Transport properties of a two-dimensional electron gas strongly coupled to light

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We show theoretically that the strong coupling of a two-dimensional electron gas (2DEG) to a dressing electromagnetic field drastically changes its transport properties. Particularly, the dressing field leads to the giant increase of conductivity (which can reach thousands of percents), results in nontrivial oscillating dependence of conductivity on the field intensity, and suppresses the weak localization of 2DEG. As a consequence, the developed theory opens an unexplored way to control transport properties of 2DEG by a strong high-frequency electromagnetic field. From experimental viewpoint, this theory is applicable directly to quantum wells exposed to a laser-generated electromagnetic wave.

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Introduction. — Transport properties of two-dimensional electron gas (2DEG) in nanostructures exposed to a high-frequency electromagnetic field have been studied in the deep past and taken deserved place in textbooks (see, e.g., Refs. [1–3]). However, the most attention in previous studies on the subject was paid to the regime of weak light-matter coupling. Following the conventional terminology of quantum optics, in this regime an electron energy spectrum is assumed to be unperturbed by photons. Correspondingly, the weak electron-photon interaction results only in electron transitions between unperturbed electron states, which are accompanied by absorption and emission of photons. As a consequence, the regime of weak electron-photon coupling in solids leads to photovoltaic effects, high-frequency conductivity and other well-known electronic transport phenomena which are accompanied with absorption of field energy by conduction electrons. However, the interaction between electrons and a strong electromagnetic field (the regime of strong light-matter coupling) cannot be described as a weak perturbation. In this case, the system “electron + electromagnetic field” should be considered as a whole. Such a bound electron-photon system, which was called “electron dressed by photons” (dressed electron), became a commonly used model in modern physics [4, 5]. For a long time, the main objects for studying the physical properties of dressed electrons were atoms and molecules. The field-induced modification of the energy spectrum and wave functions of dressed electrons — the so-called dynamic Stark effect — was discovered in atoms many years ago [6] and has been studied in details in various atomic and molecular systems. These studies of strong electron-photon processes formed up such an exciting field of modern physics as quantum optics [4, 5]. In nanostructures, the research activity in the area of quantum optics was focused on exciton-polaritonic effects in microcavities with quantum wells [7–12] and quantum dots [13–16], physical properties of dressed electrons in graphene [17, 18] and quantum wires [19], variety of technological applications [20], including novel types of the lasers [21, 22], optical switches and logic gates [23–25], all-optical integrated circuits [26] and others. As to transport properties of dressed 2DEG, they are still waiting for detailed research. The present Letter is aimed to fill partially this interdisciplinary gap in the modern physics, which takes place at the border between physics of nanostructures and quantum optics.

Model. — For definiteness, we will restrict our consideration to the case of 2DEG with a parabolic electron energy spectrum

$$
\varepsilon_k = \frac{\hbar^2 k^2}{2m},
$$

where \( k \) is the electron wave vector, and \( m \) is the electron effective mass. Let the 2DEG be subjected to a plane monochromatic electromagnetic wave propagating perpendicularly to the 2DEG plane (see Fig. 1). In what follows, we will assume that the wave frequency, \( \omega \), meets two conditions. Firstly, the wave frequency is far from resonant electron frequencies corresponding to interband electron transitions and, therefore, the interband absorption of the wave by the 2DEG is absent. Secondly, the wave frequency is high enough in order to satisfy the inequality

$$
\omega \tau_0 \gg 1,
$$

where \( \tau_0 \) is the electron relaxation time in an unirradiated 2DEG. It is well-known that the intraband (collisional) absorption of wave energy by conduction electrons is negligibly small under condition (2) (see, e.g., Refs. [22, 23]). Thus, the considered electromagnetic wave can be treated as a purely dressing (nonabsorbable) field. It follows from the basic principles of quantum optics that the strong coupling of electrons to such a dressing field leads to the renormalization of all physical quantities describing the electrons [1, 5]. Particularlly, it was recently shown that
the dressing electromagnetic field can strongly affect the scattering of conduction electrons [29]. Namely, the scattering probability of a dressed electron between electron states with wave vectors \( \mathbf{k} \) and \( \mathbf{k}' \) per unit time has the form [29]

\[
w_{k'k} = \frac{2\pi}{\hbar} J_0^2 (f_{k'k}) |U_{k'k}|^2 \delta(\varepsilon_{k'} - \varepsilon_k),
\]

where \( J_0(z) \) is the zeroth order Bessel function of the first kind, and \( U_{k'k} \) is the matrix element of the scattering potential, \( U(r) \), which arises from macroscopically large number of scatterers in the conductor. In the case of linearly polarized dressing field [29], the argument of the Bessel function is given by

\[
f_{k'k} = \frac{\epsilon E_\omega (k - k')}{m\omega^2},
\]

where \( E_\omega \) is the amplitude of the electric field of the wave, \( \mathbf{E}(t) = E_\omega \sin \omega t \). In the case of circularly polarized dressing field [30], this argument is

\[
f_{k'k} = \frac{2E_\omega \epsilon k}{m\omega^2} \sin \left( \frac{\theta_{k'k}}{2} \right),
\]

where \( \theta_{k'k} = (\mathbf{k'}, \mathbf{k}) \) is the angle between electron wave vectors \( \mathbf{k} \) and \( \mathbf{k}' \). The formal difference between the scattering probability of dressed electron [3] and the conventional expression for the scattering probability of bare electron [31] consists in the Bessel-function factor \( J_0^2 (f_{k'k}) \), where \( f_{k'k} \) depends on the dressing field amplitude \( E_\omega \) and the dressing field frequency \( \omega \) [see Eqs. (2)–(4)]. Just this factor results in nontrivial dependence of electronic transport properties on the dressing field. In what follows, we will focus our attention on the conductivity and the weak localization of dressed 2DEG.

![FIG. 1: (Color online) Sketch of the system under consideration: The 2DEG dressed by (a) linearly polarized high-frequency field \( \mathbf{E} \) and (b) circularly polarized one in the presence of a stationary in-plane electric field \( \mathbf{E} = (E_\parallel, E_\perp) \).](image)

**Conductivity of dressed 2DEG.** — Assuming the temperature to be zero, let us apply a stationary (dc) electric field \( \mathbf{E} \) to the 2DEG. It follows from the conventional Boltzmann equation for conduction electrons (see, e.g., Ref. [32]) that electric current density, \( \mathbf{J} \), is given by the well-known expression

\[
J = \frac{e^2}{2\pi^2} \int \frac{d^2 k}{[E \cdot v(k)]} \tau(k) v(k) \delta(\varepsilon_k - \varepsilon_F) d^2 k,
\]

where \( v(k) = (1/\hbar) \nabla_k \varepsilon_k \) is the electron velocity, \( \varepsilon_F = \hbar^2 k_F^2/2m \) is the electron Fermi energy, and \( \tau(k) \) is the relaxation time. In the most general case of anisotropic electron scattering, this relaxation time is given by the equation [33]

\[
\frac{1}{\tau(k)} = \sum_{k'} \left[ \frac{1 - \frac{\tau(k') E \cdot v(k')}{\tau(k) E \cdot v(k)}}{\tau(k) E \cdot v(k')} \right] w_{k'k}.
\]

Since a high-frequency field satisfying inequality [29] cannot be absorbed by conduction electrons, it does not change the electron distribution on wave vectors \( \mathbf{k} \) [29]. Therefore, the dressing field influences on the stationary (dc) transport of dressed electrons only through the renormalization of the scattering probability \( w_{k'k} \). Substituting the scattering probability of dressed electron (3) into Eq. (7), we can obtain from Eqs. (2)–(7) the conductivity of dressed 2DEG, \( \sigma_{ij} = J_i/E_j \).

To simplify calculations, let us consider the electron scattering within the \( s \)-wave approximation [31], where the matrix elements \( U_{k'k} \) do not depend on the angle \( \theta_{k'k} = (\mathbf{k'}, \mathbf{k}) \). Substituting Eq. (5) into Eqs. (3) and (6)–(7), we arrive at the isotropic conductivity of 2DEG dressed by circularly polarized field, \( \sigma_c \), which is given by the expression

\[
\frac{\sigma_c}{\sigma_0} = 2\pi \left[ \int_{0}^{2\pi} [1 - \cos \theta] J_0^2 \left( \frac{2E_\omega \epsilon k_F}{m\omega^2} \sin \frac{\theta}{2} \right) d\theta \right]^{-1},
\]

where \( \sigma_0 \) is the conductivity of an unirradiated 2DEG. In the case of linearly polarized dressing field, the conductivity tensor \( \sigma_{ij} \) has a simple diagonal form, \( \sigma_{ii} = (\sigma_{||}, \sigma_{\perp}) \), in the basis of two axes which are parallel (\( || \)) and perpendicular (\( \perp \)) to the wave field \( \mathbf{E} \), respectively. Substituting Eq. (4) into Eqs. (3) and (6)–(7), we arrive at

\[
\frac{\sigma_{||}}{\sigma_0} = \frac{1}{\pi} \int_{0}^{2\pi} \frac{\tau(\theta)}{\tau_0} \cos^2 \theta d\theta
\]

and

\[
\frac{\sigma_{\perp}}{\sigma_0} = \frac{1}{\pi} \int_{0}^{2\pi} \frac{\tau(\theta)}{\tau_0} \sin^2 \theta d\theta
\]

where \( \tau(\theta) \) and \( \tau_0(\theta) \) are the relaxation time [7] at the Fermi energy for the dc electric field \( \mathbf{E} = (E_{||}, 0) \) and \( \mathbf{E} = (0, E_{\perp}) \), respectively (see Fig. 1a), and \( \theta = (\mathbf{k}, \mathbf{E}) \). The conductivities of irradiated 2DEG (8–10), which can be easily calculated numerically, are plotted in Fig. 2. It is seen that the irradiation of 2DEG by a dressing light results in giant increase of conductivity, which can reach thousands of percents. Physically, this increase follows from the fact that the dressing field significantly decreases the scattering probability of dressed 2DEG [3]. From the mathematical viewpoint, this is a consequence of the rapid decrease of the Bessel function in Eq. (3)
versus the dressing field amplitude $E_{\omega}$. The difference between the conductivities $\sigma_0$ and $\sigma_{\perp}$ arises from the scattering anisotropy of a 2DEG dressed by a linearly polarized field, which follows directly from Eqs. (15)–(16).

As to the oscillations of the plotted conductivity, this is a formal consequence of the oscillating behavior of the Bessel function in the probability (3).

**Weak localization of dressed 2DEG.** — Multiple scattering of electrons in a conductor leads to the self-interference of electron waves propagating along a closed trajectory in mutually opposite — clockwise and counterclockwise — directions. As a result of the interference, the well-known weak localization (WL) of conduction electrons appears [34, 35]. To find the WL correction to the conductivity of dressed 2DEG, $\Delta \sigma$, one needs to consider all possible closed electron trajectories in the 2DEG irradiated by a dressing electromagnetic field. Going this way, the sought correction can be calculated using the conventional expression [34]

$$\frac{\Delta \sigma}{\sigma_0} = \frac{-\hbar}{m} \int_{\tau_F} C(t)dt, \quad (11)$$

where $C(t)$ is the probability to find an electron in the initial point of its trajectory at the time $t$ averaged over 2DEG plane, $\tau_F$ is the mean free time of conduction electron at the Fermi energy, which is given by the expression

$$\frac{1}{\tau_F} = \sum_{k_F} \tilde{w}_{k_F} k_F^2, \quad (12)$$

and $\tau_F$ is the effective breaking-time of electron phase. Generally, electron phase can be broken by both inelastic scattering and a magnetic field which destroy WL [37–42]. The breaking-time of electron phase caused by a magnetic field, $B$, is $\tau_{FB} = \hbar/4eDB$, where $D$ is the diffusion constant [42]. Correspondingly, the effective breaking-time of electron phase, $\tau_F$, can be written as $1/\tau_F = 1/\tau_{\varphi e} + 1/\tau_{\varphi B}$, where $\tau_{\varphi e}$ is the characteristic breaking-time arisen from inelastic processes. As to the mean free time of conduction electron, $\tau_F$, it can be found by substituting the scattering probability of dressed electron (3) into Eq. (11). It has been mentioned above that this scattering probability is anisotropic for 2DEG dressed by a linearly polarized field. Therefore, the time $\tau_F$ depends on the initial electron wave vector $k_F$. To describe this anisotropy, it is suitable to introduce the two mean free times: $\tau_{F||}$ and $\tau_{F\perp}$ for the cases of $k_F || \mathbf{E}$ and $k_F \perp \mathbf{E}$, respectively. Substituting these two times into Eq. (11), we arrive at the two WL corrections, $\Delta \sigma_{||}$ and $\Delta \sigma_{\perp}$. These two corrections describe the conductivity of dressed 2DEG for the cases of $\mathbf{E} = (E_||, 0)$ and $\mathbf{E} = (0, E_\perp)$, respectively (see Fig. 1a). Certainly, in the case of circularly polarized dressing field, the anisotropy vanishes and the corresponding WL correction, $\Delta \sigma_e$, does not depend on direction in the 2DEG plane.

Within the conventional theory of WL, the probability $C(t)$ in Eq. (11) can be described by the Feynman path integral [35, 43]

$$C(t) = \int D\mathbf{x}D\mathbf{y} \exp \left[ -\int_{-t/2}^{t/2} \left( \frac{\dot{x}^2}{4D_x} + \frac{\dot{y}^2}{4D_y} \right) dt + \frac{i}{\hbar} (\mathbf{r}(t') - \mathbf{r}(-t')) dt' \right], \quad (13)$$

where the integration should be performed over closed electron trajectories per unit plane of 2DEG, and $\mathbf{r} = (x, y)$ is the electron radius-vector in the 2DEG plane. In the integrand of the path integral (13), the first two terms in the exponent describe the kinetic energy of the diffusion propagation of a dressed electron along the path, where $D_x, y$ are the diffusion constants along the $x, y$ axes. Generally, these constants can be written as $D_x = v_F^2 \tau_{F||}/2$ and $D_y = v_F^2 \tau_{F\perp}/2$ (see, e.g., Ref. [42]), where $v_F$ is the Fermi velocity, and the $x$ axis is assumed to be directed along the electric field $\mathbf{E}$ of linearly polarized dressing field. In the case of circularly polarized dressing field, we have $\tau_{F||} = \tau_{F\perp}$ and, therefore, the $x, y$ axes can be chosen arbitrary. The last term in the exponent takes into account the potential energy of an electron in the dressing field and describes the self-interference of the dressed electron between the time-reversed trajectories. Performing the path integration in Eq. (13) within the conventional procedure [44] and substituting the obtained probability (13) into Eq. (11), we arrive at the WL corrections to the conductivity of dressed 2DEG, which are presented in Fig. 3.

It is seen in Fig. 3 that the irradiation of 2DEG by a
dressing field leads to decreasing WL corrections to the conductivity. Physically, this is a consequence of suppressed scattering in a dressed 2DEG, which has been mentioned above. As a result of the suppression, the probability of electron movement along a closed trajectory in the dressed 2DEG decreases and, correspondingly, WL corrections to the conductivity of dressed 2DEG decrease as well. It should be stressed that this effect differs significantly from the known suppression of WL due to intraband absorption of irradiation by conduction electrons [34–37]. Indeed, under the condition (2) there is no absorption of high-frequency field by conduction electrons. Therefore, the destruction of WL, which arises from the breaking of electron phase in inelastic electron-photon processes [45–47], is negligibly small in the 2DEG under consideration. As a consequence of decreasing WL effects, the influence of a magnetic field on WL corrections to the conductivity for a dressed 2DEG is less than one for a bare 2DEG (see the insert in Fig. 3). It should be noted that the WL correction to the conductivity of 2DEG has a logarithmic dependence on a magnetic field [34]. Although the irradiation on the 2DEG reduces WL corrections, the behavior of conductivity of dressed 2DEG in a magnetic field obeys the same law.

FIG. 3: (Color online) The dependence of weak localization corrections to conductivity of 2DEG on intensity of irradiation by a dressing field. Calculations are performed for 2DEG in a GaAs quantum well at the temperature $T = 0$ for $\varepsilon_F = 10$ meV and $\tau_{\text{ee}} = 80$ ps. The insert demonstrates the dependence of the corrections on magnetic field directed perpendicularly to the 2DEG in the presence of the irradiation with the intensity $I = 3$ mW/cm$^2$.

Conclusion.— We demonstrated theoretically that the strong coupling of 2DEG to a dressing electromagnetic field results in a set of such unexpected transport phenomena as a giant increase of conductivity, oscillating behavior of conductivity versus intensity of dressing field and suppression of weak localization effects. These phenomena open a new way to control the transport properties of 2DEG by a strong high-frequency electromagnetic field and, therefore, form a basis for novel optoelectronic nanodevices.

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SUPPLEMENTAL MATERIAL: SCATTERING OF 2DEG DRESSED BY A CIRCULARLY POLARIZED ELECTROMAGNETIC FIELD

For definiteness, we will restrict our consideration to the case of a two-dimensional electron system subjected to a circularly polarized electromagnetic wave propagating perpendicularly to the system. Let the system lie in the plane $(x, y)$ at $z = 0$ and the wave propagate along the $z$ axis. Then the electron properties of the system in the absence of scatterers are described by the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}_0 \psi$$

(14)

with the Hamiltonian $\hat{H}_0 = [\hat{p} - eA(t)]^2/2m$, where $\hat{p} = (\hat{p}_x, \hat{p}_y)$ is the operator of electron momentum, $m$ is the effective electron mass, $e$ is the electron charge, $A(t) = e_x(E_\omega/\omega) \sin \omega t + e_y(E_\omega/\omega) \cos \omega t$ is the vector potential of the wave, $E_\omega$ is the electric field amplitude of the wave, $\omega$ is the frequency of the wave, and $e_{x,y}$ are the unit vectors along the $x, y$ axis. The Schrödinger equation (14) can be solved accurately and the exact wave function of the electron is

$$\psi_k(r, t) = \exp \left[ -i \left( \frac{\varepsilon_k t}{\hbar} + \frac{E_\omega^2 e^2 t}{2m \omega^2 \hbar} - \frac{E_\omega e k_y}{m \omega^2} \sin \omega t \right. \right.$$

$$\left. + \left. \frac{E_\omega e k_x}{m \omega^2} \cos \omega t \right) \right] \varphi_k(r),$$

(15)

where $\varphi_k(r) = V^{-1/2} \exp(i kr)$ is the plane electron wave, $k = (k_x, k_y)$ is the electron wave vector, $r = (x, y)$ is the electron radius-vector, $V$ is the normalization volume, and $\varepsilon_k = \hbar^2 k^2/2m$ is the energy spectrum of a free electron. Evidently, the wave function (15) can be easily verified by direct substitution into the Schrödinger equation (14). Introducing the polar system $\{ k, \varphi \}$, we can write the electron wave vectors as $k_x = k \cos \varphi$ and $k_y = k \sin \varphi$. Then the wave function (15) takes the simplest form

$$\psi_k(r, t) = \exp \left[ -i \left( \frac{\varepsilon_k t}{\hbar} + \frac{E_\omega^2 e^2 t}{2m \omega^2 \hbar} \right. \right.$$

$$\left. + \frac{E_\omega e k_x}{m \omega^2} \cos(\omega t + \varphi) \right) \right] \varphi_k(r).$$

(16)
Let an electron move in a scattering potential $U(r)$ in the presence of the same field. Then the wave function of the electron, $\Psi(r, t)$, satisfies the Schrödinger equation

$$i\hbar \frac{\partial \Psi(r, t)}{\partial t} = [\hat{H}_0 + U(r)]\Psi(r, t).$$

(17)

Assuming the scattering potential energy $U(r)$ to be a small perturbation, we can apply the conventional perturbation theory to describe the electron scattering. Since the functions (16) with different wave vectors $k$ form the complete function system for any time $t$, we can seek solutions of the Schrödinger equation (17) as an expansion

$$\Psi(r, t) = \sum_{k'} a_{k'}(t) \varphi_{k'}(r, t).$$

(18)

Let an electron be in the state (16) with the wave vector $k$ at the time $t = 0$. Correspondingly, $a_{k'}(0) = \delta_{k', k}$, where $\delta_{k', k}$ is the Kronecker symbol. In what follows, we will assume that the wave frequency $\omega$ is large enough to satisfy the inequality

$$\omega \tau_0 \gg 1,$$

(19)

where $\tau_0$ is the characteristic relaxation time of conduction electron in the absence of the wave. Under the condition (19), we can neglect the absorption (emission) of field energy by a scattered electron (see, e.g., Refs. [27, 28]). Within this approximation, the scattering potential $U(r)$ mixes only electron states $k$ and $k'$ with the same energy, $\varepsilon_k = \varepsilon_{k'}$. Therefore, it is enough to take into account only terms with $k' = k$ in the expansion (18). Substituting the expansion (18) into the Schrödinger equation (17) and restricting the accuracy by the first order of perturbation theory, we arrive at the expression

$$a_{k'}(t) = -i \frac{U_{k'k}}{\hbar} \int_0^t e^{i(\varepsilon_{k'} - \varepsilon_k)t'/\hbar} e^{i f_{k'k} \sin(\omega t' + (\varphi + \varphi')/2)} dt',$$

(20)

where

$$U_{k'k} = \langle \varphi_{k'}(r) | U(r) | \varphi_k(r) \rangle$$

(21)

is the matrix element of the scattering potential,

$$f_{k'k} = \frac{2E_{\omega k}k}{m\omega^2} \sin \left( \frac{\theta}{2} \right),$$

(22)

and $\theta = \varphi - \varphi' = (k, k')$ is the scattering angle. Let us apply the Jacobi-Anger expansion,

$$e^{iz \sin \gamma} = \sum_{n = -\infty}^{\infty} J_n(z) e^{in\gamma},$$

in order to rewrite Eq. (20) as

$$|a_{k'}(t)|^2 = \frac{|U_{k'k}|^2}{\hbar^2} \sum_{n = -\infty}^{\infty} J_n(f_{k'k}) e^{i(\varepsilon_{k'} - \varepsilon_k + n\hbar\omega)t/2\hbar} \times e^{in(\varphi + \varphi')/2} \int_{-t/2}^{t/2} e^{i\omega t'/\hbar} dt'.$$

(23)

where $J_n(z)$ is the n-th order Bessel function of the first kind. Since the integrals in Eq. (23) for long time $t$ turn into the delta-function

$$\delta(\varepsilon) = \frac{1}{2\pi \hbar} \lim_{t \to \infty} \int_{-t/2}^{t/2} e^{i\varepsilon t'/\hbar} dt',$$

the expression (23) takes the form

$$|a_{k'}(t)|^2 = 4\pi^2 |U_{k'k}|^2 \sum_{n = -\infty}^{\infty} J_n^2(f_{k'k}) \delta(\varepsilon_{k'} - \varepsilon_k + n\hbar\omega).$$

(24)

To transform the square delta-functions in Eq. (24), we can apply the conventional procedure,

$$\delta^2(\varepsilon) = \delta(\varepsilon) \delta(0) = \frac{\delta(\varepsilon)}{2\pi \hbar} \lim_{t \to \infty} \int_{-t/2}^{t/2} e^{i\varepsilon t'/\hbar} dt' = \frac{\delta(\varepsilon)t}{2\pi \hbar}.$$

Keeping in mind that $\varepsilon_k = \varepsilon_{k'}$, the probability of the electron scattering between the states (15) with the wave vectors $k$ and $k'$ per unit time, $w_{k'k} = d|a_{k'}(t)|^2/dt$, is given by

$$w_{k'k} = \frac{2\pi}{\hbar} J_0^2(f_{k'k}) |U_{k'k}|^2 \delta(\varepsilon_{k'} - \varepsilon_k).$$

(25)

Since Eq. (22) depends only on the electron energy $\varepsilon_k = \hbar^2 k^2/2m$ and the scattering angle $\theta$, the probability (25) describes the isotropic scattering which can be tuned by the wave amplitude $E_\omega$ and the wave frequency $\omega$.

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