Crustal Failure on Icy Moons from a Strong Tidal Encounter

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ABSTRACT

Close tidal encounters among large planetesimals and moons should have been more common than grazing or normal impacts. Using a mass spring model within an \textit{N}-body simulation, we simulate the deformation of the surface of an elastic spherical body caused by a close parabolic tidal encounter with a body that has similar mass as that of the primary body. Such an encounter can induce sufficient stress on the surface to cause brittle failure of an icy crust and simulated fractures can extend a large fraction of the radius of body. Strong tidal encounters may be responsible for the formation of long graben complexes and chasmata in ancient terrain of icy moons such as Dione, Tethys, Ariel and Charon.

Keywords: Tides, solid body: Satellites, surfaces:

1 INTRODUCTION

A number of large surface features in the solar system have origins potentially due to giant impacts that occurred between planet-sized bodies or planets and large planetesimals. These include the crustal dichotomy of Mars (Wilhelms & Squyres 1984; Frey & Schultz 1988; Marinova et al. 2008) and the Moon (Jutzi & Asphaug 2011), jumbled terrain on Mercury (Schultz & Gault 1975) and numerous impact craters throughout the solar system (Melosh 1989). Because of their larger cross-section, grazing impacts among planets and planetesimals are more likely than normal angle impacts (Asphaug 2010). Likewise, strong \textit{tidal encounters}, those involving close encounters between two large bodies that do not actually touch, are more likely than grazing impacts. A fraction of a body's gravitational binding energy can be dissipated during a close tidal encounter (Press & Teukolsky 1977). While some large surface features on planets and moons have proposed origins due to large impacts, so far none have been linked to single close and strong tidal encounters.

Strong tidal encounters are unlikely now, but would have occurred in the past, during the late-heavy bombardment era and beforehand. Orbits of closely packed moons can become unstable (e.g., French & Shoemaker 2012; Cheng et al. 2014) and this too could cause close encounters between similar mass bodies. Impacts primarily cause compressive stress (Melosh 1989), but tidal stress can be tensile and many materials are weaker when subjected to tensile stress than a comparable magnitude of compressive stress (for ice see Figure 1 by Petrovic 2003 and Figure 7.2 by Collins et al. 2010 and for the Earth’s lithosphere see failure envelopes in Figure 6.24 and Yield Strength envelopes in Figure 6.27 by Watts 2001 or Figure 9.6 by Kohlstedt & Mackwell 2010). Ancient regions of planets and moons exhibit features such as chasmata, grooves, grabens or graben complexes that are associated with extension and tensile deformation (Collins et al. 2010), and some of these may have been caused by strong tidal encounters with large bodies.

Many studies of tidal encounters between two planetesimals or between a planetesimal and a planet have focused on tidal disruption (e.g., Dobrovolskis 1990; Boss 1994; Richardson et al. 1998; Sharma et al. 2006; Holsapple & Michel 2008). But tidal stresses due to close encounters between bodies can affect body rotation and shape (Bottke et al. 1999) and disturb weathered surfaces of asteroids, exposing fresh surface materials (Binzel et al. 2010; Nesvorny et al. 2010). Simulations of granular materials have predicted resurfacing in weak regions where tidal stresses cause avalanches or landslides (Yu et al. 2014).

Some icy moons exhibit global tectonic features, such as grooves or long fractures, that could be caused by varying tidal stresses exerted by their host planet (e.g., Helfenstein & Parmentier 1985; McEwen 1986; Hurford et al. 2007; Smith-Konter & Pappalardo 2008; Wahr et al. 2009; Hurford et al. 2015). The patterns and individual morphologies of parallel sets of grooves and troughs on satellites and asteroids such as Phobos, Eros, Ida, Gaspra, Epimetheus and Pandora (see Thomas & Prockter 2010 for a review) can be attributed to fracturing in weak materials caused by oscillating tidal stresses.
stresses associated with orbital eccentricity (Morrison et al. 2009) or an increase in tidal stress resulting from the orbital decay of the body itself (Soter & Harris 1977; Hurford et al. 2015). For Phobos, the length of the grooves is perpendicular to the oscillating tidal stress tensor (Morrison et al. 2009). Long (130 km) linear fractures termed “tiger stripes” on Enceladus are connected to diurnal tidal stress variations (Smith–Konerter & Pappalardo 2008; Nimmo & Matsuyama 2007; Hurford et al. 2007).

Dione, Tethys, Rhea (moons of Saturn), Titania (moon of Uranus) and Charon (moon of Pluto) have heavily cratered surfaces and also display long faults extending a significant fraction of the moon radius, and chasmas and faults in pairs interpreted as graben or graben complexes (for a review see section 6.4 by Collins et al. 2010 and for recent results on the Pluto system see Stern et al. 2015). The Ithaca Chasma on Tethys is interpreted as a large graben complex (Giese et al. 2007) and crater counts indicate that it is older than the large Odyssey impact basin (with radius 0.4 times that of the moon itself). On Dione, Cassini imagery revealed that some fault networks have vertical offsets that dissect craters, confirming their extensional tectonic origin and suggesting that they were formed early (Jaumann et al. 2009). These studies suggest that the formation of chasmas and graben complexes can take place before or during an epoch of large impacts and so during an epoch when strong tidal encounters would have occurred. Explanations for the graben complexes include heating and expansion of the interior (e.g., Hillier & Squyres 1991) and stresses induced by reorientation following a large impact (Nimmo & Matsuyama 2007). Strong tidal encounters have not yet been explored as a possible explanation for the formation of surface features such as chasmas and graben complexes on icy moons.

In this study we focus on the intersection between the works introduced above. We consider rare and close tidal encounters, that might have occurred billions of years ago, between large bodies that are not gravitationally bound (not in orbit about each other). The time of a parabolic or hyperbolic tidal encounter can be a few hours, so tidal encounters are extremely fast compared to the timescales of most geophysical processes. On such a short time scale rock and ice should deform in a brittle-elastic mode rather than a ductile, plastic or visco-elastic mode (e.g., Turcotte & Schubert 2002; Brügmann & Dresen 2008). To numerically simulate tidal deformation we use a mass-spring model to simulate both elastic response and gravity (see Frouard et al. 2016). Brittle failure is modeled by allowing springs on the surface to fail if they exceed a critical tensile strain value. Our simulations allow to us to visualize brittle crustal failure following a hypothetical strong tidal encounter. Our approach differs from the granular flow simulations by Schwartz et al. (2013) with spring-like forces between neighboring soft spheres that mimic cohesion and can simulate bulk tensile failure.

1.1 Tidal encounters

Following Press & Teukolsky (1977) (also see Ogilvie 2014) the response of a body during a tidal encounter can be estimated using an impulse approximation. The maximum tidal force, \( F_T \), on body \( M \) from body \( m \) during the encounter is approximately

\[ F_T \sim \frac{G m R}{q^3} \]  

(1)

where \( q \) is the distance between body centers at closest approach (pericenter), \( m \) is the mass of the tidal perturber, \( R \) is the radius of the primary body with mass \( M \), and \( G \) is the gravitational constant. The time scale of the encounter is

\[ t_{\text{enc}} \sim \frac{2q}{V_q} \]  

(2)

where \( V_q \) is the velocity at pericenter. Together \( F_T \) and \( t_{\text{enc}} \) cause a velocity perturbation on the surface of the primary body

\[ \Delta v \sim \frac{2G m R}{q^2 V_q} \]  

(3)

If the orbit is parabolic then \( \Delta v/\sqrt{GM/R} = \eta \), a dimensionless parameter used to characterize parabolic tidal encounters (Press & Teukolsky 1977), that is the ratio of acceleration due to self-gravity and the tidal acceleration at the body’s surface.

The extent of the tidal deformation of body \( M \) can be estimated by balancing the kinetic energy per unit mass due to the tidal impulse with elastic energy per unit mass

\[ \Delta v^2 \sim \frac{\epsilon^2 E}{\rho} \]  

(4)

with \( E \) the Young’s modulus and \( \rho \) the density of body \( M \), giving a strain of

\[ \epsilon \sim \left( \frac{\epsilon_g}{E} \right)^\frac{1}{2} \left( \frac{R}{q} \right)^2 \left( \frac{M}{m} \right) \left( \frac{V_q}{V_q} \right) \]  

(5)

where

\[ \epsilon_g \equiv \frac{G M}{R^2} \left( \frac{4 \pi}{3} \right) G R^3 \rho^2 \]  

(6)

is the velocity of a particle in a circular orbit grazing the surface of \( M \) and \( \epsilon_g \)

\[ \epsilon_g \equiv \frac{G M^2}{R^3} \left( \frac{4 \pi}{3} \right) \left( \frac{G R^3 \rho^2}{1 \text{ g cm}^{-3}} \right)^2 \]  

(7)

is approximately the gravitational binding energy density of body \( M \) (and to order of magnitude its central pressure). We have assumed that \( M \) is a homogenous and spherical body and remains so during the encounter.

A time scale for elastic response can be estimated from the speed of elastic waves and the radius of the primary body.

\[ t_{\text{elas}} \sim \frac{R}{\sqrt{E/\rho}} \left( \frac{\epsilon_g}{E} \right) \sim \frac{1}{2} t_{\text{grav}} \]  

(8)

where we have defined a gravitational time scale

\[ t_{\text{grav}} \equiv \sqrt{\frac{R^3}{G M}} \sim \frac{3}{4 \pi G \rho} \left( \frac{\rho}{1 \text{ g cm}^{-3}} \right)^{-\frac{1}{2}} \]  

(9)

equivalent to the the inverse of the angular rotation rate of a particle in a circular orbit grazing the surface of the body...
The gravitational time scale is only dependent on the body’s mean density \( \rho \). For a Poisson modulus of \( \nu = 1/4 \), the speed for P-waves is \( V_p \approx 1.1V_0E/\rho \) and that for S waves \( V_S \approx 0.6V_p \). To order of magnitude \( t_{\text{elas}} \) is equal to the frequency of the slowest vibrational mode of the body.

The strain rate (on \( M \)) during the tidal encounter can be estimated from the time scale for elastic response

\[
\dot{\epsilon} \sim \frac{\epsilon}{t_{\text{elas}}} \\
\sim \left( \frac{R}{q} \right)^2 \left( \frac{m}{M} \right) \left( \frac{\nu_c}{\nu_g} \right) t_{\text{grav}}^{-1} \\
\sim 10^{-4} \text{s}^{-1} \left( \frac{\rho}{1 \text{ g cm}^{-3}} \right)^{\frac{1}{2}} \left( \frac{R}{q} \right)^2 \left( \frac{m}{M} \right) \left( \frac{\nu_c}{\nu_g} \right) \tag{10}
\]

This expression implies that close encounters between similar mass bodies are likely to be in a high strain rate regime (geophysical strain rates tend to be 10 orders of magnitude lower, see Figure 2 by Hammond et al. 2013, Nimmo 2004b and discussion below).

The above estimate for the strain and strain rate used an impulse approximation, assuming that the body does not have time to elastically respond during the encounter. We compare the elastic response time scale to the encounter time scale

\[
t_{\text{enc}} = \frac{q}{R} \left( \frac{\nu_c}{\nu_g} \right) \left( \frac{E}{\rho} \right)^{\frac{1}{2}} \tag{11}
\]

If \( t_{\text{enc}} / t_{\text{elas}} \ll 1 \) then the impulse approximation is valid, whereas if \( t_{\text{enc}} / t_{\text{elas}} \gg 1 \) the encounter can be considered adiabatic. When the ratio of time scales is near unity, the impulse approximation can be used but the response should be reduced by a factor that depends on the ratio of the two time scales.

In Table 1 we list gravitational energy densities \( e_g \), time scales \( t_{\text{grav}} \) and densities for some moons that exhibit long faults, chasma or graben complexes. The planet Mars, which exhibits the extremely prominent chasm, Valles Marineris, is included for later discussion. The gravitational time scales in the icy bodies range from about 1000-2000 s (15 to 30 minutes). We compare the gravitational binding energy densities, \( e_g \), to an estimate for the Young’s modulus of ice. Observations of ice shelves response to tides on Earth give an effective Young’s modulus \( E_{\text{ice}} \sim 0.9 \text{ GPa} \) (Vaugan 1995), an order of magnitude below that of solid ice in the lab. On icy moons, porosity and surface fracturing may lower the effective value of the Young’s modulus (Nimmo & Schenk 2006; see Collins et al. 2010 for a review). The values of gravitational energy density \( e_g \) for icy bodies in Table 1 range from 0.3 GPa for Tethys to 2.14 for Titania. Depending upon the value adopted for the Young’s modulus for ice, the ratio \( e_g/E_{\text{ice}} \) ranges from about 1 to 0.1 for these bodies.

Using the estimate for the ratio \( e_g/E_{\text{ice}} \) we estimate the strain during an encounter. For an equal mass and density perturber, the pericenter distance must be more than twice the body’s radius; \( q > 2R \). For a parabolic grazing encounter with \( q = 2R \), the pericenter velocity is equal to the escape velocity at pericenter, and \( V_q = \sqrt{2\nu_c} \) with \( \nu_c \) defined in equation 6. Equation 5 then implies that the strain caused by the tidal encounter could be of order 10% for a parabolic equal mass encounter. The ratio \( E_{\text{ice}}/e_g \) is large enough that the time scale for the encounter could be a few times longer than the elastic time scale. This should reduce the strain on the surface compared to that estimated using the impulse approximation in equation 5 by a factor of a few (with factor that depends on the ratio \( t_{\text{enc}}/t_{\text{elas}} \)). Icy moons are likely to contain rocky higher density and strength cores and here we have neglected compositional variation in our estimate of the tidally induced surface strain. This too would reduce the estimated strain value by a factor of a few. Nevertheless our low estimated ratio of \( E_{\text{ice}}/e_g \) implies that icy crusts or mantles in these icy bodies are weak enough that deformation at the level of few percent strain is expected in close parabolic tidal encounters with similar mass objects.

Close encounters with a similar mass body would be in a regime of high strain rate, compared to most geophysical settings for ice, but lie in the mid to lower end of laboratory measurements (Lange & Ahrens 1993; Schulson 1999; Fortt & Schulson 2012). Figure 2 by Petrovic (2003) illustrates that the tensile strength of ice is relatively insensitive to the strain rate, however Lange & Ahrens (1993) found a dependence, with the strength increasing at very high strain rates. At strain rates of order \( 10^{18} \text{s}^{-1} \) the tensile strength of ice is a few MPa (Lange & Ahrens 1993; Schulson 1999; Petrovic 2003). Using a Young’s modulus of a few GPa, brittle failure is likely to take place if the strain under uniaxial tension exceeds 0.01 - 0.1%. This crude strain based estimate for brittle failure ignores a dependence of the material strength on depth or pressure. One of the modes of tensile failure causes fractures parallel to one of the axes of principal stress and is independent of confining pressure (Jaeger & Cook 1976). This is equivalent to the limit with principal stress \( \sigma_1 \sim 0 \) (as on the body’s surface), \( \sigma_2 < 0 \), corresponding to extension, and the Griffiths failure criterion that is a function only of the uniaxial tensile strength. Here fractures are expected parallel to the direction associated with \( \sigma_2 \) (on the surface and perpendicular to the principal axis of tensile stress). Based on our estimate of the induced strain during a close encounter with a massive body (equation 5) and using the values for gravitational energy density \( e_g \) for the icy bodies listed in Table 1, we estimate that tidal stress and associated body deformation could cause sufficient tension on the surfaces of these bodies to exceed the tensile strength of their icy crusts and cause widespread surface brittle failure.

### 2 MASS-SPRING MODEL SIMULATIONS

Because of the short time scale of tidal encounters we model brittle-elastic behavior alone and neglect ductile, plastic or liquid-like behavior. We model only the tidal encounter, and do not model longer time scale viscoelastic behavior, such as crustal flexure, subsidence and decompression melting. Mass-spring models or lattice spring models are a popular method for simulating soft elastic bodies (Meier et al. 2005; Nealen et al. 2006). In a mass-spring model, massive particles are connected with a network of massless springs. Mass-spring models are considered a better choice than a finite difference method when a fast, but not necessarily accurate simulation is desired. However, with enough masses and springs and an appropriate choice of spring types, behaviors and geometry for the network of connections, mass spring models can accurately represent elastic materials (Hren-
The gravitational energy density $\rho$ is computed using equation 7 and the gravitational time scale $t_{grav}$ is computed using equation 9. Also listed are the mean densities. Tethys, and Dione are moons of Saturn, Ariel and Titania are moons of Uranus, and Charon is a moon of Pluto.

| Bodies exhibiting long chasma or graben complexes | mass ($10^{21}$ kg) | radius (km) | $\epsilon_g$ (GPa) | $t_{grav}$ (s) | $\rho$ (g cm$^{-3}$) |
|-------------------------------------------------|---------------------|-------------|------------------|----------------|------------------|
| Tethys                                           | 0.62                | 531         | 0.32             | 1906           | 0.98             |
| Dione                                            | 1.1                 | 561         | 0.81             | 1551           | 1.48             |
| Rhea                                             | 2.3                 | 764         | 1.04             | 1701           | 1.24             |
| Ariel                                            | 1.3                 | 579         | 1.08             | 1468           | 1.66             |
| Titania                                          | 3.5                 | 788         | 2.14             | 1443           | 1.72             |
| Charon                                           | 1.5                 | 603         | 1.16             | 1472           | 1.65             |

We work in units of the planetesimal body radius and mass $R = 1, M = 1$. Time is specified in units of $t_{grav} = \sqrt{GM/R^3}$ (equation 9) and we refer to this as a gravitational time scale. In these units, the velocity of a particle in a circular orbit just grazing the surface of the body is 1, and the period of this orbit is $2\pi$. Velocities are given in units of $\sqrt{GM/R}$, accelerations in units of $GM/R^2$ and spring constants in units of $GM^2/R^3$. Pressure, energy density and elastic moduli are given in units of $GM^2/R^4$ or $\epsilon_g$ (equation 7). In units of $R = 1, M = 1$ the mean density of the primary body is $\bar{\rho} = 3/(4\pi) = 0.239$.

The parameters used to describe our simulations are summarized in Table 2.

### 2.1 Spring forces

The code **rebound** advances particle positions using their accelerations (Rein & Liu 2012). We add spring and hinge forces as additional forces by adding to the particle accelerations at each time step. To compute the particle accelerations, the forces from each spring and hinge on each particle are divided by the mass of each particle.

The elastic force from a spring between two particles $i, j$ with masses $m_i, m_j$ and coordinate positions $x_i, x_j$, on particle $i$ is computed as follows. The vector between the two particles $x_i - x_j$ gives a spring length $L_{ij} = |x_i - x_j|$ that we compare with the spring rest length $L_{ij,0}$. The elastic force from a spring between two particles $i, j$ on particle $i$ is computed as

$$F_{i}^{elastic} = -k_{ij}(L_{ij} - L_{ij,0})\hat{n}_{ij} \quad (12)$$

where $k_{ij}$ is the spring constant and the unit vector $\hat{n}_{ij} = (x_i - x_j)/L_{ij}$. The force has the opposite sign on particle $j$. When the spring length is longer than its rest length $L_{ij}$ the force pushes the two masses together and when it is shorter than its rest length the force pushes the two masses apart.

The strain rate of the spring is

$$\dot{\epsilon}_{ij} = \frac{L_{ij}}{L_{ij,0}} \frac{1}{L_{ij,0} - 1} (x_i - x_j) \cdot (v_i - v_j) \quad (13)$$

where $v_i$ and $v_j$ are the particle velocities, $L_{ij}$ is the rate of change of the spring length, $L_{ij}$ and $\epsilon_{ij} = (L_{ij} - L_{ij,0})/L_{ij,0}$ is the spring strain. To the elastic force on particle $i$ we add a damping force proportional to the strain rate

$$F_{i}^{damping} = -\gamma_{ij}\dot{\epsilon}_{ij} L_{ij,0} \mu_{ij} \hat{n}_{ij} \quad (14)$$

with damping coefficient $\gamma_{ij}$ that is equivalent to the inverse of a damping or relaxation time scale. The coefficient $\gamma_{ij}$ is independent of the spring constant $k_{ij}$. Here $\mu_{ij}$ is the reduced mass $\mu_{ij} \equiv m_i m_j/(m_i + m_j)$. The damping force is in parallel with the elastic term so the spring model approximates a viscously damped elastic material or a Kelvin-Voigt solid (see Frouard et al. 2016). Between each pair of particles, forces are oriented along the vector spanning the two particles and this ensures angular momentum conservation.

To maintain numerical stability, the time step should be smaller than the time it takes physical information to travel between adjacent mass nodes. The ratio of the P-wave velocity to gravitational velocity $V_P/v_g \sim \sqrt{GM/R}$ and this exceeds 1 otherwise the body would collapse due to self-gravity. In our simulations the speed of elastic waves exceeds...
the interior volume. and spring rest lengths for spring Such a spring network is isotropic, has Poisson ratio (Kot et al. 2015), where \( k_i \) nodes if the distance between nodes is less than distance sufficiently separated from other nodes (at a distance greater than minimum distance \( d_{ij} \)). Springs are added between two nodes if the distance between nodes is less than distance \( d_i \). Such a spring network is isotropic, has Poisson ratio \( \nu = 1/4 \) and has a Young’s modulus of

\[
E_I \approx \frac{1}{6V} \sum_i k_i L_i^2
\]

(Kot et al. 2015), where \( k_i \) and \( L_i \) are the spring constants and spring rest lengths for spring \( i \), and \( V \) is the total volume. In the above expression the sum is over all springs in the interior volume.

We compute the velocity of \( P \) waves in the interior with

\[
V_{r,p} = \sqrt{E_I/\rho} \left( \frac{1 - \nu}{(1 + \nu)(1 - 2\nu)} \right)^{1/2} \approx 1.1\sqrt{E_I/\rho}
\]

With non-zero damping coefficients, the stress is the sum of an elastic term proportional to the strain and a viscous term proportional to the strain rate. The bulk and shear viscosities can be estimated from the damping coefficient, \( \gamma \), and other integrated properties of the mass spring model (Frouard et al. 2016).

In a three-dimensional volume, the number of springs is proportional to the number of particles in the interior, \( N_I \). The mean length of the springs \( L \propto N_I^{-1/3} \) giving spring constant \( k \propto E_I L \propto N_I^{-1/3} \) to maintain a specific Young’s modulus. The mass of each particle is \( m \propto 1/N_I \). Altogether this gives a time step (using equation 15) \( dt \propto N_I^{-1/3} \) and, as expected, the more particles simulated, the smaller is the required time step. (Frouard et al. 2016) has tested sensitivity of the random spring model to numbers of simulated mass nodes and numbers of springs per node.

2.3 Initial conditions for the interior: Stretching the springs and strengthening the core

We first generate springs with rest length equal to the distance between the spring vertices. However this does not generate an equilibrium state for our body because of compression due to self-gravity. When the simulation is begun in this state the body shrinks and then bounces, eventually damping to a denser equilibrium state than the initial condition. To begin with the system nearer equilibrium we start with the springs initially slightly under compression so that they counter-act self-gravity. We iteratively stretch all springs by the same amount to zero the acceleration at the surface. We then increase the spring strengths in the center of the body so as to approximate a state of hydrostatic equilibrium. A constant density self-gravitating sphere in hydrostatic equilibrium has pressure as a function of radius

\[
P(r) = \frac{2}{3} \rho^2 G \pi (R^2 - r^2)
\]

(consistent with equation 2 by Dobrovolskis 1990). The pressure at any radius is approximately \( P \sim E_I \epsilon \) where \( \epsilon \) is the strain of each spring. The resulting model is nearly in equilibrium, nearly constant density, and does not bounce excessively at the beginning of the simulation.

To illustrate the elasticity of the interior we show a gravitational tidal encounter with a perturber mass \( M_2 = M \) equal to the primary body, \( M \). The simulation is shown in Figure 1 and has body parameters listed in Table 3 and encounter parameters listed in Table 4 under the row \( N \). The perturbing mass, \( M_2 \), is modeled as a solid sphere that does not deform during the encounter and it is rendered as a sphere with the same density as the primary body. The simulation view has been shifted so that it remains in the center of mass frame of the primary body. As the spring damping coefficient is low, the body is soft, like jello and the body continues to vibrate and deform after the tidal encounter. Vibrational oscillations are excited in the body by the tidal encounter. We have checked that the elasticity code conserves momentum and angular momentum, as would be expected as particle interaction forces (including damping forces) are radial. With the resolved body in a circular orbit about a larger mass and on long time scales, the spring damping causes the resolved body to spin up or spin down, as expected, and with rate matching that predicted analytically (Frouard et al. 2016).

2.4 Crustal Shell Model

We model the elastic crust with a 2-dimensional spherical lattice. Each vertex of the shell lattice is a mass node and each node has the same mass. However the shell node masses are not the same as the interior node masses. To create a sphere of particles we begin with vertices of an icosahedron. Each face is then subdivided into 4 triangles. We recursively from the body center. After subdivision, springs are placed with spring constant \( k_S \) between each adjacent vertex, creating a triangular spring lattice (see Figure 2) and giving the 2-dimensional shell elasticity (e.g. Monette & Anderson 1994; Van Gelder 1998). The vertices of each triangular face are stored to later aid in displaying (rendering) the body’s surface.

For a triangular 2D lattice, the Young’s modulus de-

1 The difference between angular momentum at the beginning and end of the simulation is less than \( 10^{-12} \) in our N-body units described in section 2.0.
Figure 1. Simulation of a near parabolic tidal encounter of a random spring elastic model with parameters listed in Table 3 and 4 for the N simulation. The perturbing body approaches from the top left, and is seen in the leftmost two panels. Different times in the simulation are shown from left to right, separated in time by $t_{\text{grav}}$ (equation 9) and labelled by time from pericenter. The leftmost panel shows a time just after closest approach. The primary body is simulated with a mass-spring model and springs are shown as pink connecting lines between particles. The body is initially spherical but is elongated by the tidal force of the perturber. Vibrational oscillations are excited in the body by the tidal encounter. The rendered spheres are shown to illustrate the random distribution of node point masses, not imply that the body behaves as a granular rubble pile (e.g., Richardson et al. 2009).

depends on the spring constant

$$E_{2D} = \frac{2}{\sqrt{3}} k_S$$

(19)

and the Poisson ratio is $\nu_{2D} = 1/3$ (see equations 3.7 by Monette & Anderson 1994 or equations 10,12 by Kot et al. 2015). The modulus is a force per unit length rather than a force per unit area as is true in 3D. The sphere locally contains a single layer of particles and approximates a thin plate with

$$E_{2D} = E_S h_S$$

(20)

where $E_S$ is the Young’s modulus of the plate and $h_S$ its thickness. For $N_S$ particles in the shell, the mean length of the springs $L_s \propto N_S^{-1/2}$, the mass of each particle $m_s \propto N_S^{-1}$ but $k_S$ is independent of the number of particles. Hence the required time step (using equation 15) $dt \propto N_S^{-1/2}$. The speed of P-waves in the shell is

$$V_{s,p} \approx \sqrt{E_{2D}/\Sigma_S} \sim \sqrt{\frac{k_S 4\pi R^2}{M_S}}$$

(21)

where $\Sigma_S$ is the mass per unit area in the shell.

2.5 Hinge Forces and Flexure of the Crustal Shell

As a 2-dimensional lattice is used to simulate the body’s crust, the simulated crust is a thin membrane and when simulated with radial elastic forces between particles, and in the presence of gravity, it would be unstable and would bend and sag. Using the triangular phases in our crustal sphere, we use a hinge model to resist bending of adjacent triangular faces.

The force on each edge depends on the dihedral angle between the two faces as described by Bridson et al. (2003) in their section 4, see Figure 3, also see Grindspun et al. (2003). An angular dependent force is applied to the two vertices in a spring edge and to two adjacent vertices forming the hinge.

The magnitude of the applied force is

$$|F| = k_s \frac{L^2}{|\mathbf{N}_1| + |\mathbf{N}_2|} \sin((\theta - \theta_{\text{rest}})/2)$$

(22)

where $L$ is the length of the edge. The dihedral angle is equivalent to $\pi - \theta$ with angle $\theta$ between the two vectors, $\mathbf{N}_1$ and $\mathbf{N}_2$, that are normal to the surfaces of each triangular face. Here $|\mathbf{N}_1|$ and $|\mathbf{N}_2|$ are the areas of the two triangular faces of the hinge (see Figure 3). The force strengths are the same on each vertex but applied with direction given in vector form given by Bridson et al. (2003) assuring momentum and angular momentum conservation. The angle $\theta_{\text{rest}}$ is a
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rest bend angle and describes the angle of the surface without stress. We adjust the sign of $\theta$ by ordering the vertices in the edge. Before the simulation starts for each hinge we compute $\theta_{\text{init}}$ from the initial configuration of the surface lattice. Here our surface lattice is spherical but rest hinge angles can be computed for any smooth 2-dimensional lattice. This ensures that the initial state of our simulated crustal shell is not under flexural stress. As is true for the spring forces, accelerations for each mass are computed from the applied forces by dividing by the mass of each vertex particle.

The coefficient $k_e$ is in units of force but the force is applied at each hinge and with opposite sign in the center and ends of the hinge. The hinge is similar to a flexed beam, held at both ends but with an applied force at its center, and with width equal to the length of the hinge edge and length equal to the distance between outer vertices. The hinge model locally approximates a thin plate with Young’s modulus $E_S$ and plate thickness $h_S$ with

$$\frac{k_e L_S}{2} \sim \frac{E_S h_S^3}{12(1-\nu^2)} \equiv D_F$$

with $L_S$ the length of an edge and $D_F$ equal to the flexural rigidity or bending stiffness of the plate (e.g., Batty et al. 2012). This is approximate as simulated static load tests have not been carried out with the membrane/hinge model (though simulated static beam tests have been done for the lattice and random spring models, see Kot et al. 2015).

2.6 Crack formation

Crack formation in the crustal shell can be simulated in a spring model by breaking springs that have a strain value above a critical threshold value, $\epsilon$, (e.g., Norton et al. 1991; Marder & Liu 1993; Hirota et al. 2000; Sadhukhan et al. 2011). The springs in our simulated spherical shell behave as perfect linear springs until the moment that they reach the threshold value. If we completely dissolve failed springs, eventually three particles in a hinge triangle face (see Figure 3) could approach a line. When this happens the hinge force (equation 22) becomes infinite and the surface is unstable. Rather than completely dissolve springs, we instead reduce the spring force constant by a factor $F_{sk}$.

When a crack forms in a solid, the stress perpendicular to the crack’s path is reduced. Subsequent deformation concentrates stress at the new crack tip, which in turn fails, allowing the crack to propagate. In a simulation, the stress is redistributed as vertex positions are updated, and this allows the crack to propagate. A simple way to allow the stress redistribution (or relaxation) to take place within the simulation is to allow only one element to rupture in a given time interval, often the simulation time step (see discussion by Pfaff et al. 2014). Here, we only allow a single shell spring, that with maximum strain, to fail in a given time interval, denoted $t_{\text{fail}}$, which we set between one and three time steps. This relaxation procedure allows linear fractures to propagate, and prevents large areas of the surface from failing simultaneously. A more sophisticated code would allow more than one crack to simultaneously propagate (and in this case additional relaxation steps must be implemented and the residual momentum propagation taken into account in each crack tip region; Busaryev et al. 2013; Pfaff et al. 2014).

The surface of the shell is displayed using the triangular faces from each lattice triangle. Each triangular face is displayed with a texture. The edges of each triangle contain connecting springs. The texture displayed on each triangular face depends on the number and orientations of failed edge springs (see Figure 4). Long connected or partially connected sets of black bars illustrate surface fractures. The surface lattice introduces directional biases in the crack rendering. Offset individual black bars in Figure 4 are artifacts from the lattice rather than en-echelon structures.

2.7 Support of the Shell

The crustal shell lattice must be supported by the interior otherwise it will collapse due to gravity. To connect the two components of our models (amorphous interior to the lattice shell membrane) we insert springs between shell particles and interior particles. We create springs for all mixed pairs of particles, a pair consisting of a particle in shell and a particle in the interior, that have inter-particle distance less than a distance $d_C$. The springs that cross between the interior and shell, we denote cross springs and they are described with a spring constant $k_C$, number $NSC$ and mean rest length $L_C$. The number of cross springs per shell particle can be increased by increasing the distance $d_C$.

To ensure that the initial model does not bounce excessively at the beginning of the simulation, these springs are initially set slightly under compression, as described in section 2.3. An illustration of our cross springs is shown in Figure 5.

We match the speed of elastic waves traveling horizontally in the shell to the speed of vertical oscillations of the cross springs. The strength of the cross springs we estimate using the dimensional scaling of equation 16

$$k_C \sim \frac{E_I N_S}{L_C NSC}$$

![Figure 3](image-url) Hinge model for crustal flexural stiffness as described by Bridson et al. (2003); Grindspun et al. (2003). For each edge in the crustal shell, the dihedral angle is used to compute the bending force on each vertex particle.

![Figure 4](image-url) Offset individual black bars in Figure 4 are artifacts from the lattice rather than en-echelon structures.
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Figure 4. Rendering of crustal failure. The crustal shell is modeled with a triangular lattice with springs connecting each lattice point. Lattice points are mass nodes in the spring network. Springs (along the edges of triangles in the mesh) that have exceeded their maximum strain are colored tan instead of pink. Each surface triangle is displayed based on the properties of its edge springs. If an edge spring has failed, a black bar is shown on the triangular face perpendicular to the failed spring. If two edge springs fail, then a bent black bar is shown on the triangle with each end touching a failed spring. If all three edge springs have failed then a three pronged black fork is shown on the triangular face. If all springs in the surface network fail, the lattice that is dual to the triangular lattice, a hexagonal one, is seen in black. Long connected or partially connected sets of black bars illustrate fractures in the surface shell. In this close-up view tidal forces from the perturber (colored magenta) has caused some springs in the surface lattice to fail.

where the Young’s modulus is that of the interior and the value for spring constant $k_C$ depends on the number of cross springs per shell particle. Compression is communicated elastically from the crust to the interior with a speed $V_{C\omega} = \sqrt{\frac{k_C N S_C}{M_S}} L_C$ (25)

where $M_S$ is the mass in the shell. We adjust $d_C$ and $k_C$ so that the coupling speed $V_{C\omega}$ is comparable to the velocity of P-waves in the interior, $V_{I,P}$.

2.8 Damping to an equilibrium state

If the crustal shell is initially under compression at the beginning of the simulation, increased extension would be needed to cause spring failure. If there are local crustal stresses present at the beginning of the simulation, they would influence the location of tensile failure during the tidal encounter. Despite adjustment of interior spring lengths and strengths (described in section 2.3), and setting the cross springs slightly under compression, the body bounces radially at the beginning of a simulation. This can affect the timing (and so location) of crustal tensile failure, as failure would be most likely to occur when the body has largest radius. We would like shell springs to be at their rest lengths at the beginning of the simulation and the body should be in equilibrium and not vibrating. To ensure equilibrium and zero the stress in the crust we run a relaxation simulation before each tidal encounter. The relaxation simulations are run without a tidal perturber, of the primary body alone, with a large damping parameter $\gamma$. During this relaxation simulation, springs are not allowed to fail and we slowly adjust the lengths of the shell springs so that they approach a zero force condition. The rest hinge angles are periodically reset to zero the flexural stress of the surface.

At the end of the relaxation, shell springs are very close to their rest lengths (under no extension or compression), the vibrations of the body have decayed, and the crustal membrane is under no flexural stress. The support of the crustal membrane by compression by the cross springs approximates hydrostatic support even though there is no density contrast. The relaxation simulations are run $T = 3$ (in units of the gravitational time scale; equation 9) under a damping coefficient (for all springs) of $\gamma = 50$. After the relaxation run is finished we store the positions and velocities of all masses and properties of all springs and hinges so that they can be read back into the code to run the tidal encounters. By simulating the relaxed body with a low damping coefficient and without an encounter, we check that the body is stable and that springs do not fail in the absence of any perturber.

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2.9 Parameter choices for the Tidal encounters

To describe the primary body a large number of parameters must be chosen (see Table 2). We adjusted the minimum distance between particles $d_s$ to be somewhat larger than the distance between shell particles as we need to well resolve the shell but not the interior. The parameter $d_{1S}$ was adjusted so that the number of springs per node in the interior exceeded 10, and so that the effective elastic coefficients (Young’s modulus and Poisson ratio) are not strongly dependent on the spring network (Kot et al. 2015). The shell mass and distance between shell and outer boundary of the interior were adjusted to give the shell approximately the same density as the interior and so that implied shell thickness $h_S$ is approximately consistent with that estimated from the flexural strength and speed of elastic waves through the crust.

The spring constant in the interior was chosen so that the Young’s modulus of the interior is approximately 3, matching an estimate for the value of ice’s Young’s modulus in units of $\epsilon_g$ for the icy bodies listed in Table 1. We adjusted spring constants in the cross springs and shell so that the speed of elastic waves in the shell is similar to that in the interior and the speed of vibrations passing from shell to cross spring $V_{S,P} \sim V_{C,S} \sim V_{I,P}$ and assumed composition of crust and interior is similar. The spring constant in the shell was adjusted so that the implied crustal thickness from equation 20 gave $h_S \sim 0.02$ (in units of radius), corresponding to 10 km crustal thickness for a body with of radius $R = 500$ km. Estimates for the thickness of the icy crust on Dione and Tethys range from 1 - 7 km, and are based on assuming that topographic features are signatures of flexure of a broken elastic plate (Giese et al. 2007; Hammond et al. 2013). Our modeled thickness exceeds these estimates by a factor of 2-3 but they are based on flexure over long time scales (Myr) and the effective elastic thickness of the crust should be larger on the shorter tidal time scale (hours). A comparison of observed and predicted flexural rigidity (proportional to effective thickness to the third power) as a function of age of geological load for seamounts and oceanic islands implies that elastic thickness is a strong function of strain rate (see section 6.7 by Watts 2001) but it would be inaccurate to extrapolate over orders of magnitude to tidal time scales. Our simulated crustal shell is connected via cross springs to the interior so vibrational waves can propagate throughout the body, similar to the way seismic waves propagate through the Earth’s mantle. Using the implied crustal thickness, $h_S$, a flexural force parameter, $k_n$, was chosen to be stronger than that estimated using equation 23) so as to maintain numerical stability (keep the hinges from collapsing) during the simulation.

The strain value for surface spring failure for most of the simulations was chosen to be $\epsilon_s = 0.003$, however weaker tidal encounters would allow the surface to fracture were we to reduce this number. The strength reduction parameter $F_{S,n}$ (setting spring strength after failure in a shell spring) was set high enough to ensure that surface triangles never collapsed to a line during the simulation. Collapse of a surface triangle causes the surface to become numerically unstable as the hinge forces become infinite when the triangular face areas drop to zero.

After the relaxation runs are done, we run the tidal encounters. During the encounter simulations no new springs are created and only shell springs are allowed to fail. The tidal encounter simulations are begun and ended with the secondary mass located at distance of approximately 3 times the radius of the primary away from the primary body center and are run for a total time of $T \sim 3$ in gravitational units. The perturber is modeled as a point mass. The inverse of our gravitational time unit is equivalent to the spin of a body with surface near the centrifugal rotational breakup velocity. Most moons rotate much more slowly than this value (e.g., Murray & Dermott 1999). As tidal encounters are fast (taking place on a time $t_{grav}$), we ignore the role of the spin of the primary body, setting it to zero. Gravitational softening is set to 1/100 of the minimum initial inter-particle spacing and we have checked that its exact value does not influence the simulations. Simulations were run at half the timestep listed in Table 3 to check that the resulting surface morphologies were similar and not dependent on the time step.
Figure 6. Simulation of tidal encounters with parameters listed in Table 3 and encounter parameters listed in Table 4. From top to bottom the simulations are T5, T1, T10, and T10b. Perturber masses are shown on the left. Each sub-panel shows a different time with time advancing from left to right with times labelled from pericenter. Pericenter is approximately at the second column from left. The line of sight is perpendicular to the encounter orbit plane. As the body deforms in response to the tidal perturbation, long linear fractures appear on the surface.

in Figure 8 in a cylindrical projection (horizontal axis corresponding to longitude and vertical axis showing latitude) for the same simulations. In these figures with longitude ranging from $-\pi$ to $\pi$ the subsatellite point (point of closest approach during the encounter) lies on the equator at a longitude of $\pi/2$ and on the right hand side.

Figure 6 shows that simulated crustal fractures extend a large fraction of the body, even for the lowest mass perturber. Cracks are oriented both perpendicular and parallel to the orbit path, and are predominantly present on a single hemisphere (see Figure 8). Cracks tend to be concentric around the point of closest approach (also called the subsatellite point). As expected, the faster encounter T10b causes fewer fractures but the fractures seem to cover the
Figure 7. The trajectory of the perturbing body in the orbital plane at different times for each of the tidal encounters shown in Figure 6. The coordinate system is with respect to the resolved body shown as a solid black circle at the origin. Red diamonds show the positions of the center of the perturbing body at times separated by 0.1. The open circles show the perturbing body at the times of snapshots shown in Figure 6. The viewer for these snapshots is located at negative $y$ and looking upwards in the figures shown here. The red arrows show the direction of motion of the perturber.

Our simulations produce crude illustrations of surface fractures that we can compare to long chasmata or graben complexes on icy bodies such as those listed in Table 1. In Figure 9 we show full hemispheres using an orthographic projection of Dione, Tethys and Charon created from maps available from the The Jet Propulsion Lab Photojournal (see http://photojournal.jpl.nasa.gov/). The images shown in Figure 9 were made using the open-GL display software Sphere-Mapper (https://github.com/dmgiannel/Sphere-Mapper), written by one of us (David Giannella), that takes cylindrical projection cartographic maps and applies them as a texture to a sphere that can be tilted to any desired angle and rotation angle and viewed using an orthographic projection. The Dione input image is the Planetary Image Atlas (PIA) 18434 that is a global 3-Color map of Dione (IR-Green-UV) posted April 2014. Its cartographic control and digital mosaic construction are by Dr. Paul Schenk (LPI, Houston). The original map has a simple cylindrical map projection at 250m/pixel at equator and is based on Cassini ISS images acquired 2004-2014. The global map of Saturn’s moon Tethys (PIA 11673) was created using images taken by NASA’s Cassini spacecraft and includes new data collected during Cassini’s Aug. 14, 2010, flyby (original Image Credit: NASA/JPL/Space Science Institute). The map of Charon we used (PIA 19866) is by NASA/Johns Hopkins University Applied Physics Laboratory/Southwest Research Institute and was created from all available resolved images of the surface acquired between
Figure 8. Surface fractures as a function of longitude and latitude (using cylindrical projection) for the same simulations as shown in Figures 6. Point of closest approach (subsatellite point) is shown as a yellow ring on the middle right in each panel.

July 7-14, 2015, at pixel resolutions ranging from 40 kilometers on the anti-Pluto facing hemisphere (left and right sides of the map), to 400 meters per pixel on portions of the Pluto-facing hemisphere.

The fractures we have simulated (Figures 6, 8) are of similar extent to those exhibited by the bodies shown in Figure 9. However our simulated bodies often exhibit more than one large fracture, and only Dione has as many chasmata. Dione has a number of large chasmata, but they are not concentric about a single point as seen on our simulated fractured bodies. The C shaped feature on Dione known as Padua Chasmata, shown on the top left in Figure 9, might originate from a very close but lower mass encounter with closest approach at the center of the C. Ithaca Chasma on Tethys, as a single set of features, might be consistent with a fracture caused by a lower mass perturber (1/10 of that of Tethys) or more distant encounter with a massive perturber (equal mass). The long sequence of chasmata on Charon forming a great arc (Macross and Serenity chasmata) might have been caused by a moderately distant encounter with a very large object, such as Pluto itself. Our simulations do show parallel sets of fractures (for example in the T10 simulation) that might correspond to a series of parallel chasmata such as Tardis and Nostromo Chasmata that are parallel to Macross and Serenity Chasmata on Charon.

3.1 Discussion

Within the bright terrain on Ganymede is a mosaic of ridges and troughs, termed grooved terrain, exhibiting abundant evidence of extensional strain (e.g., see Pappalardo & Greeley 1995 and Collins et al. 2010 section 6.1.2). Ganymede has a higher energy density, \( e_g \approx 30 \) GPa, than the icy bodies we listed in Table 1, and this lies in between the Young’s modulus of ice (a few GPa) and rocky materials 50–100 GPa. If Ganymede were approximated with an elastic solid with Young’s modulus similar to materials in the Earth’s lithosphere, \( E \approx 100 \) GPa, then equation 5 suggests that a near equal mass perturber tidal encounter would crack Ganymede’s surface. Due to Ganymede’s liquid metal core and its few hundred km deep subsurface saltwater ocean (e.g., Saur et al. 2015), Ganymede would deform more strongly to a tidal encounter than a purely elastic body. Ganymede is differentiated so a model with a multiple layer interior, comprised of both solids and liquids, is required to study its tidal response.

We have taken care in our simulations to relax the

\[ N \text{MRAS 000, 000–000 (0000)} \]
body before each encounter and we have tried to model a
crust with constant thickness, uniform elasticity and flexu-
ral rigidity that is approximately hydrostatically supported.
Even though we have used a moderate number of particles to
resolve shell and interior, the random particle distribution
of the interior and associated spring network is not even.
There are different numbers of cross springs per shell parti-
cle so some areas of the surface shell are more likely to fail
than others and this prevents us from running at low levels
of maximal strain $\epsilon_S$. Conversely icy crust on moons is un-
likely to have uniform thickness and composition and could
be under residual localized stress. Our simulations predict
symmetrical features above and below the orbital plane, but
real bodies are likely to be heterogeneous in terms of crustal
thickness, thermal state and pre-existing fractures so they
might preferentially fracture in weaker regions and only on
one side. Here we have neglected body spin. However with
a nearly parabolic (or slow) encounter with a rapidly spin-
ning body (near the breakup spin rate), tidal stresses on the
surface would not be symmetrical and this too could cause
asymmetry in the fracture distribution.

We found that our simulated bodies did not exhibit
fractures for encounters with larger pericenter distances or
higher encounter velocities. The escape velocity from Dione
is $\sim 0.5$ km/s and this is representative for the escape veloc-
ities for the other icy bodies listed in Table 1. In comparison,
the orbital velocity of Dione is approximately 10 km/s and
the orbital velocity of Saturn is 9.6 km/s. This implies that
tidal encounters with asteroids or centaurs would preferen-
tially be at higher relative velocities with ratio $V_q/v_c \sim 10$
and so higher than the parabolic encounters consider here
(see equation 5 for the estimated strain dependence on en-
counter velocity). However, encounters with moons that are
also orbiting the planet would have much lower relative ve-
locities. The orbital speed of Charon is only 0.2 km/s so an
encounter between Charon and Pluto would have been in
the nearly parabolic regime simulated here. While a fast en-
counter from an external object would be isolated, if a close
tidal encounter occurs between two satellites orbiting the
same body, then multiple close encounters are likely, each
producing a group of fractures about a different pericenter
 locus.

Vibrations are excited during a tidal encounter and
vibrational energy dissipated due to the viscoelastic body
response and surface fracture. The total energy dissipated
should be a small fraction (at most a few percent) of the
gravitational binding energy (Press & Teukolsky 1977) and
so is at most a small fraction of the orbital energy. Only if
the encounter is almost exactly parabolic would this energy

Figure 9. Images of Dione, Tethys and Charon using an orthographic projection and showing entire hemispheres. Top left: Dione using
Planetary Image Atlas 18434. The central view point has latitude $16^\circ$, longitude $35^\circ$ and up corresponds to an azimuthal heading of
$4^\circ$ from North. The C shaped feature is Padua Chasmata. Top Right: Also Dione but the central point has latitude $-42^\circ$, longitude $131^\circ$, and the heading $9^\circ$. This image shows Palatine and Eurotas Chasmata. Bottom Left: Tethys using Planetary Image Atlas 11673 and showing the Ithaca Chasma. The central point has latitude $-38^\circ$, longitude $18^\circ$, and heading $289^\circ$. Bottom Right: Charon using Planetary Image Atlas 19866. The central point has latitude $42^\circ$, longitude $19^\circ$, and heading $8^\circ$. The large fractures are informally known as Macross and Serenity chasmata.
loss allow the two bodies to become gravitational bound (in orbit about each other) after the encounter.

Here we have modeled the perturbing body as a point mass. If this body is a strong solid and nearly spherical this is a good approximation. However if the perturbing body were weak (a low cohesion rubble pile, e.g., Richardson et al. 2005) and lower mass than the primary body then it could be strongly deformed or disrupted during then encounter and its tidal field would significantly differ from that of a point mass.

Many large icy moons are believed to have global oceans which decouple the motions of their floating shells from their interiors (Schubert et al. 2004; Thomas et al. 2016) and these could make the tidal response larger than estimated from elasticity of a solid body alone (e.g., less et al. 2012). Conversely, in our simulations we have neglected the presence of a rocky core and this would have reduced the tidal response compared to that simulated here using a uniform body. The maximum strain value for spring failure $\epsilon_S = 0.003$ for most of our simulations may be an over estimate for the brittle strength of ice, so faster and more distant tidal encounters might also be able to cause crustal fractures in icy satellites. Unfortunately we cannot yet run simulations with lower levels of $\epsilon_S$ because the surfaces tend to exhibit fractures even in the absence of a perturber. We have also been unable to run encounters with larger mass bodies because in this regime, the shell can be lifted off the body, our surface triangles collapse and the hinge forces cause the shell to become unstable (and explode). So far we have only simulated elastic materials and do not have the capability to simulate simultaneously elastic and liquid materials, though we could increase the density and strength in the core by adjusting particle masses and spring constants so we could mimic the behavior of a rocky core. We would like to improve our code to improve its precision and extend the types of materials that we can simulate.

We have briefly explored the effect of a non-linear spring force law on the cross springs by multiplying the spring forces (equation 12) by a strain dependent factor (Clavet et al. 2005)

$$f(\epsilon) = \begin{cases} 
1 & \text{if } \epsilon \leq \epsilon_C \\
\max \left( 1 - \frac{\epsilon}{\epsilon_C} , 0 \right) \cdot F_{Ck} & \text{if } \epsilon > 0
\end{cases}$$

### Table 2. Simulation Parameter Descriptions

| Parameter | Description |
|-----------|-------------|
| $N_I$     | Number of particles in interior |
| $NS_I$   | Number of interconnecting springs in interior |
| $L_I$     | Mean rest spring length in interior |
| $k_I$     | Mean spring constant of interior springs |
| $E_I$     | Young’s modulus of interior |
| $d_I$     | Interior spring formation distance |
| $d_{IS}$  | Minimum initial interparticle distance |
| $V_{I,P}$ | Speed of P-waves in the interior |
| $N_S$     | Number of particles in shell |
| $NS_S$   | Number of interconnecting springs in shell |
| $M_S$    | Mass of shell |
| $L_S$    | Mean rest spring length of shell springs |
| $k_S$    | Spring constant of shell springs |
| $h_S$    | Simulated crustal thickness |
| $\epsilon_S$ | Maximum strain for spring failure in shell |
| $F_{Sk}$ | Factor spring constant $k_S$ is reduced after spring failure |
| $V_{S,P}$ | Speed of P-waves in the shell |

| Parameter | Description |
|-----------|-------------|
| $N_{SC}$ | Number of cross springs connecting shell and interior |
| $L_C$     | Mean rest spring length of cross springs |
| $k_C$     | Spring constant of cross springs |
| $d_C$     | Cross spring formation distance |
| $V_{Co}$  | Effective velocity of shell and interior |

### Table 3. Parameters for Simulations

| Parameter | Description |
|-----------|-------------|
| $N$       | Number of simulations |
| $T$       | T-type of simulations |

- $N_{SC}$ | 0.0566 |
- $L_C$   | 0.156 |
- $k_C$   | 0.005 |
- $d_C$   | 0.18 |
- $V_{Co}$ | 3.3 |

| Parameter | Description |
|-----------|-------------|
| $\gamma$ | Spring damping coefficient |
| $dt$      | Time step |
| $t_{fail}$ | Time interval between failure of individual springs |

| Parameter | Description |
|-----------|-------------|
| $M_2$     | Mass of tidal paterd |
| $q$       | Distance between body centers at closest approach |
| $v_q$     | Relative velocity at closest approach |

Here $t_{fail}$ is the time after spring failure. It is computed using equation 17, 21, 25.

### Table 3. Parameters for Simulations

| $N$ | $T$-series |
|-----|-------------|
| $N_I$ | 798 | 1869 |
| $NS_I$ | 11632 | 23663 |
| $L_I$ | 0.30 | 0.176 |
| $k_I$ | 0.0076 | 0.076 |
| $E_I$ | 4.0 | 3.73 |
| $d_I$ | 0.38 | 0.10 |
| $d_{IS}$ | 0.15 | 0.23 |
| $V_{I,P}$ | 1.4 | 4.5 |

| $N_S$ | 0 | 2562 |
| $NS_S$ | 7680 |
| $M_S$ | 0.07 |
| $L_S$ | 0.075 |
| $k_S$ | 0.06 |
| $h_S$ | 0.0075 |
| $\epsilon_S$ | 0.003 |
| $F_{Sk}$ | 0.005 |
| $V_{S,P}$ | 3.28 |
where $\epsilon_C$ sets a strain scale and the spring force is reduced as this scale is approached. Here $F_{Ck}$ is a factor that limits the minimum force under extension so that it never drops to zero. However for the strengths of the bodies we have simulated here, and using the same initial relaxed body, we saw little difference in simulated fracture morphology when we varied $\epsilon_C$ from 0.02 to 0.04 (and the maximum strain value was never approached, possibly because the cross springs begin under compression). Were we to simulate a weaker (so thinner) surface, a reduction in the connection of the surface to the interior would affect simulated fracture extent and morphology.

Lithospheric brittle failure depends on depth, tensile mode, strain and strain rate (see the review by Burov 2011). Here we have adopted a simplistic maximal strain value for brittle failure. Crack propagation is only crudely simulated here, and crack morphology is numerically dependent on the nature of relaxation and so here on the time interval used to identify failed springs in the simulation. When this interval is reduced (and comparing simulations beginning with the same relaxed body), there is some additional surface failure and in some regions wider cracks can form. To meaningfully predict fault lengths, widths and structures we would need to better simulate crack formation, propagation, decompression and subsequent relaxation. Wholesale crustal failure in a model with more realistic coupling between crust and interior might exhibit fluid flow from subsurface oceans and decompression melting.

We have not seen qualitative differences in fracture morphology if we set rest lengths of springs to their failure length when they fail. For the strongest perturber (the T10) simulation large regions of the surface were ruptured. If crack propagation were more accurately modeled with higher resolution, these might manifest as jumbled terrain or parallel sets of fractures and graben complexes.

Here we have only simulated brittle elastic response. At the high strain rates of tidal encounters there must be a depth where the ice is ductile and deforms plastically. If we more accurately modeled the strength envelope and material properties as a function of depth, then fractures would open during the encounter but they would not close all the way afterwards, giving us an estimate for their extension.

We have not modeled viscoelastic response on long time scales. While we might have identified regions where fractures originate, our work does not predict depths and widths of resulting chasmata or graben complexes that might result. Improved simulations would more accurately model crack propagation, fracture formation and explore the subsequent evolution of the resulting geophysical structures, such as magma and water escape along dilatant cracks and associated partial resurfacing.

### 3.1.1 Cracking Mars’ Lithosphere

Using the Young’s modulus of a rocky material, $E_{\text{rock}} \sim 50$–100 GPa, we notice that Mars has ratio of gravitational energy density to Young’s modulus $g/E_{\text{rock}} \sim 2$–4, only a factor of a few higher than $g/E_{\text{ice}}$ estimated for the icy bodies we have listed in Table 1. The yield strength envelope as a function of depth for a rocky lithosphere reaches a maximum at about a few hundred MPa (e.g., Figure 6.35 by Watts 2001). The maximum strength divided by a Young’s modulus of 50 GPa (for a rocky material) gives a ratio of order 0.01, similar to the value of maximal strain for uniaxial tensile brittle failure of ice that we used above as a criterion to mimic fracture in an icy crust. The ratio $g/E$ and the maximum uniaxial strain for crustal brittle failure are similar for icy moons and Mars, suggesting that the scenario we have explored here can be scaled to Mars. If Mars experienced a strong (nearly equal mass) and close tidal encounter with a rocky body, tensile deformation during the encounter could have fractured Mars’ lithosphere.

Valles Marineris is thought to have been formed by crustal extension (Tanaka and Golombek 1989) similar to the formation of rift faults like the East African Rift (Eggen et al. 1991). Valles Marineris was formed after much of the volcanic Tharsis bulge or rise was in place (see section 4.5 by Golombek and Phillips 2010). Tectonic formation scenarios for Valles Marineris associate its formation with the inability of Mars’ lithosphere to support the large load of the Tharsis bulge itself (Tanaka and Golombek 1989; Mège and Masson 1996; Nimmo & Tanaka 2005; Golombek and Phillips 2010; Andrews-Hanna 2012a,b,c). Flexural (bending and membrane) stresses in the lithosphere account for the radial grabens on the topographic rise and concentric compression wrinkle ridges around its circumference (Banerdt & Golombek 2000; Golombek and Phillips 2010). However, a stress model that is azimuthally symmetric about the center of the Tharsis rise would predict radial grabens that are of similar length and width on opposite sides of the rise. In contrast, Valles Marineris (south east of the peak) is exceptional – much wider and deeper than radial grabens to the north or west of the center of the rise.

Gravity and topography data indicate that Mars’

### Table 4. Encounter Parameters for Simulations

| Simulation | $M_2$ | $q$ | $V_q$ | Description |
|------------|-------|-----|-------|-------------|
| N          | 1.0   | 2.13| 1.84  | interior only |
| T5         | 0.5   | 1.84| 1.60  | fiducial |
| T1         | 0.1   | 1.47| 1.62  | lower mass perturber and weaker crust |
| T10        | 1.0   | 2.10| 1.61  | higher mass perturber |
| T10b       | 1.0   | 2.22| 1.94  | higher mass perturber and faster encounter |
crustal thickness is bimodal, approximately 30 km in the northern hemisphere and 60 km in the southern hemisphere, and exceeding 80 km on the Tharsis bulge (Neumann et al. 2004). Andrews-Hanna (2012b) proposed that Valles Marineris is located near and aligned with the buried crustal dichotomy boundary that bisects the Tharsis bulge. In his model, the difference in crustal thickness underlying the Tharsis bulge generates tensile stress directly along the crustal dichotomy boundary and this accounts for the exceptional width and depth of Valles Marineris compared to other radial grabens surrounding the Tharsis bulge.

We have found that a strong tidal encounter can cause long extensional fractures that extend a significant fraction of the body’s radius. So a tidal encounter might account for the exceptional depth and length of Valles Marineris. However, we expect (see Figure 8) fractures in a large ring centered at the point of closest approach. If a tidal encounter were responsible for formation of Valles Marineris, why doesn’t the valley extend further, forming a large ring? We simulated a relaxed, constant thickness, uniform elasticity and flexural rigidity crustal shell but Mars’ lithosphere is not uniform thickness and has variations in levels of localized stress. During a tidal encounter, Mars’ lithosphere could have preferentially fractured along a region of localized stress rather than in a circle centering the point of closest approach. Magma underlying the Tharsis bulge at the time of the tidal encounter may have allowed Mars’ crust to be more easily deformed during the encounter, perhaps even lifted away from the core. So the volcanic activity in the Tharsis province itself could have exacerbated tidal deformation. A tidal encounter is an intriguing alternate explanation for the extensional stress forming Valles Marineris, but a more sophisticated study would be needed to test it and contrast it with tectonic models for the formation of Valles Marineris.

4 SUMMARY AND ADDITIONAL DISCUSSION

In this paper we have explored tidal encounters with elastic bodies using a mass-spring model to simulate elasticity within the context of an N-body simulation. We have simulated crustal failure using an elastic shell model, with flexural stiffness for the crust. Brittle failure is modeled using a maximum strain value for the surface springs, and crack propagation modeled with a crude relaxation procedure, allowing only a single spring to fail in a specific time interval (a few computational time steps). Our simulations illustrate that strong, close tidal encounters can cause crustal failure on icy bodies, confirming an order of magnitude estimate for the tidally induced surface strain. Following the encounters, simulated crustal shells exhibit long fractures extending over a large fraction of a body radius. Using near parabolic encounters with a nearly equal mass body perturbing a non-spinning body, we find that surface fractures tend to be concentric around the subsatellite point (point of closest approach for the encounter) and are restricted to a single hemisphere. Tidally induced crustal fractures might provide an explanation for long chasmata and graben complexes on icy bodies such as Dione, Tethys and Charon.

We have attempted to construct simulations that represent icy bodies. However, the tidal regime, because it is at high strain rate, is different than other geophysical settings (such as studies of crustal plate flexure). If some chasmata are explained by tidal encounters we might be able to place constraints on the effective crustal elastic thickness at high strain rate and on the connection between crust and interior. Conversely uncertainty in the rheological models make it difficult for us to carry out simulations accurate enough to do this comparison.

We have focused here on icy bodies that have old crusts. Rocky satellites and asteroids such as Phobos, Eros, Ida, Gaspra, Epimetheus and Pandora can exhibit long grooves and troughs (see the review by Thomas & Prockter 2010). Single tidal encounters could be investigated as a mechanism for the formation of fractures that are suspected to underly their surface regolith. Lastly, strong close tidal encounters might have occurred with large rocky bodies such as Mars, and old regions of their surfaces may retain extensional features caused by these encounters.

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