Neutral pion electroproduction in \( p(e, e'\pi^0)p \) above \( \sqrt{s} > 2 \text{ GeV} \)

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(Dated: January 20, 2013)

Electroproduction of neutral pions in exclusive reaction \( p(e, e'\pi^0)p \) is studied above the resonance region, \( \sqrt{s} > 2 \text{ GeV} \). The reaction amplitude is described by exchanges of vector \( \omega(782) \), \( \rho(770) \) and axial-vector \( h_1(1170) \) and \( h_1(1235) \) Regge trajectories. The residual effect of \( s \)- and \( u \)-channel nucleon resonances is taken into account using a dual connection between the exclusive form factors and inclusive deep inelastic structure functions. In photoproduction at forward angles the exchange of Regge trajectories dominates and the dip region is filled by the resonances. In electroproduction the excitation of nucleon resonances explains the high \( Q^2 \) data from JLAB. The results for the beam spin azimuthal asymmetry measured at CLAS/JLAB are presented. Model calculations in the deep inelastic region at HERMES/DESY are given.

PACS numbers: 12.39.Fe, 13.40.Gp, 13.60.Le, 14.20.Dh

I. INTRODUCTION

Electroproduction of mesons in the deep inelastic scattering (DIS), that is \( \sqrt{s} > 2 \text{ GeV} \) and \( Q^2 > 1 \text{ GeV}^2 \), is a modern tool which permits to study the structure of the nucleon on the partonic level. Exclusive channels in DIS are of particular importance. In this kind of hard exclusive processes one may learn about the off-forward parton distributions that parameterize an intrinsic nonperturbative pattern of the nucleon, see Ref. [1] and references therein. Much work have been done to understand the production of pions in exclusive kinematics. For instance, in QCD at large values of \( \langle \sqrt{s}, Q^2 \rangle \) and finite value of Bjorken \( x_F \) the description of \( N(e, e'\pi)N' \) relies on the dominance of the longitudinal cross section \( \sigma_L \) [2]. The transverse cross section \( \sigma_T \) is predicted to be suppressed by power of \( \sim 1/Q^2 \). However, being a leading twist prediction the kinematic domain where this power suppression starts to dominate is not yet known for exclusive \( \pi \) production.

A somewhat different concept is used in the Regge pole models which rely on effective degrees of freedom. Here the forward \( (\gamma^*, \pi) \) production mechanism is peripheral, that is a sum of all possible \( t \)-channel meson-exchange processes. Although both partonic and Regge descriptions are presumably dual the ongoing and planned experiments have a potential to discriminate between different models. The related studies have been carried out at JLAB [3-5] and at DESY [6]. A dedicated program on exclusive production of pions is planned in the future at the JLAB upgrade [7].

A closely related phenomenon is the color transparency (CT) effect. It becomes effective in exclusive electroproduction of mesons off nuclei, see Ref. [8] for a possible observation and Refs. [9-11] for further interpretations of the CT signal in the reaction \( A(e, e'\pi) \). Presently one believes that at high values of \( Q^2 \) the exclusive pions are produced in point like configurations which may interact in the nuclear medium only weakly. However, the fate of pions inside the nucleus depends on the initial longitudinal and/or transverse production mechanisms [11]. Therefore, our understanding of the \( \pi \) production off nucleons is mandatory for a proper interpretation of the CT signal observed in electroproduction off nuclei.

On the experimental side, it would be interesting to see an onset of \( \sigma_L/\sigma_T \propto Q^2 \) scaling already at presently available energies. However, the high \( Q^2 \) data from JLAB [12] and single spin asymmetries measured in the true DIS region at HERMES [13] demonstrate the presence of nonvanishing transverse components in \( p(\gamma^*, \pi^+)n \). Moreover, the high \( Q^2 \) region at JLAB [12, 14], DESY [15-17], Cornell [18-20] and CEA [21] is even dominated by the conversion of transverse photons. For instance, the \( Q^2 \) dependence of \( \sigma_L \) and \( \sigma_T \) in the \( \pi^+ \) electroproduction above \( \sqrt{s} > 2 \text{ GeV} \) has been studied in [3]. In the charged pion case the longitudinal cross section \( \sigma_L \) at forward angles is well described by the quasi-elastic \( \pi \) knockout mechanism [22, 23]. It is driven by the pion charge form factor both at JLAB [12, 24, 25] and HERMES [26]. On the contrary, the \( \langle \sqrt{s}, Q^2 \rangle \) behavior of \( \sigma_T \) remains to be puzzling. The data demonstrate that \( \sigma_T \) is large and tends to increase relative to \( \sigma_L \) as a function of \( Q^2 \). Interestingly, the \( \langle \sqrt{s}, Q^2 \rangle \) dependence of exclusive \( \sigma_T \) exhibits features of the semi-inclusive \( p(e, e'\pi)X \) reaction in DIS in the limit \( z \rightarrow 1 \) [27]. This kind of an exclusive-inclusive connection [28] has been also observed in exclusive \( (\gamma^*, \rho^0) \) production [29].

Theoretically, hadronic models based on the meson-exchange scenario alone largely underestimate the measured \( \sigma_T \) in charged pion electroproduction, see Ref. [30] for further discussions and references therein. In the partonic models the higher twist transversity distributions might be relevant to understand the \( (\gamma_T, \pi) \) reaction [30, 31]. Several phenomenological models already attempted to describe the \( \sigma_T \) component [27, 32, 33]. For instance, the description of charged pion production proposed in Ref. [33] relies on the residual contribution of the nucleon resonances. This approach will be followed in this work. It is supposed that the excitations of nucleon resonances dominate in electroproduction. The resonances are dual to the direct partonic interactions due to the Bloom-Gilman duality connection [34, 35]. In [35] the resonances supplement the Regge based exchanges of single mesonic trajectories. Therefore, one distinguishes the peripheral \( t \)-channel meson-exchange processes and the \( s(u) \)-channel resonance/partonic contributions. For instance, in this
way all the data collected so far in the charged pion electroproduction at JLAB, DESY, Cornell and CEA can be well described [33].

In this work we continue our studies of the reaction \( N(e, e'\pi)N' \) and attempt to describe the electroproduction of neutral pions above the resonance region. Recently, the \( p(\gamma^*, \pi^0)p \) partial cross sections were measured at JLAB in the deeply virtual kinematics around the values of \( \sqrt{s} \approx 2.1 \text{ GeV} \) and \( Q^2 \approx 2 \text{ GeV}^2 \) [5]. In the \( \pi^0 \) channel there are interesting features which are worth to mention. At the real photon point the \( \pi^0 \) production is probably the simplest of all photoproduction reactions because of the limited number of allowed Reggeon exchanges. By quantum numbers prescribed [33].

The situation is very similar to the charged pion production where the Regge model also largely underestimates the \( \pi^0 \) electroproduction cross sections. Indeed, at high values of \( Q^2 \) the meson-exchange contributions are suppressed by the \( \gamma \pi \)-Reggeon transition form factors and must show a rapid decrease as a function of \( Q^2 \). Being dominated by the transverse \( \gamma^*_T \rightarrow \pi^0 \) component, the \( \pi^0 \) data, however, demonstrate no pronounced \( Q^2 \) dependence [5]. An importance of Regge-cut unitary corrections in this case has been demonstrated in [38]. Another observable that we consider in this work is the beam single spin asymmetry (SSA). Sizeable and positive beam SSA have been found at CLAS/JLAB in the reaction \( p(\vec{e}, e'\pi^0)p \) with the longitudinally polarized electron beam [39].

The outline of the manuscript is as follows. In Sec. II we briefly recall the kinematics and definitions of the cross sections in exclusive \( \pi^0 \) electroproduction off nucleons. In Sec. III we consider the contributions of vector \( \omega(782) \), \( \rho(770) \) and axial-vector \( h_1(1170) \) and \( b_1(1235) \) Regge trajectories. In Sec. IV we describe the contribution of nucleon resonances. In Sec. V we briefly discuss the \( \pi^0 \) photoproduction at forward angles. Then in Sec. VI the model results are compared with the electroproduction data from JLAB. In Sec. VII we calculate the cross sections in the kinematics at HERMES. The conclusions are summarized in Sec. VIII.

### II. KINEMATICS AND DEFINITIONS

At first, we recall briefly the kinematics in exclusive \( \pi \) electroproduction

\[
e(l) + N(p) \rightarrow e'(l') + \pi(k') + N'(p'),
\]

and specify the notations and definitions of variables. In exclusive reaction Eq. (1) we shall deal with an unpolarized target and, both unpolarized and polarized lepton beams. The differential cross section is given by

\[
\frac{d\sigma}{dQ^2 dv dt d\phi} = \Phi \left( \frac{d\sigma_T}{dt} + \varepsilon \frac{d\sigma_L}{dt} 
\right.
\]

\[
+ \sqrt{2\varepsilon(1+\varepsilon)} \frac{d\sigma_{TT}}{dt} \cos(\phi)
\]

\[
+ \varepsilon \frac{d\sigma_{LT}}{dt} \cos(2\phi)
\]

\[
+ h \sqrt{2\varepsilon(1-\varepsilon)} \frac{d\sigma_{LT}^T}{dt} \sin(\phi)
\],

\( (2) \)

where \( d\sigma_T \) is the transverse cross section, \( d\sigma_L \) is the longitudinal cross section, \( d\sigma_{TT} \) is the cross section originating from the interference between the transverse components of the virtual photon, \( d\sigma_{LT} \) is the cross section arising from the interference between the transverse and longitudinal polarizations of the virtual photon and \( d\sigma_{LT}^T \) is the beam-spin polarized cross section resulting from the interference between the transverse and longitudinal photons and helicity \( h = \pm 1 \) of the incoming electron.

In the laboratory where the target nucleon is at rest, the \( z \)-axis is directed along the three momentum \( \vec{q} = (0, 0, \sqrt{\nu^2 + Q^2}) \) of the exchanged virtual photon \( \gamma^* \) with \( q = l - l' = (\nu, \vec{q}) \), \( Q^2 = -q^2 \), \( \nu = E_e - E_e' \) and \( l(l') \) is the four momentum of incoming (deflected) electrons. In Eq. (2) \( \phi \) stands for the azimuthal angle between the electron scattering \((e, e')\) plane and \( \gamma^* N \rightarrow \pi N' \) reaction plane. \( \phi \) is zero when the pion is closest to the outgoing electron [40]. The differential cross section integrated over \( \phi \) is denoted as

\[
\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt} + \varepsilon \frac{d\sigma_L}{dt}.
\]

The virtual photon flux is conventionally defined as [41]

\[
\Phi = \frac{\pi}{E_e(E_e - \nu)} \left( \frac{\alpha_e}{2\pi^2} \frac{E_e - \nu}{E_e} \frac{K}{Q^2(1-\varepsilon)} \right),
\]

\( (4) \)

with \( \alpha_e \simeq 1/137, K = (W^2 - M_N^2)/2M_N \) and

\[
\varepsilon = \frac{1}{1 + 2\frac{\nu^2 + Q^2}{\nu(E_e - \nu) - Q^2}}.
\]

\( (5) \)

is the ratio of longitudinal to transverse polarization of the virtual photon.

The longitudinal beam SSA in \( N(\vec{e}, e'\pi)N' \) scattering is defined so that

\[
A_{LU}(\phi) \equiv \frac{d\sigma^{-\pi}(\phi) - d\sigma^{+\pi}(\phi)}{d\sigma^{-\pi}(\phi) + d\sigma^{+\pi}(\phi)},
\]

\( (6) \)

where \( d\sigma^{\pm\pi} \) refers to positive helicity \( h = +1 \) of the incoming electron. The azimuthal moment associated with the beam SSA is given by [40]

\[
A_{LU}^{\sin}(\phi) = \frac{\sqrt{2\varepsilon(1-\varepsilon)} d\sigma_{LT}^T}{d\sigma_T + \varepsilon d\sigma_L}.
\]

\( (7) \)
III. EXCHANGE OF REGGE TRAJECTORIES

At first we consider the exchange of mesonic Regge trajectories. The leading trajectories contributing to $p(e,e'\pi^0)p$ are the natural parity $P = (-1)^J$ vector $\omega(782)$, $\rho(770)$ and the unnatural parity $P = (-1)^{J+1}$ axial-vector $h_1(1170)$ and $b_1(1235)$ exchanges, with $\omega$ being the dominant trajectory.

At high energies the differential cross section $d\sigma/dt$ in the photoproduction $(\gamma,\pi^0)$ reaction is characterized by pronounced dip around $t \approx -0.5 \text{ GeV}^2$. For instance, there is no such a dip in the $(\gamma,\pi^\pm)$ reaction. There exist two explanations of this structure in $d\sigma/dt$ \cite{42}. According to one viewpoint, a coincidence of a dip with the point in $-t$ where the $\omega$-Regge trajectory (with a slope $\alpha'_\omega = 0.85 \text{ GeV}^{-2}$)

$$\alpha_\omega(t) \simeq 0.4 + \alpha'_\omega t$$

passes through nonsense value $-t = 0.47 \text{ GeV}^2$ demonstrates that the dips are nonsense wrong-signature zeros (NWSZ) of the Regge pole amplitudes. In this case the Regge cuts are supposed to be weak and modify the region around the dips only slightly. The alternative viewpoint is that the Regge trajectories are degenerated - the Regge poles are essentially featureless as function of $-t$. Then a structure of the dip in $d\sigma/dt$ is similar to the diffraction minima caused by the destructive interference between the exchange of single Regge poles and very strong absorptive Reggeon/Pomeron cuts in this case \cite{42}.

In electroproduction for finite $Q^2$ the two viewpoints should not necessarily result in identical predictions \cite{53}. If the dip is a NWSZ its position should remain fixed in $t$ since the $\omega$-exchange should be independent of the external masses. On the contrary, if an absorptive cut is responsible for a dip structure it is very likely that the dip position will move with $Q^2$ since the range of the interaction may well change \cite{53}. Interestingly, the early experiments at DESY \cite{44, 45} have shown that the dip neither remained fixed nor moved out with $Q^2$. It simply disappeared as $Q^2$ was changed from its null value. This behavior has been much discussed in \cite{42, 46} and recently in \cite{38}.

Note that, the dip problem shows up when one compares the photoproduction data obtained at higher energies which indeed exhibit a pronounced dip and then extrapolated with some assumption to the DESY region \cite{53}. Recently, at JLAB \cite{47} and ELSA \cite{48, 49} the $\pi^0$ photon-induced cross sections were measured also just above the resonance region. Interestingly, the dip which starts to develop only at high energies is not there and the experimental points are only leveling off. Therefore, the dip issue in $(\gamma^*,\pi^0)$ is rather artificial and might be related to the comparison of electro- and photo-cross section with different values of $W$.

In the following we rely on a NWSZ scenario. In this case the Regge propagator of the $J = 1$ mesons reads

$$D^{\mu \nu}_R = \left(-g^{\mu \nu} + \frac{k^\mu k^\nu}{m_R^2}\right) \left[1 - e^{-i\pi\alpha_R(t)}\right]^2 \times (-\alpha'_R) \Gamma(1 - \alpha_R(t)) e^{\ln(\alpha_R(t))(\alpha_R(t)-1)},$$

where $t = k^2$ and $k^2 = q^2 = p - p'$. In $\omega$ the $\Gamma$-function contains the pole propagator $\sim 1/\sin(\pi\alpha_R(t))$ but no zeroes and the amplitude zeroes only occur through the factor $1 - e^{-i\pi\alpha_R(t)}$. When using Eq. (9) the region around the dip will remain strongly underestimated. If one assumes that the cross section at the dip is dominated by unnatural parity exchanges then the observed dip must be filled up by the axial-vector $h_1(1235)$ and/or $h_1(1170)$ Regge trajectories. However, this assumption is in contradiction to the experimental data. The energy dependence of the dip is approximately that of the region outside the dip.

A possible way to fill the dip region is to allow absorption cuts in the model (see Ref. \cite{50} for the recent calculations and references therein). A somewhat different prescription has been used in \cite{51}. Nevertheless, as we shall see in the following in the model without strong cuts there is an alternative mechanism which allows to fill the region around the dip. Furthermore, we will show that this mechanism is at the origin of the large transverse cross section observed in the $\pi^0$ electroproduction.

\section{A. Vector $\omega(782)$ and $\rho(770)$ exchange currents}

In the following we give the expressions for the currents (amplitudes) only. Then the L/T separated virtual-photon nucleon cross sections are calculated using the expressions given in the Appendix A of Ref. \cite{53}.

The currents $J^\mu_V$ describing the exchange of $V = \rho$- and $\omega$-Regge trajectories in the reaction $p(\gamma^*,\pi^0)p$ are given by

$$-i J^\mu_V(\gamma^* p \rightarrow \pi^0 p) = -i G_{\gamma \pi}^V G_{\rho NN}^V F_{\nu \pi}^V (Q^2_e) e^{i\mu \nu \alpha \beta} q_\beta \bar{u}_s(p') \left[\left(1 + \kappa_V\right)_{\gamma \beta} \frac{-\kappa_V}{2M_p} (p + p')_\beta\right] u_s(p) \times \left[1 - e^{-i\pi\alpha_V(t)}\right]^2 (-\alpha'_V) \Gamma(1 - \alpha_V(t)) e^{\ln(\alpha'_V)(\alpha_V(t)-1)},$$

In the $V = \rho(770)$ case the $G_{\rho NN} = 3.4$ and $\kappa_\rho = 6.1$ are the vector and anomalous tensor coupling constants, respectively.

The $\rho$-trajectory adopted here reads

$$\alpha_\rho(t) = 0.53 + \alpha'_\rho t$$

in Appendix A of Ref. \cite{53}.
with a slope $\alpha'_p = 0.85$ GeV$^{-2}$. For the transition form factor $F_{\gamma\pi}(Q^2)$ we use $F_{\gamma\pi}(Q^2) = (1 + Q^2/\Lambda_{\gamma\pi}^2)^{-1}$ with $\Lambda_{\gamma\pi} = m_{\pi}(782)$. All these parameters and Eq. (10) (up to the isospin factor) are the same as have been used in the charged pion electroproduction of Ref. [33].

In the natural parity sector the dominant contribution comes from the exchange of the $\omega(782)$-Regge trajectory given by Eq. (8). From the fit to high energy photoproduction data we obtain $G_{\omega NN} = 18 \pm 1$. We also neglect the anomalous tensor component $\kappa_\omega \approx 0$ [51]. For the transition form factor $F_{\omega\pi}(Q^2)$ we use a monopole form factors $F_{\omega\pi}(Q^2) = (1+Q^2/\Lambda_{\omega\pi}^2)^{-1}$ with the cut-off $\Lambda_{\omega\pi}$ being a fit parameter.

The $\gamma\pi$ coupling constants $G_{\gamma\pi}$ can be deduced from the radiative $\gamma\pi$ decay widths of $V$:

$$\Gamma_{\gamma\rightarrow\gamma\pi} = \frac{\alpha_e G^2_{V\pi}}{24 m_V^3} (m_V^2 - m_{\pi}^2)^3. \quad (12)$$

The measured widths are $\Gamma_{\rho\rightarrow\gamma\pi} = (89 \pm 11)$ keV and $\Gamma_{\omega\rightarrow\gamma\pi} = (764 \pm 51)$ keV where the central values correspond to $G_{\rho\pi} = 0.84$ GeV$^{-1}$, $G_{\omega\pi} = 2.4$ GeV$^{-1}$.

### B. Axial-vector $b_1(1235)$ and $h_1(1170)$ exchange currents

We further take into account the exchanges of lowest lying C-parity odd $J^{PC} = 1^{-+}$ axial-vector $b_1(1235)$ and $h_1(1170)$ mesons with $F^G = 1^+$ and $F^G = 0^-$, respectively. The results reported in [39] suggest an important role of axial-vector mesons in the description of polarization observables. Together with $h_1(1380)$ state ($F^G = 0^-$) they belong to the nonet of $C = -1$ axial-vector mesons. Assuming the ideal mixing pattern between the singlet and $I = 0$ of the octet the $h_1(1380)$ meson decouples from the $\pi^0\gamma$ channel. The existing empirical information [52] does not allow to fix the coupling constant of $h_1(1170)$ to $\pi^0\gamma$ channel. From SU(3) arguments the width of $h_1(1170)$ into the $\pi^0\gamma$ decay channel should be about an order of magnitude larger than the decay width of $b_1(1235) \rightarrow \pi^0\gamma$. However, the strength of their interaction with nucleons can be estimated only indirectly. For instance, in the analysis of charged pion production we did not find any strong evidence for the contribution of $b_1(1235)$ trajectory [33]. Since, the Lorentz structure of the interaction vertices are essentially the same, both trajectories can be combined in a single current describing the exchange of axial-vector mesons. The hadronic current $-iJ_B$ describing the exchange of $B = b_1(1235)$ and $h_1(1170)$ Regge trajectories takes the form [33].

$$-iJ_B(\gamma^* p \rightarrow \pi^0 p) = G_B \left[ kq' - (qk)g_{\mu
u} \right] (p + p') u_s(p') \gamma_5 u_s(p) \times \frac{1 - e^{-i\alpha_B(t)}}{\alpha_B(t)} \Gamma[1 - \alpha_B(t)] e^{i\beta_B(t) - (\alpha_B(t-1)). \quad (13)}$$

The effective coupling constant $G_B \sim$ GeV$^{-2}$ absorbs the couplings in the $B\gamma\pi^0$ and $BNN$ vertices. The $\pi$ and $b_1(1235)$ Regge trajectories are nearly degenerate. Therefore, we assume $\alpha_B(t) \approx \alpha_B(t) = \alpha_{\pi}(t)$ where $\alpha_{\pi}(t) = 0.74(t - m_{\pi}^2)$ is the $\pi$-trajectory. In the $B\gamma^* \pi$ vertex we use the monopole form factor $F_{B\pi}(Q^2) = (1 + Q^2/\Lambda_{B\pi}^2)^{-1}$ with $\Lambda_{B\pi} = m_V$ where $m_V$ is, e.g., an average mass of vector $\omega(\rho)$ mesons. Note that, Eq. (13) does not interfere with the amplitude describing the exchange of vector mesons, see Eq. (10). That is, in the interference term the trace over the spinor indices involve the $\gamma_5$ with odd number of $\gamma$ matrices.

## IV. EFFECT OF NUCLEON RESONANCES

So far we have considered the $t$-channel meson-exchange contributions. In this section we estimate the residual effect from the propagation of the nucleon and its excitations in the the $s$- and $u$-channels. We start from the nucleon Born terms in the $s$- and $u$-channels. The contribution (the sum) of the corresponding $s$- and $u$-channel diagrams in the reaction $p(\gamma^*, \pi^0)p$ takes the form [11]

$$-iJ_s^{\mu} - iJ_u^{\mu} = g_{\pi NN} \bar{u}_s(p') \left[ F_s(Q^2, s, t) \gamma_5 (p + q)_{\sigma} \gamma_\mu + M_p \gamma^\mu \right] s - M_p^2 + i0^+$$

$$+ F_u(Q^2, u, t) \gamma_5 (p' - q)_{\sigma} \gamma_\mu + M_p \gamma^\mu \left[ F_s(Q^2, s, t) - F_u(Q^2, u, t) \frac{(k - k')^\mu}{Q^2} \right] u_s(p), \quad (14)$$

where $F_{s(u)}(Q^2, s(u), t)$ stands for the proton $s(u)$-channel transition form factors. In Eq. (14) $g_{\pi NN} = 13.4$ is the pseudoscalar $\pi N$ coupling constant. In the sum of two Born amplitude the current conservation condition, $q_{\mu}^{\nu} s_{s+u}^{\nu} = 0$, is satisfied in the presence of different form factors, $F_s$ and $F_u$, which in general can depend on values of $(Q^2, s(u), t)$. Eq. (14) has been obtained using the requirement that the modified electromagnetic vertex functions entering the amplitude obey the same Ward-Takahashi identities as the bare ones, see [11] and references therein.
The nucleon resonances are taken into account using the Bloom-Gilman duality connection \([34, 35, 59]\) between the resonance form factors and deep inelastic structure functions. This allows to absorb the contribution of an infinite sum of resonances into the form factors and deep inelastic structure functions. The details of the dual connection used here are described in Ref. \([33]\). Here we only give an explicit expression for the \(s\)- and \(u\)-channel form factors. They are given by

\[
F_s(Q^2, s) = \frac{s \ln \left( \frac{\xi Q^2 + s}{M_p^2} + 1 \right) (\frac{2\xi Q^2 + s}{\xi Q^2})^2 - s(\xi Q^2 + s) \ln \left( \frac{s - M_p^2}{M_p^2} \right) - i\pi}{(\frac{s}{\xi Q^2} + 1)^2 \left( \frac{s^2 + 2s M_p^2}{2 M_p^2} + \ln \left( \frac{s - M_p^2}{M_p^2} \right) - i\pi \right)},
\]

(15)

\[
F_u(Q^2, u) = \frac{u \ln \left( \frac{\xi Q^2 + u}{M_p^2} + 1 \right) (\frac{2\xi Q^2 + u}{\xi Q^2})^2 - u(\xi Q^2 + u) \ln \left( \frac{M_p^2 - u}{M_p^2} \right)}{u^2 + 2u M_p^2 + \ln \left( \frac{M_p^2 - u}{M_p^2} \right)},
\]

(16)

with the cut off \(\xi = 0.4\) \([33]\). Note that this parameter is fixed in the charged pion electroproduction and we do not refit it in the neutral pion channel. Furthermore, for generalized form factors in the \(s\)- and \(u\)-channels we take the same phase as determined in the charged pion production, that is

\[
F_{s(u)}(Q^2, s(u), t) = F_{s(u)}(Q^2, s(u))(t - m_{\pi}^2)\mathcal{R}(\alpha(t)),
\]

(17)

where \(F_{s(u)}\) are given by Eqs. (15) and (16) and the Regge factors \(\mathcal{R}_{s(u)}\) are given by

\[
\mathcal{R}_s(\alpha(t)) = e^{-i\pi\alpha(t)} (\alpha' - \alpha') \Gamma[\alpha(t)] \ln(\alpha's),
\]

\[
\mathcal{R}_u(\alpha(t)) = (\alpha') \Gamma[\alpha(t)] \ln(\alpha's),
\]

where \(\alpha(t) = \alpha'(t - m_{\pi}^2), \alpha' = \alpha'/1 + 2.4 Q^2/W^2\) and \(\alpha' = 0.74\) stands for the slope of the \(\pi\)-Regge trajectory.

### V. PHOTOPRODUCTION

In this section we consider briefly the reaction \(p(\gamma, \pi^0)p\) at the real photon point. In Fig. 1 we compare the model results with photoproduction data from \([53]\) for values of \(E\gamma = 6, 9, 12\) and 15 GeV in the laboratory. The dashed and dash-dotted curves in Fig. 1 describe the contributions of the \(\omega\)- and \(\rho\)-Regge trajectories, respectively. The value of \(G_{\omega\gamma NN} = 18\) has been used. Within the Regge phenomenology with NWSZ the contributions from \(\omega(782)\) and \(\rho(770)\) trajectories disappear around the value of \(-t \approx 0.5\) GeV\(^2\). Indeed, using the linear Regge trajectories \(\alpha_\omega(t)\) and \(\alpha_\rho(t)\) adopted here, see Eqs. (8) and (11), the zeroes occur at \(-t_\omega = 0.47\) GeV\(^2\) and \(-t_\rho = 0.62\) GeV\(^2\), correspondingly. Therefore, the observed dip near \(-t \approx 0.5\) GeV\(^2\) can be readily explained. However, the region around the dip remains strongly underestimated in this case.

We further add to the Regge amplitudes the amplitude resulting from the \(s\)- and \(u\)-channel contributions, see Eq. (14). Individually the \(s(u)\)-channel terms are large like in the \(\pi^\pm\) production at forward angles, see Ref. \([33]\). However, in the neutral pion channel the sum of \(s\)- and \(u\)-channel pole terms partially cancel. For instance, at forward angles the sum of pole terms practically cancel and the dash-dash-dotted curves in Fig. 2 result from the partial cancellation of two contributions. In the following we rely on this scenario where the \(s(u)\)-channel pole terms explain the region around the dip. This is in line with the results of \([54]\) where the nucleon resonance background has been considered as the main dip filling mechanism.

The solid curves in Fig. 1 describe the sum of all the production amplitudes. The very strong rise of the cross section at extreme forward directions is due to the Primakoff effect \([55]\). The Primakoff amplitude is given in Ref. \([56]\) and we do not repeat these formulae here. Note that, the dominance of Regge contributions and the cancellation of \(s(u)\)-channel pole terms at extreme forward angles simplifies the analysis of Primakoff \(\pi^0\) data off nuclei \([56]\).

In Fig. 2 we show our results for the polarized photon asymmetry \(\Sigma\) defined as

\[
\Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\perp + d\sigma_\parallel}.
\]

(18)

Here \(d\sigma_\perp\) and \(d\sigma_\parallel\) are the differential cross sections where the incoming photons are polarized in the direction perpendicular and parallel to the \(\gamma\pi^0\) reaction plane, respectively. With our definition of partial \(L/T\) cross sections the asymmetry is \(\Sigma = -\frac{d\sigma_\perp}{d\sigma_\parallel}\). This observable is known to be sensitive to the reaction mechanism around the dip. In Fig. 2 we also plot the compilation of high energy data from \([53, 57]\) in the range \(E\gamma = 2.5 \div 10\) GeV in the laboratory. In a model with \(\omega\)- and \(\rho\)-Regge trajectories only (dashed curve) this observable is essentially unity. Because of the weak energy dependence of the model results the calculations are performed for the average value of \(E\gamma = 4\) GeV. The asymmetry generated by the \(s\)- and \(u\)-channel contributions only is shown by the dash-dotted line. The solid curve is a combined effect of the \(\omega(\rho)\)-Regge...
and the $s(u)$-channel components of the cross section. As one can see, the addition of the $s$- and $u$-channel pole terms results in an asymmetry around the dip which is qualitatively in agreement with data.

In the model without $s(u)$-channel effects the polarized photon asymmetry $\Sigma$ could be generated by the axial-vector mesons. In the model with axial-vector mesons only the asymmetry is $\Sigma = -1$ (dot-dot-dashed line in Fig. 2). The dash-dash-dotted curve in Fig. 2 describe the Regge model with the vector and axial-vector exchanges. In these calculations we used the values of $G_B = \pm 11 \text{ GeV}^{-2}$. Note that, with proper $B N N$ interactions constrained by $C$-parity [33] the $C$-parity odd axial-vector and vector exchanges do not interfere with each other. Therefore, this result does not depend on the sign of the $G_B$ coupling in Eq. (13).

The shaded band in Fig. 2 describes the model results with vector, axial-vector ($G_B = \pm 11 \text{ GeV}^{-2}$) and $s(u)$-channel contributions. The axial-vector currents have a potential to improve our description of photon asymmetry data by their interference with $s(u)$-channel contributions. As one can see, however, in Fig. 2 the contribution of axial-vector-mesons (dot-dot-dashed curves) is marginally small and can be readily neglected in the rest of observables studied here.

VI. ELECTROPRODUCTION: JLAB DATA

In Fig. 3 we compare our results in the reactions with real $p(\gamma, \pi^0)p$ and virtual $p(\gamma^*, \pi^0)p$ photons. The forward photo-data (filled squares) with $E_\gamma = 3 \text{ GeV}$ are from [58]. The electroproduction data [5] (filled circles) contain both the transverse and longitudinal components, that is $d\sigma_T/dt = d\sigma_T + \epsilon d\sigma_L/dt$ and correspond to values of $Q^2 = 1.94 \text{ GeV}^2$, $x_R = 0.37$ and $E_e = 5.75 \text{ GeV}$. Again in photoproduction at forward angles the Regge-exchange contributions (upper dash-dotted curve) dominate the cross section. The Primakoff effect is relevant at extreme forward angles and is anyway cut by $-t_{\text{min}}$ in electroproduction. The resonances (upper dashed curve) take over in the region of the NWSZ dip. However,
in the electroproduction of $\pi^0$ the situation is different - at least in the context of the present model. Neglect the contribution of axial-vector mesons. Then the only model parameter to be fitted to the electroproduction data is the cut off in the excitation of resonances largely dominates the electroproduction cross section ([33]. It is interesting to see the contribution of resonances as compared to the contribution of the nucleon pole terms. Replacing the resonance form factors in Eqs. (15) and (16) by the nucleon form factors $F_{a(u)} \rightarrow F^p_\mu$ one gets the contribution of $s$- and $u$-channel nucleon Born terms (dotted curves). The parameterization of $F^p_\mu$ used here is given in [33] and follows the results of [60]. The difference between the dashed and dotted curves is the effect of nucleon resonances. As one can see, the excitation of resonances largely dominates the electroproduction cross section.

In Fig. 4 we show the $Q^2$ dependence of the longitudinal cross section $d\sigma_L/dt$ to the total unseparated cross section $d\sigma_U/dt$. The impact of $d\sigma_L/dt$ is marginal and the observed cross section is totally transverse. Furthermore, we find that the longitudinal cross section $d\sigma_L/dt$ decreases as a function of $Q^2$. This is in agreement with the two-component model of Ref. [27]. However, this is at variance with the present expectations based on pQCD models which rely on the dominance of the longitudinal cross section $d\sigma_L/dt$. To check this result against the data one needs the experimental Rosenbluth separation of $d\sigma_U/dt$ into $d\sigma_T/dt$ and $d\sigma_L/dt$ components.

In Fig. 5 we compare the model results with the interference cross section measured at JLAB [5]. Note that, in [5] the $\pi^0$ data have been presented using different conventions, see Eq. (2), for the partial cross sections. The interference term $d\sigma_{TT}/dt$ and $d\sigma_{TU}/dt$ are the same in both conventions. $d\sigma_{LT}/dt$ and $d\sigma_{LT}/dt$ are plotted according to (5).

Our calculations are in qualitative agreement with data and at forward angles result in positive values of $d\sigma_{LT}/dt$ and negative values of $d\sigma_{TT}/dt$ and $d\sigma_{LT}/dt$, interference cross section. For instance, the model of Ref. [38] agrees with the sign of $d\sigma_{LT}/dt$ and $d\sigma_{LT}/dt$ cross sections and predicts the positive values of $d\sigma_{LT}/dt$. Interestingly, the preliminary data from Hall C at JLAB [61] show also the positive $d\sigma_{LT}/dt$ at least at higher values of $-t$. Therefore, both the experimental and theoretical results concerning the sign of $d\sigma_{LT}/dt$ are still not settled.

In general, a nonzero $\sigma_{LT}$ or the corresponding beam SSA $A_{LU}(\phi)$, Eq. (6), demands interference between single helicity flip and nonflip or double helicity flip amplitudes. Furthermore, $\sigma_{LT}$ is proportional to the imaginary part of an interference between the $L/T$ photons and therefore sensitive to the relative phases of amplitudes. In Regge models the asymmetry may result from Regge cut corrections to single Reggeon exchange [30]. This way the amplitudes in the product acquire different phases and therefore relative imaginary parts. A nonzero beam SSA can be also generated by the interference pattern of amplitudes where particles with opposite parities are exchanged. In the following we discuss briefly the generic features of the beam SSA in the present model. For the recent calculations of the beam SSA at JLAB in the partonic and Regge models see Refs. [30, 38].

In Fig. 6 we present our results for the azimuthal moment $A^{\sin(\phi)}_{LU}$ associated with the beam SSA, Eq. (7), in the reaction
\[ -t + t_{\text{min}} \] dependence of \( d\sigma_U/dt = d\sigma_T + \varepsilon d\sigma_L/dt \) differential cross section in exclusive reaction \( p(\gamma^*, \pi^0)p \). The experimental data are from [5]. The numbers displayed in the plots are the average values of \((Q^2, x_B)\) for each bin. The solid curves are the model results and include the Regge and nucleon resonance contributions. The dash-dotted curves describe the Regge exchange contributions and the dashed curves describe the effect of nucleon resonances. The dotted curves are the contribution of the nucleon pole terms. The dash-dash-dotted curves describe the contribution of the longitudinal component \( \varepsilon d\sigma_L/dt \) to the total unseparated cross section \( d\sigma_U/dt \).

The CLAS/JLAB data are from [39]. Consider a mixture of \( \alpha_1(1260) \)-exchange generated by the exchange of Regge trajectories. In Fig. [6] the dashed curves describe the model results with \( \omega \)- and \( \rho \)-Regge trajectories alone. This simple Regge model results in a zero \( A_{\text{LU}}^{\text{sin}(\phi)} \) and therefore a zero beam SSA. Note that, in the charged pion production the addition of the unnatural parity \( a_1(1260) \)-exchange generates by the interference with the natural parity \( \rho(770) \) exchange a sizable \( A_{\text{LU}}^{\text{sin}(\phi)} \) in both \( \pi^+ \) and \( \pi^- \) channels. However, in the \( \pi^0 \) production the amplitude describing the unnatural parity axial-vector \( b_1 \) and \( h_1 \) exchanges does not interfere with the natural parity \( \omega(\rho) \) exchanges. Therefore, there is no way to generate the nonzero beam SSA in the model based only on the exchange of single Regge trajectories.

The \( s(u) \)-channel resonance contributions strongly influence the asymmetry parameter \( A_{\text{LU}}^{\text{sin}(\phi)} \). The solid curves describe the effect of nucleon resonances only and are in good agreement with data. The shaded bands describe the model results which include the Regge and \( s(u) \)-channel resonance contributions. The band takes into account the vector and axial-vector mesons. The width of the band reflects our estimations, see Fig. [2] of couplings and form factors of the axial-vector mesons. It indicates that in the present model the data do not require the presence of large axial-vector exchanges.

VII. DEEPLY VIRTUAL \( p(e, e'\pi^0)p \) AT HERMES

The HERMES collaboration at DESY already attempted to measure the exclusive \( \pi^0 \) electroproduction of protons in the DIS region [62]. The \( \pi^\pm \) data reported in Ref. [6] have been analyzed using the present model in Ref. [33]. In Fig. [7] we present our results for the exclusive reaction \( p(e, e'\pi^0)p \) in the kinematics at HERMES.

We choose the value of \( W = 4.21 \) GeV where the photoproduction data exist [53]. Then the behavior of the electroproduction cross section \( d\sigma_U/dt \) for the beam energy \( E_e = 27.7 \) GeV and as a function of \( Q^2 \) is shown for three bins \( Q^2 = 1, 3 \) and 5 GeV^2. These results correspond to the VMD cut-offs in the \( \gamma\omega\pi^0 \) and \( \gamma\rho\pi^0 \) form factors. As one can see, the very different \( Q^2 \) behavior of the Regge-exchange
FIG. 5. (Color online) \(-t + t_{\text{min}}\) dependence of the interference \(d\sigma_{LT}/dt\), \(d\sigma_{TT}/dt\) and \(d\sigma_{LT'}/dt\) differential cross sections in exclusive reaction \(p(\gamma^*, \pi^0)p\). The experimental data are from [5]. The dashed, dash-dash-dotted and dash-dotted curves are the model results for \(d\sigma_{LT}/dt\), \(d\sigma_{TT}/dt\) and \(d\sigma_{LT'}/dt\), respectively.

 connection between the exclusive form factors and inclusive deep inelastic structure functions. We have shown that with these components and using the same model parameters as in the charged pion production one can quantitatively describe the \(\pi^0\) cross sections and beam SSA measured at JLAB.

We have briefly discussed the real photon point where the exchanges of Regge trajectories dominate. Here the contribution of the \(\omega(782)\)-Regge trajectory plays an important role and should be considered as an indispensable part of any model. As a novel feature we demonstrated an interesting effect of \(s\)- and \(u\)-channel contributions around the region where the dip occurs.

However, in high \(Q^2\) electroproduction the importance of Regge and nucleon resonance reaction mechanisms is just opposite to that seen in photoproduction. With increasing value of \(Q^2\) the contributions of Regge exchanges decrease strongly and the reaction cross section is dominated by the excitation of nucleon resonances and corresponding large transverse contributions. The model describes the measured cross sections fairly well and clearly demonstrate the importance of resonances in the description of high \(Q^2\) electroproduction data.
FIG. 6. (Color online) The beam spin azimuthal moment $A_{LU}^{\sin(\phi)}$ in exclusive reaction $p(\gamma^*, \pi^0)p$ as a function of $-t$. The experimental data are from [39]. The dashed curves describe the results (the asymmetry is zero) based on exchange of $\omega, \rho, h_1$ and $b_1$ Regge trajectories (without the resonance contributions). The solid curves are the model results which account for the effect of nucleon resonances only. The shaded bands describe the calculations with Regge and resonances contributions and take into account the uncertainties in the couplings of the axial-vector mesons.
FIG. 7. (Color online) $-t + t_{\text{min}}$ dependence of the differential cross section $d\sigma_U/dt$ in photoproduction [53] ($Q^2 = 0$) and $d\sigma_U/dt$ in electroproduction for different values of $Q^2 = 1, 3$ and $5$ GeV in the kinematics of the HERMES experiment [62]. The solid curves are the model results and include the Regge and $s(u)$-channel resonance contributions. The dash-dotted curves describe the Regge exchange and the dash-dash-dotted curves describe the resonance components, respectively. The dashed curves describe the contribution of the longitudinal cross section $d\sigma_L/dt$.

at JLAB. Similar effects of the nucleon resonances have been already discussed in the electroproduction of charged pions.

We find that a model with $\omega$, $\rho$, $h_1$, and $b_1$ Regge exchanges does not generate any beam SSA. Since the axial-vector and vector meson exchanges do not interfere there is no way to generate the beam SSA just based on exchanges of single Regge trajectories. The positive beam SSA observed in the experiment is a result of the resonance contributions. This is again very similar to the charged pion production [33] where the nucleon resonances are at the origin of the nonzero beam SSA observed in the experiment.

ACKNOWLEDGMENTS

I am grateful to Prof. Ulrich Mosel for reading the manuscript and many useful comments. This work was supported by DFG through TR16 and by BMBF.

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