On the probability of major-axis precession in triaxial ellipsoidal potentials

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ABSTRACT
Orbits in triaxial ellipsoidal potentials precess about either the major or minor axis of the ellipsoid. In standard perturbation theory it can be shown that a circular orbit will precess about the minor axis if its angular momentum vector lies in a region bounded by two great circles which pass through the intermediate axis and which are inclined with minimum separation \( i_T \) from the minor axis, where \( i_T = \arctan([(B^2 - C^2)/(A^2 - B^2)])^{1/2} \) and \( A, B \) and \( C \) are the axis ratios, \( A \geq B \geq C \). We test the accuracy of this formula by performing orbit integrations to determine \( i_S \), the simulated turnover angle corresponding to \( i_T \).

We reach two principal conclusions: (i) \( i_S \) is usually greater than \( i_T \), by as much as 12 degrees even for moderate triaxialities, \( A/1.2 < B < C/0.8 \). This reduces the expected frequency of polar rings. (ii) \( i_S \) is not a single, well-defined number but can vary by a few degrees depending upon the initial phase of the orbit. This means that there is a reasonable probability for capture of gas onto orbits which precess about both axes. Interactions can then lead to substantial loss of angular momentum and subsequent infall to the galactic centre.

Key words: Galaxies: peculiar, Galaxies: formation

1 INTRODUCTION
Polar ring galaxies are systems in which a ring of gas (and/or young stars) is seen to orbit about the major axis of an early-type galaxy. In many cases it has been established that the host galaxy is rotating at right-angles to the ring and this is sometimes taken to be part of the definition of a polar ring system. There are just 6 confirmed polar rings but 27 good candidates and many more possibles—see Whitmore et al. (1990) for a review.

The host galaxy in polar ring systems often appears to be an S0 but this leads to theoretical problems: in an oblate axisymmetric system all orbits will precess about the minor axis at a rate which is a function of radius (typically the period is proportional to radius). This differential precession will cause the polar ring to fragment. This can be overcome if a small degree of triaxiality is assumed as orbits whose angular momentum is sufficiently close to the major axis will then precess around the major rather than the minor axis. In general accreted discs of gas will have an arbitrary orientation. In this case differential precession about a symmetry axis will lead to dissipation and collapse into the plane perpendicular to that axis.

The regions of phase space which lead to precession about the major or minor axes can be investigated using perturbation theory in Hamiltonian mechanics. It is assumed that the unperturbed potential is spherically symmetric and that the orbits are circular. The Hamilton is then expanded in spherical harmonics and the time-averaged perturbing potential calculated (see, for example, Steiman-Cameron & Durisen, 1984). This results in an expression of the form

\[ \Phi = C_{20}(3\sin^2 i - 2) + 6C_{22}\cos 2\Omega \sin^2 i \]

where \( \Omega \) and \( i \) are the node and inclination of the orbit, as illustrated in Figure 1 and \( C_{20} \) and \( C_{22} \) are constants. Orbits will precess along lines \(< \Phi > \) = constant as shown in Figure 2. If the angular momentum of the orbit, \( J \), lies within the shaded region then it will precess about the major axis, otherwise it will precess about the minor axis. The dividing lines between the two regions are great circles which pass through the intermediate axis and which are inclined at an angle

\[ i_T = \arcsin\left(\frac{C_{20} + 2C_{22}}{C_{20} - 2C_{22}}\right)^{1/2} \]

to the minor axis. Hence the probability that a randomly inclined disk will precess around the major axis is

\[ f = 1 - \frac{2i_T}{\pi} \]
2 INTEGRATION METHOD AND RESULTS

2.1 The integration method

The numerical code used in this paper is that described by Pearce & Thomas (1991). It is a simple predictor-corrector method integrates orbits to high accuracy and we refer the reader to this paper for details.

We use a potential

\[ \phi = \ln \left( \frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{z^2}{C^2} \right) \]

where \( A \geq B \geq C \) and without loss of generality we can take \( B = 1 \). The time-averaged first-order perturbation potential takes the form of Equation 1 with

\[ C_{20} = \frac{1}{A^2} + \frac{1}{B^2} - \frac{1}{C^2} \]
\[ C_{22} = \frac{1}{2} \left( \frac{1}{A^2} - \frac{1}{B^2} \right) \]

which gives, on substitution in Equation 2

\[ i_T = \arcsin \left( \frac{1}{A^2} - \frac{1}{B^2} \right) = \arctan \left( \frac{B^2 - C^2}{A^2 - B^2} \right)^{\frac{1}{2}}. \]

Note that this expression is different from that for a system with ellipsoidal density contours, \( \rho = \rho(x/a)^2 + (y/b)^2 + (z/c)^2 \), for which

\[ i_T = \arcsin \left( \frac{b^2 - c^2}{a^2 - c^2} \right)^{\frac{1}{2}}. \]

These two formulae agree for near-spherical systems but can differ by 10 degrees or more for moderate triaxialities. For example \( a = 1.2, b = 1, c = 0.8 \) gives \( i_T \approx 42,1 \) degrees whereas \( A = 1.2, B = 1, C = 0.8 \) gives \( i_T \approx 53.6 \) degrees.

Particles are given an initial tangential velocity of \( \sqrt{2} \) which would lead to a circular orbit in a spherical potential, \( A = B = C \). The subsequent behaviour depends both upon the initial orientation of the plane of the orbit and on the orbital phase of the particle. These are labelled by the co-ordinates shown in Figure 1. \( i \) is the inclination of the orbit to the \( x-y \) plane (also the angle between the angular momentum vector, \( \mathbf{J} \), and the \( z \)-axis), \( \Omega \) is the longitude of the ascending node, and \( \chi \) is the phase measured from the position of the ascending node. Because the radial velocity is initially zero \( \chi \) corresponds also to the phase of the peri- or apogee of the orbit. Note, however, that this phase is not conserved as it would be in a Keplerian potential.

There are too many free parameters to be able to investigate them all in depth so we choose to restrict \( \mathbf{J} \) to lie initially in the \( x-z \) plane (i.e. \( \Omega = \pi/2 \)). If the transition lines separating the two regions of precession about the major- and minor-axes are great circles (as in linear perturbation theory) then it is trivial to generalise our results to arbitrary \( \mathbf{J} \).

2.2 Results

The measured transition angle, \( i_T \), is a function of the initial phase, \( \chi \), as illustrated in Figure 3 for one particular choice of axis ratios, \( A = 1.1, B = 1.0 \) and \( C = 0.9 \). The minimum value of \( i_T \) occurs at \( \chi = 0 \) and the maximum at \( \chi = \pi/2 \) with an approximately sinusoidal variation between the two.
Precession in triaxial potentials

Figure 3. The measured turnover angle, $i_S$, as a function of initial phase for the case $A = 1.1$, $B = 1.0$ and $C = 0.9$. The theoretical value, $i_T$, is shown as a dotted line.

Because of this we need only present results for these two extremes, and the maximum and minimum transition angles for a range of triaxialities are given in Figure 3. The effect in each case is to lower the boundary of the shaded region in Figure 2 and to smear it out over a few degrees.

As an aside, we never see orbits which switch from precessing about the major to the minor axis, or vice versa. In general the phase of the apogee of the orbit is not conserved. However for those orbits which lie close to the transition angle, $i_S$, the phase returns almost exactly to its original value after one whole precession time. We do not know why this is the case although it is presumably a reflection of some underlying conservation law.

3 CONCLUSIONS

In this paper we calculate the simulated transition angle for precession about the major or minor axis of a triaxial potential and reach the following two conclusions: (i) $i_S$ is almost always greater than $i_T$ and (ii) $i_S$ is spread out over a few degrees depending upon the initial phase of the orbit.

The first of these results means that theoretical estimates, based on $i_T$, of the expected fraction of accreted gas disk which will settle down to give polar rings will be too high. However the error is not likely to be greater than about 10 percent, much lower than the uncertainty in the observed frequency of polar ring systems.

More interesting is the second result which may provide a mechanism for overcoming the angular momentum barrier which prevents accretion into the core of a galaxy. If material is accreted at an inclination between the measured maximum and minimum values of $i_S$, and if the accreted material is spread out over a range of phases, then precession will occur around both the minor and the major axes. Interactions between the two components can then lead to a large reduction in angular momentum. Indeed, once each component has precessed through 180 degrees then the planes of their orbits again coincide but their angular momenta are oppositely aligned. Collisions between gas clouds of similar mass may then reduce their velocity relative to the galactic centre to almost zero and they will be accreted into the core. This situation may not be as unlikely as it at first appears because the appropriate range of $i_S$ may span up to 10 degrees even for moderate triaxialities. Also accretion of a gas-rich dwarf spiral or dwarf irregular galaxy is likely to populate a fair region of phase space and a large amount of dissipation is required for this to settle into a disk of gas clouds on circular orbits, as is often assumed for simplicity.

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Figure 4.