Relativistic Electronic Dressing in Laser-assisted ionization of atomic hydrogen by electron impact.

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Abstract

Within the framework of the coplanar binary geometry where it is justified to use plane wave solutions for the study of the (e,2e) reaction and in the presence of a circularly polarized laser field, we introduce as a first step the DVRPWBA1 (Dirac-Volkov Plane Wave Born Approximation1) where we take into account only the relativistic dressing of the incident and scattered electrons. Then, we introduce the DVRPWBA2 (Dirac-Volkov Plane Wave Born Approximation2) where we take totally into account the relativistic dressing of the incident, scattered and ejected electrons. We then compare the corresponding triple differential cross sections for laser-assisted ionization of atomic hydrogen by electron impact both for the non relativistic and the relativistic regime.

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1 Introduction

Ehrhardt et al (1969) [1] were the first to conduct electron impact ionization experiments in which the two outgoing electrons are detected in coincidence after angular and energy analysis. The first theoreticians who proposed such type of experiment were Smirnov and Neudachin (1966) [2]. These experiments now are commonly referred to as (e,2e). Since then, the (e,2e) reaction has been studied extensively in the non relativistic kinematic domain (Camilloni et al 1972 [3], Weigold et al 1973 [4], van der Wiel 1973 [5], Brion 1975 [6]). All these studies show that the non relativistic (e,2e) reaction is a very sensitive test of the target electronic structure and the electron impact ionization reaction mechanism. For the relativistic domain, the first electron impact ionization experiments where conducted by Dangerfield and Spicer

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Total cross sections with relativistic electrons where measured for the K and L shells of heavy elements. Some theoreticians (Scofield 1978 [10], Moiseiwitsch and Stockman [11]) proposed models for total ionization cross sections. Finally, Fuss et al (1982) [12] proposed a model of an \( (e, 2e) \) reaction they called binary \( (e, 2e) \) reaction in which the maximum momentum transfer occurs, that is, a reaction where the outgoing electrons have equal energy. This type of reaction has been since then the most successful for probing atomic, molecular and solid state structure. In a report devoted both to experimental and theoretical developments in the study of relativistic \( (e, 2e) \) processes, Nakel and Whelan (1999) [13] reviewed the goals of these investigations aimed at gaining a better understanding of inner-shell ionization process by relativistic electrons up to the highest atomic numbers and probing the quantum mechanical Coulomb problem in the regime of high energies (up to 500 keV) and strong fields. With the advent of the laser field, many theoretical models have been proposed [14] mainly in the non relativistic domain whereas in the relativistic domain we can only quote the work of Reiss (1990) [15] and that of Crawford and Reiss (1994,1998) [16] who studied the relativistic ionization of hydrogen (without electron impact) by linearly polarized light. They focused their work on the calculations of the differential transition rates and have shown that strong field atomic stabilization is enhanced by relativistic effects.

In this article, we present a theoretical model for the relativistic electronic dressing in laser-assisted ionization of atomic hydrogen by electron impact with a circularly polarized laser field and in order to check the consistency of our calculations, we begin our study in absence of the laser field. As we devote this analysis to atomic hydrogen, all distortion effects mentioned in [13] need not be addressed since the atomic number we deal with is that of atomic hydrogen, that is \( Z = 1 \). It is checked first that, in the absence of the laser field and working in the coplanar binary geometry where the kinetic energies of the scattered electron and the ejected electron are nearly the same, it is justified to use plane wave solutions for the study of the \( (e, 2e) \) reaction. Indeed, this particular geometry is such that the ejected electron does not feel the Coulomb influence of atomic target and can be described by a plane wave in the non relativistic as well as in the relativistic domain. Then, in the presence of a circularly polarized laser field, we introduce as a first step the DVRPWBA1 (Dirac-Volkov Plane Wave Born Approximation1) where we take into account only the relativistic dressing of the incident and scattered electrons. It is shown that this approximation introduces an asymmetry in the description of the scattering process since it does not allow photon exchange between the laser field and the ejected electron and its domain of validity is only restricted to very weak fields.
and non relativistic electron kinetic energy. Then, we introduce the DVRPWBA2 (Dirac-Volkov Plane Wave Born Approximation2) where we take totally into account the relativistic dressing of the incident, scattered and ejected electrons.

The organization of this paper is as follows: in section II, we present the relativistic formalism of \((e, 2e)\) reaction in absence of the laser field RPWBA (Relativistic Plane Wave Born Approximation) and we compare it in the non relativistic domain with the NRPWBA (Non Relativistic Plane wave Born Approximation) as well as the NRCBA (Non Relativistic Coulomb-Born Approximation). In section III, we introduce the DVPWBA1 (Dirac-Volkov Plane Wave Born Approximation1). This approximation is introduced as a first step. In section IV, we introduce the DVPWBA2 (Dirac-Volkov Plane Wave Born Approximation2) in which we take fully account of the relativistic electronic dressing of the incident, scattered and ejected electrons. This approximation is more founded on physical grounds since the ejected electron can also exchange photons (absorption or emission) with the laser field. This more complete description of the incoming and outgoing electrons allows to investigate the relativistic domain. In section V, we discuss the results we have obtained and we end by a brief conclusion in Section VI. Throughout this work, atomic units (a.u) are used \((\hbar = m_e = e = 1)\) where \(m_e\) is the electron mass and TDCS stands for triple differential cross section.

## 2 The TDCS in absence of the laser field

The transition matrix element for the direct channel (we neglect exchange effects) is given by

\[
S_{fi} = \frac{i}{\hbar} \int_{-\infty}^{\infty} dx_0 \langle \psi_{pf}(x_1)\phi_f(x_2) | V_d | \psi_{pi}(x_1)\phi_i(x_2) \rangle, \tag{1}
\]

where \(V_d = 1/r_{12} - 1/r_1\) is the direct interaction potential \((t_1 = t_2 = t \implies x_1^0 = x_2^0 = x^0)\) and in the RPWBA, \(\psi_{pf}(x_1)\) is the wave function describing the scattered electron

\[
\psi_{pf}(x_1) = \frac{u(p_f, s_f)}{\sqrt{2E_fV}} e^{-ip_f\cdot x_1}, \tag{2}
\]

given by a free Dirac solution normalized to the volume \(V\). For the incident electron, we use

\[
\psi_{pi}(x_1) = \frac{u(p_i, s_i)}{\sqrt{2E_iV}} e^{-ip_i\cdot x_1}. \tag{3}
\]
For the atomic target, \( \phi_i(x_2) = \phi_i(t, r_2) \) is the relativistic wave function of atomic hydrogen in its ground state. For the ejected electron, we use again a free Dirac solution normalized to the volume \( V \) and \( \phi_f(x_2) \) is given by

\[
\phi_f(x_2) = \psi_{p_B}(x_2) = \frac{u(p_B, s_B)}{\sqrt{2E_BV}} e^{-ip_B \cdot x_2}.
\]  

The free spinor \( u(p, s) \) is such that \( \pi(p, s)u(p, s) = 2c^2 \) and \( u^\dagger(p, s)u(p, s) = 2E \). Using the standard methods of QED [17], we obtain for the unpolarized TDCS

\[
\frac{d\sigma}{dE_B d\Omega_B d\Omega_f} = \frac{|P_f||P_B|}{|P_i|c^2} \left( \sum_{s_B} \pi(p_B, s_B)u(p_B, s_B) \right) \frac{(2E_iE_f/c^2 - p_i \cdot p_f + c^2)}{|p_f - p_i|^4} \times \Phi_{1,1/2,1/2}(q_1 = \Delta - p_B) - \Phi_{1,1/2,1/2}(q_0 = -p_B) |^2.
\]  

The sum over the spins of the ejected electron gives

\[
\sum_{s_B} \pi(p_B, s_B)u(p_B, s_B) = 4E_B.
\]  

The functions \( \Phi_{1,1/2,1/2}(q) \) are the Fourier transforms of the relativistic atomic hydrogen wave functions

\[
\Phi_{1,1/2,1/2}(q) = (2\pi)^{(-3/2)} \int dr_2 e^{iq \cdot r_2} \Psi_{n=1, j=1/2, m=1/2}(r_2),
\]  

and \( \Delta = p_i - p_f \) is the momentum transfer. This TDCS is to be compared with the corresponding one in the NRPWBA (Non Relativistic Plane Wave Born Approximation)

\[
\frac{d\sigma}{dE_B d\Omega_B d\Omega_f} = \frac{2^7}{(2\pi)^2} \frac{|P_f||P_B|}{|P_i|c^4} \frac{1}{\Delta} \left( \frac{1}{(q_1^2 + 1)^2} - \frac{1}{(q_0^2 + 1)^2} \right)^2,
\]

where \( q_1 = \Delta - p_B \) and \( q_0 = -p_B \) to the TDCS in the NRCBA (Non Relativistic Coulomb Born Approximation)

\[
\frac{d\sigma^{CB}}{dE_B d\Omega_B d\Omega_f} = \frac{|P_f||P_B|}{|P_i|} \left| f^{CB}_{ion} \right|^2,
\]

where \( f^{CB}_{ion} \) is the first Coulomb-Born amplitude corresponding to the ionization of atomic hydrogen by electron impact [18].
\[ f_{ion}^{CB1} = -\frac{2}{|\Delta|^2} M_{1s}(\Delta, \mathbf{p}_B). \]  

(10)

The quantity \( M_{1s}(\Delta, \mathbf{p}_B) \) is easily deduced from the Nordsieck integral [19]

\[ I(\lambda) = \int e^{-r^2} \frac{e^{i\lambda \mathbf{r}.\mathbf{r}}}{r} \, \mathbf{F}_1 \left( i \left( \frac{i}{\mathbf{p}_B}, 1, i(p_B r + \mathbf{p}_B) \right) \right) \, d\mathbf{r} = \frac{4\pi}{q^2 + \lambda^2} \left( q^2 + \lambda^2 + 2q \mathbf{p}_B - 2i\lambda \mathbf{p}_B \right)^{-i/p_B}, \]

giving the well known result

\[ M_{1s}(\Delta, \mathbf{p}_B) = \frac{e^{\pi/2p_B}}{2\sqrt{2\pi^2}} \Gamma(1 - i/p_B) \left( -\frac{dI(\lambda)}{d\lambda} \right)_{\lambda=1}. \]  

(11)

3 The TDCS in presence of the laser field. The DVP-WBA1

We begin our study of the \((e,2e)\) reaction by considering first the DVP PWBA1 where we only take into account the dressing of the incident and the scattered electron. The laser field is circularly polarized. Again, the transition matrix element for the direct channel is

\[ S_{fi} = -\frac{i}{c} \int_{-\infty}^{\infty} dx^0 \, \langle \psi_{q_f}(x_1) \phi_f(x_2) | V_d | \psi_{q_i}(x_1) \phi_i(x_2) \rangle, \]  

(12)

where \((t_1 = t_2 = t \implies x^0_1 = x^0_2 = x^0)\) and in the DVPWBA1, \(\psi_{q_f}(x_1)\) is the Dirac-Volkov wave function normalized to the volume \(V\) describing the scattered electron

\[ \psi_{q_f}(x_1) = \left[ 1 + \frac{k A_{(1)}^{(1)}}{2c(k.p_f)} \right] u(p_f, s_f) e^{is_f(x_1)}, \]  

(13)

where \(A_{(1)} = a_1 \cos(\phi_1) + a_2 \sin(\phi_1)\) is the four potential of the laser field, \(\phi_1 = k.x_1 = k_0 x^0_1 - \mathbf{k}.x_1 = wt - \mathbf{k}.x_1\) is the phase of the laser field and \(w\) its pulsation. \(Q\) is the total energy acquired by the electron in the presence of a laser field and is given by:

\[ Q = E - \frac{a^2 w}{2c^2(k.p)}. \]  

(14)
The phase $s_f(x_1)$ is given by

$$s_f(x_1) = -q_f.x_1 - \frac{a_1.p_f}{c(k.p_f)} \sin(\phi_1) + \frac{a_2.p_f}{c(k.p_f)} \cos(\phi_1).$$

(15)

For the incident electron, we use

$$\psi_{q_i}(x_1) = [1 + \frac{k_A}{\sqrt{2c(k.p_i)}}] u(p_i, s_i) e^{i s_i(x_1)},$$

(16)

with the phase $s_i(x_1)$ given by

$$s_i(x_1) = -q_i.x_1 - \frac{a_1.p_i}{c(k.p_i)} \sin(\phi_1) + \frac{a_2.p_i}{c(k.p_i)} \cos(\phi_1),$$

(17)

where the four vector $q^\mu$ is such

$$q^\mu = p^\mu - \frac{a^2}{2c^2(k.p)} k^\mu,$$

(18)

and $a^2 = A_\mu A^\mu = a_1^2 = a_2^2$. For the atomic target, $\phi_i(x_2) = \phi_i(t, r_2) = e^{-i \epsilon_b t} \phi_i(r_2)$ is the relativistic wave function of atomic hydrogen in its ground state and $\epsilon_b = c^2(\sqrt{1 - \alpha^2} - 1)$ is the binding energy of the ground state of atomic hydrogen with $\alpha = 1/c$ the fine structure constant. For the ejected electron, we use a free Dirac solution normalized to the volume $V$

$$\phi_f(x_2) = \psi_B(x_2) = \frac{u(p_B, s_B)}{\sqrt{2E_B}} e^{-ip_B.x_2}.$$

(19)

Using the standard methods of QED, we have for the unpolarized TDCS

$$\frac{d\sigma}{dE_B d\Omega_B d\Omega_f} = \sum_{s=-\infty}^{\infty} \frac{d\sigma^{(s)}}{dE_B d\Omega_B d\Omega_f} \bigg|_{Q_f=Q_i+sw+\epsilon_b-E_B},$$

(20)

where the expression of $d\sigma^{(s)}/dE_B d\Omega_B d\Omega_f$ is

$$\frac{d\sigma^{(s)}}{dE_B d\Omega_B d\Omega_f} = 1 \frac{|q_f||p_B|}{2 |q_i| c^6} \frac{(\sum_{s_i,s_f} |M^{(s)}_{fi}|^2/2)}{(4E_B)} \left| \frac{\Phi_{1/2,1/2}(q = \Delta_s-p_B) - \Phi_{1/2,1/2}(q = -p_B)}{2} \right|^2.$$

(21)

The sum $(\sum_{s_i,s_f} |M^{(s)}_{fi}|^2/2)$ has already been evaluated in a previous work [20] and $\Delta_s = q_i-q_f + sk$ is the momentum transfer in presence of the laser field. This TDCS is
compared to the corresponding TDCSs in the non relativistic regime. On the one hand, the calculations within the framework of the NRPWBA1 (where the incident and scattered electrons are described by non relativistic Volkov plane waves whereas the ejected electron is described by a non relativistic free plane wave) give

$$\frac{d\sigma^{\text{NRPWBA1}}}{dE_Bd\Omega_Bd\Omega_f} \sum_{s=-\infty}^{\infty} \frac{d\sigma^{(s)}}{dE_Bd\Omega_Bd\Omega_f} \bigg|_{E_f=E_i+sw+i\varepsilon_{1s}-E_B},$$

with

$$\frac{d\sigma^{(s)}}{dE_Bd\Omega_Bd\Omega_f} = \frac{2\pi}{(2\pi)^2} \frac{|p_f||p_B|}{|p_i|} \frac{J^2(z_{NR})}{|p_f-p_i-sk|^4 \left( (q_{1s}^2 + 1)^2 - (q_{0s}^2 + 1)^2 \right)^2},$$

where $\varepsilon_{1s} = -0.5$ a.u is the non relativistic binding energy of atomic hydrogen in its ground state, $q_{1s} = p_i + sk - p_f - p_B = \Delta_{s}^{NR} - p_B$ and $q_{0s} = -p_B$. On the other hand, the calculations within the framework of the NRCBA1 (where the incident and scattered electrons are described by non relativistic Volkov plane waves whereas the ejected electron is described by a Coulomb wave function) give

$$\frac{d\sigma^{\text{NRCBA1}}}{dE_Bd\Omega_Bd\Omega_f} = \sum_{s=-\infty}^{\infty} \frac{d\sigma^{(s)}}{dE_Bd\Omega_Bd\Omega_f} \bigg|_{E_f=E_i+sw+i\varepsilon_{1s}-E_B},$$

with

$$\frac{d\sigma^{(s)}}{dE_Bd\Omega_Bd\Omega_f} = \frac{1}{2\pi^4} \frac{|p_f||p_B|}{|p_i|} \frac{J^2(z_{NR})}{|p_f-p_i-sk|^4 e^{\pi/p_B} \left( 1 - \frac{i}{p_B} \right)^2 I(q_s = p_i-p_f + sk - p_B)}.$$  

Note that we may write $q_s = p_i-p_f + sk - p_B = \Delta_{s}^{NR} - p_B$. The result for $I(q_s = \Delta_{s}^{NR} - p_B)$ is

$$I(q_s) = p_i-p_f + sk - p_B = \frac{16\pi}{(q_s^2 + 1)^2 - i/p_B \left( (\Delta_{s}^{NR})^2 - (p_B + i)^2 \right)^{1+i/p_B}}.$$  

In the expressions of the last two non relativistic TDCSs, the argument of the ordinary Bessel functions is given by

$$z_{NR} = \frac{|a|}{cw} \left| \Delta_{s}^{NR} \right|.  \quad (27)$$
4 The TDCS in presence of the laser field. The DVP-WBA2

We now take into account the electronic relativistic dressing of all electrons which are described by Dirac-Volkov plane waves normalized to the volume $V$. This will give rise to a new trace to be calculated but it will turn out that taking into account the relativistic electronic dressing of the ejected electron amounts simply to introduce a new sum on the $l_B$ photons that can be exchanged with the laser field. The transition amplitude in the DVPWBA2 is now given by

$$S_{fi} = -\frac{i}{c} \int_{-\infty}^{\infty} dx^0 <\psi_q(x_1)\phi_f(x_2) | V_d | \psi_q(x_1)\phi_i(x_2) > .$$

(28)

The difference between DVPWBA1 and DVPWBA2 is related to the way we choose $\phi_f(x_2)$. Now, the Dirac-Volkov wave function for the ejected electron is such that

$$\phi_f(x_2) = \psi_{qB}(x_2) = [1 + \frac{k A(2)}{2c(k,p_B)} u(p_B,s_B) e^{is_B(x_2)},$$

(29)

where $A(2) = a_1 \cos(\phi_2) + a_2 \sin(\phi_2)$ is the four potential of the laser field felt by the ejected electron, $\phi_2 = k \cdot x_2 = k_0 x_2^0 - k \cdot x_2 = wt - k \cdot x_2$ is the phase of the laser field and $w$ its pulsation.

Proceeding along the same line as before, we get for the unpolarized TDCS

$$\frac{d\sigma}{dE_B d\Omega_B d\Omega_f} = \sum_{s,l_B=-\infty}^{\infty} \frac{d\sigma^{(s,l_B)}}{dE_B d\Omega_B d\Omega_f} |_{Q_f=Q_i+(s+l_B)w+\epsilon_B-Q_B} ,$$

(30)

with

$$\frac{d\sigma^{(s,l_B)}}{dE_B d\Omega_B d\Omega_f} = \frac{1}{2} |q_f||q_B| (\sum_{s_i,s_f} |M_{fi}^{(s)}|^2/2) \sum_{s_B} |u(p_B,s_B)\Gamma_{l_B} \gamma^0|^2 \times |\Phi_{1,1/2,1/2}(q = \Delta_{s+l_B}-q_B) - \Phi_{1,1/2,1/2}(q = -q_B+(s+l_B)k)|^2 .$$

(31)

The quantity $\Delta_{s+l_B}$ is simply given by $\Delta_{s+l_B} = q_i - q_f + (s+l_B)k$. Introducing the factor $c(p_B) = 1/(2c(k,p_B))$, the symbol $\Gamma_{l_B}$ is defined as

$$\Gamma_{l_B} = B_{l_B}(z_B) + c(p_B)[\phi_1 k B_{l_B}(z_B) + \phi_2 k B_{2l_B}(z_B)],$$

(32)
where the three quantities \( B_{1B}(z_B) \), \( B_{1B}(z_B) \) and \( B_{2B}(z_B) \) are respectively given by

\[
\begin{align*}
\left\{ \begin{array}{l}
B_{1B}(z_B) \\
B_{1B}(z_B) \\
B_{2B}(z_B)
\end{array} \right\} = \left\{ \begin{array}{l}
\{ J_{l_B+1}(z_B) e^{i(l_B+1)\phi_B} + J_{l_B-1}(z_B) e^{i(l_B-1)\phi_B} \} / 2 \\
\{ J_{l_B+1}(z_B) e^{i(l_B+1)\phi_B} - J_{l_B-1}(z_B) e^{i(l_B-1)\phi_B} \} / 2i
\end{array} \right\},
\end{align*}
\tag{33}
\]

where \( z_B = \frac{|a|}{c(k.p_B)} \sqrt{(\hat{y}.p_B)^2 + (\hat{x}.p_B)^2} \) is the argument of the ordinary Bessel functions that will appear in the calculations and the phase \( \phi_{0B} \) is defined by

\[
\phi_{0B} = \arctan((\hat{y}.p_B)/(\hat{x}.p_B)).
\tag{34}
\]

The sum over the spins of the ejected electron can be transformed to traces of gamma matrices. Using REDUCE, we find

\[
\sum_{s_B} |\pi(p_B,s_B)\Gamma_{l_B}\gamma^0|^2 = 4E_B \{ J_{l_B}^2(z_B) - wc(p_B) \cos(\phi_{0B})(a_1.p_B) + \sin(\phi_{0B})(a_2.p_B) \} \\
\times \{ J_{l_B}(z_B)(J_{l_B+1}(z_B) + J_{l_B-1}(z_B)) - a^2 w(k.p_B)c^2(p_B)(J_{l_B+1}^2(z_B) + J_{l_B-1}^2(z_B)) \}.
\tag{35}
\]

In absence of the laser field, only the term \( 4E_B J_{l_B}^2(z_B = 0)\delta_{l_B,0} = 4E_B \) contributes, which was to be expected. Once again, one encounters terms proportional to \( \cos(\phi_{0B}) \) as well as to \( \sin(\phi_{0B}) \) which contributes to the sum over the spins of the ejected electron. We compare this TDCS with the corresponding cross sections in the framework of the NRPWBA2 (where the incident, scattered and ejected electrons are described by non relativistic Volkov plane waves)

### 5 Results and discussion

#### 5.1 In absence of the laser field

We begin our discussion by the kinematics of the problem. In absence of the laser field, there is no dressing of angular coordinates and we choose a geometry where \( p_i \) is along the \( Oz \) axis (\( \theta_i = \phi_i = 0 \)). For the scattered electron, we choose (\( \theta_f = 45^\circ, \phi_f = 0 \)) and for the ejected electron we choose \( \phi_B = 180^\circ \) and the angle \( \theta_B \) varies from \( 0^\circ \) to \( 360^\circ \). This is an angular situation where we have a coplanar geometry. Spin effects are fully included and we use an exact relativistic description of the electrons and the atomic target. In order to check
the validity of the coplanar binary geometry, we begin first by comparing the three TDCSs (RPWBA, NRPWBA and NRCBA) in the non relativistic domain. In the expression of the Fourier transforms $\Phi_{1,1/2,1/2}(q = \Delta - p_B)$ and $\Phi_{1,1/2,1/2}(q = -p_B)$, one has to determine the angle between $\Delta - p_B$ and $p_i$ on the one hand and the angle between $-p_B$ and $p_i$ on the other hand. Keeping in mind that $p_i$ is along the $Oz$ axis, one finds

$$\cos(\Delta - p_B, p_i) = \frac{(p_i - p_f \cos \theta_f - p_B \cos(\theta_B))}{|\Delta - p_B|},$$

(36)

while for $\cos(-p_B, p_i) = -\cos(\theta_B)$ from which one deduces the corresponding angles. To have an idea of how the ejected electron ”looses” its Coulombian behaviour, we begin by a process where the incident electron kinetic energy is $T_i = 1350$ eV, and the ejected electron kinetic energy is $T_B = 574.5$ eV.

In Fig 1, we see that both RPWBA and NRPWBA give nearly the same results whereas NRCBA gives a higher TDCS due to the fact that the ejected electron still feels the influence of the atomic field. Increasing this energy of the ejected electron from $574.5$ eV to $674.5$ eV gives rise to almost three indistinguishable curves. This situation is shown in Fig 2. As the incident electron kinetic energy is increased to $2700$ eV which corresponds to a relativistic parameter $\gamma = (1 - v^2/c^2)^{-1/2} = 1.0053$, that of the ejected electron to $1349.5$ eV, we have a very good
agreement between the three TDCSs. This is a crucial test for the models we will develop in the presence of a laser field since the adjunction of the latter cannot be done without a judicious choice of a geometry. It must be borne in mind that this coplanar binary geometry is not well suited for the study of the domain of low kinetic energy for the ejected electron. The agreement between these three approaches remains good up to $T_i = 15$ keV from which the RPWBA gives results that are a little lower than NRPWBA and NRCBA because relativistic and spin effects can no longer be ignored. Due to the relative simplicity of the model (even if the calculations are far from being obvious) and the exact non dressed relativistic description of the target, it is remarkable that such an agreement should be reached for these energies. When the laser field is introduced, the dressing of angular coordinates is not important for the non relativistic regime ($\gamma = 1.0053$, $E = 0.05$ a.u) but becomes noticeable for the relativistic regime ($\gamma = 2.0$, $E = 1.00$ a.u) where $E$ is the electric field strength. The unit of electric field strength in atomic units is $E = 5.14225 \times 10^9$ V/cm.

5.2 In presence of the laser field

5.2.1 A. The non relativistic regime ($\gamma = 1.0053$, $E = 0.05$ a.u)

The first check to be done is to take a zero electric field strength in order to recover all the results in absence of the laser field. We have done these checks for all approximations. It has been shown [23] that for a laser frequency $w = 0.043$ a.u, which corresponds to a laser photon energy of 1.17 eV, dressing effects due to the atomic target are not very important. A complete
and exact relativistic treatment of the ejected electron is not analytically possible since the non relativistic wave equation for continuum states in a Coulomb field is separable in parabolic coordinates, but the corresponding Dirac equation is not. In other words, a decomposition of the relativistic continuum wave function into partial waves is not as straightforward as for the non relativistic case and the quantum numbers of each partial wave have to be taken into account very carefully. However, a tedious numerical construction of the first few partial waves is possible.

We first compare the results obtained within the three approximations (DVPWBA1, NR-PWBA1, NRCBA1) where it is expected on physical grounds that these cannot be used to study the relativistic regime. The three summed TDCSs are all peaked around $\theta_B = 45^\circ$, $\phi_B = 180^\circ$ which was to be expected for the case of the geometry chosen since for the scattered electron, the choice we have made is $\theta_f = 45^\circ$, $\phi_f = 0^\circ$ and in the $xOy$ plane, this amounts to a scattered electron and an ejected electron having an opposite value of $\theta$. Even with no photon exchange and for an electric field strength of 0.05 $a.u$, the presence of the laser field reduces considerably the magnitude of the TDCSs. The NR-PWBA1 and NRCBA1 TDCSs are nearly indistinguishable whereas the DVPWBA1 TDCS is lower than the former ones in the vicinity of the maximum for $\theta_B = 45^\circ$. For this angle, we have $\text{TDCS(NRPWBA1)} \simeq 0.717 \times 10^{-5} a.u$, $\text{TDCS(NRCBA1)} \simeq 0.7115 \times 10^{-5} a.u$, $\text{TDCS(DVPWBA1)} \simeq 0.472 \times 10^{-5} a.u$. Two interesting cases are those corresponding to the absorption and emission of one photon. We have shown in a previous work [24] that when all electrons are described by Dirac-Volkov planes, the corresponding differential cross sections for the absorption and emission (of one photon) processes are identical. It is not the case for these three approximations since the ejected electron is described by a free Dirac plane wave. The DVPWBA1 TDCS is larger than the two other non relativistic TDCSs by a factor 4 at the maximum for $\theta_B = 45^\circ$ for the absorption process and the emission process but these relativistic TDCSs are not identical. The TDCS for the emission of one photon is smaller than the corresponding one for the absorption of one photon by a factor 2 at the same maximum. These remarks are not without interest since a crucial test of our next model (all electrons are described by Dirac-Volkov plane waves) will be to compare the two TDCSs within the framework of DVPWBA2 for these two processes. It will be a sound consistency check of our calculations. In Fig. 3, we show the three summed TDCSs for an exchange of $\pm 100$ photons and we obtain close curves for the non relativistic plane wave and Coulomb Born results with the relativistic plane wave TDCS a little lower than the former ones in the vicinity of the peak at $\theta_B = 45^\circ$. For angles lower or larger than $\theta_B = 45^\circ$, the three TDCSs give nearly the same results. The dressing of
angular coordinates being negligible, the maximum is maintained for the above mentioned value of $\theta_B$ but in the relativistic regime, this maximum is shifted. Also, we have compared the relativistic TDCS for different numbers of photons exchanged, typically ±50, ±100 and ±150 photons. The TDCSs increase when the number of the photons exchanged increases and finally, we have compared the relativistic TDCS without laser field with these summed relativistic TDCSs in order to obtain a check of the well known pseudo sumrule [25]. We have obtained results that converge to the relativistic TDCS without laser field but complete convergence is not reached since the ejected electron is not properly described.

Now, we discuss the results obtained within the framework of the three more accurate approximations (DVPWBA2, NRPWBA2, NRCBA2) in the same non relativistic regime. The description of the ejected electron by a Dirac-Volkov plane wave being more accurate and necessary on physical grounds (there are no constraints that forbid the ejected electron to exchange photons with the laser field), we have first investigated the case where no photon is exchanged at all ($s = 0, l_B = 0$). There is a shift of the location for the maximum corresponding to the relativistic TDCS while the two non relativistic TDCSs are nearly the same. The magnitude of the TDCSs is also considerably reduced. We have $\text{TDCS}(\text{DVPWBA2}) \simeq 0.347 \times 10^{-8}$ a.u for $\theta_B = 41^\circ$ while $\text{TDCS}(\text{NRPWBA2}) \simeq \text{TDCS}(\text{NRCBA2}) \simeq 0.3326 \times 10^{-8}$ a.u for $\theta_B = 43^\circ$. There are three small secondary peaks for the relativistic TDCS and two secondary peaks for the two non relativistic TDCSs. This behaviour stems in the relativistic description from the contribution of the sum over the spins of the ejected electron. This sum shows a
narrow peak for $\theta_B = 41^\circ$ and presents two minima for $\theta_B = 45^\circ$ and $\theta_B = 50^\circ$. The other peaks can also be traced back to the behaviour of this sum. Relativistic and spin effects begin at this stage to become noticeable since the shift as well as the magnitude of the relativistic TDCS with respect to the non relativistic TDCSs are clear signatures even in the non relativistic regime.

Moreover, the most crucial test of our model is the complete symmetry between the emission and absorption processes. In Fig. 4, we show the relativistic and non relativistic TDCSs for $s = 1$ and $l_B = -1$ in the non relativistic regime scaled in units of $10^{-8} \text{ a.u.}$ We obtain the same figure for $s = -1$ and $l_B = 1$.

For these two curves, the maximum shifts back to the value $\theta_B = 45^\circ$. As the number of exchanged photons $s$ is linked both to the incident and scattered electron, we simulated a process where the number $s = 2l_B$ to check the corresponding influence on the TDCSs and on the location of the maximum. The summed TDCSs for $(s = \pm 50, \ l_B = \pm 25)$ and for $(s = \pm 100, \ l_B = \pm 50)$ are almost halved when compared to the corresponding TDCSs for $s = l_B$. All curves are peaked around a maximum angle due to the behaviour of the square of the Fourier transform of the relativistic hydrogenic wave functions that falls off rapidly to zero in a small region around this maximum. Another interesting remark concerns once again the behaviour of the sum over the spins of the ejected electron with the number of photons exchanged. As $l \geq \pm 500$, this sum is almost zero and contributes also to the rapid fall off of the corresponding relativistic TDCS. To illustrate the complexity of the location of the visual cut-off of the relativistic TDCS, we consider an electric field strength $\mathcal{E} = 0.01 \text{ a.u}$ where it is expected that the numbers $s$ and $l$ of photons exchanged will not be high. The envelope of

![Figure 4: The two TDCSs for $s = 1$ and $l_B = -1$ in the non relativistic regime scaled in units of $10^{-8} \text{ a.u.}$ We obtain the same figure for $s = -1$ and $l_B = 1$](image-url)
photon energy transfer obtained is a curve in three dimensions. As in the case of the process of Mott scattering in a strong laser field, we observe in Fig. 5

![Figure 5: The enveloppe of photon energy transfer in the non relativistic regime and for an electric field strength $\mathcal{E} = 0.01 \text{ a.u.}$]

a rapid fall off of the relativistic TDCS for $s \simeq l \simeq \pm 40$ where the absolute value of the indices of the ordinary Bessel functions are close to their arguments. Also, as a side result, we see clearly in this Figure that there is a complete symmetry between $s$ and $l$. To obtain a converging envelope, one has to sum over the same numbers $s$ and $l$ of photons exchanged.

### 5.2.2 B. The relativistic regime ($\gamma = 2$, $\mathcal{E} = 1.00 \text{ a.u.}$)

In the relativistic regime, dressing of angular coordinates is not negligible and we indeed observed as in the case of excitation a shift from $\theta_B = 45^\circ$ to lower values. The relativistic parameter $\gamma = 2$ corresponds to an incident electron kinetic energy of $c^2$ in atomic units or the rest mass of the electron ($0.511 \text{ MeV}$). In Fig 6, we show the behaviour of the relativistic TDCS in absence of a laser field in order to have an idea about the reduction of the corresponding TDCS when the laser field is introduced. The maximum is well located at $\theta_B = 40^\circ$ and at this value, we have for the corresponding TDCS(RPWBA) $\simeq 0.54 \times 10^{-15} \text{ a.u.}$ For this regime and in presence of a strong laser field, the non relativistic TDCSs are no longer reliable and we will focus instead on the discussion of the results obtained within the DVPWBA1 and DVPWBA2. For the DVPWBA1, we first analyzed what happens when no photon is exchanged. There is a drastic reduction of the TDCS with a maximum shifted for $\theta_B$ lower than $40^\circ$ and at this value, we have for the corresponding TDCS(DVPWBA1) $\simeq 0.5 \times 10^{-21}$
Figure 6: The RPWBA TDCS without laser field in the relativistic regime, scaled in $10^{-16}$ a.u. Once again, there is an asymmetry between the absorption and emission processes of one photon. Due to a lack of high speed computing facilities we cannot check the pseudo sumrule but we have a TDCS that increases when the number of the photons exchanged increases. For the DVPWBA2, there is also a considerable reduction of the TDCS for $s = l_B = 0$. In Fig

Figure 7: The summed TDCS (a) for an exchange of $s = \pm 50$, $l_B = \pm 50$ (b) for an exchange of $s = \pm 100$, $l_B = \pm 100$ in the relativistic regime, scaled in $10^{-22}$ a.u.

7, we compare the summed relativistic TDCS for $(s = \pm 50, l_B = \pm 50)$ and for $(s = \pm 100, l_B = \pm 100)$ where the shift of the maximum is clearly visible. We still have a complete symmetry between the absorption and emission processes.
5.3 Conclusion

In this work, we have investigated the contribution of the relativistic electronic dressing in laser-assisted ionization of atomic hydrogen by electronic impact using the Dirac-Volkov plane wave solutions to describe the incoming and the two outgoing electrons. We have worked in the binary coplanar geometry where the description of the ejected electron by a relativistic Coulomb wave function is not necessary. The influence of the laser field is taken into account to all orders in the Dirac-Volkov description of electrons and the description of the atomic target used is the analytical relativistic hydrogenic wave functions. It turns out that all the TDCSs are well peaked around a maximum angle due to the behaviour of the Fourier transforms of the relativistic hydrogenic wave functions. Symmetry between the absorption and emission processes is obtained when all electrons are described by Dirac-Volkov plane waves both for the non relativistic and relativistic regime.

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