Multi-parameter Dynamical Dark Energy Equation of State and Present Cosmological Tensions

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We study the consequences of an enlarged four parameter dynamical dark energy (4pDE) equation of state using the latest Planck, BAO, and Pantheon supernovae data. This parameterization of the dark energy equation of state incorporates a generic non-linear monotonic evolution of the dark energy equation of state, where the four parameters are the early and the present value of the equation of state, the transition scale factor and the sharpness of the transition. In this study we use SH0ES $M_B$ prior and the KIDS/Viking $S_8$ prior while keeping the neutrino mass $\Sigma m_\nu$ as a free parameter. We show that in this case the dynamical dark energy 4pDE model can bring down the Hubble tension to $\sim 2.5\sigma$ level and the $S_8$ tension to $\sim 1.5\sigma$ level when tested against Planck, BAO and Pantheon supernovae data together. We also compare our results with the well-explored CPL model. We find that the present data can not constrain all the four dark energy equations of state parameters ensuring the fact that the present observations do not demand a complex non-linear multi-parameter evolution of the time-dependent DE equation of state. We also report that with SH0ES $M_B$ and KIDS/Viking $S_8$ prior 4pDE and CPL model favours a non-zero value for the neutrino mass parameter at the most at $\sim 1\sigma$ level ($\Sigma m_\nu \sim 0.2 \pm 0.1$ eV).

I. INTRODUCTION

The observations of type Ia supernovae show that the expansion of the Universe is accelerating. The acceleration requires the Universe to be dominated by an exotic fluid with negative pressure. The simplest explanation for dark energy is the cosmological constant or vacuum energy that explains the acceleration of the Universe. Though the cosmological constant is preferred from cosmological observations yet its theoretical understanding has been questionable [1]. The other alternatives of the cosmological constant that can act as dark energy are scalar fields such as quintessence field, modified gravity, phantom dark energy, etc [2]. There has not been any observational evidence of such alternatives but it has not been ruled out either. One such model is dynamical dark energy driven by a slowly rolling scalar field. If this is true, then it opens up many new observational windows which may shed light on the fundamental nature of this mysterious component of the Universe.

The other reason to explore beyond the $\Lambda$CDM model is the recently emerging and persistent anomalies in present high precision cosmological data. The mismatch between values of Hubble parameter inferred from CMB data and direct measurements is one of them. The SH0ES (Supernovae H0 for equation of state) team has measured the value of Hubble parameter $H_0 = 73.2 \pm 1.3$km/s/Mpc using the distance ladder method [3, 4]. However The Planck 2018 measurement of CMB (Cosmic Microwave Background) has measured the value of Hubble parameter $H_0 = 67.36 \pm 0.54$km/s/Mpc using $\Lambda$CDM model [5]. So there is a $4.2\sigma$ discrepancy between both measurements. This mismatch gained significance with various improved precision measurements see [6][10].

Similarly there is another tension related to the measured value of $S_8 \equiv \sigma_8 (\Omega_m/0.3)^{0.5}$, where $\sigma_8$ is the root mean square of matter fluctuations on a $8$ h$^{-1}$Mpc scale, and $\Omega_m$ is the total matter abundance. The latest prediction from Planck CMB data within the $\Lambda$CDM framework is $S_8 = 0.832 \pm 0.013$ [5].

Originally, observations of galaxies through weak lensing by the CFHTLenS collaboration have indicated that the $\Lambda$CDM model predicts a $S_8$ value that is larger than the direct measurement at the $2\sigma$ level [11][12]. This tension has since then been further established within the KiDS/Viking data [13][14], but is milder within the DES data [15]. However, a re-analysis of the DES data, combined with KiDS/Viking, leads to a determination of $S_8$ that is discrepant with Planck at the $3\sigma$ level, $S_8 = 0.755^{+0.019}_{-0.021}$ [17]. Recently, the combination of KiDS/Viking and SDSS data has established $S_8 = 0.766^{+0.02}_{-0.014}$ [16]. However, a study in [17] shows fainter $S_8$ tension when redshift-space distortions (RSD) data is included.

There has been a wide range of solutions proposed to solve these cosmological tensions which requires new physics/modifications in the early Universe i.e. pre-recombination era as well as in the late Universe. Not a single model yet fully solves both $H_0$ and $S_8$ tensions simultaneously. The class of solutions which invokes modifications of the late-time Universe dynamics in dark energy generally leaves $r_s$ unaffected by construction and has been studied extensively in recent times. The higher value of $H_0$ is then accommodated by a smaller value of $\Omega_{DE}$ or $\Omega_m$ redshift below $z_s$ such that $d_A(z_s)$ stays unaffected as well. This can be done for instance by invoking variations in the dark-energy equation of state [18][27] or decaying dark matter [28][29] or non-thermal
The study of the dynamical behavior of dark energy is often done in terms of its equation of state \( w(z) = \frac{p(z)}{\rho(z)} \) that can vary as a function of redshift. Equation of state \( w = -1 \) corresponds to the cosmological constant. There are some recent studies where it has been shown that solving of \( H_0 \) and \( S_8 \) tensions require \( w(z) < -1 \) at some \( z > 0 \) and time-varying dark energy equation of state which cross the phantom barrier \[51\]. Also it has been shown that a large class of quintessence \((w > -1)\) models including the ones which arise from string swampland conjecture lower the \( H_0 \) parameter and thereby makes \( H_0 \) tension worse \[52\]. From observations, it’s required that equations of state at present time should be consistent with value \( w \approx -1 \), however, constraints on the equation of state at higher redshifts are weaker. There have already been several efforts to parameterize the equation of state of dark energy. Some recent works in this direction can be found in \[53–57\].

We explore in detail the possibility of dynamical DE with a more general model-independent approach where we go beyond the CPL (Chevalier-Polarski and Linder) parameterization \[52\] \[53\] where the dark energy equation of state \( w \) evolves linearly with expansion factor \( a \). To be specific, in this paper, we study a generic non-linearly evolving equation of state. Some of the recent works on dynamical dark energy scenario such as \[56\] suggest that CPL parameterization is not sensitive at low redshifts and thus provide motivation for going beyond CPL like parameterization. It was recently pointed out that if late-time cosmology is modified through time-varying \( w \), one should use the direct \( M_0 \) data instead of \( H_0 \) prior \[54\] \[56\] \[57\]. To our knowledge, this work is the first work where we present a detailed analysis of a four parameter dynamical DE model. To do so, we use a generic four parameter model of dynamical dark energy equation of state \( w_{de}(a) \) originally proposed in \[58\] and test it against the recent Planck-2018, Pantheon and BAO datasets. In comparison to CPL parameterization, this parameterization has two extra parameters to incorporate the possible non-linear evolution of the equation of state with time. The main interest of this parameterization is that it captures possible transition in the equation of state of the dynamical dark energy during the course of its evolution, which many quintessence/K-essence and phantom dark energy models exhibit \[58\].

In this study, we find that all four parameters of the equation of state can not be constrained with current observational data. Especially, Planck 2018 data alone has poor constraining ability on dark energy parameters. Once we include the BAO and Pantheon data, the constraints improve and the Hubble tension comes down to 2.5\( \sigma \) level from SHOES measurement and \( S_8 \) tension comes down to 1.5\( \sigma \) from KIDS/Viking measurement.

An important aspect of this paper is to get neutrino mass constraints in the 4pDE model. Standard massive neutrinos play an important role in the evolution of the Universe, they leave a non-negligible impact on the cosmic microwave background (CMB) and large-scale structure (LSS) at different epochs of the evolution of the Universe. This impact is used to get a bound on neutrino mass. Some of the effects of standard model neutrinos and dark energy are the same during specific cosmic time. Therefore nature of dark energy has an important role in constraining neutrino mass. Some of the relevant studies we find in the literature are \[59\] \[60\] \[61\] \[62\] \[20\] \[63\] \[64\]. In our analysis, we detect a non-zero neutrino mass at 1\( \sigma \) level \((\Sigma m_\nu \sim 0.2 \pm 0.1 \text{ eV})\) but consistent with zero at 2\( \sigma \) level unlike a previous study \[20\] where the analysis was done with earlier (2015) Planck data and the neutrino mass \( \Sigma m_\nu \) was found to be non-zero even at \( \gtrsim 2\sigma \).

The plan of the paper is as follows. A brief description of the four parameter dynamical dark energy equation of state, \( w_{de}(a) \), is given in section II. In Section III and IV we provide a detailed description of our analysis and results. Then Section V summarizes the paper and future outlook.

II. FOUR-PARAMETER MODEL FOR DARK ENERGY

To investigate the effect of a non-linearly evolving dark energy equation of state, we use a model independent, 4 parameter dynamical dark energy equation of state \( w_{de}(a) \), suggested by \[58\],

\[
w_{de}(a) = w_0 + (w_m - w_0) \times \Gamma(a)
\]

where \( w_0 \) and \( w_m \) are 2 parameters denoting the initial and final values of the dark energy equation of state, ie., \( w_0 = w_{de}(a = 1) \) and \( w_m = w_{de}(a \ll 1) \). The factor \( \Gamma(a) \) contains the other 2 parameters describing the course of the evolution of \( w_{de}(a) \), and is given as,

\[
\Gamma(a) = \frac{1 - \exp \left( -\frac{(a - 1)}{\Delta de} \right)}{1 - \exp \left( \frac{1}{\Delta de} \right)} \times \frac{1 + \exp \left( \frac{a_t}{\Delta de} \right)}{1 + \exp \left( -\frac{(a - a_t)}{\Delta de} \right)}
\]

where \( a_t \) is the scale factor at which the transition from \( w_m \) to \( w_0 \) takes place and the \( \Delta de \) is the steepness of the transition (see Figure A1 and A4 for more details on the nature of the parameters \( a_t \) and \( \Delta de \)).

Our parameterization is generic to a class of non-interacting scalar field dynamical dark energy models only, ie., we assume \( c_{de} = 1 \). Also this parameterization can only mimic monotonically evolving dynamical dark energy models.

We will consider a homogeneous and isotropic flat background for the universe described by a FLRW metric. If we neglect the radiation density today, Friedmann equation will have the following form,

\[
\frac{H^2}{H_0^2} = \Omega_M/a^3 + \Omega_{DE} \exp \left( 3 \times \int_1^a \frac{1 + w_{de}(a')}{a'} \, da' \right)
\]

where \( \Omega_M \) and \( \Omega_{DE} \) is matter density and dark energy density parameters respectively and for a flat universe \( \Omega_{DE} + \Omega_M = 1 \).
III. NUMERICAL ANALYSIS

A. Data Sets

- Planck 2018 measurements of the low-ℓ CMB TT, EE, and high-ℓ TT, TE, EE power spectra, together with the gravitational lensing potential reconstruction 65.
- The BAO measurements from 6dFGS at $z = 0.106$ 66, SDSS DR7 at $z = 0.15$ 67, BOSS DR12 at $z = 0.38,0.51$ and 0.61 68, and the joint constraints from eBOSS DR14 Ly-α autocorrelation at $z = 2.34$ 69 and cross-correlation at $z = 2.35$ 70.
- The measurements of the growth function $f\sigma_8(z)$ (FS) from the CMASS and LOWZ galaxy samples of BOSS DR12 at $z = 0.38, 0.51$, and 0.61 68.
- The Pantheon SNIa catalogue, spanning redshifts 0.01 < $z$ < 2.3 71.
- The SH0ES result, modeled with a Gaussian likelihood centered on $H_0 = 73.2 \pm 1.3$ km/s/Mpc 4; however, choosing a different value that combines various direct measurements would not affect the result, given their small differences.
- The KIDS1000+BOSS+2dfLenS weak lensing data, compressed as a split-normal likelihood on the parameter $S_8 = 0.760^{+0.023}_{-0.014}$ 10.
- The Gaussian prior on $M_B = -19.244 \pm 0.037$ mag 56, corresponding to the SN measurements from SH0ES.

B. Methodology

Our baseline cosmology consists in the following combination of the six ΛCDM parameters ($\omega_b, \omega_{cdm}, 100 \times \theta_s, n_s, \ln(10^{10} A_s), \tau_{reio}$), plus four dark energy equation of state parameters as discussed in Sec 11, namely $w_0$, $w_m$, $a_t$, $\Delta_{de}$ and neutrino mass $\Sigma m_\nu$. We dub this model as 4pDE. We do MCMC analysis of 4pDE model against various combinations of the CMB, BAO and supernovae data sets (details of which is given in Sec III A) with the Metropolis-Hasting algorithm as implemented in the MontePython-v3 72 code interfaced with our modified version of CLASS. All reported $\chi^2_{min}$ are obtained with the python package iMINUIT 73. We make use of a Choleski decomposition to better handle the large number of nuisance parameters 74 and consider chains to be converged with the Gelman-Rubin convergence criterion $R - 1 < 0.05$ 75.

TABLE I. Comparison of $\Delta\chi^2_{min}$ and $\Delta$AIC for 4pDE and CPL models.

| Data               | Planck+Ext+MB | Planck+Ext+MB+S8 |
|--------------------|---------------|------------------|
| Model              | $\Delta\chi^2_{min}$ | $\Delta$AIC | $\Delta\chi^2_{min}$ | $\Delta$AIC |
| ΛCDM               | 0             | 0                | 0                   | 0            |
| 4pDE               | -8.46         | -0.46            | -5.61               | +2.39        |
| $\nu$4pDE          | -6.53         | +3.47            | -5.5                | +4.5         |
| $\nu$CPL           | -7.23         | -1.23            | -2.76               | +3.24        |

IV. RESULTS

We ran two sets of models, the first one is the “4pDE Model” and the second is “ΛCDM Model”. Each model is constrained with two sets of data combinations. The first data set is “Planck TT, EE, TE+Planck Lensing”, the second data set is “Planck TT, EE, TE+Planck Lensing+BAO+Pantheon”. Additionally, we also confront the CPL model for the dark energy equation of state with “Planck+BAO+Pantheon” for comparison with the 4pDE model. To see the impact on cosmological tensions, we perform our analysis with and without $M_B$ and $S_8$ priors.

The results for the 4pDE model with combined data sets for various cases are reported in Table III. 2D posterior distributions are shown in Figure 1. We find that $w_0$ is well constrained for each data set and is consistent with the cosmological constant. However, the other three DE parameters are less constrained or unconstrained. Especially in the case of parameter $\log_{10}(a_i)$ we don’t find the lower bound. However there is a upper bound on $\log_{10}(a_i)$ in Planck+Ext+S8 case. We also find 1σ bounds on parameters $w_m$ and $\log_{10}(\Delta_{de})$. The posteriors of equation of state parameters are shown in Figure 3. We also get a peak in the posterior of the neutrino mass (see Figure 1)- though in this case, the model still has the $H_0$ tension at $\sim 3.2\sigma$ and $S_8$ tension at $\sim 2.5\sigma$. The overall $\chi^2_{min}$ is -1.7 compared to ΛCDM model. The $\chi^2_{min}$ values of 4pDE model corresponding to different data sets are reported in Table III.

When using the $M_B$ prior, there is no major impact on the equation of state parameters except the values of $w_m$ shift slightly more negative. In this case, the model has $H_0$ tension at $\sim 2.6\sigma$ with SH0ES results. The overall $\chi^2_{min}$ shift is -6.5 compared to ΛCDM model.

But when we use $S_8$ prior, (see Figure 3 right panel) the impact on the equation of state is more. Especially peak of posterior density of $w_0$ has been shifted towards more negative value and with better lower and upper bounds. The posterior distribution of $\Delta_{de}$ also shifted to a more negative value and is better constrained. There is a slight change in the value of $w_0$ which also moved towards a more negative value. There is a positive correlation between the value of $w_m$ and $H_0$. We also find that value of $\sigma_8$ has been reduced significantly. This is also because of the positive correlation between $w_m$ and $\sigma_8$. But as $\Omega_M$ is poorly constrained, the overall $S_8$ also has larger error bars. If we compare $\chi^2_{min}$ values, we

\footnotetext[1]{https://iminuit.readthedocs.io/}
### TABLE II. Best-fit $\chi^2$ per experiment (and total) in the 4pDE model.

| Experiment          | 4pDE       |
|---------------------|------------|
| Planck high-$\ell$ TT, TE, EE | $\chi^2=2343.80$ | $\Delta\chi^2_{min}=2347.00$ |
| Planck low-$\ell$ EE | $\chi^2=395.65$ | $\Delta\chi^2_{min}=396.05$ |
| Planck low-$\ell$ TT | $\chi^2=22.53$  | $\Delta\chi^2_{min}=22.57$  |
| Planck lensing      | $\chi^2=9.31$  | $\Delta\chi^2_{min}=8.67$   |
| Pantheon            | $\chi^2=0$    | $\Delta\chi^2_{min}=0$      |
| BAO FS BOSS DR12    | $\chi^2=0$    | $\Delta\chi^2_{min}=0$      |
| BAO BOSS low-$z$    | $\chi^2=0$    | $\Delta\chi^2_{min}=0$      |
| SS                  | $\chi^2=0.013$ | $\Delta\chi^2_{min}=0.28$   |
| absolute M          | $\chi^2=0$    | $\Delta\chi^2_{min}=0$      |
| SHOES               | $\chi^2=0.49$  | $\Delta\chi^2_{min}=0$      |
| Total               | $\chi^2=2771.31$ | $\Delta\chi^2_{min}=2771.88$ | $\chi^2=3810.13$ | $\Delta\chi^2_{min}=3815.39$ | $\chi^2=3830.39$ | $\Delta\chi^2_{min}=3831.50$ |

### TABLE III. The mean (best-fit) $\pm 1\sigma$ error of the cosmological parameters reconstructed from the lensing-marginalized Planck+BAO+SN1a data for 4pDE model for various cases. We also report the corresponding $\Delta\chi^2_{min}$ values.

| Model | CPL |
|-------|-----|
| Parameter | Planck+Ext | Planck+Ext+S$_H$ | Planck+Ext+M$_B$ | Planck+Ext+S$_H$+M$_B$ |
| $\omega_{cdm}$ | $0.1200(1.013)$ | $0.1192(0.997)$ | $0.1185(0.995)$ | $0.1188(0.997)$ |
| $\Lambda$ | $0.000292$ | $0.000293$ | $0.000293$ | $0.000293$ |
| $\Omega_m$ | $0.9630(0.964)$ | $0.9650(0.965)$ | $0.9665(0.967)$ | $0.9690(0.968)$ |
| $\Omega_m$ | $0.0066$ | $0.0067$ | $0.0067$ | $0.0067$ |
| $\Sigma m_n$ | $0.1198$ | $0.1200$ | $0.1201$ | $0.1201$ |
| $\Delta m_n$ | $0.0117$ | $0.0118$ | $0.0118$ | $0.0118$ |
| $H_0$ | $68.21(68.53)$ | $68.88(69.21)$ | $69.22(69.41)$ | $69.44(69.87)$ |
| $\chi^2_{min}$ | $3809.86$ | $3818.34$ | $3824.43$ | $3833.08$ |
| $\Delta\chi^2_{min}$ | $-1.67$ | $-0.32$ | $-7.23$ | $-2.76$ |

### TABLE IV. The mean (best-fit) $\pm 1\sigma$ error of the cosmological parameters reconstructed from the lensing-marginalized Planck+BAO+SN1a data for CPL model for various cases. We also report the corresponding $\Delta\chi^2_{min}$ values.

| Model | CPL |
|-------|-----|
| Parameter | Planck+Ext | Planck+Ext+S$_H$ | Planck+Ext+M$_B$ | Planck+Ext+S$_H$+M$_B$ |
| $\omega_{cdm}$ | $0.1200(1.013)$ | $0.1192(0.997)$ | $0.1185(0.995)$ | $0.1188(0.997)$ |
| $\Lambda$ | $0.000292$ | $0.000293$ | $0.000293$ | $0.000293$ |
| $\Omega_m$ | $0.9630(0.964)$ | $0.9650(0.965)$ | $0.9665(0.967)$ | $0.9690(0.968)$ |
| $\Omega_m$ | $0.0066$ | $0.0067$ | $0.0067$ | $0.0067$ |
| $\Sigma m_n$ | $0.1198$ | $0.1200$ | $0.1201$ | $0.1201$ |
| $\Delta m_n$ | $0.0117$ | $0.0118$ | $0.0118$ | $0.0118$ |
| $H_0$ | $68.21(68.53)$ | $68.88(69.21)$ | $69.22(69.41)$ | $69.44(69.87)$ |
| $\chi^2_{min}$ | $3809.86$ | $3818.34$ | $3824.43$ | $3833.08$ |
| $\Delta\chi^2_{min}$ | $-1.67$ | $-0.32$ | $-7.23$ | $-2.76$ |

have got a significant improvement of $\Delta\chi^2_{min} = -8.39$ over $\Lambda$CDM model.
value, but no significant decrease in $\sigma_8$ but as $\Omega_M$ is comparatively low, we find there is a decrease in overall $S_8$ value. More importantly, we find that there is a negative correlation between parameter $S_8$ and $H_0$. This model brings down the $S_8$ tension below $\leq 1.5\sigma$ and $H_0$ tension from $\leq 2.5\sigma$. However the Overall $\chi^2_{\text{min}}$ is improved with only $-5.45$ in comparison to $\Lambda$CDM Model.

When using the prior on $H_0$ instead of $M_B$, the main impact on results is on the parameters $w_0$ and $H_0$ (both have negative correlation with each other). The parameter $H_0$ attains a slightly higher value compared to $M_B$ prior case and as a result $w_0$ shifts towards a lower value. The results are compared in Figure 4. The goodness of fit also improved over $\Lambda$CDM model by $\Delta \chi^2_{\text{min}} = -4.3$ that was -5.5 in case of $M_B$ prior.

### A. Comparison with CPL Model

We also run the CPL model with same data combinations. Results of this model reported in Table IV. 2D posterior distributions are shown in Figure 2. Both the parameters $w_0$ and $w_a$ are well constrained for CPL model. We find the posterior distribution of main cosmological parameters$(\{\omega_b, \omega_{\text{cdm}}, 100 \times \theta_s, n_s, \ln(10^{10} A_s), \tau_{\text{reio}}\})$ of this model are matched with 4pDE model. However the model parameters $(w_0$ and $w_a$) are obviously different. The parameter $w_0$ is constrained more in 4pDE model compare to CPL model. When we don’t use any prior, we do not notice a significant change in $H_0$ and $S_8$ compared to $\Lambda$CDM model. But when we use $M_b$ prior, the level of $H_0$ tension is reduced, and is within $2.5\sigma$ level with SH0ES measurement. Similarly, when we use $S_8$ prior, we notice a slight reduction in the $S_8$ parameter also. However the overall $\Delta \chi^2_{\text{min}} = -2.78$.

### B. Quantitative model comparison using Akaike Information Criterion (AIC)

Akaike Information Criterion (AIC) is one of the popular methods of estimating the relative quality of proposed models for a given data. AIC is based on using a trade-off between the goodness of fit of the model and the simplicity. AIC uses a model’s log-likelihood as a measure of fit and the number of parameters in the model as the complexity of the model. If $N_{\text{Model}}$ is the total number of parameters in a model the AIC score for that model is
FIG. 2. Reconstructed 2D posterior distributions of $(H_0, S_8, \Omega_m, \Sigma m_\nu, M_B, w_0, w_a)$ is shown for CPL model with Planck+BAO/FS+SN1a data. We also have added 68% (dark brown) and 95% (light brown) bands corresponding to a Gaussian $H_0$ prior from SH0ES and 68% (dark gray) and 95% (light gray) bands corresponding to $S_8$ prior from KIDS1000+BOSS+2dfLenS.

FIG. 3. Reconstructed 2D posterior distributions of equation of state parameters $(w_0, w_m, \log_{10}(a_t), \log_{10}(\Delta_{de}))$ is shown for 4pde model with Planck+BAO/FS+Pantheon data with a combination of priors (see legends).
The 4pDE model can bring down the Hubble tension to \( \sim 2.5\sigma \) level and the \( S_8 \) tension to \( \sim 1.5\sigma \) level when tested against Planck, BAO and Pantheon supernovae data together. More importantly, we find that there is a negative correlation between parameter \( S_8 \) and \( H_0 \) which is very interesting. However, both the 4pDE model and CPL model improves \( \Delta \chi^2 \) for the Planck+Ext data set and the recent measurements of \( H_0 \) and \( S_8 \) in comparison to \( \Lambda \text{CDM} \), this lowering of \( \chi^2 \) is achieved at the expense of adding extra parameters. So if we follow \( \Delta \text{AIC} \) criteria like recently done by [7], the level of success of these models degrades as none of the models has significantly improved \( \Delta \text{AIC} \) value over \( \Lambda \text{CDM} \) model.

We find that with Planck data alone, the equation of state parameters are poorly constrained. This may be because of the fact that some of the CMB constraints come from low multipole data of CMB power spectra and Planck data has large error bars in the low l region. Because of this, the equation of state parameter today also has large error bars that reflect in other parameters like posteriors of \( H_0 \) and \( \sigma_8 \) which attain high value and have larger error bars too (see [A1]). It is interesting to note that when we use \( H_0 \) prior, both the \( H_0 \) and \( S_8 \) tensions nearly disappear. Once we include Pantheon+BAO data into analysis, the model falls again in the category of these tensions, although the level of \( H_0 \) and \( S_8 \) tensions decreases, and the equation of state parameters have better constraints. This is in agreement with the results of [79], where they show in their analysis that the low-\( z \) data prefers models with behavior close to a cosmological constant, whereas CMB alone is more accommodating to dark energy models having both phantom and non-phantom behavior.

We also analyzed the CPL parameterization of the dark energy equation of state for similar settings as 4pDE to make a comparison between the two. We find that the best-fit values of base six parameters \( \{\omega_b, \omega_{c\text{cdm}}, 100 \times \theta_s, n_s, \ln(10^{10}A_s), \tau_{\text{reio}}\} \) are same for the CPL and the 4pDE case and over all chi square has not improved much with 4pDE.

We also see that with added \( S_8 \) prior 4pDE favours a non-zero value for the neutrino mass parameter \( \Sigma m_\nu \sim 0.2 \pm 0.1 \text{ eV} \), which is in agreement with earlier work using the CPL parameterization [80] however our analysis suggests that \( \Sigma m_\nu \) is consistent with zero at \( \sim 2\sigma \) unlike in [80] where with 2015 Planck data, the neutrino mass \( \Sigma m_\nu \) was found to be non-zero even at \( \gtrsim 2\sigma \).

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Appendix A: Characteristics of the 4 parameter dynamical dark energy model

The factor \( \Gamma(a) \) in Eq. 1 characterizes the course of the evolution of \( w_{de}(a) \). Figure A1 elaborates the nature of the parameters \( a_t \) and \( \Delta_{de} \). It can be easily shown that for the two extreme limits of \( \Delta_{de} \) Eq. 1 takes the following form,

\[
\begin{align*}
\lim_{\Delta_{de} \to -\infty} w_{de}(a) &= w_0 + (w_m - w_0) \times (1 - a) \quad (A1) \\
\lim_{\Delta_{de} \to 0} w_{de}(a) &= w_0 + \mathcal{H}(a_t - a) \times (w_m - w_0) \quad (A2)
\end{align*}
\]

ie., Equation 1 approaches to standard 2 parameter parameterization with \( w_a = w_m - w_0 \) in the limit of \( \Delta_{de} \to \infty \) (\( \Delta_{de} \gg 0.5 \)) as this is also evident from the last panel of the Figure A1. Also when \( \Delta_{de} \to 0 \) the function \( \Gamma(a) \) tend to become a step function (Heaviside function, \( \mathcal{H} \)) around \( a = a_t \).

FIG. A1. Evolution of \( w_{de} \) for different sets of values for \( a_t \) and \( \Delta_{de} \), parameters \( w_0 \) and \( w_m \) are fixed to -0.8 and -1.2 respectively.

Appendix B: Results : Without external (Pantheon+BAO) datasets

The results of Planck only data are reported in Table A1. When we confront the 4pDE model with only Planck data (i.e. Planck TT, EE, TE+Planck Lensing), we find that the equation of state parameters are poorly constrained. Parameter \( w_0 \) deviates from value “-1” with larger error bars. The rest of the three parameters remains unconstrained. The value of the Hubble parameter is also increased and has larger error bars. Similarly, \( \sigma_8 \) also has got high value and large error bars. This high increase in \( H_0 \) and \( \sigma_8 \) is since both have a negative correlation with parameter \( w_0 \).

With Planck only data after applying \( H_0 \) prior we get better constraints for most of the parameters. Especially, the value of \( w_0 \) comes much closer to the cosmological constant. The posterior of \( H_0 \) is also now more constrained and in nearly perfect agreement with SH0ES measurement. The overall fit is improved by \( \Delta \chi^2_{\text{min}} = -21.5 \) compared to the ΛCDM model.

When we use the only \( S_8 \) prior, it impacts on the equation of state parameter \( w_0 \) it is constrained more compared to no prior case but still deviates from cosmological constant behavior. The other parameters remain unconstrained. However, when we use \( S_8 \) prior also, the parameters \( H_0 \) and \( \sigma_8 \) still attains high value but error bars shrink for both the parameters this time. Apart from these parameters, the posterior of neutrino mass has also a peak when using \( S_8 \) prior. The overall fit is improved by \( \Delta \chi^2_{\text{min}} = -11.8 \) compared to ΛCDM model.

When we use the \( H_0 \) and \( S_8 \) prior together, the posteriors of \( H_0 \) and \( S_8 \) have smaller error bars and it removes the \( H_0 \) and \( S_8 \) tension completely and in fine agreement with SH0ES and KIDS/Viking respectively. The overall fit is improved by \( \Delta \chi^2_{\text{min}} = -22.9 \) compared to ΛCDM model.

Appendix C: Results : Without external (Pantheon+BAO) datasets

We also run the 4pDE model without neutrino case. The equation of state parameter of dark energy is slightly better constrained. However there is not much change in the value of Hubble parameter. The value of \( \omega_{\text{CDM}} \) is slightly higher in this case in comparison to the case with neutrino, though the \( S_8 \) parameter shifts to slightly lower value in comparison to the neutrino case. These comparison is shown in Figure 4 and the results of this run is given in Table A2. The goodness of fit improved over ΛCDM model, in without neutrino case is \( \chi^2_{\text{min}} = -5.61 \).
| Parameter | Planck | Planck + $S_8$ | Planck + $H_0$ | Planck + $H_0 + S_8$ |
|-----------|--------|----------------|----------------|----------------------|
| $100 \, \omega_b$ | 2.236(2.243) | 2.2357(2.2440) | 2.233(2.2474) | 2.234(2.2477) |
| $\omega_{cdm}$ | 0.1196(0.1198) | 0.1197(0.1185) | 0.112(0.1118) | 0.1196(0.1186) |
| $100 \, \theta_s$ | 1.0419(1.0418) | 1.0419(1.04206) | 1.042(1.042105) | 1.042(1.04219) |
| $\ln(10^{10} A_s)$ | 3.0395(3.0304) | 3.0398(3.0248) | 3.041(3.0560) | 3.041(3.0429) |
| $n_s$ | 0.9644(0.9656) | 0.9643(0.9683) | 0.9639(0.9676) | 0.9639(0.9684) |
| $\tau_{reio}$ | 0.05285(0.04901) | 0.05283(0.04583) | 0.05353(0.06124) | 0.0535(0.0577) |
| $\Sigma m_{\nu}$ [eV] | 0.1387(0.028) | 0.1290(0.0024) | 0.1357(0.0356) | 0.2146(0.0654) |
| $w_0$ | $-1.8247(-1.567)$ | $-1.394(-1.398)$ | $-0.9806(-1.024)$ | $-1.080(-1.717)$ |
| $w_\omega$ | unconstrained | unconstrained | unconstrained | unconstrained |
| $\log_{10}(a_i)$ | unconstrained | unconstrained | unconstrained | unconstrained |
| $\Delta \chi^2_{min}$ | 2771.31 | 2771.88 | 2773.46 | 2776.91 |

**TABLE A1.** The mean (best-fit) $\pm 1\sigma$ error of the cosmological parameters reconstructed from the lensing-marginalized Planck data for 4PDE model for the cases when we do not include external (Pantheon+BAO) datasets. We also report the corresponding $\Delta \chi^2_{min}$ values.

| Model without neutrino |
|------------------------|
| \begin{tabular}{|c|c|c|c|}
| Parameter & Planck + Ext + $M_B$ & Planck + Ext + $S_8 + M_B$ |
|-----------|----------------|----------------|
| $100 \, \omega_b$ | 2.238(2.2325) & 2.246(2.2431) |
| $\omega_{cdm}$ | 0.1199(0.1203) & 0.1189(0.1192) |
| $100 \, \theta_s$ | 1.042(1.0418) & 1.042(1.0418) |
| $n_s$ | 0.9642(0.9639) & 0.9639(0.9655) |
| $\ln(10^{10} A_s)$ | 3.041(3.0383) & 3.036(3.0338) |
| $\tau_{reio}$ | 0.05278(0.05116) & 0.05192(0.0506) |
| $\Sigma m_{\nu}$ [eV] | -- & -- |
| $w_0$ | $-0.9944(-1.0187)$ & $-1.004(-1.024)$ |
| $w_\omega$ | $-2.526(-4.57)$ & $-2.309(-2.98)$ |
| $\log_{10}(a_i)$ | -- & unconstrained |
| $\Delta \chi^2_{min}$ | 3823.25 & 3830.23 |

**TABLE A2.** The mean (best-fit) $\pm 1\sigma$ error of the cosmological parameters reconstructed from the lensing-marginalized Planck+BAO+SN1a data for 4PDE model without neutrino for various cases. We also report the corresponding $\Delta \chi^2_{min}$ values.