1. Introduction

In condensed matter physics, the interactions between the constituents of the system are typically known. The building blocks are mere electrons, protons and neutrons, their collective behavior has been shown to lead to a variety of astonishing phenomena. Classic examples of such quantum correlated systems are superconductivity [1], superfluidity [2] and the fractional quantum Hall effect (FQHE) [3]. In these instances, the departure from usual behavior is often symptomatic of the presence of a non trivial ground state: a ground state which cannot be described by a systematic application of perturbation theory on the noninteracting system.

Investigations on such ground states naturally lead to that of the elementary excitations of the system. One typically probes the system with an external interaction which triggers the population of excited states. The subsequent measurement of the thermodynamical properties then provides some crucial information. From a different angle, transport measurements deal with open systems connected to reservoirs. There are several ways to approach the issue of (quasi)-particle correlations using transport experiments. One can chose to study a system where interactions are most explicit, such as the FQHE, otherwise, to look for statistical interactions in Fermi and Bose systems. Noise being a multi-particle diagnosis, it can be employed to study both issues.
The aim of this chapter is to describe two situations where positive noise correlations can be directly monitored using a transport experiment, either with a superconductor or with a correlated electron system. To be more precise, the present text reflects the presentations made by the three authors during the Delft NATO workshop. Bell inequalities and quantum mechanical non-locality with electrons injected from a superconductor will be addressed first [4]. Next, noise correlations will be computed in a carbon nanotube where electrons are injected in the bulk from a STM tip [5]. The first topic is the result of an ongoing collaboration with G. Lesovik and G. Blatter over the years. The unifying theme is that in both branched quantum circuits, entanglement is explicit and can be illustrated via noise correlations. Entanglement can be achieved either for pairs of electrons in the case of superconductor sources connected to Fermi liquid leads, or alternatively for pairs of quasiparticle excitations of the correlated electron fluid.

A normal metal fork attached to a superconductor can exhibit positive correlations [6, 7], which had been attributed primarily to photonic systems in the seminal Hanbury-Brown and Twiss experiment [8]. They arise when the source of particles is a superconductor [6, 9, 10, 11, 12, 13] or they also occur in systems with floating voltage probes [14]. Here, evanescent Cooper pairs can be emitted on the normal side, due to the proximity effect [15]. These Cooper pairs can either decay in one given lead, which gives a negative contribution to noise correlations, or may split at the junction on the normal side with its two constituent electrons propagating in different leads. This latter effect constitutes a justification for positive noise correlations. If filters, such as quantum dots, are added to the leads, such a mechanism generates delocalized, entangled electron pairs [7, 16]. In order to exit from the superconductor, a Cooper pair has to be split between the two normal leads. This provides a solid state analog of Einstein-Podolsky-Rosen (EPR) states which were proposed to demonstrate the non-local nature of quantum mechanics [17]. Photon entanglement has triggered the proposition of new information processing schemes based on quantum mechanics for quantum cryptography or for teleportation [18]. Concrete proposals for quantum information processing devices based on electron transport and electron interactions in condensed matter have been recently presented. [7, 16, 19, 20]. Here we will analyze whether a non-locality test can be conceived for electrons propagating in quantum wave guides. It is indeed possible to perform the exact analog of a Bell inequalities violation [21] for photons in a condensed matter system [4].

As cited above, positive correlation do not necessarily require a superconducting source of electrons. A particular geometry using a one dimensional correlated electron liquid can be studied for the same purposes. In
particular, it allows to probe directly the underlying charges of the collective excitations in the Luttinger liquid. In order to make contact with experiments, the setup consists of a nanotube whose bulk is contacted by an STM tip which injects electrons by tunneling, while both extremities of the nanotube collect the current. The current, the noise and the noise correlations are computed, and the effective charges are determined by comparison with the Schottky formula. For an “infinite” nanotube, the striking result is that noise correlations contribute to second order in the electron tunneling, in sharp contrast with a fermionic system which requires fourth order. The noise correlations are then positive, because the tunneling electron wave function is split in two counter propagating modes of the collective excitations in the nanotube.

The transport properties of quasiparticles will be addressed from first principles. The understanding of interactions – statistical or otherwise – in noise correlations experiments will be approached here from the point of view of scattering theory and of the Keldysh technique [22].

2. Hanbury–Brown and Twiss correlations

Particularly interesting is the role of electronic correlations in quantum transport. Correlations can have several causes. First, they may originate from the interactions between the particles themselves. Second, correlations are generated by a measurement which involves two or more particles. In the latter case, non-classical correlations may occur solely because of the bosonic or fermionic statistics of particles, with or without interactions. The measurement of noise – the Fourier transform of the current–current correlation function – constitutes a two particle measurement, as implied in the average of the two current operators:

\[ S_{\alpha\beta}(\omega = 0) = \frac{1}{2} \int dt \left( \langle I_{\alpha}(t)I_{\beta}(0) + I_{\beta}(0)I_{\alpha}(t) \rangle - 2 \langle I_{\alpha} \rangle \langle I_{\beta} \rangle \right). \]  

Here \( I_{\alpha} \) is the current operator in reservoir \( \alpha \), and the time arguments on the average currents have been dropped, assuming a stationary regime.

Consider the case of photons propagating in vacuum: the archetype of a weakly interacting boson system. It was shown [8] that when a photon beam is extracted from a thermal source such as a mercury arc lamp, the intensity correlations measured in two separated photo-multipliers are always positive. On average, each photon scattering state emanating from the source can be populated by several photons at a time – due to the bunching property of bosons. As a result when a photon is detected in one of the photo-multipliers, it is likely to be correlated with another detection in the other photo-tube. The positive correlations can be considered
as a diagnosis of the statistics of the carriers performed with a quantum transport experiment.

2.1. NOISE CORRELATIONS IN NORMAL METALS

What should be the equivalent test for electrons? A beam of electrons can be viewed as a train of wave packets, each of which is populated at most by two electrons with opposite spins. If the beam is fully occupied, negative correlations are expected because the measurement of an electron in one detector is accompanied by the absence of a detection in the other one, as depicted in Fig. 1a. It was understood recently [23] that if the electrons propagate in a quantum wire with few lateral modes, maximal occupancy could be reached, and the anti-correlation signal would then be substantial.

Consider the device drawn in Fig. 1a: electrons emanating from reservoir 3 have a probability amplitude $s_{1(2)3}$ to end up in reservoir 1(2). For simplicity we assume that each lead is connected to a single electron channel. The scattering matrix describing this multi-terminal system is hermitian because of current conservation. On general grounds, it is possible to derive the following sum rule for the autocorrelation noise and the noise correlations [24]:

$$\sum_{\alpha} S_{\alpha\alpha}(\omega = 0) + \sum_{\alpha \neq \beta} S_{\alpha\beta}(\omega = 0) = 0 . \quad (2)$$

Note that this sum rule only holds for normal conductors, but it not valid for conductors which involve superconducting reservoirs. The scattering theory of electron transport [22] then specifies how to compute the current and the noise correlations between the two branches. The current in lead $\alpha$ is given by

$$\langle I_{\alpha} \rangle = \frac{e}{h} \int dE \left( \text{Tr}[1_{\alpha} - s_{\alpha\alpha}^1 s_{\alpha\alpha}] f_{\alpha}(E) - \sum_{\beta \neq \alpha} s_{\alpha\beta}^1 s_{\alpha\beta} f_{\beta}(E) \right) , \quad (3)$$

with $s_{\alpha\beta}$ the amplitude to go from reservoir $\beta$ to reservoir $\alpha$, $1_{\alpha}$ is the identity matrix for lead $\alpha$, and $f_{\alpha}$ the associated Fermi function. The zero frequency noise correlations in the zero temperature limit are found in general to be [24]:

$$S_{\alpha\beta} = \frac{e^2}{h} \sum_{\gamma \neq \delta} \int dE \text{Tr}[s_{\alpha\gamma}^1 s_{\alpha\delta} s_{\beta\delta}^1 s_{\beta\gamma}] f_{\gamma}(E)[1 - f_{\delta}(E)] . \quad (4)$$

Note that when considering the tunnel limit, the lowest non-vanishing contribution is of fourth order in the tunneling amplitude $\Gamma \sim s_{\alpha\beta}$ for fermionic
Figure 1. a) Hanbury-Brown and Twiss geometry in a normal metal fork with electrons injected from 3 and collected in reservoirs 1 and 2. Occupied (empty) electron wave packet states are identified as black (white) dots. b) Hanbury-Brown and Twiss geometry in a superconductor–normal metal fork. Cooper pairs are emitted from the superconductor, and the two constituent electrons can either propagate in the same lead, or propagate in an entangled state in both leads.

particles. Applying the above results to the three-terminal situation depicted in Fig. 1 in the presence of a symmetric voltage bias $V$ between 3 and 1(2) so that no flow occurs between 1 and 2:

$$s_{12}(0) = -\frac{2e^3|V|}{\hbar} |s_{13}s_{23}|^2.$$  \hspace{1cm} (5)

These electronic noise correlations were measured recently [25] by two groups working either in the integral quantum Hall effect regime or in the ballistic regime, with beam splitters designed with metallic gates.

Negative correlations here are most natural, because the injection of electrons is made from a degenerate Fermi gas. Yet there exist situations where they can be positive in a fermionic system.

### 2.2. FORK GEOMETRY WITH A SUPERCONDUCTOR SOURCE

If the reservoir which injects electrons in the fork is a superconductor as in Fig. 1b, both positive and negative noise correlations are possible [6]. Charge transfer between the injector and the two collectors 1 and 2 is then specified by the Andreev scattering process, where an electron is reflected as a hole. Positive correlations are linked to the proximity effect, as superconducting correlations (Cooper pairs) leak in the two normal leads. Depending on the nature of the junction in Fig. 1b, it may be more favorable for a pair to be distributed among the two arms than for a pair to enter a lead as a whole. The detection of an electron in 1 is then accompanied by the detection of an electron in 2, giving a positive correlation signal.

In the scattering theory for normal–superconductor (NS) systems [26], the fermion operators which enter the current operator are given in terms
of the quasiparticle states using the Bogolubov transformation: \( \psi_\sigma(x) = \sum_n \left( u_n(x) \gamma^*_n \sigma - v^*_n(x) \gamma^*_n \sigma \right) \). Here, \( \gamma^*_n \sigma \) are quasiparticle creation operators, \( n = (i, \alpha, E) \) contains information on the reservoir \( i \) from which the particle \( \alpha = e, h \) is incident with energy \( E \) and \( \sigma \) labels the spin.

The contraction of these two operators gives the distribution function of the particles injected from each reservoir, which for a potential bias \( V \) are:

\[
    f_{ie} \equiv f(E - eV) \quad \text{for electrons incoming from} \quad i,
    \quad f_{ih} \equiv f(E + eV) \quad \text{for holes},
    \quad f_i \equiv f(E) \quad \text{for both types of quasiparticles injected from the superconductor (} f \text{ is the Fermi–Dirac distribution}).
\]

The solution of the Bogolubov–de Gennes equations provide the electron and hole wave functions describing scattering states \( \alpha \) and \( i \) are expressed in terms of the elements \( s_{ij\alpha\beta} \) of the S–matrix which describes the whole NS ensemble:

\[
    u_{i\alpha}(x_j) = \frac{[\delta_{ij} \delta_{\alpha e} e^{ik_+ x_j} + s_{jie\alpha} e^{-ik_+ x_j}]}{\sqrt{v_+}} , \quad (6)
\]

\[
    v_{i\alpha}(x_j) = \frac{[\delta_{ij} \delta_{\alpha h} e^{-ik_- x_j} + s_{jih\alpha} e^{ik_- x_j}]}{\sqrt{v_-}} , \quad (7)
\]

where \( x_j \) denotes the position in normal lead \( j \) and \( k_{\pm} (v_{\pm}) \) are the usual momenta (velocities) of the two branches.

Specializing now to the NS junction connected to a beam splitter (inset of Fig. 2), 6 \times 6 matrix elements are sufficient to describe all scattering processes. At zero temperature, the noise correlations between the two normal reservoirs simplify to:

\[
    S_{12}(0) = \frac{2e^2}{h} \int_0^{eV} dE \sum_{i=1,2} \left[ \sum_{j=1,2} \left( s^*_{1ie\alpha} s_{1jeh} - s^*_{1ihe\alpha} s_{1jeh} \right) \left( s^*_{2jeh} s_{2ie\alpha} - s^*_{2jhh} s_{2ie\alpha} \right) + \sum_{\alpha=e,h} \left( s^*_{1ie\alpha} s_{14e\alpha} - s^*_{1ihe\alpha} s_{14h\alpha} \right) \left( s^*_{24e\alpha} s_{2ie\alpha} - s^*_{24h\alpha} s_{2ie\alpha} \right) \right] , \quad (8)
\]

where the subscript 4 denotes the superconducting lead. The first term represents normal and Andreev reflection processes, while the second term invokes the transmission of quasiparticles through the NS boundary. It was noted \cite{27} that in the pure Andreev regime the noise correlations vanish when the junction contains no disorder: electron (holes) incoming from 1 and 2 are simply converted into holes (electrons) after bouncing off the NS interface. The central issue, whether changes in the transparency can induce changes in the sign of the correlations, is now addressed.

We now consider only the subgap or Andreev regime, were \( eV \ll \Delta \), the superconducting gap, for which a simple model for a disordered NS junction
Figure 2. Noise correlation between the two normal reservoirs of the device (inset), as a function of the transmission probability of the beam splitter, showing both positive and negative correlations. Inset: the device consists of a superconductor (4)–normal (3) interface which is connected by a beam splitter (shaded triangle) to reservoirs (1) and (2). $\epsilon = 0.5$ corresponds to maximal transmission.

is readily available. The junction is composed of four distinct regions (see inset Fig. 2). The interface between 3 (normal) and 4 (superconductor) exhibits only Andreev reflection, with scattering amplitude for electrons into holes $r_A = \gamma \exp(-i\phi)$ (the phase of $\gamma = \exp[-i\arccos(E/\Delta)] \simeq -i$ is the Andreev phase and $\phi$ is the phase of the superconductor). Next, 3 is connected to two reservoirs 1 and 2 by a beam splitter which is parameterized by a single parameter $0 < \epsilon < 1/2$ identical to that of Ref. [28]: the splitter is symmetric, its scattering matrix coefficients are real, and transmission between 3 and the reservoirs is maximal when $\epsilon = 1/2$, and vanishes at $\epsilon = 0$.

Performing the energy integrals in Eq. (8):

$$S_{12}(\epsilon) = \frac{2e^2}{h} eV \frac{\epsilon^2}{2(1-\epsilon)^4} \left( -\epsilon^2 - 2\epsilon + 1 \right) . \quad (9)$$

The noise correlations vanish at $\epsilon = 0$ and $\epsilon = \sqrt{2} - 1$. The correlations (Fig. 2) are positive (bosonic) for $0 < \epsilon < \sqrt{2} - 1$ and negative (fermionic) for $\sqrt{2} - 1 < \epsilon < 1/2$. At maximal transmission into the normal reservoirs ($\epsilon = 1/2$), the correlations normalized to the noise in 1 (or 2) give the negative minimal value: electrons and holes do not interfere and propagate independently into the normal reservoirs. It is then expected to obtain the signature of a purely fermionic system. When the transmission $\epsilon$ is de-
creased, multiple Andreev processes start taking some importance. Further reducing the beam splitter transmission allows to balance the contribution of split Cooper pairs with that of Cooper pairs entering the leads as a whole. Note that inclusion of disorder or/and additional leads has been discussed by many authors, using either the scattering approach [9, 12, 13] or circuit theory [10] to treat the diffusive limit. To summarize, both positive and negative noise correlations are possible there, and such results are discussed in detail in the contributions of [29] and [30] contained in this volume. In particular, the possibility for positive noise cross-correlation is reduced for asymmetric multichannel conductors. Although sample specific, the scattering theory results of Refs. [6, 13] are found to be rather robust in the presence of disorder.

2.3. FILTERING SPIN/ENERGY

Applying spin or energy filters to the normal arms 1 and 2 (Fig. 3), it is possible to generate positive correlations only [7]. For electron emanating from a superconductors, it is possible to project either the spin or the energy with an appropriate filter, without perturbing the entanglement of the remaining degree of freedom (energy or spin). Energy filters, which are more appropriate towards a comparison with photon experiments, will have resonant energies symmetric above and below the superconductor chemical
potential which serve to select electrons (holes) in leads 1(2). The positive correlation signal then reads:

\[ S_{12}(0) = \frac{e^2}{\hbar} \sum_\zeta \int_{\epsilon=0}^{\epsilon=|V|} d\epsilon T^A_\zeta(\epsilon)[1 - T^A_\zeta(\epsilon)], \tag{10} \]

where the index \( \zeta = h, \sigma, 2 \) \((\sigma = \uparrow, \downarrow)\) identifies the incoming hole state for energy filters (positive energy electrons with arbitrary spin are injected in lead 1 here), \( \zeta = h, \uparrow, 1 \) \((h, \downarrow, 2)\) applies for spin filters (spin up electrons – with positive energy – emerging from the superconductor are selected in lead 1). \( T^A_\zeta \) is then the corresponding (reverse) crossed-Andreev reflection probability \([31]\) for each type of setup: the energy (spin) degree of freedom is frozen, while the spin (energy) degree of freedom is unspecified. \( eV < 0 \) insures that the constituent electrons of a Cooper pair from the superconductor are emitted into the leads without suffering from the Pauli exclusion principle. Moreover, because of such filters, the propagation of a Cooper pair as a whole in a given lead is prohibited. Note the similarity with the quantum noise suppression mentioned above. This is no accident: by adding constraints to our system, it has become a two terminal device, such that the noise correlations between the two arms are identical to the noise in one arm:

\[ S_{11}(\omega = 0) = S_{12}(\omega = 0). \tag{11} \]

The positive correlation and the perfect locking between the auto and cross correlations provide a serious symptom of entanglement. One can speculate (however without a rigorous proof yet) that the wave function which describes the two electron state in the case of spin filters reads:

\[ |\Phi_{\text{spin}}^{\epsilon,\sigma}\rangle = \alpha |\epsilon, \sigma; -\epsilon, -\sigma\rangle + \beta | -\epsilon, \sigma; \epsilon, -\sigma\rangle, \tag{12} \]

where the first (second) argument in \( |\phi_1; \phi_2\rangle \) refers to the quasi-particle state in lead 1 (2) evaluated behind the filters, \( \epsilon \) is the energy and \( \sigma \) is a spin index. The coefficients \( \alpha \) and \( \beta \) can be tuned by external parameters, e.g., a magnetic field. Note that by projecting the spin degrees of freedom in each lead, the spin entanglement is destroyed, but energy degrees of freedom are still entangled, and can help provide a measurement of quantum mechanical non locality nevertheless. A measurement of energy \( \epsilon \) in lead 1 (with a quantum dot) projects the wave function so that the energy \( -\epsilon \) has to occur in lead 2. On the other hand, energy filters do preserve spin entanglement, and are appropriate to make a Bell test (see below). In this case the two electron wavefunction takes the form:

\[ |\Phi_{\text{energy}}^{\epsilon,\sigma}\rangle = \alpha |\epsilon, \sigma; -\epsilon, -\sigma\rangle + \beta | -\epsilon, \sigma; -\epsilon, \sigma\rangle. \tag{13} \]
Electrons emanating from the energy filters (coherent quantum dots) could be analyzed provided a measurement can be performed on the spin of the outgoing electrons with ferromagnetic leads.

2.4. TUNNELING APPROACH TO ENTANGLEMENT

We recall a perturbative argument which supports the claim that two electrons originating from the same Cooper pair are entangled. Consider a system composed of two quantum dots (energies \( E_{1,2} \)) next to a superconductor. An energy diagram is depicted in Fig. 4. The electron states in the superconductor are specified by the BCS wave function \( \Psi_{BCS} \).

\[
H_T = \sum_{k\sigma} [t_{1k} c_{1\sigma}^\dagger + t_{2k} c_{2\sigma}^\dagger] c_{k\sigma} + \text{h.c.},
\]

(14)

with \( c_{k\sigma}^\dagger \) creates an electron with spin \( \sigma \). Now let assume that the transfer Hamiltonian acts on a single Cooper pair.

Using the T-matrix to lowest (2nd) order, the wave function contribution of the two particle state with one electron in each dot reads:

\[
|\delta \Psi_{12}\rangle = H_T \frac{1}{i\eta - H_0} H_T |\Psi_{BCS}\rangle 
= \sum_k \nu_k u_k t_{1k} t_{2k} \left( \frac{1}{i\eta - E_k - E_1} + \frac{1}{i\eta - E_k - E_2} \right) 
\times [c_{1\uparrow}^\dagger c_{2\downarrow}^\dagger - c_{1\downarrow}^\dagger c_{2\uparrow}^\dagger] |\Psi_{BCS}\rangle,
\]

(15)
where $E_k$ is the energy of a Bogolubov quasiparticle. The state of Eq. (15) has entangled spin degrees of freedom. This is clearly a result of the spin symmetry of the tunneling Hamiltonian. Given the nature of the correlated electron state in the superconductor in terms of Cooper pairs, $H_T$ can only produce singlet states in the dots.

3. Bell inequalities with electrons

In photon experiments, entanglement is identified by a violation of Bell inequalities (BI) – which are obtained with a hidden variable theory. But in the case of photons, the BIs have been tested using photo-detectors measuring coincidence rates [32]. Counting quasi-particles one-by-one in coincidence measurements is difficult to achieve in solid-state systems where stationary currents and noise are the natural observables [26]. Here, the BIs are re-formulated in terms of current-current cross-correlators (noise correlations) [4].

In order to derive Bell inequalities, we consider that a source provides two streams of particles (labeled 1 and 2) as in Fig. 5a injecting quasi-particles into two arms labelled by indices 1 and 2. Filter $F_d^{1(2)}$ are trans-
parent for electrons spin-polarized along the direction \( \mathbf{a} (\mathbf{b}) \). Assuming separability and locality \([21]\) the density matrix for joint events in the leads \( \alpha, \beta \) is chosen to be:

\[
\rho = \int d\lambda f(\lambda) \rho_\alpha(\lambda) \otimes \rho_\beta(\lambda) ,
\]

where the lead index \( \alpha \) is even and \( \beta \) is odd (or vice-versa); the distribution function \( f(\lambda) \) is positive. \( \rho_\alpha(\lambda) \) are standard density matrices for a given lead, which are hermitian. The total density matrix \( \rho \) is the most general density matrix one can build for the source/detector system assuming no entanglement and only local correlations \([33]\).

Consider the current operator \( I_\alpha(t) \) in lead \( \alpha = 1, \ldots, 6 \) (see Fig. 5) and the associated particle number operator \( N_\alpha(t, \tau) = \int_t^{t+\tau} I_\alpha(t') dt' \). Particle-number correlators are defined as:

\[
\langle N_\alpha(t, \tau)N_\beta(t, \tau) \rangle_\rho = \int d\lambda f(\lambda) \langle N_\alpha(t, \tau) \rangle_\lambda \langle N_\beta(t, \tau) \rangle_\lambda ,
\]

with indices \( \alpha/\beta \) odd/even or even/odd. The average \( \langle N_\alpha(t, \tau) \rangle_\lambda \) depends on the state of the system in the interval \([t, t + \tau]\). An average over large time periods is introduced in addition to averaging over \( \lambda \), e.g.,

\[
\langle N_\alpha(t, \tau)N_\beta(\tau) \rangle = \frac{1}{2T} \int_{-T}^{T} dt \langle N_\alpha(t, \tau)N_\beta(t, \tau) \rangle_\rho ,
\]

where \( T/\tau \rightarrow \infty \) (a similar definition applies to \( \langle N_\alpha(\tau) \rangle \)). Particle number fluctuations are written as \( \delta N_\alpha(t, \tau) = N_\alpha(t, \tau) - \langle N_\alpha(\tau) \rangle \).

Let \( x, x', y, y', X, Y \) be real numbers such that:

\[
|x/X|, |x'/X|, |y/Y|, |y'/Y| < 1 .
\]

Then \(-2XY \leq xy - x'y + x'y' \leq 2XY\). Define accordingly:

\[
\begin{align*}
x &= \langle N_5(t, \tau) \rangle_\lambda - \langle N_3(t, \tau) \rangle_\lambda , & x' &= \langle N_5'(t, \tau) \rangle_\lambda - \langle N_3'(t, \tau) \rangle_\lambda , \\
y &= \langle N_6(t, \tau) \rangle_\lambda - \langle N_4(t, \tau) \rangle_\lambda , & y' &= \langle N_6'(t, \tau) \rangle_\lambda - \langle N_4'(t, \tau) \rangle_\lambda ,
\end{align*}
\]

where the subscripts with a ‘prime’ indicate a different direction of spin-selection in the detector’s filter (e.g., let \( \mathbf{a} \) denote the direction of the electron spins in lead 5 \((-\mathbf{a} \text{ in lead 3})\), then the subscript \( 5' \) in Eqs. (20) and (21) means that the electron spins in lead 5 are polarized along \( \mathbf{a}' \) (along \(-\mathbf{a}' \) in the lead 3). The quantities \( X, Y \) are defined as

\[
\begin{align*}
X &= \langle N_5(t, \tau) \rangle_\lambda + \langle N_3(t, \tau) \rangle_\lambda = \langle N_1(t, \tau) \rangle_\lambda , \\
Y &= \langle N_6(t, \tau) \rangle_\lambda + \langle N_4(t, \tau) \rangle_\lambda = \langle N_2(t, \tau) \rangle_\lambda ,
\end{align*}
\]
where primed quantities \((5', 3')\), \((6', 4')\) also apply. The Bell inequality follows after appropriate averaging:

\[
|F(a, b) - F(a, b') + F(a', b) + F(a', b')| \leq 2,
\]

\[
F(a, b) = \frac{\langle [N_1(a, t) - N_1(-a, t)][N_2(b, t) - N_2(-b, t)] \rangle}{\langle [N_1(a, t) + N_1(-a, t)][N_2(b, t) + N_2(-b, t)] \rangle},
\]

with \(a, b\) the polarizations of the filters \(F_{1(2)}\) (electrons spin-polarized along \(a\) \((b)\) can go through filter \(F_{1(2)}\) from lead 1(2) into lead 5(6)). This is the quantity we want to test, using a quantum mechanical theory of electron transport. Here it will be written in terms of noise correlators, as particle number correlators at equal time can be expressed in general as a function of the finite frequency noise cross-correlations. Assuming short times (see below), one obtains \(\langle N_\alpha(\tau)N_\beta(\tau) \rangle \approx \langle I_\alpha \rangle \langle I_\beta \rangle \tau^2 + \tau S_{\alpha\beta}(\omega = 0)\) where \(\langle I_\alpha \rangle\) is the average current in the lead \(\alpha\) and \(S_{\alpha\beta}\) denotes the shot noise. One then gets:

\[
F(a, b) = \frac{S_{56} - S_{54} - S_{36} + S_{34} + \Lambda_-}{S_{56} + S_{54} + S_{36} + S_{34} + \Lambda_+},
\]

with \(\Lambda_\pm = \tau(\langle I_5 \rangle \pm \langle I_3 \rangle)(\langle I_6 \rangle \pm \langle I_4 \rangle).\) Consider now the solid-state analog of the Bell-device as sketched in Fig. 5b where the particle source is a superconductor (S). The test of the Bell inequality \((24)\) requires information about the dependence of the noise on the mutual orientations of the magnetizations \(\pm a\) and \(\pm b\) of the ferromagnetic spin-filters.

Consider an example of the solid-state analog of the Bell-device [Fig. 1(b)] where the particle source is a superconductor. The chemical potential of the superconductor is larger than that of the leads, which means that electrons are flowing out of the superconductor. Two normal leads 1 and 2 are attached to it in a fork geometry [7, 16] and the filters \(F_{1,2}^e\) enforce the energy splitting of the injected pairs. \(F_{1,2}^d\)-filters play the role of spin-selective beam-splitters in the detector. Quasi-particles injected into lead 1 and spin-polarized along the magnetization \(a\) enter the ferromagnet 5 and contribute to the current \(I_5\), while quasi-particles with the opposite polarization contribute to the current \(I_3\). For a biased superconductor with grounded normal leads, we find in the tunneling limit the noise

\[
S_{\alpha\beta} = e \sin^2 \left( \frac{\theta_{\alpha\beta}}{2} \right) \int_0^{\mid eV \mid} d\varepsilon T^A(\varepsilon),
\]

which integral also represents the current in a given lead (we have dropped the subscript in \(T^A(\varepsilon)\) assuming the two channels are symmetric). Here \(\alpha = 3, 5, \beta = 4, 6\) or vice versa; \(\theta_{\alpha\beta}\) denotes the angle between the magnetization of leads \(\alpha\) and \(\beta\), e.g., \(\cos(\theta_{56}) = a \cdot b\), and \(\cos(\theta_{54}) = a \cdot (-b)\). Below, we
need configurations with different settings \( a \) and \( b \) and we define the angle 
\[ \theta_{ab} \equiv \theta_{56}. \]

\( V \) is the bias of the superconductor.

The \( \Lambda \)-terms in Eq. (26) can be dropped if 
\[ \langle I_\alpha \rangle \tau \ll 1, \quad \alpha = 3, \ldots, 6, \]
which corresponds to the assumption that only one Cooper pair is present on average. The resulting BIs Eqs. (24)-(26) then neither depend on \( \tau \) nor on the average current but only on the noise, and
\[ F = -\cos(\theta_{ab}); \]
the left hand side of Eq. (24) has a maximum when 
\[ \theta_{ab} = \theta_{a'b} = \theta_{a'b'} = \pi/4 \]
and \( \theta_{ab'} = 3\theta_{ab} \). With this choice of angles the BI Eq.(24) is \textit{violated}, thus pointing to the nonlocal correlations between electrons in the leads 1,2 [see Fig. 5(b)].

If the filters have a width \( \Gamma \) the current is of order 
\[ eT_A \Gamma / h \]
and the condition for neglecting the reducible correlators becomes 
\[ \tau \ll \bar{\hbar} / \Gamma T_A. \]
On the other hand, in order to insure that no electron exchange between 1 and 2 one requires 
\[ \tau \ll \tau_{tr} / T_A \]
\( (\tau_{tr} \) is the time of flight from detector 1 to 2). The conditions for BI violation require very small currents, because of the specification that one entangled pair at a time is in the system. Yet it is necessary to probe noise cross correlations of these same small currents. The noise experiments which we propose here are closely related to coincidence measurements in quantum optics. [32]

If we allow the filters to have a finite line width, which could reach the energy splitting of the pair, the violation of BI can still occur, although violation is not maximal. Moreover, when the source of electron is a normal source, it is possible to show that in “standard device geometries”, where single electron physics is at play, Bell inequalities are not violated: in this situation the term \( \Lambda_\pm \) dominates over the noise correlation contribution. Nevertheless, it would be possible in practice to violate Bell inequalities if the normal source itself, composed of quantum dots as suggested in Ref. [34, 35], could generate entangled electron states as the result of electron-electron interactions.

Note that there are other inequalities which test entanglement for particles sources with the number of terminals two and larger than (see, e.g., Ref. [33]): tests of such inequalities can be implemented in a similar manner as discussed above. For example, one can use Clauser-Horne (CH) inequalities [36] (instead of Bell Inequalities) to test the entanglement in the solid-state systems shown in the Fig. 5. The derivation of CH-inequality is similar to that of BI; it is based on the lemma: if \( x, x', y, y', X, Y \) are real numbers such that \( x, x' \in [0, X] \) and \( y, y' \in [0, Y] \). The following inequality then holds:
\[ -XY \leq xy - x'y' + x'y + x'y' - Yx' - Xy \leq 0. \tag{28} \]
The definitions for \( x, x', y, y' \) are the same as in Eqs.(20) and (21), but
\[ X = \langle N_5^{(0)}(t, \tau) \rangle_\lambda, \ Y = \langle N_6^{(0)}(t, \tau) \rangle_\lambda, \]
where \( \langle N_\alpha^{(0)}(t, \tau) \rangle_\lambda \) is the number of particles coming into the arm of the detector \( \alpha = 5, 6 \) when it doesn’t
include the spin-filter[36, 37]. Finally we get the CH-inequality in a similar manner as the Bell inequality, with the same assumption of small times:

$$S_{56}(a, b) - S_{56}(a', b) + S_{56}(a', b') - S_{56}(a', -) - S_{56}(-, b) \leq 0,$$

(29)

where $S_{56}(a', -)$ is the shot noise when there is no spin-filter on the way of the particles coming from the source into the detector arm 6. In the tunneling limit $S_{56}(a', b)/S_{56}(a', -) = \sin^2(\theta_{56}/2)$ and Eq. (20) gives $S_{56}$. CH-inequalities are maximally violated when $\theta_{a,b} = \theta_{a',b'} = \pi/2$, $\theta_{a,b'} = \pi/4$, and $\theta_{a',b} = 3\pi/4$ [this choice of angles is different than in BI case]; then the left-hand side of Eq. (29) is $(\sqrt{2} - 1)/2$.

CH-inequalities have one working advantage compared to BIs: Eq. (29) includes only correlations between terminals 5 and 6, the number of correlators in Eq. (29) is decreased compared to the BI's. Moreover, in the case of CH the spin-filters of the detector can include only one ferromagnet in each arm rather than two as in Fig. 5b.

4. Electron injection in the bulk of a nanotube

So far, noise was computed for non-interacting systems. If one puts aside the fact that electrons are converted into holes the calculation of noise in NS systems is similar to the normal case. Recall now the classic results for the FQHE. The long wave length edge excitations along a quantum Hall bar can be described by a Luttinger liquid [38]. Backscattering can be induced by bringing together two counter-propagating edges using a point contact. In the absence of impurities or backscattering, the maximal edge current is $I_M = \nu e^2/h$, while for weak backscattering, the current voltage characteristic is highly non linear for Laughlin fractions, i.e. $\langle I_B \rangle \sim V^{2\nu-1}$ ($\langle I_B \rangle$ is the average backscattering current and $V$ is the voltage bias between the two edges). A two terminal noise measurement performed on a gated mesoscopic device in this regime provides a direct link to the quasiparticle charge. Quasiparticles are scattered from one edge to the other one by one, so the usual Schottky formula $S_B = 2e^*\langle I_B \rangle$ which relates the zero frequency backscattering noise to the average current flowing between the two edges applies [39], with an effective carrier charge $e^* = \nu e$ contains the electron filling factor [40, 41, 42]. This fractional charge was measured recently by several groups [43, 44]. Statistical interaction between quasiparticle have been addressed theoretically [45] in an Hanbury-Brown and Twiss geometry.

Attention is now turning towards conductors which are essentially free of defects and which have one-dimensional character. Carbon nanotubes can have metallic behavior, with two propagating modes at the Fermi level, and constitute good candidates to study Luttinger liquid behavior. In particu-
Figure 6. Schematic configuration of the nanotube–STM device: electrons are injected from the tip at $x = 0$: current is measured at both nanotube ends, which are set to the ground.

Nanotube Luttinger liquids are non-chiral in nature, so a straightforward transposition of the results obtained for chiral edge systems is not obvious. Nevertheless, non–chiral Luttinger liquids also have underlying chiral fields \[49, 50\]. Such chiral fields correspond to excitations with anomalous (non-integer) charge, which has eluded detection so far.

The transport geometry (Fig. 6) implies: tunneling from the tip (normal or ferromagnetic metal) to the nanotube, and subsequent propagation of collective excitations along the nanotube. In the absence of tunneling, the Hamiltonian is thus simply the sum of the nanotube Hamiltonian, described by a two mode Luttinger liquid, together with the tip Hamiltonian. Using the standard conventions \[51\], the operator describing an electron with spin $\sigma$ moving along the direction $r$, from mode $\alpha$ is specified in terms of a bosonic field:

$$\Psi_{r\alpha\sigma}(x, t) = \frac{1}{\sqrt{2\pi a}} e^{iak_Fx + irq_Fx + i\sqrt{2}\sum_j h_{\alpha\sigma c}(\phi_{j\delta}(x,t) + r\theta_{j\delta}(x,t))}, \quad (30)$$

with $a$ a short distance cutoff, $k_F$ the Fermi momentum, $q_F$ the momentum mismatch associated with the two modes, and the convention $r = \pm$, $\alpha = \pm$ and $\sigma = \pm$ are chosen for the direction of propagation, for the nanotube branch, and for the spin orientation. The non-chiral Luttinger liquid bosonic fields $\theta_{j\delta}$ and $\phi_{j\delta}$, with $j\delta \in \{c+, c-, s+, s-\}$ identifying the charge/spin and total/relative fields, have been introduced. The coefficients are defined as $h_{\alpha\sigma c+} = 1$, $h_{\alpha\sigma c-} = \alpha$, $h_{\alpha\sigma s+} = \sigma$ et $h_{\alpha\sigma s-} = \alpha\sigma$. The Hamiltonian which describes the collective excitations in the nanotube has the standard form:

$$H = \frac{1}{2} \sum_{j\delta} \int_{-\infty}^{\infty} dx \left( v_{j\delta}(x) K_{j\delta}(x)(\partial_x \phi_{j\delta}(x,t))^2 + \frac{v_{j\delta}(x)}{K_{j\delta}(x)} (\partial_x \theta_{j\delta}(x,t))^2 \right), \quad (31)$$
with an interaction parameter $K_{j\delta}(x)$ and velocity $v_{j\delta}(x)$ which allows to address both homogeneous and inhomogeneous Luttinger liquids.

For the STM tip, one assumes for simplicity that only one electronic mode couples to the nanotube, so it can be described by a semi-infinite Luttinger liquid (with interaction parameter $K = 1$) for simplicity. For the sake of generality, we allow the two spin components of the tip fields to have different Fermi velocities $u_{\sigma}^F$, which allows to treat the case of a ferromagnetic metal. The fermion operator at the tip location $x = 0$ is then:

$$c_{\sigma}(t) = \frac{1}{\sqrt{2\pi a}} e^{i\tilde{\varphi}_{\sigma}(t)}.$$  \hspace{1cm} (32)

Here, $\tilde{\varphi}_{\sigma}$ is the chiral Luttinger liquid field, whose Keldysh Green’s function (here at $x = 0$) is given in [52].

The tunneling Hamiltonian is a standard hopping term:

$$H_T(t) = \sum_{\sigma} \varepsilon \Gamma(\varepsilon)(t)[\Psi^\dagger_{\sigma\alpha}(0, t)c_{\sigma}(t)]^{\varepsilon}.$$ \hspace{1cm} (33)

Here the superscript $(\varepsilon)$ leaves either the operators in bracket unchanged $(\varepsilon = +)$, or transforms them into their hermitian conjugate $(\varepsilon = -)$. The voltage bias between the tip and the nanotube is included using the Peierls substitution: the hopping amplitude $\Gamma(\varepsilon)$ acquires a time dependent phase $exp(i\varepsilon \omega_0 t)$, with the bias voltage identified as $V = \hbar \omega_0 / e$. We use the convention $\hbar \to 1$. Similarly, one can define the tunneling current:

$$I_T(t) = ie \sum_{\sigma} \varepsilon \Gamma(\varepsilon)(t)[\Psi^\dagger_{\sigma\alpha}(0, t)c_{\sigma}(t)]^{\varepsilon}.$$ \hspace{1cm} (34)

For this problem which implies propagation of excitations along the nanotube it is also necessary to compute the (total) charge current using the bosonized fields Eq. (30):

$$I_{\rho}(x, t) = 2e v_F \sqrt{2} \bar{\phi}_{e+}(x, t).$$ \hspace{1cm} (35)

Note that the contribution from terms containing $2k_F$ oscillations has been dropped.

The Keldysh technique is employed to compute the non-equilibrium currents $\langle I_T \rangle$ and $\langle I_{\rho}(x) \rangle$, and noises $S_T$ and $S_{\rho}(x, x')$ to second order in $\Gamma$. The contribution of the nanotube fields and of the tip are regrouped into two time ordered products. Each can be related to a correlator of several exponentiated bosonic fields. Such correlators are readily expressed in terms of the Keldysh Green’s functions $G_{\theta\theta}^{\theta\theta}$, $G_{\phi\phi}^{\phi\phi}$, $G_{\theta\phi}^{\theta\phi}$, $G_{\phi\theta}^{\phi\theta}$, associated with the fields $\theta_{j\delta}$ and $\phi_{j\delta}$, as well as the tip Green’s function $g_{\sigma}$. The
following results apply to both homogeneous and inhomogeneous Luttinger liquids:

\[
\langle I_{\rho}(x) \rangle = -\frac{e v_F \Gamma^2}{2\pi^2 a^2} \sum_{\eta_1, \eta_2} \int_{-\infty}^{+\infty} d\tau \sin(\omega_0 \tau) e^{2\pi g_{\eta_1}(\eta_1-\eta_2)(\tau)} \\
\times e^{\frac{i}{\hbar} \int_{-\infty}^{+\infty} d\tau \partial_x \left( G_{c+}(\eta_1) x, 0, \tau \right) - G_{c+}(\eta_1) x, 0, \tau' \right) + r_1 G_{c+}(\eta_1) x, 0, \tau' \rangle \right),
\]

\[
S_{\rho}(x, x', \omega = 0) = -\frac{e^2 v_F^2 \Gamma^2}{(\pi a)^2} \sum_{\eta_1, \eta_2} \int_{-\infty}^{+\infty} d\tau cos(\omega_0 \tau) e^{2\pi g_{\eta_1}(\eta_1-\eta_2)(\tau)} \\
\times e^{\frac{i}{\hbar} \int_{-\infty}^{+\infty} d\tau \partial_x \left( G_{c+}(\eta_1) x, 0, \tau_1 \right) + r_1 G_{c+}(\eta_1) x, 0, \tau_1 \right) \\
- G_{c+}(\eta_1) x, 0, \tau_1 \rangle + r_1 G_{c+}(\eta_1) x, 0, \tau_1 \rangle \right) \\
\times e^{\frac{i}{\hbar} \int_{-\infty}^{+\infty} d\tau \partial_{x'} \left( G_{c+}(\eta_1) x', 0, \tau_2 \right) + r_1 G_{c+}(\eta_1) x', 0, \tau_2 \right) \\
- G_{c+}(\eta_1) x', 0, \tau_2 \rangle + r_1 G_{c+}(\eta_1) x', 0, \tau_2 \rangle \right). 
\]

Here, \(\eta, \eta_1, \eta_2 = \pm\) are indices which specify on which branch of the Keldysh contour the times \(\tau, \tau_1, \tau_2\) and 0 are attached.

5. Nanotube noise correlations and effective charges

An accepted diagnosis to detect effective or anomalous charges is to compare the noise with the associated current with the Schottky formula in mind. Consider an infinite, homogeneous nanotube characterized by interaction parameters \(K_{j_{\beta}}^N\). The current reads:

\[
\langle I_{\rho}(x) \rangle = \frac{e \Gamma^2}{\pi a} \left( \sum_{\sigma} \frac{1}{u_F^2} \right) \frac{\text{sgn}(\omega_0)|\omega_0|^\mu}{\Gamma(\mu + 1)} \left( \frac{a}{v_F} \right)^\mu \text{sgn}(x),
\]

\[
\langle I_{T}(x) \rangle = 2\langle I_{\rho}(x) \rangle, 
\]

\[
(38)
\]

\[
(39)
\]
where $\Gamma$ is the Gamma function. We thus obtain a non-linear dependence on voltage $|\omega_0|^\mu$ when interactions are present, with exponent $\mu = \sum_{j\delta}(K_{j\delta}^N + (K_{j\delta}^N)^{-1})/8$. A striking result is that despite the fact that electrons are tunneling from the STM tip to the bulk of the nanotube, the zero frequency current fluctuations are proportional to the current (for $x' = x >> a$) with an anomalous effective charge:

$$S_{\rho}(x,x,\omega = 0) = \frac{1 + (K_{c+}^N)^2}{2} e\langle|I_{\rho}(x)|\rangle, \quad (40)$$

$$S_T = e\langle I_T \rangle. \quad (41)$$

More can be learned from a measurement of the noise correlations. Indeed, our geometry can be considered as a Hanbury-Brown and Twiss [8] correlation device. Such experiments have now been completed for photons and more recently for electrons in quantum waveguides. Here the interesting aspect is that electronic excitations do not represent the right eigenmodes of the nanotube. For $x = -x' >> a$ the noise cross-correlations read:

$$S_{\rho}(x,-x,\omega = 0) = -\frac{1 - (K_{c+}^N)^2}{2} e\langle|I_{\rho}(x)|\rangle. \quad (42)$$

Note that the prefactors in Eqs. (40) and (42) can readily be interpreted using the language of Ref. [49, 50]. According to these works, a tunneling event to the bulk of a nanotube is accompanied by the propagation of two counter-propagating charges $Q_{\pm} = (1 \pm K_{c+}^N)/2$ in opposite directions. Each charge is as likely to go right or left.

The current noise and noise correlations can be interpreted as an average over the two types of excitations:

$$S_{\rho}(x,x) \sim \frac{(Q_+^2 + Q_-^2)}{2} = \frac{1 + (K_{c+}^N)^2}{4}, \quad (43)$$

$$S_{\rho}(x,-x) \sim -Q_+Q_- = -\frac{1 - (K_{c+}^N)^2}{4}. \quad (44)$$

The current operator for the nanotube charge is measured in the same positive $x$ direction at both extremities of the nanotube. If one measures the current away from the electron source (the STM), which corresponds to the standard convention for multi-terminal systems, such correlations become positive. Here the noise correlations have the added particularity that they occur to second order in a perturbative tunneling calculation. Strictly positive noise correlations are known to occur in superconducting-normal systems with filters, with applications toward entanglement [7, 19, 53]. As noted before, positive correlations do not constitute a rigorous proof for entanglement. In the present case, one single electron is injected, but
it enters a correlated system where electrons are not “welcome” as the eigenstates of the nanotube consist of a coherent superposition of bosonic modes. It therefore has to be split into left and right excitations, unless one imposes one dimensional Fermi liquid leads \((K^f_L = 1)\). Here, we are dealing with entanglement between collective excitations of the Luttinger liquid.

A drawing where the two types of charges “flow away” from the tip while propagating along the nanotube is depicted in the lower part of Fig. 7. Both charges \(Q_\pm\) are equally likely to go right or left, and they are emitted as a pair with opposite labels. Written in terms of the chiral quasiparticle fields, the addition of an electron at \(x = 0\) with given spin \(\sigma\) on a nanotube in the ground state \(|O_{LL}\rangle\) gives:

\[
\sum_{r\alpha} \Psi_{r\alpha\sigma}^\dagger(0) |O_{LL}\rangle = \frac{1}{\sqrt{2\pi a}} \sum_{\alpha} \prod_{j=0} \left\{ [\tilde{\psi}_{j\delta+}^\dagger(0)]^{Q_{j\delta+}} [\tilde{\psi}_{j\delta-}^\dagger(0)]^{Q_{j\delta-}} + [\tilde{\psi}_{j\delta+}^\dagger(0)]^{Q_{j\delta-}} [\tilde{\psi}_{j\delta-}^\dagger(0)]^{Q_{j\delta+}} \right\} |O_{LL}\rangle ,
\]

where for each sector (charge/spin, total/relative mode) the charges \(Q_{j\delta\pm} = (1 \pm K^N_{j\delta})/2\) have been introduced, and chiral fractional operators are defined as:

\[
\tilde{\psi}_{j\delta\pm}(x) = \exp \left[ i \sqrt{\frac{\pi}{2K^N_{j\delta}}} \bar{h}_{\alpha\sigma j\delta} \tilde{\varphi}_{j\delta\pm}^\dagger(x) \right] ,
\]

with \(\tilde{\varphi}_{j\delta}^r\) the chiral bosonic fields of the (nonchiral) Luttinger liquid which time evolution is simply obtained with the substitution \(\tilde{\varphi}_{j\delta}^r(x) \rightarrow \tilde{\varphi}_{j\delta}^r(x - rt)\). Consequently, quantum mechanical non-locality is quite explicit here. This entanglement is the direct consequence of the correlated state of the Luttinger liquid: the addition of an electron does not yield an eigenstate of the Luttinger liquid Hamiltonian. Electrons must be decomposed into specific modes, which happen to propagate in opposite directions. It differs significantly from its analogs which use superconductors [7, 13, 16, 19, 53].
When considering for instance the total charge sector \( j\delta = c^+ \), the wave function is similar to a triplet spin state (a symmetric combination of “up” and “down” states, or “plus” and “minus” charges) for electrons, with the electrons being replaced by chiral quasiparticle operators. Yet, each chiral field \( \tilde{\phi}_{rj\delta} \) can be written as a linear superposition of boson creation and annihilation operators, and these bosonic fields appear in an exponential. This expresses entanglement between “many–boson” states.

We mention briefly the effect of one-dimensional Fermi liquid contacts. They can be included as in Ref. [54] by connecting both nanotube ends to Luttinger liquids with interaction parameters \( K^L_{j\delta} = 1 \). Standard results proper to Fermi liquid systems are then recovered. To a first approximation, the Luttinger liquid parameters of the nanotube disappear from transport quantities (current or noise) when such quantities are evaluated in the leads. However, higher order corrections in the voltage still carry a dependence on \( K^N_{j\delta} \). The charge noise follows a Schottky formula \( S_\rho = e\langle I_\rho \rangle \), and the noise correlations to order \( \Gamma^2 \) vanish – for fermions, the leading order being in \( \Gamma^4 \).

6. Conclusion

Hanbury–Brown and Twiss geometries provide a physical test of mesoscopic transport. They can be used to check the bosonic/fermionic statistics of the carriers, or alternatively to generate entangled streams of particles. Information about statistics is necessarily contained in quantum measurements which involve two particles or more: here the zero frequency noise correlations play the role of the intensity correlator in the early quantum optics experiment of Ref. [8].

A general form of BI-tests in solid-state systems has been proposed, which is formulated in terms of current-current cross-correlators (noise correlations), the natural observables in the stationary transport regime of a solid state device. For a superconducting source injecting correlated pairs into a normal-metal fork completed with appropriate filters [7, 16], the analysis of such BIs shows that this device is a source of entangled electrons with opposite spins when the fork is weakly coupled to a superconductor.

The possibility of a Bell test for this superconductor-normal “fork” devices puts electronic entanglement in condensed matter systems on a firm footing. It is now appropriate to imagine practical information processing devices which exploit this entanglement, in a similar manner as in quantum optics. In particular, a proposal for electron teleportation consisting of 5 quantum dots (2 superconducting dots and 3 normal dots) together with a superconducting circuit was presented at the workshop. The spin state of an electron can be transferred to another dot without direct matter transfer,
and the teleportation sequence is selected with the electrostatic interactions between the dots. Details of this proposal are presented elsewhere [55].

A diagnosis for detecting the chiral excitations of a Luttinger liquid nanotube has been presented, which is based on the knowledge of low frequency current fluctuation spectrum in the nanotube. Typical transport calculations either address the propagation in a nanotube, or compute tunneling $I(V)$ characteristics. Here, both is necessary to obtain the quasiparticle charges. Also, both the noise autocorrelation and the noise cross-correlations are needed to identify the charges $Q_\pm$.

Granted, this relies on the assumption that one dimensional Fermi liquid leads are avoided. Multiple scattering at the contacts [56] may allow to preserve the contribution of the noise cross-correlations to second order in the tunneling amplitude. In particular, special circumstances such as embedded contacts [13], transport quantities seem not be renormalized by the contact parameters. This type of geometry could in fact be tested to analyze the type of contacts which one has between the nanotube and its connections. If the ratio $S_\rho(x,-x,\omega_0)/\langle I_\rho(x) \rangle$ does not depend on the tunneling distance ($\log \Gamma$), this is a clear indication that contacts do not affect this quasiparticle entanglement.

Acknowledgements

The contribution on the Bell inequality tests for electrons was performed in collaboration with G. Blatter and G. Lesovik [4]; the contribution on noise correlations in nanotubes was achieved together with R. Guyon and P. Devillard [5]. Discussions and collaborations with J. Torrè, I. Safi and D. Feinberg on normal-superconducting systems, Luttinger liquids and teleportation are also gratefully acknowledged.

References

1. P.-G. de Gennes, *Superconductivity of Metals and Alloys*, Addison Wesley, 1989; J. R. Schrieffer, *Theory of Superconductivity*, Benjamin, 1964.
2. I. M. Khalatnikov and P. C. Hohenberg, *An introduction to the Theory of Superfluidity*, Advanced Book Classics, 2000.
3. R. B. Laughlin, Rev. Mod. Phys. 71, 863 (1999); H.L. Stormer, *ibid*. 875 (1999).
4. N. Chchelkatchev, G. Blatter, G. B. Lesovik, and T. Martin cond-mat/0112094.
5. A. Crépieux, R. Guyon, P. Devillard, and T. Martin, cond-mat/0209291.
6. J. Torrè and T. Martin, Eur. Phys. J. B 12, 319 (1999).
7. G.B. Lesovik, T. Martin, and G. Blatter, Eur. Phys. J. B 24, 287 (2001).
8. R. Hanbury-Brown and Q. R. Twiss, Nature 177, 27 (1956).
9. T. Graemepacher and M. Büttiker Phys. Rev. B 61, 8125 (2000).
10. J. Börlin, W. Belzig, and C. Bruder Phys. Rev. Lett. 88, 197001 (2002).
11. M. Schechter, Y. Imry, and Y. Levinson Phys. Rev. B 64, 224513 (2001).
12. F. Taddei and R. Fazio, Phys. Rev. B 65, 134522 (2002).
13. V. Bouchiat, N. Chtchelkatchev, D. Feinberg, G.B. Lesovik, T. Martin, and J. Torres, cond-mat/0206005.
14. C. Texier and M. Büttiker, Phys. Rev. B 62, 7454 (2000).
15. V.T. Petrashov et al., Phys. Rev. Lett. 70, 347 (1993); ibid. 74, 5268 (1995); A. Dimoulas et al., ibid. 74, 602 (1995); H. Courtis et al., ibid. 76, 130 (1996); F.B. Müller-Allinger et al., ibid. 84, 3161 (2000).
16. P. Recher, E.V. Sukhorukov, and D. Loss, Phys. Rev. B 63, 165314 (2001).
17. E. Schrödinger, Naturwissenschaften 23, 807 (1935); ibid. 23, 823 (1935); ibid. 23, 844 (1935); A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. Lett. 47, 777 (1935).
18. D. Bouwmeester, A. Ekert, and A. Zeilinger, The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computation (Springer-Verlag, Berlin, 2000).
19. P. Recher and D. Loss, Phys. Rev. B 65, 165327 (2002).
20. D. Loss and D. P. DiVicenzo, Phys. Rev. A 57, 120 (1998).
21. J.S. Bell, Physics (Long Island City, N.Y.) 1, 195 (1965); J.S. Bell, Rev. Mod. Phys. 38, 447 (1966).
22. S. Datta, Electronic Transport in Mesoscopic systems (Cambridge University Press 1996); Y. Imry, Introduction to Mesoscopic Physics (Oxford University Press, Oxford, 1997).
23. T. Martin and R. Landauer, Phys. Rev. B 45, 1742 (1992).
24. M. Büttiker, Phys. Rev. Lett. 65, 2901 (1990); Phys. Rev. B 46, 12485 (1992).
25. M. Henny et al., Science 296, 284 (1999); W. Oliver et al., Science 296, 299 (1999).
26. M.J.M. de Jong and C.W.J. Beenakker, Phys. Rev. B 49, 16070 (1994); B.A. Muzylantskii and D.E. Khmelnitskii, ibid. 50, 3982 (1994); M.P. Anantram and S. Datta, ibid. 53, 16390 (1996); G. Lesovik, T. Martin, and J. Torres, ibid. 60, 11935 (1999).
27. T. Martin, Phys. Lett. A 220, 137 (1996).
28. Y. Gefen, J. Imry, and R. Landauer, Phys. Rev. Lett. 52, 139 (1984); M. Büttiker, Y. Imry, and M. Ya. Azbel, Phys. Rev. A 30, 1982 (1984).
29. M. Büttiker, NATO workshop on noise, Delft, Y. Nazarov and Y. Blanter eds. (Kluwer 2002).
30. W. Belzig et al., NATO workshop on noise, Delft, Y. Nazarov and Y. Blanter eds. (Kluwer 2002).
31. R. Melin and D. Feinberg, Eur. Phys. J. B 26, 101 (2002).
32. A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. 49, 1804 (1982).
33. R. Werner and M. Wolf, quant-ph/0107093.
34. W.D. Oliver, F. Yamaguchi, and Y. Yamamoto, Phys. Rev. Lett. 88, 037901 (2002).
35. D.S. Saraga and D. Loss cond-mat/0205553.
36. J.F. Clauser and M.A. Horne, Phys. Rev. D 10, 526 (1974).
37. Z.Y. Ou and L. Mandel, Phys. Rev. Lett. 61, 50 (1988).
38. X.G. Wen, Int. J. Mod. Phys. B 6, 1711 (1992); Adv. Phys. 44, 405 (1995).
39. W. Schottky, Ann. Phys. (Leipzig) 57, 541 (1918).
40. C. L. Kane and M.P.A. Fisher, Phys. Rev. Lett. 72, 724 (1994).
41. C. de C. Chamon, D. Freed, and X.G. Wen, Phys. Rev. B 51, 2363 (1995-II).
42. P. Fendley, A. W. W. Ludwig, and H. Saleur, Phys. Rev. Lett. 75, 2196 (1995).
43. L. Saminadayar, D.C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. 79, 2526 (1997).
44. R. de-Picciotto et al., Nature 389, 162 (1997).
45. I. Safi, P. Devillard, and T. Martin, Phys. Rev. Lett. 86, 4628 (2001).
46. M. Bockrath, D. H. Cobden, A. Rinzler, R.E. Smalley, L. Balents, and P. McEuen, Nature 397, 598 (1999).
47. C. Kane, L. Balents, and M. P. A. Fisher, Phys. Rev. Lett. 79, 5086 (1997).
48. R. Egger, A. Bachtold, M. Fuhrer, M. Bockrath, D. Cobden, and P. McEuen, in Interacting Electrons in Nanostructures, edited by R. Haug and H. Schoeller (Springer, 2001).
49. I. Safi, Ann. Phys. Fr. 22, 463 (1997).
50. K.-V. Pham, M. Gabay and P. Lederer, Phys. Rev. B 61, 16397 (2000).
51. R. Egger and A. Gogolin, Eur. Phys. J. B 3, 781 (1998).
52. C. Chamon and D. E. Freed, Phys. Rev. B 60, 1842-1853 (1999).
53. C. Bena et al., Phys. Rev. Lett. 89, 037901 (2002).
54. I. Safi and H. Schulz, Phys. Rev. B 52, 17040 (1995); D. Maslov and M. Stone, Phys. Rev. B 52, 5539 (1995); V.V. Ponomarenko, Phys. Rev. B 52, 8666 (1995).
55. O. Sauret, D. Feinberg, and T. Martin, cond-mat/0203215.
56. K.-I. Imura, K.-V. Pham, P. Lederer, and F. Piéchon, Phys. Rev. B 66, 035313 (2002).