Modelling the aerodynamic coefficients of wind turbines by using neural networks for control design purposes

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Abstract. In the present contribution, the modelling of the aerodynamic coefficients of wind turbines are obtained by using an artificial neural network (ANN) and it is analysed from the control application point of view. The obtained results show that the artificial neural network approach is appropriate to fit the data of the aerodynamic coefficients with high accuracy. The advantage of the approach is that the ANN provides an analytical equation (including its derivatives) that can be embedded in the general dynamic model, which is used for the control system design.

1. Introduction

The advanced control system design requires a well-known dynamic model of the plant, i.e. a model that captures accurately the essentially necessary dynamic behaviour of the system. In the case of three-bladed large-sized horizontal axis wind turbines, the aerodynamics is particularly difficult to be modelled maintaining the compromise between a mathematical model enough simple to facilitate the mathematical treatment required during the control system design and an acceptable level of accuracy in the representation of the machine dynamic behaviour.

The aerodynamic subsystem computes forces and moments acting on the rotor of the wind turbine depending on the wind speed, the rotor speed and the pitch angles of the blades under the consideration of the rotor design and its parameters ([44]). These dynamics is characterized by its stochastic nonlinearity and distributed parameters. Thus, the aerodynamics of the wind turbine is not particularly friendly for the application of conventional model-based control system design, which has been conceived for lineal models and lumped parameters. However, this kind of control algorithms have yielded excellent results for more than twenty years in other areas and therefore, it is desirable to find a way to apply them to the control of wind turbines.

Model-based control algorithms include some kind of optimization process either functional or parametric ([14]) that is carried out online in real time, where the objective functional to be optimized is a convex function of an analytical dynamic model of the plant. Since the optimization algorithms need the derivatives of the objective function, the dynamic model has to be differentiable, if this is not the case, derivative-free optimization methods can be used but their computational burden is very high.

The precise calculation of the aerodynamic forces and moments of a large-sized wind turbine requires the methods of the computational fluid dynamics (CFD, [31]). However, these methods are computationally very expensive to be used for real-time control. In order to overcome this problem, much effort has been done in order to find alternative solutions to this problem.

Another precise approach uses BEM theory (Blade Element Momentum, [16]). However, it is also costly in terms of computational time. A detailed implementation of this method for control purposes and the real-time analysis is given in [13]. Other more complex modelling paradigms for the aerody-
namic system behaviour, as presented in [31], have not been used for control purposes according to the best knowledge of the authors.

The most common procedure is to consider a concentrated aerodynamic moment and a thrust force acting at the centre of the hub (see e.g. [27], [41], [48]). This approach requires the aerodynamic torque coefficient \( C_p \) and the thrust force coefficient \( C_t \), which are function of the pitch angle \( \beta \) and the tip-speed ratio \( \lambda \). Both coefficients are often given by the manufacturers as numerical nonlinear tables. However, these tables are not direct useful for the optimization-based control systems because the tables have to be embedded in the control algorithms such that derivative-free optimization algorithms ([10]) can be used. This concept fails in case of a real implementation of complex control algorithms due to the high computational time.

In order to solve the problem, empiric analytical functions for the aerodynamic coefficient \( C_p \) (and indirectly for \( C_q \)) are approximated in order to obtain a model for the control system design (see e.g. [5], [35] and [33]). For the coefficient \( C_t \), there are no such functions. It is important to remark here that the derivatives \( \partial C_p / \partial \beta \), \( \partial C_p / \partial \lambda \), \( \partial C_t / \partial \beta \) and \( \partial C_t / \partial \lambda \) are also necessary. However, the derivatives of empiric fitted functions are not necessary equal to the derivatives of the real functions.

The main objective of the present work is to review the different approaches used to approximate the aerodynamic coefficients of wind turbines by using curve fitting and to propose an artificial neural network (ANN) approach to obtain a fitted analytical expression of the aerodynamic coefficients of a wind turbine. An additional advantage of an ANN is the fact that it is possible to modify the network to obtain directly the derivatives of the fitting function ([18]). As example, the well-known 5MW reference turbine of NREL ([26]) is used.

2. Simple aerodynamic model of wind turbines for control purposes

2.1. General equations

The aerodynamics of wind turbines describes the dynamic transfer from wind energy to the rotational motion of the rotor. In the simplest case, it is assumed that the rotor is an “actuator disk,” through which the wind passes at the same speed. This leads to a model, which is very oft used in the literature and it consists of the mean equations, which describes the total power, the total torque applied to the rotor and the thrust force, i.e.

\[
P = 0.5 \pi \rho R^2 C_p(\beta, \lambda)v_{\text{we}}',
\]

\[
T_r = 0.5 \pi \rho R^2 C_t(\beta, \lambda)v_{\text{we}}'
\]

\[
F_t = 0.5 \pi \rho R^2 C_t(\beta, \lambda)v_{\text{we}}'
\]

where \( \rho \), \( R \) and \( v_{\text{we}} \) are the air density, the rotor radius and the effective wind speed, respectively. \( C_p \), \( C_q \) and \( C_t \) are the power, rotor torque and thrust coefficients, which in turn depends on the pitch angle \( \beta \) and the tip-speed ratio \( \lambda = R \omega / v_{\text{we}} \). \( \omega \) is the rotor speed. Moreover, \( C_q \) is obtained as \( C_q = C_t / \lambda \).

2.2. Examples of control applications

Examples 1

The collective pitch control (CPC) with active tower damping consists in maintaining constant the rotor speed by changing the pitch angle, which in turn modifies the aerodynamic moment \( T_r \) and the thrust force \( F_t \), while the tower top for-aft oscillations are reduced by means of a tower top speed feedback, as shown in Fig.1.

**Figure 1.** Pitch control system topology for the wind turbine.
Variables $\omega_g$ and $\dot{x}_q$ are the set points for the generator speed and for the top tower speed that is represented by $\dot{x}_t$, respectively. $\beta_{pc}$ and $\beta_{adc}$ are the contributions of each controller to the final value for the pitch angle. $\beta_0$ is the value of the pitch angle corresponding to the operating point, which in turn depends on the wind speed and $\beta_0$ is the pitch angle provided by the pitch actuator. The model-based optimal control minimizes a quadratic performance index as for example

$$J = \int_0^\infty [q_1(\omega_g - \omega_q)^2 + q_2(\dot{x}_q - \dot{x}_g)^2 + r_1 \beta^2]dt,$$

(4)

where $q_1$, $q_2$ and $r_1$ are weighting parameters defined by the user. $\omega_q$ is a function of $T_a$, $\dot{x}_g$ is a function of $F_q$, which also depend on $\beta$ and $\lambda$, i.e.

$$\omega_q = f[T_a(\beta, \lambda)] \text{ and } \dot{x}_g = g[F_q(\beta, \lambda)].$$

(5)

From $dJ/d\beta = 0$, the required $\beta$ is obtained. Thus, analytical functions for $T_a(\beta, \lambda)$ and $F_q(\beta, \lambda)$ and their derivatives are needed. For this end, it is common to use (1) and (2). However, these equations require the analytical expressions of $C_q$ and $C_t$ as well as their partial derivatives, which are normally not available. This work is focused then in the way to obtain analytical expressions for $C_q$ and $C_t$.

**Examples 2**

The generator control using the approach power signal feedback (PSF) control is based on the tracking of the maximum power curves (Fig. 2).

![Generator control system for the wind turbine.](image)

The reference $T_{e,q}$ for the torque $T_e$ is computed by using (2), which in turn depends on $C_q$. In the literature (e.g. [2], [39], [24]), it is common to use (6) for this aim. However, it is shown in [15] that this approximation has a limited range of validity. Therefore, it is desired to obtain expressions with a wider applicability span.

### 3. Analytical equations for the aerodynamic coefficients

The aerodynamic coefficients $C_p$, $C_q$ and $C_t$ are normally provided at least in part by blade manufacturers in the form of 2D look-up tables. However, as it was mentioned before, a model-based control system design requires an analytical and differentiable function for the aerodynamic coefficients. In order to compute $C_p$, several empirical functions are available from the literature as a function of the pitch angle $\beta$ and the tip-speed ratio $\lambda$. Hence, $C_q$ can be obtained as $C_p/\lambda$. Although it is also possible to use a similar procedure in order to obtain $C_t$, this has not been proposed in the literature until now. In the following, the three most used empirical functions for the fitting of $C_p$ are presented.

#### 3.1. Exponential Approximation

In order to compute $C_p$, several empirical functions are available in the literature. The three most used functions are the exponential function, the trigonometric function and the polynomial function. The exponential function was proposed first in [47] and later in [5] and in [4]. The equation is generalized in [17]. In addition, many variations can be found in the literature. Hence, the formula is modified here in order to include all particular cases. Thus, the exponential approach is summarized as follow

$$C_p(\beta, \lambda) = c_1(c_2 + c_3 \beta - c_4 \beta^2 - c_5) \exp^{c_6(1/\lambda - c_7)} + c_8 \lambda,$$

(6)
with
\[
\lambda_i^{-1} = \frac{1}{c_{ii} \lambda + c_{1i} \beta} - \frac{c_{12}}{(c_{13} \beta)^3 + 1}.
\]

Some important references that use this approach are provided in Table 1. Parameters \( c_i \), for \( i = 1, \ldots, 13 \), have to be fitted for the data of a particular wind turbine if they are not set to a particular value in Table 1. \( c_1 \) and \( c_6 \) are free parameters in all references.

### Table 1. Important References for the Exponential Approach

| References | Parameters of equations (3) and (4) |
|------------|-----------------------------------|
| [5], [47]  | \( c_2 \)  \( c_3 \)  \( c_4 \)  \( c_5 \)  \( c_7 \)  \( c_8 \)  \( c_9 \)  \( c_{10} \)  \( c_{11} \)  \( c_{12} \)  \( c_{13} \) |
| [17]       | \( c_3(\lambda, \beta) \)  --  \( c_4(\lambda, \beta) \)  0  0  1  \( 1/R \)  0  0  0 |
| [4], [43], [48] | 0  --  0  0  1  \( <0 \)  1 |
| [8], [32], [42] | 2.14  0  0  1  \( <0 \)  1 |
| [6], [21], [34] | 0  --  0  #0  1  \( <0 \)  1 |
| [30]       | 0  0  --  #0  #0  1  0  0 |
| [12]       | 0  --  0  1  \( <0 \)  1 |
| [25]       | 0  --  0  #0  1  \( <0 \)  1 |
| [3]        | \( R C_f \)  0  --  0  0  0  0 |
| [46]       | 0  --  0  0  1/\lambda^2  \( <0 \)  1 |

#### 3.2. Trigonometric Approximation

In [35], a trigonometric equation has been proposed, which can be generalized as
\[
C_p = (c_1 - c_2 \beta) \sin \left( \frac{\pi(\lambda - c_4)}{(c_4 - c_5 \beta)} \right) - c_6(\lambda - c_3) \beta.
\]

This approach is also used in [1], [20] and [9]. Parameters \( c_i \), for \( i = 1, \ldots, 7 \), have the same meaning and function as the parameters of the previous subsection.

#### 3.3. Polynomial Approximation

A polynomial equation is proposed in [33], i.e.
\[
C_p(\beta, \lambda) = \sum_{i=0}^{m} c_i \beta^i \lambda^i,
\]

which is valid for \( 2<\lambda<13 \). This approximation is also used or mentioned in [12], [33] and [29]. Here, the number of parameter is \( n \times m \) (16 in many cases).

### 4. Solving the problem by using an artificial neural network approach

A neural network is a distributed processing paradigm in which the solution to a problem is learned from a set of data. The idea of artificial neural networks was initially inspired in the information processing mechanisms of the human brain. An artificial neural network consists of a group of units called “neurons”. Each neuron is connected to each other through the “weights”. Hence, ANNs are not programmed to perform specific tasks, but they can be trained by using data sets until they learn the behaviour presented to them. A review on ANNs in field of wind energy systems is given in [7].

#### 4.1. Artificial neural network formulation

The artificial neural network is a well-known concept, which has been being developed for more than thirty years. In this section, only a very short presentation of the fundamentals is introduced by the sake of completeness. An example of an ANN is illustrated in Fig. 3. The learning scheme for the curve fitting application is summarized in Fig. 4.
The output of a single neuron $j$ in the layer $k$ can be represented by

$$y^k_j = \sigma \left( \sum_{i=0}^{n_k} w^k_{ij} x^k_i + b^k_j \right) = \sigma \left( (w^k_j)^T x^k + b^k_j \right),$$

where $x$, $w$ and $b$ are the inputs to the neuron in the layer, the synaptic weights and the bias, respectively. The weights $w^k_j$ and the inputs $x_i$ can be represented in the vectors $w_j$ and $x$. Function $\sigma$ is the activation function. Assuming that every input to a neuron is the output of a neuron from the previous layer, the outputs of the layer can be written in matrix notation as

$$y^k = \sigma \left( W^k y^{k-1} + b^k \right),$$

where $W^k = (w^k_{ij})$ and $b^k = (b^k_j)$ are the weighting matrix and the bias vector of layer $k$, respectively. The first layer is computed as

$$y^1 = \sigma \left( W^1 u + b^1 \right).$$

The learning process of an ANN is the adjustment of the weights by using a learning algorithm. Many algorithms are available in the literature (see e.g. [40]). In case of functions with complicated shapes, more neurons and weights are necessary and in consequence, more computational effort is needed for finding the correct combination of weights.

A learning algorithm that is capable of handling these large learning problems is the backpropagation with momentum learning algorithm ([40]). The $n$-th correction for weight $\omega^k_j$ is given by

$$\Delta \omega^k_j(n) = -\gamma (\partial E / \partial \omega^k_j) + \alpha \Delta \omega^k_j(n-1),$$

where $\gamma$ and $\alpha$ are the learning and momentum rate, respectively. $E$ is the error function (see Fig. 4). Normally, the interest consists in accelerating the convergence to a minimum of the error function by increasing the learning rate $\gamma$ up to an optimal value. Because the backpropagation algorithm uses a gradient descendent optimization method, the activation function hast to be differentiable. One of the most studied activation function for the backpropagation algorithm is the sigmoid function defined by

$$\sigma(x) = 1 / (1 + e^{-x}).$$

4.2. Curve fitting by using artificial neural networks

The application of ANNs as a tool for curve fitting purposes is proposed in [11]. The advantage of this consists in the fact that the predefined analytical function (like those given by (3)-(6)) are not necessary. In addition, ANN can approximate nonlinear functions as well as their derivatives and the fitting goodness can be improved by increasing the number of neurons or the layers. Moreover, an ANN can be embedded as part of a dynamic model.
The combination of a multilayer perceptron with backpropagation and sigmoid activation functions was intensively studied from the convergence and approximation accuracy point of view because their universal approximation capabilities of difficult nonlinear functions ([18], [22], [23]). In addition, these networks were extended in [18], [19] and in [36] for situations where the focus attaches not only the approximation of an unknown function but also to obtain simultaneously its partial derivatives.

Although the derivatives of the aerodynamic coefficients are not included in this first study, this is clearly a need for the implementation of advanced model-based control algorithms. Hence, due to all above-mentioned advantages, this approach is adopted in this work.

4.3. Aerodynamic coefficients of wind turbines adjusted by an ANN

The ANN used for the curve fitting of the aerodynamic coefficients has two inputs (\(\beta\) and \(\lambda\)), and two outputs (\(C_q\) and \(C_t\)). In [28] and [22], it is suggested to use two hidden layers in the network topology for function approximation problems. The number of neurons \(n\) in the hidden layers depends on the desired accuracy. Thus, \(n\) was gradually increased until satisfying a tolerance of \(10^{-6}\) for all points. This condition is achieved for the example by 25 neuron per layer. If such a precision is not necessary, the number of neurons can be reduced. For training, the data sets for \(C_q\) and \(C_t\) provided in [37] are used. The implemented ANN is schematized in Fig. 5, where \(n = 25\). The high number of neurons are required due to the complex shape of \(C_t\) and therefore, it is difficult to reach a high accuracy in particular around the edges of the surface. A tool for the design and training of the ANN in order to fit the aerodynamic coefficients is currently under development (see Fig. 7).

![Figure 5. Topology of the ANN used for the estimation of the aerodynamic coefficients.](image)

5. Results

The fitting goodness for the surfaces is shown in Table 2, where it can be seen that the ANN performs better than the other approaches. The qualitative as well as the quantitative analysis shows that the ANN provides a very good fitting. The coefficient \(C_q\) is obtained from (6)-(9) by using \(C_q = C_p/\lambda\). An additional advantage of the ANN is the calculation of \(C_t\), which is not available for the other approximations. Goodness values for \(C_t\) are \(\text{SSE} = 3.24\times10^{-4}\), \(\text{R-square} = 0.9985\) and \(\text{RMSE} = 7.31\times10^{-4}\).

| Function type | SSE (ideal value: 0) | R-square (ideal value: 1) | RMSE (ideal value: 0) |
|---------------|----------------------|---------------------------|-----------------------|
| Exponential   | 0.1305               | 0.9985                    | 0.0184                |
| Trigonometric | 0.2905               | 0.9967                    | 0.0271                |
| Polynomial    | 0.2018               | 0.9977                    | 0.0229                |
| ANN           | 0.000459             | 0.9991                    | 0.00467               |

The surfaces obtained for \(C_q\) and \(C_t\) are shown in Fig. 6.
6. Conclusions

In the present contribution, the aerodynamics coefficients of a wind turbine are modelled by means of an ANN. The advantage of the approach is that no predefined function is necessary and that the obtained network can be integrated in the whole model. Results shows a good fitting, and a relative fast convergence of the training process. The next step of this project is to extend the ANN in order to obtain the partial derivatives according to [18], [19] and [36].

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