) = \ln \left( \frac{\cosh(2x)}{\cosh^2 s} \right) \left. \right|_{x \to \infty} \ln 2.
\]  

(43)

This shows on one hand that not all genuinely quantum features are necessarily destroyed by the Unruh effect, and on the other hand that the loss of a bit of classical correlations, as shown in equation (42), is somehow compensated, when Alice stays inertial, by the endurance of a bit of quantum discord, both revealed through marginal measurements on \( A \). The permanence of a nonzero amount of discord in a similar context was found in [56] for scalar fields, albeit starting from a different (non-Gaussian) state for the modes \( A \) and \( R \) in a fully inertial perspective (in that case, the instance of measurements on \( R \) could not be worked out). Here we find that, for any other setting, namely setting (a) with measurements on \( R \), and setting (b) for both directions, the discord goes asymptotically to zero when the involved acceleration parameters diverge.

Finally, let us just recall that entanglement between \( A \) and \( R \) also goes to zero asymptotically for \( r \to \infty \) in setting (a) [6, 8], while it experiences so-called sudden death in setting (b) [8, 34], i.e. it can vanish for a range of finite values of the accelerations of Alice and Rob. Explicitly,

\[
\mathcal{E}_2(\sigma_{AR}^{[a]}(r)) = \ln \left( \frac{(\cosh(2s) + 3) \cosh(2r) + 2 \sinh^2(s)}{2 \sinh^2(r) \cosh(2s) + \cosh(2r) + 3} \right),
\]

\[
\mathcal{E}_2(\sigma_{AR}^{[b]}(r)) = \begin{cases} 
0, & \text{if } \tanh s \leq \sinh w \sinh r; \\
\ln \left( \frac{-4 \sinh w \sinh r \sinh(2s) + 2 \cosh(2w) \cosh(2r) \cosh^2(s) + 3 \cosh(2s) - 1}{2(2 \sinh w \sinh r \sinh(2s) + \cosh^2(s)(\cosh(2w) + \cosh(2r)) - 2 \sinh^2(s))} \right), & \text{otherwise.}
\end{cases}
\]

(44)

**Correlations involving inaccessible modes.** We wish to stress that the measures reported in section 2 allow us to calculate explicitly all forms of correlations and entanglement between all bipartitions in the complete states \( \sigma_{ARR}^{[a]}(s, r) \) and \( \sigma_{ARR}^{[b]}(s, w, r) \), involving also those modes \( \bar{A} \) and \( \bar{R} \) confined to the causally disconnected Rindler region \( I \) and detected, in principle, by observers anti-Alice and anti-Rob. For entanglement, this was done in [8] using different measures. Here, we do not have the space (and time) to adapt and extend such a study to encompass discord and classical correlations as well, although we believe this may constitute an interesting topic to expand upon elsewhere. We do, however, care to remark that the inertial entanglement between \( A \) and \( R \) (and the quantum correlations thereof) lost to the Unruh effect, can be partially interpreted and recovered in terms of genuine multipartite entanglement (respectively, discord) distributed among the accessible modes \( A, R, \) and the inaccessible ones \( \bar{R} \) and \( \bar{A} \) from noninertial perspectives. While this aspect was explored in [8, 34], the measures adopted in [52] and in this paper are especially suited for such a task, as the Rényi-2 entanglement \( \mathcal{E}_2 \) satisfies a general ‘monogamy’ inequality [69] on entanglement sharing for multimode Gaussian states, and the Rényi-2 discord \( D_2 \) enjoys the same property in the special case of pure three-mode Gaussian states [52], such as the states with CM \( \sigma_{ARR}^{[b]} \) (equation (35)).

Now we can focus on setting (a). It turns out that the genuine tripartite entanglement distributed among \( A, R, \) and \( \bar{R} \) is equal to the, suitably calculated, genuine tripartite discord.
The definition of the genuine tripartite entanglement involves a minimization over the three possible global bipartitions $A : (\dot{R} \dot{R})$, $R : (\dot{R} \dot{R})$ and $\dot{R} : (AR)$, as detailed in [29]; similarly, the definition of the genuine tripartite discord involves a minimization over the mode on which marginal measurements are not implemented [52]. The link between the two is provided by the Koashi–Winter duality relation [70]. In the present setting, these minimizations are solved by choosing the marginal measurements by Rob, but remain finite if revealed through measurements by Alice, as shown in figures 3(c) and (a), respectively. We then obtain, precisely,

$$E_2(a_{R(AR)}) - E_2(a_{RA}) - E_2(a_{RR}) = D_2(a_{R(AR)}) - D_2(a_{AR}) - D_2(a_{RR})$$

$$= Q_2^{\text{imp}}(a_{A,R,R})$$

where we have baptized the genuine tripartite nonclassical correlations (merging residual entanglement and residual discord) with the common symbol $Q_2^{\text{imp}}(a_{A,R,R})$, also occasionally referred to as ‘arravogliament’ [69]. We see, interestingly, that although the tripartite entanglement and tripartite discord coincide according to the chosen definitions, the bipartite quantities $E_2$ and $D_2$ are distributed in a slightly different way across the relevant partitions involving mode $R$, as shown in figures 3(a) and (b). In particular, as remarked previously, $E_2(a_{RA})$ vanishes in the limit of Rob undergoing infinite acceleration $r \to \infty$ while $D_2(a_{RA})$ remains finite (see equation (43)). As a consequence, we typically get $D_2(a_{RA}) \leq E_2(a_{RA})$, although both terms increase unboundedly with $r$ (these are the correlations specifically created across the Rindler horizon by the Unruh mechanism).

The genuine tripartite nonclassical correlations $Q_2^{\text{imp}}(a_{A,R,R})$ (equation (45)) are plotted in figure 3(c) as a function of $r$ and of the inertial squeezing degree $s$, and compared with the correlations $C_2(a_{M,R})$ (equation (34)) as detectable from a fully inertial perspective. We see that in the limit of high acceleration of Rob and large inertial correlations $(r, s \to \infty)$, a gap of $\ln 2$ remains between the two quantities, meaning that not all inertial correlations are recovered as distributed nonclassical correlations among all involved modes from a noninertial perspective. In fact, the missing bit could be read as the one remaining in the guise of bipartite discord between the two observable modes, equation (43).

### 4. Conclusion and outlook

In this paper we presented a collection of targeted review material and original research, with the primary aim of showcasing the power of continuous variable methods based on Gaussian states and operations, and their natural-born relevance for RQI. We applied our framework to the now paradigmatic study of degradation of entanglement [46] and other forms of quantum and classical correlations [47, 48, 55] between two scalar field modes in noninertial frames, as a consequence of the Unruh effect [6, 38].

Among the several highlighted phenomena which could be of interest, let us remark that the state of two field modes (with a certain degree of correlations from a fully inertial perspective) as described by an inertial observer Alice and a noninertial observer Rob approaches, in the limit of infinite uniform acceleration of Rob, a so-called classical-quantum state [54, 55], for which quantum correlations in the form of discord [47] are zero if revealed through measurements by Rob, but remain finite if revealed through measurements by Alice, as explained in the previous section. This could mean that nontrivial quantum communication between Alice and Rob might be possible in one direction only [56].
While these considerations have a value from a foundational perspective, the findings we discussed here have to be regarded more as a teaser, rather than as a concrete setting in which quantum information and communication tasks might be implemented. In our examples, we dealt with the idealized setting of uniformly accelerated observers, and critically with global field modes, whose detection for the purposes of extracting and exploiting correlations as resources in quantum protocols is somewhat troublesome [43]. It is further not obvious how to relate the entanglement and correlation properties of such field modes, as discussed in this paper, to the yield of specific quantum communication settings [71].

Interesting approaches to overcome these theoretical and practical limitations, which are now surfacing in RQI literature, include the application of quantum Shannon theory to characterize the communication capacity associated to relativistic channels [23], the study of entanglement between field modes confined in cavities undergoing general spacetime trajectories [21], the analysis of localized observables as detected in different reference frames for directional quantum communication [24], and novel models of localized field and particle
detectors [44, 45]. Crucially, in most of the above settings, \textit{mutatis mutandis} one ends up dealing with Gaussian states and transformations. Therefore, the plethora of tools presented here to assess the measure and structure of general types of correlations in bosonic Gaussian states, could and should be readily applied to those more realistic setups, possibly providing new angles for understanding and new pathways for implementation of RQI processing. This will be the scope of future work.

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