Evidence for Bosonic Electroweak Corrections in the Standard Model

PAOLO GAMBINO

Department of Physics, New York University, 4 Washington Place, New York, NY 10003, USA.

ALBERTO SIRLIN**

Department of Physics, Brookhaven National Laboratory, Upton, Long Island, NY 11973, USA.

ABSTRACT

We present strong indirect evidence for the contribution of bosonic electroweak corrections in the Standard Model. Although important conceptually, these corrections give subleading contributions in current high energy experiments, and it was previously thought that they are difficult to detect. We also discuss the separate contribution of the Higgs boson.

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** Permanent address: Dept. of Physics, New York University, 4 Washington Place, New York, NY 10003, USA
It has been known for a long time that radiative corrections play an important role in the analysis of the Standard Model (SM) predictions. For example, if their contribution at low energies were not included, there would be a very large violation of the unitarity of the CKM matrix and the SM would be placed in severe jeopardy [1]. At high energies, the current estimates on $m_t$ are derived from electroweak radiative corrections. However, unlike the problem of universality, where the corrections involve virtual $W^\pm, \gamma$, and $Z^0$, the dominant corrections in current high-energy experiments are fermionic in nature. That is, at one-loop, they involve only virtual fermions. They are responsible, for instance for the large logarithms associated with the running of $\alpha$ [3, 4] and for the sizable contributions of the $t-b$ sector, from which the $m_t$ estimates are derived. An exception to this statement is the $Z^0 \rightarrow b\bar{b}$ vertex, where there are also large corrections involving virtual bosons.

However, aside from the fermionic contributions, there are also, at high energies, conceptually very important bosonic corrections mediated by virtual $W^\pm, \gamma, Z^0$, and $H$. They involve the plethora of bosonic couplings of the SM, including the well-known tri-linear vertices, and affect self-energies, vertex, and box diagrams (in four fermion processes, bosonic vertex and box diagrams also include virtual fermions). We recall that this separation into fermionic and bosonic one-loop corrections is gauge-invariant and goes back to the early papers [5]. The difficulty is that, with the exception of the $Z^0 \rightarrow b\bar{b}$ vertex mentioned before, in current high energy experiments the bosonic corrections give subleading contributions, significantly smaller than their fermionic counterparts.

The purpose of this paper is to show that the accuracy currently reached is such that strong indirect evidence for the presence in the SM of these important subleading corrections can be obtained. Specifically, our approach consists in removing the full block of bosonic electroweak corrections in two highly precise determinations of the $\overline{\text{MS}}$-parameter $\sin^2 \theta_W(m_Z) \equiv \hat{s}^2$ and showing that, for $m_t \geq 131$ GeV (the current lower bound), this leads to a sharp disagreement. In the analysis all the fermionic contributions are retained.

One precise determination of $\hat{s}^2$ can be readily derived from the result $\sin^2 \theta^\text{eff}_W = 0.2316 \pm 0.0004$, obtained from the combination of LEP and SLC asymmetry values [6]. Here $\sin^2 \theta^\text{eff}_W$, also called $\sin^2 \theta^\text{lep}_W$, is the effective mixing parameter in the $Z^0 \rightarrow \ell\ell$ on-shell amplitude. It has been recently shown that $\sin^2 \theta^\text{eff}_W = \text{Re} \hat{k}_t(m_Z^2) \hat{s}^2$, where $\hat{k}_t(m_Z^2) = 1 + \mathcal{O}(\alpha)$ represents the relevant radiative corrections [7]. Removing the bosonic contributions, one obtains $\text{Re} \hat{k}_t(m_Z^2) = 1.0060$, so that

$$ (\hat{s}^2)_{tr} = 0.2302 \pm 0.0004 \quad (\text{Asymmetries}). $$

(1)
Henceforth the subscript \( tr \) reminds us that this value corresponds to a “truncated” version of theory, with bosonic contributions removed in the electroweak corrections. In evaluating \( \Re k_\ell(m_Z^2) \) we have neglected small \( \mathcal{O}(\alpha \hat{\alpha}_s) \) corrections that were retained in Ref. [7].

A second precise determination can be obtained from \( G_\mu, \alpha, m_Z \):

\[
(\hat{s}^2)_{tr}(\hat{c}^2)_{tr} = \frac{A^2}{m_Z^2(1 - (\Delta \hat{r})_{tr})},
\]

(2)

where \( A^2 = \pi \alpha / \sqrt{2} G_\mu \) and \( \Delta \hat{r} \) is a basic correction of the electroweak theory [8], from which we have removed once more the bosonic contributions. The comparison between the two determinations of \((\hat{s}^2)_{tr}\), from Eqs. 1 and 2, is shown in Fig.1, as a function of \( m_t \). In evaluating \((\Delta \hat{r})_{tr}\), we employed the recent preliminary values \( m_Z = 91.1899 \pm 0.0044 \text{GeV} \) [9], and \( e^2 \Re \Pi^{(5)}_{\gamma\gamma}(0) - \Pi^{(5)}_{\gamma\gamma}(m_Z^2) \) = 0.00283 \pm 0.0007 for the five flavor contribution to the photon vacuum polarization function [11]. We have neglected small \( \mathcal{O}(\hat{\alpha}^2) \) and \( \mathcal{O}(\hat{\alpha} \hat{\alpha}_s) \) corrections, except for the leading two-loop effects of \( \mathcal{O}(\hat{\alpha}_s G_\mu m_t^2) \), which we have incorporated using a simple method recently discussed by one of us [11] and \( \hat{\alpha}_s(m_Z) = 0.12 \); we have also removed the irreducible two-loop effects of \( \mathcal{O}(\alpha^2 m_t^2) \) [12] as they involve virtual bosons.

It is apparent from Fig.1 that the removal of the bosonic component of the electroweak corrections leads to a sharp disagreement. At the lower bound \( m_t = 131 \text{ GeV} \), the value obtained from Eq. 2 is

\[
(\hat{s}^2)_{tr} = 0.2282 \pm 0.0003 \quad (G_\mu, \alpha, m_Z, m_t = 131 \text{GeV})
\]

(3)

and we see that the difference with Eq. 1 amounts to 4\( \sigma \) (if the recent SLC value were not included, the top curve in Fig.1 would be shifted upwards by \( \approx 0.0006 \), the error would be slightly increased, and the difference would be 4.5\( \sigma \)). As shown in Fig.1, the discrepancy rapidly increases with \( m_t \). For instance, it is 7\( \sigma \) for \( m_t = 180 \text{ GeV} \). Fig.2 shows the same comparison, but with the bosonic electroweak corrections restored, in the case \( m_H = 300 \text{ GeV} \). As expected, there is now consistency between the two determinations for a restricted range of \( m_t \) values. Comparing Fig.1 and 2, we see that the removal of the bosonic corrections leads to lower values of \( \hat{s}^2 \). The discrepancy arises because the effect is much more pronounced in the \((G_\mu, \alpha, m_Z)\) determination.

Given the sharpness of the signal, it is natural to ask whether one can use the same approach to probe specific components of the bosonic corrections. For instance, can one search for signals of Higgs boson contribution by removing them from the electroweak corrections, retaining the rest? As \( H \) does not contribute, at one-loop level, to \( k_\ell(m_Z^2) \), the value of \( \hat{s}^2 \) derived in this case from
\sin^2 \theta_W^{eff} \text{ is that of the full SM} [7]:

\hat{s}^2 = 0.2313 \pm 0.0004 \quad (\text{Asymmetries}), \quad (4)

Instead of Eq.\[4\]. In order to be physically meaningful, the removal of the Higgs contribution from \( \Delta \hat{r} \) must be done in a gauge-invariant and finite manner. Fortunately, in the SM the diagrams involving \( H \) in the self-energies contributing to \( \Delta \hat{r} \) form a gauge-invariant subset. On the other hand, they are divergent. Therefore, one must specify the renormalization prescription and the scale at which these partial contributions are evaluated. As \( \hat{s}^2 \) is the \( \overline{\text{MS}} \) parameter and the electroweak data are dominated by information at the \( Z^0 \)-peak, it is natural to subtract the \( \overline{\text{MS}} \)-renormalized Higgs boson contribution evaluated at the \( m_Z \) scale. Neglecting non-leading \( \mathcal{O}(\alpha^2) \) terms, the latter is given by

\[
(\Delta \hat{r})_{H.B.} = \frac{\alpha}{4\pi \hat{s}^2} \left\{ \frac{1}{\hat{c}^2} H(\xi) - \frac{3}{4} \xi \ln \xi - \hat{c}^2 \ln \hat{c}^2 + \frac{19}{24} + \frac{\hat{s}^2}{6\hat{c}^2} \right\}_{\overline{\text{MS}}} \quad (5)
\]

where \( \xi = m_H^2 / m_Z^2 \), \( H(\xi) \) is a function studied in Ref.[5], and the subscript \( \overline{\text{MS}} \) reminds us that the \( \overline{\text{MS}} \) renormalization has been carried out and the scale \( \mu = m_Z \) chosen. The need to specify the scale in defining the Higgs boson contribution can be most easily understood in the on-shell method of renormalization [5], where one employs \( \sin^2 \theta_W = 1 - m_W^2 / m_Z^2 \) instead of \( \hat{s}^2 \). In that case, the relevant radiative correction is \( \Delta r \), rather than \( \Delta \hat{r} \). Although \( \Delta r \) is a physical observable and is therefore \( \mu \)-independent, the Higgs-boson contribution is \( \mu \)-dependent. Thus, a specification of the scale is necessary in its definition.

Subtracting then the Higgs boson contribution one obtains a new truncated version of \( \Delta \hat{r} \), independent of \( m_H \), from which we can compute the corresponding \( (\hat{s}^2)_{tr} \) via Eq.\[2\]. The comparison with Eq.\[4\] is given in Fig.3. In contrast with Fig.1, where the complete block of bosonic contributions was removed, there are no signals of inconsistency. This is easily understood by noting that \( (\Delta \hat{r})_{H.B.} \) vanishes for \( m_H \approx 113 \) GeV. Thus, the subtraction of Eq.\[5\] is equivalent to a SM model calculation with a relatively light Higgs scalar, \( m_H \approx 113 \) GeV, and this is consistent with current electroweak data.

On the other hand, for \( m_H = 1 \) TeV, \( (\Delta \hat{r})_{H.B.} \approx 3.4 \times 10^{-3} \), which is not far from the value \( 4.0 \times 10^{-3} \) derived from the asymptotic formula \( (\Delta \hat{r})_{H.B.} \approx (\alpha/4\pi \hat{s}^2 \hat{c}^2)[5/6 - 3\hat{c}^2/4] \ln(m_H^2/m_Z^2) \). As a consequence, in the \( m_H = 1 \) TeV case, there is an additional contribution that raises the \( m_Z \) determination of \( \hat{s}^2 \) in Fig.3 by \( \approx 1.2 \times 10^{-3} \) and therefore favors a larger \( m_t \) value. The parameter \( m_H \) occurring in the asymptotic formulae has been interpreted as playing the role of a regulator in nonlinear \( \sigma \)-models that describe the heavy-Higgs-boson limit of the SM [13].
In summary, we have presented strong indirect evidence for the presence in the SM of bosonic electroweak corrections (Fig.1). If one probes just the Higgs component of these corrections, no evidence has been uncovered in our very simple analysis. However, it is likely that the signals will become sharper as the precision increases and $m_t$ is measured. It is also likely that more precise evidence for the bosonic electroweak corrections and their components will emerge if the approach we have proposed, namely their removal from the relevant corrections, is extended systematically to global analyses.

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Fig. 1 Determination of $\sin^2 \theta_W(m_Z)$ from asymmetries (horizontal lines) and $m_Z$ (bottom curves) with the bosonic contributions removed, as a function of $m_t$ (GeV). The one $\sigma$ errors are indicated.
Fig. 2 Determination of $\sin^2 \theta_W(m_Z)$ from asymmetries and $m_Z$ in the full SM for $m_H = 300$ GeV, as a function of $m_t$ (GeV).
Fig. 3 Determination of $\sin^2 \tilde{\theta}_W(m_Z)$ from asymmetries and $m_Z$ with the Higgs scalar contribution removed (see text), as a function of $m_t$ (GeV).
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