Symmetries and Renormalization of Noncommutative Field Theory

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Abstract. An overview of recent developments in the renormalization and in the implementation of spacetime symmetries of noncommutative field theory is presented, and argued to be intimately related.

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INTRODUCTION

Noncommutative spaces

Noncommutative spaces are typically defined in physics by promoting coordinates \( x^i \) of spacetime to hermitean operators \( \hat{x}^i \) satisfying commutation relations of the form

\[
[\hat{x}^i, \hat{x}^j] = i \theta^{ij},
\]

where \( \theta^{ij} \) is an antisymmetric tensor which can be position dependent. Such spaces are believed to be important for the understanding of quantum gravity. Recent realizations of noncommutativity in such contexts have been described for \( \kappa \)-Minkowski space \([1]\) and in doubly special relativity \([2]\), among many others.

A much simpler system in which we can see noncommutative spaces arising is the Landau problem \([3]\), which deals with the motion of charged particles confined to a plane and subjected to a perpendicularly applied uniform magnetic field \( B \). The lagrangian is

\[
\mathcal{L}_m = \frac{m}{2} \dot{\mathbf{x}}^2 - e \mathbf{x} \cdot \mathbf{A},
\]

which in a symmetric gauge for the vector potential \( \mathbf{A} \) reduces in the strong field limit \( B \gg m \) to \( \mathcal{L}_0 = -\frac{eB}{2} (\dot{y} x - \dot{x} y) \). This limiting lagrangian is of first order in time derivatives, implying that upon canonical quantization the coordinates describe a noncommutative space \( [\hat{x}, \hat{y}] = \frac{i}{eB} \). This model will play a prominent role in much of our subsequent discussion.

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Noncommutative field theory

The huge interest and large amount of activity came with the seminal paper [4] wherein a very natural and precise physical realization of noncommutative spaces was illustrated, in complete analogy with the Landau problem above. It was shown that string theory, in the presence of D-branes and background “magnetic” fields, reduces in a particular low-energy limit to field theory on a noncommutative space [5, 6]. A “toy” model for this scenario, and the one this overview will focus on, is the Moyal space for which the tensor $\theta^{ij}$ is constant and nondegenerate. The coordinate operators then satisfy Heisenberg commutation relations. In particular, one has the coordinate uncertainty relations $\Delta x^i \Delta x^j \geq \frac{1}{2} |\theta^{ij}|$. Fields on Moyal spaces are multiplied using the associative, noncommutative star-product, replacing the usual pointwise multiplication $\phi \cdot \psi$ with

$$\phi \star \psi := \phi \exp \left( \frac{i}{2} \partial_i \theta^{ij} \partial_j \right) \psi.$$ (1)

Whatever our motivation may be, one of the most interesting and profound prospects of noncommutative field theory is that it has the possibility of providing diffeomorphism invariant field theories, and hence models that we might wish to call noncommutative gravity [7]. However, in order to provide viable models of quantum gravity, one is first faced with the technical task of determining whether or not they make sense. The two main issues are the precise implementation of spacetime symmetries at both the classical and quantum level, and the renormalizability of the quantum field theory. This overview will focus on the very interesting recent understanding of these two technical issues, and argue that the two problems are in fact intimately related to one another.

Violations of Lorentz invariance

The constant tensor $\theta^{ij}$ gives a preferred directionality in space. In string theory, the resulting loss of Lorentz invariance is due to the expectation value of a background supergravity field. As a consequence, noncommutative field theory is not invariant under rotations or boosts of localized field configurations within a fixed observer inertial frame of reference, i.e. under particle Lorentz transformations. This observation has been exploited to construct Lorentz-violating extensions of the standard model in [8]. Some possible resolutions to this symmetry breaking are provided by:

1. Varying $\theta^{ij}$, so that one essentially integrates over all possible backgrounds to manifestly reinstate general covariance. This has been beautifully implemented in [9] within the framework of $C^*$-algebras, but it requires dealing with a host of different noncommutative spaces not of the Moyal type which makes extracting physical quantities rather difficult.

2. Dimensional reductions of noncommutative gauge theory in higher dimensions which induce teleparallel theories of gravity in lower dimensions [7, 10]. However, the formalism is somewhat limited in the types of gauge theories of gravity one can obtain in this way, and moreover the teleparallelism is embedded in a complex way.
3. Exploiting well-known quantum group techniques to reinterpret noncommutative field theory as a twist deformed quantum field theory \[11, 12\]. This realization has been a focus of intense activity in recent years and will be the route we take in the second part of this overview.

**UV/IR mixing**

In momentum space, the replacement of the pointwise products of fields with the star-product \((1)\) amounts to multiplying the usual convolution products \(\tilde{\phi}(k) \tilde{\psi}(q)\) of Fourier transforms by a momentum dependent phase factor \(e^{ik \times q}\), where \(k \times q := \frac{1}{2} k_i \theta^{ij} q_j\). In a scalar field theory with polynomial interactions, a typical interaction vertex with \(n\) incoming momenta \(k_1, \ldots, k_n\) will thus contribute

\[
\exp \left( i \sum_{I<J} k_I \times k_J \right). \tag{2}
\]

This is effective at energies \(E\) with \(E \sqrt{\theta} \ll 1\). **Planar diagrams** are those Feynman ribbon graphs which can be drawn without crossing any lines \[13, 14\], and their values coincide with those of the ordinary \(\theta = 0\) field theory up to possible phase factors which depend only on external momenta. **Non-planar diagrams**, on the other hand, contain crossings and additional phase factors depending on internal momenta such as \((2)\), whose net result is that an ultraviolet cutoff \(\Lambda\) on the graph induces an effective infrared cutoff \(\Lambda_0 = 1/\theta \Lambda\) \[15\].

This entangling of momentum scales ruins Wilsonian renormalization, and two resolutions to this problem are to modify standard noncommutative field theory to either:

1. Duality covariant noncommutative field theory \[16\]–\[18\]. This is described in the next section.
2. Twisted noncommutative field theory \[19\]. This is explained in the second part.

**RENORMALIZATION**

**Scalar field theory**

Consider interacting charged scalar fields in four dimensions with the euclidean action

\[
S = \int d^4x \left[ \phi^\dagger (\partial^2 + m^2) \phi + \frac{\lambda}{4} \phi^\dagger \phi \phi^\dagger \phi \phi \right] \tag{3}
\]

where we have used \(\int d^4x \phi \psi = \int d^4x \phi \psi\) for any pair of fields, so that the free field theory is unaffected by the noncommutative deformation. The quantum field theory is given by the Green’s functions which are defined as the \(2n\)-point correlation functions

\[
G_n(x_I, y_I) = \langle \phi^\dagger(x_1) \phi(y_1) \cdots \phi^\dagger(x_n) \phi(y_n) \rangle. \tag{4}
\]
As an example, consider the contribution of the one-loop tadpole graph to the two-point function in the case of real scalar fields \( \phi = \phi^\dagger \). If the external legs carry momentum \( p \), then the planar tadpole diagram gives the momentum space Feynman integral

\[
\frac{\lambda}{6} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m^2}
\]

which exhibits the standard ultraviolet divergence of scalar \( \phi^4 \)-theory in four dimensions. On the other hand, the non-planar tadpole diagram gives

\[
\frac{\lambda}{12} \int \frac{d^4k}{(2\pi)^4} \frac{e^{2ik\cdot p}}{k^2 + m^2} = \frac{\lambda}{48\pi^2} \sqrt{m^2/(\theta p)^2} K_1 \left( \sqrt{m^2/(\theta p)^2} \right)
\]

which diverges as \((\theta p)^{-2}\) as \( p \to 0 \). The model is thus not renormalizable, because the quantum field theory is not covariant under “UV/IR duality” [17] as we now explain.

**UV/IR duality**

To resolve the problems associated with UV/IR mixing, we introduce a covariant version of noncommutative field theory in which the ultraviolet and infrared regimes are indistinguishable [16]. It is defined by modifying the action (3) to

\[
S = \int d^4x \sqrt{g} \left[ \phi^\dagger (g^{ij} D_i D_j + m^2) \phi + \frac{\lambda}{2} (\phi^\dagger \phi)^2 \right],
\]

where \( g_{ij} \) is a constant metric tensor and \( D_i = \partial_i - \frac{i}{2} B_{ij} x^j \) are gauge covariant derivatives in a magnetic background characterized by another nondegenerate, constant antisymmetric tensor \( B_{ij} \). The new derivatives obey the commutation relations \([D_i, D_j] = -2i B_{ij}\), so that the modification of the kinetic term in (7) can be thought of as inducing a “noncommutative momentum space”. The quantum field theory now has a duality under Fourier transformation of the fields, given by the covariant transformation rules

\[
S[\phi]_{\tilde{\lambda}, \tilde{g}, \tilde{B}, \tilde{\theta}} = |\det B| S[\tilde{\phi}]_{\tilde{\lambda}, \tilde{g}, \tilde{B}, \tilde{\theta}}, \quad \tilde{G}_n(x_I, \theta^{-1} y_I)_{\tilde{\lambda}, \tilde{g}, \tilde{B}, \tilde{\theta}} = |\det B|^{n/2} G_n(x_I, y_I)_{\tilde{\lambda}, \tilde{g}, \tilde{B}, \tilde{\theta}}
\]

where the dual parameters are defined by \( \tilde{\lambda} = \lambda / \sqrt{\det \theta}, \tilde{g} = -B^{-1} g B^{-1}, \tilde{B} = B^{-1} \) and \( \tilde{\theta} = \theta^{-1} \).

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3 The case of complex scalar fields is much more subtle, as shown in [20].
Renormalization of the duality-covariant field theory

The key to the renormalization of the duality-covariant model is the use of a “matrix basis” for the expansion of fields given by \[17, 21\]

\[
\phi(x) = \sum_{\ell,n \in \mathbb{N}_0^2} \phi^\dagger_{\ell n} \varphi_{\ell n}(x),
\]

where \(\varphi_{\ell n}\) are the “Landau wavefunctions” which at \(B = \theta - 1\) are the eigenfunctions of the kinetic operator in (7) with \(-D^2 \varphi_{\ell n} = \text{Pf}(\theta)^{-1}(\ell_1 + \ell_2 - 1) \varphi_{\ell n}\). They form a complete orthonormal system of \(L^2\)-fields which multiply together like matrix units \(|\ell_1, \ell_2\rangle\langle n_1, n_2|\) under the star-product as \(\varphi_{\ell n} \star \varphi_{\ell' n'} = \text{Pf}(4\pi \theta)^{-1} \delta_{n' \ell'} \varphi_{\ell n'}\). The noncommutative field theory (7) then becomes an infinite-dimensional complex matrix model

\[
S = \text{Tr}\left(\phi^\dagger \mathcal{G} \phi + \frac{1}{2} \text{Pf}(4\pi \theta) (\phi^\dagger \phi)^2\right).
\]

One of the beautiful calculations performed in [17] is that of the propagator \(\mathcal{G}^{-1}\) as the formal inverse of the infinite matrix \(\mathcal{G} = (\phi_{\ell n})\), which uses hypergeometric Meixner \(q\)-polynomials. The natural ultraviolet cutoff is now on the matrix dimension as \(\ell_1 + \ell_2, n_1 + n_2 \leq N\). One then slices the propagator in the renormalization group with sharp bounds on the matrix indices \(\ell, n\). By using the Wilson-Polchinski renormalization group equations, one proves in this way that the duality-covariant field theory is renormalizable to all orders [17].

Beyond perturbation theory

The beta-functions of the couplings \(B\) and \(\lambda\) in the duality-covariant field theory have been computed to all orders in [22, 23]. They are of the usual sign for any magnetic background \(B \neq \theta^{-1}\). At the special point \(B = \theta^{-1}\) a number of remarkable things happen:

- \(\beta_B = \beta_\lambda = 0\), hence the renormalized coupling flows to a finite bare coupling and the field theory is asymptotically safe. This allows in principle a nonperturbative construction of the quantum field theory.
- The noncommutative quantum field theory is completely duality invariant.
- The matrix \(\mathcal{G}\) in (10) is diagonal with entries the Landau energies, and the field theory becomes an exactly solvable large \(N\) matrix model with a huge unitary symmetry \(\phi \mapsto U \phi U^{-1}, U \in U(N)\) reflecting the degeneracy of Landau levels. In terms of fields this symmetry corresponds to the transformations \(\phi(x) \mapsto (U \star \phi \star U^\dagger)(x)\) with \(U \star U^\dagger = U^\dagger \star U = 1\), which generate the infinite unitary group \(U(\infty)\) representing “deformed” canonical transformations of the spacetime [24].

Some of these considerations have been generalized to noncommutative \(\phi^3\)-theory by mapping it onto the Kontsevich matrix model [25], and to the noncommutative Gross-Neveu model [26]. However, there are difficulties associated with the nonperturbative...
renormalizability of the self-dual model [21], due to the overly large $U(\infty)$ symmetry which appears to kill all non-trivial dynamics and we must search for an alternative way to implement the symplectomorphism symmetry.

**TWISTED SYMMETRIES**

**Twist deformations**

Suppose that $X$ is an infinitesimal symmetry transformation of fields, denoted $\phi \mapsto X \triangleright \phi$. Then the action of $X$ on tensor products of fields is implemented by a coproduct $\Delta$ with $\phi \otimes \psi \mapsto \Delta(X) \triangleright (\phi \otimes \psi)$. The primitive coproduct $\Delta = \Delta_0$, with $\Delta_0(X) = X \otimes 1 + 1 \otimes X$, is covariant with respect to the pointwise product $\Delta_0(\phi \otimes \psi) = \phi \cdot \psi$

$$X \triangleright \Delta_0(\phi \otimes \psi) = \Delta_0(X) \triangleright (\phi \otimes \psi) .$$  
 equation (11)

For example, for the translation generator $X = P_i = -i \partial_i$ the covariance (11) is just the usual Leibniz rule $P_i(\phi \cdot \psi) = (P_i\phi) \cdot \psi + \phi \cdot (P_i\psi)$. While $\Delta_0$ is not covariant with respect to the star-product, the twist deformed coproduct $\Delta = \Delta_\theta$ is [11, 12, [28]–[31]. It is defined by rewriting the star-product (1) as $\phi \star \psi = m_\theta (\phi \otimes \psi)$, where

$$\mathcal{F} = \exp \left( -\frac{i}{2} \theta^{ij} P_i \otimes P_j \right)$$  
 equation (12)

is an abelian Drinfeld twist, and forming the similarity transformation

$$\Delta_\theta(X) = \mathcal{F}^{-1} \Delta_0(X) \mathcal{F} = \mathcal{F}^{-1} (X \otimes 1 + 1 \otimes X) \mathcal{F} .$$  
 equation (13)

**Twisted spacetime symmetries**

Using (13) one can straightforwardly work out twisted Poincaré transformations generated by the usual linear and angular momentum operators $P_i = -i \partial_i$ and $M_{ij} = -i (x_i \partial_j - x_j \partial_i)$. One finds $\Delta_\theta(M_{ij}) = \Delta_0(M_{ij})$, reflecting the fact that noncommutative field theory is translationally invariant, whereas

$$\Delta_\theta(M_{ij}) = \Delta_0(M_{ij}) + \frac{i}{2} \theta^{kl} (\eta_{ik} P_j - \eta_{jk} P_i) \otimes P_l + \frac{i}{2} \theta^{kl} P_k \otimes (\eta_{il} P_j - \eta_{jl} P_i) ,$$  
 equation (14)

reflecting that it is not invariant under boosts or rotations. However, from (14) one finds that noncommutative field theory is invariant under twisted particle transformations, because

$$M_{kl} \triangleright [x^i, x^j]_\star = m_\theta \Delta_\theta(M_{kl}) \triangleright (x^i \otimes x^j - x^j \otimes x^i) = 0$$  
 equation (15)

4 Of course in the real case there is no $U(\infty)$ symmetry which kills the dynamics. In [27] numerical evidence is provided for the renormalizability of a somewhat different class of noncommutative scalar field theories.
is equivalent to $M_{kl} \triangleright \theta^{ij} = 0$. This symmetry can be generalized to linear affine transformations $x \mapsto L x + a$, $\theta \mapsto L \theta L^\top$ using covariance of the Moyal star-product [32]. One can in fact generalize the symmetry to twisted diffeomorphisms, generated by arbitrary smooth vector fields $X = X^i(x) \partial_i$ [31]. One generically has $\Delta_0(X) \neq \Delta_0(X)$, but one can construct a “twisted” tensor calculus such that star-products of tensor fields transform as tensors. However, only unimodular $U(\infty)$ twisted transformations, with $\partial_i X^i = 0$, preserve action functionals of noncommutative field theories [33].

### Twisted noncommutative quantum field theory

Given a set of one-particle wavefunctions $\phi(x)$, two-particle wavefunctions are constructed as $(\phi \otimes \psi)(x_1, x_2)$. The flip map $\sigma_0(\phi \otimes \psi) = \psi \otimes \phi$ is only superselected when $\theta = 0$, as then $\sigma_0 \Delta_0 = \Delta_0 \sigma_0$. For $\theta \neq 0$, we use instead the “twisted” flip operator $\sigma_\theta = F^{-1} \sigma_0 F = F^{-2} \sigma_0$ with $\sigma_\theta^2 = 1 \otimes 1$ and $\sigma_\theta \Delta_\theta = \Delta_\theta \sigma_\theta$ ($F^{-2}$ is the corresponding R-matrix). If $c(p)$ are the usual (bosonic or fermionic) oscillators of the undeformed quantum field theory, then this modifies them to the twisted oscillators

$$a(p) = c(p) \exp \left( \frac{i}{2} \theta^{ij} p_i p_j \right)$$

with the commutation relations $a(p) a(q) = \pm e^{2i p \cdot q} a(q) a(p)$. In particular, on free quantum fields the creation parts obey

$$\phi^+(x_1) \phi^+(x_2) = \pm \exp \left( i \theta^{ij} \frac{\partial}{\partial x^i_1} \frac{\partial}{\partial x^j_2} \right) \phi^+(x_2) \phi^+(x_1).$$

This modifies the ordinary Feynman path integral measure $\int \! dx \phi(x)$ and defines a braided quantum field theory with covariant Wick expansions [34]. The two main consequences of these arguments are:

1. For a spin 0 field with interaction hamiltonian density of the form $\mathcal{H}_I(x) = \phi(x) \star \cdots \star \phi(x)$, the corresponding S-matrix is independent of $\theta$ and hence there is no UV/IR mixing in twisted quantum field theory [19, 30].
2. Even free twisted noncommutative field theory is non-local, because one has $\langle q | \phi(x), \phi(y) | p \rangle \neq 0$ for $q \neq p$ and space-like separations [35].

### Twisted Noether symmetries

Twisted diffeomorphisms do not appear to be bonafide physical symmetries, because they do not act solely on fields. They modify the usual Leibniz rule (represented by the primitive coproduct $\Delta_0$) through transformation of the star-product as

$$\delta_X (\phi \star \psi) := m_\theta \Delta_\theta(X) \triangleright (\phi \otimes \psi) = (\delta_X \phi) \star \psi + \phi \star (\delta_X \psi) + \phi(\delta_X \star) \psi.$$  

The extra variation seems to present an obstacle to application of the standard Noether procedure and of the Ward identities in the quantum field theory. One resolution, proposed in [36] (see also [33]), is to use a proper covariant noncommutative differential
calculus to perform the Noether analysis relating fields and conserved charges. However, the physical origins of these symmetries are unclear, and in particular string theory is unable to account for twisted diffeomorphisms \cite{37}. The brane-induced low-energy dynamics of closed string theory in the presence of a \( B \)-field is much richer than any noncommutative theory of gravity based solely on star-products.

**OUTLOOK**

We have described how two notorious problems of noncommutative field theory can be resolved by at least two rather drastic modifications of the underlying models, leading to the covariant and twisted noncommutative field theories. Both models are free from UV/IR mixing, possess a \( U(\infty) \) symplectomorphism symmetry, and have underlying non-local free field theories. The main outstanding problems are to find natural physical origins for covariant and twisted noncommutative field theories, and to resolve the ambiguities in the definitions of scaling limits and of correlation functions. Another open problem is to generalize these modifications to the framework of gauge theories, wherein UV/IR mixing is logarithmic and is associated to open Wilson line operators coupling gravity to D0-branes \cite{6}.

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