NONPERTURBATIVE RENORMALIZATION GROUP EQUATION

AND BETA FUNCTION IN N=2 SUSY YANG-MILLS

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ABSTRACT

We obtain the exact beta function for $N = 2$ SUSY $SU(2)$ Yang-Mills theory and prove the nonperturbative Renormalization Group Equation

$$\partial_{\Lambda} F(a, \Lambda) = \frac{\Lambda}{\Lambda_0} \partial_{\Lambda_0} F(a_0, \Lambda_0) e^{-2 \int_{\tau_0}^{\tau} dx \beta^{-1}(x)}.$$

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1. Montonen-Olive duality \([1]\) and related versions suggest the existence of deep structures underlying relevant QFT’s. As a remarkable example the Seiberg-Witten exact results about \(N = 2\) SUSY Yang-Mills \([2]\) (see \([3]\) for reviews and related aspects), extensively studied in \([4–31]\), are strictly related to topics such as uniformization theory, Whitham dynamics and integrable systems.

In the case of \(N = 2\) SUSY Yang-Mills with compact gauge group \(G\), the terms in the low-energy Wilsonian effective action with at most two derivatives and four fermions are completely described by the so-called prepotential \(F\) \([32]\)

\[
S_{\text{eff}} = \frac{1}{4\pi} \text{Im} \left( \int d^4x d^2\theta d^2\bar{\theta} \Phi_i^D \bar{\Phi}_i \right) + \frac{1}{2} \int d^4x d^2\theta \tau^{ij} W_i W_j ,
\]

where \(W_i\) is a vector multiplet, \(\Phi_i^D \equiv \partial F / \partial \Phi_i\) is the dual of the chiral superfield \(\Phi_i\), \(\tau^{ij} \equiv \partial^2 F / \partial \Phi_i \partial \Phi_j\) are the effective couplings and \(i \in [1, r]\) with \(r\) the rank of \(G\).

The prepotential \(F\) plays a central role in the theory. The most important property of \(F\) is holomorphicity \([32]\). Furthermore, it has been shown in \([32]\) that \(F\) gets perturbative contributions only up to one-loop. Higher-order terms in the asymptotic expansion comes as instanton contribution implicitly determined in \([2]\).

We stress that the exact results obtained by Seiberg-Witten concern the Wilsonian effective action in the limit considered in \([1]\). In this context it is useful to recall that when there are no interacting massless particles the Wilsonian action and the standard generating functional of one-particle irreducible Feynman diagrams are identical. In the case of supersymmetric gauge theories the situation is different. In particular due to IR ambiguities (Konishi anomaly) the 1PI effective action might suffer from holomorphic anomalies \([33]\).

An interesting question concerning the Seiberg-Witten theory is whether using their non-perturbative results it is possible to reconstruct the full quantum field theoretical structure. In this context we note that in \([17]\), where a method to invert functions was proposed, it has been derived a nonperturbative equation which relates in a simple way the prepotential and the vevs of the scalar fields. It \([20]\) J. Sonnenschein, S. Theisen and S. Yankielowicz conjectured that the above relation should be interpreted in terms of RG ideas.

In this letter we will prove this conjecture. In particular we will obtain the nonperturbative Renormalization Group Equation (RGE) and the exact expression for the beta function of \(N = 2\) SUSY \(SU(2)\) Yang-Mills.

Let us denote by \(a_i \equiv \langle \phi^i \rangle\) and \(a_i^D \equiv \langle \phi_D^i \rangle\) the vevs of the scalar component of the chiral superfield. For gauge group \(SU(2)\) the moduli space of quantum vacua, parameterized by
$u \equiv \langle \text{tr} \phi^2 \rangle$, is $\Sigma_3 = \mathbb{C} \setminus \{-\Lambda^2, \Lambda^2\}$, the Riemann sphere $\hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ with punctures at $\pm \Lambda^2$ and $\infty$, where $\Lambda$ is the dynamically generated scale. It turns out that

$$a_D(u, \Lambda) = \partial_u F = \frac{\sqrt{2}}{\pi} \int^u \frac{dx \sqrt{x - u}}{\sqrt{x^2 - \Lambda^4}}, \quad a(u, \Lambda) = \frac{\sqrt{2}}{\pi} \int_{-\Lambda^2}^{\Lambda^2} \frac{dx \sqrt{x - u}}{\sqrt{x^2 - \Lambda^4}},$$

(2)

where $F$ is the prepotential. A crucial step in recognizing the full QFT structures underlying the Seiberg-Witten theory is the fact that [8] (see also [17])

$$\left[ \frac{\partial^2}{\partial u^2} + \frac{1}{4(u^2 - \Lambda^4)} \right] a_D = 0 = \left[ \frac{\partial^2}{\partial u^2} + \frac{1}{4(u^2 - \Lambda^4)} \right] a,$$

(3)

which is the “reduction” of the uniformizing equation for $\Sigma_3$, the Riemann sphere with punctures at $\pm \Lambda^2$ and $\infty$ [3, 8, 17]

$$\left[ \frac{\partial^2}{\partial u^2} + \frac{u^2 + 3\Lambda^4}{4(u^2 - \Lambda^4)^2} \right] \sqrt{\Lambda^4 - u^2} \partial_D = 0 = \left[ \frac{\partial^2}{\partial u^2} + \frac{u^2 + 3\Lambda^4}{4(u^2 - \Lambda^4)^2} \right] \sqrt{\Lambda^4 - u^2} \partial a.$$

(4)

A related aspect concerns the transformation properties of $F$. It turns out that [17, 34]

$$\gamma \cdot F(a) = \tilde{F}(\tilde{a}) = F(a) + \frac{a_{11}a_{21}}{2} a_D^2 + \frac{a_{12}a_{22}}{2} a^2 + a_{12}a_{21} a a_D =$$

$$F(a) + \frac{1}{4} v^t \left[ G^t_\gamma \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) \right] v,$$

(5)

where $v = \left( \begin{array}{c} a_D \\ a \end{array} \right)$ and $G_\gamma = \left( \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right) \in SL(2, \mathbb{C})$. Observe that $\gamma_2 \cdot (\gamma_1 \cdot F(a)) = (\gamma_1 \gamma_2) \cdot F(a)$ and

$$G^t_\gamma \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) G_\gamma - \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) = 2 \left( \begin{array}{cc} a_{11} a_{21} & a_{12} a_{21} \\ a_{12} a_{21} & a_{12} a_{22} \end{array} \right).$$

(6)

We stress that if $G_\gamma \in \Gamma(2)$ then $\tilde{F} = F$, that is $\gamma \cdot F(a) = F(\tilde{a})$. The transformation properties of $F$ have been obtained for more general cases in [20, 34, 35]. Eq. (3) implies that $2F - a \partial_a F$ is invariant under $SL(2, \mathbb{C})$. In particular, it turns out that [17]

$$2F - a \frac{\partial F}{\partial a} = -8\pi i b_1 \langle \text{tr} \phi^2 \rangle,$$

(7)

where, as stressed in [24, 21], $b_1 = 1/4\pi^2$ is the one-loop coefficient of the beta function. Relevant generalizations of the nonperturbative relation (7) have been obtained by J. Sonnenschein, S. Theisen and S. Yankielowicz [20] and by T. Eguchi and S.-K. Yang [21].

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We note that the relation (7) turns out to be crucial in obtaining Seiberg-Witten theory from the tree-level Type II string theory in the limit $\alpha' \to 0$.

In [20] it has been suggested that Eq.(7) should be understood in terms of RG ideas. In particular, it was suggested to consider the LHS of (7) as a measure of the anomalous dimension of $F$. Actually we will see that $\langle \text{tr} \phi^2 \rangle$ involves the nonperturbative beta function in a natural way. This allows us to find the RGE for $F$.

In order to specify the functional dependence of $u$ we use the notation of [18] by setting $u = \Lambda^2 G_1(a)$ and $u = \Lambda^2 G_3(\tau)$ where $\tau = \partial^2_a F$. Eq.(3) implies

$$(1 - G_1^2)\partial^2_a G_1 + \frac{a}{4} (\partial_a G_1)^3 = 0,$$

(8)

and by (7)(8) [17, 18]

$$\partial^3_a F = \frac{(a\partial^2_a F - \partial_a F)^3}{4 \left[ 64\pi^2 b_1^2 \Lambda^4 + (a\partial_a F - 2F)^2 \right]},$$

(9)

which provides recursion relations for the instanton contribution. By (2) we have $a(u = -\Lambda^2, \Lambda) = -i4\Lambda/\pi$ and $a(u = \Lambda^2, \Lambda) = 4\Lambda/\pi$ so that the initial conditions for the second-order equation (8) are $G_1(-i4\Lambda/\pi) = -1$ and $G_1(4\Lambda/\pi) = 1$.

Eqs.(7)(9) are quite basic for our purpose. For example, by (7) we have [18]

$$F(a, \Lambda) = 8\pi i b_1 \Lambda^2 a^2 \int_{i\Lambda/\pi}^a dx G_1(x)x^{-3} - \frac{ib_1 \pi^3}{4} a^2,$$

(10)

and [18]

$$\partial_{\tau}\langle \text{tr} \phi^2 \rangle = \frac{1}{8\pi i b_1} \langle \phi \rangle^2,$$

(11)

which is the quantum version of the classical relation $u = a^2/2$. In this context we observe that $\hat{\tau} = a_D/a$ has the same monodromy of $\tau$ and their fundamental domains differ only for the values of the opening angle at the cusps [37]. These facts and (11) suggest to consider $\tau$ and $\hat{\tau}$ as dual couplings. In particular, it should exist a “dual theory” with $\hat{\tau}$ playing the role of gauge coupling.

As noticed in [20, 21], the fact that $\tau = \partial^2_a F$ is dimensionless implies

$$a(\partial_a F)_\Lambda + \Lambda (\partial_\Lambda F)_a = 2F.$$

(12)

Thus, according to (7), we have

$$\Lambda\partial_\Lambda F = -8\pi i b_1 \langle \text{tr} \phi^2 \rangle.$$
In [18] it has been shown that $\mathcal{G}_3$ satisfies the equation

$$2(1 - \mathcal{G}_3^2)^2 \{\mathcal{G}_3, \tau\} = -(3 + \mathcal{G}_3^2) (\partial_\tau \mathcal{G}_3)^2,$$

with initial conditions

$$\mathcal{G}_3(-1) = \mathcal{G}_3(1) = -1, \quad \mathcal{G}_3(0) = 1.$$  

The solution of (14) is

$$u = \Lambda^2 \mathcal{G}_3(\tau) = \Lambda^2 \left\{1 - 2 \left[\frac{\Theta_2(0|\tau)}{\Theta_3(0|\tau)}\right]^4\right\},$$

that by the “inversion formula” [7] implies [18]

$$2\mathcal{F} - a \frac{\partial \mathcal{F}}{\partial a} = 8\pi i b_1 \Lambda^2 \left\{2 \left[\frac{\Theta_2(0|\partial_a^2 \mathcal{F})}{\Theta_3(0|\partial_a^2 \mathcal{F})}\right]^4 - 1\right\},$$

showing that such a combination of theta-functions acts on $\partial_a^2 F$ as integral operators.

2. Before considering the beta function we observe that the scaling properties of $a_D$ and $a$ suggest to introduce the following notation

$$\Lambda^{-1} a_D(u, \Lambda) = a_D(v, 1) \equiv b_D(v), \quad \Lambda^{-1} a(u, \Lambda) = a(v, 1) \equiv b(v), \quad v \equiv u/\Lambda^2.$$  

We now start to evaluate the nonperturbative beta function. First of all note that in taking the derivative of $\tau$ with respect to $\Lambda$ we have to distinguish between $\partial_\Lambda \tau$ evaluated at $u$ or $a$ fixed. We introduce the following notation

$$\beta(\tau) = (\Lambda \partial_\Lambda \tau)_u, \quad \beta^{(a)}(\tau) = (\Lambda \partial_\Lambda \tau)_a.$$  

Acting with $\Lambda \partial_\Lambda$ on $\mathcal{G}_3(\tau) = u/\Lambda^2$, we have

$$\beta(\tau) \mathcal{G}_3'(\tau) = -2 \frac{u}{\Lambda^2},$$

so that

$$\beta(\tau) = -2 \frac{\mathcal{G}_3}{\mathcal{G}_3'}. $$

Integrating this expression and considering the initial condition $\mathcal{G}_3(0) = 1$ in [15] we obtain

$$\langle \text{tr} \phi^2 \rangle_\tau = \Lambda^2 e^{-2 \int_0^\tau d\tau' \beta^{-1}(\tau'), }.$$
or equivalently
\[ \langle \text{tr} \phi^2 \rangle_\tau = \left( \frac{\Lambda}{\Lambda_0} \right)^2 \langle \text{tr} \phi^2 \rangle_{\tau_0} e^{-2 \int_{\tau_0}^{\tau} dx \beta^{-1}(x)}. \]

Using once again the relation (7) we obtain
\[ (a \partial_a - 2) F(a, \Lambda) = 8\pi i b_1 \Lambda^2 e^{-2 \int_{\tau_0}^{\tau} dx \beta^{-1}(x)}, \]

or equivalently
\[ (a \partial_a - 2) F(a, \Lambda) = \left( \frac{\Lambda}{\Lambda_0} \right)^2 (a_0 \partial_{a_0} - 2) F(a_0, \Lambda_0) e^{-2 \int_{\tau_0}^{\tau} dx \beta^{-1}(x)}, \]

which provides the anomalous dimension of \( F \). Note that in (23) we used the notation \( a_0 \) to denote \( a \) at \( \tau_0 \equiv \tau(\Lambda_0) \). By Eqs. (24) (25) and (12) we obtain the nonperturbative Renormalization Group Equation
\[ \partial_\Lambda F = -8\pi i b_1 \Lambda e^{-2 \int_{\tau_0}^{\tau} dx \beta^{-1}(x)}, \]

that is
\[ \partial_\Lambda F(a, \Lambda) = \frac{\Lambda}{\Lambda_0} \partial_{\Lambda_0} F(a_0, \Lambda_0) e^{-2 \int_{\tau_0}^{\tau} dx \beta^{-1}(x)}. \]

We note that, due to the \( \tau(\Lambda) \) dependence, this equation is highly nonlinear reflecting its nonperturbative nature.

3. We now start in deriving from Eq. (3) an alternative expression for the beta function. Let us consider the differentials
\[ d\tau = (\partial_\tau \tau) a d\Lambda + (\partial_a \tau) \Lambda da = (\partial_\Lambda \tau) a d\Lambda + (\partial_a \tau) \Lambda du, \]

and
\[ da = (\partial_\Lambda a) a d\Lambda + (\partial_a a) \Lambda du = bd\Lambda + \Lambda b' dv = (b - 2vb')d\Lambda + \Lambda^{-1}b'du. \]

Eqs. (28) (29) yield
\[ (\partial_\Lambda \tau)_a = (\partial_\tau \tau)_a + (b - 2vb')(\partial_a \tau)_\Lambda. \]

By (12) we have \( \Lambda(\partial_\Lambda \tau)_a = -a(\partial_a \tau)_\Lambda \), so that
\[ \beta(\tau) = -2vb'\Lambda(\partial_a \tau)_\Lambda = 2\frac{b'}{b} \beta(\tau). \]
Let us introduce $G$ and $\sigma$ defined by $b_D = \partial b G$ and $\sigma = \partial^2_b G = b'_D/b'$. By a suitable rescaling of (9), it follows that $(\partial_b \sigma) = 1/[2\pi i b^3(1 - v^2)]$. On the other hand $G = \Lambda^{-2} F$ and $\sigma = \tau$, so that

$$(\partial_b \tau) = \frac{1}{2\pi i b^3(v^2 - 1)}. \tag{32}$$

Being $\Lambda(\partial_{\Lambda} \tau) = -b(\partial_b \tau)$, we have

$$\beta(\tau) = \frac{v}{\pi i b^2(v^2 - 1)}, \tag{33}$$

and

$$\beta^{(a)}(\tau) = \frac{b}{2\pi i b^3(v^2 - 1)}. \tag{34}$$

By (16) and using Riemann’s theta relation $\Theta_{3}^4 = \Theta_{2}^4 + \Theta_{4}^4$, where $\Theta_i \equiv \Theta_i(0|\tau)$, we obtain

$$\beta(\tau) = \frac{2\pi i (\Theta_{4}^4 - \Theta_{2}^4)}{(\Theta_{2}^8 - \Theta_{2}^4 \Theta_{4}^4)} \left[ \int_{-1}^{1} dx \sqrt{\frac{1}{(x^2 - 1)(x\Theta_{4}^4 + \Theta_{2}^4 - \Theta_{4}^4)}} \right]^2, \tag{35}$$

and

$$\beta^{(a)}(\tau) = \frac{2\pi i \int_{-1}^{1} dx \sqrt{\frac{x\Theta_{4}^4 + \Theta_{2}^4 - \Theta_{4}^4}{x^2 - 1}}}{(\Theta_{2}^8 - \Theta_{2}^4 \Theta_{4}^4)} \left[ \int_{-1}^{1} dx \sqrt{\frac{1}{(x^2 - 1)(x\Theta_{4}^4 + \Theta_{2}^4 - \Theta_{4}^4)}} \right]^3. \tag{36}$$

Let us discuss some properties of $\beta(\tau)$ and $\beta^{(a)}(\tau)$. First of all by (33) it follows that $\beta^{(a)}(\tau)$ is nowhere vanishing. This is a consequence of the fact that $|b|$ has a lower bound that, as noticed in [38], is given by $b(0) \sim 0.76$. Both $\beta(\tau)$ and $\beta^{(a)}(\tau)$ diverge at $u = \pm \Lambda^2$ where dyons and monopoles are massless. This happens at $\tau \in \mathbb{Z}$, corresponding to a divergent gauge coupling constant.

By (33) the $\beta(\tau)$ function is vanishing at $u = 0$. We can found the corresponding values of $\tau$ by (16). On the other hand, by uniformization theory we know that $u = 0$ corresponds to $\tau = (i + 2n + 1)/2$, $n \in \mathbb{Z}$.

As a byproduct of our investigation we observe that (21) and (33) yield

$$\Theta_{2}^8 \Theta_{3} - \Theta_{2} \Theta_{4} = \Theta_{2}^4 \Theta_{3} - \Theta_{2} \Theta_{4} \left[ \int_{-1}^{1} dx \sqrt{\frac{1}{(x^2 - 1)(x\Theta_{4}^4 + \Theta_{2}^4 - \Theta_{4}^4)}} \right]^2, \tag{37}$$

where $\Theta_i' \equiv \partial_{\tau} \Theta_i(0|\tau)$.

We note that, in a different context, an expression for the beta function was derived in [39] whereas very recently J. Minahan and D. Nemeschansky [40], using different techniques,
obtained an expression for the beta function which has the same critical points of $\beta(\tau)$ in (35). If one identifies (up to normalizations) $\beta(\tau)$ with that in [40] one obtains a relation involving the four $\Theta_i$’s (including $\Theta_1$).

The beta function has also a geometrical interpretation. To see this we use the Poincaré metric on $\Sigma_3$ expressed in terms of vevs in [18]. In terms of $\beta$ we have

$$ds_P^2 = \left| \frac{\beta}{2v\mathrm{Im}\tau} \right|^2 |du|^2 = e^{\varphi}|du|^2,$$ (38)

so that $\beta/v$ is the chiral block of the Poincaré metric. We observe that (3) is essentially equivalent to the Liouville equation $2\partial_u\partial_{\bar{u}}\varphi = e^{\varphi}$ (see for example [43]).

An important aspect of the Seiberg-Witten theory concerns the structure of the critical curve $C$ on which $\mathrm{Im}a_D/a = 0$. The structure and the role of this curve have been studied in [2,18,31,32,41,42]. In particular, in [18], using the Koebe 1/4-theorem and Schwarz’s lemma, inequalities involving the correlators and $\Lambda\partial_\Lambda F = -8\pi i u$ have been obtained. Expanding the beta function in the regions of weak and strong coupling one has to consider Borel summability for which the inequalities in [18] should provide estimations for convergence domains.

Finally we observe that the way the results in this paper have been obtained suggest an extension to more general cases.

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