Covariance matrix adaptation evolution strategy based optical phase control

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In this letter, we present an investigation of the use of a covariance matrix adaptation evolution strategy (CMA-ES) algorithm as a phase-locking method for multi-channel coherent beam combining (CBC) for the first time. A comprehensive numerical analysis is carried out on the operational performances of the CMA-ES based phase-locking algorithm implemented into 7- and 19-channel CBC systems in a filled-aperture format. Through numerical simulations, it can be verified that the CMA-ES algorithm can readily lead to over 0.90 of normalised beam combining efficiency with appropriate algorithm parameter sets, which can also be optimised by a combinatorial study.

Introduction: The scaling of the continuous-wave near-single-mode laser power beyond ~10 kW is not straightforward at all because of the non-linear and thermal effects such as Brillouin/Raman scattering, self-focusing, transverse mode instability, etc. [1, 2]. While a number of strategies have been proposed to mitigate such effects, there is indeed a limit for the maximum power that a single strand of fibre laser can handle [1]. Consequently, various beam-combining techniques have emerged to overcome the single-fibre limitation, which particularly include the spectral beam combining (SBC), coherent beam combining (CBC) techniques [3]. In fact, a lot of experimental demonstrations have shown that beam combining techniques are a viable route towards the next level high-power laser system [4, 5].

In particular, the CBC technique has earned a lot of attention by virtue of its high-performance characteristics when realised appropriately, e.g., excellent beam quality, high on-axis intensity, high-adaptability for wavefront correction, etc. [3]. In this case, the phase locking of each channel beam of the CBC system is the primary key to success, for which the following three well-known phase control algorithms are mainly adopted to control each channel beam's phase: optical homodyne detection (OHD), locking of optical coherence via single-detector electronic-frequency tagging (LOCSET), and stochastic parallel gradient descent (SPGD) [5, 6]. The OHD obtains the phase information of channel beams by comparing each channel's phase with that of a fixed reference channel, thereby requiring as many detectors as the channel beams. However, unlike SPGD based algorithms which determine the phase perturbations for the next iteration step by taking the derivatives of the perturbations of the preceding steps, CMA-ES algorithms often encounter. Thus, CMA-ES algorithms have been receiving high research attention in numerous fields in recent years [10].

In the case of CBC, the local intensity fluctuation grows substantially as the number of channel beams to be combined increases, because of the multiple interference effect [8]. Consequently, we note that the local extremum issue will also grow accordingly as far as CBC systems rely on derivative-based algorithms. We hence investigate CMA-ES based phase-locking algorithm implemented onto CBC.

Let us suppose that there is an N-channel CBC system, the N-channel beams of which are phase-controlled based on the CMA-ES algorithm. The initial phase of each channel is unknown, being randomly determined in between 0 to 2\pi rad. Thus, the CMA-ES algorithm should find a compensating phase for each channel beam such that all the channel beams eventually result in an in-phase combined state at the target plane.

The general procedure is based on the five steps as follows.

(i) Apply I sets of phases (samples) sequentially onto each channel beam.

\[ \Phi_1 (\phi_1, \phi_2, \ldots, \phi_N) \]

\[ \Phi_2 (\phi_1, \phi_2, \ldots, \phi_N) \]

\[ \vdots \]

\[ \Phi_I (\phi_1, \phi_2, \ldots, \phi_N) \]

(ii) Select top J (J ≤ I) sets of phases (elite samples) which give the highest combined beam intensities.

(iii) Calculate the covariance matrix and the mean value of the selected J sets of phases from (ii).

(iv) Update new I sets of phases which have the probability distribution represented by the covariance matrix calculated in (iii), and apply them onto each channel beam.

(v) Iterate the above steps until an optimal set is found.

Depending on the sizes of I and J, the optimisation performances, e.g., accuracy, time elapsed, may vary. However, it is relatively easy to make them suitable for optimisation of the given situation, because they are just numbers indicating the samples and elite samples used, unlike many other algorithms which normally rely on quite complex and sophisticated parameters [11].

Simulation results and discussion: Given that a hexagonal array of channel beams is generally applied to a CBC system, we consider two different cases of CMA-ES based phase-locking for 7- and 19-channel CBC systems.

Implementation of the CBC technique can be made in two ways: One is the tiled-aperture configuration and the other is the filled-aperture configuration [3]. In Figure 1, we illustrate conceptual schematics for the filled-aperture configuration, which can be realised based on a free-space diffractive optical element (DOE) or an all-fiberised combiner (AFC), for example. In particular, the filled-aperture configuration offers a simple and compact arrangement for CBC, which hints at a best conceptual
Fig. 1 Conceptual schematics for a filled-aperture-type CBC system based on (a) a diffractive optical element and (b) a fibre beam combiner.

Thus, in the following simulations, we assume that the N-channel CBC system is based on the filled-aperture configuration and that each channel is based on an ideal Gaussian mode with a randomly determined arbitrary initial phase. We also assume that the DOE or AFC’s efficiency is 100% and that there are no other additional loss or distortion mechanisms involved with it. We define the beam combining efficiency (BCE) as the ratio of the peak intensity of the combined beam obtained by the CMS-ES algorithm with respect to the theoretical maximum of the peak intensity of the combined beam in the ideal condition. It is noted that the theoretical maximum is calculated based on the assumption that all the N-channel beams are absolutely in phase at the target plane. We first perform beam combining simulations for various sets of algorithm parameters (i.e. # of samples, # of elite samples), based on which we determine the range of algorithm parameters for further investigation from the perspective of beam combining performance. We fix the total iteration number for the CMA-ES optimisation for a single trial to be 100, and repeat the trial 50 times with a renewed initial phase set.

Phase locking of 7-channel beams: We first applied the CMA-ES algorithm to the phase-locking of a 7-channel CBC system, trying various combinations of the algorithm parameters. In Figure 2, we show the simulation results on the peak intensity of the combined beam normalised by its theoretical maximum, i.e. the BCE, when we chose (10, 3), (10, 5), and (15, 4) for the algorithm parameter set (# of samples, # of elite samples) throughout the iteration. For 10 samples, we note that with 3 elite samples, the BCE were widely spread, having the minimum BCE of ∼0.4. When we increased the elite sample number to 5, we could obtain significantly improved phase-locking performance as one can see that the higher BCEs acquired more counts. The overall average BCEs over the 50 trials for both cases were given by 0.89 and 0.95, respectively. For the algorithm parameter set of (15, 4), we obtained an even more significant enhancement in the phase-locking performance resulting in the overall average BCE of 0.98, which implies that increasing the number of samples is more effective in improving the performance of phase-locking than increasing the number of elite samples.

Phase locking of 19-channel beams: We secondly applied the CMA-ES algorithm to the phase-locking of a 19-channel CBC system, also trying various combinations of the algorithm parameters. In Figure 3a, we show the simulation results on the phase locking of 19-channel beams with the exact same algorithm parameter sets that were used in the case of 7-channel beams as shown in Figure 2, i.e. (10, 3) and (10, 5). The results exhibit that the BCEs are much more widely spread than those for 7-channel beams even with the same algorithm parameter sets. The overall average BCEs are reduced down to 0.77 and 0.89 for the cases of 3 and 5 elite samples, respectively. The reason for the degradation of the BCE in this case must be obvious that the CMA-ES algorithm should deal with even more channel phases for optimisation. In Figure 3b, the simulation results are shown when (15, 4) and (15, 8) are chosen for the algorithm parameter sets.
the number of iterations for full convergence may vary depending on time as well. In other words, given that the time elapsed for a single iteration is fixed to 100, it is noteworthy that increasing the number of elite samples invariably yields a BCE in excess of 0.90 as far as the number of samples is set equal to or greater than 10. In addition, one can see that at a fixed number of samples, increasing the number of elite samples invariably yields a smaller standard deviation of the BCE, i.e., it results in a more consistent phase-locking characteristic for every trial. It is noteworthy that increasing the number of samples does not necessarily lead to the improvement in the BCE, because the total iteration number is fixed to 100. While a large set of phases may have an advantage in obtaining various possible combined intensities, it may give rise to generating duplicated phase sets that may eventually result in similar combined intensities. This may, in turn, give rise to too broad a distribution of the next-step phase sets represented by the covariance matrix, so that the convergence to the optimal phase set can arguably be disturbed. Thus, we need to pay attention to the optimisation of the algorithm parameter set itself with care.

In Figures 4c and 4d, the simulation results on the mean value of the BCE and the corresponding standard deviation for a 7-channel CBC system under various parameter conditions are shown, respectively. One can see that the CMA-ES phase-locking algorithm can yield the BCE in excess of 0.90 as far as the number of samples is set equal to or greater than 10. Also, one can see that increasing the number of elite samples leads to enhancement in the standard deviation of the BCE.

After all, in the case of the 7-channel CBC system, the best phase-locking performance obtained by the CMS-ES algorithm was given by the BCE of 0.98 and the standard deviation of 0.02 with the parameter set of (15, 5), (20, 4) or (25, 5). In the case of 19-channel CBC system, the best phase-locking performance was given by the BCE of 0.98 and the standard deviation of 0.02 with the parameter set of (20, 8) or (25, 7). We however note that the parameter set which yields the best performance in terms of the BCE and the corresponding standard deviation does not necessarily mean the best of all in that the CMA-ES phase-locking performance should also be justified in terms of a convergence time as well. In other words, given that the time elapsed for a single iteration is fixed to 100, it is noteworthy that increasing the number of elite samples between 15 and 20 and the number of elite samples between 5 and 7 should be of great interest.

**Conclusion:** We have proposed a novel CMA-ES based phase-locking algorithm for a CBC system. With a comprehensive numerical investigation on 7- and 19-channel CBC systems, we have verified that the CMA-ES based phase-locking algorithm is a viable alternative to the existing phase-locking methods for CBC [9]. We have also analysed the mean BCE and the corresponding standard deviation through a combinatorial study on the algorithm parameters. In general, it is well known that a derivative-based optimisation algorithm like SPGD has severe drawbacks in situations when the derivative of the object function of optimisation is unreliable or hard to evaluate [12], for example, when the object function encounters numerous local extrema or drastic changes around them. Thus, a derivative-free optimisation algorithm is often looked for and implemented to overcome such drawbacks [12, 13]. Given that the phase-fronts of laser beams can be distorted and fluctuated severely under the outdoor conditions for atmospheric turbulence [13], the objective function of optimisation for CBC can also suffer from such issues. In this light, the derivative-free CMA-ES algorithm can be a feasible, alternative phase-locking algorithm for CBC.

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