We study angular distributions of Drell-Yan leptons in the proton-proton collisions invoking the hypothesis of quark Reggeization in $t$-channel exchanges at high energy.

1 Introduction

The inclusive production of two leptons with opposite electric charge $l^+l^-$ and invariant mass $Q$ larger than a few GeV, which is usually specify as Drell-Yan pair production [1], in hadronic collisions is considered as very important process of high-energy hadronic physics. The experimental study of Drell-Yan pair production includes measurements of massive lepton pair distributions in rapidity ($y$), invariant mass ($Q$), transverse momentum ($q_T = |\vec{q}_T|$), and angular distributions of leptons in the virtual photon rest frame. The last ones are usually presented in terms of so-called angular coefficients, which is being a subject of study in this paper. The relevant data for proton-proton collisions have been obtained at the Fermilab Tevatron by NuSea Collaboration [2] at the $\sqrt{s} = 39$ GeV. Theoretical study of Drell-Yan pair production is based on the Parton Model (PM) and perturbative quantum chromodynamics (QCD) in leading-order (LO) and next-to-leading-order (NLO) approximations [3], as well as on the soft initial-state gluon resummation procedure in all orders in $\alpha_s$ [4] (see, also, references therein). The attempts of description of Drell-Yan pair production were undertaken also in the uncollinear factorization scheme, namely $k_T$–factorization approach, which takes into account off-shell properties of $t$–channel exchange partons and unintegrated parton PDFs
In this paper we study Drell-Yan pair production in the framework of the Parton Reggeization Approach (PRA), invoking the hypothesis of quark Reggeization in $t$-channel exchanges at high energy. The quark Reggeization hypothesis has been used successfully for description of different spectra of prompt photons at the Fermilab Tevatron and CERN LHC, electron deep inelastic scattering and prompt photon production cross sections at the DESY HERA.

2 Drell-Yan pair production in QCD

In the experiments under consideration, massive lepton pairs are produced with substantial transverse momentum $\vec{q}_T$. To describe nonzero transverse momentum Drell-Yan pair production we need to consider in the collinear PM the NLO partonic subprocesses $2 \rightarrow 2$: $q + \bar{q} \rightarrow g + \gamma^* \rightarrow g + l^+ + l^-$, $q + g \rightarrow q + \gamma^* \rightarrow q + l^+ + l^-$, $q + g \rightarrow q + \gamma^* \rightarrow q + l^+ + l^-$. The Drell-Yan pair production in collisions of hadrons have been study carefully in the NLO QCD and the collinear PM, excluding the region of small $q_T$ and $Q$. In the $k_T$–factorization approach the processes with the off-shell initial partons ($q^*, g^*$) having nonzero transverse momenta are considered as the source of the LO and NLO contributions: $q^* + \bar{q}^* \rightarrow \gamma^* \rightarrow l^+ + l^-$, $q^* + g^* \rightarrow g + \gamma^* \rightarrow g + l^+ + l^-$, $q^* + g^* \rightarrow q + \gamma^* \rightarrow q + l^+ + l^-$. However, the conception of off-shell quarks is not defined correctly in the $k_T$–factorization approach because it breaks the gauge invariance of relevant amplitudes and thereby the charge current conservation. This difficulty can be solved in the PRA, where the initial off-shell gluons and quarks are considered as Reggeons or Reggeized gluons and quarks, which interact with usual quarks and Yang-Mills gluons by a special way, via gauge invariant effective vertices. Our previous study of inclusive jet and inclusive prompt photon production show that in the PRA it is enough to consider only LO subprocess to describe inclusive data well.

The LO subprocess, which describe Drell-Yan pair production with nonzero transverse momentum in the PRA, is annihilation of Reggeized quark and Reggeized antiquark in lepton pair via virtual photon:

$$\mathcal{Q}(q_1) + \bar{\mathcal{Q}}(q_2) \rightarrow \gamma^* \rightarrow l^+(k_1) + l^-(k_2).$$

Four-momenta of Reggeized quarks (antiquarks) have transverse components and they read $q^\mu_i = x_i P^\mu + q_i^\mu_T$, $q_i^\mu_T = (0, \vec{q}_iT, 0)$, $q_i^2 = q_i^2 = -t_i \neq 0$. The amplitude of the subprocess reads in PRA as follows

$$M(\mathcal{Q}, \bar{\mathcal{Q}} \rightarrow l^+ l^-) = 4\pi\alpha_i \bar{V}(x_2 P_2) \Gamma_D^{\gamma^*}(q_1, q_2) U(x_1 P_1) \otimes \bar{U}(k_1) \gamma_{\mu} V(k_2),$$
where $e_i$ is the electric charge of quark $i$ (in units of electron charge), and $\Gamma^{\gamma,\mu}_{\bar{Q}Q}(q_1, q_2)$ is the gauge invariant Fadin-Sherman effective vertex \cite{9, 14},

$$\Gamma^{\gamma,\mu}_{\bar{Q}Q}(q_1, q_2) = \gamma^\mu - \frac{2q_1 P^\mu_1}{x_2 S} - \frac{2q_2 P^\mu_2}{x_1 S}. \quad (3)$$

We study the Drell-Yan pair production with nonzero transverse-momentum in the proton-proton high-energy collisions: $p(P_1) + p(P_2) \rightarrow l^+(k_1) + l^-(k_2) + X$, where 4-momenta of particles are shown in brackets, $l = e, \mu$ (electron or muon), $q = q_1 + q_2 = k_1 + k_2$ is the 4-momentum of virtual photon, $Q = \sqrt{q^2}$, $Q^2_T = Q^2 + q^2_T = x_1 x_2 S$.

Differential cross section of this process can be presented as follows:

$$\frac{d\sigma}{dQ^2 dq^2_T dy d\Omega} = \frac{\alpha^2}{64\pi^3 S Q^4} L_{\mu\nu} W_{\mu\nu}, \quad (4)$$

where $y$ is the rapidity of virtual photon (or lepton pair), $d\Omega = d\phi d\cos \theta$ is the space angle of producing positive lepton in the rest frame or virtual photon, $\alpha$ is the electromagnetic constant, $L_{\mu\nu} = 2(k_1^\mu k_2^\nu + k_2^\mu k_1^\nu) - Q^2 g_{\mu\nu}$ is the leptonic tensor, $W_{\mu\nu} = \int d^4x e^{iqx} \langle P_1P_2 | j_{\mu}(x) j_{\nu}(0) | P_1P_2 \rangle$ is the hadronic tensor, $P_{1,2} = \sqrt{S}(1, 0, 0, \pm 1)$.

The normalized angular distribution of leptons can be written using two different sets of the angular coefficients:

$$A_0 = \frac{W_L}{W_{TL}}, \quad A_1 = \frac{W_\Delta}{W_{TL}}, \quad A_2 = \frac{2W_{\Delta\Delta}}{W_{TL}}, \quad (5)$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0}, \quad \mu = \frac{2A_1}{2 + A_0}, \quad \nu = \frac{2A_2}{2 + A_0}. \quad (6)$$

Helicity structure functions $W_{TL, \Delta, \Delta\Delta}$ are obtained by the projection of hadronic tensor on the photon states with the different polarizations $\epsilon^\mu(q)$, $\lambda = \pm 1, 0$ \cite{4}.

3 Helicity structure functions in PRA

We suggest that factorization formula can be used in the PRA:

$$d\sigma(pp \rightarrow l^+l^- X) = \sum_q \int \frac{d\phi_1}{2\pi} dt_1 \frac{dx_1}{x_1} \frac{d\phi_2}{2\pi} dt_2 \frac{dx_2}{x_2} \Phi^\mu_q(x_1, t_1, \mu) \Phi^\mu_{\bar{q}}(x_2, t_2, \mu) d\sigma(Q\bar{Q} \rightarrow l^+l^-), \quad (7)$$

where $\Phi^\mu_q(x_{1,2}, t_{1,2}, \mu)$ are unintegrated over the transverse momentum PDFs of the Reggeized quarks. In our numerical analysis, we adopt the prescription proposed by Kimber, Martin, and Ryskin (KMR) \cite{15} to obtain unintegrated quark PDFs of the proton from the
conventional integrated one. The differential cross section \( \hat{\sigma}(Q\bar{Q} \rightarrow l^+l^-) \) is directly connected with squared matrix element. In the PRA we obtain for the squared amplitude of the subprocess (11)

\[
|M(Q_i\bar{Q}_i \rightarrow l^+l^-)|^2 = \frac{16\pi^2}{3Q^4} \alpha_s^2 e_i^2 L^{\mu\nu} w_{\mu\nu}^{\text{Regge}},
\]

where the partonic tensor for Reggeized quarks reads:

\[
w_{\mu\nu}^{\text{Regge}} = x_1 x_2 [-S g^{\mu\nu} + 2(P_1^\mu P_2^\nu + P_2^\mu P_1^\nu) \frac{(2x_1x_2S - Q^2 - t_1 - t_2)}{x_1x_2S} + + \frac{2}{x_2} (q_1^\mu P_1^\nu + q_1^\nu P_1^\mu) + \frac{2}{x_1} (q_2^\mu P_2^\nu + q_2^\nu P_2^\mu) + \frac{4(t_1 - x_1x_2S)}{x_1x_2S} P_1^\mu P_1^\nu + \frac{4(t_2 - x_1x_2S)}{x_1x_2S} P_2^\mu P_2^\nu].
\]

The helicity structure functions \( W_{T,...}^{\text{Regge}} \) at the fixed values of variables \( S, Q, q_T, y \) can be presented via corresponding quark helicity functions \( w_{T,...}^{\text{Regge}} \):

\[
W_{T,...}^{\text{Regge}}(S, Q, q_T, y) = \frac{8\pi^2 S}{3Q^4} \int dt_1 \int d\phi_1 \sum_q \Phi_q^p(x_1, t_1, \mu^2) \Phi_{\bar{q}}^p(x_2, t_2, \mu^2) w_{T,...}^{\text{Regge}},
\]

where

\[
w_T^{\text{Regge}} = Q^2 + \left( \bar{q}_1 T + \bar{q}_2 T \right)^2, \quad w_L^{\text{Regge}} = (\bar{q}_1 T - \bar{q}_2 T)^2,
\]

\[
w_\Delta^{\text{Regge}} = 0, \quad w_{\Delta\Delta}^{\text{Regge}} = \left( \bar{q}_1 T + \bar{q}_2 T \right)^2.
\]

Factorization scale is chosen to be \( \mu = \xi Q_T \), and \( \xi \) was varied between 1/2 and 2, to obtain the scale uncertainty. It was found, that observables \( A_0, A_2, \lambda, \nu \) are much more stable under the scale variations than values of cross sections. Their variations under scale changing in the kinematical regions under consideration are found to be less than 20%.

Our LO PRA results should be corrected by the so-called K-factor, which includes higher order (HO) QCD corrections to the LO diagrams. The main part of HO corrections arising from real gluon emission already accounted in LO PRA. Another part comes from the non-logarithmic loop corrections arising from gluon vertex corrections. Accordingly to Ref. [16], this K-factor is written as follows

\[
K(Q\bar{Q} \rightarrow \gamma^*) = \exp(C_F \frac{\alpha_s(\mu^2)}{2\pi} \pi^2),
\]

where a particular scale choice \( \mu^2 = Q_T^{4/3} Q^{2/3} \) has been used to evaluate \( \alpha_s(\mu^2) \).

### 4 Results

Recently, NuSea Collaboration from Fermilab Tevatron has published data for Drell-Yan pair production in fixed-target experiment with hydrogen and deuterium targets at
Figure 1: Angular coefficients $\lambda$ (left panel) and $\nu$ (right panel) as a function of $q_T$. The histogram corresponds to LO calculation in the PRA with KMR \cite{15} unintegrated PDFs. The data are from NuSea Collaboration \cite{2}.

$E_p = 800$ GeV proton beam ($\sqrt{s} = 39$ GeV) \cite{2}. The measurements were made in the following kinematics domain: $4.5 < Q < 15$ GeV, $0 < q_T < 4$ GeV, $0 < x_F < 0.8$. The results of measurements of angular distributions are presented in terms of angular coefficients $\lambda$ and $\nu$ as functions of virtual photon transverse momentum. We find good agreement of our LO PRA calculations with data for $\lambda$ and $\nu$ at all values of $q_T$, as it is shown in Fig. 1. Additionally, we predict $\mu = 0$ that is also in agreement with data with the experimental accuracy.

Acknowledgements

M. N. thanks Organizing Committee of HSQCD 2012 for invitation and nice accommodation during the Conference. The work was supported by the Ministry for Science and Education of the Russian Federation under Contract No. 14.740.11.0894. The work of M. N. is supported also by the Grant of the Student’s Stipend Program of the Dynasty Foundation.
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