Formation of Vortex Clusters and Giant Vortices in Mesoscopic Superconducting Disks with Strong Disorder

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Abstract. Merged, or giant, multi-quanta vortices (GVs) appear in very small superconductors near the superconducting transition due to strong confinement of magnetic flux. Here we present evidence for a new, pinning-related, mechanism for vortex merger. Using Bitter decoration to visualise vortices in small Nb disks, we show that confinement in combination with strong disorder causes individual vortices to merge into clusters/GVs well below \( T_c \) and \( H_{c2} \), in contrast to well-defined shells of individual vortices found in the absence of pinning.

1. Introduction
Mesoscopic superconductors exhibit complex and unique vortex structures due to the competition between surface superconductivity and vortex-vortex interactions [1-6]. For mesoscopic disks, theoretical studies found two kinds of superconducting states: a giant vortex (GV), i.e., a circular symmetric state with a fixed value of angular momentum that can carry several flux quanta [1,2] and multivortex states (MVS) with an effective total angular momentum corresponding to the number of vortices in the disk (vorticity \( L \)) [3]. Recently, it became possible to experimentally distinguish between a single-core GV and a MVS composed of singly quantized vortices using the multiple-small-tunnel-junction method [4]. GVs have also been inferred from an experimental observation that vortices in perforated Al films merge into large flux spots close to the superconducting transition temperature \( T_c \) [7]. In another recent experiment [5], Bitter decoration was used to directly visualise MVS in small Nb disks and circular symmetry was found to lead to the formation of concentric shells of vortices, similar to electron shells in atoms. Analysis of different vortex configurations revealed the

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rules (“periodic law”) of shell filling and also “magic-number” configurations corresponding to commensurability between the shells [6,8].

Here we show that, while weak pinning in mesoscopic disks only leads to distortions in symmetric shell configurations, the presence of strong disorder changes the situation dramatically. Using Bitter decoration to visualise vortices in small disks etched in the surface of Nb single crystals and numerical simulations to analyse the role of disorder, we found [9] that the interplay between confinement and pinning results in the formation of clusters or even GVs in relatively large samples and at temperatures well below $T_c$, due to selective enhancement of pinning strength by disks’ boundaries. Depending on the disk’s size and the applied field, either all vortices merge into a single cluster/GV (typically for $L$ ≥ 3 to 6) or clusters/GVs coexist with singly quantised vortices, for larger $L$. No clustering/GV formation was found for the same disorder in macroscopic samples.

2. Experimental observation of clustering/GV formation

Samples for this study were prepared from bulk Nb single crystals using e-beam lithography and reactive etching ($\text{CF}_4$ with added oxygen) followed by high-vacuum annealing (at $<10^{-8}$ torr) at temperatures ~ 750ºC, to remove gaseous contaminants. This produced large arrays of disks - 0.5 to 1.4 µm high, of 2 to 5µm diameters - on the surface of ~ 0.5mm thick Nb crystals (see upper-right inset in Fig. 1a). The array geometry and decoration details were the same as in [5], i.e., a whole array of over 300 disks was decorated in each experiment after field cooling to ~1.8K, allowing us to obtain simultaneous snapshots of up to a hundred vortex configurations in nominally identical disks. To assess the degree of disorder before and after annealing, the samples were decorated straight after etching, and then after annealing for $t = 1$, 2, 4 and 6 hours, and magnetisation and resistivity were measured at $T_c$ and bulk pinning, consistent with the presence of solute oxygen atoms in interstitial positions [10,11]. By varying the annealing time we were able to achieve partial removal of solute oxygen thereby changing the degree of disorder (for details, see [9]). Accordingly, no vortices were observed either in the bulk of the crystals or inside the disks for $t < 2h$, as contamination rendered the samples non-superconducting at relevant $T_c$, but annealing for $t ≥ 2h$ recovered superconductivity over the entire crystals, with the amount of disorder gradually decreasing with increasing $t$ – see Fig. 1b,c. The disordered and very inhomogeneous vortex distribution in Fig. 1b is consistent with the presence of many pins with sizes $≥ ξ$ [12]. Longer $t$ resulted in a homogeneous, although still disordered, vortex structure (Fig. 1c), very similar to that in macroscopic Nb films from our previous study [5].

Fig. 1a (bottom-right inset) shows a typical vortex configuration in disks with weak disorder. Clearly, the effect of vortex confinement in single-crystal-based disks is very similar to that in individual thin-film disks studied in [5], i.e., confinement has a mitigating effect on disorder in that, despite the presence of weak pinning, vortices form nearly perfect shell configurations [e.g., state (1,6,13) in Fig. 1a]. Furthermore, both systems show identical diamagnetic response (see $Φ/Φ_0$ vs $L$ curves in Fig. 1a, where $Φ = H S$ is the flux through the disk’s area $S$, $H$ the applied magnetic field and $Φ_0$ the flux quantum), same vortex shell states are observed over corresponding intervals of $Φ/Φ_0$, etc. Details of the observed behavior will be published elsewhere. The only important implication for the present study is that the effect of vortex confinement in single-crystal-based disks is identical to that in individual thin-film disks, even though in the former case only a small fraction of the total vortex length is subject to the additional interaction with the surface. Moreover, the response was found to be independent of the disk’s height, in agreement with theoretical predictions [13]. In contrast, in the case of strong disorder (as in Fig. 1b), instead of mitigating pinning as above, confinement was found to selectively enhance the effect of pins, ultimately leading to merger of vortices into clusters/GVs, as demonstrated below. The most striking features of vortex configurations in this case are, firstly, that some of the “vortices” inside the disks appear to be significantly larger than vortices in the bulk or in weak-disorder disks and, secondly, that there are on average noticeably fewer vortices in these disks, compared to the number expected for a given value of $Φ/Φ_0$ or found in weak-disorder samples, i.e., they show a stronger diamagnetic response and a significant variation in the observed value of $L$ for
the same $\Phi/\Phi_0$, as demonstrated in the main panel of Fig. 1a. Indeed, a wide range of $L$ values is observed for any given $\Phi/\Phi_0$ (e.g. $L = 2-6, 8$, and $9$ for $\Phi/\Phi_0 = 17-19$ while only $L = 12$ and $13$ are found for this flux interval in weak-disorder disks) and even the maximum observed $L$ is 25-30\% lower than for the weak-pinning disks, indicating that strong disorder somehow facilitates expulsion of extra vortices. The two left insets of Fig. 1a show vortex images observed in the same experiment ($H = 70$ Oe) in identical 3µm disks. Vortices in the bottom image form a slightly distorted shell configuration (1,7) while a disordered pattern of only 5 vortices is seen in the second disk of exactly the same area, with one of the vortices having a ~ 2.5 times larger diameter compared to the rest. Indeed, identifiable shell configurations containing identical (small) vortices were seen only rarely in

![Figure 1](image)

Figure 1. (a) Main panel: Diamagnetic response of single-crystal-based disks with weak (○) and strong (●) disorder; data for individual thin-film disks from Ref. [5] included for comparison (◇). Solid lines are guides to the eye. Top-left and bottom-right insets show vortex configurations in disks with strong and weak disorder, respectively ($H = 70$ Oe). Top-right inset: vortices in and around a 1.4µm high disk decorated after field-cooling in $H = 60$ Oe. (b,c) Vortex distributions in the bulk of Nb crystals with strong and weak disorder, respectively (After Ref. [9]).

these samples while most disks contained a combination of small and large vortices in a disordered pattern or just a single large vortex. Fig. 2 shows typical observations for 2, 3 and 5 µm disks at $\Phi/\Phi_0$ corresponding to different $L_{\text{max}}$ (i.e., maximum number of vortices found in disks with only small, singly-quantized, vortices). For $L_{\text{max}}=3$ most disks contained 2 or 3 standard-size vortices (top image), i.e., the diameter of a cluster of Fe particles ‘decorating’ a vortex was the same as for vortices in the bulk and in weak-disorder disks, but a significant proportion of disks showed only one vortex of about 80\% larger diameter (bottom image). For $L_{\text{max}} = 9$ only 10\% of the disks showed configurations of small vortices (top image) while most of the rest contained a combination of several small and 1 or 2 large vortices (as in bottom image) and a few disks showed only one very large vortex, as in the bottom image for $L_{\text{max}} = 11$. As the vorticity increased above $L_{\text{max}} = 11$, very few disks contained only small vortices while a typical configuration was a combination of small vortices and a number of large vortices of several different diameters, as in the image for $L_{\text{max}} = 36$. 


3. Numerical simulations and discussion

To understand the nature of large vortices observed in disks with strong disorder, we recall that there is an excellent correlation between the size of vortex images in decoration experiments and the ‘magnetic’ size of individual vortices, $\propto \lambda$ [14]. This allows us to identify the observed large vortices as several singly quantized vortices merged into GVs or clusters with vorticity $L^* \geq 2$ (as explained below). Bitter decoration does not allow us to distinguish between a single-core GV and a multi-core cluster. We emphasise that, although the presence of strong disorder appears to be the necessary condition for the formation of GVs/clusters, they are only formed in small disks and were never observed in the bulk of the same crystals (away from the disk arrays). To estimate $L^*$ associated with different clusters/GVs we analysed intensity profiles of many different vortex images [9]. We distinguish between the results obtained on disks containing only individual, only merged, or a combination of individual and merged vortices: (i) the size of individual vortices is well defined and shows only small variations between different disks and/or the bulk; (ii) merged vortices show only few typical sizes (rather than a continuous distribution), with the same sizes found in disks with only one merged vortex (where the vorticity can be determined fairly unambiguously from $L$ vs $\Phi/\Phi_0$ curves) and in disks containing combinations of individual and merged vortices. This allowed us to identify the number of vortices merged in different clusters/GVs, $L^*$ [9].

To model the role of pinning in a confined geometry theoretically, we place a superconducting disk of thickness $\delta$ and radius $R$ in a perpendicular external field $H$. The forces of vortex interactions with each other $f_{\text{vv}}$ and with the shielding currents and the edge $f_{\text{ve}}$ can then be modelled as [14,15] $f_{\text{vv}} = f_0 \Sigma_{i<k,L} \{|(\mathbf{r}_i - \mathbf{r}_k)| - r_i^2(r_i^2 r_k - r_k^2) r_k^2 r_i - r_i^2 |\}$ and $f_{\text{ve}} = f_0 \{1/(1 - r_i^2) - h\} r_i$, where $h = \pi R \mu_0 H/\Phi_0 = (H/2H_c)(R/\xi)^2$, $r_i = \rho_i / R$ is the position of the $i$th vortex, $L$ the vorticity, and $f_0 = 4\pi \lambda_0^2 H_c^2 / R$ the unit of force. Our numerical approach is based on the Langevin dynamics algorithm. Then the overdamped equations of motion become: $\eta \mathbf{v}_i = \mathbf{f}_i^{\text{vv}} + \mathbf{f}_i^{\text{ve}} + \mathbf{f}_i^{\text{vf}} + \mathbf{f}_i^{\text{f}}$. Here $\mathbf{f}_i^{\text{vv}}$ is the force due to vortex-pin interactions (see, e.g., [16,17]) which is modelled by short-range parabolic potential wells located at positions $\mathbf{r}_k^{(p)}$; $\eta$ is the viscous Bardeen-Stephen friction, which we take $\eta = 1$. The pinning force is $f_i^{\text{ve}} = \Sigma_{k=L,B} (f_p / l_p) |(\mathbf{r}_i - \mathbf{r}_k^{(p)})| \Theta (r_p - |\mathbf{r}_i - \mathbf{r}_k^{(p)}|) r_k^{(p)}$, where $N_p$ is the number of pinning sites, $f_p$ the maximum pinning force for each potential well, $r_p$ the pinning range (for a strong disorder some pinning centres can overlap forming a complex potential energy profile), $\Theta$ the Heaviside step function, and $\mathbf{r}_k^{(p)} = (\mathbf{r}_k - \mathbf{r}_p^{(p)})/|\mathbf{r}_k - \mathbf{r}_p^{(p)}|$. The $\mathbf{f}_i^{\text{f}}$ is the thermal stochastic force satisfying: $\langle \mathbf{f}_i^{\text{f}}(t) \rangle = 0$ and $\langle \mathbf{f}_i^{\text{f}}(t) \mathbf{f}_j^{\text{f}}(t') \rangle = 2\eta \delta_{ij} \delta(t - t') k_b T$. The ground state of the system is obtained by simulating field-cooled experiments.

Figure 2. Vortex configurations in disks with strong disorder for different values of $L_{\text{max}}$ (see text). $L^*$ is the vorticity of GVs/clusters estimated from their diameters (After Ref. [9]).
We now consider a disk containing, e.g., 8 vortices in a field $h = 15$ (for $R = 1.5 \mu m$, this corresponds to $H \approx 45Oe$). For a perfect disk (no pinning), vortices form a symmetric two-shell configuration (1,7) (cf. [5]) as shown in Fig. 3a. A weak pinning site placed near the disk’s centre distorts the symmetric configuration, moving away the central vortex; the rest of the vortices adjust themselves accordingly (Fig. 3b). However, the situation changes dramatically if $f_p$ is increased: (i) a stronger pin traps 3 out of 8 vortices resulting in vortex merger into a cluster with $L = 3$ coexisting with individual vortices (Fig. 3c), similar to the experimental pattern shown in Fig. 3. A stronger still pin traps 5 out of 8 vortices (Fig. 3d). Furthermore, we find that the ability of a pin to trap vortices depends on its position inside a mesoscopic disk: a pin close to the disk’s centre traps more vortices than the same pin near the boundary (see Fig. 3e,f), i.e., its pinning strength is enhanced when it is near the centre. (ii) The total vorticity $L$ in a disk with strong pins is lower than in the weak-pinning situation because vortices are repelled by the cluster stronger than by a single vortex; (e,f) $f_p/f_0 = 8$, $r_p = 0.32$: (e) the pinning site close to the boundary; combination of merged vortices with $L=7$ and $L=3$ individual vortices; (f) enhancement of the pinning strength of a pinning site near the centre: $L=8$ vortices are merged, while other vortices are expelled from the disk by strong repulsion from the cluster.

Figure 3. Vortex patterns for $L = 8$ (a-d) and $L = 10$ (e,f) in mesoscopic superconducting disks for different pinning potentials: (a) no pinning, perfect (1,7) state; (b) weak pinning, $f_p/f_0 = 2$, $r_p = 0.08$, distorted (1,7) state; the range of the pinning potential is shown by red dashed line; (c,d) strong pinning, $f_p/f_0 = 4$, $r_p = 0.16$ and $f_p/f_0 = 6$, $r_p = 0.24$, respectively; combination of merged and individual vortices; the latter are repelled by the cluster stronger than by a single vortex; (e,f) $f_p/f_0 = 8$, $r_p = 0.32$: (e) the pinning site close to the boundary; combination of merged vortices with $L=7$ and $L=3$ individual vortices; (f) enhancement of the pinning strength of a pinning site near the centre: $L=8$ vortices are merged, while other vortices are expelled from the disk by strong repulsion from the cluster.

Figure 4. (a,b) Vortex patterns for $L = 20$; formation of multiquanta vortices in disks with $f_p/f_0 = 10$, $r_p = 0.16$ and different $r_{min}$: (a) $r_{min} = 0.048$, vortices form clusters with $L = 5$ (near centre) and $L = 4$ (out-of-centre); (b) $r_{min} = 0.064$, GV's with $L = 5$ (near centre) and $L = 4$ (out-of-centre); (c) the Cooper pair density, $|\Psi|^2$, in a disk with $R = 20\xi$, for $H=0.2H_{c2}$, for two pinning sites with $U_0 = 1$, and $w = 5\xi$; (d) the phase pattern, corresponding to (c), showing a GV with $L = 5$ and a cluster with $L = 3$.
vortex cores strongly overlap [19]. To demonstrate formation of GVs in disks with strong pinning, we introduced a pinning potential \( U_{\text{pin}}(\rho) \) in the dimensionless GL equations (see, e.g., [17]): 
\[
\Psi = \Psi [1 - U_{\text{pin}}(\rho)] - \Psi \frac{\partial \Psi}{\partial r},
\]
where \( U_{\text{pin}} = U_0 \exp (-\rho/w) \), and \( \rho = \sqrt{(x-x_{\text{displ}})^2 + (y-y_{\text{displ}})^2} \). The results of calculations of \( |\Psi|^2 \) and \( \phi \) are shown in Fig. 4a,b for a disk with \( R = 20\xi \) and pins with \( U_0 = 1 \) and \( w = 5\xi \) at the centre and out-of-centre, respectively. The Cooper pair density \( |\Psi|^2 \) vanishes at pin positions, forming large spots (larger than individual vortices): The central spot is a GV with vorticity \( L = 5 \), while the one at the distance of a half of disk’s radius is a cluster consisting of \( L = 3 \) individual vortices. Thus, depending on the pinning strength, range and \( r_{\text{min}} \) (e.g., \( r_{\text{min}} > 50 \, \text{nm} \) for disks with \( R = 1 \, \mu\text{m} \), \( f_p/f_0 = 10 \), \( r_p = 0.16 \), as shown in Fig. 4a,b, where the lengths are in units of \( R \)), vortices can form either clusters (Fig. 4a) or GVs with \( L = 5 \) (centre) and \( L = 4 \) (out-of-centre), Fig. 4a. We note that the above pinning forces are of the same order of magnitude as \( f_p \) estimated for normal inclusions in Nb [20] with size \( \geq 5\xi \), with largest pins (>100nm) having \( f_p \) and \( r_p \) sufficient to form a true GV.

4. Conclusions
We present the first direct observation of pinning-induced merger of vortices in small superconducting disks with strong disorder. The interplay between confinement and disorder is shown to result in the enhancement of pinning so that up to 8 vortices can merge into a single cluster or even a GV, which can coexist with individual vortices.

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