Effects of tensor and $T=0$ pairing interactions on nuclear $\beta^+/EC$ decay

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The Hartree-Fock-Bogolyubov (HFB) plus proton-neutron quasiparticle random phase approximation (pnQRPA) approach based on Skyrme interaction is applied to study the nuclear $\beta^+/EC$ decay for nuclei near the proton magic numbers $Z=20, 28,$ and $50$. With properly selected Skyrme interactions that include the tensor terms, and the $T=0$ pairing interaction, the experimental $\beta^+/EC$ decay half-lives of these nuclei can be systematically reproduced quite well. It is shown that the tensor and $T=0$ pairing interactions play different roles in different nuclei. The specific effect is relevant to the configurations that contribute to the decay. The attractive and repulsive properties of tensor interaction for the GT state is also studied. The present results indicate that it works repulsively on the GT states dominated by the configurations from $\pi j_2$ to $\nu j_2$ orbits, while working attractively for those mainly composed of the configurations from $\pi j_2$ to $\nu j_5$ orbits.

PACS numbers: 23.40.-s, 21.10.Tg, 21.60.Jz, 21.30.Fe

I. INTRODUCTION

Nuclear $\beta$ decay plays an important role in nuclear physics. The presumably most important reason is the role played by nuclear $\beta^-$ decay in nuclear astrophysics, since the $\beta^-$ decay and neutron capture account for the synthesis of elements heavier than iron, and thereby shape the nuclear isotopic abundance. Moreover, it also provides insights for nucleon-nucleon interactions.

Theoretically, besides the macroscopic gross theory [1], two main microscopic models, i.e., the shell model [2, 3] and pnQRPA approach [4, 10], have been widely applied to describe and predict the $\beta$ decay half-lives. In these works, the isoscalar ($T=0$) proton-neutron pairing interaction was claimed to have a strong effect on the $\beta$ decay half-lives [3, 7], as it shifts the low energy Gamow-Teller (GT) state downward and, therefore, serves to reduce the half-lives. Another important interaction that may not be neglected is the tensor interaction, since it can significantly shift the low-energy GT states and influence the $\beta^-$ decay half-lives [11, 12, 19]. Recently, extensive evaluations for $\beta^-$ decay half-lives have been done in Ref. [12] by using Skyrme-type interactions with the above two interactions been taken into account.

In this work, the HFB+pnQRPA approach based on Skyrme interactions is applied to study the nuclear $\beta^+/EC$ decay for nuclei around proton magic numbers $Z=20, 28,$ and $50$. Both the effects of the tensor and $T=0$ pairing interactions will be studied.

This paper is organized as follows. In Sec. II some necessary details about the HFB+pnQRPA approach and essential formulas relevant to the calculation of half-lives for nuclear $\beta^+/EC$ decay are presented. In Sec. III we further make choice of Skyrme parameter sets. Effects of the tensor and $T=0$ pairing interactions are discussed in Sec. IV. The summary is made in Sec. V.

II. FORMULISM

In our HFB+pnQRPA calculation, the zero-range Skyrme interactions are employed, and the Skyrme tensor terms read [21]:

$$V_T = \frac{T}{2} \{[(\sigma_1 \cdot k')(\sigma_2 \cdot k') - \frac{1}{3} (\sigma_1 \cdot \sigma_2) k'^2] \delta(r) + \frac{2}{3} [(\sigma_1 \cdot k') \cdot k] \delta(r) + \frac{2}{3} (\sigma_1 \cdot k') \cdot \delta(r) k\},$$

where $T$ and $U$ are the strengths of triplet-even and triplet-odd tensor terms, respectively, and the operator $k'=(\nabla_1 - \nabla_2)/2i$ acts on the right and $k'=-((\nabla_1 - \nabla_2)/2i$ acts on the left. In HFB calculation, tensor force only contributes to the spin-orbit potential, which is given by [22, 23]

$$U_{so}^q = \frac{W_0}{2r} \left( \frac{d \rho_q}{dr} + \frac{d \rho_q'}{dr} \right) + \left( \frac{J_q}{r} + \beta \frac{J_{q'}}{r} \right),$$

where $q$ labels either proton or neutron, $\rho$ is the nucleon density, $W_0$ is the strength of the two-body spin-orbit interaction, and $J$ is the spin-orbit density. $\alpha$ and $\beta$ receive the contribution of both the central and tensor terms:

$$\alpha = \alpha_C + \alpha_T, \quad \beta = \beta_C + \beta_T,$$
with
\[
\alpha_C = \frac{1}{8}(t_1 - t_2) - \frac{1}{8}(t_1 x_1 + t_2 x_2),
\]
\[
\beta_C = -\frac{1}{8}(t_1 x_1 + t_2 x_2),
\]
\[
\alpha_T = \frac{5}{12} U, \quad \beta_T = \frac{5}{24} (T + U).
\]  

For the pairing force, in the isovector \((T = 1)\) channel, the zero-range density-dependent surface pairing interaction is adopted in both HFB and QRPA calculations as it affects the Fermi transition \([23]\), and takes the form \([24]\):
\[
V_{T=1}(r_1, r_2) = V_0 \frac{1 - P_\sigma}{2} \left(1 - \frac{\rho(r)}{\rho_0}\right) \delta(r_1 - r_2),
\]  
in which \(r = (r_1 - r_2)/2\), \(\rho_0\) is taken to be 0.16 fm\(^{-3}\), and \(P_\sigma\) is the spin exchange operator. \(V_0 = -470.0\) MeV \(\cdot\) fm\(^3\) is determined by adjusting the empirical pairing gaps of the nuclei studied in this work. The \(T = 0\) pairing interaction is employed in the QRPA calculation and takes the similar form as that in the \(T = 1\) channel \([26]\):
\[
V_{T=0}(r_1, r_2) = fV_0 \frac{1 + P_\sigma}{2} \left(1 - \frac{\rho(r)}{\rho_0}\right) \delta(r_1 - r_2),
\]  
in which \(f\) denotes the strength for the \(T = 0\) pairing interaction relative to that in the \(T = 1\) channel, and is taken to be 1.1 in this work, which is very close to the value recommended in Ref. \([27]\).

The pnQRPA calculation is based on the ground state of the parent nuclei calculated by solving the HFB equation in coordinate space \([24, 28]\). Under the canonical HFB basis \([23, 30]\), the pnQRPA equation takes the form:
\[
\begin{pmatrix} A \\ -B \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E_{QRPA} \begin{pmatrix} X \\ Y \end{pmatrix},
\]  
in which \(X\) and \(Y\) denote the forward and backward amplitudes, \(E_{QRPA}\) is the QRPA phonon energy, and
\[
A_{pn, p', n'} = E_{pp'} \delta_{n'n} + E_{nn'} \delta_{pp'} + V_{pp, p', n'}^{ph}(u_p v_{n'} + v_{p'} u_n) + v_p u_{n'} v_{p'} u_n + v_{p'} u_n v_p u_{n'} u_n)
\]  
where \(p, p', \) and \(n, n'\) denote the quasiparticle canonical states of proton and neutron, respectively. \(u\) and \(v\) are the Bogolyubov-Valatin transformation factors. \(V_{pp, p', n'}^{ph}\) and \(V_{pp, p', n'}^{pp}\) are the matrix elements for the residual interaction in the particle-hole (ph) and particle-particle (pp) channels, respectively.

After diagonalizing the QRPA matrix, the strength associated with operator \(\hat{O}_+\) in \(t_+\) channel is calculated by
\[
B^\nu_+ = \sum_{pn} (X^p_{pn} v_p u_{n'} + Y^p_{pn} u_p v_{n'}) \langle p | \hat{O}_+ | n \rangle^2,
\]  
in which \(\hat{O}_+ = \sum_i A_i \hat{\sigma}_i\) for GT transition, and \(\hat{O}_+ = \sum_i t_{+i}\) for Fermi transition. For a specific configuration \(pn\), we can define a relative transition amplitude:
\[
R_{pn} = \frac{(X^p_{pn} v_p u_{n'} + Y^p_{pn} u_p v_{n'}) \langle p | \hat{O}_+ | n \rangle^2}{B_+},
\]  
where
\[
C_{pn} = X^2_{pn} - Y^2_{pn}
\]  
is the normalization factor. In this work, we will use \(R_{pn}\) and \(C_{pn}\) to evaluate the contribution of a particular \(pn\) configuration to a given collective excited state.

The half-life for allowed \(\beta\) decay is given by \([28]\)
\[
T_{1/2} = \frac{D}{\sum \left| \langle x | V_{eff} | f \rangle \right|^2 B_{GT}(E_\nu) + B_F(E_\nu) f(Z, E_\nu)},
\]  
in which \(D = 6163.4\) s, and \((g_A/g_V)_{eff} = 1\) is adopted in our calculation. \(B_F(E_\nu)\) and \(B_{GT}(E_\nu)\) are the transition strengths for the allowed Fermi (F) and GT transition, which are calculated by the pnQRPA. In \(\beta^+ /\beta^-\) decay, \(f(Z, E_\nu)\) consists of two parts: positron emission \((f^{\beta^-})\) and electron capture \((f^{EC})\). For positron emission, the Fermi integrals \(f^{\beta^-}\) is given by
\[
f^{\beta^-}(Z, E_m) = \int_{m_e}^{E_m} p_e E_e (E_m - E_e)^2 F_0(Z, E_e) dE_e,
\]  
where \(p_e\) and \(E_e\) are the emitted positron momentum and energy, \(m_e\) is the positron mass, and \(F_0(Z, E_e)\) is the Fermi function. \(E_m\) is the \(\beta^+\) decay energy, which can be estimated by
\[
E_m = -\Delta_n H - m_e - E_x,
\]  
in which \(\Delta_n H = 0.78227\) MeV is the mass difference between the neutron and hydrogen atom. \(E_x = E_{QRPA} - (\lambda_p - \lambda_n)\) is the QRPA energy with respect to the ground state of the parent nucleus corrected by the difference of the proton and neutron Fermi energy in the parent nucleus. The \(\beta^+\) decay energy must be higher than the rest mass of the positron, therefore, the energy threshold for \(\beta^+\) decay is \(E_x < -\Delta_n H - 2m_e\). The decay function \(f^{EC}\) for electron capture is given by
\[
f^{EC} = \frac{\pi}{2} \sum \frac{q_x^2 q_z^2 B_x}{x}.
\]  
where \(x\) denotes the atomic subshell from which the electron is captured, \(q\) is the emitted neutrino energy, \(g\) and \(B\), which are calculated by the Dirac-Hartree-Fock approach \([31, 32]\), are the radial component of the bound-state electron wave function at the nuclear surface, and the correction factor that includes the exchange and overlap corrections \([33, 34]\), respectively. And the threshold for EC is \(E_x < -\Delta_n H\).
III. CHOICE OF THE SKYRME PARAMETER SETS

This work is devoted to study the \( \beta^+ / EC \) decay half-lives of nuclei around the proton magic numbers \( Z = 20, 28, \) and \( 50 \), as our present HFB+pnQRPA are done in spherical approximation. In the present calculations, the Skyrme parameter \( T21, T32, T43, \) and \( T54 \) from \( TJJ \) family are employed \([35]\), since they include the central and tensor terms on equal footing, and were checked to give reasonable results for the centroid energies of GT and charge-exchange spin-dipole (SD) transitions in \( ^{90} \)Zr and \( ^{208} \)Pb as shown in our previous work \([36]\).

Fig. 1 shows the calculated half-lives with these four Skyrme parameter sets as well as the experimental results. It is seen that the four parameter sets give similar half-lives for Ar, Ca, Ti, Fe, Ni, and Zn isotopes. However, in Cd and Sn isotopes, there is great discrepancy among the results of these four Skyrme parameter sets. As we will show in Sec. IV, the main configurations that contribute to the decay of Ar, Ca, Ti, Fe, Ni, and Zn isotopes are of \((\pi j_2 \rightarrow \nu j_2)\) type, while they are of \((\pi j_3 \rightarrow \nu j_3)\) type for Cd and Sn isotopes. It is known that the \( TJJ \) parameter sets are fitted by the definition that \( \alpha = 60( J - 2) \) MeV \( \cdot \) fm\(^2\) and \( \beta = 60( I - 2) \) MeV \( \cdot \) fm\(^2\). The spin-orbit splitting in the four interactions are quite different due to the different values of \( \alpha \) and \( \beta \). The excitation energies of configurations from \( \pi j_2 \) to \( \nu j_2 \) orbits are more sensitive to the change of spin-orbit splitting as the energy difference between the \( \pi j_2 \) and \( \nu j_2 \) orbitals is readily to be amplified in these nuclei, in which the spin-orbit splittings for proton and neutron are increased or decreased simultaneously. However, the excitation energies of configurations from \( \pi j_2 \) to \( \nu j_2 \) orbits are quite stable as the energy changes of \( \pi j_2 \) and \( \nu j_2 \) orbitals due to the spin-orbit splitting altering tend to be cancelled. This indicates that reasonable choice of \( \alpha \) and \( \beta \), and hence the tensor interaction is essential for theoretical study of \( \beta \) decay. Specifically, among these four parameter sets, Fig. 1 suggests that T32 and T43 are better choices for the present topic.

IV. EFFECTS OF THE TENSOR AND \( T = 0 \) PAIRING INTERACTIONS

In this section, the effects of the tensor and \( T = 0 \) pairing interactions on the \( \beta^+ / EC \) decay half-lives will be studied based on the results calculated with T32 and T43. As T32 produces quite similar results, we only show the results produced by T43 for simplicity in the following discussion. The \( \beta^+ / EC \) decay half-lives for Ar, Ca, Ti, Fe, Ni, Zn, Cd, and Sn isotopes calculated by HFB+pnQRPA with T43 are shown in Fig. 2. Since both the tensor and \( T = 0 \) pairing interactions may affect the half-lives, three kinds of results are presented for comparison: 1) neither the tensor nor the \( T = 0 \) pairing interactions are included, and the results are labelled by “np nt”; 2) only the \( T = 0 \) pairing interaction is taken into account, and the results are denoted as “wp nt”; 3) both the tensor and \( T = 0 \) pairing interactions are included, and the relevant results are labeled by “wp wt”.

From Fig. 2, one can see that the \( T = 0 \) pairing interaction plays an important role in the evaluation of half-lives for Ar, Ti, and Zn isotopes, while the effect of tensor interaction on the half-lives of these isotopes is quite trivial. The dominant configurations and relevant quantities of the GT states that are important for the evaluation of \( \beta^+ / EC \) decay half-lives of these isotopes are listed in Table I, in which both the \( T = 0 \) pairing interaction (with \( f = 1.1 \)) and the tensor interaction have been taken into account. It is shown that the important GT states of these isotopes are mainly composed of configurations that receive contribution from the pp channel more easily. As a result, the \( T = 0 \) pairing interaction plays a determinant role for the decay rates of these isotopes. Moreover, the good agreement between our theoretical results and the measured half-lives of these nuclei indicates that the present choice for the strength of the \( T = 0 \) pairing interaction is reasonable.

On the contrary, for \( ^{50} \)Ni and Sn isotopes, the tensor interaction plays a dominant role while the impact of \( T = 0 \) pairing interaction is quite small. As tabulated
in Table I, the configurations that are important for the decay can more readily receive contribution from the ph channel. Therefore, interactions in the ph channel, specially the tensor interaction, play an important role in the evaluation for half-lives of these nuclei.

Apart from the above two situations, there are also many nuclei in which both the tensor and $T = 0$ pairing interactions have non-negligible impact. As listed in Table III, the main configurations that are important for the decay can conveniently receive contributions from both the pp and ph channel. Thus, both the $T = 0$ pairing and tensor interactions can obviously influence the calculated half-lives of these nuclei. Specially, by comparing the results of Fe or Ni isotopes, one can find that as more neutrons occupy in the $\nu 1f_{7/2}$ orbital, the effect of $T = 0$ pairing interaction relative to that of the tensor force gradually become larger. This indicates that the $T = 0$ and tensor interactions compete with each other in some nuclei whose decay are mainly built from some specific configurations.

Furthermore, as shown in Fig. 2 the attractive $T = 0$ pairing interaction universally helps to reduce the half-lives as it can shift the excited states downward to low energy region, which has already been demonstrated in many works. The effects of the tensor interaction on the half-lives are not so monotonous: it reduces the calculated half-lives in Cd and Sn isotopes, while increases the calculated half-lives in Ca, Fe, and Ni isotopes. This indicates that tensor interaction can work attractively and repulsively. Moreover, it is seen that the inclusion of tensor interaction generally helps to improve the theoretical results: it increases the half-lives when the $T = 0$ pairing interaction reduces it too much, while decreases the half-lives when the $T = 0$ pairing interaction does not reduce it sufficiently.

The excitation energies ($E_x$) of the GT states important for the $\beta^+/EC$ decay of Ca, Fe, Ni, Cd, and Sn isotopes calculated with and without the tensor interaction are listed in Table IV. From Table IV, one may readily see that the tensor interaction shifts the low energy GT states of Ca, Fe, and Ni isotopes upward, while

![Figure 2](image_url)
TABLE III: Same as Table II but for Ca, Fe, $^{52}$Ni, $^{54}$Ni, and Cd isotopes.

| nuclei | $E_x$ | $B$(GT) | $\pi(v_j^2)$ | $\nu(v_j^2)$ | $R$ | $C$ |
|--------|-------|---------|--------------|--------------|-----|-----|
| $^{46}$Ca | -9.03 | 1.57 | $2s_{1/2}(1.000)$ | $2s_{1/2}(0.841)$ | 0.211 | 0.353 |
| $^{38}$Ca | -6.48 | 1.58 | $1d_{3/2}(0.806)$ | $1d_{3/2}(0.443)$ | 0.526 | 0.879 |
| $^{46}$Fe | -11.56 | 7.53 | $1f_{7/2}(0.667)$ | $1f_{7/2}(0.000)$ | 0.861 | 0.950 |
| $^{48}$Fe | -9.86 | 5.44 | $1f_{7/2}(0.681)$ | $1f_{7/2}(0.232)$ | 0.787 | 0.906 |
| $^{50}$Fe | -7.87 | 2.70 | $1f_{7/2}(0.695)$ | $1f_{7/2}(0.463)$ | 1.030 | 0.972 |
| $^{52}$Ni | -10.48 | 3.05 | $1f_{7/2}(1.000)$ | $1f_{7/2}(0.467)$ | 1.811 | 1.012 |
| $^{54}$Ni | -8.79 | 1.37 | $1f_{7/2}(1.000)$ | $1f_{7/2}(0.715)$ | 2.190 | 1.023 |
| $^{98}$Cd | -3.22 | 6.66 | $1g_{9/2}(0.729)$ | $1g_{9/2}(0.000)$ | 0.926 | 0.481 |
| $^{100}$Cd | -3.32 | 3.26 | $1g_{9/2}(0.731)$ | $1g_{9/2}(0.055)$ | 0.498 | 0.135 |

TABLE IV: The excitation energies ($E_x$) of the GT states that play the most important role in the evaluation of $\beta^+/EC$ decay half-lives of Ca, Fe, Ni, Cd, and Sn isotopes, in unit of MeV. The HFB+QRPA calculations are done with and without tensor interaction, and the results are labelled by $ut$ and $nt$, respectively. The $T = 0$ pairing interaction is included with $f = 1.1$.

| nuclei | $E_{ut}$ | $E_{nt}$ | nuclei | $E_{ut}$ | $E_{nt}$ |
|--------|---------|---------|--------|---------|---------|
| $^{36}$Ca | -10.95 | -9.03 | $^{54}$Ni | -9.41 | -8.79 |
| $^{38}$Ca | -6.92 | -6.48 | $^{98}$Cd | -3.56 | -4.19 |
| $^{46}$Fe | -12.86 | -11.56 | $^{100}$Cd | -3.11 | -3.32 |
| $^{48}$Fe | -10.72 | -9.86 | $^{100}$Sn | -4.32 | -5.33 |
| $^{50}$Fe | -8.42 | -7.87 | $^{102}$Sn | -3.15 | -4.05 |
| $^{52}$Ni | -13.80 | -12.26 | $^{104}$Sn | -2.30 | -3.25 |

The HFB+puQRPA approach based on Skyrme-type interaction has been applied to study the nuclear $\beta^+/EC$ decay for nuclei around the proton magic numbers $Z = 20, 28, 50$. With properly selected $T = 0$ pairing interaction and Skyrme interaction that includes the tensor terms, the experimental half-lives of these nuclei can be well reproduced.

It is shown that the tensor and $T = 0$ pairing interactions play different roles in different nuclei. The specific effect is relevant to the configurations that contribute to the decay. For Ar, Ti, Zn isotopes, the $T = 0$ pairing interaction plays a dominant role, while for $^{50}$Ni, and Sn isotopes, the tensor force is much more important. Besides, for Ca, Fe, $^{52}$Ni, $^{54}$Ni, and Cd isotopes, both the $T = 0$ pairing and tensor interactions may make the differences. Moreover, our present studies indicate that the tensor interaction works repulsively for the GT states dominated by the configurations from $\pi j_{\geq 2}$ to $\nu j_{\leq 2}$ orbitals, while working attractively for those mainly composed of the configurations from $\pi j_{\leq 2}$ to $\nu j_{\geq 2}$ orbitals. This property of the tensor interaction is consistent with the one in which the tensor interaction works on the single-particle states, as shown in shell model and mean field calculations.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grant Nos. 11575120 and 11375266.

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