Standard Model at Intersecting D5-branes:
Lowering the String Scale

D. Cremades, L. E. Ibáñez and F. Marchesano
Departamento de Física Teórica C-XI and Instituto de Física Teórica C-XVI,
Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain.

Abstract

Recently a class of Type IIA orientifold models was constructed yielding just the fermions of the SM at the intersections of D6-branes wrapping a 6-torus. We generalize that construction to the case of Type IIB compactified on an orientifold of $T^4 \times (C/Z_N)$ with D5-branes intersecting at angles on $T^4$. We construct explicit models in which the massless fermion spectrum is just the one of a three-generation Standard Model. One of the motivations for these new constructions is that in this case there are 2 dimensions which are transverse to the SM D5-brane configuration. By making those two dimensions large enough one can have a low string scale $M_s$ of order 1-10 TeV and still have a large $M_{Planck}$ in agreement with observations. From this point of view, these are the first explicit D-brane string constructions where one can achieve having just the fermionic spectrum and gauge group of the SM embedded in a Low String Scale scenario. The cancellation of $U(1)$ anomalies turns out to be quite analogous to the toroidal D6-brane case and the proton is automatically stable due to the gauging of baryon number. Unlike the D6-brane case, the present class of models has $\mathcal{N} = 0$ SUSY both in the bulk and on the branes and hence the spectrum is simpler.
1 Introduction

The brane-world idea has become popular in the last couple of years. In this scheme it is assumed that the standard model (SM) fields and interactions are confined to a \((p + 1)\) dimensional submanifold of a larger \(D\)-dimensional \((D > (p + 1))\) manifold in which gravitational fields propagate. Dp-branes in string theory provide a natural setting in which this scenario arises, since gauge interactions are confined to the world-volume of branes. However, in the brane-world scenario studies a crucial property of the SM is often ignored, the fact that its spectrum is chiral. Dp-branes on a smooth space have non-chiral extended SUSY on their worldvolume.

In order to obtain explicit D-brane realizations of the SM it is thus necessary to do something to obtain chiral theories on their worldvolume. One of the simplest options to obtain chirality is locating stacks of branes on top of some, e.g., orbifold singularity on transverse space. For example, three generation models may be obtained by locating sets of D3-branes on top of a \(\mathbb{Z}_3\) singularity [1, 2] (see also [3, 4, 5, 6, 7, 8]). Local tadpole cancellation in general requires the immersion of those D3-branes on some D7-branes. These are simple theories with phenomenological interest. However the spectrum in general goes beyond the minimal content of the SM (or the MSSM), since extra doublet fermions appear in the spectrum due to the structure of \(U(1)\) anomaly cancellation \(^1\).

Another option in order to get \(D = 4\) chirality is to consider intersecting D-branes [10, 11] (for somewhat related work see also [12, 13, 14]). Recently, a class of intersecting D-brane configurations yielding just the fermionic spectrum of the SM was for the first time constructed [15]. They are obtained from four stacks of D6-branes wrapping an orientifolded 6-torus and intersecting at angles [16, 17, 18, 19] (see also [20, 21, 22, 23, 24, 25, 26, 27, 28] for further developments). In the bulk there is \(\mathcal{N} = 4\) supersymmetry but the spectrum is chiral at the brane intersections. The models are in general non-supersymmetric but for certain choices of the compact radii one can preserve some \(\mathcal{N} = 1\) SUSY at each intersection [22, 26, 27]. One of the nice features of the simplest such constructions is that the existence of three quark-lepton generations is related (via \(U(1)\) anomaly cancellation) to the presence of three colours in QCD [15]. Another interesting feature is that one may expect the appearance of some exponential suppression in certain Yukawa couplings, providing a means to understand the hierarchical structure of fermion masses [18]. The SM Higgs mechanism has an

\(^1\)For attempts to obtain models of D3-branes on a \(\mathbb{Z}_N\) singularity without extra fermionic doublets see ref.[9].
interpretation as a brane recombination process in which the branes supporting the SM
gauge group are recombined into a single brane related to electromagnetism [18, 27].

One point which remains to be understood in those brane configurations is the hier-
archy between the Planck scale and the weak scale. The models are non-supersymmetric
and in order to avoid the standard gauge hierarchy problem of the Higgs scalars a na-
tural option is to assume the string scale $M_s$ not much above the electroweak scale,
i.e., $M_s \sim 1 - 10$ TeV. Then a possibility for understanding the observed smallness of
gravitational interactions would be to have some compact dimensions (transverse to
the brane) very large, as suggested in [29, 30]. However in the case of these intersect-
ing D6-brane models one can see there is no compact direction transverse to all SM
stacks of branes [16]. Thus one has to look for other possible sources of suppression
for gravitational interactions like e.g., that in ref.[31].

A natural alternative is to consider instead of D6-branes lower dimensional ones,
intersecting D5- and D4-branes. This would be interesting since, as pointed in ref.[17],
then there are more transverse dimensions to the branes which can be made large, allow-
ning for a low string scale $M_s << M_p$. In the present paper we extend the work of ref.[15]
to the case of intersecting D5- and D4-branes wrapping cycles on $T^2 \times T^2 \times T^2 / \mathbb{Z}_N$
and $T^2 \times (T^4 / \mathbb{Z}_N)$ respectively. In the case of D5-branes, these are localized on a
fixed point of the orbifold $T^2 / \mathbb{Z}_N$ and wrap 2-cycles on $T^2 \times T^2$. This class of con-
structions was already introduced in [17], but in order to obtain just the spectrum of
the SM we will be now considering orientifolds of such constructions.

We will be able to obtain intersecting D5-brane models with the fermionic spectrum
and gauge group of the SM. The $U(1)$ anomaly structure is identical to that of the D6-
brane models of ref.[15]. There are however many differences between both classes of
models. The present class of models have $\mathcal{N} = 0$ SUSY both in the bulk and on the
branes and none of the quarks, leptons or gauge bosons have any SUSY partner. Thus
the massless spectrum is closer to that of the non-SUSY SM spectrum. The only light
particles beyond the SM spectrum will be some extra scalars (often coloured) and a
minimal Higgs system analogous to that of the MSSM.

The structure of the paper is as follows. In the next section we describe the con-
struction of intersecting D5-branes wrapping cycles on $T^2 \times T^2 \times C / \mathbb{Z}_N$. We derive
the RR-tadpole cancellation conditions for the orientifold case and obtain the lightest
spectrum. The cancellation of mixed $U(1)$ anomalies through a generalized Green-
2 Unlike the case of D6-branes, D5- or D4-branes wrapping cycles on $T^2 \times T^2 \times T^2$ do not lead to
D=4 chiral theories. This is why in order to achieve chirality an additional $\mathbb{Z}_N$ twist in transverse
dimensions is performed in the constructions of the present paper.
Schwarz mechanism is presented in some detail. In Section 3 we present the general strategy to obtain intersecting D5-brane configurations with the spectrum of the SM. We present a particular example in some detail and leave further examples for Appendix II. Examples of left-right symmetric models are provided as well. We also show how to construct a left-right symmetric model free of open string tachyons for any odd value $N$ of the twist $\mathbb{Z}_N$. Some general physical issues like the form of the lightest spectrum beyond the SM and the lowering of the string scale are discussed in Section 4. We leave some general comments and conclusions for Section 5.

In Appendix I we analyze the case of intersecting D4-branes wrapping one-cycles on $T^2 \times T^4 / \mathbb{Z}_N$. These configurations turn out to be less flexible for model building purposes. In particular there is no obvious way to obtain just the SM fermion spectrum at the intersections. We nevertheless provide some semi-realistic example based on intersecting D4-branes in that appendix.

## 2 Intersecting D5-branes on $T^4 \times C / \mathbb{Z}_N$ orientifolds

Let us describe the general intersecting branes setup that we will be considering. As was explained in [17], chiral compactifications may naturally arise from considering configurations of D$(3 + n)$-branes filling four-dimensional Minkowski space, wrapping $n$-cycles of a $2n$-dimensional compact manifold $A_{2n}$ and sitting at a point in a transverse $(6 - 2n)$-dimensional manifold $B_{6-2n}$. In order to obtain a chiral spectrum from the open string sector, the cycles the branes wrap should have nontrivial intersection numbers, while the point they sit in $B_{6-2n}$ must be singular. Lowering the string scale in a natural way implies, in turn, having $n < 3$, so that we can consider a nontrivial transverse space $B_{6-2n}$ whose global properties (as its volume) do not directly affect our open string sector (where our chiral gauge theory arises), but only the closed string sector.

The special case $n = 0$, that corresponds to D3-branes on top of a singularity, was already considered in [2] (and more recently in [9]), yielding semi-realistic gauge theories in $D = 4$. The cases $n = 1, 2, 3$ were then considered in [17, 18] in a simple setup where $A_{2n} = T^{2n}$ and the branes sit in an orbifold singularity $C^{3-n} / \mathbb{Z}_N$. However, as was explicitly shown in [15], considering orientifold models may greatly simplify our chiral spectrum, being possible to attain configurations where the matter content just reduces to the SM fermion content. These models were constructed in a particular setup of the case $n = 3$, where D6-branes wrap 3-cycles of $A_{2n} = T^2 \times T^2 \times T^2$, as previously
considered in [16, 19].

This fact naturally lead us to consider orientifold models of branes at angles. In particular, we will consider the orientifold version of the compactifications already constructed in [17], focusing on the cases $n = 1, 2$ that allow us to obtain low string scale scenarios [29, 30]. Some related constructions of branes at angles have been considered in [24, 25]. Notice, however, that the class of models constructed in the present paper are more general, in the sense that, following the Bottom-Up approach described in [2], we will only bother about the local physics arising from the singular point in $B_{6-2n}$ where the D-brane content lies. The specific models presented as explicit examples of such constructions are also new, as well as their associated phenomenology.

2.1 Construction

Let us consider some specific D-brane setting where the above scenario can be naturally realized. As previously stated, this will imply considering configurations of D5(D4)-branes wrapping 2(1)-cycles of a 4(2)-dimensional compact manifold $\mathcal{A}$ which is in turn embedded in some 6-dimensional manifold $\mathcal{M}$ as the ‘tip’ of an orientifold singularity. We will consider in what follows the D5-brane case, leaving the discussion of D4-brane constructions for an appendix. Following [17], we will choose a fairly simple realization of this setup, given by

$$\text{Type IIB on } M_4 \times \frac{T^4 \times \mathbb{C}/\mathbb{Z}_N}{\{1 + \Omega \mathcal{R}\}},$$

where $\mathcal{R}$ stands for an involution associated with the parity reversal operation $\Omega$. In our case, $\mathcal{R} = \mathcal{R}_{(5)} \mathcal{R}_{(7)} \mathcal{R}_{(8)} \mathcal{R}_{(9)} (-1)^{F_L}$, $\mathcal{R}_{(i)}$ standing for a reflection in the $i^{th}$ coordinate and $F_L$ being the left fermion number. More specifically, if we describe our internal coordinates by complex variables $Z_i = X_{2i+2} + iX_{2i+3}$, then $\mathcal{R}$ is given by the geometrical action

$$\mathcal{R}_g: \begin{align*}
Z_i &\mapsto \bar{Z}_i, \quad i = 1, 2 \\
Z_3 &\mapsto -Z_3.
\end{align*}$$

Such orientifold singularity will induce, as usual, a non-vanishing Klein-bottle amplitude, signaling the presence of an O5-plane in our configuration. In order to cancel the associated RR tadpoles, we will introduce $K$ stacks of $N_a$ D5-branes filling $M_4$ and wrapping 2-cycles $[\Pi_a] \in H_2(T^4, \mathbb{Z})$ ($a = 1, \ldots, K$), while sitting at the origin of $\mathbb{C}/\mathbb{Z}_N$, $N$ being an odd integer. Furthermore, we will consider a particularly simple

\[\text{In the compact case of interest the D5-branes will be sitting at a generic } \mathbb{Z}_N \text{ singularity in the third complex compact dimension.}\]
Figure 1: Intersecting brane world setup. We consider configurations of D5-branes filling four-dimensional Minkowski spacetime, wrapping factorizable 2-cycles of $T^2 \times T^2$ and sitting at a singular point of some compact two-dimensional space $B_2$. In the figure, two such branes are depicted, with wrapping numbers $(1, 2)(1, \frac{3}{2})$ (solid line) and $(1, -1)(1, \frac{1}{2})$ (dashed line). The fractional wrapping numbers arise from a tilted complex structure: $b^{(1)} = 0$, $b^{(2)} = \frac{1}{2}$.

subclass of configurations where $T^4$ is a factorizable torus $T^2 \times T^2$, and the 2-cycles the branes wrap can be decomposed as a product of two 1-cycles $[(n_1, m_1)] \otimes [(n_2, m_2)]$, each wrapping a different $T^2$ (see figure 1 for an example).

The $Z_N$ orbifold twist on the third complex dimension is generated by a geometric action $\omega$, encoded in a twist vector of the form $v_\omega = \frac{1}{N}(0, 0, -2, 0)$ for modular invariance requirements and for the variety to be spin. This same $Z_N$ action may be embedded in the $U(N_a)$ degrees of freedom arising from the $a^{th}$ stack of D5-branes, through a unitary matrix of the form

$$\gamma_{\omega, a} = \text{diag} \left( 1_{N_0^a}, \alpha 1_{N_1^a}, \ldots, \alpha^{N-1} 1_{N_{N-1}^a} \right) ,$$

with $\sum_{i=0}^{N-1} N_i^a = N_a$, and where we have defined $\alpha \equiv \exp(2\pi i/N)$.

This same class of configurations can be analyzed in a T-dual picture, in terms of Type IIB D7-branes with non-trivial wrapping numbers and fluxes in the first two compact complex dimensions, while again localized in the orientifold singularity. Furthermore, when embedding such singularity in a simple toroidal orbifold as $T^2/Z_N$, any configuration can be easily related to Type I compactified on $T^2 \times T^2 \times T^2/Z_N$, with some $F$ and $B$-fluxes in the first two tori. As discussed in [19] (see also [32]),
in such compactifications only discrete values of the $b$-field are allowed by the presence of $\Omega$, namely $b = 0, \frac{1}{2}$. In our T-dual picture of branes at angles, this can be seen by noticing that the geometric action of $\mathcal{R}$ restricts the generic toroidal complex structures of $T^2 \times T^2$ to those that are invariant under complex conjugation. This allows us to consider either rectangular ($b = 0$) or special tilted ($b = \frac{1}{2}$) lattices when defining our $T^2$ (see figure 1). In order to describe configurations with non-vanishing $b$, it is convenient to define effective wrapping numbers

$$(n^i, m^i)_{\text{eff}} := (n^i, m^i) + b^{(i)}(0, n^i),$$

(2.4)

where $b^{(i)}$ stands for the value of $b$ on the $i$th torus $T^2$. This simple redefinition of the wrapping numbers allows us to simply describe the action of $\Omega \mathcal{R}$ in the open string sector. Indeed, in order to have a consistent compactification we should always consider either D5-branes invariant under $\Omega \mathcal{R}$ or pairs of branes related by its action. If a D5$_a$-brane is described by

$$(n^1_a, m^1_a) \otimes (n^2_a, m^2_a),$$

$$\gamma_{\omega,a} = \text{diag} \left(1_{N^0_a}, \alpha 1_{N^1_a}, \ldots, \alpha^{N-1} 1_{N^{N-1}_a} \right),$$

(2.5)

then the sector $\Omega \mathcal{R}$D5$_a$ or D5$_{a^*}$ will be given by

$$(n^1_a, -m^1_a) \otimes (n^2_a, -m^2_a),$$

$$\gamma_{\omega,a^*} = \text{diag} \left(1_{N^0_a}, \alpha^{N-1} 1_{N^1_a}, \ldots, \alpha 1_{N^{N-1}_a} \right),$$

(2.6)

where we consider the effective wrapping numbers defined in (2.4). Both branes $a$ and its mirror partner $a^*$ should be included in a consistent configuration.

Let us now analyze the low energy spectrum arising from such generic class of configurations. First let us consider the closed string sector, which can be computed using standard orbifold techniques. Such techniques have been recently used in a systematic study of non-supersymmetric Type II and Heterotic toroidal orbifolds in [33]. In particular, the closed Type IIB spectrum has been explicitly computed for the toroidal embedding of the $N = 3$ orbifold singularity, so we refer the reader to the appendix of [33] for further details.

Let us first notice that, since we are only concerned with the physics arising at the orientifold singularity, it is pointless to compute the untwisted sector of the theory, which concerns the whole compactification. However, when embedding this $\mathbb{Z}_N$ singularity in a compact six-dimensional manifold $\mathcal{M}$, this sector should give rise to four dimensional gravitation plus some other extra massless particles. At this level we
will only state that, since the twist \( v_\omega \) is explicitly non-supersymmetric, the spectrum in the bulk will necessarily present \( N = 0 \), thus yielding a more economical spectrum that the one obtained by plain dimensional reduction on a torus.

On the contrary, the closed string twisted sector of the theory will play a relevant role with respect to the local physics on the singularity. As expected, Type IIB theory on such singularity will give rise to RR twisted \( p \)-forms of even \( p \):

\[
A_{0}^{(k)}, \ A_{2}^{(k)}, \ A_{4}^{(k)}, \ A_{6}^{(k)} \tag{2.7}
\]

where \( k = 1, \ldots, N - 1 \) denotes the \( k^{th} \)-twisted sector of the theory. \( A_{p}^{(k)} \) and \( A_{6-p}^{(k)} \) field strengthes are related by Hodge duality in \( D = 8 \), while the orientifold action identifies \( k \) and \( N - k \) sectors. In addition, each NSNS \( k^{th} \)-twisted sector will lead to a closed string tachyon of \( \alpha'(mass)^2 = -\frac{4k}{N} \) (actually, due to the \( \Omega R \) modding, only \( \frac{1}{2}(N-1) \) real such tachyons do actually appear). The physical interpretation of analogous type of closed string tachyons have been recently discussed in ref.[34] for the case of non-compact orbifolds. A similar discussion in the case of compact orbifolds and orientifolds is still lacking and goes beyond the scope of the present paper. Note in particular that in the \( C/Z_N \) case considered in [34] the tachyons are complex and their vev signal the smoothing of the singularity. In the present orientifold case the tachyons are real fields and the analysis should be different.

Let us now focus on the open string sector of the theory. Whenever a \( D5_a \)-brane is not invariant under the orientifold action \( \Omega R \), the massless spectrum arising from the \( D5_aD5_a \) sector is identical to the one computed for the orbifold case in [17], since this sector will be mapped to the \( D5_a\ast D5_a\ast \) sector, and there will not be any associated \( \Omega \) projection. This spectrum can be easily described in bosonic language. Indeed, to each open string excitation we can associate a four-dimensional vector \( r \in (\mathbb{Z}^4 + \nu) \), where \( \nu \) distinguishes between the Ramond (\( \nu = \frac{1}{2} \)) and Neveu-Schwarz (\( \nu = 0 \)) sectors of the theory. The GSO projection is imposed by requiring \( \sum_i r^i = \text{odd} \), and the massless states are those that satisfy \( \sum_i (r^i)^2 = 1 \). Namely, the massless states surviving the GSO projection in both R and NS sectors are

| NS State | \( Z_N \) phase | R State | \( Z_N \) phase |
|----------|----------------|---------|----------------|
| \( (\pm 1, 0, 0, 0) \) | 1 | \( \pm \frac{1}{2}(-,+,+,-) \) | \( e^{\mp 2\pi i \frac{1}{N}} \) |
| \( (0, \pm 1, 0, 0) \) | 1 | \( \pm \frac{1}{2}(+,-,+,+) \) | \( e^{\mp 2\pi i \frac{1}{N}} \) |
| \( (0, 0, \pm 1, 0) \) | \( e^{\mp 4\pi i \frac{1}{N}} \) | \( \pm \frac{1}{2}(+,+,+,+) \) | \( e^{\pm 2\pi i \frac{1}{N}} \) |
| \( (0, 0, 0, \pm 1) \) | 1 | \( \pm \frac{1}{2}(+,+,+,+) \) | \( e^{\mp 2\pi i \frac{1}{N}} \) |

where its behaviour under the \( Z_N \) orbifold twist has also been indicated. As usual, the open string spectrum is computed by keeping states invariant under the combined
geometric plus Chan-Paton (CP) $Z_N$ action [35], so after this projection we are led to a spectrum of the form

\[\begin{align*}
\text{Gauge Bosons} & \quad \prod_a \prod_{i=1}^N U(N_a^i) \\
\text{Complex Scalars} & \quad \sum_a \sum_{i=1}^N \left( (N_a^i, N_a^{i-2}) + 2 \times \text{Adj}_a^i \right) \\
\text{Left Fermions} & \quad \sum_a \sum_{i=1}^N 2 \times (N_a^i, N_a^{i-1}) \\
\text{Right Fermions} & \quad \sum_a \sum_{i=1}^N 2 \times (N_a^i, N_a^{i-1})
\end{align*}\]

(2.9)

where the index $i$ is defined mod $N$. Notice that this spectrum is explicitly non-chiral and also non-supersymmetric, since all the gauginos have been projected out. Notice, as well, that when considering D5-branes invariant under the $\Omega R$ action, $SO(N)$ and $USp(N)$ gauge groups will also arise. Since we are not interested in constructing configurations with these groups, we will not consider this option any longer.

Both the chiral matter and the tachyonic content of our configuration will arise from the sectors $D5_a D5_b$, $D5_a D5_{b^*}$ and $D5_a D5_{a^*}$. Let us compute this spectrum explicitly for the $D5_a D5_b$ sector. Just as in the $D5_a D5_a$ case, this sector is not constrained by the $\Omega R$ projection, so its associated spectrum is computed in the same way as in an orbifold compactification. In order to properly describe it, we can introduce the twist vector $v_\vartheta = (\vartheta_1^{ab}, \vartheta_2^{ab}, 0, 0)$, $\pi \vartheta_i^{ab}$ being the angle that both branes form on the $i$th torus [17]. The states living in the $ab$ intersection are then characterized by four-dimensional vectors of the form $r + v_\vartheta$, where $r$ stands for the set of discrete vectors introduced above. The mass formula is also modified to [11, 17]

\[
\alpha' M^2_{ab} = \frac{Y^2}{4\pi \alpha'} + N_{\text{bos}}(\vartheta) + \frac{(r + v_\vartheta)^2}{2} - \frac{1}{2} + E_{ab},
\]

(2.10)

where $Y$ stands for any transversal separation between both branes, $N_{\text{bos}}(\vartheta)$ is the bosonic oscillator contribution and $E_{ab}$ is the vacuum energy:

\[
E_{ab} = \sum_{i=1}^3 \frac{1}{2} |\vartheta^i|(1 - |\vartheta^i|)
\]

(2.11)

The massless and tachyonic states will now be

| Sector | State | $Z_N$ phase | $\alpha' \text{Mass}^2$ |
|--------|-------|-------------|-----------------------|
| NS     | $(-1 + \vartheta^1, \vartheta^2, 0, 0)$ | 1 | $-\frac{1}{2} (\vartheta^1 - \vartheta^2)$ |
|        | $(\vartheta^1, -1 + \vartheta^2, 0, 0)$ | 1 | $\frac{1}{2} (\vartheta^1 - \vartheta^2)$ |
| R      | $(-\frac{1}{2} + \vartheta^1, -\frac{1}{2} + \vartheta^2, -\frac{1}{2}, +\frac{1}{2})$ | $e^{2\pi i \frac{1}{N}}$ | 0 |
|        | $(-\frac{1}{2} + \vartheta^1, -\frac{1}{2} + \vartheta^2, +\frac{1}{2}, -\frac{1}{2})$ | $e^{-2\pi i \frac{1}{N}}$ | 0 |

where $\vartheta^i \equiv \vartheta_{ab}^i$ and we have supposed $0 < \vartheta^i < 1, i = 1, 2$. In any case, one of the NS states will be necessarily tachyonic, unless $|\vartheta^1| = |\vartheta^2|$ and both are massless.
The spectrum can be found again by projecting out non-invariant states. In this case, however, we must also consider the intersection number of both branes

\[ I_{ab} \equiv [\Pi_a] \cdot [\Pi_b] = I_{ab}^1 I_{ab}^2 = (n_a^1 m_b^1 - m_a^1 n_b^1)(n_a^2 m_b^2 - m_a^2 n_b^2). \]  

(2.13)

This number is a topological invariant associated to the two 2-cycles the branes wrap. Its absolute value counts the net number of intersection between such cycles, thus telling us how many replicas of (2.12) are present, and its sign indicates the chirality of the fermions living at the intersection [17, 26]. The final spectrum arising from this sector is thus

\[
\begin{align*}
\text{Tachyons} & \quad \sum_{a<b} \sum_{i=1}^N I_{ab} \times (N_a^i, N_b^i) \\
\text{Left Fermions} & \quad \sum_{a<b} \sum_{i=1}^N I_{ab} \times (N_a^i, N_b^{i+1}) \\
\text{Right Fermions} & \quad \sum_{a<b} \sum_{i=1}^N I_{ab} \times (N_a^i, N_b^{i-1})
\end{align*}
\]  

(2.14)

In the same manner, we can compute the massless and tachyonic spectrum arising from the $D5_a D5_b^*$ and $D5_a D5_a^*$ sectors, taking account of their respective wrapping numbers and twist vectors. The important point to notice is that fermions arising from $D5_a D5_b^*$ will transform as bifundamentals in some $(N_a^i, N_b^{-i})$ instead of $(N_a^i, N_b^{i+1})$. This simple fact will allow us to achieve a much more economical chiral spectrum, as already noted in [15]. The $D5_a D5_a^*$ sector, in turn, will generically have some fixed points under the orientifold action, giving rise to fermions and scalars in symmetric (S) and antisymmetric (A) representations. The complete spectrum is given by

**Complex Scalars**

\[
\sum_{a<b} \sum_{i=1}^N \left[ I_{ab} (N_a^i, N_b^i) + I_{ab} (N_a^i, N_b^{-i}) \right] \sum_a \left[ 2 |m_a^1 m_a^2| (n_a^1 n_a^2 + 1) |A_a^0| + 2 |m_a^1 m_a^2| (|n_a^1 n_a^2| - 1) |S_a^0| \right] \\
\sum_a \sum_{j,i=1}^N \delta_{j,-i-1} \left[ 2m_a^1 m_a^2 (n_a^1 n_a^2 + 1) |A_a^j| + 2m_a^1 m_a^2 (n_a^1 n_a^2 - 1) |S_a^j| \right]
\]  

(2.15)

**Left Fermions**

\[
\sum_{a<b} \sum_{i=1}^N \left[ I_{ab} (N_a^i, N_b^{i+1}) + I_{ab} (N_a^i, N_b^{-i-1}) \right] \sum_a \sum_{j,i=1}^N \delta_{j,-i-1} \left[ 2m_a^1 m_a^2 (n_a^1 n_a^2 + 1) |A_a^j| + 2m_a^1 m_a^2 (n_a^1 n_a^2 - 1) |S_a^j| \right]
\]

**Right Fermions**

\[
\sum_{a<b} \sum_{i=1}^N \left[ I_{ab} (N_a^i, N_b^{i-1}) + I_{ab} (N_a^i, N_b^{-i+1}) \right] \sum_a \sum_{j,i=1}^N \delta_{j,-i+1} \left[ 2m_a^1 m_a^2 (n_a^1 n_a^2 + 1) |A_a^j| + 2m_a^1 m_a^2 (n_a^1 n_a^2 - 1) |S_a^j| \right]
\]

Let us also mention that, in case $I_{aa^*} = 0$, some care should be taken when considering the scalar spectrum arising from the $aa^*$ sector. If, for instance, $I_{aa^*}^1 = 0$, in order to obtain such spectrum we must ‘forget’ about this $(T^2)_j$ and compute it from a system of D4-branes wrapping as $(m_a^i, m_b^j)$ on $(T^2)_j$, $j \neq i$ (see formulae (6.9)). Notice that if $m_a^i = 0$ there is an extra contribution to the mass^2 of the whole spectrum arising
from $aa^*$, coming from the separation $Y$ that both mirror branes may have in the $i^{th}$ torus.

### 2.2 Tadpoles and Anomalies

When dealing with a full consistent configuration of D5-branes, RR tadpole cancellation conditions should always be satisfied. These can be easily computed from usual factorization of one-loop amplitudes. As mentioned before, the presence of the $\Omega R$ factor will induce non-vanishing Klein bottle and Moebius strip contributions to such amplitudes, so the conditions computed in [17] for D5-branes sitting on an orbifold singularity will be slightly modified to

\[
c^2_k \sum_a n_a m_a^2 (\text{Tr} \gamma_{k,a} + \text{Tr} \gamma_{k,a^*}) = 16 \sin \left( \frac{\pi k}{N} \right)
\]

\[
c^2_k \sum_a m_a n_a^2 (\text{Tr} \gamma_{k,a} + \text{Tr} \gamma_{k,a^*}) = 0
\]

\[
c^2_k \sum_a n_a m_a^2 (\text{Tr} \gamma_{k,a} - \text{Tr} \gamma_{k,a^*}) = 0
\]

\[
c^2_k \sum_a m_a n_a^2 (\text{Tr} \gamma_{k,a} - \text{Tr} \gamma_{k,a^*}) = 0
\]

(2.16)

where $c^2_k = \sin \frac{2\pi k}{N}$ is a weight for each $k^{th}$ twisted sector usually arising in $\mathbb{Z}_N$ orientifold compactifications [36]. As can easily be seen, the difference with the orbifold case amounts to consider the presence of mirror branes $a^*$ in our configuration and including a constant term in the first equation. This constant term can be interpreted as a negative RR charge induced by the presence of an O5-plane. Indeed, in the more general context of D5$_a$-branes wrapping general 2-cycles $[\Pi_a]$ on $\mathbb{T}^4$ these conditions can be expressed as

\[
c^2_k \sum_a ([\Pi_a] \text{Tr} \gamma_{k,a} + [\Pi_{a^*}] \text{Tr} \gamma_{k,a^*}) = [\Pi_{O5}] 16 \beta^1 \beta^2 \sin \left( \frac{\pi k}{N} \right),
\]

(2.17)

where $[\Pi_{O5}]$ describes the 2-cycle the O5-plane wraps, and $\beta^i = 1 - b^{(i)}$. Notice that the factor of $16\beta^1 \beta^2$ can be interpreted as the number of O5-planes, which is $4\beta^1 \beta^2$, times their relative charge to a D5-brane, which is $-4$. We thus see that RR conditions can be interpreted, as usual, as the vanishing of the total RR charge in a compact space (in our case $\mathbb{T}^4$). In this token, notice that $c^2_0 = 0$, so we are not imposing any condition in the untwisted sector, whose associated RR form can escape the singularity. This implies that we are not fixing the total number of branes. In this sense, we are being less restrictive than in a simple toroidal orbifold (see, e.g., the related constructions considered in [24]). When embedding our orientifold singularity in a full compact variety $\mathcal{M}$, however, these RR untwisted conditions should also be taken into account. The cancellation of these untwisted tadpoles is easy to achieve by adding appropriate...
D5-branes at locations in the third torus away from the \( \mathbb{Z}_N \) singularity at which the SM branes sit. This is why we will not discuss them explicitly in the rest of the paper.

Although quite general, the expression (2.17) is not very useful for our model-building purposes. We will make use instead of (2.16), which can be also be converted into a more tractable expression. Indeed, notice that the upper set of equations in (2.16) is equivalent to

\[
\sum a^n_1 a^n_2 (\text{Tr} \gamma_{2k,a} + \text{Tr} \gamma_{2k,a^*}) = \frac{16}{\alpha^k + \alpha^{-k}},
\]

where we have again used \( \alpha = e^{2\pi i/N} \). Taking \( 2k \equiv 1 \mod N \), we can easily read the condition that has to be imposed to the Chan-Paton matrix \( \gamma_{\omega,a} \)

\[
\sum a^n_1 a^n_2 (\text{Tr} \gamma_{\omega,a} + \text{Tr} \gamma_{\omega,a^*}) = \frac{16}{\alpha^N + 1} + \frac{16}{\alpha^N - 1} = 16 \sum_{l=1}^r (\alpha^{2l-1} + \alpha^{2l-1}),
\]

\[
\eta = \begin{cases} 
+1 & \text{if } N = 4r - 1 \\
-1 & \text{if } N = 4r + 1
\end{cases}
\]

Cancellation of RR tadpoles has, as usual, very important consequences from the point of view of the effective four-dimensional field theory. Indeed, when considering a chiral spectrum as the one considered in (2.15) potential chiral anomalies may arise. RR tadpole conditions (2.16) insure the cancellation of such anomalies, as we will now see. Let us first consider the cancellation of the cubic non-Abelian anomaly for the gauge group \( SU(N_a^i) \), which in our configurations reads

\[
A_{SU(N_a^i)^3} = \sum_{b,j} N_b^j (I_{ab} \delta(i, j) + I_{ab^*} \delta(i, -j)) + 16\beta^1 \beta^2 I_{a,O5} \delta(i, -i),
\]

where \( \delta(i, j) = \delta_{i+1,j} - \delta_{i-1,j} \) (the indexes \( i, j \) are again defined \( \mod N \)) and, in case of factorizable branes (2.5), \( \beta^1 \beta^2 I_{a,O5} = m_a^1 m_a^2 \).

Just as done in [37, 17], we can use the discrete Fourier transform \( \delta_{ij} = \frac{1}{N} \sum_{k=1}^N e^{2\pi i k (j-i)} \) to rewrite (2.21) as

\[
A_{SU(N_a^i)^3} = -\frac{4}{N} \sum_{k=1}^N e^{2\pi i k j/N} c_k \left( \sum_b I_{ab} \text{Tr} \gamma_{k,a} + I_{ab^*} \text{Tr} \gamma_{k,a^*} \right) + 16m_a^1 m_a^2 \delta(i, -i),
\]

which after some simple manipulations, can be seen to vanish whenever the tadpoles conditions (2.16) are satisfied. As usual, the latter turn out to be more restrictive that the vanishing of (2.22).

We can also consider mixed and cubic \( U(1) \) anomalies, both involving a generalized Green-Schwarz mechanism mediated by RR twisted fields. Indeed, the full expression
for the mixed $U(1)_{a,i} - SU(N_b^i)^2$ anomaly is given by

$$\mathcal{A}_{U(1)_{a,i} - SU(N_b^i)^2} = \frac{1}{2} \delta_{ab} \delta_{ij} \left( \sum_{c,l} N_c^i [I_{ac} \delta(i, l) + I_{ac^*} \delta(i, -l)] + 16 m_a^1 n_a^2 \delta(i, -i) \right)$$

$$+ \frac{1}{2} N_a^i (I_{ab} \delta(i, j) + I_{ab^*} \delta(i, -j)),$$

the first term in brackets being proportional to the cubic chiral anomaly of $SU(N_a^i)$, thus vanishing when imposing tadpoles. The remaining contribution can then be canceled by means of a generalized Green-Schwarz mechanism. Indeed, by use of the discrete Fourier transform, we can rewrite the residual anomaly in (2.23) as

$$\mathcal{A}_{U(1)_{a,i} - SU(N_b^i)^2} = \frac{-2 N_a^i}{N} \sum_{k=1}^{N} e^{2\pi i k b N} c_k^2 \left( I_{ab} e^{-2\pi i k a N} + I_{ab^*} e^{2\pi i k b N} \right).$$

As explained in [17] for the orbifold (non-orientifold) case, this quantity can be canceled by exchange of four-dimensional fields, which arise upon dimensional reduction of the RR twisted forms living on the singularity. For showing this, let us consider the T-dual picture of fractional D7-branes wrapping the first two tori, and with non-trivial $F$ and $B$-fluxes on them. On the worldvolume of each D7-brane, there will appear some couplings to the twisted RR forms in (2.7), and by integrating such couplings on the compact toroidal dimensions $(T^2)_1 \times (T^2)_2$ we will obtain four-dimensional couplings that will be relevant to our low-energy theory. Indeed, if we define

$$B_0^{(k)} = A_0^{(k)}, \quad B_2^{(k)} = f_{(T^2)_1 \times (T^2)_2} A_6^{(k)},$$

$$C_0^{(k)} = f_{(T^2)_1 \times (T^2)_2} A_4^{(k)}, \quad C_2^{(k)} = A_2^{(k)},$$

$$D_0^{(k)} = f_{(T^2)_2} A_2^{(k)}, \quad D_2^{(k)} = f_{(T^2)_1} A_4^{(k)},$$

$$E_0^{(k)} = f_{(T^2)_1} A_2^{(k)}, \quad E_2^{(k)} = f_{(T^2)_2} A_4^{(k)},$$

then these four dimensional couplings can be computed to be

$$c_k N_a^i n_a^l m_a^2 \int_{M_4} \text{Tr} (\gamma_{k, a} - \gamma_{k, a^*}) \lambda_l \ B_2^{(k)} \wedge \text{Tr} F_{a,i},$$

$$c_k N_a^i m_a^1 n_a^2 \int_{M_4} \text{Tr} (\gamma_{k, a} - \gamma_{k, a^*}) \lambda_i \ C_2^{(k)} \wedge \text{Tr} F_{a,i},$$

$$c_k N_a^i n_a^l m_a^2 \int_{M_4} \text{Tr} (\gamma_{k, a} + \gamma_{k, a^*}) \lambda_l \ D_2^{(k)} \wedge \text{Tr} F_{a,i},$$

$$c_k N_a^i m_a^1 n_a^2 \int_{M_4} \text{Tr} (\gamma_{k, a} + \gamma_{k, a^*}) \lambda_i \ E_2^{(k)} \wedge \text{Tr} F_{a,i},$$

$$c_k m_b^1 n_b^2 \int_{M_4} \text{Tr} (\gamma_{k, b} + \gamma_{k, b^*}) \lambda_2^2 \ B_0^{(k)} \wedge \text{Tr} (F_{b,j} \wedge F_{b,j}),$$

$$c_k n_b^1 m_b^2 \int_{M_4} \text{Tr} (\gamma_{k, b} + \gamma_{k, b^*}) \lambda_2^2 \ C_0^{(k)} \wedge \text{Tr} (F_{b,j} \wedge F_{b,j}),$$

$$c_k m_b^1 n_b^2 \int_{M_4} \text{Tr} (\gamma_{k, b} - \gamma_{k, b^*}) \lambda_2^2 \ D_0^{(k)} \wedge \text{Tr} (F_{b,j} \wedge F_{b,j}),$$

$$c_k n_b^1 m_b^2 \int_{M_4} \text{Tr} (\gamma_{k, b} - \gamma_{k, b^*}) \lambda_2^2 \ E_0^{(k)} \wedge \text{Tr} (F_{b,j} \wedge F_{b,j}),$$

where $\lambda$ denotes the Chan-Paton wavefunction for the gauge boson state, and the $N_a^i$ factor arises from normalization of the $U(1)_{a,i}$ generator (see [38]). Since $B_2^{(k)}$ and
$B_0^{(k)}$ are four-dimensional Hodge duals, same for $C$, $D$ and $E$, the sum over the GS diagrams will provide a counterterm with the structure (2.24), just as required to cancel the residual mixed anomaly in (2.23). Cancellation of cubic $U(1)$ anomalies works in a similar way.

An important consequence of this anomaly cancellation mechanism is the Abelian gauge structure of the low-energy effective action. It can be shown that, as a result of the couplings (2.26) the gauge bosons of the potentially anomalous $U(1)$ get massive, decoupling from the low energy spectrum of the theory. More generally, any $U(1)$ gauge boson (anomalous or not) with a non-vanishing axionic coupling of the form (2.26) will have an induced mass term of the order of the string scale. The associated gauge symmetry will no longer be present, although such $U(1)$ will remain as an exact perturbative global symmetry.

A similar analysis regarding the construction of intersecting D4-branes configurations wrapping $T^2$ and sitting in a $C^2/Z_N$ orientifold singularity can also be performed, the general formalism being much alike as the one just presented for the case of D5-branes. These D4-branes constructions are also of interest from the model-building point of view, and some non-orientifold examples were built in [18] (see also related models in [24]). However, it turns out to be difficult to obtain D4-brane models with just the SM fermion spectrum. That is why we leave the presentation of the D4-brane formalism for an appendix.

## 3 The Standard Model at intersecting D5-branes

In the present section we will be interested in finding intersecting D5-branes models whose gauge group and matter content correspond to either the Standard Model (SM) or some Left-Right symmetric (LR) extension of it [39]. Such low energy spectra must contain the following gauge group and fermionic content:

| Standard Model                              | Left-Right Model                        |
|---------------------------------------------|----------------------------------------|
| $SU(3)_c \times SU(2)_L \times U(1)_Y$     | $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ |
| $Q^i_L = (3, 2)_{\frac{1}{6}}$             | $Q^i_L = (3, 2, 1)_{1/3}$               |
| $U^i_R = (3, 1)_{-\frac{2}{3}}$            | $Q^i_R = (3, 2, 1)_{-1/3}$              |
| $D^i_R = (3, 1)_{\frac{1}{3}}$             | $L^i_L = (1, 2, 1)_{-1}$                |
| $L^i = (1, 2)_{-\frac{1}{3}}$              |                                       |
| $E^i_R = (1, 1)_{1}$                       | $L^i_R = (1, 1, 2)_{1}$                 |
| $N^i_R = (1, 1)_{0}$                       |                                      |
where $i = 1, 2, 3$ indexes the three different generations that have to be considered in each model.

Following the general philosophy described in [15], we will be considering a class of configurations where the chiral fermions arise only in bifundamental representations

$$\sum_{a, b} n_{ab}(N_a, \overline{N}_b) + m_{ab}(N_a, N_b) + n_{ab}^*(\overline{N}_a, N_b) + m_{ab}^*(\overline{N}_a, \overline{N}_b),$$

(3.2)

where $n_{ab}, n_{ab}^*, m_{ab}, m_{ab}^*$ are model dependent and non-negative integer numbers. In this particular class of models, cubic anomaly cancellation for a non-Abelian gauge group $SU(N_a)$ reduces to having the same number of fundamental representations $N_a$ as anti-fundamental representations $\overline{N}_a$. Notice also that, from the point of view of Left-Right unification, right-handed neutrinos must exist, as they complete the $SU(2)_R$ leptonic doublet that contains the charged right-handed leptons $E_R^i$. From the point of view of SM building, though, there is no reason why we should consider having such representations in our fermionic content. However, as was emphasized in [15], when obtaining the chiral content of our theory just from fields transforming in bifundamental representations, such right-handed neutrinos naturally appear from anomaly cancellation conditions. Since in the present paper we will construct our models from such “bifundamental” fermions, we will include these particles right from the start.

In general, it can also be shown that in this case where chiral fields transform in bifundamentals the simplest embedding of the SM (or the LR extension) will consist in a configuration of four stack of branes, as presented in table 1.

Given this brane content is relatively easy to figure out how to realize the specific fermion content of both SM and LR models. Indeed, let us for instance consider strings coming from the $ab$ and $ab^*$ sectors. Their (left-handed) massless modes will transform as either $(3, \overline{2})$ or $(3, 2)$ under the gauge group $SU(3) \times SU(2)_L$ and hence can be naturally identified with the left-handed quarks $Q_L^i$. The fermion content of both classes of models are shown in tables 2 and 3, where each chiral fermion in (3.1) is associated to a definite sector.

In order to realize such spectra as the chiral content of a concrete configuration of D5-branes we must impose some topological constraints on our models. Unlike the case of D6-branes, where all the spectrum information is encoded on the intersection numbers, we must now also consider the orbifold structure of our configuration. Such structure can be easily encoded in a quiver diagram, as shown in figure 2.

---

4For some intersecting branes SM constructions without right-handed neutrinos see [21, 27].

5These are quivers in the sense of ref.[35, 40, 6], not in the sense of the SUSY-quivers discussed in ref.[26, 27] in which no $\mathbb{Z}_N$ twist is present.
| Label  | Multiplicity | Gauge Group             | Name            |
|--------|--------------|-------------------------|-----------------|
| stack $a$ | $N_a = 3$       | $SU(3) \times U(1)_a$  | Baryonic brane  |
| stack $b$ | $N_b = 2$       | $SU(2)_L \times U(1)_b$ | Left brane      |
| stack $c$ | $N_c = \begin{cases} 2 \\ 1 \end{cases}$ | $SU(2)_R \times U(1)_c$ | Right brane     |
| stack $d$ | $N_d = 1$       | $U(1)_d$                | Leptonic brane  |

Table 1: Brane content yielding the SM or LR spectrum.

| Intersection | Matter fields | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ | $Y$ |
|--------------|---------------|------|------|------|------|-----|
| (ab)         | $Q_L$         | $2(3,2)$ | 1    | -1   | 0    | 0/6 |
| (ab*)        | $q_L$         | $2(3,2)$ | 1    | 1    | 0    | 0/6 |
| (ac)         | $U_R$         | $3(3,1)$ | -1   | 0    | 1    | 2/3 |
| (ac*)        | $D_R$         | $3(3,1)$ | -1   | 0    | -1   | 0   |
| (bd)         | $L$           | $3(1,2)$ | 0    | -1   | 0    | 1/3 |
| (cd)         | $N_R$         | $3(1,1)$ | 0    | 1    | 0    | 1/2 |
| (cd*)        | $E_R$         | $3(1,1)$ | 0    | 0    | -1   | 0   |

Table 2: Standard model spectrum and $U(1)$ charges. The hypercharge generator is defined as $Q_Y = \frac{1}{6}Q_a - \frac{1}{2}Q_c - \frac{1}{2}Q_d$.

In general, a D-brane configuration living on an orbifold singularity can be locally described by quotienting the theory by a discrete group $\Gamma$, which is acting on both an ambient space $\mathbb{C}^n$ and on the CP degrees of freedom. To each $\Gamma$ action we can associate a quiver diagram [35, 40, 6]. Each node of such diagram will represent an irreducible representation (irreps) of $\Gamma$, whereas the arrows connecting the nodes represent invariant fields under combined geometric and gauge actions. In general, the $\Gamma$ action $\gamma_g$ on the Chan-Paton degrees of freedom can be written as a direct sum of such irreps, and the gauge groups that will arise from it will correspond to a product of unitary groups, each one associated to a definite irreps. In our specific setup $n = 1$ and $\Gamma = \mathbb{Z}_N$, so each irreps of $\Gamma$ is one-dimensional and can be associated to a $N^{\text{th}}$-root
Table 3: Left Right symmetric chiral spectrum and $U(1)$ charges. The $U(1)_{B-L}$ generator is defined as $Q_{B-L} = \frac{1}{3}Q_a - Q_d$.

| Intersection | Matter fields | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ | $B - L$ |
|--------------|--------------|-------|-------|-------|-------|---------|
| (ab)         | $Q_L$        | 1     | -1    | 0     | 0     | 1/3     |
| (ab*)        | $q_L$        | 2     | 1     | 0     | 0     | 1/3     |
| (ac)         | $Q_R$        | -1    | 0     | 1     | 0     | -1/3    |
| (ac*)        | $q_R$        | 2     | -1    | 0     | -1    | -1/3    |
| (bd)         | $L_L$        | 3     | -1    | 0     | 1     | -1      |
| (cd)         | $L_R$        | 3     | 0     | 1     | -1    | 1       |

Figure 2: Quiver diagram of a $\mathbb{Z}_N$ orbifold singularity. The nodes of such diagram represent the phases associated to each different gauge group in the theory, whereas each arrow represents a chiral fermion transforming in a bifundamental of the two groups it links.

of unity. Indeed, any $\mathbb{Z}_N$ generator action on the Chan-Paton degrees of freedom can be written on the form (2.3), where several such phases are involved. Without loss of generality, we will consider that each brane $a, b, c, d$ has a $\gamma_\omega$ matrix proportional to the identity, that is $\gamma_{\omega,i} = \alpha^n 1_{N_i}$, so that it will give rise to just one unitary gauge group $U(N_i)$. We will represent this by locating that brane $i$ on the node corresponding to the irreps $\alpha^n$. Notice that, in an orientifold theory, the mirror brane $i^*$ will then be placed in the node $\bar{\alpha}^n$.

Chiral fields can also be easily identified in this diagram by arrows connecting the
nodes. These arrows will always link two different nodes, so that if there is some brane content in both of them we will find a fermion transforming under the corresponding gauge groups. The sense of the arrow will denote the chirality that such representation has. In our conventions the positive sense represents left fermions. This arrow structure can be easily read from the chiral spectrum in (2.14), giving rise to the cyclic quiver depicted in figure 2. Notice that this simple spectrum comes from a plain orbifold singularity. In this case every chiral field will transform in bifundamental representations of two gauge groups with contiguous phases. When considering orientifold singularities, however, we should also include the mirror branes on the picture, and more “exotic” representations may arise.

There are, in principle, many different ways of obtaining the desired chiral spectrum (3.1) from the brane content of table 1. Furthermore, the details of the construction will depend on the specific model (SM or LR) and on the $\mathbb{Z}_N$ quiver under consideration. There are, however, some general features of the construction that can be already addressed at this level.

- **In both SM and LR models, chiral fermions must arise in a very definite pattern.** Namely, we need left and right-handed quarks, so we must consider matter arising from the intersections of the baryonic brane with both the left and right branes. We must avoid, however, lepto-quarks which may arise from some intersection with the leptonic brane. The same considerations must be applied to the latter. This pattern can be easily achieved in D5-branes configurations by placing both $b$ and $c$ (or $c^*$) branes on the same node of the $\mathbb{Z}_N$ quiver, while $a$ and $d$ in some contiguous node. Since, in order to achieve the spectra of tables 2 and 3, we must consider non-trivial $ab$, $ab^*$, $ac$ and $ac^*$ sectors, we must place the stack $a$ either in the phase 1 or in the phase $\alpha$, while stacks $b$, $c$ must be in the other one. This restricts our search to essentially two different distributions of branes, which are shown in figure 3.

- **Given these two possibilities, it is now easy to guess which intersection numbers must we impose in order to achieve the desired spectra.** Indeed, the modulus of an intersection number, say $I_{ab}$, will give us the multiplicity of this sector. This implies that, in order to have the desired number of left-handed quarks, we must impose $|I_{ab}| = 1$, $|I_{ab^*}| = 2$ as can be read directly from tables 2 and 3. On the other hand, we will have to choose the sign of these intersection numbers in order

---

6We could have alternatively imposed $|I_{ab}| = 2$, $|I_{ab^*}| = 1$, giving an equivalent spectrum.
to properly fix the chirality of our fermions. These signs will be different for each
distribution of branes considered in figure 3, since chirality also depends on the
arrow structure of the quiver diagram. For instance, we should impose $I_{ab} = 1,$
$I_{ab^*} = -2$ in the $a$)-type of quiver in this figure, while $I_{ab} = -1,$
$I_{ab^*} = -2$ in the $b$)-type. Similar reasoning applies to other intersections involving branes $b$ and $c$.

Finally, we are interested in getting all of our chiral matter from bifundamental representations. Thus, we must avoid the appearance of Symmetrics and Antisymmetrics that might appear from the general spectrum (2.15). This will specially arise in $\mathbb{Z}_3$ models, where we will have to impose $I_{ii^*} = 0$ for those branes in the $\alpha$ node.

3.1 D5 Standard Models

Let us give an example that shows how the SM structure can be implemented on
D5-branes configurations. The simplest choice for such example is the $\mathbb{Z}_3$ singularity,
which is the smallest $\mathbb{Z}_N$ quiver that provides non vector-like spectra. Imposing the
chirality pattern discussed above give us four different ways of embedding the SM
spectrum, each of them depicted in figure 4. In order to achieve a SM configuration,
we must impose the intersection numbers that will give us the desired matter content.
As discussed above, these will depend on the particular $\mathbb{Z}_3$ quiver considered. Let us
first consider the quiver $a_1$). In table 4 we show the general class of solutions for the
wrapping numbers that will provide us with such fermionic spectrum.

Notice that for the sake of generality we have added a new stack of $N_h$ branes to
our initial configuration, yielding an extra $U(N_h)$ gauge group. However, the wrapping
numbers and the CP phase of such brane have been chosen in such a way that no extra
chiral matter arises from its presence. Since no chiral fermion is charged under the
gauge group of this brane, the stack \( h \) is a sort of hidden sector of the theory. This
is strictly true, however, only from the fermion content point of view, and generically
some scalars with both SM and \( U(N_h) \) quantum numbers may appear.

Having achieved the fermionic spectrum of table 2, our low energy field theory will
be automatically free of cubic chiral anomalies. In order to have a consistent comp-
actification, however, we must impose the stronger tadpole cancellation conditions.
Interestingly enough, most of the conditions in (2.16) turn out to be trivially satisfied
by this brane content, the only non-trivial one being the first condition, that now reads

\[
9n_a^1 + n_d^1 - \frac{\hat{e}}{\beta^4} + 2N_h \epsilon_h \frac{\epsilon_h}{\beta^4} = -8.
\]  

(3.3)

Let us now analyze the \( U(1) \) structure of such model. As described in the previous
section, couplings of gauge bosons to twisted RR fields will give rise to GS counterterms
that will cancel the residual \( U(1) \) anomalies. We are particularly interested in couplings
(2.26), that tell us which gauge bosons are becoming massive by this mechanism. In the
\( \mathbb{Z}_3 \) orientifold case there is only one independent twisted sector, so only four couplings
are relevant. By considering the brane content above we find that these couplings are

\[
\begin{align*}
B_2^{(1)} &\wedge c_1 \left( \alpha - \alpha^2 \frac{2\tilde{\epsilon}}{3} F^b \right) \\
D_2^{(1)} &\wedge c_1 \left( \tilde{\epsilon} \left( -3n_1^a F^a + n_1^d F^d \right) + \frac{6}{3} (F^b - F^c) \right) \\
E_2^{(1)} &\wedge c_1 6\epsilon \beta^1 (3F^a + F^d)
\end{align*}
\]

the coupling to the $C_2^{(1)}$ field being trivially null. In general, such couplings will give mass to three linearly independent combinations of $U(1)$’s, leaving just one $U(1)$ as a true Abelian gauge symmetry of the spectrum. Among these massive $U(1)$’s, two are model-independent, and correspond to the ‘anomalous’ combinations $U(1)_b$ and $3U(1)_a + U(1)_d$ characteristic of this fermionic spectrum. The third one, however, will depend on the specific model considered. Indeed, we find that the generator of the massless $U(1)$ is given by

\[
Q_0 = Q_a - 3Q_d - 3\tilde{\epsilon}\beta^1 (n_1^a + n_1^d)Q_c,
\]

so if we further impose to our class of models the condition

\[
\tilde{\epsilon}\beta^1 (n_1^a + n_1^d) = 1,
\]

then we find that this massless Abelian gauge group precisely corresponds to the hypercharge, which in these models is given by $U(1)_Y = \frac{1}{6}U(1)_a - \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d$.

Table 4: D5-branes wrapping numbers and CP phases giving rise to a SM spectrum the $\mathbb{Z}_3$ quiver of fig 4.a1. The solution is parametrized by $n_1^a, n_1^d \in \mathbb{Z}, \epsilon, \tilde{\epsilon} = \pm 1$ and $\beta^1 = 1 - b^{(1)} = 1, 1/2$. Notice that the second torus has to be tilted, hence $\beta^2 = 1/2$.

| $N_i$ | $(n_1^i, n_2^i)$ | $(m_1^i, m_2^i)$ | $\gamma_{\omega,i}$ |
|-------|----------------|----------------|-----------------|
| $N_a$ | $(n_1^a, \epsilon \beta^1)$ | $(3, -\frac{1}{2}\tilde{\epsilon})$ | 1 |
| $N_b$ | $(1/\beta^1, 0)$ | $(\tilde{\epsilon}, -\frac{1}{2} \epsilon)$ | $\alpha_{12}$ |
| $N_c$ | $(1/\beta^1, 0)$ | $(0, \epsilon)$ | $\alpha$ |
| $N_d$ | $(n_1^d, 3\epsilon \beta^1)$ | $(1, \frac{1}{2} \epsilon \tilde{\epsilon})$ | 1 |
| $N_h$ | $(\epsilon h/\beta^1, 0)$ | $(2, 0)$ | $1_{N_h}$ |
regarding tadpoles and the Abelian gauge structure, we are finally led to a compactification yielding just the gauge and fermionic spectrum of the Standard Model (and possibly some hidden sector of the theory given by the brane $h$).

### 3.1.1 Scalars and tachyons in the spectrum

As explained in Section 2.1, at the intersection of pairs of D5-branes with the same CP phase there may appear scalar tachyons with masses given in eq.(2.12). Since branes $b, c$ and their mirrors are parallel along the first 2-torus, they generically do not intersect. On the other hand there may be tachyons at the intersections $(aa^*), (dd^*), (ad), (ad^*)$ plus possibly others involving the hidden branes $h$. One can get rid of many of these tachyons by appropriately choosing some discrete parameters and the compactification radii. Consider for instance the following choice of parameters:

$$n_a^1 = n_d^1 = -1, \ N_h = 0, \ \bar{\epsilon} = -1, \ \beta^1 = \frac{1}{2}.$$  \hspace{1cm} (3.7)

With this choice it is easy to check that the tadpole cancellation conditions (3.3) are verified and the standard hypercharge is the only $U(1)$ remaining at the massless level. Furthermore, the $h$ brane is not needed in order to cancel tadpoles, this hidden sector thus being absent. Now, the angles formed by the branes $d, a$ with the orientifold plane on the two tori are given by

$$\theta_a^1 = \epsilon \left( \pi - tg^{-1}\left( \frac{U_1^1}{2} \right) \right) ; \ \ \theta_a^2 = \epsilon \left( \pi - tg^{-1}\left( \frac{U_2^2}{6} \right) \right)$$

$$\theta_d^1 = \epsilon \left( \pi - tg^{-1}\left( \frac{3U_1^1}{2} \right) \right) ; \ \ \theta_d^2 = -\epsilon \left( \pi - tg^{-1}\left( \frac{U_2^2}{6} \right) \right)$$ \hspace{1cm} (3.8)

respectively. Here $U^i = R^i_2/R^i_1, \ i = 1, 2$. Now, the angles formed by such branes with their mirrors is given by $\vartheta_{a,d}^i \equiv -2\theta_{a,d}^i \mod 2\pi$, so for $U_1 = U_2/3$ one gets $|\vartheta_{a,d}^1| = |\vartheta_{a,d}^2|$, and according to eq.(2.12) the scalars in $(aa^*)$ and $(dd^*)$ cease to be tachyonic and become massless $^7$. The only tachyonic scalars in the spectrum persist in the $ad$ and $ad^*$ intersections which have mass$^2$:

$$m_{ad}^2 = m_{ad^*}^2 = -\frac{1}{\pi}tg^{-1}\left( \frac{U_2^2}{6} \right) M_s^2.$$  \hspace{1cm} (3.9)

In table 5 we present the lightest scalar spectrum arising from branes $a, d$ and their mirrors when the particular choice (3.7) is made.

$^7$Actually, according to (2.15), scalars in the sector $(dd^*)$ transform in the antisymmetric representation of $U(N_d) = U(1)$, thus being absent for any choice of angles.
Table 5: Lighter scalar excitations arising from the brane content with phase 1 in table 4, under the choice of parameters (3.7).

Note however that all the above scalar masses are tree level results and that, since the models are non-SUSY, there are in general important one-loop contributions to the scalar masses. Those will be particularly important for the coloured objects like the scalars in (ad), (ad*) sectors which are color triplets. Those one-loop corrections may be estimated from the effective field theory (one gauge boson exchange) and yield [18]

$$\Delta m^2(\mu) = \sum_a \frac{4C_a^2\alpha_a(M_s)}{4\pi} M_s^2 f_a \log(M_s/\mu) + \Delta M^2_{KK/W}$$  (3.10)

where the sum on $a$ runs over the different gauge interactions under which the scalar transforms and $C_a^2$ is the eigenvalue of the quadratic Casimir in the fundamental representation. Here $\Delta M^2_{KK/W}$ denotes further contributions which may appear from the Kaluza-Klein, winding and string excitations if they are substantially lighter than the string scale $M_s$. The function $f_a$ is given by

$$f_a = \frac{2 + b_a \frac{\alpha_a(M_s)}{4\pi} t}{1 + b_a \frac{\alpha_a(M_s)}{4\pi} t}$$  (3.11)

where $t = 2\log(M_s/\mu)$ and $b_a$ are the coefficients of the one-loop $\beta$-functions. These corrections are positive and may easily overcome the tree level result if $U^2$ is not too large. This is analogous to the one-loop contribution to squark masses in the MSSM in which for large gaugino masses the one-loop contribution clearly dominates over the tree-level soft masses (see e.g. ref.[41] and references therein). Thus in this class of models, apart from the fermion spectrum of the SM, one expects the presence of some extra relatively light (of order the electroweak scale) coloured scalars.

### 3.1.2 Electroweak symmetry breaking

The Higgs sector in this class of theories is relatively similar to the one in the models in [15]. Consider in particular the SM configuration described in the previous subsections.
Here, the only light scalar with the quantum numbers of a Higgs boson lives in the $bc$ sector. Branes $b$ and $c$ are parallel in the first torus, but if the distance $X_{bc}$ between the branes in that torus is set to zero the branes intersect at an angle

$$\pi \vartheta_{bc}^2 = \epsilon \tilde{\epsilon} \left( \frac{\pi}{2} + \tan^{-1} \left( \frac{U^2}{2} \right) \right), \quad (3.12)$$

and at those intersections complex scalar doublets appear with masses

$$m_{H^\pm}^2 = \frac{X_{bc}^2}{4\pi} M_s^2 \pm \frac{M_s^2}{2} |\vartheta_{bc}^2|; \quad (3.13)$$

There are in fact two scalar doublets with quantum numbers as in table 6,

| Higgs | $Q_b$ | $Q_c$ | Y |
|-------|------|------|---|
| $H_1$ | 1    | -1   | 1/2 |
| $H_2$ | -1   | 1    | -1/2 |

Table 6: Electroweak Higgs fields

and defined as

$$H^\pm = \frac{1}{2}(H_1^* \pm H_2). \quad (3.14)$$

The intersection number of these branes in the second torus is equal to $\pm 1$ so that only one copy of this Higgs system appears. Thus in the present model we have the same minimal Higgs sector as in the MSSM. As may be seen from eq.(3.13) as the distance $X_{bc}$ decreases the Higgs doublets become tachyonic, giving rise to EW symmetry breaking. This is quite similar to the process of EW symmetry breaking in the D6-brane models of ref.[15, 27], in which it may be described as brane recombination of a $b$ brane and a $c$ brane into a single recombined brane $e$. Note that, although one-loop positive corrections as given in eq.(3.10) will in general be present also for the Higgs fields, one also expects large negative contributions from the usual one-loop top-quark contribution which will again favour EW symmetry breaking [42].

To sum up, the brane content of table 4 give us an example of how an SM construction can be achieved by means of intersecting D5-branes. This particular class of models shares many features already present in the D6-branes models of [15], whereas some important novelties do also appear. Notice that in this section we have restricted ourselves to only one possible quiver configuration of fig. 4. Some other inequivalent constructions can also be performed from the rest of the quivers in that figure, their discussion being postponed to Appendix II.
3.2 D5 Left-Right Symmetric Models

Quite analogously, the LR structure can also be implemented in a D5-brane construction. To show this, let us again consider a $Z_3$ orbifold. Since the chirality pattern is the same for both SM and LR configurations, the possible brane distributions will again be those of figure 4. Let us consider now the quiver $a_2)$. The brane content with LR spectrum for such quiver is shown in table 7.

![Table 7: D5-branes wrapping numbers and CP phases yielding a LR spectrum in the $Z_3$ orbifold of fig.4.a2). Solutions are parametrized by $n_a^1, n_d^1 \in Z, \epsilon, \tilde{\epsilon} = \pm 1, \beta^1 = 1 - b^{(1)} = 1, 1/2$ and $\rho = 1, 1/3$.]

Notice that branes $b$ and $c$ belong in fact to the same stack of four branes, with a non-trivial CP action on it. From the point of view of gauge fields, however, each one is a separate sector. Tadpole cancellation conditions are, as usual, almost satisfied when imposing this wrapping numbers. The only non-trivial conditions that remains is

$$\frac{3n_a^1}{\rho} - \frac{2\tilde{\epsilon}}{\beta^1} + n_d^1 + 2N_h \epsilon_h \beta^1 = -8. \quad (3.15)$$

On the other hand, we must also compute the couplings to RR twisted fields, which in this case are

$$B_2^{(1)} \wedge c_1 (\alpha - \alpha^2) \frac{2\tilde{\epsilon}}{\beta^1} (F^b - F^c)$$

$$D_2^{(1)} \wedge c_1 \left( \frac{3\epsilon}{\beta^1} (F^b + F^c) - 3\epsilon \tilde{\epsilon} (n_a^1 F^a - \rho n_d^1 F^d) \right)$$

$$E_2^{(1)} \wedge c_1 \frac{2\epsilon\beta^1}{\rho} (3F^a + F^d) \quad (3.16)$$

This $B \wedge F$ couplings will again give mass to three of the four $U(1)$ gauge bosons
initially present in our spectrum. If we impose the condition

\[ n_a^1 = -3\rho n_d^1, \]  

then the only generator with null coupling to these fields is \( Q_0 = Q_a - 3Q_d \), which corresponds to \( U(1)_{B-L} \). After imposing this condition, tadpoles (3.15) become

\[ 4n_d^1 + \frac{1}{\beta_1^1} (\bar{\epsilon} - N_h \epsilon_h) = 4, \]  

so the extra brane \( h \) will be generically necessary in order to satisfy tadpoles.

For completeness, let us give an explicit solution of (3.18). Consider the following choice of parameters:

\[ n_d^1 = N_h = 1 \quad \overset{(3.17)}{\Rightarrow} \quad n_a^1 = -3\rho \quad \epsilon_h = \bar{\epsilon}, \]

which now give us a non-trivial \( h \) sector with gauge group \( U(1) \). Following the same considerations as in the previous SM construction, we see that the angles the branes \( a, d \) and \( h \) form with the orientifold plane are

\[ \begin{align*}
\theta^1_a &= \epsilon \left( \pi - tg^{-1} \left( \frac{\beta^1_1}{3\rho} U^1 \right) \right) \\
\theta^1_d &= \epsilon \left( \pi - tg^{-1} \left( \frac{\beta^1_1}{3\rho} U^1 \right) \right) \\
\theta^1_h &= \frac{\pi}{2} (1 - \bar{\epsilon}) \\
\theta^2_a &= -\epsilon \bar{\epsilon} \left( \pi -tg^{-1} \left( \frac{\beta^2_1}{2\rho} U^2 \right) \right) \\
\theta^2_d &= \epsilon \bar{\epsilon} \left( \pi -tg^{-1} \left( \frac{\beta^2_1}{2\rho} U^2 \right) \right) \\
\theta^2_h &= 0
\end{align*} \]

where again \( U^i = R^i_2/R^i_1, i = 1, 2 \). Under the choice \( U^1 = \frac{3\rho^2}{2\beta^1_1} U^2 \), some of the potential tachyons in these sectors will become massless, as for instance those arising from \((aa^*)\) intersections. However, just as in the previously discussed SM construction some tachyons will remain at \((ad)\), \((ad^*)\) intersections, and some other new tachyons involving the brane \( h \). Again, as in the previous SM case, one-loop contributions to the scalar masses may easily overcome the tachyonic contribution.

One can also find an interesting family of left-right symmetric models with no open string tachyons already at the tree-level. Indeed, it is quite easy to generalize the Left-Right symmetric spectrum for a \( \mathbb{Z}_N \) orbifold with odd \( N > 3 \). As an example, let us take the brane content of table 8, which corresponds to a particular case of fig. 3.a), and that will again give us the spectrum of table 3. As in our previous LR example, tadpoles will be cancel by means of a hidden-brane sector, which in this \( \mathbb{Z}_N \) case will consist of a brane system as shown in table 9. There one has \( \epsilon_{hi} = \pm 1 \) and the value of \( s \) is fixed by tadpole conditions. Consistency conditions in (2.16) are now easily
Table 8: D5-branes wrapping numbers and CP phases yielding a LR spectrum in a $\mathbb{Z}_N$. Solutions are parametrized by $n^1_b, n^1_c \in \mathbb{Z}$, $\epsilon, \tilde{\epsilon} = \pm 1$ and $\beta^1 = 1 - b^{(1)} = 1,1/2$.

| $N_i$ | $(n^1_i, m^1_i)$ | $(n^2_i, m^2_i)$ | $\gamma_{w,i}$ |
|-------|-----------------|-----------------|----------------|
| $N_a = 3$ | $1/\beta^1, 0$ | $(\epsilon, \frac{1}{2}\tilde{\epsilon})$ | 13 |
| $N_b = 2$ | $(n^1_b, -\tilde{\epsilon} \beta^1)$ | $(3, \frac{1}{2}\epsilon \tilde{\epsilon})$ | $\alpha_{12}$ |
| $N_c = 2$ | $(n^1_c, \epsilon \beta^1)$ | $(3, \frac{1}{2}\epsilon \tilde{\epsilon})$ | $\alpha_{12}$ |
| $N_d = 1$ | $(1/\beta^1, 0)$ | $(-3\epsilon, \frac{1}{2}\tilde{\epsilon})$ | 1 |

Table 9: Hidden brane system in a $\mathbb{Z}_N$ orbifold singularity.

| $N_i$ | $(n^1_i, m^1_i)$ | $(n^2_i, m^2_i)$ | $\gamma_{w,i}$ |
|-------|-----------------|-----------------|----------------|
| $N_{h1}$ | $(\epsilon_{h1}/\beta^1, 0)$ | $(n^2_h, m^2_h)$ | $\alpha$ |
| $N_{h2}$ | $(\epsilon_{h2}/\beta^1, 0)$ | $(2, 0)$ | $\alpha^3$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $N_{hs}$ | $(\epsilon_{hs}/\beta^1, 0)$ | $(2, 0)$ | $\alpha^{2s-1}$ |

satisfied. Indeed, second and fourth conditions are already satisfied with this brane content, while the third amounts to imposing

$$\epsilon \tilde{\epsilon} (n^1_b + n^1_c) + \epsilon_{h1} N_{h1} \frac{m_h^2}{\beta^1} = 0. \quad (3.21)$$

As mentioned above, the first of these conditions can be expressed as (2.19), from where we can read that we must also impose

$$6(n^1_b + n^1_c) + \epsilon_{h1} N_{h1} \frac{n_h^2}{\beta^1} = \eta 16 \quad (3.22)$$

$$\epsilon_{hi} N_{hi} \frac{2}{\beta^1} = \eta 16, \quad (i = 2, \cdots, r), \quad (3.23)$$

where $r$ and $\eta$ have been defined in (2.20). Last condition actually implies $s = r$, $\epsilon_{hi} = \eta$ and $N_{hi} = 8\beta^1$, for $i > 1$. Let us also compute the couplings to RR 2-form
twisted fields which will render some of these $U(1)$ gauge bosons massive. Even if there are in principle $2(N-1)$ such fields, most of their couplings are redundant, so we will still have some massless $U(1)$’s in our gauge group. Indeed, these couplings are

$$B_2^{(k)} \wedge c_k \left( (\alpha^k - \bar{\alpha}^k) \left[ 6(n_b^1 F_b + n_c^1 F_c) + \epsilon_{h_1} N_{h_1} \frac{n_b^2}{\beta^1} F^{h_1} \right] + \sum_{i=2}^r (\alpha^{ik} - \bar{\alpha}^{ik}) \eta_1 6 F^{h_i} \right),$$

$$C_2^{(k)} \wedge c_k (\alpha^k - \bar{\alpha}^k) \epsilon \beta^1 (-F^b + F^c),$$

$$D_2^{(k)} \wedge c_k \left( \frac{2\epsilon}{\beta^1} (3F^a + F^d) + (\alpha^k + \bar{\alpha}^k) \left[ \epsilon \bar{\epsilon} (n_b^1 F_b + n_c^1 F_c) + \epsilon_{h_1} N_{h_1} \frac{m_d^2}{\beta^1} F^{h_1} \right] \right),$$

$$E_2^{(k)} \wedge c_k (\alpha^k + \bar{\alpha}^k) \epsilon \beta^1 \bar{\epsilon} (-F^b + F^c).$$

(3.24)

Imposing tadpole conditions (3.21), (3.22) and (3.23) is easy to see that the only linear combination of abelian groups that does not couple to any RR field is just $U(1)_{B-L} = \frac{1}{3} U(1)_a - U(1)_d$, providing us with another example of Left-Right symmetric model. This family of configurations yielding the same spectrum for arbitrary odd-ordered $Z_N$ orientifold seems quite interesting, since it gives us a family of $Z_N$ models with $N$ arbitrarily large. In addition they may have an open-string tachyonless spectrum. For instance, by the choice of discrete parameters

$$n_b^1 = n_c^1 = \eta, \quad N_{h_1} = 4 \beta^1,$$

$$\epsilon_{h_1} = \eta, \quad (n_b^2, m_h^2) = (1, -\frac{1}{2} \bar{\epsilon} \bar{\epsilon}),$$

(3.25)

conditions (3.21), (3.22) and (3.23) are satisfied, and the compactification radii can also be chosen to avoid any tachyonic excitation. Indeed, our potential tachyons will arise only from $(bh)_1$ and $(ch)_1$ intersections whose characteristic angles are

$$\pi |\vartheta_{bh_1}^1| = \pi |\vartheta_{ch_1}^1| = tg^{-1} \left( \beta^1 U^1 \right); \quad \pi |\vartheta_{bh_1}^2| = \pi |\vartheta_{ch_1}^2| = tg^{-1} \left( \frac{U^2}{6} \right) + tg^{-1} \left( \frac{U^2}{2} \right),$$

(3.26)

so by appropriately choosing the complex structure moduli we can achieve $|\vartheta_{bh_1}^1| = |\vartheta_{bh_1}^2|$ and $|\vartheta_{ch_1}^1| = |\vartheta_{ch_1}^2|$, finding a one-parameter family of tachyonless open-string spectra.

Let us end this subsection by recalling an apparent phenomenological shortcoming of the class of left-right symmetric models built here. Eventually we would like to break the gauge symmetry down to the Standard Model one and, in order to do that, we need to give a vev to a right-handed doublet of scalars with non-vanishing lepton number. No such scalars are present in the lightest spectrum of the particular models constructed here. It would be interesting to find other examples in which correct gauge symmetry breaking is feasible.
4 Some physics issues

4.1 Low-energy spectrum beyond the SM

Let us summarize the lightest (open string) spectra in the class of SM D5-brane constructions:

- **Fermions**
  The only massless fermions are the ones of the SM (plus right-handed neutrinos). In particular, unlike the case of D6-branes, there are no gauginos in the lightest spectrum.

- **Gauge bosons**
  There are only the ones of the SM (or its left-right extension). There are in addition three extra massive (of order the string scale) $Z_0$’s, two of them anomalous and the other being the extra $Z_0$ of left-right symmetric models. As discussed in ref.[43] for a string scale of order a few TeV the presence of these extra $U(1)$’s may be amenable to experimental test. In fact already present constraints from electroweak precision data (i.e., $\rho$-parameter) put important bounds on the mass of these extra gauge bosons.

- **Scalars in the D5-branes bulk**
  There are two copies of scalars in the adjoint representation of $SU(3) \times SU(2) \times U(1)_a \times U(1)_b \times U(1)_c \times U(1)_d$, as given in eq.(2.9). These will include a couple of colour octets and $SU(2)_L$ triplets plus eight singlets. The vevs of the latter parametrize the locations of the four stacks of branes along the two tori ($4 \times 2$ parameters) and hence are moduli at the classical level. The colour octets and $SU(2)$ triplets get masses at one loop as given in eq.(3.10).

- **Scalars at the intersections**
  These are model dependent. In the SM example described in some detail in section (3.1) there are colour triplets and sextets (from $(aa\ast)$) and colour triplets ‘leptoquarks’ (from $(ad), (ad\ast)$) (see table 5). Again their leading contribution to their masses should come from eq.(3.10). These scalars are not stable particles, they decay into quarks and leptons through Yukawa couplings. In the SM examples in Appendix II the scalars at the intersections are colour singlets.
• **SM Higgs doublets**

There are sets of Higgs doublets as in table 6 with a multiplicity which is model dependent. In the example of section (3.1) the multiplicity is one and hence we have the same minimal Higgs sector as in the MSSM.

The above states constitute the lightest states in the brane configuration. At the massive level there will appear Kaluza-Klein replicas for the gauge bosons as well as stringy winding and oscillator states (gonions). Compared to the spectra of D6-brane intersection models [15, 26, 27] the present spectrum is quite simpler, since the fermions and gauge bosons of the SM do not have any kind of SUSY partner.

Note that the structure of the $U(1)$ gauge bosons in D5-brane models is remarkably similar to that of the D6-brane models of ref.[15, 26, 27]. This similarity is dictated by the massless chiral fermion spectrum in both classes of models which is identical, i.e., the fermions of the SM. In particular baryon number is a gauged symmetry ($U(1)_a$) which remains as a global symmetry in perturbation theory once the corresponding $U(1)$’s become massive. This naturally guarantees proton stability.

Concerning the closed string sector, the $Z_N$ projection kills all fermionic partners of the untwisted sector. We will have the graviton plus a number of untwisted moduli field as well as untwisted RR-fields. The twisted closed string sector is relevant to anomaly cancellation.

### 4.2 Lowering the string scale

The D5-brane models here constructed are non-supersymmetric. In order not to have the standard hierarchy problem for Higgs scalars the most obvious possibility is to have the fundamental string scale not much above the weak scale. Thus we should have $M_s \propto 1 - 10$ TeV.

Interestingly enough, in the intersecting D5-brane models here studied one can have a low string scale $M_s \propto 1 - 10$ TeV while maintaining the experimentally measured four-dimensional Planck mass $M_p = 1.18 \times 10^{19}$ GeV by some dimensions getting very large [30]. Indeed, in the present examples the compact space has the form $T^4 \times B_2$, and the D5-branes sit at a $C/Z_N$ singularity in $B_2$ and wrap two-cycles on $T^4$. Let us denote by $V_4$ the volume of $T^4$ and by $V_2$ that of the manifold $B_2$. Then the Planck scale is given by

$$M_p = \frac{2}{\lambda} M_s \sqrt{V_4 V_2} \quad (4.1)$$
Figure 5: Intersecting D5-world set up. The $Z_i$, $i = 1, 2, 3$ represent complex compact dimensions. The D5-branes $a, b, c, d$ (corresponding to the gauge group $U(3) \times U(2) \times U(1) \times U(1)$) wrap cycles on $T^2 \times T^2$. At the intersections lie quarks and leptons. This system is transverse to a 2-dimensional compact space $B_2$ (e.g., $T^2/\mathbb{Z}_N$) whose volume may be quite large so as to explain $M_p >> M_s$. This would be a D-brane realization of the scenario in ref.[30].
In order to avoid too light KK/Winding modes in the worldvolume of the D5-branes let us assume $V_4 \propto 1/M_s^4$. Then one has

$$V_2 = \frac{M_s^2 \lambda^2}{4M_s^4}$$

and one can accommodate a low string scale $M_s \propto 1$ TeV by having the volume $V_2$ of the 2-dimensional manifold $B_2$ large enough (i.e., of order $(mm.)^2$). For a pictorial view of this explicit D-brane realization of the proposal in [30] see fig.5.

5 Final comments

In this paper we have presented D5-brane configurations wrapping cycles on $T^2 \times T^2 \times (C/Z_N)$ yielding the massless fermionic spectrum of the three-generation SM. This is a generalization of the work in ref.[15] in which it was obtained the SM spectrum from D6-branes wrapping cycles on $T^2 \times T^2 \times T^2$. We have also presented for completeness the case of D4-branes wrapping cycles on $T^2 \times (T^4/Z_N)$, which turns out to be less flexible from the model-building point of view.

One of our main motivations to consider the case of D5-branes is the fact that in this case there are 2 dimensions which are transverse to the SM D5-brane configuration. By making those two dimensions large enough one can have a low string scale $M_s$ of order 1-10 TeV and still have a large $M_{Planck}$ in agreement with observations. From this point of view these are the first explicit D-brane string constructions in which one has just the fermionic spectrum of the SM at low energies and the mechanism for lowering the string scale in [30] simultaneously at work.

There are a number of questions both theoretical and phenomenological which we have not addressed in this paper and should be the subject of further research. These D5-brane constructions are non-supersymmetric and it remains to be seen if such configurations can be rendered stable. One source of instability may be the presence of closed string tachyons in the twisted spectrum. An analogous class of tachyons have been studied recently in ref.[34] in the non-compact orbifold case. That analysis cannot be directly translated to the compact orientifold case considered here in which e.g. the tachyons are real rather than complex fields. It remains to be seen whether in the compact orientifold case a stabilization of the closed string tachyons may be feasible. This is also relevant to the question whether one can obtain a stable minimum in which the two dimensions transverse to the SM brane configuration are very large compared to the rest, thus providing for a dynamical explanation of the smallness of the string
scale compared to the Planck mass.

The only fermions in the light spectrum are those of the non-SUSY SM. The fermions do not have any SUSY-partners, no squarks or sleptons appear. There are however some scalars in the lightest spectrum. There are some with the quantum numbers of electroweak Higgs fields which may become tachyonic and trigger electroweak symmetry breaking if certain branes are sufficiently close. On the other hand there are further scalars which may be tachyonic at the tree level. In the simple SM example in the text those are coloured particles and we argue that their full mass\(^2\) including one-loop effects will in general be positive. Those coloured (triplets and sextets ) should then be relatively light with masses close to the electroweak scale. They are unstable and decay into ordinary quarks and leptons. In addition there are three extra \(Z_0\)'s beyond the ordinary one with masses of order the string scale (i.e., 1-10 TeV in low string models). These may lead already to observable effects as recently argued in ref.[43]. The fact that baryon number is gauged will guarantee that in these constructions the proton is perturbatively stable. We leave a more systematic study of the phenomenological aspects of this class of brane models for future work.

**Acknowledgements**

We are grateful to G. Aldazábal, A. Font, C. Kokorelis, R. Rabadán and A. Uranga for useful discussions. The research of D.C. and F.M. was supported by the Ministerio de Educación, Cultura y Deporte (Spain) through FPU grants. This work is partially supported by CICYT (Spain) and the European Commission (RTN contract HPRN-CT-2000-00148).
6 Appendix I: D4-branes wrapping on $T^2 \times C^2/Z_N$ orientifolds

For the sake of completeness, in this appendix we present the general construction involving intersecting D4-branes in an orientifold singularity. This general class of models is both of theoretical and phenomenological interest since they also provide a natural setup for considering chiral compactifications with low string scale scenarios. To be concrete, we will consider the compactification

$$\text{Type IIA on } M_4 \times \frac{T^2 \times C^2}{1 + \Omega R},$$

where $R$ now stands for $R_{(5)}R_{(6)}R_{(7)}R_{(8)}R_{(9)}$. In terms of its action on complex coordinates this involution is given by

$$R: Z_1 \mapsto \bar{Z}_1, \quad Z_i \mapsto -Z_i, \quad i = 2, 3.$$

This theory will contain a O4-plane wrapping a 1-cycle in $T^2$ (the one invariant under $R_{(5)}$), and in order to cancel its negative RR charge we will have to include an open string sectors involving D4-branes wrapping 1-cycles $[\Pi] = [(n, m)]^8$ of this same $T^2$, while sitting at the origin of $C^2/Z_N$.

The geometric action of the orbifold group $Z_N$ can be described by a twist vector $v_{\omega} = \frac{1}{N}(0, b_1, b_2, 0), b_1 = b_2 \text{ mod 2}$ for the variety to admit spinors. This twist will preserve some bulk supersymmetry whenever $b_1 = \pm b_2 \text{ mod } N$. Just as in the case of D5-branes, the orbifold action on the Chan-Paton degrees of freedom can be described by a matrix of the form (2.3), and the orientifold action can also be implemented by adding a mirror sector $a^*$ for every D4-brane $a$ in the configuration. If we again consider effective wrapping number for describing our 1-cycles, mirror branes will be related in an analogous way that the one described in (2.5) and (2.6) for the case of D5-branes.

Let us now describe the low energy spectrum of the theory

- Closed string sector

  The twisted closed string sector will consist, in the supersymmetric case, of a $D = 4$ $N = 4 U(\frac{N-1}{2})$ gauge multiplet for odd $N$, the gauge group being $U(\frac{N}{2})$ if $N$ is even. When dealing with the non-supersymmetric $|b_1| \neq |b_2|$ case, however, the twisted closed string spectrum will be much similar to the case of D5-branes, a closed string tachyon appearing for each twisted sector.

---

8 Notice that in this particular class of compactifications every cycle is factorizable.
• $D_{4a}D_{4a}$ sector

This sector gets mapped to $D_{4a^*}D_{4a^*}$, which usually is a different sector of the theory. The computation of its massless spectrum will be the same as in the orbifold case, already computed in [17]. However, we present its computation for completeness. The massless GSO projected states in both R and NS sectors are

\[
\text{NS Sector} \quad Z_N \text{ phase} \quad \text{R Sector} \quad Z_N \text{ phase}
\begin{align*}
(\pm 1, 0, 0, 0) & \quad 1 & \pm \frac{1}{2}(-, +, +, +) & \quad e^{\pm \pi i \frac{b_1+b_2}{N}} \\
(0, \pm 1, 0, 0) & \quad e^{\pm 2\pi i \frac{b_0}{N}} & \pm \frac{1}{2}(+, -, +, +) & \quad e^{\pm \pi i \frac{b_0-b_2}{N}} \\
(0, 0, \pm 1, 0) & \quad e^{\pm 2\pi i \frac{b_0}{N}} & \pm \frac{1}{2}(+, +, +, -) & \quad e^{\pm \pi i \frac{b_0-b_2}{N}} \\
(0, 0, 0, \pm 1) & \quad 1 & \pm \frac{1}{2}(+, +, +, -) & \quad e^{\pm \pi i \frac{b_0-b_2}{N}}
\end{align*}
\]

where the behaviour of such states under the $Z_N$ action has been indicated. Keeping states invariant under combined geometrical and Chan-Paton action we are left with the following spectrum

- **Gauge Bosons** \( \prod_a \Pi_{i=1}^N U(N_a^i) \)
- **Complex Scalars** \( \sum_a \sum_{i=1}^N [(N_a^i, \overline{N_a^i}^b_1^i + b_1) + (N_a^i, \overline{N_a^i}^1^i + b_2) + \text{Adj}_a^i] \)
- **Left Fermions** \( \sum_a \sum_{i=1}^N [(N_a^i, \overline{N_a^i}^{b_1-b_2}_a) + (N_a^i, \overline{N_a^i}^{b_1-b_2}_a)] \)
- **Right Fermions** \( \sum_a \sum_{i=1}^N [(N_a^i, \overline{N_a^i}^{b_1+b_2}_a) + (N_a^i, \overline{N_a^i}^{b_1+b_2}_a)] \)

which is generically non-supersymmetric and always non-chiral. The supersymmetric twist give us the $\mathcal{N} = 2$ theory

- **Vector Multiplet** \( \prod_a \Pi_{i=1}^N U(N_a^i) \)
- **Hypermultiplet** \( \sum_a \sum_{i=1}^N (N_a^i, \overline{N_a^i}^{i+1}_a) \)

• $D_{4a}D_{4b}$, $D_{4a}D_{4b^*}$ and $D_{4a}D_{4a^*}$ sectors

These three sectors will contain the chiral spectrum of the theory. Let us analyze the $D_{4a}D_{4b}$ spectrum, whose associated twisted vector is given by \( v_\theta = (0, \vartheta_{ab}, 0, 0) \). Being mapped into $D_{4a^*}D_{4a^*}$ under the action of $\Omega \mathcal{R}$, we only have to consider the orbifold action. The massless states are

\[
\begin{array}{cccccc}
\text{Sector} & \text{State} & Z_N \text{ phase} & \alpha'\text{Mass}^2 \\
\hline
\text{NS} & (-1 + \vartheta, 0, 0, 0) & 1 & -\frac{1}{2}|\vartheta_{ab}| \\
\text{R} & (-\frac{1}{2} + \vartheta, +\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}) & e^{\pi i \frac{(\vartheta_1-\vartheta_2)}{N}} & 0 \\
& (-\frac{1}{2} + \vartheta, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}) & e^{-\pi i \frac{(\vartheta_1-\vartheta_2)}{N}} & 0 \\
& (-\frac{1}{2} + \vartheta, -\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}) & e^{-\pi i \frac{(\vartheta_1+\vartheta_2)}{N}} & 0 \\
& (-\frac{1}{2} + \vartheta, +\frac{1}{2}, +\frac{1}{2}, +\frac{1}{2}) & e^{\pi i \frac{(\vartheta_1+\vartheta_2)}{N}} & 0 \\
\end{array}
\]
where we have supposed $0 < \vartheta_{ab} < 1$. This spectrum is explicitly non-supersymmetric, even for a supersymmetric twist. Keeping the invariant states we are left with the spectrum

\[
\begin{align*}
\text{Tachyons} & : \sum_{a<b} \sum_{i=1}^{N_i} I_{ab} \times (N_a^i, N_b^i) \\
\text{Left Fermions} & : \sum_{a<b} \sum_{i=1}^{N_i} I_{ab} \times \left[ (N_a^i, N_b^{-i}) + (N_a^{-i}, N_b^i) + (N_a^{-i}, N_b^{-i}) \right] \\
\text{Right Fermions} & : \sum_{a<b} \sum_{i=1}^{N_i} I_{ab} \times \left[ (N_a^i, N_b^{-i}) + (N_a^{-i}, N_b^i) + (N_a^{-i}, N_b^{-i}) \right]
\end{align*}
\]

(6.8)

which is generically supersymmetric. Notice that the intersection number is now given by $I_{ab} \equiv [\Pi_a] \cdot [\Pi_b] = n_a m_b - m_a n_b$. Similarly, we can compute the other two chiral sectors of the theory, the complete spectrum being \(^9 \)

\[
\begin{align*}
\text{Tachyons} & : \sum_{a<b} \sum_{i=1}^{N_i} \left[ I_{ab} (N_a^i, N_b^i) + I_{ab} (N_a^{-i}, N_b^i) \right] \sum_a [2|m_a| (|n_a| + 1) (A_a^0) + 2|m_a| (|n_a| - 1) (S_a^0)] \\
\text{Left Fermions} & : \sum_{a<b} \sum_{i=1}^{N_i} I_{ab} \left[ (N_a^i, N_b^{-i}) + (N_a^{-i}, N_b^i) + (N_a^{-i}, N_b^{-i}) \right] \\
\text{Right Fermions} & : \sum_{a<b} \sum_{i=1}^{N_i} I_{ab} \left[ (N_a^i, N_b^{-i}) + (N_a^{-i}, N_b^i) + (N_a^{-i}, N_b^{-i}) \right] \\
\end{align*}
\]

(6.9)

The construction of these configurations are, as usual, constrained by tadpole cancellation conditions, which in this case read

\[
\begin{align*}
& c_k^2 \sum_a n_a (\text{Tr} \gamma_{k,a} + \text{Tr} \gamma_{k,a^*}) = 8 \prod_{r=1}^{2N} \sin \left( \frac{\pi k b_r}{2N} \right), \quad (6.10) \\
& c_k^2 \sum_a m_a (\text{Tr} \gamma_{k,a} - \text{Tr} \gamma_{k,a^*}) = 0, \quad (6.11)
\end{align*}
\]

where now $c_k^2 = \prod_{r=1}^{2N} \sin(\pi k b_r / N)$, and we are using effective wrapping numbers. These two conditions can be expressed more elegantly as

\[
\begin{align*}
& c_k^2 \sum_a ([\Pi_a] \text{Tr} \gamma_{k,a} + [\Pi_{a^*}] \text{Tr} \gamma_{k,a^*}) = [\Pi_{O4}] \sum_{r=1}^{2N} \sin \left( \frac{\pi k b_r}{2\beta N} \right), \quad (6.12)
\end{align*}
\]

\(^9\)In case $n_a = 0$, there is just one tachyon coming from the $aa^*$ sector, transforming in the antisymmetric representation $(A_a^0)$ of the $U(N_a^0)$ gauge group. This is just a T-dual orbifolded version of the non-BPS systems constructed in [44].
where $\beta = 1 - b$ discriminates between rectangular and tilted tori. In the same manner as sketched for the case of D5-branes in section 2, tadpole conditions will directly imply cancellation of cubic chiral anomalies, whose expression is now given by

$$A_{SU(N_j)^3} = \sum_{b,k} N_b^k (I_{ab} \delta(j,k) + I_{ab^*} \delta(j,-k)) + 8\beta I_{a,O4} \delta(j,-j) \quad (6.13)$$

$$\delta(j,k) \equiv \delta_{j,k}^{1/2} + \frac{1}{\sqrt{2}} \delta_{j,k}^{1/2} + \frac{1}{\sqrt{2}} \delta_{j,k}^{-1/2} - \delta_{j,k}^{-1/2} - \delta_{j,k}^{1/2} - \delta_{j,k}^{-1/2}. \quad (6.14)$$

On the other hand, the mixed anomalies analysis mimics the one performed in Section 2 for D5-branes. In fact, expressions (2.23) and (2.24) are also valid for this case if we just substitute $16m_a^1m_a^2$ by $8m_a$ and consider the definitions of $\delta(i,j)$ and $c_k$ used in this appendix. The only difference comes from the details of the GS mechanism which now only involves $(N-1)$ RR twisted fields. For completeness, we present the four-dimensional couplings that give rise to such mechanism

$$c_k N_a n_a \int_{M_4} \text{Tr} (\gamma_{k,a} - \gamma_{k,a^*}) \lambda_i C_2^{(k)} \wedge \text{Tr} F_{a,i},$$
$$c_k N_a m_a \int_{M_4} \text{Tr} (\gamma_{k,a} + \gamma_{k,a^*}) \lambda_i B_2^{(k)} \wedge \text{Tr} F_{a,i}, \quad (6.15)$$
$$c_k N_b m_b \int_{M_4} (\gamma_{k,b} - \gamma_{k,b^*}) \lambda_j^2 C_0^{(k)} \wedge \text{Tr} (F_{b,j} \wedge F_{b,j}),$$
$$c_k N_b n_b \int_{M_4} (\gamma_{k,b} + \gamma_{k,b^*}) \lambda_j^2 B_0^{(k)} \wedge \text{Tr} (F_{b,j} \wedge F_{b,j}). \quad (6.16)$$

Of special interest are the couplings (6.15), which encode the massive $U(1)$’s of the theory.

Let us also sketch some model-building features regarding D4-branes orientifold models. For simplicity, we will constrain ourselves to the supersymmetric case $b_1 = -b_2 = 1$. Since the $\mathbb{Z}_3$ orbifold case has already been considered in [24], we will focus on odd $N > 3$ orientifolds. In the same way as performed for D5-branes tadpoles, we can express the tadpole condition (6.10) as

$$\sum_a n_a (\text{Tr} \gamma_{\omega,a} + \text{Tr} \gamma_{\omega,a^*}) = \frac{8}{(\alpha_{N+1}^{N+1} + \bar{\alpha}_{N+1}^{N+1})^2}, \quad (6.17)$$

where $\alpha = e^{2\pi i/N}$. Again we can reexpress (6.17) as a sum of orbifold phases by using

$$\frac{1}{\alpha_{N+1}^{N+1} + \bar{\alpha}_{N+1}^{N+1}} = \tau \left( 1 + \sum_{l=1}^r (\alpha^r + \bar{\alpha}^r) \right), \quad (6.18)$$

$$\tau = \begin{cases} 
+1 & \text{if } N = 4r + 1 \\
-1 & \text{if } N = 4r + 3 \end{cases} \quad (6.19)$$

Let us, for instance, consider the $\mathbb{Z}_5$ orientifold model whose brane content is shown in table 10. As usual, the brane content of this model consist of four D4-branes $a, b, c, d$,
again identified with those of table 1, plus some hidden brane $h$. The gauge group is $SU(3) \times SU(2) \times SU(2) \times U(1)^4 \times [U(4)_h]$, which is the LR gauge group extended by three abelian groups and one hidden $U(4)_h$. The chiral matter content of such model is given in table 11

| Intersection | Matter fields | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ | $B - L$ |
|--------------|--------------|------|------|------|------|--------|
| $(ab)$       | $Q_L$        | 1    | -1   | 0    | 0    | 1/3    |
| $(ac)$       | $Q_R$        | 3(3, 1, 2) | 0    | 0    | 1    | -1/3   |
| $(bd)$       | $L_L$        | 3(1, 2, 1) | 0    | 0    | 1    | -1     |
| $(cd)$       | $L_R$        | 3(1, 1, 2) | 0    | 0    | 1    | -1     |
| $(bc^*)$     | $H$          | 3(1, 2, 2) | 0    | 1    | 0    | 0      |
| $(bb^*)$     | $A_i$        | 3(1, 1, 1) | 0    | -2   | 0    | 0      |
| $(cc^*)$     | $S_i$        | 3(1, 1, 3) | 0    | 0    | -2   | 0      |

Table 11: Extended Left-Right symmetric chiral spectrum arising from the $Z_5$ D4-branes model of table 10. The $U(1)_{B-L}$ generator is defined as $Q_{B-L} = \frac{1}{3}Q_a - Q_d$.

Notice that this particular example does not follow the general philosophy described in Section 3, where every chiral fermion arised from a bifundamental representation and the matter content was thus described by table 3. Instead, we now find some extra chiral fermions that can be identified with three Higgsino-like particles, whereas some exotic matter transforming as singlets ($A_i$) and symmetrics of $SU(2)_R$ ($S_i$) do also appear. The only light bosonic sector arises from branes $b$ and $c$ giving us a Higgs-like
particle that can become tachyonic if we approach both branes close enough. No extra chiral matter nor scalars arise from the hidden sector of the theory.

It is easy to see that this brane content satisfies both twisted tadpole conditions (6.10) and (6.11). Interestingly enough, it also satisfies untwisted tadpoles conditions, so when embedding such model in a compact four-dimensional manifold $B$ no extra brane content would be needed.

Finally, by computing the couplings (6.15) that mediate the GS mechanism, we can check that two of the abelian gauge groups are in fact massive, the only massless linear combinations being $U(1)_{B-L} = \frac{1}{3} U(1)_a - U(1)_d$ and $U(1)_b + U(1)_c$, just as in our $\mathbb{Z}_N$ D5-branes constructions of Section 3.2.

Note that the $\mathbb{Z}_5$ twist in this model preserves a $\mathcal{N} = 2$ supersymmetry of the gravitational bulk. Due to this fact there are no closed string twisted tachyons.


7 Appendix II: Other D5-brane configurations yielding SM spectra

Although in Section 3 we have focussed on a very particular class of D5-branes configurations in a $\mathbb{Z}_3$ orbifold, there are other possibilities when constructing models giving rise to just the SM fermionic spectrum. Indeed, the brane content of table 4 corresponds to the brane distribution of fig. 4.1, while in principle any of these four figures is valid. For completeness, in this appendix we consider the other three possibilities.

After imposing the analogous constraints to the rest of the $\mathbb{Z}_3$ quivers of figure 4, we find that the distribution $a_2$) give us a totally equivalent class of models to the one already presented, whereas $b_1$) and $b_2$) give us two new different families of configurations. Let us first consider the $\mathbb{Z}_3$ quiver in fig. 4.1. The wrapping numbers giving the same SM spectrum of table 2 are shown in table 12.

| $N_i$ | $(n_i^1, m_i^1)$ | $(n_i^2, m_i^2)$ | $\gamma_{\omega,i}$ |
|-------|-----------------|-----------------|---------------------|
| $N_a = 3$ | $(1/\beta^1, 0)$ | $(\epsilon, -\frac{1}{2}\bar{\epsilon})$ | $\alpha 1_3$ |
| $N_b = 2$ | $(n_b^1, \bar{\epsilon}\beta^1)$ | $(1, -\frac{2}{\epsilon}\bar{\epsilon})$ | $1_2$ |
| $N_c = 1$ | $(n_c^1, 3\epsilon\beta^1)$ | $(0, 1)$ | $1$ |
| $N_d = 1$ | $(1/\beta^1, 0)$ | $(\epsilon, \frac{3}{2}\bar{\epsilon})$ | $\alpha$ |
| $N_h$ | $(\epsilon_h/\beta^1, 0)$ | $(2, 0)$ | $1_{N_h}$ |

Table 12: D5-branes wrapping numbers and CP phases giving rise to a SM spectrum in the $\mathbb{Z}_3$ quiver of fig. 4.1. The solution is now parametrized by $n_b^1, n_c^1 \in \mathbb{Z}$, $\epsilon, \bar{\epsilon} = \pm 1$, and $\beta^1 = 1 - b^{(1)} = 1, 1/2$.

Just as before, tadpoles are almost automatically satisfied, and the only condition to be imposed is

$$n_b^1 = -4 + \frac{1}{\beta^1}(\epsilon - N_h \epsilon_h). \quad (7.1)$$

The $U(1)$ structure is quite similar as well, again with three non-trivial couplings to RR twisted fields, now given by

$$B_2^{(1)} \wedge c_1 \left( \alpha - \alpha^2 \right) \frac{1}{\beta^1} \left( 3F^a + F^d \right)$$
$$D_2^{(1)} \wedge c_1 \left( \frac{2\bar{\epsilon}}{\beta^1} (F^a - F^d) - 6n_b^1 \epsilon \bar{\epsilon} F^b + 2n_c^1 F^c \right)$$
$$E_2^{(1)} \wedge c_1 4\epsilon \beta^1 F^b \quad (7.2)$$
The massless $U(1)$ will again be model-dependent

$$Q_0 = Q_a - 3Q_d - \frac{3\tilde{\epsilon}}{n_c^1\beta^1}Q_c, \quad (7.3)$$

and getting the hypercharge as the unique massless $U(1)$ amounts to requiring

$$n_c^1 = \frac{\tilde{\epsilon}}{\beta^1} \Rightarrow \beta^1 = 1, \quad (7.4)$$

since $n_c^1$ has to be an integer. A simple solution is $N_h = 3, \epsilon = -\epsilon_h = 1$. This implies setting $n_h^1 = 0$, and then we have a single Higgs system as in table 6.

Considering now the quiver in fig. 4.b2) give us another family of configurations. Looking for the same spectrum than in table 2, we find the following wrapping numbers:

| $N_i$ | $(n_i^1, m_i^1)$ | $(n_i^2, m_i^2)$ | $\gamma_{\omega,i}$ |
|-------|-----------------|-----------------|-----------------|
| $N_a = 3$ | $1/\beta^1, 0$ | $(\epsilon, -\frac{1}{2}\tilde{\epsilon})$ | $\alpha_{13}$ |
| $N_b = 2$ | $(n_b^1, \tilde{\epsilon}\beta^1)$ | $(1, -\frac{3}{2}\epsilon\tilde{\epsilon})$ | $1_2$ |
| $N_c = 1$ | $(n_c^1, 3\epsilon\beta^1)$ | $(0, 1)$ | $1$ |
| $N_d = 1$ | $(1/\beta^1, 0)$ | $(-\epsilon, -\frac{3}{2}\epsilon)$ | $\alpha^2$ |
| $N_h$ | $(\epsilon_h/\beta^1, 0)$ | $(2, 0)$ | $1_{N_h}$ |

Table 13: D5-branes wrapping numbers and CP phases giving rise to a SM spectrum in the $\mathbb{Z}_3$ quiver of fig. 4.b2). The solution is now parametrized by $n_b^1, n_c^1 \in \mathbb{Z}, \epsilon, \tilde{\epsilon} = \pm 1$, and $\beta^1 = 1 - b^{(1)} = 1, 1/2$.

Tadpoles read:

$$2n_b^1 = -8 + \frac{1}{\beta^1}(\epsilon - 2N_h \epsilon_h) \Rightarrow \beta^1 = \frac{1}{2}, \quad (7.5)$$

The $U(1)$ couplings are:

$$B_2^{(1)} \wedge c_1 (\alpha - \alpha^2) \frac{\tilde{\epsilon}}{\beta^1} (3F^a + F^d)$$
$$D_2^{(1)} \wedge c_1 \left(\frac{3\tilde{\epsilon}}{2\beta^1}(F^a + F^d) - 6n_b^1\epsilon\tilde{\epsilon}F^b + 2n_c^1F^c\right) \quad (7.6)$$
$$E_2^{(1)} \wedge c_1 4\tilde{\epsilon}\beta^1F^b$$

The massless $U(1)$ will now be

$$Q_0 = Q_a - 3Q_d + \frac{3\tilde{\epsilon}}{2n_c^1\beta^1}Q_c, \quad (7.7)$$
and getting the hypercharge as the unique massless $U(1)$ amounts to requiring

$$n_c^1 = -\frac{\tilde{\epsilon}}{2\beta^1} = -\tilde{\epsilon}. \quad (7.8)$$

Unlike the SM D5-brane constructions in the main text, the lightest scalars and/or tachyons are now colour singlets.
References

[1] G. Aldazábal, L. E. Ibáñez and F. Quevedo, "Standard-like models with broken supersymmetry from type I string vacua," JHEP 0001, 031 (2000), hep-th/9909172. "A D-brane alternative to the MSSM," JHEP 0002, 015 (2000), hep-ph/0001083.

[2] G. Aldazábal, L. E. Ibáñez, F. Quevedo and A. M. Uranga, "D-branes at singularities: A bottom-up approach to the string embedding of the standard model," JHEP 0008, 002 (2000), hep-th/0005067.

[3] M. Cvetič, A. Uranga and J. Wang, "Discrete Wilson lines in N=1, D=4, Type IIB orientifolds: a systematic exploration for Z_6 orientifold," Nucl. Phys. B 595, 63 (2001), hep-th/0010091.

[4] D. Bailin, G. V. Kraniotis and A. Love, "Supersymmetric standard models on D-branes," Phys. Lett. B 502, 209 (2001), hep-th/0011289.

[5] D. Berenstein, V. Jejjala and R. G. Leigh, "The standard model on a D-brane," Phys. Rev. Lett. 88, 071602 (2002), hep-ph/0105042.

[6] A. Uranga, "From quiver diagrams to particle physics," hep-th/0007173.

[7] T. W. Kephart and H. Pas, "Three family N = 1 SUSY models from Z_n orbifolded AdS/CFT," Phys. Lett. B 522, 315 (2001), hep-ph/0109111.

[8] H. X. Yang, "Standard-like model from D = 4 type IIB orbifolds," hep-th/0112259.

[9] L. F. Alday and G. Aldazábal, "In quest of 'just' the standard model on D-branes at a singularity," JHEP 0205, 022 (2002), hep-th/0203129.

[10] M. Berkooz, M. R. Douglas and R. G. Leigh, "Branes intersecting at angles," Nucl. Phys. B 480, 265 (1996), hep-th/9606139; V. Balasubramanian and R. G. Leigh, "D-branes, moduli and supersymmetry," Phys. Rev. D 55, 6415 (1997), hep-th/9611165.

[11] H. Arfaei and M. M. Sheikh Jabbari, "Different D-brane interactions," Phys. Lett. B 394, 288 (1997), hep-th/9608167. M. M. Sheikh Jabbari, "Classification of different branes at angles," Phys. Lett. B 420, 279 (1998), hep-th/9710121.

[12] C. Bachas, "A Way to Break Supersymmetry," hep-th/9503030.
[13] R. Blumenhagen, L. Görlich and B. Körs, “Supersymmetric orientifolds in 6D with D-branes at angles,” Nucl. Phys. B 569, 209 (2000), hep-th/9908130. “Supersymmetric 4D orientifolds of type IIA with D6-branes at angles,” JHEP 0001, 040 (2000), hep-th/9912204. “A new class of supersymmetric orientifolds with D-branes at angles,” hep-th/0002146.

S. Förste, G. Honecker and R. Schreyer, “Supersymmetric $Z_N \times Z_M$ Orientifolds in 4D with D-Branes at Angles,” Nucl. Phys. B 593, 127 (2001), hep-th/0008250.

[14] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, “Asymmetric orbifolds, noncommutative geometry and type I string vacua,” Nucl. Phys. B 582, 44 (2000), hep-th/0003024.

C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, “Type-I strings on magnetised orbifolds and brane transmutation,” Phys. Lett. B 489, 223 (2000), hep-th/0007090.

C. Angelantonj and A. Sagnotti, “Type-I vacua and brane transmutation,” hep-th/0010279.

[15] L. E. Ibáñez, F. Marchesano and R. Rabadán, “Getting just the standard model at intersecting branes,” JHEP 0111, 002 (2001), hep-th/0105155.

L. E. Ibáñez, “Standard Model Engineering with Intersecting Branes,” hep-ph/0109082.

[16] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, “Noncommutative compactifications of type I strings on tori with magnetic flux”, JHEP 0010, 006 (2000), hep-th/0007024. “Magnetic flux in toroidal type I compactification,” Fortsch. Phys. 49, 591 (2001), hep-th/0010198.

[17] G. Aldazábal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, “$D = 4$ chiral string compactifications from intersecting branes,” J. Math. Phys. 42, 3103 (2001), hep-th/0011073.

[18] G. Aldazábal, S. Franco, L. E. Ibáñez, R. Rabadán and A. M. Uranga, “Intersecting brane worlds,” JHEP 0102, 047 (2001), hep-ph/0011132.

[19] R. Blumenhagen, B. Körs and D. Lüst, “Type I strings with F- and B-flux,” JHEP 0102, 030 (2001), hep-th/0012156.

[20] S. Förste, G. Honecker and R. Schreyer, “Orientifolds with branes at angles,” JHEP 0106, 004 (2001), hep-th/0105208.
[21] R. Blumenhagen, B. Körs, D. Lüst and T. Ott, “The standard model from stable intersecting brane world orbifolds,” Nucl. Phys. B 616, 3 (2001), hep-th/0107138. “Intersecting brane worlds on tori and orbifolds,” hep-th/0112015.

[22] M. Cvetič, G. Shiu and A. M. Uranga, “Three-family supersymmetric standard like models from intersecting branes,” Phys. Rev. Lett. 87, 201801 (2001), hep-th/0107143. “Chiral four-dimensional N = 1 supersymmetric type IIA orientifolds from intersecting branes,” Nucl. Phys. B 615, 3 (2001), hep-th/0107166. “Chiral type II orientifold constructions as M theory on G(2) holonomy spaces,” hep-th/0111179.

[23] D. Bailin, G. V. Kraniotis and A. Love, “Standard-like models from intersecting D4-branes,” Phys. Lett. B 530, 202 (2002), hep-th/0108131.

[24] G. Honecker, “Non-supersymmetric orientifolds with D-branes at angles,” hep-th/0112174. “Intersecting brane world models from D8-branes on (T^2 × T^4/Z_3)/ΩR_1 type IIA orientifolds,” JHEP 0201, 025 (2002), hep-th/0201037.

[25] H. Kataoka and M. Shimojo, “SU(3)×SU(2)×U(1) chiral models from intersecting D4/D5 branes,” hep-th/0112247.

[26] D. Cremades, L. E. Ibáñez and F. Marchesano, “SUSY quivers, intersecting branes and the modest hierarchy problem,” JHEP 0207, 009 (2002), hep-th/0201205.

[27] D. Cremades, L. E. Ibáñez and F. Marchesano, “Intersecting Brane Models of Particle Physics and the Higgs Mechanism,” JHEP 0207, 022 (2002), hep-th/0203160.

[28] C. Kokorelis, “GUT model hierarchies from intersecting branes”, hep-th/0203187.

[29] J. D. Lykken, “Weak Scale Superstrings,” Phys. Rev. D 54, 3693 (1996), hep-th/9603133.

[30] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, “The hierarchy problem and new dimensions at a millimeter,” Phys. Lett. B 429, 263 (1998), hep-ph/9803315. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, “New dimensions at a millimeter to a Fermi and superstrings at a TeV,” Phys. Lett. B 436, 257 (1998) hep-ph/9804398.

[31] L. Randall and R. Sundrum, “An alternative to compactification,” Phys. Rev. Lett. 83, 4690 (1999), hep-th/9906064.
[32] M. Bianchi, G. Pradisi and A. Sagnotti, “Toroidal compactification and symmetry breaking in open string theories,” Nucl. Phys. B 376, 365 (1992).
M. Bianchi, “A note on toroidal compactifications of the type I superstring and other superstring vacuum configurations with 16 supercharges,” Nucl. Phys. B 528, 73 (1998), hep-th/9711201.
E. Witten, “Toroidal compactification without vector structure,” JHEP 9802, 006 (1998), hep-th/9712028.
C. Angelantonj, “Comments on Open String Orbifolds with a Non-Vanishing $B_{ab}$,” Nucl. Phys. B 566, 126 (2000), hep-th/9908064.
C. Angelantonj and R. Blumenhagen, “Discrete deformations in type I vacua,” Phys. Lett. B 473, 86 (2000) hep-th/9911190.
Z. Kakushadze, “Geometry of orientifolds with NS-NS B-flux,” Int. J. Mod. Phys. A 15, 3113 (2000), hep-th/0001212.

[33] A. Font and A. Hernández, “Non-supersymmetric orbifolds”, hep-th/0202057.

[34] A. Adams, J. Polchinski and E. Silverstein, “Don’t panic! Closed string tachyons in ALE space-times,” JHEP 0110, 029 (2001), hep-th/0108075.

[35] M. R. Douglas and G. W. Moore, “D-branes, Quivers, and ALE Instantons,” hep-th/9603167.
C. V. Johnson and R. C. Myers, “Aspects of type IIB theory on ALE spaces,” Phys. Rev. D 55, 6382 (1997), hep-th/9610140.

[36] E. G. Gimon and C. V. Johnson, “K3 Orientifolds,” Nucl. Phys. B 477, 715 (1996), hep-th/9604129.

[37] R. G. Leigh and M. Rozali, “Brane boxes, anomalies, bending and tadpoles,” Phys. Rev. D 59, 026004 (1999), hep-th/9807082.

[38] L. E. Ibáñez, R. Rabadán and A. M. Uranga, “Anomalous $U(1)$’s in type I and type IIB $D = 4$, $N = 1$ string vacua,” Nucl. Phys. B 542, 112 (1999), hep-th/9808139.

[39] R.N. Mohapatra and J. C. Pati, “Left-Right Gauge Symmetry And An “Isoconjugate” Model Of CP Violation”; Phys. Rev. D 11, 566 (1975).
G. Senjanovic and R.N. Mohapatra, “Exact Left-Right Symmetry And Spontaneous Violation Of Parity”; Phys. Rev. D 12, 1502 (1975).
R.N. Mohapatra, “Left-right symmetry just beyond minimal supersymmetric standard model, electric dipole moment of the neutron, and HERA leptoquarks”; Phys.
[40] M. R. Douglas, B. R. Greene and D. R. Morrison, “Orbifold resolution by D-branes,” Nucl. Phys. B 506, 84 (1997), hep-th/9704151.

[41] S.P. Martin, “A supersymmetry primer”, hep-ph/9709356;
L. E. Ibáñez and G. G. Ross, “Electroweak breaking in supersymmetric models,” hep-ph/9204201. In ‘Perspectives on Higgs Physics’, G. Kane ed., World Scientific (1993).

[42] L. E. Ibáñez and G. G. Ross, “SU(2)_L × U(1) symmetry breaking as a radiative effect of supersymmetry breaking in GUT’s,” Phys. Lett. B 110, 215 (1982).

[43] D. M. Ghilencea, L. E. Ibáñez, N. Irges and F. Quevedo, “TeV-scale Z' bosons from D-branes,” hep-ph/0205083.

[44] E. Witten, “D-branes and K-theory,” JHEP 9812, 019 (1998), hep-th/9810188.
R. Rabadan and A. M. Uranga, “Type IIB orientifolds without untwisted tadpoles, and non-BPS D-branes,”, JHEP 0101, 029 (2001) hep-th/0009135.
O. Loaiza-Brito and A. M. Uranga, “The fate of the type I non-BPS D7-brane,” Nucl. Phys. B 619, 211 (2001), hep-th/0104173.