Transformations of CCP programs

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We introduce a transformation system for concurrent constraint programming (CCP). We define suitable applicability conditions for the transformations which guarantee that the input/output CCP semantics is preserved also when distinguishing deadlocked computations from successful ones and when considering intermediate results of (possibly) non-terminating computations.

The system allows us to optimize CCP programs while preserving their intended meaning: In addition to the usual benefits that one has for sequential declarative languages, the transformation of concurrent programs can also lead to the elimination of communication channels and of synchronization points, to the transformation of non-deterministic computations into deterministic ones, and to the crucial saving of computational space. Furthermore, since the transformation system preserves the deadlock behavior of programs, it can be used for proving deadlock freeness of a given program with respect to a class of queries. To this aim it is sometimes sufficient to apply our transformations and to specialize the resulting program with respect to the given queries in such a way that the obtained program is trivially deadlock free.

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1. INTRODUCTION

Optimization techniques, in the case of logic-based languages, fall into two main categories: on one hand, there exist methods for compile-time and low-level optimizations such as the ones presented for constraint logic programs by [Jørgensen et al. 1991], which are usually based on program analysis methodologies (e.g. abstract interpretation). On the other hand, we find source to source transformation techniques such as partial evaluation [Mogensen and Sestoft 1997] and more general techniques based on the unfold and fold or on the replacement operation.

Unfold/fold transformation techniques were first introduced for functional programs in [Burstall and Darlington 1977], and then adapted to logic programming (LP) both for program synthesis [Clark and Sickel 1977; Hogger 1981], and for program specialization and optimization [Komorowski 1982]. Tamaki and Sato [1984] proposed a general framework for the unfold/fold transformation of logic programs, which has remained in the years the main historical reference of the field, and has later been extended to constraint logic programming (CLP) in [Maher 1993; Etalle and Gabbrielli 1996; Bensaou and Guessarian 1998] (for an overview of the subject, see the survey by Pettorossi and Proietti [1994]). As shown by a number of applications, these techniques provide a powerful methodology for the development and optimization of large programs, and can be regarded as the basic transformations techniques, which might be further adapted to be used for partial evaluation.

Despite a large literature in the field of declarative sequential languages, unfold/fold transformation sequences have hardly been applied to concurrent languages. Notable exceptions are the papers of Ueda and Fukurawa [1988], Sahlin [1995], and of de Francesco and Santone [1996] (their relations with this paper are discussed in Section 7). Also when considering partial evaluation we find only very few recent attempts [Hosoya et al. 1996; Marinescu and Goldberg 1997; Gengler and Martel 1997] to apply it in the field of concurrent languages.

This situation is partially due to the fact that the non-determinism and the synchronization mechanisms present in concurrent languages substantially complicate their semantics, thus complicating also the definition of correct transformation systems. Nevertheless these transformation techniques can be very useful also for concurrent languages, since they allow further optimizations related to the simplification of synchronization and communication mechanisms.

In this paper we introduce a transformation system for concurrent constraint programming (CCP) [Saraswat 1989; Saraswat and Rinard 1990; Saraswat et al. 1991]. This paradigm derives from replacing the store-as-evaluation concept of von Neumann comput- ing by the store-as-constraint model: Its computational model is based on a global store, which consists of the conjunction of all constraints established until that moment and expresses some partial information on the values of the variables involved in the computation. Concurrent processes synchronize and communicate asynchronously via the store by using elementary actions (ask and tell) which can be expressed in a logical form (essentially implication and conjunction [Boer et al. 1997]). On one hand, CCP enjoys a clean logical semantics, avoiding many of the complications arising in the concurrent imperative setting; as argued in the position paper [Etalle and Gabbrielli 1998] this aspect is of great help in the development of effective transformation tools. On the other hand,
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differently from the case of other theoretical models for concurrency (e.g. the \(\pi\)-calculus), there exist “real” implementations of concurrent constraint languages (notably, the Oz language [Smolka 1995] and the related ongoing Mozart project [Smolka 1995] and the related ongoing Mozart project http://www.mozart-oz.org/); thus, in contrast to other models for concurrency, in this framework transformation techniques can be readily applied to practical problems.

The transformation system we are going to introduce is originally inspired by the system of Tamaki and Sato [1984]. Compared to its predecessors, it improves in three ways: Firstly, we managed to eliminate the limitation that in a folding operation the folding clause has to be non-recursive, a limitation which is present in many other unfold/fold transformation systems, this improvement possibly leads to the use of new more sophisticated transformation strategies. Secondly, the applicability conditions we propose for the folding operation are now independent from the transformation history, making the operation much easier to understand and to implement. In fact, following [Francesco and Santone 1996], our applicability conditions are based on the notion of “guardedness” and can be checked locally on the program to be folded. Finally, we introduced several new transformation operations. It is also worth mentioning that the declarative nature of CCP allows us to define reasonably simple applicability conditions which ensure the correctness of our system.

We will illustrate with a practical example how our transformation system for CCP can be even more useful than its predecessors for sequential logic languages. Indeed, in addition to the usual benefits, in this context the transformations can also lead to the elimination of communication channels and of synchronization points, to the transformation of non-deterministic computations into deterministic ones, and to the crucial saving of computational space. These improvements were possible already in the context of GHC programs by using the system defined in [Ueda and Furukawa 1988].

Our results show that the original and the transformed program have the same input/output semantics in a rather strong sense, which distinguishes successful, deadlocked and failed derivations. As a corollary, we obtain that the original program is deadlock free if and only if the transformed one is and this allows us to employ the transformation system as an effective tool for proving deadlock-freeness: if, after the transformation, we can prove or see that the process we are considering never deadlocks (in some cases the transformation simplifies the program’s behavior so that this can be immediately checked), then we are also sure that the original process does not deadlock either. We also consider non-terminating computations by proving three further correctness results. The first one shows that the intermediate results of (possibly non-terminating) computations are preserved up to logical implication, while the second one ensures full preservation of (traces of) intermediate results, provided we slightly restrict the applicability conditions for our transformations. The third result shows that this restricted transformation system preserves a certain kind of infinite computations (active ones). We discuss the extension of this result to the general case, claiming that our system does not introduce any new infinite computation.

This paper is organized as follows: in the next section we present the notation and
the necessary preliminary definitions, most of them regarding the CCP paradigm. In Section 3 we define the transformation system, which consists of various different operations (for this reason the section is divided in a number of subsections). We will also use a working example to illustrate the application of our methodology. Section 4 states the first main result, concerning the correctness of the transformation system, while Section 5 contains the results for non-terminating computations. Further examples are contained in Section 6. Section 7 compares this paper to related work in the literature and Section 8 concludes. For the sake of readability we include in this paper only proof sketches of several results, the (rather long) technical details being deferred to the (on-line only) Appendix.

A preliminary version of this paper appeared in [Etalle et al. 1998].

2. PRELIMINARIES

The basic idea underlying the CCP paradigm is that computation progresses via monotonic accumulation of information in a global store. The information is produced (in form of constraints) by the concurrent and asynchronous activity of several agents which can add a constraint $c$ to the store by performing the basic action tell$(c)$. Dually, agents can also check whether a constraint $c$ is entailed by the store by using an ask$(c)$ action. This allows the synchronization of different agents.

Concurrent constraint languages are defined parametrically with respect to the notion of constraint system, which is usually formalized in an abstract way following the guidelines of Scott’s treatment of information systems (see [Saraswat and Rinard 1990]). Here, we consider a more concrete notion of constraint which is based on first-order logic and which coincides with the one used for constraint logic programming (e.g. see [Jaffar and Maher 1994]). This will allow us to define the transformation operations in a more comprehensible way, while retaining a sufficient expressive power. We could equally well define the transformations in terms of the abstract notion of constraint system given in [Saraswat and Rinard 1990]$^1$.

Thus, assume given a signature $\Sigma$ defining a set of function and predicate symbols and associating an arity with each symbol. A constraint $c$ is a first-order $\Sigma$-formula built by using symbols of $\Sigma$, variables from a given (countable) set $V$ and the logical connectives and quantifiers ($\land, \lor, \neg, \exists$) in the usual way. The interpretation for the symbols in $\Sigma$ is provided by a $\Sigma$-structure $D$ consisting of a set $D$ and an assignment of functions and relations on $D$ to the symbols in $\Sigma$ which respect the arities. So, $D$ defines the computational domain on which constraints are interpreted. Usually, in order to model parameter passing, $\Sigma$ is assumed to contain the binary predicate symbol $=$ which is interpreted as the identity in $D$. We will follow this assumption, which allows us to avoid the use of most general unifiers (indeed, for many computation domains $D$ the most general unifier of two terms does not exist).

The formula $D \models c$ states that $c$ is valid in the interpretation provided by $D$, i.e. that it is true for every valuation of the free variables of $c$. The empty conjunction of primitive constraints will be identified with true. We also denote by $\text{Var}(e)$ the set of free variables occurring in the expression $e$.

$^1$To this aim, essentially we should replace equations of the form $X = Y$ for diagonal elements $d_{XY}$.

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In the sequel, constraints will be considered up to equivalence in the domain \( \mathcal{D} \), i.e. we write \( c_1 = c_2 \) in case \( \mathcal{D} \models c_1 \leftrightarrow c_2 \). Terms will be denoted by \( t, s, \ldots \), variables with \( X, Y, Z, \ldots \), further, as a notational convention, \( \bar{t} \) and \( \bar{X} \) denote a tuple of terms and a tuple of distinct variables, respectively. \( \exists_{\bar{X}} c \) stands for the existential closure of \( c \) except for the variables in \( \bar{X} \) which remain unquantified. We also assume that the reader is acquainted with the notion of substitution and of most general unifier (see [Lloyd 1987]). We denote by \( \sigma \) the result application of a substitution \( \sigma \) to an expression \( e \). Given a substitution \( \sigma \), the domain of \( \sigma \), \( \text{Dom}(\sigma) \), is the finite set of variables \( \{X \mid X \sigma \neq X\} \), the range of \( \sigma \) is defined as \( \text{Ran}(\sigma) = \bigcup_{X \in \text{Dom}(\sigma)} \text{Var}(X \sigma) \).

The notation and the semantics of programs and agents is virtually the same one of [Saraswat and Rinard 1990]. In particular, the \( \parallel \) operator allows one to express parallel composition of two agents and it is usually described in terms of interleaving, while non-determinism arises by introducing a (global) choice operator \( \sum_{i} \text{ask}(c_i) \rightarrow A_i \); the agent \( \sum_{i} \text{ask}(c_i) \rightarrow A_i \) nondeterministically selects one \( \text{ask}(c_i) \) which is enabled in the current store, and then behaves like \( A_i \). Thus, the syntax of CCP declarations and agents is given by the following grammar:

\[
\begin{align*}
\text{Declarations} & \quad \mathcal{D} ::= \epsilon \mid p(\bar{t}) \leftarrow A \mid D, D \\
\text{Agents} & \quad A ::= \text{stop} \mid \text{tell}(c) \mid \sum_{i=1}^{n} \text{ask}(c_i) \rightarrow A_i \mid A \parallel A \mid p(\bar{t}) \\
\text{Processes} & \quad \text{Proc} ::= D, A
\end{align*}
\]

where \( c \) and \( c_i \)'s are constraints. Note that here we allow terms both as formal and actual parameters.

Usually this is not the case, since the procedure call \( p(\bar{t}) \) can be equivalently written as \( p(\bar{X}) \parallel \text{tell}(\bar{X} = \bar{t}) \), while the declaration \( p(\bar{X}) \leftarrow A \) is equivalent to \( p(\bar{X}) \leftarrow A \parallel \text{tell}(\bar{X} = \bar{t}) \). We make this assumption only because this simplifies the writing of programs in the examples.

Due to the presence of an explicit choice operator, as usual we assume (without loss of generality) that each predicate symbol is defined by exactly one declaration. A program \( \sum \) is a set of declarations. In the following examples we assume that the operator \( \sum \) binds tighter than \( \parallel \) (so, \( \text{ask}(a) \rightarrow A \parallel \text{ask}(b) \rightarrow B + \text{ask}(d) \rightarrow C \) means \( (\text{ask}(c) \rightarrow A) \parallel (\text{ask}(b) \rightarrow B + \text{ask}(d) \rightarrow C) \)). In case some ambiguity arises we will use brackets to indicate the scope of the operators.

An important aspect for which we slightly depart from the usual formalization of CCP regards the notion of locality. In [Saraswat and Rinard 1990] locality is obtained by using the operator \( \exists_{\bar{X}} \), and the behavior of the agent \( \exists_{\bar{X}} A \) is defined like the one of \( A \), with the variable \( X \) considered as local to it. Here we do not use such an explicit operator: analogously to the standard CLP setting, locality is introduced implicitly by assuming that if a process is defined by \( p(\bar{t}) \leftarrow A \) and a variable \( Y \) occurs in \( A \) but not in \( \bar{t} \), then \( Y \) has to be considered local to \( A \).

The operational model of CCP is described by a transition system \( \mathcal{T} = (\mathcal{Conf}, \rightarrow) \) where configurations (in) \( \mathcal{Conf} \) are pairs consisting of a process and a constraint (representing the common store), while the transition relation \( \rightarrow \subseteq \mathcal{Conf} \times \mathcal{Conf} \) is described by the (least relation satisfying the) rules \( \text{R1-R4} \) of Table I which should be self-explanatory. Here and in the following we assume given a set \( D \)
of declarations and we denote by $\text{defn}_D(p)$ the set of variants\(^2\) of the (unique) declaration in $D$ for the predicate symbol $p$. Due to the presence of terms as arguments to predicates symbols, differently from the standard setting in rule $R4$ parameter passing is performed by a tell action. We also assume the presence of a renaming mechanism that takes care of using fresh variables each time a declaration is considered\(^3\).

We denote by $\rightarrow^*$ the reflexive-transitive closure of the relation $\rightarrow$ defined by the transition system, and we denote by $\text{Stop}$ any agent which contains only $\text{stop}$ and $\parallel$ constructs. A finite derivation (or computation) containing only satisfiable constraints is called successful if it is of the form $\langle D,A,c \rangle \rightarrow^* \langle D,\text{Stop},d \rangle \not\rightarrow$ while it is called deadlocked if it is of the form $\langle D,A,c \rangle \rightarrow^* \langle D,B,d \rangle \not\rightarrow$ with $B$ different from $\text{Stop}$ (i.e., $B$ contains at least one suspended agent). A derivation producing eventually false is called failed. Note that we consider here the so called “eventual tell” CCP, i.e. when adding constraints to the store (via tell operations) there is no consistency check. Our results could be adapted to the CCP language with consistency check (“atomic tell” CCP) by minor modifications of the transformation operations.

3. THE TRANSFORMATION

In order to illustrate the application of our method we will adopt a working example. We consider an auction problem in which two bidders participate: bidder\(_a\) and bidder\(_b\); each bidder takes as input the list of the bids of the other one and produces as output the list of his own bids. When one of the two bidders wants to quit the auction, it produces in its own output stream the token quit. This protocol is implemented by the following program $\text{AUCTION}$. Here and in the following examples we do not make any assumption on the specific constraint domain being used, apart from the fact that it should allow us to use lists of elements. This is the case for most existing general purpose constraint languages, which usually incorporate also some arithmetic domain (see [Jaffar and Maher 1994]).

\begin{align}
\text{auction}(\text{LeftBids},\text{RightBids}) &\leftarrow \text{bidder\(_a\)[([0]|\text{RightBids}),\text{LeftBids}] \parallel \text{bidder\(_b\)[\text{LeftBids},\text{RightBids}]}
\end{align}

\(^2\)A variant of a declaration $d$ is obtained by replacing the tuple $\tilde{X}$ of all the variables appearing in $d$ for another tuple $\tilde{Y}$.

\(^3\)For the sake of simplicity we do not describe this renaming mechanism in the transition system. The interested reader can find in [Saraswat and Rinard 1990; Saraswat et al. 1991] various formal approaches to this problem.

Table I. The (standard) transition system.

| Rule | Transition |
|------|------------|
| $R1$ | $\langle D,\text{tell}(c),d \rangle \rightarrow \langle D,\text{stop},c \land d \rangle$ |
| $R2$ | $\langle D,\sum_{j=1}^n \text{ask}(c_j) \rightarrow A_j,d \rangle \rightarrow \langle D,\sum_{j=1}^n A_j,d \rangle$ if $j \in [1,n]$ and $D \models d \rightarrow c_j$ |
| $R3$ | $\langle D,A,c \rangle \rightarrow \langle D,A',c' \rangle$ if $j \in [1,n]$ and $D \models d \rightarrow c_j$ |
| $R4$ | $\langle D,\text{p}(\tilde{t}),c \rangle \rightarrow \langle D,\text{tell}(\tilde{t} = \tilde{s}),c \rangle$ if $p(\tilde{t}) \leftarrow A \in \text{defn}_D(p)$ |

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bidder_a(HisList, MyList) ←
  ask(∃HisBid. HisList = [HisBid|HisList’] ∧ HisBid = quit) → stop
  + ask(∃HisBid. HisList = [HisBid|HisList’] ∧ HisBid ≠ quit) →
    (tell(HisList = [HisBid|HisList’]) ||
     make_new_bid_a(HisBid,MyBid) ||
     ask(MyBid = quit) → (tell(MyList = [MyBid|MyList’]) || broadcast(“a quits”))
    + ask(MyBid ≠ quit) → (tell(MyList = [MyBid|MyList’]) ||
     tell(MyBid ≠ quit) ||
     bidder_a(HisList’,MyList’)))

plus an analogous definition for bidder_b.

Here, the agent make_new_bid_a(HisBid,MyBid) is in charge of producing a new offer in presence of the competitor’s offer HisBid; the agent will produce MyBid = quit if it evaluates that HisBid is too high to be topped, and decides to leave the auction. This agent could be further specified by using arithmetic constraints. In order to avoid deadlock, auction initializes the auction by inserting a fictitious zero bid in the input of bidder a. Notice that in the above program the agent tell(HisList = [HisBid|HisList’]) is needed to bind the local variables (HisBid, HisList’) to the global one (HisList): In fact, as a result of the operational semantics, such a binding is not performed by the ask agent. On the contrary the agent tell(MyBid ≠ quit) is redundant: We have introduced it in order to slightly simplify the following transformations (the transformations remain possible also without such a tell). The introduction of redundant tell’s is a transformation operation which will be formally defined in Subsection 3.4.

3.1 Introduction of a new definition

The introduction of a new definition is virtually always the first step of a transformation sequence. Since the new definition is going to be the main target of the transformation operation, this step will actually determine the very direction of the subsequent transformation, and thus the degree of its effectiveness.

Determining which definitions should be introduced is a very difficult task which falls into the area of strategies. To give a simple example, if we wanted to apply partial evaluation to our program with respect to a given agent A (i.e. if we wanted to specialize our program so that it would execute the partially instantiated agent A in a more efficient way), then a good starting point would most likely be the introduction of the definition $p(\tilde{X}) \leftarrow A$, where $\tilde{X}$ is an appropriate tuple of variables and p is a new predicate symbol. A different strategy would probably determine the introduction of a different new definition. For a survey of the other possibilities we refer to [Pettorossi and Proietti 1994].

In this paper we are not concerned with the strategies, but only with the basic transformation operations and their correctness: we aim at defining a transformation system which is general enough so to be applied in combination with different strategies.

In order to simplify the terminology and the technicalities, we assume that these new declarations are added once for all to the original program before starting the transformation itself. Note that this is clearly not restrictive. As a notational convention we call $D_0$ the program obtained after the introduction of new definitions.

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In the case of program AUCTION, we assume that the following new declarations are added to the original program.

\[
\text{auction\_left}(\text{LastBid}) \leftarrow \text{tell}(\text{LastBid} \neq \text{quit}) \parallel \text{bidder\_a}[[\text{LastBid}\|\text{Bs}], \text{As}) \parallel \text{bidder\_b}(\text{As}, \text{Bs}) .
\]

\[
\text{auction\_right}(\text{LastBid}) \leftarrow \text{tell}(\text{LastBid} \neq \text{quit}) \parallel \text{bidder\_a}(\text{Bs}, \text{As}) \parallel \text{bidder\_b}([\text{LastBid}\|\text{As}], \text{Bs}) .
\]

The agent \text{auction\_left}(\text{LastBid}) engages an auction starting from the bid \text{LastBid} (which cannot be \text{quit}) and expecting the bidder “a” to be the next one in the bid. The agent \text{auction\_right}(\text{LastBid}) is symmetric.

3.2 Unfolding

The first transformation we consider is the unfolding. This operation consists essentially in the replacement of a procedure call by its definition. The syntax of CCP agents allows us to define it in a very simple way by using the notion of context. A context, denoted by \( C[ ] \), is simply an agent with a “hole”, where the hole can contain any expression of type agent. So, for example, \([ ] \parallel A\) and \(\text{ask}(c) \rightarrow A + \text{ask}(b) \rightarrow [ ]\) are contexts, while \(\text{ask}(a) \rightarrow A + [ ]\) is not. \(C[A]\) denotes the agent obtained by replacing the hole in \(C[ ]\) for the agent \(A\), in the obvious way.

**Definition 3.1 (Unfolding).** Consider a set of declarations \(D\) containing

\[
d : H \leftarrow C[p(\tilde{t})]
\]

\[
u : p(\tilde{s}) \leftarrow B
\]

Then unfolding \(p(\tilde{t})\) in \(d\) consists in replacing \(d\) by

\[
d' : H \leftarrow C[B \parallel \text{tell}(\tilde{t} = \tilde{s})]
\]

in \(D\). Here \(d\) is the unfolded definition and \(u\) is the unfolding one; \(d\) and \(u\) are assumed to be renamed so that they do not share variables.

After an unfolding we often need to simplify some of the newly introduced tell’s in order to “clean up” the resulting declarations. This is accomplished via a tell elimination. Recall that a most general unifier \(\sigma\) of the terms \(t\) and \(s\) is called relevant if \((\text{Dom}(\sigma) \cup \text{Ran}(\sigma)) \subseteq \text{Var}(t,s)\).

**Definition 3.2 (Tell Elimination and Tell Introduction).** The declaration

\[
d : H \leftarrow C[\text{tell}(\tilde{s} = \tilde{t}) \parallel B]
\]

can be transformed via a tell elimination into

\[
d' : H \leftarrow C[B\sigma]
\]

where \(\sigma\) is a relevant most general unifier of \(\tilde{s}\) and \(\tilde{t}\), provided that the variables in the domain of \(\sigma\) do not occur neither in \(C[ ]\) nor in \(H\). This operation is applicable either when the computational domain \(D\) admits a most general unifier, or when \(\tilde{s}\) and \(\tilde{t}\) are sequences of distinct variables, in which case \(\sigma\) is simply a renaming. On the other hand, the declaration

\[
d : H \leftarrow C[B\sigma]
\]

can be transformed via a tell introduction into

\[
d' : H \leftarrow C[\text{tell}(\bar{X} = \bar{X}\sigma) \parallel B]
\]
provided that $\sigma$ is a substitution such that $\tilde{X} = \text{Dom}(\sigma)$ and $\text{Dom}(\sigma) \cap (\text{Var}(\text{C}[\cdot], H) \cup \text{Ran}(\sigma)) = \emptyset$.

Notice that, in particular, we can always exchange $\text{C}[\text{tell}(\text{true}) \parallel A]$ with $\text{C}[A]$ and vice-versa. The presence of $\text{Ran}(\sigma)$ in the above condition is needed to ensure that $\sigma$ is idempotent: in fact, using substitutions $\sigma$ of the form $X/f(X)$ would not be correct in general. In practice, the constraints on the domain of $\sigma$ can be weakened by appropriately renaming some local variables; this is also shown in the upcoming example. In fact, if all the occurrences of a local variable in $\text{C}[\cdot]$ are in choice branches different from the one the “hole” lies in, then we can safely rename apart each one of these occurrences.

In our AUCTION example, we start working on the definition of $\text{auction\_right}$, and we unfold the agent $\text{bidder\_b}([\text{LastBid}|A], B)$ and then we perform the subsequent tell eliminations (we eliminate the tells introduced by the unfolding). The result of these operations is the following program.

```plaintext
\text{auction\_right}(\text{LastBid}) \leftarrow \text{tell}(\text{LastBid} \neq \text{quit}) \parallel \\
\text{bidder\_a}(B, A) \parallel \\
\text{ask}(\exists \text{HisBid, HisList}', \text{LastBid}|A) = \text{HisList'} \land \text{HisBid} = \text{quit} \rightarrow \text{stop} \\
+ \text{ask}(\exists \text{HisBid, HisList}', \text{LastBid}|A) = \text{HisList'} \land \text{HisBid} \neq \text{quit} \rightarrow \\
\text{tell}(\exists \text{HisBid|HisList'}) \parallel \\
\text{make\_new\_bid}(\text{HisBid, MyBid}) \parallel \\
\text{ask}(\text{MyBid} = \text{quit}) \rightarrow \text{tell}(B = \text{MyBid|Bs'}) \parallel \text{broadcast}("b quits") \\
+ \text{ask}(\text{MyBid} \neq \text{quit}) \rightarrow \text{tell}(B = \text{MyBid|Bs'}) \parallel \\
\text{tell}(\text{MyBid} \neq \text{quit}) \parallel \\
\text{bidder\_b}(\text{HisList}', Bs')
```

### 3.3 Backward Instantiation

The new operation of \textit{backward instantiation}, is somehow similar to the one of unfolding. We immediately begin with its definition.

\textit{Definition 3.3 (Backward instantiation).} Let $D$ be a set of definitions and

- $d : H \leftarrow C(p(\tilde{t}))$
- $b : p(\tilde{s}) \leftarrow \text{tell}(c) \parallel B$

be two definitions of $D$. The \textit{backward instantiation} of $p(\tilde{t})$ in $d$ consists in replacing $d$ by $d'$, which is either

$$d' : H \leftarrow C[p(\tilde{t})] \parallel \text{tell}(c) \parallel \text{tell}(\tilde{t} = \tilde{s})$$

or

$$d' : H \leftarrow C[p(\tilde{t})] \parallel \text{tell}(\tilde{t} = \tilde{s})$$

(it is assumed here that $d$ and $b$ are renamed so that they have no variables in common).

More generally, the operation can also be applied when $b$ is not of the form $p(\tilde{s}) \leftarrow \text{tell}(c) \parallel B$ by considering $c$ to be true.

Intuitively, this operation can be regarded as a “half-unfolding” for the following reason: performing an unfolding is equivalent to applying a derivation step to the
atomic agent under consideration, here we do not quite do it, yet we carry out (part
of) the two first phases that the derivation step requires.
In the Section 6 we will show an application of this operation (Example 6.2).

3.4 Ask and Tell Simplification

A new important operation is the one which allows us to modify the ask guards
and the tell's occurring in a program. Let us call produced constraint of \( C[ ] \) the
conjunction of all the constraints appearing in ask and tell actions which can be
evaluated before \( [ ] \) is reached (in the context \( C[ ] \)). Now, if \( a \) is the produced
constraint of \( C[ ] \) and \( D \models a \rightarrow c \), then clearly we can simplify an agent of the form
\( C[\text{ask}(c) \rightarrow A + \text{ask}(d) \rightarrow B] \) to \( C[\text{ask}(\text{true}) \rightarrow A + \text{ask}(d) \rightarrow B] \). Moreover, under the
previous hypothesis, we can clearly transform \( C[\text{tell}(c) \parallel A] \) to \( C[A] \) and, conversely,
\( C[A] \) to \( C[\text{tell}(c) \parallel A] \) (as previously mentioned, this latter transformation consisting in
the introduction of a redundant tell might be needed to prepare a program for
the folding operation).

In general, if \( a \) is the produced constraint of \( C[ ] \) and for some constraint \( c' \) we
have that \( D \models \exists_{\tilde{Z}} (a \land c) \leftrightarrow (a \land c') \) (where \( \tilde{Z} = \text{Var}(C, A) \)), then we can replace
\( c \) with \( c' \) in \( C[\text{ask}(c) \rightarrow A] \) and in \( C[\text{tell}(c)] \). In particular, if we have that \( a \land c \)
is unsatisfiable, then \( c \) can immediately be replaced with false (the unsatisfiable
constraint).
In order to formalize this intuitive idea, we start with the following
definition.

**Definition 3.4.** Given an agent \( A \), the produced constraint of \( A \) is denoted by
\( pca(A) \) and is defined by structural induction as follows:

\[
\begin{align*}
pca(\text{tell}(c)) &= c \\
pca(A \parallel B) &= pca(A) \land pca(B) \\
pca(A) &= \text{true} \quad \text{for any agent } A \text{ which is neither of the form } \text{tell}(c) \text{ nor a parallel composition.}
\end{align*}
\]

By extending the definition we use for agents to contexts, given a context \( C[ ] \)
the produced constraint of \( C[ ] \) is denoted by \( pc(C[ ]) \) and is inductively defined as follows:

\[
\begin{align*}
pc([ ]) &= \text{true} \\
pc(C'[ ] \parallel B) &= pc(C'[ ]) \land pca(B) \\
pc(\sum_{i=1}^{n} \text{ask}(c_i) \rightarrow A_j) &= c_j \land pc(C'[ ]) \text{ where } j \in [1, n] \text{ and } A_j = C'[ ]
\end{align*}
\]

The following definition allows us to determine when two constraints are equivalent
within a given context \( C[ ] \).

**Definition 3.5.** Let \( c, c' \) be constraints, \( C[ ] \) be a context, and \( \tilde{Z} \) be a set
of variables. We say that \( c \) is equivalent to \( c' \) within \( C[ ] \) and w.r.t. the variables in \( \tilde{Z} \)
iff \( D \models \exists_{\tilde{Z}} (pc(C[ ]) \land c) \leftrightarrow \exists_{\tilde{Z}} (pc(C[ ]) \land c') \)

This definition is employed in the following operation, which allows us to simplify
the constraints in the ask and tell guards.

\(^{4}\text{Note that in general the further simplification to } C[\text{A + ask(d) \rightarrow B}] \text{ is not correct, although we can transform } C[\text{ask(true) \rightarrow A}] \text{ into } C[A].\)
Definition 3.6 (Ask and Tell Simplification). Let \( D \) be a set of declarations.

1. Let \( d : H \leftarrow C[\sum_{i=1}^{n} \text{ask}(c_i) \rightarrow A_i] \) be a declaration of \( D \). Suppose that \( c_1', \ldots, c_n' \) are constraints such that for \( j \in [1, n] \), \( c_j' \) is equivalent to \( c_j \) within \( C[ ] \) and w.r.t. the variables in \( \text{Var}(C, H, A_j) \).
   
   Then we can replace \( d \) with \( d' : H \leftarrow C[\sum_{i=1}^{n} \text{ask}(c_i') \rightarrow A_i] \) in \( D \). We call this an ask simplification operation.

2. Let \( d : H \leftarrow C[\text{tell}(c)] \) be a declaration of \( D \). Suppose that the constraint \( c' \) is equivalent to \( c \) within \( C[ ] \) and w.r.t. the variables in \( \text{Var}(C, H) \).
   
   Then we can replace \( d \) with \( d' : H \leftarrow C[\text{tell}(c')] \) in \( D \). We call this a tell simplification operation.

In our AUCTION example, we can consider the produced constraint of \( \text{tell}(\text{LastBid} \neq \text{quit}) \), and modify the subsequent ask constructs as follows:

\[
\text{auction\_right}(\text{LastBid}) \leftarrow \text{tell}(\text{LastBid} \neq \text{quit}) \parallel \text{bidder\_a}(\text{Bs}, \text{As}) \parallel \text{ask(false)} \rightarrow \text{stop} + \text{ask(true)} \rightarrow \text{tell}([\text{LastBid} | \text{As}] = [\text{HisBid} | \text{HisList}']) \parallel \ldots
\]

Via the same operation, we can immediately simplify this to:

\[
\text{auction\_right}(\text{LastBid}) \leftarrow \text{tell}(\text{LastBid} \neq \text{quit}) \parallel \text{bidder\_a}(\text{Bs}, \text{As}) \parallel \text{ask(false)} \rightarrow \text{stop} + \text{ask(true)} \rightarrow \text{tell}([\text{LastBid} | \text{As}] = [\text{HisBid} | \text{HisList}']) \parallel \ldots
\]

3.5 Branch Elimination and Conservative Ask Elimination

In the above program we have a guard \( \text{ask(false)} \) which of course will never be satisfied. The first important application of the guard simplification operation regards then the elimination of unreachable branches.

Definition 3.7 (Branch Elimination). Let \( D \) be a set of declarations and let

\[
d : H \leftarrow C[\sum_{i=1}^{n} \text{ask}(c_i) \rightarrow A_i]
\]

be a declaration of \( D \). Assume that \( n > 1 \) and that for some \( j \in [1, n] \), we have that \( c_j = \text{false} \), then we can replace \( d \) with

\[
d' : H \leftarrow C[(\sum_{i=1}^{j-1} \text{ask}(c_i) \rightarrow A_i) + (\sum_{i=j+1}^{n} \text{ask}(c_i) \rightarrow A_i)].
\]

The condition that \( n > 1 \) means that we cannot eliminate all the branches of a choice and it is needed to ensure the correctness of the system (otherwise one could transform a deadlock into a success: For example, the agent \( \text{tell}(c) \parallel \text{ask(false)} \rightarrow \text{stop} \) when evaluated in the empty store produces the constraint \( c \) and deadlocks, while the agent \( \text{tell}(c) \) produces \( c \) and succeeds).

By applying this operation to the above piece of example, we can eliminate \( \text{ask(false)} \rightarrow \text{stop} \), thus obtaining

\[
\text{auction\_right}(\text{LastBid}) \leftarrow \text{tell}(\text{LastBid} \neq \text{quit}) \parallel \text{bidder\_a}(\text{Bs}, \text{As}) \parallel \ldots
\]
ask(true) → tell([LastBid|As] = [HisBid|HisList']) ∥ ...

Now we do not see any reason for not eliminating the guard ask(true) altogether. This can indeed be done via the following operation.

**Definition 3.8 (Conservative Ask Elimination).** Consider the declaration
d : H ← C[ask(true) → B]

We can transform d into the declaration
d' : H ← C[B].

This operation, although trivial, is subject of debate. In fact, Sahlin [1995] defines a similar operation, with the crucial distinction that the choice might still have more than one branch, in other words, in the system of [Sahlin 1995] one is allowed to simplify the agent C[ask(true) → A + ask(b) → B] to the agent C[A], even if b is satisfiable. Ultimately, one is allowed to replace the agent C[ask(true) → A + ask(true) → B] either with C[A] or with C[B], indifferently. Such an operation is clearly more widely applicable than the one we have presented but is bound to be incomplete, i.e. to lead to the loss of potentially successful branches. Nevertheless, Sahlin argues that an ask elimination such as the one defined above is potentially too restrictive for a number of useful optimization. We agree with the statement only partially; nevertheless, the system we propose could easily be equipped also with an ask elimination as the one proposed by Sahlin (which of course, if employed, would lead to weaker correctness results).

In our example program, the application of these branch elimination and conservative ask elimination leads to the following:
auction_right(LastBid) ← tell(LastBid ≠ quit) ∥
  bidder_a(Bs, As) ∥
  tell([LastBid|As] = [HisBid|HisList']) ∥
  make_new_bid_b(HisBid,MyBid) ∥
    ask(MyBid = quit) → (tell(Bs = [quit|Bs']) ∥ broadcast("b quits"))
  + ask(MyBid ≠ quit) → (tell(Bs = [MyBid|Bs']) ∥
     tell(MyBid ≠ quit) ∥
     bidder_b(HisList'.Bs'))

Via a tell elimination of tell([LastBid|As] = [HisBid|HisList']), this simplifies to:
auction_right(LastBid) ← tell(LastBid ≠ quit) ∥
  bidder_a(Bs, As) ∥
  make_new_bid_b(LastBid,MyBid) ∥
    ask(MyBid = quit) → (tell(Bs = [quit|Bs']) ∥ broadcast("b quits"))
  + ask(MyBid ≠ quit) → (tell(Bs = [MyBid|Bs']) ∥
     tell(MyBid ≠ quit) ∥
     bidder_b(As,Bs'))

### 3.6 Distribution

A crucial operation in our transformation system is the distribution, which consists of bringing an agent inside a choice as follows: from the agent A ∥ ∑ i ask(c_i) → B_i, we want to obtain the agent ∑ i ask(c_i) → (A ∥ B_i). This operation requires delicate
applicability conditions, as it can easily introduce deadlocks: consider for instance the following contrived program $D$.

\[
p(Y) \leftarrow q(X) \parallel \text{ask}(X > = 0) \rightarrow \text{tell}(Y=0)
\]

\[
q(0) \leftarrow \text{stop}
\]

In this program, the process $D,p(Y)$ originates the derivation $\langle D,p(Y), \text{true} \rangle \rightarrow^* \langle D,\text{Stop}, Y = 0 \rangle$. Now, if we blindly apply the distribution operation to the first definition we would change $D$ into:

\[
p(Y) \leftarrow \text{ask}(X > = 0) \rightarrow (q(X) \parallel \text{tell}(Y=0))
\]

and now we have that $\langle D,p(Y), \text{true} \rangle$ generates only deadlocking derivations. This situation is avoided by demanding that the agent being distributed will not be able to produce any output, unless it is completely determined which branches of the choices might be entered.

To define the applicability conditions for the distribution operation we then need the notion of productive configuration. Here and in the following we say that a derivation $\langle D,A,c \rangle \rightarrow^* \langle D,A',c' \rangle$ is maximal if $\langle D,A',c' \rangle \neq \langle D.A,c \rangle$.

**Definition 3.9 (Productive).** Given a process $D,A$ and a satisfiable constraint $c$, we say that $\langle D,A,c \rangle$ is productive iff either it has no (finite) maximal derivations or there exists a derivation $\langle D,A,c \rangle \rightarrow^* \langle D,A',c' \rangle$ such that $D \models \neg(\exists \_ \_ Z c \rightarrow \exists \_ \_ Z c')$, where $\_ \_ Z = \text{Var}(A)$.

So, a configuration is productive if its evaluation can (strictly) augment the information contained in the global store. For technical reasons which will be clear after the next definition, we call productive also those configurations which have no finite maximal derivations.

We can now provide the definition of the distribution operation.

**Definition 3.10 (Distribution).** Let $D$ be a set of declarations and let

\[
d : H \leftarrow C[A \parallel \sum_{i=1}^{n} \text{ask}(c_i) \rightarrow B_i]
\]

be a declaration in $D$, where $e = \text{pc}(C[\_])$. The distribution of $A$ in $d$ yields as result the definition

\[
d' : H \leftarrow C[\sum_{i=1}^{n} \text{ask}(c_i) \rightarrow (A \parallel B_i)]
\]

provided that for every constraint $c$ such that $\text{Var}(c) \cap \text{Var}(d) \subseteq \text{Var}(H, C)$, if $\langle D.A,c \wedge e \rangle$ is productive then both the following conditions hold:

(a) There exists at least one $i \in [1, n]$ such that $D \models (c \wedge e) \rightarrow c_i$,
(b) for each $i \in [1, n]$, either $D \models (c \wedge e) \rightarrow c_i$ or $D \models (c \wedge e) \rightarrow \neg c_i$.

Intuitively, the constraint $c$ models the possible ways of “calling” $A \parallel \sum_{i=1}^{n} \text{ask}(c_i) \rightarrow B_i$. Condition (b) basically requires that if the store $c$ is such that $A$ might produce some output (that is, the configuration $\langle D.A,c \wedge e \rangle$ is productive), then for each branch of the choice it is already determined whether we can follow it or not. This guarantees that the constraints possibly added to the store by the evaluation of $A$ cannot influence the choice. Moreover, condition (a) guarantees that we do not apply the operation to a case such as $\text{tell}(X = a) \parallel \text{ask}(\text{false}) \rightarrow \text{stop}$, which would clearly be wrong. If $\langle D.A,c \wedge e \rangle$ is not productive then we do not impose any
condition, since the evaluation of \( \langle D.A, c \land e \rangle \) cannot affect the choice. As previously mentioned, we call productive also those configurations which have no finite maximal derivations, that is, those configurations which originate non-terminating computations only (possibly with no output). In fact, also in this case we need conditions (a) and (b), since otherwise bringing \( A \) inside the choice might transform a looping program into a deadlocking one.

The above applicability conditions are a strict improvement on the ones we presented in [Etalle et al. 1998], in which we used the concept of required variable. We now report this definition, both for simplifying the explanation for some examples and for comparing the above definition of distribution with the one in [Etalle et al. 1998].

**Definition 3.11 (Required Variable).** We say that the process \( D.A \) requires the variable \( X \) if, for each satisfiable constraint \( c \) such that \( D \models \exists X \iff c \), \( \langle D.A, c \rangle \) is not productive.

In other words, the agent \( A \) requires the variable \( X \) if, in the moment that the global store does not contain any information on \( X \), \( A \) cannot produce any information which affects the variables occurring in \( A \) and has at least one finite maximal derivation. Even though the above notion is not decidable in general, it is easy to find wide-applicable (decidable) sufficient conditions guaranteeing that a certain variable is required. For example it is immediate to see that, in our program, \( \text{bidder}_a(Bs, As) \) requires \( Bs \): in fact the derivation starting in \( \text{bidder}_a(Bs, As) \) suspends (without having provided any output) after one step and resumes only when more information for the variable \( Bs \) has been produced.

The following remark clarifies how the concept of required variable might be used for ensuring the applicability of the distributive operation. Its proof is straightforward.

**Remark 3.12.** Referring to Definition 3.10. If \( A \) requires a variables which does not occur in \( H, C[\cdot] \), then the distribution operation is applicable.

**Proof.** In this case, there exists no constraint \( c \) such that \( Var(c) \cap Var(d) \subseteq Var(H, C) \) and \( \langle D.A, c \land e \rangle \) is productive.

In our example, since the agent \( \text{bidder}_a(Bs, As) \) requires the variable \( Bs \), which occurs only inside the \( \text{ask} \) guards, we can safely apply the distributive operation. The result is the following program.

```
auction_right(LastBid) ← tell(LastBid \neq quit) || make_new_bid_b(LastBid,MyBid) ||
   ask(MyBid = quit) → tell(Bs = [quit|Bs']) || broadcast("b quits") || bidder_a(Bs, As)
+ ask(MyBid \neq quit) → (tell(Bs = [MyBid|Bs']) ||
   tell(MyBid \neq quit) ||
   bidder_a(Bs, As) ||
   bidder_b(As, Bs'))
```

In this program we can now eliminate the construct \( \text{tell}(Bs = [MyBid|Bs']) \): In fact, even though the variable \( Bs \) here occurs also elsewhere in the definition, we can assume it to be renamed since it occurs only on choice-branches different than the one on which the considered agent lies. Thus we obtain:

```
auction_right(LastBid) ← tell(LastBid \neq quit) || make_new_bid_b(LastBid,MyBid) ||
```

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Transformations of CCP programs

Before we introduce the fold operation, let us clean up the program a bit further: we can now first apply a tell elimination to \( \text{tell}(\text{Bs} = [\text{quit} | \text{Bs'}]) \), and then properly transform (by unfolding, and simplifying the result) the agent \( \text{bidder}_a([\text{quit} | \text{Bs'}], \text{As}) \) in the first ask branch. We easily obtain:

\[
\text{auction}_\text{right}(\text{LastBid}) \leftarrow \text{tell}(\text{LastBid} \neq \text{quit}) \parallel \text{make}_\text{new}_\text{bid}_b(\text{LastBid}, \text{MyBid}) \parallel \\
\text{ask}(\text{MyBid} = \text{quit}) \rightarrow \text{broadcast}("b quits") \parallel \text{stop} \\
+ \text{ask}(\text{MyBid} \neq \text{quit}) \rightarrow (\text{tell}(\text{MyBid} \neq \text{quit}) \parallel \\
\text{bidder}_a([\text{MyBid} | \text{Bs'}], \text{As}) \parallel \\
\text{bidder}_b(\text{As}, \text{Bs'}))
\]

The just introduced \text{stop} agent can safely be removed (see Proposition 4.2) and we are left with:

\[
\text{auction}_\text{right}(\text{LastBid}) \leftarrow \text{tell}(\text{LastBid} \neq \text{quit}) \parallel \text{make}_\text{new}_\text{bid}_b(\text{LastBid}, \text{MyBid}) \parallel \\
\text{ask}(\text{MyBid} = \text{quit}) \rightarrow \text{broadcast}("b quits") \\
+ \text{ask}(\text{MyBid} \neq \text{quit}) \rightarrow (\text{tell}(\text{MyBid} \neq \text{quit}) \parallel \\
\text{bidder}_a([\text{MyBid} | \text{Bs'}], \text{As}) \parallel \\
\text{bidder}_b(\text{As}, \text{Bs'}))
\]

3.7 Folding

The folding operation has a special role in the suite of the transformation operations. This is due to the fact that it allows us to introduce recursion in a definition, often making it independent from the definitions it depended on. As previously mentioned, the applicability conditions that we use here for the folding operation do not depend on the transformation history, nevertheless, we require that the declarations used to fold an agent appear in the initial program. Thus, before defining the fold operation, we need the following.

**Definition 3.13.** A transformation sequence is a sequence of programs \( D_0, \ldots, D_n \), in which \( D_0 \) is an initial program and each \( D_{i+1} \) is obtained from \( D_i \) via one of the following transformation operations: unfolding, backward instantiation, tell elimination, tell introduction, ask and tell simplification, branch elimination, conservative ask elimination, distribution and folding.

Recall that we assume that the new declarations introduced by using the definition introduction operation are added once for all to the original program \( D_0 \) before starting the transformation itself. We also need the notion of *guarding context*. Intuitively, a context \( C[ \ ] \) is guarding if the “hole” appears in the scope of an ask guard.

**Definition 3.14 (Guarding Context).** We call \( C[ \ ] \) is a guarding context iff

\[
C[ \ ] = C[\sum_{i=1}^{n} \text{ask}(c_i) \rightarrow A_i] \quad \text{and} \quad A_j = C'[ \ ] \quad \text{for some} \ j \in [1, n].
\]

So, for example, \( \text{ask}(c) \rightarrow (A \parallel [ \ ]) \) is a guarding context, while \( (\text{ask}(c) \rightarrow A) \parallel [ \ ] \) is not. We can finally give the definition of folding:
Definition 3.15 (Folding). Let $D_0,\ldots,D_i, i \geq 0$, be a transformation sequence. Consider two definitions.

\[
d: \quad H \leftarrow C[A] \quad \in D_i
\]

\[
f: \quad B \leftarrow A \quad \in D_0
\]

If $C[]$ is a guarding context, $B$ contains only distinct variables as arguments and $\text{Var}(A) \cap \text{Var}(C,H) \subseteq \text{Var}(B)$ then folding $A$ in $d$ consists of replacing $d$ by

\[
d': \quad H \leftarrow C[B] \quad \in D_{i+1}
\]

(it is assumed here that $d$ and $f$ are suitably renamed so that the variables they have in common are only the ones occurring in $A$).

In many situations this operation is actually applicable also in absence of a guarding context as discussed below.

Remark 3.16. We can apply the fold operation also in case $C[]$ is not guarding context (referring to the notation of the previous definition), provided that the definition $H \leftarrow C[A]$ was not modified nor used during the transformation. In fact, in this case we can simply assume that the original definition of $H \leftarrow C[A]$ contained a dummy ask guard as in

\[
H \leftarrow \text{ask(true)} \rightarrow C[A]
\]

that the folding operation is applied to this definition, and that the guard $\text{ask(true)}$ will eventually be removed by an ask elimination operation.

Actually, in many cases this reasoning can be applied also to definitions that are used during the transformation. This kind of folding is called propagation folding (as opposed to the recursive folding): it is not employed to introduce recursion, but to propagate to other contexts the efficiency that was hopefully gained by the transformation. Usually, transformation systems provide a special condition for the propagation folding operation. For instance, in [Tamaki and Sato 1984], a distinction is made between new and old predicates. Here we decided not to do so. This allows us to have a definition of folding operation which is particularly simple.

We refer to the end of Example 6.2 for an example of application of folding without guarding context.

The reach of the folding operation is best shown via our example. We can now fold $\text{auction_left(MyBid)}$ in the above definition, and obtain:

\[
\text{auction_right(LastBid)} \leftarrow \text{tell}((\text{LastBid} \neq \text{quit}) \; \parallel \; \text{make_new_bid}_b(\text{LastBid,MyBid}) \; \parallel \; \text{ask(MyBid} = \text{quit}) \rightarrow \text{broadcast}(\text{"b quits"})
\]

\[
+ \; \text{ask(MyBid} \neq \text{quit}) \rightarrow \text{auction_left(MyBid)}
\]

Now, by performing an identical optimization on $\text{auction_left}$, we can also obtain:

\[
\text{auction_left(LastBid)} \leftarrow \text{tell}((\text{LastBid} \neq \text{quit}) \; \parallel \; \text{make_new_bid}_a(\text{LastBid,MyBid}) \; \parallel \; \text{ask(MyBid} = \text{quit}) \rightarrow \text{broadcast}(\text{"a quits"})
\]

\[
+ \; \text{ask(MyBid} \neq \text{quit}) \rightarrow \text{auction_right(MyBid)}
\]

This part of the transformation shows in a striking way one of the main benefits of the folding operation: the saving of synchronization points. Notice that in the initial program the two bidders had to “wait” for each other. In principle they were working in parallel, but in practice they were always acting sequentially, since...
one had always to wait for the bid of the competitor. The transformation allowed us to discover this sequentiality and to obtain an equivalent program in which the sequentiality is exploited to eliminate all suspension points, which are known to be one of the major overhead sources. Furthermore, the transformation allows a drastic saving of computational space. In fact, in the initial definition the parallel composition of the two bidders leads to the construction of two lists containing all the bids done so far. After the transformation we have a definition which does not build the list any longer, and which, by exploiting a straightforward optimization can employ only constant space.

Concerning the syntax of the operation, in our setting the folding operation reduces to a mere replacement. To people familiar with this operation, this might seem restrictive: one might wish to apply the folding also in the case that the definition to be folded contains an instance of $A$, i.e. when $d$ has the form $H ← C[A_\sigma]$ (in this case the folding operation is applicable only if $\sigma$ satisfies specific conditions described in [Tamaki and Sato 1984] for logic programs and in [Etalle and Gabelli 1996] for CLP). This extended operation would actually correspond to the (most) usual definition of folding as in [Etalle and Gabbrielli 1996; Bensaou and Guessarian 1998]. In our system such an extended operation is formally not needed, as it can be obtained by combining together the folding operation with the tell introduction.

In fact, assume that we would like to fold the definition

$$d : H ← C[A_\sigma] \in D_1,$$

by using the definition

$$f : B ← A \in D_0.$$

In the first place, via a tell introduction, we can modify definition $d$ as follows

$$d^* : H ← C[A \parallel \text{tell}(\tilde{X} = \tilde{X}_\sigma)],$$

Clearly, we assume here that $\tilde{X}$ and $\sigma$ fulfill the applicability conditions given in Definition 3.2. Then, via a normal folding operation we obtain

$$d^{**} : H ← C[B \parallel \text{tell}(\tilde{X} = \tilde{X}_\sigma)]$$

(provided that the applicability conditions for the folding are satisfied) which is equivalent to the definition

$$d' : H ← C[B_\sigma].$$

obtained in the case of the folding operation as defined in [Tamaki and Sato 1984; Etalle and Gabbrielli 1996; Bensaou and Guessarian 1998]. Actually, in case the constraint domain admit most general unifiers, the definition $d'$ can be obtained from $d^{**}$ by using a tell elimination operation (also in this case we assume that the applicability conditions for the tell elimination are satisfied).

For the sake of simplicity, we do not give the explicit definition of this (derived) extended folding operation and of its applicability conditions. Therefore, the occurrences of this operation in the last example of Section 6 have to be considered as a shorthands for the sequence of operations described above.
4. CORRECTNESS

Any transformation system must be useful (i.e. allow useful transformations and optimization) and – most importantly – correct, i.e., it must guarantee that the resulting program is in some sense equivalent to the one we have started with.

Having at hand the transition system in Table I, we provide now the intended semantics to be preserved by the transformation system by defining a suitable notion of “observables”. We start with the following definition which takes into account terminating and failed computations only. In the next Section we will consider also non-terminating computations. Here and in the sequel we say that a constraint \(c\) is satisfiable iff \(\mathcal{D} \models \exists \ c\).

**Definition 4.1 (Observables).** Let \(D.A\) be a CCP process. We define

\[
\mathcal{O}(D.A) = \{ \langle c, \exists \_ \text{Var}(A,c) d, ss \rangle \mid c \text{ and } d \text{ are satisfiable, and there exists a derivation } \langle D.A, c \rangle \rightarrow^* \langle D.\text{Stop}, d \rangle \}
\]

\[
\cup \{ \langle c, \exists \_ \text{Var}(A,c) d, dd \rangle \mid c \text{ and } d \text{ are satisfiable, and there exists a derivation } \langle D.A, c \rangle \rightarrow^* \langle D.B, d \rangle \not\rightarrow, B \neq \text{Stop} \}
\]

\[
\cup \{ \langle c, \text{false, ff} \rangle \mid c \text{ is satisfiable, and there exists a derivation } \langle D.A, c \rangle \rightarrow^* \langle D.B, \text{false} \rangle \}.
\]

Thus what we observe are the results of terminating computations (if consistent), abstracting from the values of the local variables in the results, and distinguishing the successful computations from the deadlocked ones (by using the termination modes \(ss\) and \(dd\), respectively). We also observe failed computations, i.e. those computations which produce an inconsistent store.

Having defined a formal semantics for our paradigm, we can now define more precisely the notion of correctness for the transformation system: we say that a transformation sequence \(D_0, \ldots, D_n\) is partially correct iff, for each agent \(A\), we have that

\[
\mathcal{O}(D_0.A) \supseteq \mathcal{O}(D_n.A)
\]

holds, that is, nothing is added to the semantics of the initial program. Dually, we say that \(D_0, \ldots, D_n\) is complete iff, for each agent \(A\), we have that

\[
\mathcal{O}(D_0.A) \subseteq \mathcal{O}(D_n.A)
\]

holds, that is, no semantic information is lost during the transformation. Finally a transformation sequence is called totally correct iff it is both partially correct and complete.

In the following we prove that the our transformation system is totally correct. As previously mentioned, for the sake of readability some proofs are only sketched and their full versions can be found in the Appendix.

The proof of this result is originally inspired by the one of Tamaki and Sato for pure logic programs [Tamaki and Sato 1984] and has retained some of its notation, in particular we also use the notions of weight and of split derivation. Of course the similarities do not go any further, as demonstrated by the fact that in our transformation system the applicability conditions of folding operation do not depend on...
the transformation history (while allowing the introduction of recursion), and that
the folding definitions are allowed to be recursive (the distinction between \(P_{\text{new}}\)
and \(P_{\text{old}}\) of [Tamaki and Sato 1984] is now superfluous).

We start with the following proposition allows us to eliminate stop agents in
programs.

**Proposition 4.2.** For any agent \(A\) and set of declarations \(D\), \(O(D.A \parallel \text{stop}) = O(D.A)\).

**Proof.** The proof follows immediately from the definition of observables by
noting that, according to rules R1-R4, the agent \(\text{stop}\) has no transition and \((D.A \parallel \text{stop}, c) \rightarrow^* (D.B \parallel \text{stop}, d)\) if \((D.A, c) \rightarrow^* (D.B, d)\), where obviously \(B \parallel \text{stop}\) is
equal to Stop iff \(B = \text{Stop}\) (recall that \(\text{Stop}\) is the generic agent containing only \(\parallel\)
and \(\text{stop}\)).

The following notion of mode will be useful to shorten the notation.

**Definition 4.3.** Let \(D_0, \ldots, D_n\) be a transformation sequence, \(A\) be an agent and \(d\) be a constraint. We define the mode \(m(A, d)\) of the agent \(A\) w.r.t. the constraint \(d\) as follows

\[
m(A, d) = \begin{cases} ss & \text{if } d \text{ is satisfiable and } A = \text{Stop} \\ dd & \text{if } d \text{ is satisfiable, } (D_0, A, d) \not\rightarrow^* \text{ and } A \neq \text{Stop} \\ ff & \text{if } d \text{ is not satisfiable} \end{cases}
\]

Note that the notion of mode does not depend on the set of declarations \(D_i\) we
are considering, that is, in the above definition we could equivalently use \(D_i\) rather
than \(D_0\). This is the content of the following.

**Proposition 4.4.** Let \(D_0, \ldots, D_n\) be a transformation sequence, \(A\) be an agent and \(d\) be a constraint. Then \((D_0, A, d) \not\rightarrow^*\) iff \((D_i, A, d) \not\rightarrow^*\), for any \(i \in [1, n]\).

**Proof.** Immediate by observing that a procedure call can be evaluated in \(D_0\) iff
it can be evaluated in \(D_i\), for any \(i \in [1, n]\).

In what follows, we are going to refer to a fixed transformation sequence \(D_0, \ldots, D_n\).

We start with the following result, concerning partial correctness.

**Proposition 4.5 (Partial Correctness).** If, for each agent \(A\), \(O(D_0, A) = O(D_i, A)\) holds then, for each agent \(A, O(D_0, A) \supseteq O(D_{i+1}, A)\).

**Proof.** (Sketch). We show that given an agent \(A\) and a satisfiable constraint \(c_i\), if there exists a derivation \(\xi = (D_{i+1}, A, c_i) \rightarrow^* (D_{i+1}, B, c_F, t)\), with \(m(B, c_F) \in \{ss, dd, ff\}\), then there exists also a derivation \(\xi' = (D, A, c_i) \rightarrow^* (D, B', c'_F, t)\) with \(\exists_{\text{Var}(A, c_i)} c'_F = \exists_{\text{Var}(A, c_i)} c_F\) and \(m(B', c'_F) = m(B, c_F)\). By Definition 4.1, this will imply the thesis. The proof is by induction on the length \(l\) of the derivation.

\((l = 0)\). In this case \(\xi = (D_{i+1}, A, c_i)\). By the definition \((D_{i+1}, A, c_i)\) is also a
derivation of length 0 and then the thesis holds.

\((l > 0)\). If the first step of derivation \(\xi\) does not use rule R4, then the proof
follows from the inductive hypothesis.

Now, assume that the first step of derivation \(\xi\) uses rule R4 and let \(d' \in D_{i+1}\)
be the declaration used in the first step of \(\xi\). If \(d'\) was not modified in the transform-
}
inductive hypothesis. We assume then that \( d' \notin D_1 \); \( d' \) is then the result of the transformation operation applied to obtain \( D_{i+1} \). The proof proceeds by distinguishing various cases according to the operation itself. Here we consider only the operations of unfolding, tell elimination, tell introduction and folding. The other cases are deferred to the Appendix.

**Unfolding:** If \( d' \) is the result of an unfolding operation then proof is immediate.

**Tell elimination and introduction:** If \( d' \) is the result of a tell elimination or of a tell introduction the thesis follows from a straightforward analysis of the possible derivations which use \( d \) or \( d' \). First, observe that for any derivation which uses a declaration \( H \leftarrow C[tell(\bar{s} = \bar{t}) \parallel B] \), we can construct another derivation such that the agent \( tell(\bar{s} = \bar{t}) \) is evaluated before \( B \). Moreover for any constraint \( c \) such that \( \exists_{dom(\sigma)} c = \exists_{dom(\sigma)} c \sigma \), (where \( \sigma \) is a relevant most general unifier of \( \bar{s} \) and \( \bar{t} \)), there exists a derivation step \( \langle D_iB_1\sigma, c \sigma \rangle \rightarrow \langle D_iB_2\sigma, c' \sigma \rangle \) if and only if there exists a derivation step \( \langle D_iB_1, c \land (\bar{s} = \bar{t}) \rangle \rightarrow \langle D_iB_2, c'' \rangle \), where, for some constraint \( e \), \( c' = \sigma \), \( c'' = e \land (\bar{s} = \bar{t}) \) and therefore \( c' = \exists_{dom(\sigma)} c'' \). Finally, since by definition \( \sigma \) is idempotent and the variables in the domain of \( \sigma \) do not occur neither in \( C[\] \) nor in \( H \), for any constraint \( e \) we have that \( \exists_{-Var(A,c)} e \sigma = \exists_{-Var(A,c)} (e \land (\bar{s} = \bar{t})) \).

**Folding:** If \( d' \) is the result of a folding then let
- \( d: \quad q(\bar{r}) \leftarrow C[H] \) be the folded declaration \((\in D_i)\),
- \( f: \quad p(\bar{X}) \leftarrow H \) be the folding declaration \((\in D_0)\),
- \( d': \quad q(\bar{r}) \leftarrow C[p(\bar{X})] \) be the result of the folding operation \((\in D_{i+1})\) where, by hypothesis, \( Var(d) \cap Var(\bar{X}) \subseteq Var(H) \) and \( Var(H) \cap Var(\bar{X}) \subseteq Var(H) \). In this case \( \xi = \langle D_{i+1}, C[q(\bar{r})], c_i \rangle \rightarrow \langle D_{i+1}, C[p(\bar{X})], c_i \rangle \rightarrow^* \langle D_{i+1}B, c_f \rangle \) and we can assume, without loss of generality, that \( Var(C[q(\bar{r})], c_i) \cap Var(H) = \emptyset \).

By the inductive hypothesis, there exists a derivation

\[
\chi = \langle D_iC_i[C[p(\bar{X})] \parallel tell(\bar{v} = \bar{r}), c_i] \rightarrow^* \langle D_iB''', c''_f \rangle, \quad (1)
\]

with \( \exists_{-Var(C_i[C[p(\bar{X})] \parallel tell(\bar{v} = \bar{r}), c_i]} c''_f = \exists_{-Var(C_i[C[p(\bar{X})] \parallel tell(\bar{v} = \bar{r}), c_i] c_f} \) and

\[
m(B''', c''_f) = m(B, c_f). \quad (2)
\]

Since \( Var(C_i[q(\bar{v})], c_i) \subseteq Var(C_i[C[p(\bar{X})] \parallel tell(\bar{v} = \bar{r}), c_i] \), we have that

\[
\exists_{-Var(C_i[q(\bar{v})], c_i]} c''_f = \exists_{-Var(C_i[q(\bar{v}), c_i] c_f}. \quad (3)
\]

Since by hypothesis for any agent \( A' \), \( O(D_0A') = O(D_iA') \), there exists a derivation

\[
\zeta_0 = \langle D_0C_0[C[p(\bar{X})] \parallel tell(\bar{v} = \bar{r}), c_i] \rightarrow^* \langle D_0B_0, c_0 \rangle
\]

such that \( \exists_{-Var(C_i[C[p(\bar{X})] \parallel tell(\bar{v} = \bar{r}), c_i]} c_0 = \exists_{-Var(C_i[C[p(\bar{X})] \parallel tell(\bar{v} = \bar{r}), c_i] c_f} \) and \( m(B_0, c_0) = m(B'', c''_f). \)

By (1), (2) and since \( Var(C_i[q(\bar{v})], c_i) \subseteq Var(C_i[C[p(\bar{X})] \parallel tell(\bar{v} = \bar{r}), c_i] \), we have that

\[
\exists_{-Var(C_i[q(\bar{v}), c_i] c_0 = \exists_{-Var(C_i[q(\bar{v}), c_i] c_f} \) and \( m(B_0, c_0) = m(B, c_f). \quad (3)
\]

Let \( f': \quad p(\bar{X}') \leftarrow H' \) be an appropriate renaming of \( f \), which renames only the variables in \( \bar{X} \), such that \( Var(d) \cap Var(f') = \emptyset \) (note that this is possible, since

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Var(\(H\)) \cap (Var(\overline{r}) \cup Var(C)) \subseteq Var(\overline{X})

By hypothesis, Var(\(C'[\overline{v}]\), \(c_i\)) \cap Var(\(H\)) = \emptyset. Then, without loss of generality we can assume that Var(\(\xi\)) \cap Var(\(f'\)) \neq \emptyset if and only if the procedure call \(p(\overline{X})\) is evaluated, in which case declaration \(f'\) is used.

Thus there exists a derivation
\[ \langle D_0, C_i[C[H' \parallel \text{tell}(\overline{X} = \overline{X'})] \parallel \text{tell}(\overline{v} = \overline{r})], c_i \rangle \rightarrow^* \langle D_0, B_0', c_0 \rangle, \]

where \(m(B_0', c_0) = m(B_0, c_0)\). By (3) we have
\[ m(B_0', c_0') = m(B, c_f). \quad (4) \]

We now show that we can substitute \(H\) for \(H'\) \parallel \text{tell}(\overline{X} = \overline{X'})\) in the previous derivation. Since \(f' : p(\overline{X'}) \leftarrow H'\) is a renaming of \(f : p(\overline{X}) \leftarrow H\), the equality \(\overline{X} = \overline{X'}\) is conjunction of equations involving only distinct variables. Then, by replacing the variables \(\overline{X}\) and \(\overline{X'}\) in the previous derivation we obtain the derivation \(\chi_0 = \langle D_0, C_i[C[H \parallel \text{tell}(\overline{X} = \overline{X})] \parallel \text{tell}(\overline{v} = \overline{r})], c_i \rangle \rightarrow^* \langle D_0, B_0', c_0 \rangle\)

where \(\exists \neg \text{Var}(C_i[C[H \parallel \text{tell}(\overline{X} = \overline{X})] \parallel \text{tell}(\overline{v} = \overline{r})]), c_i \rangle \rightarrow^* (D_0, B_0, c_0)\) which performs exactly the same steps of \(\chi_0\), (possibly) except for the evaluation of \(\text{tell}(\overline{X} = \overline{X})\), and such that \(\exists \neg \text{Var}(C_i[C[H] \parallel \text{tell}(\overline{v} = \overline{r})]), c_i \rangle \rightarrow^* \langle D_0, B_0, c_0 \rangle\) and \(m(B_0, c_0) = m(B_0', c_0)\). From (5), (6) and since \(Var(C_i[q(\overline{v})], c_i) \subseteq Var(C_i[C[H] \parallel \text{tell}(\overline{v} = \overline{r})], c_i)\), it follows that
\[ m(B_0, c_0) = m(B, c_f) \quad \text{and} \quad m(B_0', c_0') = m(B', c_f). \quad (7) \]

Since \(O(D_0, A') = O(D, A')\) holds by hypothesis for any agent \(A'\), there exists a derivation
\[ \langle D_0, C_i[C[H] \parallel \text{tell}(\overline{v} = \overline{r})], c_i \rangle \rightarrow^* \langle D_0, B', c_f \rangle \]

where
\[ \exists \neg \text{Var}(C_i[C[H] \parallel \text{tell}(\overline{v} = \overline{r})], c_f) \rightarrow^* \langle D_0, B_0, c_0 \rangle, \]

and \(m(B', c_f') = m(B_0, c_0)\).

Finally, since \(d : q(\overline{r}) \leftarrow C[H] \in D_0\), there exists a derivation
\[ \epsilon' = \langle D_0, C_i[q(\overline{v})], c_i \rangle \rightarrow \langle D_0, C_i[C[H] \parallel \text{tell}(\overline{v} = \overline{r})], c_i \rangle \rightarrow^* \langle D_0, B', c_f \rangle \]

and then the thesis follows from (8). 

\[ \square \]
In order to prove total correctness we need the following.

**Definition 4.6 (Weight).** Let $\xi$ be a derivation. We denote by $wh(\xi)$ the number of derivation steps in $\xi$ which use rule $R2$. Given an agent $A$ and a pair of satisfiable constraints $c,d$, we then define the **success weight** $w_{ss}(A,c,d)$ of the agent $A$ w.r.t. the constraints $c$ and $d$ as follows

$$w_{ss}(A,c,d) = \min \{ n \mid n = wh(\xi) \text{ and } \xi \text{ is a derivation} \langle D_0, A, c \rangle \rightarrow^* \langle D_0, B, d' \rangle \not\leftrightarrow$$

$$\text{with } B \neq \text{Stop and } \exists_{\neg Var(A,c)} d' = \exists_{\neg Var(A,c)} d \}$$

Analogously, we define the **deadlock weight** $w_{dd}(A,c,d)$ of the agent $A$ w.r.t. the constraints $c$ and $d$

$$w_{dd}(A,c,d) = \min \{ n \mid n = wh(\xi) \text{ and } \xi \text{ is a derivation} \langle D_0, A, c \rangle \rightarrow^* \langle D_0, B, d' \rangle \not\leftrightarrow$$

$$\text{with } B \neq \text{Stop and } \exists_{\neg Var(A,c)} d' = \exists_{\neg Var(A,c)} d \}$$

and the **failure weight** $w_{ff}(A,c,d')$ of the agent $A$ w.r.t. the constraints $c$ and $d'$

$$w_{ff}(A,c,d') = \min \{ n \mid n = wh(\xi) \text{ and } \xi \text{ is a derivation} \langle D_0, A, c \rangle \rightarrow^* \langle D_0, B, d' \rangle \not\leftrightarrow$$

$$\text{with } d' = \text{false} \}$$

Notice that $w_{ss}(A,c,d')$ is undefined in case there is no successful derivation corresponding to the given constraints (and analogously for $w_{dd}$ and $w_{ff}$). Also, both $w_{ss}(A,c,\text{false})$ and $w_{dd}(A,c,\text{false})$ are undefined, as the success and deadlock weight consider only non failed derivations (i.e. derivations which do not produce the constraint false).

As previously mentioned, this notion of weight is rather different from the one in [Tamaki and Sato 1984], since the latter is based on the number of nodes in a proof tree for an atom, by taking into account the fact that the predicate symbol appearing in that atom is “new” or “old”.

In the total correctness proof we also make use of the concept of **split derivations**. Intuitively, these are derivations which can be split into two parts: the first one, up to the first ask evaluation, is performed in the program $D_i$ while the second one is carried out in $D_0$.

**Definition 4.7 (Split Derivation).** Let $D_0, \ldots, D_i$ be a transformation sequence. We call a derivation in $D_i \cup D_0$ a **successful split derivation** if it has the form

$$\langle D_i, A_1, c_1 \rangle \rightarrow^* \langle D_i, A_m, c_m \rangle \rightarrow \langle D_0, A_{m+1}, c_{m+1} \rangle \rightarrow^* \langle D_0, \text{Stop}, c_n \rangle \not\leftrightarrow$$

where $c_m$ is a satisfiable constraint, $m \in [1,n]^5$ and the following conditions hold:

(a) the first $m - 1$ derivation steps do not use rule $R2$;

(b) the $m$-th derivation step $\langle D_i, A_m, c_m \rangle \rightarrow \langle D_0, A_{m+1}, c_{m+1} \rangle$ uses rule $R2$;

(c) $w_{ss}(A_1, c_1, c_n) > w_{ss}(A_{m+1}, c_{m+1}, c_n)$.

A **deadlocked split derivation** is defined analogously, by replacing $w_{ss}$ for $w_{dd}$ and $\text{Stop}$ for a generic agent $B \neq \text{Stop}$ in the last configuration of the derivation above.

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5If $m = n$ we can write indifferently $\langle D_i, \text{Stop}, c_n \rangle$ or $\langle D_0, \text{Stop}, c_n \rangle$ to denote the last configuration of the derivation.

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Finally a failed split derivation is defined by replacing \( w_{ss} \) for \( w_{ff} \) and \( \text{Stop} \) for a
generic agent (which is not necessarily terminated) and by assuming that \( c_n = \text{false} \)
in the last configuration of the derivation above.

In the following we call split derivations both successful, deadlocked and failed
split derivations. The previous definition is inspired by the definition of descent clause of [Kawamura and Kanamori 1988]; however, here we use a different notion of weight and rather different conditions on them. We need one first concept.

**Definition 4.8.** We call the program \( D_i \) weight complete iff, for any agent \( A \), for
any satisfiable constraint \( c \) and for any constraint \( d \), the following hold: if there
exists a derivation

\[
(D_0,A,c) \rightarrow^* (D_0,B,d)
\]

such that \( m(B,d) \in \{ ss, dd, ff \} \) then there exists a split derivation in \( D_i \cup D_0 \)

\[
(D_i,A,c) \rightarrow^* (D_0,B',d')
\]

where \( \exists_{-\text{Var}(A,c)} d' = \exists_{-\text{Var}(A,c)} d \) and \( m(B',d') = m(B,d) \).

So \( D_i \) is weight complete if we can reconstruct the semantics of \( D_0 \) by using only (successful, deadlocked and failed) split derivations in \( D_i \cup D_0 \). We now show that if \( D_i \) is weight complete then no observables are lost during the transformation (i.e.,
the transformation is complete). This is the content of the following.

**Proposition 4.9.** If \( D_i \) is weight complete then, for any agent \( A \), \( O(D_0,A) \subseteq O(D_i,A) \).

**Proof.** We consider only the case of successful derivations, since the case of
deadlocked (failed) derivations can be proved analogously by considering the notions of deadlock (failure) weight and deadlocked (failed) split derivation. Assume that there exists a (finite, successful) derivation \( (D_0,A,c) \rightarrow^* (D_0,\text{Stop},d) \).

We show, by induction on the success weight of \( (A,c,d) \), that there exists a derivation

\[
(D_i,A,c) \rightarrow^* (D_i,\text{Stop},d'),
\]

where \( \exists_{-\text{Var}(A,c)} d' = \exists_{-\text{Var}(A,c)} d \).

**Base Case.** If \( w_{ss}(A,c,d) = 0 \) then, since \( D_i \) is weight complete, from Definition 4.7
and Definition 4.8 it follows that there exists a (successful) split derivation in \( D_i \cup D_0 \)
of the form \( (D_i,A,c) \rightarrow^* (D_i,\text{Stop},d') \) where \( \exists_{-\text{Var}(A,c)} d' = \exists_{-\text{Var}(A,c)} d \), rule \( \text{R2} \) is not used and therefore each derivation step is done in \( D_i \).

**Inductive Case.** Assume that \( w_{ss}(A,c,d) = n \). Since \( D_i \) is weight complete there exists a (successful) split derivation in \( D_i \cup D_0 \)

\[
\xi : (D_i,A,c) \rightarrow^* (D_0,\text{Stop},d'),
\]

where \( \exists_{-\text{Var}(A,c)} d' = \exists_{-\text{Var}(A,c)} d \). If rule \( \text{R2} \) is not used in \( \xi \) then the proof is the same as in the previous case. Otherwise \( \xi \) has the form

\[
(D_i,A,c) \rightarrow^* (D_i,A_m,c_m) \rightarrow (D_0,A_{m+1},c_{m+1}) \rightarrow^* (D_0,\text{Stop},d')
\]

where \( w_{ss}(A,c,d') > w_{ss}(A_{m+1},c_{m+1},d') \). Let \( \xi' \) be the derivation

\[
\xi' : (D_i,A,c) \rightarrow^* (D_i,A_m,c_m) \rightarrow (D_i,A_{m+1},c_{m+1}).
\]
By the inductive hypothesis, there exists a derivation 
\[ \xi'' : \langle D_i, A_{m+1}, c_{m+1} \rangle \rightarrow^* \langle D_i, \text{Stop}, d'' \rangle \]
where \( \exists_{\text{Var}(A_{m+1}, c_{m+1})} d'' = \exists_{\text{Var}(A_{m+1}, c_{m+1})} d' \). Without loss of generality, we can assume that \( \text{Var}(\xi') \cap \text{Var}(\xi'') = \text{Var}(A_{m+1}, c_{m+1}) \) and hence there exists a derivation 
\[ \langle D_i, A, c \rangle \rightarrow^* \langle D_i, \text{Stop}, d'' \rangle. \]

Finally, by our hypothesis on the variables and by construction,
\[
\exists_{\text{Var}(A, c)} d'' = \\
\exists_{\text{Var}(A, c)} (c_{m+1} \land \exists_{\text{Var}(A_{m+1}, c_{m+1})} d'') = \\
\exists_{\text{Var}(A, c)} d = \\
\exists_{\text{Var}(A, c)}
\]
which concludes the proof. □

Before proving the total correctness result we need some technical lemmata. Here and in the following we use the notation \( w_t \) (with \( t \in \{\text{ss}, \text{dd}, \text{ff} \} \)) as a shorthand for indicating the success weight \( w_{\text{ss}} \), the deadlock weight \( w_{\text{dd}} \) and the failure weight \( w_{\text{ff}} \).

**Lemma 4.10.** Let \( q(\overline{r}) \rightarrow H \in D_0 \), \( t \in \{\text{ss}, \text{dd}, \text{ff} \} \) and let \( C[ ] \) be a context. For any satisfiable constraint \( c \) and for any constraint \( c' \), such that \( \text{Var}(C[q(\overline{t})], c) \cap \text{Var}(\overline{t}) = \emptyset \) and \( w_t(C[q(\overline{t})], c, c') \) is defined, there exists a constraint \( d' \) such that \( w_t(C[q(\overline{t})] \| \text{tell}(\overline{t} = \overline{r}), c, d') \leq w_t(C[q(\overline{t})], c, c') \) and \( \exists_{\text{Var}(C[q(\overline{t})], c)} d' = \exists_{\text{Var}(C[q(\overline{t})], c)} c' \).

**Proof.** Immediate. □

**Lemma 4.11.** Let \( q(\overline{r}) \rightarrow H \in D_0 \) and \( t \in \{\text{ss}, \text{dd}, \text{ff} \} \). For any context \( C[ ] \), any satisfiable constraint \( c \) and for any constraint \( c' \), the following holds.

1. If \( \text{Var}(H) \cap \text{Var}(C_1, c) \subseteq \text{Var}(\overline{r}) \) and \( w_t(C_1[q(\overline{r})], c, c') \) is defined, then there exists a constraint \( d' \), such that \( \text{Var}(d') \subseteq \text{Var}(C_1[H], c) \), \( w_t(C_1[H], c, d') \leq w_t(C_1[q(\overline{r})], c, c') \) and \( \exists_{\text{Var}(C_1[q(\overline{r})], c)} d' = \exists_{\text{Var}(C_1[q(\overline{r})], c)} c' \).

2. If \( \text{Var}(H) \cap \text{Var}(C_1, c) \subseteq \text{Var}(\overline{r}) \), \( \text{Var}(c') \cap \text{Var}(\overline{r}) \subseteq \text{Var}(C_1[H], c) \) and \( w_t(C_1[H], c, c') \) is defined, then there exists a constraint \( d' \), such that \( w_t(C_1[q(\overline{r})], c, d') \leq w_t(C_1[H], c, c') \) and \( \exists_{\text{Var}(C_1[q(\overline{r})], c)} d' = \exists_{\text{Var}(C_1[q(\overline{r})], c)} c' \).

**Proof.** Immediate. □

The following Lemma is crucial in the proof of completeness.

**Lemma 4.12.** Let \( 0 \leq i \leq n \), \( t \in \{\text{ss}, \text{dd}, \text{ff} \} \), \( \text{cl} : q(\overline{r}) \rightarrow H \in D_i \), and let \( c' : q(\overline{r}) \rightarrow H' \) be the corresponding declaration in \( D_{i+1} \) (in the case \( i < n \)). For any context \( C[ ] \) and any satisfiable constraint \( c \) and for any constraint \( c' \) the following holds:

1. If \( \text{Var}(H) \cap \text{Var}(C_1, c) \subseteq \text{Var}(\overline{r}) \) and \( w_t(C_1[q(\overline{r})], c, c') \) is defined, then there exists a constraint \( d' \), such that \( \text{Var}(d') \subseteq \text{Var}(C_1[H], c) \), \( w_t(C_1[H], c, d') \leq w_t(C_1[q(\overline{r})], c, c') \) and \( \exists_{\text{Var}(C_1[q(\overline{r})], c)} d' = \exists_{\text{Var}(C_1[q(\overline{r})], c)} c' \);
∃-Var(Cₗ[H], c) ⊆ Var(∅), Var(c’) ∩ Var(∅) ⊆ Var(Cₗ[H], c) and
wₗ(Cₗ[H], c, c’) is defined, then there exists a constraint d’, such that Var(d’) ⊆ Var(Cₗ[H], c, c’) and
∃-Var(Cₗ[∅], c) d’ = ∃-Var(Cₗ[∅], c) c’.

Proof. (Sketch). Observe that, for i = 0, the proof of 1 follows from the first part of Lemma 4.11. We prove here that, for each i ≥ 0,
a). If 1 holds for i then 2 holds for i;
b). If 1 and 2 hold for i then 1 holds for i + 1.

The proof of the Lemma then follows from straightforward inductive argument.
a). If cl was not affected by the transformation step from Dᵢ to Dᵢ₊₁ then the result is obvious by choosing d’ = ∃-Var(Cₗ[∅], c) c’.
Assume that cl is affected when transforming Dᵢ to Dᵢ₊₁. We have various cases according to the operation used to perform the transformation. Here we show only the proofs for the unfolding and the folding operations, the other cases being deferred to the Appendix.

Unfolding: Assume c’ ∈ Dᵢ₊₁ was obtained from Dᵢ by unfolding. In this case, the situation is the following:
- cl : q(∅) ← C[p(∅)] ∈ Dᵢ,
- u : p(∅) ← B ∈ Dᵢ,
- c’ : q(∅) ← C[B || tell(∅ = 5)] ∈ Dᵢ₊₁

where cl and u are assumed to be renamed so that they do not share variables.
Let n = wₗ(Cₗ[C[p(∅)]], c, c’). By the definition of transformation sequence, there exists a declaration p(∅) ← B₀ ∈ D₀. Moreover, by the hypothesis on the variables, Var(Cₗ[C[p(∅)]], C[B || tell(∅ = 5)]) ∩ Var(Cₗ[c]) ⊆ Var(∅) and then Var(Cₗ[C[p(∅)]], c) ∩ Var(∅) = ∅. Therefore, by Lemma 4.10, there exists a constraint d₁, such that
wₗ(Cₗ[C[p(∅)] || tell(∅ = 5)], c, d₁) ≤ wₗ(Cₗ[C[p(∅)]], c, c’) = n
(9)

and
∃-Var(Cₗ[C[p(∅)]], c) d₁ = ∃-Var(Cₗ[C[p(∅)]], c) c’.
(10)

By the hypothesis on the variables and since u is renamed apart from cl, Var(B) ∩ Var(Cₗ, c, c) = ∅ and therefore Var(B) ∩ Var(Cₗ, c) is empty. Then, by Point 1, there exists a constraint d’, such that
Var(d’) ⊆ Var(Cₗ[C[B || tell(∅ = 5)]], c)
wₗ(Cₗ[C[B || tell(∅ = 5)]], c, d’) ≤ wₗ(Cₗ[C[p(∅)] || tell(∅ = 5)], c, d₁)
∃-Var(Cₗ[C[p(∅)] || tell(∅ = 5)], c) d’ = ∃-Var(Cₗ[C[p(∅)] || tell(∅ = 5)], c) d₁.

By (9), wₗ(Cₗ[C[B || tell(∅ = 5)]], c, d’) ≤ n. Furthermore, by hypothesis and construction,
Var(c’, d’) ∩ Var(∅) ⊆ Var(Cₗ[C[p(∅)]], c)

and, without loss of generality, we can assume that
Var(d₁) ∩ Var(∅) ⊆ Var(Cₗ[C[p(∅)]], c).

Then, by (10) and since Var(Cₗ[C[p(∅)]], c) ⊆ Var(Cₗ[C[p(∅)] || tell(∅ = 5)], c), we have that ∃-Var(Cₗ[∅], c) d’ = ∃-Var(Cₗ[∅], c) c’ and this completes the proof.

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Folding: Let
\[ \text{- cl : } q(\overline{r}) \leftarrow C[B] \text{ be the folded declaration (} \in D_i, \right. \]
\[ \text{- f : } p(X) \leftarrow B \text{ be the folding declaration (} \in D_0, \right. \]
\[ \text{- cl' : } q(\overline{r}) \leftarrow C[p(\overline{X})] \text{ be the result of the folding operation (} \in D_{i+1}, \right. \]
where, by hypothesis, \( \text{Var(cl)} \cap \text{Var(\bar{X})} \subseteq \text{Var(B)}, \text{Var(B)} \cap \text{Var(\bar{r}, C)} \subseteq \text{Var(\bar{X})}, \)
\( \text{Var(C[B], C[p(\overline{X})])} \subseteq \text{Var(\bar{r}, C)}, \text{Var(c')} \cap \text{Var(\bar{r})} \subseteq \text{Var(C_i[C[B]], c)} \)
and there exists \( n \) such that \( w_i(C_i[C[B]], c, c') = n. \) Then,
\[ \text{Var(B)} \cap \text{Var(C_i[C[B]], c)} \subseteq \text{Var(B)} \cap \text{Var(\bar{r}, C)} \subseteq \text{Var(\bar{X})} \]
and
\[ \text{Var(c')} \cap \text{Var(\bar{r})} \subseteq \text{Var(C_i[C[B]], c)} \cap \text{Var(\bar{r})} \subseteq \text{Var(C_i[C[B]], c)} \]
hold. Moreover, we can assume without loss of generality that \( \text{Var(c')} \cap \text{Var(\bar{X})} \subseteq \text{Var(C_i[C[B]], c)}. \)
Since \( f \in D_0, \) from (11) and Point 2 of Lemma 4.11 it follows that there exists a constraint \( d' \) such that \( w_i(C_i[C[p(\overline{X})]], c, d') \leq w_i(C_i[C[B]], c, c') \)
\[ \exists \text{- Var(C_i[C[p(\overline{X})]], c)} d' = \exists \text{- Var(C_i[C[p(\overline{X})]], c)} c'. \] (13)
We can assume, without loss of generality, that \( \text{Var(d')} \subseteq \text{Var(C_i[C[p(\overline{X})]], c)}. \) Then, by using (12) and (13) we obtain that \( \exists \text{- Var(C_i[q(\overline{r})], c)} d' = \exists \text{- Var(C_i[q(\overline{r})], c)} c' \) which concludes the proof of a).

b). Assume that the parts 1 and 2 of this Lemma hold for \( i \geq 0. \) We prove that
1 holds for \( i + 1 > 0. \)
Let \( cl : q(\overline{r}) \leftarrow H \in D_{i+1}, \) and let \( cl : q(\overline{r}) \leftarrow \bar{H} \) be the corresponding declaration in \( D_i. \) Moreover let \( C_i[\ ] \) be a context, \( c \) a satisfiable constraint and let \( c' \) be a constraint, such that \( \text{Var(H)} \cap \text{Var(C_i, c)} \subseteq \text{Var(\bar{r})} \) and \( w_i(C_i[q(\overline{r})], c, c') \) is defined. Without loss of generality, we can assume that \( \text{Var(H)} \cap \text{Var(C_i, c)} \subseteq \text{Var(\bar{r})}. \) Then, since by inductive hypothesis, part 1 holds for \( i, \) there exists a constraint \( d_1 \) such that
\[ \text{Var(d_1)} \subseteq \text{Var(C_i[H]), c, c'), \}
\[ w_i(C_i[H], c, d_1) \leq w_i(C_i[q(\overline{r})], c, c') \] and \( \exists \text{- Var(C_i[q(\overline{r})], c)} d_1 = \exists \text{- Var(C_i[q(\overline{r})], c)} c'. \) (14)
Since by inductive hypothesis part 2 holds for \( i, \) there exists a constraint \( d' \), such that \( \text{Var(d')} \subseteq \text{Var(C_i[H], c)}, \text{Var(C_i[H], c, d')} \leq w_i(C_i[H], c, d_1) \) and \( \exists \text{- Var(C_i[q(\overline{r})], c)} d' = \exists \text{- Var(C_i[q(\overline{r})], c)} d_1. \) By (14), \( w_i(C_i[H], c, d') \leq w_i(C_i[q(\overline{r})], c, c') \) and \( \exists \text{- Var(C_i[q(\overline{r})], c)} d' = \exists \text{- Var(C_i[q(\overline{r})], c)} c'. \) and then the thesis follows.

We finally obtain our first main theorem.

THEOREM 4.13 (TOTAL CORRECTNESS). Let \( D_0, \ldots, D_n \) be a transformation sequence. Then, for any agent \( A, O(D_0,A) = O(D_n,A). \)

PROOF. (Sketch). The proof proceeds by showing simultaneously, by induction on \( i, \) that for \( i \in [0, n]: \)
(1) for any agent \( A, O(D_0,A) = O(D_i,A); \)
(2) \( D_i \) is weight complete.

Base case. We just need to prove that \( D_0 \) is weight complete. Assume that there exists a derivation \( \langle D_0,A, q_i \rangle \rightarrow^* \langle D_0,B, c_r \rangle, \) where \( q_i \) is a satisfiable constraint and
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Without loss of generality, we can assume that \( \chi \) where \( d \) also assume that the declaration 
there exists a split derivation 
\( \xi = \langle D_{i-1}, A, c_i \rangle \rightarrow^* \langle D_{i-1}, A_m, c_m \rangle \rightarrow \langle D_0, A, c_i \rangle \rightarrow^* \langle D_0, B'', c''_F \rangle \)
which performs the same steps of \( \chi \) and then the thesis holds.

Otherwise, assume without loss of generality that \( R4 \) is the rule used in the first step of derivation \( \chi \) and that \( d \) is the clause employed in the first step of \( \chi \). We also assume that the declaration \( d \) is used only once in \( \chi \), since the extension to the general case is immediate.

We have to distinguish various cases according to what happens to the clause \( d \) when moving from \( D_{i-1} \) to \( D_i \). As before, we consider here only the unfolding and the folding cases, the others being deferred to the Appendix.

**Unfolding:** Assume that \( d \) is unfolded and let \( d' \) be the corresponding declaration in \( D_i \). The situation is the following:
- \( -d' : q(\overline{t}) \leftarrow C[p(\overline{t})] \in D_{i-1}, \)
- \( -u : p(\overline{s}) \leftarrow H \in D_{i-1}, \) and
- \( -d'' : q(\overline{t}) \leftarrow C[H \parallel \text{tell}(\overline{t} = \overline{s})] \in D_i, \)

where \( d \) and \( u \) are assumed to be renamed apart. By the definition of split derivation, \( \chi \) has the form

\[
\langle D_{i-1}, C_i[q(\overline{v})], c_i \rangle \rightarrow \langle D_{i-1}, C_i[p(\overline{t})] \parallel \text{tell}(\overline{v} = \overline{t}), c_i \rangle \rightarrow^* \langle D_{i-1}, A_m, c_m \rangle \rightarrow \langle D_0, A_{m+1}, c_{m+1} \rangle \rightarrow \langle D_0, B'', c''_F \rangle.
\]

Without loss of generality, we can assume that \( \text{Var}(\chi) \cap \text{Var}(u) \neq \emptyset \) if and only if \( p(\overline{t}) \) is evaluated in the first \( m \) steps of \( \chi \), in which case \( u \) is used for evaluating it.

We have to distinguish two cases.

1) There exists \( k < m \) such that the \( k \)-th derivation step of \( \chi \) is the procedure call \( p(\overline{t}) \). In this case \( \chi \) has the form

\[
\langle D_{i-1}, C_i[q(\overline{v})], c_i \rangle \rightarrow \langle D_{i-1}, C_i[p(\overline{t})] \parallel \text{tell}(\overline{v} = \overline{t}), c_i \rangle \rightarrow^* \langle D_{i-1}, C_k[p(\overline{f})], c_k \rangle \rightarrow \langle D_{i-1}, C_k[H \parallel \text{tell}(\overline{t} = \overline{s}), c_k \rangle \rightarrow^* \langle D_{i-1}, A_m, c_m \rangle \rightarrow \langle D_0, A_{m+1}, c_{m+1} \rangle \rightarrow \langle D_0, B'', c''_F \rangle.
\]

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Then there exists a corresponding derivation in $D_i \cup D_0$

$$\xi = \langle D_i, C_i[q(\bar{v})], \bar{c}_i \rangle \rightarrow \langle D_i, C_i[\bar{c}[H \parallel \text{tell}(\bar{t} = \bar{s})] \parallel \text{tell}(\bar{v} = \bar{r})], \bar{c}_i \rangle \rightarrow^* \langle D_i, C_i[H \parallel \text{tell}(\bar{t} = \bar{s})], \bar{c}_i \rangle \rightarrow^* \langle D_i, A_m, \bar{c}_m \rangle \rightarrow \langle D_0, A_{m+1}, \bar{c}_{m+1} \rangle \rightarrow^* \langle D_0, B''', \bar{c}_F'' \rangle,$$

which performs exactly the same steps of $\chi$ except for a procedure call to $p(\bar{t})$. In this case the proof follows by observing that, since by the inductive hypothesis $\chi$ is a split derivation, the same holds for $\xi$.

2) There is no procedure call to $p(\bar{t})$ in the first $m$ steps. Therefore $\chi$ has the form

$$\langle D_{i-1}, C_i[q(\bar{v})], \bar{c}_i \rangle \rightarrow \langle D_{i-1}, C_i[p(\bar{t})] || \text{tell}(\bar{v} = \bar{r}), \bar{c}_i \rangle \rightarrow^* \langle D_{i-1}, C_m[p(\bar{t})], \bar{c}_m \rangle \rightarrow \langle D_0, C_{m+1}[p(\bar{t})], \bar{c}_m \rangle \rightarrow^* \langle D_0, B''', \bar{c}_F'' \rangle.$$

Then, by the definition of $D_i$, there exists a derivation

$$\xi_0 = \langle D_{i-1}, C_i[q(\bar{v})], \bar{c}_i \rangle \rightarrow \langle D_{i-1}, C_i[H || \text{tell}(\bar{t} = \bar{s})] || \text{tell}(\bar{v} = \bar{r}), \bar{c}_i \rangle \rightarrow^* \langle D_{i-1}, C_m[H || \text{tell}(\bar{t} = \bar{s})], \bar{c}_m \rangle \rightarrow \langle D_0, C_{m+1}[H || \text{tell}(\bar{t} = \bar{s})], \bar{c}_m \rangle.$$

Observe that from the derivation $\langle D_0, C_{m+1}[p(\bar{t})], \bar{c}_m \rangle \rightarrow^* \langle D_0, B''', \bar{c}_F'' \rangle$ and (15) it follows that

$$w_t(C_{m+1}[p(\bar{t})], \bar{c}_m, c_F')$$

and

$$\exists_{\bar{c}} \exists_{\bar{c}_m} \exists_{\bar{c}_F'} \left( m(B', c_F') \right).$$

Therefore, by the definition of $w_t$, by (17) and since $w_t(C_{m+1}[p(\bar{t})], \bar{c}_m, c_F')$ is defined, there exists a derivation

$$\xi_1 = \langle D_0, C_{m+1}[H || \text{tell}(\bar{t} = \bar{s})], \bar{c}_m \rangle \rightarrow^* \langle D_0, B', c_F' \rangle,$$

where $\exists_{\bar{c}} \exists_{\bar{c}_m} \exists_{\bar{c}_F'} = \exists_{\bar{c}} \exists_{\bar{c}_m} \exists_{\bar{c}_F'} \left( m(B', c_F') \right)$ and, by (16),

$$m(B', c_F') = m(B, c_F).$$

By (18)

$$\exists_{\bar{c}} \exists_{\bar{c}_m} \exists_{\bar{c}_F'} = \exists_{\bar{c}} \exists_{\bar{c}_m} \exists_{\bar{c}_F'}$$

holds and, by definition of weight, we obtain

$$w_t(C_{m+1}[H || \text{tell}(\bar{t} = \bar{s})], \bar{c}_m, c_F') = w_t(C_{m+1}[H || \text{tell}(\bar{t} = \bar{s})], \bar{c}_m, d_r).$$

Moreover, we can assume without loss of generality that $\text{Var}(\xi_0) \cap \text{Var}(\xi_1) = \text{Var}(C_{m+1}[H || \text{tell}(\bar{t} = \bar{s})], \bar{c}_m)$. Then, by the definition of procedure call

$$\text{Var}(C_i[q(\bar{v})], \bar{c}_i) \cap \left( \text{Var}(c_F') \cup \text{Var}(c_F') \right) \subseteq \text{Var}(C_{m+1}, \bar{c}_m)$$
and there exists a derivation
\[ \xi = \langle D_i, C_i[q(\overline{y})], c_i \rangle \rightarrow \langle D_i, C_i[H \parallel tell(\hat{t} = \hat{y})] \parallel tell(\hat{v} = \hat{t})], c_i \rangle \rightarrow^* \langle D_0, B', c_f' \rangle \]
such that the first \( n - 1 \) derivation steps do not use rule \textbf{R2} and the \( m \)-th derivation step uses the rule \textbf{R2}. Now, we have the following equalities
\[
\begin{align*}
\exists \text{var}(C_i[q(\overline{y})], c_i) c_f & = (\text{by } (22) \text{ and by construction}) \\
\exists \text{var}(C_i[q(\overline{y})], c_i) (c_m \land \exists \text{var}(C_{m+1}, c_m) c_f') & = (\text{by } (20)) \\
\exists \text{var}(C_i[q(\overline{y})], c_i) (c_m \land \exists \text{var}(C_{m+1}, c_m) c_f') & = (\text{by } (22) \text{ and by construction}) \\
\exists \text{var}(C_i[q(\overline{y})], c_i) c_f' & = (\text{by the first statement in } (15))
\end{align*}
\]
By the definition of weight, \( w_t(C_i[q(\overline{y})], c_i, c_f') = w_t(C_i[q(\overline{y})], c_i, c_f') \), by (21) and (17), \( w_t(C_{m+1}[H \parallel tell(\hat{t} = \hat{y})], c_m, c_f') \leq w_t(C_{m+1}[p(\hat{t})], c_m, c_f') \) and \( w_t(C_{m+1}[p(\hat{t})], c_m, c_f') < w_t(C_i[q(\overline{y})], c_i, c_f') \), since \( c \) is a split derivation. Therefore \( w_t(C_{m+1}[H \parallel tell(\hat{t} = \hat{y})], c_m, c_f') < w_t(C_i[q(\overline{y})], c_i, c_f') \) and then, by definition, \( \xi \) is a split derivation in \( D_i \cup D_0 \). This, together with (19), implies the thesis.

**Folding:** Assume that \( d \) is folded and let
- \( d : q(\overline{y}) \rightarrow C[H] \) be the folded declaration \((\in D_{i-1})\),
- \( f : p(X) \rightarrow H \) be the folding declaration \((\in D_0)\),
- \( d' : q(\overline{r}) \rightarrow C[p(X)] \) be the result of the folding operation \((\in D_i)\),
where, by definition of folding, \( \text{var}(d) \cap \text{var}(X) \subseteq \text{var}(H) \) and \( \text{var}(H) \cap (\text{var}(\overline{r}) \cup \text{var}(C)) \subseteq \text{var}(X) \). Since \( C[\ ] \) is a guarding context, the agent \( H \) in \( C[H] \) appears in the scope of an \textit{ask} guard. By definition of split derivation \( \chi \) has the form
\[
\langle D_{i+1}, C_i[q(\overline{y})], c_i \rangle \rightarrow \langle D_{i+1}, C_i[H \parallel tell(\overline{v} = \overline{t})], c_i \rangle \rightarrow^* \langle D_{i+1}, C_{m+1}[H], c_m \rangle \rightarrow \langle D_0, C_m[H], \text{var} \rangle \rightarrow^* \langle D_0, B'', c_f' \rangle,
\]
where \( C_m[\ ] \) is a guarding context. Without loss of generality we can assume that \( \text{var}(\chi) \cap \text{var}(X) \subseteq \text{var}(H) \). Then, from the definition of \( D_i \) it follows that there exists a derivation
\[
\xi_0 = \langle D_i, C_i[q(\overline{y})], c_i \rangle \rightarrow \langle D_i, C_i[H \parallel tell(\overline{v} = \overline{t})], c_i \rangle \rightarrow^* \langle D_0, C_{m+1}[H], c_m \rangle,
\]
which performs exactly the first \( m \) steps as \( \chi \). Since \( \langle D_0, C_{m+1}[H], c_m \rangle \rightarrow^* \langle D_0, B'', c_f' \rangle \), the definition of weight implies that \( w_t(C_{m+1}[H], c_m, c_f') \) is defined, where \( t = m(B'', c_f') \). Then, by (15), we have that
\[
t = m(B, c_f).
\]
(23)
The definitions of derivation and folding imply that \( \text{var}(H) \cap \text{var}(C_i[H]) \subseteq \text{var}(\overline{r}) \cap \text{var}(\overline{X}) \cap \text{var}(H) \cap \text{var}(\overline{r}) \subseteq \text{var}(X) \) holds. Moreover, from the assumptions on the variables, we obtain that \( \text{var}(c_f') \cap \text{var}(X) \subseteq \text{var}(H) \). Thus, from part 2 of Lemma 4.11 it follows that there exists a constraint \( d' \) such that
\[
w_t(C_{m+1}[p(X)], c_m, d') \leq w_t(C_{m+1}[H], c_m, c_f') \quad \text{and} \quad \exists \text{var}(C_{m+1}[p(X)], c_m) d' = \exists \text{var}(C_{m+1}[p(X)], c_m) c_f'.
\]
(24) (25)
\( \xi \) from the definition of weight and the fact that \( w_t(C_{m+1}[H], c_m, c_f') \) is defined it follows that there exists a derivation \( \xi_1 = \langle D_0, C_{m+1}[p(X)], c_m \rangle \rightarrow^* \langle D_0, B', c_f' \rangle \), where
Thus concluding the proof.

\( D \) is a derivation in \( \text{Var}(C_{m+1}[p(X)], c_m) \). Then, by the definition of weight, \( w_t(C_{m+1}[p(X)], c_m, c'_F) = w_t(C_{m+1}[p(X)], c_m, d') \) and therefore, by (24) and (25),

\[
\exists \text{Var}(C_{m+1}[p(X)], c_m) c'_F = \exists \text{Var}(C_{m+1}[p(X)], c_m) c''_F \quad \text{and} \quad w_t(C_{m+1}[p(X)], c_m, c'_F) \leq w_t(C_{m+1}[H], c_m, c''_F) \tag{27}
\]

hold. Moreover, from (23) we obtain

\[
m(B', c'_F) = m(B, c_F). \tag{28}
\]

Without loss of generality, we can now assume that

\[
\text{Var}(\xi_0) \cap \text{Var}(\xi_1) = \text{Var}(C_{m+1}[p(X)], c_m)
\]

Then, by (26), (27) and (15) it follows that

\[
\exists \text{Var}(C_{i}[q(\tilde{v})], c_i) c'_F = \exists \text{Var}(C_{i}[q(\tilde{v})], c_i) (c_m \wedge \exists \text{Var}(C_{m+1}[p(X)], c_m) c'_F) = \\
\exists \text{Var}(C_{i}[q(\tilde{v})], c_i) c''_F = \exists \text{Var}(C_{i}[q(\tilde{v})], c_i) c_F.
\tag{29}
\]

From the definition of weight \( w_t(C_{i}[q(\tilde{v})], c_i, c'_F) = w_t(C_{i}[q(\tilde{v})], c_i, c''_F) \) and since \( \chi \) is a split derivation we obtain \( w_t(C_{i}[q(\tilde{v})], c_i, c''_F) > w_t(C_{m+1}[H], c_m, c''_F) \). Then, from (29) it follows that

\[
w_t(C_{i}[q(\tilde{v})], c_i, c'_F) > w_t(C_{m+1}[p(X)], c_m, c''_F) \tag{30}
\]

and therefore, by construction,

\[
\xi = \langle D, C_{i}[q(\tilde{v})], c_i \rangle \rightarrow \langle D, C_{i}[C[p(X)] \ | \ \text{tell}(\tilde{v} = \tilde{v}), c_i \rangle \rightarrow^* \langle D, C_{m}[p(\tilde{X})], c_m \rangle \rightarrow \langle D_{0}, C_{m+1}[p(\tilde{X})], c_m \rangle \rightarrow^* \langle D_{0}, B', c'_F \rangle
\]

is a derivation in \( D_{0} \cup D_{0} \) such that: (a) rule \textbf{R2} is not used in the first \( m - 1 \) steps; (b) rule \textbf{R2} is used in the \( m \)-th step. The thesis then follows from (29), (28) and (30) thus concluding the proof. \( \square \)

It is important to notice that – given the definition of observables we are adopting (Definition 4.1) – the initial program \( D_0 \) and the final one \( D_n \) have exactly the same successful derivations, the same deadlocked derivations and the same failed derivations. The first feature (regarding successful derivations) is to some extent the one we expect and require from a transformation, because it corresponds to the intuition that \( D_n \) “produces the same results” as \( D_0 \). Nevertheless, also the second feature (preservation of deadlock derivations) has an important role. Firstly, it ensures that the transformation does not introduce deadlock points, which is of crucial importance when we are using the transformation for optimizing a program. Secondly, as exemplified in the Section 6, this feature allows us to use the transformation as a tool for proving deadlock freeness (i.e., absence of deadlock). In fact, if, after the transformation we can prove or see that the process \( D_n, A \) does never deadlock, then we are also sure that \( D_0, A \) does not deadlock either.
5. CORRECTNESS FOR NON-TERMINATING COMPUTATIONS

The correctness results obtained so far consider terminating (successful and dead-locked) and failed computations only. This is satisfactory for many applications of concurrent constraint programming which have a “transformational” behaviour, i.e. which are supposed to produce a (finite) output for a given (finite) input. In this respect, it is worth noting that the two main semantic models of CCP consider essentially the same notion of observables we used. In fact, the model based on linear sequences defined in [de Boer and Palamidessi 1991] characterizes (in a fully abstract way) the results of terminating computations, together with a termination mode indicating success, deadlock or failure\(^6\). Such a model has been proved ([de Boer and Palamidessi 1992]) to be isomorphic to the semantics based on (bounded) closure operators introduced in [Saraswat et al. 1991], provided that the termination mode and the consistency checks are eliminated.

So, our correctness results are adequate in the sense that they ensure that the standard semantics of CCP is preserved. On the other hand, as in the case of any other concurrent programming paradigm, CCP programs may have a “reactive” nature: rather than producing a final result they produce a (possibly non-terminating) sequence of intermediate results in response to some external stimuli. For these programs the notion of observables employed in Theorem 4.13 and the related results are not adequate, since they exclude non-terminating computations.

When considering non-terminating computations one is interested in observing (possibly in terms of traces) the intermediate results, that is the constraints produced also by non-maximal derivations, rather than the final limit of the computation (note however that in CCP such a notion of limit makes sense, as the store grows monotonically). Therefore, in the remainder of this section we first discuss the correctness of our system w.r.t. this new class of observables. Then, we show a modification of our transformation system and we present a stronger correctness result, which guarantees that (traces of) intermediate results are preserved.

5.1 Partial preservation of intermediate results

It is easy to see that the system we have proposed does not preserve the intermediate results of computations. More precisely, let us define these observables as follows:

\[
\mathcal{O}_i(D.A) = \{ (c, \exists \_\text{Var}(A,c)d, \text{pp}) \mid c \text{ and } d \text{ are satisfiable, and there exists a derivation } (D.A, c) \rightarrow^* (D.B, d) \}
\]

(the symbol \text{pp} indicates here that we consider results obtained from “partial”, that is, possibly not maximal, derivations). Now, it is easy to see that the operations of ask and tell simplification are neither partially nor totally correct w.r.t. the semantics \(O_i(D.A)\). In fact, the ask simplification allows one to transform the agent

\[
A : \text{tell}(c) \parallel \text{ask(true)} \rightarrow \text{tell(d)}
\]

\(^6\)There are irrelevant differences between the observables considered in [de Boer and Palamidessi 1991] and the ones we used, due to the treatment of failure and to the existential quantification on local variables.
into the agent
\[ A' : \text{tell}(c) \parallel \text{ask}(c) \rightarrow \text{tell}(d). \]

While the agent \( A \), when evaluated in the empty store, produces the intermediate result \( d \), this is not the case for the agent \( A' \) (we assume that \( c \wedge d \neq d \)). Analogously, assuming that \( D \models d \rightarrow c \) and \( D \models d \rightarrow c' \), the tell simplification allows one to transform
\[ B : \text{tell}(c) \parallel \text{tell}(d) \]
into the agent
\[ B' : \text{tell}(c') \parallel \text{tell}(d) \]
and the agents \( B \) and \( B' \) have different intermediate results. Other operations which are not correct w.r.t. the above semantics are the distribution and the tell elimination and introduction.

Nevertheless, the system we have defined does preserve already a form of intermediate results. This is shown by the following theorem.

**Theorem 5.1 (Total Correctness 2).** Let \( D_0, \ldots, D_n \) be a transformation sequence, and \( A \) be an agent.

- If there exists a derivation \( \langle D_0, A, c \rangle \rightarrow^* \langle D_0, B, d \rangle \) then there exists a derivation \( \langle D_n, A, c \rangle \rightarrow^* \langle D_n, B', d' \rangle \) such that \( D \models \exists_{\text{Var}(A,c)}d' \rightarrow \exists_{\text{Var}(A,c)}d \).

- Converse, if there exists a derivation \( \langle D_n, A, c \rangle \rightarrow^* \langle D_n, B, d \rangle \) then there exists a derivation \( \langle D_0, A, c \rangle \rightarrow^* \langle D_0, B', d' \rangle \) with \( D \models \exists_{\text{Var}(A,c)}d' \rightarrow \exists_{\text{Var}(A,c)}d \).

**Proof.** The proof of this result is essentially the same as that one of the total correctness Theorem 4.13 provided that in such a proof, as well as in the proofs of the related preliminary results, we perform the following changes:

1. Rather than considering terminating derivations, we consider any (possibly non-maximal) finite derivation.
2. Whenever in a proof we write that, given a derivation \( \xi \), a derivation \( \xi' \) is constructed which performs the same steps \( \xi \) does, possibly in a different order, we now write that a derivation \( \xi'' \) is constructed which performs the same step of \( \xi \) (possibly in a different order) plus some other additional steps. Since the store grows monotonically in CCP derivations, clearly if a constraint \( c \) is the result of the derivation \( \xi \), then a constraint \( c'' \) is the result of \( \xi'' \) such that \( D \models c'' \rightarrow c \) holds. For example, for case 2 in the proof of Proposition 4.5, when considering a (non-maximal) derivation \( \xi \) which uses the declaration \( H \leftarrow C[\text{tell}(\tilde{s} = \tilde{t})] \parallel B \) we can always construct a derivation \( \xi'' \) which performs all the steps of \( \xi \) (possibly plus others) and such that the \( \text{tell}(\tilde{s} = \tilde{t}) \) agent is evaluated before \( B \). Differently from the previous proof, now we are not ensured that the result of \( \xi \) is the same as that one of \( \xi'' \), since \( \xi \) is non-maximal (thus, \( \xi \) could also avoid the evaluation of \( \text{tell}(\tilde{s} = \tilde{t}) \)). However, we are ensured that the result of \( \xi'' \) is stronger (i.e. implies) that one of \( \xi \).
This result ensures that the original and the transformed program have the same intermediate results up to logical implication: If the evaluation of an agent in the original program produces a constraint $d$, then a constraint stronger than $d$ is produced in the transformed program and vice versa. The vice versa is important, as it ensures that the transformed program will never produce something that could not be produced by the original program, up to implication. Clearly, this result is relevant in presence of non-terminating computations (which were not covered by Theorem 4.13).

In order to maintain a consistent notation throughout the paper, the above result can be reformulated in terms of the following class of observables

$$O_{ic}(D,A) = \{ \langle c, \exists_{Var(A,c)}d, pp \rangle \mid c \text{ is satisfiable, there exists a derivation } (D,A,c) \rightarrow^* (D,B,d') \text{ and } D \models \exists_{Var(A,c)}d' \rightarrow \exists_{Var(A,c)}d \}$$

where the subscript $ic$ stands for implication closure (of intermediate results). We then have following Corollary whose proof is immediate.

**Corollary 5.2.** Let $D_0, \ldots, D_n$ be a transformation sequence. Then, for any agent $A$, $O_{ic}(D_0,A) = O_{ic}(D_n,A)$.

This result guarantees a degree of correctness which should be sufficient for many reactive programs employing non-terminating computations. In fact, when transforming a program, probably one should not expect to be able to preserve exactly each intermediate result the original program was producing.

Nevertheless, it is of interest to check if it is possible to modify the system in order to obtain stronger correctness results. We do this in the following section.

### 5.2 Full preservation of intermediate results

In this section we introduce a few restrictions on our transformation system and we prove that they guarantee the preservation of the whole sequence of intermediate results of a program.

As previously mentioned, the only operations not preserving the intermediate results are the ask and tell simplification, the distribution and the tell elimination and introduction. As it possibly appears from the example above, the problem using the ask and tell simplification lies in the fact that one can modify the arguments of ask and tell agents by taking into account (via the “produced constraint”) also the constraints introduced by tell actions appearing in the parallel context (see Definitions 3.4 and 3.6). This clearly can affect the intermediate results of the computations, since no order is imposed on the evaluation of parallel agents. This reasoning applies to the distribution operation as well.

We have then to modify the ask and tell simplification and the distribution by considering a weaker notion of “produced constraint”, which includes only those constraints which have *certainly* been produced before reaching the ask or tell agent we are simplifying. Such a notion is defined as follows.

**Definition 5.3.** Given a context $C[\ ]$ the weakest produced constraint $wpc(C[\ ])$ of $C[\ ]$ is inductively defined as follows:

- $wpc(C_1 \oplus C_2) = \min\{wpc(C_1), wpc(C_2)\}$
- $wpc(\alpha C) = \min\{\alpha, wpc(C)\}$
- $wpc(\varsigma C) = \max\{\varsigma, wpc(C)\}$
- $wpc(\exists_{\text{Var}} C) = \max\{\exists_{\text{Var}}, wpc(C)\}$
- $wpc(\bot) = \bot$
- $wpc(\top) = \top$
- $wpc(C) \leq wpc(D)$ if $C \leq D$.

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weakest produced constraint of \[ \text{tell}(c) \] is true, while the weakest produced constraint of \[ \text{tell}(c) \parallel \text{ask}(d) \rightarrow (\text{ask}(e) \rightarrow \Box) \] is \( d \land e \). We can then define the weak equivalence of two constraints within a given context \( C[ ] \) as follows.

**Definition 5.4.** Let \( c, c' \) be constraints, \( C[ ] \) be a context, and \( \bar{Z} \) be a set of variables. We say that \( c \) is *weakly equivalent to* \( c' \) *within* \( C[ ] \) and w.r.t. the variables in \( \bar{Z} \) iff \( D \models \exists_{\bar{Z}} (\text{wpc}(C[ ] \land c) \leftrightarrow \exists_{\bar{Z}} (\text{wpc}(C[ ] \land c')) \).

Using this definition we can modify the operations of ask and tell simplification and of distribution by simply replacing the context equivalence used in Definition 5.6 with the above notion of weak context equivalence. For the sake of clarity we state below the resulting definitions.

**Definition 5.5 (Restricted Ask and Tell Simplification).** Let \( D \) be a set of declarations.

1. Let \( d : H \leftarrow C[\sum_{i=1}^{n} \text{ask}(c_i) \rightarrow A_i] \) be a declaration of \( D \). Suppose that \( c'_1, \ldots, c'_n \) are constraints such that for \( j \in [1,n] \), \( c'_j \) is weakly equivalent to \( c_j \) within \( C[ ] \) and w.r.t. the variables in \( \text{Var}(C,H,A_i) \). Then we can replace \( d \) with \( d' := H \leftarrow C[\sum_{i=1}^{n} \text{ask}(c'_i) \rightarrow A_i] \) in \( D \). We call this a *restricted ask simplification* operation.

2. Let \( d : H \leftarrow C[\text{tell}(c)] \) be a declaration of \( D \). Suppose that the constraint \( c' \) is weakly equivalent to \( c \) within \( C[ ] \) and w.r.t. the variables in \( \text{Var}(C,H) \). Then we can replace \( d \) with \( d' := H \leftarrow C[\text{tell}(c')] \) in \( D \). We call this a *restricted tell simplification* operation.

**Definition 5.6 (Restricted Distribution).** Let \( D \) be a set of declarations and let

\[
d : H \leftarrow C[A \parallel \sum_{i=1}^{n} \text{ask}(c_i) \rightarrow B_i]
\]

be a declaration in \( D \). Let also \( e = \text{wpc}(C[ ]) \). The *restricted distribution* of \( A \) in \( d \) yields the definition

\[
d' : H \leftarrow C[\sum_{i=1}^{n} \text{ask}(c_i) \rightarrow (A \parallel B_i)]
\]

provided that for every constraint \( c \) such that \( \text{Var}(c) \cap \text{Var}(d) \subseteq \text{Var}(H,C) \), if \( \langle D.A, c \land e \rangle \) is productive then both the following conditions hold:

(a) There exists at least one \( i \in [1,n] \) such that \( D \models (c \land e) \rightarrow c_i \),

(b) for each \( i \in [1,n] \), either \( D \models (c \land e) \rightarrow c_i \) or \( D \models (c \land e) \rightarrow \neg c_i \).

Remark 3.12 is also sufficient for guaranteeing that the restricted distribution operation is applicable. Thus we have the following.

**Remark 5.7.** Referring to Definition 5.6. If \( A \) requires a variables which does not occur in \( H, C[ ] \), then the restricted distribution operation is applicable.
Also the tell elimination and the tell introduction operations do not preserve the intermediate results of computations. This is not due to the presence of the produced constraint, but rather to the very nature of the operation which can eliminate or introduce constraints which, via the local variables, can (temporarily) affect also the values of global variables. For example, the declaration
\[
d : p(Y) \leftarrow \text{tell}(Z = a) \parallel \text{tell}(Y = f(Z))
\]
can be transformed via a tell elimination into
\[
d' : p(Y) \leftarrow \text{tell}(Y = f(a))
\]
The evaluation of \(p(Y)\) in the empty store and using \(d\) produces the (intermediate) result \(Y = f(Z)\), while this is not the case if one uses the declaration \(d'\). We can solve this problem by simply requiring that if we eliminate a tell by applying the resulting substitution to the parallel context \(B\), then \(B\) does not contain any variable appearing the head or in the outer context. Thus we have the following.

**Definition 5.8 (Restricted Tell Elimination and Tell Introduction).** The declaration
\[
d : H \leftarrow C[\text{tell}(\tilde{X} = \tilde{X}) \parallel B]
\]
can be transformed via a restricted tell elimination into
\[
d' : H \leftarrow C[B\sigma]
\]
where \(\sigma\) is a relevant most general unifier of \(\tilde{X}\) and \(\tilde{X}\), provided that the variables in the domain of \(\sigma\) do not occur neither in \(C[\] nor in \(H\), and that \(\text{Var}(B) \cap \text{Var}(H, C) = \emptyset\). Again, this operation is applicable either when the computational domain admits a most general unifier, or when \(\tilde{X}\) and \(\tilde{X}\) are sequence of distinct variables, in which case \(\sigma\) is simply a renaming. On the other hand, the declaration
\[
d : H \leftarrow C[B\sigma]
\]
can be transformed via a restricted tell introduction into
\[
d' : H \leftarrow C[\text{tell}(\tilde{X} = \tilde{X}\sigma) \parallel B]
\]
provided that \(\sigma\) is a substitution such that \(\tilde{X} = \text{Dom}(\sigma)\) and \(\text{Dom}(\sigma) \cap (\text{Var}(C[ ], H) \cup \text{Ran}(\sigma)) = \emptyset\), and that \(\text{Var}(B) \cap \text{Var}(H, C) = \emptyset\).

At this point it is worth recalling that the tell elimination is often used for making variable bindings explicit after an unfolding operation: In fact we start from a definition of the form \(d : H \leftarrow C[p(\tilde{t})]\) and by unfolding \(p(\tilde{t})\) we end with \(d' : H \leftarrow C[B \parallel \text{tell}(\tilde{t} = \tilde{s})]\) (provided that \(p\) is defined by \(u : p(\tilde{s}) \leftarrow B\)). Then we want to eliminate \(\text{tell}(\tilde{t} = \tilde{s})\) from \(d'\) in order to perform the “parameter passing”. Since \(d\) and \(u\) are always renamed apart, clearly the additional condition of the restricted tell elimination (\(\text{Var}(B) \cap \text{Var}(H, C) = \emptyset\)) is always satisfied here. So, in general, this operation is applicable every time that \(\tilde{t}\) is an instance of \(\tilde{s}\).

We can finally define the restricted transformation system as follows.

**Definition 5.9.** A restricted transformation sequence is a sequence of programs \(D_0, \ldots, D_n\) in which \(D_0\) is a initial program and each \(D_{i+1}\) is obtained from \(D_i\) via one of the following operations: unfolding, backward instantiation, restricted...
tell elimination, restricted tell introduction, restricted ask and tell simplification, branch elimination, conservative ask elimination, restricted distribution and folding.

Clearly, the restricted transformation operations are applicable in fewer situations than their non-restricted counterparts, yet they are useful in many cases. Example 6.1 shows a case of an unfold-fold transformation sequence using only restricted operations and the other examples contain several occurrences of them. We now prove that the restricted system is correct w.r.t. the trace semantics of CCP. Here and in the following we denote by $c_1; c_2; \ldots; c_n$ a sequence of constraints, also called trace.

**Definition 5.10 (Traces).** Let $D.A$ be a CCP process. We define $O_t(D.A) =$

\[
\{ \langle c_1; c_2; \ldots; c_n, ss \rangle \mid \text{there exists a derivation } \langle D.A, d_1 \rangle \rightarrow \langle D.A_2, d_2 \rangle \rightarrow \ldots \rightarrow \langle D.\text{Stop}, d_n \rangle \\
    \text{d}_i \text{ is satisfiable for each } i \in [1, n], \\
    c_1 = d_1 \text{ and } c_j = \exists_{\text{Var}(A.c_j)}d_j \text{ for each } j \in [2, n] \}
\] 

\union

\[
\{ \langle c_1; c_2; \ldots; c_n, dd \rangle \mid \text{there exists a derivation } \langle D.A, d_1 \rangle \rightarrow \langle D.A_2, d_2 \rangle \rightarrow \ldots \rightarrow \langle D.A_n, d_n \rangle \}
\]

\[A_n \neq \text{Stop}, \text{ d}_i \text{ is satisfiable for each } i \in [1, n],
\]

\[
c_1 = d_1 \text{ and } c_j = \exists_{\text{Var}(A,c_j)}d_j \text{ for each } j \in [2, n] \}
\] 

\union

\[
\{ \langle c_1; c_2; \ldots; c_n, pp \rangle \mid \text{there exists a derivation } \langle D.A, d_1 \rangle \rightarrow \langle D.A_2, d_2 \rangle \rightarrow \ldots \rightarrow \langle D.A_n, d_n \rangle \\
    \text{d}_i \text{ is satisfiable for each } i \in [1, n], \\
    c_1 = d_1 \text{ and } c_j = \exists_{\text{Var}(A,c_j)}d_j \text{ for each } j \in [2, n] \}
\] 

\union

\[
\{ \langle c_1; c_2; \ldots; c_n, ff \rangle \mid \text{there exists a derivation } \langle D.A, d_1 \rangle \rightarrow \langle D.A_2, d_2 \rangle \rightarrow \ldots \rightarrow \langle D.A_n, d_n \rangle \\
    \text{d}_i \text{ is satisfiable for each } i \in [1, n - 1], \text{ d}_n = \text{false}
\]

\[
c_1 = d_1 \text{ and } c_j = \exists_{\text{Var}(A,c_j)}d_j \text{ for each } j \in [2, n] \}
\] 

Thus what we observe are the finite traces consisting of the constraints produced by any (possibly non-terminating) derivation. As before, we abstract from the values for the local variables in the results, and we make distinction between the successful traces (termination mode $ss$), the deadlocked ones ($dd$), the partial (i.e. possibly non maximal) traces ($pp$) and the failed ones ($ff$). Note that, due to the monotonic computational model of CCP which does not allow us to retract information from the global store, the traces we observe are monotonically increasing. That is, given a trace $c_1; c_2; \ldots; c_n$ appearing in the observables, we have that $D \models c_i \rightarrow c_j$ for each $i, j \in [1, n]$ such that $i \geq j$. Before giving the correctness result, we need one last definition.

**Definition 5.11.** We say that a trace $c_1; c_2; \ldots; c_n$ is *simulated by* a trace $d_1; d_2; \ldots; d_m$, notation $c_1; c_2; \ldots; c_n \preceq d_1; d_2; \ldots; d_m$, iff there exists $\{j_1, \ldots, j_n\} \subseteq \{1, 2, \ldots, m\}$ such that

1. $c_i = d_j$ for each $i \in [1, n]$;
2. $j_1 = 1, j_n = m$ and $j_i \leq j_k$ iff $i < k$. 

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So, a trace \( s \) is simulated by a trace \( s' \) iff they have the same first and last element and, all components appearing in \( s \) appear, in the same order, in \( s' \).

We can now state our strongest correctness result. Its proof, contained in the Appendix, follows the guidelines of that one of Theorem 4.13. In fact, the definitions of mode, weight, split derivation and weight complete program can readily be extended to consider traces and weakest produced constraints, rather than input/output pairs and produced constraints. Then it is easy to extend all the technical lemmata needed for Theorem 4.13 in order to obtain the preliminary results needed in the proof of the following.

**Theorem 5.12 (Strong Total Correctness).** Let \( D_0, \ldots, D_n \) be a restricted transformation sequence, and \( A \) be an agent.

—If \( \langle s, x \rangle \in O_t(D_0, A) \) (with \( x \in \{ss, dd, pp, ff\} \)) then there exists \( \langle s', x \rangle \in O_t(D_n, A) \) such that \( s \preceq s' \).

—Conversely, if \( \langle s, x \rangle \in O_t(D_n, A) \) then there exists \( \langle s', x \rangle \in O_t(D_0, A) \) such that \( s \preceq s' \).

As it results from the definition of \( \preceq \), we do not have exactly the equality of traces since in some traces we might introduce some intermediate steps. However, notice that these additional steps do not introduce new values, rather they can be seen as different “approximation” to obtain a given constraint, since we consider here monotonically increasing traces. This can best be explained by means of an example. Consider the following one-line program \( D_0: p(Y) \leftarrow \text{tell}(X = f(a,W)) \parallel \text{tell}(X = f(Z,b)) \parallel \text{tell}(X = Y) \). Its trace semantics \( O_t(D_0, p(Y)) \) contains \( (t, ss) \), where \( t \) is the trace \( (true; true; true; Y = f(a,b)) \). If we apply here a restricted tell evaluation to \( \text{tell}(X = Y) \) we obtain the program \( D_1: p(Y) \leftarrow \text{tell}(Y = f(a,W)) \parallel \text{tell}(Y = f(Z,b)) \). Now, \( O_t(D_1, p(Y)) \) does not contain \( t \): one cannot obtain \( Y = f(a,b) \) from \( true \) in one step. On the other hand, \( O_t(D_1, p(Y)) \) contains \( (\langle true; \exists W. Y = f(a,W); Y = f(a,b) \rangle, ss) \) and \( (\langle true; \exists Z. Y = f(Z,b); Y = f(a,b) \rangle, ss) \) and both the two traces appearing in these pairs simulate \( t \). Notice also that the intermediate results semantics is now preserved. In fact, the following is an immediate consequence of Theorem 5.12.

**Corollary 5.13.** Let \( D_0, \ldots, D_n \) be a restricted transformation sequence, and \( A \) be an agent. Then \( O_t(D_0, A) = O_t(D_n, A) \).

### 5.3 Preservation of infinite traces

It is worth noting that Theorem 5.12 can be extended to consider also infinite traces, as we show below.

In the following we indicate by \( |s| \) the length of a trace \( s \), and we say that a configuration \( \langle D, A, c_1 \rangle \) produces the trace \( c_1; c_2; \ldots ; c_n \) iff there exists a derivation \( \langle D, A, d_1 \rangle \rightarrow \langle D, A_2, d_2 \rangle \rightarrow \ldots \rightarrow \langle D, A_n, d_n \rangle \) such that \( c_1 = d_1 \) and \( c_j = \exists_{\text{Var}(A,c_1)} d_j \) for each \( j \in [2, n] \). This notion can be extended to consider infinite computations (and infinite traces) in the obvious way. We also call an infinite trace \( c_1; c_2 \ldots \) “active” iff, for any \( i \geq 1 \), there exists \( j > i \) such that \( D \models \neg (c_i \rightarrow c_j) \) (on the other hand, the implication \( D \models (c_j \rightarrow c_i) \) holds for any \( j \geq i \) when considering traces produced by CCP derivations, since they are monotonically increasing). So, an active trace is that one produced by a computation which continuously updates the
store by adding new constraints. Clearly, when considering infinite computations, one is interested mainly in those producing active traces, as the others are essentially pure loops which stop producing new results after a finite number of steps.

The essential result we use for extending Theorem 5.12 to infinite traces is the following: If a CCP configuration can produce all the finite prefixes of an infinite trace, then it can produce the infinite trace itself. The following Lemma contains a slightly stronger version of it. With a minor abuse of notation, in the following we denote by ; also the operator which concatenates traces. Thus, if \( s_i \) are traces and \( c_i \) are constraints, for \( i \in [1, n] \), then \( c_1; s_1; c_2; s_2; \ldots; s_{n-1}; c_n \) denotes the trace obtained by concatenating the \( s_i \)’s in the obvious way.

**Lemma 5.14.** Let \( D.A \) be a CCP process and \( c_0 \) be a constraint. Assume that \( \langle D.A, c_0 \rangle \) produces the (infinitely many) finite traces

\[
\begin{align*}
  &c_0 \\
  &c_0; s_{1,1}; c_1 \\
  &c_0; s_{2,1}; c_1; s_{2,2}; c_2 \\
  &c_0; s_{3,1}; c_1; s_{3,2}; c_2; s_{3,3}; c_3 \\
  &\vdots
\end{align*}
\]

where the \( c_0, c_1, c_2 \ldots \) are different constraints (i.e. for any \( i, D \models \neg(c_i \rightarrow c_{i+1}) \)) and the \( s_j \) are (finite) sub-traces such that, for each \( j \geq 1 \), the (infinite) set containing the lengths \( \{s_{1,j} |, s_{2,j} |, s_{3,j} |, \ldots \} \) admits a (finite) maximal element. Then \( \langle D.A, c_0 \rangle \) produces also the infinite trace \( c_0; s_1; c_1; s_2; s_3; c_3; \ldots \) where, for each \( j \geq 1 \), \( s_i = s_{i,j} \) for some \( i \geq 1 \).

**Proof.** The proof uses the Koenig Lemma and the fact that the transition system defining the CCP operational semantics is finitely branching.

Let us denote by \( m_j \) the maximal element appearing in the set \( \{s_{1,j} |, s_{2,j} |, s_{3,j} |, \ldots \} \), for each \( j \geq 1 \), that is, \( m_j \) is the maximal length of the sub-traces \( s_{i,j} \) for a fixed \( j \) and \( i = 1, 2, \ldots \). We now construct a tree \( T \) representing the (infinitely many) finite traces

\[
\begin{align*}
  &c_0 \\
  &c_0; s_{1,1}; c_1 \\
  &c_0; s_{2,1}; c_1; s_{2,2}; c_2 \\
  &c_0; s_{3,1}; c_1; s_{3,2}; c_2; s_{3,3}; c_3 \\
  &\vdots
\end{align*}
\]

produced by \( \langle D.A, c_0 \rangle \). The nodes of the tree \( T \) are labeled by configurations of the form \( \langle D.B, c_i \rangle \), for some \( i \), and the edges are labeled by the sub-traces \( s_{i,j} \). More precisely, the tree \( T \) is defined inductively as follows:

(Base step). The root (level 0) of \( T \) is labeled by \( \langle D.A, c_0 \rangle \). For each derivation of the form \( \langle D.A, c_0 \rangle \rightarrow^* \langle D.A_{i,1}, c_1 \rangle \) which performs at most \( m_1 + 1 \) transition steps and which produces the trace \( c_0; s_1 \) we add a son \( N \) of the root (at level 1) labeled by \( \langle D.A_{i,1}, c_1 \rangle \) and an edge, labeled by \( s_{1,1} \), connecting the root and \( N \).

(Inductive step). Assume that \( T \) has depth \( n - 1 \) and let \( \langle D.A_{i,n-1}, c_{n-1} \rangle \) be a configuration labeling a node \( N \) at level \( n - 1 \). For each derivation of the form \( \langle D.A_{i,n-1}, c_{n-1} \rangle \rightarrow^* \langle D.A_{i,n}, c_n \rangle \) which performs at most \( m_n + 1 \) transition steps
we add a son \(N'\) of \(N\) labeled by \(\langle D, A, n, c_n \rangle\) and we add an edge labeled by \(s_{n, n}\), connecting \(N\) and \(N'\).

Note that the number of the configurations \(\langle D, A, n, c_n \rangle\) obtained in this way is finite, since we allow at most \(m_n + 1\) transition steps. Therefore we construct a finitely branching tree.

On the other hand, such a tree contains infinitely many nodes, as it contains all the (different) constraints \(c_i\) with \(i \geq 1\). Then, from the Koenig Lemma it follows that the tree contains an infinite branch and this, by construction of the tree, implies that \(\langle D, A, c_0 \rangle\) produces the infinite trace

\[
s_0; c_1; s_1; c_2; s_2; c_3; s_3; \ldots
\]

where, for each \(j \geq 1\), \(s_j = s_{i, j}\) for some \(i \geq 1\).

We also need the following Lemma.

**Lemma 5.15.** Let \(D_0, \ldots, D_n\) be a restricted transformation sequence, and \(A\) be an agent. If \(\langle D_0, A, c_0 \rangle\) produces the trace

\[
s_0; c_1; s_1; c_2; s_2; \ldots; c_m; s_m,
\]

where the \(c_i\) are different constraints and the \(s_i\) are sub-traces of constraints all equal to \(c_{i-1}\), then \(\langle D_n, A, c_0 \rangle\) produces the trace

\[
s_0; s'_1; c_1; s'_2; c_2; \ldots; s'_m; c_m,
\]

such that, for any \(i \in [1, m]\), there exists \(k_i\) such that \(|s'_i| \leq |s_i| + k_i\). Furthermore, the vice versa (obtained by exchanging \(D_0\) with \(D_n\) in the previous statement) holds as well.

**Proof.** The first part follows from Theorem 5.12. The part concerning the length is a direct consequence of the definition of the transformation sequence, since each transformation operation can at most add or delete a finite number of computation steps.

We then obtain the following extension of Theorem 5.12. Here we consider the obvious extension of the relation \(\preceq\) to the case of infinite traces.

**Theorem 5.16.** Let \(D_0, \ldots, D_n\) be a restricted transformation sequence and \(A\) be an agent.

\(\langle D_0, A, c_0 \rangle\) produces the infinite active trace \(s\), then \(\langle D_n, A, c_0 \rangle\) produces an infinite trace \(s'\) such that \(s \preceq s'\).

Conversely, if \(\langle D_n, A, c_0 \rangle\) produces the infinite active trace \(s\), then \(\langle D_0, A, c_0 \rangle\) produces an infinite trace \(s'\) such that \(s \preceq s'\).

**Proof.** Assume that \(\langle D_0, A, c_0 \rangle\) produces the infinite active trace

\[
t : c_0; s_1; c_1; s_2; c_2; s_3; c_3; \ldots
\]

where, in order to simplify the notation, we assume that the \(c_i\) are different constraints while the \(s_i\) are sequences of constraints all equal to \(c_{i-1}\) (so the \(s_i\) are sequences of stuttering steps). Clearly, by definition of produced sequence, \(\langle D_0, A, c_0 \rangle\)
produces also the (infinitely many) finite prefixes of \( t \)

\[
\begin{align*}
&c_0 \\
&c_0; s_1; c_1 \\
&c_0; s_1; c_1; s_2; c_2 \\
&c_0; s_1; c_1; s_2; c_2; s_3; c_3 \\
&\vdots
\end{align*}
\]

¿From Lemma 5.15 it follows that \( \langle D_n.A, c_0 \rangle \) produces the traces

\[
\begin{align*}
&c_0 \\
&c_0; s'_1; c_1 \\
&c_0; s'_1; c_1; s'_2; c_2 \\
&c_0; s'_1; c_1; s'_2; c_2; s'_3; c_3 \\
&\vdots
\end{align*}
\]

where, for any \( j \geq 1 \), there exists \( k_j \) such that for any \( i \in [1, j] \) we have that \(|s'_i| \leq |s_j| + k_j\). Therefore the set \( \{ |s_1|, |s_2|, |s_3|, \ldots \} \) admits a (finite) maximal element for each \( j \). Lemma 5.14 then implies that \( \langle D_n.A, c_0 \rangle \) produces the infinite trace \( t' : c_0; s'_1; c_1; s'_2; c_2; s'_3; c_3 \ldots \) and clearly, by construction, \( t \preceq t' \) holds. Analogously for the vice versa.

5.3.1 Preservation of Termination. The results we have presented guarantee the correctness of the transformation system w.r.t. various semantics based on produced constraints. We should mention however that these results do not imply that the system preserves non-declarative properties such as termination. In fact, in case of non-active traces (that from a certain point do not generate any new constraint), the semantics we have considered equate infinite and finite traces.

A full treatment of infinite computations is beyond the scope of this paper and is left for future work.

Nevertheless, we claim that the transformation system we have proposed here cannot introduce non-termination. That is, if the initial program, for a given configuration, does not produce any infinite computations then this is the case also for the transformed program.

We now provide a sketch of a proof of this claim by considering a specific class of declarations, and by showing the intuitive, informal, argument that indicates the proof methodology to be used for the general case.

Let us then assume that declarations does not contain mutually recursive definitions (note that mutually recursive definitions can usually be eliminated by means of unfolding). We also concentrate on the restricted system, which preserves active traces. In the following we say that a configuration \( \langle D.A, c \rangle \) terminates if it produces only finite computations, while we say that it does not terminate if it produces also at least one infinite derivation.

Let \( D_0, \ldots, D_n \) be a transformation sequence, and assume that \( \langle D_n.A, c \rangle \) has an infinite (non active)\(^7\) trace. This implies that there exists a derivation \( \xi = \)

\(^7\)In case of active traces, our result on the preservation of intermediate results guarantees the preservation of termination.
\[ \langle D_n, A, c \rangle \rightarrow \langle D_n, A_1, c_1 \rangle \rightarrow^* \ldots \rightarrow^* \langle D_n, A_j, c_j \rangle \rightarrow^* \ldots, \] where for some \( k \), for each \( i \geq k \), \( \exists \forall_{\text{Var}(A, c)} c_i = \exists \forall_{\text{Var}(A, c)} c_k \) holds. Assume also that for each \( i \in \{0, n-1\}, \langle D_i, A, c \rangle \) terminates.

It is easy to see that the only operation that might introduce non-termination is the folding one (all other operations are clearly “safe” in this respect). So the situation is the following:

\[
\begin{align*}
  d : & \quad H \leftarrow C[A'] \quad \in D_{n-1} \\
  f : & \quad B \leftarrow A' \quad \in D_0 \\
  d' : & \quad H \leftarrow C[B] \quad \in D_n
\end{align*}
\]

This operation can introduce non-termination only when it introduces recursion, i.e., when the definition of \( B \) depends on the one of \( H \). The typical case is when \( B \) and \( H \) have the same predicate and in the following, for the sake of simplicity, we assume that this is the case, so we assume that:

\[
\begin{align*}
  d : & \quad p(\bar{X}) \leftarrow C[A'] \quad \in D_{n-1} \\
  f : & \quad p(\bar{Y}) \leftarrow A' \quad \in D_0 \\
  d' : & \quad p(\bar{X}) \leftarrow C[p(\bar{Y})] \quad \in D_n
\end{align*}
\]

\( f \) From the definition of folding we have that \( C[\ ] \) is a guarding context and \( \text{Var}(A') \cap \text{Var}(C, \bar{X}) \subseteq \bar{Y} \) (\( f \) and \( d \) are suitably renamed so that the variables they have in common are only those occurring in \( A' \)). Since \( C \) is a guarding context let us assume that \( C[\ ] = C'[\sum_{i=1}^{n} \text{ask}(c_i)] \rightarrow A_i' \), where \( A_i' = C''[\ ] \) and \( C'[\ ] \) and \( C''[\ ] \) are non-guarding contexts. If the infinite computation is due to the folding operation then the derivation \( \xi \) must contain an infinite number of calls of the form \( p(\bar{Y})\sigma_i \), where, for each \( i \geq 1 \), \( \sigma_i \) is a renaming and the current the store \( d_i \) entails \( c_i \sigma_i \). Moreover, assume that \( A \) is of the form \( C_0[p(\bar{v})] \).

Now, by the definition of transformation sequence, the unfolding is the only operation which can introduce a new ask action, thus the guard \( c_i' \) in the context \( C_i' \) was certainly introduced during an unfolding operation of an agent in \( A' \) with a recursive definition (recall that \( d \) must be obtained from \( f \), thus, by unfolding \( A' \) we must obtain \( C[A'] \), and that we are restricting to the case of direct recursion). Therefore \( A' \) must contain an atom \( q \), whose definition in \( D_0 \) is

\[
\begin{align*}
  d : & \quad q(\bar{Z}) \leftarrow D[q(\bar{W})] \quad \in D_0
\end{align*}
\]

where the weakest produced constraint of \( D \) is precisely \( c_i' \rho \), for some appropriate renaming. Notice also that all tell actions present in \( D \) can be skipped (they are always in parallel with the rest, they don’t form a guard). Because of this, by taking \( c \) as initial store, one can show that there exist an infinite derivation starting from \( \langle D_0, C_0[p(\bar{V})], c \rangle \) where, from a certain point of the derivation \( j \), the current store \( d \) satisfies \( \exists \forall_{\text{Var}(C_0[p(\bar{v})], c)} d_j = \exists \forall_{\text{Var}(C_0[p(\bar{v})], c)} c_i' \).

This is in contrast with the hypothesis made on the original program, thus showing that no new infinite computation is generated.

In the rest of the paper we are going to provide some extra examples of transformations and – in the Appendix – the technical proofs of the correctness results.
6. MORE EXAMPLES

The following example is inspired by the one in [Etalle et al. 1998]. It shows that the transformation system can be used to simplify the dynamic behavior of a program to the point that it can be used to prove deadlock freeness. All the operations used in it are of the restricted sort; the transformation preserves thus the semantics of the intermediate results as well as that of terminating derivations.

Here and in the following we say that a variable $X$ is instantiated to a term $t$ in case the current store entails $X = t$. Accordingly, we also say that an agent instantiates a variable $X$ to $t$ in case that the agent adds the constraint $X = t$ to the store. Finally, we say that $X$ is instantiated if the store entails $X = t$ for some non variable term $t$.

Example 6.1. Consider the following simple Collect-Deliver program, which uses a buffer of length one:

$$
\text{collect} \leftarrow \text{collect}(Xs) \ || \ \text{deliver}(Xs).
$$

$$
\text{collect}(Xs) \leftarrow \% \text{collects tokens and puts them in the queue } Xs
\text{ask} (\exists_X Xs = [X|Xs']) \rightarrow \text{tell}(Xs = [X|Xs']) \ || \ \text{get_token}(X) \ || \ \text{collect}(Xs')
+ \ \text{ask}(Xs = [\ ]) \rightarrow \text{stop}.
$$

$$
\text{deliver}([Y|Ys]) \leftarrow \% \text{delivers the tokens in the queue } Xs
\text{ask}(Y = \text{eof}) \rightarrow \text{tell}(Ys = [\ ]) + \ \text{ask}(Y \neq \text{eof}) \rightarrow \text{deliver_token}(Y) \ || \ \text{deliver}(Ys).
$$

The dynamic behavior of this program is not elementary. $\text{collect}(Xs)$ behaves as follows: (a) it waits until more information for the variable $Xs$ is produced, (b1) if $Xs$ is instantiated to $[X|Xs']$ (i.e. when the store entails $\exists_X Xs = [X|Xs']$) then (by using $\text{get_token}(X)$) it instantiates $X$ with the value it collects, (b2) if $Xs$ is instantiated to $[\ ]$ it stops. On the other hand, the actions $\text{deliver}(Xs)$ performs are: (a) it instantiates $Xs$ to $[Y|Ys]$ (this activates $\text{collect}(Xs)$), then (b) it waits until $Y$ is instantiated. Now there are two possibilities: (c1) if $Y$ is the end of file character then it instantiates $Ys$ to $[\ ]$ (this will also stop the collector), (c2) otherwise it delivers $Y$ (by using $\text{deliver_token}(Y)$) and proceeds with the recursive call (which will further activate $\text{collect}$).

Thus, $\text{collect-deliver}$ actually implements a communication channel with a buffer of length one, and $Xs$ is a bidirectional communication channel. Note also that proving that this program is deadlock-free is not trivial.

We now proceed with the transformation. First we unfold $\text{deliver}(Xs)$ in the body of the first definition. The result, after cleaning up the definition via a (restricted) tell elimination is:

$$
\text{collect_deliver} \leftarrow \text{collect}([Y|Ys]) \ ||
\text{ask}(Y = \text{eof}) \rightarrow \text{tell}(Ys = [\ ])
+ \ \text{ask}(Y \neq \text{eof}) \rightarrow \text{deliver_token}(Y) \ || \ \text{deliver}(Ys).
$$

Then, we unfold $\text{collect}([Y|Ys])$ in the resulting definition; we obtain

$$
\text{collect_deliver} \leftarrow
\text{collect}([Y|Ys]) \ ||
\text{ask}(Y = \text{eof}) \rightarrow \text{tell}(Ys = [\ ])
+ \ \text{ask}(Y \neq \text{eof}) \rightarrow \text{deliver_token}(Y) \ || \ \text{deliver}(Ys).
$$
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This definition can be simplified: first, by an ask simplification, we obtain.

\[
\text{collect_deliver} \leftarrow \\
\begin{align*}
( & \text{ask}(Y = \mathrm{eof}) \rightarrow \text{tell}(Ys = []) \\
& + \text{ask}(Y \neq \mathrm{eof}) \rightarrow \text{deliver_token}(Y) \parallel \text{deliver}(Ys))
\end{align*}
\]

Next, we can eliminate the branch \( \text{ask}(\text{false}) \rightarrow \text{stop} \), eliminate \( \text{tell}([Y|Ys]=[X|Xs]) \) and eliminate the \( \text{ask}((true) \); the result is

\[
\text{collect_deliver} \leftarrow \text{get_token}(Y) \parallel \text{collect}(Ys) \\
\begin{align*}
( & \text{ask}(Y = \mathrm{eof}) \rightarrow \text{tell}(Ys = []) \\
& + \text{ask}(Y \neq \mathrm{eof}) \rightarrow \text{deliver_token}(Y) \parallel \text{deliver}(Ys))
\end{align*}
\]

Then we apply the restricted distributive operation in order to bring \( \text{collect}(Ys) \) inside the scope of the \( \text{ask} \) construct. Notice that \( \text{collect}(Ys) \) requires \( Ys \). Remark 5.7 allows us to apply the operation.

\[
\begin{align*}
\text{collect_deliver} & \leftarrow \text{get_token}(Y) \parallel \text{collect}(Ys) \\
& \begin{align*}
( & \text{ask}(Y = \mathrm{eof}) \rightarrow \text{collect}(Ys) \parallel \text{tell}(Ys = []) \\
& + \text{ask}(Y \neq \mathrm{eof}) \rightarrow \text{deliver_token}(Y) \parallel \text{collect}(Ys) \parallel \text{deliver}(Ys))
\end{align*}
\end{align*}
\]

We can now fold \( \text{collect}(Ys) \parallel \text{deliver}(Ys) \), using the original definition \( \text{collect_deliver} \leftarrow \text{collect}(Xs) \parallel \text{deliver}(Xs) \). We obtain.

\[
\text{collect_deliver} \leftarrow \text{get_token}(Y) \parallel \text{collect}(Ys) \parallel \text{tell}(Ys = []) \\
\begin{align*}
( & \text{ask}(Y = \mathrm{eof}) \rightarrow \text{collect}(Ys) \parallel \text{tell}(Ys = []) \\
& + \text{ask}(Y \neq \mathrm{eof}) \rightarrow \text{deliver_token}(Y) \parallel \text{collect_deliver})
\end{align*}
\]

To clean up the result, we can now eliminate \( \text{tell}(Ys = []) \), unfold the obtained \( \text{collect}([|]) \) agent, and perform the usual clean-up operations on the result. Our final program is the simple

\[
\text{collect_deliver} \leftarrow \text{get_token}(Y) \\
\begin{align*}
( & \text{ask}(Y = \mathrm{eof}) \rightarrow \text{stop} \\
& + \text{ask}(Y \neq \mathrm{eof}) \rightarrow \text{deliver_token}(Y) \parallel \text{collect_deliver})
\end{align*}
\]

It is important to compare this to the initial program. In particular, three aspects are worth noticing.

First, that – as opposed to the initial program – the resulting one has a straightforward dynamic behavior. For instance if we consider the agent \( \text{collect_deliver} \), one can easily see it to be deadlock-free in the latter program while in the original program this is not at all immediate. After proving that the transformation does not introduce nor eliminate any deadlocking branch in the semantics of the program, we are able to state that “since the resulting program is deadlock-free then also the initial program is deadlock-free”. Thus program’s transformations can be profitably used as analysis tool: it is in fact often easier to prove deadlock freeness for a transformed version of a program than for the original one.

Secondly, that the resulting program is more efficient than the initial one: in fact it does not need to use the global store as heavily as the initial one for passing the parameters between \( \text{collect} \) and \( \text{deliver} \).
Finally, it is straightforward to check that all transformation operations used here are of the restricted kind, therefore, by the Strong Total Correctness Theorem 5.12 this transformation is correct wrt the sequence of intermediate results.

We show now an application of our methodology with a third example, containing an extended folding operation (see discussion after Definition 3.15): this is the case when the replaced agent coincides with an instance of the body of the folding definition.

Example 6.2. We consider a stream protocol problem where two input streams are merged into an output stream. An input stream consists of lines of messages, and each line has to be passed to the output stream without interruption. Input and output streams are dynamically constructed by a reader and a monitor process, respectively. A reader communicates with the monitor by means of a buffer of length one, and is synchronized in such a way that it can read a new message only when the buffer is empty (i.e., when the previous message has been processed by the monitor). On the other hand, the monitor can access a buffer only when it is not empty (i.e., when the corresponding reader has put a message into its buffer). This protocol is implemented by the following program STREAMER:

```
streamer ← reader(left,Ls) || reader(right,Rs) || monitor(Ls,Rs,idle)

reader(Channel,Xs) ←
   ask(∃X,Xs:Xs=[X|Xs']) → tell(Xs=[X|Xs']) || read(Channel,X) || reader(Channel,Xs')
   + ask(Xs=[ ]) → stop.

monitor([L|Ls],[R|Rs],State) ←
   ask(State=idle) → % waiting for an input
   (  ask(char(L)) → monitor([L|Ls],[R|Rs],left)  
      + ask(char(R)) → monitor([L|Ls],[R|Rs],right))
   + ask(State=left) → ask(char(L)) → write(L) || % processing the left stream
      (  ask(L=eof) → tell(Ls=[ ]) || onestream([R|Rs])  
         + ask(L=eol) → monitor(Ls,[R|Rs],idle)  
         + ask(L ≠ eol AND L ≠ eof) → monitor(Ls,[R|Rs],left))
   + ask(State=right) → ask(char(R)) → → . . . ) % analogously for the right stream

onestream([X|Xs]) ←
   ask(char(X)) →
   (  ask(X=eof) → tell(Xs=[ ])  
         + ask(X ≠ eof) → write(X) || onestream(Xs))
```

Here, the primitive agent `read(Channel,X)` is supposed to read an input token from channel `Channel` and instantiate `X` with the read value; similarly, `write(X)` writes the value of `X` to the (unique) output stream. The primitive constraint predicate `char` is true if its argument is either a printable (e.g. ASCII) character or if it is equal to `eol` or `eof`, which are constants denoting the end of line and the end of file characters, respectively. Furthermore, the agent `reader(Channel,Xs)` waits to process `Channel` until `Xs` is instantiated; `monitor(Ls,Rs,State)` takes care of merging `Ls` and `Rs` and of writing to the output; the agent `onestream(Xs)` takes care of handling the single stream `Xs` (when one of the streams has finished). Finally, the constants `left`, `right`
and idle describe the state of the monitor, i.e., if it is processing a message from
the left stream, from right stream, or if it is in an idle situation, respectively.

Notice that reader(\text{Channel}, \text{Xs}) suspends until \text{Xs} is instantiated and that \text{Xs} will
eventually be instantiated by the monitor process.

We can now transform the \text{STREAMER} program in order to improve its efficiency.

First we add the following new declaration to the original program.

\begin{verbatim}
handle\_two(L,R,State) ← reader(left,Ls) ∥ reader(right,Rs) ∥ \text{monitor}([L|Ls],[R|Rs],State)
\end{verbatim}

Next, we unfold the agent \text{monitor}([L|Ls],[R|Rs],State) in the new declaration and
then we perform the subsequent tell eliminations (these are restricted in virtue of
the argument presented after Definition 5.8). The result of these operations is the
following program.

\begin{verbatim}
handle\_two(L,R,State) ← ( ask(State=idle) → % waiting for an input
    ( ask(char(L)) → monitor([L|Ls],[R|Rs],left)
    + ask(char(R)) → monitor([L|Ls],[R|Rs],right))
+ ask(State=left) → ask(char(L)) → write(L) ∥ % processing the left stream
    ( ask(L=eof) → tell([Ls=[ ]]) ∥ onestream([R|Rs])
    + ask(L=eol) → monitor(Ls,[R|Rs],idle)
    + ask(L ≠ eol AND L ≠ eof) → monitor(Ls,[R|Rs],left))
+ ask(State=right) → ask(char(R)) → ... ) % analogously for the right stream
\end{verbatim}

According to Definition 3.11, the agent reader(left,Ls) requires the variable \text{Ls}
and reader(right,Rs) requires the variable \text{Rs}. By Remark 3.12 it is possible for us
to apply twice the distribution operation\(^8\) and bring them inside the ask constructs.
The result is the following program.

\begin{verbatim}
handle\_two(L,R,State) ← ( ask(State=idle) → % waiting for an input
    ( ask(char(L)) → reader(left,Ls) ∥ reader(right,Rs) ∥
        monitor([L|Ls],[R|Rs],left)
    + ask(char(R)) → reader(left,Ls) ∥ reader(right,Rs) ∥
        monitor([L|Ls],[R|Rs],right))
+ ask(State=left) → ask(char(L)) → write(L) ∥ % processing the left stream
    ( ask(L=eof) → reader(left,Ls) ∥ reader(right,Rs) ∥ tell([Ls=[ ]])
        ∥ onestream([R|Rs])
    + ask(L=eol) → reader(left,Ls) ∥ reader(right,Rs) ∥ monitor(Ls,[R|Rs],idle)
    + ask(L ≠ eol AND L ≠ eof) → reader(left,Ls) ∥ reader(right,Rs) ∥
        monitor(Ls,[R|Rs],left))
+ ask(State=right) → ask(char(R)) → ... ) % analogously for the right stream
\end{verbatim}

In this program we can now eliminate tell(Ls = [ ]) in the agent reader(left,Ls) ∥
reader(right,Rs) ∥ tell(Ls = [ ]) ∥ onestream([R|Rs]) thus obtaining\(^9\):

\begin{verbatim}
handle\_two(L,R,State) ←
\end{verbatim}

\(^8\)Remark 5.7, guarantees also in both cases it is a restricted distribution operation.
\(^9\)Again, it is true that the variable \text{Ls} here occurs also elsewhere in the definition, but since it
occurs only on choice-branches different than the one on which the considered agent lies, we can
assume it to be renamed.
\begin{verbatim}
( ask(State=idle) → % waiting for an input
  ( ask(char(L)) → reader(left,Ls) || reader(right,Rs) ||
    monitor([L|Ls],[R|Rls],left)
    + ask(char(R)) → reader(left,Ls) || reader(right,Rs) ||
    monitor([L|Ls],[R|Rls],right))
  + ask(State=left) → ask(char(L)) → write(L) || % processing the left stream
    ( ask(L= eof) → reader(left,[]) || reader(right,Rs) || onestream([R|Rls])
      + ask(L=eol) → reader(left,Ls) || reader(right,Rs) ||
      monitor(Ls,[R|Rls],idle)
      + ask(L̸=eol AND L̸=eof) → reader(left,Ls) ||
      reader(right,Rs) ||
      monitor(Ls,[R|Rls],left))
  + ask(State=right) → ask(char(R)) → ... ) % analogously for the right stream
\end{verbatim}

In this program, the unfolding of the agent reader(left,[ ]) yields as result the agent
\begin{verbatim}
ask(∃X,Xs'.[ ] = [X|Xs']) → tell([ ] = [X|Xs']) || read(Channel,X)
|| reader(Channel,Xs')
\end{verbatim}

By (trivial) guard simplification, this can become
\begin{verbatim}
ask(false) → tell([ ] = [X|Xs']) || read(Channel,X) || reader(Channel,Xs')
\end{verbatim}

+ ask(true) → stop.

Now, by using branch elimination we can eliminate the first branch and by applying
the conservative ask elimination we can transform the second branch into stop. The
application of these operations yields:
\begin{verbatim}
handle_two(L,R,State) ←
  ( ask(State=idle) → % waiting for an input
    ( ask(char(L)) → reader(left,Ls) || reader(right,Rs) ||
      monitor([L|Ls],[R|Rls],left)
      + ask(char(R)) → reader(left,Ls) || reader(right,Rs) ||
      monitor([L|Ls],[R|Rls],right))
    + ask(State=left) → ask(char(L)) → write(L) || % processing the left stream
      ( ask(L= eof) → reader(left,[]) || reader(right,Rs) || onestream([R|Rls])
        + ask(L=eol) → reader(left,Ls) || reader(right,Rs) ||
        monitor(Ls,[R|Rls],idle)
        + ask(L̸=eol AND L̸=eof) → reader(left,Ls) ||
        reader(right,Rs) ||
        monitor(Ls,[R|Rls],left))
    + ask(State=right) → ask(char(R)) → ... ) % analogously for the right stream
\end{verbatim}

We now apply the backward instantiation operation to monitor(Ls,[R|Rls],idle) and
to monitor(Ls,[R|Rls],left). By cleaning up the result with a tell elimination\footnote{This is the first operation in this example that is not restricted.},
this amounts to instantiating Ls to [L'|Ls']. Therefore, we have obtained.
\begin{verbatim}
handle_two(L,R,State) ←
  ( ask(State=idle) → % waiting for an input
    ( ask(char(L)) → reader(left,Ls) || reader(right,Rs) ||
      monitor([L|Ls],[R|Rls],left)
      + ask(char(R)) → reader(left,Ls) || reader(right,Rs) ||
      monitor([L|Ls],[R|Rls],right))
\end{verbatim}
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+ ask(State=left) → ask(char(L)) → write(L) || % processing the left stream
  (  ask(L=eof) → reader(right,Rs) || onestream([R|Rs])
  + ask(L=eol) → reader(left,[L'|Ls']) || reader(right,Rs) ||
      monitor([L'|Ls'],[R|Rs],idle)
  + ask(L̸=eol AND L̸=eof) → reader(left,[L'|Ls']) || reader(right,Rs) ||
      monitor([L'|Ls'],[R|Rs],left))
+ ask(State=right) → ask(char(R)) → . . . % analogously for the right stream

In order to prepare the program for the folding operation we need one more clean up phase: using the unfolding and some simplification operations, we can replace each call reader(left,[L'|Ls']) with read(left,L'). The result of these operations is the program:

def handle_two(L,R,State):
    (  ask(State=idle) → % waiting for an input
      (  ask(char(L)) → handle_two(L,R,left)
      + ask(char(R)) → handle_two(L,R,right)
      )
    + ask(State=left) → ask(char(L)) → write(L) || % processing the left stream
      (  ask(L=eof) → reader(right,Rs) || onestream([R|Rs])
      + ask(L=eol) → reader(left,L') || reader(left,Ls') || reader(right,Rs) ||
          monitor([L'|Ls'],[R|Rs],left)
      + ask(L̸=eol AND L̸=eof) → reader(left,L') || reader(left,Ls') || reader(right,Rs) ||
          monitor([L'|Ls'],[R|Rs],right))
    + ask(State=right) → ask(char(R)) → . . . % analogously for the right stream

We can now apply twice the extended folding operation. The first folding allows us to replace reader(left,Ls') || reader(right,Rs) || monitor([L'|Ls'],[R|Rs],right) with handle_two(L,R,left). With the second one we replace reader(left,Ls') || reader(right,Rs) || monitor([L'|Ls'],[R|Rs],idle) with handle_two(L,R,right). Recall that the extended folding operation, as described in Subsection 3.7, occurs when the replaced agent coincides with a non-trivial instance of the body of the folding definition; as already explained in the discussion after Definition 3.15 this is only a shorthand for a sequence of tell introduction, folding and tell elimination, as described in Subsection 3.7. The resulting program after these two operations is:

def handle_two(L,R,State):
    (  ask(State=idle) → % waiting for an input
      (  ask(char(L)) → handle_two(L,R,left)
      + ask(char(R)) → handle_two(L,R,right)
      )
    + ask(State=left) → ask(char(L)) → write(L) || % processing the left stream
      (  ask(L=eof) → reader(right,Rs) || onestream([R|Rs])
      + ask(L=eol) → reader(left,L') || reader(left,Ls') || reader(right,Rs) ||
          monitor([L'|Ls'],[R|Rs],left)
      + ask(L̸=eol AND L̸=eof) → reader(left,L') || reader(left,Ls') || reader(right,Rs) ||
          monitor([L'|Ls'],[R|Rs],right))
    + ask(State=right) → ask(char(R)) → . . . % analogously for the right stream

Then, we perform two more extended foldings: with the first one we replace the agent reader(left,Ls') || reader(right,Rs) || monitor([L'|Ls'],[R|Rs],idle) with the agent

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handle_two(L',R,idle), with the latter we replace reader(left,Ls') \parallel reader(right,Rs) \parallel monitor([L'[|Ls'],[R|Rs],left) with handle_two(L',R,left). The resulting program is

\[
\text{handle_two}(L,R,\text{State}) \leftarrow \begin{cases} 
\text{ask}(\text{State}=\text{idle}) & \text{% waiting for an input} \\
& ( \text{ask(char(L))} \rightarrow \text{handle_two}(L,R,\text{left}) \\
& + \text{ask(char(R))} \rightarrow \text{handle_two}(L,R,\text{right}) \\
& + \text{ask}(\text{State}=\text{left}) \rightarrow \text{ask(char(L))} \rightarrow \text{write}(L) \parallel \text{% processing the left stream} \\
& ( \text{ask}(L=\text{eof}) \rightarrow \text{reader}(\text{right},Rs) \parallel \text{onestream}([R|Rs]) \\
& + \text{ask}(L=\text{eol}) \rightarrow \text{read}(\text{left},L') \parallel \text{handle_two}(L',R,\text{idle}) \\
& + \text{ask}(L \neq \text{eof} \text{ AND } L \neq \text{eof}) \rightarrow \text{read}(\text{left},L') \parallel \text{handle_two}(L',R,\text{left}) \\
& + \text{ask}(\text{State}=\text{right}) \rightarrow \text{ask(char(R))} \rightarrow \ldots \) \text{% analogously for the right stream} 
\end{cases}
\]

Notice that now the definition of handle_two is recursive. Moreover, the above program is almost completely independent from the definition of reader. In order to eliminate the atom reader(right,Rs) as well, we use an unfold/fold transformation similar to (but simpler than) the previous one. This transformation starts with the new definition 11:

\[
\text{handle_one}(X, \text{Channel}) \leftarrow \text{reader}(\text{Channel},Xs) \parallel \text{onestream}([X|Xs])
\]

After the transformation, we end up with the definition:

\[
\text{handle_one}(X, \text{Channel}) \leftarrow \text{ask(char(X))} \rightarrow \begin{cases} 
\text{ask}(X=\text{eof}) \rightarrow \text{stop} \\
& + \text{ask}(X \neq \text{eof}) \rightarrow \text{write}(X) \parallel \text{read}(\text{Channel},X') \parallel \text{handle_one}(X',\text{Channel}) 
\end{cases}
\]

Also in this case the folding operation allows us to save computational space. In fact, the parallel composition of reader and of onestream in the original definition leads to the construction of a list containing all the data read so far. In a concurrent setting this list could easily be of unbounded size and monotonically increasing. The initial definition employs a computational space which is linear in the input. After the transformation we have a definition which does not build the list any longer, and which could be optimized to employ only constant space (this could be achieved by a using a garbage collection mechanism which allows one to re-use the space allocated for local variables).

We now continue with the last steps of our example. By folding handle_one into the last definition of handle_two, we obtain

\[
\text{handle_two}(L,R,\text{State}) \leftarrow \begin{cases} 
( \text{ask}(\text{State}=\text{idle}) & \text{% waiting for an input} \\
& ( \text{ask(char(L))} \rightarrow \text{handle_two}(L,R,\text{left}) \\
& + \text{ask(char(R))} \rightarrow \text{handle_two}(L,R,\text{right}) \\
& + \text{ask}(\text{State}=\text{left}) \rightarrow \text{ask(char(L))} \rightarrow \text{write}(L) \parallel \text{% processing the left stream} \\
& ( \text{ask}(L=\text{eof}) \rightarrow \text{handle_one}(R,\text{right}) \\
& + \text{ask}(L=\text{eol}) \rightarrow \text{read}(\text{left},L') \parallel \text{handle_two}(L',R,\text{idle}) \\
& + \text{ask}(L \neq \text{eof} \text{ AND } L \neq \text{eof}) \rightarrow \text{read}(\text{left},L') \parallel \text{handle_two}(L',R,\text{left}) \\
& + \text{ask}(\text{State}=\text{right}) \rightarrow \text{ask(char(R))} \rightarrow \ldots \) \text{% analogously for the right stream} 
\end{cases}
\]

11This definition is presented here for the sake of clarity; however recall that we assume that it is added to the original program at the beginning of the transformation.

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We now want to let streamer benefit from the improvements we have obtained via this transformation. First, we transform its definition by applying the backward instantiation to monitor(Ls, Rs, idle), and obtain:

\[
\text{streamer} \leftarrow \text{reader}(\text{left}, [L | Ls]) \parallel \text{reader}(\text{right}, [R | Rs]) \parallel \text{monitor}([L | Ls], [R | Rs], \text{idle}).
\]

Next, we unfold the two reader atoms, and eliminate the redundant ask and tell guards.

\[
\text{streamer} \leftarrow \text{read}(\text{left}, L) \parallel \text{reader}(\text{left}, Ls) \parallel \text{read}(\text{right}, R) \parallel \text{reader}(\text{right}, Rs) \parallel \text{monitor}([L | Ls], [R | Rs], \text{idle}).
\]

We can now fold handle_two in it (via an extended folding operation), obtaining:

\[
\text{streamer} \leftarrow \text{read}(\text{left}, L) \parallel \text{read}(\text{right}, R) \parallel \text{handle_two}(L, R, \text{idle}).
\]

Note that this last folding operation is applied to a non-guarding context. As discussed in Remark 3.16, we can apply the folding also in this case because the definition of streamer is never modified nor used by the transformation. So we can simply assume that the original definition of streamer contained a dummy ask guard as in

\[
\text{streamer} \leftarrow \text{ask}(\text{true}) \rightarrow ( \text{read}(\text{left}, L) \parallel \text{reader}(\text{left}, Ls) \parallel \text{read}(\text{right}, R) \parallel \text{reader}(\text{right}, Rs) \parallel \text{monitor}([L | Ls], [R | Rs], \text{idle})).
\]

Then we assume that the folding operation is applied to this definition, and that the guard ask(true) will eventually be removed by an ask elimination operation.

In the final program, we only need the definitions of streamer and of handle_two together with the ones of the built-in predicates. Observe that the definition of streamer is much more efficient than the original one. Firstly, it now benefits from a straightforward left-to-right dataflow. In the initial program the variables Ls and Rs are employed as bidirectional communication channels, in fact there exist two agents (reader and monitor) which alternate in “instantiating” them further. This is not the case in the final program, where for each variable it is clear which is the agent that is supposed to “instantiate” it (i.e. to progressively add information to the store about it). This fact implies that on the final program are possible a number of powerful compile-time (low-level) optimizations which in the first program are not possible.

Secondly, the number of suspension points is dramatically reduced: in the original program reader had to suspend and awaken itself at each input token. In the final one streamer is independent from reader and has to suspend less often.

Lastly but certainly not least, as previously mentioned streamer now does not construct the list and could be optimized to employ a constant computational space, while in its initial version it employed a space linear in the input, that is, possibly unbounded. It is worth remarking that in a concurrent setting processes are often not meant to end their computation, in which case it is of vital importance that the computational space remains bounded in size; thus in this context a space gain like the one obtained in the above example makes the difference between a viable and a non-viable definition.

Example 6.3. This is a variation on a standard example for unfold/fold transformations: a program computing the sum and the length of the elements in a list. The variation consists in the fact that we consider only the elements of the list.
which are larger than the given parameter Limit. We assume here that the constraint system being used incorporates some arithmetic domain. Therefore, in the following program we use also arithmetic constraints, with the obvious intended meaning.

\[
\text{sumlen}(Xs, \text{Limit}, S, L) \leftarrow \text{sum}(Xs, \text{Limit}, S) \parallel \text{len}(Xs, \text{Limit}, L)
\]

\[
\text{sum}(Xs, \text{Limit}, S) \leftarrow
\begin{cases}
( \text{ask}(Xs=[\emptyset]) \rightarrow \text{tell}(S=0) ) \\
+ ( \text{ask}(\exists Y, Ys. (Xs=[Y]Ys \land Y \leq \text{Limit})) \rightarrow \text{tell}(Xs=[Y]Ys) ) \\
+ ( \text{sum}(Ys, \text{Limit}, S) )
\end{cases}
\]

\[
\text{len}(Xs, \text{Limit}, L) \leftarrow
\begin{cases}
( \text{ask}(Xs=[\emptyset]) \rightarrow \text{tell}(L=0) ) \\
+ ( \text{ask}(\exists Y, Ys. (Xs=[Y]Ys \land Y \leq \text{Limit})) \rightarrow \text{tell}(Xs=[Y]Ys) ) \\
+ \text{len}(Ys, \text{Limit}, L) \\
+ ( \text{len}(Ys, \text{Limit}, L') ) \\
\text{tell}(L=L'+1))
\end{cases}
\]

With two unfoldings we obtain:

\[
\text{sumlen}(Xs, \text{Limit}, S, L) \leftarrow
\begin{cases}
( \text{ask}(Xs=[\emptyset]) \rightarrow \text{tell}(S=0) ) \\
+ ( \text{ask}(Xs=[\emptyset]) \rightarrow \text{tell}(L=0) ) \\
+ ( \text{ask}(\exists Y, Ys. (Xs=[Y]Ys \land Y \leq \text{Limit})) \rightarrow \text{tell}(Xs=[Y]Ys) ) \\
+ \text{len}(Ys, \text{Limit}, L) \\
+ \text{len}(Ys, \text{Limit}, L') \\
\text{tell}(L=L'+1))
\end{cases}
\]

We now apply the (restricted) distribution operation; in practice, we now bring one choice inside the other one.

\[
\text{sumlen}(Xs, \text{Limit}, S, L) \leftarrow
\begin{cases}
( \text{ask}(Xs=[\emptyset]) \rightarrow \text{tell}(S=0) ) \\
( \text{ask}(Xs=[\emptyset]) \rightarrow \text{tell}(L=0) ) \\
+ ( \text{ask}(\exists Y, Ys. (Xs=[Y]Ys \land Y' \leq \text{Limit})) \rightarrow \text{tell}(Xs=[Y]Ys) ) \\
+ \text{len}(Ys, \text{Limit}, L) \\
+ \text{len}(Ys, \text{Limit}, L') \\
\text{tell}(L=L'+1))
\end{cases}
\]

\[
( \text{ask}(Xs=[\emptyset]) \rightarrow \text{tell}(L=0)
\]

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It is worth noticing that the applicability conditions of Definition 3.10 are trivially satisfied thanks to the fact that both choices depend on the same variable \( X_s \). Notice also that in this case we cannot apply Remark 3.12, in fact this is an example of a distribution operation which is not possible with the tools of [Etalle et al. 1998].

By using the ask simplification followed by a branch elimination and by a conservative ask elimination we obtain the following program. Notice that the ask simplification is possible here because we can take arithmetic constraints into account.

\[
\text{sumlen}(X_s, \text{Limit}, S, L) \leftarrow \\
\begin{cases}
\text{ask}(X_s = [\_]) & \rightarrow \text{tell}(S = 0) \parallel \text{tell}(L = 0) \\
+ \text{ask}(\exists Y, Y_s (X_s = [Y | Y_s] \land Y \leq \text{Limit})) & \rightarrow \text{tell}(X_s = [Y | Y_s]) \\
\quad \text{sum}(Y_s, \text{Limit}, S') \parallel \\
\quad \text{tell}(S = S' + Y) \parallel \\
\quad \text{tell}(X_s = [Y' | Y'_s]) \parallel \\
\quad \text{len}(Y'_s, \text{Limit}, L') \parallel \\
\quad \text{tell}(L = L' + 1))
\end{cases}
\]

Via a tell simplification (first and last non-restricted operation of this example), we can transform \( \text{tell}(X_s = [Y' | Y'_s]) \) into \( \text{tell}([Y | Y_s] = [Y' | Y'_s]) \), and subsequently apply a tell elimination we obtain:

\[
\text{sumlen}(X_s, \text{Limit}, S, L) \leftarrow \\
\begin{cases}
\text{ask}(X_s = [\_]) & \rightarrow \text{tell}(S = 0) \parallel \text{tell}(L = 0) \\
+ \text{ask}(\exists Y, Y_s (X_s = [Y | Y_s] \land Y \leq \text{Limit})) & \rightarrow \text{tell}(X_s = [Y | Y_s]) \\
\quad \text{sum}(Y_s, \text{Limit}, S) \parallel \\
\quad \text{tell}(X_s = [Y' | Y'_s]) \parallel \\
\quad \text{len}(Y'_s, \text{Limit}, L) \parallel \\
\quad \text{tell}(S = S' + Y) \parallel \\
\quad \text{tell}(X_s = [Y' | Y'_s]) \parallel \\
\quad \text{len}(Y'_s, \text{Limit}, L') \parallel \\
\quad \text{tell}(L = L' + 1))
\end{cases}
\]

We can now apply the folding operation.
sumlen(Xs, Limit, S, L) ←
( ask(Xs=[[]]) → tell(S=0) || tell(L=0)
+ ask(∃Y,Ys (Xs=[Y|Ys] ∧ Y ≤ Limit)) → tell (Xs=[Y|Ys]) || sumlen(Ys, Limit, S, L)
+ ask(∃Y,Ys (Xs=[Y|Ys] ∧ Y > Limit)) → tell (Xs=[Y|Ys]) ||
    sumlen(Ys, Limit, S’, L’)
    ||
    tell(S=S’+ Y)
    ||
    tell(L=L’+ 1))

Again, we have reached a point in which the main definition is directly recursive. Moreover, the number of choice-points encountered while traversing a list is now half of what it was initially.

7. RELATED WORK

As mentioned in the introduction, this is one of the few attempts to apply fold/unfold techniques in the field of concurrent languages. In fact, in the literature we find only three papers which are relatively closely related to the present one: Ueda and Furukawa [1988] defined transformation systems for the concurrent logic language GHC [Ueda 1986], Sahlin [1995] defined a partial evaluator for AKL, while de Francesco and Santone in [1996] presented a transformation system for CCS [Milner 1989].

The transformation system we are proposing builds on the systems defined in the papers above and can be considered an extension of them. Differently from the previous cases, our system is defined for a generic (concurrent) constraint language. Thus, together with some new transformations such as the distribution, the backward instantiation and the branch elimination, we introduce also specific operations which allow constraint simplification and elimination (though, some constraint simplification is done in [Sahlin 1995] as well).

It is interesting and not straightforward to compare our system with the one of Ueda and Furukawa [1988]. This is specific for the GHC language, which has a different syntactic structure from CCP and uses the Herbrand universe as computational domain. Also because of this, [Ueda and Furukawa 1988] employs operations which are completely different from ours. In particular, our operation of unfolding is replaced by immediate execution and case splitting in [Ueda and Furukawa 1988]. Our unfolding is a weaker operation which has a broader applicability than case splitting, since the latter operation involves the moving of synchronization points and therefore requires suitable applicability conditions. Furthermore, the distribution operation is not present in [Ueda and Furukawa 1988], as it would not be possible in the syntactic structure of GHC. However, in many cases the effect of distribution can be achieved in [Ueda and Furukawa 1988] by introduction of a new clause followed by case splitting. In order to clarify this, below we report how the transformations of the Example 6.1 could be mimicked in GHC by using the operations of [Ueda and Furukawa 1988].

Example 7.1. The initial program \texttt{collect_deliver} considered in Example 6.1, in terms of the GHC syntax is

1: collect_deliver :- | collect(Xs), deliver(Xs).

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2: collect([X|Xs]) :- get_token(X), collect(Xs).
3: collect([]) :- true.

4: deliver(Ys0) :- Ys0=[Y|Ys], deliver_2(Y,Ys).
5: deliver_2(eof,Ys) :- Ys=[].
6: deliver_2(Y,Ys) :- Y\=eof | deliver_token(Y), deliver(Ys).

The presence of deliver_2 is due to the fact that GHC does not allow nested guards. The first operation to be used is an immediate execution, applied to clause (1). The result is

7: collect_deliver :- collect(Xs), Xs=[Y|Ys], deliver_2(Y,Ys).

By normalizing this, we obtain
8: collect_deliver :- collect([Y|Ys]), deliver_2(Y,Ys).

Another immediate execution operation yields
9: collect_deliver :- get_token(Y), collect(Ys), deliver_2(Y,Ys).

Now, we need to introduce a new definition.

10: collect_deliver_2(Y) :- collect(Ys), deliver_2(Y,Ys).

By applying to this the case splitting operation, we obtain

11: collect_deliver_2(eof) :- collect(Ys), Ys=[].
12: collect_deliver_2(Y) :- Y\=eof | collect(Ys), deliver_token(Y), deliver(Ys).

By normalizing clause 11, and subsequently applying an immediate execution operation, we obtain

13: collect_deliver_2(eof) :- true.

To (12) and (9) we can apply the folding operation, and the resulting program is thus

collect_deliver :- get_token(Y), collect_deliver_2(Y).
collect_deliver_2(eof) :- true.
collect_deliver_2(Y) :- Y\=eof | deliver_token(Y), collect_deliver.

collect_deliver :- get_token(Y), collect_deliver_2(Y).
collect_deliver_2(eof) :- true.
collect_deliver_2(Y) :- Y\=eof | deliver_token(Y), collect_deliver.

It is worth noting how it is possible to achieve a resulting program which is basically identical to the one of Example 6.1, despite the completely different nature of the operation used.

Compared to [Ueda and Furukawa 1988] we also provide a more flexible definition for the folding operation which allows the folding clause to be recursive (which is really a step forward in the context of folding operations which are themselves capable of introducing recursion) and frees the initial program from having to be partitioned in $P_{new}$ and $P_{old}$. In fact, as opposed to virtually all fold operations which enable to introduce recursion presented so far (the only exception being [Francesco and Santone 1996]), the applicability of the folding operation does not...
depend on the transformation history, (which has always been one of the “obscure
sides” of it) but it relies on plain syntactic criteria. The idea of using a guarded
folding in order to obtain applicability conditions independent of the transformation
history was first introduced by de Francesco and Santone [1996] in the CCS setting.
However, their technical development is rather different from ours, in particular
our correctness results and proofs are completely different from those sketched in
[Francesco and Santone 1996].

As previously mentioned, differently from our case in [Sahlin 1995] it is consid-
ered a definition of ask elimination which allows us to remove potentially selectable
branches; the consequence is that the resulting transformation system is only par-
tially (thus not totally) correct. We should mention that in [Sahlin 1995] two
preliminary assumptions on the “scheduling” are made in such a way that this
limitation is actually less constraining than it might appear.

8. CONCLUSIONS

We have introduced an unfold/fold transformation system for CCP and we have
proved its total correctness w.r.t. the input/output semantics defined by the ob-
servables $O$, which takes into account also the termination modes. This semantics
Corollaries, modulo irrelevant differences due to the treatment of failure and of
local variables) to that one proposed in [de Boer and Palamidessi 1991]. This is
one of the two fully abstract “standard” semantics for CCP, the other being that
one defined in [Saraswat et al. 1991]. (Actually, these two semantic models have
been proved to be isomorphic ([de Boer and Palamidessi 1992]), provided that the
termination mode and the consistency checks are eliminated.)

We have also shown that the proposed transformation system preserves another,
stronger semantics which takes into account the intermediate results of computa-
tions up to logical implication (Theorem 5.1). We argued that this result should be
strong enough for transforming also programs which might not terminate, in
particular for transforming reactive programs. Nevertheless, in addition to this
we have presented a restricted transformation system, obtained from the initial
one by adding some (relatively mild) restrictions on some operations. We have
shown that this second system preserves the trace semantics of programs (up to
simulation, Theorem 5.12) and therefore it is totally correct w.r.t. the semantics
$O_i$ which takes into account all the intermediate results (Corollary 5.13). We have
also proved that this system preserves active infinite computations and we claim
that, more generally, this system does not introduce in the transformed program
any new infinite computation which was not present in the original one.

As shown by the examples, this system can be used for the optimization of con-
current constraint programs both in terms of time and of space. In fact, it allows us
to eliminate unnecessary suspension points (and therefore to reduce sequentiality),
to reduce the number of communication channels and to avoid the construction of
some global data structures. The system can also be used to simplify the dynamic
behavior of a program, thus allowing us to prove directly absence of deadlock.

Concerning future work, there exist other techniques for proving deadlock freeness
for CCP programs, notably in [Codish et al. 1994] a methodology based on abstract
interpretation has been defined. It could be interesting to investigate an integration
of our methodology with abstract interpretation tools. We are also considering a formal comparison of some different transformation systems (in particular our system and that of [Ueda and Furukawa 1988]) to assess their relative strength. This task is not immediate, since the target languages are different.

A. DETAILED PROOFS

Appendix A is available only online. You should be able to get the online-only toplas from the citation page for this article:

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Transformations of CCP programs

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In this Appendix we provide the detailed proofs for the results which ensure that the transformation system we have defined is totally correct. In particular, we provide the detailed proofs for Theorems 4.13 and 5.12. In order to obtain a self contained Appendix some technical Lemmata contained also in the paper are repeated here. In what follows, we are going to refer to a fixed transformation sequence $D_0, \ldots, D_n$.

**Lemma A.1.** Assume that there exists a derivation $\langle D, C[A], c \rangle \xrightarrow{*} \langle D, C'[A], c' \rangle$ where $c$ is a satisfiable constraint and the context $C'[\ ]$ has the form $A_1 \parallel \ldots \parallel \bar{C}[\ ] \parallel \ldots \parallel A_n$ and, for each $j \in [1, n]$, $A_j$ is either a choice agent, or a procedure call or the agent $\text{Stop}$. Then $D\models (\text{pc}(\bar{C}[\ ] \land c') \rightarrow \text{pc}(C'[\ ]))$ holds and in case $\bar{C}[\ ]$ is the empty context also $D\models c' \rightarrow \text{pc}(C'[\ ])$ holds.

**Proof.** By a straightforward inductive argument it follows that if there exists a derivation $\langle D, C[A], c \rangle \xrightarrow{*} \langle D, C'[A], c' \rangle$, then $D\models (\text{pc}(C'[\ ] \land c') \rightarrow \text{pc}(C'[\ ]))$. Now, if $C'[\ ]$ has the form $A_1 \parallel \ldots \parallel \bar{C}[\ ] \parallel \ldots \parallel A_n$, where each $A_j$ is either a choice agent or a procedure call or $\text{Stop}$, then $\text{pc}(C'[\ ] \land c') = \text{pc}(C'[\ ])$ which implies $D\models (\text{pc}(C'[\ ] \land c') \rightarrow \text{pc}(C'[\ ]))$. Obviously if $\bar{C}[\ ]$ is the empty context then $\text{pc}(\bar{C}[\ ] \land c') = \text{true}$, from which the second part of the Lemma follows. \qed

We prove now Proposition 4.5.

**Proposition 4.5 (Partial Correctness).** If, for each agent $A$, $O(D_0, A) = O(D_i, A)$ then, for each agent $A$, $O(D, A) \supseteq O(D_{i+1}, A)$.

**Proof.** We now show that given an agent $A$ and a satisfiable constraint $c_i$, if there exists a derivation $\xi = \langle D_{i+1}, A, c_i \rangle \xrightarrow{*} \langle D_{i+1}, B, c_F \rangle$, with $m(B, c_F) \in \{ss, dd, ff\}$, then there exists also a derivation $\xi' = \langle D_i, A, c_i \rangle \xrightarrow{*} \langle D_i, B', c_F' \rangle$ with
\begin{align*}
\exists \forall \text{Var}(A,c)\xi' = \exists \forall \text{Var}(A,c)\xi \land m(B',\xi') = m(B,\xi). \text{ By Definition 4.1, this will imply the thesis. The proof is by induction on the length } l \text{ of the derivation.}
\end{align*}

(l = 0). In this case \(\xi = \langle D_{i+1},A,c \rangle\). By the definition \(\langle D_i,A,c \rangle\) is also a derivation of length \(0\) and then the thesis holds.

(l > 0). If the first step of derivation \(\xi\) does not use rule R4, then the proof follows from the inductive hypothesis: In fact, if \(\xi = \langle D_{i+1},A,c \rangle \rightarrow \langle D_{i+1},A_1,c_1 \rangle \rightarrow^* \langle D_{i+1},B,c_\xi \rangle\) then by the inductive hypothesis, there exists a derivation

\(\xi'' = \langle D_1,A_1,c_1 \rangle \rightarrow^* \langle D_1,B',\xi'_F \rangle\)

with \(\exists \forall \text{Var}(A_1,c_1)\xi'_F = \exists \forall \text{Var}(A,c)\xi\) and \(m(B',\xi'') = m(B,\xi_\tau)\). We can assume, without loss of generality, that \(\text{Var}(A,c) \land \text{Var}(\xi'') \subseteq \text{Var}(A_1,c_1)\). Therefore, there exists a derivation \(\xi' = \langle D_1,A,c_1 \rangle \rightarrow^* \langle D_1,B',\xi''_F \rangle\). Now, to prove the thesis it is sufficient to observe that, by the hypothesis on the variables, \(\exists \forall \text{Var}(A_1,c_1)\xi''_F = \exists \forall \text{Var}(A_1,c_1)\xi''\).

Now, assume that the first step of derivation \(\xi\) uses rule R4 and let \(d' \in D_{i+1}\) be the declaration used in the first step of \(\xi\). If \(d'\) was not modified in the transformation step from \(D_i\) to \(D_{i+1}\) (that is, \(d' \in D_i\)), then the result follows from the inductive hypothesis. We assume then that \(d' \notin D_i\). Then \(d'\) is then the result of the transformation operation applied to obtain \(D_{i+1}\), and we now distinguish various cases according to the operation itself.

**Case 1**: \(d'\) is the result of an unfolding operation.

In this case the proof is straightforward.

**Case 2**: \(d'\) is the result of a tell elimination or of a tell introduction.

In this case the thesis follows from a straightforward analysis of the possible derivations which use \(d\) or \(d'\). First, observe that for any derivation which uses a declaration \(H \leftarrow C[tell(\{s = t\} | B)]\), we can construct another derivation such that the agent \(tell(s = t)\) is evaluated before \(B\). Moreover for any constraint \(c\) such that \(\exists \text{dom}(\sigma)c = \exists \text{dom}(\sigma)c\sigma\), (where \(\sigma\) is a relevant most general unifier of \(s\) and \(t\)), there exists a derivation step \(\langle D_1,B_1(\sigma,c) \rangle \rightarrow \langle D_1,B_2,\xi' \rangle\) if and only if there exists a derivation step \(\langle D_1,B_1,\xi' \rangle \rightarrow \langle D_1,B_2,\xi'' \rangle\), where, for some constraint \(e, c' = e \land \xi'\) and therefore \(c' = \exists \text{dom}(\sigma)c''\). Finally, since by definition \(\sigma\) is idempotent and the variables in the domain of \(\sigma\) do not occur neither in \(C\) nor in \(H\), for any constraint \(e\) we have that \(\exists \forall \text{Var}(A,c)\sigma = \exists \forall \text{Var}(A,c)(e \land \xi')\).

**Case 3**: \(d'\) is the result of a backward instantiation.

Let \(d\) be the corresponding declaration in \(D_i\). The situation is the following:

- \(d : q(\bar{t}) \leftarrow C[p(\bar{t})]\)
- \(d' : q(\bar{t}) \leftarrow C[p(\bar{t})] \land \text{tell}(b) \land \text{tell}(\bar{t} = s)\)

where \(f : p(s) \leftarrow \text{tell}(b)\) when \(H \in D_i\) has no variable in common with \(d\) (the case \(d' : q(\bar{t}) \leftarrow C[p(\bar{t})] \land \text{tell}(\bar{t} = s)\) is analogous and hence omitted). In this case

\(\xi = \langle D_{i+1},C_1[q(\bar{t})],c_1 \rangle \rightarrow \langle D_{i+1},C_1[C[p(\bar{t})] \land \text{tell}(b) \land \text{tell}(\bar{t} = s)] \land \text{tell}(\bar{v} = \bar{r})] \land c_1 \rangle \rightarrow^* \langle D_{i+1},B,c_\xi \rangle\).

By the inductive hypothesis, there exists a derivation

\(\chi = \langle D_i,C_1[C[p(\bar{t})] \land \text{tell}(b) \land \text{tell}(\bar{r} = s)] \land \text{tell}(\bar{v} = \bar{r})] \land c_1 \rangle \rightarrow^* \langle D_i,B',\xi''_F \rangle\).
there exists a derivation $C$ whenever the choice agent inside $\text{ask}$ an ask action of the form $\text{ask}$. Consider also inconsistent stores resulting from non-terminated computations; (b) The proof is straightforward by noting that: (a) according to Definition 4.1 we have that $\exists - \text{Var}(C_i[q(\tilde{v})], c_i) \subseteq \text{Var}(C_i[p(\tilde{t})] \parallel \text{tell}(\tilde{t} = \tilde{s})] \parallel \text{tell}(\tilde{v} = \tilde{r})], c_i)$, we have that $\exists - \text{Var}(C_i[q(\tilde{v})], c_i) \subseteq \text{Var}(C_i[p(\tilde{t})] \parallel \text{tell}(\tilde{t} = \tilde{s})] \parallel \text{tell}(\tilde{v} = \tilde{r})], c_i)$. If $p(\tilde{t})$ is not evaluated in $\chi$, then the proof is immediate. Otherwise, by the definition of $\chi$ and since $f \in D_i$, there exists also a derivation $\chi' = \langle D_i, C_i[p(\tilde{t})] \parallel \text{tell}(\tilde{v} = \tilde{r})], c_i \rangle \rightarrow^* \langle D_i, B', c'_f \rangle$ such that $\exists - \text{Var}(C_i[C[p(\tilde{t})] \parallel \text{tell}(\tilde{v} = \tilde{r})], c_i) \subseteq \text{Var}(C_i[C[p(\tilde{t})] \parallel \text{tell}(\tilde{v} = \tilde{r})], c_i)\subseteq \text{Var}(C_i[C[p(\tilde{t})] \parallel \text{tell}(\tilde{v} = \tilde{r})], c_i)$ and $m(B', c'_f) = m(B, c_f)$. By the definition of derivation and since $d \in D_i$, $\langle D_i, C_i[p(\tilde{t})] \parallel \text{tell}(\tilde{v} = \tilde{r})], c_i \rangle \rightarrow^* \langle D_i, B', c'_f \rangle$ and the thesis follows from (33).

Case 4: $d'$ is obtained from $d$ by either an ask simplification or a tell simplification. We consider only the first case (the proof of the other one is analogous and hence it is omitted). Let $d': q(\tilde{t}) \leftarrow C[\sum_{j=1}^{n} \text{ask}(c'_j) \rightarrow A_j]$, and $d: q(\tilde{t}) \leftarrow C[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow A_j]$, where for $j \in [1, n], D' = \exists - \text{Var}(q(\tilde{t}), c_A) (pc(C[ ]) \land c_j) \leftrightarrow (pc(C[ ]) \land c'_j)$. According to the definition of $pc$ and by Lemma A.1, for any derivation $\chi$ for $\langle D_i, C_i[C[\sum_{j=1}^{n} \text{ask}(c'_j) \rightarrow A_j] \parallel \text{tell}(\tilde{v} = \tilde{r})], c_i \rangle$ there exists a derivation $\chi'$ for $\langle D_i, C_i[C[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow A_j] \parallel \text{tell}(\tilde{v} = \tilde{r})], c_i \rangle$ which performs the same steps of $\chi$ (possibly in a different order) and such that whenever the choice agent inside $C[ ]$ is evaluated the current store implies $pc(C[ ])$. Therefore the thesis follows from the above equivalence.

Case 5: $d'$ is the result of a branch elimination or of a conservative ask elimination. The proof is straightforward by noting that: (a) according to Definition 4.1 we consider also inconsistent stores resulting from non-terminated computations; (b) an ask action of the form $\text{ask}$ always succeeds.
Let $d$ be the result of a distribution operation. Let
- $\mathit{d} \colon \mathcal{F}(\bar{r}) \rightarrow \mathcal{C}(\bar{r} \parallel \sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow B_j) \in \mathcal{D}_i$
- $\mathit{d}' \colon \mathcal{F}(\bar{r}) \rightarrow \mathcal{C}(\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)) \in \mathcal{D}_{i+1}$

where $e = \mathit{pc}(\mathcal{C}[\bar{r}])$ and for every constraint $c$ such that $\mathit{Var}(c) \cap \mathit{Var}(d) \subseteq \mathit{Var}(\mathcal{F}(\bar{r}), \mathcal{C})$, if $\langle \mathcal{D}_i, \mathcal{H}, c \land e \rangle$ is productive then both the following conditions hold:

- there exists at least one $j \in [1, n]$ such that $\mathcal{D}_i \models (c \land e) \rightarrow c_j$
- for each $j \in [1, n]$, either $\mathcal{D}_i \models (c \land e) \rightarrow c_j$ or $\mathcal{D}_i \models (c \land e) \rightarrow \neg c_j$.

In this case $\xi = \langle \mathcal{D}_{i+1}, \mathcal{C}_i[q(\bar{v})], c_i \rangle \rightarrow \langle \mathcal{D}_{i+1}, \mathcal{C}_i[\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)] \parallel \mathit{tell}(\bar{v} = \bar{r}), c_i \rangle \rightarrow^{*} \langle \mathcal{D}_{i+1}, \mathcal{B}'', c''_F \rangle$. By the inductive hypothesis, there exists a derivation

$$
\chi = \langle \mathcal{D}_i, \mathcal{C}_i[\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)] \parallel \mathit{tell}(\bar{v} = \bar{r}), c_i \rangle \rightarrow^{*} \langle \mathcal{D}_i, \mathcal{B}'', c''_F \rangle
$$

with

$$
\exists \neg \mathit{Var}(\mathcal{C}_i[q(\bar{v})], c_i) \mathit{CF}' = \exists \neg \mathit{Var}(\mathcal{C}_i[\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)] \parallel \mathit{tell}(\bar{v} = \bar{r}), c_i) \mathit{CF}'
$$

and

$$
m(B'', c''_F) = m(B, c_F). \tag{34}
$$

Moreover, since $\mathit{Var}(\mathcal{C}_i[q(\bar{v})], c_i) \subseteq \mathit{Var}(\mathcal{C}_i[\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)] \parallel \mathit{tell}(\bar{v} = \bar{r}), c_i)$, we have that

$$
\exists \neg \mathit{Var}(\mathcal{C}_i[q(\bar{v})], c_i) \mathit{CF}' = \exists \neg \mathit{Var}(\mathcal{C}_i[q(\bar{v})], c_i) \mathit{CF}. \tag{35}
$$

Now, we distinguish two cases:

1. $\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)$ is not evaluated in $\chi$. In this case the proof is obvious.
2. $\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)$ is evaluated in $\chi$. We have two more possibilities:
   1. There exists $h \in [1, n]$, such that
      $$
      \chi = \langle \mathcal{D}_i, \mathcal{C}_i[\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)] \parallel \mathit{tell}(\bar{v} = \bar{r}), c_i \rangle \rightarrow^{*} \langle \mathcal{D}_i, \mathcal{C}_m[\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)], c_m \rangle \rightarrow \langle \mathcal{D}_i, \mathcal{C}_m[H \parallel B_h], c_m \rangle \rightarrow^{*} \langle \mathcal{D}_i, \mathcal{B}'', c''_F \rangle
      $$
      where $\mathcal{D}_i \models c_m \rightarrow c_h$. In this case the thesis follows immediately, since using $d$ one can obtain the agent $\mathcal{C}_m[H \parallel B_h]$ after having evaluated the choice agent in $\mathcal{C}[\bar{r}]$.
   2. There is no $h \in [1, n]$, such that $\mathcal{D}_i \models c''_F \rightarrow c_h$. In this case
      $$
      c''_F \text{ is satisfiable, } m(B'', c''_F) = dd. \tag{36}
      $$

$B''$ is the agent $\mathcal{C}_F[\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)]$ and

$$
\chi = \langle \mathcal{D}_i, \mathcal{C}_F[\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)] \parallel \mathit{tell}(\bar{v} = \bar{r}), c_i \rangle \rightarrow^{*} \langle \mathcal{D}_i, \mathcal{C}_F[\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)], c''_F \rangle
$$

From the definition of derivation, the definition of $B''$ and the hypothesis that

$\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)$ is evaluated in $\chi$, it follows that $\mathcal{C}_F[\sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j)]$

is of the form $A_1 \parallel \ldots \parallel \sum_{j=1}^{n} \mathit{ask}(c_j) \rightarrow (H \parallel B_j) \parallel \ldots \parallel A_l$, where either $A_k$ is a choice agent or $A_k = \text{Stop}$. By Lemma A.1, $\mathcal{D}_i \models c''_F \rightarrow \mathit{pc}(\mathcal{C}[\bar{r}])$ and by

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definition of derivation $\text{Var}(c'_F) \cap \text{Var}(d) \subseteq \text{Var}(q(\overline{r}), C)$. Then, since there is no $j \in [1,n]$ such that $D \models c'_F \rightarrow c_i$, by definition of distribution, $\langle D_i'H', c''_F \rangle$ is not productive. Then, by definition, $\langle D_i'H', c''_F \rangle$ has at least one finite derivation $\chi_1 = \langle D_i'H', c''_F \rangle \rightarrow^{*} \langle D_i'H', c'_F \rangle \not\models \langle D_i'H', c''_F \rangle$ such that $D \models \exists_{-2} c'_F \rightarrow \exists_{-2} c'_F$, where $\hat{Z} = \text{Var}(H)$. Moreover, since in a derivation we can add to the store only constraints on the variables occurring in the agents, $c'_F = \exists_{-\text{Var}(H), c''_F} c'_F = \exists_{-\text{Var}(H), c''_F} c'_F$ holds.

Without loss of generality, we can assume that $\text{Var}(\chi_1) \cap \text{Var}(\chi) \subseteq \text{Var}(H, c''_F)$. Therefore, by the previous observation,

$$\exists_{-\text{Var}(C_i) \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j), c'_F} c'_F = c'_D \tag{37}$$

and since $\langle D_i, C_F [\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j), c'_F] \rangle \not\models \langle D_i, C_F [\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j), c''_F] \rangle$, there exists a derivation

$$\chi' = \langle D_i, C_F [H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j] || \text{tell}(\overline{v} = \overline{r})], c_j \rightarrow^{*} \langle D_i, C_F [H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c''_F \rangle \not\models \langle D_i, C_F [H' \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c'_F \rangle \not\models$$

Moreover, since $d \in D_i$ there exists a derivation

$$\xi' = \langle D_i, C_F [\xi(v)], c_i \rightarrow (D_i, C_F [H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j] || \text{tell}(\overline{v} = \overline{r})], c_j \rightarrow^{*} \langle D_i, C_F [H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c''_F \rangle \not\models \langle D_i, C_F [H' \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c'_F \rangle \not\models$$

Finally, to prove the thesis it is sufficient to observe that from (34), (36), (37) and from the definition of $B' = C_F [H' \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j]$ it follows that $m(B', c''_F) = m(B, c_F) = dd$. Moreover

$$\exists_{-\text{Var}(C_i) \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j), c'_F} c''_F = (by \ construction)$$

$$\exists_{-\text{Var}(C_i) \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j), c'_F} c''_F = (by \ (37))$$

$$\exists_{-\text{Var}(C_i) \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j), c'_F} c''_F = (by \ (35))$$

which concludes the proof of this case.

**Case 7: $d'$ is the result of a folding.**

Let $d : q(\overline{r}) \leftrightarrow C[H]$ be the folded declaration ($\in D_i$), $f : p(\overline{X}) \leftrightarrow H$ be the folding declaration ($\in D_0$), $d' : q(\overline{r}) \leftrightarrow C[p(\overline{X})]$ be the result of the folding operation ($\in D_{i+1}$) where, by hypothesis, $\text{Var}(d) \cap \text{Var}(\overline{X}) \subseteq \text{Var}(H)$ and $\text{Var}(H) \cap \text{Var}(\overline{r}) \cap \text{Var}(C_i) \subseteq \text{Var}(\overline{X})$. In this case $\xi = \langle D_i+1, C_i[q(\overline{v})], c_i \rightarrow (D_i+1, C_i[p(\overline{X})] || \text{tell}(\overline{v} = \overline{r})], c_i \rightarrow^{*} (D_i+1, B, C_F)$ and we can assume, without loss of generality, that $\text{Var}(C_i[q(\overline{v})], c_i \cap \text{Var}(H) = \emptyset$.

By the inductive hypothesis, there exists a derivation

$$\chi = \langle D_i, C_i[C[p(\overline{X})] \parallel \text{tell}(\overline{v} = \overline{r})], c_i \rightarrow^{*} (D_i, B'', c''_F)\rangle$$

with $\exists_{-\text{Var}(C_i[C[p(\overline{X})] \parallel \text{tell}(\overline{v} = \overline{r})], c_i} c''_F = \exists_{-\text{Var}(C_i[C[p(\overline{X})] \parallel \text{tell}(\overline{v} = \overline{r})], c_i} c''_F$ and

$$m(B'', c''_F) = m(B, c_F). \tag{38}$$

Since $\text{Var}(C_i[q(\overline{v})], c_i) \subseteq \text{Var}(C_i[C[p(\overline{X})] \parallel \text{tell}(\overline{v} = \overline{r})], c_i)$, we have that

$$\exists_{-\text{Var}(C_i[q(\overline{v})], c_i} c''_F = \exists_{-\text{Var}(C_i[q(\overline{v})], c_i} c''_F. \tag{39}$$
Since by hypothesis for any agent $A'$, $O(D_0.A') = O(D_i.A')$, there exists a derivation

$$c_0 = (D_0, C|I[p(X)]) \parallel \text{tell}(\bar{v} = \bar{r}], c_1) \rightarrow^* (D_0, B_0, c_0)$$

such that $\exists - \text{Var}(C|I[p(X)]) \parallel \text{tell}(\bar{v} = \bar{r}], c_1)c_0 = \exists - \text{Var}(C|I[p(X)]) \parallel \text{tell}(\bar{v} = \bar{r}], c_1)c'_0$ and $m(B_0, c_0) = m(B'', c'_0)$. By (38), (39) and since $\text{Var}(C|I[q(\bar{v})], c_1) \subseteq \text{Var}(C|I[p(X)]) \parallel \text{tell}(\bar{v} = \bar{r}], c_1)$, we have that

$$\exists - \text{Var}(C|I[q(\bar{v})], c_1)c_0 = \exists - \text{Var}(C|I[q(\bar{v})], c_1)c_F$$

and $m(B_0, c_0) = m(B, c_F)$. (40)

Let $f : p(\bar{X}^t) \leftarrow H'$ be an appropriate renaming of $f$, which renames only the variables in $\bar{X}$, such that $\text{Var}(d) \cap \text{Var}(f') = \emptyset$ (note that this is possible, since $\text{Var}(H) \cap (\text{Var}(T) \cup \text{Var}(C)) \subseteq \text{Var}(\bar{X})$). Moreover by hypothesis, $\text{Var}(C|I[q(\bar{v})], c_1) \cap \text{Var}(H) = \emptyset$. Then, without loss of generality we can assume that $\text{Var}(\bar{v}) \cap \text{Var}(H) \neq \emptyset$ if and only if the procedure call $p(\bar{X})$ is evaluated, in which case declaration $f'$ is used.

Thus there exists a derivation

$$(D_0, C|I[C|H'] \parallel \text{tell}(\bar{X} = \bar{X}^t]) \parallel \text{tell}(\bar{v} = \bar{r}], c_1) \rightarrow^* (D_0, B_0', c_0),$$

where $m(B_0', c_0) = m(B_0, c_0)$. By (40) we have

$$m(B_0', c_0) = m(B, c_F).$$

(41)

We show now that we can substitute $H$ for $H'$ in the previous derivation. Since $f' : p(\bar{X}^t) \leftarrow H'$ is a renaming of $f : p(\bar{X}) \leftarrow H$, the equality $\bar{X} = \bar{X}^t$ is a conjunction of equations involving only distinct variables. Then, by replacing $\bar{X}$ with $\bar{X}^t$ and vice versa in the previous derivation we obtain the derivation $\chi_0 = (D_0, C|I[C|H] \parallel \text{tell}(\bar{X} = \bar{X}^t]) \parallel \text{tell}(\bar{v} = \bar{r}], c_1) \rightarrow^* (D_0, B_0', c_0)$ where

$$\exists - \text{Var}(C|I[C|H] \parallel \text{tell}(\bar{X} = \bar{X}^t]) \parallel \text{tell}(\bar{v} = \bar{r}], c_1)c'_0 = \exists - \text{Var}(C|I[C|H] \parallel \text{tell}(\bar{X} = \bar{X}^t]) \parallel \text{tell}(\bar{v} = \bar{r}], c_1)c_0$$

and $m(B_0', c_0) = m(B_0, c_0)$.

From (41) it follows that

$$m(B_0'', c'_0) = m(B, c_F).$$

(42)

Then, from (40) and since $\text{Var}(C|I[q(\bar{v})], c_1) \subseteq \text{Var}(C|I[C|H] \parallel \text{tell}(\bar{X} = \bar{X}^t]) \parallel \text{tell}(\bar{v} = \bar{r}], c_1)$ we obtain

$$\exists - \text{Var}(C|I[q(\bar{v})], c_1)c'_0 = \exists - \text{Var}(C|I[q(\bar{v})], c_1)c_F.$$  

(43)

Moreover, we can drop the constraint $\text{tell}(\bar{X}^t = \bar{X})$, since the declarations used in the derivation are renamed apart and, by construction, $\text{Var}(C|I[C|H] \parallel \text{tell}(\bar{v} = \bar{v}], c_1) \cap \text{Var}(\bar{X}^t) = \emptyset$. Therefore there exists a derivation $(D_0, C|I[C|H] \parallel \text{tell}(\bar{v} = \bar{v}], c_1) \rightarrow^* (D_0, B_0, c_0)$ which performs exactly the same steps of $\chi_0$, (possibly) except for the evaluation of $\text{tell}(\bar{X} = \bar{X}^t)$, and such that $\exists - \text{Var}(C|I[C|H] \parallel \text{tell}(\bar{v} = \bar{v}], c_1)c_0 = \exists - \text{Var}(C|I[C|H] \parallel \text{tell}(\bar{v} = \bar{v}], c_1)c_0$ and $m(B_0, c_0) = m(B_0', c_0)$. (42), (43) and since $\text{Var}(C|I[q(\bar{v})], c_1) \subseteq \text{Var}(C|I[C|H] \parallel \text{tell}(\bar{v} = \bar{r}], c_1)$, it follows that

$$m(B_0, c_0) = m(B, c_F)$$

and $\exists - \text{Var}(C|I[q(\bar{v})], c_1)c_0 = \exists - \text{Var}(C|I[q(\bar{v})], c_1)c_F$. (44)

Since $O(D_0.A') = O(D_i.A')$ holds by hypothesis for any agent $A'$, there exists a derivation

$$(D_1, C|I[C|H] \parallel \text{tell}(\bar{v} = \bar{r}], c_1) \rightarrow^* (D_i, B', c'_1)$$

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where
\[ \exists^{-\Var(C_i[H])} \parallel \text{tell}(\bar{v} = \bar{r}), \alpha_i \epsilon F' = \exists^{-\Var(C_i[H])} \parallel \text{tell}(\bar{v} = \bar{r}), \alpha_i \epsilon F_0 \]
and \(m(B', c_f') = m(\bar{B}_0, \bar{c}_0)\). From (44) and since \(\Var(C_i[q(\bar{v})], \alpha_i) \subseteq \Var(C_i[H]) \parallel \text{tell}(\bar{v} = \bar{r}), \alpha_i\), we obtain
\[ m(B', c_f') = m(B, c_f) \quad \text{and} \quad \exists^{-\Var(C_i[q(\bar{v})], \alpha_i)} c_f' = \exists^{-\Var(C_i[q(\bar{v})], \alpha_i)} c_f. \quad (45) \]
Finally, since \(d : q(\bar{r}) \leftarrow C[H] \in D_i\), there exists a derivation
\[ c' = (D_i)_{C_i[q(\bar{v})], \alpha_i} \rightarrow (D_i)_{C_i[H] \parallel \text{tell}(\bar{v} = \bar{r}), \alpha_i} \rightarrow (D_i, B', c_f') \]
and then the thesis follows from (45). \(\square\)

Before proving the total correctness result we need some technical lemmata. Here and in the following we use the notation \(w_t\) (with \(t \in \{ss, dd, ff\}\)) as a shorthand for indicating the success weight \(w_{ss}\), the deadlock weight \(w_{dd}\) and the failure weight \(w_{ff}\).

**Lemma A.3.** Let \(q(\bar{r}) \leftarrow H \in D_0\), \(t \in \{ss, dd, ff\}\) and let \(C[\cdot]\) be context. For any satisfiable constraint \(c\) and for any constraint \(c'\), such that \(\Var(C[q(\bar{r})], c) \cap \Var(\bar{r}) = \emptyset\) and \(w_t(C[q(\bar{r})], c, c')\) is defined, there exists a constraint \(d'\) such that \(w_t[C[q(\bar{r})] \parallel \text{tell}(\bar{r})], c, d'] \leq w_t(C[q(\bar{r})], c, c')\) and \(\exists^{-\Var(C[q(\bar{r})], c)} d' = \exists^{-\Var(C[q(\bar{r})], c)} c'.\)

**Proof.** Immediate. \(\square\)

**Lemma A.4.** Let \(q(\bar{r}) \leftarrow H \in D_0\) and \(t \in \{ss, dd, ff\}\). For any context \(C[\cdot]\), any satisfiable constraint \(c\) and for any constraint \(c'\), the following holds.

1. If \(\Var(H) \cap \Var(C_i, c) \subseteq \Var(\bar{r})\) and \(w_t(C_i[q(\bar{r})], c, c')\) is defined, then there exists a constraint \(d'\), such that \(\Var(d') \subseteq \Var(C_i[H], c)\), \(w_t(C_i[H], c, d'] \leq w_t(C_i[q(\bar{r})], c, c')\) and \(\exists^{-\Var(C_i[q(\bar{r})], c)} d' = \exists^{-\Var(C_i[q(\bar{r})], c)} c'.\)
2. If \(\Var(H) \cap \Var(C_i, c) \subseteq \Var(\bar{r})\), \(\Var(c') \cap \Var(H) \subseteq \Var(C_i[H], c)\) and \(w_t(C_i[H], c, c')\) is defined, then there exists a constraint \(d'\), such that \(w_t(C_i[q(\bar{r})], c, d') \leq w_t(C_i[H], c, c')\) and \(\exists^{-\Var(C_i[q(\bar{r})], c)} d' = \exists^{-\Var(C_i[q(\bar{r})], c)} c'.\)

**Proof.** Immediate. \(\square\)

The following Lemma is crucial in the proof of completeness.

**Lemma A.5.** Let \(0 \leq i \leq n\), \(t \in \{ss, dd, ff\}\), \(c : q(\bar{r}) \leftarrow H \in D_i\), and let \(c' : q(\bar{r}) \leftarrow H'\) be the corresponding declaration in \(D_{i+1}\) (in the case \(i < n\)). For any context \(C[\cdot]\) and any satisfiable constraint \(c\) and for any constraint \(c'\) the following holds:

1. If \(\Var(H) \cap \Var(C_i, c) \subseteq \Var(\bar{r})\) and \(w_t(C_i[q(\bar{r})], c, c')\) is defined, then there exists a constraint \(d'\), such that \(\Var(d') \subseteq \Var(C_i[H], c)\), \(w_t(C_i[H], c, d'') \leq w_t(C_i[q(\bar{r})], c, c')\) and \(\exists^{-\Var(C_i[q(\bar{r})], c)} d'' = \exists^{-\Var(C_i[q(\bar{r})], c)} c'.\)
2. If \(\Var(H', H) \cap \Var(C_i, c) \subseteq \Var(\bar{r})\), \(\Var(c') \cap \Var(H) \subseteq \Var(C_i[H], c)\) and \(w_t(C_i[H], c, c')\) is defined, then there exists a constraint \(d'\), such that \(\Var(d') \subseteq \Var(C_i[H'], c)\), \(w_t(C_i[H'], c, d'') \leq w_t(C_i[H], c, c')\) and \(\exists^{-\Var(C_i[q(\bar{r})], c)} d'' = \exists^{-\Var(C_i[q(\bar{r})], c)} c'.\)
Proof. Observe that, for $i = 0$, the proof of 1 follows from the first part of Lemma A.4. We prove here that, for each $i \geq 0$,

a) if 1 holds for $i$ then 2 holds for $i$;

b) if 1 and 2 hold for $i$ then 1 holds for $i + 1$.

The proof of the Lemma then follows from straightforward inductive argument.

a) If $c_l$ was not affected by the transformation step from $D_i$ to $D_{i+1}$ then the result is obvious by choosing $d' = 3_{-\text{Var}(C_i, c)}c'$. Assume then that $c_l$ is affected when transforming $D_i$ to $D_{i+1}$ and let us distinguish various cases.

Case 1: $c_l' \in D_{i+1}$ was obtained from $D_i$ by unfolding.

In this case, the situation is the following:

- $c_l : q(\bar{t}) \leftarrow C[p(\bar{t})] \in D_i$
- $u : p(\bar{s}) \leftarrow B \in D_i$
- $c_l' : q(\bar{r}) \leftarrow C[B \parallel \text{tell}(\bar{t} = \bar{s})] \in D_{i+1}$

where $c_l$ and $u$ are assumed to be renamed so that they do not share variables. Let $n = w_t(C_i[C[p(\bar{t})]], c, c')$. By the definition of transformation sequence, there exists a declaration $p(\bar{s}) \leftarrow B_2 \in D_0$. Moreover, by the hypothesis on the variables, $Var(C_i[p(\bar{t})], C[B \parallel \text{tell}(\bar{t} = \bar{s})]) \cap \text{Var}(C_i, c) \subseteq \text{Var}(\bar{r})$ and then $Var(C_i[C[p(\bar{t})]], c) \cap \text{Var}(\bar{r}) = \emptyset$. Therefore, by Lemma A.3, there exists a constraint $d_1$, such that

$$w_t(C_i[C[p(\bar{r})] \parallel \text{tell}(\bar{t} = \bar{s})], c, d_1) \leq w_t(C_i[C[p(\bar{t})]], c, c') = n \quad (46)$$

and

$$3_{-\text{Var}(C_i[C[p(\bar{t})]], c)}c d_1 = 3_{-\text{Var}(C_i[C[p(\bar{t})]], c)}c' \quad (47)$$

By the hypothesis on the variables and since $u$ is renamed apart from $c_l$, $Var(B) \cap Var(C_i, C, \bar{r}, c) = \emptyset$ and therefore $Var(B) \cap Var(C_i[C[B \parallel \text{tell}(\bar{t} = \bar{s})]], c) \subseteq Var(\bar{r})$.

Then, by Point 1, there exists a constraint $d''$, such that $Var(d'') \subseteq Var(C_i[C[B \parallel \text{tell}(\bar{t} = \bar{s})]], c)$, $w_t(C_i[C[B \parallel \text{tell}(\bar{t} = \bar{s})]], c, d'') \leq w_t(C_i[C[p(\bar{r})] \parallel \text{tell}(\bar{t} = \bar{s})], c, d_1)$ and

$$3_{-\text{Var}(C_i[C[p(\bar{r})] \parallel \text{tell}(\bar{t} = \bar{s})]], c)}d'' = 3_{-\text{Var}(C_i[C[p(\bar{r})] \parallel \text{tell}(\bar{t} = \bar{s})]], c)}d_1.$$ 

By (46), $w_t(C_i[C[B \parallel \text{tell}(\bar{t} = \bar{s})]], c, d'') \leq n$.

Furthermore, by hypothesis and construction, $Var(c', d'') \cap \text{Var}(\bar{r}) \subseteq Var(C_i[C[p(\bar{r})]], c)$ and, without loss of generality, we can assume that $Var(d_1) \cap \text{Var}(\bar{r}) \subseteq Var(C_i[C[p(\bar{r})]], c)$.

Then, by (47) and since $Var(C_i[C[p(\bar{r})]], c) \subseteq Var(C_i[C[p(\bar{r})] \parallel \text{tell}(\bar{t} = \bar{s})], c)$, we have that $3_{-\text{Var}(C_i[C[p(\bar{r})] \parallel \text{tell}(\bar{t} = \bar{s})]], c)}d'' = 3_{-\text{Var}(C_i[C[p(\bar{r})] \parallel \text{tell}(\bar{t} = \bar{s})]], c)}c'$ and this completes the proof.

Case 2: $c_l'$ is the result of a tell elimination or introduction.

The proof is analogous to that one given for Case 2 of Proposition 4.5 and it is omitted.

Case 3: $c_l'$ is the result of a backward instantiation.

Let $c_l$ be the corresponding declaration in $D_i$. The situation is then the following:

- $c_l : q(\bar{t}) \leftarrow C[p(\bar{t})]$
- $c_l' : q(\bar{r}) \leftarrow C[p(\bar{t}) \parallel \text{tell}(b) \parallel \text{tell}(\bar{t} = \bar{s})]$

where $f : p(\bar{s}) \leftarrow b \parallel H \in D_i$ has no variable in common with $c_l$ (the case $c_l' : q(\bar{r}) \leftarrow C[p(\bar{t})] \parallel \text{tell}(\bar{t} = \bar{s})]$ is analogous and hence omitted). By the hypothesis, $Var(C_i[p(\bar{t})], C[p(\bar{t}) \parallel \text{tell}(b) \parallel \text{tell}(\bar{t} = \bar{s})]) \cap \text{Var}(C_i, c) \subseteq \text{Var}(\bar{r})$, $Var(c', \bar{r}) \cap \text{Var}(\bar{r}) \subseteq Var(C_i[C[p(\bar{t})]], c)$ and there exists $n$ such that $w_t(C_i[C[p(\bar{r})]], c, c') = n$. Then $Var(C_i[C[p(\bar{r})]], c) \cap \text{Var}(\bar{s}) = \emptyset$ and, without loss of generality, we can assume that $Var(H) \cap \text{Var}(C_i, c) = \emptyset$. 

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Moreover, by the definition of transformation sequence, there exists a declaration $p(\tilde{s}) \leftarrow B_0 \in D_0$ and then, by Lemma A.3, there exists a constraint $d_1$ such that

$$w_t(C_i[p(\tilde{s})] \parallel \text{tell}(\tilde{t} = \tilde{s})), c, d_1) \leq w_t(C_i[p(\tilde{t})], c, c') = n \tag{48}$$

and

$$\exists_{\text{Var}(C_i[C[p(\tilde{s})]], c)} d_1 = \exists_{\text{Var}(C_i[C[p(\tilde{t})]], c)} c'. \tag{49}$$

Using the hypothesis on the variables and since $f$ is renamed apart from $\text{Var}(\tilde{r})$, we have that

$$\text{Var}(\text{tell}(b) \parallel H) \cap \text{Var}(C_i[C[\text{tell}(\tilde{t} = \tilde{s})]], c) \subseteq \text{Var}(\tilde{s}).$$

Then, from Point 1 of the Lemma (assumed as hypothesis) and (48) it follows that there exists a constraint $d_2$ such that

$$w_t(C_i[C[\text{tell}(b) \parallel H] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c, d_2) \leq w_t(C_i[C[p(\tilde{t})] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c, d_1) \leq n \tag{50}$$

and

$$\exists_{\text{Var}(C_i[C[p(\tilde{s})] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c} d_2 = \exists_{\text{Var}(C_i[C[p(\tilde{t})] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c} d_1 \tag{51}$$

hold. By definition of weight, we can assume that $\text{Var}(d_1) \subseteq \text{Var}(C_i[C[p(\tilde{s})] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c)$ and therefore, we have that $\text{Var}(b) \cap \text{Var}(C_i[C[p(\tilde{t})] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c, d_1) \subseteq \text{Var}(\tilde{s}).$

We have now two cases:

1) $D \models \exists_{\text{Var}(\tilde{s})} d_1 \rightarrow \exists_{\text{Var}(\tilde{s})} b$. In this case, by (48), there exists a derivation

$$\xi = \langle D_0, C_i[C[p(\tilde{s})] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c \rangle \rightarrow^* \langle D_0, B_F, c_F \rangle,$$

such that $m(B_F, c_F) = t, \text{wh}(\xi) \leq n$ and

$$\exists_{\text{Var}(C_i[C[p(\tilde{s})] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c} c_F = \exists_{\text{Var}(C_i[C[p(\tilde{t})] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c} d_1.$$

By the hypothesis on the variables, we can build a derivation

$$\chi = \langle D_0, C_i[C[p(\tilde{t})] \parallel \text{tell}(b) \parallel \text{tell}(\tilde{t} = \tilde{s})]), c \rangle \rightarrow^* \langle D_0, B_F, d_3 \rangle$$

which performs exactly the same steps of $\xi$, plus possibly a tell action, such that $\text{wh}(\chi) \leq n, m(B_F, d_3) = m(B_F, c_F)$ and

$$\exists_{\text{Var}(C_i[C[p(\tilde{t})] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c} d_3 = \exists_{\text{Var}(C_i[C[p(\tilde{t})] \parallel \text{tell}(\tilde{t} = \tilde{s})]), c} d_1. \tag{52}$$

Let $d' = \exists_{\text{Var}(C_i[C[p(\tilde{s})] \parallel \text{tell}(b) \parallel \text{tell}(\tilde{t} = \tilde{s})]), c} d_3$. By the previous result and by definition of weight $w_t(C_i[C[p(\tilde{t})] \parallel \text{tell}(b) \parallel \text{tell}(\tilde{t} = \tilde{s})]), c, d') \leq n$.

Moreover, by hypothesis, $\text{Var}(c', d') \cap \text{Var}(\tilde{r}) \subseteq \text{Var}(C_i[C[p(\tilde{t})]], c)$ and we can assume, without loss of generality, that $\text{Var}(d_1, d_2) \cap \text{Var}(\tilde{r}) \subseteq \text{Var}(C_i[C[p(\tilde{t})]], c)$. Then, by (49), (52) and by definition of $d'$, it follows that $\exists_{\text{Var}(C_i[q(\tilde{t})], c)} d' = \exists_{\text{Var}(C_i[q(\tilde{t})], c)} c'$ and then the thesis holds.

2) $D \not\models \exists_{\text{Var}(\tilde{s})} d_1 \rightarrow \exists_{\text{Var}(\tilde{s})} b$. In this case, by (51), $D \not\models \exists_{\text{Var}(\tilde{s})} d_2 \rightarrow \exists_{\text{Var}(\tilde{s})} b$.

By (50) this means that there exists a derivation

$$\xi = \langle D_0, C_i[C[\text{tell}(b) \parallel H \parallel \text{tell}(\tilde{t} = \tilde{s})]), c \rangle \rightarrow^* \langle D_0, B_F, c_F \rangle \not\models$$

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such that \( \text{tell}(b) \parallel H \parallel \text{tell}(\bar{t} = \bar{s}) \) is not evaluated in \( \xi \), \( m(\text{B}_F, \text{C}_F) = t \), \( wh(\xi) \leq n \) and

\[ \exists_{\text{Var}(\text{C}_i[\text{tell}(b)] \parallel H \parallel \text{tell}(\bar{t} = \bar{s})) \text{C}_F} = \exists_{\text{Var}(\text{C}_i[\text{tell}(b)] \parallel H \parallel \text{tell}(\bar{t} = \bar{s})) \text{C}_F}^{d_2}. \]

By definition, we can construct another derivation

\[ \chi = \langle D_0, C_i[\text{C[p(\bar{t})] \parallel \text{tell}(b) \parallel \text{tell}(\bar{t} = \bar{s})}], c \rangle \rightarrow^* \langle D_0, \text{B}_F, \text{C}_F \rangle \neq \]

which performs exactly the same steps of \( \xi \) (and therefore \( wh(\chi) \leq n \)) and such that

\[ m(\text{B}_F, \text{C}_F) = m(\text{B}_F, \text{C}_F). \]

Let \( d' = \exists_{\text{Var}(\text{C}_i[\text{C[p(\bar{t})] \parallel \text{tell}(b) \parallel \text{tell}(\bar{t} = \bar{s})]), c \rangle \text{C}_F}. \]

By definition of derivation

\[ \text{Var}(\text{C}_F) \cap \text{Var}(\text{C}_i[\text{tell}(b)] \parallel H \parallel \text{tell}(\bar{t} = \bar{s})], c \rangle \subseteq \text{Var}(C_i, C, c) \]

and therefore

\[ \exists_{\text{Var}(\text{C}_i[\text{tell}(b)] \parallel H \parallel \text{tell}(\bar{t} = \bar{s})], c \rangle \text{d}' = \exists_{\text{Var}(\text{C}_i[\text{tell}(b)] \parallel H \parallel \text{tell}(\bar{t} = \bar{s})], c \rangle \text{d}_2}. \]

The remainder of the proof is now analogous to that one of the previous case.

**Case 4:** Either \( cl' \) is the result of an ask simplification or \( cl'' \) is the result of a tell simplification. The proof is analogous to that one given for Case 4 of Proposition 4.5 and hence it is omitted.

**Case 5:** \( cl' \) is the result of a branch elimination or of a conservative ask elimination. The proof is straightforward by noting that: (a) according to Definition 4.1 we consider also inconsistent stores resulting from non-terminated computations; (b) an ask action of the form \( \text{ask}(\text{true}) \) always succeeds; (c) if we delete an \( \text{ask}(\text{true}) \) action we obtain a derivation whose weight is smaller.

**Case 6:** \( cl' \) is the result of a distribution.

Let

\[ -cl : q(\bar{t}) \leftarrow C[H \parallel \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j] \in D_i \]

\[ -cl' : q(\bar{t}) \leftarrow C[\sum_{j=1}^n \text{ask}(c_j) \rightarrow (H \parallel B_j)] \in D_i+1 \]

where \( e = \text{pc}(C[\]) \) and for every constraint \( e' \) such that \( \text{Var}(e') \cap \text{Var}(cl) \subseteq \text{Var}(q(\bar{t}), C) \), if \( (D_i, H, e' \land e) \) is productive then both the following conditions hold:

--- there exists at least one \( j \in [1, n] \) such that \( D \models (e' \land e) \rightarrow c_j \)

--- for each \( j \in [1, n] \), either \( D \models (e' \land e) \rightarrow c_j \) or \( D \models (e' \land e) \rightarrow \neg c_j \)

We prove that, for any derivation

\[ \xi = \langle D_0, C_i[C[H \parallel \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j]], c \rangle \rightarrow^* \langle D_0, B, d \rangle \]

with \( m(B, d) \in \{\text{ss}, \text{dd}, \text{ff}\} \), there exists a derivation

\[ \xi' = \langle D_0, C_i[C[\sum_{j=1}^n \text{ask}(c_j) \rightarrow (H \parallel B_j)], c \rangle \rightarrow^* \langle D_0, B', d' \rangle \]

such that

\[ \exists_{\text{Var}(C_i[C[H \parallel \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j]], c \rangle d'} = \exists_{\text{Var}(C_i[C[H \parallel \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j]], c \rangle d} \]

where also \( wh(\xi') \leq wh(\xi) \), and \( m(B', d') = m(B, d) \). This together with the definition of weight implies the thesis.
If $H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j$ is not evaluated in $\xi$, then the proof is immediate. Otherwise we have to distinguish two cases:

1) There exists an $h \in [1, n]$, such that

$$\xi = (D_0.\text{c}_i[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j]), c) \rightarrow^* (D_0.\text{c}_m[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_m)$$

$$\rightarrow (D_0.\text{c}_m[H || B_h], d_m) \rightarrow^* (D_0.\text{B}, d)$$

and $D \models d_m \rightarrow c_n$. In this case we can construct the derivation

$$\chi = (D_0.\text{c}_i[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j]), c)$$

$$\rightarrow^* (D_0.\text{c}_m[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_m)$$

$$\rightarrow (D_0.\text{c}_m[H || B_h], d_m) \rightarrow^* (D_0.\text{B}, d)$$

which performs the same steps of $\xi$ and then the thesis holds.

2) $\xi$ is of the form

$$\xi = (D_0.\text{c}_i[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j]), c) \rightarrow^* (D_0.\text{c}_m[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_m)$$

$$\rightarrow (D_0.\text{c}_m[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_{m+1}) \rightarrow^* (D_0.\text{B}, d).$$

By Lemma A.1 and by definition of $\text{pc}$, we can construct another derivation

$$\chi = (D_0.\text{c}_i[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j]), c) \rightarrow^* (D_0.\text{c}_k[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_k)$$

$$\rightarrow^* (D_0.\text{c}_k[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_k) \rightarrow^* (D_0.\text{B}, d)$$

which performs the same steps of $\xi$ (possibly in a different order) and such that the agent $H$ is not evaluated in the first $k$ steps, where $\text{Var}(d_k) \cap \text{Var}(c_i) \subseteq \text{Var}(\text{q}(\tilde{r}), C)$ and $D \models d_k \rightarrow (\text{pc}(C) \cap C).$ Let $\chi_1 = (D_0.\text{c}_k[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_k) \rightarrow^* (D_0.\text{B}, d).$ Now, if $(D_0.\text{H}, d_k)$ is not productive, the proof is analogous to that one of Case 6 of Proposition 4.5 and hence it is omitted. Then assume that $(D_0.\text{H}, d_k)$ is productive. By definition of distribution there exists at least one $j \in [1, n]$ such that $D \models d_k \rightarrow c_j$ and for each $j \in [1, n]$, either $D \models d_k \rightarrow c_j$ or $D \models d_k \rightarrow \neg c_j$. Then, by definition, there exists a derivation $\xi_1 = (D_0.\text{c}_i[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_k) \rightarrow^* (D_0.\text{B}, d)$, which performs the same steps of $\chi_1$ (possibly in a different order).

Therefore there exists a derivation

$$\xi' = (D_0.\text{c}_i[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j]), c) \rightarrow^* (D_0.\text{c}_m[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_m)$$

$$\rightarrow^* (D_0.\text{c}_m[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_m) \rightarrow^* (D_0.\text{B}, d)$$

which performs the same steps of $\chi$ (in a different order). By construction $\text{wh}(\xi') = \text{wh}(\chi) = \text{wh}(\xi)$ and then the thesis holds.

**Case 7:** $\text{cf}'$ is the result of a folding.

Let

- $\text{cl'}: \text{q}(\tilde{r}) \rightarrow C[B]$ be the folded declaration ($\in D_1$),
- $\text{f}: p(\tilde{X}) \rightarrow B$ be the folding declaration ($\in D_0$),
- $\text{cf}: \text{q}(\tilde{r}) \rightarrow C[p(\tilde{X})]$ be the result of the folding operation ($\in D_1$),

where, by hypothesis, $\text{Var}(cl') \cap \text{Var}(\tilde{X}) \subseteq \text{Var}(B)$, $\text{Var}(B) \cap \text{Var}(\tilde{r}, C) \subseteq \text{Var}(\tilde{X})$, $\text{Var}(C[B], C[p(\tilde{X})]) \cap \text{Var}(C_i, c) \subseteq \text{Var}(\tilde{r})$, $\text{Var}(c') \cap \text{Var}(\tilde{r}) \subseteq \text{Var}(C_i[C[B]], c)$ and there exists $n$ such that $w_t(C_i[C[B]], c, c') = n$. Then,

$$\text{Var}(B) \cap \text{Var}(C_i[C[B]], c) \subseteq \text{Var}(B) \cap \text{Var}(\tilde{r}, C) \subseteq \text{Var}(\tilde{X})$$

(53)
and

\[ \text{Var}(c') \cap \text{Var} (\bar{\alpha}) \subseteq \text{Var}(C_i[C[B]], c) \cap \text{Var} (\bar{\beta}) \subseteq \text{Var}(C_i[C[p(\bar{X})]], c) \] (54)

hold. Moreover, we can assume without loss of generality that \( \text{Var}(c') \cap \text{Var}(\bar{X}) \subseteq \text{Var}(C_i[C[B]], c) \).

Since \( f \in D_0 \), from (53) and Point 2 of Lemma A.4 it follows that there exists a constraint \( d' \) such that \( w_t(C_i[C[p(\bar{X})]], c, d') \leq w_t(C_i[C[B]], c, c') \) and

\[ \exists \text{Var}(C_i[C[p(\bar{X})]], c) d' = \exists \text{Var}(C_i[C[p(\bar{X})]], c) c'. \] (55)

We can assume, without loss of generality, that \( \text{Var}(d') \subseteq \text{Var}(C_i[C[p(\bar{X})]], c) \). Then by using (54) and (55) we obtain that \( \exists \text{Var}(C_i[q(\bar{r})], c) d' = \exists \text{Var}(C_i[q(\bar{r})], c) c' \) which concludes the proof of a).

b) Assume that the parts 1 and 2 of this Lemma hold for \( i \geq 0 \). We prove that 1 holds for \( i + 1 > 0 \).

Let \( \bar{c} : q(\bar{r}) \to H \in D_{i+1} \), and let \( \bar{c} : q(\bar{r}) \to H \) be the corresponding declaration in \( D_i \). Moreover let \( C_i[\ ] \) be a context, \( c \) a satisfiable constraint and let \( c' \) be a constraint, such that \( \text{Var}(H) \cap \text{Var}(C_i, c) \subseteq \text{Var} (\bar{r}) \) and \( w_t(C_i[q(\bar{r})], c, c') \) is defined. Without loss of generality, we can assume that \( \text{Var}(H) \cap \text{Var}(C_i, c) \subseteq \text{Var} (\bar{r}) \).

Then, since by inductive hypothesis, part 1 holds for \( i \), there exists a constraint \( d_1 \) such that \( \text{Var}(d_1) \subseteq \text{Var}(C_i[H], c) \),

\[ w_t(C_i[H], c, d_1) \leq w_t(C_i[q(\bar{r})], c, c') \] and \( \exists \text{Var}(C_i[q(\bar{r})], c) d_1 = \exists \text{Var}(C_i[q(\bar{r})], c) c'. \] (56)

Since by inductive hypothesis part 2 holds for \( i \), there exists a constraint \( d' \), such that \( \text{Var}(d') \subseteq \text{Var}(C_i[H], c) \), \( w_t(C_i[H], c, d') \leq w_t(C_i[H], c, d_1) \) and \( \exists \text{Var}(C_i[q(\bar{r})], c) d' = \exists \text{Var}(C_i[q(\bar{r})], c) c' \).

By (56) we obtain \( w_t(C_i[H], c, d') \leq w_t(C_i[q(\bar{r})], c, c') \) and

\[ \exists \text{Var}(C_i[q(\bar{r})], c) d' = \exists \text{Var}(C_i[q(\bar{r})], c) c' \]

and then the thesis holds. \( \square \)

**Lemma A.6.** Let \( 0 \leq i \leq n \), \( c_1, c_m \) satisfiable constraints, \( c_k \) a constraint and assume that there exists a derivation \( \xi : \langle D_i, A_1, c_1 \rangle \to^+ \langle D_i, A_m, c_m \rangle \to^+ \langle D_i, A_k, c_k \rangle \), such that

i) in the first \( m - 1 \) steps of \( \xi \) rule R2 is used only for evaluating agents of the form ask(c) \( \to B \),

ii) \( w_t(A_1, c_1, c_k) \) is defined (for \( t = m(A_k, c_k) \in \{ss, dd, ff\} \)).

Then there exists a constraint \( c' \) such that \( \text{Var}(c') \subseteq \text{Var}(A_m, c_m) \), \( \exists \text{Var}(A_t, c_1) c_k = \exists \text{Var}(A_t, c_1) c' \) and \( w_t(A_m, c_m, c') \leq w_t(A_1, c_1, c_k) \).

**Proof.** We prove the thesis for one derivation step. Then the proof of the Lemma follows by using a straightforward inductive argument. Assume that \( c_1, c_2 \) are satisfiable constraints, \( c_k \) is a constraint and that there exists a derivation

\[ \langle D_i, A_1, c_1 \rangle \to \langle D_i, A_2, c_2 \rangle \to^+ \langle D_i, A_k, c_k \rangle \]

such that \( m(A_k, c_k) \in \{ss, dd, ff\} \) and the first step can use rule R2 only for evaluating agents of the form ask(c) \( \to B \). By the definition of derivation we have \( A_1 = C_i[\ ], \) where \( C_i[\ ] \) is not a guarding context. We have now three cases:
1) \( A = \text{tell}(c) \). In this case
\[
\langle D_i, C_1 [\text{tell}(c)], c_1 \rangle \rightarrow \langle D_i, C_1 [\text{Stop}], c_1 \land c \rangle \rightarrow^* \langle D_i, A_k, c_k \rangle.
\]
Since \( C_1[\_] \) is not a guarding context the definition of weight implies that
\[
W_i(C_1 [\text{Stop}], c_1 \land c, \exists \exists_{\text{Var}(C_1 [\text{Stop}], c_1 \land c) c_k}) = W_i(C_1 [\text{tell}(c)], c_1, c_k)
\]
where \( t = m(A_k, c_k) \). Then the thesis holds

2) \( A = q(\bar{v}) \) and there exists a declaration \( cl : q(\bar{v}) \leftarrow B \in D_i \). In this case
\[
\langle D_i, C_1[q(\bar{v})], c_1 \rangle \rightarrow \langle D_i, C_1 [B \parallel \text{tell}(\bar{v} = \bar{r})], c_1 \rangle \rightarrow^* \langle D_i, A_k, c_k \rangle.
\]
From the definition of derivation it follows that \( \text{Var}(C_1[q(\bar{v})], c_1) \cap \text{Var}(q(\bar{r})) = \emptyset \). Furthermore, by definition of transformation sequence, there exists a declaration \( q(\bar{r}) \leftarrow H \in D_0 \). Since \( W_i(C_1[q(\bar{v})], c_1, c_k) \) is defined by hypothesis (where \( t = m(A_k, c_k) \)), from Lemma A.3 it follows that there exists a constraint \( d' \) such that
\[
W_i(C_1[q(\bar{r})] \parallel \text{tell}(\bar{v} = \bar{r}), c_1, d') \leq W_i(C_1[q(\bar{v})], c_1, c_k) \text{ and } \exists \exists_{\text{Var}(C_1[q(\bar{v})], c_1) c_k}.
\]
From the definition of derivation it follows that \( \text{Var}(B) \cap \text{Var}(C_1[\_] \parallel \text{tell}(\bar{v} = \bar{r}), c_1) \subseteq \text{Var}(\bar{r}) \). Part 1 of Lemma A.5 implies that there exists a constraint \( c' \) such that \( \text{Var}(c') \subseteq \text{Var}(C_1[B \parallel \text{tell}(\bar{v} = \bar{r}), c_1), \) \( W_i(C_1[B \parallel \text{tell}(\bar{v} = \bar{r}), c_1, c') \leq W_i(C_1[q(\bar{v})], c_1, c_k) \) and
\[
\exists \exists_{\text{Var}(C_1[q(\bar{v})], c_1) c'} = \exists \exists_{\text{Var}(C_1[q(\bar{v})], c_1) c_k},
\]
thus concluding the proof for this case.

3) \( A = \text{ask}(c) \rightarrow B \) and \( D \models c_1 \rightarrow c \). In this case
\[
\langle D_i, C_1 [\text{ask}(c) \rightarrow B], c_1 \rangle \rightarrow \langle D_i, C_1 [B], c_1 \rangle \rightarrow^* \langle D_i, A_k, c_k \rangle.
\]
Since \( C_1[\_] \) is not a guarding context and \( D \models c_1 \rightarrow c \) we obtain
\[
W_i(C_1[B], c_1, \exists \exists_{\text{Var}(C_1[B], c_1) c_k}) \leq W_i(C_1[\text{ask}(c) \rightarrow B], c_1, c_k)
\]
where \( t = m(A_k, c_k) \), which concludes the proof.

We need one last lemma.

**Lemma A.7.** Let \( C \) be a satisfiable constraint, \( A \) be the agent \( A_1 \parallel \ldots \parallel A_l \), where for any \( j \in [1, l] \) either \( A_j \) is a choice agent or \( A_j = \text{Stop} \) and assume there exists a split derivation \( \nu \) in \( D_0 \),
\[
\nu = \langle D_0, A, c \rangle \rightarrow \langle D_0, A', c' \rangle \rightarrow^* \langle D_0, B, d \rangle,
\]
where \( m(B, d) \in \{ss, dd, ff\} \). Then \( \langle D_i, A, c \rangle \rightarrow \langle D_0, A', c' \rangle \rightarrow^* \langle D_0, B, d \rangle \) is a split derivation in \( D_i \cup D_0 \).

**Proof.** The proof is straightforward, by observing that by the hypothesis on \( A \) the first step of \( \nu \) uses the rule R2 (in case such a step exists) and therefore, by definition of split derivation, \( W_i(A, c, d) > W_i(A', c', d) \), where \( t = m(B, d) \). Then by definition, \( \langle D_i, A, c \rangle \rightarrow \langle D_0, A', c' \rangle \rightarrow^* \langle D_0, B, d \rangle \) is a split derivation in \( D_i \cup D_0 \).
We can now prove our main theorem.

**Theorem 4.13 (Total Correctness).** Let $D_0, \ldots, D_n$ be a transformation sequence. Then, for any agent $A$,

$$\neg \mathcal{O}(D_0, A) = \mathcal{O}(D_n, A).$$

**Proof.** The proof proceeds by showing simultaneously, by induction on $i$, that for $i \in [0, n]$:

1. for any agent $A$, $\mathcal{O}(D_0, A) = \mathcal{O}(D_i, A)$;
2. $D_i$ is weight complete.

**Base case.** We just need to prove that $D_0$ is weight complete. Assume that there exists a derivation $(D_0, A, c_i) \rightarrow^* (D_0, B, c_f)$, where $c_i$ is a satisfiable constraint and $m(B, c_f) \in \{\text{ss}, \text{dd}, \text{ff}\}$. Then there exists a derivation $\xi : (D_0, A, c_i) \rightarrow^* (D_0, B', c_f')$, such that $m(B', c_f') = m(B, c_f)$, whose weight is minimal and where $\exists_{\text{Var}(A, c_i)} c_f' = \exists_{\text{Var}(A, c_i)} c_f$. It follows from Definition 4.7 that $\xi$ is a split derivation.

**Induction step.**

By the inductive hypothesis for any agent $A$, $\mathcal{O}(D_0, A) = \mathcal{O}(D_{i-1}, A)$ and $D_{i-1}$ is weight complete. From propositions 4.5 and 4.9 it follows that if $D_i$ is weight complete then for any agent $A$, $\mathcal{O}(D_0, A) = \mathcal{O}(D_i, A)$. So, in order to prove parts 1 and 2, we only have to show that $D_i$ is weight complete.

Assume then that there exists a derivation $(D_0, A, c_i) \rightarrow^* (D_0, B, c_f)$ such that $c_i$ is a satisfiable constraint and $m(B, c_f) \in \{\text{ss}, \text{dd}, \text{ff}\}$. From the inductive hypothesis it follows that there exists a split derivation

$$\chi = (D_{i-1}, A, c_i) \rightarrow^* (D_{i-1}, A_m, c_m) \rightarrow (D_0, A_{m+1}, c_{m+1}) \rightarrow^* (D_0, B'', c_f'')$$

where

$$\exists_{\text{Var}(A, c_i)} c_f'' = \exists_{\text{Var}(A, c_i)} c_f \text{ and } m(B'', c_f'') = m(B, c_f). \tag{57}$$

Let $d \in D_{i-1} \setminus D_i$ be the modified clause in the transformation step from $D_{i-1}$ to $D_i$.

If in the first $m$ steps of $\chi$ there is no procedure call which uses $d$ then clearly there exists a split derivation $\xi$ in $D_i \cup D_0$,

$$\xi = (D_i, A, c_i) \rightarrow^* (D_i, A_m, c_m) \rightarrow (D_0, A_{m+1}, c_{m+1}) \rightarrow^* (D_0, B'', c_f'')$$

which performs the same steps of $\chi$ and then the thesis holds.

Otherwise, assume without loss of generality that $R4$ is the rule used in the first step of derivation $\chi$ and that $d$ is the clause employed in the first step of $\chi$. We also assume that the declaration $d$ is used only once in $\chi$, since the extension to the general case is immediate.

We have to distinguish various cases according to what happens to the clause $d$ when moving from $D_{i-1}$ to $D_i$.

**Case 1:** $d$ is unfolded.

Let $d'$ be the corresponding declaration in $D_i$. The situation is the following:

- $d : q(\bar{r}) \leftarrow C[p(\bar{t})] \in D_{i-1}$,
- $u : p(\bar{s}) \leftarrow H \in D_{i-1}$, and
- $d' : q(\bar{f}) \leftarrow C[H \parallel \text{tell}(\bar{f} = \bar{s})] \in D_i$,

where $d$ and $u$ are assumed to be renamed apart. By the definition of split deriva-
tion, \( \chi \) has the form
\[
\langle D_{i-1}.C_i[q(\tilde{v})], c_i \rangle \rightarrow \langle D_{i-1}.C_i[p(\tilde{t})] \parallel \text{tell}(\tilde{v} = \tilde{r}), c_i \rangle \rightarrow^* \langle D_{i-1}.A_m, c_m \rangle \rightarrow \langle D_0.A_{m+1}, c_{m+1} \rangle \rightarrow^* \langle D_0.B'', c''_f \rangle.
\]
Without loss of generality, we can assume that \( \text{Var}(\chi) \cap \text{Var}(u) \neq \emptyset \) if and only if \( p(\tilde{t}) \) is evaluated in the first \( m \) steps of \( \chi \), in which case \( u \) is used for evaluating it. We have to distinguish two cases.

1) There exists \( k < m \) such that the \( k \)-th derivation step of \( \chi \) is the procedure call \( p(\tilde{t}) \). In this case \( \chi \) has the form
\[
\langle D_{i-1}.C_i[q(\tilde{v})], c_i \rangle \rightarrow \langle D_{i-1}.C_i[p(\tilde{t})] \parallel \text{tell}(\tilde{v} = \tilde{r}), c_i \rangle \rightarrow^* \langle D_{i-1}.C_k[p(\tilde{t})], c_k \rangle \rightarrow \langle D_{i-1}.C_k[H \parallel \text{tell}(\tilde{r} = \tilde{s}), c_k \rangle \rightarrow^* \langle D_{i-1}.A_m, c_m \rangle \rightarrow \langle D_0.A_{m+1}, c_{m+1} \rangle \rightarrow^* \langle D_0.B'', c''_f \rangle.
\]

Then there exists a corresponding derivation in \( D_0 \cup D_0 \)
\[
\xi = \langle D_i.C_i[q(\tilde{v})], c_i \rangle \rightarrow \langle D_i.C_i[C[H \parallel \text{tell}(\tilde{r} = \tilde{s})] \parallel \text{tell}(\tilde{v} = \tilde{r}), c_i \rangle \rightarrow^* \langle D_i.C_m[H \parallel \text{tell}(\tilde{r} = \tilde{s}), c_m \rangle \rightarrow \langle D_0.A_m, c_m \rangle \rightarrow \langle D_0.A_{m+1}, c_{m+1} \rangle \rightarrow^* \langle D_0.B'', c''_f \rangle,
\]
which performs exactly the same steps of \( \chi \) except for a procedure call to \( p(\tilde{t}) \). In this case the proof follows by observing that, since by the inductive hypothesis \( \chi \) is a split derivation, the same holds for \( \xi \).

2) There is no procedure call to \( p(\tilde{t}) \) in the first \( m \) steps. Therefore \( \chi \) has the form
\[
\langle D_{i-1}.C_i[q(\tilde{v})], c_i \rangle \rightarrow \langle D_{i-1}.C_i[C[H \parallel \text{tell}(\tilde{r} = \tilde{s})] \parallel \text{tell}(\tilde{v} = \tilde{r}), c_i \rangle \rightarrow^* \langle D_{i-1}.A_{m+1}, c_{m+1} \rangle \rightarrow \langle D_0.A_{m+1}, c_{m+1} \rangle \rightarrow^* \langle D_0.B'', c''_f \rangle.
\]
Then, by the definition of \( D_i \), there exists a derivation
\[
\xi_0 = \langle D_{i-1}.C_i[q(\tilde{v})], c_i \rangle \rightarrow \langle D_{i-1}.C_m[H \parallel \text{tell}(\tilde{r} = \tilde{s})] \parallel \text{tell}(\tilde{v} = \tilde{r}), c_i \rangle \rightarrow^* \langle D_i.C_m[H \parallel \text{tell}(\tilde{r} = \tilde{s}), c_m \rangle \rightarrow \langle D_0.A_{m+1}[H \parallel \text{tell}(\tilde{r} = \tilde{s}), c_{m+1} \rangle.
\]
Observe that from the derivation \( \langle D_0.A_{m+1}, c_{m+1} \rangle \rightarrow^* \langle D_0.B'', c''_f \rangle \) and (57) it follows that
\[
\omega_t(C_{m+1}[p(\tilde{t})], c_m, c''_f) \text{ is defined, where } t = m(B, c_f).
\]
(58)
The hypothesis on the variables implies that \( \text{Var}(C_{m+1}[p(\tilde{t})], c_m) \cap \text{Var}(u) = \emptyset \).

Then, by the definition of transformation sequence and since \( u \in D_{i-1}, \), there exists a declaration \( p(\tilde{s}) = H_0 \in D_0 \). By Lemma A.3 and part 1 of Lemma A.5 it follows that there exists a constraint \( d_f \) such that
\[
\omega_t(C_{m+1}[H \parallel \text{tell}(\tilde{r} = \tilde{s})], c_m, d_f) \leq \omega_t(C_{m+1}[p(\tilde{t})], c_m, c''_f)
\]
and
\[
\exists_{\text{Var}(C_{m+1}[p(\tilde{t})], c_m)} d_f = \exists_{\text{Var}(C_{m+1}[p(\tilde{t})], c_m)} c''_f.
\]
(60)
Therefore, by the definition of \( \omega_t \), by (59) and since \( \omega_t(C_{m+1}[p(\tilde{t})], c_m, c''_f) \) is defined, there exists a derivation
\[
\xi_1 = \langle D_0.C_{m+1}[H \parallel \text{tell}(\tilde{r} = \tilde{s}), c_m] \rightarrow^* \langle D_0.B', c'_f \rangle,
\]
where \( \exists_{\text{Var}(C_{m+1}[H \parallel \text{tell}(\tilde{r} = \tilde{s}), c_m]} c''_f = \exists_{\text{Var}(C_{m+1}[H \parallel \text{tell}(\tilde{r} = \tilde{s}), c_m]} d_f \) and, by (58),
\[
\omega_t(B', c'_f) = \omega_t(B, c_f).
\]
(61)
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By (60)
\[ \exists_{\text{Var}}(C_{m+1}, c_m) c_F = \exists_{\text{Var}}(C_{m+1}, c_m) c_F' \]  

holds and, by definition of weight, we obtain
\[ w_t(C_{m+1}[H || \text{tell}(\bar{t} = s)], c_m, c_F') = w_t(C_{m+1}[H || \text{tell}(\bar{t} = s)], c_m, d_F). \]  

Moreover, we can assume without loss of generality that \( \text{Var}(\xi_2) \cap \text{Var}(\xi_1) = \text{Var}(C_{m+1}[H || \text{tell}(\bar{t} = s)], c_m) \). Then, by the definition of procedure call
\[ \text{Var}(C_1[q(\bar{v})], c_1) \cap (\text{Var}(c'_F) \cup \text{Var}(c''_F)) \subseteq \text{Var}(C_{m+1}, c_m) \]  

and there exists a derivation
\[ \xi = \langle D_1, C_1[q(\bar{v})], c_1 \rangle \Rightarrow \langle D_1, C_1[C[H || \text{tell}(\bar{t} = s)] || \text{tell}(\bar{y} = \bar{t})], c_1 \rangle \rightarrow^* \langle D_1, C_1[H || \text{tell}(\bar{t} = s)], c_m \rangle \Rightarrow \langle D_0, C_{m+1}[H || \text{tell}(\bar{t} = s)], c_m \rangle \rightarrow^* \langle D_0, B', c'_F \rangle \]  

such that the first \( m-1 \) derivation steps do not use rule \( \text{R2} \) and the \( m \)-th derivation step uses the rule \( \text{R2} \). Now, we have the following equalities
\[ \exists_{\text{Var}}(C_1[q(\bar{v})], c_1) c_F = (\text{by } (64) \text{ and by construction}) \]
\[ \exists_{\text{Var}}(C_1[q(\bar{v})], c_1) (c_m \land \exists_{\text{Var}}(C_{m+1}, c_m) c_F') = (\text{by } (62)) \]
\[ \exists_{\text{Var}}(C_1[q(\bar{v})], c_1) (c_m \land \exists_{\text{Var}}(C_{m+1}, c_m) c_F') = (\text{by } (64) \text{ and by construction}) \]
\[ \exists_{\text{Var}}(C_1[q(\bar{v})], c_1) c_F = (\text{by the first statement in } (57)) \]

By the definition of weight, \( w_t(C_1[q(\bar{v})], c_1, c_F') = w_t(C_1[q(\bar{v})], c_1, c_F''), \) by (63) and (59), \( w_t(C_{m+1}[H || \text{tell}(\bar{t} = s)], c_m, c_F') \leq w_t(C_{m+1}[p(\bar{t})], c_m, c_F'') \) and \( w_t(C_{m+1}[p(\bar{t})], c_m, c_F') < w_t(C_1[q(\bar{v})], c_1, c_F'), \) since \( \chi \) is a split derivation. Therefore \( w_t(C_{m+1}[H || \text{tell}(\bar{t} = s)], c_m, c_F') \) is a split derivation. Then \( w_t(C_{m+1}[H || \text{tell}(\bar{t} = s)], c_m, c_F') \) and then, by definition, \( \xi \) is a split derivation in \( D_1 \cup D_0 \). This, together with (61), implies the thesis.

**Case 2:** A tell constraint in \( d \) is eliminated or introduced.

In the first case, let \( d' \) be the corresponding declaration in \( D_1 \). Therefore the situation is the following:
- \( d: \ q(\bar{t}) \leftarrow \text{C[tell}(\bar{t} = \bar{s}) \ || H] \)
- \( d': \ q(\bar{t}) \leftarrow \text{C[H} \sigma] \)

where \( \sigma \) is a relevant most general unifier of \( \bar{s} \) and \( \bar{t} \) and the variables in the domain of \( \sigma \) do not occur neither in \( C[ ] \) nor in \( q(\bar{t}) \). Observe that for any derivation which uses the declaration \( d \), we can construct another derivation such that the agent \( \text{tell}(\bar{t} = \bar{s}) \) is evaluated before \( H \). Then the thesis follows from Lemma A.5 and from the argument used in the proof of Case 2 of Proposition 4.5. The proof for the tell introduction is analogous and hence it is omitted.

**Case 3:** \( d \) is backward instantiated.

Let \( d' \) be the corresponding declaration in \( D_1 \). The situation is the following:
- \( d: \ q(\bar{t}) \leftarrow \text{C[p}(\bar{t})] \in D_{i-1}, \)
- \( d': \ q(\bar{t}) \leftarrow \text{C[p}(\bar{t}) \ || \text{tell}(b) \ || \text{tell}(\bar{t} = \bar{s})] \in D_1, \)

where \( c: \ p(\bar{s}) \leftarrow \text{tell}(b) \ || B \in D_{i-1} \) has no variable in common with \( d \) (the case \( d': \ q(\bar{t}) \leftarrow \text{C[p}(\bar{t}) \ || \text{tell}(\bar{t} = \bar{s})] \) is analogous and hence omitted). We distinguish two cases:

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1) There is no procedure call to $p(\tilde{t})$ in the first $m$ steps. Therefore $\chi$ has the form

$$
(D_{i-1}.C_i[q(\bar{v})], c_i) \rightarrow (D_{i-1}.C_i[C[p(\tilde{t})] \parallel \text{tell}(\bar{v} = \bar{r})], c_i) \rightarrow^* (D_{i-1}.C_m[p(\tilde{t})], c_m) \rightarrow (D_0.C_{m+1}[p(\tilde{t})], c_m) \rightarrow^* (D_0.B'', c''_f).
$$

Without loss of generality, we can assume that $\text{Var}(\chi) \cap \text{Var}(p(\tilde{t}) \parallel \text{tell}(b)) \parallel \text{tell}(\bar{t} = \bar{s})) = \text{Var}(\tilde{t})$. Then, by the definition of $D_i$, there exists a derivation corresponding to $\chi$.

$$
\xi_0 = (D_i.C_i[q(\bar{v})], c_i) \rightarrow (D_i.C_i[C[p(\tilde{t})] \parallel \text{tell}(b) \parallel \text{tell}(\bar{t} = \bar{s})] \parallel \text{tell}(\bar{v} = \bar{r}), c_i) \rightarrow^* (D_i.C_m[p(\tilde{t})] \parallel \text{tell}(b) \parallel \text{tell}(\bar{t} = \bar{s}), c_m) \rightarrow (D_0.C_{m+1}[p(\tilde{t})] \parallel \text{tell}(b) \parallel \text{tell}(\bar{t} = \bar{s}), c_m).
$$

Following the same reasoning as in Case 3 of Lemma A.5, we can prove that there exists a constraint $d_f$ such that

$$
\omega_t(C_{m+1}[p(\tilde{t})] \parallel \text{tell}(b) \parallel \text{tell}(\bar{t} = \bar{s}), c_m, d_f) \leq \omega_t(C_{m+1}[p(\tilde{t})], c_m, c''_f)
$$

where $\exists \text{Var}(C_{m+1}[p(\tilde{t})], c_m, d_f) = \exists \text{Var}(C_{m+1}[p(\tilde{t}), c_m]c''_f$ and $t = m(B'', c''_f)$. The rest of the proof is analogous to Case 1 (unfolding) and hence it is omitted.

2) There exists $k < m$ such that the $k$-th derivation step of $\chi$ is the procedure call $p(\tilde{t})$. We distinguish two more cases:

2a) $p \neq q$. In this case we can assume, without loss of generality, that $\chi$ has the form

$$
(D_{i-1}.C_i[q(\bar{v})], c_i) \rightarrow (D_{i-1}.C_i[C[p(\tilde{t})] \parallel \text{tell}(\bar{v} = \bar{r}), c_i) \rightarrow^* (D_{i-1}.C_k[p(\tilde{t})], c_k) \rightarrow (D_{i-1}.C_k[\text{tell}(b) \parallel \bar{B} \parallel \text{tell}(\bar{t} = \bar{s}'), c_k] \rightarrow^* (D_{i-1}.A_m, c_m) \rightarrow (D_0.A_{m+1}, c_m) \rightarrow^* (D_0.B'', c''_f)
$$

where $c' = p(\bar{s}') \leftarrow \text{tell}(\bar{b}) \parallel \bar{B}$ is a renaming of $c$ such that $\text{Var}(c') \cap \text{Var}(d') = \emptyset$.

In this case there exists a derivation

$$
(D_i.C_i[q(\bar{v})], c_i) \rightarrow (D_i.C_i[C[p(\tilde{t})] \parallel \text{tell}(b) \parallel \text{tell}(\bar{t} = \bar{s})] \parallel \text{tell}(\bar{v} = \bar{r}), c_i) \rightarrow^* (D_i.C_k[\text{tell}(b) \parallel \bar{B} \parallel \text{tell}(\bar{t} = \bar{s'}), c_k).\n$$

Observe now that, given any set of declarations, if there exists a derivation $\chi'$ for the configuration $\langle C'[\text{tell}(b) \parallel \bar{B} \parallel \text{tell}(\bar{t} = \bar{s'}), c'\rangle$ where $c'$ is satisfiable and $\text{Var}(C', c') \cap \text{Var}(b, \bar{s}) = \emptyset$, then there exists a derivation for $\langle C'[\text{tell}(b) \parallel \bar{B} \parallel \text{tell}(\bar{t} = \bar{s'}) \parallel \text{tell}(\bar{t} = \bar{s}), c'\rangle$ which performs the same steps of $\chi'$ plus (possibly) two steps corresponding to the evaluation of $\text{tell}(b)$ and $\text{tell}(\bar{t} = \bar{s})$. Since $(\bar{t} = \bar{s'}) \land (\bar{t} = \bar{s})$ is logically equivalent to $(\bar{t} = \bar{s'}) \land (\bar{t} = \bar{s})$, we can substitute $\text{tell}(\bar{t} = \bar{s'}) \parallel \text{tell}(\bar{t} = \bar{s})$ for $\text{tell}(\bar{t} = \bar{s'}) \parallel \text{tell}(\bar{t} = \bar{s})$. Moreover, since $p(\bar{s'}) \leftarrow \text{tell}(b) \parallel \bar{B}$ is a renaming of $c$ and therefore $D \models (\bar{b} \land (\bar{t} = \bar{s'})) \rightarrow \bar{b}$ holds, we can drop the agent $\text{tell}(b)$.

Finally, observe that $\bar{s'} = \bar{s}$ can be reduced to a conjunction of equations of the form $X = \bar{y}$, where $X \subseteq \text{Var}(\bar{s})$ and $\bar{y} \subseteq \text{Var}(\bar{s'})$ are distinct variables. Therefore, we can drop the constraint $\text{tell}(\bar{s'} = \bar{s})$, since the declarations used in the derivation are renamed apart and $\text{Var}(C'[\text{tell}(b) \parallel \bar{B} \parallel \text{tell}(\bar{t} = \bar{s'}), c') \cap \text{Var}(\bar{s}) = \emptyset$. Then the thesis holds for this case.

2b) $p = q$. In this case, the situation is the following:

- $d : p(\bar{r}) \leftarrow \text{tell}(b') \parallel C''[p(\tilde{t})] \in D_{i-1}$.
- $d'$: $p(\bar{r}) \leadsto \text{tell}(b') \parallel C'[p(\bar{t})] \parallel \text{tell}(\bar{t} = \bar{s})] \in D_i$,
where $c: p(\bar{s}) \leadsto \text{tell}(b) \parallel C'[p(\bar{u})]$ is a renaming of $d$ which has no variables in common with $d$.

Let $c' = p(\bar{s}') \leadsto \text{tell}(b) \parallel C'[p(\bar{u}')]$ be a renaming of $c$ such that $\text{Var}(c') \cap \text{Var}(d') = \emptyset$.

Now the proof is analogous to the previous one by observing that, for any set of declarations, if there exists a derivation $\chi'$ for $\langle C'[\text{tell}(b) \parallel C'[p(\bar{u}')] \parallel \text{tell}(\bar{t} = \bar{s}')] \parallel \text{tell}(b) \parallel \text{tell}(\bar{t} = \bar{s}) \rangle$, $c'$ which performs the same steps of $\chi'$, plus some tell actions (analogously to the previous case, we can drop the tell agents $\text{tell}(b)$ and $\text{tell}(\bar{t} = \bar{s})$). This concludes the proof of this case.

**Case 4:** An ask guard in $d$ is simplified. Let
- $d$: $q(\bar{r}) \leadsto C[\sum_{i=1}^{n} \text{ask}(c_i) → B_j],$
- $d'$: $q(\bar{r}) \leadsto C[\sum_{i=1}^{n} \text{ask}(c_i') → B_j] \in D_i$,
where for $j \in [1, n]$, $D_i \models \exists_{\text{Var}(q(\bar{r}), C, B_j)} (pc(C[ ]]) \land c_j) \iff (pc(C[ ]]) \land c_j)$ and $d \in D_{i-1}$ is the declaration to which the guard simplification was applied.

By the definition of split derivation $\chi$ has the form

$$
\chi = \langle D_{i-1}.C_i[q(\bar{v})], c_i \rangle \rightarrow \langle D_{i-1}.C_i[C[\sum_{j=1}^{n} \text{ask}(c_j) → B_j] \parallel \text{tell}(\bar{v} = \bar{r})], c_i \rangle \rightarrow^*
\langle D_{i-1}.C_i[C[\sum_{j=1}^{n} \text{ask}(c_j) → B_j], c_m] \rightarrow \langle D_i.A_{m+1}, c_m \rangle \rightarrow^* \langle D_i.B', c'', \bar{c}_F \rangle
$$

Since by the inductive hypothesis for any agent $A$, $O(D_0.A) = O(D_{i-1}.A)$, it is easy to check that there exists a derivation

$$
\chi' = \langle D_{i-1}.C_i[q(\bar{v})], c_i \rangle \rightarrow \langle D_{i-1}.C_i[C[\sum_{j=1}^{n} \text{ask}(c_j) → B_j] \parallel \text{tell}(\bar{v} = \bar{r})], c_i \rangle \rightarrow^*
\langle D_{i-1}.C_i[C[\sum_{j=1}^{n} \text{ask}(c_j) → B_j], c_m] \rightarrow^* \langle D_{i-1}.C_i[\sum_{j=1}^{n} \text{ask}(c_j) → B_j], c_m+h] \rightarrow^* \langle D_{i-1}.B, \bar{c}_F \rangle
$$

such that $\exists_{\text{Var}(C_i[q(\bar{v})], c_i)} \bar{c}_F = \exists_{\text{Var}(C_i[q(\bar{v})], c_i)} c''_F$ and $m(B, \bar{c}_F) = m(B', c'', \bar{c}_F)$. From (57) it follows that

$$
\exists_{\text{Var}(C_i[q(\bar{v})], c_i)} \bar{c}_F = \exists_{\text{Var}(C_i[q(\bar{v})], c_i)} c''_F \land m(B, \bar{c}_F).
$$

Without loss of generality, we can assume that $\chi'$ is chosen in such a way that the first $m + h$ steps of $\chi'$ do not use rule R2 and that $h$ is maximal, in the sense that either $c_{m+h}$ is not satisfiable or in the $m + h + 1$-th step we can only use rule R2.

In the first case, let $C_{m+h}$ be the context obtained from $C_{m+h}$ as follows: any (renamed) occurrence of the agent $\sum_{j=1}^{n} \text{ask}(c_j) → B_j$ in $C_{m+h}$, introduced in $\chi_0$ by a procedure call of the form $q(\bar{s})$, is replaced by a (suitably renamed) occurrence of the agent $\sum_{j=1}^{n} \text{ask}(c_j) → B_j$. Then, by definition of $D_i$, we have that

$$
\xi = \langle D_i.C_i[q(\bar{v})], c_i \rangle \rightarrow \langle D_i.C_i[C[\sum_{j=1}^{n} \text{ask}(c_j) → B_j] \parallel \text{tell}(\bar{v} = \bar{r})], c_i \rangle \rightarrow^*
\langle D_i.C_i'[\sum_{j=1}^{n} \text{ask}(c_j) → B_j], c_{m+h} \rangle
$$

is a derivation in $D_i$ which does not use rule R2 and such that

$$
m(C_{m+h}[\sum_{j=1}^{n} \text{ask}(c_j) → B_j], c_{m+h}) = m(B, c_F) = ff.
$$

Then the thesis follows by definition of split derivation.

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Now assume that $c_{m+h}$ is satisfiable. By Lemma A.6 and (65), there exists a constraint $d$, such that $\text{Var}(d) \subseteq \text{Var}(C_{m+h}[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_{m+h})$ and

$$w_t(C_{m+h}[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_{m+h}, \bar{d}) \leq w_t(C_t[q(\bar{v})], c_t, c_F)$$  \hspace{1cm} (66)

where

$$\exists_{- \text{Var}(C_t[q(\bar{v})], c_t)} \bar{d} = \exists_{- \text{Var}(C_t[q(\bar{v})], c_t)} c_F$$ and $t = m(B, c_F)$. \hspace{1cm} (67)

By definition of weight, by (66) and since $\text{Var}(d) \subseteq \text{Var}(C_{m+h}[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_{m+h})$, there exists a derivation

$$\langle D_0, C_{m+h}[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_{m+h} \rangle \rightarrow^* \langle D_0, \tilde{B}', \bar{d}' \rangle$$

such that $\exists_{- \text{Var}(C_{m+h}[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_{m+h})} \bar{d}' = \bar{d}$ and $m(\tilde{B}', \bar{d}') = t$. Then, by the definition of weight and by (66),

$$w_t(C_{m+h}[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_{m+h}, \bar{d}') \leq w_t(C_t[q(\bar{v})], c_t, c_F)$$ \hspace{1cm} (68)

holds. Without loss of generality, we can assume that $\text{Var}(\bar{d}') \cap \text{Var}(C_t[q(\bar{v})], c_t) \subseteq \text{Var}(C_{m+h}, c_{m+h})$. Therefore, from (67) it follows that

$$\exists_{- \text{Var}(C_t[q(\bar{v})], c_t)} \bar{d}' = \exists_{- \text{Var}(C_t[q(\bar{v})], c_t)} c_F$$ and $m(\tilde{B}', \bar{d}') = m(B, c_F)$.

Let $\tilde{B}' = C'_{m+h}[\sum_{j=1}^{n} \text{ask}(c'_j) \rightarrow B_j]$ be the agent obtained from

$$\tilde{B} = C_{m+h}[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j]$$

as follows: any (renamed) occurrence of the agent $\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j$ in $C_{m+h}[\ ]$, introduced in $\chi_0$ by a procedure call of the form $q(\bar{s})$, is replaced by a (suitably renamed) occurrence of the agent $\sum_{j=1}^{n} \text{ask}(c'_j) \rightarrow B_j$. By the definition of $D_t$ and since $\langle D_{t-1}, C_t[q(\bar{v})], c_t \rangle \rightarrow^* \langle D_{t-1}, C_{m+h}[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_{m+h} \rangle$, there exists a derivation

$$\xi_0 = \langle D_t, C_t[q(\bar{v})], c_t \rangle \rightarrow^* \langle D_t, C_{m+h}[\sum_{j=1}^{n} \text{ask}(c'_j) \rightarrow B_j], c_{m+h} \rangle,$$

which does not use rule R2. Observe that, by construction, $\tilde{B}$ has the form $A_1 || \ldots || A_l$, where $A_j$ is either a choice agent or $\text{Stop}$ for each $j \in [1, l]$. Moreover, since the first $m + h$ steps of $\chi_0$ do not use rule R2 (and therefore, it is not possible to evaluate a procedure call of the form $q(\bar{s})$ inside a guarding context), $\tilde{B}'$ will have the form $A'_1 || \ldots || A'_l'$, where either $A'_j = A_j$ or $A'_j$ is a (renamed) occurrence of the agent $\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j$ while $A'_j'$ is a (suitably renamed) occurrence of the agent $\sum_{j=1}^{n} \text{ask}(c'_j) \rightarrow B_j$. By Lemma A.1, $D \vdash c_{m+h} \rightarrow pc(C'[\ ])$, where $C'[\ ]$ is a renamed version of the context $C[\ ]$ in $\tilde{B}$, which was introduced in $\chi_0$ by a procedure call of the form $q(\bar{s})$. 
Now from the definition of derivation and of ask simplification it follows that, if \( \text{ask}(\xi_j) \rightarrow \bar{B}_j \) is a choice branch in \( \bar{B} \) and \( \text{ask}(\xi'_j) \rightarrow \bar{B}_j \) is the corresponding choice branch in \( \bar{B}' \), then

\[
\mathcal{D} \models \exists_{- \text{Var}(\bar{B}_j, c_{m+h})} (c_{m+h} \land \bar{z}_j) \rightarrow (c_{m+h} \land \bar{z}'_j)
\]

holds. Therefore, by using the same arguments as in Case 4 of Proposition 4.5, since (by inductive hypothesis) \( D_0 \) is weight complete and \( \langle D_0, C_{m+h} \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j, c_{m+h} \rangle \) is weight complete, we obtain that there exists a split derivation in \( D_0 \) of the form

\[
\nu = \langle D_0, C'_{m+h} \sum_{j=1}^n \text{ask}(c'_j) \rightarrow B_j, c_{m+h} \rangle \rightarrow \langle D_0, B_{m+h+1}, c_{m+h} \rangle \rightarrow^* \langle D_0, B', c_F' \rangle
\]

such that \( \exists_{- \text{Var}(C'_{m+h} \sum_{j=1}^n \text{ask}(c'_j) \rightarrow B_j, c_{m+h})} c_F' = \exists_{- \text{Var}(C_{m+h} \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j, c_{m+h})} d' \) and \( m(B', c_F') = m(B', d') \).

Then, by using the same arguments as in Case 4 of Proposition 4.5, from the definition of weight and from (68) it follows that

\[
\begin{align*}
    w_t(C'_{m+h} \sum_{j=1}^n \text{ask}(c'_j) \rightarrow B_j, c_{m+h}, c_F') &= \sum_{j=1}^n w_t(C_{m+h} \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j, c_{m+h}, d') = \sum_{j=1}^n w_t(C_{m+h} \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j, c_{m+h}, d') \leq w_t(C \tilde{q}(\tilde{v}), c_1, c_F),
\end{align*}
\]

where \( t = m(B', c_F') \). Moreover, we can assume without loss of generality that

\[
\text{Var}(\xi_0) \cap \text{Var}(\nu) = \text{Var}(C'_{m+h} \sum_{j=1}^n \text{ask}(c'_j) \rightarrow B_j, c_{m+h}).
\]

Then by (69) we obtain

\[
\exists_{- \text{Var}(C_1 \tilde{q}(\tilde{v}), c_1, c_F') = \exists_{- \text{Var}(C_1 \tilde{q}(\tilde{v}), c_1, c_F)} c_F' = m(B, c_F)},
\]

and therefore, by definition of weight,

\[
\begin{align*}
    w_t(C \tilde{q}(\tilde{v}), c_1, c_F') &= w_t(C \tilde{q}(\tilde{v}), c_1, c_F),
\end{align*}
\]

holds. By Lemma A.7 and by construction of \( C'_{m+h} \sum_{j=1}^n \text{ask}(c'_j) \rightarrow B_j \)

\[
\xi_1 = \langle D_1, C'_{m+h} \sum_{j=1}^n \text{ask}(c'_j) \rightarrow B_j, c_{m+h} \rangle \rightarrow \langle D_0, B_{m+h+1}, c_{m+h} \rangle \rightarrow^* \langle D_0, B', c_F' \rangle
\]

is a split derivation in \( D_1 \cup D_0 \). By the definition of split derivation \( w_t(B_{m+h+1}, c_{m+h}, c_F') < w_t((C'_{m+h} \sum_{j=1}^n \text{ask}(c'_j) \rightarrow B_j, c_{m+h}, c_F'), \text{where } t = m(B', c_F') \). Then, by (72) and (70), we have that

\[
\begin{align*}
    w_t(B_{m+h+1}, c_{m+h}, c_F') < w_t(C \tilde{q}(\tilde{v}), c_1, c_F').
\end{align*}
\]

Finally,

\[
\begin{align*}
    \xi &= \langle D_1, C_1 \tilde{q}(\tilde{v}), c_1 \rangle \rightarrow^* \langle D_1, C'_{m+h} \sum_{j=1}^n \text{ask}(c'_j) \rightarrow B_j, c_{m+h} \rangle \rightarrow \langle D_0, B_{m+h+1}, c_{m+h} \rangle \rightarrow^* \langle D_0, B', c_F' \rangle.
\end{align*}
\]
is a derivation in $D_i \cup D_0$. By construction the first $m + h$ steps of $\xi$ do not use rule $R2$, the $m + h + 1$-th step uses rule $R2$. Thus the thesis follows from (73) and (71).

**Case 5:** $d$ is the declaration to which either a branch elimination or an ask elimination was applied. In the case of branch elimination the proof follows immediately from the fact that we consider also the inconsistent results of non-terminated computations. As for the ask elimination case, let us assume that

- $d : q(\bar{r}) \leftarrow C[\text{ask(true)} \rightarrow H] \in D_{i-1}$ and
- $d' : q(\bar{r}) \leftarrow C[H] \in D_i$.

We show, by induction on the weight $w_t(C_i[q(\bar{v})], c_i, c_i')$, that there exists a split derivation $\xi = \langle D_i, C_i[q(\bar{v})], c_i \rangle \rightarrow^* \langle D_0, B', c_i' \rangle$ in $D_i \cup D_0$, such that $\exists_{\text{Var}(C_i[q(\bar{v})], c_i)} c_i' = \exists_{\text{Var}(C_i[q(\bar{v})], c_i)} c_i'. \quad (71)$

Moreover, by definition of split derivation, $B'' = C_i[\text{ask(true)} \rightarrow H]$, $\chi$ has the form

$$\chi = \langle D_{i-1}, C_i[q(\bar{v})], c_i \rangle \rightarrow \langle D_{i-1}, C_i[C[\text{ask(true)} \rightarrow H] \parallel \text{tell}(\bar{v} = \bar{r})], c_i \rangle \rightarrow^* \langle D_{i-1}, C_i[\text{ask(true)} \rightarrow H], c_i' \rangle,$$

rule $R2$ is not used and therefore each derivation step is done in $D_{i-1}$. Moreover, observe that since $t \in \{ss, dd, ff\}$, if $c_i'$ is satisfiable, then $C_k$ is a guarding context. Then, it is easy to check that

$$\xi = \langle D_i, C_i[q(\bar{v})], c_i \rangle \rightarrow \langle D_i, C_i[C[H] \parallel \text{tell}(\bar{v} = \bar{r})], c_i \rangle \rightarrow^* \langle D_i, C_k[H], c_i' \rangle$$

is a split derivation in $D_i \cup D_0$, such that $m(C_k[H], c_i') = m(C_i[\text{ask(true)} \rightarrow H], c_i') \in \{dd, ff\}$ and then the proof follows by (57).

**Base case.** In this case $w_t(C_i[q(\bar{v})], c_i, c_i') = 0$ and by definition of split derivation, $B'' = C_i[\text{ask(true)} \rightarrow H]$, $\chi$ has the form

$$\chi = \langle D_{i-1}, C_i[q(\bar{v})], c_i \rangle \rightarrow \langle D_{i-1}, C_i[C[\text{ask(true)} \rightarrow H] \parallel \text{tell}(\bar{v} = \bar{r})], c_i \rangle \rightarrow^* \langle D_{i-1}, C_i[\text{ask(true)} \rightarrow H], c_i' \rangle,$$

and therefore, by inductive hypothesis there exists a split derivation in $D_i \cup D_0$,

$$\xi_0 = \langle D_i, C_i[q(\bar{v})], c_i \rangle \rightarrow \langle D_i, C_i[C[H] \parallel \text{tell}(\bar{v} = \bar{r})], c_i \rangle \rightarrow^* \langle D_i, C_m[H], c_m \rangle,$$

which does not use rule $R2$. Moreover, by definition of split derivation

$$w_t(C_m[H], c_m, c_i') < w_t(C_i[q(\bar{v})], c_i, c_i')$$

and therefore, by inductive hypothesis there exists a split derivation in $D_i \cup D_0$,

$$\xi_1 = \langle D_i, C_m[H], c_m \rangle \rightarrow^* \langle D_0, B'', c_i' \rangle,$$

such that

$$m(B'', c_i') = m(B'', c_i') \in \{t, ss, dd, ff\} \quad (74)$$
Without loss of generality, we can assume that \( \text{Var}(\xi_0) \cap \text{Var}(\xi_1) \subseteq \text{Var}(C_m[H], c_m) \). Therefore, by (74) and by definition of \( c_F^P \) and \( c_F^{P'} \),

\[
\exists \neg \text{Var}(C_i(q(\bar{v})), c_i) c_F^P = \\
\exists \neg \text{Var}(C_i(q(\bar{v})), c_i)(c_m \wedge \exists \neg \text{Var}(C_m[H], c_m) c_F^P) = \\
\exists \neg \text{Var}(C_i(q(\bar{v})), c_i)(c_m \wedge \exists \neg \text{Var}(C_m[H], c_m) c_F^{P'}) = \\
\exists \neg \text{Var}(C_i(q(\bar{v})), c_i) c_F^{P'}.
\]

Then by definition of weight, since \( w_t(C_m[H], c_m, c_F^{P'}) < w_t(C_i[q(\bar{v})], c_i, c_F^P) \) and by (74)

\[
w_t(C_m[H], c_m, c_F^{P'}) < w_t(C_i[q(\bar{v})], c_i, c_F^P).
\]

Moreover, by our hypothesis on the variables of \( \xi_0 \) and of \( \xi_1 \), there exists a derivation \( D_i \cup D_0 \),

\[
\xi = \langle D_i, C_i[q(\bar{v})], c_i \rangle \Rightarrow \langle D_i, C_i[C[H] || \text{tell}(\bar{v} = \bar{r})], c_i \rangle \Rightarrow^* \langle D_i, C_m[H], c_m \rangle \Rightarrow^* \langle D_0, B', c_F^P \rangle.
\]

By (74), (76), since \( \xi_0 \) do not use Rule R2 and \( \xi_1 \) is a split derivation in \( D_i \cup D_0 \), we have that \( \xi \) is a split derivation in \( D_i \cup D_0 \), such that \( m(B', c_F^P) = m(B'', c_F^{P'}) \). Now, the thesis follows by (75).

**Case 6:** An ask guard in \( d \) is distributed. Let

- \( d : q(\bar{r}) \leftrightarrow C[H || \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j] \in D_{i-1} \)
- \( d' : q(\bar{r}) \leftarrow C[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H || B_j)] \in D_i \),

where, for every constraint \( c' \) such that \( \exists \neg \text{Var}(c') \cap \text{Var}(d) \subseteq \text{Var}(q(\bar{r}), C_i) \), if \( \langle D_{i-1}, H, c', \text{pc}(C_i[[i]]) \rangle \) is productive then there exists at least one \( j \in [1, n] \) such that \( D \models (c' \wedge \text{pc}(C_i[[i]])) \rightarrow c_j \) and for each \( j \in [1, n] \), either \( D \models (c' \wedge \text{pc}(C_i[[i]])) \rightarrow -c_j \) or \( D \models (c' \wedge \text{pc}(C_i[[i]])) \rightarrow \neg c_j \).

By the definition of split derivation, \( \chi \) has the form

\[
\chi = \langle D_{i-1}, C_i[q(\bar{v})], c_i \rangle \Rightarrow \langle D_{i-1}, C_i[C[H || \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j] || \text{tell}(\bar{v} = \bar{r})], c_i \rangle \Rightarrow^* \langle D_{i-1}, C_m[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_m \rangle \Rightarrow \langle D_0, A_{m+1}, c_m \rangle \Rightarrow^* \langle D_0, B'', c_F^{P'} \rangle.
\]

If the first \( m-1 \) steps of \( \chi \) do not evaluate the agent \( H \) then the proof is analogous to that one of Case 6 of Lemma A.5. Otherwise, let us assume that

\[
\chi = \langle D_{i-1}, C_i[q(\bar{v})], c_i \rangle \Rightarrow \langle D_{i-1}, C_i[C[H || \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j] || \text{tell}(\bar{v} = \bar{r})], c_i \rangle \Rightarrow^* \langle D_{i-1}, C_i[C[H || \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_m \rangle \Rightarrow \langle D_0, A_{m+1}, c_m \rangle \Rightarrow^* \langle D_0, B'', c_F^{P'} \rangle.
\]

Since by the inductive hypothesis for any agent \( A \), \( O(D_0.A) = O(D_{i-1}.A) \) there exists a derivation

\[
\chi' = \langle D_{i-1}, C_i[q(\bar{v})], c_i \rangle \Rightarrow \langle D_{i-1}, C_i[C[H || \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j] || \text{tell}(\bar{v} = \bar{r})], c_i \rangle \Rightarrow^* \langle D_{i-1}, C_i[C[H || \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_m \rangle \Rightarrow \langle D_0, B', c_F^{P'} \rangle,
\]

where \( \exists \neg \text{Var}(C_i[q(\bar{v})], c_i) c_F = \exists \neg \text{Var}(C_i[q(\bar{v})], c_i) c_F^{P'} \) and \( m(B', c_F) = m(B'', c_F') \). By (57),

\[
\exists \neg \text{Var}(C_i[q(\bar{v})], c_i) c_F = \exists \neg \text{Var}(C_i[q(\bar{v})], c_i) c_F^{P'} \text{ and } m(B', c_F) = m(B', c_F').
\]

Without loss of generality we can assume that the first \( k \) steps of \( \chi' \) neither use Rule R2 nor contain the evaluation of any (renamed) occurrence \( H \) of the agent \( H \), where \( q(\bar{r}) \leftrightarrow C[H || \sum_{j=1}^{n} \text{ask}(\bar{e}_j) \rightarrow B_j] \) is a renamed version of the declaration \( d \)
and \(\tilde{C} | H \parallel \sum_{j=1}^{n} \text{ask}(\tilde{c}_j) \rightarrow B_j\) has been introduced by the evaluation of a procedure call of the form \(q(\tilde{s})\). Moreover, we can assume that \(k\) is maximal, in the sense that either \(c_k\) is not satisfiable or the \(k+1\)-th step can only either use rule \(R2\) or evaluate a (renamed) occurrence of \(H\) introduced by a procedure call of the form \(q(\tilde{s})\). If \(c_k\) is not satisfiable, then the proof is analogous to that one of the previous Case 4.

Assume then that \(c_k\) is satisfiable. By Lemma A.6 and (77), there exists a constraint \(\tilde{d}\), such that \(\text{Var}(\tilde{d}) \subseteq \text{Var}(C_k[H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_k)\) and

\[
\omega_t(C_k[H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_k, \tilde{d}) \leq \omega_t(C_i[q(\tilde{v})], c_i, c_f),
\]

(78)

where

\[
\exists_{- \text{Var}(C_i[q(\tilde{v})], c_i)} \tilde{d} = \exists_{- \text{Var}(C_i[q(\tilde{v})], c_i)} c_f \text{ and } t = m(B, c_f).
\]

(79)

By definition of weight, by (78) and since \(\text{Var}(\tilde{d}) \subseteq \text{Var}(C_k[H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_k)\), there exists a derivation

\[
\langle D_0, C_k[H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_k \rangle \rightarrow^* \langle D_0, \tilde{B}', \tilde{d}' \rangle
\]

such that \(\exists_{- \text{Var}(C_i[H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_i)} \tilde{d}' = \tilde{d}\) and \(m(\tilde{B}', \tilde{d}') = t\). Then, by the definition of weight and by (78),

\[
\omega_t(C_k[H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_k, \tilde{d}') \leq \omega_t(C_i[q(\tilde{v})], c_i, c_f).
\]

(80)

Without loss of generality, we can assume that \(\text{Var}(\tilde{d}') \cap \text{Var}(C_i[q(\tilde{v})], c_i) \subseteq \text{Var}(C_k, c_k)\). Therefore from (79) it follows that

\[
\exists_{- \text{Var}(C_i[q(\tilde{v})], c_i)} \tilde{d}' = \exists_{- \text{Var}(C_i[q(\tilde{v})], c_i)} c_f \text{ and } m(\tilde{B}', \tilde{d}') = m(B, c_f).
\]

(81)

Let \(C_k[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j)]\) be the agent obtained from \(C_k[H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j]\) as follows: any (renamed) occurrence of the agent \(H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j\) in \(C_k\) which has been introduced by a procedure call of the form \(q(\tilde{s})\) is replaced by a (suitably) renamed occurrence of the agent \(\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j)\).

By the definition of \(D_i\) and since \(\langle D_{i-1}, C_i[q(\tilde{v})], c_i \rangle \rightarrow^* \langle D_{i-1}, C_i[H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_k \rangle\), there exists a derivation

\[
\xi_0 = \langle D_1, C_i[q(\tilde{v})], c_i \rangle \rightarrow^* \langle D_1, C_i[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j)], c_k \rangle
\]

which does not use rule \(R2\).

Now, by construction, \(C_i[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j)]\) has the form \(A_1 \parallel \ldots \parallel A_i\), where \(A_i\) is either a choice agent or \textbf{Stop}.

Moreover, since \(D_0\) is weight complete, \(\langle D_0, C_k[H \parallel \sum_{j=1}^{n} \text{ask}(c_j) \rightarrow B_j], c_k \rangle \rightarrow^* \langle D_0, \tilde{B}', \tilde{d}' \rangle\) and analogously to the Case 6 of Lemma A.5, there exists a split derivation

\[
\xi_1 = \langle D_0, C_k[\sum_{j=1}^{n} \text{ask}(c_j) \rightarrow (H \parallel B_j)], c_k \rangle \rightarrow^* \langle D_0, \tilde{B}', c_f \rangle,
\]
where $C$ exists a derivation

where, by definition of folding,

Case 7: and therefore it is omitted.

which performs exactly the first $m$ steps as $\chi$. Since $(D_0, C_m+1[H], c_m) \xrightarrow{*} (D_0, B'', c_F')$, the definition of weight implies that $w_t(C_{m+1}[H], c_m, c_F')$ is defined, where $t = m(B'', c_F')$. Then, by (57), we have that

$t = m(B, c_F')$.  

(82)

The definitions of derivation and folding imply that $Var(H) \cap Var(C_{m+1}, c_m) \subseteq Var(H) \cap Var(C, \bar{r}) \subseteq Var(H)$ holds. Moreover, from the assumptions on the variables, we obtain that $Var(c_F') \cap Var(\bar{X}) \subseteq Var(H)$. Thus, from part 2 of Lemma A.4 it follows that there exists a constraint $d'$ such that

\[ w_t(C_{m+1}[p(\bar{X})], c_m, d') \leq w_t(C_{m+1}[H], c_m, c_F') \]  

and

\[ \exists_{Var(C_{m+1}[p(\bar{X})], c_m)} d' \]  

(83)

\[ \exists_{Var(C_{m+1}[H], c_m)} d' \]
Thus concluding the proof.

(b) Rule \( R_2 \) is a derivation in \( D \) and (86) it follows that \( w_t(C_{m+1}[p(\bar{X})], c_m, c'_F) = w_t(C_{m+1}[p(\bar{X})], c_m, d') \) and therefore, by (83),

\[
\exists_{Var(C_{m+1}[p(\bar{X})], c_m)} c'_F = \exists_{Var(C_{m+1}[p(\bar{X})], c_m)} c''_F
\]

\[
w_t(C_{m+1}[p(\bar{X})], c_m, c'_F) \leq w_t(C_{m+1}[p(\bar{X})], c_m, c''_F) \tag{84}
\]

holds. Moreover, from (82) we obtain

\[
m(B', c'_F) = m(B, c_F). \tag{85}
\]

Without loss of generality, we can now assume that

\[
Var(\xi_0) \cap Var(\xi_1) = Var(C_{m+1}[p(\bar{X})], c_m).
\]

Then, by (84) and (57) it follows that

\[
\exists_{Var(C_{i}[q(\bar{v})], c_i)} c'_F = \exists_{Var(C_{i}[q(\bar{v})], c_i)} (c_m \land \exists_{Var(C_{m+1}[p(\bar{X})], c_m)} c'_F) = \exists_{Var(C_{i}[q(\bar{v})], c_i)} c'_F = \exists_{Var(C_{i}[q(\bar{v})], c_i)} c'_F. \tag{86}
\]

From the definition of weight \( w_t(C_i[q(\bar{v})], c_i, c'_F) = w_t(C_i[q(\bar{v})], c_i, c''_F) \) and since \( \chi \) is a split derivation we obtain \( w_t(C_i[q(\bar{v})], c_i, c'_F) > w_t(C_{m+1}[H], c_m, c''_F) \). Then, from (84) and (86) it follows that

\[
w_t(C_i[q(\bar{v})], c_i, c'_F) > w_t(C_{m+1}[p(\bar{X})], c_m, c'_F) \tag{87}
\]

and therefore, by construction,

\[
\xi = \langle D_1, C_i[q(\bar{v})], c_i \rangle \rightarrow \langle D_1, C_i[C[p(\bar{X})] \parallel \text{tell}(\bar{v} = \bar{r}), c_i \rangle \rightarrow^* \langle D_1, C_m[p(\bar{X})], c_m \rangle \rightarrow \langle D_0, C_{m+1}[p(\bar{X})], c_m \rangle \rightarrow^* \langle D_0, B', c'_F \rangle
\]

is a derivation in \( D_1 \cup D_0 \) such that: (a) rule \( R_2 \) is not used in the first \( m - 1 \) steps; (b) rule \( R_2 \) is used in the \( m \)-th step. The thesis then follows from (86), (85) and (87) thus concluding the proof. \( \square \)

A.1 Proof of correctness for intermediate results and traces

In this subsection we show how the previous proofs can be adapted when considering intermediate results and traces as observables. We first consider Theorem 5.1. Since its proof is essentially the same of that one already given for the total correctness theorem, here we provide only the intuition illustrating the (minor) modifications needed.

Theorem 5.1 (Total Correctness 2). Let \( D_0, \ldots, D_n \) be a transformation sequence, and \( A \) be an agent.

---If there exists a derivation \( (D_0, A, c) \rightarrow^* (D_0, B, d) \) then there exists a derivation \( (D_n, A, c) \rightarrow^* (D_n, B', d') \) such that \( D \models \exists_{Var(A,c)} d' \rightarrow \exists_{Var(A,c)} d \).

---Conversely, if there exists a derivation \( (D_n, A, c) \rightarrow^* (D_n, B, d) \) then there exists a derivation \( (D_0, A, c) \rightarrow^* (D_0, B', d') \) with \( D \models \exists_{Var(A,c)} d' \rightarrow \exists_{Var(A,c)} d \).

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Proof. The proof of this result is essentially the same as that one of the total correctness Theorem 4.13 provided that in such a proof, as well as in the proofs of the related preliminary results, we perform the following changes:

(1) Rather than considering terminating derivations, we consider any (possibly non-maximal) finite derivation.

(2) Whenever in a proof we write that, given a derivation $\xi$, a derivation $\xi'$ is constructed which performs the same steps of $\xi$, possibly in a different order, we now write that a derivation $\xi''$ is constructed which performs the same steps as $\xi$ (possibly in a different order) plus some other additional steps. Since the store grows monotonically in ccp derivations, clearly if a constraint $c$ is the result of the derivation $\xi$, then a constraint $c''$ is the result of $\xi''$ such that $D |\Rightarrow c'' \rightarrow c$ holds. For example, for case 2 in the proof of Proposition 4.5 (in the Appendix), when considering a (non-maximal) derivation $\xi$ which uses the declaration $H \leftarrow C[tell(\bar{s} = \bar{t}) ] \parallel B$ we can always construct a derivation $\xi''$ which performs all the steps of $\xi$ (possibly plus others) and such that the $tell(\bar{s} = \bar{t})$ agent is evaluated before $B$. Differently from the previous proof, now we are not ensured that the result of $\xi$ is the same as that one of $\xi''$, since $\xi$ is non-maximal (thus, $\xi$ could also avoid the evaluation of $tell(\bar{s} = \bar{t})$). However, we are ensured that the result of $\xi''$ is stronger (i.e. implies) that one of $\xi$.

We now consider the correctness results given for the restricted transformation system with respect to the traces. Also in this case, the proofs follow the guidelines of that one already presented in Section 4 and in the previous part of this Appendix. We then sketch the proofs by showing which are the relevant new notions and differences with respect to the previous ones.

In the remainder of this section we will always refer to the restricted transformation system and to a given restricted transformation sequence $D_0, \ldots, D_n$.

We start with the following definition.

**Definition A.10.** Let $D$ be a set of declarations and let $\xi$ be the derivation $(D.A_1, c_1) \rightarrow^* (D.A_m, c_m) \rightarrow^* (D.A_n, c_n)$.

We define $\text{tr}(\xi) =$

\[ \exists \neg \text{Var}(A_1, c_1) (c_1; c_2; \ldots; c_n) = (c_1; (\exists \neg \text{Var}(A_1, c_1) c_2); \ldots; (\exists \neg \text{Var}(A_1, c_1) c_n)). \]

The function $\text{mode}$ ($m(A, d)$) is extended to consider also non-terminated derivations in the obvious way. We then extend the notion of weight, split derivation and weight complete programs to the case of traces. Here and in the following the subscript $t$ will denote a generic termination mode, that is, we assume $t \in \{ss, dd, pp, ff\}$. We also say that a trace starts with $c$ in case $c$ is the first constraint appearing in that trace.

**Definition A.11 (Weight for Traces).** Given an agent $A$, a satisfiable constraint $c$ and a trace $s$ starting with $c$, we define the weight of the agent $A$ w.r.t. the trace $s$, notation $\text{wt}(A, s)$, as follows:

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If there exists a derivation $O \models \langle D_0, A, c \rangle \rightarrow^* \langle D_0, B, d \rangle$

such that $\exists_{\forall \alpha(A,c)s \leq \text{tr}(\xi)} t = m(B,d)$. 

**Definition A.12 (Split Derivation for Traces).** Let $D_0, \ldots, D_i$ be a transformation sequence. We call a derivation in $D_i \cup D_0$ a split derivation for traces if it has the form

$\langle D_i, A_1, c_1 \rangle \rightarrow^* \langle D_i, A_m, c_m \rangle \rightarrow \langle D_0, A_{m+1}, c_{m+1} \rangle \rightarrow^* \langle D_0, A_n, c_n \rangle$

where $m \in [1, n]$ and the following conditions hold:

(a) the first $m - 1$ derivation steps do not use rule R2;
(b) the $m$-th derivation step $\langle D_i, A_m, c_m \rangle \rightarrow \langle D_0, A_{m+1}, c_{m+1} \rangle$ uses rule R2;
(c) $w_t(A_1, (c_1; c_2; \ldots; c_n)) > w_t(A_{m+1}, (c_{m+1}; \ldots; c_n))$, where $t = m(A_n, c_n)$.

**Definition A.13.** We call the program $D_i$ weight complete for traces iff, for any agent $A$ and any satisfiable constraint $c$ the following hold: If there exists a derivation

$\chi = \langle D_0, A, c \rangle \rightarrow^* \langle D_0, B, d \rangle$

such that $m(B,d) \in \{ss, dd, pp, ff\}$ then there exists a split derivation in $D_i \cup D_0$

$\xi = \langle D_i, A, c \rangle \rightarrow^* \langle D_i, B', d' \rangle$

where $\text{tr}(\chi) \leq \text{tr}(\xi)$ and $m(B', d') = m(B,d)$.

Proposition 4.9 holds also when considering as observables $O_i$ rather than $O$ and its proof is essentially the same, thus we omit it.

The following Lemma is obtained from Lemma A.1 by considering the weakest produced constraint wpc rather than the produced constraint. The proof is analogous to that one given for Lemma A.1 and hence it is omitted.

**Lemma A.14.** Assume that there exists a derivation $(D, C[A], c) \rightarrow^* (D, C'[A], c')$ where $c$ is a satisfiable constraint and the context $C[ ]$ has the form

$A_1 \| \ldots \| \tilde{C}[ ] \| \ldots \| A_n$.

Then $D \models (\text{wpc}(\tilde{C}[ ]) \land c') \rightarrow \text{wpc}(C[ ])$ holds and in case $\tilde{C}[ ]$ is the empty context also $D \models c' \rightarrow \text{wpc}(C[ ])$. holds.

In the following we extend to set of observables the (pre-order) relation $\preceq$ in the expected way: Given two sets of observables $O_i(D, A)$ and $O_i(D, A)$, we say that $O_i(D, A) \preceq O_i(D, A)$ iff, for any $(s, x) \in O_i(D, A)$, (with $x \in \{ss, dd, pp, ff\}$), there exists $(s', x) \in O_i(D, A)$ such that $s \preceq s'$. We denote by $\equiv$ the equivalence relation induced by $\preceq$ on sets of observables, that is, $O_i(D, A) \equiv O_i(D, A)$ iff $O_i(D, A) \preceq O_i(D, A)$ and $O_i(D, A) \preceq O_i(D, A)$.

The following is analogous of Proposition 4.5 for traces.

**Proposition A.15 (Partial Correctness for Traces).** If, for each agent $A$, $O_i(D_0, A) \equiv O_i(D_i, A)$ then, for each agent $A$, $O_i(D_{i+1}, A) \preceq O_i(D, A)$.

**Proof.** We have to show that, given an agent $A$ and a satisfiable constraint $c_i$, if there exists a derivation $\xi = \langle D_{i+1}, A, c_i \rangle \rightarrow^* \langle D_{i+1}, B, c_f \rangle$, then there exists also
a derivation \( \xi' = \langle D_i, A, c_i \rangle \rightarrow^* \langle D_i, B', c_i' \rangle \) such that \( \text{tr}(\xi) \leq \text{tr}(\xi') \) and \( m(B', c_i') = m(B, c_i) \).

The proof is analogous to that one given for Proposition 4.5, therefore we illustrate only the modifications needed to adapt such a proof.

Assume that the first step of derivation \( \xi \) uses rule \( R4 \) and let \( d' \in D_i+1 \) be the declaration used in the first step of \( \xi \). Assume also that \( d' \not\in D_i \) and that \( d' \) is the result of the transformation operation applied to obtain \( D_i+1 \). As usual, we distinguish various cases according to the kind of operation performed. Here we consider only those cases whose proof is different from that one of Proposition 4.5, due to the fact that here we consider traces (consisting of intermediate results) rather than the final constraints.

Case 2. In this case \( d : H \leftarrow C[\text{tell}(\bar{s} = \bar{t}) \parallel B] \in D_i, d' : H \leftarrow C[B\sigma] \in D_i+1 \), where \( \sigma \) is a relevant most general unifier of \( \bar{s} \) and \( \bar{t} \) (or a renaming, in case of \( \bar{s} \) and \( \bar{t} \) consist of distinct variables). From the definition of the operation we know that the variables in the domain of \( \sigma \) do not occur neither in \( C[\ ] \) nor in \( H \) and, differently from the case of Proposition 4.5, that \( \text{Var}(B) \cap \text{Var}(H, C) = \emptyset \).

For any derivation which uses a declaration \( H \leftarrow C[\text{tell}(\bar{s} = \bar{t}) \parallel B] \), if the agent \( \text{tell}(\bar{s} = \bar{t}) \) is evaluated before \( B \) then the proof is analogous to that one given for Case 2 of Proposition 4.5. Otherwise, if the agent \( \text{tell}(\bar{s} = \bar{t}) \) is not evaluated before \( B \), then by using the condition \( \text{Var}(B) \cap \text{Var}(H, C) = \emptyset \) we obtain that the evaluation of the agent \( B \) can add to the store only constraints on variables which do not occur neither in the global store (before the evaluation of \( B \)) nor in \( \text{Var}(A, c_i) \). Therefore the contribution to the global store of the agent \( B \) (before the evaluation of the agent \( \text{tell}(\bar{s} = \bar{t}) \)) when restricted to \( \text{Var}(A, c_i) \) is equivalent either to the constraint \( \text{true} \) or to the constraint \( \text{false} \).

In the first case the global store is the same as that one existing before the evaluation of \( B \). In the second case we can obtain the constraint \( \text{false} \) by evaluating the same agents evaluated in \( B \) also in \( B\sigma \).

Case 3. In this case the proof is analogous to that one given for Case 3 of Proposition 4.5 by observing the following: If in the derivation \( \chi \) in \( D_i \) either the agent \( \text{tell}(b) \) or the agent \( \text{tell}(\bar{t}) \) are evaluated, then in the derivation \( \chi' \) the agent \( p(\bar{t}) \) can be evaluated and then one performs exactly the same steps of \( \chi \), except for the evaluation of a renamed version of the agents \( \text{tell}(b) \) and \( \text{tell}(\bar{t}) \).

Cases 4. For the ask simplification the proof of Case 4 of Proposition 4.5 is simplified by using Lemma A.14 and by observing that, for any derivation, when the choice agent inside \( C[\ ] \) is evaluated the current store certainly implies \( \text{wpc}(C[]) \). Therefore we do not need to construct the new derivation \( \chi' \). The same holds for the tell simplification.

Case 7. In this case the proof is analogous to that given for the previous Case 2, by observing that in the derivation

\[
\beta = \langle D_0, C, I \mid C[H \parallel \text{tell}(\bar{X} = \bar{X}')] \mid \text{tell}(\bar{v} = \bar{r}), c_i \rangle \rightarrow^* \langle D_0, B', c_0 \rangle,
\]

\( \text{Var}(H') \cap \text{Var}(C, C, I, \bar{X}, \bar{v}, \bar{r}) = \emptyset \). Therefore we can construct a derivation

\[
\chi_0 = \langle D_0, C \mid C[H \parallel \text{tell}(\bar{X}' = \bar{X}')] \mid \text{tell}(\bar{v} = \bar{r}), c_i \rangle \rightarrow^* \langle D_0, B', c_0 \rangle
\]
where $\text{tr}(\beta) \preceq \text{tr}(\chi_0)$ and $m(B_0^c, c_0) = m(B_0^c, c_0)$. Moreover, we can drop the constraint $\text{tell}(\bar{X}' = \bar{X})$, since the declarations used in the derivation are renamed apart and, by construction, $\text{Var}(C_i[H] \parallel \text{tell}(\bar{v} = \bar{r})), c_i) \cap \text{Var}(\bar{X'}) = \emptyset$. We then obtain that there exists a derivation $\beta' = (D_0, C_i[H] \parallel \text{tell}(\bar{v} = \bar{r})), c_i) \rightarrow^* (D_0, \bar{B}_0, \bar{c}_0)$ which performs exactly the same steps of $\chi_0$ except for (possibly) the evaluation of $\text{tell}(\bar{X}' = \bar{X})$ and such that $\exists_{-\text{Var}(C_i[H] \parallel \text{tell}(\bar{v} = \bar{r})), c_i)} \text{tr}(\chi_0) \preceq \text{tr}(\beta')$ and $m(\bar{B}_0, \bar{c}_0) = m(B_0^c, c_0)$. Now, the proof is the same to that given for Case 7 of Proposition 4.5, since the evaluation of $\text{tell}(\bar{X}' = \bar{X})$ does not modify the current store with respect to the variables not in $\text{Var}(\bar{X'})$.

The following Lemmata are the counterpart of previous Lemma A.3 and Lemma A.4, when considering the observable $O_i(D_i, A_i)$.

**Lemma A.16.** Let $q(\bar{r}) \leftarrow H \in D_0$ and let $C[\ ]$ be context. For any satisfiable constraint $c$ and for any trace $s$ starting with $c$, such that $\text{Var}(C(q(\bar{r})), c) \cap \text{Var}(\bar{r}) = \emptyset$ and $w_i(C[q(\bar{r})], s)$ is defined, there exists a trace $s'$ such that $w_i(C[H], c), s' \leq w_i(C[q(\bar{r})], s)$ and $\exists_{-\text{Var}(C[q(\bar{r})], c)} s \preceq \exists_{-\text{Var}(C[q(\bar{r})], c)} s'$.

**Proof.** Immediate. \qed

**Lemma A.17.** Let $q(\bar{r}) \leftarrow H \in D_0$. For any context $C[\ ]$, any satisfiable constraint $c$ and for any sequence $s$ starting in $c$, the following holds:

1. If $\text{Var}(H) \cap \text{Var}(C_i, c) \subseteq \text{Var}(\bar{r})$ and $w_i(C[q(\bar{r})], s)$ is defined, then there exists a sequence $s'$, such that $\text{Var}(s') \subseteq \text{Var}(C_i[H], c), w_i(C_i[H], s') \leq w_i(C[q(\bar{r})], s)$ and $\exists_{-\text{Var}(C[q(\bar{r})], c)} s \preceq \exists_{-\text{Var}(C[q(\bar{r})], c)} s'$.
2. If $\text{Var}(H) \cap \text{Var}(C_i, c) \subseteq \text{Var}(\bar{r}), \text{Var}(s) \cap \text{Var}(\bar{r}) \subseteq \text{Var}(C_i[H], c)$ and $w_i(C_i[H], s)$ is defined, then there exists a sequence $s'$, such that $w_i(C[q(\bar{r})], s') \leq w_i(C_i[H], s)$ and $\exists_{-\text{Var}(C[q(\bar{r})], c)} s \preceq \exists_{-\text{Var}(C[q(\bar{r})], c)} s'$.

**Proof.** Immediate. \qed

Analogously to the case of the previous results, the following Lemma is crucial in the proof of completeness for traces.

**Lemma A.18.** Let $0 \leq i < n$, cl : $q(\bar{r}) \leftarrow H$ be a declaration in $D_i$ and let $cl' : q(\bar{r}) \leftarrow H'$ be the corresponding declaration in $D_{i+1}$ (in case $i < n$). For any context $C_i[\ ]$, any satisfiable constraint $c$ and for any sequence $s$ starting in $c$ the following holds:

1. If $\text{Var}(H) \cap \text{Var}(C_i, c) \subseteq \text{Var}(\bar{r})$ and $w_i(C_i[q(\bar{r})], s)$ is defined, then there exists a sequence $s'$, such that $\text{Var}(s') \subseteq \text{Var}(C_i[H], c), w_i(C_i[H], s') \leq w_i(C_i[q(\bar{r})], s)$ and $\exists_{-\text{Var}(C_i[q(\bar{r})], c)} s \preceq \exists_{-\text{Var}(C_i[q(\bar{r})], c)} s'$;
2. If $\text{Var}(H', H') \cap \text{Var}(C_i, c) \subseteq \text{Var}(\bar{r}), \text{Var}(c') \cap \text{Var}(\bar{r}) \subseteq \text{Var}(C_i[H], c)$ and $w_i(C_i[H], s)$ is defined, then there exists a sequence $s'$, such that $\text{Var}(s') \subseteq \text{Var}(C_i[H'], c), w_i(C_i[H'], s') \leq w_i(C_i[H], s)$ and $\exists_{-\text{Var}(C_i[q(\bar{r})], c)} s \preceq \exists_{-\text{Var}(C_i[q(\bar{r})], c)} s'$.

**Proof.** The proof is analogous to that given for Lemma A.5, by using Lemma A.17 and A.16 instead of Lemma A.4 and A.3, respectively. We have only to observe the following facts:
For Case 3, Point (1) we can evaluate the agent \texttt{tell(b)} after the global store implies $\exists_{\var{A_1}} b$. In this way the new derivation has the same sequence of intermediate results.

For Case 6, Point (2), by using Lemma A.14, if there exists a derivation $\xi = (D_0.C_1[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], c) \rightarrow^* (D_0.C_m[H || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_m) \rightarrow (D_0.C_m[H' || \sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j], d_{m+1}) \rightarrow^* (D_0.B, d)$,

then $D \models d_m \rightarrow e (= \text{wpc}(C[ ]))$. If $(D_0.H, d_m)$ is not productive then the proof is straightforward. Otherwise, assume that $(D_0.H, d_m)$ is productive. By definition of distribution there exists at least one $j \in [1, n]$ such that $D \models d_m \rightarrow c_j$ and, for each $j \in [1, n]$, either $D \models d_m \rightarrow c_j$ or $D \models d_m \rightarrow \neg c_j$. Then, by definition, there exists a derivation $\xi_1 = (D_0.C_m[\sum_{j=1}^n \text{ask}(c_j) \rightarrow (H || B_j)], d_m) \rightarrow^* (D_0.B, d)$ which performs the same steps of $\chi_1$ in the same order, except for one step of evaluation of the agent $\sum_{j=1}^n \text{ask}(c_j) \rightarrow B_j$ which is performed before evaluating the agent $H$. Then the thesis follows by definition of the relation $\preceq$. $\square$

Also the proof of the following Lemma is analogous to that of its previous counterpart (Lemma A.6) and hence it is omitted.

**Lemma A.19.** Let $0 \leq i \leq n$, $\beta_i$ be a satisfiable constraint and assume that there exists a derivation $\xi : (D_i.A_i, c_i) \rightarrow^* (D_i.A_m, c_m) \rightarrow^* (D_i.A_k, c_k)$, such that $c_m$ is satisfiable. If

i) in the first $m - 1$ steps of $\xi$ rule R2 is used only for evaluating agents of the form $\text{ask}(c) \rightarrow B$,

ii) $w_t(A_1, \text{tr}(\xi))$ is defined (for $t = m(A_k, c_k) \in \{ss, dd, pp, ff\}$).

then there exists a sequence $s'$ starting in $c_m$, such that $\var(A_m, c_m)$, $\exists_{\var(A_i, c_i)}(c_m; \ldots; c_k) \preceq \exists_{\var(A_i, c_i)} s'$ and $w_t(A_m, s') \leq w_t(A_1, \text{tr}(\xi))$.

Finally we have the following.

**Theorem 5.12 (Strong Total Correctness).** Let $D_0, \ldots, D_n$ be a restricted transformation sequence, and $A$ be an agent.

If $(s, x) \in O_t(D_0.A)$ (with $x \in \{ss, dd, pp, ff\}$) then there exists $(s', x) \in O_t(D_n.A)$ such that $s \preceq s'$.

Conversely, if $(s, x) \in O_t(D_n.A)$ then there exists $(s', x) \in O_t(D_0.A)$ such that $s \preceq s'$.

**Proof.** The proof is analogous to that given for Theorem 4.13 and proceeds by showing simultaneously, by induction on $i$, that for $i \in [0, n]$ and for any agent $A$:

1) $O_t(D_0.A) \equiv O_t(D_i.A)$;

2) $D_i$ is weight complete for the traces.

The proof of the base case is analogous to that given for the base case of Theorem 4.13 and hence it is omitted. For the induction step we have that, by induction hypothesis, for any agent $A$, $O_t(D_0.A) \equiv O_t(D_{i-1}.A)$ and $D_{i-1}$ is weight complete for the traces. Proposition 4.9 holds also when considering $O_t$ rather than $O$. From Proposition A.15 and (the counterpart for traces of) Proposition 4.9
then it follows that if $D_i$ is weight complete for traces then, for any agent $A$, $O_t(D_0, A) = O_t(D_i, A)$. So, in order to prove parts 1 and 2, we have only to show that, for any derivation $\beta = \langle D_0, A, c_i \rangle \rightarrow^* \langle D_0, B, c_f \rangle$ such that $c_i$ is a satisfiable constraint and $m(B, c_f) \in \{ss, dd, pp, ff\}$, there exists a split derivation in $D_i \cup D_0$, $\xi = \langle D_i, A, c_i \rangle \rightarrow^* \langle D_0, B', c_f' \rangle$, such that $tr(\beta) \leq tr(\xi)$ and $m(B', c_f') = m(B, c_f)$.

¿From the inductive hypothesis it follows that there exists a split derivation $\chi = \langle D_{i-1}, A, c_i \rangle \rightarrow^* \langle D_0, B'', c_f'' \rangle$ where $tr(\beta) \leq tr(\chi)$ and $m(B'', c_f'') = m(B, c_f)$. Now, let $d \in D_{i-1} \setminus D_i$ be the modified clause in the transformation step from $D_{i-1}$ to $D_i$. The rest of the proof is essentially analogous to that given for Theorem 4.13. The only points which require some case are the following:

Case 2. In this case, the proof is analogous to that given for Case 2 of Proposition A.15.

Case 3. In this case the proof is analogous to that given for Case 3 of Proposition A.15, provided we observe the following fact for case 2a) in such a proof: Given any set of declarations, if there exists a derivation $\chi'$ for the configuration $\langle C'[\text{tell}(\bar{b}) \parallel B \parallel \text{tell}(\bar{t} = \bar{s}'\prime)] \parallel C[r \parallel \text{tell}(\bar{t} = \bar{s})], c' \rangle$ where $c'$ is satisfiable and $Var(C'[\text{tell}(\bar{b}) \parallel B \parallel \text{tell}(\bar{t} = \bar{s}')]) \cap Var(r \parallel \text{tell}(\bar{t} = \bar{s})) = \emptyset$, then there exists a derivation for $\langle C'[\text{tell}(\bar{b}) \parallel B \parallel \text{tell}(\bar{t} = \bar{s}') \parallel \text{tell}(\bar{b}) \parallel \text{tell}(\bar{t} = \bar{s}'), c' \rangle$ which performs the same steps of $\chi'$ plus (possibly) two steps corresponding to the evaluation of $\text{tell}(\bar{t} = \bar{s})$ and $\text{tell}(\bar{b})$, after the evaluation of $\text{tell}(\bar{t} = \bar{s}')$.

Case 4. Analogously to the proof of Case 4 of Proposition A.15, it is sufficient to observe the following. From Lemma A.14 it follows that, for any derivation, when the choice agent inside a context $C[\parallel]$ is evaluated the current store implies $wpc(C[\parallel])$. Then, by definition of ask simplification, the constraint $c_i$ and $c_i'$ are equivalent with respect to the current store (and therefore we do not need to construct the new derivation $\chi'$). The same reasoning applies to the case of tell simplification.

Case 6. The proof is analogous to that of Case 6 of Lemma A.18.

\[\square\]