Research and Comparison of Brillouin Characteristics of Single-Mode Fiber, Few-Mode Fiber and Multi-Mode Fiber

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Abstract. This article systematically studies and compares the Brillouin characteristics of single-mode fiber (SMF), few-mode fiber (FMF) and multi-mode fiber (MMF). The Brillouin frequency shift (BFS), Brillouin linewidth, gain peak, and Brillouin gain spectrum of different types of optical fibers in the case of single mode and mode superposition are investigated. The results show that in single mode case, the BFS, Brillouin linewidth, and gain peak of the FMF and MMF decrease as the mode order increases. Considering the mode superposition case, the SMF has the largest BFS, followed by the MMF, and the smallest is FMF. The MMF has the largest Brillouin linewidth, followed by the FMF, and the smallest is SMF. The SMF has the largest gain peak, followed by FMF, and the smallest for MMF. This work provides an important reference value for the further study of Brillouin distributed fiber optical sensors.

1. Introduction

At present, the Brillouin distributed fiber optical sensors [1-3] can measure both the temperature and strain simultaneously, having higher accuracy and spatial resolution. The information transmission of single-mode fiber (SMF) is approaching the limit of Shannon's theorem, while few-mode fiber (FMF) and multi-mode fiber (MMF) can transmit many modes, the Brillouin scattering parameters change with the mode. The Brillouin scattering parameters of different modes caused by different physical quantities may be different. There are potential applications in advanced optical communications and multi-parameter fiber sensing. Many researchers studied the BGS characteristics of SMF [5, 6], FMF [7-9] and MMF [7, 10], and based on this, they have carried out a lot of temperature and strain sensing works, which promoted the progress of optical fiber sensing field [11-13]. But they have not systematically studied the differences and connections between the Brillouin characteristics of the three types of optical fibers. Comparison and selection of optical fibers are lack of theoretical and modeling supports. Therefore, it is necessary to study and compare the Brillouin characteristics of SMF, FMF and MMF.

The waveguide models of SMF, FMF and MMF, the electric field distributions and the effective refractive indexs (ERIs) in different propagation modes are obtained by using finite element method (FEM) [14]. The Brillouin parameters of different optical fibers and different modes of the same optical fiber are investigated. The Brillouin gain spectrum (BGS) of SMF, FMF and MMF are studied
in the case of mode superposition. This work provides theoretical and modeling support for the optimization and selection of Brillouin distributed fiber optical sensors.

2. Theoretical analysis
Brillouin scattering in an optical fiber is a phenomenon caused by the interaction between a lightwave and an acoustic wave. In order to simplify the analysis, this paper ignores the coupling effects between modes, assuming that the SMF, FMF and MMF are ideal optical waveguides, that is, each mode transmitted in the optical fiber independently and coupled with the acoustic wave field to generate its own Brillouin scattering.

2.1. Single mode
In the case of single mode, the Brillouin scattering of FMF and MMF has the same transmission and statistical characteristics as the SMF.

(1) Brillouin frequency shift (BFS)
The BFS $v_B$ depends on the acoustic wave velocity and is given by

$$v_B = \frac{2n_{ef} V_A}{\lambda}$$

where $n_{ef}$ is the ERI of the propagation mode in the fiber, $V_A$ is the acoustic phonon velocity and $\lambda$ is the pump wavelength.

(2) Brillouin linewidth
The Brillouin linewidth $\Delta v_B$ is also called the full width at half maximum (FWHM). The calculation formula of Brillouin linewidth is as follows

$$\Delta v_B = \frac{16\pi n_{ef}^2 \eta}{\lambda^2 \rho}$$

(3) Gain peak
The BGS peaks at the BFS, and the peak value is given by the Brillouin gain coefficient $g_0$

$$g_n(v_B) = g_0 = \frac{2\pi n_{ef}^2 p_{12}^2}{c \lambda^2 \rho V_A \Delta v_B}$$

where $c$ is the vacuum velocity of light, and $p_{12}$ is the longitudinal elasto-optic coefficient.

(4) BGS
The BGS has a Lorentzian spectral profile given by

$$g_n(v) = g_0 \frac{(\Delta v_B / 2)^2}{(v - v_B)^2 + (\Delta v_B / 2)^2}$$

where $v$ is the frequency.

2.2. Mode superposition
In practical optical fiber, Brillouin scattering of FMF and MMF includes the superposition of many modes. Therefore, another method should be used to study the transmission characteristics of FMF and MMF.

(1) Brillouin linewidth
\[ \Delta v_{B0} = \sqrt{\Delta v_{B0}^2 + v_{Bmax}^2 \left(\frac{NA}{n_{co}}\right)^4} \]  
(5)

where \( \Delta v_{B0} \) is the Brillouin linewidth of bulk silica, \( v_{Bmax} \) is the maximum BFS of different modes coupled into the fiber, \( NA \) is the numerical aperture, and \( n_{co} \) is the refractive index of core.

2. The inhomogeneous broadening of BGS

There are different Brillouin scattering angles in different modes of SMF and MMF. Due to the limitation of the \( NA \), the scattering angle changes within a certain range. The maximum scattering angle \( \theta_{max} = \pi \), the minimum scattering angle \( \theta_{min} = \pi - 2\theta_c \), where \( \theta_c \) is the complement of the critical angle of a fiber, defined by

\[ \theta_c = \arcsin\left(1 - \frac{n_{cl}^2}{n_{co}^2}\right)^{1/2} = \arcsin\left(\frac{NA}{n_{co}}\right) \]
(6)

where \( n_{cl} \) is the refractive index of cladding.

When \( \theta \) takes the maximum value, the BFS reaches the maximum value

\[ v_{Bmax} = \frac{2n_{co}V_A}{\lambda} \]
(7)

When \( \theta \) takes the minimum value, the BFS reaches the minimum value

\[ v_{Bmin} = \frac{2n_{co}V_A}{\lambda} \left[1 - \left(\frac{NA}{n_{co}}\right)^2\right]^{-1/2} \]
(8)

We express the homogeneous broadening of the Brillouin scattering spectrum

\[ g_{B}(v) = G_0 \frac{\Delta v_{B0}}{v_{Bmax} - v_{Bmin}} \left[\arctan\left(\frac{v_{Bmax} - v}{\Delta v_{B0}/2}\right) - \arctan\left(\frac{v_{Bmin} - v}{\Delta v_{B0}/2}\right)\right] \]
(9)

where \( G_0 \) is the Brillouin gain coefficient of bulk silica, \( v_{Bmax} \) and \( v_{Bmin} \) are respectively the maximum and minimum values of the BFS.

3. Simulation and discussion

3.1. Fiber mode distribution

From the Maxwell equations, it can be obtained that all the \( x \), \( y \), and \( z \) components of the electric field in the optical fiber meet the scalar Helmholtz equation:

\[ \nabla^2 E + k_0^2 n^2 E = 0 \]
(10)

where \( E \) represents each component of the electric field, \( k_0 = 2\pi/\lambda \) is a wave number in a vacuum, \( \lambda \) is the wavelength of a light wave in a vacuum, and \( n \) is the refractive index of a fiber.

Waveguide models of SMF, FMF, and MMF are established and solved by FEM, and the fiber boundary is a perfect electrical conductor and conforms to: \( n \times E = 0 \). The ERI and electric field distribution of each mode can be obtained by solving Eq. (10), and then the Brillouin characteristics of SMF, FMF, and MMF are obtained and compared by use of Eqs. (1) ~ (9).
The SMF, FMF and MMF fiber parameters used in this paper are shown in Table 1. The electric field distribution of the SMF, FMF and MMF are shown in Figure 1. There is not enough space to study all the transmission modes of MMF, and only the following five typical propagation modes are shown in Figure 1.

The results show that the SMF can only transmit one mode, that is \( \text{LP}_{01} \), and its ERI is 1.4426. The FMF can transmit five modes, that is \( \text{LP}_{01}, \text{LP}_{11}, \text{LP}_{21}, \text{LP}_{02}, \text{LP}_{31} \). The ERIs corresponding to the five modes are 1.4675, 1.4662, 1.4646, 1.4641 and 1.4627, respectively. Five main propagation modes of MMF are \( \text{LP}_{01}, \text{LP}_{11}, \text{LP}_{21}, \text{LP}_{02}, \text{LP}_{31} \) corresponding to the ERIs of 1.4498, 1.4496, 1.4492, 1.4491, 1.4488, respectively.

By observing the results of three types of optical fibers, it can be shown that the electric field distributions of the three types of optical fibers in the same propagation mode are very similar. However, the ERI values are different in the same propagation mode.

### Table 1. Fiber parameters.

|          | Core radius/μm | Cladding radius/μm | Core refractive index | Refractive index of cladding |
|----------|----------------|--------------------|-----------------------|-----------------------------|
| SMF      | 4.5            | 62.5               | 1.4457                | 1.4378                      |
| FMF      | 10             | 62.5               | 1.4683                | 1.4625                      |
| MMF      | 25             | 62.5               | 1.45                  | 1.436                       |

3.2. Brillouin characteristics comparison

The \( \text{LP}_{01} \) mode of SMF, \( \text{LP}_{01}, \text{LP}_{11}, \text{LP}_{21}, \text{LP}_{02} \) and \( \text{LP}_{31} \) modes of FMF and MMF are analyzed in this section. The BFS, Brillouin linewidth, gain peak and BGS for the three types of optical fibers are obtained and compared. From Eqs. (1) ~ (3), the BFS, the Brillouin linewidth and the gain peak of each propagation mode are related to its ERI. Substituting the relevant parameters of different modes for different types of optical fibers into Eqs. (1) ~ (3), the relationship between the BFS, Brillouin linewidth and gain peak of different types of fiber and ERI is shown in the Figure 2(a) ~ (c). The Comparison of BGS of SMF, FMF and MMF are shown in Figure 2(d). The BGS parameters of the three types of optical fibers are shown in Table 2.

### Table 2. BGS parameters of the three types of optical fibers.

|          | Brillouin frequency shift / GHz | Brillouin linewidth / MHz | Gain peak /m/W |
|----------|--------------------------------|--------------------------|----------------|
| SMF      | 10.67                          | 21.76                    | 9.4501×10⁻¹¹    |
| FMF      | 9.2178                         | 41.35                    | 3.2642×10⁻¹¹    |
| MMF      | 10.323                         | 100.68                   | 1.3836×10⁻¹¹    |
From Figure 2(a), the BFS of SMF is 10.67GHz, the BFS of FMF is around 9.2GHz, the BFS of MMF is around 10.3GHz. SMF has the largest BFS, followed by MMF, and the BFS of FMF is the smallest. According to the local enlarged drawing, in the case of single mode, the BFS of FMF and MMF increases with decreasing mode order.

From Figure 2(b), the Brillouin linewidth of SMF is 21.76MHz, the Brillouin linewidth of FMF different modes ranges from 22.37MHz ~ 22.52MHz and increases with decreasing mode order, and the Brillouin linewidth of MMF different modes ranges from 21.95MHz ~ 21.98MHz and increases with decreasing mode order.

From Figure 2(c), the gain peak value of SMF is $9.46 \times 10^{-11}$ m/W, and the gain peak values of different modes of FMF varies from $1.19 \times 10^{-11}$ m/W to $1.21 \times 10^{-11}$ m/W and increases with decreasing mode order. The gain peak values of MMF in different modes is in the range of $9.99 \times 10^{-11}$ m/W ~ $1.0 \times 10^{-10}$ m/W and increases with the decrease of the mode order. As shown in Figure 2(c), the gain peak of FMF is the largest, MMF is the second, and SMF is the smallest.

From Figure 2(d), the BGS of FMF and MMF is broadened due to the mode superposition. The broadening phenomenon is consistent with the discussion in Ref. [15]. The Brillouin linewidth of FMF is about twice that of SMF, but its gain peak is about a third of the SMF, and its BFS is about 1460MHz less than that of SMF. The Brillouin linewidth of MMF is about five-fold of SMF, however, its gain peak is about a sixth of the SMF, and its BFS is about 347MHz less than that of SMF. If the number of sweeps and the ratio of frequency sweep span and Brillouin linewidth are fixed, the BFS error is proportional to Brillouin linewidth, while the relationship between BFS and temperature or...
strain is linear [16]. Therefore, the accuracy of temperature and strain measurements based on Brillouin scattering can be improved by selecting a fiber with smaller Brillouin linewidth under the same conditions.

4. Conclusion
We conduct mode analysis on SMF, FMF and MMF, and obtain the electric field distribution of different modes. Furthermore, the Brillouin characteristics of SMF, FMF and MMF are studied and compared based on the calculation method of Brillouin scattering parameters in the case of single mode and superposition mode. The conclusions are as follows:
(1) The BFS and Brillouin linewidth of different modes of FMF and MMF increase as the mode order decreases;
(2) The SMF has the largest BFS, followed by the MMF, and the smallest is FMF.
(3) The MMF has the largest Brillouin linewidth, followed by the FMF, and the smallest is SMF.
(4) The SMF has the largest gain peak, followed by FMF, and the smallest for MMF.
This work provides theoretical and modeling support for Brillouin distributed fiber optical sensors, and a theoretical basis for the selection and optimization of sensing parameters.

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