Monte Carlo calculations for the hard Pomeron

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Starting from the same input as the standard BFKL Pomeron, we directly calculate the “hard” Pomeron as a gluonic ladder by using Monte Carlo methods. We reproduce the characteristic features of the the BFKL Pomeron and are now also able to evaluate new observables. The applicability of the BFKL approach under realistic kinematical conditions can be tested and the influence of the running coupling constant examined.

I. INTRODUCTION

A large amount of data for reactions at high energies could be explained very succesfusly in terms of the exchange of the Pomeron. It was introduced into the general framework of reggeon exchanges in an ad hoc fashion and provides the dominant contribution. It is therefore a major challenge to understand the nature of this Pomeron microscopically in terms of QCD. Most of the data so far involved small momentum transfers, and the exchanged Pomeron was referred to as a “soft” Pomeron. In trying to understand this phenomenology on the basis of QCD, it can be argued that the underlying mechanism mainly consists of the exchange of gluons, is of long range and thus involves non-perturbative dynamics.

Attempts to understand the Pomeron from a perturbative QCD point of view lead to the concept of a “hard” Pomeron that is involved in small distance, high momentum transfer processes. The interest in this Pomeron was renewed when recent data from HERA at low \(x\) and large \(Q^2\) showed features that could be explained with the exchange of such a hard Pomeron, which has a behavior quite different from the soft one. The theoretical approach to understand the hard Pomeron goes back to the work of Lipatov et al. (BFKL) \cite{1,2}. These authors were able to infer some properties of the hard Pomeron perturbatively, describing it in terms of a reggeized gluon ladder. Their calculation was performed in leading logarithmic order (\(\alpha_s \log s \sim 1\) for a fixed \(\alpha_s \ll 1\)) where the multiregge kinematics gives the dominant contribution.

In this paper, we use a numerical method to better understand the features of the hard Pomeron and to see to what extent the concept of a perturbative Pomeron as embodied in the BFKL approach remains valid. Rather than proceeding as BFKL and solving analytically under the conditions mentioned above, where \(\log s\) goes to infinity, we use a Monte Carlo method to directly evaluate reggeized gluon ladders over a broad kinematical range and for different values of \(\alpha_s\). This allows us to study the ladder structure explicitly and in more detail. In particular, we are interested to see to what extent the kinematics within the ladder stays in a region where perturbative QCD can be applied. Our method also allows us to examine the consequences of letting the coupling constant run. Our approach differs from previous numerical studies which use angular ordering \cite{3} or start from the BFKL equations \cite{4}.
II. THE BFKL POMERON

We will shortly review the approach leading to the BFKL Pomeron. The BFKL Pomeron can be viewed as the exchange of gluonic ladders between two particles \( A \) and \( B \) as shown in Fig.1. The exchanged ladder must be a color singlet state. Working in the leading logarithmic approximation, the main contribution then comes from the multiregge region. In terms of the rapidities \( y_i \) of the emitted gluons with momenta \( k_i \), in this kinematical region the rapidity intervals, \( \delta y_i \), between two neighboring emitted gluons with rapidities \( y_i \) and \( y_i+1 \), satisfy \( \delta y_i \gg 1 \). The BFKL Pomeron thus is a gluon ladder where the rapidities along the ladder increase monotonically. The total rapidity interval, \( Y = Y_B - Y_A \), between the particles \( A \) and \( B \) can be simply expressed as the sum of the individual rapidity intervals:

\[
Y = \sum_{i=1}^{n+1} \delta y_i .
\]

The main features of the BFKL Pomeron can be discussed by starting from the expression for the total inclusive cross section for the reaction \( A + B \rightarrow X \), proportional to the imaginary part of the elastic amplitude with a gluon ladder exchange; an example with \( n \) rungs is shown in Fig.1. The differential cross section has the form

\[
d\sigma_n(A + B \rightarrow X) = \frac{\pi}{s^2} \prod_{i=1}^{n+1} \frac{dy_i d^2 k_i^\perp}{2(2\pi)^3} \delta \left( Y - \sum_{i=1}^{n+1} \delta y_i \right) |T_n|^2 ,
\]

where \( T_n \) is the amplitude for emission of \( n \) hard gluons (gluon jets). The Pomeron exchange consists of the sum over all such ladders with different numbers of gluons. The main ingredients in this gluon emission amplitude are the non-local, gauge invariant effective gluon vertices, \( \Gamma^\mu \), and the reggeized gluon propagators. In the cross section, the vertices appear only in a form contracted over color indices and gluon polarizations, i.e. as a contraction of two “Lipatov vertices”,

\[
\Gamma^\mu(q_k, q_{k+1}) \cdot \Gamma^\nu(q_k, q_{k+1}) = \frac{(q_{k+1,\perp}^2 + \mu^2)(q_k^2 + \mu^2)}{(q_{k,\perp}^2 - q_{k+1,\perp}^2)^2 + \mu^2} .
\]

The gluon mass \( \mu \) is introduced in order to regulate the infrared behaviour. The Lipatov vertices can only be used in the case of large rapidity intervals because otherwise higher-order corrections to the vertices become important. The reggeization of the gluons is taken into account by replacing the free gluon propagators by the reggeized ones:

\[
\frac{1}{t_i - \mu^2} \rightarrow \frac{e^{\delta y_i \epsilon(t_i) \frac{\mu}{\alpha_s N_c}}}{t_i - \mu^2} ,
\]

where \( t_i = q_i^2 \approx -q_{i,\perp}^2 \) and \( \epsilon \) the gluon trajectory. For large momenta \( \epsilon \) is given by

\[
\epsilon(q_{i,\perp}^2) \sim -\frac{\alpha_s N_c}{4\pi} \log \left( \frac{q_{i,\perp}^2}{\mu^2} \right) .
\]

The reggeized gluons represent the exchange of color octet gluon ladders. Their use can be interpreted as taking into account Sudakov-like suppression; the inclusion of radiative corrections corresponds to the probability that there is no emission of additional gluons in rapidity interval \( \delta y_i \). In this way any double counting is avoided. Since the infrared divergences coming from the vertices and the reggegeized propagators cancel each other, one can safely take the limit \( \mu \rightarrow 0 \) as is done in the original BFKL Pomeron. It can be shown that the colour octet part of gluon ladder exchanges as in Fig.1 indeed yields the reggeized gluon, proving the self-consistency of this approach.

The reaction mechanism that is independent of the properties of the initial particles \( A \) and \( B \), the exchanged Pomeron, is indicated by the dashed box in Fig.1. For the dashed box one can derive a recursion
relation in the number of the emitted gluons which leads to the well-known BFKL equation \[ [1,2] \]. For the cross section for this subprocess, the interaction of two gluons (with transverse momenta \( q_{A\perp} \) and \( q_{B\perp} \)), the BFKL equation predicts in the asymptotic limit

\[
\frac{d\sigma}{dq_{A\perp}dq_{B\perp}} \sim \frac{1}{\sqrt{Y}} \exp \left( Y\Delta - \frac{\log^2(q_{A\perp}^2/q_{B\perp}^2)}{4BY} \right),
\]

where

\[
\Delta = \alpha_s \frac{4N_c}{\pi} \log 2 \approx 2.65 \cdot \alpha_s, \quad B = 14 \zeta(3) \frac{\alpha_s N_c}{\pi} \approx 12.6 \cdot \alpha_s.
\]

It is necessary that the initial and final transverse momenta of the ladder are large such that perturbation theory can be expected to hold. This means that the dimensions of the hadrons \( A \) and \( B \) (\( R_A \) and \( R_B \)) should be small. From Eq.(5) one can now see several characteristic features of the BFKL Pomeron exchange:

- **Energy dependence of the total cross section:**
  After integration over the gluon transverse momenta, one obtains for \( \sigma \)

\[
\sigma \sim \frac{1}{\sqrt{Y}} \exp(\Delta Y) \sim s^{\Delta}.
\]

For a realistic value of the fixed coupling constant, \( \alpha_s = 0.2 \), one obtains for \( \Delta = 0.53 \), which is much larger than for the soft Pomeron, while the low-\( x \) data indicate \( \Delta \approx 0.3 \). So it will be interesting to find out what will be the effect of including a running coupling constant in the BFKL Pomeron on the intercept.

- **Diffusion pattern of the transverse momentum distribution:**
  From Eq.(5) one also immediately recognizes the diffusion behavior in the logarithm of the transverse momenta \[4\]. For a gluon with transverse momentum \( q_{\perp} \) and with rapidity \( Y_A + y \) the logarithm of the transverse momentum will under the condition \( 1 \ll y \ll Y \) fluctuate around its initial value \( \log(q_{A\perp}^2) \) according to

\[
< |\log(q_{A\perp}^2) - \log(q_{A\perp}^2)| >^2 \approx Cy, \quad C = \frac{4B}{\pi} \approx 16.0 \alpha_s.
\]

The transverse momenta are not ordered like in the Altarelli-Parisi evolution, but they perform a random walk. This means that there can be contributions to the ladder from configurations both with low and high transverse momenta. For the relatively small starting \( q_{A\perp}^2 \) we consider below, the momenta diffuse quite rapidly to larger values of \( q_{A\perp}^2 \).

- **Rapidity distribution:**
  Eq.(5) can also be used to obtain the gluon density in the ladder at a given rapidity difference \( y \). In Refs. \([1,2]\) the inclusive cross section for the production of a gluon (gluon jet) with transverse momentum \( k_{\perp} \) and rapidity \( Y_A + y \) in the exchange of a BFKL Pomeron was derived for the case \( Y - y \gg 1 \) and \( y \gg 1 \). Under these conditions the emitted gluon is not too close to the ends of the ladder. This cross section corresponds to fixing the momentum of a rung in the gluon ladder of Fig.1 and can be described by Eq.(5) for the parts of the ladder above and below the gluon in question. Upon integration over \( k_{\perp} \), one can obtain the rapidity distribution \( \rho(y) \), the gluon density per unit rapidity interval of the emitted gluons at a given rapidity difference \( y \),

\[
\rho(y) \equiv \int dk_{\perp}^2 \frac{1}{\sigma} \frac{d\sigma}{dydk_{\perp}^2} [gg \to Xg(y, k_{\perp}^2)]
\]

\[
\simeq 1.6 \alpha_s^{3/2} \sqrt{\frac{y(Y - y)}{Y}}.
\]
In the center of the spectrum one has $\rho(y = Y/2) \simeq 0.8 \cdot \alpha_s^{3/2} \cdot \sqrt{\gamma}$. The BFKL results are valid only in multiregge kinematics, where $<\delta y>_y = \rho^{-1} \gg 1$. To fulfill this condition one should have $\alpha_s Y \ll 1.2 \alpha_s^{-2}$. At the edge of the inclusive spectrum ($y \ll Y$) one finds

$$\rho(y) \simeq 1.6 \ \alpha_s^{3/2} \ \sqrt{\gamma} + D(\alpha_s) \ , \quad (10)$$

where an unknown $y$ independent function $D$ which determines the density at small $y$, has been added. The rapidity interval between neighboring gluons $<\delta y>_y \simeq 0.6/(\alpha_s^{3/2} \sqrt{\gamma})$ becomes thus smaller with $y$ towards the middle of the ladder. Note that the analytical result in Eq. (10) can be simply understood. As the only dependence of the amplitude $T_n$ on $\delta y$ is due to the propagator, Eq. (3), this $\rho(y)$ can be estimated as $\rho(y) \simeq 1/ <\delta y>_y \sim 2\epsilon(\epsilon) \sim \alpha_s^{3/2} \sqrt{\gamma}$, where we used Eq. (8) to approximate $<\log q_\perp^2 > \sim \alpha_s \sqrt{\gamma}$.

III. MONTE CARLO APPROACH FOR THE POMERON

To get a more detailed understanding of the hard Pomeron, a direct evaluation of the multidimensional integrals in Eq. (10) would be necessary. The expected gluon multiplicities for large total rapidity intervals are large, $n \sim 10^5 \div 100$, and thus the integrals of such high dimensionality that straightforward evaluation is impossible. We therefore have developed a Monte Carlo approach which enables us to deal explicitly with the multidimensional integrals. By keeping track of the kinematics, we can then examine to what extent the BFKL Pomeron indeed is made up from hard gluons and, for example, if the assumptions about the large rapidity intervals, $\delta y_i \gg 1$, are justified throughout the ladder. An advantage of working numerically is that we can investigate what effect the use of a running coupling constant will have on the structure of the gluon ladder. We use for the running coupling constant

$$\alpha_s(q_\perp^2) = \frac{1}{b_0 \log (a + q_\perp^2 / \Lambda^2)} \ , \quad (11)$$

i.e. the standard expression with $\Lambda = 250$ MeV, but extended by an additional parameter $a$ that fixes the coupling constant at $|q_\perp| = 0$, thus regularizing the infrared singularity. The parameter $a$ can be interpreted as the interaction with non-perturbative vacuum fluctuations which freezes the coupling constant at low momentum scales $\Lambda$. The value of $a$ should be such that $\alpha_s(0) \sim 0.4 \div 0.7$. In our calculations we took $a = 10.3$, corresponding to $\alpha_s(0) = 0.6$.

As in the discussion in the last section, we will focus on the gluon subprocess indicated by the box in Fig. 1. Therefore, we will generate for our Monte Carlo evaluation configurations $\{n, y_i, k_{i\perp}\}$ for the emitted gluons or, equivalently, for the intermediate gluons along the ladder, separated in rapidity by at least $\delta y_{min} \geq 0$, which is an input parameter. This is done in two steps. First, we generate ladders with a certain distribution in the momenta and in the number of rungs by an approximation method as described below. Each of these configurations is then used as the starting point for a Metropolis procedure to generate new configurations. This serves to correct for errors inherent in the approximation in the first step. This procedure leaves the distribution in $n$ unchanged.

In generating these multidimensional configurations, we make use of the specific dependence of the integrand on the kinematical variables,

$$|T_n|^2 \sim \prod_{i=1}^{n} f(\delta y_{i+1}, q_{i\perp}, q_{i+1\perp}) \ , \quad (12)$$

$$f(\delta y_{i+1}, q_{i\perp}, q_{i+1\perp}) = \frac{3\alpha_s(q_{i+1\perp}^2)}{\pi^2} \cdot \frac{1}{(q_{i\perp} - q_{i+1\perp})^2 + \mu^2} \ , \quad (13)$$
Note that the external kinematics requires $q_{1\perp} = q_{A\perp}$ and $q_{n+1\perp} = q_{B\perp}$. Further we take for the gluon trajectory the usual form
\begin{equation}
\epsilon(q_{\perp}^2) = -(q_{\perp}^2 + \mu^2) \int \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{N_c \alpha(k_{\perp}^2)}{(k_{\perp}^2 + \mu^2)(|k_{\perp} - q_{\perp}|^2 + \mu^2)} ,
\end{equation}
but now with a running coupling constant and which at high $q_{\perp}^2$ for fixed coupling constant leads to the asymptotic form of Eq. (4).

The fact that Eq. (12) is a product of functions of pairs of kinematical variables referring to subsequent rungs in the ladder allows one to generate the ensemble of configurations in a sequential fashion, for which we chose the von Neumann rejection method. It is essential that the factors $f(\delta y_{i+1}, q_{i\perp}, q_{i+1\perp})$ are positive definite and thus can be interpreted as probabilities for the production of the $(i + 1)$th rung with rapidity difference $\delta y_{i+1}$ and linked to the former rung by a gluon with momentum $q_{i+1\perp}$, when the previous rungs have already been generated. In fact, the conditional probability to produce the $(i + 1)$th rung with $(\delta y_{i+1}, q_{i+1\perp})$ as part of the whole ladder is given by
\begin{equation}
f(\delta y_{i+1}, q_{i\perp}, q_{i+1\perp}) \cdot \psi(Y - Y_{i+1}, q_{i+1\perp}) ,
\end{equation}
where $Y_{i+1} = \sum_{k=1}^{i+1} \delta y_k$ is the rapidity interval between the ‘zeroth’ (i.e. particle A) and $(i + 1)$th rung and
\begin{equation}
\psi(Y - Y_{i+1}, q_{i+1\perp}) = \prod_{n=i+1}^{\infty} \int \prod_{l=i+2}^{n+1} \frac{d\delta y_l}{2\pi} \frac{d^2 q_{l\perp}}{(2\pi)^2} f(\delta y_l, q_{l-1\perp}, q_{l\perp}) \delta \left( Y - Y_{i+1} - \sum_{l=i+2}^{n+1} \delta y_l \right) ,
\end{equation}
is the total probability of configurations ‘after’ this $(i + 1)$th rung. Since the integrand in $\psi$ is given by a similar factorized form as Eq. (13), we can use Eq. (5) to see that at very large $Y - Y_{i+1}$
\begin{equation}
\psi(Y - Y_{i+1}, q_{i+1\perp}) \sim \exp \left( -\frac{\log^2(q_{i+1\perp}^2/q_{B\perp}^2)}{4B(Y - Y_{i+1})} \right) ,
\end{equation}
where $q_{B\perp}$ is the transverse momentum of the final gluon in the ladder which is taken to be fixed and not too large. So for $Y - Y_{i+1} \gg Y_{i+1}$ the function $\psi$ is constant for values of $\log q_{i+1\perp}^2 \sim \sqrt{Y_{i+1}}$ which we expect to be essential in the generation of the quantities $(\delta y_{i+1}, q_{i+1\perp})$ at the $(i + 1)$th rung. Therefore when generating the initial part of the ladder, it is a good approximation to use only the first factor $f(\delta y_{i+1}, q_{i\perp}, q_{i+1\perp})$ in Eq. (13). Starting with the first rung, for which we take $|q_{A\perp}| = R_A^{-1}$, we can therefore generate subsequently for every rung the variables $(\delta y_i, q_{i\perp})$. We repeat this until $Y_{n+1} \approx Y$, which in our approach then fixes the gluon multiplicity $n$ in this ladder. We stress that we will only use the first part of the ladder for the results below.

It should be noted that when using the weight functions generated with the von Neumann rejection method, we have to introduce an ultraviolet cut in $k_{\perp}$. This is due to neglecting $\psi$ in Eq. (13). The dependence on this cutoff parameter $Q_{\text{max}}$ is of type $\log \log (Q_{\text{max}}^2/\mu^2)$ and thus weak. Since we fix the end momentum of the ladder, $|q_{B\perp}| \sim R_B^{-1}$, the dependence on $Q_{\text{max}}$ vanishes.

For the Metropolis routine, we use the configurations obtained by the von Neumann method as starting configurations to improve the sampling in the phase space, producing a series of new configurations $\{n, y'_i, k'_{i\perp}\}$. Note that the number of runs $n$ is not changed. A more detailed discussion will be published elsewhere.

To calculate transverse momenta distributions, mean rapidity intervals etc., we take a very large total rapidity interval $Y$ and consider only a part of the ladder for which $y \ll Y$ in accordance to our approximation $\psi \sim \text{constant}$. That this yields predictions independent of $Y$ and other quantities characterizing
the particle $B$ at the end of the gluon chain can also be seen, for example, from the asymptotic expression for the inclusive cross section, Eq. (1). So one should realize that using our method we can investigate only distributions in the growing part of the gluon spectrum.

For the dependence of the total cross section on the energy $s$ we have to proceed differently. In order to obtain the Pomeron intercept it is sufficient for the total cross section to integrate (12) over the full phase space with logarithmic precision. This was done in two ways. In the first method, we used the ratio of accepted to total events in the von Neumann procedure at different values of $Y \sim \log s$. Fitting this to an exponential function, we obtain the exponent $\Delta$ in Eq. (7). In the second method, we first calculate for a given energy $s$ (or rapidity interval $Y$) the mean multiplicity of the ladder and then approximate the Pomeron by a ladder containing that many rungs. In the corresponding amplitude, Eq. (12), we then replace the transverse momenta of the previous rungs in the products of the functions $f(\delta y_{i+1}, q_{i+1\perp}, q_{i\perp})$ by the already determined mean value for that rung. This approximation enables us to compute all integrals needed for the total cross section separately. Performing this again for different values of $Y$ the exponent $\Delta$ can be extracted. Both methods were found to agree reasonably well.

### IV. RESULTS AND DISCUSSION

We first show what our Monte Carlo calculations yield for the properties of the gluon ladder discussed in Section 2 and compare them to the predictions of the BFKL theory in the asymptotic limit. In all our calculations presented in this section we chose for the “constituent” gluon mass $\mu$ one half of the gluonium mass: $\mu \simeq 0.5$ GeV. We have checked that changes by a factor of two in this parameter had a negligible effect on the results below. Furthermore, we established in all cases that there is indeed no dependence on the ultraviolet cutoff introduced in the von Neumann step.

First we examined the exponent $\Delta$, which fixes the Pomeron intercept. We performed calculations with a fixed coupling constant in the range 0.01–0.20 and for a total rapidity interval $Y = 50$. For the lowest values of $\alpha_s$ this yields rather short ladders ($n \sim 6$) for which our method of generating rungs in the ladder is less reliable. For $\alpha_s = 0.05$, we deal with ladders of total length $n \sim 20$ and obtain $\Delta = 0.11$, close to the BFKL result of Eq. (6) which gives 0.13. Taking $\alpha_s = 0.1$, we get numerically 0.27, compared to the BFKL value of 0.27. This reasonable agreement indicates that our numerical approach is quite reliable.

To use our samples to extract the gluon transverse momentum and rapidity distributions, we should restrict ourselves to the first part of the ladder, i.e. up to rapidity intervals of $Y \sim 20$ in our example. For the final gluon we took $|q_{B\perp}| = R_B^{-1} \simeq R_A^{-1}$. In this special case with similar transverse momenta for both ends the last part of the ladder is identical to the first part and, aside from some uncertainty in the middle of the ladder, the complete ladder structure can be obtained. Results for the mean logarithm of the transverse momenta for an initial gluon momentum of $|q_{A\perp}| = R_A^{-1} = 10$ fm$^{-1}$ and for two values of the coupling constant $\alpha_s$ are shown in Fig. 2. Indeed we see that $<\log(q_{i\perp}^2/q_{i+1\perp}^2)>^2$ follows the behavior predicted by the BFKL theory, Eq. (6); a linear rise with the rapidity difference $y$. The slopes also agree reasonably well with the BFKL prediction. For $\alpha_s = 0.05$ and $\alpha_s = 0.1$ we find respectively 0.31 and 0.51 compared to the analytic BFKL values of 0.8 and 1.6. The gluon density $\rho(y)$ is shown in Fig. 3. Here our results also reproduce the BFKL predictions well, both the $y$ dependence and the coefficients: we fitted both curves to the function $\rho(y) = C_0 + C_1 \sqrt{g}$ and found for $C_1$ a value of $2.2 \times 10^{-2}$ for $\alpha_s = 0.05$ and $5.8 \times 10^{-2}$ for $\alpha_s = 0.1$ whereas BFKL in the asymptotic limit predicts $1.8 \times 10^{-2}$ and $5.1 \times 10^{-2}$, respectively.

This good agreement with the main BFKL features confirms the validity of our method. We now use it to examine several aspects that go beyond the original framework of the BFKL Pomeron. The first concerns the effect of a running coupling constant. We repeated the calculation for the total cross section with the $\alpha_s$ given in Eq. (1). If we try to fit the energy dependence according to Eq. (6), the effective value for $\Delta$ clearly decreases compared to that obtained with a fixed coupling constant $\alpha_s = \alpha_s(q_{2\perp}^2)$, but our method of calculating is not reliable enough to give its precise value. This decrease can be understood from Fig. 2, where we also show a curve for a running coupling constant. We observe that $q_{2\perp}^2$ continues to grow as we go down the ladder (just as for fixed $\alpha_s$); this is due to our choice of $q_{A\perp}^2$ which leads to
a rapid diffusion from \( \log(q_{\perp}^2) \) mainly to larger values of \( \log(q_{\perp}^2) \). This leads to contributions with lower coupling constants, resulting in a lower \( \Delta \). The effect of a running coupling constant on the gluon density is shown in Fig.3. While there are some changes, it has no essential influence on the qualitative features of the distribution of gluons in the hard Pomeron. There have been previous studies of the effect of a running coupling constant \[2,8,9\]. They showed the decrease of \( \Delta \) and its influence on the character of the Pomeron singularity.

Another important aspect we can investigate is the rapidity interval, \( \delta y \), between successive rungs of the gluon ladder. What we show in Fig.4 is the average rapidity interval as function of the rung number in the ladder. Our results show a rapid drop of \( \langle \delta y \rangle \) for the first rungs, going over in a much slower decrease which continues to the end of the ladder. In our calculations we imposed a lower bound of \( \delta y_{\text{min}} = 0 \), since the leading contribution is assumed to come from high \( \delta y \) configurations. Fig.4 makes clear how important the value of this cut is: a major part of the ladder consists of rungs with only a relatively small rapidity separation, typically of order 1.5 for \( \alpha_s = 0.1 \). Thus had we chosen a large \( \delta y_{\text{min}} \), entirely different results would have been obtained. The use of a running coupling constant leads to the same observations. For a smaller fixed coupling constant (e.g. \( \alpha_s \sim 0.05 \)) we find rapidity intervals mainly around 3.0, which means imposing \( \delta y_{\text{min}} > 0 \) would have a smaller effect on the ladder structure. From this analysis we see that configurations with small rapidity intervals in the ladder play an important role and can influence the results for larger values of the fixed coupling constant and for the running one.

Our Monte Carlo study of the hard Pomeron in terms of explicit calculations of gluon ladders reproduced the main predictions of the BFKL Pomeron over a wider range of kinematical conditions. This seems to confirm the validity of the kinematical assumptions that go into the BFKL derivation. However, when one looks in more detail at the gluon ladder, which can be done with our numerical method, it becomes clear that the multiregge kinematics are not applicable anymore towards the middle of the ladder, especially for not too small values of \( \alpha_s \). In a forthcoming publication, we will therefore study the validity of the BFKL Pomeron in more detail for different kinematical conditions and the importance of higher order corrections through numerical simulations. Theoretical progress has been made in computing the next-to-leading order terms \[9\], but the expressions are rather complicated and there is presently little hope that higher order terms can be computed exactly. More intuitive pictures for the hard Pomeron such as our direct ladder calculations or the color dipole model \[11\] can show the way towards a better understanding of the nature of the “hard” Pomeron.

ACKNOWLEDGEMENTS

We thank K. Boreskov and A. Kaidalov for stimulating discussions, and Y. Simonov and K. Ter Martirosyan for interesting remarks. The work of L.H. and J.K. is part of the research program of the Foundation for Fundamental Research of Matter (FOM) and the National Organization for Scientific Research (NWO). The collaboration with between NIKHEF and ITEP was supported by a grant from NWO and by grant 93-79 of INTAS. O.K. also acknowledges support from grant J74100 of the International Science Foundation and the Russian Government.

[1] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP 45 (1977) 199; Ya. Ya. Balitskii and L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822;
[2] L.N. Lipatov, Sov. Phys. JETP 63 (1986) 904; L.N. Lipatov in Perturbative Quantum Chromodynamics Ed. A.H. Mueller, World Scientific Singapore
[3] G. Marchesini and B.R. Webber, Nucl. Phys. B349 (1991) 617
[4] J.Bartels, H. Lotter and M. Vogt, Phys. Lett. B373 (1996) 215
[5] V. Del Duca, M.E. Peskin and W-K. Tang, Phys. Lett. B406 (1993) 259
[6] M.G. Ryskin, Sov. J. Nucl. Phys. 32 (1980) 133
[7] Y.A. Simonov, private communication
[8] R.E. Hancock and D.A. Ross, Nucl. Phys. B383 (1992) 575
[9] A.J. Askew, J. Kwiecinski, A.D. Martin and P.J. Sutton, Phys. Rev. D47 (1993) 377
[10] V.S. Fadin and L.N. Lipatov, DESY Report 96-020 (1996)
[11] A. Mueller, Nucl. Phys. B415 (1994) 373; N.N. Nikolaev, B.G. Zaharov and V. Zoller, Phys. Lett. B327 (1994) 149
FIGURE CAPTIONS

**Fig. 1** The contribution of a gluonic ladder with $n$ rungs to the Pomeron exchange. The Lipatov vertices are denoted by the dots and the reggeized gluon propagators by the thick, vertical gluon lines.

**Fig. 2** The square of the mean for the absolute value of the logarithm of the gluon transverse momentum along the ladder, $|\log(q_{\perp}^2/q_{A,\perp}^2)|$, as function of rapidity interval $y$ for different coupling constants; the dashed line for $\alpha_s = 0.05$, the solid line for $\alpha_s = 0.1$ and the dot-dashed line for a running coupling constant.

**Fig. 3** The gluon density, $\rho$, as a function of rapidity interval $y$ for different coupling constants; the dashed line for $\alpha_s = 0.05$, the solid line for $\alpha_s = 0.1$ and the dot-dashed line for a running coupling constant.

**Fig. 4** The average rapidity interval between two neighboring rungs as function of the rung number $N$ for different coupling constants; the dashed line for $\alpha_s = 0.05$, the solid line for $\alpha_s = 0.1$ and the dot-dashed line for a running coupling constant.
Figure 1
Figure 2
Figure 3
