Magnetic quadrupole transitions in the relativistic energy density functional theory

Goran Kružić1,a, Tomohiro Oishi2,b, Nils Paar3,c

1 Research Department, Ericsson-Nikola Tesla, Krapinska 45, 10000 Zagreb, Croatia
2 Yukawa Institute for Theoretical Physics, Kyoto University, Kitashirakawa Oiwakechou, Sakyou, Kyoto, Japan
3 Department of Physics, Faculty of Science, University of Zagreb, Bijenička c. 32, 10000 Zagreb, Croatia

Received: 25 December 2022 / Accepted: 9 February 2023 / Published online: 17 March 2023
© The Author(s), under exclusive licence to Società Italiana di Fisica and Springer-Verlag GmbH Germany, part of Springer Nature 2023
Communicated by Takashi Nakatsukasa

Abstract Magnetic quadrupole (M2) excitation represents a fundamental feature in atomic nucleus associated to nuclear magnetism induced by spin and orbital transition operator. Since it has only been investigated within the non-relativistic theoretical approaches, and available experimental data are rather limited, it is interesting to study the properties of M2 transitions using the framework of relativistic nuclear energy density functional. In this work the nuclear ground state is calculated with relativistic Hartree-Bogoliubov model, while the M2 excitations are described using the relativistic quasi-particle random phase approximation with the residual interaction extended with the isovector-pseudovector term. The M2 transition strength distributions are described and analyzed for closed shell nuclei 16O, 48Ca, 208Pb, open-shell 18O, 42Ca, 56Fe, and semi-magic 90Zr. The evolution of M2 transition properties has been investigated within the 36−64Ca isotope chain. The main M2 transitions have rather rich underlying structure and detailed analysis shows that collectivity increases with the mass number due to larger number of contributing particle-hole configurations. Pairing correlations in open shell nuclei have strong effect, causing the reduction of M2 strength and shifts of the centroid energies to higher values. The analysis of M2 transition strengths indicate that considerable amount of experimental strength may be missing, mainly due to limitations to rather restricted energy ranges. The calculated M2 strengths for Ca isotopes, together with the future experimental data will allow constraining the quenching of the g factors in nuclear medium.

1 Introduction

Magnetic transitions probe the spin and isospin degrees of freedom of atomic nucleus and provide valuable information for a variety of nuclear properties. A major interest in nuclear magnetic transitions is strongly focused on dipole (M1) excitations which was subject of a variety of previous studies: for details, see Refs. [1–4] and references therein. However, the knowledge on higher multipole magnetic transitions, especially magnetic quadrupole (M2) transitions and respective giant-quadrupole resonances (GQR) is rather limited, both from theoretical and experimental side. Several experimental studies [5–16] have elaborated highly fragmented M2 structure whose strength ∑ B_M2(E) is strongly suppressed compared to the theoretical results obtained either in shell model or random-phase approximation (RPA) [6, 11, 17–24].

Theoretical models which investigate excitations based on nuclear fluid-dynamics [25–28] predicted activation of the twist mode, attributed to the orbital transitions caused by an effective rotation operator around z-axis, that has also been experimentally studied in Ref. [29]. In this mode, nucleon orbitals as different fluid layers rotate in opposite directions, namely for z > 0 layers rotate counterclockwise while for z < 0 they rotate clockwise. Consistency check and comparison between different theoretical models can be done using sum-rule approaches, e.g., those developed in Refs. [30–34]. The M2 giant resonances have also been studied in early study both theoretically and experimentally in 36Si, 90Zr, and 208Pb, using simplified particle-hole model, indicating mass-dependent quenching of the spin gyromagnetic factor g_s [35]. Mass-dependent quenching of magnetic transitions in nuclei has also been addressed in the model where spin-isospin core polarization is treated using dimesic function techniques [36]. It has been shown that the quenching effect increases with increasing mass number, in agreement
with the experimental findings [36]. In high resolution electron scattering the isovector M2 transitions have been studied in $^{42,44}$Ca [37]. In Ref. [38] quasiparticle phonon model has been employed in studies of M2 transition probabilities in odd-A Sn isotopes. Apart of M2 excitations, higher multipoles $λ > 2$ [39], even going toward M8 [40] or higher spin states like M12 and M14 have been experimentally studied [41,42]. In several theoretical studies with the non-relativistic methods [11], a wide fragmentation of the M2 mode can be described only by considering the two-particle-two-hole (2p2h) components. The aim of this work is to provide the first relativistic-microscopic analysis of nuclear M2 transitions in the energy density functional (EDF) framework, and explore their relevance for possible constraining the quenching of the spin gyromagnetic factor. We have focused on M2 electromagnetcance for possible constraining the quenching of the spin density functional (EDF) framework, and explore their microscopic analysis of nuclear M2 transitions in the energy (2p2h) components.

For open shell nuclei, pairing correlations are described by using phenomenological Gogny interaction [48] implemented as in Ref. [49],

$$V^{pp}(1, 2) = \sum_{i=1,2} e^{(r_1-r_2)/\mu_i} \left( W_i + B_i \hat{\rho}^\sigma - H_i \hat{\rho}^r \right) + \left. M_i \hat{\rho}^\sigma \hat{\rho}^r \right|_{\mu_i(i=1,2)}$$

with parameters $\mu_i$, $W_i$, $B_i$, $H_i$ and $M_i$ (i = 1,2) from the D1S set [48]. The $V^{pp}$ particle - particle correlations in the RQRPA residual interaction has the same phenomenological Gogny form and parameterization as in the ground state calculation using the RHB model.

### 2 Formalism

Theory framework employed in this work follows the formalism based on relativistic energy density functional recently developed in studies of M1 transitions [44,45]. The nuclear ground state represents the basis for studies of excitations, and it is described by employing self-consistent RHB model as given in Ref. [43], with point-coupling interactions. The corresponding parameterizations employed in this study are DD-PC1 [46] and more recent parameterization DD-PCX [47] constrained not only by the nuclear ground state properties, but also by using the isoscalar giant monopole resonance excitation energy and dipole polarizability in $^{208}$Pb.

To describe M2 transitions in nuclei, we employ the RQRPA based on the point coupling interaction, that contains an additional relativistic isovector-pseudovector (IV-PV) contact type of residual interaction to account for the unnatural parity transitions [44], given as an effective Lagrangian density,

$$\mathcal{L}_{IV-PV} = -\frac{1}{2} \alpha_{IV-PV} [\bar{\Psi}_{N} i\gamma^5 \gamma^\mu \tau \Psi_{N}] [\bar{\Psi}_{N} i\gamma^5 \gamma^\mu \tau \Psi_{N}]$$

For the DD-PC1 interaction, the IV-PV coupling constant has previously been constrained to $\alpha_{IV-PV} = 0.53 \cdot 197 = 104.41$ MeVfm$^3$ by minimizing relative error $\Delta \gtrsim 1$ MeV between experimental M1 excitation peak and theoretical centroid energies for magic nuclei $^{48}$Ca and $^{208}$Pb [44]. The same procedure is performed in this study for the DD-PCX interaction, resulting with $\alpha_{IV-PV} = 0.63 \cdot 197 = 124.11$ MeVfm$^3$. In this way all the model parameters of the relativistic effective Lagrangian density are fixed for M2 transitions.

In the general (Q)RPA framework, the $\lambda$-mode strength $B(\lambda, \omega_i)$ is represented by a discrete spectrum, where $\hbar \omega_i$ indicates the $i$th eigenenergy. The discrete strength $B(\lambda, \omega_i)$ of operator $\hat{q}_{\lambda,\nu} (-\lambda \leq \nu \leq \lambda)$ for spherical systems is calculated as follows [49]:

$$B(\lambda, \omega_i) = \left| \sum_{j_k j'_{k'}} \left( X^{\lambda,\nu}_{j_k j_k'} | j_k \hat{\rho} j_{k'} \right| \right|_{\lambda,\nu} \left( \sum_{j_k} \left( X^{\lambda,\nu}_{j_k j_k'} | j_k \hat{\rho} j_{k'} \right| \right) \right. \left. \times \left( u_{j_k} v_{j_k'} + (-1)^{\lambda-\nu} u_{j_k'} v_{j_k} \right) \right|^2$$

where $j_k$ and $j_{k'}$ are quantum numbers of angular momentum for single-particle (SP) states in the canonical basis. The $u_{j_k}$ and $v_{j_k}$ are the RHB occupation coefficients, whereas $X^{\lambda,\nu}_{j_k j_k'}$ and $Y^{\lambda,\nu}_{j_k j_k'}$ are the corresponding (Q)RPA amplitudes [49].

The $\lambda\nu$ operator in the relativistic formalism, which acts on Hilbert space with mixed spin-isospin basis, is given in a block diagonal form [44]. That is,

$$\hat{\mu}_{\lambda,\nu} = \sum_{k=1}^{A} \left( \hat{\mu}_{\lambda,\nu}^{(IS)}(11)_{k} \quad 0 \quad \hat{\mu}_{\lambda,\nu}^{(IS)}(22)_{k} \right) \otimes 1_{\tau}(k)$$

$\otimes$ Springer
\[
- \sum_{k=1}^{A} \left( \tilde{\mu}_{k\nu}^{(IV)}(11)_k \quad 0 \\
0 \quad \tilde{\mu}_{k\nu}^{(IV)}(22)_k \right) \otimes \hat{t}_3(k),
\]  
\tag{4}
\]

where \(1_3\) and \(\hat{t}_3\) are the unit and Pauli matrices in the isospin space. The \(\tilde{\mu}_{k\nu}^{(IS)}\) and \(\tilde{\mu}_{k\nu}^{(IV)}\) correspond to the isoscalar (IS) and isovector (IV) part of the \(M\lambda\) operator for \(k\)th nucleon [50]. That is, 
\[
\tilde{\mu}_{k\nu}^{(IS, IV)}(11)_k = \tilde{\mu}_{k\nu}^{(IS, IV)}(22)_k = \frac{\mu_N}{\hbar} \left( \frac{2}{\lambda + 1} g_{\lambda\nu}^{(IS, IV)} \hat{t}_k + g_s^{(IS, IV)} \hat{s}_k \right) \nabla [r^2 Y_{\lambda\nu}(\Omega_k)].
\]
\tag{5}

The empirical gyromagnetic factors for the bare proton (neutron) are given as \(g_s^{(p)} = 1\) (0), and \(g_s^{(n)} = 5.586\) (−3.826) [51], expressed in the nuclear-magneton unit \(\mu_N = e\hbar / 2m_N\). Since calculations are performed in the mixed spin-isospin basis, the isoscalar and isovector gyromagnetic factors are given separately for the orbital and spin components. Namely,
\[
g_{\ell\nu}^{IS} = \frac{g_\ell^p + g_\ell^n}{2} = 0.5, \quad g_{\ell\nu}^{IV} = \frac{g_\ell^p + g_\ell^n}{2} = 0.880,
\]
for the IS mode, whereas
\[
g_{\ell\nu}^{IV} = \frac{g_\ell^p - g_\ell^n}{2} = 0.5, \quad g_{\ell\nu}^{IV} = \frac{g_\ell^p - g_\ell^n}{2} = 4.706,
\]
for the IV mode.

The M2 transition strength from the R(Q)RPA is given as
\[
B_{M2}(E_i) = \left| \beta_{M2}^{IS}(E_i) + \beta_{M2}^{IV}(E_i) \right|^2.
\]
\tag{6}

Here the IV-M2 amplitude is determined as
\[
\beta_{M2}^{IV}(E_i) = \sum_{j_kj_{k'}} \left( X_{j_kj_{k'}}^j + (-1)^{j_k-j_{k'}} Y_{j_kj_{k'}}^j \right) \times \left( u_{j_k} v_{j_{k'}} + v_{j_k} u_{j_{k'}} \right) | \hat{\mu}_{\lambda=2}^{(IV)} \rangle | \hat{\mu}_{\lambda=2}^{(IV)} \rangle.
\]
\tag{7}

with \(\tau_{IV} = 1\) (−1) for neutrons (protons). On the other side, the IS-M2 amplitude reads
\[
\beta_{M2}^{IS}(E_i) = \sum_{j_kj_{k'}} \left( X_{j_kj_{k'}}^j + (-1)^{j_k-j_{k'}} Y_{j_kj_{k'}}^j \right) \times \left( u_{j_k} v_{j_{k'}} + v_{j_k} u_{j_{k'}} \right) | \hat{\mu}_{\lambda=2}^{(IS)} \rangle | \hat{\mu}_{\lambda=2}^{(IS)} \rangle.
\]
\tag{8}

In addition, the spin-M2 strength \(B_{S{M2}}^s\) and the orbital-M2 strength \(B_{O{M2}}^o\) are obtained by retaining only the spin and orbital part of the M2 transition operator, respectively.

The moments \(m_k\) of discrete-strength distributions are defined as [49]
\[
m_k = \sum_i B_{M2}(E_i) E_i^k.
\]
\tag{9}

The non-energy-weighted (NEW) moment, \(m_0\), corresponds to the total summation of the transition strength, whereas \(m_1\) is the energy-weighted (EW) summation of \(B_{M2}\) values. The centroid energy is obtained as \(\bar{E} = m_1/m_0\), which represents an average energy of calculated strength distribution. The (Q)RPA response function \(R_{M2}(E)\) is defined as [49],
\[
R_{M2}(E) = \frac{\sum_i B_{M2}(E_i)}{\pi} \frac{\Gamma/2}{(E - \hbar \omega)^2 + (\Gamma/2)^2},
\]
\tag{10}

where the discrete spectrum is smoothed by the Lorentzian \(\Gamma = 1.0\) MeV. Note that the width of the individual excitation can not be considered in this framework, and further developments going beyond the (Q)RPA are required [52,53].

For the discussion of our results, we also mention the Weisskopf estimate for the transition strength, that corresponds to the single-particle (SP) electric or magnetic transition [54,55]. The Weisskopf estimate for the proton’s \(M\lambda\) strength is
\[
B_{SP-M\lambda} \approx 1 + \left( \frac{g_s^p / 2}{\pi} \right)^2 \left( \frac{3}{\lambda + 3} \right)^2 R_{0}^{2A-2} \mu_N^2
\]
\tag{11}

with \(R_0 \equiv 1.2A^{1/3}\) fm. Note that the first factor is replaced to \((g_s^p / 2)^2 / \pi\) for neutrons. Thus, the SP-M2 strength is roughly proportional to \(A^{2/3}\), where \(A\) is the mass number. That is,
\[
B_{SP-M\lambda} \approx 1.45A^{2/3} \mu_N^2 \text{fm}^2.
\]
\tag{12}

This is used as the M2 Weisskopf unit. For one excited state with the strength \(B_{M2}(E)\), the ratio of \(B_{M2}(E) / B_{SP-M2}\) gives a rough estimate of the number of excited nucleons.

3 Results and discussions

In this work the \(0^+ \rightarrow 2^-\) transitions induced by the M2 operator have been evaluated for spherical nuclei. Calculated results are compared either with available experimental data, different theoretical approaches or both.

3.1 M2 transitions in \(^{16}\text{O}\)

As the first example, we investigate the doubly magic nucleus \(^{16}\text{O}\) with the RRPDA with DD-PC1 interaction [46]. Figure 1a shows the corresponding M2 strength distributions, including \(R_{M2}\) (full), \(R_{IS-M2}^{(IS)}\), and \(R_{IV-M2}^{(IV)}\). The centroid energy of the full response amounts \(\bar{E} = 16.55\) MeV. The full M2 strength distribution is composed of two main excitation peaks, namely, low-energy peak at \(\approx 8\) MeV, and higher one at \(\approx 17.5\) MeV. One can also observe that the transition strength is dominated by the IV response, while the IS transitions have small contribution being visible only at high-energy region of the spectra. This dominance is mainly due
to the spin $g$ factors: $g^{IV}_s > g^{IS}_s$. In order to assess the role of the residual RRPA interaction, in Fig. 1a, the unperturbed Hartree response is also shown for comparison with the full calculation. Clearly, the residual interaction considerably modifies the unperturbed spectra, resulting in shifts of the energies of the two main peaks. We also calculate the NEW sum of the transition strength up to 50 MeV. That reads $m_0 = \sum B_{M2} = 1534.58$, whereas $\sum B^{(IV)}_{M2} = 1453.83$, and $\sum B^{(IS)}_{M2} = 73.59$ in the unit of $\mu_N^2 \text{fm}^2$. This again confirms the dominance of IV component.

We decompose the full M2 strength distribution into the spin and orbital components in order to assess the relevance of their contributions. In Fig. 1b, the corresponding spin and orbital response functions, $R_{M2}^{(S)}(E)$ and $R_{M2}^{(I)}(E)$, are shown. One clearly reads that the spin response dominates over the orbital one through the whole energy range. However, the interference between those contributions is complicated. At the low-lying peak at 8.6 MeV, the spin-M2 response is larger than the total response. At the higher energies, the opposite result is obtained. By symbolically writing, the total response is determined as the absolute square of the two transition amplitudes, i.e.,

$$R_{M2}(E) = \left| \beta^\sigma_{M2}(E) + \beta^\ell_{M2}(E) \right|^2,$$

whereas $R_{M2}^{(S)}(E) = \left| \beta^\sigma_{M2}(E) \right|^2$. Thus, at the low-lying (high-energy) M2 peak, the spin-M2 and orbital-M2 amplitudes have the opposite (same) phase to realize the reduced (enhanced) total $R_{M2}(E)$ value. As the result, our NEW summations up to 50 MeV read $m_0 = \sum B_{M2} = 1534.6$ (full), $\sum B^{(IV)}_{M2} = 1492.2$ (spin), and $\sum B^{(IS)}_{M2} = 44.3$ (orbital) in the unit of $\mu_N^2 \text{fm}^2$.

The available experimental data for $^{16}$O from Refs. [7,9,10] are listed in Table 1, and also plotted in Fig. 1c. These data indicate a fragmentation of the M2-excited states among 12-20 MeV. Note that the M2-Weisskopf unit is $B_{SP-M2} = 9.21 \mu_N^2 \text{fm}^2$ in this case [54,55]. In contrast, the experimental M2 strength shows noticeably larger values than this Weisskopf unit. Thus, the M2 transition is expected as a collective process, where several SP transitions simultaneously contribute. These data also suggest that the Weisskopf assumption is not definitely applicable. For example, at

| Energy [MeV] | $B_{M2}(E) [\mu_N^2 \text{fm}^2]$ | Reference |
|-------------|----------------------------------|-----------|
| 12.53       | 38 ± 9                           | [9,10]    |
| 12.96       | 121 ± 24                         | [9,10]    |
| 16.82       | 19 ± 2                           | [7]       |
| 17.78       | 13 ± 2                           | [7]       |
| 18.50       | 59 ± 7                           | [7]       |
| 19.0        | 341 ± 51                         | [7]       |
| 20.30       | 461 ± 162                        | [9,10]    |
| 12.53-20.30 | 1052 ± 257                       | [7]+ [9,10]|
| 0-50        | 1534.58                          | (This work)|
19.0 MeV with $B_{M_2}^{\text{expt.}} \cong 341 \mu N^2 fm^2$ [7], the mean number of excited nucleons is estimated as $341/9.21 = 37$, being larger than the total mass number of $^{16}$O. Thus, the actual SP-M2 strength is expected to be more dependent on the quantum numbers of relevant orbits. Table 1 also displays the total summation of the experimental data.

As shown in Table 1, the M2-NEW summation of transition strength from the present RRPA calculation is somewhat larger than the experimental value, indicating that some strength could be missing from experiments. Especially, the calculated low-lying M2 state has not been reported in experimental studies.

In order to analyze the underlying structure of each pronounced M2 state obtained in the RRPA calculation, Table 4 in the Appendix is prepared. There, the major $ph$-transition components contributing to the M2 transition strength are listed. These components, denoted as $b_{ph}^{\pi,\nu}$, correspond to the contribution of each $ph$ configuration to the summation in the transition strength, Eq. (3), for protons ($\pi$) and neutrons ($\nu$), respectively. Thus, the overall transition strength is given as

$$B_{M_2}(E) = \left| \sum_{ph} b_{ph}^{\pi}(E) + \sum_{ph} b_{ph}^{\nu}(E) \right|^2.$$  

(16)

The analysis of $b_{ph}^{\pi,\nu}$ provides useful information on the structure of RPA states and their collective properties, as shown in previous studies [56,57]. In particular, if more $ph$ components have sizable contributions, thus involving excitations of considerable number of nucleons, one can conclude on collective nature of the RPA state. On the other side, if e.g., only one $ph$ component is relevant, the RRPA state is of pure single-particle nature. By inspecting M2 partial contributions for $^{16}$O in Table 4, one can conclude that, all the protons and neutrons in the $1p_{3/2}$ and $1p_{1/2}$ orbits simultaneously contribute, whereas the deepest $1s_{1/2}$ orbit is not active. The most collective state is obtained at $17.81$ MeV, with $7$ $ph$ configurations contributing to the overall transition strength. This collectivity is different than for the M1 transitions, where only a few $LS$-partner orbits can be active [44,45]. There are also other M2 states which are composed of one or two $ph$ configurations, such as those at $19.04$ and $19.99$ MeV.

The low-lying M2 peak predicted at $8.6$ MeV is mainly from the $1p_{3/2} \rightarrow 1d_{5/2}$ transitions of both protons and neutrons. For the absence of this peak in experiments, from the theory side one possible reason is the ambiguity of IV-PV residual interaction in QRPA. The M2-excitation energy is sensitive to the IV-PV coupling, where the finite ambiguities remain as discussed in our previous study on M1 and Gamow-Teller transitions [58].

3.2 M2 transitions in $^{48}$Ca

The M2-response functions calculated with the RRPA with DD-PC1 interaction for $^{48}$Ca are shown in Fig. 2a. The transition strength is characterized by the main peak at excitation energy $\approx 15$ MeV, and two additional peaks, i.e., low-lying one at $\approx 7$ MeV and high-lying one at $\approx 21$ MeV. The calculated centroid energy of the full response amounts $\bar{E} = 15.48$ MeV. The corresponding NEW summations up to 50 MeV read $m_0 = \sum B_{M_2} = 4915.5$ (full), $\sum B_{M_2}^{(IV)} = 5136.4$, and $\sum B_{M_2}^{(IS)} = 494.2$ in the unit of $\mu N^2 fm^2$. Figure 2b shows the full, spin and orbital M2 response functions for $^{48}$Ca.
Fig. 3 The M2 transition strength distributions for $^{48}$Ca by changing the IV-PV coupling strength parameter, $\alpha_{\text{IV-PV}} = \beta_{\text{IV-PV}} \cdot \hbar c$. Note that $\beta_{\text{IV-PV}} = 0.53 \text{ fm}^2$ is the default setting when combined with the DD-PC1 interaction.

corresponding summations are given as $\sum B_{M2}^\sigma = 4575.6$, and $\sum B_{M2}^\ell = 319.1$ in the unit of $\mu_N^2 \text{ fm}^2$. It shows that the spin-M2 strength is significant, as similarly confirmed in the previous $^{16}$O case. In addition, one can find that, at the low-lying (high-energy) M2 peak, the spin-M2 and orbital-M2 amplitudes have the opposite (same) phase to realize the reduced (enhanced) total $R_{M2}(E)$ value. This conclusion is consistent to the $^{16}$O case.

The calculated EW moment amounts $m_1 = 76.1 \times 10^3 \text{ MeV} \mu_N^2 \text{ fm}^2$ up to 50 MeV. This is larger than the RPA result in Ref. [11], $52.4 \times 10^3 \text{ MeV} \mu_N^2 \text{ fm}^2$. Note that the result in Ref. [11] includes the quenching effect, $g_s \rightarrow 0.64g_s$, in order to reproduce the experimental data of M1 transitions [45]. For comparison, the experimental EW-M2 summation amounts $15.7 \times 10^3 \text{ MeV} \mu_N^2 \text{ fm}^2$ up to 15 MeV [11]. For the observed discrepancy between theoretical and experimental values in the present study, one explanation is the quenching effect on $g$ factors. For adjusting our EW-M2 summation to the experimental one, the quenching factor of $\zeta = \sqrt{15.7/76.1} \approx 0.45$ is necessary for all the $g$ factors. However, this strong quenching is not consistent to the M1 result in our previous work [44], where the minor quenching of $\zeta = 0.8-0.9$ was appropriate to reproduce the experimental M1 summation. Another possible reason is the effect going beyond the RPA. Indeed in Ref. [11], the second RPA calculation for M2 transitions was also performed, where the better agreement with experiment than the pure RPA calculation was confirmed. This also suggests that beyond-RPA effects may be more (less) essential for the M2 (M1) transition, in which the collectivity is relatively strong (weak). However, similar beyond-RPA calculation but of relativistic version is still challenging, and going beyond the scope of this work. Note also that considerable M2 data especially of high energies could be missing from the experimental side due to the limited energy range [11].

Details on the composition of M2 transitions in $^{48}$Ca are given in Table 5 in the Appendix. The analysis of the relevant $ph$ contributions to the main 10 excited states indicates that, both for protons and neutrons, the $2s_{1/2}$, $1d_{5/2}$, and $1d_{3/2}$ orbits are active for M2 transitions. In addition, the neutrons in $v1f_{7/2}$ are also contributing. On the other side, several transitions, e.g. $1p_{3/2} \rightarrow 1d_{3/2}$, are forbidden, since their final states are occupied. Notice that the collectivity of M2 transition is shown as larger for $^{48}$Ca than previously analyzed $^{16}$O, simply because, in $^{48}$Ca, the initial state has...
The RQRPA full M2 response function for $^{18}$O, $R_{M2}(E)$, based on the DD-PC1 parameterization and Gogny-pairing interaction. The response functions without pairing correlations and with full pairing in the residual RQRPA interaction are shown separately.

From Table 5 in the Appendix, the low-lying peak at 7.17 MeV is predicted with $B_{M2} = 834.41 \mu_{N}^2$fm$^2$. There, the most dominant contribution is from the proton transition $1d_{5/2} \rightarrow 1f_{7/2}$. This transition is natural for the M2-selection rule. In comparison, however, the corresponding data in Ref. [11] between 6-8 MeV yields only $\approx 25 \mu_{N}^2$fm$^2$. The RPA and second RPA calculations in Ref. [11] commonly predict the similar, minor strengths in this low-lying region of $^{48}$Ca. Thus, the discrepancy in M2 transition strengths does not seem to be cured even if the beyond-QRPA effects are taken into account. Alternative reason is possibly the sensitivity of the RRPA energies on the IV-PV part of the residual interaction. By assuming that our RRPA energy, 7.17 MeV, is over-estimated, the low-lying M2 peak may exist but below the minimum of experimental accessibility. In Fig. 3, the M2 response as a function of the strength parameter of the IV-PV coupling is presented. There, one can find that the low-lying M2 peak is sensitive to the IV-PV interaction, similar to the sensitivity that was also obtained in the study of M1 and Gamow-Teller transitions [58]. Further experimental studies of magnetic transitions could provide additional constraints for the IV-PV interaction. From the theory side, improvement of the relevant SP energies could also result in modifications of the present results.

3.3 M2 transitions in $^{208}$Pb

Figure 4a shows the full, IS and IV M2-response functions for the $^{208}$Pb nucleus, where one dominant peak and simultaneously several finite structures are confirmed. The centroid energy of full response amounts $E^{th} = 11.87$ MeV, that is slightly above the main peak excitation energy due to the tail of the transition strength extending toward 20 MeV. The NEW summations up to 50 MeV yield $m_0 = \sum B_{M2} = 35,648 \mu_{N}^2$fm$^2$ (full), $\sum B_{M2}^{(IV)} = 35,459 \mu_{N}^2$fm$^2$, and $\sum B_{M2}^{(IS)} = 6358 \mu_{N}^2$fm$^2$. Figure 4b displays the decomposition of the full M2 strength into the spin and orbital M2 components. In addition to the dominant spin response, a non-negligible contribution from the orbital response is observed, that is, the peaked around 10 MeV. There, a con-
Fig. 7 The M2 full RQRPA response functions for Ca isotopes, shown for the DD-PC1 and DD-PCX interactions.

Fig. 8 The M2 spin and orbital RQRPA response functions supplemented with the unperturbed RHB response for Ca isotopes, using DD-PC1 interaction.

Table 2 Experimental M2 data for $^{208}$Pb. Note that the M2-Weisskopf unit is $B_S P_{M2} = 50.9 \mu_2^2$fm$^2$.

| Energy [MeV] | $B_{M2}(E)$ [$\mu_2^2$fm$^2$] | Reference |
|--------------|-------------------------------|-----------|
| 7.40         | 449 ± 89                      | [12]      |
| 7.91         | 614 ± 92                      | [12]      |
| $\sum B_{M2}$ | 6.428–8.008                   | 8500 ± 750 | [13]      |
| 7.457–8.008  | 5230 ± 130                    | [13]      |
| 0-50         | 35.648                        | (This work) |
| 0-9.88       | 11.785                        | (This work) |
| 0-8.97       | 3694                          | (This work) |

Table 3 The NEW summation, $m_0 = \sum E B_{M2}(E)$ up to 50 MeV, and the centroid energy, $\bar{E} = m_1/m_0$, calculated with the present RQRPA for $^{18}$O, $^{42}$Ca, $^{56}$Fe, and $^{90}$Zr. Results with and without pairing interaction are compared.

| Pairing | $m_0 [10^3 \mu_2^2$fm$^2]$ | $\bar{E}$ [MeV] |
|---------|-----------------------------|----------------|
| $^{18}$O | No                          | 16.17          | 16.87       |
|         | Full                        | 15.40          | 17.91       |
| $^{42}$Ca | No                          | 6.89           | 14.06       |
|         | Full                        | 4.62           | 14.81       |
| $^{56}$Fe | No                          | 6.29           | 16.47       |
|         | Full                        | 5.87           | 17.80       |
| $^{90}$Zr | No                          | 11.78          | 13.09       |
|         | Full                        | 11.02          | 14.53       |

Fig. 9 The evolution of the $m_0$ and $m_1$ RQRPA moments for M2 transitions in $^{36}$–$^{64}$Ca isotope chain, for DD-PC1 and DD-PCX interactions. Higher limit values correspond to the quenching of $g$ factors $\zeta = 0.93$, and lower limit values to $\zeta = 0.8$. 
Structural interference between the spin and orbital components exists as $R_{M2} > R_{M2}$ / $R_{2}$. On the other hand, the destructive interference is also found around 7 MeV, but its effect is minor in this $^{208}$Pb nucleus. The corresponding summations up to 50 MeV are $\sum B_{M2} = 35,648 \mu_{N}^{2}$fm$^{2}$ (full), $\sum B'_{M2} = 25,515 \mu_{N}^{2}$fm$^{2}$ (spin), and $\sum B_{M2} = 6936 \mu_{N}^{2}$fm$^{2}$ (orbital).

In Tables 7 and 8 in the Appendix, particle-hole transition components of the main M2 peaks for $^{208}$Pb are presented. In comparison to other nuclei previously discussed, for $^{208}$Pb more complicated composition is obtained, especially for the two main peaks at 9.88 and 10.18 MeV. This is expected due to larger number of M2-active nucleons.

The experimental data on M2 excitations in $^{208}$Pb are summarized in Table 2. In Ref. [13], if the experimental M2 strengths between 6-8 MeV are summed, the result reads $\sum B_{M2} = 8500 \pm 750 \mu_{N}^{2}$fm$^{2}$ [13]. Note that, in Ref. [13], the analyzing procedure with RPA refers to the experimentally observed $J^\pi = 2^-$ peak at 7.47 MeV, and its result is confirmed by the MSI-RPA in Ref. [18]. The other RPA study in Ref. [19] gives 9700–12,600 $\mu_{N}^{2}$fm$^{2}$ within the excitation energies of 6.1–8.4 MeV. Other theoretical investigations have concluded 11,000 $\mu_{N}^{2}$fm$^{2}$ [20] or 11,600 $\mu_{N}^{2}$fm$^{2}$ [21]. Indeed the M2 summation depends on the energy window available in measurements. By considering the present results with the DD-PC1 interaction outlined in our analysis in Tables 7 and 8, if we sum up $B_{M2}$ values at excitation energies of $E_x = 6.46, 7.45, 7.61, 8.69, 8.97$ and 9.88 MeV, the result is $\sum B_{M2}(E) = 11785 \mu_{N}^{2}$fm$^{2}$. This value is in agreement with other theoretical predictions and also comparable with the experimental data in Table 2. For another example, if we limit our summation up to 8.97 MeV, then the total strength reduces as $\sum B_{M2}(E) = 3694 \mu_{N}^{2}$fm$^{2}$, which is smaller than the experimental data. If the higher-energy M2 measurement becomes available, the comparison could improve the agreement.

We again mention the collectivity of M2 and its dependence on mass numbers. The Weisskopf’s estimate gives $B_{SP-M2} \equiv 1.45 \times A^{3/4} = 9.21, 19.2$, and 50.9 $\mu_{N}^{2}$fm$^{2}$ for $^{16}$O, $^{48}$Ca, and $^{208}$Pb nuclei, respectively [54,55]. On the other side, the present RRPA predicts the most excited state in each nucleus as shown in Tables in the Appendix: $B_{M2} = 487.57 \mu_{N}^{2}$fm$^{2}$ at 17.81 MeV in $^{16}$O; $B_{M2} = 1569.21 \mu_{N}^{2}$fm$^{2}$ at 15.08 MeV in $^{48}$Ca; $B_{M2} = 8020.14 \mu_{N}^{2}$fm$^{2}$ at 9.88 MeV in $^{208}$Pb. Their ratios thus read $B_{M2}/B_{SP-M2} = 52.9, 81.7$, and 157.4 for $^{16}$O, $^{48}$Ca, and $^{208}$Pb nuclei, respectively, indicating the mean numbers of excited nucleons, that seem to be overestimated. Thus, as reliable measure of collective properties of M2 transitions we consider our detailed analysis of the $ph$ composition in relevant excited states, as given in Appendix.

3.4 Pairing effect in $^{18}$O, $^{42}$Ca, $^{56}$Fe, and $^{90}$Zr

Of particular interest here are the effects of pairing correlations on the M2 transitions. Our calculations are based on the DD-PC1 parametrization for the relativistic EDF [46], and pairing correlations are described using the pairing part of the Gogny interaction [48].

Figure 5 shows the RQRPA results for the full response function for M2 transitions of $^{16}$O. The response function without pairing correlations in the residual RQRPA interaction is also shown. In comparison to previously studied nucleus $^{16}$O, open-shell system $^{18}$O displays rather different response. In particular, its high-energy part is fragmented into two strong peaks. The third peak at 21 MeV is indeed from valence neutrons in the $d_{5/2}$ orbit, which was empty in the previous $^{16}$O case. By comparing two cases with and without pairing correlations, one can observe considerable pairing effects on the transition strengths, whereas their excitation energies are less sensitive. We perform similar analysis for $^{42}$Ca and $^{56}$Fe as shown in Fig. 6a, b, respectively. The response function for $^{42}$Ca displays a triple-hump structure, and its strength is strongly sensitive on pairing correlations. For the $^{56}$Fe case in Fig. 6b, the low-energy peak is absent in the full-pairing calculation, and the overall transition strength is rather fragmented.

We next investigate the $^{90}$Zr nucleus, because experimental data on M2 transitions have been reported up to the excitation energy of about 12 MeV in Ref. [11]. The calculated M2 response function for $^{90}$Zr is shown in Fig. 6c. There, three main structures are obtained in the strength distribution, with the most pronounced peak around 13 MeV. The RQRPA result for the EW summation amounts $m_{1}(E) = 160.10 \times 10^{3}$ MeV/$\mu_{N}^{2}$fm$^{2}$ up to 50 MeV. For comparison, the experimental data from Ref. [11] are available up to 12 MeV, resulting in the value as $m_{1}^{\exp}(E) = 23.6 \times 10^{3}$ MeV/$\mu_{N}^{2}$fm$^{2}$ for $^{90}$Zr. This value is considerably lower than our prediction. If we limit our theoretical consideration up to 12 MeV, the EW summation gives $m_{1}(E \leq 12 \text{ MeV}) = 13.67 \times 10^{3}$ MeV/$\mu_{N}^{2}$fm$^{2}$, which is of the reasonable order with respect to the experimental value. Since several M2 peaks are predicted above 12 MeV, future experiments may additionally find higher transitions for complete information.

Another experimental investigation with the inelastic electron scattering [59], whose data were analyzed with the distorted-wave Born approximation, provides the NEW-M2 summation. $\sum B_{M2}^{\text{NEW}}(E) = 950 \pm 100 \mu_{N}^{2}$fm$^{2}$ for the M2-excited energies between 8 and 10 MeV for $^{90}$Zr. For comparison, from our RQRPA, the same summation amounts 291 $\mu_{N}^{2}$fm$^{2}$ in the case with pairing correlation. Indeed in our RQRPA results between 8–10 MeV, there is only one M2-excited peak predicted, and thus, the NEW summation becomes smaller than the experimental one.
The non-relativistic RPA study for $^{90}$Zr exists also in Ref. [11], where the EW sum of the B(M2) strength up to 12 MeV yields $m_1 = 112.3 \times 10^3$ MeV$\mu^2_N$ (including the quenching of 0.6 in the spin g factor), being higher than the experimental value. Similar to the present study, the second RPA calculation [11], that is based on rather different effective nuclear interactions and it does not include pairing, also result in pronounced low-energy M2 spin excitation, at $\approx 5$ MeV. Therefore, one can expect that by extending presently experimentally covered energy range between 7 and 12 MeV, both to low-energy and high-energy regions, would result in more M2 transitions that would reduce presently existing discrepancies between the overall measured and calculated strengths. We note that by including couplings to complex configurations such as $2p2h$ in the second RPA [11], considerable fragmentation of the transition strength is obtained when compared to calculation based only at the RPA level. Therefore, it is expected that the structure predicted by the RQRPA would become much more fragmented if complex configurations would be included.

In Table 3 we summarize the NEW summations and centroid energies for open-shell nuclei with and without full consideration of the pairing correlation. In general, the pairing interaction reduces the NEW summation and shifts the centroid energy to the higher region.

### 3.5 M2 transitions in $^{36-64}$Ca isotopes

In order to assess how the M2 transition evolves with the increase of the neutron number along the isotope chain, we have calculated the RQRPA response functions in $^{36-64}$Ca isotopes. Calculated predictions on M2 excitation properties for Ca isotopes could be useful for the future experimental studies. Two relativistic density-dependent interactions have been used, DD-PC1 [46] and DD-PCX [47], supplemented with the Gogny force [48] for the pairing correlations.

The calculated M2 response functions are shown for $^{36-64}$Ca in Fig. 7. The transition strength distributions show no significant differences when comparing DD-PC1 versus DD-PCX results, neither in terms of fragmentation or transition strength. With increasing the neutron number, the overall M2 transition strength increases. Qualitative changes are especially pronounced in the low-energy part of the spectra which becomes more fragmented, and in neutron-rich Ca isotopes the main M2 peak becomes split into two peaks, which are of comparable strength for $^{60,64}$Ca.

In Fig 8 the M2 response is shown separately for the $R^S_{M2}(E)$ (spin) and $R^L_{M2}(E)$ (orbital) transitions. Similarly in the case of M1 transitions for spherical symmetric nuclei [44], the spin M2 transition is much stronger in comparison to almost-negligible orbital one. Thus the evolution of the M2 response in Ca isotopes is governed mainly by changes in the spin-response, which also determines the increase of the fragmentation of the spectra when moving toward neutron-rich isotopes. Due to small contribution of the orbital-response, it is difficult to conclude on its evolution along the isotope chain.

Finally, in Fig. 9, the $m_0$ (NEW) and $m_1$ (EW) M2 summations are shown for $^{36-64}$Ca isotope chain, using the RQRPA with the DD-PC1 and DD-PCX interactions. In this way we can observe the evolution of the M2 properties with increasing the neutron number in Ca isotopes. The model calculations are performed using two limit values for the quenching of the spin and orbital g factors, $\zeta = 0.93$ and $\zeta = 0.8$, which were previously discussed in the study of M1 transitions [44]. Therefore, it allows one to infer how these limits in the quenching factors fit into the future experimental studies of M2 transitions in Ca isotopes. The figure also demonstrates rather weak model dependence over the whole isotope chain when using DD-PC1 and DD-PCX interactions. In Fig. 9 we have denoted the overlap of the results obtained from the two interactions, that represents a theoretical prediction of this study for $m_0$ and $m_1$ moments, to be compared with forthcoming experiments. In this way, the analysis of M2 transitions, simultaneously with those of M1 transitions, can provide deeper insight into the quenching of gyromagnetic factors.

### 4 Summary

In this work we have investigated the properties of M2 transitions from $0^+$ ground state to $2^-$ excited states in even-even nuclei, by using the RHB+RQRPA framework. Two density-dependent relativistic point coupling interactions have been used, DD-PC1 and DD-PCX, supplemented with the IV-PV term in the residual RQRPA interaction. The pairing correlations have been described with the pairing part of the Gogny force. Model calculations of M2 transitions in $^{16}$O, $^{48}$Ca, and $^{208}$Pb provided insight into their structure and collective properties. The analysis of relevant particle-hole configurations confirmed that the main M2 peaks have rather complicated structure, different than single-particle transition, where nucleons in different orbits simultaneously contribute.

With increase of the mass number the collectivity of main M2 states increases due to larger number of possible $ph$ transitions. The calculated M2 transition strengths appear larger than available experimental data, possibly because (i) measurements are available only within rather restricted energy ranges, and/or (ii) beyond-QRPA effects are missing in the present approach. Similar results are obtained in earlier RPA and shell-model calculations, if no additional quenching is imposed on the g factors. For the missing low-lying M2 peak in measurements for $^{48}$Ca, we point out that further experimental studies of low-energy M2 transitions could provide...
additional constraints for the IV-PV interaction, and clarify if additional M2 states exist at energies lower than presently available from the experiment. More complete experimental studies are expected as reference for benchmarking the model calculations. We especially emphasize the interest in two cases, i.e., (i) the low-lying region of $^{48}$Ca, where another peak of proton’s $1d_{3/2}^{-1} \rightarrow 1f_{7/2}$ transition is predicted, and (ii) the region higher than 9 MeV of $^{208}$Pb, where its M2 response is predicted to have rather distributed structure.

We have also demonstrated that pairing effects in open-shell nuclei $^{18}$O, $^{42}$Ca, $^{56}$Fe, and semi-magic $^{90}$Zr have an impact on M2 transition spectra, by reducing the strength and shifting the centroid energy to the higher values. The evolution of the $n_0$ and $n_1$ moments in $^{36-64}$Ca isotope chain has also been investigated. By imposing the limit values for the quenching of the $g$ factors consistent with the analysis of M1 transitions [44], we provide the limits for $n_0$ and $n_1$ moments for M2 transitions within the Ca isotopes, that could provide useful constraints for the quenching of $g$ factors in nuclear medium. To our present knowledge, there is no systematic experimental data on properties of M2 transitions available for Ca isotopes except $^{48}$Ca. Thus, we hope that the predictions of this work could motivate future experimental studies on M2 transitions in Ca isotopes. Some preparations along this line of research are already in progress [60].

Acknowledgements We thank Peter von Neumann-Cosel for useful discussion on magnetic transitions. This work is supported within the Tenure Track Pilot Programme of the Croatian Science Foundation and the École Polytechnique Fédérale de Lausanne, and the Project TTP-2018-07-3554 Exotic Nuclear Structure and Dynamics, with funds of the Croatian-Swiss Research Programme. This work is supported by the “QuantiXLie Centre of Excellence” project co-financed by the Croatian Government and European Union through the European Regional Development Fund, the Competitiveness and Cohesion Operational Programme (KK.01.1.1.01.00004). We acknowledge support by the Multidisciplinary Cooperative Research Program of the Center for Computational Sciences, University of Tsukuba, using Oakforest-PACS Systems (Project no. xg21i064, FY2021), and T.O. acknowledges the support by Takashi Nakatsukasa and Hiroyuki Kobayashi in this program.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: Data supporting this study are included within the article.]

Appendix A: The analysis of contributing particle-hole transitions in M2 excited states

Here we display the components of M2 transitions for $^{16}$O, $^{48}$Ca, and $^{208}$Pb nuclei, on which discussions are presented in the main text. See Tables 4, 5, 6, 7, 8.
### Table 5 continued

| $E_{\text{peak}}^{\text{ph}}$ [MeV] | $B_{M2}(E)$ | $b_{\text{ph}}^{\mu}$ | $b_{\text{ph}}^{\nu}$ | $b_{\text{ph}}^{\pi}$ | $\phi$ transition |
|-----------------------------------|--------------|---------------------|---------------------|---------------------|------------------|
| 14.41                            | 223.95       | –5.05               | 1.47                | –1.73               | 8.68             |
|                                  |              | $\pi 2s_{1/2} \rightarrow 2p_{3/2}$ | $(\pi 1f_{7/2} \rightarrow 1g_{9/2})$ | $(\pi 1d_{5/2} \rightarrow 1f_{7/2})$ | $(\pi 1d_{3/2} \rightarrow 1f_{7/2})$ |
|                                  |              | –5.59               | –1.07               | 0.21                | 1.57             |
|                                  |              | $(v \ell 1f_{7/2} \rightarrow 1g_{9/2})$ | $(\pi 1d_{5/2} \rightarrow 2p_{3/2})$ | $(v \ell 1f_{7/2} \rightarrow 2d_{5/2})$ | $(\pi 1d_{5/2} \rightarrow 1f_{7/2})$ |
|                                  |              | 0.27                | 2.29                |                      |                  |
|                                  |              | $(v \ell 1d_{5/2} \rightarrow 2p_{3/2})$ | $(\pi 1d_{5/2} \rightarrow 1f_{7/2})$ |                     |                  |

### Table 6 continued

| $E_{\text{peak}}^{\text{ph}}$ [MeV] | $B_{M2}(E)$ | $b_{\text{ph}}^{\mu}$ | $b_{\text{ph}}^{\nu}$ | $b_{\text{ph}}^{\pi}$ | $\phi$ transition |
|-----------------------------------|--------------|---------------------|---------------------|---------------------|------------------|
|                                  |              | –2.38               | –1.46               | –2.32               | 0.74             |
|                                  |              | $(v \ell 1d_{3/2}^{1} \rightarrow 2p_{3/2})$ | $(\pi 1d_{3/2}^{1} \rightarrow 1f_{7/2})$ | $(v \ell f_{7/2}^{1} \rightarrow 2g_{9/2})$ | $(\pi 1d_{3/2}^{1} \rightarrow 1f_{7/2})$ |
|                                  |              | 21.59               | 165.43              | –1.49               | 2.76             |
|                                  |              | $(v \ell f_{7/2}^{1} \rightarrow 2g_{9/2})$ | $(\pi 1d_{3/2}^{1} \rightarrow 2p_{3/2})$ |                     |                  |

### Table 7

The same as Table 4 but for $^{208}$Pb

| $E_{\text{peak}}^{\text{ph}}$ [MeV] | $B_{M2}(E)$ | $b_{\text{ph}}^{\mu}$ | $b_{\text{ph}}^{\nu}$ | $b_{\text{ph}}^{\pi}$ | $\phi$ transition |
|-----------------------------------|--------------|---------------------|---------------------|---------------------|------------------|
|                                  |              | 4.64                | 1815.74             | 1.86                | 7.67             |
|                                  |              | $(v \ell 3p_{1/2} \rightarrow 3d_{5/2})$ | $(\pi 2d_{5/2} \rightarrow 2f_{7/2})$ |                     |                  |
|                                  |              | 0.39                | 32.11               | –1.88               | 3.35             |
|                                  |              | $(v \ell f_{7/2}^{1} \rightarrow 2g_{9/2})$ | $(\pi 1f_{11/2}^{1} \rightarrow 1f_{13/2})$ |                     |                  |
|                                  |              | –1.11               | –0.48               |                      |                  |
|                                  |              | $(v \ell 1f_{11/2}^{1} \rightarrow 1f_{13/2})$ |                     |                     |                  |
### Table 7 continued

| $E^{th}_{\text{peak}}$ [MeV] | $B_{M2}(E)$ [$\mu^2N\text{fm}^2$] | $b^\nu_{ph}$ [$\mu_N\text{fm}$] | $b^\pi_{ph}$ [$\mu_N\text{fm}$] | $\phi$ transition |
|-----------------------------|-------------------------------|----------------|----------------|----------------|
| -1.62                       | -1.69                         | (\nu f^3 p_{1/2} \rightarrow 3d_{5/2}) | | |
| -1.24                       | 0.37                          | (\pi h^1_{1/2} \rightarrow 1i_{1/2}) | | |
| (\nu f^3 p_{1/2} \rightarrow 2g_{9/2}) | (\pi 2d^3 s_{5/2} \rightarrow 3p_{3/2}) | | | |

### Table 8 Continuation of Table 7 for 208Pb

| $E^{th}_{\text{peak}}$ [MeV] | $B_{M2}(E)$ [$\mu^2N\text{fm}^2$] | $b^\nu_{ph}$ [$\mu_N\text{fm}$] | $b^\pi_{ph}$ [$\mu_N\text{fm}$] | $\phi$ transition |
|-----------------------------|-------------------------------|----------------|----------------|----------------|
| 9.88                        | 8020.14                       | 1.58           | -1.62         | (\nu f^2 s_{1/2} \rightarrow 1g_{9/2}) |
| 8.94                        | -1.16                         | (\nu f^3 p_{1/2} \rightarrow 1g_{9/2}) | (\pi 2d^3 s_{5/2} \rightarrow 1f_{7/2}) | |
| -2.16                       | -20.72                       | (\nu h^1_{9/2} \rightarrow 1i_{1/2}) | (\pi 2d^3 s_{5/2} \rightarrow 1f_{7/2}) | |
| 11.55                       | 92.71                        | (\nu i_{13/2} \rightarrow 1j_{15/2}) | (\pi h^1_{1/2} \rightarrow 1i_{13/2}) | |
| 10.18                       | 7843.68                      | -3.91          | 4.62           | (\nu f^2 s_{1/2} \rightarrow 1g_{9/2}) |
| 10.78                       | 1926.70                      | 1.18           | 32.23          | (\nu f^2 s_{1/2} \rightarrow 2g_{9/2}) |
| -1.24                       | 4.75                         | (\nu h^1_{9/2} \rightarrow 1i_{11/2}) | (\pi 2d^3 s_{5/2} \rightarrow 2f_{7/2}) | |
| 2.23                        | 4.14                         | (\nu i_{13/2} \rightarrow 1j_{15/2}) | (\pi h^1_{1/2} \rightarrow 1i_{13/2}) | |
| 16.24                       | 1094.44                      | 2.72           | -4.61          | (\nu f^2 s_{1/2} \rightarrow 4d_{3/2}) |
| 13.69                       | 0.68                         | (\nu h^1_{9/2} \rightarrow 1i_{11/2}) | (\pi 1g_{9/2} \rightarrow 2f_{5/2}) | |
| 6.30                        | 15.49                        | (\nu i_{13/2} \rightarrow 1j_{15/2}) | (\pi h^1_{1/2} \rightarrow 1i_{11/2}) | |
| 19.33                       | 1888.20                      | 16.05          | 5.45           | (\nu h^1_{1/2} \rightarrow 2g_{7/2}) |
| 8.58                        | 5.01                         | (\nu i_{13/2} \rightarrow 3b_{9/2}) | (\pi 1g_{9/2} \rightarrow 2f_{5/2}) | |

References

1. K. Heyde, P. von Neumann-Cosel, A. Richter, Rev. Mod. Phys. 82, 2365 (2010), (and references therein)
2. A. Richter, Progress in Particle and Nuclear Physics 34, 261 (1995)
3. Y. Fujita, B. Rubio, W. Gelletly, Progress in Particle and Nuclear Physics 66(3), 549 (2011)
4. R. Schwengner, S. Frauendorf, B.A. Brown, Phys. Rev. Lett. 118, 092502 (2017)
5. G.E. Dogotar, R.A. Ermazhyan, M. Gmitro, H.R. Kissener, E. Tintkova, Journal of Physics G: Nuclear Physics 5(12), L221 (1979)
6. R. Ermazhyan, M. Gmitro, H. Kissener, Nucl. Phys. A 338(2), 436 (1980)
7. G. Küchler, A. Richter, E. Spamer, W. Steffen, W. Knüpfel, Nucl. Phys. A 406(3), 473 (1983)
8. J. Kokame, K. Fukunaga, N. Inoue, H. Nakamura, Physics Letters 8(5), 342 (1964)
9. M. Stroetzell, A. Goldmann, Zeitschrift für Physik A Hadrons and nuclei 233(3), 245 (1970)
10. A. Goldmann, M. Stroetzell, Zeitschrift für Physik A Hadrons and nuclei 239(3), 235 (1970)
11. P. von Neumann-Cosel, F. Neumeyer, S. Nishizaki, V.Y. Ponomarev, C. Rangacharyulu, B. Reitz, A. Richter, G. Schrieder, D.J. Sober, T. Waindzech, J. Wambach, Phys. Rev. Lett. 82, 1105 (1999)
12. R.A. Lindgren, W.L. Bendel, L.W. Fagg, E.C. Jones, Phys. Rev. Lett. 36, 116 (1976)
13. R. Frey, A. Richter, A. Schwierzczinski, E. Spamer, O. Titze, W. Knüpfel, Phys. Lett. B 74(1), 45 (1978)
14. G. Peterson, J. Ziegler, Physics Letters 21(5), 543 (1966)
15. E. Boridy, J. Pearson, Nuclear Physics A 185(2), 593 (1972)
16. J. Friedrich, N. Voegler, H. Euteneuer, Physics Letters B 64(3), 269 (1976)
17. W. Knüpfel, R. Frey, A. Friebel, W. Mettner, D. Meier, A. Richter, E. Spamer, O. Titze, Phys. Lett. B 77(4), 367 (1978)
18. W. Knüpfel, M.G. Huber, Phys. Rev. C 14, 2254 (1976)
19. V. Ponomarev, V. Soloviov, C. Stoyanov, A. Vdovin, Nucl. Phys. A 323(2), 446 (1979)
20. B. Castel, I. Hamamoto, Phys. Lett. B 65(1), 27 (1976)
21. J.S. Dehesa, Ph.D. thesis, Rheinischen Friedrich-Wilhelms-Universität zu Bonn (1977)
22. P. Ring, J. Speth, Nuclear Physics A 235(2), 315 (1974)
23. P. Ring, J. Speth, Physics Letters B 46(6), 477 (1973)
24. S. Krewald, J. Speth, Physics Letters B 52(3), 295 (1974)
25. G. Holzwarth, G. Eckart, Z. Physik A 283, 219 (1977)
26. G. Holzwarth, G. Eckart, Nucl. Phys. A 325, 1 (1979)
27. G. Holzwarth, G. Eckart, Nucl. Phys. A 396, 171 (1983)
28. G. Holzwarth, G. Eckart, Phys. Lett. B 118, 9 (1982)
29. B. Schwesinger, K. Pintig, G. Holzwarth, Nucl. Phys. A 341(1), 1 (1980)
30. M. Traini, Phys. Rev. Lett. 41, 1535 (1978)
31. T. Suzuki, Physics Letters B 83(2), 147 (1979)
32. D. Kurath, Argonne National Laboratory Report No. 97/14 (1977)
33. E. Lipparini, S. Stringari, Physics Reports 175(3), 103 (1989)
34. N. Shafigu, A. Kuzuhiko, Prog. Theor. Phys. 63(5), 1599 (1980)
35. W. Knüpfel, R. Frey, A. Friebel, W. Mettner, D. Meier, A. Richter, E. Spamer, O. Titze, Physics Letters B 77(4), 367 (1978)
36. H. Toki, W. Weise, Physics Letters B 97(1), 12 (1980)
37. J. Rangacharyulu, W. Steffen, A. Richter, E. Spamer, O. Titze, Physics Letters B 335(1), 29 (1984)
38. A. Vdovin, C. Stoyanov, W. Andrejevski, Nuclear Physics A 440(3), 437 (1985)
39. J. Speth, Electric and Magnetic Giant Resonances in Nuclei (World Scientific, New York, 1991)
40. D.F. Geesaman, R.D. Lawson, B. Zeidman, G.C. Morrison, A.D. Bacher, C. Olmer, G.R. Burleson, W.B. Cottingham, S.J. Greene,
41. A.M. Lallena, Nuclear Physics A 489(1), 70 (1988)
42. A. Bacher, G. Emery, W. Jones, D. Miller, G. Adams, F. Petrovich, W. Love, Physics Letters B 97(1), 58 (1980)
43. T. Nikšić, N. Paar, D. Vretenar, P. Ring, Comp. Phys. Comm. 185(6), 1808 (2014)
44. G. Kružič, T. Oishi, D. Vale, N. Paar, Phys. Rev. C 102, 044315 (2020)
45. G. Kružič, T. Oishi, N. Paar, Phys. Rev. C 103, 054306 (2021)
46. T. Nikšić, D. Vretenar, P. Ring, Phys. Rev. C 78, 034318 (2008)
47. E. Yüksel, T. Marketin, N. Paar, Phys. Rev. C 99, 034318 (2019)
48. J.F. Berger, M. Girod, D. Gogny, Nucl. Phys. A 428 (1984)
49. N. Paar, P. Ring, T. Nikšić, D. Vretenar, Phys. Rev. C 67, 034312 (2003)
50. J. Suhonen, From Nucleons to Nucleus: Concepts of Microscopic Nuclear Theory (Springer-Verlag, Berlin-Heidelberg, 2007)
51. Y. Fujita, B. Rubio, W. Gelletly, Prog. Part. Nucl. Phys. 66, 549 (2011)
52. S. Kamerdzhev, J. Speth, G. Tertychny, J. Wambach, Zeitschrift für Physik A 346, 253 (1993)
53. R. Schwengner, S. Frauendorf, B.A. Brown, Phys. Rev. Lett. 118, 092502 (2017)
54. V.F. Weisskopf, Phys. Rev. 83, 1073 (1951)
55. P. Ring, P. Schuck, The Nuclear Many-Body Problems (Springer-Verlag, Berlin and Heidelberg, Germany, 1980)
56. X. Roca-Maza, G. Pozzi, M. Brenna, K. Mizuyama, G. Colò, Phys. Rev. C 85, 024601 (2012)
57. X. Roca-Maza, G. Pozzi, M. Brenna, K. Mizuyama, G. Colò, Phys. Rev. C 85, 024601 (2012)
58. T. Oishi, A. Ravlić, N. Paar, Phys. Rev. C 105, 064309 (2022)
59. S. Müller, F. Beck, D. Meuer, A. Richter, Phys. Lett. B 113(5), 362 (1982)
60. P. von Neumann-Cosel, private communications (2021)