Natural and unnatural parity resonance states in positron–hydrogen scattering

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Abstract
We present an investigation of resonances with natural and unnatural parities in positron scattering with atomic hydrogen. The complex scaling method has been used. Resonance states for natural parity \( \pi = -1 \) with total angular momenta \( J = 0 - 2 \) and unnatural parity \( \pi = -J^{1} \) with \( J = 1, 2 \) are calculated. Resonance energies and widths are reported and compared with other theoretical calculations.

Keywords: positron–hydrogen scattering, positronium, resonances

1. Introduction
Over the past few decades, the study of resonance phenomena in positron–hydrogen systems has gained interest [1–8]. These resonances could play an important role in the production of anti-hydrogen atoms through rearrangement scattering of positronium (Ps) by anti-protons [10]. This system consists of three particles: (1) a positron, (2) an electron, and (3) a proton. There is no truly bound state of this system, but there exist many quasi bound states that lie in the \( e^+H \) scattering continuum. A number of theoretical methods have been used to find the positions and widths of the resonances in \( e^+H \) scattering, even though there are no experimental observations [10–12]. The majority of these results are limited to S-, P- and D-wave resonances. Ho and Yan [4, 8] comprehensively analyzed high-partial-wave resonances in positron–hydrogen scattering using the complex-coordinate rotation method. They used Hylleraas type wave functions, in which the coordinates of the positronium atom were included explicitly.

Both resonances with natural parity \( \pi = -1 \), where \( J \) is the total angular momentum of the system, and resonance states with unnatural parity \( \pi = -J^{1} \) are of interest. The latter type of states can exist because the angular momenta of the electron and positron, \( l_1 \) and \( l_2 \), couple to the total angular momentum \( J \) according \( |l_1 - l_2| \leq J \leq l_1 + l_2 \). While the parity is \( \pi = -J^{1} \), Thus, if \( l_1 \) and \( l_2 \) are both larger than 0, \( J \) can take both even and odd values for any \( l_1 \) and \( l_2 \). Since \( l_1 \) and \( l_2 \) are both non-zero these states are, to first order, stable against \( e^+e^- \) annihilation, which is blocked by the centrifugal barrier. A few theoretical works have investigated \( e^+H \) resonances with unnatural parity [16–18]. Again using Hylleraas functions and complex scaling, Yan and Ho [16] reported an examination of resonances with unnatural parities in a positron–hydrogen system. Shimamura et al [17] also investigated unnatural parity resonance states in the positron–hydrogen system using hyper-spherical close coupling calculations. Since ground state H is an S-state, unnatural parity states can only be accessed in positron scattering with an excited state of hydrogen, or excited Ps colliding with a proton [8].

The present study reports results for resonance states with natural parity for S-, P- and D-states and unnatural parity for \( P_e \) and \( D_o \) state using explicitly correlated Gaussian wave functions. In addition to the resonances calculated in [16, 17] we also identify the unnatural parity resonances below Ps \((n = 4)\) which we have not found in previous literature.

2. Method
The three-body systems were calculated using the coupled rearrangement channels method developed by Kamimura and co-workers [20, 21]. In this method the wave function is expanded using Jacobi coordinates \( \{r_i, R_i\}, i = 1, 2, 3 \) in all three possible rearrangement channels. This gives a very
The three rearrangement channels of the $e^+H$ system and their Jacobian coordinates. The second rearrangement channel above corresponds for large $R_e$ to positronium-like states, while the third channel corresponds to hydrogen-like states.

Versatile basis set, capable of adapting to states close to breakup thresholds of any pair of particles. We also note that by using Jacobi coordinates we automatically include all so-called mass-polarization terms which, in atomic calculations using coordinates centred on the nuclei, arise as corrections which are non-trivial to calculate.

Within this coordinate system we represent the wave function using a partial wave expansion of the angular variables and Gaussians in the radial variables. That is, for a state with total orbital angular momentum $J$, $M$ the wave function has the form

$$\Psi_{JM} = \sum_{a,l_1=1}^{3} \sum_{l_2=0}^{l_{\text{max}}} \sum_{l_3=0}^{l_{\text{max}}} \sum_{l_{\text{max}}}^{l_{\text{max}}} \phi_{a,l_1l_2l_3} \hat{\phi}_{a,l_1l_2l_3} \chi_{a,l_1l_2l_3} \chi_{M,l_1l_2l_3},$$

(1)

$$\phi_{a,l_1l_2l_3} = N_{a,l_1l_2l_3} \sum_{i_1=0}^{l_{\text{max}}} r_{a,l_1l_2l_3} R_{a,l_2l_3} e^{-i(i_1/a_{l_1l_2l_3})r} e^{-iR_{a,l_1}r} \times \left[ Y_{a,l_1} (\hat{r}_a) \otimes Y_{a,l_2} (\hat{R}_a) \right]_{JM}.$$  

(2)

Here $a$ denotes the three rearrangement channels, $l_a$ and $L_a$ are the angular momenta along $\mathbf{r}_a$ and $\mathbf{R}_a$, respectively, and $i, I$ numbers the Gaussians along the two radial coordinates (see figure 1). The angular momenta $l_a$ and $L_a$ are chosen consistent with the total $J$ (i.e. $|l_a - L_a| \leq l \leq l_a + L_a$), up to some maximum values $l_{\text{max}}$ and $L_{\text{max}}$, which may be different for different rearrangement channels.) The total number of Gaussian trial functions for each rearrangement channel and angular momentum are given by $l_{\text{max}}$ and $L_{\text{max}}$. $N_{a,l_1l_2l_3}$ is a normalization constant ensuring that $\left\langle \phi_{a,l_1l_2l_3} | \phi_{a,l_1l_2l_3} \right\rangle = 1$.

The widths of the Gaussians $r_{a,l_1}$ and $R_{a,l_2l_3}$ are, for each channel and set of angular momenta, chosen as geometric progressions

$$r_{a,l_1} = r_{a,l_1} \left( \frac{r_{a,l_1}^{\text{max}}}{r_{a,l_1}} \right)^{\frac{l_1}{l_{\text{max}}}},$$

$$R_{a,l_2l_3} = R_{a,l_2l_3} \left( \frac{R_{a,l_2l_3}^{\text{max}}}{R_{a,l_2l_3}} \right)^{\frac{l_2l_3}{l_{\text{max}}}}.$$

(3)

where the smallest and largest values $r_{a,l_1}^{\text{min}}, r_{a,l_1}^{\text{max}}, R_{a,l_2l_3}^{\text{min}}, R_{a,l_2l_3}^{\text{max}}$ are set explicitly and used as non-linear variational parameters. In this way most Gaussians will span the short-to medium range, while a few more diffuse Gaussians capture the long range part of the wave function. In this work, resonances with very small binding energies are calculated, and hence it was essential to set a large enough value for the outer radius $R_e$.

Resonances are calculated using the complex scaling method [22–25]. The complex dilation operator $U(\theta)$ acting on a function $f(\mathbf{r})$ is defined through

$$U(\theta) f(\mathbf{r}) = e^{i\theta/2} f\left( e^{i\theta/2} \mathbf{r} \right),$$

(4)

where the exponential prefactor ensures that the complex scaled function satisfies the normalization condition

$$\int U(\theta) f^n(\mathbf{r}) U(\theta) f(\mathbf{r}) d^3r = 1.$$  

The corresponding transformation of the Hamiltonian is, for the special case $V \propto 1/r$,

$$H(\theta) = U(\theta) H U^{-1}(\theta) = e^{-i\theta/2} H + e^{-i\theta} V.$$

(5)

Stationary eigenvalues of the complex scaled generalized eigenvalue problem

$$H(\theta) \tilde{c}^\theta = E \tilde{S}(\theta) \tilde{c}^\theta, \quad \tilde{S}(\theta) = \left\langle \psi_f | H(\theta) | \psi_i \right\rangle,$$

(6)

correspond to complex energies $E = E_r - i\Gamma/2$ of the system, where $E_r$ is the resonance energy and $\Gamma$ the width.

3. Results and discussion

3.1. The $e^+H$ system with natural parity

The resonance positions and widths found in the $e^+H$ system with natural parity are summarized for S-, P- and D-waves in tables (1–3). Most of the resonances are comparable with those found in literature, which are reproduced in these tables. Our results were obtained using 7000 Gaussians basis function with different possible combinations for angular momentum, with maximum values $l_{\text{max}} = l_{\text{max}}^{\text{max}} = l_{\text{max}}^{\text{max}} = 4$ used for S-, P- and D-waves. A larger basis with more flexibility better represents the inter-particle correlation and often leads to lower resonance energies (though strictly speaking the variational principle does not apply to resonances). The thresholds in the tables were calculated using the exact reduced mass.

S-wave resonances in positron hydrogen scattering below the $H(n = 2)$ threshold are shown in table 1. For $H(n = 2)$ we found two resonances, which both agree very well with earlier calculations. For $Ps(n = 2)$ we found four resonances both for infinite and real proton mass. In the latter case, a resonance with $E = -0.062 737$ au and width $\Gamma = 1.10 \times 10^{-5}$ au has not been previously reported. Similarly, for other states, we conclude that our results for resonances in $e^+H$ is good to at least about $10^{-5}$ atomic units for lower $n$ states but the uncertainty is larger for large $n$ in both physical and infinite proton mass calculations. Most of the resonances are similar to previous literature values, few of them have better values and some resonances not previously reported are also presented in table 1. The resonances from
Table 1. S-wave resonance energies and widths in e^4H system (a.u). Left-hand side ($^4E$) of the table shows the physical proton mass results and right-hand side ($^\infty E$) for infinite mass. The notation $x[y]$ means $10^x$.

| Threshold | This work $^4E$ | This work $^\infty E$ | This work $^\infty \Gamma$ | Literature $^4E$ | Literature $^\infty E$ | Literature $^\infty \Gamma$ | reference |
|-----------|----------------|-----------------------|---------------------------|----------------|-----------------------|---------------------------|-----------|
| H(n = 2), | -0.128 623 3.33[-5] | -0.128 623 3.33[-5] | [4] | -0.128 687 3.34[-5] | -0.128 687 3.34[-5] | [8] |          |
| -0.124 932 | -0.125 132 2.50[-6] | -0.125 131 2.40[-6] | [7] | -0.125 200 2.50[-6] | -0.125 200 2.0 [-6] | [8] |          |
| Ps(n = 2), | -0.075 140 1.67[-4] | -0.075 150 1.56[-4] | [7] | -0.075 159 1.67[-4] | -0.075 169 | |          |
| -0.062 500 | -0.065 830 8.0[-5] | -0.065 830 8.15[-5] | [4] | -0.065 838 8.12[-5] | -0.065 852 8.12[-5] | [12] |          |
| -0.063 387 | -0.063 387 2.48[-5] | -0.063 387 2.4[-5] | [4] | -0.063 390 2.50[-5] | -0.063 403 2.52[-5] | [12] |          |
| -0.062 737 | 1.10[-5] | — | — | -0.062 738 1.09[-5] | -0.062 735 7.9 [-6] | [12] |          |
| H(n = 3), | -0.058 059 2.86[-4] | -0.058 047 3.21[-4] | [7] | -0.058 061 3.04[-4] | -0.058 076 | |          |
| -0.055 525 | -0.056 034 6.40[-5] | -0.056 03 7.0[-5] | [4] | -0.056 064 6.48[-5] | -0.056 068 | |          |
| -0.055 571 | 9.45[-5] | — | — | -0.055 583 7.02[-5] | — | |          |
| H(n = 4), | -0.038 536 2.50[-5] | -0.038 532 2.83[-5] | [4] | -0.038 548 2.52[-5] | -0.038 55 2.38[-5] | [12] |          |
| -0.031 233 | -0.033 942 2.80[-5] | -0.033 928 2.4[-5] | [4] | -0.033 956 2.86[-5] | -0.033 951 2.39[-5] | [12] |          |
| -0.032 294 | 1.29[-5] | -0.032 341 2.17[-5] | [4] | -0.032 307 8.52[-6] | -0.032 312 8.13[-6] | [12] |          |
| -0.031 843 | 2.58[-5] | -0.031 849 2.5[-5] | [4] | -0.031 855 2.63[-5] | -0.031 863 2.5[-5] | [8] |          |
| -0.031 617 | 2.32[-5] | — | — | -0.031 641 2.13[-5] | -0.031 657 2.01[-6] | [8] |          |
| H(n = 5), | -0.023 617 8.75[-6] | -0.023 26 4.05[-5] | [4] | -0.023 630 9.58[-6] | -0.023 634 2.3[-6] | [8] |          |
| -0.019 989 | -0.021 565 4.77[-5] | — | — | -0.021 577 4.70[-5] | -0.021 585 3.01[-6] | [8] |          |
| -0.021 089 | 1.23[-5] | -0.021 185 1.01[-5] | [4] | -0.021 114 1.14[-5] | -0.021 198 8.51[-6] | [8] |          |
| -0.020 956 | 9.42[-5] | -0.020 24 1.15[-4] | [4] | -0.020 674 5.61[-5] | -0.020 70 1.01[-6] | [8] |          |
| -0.020 145 | 3.37[-5] | — | — | -0.020 151 3.23[-5] | -0.020 259 3.25[-6] | [8] |          |
| Ps(n = 4), | -0.018 333 3.27[-4] | — | — | -0.018 719 2.80[-4] | — | |          |
| -0.015 625 | -0.017 940 2.69[-4] | — | — | -0.017 965 2.77[-5] | — | |          |
| -0.017 438 | 3.78[-5] | -0.0175 4.9[-5] | [4] | -0.017 430 4.05[-5] | -0.017 48 4.8[-5] | [8] |          |
| -0.016 268 | 2.69[-5] | — | — | -0.016 248 3.46[-5] | -0.016 29 2.25[-5] | [8] |          |
| -0.015 973 | 7.20[-4] | — | — | -0.015 973 7.20[-4] | — | |          |
with other calculations [6]. As was pointed out in [4], the agreement with [8, 12]; the uncertainty is about 10−6.

Table 2. P-wave resonance energies and widths in e+H system (au). Left-hand side (H) of the table shows the physical proton mass results and right-hand side (ω) for infinite mass. The notation x[y] means 10^y.

| Threshold | H | 1/2 | ω | 1/2 | Literature |
|-----------|---|-----|---|-----|------------|
| H(n = 2), | −0.126 990 | 4.12[-6] | −0.127 055 | 4.12[-6] | −0.127 066 | 4.08[-5] | [12] |
|           | −0.124 932 | 2.74[-6] | −0.125 063 | 4.29[-6] | −0.125 055 | 1.05[-7] | [12] |
| Ps(n = 2), | −0.074 072 | 1.51[-4] | −0.074 091 | 1.50[-4] | −0.074 091 | 1.48[-4] | [8] |
|           | −0.065 366 | 7.56[-5] | −0.065 374 | 7.59[-5] | −0.065 365 | 8.2[-5]  | [8] |
|           | −0.063 664 | 3.52[-5] | −0.063 676 | 3.51[-5] | −0.063 676 | 3.44[-5] | [8] |
|           | −0.063 191 | 1.22[-5] | −0.063 194 | 1.22[-5] | — | — | — |
|           | −0.062 667 | 1.01[-5] | −0.062 681 | 1.79[-5] | — | — | — |
| H(n = 3), | −0.057 771 | 2.85[-4] | −0.057 809 | 2.83[-4] | −0.057 813 | 2.86[-4] | [12] |
|           | −0.055 952 | 5.61[-5] | −0.055 982 | 5.83[-5] | −0.055 978 | 5.5[-5]  | [8] |
|           | −0.055 806 | 3.75[-6] | −0.055 835 | 3.51[-6] | −0.055 85 | 3.31[-6] | [12] |
|           | −0.055 553 | 2.67[-5] | −0.055 571 | 2.48[-5] | −0.055 61 | 3.0[-5]  | [8] |
| H(n = 4), | −0.038 282 | 3.10[-5] | −0.038 295 | 2.48[-5] | −0.038 337 | 2.64[-5] | [12] |
|           | −0.035 545 | 3.13[-5] | −0.035 556 | 3.03[-5] | −0.035 601 | 2.85[-5] | [12] |
|           | −0.033 783 | 4.92[-5] | −0.033 769 | 2.08[-5] | −0.033 793 | 2.33[-5] | [12] |
|           | −0.032 160 | 3.81[-6] | −0.032 225 | 4.73[-6] | −0.032 26 | 6.55[-6] | [12] |
|           | −0.031 954 | 2.65[-5] | −0.032 124 | 1.46[-5] | −0.032 18 | 1.41[-5] | [12] |
|           | −0.031 546 | 6.04[-5] | −0.031 638 | 9.89[-5] | −0.031 637 | 5.5[-6]  | [8] |
| Ps(n = 3), | −0.029 806 | 2.28[-4] | −0.029 904 | 2.40[-4] | −0.029 902 | 2.15[-4] | [8] |
|           | −0.028 450 | 5.89[-5] | −0.028 449 | 5.95[-5] | −0.028 456 | 6.63[-5] | [8] |
|           | −0.027 989 | 5.17[-5] | −0.027 999 | 4.98[-5] | — | — | — |
|           | −0.027 820 | 2.13[-5] | — | — | — | — | — |
| H(n = 5), | −0.023 496 | 7.40[-5] | −0.023 509 | 1.61[-5] | −0.023 558 | 4.1[-6]  | [8] |
|           | −0.022 400 | 6.56[-5] | −0.022 396 | 4.95[-5] | −0.022 537 | 2.81[-5] | [8] |
|           | −0.021 453 | 4.02[-5] | −0.021 476 | 2.97[-5] | −0.021 531 | 3.05[-5] | [8] |
|           | −0.020 798 | 1.20[-5] | −0.020 812 | 1.70[-5] | −0.020 863 | 1.43[-5] | [8] |
|           | −0.020 496 | 8.20[-6] | −0.020 357 | 2.10[-5] | −0.020 305 | 9.5[-6]  | [8] |
|           | −0.020 043 | 1.80[-5] | −0.020 055 | 2.22[-5] | −0.020 222 | 3.0[-6]  | [8] |
| Ps(n = 4), | −0.018 506 | 1.69[-4] | −0.018 598 | 1.44[-4] | −0.018 555 | 2.17[-4] | [8] |
|           | −0.017 085 | 6.33[-4] | −0.017 161 | 1.49[-4] | −0.017 404 | 4.4[-5]  | [8] |
|           | −0.016 132 | 2.69[-5] | −0.016 129 | 6.10[-5] | −0.017 13 | 9.0[-5]  | [8] |
|           | −0.015 831 | 3.44[-5] | −0.015 887 | 5.47[-5] | −0.015 934 | 2.25[-5] | [8] |

dipole series converging to the atomic thresholds. According to [9] the ratio of binding energies of successive resonances should then be a constant for each threshold. We calculated these ratios but found considerable variation, in agreement with other calculations [6]. As was pointed out in [4], the dipole formula only works well for resonances dominated by the long-range forces, i.e. very close to threshold. Most likely, resonances closer to the threshold should follow the dipole formula, but unfortunately these become increasingly difficult to calculate.

For states with total angular momentum J = 1, we have different possible combinations of the individual angular momenta (l1, l2, l3). The total number of configurations used was 24. For P-wave, our results in table 2 are in good agreement with [8, 12]; the uncertainty is about 10−6 atomic units again larger for large n. We found no literature values for the physical proton mass calculation. We found some new resonances for infinite proton mass, below the Ps(n = 2) threshold, at position 0.063 194 au and width 1.22 × 10−5 au, 0.062 681 au and 1.79 × 10−5 au. Also, one resonance is found below the Ps(n = 3) threshold at position 0.027 999 au and width 4.98 × 10−5 au.

The results for states with total angular momentum J = 2 are summarized in table 3. The total number of configuration used was 24. We found no literature values for physical proton mass calculation for D-waves. We found some new resonances for infinite proton mass, below the Ps(n = 2) threshold, at position 0.063 194 au and width 1.22 × 10−5 au, 0.062 681 au and 1.79 × 10−5 au. Also, one resonance is found below the Ps(n = 3) threshold at position 0.027 999 au and width 4.98 × 10−5 au.

The results for states with total angular momentum J = 2 are summarized in table 3. The total number of configuration used was 24. We found no literature values for physical proton mass calculation for D-waves. We found some new resonances for infinite proton mass, below the Ps(n = 2) threshold, at position 0.063 194 au and width 1.22 × 10−5 au, 0.062 681 au and 1.79 × 10−5 au. Also, one resonance is found below the Ps(n = 3) threshold at position 0.027 999 au and width 4.98 × 10−5 au.
In the present work, also resonance states with unnatural parities have been calculated for positron-excited hydrogen.

\[
\Gamma_{\alpha} = \frac{\alpha^2 e^4}{m_e c^2} \times 10^7
\]

\(\alpha\) is the number of configurations used.

For the \(P^+\) states, both the electron and positron have the same angular momentum which couple to form the total angular momentum \(J = 1\) shown in table 4. The numerical results are shown in table 5, along with the results obtained by Yan and Ho and by Shimamura [16, 17]. In [16], Yan and Ho used infinite proton mass while [17] used the real proton mass. The lowest unnatural parity resonance state is located at energy position \(E = -0.063\, 66\, \text{au}\) with a width \(\Gamma = 2.06 \times 10^{-7}\, \text{au}\) lying below the \(Ps(\alpha = 2)\) threshold, which is almost the same as in [16], but for real proton mass at energy \(E = -0.063\, 65\, \text{au}\) and width \(\Gamma = 2.26 \times 10^{-7}\), the width we get is much smaller than [17]. Specifically, it is a result of an electron in its 2p positronium state interacting with a positively charged and infinitely massive proton [16]. Most resonances belong to a dipole series, with dipole moments arising from the L-degeneracy of the hydrogen states. However, for unnatural-parity state the S-states of H do not contribute. Thus, for \(\alpha = 2\) there is no dipole interaction and no resonances exist. It is pointed out in [16], that resonances

| State | \(J\) | \((l_1, l_2, l_3)\) | \(N\) | Total basis |
|-------|-------|-----------------|------|-------------|
| \(P^+\) | 1     | (1,1), (2,2), (3,3), (4,4) | 12   | 6500        |
| \(D^0\) | 2     | (2,1),(1,2), (3,2),(2,3), (4,3),(3,4) | 18   | 6800        |
Table 5. P° wave unnatural parity resonance energies and width in the e+H system (au). Left-hand side (\(\alpha E\)) of the table shows the physical proton mass results and right-hand side (\(\alpha E\)) for infinite mass. The notation x\(\gamma\) means 10\(\gamma\).

| Threshold | This work \(\alpha E\) | \(\Gamma/2\) | Yan and Ho [16] \(\alpha E\) | \(\Gamma/2\) | This work \(\beta\) | \(\Gamma/2\) | Shimamura et al [17] \(\beta\) | \(\Gamma/2\) |
|-----------|-----------------|-----------|-----------------|-----------|-----------------|-----------|-----------------|-----------|
| \(P_s(n = 2)\), \(-0.062\) | 0.063 66 | 2.06[-7] | 0.063 66 | 2.05[-6] | 0.063 65 | 2.26[-7] | 0.063 64 | 2.15[-6] |
| \(H(n = 3)\), \(-0.055\) | 0.055 83 | 7.04[-7] | 0.055 83 | 6.0[-7] | 0.055 80 | 5.78[-7] | 0.055 80 | 6.0[-7] |
| \(H(n = 4)\), \(-0.031\) | 0.035 57 | 2.30[-5] | 0.035 58 | 2.3[-5] | 0.035 56 | 2.34[-5] | 0.035 52 | 2.3[-5] |
| \(P_s(n = 3)\), \(-0.027\) | 0.029 92 | 2.20[-4] | — | — | 0.029 91 | 2.19[-4] | 0.029 88 | 2.04[-4] |
| \(H(n = 5)\), \(-0.019\) | 0.024 50 | 5.12[-5] | — | — | 0.022 43 | 6.43[-5] | 0.022 36 | 2.30[-5] |
| \(P_s(n = 4)\), \(-0.015\) | — | — | — | — | 0.017 09 | 9.81[-5] | — | — |

Table 6. D° wave unnatural parity resonance energies and width in the e+H system (au). Left-hand side (\(\beta E\)) of the table shows the physical proton mass results and right-hand side (\(\alpha E\)) for infinite mass. The notation x\(\gamma\) means 10\(\gamma\).

| Threshold | This work \(\beta E\) | \(\Gamma/2\) | Yan and Ho [16] \(\alpha E\) | \(\Gamma/2\) | This work \(\alpha E\) | \(\Gamma/2\) | Shimamura et al [17] \(\beta\) | \(\Gamma/2\) |
|-----------|-----------------|-----------|-----------------|-----------|-----------------|-----------|-----------------|-----------|
| \(H(n = 3)\) | 0.055 58 | 4.46[-6] | 0.055 57 | 5.0[-7] | 0.055 55 | 4.56[-6] | 0.055 55 | 1.9[-6] |
| \(H(n = 4)\) | 0.034 87 | 1.52[-5] | 0.034 92 | 1.5[-5] | 0.034 86 | 1.52[-5] | 0.034 85 | 1.5[-5] |
| \(P_s(n = 3)\) | 0.029 60 | 1.35[-4] | — | — | 0.029 58 | 1.31[-4] | 0.029 57 | 1.25[-4] |
| \(H(n = 5)\) | 0.022 18 | 2.99[-5] | — | — | 0.022 18 | 3.06[-5] | 0.022 17 | 2.6[-5] |
| \(P_s(n = 4)\) | 0.018 31 | 1.18[-5] | — | — | 0.018 30 | 1.18[-4] | — | — |

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under the Ps thresholds can instead arise from the polarizability of positronium creating an effective $1/r^4$ potential. The resonances below Ps($n = 4$) in table 5, to the best of our knowledge, have not been discussed previously. All other resonances are in good agreement with the results of [16, 17].

For the D' state resonances, present results are displayed in table 6, along with the other available theoretical results from [16, 17]. The resonances below the $H(n = 3)$ and $H(n = 4)$ thresholds are in good agreement with literature values. The resonances below Ps($n = 3$) threshold were not presented in [17].

4. Conclusion

In summary, we found resonances with natural as well as unnatural parities in positron-excited hydrogen scattering. Our results are in line with prior calculations. This study reports some resonances for the first time. On the experimental front, there is no report on resonances in $e^+H$ scattering. Our predictions will be useful in any future experimental search for atomic resonances involving positrons.

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