Macroscopic Modeling and Simulation of Managed Lane-Freeway Networks

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Abstract

To help mitigate road congestion caused by the unrelenting growth of traffic demand, many transit authorities have implemented managed lane policies. Managed lanes typically run parallel to a freeway’s standard, general-purpose (GP) lanes, but are restricted to certain types of vehicles. It was originally thought that managed lanes would improve the use of existing infrastructure through incentivization of demand-management behaviors like carpooling, but implementations have often been characterized by unpredicted phenomena that is often to detrimental system performance. Development of traffic models that can capture these sorts of behaviors is a key step for helping managed lanes deliver on their promised gains.

Towards this goal, this paper presents several macroscopic traffic modeling tools we have used for study of freeways equipped with managed lanes, or “managed lane-freeway networks.” The proposed framework is based on the widely-used first-order kinematic wave theory. In this model, the GP and the managed lanes are modeled as parallel links connected by nodes, where certain type of traffic may switch between GP and managed lane links. Two types of managed lane configuration are considered: full-access, where vehicles can switch between the GP and the managed lanes anywhere; and separated, where such switching is allowed only at certain locations called gates.

We incorporate two phenomena into our model that are particular to managed lane-freeway networks: the inertia effect and the friction effect. The inertia effect reflects drivers’ inclination to stay in their lane as long as possible and switch only if this would obviously improve their travel condition. The friction effect reflects the empirically-observed driver fear of moving fast in a managed lane while traffic in the adjacent GP links moves slowly due to congestion.

Calibration of models of large road networks is difficult, as the dynamics depend on many parameters whose numbers grow with the network’s size. We present an iterative learning-based approach to calibrating our model’s physical and driver-behavioral parameters. Finally, we validate our model and calibration methodology with case studies of simulations of two managed lane-equipped California freeways.

Keywords: macroscopic first order traffic model, first order node model, multi-commodity traffic, managed lanes, HOV lanes, dynamic traffic assignment, dynamic network loading, inertia effect, friction effect

1 Introduction

Traffic demand in the developed and developing worlds shows no sign of decreasing, and the resulting congestion remains a costly source of inefficiency in the built environment. One study (Lomax et al., 2015) estimated that, in 2014, delays due to congestion cost drivers 7 billion hours and $160B in the United States alone, leading to the burning of 3 billion extra gallons of fuel. The historical strategy for accommodating more demand has been construction of additional infrastructure, but in recent years planners have also developed
strategies to improve the performance of existing infrastructure, both through improved road operations and demand management, which seeks to lower the number of vehicles on the road (Kurzhanskiy and Varaiya, 2015). One such strategy that has been widely adopted in the United States and other developed countries is the creation of so-called managed lanes (Obenberger, 2004). Managed lanes are implemented on freeways by restricting the use of one or more lanes to certain vehicles. As an example, high-occupancy-vehicle (HOV) lanes are intended to incentivize carpooling, which reduces the total number of cars on the road as a demand management outcome (Chang et al., 2008).

In addition to demand management, managed lanes provide an opportunity for improved road operations through real-time, responsive traffic control. For example, tolled express lanes give drivers the opportunity to pay a toll to drive parallel to the general-purpose lanes on a (presumably less-congested) express lane. Traffic management authorities here have an opportunity to adjust the toll amount in response to the real-time state of traffic on the network. The potential for managed lanes as instruments for reactive, real-time traffic operations—in addition to their demand-management purpose—has made them popular among transportation authorities (Kurzhanskiy and Varaiya, 2015).

However, the traffic-operational effects of managed lanes are not always straightforward or as rehabilitative as expected, as their presence can create complex traffic dynamics (Jang and Cassidy, 2012). Even in a freeway with simple geometry, the dynamics of traffic flow are complex and not fully understood, and adding managed lanes alongside the non-managed, general-purpose (GP) lanes only exacerbates this. In effect, adding a managed lane creates two parallel and distinct, but coupled, traffic flows on the same physical structure. When used as intended, managed lanes carry flows with different density-velocity characteristics and vehicle-type (e.g., strictly HOVs) compositions than the freeway. When vehicles move between the two lane flows, complex phenomena that are unobserved in GP-only freeways can emerge (see e.g. Daganzo and Cassidy (2008); Liu et al. (2011); Jang and Cassidy (2012); Cassidy et al. (2015), and others).

Making better use of managed lanes requires an understanding of the macroscopic behavior they induce. One widely-used tool for understanding macroscopic traffic flow behavior is the macroscopic traffic flow model. A rich literature exists on macroscopic models for flows on long roads, and at junctions where those roads meet, but an extension to the parallel-flows situation created by placing a managed lane in parallel with a freeway (a “managed lane-freeway network”) is not straightforward. Such a model should describe when vehicles enter or exit the managed lane, and capture the interactions caused by the two flows’ proximity, such as the so-called “friction effect” (Liu et al., 2011; Jang and Cassidy, 2012).

This paper presents macroscopic flow modeling tools we have used for simulation of managed lane-freeway networks. We begin in Section 2 with a discussion of relevant modeling tools from the literature, and how we make use of them. Section 3 describes network structures for the two common managed lane configurations: gated-access and full-access (ungated) lanes. This Section also describes models for friction and inertia effects that arise in managed lanes. Section 4 outlines how one may take these managed lane-specific constructions and add them to a general macroscopic traffic simulation process. Section 5 describes calibration methodologies for both network configurations. Section 6 presents case studies of two networks, one each of gated- and full-access, and typical macroscopic simulation results.

2 Managed Lane Modeling

The modeling techniques presented in this paper are based on the first-order “kinematic wave” macroscopic traffic flow model. These models describe aggregate traffic flows as fluids following a one-dimensional conservation law. We briefly introduce our notation here, but do not discuss the basics of this class of models. Detailed reviews are available in many references.
2.1 Modeling basics

In this simulation framework, a road is divided into discrete cells, which we refer to as links. Links are drawn between nodes: a link begins at one node and ends at another. Many links may begin and end at each node. Each link \( l \) is characterized by density \( n_l \), the number of cars in the link. In a first-order model, the traffic flows are fully prescribed by the density. From timestep \( t \) to \( t + 1 \), link \( l \)'s density updates according to the equation

\[
n_l(t + 1) = n_l(t) + \frac{1}{L_l} \left( \sum_{i=1}^{M} f_{il}(t) - \sum_{j=1}^{N} f_{lj}(t) \right),
\]

where \( L_l \) is the length of link \( l \), \( f_{il}(t) \) is the flow (number of vehicles) leaving link \( i \) and entering link \( l \) at time \( t \), \( f_{lj}(t) \) is the flow leaving link \( l \) and entering link \( j \) at time \( t \), \( M \) is the number of links that end at link \( l \)'s beginning node, and \( N \) is the number of links that begin at link \( l \)'s ending node.

Computing the inter-link flows requires the use of two intermediate quantities for each link. These are the link demand \( S_l(t) \), which is the number of vehicles that wish to exit link \( l \) at timestep \( t \); and the link supply, \( R_l(t) \), which is the number of vehicles link \( l \) can accept at time \( t \). Both \( S_l \) and \( R_l \) are functions of the density \( n_l \). The model that computes \( S_l \) and \( R_l \) from \( n_l \) is often called the “fundamental diagram” or “link model,” and the model that computes the flows from all links’ supplies and demands is often called the “node model.”

A brief outline of how first-order macroscopic simulation of a road network (sometimes called a dynamic network loading simulation) is performed could be:

1. At time \( t \), use the link model for each link \( l \) to compute the link’s demand \( S_l(t) \) and supply \( R_l(t) \) as a function of its density \( n_l(t) \).
2. Use the node model for each node to compute the inter-link flows \( f_{ij} \) for all incoming links \( i \) and outgoing links \( j \) as functions of \( S_i(t) \), \( R_j(t) \), and information about vehicles’ desired movements \( ij \).
3. Update the state of each link using (2.1).
4. Increment \( t \) and repeat until the desired simulation end time is reached.

The tools described in this paper are compatible with any such link model. We make use of a particular node model that we have studied in Wright et al. (2016a,b). An aspect of this node model of particular relevance to managed lane modeling is our “relaxed first-in-first-out (FIFO) rule” construction (Wright et al., 2016a). This is necessary for modeling the flows between the GP and managed lanes. Without a FIFO relaxation, congestion in one of the two lane groups could block traffic in the other when that may be unrealistic (see Wright et al. (2016a, Section 2.2) for a detailed discussion).

So far, we have presented ingredients for a model that, while able to express many simple network topologies by joining links and nodes, does not capture several important behaviors in managed lane-freeway networks. The next three Sections briefly overview the additions to the standard model that will be explained in greater detail in the remainder of the paper.

2.2 Multiple classes of vehicles and drivers

In (2.1), we describe the number of vehicles in a link as a single number, \( n_l \). In this formulation, all vehicles are treated the same. However, for simulation in a managed lane-freeway network, it makes sense to break
introduce different classes of vehicles and/or drivers. For example, for a freeway with an HOV lane facility, we might consider two classes: HOVs and non-HOVs. To this end, (2.1) can be rewritten as

\[ n_i(t + 1) = n_i(t) + \frac{1}{L_i} \left( \sum_{i=1}^{M} f_{u_i}^c(t) - \sum_{j=1}^{N} f_{ij}^c(t) \right), \]

where \( c \in \{1, \ldots, C\} \) indexes vehicle classes (often called "commodities" in the traffic literature).

Extending the density update equation to multiple classes means that the link and node models must also be extended to produce per-class flows \( f_{ij}^c \). In this paper, we will not specify a particular link model, but assume use of one that produces per-class demands \( S_i^c \) and overall supplies \( R_j \) (the node model, in computing \( f_{ij}^c \), is responsible for splitting the available supply \( R_j \) among the different demanding vehicle classes). Examples of this type of link model include those considered in Wong and Wong (2002), Daganzo (2002), and van Lint et al. (2008) (examples of multi-class link models of second- or higher-order include those of Hoogendoorn and Bovy (2000). These types of models have higher-order analogs of supply and demand).

### 2.3 Topological expression of managed lane-freeway networks

In both (2.1) and (2.2), we describe a link in terms of its total density \( n_i \) and its breakdown into per-commodity portions, \( n_i^c \). By discretizing the road into these one-dimensional links, we lose information about differences between vehicle proportions across lanes, as well as inter-lane and lane-changing behavior. This becomes a problem if such unmodeled behavior is of interest. In our setting, this means that modeling a freeway with a managed lane should not be done with a single link following (2.1) or (2.2), as it would be impossible to study the managed lane-freeway network behavior of interest.

Modeling differences in vehicle density across lanes is natural in microscopic and mesoscopic (see, for example, Treiber et al. (1999); Hoogendoorn and Bovy (1999); Ngoduy (2006), and others) models, but macroscopic models, in their simplicity, have less readily-accessible avenues for including these differences. One straightforward method is to model each lane as a separate link, as in, e.g. Bliemer (2007) or Shiomi et al. (2015). However, this method has a few drawbacks. First, it requires the addition of some lane-assignment method to prescribe the proportions of each vehicle class \( c \) for each lane (such as a logit model as used in Farhi et al. (2013) and Shiomi et al. (2015)), which requires not-always-accessible data for calibration. Second, drastically increasing the number of links in a macroscopic model will necessarily increase the size of the state space and model complexity, which is, in a sense, incompatible with the overall goal of selecting a macroscopic model over a micro- or mesoscopic model: some of the “macro” in the macroscopic model is lost.

Instead, in this paper we choose to model the GP lanes (or “GP lane group”) as one link and the parallel managed lanes (or “managed lane group”) as another link. Applied to an entire length of road, this creates a network topology of two “parallel chains” of links - one GP and one managed. The two chains will share nodes, but cross-flows between the chains are permitted only in locations where there is physical access (i.e., no physical barriers) and policy access (i.e., no double solid lines under U.S. traffic markings). Where cross-flows are possible, we do not use a logit model, but instead a driver behavior model first introduced in Wright et al. (2016a). This two-chain model is similar to the one described in Liu et al. (2012), though in this reference, GP-managed lane crossflows were not considered.

### 2.4 Observed phenomena in managed lane-freeway networks

The above-mentioned friction effect is one of several emergent behaviors thought to be important for understanding traffic phenomena present on managed lane-freeway networks, and has been blamed for some of the “underperformance” of managed lanes (Jang and Cassidy, 2012). To expand on our earlier description, the
friction effect describes a tendency of vehicles in an HOV lane to reduce their speed when the vehicles in the adjacent GP lane(s) congest to the point where they slow down. It has been hypothesized that this occurs due to HOV drivers being uncomfortable when traveling at a drastically higher speed than the adjacent GP vehicles, and slowing down to reduce the speed differential (Jang and Cassidy, 2012).

In Liu et al. (2012), the authors created a macroscopic model of a managed lane freeway network with a friction effect model. They proposed modeling the friction effect by having two separate link models for the managed lanes – one each for when the GP lane is above or below some threshold density value.

In Section 3, we propose a more expressive model of the friction effect by replacing this piecewise implementation with a linear implementation, where the reduction in demands $S_c^l$ in the managed lane is proportional to the speed differential between the managed lane(s) and GP lane(s). A linear relationship like this was hypothesized in, e.g., Jang and Cassidy (2012, Fig. 5(c)). The magnitude of the friction effect (i.e., the degree to which managed lane drivers slow down to match GP lane drivers’ speed) is thought to be dependent on the type of separation between the managed lane(s) and GP lane(s) (e.g., painted lines vs. a concrete barrier) (Jang et al., 2012), which can be encoded by selecting the linear coefficient to this speed differential.

3 Full- and Gated-Access Managed Lane-Freeway Network Topologies

We will consider two types of managed lane freeway network configurations: full access and separated with gated access. In a full-access configuration, the managed lane(s) are not physically separated from the GP lane(s), and eligible vehicles may switch between the two lane groups at any location. Often, full-access managed lane(s) are special-use only during certain periods of the day, and at other times they serve as GP lane(s) (e.g., HOV lanes are often accessible to non-HOVs outside of rush hour). On the other hand, in a gated-access configuration, traffic may switch between the managed lane(s) and GP lane(s) only at certain locations, called gates; at non-gate locations, the two lane groups are separated by road markings (i.e., a double solid line in the U.S.) or a physical barrier. Usually, gated-access managed lanes are special-use at all times. The implemented managed lane access scheme depends on jurisdiction. For example, full-access lanes are common in Northern California, and separated lanes are common in Southern California.

The differences in physical geometry and access points between the two access types requires two different types of topology in constructing a network for a macroscopic model.

3.1 Note on link and node models used in this section

As discussed in the previous Section, we attempt to be agnostic with regards to the particular link model (e.g., first-order fundamental diagram) used in our implementations. However, in our model of the friction effect in Section 3.3, we parameterize friction being in effect on a particular link $l$ for a particular vehicle class $c$ at time $t$ by adjusting that link’s demand $S_l(t)$. For that discussion only, we specify a particular link model. This link model is reviewed in Appendix A. This link model is also the one used in the simulations presented in Section 6.

The node model used here and for the remainder of this paper, when a particular form is necessary, is the one discussed in Wright et al. (2016a) and Wright et al. (2016b). We use this node model because it handles multi-commodity traffic, optimizes the utilization of downstream supply, makes use of input link priorities and has a relaxation of the “conservation of turning ratios” or “first-in-first-out” (FIFO) constraint of most node models. This last feature allows us to describe a set of GP lanes just upstream of an offramp with one link, and handle a condition of a congested offramp by having the congestion spill back onto only the offramp-serving lanes of the GP link (as opposed to the entirety of the GP link). See Wright et al. (2016a, Section 3) and Wright et al. (2016b) for more discussion.
3.2 Full-access managed lanes

A full-access managed lane configuration is presented in Figure 1: GP and managed links (recall, as discussed above, all GP lanes and all managed lanes are collapsed into one link each) are parallel with the same geometry and share the same beginning and ending node pairs; traffic flow exchange between GP and managed lanes can happen at every node. Note that in Figure 1, we use a slightly irregular numbering scheme so that it is clear whether a link is a GP link, managed lane link, or ramp link. Parallel links in the graph have numbers made up of the digit of their terminating node, with GP links having one digit (i.e., link 1), managed lane links having two (i.e., link 11), and ramp links having three (i.e., link 111). Note also that we use a U.S.-style, driving-on-the-right convention here, with the managed lane(s) on the left of the GP lanes and the ramps on the far right.

Links that are too long for modeling purposes (i.e., that create too low-resolution a model) may be broken up into smaller ones by creating more nodes, such as nodes 2 and 3 in Figure 1. Fundamental diagrams for parallel GP and managed lane links may be different (Liu et al., 2011).

We introduce two vehicle classes $\mathcal{C} = 2$: $c = 1$ corresponds to the GP-only traffic and $c = 2$ corresponds to the special traffic. When the managed lane(s) is (are) active, $c = 1$-traffic is confined to the GP link, whereas $c = 2$-traffic can use both the GP and managed lane links. We denote the portion of vehicles of class $c$ in link $i$ that will attempt to enter link $j$ as $\beta_{c,i,j}$. This quantity is called the split ratio.

3.2.1 Split ratios for full-access managed lanes

We make an assumption that both vehicle classes take offramps at the same rate. For example, for node 1 in Figure 1, we might say that $\beta_{1,222} = \beta_{1,222} = \beta_{1,222}$. Strictly speaking, it is not necessary to assume that the $\beta_{c,i,222}$ are equal for all $c$. However, in practice the oframp split ratios are typically estimated from flow count data taken from detectors on the offramp and freeway. Generally speaking, these detectors cannot identify vehicle type, so the only quantity estimable is a flow-weighted average of the quantities $\beta_{c,i,222}$. To estimate the class-specific split ratios, one needs some extra knowledge of the tendency of each class to take each offramp (for example, that GP-only vehicles are half as likely as special traffic to take a certain offramp). Assuming that each class exits the freeway at the same rate is a simple and reasonable-seeming assumption.

This same problem of unidentifiability from typical data appears in several other split ratios. First, it may not be possible to tell how many vehicles taking an oframp link come from the upstream GP, managed lane, or (if present) onramp link. In this case, some assumptions must then be made. For example, three different assumptions that may be reasonable are (1) that vehicles in each link $i$ take the oframp at the same rate; or (2) that no vehicles from the managed lane(s) are able to cross the GP lanes to take the oframp at this node, and that no vehicles entering via the onramp, if one is present, exit via the oframp at the same node; or (3) that vehicles in GP and managed links take the oframp at the same rate, while no vehicles coming from the onramp are directed to the oframp. Looking back at node 1 in Figure 1 again, assumption (1) would say the $\beta_{1,222}$ are equal for all $i$; assumption (2) would say $\beta_{11,222} = \beta_{111,222} = 0$; and assumption (3)
would say $\beta_{1,222} = \beta_{11,222}$ and $\beta_{111,222} = 0$. The best assumption for each node will depend on the road geometry for that particular part of the road (how near the offramp is to any onramps, how many GP lanes a vehicle in a managed lane would have to cross, etc.).

Second, the crossflows between the GP and managed lane links are not observable. Even if there exist detectors immediately upstream and downstream of the node where traffic can switch between GP and managed lanes, it is impossible to uniquely identify the crossflows. In a simulation, these crossflows must be governed by some driver choice model.

Putting together these assumptions and the special-traffic-only policy for the managed lane, we can summarize most of the necessary split ratios needed for computing flows in a node model. For example, for node 1 in Figure 1,

$$
\beta^1_{i,j} = \begin{cases}
  j = 1 & i = 111 \\
  j = 2 & 1 - \beta_{1,222} \\
  j = 22 & 0 \\
  j = 222 & \beta_{1,222}
\end{cases}
$$

and

$$
\beta^2_{i,j} = \begin{cases}
  j = 1 & i = 111 \\
  j = 2 & 1 - \beta_{111,222} \\
  j = 22 & 0 \\
  j = 222 & \beta_{111,222}
\end{cases}
$$

where “n/a” means that the split ratios $\beta_{11,j}$ are not applicable, as there should be no vehicles of class $c = 1$ in the managed lane. The split ratios marked with a dash are those above-mentioned flows that are unobservable and come from some driver choice model. Of course, whatever method is chosen to compute these unknown split ratios, we must have $\beta_{i,2} + \beta_{i,22} = 1 - \beta_{i,222}$.

Similarly, for node 2, which does not have an onramp or an offramp,

$$
\beta^1_{i,j} = \begin{cases}
  j = 2 & i = 2 \\
  j = 3 & 1 - \beta_{1,22} \\
  j = 33 & 0
\end{cases}
$$

and

$$
\beta^2_{i,j} = \begin{cases}
  j = 2 & i = 22 \\
  j = 3 & 1 - \beta_{111,222} \\
  j = 33 & 0
\end{cases}
$$

with “n/a” and the dash meaning the same as above.

As previously mentioned, full-access managed lanes often have certain time periods during which nonspecial ($c = 1$) traffic is allowed into the managed lane. We can model this change in policy simply by changing the split ratios at the nodes. For node 1, for example, the nonrestrictive policy is encoded as

$$
\beta^c_{i,j} = \begin{cases}
  j = 2 & i = 111 \\
  j = 22 & 1 - \beta_{1,222} \\
  j = 222 & \beta_{1,222}
\end{cases}
$$

for $c = \{1, 2\}$,

and for node 2 as

$$
\beta^c_{i,j} = \begin{cases}
  j = 3 & i = 111 \\
  j = 33 & 1 - \beta_{111,222} \\
  j = 33 & \beta_{111,222}
\end{cases}
$$

for $c = \{1, 2\}$.

In other words, the managed lane link is treated as additional GP lane(s), and the split ratios governing the crossflows between the two links should be found from the driver choice model for both vehicle classes.
3.2.2 Node model for full-access managed lane-freeway networks

As mentioned in Section 3.1, we make use of the node model discussed in Wright et al. (2016a) and Wright et al. (2016b) to describe a freeway network with managed lanes. This node model differentiates itself from others in that it deals with multi-commodity traffic flow, optimally utilizes the available supply, makes use of input link priorities, and has a relaxation of the common FIFO constraint. By default, link priorities can be taken proportional to link capacities. To explain relaxed FIFO, say that some link \( i \) has vehicles that wish to enter both links \( j \) and \( j' \). If link \( j' \) is jammed and cannot accept any more vehicles, a strict FIFO constraint would say that the vehicles in \( i \) that wish to enter \( j' \) will queue at \( i \)'s exit, and block the vehicles that wish to enter \( j \). In a multi-lane road, however, only certain lanes may queue, and traffic to \( j \) may still pass through other lanes. The relaxation is encoded in so-called “mutual restriction intervals” \( \eta_{i,j,j'} \subseteq [0,1] \).

This interval partly describes the overlapping regions of link \( i \)'s exit that serve both links \( j \) and \( j' \). For \( \eta_{i,j,j'} = [y,z] \), a \( z - y \) portion of \( i \)'s lanes that serve \( j \) also serve \( j' \), and will be blocked by the cars queueing to enter \( j' \) when \( j' \) is congested. For example, if \( j \) is served by three lanes of \( i \), and of those three lanes, the leftmost also serves \( j' \), we would have \( \eta_{i,j,j'} = [0,1/3] \).

As an example, we consider again node 1 in Figure 1. Say that the GP links (1 and 2) have four lanes, that the managed lane links (11 and 22) have two lanes, and that the onramp merges into and the offramp diverges from the rightmost GP lane. Further, we say that when the managed lane link is congested, vehicles in the GP lanes that wish to enter the managed lanes will queue only in the leftmost GP lane. On the other hand, when the GP link is congested, vehicles in the managed lanes that wish to enter the GP lanes will queue only in the rightmost managed lane. Finally, we suppose that jammed offramp (222) will cause vehicles to queue only in the rightmost GP lane. Taking together all of these statements, our mutual restriction intervals for this example are:

\[
\eta_{j,j'}^{1} = \begin{cases} 
  j' = 2 & j = 2 \\
  j' = 22 & j = 22 \\
  j' = 222 & j = 222 
\end{cases}
\]

\[
\eta_{j,j'}^{11} = \begin{cases} 
  j' = 2 & j = 2 \\
  j' = 22 & j = 22 \\
  j' = 222 & j = 222 
\end{cases}
\]

\[
\eta_{j,j'}^{111} = \begin{cases} 
  j' = 2 & j = 2 \\
  j' = 22 & j = 22 \\
  j' = 222 & j = 222 
\end{cases}
\]

To read the above tables, recall that as written, \( j' \) is the congested, restricting link, and \( j \) is the restricted link. These chosen restriction intervals allow for expected behavior in this network, such as a congested GP link causing possible queueing in the right managed lane (if some drivers are trying to enter the GP link), but no spillback into the left managed lane.\(^1\)

For a detailed discussion on how mutual restriction intervals are included in the node model’s flow calculations and solution algorithms, see Wright et al. (2016a) and Wright et al. (2016b).

3.3 Friction effect

The friction effect is an empirically-observed phenomenon in situations where managed lanes are relatively uncongested, but the managed-lane traffic will still slow down when the adjacent GP lanes congest and

\(^1\) In this example, we assume that the managed lane has two sublanes — left and right.
slow down (see Daganzo and Cassidy (2008), Liu et al. (2011), Cassidy et al. (2015), etc.). It has been hypothesized (Jang and Cassidy, 2012) that this phenomenon arises from the managed-lane drivers’ fear that slower-moving vehicles will suddenly and dangerously change into the managed lane ahead of them.

We suggest modeling the friction effect based on a feedback mechanism that uses the difference of speeds in the parallel GP and managed lane links to scale down the flow (and therefore the speed) out of the managed lane link if necessary.

To explain the concept, we again refer to Figure 1 and consider parallel links 1 (GP) and 11 (managed lane). Recall that, under a first-order model (2.2), the speed of traffic in link $l$ at time $t$ is

$$v_l(t) = \begin{cases} \frac{\sum_{c=1}^{C} \sum_{j=1}^{N} f_{cj}(t)}{\sum_{c=1}^{C} n_{cl}(t)} & \text{if } \sum_{c=1}^{C} n_{cl}(t) > 0, \\ v_f^l(t) & \text{otherwise}, \end{cases} \quad (3.1)$$

where $v_f^l(t)$ is the theoretical free flow speed of link $l$ at time $t$.

We say that the friction effect is present in managed lane link 11 (following the notation of Figure 1) at time $t$ if

$$v_1(t-1) < \min \left\{ v_f^1, v_{11}(t-1) \right\}, \quad (3.2)$$

which means that (1) the GP link is in congestion (its speed is below its current free flow speed), and (2) the speed in the GP link is less than the speed in the managed lane link. We denote this speed differential as:

$$\Delta_{11}(t) = v_f^{11} - v_1(t-1). \quad (3.3)$$

It has been observed (Jang et al., 2012) that the magnitude of the friction effect — the degree to which managed-lane drivers slow down towards the GP lane’s traffic speed — depends on the physical configuration of the road. For example, less of a friction effect will be present on managed lanes that are separated from the GP lanes by a buffer zone than those that are contiguous with the GP lanes (Jang et al., 2012), and the presence of a concrete barrier would practically eliminate the friction effect. Other factors that may affect this magnitude include, for example, whether there is more than one managed lane, or whether there is a shoulder lane to the left of the managed lane that drivers could swerve into if necessary.

To encode this variability in the magnitude of the friction effect in managed lane link 11, we introduce $\sigma_{11} \in [0,1]$ the friction coefficient of this link. The friction coefficient reflects the strength of the friction. Its value depends on the particular managed configuration and is chosen by the modeler. A value of $\sigma_{11} = 0$ means there is no friction (which may be appropriate if, perhaps, the managed lane(s) are separated from the GP lanes by a concrete barrier), and $\sigma_{11} = 1$ means that the managed lane link speed tracks the GP link speed exactly.

When the friction effect is active (i.e., when (3.2) is true), we adjust the fundamental diagram of the managed lane link by scaling down its theoretical free flow speed $v_f^l(t)$, and propagate that change through the rest of the fundamental diagram parameters. The exact mathematical changes will of course be different for every different form of fundamental diagram. For the particular fundamental diagram discussed in Appendix A, this means adjusting the free flow speed and capacity as follows:

$$\hat{v}_{11}(t) = v_f^{11}(t) - \sigma_{11} \Delta_{11}(t); \quad (3.4)$$

$$\hat{F}_{11}(t) = \hat{v}_{11}(t) n_{11}^+; \quad (3.5)$$

where $n_{11}^+$ is the high critical density (see Appendix A for its definition), and using these adjusted values in the calculation of the sending function (A.1),

$$S_{11}(t) = \hat{v}_{11}(t) n_{11}^c \min \left\{ 1, \frac{\hat{F}_{11}(t)}{\hat{v}_{11}(t) \sum_{c=1}^{C} n_{11}^c(t)} \right\}. \quad (3.6)$$
For the fundamental diagram of Appendix A, we must also check whether

$$\sum_{c=1}^{C} n_c^{\circ 11}(t) < \frac{\dot{F}_{11}(t)}{v_{11}(t) - \Delta_{11}(t)} = \frac{\dot{v}_{11}(t)n_{11}^{\circ}(t)}{v_{11}(t) - \Delta_{11}(t)}.$$  (3.7)

If not, then applying friction will lead to the managed lane link speed falling below the GP link speed, and the unadjusted sending function should be used (the possibility of this happening is due to the use of two critical densities to create Appendix A’s fundamental diagram’s “backwards lambda” shape).

Evidence of a friction effect reducing managed lane speed in a form that is linear in the speed differential (i.e., (3.4)) can be found in e.g., Jang et al. (2012). Again, the exact form of (3.6), the sending function with friction, will depend on the link’s original fundamental diagram model.

### 3.4 Separated managed lanes with gated access

A separated, gated-access managed lane configuration is presented in Figure 2. Unlike the full-access configuration, the GP and managed lane link chains do not necessarily meet at every node. Instead, they need only meet at a few locations, where vehicles can move into and out of the managed lane(s). Note that, unlike the full-access configuration, there is no need for GP and managed lane links to be aligned.

As labeled in Figure 2, the nodes where the two link chains meet are called **gates** (as an aside, one way to describe the full-access managed lane configuration would be that every node is a gate). Similar to our construction of excluding GP-only traffic from the managed lane in the full-access case, we can disable flow exchange at a given gate by fixing split ratios so that they keep traffic in their lanes. For example, to disable the gate (the flow exchange between the two lanes) at node 2 in Figure 1, we set $\beta_{c2,3} = 1$ and $\beta_{c22,33} = 1$ (this means that $\beta_{c233} = 0$ and $\beta_{c223} = 0$), $c = 1, 2$. Thus, the full-access managed lane can be easily converted into the separated managed lane by setting non-exchanging split ratios everywhere but designated gate-nodes.

In practice, a gate is stretch of freeway that may be a few hundreds of meters long (Cassidy et al., 2015), and, potentially, we can designate two or three sequential nodes as gates. In this paper, however, we model a gate as a single node.

For the gated-access configuration, we suggest setting mutual restriction coefficients in the same manner as full-access managed lanes, in Section 3.2.2.

Compared to the full-access managed lane configuration, the gated-access configuration has a smaller friction effect (Jang et al., 2012): drivers in the separated managed lane feel somewhat protected by the buffer, whether it is virtual (double solid line) or real (concrete), from vehicles changing abruptly from the slow moving GP lane and, therefore, do not drop speed as dramatically. The degree to which the friction effect is mitigated is disputed (e.g., see Footnote 3 in Cassidy et al. (2015)), but overall the bottlenecks created by the gates are much greater instigators of congestion (Cassidy et al., 2015). Inclusion of the friction effect in modeling separated managed lane configurations is thus not as essential as in modeling the full-access case.
3.4.1 Modeling a flow of vehicles from the managed lanes to the offramps

Recall from Section 3.2.1 that, in the full-access managed lane model, we can model vehicles moving from the managed lane link to offramps in a straightforward manner, by setting corresponding split ratios (for example, $\beta_{11,222}^0 = 1.2$, for node 1 in configuration from Figure 1). For the gated-access configuration, however, modeling traffic as moving from the managed lanes to offramps is more complicated: generally, gates do not coincide with offramp locations. In fact, there are typically between two and five offramps between two gate locations. We denote the offramps in the GP road segment connecting two gates as exits $e_1, e_2, \ldots, e_K$ (see Figure 2). These offramps cannot be accessed directly from the managed lane. Instead, vehicles traveling in the managed lane link that intend to take one of the exits $e_1, \ldots, e_K$, must switch from the managed lane link to the GP link at gate-node 1 and then be directed to the correct offramp. Creating a simulation where vehicles behave like this requires a more involved modeling construction.

To resolve this challenge, our gated-access model introduces new vehicle classes in addition to the $c = 1$ (GP-only) and $c = 2$ (special) traffic used in the full-access model of Section 3.2. These additional classes will be used to distinguish subsets of the special traffic population by its destination offramp. If $K$ is the largest number of offramps between two adjacent gates, then altogether we have $C = K + 2$ vehicle classes: $c = 1, 2, e_1, \ldots, e_K$, where $e_k$ indicates the class of vehicles that will exit through the $k$-th offramp after leaving the managed lane link through the gate. By definition, traffic of type $c = e_k$ may exist in the GP lane segment between gate 1 and offramp $e_k$, but there is no traffic of this type either in the GP link segment between offramp $e_k$ and gate 2 or in the managed lane link. This movement pattern is ensured by setting constant split ratios:

$$
\begin{align}
\beta_{x_1}^{c_k} &= 1, \quad i = 1, 11, 111, \quad \text{direct all } e_k\text{-type traffic to the GP link at gate 1;} \\
\beta_{x_k}^{c_k} &= 1, \quad \text{direct all } e_k\text{-type traffic to offramp } e_k; \\
\beta_{x_k,e_k'}^{c_k} &= 0, \quad k' \neq k, \quad \text{do not send any } e_k\text{-type traffic to other offramps},
\end{align}
$$

where $k = 1, \ldots, K$, and $x_k$ denotes the input GP link for the node that has the output link $e_k$ (see Figure 2).

Vehicles of class $c = e_k$ do not enter the network via onramps or the upstream boundary, but instead are converted from special $c = 2$ traffic as it leaves the managed lane(s) through the gate. We perform this conversion as part of the link model computation, such that the total demand for the switching link, $\sum_{c=1}^C S_t^c$, remains the same before and after the switch. The exact link on which this switching takes place is the managed lane link immediately upstream of the gate (e.g., link 11 in Figure 2).

We say that the amount of traffic that should change from vehicle class $c = 2$ to vehicle class $e_k$ at time $t$ is (using link 11 as an example) $n_{11}^c(t)\beta_{x_k}^{c_k}(t)v_{11}(t)$, where $v_{11}(t)$ is in units of vehicles per simulation timestep (this factor is included so that the switching done is proportional to link 11’s outflow, rather than its density), and $\beta_{x_k}^{c_k}(t)$ is the split ratio from $x_k$, the GP link immediately upstream of exit $e_k$, and the exit $e_k$ at time $t$. This statement is based on the assumption that the vehicles in the managed lane link will exit the freeway through exit $e_k$ at the same rate as special ($c = 2$) vehicles that happened to stay in the GP lanes. That is, if a $\beta_{x_k}^{c_k}$ portion of $c = 2$ vehicles intend to leave the GP lanes through exit $e_k$, then a $\beta_{x_k}^{c_k}$ portion of the $c = 2$ vehicles in the managed lane(s) will leave the managed lane link at the closest upstream gate and leave the network at exit $e_k$ when they reach it. Note that if there are $K' < K$ offramps between two particular gates, then no vehicles should switch to type $c = e_k$, $k \in \{K' + 1, \ldots, K\}$ at the upstream gate, as they would have no ramp to exit through.

Using Figure 2 as a reference, we can now formally describe the procedure for destination assignment to traffic in the managed lane link.

1. Given are vehicle counts per commodity $n_{11}^c$, $c = 1, 2, e_1, \ldots, e_K$; free flow speed $v_{11}$; and offramp split ratios $\beta_{x_k}^{c_k}$ and $\beta_{x_k,e_k}^{c_k}$, $k = 1, \ldots, K$.\(^2\)

\(^2\)If a given GP segment connecting two adjacent gates has $K'$ offramps, where $K' < K$, then assume $\beta_{x_k}^{c_k} = \beta_{x_k,e_k}^{c_k} = 0$ for $k \in \{K', K\}$.
2. Initialize:
\[
\hat{n}_{11}^c(0) := n_{11}^c, \quad c = 1, 2, e_1, \ldots, e_K; \\
k := 1.
\]

3. Assign \(e_k\)-type traffic:
\[
\hat{n}_{11}^c(k) = \hat{n}_{11}^c(k-1) + \beta_{x_k,e_k}^1 v_{11} \hat{n}_{11}^c(k-1) + \beta_{x_k,e_k}^2 v_{11} \hat{n}_{11}^c(k-1); \quad (3.9)
\]
\[
\hat{n}_{11}^c(k) = \hat{n}_{11}^c(k-1) - \beta_{x_k,e_k}^1 v_{11} \hat{n}_{11}^c(k-1); \quad (3.10)
\]
\[
\hat{n}_{11}^c(k) = \hat{n}_{11}^c(k-1) - \beta_{x_k,e_k}^2 v_{11} \hat{n}_{11}^c(k-1). \quad (3.11)
\]

4. If \(k < K\), then set \(k := k + 1\) and return to step 3.

5. Update the state:
\[
n_{11}^c = \hat{n}_{11}^c(K), \quad c = 1, 2, e_1, \ldots, e_K.
\]

After the switches to \(c = e_k\) class traffic have been done, there may be unresolved split ratios for both classes \(c = 1\) and \(c = 2\) at the gates (similar to the dashed split ratios in the tables in Section 3.2.1). These split ratios should be filled in with the same tools as those in Section 3.2.1: some sort of driver lane choice behavior.

### 3.5 Inertia effect

At the end of the previous Section, we mentioned that some split ratios will likely be undefined after switching \(c = 2\) class vehicles to the \(c = e_k\) classes. In particular, \(c = 2\) class vehicles in the upstream GP link (e.g., link 1 in Figure 2) and remaining \(c = 2\) vehicles in the upstream managed lane link (e.g. link 11 in Figure 2) will need to decide whether to pass through the gate or remain in their current lane. Sample tools for modeling driver lane choices such as these include the class of “logit” logistic regression models (McFadden, 1973), or dynamic split ratio solvers such as the one presented in Wright et al. (2016a) and reviewed in Appendix B.

When applied to lane choice (e.g., as in Farhi et al. (2013)), logit models produce a set of portions in \([0, 1]\), one for each lane, that sum to one. Each lane’s value is the equilibrium portion of vehicles that will travel on that lane. In a dynamic simulation context, such as considered in this article, the differences between the logit model’s equilibrium portions and the current distribution of vehicles across lanes at time \(t\) are used to select split ratios at time \(t\) such that the actual distribution approaches the logit equilibrium. We argue, though, that use of this sort of split ratio solver in unmodified form might be inappropriate for computing gate split ratios in the gated-access managed lane configuration. Unmodified, a value function for either the GP or managed lane link might include terms such as the link’s speed of traffic, density, etc. However, often a gated-access managed lane is separated from the GP lanes by some buffer zone or visibility-obstructing barrier that makes switching between the two links more hazardous than switching between two contiguous lanes. Therefore, if one uses a logit-based model, it would be appropriate to modify the logit model’s value function such that staying in the current link (e.g., the movements (1,2) and (11,22) in Figure 3) has some positive value for drivers, and the gains (e.g., in travel time) for ingress/egress movements (e.g., (1,22) and (11,2) in Figure 3) must be of more value than the staying-in-the-lane value. We refer to this model as the inertia effect.

We may also incorporate the inertia effect into dynamic split ratio solvers, such as the one introduced in Wright et al. (2016a) and reviewed in Appendix B. At time \(t\), this particular algorithm selects split ratios in an attempt to balance the density ratio at the next timestep \(t+1\), \[ \sum_c n_{11}^c(t+1)/n_1^f, \] where \(n_1^f\) is the jam
density, or the maximum number of vehicles that link \( l \) can hold. For example, if applied to the node in Figure 3, this algorithm would attempt to make \( \sum_c n_{ij}^c(t+1)/n_{ij}^l \) and \( \sum_c n_{22}^c(t+1)/n_{22}^l \) as equal as possible. For more details, see Appendix B and Wright et al. (2016a).

Here, we modify several steps of the solver so that this equality-seeking goal is balanced with a goal towards enforcing the inertia effect. We illustrate these changes with the particular example of the node in Figure 3. Ensuring that the split ratio assignment algorithm gives preferences to movements \((1,2)\) and \((11,22)\) over enforcing the inertia effect. We illustrate these changes with the particular example of the node in Figure 3. Here, we modify several steps of the solver so that this equality-seeking goal is balanced with a goal towards the inertia effect.

For each input link \( i \), we modify (B.4). For this particular example, the original formula gives us:

\[
\begin{align*}
\gamma_{i,1}(k) &= \tilde{\beta}_{i,1}^c(k) + \frac{\bar{\gamma}_i(k)}{2} + \frac{\gamma_{i,2}(k)}{2}, \\
\gamma_{i,2}(k) &= \tilde{\beta}_{i,2}^c(k) + \frac{\bar{\gamma}_i(k)}{2} + \frac{\gamma_{i,2}(k)}{2}.
\end{align*}
\]

Since for \( k = 0 \) \( \tilde{\beta}_{i,j}(0) = 0 \), \( i = 1, 11, j = 2, 22 \), we get \( \gamma_{i,1}(0) = \gamma_{i,2}(0) = \frac{\bar{\gamma}_i(k)}{2} \) and \( \gamma_{i,12}(0) = \gamma_{i,22}(0) = \frac{\bar{\gamma}_i(k)}{2} \), which, according to (B.3), yields \( \tilde{p}_{i,12}(0) = \tilde{p}_{i,22}(0) \) and \( \tilde{p}_{11,22}(0) = \tilde{p}_{11,22}(0) \).

For each input link \( i \) that forms one lane with an output link \( j \) and class \( c \), such that \( j \in V^c_i \), (B.4) can be modified as follows:

\[
\gamma^c_{ij}(k) = \begin{cases} 
\beta^c_{ij}(k) + \bar{\gamma}_i(k)\lambda^c_i, & \text{if split ratio is defined a priori: } \{i,j,c\} \in \mathcal{B}, \\
\tilde{\beta}^c_{ij}(k) + \bar{\gamma}_i(k)\frac{k}{1-\lambda^c_i}, & \text{if } i \text{ and } j \text{ form one lane: } j = j, \\
\tilde{\beta}^c_{ij}(k) + \bar{\gamma}_i(k)\frac{k}{1-\lambda^c_i}, & \text{if } i \text{ and } j \text{ are in different lanes: } j \neq j,
\end{cases}
\]  

(3.12)

where the parameter \( \lambda^c_i \in \left[ \frac{1}{\vert V^c_i \vert}, 1 \right] \) is called the inertia coefficient, and indicates how strong the inertia effect is. With \( \lambda^c_i = \frac{1}{\vert V^c_i \vert} \), (3.12) reduces to the original formula, (B.4). With \( \lambda^c_i = 1 \), all the a priori unassigned traffic from link \( i \) must stay in its lane — be directed to output link \( j \). The choice of \( \lambda^c_i \) lies with the modeler.

In the case example from Figure 3, the modified formula (3.12) yields:

\[
\begin{align*}
\gamma^c_{i,1}(k) &= \beta^c_{i,1}(k) + \bar{\gamma}_i(k)\lambda^c_i, \\
\gamma^c_{i,2}(k) &= \beta^c_{i,2}(k) + \bar{\gamma}_i(k)\lambda^c_i, \\
\gamma^c_{i,12}(k) &= \beta^c_{i,12}(k) + \bar{\gamma}_i(k)(1 - \lambda^c_i), \\
\gamma^c_{i,22}(k) &= \beta^c_{i,22}(k) + \bar{\gamma}_i(k)(1 - \lambda^c_i).
\end{align*}
\]  

(3.13)

where \( \lambda_{i1}, \lambda_{11} \in \left[ \frac{1}{2}, 1 \right] \), and picking \( \lambda_1 > \frac{1}{2} \) \( (\lambda_{11} > \frac{1}{2}) \) would give preference to movement \((1,2)\) over \((1,22)\) and \((11,22)\) over \((11,2)\). The way of choosing \( \lambda^c_i \) for multiple input links is not obvious and an arbitrary choice may result in an unbalanced flow distribution among output links. Therefore, we suggest picking just one input-output pair \((i,j)\), and for that input link setting \( \lambda^c_i = 1 \), while for other input links \( i \) setting \( \lambda^c_i = \frac{1}{\vert V^c_i \vert}, c = 1, \ldots, C \). The input link \( i \) must be from the lane that is expected to have a positive net inflow of vehicles as a result of the split ratio assignment and flows computed by the node model. So,

\[
i = \arg \min_{i \in \mathcal{U}} \frac{\sum_{c \in \{i,j,c\} \in \mathcal{B}} \tilde{S}^c_i + \sum_{i \in \{i,j,c\} \in \mathcal{B}} \sum_{c \in \{i,j,c\} \in \mathcal{B}} \beta^c_{ij} S^c_i}{R_j}.
\]  

(3.14)
where
\[ \hat{U} = \{ \text{input links } i : \exists j, c, \text{ s.t. } j \in V_i^c \text{ and the pair of links } (i, j) \text{ belongs to the same lane} \} \]
and \( j \) denotes the output link that is in the same lane as input link \( i \).

For the particular example node in Figure 3, we need to determine whether flow from link 1 will proceed to link 2 or flow from link 11 to link 22, while other \textit{a priori} undefined split ratios will be computed according to the split ratio assignment algorithm. If \( i = 1 \), then \( \lambda_{11} = 1, \lambda_1 = \frac{1}{2} \), and \textit{a priori} unassigned traffic in the managed lane will stay in the managed lane (\( \beta_{1,12} = 0 \)), while \textit{a priori} unassigned traffic coming from links 1 and 111, will be distributed between links 2 and 22 according to the dynamic split ratio solver.

On the other hand, if \( i = 1 \), then \( \lambda_1 = 1, \lambda_{11} = \frac{1}{2} \), and \textit{a priori} unassigned traffic in the GP lane will stay in the GP lane (\( \beta_{1,22} = 0 \)), while \textit{a priori} unassigned traffic coming from links 11 and 111, will be distributed between links 2 and 22 according to the dynamic split ratio solver.

### 4 Putting Together the Pieces: A Managed Lane-Freeway Network Simulation Model

In this Section, we present a unified simulation algorithm for managed lane-freeway networks. This can be considered a fleshing-out of the simplified first-order macroscopic simulation method briefly outlined in Section 2.1, with extensions made by incorporating the additional items we have described in the sections between that and this one.

#### 4.1 Definitions

- We have a network consisting of set of links \( \mathcal{L} \) and a set of nodes \( \mathcal{N} \).
  - A node always has at least one incoming link and one outgoing link.
  - A link may have an upstream node, a downstream node, or both.
- We have \( C \) different vehicle classes traveling in the network, with classes indexed by \( c \in \{1, \ldots, C\} \).
- Let \( t \in \{0, \ldots, T\} \) denote the simulation timestep.
- In addition, while we have not covered them here, the modeler may optionally choose to define control inputs to the simulated system that modify system parameters or the system state. Such control inputs may represent operational traffic control schemes such as ramp metering, changeable message signs, a variable managed-lane policy, etc. In the context of this paper, we suggest including the parameterization of the friction effect and the class-switching construction of the separated-access managed lane model as control actions.

#### 4.1.1 Link model definitions

- For each link \( l \in \mathcal{L} \), let there be a time-varying \( C \)-dimensional state vector \( \mathbf{n}_l(t) \), which denotes the density of the link of each of the \( C \) vehicle classes at time \( t \).
  - Each element of this vector, \( n^c_l(t) \), updates between timestep \( t \) and timestep \( t + 1 \) according to (4.1).
  - Also define \( n^c_{l,0} \) for all \( l, c \), the initial condition of the system.
• We define three types of links:
  – Ordinary links are those links that have both beginning and ending nodes.
  – Origin links are those links that have only an ending node. These links represent the roads that vehicles use to enter the network.
  – Destination links are those links that have only a beginning node. These links represent the roads that vehicles use to exit the network.

• For each link $l \in L$, define a “link model” that computes the per-class demands $S^c_l(t)$ and link supply $R_l(t)$ as a function of $t$ and $\vec{n}_l(t)$. Appendix A describes a particular example link model that will be used in the example simulations in Section 6.

• For each origin link $l \in L$, define a $C$-dimensional time-varying vector, $\vec{d}_l(t)$, where the $c$-th element $d^c_l(t)$ denotes the exogenous demand of class $c$ into the network at link $l$.

4.1.2 Node model definitions

• For each node $\nu \in \mathcal{N}$, let $i \in \{1, \ldots, M_\nu\}$ denote the incoming links and $j \in \{1, \ldots, N_\nu\}$ denote the outgoing links.

• For each node, define $\{\beta^c_{ij}(t) : \sum_j \beta^c_{ij}(t) = 1 \forall i, c\}$ the time-varying split ratios for each triplet $\{i, j, c\}$. Each split ratio may be fully defined, partially defined, or fully undefined. If some split ratios for a node are undefined, then also define for that node some split ratio solver (e.g., logit or dynamic) to fill in undefined split ratios $\beta^c_{ij}(t)$ at time $t$.

• For each node, define a “node model” that, at each time $t$, takes its incoming links’ demands $S^c_i(t)$ and split ratios $\beta^c_{ij}(t)$, its outgoing links’ supplies $R_j(t)$, and other nodal parameters, and computes the flows $f^c_{ij}(t)$. In this paper, we refer to a specific node model with a relaxed first-in-first-out (FIFO) construction that has additional parameters $\eta^t_{ij}(t)$ (mutual restriction intervals) and $p_i(t)$ (the incoming links’ priorities).

4.1.3 State update equation definitions

• All links $l \in L$ update their states according to the equation

$$n^c_l(t + 1) = n^c_l(t) + \frac{1}{L_l} \left( f^c_{l,\text{in}}(t) - f^c_{l,\text{out}}(t) \right) \quad \forall c \in \{1, \ldots, C\},$$

which is a slightly generalized form of (2.2).

• For all ordinary and destination links,

$$f^c_{l,\text{in}}(t) = \sum_{i=1}^{M_\nu} f^c_{i\nu}(t),$$

where $\nu$ is the beginning node of link $l$.

• For all origin links,

$$f^c_{l,\text{in}}(t) = d^c_l(t).$$

• For all ordinary and origin links,

$$f^c_{l,\text{out}}(t) = \sum_{j=1}^{N_\nu} f^c_{lj}(t),$$

where $\nu$ is the ending node of link $l$. 

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For all destination links,
\[ f^l_{\text{out}}(t) = S^l(t). \] (4.5)

4.2 Simulation algorithm

1. Initialization:
\[
\begin{align*}
n^c_l(0) &:= n^c_{l,0} \\
t &:= 0,
\end{align*}
\]
for all \( l \in L, c \in \{1, \ldots, C\} \).

2. Perform all control inputs that have been (optionally) specified by the modeler. For our purposes, this includes:
   (a) For each managed lane link, modify the sending function of the link model in accordance with the friction effect model. Recall that, for the particular link model of Appendix A, we use (3.1)-(3.7).
   (b) For each managed lane link whose downstream node is a gate node in a gated-access managed lane configuration, perform class switching as detailed in Section 3.4.1

3. For each link \( l \in L \) and commodity \( c \in \{1, \ldots, C\} \), compute the demand, \( S^c_l(t) \) using the link’s link model.

4. For each ordinary and destination link \( l \in L \), compute the supply \( R^l_l(t) \) using the link model. For origin links, the supply is not used.

5. For each node \( \nu \in N \) that has one or more undefined split ratios \( \beta^c_{ij}(t) \), use the node’s split ratio solver to complete a fully-defined set of split ratios. Note that if an inertia effect model is being used, the modified split ratio solver, e.g. the one described in Section 3.5, should be used where appropriate.

6. For each node \( \nu \in N \), use the node model to compute throughflows \( f^c_{ij}(t) \) for all \( i, j, c \).

7. For every link \( l \in L \), compute the updated state \( n^l_i(t + 1) \):
   - If \( l \) is an ordinary link, use (4.1), (4.2), and (4.4).
   - If \( l \) is an origin link, use (4.1), (4.3), and (4.4)
   - If \( l \) is a destination link, use (4.1), (4.2), and (4.5).

8. If \( t = T \), then stop. Otherwise, increment \( t := t + 1 \) and return to step 2.

5 Calibrating the Managed Lane-Freeway Network Model

Typically, a traffic modeler will have some set of data collected from traffic detectors (e.g., velocity and flow readings), and will create a network topology with parameter values that allow the model to reproduce these values in simulation. Then, the parameters can be tweaked to perform prediction and analysis. For our managed lane-freeway networks, the parameters of interest are:

1. Fundamental diagram parameters for each link. Calibration of a fundamental diagram is typically agnostic to the node model and network topology, and there exists an abundant literature on this topic. For the purposes of the simulations in the following Section, we used the method of Dervisoglu et al. (2009), but any other method is appropriate.
2. Percentage of special (that is, able to access the managed lane) vehicles in the traffic flow entering the system. This parameter depends on, e.g., the time of day and location as well as on the type of managed lane. It could be roughly estimated as a ratio of the managed lane vehicle count to the total freeway vehicle count during periods of congestion at any given location.

3. Inertia coefficients. These parameters affect only how traffic of different classes mixes in different links, but they have no effect on the total vehicle counts produced by the simulation.

4. Friction coefficients. How to tune these parameters is an open question. In Jang and Cassidy (2012) the dependency of a managed lane’s speed on the GP lane speed was investigated under different densities of the managed lane, and the presented data suggests that although the correlation between the two speeds exists, it is not overwhelmingly strong, below 0.4. Therefore, we suggest setting friction coefficients to values not exceeding 0.4.

5. Mutual restriction intervals. It is also an open question how to estimate mutual restriction intervals from the measurement data. See the discussion in Section 3.2.2 for some guidelines.

6. Offramp split ratios.

Calibrating a traffic model, or identifying the best values of its parameters to match real-world data, is typically an involved process for all but the simplest network topologies. In particular, once we consider more than a single, unbroken stretch of freeway, the nonlinear nature and network effects of these systems mean that estimating each parameter in isolation might lead to unpredictable behavior. Instead, nonlinear and/or non-convex optimization techniques such as genetic algorithms (Poole and Kotsialos, 2012), particle swarm methods (Poole and Kotsialos, 2016), and others (Ngoduy and Maher (2012); Fransson and Sandin (2012), etc.) are employed.

In the managed lane-freeway networks we have discussed, the key unknown parameters we have introduced are the offramp split ratios, item 6, which may be particularly hard to estimate as they are typically time-varying and explicitly represent driver behavior, rather than physical parameters of the road. The other items are not too difficult to identify using methods from the literature. The remainder of this section describes iterative methods for identification of the offramp split ratios for both the full-access and gated-access configurations.

5.1 Split ratios for a full-access managed lane

Consider a node, one of whose output links is an offramp, as depicted in Figure 3. We shall make the following assumptions.

1. The total flow entering the offramp, $\hat{f}_{222}^{in}$, at any given time is known (from measurements) and is not restricted by the offramp supply: $\hat{f}_{222}^{in} < R_{222}$.

2. The portions of traffic sent to the offramp from the managed lane and from the GP lane at any given time are equal: $\beta_{1,222}^c = \beta_{11,222}^c \triangleq \beta, c = 1, \ldots, C$.

3. None of the flow coming from the onramp (link 111), if such flow exists, is directed toward the offramp. In other words, $\beta_{111,222}^c = 0, c = 1, \ldots, C$.

4. The distribution of flow portions not directed to the offramp between the managed lane and the GP output links is known. This can be written as: $\beta_{ij}^c = (1 - \beta)\delta_{ij}^c$, where $\delta_{ij}^c \in [0, 1]$, as well as $\beta_{111,j}$, $i = 1, 11$, $j = 2, 22$, $c = 1, \ldots, C$, are known.

5. The demand $S_i^c, i = 1, 11, 111, c = 1, \ldots, C$, and supply $R_j, j = 2, 22$, are given.
At any given time, β is unknown and is to be found.

If β were known, the node model would compute the input-output flows, in particular, \( f_{i,222} = \sum_{c=1}^{C} f_{i,c}^{c} \), \( i = 1,11 \). Define
\[
\psi(\beta) = f_{1,222} + f_{11,222} - \hat{f}_{222}^{\text{in}}.
\]

Our goal is to find β from the equation
\[
\psi(\beta) = 0,
\]
such that \( \beta \in \left[ \frac{\hat{f}_{222}^{\text{in}}}{S_{1} + S_{11}}, 1 \right] \), where \( S_{i} = \sum_{c=1}^{C} S_{i}^{c} \). Obviously, if \( S_{1} + S_{11} < \hat{f}_{222}^{\text{in}} \), the solution does not exist, and the best we can do in this case to match \( \hat{f}_{222}^{\text{in}} \) is to set \( \beta = 1 \), directing all traffic from links 1 and 11 to the offramp.

Suppose now that \( S_{1} + S_{11} \geq \hat{f}_{222}^{\text{in}} \). For any given \( \hat{f}_{222}^{\text{in}} \), we assume \( \psi(\beta) \) is a monotonically increasing function of \( \beta \) (this assumption is true for the particular node model of Wright et al. (2016a)). Moreover, \( \psi \left( \frac{\hat{f}_{222}^{\text{in}}}{S_{1} + S_{11}} \right) \leq 0 \), while \( \psi(1) \geq 0 \). Thus, the solution of (5.2) within the given interval exists and can be obtained using the bisection method.

The algorithm for finding β follows.

1. Initialize:
   \[
   \underline{b}(0) := \frac{\hat{f}_{222}^{\text{in}}}{S_{1} + S_{11}}; \quad \bar{b}(0) := 1; \quad k := 0.
   \]
2. If \( S_{1} + S_{11} \leq \hat{f}_{222}^{\text{in}} \), then are not enough vehicles to satisfy the offramp demand. Set \( \beta = 1 \) and stop.
3. Use the node model with \( \beta = \underline{b}(0) \) and evaluate \( \psi(\beta) \). If \( \psi(\underline{b}(0)) \geq 0 \), then set \( \beta = \underline{b}(0) \) and stop.
4. Use the node model with \( \beta = \frac{\underline{b}(k) + \bar{b}(k)}{2} \) and evaluate \( \psi(\beta) \). If \( \psi \left( \frac{\underline{b}(k) + \bar{b}(k)}{2} \right) = 0 \), then set \( \beta = \frac{\underline{b}(k) + \bar{b}(k)}{2} \) and stop.
5. If \( \psi \left( \frac{\underline{b}(k) + \bar{b}(k)}{2} \right) < 0 \), then update:
   \[
   \underline{b}(k+1) = \frac{\underline{b}(k) + \bar{b}(k)}{2}; \quad \bar{b}(k+1) = \bar{b}(k).
   \]
   Else, update:
   \[
   \underline{b}(k+1) = \underline{b}(k); \quad \bar{b}(k+1) = \frac{\underline{b}(k) + \bar{b}(k)}{2}.
   \]
6. Set \( k := k + 1 \) and return to step 4.

Here, \( \underline{b}(k) \) represents the lower bound of the search interval at iteration \( k \) and \( \bar{b}(k) \) the upper bound.
5.2 Split ratios for the separated managed lane with gated access

The configuration of a node with an offramp as one of the output links is simpler in the case of a separated HOV lane, as shown in Figure 4. Here, traffic cannot directly go from the HOV lane to link 222, and, thus, we have to deal only with the 2-input-2-output node. There is a caveat, however. Recall from Section 3.4 that in the separated managed lane case we have destination-based traffic classes, and split ratios for destination-based traffic are fixed.

![Diagram](image)

**Figure 4:** A node with a GP link and an onramp as inputs, and a GP link and an offramp as outputs.

We shall make the following assumptions:

1. The total flow entering the offramp, \( \hat{f}_{in}^{222} \), at any given time is known (from measurements) and is not restricted by the offramp supply: \( \hat{f}_{in}^{222} < R_{222} \).
2. All the flow coming from the onramp (link 111), if such flow exists, is directed toward the GP link 2. In other words, \( \beta_{111,2} = 1 \) and \( \beta_{111,222} = 0 \), \( c = 1, \ldots, C \).
3. The demand \( S_i^c \), \( i = 1, 111 \), \( c = 1, \ldots, C \), and supply \( R_2 \) are given.
4. We denote the set of destination-based classes as \( D \). The split ratios \( \beta_{ij}^c \) for \( c \in D \) are known. Let the split ratios \( \beta_{ij}^c = \beta \) for \( c \in \{1, \ldots, C\} \setminus D \), where \( \beta \) is to be determined (i.e., we assume all non-destination-based classes exit at the same rate).

The first three assumptions here reproduce assumptions 1, 3 and 5 made for the full-access managed lane case. Assumption 4 is a reminder that there is a portion of traffic flow that we cannot direct to or away from the offramp, but we have to account for it.

Similarly to the full-access managed lane case, we define the function \( \psi(\beta) \):

\[
\psi(\beta) = \sum_{c \in D} f_{i,222}^c + \sum_{c \in D} f_{i,222}^c - \hat{f}_{in}^{222},
\]

where \( f_{i,222}^c \), \( c = 1, \ldots, C \) are determined by the node model. The first term of the right-hand side of (5.3) depends on \( \beta \). As before, we assume \( \psi(\beta) \) is a monotonically increasing function. We look for the solution of equation (5.2) on the interval \([0, 1]\). This solution exists iff \( \psi(0) \leq 0 \) and \( \psi(1) \geq 0 \). The algorithm for finding \( \beta \) is the same as the one presented in the previous section, except that \( b(0) \) should be initialized to 0, and \( S_{11} \) is to be assumed 0.

5.3 An iterative full calibration process

For the purposes of the simulations presented in the following Section, we placed the iterative split ratio identification methods of Sections 5.1 and 5.2 within a larger iterative loop for the remaining parameters. The model calibration follows the flowchart shown in Figure 5.
1. We start by assembling the available measurement data. Fundamental diagrams are assumed to be given. Mainline and onramp demand are specified per 5-minute periods together with the special vehicle portion parameter indicating the fraction of the input demand that is able to access the managed lane. Initially, we do not know offramp split ratios as they cannot be measured directly. Instead, we use some arbitrary values to represent them and call these values “initially guessed offramp split ratios”. Instead of the offramp split ratios, we have the flows directed to offramps, to which we refer to as offramp demand.

2. We run our network simulation outlined in Section 4.2 for the entire simulation period. At this point, in step 5 of the simulation, the a priori undefined split ratios between traffic in the GP and in the managed lanes are assigned using a split ratio solver.

3. Using these newly-assigned split ratios, we run our network simulation again, only this time, instead of using the initially guessed offramp split ratios, we compute them from the given offramp demand as described in Sections 5.1 and 5.2. As a result of this step, we obtain new offramp split ratios.

4. Now we run the network simulation as we did originally, in step 2, only this time with new offramp split ratios, and record the simulation results — density, flow, speed, as well as performance measures such as vehicle miles traveled (VMT) and vehicle hours traveled (VHT).

5. Check if the resulting offramp flows match the offramp demand. If yes, proceed to step 6, otherwise, repeat steps 2-5. In our experience (i.e., the case studies in the following Section), it takes the process described in steps 2-5 no more than two iterations to converge.
6. Evaluate the simulation results:
   • correctness of bottleneck locations and activation times;
   • correctness of congestion extension at each bottleneck;
   • correctness of VMT and VHT.

If the simulation results are satisfactory, stop. Otherwise, proceed to step 7.

7. Tune/correct input data in the order shown in block 7 of Figure 5.

6 Simulation Results

6.1 Full-access managed lane case study: Interstate 680 North

We consider a 26.8-mile stretch of I-680 North freeway in Contra Costa County, California, from postmile 30 to postmile 56.8, shown in Figure 6, as a test case for the full-access managed lane configuration. This freeway’s managed lane is a high-occupancy-vehicle (HOV) lane, which allows entry to vehicles with two or more passengers. This stretch contains two HOV lane segments whose beginning and ending points are marked on the map. The first HOV segment is 12.3 miles long and will be converted to HOT in spring 2017 (Metropolitan Transportation Commission), and the second HOV segment is 4.5 miles long. There are 26 onramps and 24 offramps. The HOV lane is active from 5 to 9 AM and from 3 to 7 PM. The rest of the time, the HOV lane is open to all traffic, and behaves as a GP lane.

To build the model, we used data collected for the I-680 Corridor System Management Plan (CSMP) study (System Metrics Group, Inc., 2015). The bottleneck locations as well as their activation times and congestion extension were identified in that study using video monitoring and tachometer vehicle runs. On- and offramp flows were given in 5-minute increments. Here, we assume that the HOV portion of the input demand is 15%. The model was calibrated to a typical weekday, as suggested in the I-680 CSMP study.

For this simulation, we used the fundamental diagram described in Appendix A, with parameters as follows:
• The capacity of the ordinary GP lane is 1,900 vehicles per hour per lane (vphl);
• The capacity of the auxiliary GP lane is 1,900 vphl;
• The capacity of HOV lane is 1,800 vphl while active and 1,900 vphl when it behaves as a GP lane;
• The free flow speed varies between 63 and 70 mph — these measurements came partially from the California Performance Measurement System (PeMS) (California Department of Transportation, 2016) and partially from tachometer vehicle runs.
• The congestion wave speed for each link was taken as 1/5 of the free flow speed.

The modeling results are presented in Figures 7, 8 and 9 showing density, flow and speed contours, respectively, in the GP and the HOV lanes. In each plot, the top contour corresponds to the HOV lanes, and the bottom to the GP lanes. In all the plots traffic moves from left to right along the “Absolute Postmile” axis, while the vertical axis represents time. Bottleneck locations and congestion areas identified by the I-680 CSMP study are marked by blue boxes in GP lane contours. The HOV lane does not get congested, but there is a speed drop due to the friction effect. The friction effect, when vehicles in the HOV lane slow down because of the slow moving GP lane traffic, can be seen in the HOV lane speed contour in Figure 9.

Figure 10 shows an example of how well the offramp flow computed by the simulation matches the target, referred to as offramp demand, as recorded by the detector on the offramp at Crow Canyon Road. We can see that in the beginning and in the end of the day, the computed flow falls below the target (corresponding areas are marked with red circles). This is due to the shortage of the mainline traffic in the simulation — the offramp demand cannot be satisfied.

Finally, Table 1 summarizes the performance measurements — vehicle miles traveled (VMT), vehicle hours traveled (VHT) and delay in vehicle-hours — computed by simulation versus those collected in the course of the I-680 CSMP study. Delay is computed for vehicles with speed below 45 mph.

|                      | Simulation result | Collected data |
|----------------------|-------------------|----------------|
| GP Lane VMT         | 1,687,618         | -              |
| HOV Lane VMT        | 206,532           | -              |
| Total VMT           | 1,894,150         | 1,888,885      |
| GP Lane VHT         | 27,732            | -              |
| HOV Lane VHT        | 3,051             | -              |
| Total VHT           | 30,783            | 31,008         |
| GP Lane Delay       | 2,785             | -              |
| HOV Lane Delay      | 6                 | -              |
| Total Delay         | 2,791             | 2,904          |

Table 1: Performance measures for I-680 North.
Figure 7: I-680 North density contours for GP and HOV lanes produced by simulation. Density values are given in vehicles per mile per lane. Blue boxes on the GP lane speed contour indicate congested areas as identified by the I-680 CSMP study.
Figure 8: I-680 North flow contours for GP and HOV lanes produced by simulation. Flow values are given in vehicles per hour per lane. Blue boxes on the GP lane speed contour indicate congested areas as identified by the I-680 CSMP study.
Figure 9: I-680 North speed contours for GP and HOV lanes produced by simulation. Speed values are given in miles per hour. Blue boxes on the GP lane speed contour indicate congested areas as identified by the I-680 CSMP study.
Figure 10: Flow at the Crow Canyon Road offramp over 24 hours — collected (offramp demand) vs. computed by simulation (offramp flow).
6.2 Gated-access managed lane case study: Interstate 210 East

We consider a 20.6-mile stretch of SR-134 East/I-210 East in Los Angeles County, California, shown in Figure 11, as a test case for the separated managed lane configuration. This freeway’s managed lane is also an HOV lane. This freeway stretch consists of 3.9 miles of SR-134 East from postmile 9.46 to postmile 13.36, which merges into 16.7 miles of I-210 East from postmile 25 to postmile 41.7. Gate locations where traffic can switch between the GP and the HOV lanes are marked on the map. At this site, the HOV lane is always active. There are 28 onramps and 25 offramps. The largest number of offramps between two gates is 5. Thus, our freeway model has 7 vehicle classes - LOV, HOV and 5 destination-based.

To build the model, we used PeMS data for the corresponding segments of the SR-134 East and I-210 East for Monday, October 13, 2014 (California Department of Transportation, 2016). Fundamental diagrams were calibrated using PeMS data following the methodology of Dervisoglu et al. (2009). As in the I-680 North example, we assume that HOV portion of the input demand is 15%.

The modeling results are presented in Figures 12, 13 and 14 showing density, flow and speed contours, respectively, in the GP and the HOV lanes. In each plot, the top contour corresponds to the HOV lanes, and the bottom to the GP lanes. As before, in all the plots traffic moves from left to right along the “Absolute Postmile” axis, while the vertical axis represents time. The HOV lane does not get congested. Dashed blue lines on the contour plots indicate HOV gate locations.

Figure 15 shows the PeMS speed contours for the SR-134 East/I-210 East GP and HOV lanes that were used as a target for our simulation model. In these plots, traffic also travels from left to right, with the horizontal axis representing postmiles, while the vertical axis represents time.

Figure 16 shows an example of how well the offramp flow computed by the simulation matches the target, referred to as offramp demand, as recorded by the detector on the offramp at North Hill Avenue. The simulated offramp flow matches the offramp demand fairly closely. Similar results were found for the other offramps.

Finally, Table 2 summarizes the performance measurements — VMT, VHT and delay — computed by simulation versus those values obtained from PeMS. The PeMS data come from both SR-134 East and I-210 East, and VMT, VHT and delay values are computed as sums of the corresponding values from these two freeway sections. Delay values are computed in vehicle-hours for those vehicles traveling slower than 45 mph.
|                          | Simulation result | PeMS data          |
|--------------------------|-------------------|--------------------|
| GP Lane VMT              | 2,017,322         |                    |
| HOV Lane VMT             | 378,485           |                    |
| Total VMT                | 2,395,807         | \(414,941 + 2,006,457 = 2,421,398\) |
| GP Lane VHT              | 33,533            |                    |
| HOV Lane VHT             | 6,064             |                    |
| Total VHT                | 39,597            | \(6,416 + 36,773 = 43,189\) |
| GP Lane Delay            | 3,078             |                    |
| HOV Lane Delay           | 584               |                    |
| Total Delay              | 3,662             | \(1 + 3,802 = 3,803\) |

Table 2: Performance measures for SR-134 East/ I-210 East.
Figure 12: SR-134 East / I-210 East density contours for GP and HOV lanes produced by simulation. Density values are given in vehicles per mile per lane.
Figure 13: SR-134 East/ I-210 East flow contours for GP and HOV lanes produced by simulation. Flow values are give in vehicles per hour per lane.
Figure 14: SR-134 East/ I-210 East speed contours for GP and HOV lanes produced by simulation. Speed values are given in miles per hour.
Figure 15: SR-134 East/ I-210 East speed contours for GP and HOV lanes obtained from (California Department of Tranportation, 2016) for Monday, October 13, 2014. The horizontal axis represents absolute postmile, and the vertical axis represents time in hours. The four contours share the same color scale.
Figure 16: Flow at the North Hill Avenue offramp over 24 hours — PeMS data (offramp demand) vs. computed by simulation (offramp flow).
7 Conclusion

In this paper we discussed modeling procedures for two managed lane configurations: (1) full access, where special traffic can switch between the GP and the managed lanes at any node; and (2) separated, where special traffic can switch between the two lanes only at specific nodes, called gates. We have introduced the friction effect (Section 3.3) and the inertia effect (Section 3.5). The friction effect reflects the empirically-observed drivers’ fear of moving fast in the managed lane while traffic in the adjacent GP links moves slowly due to congestion. The inertia effect reflects drivers’ inclination to stay in their lane as long as possible and switch only if this would obviously improve their travel condition.

The presence of interchanges in freeways with managed lanes produces modeling complications beyond what one would see in a generic freeway model. For example, the separated managed lane requires a special modeling trick for sending vehicles traveling in the managed lane to offramps, as locations of gates and offramps generally do not coincide. In this paper we proposed such a mechanism. It employs destination-based traffic classes, which are populated at modeling links approaching gates using offramp split ratios (Section 3.4). This approach is a natural use of the multi-class feature of many traffic models.

Freeways with managed lanes feature many parameters, and calibrating them can be difficult. We presented an iterative learning approach for estimating some of the harder-to-estimate parameters. Our simulation results comparing our model and calibration results showed good agreement in two case studies, validating both our full-access and gated-access modeling techniques. In the sequel to this paper, we will further extend these results to include traffic control, with simulation of a reactive tolling controller on the managed lane.

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A Link Model

For the majority of this paper, we remain agnostic as to the particular functional relationship between density $n_l$, demand $S_l$ (per commodity, $n^c_l$, $S^c_l$) and supply $R_l$, and flow $f_l$ (also called the fundamental diagram)
used in our first-order macroscopic model (2.1). Where a particular fundamental diagram is required, i.e. for the simulation results presented in Section 6 and the example implementation of the friction effect in (3.4), (3.5), (3.6), and (3.7), we use a fundamental diagram from Horowitz et al. (2016), shown in Figure 17.

This fundamental diagram captures the traffic hysteresis behavior with the “backwards lambda” shape often observed in detector data (Koshi et al., 1983):

\[
S_l^f(t) = v_l^f(t) n_l^f(t) \min \left\{ 1, \frac{F_l(t)}{v_l^f(t) \sum_{c=1}^{C} n_{l}^c(t)} \right\}, \quad S_l(t) = \sum_{c=1}^{C} S_l^c(t),
\]

\[
R_l(t) = (1 - \theta_l(t)) F_l(t) + \theta_l(t) w_l(t) \left( n_l^f(t) - \sum_{c=1}^{C} n_l^c(t) \right),
\]

where, for link \( l \), \( F_l \) is the capacity, \( v_l^f \) is the free flow speed, \( w_l \) is the congestion wave speed, \( n_l^f \) is the jam density, and \( n_l^m = \frac{w_l n_l^f}{v_l^f + w_l} \) and \( n_l^h = \frac{F_l}{v_l^f} \) are called the low and high critical densities, respectively. As written here and used in this paper, \( F_l, v_l^f, w_l \) are in units per simulation timestep. The variable \( \theta_l(t) \) is a congestion metastate of \( l \), which encodes the hysteresis:

\[
\theta_l(t) = \begin{cases} 
0 & n_l(t) \leq n_l^m, \\
1 & n_l(t) > n_l^m, \\
\theta_l(t - 1) & n_l^m < n_l(t) \leq n_l^h,
\end{cases}
\]

where \( n_l(t) = \sum_{c=1}^{C} n_l^c(t) \).

Examining (A.3) and (A.2), we see that when a link’s density goes above \( n_l^h \) (i.e., when it becomes congested), its ability to receive flow is reduced until the density falls below \( n_l^m \).

An image of (A.1) and (A.2) overlaid on each other, giving a schematic image of the fundamental diagram, is shown in Figure 17. Unless \( n_l^m = n_l^h \), when it assumes triangular shape, the fundamental diagram is not a function of density alone (i.e., without \( \theta_l(t) \)): \( n_l(t) \in (n_l^m, n_l^h) \) admits two possible flow values.

**B Dynamic Split Ratio Solver**

Throughout this article, we have made reference to a dynamic-system-based method for solving for partially- or fully-undefined split ratios from Wright et al. (2016a). This split ratio solver is designed to implicitly solve the logit-based split ratio problem

\[
\beta_{ij}^c = \frac{\exp \left( \frac{\sum_{i=1}^{M} \sum_{c=1}^{C} S_{ij}^c}{R_{ij}} \right)}{\sum_{j' \neq i}^{N} \exp \left( \frac{\sum_{i=1}^{M} \sum_{c=1}^{C} S_{ij'}^c}{R_{ij'}} \right)},
\]

which cannot be solved explicitly, as the \( S_{ij}^c \)'s are also functions of the \( \beta_{ij}^c \)'s. The problem (B.1) is chosen to be a node-local problem that does not rely on information from the link model (beyond supplies and demands), and is thus independent of the choice of link model (Wright et al., 2016a).

The solution algorithm is as follows, reproduced from Wright et al. (2016a). More discussion is available in the reference.

- Define the set of commodity movements for which split ratios are known as \( B = \{ (i, j, c) : \beta_{ij}^c \in [0, 1] \} \), and the set of commodity movements for which split ratios are to be computed as \( B = \{ (i, j, c) : \beta_{ij}^c \text{ are unknown} \} \).
• For a given input link $i$ and commodity $c$ such that $S^r_i = 0$, assume that all split ratios are known: \( \{i, j, c\} \in B \).\(^3\)

• Define the set of output links for which there exist unknown split ratios as \( V = \{ j : \exists \{i, j, c\} \in B \} \).

• Assuming that for a given input link $i$ and commodity $c$, the split ratios must sum up to 1, define the unassigned portion of flow by \( \overline{\beta}_i^r = 1 - \sum_{j \in V} \beta_{ij} \).

• For a given input link $i$ and commodity $c$ such that there exists at least one commodity movement \( \{i, j, c\} \in B \), assume \( \overline{\beta}_i^r > 0 \), otherwise the undefined split ratios can be trivially set to 0.

• For every output link $j \in V$, define the set of input links that have an unassigned demand portion directed toward this output link by \( U_j = \{ i : \exists \{i, j, c\} \in B \} \).

• For a given input link $i$ and commodity $c$, define the set of output links for which split ratios for which are to be computed as \( V^c_i = \{ j : \exists i \in U_j \} \), and assume that if nonempty, this set contains at least two elements, otherwise a single split ratio can be trivially set equal to \( \overline{\beta}_i^r \).

• Assume that input link priorities are nonnegative, \( p_i \geq 0 \), $i = 1, \ldots, M$, and \( \sum_{i=1}^M p_i = 1 \).

• Define the set of input links with zero priority: \( U_{zp} = \{ i : p_i = 0 \} \). To enable split ratio assignment for inputs with zero priorities, perform regularization:

\[
\tilde{p}_i = p_i \left(1 - \frac{|U_{zp}|}{M}\right) + \frac{1}{M} \frac{|U_{zp}|}{M} = p_i \frac{M - |U_{zp}|}{M} + \frac{|U_{zp}|}{M^2},
\]

where \( |U_{zp}| \) denotes the number of elements in set \( U_{zp} \). Expression (B.2) implies that the regularized input priority \( \tilde{p}_i \) consists of two parts: (1) the original input priority \( p_i \) normalized to the portion of input links with positive priorities; and (2) uniform distribution among \( M \) input links: \( \frac{1}{M} \), normalized to the portion of input links with zero priorities.

Note that the regularized priorities \( \tilde{p}_i > 0 \), $i = 1, \ldots, M$, and \( \sum_{i=1}^M \tilde{p}_i = 1 \).

The algorithm for distributing \( \overline{\beta}_i^r \) among the commodity movements in \( B \) (that is, assigning values to the a priori unknown split ratios) aims at maintaining output links as uniform in their demand-supply ratios as possible. At each iteration $k$, two quantities are identified: 

\[ \mu^+(k) \text{ is the largest oriented demand-supply ratio produced by the split ratios that have been assigned so far, and} \]

\[ \mu^-(k) \text{ is the smallest oriented demand-supply ratio whose input link, denoted } i^- \text{, still has some unclaimed split ratio. Once these two quantities are found, the commodity } c^- \text{ in } i^- \text{ with the smallest unallocated demand has some of its demand directed to the } j \text{ corresponding to } \mu^-(k) \text{ to bring } \mu^-(k) \text{ up to } \mu^+(k) \text{ (or, if this is not possible due to insufficient demand, all such demand is directed).} \]

To summarize, in each iteration $k$, the algorithm attempts to bring the smallest oriented demand-supply ratio \( \mu^-(k) \) up to the largest oriented demand-supply ratio \( \mu^+(k) \). If it turns out that all such oriented demand-supply ratios become perfectly balanced, then the demand-supply ratios \( \frac{\sum_i \sum_c S_{ij}^r}{R_j} \) are as well.

The algorithm is:

1. Initialize:

\[
\beta_{ij}(0) := \begin{cases}
\beta_{ij}, & \text{if } \{i, j, c\} \in B, \\
0, & \text{otherwise};
\end{cases}
\]

\[
\overline{\beta}_i^r(0) := \overline{\beta}_i^r;
\]

\[
\overline{U}_j(0) = U_j;
\]

\[
\overline{V}(0) = V;
\]

\[
k := 0,
\]

\(^3\)If split ratios were undefined in this case, they could be assigned arbitrarily.
Here $\tilde{U}_j(k)$ is the remaining set of input links with some unassigned demand, which may be directed to output link $j$; and $\tilde{V}(k)$ is the remaining set of output links, to which the still-unassigned demand may be directed.

2. If $\tilde{V}(k) = \emptyset$, stop. The sought-for split ratios are $\{\tilde{\beta}_{ij}^c(k)\}, \ i = 1, \ldots, M, \ j = 1, \ldots, N, \ c = 1, \ldots, C$.

3. Calculate the remaining unallocated demand:

$$\tilde{S}_i^c(k) = \tilde{\beta}_{ij}^c(k)S_i^c, \ i = 1, \ldots, M, \ c = 1, \ldots, C.$$ 

4. For all input-output link pairs, calculate oriented demand:

$$\tilde{S}_{ij}^c(k) = \tilde{\beta}_{ij}^c(k)\tilde{S}_{ij}^c.$$ 

5. For all input-output link pairs, calculate oriented priorities:

$$\tilde{p}_{ij}^c(k) = \tilde{p}_{ij}^c(k)\tilde{\beta}_{ij}^c(k) + \beta_{ci}(k)\frac{\gamma_{ij}^c}{|V_c^i|},$$ 

where $|V_c^i|$ denotes the number of elements in the set $V_c^i$. Examining the expression (B.3)-(B.4), one can see that the split ratios $\tilde{\beta}_{ij}^c(k)$, which are not fully defined yet, are complemented with a fraction of $\tilde{\beta}_{ij}^c(k)$ inversely proportional to the number of output links among which the flow of commodity $c$ from input link $i$ can be distributed.

Note that in this step we are using regularized priorities $\tilde{p}_i$ as opposed to the original $p_i$, $i = 1, \ldots, M$. This is done to ensure that inputs with $p_i = 0$ are not ignored in the split ratio assignment.

6. Find the largest oriented demand-supply ratio:

$$\mu^+(k) = \max_j \max_i \frac{\sum_{c=1}^C \tilde{S}_{ij}^c(k)}{\tilde{p}_{ij}(k)R_j} \sum_{i \in U_j} \tilde{p}_{ij}(k).$$

7. Define the set of all output links in $\tilde{V}(k)$, where the minimum of the oriented demand-supply ratio is achieved:

$$Y(k) = \arg \min_{j \in \tilde{V}(k)} \min_{i \in \tilde{U}_j(k)} \frac{\sum_{c=1}^C \tilde{S}_{ij}^c(k)}{\tilde{p}_{ij}(k)R_j} \sum_{i \in U_j} \tilde{p}_{ij}(k),$$

and from this set pick the output link $j^-$ with the smallest output demand-supply ratio (when there are multiple minimizing output links, any of the minimizing output links may be chosen as $j^-$):

$$j^- = \arg \min_{j \in Y(k)} \sum_{i=1}^M \frac{\sum_{c=1}^C \tilde{S}_{ij}^c(k)}{R_j}.$$

8. Define the set of all input links, where the minimum of the oriented demand-supply ratio for the output link $j^-$ is achieved:

$$W_{j^-}(k) = \arg \min_{i \in \tilde{U}_{j^-}(k)} \frac{\sum_{c=1}^C \tilde{S}_{ij^-}^c(k)}{\tilde{p}_{ij^-}(k)R_{j^-}} \sum_{i \in U_{j^-}} \tilde{p}_{ij^-}(k),$$

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and from this set pick the input link \( i^- \) and commodity \( c^- \) with the smallest remaining unallocated demand:

\[
\{i^-, c^-\} = \arg \min_{i \in W_j^-(k), \ c : \overline{S}_i^-(k) > 0} \overline{S}_i^-(k).
\]

9. Define the smallest oriented demand-supply ratio:

\[
\mu^-(k) = \frac{\sum_{c=1}^C \hat{S}_{i^-j^-}^c(k)}{\sum_{i \in U_{i^-}} \hat{p}_{i^-j^-}(k)}.
\]

- If \( \mu^-(k) = \mu^+(k) \), the oriented demands created by the split ratios that have been assigned as of iteration \( k \), \( \beta_{ij}^c(k) \), are perfectly balanced among the output links, and to maintain this, all remaining unassigned split ratios should be distributed proportionally to the allocated supply:

\[
\beta_{ij}^c(k + 1) = \frac{R_j}{\sum_{j' \in V_i^+(k)} R_{j'}} \beta_{ij}^c(k), \quad c : \overline{S}_i^-(k) > 0, \quad i \in \hat{U}_j(k), \quad j \in \hat{V}(k); \quad (B.5)
\]

\[
\overline{S}_i^-(k + 1) = 0, \quad c : \overline{S}_i^-(k) > 0, \quad i \in \hat{U}_j(k), \quad j \in \hat{V}(k);
\]

\[
\hat{U}_j(k + 1) = \emptyset, \quad j \in \hat{V}(k);
\]

\[
\hat{V}(k + 1) = \emptyset.
\]

If the algorithm ends up at this point, we have emptied \( \hat{V}(k + 1) \) and are done.

- Else, assign:

\[
\Delta \hat{\beta}_{i^-j^-}^c(k) = \min \left\{ \overline{S}_i^-(k), \left( \frac{\mu^+(k)\hat{p}_{i^-j^-}(k)R_{j^-}}{\overline{S}_i^-(k) \sum_{i \in U_{i^-}} \hat{p}_{i^-j^-}(k)} - \sum_{c=1}^C \hat{S}_{i^-j^-}^c(k) \right) \right\}; \quad (B.6)
\]

\[
\hat{\beta}_{i^-j^-}^c(k + 1) = \hat{\beta}_{i^-j^-}^c(k) + \Delta \hat{\beta}_{i^-j^-}^c(k); \quad (B.7)
\]

\[
\overline{S}_i^-(k + 1) = \overline{S}_i^-(k) + \Delta \overline{S}_i^-(k); \quad (B.8)
\]

\[
\hat{S}_{i^-j^-}^c(k + 1) = \hat{\beta}_{ij}^c \text{ for } \{i, j, c\} \neq \{i^-, j^-, c^-\};
\]

\[
\overline{S}_i^-(k + 1) = \overline{S}_i^-(k) \text{ for } \{i, c\} \neq \{i^-, c^-\};
\]

\[
\hat{U}_j(k + 1) = \hat{U}_j(k) \setminus \left\{ i : \overline{S}_i^-(k + 1) = 0, \ c = 1, \ldots, C \right\}, \quad j \in \hat{V}(k);
\]

\[
\hat{V}(k + 1) = \hat{V}(k) \setminus \left\{ j : \hat{U}_j(k + 1) = \emptyset \right\}.
\]

In (B.6), we take the minimum of the remaining unassigned split ratio portion \( \overline{S}_i^-(k) \) and the split ratio portion needed to equalize \( \mu^-(k) \) and \( \mu^+(k) \). To better understand the latter, the second term in \( \min \{ \cdot \} \) can be rewritten as:

\[
\frac{\mu^+(k)\hat{p}_{i^-j^-}(k)R_{j^-}}{\overline{S}_i^-(k) \sum_{i \in U_{i^-}} \hat{p}_{i^-j^-}(k)} - \sum_{c=1}^C \hat{S}_{i^-j^-}^c(k) \overline{S}_i^-(k) = \left( \frac{\mu^+(k)}{\mu^-(k)} - 1 \right) \left( \sum_{c=1}^C \hat{S}_{i^-j^-}^c(k) \right) \frac{1}{\overline{S}_i^-(k)}.
\]

The right hand side of the last equality can be interpreted as: flow that must be assigned for input \( i^- \), output \( j^- \) and commodity \( c^- \) to equalize \( \mu^-(k) \) and \( \mu^+(k) \) minus flow that is already assigned for \( \{i^-, j^-, c^-\} \), divided by the remaining unassigned portion of demand of commodity \( c^- \) coming from input link \( i^- \).

In (B.7) and (B.8), the assigned split ratio portion is incremented and the unassigned split ratio portion is decremented by the computed \( \Delta \hat{\beta}_{i^-j^-}^c(k) \).

10. Set \( k := k + 1 \) and return to step 2.
References

M. Bliemer. Dynamic queueing and spillback in an analytical multiclass dynamic network loading model. *Transportation Research Record*, 2029:14–21, 2007.

California Department of Transportation. PeMS homepage, 2016. http://pems.dot.ca.gov.

M. J. Cassidy, K. Kim, W. Ni, and W. Gu. A problem of limited-access special lanes. Part I: Spatiotemporal studies of real freeway traffic. *Transportation Research Part A: Policy and Practice*, 80:307 – 319, 2015. ISSN 0965-8564. doi: http://dx.doi.org/10.1016/j.tra.2015.07.001. URL http://www.sciencedirect.com/science/article/pii/S0965856415001834.

M. Chang, J. Wiegmann, A. Smith, and C. Bilotto. A Review of HOV Lane Performance and Policy Options in the United States. Report FHWA-HOP-09-029, Federal Highway Administration, 2008.

C. F. Daganzo. A behavioral theory of multi-lane traffic flow. Part i: Long homogeneous freeway sections. *Transportation Research Part B: Methodological*, 36(2):131 – 158, 2002. ISSN 0191-2615. doi: http://dx.doi.org/10.1016/S0191-2615(00)00042-4. URL http://www.sciencedirect.com/science/article/pii/S0191261500000424.

C. F. Daganzo and M. J. Cassidy. Effects of high occupancy vehicle lanes on freeway congestion. *Transportation Research Part B: Methodological*, 42(10):861–872, Dec. 2008. ISSN 01912615. doi: 10.1016/j.trb.2008.03.002. URL http://linkinghub.elsevier.com/retrieve/pii/S0191261508000325.

G. Dervisoglu, G. Gomes, J. Kwon, A. Muralidharan, P. Varaiya, and R. Horowitz. Automatic calibration of the fundamental diagram and empirical observations on capacity. 88th Annual Meeting of the Transportation Research Board, Washington, D.C., USA, 2009.

N. Farhi, H. Haj-Salem, M. Khoshyaran, J.-P. Lebacque, F. Salvarani, B. Schnetzer, and F. De Vyust. The Logit lane assignment model: first results. In TRB 92nd Annual Meeting Compendium of Papers, 2013.

M. Fransson and M. Sandin. *Framework for Calibration of a Traffic State Space Model*. Masters Thesis, Linkoping University, Norrkoping, Sweden, Oct. 2012.

S. Hoogendoorn and P. Bovy. Gas-Kinetic Model for Multilane Heterogeneous Traffic Flow. *Transportation Research Record: Journal of the Transportation Research Board*, 1678:150–159, 1999. doi: 10.3141/1678-19.

S. P. Hoogendoorn and P. H. Bovy. Continuum modeling of multiclass traffic flow. *Transportation Research Part B: Methodological*, 34(2):123 – 146, 2000. ISSN 0191-2615. doi: http://dx.doi.org/10.1016/S0191-2615(99)00017-X. URL http://www.sciencedirect.com/science/article/pii/S019126159900017X.

R. Horowitz, A. A. Kurzhanskiy, A. Siddiqui, and M. A. Wright. Modeling and control of hot lanes. Technical report, Partners for Advanced Transportation Technologies, University of California, Berkeley, Berkeley, CA, 2016.

K. Jang and M. Cassidy. Dual influences on vehicle speed in special-use lanes and critique of US regulation. *Transportation Research, Part A*, 46(2012):1108–1123, 2012.

K. Jang, S. Oum, and C.-Y. Chan. Traffic Characteristics of High-Occupancy Vehicle Facilities: Comparison of Contiguous and Buffer-Separated Lanes. *Transportation Research Record: Journal of the Transportation Research Board*, 2278:180–193, 2012. doi: 10.3141/2278-20.

M. Koshi, M. Iwasaki, and I. Ohkura. Some findings and an overview on vehicular flow characteristics. In *Proceedings of the 8th International Symposium on Transportation and Traffic Flow Theory*, volume 198, pages 403–426, 1983.

A. A. Kurzhanskiy and P. Varaiya. Traffic management: An outlook. *Economics of Transportation*, 4(3):135–146, Sept. 2015. ISSN 22120122. doi: 10.1016/j.ecotra.2015.03.002.

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X. Liu, B. Schroeder, T. Thomson, Y. Wang, N. Rouphail, and Y. Yin. Analysis of operational interactions between freeway managed lanes and parallel, general purpose lanes. *Transportation Research Record: Journal of the Transportation Research Board*, 2262:62–73, 2011.

X. Liu, G. Zhang, Y. Lao, and Y. Wang. Modeling Traffic Flow Dynamics on Managed Lane Facility: Approach Based on Cell Transmission Model. *Transportation Research Record: Journal of the Transportation Research Board*, 2278:163–170, 2012.

T. Lomax, D. Schrank, and B. Eisele. The 2015 Annual Urban Mobility Scorecard. Technical report, Texas Transportation Institute, 2015. [http://mobility.tamu.edu](http://mobility.tamu.edu).

D. McFadden. Conditional Logit Analysis of Qualitative Choice Behavior. In P. Zarembka, editor, *Frontiers in Econometrics*, pages 105–142. Academic Press, New York, 1973.

Metropolitan Transportation Commission. Bay Area Express Lanes. [http://bayareaexpresslanes.org](http://bayareaexpresslanes.org).

D. Ngoduy. *Macroscopic discontinuity modeling for multiclass multilane traffic flow operations*. PhD thesis, Delft University of Technology, 2006.

D. Ngoduy and M. Maher. Calibration of second order traffic models using continuous cross entropy method. *Transportation Research Part C: Emerging Technologies*, 24:102–121, Oct. 2012. ISSN 0968090X. doi: 10.1016/j.trc.2012.02.007.

J. Obenberger. Managed lanes: Combining access control, vehicle eligibility, and pricing strategies can help mitigate congestion and improve mobility on the nation’s busiest roadways. *Public Roads*, 68(3):48–55, 2004. URL [http://www.fhwa.dot.gov/publications/publicroads/04nov/08.cfm](http://www.fhwa.dot.gov/publications/publicroads/04nov/08.cfm).

A. Poole and A. Kotsialos. METANET Model Validation using a Genetic Algorithm. *IFAC Proceedings Volumes*, 45(24):7–12, Sept. 2012. ISSN 14746670. doi: 10.3182/20120912-3-BG-2031.00002.

A. Poole and A. Kotsialos. Second order macroscopic traffic flow model validation using automatic differentiation with resilient backpropagation and particle swarm optimisation algorithms. *Transportation Research Part C: Emerging Technologies*, 71:356–381, Oct. 2016. ISSN 0968090X. doi: 10.1016/j.trc.2016.07.008. URL [http://linkinghub.elsevier.com/retrieve/pii/S0968090X16301152](http://linkinghub.elsevier.com/retrieve/pii/S0968090X16301152).

Y. Shiomi, T. Taniguchi, N. Uno, H. Shimamoto, and T. Nakamura. Multilane first-order traffic flow model with endogenous representation of lane-flow equilibrium. *Transportation Research Part C: Emerging Technologies*, 59:198–215, Oct. 2015. ISSN 0968090X. doi: 10.1016/j.trc.2015.07.002. URL [http://linkinghub.elsevier.com/retrieve/pii/S0968090X15002429](http://linkinghub.elsevier.com/retrieve/pii/S0968090X15002429).

System Metrics Group, Inc. Contra Costa County I-680 Corridor System Management Plan Final Report. Technical report, Caltrans District 4, 2015. [http://dot.ca.gov/hq/tpp/corridor-mobility/CSMPs/D4_CSMPs/D04_I680_CSMP_Final_Revised_Report_2015-05-29.pdf](http://dot.ca.gov/hq/tpp/corridor-mobility/CSMPs/D4_CSMPs/D04_I680_CSMP_Final_Revised_Report_2015-05-29.pdf).

M. Treiber, A. Hennecke, and D. Helbing. Derivation, properties, and simulation of a gas-kinetic-based, nonlocal traffic model. *Phys. Rev. E*, 59:239–253, Jan 1999. doi: 10.1103/PhysRevE.59.239. URL [http://link.aps.org/doi/10.1103/PhysRevE.59.239](http://link.aps.org/doi/10.1103/PhysRevE.59.239).

J. van Lint, S. Hoogendoorn, and M. Schreuder. FastLane: New Multiclass First-Order Traffic Flow Model. *Transportation Research Record: Journal of the Transportation Research Board*, 2088:177–187, 2008. doi: 10.3141/2088-19.

G. Wong and S. Wong. A multi-class traffic flow model - an extension of LWR model with heterogeneous drivers. *Transportation Research Part A: Policy and Practice*, 36(9):827 – 841, 2002. ISSN 0965-8564. doi: [http://dx.doi.org/10.1016/S0965-8564(01)00042-8](http://dx.doi.org/10.1016/S0965-8564(01)00042-8). URL [http://www.sciencedirect.com/science/article/pii/S0965856401000428](http://www.sciencedirect.com/science/article/pii/S0965856401000428).

M. A. Wright, G. Gomes, R. Horowitz, and A. A. Kurzhanskiy. On node and route choice models for high-dimensional road networks. *Submitted to Transportation Research Part B*, 2016a. Online: [http://arxiv.org/abs/1601.01054](http://arxiv.org/abs/1601.01054).
M. A. Wright, R. Horowitz, and A. A. Kurzhanskiy. A dynamic system characterization of road network node models. In Proceedings of the 10th IFAC Symposium on Nonlinear Control Systems, pages 1053–1058, August 2016b. Online: http://arxiv.org/abs/1608.07623.