Effect of Nonlinearity by the Amplitude Variation in coherent transmission in Laser Heterodyne Interferometric

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Abstract. To reduce the nonlinearity of nanometer measurement in laser heterodyne interferometric, the influence mechanics of the amplitude variation in coherent transmission upon nonlinearity must be confirmed. Based on the mechanics of nonlinearity, the models about how first-harmonic and second-harmonic nonlinearity caused by the amplitude variation in coherent transmission are proposed. The emulation result shows that different amplitude between measurement arm and reference arm increases the first-harmonic nonlinearity when laser beams nonorthogonality errors exist, but it doesn’t change the relationship between nonlinearity and half wavelength. When the rotation angle error \( \beta \) of polarizing beam splitter (PBS) exists, amplitude variation only affects the first-harmonic nonlinearity. With a constant rotation angle of PBS \( \beta=4^\circ \), when the amplitude factor of measurement arm reduces from 1 to 0.6, the nonlinearity increases from 0.25 nm to 3.81 nm, and the nonlinearity is simple superposition of first-harmonic and second-harmonic. Theoretic analysis and emulation show that the reduction of amplitude variation in coherent transmission can reduce influence on nonlinearity.

1. Introduction

With the development of aerospace, ultra-precision machining, micro machinery, microelectronic technology, nanometer measurement technology has been the hot research around the world. Laser heterodyne interferometer is applied broadly in the field of precision and ultra-precision nanometer measurement for its special advantages, such as large measurement scale, high resolution and measurement precision and so on [1,2]. However, it is difficult to improve the measurement precision fatherly because of the nonlinearity [3-11].

The nonlinearity is studied in many theory and experimental researches all over the world. In literature [3], it is indicated that beam amplitude variation in coherent transmission increases the nonlinearity caused by laser beam nonorthogonality, but there is no analyses about how beam amplitude variation influencing upon first-harmonic and second-harmonic. In literatures [4-11], several methods are proposed to measure and adjust the nonlinearity, but the influence of amplitude variation on the nonlinearity in coherent transmission is not considered.

Based on the mechanism of nonlinearity, the models about how first-harmonic and second-harmonic nonlinearity affected by the amplitude variation in coherent transmission are established in...
this thesis. And how the amplitude variation in coherent transmission affecting the nonlinearity is analyzed, and emulated at the same time.

2. Mechanism of nonlinearity

The prototype principle of laser heterodyne interferometric measurement is shown in Figure 1. Two orthogonally polarized beams with different frequency are emitted by a laser source and then divided by a non-polarizing beam splitter (BS) in two parts: the reference part served as reference signal \( f_r \) passes an analyzer and a detector. The transmission part is split by polarizing beam splitter (PBS) into two separated beams. The two beams are reflected by a reference cube-corner retroreflector \( M_r \) and a measuring cube-corner retroreflector \( M_m \), respectively, and then recombined in the PBS. The combined beam is reflected by Mirror and served as the measurement signal \( f_m \).

In the process of measurement, laser source dichroism and birefringence in the laser cavity make the two linearly polarized beams be in elliptically polarized and nonorthogonal. Literature [6] points out that elliptical polarized and nonorthogonal laser beams cause first-harmonic nonlinearity, and the nonlinearity model is also proposed.

Angle rotation error of PBS can cause second-harmonic nonlinearity. In literature [7], the nonlinearity model caused by angle rotation error of PBS is proposed.

![Figure 1. Laser heterodyne interferometric principle.](image)

3. The influence of amplitude variation in coherent transmission on the nonlinearity

In the laser heterodyne interferometric measurement, the amplitude is attenuated because of the move of measurement cube-corner retroreflector or the absorption of optic element. The attenuation leads the difference of amplitude between measurement arm and reference arm and effects the nonlinearity when there is already incorrect frequency mixing.

3.1. The influence on first-harmonic

In order to simplify the analysis, only first-harmonic nonlinearity caused by non-orthogonal polarized laser beams is considered. The two polarized beams of a laser source with different frequency can be represented as:

\[
\begin{align*}
\vec{E}_1 &= iE_{01}\cos(2\pi f_1) \\
\vec{E}_2 &= jE_{02}\cos(2\pi f_2)
\end{align*}
\]

where \( i \) and \( j \) represent unit vector of \( \vec{E}_1 \) and \( \vec{E}_2 \), \( E_{01} \) and \( E_{02} \) represent amplitude, \( f_1 \) and \( f_2 \) represent the frequency of \( \vec{E}_1 \) and \( \vec{E}_2 \), \( \Delta f = f_1 - f_2 \).

It is assumed that the departure angle of polarized direction of \( \vec{E}_1 \) with frequency of \( f_1 \) is \( \alpha \). The attenuation of laser beam in measurement arm is described by the amplitude factor \( k \). The following expressions are the beams that enter measurement arm and reference arm after splitting in the PBS:

\[
\begin{align*}
\vec{E}'_1 &= ikE_{01}\cos\alpha\cos(2\pi f_1 t + \varphi_1)
\end{align*}
\]
\[ \vec{E}_2 = j \left[ E_{02} \cos(2 \pi f_2 t + \varphi_2) + E_{01} \sin \alpha \cos(2 \pi f_1 t + \varphi_2) \right] \]  \hspace{1cm} (4)

where \( \varphi_1 \) is the phase shift, which varies with the path difference in the measurement arm, \( \varphi_2 \) is the phase shift, which varies with the path difference in the reference arm, \( \Delta \varphi = \varphi_1 - \varphi_2 \).

\( \vec{E}_1 \) and \( \vec{E}_2 \) are recombined in the PBS and detected by photodetector. The measurement signal \( I_m \) is obtained:

\[ I_m = I_0 A^* \cos \left[ 2 \pi \Delta f t + \Delta \varphi + \Delta \varphi_{\text{nonlin}} \right] \]  \hspace{1cm} (5)

Where \( \Delta \varphi_{\text{nonlin}} \) is the effect on first-harmonic nonlinearity cased by amplitude variation in coherent transmission:

\[ \Delta \varphi_{\text{nonlin}} = -\arctan \left( \frac{\sin \alpha \sin(\Delta \varphi)}{k \cos \alpha + \sin \alpha \cos(\Delta \varphi)} \right) \]  \hspace{1cm} (6)

Assuming \( \alpha = 6^\circ \), the simulation result of the nonlinearity obtained with different amplitude factor \( k \) in equation (6) is shown in Figure 2. In Figure 2 it can be seen that the first-harmonic nonlinearity increases as \( k \) decreases with the same \( \alpha \). The first-harmonic nonlinearity will increase from 5.30 nm to 8.87 nm when \( k \) decreases from 1 to 0.6.

![Figure 2](image)

**Figure 2.** The nonlinearity resulting from different \( k \) and constant laser beams nonorthogonality errors (\( \alpha = 6^\circ \))

4. **The influence on second-harmonic**

It is assumed that the angle of rotation error of PBS is \( \beta \) and there is no other nonlinearity source. The following expressions are the beams that enter measurement arm and reference arm after splitting in the PBS:

\[ \vec{E}_1' = i \left[ k E_{01} \cos \beta \cos(2 \pi f_1 t + \varphi_1) + k E_{02} \sin \beta \cos(2 \pi f_2 t + \varphi_1) \right] \]  \hspace{1cm} (7)

\[ \vec{E}_2' = j \left[ E_{02} \cos \beta \cos(2 \pi f_2 t + \varphi_2) - E_{01} \sin \beta \cos(2 \pi f_1 t + \varphi_2) \right] \]  \hspace{1cm} (8)

\( \vec{E}_1' \) and \( \vec{E}_2' \) are recombined in the PBS and detected by photodetector. The measurement signal \( I_m \) is obtained:

\[ I_m = I_0 B^* \cos \left[ 2 \pi \Delta f t + \Delta \varphi + \Delta \varphi_{\text{nonlin2}} \right] \]  \hspace{1cm} (9)

Where \( \Delta \varphi_{\text{nonlin2}} \) is the effect on second-harmonic nonlinearity cased by amplitude variation in coherent transmission:
\[
\Delta \varphi_{\text{nonlin}} = -\arctan \frac{(1 - k^2)\sin \beta \cos \sin (\Delta \varphi) - k \sin^2 \beta \sin (2 \Delta \varphi)}{k \cos^2 \beta + (1 - k^2) \sin \beta \cos \cos (\Delta \varphi) - k \sin^2 \beta \cos (2 \Delta \varphi)}
\] (10)

Assuming \( \beta = 4^\circ \), the simulation result of the nonlinearity obtained with different amplitude factor \( k \) in equation (10) is shown in Figure 3. In Figure 3 it can be seen that the nonlinearity increases as \( k \) decreases with the same \( \beta \). The nonlinearity will increase from 0.25 nm to 3.81 nm when \( k \) decreases from 1 to 0.6 and the amplitude attenuation will change the relationship between nonlinearity and half wavelength.

**Figure 3.** The nonlinearity resulting from different \( k \) and constant rotation angle \((\beta = 4^\circ)\) of PBS.

In order to analyze the influence of the amplitude attenuation on second-harmonic nonlinearity, the nonlinearity with different \( k \) is subtracted from the nonlinearity with \( k=1 \) \((\beta = 4^\circ)\) as shown in Figure 4. It can be seen that the amplitude attenuation increases first-harmonic, but it has no effect on second-harmonic. It also shows that the nonlinearity caused by angle rotation error of PBS and amplitude variation is the simple superposition of first-harmonic and second-harmonic.

**Figure 4.** The difference of nonlinearity resulting from different \( k \) with a constant rotation angle \((\beta = 4^\circ)\) of PBS.
5. Conclusion
The influence of amplitude variation in coherent transmission to nonlinearity is analyzed. Theoretical analysis and simulation experiment show that: (1) Different amplitude between measurement arm and reference arm increases the first-harmonic nonlinearity when beam nonorthogonality errors exist, but they do not change relationship between nonlinearity and half wavelength. With a constant nonorthogonal error $\alpha=6^\circ$, when the amplitude factor of measurement arm $k$ reduces from 1 to 0.6, the first-harmonic nonlinearity increases from 5.30 nm to 8.87 nm. (2) When angle rotation error exists in PBS, nonlinearity caused by amplitude variation in coherent transmission is simple superposition of first-harmonic and second-harmonic nonlinearity, the different amplitude between measurement arm and reference arm does not increase the second-harmonic nonlinearity but first-harmonic nonlinearity and the nonlinearity is affected seriously. With a constant rotation angle of PBS $\beta=4^\circ$, when the amplitude factor $k$ of measurement arm reduces from 1 to 0.6, the nonlinearity increases from 0.25 nm to 3.81 nm.

In the adjustment and attenuation of nonlinearity, the difference between the measurement arm and reference arm due to movement of measurement cube-corner retroreflector should be avoided. At the same time, the use of prime optical element can reduce the absorption of beams in transmission, and the amplitude attenuation and affection on nonlinearity. The paper provides the theoretical basis for the reduction of nonlinearity.

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