Influence of aspect ratio on heat transfer in non-uniformly heated cylindrical fluid layers

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Abstract. In this paper experimental study of heat transfer processes in a cylindrical container non-homogeneously heated from below is presented. The heater has a circular form and was placed in the center of the vessel. The experiments were performed in the range of Rayleigh numbers from $4.4 \cdot 10^6$ to $1.2 \cdot 10^7$ and fixed Prandtl number ($Pr = 67$). Special attention was paid to the influence of the aspect ratio (the ratio of the height of the fluid layer to the diameter of the heater) on the flow structure and the intensification of heat transfer.

1. Introduction

It is known that convection is the main mechanism of heat transfer in a number of technological and geophysical processes. In thermal convection, the control parameters are the Rayleigh number $Ra$ and the Prandtl number $Pr$. The intensity of heat exchange in convective systems is described by the Nusselt number $Nu$ (the dimensionless heat flux). And the key question in heat transfer studies is to obtain and understand the dependences $Nu(Ra, Pr)$.

So far the convective heat transfer are studied mostly for Rayleigh-Benard convection when the entire top surface is cooled and the entire bottom surface is heated (vertical temperature gradient) [1-4]. Another interesting case that attracts a lot of attention is a horizontal convection when the flow is initiated by a horizontal temperature gradient [5, 6].

Convection from a localized heat source, which will be discussed in this article, is the result of both vertical and horizontal temperature gradients. The flow in such a system is a complex combination of multiscale convective structures. It consists of a large-scale advective cell, which occupies the entire cavity, and secondary flows forming over the heating region. The formation and structure of large-scale circulation, laminar or turbulent, were considered in [7-9]. The formation of secondary structures in thermal boundary layer over a localized heat source was observed in [9,10] and their dynamics was described in [11-13]. At the same time, in various studies it was shown that the structure of convective flow and their dynamics are sensitive to the heating conditions, the shape and size of the heat source, to the properties of the working fluids. This leads to the need for a systematic approach for study of heat and mass transfer processes in such systems.

Heat transfer in a horizontal fluid layer over an nonuniform heated surface was investigated in [14] for a wide range of Rayleigh and Prandtl numbers. It was shown that the decrease in the viscosity of the fluid under study leads to a significant change in the dynamics of secondary flows, but does not have a significant effect on the intensity of heat exchange. The influence of different boundary conditions (constant temperature or a constant heat flux) on heat transfer in
the same configuration also was considered. The next logical step is to study the effect of aspect ratio (the ratio of the height of the liquid layer to the size of the heater) on the heat exchange processes. Results of this investigation are presented in this paper.

Despite the fact that we considered only a particular configuration we strongly believe that our results will be helpful in solving of various practical problems such as optimization of cooling systems for electronic equipment, semiconductor industry, design of nuclear reactors.

2. Experimental setup

Two cylindrical vessels were used. The first vessel (I) has diameter $D = 300$ mm (figure 1(a)). The heater is a brass cylindrical plate mounted flush with the bottom. The diameter of the plate $2R$ is 104 mm, and its thickness is 10 mm. The second vessel (II) has diameter $D = 690$ mm. The bottom of the larger vessel is the textolite plate with thickness of 20 mm. In the center of plate the copper heater of diameter $2R = 195$ mm was placed flush with the bottom. The temperature of the heaters was controlled by computer. Silicon oil PMS-5 ($Pr=67$ at $T = 25^\circ$C) was used as a working fluid. The surface of the fluid was always open. The depth of the layer for the vessel I was fixed $h = 30$ mm. For the vessel II a series of experiments was carried out for different values of $h$. The Table 1 provides the geometric parameters of the experiments for convenience of the readers, where $\alpha$ is the aspect ratio (the ratio of layer height to the diameter of the heater). The room temperature was kept constant by air-conditioning system, and cooling of the fluid was provided mainly by the heat exchange with a surrounding air on the free surface and some heat losses through sidewalls. The temperature of the fluid was measured by a horizontal row of 12 copperconstantan thermocouples. The location of the thermocouples is shown in figure 1(b). The coordinate origin is at the center of the bottom. The thermocouples can be moved through the whole layer depth with a step of 1 mm by a motorized translation stage. The data from the thermocouples were obtained by an Agilent 34970A data acquisition switch unit with a 16-channel multiplexer module 34902A. Temperature was measured in the central vertical cross-section above the heating area. To estimate the mean temperature of the fluid, one thermocouple measured temperature inside the fluid layer near the periphery.

We use the set of the non-dimensional parameters which are commonly used for similar problems. These are Prandtl number $Pr$ and Rayleigh number $Ra$:

$$Pr = \frac{\nu}{\kappa}$$

$$Ra = \frac{g\beta h^3 \Delta T}{\nu^2}$$

![Figure 1. Experimental setup, dimensions and location of the coordinate system.](image-url)
where $g$ is the gravitational acceleration, $h$ is the layer depth, $\beta$ is the coefficient of thermal expansion, $\Delta T$ - temperature difference between temperatures of the heater and the room, $\nu$ is the coefficient of kinematic viscosity.

Heat transfer was characterized by the Nusselt number defined as

$$Nu = \frac{q}{q_\lambda}$$

where $q_\lambda$ is heat flux due to conduction, and $q$ is overall heat flux. The heat fluxes $q$ and $q_\lambda$ were calculated as

$$q = \frac{P}{S}; q_\lambda = \frac{\lambda(T_h - T_s)}{h}$$

where $P$ is the power of the heater, $S$ is the heater surface area, $T_h$ is the temperature of the heater, and $T_s$ is the temperature of the fluid surface. The temperature of the fluid surface $T_s$ was received by the following way. The row of thermocouples measured the temperature of fluid above the heater consistently at $z = 28, 29$ and $30$ mm. The time of measurements was about 1000 seconds for each height. Then, the temperature was averaged over time and space in this thin near-surface layer, and the resulting temperature ($T_s$) was used to calculate the heat flux $q_\lambda$. Here, we neglect heat losses through the bottom and side walls.

Figure 2. Thermal image of the upper surface of the fluid for $Ra = 4.2 \cdot 10^6$ (model I), $T_h = 32$ $^\circ$C.

The temperature $T_s$ is the mean value of temperature along the thermocouple row, which is placed in the central part of the vessel above the heater. Temperature fields on a fluid surface were obtained by a thermal imager Fluke Ti32 system for different regimes. Temperature sensitivity of the Fluke Ti32 is high (about 0.05 $^\circ$C), but temperature measurement accuracy is $\pm 2$ $^\circ$C and it needs to be calibrated for every regime. Therefore it was used only for getting information about homogeneity of temperature distribution over the entire fluid surface. On figure 2 the thermal image of free surface of fluid is shown for $Ra = 4.2 \cdot 10^6$. It is clearly seen that temperature field is not homogeneous in both radial and azimuthal directions and this the main source of error in $T_s$ measurements. However according to our estimations, the inaccuracy of $T_s$ measurements does not exceed 5 % and the one for $Nu$ calculation is less than 7 % (taking into account the heat leakage through the sidewalls).
### Table 1.

|   | h, mm | D, mm | 2R, mm | α = h/2R |
|---|-------|-------|--------|----------|
| I | 30    | 300   | 104    | 0.3      |
| II| 30    | 690   | 100    | 0.3      |
|   | 40    | 690   | 100    | 0.4      |
| III| 50   | 690   | 100    | 0.5      |
| IV| 60    | 690   | 100    | 0.6      |

3. Results

Detailed description of the basic flow structure can be found in [12] but brief description of the general structure of the large-scale flow is necessary for a better understanding of the results. The heat flux in the central part of the bottom is a source of the intensive upward motion above the heater. Warm fluid cools at the free surface and moves toward the periphery where the cooled fluid moves downward along the side wall. Large-scale advective flow occupies the whole vessel (figure 3). Along with the main updraft in the center there are less intensive but pronounced upgoing convective flows close to the periphery of the heater.

![Figure 3. Large-scale circulation.](image)

The convergent flow in the lower part of the layer leads to the formation of a boundary layer with potentially unstable temperature stratification above the heater. Examples of the experimental temperature profiles for different values of Ra and aspect ratio are presented in figure 4. In this boundary layer secondary flows appear. The structure and dynamics of secondary flows are strongly dependent on heating intensity. Weak heating regimes are characterized by the appearance of ring-like rolls. An increase of the heat flux leads to more complex convective patterns - the superposition of a spiral and a system of radial rolls on the periphery. For the more detailed information about dynamics and structure of secondary flows we refer the reader to [12].

One of the most remarkable results obtained in [12] is that ring-like rolls and spirals appear periodically. Dependence of characteristic frequency of secondary structures formation $F$ on Rayleigh number for fixed $Pr$ and different aspect ratio is presented in figure 5. The best fitting for all data is the power law ($F \propto Ra^{0.72}$) but there is significant quantitative difference between measurements for different values of layer depth (aspect ratios). The formation of secondary structures occurs in an unstable boundary layer over the heated area. Its thickness $\delta$ is determined by the properties of the working fluid, the value of the aspect ratio and the heating intensity. Changing the depth of the fluid layer for the fixed $Ra$ leads to an increase in the thickness of the boundary layer and to the reduction temperature gradient. Thus, for fixed values of Prandtl and Rayleigh numbers, the increase of aspect ratio leads to a decrease in the frequency of secondary structures formation. It is also interesting that remoteness of lateral boundaries have no substantial impact on secondary flows formation.
In figure 6(a) the dependence of the Nusselt number on the Rayleigh number is presented in log-log scale. One can see that the heat transfer does not depend on the aspect ratio and increases with increasing Rayleigh number ($Nu \propto Ra^{0.28}$). This exponent value defines heat transfer for quite different flow configurations [14]. By definition the Nusselt number ($Nu$) is the ratio of convective to conductive heat transfer across (normal to) the boundary. In other words, the Nusselt number is proportional to the ratio of the thickness of the fluid layer to the thickness of the temperature boundary layer. Our results show that for the described configuration the Nusselt number for the fixed $Ra$ does not depend on the aspect ratio. This means that the thickness of the boundary layer varies linearly with the fluid layer depth. The characteristic time between formation of two successive plumes can be estimated as follows $\tau \sim \delta^2/\chi$ [15]. Thus using relation $\delta \sim h$ we proposed a new parameter $f = F \cdot h^2/\chi$, which is the dimensionless frequency of secondary structures formation. The dependence of $f$ on the Rayleigh number is shown in figure 6 (b). We see that all experimental points lie on the same curve, which proves our assumptions.
Figure 6. Nusselt number (a) and dimensionless characteristic frequency F (b) versus Rayleigh number for different aspect ratio

4. Conclusions
Heat transfer from the localized heat source in a cylindrical layer was studied experimentally for different values of aspect ratio. The flow in such a system consists of large-scale advective circulation and secondary structures appeared in the thermal boundary layer above the heated area. Our temperature measurements showed that the aspect ratio do not cause any substantial changes in the dependence of heat transfer on the Rayleigh number. Based on this result we proposed a new parameter that characterizes the formation of secondary structures.

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