MSSM inflation

David H Lyth

Physics Department, Lancaster University, Lancaster LA1 4YB, UK
E-mail: d.lyth@lancaster.ac.uk

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Abstract. Variants of the $A$-term model of hep-ph/0605035 are considered. They are equally successful, indicating that the model is quite robust once the relation between $A$ and the soft mass is regarded as tunable. Alternatively a flat direction might support modular inflation.

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1. The $A$-term model

It has been pointed out recently [1] that a flat direction might support inflation, with a field value $\phi \ll M_P$ and the tree-level potential

$$V = \frac{1}{2} m^2 \phi^2 - A \frac{\lambda_p \phi^p}{p M_P^{p-3}} + \lambda_p^2 \frac{\phi^{2(p-1)}}{M_P^{2(p-3)}}.$$  

This potential corresponds to soft supersymmetry breaking, keeping only a term $W = \lambda_p \phi^p / M_P^{p-3}$ in the superpotential with $p > 3$.

Specifically, it is suggested in [1] that we are dealing with one of the flat directions of the MSSM, which provides two candidates [2]–[4] with $p = 6$. This is a new type of inflation model though, which might work with any flat direction (gauge or singlet) that generates an $A$ term, and the value of $p$ will not be specified in the following.

Slow-roll inflation requires [5] the flatness conditions $\epsilon \ll 1$, $|\eta| \ll 1$ and $|\xi^2| \ll 1$

where

$$\epsilon = \frac{1}{2} M_P^{-2} \left( \frac{V''}{3H_*^2} \right)^2$$  \hspace{1cm} (2)$$

$$\eta = \frac{V''}{3H_*^2}$$  \hspace{1cm} (3)$$

$$\xi^2 = \frac{V''V''''}{9H_*^4}$$  \hspace{1cm} (4)$$

and $H_*$ is the inflationary Hubble parameter.

In the small-field regime each of the three terms in equation (1) (in fact any monomial) violates all of the flatness conditions [5]. Two terms might combine to generate a maximum but $\eta$ will still be too big. All three terms must conspire to satisfy the conditions, and this is achieved if

$$8(p-1)m^2 = A^2.$$  \hspace{1cm} (5)

1 Higher-order generalizations of $\xi^2$ should in general also be small but this is automatic in the cases we consider. Near a maximum one can allow $|\eta| \sim 1$ if the inflaton perturbation is not required to generate the curvature perturbation.
Indeed, this gives $V' = V'' = 0$ at $\phi = \phi_0$,

$$\frac{\phi_0}{M_P} = \left(\frac{m}{\lambda_p \sqrt{2p - 2M_P}}\right)^{1/(p-2)} \sim \left(\frac{m}{\lambda_p M_P}\right)^{1/(p-2)},$$

with

$$V(\phi_0) \sim m^2 \phi_0^2$$

$$V'''(\phi_0) \sim m^2 \phi_0.$$  \hspace{1cm} (7)

In the regime $|\phi - \phi_0| \ll |\phi_0|$ a good approximation is

$$V = V(\phi_0) + \frac{1}{6} V'''(\phi_0)(\phi - \phi_0)^3.$$

Slow-roll inflation ends when $|\eta| \sim 1$ at

$$\frac{\phi_0 - \phi}{\phi_0} \sim \left(\frac{\phi_0}{M_P}\right)^3,$$

which is within the regime of validity of (9).

Suppose now that a term $-\frac{1}{2}\delta m^2 \phi^2$ is added to the right-hand side of (1), while keeping the relation (5) between $A$ and the original $m$. Retaining the approximation (9) for the original potential we arrive at

$$V = V(\phi_0) + \frac{1}{6} V'''(\phi_0)(\phi - \phi_0)^3 - \frac{1}{2}\delta m^2 \phi^2.$$  \hspace{1cm} (11)

There is now a maximum slightly below $\phi_0$, at which $|\eta| \sim \delta m^2 / H_*^2$, where $3M_P^2 H_*^2 = V(\phi_0)$ is the Hubble parameter. Inflation can occur provided that

$$\frac{\delta m}{m} \ll \frac{H_*}{m}.$$  \hspace{1cm} (12)

From equations (6) and (7)

$$\frac{H_*}{m} \sim \frac{\phi_0}{M_P} \sim \left(\frac{m}{\lambda_p M_P}\right)^{1/(p-2)} \gtrsim 10^{-8}. $$

The inequality comes from setting $m > \text{TeV}$ and $p \geq 4$, assuming $\lambda_p^{1/(p-2)} \sim 1$.

The condition (5) is consistent with the expectation that $m$ and $A$ have the same order of magnitude, but the accurate satisfaction of that condition implied by equation (12) represents significant fine-tuning. However, slow-roll inflation will presumably be preceded by an era of eternal inflation with $\phi$ very close to the point where $V' = 0$. If a landscape is available with essentially continuous values for $m$ and $A$, this could justify the fine-tuning implied by equation (12). Alternatively one can just accept the fine-tuning.

Using the potential (11), the prediction for the spectrum of the curvature perturbation generated by the inflaton perturbation depends on the parameters $m$ and $\delta m$ (with $A$ fixed at the value given by equation (5)). Only the case $\delta m = 0$ has been considered so far [1], and the matter will not be pursued here.

2 This is stronger than the condition $\eta \sim 1$ invoked in [1].
2. Robustness of the $A$-term model

The flat inflationary potential is usually very sensitive to \textquoteleft corrections', which can alter a model or prevent inflation altogether. Such corrections can come from high-dimension terms in the tree-level potential \cite{5,6} and from loop corrections \cite{5,7}. As we now see, $A$-term inflation is essentially immune from these corrections.

Consider first the likely value of $\lambda_p$. If the ultra-violet cut-off for the MSSM is $M \ll M_P$, it is appropriate to replace $M_P$ in equation (1) by $M$. Then \cite{6} one may expect $\lambda_p \sim 1/p!$. Retaining $M_P$ in equation (1) these considerations give the estimate \cite{6,8}

$$\lambda_p \sim \left( \frac{M_P}{M} \right)^{p-3} \frac{1}{p!}. \tag{14}$$

If there is a GUT one may identify $M$ with the unification scale $M \approx 10^{-2}M_P$. Then equation (14) gives $\lambda_6 \sim 10^3$ and $\lambda_9 \sim 10^6$. If instead one takes $M \approx M_P$ the estimate becomes $\lambda_6 \sim 10^{-3}$ and $\lambda_9 \sim 10^{-6}$.

A similar estimate will apply to higher-order terms in $V$, of the form\footnote{The most general possibility is $\Delta V \propto \phi^m$. This does not essentially change the following discussion.}

$$\Delta V \sim \lambda^2 \phi^{2q(p-1)} \frac{2q^2}{M_P^{2q(3-p)}}, \tag{15}$$

with $q > p$. Whatever the value of $M$, one presumably needs $\phi_0 \lesssim M$ for such terms to be under control.

Given the condition $\phi_0 \ll M$ it is reasonable to expect each higher-order term to give $\Delta V \ll V$. But one or more of them can still have a significant effect on the very flat inflationary potential. Indeed, the effect of $\Delta V$ on the flatness parameter $\eta$ (and $\xi^2$) is

$$\Delta \eta \sim \left( \frac{\lambda^2}{\lambda_P^2} \right) \left( \frac{\phi}{M_P} \right)^{2(q-p-1)}. \tag{16}$$

(To obtain this estimate we used $\Delta V'' \sim A \lambda_6^2 \phi^{2q-4}$ and $V \sim \lambda^2 \phi^{2q-2}$.)

At least if $q = p + 1$ one may expect $\Delta \eta \gtrsim 1$. With the relation between $m$ and $A$ fixed by equation (5) this would prevent inflation, but there is no need to insist on this relation. We can make a small change in $m^2$ (at fixed $A$) to recover again the condition $V' = V'' = 0$ at some point $\phi_0$. Unless $\Delta \eta \lesssim 1$ the change will take $m^2$ outside the original range (12), but that does not matter as long as we are taking the view that the relation between $m$ and $A$ is to be specified in order to achieve inflation. Provided that $\Delta V \ll V$, the fractional change in $m^2$ will anyway be small.

Now consider the one-loop correction to $V$. At the one-loop level $m^2(\ln \phi)$ becomes a linear function of $\ln \phi$ \cite{5}, with

$$\frac{d \ln (m^2)}{d \ln \phi} \equiv \gamma \sim \alpha \tilde{m}^2 / m^2. \tag{17}$$

Here $\tilde{m}$ is the mass of the particle giving the dominant contribution and $\alpha$ is its coupling strength (gauge or Yukawa) to $\phi$. To justify the neglect of higher loops one needs $|\gamma| \ll 1$.

The derivatives of the potential are now

\begin{align*}
V' / \phi &= m^2 \left( 1 + \frac{3}{2} \gamma \right) - A \lambda_p \phi^{p-2} + 2 \lambda_p^2 (p-1) \phi^{2(p-2)} \tag{18} \\
V'' &= m^2 \left( 1 + \frac{3}{2} \gamma \right) - A \lambda_p (p-1) \phi^{p-2} + 2 \lambda_p^2 (p-1)(2p-3) \phi^{2(p-2)}. \tag{19}
\end{align*}
With $\gamma = 0$, the requirement $V' = V'' = 0$ leads to the relation (5) between $A$ and $m$ (and to equation (6) for $\phi_0$). If we insist on that relation, the inclusion of $\gamma$ will spoil inflation unless $\gamma$ is very small. But, analogously with the previous discussion for a higher-order term in the tree-level potential, we can instead make a change in $m^2$, to achieve $V' = V'' = 0$ in the presence of $\gamma$. Once the change has been made the loop correction has a negligible effect, as a consequence of the perturbative condition $|\gamma| \ll 1$.

Finally, there may be a rather different possibility for $A$-term inflation, which is to use a $D$-flat but not $F$-flat direction. The relevant Yukawa coupling might be many orders of magnitude below unity, as has been discussed in another context [9], and then an $A$-term model might be possible with $m^2 \phi^2$ replaced by $\lambda \phi^4$.

3. Parameter choices

We focus now on a flat direction of the MSSM, corresponding to $p = 6$. To minimize the effect of multi-loop corrections, $\phi_0$ should be roughly of order the renormalization scale $Q$. Given this identification there are at least two possible attitudes regarding the soft supersymmetry breaking parameters $m$, $A$ and $\tilde{m}$.

As this scale is high ($\phi_0 \gtrsim 10^{-4} M_P$) one can suppose that the parameters are completely different from the corresponding quantities evaluated at the TeV scale, which are to be compared directly with observation. String theory nowadays offers so many possibilities that such a view is quite tenable, but it loses the direct connection with observation. Therefore it is more attractive to suppose that the parameters are those measured at the TeV scale. This view (assuming that supersymmetry is already broken at the high scale) is perhaps supported by the compatibility of the observed gauge couplings with naive unification.

With ordinary supersymmetry, the parameters are then $\alpha \sim 0.1$ and $m \sim \tilde{m} \sim$ TeV (with $\tilde{m}$ the gaugino mass). Keeping the gauge unification one might consider instead split supersymmetry [10], which makes the scalar masses very large while keeping the gaugino masses of order TeV. But the phenomenology then requires $A \ll m$ making the $A$-term inflation unviable.

Throughout this discussion we have taken the view that the parameters are to be specified directly at the scale $\phi_0$. Instead one might suppose that they are defined at the GUT scale $M$. In that case the parameters at the scale $\phi_0$ will have to be calculated from the RGEs. To achieve the required relations (5) and (6) between $m^2$, $A$ and $\phi_0$ will require fine-tuning between the values of $m^2$ and $A$ at the GUT scale, analogous to but not identical with the relation equation (5).

4. Modular MSSM inflation

Instead of invoking an $A$-term, one can suppose that a flat direction supports inflation with a potential of the form

$$V(\phi) = V_0 f(\phi/M_P),$$

(20)

where $f$ and its low derivatives have magnitude of order 1 at a typical point in the regime $\phi/ \lesssim M_P$. In the context of string theory, $\phi$ might be a modulus with the vacuum a point of enhanced symmetry.
The possibility that an MSSM flat direction could have the potential (20) has been mentioned before (see for instance [2]) but not in connection with inflation. Inflation with the potential (20) is indeed a quite attractive possibility (see for instance [11]). Given the minimum at $\phi = 0$, there will typically be a maximum at $\phi \sim M_P$, with $|\eta| \sim 1$. Following the philosophy of modular inflation one either hopes to get lucky so that actually $|\eta| \ll 1$, or else to generate the curvature perturbation through a curvaton-type mechanism.

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