Inline holography of miniaturized objects with an intrinsic reference angle determined by the sagitta

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Abstract
The twin image problem, well known in Gabor holography, greatly obstructs the output of high-quality holograms. Previous solutions include numerical and instrumental means to eliminate or mitigate the issue. The proposed method demonstrates the feasibility of using the sagitta angle in a spherical reference beam as an intrinsic reference angle within in-line holography. Along with a miniaturized version of the object, to allow for a wide range of object sizes, from millimeters to meters, with a small variation within the original optical system. The inherent reference angle allows for a separation of the twin images in the reconstruction of a Gabor hologram, while maintaining the system on axis. Under lens-less Fourier criteria, the peripheral information of the visual field interferes with a spherical wave to generate an interference pattern that results in a hologram with separated images.

Keywords Holography · Computer generated holograms · Gabor · Inline holography

1 Introduction
The concept of holography was proposed and demonstrated by Denis Gabor in the forties. While trying to expand the resolution limits of electron microscopy, he devised a lensless microscope capable of recording the complex phase and amplitude of a weakly diffracting sample [1].

Gabor’s inline technique precedes the apparition of the laser, and its true power was not appreciated until after its invention. It was quickly applied to biology by measuring growth rates of plants and to other sciences by proposing methods to measure positions and sizes of droplets [2] or other particles. As soon as 1986, a computer algorithm that allowed for a digitized hologram capable of being reconstructed by computer processing was reported [3]. Nevertheless its multiple applications, inline holography has always been limited by the well-known twin image problem where both virtual and real images overlap in the reconstructed image.

Over the years, different techniques have been developed for the mitigation and complete elimination of the twin image in inline holography [4, 5]. It is often needed a second holographic recording to cancel the obscuring object or iterative algorithms that retrieve absorption and phase distributions from a single holographic record. The method proposed in this work complements the existing solutions, that require heavy computational power and many iterations. This technique completely circumvents the need for any post processing of the hologram as the twin images are separated within the experimental arrangement.

We propose a method for recording inline Gabor-type holograms that satisfy lens-less Fourier techniques and thus can be reconstructed by applying the Fast Fourier Transform (FFT). The proposed technique requires a spherical reference angle. The spherical wavefront will form an intrinsic reference angle that produces a separation between the real image and its conjugate, resulting in a clean recovery of
the original wavefront without any post-processing of the hologram or filtering of the conjugate image.

The important particularity is that the propagated field of the object corresponds to an image located in the periphery of the visual field of the optical system used to reduce the dimensions of the object. The concept is simple, but powerful, since this way of propagating the image forms a field that exists outside of the optical axis; allowing us to remove the superposition of the conjugate pair of images, as well as zero order and the propagation halo. This is possible thanks to the sagitta within the radius of curvature of the spherical reference wave. Interference with the propagated object plane, captured by the CCD, introduces a fringe modulation at a specific point in the periphery of the visual field; forming a hologram capable of being reconstructed by FFT and lacking the superimposed images.

2 Theory

2.1 Wavefront for Huygens–Fresnel–Kirchhoff

The wavefront is constructed using the Huygens–Fresnel–Kirchhoff equation [6].

\[
W(u,v,z) = z \sum_{k=1}^{p} \sum_{j=1}^{m} \sum_{i=1}^{n} O(x_i,y_j,z_k) \exp(iKr) \frac{1}{r^2}
\]  

(1)

where \( r \) is defined as \( r = \sqrt{(u-x)^2 + (v-y)^2 + z^2} \) and corresponds to the distances from each point of the object to the holographic plane. The objects \( O(x,y,z) \), can be arbitrary flat or 3D objects, and \( K \) is the wavenumber. The wavefront of the object represented by Eq. (1) was implemented using optimized calculation techniques [7, 8]. We take this approach in order to generalize the concept for 3D and 2D objects. Our particular case is equivalent to a 3D object sectioned into slices of arbitrary distances; when restricted to 2D flat objects, it is feasible to apply angular spectrum theory [9].

2.2 Hologram formation

The wavefront of an object represented by Eq. (1), can be superimposed with a spherical reference wave \( R(x,y,z) \), with a radius of curvature equal to the distance from the object to the propagation plane. This condition is known as lens-less Fourier.

\[
H(u,v,z) = \{ R(x,y,z) \exp(iKr) + W(u,v,z) \}
\]  

(2)

Equation (2) represents the hologram \( H(u,v,z) \), with the general shape of the wavefront, and the reference beam with spherical phase, at equal \( r \) distances. Both exposures are superimposed with an angle equal to zero. That is, they are superimposed on axis, generating a Gabor hologram. This physical condition is essential to the formation of lens-less Fourier holograms, through Huygens–Fresnel–Kirchhoff or angular spectrum rules [10].

2.3 Sagitta

The importance of a spherical reference beam lies within the sagitta \( S \) [11]. This concept, illustrated in Figure 1, is used to determine the inherent reference angle \( \alpha \). The hologram incident on the CCD surface is essential in the work zone \((a, b)\). In Gabor hologram formation, \( \alpha \) is determined by the size of the wavefront zone; delimited by \((a, b)\) [12].

The peripheral information of the system. The sagitta becomes a major value as an implicit reference angle \( \alpha \); determined between the distance of origin \((ao - bo) / 2 + bo\), that is same as chord \( L/2 \), and the sagitta \( S \), parameters.

\[
S = r - \left( r^2 - \left( \frac{L}{2} \right)^2 \right)^{1/2}
\]  

(3)

\[
\alpha = \arctan \left( \frac{S}{\left( \frac{ao-bo}{2} \right) + bo} \right) = \arctan \left( \frac{2L}{L} \right)
\]  

(4)

One of the peculiarities of the Gabor holograms can be appreciated when illuminated with a spherical reference wave. One can observe a circular diffraction pattern known as Newton rings, defined in Eq. (5), and further explained in Supplement 1. Corresponding to the family of stripes of equal thickness [13], this concept is closely related to the sagitta and chord. \( \rho_n \) corresponds to the \( n \)th radius of the Newton ring, \( r \) is the radius of curvature of the spherical wave front, \( \lambda \) represents the wavelength, and \( n \) is an integer designating the ring number.

\[
\rho_n = \left[ \left( n + \frac{1}{2} \right) \lambda r \right]^{1/2}
\]  

(5)

To obtain the value of the \( n \)th ring, as a function of the position of the object at the tangent point, we substitute \( \rho_n \) of Eq. (5) for the chord \( \frac{L}{2} \), and developing for \( n \), we obtain.
With Eqs. (5) and (6) and knowing the value of \( n \) and \( n + 1 \), we obtain the period \( p = (n + 1 - n) \), which defines the spatial frequency of the rings, as \( f = \frac{1}{p} \). This value is important because it determines the resolution capability of our CCD digital capture system, and it is related to the intrinsic reference angle of Eq. (4).

In this case, the CCD has a maximum resolution of 5196 x 3464 pixels, giving a pixel size (ps) of 4.29\( \mu m \). The period (p) of the rings decreases as one moves away from the center of the photo sensor. The fringes introduced by the sagitta will be resolvable as long as the condition \( ps < p \) is satisfied. Please see Appendix B for more information on the behavior of the rings on the sensor.

3 Results

3.1 Simulation results

A series of simulations where run in MATLAB to further support the concept of the intrinsic reference angle for Gabor holography. Figure 2a shows several letters used as independent objects comprising a composite image. Which can be taken as independent elements such as (b) and (d). These elements show that the off-center objects can be reconstructed cleanly without the superposition of their conjugate image, zero order and halo. To the contrary, Fig. 2g presents the basic problems of Gabor holograms. The propagation of (a,b,c,d) was performed with Eqs. (1) and (2), while the images (e,f,g,h) were reconstructed with the FFT algorithm, filtering out the zero order, and applying logarithm to highlight the diffracted images.

The first row of Fig. 3 shows a series of flat objects, (a) shows a 141x155 pixel photograph of Gustav Kirchhoff, as a flat object in the center of the field of view. Fig. 3b shows the displacement of the photograph to the left by 100 pixels. Figure 3c shows a displacement in the same direction with 200 pixels, and finally Fig. 3d, with a displacement of 300 pixels. Each of these images was propagated with Eq. (1) forming its respective wavefront, which was used to form the holograms with a spherical wave reference beam as in Eq. (2). Hence, the reconstruction of the holograms is shown applying FFT. Figure 3i corresponds to the reconstruction of the object centered on axis, where it is observed that the conjugate pair of images are superimposed with the zero order and the halo. Since a spherical wave is used as a reference, the pattern of rings forming in the center of the field of view is evident. Figure 3j shows a partial separation of the image, with visible rings in the center, and the superimposed images. Figure 3k demonstrates a total separation of the reconstructed images, but still close to the halo zone. At last, Fig. 3l represents the optimal reconstruction for a clean visualization of the hologram, where the pair of conjugate images are satisfactorily reconstructed with a higher quality than in previous attempts. The zero-order noise and the halo remain attenuated in the center of the reconstructed image.

The second row in Fig. 3 shows the holograms of the images above, whereas in Fig. 3e the rings are seen centered in the visual field. The next object, Fig. 3f, is displaced 100 pixels to the left and the ring begins to deform. The next

![Fig. 2](image)

**Fig. 2** a Image composed of nine letters. b Image of the letter A of the composition. c Image of letter R. d Image of the letter T. e Reconstruction of the composition, where the superposition of images is observed. f Twin-image free reconstruction of letter A (g) Reconstruction of R with superposition issue. h Reconstruction of T with no superposition problem

![Fig. 3](image)

**Fig. 3** a Centered object. b Object displaced 100 pixels to the left. c Object displaced 200 pixels to the left. d Object displaced 300 pixels to the left. e Phase distribution of propagated object (a) interfered with reference wave (hologram). f Phase distribution of b. g Phase distribution of c. h Phase distribution of d. i FFT reconstruction of a. j FFT reconstruction of b. k FFT reconstruction of c. l FFT reconstruction of d. Portrait of Gustav Kirchhoff courtesy of the Smithsonian Libraries and Archives [14]
step, Fig. 3g, exemplifies how the rings are deformed in the presence of the object by the 200 pixel displacement. Fig. 3h demonstrates that by a 300 pixel displacement the spatial modulation is of a higher frequency, which is noticeable in the image. In this figure, the influence of the sagitta on the spatial modulation in the interference fringes is evident. The last line of figures depicts the FFT reconstruction of the Kirchhoff portrait, with Fig. 3i showing a complete overlap of the images, Fig. 3j showing a partial overlap, and Fig. 3j, k demonstrating a sufficient separation between the images.

### 3.2 Experimental results

Figure 4 shows the experimental setup employed to record Gabor type holograms under lens-less Fourier. The interferometric setup requires a laser, three cube prism beam splitters (CBS), two rectangular prisms, a lens, a CCD camera, and at least 7 mirrors. The camera is of high resolution for optimal results. We used a Canon EOS Rebel T3i camera with an 18.0MP CMOS (Complementary Metal Oxide Semiconductor) sensor. The sensor is 22.3 x 14.9 mm in size with 5196 x 3464 resolution, and a 4.29/μm pixel pitch.

The three objects in Fig. 5 were used for this study: a copper cent US dollar coin, a metallic pin with the INAOE logo, and a wooden chess piece.

The first image in Fig. 6 shows a reconstruction of the coin, the twin images are superimposed and the object cannot be clearly appreciated. The next step shows an instance where with the object was displaced to the right by a distance of about half its size. The two images in the reconstruction appear to shift resulting in only a small overlap of the objects. Increasing the original object’s shift further separates the twin images until the object can be fully resolved; greatly simplifying post-image processing. The difference between Fig. 6a and Fig. 6c corresponds to the difference in spatial modulation of interference introduced by the sagitta. A live video of the separation of the twin images of this object can be appreciated in Visualization 1.

Figure 7 shows a sequence of images, from the INAOE pin, Fig. 7a shows the centered flat brass metal object, whose FFT reconstruction shows the overlapping of images. Fig. 7b shows the reconstruction with a slight displacement of the object. Figure 7c shows the reconstruction of the object which was displaced a little more than half of the object width. Figure 7d reconstruction of the object that was displaced a little less than object width. The last two images show Newton’s rings, in this case the halo is relatively small. The reflection of the object changes when it is displaced, due to the angle at which it is incident with respect to the center of the optical axis.
3.2.1 Reconstruction of 3D diffuse objects

This method includes the capability of obtaining holograms of volumetric 3D objects. Figure 8 shows a wooden chess pawn with a more considerable volume than objects Fig. 5a, b. The first reconstruction of the slightly displaced wooden pawn is shown in Figure 8a, where the twin images are overlapped and the object cannot be observed. The next reconstruction presents the two images separated, due to a larger displacement of the chess pawn. A last reconstruction is shown, where the contrast is enhanced to further demonstrate the capabilities of volumetric object visualization. In the first two scenes, the Newton rings and the halo scattering are visible, due to the fact that the object is diffuse. The rings are also present in the third image but too faint to be appreciated.

4 Conclusion

The use of a spherical wave as a reference beam with zero reference angle to form a Gabor-type hologram is essential to the technique presented in this manuscript. The sagitta is an intrinsic parameter of the sphericity of the wave, which forms an inherent modulation of the reference beam, allowing for a separation of the twin images in the reconstruction of the Gabor hologram, while maintaining the system on axis. The size of objects to be recreated range from a few millimeters to a couple of meters only by adjusting the miniaturization within the system. The original object is reduced in dimension by the optical system, to be closer to the optical axis and to implement more accurately the lens-less Fourier condition. The compensated trajectories increase the ability to obtain holograms of larger 3D objects, which is not usual with similar techniques. It should be noted that this proposal is not evident and is not applicable when plane waves are used as reference beam, since there is no sagitta, it would not have an inherent reference beam (see Appendix A for further comment).

This on-axis setup shares the trade-off in field of view (FOV) associated with off-axis holography. While working off-axis, the FOV is sacrificed in favor of improving temporal resolution. In the proposed method the FOV is limited by the size of the elements within the optical system, as there must be space for the object to be displaced and remain within the CBS. As a good rule of thumb, framing the object in one third of the FOV will result in sufficient displacement for the twin images to separate in a single shot hologram.

Appendix A Flat reference waves

The use of plane waves does not meet the Fourier criterion without lenses, since the distance $z$ of the object at a given position does not correspond to $z$ of the plane waves. Plane waves derived from the algebra of the cosines directors, by construction $z$ has very large values, to induce that the wave face is constant.

The second row of Fig. 9 shows the holograms of the objects in the first row, using as reference beam a plane wave, where it is clearly observed that there is no spatial fringe modulation to separate the superposition of the conjugate pair of images with a shift. Figure 9a shows the characteristic Gabor-like hologram, in the following figures (b), (c), and (d), the pattern is identical to (a), but shifted. This result is as expected, since there is no sagitta.

Figure 10a shows the object of 256 gray levels, displaced by 100 pixels. Fig. 10b displays a hologram generated with plane wave, which is later reconstructed in 10c using the FFT algorithm and applying the logarithm function, to amplify the visualization; where it is observed that it is not possible to reconstruct it by this way. The only way to extract the information is by reconstructing plane by plane with the Huygens–Fresnel–Kirchhoff equation to obtain the image, which will be modulated by the zero order, the halo, and the conjugate pair information.
Appendix B Field of view analysis

The Newton’s rings in Fig. 11 were captured by the CCD sensor by substituting the object with a plane mirror at a 90° angle, overlapping with the spherical reference beam in the CBS. Newton’s rings appear because of the difference in radii of the two, since the spherical waves emerge at different distances from the microscope objectives [13]. The appearance of Newton’s rings further support the notion that the full system is aligned with the optical axis.

Table 1 shows the change of the ring frequency \((f)\) when influenced by the radius of curvature of the reference spherical wave \((r)\) varying from 300\(\text{mm}\) to 165\(\text{mm}\). As previously mentioned, the CCD has a pixel size \((ps)\) of 4.29\(\mu\text{m}\), and the photo sensor can resolve the fringes caused by the sagitta as long as the condition \(ps < p\) is satisfied. The period \((p)\) of the rings decreases as one moves away from the center of the photo sensor. The working wavelength was 632.8\(\times10^{-6}\text{mm}\) and the \(L\) chord was taken as the maximum diagonal of the sensor \(L = 26.82\text{mm}\).

For each \(r\) there is a number of rings over the sensor area. We calculated the radius of the \(n\)th ring, the periods, and the frequencies as described in Eqs. 5 and 6. The sagitta \((S)\) from Eq. 4 was also included to visualize the phase delay of the spherical wave at the edges of the photo sensor. The data in bold indicates that parts of the CCD will not be able to resolve the interference fringes produced by the sagitta. The rest of the data corresponds to an \(r\) suitable for working in any part of the sensor area.

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Declarations

Competing Interests The authors declare no conflicts of interest.

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