Is there a risk and return relation?

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ABSTRACT

Traditional finance theory posits that the relation between the risk and return of stocks is positive. Equally, investment practice is often based on the contention that high (low) beta stocks earn higher (lower) returns. However, this fundamental relation is questioned by several researchers, who present mixed evidence. The purpose of this paper is to shed further light on this question by examining both market- and firm-level price data; employing a battery of tests, including individual market, panel and quantile regressions; analysing the nature of the relation during periods of high and low volatility and in bull and bear markets. The results indicate that there is no single robust relation between risk and return. Notably, the results suggest a positive relation when returns are high and during bear markets. Further, the finding of a positive relation is stronger at the market-level than the firm-level and over long time periods. However, a negative relation exists at low return levels, during bull markets and, even more so, at the individual firm level. Overall, the results suggest that the risk-return relation is switching in nature and is primarily driven by changing risk preferences. A positive relation exists when macroeconomic risk plays a larger role.

1. Introduction

The presence of a positive risk and return relation lies at the heart of finance theory that underpins our view of asset valuation. This idea contends that investors are risk averse and thus demand a premium for bearing risk, which generates a positive risk-return trade-off. This positive relation underpins a range of asset pricing models that link the expected return on an asset to a proxy for risk, either through its own variance (volatility) or its covariance with a wider market portfolio. In a static single-period framework, this arises through the Capital Asset Pricing Model (CAPM; Sharpe 1964; Lintner, 1965; Mossin 1966), which relates the required (or expected) rate of return on a stock to the market portfolio, as measured by the stock’s beta. Equally, a positive relation arises in a time-varying multi-period context through the Intertemporal Capital Asset Pricing Model (ICAPM; Merton 1973), which posits that an asset’s return is a function of its own conditional variance (risk) as well as the covariance with other common factors relating to the marginal utility of investors. Thus, both approaches lead to the fundamental belief of a positive risk and return relation.

However, this view has been repeatedly questioned over time. For example, research dating back to Black (1976), Christie (1982), French, Schwert, and Stambaugh (1987) and Bekaert and Wu (2000) question the existence of a positive risk-return relation. Indeed, this stream of research argues that returns and volatility are negatively related, and that the nature of the relation may be asymmetric across market states. Since this early research, the nature of the risk-return relation has attracted extensive scrutiny in the academic literature.\textsuperscript{1} Indeed, the risk-return trade-off remains one of the most hotly-debated puzzles in finance (Markowitz and Blay 2013; Sevi 2013).
Thus, the current state of the literature is mixed between research that supports a positive relation (for example, Jiang and Lee 2014; Hedegaard and Hodrick 2016) and studies that suggest a negative relation (for example, Aslanidis, Christiansen, and Savva 2016; Badshah et al. 2016). Moreover, a strand of research argues that the nature of the relation is regime dependent and varies according to economic or market conditions and volatility. Notably, Christensen, Nielsen, and Zhu (2015) argue that the US risk-return relation is significantly positive only during crisis periods. Whitelaw (1994) and Ludvigson and Ng (2007) argue that the strength of the risk and return relation should be conditional upon a set of macroeconomic variables, while Liu (2017) notes that the relation generally moves procyclically with the business cycle. In a different vein, Kinnunen (2014), while generally supportive of a positive risk-return relation, argues its nature varies with volatility, such that the strength of the relation weakens in low volatility periods.

The purpose of this paper is to shed further light on the risk-return trade-off puzzle. The paper is innovative in several ways. First, the paper bridges the gap in the substantive literature by examining the nature of the risk-return relation using both market- and firm-level data. Second, the paper examines the relation for a broad cross-section of developed and emerging stock markets. As emerging markets typically exhibit high expected returns and high volatility (Bekaert et al. 1998), it is reasonable to expect that the risk-return relation in these markets is different from that in more mature markets. Furthermore, the nature of the risk-return relation for emerging stock markets has received relatively little attention in the academic literature to date, despite their economic importance and the efforts many of these countries are making towards financial market integration. Third, the paper investigates whether the nature of the risk-return relation is linked to different market states, including periods of high and low volatility and bull and bear markets. Finally, a number of empirical approaches are employed to model the relation. Specifically, the paper utilises standard linear, panel and quantile regression analysis, as well as different measures of risk and return. Thus, the paper seeks to provide firm evidence on the nature of the risk-return relation. The use of alternative data sets and methodologies is designed to address the issues highlighted by Hansen and Richard (1987), Harvey (2001) and Ludvigson and Ng (2007) who argue that limited degrees of freedom and reliance on a given model for conditioning the mean or variance can result in misleading results. Thus, the consideration of a range of data and modelling approaches should provide more robust results.

The remainder of the paper is organised as follows. To establish the background for the analysis, a brief review of the literature examining the risk-return relation is presented in Section 2. Section 3 introduces the dataset, while Section 4 discusses the empirical methods employed in the paper. The empirical results are discussed Sections 5 and 6. The final section offers several concluding observations and outlines the implications of the results for asset pricing.

2. Review of the literature.

The underlying concept that motivates the paper is the assertion that holds in our main asset pricing models of a positive relation between return and risk (volatility or standard deviation). This arises from our view of how investors behave in respect of forming mean-variance efficient portfolios based on a range of (standard) assumptions including risk-aversion. The result of investors forming such portfolios is the above noted positive relation, with the combination of individual portfolios resulting in the market portfolio, which (in equilibrium) will produce the highest available Sharpe ratio. This view of investor behaviour in forming portfolios leads to both the CAPM and ICAPM approaches noted in the Introduction. The CAPM is essentially a static (one-period) model that relates an individual asset or portfolio to the market portfolio, while the ICAPM is a multi-period model that relates the behaviour of the market portfolio to its own variance. Although both approaches support a positive risk and return trade-off, as they both assume mean-variance efficiency on behalf of investors, the empirical modelling in this paper builds on the Merton approach by considering the behaviour of the risk and return relation over time. Briefly, Merton (1973) states the following relation:

$$E_{t-1}(r_t) = \mu + \gamma Var_{t-1}(r_t)$$

where $\gamma$ is the time-invariant price of risk and determines the nature of the relation between expected returns, $E_{t-1}(r_t)$, and their conditional variance, $Var_{t-1}(r_t)$. A key issue in this literature is that both sides of equation (1)
are unobservable. While much of the literature focuses on the conditional variance, for which a range of empirical models are presented and discussed below, the expected return is also unobservable. In an attempt to address this, Whitelaw (1994) and Ludvigson and Ng (2005) model the conditional mean and variance as functions of other financial and macroeconomic variables, while Brandt and Wang (2010) use the cross-sectional information from a Fama-French three-factor model to derive the time-varying risk and return relation. Brandt and Wang (2010) note that the modelling assumptions used in estimating an empirical version of equation (1) can affect the nature of the coefficient defining the relation. This point is further considered by Adcock (2013) who shows that the shape of the efficient frontier can change even when assuming the covariance matrix is known but replacing the expected returns with an estimated value.

Two empirical approaches are adopted in the substantive literature to examine the risk-return relation, which accord with the static (CAPM) and time-varying (ICAPM) asset pricing models noted above. The first approach, which is most closely related to this paper, employs the ICAPM model and tends to focus on index-level data. It employs relatively sophisticated econometric techniques to examine the behaviour of, and interaction between, the conditional mean and conditional variance. By contrast, the second strand, which focuses on the CAPM, analyses firm-level stock price data and typically examines the difference in the returns of portfolios characterised by different levels of volatility.

In terms of the first approach, a recent study by Badshah et al. (2016) employs a quantile regression approach to examine the intra-day return-volatility relation at return horizons of 1, 5, 10, 15, 60 min and one day using data for the S&P 500 over the period September 2003 to December 2011. They find evidence of a strong negative relation between risk and return. Moreover, they find evidence of an asymmetric relation, whereby the effects of positive and negative returns on volatility are different and more pronounced for negative returns and in the tails of the conditional distribution of volatility changes. However, they note that this asymmetry tends to disappear at the daily return horizon. The finding of a negative relation between risk and return is also supported by Aslanidis, Christiansen, and Savva (2016) who employ a Markov-switching approach to study 13 European stock markets over the period 1986–2012. They find evidence of a negative risk-return trade-off that is strongest at the lowest quantile. The authors also document time variation in the trade-off that is linked to the state of the economy.²

However, several studies argue that the relation between risk and return is positive. For example, Frazier and Liu (2016) use a copula approach and find evidence of a positive risk-return trade-off for four international stock market indices that is driven by market timing and skewness. Similarly, Bali, Demirtas, and Levy (2009) find evidence of a significant and positive relation between downside risk and return for a portfolio of US equities, while Breckenfelder and Tédongap (2012) and Sevi (2013) validate this finding using intra-day high-frequency stock returns. Bollerslev et al. (2013) argue that the positive risk and return relation is revealed when utilising fractional integration models that capture longer-horizon information. In a more recent analysis, Chang (2016) documents a positive relation between risk and return, the strength of which varies according to different stock market conditions; specifically, the relation is stronger during bear markets as compared to bull market periods. By employing a common information set to measure expected excess return and conditional variance, Jiang and Lee (2014) detect a positive relation that is robust across different time intervals. Hedegaard and Hodrick (2016) confirm this finding using an overlapping data inference approach to the relation using data for the US market. Similarly, Ghysels, Plazzi, and Valkanov (2016) confirm the existence of the traditional risk-return relation using a MIDAS approach. However, they find evidence of fundamental changes in the relation during periods of financial crisis. Notably, the strength of the relation varies with the level of volatility, and is also documented by Salvador, Floros, and Arago (2014) for a sample of 11 European markets and by Wu and Lee (2015) for the US market.³ Christensen, Nielsen, and Zhu (2015) find that the risk-return relation is significantly positive only during crisis periods for the US (for example, the 1970s oil price shock, the stock market crashes of 1987 and 2000, the 9/11 terrorist attacks and the 2007/08 global financial crisis). In a similar vein, Whitelaw (1994), Ludvigson and Ng (2007) and Liu (2017) seek to condition the risk and return relation on macroeconomic conditions and report more supportive evidence. Liu (2017) notes that the relation generally moves procyclically with the business cycle, indicating the existence of negative trade-off periods, notably during recessions of the early 1980s, 2000s and the 2007/08 financial crisis period. Liu also reports shorter periods where the risk and return relation moves counter-cyclically and strengthens during the 1990s recession.
Of course, some authors report mixed results. For example, in a comprehensive study of 37 stock markets, Bali and Cakici (2010) find that the risk-return trade-off varies across countries. Galagedera, Maharaj, and Brooks (2008) find evidence of a positive risk-return relation only (i) when risk is measured by downside co-skewness; and (ii) for longer timescales. Kinnunen (2014) generally supports a positive relation but argues its nature varies with volatility, the strength of which weakens in low volatility periods. Some authors attempt to provide clarity on the mixed nature of results. For example, Fenou, Jahan-Parver, and Tedongap (2013) and Cheng and Jahan-Parver (2014) suggest that the failure to find a positive relation may result from a need to model skewness behaviour in stocks as well as means and variances. In a different vein, Wang and Yang (2013) argue that the offsetting nature of a volatility feedback mechanism may lead to confounding effects on the positive risk-return relation.

Jia and Yang (2017) argue that the extent of disagreement in the market is linked to the nature of the risk and return trade-off. Where disagreement is based on whether trades are buyer or seller initiated, Jia and Yang note that an increase in disagreement is associated with a positive relation, while the converse is true with a decrease.

The second approach to investigating the risk-return relation typically focuses on individual stock level data and is based on the static CAPM. This body of work typically involves constructing portfolios of stocks according to their degree of volatility and examining whether portfolios that are characterised by high volatility are associated with higher returns. A series of papers have identified a negative risk-return relation for US stock markets. For example, Haugen and Baker (1991) find that low risk portfolios earn higher returns relative to a market capitalisation-weighted benchmark. The issue has attracted renewed interest in recent years and the finding of a negative relation between risk and return has been confirmed for the US stock market (Jagannathan and Ma 2003; Blitz and van Vliet 2007), as well as other developed markets (Ang et al. 2006; Blitz et al., 2013; de Carvalho, Xiao, and Moulin 2012). More recently, Baker and Haugen (2012) document comprehensive evidence of a negative risk-return relation for a sample of 33 developed and emerging markets that has been evident since 1990. Notably, Baker and Haugen construct portfolios according to past volatility for nearly all firms in each of the 33 markets and show that a hedged portfolio of low volatility minus high volatility stocks earns a positive return. They further argue for, and demonstrate, an inverse risk and return trade-off. This literature also ties in with the recent discussion surrounding a low volatility anomaly, which states that low volatility stocks earn higher returns than high volatility stocks (Baker, Bradley, and Wurgler 2011). Further insight into this anomaly has recently been provided by Wang, Yan, and Yu (2017) who find evidence of heterogeneity in the risk-return trade-off. Specifically, they note a positive risk-return relation for firms in which investors face capital gains and a significant inverted risk-return relation among firms for which investors face a capital loss.

Overall, this review indicates that a dichotomy exists between studies that examine the risk-return relation at the market-level, which exploit the time-varying ICAPM, and those that analyse firm-level data, which make use of the static CAPM. Notably, the results from studies of market-level data are mixed; while several studies find evidence of a positive risk-return relation, others document a negative relation. In addition, some argue that the nature of the trade-off varies across, for example, market states or according to other related factors. By contrast, analyses that employ firm-level data are more conclusive and assert that there is a negative relation between risk and return. However, despite this evidence, there remains a belief that a positive relation should exist. Indeed, Baker and Haugen (2012) argue that the positive risk-return paradigm should fall but remains rooted in our understanding of finance.

This paper provides a comprehensive analysis of the risk-return relation by examining both market index and firm-level data and utilising tools from both strands of the literature. While the ICAPM and CAPM approaches imply different empirical specifications, they both rely on the underlying assumptions with regard to risk-aversion and the construction of mean-variance efficient portfolios and a positive risk-return relation. Thus, this paper furthers our understanding of both the existence and the nature of the risk-return relation and attempts to provide clarity with respect to a key puzzle in empirical finance.

3. Data

Three broad datasets are employed to examine the stock market risk and return relation. The first dataset consists of monthly observations for a broad cross-section of 43 international stock markets over the period January
1973 to December 2014. In particular, Datastream is used to obtain the total market index for each of the sample countries: Australia, Austria, Belgium, Brazil, Canada, China, Cyprus, the Czech Republic, Denmark, Finland, France, Germany, Greece, Hong Kong, India, Indonesia, Ireland, Italy, Japan, Luxembourg, Malaysia, Mexico, the Netherlands, Norway, New Zealand, Pakistan, the Philippines, Poland, Portugal, Russia, Singapore, South Africa, South Korea, Spain, Sri Lanka, Sweden, Switzerland, Taiwan, Thailand, Turkey, the UK, the US, and Venezuela. Stock returns are calculated as the first-difference of the natural logarithm of the price index series. The risk series is calculated as the two-year moving average of the standard deviation of returns. The selection of markets is motivated by the desire to consider a wide range of both developed and emerging markets. Notably, should the positive risk-return relation be a pervasive phenomenon within financial markets then we would expect to find it regardless of the market concerned. The second dataset used in the study is obtained from Dimson, Marsh and Staunton (2011). This dataset includes annual price data for 17 markets over the period 1900–2010: Australia, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, South Africa, Spain, Sweden, Switzerland, the UK and the US. Stock returns and risk are calculated in the same fashion as outlined above, although a five-year moving average of the standard deviation is calculated due to the annual data frequency.

In contrast to the first two market-level datasets, the third dataset consists of individual firm data. Specifically, daily data is obtained from Datastream over the period 3rd January 2000 to 31st December 2013 for all available firms in the S&P 500 for the US, the FTSE 350 for the UK, the DAX, MDAX and SDAX for Germany, the SBF 120 for France, the MIBTEL for Italy, the TTOCOMP for Canada, the Nikkei 225 for Japan, the ASX 200 for Australia, the Hang Seng for Hong Kong and the KOSPI for South Korea. The individual stocks are selected based on their inclusion in the main market indices of each market. Thus, the data consist of firms that are broadly equivalent across the sample markets. In addition, the data selection criterion excludes the potential for very small firms to be included in the analysis. Returns and risk for each sample firm are calculated in the manner outlined above, although a four-year rolling window is used to compute standard deviation.

The three datasets are included in the analysis in order to provide a comprehensive examination of the nature of the risk-return relation. The two market-level datasets facilitate an examination of the risk-return relation across both a recent and extended time period and for a wide range of both developed and emerging markets. These data are sampled at a frequency that is consistent with the time horizon that a portfolio manager would typically adopt (monthly data), as well as a longer time horizon that encapsulates a greater number of market phases (annual data). Further, the use of a long-time span is also in keeping with the argument of Lunblad (2007) that a lengthy time-series of data is needed to reliably estimate the risk-return relation. The annual data also serves to reduce the influence of noise that is often apparent at higher frequencies. The final dataset, which consists of individual firm-level data, captures different information. That is, while investor behaviour is likely to be dominated by expected future economic performance and macroeconomic risk at the market level, at the firm level, information regarding cashflows, earnings, dividends and the future prospects of the firm is pertinent to investors (Jung and Shiller 2005). Thus, market- and firm-level data capture different information and, consequently, provide a more complete picture regarding risk-return behaviour and its relation. Finally, the use of daily, monthly and annual data allows for the fact that shocks at different levels of frequency may have a differential impact on the market.

4. Empirical methodology

4.1. Model development

This paper analyses the relation between stock market returns and risk (typically defined as the standard deviation). Conventional finance theory posits that investors are risk-averse and that a positive relation exists between risk and return. While this view holds with both the CAPM and ICAPM, our main empirical analysis builds upon the Merton approach outlined in equation (1). The empirical regression model used to examine this contention is given by:

$$r_{it} = \alpha_i + \beta_i v_{it} + \varepsilon_{it}$$ (2)
where \( r_{it} \) denotes stock returns for market or firm \( i \), \( v_{it} \) is the measure of risk, or volatility, for market or firm \( i \) and \( \epsilon_{it} \) refers to the random error term. Of key interest is the sign and significance of the parameter \( \beta_i \), which determines the risk-return relation. Equation (2) is estimated for each of the sample markets and firms included in the analysis. In addition, a fixed effects panel model is estimated, such that equation (2) extends to:

\[
 r_{ti} = \alpha_i + \gamma_i + \beta_i v_{ti} + \epsilon_{ti} 
\]

(3)

where the subscript \( i \) refers to the individual markets or firms and \( \gamma_i \) is the cross-sectional fixed effects term. The use of a panel regression in this context is to enhance the available degrees of freedom and thus the statistical accuracy of the estimated coefficients. This is motivated by the arguments in Harvey (2001) and Ludvigson and Ng (2007) that traditional risk-return analyses can suffer from a degrees of freedom problem that restricts the modelling approach used. The coefficient \( \beta_i \) represents the average relation between market returns and risk across the sampled markets.

After estimating equations (2) and (3), the nature of the risk-return relation is examined to consider whether it varies according to (i) the level of past returns; (ii) alternative measures of risk and return; and (iii) market conditions, including periods of high and low volatility and bull and bear markets. This analysis involves conducting quantile regressions in which the risk-return relation varies with the level of returns, as well as estimating equations (2) and (3) but with the risk series based on VIX and VaR (value-at-risk) measures and the realised return series replaced by expected returns, which are calculated based on standard asset pricing models. Equations (2) and (3) are also estimated after the data is partitioned according to the level of volatility and whether the market is in a bull or bear phase.

A quantile regression models the quantiles (partitions or sub-sets) of the dependent variable given the set of potential explanatory variables (Koenker and Bassett 1978; Koenker and Hallock 2001). The quantile regression therefore extends the linear model in equation (2) by allowing a different coefficient for each specified quantile:

\[
 r_t = \alpha^{(q)} + \beta^{(q)} v_t + \epsilon_t 
\]

(4)

where \( \alpha^{(q)} \) represents the constant term for each estimated quantile \( (q) \), \( \beta^{(q)} \) is the slope coefficient that reveals the relation between risk and return at each quantile, and \( \epsilon_t \) is the error term. A quantile regression approach is considered by Badshah et al. (2016), who report a negative risk-return relation over a range of intra-day frequencies.

### 4.2. Alternative approaches for expected returns and risk

The above analysis utilises the realised return and standard deviation. However, alternative approaches can be considered, which may capture different aspects of the data to reveal the nature of the risk-return relation. Arguments made by, for example, Hansen and Richard (1987), Harvey (2001) and Ludvigson and Ng (2007) note that reliance on one approach for conditioning the data may lead to misleading results. Moreover, as the risk-return relation is designed to capture the behaviour of expected returns, as opposed to realised returns, we reconsider the equations detailed above using two approaches to obtain expected returns. First, from an asset pricing model for the market index data, a GARCH(1,1)-in-mean model is estimated for the excess stock return and the fitted value obtained. This process is based on the ICAPM view that the expected market index return is given by:

\[
 E_{t-1}(r_t) = r_{ft} + \lambda \sigma_t 
\]

(5)

where the risk-free rate \( (r_{ft}) \) is based on a short-term (three-month) Treasury bill, \( \sigma_t \) is the market standard deviation and \( \lambda \) is the time invariant market price of risk, where the size and sign of \( \lambda \) indicate the size and direction of the risk-return trade-off. Thus, the following GARCH(1,1)-M model is estimated:

\[
 r_{ti} = \mu_{ti} + \lambda_i \sigma_{ti} + \epsilon_{ti} 
\]

(6)
where the return process is defined as a function of the conditional mean, \( \mu_{i,t} \), the estimated conditional standard deviation, \( \sigma_{i,t} \), and the disturbance term, \( \epsilon_{i,t} \). The conditional variance (\( \sigma_{i,t}^2 \)) of the return series is given by the variance of the random error term (\( \epsilon_{i,t} \)) conditional on the past information set \( \Omega_{t-1} \), such as:

\[
\sigma_{i,t}^2 = \omega_i + \alpha_i \epsilon_{t-1,i}^2 + \beta \sigma_{t-1,i}^2
\]

where the non-negativity constraint must hold for all parameters in the model \((\alpha, \beta, \omega)\) and the measure of persistence of shocks to volatility is given by \( \alpha + \beta < 1 \). The GARCH-M methodology has previously been adopted with a range of results reported, including evidence of both a positive relation (for example, French, Schwert, and Stambaugh 1987; Müller, Durand, and Maller 2011) and a negative relation (for example, Nelson 1991; Jensen and Lunde 2001). Recently, Kanas (2013) augments the conditional variance equation with the squared VIX and reports a positive relation, while Wang and Yang (2013) argue that any positive relation can be obscured by a volatility feedback effect. However, Christensen, Nielsen, and Zhu (2015), using an extended GARCH-M model, report a positive relation only during crisis periods.

For the stock-level data, expected returns are obtained from the standard CAPM, as well as the Fama-French three factor model (FF3). Thus, the fitted values from the following two equations are obtained:

\[
R_{ti} - r_{ft} = \alpha_i + \beta_i (r_{mt} - r_{ft}) + \epsilon_{i,t}
\]

\[
R_{i,t} - r_{ft} = \alpha_i + \beta_m (r_{mt} - r_{ft}) + \beta_S SMB_t + \beta_H HML_t + \epsilon_{i,t}
\]

where \( r_{mt} \) is the market return, and \( SMB_t \) and \( HML_t \) are the small minus big and high minus low Fama-French market capitalisation and book-to-market factors, respectively.

Second, we follow the stock return predictability literature and estimate expected returns and standard deviation using the dividend-yield and interest rates. The choice of these variables follows the extensive predictability literature (for example, Campbell and Thompson 2008; Welch and Goyal 2008; Hjalmarsson 2010). For both the stock return and standard deviation series, we estimate the model:

\[
y_{i,t} = \alpha_i + \beta_{1,i} \text{ld}y_{i,t} + \beta_{2,i} ir_{i,t} + \epsilon_{i,t}
\]

where \( y_{i,t} \) refers to the stock return and standard deviation series, \( \text{ld}y_{i,t} \) is the log dividend-yield series and \( ir_{i,t} \) is the interest rate series.\(^{12}\) After estimating the model for each series and market, we obtain the fitted value and use these values in estimating equation (2).

The analysis is also extended to consider alternative risk proxies. First, we consider the volatility index based upon option price implied volatility as an alternative measure of risk for the markets for which suitable data are available. Often referred to as the VIX measure, the volatility index is colloquially regarded as the market fear index and represents risk derived from the behaviour of investors trading in options relating to a particular index. As noted above, the VIX is considered by Kanas (2013) in an augmented GARCH model, but more directly by Badshah et al. (2016). We obtain VIX measures for five markets, including three measures based on different US indexes.

The use of the standard deviation, which covers the full distribution of the data, in measuring risk can be criticised as it does not necessarily accord with our perception of risk. Specifically, risk is more associated with negative outcomes rather than positive outcomes (March and Shapira 1987; Tversky and Kahneman 1992; Unser 2000)\(^{13}\) and, particularly, if the distribution is skewed, the use of the standard deviation may not be appropriate (Barberis and Huang 2008).\(^{14}\) Indeed, work dating back to Roy (1952) argues that investors are concerned about safety and the avoidance of a severely negative outcome. One solution would be to use the semi-standard deviation where the standard deviation for observations below a given threshold value (such as a return of zero) is calculated. However, even this value may include small (albeit negative) return values, whereas we can consider risk as being associated with larger losses. Bali, Demirtas, and Levy (2009) utilise the Value-at-Risk (VaR)
measure to examine the risk-return relation for US stocks. Therefore, we calculate the 95 per cent VaR for the return series and consider its relation with returns. The 95 per cent VaR is the value associated with cutting-off the five per cent left hand side tail. To obtain a time-varying VaR, we utilise the GARCH model in equation (7) and calculate the VaR as:

\[
VaR_\alpha = \mu_t(r) + (\varphi(\alpha)\sqrt{\sigma_t^2})
\]  

(11)

where \( \mu_t(r) \) is the conditional mean of the return series, \( \varphi \) is the cumulative distribution function, with a significance level given by \( \alpha \) and \( \sqrt{\sigma_t^2} \) is the standard deviation obtained from the GARCH variance model.

A further issue that we consider is the interrelated nature of stock markets. A range of work, dating back to Eun and Shin (1989) and including the work of Engle, Ito, and Lin (1990), Bekaert and Harvey (1997) and Diebold and Yilmaz (2009, 2012), demonstrates the potential for spillover effects between financial markets. Following Rapach, Strauss, and Zhou (2013), we incorporate this by including lagged values of the US return and risk in equation (2). Rapach, Strauss, and Zhou (2013) argue that US stock returns have a conditioning effect on global stock returns and, thus, ignoring such an effect may result in an omitted variable bias that can affect the risk-return coefficient.

5. Risk-return results

5.1. Evidence from 43 stock market indexes

We begin our examination of whether there exists a positive risk-return relation by considering the simple cross-sectional relation between risk and return for the 43 international stock market indexes. Such an analysis is more akin to the static CAPM approach and reveals the nature of the relation. Figure 1 presents a scatter plot of the average monthly return and standard deviation for the 43 markets, along with the OLS regression line.\(^\text{15}\) The graph supports a positive relation between returns and risk; the regression coefficient is 0.116, with

![Figure 1. Average Return and Standard Deviation for 43 Market Indexes.](image)

Note: The figure shows a scatter plot of average returns (differenced log prices) and standard deviation, together with the fitted OLS regression line, for 43 stock market indexes using monthly data for the period January 1973 – December 2012.
Figure 2. Average Return and Standard Deviation for 38 Market Indexes.

Note: The figure shows a scatter plot of average returns (differenced log prices) and standard deviation, together with the fitted OLS regression line, for 38 stock market indexes, using monthly data for the period January 1973 – December 2012. The graph excludes all markets that have a negative return or an average monthly return in excess of 1.60 per cent.

A White (heteroscedasticity) corrected $t$-statistic of 2.05. The correlation coefficient between returns and standard deviation is 0.47. Within this graph, it is apparent that there is a small group of markets located in the North East corner of the plot that appear to exhibit different characteristics from the majority of the series, with noticeably higher returns. Potentially, this grouping of markets may be driving the overall results. Therefore, Figure 2 presents results excluding the four high return markets from the analysis, together with the market that presents a negative return (Cyprus). Figure 2 shows that although the scatter plot and regression line still indicate a positive relation between return and risk, the regression coefficient is 0.039, with a White corrected $t$-statistic of only 1.59. In addition, the correlation between return and risk declines to 0.28. Notwithstanding this, the preliminary analysis indicates that across average return and standard deviation values for a range of international stock indexes, a positive relation exists, albeit one that may be driven by the behaviour of only a few markets.

We now seek to introduce a time dimension into the analysis, which is related to the ICAPM approach, and examine the risk-return trade-off for each market. We do this by calculating the two-year rolling standard deviation for each individual market and estimating the regression model in equation (2) for each market and across all markets in a panel regression, according to equation (3). An obvious question concerns the choice of window in calculating the time-varying standard deviation. Blitz and van Vliet (2007) consider a three-year window, while annualised standard deviations are often considered in the practitioner literature. Experimentation with these alternatives indicated that there was no discernible difference in the nature of the results. Table 1 presents the results from this analysis. It is apparent from the table that there is very little evidence of a positive and significant relation between return and risk, even in the panel regression analysis, which should provide greater statistical reliability. Indeed, for 40 of the markets, the coefficient on the standard deviation is not significant at any conventional level, while the slope coefficient is positive for 20 markets and negative for 23 markets. Furthermore, there is a statistically significantly positive relation between risk and return in only one market (China). Thus, the results suggest a minimal relation between stock returns and their associated risk that, at best, is weakly positive.
Table 1. Return-Risk Regression Coefficients – 43 Markets.

| Market      | Start Date | Beta (t-stat) | Market      | Start Date | Beta (t-stat) |
|-------------|------------|---------------|-------------|------------|---------------|
| Australia   | 1973:1     | -0.075       | Mexico      | 1988:2     | -0.042        |
| Austria     | 1973:1     | -0.055       | Netherlands | 1973:1     | -0.136        |
| Belgium     | 1973:1     | 0.046        | Norway      | 1980:2     | -0.039        |
| Brazil      | 1994:8     | -0.015       | New Zealand | 1988:2     | 0.017         |
| Canada      | 1973:2     | -0.047       | Pakistan    | 1992:8     | -0.106        |
| China       | 1991:9     | 0.383        | Philippines | 1987:10    | 0.053         |
| Cyprus      | 1993:1     | -0.358       | Poland      | 1994:3     | 0.045         |
| Czech Rep.  | 1993:12    | -0.155       | Portugal    | 1990:2     | -0.107        |
| Denmark     | 1973:1     | 0.037        | Russia      | 1998:2     | 0.255         |
| Finland     | 1988:4     | 0.036        | South Africa| 1973:1     | -0.102        |
| France      | 1973:1     | 0.057        | Singapore   | 1973:1     | 0.147         |
| Germany     | 1973:1     | -0.161       | Spain       | 1987:3     | -0.094        |
| Greece      | 1988:2     | 0.029        | Sri Lanka   | 1987:6     | 0.577         |
| Hong Kong   | 1973:1     | -0.003       | Sweden      | 1982:2     | -0.094        |
| India       | 1990:1     | -0.091       | Switzerland | 1973:1     | -0.051        |
| Indonesia   | 1990:4     | -0.040       | Taiwan      | 1987:10    | 0.0876        |
| Ireland     | 1973:1     | -0.201       | Thailand    | 1987:2     | -0.111        |
| Italy       | 1973:1     | 0.076        | Turkey      | 1988:2     | 0.216         |
| Japan       | 1973:1     | -0.333       | UK          | 1973:1     | 0.169         |
| Korea       | 1987:10    | 0.330        | US          | 1973:2     | 0.007         |
| Luxembourg  | 1992:2     | -0.418       | Venezuela   | 1990:2     | 0.283         |
| Malaysia    | 1983:11    | 0.067        |            |            |               |

Fixed Effects Panel 0.053(1.31)

Notes: The table shows the slope coefficient (beta) and accompanying Newey-West t-statistic from the regression of returns on a two-year rolling standard deviation: \( r_t = \alpha + \beta sdt + \epsilon_t \) as given in Equation [1]. The panel regression is given by Equation [2]. The values in parentheses are autocorrelation and heteroscedasticity adjusted t-statistics.

5.2. Dimson-Marsh-Staunton data

In the second part of the analysis, the Dimson, Marsh and Staunton (2002, 2008, 2011) dataset is used to examine the risk and return trade-off. This dataset expands the time horizon of available data with which to examine the risk-return relation; specifically, the dataset consists of annual returns for 17 markets over the period 1900–2010. The longer time frame may provide more robust results as it captures a greater number of market cycles. Lunblad (2007) also considers a long history of data and notes that the span of the data is arguably more important than the frequency of data in estimating the returns behaviour. Thus, while Lunblad uses monthly data, albeit for only one market, we are satisfied that the use of annual data will be informative in the current context.
Again, as we are interested in revealing where any positive risk-return relation lies, we consider both the static and time-varying evidence. Figure 3 presents a scatter plot of the mean return and standard deviation for all markets over the sample period and is comparable with Figures 1 and 2 detailed above. It is apparent from the figure that there is a positive relation between return and risk; the regression coefficient of returns on risk is 0.186, with a White corrected $t$-statistic of 3.32, and the correlation between the average return and standard deviation is 0.64. Therefore, this result is more supportive of a positive risk-return relation, which may arise from the longer time span considered. In addition, the use of annual rather than monthly data is likely to smooth out shorter-term fluctuations and, thus, may reveal in greater clarity the nature of the longer-term relation.

Table 2 presents the regression results using time-varying standard deviations that are obtained by constructing five-year rolling values. Again, the choice of window length is largely arbitrary, although a ten-year window produces qualitatively similar results. In contrast to the evidence reported in Figure 3, the results are not overwhelmingly in favour of a significant positive relation; 14 of the 17 sample markets have a positive risk-return relation, although this is significant at the five (ten) per cent level for only two (two) markets. Thus, there is no evidence of a significant and positive risk-return relation in 13 of the 17 sample markets. The results from the fixed effects panel model show a positive risk-return relation; these results could be considered as more reliable due to the increase in the degrees of freedom or, alternatively, they could be driven by a subset of the markets considered. Hence, even with a longer time series of data, the view of a universal positive risk-return relation seems doubtful.

The nature of the results presented above broadly reflects those within the current literature and suggests that there is no convincing evidence of a positive risk-return relation. Within the literature, this finding motivates a deeper look at the nature of the relation by considering regimes where a positive (or negative) trade-off may appear and, likewise, we consider this, first, over different return quantiles.

### 5.3. Quantile regression analysis

The above regression analyses utilise OLS and thus focuses on the conditional mean point estimate in order to garner information about the risk-return relation. However, this approach ignores the possibility that deviations...
Table 2. Return-Risk Regression Coefficients – Dimson-Marsh-Staunton Data.

| Market         | Beta (t-stat) | Market         | Beta (t-stat) |
|----------------|--------------|----------------|--------------|
| Australia      | -0.173       | Netherlands    | 0.411        |
|                | (-1.01)      |                | (1.56)       |
| Belgium        | 0.379        | Norway         | 0.239        |
|                | (1.04)       |                | (1.48)       |
| Canada         | 0.175        | South Africa   | -0.350       |
|                | (0.40)       |                | (-1.80)      |
| Denmark        | 0.293        | Spain          | 0.145        |
|                | (1.70)       |                | (0.50)       |
| France         | 0.454        | Sweden         | 0.203        |
|                | (2.49)       |                | (0.86)       |
| Germany        | 0.395        | Switzerland    | 0.226        |
|                | (1.51)       |                | (1.10)       |
| Ireland        | 0.231        | UK             | 0.412        |
|                | (1.40)       |                | (2.15)       |
| Italy          | 0.369        | US             | -0.023       |
|                | (1.94)       |                | (-0.06)      |
| Japan          | 0.474        |                |              |
|                | (1.55)       |                |              |
| Fixed Effects Panel | 0.292 (6.68) |                |              |

Note: The table shows the slope coefficient (beta) and accompanying Newey-West t-statistic from a regression of returns on a five-year rolling standard deviation: $r_t = \alpha + \beta \cdot sdt_t + \epsilon_t$, as given in Equation [1]. The panel regression is given by Equation [2]; the values in parentheses are autocorrelation and heteroscedasticity adjusted t-statistics.

in the risk-return relation may occur in the tails of the distribution due to heterogeneity in investor beliefs. Thus, in order to allow for such heterogenous investor beliefs and, therefore, differences in the trade-off across the distribution, a quantile regression analysis is conducted. Badshah et al. (2016) note a negative risk and return relation across all quantiles in their analysis of intra-day data. Thus, our results using lower frequency data will provide an interesting point of comparison.

The results from conducting this exercise are reported in Table 3 for the monthly returns data across 43 markets and in Table 4 for the Dimson-Marsh-Staunton annual data for 17 markets. A visual inspection of Table 3 reveals an interesting pattern in the risk-return relation for different return values. In particular, the table shows that there is an exclusively negative relation at the lowest return quantile (Q1), which is statistically significant at the five per cent level for 28 of the 43 markets considered, and for a further six markets at the ten per cent level. As a mirror image, there is an (almost) exclusively positive risk-return relation at the highest return quantile (Q9). This positive relation is statistically significant at the five per cent level for 34 markets, and significant at the ten per cent level for an additional three markets. By contrast, the pattern of results for the middle return quantile (Q5) is very mixed. Of the 43 markets, 24 exhibit a negative relation, while 19 show a positive relation. Furthermore, the relation is significant at the five per cent level for only two markets (Cyprus and Greece); in both cases, the risk-return trade-off is negative. This result for Q5 is consistent with the conditional mean results reported in Table 1, which showed scant evidence of a significant risk-return relation.

A similar picture emerges from Table 4, which shows the results for the Dimson, Marsh and Staunton data. For the lower return quantile (Q1), a negative risk-return relation is observed for all of the 17 sample markets; the results for 13 of these markets are significant at the five per cent level. The relation is positive for all the 17 markets at the highest return quantile (Q9), and this is statistically significant for 15 of the 17 markets (with a further market significant at the ten per cent level). The results for the middle quantile (Q5) indicate a positive (negative) relation for 13 (four) of the markets, however, the relation is positively (negatively) statistically significant at the five (ten) per cent level for only the Irish (South African) market.

Overall, these results constitute compelling evidence that the risk-return relation is negative at low levels of return and positive at high levels of return. This pattern could be seen to be consistent with the disposition effect that underlies prospect theory. The disposition effect contends that investors are likely to take profit when
The results from conducting a similar analysis using firm-level data are reported in Table 5. In many respects, this analysis is indicative rather than exact as we are not specifying the asset pricing model but simply asking whether, at a stock level, there is a positive relation between risk and return. Column two of the table reports the results from the fixed effects panel regression analysis, while columns three to five show the results from the quantile regressions. The results for the full sample are very mixed, with half of the sample exhibiting a negative relation and the other half exhibiting a positive relation. This risk-taking behaviour can generate positive returns. However, when returns are low, investors are more likely to maintain their position in the hope that subsequent returns will increase, leading to a profitable trading position. This risk-taking behaviour can induce a negative relation.  

5.4. Individual firms

The results from conducting a similar analysis using firm-level data are reported in Table 5. In many respects, this analysis is indicative rather than exact as we are not specifying the asset pricing model but simply asking whether, at a stock level, there is a positive relation between risk and return. Column two of the table reports the results from the fixed effects panel regression analysis, while columns three to five show the results from the quantile regressions. The results for the full sample are very mixed, with half of the sample exhibiting a negative relation and the other half exhibiting a positive relation. This risk-taking behaviour can generate positive returns. However, when returns are low, investors are more likely to maintain their position in the hope that subsequent returns will increase, leading to a profitable trading position. This risk-taking behaviour can induce a negative relation.
Table 4. Return-Risk Quantile Regression Coefficients – Dimson-Marsh-Staunton Data.

| Market    | Q1     | Q5     | Q9     | Market    | Q1     | Q5     | Q9     |
|-----------|--------|--------|--------|-----------|--------|--------|--------|
| Australia | -1.305 | -0.202 | 1.103  | Netherlands| -0.317 | 0.094  | 1.920  |
|           | (-3.79)| (-0.57)| (3.58) |           | (-0.91)| (0.42) | (3.19) |
| Belgium   | -0.580 | 0.120  | 2.266  | Norway    | -1.089 | -0.191 | 1.655  |
|           | (-2.02)| (0.49) | (4.70) |           | (-2.73)| (-0.65)| (2.56) |
| Canada    | -1.157 | 0.728  | 1.337  | South     | -0.922 | -0.316 | 0.216  |
|           | (-2.86)| (1.62) | (3.05) |           | (-3.80)| (-1.94)| (0.08) |
| Denmark   | -0.600 | 0.033  | 1.643  | Spain     | -0.905 | 0.194  | 1.458  |
|           | (-3.47)| (0.07) | (1.95) |           | (-3.53)| (0.68) | (3.75) |
| France    | -0.743 | 0.187  | 1.752  | Sweden    | -1.139 | 0.230  | 1.358  |
|           | (-2.89)| (0.62) | (7.17) |           | (-2.87)| (0.56) | (3.01) |
| Germany   | -0.573 | 0.247  | 1.838  | Switzerland| -0.759 | 0.269  | 1.644  |
|           | (-3.70)| (0.88) | (6.25) |           | (-3.08)| (1.02) | (4.56) |
| Ireland   | -0.940 | 0.678  | 1.351  | UK        | -0.944 | 0.216  | 1.716  |
|           | (-2.63)| (1.98) | (7.60) |           | (-1.23)| (0.63) | (3.87) |
| Italy     | -0.531 | 0.120  | 1.982  | US        | -1.326 | -0.274 | 0.980  |
|           | (-1.63)| (0.60) | (5.12) |           | (-2.10)| (-0.45)| (2.98) |
| Japan     | -0.308 | 0.231  | 2.025  |           |        |        |        |
|           | (-0.99)| (0.82) | (4.08) |           |        |        |        |

Note: The table shows the coefficient values and accompanying t-statistic from a quantile regression of returns on a two-year rolling standard deviation: \( r_t = \alpha^{(q)} + \beta^{(q)} v_t + \epsilon_t \), as given in Equation [3]. The terms Q(1,5,9) refer to the selected quantiles.

Table 5. Panel and Quantile Estimation Across Individual Companies.

| Market    | Full Sample | Q1     | Q5     | Q9     |
|-----------|-------------|--------|--------|--------|
| Australia | 0.038       | -0.853 | -0.012 | 0.932  |
|           | (1.84)      | (-5.20)| (-0.55)| (8.45) |
| Canada    | 0.217       | -0.770 | 0.100  | 1.066  |
|           | (7.88)      | (-6.45)| (2.31) | (20.35)|
| France    | 0.023       | -1.802 | -0.012 | 0.940  |
|           | (1.18)      | (-5.80)| (-0.08)| (9.90) |
| Germany   | -0.071      | -1.977 | 0.046  | 0.861  |
|           | (-0.69)     | (-6.91)| (0.30) | (8.60) |
| Hong Kong | -0.009      | -0.988 | 0.157  | 1.360  |
|           | (-0.09)     | (-8.65)| (0.89) | (5.04) |
| Italy     | -0.062      | -1.869 | -0.061 | 0.793  |
|           | (-2.84)     | (-11.31)| (-1.15)| (35.69)|
| Japan     | -0.005      | -1.562 | 0.032  | 1.440  |
|           | (-0.12)     | (-7.03)| (0.42) | (21.09)|
| South Korea| 0.035      | -1.151 | 0.106  | 1.242  |
|           | (0.73)      | (-15.04)| (2.20)| (12.32)|
| UK        | -0.051      | -1.521 | 0.072  | 0.892  |
|           | (-2.19)     | (-8.97)| (2.63) | (11.63)|
| US        | 0.059       | -1.313 | 0.190  | 1.269  |
|           | (2.49)      | (-22.05)| (3.75)| (16.75)|

Note: The table shows the coefficient values and autocorrelation and heteroscedasticity adjusted t-statistics from Equation [2] for the panel analysis and Equation [3] for the quantile regression.

**risk-return relation and the other half showing a positive relation (with two markets statistically significant at the five per cent level in each case).26** The results from the quantile analysis are consistent with the market-level analysis and suggest a distinction between high and low returns in their relation with risk. Notably, there is a strong negative risk-return relation at low levels of return and a positive relation at high levels of return, both of which are statistically significant for all markets. At the median level, seven markets exhibit a positive relation, four of which are statistically significant. Overall, the results from the analysis of individual firm-level data are broadly in line with those reported from the analysis that utilised index level data; both sets of results provide mixed evidence over their full samples but, on a sub-sample (quantile) basis, the results suggest a strong risk-return relation that varies according to the level of return Table 6.
Table 6. Risk-Return Regression Coefficients – VIX.

| Market     | Beta (t-stat) | Market     | Beta (t-stat) |
|------------|--------------|------------|--------------|
| France     | -0.250 (-6.26) | US – VIX   | -0.235 (-4.49) |
| Germany    | -0.258 (-6.29) | US – VXD   | -0.264 (-4.96) |
| Netherlands| -0.259 (-4.30) | US – VSN   | -0.156 (-4.08) |
| Switzerland| -0.267 (-8.07) |            |              |
| UK         | -0.229 (-5.88) |            |              |

Note: The table shows the slope coefficient (beta) and accompanying Newey-West t-statistic from the regression given by Equation [1]. However, the rolling standard deviation is replaced with the VIX measure for each market.

6. Further tests

6.1. Alternative measures of risk and return

Given the absence of a robust relation between risk and return detected so far across both individual market- and firm-level data, we now extend the analysis to consider alternative measures of risk and return. As alternative measures of risk, we consider both VIX and VaR values, while we use an asset pricing model to obtain expected (as opposed to realised) returns. The results for the return and VaR regressions are reported in Table 7, in the column headed VaR. These results present a similar picture to those observed above, in which there is very little evidence of a significant relationship between return and risk, regardless of the sign of that relation. We can observe a positive coefficient sign for 25 markets and a negative sign for 18 markets. However, the positive (negative) coefficient is statistically significant at the five (ten) percent level for only one (two) market(s).27

To further consider the behaviour of the return and risk relation in the left tail of the distribution, the results under the column headed exceedances are based on the return and risk regression in which we only include observations from the left five per cent tail of the distribution (that is, we only include observations that exceed the five per cent VaR, which is akin to the expected loss measure). The table shows that the signs of the coefficients are universally negative and statistically significant for 37 markets at the five per cent level (with three markets significant at the ten per cent level). These results confirm those reported in Table 3 in which low return values exhibit a negative relation with risk.

In line with most of the literature, the foregoing analysis is based on realised returns. However, the theoretical risk-return relation is based on expected, rather than realised, returns. Thus, we seek to derive the expected returns for the 43 monthly markets, first, using the ICAPM approach, in accordance with equations (5) to (7), and, second, using a predictive regression model of the kind employed by Welch and Goyal (2008). In addition, expected returns are calculated at the firm level using the CAPM, as shown in equation (8), as well as the FF3 model given by equation (9).

Table 8 presents the results from examining the relation between expected returns and risk for the 43 international stock indexes. It is apparent from the table that, relative to the results reported in Table 1 for the realised returns of the stock market series, there is now greater precision in the estimates, with a much higher degree of statistical significance. Nevertheless, this increased precision does not translate into any greater evidence in favour of a positive risk-return relation; the results from using both methods to obtain expected returns are mixed. Specifically, following the ICAPM-GARCH approach, while 20 markets exhibit a positive and significant relation, 18 markets reveal a negative and significant relation. Equally, for the predictive regression model, 23 markets exhibit a positive and significant relation, but 15 markets still exhibit a negative and significant relation. In both cases, five markets exhibit no significant relation.28 Furthermore, in both cases the fixed effects panel regression is no longer statistically significant and is now negative for the former approach.

The results from estimating expected returns for the individual stocks using both a CAPM and FF3 model are reported in Table 9. Similar to the results for the stock market indexes, the pattern that emerges is one that
Table 7. Return-VaR Regression Coefficients – 43 Markets.

| Market      | VaR | Exceedances | Market      | VaR   | Exceedances |
|-------------|-----|-------------|-------------|-------|-------------|
| Australia   | -0.013 | -1.564     | Mexico      | 0.110 | -1.682      |
|             | (-0.17) | (-4.03)    |             | (1.28) | (-4.97)     |
| Austria     | 0.056   | 1.263       | Netherlands | -0.196 | -0.889      |
|             | (0.54)  | (-16.3)     |             | (-0.92) | (-2.65)     |
| Belgium     | -0.112  | -1.981      | Norway      | -0.235 | -0.982      |
|             | (-0.54) | (-5.37)     |             | (-0.81) | (-2.97)     |
| Brazil      | 0.044   | 3.011       | New Zealand | -0.037 | -0.787      |
|             | (0.24)  | (-3.71)     |             | (-0.38) | (-3.68)     |
| Canada      | -0.006  | 1.518       | Pakistan    | 0.484  | -14.669     |
|             | (-0.03) | (-4.54)     |             | (0.84)  | (-1.85)     |
| China       | 0.201   | -1.155      | Philippines | 0.003  | -1.606      |
|             | (2.24)  | (-6.71)     |             | (0.02)  | (-6.72)     |
| Cyprus      | -0.016  | -1.549      | Poland      | -0.222 | -2.451      |
|             | (-0.13) | (-3.89)     |             | (-1.46) | (-2.08)     |
| Czech Rep.  | 0.091   | 0.746       | Portugal    | 0.074  | 1.876       |
|             | (0.28)  | (-1.15)     |             | (0.52)  | (-5.72)     |
| Denmark     | 0.128   | -1.981      | Russia      | 0.201  | -1.474      |
|             | (0.62)  | (-5.78)     |             | (1.10)  | (-2.63)     |
| Finland     | 0.079   | -0.810      | South Africa| 0.053  | -1.420      |
|             | (0.52)  | (-3.47)     |             | (0.57)  | (-3.55)     |
| France      | 0.148   | -0.999      | Singapore   | -0.031 | -1.159      |
|             | (1.06)  | (-1.70)     |             | (-0.33) | (-4.84)     |
| Germany     | -0.049  | -1.290      | Spain       | 0.171  | -1.591      |
|             | (-0.48) | (-4.47)     |             | (1.38)  | (-2.36)     |
| Greece      | 0.059   | -1.359      | Sri Lanka   | 0.188  | -2.803      |
|             | (0.41)  | (-3.25)     |             | (0.60)  | (-1.21)     |
| Hong Kong   | -0.164  | 0.780       | Sweden      | -0.052 | -0.542      |
|             | (-1.56) | (-3.27)     |             | (-0.29) | (-1.35)     |
| India       | -0.227  | -1.168      | Switzerland | -0.076 | -0.854      |
|             | (-1.70) | (-15.56)    |             | (-0.56) | (-3.92)     |
| Indonesia   | 0.001   | -1.742      | Taiwan      | -0.011 | -1.063      |
|             | (0.01)  | (-1.97)     |             | (-0.10) | (-7.61)     |
| Ireland     | 0.001   | -0.907      | Thailand    | -0.236 | -1.535      |
|             | (0.01)  | (-4.79)     |             | (-1.69) | (-4.83)     |
| Italy       | 0.130   | -1.404      | Turkey      | 0.073  | -1.612      |
|             | (0.76)  | (-3.47)     |             | (0.72)  | (-3.57)     |
| Japan       | -0.051  | -1.953      | UK          | 0.146  | -0.982      |
|             | (-0.42) | (-6.41)     |             | (1.11)  | (-3.52)     |
| Korea       | 0.089   | -0.426      | US          | 0.103  | -1.341      |
|             | (0.44)  | (-1.77)     |             | (1.03)  | (-7.16)     |
| Luxembourg  | -0.257  | -1.806      | Venezuela   | 0.211  | -1.561      |
|             | (-1.63) | (-2.69)     |             | (1.42)  | (-4.78)     |
| Malaysia    | 0.060   | -1.647      |             | (0.62)  | (-10.45)    |

Notes: The table shows the slope coefficient (beta) and accompanying Newey-West t-statistic from the regression given by Equation [1]. However, the rolling standard deviation is replaced with the VaR measure for each market. Entries under the Exceedances heading include only returns located in the five per cent left tail of the distribution.

does not support a positive risk-return relation. Only three (one) of the nine sample markets exhibit a positive (negative) and significant risk-return relation, while no significant relation is detected for five markets. Similarly, using FF3 derived expected returns, only three (four) markets exhibit a positive (negative) and significant relation, and two markets do not exhibit a significant relation. 30,31

6.2. High vs low volatility

We examine the nature of the risk-return relation across different volatility states using individual stock level data. Salvador, Floros, and Arago (2014) note a positive risk-return relation in low volatility states that disappears in higher states. In a similar vein, Chiang, Li, and Zheng (2015) argue that any positive relation is more
Table 8. Expected Return-Risk Regression Coefficients – 43 Markets.

| Market     | CAPM  | Predictive Regr | Market     | CAPM  | Predictive Regr |
|------------|-------|-----------------|------------|-------|-----------------|
| Australia  | 0.070 | 0.415           | Netherlands| −0.037| 0.187           |
|            | (4.23)| (5.99)          |            | (−3.09)| (6.62)          |
| Austria    | 0.156 | −0.053          | Norway     | 0.043 | 1.096           |
|            | (5.18)| (−2.12)         |            | (−4.14)| (10.01)         |
| Belgium    | 0.021 | 0.525           | New Zealand| −0.079| −0.211          |
|            | (2.19)| (29.99)         |            | (−12.57)| (−6.55)        |
| Brazil     | −0.006| 0.312           | Pakistan   | −0.131| 0.547           |
|            | (−0.28)| (5.89)         |            | (−0.75)| (6.48)          |
| Canada     | 0.046 | 0.515           | Philippines| −0.081| 0.179           |
|            | (1.41)| (2.59)          |            | (−2.65)| (0.36)          |
| China      | 0.368 | −1.683          | Poland     | −0.141| −0.334          |
|            | (11.18)| (−12.89)     |            | (−5.94)| (10.01)         |
| Cyprus     | −0.140| 1.071           | Portugal   | 0.019 | 4.540           |
|            | (−3.57)| (2.06)         |            | (2.27) | (11.03)         |
| Czech      | −0.030| −0.481          | Russia     | 0.281 | 0.059           |
| Republic   | (−3.33)| (−11.57)     |            | (5.33) | (3.23)          |
| Denmark    | −0.023| −1.754          | Singapore  | −0.026| 0.414           |
|            | (−6.08)| (−5.33)       |            | (−1.95)| (3.81)          |
| Finland    | 0.115 | 1.047           | South Africa| 0.091| 0.337           |
|            | (1.56)| (12.37)        |            | (4.63) | (5.61)          |
| France     | 0.149 | 0.376           | South Korea| 0.153| −0.311          |
|            | (4.75)| (4.79)         |            | (4.70) | (−3.65)         |
| Germany    | 0.017 | 0.127           | Spain      | 0.098 | −0.009          |
|            | (7.71)| (2.50)         |            | (4.06) | (−0.28)         |
| Greece     | 0.055 | 0.273           | Sri Lanka  | 0.071 | 2.605           |
|            | (2.46)| (8.49)         |            | (4.41) | (5.62)          |
| Hong Kong  | −0.156| 0.083           | Sweden     | 0.043 | −1.801          |
|            | (−5.67)| (0.47)        |            | (0.83) | (2.01)          |
| India      | −0.044| −0.542          | Switzerland| −0.059| 0.187           |
|            | (−2.39)| (−16.88)     |            | (−2.21)| (1.17)          |
| Indonesia  | −0.067| −1.755          | Taiwan     | −0.005| −0.486          |
|            | (−7.72)| (−2.76)       |            | (−9.04)| (−6.02)         |
| Ireland    | 0.079 | 1.065           | Thailand   | −0.117| −2.206          |
|            | (3.73)| (6.16)         |            | (−4.63)| (−6.97)         |
| Italy      | 0.056 | 0.674           | Turkey     | −0.021| −0.489          |
|            | (2.05)| (3.27)         |            | (−2.06)| (3.67)          |
| Japan      | −0.096| −0.369          | UK         | 0.233 | 0.880           |
|            | (−5.48)| (−2.63)      |            | (4.25) | (23.62)         |
| Luxembourg | −0.171| 0.385           | US         | 0.057 | 0.622           |
|            | (−3.46)| (3.67)        |            | (2.68) | (8.35)          |
| Malaysia   | 0.035 | −1.342          | Venezuela  | 0.204 | 0.203           |
|            | (3.37)| (−4.64)        |            | (9.28) | (2.25)          |
| Mexico     | 0.113 | 0.547           |            |       |                 |
|            | (2.61)| (12.40)        |            |       |                 |

Fixed Effects Panel

−0.055 (−0.46) 0.004 (0.01)

Note: The table shows the slope coefficient (beta) and accompanying Newey-West t-statistic from a regression of expected returns on a two-year rolling standard deviation for 43 stock markets. The panel regression is given by Equation [2]; the values in parentheses are autocorrelation and heteroscedasticity adjusted t-statistics. Expected returns are calculated according to Equations [4]-[6].

pronounced in tranquil periods. To that end, the data are partitioned according to the level of volatility, as measured by standard deviation, for each stock in each year, and the risk-return relation reconsidered. Table 10 presents the returns earned by portfolios of firms that are organised according to the level of volatility for each year. Following Blitz, Pang, and van Vliet (2013), the portfolios are equally-weighted and volatility is divided according to quartiles (Q). This analysis is useful for providing evidence on the low volatility anomaly that is reported in the literature and which finds that returns are higher (lower) for low (high) volatility stocks (Ang et al. 2006, 2009; Blitz and van Vliet 2007; Baker, Bradley, and Wurgler 2011; Baker and Haugen 2012). The results from the analysis reported in Table 10 are inconclusive. That is, although they indicate that returns increase with volatility for four of the sample markets, there is evidence that returns decrease with volatility in three markets. Furthermore, returns for three markets are highest at medium levels of volatility, suggesting the presence of a
Table 9. Expected Return-Risk Regression Coefficients – Individual Firms.

| Market  | CAPM  | FF3  |
|---------|-------|------|
|         | 0.054 | 0.049 |
|         | (8.52)| (6.89)|
| Canada  | 0.011 | 0.014 |
|         | (3.16)| (2.96)|
| France  | 0.011 |−0.050 |
|         | (1.39)| (−1.60)|
| Germany | 0.001 |−0.116 |
|         | (0.32)| (−3.19)|
| Hong Kong| 0.002 |−0.059 |
|         | (3.43)| (−2.19)|
| Italy   | 0.001 |−0.017 |
|         | (1.00)| (−2.78)|
| Japan   |−0.001 | 0.001 |
|         | (−0.09)| (0.27)|
| UK      |−0.053 |−0.056 |
|         | (−8.48)| (−4.24)|
| US      | 0.004 | 0.011 |
|         | (1.26)| (2.14)|

Note: The table shows the results from a panel regression given by Equation [2] where the realised return is replaced by the expected return given by the CAPM Equation [7] or the Fama-French three-factor model in Equation [8].

Table 10. Average Return by Volatility Quartile.

| Market   | Q1   | Q2−Q3 | Q4   |
|----------|------|-------|------|
| Australia| 4.52 | 4.43  | 1.97 |
| Canada   | 4.94 | 8.53  | 12.76|
| France   | 3.95 | 5.15  | 0.05 |
| Germany  | 4.96 | 4.69  | 1.70 |
| Hong Kong| 4.26 | 6.90  | 15.68|
| Italy    |−0.94 |−2.75  |−7.03|
| Japan    | 1.36 | 2.10  | 2.43 |
| South Korea| 6.52 |10.72 | 9.71 |
| UK       | 4.89 | 8.15  | 4.38 |
| US       | 4.38 | 7.76  | 10.33|

Note: The table shows the average return across all companies within each quartile arranged by volatility in each year.

hump-shaped risk-return profile. Figure 4 presents graphical evidence of the results in Table 10. If a positive risk-return relation is present, we would see all the lines moving upwards from Q1 to Q4. However, it is apparent that the profile for most countries is broadly flat. These findings are consistent with the previous results of Blitz and van Vliet (2007) and Blitz, Pang, and van Vliet (2013).

To further analyse the risk and return profile across portfolios sorted by volatility, we estimate equation (2) but expand it to include the Fama-French three-factors, as shown in equation (8). The results of this exercise are reported in Table 11. The table shows that the results continue to portray a similar mixed picture. Specifically, the results in the table show that there is no pattern in the coefficient value on the market risk premium (MRP) whereby it increases as we move from low to high volatility portfolios. Again, this pattern (or lack thereof) of a broadly flat coefficient value across volatility states is consistent with Blitz and van Vliet (2007) and Blitz, Pang, and van Vliet (2013). The Fama-French factors, size (SMB) and value (HML) premiums, are designed to proxy for systematic risk and, thus, we might expect to see a discernible pattern in the behaviour of the coefficients. However, the results reveal no such evidence as the parameters are broadly flat across the portfolios.
Similarly, the volatility term also shows no pattern of an increasing coefficient value over the different portfolios. Moreover, the results continue to present a mixed picture in terms of the sign of the coefficient. For the low volatility portfolios, a positive risk and return relation is noted three times, as compared to six that indicate a negative relation. Furthermore, only one positive coefficient is statistically significant, while four negative coefficients are significant. For the portfolios covering the middle two volatility quartiles, four exhibit a positive relation (all statistically significant) and five show a negative relation (three are statistically significant). For the high volatility portfolios, a positive coefficient is reported for five markets (although this is only statistically significant for one), with a negative coefficient for four (which are not statistically significant).

The alpha term within the asset pricing literature indicates an abnormal return component. As such, we may expect the alpha associated with the high volatility portfolios to be larger than that associated with the low volatility portfolios. However, the table shows that there is no consistent pattern across the markets and portfolios. For example, for the low and mid-level volatility portfolios, the alpha term is positive for seven markets and negative for two while, for the high volatility portfolios, more alphas have negative values (five) rather than positive (four). Moreover, the statistical significance of the alphas is limited.

6.3. Bull vs bear markets

In this part of the analysis, the risk-return relation is examined to consider if it varies with the financial cycle of bull and bear markets. A three-year moving average of the stock index is used to define bull and bear regimes for each market; if the change in this moving average is positive then the market is characterised as a bull market, while if the change in the three-year moving average is negative, the market is in a bear phase. This definition is consistent with Chauvet and Potter (2000) who defined bull (bear) markets as those that correspond to periods of increasing (decreasing) market prices. This approach eschews measures based around some (arbitrary) threshold for price rises or falls that fail to take account of market trends. This general approach is also consistent with Wu and Lee (2015), while the specific use of a three-year average follows Cooper, Gutierrez, and Hameed (2004).
The results for the 43 international markets are presented in Table 12, while Table 13 reports the results obtained from the analysis of individual stock level data. In terms of bull market conditions, Table 12 shows that there is a positive risk-return relation for 15 of the 43 markets and that this relation is significant at the five per cent level for only one market (Sri Lanka). Hence, under bull market conditions, a majority of the markets (28) exhibit a negative risk-return relation, albeit with limited statistical significance (only three markets are significant at the five per cent level: Cyprus, Germany and the UK). The opposite pattern emerges under bear market conditions, where a majority of markets (32) exhibit a positive risk-return relation, although this relation is statistically significant for only three markets at the five per cent level (Italy, South Korea and the US) and a further four markets at the ten per cent level (Australia, France, Spain and Sweden).

A similar picture emerges from the analysis of firm-level data. Table 13 shows that, in bull market conditions, the risk-return relation is predominantly negative. Indeed, eight of the ten sample markets exhibit a negative risk-return trade-off, and this relation is significant at the five (ten) per cent level in six (one) case(s). By contrast, with the exception of Germany, the risk-return relation is positive in bear markets. Furthermore, this relation is positive and significant for five and two markets at the five and ten per cent levels, respectively. Taken together, the results from the analysis of bull and bear markets suggest strongly that risk aversion is state dependent. Furthermore, the results lend some credence to the countercyclical risk premium hypothesis according to which investors demand a higher compensation from holding a risky asset in a bear market. A similar finding is reported for the US market by Chang (2016) who notes that the magnitude of compensation for enduring risk is stronger during periods of unfavourable financial conditions.
Table 12. Return-Risk Regression Coefficients – Bull vs Bear Markets.

| Market         | Bull Beta | Bear Beta | Market         | Bull Beta | Bear Beta |
|----------------|-----------|-----------|----------------|-----------|-----------|
| Australia      | -0.242    | 0.473     | Mexico         | 0.181     | -4.062    |
| (1.84)         | (1.69)    | (0.97)    | (2.81)         |           |           |
| Austria        | -0.297    | 0.034     | Netherlands    | -0.176    | 0.235     |
| (-1.77)        | (0.29)    |           | (0.97)         |           |           |
| Belgium        | -0.091    | 0.342     | Norway         | 0.003     | 0.453     |
| (-0.46)        | (1.50)    |           | (0.01)         |           | (1.09)    |
| Brazil         | 0.162     | -0.454    | New Zealand    | 0.014     | 0.371     |
| (0.67)         | (-0.60)   |           | (0.07)         |           | (1.09)    |
| Canada         | -0.247    | 0.442     | Pakistan       | 0.392     | 0.113     |
| (-1.33)        | (1.45)    |           | (1.20)         |           | (0.27)    |
| China          | 0.114     | 0.202     | Philippines    | -0.033    | -0.230    |
| (0.51)         | (0.86)    |           | (-0.10)        |           | (-0.42)   |
| Cyprus         | -0.415    | -0.015    | Poland         | -0.133    | 0.539     |
| (-2.01)        | (-0.04)   |           | (0.53)         |           | (0.73)    |
| Czech Rep.     | -0.236    | 0.071     | Portugal       | -0.322    | 0.320     |
| (-0.72)        | (0.15)    |           | (1.43)         |           | (0.76)    |
| Denmark        | -0.295    | 0.385     | Russia         | 0.454     | 1.495     |
| (-1.05)        | (1.07)    |           | (1.71)         |           | (1.50)    |
| Finland        | -0.098    | 0.378     | South Africa   | -0.016    | -0.742    |
| (-0.41)        | (1.01)    |           | (-0.10)        |           | (-0.85)   |
| France         | -0.060    | 0.734     | Singapore      | -0.069    | 0.262     |
| (-0.27)        | (1.86)    |           | (-0.47)        |           | (1.26)    |
| Germany        | -0.409    | 0.244     | Spain          | -0.139    | 1.708     |
| (-2.10)        | (1.02)    |           | (-0.58)        |           | (1.82)    |
| Greece         | -0.094    | 0.171     | Sri Lanka      | 0.798     | -0.451    |
| (-0.39)        | (0.30)    |           | (2.15)         |           | (-0.83)   |
| Hong Kong      | -0.332    | 0.469     | Sweden         | -0.208    | 2.068     |
| (-2.05)        | (1.50)    |           | (-0.92)        |           | (1.78)    |
| India          | 0.047     | -0.806    | Switzerland    | -0.130    | 0.341     |
| (0.27)         | (-1.34)   |           | (-0.86)        |           | (1.06)    |
| Indonesia      | 0.351     | -1.098    | Taiwan         | 0.349     | -0.274    |
| (1.81)         | (-2.67)   |           | (1.95)         |           | (1.18)    |
| Ireland        | -0.011    | 0.109     | Thailand       | 0.084     | 0.416     |
| (-0.06)        | (0.37)    |           | (0.29)         |           | (1.27)    |
| Italy          | -0.127    | 1.561     | Turkey         | 0.228     | 0.060     |
| (-0.61)        | (3.01)    |           | (1.20)         |           | (0.04)    |
| Japan          | -0.078    | -0.047    | UK             | -0.001    | 0.165     |
| (-0.33)        | (-0.14)   |           | (-0.01)        |           | (0.75)    |
| Korea          | 0.071     | 1.128     | US             | -0.162    | 0.837     |
| (0.41)         | (2.44)    |           | (-0.97)        |           | (2.17)    |
| Luxembourg     | -0.397    | -0.037    | Venezuela      | 0.143     | 0.067     |
| (-1.16)        | (-0.68)   |           | (0.71)         |           | (0.46)    |
| Malaysia       | -0.011    | 0.445     |                |           |           |
| (-0.07)        | (1.53)    |           |                |           |           |

Note: The table shows the slope coefficient (beta) and accompanying Newey-West $t$-statistic from a regression of returns on a two-year rolling standard deviation as given in Equation [1]. Bull and bear markets are defined according to whether the market three-year moving average is rising (bull) or falling (bear).

Overall, the results from a battery of tests on market- and firm-level data indicate that the relation between risk and return is not uniform across countries and different market states. Evidence is detected to suggest a positive risk-return trade-off over long horizons, when returns are high, and during bear markets. By contrast, the relation is negative at the individual firm level, low return levels and during bull markets. Thus, the paper finds evidence to support a risk-return relation that switches according to the level of market aggregation, the level of returns and the overall market state. 38

7. Summary and conclusion.

A central core of finance theory retains the belief in a positive risk-return trade-off. This belief arises from the behaviour of risk-averse investors who build mean-variance efficient portfolios such that expected returns are an
Table 13. Return-Risk Coefficients Across Individual Firms – Bull vs Bear Markets.

| Market      | Bull Beta | Bear Beta |
|-------------|-----------|-----------|
| Australia   | −0.061    | 0.126     |
|             | (−3.32)   | (5.34)    |
| Canada      | 0.411     | 0.457     |
|             | (1.93)    | (14.64)   |
| France      | −0.024    | 0.043     |
|             | (−0.13)   | (0.22)    |
| Germany     | −0.480    | −0.080    |
|             | (−2.23)   | (−0.23)   |
| Hong Kong   | −0.345    | 0.774     |
|             | (−2.68)   | (3.52)    |
| Italy       | 0.169     | 0.063     |
|             | (3.25)    | (1.33)    |
| Japan       | −0.316    | 0.177     |
|             | (−4.51)   | (3.04)    |
| South Korea | −0.097    | 0.292     |
|             | (−1.61)   | (1.95)    |
| UK          | −0.203    | 0.075     |
|             | (−6.76)   | (1.87)    |
| US          | −0.115    | 0.139     |
|             | (−3.54)   | (3.83)    |

Note: The table shows the coefficient values and autocorrelation and heteroscedasticity adjusted t-statistics from Equation [2]. Bull and bear markets are defined according to whether the market three-year moving average is rising (bull) or falling (bear).

Increasing function of volatility. Despite mounting empirical evidence against this view, there remains a belief that the failure to report a positive relation is due to modelling issues or the use of small data samples rather than the absence of an intrinsically positive relation (Lunblad 2007; Wu and Lee 2015). Consequently, this paper has sought to examine the risk-return relation across a range of international stock markets at both the market and individual stock level and for different frequencies and time horizons; existing work in the area typically focusses on only one type of data. Further, this paper examines the nature of the relation using a range of different modelling techniques, including standard linear, panel and quantile regression analysis, models designed to capture expected returns behaviour, different definitions of risk and models that vary according to the level of volatility and market conditions. This paper seeks to provide an answer to the question of not only whether there is a positive risk-return relation but also where it can be found. We believe this broad set of evidence will provide a robust answer to this question.

Overall, and employing a global view, the results suggest that there is no systematic relation between risk and return. Notwithstanding this general conclusion, it is apparent that there is an emergence of characteristics that define where a systematic relation does (and does not) occur. For example, the market index results indicate that a positive relation exists across markets, in a static context, using both shorter monthly and longer annual data. These results suggest a cross-sectional positive risk and return relation. However, this positive relation is not evident when individual markets are examined over time; this result holds for both market- and firm-level data. When examining the behaviour across sub-samples of the data, a systematic relation according to return size and bull or bear market phase is observed. Specifically, examining the return quantile results reveals a positive relation at the low return quantile and a negative relation at the high quantile. A positive relation is also identified during a bear market, with a negative relation occurring during a bull run. Elsewhere, such as according to high and low volatility states or utilising alternative measures of risk and return, there is no obvious systematic relation. An exception to this generalisation is when the VIX is used as a measure of risk. Specifically, in this case, the results confirm a negative relation with returns that was previously reported.

In sum, a central view is that risk-averse investors build a portfolio such that there exists a positive risk-return relation, both in a static CAPM and time-varying ICAPM framework. However, our results suggest that there
is no single straightforward systematic risk-return relation of the kind suggested in the standard asset pricing literature. Instead, the results support the view that the sign of the risk-return relation switches across the level of returns and whether the market is in a bull or bear state. Furthermore, and in contrast to earlier work, volatility regimes do not appear to define a risk-return relation, with evidence of increasing and decreasing returns across volatility-ranked portfolios. Thus, the paper is supportive of the recent, and increasing, view that a universal notion of a positive risk-return relation does not exist. Instead, the relation can be more accurately described as switching in nature. One possible explanation for the results is the changing risk preferences of investors, who may be more prone to act in a risk averse or risk-taking manner according to the level of returns and the general market state. The presence of changing risk preferences on the part of investors has important implications for financial activities such as asset pricing and portfolio selection. In particular, conventional finance theory assumes time-invariant risk-aversion, which then generates the traditional positive risk-return relation. Therefore, the future development of asset pricing models must seek to incorporate the changing risk appetite of investors. Future research could usefully expand upon the work presented here by examining the role of higher moments, such as skewness, kurtosis and downside risk, in determining the risk-return relation, with a view to constructing more accurate risk management tools. Equally, another fruitful line of enquiry would be to examine state variables, such as macroeconomic factors, to adequately capture the changing environment that confronts investors.

Notes

1. Most of the recent research effort investigates time series variation in the risk-return trade-off (Salvador, Floros, and Arago 2014; Wu and Lee 2015; Frazer and Liu 2016; Ghysels, Plazzi, and Valkanov 2016). Recent studies that have focused on the cross-sectional risk-return relation include Ang et al. (2006, 2009), Bali and Cakici (2008), Huang et al. (2010, 2012), and Wang, Yan, and Yu (2017).
2. They further find that the state of the economy depends on more than the business cycle alone.
3. In a forecasting context, Christoffersen and Diebold (2006) and Anatolyev and Gospodinov (2010) argue that volatility and return sign dependence are linked.
4. The volatility feedback hypothesis was proposed by French, Schwert, and Stambaugh (1987) and Campbell and Hentschel (1992). Under this hypothesis, higher volatility in the current period induces expectations of higher volatility in the future, a higher expected return and a higher discount rate. The higher discount rate reduces the present value of future cashflows and, thus, causes the current stock price to fall. As a result, price falls tend to be contemporaneously related to high volatility. For studies that have attempted to decompose the risk-return relationship into a risk premium and a volatility feedback mechanism, the reader is referred to Yang (2011) and Wang and Yang (2013).
5. This issue further relates to a separate line of literature that examines idiosyncratic volatility and whether this is a price risk for stock returns (see, for example, Bali and Cakici 2008; Ang et al. 2009). Idiosyncratic volatility is not examined in the current paper.
6. Given the large number of existing tables in the paper, we do not present summary statistics, but these are available upon request. Moreover, the characteristics of the data are typical of those observed for stock price data.
7. The Datastream total market indices are highly correlated with their respective national indexes. For example, the correlation between the S&P500 and the Datastream Total Market Index for the US is 0.997.
8. This data set also straddles the 2007/08 global financial crisis and, thus, facilitates an examination of whether the crisis affected the risk-return relation. However, the inclusion of both an intercept and slope dummy variable does not affect the results reported in Table 1. Full results are available from the authors upon request.
9. As we are using firm-level data, one issue is whether to winsorise the data to remove the potential effects of outliers. We do not do this routinely for two reasons. First, the firms are from a major index in each country and are thus relatively large and, second, because we consider the quantile regression approach that directly captures different parts of the distribution.
10. A mix of different data frequencies is employed in the existing literature to examine the risk-return relationship. While some studies use low frequency data, at quarterly and monthly intervals, to eliminate short-term noise (Frazer and Lu, 2016; Wu and Lee 2015; Aslanidis, Christiansen, and Savva 2016), others utilise weekly (Guo and Neely 2008), daily (Darrat et al. 2011) or even intraday data (Sevi 2013; Badshah et al. 2016). A small number of studies employ a MIDAS specification that allows for mixed data frequencies (Salvador, Floros, and Arago 2014; Ghysels, Plazzi, and Valkanov 2016).
11. Indeed, Galagedera, Maharaj, and Brooks (2008) emphasise the importance of considering other time horizons besides the typical daily and monthly frequencies when investigating the risk-return relation.
12. For each country we use a short-term interest rate, typically a three-month Treasury bill or an approximate series, depending on the country.
13. The use of standard deviation as a measure of risk in practice has been studied extensively. For example, the behavioural approach to risk focuses on the notion of loss aversion and documents that, in practice, individuals weight losses twice as
much as gains (Tversky and Kahneman 1992). Similarly, in an investor context, Unser (2000) and Veld and Veld-Merkoulova (2008) find that investors prefer to use shortfall risk or semi-variance, rather than standard deviation as a measure of risk. Finally, several studies have focused on managerial attitudes to risk and report that, in practice, risk is associated with negative outcomes and uncertainty regarding positive outcomes is typically not perceived as ‘risk’ (March and Shapira 1987; Helliar et al. 2002).

14. Barberis and Huang (2008, 2069) argue that some investors place more value on positively skewed securities as it makes the distribution of their overall wealth more ‘lottery-like’. Thus, the authors argue that some investors may be willing to pay a high price for positively skewed stocks and to accept a negative average excess return.

15. Specifically, these values are based on the sample mean and standard deviation and not the rolling standard deviation described in the text.

16. Returns for these markets (Mexico, Russia, and Turkey in Venezuela) are all in excess of 1.60 per cent.

17. Specifically, the results are significant at the five per cent level for France and the UK, and at the ten per cent level for Denmark and Italy.

18. The importance of heterogeneity in the beliefs of investors to asset pricing is cogently argued by Shefrin (2008). Here, expected stock returns in the US are bi-modal and fat-tailed due to the extreme beliefs of optimistic and pessimistic investors.

19. Specifically, there is a negative risk-return relationship that is statistically significant at the five per cent level for Austria, China, Cyprus, the Czech Republic, Finland, Germany, Greece, India, Indonesia, Ireland, Italy, Japan, Luxembourg, Malaysia, Mexico, the Netherlands, New Zealand, the Philippines, Poland, South Africa, South Korea, Sweden, Taiwan, Thailand, Turkey, the UK, the US and Venezuela. The relationship is significant at the ten per cent level for Australia, Belgium, Brazil, Norway, Pakistan, and Singapore.

20. The Czech Republic is the only market that has a negative risk-return relationship at the highest return quantile, although it is not statistically significant.

21. The 34 markets that show a positive risk-return relationship that is significant at the five per cent level include Austria, Australia, Belgium, Brazil, China, Denmark, Finland, France, Germany, Greece, Hong Kong, Indonesia, Italy, Japan, Malaysia, Mexico, the Netherlands, Norway, New Zealand, Pakistan, the Philippines, Poland, Russia, South Africa, South Korea, Singapore, Sri Lanka, Sweden, Taiwan, Turkey, the UK, the US and Venezuela. The relationship is positive and significant at the ten per cent level in Canada, Cyprus and Spain.

22. The results are significant at the five per cent level for Australia, Belgium, Canada, Denmark, France, Germany, Ireland, Norway, South Africa, Spain, Sweden, Switzerland, and the US.

23. Only South Africa shows a positive relation that is not statistically significant, while Denmark is significant at the ten per cent level.

24. Prospect Theory was pioneered by Kahneman and Tversky (1979) and later extended by Tversky and Kahneman (1992). The theory has been used to explain the disposition effect, as well as other phenomena in finance, such as momentum and the equity premium puzzle. For a review of Prospect Theory and its application to economics, the reader is referred to Barberis (2013).

25. As we are looking at firm-level returns, it can be argued that Equation [3] could be expanded to include additional explanatory variables. For example, Blitz and van Vliet (2007) and Blitz, Pang, and van Vliet (2013) consider the Fama and French three factor model. We consider this explicitly below but note that their inclusion does not affect the qualitative nature of the results reported here.

26. If we winorise at the one per cent level, the only material difference is that the result for Australia becomes statistically significant at the five per cent level, as opposed to ten per cent, as reported in Table 5.

27. Specifically, the coefficient was statistically significantly positive for China at the five per cent level, and statistically significantly negative for India and Thailand at the ten per cent level.

28. The increased evidence of positive and significant results is consistent with the results reported for the US market by Ludvigson and Ng (2007), who also consider predictive regressions. Nevertheless, our results suggest that the positive relation is not found across all markets.

29. South Korea was excluded from this analysis due to the unavailability of data needed to estimate the FF3 model.

30. A significant positive risk-return relationship is documented for Australia, Canada and Hong Kong using the CAPM, and for Australia, Canada and the US using the FF3 model. By contrast, there is a significant negative relationship for the UK using the CAPM, and for Germany, Hong Kong, Italy and the UK using the FF3 model.

31. As a further robustness check, we include US lagged returns and standard deviation in the regression of equation [2]. This follows the general literature on spillover effects and, specifically, Rapach, Strauss, and Zhou (2013) who argue that US stock returns condition international market returns. The results are identical to those reported in Table 1 and, thus, are not reported separately. Specifically, only China reveals a positive and significant risk-return relation, while a significant and negative relation is found for Japan. All the remaining markets have an insignificant coefficient at the five per cent level.

32. The coefficient is statistically significantly positive for only the UK, and statistically significantly negative for Canada, Italy, Japan and the US.

33. The markets with a statistically significant positive coefficient include Australia, Germany, Hong Kong and the UK, while the markets with a statistically significant negative coefficient are Italy, Japan and the US.

34. The UK is the only market with a statistically significant coefficient. This coefficient is positive.
35. In particular, the relation is significantly negative at the five per cent level for Australia, Germany, Hong Kong, Japan, the UK and the US, and at the ten per cent level for South Korea.
36. Statistically significantly positive coefficients at the five per cent level are reported for Australia, Canada, Hong Kong, Japan, and the US. The coefficient is significantly positive at the ten per cent level for South Korea and the UK.
37. By contrast, Wu and Lee (2015) find evidence to suggest that the risk-return relationship in the US market is significantly positive in bull markets and significantly negative in bear markets.
38. That the risk-return relationship varies across different regimes is also supported by Ghysels, Guerin, and Marcellino (2014) for the US stock market. Similarly, in a study of 13 European stock markets, Aslanidis, Christiansen, and Savva (2016) report that the strength of the risk-return relationship varies according to the state of the economy.

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References
Adcock, C. 2013. “Ex Post Efficient Set Mathematics.” Journal of Mathematical Finance 3: 201–210.
Anatolyev, S., and N. Gospodinov. 2010. “Modeling Financial Return Dynamics via Decomposition.” Journal of Business and Economic Statistics 28 (2): 232–245.
Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang. 2006. “The Cross-Section of Volatility and Expected Returns.” The Journal of Finance 61 (1): 259–299.
Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang. 2009. “High Idiosyncratic Volatility and low Returns: International and Further U.S. Evidence.” Journal of Financial Economics 91 (1): 1–23.
Aslanidis, N., C. Christiansen, and C. S. Savva. 2016. “Risk-return Trade-off for European Stock Markets.” International Review of Financial Analysis 46: 84–103.
Badshah, I., B. Frijns, J. Knif, and A. Tourani-Rad. 2016. “Asymmetries of the Intraday Return-Volatility Relation.” International Review of Financial Analysis 48: 182–192.
Baker, M., B. Bradley, and J. Wurgler. 2011. “Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly.” Financial Analysts Journal 67 (1): 40–54.
Baker, N. L., and R. A. Haugen. 2012. “Low Risk Stocks Outperform within All Observable Markets of the World.” Working Paper. Accessed 21 June 2018. https://www.lowvolatilitystocks.com/wp-content/uploads/Low_Risk_Stocks_Outperform.pdf, April.
Bali, T. G., and N. Cakici. 2008. “Idiosyncratic Volatility and the Cross Section of Expected Stock Returns.” Journal of Financial and Quantitative Analysis 43 (1): 29–58.
Bali, T. G., and N. Cakici. 2010. “World Market Risk, Country-Specific Risk and Expected Returns in International Stock Markets.” Journal of Banking & Finance 34 (6): 1152–1165.
Bali, T. G., K. O. Demirtas, and H. Levy. 2009. “Is There an Intertemporal Relation Between Downside Risk and Expected Returns.” Journal of Financial and Quantitative Analysis 44 (4): 883–909.
Barberis, N. 2013. “Thirty Years of Prospect Theory in Economics: A Review and Assessment.” Journal of Economic Perspectives 27 (1): 173–196.
Barberis, N., and M. Huang. 2008. “Stocks as Lotteries: The Implications of Probability Weighting for Security Prices.” American Economic Review 98 (5): 2066–2100.
Bekaert, G., C. B. Erb, C. R. Harvey, and T. E. Viskanta. 1998. “Distributional Characteristics of Emerging Market Returns and Asset Allocation.” The Journal of Portfolio Management 24 (2): 102–116.
Bekaert, G., and C. R. Harvey. 1997. “Emerging Equity Market Volatility.” Journal of Financial Economics 43 (1): 29–77.
Bekaert, G., and G. Wu. 2000. “Asymmetric Volatility and Risk in Equity Markets.” Review of Financial Studies 13 (1): 1–42.
Black, F. 1976. “Studies of stock price volatility changes.” Proceedings of the 1976 meeting of the business and economic statistics section, American Statistical Association, 177–181.
Blitz, D., J. Pang, and P. van Vliet. 2013. “The Volatility Effect in Emerging Markets.” Emerging Markets Review 16: 31–45.
Blitz, D. C., and P. van Vliet. 2007. “The Volatility Effect.” The Journal of Portfolio Management 34 (1): 102–113.
Bollerslev, T., D. Osterrieder, N. Sizova, and G. Tauchen. 2013. “Risk and Return: Long run Relations, Fractional Cointegration, and Return Predictability.” Journal of Financial Economics 108 (2): 409–424.
Brandt, M. W., and L. Wang. 2010. “Measuring the Time-Varying Risk-Return Relation from the Cross-Section of Equity Returns.” Working Paper, Duke University.
Breckenfelder, H. J., and R. Tédongap. 2012. “Asymmetry Matters: A High-Frequency Risk-Reward Trade-Off.” Working Paper. Accessed 21 June 2018. http://romeo-tedongap.com/medias/hf-gda-july2012.pdf, June.
Campbell, J. Y., and L. Hentschel. 1992. “No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns.” Journal of Financial Economics 31 (3): 281–318.
Campbell, J. Y., and S. Thompson. 2008. Predicting excess stock returns out of sample: Can anything beat the historical average? Review of Financial Studies, 21, 1509-1531.
Chang, K.-L. 2016. “Does the Return-State-Varying Relationship Between Risk and Return Matter in Modeling the Time Series Process of Stock Return?” International Review of Economics & Finance 42: 72–87.
Chauvet, M., and S. Potter. 2000. “Coincident and Leading Indicators of the Stock Market.” *Journal of Empirical Finance* 7 (1): 87–111.

Cheng, A.-R., and M. R. Jahan-Parver. 2014. “Risk–Return Trade-off in the Pacific Basin Equity Markets.” *Emerging Markets Review* 18: 123–140.

Chiang, T. C., H. Li, and D. Zheng. 2015. “The Intertemporal Risk–Return Relationship: Evidence From International Markets.” *Journal of International Financial Markets, Institutions and Money* 39: 156–180.

Christensen, B. J., MØ Nielsen, and J. Zhu. 2015. “The Impact of Financial Crises on the Risk-Return Tradeoff and the Leverage Effect.” *Economic Modelling* 49: 407–418.

Christie, A. 1982. “The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects.” *Journal of Financial Economics* 10 (4): 407–432.

Christoffersen, P. F., and F. X. Diebold. 2006. “Financial Asset Returns, Direction-of-Change Forecasting, and Volatility Dynamics.” *Management Science* 52 (8): 1273–1287.

Cooper, M., R. Gutierrez, Jr., and A. Hameed. 2004. “Market States and Momentum.” *The Journal of Finance* 59 (3): 1345–1365.

Darrat, A. F., O. W. Gilley, B. Li, and Y. Wu. 2011. “Visiting the Risk/Return Relations in the Asian Pacific Markets: New Evidence From Alternative Models.” *Journal of Business Research* 64 (2): 199–206.

de Carvalho, R. L., L. Xiao, and P. Moulin. 2012. “Demystifying Equity Risk-Based Strategies: A Simple Alpha Plus Beta Description.” *The Journal of Portfolio Management* 38 (3): 56–70.

Diebold, F. X., and K. Yilmaz. 2009. “Measuring Financial Asset Return and Volatility Spillovers, with Application to Global Equity Markets.” *The Economic Journal* 28 (1): 57–66.

Diebold, F. X., and K. Yilmaz. 2012. “Better to Give Than to Receive: Predictive Directional Measurement of Volatility Spillovers.” *International Journal of Forecasting* 119 (534): 158–171.

Dimson, E., P. Marsh, and M. Staunton. 2002. *Triumph of the Optimists: 101 Years of Global Investment Returns*. Princeton, New Jersey: Princeton University Press.

Dimson, E., P. Marsh, and M. Staunton. 2008. “The Worldwide Equity Premium: A Smaller Puzzle.” In *Handbook of the Equity Risk Premium*, edited by R. Mehra, 467–514, Chapter 11. Amsterdam: Elsevier.

Dimson, E., P. Marsh, and M. Staunton. 2011. “Equity Premiums around the World.” In *Rethinking the Equity Risk Premium*, CFA Institute Research Foundation, December, 32–52.

Engle, R. F., T. Ito, and W. Lin. 1990. “Meteor Showers or Heat Waves? Heteroskedastic Intra-Daily Volatility in the Foreign Exchange Market.” *Econometrica* 58: 525–542.

Eun, C. S., and S. Shin. 1989. “International Transmission of Stock Market Movements.” *The Journal of Financial and Quantitative Analysis* 24 (2): 241–256.

Fenou, B., M. R. Jahan-Parver, and R. Tedongap. 2013. “Modeling Market Downside Volatility.” *Review of Finance* 17 (1): 443–481.

Frazier, D. T., and X. Liu. 2016. “A new Approach to Risk-Return Trade-off Dynamics via Decomposition.” *Journal of Economic Dynamics and Control* 62: 43–55.

French, K. R., G. W. Schwert, and R. F. Stambaugh. 1987. “Expected Stock Returns and Volatility.” *Journal of Financial Economics* 19 (1): 3–29.

Galagedera, D. U. A., E. A. Maharaj, and R. Brooks. 2008. “Relationship Between Downside Risk and Return: new Evidence Through a Multiscaling Approach.” *Applied Financial Economics* 18 (20): 1623–1633.

Ghysels, E., P. Guerin, and M. Marcellino. 2014. “Regime Switches in the Risk-Return Trade-off.” *Journal of Empirical Finance* 28: 118–138.

Ghysels, E., A. Plazzi, and R. I. Valkanov. 2016. “The Risk-Return Relationship and Financial Crises.” Working Paper. Accessed 21 June 2018. [https://papers.ssrn.com/sol3/papers.cfm?abstract_id = 2776702](https://papers.ssrn.com/sol3/papers.cfm?abstract_id = 2776702)

Guo, H., and C. J. Neely. 2008. “Investigating the Inter-Temporal Risk-Return Relation in International Stock Markets with the Component GARCH Model.” *Economics Letters* 99 (1): 2–9.

Hansen, L. P., and S. F. Richard. 1987. “The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models.” *Econometrica* 55 (3): 587–613.

Harvey, C. R. 2001. “The Specification of Conditional Expectations.” *Journal of Empirical Finance* 8 (5): 573–637.

Haugen, R. A., and N. L. Baker. 1991. “The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios.” *The Journal of Portfolio Management* 17 (3): 35–40.

Hedegaard, E., and R. J. Hodrick. 2016. “Estimating the Risk-Return Trade-off with Overlapping Data Inference.” *Journal of Banking & Finance* 67: 135–145.

Helliar, C. V., A. A. Lonie, D. M. Power, and C. D. Sinclair. 2002. “Managerial Attitudes to Risk: A Comparison of Scottish Chartered Accountants and U.K. Managers.” *Journal of International Accounting, Auditing and Taxation* 11 (2): 165–190.

Hjalmarsson, E. 2010. “Predicting Global Stock Returns.” *Journal of Financial and Quantitative Analysis* 45 (1): 49–80.

Huang, W., Q. Liu, S. G. Rhee, and F. Wu. 2012. “Extreme Downside Risk and Expected Stock Returns.” *Journal of Banking & Finance* 36 (5): 1492–1502.

Huang, W., Q. Liu, S. G. Rhee, and L. Zhang. 2010. “Return Reversals, Idiosyncratic Risk, and Expected Returns.” *Review of Financial Studies* 23 (1): 147–168.

Jagannathan, R., and T. Ma. 2003. “Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps.” *The Journal of Finance* 58 (4): 1651–1683.
Jensen, M. B., and A. Lunde. 2001. “The NIG-S&ARCH Model: A fat-Tailed, Stochastic, and Autoregressive Conditional Heteroskedastic Volatility Model.” The Econometrics Journal 4 (2): 319–342.

Jia, Y., and C. Yang. 2017. “Disagreement and the Risk-Return Relation.” Economic Modelling 64: 97–104.

Jiang, X., and B.-S. Lee. 2014. “The Intertemporal Risk-Return Relation: A Bivariate Model Approach.” Journal of Financial Markets 18: 158–181.

Jung, J., and R. J. Shiller. 2005. “Samuelson’s Dictum and the Stock Market.” Economic Inquiry 43 (2): 221–228.

Kahneman, D., and A. Tversky. 1979. “Prospect Theory: An Analysis of Decision Under Risk.” Econometrica 47 (2): 263–292.

Kanas, A. 2013. “The Risk-Return Relation and VIX: Evidence From the S&P500.” Empirical Economics 44 (3): 1291–1314.

Kinnunen, J. 2014. “Risk-return Trade-off and Serial Correlation: Do Volume and Volatility Matter?” Journal of Financial Markets 20: 1–19.

Koene, K., and G. Bassett, Jr. 1978. “Regression Quantiles.” Econometrica 46 (1): 33–50.

Koenker, R., and K. F. Hallock. 2001. “Quantile Regression.” Journal of Economic Perspectives 15 (4): 143–156.

Lintner, J. 1965. “The valuation of risk assets on the selection of risky investments in stock portfolios and capital budgets.” Review of Economics and Statistics 47: 13–37.

Liu, X. 2017. “Can Macroeconomic Dynamics Explain the Time Variation of Risk-Return Trade-Offs in the U.S. Financial Market?” The Quarterly Review of Economics and Finance 66: 275–293.

Ludvigson, S.C., and S. Ng. 2005. The Empirical Risk-Return Relation: A Factor Analysis Approach. NBER Working Paper No. 11477.

Merton, R. C. 1973. “An Intertemporal Asset Pricing Model.” Econometrica 41 (5): 867–888.

Mossin, J. 1966. “Equilibrium in a Capital Asset Market.” Econometrica 34 (4): 768–783.

Müller, G., R. B. Durand, and R. A. Maller. 2011. “The Risk-Return Tradeoff: A COGARCH Analysis of Merton’s Hypothesis.” Journal of Empirical Finance 18 (2): 306–320.

Nelson, D. B. 1991. “Conditional Heteroskedasticity in Asset Returns: A New Approach.” Econometrica 59 (2): 347–370.

Sharpe, W. F. 1964. “Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk.” Journal of Finance 19 (3): 425–442.

Shefrin, H. 2008. A Behavioral Approach to Asset Pricing. 2nd ed. Burlington: Academic Press.

Tversky, A., and D. Kahneman. 1992. “Advances in Prospect Theory: Cumulative Representation of Uncertainty.” Journal of Risk and Uncertainty 5 (4): 297–323.

Unser, M. 2000. “Lower Partial Moments as Measures of Perceived Risk: An Experimental Study.” Journal of Economic Psychology 21 (3): 253–280.

Veld, C., and Y. Veld-Merkoulova. 2008. “The Risk Perceptions of Individual Investors.” Journal of Economic Psychology 29 (2): 226–252.

Wang, H., J. Yan, and J. Yu. 2017. “Reference-dependent Preferences and the Risk-Return Trade-off.” Journal of Financial Economics 123 (3): 395–414.

Wang, J., and M. Yang. 2013. “On the Risk Return Relationship.” Journal of Empirical Finance 21: 132–141.

Welch, I., and A. Goyal. 2008. “A Comprehensive Look at the Empirical Performance of Equity Premium Prediction.” Review of Financial Studies 21 (4): 1455–1508.

White, R. F. 1994. “Time Variations and Covariations in the Expectation and Volatility of Stock Market Returns.” The Journal of Finance 49 (2): 515–541.

Wu, S.-J., and W.-M. Lee. 2015. “Intertemporal Risk–Return Relationships in Bull and Bear Markets.” International Review of Economics & Finance 38: 308–325.

Yang, M. 2011. “Volatility Feedback and Risk Premium in GARCH Models with Generalized Hyperbolic Distributions.” Studies in Nonlinear Dynamics & Econometrics 15 (3): 1–21.