**Abstract**

Work in information retrieval has largely been centered around ranking and relevance: given a query, return some number of results ordered by relevance to the user. The problem of result list truncation, or where to truncate the ranked list of results, however, has received less attention despite being crucial in a variety of applications. Such truncation is a balancing act between the overall relevance, or usefulness of the results, with the user cost of processing more results. Result list truncation can be challenging because relevance scores are often not well-calibrated. This is particularly true in large-scale IR systems where documents and queries are embedded in the same metric space and a query’s nearest document neighbors are returned during inference. Here, relevance is inversely proportional to the distance between the query and candidate document, but what distance constitutes relevance varies from query to query and changes dynamically as more documents are added to the index. In this work, we propose *Surprise* scoring, a statistical method that leverages the Generalized Pareto Distribution that arises in Extreme Value Theory to produce interpretable and calibrated relevance scores at query time using nothing more than the ranked scores. We demonstrate its effectiveness on the result list truncation task across image, text, and IR datasets and compare it to both classical and recent baselines. We draw connections to hypothesis testing and $p$-values.

1 Introduction

Work in information retrieval systems has traditionally focused on improving ranking while the problem of determining how many results to return has received less attention. This task, referred to as result list truncation, is defined as follows: for a specific query and its ranked result list, determine the number of candidates that should be returned such that some evaluation metric, like F1, is optimized. If the relevance scores are well-calibrated across queries, then one effective strategy is to pick a single global decision threshold on the scores. If they are not, then a parametric model can be learned to find the optimal cutoff conditioned on the query. As we will see however, in common IR systems based on “metric learning”, this latter approach is challenging or impossible.

In this work we propose a statistical method called *Surprise* scoring for calibrating a single result list by capturing how surprising or unexpected each candidate score is under the null hypothesis that it comes from a non-relevance distribution we fit. Our method remedies many of the concerns of existing approaches and is broadly applicable. It is premised on a single condition — that the IR system has good ranking performance (i.e., it scores relevant results over non-relevant ones with high probability) and that, naturally, the system returns the best results it has for a particular query.

In metric learning, an encoder model is machine-learned to embed documents and queries in the same metric space such that the most relevant documents for a query are its nearest neighbors in this space. Typically, the encoder is trained using (query, document) pairs, where positive examples comprise queries with their relevant documents and negative ones comprise queries with a random sample of non-relevant documents. The loss function, usually contrastive or triplet loss, encourages positive pairs to be closer together than negative ones. After learning an encoder, an input dataset is encoded and indexed in a scalable retrieval system that supports efficient, exact or approximate nearest neighbor lookup. During serving, the query’s $k$ nearest results in the embedding space are fetched and returned in ranked order of closeness, where relevance is inversely proportional to the distance between the query and result. Unfortunately however, as shown in Figure 1, what distance constitutes relevance is highly query dependent and so choosing a single global distance threshold to separate relevant and non-relevant results is prone to error. Existing losses only care about ranking order or relative distances within a mini-batch of examples and achieving calibrated distance scores across all queries involves joint optimization over all examples, which is quite often infeasible. Moreover, the distance distributions are constantly changing as documents are added to the index. These reasons
Figure 1: Distance distributions across queries for different datasets. Blue distributions capture distance to the first relevant sample while red distributions capture distance to the closest 10 non-relevant samples. Cosine and dot product similarities are negated to ensure lower means closer. When aggregated together, it is hard to distinguish between relevant and non-relevant results from the distances alone. This is a result of distances being uncalibrated across the embedding space.

make list truncation particularly challenging in this setting.

To this end, we present Surprise scoring, a broadly applicable method for constructing calibrated scores using nothing more than a single ranked list of raw distance or relevance scores. The main contributions of this paper are:

1. We present a statistically-grounded method for resoring a ranked list of relevance or distance-based scores. Our method consists of (1) a novel rescoring function and (2) a new greedy method for selecting its hyper-parameters. Unlike many other parametric methods, the rescoring is learned on a single example and is thus particularly useful in applications where training a parametric model on all examples is challenging due to, for example, dataset size or frequent distribution shifts. Furthermore, much like the concept of p-values in hypothesis testing, our scores are interpretable and well-calibrated across settings.

2. We apply our proposed method to the result list truncation task in two distinct settings over four datasets:

- When queries and results are represented in an embedding space and retrieval is efficient nearest-neighbor search that fetches a small neighborhood of the query from a massive index. In this case, we leverage only the local neighborhood to select a cutoff threshold for truncating the ordered list of rescoped candidates. This large-scale setting is becoming increasingly prevalent in the IR community.
- In the more general setting where the indexing and retrieval system can take any form. In this case, our rescoring algorithm remains the same, but we leverage global training data to pick the cutoff threshold for the rescoped candidates.

We show that our proposed method often outperforms both classical and recent competitive baselines for result list truncation.

2 Related Work

2.1 Modeling Relevance Scores

There has been extensive research focused on understanding and modeling score distributions of retrieval systems. Most of the early work in this area focused on fitting parametric probability distributions to score distributions [Manmatha et al. 2001; Arampatzis et al. 2009]. This is often accomplished by assuming that score distributions can be represented as a mixture of a relevance distribution and a non-relevance (or background or noise) distribution. The parameters of these distributions are typically fit for a given system using the expectation-maximization (EM) algorithm.

More recently, there has been research into how to leverage machine learning to optimally truncate a ranked list with respect to some metric of interest. For example, in cascade-style ranking systems [Wang et al. 2011b] the goal is balance between effectiveness and efficiency. Recent work investigated a number of machine learning approaches for dynamically determining cutoffs within cascade-style ranking systems [Culpepper et al. 2016]. In addition, the Transformer and Bi-directional Long Short-Term Memory (LSTM) neural architectures have been utilized to identify the best position to truncate a given ranked list [Bahri et al. 2020, Lien et al. 2019].

The work presented here is also closely related to the task of query performance prediction [Cronen-Townsend et al. 2002]. The goal of this task is to automatically determine the effectiveness of a given query. If there existed a system that could perfectly predict the effectiveness of a query with respect to a set of results, it could be leveraged to determine the best set of results to the user for any given effectiveness measure. Approaches to query performance prediction include pre-retrieval approaches [Hauff et al. 2008],
relevance modeling-based approaches [Cronen-Townsend et al., 2002; Zhou and Croft, 2007], and more recently neural network-based approaches [Zamani et al., 2018].

Determining how many results to return to users arises in a number of practical applications. For example, in sponsored search, displaying too many irrelevant ads to users may frustrate them and result in so-called "query blindness". As a result, there has been research that investigated whether it is possible to determine whether any ads should be returned to the user or not (Broder et al., 2008). A similar problem formulation investigated how many ads should be returned to the users (Wang et al., 2011a). Determining the optimal number of results to return is also important in a number of other search tasks, including legal e-discovery (Tomlinson et al., 2007), where there is an immense cost associated with reviewing results.

The ability to effectively calibrate scores across queries and corpora has also been studied in the context of federated search tasks (Shokouhi and Si, 2011), such as meta-search (Montague and Aslam, 2001).

2.2 Extreme Value Theory

Extreme value theory (EVT) (Pickands III et al., 1975), also referred to as extreme value analysis, is a subfield of statistics dealing with extreme deviations from the mean of probability distributions. More precisely, given an ordered sample of some random variable, it attempts to estimate the probability of events that are more extreme than any observed earlier. Usage of EVT spans many disciplines, including finance / risk management, traffic prediction, geological engineering, structural engineering, and earth sciences. Some concrete examples include modeling extreme floods and the magnitude of freak waves, financial market risk, mutational events during evolution, and large wildfires (Abarbanel et al., 1992; Alvarado et al., 1998; Castillo, 2012; Embrechts et al., 2013).

Within the context of machine learning, Extreme Value Machines (Rudd et al., 2017) were proposed, which are able to perform incremental learning in the presence of unknown query classes. Vignotto and Engelke (2020) suggest techniques for anomaly detection that improve the Extreme Value Machine. Meanwhile, Weng et al. (2018) leverage EVT in “Cross Lipschitz Extreme Value for Network Robustness”, a new measure of a neural network’s robustness to adversarial examples. EVT has also been used in applications like learning to rank (Zong and Huang, 2014).

3 Preliminaries

We begin by reviewing the statistical underpinnings of our method.

Definition 1 (Conditional Excess Distribution) The conditional excess distribution of a random variable Y is F_{u}, where

\[ F_{u}(x) = P(Y - u \leq x | Y > u) . \]

Definition 2 (Maximum Domain of Attraction) Let F denote a distribution such that F \in MDA(H) for some Generalized Extreme Value distribution H of a Generalized Pareto distribution (GPD) with shape parameter c and scale parameter \( \alpha \) if and only if there exists constants \( c_{n} > 0 \) and \( d_{n} \) such that if \( X_{i} \sim F \),

\[ \max(X_{1}, \ldots, X_{n}) - d_{n} \xrightarrow{\text{dist.}} H, \text{ as } n \to \infty. \]

Definition 3 (Generalized Pareto Distribution) A random variable X is said to have a Generalized Pareto distribution (GPD) with shape parameter c and scale parameter \( \alpha \) if its cumulative distribution function is given by:

\[ G_{c,\alpha}(x) = \begin{cases} 1 - \left( 1 - \frac{x}{\alpha} \right)^{1/c}, & c \neq 0 \\ 1 - e^{-x/\alpha}, & c = 0 \end{cases} \]

The support is \( x > 0 \) for \( c \leq 0 \) and \( 0 < x < \alpha/c \) for \( c > 0 \).

Theorem 1 (Pickands-Balkemade Haan) Let F be a distribution such that F \in MDA(H) for some Generalized Extreme Value distribution H, and let \( F_{u} \) be its conditional distribution function. Then,

\[ F_{u} \xrightarrow{\text{dist.}} G_{c,\alpha}, \text{ as } u \to \infty, \]

for some \( c \) and \( \alpha \). In other words, a large family of distributions can have its conditional excess well-approximated by a GPD for sufficiently large threshold \( u \).

With the necessary machinery now in place, let us suppose our retrieval system returns \( n \) results with ascending relevance scores \( (s_{0}, \ldots, s_{n-1}) \), each with some binary relevance label \( l \). Since these scores represent the tail end of relevance scores for the entire, and presumably large, index, we can pretend that they were the result of the following generative process: the scores of observed results with relevance label \( l \) are generated by first drawing an i.i.d sample \( S_{i} \) from an unknown distribution \( F_{l} \) and then discarding it if \( S_{i} < u \), for some \( u \). In other words, we are effectively drawing from the conditional excess distribution of some unknown \( F_{l} \). Theorem 1, the Pickands-Balkemade Haan theorem (Balkema and De Haan, 1974), also referred to as the Second Extreme Value theorem, provides a theoretical justification for using a Generalized Pareto distribution to model the excess distribution of observed relevance scores. This is the basis of our proposed method, Surprise scoring.

4 Surprise Scores

We now present our proposed method.
The key idea behind our method is to model the conditional excesses of non-relevant scores using a GPD and assign a new score to each result that captures how unexpected, or surprising, the result’s original score was under the null hypothesis that it came from the non-relevance distribution we fit. Surprise scoring does not change the ranking of the results. Instead, it provides an interpretable and calibrated measure of how relevant each result is compared to non-relevant ones from the result set, much like p-values in hypothesis testing. Once each result is rescored, we can truncate the list by picking a threshold that optimizes the target metric of choice. We take as given that the retrieval system has good ranking performance - that is, $S_{\text{relevant}} > S_{\text{non-relevant}}$ with high probability - and that a large fraction of the original, non-truncated result list is non-relevant. We can break our algorithm down into four steps.

1. We remove scores of results we suspect are relevant (i.e. from $F_{\text{relevant}}$). We do this by dropping the $n - \hat{j}$ largest scores that hurt the overall model fit. $\hat{j}$ is identified using an iterative greedy procedure that we will describe later.

2. Now having scores only from $F_{\text{non-relevant}}$, we proceed to pick the excess cutoff $\hat{u} = s_{\hat{i}}$. To find $\hat{i}$, we apply the same method from step 1, but start with the smallest score and work upwards.

3. We fit a GPD $G_{\hat{c}, \hat{\alpha}}$ to the selected samples using maximum likelihood estimation (MLE), constraining $c \leq 0$ so that the GPD has infinite positive support. This is important since we rescore the relevant results we dropped in step 1, whose scores can exceed the ones used in the fit. The constraint has the added benefit that the maximum likelihood estimate is guaranteed to exist and is asymptotically normal and asymptotically efficient (Smith, 1984).

### Algorithm 1 Surprise Scoring

```
function GPDRESCORE(scores)
    Ascending score list of length n
    i ← 0, j ← n.
    while CVMTest(scores[i : j] − scores[i]) decreasing do
        j ← j − 1.
    end while
    while CVMTest(scores[i : j] − scores[i]) decreasing do
        i ← i + 1
    end while
    excess = scores[i : j] − scores[i].
    gpd = FitGPDEXCESS(excess).
    surprise = zero array of length n.
    surprise[i : j] = −log(1 − gpd.cdf(excess)).
    return surprise.
end function
```

```
function CVMTEST(excess)
    Ascending score excess list of length m
    gpd = FitGPDEXCESS(excess).
    cdf = gpd.cdf(excess).
    linear = [1/2m, 3/2m, ..., (2m − 1)/2m].
    T = ∑(cdf − linear)² + 1/(12m).
    return T. > Cramer-von-Mises statistic testing GPD fitness of excess values. Lower is better.
end function
```

### Definition 4 (Surprise Score)

Given Generalized Pareto distribution $G_{c,\alpha}$ with $c \leq 0$ and threshold $u$, define the function surprise$_{c,\alpha,u}(s)$ as:

\[
\begin{cases}
-\log(1 - G_{c,\alpha}(s - u)), & s \geq u \\
0, & otherwise
\end{cases}
\]
4. Rescore each score $s$:

$$s \leftarrow \text{Surprise}_{c, \hat{\alpha}, \hat{\beta}}(s).$$

We now describe the greedy procedure for selecting $i$ and $j$. The goal is to measure how well each candidate set of excess scores fits a GPD. Various methods for selecting optimal threshold $\hat{u}$ have been suggested (see [1] for a survey), and although each has its own strengths and weaknesses, the goodness-of-fit tests posed by [2] (2001) fares well in many practical settings. Given a candidate set of excesses $\{e_0, \ldots, e_m\}$, they suggest:

1. Fit a GPD $G_{c, \hat{\alpha}}$ to $\{e_0, \ldots, e_m\}$ using MLE.
2. Compute the Cramer-von-Mises statistic:

$$W^2 = \sum_{i=1}^{m} \left( G_{c, \hat{\alpha}}(e_i) - \frac{2i - 1}{2m} \right)^2 + 12m.$$

3. Look up in a pre-computed table the $W^2$ expected under the null hypothesis the excesses come from a GPD with shape $\hat{\alpha}$. The scale $\hat{\alpha}$ does not matter.

They suggest setting $u = s_k$ with $k = 0$ initially (so that the initial candidate excesses are $\{0, \ldots, s_{n-1} - s_0\}$), and increasing $k$ until the $p$-value of $W^2$ is within some level of significance. While statistically sound, this method has practical limitations for our setting. Firstly, it requires computing $W^2$ values for all parameters $c$ and sample size $n$. The authors suggest pre-computing $W^2$ for a finite number of $c$ and then linearly interpolating between known values, but this incurs some error, and since $c$ is unbounded it need not fall near a pre-computed point. Furthermore, in our use case $n$ can vary from query to query and need not be sufficiently high that falling back to the asymptotic $W^2$ values is justified. Last but not least, their method does not account for the presence of tail, relevant scores that need not come from the hypothesized GPD. Our greedy approach first computes $W^2$ on the entire score list following steps 1 and 2. It then iteratively drops the largest scores and recomputes $W^2$ until the statistic begins to increase. It then applies the same approach to the other side, but dropping the lowest score each iteration. Algorithm 1 shows our method end to end. Figure 2 illustrates the GPD fit and resulting Surprise scores for a sample query.

5 Experimental Setup

We now describe the experimental setup used to evaluate our proposed Surprise scoring method for the result list truncation task in diverse settings.

5.1 Datasets

5.1.1 Same Document Detection

We would like to evaluate how well our method is able to decide when to swing or not to swing - that is, when to return results and when not to. To that end, we construct the following problem using the Reuter’s corpus of 10,788 news documents (total of 1.3 million words). First, we combine the canonical train and test splits with more than 800 characters and truncate each document to 800 characters. This leaves us with 3,342 documents. We then fit a TF-IDF featurizer using the 1,024 most frequent 1-gram to 6-gram. Each document is subsequently split into a top and bottom half, each consisting of 400 characters, and then featurized. Next, we train a siamese network on the (top, bottom) example pairs using Euclidean distance and contrastive loss with a margin of 1. The encoder is a neural network with 2 1024-unit ReLU layers. We use Adam [3] optimization with default learning rate 0.001. After training the encoder and encoding all examples, we select a random 30% of the example pairs. Half of the bottom halves are discarded, while the rest are combined with all the examples from the remaining 70%. We index over this latter set and use the top halves of the 30% split as a test set. The end result is that half of our test examples will have its matching pair in the index and half will not. Furthermore, since we trained on the combined set, when there is a match, it’s particularly close to the query. We want our thresholding methods to return no results when there is no match in the index and to capture it in the result list when there is. Our target metric is accuracy at this task.

5.1.2 News Authorship

News articles written by 1,256 authors are partitioned into three sets: train, index, and test. Train (test) consist of 50 (10) articles per author. The index is a subset of train, built as follows: the authors are partitioned into 5 nearly equal sets and for each set a predetermined number of articles are selected for authors in the set. These numbers are 10, 20, 30, 40, 50. Each article has at least 500 words. We featurize each article as a normalized bag of 2,000-most-frequent 4-grams occurring in the training data, and train a siamese network on the character 4-grams features using dot product similarity and batch-based cross-entropy loss. The encoder is a neural network with 1 hidden ReLU layer of 256 units and 1 linear output layer of 128 units. We generate positive article pairs by randomly sampling 5 articles with the same label and generate negative ones using batch-based negative sampling. More concretely, within each batch of 256 (left, right) example pairs, we have the classification task of predicting each left example’s matching right example via softmax over dot product similarities. The loss is cross-entropy, summed over left examples. We
|                     | Same Document | Omniglot | News Authorship |
|---------------------|---------------|----------|-----------------|
|                     | Accuracy      | F1       | DCG             | F1   | DCG    |
| Oracle              | 1.0           | 0.3191   | 0.6659          | 0.1122 | 0.1906 |
| Global-k            | 0.5005        | 0.2259   | 0.0             | 0.0798 | 0.0    |
| Local-k             | 0.4905        | 0.2265   | 0.0239          | 0.0813 | 0.0005 |
| Isotonic            | 0.5723        | 0.2126   | 0.0383          | 0.0689 | -0.0019|
| BiCut score only    | 0.3430        | 0.2175   | -               | 0.0789 | 0.0    |
| BiCut w/ encoding   | 0.4895        | 0.2225   | -               | 0.0733 | 0.0    |
| Surprise            | **0.6112**    | **0.2285** | **0.0948**   | **0.0814** | **-0.0213** |

Table 1: Results for the metric learning setting. When matches are present, they are unusually close to their query and Surprise scores are able to capture this well. Surprise outperforms the baselines on everything except News Authorship DCG.

Figure 3: **Left:** relevance scores for 10 test queries from the Robust04 DRMM test split (relevance decreasing from left to right). **Right:** Surprise scores for the same 10 queries, along with the thresholds that achieve $p$-values of 0.1 and 0.01. These correspond to weak and very strong grounds to reject the null hypothesis respectively, where the null hypothesis here is: “score is non-relevant”.

train the siamese network using Adam with learning rate 0.001 for 300 epochs, and then use the trained encoder to encode the index set, chosen in the aforementioned way so that test queries have a varying number of relevant examples.

5.1.3 Omniglot

Omniglot ([Lake et al., 2015]) is a dataset of hand-drawings of 1,623 distinct characters from 50 alphabets, with 20 image examples per character. The dataset is partitioned into train, index and test sets. The train set contains 16 images of each character and the test set contains the remaining 4 images of each character. Index is a subset of train, built the same way as News Authorship: the characters are partitioned into 5 equal sets and for each set a predetermined number of images are selected for characters in the set. These numbers are 4, 7, 10, 13, 16. We train a siamese network using a ResNet-50 ([He et al., 2015]) image encoder and Euclidean distance. We train the encoder using Adam with learning rate 0.001 and then use it to encode the index set.

5.1.4 Robust04

To evaluate the effectiveness of our method beyond nearest-neighbor-based retrieval, we use the TREC collection Robust04 from the TREC 2004 Robust Track. It consists of 250 queries over 528k news articles, where each query has 1000 total results and an average of 70 relevant ones. This is the same dataset used in Lien et al. ([2019]). We use a random 80/20 train/test that achieves comparable performance to the reported results in Lien et al. ([2019]). We evaluate the efficacy of our truncation model using two different retrieval approaches - BM25, a traditional tf-idf based model, and DRMM ([Guo et al., 2016]), a neural model.

5.2 Baseline Methods

We evaluate our method against the following baselines:

- **Global-k** returns the top-$k$ results, where $k$ is fixed across test queries. The optimal $k$ across training is
selected.

- **Local-\( k \)** returns the top-\( k \) results, where \( k \) is chosen on a per-query basis. The \( k \) that achieves the best average metric in the query’s neighborhood is chosen.

- **Oracle** uses knowledge of each test query’s true label to choose the cutoff. This represents an upper bound on the achievable performance.

- **Isotonic Regression** gathers the scores and labels of the query’s neighbors and learns a isotonic regression model that regresses positive scores to 1 and negative ones to 0. It uses the same neighborhood to pick the optimal threshold and applies the model with this threshold to the query’s result scores.

- **BiCut** (Lien et al., 2019) learns a multi-layer bidirectional LSTM model on the entire training set using the relevance scores and optionally the result’s embeddings as inputs. At position \( i \) of the result list, the model predicts probability \( p_i \) to “continue” and probability \( 1 - p_i \) to “end”. At inference time, the cutoff is made right before the first occurrence of “end”. The training loss is defined as:

\[
\sum_i w_{y_i} |y_i - p_i|,
\]

where \( y \) is the relevance label. \( w_y \) is a hyperparameter which we optimized over the training split following the implementation in the paper.

### 5.3 Setting

For every dataset except Robust04, which is not nearest-neighbor based, we fetch the 200 closest examples from the index set for each query. We then apply each method, which truncate the lists at some point without changing the ordering of the example. We assign a relevance label \( rel \) of 1 if the example’s label matches the query’s and \(-1\) otherwise. We report F1 and Discounted Cumulative Gain (DCG) scores in this binary setting, where we define DCG to penalize negative, or non-relevant results as follows:

\[
DCG_p := \sum_{i=1}^{p} \frac{rel_i}{\log_2(i+1)},
\]

where \( p \) is the length of the result list. DCG is not supported by BiCut and is thus omitted from results. No feature normalization was applied to the scores with the exception of max normalization for Isotonic Regression on Robust04 (performance is abysmal without it). In all experiments, the best Surprise score threshold was found by searching the range \([0, 8]\). All methods were implemented using the scipy and scikit-learn python packages.

### 6 Experimental Results

#### 6.1 Same Document, Omniglot, News Authorship

Results are shown in Table 1. For the Same Document task, since there are 502 out of 1003 examples with a match in the training set (and in which case the match is always the nearest neighbor), the results for Global-\( k \) and Oracle methods perform as expected. Local-\( k \) does slightly worse, which is due to the fact that it uses the neighboring training examples to pick \( k \) but there is a mismatch between the train and test distributions. In particular, the train set has far more examples with matching pairs than non-matching pairs, while the split is 50/50 for test. Meanwhile, Isotonic Regression performs better than Global-\( k \), which is due to the separability of the match and non-match results. Surprise performs the best as it amplifies this separation. Furthermore, unlike Isotonic Regression which captures the separability over aggregated relevant and non-relevant scores, Surprise scoring does so on an individual query basis. In other words, what matters for Surprise is how the relevant scores are distributed relative to non-relevant ones for *each* query independently, and so it is robust to cases where the variance in distance distribution across examples, even

|                | BM25        | DRMM        |
|----------------|-------------|-------------|
|                | F1  | DCG   | F1  | DCG   |
| Oracle         | 0.375 | 1.292 | 0.375 | 1.292 |
| Global-\( k \) | 0.263 | 0.266 | 0.263 | 0.266 |
| Isotonic       | 0.217 | 0.0   | 0.217 | 0.0   |
| BiCut          | 0.262 | 0.0   | 0.262 | 0.0   |
| Surprise       | 0.251 | 0.151 | 0.251 | 0.151 |

Table 2: Average F1 and DCG performance on Robust04. Surprise achieves competitive results across the board. Since there is no concept of a local neighborhood for this dataset, Local-\( k \) is omitted.

Figure 4: Test F1 score for different Surprise thresholds on Robust04 DRMM. The optimal test threshold is close to the optimal training one and F1 varies smoothly with threshold.
in the local neighborhood, is high.

In a similar vein, Surprise outperforms all other baselines on Omniglot for both F1 and DCG metrics. For the News Authorship data, Surprise fares the best on F1 but struggles with DCG, as does Isotonic Regression. One possible explanation here is that in light of the low Oracle DCG and 0 / near 0 DCG for Local and Global-$k$, for many test examples the optimal strategy is to return no results (this gives a DCG of 0.). This can be challenging for both Surprise and Isotonic Regression since it would require the methods to select a very large decision threshold that rejects all candidate scores.

6.2 Robust04 Results and Analysis

Surprise consistently beats the baselines on Robust04. The score distributions from Robust04 are more well-behaved than News Authorship and typical of what one might expect in most retrieval applications. In Figure 4 we plot the original relevance scores along with Surprise scores for 10 test queries from DRMM. We see that the original scores occupy different ranges but that Surprise calibrates them to be in a consistent range and furthermore amplifies the prominence of highly relevant results. If we consider the hypothesis testing view of the Surprise method – that is, for every relevance score we test the null hypothesis “score is non-relevant” per the non-relevance score distribution we fit using the GPD – then, quite interestingly, the head of the Surprise score distribution lies in a numerical range that corresponds to $p$-values of 0.1 to 0.01. The mapping between $p$-values and Surprise scores is given by:

$$p\text{-value}(s) = \exp(-s),$$

where $s$ is the Surprise score. A common evidence scale used in hypothesis testing is to take 0.1 to be weak evidence to reject the null and 0.01 to be strong grounds for rejection, i.e. to deem the result not non-relevant. In applications without well-defined target metrics to calibrate against, one can resort to selecting thresholds based on $p$-values. For example, the $p$-value range of $[0.1, 0.01]$ maps to a Surprise score threshold range of $[2.3, 4.6]$.

In Figure 4 we plot the test F1 score for different Surprise score thresholds on DRMM. We note that (1) the optimal test threshold is very close to the optimal training one and (2) the performance profile is a smooth, well-behaved function of the threshold.

7 Conclusion

To summarize, in many practical applications, particularly in search and recommenders, the system returns the best results ranked by some machine-learned relevance score. By design, the ranking and overall quality of these top results are good, but their accompanying scores are often miscalibrated with true relevance. Having a calibrated and understandable ranking score can be useful for users in some applications, enabling them to make statements like “results with calibrated score value [X] all have about the same relevance to me, regardless of my query”. In addition to being useful to the end user, such calibrated scores are useful programatically, in the result list truncation task, by allowing the system to truncate the result list using a cutoff on the calibrated scores.

To this end, we proposed Surprise score, a statistical method that captures how relevant or surprising a result is by comparing it to non-relevant ones. It does so by leveraging the Generalized Pareto distribution that arises in Extreme Value theory. We apply Surprise to the important task of result list truncation, testing it on not only three distinct nearest-neighbor retrieval-based datasets but also on the benchmark Robust04 dataset. We show that Surprise outperforms both classical and recent baselines in nearly all settings. We illustrate its mechanism of action, its stable performance across thresholds, and show its connection to hypothesis testing and $p$-values.

Potential future work involves testing performance of other distribution-fitting methods and designing experiments to better understand the relationship between retrieval model performance and result list truncation model performance.

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