Bose-Einstein and Colour Interference in W-pair Decays

Jari Häkkinen and Markus Ringnér
Department of Theoretical Physics, Lund University,
Sölvegatan 14A, S-223 62 Lund, Sweden

Abstract
We study effects on the W mass measurements at LEP2 from non-perturbative interference effects in the fully hadronic decay channel. Based on a model for Bose-Einstein interference, which is in agreement with LEP1 data, we argue that there are no Bose-Einstein correlations between bosons coming from the different W’s. For small reconnection probabilities we rule out the possible experimental signal of colour interference at LEP2, suggested in [1]. The conclusions from this paper are that the theoretical uncertainties in the W mass determination should be smaller than the experimental statistical error.
1 Introduction

One of the main goals of LEP2 is to perform high quality precision measurements of the W mass. In order to obtain the projected statistical error of 30–40 MeV, all decay channels – the leptonic, the semi-leptonic, and the hadronic – have to be used. The purely leptonic decays will however be rare and they will not have a large impact on the measurements. In the two cases involving hadronic systems non-perturbative effects, such as colour- and Bose-Einstein interference, can occur and the measured W mass may be affected. The interference effects within the hadronic system can in the semi-leptonic case be estimated from LEP1 studies. From these studies we understand the effects of Bose-Einstein (BE) correlations quite well and we have also learnt that the colour interference (CI) effects are probably small. This means that the semi-leptonic case can be reconstructed using a Monte Carlo tuned to LEP1 data and that the theoretical uncertainties due to interference effects will only influence the fully hadronic channel. These uncertainties arise since the interference effects may have impact on the identity of the two decaying W’s. The fully hadronic channel is very nice since we can, in principle, observe all the momentum of the event. However even if LEP2 provides enough statistics for a sub 30 MeV error the interference effects have to be taken into account, or at least be under theoretical control.

That Bose-Einstein correlations might affect the measurement of the mass of the W at LEP2 was first suggested in [2]. The typical separation in space and time between the W$^+$ and W$^-$ decay vertices is smaller than 0.1 fm in fully hadronic events, i.e. $e^+e^- \rightarrow W^+W^- \rightarrow q_1\bar{q}_2q_3\bar{q}_4$, at LEP2 energies [3, 4]. Since this distance is much smaller than typical hadronic sizes and the correlation lengths associated with Bose-Einstein effects, pions from different W’s are argued to be subject to Bose-Einstein symmetrisation. The effect on the W mass has been estimated in a number of models with widely varying results [2, 5, 6]. In this paper we will based on the model in [7] argue that there are no Bose-Einstein correlations between particles stemming from different W’s at LEP2. We will also discuss the consequences of the symmetrisation for various ways of reconstructing the W mass.

Colour interference can occur in the W-pair decays at LEP2 but the probability for reconnections is unknown. In this study we use an improved Monte Carlo implementation of the model described in [1] to address the possibility to experimentally detect effects from CI at LEP2. We will also use it to estimate the effect of CI on the W mass determination.

After a short description of the various mass reconstruction schemes we use, we will in section 3 describe the important features of our interference models. We will in particular review how the correlation length in our BE model arises stressing the parts relevant to understand correlations between particles from different W’s. This is followed by the results for the reconstruction of the W mass and conclusions.

2 Mass reconstruction

If every final particle in the fully hadronic case can be uniquely and correctly assigned to either the W$^+$ or the W$^-$ decay, the W$^\pm$ four-momenta can be reconstructed and squared
to give the W$^\pm$ masses. There are however many complications which have to be taken into account in practice. It is not our intention to cover these complications here, but a detailed discussion in can be found in [3] together with a discussion about various ways to reconstruct the W mass in order to avoid complications. Reconstruction schemes are devised in [3] to study the effects of interference and we have adopted some of them in the our analysis. We will only give a brief sketch of how it is done and the reader is referred to the original work for details.

Four jet events are selected using the LUCLUS algorithm [8], with the jet distance parameter $d_{\text{join}} = 8$ GeV. This rejection of events with hard gluon jets is done since they give a much worse W mass resolution. In addition, we require the jets to have energies above 20 GeV and that the angle between any two jets is greater than 0.5 radians, to reduce the number of misassignments. The four jets can be paired in three different ways giving different results for the W mass. We use three different criteria to single out one combination.

1 : The pairs are chosen so that the deviation of the average reconstructed W mass from the used mass is minimized;

$$\min \left| \frac{M_{W^+} + M_{W^-}}{2} - M_W \right| .$$

This is not measurable in an experimental situation since we cannot know with which masses the W’s were produced, but it is included for comparison.

2 : The pairs are chosen so that the deviation of the sum of the reconstructed masses from a known nominal mass is minimized;

$$\min (|M_{W^+} - M_W| + |M_{W^-} - M_W|).$$

3 : The pairs are chosen so that the sum of their opening angles is maximized. This makes sense close to threshold where the jets from the same W should be almost back-to-back.

To investigate the effects of the interference models we compare the reconstructed W mass with interference with the reconstructed mass without interference.

3 Models

Before going into the details of our models we will shortly discuss some general features of $WW \rightarrow q_1 \bar{q}_2 q_3 \bar{q}_4$ events, which provide a motivation for some of our assumptions. As will be made clear, our models for the interference effects and in particular some of their major consequences are based upon the picture of singlet strings fragmenting. This may however not be the full story, since there could be an important non-singlet component of hadronisation, especially in the scenario when two strings are formed close to each other. The only hadronisation model which includes a non-singlet component is that of Ellis and Geiger [5]. In the case of a non-singlet component in $WW \rightarrow q_1 \bar{q}_2 q_3 \bar{q}_4$ one would expect that the
multiplicity in W-pair events is different from twice the multiplicity in single string events. This is manifested in particular in the colour reconnection scheme of Ellis and Geiger, where not only the W mass shift is much larger than in their singlet models, but it also results in a substantial reduction of the number of hadrons coming from the overlap region of the two W’s.

Three of the LEP experiments (DELPHI/L3/OPAL) have measured the mean charged hadronic multiplicity in $W^+W^- \rightarrow q_1q_2q_3q_4$ events, $\langle N_{ch}^{4q} \rangle$, and in $W^+W^- \rightarrow q\bar{q}\nu\ell$ events, $\langle N_{ch}^{\ell\nu\bar{q}q} \rangle$ [9, 10, 11]. Summarizing their results gives [12]

$$\frac{\langle N_{ch}^{4q} \rangle}{2\langle N_{ch}^{\ell\nu\bar{q}q} \rangle} = 1.04 \pm 0.03$$

which gives no support for models leading to a reduction of the hadronic multiplicity in W-pair events. This suggests that singlet strings provide a good description of $W^+W^- \rightarrow q_1q_2q_3q_4$ hadronisation.

### 3.1 Colour interference at LEP2 energies

The CI model in this paper is an improved Monte Carlo implementation of the model described in [1]. The model for recoupling is quite simple and its features are described in detail in [1]. Here we give a summary of the model with emphasis on the improvements.

The space–time distance between the W decay points in $e^+e^- \rightarrow W^+W^- \rightarrow q_1\bar{q}_2q_3\bar{q}_4$ is about $1/\Gamma_W$ and hard gluons with energies above $\Gamma_W$ are therefore emitted incoherently by the two quark systems early in the event [13]. This means that there are two sets of partons before any possible colour interference can occur. The two sets $q_1g_1g_2\ldots g_n\bar{q}_4$ and $q_3g'_1g'_2\ldots g'_m\bar{q}_4$ have a lot of different recoupling possibilities since every set of particles $q\ldots g$ is a colour-triplet. Recoupling of a $q\ldots g$ with any $g\ldots \bar{q}$ from the other set can occur with the probability $1/N_c^2$ so the total probability for recoupling can in principle be very large. The estimation of the total recoupling probability is non-trivial. In [1] a discussion is made about what kind of probabilities to expect. No real conclusion was or can be made, and the probability remains a free parameter of the model.

Perturbative QCD favours states which correspond to short strings i.e. parton states which produce few hadrons. The $\lambda$ measure was introduced in [14] and is a measure of the effective rapidity range inside which the decay products of a particular colour-singlet string are distributed. In this way it is related to the multiplicity. In [1] it is argued that states with smaller $\lambda$’s could be dynamically enhanced, and that this choice also gives reconnected events that differ most from non-reconnected systems. Reconnected states with the smallest $\lambda$ measure are therefore chosen in the model.

All of this is still true in the CI model in this paper. We have however made significant improvements in the MC implementation. The Ariadne MC v4.08 [15] allows the user to stop the production of gluons below some given energy. This feature was not available in the original work, where gluons with energy below $\Gamma_W$ where simply neglected (leading to a 3% loss of energy). Furthermore, the W-pairs were incorrectly generated in the original work since no spin information was preserved and the W’s were therefore allowed to decay
isotropically. In order to take the full angular correlations into account we now use Pythia v5.7, where the full $2 \rightarrow 2 \rightarrow 4$ matrix elements are included for the W-pair production and decay.

These improvements will lead to consequences for the results obtained in [1]. In addition to studying possible experimental signals at LEP2 of recoupled events we also extend the analysis of [1] to study CI effects on W mass determination.

### 3.2 Bose-Einstein correlations in W-pair production

A model for Bose-Einstein correlations based upon a possible quantum-mechanical framework for the Lund Fragmentation Model [16] has been proposed [17] and it has been extended to the multi-particle correlations needed at LEP energies [7]. An important feature of the model is that it can be used as an extension of the probability based Lund Model, implementing the correlations as event weights.

The interpretation of the Lund Fragmentation Model in [7] gives an explicit form for the transition matrix element for a string fragmenting into hadrons. The resulting matrix element depends only on the space-time history of the string and the model therefore uniquely predicts the relative amplitudes for different particle configurations, and therefore also the magnitude of the Bose-Einstein effect. To understand how the correlation length between pions arise in the model we will in the following shortly discuss the basic steps leading to the specific form of the transition matrix element.

A unique breakup rule for a string can be derived inside the Lund Model, which results in the following probability for a string to decay into hadrons $(p_1, \ldots, p_n)$,

$$dP(p_1, \ldots, p_n) = \prod_i (Nd p_i \delta(p_i^2 - m_i^2)) \delta(\sum_j p_j - P_{tot}) \exp(-bA) \quad \text{(2)}$$

where $A$ is the space-time area spanned by the string during its break-up into q$\bar{q}$-pairs, and $N$ and $b$ are two free parameters.

The production of hadrons from a single string in the Lund Model can be given a quantum mechanical interpretation inside a non-Abelian field theory. The transition matrix element, $M$, can, to obtain the result in Eq (2) be identified with (note the similarity with Fermi's golden rule)

$$M = \exp(i\xi A) \quad \text{with} \quad \xi = \frac{1}{2\kappa} + \frac{ib}{2} \quad \text{(3)}$$

where the decay surface area, $A$, is in energy-momentum units in the light-cone metric. The imaginary part of the quantity $\xi$ is related to the pair production probability. As discussed in [7] the phase for $M$, as given by the real part of $\xi$, is found by observing how gauge invariance will constrain the production of q$\bar{q}$-pairs along the colour force fields. The main observation is that a final state hadron stems from a q from one vertex and a $\bar{q}$ from the adjoining vertex. This implies that in order to keep gauge-invariance it is necessary that the production matrix element contains at least a gauge connector, $\exp(i g f_{j+1}^{j+1} A^\mu dx_\mu)$, between the two vertices, denoted $j$ and $j+1$. The total matrix amplitude for a single string must
contain at least one gauge connector for each hadron and we get a Wilson Loop Operator as a minimal requirement for gauge invariance

\[ \mathcal{M} = \exp(ig \oint A^\mu dx^\mu) \]  

where the integral is around the decay surface of the string. Using Wilson’s confinement criteria for the behaviour of such a loop operator we get the real part of \( \xi \), as in Eq (3).

The transverse momentum generation will also contribute to the total matrix element. This contribution is discussed in detail in [7] and is found to be

\[ \propto \exp\left(-\frac{1}{4\sigma^2}k_\perp^2\right) \]  

where \( \pm k_\perp \) are the compensating transverse momenta generated in a \( q\bar{q} \)-vertex and \( \sigma \) is the width of the Gaussian suppression of the quarks transverse momenta.

In order to see the main mechanism for BE-correlations in the Lund Model we consider Fig 1, in which two of the produced hadrons, denoted (1, 2), are assumed to be identical bosons and the state in between them is denoted \( I \). There are two ways to produce the entire state, corresponding to exchange of the two identical bosons. The two configurations, \((\ldots, 1, I, 2, \ldots)\) and \((\ldots, 2, I, 1, \ldots)\), are shown in the figure and in general they correspond to different areas \( A \).

The area difference, \( \Delta A \), depends not only on the energy momentum vectors \( p_1 \) and \( p_2 \), but also on the four-momentum of the intermediate state, \( p_I \). The difference can be written as

\[ \frac{\Delta A}{2\kappa} = \delta p \delta x \]  

Figure 1: The two possible ways, \((1, I, 2)\) and \((2, I, 1)\), to produce the entire state when 1 and 2 are identical bosons. The space-time area, \( A \), spanned by the string during its break-up is shaded.
where $\delta p = p_2 - p_1$ and $\delta x = (\delta t, 0, 0, \delta z)$ is a reasonable estimate of the space–time difference, along the string surface, between the production points. This means that the correlation length, which is being measured by the four-momentum difference between pairs, is in the model dynamically implemented as $\delta x$. The correlation length is therefore not the direct distance between production points. Instead it is the distance along the string surface, i.e. the distance along the colour force field. This is not surprising if we consider how the quantum-mechanical process corresponding to the Lund Model was derived; to keep gauge-invariance we got a gauge-connector between adjacent vertices and this is what provides us with the $A/(2\kappa)$ factor in the matrix element, from which the correlation length in the model stems.

In the case of production of two strings, i.e. a $q_1\overline{q}_2q_3\overline{q}_4$ system, there is no reason for a gauge-connector between vertices belonging to different strings. We will therefore assume that the distance along the gauge-field between them is infinite even though the direct space–time distance may be very small. This implies that there is no interference between production vertices belonging to different strings. This means that in this model each string can be considered a system of its own, with separate Bose-Einstein effects. The resulting event weight is then of course the product of the weights for each system separately. In [7] it is explained how the BE interference can be incorporated in a probabilistic event generation scheme by weighting the produced events. In particular using the amplitudes Eq (3) and Eq (5) results in the weight

$$w = \prod_{n=1}^{2} \left( 1 + \sum_{P_n' \neq P_n} \frac{\cos \Delta A_n}{2\kappa} \frac{b\Delta A_n}{\cosh \left( \frac{b\Delta A_n}{2} + \frac{\Delta(\sum_{q} p_{\perp q}^2)}{2\sigma_{p\perp}^2} \right)} \right)^2$$

(7)

for a fully hadronic WW event, where $\Delta$ denotes the difference with respect to configurations $P_n$ and $P_n'$ of the string $n$ and the sum of $p_{\perp q}^2$ is over all the vertices of string $n$. We have introduced $\sigma_{p\perp}$ as the width of the transverse momenta for the generated hadrons, (i.e. $\sigma_{p\perp}^2 = 2\sigma^2$).

It should be emphasized that if only colour-singlet combinations of partons are allowed to be formed there is no model consistent way to get correlations between particles stemming from the different W’s. In comparison to most of the models using event weights to implement BE-correlations [3] we have a physical picture of how the correlation length in our model arises and it describes data well in single string fragmentation [7]. Taken together with our previous discussion of multiplicities in WW events this supports our conclusion that there are no correlations between particles from the different W’s.

4 Results

All the results are for W-pairs generated at 170 GeV. We have checked the effects of our models on the mean charged multiplicity and the results are shown in Table 1. We get
Table 1: The mean charged multiplicity for the two interference models. For the CI model we show the results for 100% recoupled events and for an admixture of recoupled and non-recoupled events of the order 10%. The lower multiplicities for the BE results are due to that no parton cascade has been used in this case.

small effects on the mean multiplicity and they are compatible with the experimental result, Eq (1).

4.1 Colour interference results

It is natural to divide the CI results into two independent parts. First we discuss the possibility to detect signals of CI at LEP2 in the same way as it was done in [1] and then we will study mass reconstruction effects.

4.1.1 CI signal search at LEP2

The search for signals of CI at LEP2 in [1] give numbers which are very close to what will be statistically significant with the expected number of events from LEP2, when a 10% recoupling probability is assumed. The improvements made here will dilute the signal proposed in [1]. Multiplicity distributions (including \( \pi^0 \)) in the central rapidity region, \(|y| < 0.5\), for recoupled and non-recoupled events are shown in Fig 2. Comparing these with the original results from [1] we note that the signal-to-background ratio is significantly reduced. In Table 2 we have compiled the number of events without particles in a central rapidity region at LEP2 using two different thrust cuts and an expected 5000 fully hadronic events. We note that the signal decreases and if this signal is to be seen at LEP2 there must
be a larger recoupling probability than 10%. A larger recoupling probability would increase the number of events without particles in the central rapidity bin. A closer examination of the improvements of the MC implementation in this paper reveals that the conservation of energy will not change the result too much from the original work. Almost all of the suppression of the signal comes from taking the anisotropy of the W decays into account.

From this study we conclude that the statistics from LEP2 will make it hard to use the signal proposed in [1].

4.1.2 W mass reconstruction results

We have studied the effects of CI on the W mass measurement to estimate the size of the theoretical error on the mass implied by our model.

In Fig 3 we show the generated W mass and the reconstructed masses with and without CI interference. We see that the difference between the reconstructed distributions is small. The mass shifts for 100% reconnected events are shown in Table 3 and if the reconnection probability is assumed to be 10% the shifts should be scaled down with a factor 10. $\Delta M$ denotes the mass shift due to the reconstruction method as compared with the generated W mass and the additional shift due to the interference is denoted by $\delta M$.

| Method | $\Delta M$ [MeV] | $\delta M$ [MeV] |
|--------|------------------|-----------------|
| 1      | -279 ± 15        | -3 ± 21         |
| 2      | -1238 ± 19       | -90 ± 27        |
| 3      | -75 ± 16         | -27 ± 23        |

Table 3: Shifts in the reconstructed W masses using the different methods from Section 3. $\Delta M$ denotes the mass shift due to the reconstruction method and $\delta M$ denotes the additional shift due to the colour interference.
Assuming a 10% reconnection probability the shifts will be small and negligible from the experimental mass reconstruction point of view. However, in a worse case scenario with a 100% probability the shifts can be quite large but the experimental signal suggested in [1] would then on the other hand be observable.

4.2 Bose-Einstein interference results

We have studied how the inclusion of Bose-Einstein correlations, implemented as event weights, affect the results from various mass reconstruction schemes. The main concern in [2] was that the BE effects in the hadronisation stage can couple identical particles from the W⁺ and W⁻. They used the LUBOEI algorithm [2] in which the momenta of the produced bosons are reshuffled to reproduce a chosen BE-correlation. The momenta are then rescaled by a common factor to keep energy-momentum conservation for the event as a whole. This procedure might result in a redistribution of momenta in such a way that the hadrons which come from the W⁺(W⁻) decay don’t add up to the same invariant mass as the original W⁺(W⁻) had. It should be noted that the rescaling procedure needed afterwards introduces shifts in the W mass even if there are no BE-correlations between particles stemming from different W’s. After corrections for this ‘spurious’ mass shift a shift of about +100 MeV at 170 c.m. energy was found in [2].

The main feature of our model is that we don’t expect a coupling between particles coming from different W’s. The inclusion of correlations may however affect for example multiplicities and event shape variables and therefore it may affect the reconstruction of the W boson mass. Such an artificial mass shift is hoped to be taken into account by the tuning of the JETSET MC [8] to the experimental LEP1 data. Using the MC implementation of our model, we have tuned multiplicity distributions and some event shape variables to the cor-
responding results as obtained from JETSET for a single string at LEP1 energies. To study the effect of the symmetrisation we have then analysed and compared the reconstructed W mass of W’s generated by Pythia with and without symmetrisation included. We have used our tuning to LEP1 energies for the symmetrised events. The events are generated without a parton cascade, i.e. pure $q_1\bar{q}_2q_3\bar{q}_4$ events, in this analysis since the MC implementation of the BE-model has not been extended to general parton configurations. We believe that the inclusion of gluons will affect the mass reconstruction, but it will do it in the same way whether BE symmetrisation is included or not.

The use of event weights introduces statistical fluctuations which require the generation of many events. We have generated a sufficient number of events in order to get reasonable statistical errors for the mass distributions. In Fig 4 we show the reconstructed mass for symmetrised and non-symmetrised events. As can be seen the difference between the distributions is very small.

![Figure 4: The distribution of the reconstructed W mass with (diamonds) and without (solid) Bose-Einstein symmetrisation turned on. The results are for reconstruction method 2.](image)

| Method | $\Delta M \ [MeV]$ | $\delta M \ [MeV]$ |
|--------|-----------------|-----------------|
| 1      | -306 ± 1        | -6 ± 8          |
| 2      | -83 ± 1         | -5 ± 7          |
| 3      | -601 ± 4        | -6 ± 6          |

Table 4: Shifts in the reconstructed W masses using the different methods from Section 3. $\Delta M$ denotes the mass shift due to the reconstruction method and $\delta M$ denotes the additional shift due to the Bose-Einstein symmetrisation.

Using the same notation for the mass shifts as in Section 4.1.2 we have compiled the results for the different reconstruction schemes in Table 4. The mass shifts due to BE interference are all very small and compatible with zero. They will therefore not affect
the LEP2 measurement and in particular we conclude that the inclusion of Bose-Einstein correlations will compared to a carefully tuned conventional Monte-Carlo not affect the reconstruction of the W mass. It is important to note that using event weights can in principle affect the W mass even though we don’t have any interference between the two W’s. This is however not the case with our model.

5 Conclusions

The previous work on the effects of BE correlations on the W mass, with the exception of [18], are all based on the observation that the BE effect packs identical particles closer together. The local model [2] as well as the global event weight models [3, 4] are all phenomenological models used to estimate the influence of such a close-packing on the masses of the two quark systems, if the two W systems cross-talk. Our model starts from a completely different point of view, i.e. with a quantum mechanical scenario for the particle production dynamics, and at LEP1 energies the results obtained with our model are in agreement with the observables on which the other models are based. A natural consequence of our model is that we do not expect any cross-talk due to BE effects between the W’s. The correlations between pions from different W’s have been investigated by two of the LEP experiments. The DELPHI experiment has at their present level of statistics found no enhancement of the correlations between pions from different W’s, compared to what is expected from a pair of uncorrelated W’s [19] (confirmed in [9]) and ALEPH draws a similar conclusion from their data [20]. Their statistics are rather poor but if the results are confirmed when more data becomes available, it would rule out mass shifts due to cross-talk between the two W’s, in agreement with our model.

The reconnection probability of our CI model, as in other models, remains a free parameter. Assuming a moderate probability of 10% the mass shift due to CI will be very small. If we however assume a 100% probability the mass shift can be important, but in this case the experimental signal of [1] should be visible. The magnitude of the signal is a measure of the reconnection probability in our model, and if the signal is found it can be used to estimate the theoretical uncertainty in the mass determination.

To summarize, we conclude that neither colour nor Bose-Einstein interference is expected to affect the W mass reconstruction at LEP2 and in particular that the theoretical uncertainties, as estimated by our models, are much smaller than the expected experimental statistical error.

References

[1] G. Gustafson, J. Härkinen, Z. Phys. C64, 659 (1994)

[2] L. Lönnblad and T. Sjöstrand, Phys. Lett. B351, 293 (1995)

[3] T. Sjöstrand and V.A. Khoze, Z. Phys. C62, 281 (1994)
[4] T. Sjöstrand and V.A. Khoze, *Phys. Rev. Lett.* **72**, 22 (1994)

[5] J. Ellis and K. Geiger, *Phys. Rev* **D54**, 1967 (1996)

[6] S. Jadach and K. Zalewski, *Acta. Phys. Pol. B* **28**, 1363 (1997)

V. Kartvelishvili, R. Kvadze and R. Møller, *e-print, hep-ph/9704424* (1997)

K. Fialkowski and R. Wit, *e-print, hep-ph/9709205* (1997)

[7] B. Andersson and M. Ringnér, *LU TP 97-07* and *hep-ph/9704383* (1997)

[8] T. Sjöstrand, *Comp. Phys. Comm.* **82**, 74 (1994)

[9] DELPHI Coll., * Contribution to the EPS conference, Jerusalem, EPS-HEP/97-307, DELPHI 97-84 CONF 70* (1997)

[10] L3 Coll., * Contribution to the EPS conference, Jerusalem, EPS-HEP/97-814, L3 NOTE 2134* (1997)

[11] OPAL Coll., *CERN-PPE/97-116* (1997)

[12] D.R. Ward, *Talk given at the EPS conference, Jerusalem* (1997)

[13] Yu.L. Dokshitzer, V.A. Khoze, L.H. Orr, W.J. Stirling, *Nucl. Phys. B* **403**, 65 (1993)

T. Sjöstrand, V.A. Khoze, *Z. Physik C* **62**, 281 (1994)

[14] B. Andersson, P. Dahlqvist, G. Gustafson, *Phys. Lett. B* **214**, 604 (1988), *Z. Phys. C* **44**, 455 (1989)

[15] L. Lönnblad, *Comp. Phys. Comm.* **71**, 15 (1992)

[16] B. Andersson, G. Gustafson, G. Ingelman and T. Sjöstrand, *Phys. Rep.* **97**, 31 (1983)

[17] B. Andersson and W. Hofmann, *Phys. Lett. B* **169**, 364 (1986)

[18] Š. Todorova-Nová and J. Rameš, *hep-ph/9710280* (1997)

[19] DELPHI Coll., *Phys. Lett. B* **401**, 181 (1997)

[20] ALEPH Coll., * Contribution to the EPS conference, Jerusalem, EPS-HEP/97-590* (1997)