Section – 1

Effect of Size on Mechanical Properties of Bonded Bilayer Germanene Sheets:

![Graphs showing stress-strain curves]

**Figure S1:** Effect of sample size on stress-strain curves at 300 K temperature for a) armchair loading & b) zigzag loading.

![Graphs showing ultimate tensile strength]

**Figure S2:** Convergence of UTS along with size for a) armchair loading & b) zigzag loading direction.
Section 2

Optimized Potential Parameters:

In this study, Ge-Ge interaction has been modeled by optimized Tersoff potential\(^{1}\) as below:

\[
E = \sum E = \frac{1}{2} \sum_{i \neq j} V_j, \quad V_j = f(r_j) \left[ f(r_j) + b_j f(r_j) \right]; \quad 1(a)
\]

\[
f(r_j) = A e^{-\lambda_1 r_{ij}}, \quad f(r_j) = -B e^{-\lambda_2 r_{ij}}; \quad 1(b)
\]

\[
f(r_j) = \begin{cases} 
1, & r_j < R - D \\
\frac{1}{2} - \frac{1}{2} \sin \left( \frac{\pi (r_{ij} - R)}{2} \right), & R - D < r_j < R + D; \\
0, & r_j > R + D
\end{cases} \quad 1(c)
\]

\[
ij = \frac{1}{(1 + \beta^n \xi_j^n)^{2n}}, \quad \zeta_j = \sum_{k \neq i} f(r_{kj}) g(\theta_{jk}) e^{\lambda m (r_{ij} - r_{ik})^m}; \quad 1(d)
\]

\[
g(\theta_{jk}) = \gamma_{jk} \left( 1 + \frac{c^2}{d^2} \right)^{\frac{c^2}{d^2} - \frac{c^2}{d^2}} \quad 1(e)
\]

where \(E\) is the total potential energy of the system. Further parameter notations of Tersoff potential and values of force field parameters for Ge-Ge interactions are given in\(^{1}\). In this study, the MD (Molecular Dynamics) analysis have been performed with a wide range of cutoff distance. This range stretches from R-D to R+D as shown in Tersoff formulation equation 1(c). The parameter of D has been increased to 1.25 Å keeping the R fixed to 2.95 Å, and thus the value of smaller cutoff radius R-D is decreased to 1.7 Å and the value of larger cutoff radius R+D is increased to 4.2 Å. This parametric analysis is adopted to match the derived mechanical properties of monolayer germanene (UTS and fracture strain) with the DFT results.\(^{2}\)

In Tersoff potential, the cutoff function introduces a dramatic increase in the interatomic force which rises sharply with a peak at around 30\% strain\(^{3-5}\). Similar phenomenon has been observed in this study when the parameter of D=0.15 was used with optimized Tersoff potential following Mahdizadeh\(^{1}\), and also with D=0.30 maintaining the original Tersoff potential parameter\(^{6}\) (as shown in Figure S3). Such an unphysical strain hardening is attributed to the sharply discontinuous second derivative of the Tersoff cutoff function at the cutoff activation point.
To solve this overestimation, at first a hard cutoff of R-D=R+D=2.95 Å was chosen (considering D=0) following the methodology of many studies\(^3\,^7\). In order to validate the properties obtained from this approach with DFT study on germanene\(^2\), tensile simulation in a single layer of germanene was performed at a strain rate of 0.003 ps\(^{-1}\) and at 300 K temperature. With this approach, the abrupt strain hardening was avoided but the mechanical properties (strength and fracture strain) were found to be much lower than the DFT calculations as shown in Table S1.

**Table S1: Mechanical properties of monolayer germanene obtained from different methods**

| Methodology | UTS | Fracture Strain |
|-------------|-----|-----------------|
|             | Armchair | Zigzag | Armchair | Zigzag |
| D= 0        | 2.91 | 2.95 | 12.69 | 15.1 |
| D=1.25      | 5.66 | 6.02 | 16.32 | 17.73 |
| DFT\(^2\)   | 4.7 | 4.1 | 20 | 20.5 |
| M.Qui.Le \(^8\) | 4.6 | 5.1 | 15.9 | 21.4 |

This type of reduction in mechanical properties resulting from using a hard cutoff has also been reported in the study of Gamboa et al\(^9\). Moreover, even with the hard cutoff of 2.95 Å, the unexpected strain hardening was observed when we performed tensile simulations at any temperature below 300 K. The similar scenario also occurred when the value of hard cutoff was enhanced from R=2.95 Å to R=3.50 Å as shown in Figure S4. This motivated us to prepare a detail mapping of the variability of mechanical properties (UTS) with the variation of the cutoff range as shown in Figure S5. Such a range (=2D), when short as in D=0.6\(^4\) and D=0.3\(^6\), abruptly alters mechanical property (UTS) due to a highly steep slope in stress strain diagram near the cutoff activation point\(^10\). Therefore, when the range is widened (the value of D is increased), this unexpected strain hardening gradually goes away\(^11\). Moreover, increment in the larger cutoff, also helps to decrease the force overestimation\(^12\). It is also observed a similar conclusion from the mapping of UTS with a wide range of cutoff distance. It was also observed from Figure S5 that
the UTS value first shoots up as the value of D is changed from D=0. Then, the value of UTS gradually decreases with increasing D with converging the results for the values D>1. Moreover, this study finds that with D=1.25 the UTS and the fracture strain results match with that of the DFT calculation. It is further noted that, the value of Young’s Modulus does not change significantly with variation of D because the influence of change of D is basically observed after the cutoff activation point. This approach only suppresses the spurious strength otherwise achieved through the conventional parameters.

![Stress-strain curves for armchair loading with different values of hard cutoff at 5 K.](image1.png)

**Figure S4:** stress-strain curves for armchair loading with different values of hard cutoff at 5 K.

![Effect of Tersoff parameter D on the UTS at a) armchair loading & b) zigzag loading.](image2.png)

**Figure S5:** Effect of Tersoff parameter D on the UTS at a) armchair loading & b) zigzag loading.
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Section – 3

Error Analysis:

**Figure S6**: Stress-strain curves for six different samples with different initial velocity seeds at 200 K for a) armchair loading & b) zigzag loading.

**Figure S7**: Stress-strain curves for six different samples with different initial velocity seeds at 400 K for a) armchair loading & b) zigzag loading.
Figure S8: Stress-strain curves for six different samples having monovacancy with different initial velocity seeds at 25 K for a) armchair loading & b) zigzag loading.

Figure S9: Errors associated to ultimate tensile strength with different initial velocity seeds at various temperatures for (a) armchair loading & (b) zigzag loading.