A QUEST FOR A “DIRECT” OBSERVATION OF THE UNRUH EFFECT WITH CLASSICAL ELECTRODYNAMICS: IN HONOR OF ATSUSHI HIGUCHI 60th ANNIVERSARY

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The Unruh effect is essential to keep the consistency of quantum field theory in inertial and uniformly accelerated frames. Thus, the Unruh effect must be considered as well tested as quantum field theory itself. In spite of it, it would be nice to realize an experiment whose output could be directly interpreted in terms of the Unruh effect. This is not easy because the linear acceleration needed to reach a temperature 1 K is of order \(10^{20}\) m/s\(^2\). We discuss here a conceptually simple experiment reachable under present technology which may accomplish this goal. The inspiration for this proposal can be traced back to Atsushi Higuchi’s Ph.D. thesis, which makes it particularly suitable to pay tribute to him on occasion of his 60th anniversary.

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1. Introduction

In November 1972, Stephen Fulling submitted his famous paper \(^\text{1}\) “Nonuniqueness of canonical field quantization in Riemannian space-time”, where he presented the Bogoliubov coefficients relating Minkowski and Rindler vacua and stated that “The notion of particle is completely different in the two theories”. It would be shown later that this was closely connected to the seminal paper \(^\text{2}\) submitted by Stephen Hawking sixteen months later on the evaporation of black holes. In the beginning, Hawking’s effect was not consensual at all. The fact that Hawking’s analysis implied an arbitrary large density of particles near the horizon was disturbing, since it seemed to impact on the hole’s integrity itself. An intense discussion was on the verge of beginning. In August 1974, Paul Davies submitted his paper \(^\text{3}\) “Scalar particle production in Schwarzschild and Rindler metrics”, where the procedure used by Hawking in his 1975 paper \(^\text{4}\) “Particle creation by black holes”, was applied in flat spacetime to a Rindler coordinate system. In Ref. \(^\text{3}\) we find the famous acceleration temperature formula,

\[ T_U = \frac{a\hbar}{(2\pi k_Bc)}, \]

but it was derived in the presence of a mirror and its physical content was quite unclear: “The apparent production of particles is somewhat paradoxical, because there is no obvious source of energy for such a production”. The fact that a uniformly accelerated observer with proper acceleration \(a\) in the Minkowski vacuum sees a thermal bath of particles at a temperature \((1)\) was communicated by William Unruh \(^\text{5}\) in the 1st Marcel Grossmann meeting on General Relativity held in Trieste in July 1975. Unfortunately, the tradition of the Marcel Grossmann meeting of publishing proceedings late can be traced back to its first edition. \(^\text{5}\) In the meantime, Unruh published his renowned paper \(^\text{6}\) “Notes on black hole evaporation” in 1976 announcing the effect named after him. His motivation was twofold. On the one hand, he wanted to understand better Fulling’s 1972 result and on the other one Hawking’s effect. It became clear, afterwards, that the thermal bath experienced by uniformly accelerated observers in Minkowski spacetime is composed of real particles (similarly to the ones experienced by stationary observers outside evaporating black holes) as well as that their presence is consistent with a negligible backreaction effect. As a result, black holes would not be disrupted at all by the arbitrarily large temperature of Hawking radiation near its horizon. Interestingly enough, Geoffrey Sewell \(^\text{7}\) realized in 1982 that the Unruh effect was also codified in Bisognano and Wichmann 1976 work \(^\text{8}\). No matter how nonintuitive the Unruh effect is \(^\text{5}\), it should be clear since the mid 80s that it is necessary to keep the consistency of quantum field theory in inertial and uniformly accelerated frames and, thus, that it must be considered as well tested as quantum field theory itself \(^\text{9}\) (see also Ref. \(^\text{10}\)). In spite

\(^\text{a}\) Actually, this was clear to Unruh before April 1974 – private communication.

\(^\text{b}\) One could see “accel. temp.” listed among the four issues “to learn” at Richard Feynman’s blackboard by the time he passed away in 1988 (see Fig. 1 of Ref. \(^\text{14}\)).
of this, claims that the Unruh effect either does not exist or, more often, requires experimental confirmation can still be found in the literature (see, e.g., Ref. [11] for a recent instance). Here, we bow in submission to what seems to be a demand to make the Unruh effect consensual, although some may see it as an unnecessary concession.[12]

Any observation of the Unruh effect must rely on resilient probes entrusted to (i) record the Unruh effect and (ii) make the information available to us, quasi-inertial observers. The very origin of the difficulty of “directly” observing the Unruh effect stems from the fact that the linear acceleration needed to reach a temperature 1 K is of order $10^{20}$ m/s$^2$. Item (i) drives our attention to massless rather than massive particles of the Unruh thermal bath. This is so because massive Rindler particles concentrate closer to the horizon than massless ones, making their observation a harder enterprise.[16,17] This is why using accelerated observers to investigate, e.g., Planck scale particles (with masses of order $10^{19}$ GeV), would be a terrible idea in practice. Among the massless particles, photons are much more promising than, say, gravitons, because their coupling to physical detectors is typically much larger.

Bell and Leinaas[13] were the first ones to say that the electron depolarization in storage rings could be explained in terms of the Unruh effect. They achieved partial success because strictly speaking the Unruh effect is not valid for circularly moving observers. Considering linear accelerators, rather than circular storage rings, is not an option, because the spins do not have enough time to thermalize in the Unruh thermal bath in the ultrashort lapse of time they get accelerated. In other proposals,[18–20] it is argued that one could relate the pairs of correlated photons emitted by accelerated charges and the corresponding charge quivering as seen by inertial observers with the scattering of Rindler photons of the Unruh thermal bath as defined by accelerated observers. The difficulty with this strategy is that the radiation of such correlated photons, usually denominated quantum radiation, would require still unavailable ultraintense lasers.

In a recent paper,[21] however, we have proposed a simple electromagnetic experiment feasible with present technology and free of unfamiliar concepts, which should make clear that the Unruh thermal bath can be already seen in the much stronger signal of Larmor radiation (i.e., one-photon emission at the tree level), once one accepts the “indisputable” quantum formula: $E = h\omega$. This is the single quantum ingredient we require to identify a signal of the Unruh effect in the classical Larmor radiation (see Ref. [22] for a comprehensive discussion). The source of inspiration for such an experiment can be traced back to Ref. [23] which, for its turn, is related to Atsushi Higuchi’s Ph.D. paper[24] making it particularly suitable to pay tribute to him on occasion of his 60th anniversary. Here, we focus on the main challenges to realize such an experiment.

The paper is organized as follows. In Sec. 2 we present the experimental setup. In Sec. 3 we explain the experimental strategy to confirm the Unruh effect. Because, after all, the experiment is based on plain classical electrodynamics, we are confident to anticipate its output using Maxwell equations in Sec. 4. In Sec. 5 we discuss some
experimental challenges to perform the real experiment. Our final conclusions are in Sec. 6. We adopt metric signature $(+,-,-,-)$ and natural units, $G = c = k_B = 1$, unless stated otherwise.

2. The experimental apparatus

Let us begin considering a pair of homogeneous and constant electric and magnetic fields, $E^z$ and $B^z$, respectively, lying along the $z$ axis. We will assume here usual cylindrical coordinates $(t, z, r, \phi)$ as in Ref. [21]. In general, a classical charge $q$ with mass $m_q$ in this environment will loop around the $z$ axis with some constant radius $r \equiv R$ and non constant pitch because of the presence of the electric field. We assume that the charge moves initially against the direction of the electric field, turning over at some point.

A properly chosen linearly accelerated observer (named here, Atsushi, for brevity) moving along the $z$ axis according to the worldline

$$t = a^{-1} \sinh a \tau, \quad z = a^{-1} \cosh a \tau, \quad x = y = 0,$$

with proper time $\tau$, and constant proper acceleration

$$a = \frac{qE^z}{m_q \gamma},$$

where $\gamma \equiv 1/\sqrt{1 - R^2 \Omega^2}$, will describe the charge as having closed circular trajec-

Fig. 1. Electrons are injected with velocity $\vec{v}$ in a cylinder containing linear electric and magnetic fields, $E^z$ and $B^z$, respectively. Radiation emitted where the charges make their U turn (look at the center of the figure) is released through an open window and collected by detectors lying on the sphere.
tory with constant angular velocity
\[ \Omega = \frac{d\phi}{d\tau} = \frac{B^z}{[m_\phi^2/q^2 + (RB^z)^2]^{1/2}}. \]

A prototype experimental apparatus is shown in Fig. 1. The charges are injected in a cylinder containing the linear electric, \( E^z \), and magnetic, \( B^z \), fields. The radiation emitted where the charges make the U turn is released through an open window of length \( L \) and collected by detectors set on a sphere with radius \( R_S \gg L \). Now, since the Unruh effect concerns the Minkowski vacuum (free of boundaries), we must demand that the finite size of the window does not influence the results. Since the wavelengths of the emitted radiation goes as \( \lambda \sim 1/a_{\text{tot}} \), where
\[ a_{\text{tot}} = \gamma^2 \sqrt{a^2 + R^2 \Omega^4} \]
is the charge total proper acceleration, we must require
\[ a_{\text{tot}} \sim 1/\lambda \gg 1/L \approx 10^{17} \text{ m/s}^2 \times (1 \text{ m}/L). \] (3)

Typical values achievable under present technology for the magnetic and electric fields, \( B^z \approx 10^{-1} \text{ T} \) and \( E^z \approx 1 \text{ MV/m} \), respectively, produce accelerations \( a \sim 10^{17} \text{ m/s}^2 \) and \( a_{\text{tot}} \sim 10^{19} \text{ m/s}^2 \), where we have assumed \( R \sim 10^{-1} \text{ m} \).

It is also worthwhile to note that we can neglect any radiation reaction effects on the charge trajectory. By using Larmor formula, we see that, under the conditions above, a bunch with a total charge of \( 10^7 e \) will emit (coherently) about

![Image](image_url)

Fig. 2. We plot \( dN_{k_\perp}^R / dk_\perp \) for different values of \( T \) assuming \( E^z = 1 \text{ MV/m} \), \( B^z = 10^{-1} \text{ T} \), \( R = 10^{-1} \text{ m} \), and injection energy 3.5 MeV. The right-hand side of Eq. (5) corresponding to the very prediction using the Unruh effect, is given by the solid line.
$10 \times L/(1 \text{ m}) \text{ GeV}$, which is, indeed, neglctable even in comparison to its rotational energy of about $10^4 \text{ GeV}$.

3. Experimental proposal to confirm the Unruh effect

According to the Unruh effect, uniformly accelerated observers in the Minkowski vacuum will experience a thermal bath at the Unruh temperature. Then, Atsushi, who sees the charge making closed circles around him, shall also witness it emitting and absorbing Rindler photons to and from the underlying thermal bath, respectively. A straightforward calculation using quantum field theory in uniformly accelerated frames allows Atsushi to calculate the corresponding emission and absorption rates. The combined distribution rate, i.e., emission plus absorption rates, of Rindler photons per transverse momentum $k_\perp \in [0, +\infty)$ and fixed magnetic quantum number $m \in \mathbb{Z}$, is computed to be

$$d\Gamma_{R \text{tot}}^{k_\perp m}dk_\perp = \frac{q^2k_\perp}{\pi^2ha} \left| \frac{K_{im}\Omega/a}{a} \left( \frac{k_\perp}{a} \right) \right|^2 |J_m(k_\perp R)|^2$$

$$+ (R\Omega)^2 \left| \frac{K_{im}\Omega/a}{a} \left( \frac{k_\perp}{a} \right) \right|^2 |J'_m(k_\perp R)|^2$$

$$\times \sinh \left( \frac{\pi m\Omega}{a} \right) \coth \left( \frac{m\Omega h}{2T} \right) \Theta(m), \quad (4)$$

where $J_n(x)$ and $K_n(x)$ are the first kind and modified Bessel functions, respectively, “’" means derivative with respect to the argument, $\Theta(m) \equiv 0, 1/2, 1$ for $m < 0, m = 0$, and $m > 0$, respectively, and $T$ is the temperature of the thermal bath.

The issue to be experimentally decided is whether or not $T = T_U$. To settle the issue, Atsushi uses the fact that each absorption and emission of a Rindler photon from and to the Unruh thermal bath, respectively, will correspond to the emission of a Minkowski photon according to inertial observers. Therefore, the validity of the Unruh effect demands that inertial experimentalists must measure the following corresponding rate of emitted photons:

$$\frac{dN^M_{k_\perp}}{dk_\perp} \propto \sum_{m=-\infty}^{\infty} \left. \frac{d\Gamma_{R \text{tot}}^{k_\perp m}}{dk_\perp} \right|_{T=T_U}. \quad (5)$$

The proportionality appears because the total number of emitted photons depends on how long the experiment is run. In Fig. 2 we plot $\sum_{m=-\infty}^{\infty} \frac{d\Gamma_{k_\perp m}}{dk_\perp}$ for different values of $T$ assuming $E^z = 1 \text{ MV/m}$, $B^z = 10^{-1} \text{ T}$, $R = 10^{-1} \text{ m}$, and injection energy 3.5 MeV. The prediction given by Eq. (5) ($T = T_U$) is represented by the solid line. We recall that we should focus on the region $k_\perp \gtrsim 1/L$ in order to guarantee that the finite size of the window may be neglected.

Rather than asking experimentalists to measure photons individually, it is
enough to ask them to measure the spectral-angular distribution

$$I(\omega, \theta, \phi) \equiv \frac{dE(\omega, \theta, \phi)}{d\omega d(\cos \theta) d\phi}$$

with $E$ being the energy radiated away, since

$$\frac{dN^{M}_{k_{\perp}}}{dk_{\perp}} = \frac{k_{\perp}}{\hbar} \int_{0}^{2\pi} d\phi \int_{-\infty}^{\infty} \frac{dk_{z}}{(k_{\perp}^{2} + k_{z}^{2})^{3/2}} I(\omega, \theta, \phi), \quad (6)$$

where $\omega^{2} = k_{\perp}^{2} + k_{z}^{2}$ and $k_{\perp} = \omega \sin \theta$. We note that the classical (Larmor) radiation corresponds to a coherent emission of photons and should be associated with a tree-level Feynman diagram with one single photon at the final state. We also note that the appearance of the $\hbar$ is thanks to the use of the one-photon relation $E = \hbar \omega$. This is the crux of how the classical quantity $I(\omega, \theta, \phi)$ is translated into the quantum one $dN^{M}_{k_{\perp}}/dk_{\perp}$ used to test the Unruh effect.

In summary, the idea is to perform the experiment above, measure $I(\omega, \theta, \phi)$, plug it in Eq. (6), and see whether or not $dN^{M}_{k_{\perp}}/dk_{\perp}$ is in agreement with Eq. (5).

4. Virtual confirmation of the Unruh effect

It happens, however, that we can anticipate instrumentalists and calculate the spectral-angular distribution

$$I(\omega, \theta, \phi) = \frac{R_{S}^{2}}{\pi} \left| \int_{-\infty}^{\infty} dt \, \tilde{E}_{\text{rad}}(t, \theta, \phi)e^{-i\omega t} \right|^{2} \quad (7)$$

using Maxwell equations where $\tilde{E}_{\text{rad}}(t, \theta, \phi)$ is the electric field, which reaches the detectors on the sphere with radius $R_{S}$.

A straightforward calculation assuming our accelerated point-like charge and Eq. (6) leads to

$$\frac{dN^{M}_{k_{\perp}}}{dk_{\perp}} = \left( \frac{4\pi}{a} \int_{-\infty}^{\infty} \frac{dk_{z}}{(1 + k_{z}^{2})^{1/2}} \right) \sum_{m=-\infty}^{\infty} \frac{dT^{R}_{k_{\perp}m}}{dk_{\perp}k_{\perp}} \bigg|_{T=T_{U}}, \quad (8)$$

which is in perfect agreement with Eq. (5). The fact that the term between parentheses diverges is because the calculation above has assumed a charge accelerating for infinite time, in which case an infinite number of photons is emitted for fixed $k_{\perp}$ element. Of course, in real experiments no divergence appears.

5. Do we need to perform the real experiment at all?

The natural question which follows is: “To what extent the theoretical calculation above can be seen as a virtual observation of the Unruh effect?” A reasonable answer would be: “Since there is neither a reason to doubt Maxwell equations in the regime considered above nor any argument to distrust the golden quantum formula $E = \hbar \omega$, the derivation above provides a sound anticipation of the output of the real physical experiment and, thus, it should be seen as a very confirmation of the Unruh effect.
On the other hand, we cannot dismiss the existence of “hard-core” practitioners who may argue, e.g., that the fact that Maxwell equations have wonderfully succeeded up to now does not logically imply that, for some reason, they will not fail in the particular case here considered (no matter how conservative is the regime where they are being applied). This would be as strange as raising the possibility that pink apples could float rather than fall down in the south pole against all odds just because such an experiment was never performed. Anyway, for these ones who argue in this way, we must concede that, to the best of our knowledge, no experiment has ever produced a graph like our Fig. 2 for the Larmor radiation emitted by linearly accelerated charges with constant proper acceleration. This may sound surprising at first sight but it has an explanation: although doable under present technology, the proposed experiment is not trivial, as we will see next.

Before we begin discussing some experimental challenges to perform the experiment above, we introduce a simplification, namely, we set the magnetic field to zero. This drives us straightforwardly to Ref. 23. In this instance, all radiation emitted as described by inertial observers corresponds to the emission and absorption of zero-energy Rindler photons as described by uniformly accelerated observers. In Ref. 23 a regularization procedure was implemented to deal with indeterminacies which appeared by the presence of these zero-energy Rindler photons. This was circumvented in Ref. 21 by introducing the magnetic field, avoiding, thus, raising

Fig. 3. It is shown what is the minimum electric field required in order that the typical photon wavelength be short enough to pass the window undisturbed by its finite size. It also shows what is the vacuum level necessary in order to avoid sparking and scattered of the charges by air molecules.
unnecessary concerns with regularization procedures. Notwithstanding, because the crux of the Unruh effect is the linear acceleration, this simplification does not jeopardize the goal of this section by any means.

Firstly, we must recall that in order to avoid the finite size of the window to significantly influence the emitted radiation, we must keep a compromise between the window size and the minimum value of the electric field. Fig. 3 plots the minimum electric field which is necessary to comply with Eq. (3) as a function of the window size. E.g., for a 1 m window, the electric field should be at least about 1 MV/m.

Next, we should guarantee the free mean path of the charges to be at least larger than the window size. We do not want our charges to be scattered by air molecules. In Fig. 3 we also show the necessary vacuum to accomplish this. It is clear that making vacuum is not an issue. A trivial vacuum of 0.025 Pa would be enough to perform the experiment with windows as large as 1 m long.

From Fig. 3 we may be induced to believe that mild electric fields would be just fine, once the window is chosen large enough. This is not the case, however, because the detectors must lie at the radiation zone, where, strictly speaking, \( R_S \gg L \).

Fig. 4. It is shown the expected experimental output against the result obtained with the Unruh effect for a window size of \( L = 1 \text{m} \), an electric field \( |E^z| = 1 \text{MV/m} \), a charged bunch containing \( 10^7 \text{e}^- \), and detectors lying \( R_S = 10 \text{m} \) far from the window.
Fig. 4, we show the expected photon detection rate per transverse momentum $k_{\perp}$ assuming a window size $L = 1\text{m}$, an electric field $|E^z| = 1\text{MV/m}$, and a charged bunch containing $10^7 \text{e}^-$, which should be measured by detectors lying $R_S = 10 \text{m}$ far from the window. (The plot in Fig. 4 differs from the one shown in Fig. 2 because of the absence of the magnetic field.) In Fig. 4, the left-hand side of the vertical dotted line $k_{\perp} = 2\pi/L$ was excluded because the experiment would be sensitive to the window size. We also exclude signal contamination due to a background 300 K thermal bath and cell-phone communication (assuming Brazilian regulation). At the end, we are left with a small observation region, where, nevertheless, we expect (i) the prediction performed via the Unruh effect and (ii) the experimental signal to fit each other very well.

6. Conclusions

We have discussed a doable experiment whose output can be directly interpreted in terms of the Unruh effect. Although it is based on simple classical electrodynamics, it has not been performed yet. Rather than waiting experimentalists to do it, we have calculated the output theoretically and obtained full agreement with the Unruh effect. This should be enough for most scientists to consider it as a virtual observation of the Unruh effect; who does doubt Maxwell equations when applied to regular regimes? Despite it, we cannot dismiss the existence of radical contenders arguing, e.g., that the fact that Maxwell equations have worked so nicely until today does not imply, strictly speaking, they will work as desired in the particular case considered here no matter how conservative is the regime where they are being applied. For these ones, we dedicate Sec. 5 where the challenges to realize the physical experiment above are discussed.

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