Effect of Local Population Uncertainty on Cooperation in Bacteria

(Invited Paper)

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Abstract—Bacteria populations rely on mechanisms such as quorum sensing to coordinate complex tasks that cannot be achieved by a single bacterium. Quorum sensing is used to measure the local bacteria population density, and it controls cooperation by ensuring that a bacterium only commits the resources for cooperation when it expects its neighbors to reciprocate. This paper proposes a simple model for sharing a resource in a bacterial environment, where knowledge of the population influences each bacterium’s behavior. Game theory is used to model the behavioral dynamics, where the net payoff (i.e., utility) for each bacterium is a function of its current behavior and that of the other bacteria. The game is first evaluated with perfect knowledge of the population. Then, the unreliability of diffusion introduces uncertainty in the local population estimate and changes the perceived payoffs. The results demonstrate the sensitivity to the system parameters and how population uncertainty can overcome a lack of explicit coordination.

I. INTRODUCTION

Game theory, a formal mathematical method for modeling interactive behavior, has a long history in fields as diverse as economics, sociology, political science, and wireless communications [1]. Mathematical games permit the theoretical study of complex behavior such as competition and cooperation. These games work the same way as, for example, a game of chess: players have certain allowed actions (or “moves”), and the moves of all the players interact, resulting in specified rewards or losses for the players.

Game theory has also long been useful in biology, as it provides a mathematical framework to show how species in competition can arrive at interdependent optimal solutions for their evolutionary fitness [2]. Mathematical games permit the theoretical study of complex behavior such as competition and cooperation. These games work the same way as, for example, a game of chess: players have certain allowed actions (or “moves”), and the moves of all the players interact, resulting in specified rewards or losses for the players.

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In this section, we present a model for resource (e.g., food) access and sharing within a single population. We frame the model from the perspective of bacteria, but it could also be applied to other populations that share a common resource. Our goal in this work is not quantitative precision, but instead
to have a simple analytical model whose output is qualitatively
consistent with real-world observations of bacteria.

We consider a population with \( n_T \) members in an environment
with a total resource availability (or availability rate) \( R \). At any given moment, there are \( n_c \) cooperating members
\((1 \leq n_c \leq n_T)\) and \( n_s = n_T - n_c \) selfish (i.e., competing)
members. Every member pays a cost for its behavior, which
members represents the consumption of energy needed to act. Coop-
erating and selfish members pay \( \gamma_c \) and \( \gamma_s \), respectively. We
assume that cooperation is more expensive than selfishness,
\( \gamma_c > \gamma_s \), as supported by models in [7], [11], [13].

All members of the population, whether they are coop-
erating or selfish, divide the available resource \( R \) among
themselves. We impose that each type of member accesses
the resource with a different efficiency \( \epsilon \), where \( 0 \leq \epsilon \leq 1 \),
and the value depends on the population distribution. We
define the efficiency as follows. We assume that cooperating
members work together to improve their efficiency, such that
they become more efficient when more cooperators work

\[
\epsilon_c = 1 - \exp \left( -\alpha_cn_c \right),
\]

where \( \alpha_c \) is a cooperation multiplier and \( \alpha_c > 0 \).

For the selfish members, we consider two different ef-
ficiency models. If the selfish members are able to take
advantage of the cooperators, e.g., by acquiring better access to
the resource, then their efficiency improves with the presence
of cooperators. This behavior is known as defection, which
underlies the classical Prisoner’s Dilemma game (see [13]
Ch. 4)) and has also been observed within bacteria populations
including \textit{E. coli} and \textit{Myxococcus}; see [4]. In this case, a simple model that satisfies these assumptions is exponential, and we write the cooperating efficiency \( \epsilon_c \) as

\[
\epsilon_c^+ = \frac{\epsilon_c}{\beta^+_s},
\]

where \( \beta^+_s \) is a selfishness penalty. We assume that \( \beta^+_s > 1 \), but
this does not prevent a selfish member from having a higher net
payoff, since cooperation has a higher cost.

It is also possible that non-cooperative members of a population
are prevented from accessing the benefits of cooperation,
\textit{i.e., the selfish members are punished}. For example, coop-
erators may generate toxins that are only harmful to selfish
members; see [8] for details and other possible punishment
mechanisms for microbes. We model this by decreasing the
selfish efficiency \( \epsilon^+_s \) as cooperation increases and write

\[
\epsilon^+_s = \frac{1 - \epsilon_c}{\beta^+_s},
\]

where again \( \beta^+_s \) is a selfishness penalty and \( \beta^+_s > 1 \).

Given a member’s efficiency \( \epsilon \), we define its revenue \( A \) as

\[
A = \frac{\epsilon}{n_T} R.
\]

### Table I

| Parameter               | Symbol | Value |
|-------------------------|--------|-------|
| Resource Availability   | \( R \) | 100   |
| Total Population        | \( n_T \) | 20    |
| Cooperation Cost        | \( \gamma_c \) | 1     |
| Selfish Cost            | \( \gamma_s \) | 0.5   |
| Cooperation Multiplier  | \( \alpha_c \) | 0.1   |
| Selfishness Penalty     | \( \beta^-_s, \beta^-_c \) | 2     |

The net payoff \( U \), which we will hereafter refer to as the
payoff, is the difference between the revenue and the cost.
Thus, a cooperating member’s payoff \( U_c \) is

\[
U_c = \frac{\epsilon_c}{n_T} R - \gamma_c = \frac{1 - \exp \left( -\alpha_cn_c \right)}{n_T} R - \gamma_c,
\]

and a selfish member’s payoff \( U_s \) is

\[
U_s = \frac{\epsilon_s}{n_T} R - \gamma_s = \left\{ \begin{array}{ll}
\frac{\epsilon_s}{\beta^+_s} R - \gamma_s, & \text{if gain from cooperation} \\
\frac{\epsilon_s}{\beta^-_s} R - \gamma_s, & \text{if lose from cooperation}.
\end{array} \right.
\]

The total payoff \( U_{tot} \) for the entire population is then

\[
U_{tot} = n_c \left( \frac{\epsilon_c}{n_T} R - \gamma_c \right) + n_s \left( \frac{\epsilon_s}{n_T} R - \gamma_s \right).
\]

As an example of member payoffs, consider the system
whose parameters are listed in Table I and which are the
system defaults for the remainder of this paper. For this
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### III. BACTERIA COOPERATION GAME

In this section, we present the bacteria game dynamics
that we will simulate in Section IV. The members of the
population are hereafter referred to as the players, and each
player’s strategy may change over time. We do not consider
bacteria to be rational decision-makers, as is typical in classical
game theory. Nevertheless, game theoretic analysis is suitable,
as emphasized in [2], [4], because bacteria do change their
behavior in response to external signals. From this perspective,
each player’s strategy is whether it cooperates or acts selfishly.

We model a dynamic game as follows. Every player has
some fixed initial behavior, \textit{i.e., to be selfish or to cooperate}.
Then, every player has the opportunity to maintain its current
behavior or switch to the alternate behavior. These assessments
are made simultaneously, independently of the other players, and “selfishly”, i.e., a player will only cooperate if there is a perceived individual benefit, based on the information available. The game reaches a Nash equilibrium when no member can benefit by changing behaviors (see [13] Ch. 3), i.e., when the population distribution converges. In the following, we consider perfect knowledge as an ideal case and then imperfect knowledge due to the randomness of diffusive signaling.

A. Perfect Population Knowledge

In the perfect (ideal) case, every player knows the current population size $n_T$ and the number of cooperators $n_c$. If a player is currently cooperating, then it compares its current payoff $U_c$, found via (5), with the potential selfish payoff $U_s$, found via (6) and where the number of cooperators when finding $\epsilon_c$ is decremented by 1. Analogously, if a player is currently selfish, then it compares its current payoff $U_s$, found via (6), with the potential cooperating payoff $U_c$, found via (5) and where the number of cooperators when finding $\epsilon_c$ is incremented by 1. When no player can benefit by changing its behavior, then a Nash equilibrium is achieved.

Both Figs. 1a) and 1b) show that there are two Nash equilibria (zero cooperators or all cooperators), even though there is clearly one optimal strategy, since all players cooperating leads to the largest average payoff. The equilibria can be verified by confirming that, at equilibrium, a player’s payoff will decrease if it switches strategies. We also note that the optimal solution is not always a Nash equilibrium. For example, consider Fig. 2 where we decrease the total resource availability $R$ and the selfishness penalty $\beta_s^i$. The highest average (i.e., mean) payoff is when there are 7 cooperators, but this will not be maintained in a game because the cooperating players can always increase their individual payoffs by becoming selfish.

B. Noisy Population Detection

In the noisy case, each player has to estimate the size of the population and the number of cooperators. We assume that the other payoff parameters are known. To simplify our model, we propose a proxy for bacteria quorum sensing. We define a probability of detection that increases as the distance between two players decreases. This is appropriate because the energy in a diffusing pulse of molecules is inversely proportional to the distance traveled, and a pulse with more energy is easier to detect; see [14]. We write the probability of the $i$th player detecting the presence of the $j$th player, $p_{d,i,j}$, as

$$p_{d,i,j} = 1 - \exp \left( -\frac{\eta}{d_{i,j}} \right),$$  \hspace{1cm} (8)

where $d_{i,j}$ is the distance between the two players and $\eta$ is a detection scaling factor. We assume that cooperative players are easier to detect (e.g., by releasing more molecules), since they are motivated to attract other cooperators, so their detection scaling factor is larger than that of selfish players, i.e., $\eta_c > \eta_s$.

The $i$th player’s estimate of the population size, $\hat{n}_{T,i}$, is then

$$\hat{n}_{T,i} = 1 + \sum_{j \neq i} B(p_{d,i,j}),$$  \hspace{1cm} (9)

where $B(p_{d,i,j})$ is a binary number drawn from a Bernoulli distribution with probability $p_{d,i,j}$, and $p_{d,i,i} = 1$. To estimate the number of cooperating players, $\hat{n}_c,i$, the $i$th player classifies every other player that they detected using (9). For simplicity, we assume that classifying cooperators is as unreliable as their detection, such that they are classified with probability $p_{d,i,j}$ (i.e., they are misclassified as selfish with probability $1 - p_{d,i,j}$). It is less intuitive for distant selfish players to be
misclassified as cooperators with a high probability, so we misclassify detected selfish players with a constant “false alarm” probability  \( p_{FA} \).

Given each player’s estimates  \( \hat{n}_{T,i} \)  and  \( \hat{n}_{c,i} \), the game proceeds analogously to the perfect knowledge case. The current and prospective payoffs are estimated using (5) and (6), where  \( n_T \)  and  \( n_c \)  are replaced with their corresponding estimates. However, as we will demonstrate in the following section, the progress of the noisy detection game can be quite distinct from the perfect knowledge game, such that the uncertainty can overcome a lack of explicit coordination.

### IV. Numerical Results

In this section, we execute examples of the bacteria cooperation game. Unless otherwise noted, we consider the payoff parameters listed in Table I and use  \( \beta_k \)  to evaluate the selfish efficiency  \( \epsilon_k \). Every game is run for 100 rounds, and in each round every player has the opportunity to switch behaviors. In the perfect knowledge game, we do not need to account for the locations of the players. In the noisy detection game, we assume independent random motion and place the players as follows. We define the environment as a cube of width  \( h \). In each round, a uniformly-distributed location is generated within the cube for every player. From these locations, the distance  \( d_{i,j} \)  terms for the evaluation of (5) can be found. The system width  \( h \)  and the other detection parameters are as specified in Table II.

In Fig. 3, we consider the progression of a game with the default system parameters. The perfect knowledge game is run with all players initially selfish and again with all players initially cooperating. As we discussed in Section III, these two cases are Nash equilibria, so no player modifies its behavior and the population remains stable for the entire duration of the game. We also consider the noisy detection game where all players are initially selfish. In this game, the imperfect knowledge leads to cases of misclassification, such that the players expect payoffs from cooperation that are larger than what is currently possible, but after 60 rounds the population has been “pushed” towards the equilibrium where all of the players are cooperating and the total payoff is maximized (as shown in Fig. 4a)). Thus, noisy signaling is shown to promote optimal behavior among the bacteria, even though the players are not explicitly coordinating their strategies.

In the remaining figures, we focus on the noisy detection game and its sensitivity to individual system parameters. We vary one parameter at a time and for each value we run at least 10 games of 100 rounds each. We plot the mean number of cooperating players  \( n_c \)  and mean total payoff  \( U_{tot} \) over the entire duration of all of the games. We separately consider the cases where all players are initially cooperating and where all players are initially selfish. In many of the cases considered, the population reaches an equilibrium where either all or none of the players cooperate, independent of the initial state.

In Fig. 4a), we vary the resource availability  \( R \). We observe that when  \( R \)  is sufficiently low, the players cannot benefit from cooperation. This is a common occurrence in biology when food is scarce; see [13, Ch. 17]. As  \( R \)  increases, the higher efficiency in cooperation overcomes its cost. Similar results are observed when using  \( \epsilon_k^+ \)  as when using  \( \epsilon_k^- \); in the remainder of this work, we use  \( \epsilon_k^+ \)  and assume that the selfish efficiency decreases with the efficiency of the cooperating players.

In Fig. 4b), we vary the total population  \( n_T \). When there are only a few players, they cannot benefit from cooperation and adding players reduces the total payoff  \( U_{tot} \). However, consis-

### Table II: Default Detection Parameters

| Parameter                | Symbol | Value          |
|--------------------------|--------|----------------|
| System Width             |  \( h \) | 10 \( \mu m \) |
| Cooperation Detection Factor |  \( \eta_c \) | 20 \( \times 10^{-6} \) |
| Selfish Detection Factor  |  \( \eta_s \) | 5 \( \times 10^{-6} \) |
| False Alarm Probability   |  \( p_{FA} \) | 0.2            |

![Graph](image-url)
In this paper, we presented a simple payoff model for bacteria consuming a common resource and evaluated the model using game theory. We observed how cooperative behavior could be promoted or discouraged by the modification of system parameters, and that uncertainty in the local population can lead to higher payoffs than perfect knowledge. Future work could consider diverse populations with non-uniform payoff structures, more refined models to describe the payoffs and the estimation process, and the effects of evolution where successful strategies are preserved across generations of players.

V. CONCLUSIONS

In this paper, we presented a simple payoff model for bacteria consuming a common resource and evaluated the model using game theory. We observed how cooperative behavior could be promoted or discouraged by the modification of system parameters, and that uncertainty in the local population can lead to higher payoffs than perfect knowledge. Future work could consider diverse populations with non-uniform payoff structures, more refined models to describe the payoffs and the estimation process, and the effects of evolution where successful strategies are preserved across generations of players.

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REFERENCES

[1] V. Srivastava, J. Neel, A. Mackenzie, R. Menon, L. Dasilva, J. Hicks, J. Reed, and R. Gilles, “Using game theory to analyze wireless ad hoc networks,” IEEE Commun. Surv. Tutorials, vol. 7, no. 4, pp. 46–56, 2005.
[2] S. Schuster, J.-U. Kreft, A. Schroeter, and T. Pfeiffer, “Use of game-theoretical methods in biochemistry and biophysics.” J. Biol. Phys., vol. 34, no. 1-2, pp. 1–17, Apr. 2008.
[3] H. C. Berg, E. coli in Motion. Springer, 2004.
[4] G. J. Velicer, “Social strife in the microbial world,” Trends Microbiol., vol. 11, no. 7, pp. 330–337, Jul. 2003.
[5] S. Hummert, K. Bohl, D. Basanta, A. Deutsch, S. Werner, G. Theißen, A. Schroeter, and S. Schuster, “Evolutionary game theory: Cells as players,” Mol. BioSyst., vol. 10, no. 12, pp. 3034–3065, Aug. 2014.
[6] S. Atkinson and P. Williams, “Quorum sensing and social networking in the microbial world,” J. R. Soc. Interface, vol. 6, no. 40, pp. 959–978, Nov. 2009.
[7] G. Lambert, S. Vyawahare, and R. H. Austin, “Bacteria and game theory: The rise and fall of cooperation in spatially heterogeneous environments,” Interface Focus, vol. 4, no. 4, p. 20140029, Jun. 2014.
[8] M. Travisano and G. J. Velicer, “Strategies of microbial cheater control,” Trends Microbiol., vol. 12, no. 2, pp. 72–78, Feb. 2004.
[9] S. P. Brown and R. A. Johnstone, “Cooperation in the dark: Signalling and collective action in quorum-sensing bacteria,” Proc. R. Soc. London B Biol. Sci., vol. 268, no. 1470, pp. 961–965, May 2001.
[10] N. Michelusi, “On population density estimation via quorum sensing,” in Proc. Can. Work. Inf. Theory. IEEE, Jun. 2017, pp. 1–5.
[11] L. Cao, K. Zhao, G. C. L. Wong, and M. van der Schaar, “A dynamic network formation model for understanding bacterial self-organization into micro-colonies,” IEEE Trans. Mol. Biol. Multi-Scale Commun., vol. 1, no. 1, pp. 76–89, Mar. 2015.
[12] C. Koca and O. B. Akan, “Anarchy vs. cooperation on internet of molecular things,” IEEE Internet Things J., to appear.
[13] M. Broom and J. Rychtar, Game-Theoretical Models in Biology. CRC Press, 2013.
[14] I. Llatser, A. Cabellos-Aparicio, M. Pierobon, and E. Alarcón, “Detection techniques for diffusion-based molecular communication.” IEEE J. Sel. Areas Commun., vol. 31, no. 12, pp. 726–734, Dec. 2013.