Simulation and analysis of multi-frequency signal superposition with scattering center model

R P Yang¹, X Wang¹, C H Liu², Y Liu² and Y Liu¹

¹National Key Laboratory of Antenna and Microwave Technology, Xidian University, Xi’an, Shaan Xi, China
²National Key Laboratory of Science and Technology on Information System Security, Beijing, China

E-mail: wangxing@mail.xidian.edu.cn

Abstract. In this paper, the superposition of time domain scattering signal with different frequency is analyzed. Simulation results shows that the peak value of the sum signal is able to be approximately estimated from scattering center model. Besides, the results show that energy of multi-frequency scattering signal has the potential to accumulate in time domain, which might help to design new kinds of signal to recognize target and resist noise at the same time.

1. Introduction

Target recognition is of great importance for both civil and military. The key to recognize an object with high-frequency electromagnetic wave is analysis of scattering field of the object. To analyze scattering field intuitively from the perspective of geometric structure, scattering center model might be a reasonable approach. It allows one intuitively understand the position of dominant contributors of scattering field as an image. Michael P [1] described two-dimensional objects and the scattering centers extracted from them, presenting a clear Fourier transform (FT) relationship between source distribution and scattered field. In the same paper, the difference between inverse Fourier transform (IFT) of electric scattered field and physical current is also mentioned. Kun yi Guo et al. [2] divided scattering centers (SC) into three types: localized scattering centers (LSC), distributed scattering centers (DSC) and sliding scattering centers (SSC). The spatial distribution of different kinds of scattering centers according to geometric structure of a 3-D UAV is analyzed. Brian D. Rigling [3] discussed a bistatic approximation of 2-D flat plate from monostatic scattering center mechanism. And they combined 2-D scattering center model (when analyzing flat plate) and geometry optics model (when analyzing right angle structure), as separable azimuth and elevation components, to model 3-D canonical reflectors, such as cylinder, tophat and trihedral. L.C. Potter et al. [4] present an approximate maximum likelihood (ML) algorithm to estimate parameter of GTD based scattering center model. Compared to Prony-based algorithm, GTD algorithm is shown to be more accurate to solve problem with a large relative bandwidth. But for small bandwidth simulation, it is not clear that parameter estimation is any more reliable. Julie Ann Jackson applied GTD scattering center model to automatic target recognition (ATR) of synthetic aperture radar (SAR) image in [5]. But references lack on time domain signal simulation using scattering center method.

The relationship between scattering field and time domain signal is found with the help of the work presented by Mark A. Richards [6]. In [6], it is discussed that received complex voltage of radar echoes can be viewed as the output of a linear filter with weighted variation in reflectivity over range or angle.
as its input. In short, observed wideband radar signal is filtered by the reflectors. And the sweep frequency scattering field computed by EM code, such as MOM or PO, could be regarded as frequency response of the linear filter - the reflectors – to the incident wave.

The relationship between scattering field and space domain inverse image is detailly discussed in work presented by C. Özdemir [7]. According to [7], scattering centers can be extracted from ISAR image or directly from reflected field using “matching pursuit” method [8]. This approach allows us find dominant contributors to scattering field.

According to [9], specular points are dominant contributors to reflected field. So, it helps understand relationship between the amplitude of scattering centers and the specular orientation of reflector surface.

In this paper, the superposition of time domain scattering signal with different frequency is analyzed. The main contributions of this paper are:

- Simulated results we presented shows that the peak value of the sum signal is able to be approximately estimated from scattering center model.
- The spatial position of scattering centers depends on the geometry structure of the reflector, it helps one intuitively understand the time when peak value of multi-frequency signal occurs.
- Results show that energy of multi-frequency scattering signal has the potential to accumulate in time domain, which might help to design new kinds of signal to resist noise.

The paper is organized as follows. Section 2 describes the formulation to show the relationship between scattering center and peak value of multi-frequency signal. In Section 3, we present simulation results with two different reflectors. Simulated signal is the sum of signal 1 and signal 2. Signal 1 consists of 5 single-frequency scattering signals at 10GHz~10.3GHz, 0.75GHz frequency interval. Signal 2 consists of another 5 single-frequency scattering signals at 11GHz~11.3GHz, 0.75GHz frequency interval. And conclusion is drawn in Section 4.

2. Theory

The relationship between time domain scattering field and the frequency response of the reflectors is given as:

$$E_{\text{time}}(\omega) = \int E(t)e^{-j\omega t}{dt}$$

According to scattering center theory presented by Michael. P. Hurst [1], the frequency response of reflectors can also be written as:

$$E_{\text{space}}(k) = \int A(z)e^{jkz}{dz}$$

where $k$ is the wavenumber in free space, $A$ is the amplitude of scattering center, and $z$ is the distance between scattering center and origin, since the scattering center theory in [1] is a 1-D theory.

$$\begin{cases} \omega = \frac{2\pi c}{\lambda} \\ k = \frac{2\pi}{\lambda} \\ k = \frac{\omega}{c} \end{cases}$$

where $c$ is lightspeed.

Thus, the frequency response of reflectors in (1) and (2) meet the following equation:

$$E_{\text{space}}(k) = E_{\text{space}}(\omega/c) = E_{\text{time}}(\omega)$$

We define (1) as space domain inverse Fourier transform (space domain IFT), and (2) as time domain inverse Fourier transform (time domain IFT).

So, the relationship between scattering center and electric field in time domain can be defined as follow:

$$A(z) \overset{\text{IFT}_{\text{space}}}{\longrightarrow} E_{\text{space}}(k) \overset{\omega/c}{\longrightarrow} E_{\text{time}}(\omega)$$

According to [6], observed complex voltage can be viewed as the output of a linear filter with weighted variation in reflectivity over range or angle as its input. The reflectivity here refers to $E(\omega)$. 


The relationship between frequency response \( E(\omega) \), wideband incident signal \( s_{inc}(t) \) and scattering signal \( s_{sc}(t) \) can be described as follow:

\[
s_{inc}(t) \xrightarrow{\varphi E_{inc}(\omega)} s_{sc}(t)
\]

(6) is equivalent to:

\[
S_{inc}(\omega) = S_{sc}(\omega) \cdot E_{inc}(\omega)
\]

(7)

where \( S_{inc}(\omega) \) is the time domain FT of \( s_{inc}(t) \), and \( S_{sc}(\omega) \) is the time domain FT of \( s_{sc}(t) \).

Back to definition in (2), the amplitude of scattering centers does not vary as wavenumber \( k \) changes. At the same time, the location of the scattering centers is not a function of frequency when solving high-frequency problem [1].

According to [7], dominant contributors to \( E(k) \) can be extracted as scattering centers from \( A(z) \) using matching pursuit method [8].

Once scattering center is extracted, the change of electric scattering field as frequency increasing is determined by the phase term \( e^{j\phi} = e^{jkr} \) (as is shown in Figure 1).

The position of scattering center is related to direction of specular reflection from the reflector surface. According to [9], it is a well-known phenomenon that waves of short wavelength, compared with object dimensions, are scattered almost entirely from those surface points which are specular oriented. Analysis using stationary phase principle is interpreted, in showing that specular points are dominant contributors to reflected field.

It is discussed in [1] that the extracted scattering centers are not the real sources. Physical currents distributed everywhere on the reflector. However scattering centers usually located at an aft edge or tip vertex. This phenomenon could be related to that the swept frequency stationary phase point is only at the geometry end region. And probably because of that, in high-frequency scattering, the location of these scattering centers is not a function of frequency [1].

We use PO method in FEKO to compute scattering field of 10 different discrete frequency. We assume each incident wave is a single-frequency signal. Besides, we assume \( S_{inc}(\omega) = 1(V) \), and each incident wave reach the reflector at the same time. So that the sum of them meets the following equation:

\[
S_{sc}(t) = \sum_{i=1}^{N} \text{IFT}_{time}\left(S_{inc}(\omega) \cdot E_{inc}(\omega)\right) = \sum_{i=1}^{N} \text{IFT}_{time}\left(E_{space}(k, c)\right)
\]

(8)

where \( \text{IFT}_{time} \) is inverse Fourier Transform in time domain.

The peak value of (8) could be asymptotically estimated by scattering centers:

\[
t_{peak} \approx \left( z_{sc} + n \Delta z \right) / c, n = 1, 2, 3, ...
\]

(9)
where $t_{\text{peak}}$ is the time when dominant peak value of signal occurs, $z_{SC}$ is the spatial location of scattering centers extracted from electric field of 5 frequency (10GHz~10.3GHz, frequency interval 0.75GHz), $\Delta z$ is the total range of inverse image of the electric field of 5 frequency mentioned above, and $c$ is light speed.

Then we compare the time of peak value we predict from 5 frequency point and simulated sum signal of all incident wave of 10 different frequency. Flaw chart is shown in Figure 2.

Figure 2. Multi-frequency signal superposition flow chart.

3. Numerical Result

The first example we present is a 2m*1.5m*1.5m cube as shown in Figure 3. It is meshed into 547660 triangles with lambda/3 at 12GHz. Incident angle is $\theta=0^\circ$, $\phi=0^\circ$, polarization angle is $0^\circ$. Receive angle is $\theta=90^\circ$, $\phi=0^\circ$, and only main polarization component is considered.

Figure 3. 2m*1.5m*1.5m cube.

Signal 1 consists of 5 single-frequency scattering signals at 10GHz~10.3GHz, 0.75GHz frequency interval. Signal 2 consists of another 5 single-frequency scattering signals at 11GHz~11.3GHz, 0.75GHz frequency interval. We assume that all of those signals reach the reflector at the same time and the amplitude of each signal is 1 V.

Simulated results are shown in Figure 4.

As shown in Figure 4(a), two dominant scattering centers are extracted at -1.5m and 0.5m, which depends on the positions of two edges of the cube. The total range of the inverse image of scattering field is 4m. So, the predicted time of signal peak value meets the following equation:

$$t_{\text{peak}}^1 \approx (-1.5+n \times 4)/c, \; n = 1, 2, 3, \ldots$$

$$t_{\text{peak}}^2 \approx (0.5+n \times 4)/c, \; n = 1, 2, 3, \ldots$$

As shown in Figure 4(b), though it might because of limited bandwidth and number of frequency points, the location of scattering centers doesn’t lie on the exact position of the edges of cube, the predicted time of peak value and simulated one are approximately the same.
The second example we present is a 3.5m*3m*0.74m airplane as shown in Figure 5. It is meshed into 226674 triangles with lambda/3 at 12GHz. Incident angle is $\theta=0^\circ$, $\phi=0^\circ$, polarization angle is $0^\circ$. Receive angle is $\theta=90^\circ$, $\phi=0^\circ$, and only main polarization component is considered.

Signal 1 consists of 5 single-frequency scattering signals at 10GHz~10.3GHz, 0.75GHz frequency interval. Signal 2 consists of another 5 single-frequency scattering signals at 11GHz~11.3GHz, 0.75GHz frequency interval. We assume that all of those signals reach the reflector at the same time and the amplitude of each signal is 1 V.

Simulated results are shown in Figure 6.
As shown in Figure 6(a), one dominant scattering center is extracted at -0.25m, which is the position of wing leading edge. The total range of the inverse image of scattering field is 4m. So, the predicted time of signal peak value meets the following equation:

$$t_{\text{peak}} \approx \frac{0.25+n \times 4}{c}, \quad n = 1, 2, 3, \ldots$$

(11)

As shown in Figure 6(b), the predicted time of peak value and simulated one are approximately the same.

4. Conclusion
In this paper, the superposition of time domain scattering signal with different frequency is analyzed.

Simulation results shows that the peak value of the sum signal is able to be approximately estimated from scattering center model.

Furthermore, since the spatial position of scattering centers depends on the geometry structure of the reflector, it helps one intuitively understand the time when peak value of multi-frequency signal occurs.

Besides, the results show that energy of multi-frequency scattering signal has the potential to accumulate in time domain, which might help to design new kinds of signal to recognize target and resist noise at the same time.

References
[1] M. P. Hurst and R. Mittra, Scattering center analysis via Prony's method, *IEEE Trans. Antennas Propagat.*, vol. 35, no. 8, pp. 986-988, 1987.
[2] K. Guo, Q. Qu and X. Sheng, Geometry Reconstruction Based on Attributes of Scattering Centers by Using Time-Frequency Representations, *IEEE Trans. Antennas Propagat.*, vol. 64, no. 2, pp. 708-720, 2016.
[3] B. D. Rigling, “Signal processing strategies for bistatic synthetic aper-ture radar,” Ph.D. dissertation, Dept. Elect. Eng., The Ohio State Univ., 2003.
[4] L. C. Potter, Da-Ming Chiang, R. Carriere and M. J. Gerry, A GTD-based parametric model for radar scattering, *IEEE Trans. Antennas Propagat.*, vol. 43, no. 10, pp. 1058-1067, 1995.
[5] J. A. Richards, “Target Model Generation from Multiple Synthetic Aperture Radar Images,” Ph.D. dissertation, Dept. Elect. Eng., Massachusetts Institute of Technology, 1996.
[6] M. A. Richards, “Fundamentals of Radar Signal Processing,” 2nd ed., McGraw-Hill, 2005, pp. 90-92.
[7] C. Özdemir, “Inverse Synthetic Aperture Radar Imaging with MATLAB Algorithms,” John Wiley & Sons, 2012, pp. 283-297.
[8] S. G. Mallat and Z. Zhang, Matching pursuits with time-frequency dictionaries, *IEEE Trans. Signal Process.*, vol. 41, no. 12, pp. 3397-3415, 1993.
[9] George T. Ruck, “Radar Cross Section Handbook,” Peninsula, 1970, pp. 59-64.