NMR in the quantum Hall effect

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Abstract.

We report that the rate of field sweep is important for the observation of the zeroes in the transverse resistivity in the quantum Hall effect. The resistivity also shows resonances at the nuclear magnetic resonance (NMR) frequencies when the r.f. coil is placed along the x direction. The relationship of the eigen values with the resistivity is pointed out which shows that even if the eigen values are largely corresponding to that of the single-particle, the plateau widths, shifts and shape require many-body interactions.
Recently, Kronmüller et al[1] have found that the longitudinal resistivity shows maxima in samples of GaAs/Al\textsubscript{x}Ga\textsubscript{1-x}As which have small well thickness and when the field sweep rate is small, 0.002 T/min, instead of zeros at the same values of the magnetic field at which the transverse resistivity, $\rho_{xy}$ shows plateaus. Usually, at a large sweep rate in samples of large thickness, the longitudinal resistivity, $\rho_{xx}$, shows zeros at certain values of the field, $B_z$, at which $\rho_{xy}$ shows plateaus. The optically pumped[2] NMR studies have been performed at the Landau level filling factor of $\nu = 1/2$ which show that at the points where $\rho_{xx}$ shows zeros and $\rho_{xy}$ plateaus, the maxima in $\rho_{xx}$ occur instead of zeros and the existence of field is proved by performing the NMR of $^{71}$Ga. According to Maxwell equations, the zero in the resistivity is consistent with the zero in the magnetic field. However, in the quantum Hall effect $\rho_{xx} = 0$ points have large finite fields, $B_z$ which requires that ordinary Maxwell equations may be modified to accomodate the experimentally observed fields when $\rho_{xx} = 0$. The consistency in the Maxwell equations may be obtained by a suitable shift in the field. The entire field is subject to the flux quantization but it is not necessary to split it into two terms, one flux quantized and the other not quantized. It is also not necessary to attach fluxex even number or odd, to the electrons.

In the present letter, we find that both the electric as well as the magnetic fields require modification and not only the vector potential but also the scalar potential has to be corrected to understand the NMR at such points where $\rho_{xy}$ has plateaus and $\rho_{xx}$ has zeros. Since the NMR shows that the magnetic field is not zero, it is clear that $\rho_{xx}$ should also not be zero. Therefore, we report modifications in both the electric as well as the magnetic fields and hence in the scalar as well as the vector potential of the electromagnetic fields.

In quantum Hall effect, the magnetic field is applied along the $z$ direction, the electric field along the $x$ direction and the Hall voltage is measured along the $y$ direction, in a layered semiconductor with layers in the $x - z$ plane. It is found that the longitudinal resistivity, $\rho_{xx}$, as a function of magnetic field, $B_z$, becomes zero
at several values of $B_z$ and oscillates between zeros and maxima. The transverse resistivity in the $xy$ plane, $\rho_{xy}$, shows plateaus at the same values of field, $B_z$, at which $\rho_{xx}$ shows zero values. In the samples of GaAs/Al$_x$Ga$_{1-x}$As with reduced well thickness, 15 nm, as compared with the conventional ones, the transverse resistivity shows maxima, at the same values of the fields where zeros occur in the large well samples. At integer filling factors the resistance, $\rho_{xx}$ goes to zero and at filling factor $\nu = 2/3$ there is a minimum but when the sweep rate of the magnetic field is reduced to 0.002 T/min a huge maximum (HLR) is found instead of a minimum or zero. Kronmüller et al[1] have performed the nuclear magnetic resonance measurement of $^{75}$As at the same field as that at which $\rho_{xx}$ is a maximum. Thus it is proved that there is a field at which $\rho_{xx} = 0$. We make an effort to understand the NMR at a field at which there is a huge longitudinal resistance (HLR). This is the same point at which $\rho_{xx}$ shows zero in thick well samples and $\rho_{xy}$ shows plateaus. In superconductors, the zero resistivity is consistent with zero field but in the present case there is always a field along z direction. Therefore, we develop a theory which can give a zero resistivity at a finite magnetic field without superconductivity. Such a theory does not exist in the literature.

We assume that magnetic induction is independent of time so that $dB/dt = 0$. According to a Maxwell equation,\n
$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{1}$$

so that $\nabla \times E = 0$, which gives $E = 0$. Since the current $j = (1/\rho)E$, the value $E = 0$ is consistent with $\rho = 0$. This is true in superconductivity where all components of $B$ are zero. In the quantum Hall effect, (QHE) when $\rho_{xx} = 0$, there is a large magnetic field along the $z$ direction. Therefore $B = 0$ and $\rho_{xx} = 0$ type theory is not applicable. In order to understand the QHE, we suggest that both $E$ and $B$ are shifted. The quasiparticles of charge $\nu e$ with a finite charge density, produce a field
due to Maxwell equations

\[ \nabla \cdot E_o = 4\pi \rho_o \]
\[ \nabla \times B_o - c^{-1}(\partial E_o/\partial t) = (4\pi/c)j \] (2)

so that we can assume that there are fields \( E_o \) and \( B_o \) which shift the electric field and the magnetic induction. Now, \( B_z \) is replaced by \( B_z - B_o \) and

\[ \frac{d}{dt}(B_z - B_o) = 0 \] (3)

so that there is no field inside the semiconductor. Substituting this result into the Maxwell equation and replacing \( E_x \) by \( E_x - E_o \), we find,

\[ \nabla \times (E_x - E_o) = 0 \] (4)

so that \( E_x - E_o = 0 \). The Ohm’s law is

\[ \rho_{xx}j = E_x - E_o \] (5)

which is consistant with \( \rho_{xx} = 0 \) for \( E_x - E_o = 0 \). This result gives the usual QHE. The magnetic induction inside the semiconductor is

\[ B_z - B_o + 4\pi M = 0 \] (6)

because \( \rho_{xx} = 0 \). Here \( M \) is the magnetization of the sample. The above result shows that the points where \( \rho_{xx} = 0 \) have diamagnetic magnetization,

\[ \frac{M}{B_z - B_o} = -\frac{1}{4\pi} \] . (7)

The \( \rho_{xx} = 0 \) is thus consistant with a shift \( B_o \) in the field. Since there is a sweep rate dependence, the above processes occur in a time \( \tau \) such that the field shift is given by

\[ B_o = \tau \frac{d}{dt}B_o = \tau \alpha_o \] (8)
where $\alpha_o$ is the sweep rate. The effective field is thus given by

$$B_{\text{eff}} = B_z - \tau \frac{d}{dt} B_o = B_z - \tau \alpha_o .$$  \hspace{1cm} (9)

For small values of $\alpha_o$, the condition

$$f(\alpha_o) = B_z - \tau \alpha_o + 4\pi M = 0$$  \hspace{1cm} (10)

is not satisfied so that $\rho_{xx} = 0$ points do not occur. Thus the maximum value of $\alpha_o$ for which $f(\alpha_o) = 0$ is

$$\alpha_{o,\text{min}} = (B_z + 4\pi M)/\tau .$$  \hspace{1cm} (11)

If $\alpha_o$ is less than $\alpha_{o,\text{min}}$, the value of $f(\alpha_o)$ is positive and $\rho_{xx} = 0$ does not occur. Large values of $\alpha_o$ give negative $f(\alpha_o)$ while small values of $\alpha_o$ give positive $f(\alpha_o)$. Thus there is a particular value of $\alpha_o$ for which $\rho_{xx} = 0$ points occur. For small values of $\alpha_o$, finite value of $\rho_{xx}$ occurs. Thus the quantum Hall effect requires an effective charge $\nu e$ of the quasiparticles which shifts both $E$ and $B$ and the sweep rate of the field is important for the observation of zeros in the $\rho_{xx}$ at certain finite values of the magnetic field along the $z$ direction. Since $\vec{B} = \vec{\nabla} \times \vec{A}$ a shift in $\vec{B}$ amounts to a shift in the vector potential $\vec{A}$, but there is a time dependent term in the above so that $\partial A/\partial t$ is not zero. Since the scalar potential $\phi$ depends on $E$ as well as on $\partial A/\partial t$, by means of the relation $E + (1/c)(\partial A/\partial t) = -\nabla \phi$, it is clear that $\phi$ also requires a correction due to shift $E_o$ in $E$ and the finite sweep rate dependence in $B$ and hence in $\partial A/\partial t$.

According to the Chern-Simons transformation\cite{3,4} in three dimensions, one can add a term $\epsilon_{ijk} A_i F_{jk}$ to the vector potential $\vec{A}$ in the expression for the linear momentum $p - \epsilon \vec{A}$ without disturbing the gauge and Lorentz invariance. Our calculation clearly shows that additional shift in the magnetic field is consistent with the Chern-Simons (CS) transformation and hence the vector potential is shifted. However, it is also clear that the scalar potential should also be corrected. Thus not only the vector potential (CS) but also the scalar potential is shifted.
For $\alpha_o < \alpha_{o,\min}$ the $\rho_{xx} = 0$ point does not occur but the $\rho_{xy}$ has plateaus. The field $B_z$ at this point may be called $B_{\text{eff}}$ so that NMR may be used to measure this field. The radio-frequency coil is fixed such that the r.f. oscillating field is along the $x$ direction with $B_z$ along the $z$ direction. The NMR transition occurs when the resonance frequency matches with the effective field,

$$\omega = \gamma B_{\text{eff}}$$

where $\gamma = g_N \mu_N / \hbar$ is the nuclear gyromagnetic ratio. Thus the NMR is shifted according to eq.(9). The quadrupole interaction is given by the Hamiltonian,

$$\mathcal{H} = g_N \mu_N B_{\text{eff}} I_z + Q' \left( I_z^2 - \frac{1}{3} I(I + 1) \right) + Q'' \left( I_x^2 - I_y^2 \right)$$

where $g_N$ is the nuclear $g$ factor, $\mu_N$ is the nuclear magneton, $I$ is the nuclear spin and $Q'$ and $Q''$ are the nuclear quadrupole interaction constants. Usually, the transitions $-3/2 \rightarrow -1/2$, $-1/2 \rightarrow +1/2$ and $1/2 \rightarrow 3/2$ are superimposed on each other so that only one spectral line occurs. However, in the present case $-3/2 \rightarrow 1/2$ and $-1/2 \rightarrow 3/2$ transitions are also quite strong. The separation between these two transitions is

$$E_{-3/2 \rightarrow 1/2} - E_{-1/2 \rightarrow 3/2} = 2\left[ (Q' + g_N \mu_N B_{\text{eff}})^2 + 3Q''^2 \right]^{1/2}$$

$$+\left[ (Q' - g_N \mu_N B_{\text{eff}})^2 + 3Q''^2 \right]^{1/2}.$$  

In addition $-3/2 \rightarrow +3/2$ is also possible so that there are four lines in the NMR spectrum of $^{75}\text{As}$ with $I = 3/2$ as clearly seen in the spectra of $\rho_{xx}$ as a function of r.f. frequency. In the NMR experiments, it is sufficient to have a large field along the $z$ direction and a small r.f. oscillating field along the $x$ direction. However, in the present case, there is a large electric field along the $x$ direction. Therefore, the theory of usual NMR requires correction for the electric field. The wave functions are mixed $|a'\rangle = a|3/2 > +b|1/2 > +c| -1/2 > +d| -3/2 >$ due to interactions of the form $\sum_i \sum_{j \leq k} \frac{1}{2} R_{ijk} E_i (I_j I_k + I_k I_j)$ where $E_i$ are the components along the $x, y$
and z axis which correct the nuclear quadrupole interaction as given in an analogous problem[5, 6]. The relaxation at very low temperatures such as 0.3 K is caused by the absorption and emission of photons which gives rise to very long relaxation times, of the order of minutes. The details of the calculation of relaxation times were given long time ago. The relative change in resistivity is given by,

$$\frac{\delta R_{xx}}{R_{xx}} = \left( \frac{L_o \omega}{R_o} \right) 4\pi \chi'' = 4\pi \chi'' Q$$

where the $\chi''$ is the imaginary part of the susceptibility, $\chi = \chi' (\omega) - i \chi'' (\omega)$, $Q$ is the quality factor of the coil and the inductance is $L_o (1 + 4\pi \chi_o)$. The coil of inductance $L_o$ is filled with the material of susceptibility, $\chi_o$. The imaginary part of the susceptibility is given by,

$$\chi'' = \frac{\chi_o}{2} \frac{\omega_o T_2}{1 + (\omega - \omega_o)^2 T_2^2}$$

where $T_2$ determines the life time which in this case is caused[7] by the radiative process. At $\omega = \omega_o$ there is a resonance detected by the resistivity, $\rho_{xx}$. The hyperfine splitting has not been seen by Kronmüller et al[1]. However, in general the isotropic value of the hyperfine constant, $A$, in the hyperfine interaction A.I.S is determined by

$$A_s = \frac{8\pi}{3} g g_N \mu_B \mu_N |\psi(o)|^2.$$ 

If $e$ is changed to $\nu e$, the change may be introduced as in previous studies [8, 9]. Hence it will be of interest to observe hyperfine interactions with fractional charge. Kronmüller et al have thus opened the doors to a whole variety of new NMR measurements.

It turns out that the values of the fractions given in Fig.18 of Störmer’s Nobel lecture [10] are the same as those given by Shrivastava[11] in 1986. The equality of masses of some of the quasiparticles is also well explained [8] by the mechanism of Shrivastava [11]. The high Landau levels are also understood by this mechanism [12,13]. Considerable amount of the experimental data agrees very well with Shrivastava’s theory [14]. It is found that there is a correction to the value of the Bohr
magneton[15]. From this study it is clear that as far as determining the centres of the plateus in the ρ_{xy} and the zeroes in ρ_{xx} is concerned, there is little role of the many-body theory. However, electrons interact with radiation and with phonons, so that many-body theory enters for the determination of the plateau length and shape of the minima curves.

In conclusion, we find that both the electric as well as the magnetic fields are subject to a shift in the quantum Hall effect so that a new mode is predicted. The NMR can be detected by measuring the resonances in the resistivity, ρ_{xx}. 
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