Flavour Dynamics & CP Violation in the SM*: A Tale in Five Parts of Great Successes, Little Understanding – and Promise for the Future!

I.I. Bigi

Department of Physics, University of Notre Dame du Lac
Notre Dame, IN 46556, USA
email: ibigi@nd.edu

Abstract

Our knowledge of flavour dynamics has undergone a ‘quantum jump’ since just before the turn of the millenium: direct CP violation has been firmly established in $K_L \to \pi\pi$ decays in 1999; the first CP asymmetry outside $K_L$ decays has been discovered in 2001 in $B_d \to \psi K_S$, followed by $B_d \to \pi^+\pi^-$ and $B \to K^\pm\pi^\mp$, the latter establishing direct CP violation also in the beauty sector. Furthermore CKM dynamics allows a description of CP insensitive and sensitive $B$, $K$ and $D$ transitions that is impressively consistent also on the quantitative level. Theories of flavour dynamics that could serve as alternatives to CKM have been ruled out. Yet these novel successes of the Standard Model (SM) do not invalidate any of the theoretical arguments for the incompleteness of the SM. In addition we have also more direct evidence for New Physics, namely neutrino oscillations, the observed baryon number of the Universe, dark matter and dark energy. While the New Physics anticipated at the TeV scale is not likely to shed any light on the SM’s mysteries of flavour, detailed and comprehensive studies of heavy flavour transitions will be essential in diagnosing salient features of that New Physics. Strategic principles for such studies will be outlined.
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In my lecture series I will sketch the past evolution of central concepts of the Standard Model (SM), which are of particular importance for its flavour dynamics. The reason is not primarily of a historical nature. I hope these sketches will illuminate the main message I want to convey, namely that we find ourselves in the midst of a great intellectual adventure: even with the recent novel successes of the SM the case for New Physics at the TeV and at higher scales is as strong as ever; yet at the same time we cannot count on such New Physics having a massive impact on $B$ decays. Furthermore while there is a crowd favourite for the TeV scale New Physics, namely some implementation of Supersymmetry (SUSY) – an expectation I happen to share – we better allow for many diverse scenarios. Accordingly I will emphasize general principles for designing search strategies for New Physics over specific and detailed examples. As Prof. Sanda set as an essential goal for this school: we want to help you prepare yourself for a future leadership role; that requires that you do your own thinking rather than ‘out-source’ it.

Let me add a personal comment: the lecture room at Villa Monastero – not surprisingly – used to be a chapel. This should present no problem for experimentalists; after all they talk about empirical facts. Yet for a theorist it constitutes a more dangerous environment. I was thus greatly relieved when Prof. Manelli assured me he does not entertain the illusion that a theorist can speak the truth all the time; speaking in good faith is all he expects from a theorist. This I can promise.

The outline of my five lectures is as follows:

- Lecture I: Introduction of the SM* – Renormalizability, Neutral Currents, Mass Generation, GIM mechanism, CP violation a la CKM
- Lecture II: CKM Phenomenology
- Lecture III: CP Violation in $B$ Decays – the ‘Expected’ Triumph of a Peculiar Theory
- Lecture IV: Adding High Accuracy to High Sensitivity
- Lecture V: Searching for a New Paradigm 2005 & Beyond Following Sam Beckett’s Dictum
1 Lecture I: Introduction of the SM∗ – Renormalizability, Neutral Currents, Mass Generation, GIM mechanism, CP violation a la CKM

1.1 Introduction

A famous American Football coach once declared:’’Winning is not the greatest thing – it is the only thing!’’ This quote provides some useful criteria for sketching the status of the different components of the Standard Model (SM). It can be characterized by the carriers of its strong and electroweak forces described by gauge dynamics and the mass matrices for its quarks and leptons as follows:

\[ \text{SM}^* = SU(3)_C \times SU(2)_L \oplus \text{’CKM’}(\oplus \text{’PMNS’}) \]  

I have attached the asteriks to ‘SM’ to emphasize the SM contains a very peculiar pattern of fermion mass parameters that is not illuminated at all by its gauge structure. I will address the status of these components in my first lecture.

1.2 QCD – the ‘Only’ Thing

1.2.1 ‘Derivation’ of QCD

While it is important to subject QCD again and again to quantitative tests as the theory for the strong interactions, one should note that these serve more as tests of our theoretical control over QCD dynamics than of QCD itself. For its features can be inferred from a few general requirements and basic observations. A simplified list reads as follows:

- Our understanding of chiral symmetry as a spontaneously realized one – which allows treating pions as Goldstone bosons implying various soft pion theorems – requires vector couplings for the gluons.
- The rates for \( e^+e^- \to \text{had.}, \pi^0 \to \gamma\gamma \) etc. etc. point to the need for three colours.
- Colour has to be implemented as an unbroken symmetry. Local gauge theories are the only known way to couple massless spin-one fields in a Lorentz invariant way. The basic challenge is easily stated: 4 \( \neq \) 2; i.e., while Lorentz covariance requires four component to describe a spin-one field, the latter contains only two physical degrees of freedom for massless fields.
- Combining confinement with asymptotic freedom requires a non-abelian gauge theory.

In summary: for describing the strong interactions QCD is the unique choice among local quantum field theories. A true failure of QCD would thus create a genuine paradigm shift, for one had to adopt an intrinsically non-local description. It should be remembered that string theory was first put forward for providing a framework for describing the strong interactions.
1.2.2 ‘Fly-in-the-Ointment’: the Strong CP Problem of QCD

A theoretical problem arises for QCD from an unexpected quarter that is however very relevant for our context here: QCD does not automatically conserve $P$, $T$ and $CP$. To reflect the nontrivial topological structure of QCD’s ground state one employs an effective Lagrangian containing an additional term to the usual QCD Lagrangian [1]:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \theta \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}, \quad \tilde{G}_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$$

(2)

Since $G_{\mu\nu} \tilde{G}^{\mu\nu}$ is a gauge invariant operator, its appearance in general cannot be forbidden, and what is not forbidden has to be considered allowed in a quantum field theory. It represents a total divergence, yet in QCD – unlike in QED – it cannot be ignored due to the topological structure of the ground state.

Since under parity $P$ and time reversal $T$ one has

$$G_{\mu\nu} \tilde{G}^{\mu\nu} \overset{P,T}{\rightarrow} -G_{\mu\nu} \tilde{G}^{\mu\nu};$$

(3)
i.e., the last term in Eq.(2) violates $P$ as well as $T$! Since $G_{\mu\nu} \tilde{G}^{\mu\nu}$ is flavour-diagonal, it generates an electric dipole moment (EDM) for the neutron [2]. From the upper bound on the latter one infers [3]

$$\theta < 10^{-9}. \quad (4)$$

Being the coefficient of a dimension-four operator $\theta$ can be renormalized to any value, even zero. Yet the modern view of renomalization is more demanding: requiring the renormalized value to be smaller than its ‘natural’ one by orders of magnitude is frowned upon, since it requires finetuning between the loop corrections and the counterterms. This is what happens here. For purely within QCD the only intrinsically ‘natural’ scale for $\theta$ is unity. If $\theta \sim 0.1$ or even 0.01 were found, one would not be overly concerned. Yet the bound of Eq.(4) is viewed with great alarm as very unnatural – unless a symmetry can be called upon. If any quark were massless – most likely the $u$ quark – chiral rotations representing symmetry transformations in that case could be employed to remove $\theta$ contributions. Yet a considerable phenomenological body rules against such a scenario.

A much more attractive solution would be provided by transforming $\theta$ from a fixed parameter into the manifestation of a dynamical field – as is done for gauge and fermion masses through the Higgs-Kibble mechanism, see below – and imposing a Peccei-Quinn symmetry would lead naturally to $\theta \ll \mathcal{O}(10^{-9})$. Alas – this attractive solution does not come ‘for free’: it requires the existence of axions. Those have not been observed despite great efforts to find them.

This is a purely theoretical problem. Yet I consider the fact that it remains unresolved a significant chink in the SM’s armour. I still have not given up hope that ‘victory can be snatched from the jaws of defeat’: establishing a Peccei-Quinn-type solution would be a major triumph for theory.
1.2.3 Theoretical Technologies for QCD

True theorists tend to think that by writing down, say, a Lagrangian one has defined a theory. Yet to make contact with experiment one needs theoretical technologies to infer observable quantities from the Lagrangian. That is the task that engineers and plumbers like me have set for themselves. Examples for such technologies are:

- perturbation theory;
- chiral perturbation;
- QCD sum rules;
- heavy quark expansions (which will be described in some detail in Lecture IV).

Except for the first one they incorporate the treatment of nonperturbative effects. None of these can claim universal validity; i.e., they are all ‘protestant’ in nature. There is only one ‘catholic’ technology, namely lattice gauge theory ¹:

- It can be applied to nonperturbative dynamics in all domains (with the possible practical limitation concerning strong final state interactions).
- Its theoretical uncertainties can be reduced in a systematic way.

Chiral perturbation theory is QCD at low energies describing the dynamics of soft pions and kaons. The heavy quark expansions treating the nonperturbative effects in heavy flavour decays through an expansion in inverse powers of the heavy quark mass are tailor made for describing $B$ decays; to which degree their application can be extended down to the charm scale is a more iffy question, to which I will return in Lecture IV. Different formulations of lattice QCD can approach the nonperturbative dynamics at the charm scale from below as well as from above. The degree to which they yield the same results for charm provides an essential cross check on their numerical reliability. In that sense the study of charm decays serves as an important bridge between our understanding of nonperturbative effects in heavy and light flavours.

1.3 $SU(2)_L \times U(1)$ – not even the Greatest Thing

1.3.1 Prehistory

It was recognized from early on that the four-fermion-coupling of Fermi’s theory for the weak forces yields an effective description only that cannot be extended to very high energies. The lowest order contribution violates unitarity around 250 GeV. Higher order contributions cannot be called upon to remedy the situation, since due to the theory being non-renormalizable those come with more and more untamable infinities. Introducing

¹I hasten to add that lattice gauge theory – while catholic in substance – exhibits a different sociology: it has not developed an inquisition and deals with heretics in a rather gentle way.
massive charged vector bosons softens the problem, yet does not solve it. Consider the propagator for a massive spin-one boson carrying momentum $k$:

$$\frac{-g_{\mu\nu} + \frac{k_{\mu}k_{\nu}}{M_W^2}}{k^2 - M_W^2}$$

The second term in the numerator has great potential to cause trouble. For it can act like a coupling term with dimension $1/(\text{mass})^2$; this is quite analogous to the original ansatz of Fermi’s theory and amounts to a non-renormalizable coupling. It is actually the longitudinal component of the vector boson that is at the bottom of this problem.

This potential problem is neutralized, if these massive vector bosons couple to conserved currents. To guarantee the latter property, one needs a non-abelian gauge theory, which implies the existence of neutral weak currents.

### 1.3.2 Strong Points

The requirements of unitarity, which is nonnegotiable, and of renormalizability, which is to some degree, severely restrict possible theories of the electroweak interactions. It makes the generation of mass a highly nontrivial one, as sketched below. There are other strong points as well:

⊕ Since there is a single $SU(2)_L$ group, there is a single set of gauge bosons. Their self-coupling controls also, how they couple to the fermion fields. As explain later in more detail, this implies the property of ‘weak universality’.

⊕ The SM truly predicted the existence of neutral currents characterized by one parameter, the weak angle $\theta_W$ and the masses of the $W$ and $Z$ bosons.

⊕ Most remarkably the $SU(2)_L \times U(1)$ gauge theory combines QED with a pure parity conserving vector coupling to a massless neutral force field with the weak interactions, where the charged currents exhibit maximal parity violation due to their $V-A$ coupling and a very short range due to $M_Z > M_W \gg m_\pi$.

### 1.3.3 Generating Mass

A massive spin-one field with momentum $k_{\mu}$ and spin $s_{\mu}$ has four (Lorentz) components. Going into its rest frame one realizes that the Lorentz invariant constraint $k \cdot s = 0$ can be imposed, which leaves three independent components, as it has to be.

A massless spin-one field is still described by four components, yet has only two physical degrees of freedom. It needs another physical degree of freedom to transmogrify itself into a massive field. This is achieved by having the gauge symmetry realized spontaneously. For the case at hand this is implemented through an ansatz that should be – although rarely is – referred to as Higgs-Brout-Englert-Guralnik-Hagen-Kibble mechanism (HBEGHK). Consider as simplest case a complex scalar field $\phi$ with a potential invariant under $\phi(x) \rightarrow e^{i\alpha(x)}\phi(x)$:

$$V(\phi) = \lambda|\phi|^4 - \frac{m^2}{2}|\phi|^2$$

(6)
Its minimum is obviously not at $|\phi| = 0$, but at $\sqrt{m^2/4\lambda}$. Thus rather than having a unique ground state with $|\phi| = 0$ one has an infinity of different, yet equivalent ground states with $|\phi| = \sqrt{m^2/4\lambda}$. To understand the physical content of such a scenario, one considers oscillations of the field around the minimum of the potential: oscillations in the radial direction of the field $\phi$ represent a scalar particle with mass; in the polar direction (i.e. the phase of $\phi$) the potential is at its minimum, i.e. flat, and the corresponding field component constitutes a massless field.

It turns out that this massless scalar field can be combined with the two transverse components of a $M = 0$ spin-one gauge field to take on the role of the latter’s longitudinal component leading to the emergence of a massive spin-one field. Its mass is thus controlled by the nonperturbative quantity $\langle 0|\phi|0 \rangle$.

Applying this generic construction to the SM one finds that a priori both $SU(2)_L$ doublet and triplet Higgs fields could generate masses for the weak vector bosons. The ratio observed for the $W$ and $Z$ masses is fully consistent with only doublets contributing. Intriguingly enough such doublet fields can eo ipso generate fermion masses as well.

In the SM one adds a single complex scalar doublet field to the mix of vector boson and fermion fields. Three of its four components slip into the role of the longitudinal components of $W^\pm$ and $Z^0$; the fourth one emerges as an independent physical field – ‘the’ Higgs field. Fermion masses are then given by the product of the single vacuum expectation value (VEV) $\langle 0|\phi|0 \rangle$ and their Yukawa couplings – a point we will return to soon.

### 1.3.4 Triangle or ABJ Anomaly

The diagram with an internal loop of only fermion lines, to which three external axial vector (or one axial vector and two vector) lines are attached, generates a ‘quantum anomaly’ ²: it removes a classical symmetry as expressed through the existence of a conserved current. In this specific case it affects the conservation of the axialvector current $J^5_\mu$. Classically we have $\partial^\mu J^5_\mu = 0$ for massless fermions; yet the triangle anomaly leads to

$$\partial^\mu J^5_\mu = \frac{g^2 S}{16\pi^2} G \cdot \tilde{G} \neq 0$$

(7)
even for massless fermions; $G$ and $\tilde{G}$ denote the gluonic field strength tensor and its dual, respectively, as introduced in Eq.(2).

While by itself it yields a finite result on the right hand side of Eq.(7), it destroys the renormalizability of the theory. It cannot be ‘renormalized away’ (since in four dimensions it cannot be regularized in a gauge invariant way). Instead it has to be neutralized by requiring that adding up this contribution from all types of fermions in the theory yields a vanishing result.

²It is referred to as ‘triangle’ anomaly due to the form of the underlying diagram or A(dler)B(ell)J(ackiw) anomaly due to the authors that identified it [4].
For the SM this requirement can be expressed very concisely that all electric charges of
the fermions of a given family have to add up to zero. This imposes a connection between
the charges of quarks and leptons, yet does not explain it.

1.3.5 Theoretical Deficiencies

With all the impressive, even amazing successes of the SM, it is natural to ask why is the
community not happy with it. There are several drawbacks:
- Since the gauge group is $SU(2)_L \times U(1)$, only partial unification has been achieved.
- The HBEGHK mechanism is viewed as providing merely an ‘engineering’ solution, in
  particular since the physical Higgs field has not been observed yet. Even if or when it
  is, theorists in particular will not feel relieved, since scalar dynamics induce quadratic
  mass renormalization and are viewed as highly ‘unnatural’, as exemplified through the
gauge hierarchy problem. This concern has lead to the conjecture of New Physics entering
around the TeV scale, which has provided the justification for the LHC and the motivation
for the ILC.
- maximal violation of parity is implemented for the charged weak currents ‘par ordre
du mufti’$^3$, i.e. based on the data with no deeper understanding.
- Likewise neutrino masses had been set to zero ‘par ordre du mufti’.
- The observed quantization of electric charge is easily implemented and is instrumental
  in neutralizing the triangle anomaly – yet there is no understanding of it.

One might say these deficiencies are merely ‘warts’ that hardly detract from the beauty
of the SM. Alas – there is the whole issue of family replication!

1.4 CKM – an ‘Accidental’ Miracle

The twelve known quarks and leptons are arranged into three families. Those families
possess identical gauge couplings and are distinguished only by their mass terms, i.e.
their Yukawa couplings. We do not understand this family replication or why there are
three families. It is not even clear whether the number of families represents a funda-
mental quantity or is due to the more or less accidental interplay of complex forces as
one encounters when analyzing the structure of nuclei. The only hope for a theoretical
understanding we can spot on the horizon is superstring or M theory – which is merely a
euphemistic way of saying we have no clue.

Yet the circumstantial evidence that we miss completely a central element of Nature’s
‘Grand Design’ is even stronger.

1.4.1 Quark Masses, the GIM Mechanism & CP

Let us consider the mass terms for the up- and down-type quarks as expressed through
matrices $\mathcal{M}_{U/D}$ and vectors of quark fields $U^F = (u, c, t)^F$ and $D^F = (d, s, b)^F$ in terms

$^3$A French saying describing a situation, where a decision is imposed on someone with no explanation
and no right of appeal.
of the *flavour* eigenstates denoted by the superscript $F$:

$$\mathcal{L}_M \propto \bar{U}_L^F \mathcal{M}_U U_R^F + \bar{D}_L^F \mathcal{M}_D D_R^F.$$  

(8)

A priori there is no reason why the matrices $\mathcal{M}_{U/D}$ should be diagonal. Yet applying bi-unitary rotations $\mathcal{J}_{U/D,L}$ will allow to diagonalize them

$$\mathcal{M}^{\text{diag}}_{U/D} = \mathcal{J}_{U/D,L} \mathcal{M}_{U,D} \mathcal{J}_{U/D,R}^\dagger$$  

(9)

and obtain the *mass* eigenstates of the quark fields:

$$U^m_{L/R} = \mathcal{J}_{U,L/R} \bar{U}^F L/R, \quad D^m_{L/R} = \mathcal{J}_{D,L/R} \bar{D}^F L/R.$$  

(10)

The eigenvalues of $\mathcal{M}_{U/D}$ represent the masses of the quark fields. The measured values exhibit a very peculiar pattern that hardly appears to be arbitrary being so hierarchical for up- and down-type quarks, charged and neutral leptons.

Yet again, there is much more to it. Consider the neutral current coupling

$$\mathcal{L}^{U[D]}_{NC} \propto \bar{g} Z \bar{U}^F \gamma_\mu U^F Z^\mu.$$  

(11)

It keeps its form when expressed in terms of the mass eigenstates

$$\mathcal{L}^{U[D]}_{NC} \propto \bar{g} Z \bar{U}^m \gamma_\mu U^m Z^\mu;$$  

(12)

i.e., there are *no* flavour changing neutral currents. This important property is referred to as the 'generalized' GIM mechanism [5].

However for the charged currents the situation is quite different:

$$\mathcal{L}_{CC} \propto \bar{g}_W U^F \gamma_\mu D^\mu W^\mu = \bar{g}_W \bar{U}^m_L \gamma_\mu V_{CKM} D^m W^\mu$$  

(13)

with

$$V_{CKM} = \mathcal{J}_{U,L} J_{D,L}^\dagger.$$  

(14)

There is no reason, why the matrix $V_{CKM}$ should be the identity matrix or even diagonal

4Even if some speculative dynamics were to enforce an alignment between the $U$ and $D$ quark fields at some high scales causing their mass matrices to get diagonalized by the same bi-unitary transformation, this alignment would be likely to get upset by renormalization down to the electroweak scales.

Consider $N$ families. $V_{CKM}$ then represents an $N \times N$ matrix that has to be unitary based on two facts:

- The transformations $\mathcal{J}_{U/D,L/R}$ are unitary by construction.
As long as the carriers of the weak force are described by a single local gauge group – \( SU(2)_L \) in this case – they have to couple to all other fields in a way fixed by their self-coupling. This was already implied by Eq.(13), when writing the weak coupling \( \bar{g}_W \) as an overall factor.

The unitarity of \( V_{CKM} \) implies weak universality, as addressed later in more detail. There are actually \( N \) such relations characterized by

\[
\sum_j |V(ij)|^2 = 1 , \quad i = 1, \ldots, N
\]

These relations are important, yet insensitive to weak phases; thus they provide no direct information on CP violation.

Violations of weak universality can be implemented by adding dynamical layers to the SM. So-called horizontal gauge interactions, which differentiate between families and induce flavour-changing neutral currents, will do it. Another admittedly ad-hoc possibility is to introduce a separate \( SU(2)_L \) group for each quark family while allowing the gauge bosons from the different \( SU(2)_L \) groups to mix with each other. This mixing can be set up in such a way that the lightest mass eigenstates couple to all fermions with approximately universal strength. Weak universality thus emerges as an approximate symmetry. Flavour changing neutral currents are again induced, and they can generate electric dipole moments.

After this aside on weak universality let us return to \( V_{CKM} \). There are \( N^2 - N \) orthogonality relations:

\[
\sum_j V^*(ij)V(jk) = 0 , \quad i \neq k
\]

Those are very sensitive to complex phases and tell us directly about CP violation.

An \( N \times N \) complex matrix contains \( 2N^2 \) real parameters; the unitarity constraints reduce it to \( N^2 \) independent real parameters. Since the phases of quark fields like other fermion fields can be rotated freely, \( 2N-1 \) phases can be removed from \( \mathcal{L}_{CC} \) (a global phase rotation of all quark fields has no impact on \( \mathcal{L}_{CC} \)). Thus we have \( (N-1)^2 \) independent physical parameters. Since an \( N \times N \) orthogonal matrix has \( N(N - 1)/2 \) angles, we conclude that an \( N \times N \) unitary matrix contains also \( (N - 1)(N - 2)/2 \) physical phases. This was the general argument given by Kobayashi and Maskawa. Accordingly:

- For \( N = 2 \) families we have one angle – the Cabibbo angle – and zero phases.
- For \( N = 3 \) families we obtain three angles and one irreducible phase; i.e. a three family ansatz can support CP violation with a single source – the ‘CKM phase’.

PDG suggests a ”canonical” parametrization for the \( 3 \times 3 \) CKM matrix:

\[
V_{CKM} = \begin{pmatrix}
V(ud) & V(us) & V(ub) \\
V(cd) & V(cs) & V(cb) \\
V(td) & V(ts) & V(tb)
\end{pmatrix}
\]

11
\[
\begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{13}s_{23} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{13}c_{23}
\end{pmatrix}
\] (17)

where
\[
c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij}
\] (18)

with \(i, j = 1, 2, 3\) being generation labels.

This is a completely general, yet not unique parametrisation: a different set of Euler angles could be chosen; the phases can be shifted around among the matrix elements by using a different phase convention.

- For even more families we encounter a proliferation of angles and phases, namely six angles and three phases for \(N = 4\).

These results obtain by algebraic means can be visualized graphically:

- For \(N = 2\) we have two weak universality conditions and two orthogonality relations:
\[
V^*(ud)V(us) + V^*(cd)V(cs) = 0
\]
\[
V^*(us)V(ud) + V^*(cs)V(cd) = 0
\] (19)

While the CKM angles can be complex, there can be no nontrivial phase \((\neq 0, \pi)\) between their observable combinations; i.e., there can be no CP violation for two families in the SM.

- For three families the orthogonality relations read
\[
\sum_{j=1}^{3} V^*(ij)V(jk) = 0, \quad i \neq k
\] (20)

There are six such relations, and they represent triangles in the complex plane with in general nontrivial relative angles.

- While these six triangles can and will have quite different shapes, as we will describe later in detail, they all have to possess the same area, namely
\[
\text{area(every triangle)} = \frac{1}{2}J
\]
\[
J = \text{Im}[V(ud)V(cs)V^*(us)V^*(cd)]
\] (21)

If \(J = 0\), one has obviously no nontrivial angles, and there is no CP violation. The fact that all triangles have to possess the same area reflects the fact that for three families there is but a single CKM phase.

- Only the angles, i.e. the relative phases matter, but not the overall orientation of the triangles in the complex plane. That orientation merely reflects the phase convention for the quark fields.
If any pair of up-type or down-type quarks were mass degenerate, then any linear combination of those two would be a mass eigenstate as well. Forming different linear combinations thus represents symmetry transformations, and with this additional symmetry one can further reduce the number of physical parameters. For $N = 3$ it means CP violation could still not occur.

The CKM implementation of CP violation depends on the form of the quark mass matrices $M_{U,D}$, not so much on how those are generated. Nevertheless something can be inferred about the latter: within the SM all fermion masses are driven by a single VEV; to obtain an irreducible relative phase between different quark couplings thus requires such a phase in quark Yukawa couplings; this means that in the SM CP violation arises in dimension-four couplings, i.e., is ‘hard’.

1.4.2 ‘Maximal’ CP Violation?

As already mentioned charged current couplings with their $V - A$ structure break parity and charge conjugation maximally. Since due to CPT invariance CP violation is expressed through couplings with complex phases, one might say that maximal CP violation is characterized by complex phases of 90°. However this would be fallacious: for by changing the phase convention for the quark fields one can change the phase of a given CKM matrix element and even rotate it away; it will of course re-appear in other matrix elements. For example $|s⟩ → e^{iδ_s}|s⟩$ leads to $V_{qs} → e^{iδ_s}V_{qs}$ with $q = u, c, t$. In that sense the CKM phase is like the ‘Scarlet Pimpernel’: ”Sometimes here, sometimes there, sometimes everywhere.”

One can actually illustrate with a general argument, why there can be no straightforward definition for maximal CP violation. Consider neutrinos: Maximal CP violation means there are $ν_L$ and $\bar{ν}_R$, yet no $ν_R$ or $\bar{ν}_L$. Likewise there are $ν_L$ and $\bar{ν}_R$, but not $\bar{ν}_L$ or $ν_R$. One might then suggest that maximal CP violation means that $ν_L$ exists, but $\bar{ν}_R$ does not. Alas – CPT invariance already enforces the existence of both.

Similarly – and maybe more obviously – it is not clear what maximal T violation would mean although some formulations have entered daily language like the ‘no future generation’ and the ‘woman without a past’.

1.4.3 Some Historical Remarks

CP violation was discovered in 1964 through the observation of $K_L → π^+π^-$, yet it was not realized for a number of years that dynamics known at that time could not generate it. We should not be too harsh on our predecessors for that oversight: as long as one did not have a renormalizable theory for the weak interactions and thus had to worry about infinities in the calculated rates, one can be excused for ignoring a seemingly marginal rate with a branching ratio of $2 · 10^{-3}$. Yet even after the emergence of the renormalizable Glashow-Salam-Weinberg model its phenomenological incompleteness was not recognized

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5To be more precise: $ν_L$ and $\bar{ν}_R$ couple to weak gauge bosons, $ν_R$ or $\bar{ν}_L$ do not.
right away. There is a short remark by Mohapatra in a 1972 paper invoking the need for right-handed currents to induce CP violation.

It was the 1973 paper by Kobayashi and Maskawa [6] that fully stated the inability of even a two-family SM to produce CP violation and that explained what had to be added to it: right-handed charged currents, extra Higgs doublets – or (at least) a third quark family. Of the three options Kobayashi and Maskawa listed, their name has been attached only to the last one as the CKM description.

As pointed out by Sanda being at Nagoya University at that time gave Kobayashi and Maskawa a ‘competitive edge’ through ‘insider information’. Physicists at most other places analyzed nature in terms of three quarks only – $u$, $d$ and $s$ – (if at all), i.e. without the benefit of having two complete families; they also tended to view quarks as convenient mathematical entities rather than physical objects. It might be hard to grasp this in retrospect. For it appears straightforward now that charm quarks had to exist. Embedding weak charged currents with their Cabibbo couplings

\begin{align}
J^{(+)}_\mu &= \cos\theta_C \bar{d}_L \gamma_\mu u_L + \sin\theta_C \bar{s}_L \gamma_\mu u_L \\
J^{(-)}_\mu &= \cos\theta_C \bar{u}_L \gamma_\mu d_L + \sin\theta_C \bar{d}_L \gamma_\mu s_L
\end{align}

(22)

into an $SU(2)$ gauge theory to arrive at a renormalizable theory requires neutral currents of a structure as obtained from the commutator of $J^{(+)}_\mu$ and $J^{(-)}_\mu$. Using for the latter the expressions of Eq.(22) one arrives unequivocally at

\begin{align}
J^{(0)}_\mu = \ldots + \cos\theta_C \sin\theta_C (\bar{s}_L \gamma_\mu d_L + \bar{d}_L \gamma_\mu s_L),
\end{align}

(23)

i.e., strangeness changing neutral currents. Yet their Cabibbo suppression is not remotely sufficient to make them compatible with the observed super-tiny branching ratios for $K_L \to \mu^+\mu^-, 2\gamma$ etc. 6. The huge discrepancy between observed and expected branching ratios lead some daring spirits [5] to postulate a fourth quark with quite specific properties to complete a second quark family in such a way that no strangeness changing neutral currents arise at tree level. Yet I remember there was great skepticism felt in the community maybe best expressed by the quote: "Nature is smarter than Shelley (Glashow) – she can do without charm quarks." 7 These remarks can indicate how profound a shift in paradigm were begun by the observation of scaling in deep inelastic lepton-nucleon scattering and completed by the discovery of the $J/\psi$ in 1974 and its immediate aftermath.

The ‘genius loci’ of Nagoya University anticipated these developments:

- Since it was the home of the Sakata school and the Sakata model of elementary particles quarks were viewed as physical degrees of freedom from the start.

- It was also the home of Prof. Niu who in 1971 had observed [7] a candidate for a charm decay in emulsion exposed to cosmic rays and actually recognized it as such. The existence of charm, its association with strangeness and thus of two complete quark families were thus taken for granted at Nagoya.

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6The observed huge suppression of strangeness changing neutral currents actually led to some speculation that also flavour conserving neutral currents are greatly suppressed.

7The fact that nature needed charm after all does not prove the inverse of this quote, of course.
1.5 Summary of Lecture I

The SM

$$\text{SM}^* = SU(3)_C \times SU(2)_L \times U(1) \oplus 'CKM' \oplus 'PMNS' \quad (24)$$

is based on two pillars:

- The observed forces are described by gauge bosons of $SU(3)_C \times SU(2)_L \times U(1)$.
  - $SU(3)_C$ is the only possible solution for the strong interactions among local quantum field theories due to very general considerations. If it ever were to fail, one had to adopt an intrinsically non-local description. It is at least amusing to remember that string theory was initially conceived as a theory for the strong interactions.
  - The $SU(2)_L \times U(1)$ gauge structure is inferred from general considerations like renormalizability and from the data with some ‘theoretical engineering’ required for generating masses for the gauge bosons – and at least a whiff of incompleteness.

- Matter fields – quarks (& leptons) – obtain their masses from Yukawa couplings to a $SU(2)$ doublet scalar field that generates masses also for the gauge bosons; their charged current couplings are described in terms of the CKM (& PMNS) matrices, the origins of which lie in the quark (& lepton) mass matrices, which are a priori non-diagonal.
  - Its status can be described by saying: ”All it does, it works.” I.e., it describes the diverse body of electroweak data amazingly well, as we will discuss in the next lecture and
  - it achieves this phenomenological success for no understood reason,
  - yet we have reason to believe such deeper reason has to exist.

It is for this lack of understanding of some conjectured deeper reason that I attach an asteriks * to SM.

2 Lecture II: CKM Phenomenology

2.1 The Phenomenological Landscape through 1998

By 1998 we had observed a multifaceted phenomenological landscape in flavour dynamics [8]:

- The ‘$\theta - \tau$ puzzle’ – i.e., the observation that two particles decaying into final states of opposite parity: $\theta \rightarrow 2\pi$, $\tau \rightarrow 3\pi$, exhibited the same mass and lifetime – lead to the realization that parity was violated in weak interactions, and actually to a maximal degree in charged currents. This can be implemented in a gauge theory through $V - A$ currents.
The observation that the production rate of strange hadrons exceeded their decay rates by many orders of magnitude—a feature that gave rise to the term ‘strangeness’—was attributed to ‘associate production’ meaning the strong and electromagnetic forces conserve this new quantum number ‘strangeness’, while weak dynamics do not. Subsequently it gave rise to the notion of quark families. Another aspect of this notion was Cabibbo universality as already explained in Lecture I: $|V(ud)|^2 + |V(us)|^2 = 1$.

The observation of $\Gamma(K^+ \to \pi^+\pi^0) \ll \Gamma(K_S \to \pi^+\pi^-)$ was attributed to a $\Delta I = 1/2$ rule postulating that in strange decays the amplitude for $\Delta I = 1/2$ transitions are enhanced relative to those for $\Delta I = 3/2$ by a factor of about twenty. While various enhancements have been found, the full strength of this effect has not been derived from QCD yet.

$K^0 - \bar{K}^0$ oscillations including CP violation had been analyzed and characterized by the observables $\Delta \Gamma_K$, $\Delta M_K$ and $\epsilon_K$. Likewise $B_d - \bar{B}_d$ oscillations had been discovered and $\Delta M(B_d)$ been measured with good accuracy.

The existence of charm introduced specifically to produce the observed suppression of strangeness changing neutral currents in $K_L \to \mu^+\mu^-$, $\gamma\gamma$ and $K^0 - \bar{K}^0$ oscillations had been established with all its predicted properties.

Beauty, the existence of which had been telegraphed by the discovery of the $\tau$ as the third charged lepton was indeed observed exhibiting a surprising feature, namely its ‘long’ lifetime of about $10^{-12}$ sec. One gets a rough estimate for $\tau(B)$ by relating it to the muon lifetime:

$$\tau(B) \simeq \tau_\mu \sim \tau(\mu) \left(\frac{m(\mu)}{m(b)}\right)^5 \frac{1}{9 |V(cb)|^2} \simeq 3 \cdot 10^{-14} \left|\frac{\sin \theta_C}{V(cb)}\right|^2 \text{sec} \quad (25)$$

One had expected $|V(cb)|$ to be suppressed, since it represents an out-of-family coupling. Yet one had assumed without deeper reflection that $|V(cb)| \sim \sin \theta_C$—what else could it be? The measured value for $\tau(B)$ however pointed to $|V(cb)| \sim |\sin \theta_C|^2$.

That top quarks are unusually heavy—i.e. heavier than even the weak vector bosons—was first inferred from the speed of $B_d - \bar{B}_d$ oscillations and later—with a higher degree of accuracy—from radiative corrections to $Z^0$ decays at LEP. In the early nineties they were finally observed directly. The most recent data yield

$$m_t = 172.7 \pm 2.9 \text{ GeV} \quad (26)$$

## 2.2 On the Theoretical Technologies

Before quantifying the preceding remarks I want to introduce some of the central theoretical technologies that are employed.
2.2.1 Electroweak Dynamics

Electroweak forces can be dealt with perturbatively. Consider the $\Delta S = 1$ four-fermion transition operator: $(\bar{u}_L \gamma^\mu s_L)(\bar{d}_L \gamma^\mu u_L)$. It constitutes a dimension-six operator. Yet placing such an operator – or any other operator with dimension larger than four – into the Lagrangian creates nonrenormalizable interactions. What happened is that we have started out from a renormalizable Lagrangian

$$\mathcal{L}_{CC} = g_W q^{(i)}_L \gamma^\mu q^{(j)}_L W^\mu,$$  \hfill (27)

iterated it to second order in $g_W$ with $(q^{(i)}, q^{(j)}) = (u, s) \& (u, d)$ and then ‘integrated out’ the heavy field, namely in this case the vector boson field $W^\mu$. That way one arrives at an effective Lagrangian containing only light quarks as ‘active’ fields.

Such effective field theories have experienced a veritable renaissance in the last ten years. Constructing them in a self-consistent way is greatly helped by adopting a Wilsonian prescription:

- First one defines a field theory $\mathcal{L}(\Lambda_{UV})$ at a high ultraviolet scale $\Lambda_{UV} \gg$ germane scales of theory like $M_W$, $m_Q$ etc.

- For applications characterized by physical scales $\Lambda_{phys}$ one renormalizes the theory from the cutoff $\Lambda_{UV}$ down to $\Lambda_{phys}$. In doing so one integrates out the heavy degrees of freedom, i.e. with masses exceeding $\Lambda_{phys}$ – like $M_W$ – to arrive at an effective low energy field theory using the operator product expansion (OPE) as a tool:

$$\mathcal{L}(\Lambda_{UV}) \Rightarrow \mathcal{L}(\Lambda_{phys}) = \sum_i c_i(\Lambda_{phys}, \Lambda_{UV}, M_W, ...) \mathcal{O}_i(\Lambda_{phys})$$ \hfill (28)

- The local operators $\mathcal{O}_i(\Lambda_{phys})$ contain the active dynamical fields, i.e. those with frequencies below $\mathcal{O}_i(\Lambda_{phys})$.

- Their $c$ number coefficients $c_i(\Lambda_{phys}, \Lambda_{UV}, M_W, ...)$ provide the gateway for heavy degrees of freedom with frequencies exceeding $\mathcal{O}_i(\Lambda_{phys})$ to enter. They are shaped by short-distance dynamics and therefore usually computed perturbatively.

- Lowering the value of $\mathcal{O}_i(\Lambda_{phys})$ in general changes the form of the Lagrangian: $\mathcal{L}(\Lambda_{phys}^{(1)}) \neq \mathcal{L}(\Lambda_{phys}^{(2)})$ for $\Lambda_{phys}^{(1)} \neq \Lambda_{phys}^{(2)}$. In particular integrating out heavy degrees of freedom will induce higher-dimensional operators to emerge in the Lagrangian. In the example above integrating the $W$ field from the dimension-four term in Eq.(27) produces dimension six four-quark operators.

- As a matter of principle observables cannot depend on the choice of $\Lambda_{phys}$; the latter primarily provides just a demarkation line:

$$\text{short distances} < 1/\Lambda_{phys} < \text{long distances}$$ \hfill (29)
In practice, however, its value must be chosen judiciously due to limitations of our (present) computational abilities: on one hand we want to be able to calculate radiative corrections perturbatively and thus require \( \alpha_S(\Lambda_{\text{phys}}) < 1.0 \). Taken by itself it would suggest to choose \( \Lambda_{\text{phys}} \) as large as possible. Yet on the other hand we have to evaluate hadronic matrix elements; there \( \Lambda_{\text{phys}} \) can provide an UV cutoff on the momenta of the hadronic constituents. Since the tails of hadronic wave functions cannot be obtained from, say, quark models in a reliable way, one wants to pick \( \Lambda_{\text{phys}} \) as low as possible. More specifically for heavy flavour hadrons one can expand their matrix elements in powers of \( \Lambda_{\text{phys}}/m_Q \). Thus one encounters a Scylla & Charybdis situation. A reasonable middle course can be steered by picking \( \Lambda_{\text{phys}} \sim 1 \text{ GeV} \), and hence I will denote this quantity and this value by \( \mu \).

Some concrete examples might illuminate these remarks. Consider \( K^0 - \bar{K}^0 \) oscillations, which represent \( \Delta S = 2 \) transitions. As explained in more detail in Prof. Sanda’s lectures, those are driven by the off-diagonal elements of a ‘generalized mass matrix’:

\[
M_{12} = M_{12} + i \frac{\Gamma_{12}}{2} = \langle K^0 | \mathcal{L}_{\text{eff}}(\Delta S = 2) | \bar{K}^0 \rangle
\]  

(30)

The observables \( \Delta M_K \) and \( \epsilon_K \) are given in terms of \( \text{Re}M_{12} \) and \( \text{Im}M_{12} \), respectively. In the SM \( \mathcal{L}_{\text{eff}}(\Delta S = 2) \), which generates \( M_{12} \), is produced by iterating two \( \Delta S = 1 \) operators:

\[
\mathcal{L}_{\text{eff}}(\Delta S = 2) = \mathcal{L}(\Delta S = 1) \otimes \mathcal{L}(\Delta S = 1)
\]  

(31)

This leads to the well known quark box diagrams, which generate a local \( \Delta S = 2 \) operator. The contributions that do not depend on the mass of the internal quarks cancel against each other due to the GIM mechanism. Integrating over the internal fields, namely the \( W \) bosons and the top and charm quarks \(^8\) then yields a convergent result:

\[
\mathcal{L}_{\text{eff}}^{\text{box}}(\Delta S = 2, \mu) = \left( \frac{G_F}{4\pi} \right)^2 \left[ \xi_c^2 E(x_c) \eta_{cc} + \xi_t^2 E(x_t) \eta_{tt} + 2\xi_c \xi_t E(x_c, x_t) \eta_{ct} \right] \left[ \alpha_S(\mu^2) \right]^{-\frac{6}{27}} \left( \bar{d} \gamma_{\mu} (1 - \gamma_5) s \right)^2 + \text{h.c.}
\]  

(32)

with \( \xi_i \) denoting combinations of KM parameters

\[
\xi_i = V(is) V^*(id) , \ i = c, t ;
\]  

(33)

\( E(x_i) \) and \( E(x_c, x_t) \) reflect the box loops with equal and different internal quarks, respectively \(^9\):

\[
E(x_i) = x_i \left( \frac{1}{4} + \frac{9}{4(1 - x_i)} - \frac{3}{2(1 - x_i)^2} \right) - \frac{3}{2} \left( \frac{x_i}{1 - x_i} \right)^3 \log x_i
\]  

(34)

\[
E(x_c, x_t) = x_c x_t \left[ \left( \frac{1}{4} + \frac{3}{2(1 - x_t)} - \frac{3}{4(1 - x_t)^2} \right) \log x_t \frac{x_c}{x_t} - x_c \leftrightarrow x_t \right] - \frac{3}{2} \left( \frac{x_c}{1 - x_c} \right)^3 \log x_c
\]

\(^8\)The up quarks act merely as a subtraction term here.
The $\eta_{ij}$ represent the QCD radiative corrections from evolving the effective Lagrangian from $M_W$ down to the internal quark mass. The factor $[\alpha_S(\mu^2)]^{-6/27}$ reflects the fact that a scale $\mu$ must be introduced at which the four-quark operator $(\bar{s}\gamma_\mu(1-\gamma_5)d)^2$ is defined. This dependance on the auxiliary variable $\mu$ drops out when one takes the matrix element of this operator (at least when one does it correctly). Including next-to-leading log corrections one finds (for $m_t \simeq 180$ GeV) \cite{10}:

$$\eta_{cc} \simeq 1.38 \pm 0.20, \quad \eta_{tt} \simeq 0.57 \pm 0.01, \quad \eta_{cc} \simeq 0.47 \pm 0.04$$

\textbf{First dish of ‘Food for thought’ a.k.a. Homework assignment # 1}

When one calculates $\Delta M(B)$ as a function of the top mass employing the quark box diagram, one finds, see Eq.(34)

$$\Delta M(B) \propto \left( \frac{m_t}{M_W} \right)^2 \quad \text{for} \quad m_t \gg M_W$$

The factor on the right hand side of Eq.(39) for $m_t \ll M_W$ reflects the familiar GIM suppression; yet for $m_t \gg M_W$ it constitutes a (huge) enhancement! It means that a low energy observable, namely $\Delta M(B)$, is controlled more and more by a state or field at asymptotically high scales. Does this not violate decoupling theorems and even common sense? Does it violate decoupling – and if so, why is it allowed to do so – or not?

\textbf{End of Homework # 1}

\footnote{There is also a non-local $\Delta S = 2$ operator generated from the iteration of $\mathcal{L}(\Delta S = 1)$. While it presumably provides a major contribution to $\Delta m_K$, it is not sizeable for $\epsilon_K$ within the KM ansatz, as be inferred from the observation that $|\epsilon'/\epsilon_K| \ll 0.05$.}
While quark box diagrams contribute also to $\Gamma_{12}(\Delta S = 2)$, it would be absurd to assume they are significant. For $\Gamma_K$ is dominated by the impact of hadronic phase space causing $\Gamma(K_{\text{neut}} \to 2\pi) \gg \Gamma(K_{\text{neut}} \to 3\pi)$. Yet even beyond that it is unlikely that such a computation would make much sense: to contribute to $\Delta\Gamma_K$ the internal quark lines in the quark box diagram have to be $u$ and $\bar{u}$ quarks, i.e. lighter than the external quarks $s$ and $\bar{s}$. That means calculating this Feynman diagram does not correspond to integrating out the heavy degrees of freedom. For the same reason (and others as explained later in more detail) computing quark box diagrams tells us little of value concerning $D^0 - \bar{D}^0$ oscillations, since the internal quarks on the leading CKM level – $s$ and $\bar{s}$ – are lighter than the external charm quarks.

A new and more intriguing twist concerning quark box diagrams occurs when addressing $\Delta\Gamma$ for $B^0$ mesons. Those diagrams again do not generate a local operator, since the internal charm quarks carry less than half the mass of the external $b$ quarks. Nevertheless it can be conjectured that the on-shell $\Delta B = 2$ transition operator generating $\Delta\Gamma_B$ is largely shaped by short distance dynamics.

QCD radiative corrections affect the strength of these effective weak transition operators – and create different types of such operators. Consider $\Delta S = 1$ transitions. On the tree graph level there is one operator, namely $(\bar{u}_L\gamma_{\mu}s_L)(\bar{d}_L\gamma^{\mu}u_L)$. Including one-loop diagrams where a gluon is exchanged between quark lines one obtains $O(\alpha_S)$ contributions to the original $(\bar{u}_L\gamma_{\mu}s_L)(\bar{d}_L\gamma^{\mu}u_L)$ operator – and to the new coupling $(\bar{u}_L\gamma_{\mu}t^is_L)(\bar{d}_L\gamma^{\mu}t^iu_L)$, where the $t^i$ denote the generators of colour $SU(3)$. I.e., the two operators $O^{1\times1} = (\bar{u}_L\gamma_{\mu}s_L)(\bar{d}_L\gamma^{\mu}u_L)$ and $O^{8\times8} = (\bar{u}_L\gamma_{\mu}t^is_L)(\bar{d}_L\gamma^{\mu}t^iu_L)$, where the former [latter] represents the product of two colour-singlet[octet] currents, mix under QCD renormalization already on the one-loop level:

\[
(\bar{u}_L\gamma_{\mu}s_L)(\bar{d}_L\gamma^{\mu}u_L) \xrightarrow{\text{QCD 1-loop renormalization}} c_{1\times1} O^{1\times1} + c_{8\times8} O^{8\times8}
\]

with $c_{1\times1} = 1 + O(\alpha_S)$, whereas $c_{8\times8} = O(\alpha_S)$. Since some of these $\alpha_S$ corrections are actually enhanced by numerically sizeable $\log(M_W/\mu)$ factors, they are quite significant. Therefore one wants to identify the multiplicatively renormalized transition operators with

\[
\tilde{O} \xrightarrow{\text{QCD 1-loop renormalization}} \tilde{c} \tilde{O}
\]

This can be done even without brute-force computations by relying on isospin arguments: consider the weak scattering process between quarks

\[
s_L + u_L \rightarrow u_L + d_L
\]

proceeding in an S wave. It can be driven by two $\Delta S = 1$ operators, namely

\[
O_{\pm} = \frac{1}{2} \left[ (\bar{u}_L\gamma_{\mu}s_L)(\bar{d}_L\gamma^{\mu}u_L) \pm (\bar{d}_L\gamma_{\mu}s_L)(\bar{u}_L\gamma^{\mu}u_L) \right]
\]

The operator $O_+ [O_-]$ produces an $ud$ pair in the final state that is [anti]symmetric in isospin and thus carries $I = 1/2[I = 0]$; since the initial $su$ pair carries $I = 1/2$, $O_+ [O_-]$ generates $\Delta I = 1/2 \& 3/2$ [only $\Delta I = 1/2$] transitions.
With QCD conserving isospin, its radiative corrections cannot mix the operators \( O_\pm \), which therefore are multiplicatively renormalized:

\[
O_+ \ [O_-] \xrightarrow{\text{QCD 1-loop renormalization}} c_+ O_+ \ [c_- O_-].
\]  

(44)

and therefore

\[
\mathcal{L}_{\text{eff}}^{(0)}(\Delta S = 1) = O_+ + O_- \xrightarrow{\text{QCD 1-loop renormalization}} \mathcal{L}_{\text{eff}}(\Delta S = 1) = c_+ O_+ + c_- O_- \]  

(45)

with \( c_\pm = 1 + \mathcal{O}(\alpha_S) \).

Integrating out those loops containing a \( W \) line in addition to the gluon line and two quark lines yields terms \( \propto \alpha_S \log(M_W^2/\mu^2) \), which are not necessarily small. Using the renormalization group equation to sum those terms terms one finds on the leading log level

\[
c_\pm = \left[ \frac{\alpha_S(M_W^2)}{\alpha_S(\mu^2)} \right]^{\gamma_\pm}, \quad \gamma_+ = \frac{6}{33 - 2N_F} = -\frac{1}{2} \gamma_-.
\]  

(46)

I.e., \( c_- > 1 > c_+ \), \( c_- c_+^2 = 1 \)

That means that QCD radiative corrections provide a quite sizeable \( \Delta I = 1/2 \) enhancement. Corresponding effects arise for \( \mathcal{L}_{\text{eff}}(\Delta C/B = 1) \).

QCD radiative corrections create yet another effect, namely they lead to the emergence of ‘Penguin’ operators. Without the gluon line their diagram would decompose into two disconnected parts and thus not contribute to a transition operator. These Penguin diagrams can drive only \( \Delta I = 1/2 \) modes. Furthermore in the loop all three quark families contribute; the diagram thus contains the irreducible CKM phase – i.e. it generates direct CP violation in strange decays. Similar effects arise in beauty, but not necessarily in charm decays.

The main message of these more technical considerations was to show that while QCD conserves flavour, it has a highly nontrivial impact on flavour transitions by not only affecting the strength of the bare weak operator, but also inducing new types of weak transition operators already on the perturbative level. In particular, QCD creates a source of direct CP violation in strange decays naturally, albeit with a significantly reduced strength.

### 2.2.2 Nonperturbative Dynamics

Perturbative dynamics does of course not suffice to calculate decays rates for hadrons. Applying the OPE to the description of a transition \( H \rightarrow f \) we need to evaluate on-shell hadronic matrix elements:

\[
T(H \rightarrow f) \propto \langle f|\mathcal{L}_{\text{eff}}|H \rangle \propto \sum_i c_i(\mu)\langle f|O_i(\mu)|H \rangle
\]  

(48)

\(^{10}\)The expressions of Eq.(46) hold in the ‘leading log approximation’; including terms \( \sim \alpha_S^{n+1} \log^n(M_W^2/\mu^2) \) modifies them, yet \( c_- > 1 > c_+ \) and \( c_- c_+^2 \approx 1 \) still hold.
where μ denotes the demarkation line between long and short distance dynamics.

We can call on several allies to take up the challenge of nonperturbative dynamics posed by \( \langle f | O_i(\mu) | H \rangle \).

- **Quark Models**: We can still get considerable mileage out of this ‘old war horse’, if we do not overburden it. They are an excellent tool to train our intuition and arrive at first answers – yet are unsatisfactory for final answers.

- **Chiral Perturbation Theory**: It represents QCD at low energies – yet does not provide a fool proof algorithm.

- **Heavy Quark Theory**: It reflects QCD for heavy flavour hadrons and will be described in more detail later.

- **QCD Sum Rules**: They are genuinely based on QCD, yet there is typically a bound, below which their intrinsic uncertainties cannot be reduced.

- **Lattice QCD (LQCD)**: the perceived panacea.

Let me present just one example, albeit one of central interest. The quantities

\[
\langle K | (\bar{s}_L \gamma_\mu d_L) (\bar{s}_L \gamma^\mu d_L) | \bar{K} \rangle, \quad \langle B | (\bar{b}_L \gamma_\mu d_L) (\bar{b}_L \gamma^\mu d_L) | \bar{B} \rangle
\]

control \( K^0 - \bar{K}^0 \) and \( B_d - \bar{B}_d \) oscillations, respectively, in the SM. One can write:

\[
\langle K | (\bar{s}_L \gamma_\mu d_L) (\bar{s}_L \gamma^\mu d_L) | \bar{K} \rangle = \frac{4}{3} B_K f_K^2 M_K^2
\]

with \( f_K \) denoting the \( K \) meson decay constant. This expression does not entail any loss of generality as long as the parameter \( B_K \) is left open \(^{11}\). The parametrization is convenient since on general grounds one expects \( B_K \sim \mathcal{O}(1) \) based on the following argument.

The Hilbert space of all hadronic states can be spanned by the states with no hadrons – \( |0\rangle \) – and with an increasing number of hadrons – \( |n_{\text{had}}\rangle \). The identity in this Hilbert space can then be written as follows:

\[
1 = |0\rangle \langle 0| + \sum_n |n_{\text{had}}\rangle \langle n_{\text{had}}|
\]

Inserting it into Eq.(49) one obtains

\[
\langle K | (\bar{s}_L \gamma_\mu d_L) (\bar{s}_L \gamma^\mu d_L) | \bar{K} \rangle = \langle K | (\bar{s}_L \gamma_\mu d_L) | 0 \rangle \langle 0 | (\bar{s}_L \gamma^\mu d_L) | \bar{K} \rangle + \sum_n \langle K | (\bar{s}_L \gamma_\mu d_L) | n_{\text{had}} \rangle \langle n_{\text{had}} | (\bar{s}_L \gamma^\mu d_L) | \bar{K} \rangle
\]

\(^{11}\)\( B_K \) is often called the kaon ‘bag factor’. This name goes back to the heydays of the MIT bag model used to calculate a host of hadronic matrix elements. I admit freely and without shame that I have used the MIT bag model myself.
The ‘vacuum saturation’ (VS) or ‘factorization’ approximation consists of assuming that the overall contribution from all nontrivial hadronic intermediate states, i.e. the second line in Eq.(52), is zero or at least small:

\[ \langle K | (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma_\mu d_L) | \bar{K} \rangle_{VA} = \langle K | (\bar{s}_L \gamma_\mu d_L) | 0 \rangle \langle 0 | (\bar{s}_L \gamma_\mu d_L) | \bar{K} \rangle \]

(53)

Since \( \langle 0 | (\bar{s}_L \gamma_\mu d_L) | \bar{K} (p) \rangle = i f_K p_\mu \) we get

\[ \langle K | (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma_\mu d_L) | \bar{K} \rangle_{VA} = \frac{4}{3} f_K^2 M_K^2 \]

(54)

with the colour factor \( \frac{4}{3} = (1 + \frac{1}{N_C}) \) reflecting the two possible quark line contractions. The VS hypothesis \( B_K \simeq 1 \) itself does not assume contributions from individual hadronic intermediate states to be small, only that their sum is smallish, since different \( |n_{had} \rangle \) contribute with alternating signs. This expectation was first supported by analyses based on \( 1/N_C \) arguments, QCD sum rules and later more quantitatively by LQCD:

\[ B_K^{\text{theor.}} = 0.79 \pm 0.04 \pm 0.09 \]

(55)

Some technical subtleties have to be treated properly to arrive at a well defined quantity. One has to keep in mind that when assuming VS, one has to specify at which scale VS is assumed. For contributions that are factorizable at one scale \( \mu_1 \) are in general not purely factorizable at a different scale \( \mu_2 \). This is particularly relevant for \( \langle B | (\bar{b}_L \gamma_\mu d_L)(\bar{b}_L \gamma_\mu d_L) | \bar{B} \rangle \): does one assume VS at a high scale like \( M_B \) and then evolve merely the factorized expression down to ordinary hadronic scales \( \mu \sim 1 \) GeV. Or does one evolve the full expression down to \( \mu \) and assume factorization at this low scale? The two procedure actually yield quite different results; the former actually makes little sense.

2.3 The CKM Paradigm of Large CP Violation in B Decays

2.3.1 Basics

As explained in detail in Prof. Sanda’s lectures here at the school, the decays \( K_L \rightarrow \pi \pi \) are controlled by two types of coherent processes, namely

- \( \Delta S = 2 \) dynamics, which generates the two mass eigenstates \( K_L \) and \( K_S \);
- \( \Delta S = 1 \) reactions that cause the decay of the kaon into a final state without strangeness:

\[ [K^0 \xleftrightarrow{\Delta S=2} \bar{K}^0] \Rightarrow K_L^{\Delta S=1} \rightarrow \pi \pi \]

(56)

Thus there are a priori two classes of gateways, through which CP violation can enter. This is made explicit in the usual notation:

\[
\begin{align*}
\eta_{++,-0} &= \frac{T(K_L \rightarrow \pi^+\pi^-)}{T(K_S \rightarrow \pi^+\pi^-)} \\
\eta_{+-,00} &= \epsilon_K + \epsilon' \\
\eta_{-+,00} &= \epsilon_K - 2\epsilon'
\end{align*}
\]

(57)
η_{+-} or \( \eta_{00} \neq 0 \) constitutes CP violation. More specifically we see that \( \epsilon_K \) parametrizes CP violation common to both decay modes; it thus reflects CP violation in the composition of the decaying state - \( K_L \) - as produced by \( \Delta S = 2 \) forces; \( \epsilon' \) on the other hand differentiates between the two channels \( \pi^+\pi^- \) and \( \pi^0\pi^0 \) and is thus the result of \( \Delta S = 1 \) dynamics. With less than Shakespearean flourish one says \( \epsilon_K \) and \( \epsilon' \) describe indirect and direct CP violation, respectively.

This classification can be extended in a straightforward way to the decays of any hadron carrying flavour quantum number \( F \) like beauty and charm hadrons. CP violation in \( \Delta F = 1 \) and in \( \Delta F = 2 \) transitions are of the ‘direct’ and ‘indirect’ variety, respectively. Indirect CP violation can thus arise only for neutral mesons\(^{12}\). I want to stress the following: the processes \( K_L \rightarrow \pi\pi \) or \( B_d \rightarrow \psi K_S \) involve two phenomenologically distinct dynamics, namely \( \Delta F = 1\&2 \). It is important to deduce from the data, to which degree both contribute to the observed modes – yet in the end an underlying theory has to explain both. One should also note that the ‘superweak’ model is not a theory, actually not even a model – it is merely a convenient classification scheme. For a given theory one has to analyze, whether it is a dynamical implementation – exact or approximate – of a superweak scenario. As explained in detail later on, CKM theory is not of the superweak variety.

### 2.3.2 Prelude: Before 1973

The discovery of P violation in weak decays in 1957 caused great shock in the physics community – yet even the theorists quickly recovered by arguing they had placed unnecessary demands on nature. Look at politics for example: ‘left’ and ‘right’ is often defined – at least on the gut level – in terms of ‘good’ and ‘bad’ or ‘positive’ and ‘negative’. There is a slight problem, though. There is no universal consensus about who the good and bad guys are. Consider pion decay: \( \pi \rightarrow e\nu \). Maximal P violation means that the emerging electrons are purely left-handed. This statement assumes implicitly one is considering the decays of negative pions:

\[
\pi^- \rightarrow e^- (L) \bar{\nu}
\] (58)

The decay of positive pions on the other hand produce purely right-handed positrons:

\[
\pi^+ \rightarrow e^+ (R) \nu
\] (59)

Those two transitions are related by a CP transformation; thus they are equivalent as long as CP is conserved. One can say that a pion of a given charge will produce leptons of only one helicity; yet what one means by ‘left’ depends on the definition of negative charge and vice versa:

\[
'L' = f ('-')
\] (60)

This is like saying: ”The thumb is left on the right hand.” – a correct as well as useless statement since circular. Thus – to address a question raised by Pauli after 1957 – even

\(^{12}\)There is one subtlety to be noted: as explained later, the distinction between direct and indirect CP violation is not always unambiguous in the decays of neutral mesons.
maximal parity violation when coupled with CP symmetry does not signal an absolute preference of nature for ‘left’ – only a correlation of ‘left’ and ‘right’ with the sign of the electric charge. Even Landau, who was a late convert to parity violation made his peace with this situation.

Alas, the ‘fall back’ position was shattered in 1964 by the discovery of CP violation through $K_L \to \pi^+\pi^-$ and subsequently through

$$\Gamma(K_L \to l^+(R)\nu\pi^-) > \Gamma(K_L \to l^-(L)\nu\pi^+) ;$$

(61)

the latter allows to define the sign of electric charge based on data rather than a convention and likewise for the handedness of the charged lepton.

How much this discovery shook the HEP community is best gauged by noting the efforts made to reconcile the observation of $K_L \to \pi^+\pi^-$ with CP invariance:

- To infer that $K_L \to \pi\pi$ implies CP violation one has to invoke the superposition principle of quantum mechanics. One can introduce [11] nonlinear terms into the Schrödinger equation in such a way as to allow $K_L \to \pi^+\pi^-$ with CP invariant dynamics. While completely ad hoc, it is possible in principle. Such efforts were ruled out by further data, most decisively by $\Gamma(K^0(t) \to \pi^+\pi^-) \neq \Gamma(\bar{K}^0(t) \to \pi^+\pi^-)$.

- One can try to emulate the success of Pauli’s neutrino hypothesis. An apparent violation of energy-momentum conservation had been observed in $\beta$ decay $n \to p e^-$, since the electron exhibited a continuous momentum spectrum. Pauli postulated that the reaction actually was

$$n \to p e^- \bar{\nu}$$

(62)

with $\bar{\nu}$ a neutral and light particle that had escaped direct observation, yet let to a continuous spectrum for the electron. I.e., Pauli postulated a new particle – and a most whimsical one at that – to save a symmetry, namely the one under translations in space and time responsible for the conservation of energy and momentum. Likewise it was suggested that the real reaction was

$$K_L \to \pi^+\pi^- U$$

(63)

with $U$ a neutral and light particle with odd intrinsic CP parity. I.e., a hitherto unseen particle with presumably whimsical properties was introduced to save a symmetry. This attempt at evasion was also soon rejected experimentally (see Homework #2). This represents an example of the ancient Roman saying:

"Quod licet Jovi, non licet bovi."
"What is allowed Jupiter, is not allowed a bull."

I.e., we mere mortals cannot get away with speculations like ‘Jupiter’ Pauli.
Homework assignment # 2

What was the conclusive argument to rule out the reaction of Eq.(63) taking place even for a very tiny $U$ mass?

End of Homework # 2

There are more special features for CP violation not shared by P violation:

- Due to CPT invariance – an almost inescapable consequence of local relativistic quantum field theories – CP violation implies a commensurate violation of microscopic time reversal invariance.
- As explained above CP violation allows to define ‘positive’ vs. ‘negative’, ‘left-’ vs. ‘right-’handed, ‘matter’ vs. ‘antimatter’ in a convention independent way.
- If one wants to understand the observed baryon number of the Universe not as an arbitrary initial condition, but as a dynamically generated quantity – similar to the success one has achieved in understanding the abondances of light nuclei in the Universe – three ingredients are needed as pointed out by Sakharov just a year after the discovery of CP violation [12]:
  1. Dynamics that can change baryon number;
  2. CP violation, since otherwise baryon production or destruction is matched by the corresponding processes for antibaryons;
  3. the Universe has to be out of thermal equilibrium, since otherwise CPT invariance acts globally like CP symmetry.

How these ingredients work together and how they can be implemented in specific models is discussed in Prof. Dolgov’s lectures at this school.

- It is the smallest observed violation of a symmetry as characterized by the off-diagonal element in the generalized $K^0 - \bar{K}^0$ mass matrix:

$$\text{Im}M_{12}^K \simeq 1.1 \cdot 10^{-8} \text{eV}^{13} \quad (64)$$

Since a world with CP symmetry is fundamentally different from one without it, such a ‘near miss’ seems peculiar when contrasted with maximal P violation.

The phenomenology of CP violation was quickly developed. Yet, as outlined in Lecture I, the lack of a theory was not realized for a number of years even after the renormalizibility of the $SU(2)_L \times U(1)$ electroweak SM was recognized. The so-called ‘superweak’ model put forward by Wolfenstein already in 1965 is not a theory; it can hardly qualify even as a model – it is basically a classification scheme.

---

13This number is often expressed through the dimensionless ratio $\text{Im}M_{12}^K / M_K \simeq 2.2 \cdot 10^{-17}$; however using the kaon mass as a yardstick is arbitrary, since it is mainly produced by the strong interactions rather than the (super)weak forces behind $M_{12}$. 

26
2.3.3 ‘Growing up’: 1973 – 1994

The early 1970’s marked a turning point culminating in the ‘October revolution of 1974’, the discovery of the $J/\psi$ and $\psi'$. Like any true revolution, it had several events leading up to it, chiefly among them the observation of Bjorken scaling and approximate scale invariance in deep inelastic lepton nucleon scattering and the large cross section for $e^+e^- \rightarrow \text{had}$. Crucial further developments were the realization of QCD being asymptotically free, the discovery of the $\tau$ lepton and in 1976 of the $\Upsilon$ resonance. The latter was for most authors, who had lived through the charmonium revolution, ‘deja vue all over again’. It was readily accepted that hadrons with the new quantum number beauty had to exist with masses $\sim 1/2 M(\Upsilon)$; that they together with $\tau$ leptons presumably are part of a third quark-lepton family and that they decay weakly preferably to charm hadrons with a reduced CKM parameter $V(cb)$ with a typical guestimate $|V(cb)| \sim |V(us)|$.

In 1979 it was predicted that the channels $B_d \rightarrow K^+\pi^-$ should reveal sizable direct CP violation [13] and in 1980 that a host of $B$ decays should exhibit sizable or even large CP asymmetries involving quantum mechanical state mixing and oscillations [14, 15] in particular in the mode $B_d \rightarrow \psi K_S$ [15]. For proper perspective – and not merely for establishing bragging rights for theorists – one should note that these predictions were made, before a single $B$ decay mode had been identified and before their lifetime was measured.

The first indication that the $B$ lifetime is significantly longer and thus $|V(cb)|$ smaller than anticipated came in 1982. It was then confirmed that $B$ mesons live about 1 psec. This pointed to $|V(cb)| \sim \mathcal{O}(\lambda^2)$ with $\lambda = \sin \theta_C$. Together with the expected observation $|V(ub)| \ll |V(cb)|$ and coupled with the assumption of three-family unitarity this allows to expand the CKM matrix in powers of $\lambda$, which yields the following most intriguing result through order $\lambda^5$, as first recognized by Wolfenstein:

$$V_{\text{CKM}} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & -\lambda & A\lambda^3(\rho - i\eta + \frac{i}{2} \eta \lambda^2) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 - i\eta A\lambda^4 & A\lambda^3(1 + i\eta \lambda^2) \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}$$

(65)

The three Euler angles and one complex phase of the representation given in Eq.(17) is taken over by the four real quantities $\lambda$, $A$, $\rho$ and $\eta$; $\lambda$ is the expansion parameter with $\lambda \ll 1$, whereas $A$, $\rho$ and $\eta$ are a priori of order unity, as will be discussed in some detail later on. I.e., the ‘long’ lifetime of beauty hadrons of around 1 psec together with beauty’s affinity to transform itself into charm and the assumption of only three quark families tell us that the CKM matrix exhibits a very peculiar hierarchical pattern in powers of $\lambda$:

$$V_{\text{CKM}} = \begin{pmatrix}
1 & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\
\mathcal{O}(\lambda) & 1 & \mathcal{O}(\lambda^3) \\
\mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1
\end{pmatrix}, \quad \lambda = \sin \theta_C$$

(66)

14By ‘state mixing’ I mean the fact that different states can contribute coherently and by ‘oscillations’ that one pure state can evolve into another pure state in time and evolve back again. The superposition principle of quantum mechanics is central to both effects, yet you can have the former without the latter.
As explained in Lecture I, we know this matrix has to be unitary. Yet in addition it is almost the identity matrix, almost symmetric and the moduli of its elements shrink with the distance from the diagonal. It has to contain a message from nature – albeit in a highly encoded form.

My view of the situation is best described by a poem by the German poet Joseph von Eichendorff from the late romantic period 15:

Schlafen Lied in allen Dingen, There sleeps a song in all things
die da traumen fort und fort, that dream on and on,
und die Welt hebt an zu singen, and the world will start to sing,
findst Du nur das Zauberwort. if you find the magic word.

The six triangle relations obtained from the unitarity condition fall into three categories:

1. $K^0$ triangle:

\[
V^*(ud)V(us) + V^*(cd)V(cs) + V^*(td)V(ts) = \delta_{ds} = 0
\]

\[
\mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5) \tag{67}
\]

$D^0$ triangle:

\[
V^*(ud)V(cd) + V^*(us)V(cs) + V^*(ub)V(cb) = \delta_{uc} = 0
\]

\[
\mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5), \tag{68}
\]

where below each product of matrix elements I have noted their size in powers of $\lambda$. These two triangles are extremely ‘squashed’: two sides are of order $\lambda$, the third one of order $\lambda^5$ and their ratio of order $\lambda^4 \approx 2.3 \times 10^{-3}$; Eq.(67) and Eq.(68) control the situation in strange and charm decays; the relevant weak phases there are obviously tiny.

2. $B_s$ triangle:

\[
V^*(us)V(ub) + V^*(cs)V(cb) + V^*(ts)V(tb) = \delta_{sb} = 0
\]

\[
\mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2) \tag{69}
\]

tc triangle:

\[
V^*(td)V(cd) + V^*(ts)V(cs) + V^*(tb)V(cb) = \delta_{ct} = 0
\]

\[
\mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2) \tag{70}
\]

The third and fourth triangles are still rather squashed, yet less so: two sides are of order $\lambda^2$ and the third one of order $\lambda^4$.

3. $B_d$ triangle:

\[
V^*(ud)V(ub) + V^*(cd)V(cb) + V^*(td)V(tb) = \delta_{db} = 0
\]

\[
\mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) \tag{71}
\]

15I have been told that early romantic writers would have used the term ‘symmetry’ instead of ‘song’. 

9
ut triangle:

\[
V^*(td)V(ud) + \mathcal{O}(\lambda^3) + V^*(ts)V(us) + \mathcal{O}(\lambda^3) + V^*(tb)V(ub) = \delta_{ut} = 0
\]

(72)

The last two triangles have sides that are all of the same order, namely \(\lambda^3\). All their angles are therefore naturally large, i.e. \(\sim \text{several} \times 10\) degrees! Since to leading order in \(\lambda\) one has

\[
V(ud) \simeq V(tb), \ V(cd) \simeq -V(us), \ V(ts) \simeq -V(cb)
\]

(73)

we see that the triangles of Eqs.(71, 72) actually coincide to that order.

The sides of this triangle having naturally large angles are given by \(\lambda \cdot V(cb), V(ub)\) and \(V^*(td)\); these are all quantities that control important aspects of \(B\) decays, namely CKM favoured and disfavoured \(B_{u,d}\) decays and \(B_d - \bar{B}_d\) oscillations. The \(B_d\) triangle of Eq.(71) is usually referred to as ‘the’ CKM unitarity triangle.

Let the reader be reminded that all six triangles, despite their very different shapes, have the same area, see Eq.(21), reflecting the single CKM phase for three families.

Some comments on notation might not be completely useless. The BABAR collaboration and its followers refer to the three angles of the CKM unitarity triangle as \(\alpha, \beta\) and \(\gamma\); the BELLE collaboration instead has adopted the notation \(\phi_1, \phi_2\) and \(\phi_3\). While it poses no problem to be conversant in both languages, the latter has not only historical priority on its side [16], but is also more rational. For the angles \(\phi_i\) in the ‘bd’ triangle of Eq.(71) are always opposite the side defined by \(V^*(id)V(ib)\). Furthermore this classification scheme can readily be generalized to all six unitarity triangles; those triangles can be labeled by \(kl\) with \(k \neq l = d,s,b\) or \(k \neq l = u,c,t\), see Eqs.(67) – (72). Its 18 angles can then be unambiguously denoted by \(\phi_{i}^{(kl)}\); it is the angle in triangle \(kl\) opposite the side \(V^*(ik)V(il)\) or \(V^*(ki)V(li)\), respectively. Therefore I view the notation \(\phi_{i}^{(kl)}\) as the only truly Cartesian one.

For complex phases to become observable, we need two different, yet coherent amplitudes to contribute to the same process. The best and most spectacular implementation of this requirement is provided by \(B^0 - \bar{B}^0\) oscillations. Such oscillations for \(B_d\) mesons were discovered by the ARGUS collaboration [17] in 1986 with \(\Delta M(B_d)/\Gamma(B_d) \simeq 0.75\); (74)

\[
x(B_d) = \Delta M(B_d) / \Gamma(B_d) \simeq 0.75 ;
\]

i.e., the oscillation rate \(\Delta M(B_d)\) and decay rate \(\Gamma(B_d)\) are very close to each other, which is optimal. This observation was the first experimental hint (albeit an indirect one) that

\[\text{ARGUS' discovery came as a surprise, since the size of its signal exceeded most theoretical expectations. In fairness it should be remembered that the discovery of top quarks had been claimed with } m_t < 40\,\text{GeV. Since the observable signal, the ratio of 'wrong' to 'right' sign leptons in semileptonic } B \text{ decays, is given by } x^2/(2 + x^2) \text{ and } x(B_d) \text{ very roughly scales like } m_t^2 \text{ this ratio is enhanced by two orders of magnitude when going from } m_t = 40\,\text{GeV to 160 GeV.}\]
top quarks had to be super-heavy, namely \( m_t > 100 \text{ GeV} \). This is very similar to though less precise than later LEP I findings on \( m_t \). The discovery of \( B_d - \bar{B}_d \) oscillations defined the ‘CKM Paradigm of Large CP Violation in \( B \) Decays’ that had been anticipated in 1980:

- A host of nonleptonic \( B \) channels has to exhibit sizable CP asymmetries.
- For \( B_d \) decays to flavour-nonspecific final states (like CP eigenstates) the CP asymmetries depend on the time of decay in a very characteristic manner; their size should typically be measured in units of 10% rather than 0.1%.
- There is no plausible deniability for the CKM description, if such asymmetries are not found.
- For \( m_t \geq 150 \text{ GeV} \) the SM prediction for \( \epsilon_K \) is dominated by the top quark contribution like \( \Delta M(B_d) \). It thus drops out from their ratio, and \( \sin 2\phi_1 \) can be predicted within the SM irrespective of the (superheavy) top quark mass. In the early 1990’s, i.e., before the direct discovery of top quarks, it was predicted \[18\]

\[
\frac{\epsilon_K}{\Delta M(B_d)} \propto \sin 2\phi_1 \sim 0.6 - 0.7
\]

with values for \( B_B f_B^2 \) inserted as now estimated by LQCD.

- The CP asymmetry in the Cabibbo favoured channels \( B_s \to \psi \phi / \psi \eta \) is Cabibbo suppressed, i.e. below 4%, for reasons very specific to CKM theory, as pointed out already in 1980 \[15\].

In 1974 finally top quarks were observed directly with a mass fully consistent with the indirect estimates given above; the most recent analyses from CDF & D0 list

\[
m_t = 172.7 \pm 2.9 \text{ GeV}
\]  

2.3.4 Data in 1998

CP violation had been observed only in the decays of neutral kaons, and all its manifestations – \( K_L \to \pi^+\pi^- \), \( \pi^0\pi^0 \), \( K^0 \to \pi^+\pi^- \) vs. \( \bar{K}^0 \to \pi^+\pi^- \), \( K_L \to l^+\nu\pi^- \) vs. \( K_L \to l^-\bar{\nu}\pi^+ \) – could be described for 35 years with a single real number, namely \( |\eta_{+-}| \) or \( \Phi(\Delta S = 2) = \arg(M_{12}/T_{12}) \).

There was intriguing, though not conclusive evidence for direct CP violation:

\[
\frac{\epsilon'}{\epsilon_K} = \begin{cases} 
(2.30 \pm 0.65) \cdot 10^{-3} & \text{NA31} \\
(0.74 \pm 0.59) \cdot 10^{-3} & \text{E731}
\end{cases}
\]

These measurements were made in the 1980’s and had been launched by theory estimates suggesting values for \( \epsilon' \) that would be within the reach of these experiments. Theory, however, had ‘moved on’ favouring values \( \leq 10^{-3} \) – or so it was claimed.
2.3.5 The Completion of a Heroic Era

*Direct* CP violation has been unequivocally established in 1999. The present world average dominated by the data from NA48 and KTeV reads as follows [8]:

\[
\langle \epsilon'/\epsilon_K \rangle = (1.63 \pm 0.22) \cdot 10^{-3}
\]

(78)

Quoting the result in this way does not do justice to the experimental achievement, since \(\epsilon_K\) is a very small number itself. The sensitivity achieved becomes more obvious when quoted in terms of actual widths [8]:

\[
\frac{\Gamma(K^0 \rightarrow \pi^+\pi^-) - \Gamma(\bar{K}^0 \rightarrow \pi^+\pi^-)}{\Gamma(K^0 \rightarrow \pi^+\pi^-) + \Gamma(\bar{K}^0 \rightarrow \pi^+\pi^-)} = (5.04 \pm 0.82) \cdot 10^{-6}
\]

(79)

This represents a discovery of the very first rank \(^{17}\). Its significance does not depend on whether the SM can reproduce it or not – which is the most concise confirmation of how important it is. The HEP community can take pride in this achievement; the tale behind it is a most fascinating one about imagination and perseverance. The two groups and their predecessors deserve our respect; they have certainly earned my admiration.

The experimental findings are consistent with CKM theory on the qualitative level, since the latter does not represent a superweak scenario even for strange decays due to the existence of Penguin operators. It is not inconsistent with it even quantitatively. One should keep in mind that within the SM \(\epsilon'/\epsilon_K\) has to be considerably suppressed. \(\epsilon'\) requires interference between \(\Delta I = 1/2 \& 3/2\) amplitudes and is thus reduced by the ‘\(\Delta I = 1/2\) rule’:

\[|T(\Delta I = 3/2)/T(\Delta I = 1/2)| \sim 1/20.\]

Furthermore \(\epsilon'\) is generated by loop diagrams – as is \(\epsilon_K\); yet the top quark mass enhances \(\epsilon_K\) powerlike – \(|\epsilon_K| \propto m_t^2/M_W^2\) – whereas \(\epsilon'\) only logarithmically. When there is only one weak phase – as is the case for CKM theory – one has \(|\epsilon'/\epsilon_K| \propto \log m_t^2/m_b^2\), i.e. greatly reduced again for superheavy top quarks (revisit HW # 1).

CKM theory can go beyond such semiquantitative statements, but one should not expect a precise prediction from it in the near future. For the problem of uncertainties in the evaluation of hadronic matrix elements is compounded by the fact that the two main contributions to \(\epsilon'\) are similar in magnitude, yet opposite in sign [19].

2.3.6 CKM Theory at the End of the 2nd Millenium

It is indeed true that large fractions of the observed values for \(\Delta M_K\), \(\epsilon_K\) and \(\Delta M_B\) and even most of \(\epsilon'\) could be due to New Physics given the limitations in our theoretical control over hadronic matrix elements. Equivalently constraints from these and other data translate into ‘broad’ bands in plots of the unitarity triangle, see Fig.1. The problem with this statement is that it is not even wrong – it misses the real point. Let me illustrate it by a local example first. If you plot the whereabouts of the students at this school on a local map of Varenna, you would find a seemingly broad band; however when you

\(^{17}\)As a consequence of Eq.(79) I am not impressed by CPT tests falling short of the \(10^{-6}\) level.
look at the ‘big’ picture – say a map of Europe – you realize these students are very closely bunched together in one tiny spot on the map. This cannot be by accident, there has to be a good reason for it, which, I hope, is obvious in this specific case of students being concentrated in Varenna. Likewise for the problem at hand: Observables like \( \Gamma(B \to l\nu X_{c,u}) \), \( \Gamma(K \to l\nu\pi) \), \( \Delta M_K \), \( \Delta M_B \), \( \epsilon_K \) and \( \sin 2\phi_1 \) etc. represent very different dynamical regimes that proceed on time scales that span several orders of magnitude. The very fact that CKM theory can accommodate such diverse observables always within a factor two or better and relate them in such a manner that its parameters can be plotted as meaningful constraints on a triangle is highly nontrivial and – in my view – must reflect some underlying, yet unknown dynamical layer. Furthermore the CKM parameters exhibit an unusual hierarchical pattern – |\( V(ud) \)| ~ |\( V(cs) \)| ~ |\( V(tb) \)| ~ 1, |\( V(us) \)| ~ |\( V(cd) \)| ~ \( \lambda \), |\( V(cb) \)| ~ |\( V(ts) \)| ~ \( \mathcal{O}(\lambda^2) \), |\( V(ub) \)| ~ |\( V(td) \)| ~ \( \mathcal{O}(\lambda^3) \) – as do the quark masses culminating in \( m_t \approx 175 \) GeV. Picking such values for these parameters would have been seen as frivolous at best – had they not been forced upon us by (independent) data. Thus I view it already as a big success for CKM theory that the experimental constraints on its parameters can be represented through triangle plots in a meaningful way.

**Interlude: Singing the Praise of Hadronization**

Hadronization and nonperturbative dynamics in general are usually viewed as unwelcome complication, if not outright nuisances. A case in point was already mentioned: while I view the CKM predictions for \( \Delta M_K \), \( \Delta M_B \), \( \epsilon_K \) to be in remarkable agreement with the data, significant contributions from New Physics could be hiding there behind the theoretical uncertainties due to lack of computational control over hadronization. Yet *without* hadronization bound states of quarks and antiquarks will not form; without the existence of kaons \( K^0 - \bar{K}^0 \) oscillations obviously cannot occur. It is hadronization that provides the ‘cooling’ of the (anti)quark degrees of freedom, which allows subtle quantum mechanical effects to add up coherently over macroscopic distances. Otherwise one would
not have access to a super-tiny energy difference \( \text{Im}\mathcal{M}_{12} \sim 10^{-8} \text{eV} \), which is very sensitive to different layers of dynamics, and indirect \( \text{CP} \) violation could not manifest itself. The same would hold for \( B \) mesons and \( B^0 - \bar{B}^0 \) oscillations.

To express it in a more down to earth way:

- Hadronization leads to the formation of kaons and pions with masses exceeding greatly (current) quark masses. It is the hadronic phase space that suppresses the \( \text{CP} \) conserving rate for \( K_L \rightarrow 3\pi \) by a factor \( \sim 500 \), since the \( K_L \) barely resides above the three pion threshold.

- It awards ‘patience’; i.e. one can ‘wait’ for a pure \( K_L \) beam to emerge after starting out with a beam consisting of \( K^0 \) and \( \bar{K}^0 \).

- It enables \( \text{CP} \) violation to emerge in the existence of a reaction, namely \( K_L \rightarrow 2\pi \) rather than an asymmetry; this greatly facilitates its observation.

For these reasons alone we should praise hadronization as the hero in the tale of \( \text{CP} \) violation rather than the villain it is all too often portrayed.

**End of Interlude**

Looking at the present CKM triangle fit shown in Fig. 1 one realizes another triumph of CKM theory appears imminent: if one removed at present the constraints from \( \epsilon_K \) and \( \sin^2 \phi_1 \), i.e. \( \text{CP} \) constraints, then a ‘flat’ CKM ‘triangle’ is barely compatible with the constraints on \( |V(td)| \) from \( \Delta M(B_d) \) and on \( |V(ub)|/|V(cb)| \) from semileptonic \( B \) decays – processes not sensitive to \( \text{CP} \) violation per se. However a measurement of \( \Delta M(B_s) \) through resolving \( B_s - \bar{B}_s \) oscillations in the near future would definitely require this triangle to be nontrivial: ‘\( \text{CP} \) insensitive observables would imply \( \text{CP} \) violation’!

The first unequivocal manifestation of a Penguin contribution surfaced in radiative \( B \) decays, first the exclusive channel \( B \rightarrow \gamma K^* \) and subsequently the inclusive one \( B \rightarrow \gamma X_s \) [27]. These transitions represent flavour changing neutral currents and as such represent a one-loop, i.e. quantum process.

By the end of the second millennium a rich and diverse body of data on flavour dynamics had been accumulated, and CKM theory provided a surprisingly successful description of it. This prompted some daring spirits to perform detailed fits of the CKM triangle to infer a rather accurate prediction for the \( \text{CP} \) asymmetry in \( B_d \rightarrow \psi K_S \):

\[
\sin^2 \phi_1 = 0.72 \pm 0.07
\]  

(80)

### 2.3.7 CKM ‘Exotica’

Here I list three classes of effects that are sensitive to \( \text{CP} \) violation.

**Electric dipole moments:**

The energy shift of a system placed inside a weak electric field can be expressed through an expansion in terms of the components of that field \( \vec{E} \):

\[
\Delta E = d_i E_i + d_{ij} E_i E_j + \mathcal{O}(E^3)
\]  

(81)
The coefficients $d_i$ of the term linear in the electric field form a vector $\vec{d}$, called an electric dipole moment (EDM). For a non-degenerate system – it does not have to be elementary – one infers from symmetry considerations that this vector has to be proportional to that system’s spin:

$$\vec{d} \propto \vec{s}$$  \hspace{1cm} (82)

Yet, since

$$E_i \xrightarrow{T} E_i, \quad s_i \xrightarrow{T} -s_i$$  \hspace{1cm} (83)

under time reversal $T$, a non-vanishing EDM constitutes $T$ violation.

No EDM has been observed yet; the upper bounds of the neutron and electron EDM read as follows [3]:

$$d_N < 5 \cdot 10^{-26} \text{ e cm} \quad [\text{from ultracold neutrons}] \hspace{1cm} (84)$$

$$d_e < 1.5 \cdot 10^{-27} \text{ e cm} \quad [\text{from atomic EDM}] \hspace{1cm} (85)$$

The experimental sensitivity achieved can be illustrated as follows: (i) An neutron EDM of $5 \cdot 10^{-26} \text{ e cm}$ of an object with a radius $r_N \sim 10^{-13} \text{ cm}$ scales to a displacement of about 7 micron, i.e. less than the width of human hair, for an object of the size of the earth. (ii) Expressing the uncertainty in the measurement of the electron’s magnetic dipole moment $\delta((g−2)/2) \sim 10^{-11}$ in analogy to its EDM, one finds a sensitivity level of $\delta(F_2(0)/2m_e) \sim 2 \cdot 10^{-22} \text{ e cm}$ compared to $d_e < 2 \cdot 10^{-26} \text{ e cm}$.

Despite the tremendous sensitivity reached – the tour de force required is nicely described in Prof. Ramsey’s lectures [3] – these numbers are still several orders of magnitude above what is expected in CKM theory:

$$d_{N}^{\text{CKM}} \leq 10^{-30} \text{ e cm}$$  \hspace{1cm} (86)

$$d_{e}^{\text{CKM}} \leq 10^{-36} \text{ e cm}$$  \hspace{1cm} (87)

where in $d_{N}^{\text{CKM}}$ I have ignored any contribution from the strong CP problem. These numbers are so tiny for reasons very specific to CKM theory, namely its chirality structure and the pattern in the quark and lepton masses. Yet New Physics scenarios with right-handed currents, flavour changing neutral currents, a non-minimal Higgs sector, heavy neutrinos etc. are likely to generate considerably larger numbers: $10^{-28} − 10^{-26} \text{ e cm}$ represents a very possible range there quite irrespective of whether these new forces contribute to $\epsilon_K$ or not. This range appears to be within reach in the foreseeable future. It requires tremendous efforts – yet the potential insights to be gained by finding a nonzero EDM somewhere are tremendous.

$\text{Pol}_\perp(\mu)$ in $K_{\mu3}$ decays:

The correlation

$$\text{Pol}_\perp(\mu) \equiv \frac{\langle \vec{s}(\mu) \cdot (\vec{p}(\mu) \times \vec{p}(\pi)) \rangle}{|\vec{p}(\mu) \times \vec{p}(\pi)|}$$  \hspace{1cm} (88)

between momenta $\vec{p}$ and spin $\vec{s}$ for the mode $K \rightarrow \mu^+\nu\pi$ is called $T$ odd or a $T$ odd moment, since it changes sign under time reversal $T$:

$$\vec{s} \rightarrow -\vec{s}, \quad \vec{p} \rightarrow -\vec{p} \implies \text{Pol}_\perp(\mu) \rightarrow -\text{Pol}_\perp(\mu)$$  \hspace{1cm} (89)
It should be noted that \( \text{Pol}_\perp(\mu) \neq 0 \) does not necessarily imply \( T \) violation. In \( K_L \to \mu^+\nu\pi^- \) one expects \( \text{Pol}_\perp(\mu) \leq \mathcal{O}(\alpha_S/\pi) \sim 10^{-3} \) even when \( T \) is conserved due to Coulomb interaction between the two charged particles in the final state.

The reason that a parity odd moment implies \( P \) violation, whereas a non-zero \( T \) odd moment can arise even with \( T \) invariant dynamics, is due to the fact that the \( P \) operator is linear, while \( T \) is antilinear:
\[
T(\alpha|a\rangle) = \alpha^*T|a\rangle
\] (90)

This property of \( T \) is enforced by the commutation relation \([X,P] = i\hbar\), since
\[
T^{-1}[X,P]T = -[X,P]
\] (91)
\[
T^{-1}i\hbar T = -i\hbar
\] (92)

This antilinearity comes into play when the transition amplitude is described through second (or even higher) order in the effective interaction, i.e. when final state interactions are included denoted symbolically by
\[
T^{-1}(\mathcal{L}_{\text{eff}}\Delta t + \frac{i}{2}(\mathcal{L}_{\text{eff}}\Delta t)^2 + ... )T = \mathcal{L}_{\text{eff}}\Delta t - \frac{i}{2}(\mathcal{L}_{\text{eff}}\Delta t)^2 + ... \neq \mathcal{L}_{\text{eff}}\Delta t + \frac{i}{2}(\mathcal{L}_{\text{eff}}\Delta t)^2 + ... 
\] (93)
even for \([T,\mathcal{L}_{\text{eff}}] = 0\).

For \( K^+ \to \mu^+\nu\pi^0 \) on the other hand there are no strong and only highly suppressed electromagnetic final state interactions; \( \text{Pol}_\perp(\mu) \geq 10^{-5} \) represents a genuine \( T \) violation that has to be matched by a commensurate (direct) CP violation. Data show
\[
\text{Pol}_\perp(\mu)(K^+ \to \mu^+\nu\pi^0) = (-1.7 \pm 2.3 \pm 1.1) \cdot 10^{-3} \cdot 10^{-3}.
\] (94)

CKM dynamics can produce merely \( \text{Pol}^{CKM}_\perp(\mu)(K^+ \to \mu^+\nu\pi^0) \sim 10^{-7} \). The effect is so tiny, since interference between helicity changing and conserving amplitudes is needed to induce \( \text{Pol}_\perp(\mu) \); such a contribution is highly suppressed in the SM with its purely left chiral charged currents.

Charged Higgs exchange on the other hand can naturally produce \( \text{Pol}_\perp(\mu) \) through interference with \( W \) exchange.

\( \tau \) Decays:
They provide a very intriguing laboratory to search for CP violation. That would be caused by New Physics except for \( \tau^\pm \to \nu\pi^\pm K_S \), where \( K_S \)’s slight tilt towards antimatter creates a CP asymmetry of \( 3.27 \cdot 10^{-3} \), as explained later [22].

2.4 Summary of Lecture II
Just before the turn of the millenium the situation could be characterized as follows:

- CP violation had been established in 1964 through the observation of
\[
\text{BR}(K_L \to \pi^+\pi^-) = 2.3 \cdot 10^{-3} \neq 0
\] (95)
• $K^- - \bar{K}^0$ oscillations exhibit a commensurate $T$ violation. Its most direct, although not most significant manifestation is given by the ‘Kabir Test’ performed by the CPLEAR collaboration [21]:

$$A_T = \frac{\Gamma(K^0 \rightarrow \bar{K}^0) - \Gamma(\bar{K}^0 \rightarrow K^0)}{\Gamma(K^0 \rightarrow K^0) + \Gamma(\bar{K}^0 \rightarrow \bar{K}^0)} = (6.3 \pm 2.1 \pm 1.8) \cdot 10^{-3} \quad \text{CPLEAR} \quad (96)$$

which is fully consistent with the expectation $A_T = 4\text{Re}\epsilon_K = 6.48 \cdot 10^{-3}$.

• Other manifestations had been found:

- $\text{BR}(K_L \rightarrow l^+\nu\pi^-)/\text{BR}(K_L \rightarrow l^-\nu\pi^+) \approx 1.00654 \neq 1$ (97) again consistent with $1 + 4\text{Re}\epsilon_K = 1.00648$.

- A large $T$ odd moment was found in the rare $K_L$ mode $\text{BR}(K_L \rightarrow \pi^+\pi^-e^+e^-) = (3.32 \pm 0.14 \pm 0.28) \cdot 10^{-7}$: With $\phi$ defined as the angle between the planes spanned by the two pions and the two leptons in the $K_L$ restframe:

$$\phi \equiv \angle(\vec{n}_t, \vec{n}_\pi)$$

one analyzes the decay rate as a function of $\phi$:

$$\frac{d\Gamma}{d\phi} = \Gamma_1\cos^2\phi + \Gamma_2\sin^2\phi + \Gamma_3\cos\phi\sin\phi \quad (99)$$

Since

$$\cos\phi\sin\phi = (\vec{n}_t \times \vec{n}_\pi) \cdot (\vec{p}_{\pi^+} + \vec{p}_{\pi^-})(\vec{n}_t \cdot \vec{n}_\pi)/|\vec{p}_{\pi^+} + \vec{p}_{\pi^-}| \quad (100)$$

one notes that

$$\cos\phi\sin\phi \xrightarrow{T, CP} - \cos\phi\sin\phi \quad (101)$$

under both $T$ and $CP$ transformations; i.e. the observable $\Gamma_3$ represents a $T$ - and $CP$ -odd correlation. It can be projected out by comparing the $\phi$ distribution integrated over two quadrants:

$$A = \frac{\int_0^{\pi/2} d\phi \frac{d\Gamma}{d\phi} - \int_{\pi/2}^{\pi} d\phi \frac{d\Gamma}{d\phi}}{\int_0^{\pi} d\phi \frac{d\Gamma}{d\phi}} = \frac{2\Gamma_3}{\pi(\Gamma_1 + \Gamma_2)} \quad (102)$$

It was first measured by KTEV and then confirmed by NA48:

$$A = (13.8 \pm 2.2)\% \quad (103)$$

$A \neq 0$ is induced by $\epsilon_K$, the $CP$ violation in the $K^0 - \bar{K}^0$ mass matrix, leading to the prediction [23]

$$A = (14.3 \pm 1.3)\% \quad (104)$$

The observed value for the $T$ odd moment $A$ is fully consistent with $T$ violation. Yet $A \neq 0$ by itself does not establish $T$ violation [24].
• Very sensitive searches for direct CP violation had been undertaken with the following measurements:

\[
\text{Re} \frac{\epsilon'}{\epsilon_K} = \begin{cases} 
2.3 \pm 0.7 \cdot 10^{-3} & \text{NA 31} \\
0.6 \pm 0.58 \pm 0.32 \pm 0.18 \cdot 10^{-3} & \text{E 731} 
\end{cases}
\] (105)

While the NA 31 number represented strong evidence for direct CP violation, the E 731 did not. The experimental situation was thus not conclusive.

• An impressive amount of experimental ingenuity, acumen and commitment went into producing this list. We had known since 1964 that CP violation unequivocally exists in nature. After 34 years of dedicated experimentation all its direct manifestations (i.e. ignoring the baryon number of the Universe) could still be characterized by a single non-vanishing quantity:

\[
\text{Im} M_{12} \approx 1.1 \cdot 10^{-8} \text{ eV} \neq 0 \equiv \Phi(\Delta S = 2) \equiv \text{arg} \frac{M_{12}}{\Gamma_{12}} = (6.54 \pm 0.24) \cdot 10^{-3}
\] (106)

• The KM ansatz allows us to incorporate CP violation into the Standard Model. Yet it does not regale us with an understanding. Instead it relates the origins of CP violation to central mysteries of the Standard Model: Why are there families? Why are there three of those? What is underlying the observed pattern in the fermion masses?

Just before the turn of the millenium, in 1999, the situation was clarified, direct CP violation was established in $K_L$ decays on both sides of the Atlantic, at CERN as well as at FNAL with the world average

\[
\text{Re} \frac{\epsilon'}{\epsilon_K} \bigg|_{W A} = (1.63 \pm 0.22) \cdot 10^{-3}
\] (107)

Also other manifestations of CP and T violation have been pursued with great vigour:

\[
\text{Pol}^{K^+}_\perp (\mu) = (-1.85 \pm 3.60) \cdot 10^{-3}
\] (108)

\[
d_N < 12 \cdot 10^{-26} \text{ e cm}
\] (109)

\[
d_{T1} = (1.6 \pm 5.0) \cdot 10^{-24} \text{ e cm} \Rightarrow d_e = (-2.7 \pm 8.3) \cdot 10^{-27} \text{ e cm}
\] (110)

No effect has been observed yet – and CKM theory predicts that none could realistically be ever observed in these cases.

The situation at the beginning of the new millenium could then be sketched as follows:

• The KM ansatz succeeds in accommodating the data in an unforced way: $\epsilon_K$ emerges to be naturally small, $\epsilon'$ naturally tiny (once the huge top mass is accounted for), the EDM’s for neutrons [electrons] naturally (tiny)$^2$ [(tiny)$^3$] etc.
• T violation manifesting itself in $\text{Pol}^{K^+}_{\perp}(\mu) \neq 0$ requires dynamics involving both chirality conserving and violating weak couplings combined with a relative phase between them. $W$ exchange from the SM provides the former, while exchange of a (charged) Higgs can generate the latter. Searching for $\text{Pol}^{K^+}_{\perp}(\mu)$ thus represents a sensitive probe for nonminimal Higgs dynamics and therefore should be encouraged.

• CKM theory can be characterized as follows: One takes a model with a set of mass related basic quantities – fermion masses, CKM parameters – and assigns them values that any sober person would view as frivolous, were those not forced upon us by data, in particular since we have no deeper understanding of mass generation, especially for fermions. One would have little reason to expect success in describing flavour dynamics proceeding in diverse environments on vastly different scales. Yet it did seem to work, at least on a semi-quantitative level.

• Cynics might feel that reproducing a single number after the fact does not pose a stiff challenge to a determined theorist. Yet CKM theory did more. Due to the unexpectedly ‘long’ $B$ meson lifetimes and the huge top quark mass it predicted that a phenomenon that had been tiny in kaon decays, namely $\text{CP}$ violation, had to be close to maximal, i.e 100 %, in certain $B$ decay modes, in particular, but not exclusively, in $B_d \to \psi K_S$. There was, to steal a phrase from the US political scene in the1980’s ‘no plausible deniability’ or following an older tradition: ”Hic Rhodos, hic salta!” for CKM theory.

• The fact that this ‘CKM paradigm of large $\text{CP}$ violation’ exists and can be probed experimentally is due to several favourable factors, whose confluence must be seen as a gift from nature, who had

1. arranged for a huge top quark mass,
2. a ‘long’ $B$ lifetime,
3. the $\Upsilon(4S)$ resonance being above $B\bar{B}$, yet below $BB^*$ thresholds and
4. had presented us previously with charm hadrons, which prompted the development of microvertex detectors with an effective resolution that was needed for $B$ decays.

• One of the central theoretical tools are effective field theories: after writing down the Lagrangian of the ‘true’ theory at some high energy scale $\Lambda_{UV}$, one evolves it down to lower energy scales $\mu$ by integrating out all ‘heavy’ fields, i.e. those with frequencies higher than $\mu$ and retaining only the ‘light’ fields:

$$\mathcal{L}(\Lambda_{UV} \longrightarrow \mathcal{L}_{\text{eff}}(\mu) = \sum_i c_i(\mu, \Lambda_{UV})O_i(\mu),$$

where the local operators $O_i$ are built exclusively from the light fields. The $c$ number coefficients $c_i$ are shaped by the heavy degrees of freedom; thus they provide the gateway for New Physics.
Some of us had concluded that while the observed pattern of the CKM theory strongly points to the existence of a deeper level of dynamics underlying it, its phenomenological successes suggest that one cannot count on New Physics intervening in $B$ decays in a numerically massive manner.

3 Lecture III: CP Violation in $B$ Decays – the ‘Expected’ Triumph of a Very Peculiar Theory

As explained in the previous lecture, within CKM theory one is unequivocally lead to a paradigm of large CP violation in $B$ decays. This realization became so widely accepted that two $B$ factories employing $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ were constructed – one at KEK in Japan and one in Stanford in the US – together with specialized detectors, around which two collaborations gathered, the BELLE andBABAR collaborations, respectively. Details can be find in the talks by Aihara, Lanceri, Giorgi and Hitlin at this school.

3.1 Establishing the CKM Description as a Theory – CP Violation in $B$ Decays

The three angles $\phi_{1,2,3}$ in the CKM unitarity triangle (see Fig.2 for notation) can be determined – or at least probed – through CP asymmetries in the three modes $B_d(t) \rightarrow \psi K_S, \pi^+\pi^-$ and $B_d \rightarrow K^+\pi^-$. Those will be addressed in three acts plus two interludes.

![Figure 2: The CKM Unitarity Triangle](image)

3.1.1 Act 1: $B_d(t) \rightarrow \psi K_S$ and $\phi_1$ (a.k.a. $\beta$)

The first published result on the CP asymmetry in $B_d \rightarrow \psi K_S$ was actually obtained by the OPAL collaboration at LEP I [25]:

$$\sin 2\phi_1 = 3.2^{+1.8}_{-2.0} \pm 0.5,$$

(112)
where the ‘unphysical’ value of $\sin2\phi_1$ is made possible, since a large background subtraction has to be performed. The first value inside the physical range was obtained by CDF [26]:

$$\sin2\phi_1 = 0.79 \pm 0.44 \quad (113)$$

In 2000 the two $B$ factory collaborations BABAR and BELLE presented their first measurements [27]:

$$\sin2\phi_1 = \begin{cases} 
0.12 \pm 0.37 \pm 0.09 & \text{BABAR '00} \\
0.45 \pm 0.44 \pm 0.09 & \text{BELLE '00} 
\end{cases} \quad (114)$$

Already one year later these inconclusive numbers turned into conclusive ones, and the first CP violation outside the $K^0 - \bar{K}^0$ complex was established:

$$\sin2\phi_1 = \begin{cases} 
0.59 \pm 0.14 \pm 0.05 & \text{BABAR '01} \\
0.99 \pm 0.14 \pm 0.06 & \text{BELLE '01} 
\end{cases} \quad (115)$$

By 2003 the numbers from the two experiments had well converged

$$\sin2\phi_1 = \begin{cases} 
0.741 \pm 0.067 \pm 0.03 & \text{BABAR '03} \\
0.733 \pm 0.057 \pm 0.028 & \text{BELLE '03} 
\end{cases} \quad (116)$$

allowing one to state just the world averages, which is actually a BABAR/BELLE average:

$$\sin2\phi_1 = \begin{cases} 
0.726 \pm 0.037 & \text{WA '04} \\
0.685 \pm 0.032 & \text{WA '05} 
\end{cases} \quad (117)$$

The CP asymmetry in $B_d \rightarrow \psi K_S$ is there, is huge and as expected even quantitatively. For CKM fits based on constraints from $|V(ub)/V(cb)|$, $B^0 - \bar{B}^0$ oscillations and – as the only CP sensitive observable – $\epsilon_K$ yield

$$\sin2\phi_1|_{\text{CKM}} = 0.725 \pm 0.065 \quad (118)$$

which is in impressive agreement with the data. This is illustrated by Fig.1 showing these constraints.

Before turning to Acts Two and Three, I will present an Interlude.

3.1.2 Interlude 1: ”Praise the Gods Twice for EPR Correlations”

The BABAR and BELLE analyses are based on a glorious application of quantum mechanics and in particular EPR correlations [28]. The CP asymmetry in $B_d \rightarrow \psi K_S$ had been predicted to exhibit a peculiar dependence on the time of decay, since it involves $B_d - \bar{B}_d$ oscillations in an essential way:

$$\text{rate}(B_d(t)[\bar{B}_d(t)] \rightarrow \psi K_S) \propto e^{-t/\tau_B}(1 - [+]-A\sin\Delta m_B t) \quad (119)$$

At first it would seem that an asymmetry of the form given in Eq.(119) could not be measured for practical reasons. For in the reaction

$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_d\bar{B}_d \quad (120)$$
the point where the $B$ meson pair is produced is ill determined due to the finite size of the
electron and positron beam spots: the latter amounts to about 1 mm in the longitudinal
direction, while a $B$ meson typically travels only about a quarter of that distance before
it decays. It would then seem that the length of the flight path of the $B$ mesons is poorly
known and that averaging over this ignorance would greatly dilute or even eliminate the
signal.

It is here where the existence of an EPR correlation comes to the rescue. While the
two $B$ mesons in the reaction of Eq.(120) oscillate back and forth between a $B_d$ and $\bar{B}_d$,
they change their flavour identity in a completely correlated way. For the $BB\bar{B}$ pair forms
a $C$ odd state; Bose statistics then tells us that there cannot be two identical flavour
hadrons in the final state:

$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_d\bar{B}_d \not\leftrightarrow B_dB_d, \bar{B}_d\bar{B}_d$$

(121)

Once one of the $B$ mesons decays through a flavour specific mode, say $B_d \rightarrow l^+\nu X$
[$\bar{B}_d \rightarrow l^-\bar{\nu}X$], then we know unequivocally that the other $B$ meson was a $\bar{B}_d$ [$B_d$] at that
time. The time evolution of $\bar{B}_d(t)|B_d(t)| \rightarrow \psi K_S$ as described by Eq.(119) starts at that
time as well; i.e., the relevant time parameter is the interval between the two times of
decay, not those times themselves. That time interval is related to – and thus can be
inferred from – the distance between the two decay vertices, which is well defined and can
be measured.

The great practical value of the EPR correlation is instrumental for another consid-
eration as well, namely how to see directly from the data that $CP$ violation is matched
by $T$ violation. Fig.3 shows two distributions, one for the interval $\Delta t$ between the times
of decays $B_d \rightarrow l^+X$ and $\bar{B}_d \rightarrow \psi K_S$ and the other one for the $CP$ conjugate process
$\bar{B}_d \rightarrow l^-X$ and $B_d \rightarrow \psi K_S$. They are clearly different proving that $CP$ is broken. Yet they
show more: the shape of the two distributions is actually the same (within experimental
uncertainties) the only difference being that the average of $\Delta t$ is positive for ($l^-X)_B(\psi K_S)$
and negative for ($l^+X)_B(\psi K_S)$ events. I.e., there is a (slight) preference for $B_d \rightarrow \psi K_S$
[$\bar{B}_d \rightarrow \psi K_S$] to occur after [before] and thus more [less] slowly (rather than just more
rarely) than $\bar{B} \rightarrow l^-X$ [$B \rightarrow l^+X$]. Invoking $CPT$ invariance merely for semileptonic
$B$ decays – yet not for nonleptonic transitions – synchronizes the starting point of the
$B$ and $\bar{B}$ decay ‘clocks’, and the EPR correlation keeps them synchronized. We thus see
that $CP$ and $T$ violation are ‘just’ different sides of the same coin. As explained above,
EPR correlations are essential for this argument!

The reader can be forgiven for feeling that this argument is of academic interest only,
since $CPT$ invariance of all processes is based on very general arguments. Yet the main
point to be noted is that EPR correlations, which represent some of quantum mechanics’
most puzzling features, serve as an essential precision tool, which is routinely used in these
measurements. I feel it is thus inappropriate to refer to EPR correlations as a paradox.

3.1.3 Act 2: $B_d(t) \rightarrow \pi^+\pi^-$ and $\phi_2$ (a.k.a. $\alpha$)

The situation is theoretically more complex than for $B_d(t) \rightarrow \psi K_S$ due to two reasons:
While both final states $\pi\pi$ and $\psi K_S$ are CP eigenstates, the former unlike the latter is not reached through an isoscalar transition. The two pions can form an $I = 0$ or $I = 2$ configuration (similar to $K \rightarrow 2\pi$), which in general will be affected differently by the strong interactions.

For all practical purposes $B_d \rightarrow \psi K_S$ is described by two tree diagrams representing the two effective operators $(\bar{c}_L \gamma_\mu b_L)(\bar{s}_L \gamma_\mu c_L)$ and $(\bar{c}_L \gamma_\mu \lambda_i b_L)(\bar{s}_L \gamma_\mu \lambda_i c_L)$ with the $\lambda_i$ representing the $SU(3)_C$ matrices. Yet for $B \rightarrow \pi\pi$ we have effective operators $(\bar{d}_L \gamma_\mu \lambda_i b_L)(\bar{q}\gamma_\mu \lambda_i q)$ generated by the Cabibbo suppressed Penguin loop diagrams in addition to the two tree operators $(\bar{u}_L \gamma_\mu b_L)(\bar{d}_L \gamma_\mu u_L)$ and $(\bar{u}_L \gamma_\mu \lambda_i b_L)(\bar{d}_L \gamma_\mu \lambda_i u_L)$.

This greater complexity manifests itself already in the phenomenological description of the time dependent CP asymmetry:

$$\frac{R_+ (\Delta t) - R_- (\Delta t)}{R_+ (\Delta t) + R_- (\Delta t)} = S \sin(\Delta M_d \Delta t) - C \cos(\Delta M_d \Delta t), \quad S^2 + C^2 \leq 1 \quad (122)$$

Figure 3: The observed decay time distributions for $B^0$ (red) and $\bar{B}^0$ (blue) decays.
where $R_{+[-]}(\Delta t)$ denotes the rate for $B^{\text{tag}}(t)\bar{B}_d(t+\Delta)[B^{\text{tag}}(t)B_d(t+\Delta)]$ and

$$S = \frac{2\text{Im} \rho_{\pi^+\pi^-}}{1 + \left|\rho_{\pi^+\pi^-}\right|^2}, \quad C = \frac{1 - \left|\rho_{\pi^+\pi^-}\right|^2}{1 + \left|\rho_{\pi^+\pi^-}\right|^2}$$

(123)

As before, due to the EPR correlation between the two neutral $B$ mesons, it is the relative time interval $\Delta t$ between the two $B$ decays that matters, not their lifetime. The new feature is that one has also a cosine dependence on $\Delta t$.

$C \neq 0$ obviously represents direct CP violation. Yet what is often overlooked is that also $S$ can reveal such CP violation. For if one studies $B_d$ decays into two CP eigenstates $f_a$ and $f_b$ and finds

$$S(f_a) \neq \eta(f_a)\eta(f_b)S(f_b)$$

(124)

with $\eta$ denoting the CP parities of $f_i$, then one has established direct CP violation. For the case under study that means even if $C(\pi\pi) = 0$, yet $S(\pi^+\pi^-) = -S(\psi K_S)$, one has observed unequivocally direct CP violation. One should note that such direct CP violation might not necessarily induce $C \neq 0$. For the latter requires, as explained in Prof. Sanda’s lectures and below in Sect. 3.1.5 (see Eq.(133)), that two different amplitudes contribute coherently to $B_d \to f_b$ with non-zero relative weak as well as strong phases. $S(f_a) \neq \eta(f_a)\eta(f_b)S(f_b)$ on the other hand only requires that the two overall amplitudes for $B_d \to f_a$ and $B_d \to f_b$ possess a relative phase. This can be illustrated with a familiar example from CKM dynamics: If there were no Penguin operators for $B_d \to \pi^+\pi^-$ (or it could be ignored quantitatively), one would have $C(\pi^+\pi^-) = 0$, yet at the same time $S(\psi K_S) = \sin(2\phi_1)$ together with $S(\pi^+\pi^-) = \sin(2\phi_2)$.

BELLE finds

$$S = -0.67 \pm 0.16 \pm 0.06, \quad C = 0.56 \pm 0.12 \pm 0.06,$$

(125)

which amounts to a 5.2 $\sigma$ CP asymmetry and direct CP violation with 3.3 $\sigma$ significance. For without direct CP violation one would have to find $C = 0$ and $S = -\sin 2\phi_1$ [29]. However this signal has so far not been confirmed by BABAR, which has found $S = -0.30 \pm 0.17 \pm 0.03, C = -0.09 \pm 0.15 \pm 0.04$. Future data will hopefully clarify this unsettled situation.

Once this has been settled, one can take up the challenge of extracting a value for $\phi_2$ from the data \(^{18}\). This can be done in a model independent way by analyzing $B_d(t) \to \pi^+\pi^-, \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ transitions and performing an isospin decomposition. For the Penguin contribution cannot affect $B_d(t) \to [\pi\pi]_{I=2}$ modes. Unfortunately there is a serious experimental bottle neck, namely to study $B_d(t) \to \pi^0\pi^0$ with sufficient accuracy [27]. Therefore alternative decays have been suggested, in particular $B \to \rho\pi$ and $\rho\rho$. I will comment on them later.

\(^{18}\)The complications due to the presence of the Penguin contribution are all too often referred to as ‘Penguin pollution’. Personally I find it quite unfair to blame our lack of theoretical control on water fowls rather than on the guilty party, namely us.
3.1.4 Act 3: $B_d \to K^+\pi^-$

It was pointed out in a seminal paper [30] that (rare) transitions like $\bar{B}_d \to K^-\pi^+$’s have the ingredients for sizeable direct $\text{CP}$ asymmetries:

- Two different amplitudes can contribute coherently, namely the highly CKM suppressed tree diagram with $b \to u\bar{u}s$ and the Penguin diagram with $b \to s\bar{q}q$.
- The tree diagram contains a large weak phase from $V(u\bar{b})$.
- The Penguin diagram with an internal charm quark loop exhibits an imaginary part, which can be viewed at least qualitatively as a strong phase generated by the production and subsequent annihilation of a $c\bar{c}$ pair (the diagram with an internal $u$ quark loop acts merely as a subtraction point allowing a proper definition of the operator).
- While the Penguin diagram with an internal top quark loop is actually not essential, the corresponding effective operator can be calculated quite reliably, since integrating out first the top quarks and then the $W$ boson leads to a truly local operator. Determining its matrix elements is however another matter.

To translate these features into accurate numbers represents a formidable task, we have not mastered yet. In Ref. [31] an early and detailed effort was made to treat $\bar{B}_d \to K^-\pi^+$ theoretically with the following results:

$$\text{BR}(\bar{B}_d \to K^-\pi^+) \sim 10^{-5}, \quad A_{\text{CP}} \sim -0.10$$ (126)

Those numbers turn out to be rather prescient, since they are in gratifying agreement with the data

$$\text{BR}(\bar{B}_d \to K^-\pi^+) = (1.85 \pm 0.11) \cdot 10^{-5}$$

$$A_{\text{CP}} = \begin{cases} 
-0.133 \pm 0.030 \pm 0.009 & \text{BABAR} \\
-0.113 \pm 0.021 & \text{BELLE} 
\end{cases}$$ (127)

Cynics might point out that the authors in [31] did not give a specific estimate of the theoretical uncertainties in Eq.(126). More recent authors have been more ambitious with somewhat mixed success. I list the predictions inferred from pQCD [32] and QCD Factorization [33] and the data for the three modes $\bar{B}_d \to K^-\pi^+$ and $B^- \to K^-\pi^0$, $\bar{K}^0\pi^-$:

$$A_{\text{CP}}(B_d \to K^-\pi^+) = \begin{cases} 
-0.133 \pm 0.030 \pm 0.009 & \text{BABAR} \\
-0.113 \pm 0.021 & \text{BELLE} \\
-0.09^{+0.05+0.04}_{-0.08-0.06} & \text{pQCD} \\
+0.05 \pm 0.09 & \text{QCD Fact.} 
\end{cases}$$ (128)

$$A_{\text{CP}}(B^- \to K^-\pi^0) = \begin{cases} 
+0.06 \pm 0.06 \pm 0.01 & \text{BABAR} \\
+0.04 \pm 0.04 \pm 0.02 & \text{BELLE} \\
-0.01^{+0.03+0.03}_{-0.05-0.05} & \text{pQCD} \\
+0.07 \pm 0.09 & \text{QCD Fact.} 
\end{cases}$$ (129)
\[ A_{\text{CP}}(B^- \rightarrow \bar{K}^0\pi^-) = \begin{cases} 
-0.09 \pm 0.05 \pm 0.01 & \text{BABAR} \\
+0.05 \pm 0.05 \pm 0.01 & \text{BELLE} \\
+0.00 & \text{pQCD} \\
+0.01 \pm 0.01 & \text{QCD Fact.} 
\end{cases} \]

(130)

3.1.5 Interlude 2: On Final State Interactions and CPT Invariance

Due to CPT invariance CP violation can be implemented only through a complex phase in some effective couplings. For it to become observable two different, yet coherent amplitudes have to contribute to an observable. There are two types of scenarios for implementing this requirement:

1. When studying a final state \( f \) that can be reached by a \( \Delta B = 1 \) transition from \( B^0 \) as well as \( B^0 \), then \( B^0 - \bar{B}^0 \) oscillations driven by \( \Delta B = 2 \) dynamics provide the second amplitude, the weight of which varies with time.

2. Two different \( \Delta B = 1 \) amplitudes \( M_{a,b} \) of fixed ratio – distinguished by, say, their isospin content – exist leading coherently to the same final state:

\[ T(B \rightarrow f) = \lambda_a M_a + \lambda_b M_b \]

(131)

I have factored out the weak couplings \( \lambda_{a,b} \) while allowing the amplitudes \( M_{a,b} \) to be still complex due to strong or electromagnetic FSI. For the CP conjugate reaction one has

\[ T(\bar{B} \rightarrow \bar{f}) = \lambda_a^* M_a + \lambda_b^* M_b \]

(132)

It is important to note that the reduced amplitudes \( M_{a,b} \) remain unchanged, since strong and electromagnetic forces conserve CP. Therefore we find

\[ \Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f) = \frac{2\text{Im}\lambda_a \lambda_b^* \cdot \text{Im}M_a M_b^*}{|\lambda_a|^2|M_a|^2 + |\lambda_b|^2|M_b|^2 + 2\text{Re}\lambda_a \lambda_b^* \cdot \text{Re}M_a M_b^*} \]

(133)

i.e. for a CP asymmetry to become observable, two conditions have to satisfied simultaneously irrespective of the underlying dynamics:

- \( \text{Im}\lambda_a \lambda_b^* \neq 0 \), i.e. there has to be a relative phase between the weak coulings \( \lambda_{a,b} \).

- \( \text{Im}M_a M_b^* \neq 0 \), i.e. final state interactions (FSI) have to induce a phase shift between \( M_{a,b} \).

While this is well known, it is often not fully appreciated that CPT invariance places constraints on the phases of the \( M_{a,b} \). For it implies much more than equality of masses and lifetimes of particles and antiparticles. It tells us that the widths for sub-classes of transitions for particles and antiparticles have to coincide already, either identically or at least practically. Just writing down strong phases in an equation like Eq.(131) does not automatically satisfy CPT constraints.

I will illustrate this feature first with two simple examples and then express it in more general terms.

45
• CPT invariance already implies $\Gamma(K^- \to \pi^-\pi^0) = \Gamma(K^+ \to \pi^+\pi^0)$ up to small electromagnetic corrections, since in that case there are no other channels it can rescatter with.

• While $\Gamma(K^0 \to \pi^+\pi^-) \neq \Gamma(\bar{K}^0 \to \pi^+\pi^-)$ and $\Gamma(K^0 \to \pi^0\pi^0) \neq \Gamma(\bar{K}^0 \to \pi^0\pi^0)$ one has $\Gamma(K^0 \to \pi^+\pi^- + \pi^0\pi^0) = \Gamma(\bar{K}^0 \to \pi^+\pi^- + \pi^0\pi^0)$.

• Let us now consider a scenario where a particle $P$ and its antiparticle $\bar{P}$ can each decay into two final states only, namely $a,b$ and $\bar{a},\bar{b}$, respectively [34, 35]. Let us further assume that strong (and electromagnetic) forces drive transitions among $a$ and $b$ – and likewise for $\bar{a}$ and $\bar{b}$ – as described by an $S$ matrix $S$. The latter can then be decomposed into two parts

$$S = S^{diag} + S^{off-diag},$$  \hspace{1cm} (134)

where $S^{diag}$ contains the diagonal transitions $a \Rightarrow a, b \Rightarrow b$

$$S^{diag}_{ss} = e^{2i\delta_s}, s = a,b$$  \hspace{1cm} (135)

and $S^{off-diag}$ the off-diagonal ones $a \Rightarrow b, b \Rightarrow a$:

$$S^{off-diag}_{ab} = 2i T^{resc}_{ab} e^{i(\delta_a + \delta_b)}$$  \hspace{1cm} (136)

with

$$T^{resc}_{ab} = T^{resc}_{ba} = (T^{resc}_{ab})^*,$$  \hspace{1cm} (137)

since the strong and electromagnetic forces driving the rescattering conserve CP and T. The resulting $S$ matrix is unitary to first order in $T^{resc}_{ab}$. CPT invariance implies the following relation between the weak decay amplitude of $\bar{P}$ and $P$:

$$T(P \to a) = e^{i\delta_a} [T_a + T_b T^{resc}_{ab}]$$  \hspace{1cm} (138)

$$T(\bar{P} \to \bar{a}) = e^{i\delta_a} [T^*_a + T^*_b i T^{resc}_{ab}]$$  \hspace{1cm} (139)

and thus

$$\Delta\gamma(a) \equiv |T(\bar{P} \to \bar{a})|^2 - |T(P \to a)|^2 = 4T^{resc}_{ab} \text{Im} T^*_a T_b;$$  \hspace{1cm} (140)

likewise

$$\Delta\gamma(b) \equiv |T(\bar{P} \to \bar{b})|^2 - |T(P \to b)|^2 = 4T^{resc}_{ab} \text{Im} T^*_b T_a$$  \hspace{1cm} (141)

and therefore as expected

$$\Delta\gamma(b) = -\Delta\gamma(b)$$  \hspace{1cm} (142)

Some further features can be read off from Eq.(140):

1. If the two channels that rescatter have comparable widths – $\Gamma(P \to a) \sim \Gamma(P \to b)$ – one would like the rescattering $b \leftrightarrow a$ to proceed via the usual strong forces; for otherwise the asymmetry $\Delta\Gamma$ is suppressed relative to these widths by the electromagnetic coupling.
2. If on the other hand the channels command very different widths – say $\Gamma(P \to a) \gg \Gamma(P \to b)$ – then a large relative asymmetry in $P \to b$ is accompanied by a tiny one in $P \to a$.

This simple scenario can easily be extended to two sets $A$ and $B$ of final states s.t. for all states $a$ in set $A$ the transition amplitudes have the same weak coupling and likewise for states $b$ in set $B$. One then finds

$$\Delta \gamma(a) = 4 \sum_{b \in B} T_{ab}^{resc} \text{Im} T_a^* T_b$$

The sum over all CP asymmetries for states $a \in A$ cancels the corresponding sum over $b \in B$:

$$\sum_{a \in A} \Delta \gamma(a) = 4 \sum_{b \in B} T_{ab}^{resc} \text{Im} T_a^* T_b = - \sum_{b \in B} \Delta \gamma(b)$$

These considerations tell us that the CP asymmetry averaged over certain classes of channels defined by their quantum numbers has to vanish. Yet these channels can still be very heterogenous, namely consisting of two- and quasi-two-body modes, three-body channels and other multi-body decays. Hence we can conclude:

- If one finds a direct CP asymmetry in one channel, one can infer – based on rather general grounds – which other channels have to exhibit the compensating asymmetry as required by CPT invariance. Observing them would enhance the significance of the measurements very considerably.

- Typically there can be several classes of rescattering channels. The SM weak dynamics select a subclass of those where the compensating asymmetries have to emerge. QCD frameworks like generalized factorization can be invoked to estimate the relative weight of the asymmetries in the different classes. Analyzing them can teach us important lessons about the inner workings of QCD.

- If New Physics generates the required weak phases (or at least contributes significantly to them), it can induce rescattering with novel classes of channels. The pattern in the compensating asymmetries then can tell us something about the features of the New Physics involved.

I want to end this Interlude by adding that Penguins (diagrams) are rather smart beings: they know about these CPT constraints. For when one considers the imaginary parts of the Penguin diagrams, which are obtained by cutting the internal quark lines, namely the up and charm quarks (top quarks do not contribute there, since they cannot reach their mass shell in $b$ decays), one realizes that CP asymmetries in $B \to K + \pi$’s are compensated by those in $B \to D\bar{D}_s + \pi$’s ,...
3.2 On Measuring other CP Observables in $B$ Decays

An obvious, yet still useful criterion for CP observables is that they must be ‘re-phasing’ invariant under $|\bar{B}^0\rangle \rightarrow e^{-i\xi}|\bar{B}^0\rangle$. There are three classes of such observables:

1. $|T(B \rightarrow f)| \neq |T(\bar{B} \rightarrow \bar{f})|$;
   it reflects pure $\Delta B = 1$ dynamics and thus amounts to direct CP violation.
2. $|q| \neq |p|$;
   it requires CP violation in $\Delta B = 2$ dynamics.
3. $\text{Im} \frac{q}{p} \bar{\rho}(f) \neq 0$, $\bar{\rho}(f) = \frac{T(\bar{B} \rightarrow f)}{T(B \rightarrow \bar{f})}$\(^{19}\);
   such an effect requires the interplay of $\Delta B = 1$ & $\Delta B = 2$ forces.

Decays of beauty baryons and of charged $B$ mesons can realize only the first scenario, whereas semileptonic transitions $\bar{B}^0 \rightarrow l^+X$ vs. $B^0 \rightarrow l^-X$ only the second one due to the SM’s $\Delta B = \Delta Q_l$ selection rule.

The third scenario with its interplay of direct and indirect CP violation requires a nonleptonic mode. As already stated for the example $B \rightarrow \pi^+\pi^-$ one has in general
\[ A_{\text{CP}}(t) = S \sin(\Delta M_B t) + C \cos(\Delta M_B t) . \] (145)
with the following features:

- \[ C = \frac{1 - |(q/p)\bar{\rho}(f_{\text{CP}})|^2}{1 + |(q/p)\bar{\rho}(f_{\text{CP}})|^2} \neq 0 \] (146)
  unambiguously reflects direct CP violation.
- On the other hand the interpretation of
  \[ S = \frac{2\text{Im}(q/p)\bar{\rho}(f_{\text{CP}})}{1 + |(q/p)\bar{\rho}(f_{\text{CP}})|^2} \neq 0 \] (147)
  requires some subtlety:
  - As long as it is observed in a single channel only, there is no true distinction between direct and indirect CP violation. For changing the phase convention of quark fields allows to shift the observed phase from the $\Delta B = 2$ to the $\Delta B = 1$ sector and vice versa.
  - Once it is seen in at least two channels $f_a$ and $f_b$ of the same $B^0$, the verdict becomes clearer. If one finds
    \[ S(f_a) \neq \eta(f_a)\eta(f_b)S(f_b) \] (148)

\(^{19}\)This condition is formulated for the simplest case of $f$ being a CP eigenstate.
with $\eta_i$ denoting the CP parity of the final states $f_i$, then one has established the presence also of direct CP violation. One should that such a manifestation of direct CP violation might not surface through $C(f_i) \neq 0$. For the latter requires, as explained above, the presence of two different, yet still coherent transitions in $B \to f_i$; this might not be the case.

3.2.1 $\phi_2$ from CP Violation in $B_d \to$ Multi-Pions

In Sect. 3.1.3 I have already mentioned the principal as well as practical complications in determining $\phi_2$ from $B \to \pi\pi$. An isospin decomposition can be undertaken also in $B \to 3\pi$ and $4\pi$ to disentangle the Penguin contribution.

- $B_{d,u} \to \rho\pi$

These channels are less challenging experimentally than $B_{d,u} \to 2\pi$, yet they pose some complex theoretical problems. For going from the experimental starting point $B \to 3\pi$ to $B \to \pi\rho$ configurations is quite nontrivial. There are other contributions to the three-pion final state like $\sigma\pi$, and cutting on the dipion mass provides a rather imperfect filter due to the large $\rho$ width. It hardly matters in this context whether the $\sigma$ is a bona fide resonance or some other dynamical enhancement. This actually leads to a further complication, namely that the $\sigma$ structure cannot be described adequately by a Breit-Wigner shape. As analyzed first in [36] and then in more detail in [37] ignoring such complications can induce a systematic uncertainty in the extracted value of $\phi_2$.

- $B_{d,u} \to \rho\rho$

These transitions while providing better rates experimentally, contain even more theoretical complexities, since they have to be extracted from $B \to 4\pi$ final states, where one has to allow for $\sigma\rho, 2\sigma, \rho 2\pi$ etc. in addition to $2\rho$.

My point here is one of caution rather than of agnosticism. The concerns sketched above might well be more academic than practical with the present statistics. My main conclusions are the following: (i) I remain unpersuaded that averaging over the values for $\phi_2$ obtained so far from the three methods listed above provides a reliable value, since I do not think that the systematic uncertainties have sufficiently been analyzed. (ii) It will be mandatory to study those in a comprehensive way, before we can make full use of the even larger data sets that will become available in the next few years. As I have emphasized repeatedly, our aim has to be to reduce the uncertainty down to at least the 5% level in a way that can be defended. (iii) In the end we will need

- to perform time dependent Dalitz plot analyses (and their generalizations) and
• involve the expertise that already exists or can obtained concerning low-energy hadronization processes like final state interactions among low energy pions and kaons; valuable information can be gained on those issues from \( D_{(s)} \rightarrow \pi \)'s, kaons etc. as well as \( D_{(s)} \rightarrow l\nu K\pi/\pi\pi/KK \), in particular when analyzed with state-of-the-art tools of chiral dynamics.

3.2.2 \( \phi_3 \) from \( B^+ \rightarrow D_{\text{neut}}K^+ \) vs. \( B^- \rightarrow D_{\text{neut}}K^- \)

As first mentioned in 1980 [38], then explained in more detail in 1985 [39] and further developed in [40], the modes \( B^\pm \rightarrow D_{\text{neut}}K^\pm \) should exhibit direct CP violation driven by the angle \( \phi_3 \), if the neutral \( D \) mesons decay to final states that are common to \( D_0 \) and \( \bar{D}_0 \). Based on simplicity the original idea was to rely on two-body modes like \( K_S\pi^0 \), \( K^+K^- \), \( \pi^+\pi^- \), \( K^\pm\pi^\mp \). One drawback of that method are the small branching ratios and low efficiencies.

A new method was pioneered by BELLE and then implemented also by BABAR, namely to employ \( D_{\text{neut}} \rightarrow K_S\pi^+\pi^- \) and perform a full Dalitz plot analysis. This requires a very considerable analysis effort – yet once this initial investment has been made, it will pay handsome profit in the long run. For obtaining at least a decent description of the full Dalitz plot population provides considerable cross checks concerning systematic uncertainties and thus a high degree of confidence in the results. BELLE and BABAR find [27]:

\[
\phi_3 = \begin{cases} 
68^\circ \pm 15^\circ (\text{stat}) \pm 13^\circ (\text{syst}) \pm 11^\circ (\text{model}) & \text{BELLE} \\
79^\circ \pm 31^\circ (\text{stat}) \pm 12^\circ (\text{syst}) \pm 14^\circ (\text{model}) & \text{BABAR}
\end{cases}
\] (151)

I view it still a pilot study, yet a most promising one. It exemplifies how the complexities of hadronization can be harnessed to establish confidence in the accuracy of our results. I consider this to be the way of the future.

3.2.3 \( \phi_1 \) from CP Violation in \( B_d \rightarrow 3 \) Kaons

Analysing CP violation in \( B_d \rightarrow \phi K_S \) decays is a most promising way to search for New Physics. For the underlying quark-level transition \( b \rightarrow s\bar{s}s \) represents a pure loop-effect in the SM, it is described by a single \( \Delta B = 1 \& \Delta I = 0 \) operator (a ‘Penguin’), a reliable SM prediction exists for it [41] – \( \sin 2\phi_1(B_d \rightarrow \psi K_S) \approx \sin 2\phi_1(B_d \rightarrow \phi K_S) \) – and the \( \phi \) meson represents a narrow resonance.

Great excitement was created when BELLE reported a large discrepancy between the predicted and observed CP asymmetry in \( B_d \rightarrow \phi K_S \) in the summer of 2003:

\[
\sin 2\phi_1(B_d \rightarrow \phi K_S) = \begin{cases} 
-0.96 \pm 0.5 \pm 0.10 & \text{BELLE} \\
0.45 \pm 0.43 \pm 0.07 & \text{BABAR}
\end{cases}
\] (152)

Based on more data taken, this discrepancy has shrunk considerably: the BABAR/BELLE average for 2005 yields [27]

\[
\sin 2\phi_1(B_d \rightarrow \psi K_S) = 0.685 \pm 0.032
\] (153)
compared to
\[
\sin^2 \phi_1(B_d \to \phi K_S) = \begin{cases} 
0.50 \pm 0.25^{+0.07}_{-0.04} & \text{BABAR} \\
0.44 \pm 0.27 \pm 0.05 & \text{BELLE}
\end{cases} ;
\]

BABAR’s as well as BELLE’s numbers are below the prediction, albeit by one sigma only. It is ironic that such a smaller deviation, although not significant, is actually more believable as signaling an incompleteness of the SM than the large one originally reported by BELLE.

This issue has to be pursued with vigour, since this reaction provides such a natural portal to New Physics. One complication has to be studied, though, in particular if the observed value of \(\sin^2 \phi_1(B_d \to \phi K_S)\) falls below the predicted one by a moderate amount only. For one is actually observing \(B_d \to K^+K^-K_S\). If there is a single weak phase like in the SM one finds
\[
\sin^2 \phi_1(B_d \to \phi K_S) = -\sin^2 \phi_1(B_d \to f_0(980)K_S),
\]
where \(f_0(980)\) denotes any scalar \(K^+K^-\) configuration with a mass close to that of the \(\phi\), be it a resonance or not. A smallish pollution by such a \(f_0(980)K_S\) – by, say, 10\% in amplitude – can thus reduce the asymmetry assigned to \(B_d \to \phi K_S\) significantly – by 20\% in this example.

In the end it is therefore mandatory to perform a full time dependent Dalitz plot analysis for \(B_d \to K^+K^-K_S\) and compare it with that for \(B_d \to 3K_S\) and \(B^+ \to K^+K^-K^+, K^+K_SK_S\) and also with \(D \to 3K\). This is a very challenging task, but in my view essential. There is no ‘royal’ way to fundamental insights.

An important intermediate step in this direction is given by one application of Bianco’s Razor [42], namely to analyze the CP asymmetry in \(B_d \to [K^+K^-]_M K_S\) as a function of the cut \(M\) on the \(K^+K^-\) mass.

### 3.3 Loop Induced Rare \(B_{u,d}\) Transitions

Processes that require a loop diagram to proceed – i.e. are classically forbidden – provide a particularly intriguing stage to probe fundamental dynamics.

It marked a tremendous success for the SM, when radiative \(B\) decays were measured, first in the exclusive mode \(B \to \gamma K^*\) and subsequently also inclusively: \(B \to \gamma X\). Both the rate and the photon spectrum are in remarkable agreement with SM prediction; they have been harnessed to extracting heavy quark parameters, as explained in Lecture IV.

More recently the next, i.e. even rarer level has been reached with transitions to final states containing a pair of charged leptons:
\[
\text{BR}(B \to l^+l^-X) = \begin{cases} 
(6.2 \pm 1.1 \pm 1.5) \cdot 10^{-6} & \text{BABAR/BELLE} \\
(4.7 \pm 0.7) \cdot 10^{-6} & \text{SM}
\end{cases} .
\]

\(^{20}\)The ruler of a Greek city in southern Italy once approached the resident sage with the request to be educated in mathematics, but in a ‘royal way’, since he was very busy with many obligations. Whereupon the sage replied with admirable candor: ”There is no royal way to mathematics.”
Again the data are consistent with the SM prediction [43], yet the present experimental uncertainties are very sizable. We are just at the beginning of studying $B \to l^+l^-X$, and it has to be pursued in a dedicated and comprehensive manner for the following reasons:

- With the final state being more complex than for $B \to \gamma X$, it is described by a larger number of observables: rates, spectra of the lepton pair masses and the lepton energies, their forward-backward asymmetries and $CP$ asymmetries.

- These observables provide independent information, since there is a larger number of effective transition operators than for $B \to \gamma X$. By the same token there is a much wider window to find New Physics and even diagnose its salient features.

- It will take the statistics of a Super-B factory to mine this wealth of information on New Physics.

- Essential insights can be gained also by analyzing the exclusive channel $B \to l^+l^-K^*$ at hadronic colliders like the LHC, in particular the position of the zero in the lepton forward-backward asymmetry. For the latter appears to be quite insensitive to hadronization effects in this exclusive mode [44].

Let me present a brief case study that can illustrate how various analyses get intertwined. While the electromagnetic Penguin operator drives $B \to \gamma X_s$, its strong counterpart generates $B_d \to s\bar{s}s\bar{d} \Rightarrow \phi K_S$. The observed rate for the former, which is very consistent with the SM prediction, will therefore constrain deviations from the SM in the latter. There is a loop hole in the argument, though: if New Physics produces a photon in $B \to \gamma X$ that carries a helicity opposite to that predicted in the SM, then it will contribute quadratically to $\Gamma(B \to \gamma X_s)$, yet linearly to the $CP$ asymmetry in $B_d \to \phi K_S$. Thus it could well hide in the noise level of the former while having a substantial impact on the latter.

How could one check such a scenario? One could attempt to determine the photon polarization in the radiative $B$ decays. This can be done by measuring angular correlations in $B \to \gamma K^{*+} \to \gamma (K\pi\pi)$ modes [45]. The same operator will contribute also to $B \to l^+l^-X$, and its contribution can be disentangled there. This can even be achieved by analyzing angular correlations in the exclusive channel $B \to l^+l^-K^* \to l^+l^-(K\pi)$ [46].

The analogous decays with a $\nu\bar{\nu}$ instead of the $l^+l^-$ pair is irresistibly attractive to theorists – although quite resistibly so to experimentalists:

\[
\begin{align*}
\text{BR}(B \to \nu\bar{\nu}X) & \begin{cases} 
\leq 7.7 \cdot 10^{-4} & \text{ALEPH} \\
= 3.5 \cdot 10^{-5} & \text{SM}
\end{cases} \\
\text{BR}(B \to \nu\bar{\nu}K) & \begin{cases} 
\leq 7.0 \cdot 10^{-5} & \text{BABAR} \\
= (3.8^{+1.2}_{-0.8}) \cdot 10^{-6} & \text{SM}
\end{cases}
\end{align*}
\]

(157)

(158)

where the SM predictions are taken from Refs. [47] and [48], respectively. To measure such a rare transition with no striking experimental signature requires a most hermetic detector. The justification for taking on this challenge lies in the fact that $B \to l^+l^-X$ and $B \to \nu\bar{\nu}X$ are complementary for diagnosing New Physics couplings with $l^\pm$ and $\nu$ representing the ‘down’ and ‘up’ members, respectively, of an $SU(2)$ doublet [49].
3.4 Other Rare Decays

There are some relatively rare $B$ decays that could conceivably reveal New Physics, although they proceed already on the tree level. One well known example is $B^+ \rightarrow \tau \nu$ that is sensitive to charged Higgs fields. This applies also to semileptonic $B$ decays. As will be described in Lecture IV, the Heavy Quark Expansion (HQE) has provided a sturdy and accurate description of $B \rightarrow l \nu X_c$ that allowed to extract $|V(cb)|$ with less than 2% uncertainty. With it and other heavy quark parameters determined with considerable accuracy one can predict $\Gamma(B \rightarrow \tau \nu X_c)$ within the SM and compare it with the data. A discrepancy can be attributed to New Physics, presumably in the form of a charged Higgs field. Measuring also its hadronic mass moments can serve as a valuable cross check. Such studies will probably require the statistics of a Super-B factory.

This is true also for studying the exclusive channel $B \rightarrow \tau \nu D$. As pointed out in Ref. [50], one could find that the ratio $\Gamma(B \rightarrow \tau \nu D)/\Gamma(B \rightarrow \mu \nu D)$ deviates from its SM value due to the exchange of a charged Higgs boson with a mass of even several hundred GeV. This is the case in particular for 'large $\tan \beta$ scenarios' of two-Higgs-doublet models. There is a complication, though. Contrary to the suggestion in the literature the hadronic form factors do not drop out from this ratio. One should keep in mind that (i) the contribution from the second form factor $f_-$, which is proportional to the square of the lepton mass, cannot be ignored for $B \rightarrow \tau \nu D$ and (ii) the form factors are not taken at the same momentum transfer in the two modes.

These complications can be overcome by Uraltsev’s BPS approximation [51]. Relying on it one can extract $|V(cb)|$ from $B \rightarrow e/\mu \nu D$ and compare it with the ‘true’ value obtained from $\Gamma_{SL}(B)$. If this comparison is successful and our theoretical control over $B \rightarrow l \nu D$ thus validated, one can apply the BPS approximation to $B \rightarrow \tau \nu D$. Since, as mentioned above, the second form factor $f_-$ can be measured there, one has another cross check.

3.5 $B_s$ Decays – an Independent Chapter in Nature’s Book

When the program for the $B$ factories was planned, it was thought that studying $B_s$ transitions will be required to construct the CKM triangle, namely to determine one of its sides and the angle $\phi_3$. As discussed above a powerful method has been developed to extract $\phi_3$ from $B^\pm \rightarrow D^{\text{new}} K^\pm$, and the effort has started to obtain $|V(td)/V(ts)|$ from $\Gamma(B \rightarrow \gamma\rho/\omega)/\Gamma(B \rightarrow \gamma K^*)$. None of this, however, reduces the importance of a comprehensive program to study $B_s$ decays – on the contrary! With the basic CKM parameters fixed or to be fixed in $B_{u,d}$ decays, $B_s$ transitions can be harnessed as powerful probes for New Physics and its features.

In this context it is essential to think ‘outside the box’ – pun intended. The point here is that several relations that hold in the SM (as implemented through quark box and other loop diagrams) are unlikely to extend beyond minimal extensions of the SM. In that sense $B_{u,d}$ and $B_s$ decays constitute two different and complementary chapters in Nature’s book on fundamental dynamics.
3.5.1 $B_s - \bar{B}_s$ Oscillations

There is hope, even optimism, that we are about to establish $B_s - \bar{B}_s$ oscillations as characterized by the two observables $\Delta M(B_s)$ and $\Delta \Gamma(B_s)$.

$\Delta M(B_s)$: The experimental sensitivity has reached a domain, where the SM most likely puts it:

$$\Delta M(B_s) \begin{cases} > 14.5 \text{ psec}^{-1} \text{ exp.} \\ \sim 15 - 30 \text{ psec}^{-1} \text{ SM - CKM} \end{cases} \quad (159)$$

the numerical input for the SM prediction comes from the observed rate of $B_d - \bar{B}_d$ oscillations via the expression:

$$\frac{\Delta M(B_s)}{\Delta M(B_d)} \approx \frac{B_s f^2(B_s) |V(ts)|^2}{B_d f^2(B_d) |V(td)|^2} \quad (160)$$

This relation also exhibits the phenomenological interest in measuring $\Delta M(B_s)$, namely to obtain an accurate value for $|V(td)|$. Lattice QCD is usually invoked to gain theoretical control over the first ratio of hadronic quantities. I fully subscribe to the importance of this measurement, irrespective of the validity of the SM.

$\Delta \Gamma(B_s)$: In June 2004 the CDF collaboration first presented an intriguing analysis exhibiting two surprisingly large lifetimes controlling $B_s \rightarrow \psi \phi |_{CP=\pm} [52]$:

$$\tau[CP = +] = (1.05^{+0.10}_{-0.13} \pm 0.02) \text{ps} \quad \text{vs.} \quad \tau[CP = -] = (2.07^{+0.58}_{-0.46} \pm 0.03) \text{ps} \quad (161)$$

$$\frac{\Delta \Gamma_s}{\Gamma_s} \equiv \frac{\Gamma(B_s[CP = +]) - \Gamma(B_s[CP = -])}{\frac{1}{2}(\Gamma(B_s[CP = +]) + \Gamma(B_s[CP = -]))} = (65^{+25}_{-33} \pm 1)\% \quad (162)$$

More recently D0 has presented a similar analysis [53]:

$$\frac{\Delta \Gamma_s}{\Gamma_s} = 25^{+14}_{-15}\% \quad (163)$$

My heart wishes that $\frac{\Delta \Gamma_s}{\Gamma_s}$ were indeed as large as 0.5 or even larger. For it would open up a whole new realm of CP studies in $B_s$ decays with a great potential to identify New Physics. Yet my head tells me that values exceeding 0.25 or so are very unlikely; it would point at a severe limitation in our theoretical understanding of $B$ lifetimes. For only on-shell intermediate states $f$ in $B^0 \rightarrow f \rightarrow \bar{B}^0$ can contribute to $\Delta \Gamma(B)$, and for $B^0 = B_s$ these are predominantly driven by $b \rightarrow c\bar{c} s$. Let $R(b \rightarrow c\bar{c} s)$ denote their fraction of all $B_s$ decays. If these transitions contribute only to $\Gamma(B_s[CP = +])$ one has $\Delta \Gamma_s/\Gamma_s = 2R(b \rightarrow c\bar{c} s)$. Of course this is actually an upper bound quite unlikely to be even remotely saturated. With the estimate $R(b \rightarrow c\bar{c} s) \simeq 25\%$, which is consistent with the data on the charm content of $B_{u,d}$ decays this upper bound reads 50%. More realistic calculations have yielded considerably smaller predictions:

$$\frac{\Delta \Gamma_s}{\Gamma_s} = \begin{cases} 22\% \cdot \left( \frac{f(B_s)}{220 \text{ MeV}} \right)^2 \\ 12 \pm 5\% \end{cases} \quad (164)$$
where the two predictions are taken from Refs. [54] and [55], respectively. A value as high as $20 - 25\%$ is thus not out of the question theoretically, and Eq.(162) is still consistent with it. One should note that invoking New Physics would actually ‘backfire’ since it leads to a lower prediction. If, however, a value exceeding $25\%$ were established experimentally, we had to draw at least one of the following conclusions: (i) $R(b \rightarrow c\bar{c}s)$ actually exceeds the estimate of $25\%$ significantly. This would imply at the very least that the charm content is higher in $B_s$ than $B_{u,d}$ decays by a commensurate amount and the $B_s$ semileptonic branching ratio lower. (ii) Such an enhancement of $R(b \rightarrow c\bar{c}s)$ would presumably – though not necessarily – imply that the average $B_s$ width exceeds the $B_d$ width by more than the predicted 1-2\% level. That means in analyzing $B_s$ lifetimes one should allow $\bar{\tau}(B_s)$ to float freely. (iii) If in the end one found the charm content of $B_s$ and $B$ decays to be quite similar and $\bar{\tau}(B_s) \simeq \tau(B_d)$, yet $\Delta \Gamma_s/\Gamma_s$ to exceed 0.25, we had to concede a loss of theoretical control over $\Delta \Gamma$. This would be disappointing, yet not inconceivable: the a priori reasonable ansatz of evaluating both $\Delta \Gamma_B$ and $\Delta M_B$ from quark box diagrams – with the only manifest difference being that the internal quarks are charm in the former and top in the latter case – obscures the fact that the dynamical situation is actually different. In the latter case the effective transition operator is a local one involving a considerable amount of averaging over off-shell transitions; the former is shaped by on-shell channels with a relatively small amount of phase space: for the $B_s$ resides barely 1.5 GeV above the $D_s\bar{D}_s$ threshold. To say it differently: the observable $\Delta \Gamma_s$ is more vulnerable to limitations of quark-hadron duality than $\Delta M_s$ and even beauty lifetimes.

In summary: establishing $\Delta \Gamma_s \neq 0$ amounts to important qualitative progress in our knowledge of beauty hadrons; it can be of great practical help in providing us with novel probes of CP violations in $B_s$ decays, and it can provide us theorists with a reality check concerning the reliability of our theoretical tools for nonleptonic $B$ decays.

### 3.5.2 CP Violation in Nonleptonic $B_s$ Modes

One class of nonleptonic $B_s$ transitions does not follow the paradigm of large CP violation in $B$ decays [15]:

$$ A_{\text{CP}}(B_s(t) \rightarrow [\psi \phi]_{l=0}/\psi \eta) = \sin 2\phi(B_s) \sin \Delta M(B_s)t $$

$$ \sin 2\phi(B_s) = \text{Im} \left[ \left( V^\ast(tb)V(ts) \right)^2 \left( V(cb)V^\ast(cs) \right)^2 \right] \simeq 2\lambda^2 \eta \sim 0.02 . \quad (165) $$

This is easily understood: on the leading CKM level only quarks of the second and third families contribute to $B_s$ oscillations and $B_s \rightarrow \psi \phi$ or $\psi \eta$; therefore on that level there can be no CP violation making the CP asymmetry Cabibbo suppressed. Yet New Physics of various ilks can quite conceivably generate $\sin 2\phi(B_s) \sim \text{several} \times 10\%$.

---

21These are all dominated by nonleptonic transitions, where duality violations can be significantly larger than for semileptonic modes.
Analyzing the decay rate evolution in proper time of

\[ B_s(t) \rightarrow \phi\phi \] (166)

with its direct as well as indirect CP violation is much more than a repetition of the \( B_d(t) \rightarrow \phi K_S \) saga:

- \( \mathcal{M}_{12}(B_s) \) and \( \mathcal{M}_{12}(B_d) \) – the off-diagonal elements in the mass matrices for \( B_s \) and \( B_d \) mesons, respectively – provide in principle independent pieces of information on \( \Delta B = 2 \) dynamics.

- While the final state \( \phi K_S \) is described by a single partial wave, namely \( l = 1 \), there are three partial waves in \( \phi\phi \), namely \( l = 0, 1, 2 \). Disentangling the three partial rates and their CP asymmetries – or at least separating \( l = \) even and odd contributions – provides a new diagnostics about the underlying dynamics.

### 3.5.3 Leptonic, Semileptonic and Radiative Modes

The decays into a lepton pair and to ‘wrong-sign’ leptons should be studied also for \( B_d \) mesons; however here I discuss only \( B_s \) decays, where one can expect more dramatic effects.

- The mode \( B_s \rightarrow \mu^+\mu^- \) is necessarily very rare since it suffers from helicity suppression \( \propto (m(\mu)/M(B_s))^2 \) and ‘wave function suppression’ \( \propto (f_B/M(B_s))^2 \), which reflects the practically zero range of the weak interactions. These tiny factors can be partially compensated in some large \( \tan \beta \) SUSY scenarios, where an enhancement factor of \( \tan^6 \beta \) arises. Upper bounds on the branching ratio established at FNAL already cut into the a priori allowed parameter space.

- Due to the rapid \( B_s \) oscillations those mesons have a practically equal probability to decay into ‘wrong’ and ‘right’ sign leptons. One can then search for an asymmetry in the wrong sign rate for mesons that initially were \( B_s \) and \( \bar{B}_s \):

\[
a_{SL}(B_s) \equiv \frac{\Gamma(\bar{B}_s \rightarrow l^+X) - \Gamma(B_s \rightarrow l^-X)}{\Gamma(\bar{B}_s \rightarrow l^+X) + \Gamma(B_s \rightarrow l^-X)} \] (167)

This observable is necessarily small; among other things it is proportional to \( \frac{\Delta \Gamma(B_s)}{\Delta M(B_s)} \ll 1 \). Yet within CKM theory it is truly tiny:

\[
a_{SL}(B_s) \sim \mathcal{O}(10^{-4}) \] (168)

due to a feature quite specific to CKM; analogous to \( B_s \rightarrow \psi\phi \) quarks of only the second and third family participate on the leading CKM level, which therefore cannot exhibit CP violation. Yet again, New Physics can enhance \( a_{SL}(B_s) \), this time by two orders of magnitude to the 1% level.

- As already emphasized \( B_s \rightarrow \gamma X \) and \( B_s \rightarrow l^+l^- X \) should be studied in a comprehensive manner.
3.6 Summary of Lecture III

Not only has the first CP asymmetry outside the $K^0 - \bar{K}^0$ complex been observed – the CKM paradigm has become a tested theory. The term ‘ansatz’ with its patronizing flavour is no longer adequate. The predicted ‘Paradigm of large CP Violation in B Decays’ has been established now in qualitative as well as quantitative agreement with CKM theory in three quite distinct $B_d$ channels: $B_d \to \psi K_S$ \textsuperscript{22}, $\pi^+\pi^-$ and $K^+\pi^-$. The observed effects in the first two transitions that involve $B_d - \bar{B}_d$ oscillations are fully commensurate with T violation, and the last two modes exhibit large direct CP violation.

My referring to it as the ‘expected triumph of a very peculiar theory’ should not be seen as downgrading these observations. On the contrary – the discovery of large CP asymmetries in $B$ decays is a very momentous one validating an a priori unlikely theory, namely CKM dynamics, and demonstrating a more general point, namely that CP violating phases are not intrinsically small. I maintain that this novel success of CKM theory should not have come as a shock after the previous successes. Yet even for a theorist like me the strongest fascination of physics lies in the fact that data are the final arbiter. Verifying something empirically thus always adds a higher quality. In a similar vein I would argue that the discovery of the weak bosons marks one of the major intellectual human triumphs of the second half of the twentieth century; the fact that those states had been predicted by the SM does not detract from it.

It is more than a bon mot to say "$K$ and $B$ decays are exactly the same, only different". Table 1 sketches the oscillation parameters for neutral mesons carrying strangeness, charm and beauty. There are at least qualitative similarities in the patterns. Furthermore there are actually domains in $K_L$ decays that exhibit truly large CP asymmetries, namely the angular correlation between the di-pion and the di-electron planes in $K_L \to \pi^+\pi^-e^+e^-$, see Eq.(103), and the difference between $K^0 \to \pi^+\pi^-$ and $\bar{K}^0 \to \pi^+\pi^-$ for intermediate times of decay.

Nevertheless the statement that "CP violation in $B$ decays is much larger than in $K$ decays" is an empirical fact. Suffice it to say that the mass eigenstates of neutral kaons are very well approximated by CP eigenstates as well – in marked contrast to the situation in $B$ decays \textsuperscript{23}.

Even beyond the success of the specific CKM theory a ‘demystification’ of CP violation has occurred: if the dynamics are sufficiently multi-layered to support CP violation – for the case under study it means there are at least three families –, the latter can be large, i.e. the observable phases can be of order unity. Having been raised by my mother in the spirit of the Enlightenment I welcome this development wholeheartedly. This demystification will be completed, if CP violation is found anywhere in leptodynamics – a point I will return to in Lecture V.

\textsuperscript{22}The evolution of the experimental measurements of $\sin2\phi_1$ as sketched above should illustrate for theorists that when experimentalists quote uncertainties those mean something very real. Shifts by more than one sigma are quite possible.

\textsuperscript{23}CKM theory explains why CP invariance is a ‘near miss’ in $K_L$ decays in the following sense: the first and second families are almost decoupled there from the third.
There is no longer a need for – and at present little sense in – searching for an alternative theory for the observed CP violation. Yet the decisive success of the SM description does not at all weaken the need to search for New Physics in B decays as corrections to SM effects.

- It makes the peculiar features of the SM in general and the CKM dynamics in particular even more mysterious thus suggesting an even stronger need for a more fundamental explanation.

- We know, as explained in Dolgov’s lectures [56] that CKM dynamics are irrelevant for baryogenesis in the Universe. I see that actually as a rather positive statement in the sense that the assumed baryogenesis – i.e., the conjecture that the observed baryon number is a dynamically generated quantity rather than an arbitrary initial value – implies the existence of New Physics. This is further strengthened by the aforementioned demystification telling us there is no general intrinsic reason why CP phases should be small.

- On a more pragmatic level, CP studies provide highly sensitive probes of the underlying dynamics. A comprehensive program of precision studies will be essential in diagnosing the New Physics anticipated for the TeV scale. I will return to this point in more detail in Lecture V. In this context it is essential to make the utmost efforts to bring hadronization effects under quantitative control [42].

- In most cases – with notable exceptions like $B_s(t) \to \psi\phi$ – we cannot count on New Physics inducing large deviations from SM expectations. Accordingly it is only now that we are reaching ‘territory’, where significant discrepancies with SM predictions can ‘realistically’ be hoped for.

- Studying $B_s$ transitions in a dedicated fashion will allow us to ‘read’ an independent chapter in Nature’s book on fundamental dynamics.

## 4 Lecture IV: Adding High Accuracy to High Sensitivity

None of the impressive successes of the SM weaken the case for it being incomplete, i.e. that New Physics has to exist, quite conceivably already at the TeV scale. Apart from

| $K^0$ | $D^0$ | $B_d$ | $B_s$ |
|-------|-------|-------|-------|
| $\Delta M_K \approx \Gamma_K$ | $\Delta M_D \ll \Gamma_D$ | $\Delta M(B_d) \sim \Gamma(B_d)$ | $\Delta M(B_s) \gg \Gamma(B_s)$ |
| $\Delta \Gamma_K \approx 2 \Gamma_K$ | $\Delta \Gamma_D \ll \Gamma_D$ | $\Delta \Gamma(B_d) \ll \Gamma(B_d)$ | $\Delta \Gamma(B_s) \sim O(\Gamma(B_s))$ |
| $\Delta \Gamma_K \sim \Delta M_K$ | $\Delta \Gamma_D \sim \Delta M_D$ | $\Delta \Gamma(B_d) \ll \Delta M(B_d)$ | $\Delta \Gamma(B_s) \ll \Delta M(B_s)$ |

Table 1: Comparing oscillation parameters for neutral $K$, $D$ and $B$ mesons
the general observation that the CKM structure looks very peculiar thus suggesting an underlying layer of hitherto unknown dynamics, we have some more specific theoretical arguments and data of mostly celestial origin:

- theoretical shortcomings:
  - the UV instability of Higgs dynamics and the gauge hierarchy problem;
  - the Strong CP Problem, see Sect. 1.2.2.

- ‘Heavenly pointers’:
  - the baryon number of the Universe;
  - neutrino oscillations: the solar neutrino deficit and the ‘atmospheric anomaly’ together with the earthly KAMLAND and K2K data [57];
  - the evidence for ubiquitous dark matter;
  - the most intriguing evidence for a mysterious dark energy.

I harbour few doubts that New Physics will be discovered at the LHC ‘directly’, i.e. through the observation of the production of new quanta. Yet our goal has to be to identify also the nature of this anticipated New Physics. I am convinced that it will be essential in this context to search for its indirect impact on low energy processes. Among such indirect searches for New Physics one usually distinguishes between two classes, namely those based

- on high accuracy with the most celebrated example that of the $g - 2$ of muons and
- on high sensitivity as in $\epsilon_K$, $\Delta M(B^0)$ and EDM’s.

The present successes of the SM suggest that manifestations of New Physics on heavy flavour transitions might be subtle rather than numerically massive. Therefore I formulate the

**New Paradigm of Heavy Flavour Studies:**

- With the impact of New Physics likely to be mostly subtle and
- our goal being to identify the salient features of New Physics rather than ‘merely’ establishing its existence

we have to strive for high numerical accuracy on the experimental as well as theoretical side.

To say it differently: we have to combine the elements of high sensitivity and high accuracy. The spectacular success of the $B$ factories and the emerging successes of CDF and D0 to obtain high quality data on beauty transitions in a hadronic environment give us confidence that such an experimental goal will be achieved. In this lecture I want to
describe why I think that theory will be able to hold up its side of the bargain as well and what the required elements for such an undertaking have to be.

The question is: ”Can we answer the challenge of ∼ % accuracy?” One guiding principle will be in Lenin’s concise words:

‘Trust is good – control is better!’

Table 2 provides a sketch of the theoretical control we have achieved over some aspects of $B$ decays. I hope it will excite the curiosity of the reader and fortify her/him to read the more technical discussion of Lecture IV.

### 4.1 Heavy Quark Theory

While QCD is the only candidate among *local* quantum field theories to describe the strong interactions, as explained in Lecture I in Sects. 1.2 & 1.3, $SU(2)_L \times U(1)$ is merely the minimal theory for the electroweak forces. Obtaining reliable information about the latter is, however, limited by our lack of full calculational control over the former.

It had been conjectured for more than thirty years that the theoretical treatment of heavy flavour hadrons should be facilitated, when the heavy quark mass greatly exceeds the nonperturbative scale of QCD $^{24}$:

$$m_Q \gg \Lambda_{QCD}.$$  \hfill (169)

This conjecture has been transformed into a reliable theoretical framework only in the last fifteen years, as far as beauty hadrons are concerned. I refer to it as Heavy Quark Theory (mentioned already in Sect. 1.2.3); comprehensive reviews with references to the original literature can be found in Refs. [58] and [59]. Its goal is to treat nonperturbative

\[24\] A striking prediction has been that super-heavy top quarks – i.e. with $m_t \geq 150$ GeV – would decay, before they could hadronize [61] thus bringing top quarks under full theoretical control. For then the decay width of top quarks is of order 1 GeV and provides an infrared cutoff for QCD corrections. This feature comes with a price, though, in so far as CP studies are concerned: without hadronization as a ‘cooling’ mechanism, the degree of coherence between different transition amplitudes – a necessary condition for CP violation to become observable – will be rather tiny.
dynamics quantitatively, as it affects heavy flavour hadrons, in full conformity with QCD and without model assumptions. It has achieved this goal already for several classes of beauty meson transitions with a reliability and accuracy that before would have seemed unattainable.

Heavy Quark Theory is based on a two-part strategy analogous to the one adopted in chiral perturbation theory – another theoretical technology to deal reliably with non-perturbative dynamics in a special setting. Like there Heavy Quark Theory combines two basic elements, namely an asymptotic symmetry principle and a dynamical treatment telling us how the asymptotic limit is approached:

1. The symmetry principle is **Heavy Quark Symmetry** stating that all sufficiently heavy quarks behave identically under the strong interactions without sensitivity to their spin. This can easily be illustrated with the Pauli Hamiltonian describing the interaction of a quark of mass $m_Q$ with a gauge field $A_\mu = (A_0, \vec{A}):$

$$H_{\text{Pauli}} = -A_0 + \frac{(i\vec{\sigma} - \vec{A})^2}{2m_Q} + \frac{\vec{\sigma} \cdot \vec{B}}{2m_Q} \Rightarrow -A_0 \text{ as } m_Q \to \infty; \quad (170)$$
i.e., in the infinite mass limit, quarks act like static objects without spin dynamics and subject only to colour Coulomb fields.

This simple consideration illustrates a general feature of heavy quark theory, namely that the spin of the heavy quark $Q$ decouples from the dynamics in the heavy quark limit. Hadrons $H_Q$ can therefore be labeled by the angular momentum $j_q$ carried by its ‘light’ components – light valence quarks, gluons and sea quarks – in addition to its total spin $S$. The $S$ wave pseudoscalar and vector mesons – $B$ & $B^*$ and $D$ & $D^*$ – then form the ground state doublet of heavy quark symmetry with $[S,j_q] = [0, \frac{1}{2}],[1, \frac{1}{2}];$ a quartet of $P$ wave configurations form the first excited states with $[S,j_q] = [0, \frac{1}{2}],[1, \frac{1}{2}],[1, \frac{3}{2}],[2, \frac{3}{2}].$

Heavy quark symmetry can be understood in an intuitive way: consider a hadron $H_Q$ containing a heavy quark $Q$ with mass $m_Q \gg \Lambda_{QCD}$ surrounded by a “cloud” of light degrees of freedom carrying quantum numbers of an antiquark $\bar{q}$ or diquark $qq$ \(^{25}\). This cloud has a rather complex structure: in addition to $\bar{q}$ (for mesons) or $qq$ (for baryons) it contains an indefinite number of $q\bar{q}$ pairs and gluons that are strongly coupled to and constantly fluctuate into each other. There is, however, one thing we know: since typical frequencies of these fluctuations are $\sim \mathcal{O}(\text{few}) \times \Lambda_{QCD}$, the normally dominant soft dynamics allow the heavy quark to exchange momenta of order few times $\Lambda_{QCD}$ only with its surrounding medium. $Q\bar{Q}$ pairs then cannot play a significant role, and the heavy quark can be treated as a quantum mechanical object rather than a field theoretic entity requiring second quantization. This provides a tremendous computational simplification even while maintaining a field theoretic description for the light degrees of freedom. Furthermore techniques developed long ago in QED can profitably be adapted here.

\(^{25}\)This cloud is often referred to – somewhat disrespectfully – as ‘brown muck’, a phrase coined by the late Nathan Isgur.
2. We can go further and describe the interactions between $Q$ and its surrounding light
degrees of freedom through an expansion in powers of $1/m_Q$ – the Heavy Quark
Expansion (HQE). This allows us to analyze pre-asymptotic effects, i.e. effects that
fade away like powers of $1/m_Q$ as $m_Q \to \infty$.

Let me anticipate the lessons we have learnt: we have

- identified the sources of the non-perturbative corrections;
- found them to be smaller than they could have been;
- succeeded in relating the basic quantities of the Heavy Quark Theory – KM parameters,
  masses and kinetic energy of heavy quarks, etc. – to various a priori independant
  observables with a considerable amount of redundancy;
- developed a better understanding of incorporating perturbative and nonperturbative
  corrections without double-counting.

In the following I will sketch the concepts on which the Heavy Quark Expansions are
based, the techniques employed, the results obtained and the problems encountered. It
will not constitute a self-sufficient introduction into this vast and ever expanding field.
My intent is to provide a guide through the literature for the committed student.

4.2 H(eavy) Q(uark) E(xpansions), Fundamentals

In describing weak decays of heavy flavour hadrons one has to incorporate perturbative
as well as nonperturbative contributions in a self-consistent and complete way. The only
known way to tackle such a task invokes the Operator Product Expansion a la Wilson
involving an effective Lagrangian. Further conceptual insights as well as practical results
can be gained by analysing sum rules; in particular they shed light on various aspects and
formulations of quark-hadron duality.

4.2.1 Operator Product Expansion (OPE) for Inclusive Weak Decays

Similar to the well-known case of $\sigma(e^+e^- \to had)$ one invokes the optical theorem to
describe the decay into a sufficiently inclusive final state $f$ through the imaginary part of
the forward scattering operator evaluated to second order in the weak interactions

$$\hat{T}(Q \to Q) = \text{Im} \int d^4x i\{\mathcal{L}_W(x)\mathcal{L}_W^\dagger(0)\}_T$$

with the subscript $T$ denoting the time-ordered product and $\mathcal{L}_W$ the relevant weak La-
grangian $^{26}$. The expression in Eq.(171) represents in general a non-local operator with

$^{26}$There are two qualitative differences to the case of $e^+e^- \to had$: in describing weak decays of a
hadron $H_Q$ (i) one employs the weak rather than the electromagnetic Lagrangian, and (ii) one takes the
expectation value between the $H_Q$ state rather than the hadronic vacuum.
the space-time separation $x$ being fixed by the inverse of the energy release. If the latter is large compared to typical hadronic scales, then the product is dominated by short-distance physics, and one can apply an operator product expansion a la Wilson on it yielding an infinite series of local operators of increasing dimension. The width for the decay of a hadron $H_Q$ containing $Q$ is then obtained by taking the $H_Q$ expectation value of the operator $\hat{T}$:

$$\frac{\langle H_Q | \text{Im} \hat{T}(Q \to f \to Q) | H_Q \rangle}{2M_{H_Q}} \propto \Gamma(H_Q \to f) = \frac{G_F^2 m_Q^3(\mu)}{192\pi^3} |V_{CKM}|^2.$$  

Eq.(172) exhibits the following important features:

- An auxiliary scale $\mu$ has been introduced to consistently separate short and long distance dynamics:

$$\text{short distance} < \mu^{-1} < \text{long distance}$$

with the former entering through the coefficients and the latter through the effective operators; their matrix elements will thus depend on $\mu$.

In principle the value of $\mu$ does not matter: it reflects merely our computational procedure rather than how nature goes about its business. The $\mu$ dependance of the coefficients thus has to cancel against that of the corresponding matrix elements.

In practise however there are competing demands on the choice of $\mu$:

- On one hand one has to choose

$$\mu \gg \Lambda_{QCD} ;$$

otherwise radiative corrections cannot be treated within perturbative QCD.

- On the other hand many computational techniques for evaluating matrix elements – among them the Heavy Quark Expansions – require

$$\mu \ll m_b$$

\[27\] I will formulate the expansion in powers of $1/m_Q$, although it has to be kept in mind that it is really controlled by the inverse of the energy release. While there is no fundamental difference between the two for $b \to c/ul\nu$ or $b \to c/uid$, since $m_b, m_b - m_{c,u} \gg \Lambda_{QCD}$, the expansion becomes of somewhat dubious reliability for $b \to c\bar{c}s$. It actually would break down for a scenario $Q_2 \to Q_1l\bar{\nu}$ with $m_{Q_2} \approx m_{Q_1}$, - in contrast to HQET!
The choice \( \mu \sim 1 \text{ GeV} \) satisfies both of these requirements. It is important to check that the obtained numerical results do not exhibit a significant sensitivity to the exact value of \( \mu \) when varying the latter in a reasonable range.

- **Short-distance** dynamics shape the c number coefficients \( c_i^{(f)} \). In practise they are evaluated in *perturbative* QCD. It is quite conceivable, though, that also *nonperturbative* contributions arise there; yet they are believed to be fairly small in beauty decays \([62]\).

By the same token these short-distance coefficients provide also the portals, through which New Physics can enter in a natural way.

- Nonperturbative contributions on the other hand enter through the *expectation values* of operators of dimension higher than three – \( \bar{Q} \sigma \cdot GQ \) etc. – and higher order corrections to the expectation value of the leading operator \( \bar{Q}Q \), see below.

- In practice we cannot go beyond evaluating the first few terms in this expansions. More specifically we are limited to contributions through order \( 1/m_Q^3 \); those are described in terms of six heavy quark parameters, namely two quark masses – \( m_{b,c} \) –, two expectation values of dimension-five operators – \( \mu_\pi^2 \) and \( \mu_G^2 \) – and of dimension-six operators – the Darwin and ‘LS’ terms, \( \rho_D^2 \) and \( \rho_{LS}^2 \), respectively \(^{28}\).

  - This small and universal set of nonperturbative quantities describes a host of observables in \( B \) transitions. Therefore their values can be determined from some of these observables and still leave a large number of predictions.

  - It opens the door to a novel symbiosis of different theoretical technologies for heavy flavour dynamics – in particular between HQE and lattice QCD. For the HQP can be inferred from lattice studies. This enhances the power of and confidence in both technologies by

    * increasing the range of applications and
    * providing more validation points.

I will give some examples later on.

- Expanding the expectation value of the leading operator \( \bar{Q}Q \) for a pseudoscalar meson \( P_Q \) with quantum number \( Q \) in powers of \( 1/m_Q \) yields

\[
\frac{1}{2M_{P_Q}} \langle P_Q | \bar{Q}Q | P_Q \rangle = 1 - \frac{\mu_\pi^2}{2m_Q^2} + \frac{\mu_G^2}{2m_Q^2} + \mathcal{O}(1/m_Q^3) ; \tag{177}
\]

\(^{28}\)For simplicity I ignore here so-called ‘Intrinsic Charm’ contributions, see \([63]\).
\( \mu^2_\pi(\mu) \) and \( \mu^2_G(\mu) \) denote the expectation values of the kinetic and chromomagnetic operators, respectively:

\[
\mu^2_\pi(\mu) = \frac{1}{2M_{HQ}} \langle H_Q | \bar{Q} \vec{\pi}^2 Q | H_Q \rangle (\mu), \quad \mu^2_G(\mu) = \frac{1}{2M_{HQ}} \langle H_Q | \bar{Q} \frac{i}{2} \sigma \cdot GQ | H_Q \rangle (\mu); \quad (178)
\]

for short they are often called the kinetic and chromomagnetic moments.

Eq. (177) implies that one has \( \langle H_Q | \bar{Q} Q | H_Q \rangle (\mu)/2M_{HQ} = 1 \) for \( m_Q \to \infty \); i.e., the free quark model expression emerges asymptotically for the total width.

- The *leading* nonperturbative corrections arise at order \( 1/m_Q^2 \) only. That means they are rather small in beauty decays since \( (\mu/m_Q)^2 \sim \text{few \%} \) for \( \mu \leq 1 \text{ GeV} \).

- This smallness of nonperturbative contributions explains a posteriori, why partonic expressions when coupled with a ‘smart’ perturbative treatment often provide a decent approximation.

- These nonperturbative contributions which are power suppressed can be described only if considerable care is applied in treating the *parametrically larger* perturbative corrections.

- Explicitly flavour dependant effects arise in order \( 1/m_Q^3 \). They mainly drive the differences in the lifetimes of the various mesons of a given heavy flavour.

- An important practical distinction to the OPE treatment of \( e^+e^- \to \text{had} \) or deep-inelastic lepton nucleon scattering is the fact that the weak width depends on the fifth power of the heavy quark mass, see Eq. (172), and thus requires particular care in dealing with the delicate concept of quark masses.

One general, albeit subtle point has to be kept in mind here: while everybody these days invokes the OPE it is often not done employing Wilson’s prescription with the auxiliary scale \( \mu \), and different definitions of the relevant operators have been suggested. While results from one prescription can be translated into another one order by order, great care has to be applied. I will adopt here the socalled ‘kinetic scheme’ with \( \mu \approx 1 \text{ GeV} \). It should be noted that the quantities \( \mu^2_\pi(\mu) \) and \( \mu^2_G(\mu) \) are quite distinct from the socalled HQET parameters \( \lambda_1 \) and \( \lambda_2 \) although the operators look identical. Furthermore the fact that perturbative corrections are rather smallish in the kinetic scheme does generally *not* hold in other schemes.

The absence of corrections of order \( 1/m_Q \) [64] is particularly noteworthy and intriguing since such corrections do exist for hadronic masses \( M_{HQ} = m_Q(1 + \Lambda/m_Q + \mathcal{O}(1/m_Q^2)) \) – and those control the phase space. Technically this follows from the fact that there is no *independant* dimension-four operator that could emerge in the OPE 29. This result can be illuminated in more physical terms as follows. Bound-state effects in the initial state like mass shifts do generate corrections of order \( 1/m_Q \) to the total width; yet so does

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29The operator \( \bar{Q} i \not\!{D} Q \) can be reduced to the leading operator \( \bar{Q} Q \) through the equation of motion.
hadronization in the final state. *Local* colour symmetry demands that those effects cancel each other out. *It has to be emphasized that the absence of corrections linear in $1/m_Q$ is an unambiguous consequence of the OPE description.* If their presence were forced upon us, we would have encountered a *qualitative* change in our QCD paradigm. A discussion of this point has arisen recently phrased in the terminology of quark-hadron duality. I will return to this point later.

### 4.2.2 Sum Rules

There are classes of sum rules derived from QCD proper that relate the heavy quark parameters appearing in the OPE for inclusive $B \to l\nu X_c$ – like $\mu_r^2$, $\mu_G^2$ etc. – with restricted sums over exclusive channels. They provide rigorous definitions, inequalities and experimental constraints [65]; e.g.:

\[
\mu_r^2(\mu)/3 = \sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}(1)|^2 + 2 \sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}(1)|^2 \\
\mu_G^2(\mu)/3 = -2 \sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}(1)|^2 + 2 \sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}(1)|^2
\]

(179)

(180)

where $\tau_{1/2}$, $\tau_{3/2}$ are the amplitudes for $B \to l\nu D(j_q)$ with $D(j_q)$ a hadronic system beyond the $D$ and $D^*$, $j_q = 1/2 & 3/2$ the angular momentum carried by the light degrees of freedom in $D(j_q)$, as explained in the paragraph below Eq.(170), and $\epsilon_m$ the excitation energy of the $m$th such system above the $D$ with $\epsilon_m \leq \mu$.

These sum rules have become of great practical value. I want to emphasize here one of their conceptual features: they show that *the heavy quark parameters in the kinetic scheme are observables themselves.*

### 4.2.3 Quark-hadron Duality

The concept of quark-hadron duality (or duality for short), which goes back to the early days of the quark model, refers to the notion that a *quark*-level description should provide a good description of transition rates that involve *hadrons*, if one sums over a sufficient number of channels. This is a rather vague formulation: How many channels are "sufficiently" many? How good an approximation can one expect? How process dependent is it? Yet it is typical in the sense that no precise definition of duality had been given for a long time, and the concept has been used in many different incarnations. A certain lack of intellectual rigour can be of great eruristic value in the ‘early going’ – but not forever.

A precise definition requires theoretical control over perturbative as well as nonperturbative dynamics. For limitations to duality have to be seen as effects over and beyond uncertainties due to truncations in the perturbative and nonperturbative expansions. To be more explicit: duality violations are due to corrections *not* accounted for due to

- truncations in the expansion and
- limitations in the algorithm employed.
One important requirement is to have an OPE treatment of the process under study, since otherwise we have no unambiguous and systematic inclusion of nonperturbative corrections. This is certainly the case for inclusive semileptonic and radiative $B$ decays.

While we have no complete theory for duality and its limitations, we have certainly moved beyond the folkloric stage in the last few years. We have developed a better understanding of the physics effects that can generate duality violations – the presence of production thresholds for example – and have identified mathematical portals through which duality violations can enter. The fact that we construct the OPE in the Euclidean range and then have to extrapolate it to the Minkowskian domain provides such a gateway.

The problem with the sometimes heard statement that duality represents an additional ad-hoc assumption is that it is not even wrong – it just misses the point.

More details on this admittedly complex subject can be found in Ref. [66] and for the truly committed student in Ref. [67]. Suffice it here to say that it had been predicted that duality violation in $\Gamma_{SL}(B)$ can safely be placed below 0.5% [67]. The passion in the arguments over the potential size of duality violations in $B \to l\nu X$ has largely faded away, since, as I discuss later on, the experimental studies of it have shown no sign of such limitations.

4.2.4 Heavy Quark Parameters

Through order $1/m_Q^3$ there are six heavy quark parameters (HQP), which fall into two different classes:

1. The heavy quark masses $m_b$ and $m_c$; they are ‘external’ to QCD; i.e. they can never be calculated by lattice QCD without experimental input.

2. The expectation values of the dimension five and six operators: $\mu_\pi^2$, $\mu_G^2$, $\rho_D^3$ and $\rho_{LS}^3$. They are ‘intrinsic’ to QCD, i.e. can be calculated by lattice QCD without experimental input.

Since weak decay widths depend on the fifth power of the heavy quark mass, great care has to be applied in defining this somewhat elusive entity in a way that can pass full muster by quantum field theory. To a numerically lesser degree this is true for the other HQP as well. Their dependence on the auxiliary scale $\mu$ has to be carefully tracked.

- **Quark masses**: There is no quark mass *per se* – one has to specify the renormalization scheme used and the scale, at which the mass is to be evaluated. The *pole* mass – i.e. the position of the pole in the perturbative Green function – has the convenient features that it is gauge invariant and infrared finite in perturbation theory. Yet in the complete theory it is infrared unstable [58] due to ‘renormalon’ effects. Those introduce an *irreducible intrinsic* uncertainty into the quark mass: $m_Q(1+\delta(m_Q)/m_Q)$, with $\delta(m_Q)$ being roughly $\sim \Lambda_{QCD}$. For the weak width it amounts to an uncertainty $\delta(m_Q^5) \sim 5\delta(m_Q)/m_Q$; i.e., it is parametrically larger than the power suppressed terms $\sim O(1/m_Q^2)$ one is striving to calculate. The pole mass is thus ill suited when including nonperturbative contributions. Instead one needs a running mass with an infrared cut-off $\mu$ to ‘freeze out’ renormalons.
The \( \overline{MS} \) mass, which is a rather ad hoc expression convenient in perturbative computations rather than a parameter in an effective Lagrangian, would satisfy this requirement. It is indeed a convenient tool for treating reactions where the relevant scales exceed \( m_Q \) in production processes like \( Z^0 \rightarrow b\bar{b} \). Yet in decays, where the relevant scales are necessarily below \( m_Q \) the \( \overline{MS} \) mass is actually inconvenient or even inadequate. For it has a handmade infrared instability:

\[
\overline{m}_Q(\mu) = \overline{m}_Q(m_Q) \left[ 1 + \frac{2\alpha_s}{\pi} \log \frac{\overline{m}_Q}{\mu} \right] \to \infty \text{ as } \frac{\mu}{\overline{m}_Q} \to 0 \tag{181}
\]

It is much more advantageous to use the ‘kinetic’ mass instead with

\[
\frac{d\overline{m}_Q(\mu)}{d\mu} = -\frac{16\alpha_s(\mu)}{3\pi} - \frac{4\alpha_s(\mu)}{3\pi} \frac{\mu}{m_Q} + ..., \tag{182}
\]

which has a linear scale dependence in the infrared. It is this kinetic mass I will use in the following. Its value had been extracted from

\[
e^+e^- \rightarrow \Upsilon(4S) \rightarrow H_bH'_bX \tag{183}
\]

before 2002 by different authors with better than about 2% accuracy [68] based on an original idea of M. Voloshin. Their findings expressed in terms of the kinetic mass can be summarized as follows:

\[
\langle m_b(1 \text{ GeV})\rangle_{\Upsilon(4S)\rightarrow bb} = 4.57 \pm 0.08 \text{ GeV} \tag{184}
\]

Charmonium sum rules yield

\[
m_c(m_c) \approx 1.25 \pm 0.15 \text{ GeV}. \tag{185}\]

The HQE allows to relate the difference \( m_b - m_c \) to the ‘spin averaged’ beauty and charm meson masses and the higher order HQP [58]:

\[
m_b - m_c = \langle M_B \rangle - \langle M_D \rangle + \left( \frac{1}{2m_c} - \frac{1}{2m_b} \right) \mu^2 + ... \simeq 3.50 \text{ GeV} + 40 \text{ MeV} \cdot \frac{\mu^2 - 0.5 \text{ (GeV)}^2}{0.1 \text{ (GeV)}^2} ... \tag{186}
\]

Yet this relation is quite vulnerable since it is dominantly an expansion in \( 1/m_c \) rather than \( 1/m_b \) and nonlocal correlators appear in order \( 1/m_c^2 \). Therefore one is ill-advised to impose this relation a priori. One is of course free to consider it a posteriori.

- **Chromomagnetic moment**: Its value can be inferred quite reliably from the hyperfine splitting in the \( B^* \) and \( B \) masses:

\[
\mu^2_G(1 \text{ GeV}) \simeq \frac{3}{2} \left[ M^2(B^*) - M^2(B) \right] \simeq 0.35 \pm 0.03 \text{ (GeV)}^2 \tag{187}
\]

- **Kinetic moment**: The situation here is not quite so definite. We have a rigorous lower bound from the SV sum rules [69]:

\[
\mu^2_K(\mu) \geq \mu^2_G(\mu) \tag{188}
\]

68
for any $\mu$; QCD sum rules yield
\[ \mu_s^2(1 \text{ GeV}) \simeq 0.45 \pm 0.1 \text{ (GeV)}^2 \] (189)

- *Darwin and LS terms:* The numbers are less certain still for those. The saving grace is that their contributions are reduced in weight, since they represent $O(1/m_Q^3)$ terms.
\[ \rho_D^3(1 \text{ GeV}) \sim 0.1 \text{ (GeV)}^3 \]

4.3 First Tests: Weak Lifetimes and SL Branching Ratios

Let me begin with three general statements:

- Within the SM the semileptonic widths have to coincide for $D^0$ and $D^+$ mesons and for $B_d$ and $B_u$ mesons up to small isospin violations, since the semileptonic transition operators for $b \to l\nu c$ and $c \to l\nu s$ are isosinglets. The ratios of their semileptonic branching ratios are therefore equal to their lifetime ratios to a very good approximation:
\[
\frac{\text{BR}_{SL}(B^+)}{\text{BR}_{SL}(B_d)} = \frac{\tau(B^+)}{\tau(B_d)} + O \left( \frac{|V_{ub}|}{|V_{cb}|} \right)^2,
\]
\[
\frac{\text{BR}_{SL}(D^+)}{\text{BR}_{SL}(D_0)} = \frac{\tau(D^+)}{\tau(D_0)} + O \left( \frac{|V_{cd}|}{|V_{cs}|} \right)^2.
\] (191)

For dynamical rather than symmetry reasons such a relation can be extended to $B_s$ and $D_s$ mesons [70]:
\[
\frac{\text{BR}_{SL}(B_s)}{\text{BR}_{SL}(B_d)} \simeq \frac{\tau(B_s)}{\tau(B_d)},
\] (192)

where $\bar{\tau}(B_s)$ denotes the average of the two $B_s$ lifetimes.

- Yet the semileptonic widths of heavy flavour baryons will not be universal for a given flavour. The ratios of their semileptonic branching ratios will therefore not reflect their lifetime ratios. In particular for the charmed baryons one predicts large differences in their semileptonic widths [71].

- It is more challenging for theory to predict the absolute value of a semileptonic branching ratio than the ratio of such branching ratios.

4.3.1 Charm lifetimes

The lifetimes of all seven $C = 1$ charm hadrons have been measured now with the FOCUS experiment being the only one that has contributed to all seven lifetimes. In Table 3 the predictions based on the HQE (together with brief theory comments) are juxtaposed to the data [72]. While a priori the HQE might be expected to fail even on the semiquantitative level since $\mu_{\text{had}}/m_c \sim 1/2$ is an uncomfortably large expansion parameter, it works surprisingly well in describing the lifetime ratios even for baryons except for $\tau(\Xi_c^+)$ being
about 50% longer than predicted. This agreement should be viewed as quite nontrivial, since these lifetimes span more than an order of magnitude between the shortest and longest: $\tau(D^+)/\tau(\Omega_c) \approx 14$. It provides one of the better arguments for charm acting like a heavy quark at least in cases, when the leading nonperturbative correction is of order $1/m_c^2$ rather than $1/m_c$.

The SELEX collaboration has reported candidates for weakly decaying double charm baryons. It is my judgment that those candidates cannot be $C = 2$ baryons since their reported lifetimes are too short and do not show the expected hierarchy [72].

### 4.3.2 Beauty lifetimes

Theoretically one is on considerably safer ground when applying the HQE to lifetime ratios of beauty hadrons, since the expansion parameter $\mu_{\text{had}}/m_b \sim 1/7$ is small compared to unity. The HQE provided predictions in the old-fashioned sense; i.e., it produced them before data with significant accuracy were known.

Several comments are in order to interpret the results:

- The $B^+ - B_d$ lifetime ratio has been measured now with better than 1% accuracy – and the very first prediction based on the HQE was remarkably on target [73].

- The most dramatic deviation from a universal lifetime for $B = 1$ hadrons has emerged in $B_c$ decays. Their lifetime is only a third of the other beauty lifetimes – again in full agreement with the HQE prediction. That prediction is actually less obvious than it might seem. For the observed $B_c$ lifetime is close to the charm lifetime as given by $\tau(D^0)$, and that is what one would expect already in a naive parton model treatment, where $\Gamma(b\bar{c}) \approx \Gamma(c) \cdot [1 + \Gamma(b)/\Gamma(c)]$. However it had been argued that inside such a tightly bound state the $b$ and $c$ quark masses had to be replaced by effective masses reduced by the (same) binding energy: $m_{\text{f}}^{\text{eff}} = \ldots$

| $1/m_c$ expect. | theory comments | data     |
|-----------------|----------------|----------|
| $\tau(D^+)/\tau(D^0)$ | $\sim 1 + \left(\frac{\Gamma_{D^0}}{200 \text{ MeV}}\right)^2 \sim 2.4$ | PI dominant |
| $\tau(D^+)/\tau(D^0)$ | $0.9 - 1.3[1.0 - 1.07]$ | with [without] WA |
| $\tau(\Lambda_c^+)/\tau(D^0)$ | $\sim 0.5$ | quark model matrix elements |
| $\tau(\Xi_c^+)/\tau(D^0)$ | $\sim 1.3 - 1.7$ | ditto |
| $\tau(\Xi_c^+)/\tau(\Xi_c^0)$ | $\sim 1.6 - 2.2$ | ditto |
| $\tau(\Omega_c)/\tau(\Xi_c^0)$ | $\sim 2.8$ | ditto |
| $\tau(\Omega_c)/\tau(\Omega_c)$ | $\sim 4$ | ditto |
| $\tau(\Omega_c)/\tau(\Xi_c^0)$ | $\sim 1.4$ | ditto |

Table 3: The weak lifetime ratios of $C = 1$ hadrons
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
 & \textit{1/m}_b & \textit{expect.} & \textit{theory comments} & \textit{data} \\
\hline
\nicefrac{\tau(B^+)}{\tau(B_d)} & \sim 1 + 0.05 \left( \frac{f_{B^+}}{200 \text{ MeV}} \right)^2 & '92 [73] & \text{PI in } \tau(B^+) & 1.076 \pm 0.008 [77] \\
 & 1.06 \pm 0.02 [55] & & \text{fact. at low scale 1GeV} & \\
\hline
\nicefrac{\tau(B_s)}{\tau(B_d)} & 1 \pm \mathcal{O}(0.01) & '94 [70] & & 0.920 \pm 0.030 [77] \\
\hline
\nicefrac{\tau(A_b)}{\tau(B_d)} & \geq 0.9 & '93 [74] & \text{quark model} & 0.806 \pm 0.047 '04 [77] \\
 & \simeq 0.94 & \geq 0.88 & '96 [75,76] & \text{matrix elements} & 0.944 \pm 0.089 '05 [78] \\
\hline
\tau(B_c) & \sim (0.3 - 0.7) \text{ psec} & '94ff [79] & \text{largest lifetime diff.} & 0.45 \pm 0.12 \text{ psec} [77] \\
 & & \text{no } 1/m_Q \text{ term crucial} & & \\
\hline
\frac{\Delta \Gamma(B_s)}{\Gamma(B_s)} & 22\% \cdot \left( \frac{f(B_s)}{220 \text{ MeV}} \right)^2 & '87 [54] & \text{less reliable} & 0.65 \pm 0.3 \text{ CDF} \\
 & 12 \pm 5\% '04 [55] & & \text{than } \Delta M(B_s) & 0.23 \pm 0.17 \text{ D0} \\
\hline
\end{tabular}
\caption{The weak lifetime ratios of } B = 1 \text{ hadrons}
\end{table}

\( m_b - \text{B.E.}, \ m_c^{\text{eff}} = m_c - \text{B.E.} \) with \( \text{B.E.} \sim \mathcal{O}(\Lambda_{QCD}) \). This would prolong the weak lifetimes of the two quark greatly, since those depend on the fifth power of the quark masses and would do so much more for the charm transition than for the beauty one. Yet such an effect would amount to a correction of order \( 1/m_Q \), which is not allowed by the OPE, as explained above at the end of Sect.4.2.1; the more detailed argument can be found in Ref. [80].

- A veritable saga is emerging with respect to \( \tau(\Lambda_b) \). The first prediction stated [74] that \( \tau(\Lambda_b)/\tau(B_d) \) could not fall below 0.9. A more detailed analysis led to two conclusions [75], namely that the HQE most likely leads to

\[
\frac{\tau(\Lambda_b)}{\tau(B_d)} \simeq 0.94
\]

with an uncertainty of a few percent, while a lower bound had to hold

\[
\frac{\tau(\Lambda_b)}{\tau(B_d)} \geq 0.88 .
\]

A violation of this bound would imply that we need a new paradigm for evaluating at least baryonic matrix elements.

There are actually two questions one can ask concerning \( \tau(\Lambda_b)/\tau(B_d) \):

1. What is theoretically the most likely value for \( \tau(\Lambda_b)/\tau(B_d) \)?
2. How much lower can one reasonably push it?

While there is a connection between those two questions, they clearly should be distinguished. Most theoretical analyses – employing quark models, QCD sum rules or lattice studies – agree on the first question, namely that the ratio is predicted...
to lie above 0.90. Yet the data have for many years pointed to a significantly lower value $\sim 0.80$. This apparent discrepancy has given rise to the second question listed above. Ref. [75] provided a carefully reasoned answer to it. Ref. [?] stated a value of $0.86 \pm 0.05$, which is sometimes quoted as the theory prediction. I strongly object to viewing this value as the answer to the first question above; one might consider it as a response to the second question, although even then I remain skeptical of it.

The new CDF result seems to reshuffle the cards. The question is whether it is just a high fluctuation – implying a worrisome discrepancy between theory and experiment – or represents a new trend to be confirmed in the future, which would represent an impressive ‘comeback’ success for the HQE.

No matter what the final verdict will be on $\tau(\Lambda_b)$, it is important to measure also $\tau(\Xi^0_b)$ and $\tau(\Xi^-_b)$ – either to confirm success or diagnose failure. One expects [82]:

$$\tau(\Xi^0_b) \simeq \tau(\Lambda_b) < \tau(B_d) < \tau(\Xi^-_b) \,
\tag{195}$$

where the ‘$<$’ signs indicate an about 7% difference. If the $\Lambda_b$-$B_d$ lifetime difference were larger than predicted, one would like to know whether the whole lifetime hierarchy of Eq.(195) is stretched out – say ‘$<$’ in $\tau(\Lambda_b) < \tau(B_d) < \tau(\Xi^-_b)$ represents differences of 10 % or even more – or whether the splittings in the baryon lifetimes are as expected, yet their overall values reduced relative to $\tau(B_d)$.

- The original prediction that $\tau(B_d)/\tau(B_s)$ is unity within 1 - 2 % [70, 74] has been confirmed by subsequent authors. Yet the data have stubbornly remained somewhat low. This measurement deserves great attention and effort. While I consider the prediction to be on good footing, it is based on an evaluation of a complex dynamical situation rather than a theorem or even symmetry. Establishing a discrepancy between theory and experiment here would raise some very intriguing questions.

- The theoretical evaluation of $\Delta \Gamma(B_s)$ and the available data has already been given in Sect. 3.5.1.

4.4 The $V(cb)$ ‘Saga’ – A Case Study in Accuracy

4.4.1 Inclusive Semileptonic $B$ Decays

The value of $|V(cb)|$ is extracted from $B \rightarrow l\nu X_c$ in two steps.

**A:** One expresses $\Gamma(B \rightarrow l\nu X_c)$ in terms of the HQP – quark masses $m_b$, $m_c$ and the expectation values of local operators $\mu^2_b$, $\mu^2_c$, $\rho^2_b$ and $\rho^2_{LS}$ – as accurately as possible, namely through $O(1/m_Q^3)$ and to all orders in the BLM treatment for the partonic contribution. Having precise values for these HQP is not only of obvious use for extracting $|V(cb)|$ and $|V(ub)|$, but also yields benchmarks for how much numerical control lattice QCD provides us over nonperturbative dynamics.

**B:** The numerical values of these HQP are extracted from the shapes of inclusive lepton distributions as encoded in their normalized moments. Two types of moments
have been utilized, namely lepton energy and hadronic mass moments. While the former are dominated by the contribution from the ‘partonic’ term $\propto \langle B|\bar{b}b|B \rangle$, the latter are more sensitive to higher nonperturbative terms $\mu^2_x$ and $\mu^2_G$ and thus have to form an integral part of the analysis.

Executing the first step in the so-called kinetic scheme and inserting the experimental number for $\Gamma(B \to l\nu X_c)$ one arrives at [83]

$$\frac{|V(cb)|}{0.0417} = D_{exp} \cdot (1 + \delta_{th})[1 + 0.3(\alpha_S(m_b) - 0.22)] [1 - 0.66(m_b - 4.6) + 0.39(m_c - 1.15) + 0.013(\mu^2_x - 0.4) + 0.05(\mu^2_G - 0.35) + 0.09(\rho^3_D - 0.2) + 0.01(\rho^3_{LS} + 0.15)] ,$$

$$D_{exp} = \sqrt{\frac{BR_{SL}(B)}{0.105}} \frac{1.55 \text{ ps}}{\tau_B}$$

(196)

where all the HQP are taken at the scale 1 GeV and their ‘seed’ values are given in the appropriate power of GeV; the theoretical error at this point is given by

$$\delta_{th} = \pm 0.5%|_{\text{pert}} \pm 1.2%|_{\text{hwc}} \pm 0.4%|_{\text{hpc}} \pm 0.3%|_{\text{IC}}$$

(197)

reflecting the remaining uncertainty in the Wilson coefficient of the leading operator $\bar{b}b$, as yet uncalculated perturbative corrections to the Wilson coefficients of the chromomagnetic and Darwin operators, higher order power corrections including duality violations in $\Gamma_{SL}(B)$ and nonperturbative effects due to operators containing charm fields, respectively. Concerning the last item, in Ref. [83] an error of 0.7 % was stated. A dedicated analysis of such IC effects allowed to reduce this uncertainty down to 0.3 % [63].

BaBar has performed the state-of-the-art analysis of several lepton energy and hadronic mass moments [84] obtaining an impressive fit with the following HQP in the kinetic scheme [85]:

$$m_b(1 \text{ GeV}) = (4.61 \pm 0.068) \text{ GeV}, \ m_c(1 \text{ GeV}) = (1.18 \pm 0.092) \text{ GeV}$$

(198)

$$m_b(1 \text{ GeV}) - m_c(1 \text{ GeV}) = (3.436 \pm 0.032) \text{ GeV}$$

(199)

$$\mu^2_x(1 \text{ GeV}) = (0.447 \pm 0.053) \text{ GeV}^2, \ \mu^2_G(1 \text{ GeV}) = (0.267 \pm 0.067) \text{ GeV}^2$$

(200)

$$\rho^3_D(1 \text{ GeV}) = (0.195 \pm 0.029) \text{ GeV}^3$$

(201)

$$|V(cb)|_{incl} = 41.390 \cdot (1 \pm 0.021) \times 10^{-3}$$

(202)

The DELPHI collab. has refined their pioneering study of 2002 [86] obtaining [87]:

$$|V(cb)|_{incl} = 42.1 \cdot (1 \pm 0.014)\text{meas} \pm 0.014|h_{fit} \pm 0.015|h_{th}) \times 10^{-3}$$

(203)

A comprehensive analysis of all relevant data from $B$ decays, including from $B \to \gamma X$ yields the results listed in Table 5 [60], where they are compared to their predicted values. Some had already been given in Table 2. With these HQP one arrives at

$$\langle |V(cb)|_{incl} \rangle = 41.96 \cdot (1 \pm 0.0055)_\text{exp} \pm 0.0083|h_{HQE} \pm 0.014|h_{SL}) \times 10^{-3}$$

(204)

For a full appreciation of these results one has to note the following:
| Heavy Quark Parameter | value from \( B \to l\nu X_c/\gamma X \) predict. from other observ. |
|------------------------|---------------------------------------------------------------|
| \( m_b(1 \text{ GeV}) \) | \((4.59 \pm 0.025)_{\text{exp}} \pm 0.030|_{\text{HQE} \text{ GeV}} \) \( = (4.57 \pm 0.08) \text{ GeV, Eq.}(184) \) |
| \( m_c(1 \text{ GeV}) \) | \((1.142 \pm 0.037)_{\text{exp}} \pm 0.045|_{\text{HQE} \text{ GeV}} \) \( = (1.25 \pm 0.15) \text{ GeV, Eq.}(185) \) |
| \( [m_b - m_c](1 \text{ GeV}) \) | \((3.446 \pm 0.025) \text{ GeV} \) \( = (3.46 \pm X) \text{ GeV, Eq.}(186) \) |
| \( [m_b - 0.67m_c](1 \text{ GeV}) \) | \((3.82 \pm 0.017) \text{ GeV} \) |
| \( \mu_{\pi}^2(1 \text{ GeV}) \) | \((0.297 \pm 0.024)_{\text{exp}} \pm 0.046|_{\text{HQE} \text{ GeV}^2} \) \( = (0.35 \pm 0.03) \text{ GeV}^2, \text{ Eq.}(187) \) |
| \( \mu_{\pi}^2(1 \text{ GeV}) \) | \((0.401 \pm 0.019)_{\text{exp}} \pm 0.035|_{\text{HQE} \text{ GeV}^2} \geq \mu_{\pi}^2(1 \text{ GeV}), \text{ Eq.}(188) \) |
| \( \rho_D^3(1 \text{ GeV}) \) | \((0.174 \pm 0.009)_{\text{exp}} \pm 0.022|_{\text{HQE} \text{ GeV}^3} \) \( \sim +0.1 \text{ GeV}^3 \), Eq.(190) |
| \( \rho_{L_S}^3(1 \text{ GeV}) \) | \(-(0.183 \pm 0.054)_{\text{exp}} \pm 0.071|_{\text{HQE} \text{ GeV}^3} \) \( \sim -0.1 \text{ GeV}^3 \), Eq.(190) |

Table 5: The 2005 values of the HQP obtained from a comprehensive analysis of \( B \to l\nu X_c \) and \( B \to \gamma X \) [60] and compared to predictions

- With just these six parameters one obtains an excellent fit to several energy and hadronic mass moments even for different values of the lower cut on the lepton or photon energy, as explained in Prof. Lanceri’s lectures. Varying those lower cuts also provides more direct information on the respective energy spectra beyond the moments.

- Even better the fit remains very good, when one ‘seeds’ two of these HQP to their predicted values, namely \( \mu_{\pi}^2(1 \text{ GeV}) = 0.35 \pm 0.03 \text{ GeV}^2 \) as inferred from the \( B^* - B \) hyperfine mass splitting and \( \rho_{L_S}^3 = -0.1 \text{ GeV}^3 \) allowing only the other four HQP to float.

- These HQP are treated as free fitting parameters. It could easily have happened that they assume unreasonable or even unphysical values. Yet they take on very special values fully consistent with all constraints that can be placed on them by theoretical means as well as other experimental input. To cite but a few examples:

  - The value for \( m_b \) inferred from the weak decay of a \( B \) meson agrees completely within the stated uncertainties with what has been derived from the electromagnetic and strong production of \( b \) hadrons just above threshold.
  
  - The rigorous inequality \( \mu_{\pi}^2 > \mu_{\pi}^2 \), which had not been imposed as a constraint, is satisfied.
  
  - \( \mu_{\pi}^2 \) indeed emerges with the correct value, as does \( \mu_{\pi}^2 \).

- \( m_b - m_c \) agrees very well with what one infers from the spin-averaged \( B \) and \( D \) meson masses. However this \textit{a posteriori} agreement does \textit{not} justify imposing it as an \textit{a priori} constraint. For the mass relation involves an expansion in \( 1/m_c \), which is of less than sterling reliability. Therefore I have denoted its uncertainty by \( X \).
The 1\% error in $m_b$ taken at face value might suggest that it alone would generate more than a 2.5\% uncertainty in $|V(cb)|$, i.e. by itself saturating the total error given in Eq.(204). The resolution of this apparent contradiction is as follows. The dependance of the total semileptonic width and also of the lowest lepton energy moments on $m_b$ & $m_c$ can be approximated by $m_b^2(m_b - m_c)^3$ for the actual quark masses; for the leading contribution this can be written as $\Gamma_{SL}(B) \propto (m_b - \frac{2}{3}m_c)^5$. From the values for $m_b$ and $m_c$, Eq.(198), and their correlation given in [84] one derives

$$m_b(1\text{ GeV}) - 0.67m_c(1\text{ GeV}) = (3.819\pm 0.017)\text{ GeV} = 3.819\cdot (1\pm 0.45\%)\text{ GeV.} \quad (205)$$

I.e., it is basically this peculiar combination that is measured directly through $\Gamma_{SL}(B)$, and thus its error is so tiny. It induces an uncertainty of 1.1\% into the value for $|V(cb)|$.

Eq.(205) has another important use in the future, namely to provide a very stiff validation challenge to lattice QCD’s determinations of $m_b$ and $m_c$.

With all these cross checks we can defend the smallness of the stated uncertainties. The analysis of Ref. [88] arrives at similar numbers (although I cannot quite follow their error analysis).

More work remains to be done: (i) The errors on the hadronic mass moments are still sizable; decreasing them will have a significant impact on the accuracy of $m_b$ and $\mu_2^2$. (ii) As discussed in more detail below, imposing high cuts on the lepton energy degrades the reliability of the theoretical description. Yet even so it would be instructive to analyze at which cut theory and data part ways. I will return to this point below. (iii) As another preparation for $V(ub)$ extractions one can measure $q^2$ moments or mass moments with a $q^2$ cut to see how well one can reproduce the known $V(cb)$.

4.4.2 Exclusive Semileptonic $B$ Decays

While it is my judgment that the most precise value for $|V(cb)|$ can be extracted from $B \to l\nu X_c$, this does not mean that there is no motivation for analyzing exclusive modes. On the contrary: the fact that one extracts a value for $|V(cb)|$ from $B \to l\nu D^*$ at zero recoil fully consistent within a smallish uncertainty represents a great success since the systematics experimentally as well as theoretically are very different:

$$|V(cb)|_{B\to D^*} = 0.0416 \cdot (1 \pm 0.022_{[\text{exp}]} \pm 0.06_{[\text{th}}) \quad \text{for} \quad F_{B\to D^*}(0) = 0.90 \pm 0.05 \quad (206)$$

It has been suggested [51] to treat $B \to l\nu D$ with the ‘BPS expansion’ based on $\mu_2^2 \simeq \mu^2_{\pi}$ and extract $|V(cb)|$ with a theoretical error not larger than $\sim 2\%$. It would be most instructive to compare the formfactors and their slopes found in this approach with those of LQCD [89].

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4.5 The Adventure Continues: $V(ub)$

There are several lessons we can derive from the $V(cb)$ saga: (i) Measuring various moments of $B \to l\nu X_u$ and extracting HQP from them is a powerful tool to strengthen confidence in the analysis. Yet it is done for validation purposes only. For there is no need to ‘reinvent the wheel’: When calculating the width and (low) moments of $B \to l\nu X_u$ one has to use the values of the HQP as determined in $B \to l\nu X_c$. (ii) $\Gamma(B \to l\nu X_u)$ is actually under better theoretical control than $\Gamma(B \to l\nu X_c)$ since the expansion parameter is smaller $\frac{\mu_{had}}{m_b}$ vs. $\frac{\mu_{had}}{m_b-m_c}$ – and $\mathcal{O}(\alpha_s^2)$ corrections are known exactly.

On the Impact of Cuts: In practice there arises a formidable complication: to distinguish $b \to u$ from the huge $b \to c$ background, one applies cuts on variables like lepton energy $E_l$, hadronic mass $M_X$, the lepton-pair invariant mass $q^2$. As a general rule the more severe the cut, the less reliable the theoretical calculation becomes. More specifically the imposition of a cut introduces a new dimensional scale called ‘hardness’ $Q$ [90]. Nonperturbative contributions emerge scaled by powers of $1/Q$ rather than $1/m_b$. If $Q$ is much smaller than $m_b$ such an expansion becomes unreliable. Furthermore the OPE cannot capture terms of the form $e^{-Q/\mu}$. While these are irrelevant for $Q \sim m_b$, they quickly gain relevance when $Q$ approaches $\mu$. Ignoring this effect would lead to a ‘bias’, i.e. a systematic shift of the HQP away from their true values.

This impact has been studied for radiative $B$ decays with their simpler kinematics in a pilot study [90] and a detailed analysis [91] of the average photon energy and its variance. The first provides a measure mainly of $m_b/2$, the latter of $\mu_\pi^2/12$. These biases were found to be relevant down to $E_{cut} = 1.85$ GeV and increasing quickly above 2 GeV. While the existence of such effects is of a general nature, the estimate of their size involves model dependent elements. Yet as long as those corrections are of moderate size, they can be considered reliable. Once they become large, we are losing theoretical control. Fig.4 shows data for the average photon energy and its variance for different lower cuts on the photon energy from CLEO, BABAR and BELLE compared to the OPE predictions without and with bias corrections on the left and right, respectively. The comparison shows the need for those bias corrections and them being under computational control over a sizable range of $E_{cut}$. Even more important than providing us with possibly more accurate values for $m_b$ and $\mu_\pi^2$, these studies enhance confidence in our theoretical tools.

These findings lead to the following conclusions: (i) As far as theory is concerned there is a high premium on keeping the cuts as low as possible. (ii) Such cuts introduce biases in the HQP values extracted from the truncated moments; yet within a certain range of the cut variables those biases can be corrected for and thus should not be used to justify inflating the theoretical uncertainties. (iii) In any case measuring the moments as functions of the cuts provides powerful cross checks for our theoretical control.

‘Let a Thousand Blossoms Bloom’: Several suggestions have been made for cuts to suppress the $b \to c$ background to managable proportions. None provides a panacea. The most straightforward one is to focus on the lepton energy endpoint region; however it captures merely a small fraction of the total $b \to u$ rate, which can be estimated only with considerable model dependance. This model sensitivity can be moderated with
Figure 4: The first and second moments of the photon energy in $B \to \gamma X$ compared to OPE predictions without and with bias corrections. The inner band indicates the experimental uncertainties only; the outer bands add the theoretical ones; from Ref. [60].

Information on the heavy quark distribution function inferred from $B \to \gamma X$. Furthermore weak annihilation contributes only in the endpoint region and with different weight in $B_d$ and $B_u$ decays [70]. Thus the lepton spectra have to be measured separately for charged and neutral $B$ decays.

Measuring the hadronic recoil mass spectrum up to a maximal value $M_X^{\text{max}}$ captures the lion share of the $b \to u$ rate if $M_X^{\text{max}}$ is above 1.5 GeV; yet it is still vulnerable to theoretical uncertainties in the very low $q^2$ region. This problem can be addressed in two different ways: adopting Alexander the Great’s treatment of the Gordian knot one can impose a lower cut on $q^2$ or one can describe the low $q^2$ region with the help of the measured energy spectrum in $B \to \gamma X$ for $1.8 \text{ GeV} \leq E_\gamma \leq 2.0 \text{ GeV}$. Alternatively one can apply a combination of cuts. Studying $B_d$ and $B_u$ decays is still desirable, yet not as essential as for the previous case.

In any case one should not restrict oneself to a fixed cut, but vary the latter over some reasonable range and analyze to which degree theory can reproduce this cut dependence to demonstrate control over the uncertainties.
There is not a single ‘catholic’ path to the promised land of a precise value for $|V(ub)|$; presumably many paths will have to be combined [94]. Yet it seems quite realistic that the error can be reduced to about 5% over the next few years.

4.6 Summary of Lecture IV

The central theme of this lecture was the New Heavy Flavour Paradigm, namely adding high accuracy to high sensitivity in our studies of the decays of heavy flavour hadrons. The motivation for this ambitious goal was not to establish bragging rights, but the realization that we cannot count on New Physics affecting $B$ decays in a numerically massive way. We see now that this goal is attainable. We are extracting CKM parameters with an accuracy that would have seemed unrealistic, if not even frivolous less than ten years ago:

- $\delta |V(cb)| \simeq 2.5\%$ ‘now’ and $\simeq 1\%$ ‘soon’;
- $\delta |V(ub)| \simeq 5\%$ conceivable in the foreseeable future.

These numbers are based on detailed error budgets given even by theorists. Even more importantly those estimates of the uncertainties can be defended. That ability rests on two pillars:

- A large number of a priori independent observables in $B$ decays is described in terms of a rather small number of basic parameters; i.e., there are many overconstraints.
- The fit values for these parameters are not arbitrary, but satisfy reliable theoretical relations without those being imposed and also match up with their determinations in independent systems.

The progress we have achieved was based on two key elements:

- a robust theoretical framework subjected to the challenges of
- high quality data.

It was characterized by a painstaking and comprehensive analysis of the theoretical tools available rather than revolutionary breakthroughs. Those are of course always welcome – yet are not a conditio sine qua non for further progress.

The general lesson learnt from this development of heavy quark theory can be expressed also in a less scholarly way: If you rub intriguing data sufficiently long theorists under their noses, some of them will take up the challenge; once they do and score the first success, others will follow suit. This process, due to the competitive environment, might not play out like the game of cricket was supposed to be played – but it works!
5  Lecture V: Searching for a New Paradigm in 2005 & beyond Following Samuel Beckett’s Dictum

5.1  On the Incompleteness of the SM

As described in the previous lectures the SM has scored novel – i.e., qualitatively new – successes in the last few years in the realm of flavour dynamics. Due to the very peculiar structure of the latter they have to be viewed as amazing. Yet even so the situation can be characterized with a slightly modified quote from Einstein:

"We know a lot – yet understand so little."

I.e., these successes do not invalidate the general arguments in favour of the SM being incomplete – the search for New Physics is as mandatory as ever.

You have heard about the need to search for New Physics before and what the outcome has been of such efforts so far, have you not? And it reminds you of a quote by Samuel Beckett:

"Ever tried? Ever failed?
No matter.
Try again. Fail again. Fail better."

Only an Irishman can express profound skepticism concerning the world in such a poetic way. Beckett actually spent most of his life in Paris, since Parisians like to listen to someone expressing such a world view, even while they do not share it. Being in the service of Notre Dame du Lac, the home of the ‘Fighting Irish’, I cannot just ignore such advice.

My colleague and friend Antonio Masiero likes to say: ”You have to be lucky to find New Physics.” True enough – yet let me quote someone who just missed by one year being a fellow countryman of Masiero, namely Napoleon, who said: ”Being lucky is part of the job description for generals.” Quite seriously I think that if you as an high energy physicist do not believe that someday somewhere you will be a general – maybe not in a major encounter, but at least in a skirmish – then you are frankly in the wrong line of business.

My response to these concerns is: "Cheer up – we know there is New Physics – we will not fail forever!” I will marshall the arguments – compelling ones in my judgment – that point to the existence of New Physics.

5.1.1  Theoretical Shortcomings

These arguments have been given already in the beginning of Lecture I.

- Quantization of electric charge: While electric charge quantization

\[ Q_e = 3Q_d = -\frac{3}{2}Q_u \]  

(207)
is an essential ingredient of the SM – it allows to vitiate the ABJ anomaly – it
does not offer any understanding. It would naturally be explained through Grand
Unification at very high energy scales implemented through, e.g., $SO(10)$ gauge
dynamics. I call this the ‘guaranteed New Physics’ or $gNP$.

- **Family Replication and CKM Structure**: We infer from the observed width of $Z^0$
decays that there are three (light) neutrino species. The hierarchical pattern of
CKM parameters as revealed by the data is so peculiar as to suggest that some
other dynamical layer has to underlie it. I refer to it as ‘strongly suspected New
Physics’ or $ssNP$. We are quite in the dark about its relevant scales. Saying we
pin our hopes for explaining the family replication on Super-String or M theory is
a scholarly way of saying we have hardly a clue what that $ssNP$ is.

- **Electroweak Symmetry Breaking and the Gauge Hierarchy**: What are the dynamics
driving the electroweak symmetry breaking of $SU(2)_L \times U(1) \rightarrow U(1)_{QED}$. How
can we tame the instability of Higgs dynamics with its quadratic mass divergences?
I find the arguments compelling that point to New Physics at the $\sim 1$ TeV scale –
like low-energy SUSY; therefore I call it the ‘confidently predicted’ New Physics or
$cpNP$.

- Furthermore the more specific ‘Strong CP Problem’ of QCD has not been resolved.
Similar to the other shortcomings it is a purely theoretical problem in the sense that
the offending coefficient for the $P$ and $CP$ odd operator $\bar{G} \cdot G$ can be fine-tuned to
zero , see Sect.1.2.2, – yet in my eyes that is not a flaw.

**5.1.2 Experimental Signs**

Strong, albeit not conclusive (by itself) evidence for neutrino oscillations comes from the
KAMLAND and K2K experiments in Japan studying the evolution of neutrino beams on
earth.

Yet compelling experimental evidence for the SM being incomplete comes from ‘heavenly
signals’, namely from astrophysics and cosmology.

- **The Baryon Number of the Universe**: as explained in Prof. Dolgov’s lectures one
finds only about one baryon per $10^9$ photons with the latter being mostly in the
cosmic background radiation; there is no evidence for primary antimatter.

  ⊗ We know standard CKM dynamics is irrelevant for the Universe’s baryon number.

  ⊕ Therefore New Physics has to exist.

  ⊕ The aforementioned New $CP$ Paradigm tells us that $CP$ violating phases can be
large.

- **Dark Matter**: Analysis of the rotation curves of stars and galaxies reveal that there
is a lot more ‘stuff’ – i.e. gravitating agents – out there than meets the eye. About
a quarter of the gravitating agents in the Universe are such dark matter, and they have to be mostly nonbaryonic.
⊕ The SM has no candidate for it.

• **Solar and Atmospheric $\nu$ Anomalies:** The sun has been ‘seen’ by Super-Kamiokande in the light of neutrinos, as shown in Fig.5. Looking carefully one realizes that the sun looks paler than it should: more than half of the originally produced $\bar{\nu}_e$ disappear on the way to the earth by changing their identity. Muon neutrinos produced in the atmosphere perform a similar disappearance act.

![Figure 5: The sun in the light of its neutrino emission as seen by the Super-Kamiokande detector; from Ref. [95].](image)

These disappearances have to be attributed predominantly to neutrino oscillations (rather than neutrino decays). This requires neutrinos to carry nondegenerate masses.

• **Dark Energy:** Type 1a supernovae are considered ‘standard candles’; i.e. considering their real light output known allows to infer their distance from their apparent brightness. When in 1998 two teams of researchers studied them at distance scales of about five billion light years, they found them to be fainter as a function of their redshift than what the conventional picture of the Universe’s decelerating expansion would yield. Unless gravitational forces are modified over cosmological distances, one has to conclude the Universe is filled with an hitherto completely unknown
Observation                     Lesson learnt
\(\tau - \theta\) Puzzle        \(P\) violation
production rate \(\gg\) decay rate concept of families
suppression of flavour changing neutral currents GIM mechanism & existence of charm
\(K_L \to \pi\pi\)                  \(CP\) violation & existence of top

Table 6: On the History of \(\Delta S \neq 0\) Studies

agent *accelerating* the expansion. A tiny, yet non-zero cosmological constant would apparently ‘do the trick’ – yet it would raise more fundamental puzzles.

These heavenly signals are unequivocal in pointing to New Physics, yet leave wide open the nature of this New Physics.

Thus we can be assured that New Physics exists ‘somehow’ ‘somewhere’, and quite likely even ‘nearby’, namely around the TeV scale; above I have called the latter \(cpNN\). The LHC program and the Linear Collider project are justified – correctly – to conduct campaigns for \(cpNP\). That is unlikely to shed light on the \(ssNP\), though it might. Likewise I would not *count* on a comprehensive and detailed program of heavy flavour studies to shed light on the \(ssNP\) behind the flavour puzzle of the SM. Yet the argument is reasonably turned around: such a program will be essential to elucidate salient features of the \(cpNP\) by probing the latter’s flavour structure and having sensitivity to scales of order 10 TeV. One should keep in mind the following: one very popular example of \(cpNP\) is supersymmetry; *yet it represents an organizing principle much more than even a class of theories*. I find it unlikely we can infer all required lessons by studying only flavour diagonal transitions. Heavy flavour decays provide a powerful and complementary probe of \(cpNP\). Their potential to reveal something about the \(ssNP\) is a welcome extra not required for justifying efforts in that direction.

Accordingly I see a dedicated heavy flavour program as an essential complement to the studies pursued at the high energy frontier at the TEVATRON, LHC and hopefully ILC. I will illustrate this assertion in the remainder of this lecture.

5.2 \(\Delta S \neq 0\) – the ‘Established Hero’

The chapter on \(\Delta S \neq 0\) transitions is a most glorious one in the history of particle physics, as sketched in Table 6. We should note that all these features, which now are pillars of the SM, were New Physics *at that time*!

5.2.1 Future ‘Bread & Butter’ Topics

Detailed studies of radiative decays like \(K \to \pi\gamma\gamma\) and \(K \to \pi\pi\gamma\) will allow deeper probes of chiral perturbation theory. The lessons thus obtained might lead to a better treatment of long distance dynamics’ impact on the \(\Delta I = 1/2\) rule, \(\Delta M_K, \epsilon_K\) and \(\epsilon'\).
5.2.2 The ‘Dark Horse’

As discussed at the end of Lecture II in Sect. 2.3.7 a non-zero value for the $T$ odd moment

$$\text{Pol}_\perp(\mu) \equiv \frac{\langle \vec{s}(\mu) \cdot (\vec{p}(\mu) \times \vec{p}(\pi)) \rangle}{|\vec{p}(\mu) \times \vec{p}(\pi)|}$$

measured in $K^+ \to \mu^+\nu\pi^0$ would

- represent genuine $T$ violation (as long as it exceeded the order $10^{-6}$ level) and
- constitute prima facie evidence for $CP$ violation in scalar dynamics.

5.2.3 ‘Heresy’

The large $T$ odd correlation found in $K_L \to \pi^+\pi^-e^+e^-$ for the relative orientation of the $\pi^+\pi^-$ and $e^+e^-$ decay planes, see the discussion in Sect.2.4, is fully consistent with a $T$ violation as inferred from the $CP$ violation expressed through $\epsilon_K$ – yet it does not prove it [24]. In an unashamedly contrived scenario – something theorists usually avoid at great pains – one could reconcile the data on $K_L \to \pi^+\pi^-e^+e^-$ with $T$ invariance without creating a conflict with known data. Yet the $CPT$ violation required in this scenario would have to surface through [24]

$$\frac{\Gamma(K^+ \to \pi^+\pi^0) - \Gamma(K^- \to \pi^-\pi^0)}{\Gamma(K^+ \to \pi^+\pi^0) + \Gamma(K^- \to \pi^-\pi^0)} > 10^{-3}$$

5.2.4 The ‘Second Trojan War’: $K \to \pi\nu\bar{\nu}$

According to Greek Mythology the Trojan War described in Homer’s Iliad was actually the second war over Troja. In a similar vein I view the heroic campaign over $K^0 - \bar{K}^0$ oscillations – $\Delta M_K$, $\epsilon_K$ and $\epsilon'$ – as a first one to be followed by a likewise epic struggle over the two ultra-rare modes $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. This campaign has already been opened through the observation of the first through three events very roughly as expected within the SM. The second one, which requires $CP$ violation for its mere existence, so far remains unobserved at a level well above SM predictions. These reactions are like ‘standard candles’ for the SM: their rates are functions of $V(td)$ with a theoretical uncertainty of about 5% and 2% respectively, which is mainly due to the uncertainty in the charm quark mass as discussed in Prof. Littenberg’s lectures.

While their rates could be enhanced by New Physics greatly over their SM expectation, I personally find that somewhat unlikely for various reasons. Therefore I suggest one should aim for collecting ultimately about 1000 events of these modes to extract the value of $V(td)$ and/or identify likely signals of New Physics.
5.3 The ‘King Kong’ Scenario for New Physics Searches

This scenario can be formulated as follows: “One is unlikely to encounter King Kong; yet once it happens one will have no doubt that one has come across something quite out of the ordinary!”

What it means can be best illustrated with the historical precedent of $\Delta S \neq 0$ studies sketched above: the existence of New Physics can unequivocally be inferred if there is a qualitative conflict between data and expectation; i.e., if a theoretically ‘forbidden’ process is found to proceed nevertheless – like in $K_L \rightarrow \pi \pi$ – or the discrepancy between expected and observed rates amounts to several orders of magnitude – like in $K_L \rightarrow \mu^+\mu^-$ or $\Delta M_K$.

History might repeat itself in the sense that future measurements might reveal such qualitative conflicts, where the case for the manifestation of New Physics is easily made. This does not mean that such measurements will be easy – far from it, as will become obvious.

I have already mentioned one potential candidate for revealing such a qualitative conflict, namely the muon transverse polarization in $K_{\mu3}$ decays.

In $B$ decays on the other hand we cannot count on massive divergences between SM predictions and data, as explained in Lecture IV, and we will have to deal with more moderate quantitative differences. For one, many CP asymmetries are predicted to fall above 10%. It is unlikely that an experiment could ever establish any of those to be well below 1%.

5.3.1 Charm Decays

Charm dynamics is often viewed as physics with a great past – it was instrumental in driving the paradigm shift from quarks as mathematical entities to physical objects and in providing essential support for accepting QCD as the theory of the strong interactions – yet one without a future since the electroweak phenomenology for $\Delta C \neq 0$ transitions is decidedly on the ‘dull’ side: ‘known’ CKM parameters, slow $D^0 - \bar{D}^0$ oscillations, small CP asymmetries and extremely rare loop driven decays.

Yet more thoughtful observers have realized that the very ‘dullness’ of the SM phenomenology for charm provides us with a dual opportunity, namely to

- probe our quantitative understanding of QCD’s nonperturbative dynamics thus calibrating our theoretical tools for $B$ decays and
- perform almost ‘zero-background’ searches for New Physics.

Yet the latter statement of ‘zero-background’ has to be updated carefully since experiments over the last ten years have bounded the oscillation parameters $x_D, y_D$ to fall below very few % and direct CP asymmetries below several %.

One should take note that charm is the only up-type quark allowing the full range of probes for New Physics, including flavour changing neutral currents: while top quarks do not hadronize [61], in the $u$ quark sector you cannot have $\pi^0 - \pi^0$ oscillations and many CP asymmetries are already ruled out by CPT invariance. My
basic contention is the following: *Charm transitions are a unique portal for obtaining a novel access to flavour dynamics with the experimental situation being a priori favourable (except for the lack of Cabibbo suppression)!*

I will sketch such searches for New Physics in the context of $D^0 - D^0$ oscillations and CP violation.

1. Like for $K^0$ and $B^0$ mesons the oscillations of $D^0$ mesons represent a subtle quantum mechanical phenomenon of practical importance: it provides a probe for New Physics, albeit an ambiguous one, and constitutes an important ingredient for CP asymmetries arising in $D^0$ decays due to New Physics.

In qualitative analogy to the $K^0$ and $B^0$ cases these phenomena can be characterized by two quantities, namely $x_D = \frac{\Delta M_D}{\Gamma_D}$ and $y_D = \frac{\Delta \Gamma_D}{2 \Gamma_D}$. Oscillations are slowed down in the SM due to GIM suppression and $SU(3)_{fl}$ symmetry. Comparing a conservative SM bound with the present data

$$x_D(SM), y_D(SM) < \mathcal{O}(0.01) \text{ vs. } x_D|_{exp} < 0.03, \quad y_D|_{exp} = 0.01 \pm 0.005$$

we conclude that the search has just now begun. There exists a considerable literature – yet typically with several ad-hoc assumptions concerning the nonperturbative dynamics. It is widely understood that the usual quark box diagram is utterly irrelevant due to its untypically severe GIM suppression $(m_s/m_c)^4$. A systematic analysis based on an OPE treatment has been given in Ref. [96] in terms of powers of $1/m_c$ and $m_s$. Contributions from higher-dimensional operators with a much softer GIM reduction of $(m_s/\mu_{had})^2$ (even $m_s/\mu_{had}$ terms could arise) due to ‘condensate’ terms in the OPE yield

$$x_D(SM)|_{OPE}, y_D(SM)|_{OPE} \sim \mathcal{O}(10^{-3}).$$

Ref. [97] finds very similar numbers, albeit in a quite different approach.

While one predicts similar numbers for $x_D(SM)$ and $y_D(SM)$, one should keep in mind that they arise in very different dynamical environments. $\Delta M_D$ is generated from off-shell intermediate states and thus is sensitive to New Physics, which could produce $x_D \sim \mathcal{O}(10^{-2})$. $\Delta \Gamma_D$ on the other hand is shaped by on-shell intermediate states; while it is hardly sensitive to New Physics, it involves much less averaging or ‘smearing’ than $\Delta M_D$ making it thus much more vulnerable to violations of quark-hadron duality. Observing $y_D \sim 10^{-3}$ together with $x_D \sim 0.01$ would provide intriguing, though not conclusive evidence for New Physics, while $y_D \sim 0.01 \sim x_D$ would pose a true conundrum for its interpretation.

2. Since the baryon number of the Universe implies the existence of New Physics in CP violating dynamics, it would be unwise not to undertake dedicated searches for CP asymmetries in charm decays, where the ‘background’ from known physics is small: within the SM the effective weak phase is highly diluted, namely $\sim \mathcal{O}(\lambda^4)$, and it can arise only in singly Cabibbo suppressed transitions, where one expects
them to reach the 0.1 % level; significantly larger values would signal New Physics. Any asymmetry in Cabibbo allowed or doubly suppressed channels requires the intervention of New Physics – except for $D^\pm \rightarrow K_S\pi^\pm$ [72], where the CP impurity in $K_S$ induces an asymmetry of $3.3 \cdot 10^{-3}$. Several facts actually favour such searches: strong phase shifts required for direct CP violation to emerge in partial widths are in general large as are the branching ratios into relevant modes; finally CP asymmetries can be linear in New Physics amplitudes thus enhancing sensitivity to the latter. As said above, the benchmark scale for KM asymmetries in singly Cabibbo suppressed partial widths is 0.1%. This does not exclude the possibility that CKM dynamics might exceptionally generate an asymmetry as ‘large’ as 1% in some special cases. It is therefore essential to analyze a host of channels.

Decays to final states of more than two pseudoscalar or one pseudoscalar and one vector meson contain more dynamical information than given by their widths; their distributions as described by Dalitz plots or T-odd moments can exhibit CP asymmetries that can be considerably larger than those for the width. Final state interactions while not necessary for the emergence of such effects, can fake a signal; yet that can be disentangled by comparing T-odd moments for CP conjugate modes. I view this as a very promising avenue, where we still have to develop the most effective analysis tools for small asymmetries.

CP violation involving $D^0 - \bar{D}^0$ oscillations can be searched for in final states common to $D^0$ and $\bar{D}^0$ decays like CP eigenstates $- D^0 \rightarrow K_S\phi$, $K^+K^-$, $\pi^+\pi^-$ – or doubly Cabibbo suppressed modes $- D^0 \rightarrow K^+\pi^-$. The CP asymmetry is controlled by $\sin\Delta m_D t \cdot \text{Im}(q/p)\bar{\rho}(D \rightarrow f)$; within the SM both factors are small, namely $\sim O(10^{-3})$, making such an asymmetry unobservably tiny – unless there is New Physics! One should note that this observable is linear in $x_D$ rather than quadratic as for CP insensitive quantities. $D^0 - \bar{D}^0$ oscillations, CP violation and New Physics might thus be discovered simultaneously in a transition.

One wants to reach the level at which SM effects are likely to emerge, namely down to time-dependent CP asymmetries in $D^0 \rightarrow K_S\phi$, $K^+K^-$, $\pi^+\pi^- [K^+\pi^-]$ down to $10^{-5}$ [$10^{-4}$] and direct CP asymmetries in partial widths and Dalitz plots down to $10^{-3}$.

5.3.2 CP Violation in the Lepton Sector

I find the conjecture that baryogenesis is a secondary phenomenon driven by primary leptogenesis a most intriguing and attractive one also for philosophical reasons. Yet then it becomes mandatory to search for CP violation in the lepton sector in a most dedicated fashion.

30For it would complete what is usually called the Copernican Revolution [98]: first our Earth was removed from the center of the Universe, then in due course our Sun, our Milky Way and local cluster; few scientists believe life exists only on our Earth. Realizing that the stuff we are mostly made out of – protons and neutrons – are just a cosmic ‘afterthought’ fits this pattern, which culminates in the dawning realization that even our Universe is just one among innumerable others, albeit a most unusual one.
In Lecture I, Sect. 2.3.7, I have sketched the importance of measuring *electric dipole moments* as accurately as possible. The electron’s EDM is a most sensitive probe of CP violation in leptodynamics. Comparing the present experimental and CKM upper bounds, respectively

$$d_{e}^{exp} \leq 1.5 \cdot 10^{-27} \text{ e cm } \text{ vs. } d_{e}^{CKM} \leq 10^{-36} \text{ e cm}$$  \hspace{1cm} (212)$$

we see there is a wide window of several orders of magnitude, where New Physics could surface in an unambiguous way. This observation is reinforced by the realization that New Physics scenarios can naturally generate $d_{e} > 10^{-28}$ e cm, while of only secondary significance in $\epsilon_K$, $\epsilon'$ and $\sin2\phi_i$.

The importance that at least part of the HEP community attributes to finding CP violation in leptodynamics is best demonstrated by the efforts contemplated for observing CP asymmetries in *neutrino oscillations*. Clearly hadronization will be the least of the concerns, yet one has to disentangle genuine CP violation from matter enhancements, since the neutrino oscillations can be studied only in a matter, not an antimatter environment. Our colleagues involved in such endeavours will rue their previous complaints about hadronization and remember the wisdom of an ancient Greek saying:

"When the gods want to really harm you, they fulfill your wishes."

### 5.3.3 The Decays of $\tau$ Leptons – the Next ‘Hero Candidate’

Like charm hadrons the $\tau$ lepton is often viewed as a system with a great past, but hardly a future. Again I think this is a very misguided view and I will illustrate it with two examples.

Searching for $\tau^{\pm} \rightarrow \mu^{\pm}\mu^{+}\mu^{-}$ (and its variants) – processes forbidden in the SM – is particularly intriguing, since it involves only ‘down-type’ leptons of the second and third family and is thus the complete analogy of the quark lepton process $b \rightarrow s\bar{s}s$, driving $B_s \rightarrow \phi K_S$, which has recently attracted such strong attention. Following this analogy literally one guestimates $\text{BR}(\tau \rightarrow 3\mu) \sim 10^{-8}$ to be compared with the present bound from BELLE $[27]$

$$\text{BR}(\tau \rightarrow 3\mu) \leq 2 \cdot 10^{-7}.$$  \hspace{1cm} (213)$$

It would be very interesting to know what the $\tau$ production rate at the hadronic colliders is and whether they could be competitive or even superior with the $B$ factories in such a search.

In my judgment $\tau$ decays – together with electric dipole moments for leptons and possibly $\nu$ oscillations referred to above – provide the best stage to search for manifestations of CP breaking leptodynamics.

The most promising channels for exhibiting CP asymmetries are $\tau \rightarrow \nu K\pi$, since due to the heaviness of the lepton and quark flavours they are most sensitive to nonminimal Higgs dynamics, and they can show asymmetries also in the final state distributions rather than integrated rates $[99]$. 

87
There is also a *unique* opportunity in $e^+e^- \rightarrow \tau^+\tau^-$: since the $\tau$ pair is produced with its spins aligned, the decay of one $\tau$ can ‘tag’ the spin of the other $\tau$. I.e., one can probe *spin-dependent* CP asymmetries with *unpolarized* beams. This provides higher sensitivity and more control over systematic uncertainties.

I feel these features are not sufficiently appreciated even by proponents of Super-B factories. It has been recently pointed [100] out that based on known physics one can actually predict a CP asymmetry:

$$\frac{\Gamma(\tau^+ \rightarrow K_S\pi^+\nu) - \Gamma(\tau^- \rightarrow K_S\pi^-\nu)}{\Gamma(\tau^+ \rightarrow K_S\pi^+\nu) + \Gamma(\tau^- \rightarrow K_S\pi^-\nu)} = (3.27 \pm 0.12) \times 10^{-3} \quad (214)$$

due to $K_S$’s preference for antimatter.

**5.4 Instead of a Summary: On the Future HEP Landscape – a Call to Well-Reasoned Action**

The situation of the SM, as it enters the third millenium, can be characterized through several statements:

1. The SM is nontrivially consistent with all observations – except:
   - neutrino oscillations,
   - dark matter,
   - presumably dark energy,
   - probably the baryon number of our Universe and
   - possibly the Strong CP Problem as last and least exception.

There is a new dimension due to the findings on $B$ decays: there are the first decisive tests of the CKM description of CP violation: in $B_d(t) \rightarrow \psi K_S$ one has observed the first CP violation outside the $K_L$ complex; it is huge – as predicted qualitatively as well as quantitatively. A second such success has been scored in $B_d \rightarrow K^+\pi^-$ and probably in $B_d(t) \rightarrow \pi^+\pi^-$ as well.

The CKM description thus has become a *tested* theory and should no longer be referred to as an Ansatz with the latter’s patronizing flavour.

2. Flavour dynamics has become even more intriguing due to the emergence of neutrino oscillations. We do not understand the structure of the CKM matrix in any profound way – and neither the PMNS matrix, its leptonic counterpart. Presumably we do understand why they look different, since only neutrinos can possess Majorana masses, which can give rise to the ‘see-saw’ mechanism.

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31 Originally I had intended to name this Section ‘A call to Arms’. Yet recent events have reminded us that when the drums of war sound, reason all to often is left behind.
Sometimes it is thought that the existence of two puzzles makes their resolution harder. I feel the opposite way: having a larger set of observables allows us to direct more questions to Nature, if we are sufficiently persistent, and learn from her answers. 32

3. The next ‘Grand Challenge’ after studying the dynamics behind the electroweak phase transition is to find CP violation in the lepton sector – anywhere.

4. The SM’s success in describing flavour transitions is not matched by a deeper understanding of the flavour structure, namely the patterns in the fermion masses and CKM parameters. For those do not appear to be of an accidental nature. These central mysteries of the SM strongly suggest that the SM is incomplete. I have referred to the dynamics generating the flavour structure as the ‘strongly suggested’ New Physics (ssNP).

5. The physics driving the electroweak phase transition is confidently expected around the $\sim$ TeV scale: $c_p\text{NP}$. Discovering it has been the justification for the LHC program, which will come online soon. Personally I am very partisan to the idea that the $c_p\text{NP}$ will be of the SUSY type. Yet SUSY is an organizing principle rather than a class of theories, let alone a theory. We are actually quite ignorant about how to implement the one empirical feature of SUSY that has been established beyond any doubt, namely that it is broken.

6. The LHC is likely, I believe, to uncover the $c_p\text{NP}$, and I have not given up hope that the TEVATRON will catch the first glimpses of it. Yet the LHC and a forteriori the TEVATRON are primarily discovery machines. The ILC project is motivated as a more surgical probe to map out the salient features of that $c_p\text{NP}$.

7. This $c_p\text{NP}$ is unlikely to shed light on the ssNP behind the flavour puzzle of the SM, although one should not rule out such a most fortunate development. On the other hand New Physics even at the $\sim$ 10 TeV scale could well affect flavour transitions significantly through virtual effects. A comprehensive and dedicated program of heavy flavour studies might actually elucidate salient features of the $c_p\text{NP}$ that could not be probed in any other way. Such a program is thus complementary to the one pursued at the TEVATRON, the LHC and hopefully at the ILC and – I firmly believe – actually necessary rather than a luxury to identify the $c_p\text{NP}$.

To put it in more general terms: Heavy flavour studies

• are of fundamental importance,
• many of its lessons cannot be obtained any other way and

32Allow me a historical analogy: in the 1950’s it was once suggested to a French politician that the French government’s lack of enthusiasm for German re-unification showed that the French had not learnt to overcome their dislike of Germany. He replied with aplomb: "On the contrary, Monsieur! We truly love Germany and are therefore overjoyed that there are two Germanies we can love. Why would we change that?"
they cannot become obsolete.

I.e., no matter what studies of high $p_{\perp}$ physics at the TEVATRON, LHC and ILC will or will not show – comprehensive and detailed studies of flavour dynamics will remain crucial in our efforts to reveal Nature’s Grand Design.

8. Yet a note of caution has to be expressed as well. Crucial manifestations of New Physics in flavour dynamics are likely to be subtle. Thus we have to succeed in acquiring data as well as interpreting them with high precision. Obviously this represents a stiff challenge – however one that I believe we can meet, if we prepare ourselves properly as I exemplified in Lecture IV.

One of three possible scenarios will emerge in the next several years.

1. The optimal scenario: New Physics has been observed in "high $p_{\perp}$ physics", i.e. through the production of new quanta at the TEVATRON and/or LHC. Then it is imperative to study the impact of such New Physics on flavour dynamics; even if it should turn out to have none, this is an important piece of information. Knowing the typical mass scale of that New Physics from collider data will be of great help to estimate its impact on heavy flavour transitions.

2. The intriguing scenario: Deviations from the SM have been established in heavy flavour decays – like the asymmetry in $B \rightarrow \phi K_S$ – without a clear signal for New Physics in high $p_{\perp}$ physics. A variant of this scenario has already emerged through the observations of neutrino oscillations.

3. The frustrating scenario: No deviation from SM predictions have been identified.

I am optimistic it will be the ‘optimal’ scenario, quite possibly with some elements of the ‘intriguing’ one. Of course one cannot rule out the ‘frustrating’ scenario; yet we should not treat it as a case for defeatism: a possible failure to identify New Physics in future experiments at the hadronic colliders (or the $B$ factories) does not – in my judgment – invalidate the persuasiveness of the theoretical arguments and experimental evidence pointing to the incompleteness of the SM. It ‘merely’ means we have to increase the sensitivity of our probes. I firmly believe a Super-B factory with a luminosity of order $10^{36}$ cm$^{-2}$ s$^{-1}$ or more [101] has to be an integral part of our future efforts towards deciphering Nature’s basic code. For a handful of even perfectly measured transitions will not be sufficient for the task at hand – a comprehensive body of accurate data will be essential.

I will finish with a poem I have learnt from T.D. Lee a number of years ago. It was written by A.A. Milne, who is best known as the author of Winnie-the-Pooh in 1926:

\[
\text{Wind on the Hill}
\]

\[
\text{No one can tell me}
\]

\[
\text{Nobody knows}
\]
Where the wind comes from,
Where the wind goes.

But if I stopped holding
The string of my kite,
It would blow with the wind
For a day and a night.

And then when I found it,
Wherever it blew,
I should know that the wind
Had been going there, too.

So then I could tell them
Where the wind goes ...
But where the wind comes from
Nobody knows.

One message from the poem is clear: we have to let our ‘kite’ respond to the wind, i.e. we have to perform experiments. Yet the second message ‘... Nobody knows.’ is overly agnostic: Indeed experiments by themselves will not provide us with all these answers. It means one will still need ‘us’, the theorists, to figure out ‘where the wind comes from’.

In any case, we are at the beginning of an exciting adventure – and we are most privileged to participate.

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References

[1] A more detailed and comprehensive discussion of all aspects of CP violation can be found in: I.I. Bigi, A.I. Sanda, ‘CP Violation’, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, Cambridge University Press, 2000.

[2] See lectures by A. Sanda at this school.

[3] See lectures by N. Ramsey at this school.
[4] S.L. Adler, *Phys. Rev.* **177** (1969) 2426; J.S. Bell and R. Jackiw, *Nuov. Cim.* **60** (1969) 47; W.A. Bardeen, *Phys. Rev.* **184** (1969) 1848.

[5] S. Glashow, J. Illiopolous and L. Maiani, *Phys. Rev.* **D2** (1970) 1285.

[6] M. Kobayashi, T. Maskawa, *Prog. Theor. Phys.* **49** (1973) 652.

[7] K. Niu, E. Mikumo and Y. Maeda, *Prog. Theor. Phys.* **46** (1971) 1644.

[8] For a more detailed description of this fascinating tale, see the lectures by M. Sozzi at this school.

[9] T. Inami, C.S. Lim, *Prog. Theor. Phys.* **65** (1981) 297.

[10] G. Buchalla, A.J. Buras, M.E. Lautenbacher, *Rev. Mod. Phys.* **68** (1996) 1125.

[11] B. Laurent and M. Roos, *Phys.Lett.* **13**, 269 (1964); *ibidem** 15, 104.

[12] A.D. Sakharov, *JETP Lett.* **5** (1967) 24.

[13] M. Bander, D. Silverman, and A. Soni, *Phys.Rev.Lett.* **43**, 242 (1979).

[14] A. B. Carter, A. I. Sanda, *Phys. Rev.* **D 23** (1981) 1567.

[15] I.I. Bigi, A.I. Sanda, *Nucl. Phys.* **B 193** (1981) 85.

[16] C. Hamzaoui, J. L. Rosner, A. I. Sanda, Proceedings of the Fermilab Workshop on High Sensitivity Beauty Physics at Fermilab, Edited by A. J. Slaughter, N. Lockyer, and M. Schmidt, 1987.

[17] H. Albrecht et al., *Phys.Lett.* **B192**2451987.

[18] I.I. Bigi, in: Proceed. of ‘Les Rencontres de Physique de la Vallee d’Aoste, La Thuile, Italy, 1991; in: Proceed. of ‘Les Rencontres de Moriond, Les Arcs, France, 1992.

[19] For a recent review, see, e.g.: A. Buras, ‘Flavour Physics and CP Violation’, hep-ph/0505175.

[20] See lectures by L. Lanceri for details, these Proceedings.

[21] See lectures by P. Bloch, these Proceedings.

[22] I.I. Bigi, A.I. Sanda, *Phys.Lett.* **B 625** (2005) 47.

[23] L.M. Sehgal and M. Wanninger, *Phys. Rev.* **D46** (1992) 1035; *Phys. Rev.* **D46** (1992) 5209 (E); see also the earlier papers: A.D. Dolgov and L.A. Ponomarev, *Sov. J. Nucl. Phys.* **4** (1967) 262; D.P. Majumdar and J. Smith; *Phys.Rev.* **187** (1969) 2039.

[24] I.I. Bigi, A.I. Sanda, *Phys.Lett.* **B 466** (1999) 33.
[25] The OPAL Collab., K. Akerstaff et al., *Eur.Phys.J.* **C5** (1998) 379.

[26] CDF Collab., *Phys. Rev.* **D61** (2000) 072005.

[27] L. Lanceri, these Proceedings.

[28] A. Einstein, B. Podolsky, N. Rosen, *Phys.Rev.* **47** (1935) 777.

[29] I.I. Bigi, *Phys.Lett.B* **535** (2002) 155.

[30] M. Bander, D. Silverman, A. Soni, *Phys.Rev.Lett.* **43** (1979) 242.

[31] I.I. Bigi, V.A. Khoze, N.G. Uraltsev, A.I. Sanda, in: *CP Violation*, ed. C Jarlskog (World Scientific, Singapore, 1988), p. 218.

[32] Y.Y. Keum, H-n. Li, A.I. Sanda, *Phys.Lett.* **B504** (2001) 6; *Phys.Rev.* **D63** (2001) 054008; H-n. Li, S. Mishima, A.I. Sanda, *Phys.Rev.* **D72** (2005) 094005.

[33] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, *Nucl.Phys.* **B606** (2001) 245; M. Beneke, hep-ph/0509297.

[34] L. Wolfenstein, *Phys.Rev.* **D43** (1991) 151.

[35] N. Uraltsev, hep-ph/9212233.

[36] A. Deandrea, A.D. Polosa, *Phys.Rev.Lett.* **86** (2001) 216.

[37] S. Gardner, Ulf-G. Meissner, *Phys.Rev.* **D65** (2002) 094004.

[38] A. Carter, A.I. Sanda, *Phys.Rev.* **D23** (1981) 1567.

[39] I.I. Bigi, A.I. Sanda, *Phys.Lett.* **B211** (1985) 213.

[40] M. Gronau, D. Wyler, *Phys.Lett.* **B265** (1991) 172; I. Dunietz, *Phys.Lett.* **B270** (1991) 75.

[41] See, for example: Y. Grossman et al., *Phys.Rev.* **D68** (2003) 015004.

[42] I.I. Bigi, hep-ph/0509153.

[43] A. Ali, E. Lunghi, C. Greub, G. Hiller, *Phys.Rev.* **D66** (2002) 034002; A. Ghinculov, T. Hurth, G. Isidori, Y.-P. Yao, *Nucl.Phys.* **B685** (2004) 351.

[44] A. Ali, P. Ball, L.T. Handoko, G. Hiller, *Phys.Rev.* **D61** (2000) 074024.

[45] D. Pirjol, hep-ph/0207095.

[46] D. Melikhov, N. Nikitin, S. Simula, *Phys.Lett.* **B442** (1998) 381.

[47] G. Buchalla, A. Buras, *Nucl.Phys.* **B400** (1993) 225.
[48] G. Buchalla, G. Hiller, G. Isidori, *Phys. Rev.* **D63** (2001) 014015.

[49] Y. Grossman, Z. Ligeti, E. Nardi, *Nucl.Phys.* **B465** (1996) 369; D. Melikhov, N. Nikitin, S. Simula, *Phys.Lett.* **B428** (1998) 171.

[50] T. Miki, T. Miura, M. Tanaka, hep-ph/0210051.

[51] N. Uraltsev, *Phys. Lett.* **B585** (2004) 253; *ibid.* **B545** (2002) 337.

[52] D. Acosta *et al.* (CDF Collab.), *Phys.Rev.Lett.* **94** (2005) 101803.

[53] V. M. Abazov *et. al.*, (D0 Collab.), *Phys.Rev.Lett.* **95** (2005) 171801.

[54] M. Voloshin *et al.*, *Sov. J. Nucl. Phys.* **46** (1987) 112.

[55] A. Lenz, talk given at FPCP04, Daegu, Korea, Oct. 2004, hep-ph/0412007.

[56] See lectures by A. Dolgov at this school.

[57] See lectures by J. J. Gomez-Cadenas at this school.

[58] I.I. Bigi, M. Shifman, N.G. Uraltsev, *Annu. Rev. Nucl. Part. Sci.* **47** (1997) 591.

[59] N. Uraltsev, in: Boris Ioffe Festschrift “At the Frontier of Particle Physics/Handbook of QCD”, M. Shifman (ed.), World Scientific, Singapore, 2001, hep-ph/0010328.

[60] O. Buchmüller, H. Flächer, hep-ph/0507253.

[61] I.I. Bigi, Y. Dokshitzer, V. Khoze, J. Kühn, P. Zerwas, *Phys. Lett.* **B181** (1986) 157.

[62] B. Chibisov, R. Dikeman, M. Shifman and N. Uraltsev, *Int. J. Mod. Phys.* **A12** (1997) 2075.

[63] I.I. Bigi, N. Uraltsev, R. Zwicky, hep-ph/0511158.

[64] I.I. Bigi, N.G. Uraltsev and A. Vainshtein, *Phys. Lett.* **B293** (1992) 430.

[65] N.G. Uraltsev, *Phys.Lett.* **B501** (2001) 86.

[66] I.I. Bigi, Th. Mannel, hep-ph/0212021.

[67] I. I. Bigi, N.G. Uraltsev, *Int. J. of Mod. Physics* **A16** (2001) 5201; I.I. Bigi, N.G. Uraltsev, M. Shifman, A. Vainshtein, *Phys.Rev.* **D56** (1997) 4017.

[68] K. Melnikov, A. Yelkhovsky, *Phys.Rev.* **D59** (1999) 114009; M. Beneke, A. Signer, *Phys.Lett.* **B471** (1999) 233; A. Hoang, *Phys.Rev.* **D61** (2000) 034005; J.H. Kühn, M. Steinhauser, *Nucl.Phys.* **B619** (2001) 588; *ibid.* **B640** (2002) 415(E).

[69] I.I. Bigi, M. Shifman, N.G. Uraltsev and A. Vainshtein, *Phys. Rev.* **D52** (1995) 196.
[70] I.I. Bigi, N.G. Uraltsev, *Nucl.Phys.* B**423** (1994) 33; *Z.Phys.* C**62** (1994) 623.

[71] M. Voloshin, *Phys.Lett.* B**385** (1996) 369.

[72] S. Bianco, F. Fabbri, I. Bigi, D. Benson, *La Rivista del Nuov. Cim.* 26, # 7 - 8 (2003).

[73] I.I. Bigi, N.G. Uraltsev, *Phys.Lett.* B**280** (1992) 271.

[74] I. Bigi, B. Blok, M. Shifman, N. Uraltsev, A. Vainshtein, in: ‘B Physics’, S. Stone (ed.), 2nd edition.

[75] N. Uraltsev, *Phys. Lett.* B**376** (1996) 303; D. Pirjol and N. Uraltsev, *Phys. Rev.* D**59** (1999) 034012.

[76] see also: P. Colangelo and F. De Fazio, *Phys. Lett.* B**387** (1996) 371; M. Di Pierro, C. Sachrajda, C. Michael, *Phys. Lett.* B**468** (1999) 143.

[77] Heavy Flavor Averaging Group, hep-ex/0505100.

[78] CDF note 7867.

[79] I.I. Bigi, *Nucl.Inst. & Meth. in Physics Res.* A**351** (1994)240; *Phys.Lett.* B**371** (1996)105; M. Beneke, G. Buchalla, *Phys.Rev.* D**53**(1996)4991.

[80] G. Bellini, I.I. Bigi, P.J. Dornan, *Phys.Rep.* 289 (1&2) (1997) 1.

[81] F. Gabbiani, A. Onishchenko and A. Petrov, *Phys. Rev.* D**70** (2004) 094031.

[82] M.B. Voloshin, hep-ph/0004257.

[83] D. Benson et al., *Nucl.Phys.* B**665** (2003) 367.

[84] The BaBar Collab., B. Aubert et al., *Phys.Rev.Lett.* 93 (2004) 011803, hep-ex/0404017.

[85] Translations into other schemes can be found in: M. Battaglia et al., hep-ph/0304132.

[86] M. Battaglia et al., *Phys.Lett.* B**556** (2003) 41.

[87] The DELPHI Collab., J. Abdallah et al, *Eur.Phys.J.* C**45** (2006) 35.

[88] C. Bauer et al., *Phys.Rev.* D**70** (2004) 094017.

[89] M. Okamoto, PoS(LAT2005)013, Plenary talk presented at ‘Lattice 2005’, Dublin, July 25-30, 2005; hep-lat/0510113.

[90] I.I. Bigi, N. Uraltsev, *Phys.Lett.* B**579** (2004) 340.

[91] D. Benson, I.I. Bigi, N. Uraltsev, hep-ph/0410080, accept. f. publ. in *Nucl.Phys.* B.
[92] C. Bauer, Z. Ligeti, M. Luke, *Phys.Lett.* **B479** (2000) 395.

[93] I.I. Bigi, N. Uraltsev, *Int.J.Mod.Phys.* **A17** (2002) 4709.

[94] C. Bauer, Invited talk given at ‘Heavy Quarks & Leptons 2004’, Puerto Rico, June 1 - 5, 2004, hep-ph/0408100.

[95] http://elvis.phys.lsu.edu/svoboda/superk.html.

[96] I.I. Bigi, N.G. Uraltsev, *Nucl.Phys.* **B592** (2001) 92.

[97] A. Falk *et al.*, *Phys.Rev.* **D65** (2002) 054034.

[98] The usual tale that the Dark Ages of the Middle Ages were overcome by the Copernican Revolution being born like the goddess Athena jumping out of the head of her father Zeus fully developed and in full armor is unfair to the Middle Ages. Yet more importantly it completely overlooks the immeasurable service to Human culture rendered by Arab Science. For the truly committed student I recommend reading: Ahmed Djebbar, *Une histoire de la science arabe*, Editions du Seuil, 2001.

[99] J.H. Kühn, E. Mirkes, *Phys.Lett.* **B398** (1997) 407.

[100] I.I. Bigi, A.I. Sanda, *Phys.Lett.* **B625** (2005) 47.

[101] See lecture by M. Giorgi at this school.