π± ↔ K± Meson Vacuum Transitions (Oscillations) in Diagram Approach in the Model of Dynamical Analogy of the Cabibbo-Kobayashi-Maskawa Matrices

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Abstract

The elements of the theory of vacuum oscillations and the model of dynamical expansion of the theory of weak interactions works at the tree level, i.e. the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices and its further development, are given. It is shown that the quarks and massive vector bosons must be structural and these structural particles (subparticles) must interact to generate quark and vector boson masses. In this case the problem of singularity cancellations does not arise in this model. It is also shown that, for self consistency of the theory, the weak decays of K mesons must go through massive vector boson B but not W boson.

In the framework of this model the probability of π ↔ K transitions (oscillations) in the diagram approach is computed. These transitions are virtual since masses of π and K mesons differ considerably. These transitions (oscillations) can be registered through K decays after transitions of virtual K mesons to their own mass shell by using their quasielastic strong interactions.

PACS: 12.15 Ff Quark and lepton masses and mixing.
PACS: 12.15 Ji Application of electroweak model to specific processes.

1 Introduction

The vacuum oscillation of neutral $K$ mesons is well investigated at the present time [1]. This oscillation is the result of $d, s$ quark mixings and is described by Cabibbo-Kobayashi-Maskawa matrices [2]. The angle mixing $\theta$ of neutral $K$ mesons is $\theta = 45^\circ$ since $K^o, \bar{K}^o$ masses are equal (see CPT theorem). Besides, since their masses are equal, these oscillations are real, i.e. their transitions to each other go without suppression. Oscillations of two particles having the masses overlapping their widths were discussed in works [3]. Then we calculated probabilities of $\pi \leftrightarrow K$ oscillations in an approach where the phase volume of particles at these transitions is taken into account [4,5].

This work is devoted to the development of the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices [6] and to the calculation of probabilities of $\pi \leftrightarrow K$ oscillations in framework of this model in the diagram approach [7] which was used while calculation of $K^o \leftrightarrow \bar{K}^o$ oscillations.

At first, we will consider the general elements of the theory of oscillations, elements of the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices and its further development, then come to the calculation of probabilities of $\pi \leftrightarrow K$ transitions.

As it is stressed in previous works [4,5] these transitions are virtual since masses of $\pi$ and $K$ mesons differ considerably. And we can make these virtual transitions real through their strong interactions, i.e. bring them up on the own mass shell through strong interactions after the weak interaction transforming $\pi$ mesons in virtual $K$ mesons.

Let us to consider the general elements of the theory of oscillations.
2 Probabilities of Real and Virtual Vacuum $\pi \leftrightarrow K$ Oscillations (Transitions)

The mass matrix of $\pi$ and $K$ mesons has the form

\[
\begin{pmatrix}
  m_\pi & 0 \\
  0 & m_K
\end{pmatrix}.
\] (1)

Due to the presence of strangeness violation in the weak interactions, a nondiagonal term appears in this matrix and then this mass matrix is transformed in the following nondiagonal matrix:

\[
\begin{pmatrix}
  m_\pi & m_\pi K \\
  m_\pi K & m_K
\end{pmatrix},
\] (2)

which is diagonalized by turning through the angle $\beta$ and then

\[
\begin{pmatrix}
  m_\pi & m_\pi K \\
  m_\pi K & m_K
\end{pmatrix} \rightarrow \begin{pmatrix}
  m_1 & 0 \\
  0 & m_2
\end{pmatrix}
\] (3)

where

\[
tg2\beta = \frac{2m_\pi K}{|m_\pi - m_K|},
\]

\[
sin2\beta = \frac{2m_\pi K}{\sqrt{(m_\pi - m_K)^2 + (2m_\pi K)^2}}.
\] (4)

\[
m_{1,2} = \frac{1}{2}((m_\pi - m_K) \pm \sqrt{(m_\pi - m_K)^2 + 4(m_\pi K)^2}).
\]

It is interesting to remark that expression (4) can be obtained from the Breit-Wigner distribution [8]

\[
P \sim \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2}
\] (5)

by using the following substitutions:

\[
E = m_K, \ E_0 = m_\pi, \ \Gamma/2 = 2m_\pi K,
\] (6)

where $\Gamma \equiv W(...)$. 

3
If the mass matrix contains masses in a squared form, then oscillations (or mixings) will be described by the expressions (3)-(6) with the following substitutions:

\[ m_\pi \rightarrow m_\pi^2, m_K \rightarrow m_K^2, m_{\pi K} \rightarrow m_{\pi K}^2 \]

Here two cases of \( \pi, K \) oscillations [4] take place: real and virtual oscillations.

1. If we consider the real transition of \( \pi \) into \( K \) mesons, then

\[ \sin^2 2\beta \approx \frac{4m_{\pi K}^2}{(m_\pi - m_K)^2} \approx 0, \]

i.e. the probability of the real transition of \( \pi \) mesons into \( K \) mesons through weak interactions is very small since \( m_{\pi K} \) is very small.

How can we understand this real \( \pi \rightarrow K \) transition?

If \( 2m_{\pi K} = \frac{1}{2} \) is not zero, then it means that the mean mass of \( \pi \) meson is \( m_\pi \) and this mass is distributed by \( \sin^2 2\beta \) (or by the Breit-Wigner formula) and the probability of the \( \pi \rightarrow K \) transition differs from zero. So, this is a solution of the problem of origin of mixing angle in the theory of vacuum oscillation.

In this case the probability of \( \pi \rightarrow K \) transition (oscillation) is described by the following expression:

\[ P(\pi \rightarrow K, t) = \sin^2 2\beta \sin^2 \left[ \pi t \frac{m_K^2}{2p} \right], \]

where \( p \) is momentum of \( \pi \) meson.

2. If we consider the virtual transition of \( \pi \) into \( K \) meson, then, since \( m_\pi = m_K \),

\[ \tan 2\beta = \infty, \]

i.e. \( \beta = \pi/4 \), then

\[ \sin^2 2\beta = 1. \]

In this case the probability of \( \pi \rightarrow K \) transition (oscillation) is described by the following expression:

\[ P(\pi \rightarrow K, t) = \sin^2 \left[ \pi \frac{L}{L_{osc}} \right], \]
where $L = vt$, $v$- is a velocity of $\pi$ meson, at $v \approx c \ L \approx ct$,

$$L_{osc} = \frac{2.48\rho_\pi(MeV)}{|m_1^2 - m_2^2| (eV^2)m}.$$  

Let us consider elements of the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices and its development.

### 3 Elements of the Model of Dynamical Analogy of the Cabibbo-Kobayashi-Maskawa Matrices and Its Development

In the case of three families of quarks, the current $J^\mu$ has the following form:

$$J^\mu = (\bar{u}c\ell)^L \gamma^\mu V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$  

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$

where $V$ is Kobayashi-Maskawa matrix [2].

Mixings of the $d, s, b$ quarks are not connected with the weak interaction (i.e., with $W^\pm, Z^0$ bosons exchanges). From equation (1) it is well seen that mixings of the $d, s, b$ quarks and exchange of $W^\pm, Z^0$ bosons take place in an independent manner (i.e., if matrix $V$ were diagonal, mixings of the $d, s, b$ quarks would not have taken place).

If the mechanism of this mixings is realized independently of the weak interaction ($W^\pm, Z^0$ boson exchange) with a probability determined by the mixing angles $\theta, \beta, \gamma, \delta$ (see below), then this violation could be found in the strong and electromagnetic interactions of the quarks as a clear violations of isospin, strangeness and beauty. But, the available experimental results have shown, that there is no clear violations of the number conservations in strong and electromagnetic interactions of the quarks. Then we must connect the non-conservation of isospins, strangeness and
beauty (or mixings of the $d,s,b$ quarks) with some type of interaction mixings of the quarks. We can do it introducing (together with the $W^\pm,Z^0$ bosons) the heavier vector bosons $B^\pm,C^\pm,D^\pm,E^\pm$ which interact with the $d,s,b$ quarks with violation of isospin, strangeness and beauty.

We shall choose parametrization of matrix $V$ in the form offered by Maiani [9]

$$
V = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_\gamma & s_\gamma \\
0 & -s_\gamma & c_\gamma
\end{pmatrix}
\begin{pmatrix}
c_\beta & 0 & s_\beta \exp(-i\delta) \\
0 & 1 & 0 \\
-s_\beta \exp(i\delta) & 0 & c_\beta
\end{pmatrix}
\begin{pmatrix}
c_\theta & s_\theta & 0 \\
-s_\theta & c_\theta & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

$$c_\theta = \cos \theta, s_\theta = \sin \theta, \exp(i\delta) = \cos \delta + i \sin \delta.$$  \tag{12}

To the nondiagonal terms in (12), which are responsible for mixing of the $d,s,b$-quarks and $CP$-violation in the three matrices, we shall make correspond four doublets of vector bosons $B^\pm,C^\pm,D^\pm,E^\pm$ whose contributions are parametrized by four angles $\theta,\beta,\gamma,\delta$. It is supposed that the real part of $Re(s_\beta \exp(i\delta)) = s_\beta \cos \delta$ corresponds to the vector boson $C^\pm$, and the imaginary part of $Im(s_\beta \exp(i\delta)) = s_\beta \sin \delta$ corresponds to the vector boson $E^\pm$ (the couple constant of $E$ is an imaginary value!). Then, when $q^2 << m_W^2$, we get:

$$
\tan \theta \approx \frac{m_W^2 g_B^2}{m_B^2 g_W^2}, \quad \tan \beta \approx \frac{m_W^2 g_C^2}{m_C^2 g_W^2},
$$

$$
\tan \gamma \approx \frac{m_W^2 g_D^2}{m_D^2 g_W^2}, \quad \tan \delta \approx \frac{m_W^2 g_E^2}{m_E^2 g_W^2}. \tag{13}
$$

If $g_B^\pm \approx g_C^\pm \approx g_D^\pm \approx g_E^\pm \approx g_W^\pm$, then

$$
\tan \theta \approx \frac{m_W^2}{m_B^2}, \quad \tan \beta \approx \frac{m_W^2}{m_C^2},
$$

$$
\tan \gamma \approx \frac{m_W^2}{m_D^2}, \quad \tan \delta \approx \frac{m_W^2}{m_E^2}. \tag{14}
$$

Concerning the neutral vector bosons $B^0,C^0,D^0,E^0$, the neutral scalar bosons $B^{'0},C^{'0},D^{'0},E^{'0}$ and the GIM mechanism [10], can repeat the same arguments given in the previous work [6].
The proposed Lagrangian for expansion of the weak interaction theory (without CP-violation) has the following form:

\[ L_{\text{int}} = i \sum_i g_i (J^{i,\alpha} A^i_\alpha + \text{c.c.}), \]

where \( J^{i,\alpha} = \bar{\psi}_{i,L} \gamma^\alpha T \varphi_{i,L}, \)

\[ T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \]

\( i = 1 \quad i = 2 \quad i = 3 \)

\[ \psi_{i,L} = \begin{pmatrix} u \\ c \end{pmatrix}_L, \begin{pmatrix} u \\ t \end{pmatrix}_L, \begin{pmatrix} c \\ t \end{pmatrix}_L, \]

\( i = 1 \quad i = 2 \quad i = 3 \)

\[ \varphi_{i,L} = \begin{pmatrix} d \\ s \end{pmatrix}_L, \begin{pmatrix} d \\ b \end{pmatrix}_L, \begin{pmatrix} s \\ b \end{pmatrix}_L, \]

\( i = 1 \quad i = 2 \quad i = 3 \)

The weak interaction carriers \( A^i_\alpha \), which are responsible for the weak transitions between different quark families are connected with the \( B, C, D \) bosons in the following manner:

\[ A^1_\alpha \rightarrow B^\pm_\alpha, A^2_\alpha \rightarrow C^\pm_\alpha, A^3_\alpha \rightarrow D^\pm_\alpha. \]

Using the data from [1] and equation (14) we have obtained the following masses for \( B^\pm, C^\pm, D^\pm, E^\pm \) bosons:

\[ m_{B^\pm} \approx 169.5 \div 171.8 \text{ GeV}. , \]

\[ m_{C^\pm} \approx 345.2 \div 448.4 \text{ GeV}. , \]

\[ m_{D^\pm} \approx 958.8 \div 1794 \text{ GeV}. , \]

\[ m_{E^\pm} \approx 4170 \div 4230 \text{ GeV}. . \]

Now consider some development of our model.

\[ a. \] It is clear that the masses of quarks and \( B, C, D, E \) bosons can be introduced using the Higgs’s mechanism. Here arises a question about
correspondence of the physical picture given by Higgs’s mechanism to the real physical picture of quarks and vector bosons. In the Higgs’s mechanism the quarks and vector bosons get their masses through their interactions with Higgs’s bosons [11] (in an analogy with the mechanism of superconductivity), i.e. in the presence of Higgs’s fields the quarks and vector bosons are massive. It is clear that free quarks and vector bosons (in reality, we have free quarks and vector bosons) must be massless. Then we see that Higgs’s mechanism is for introducing masses in the theory without singularity (i.e. without straight violation gauge invariance), but not a mechanism of masses generation.

On the other side, the standard weak interaction cannot generate masses for its $\gamma_5$ invariance.

Then the following question arises: how are masses of these particles generated?

It is obvious that these quarks and bosons must have a structure i.e., they consist of subparticles which take part in some interactions which generate masses. So, we see that it goes in an analogy with the strong interactions, where the fundamental interaction is the chromodynamics and the hadrons consist of the quarks. It is clear that if the quarks and massive bosons consist of subparticles, then in our approach (the Model of Dynamical Analogy of the Cabibbo-Kobayashi-Maskawa matrices) the problem of singularity does not appear since at small distances interact subparticles but not quarks and massive bosons. And then the problem of singularity must be solved in the theory of subparticle interactions in full analogy with the strong interactions theory. It is obvious that in the framework of our model it is not needed to use GIM mechanism [10] to cancel the singularity.

b. Let us have $K^\pm$ which is produced in strong interactions and we want to consider its decay. Since $K$ meson includes $s$ quark, then when we take into account the weak interaction, we must use the Cabibbo matrix [2] mixing $s, d$ quarks:

\[
\begin{align*}
    d' &= d\cos\theta + s\sin\theta \\
    s' &= -d\sin\theta + s\cos\theta
\end{align*}
\]
i.e., s quark transforms in superpositions of s, d quarks

\[ s \rightarrow -d\sin\theta + s\cos\theta \] (19)

The matrix element of K meson decay [7] is proportional to \( \sin\theta \), i.e., we take into account only the \( \sin\theta \) part from expression (19) and then the term proportional to \( \cos\theta \) is remained. It means that only the part proportional to \( \sin\theta \) decays. However, from the current experiments we know that \( K \) mesons decay fully. It can happen only if \( K \) mesons decay through massive bosons \( B \) but not \( W \) boson and the \( \sin\theta \) term of Cabibbo matrix. Then the mass of this massive boson \( B \) must be determined through the following expression:

\[ m_b^2 \sim \frac{m_W^2}{\sin\theta} \] (20)

We see that this massive boson is like \( B \) boson which appears in the above considered model of dynamical analogy of Kabibbo-Kobayashi-Maskawa matrices [6].

Let us pass to a more detailed consideration of the virtual oscillation case since it is of a real interest (i.e. we compute nondiagonal term of the mass matrix).

4 The \( \pi \overset{B}{\rightarrow} K \) Meson Transitions in Diagram Approach in the Model of Dynamical Analogy of Kabibbo-Kobayashi-Maskawa Matrices

When one takes into account \( d, s \) quark mixings and \( B \) exchange, the diagram for \( \pi \overset{B}{\rightarrow} K \) transitions has the form
It is clear that at $d, s$ mixings the transition of $\pi$ meson mass shell does not take place, i.e. $K$ meson produced from $\pi$ meson remains on the mass shell of $\pi$ meson.

The amplitude of this process has the following form (we use Feynman rules):

\[
M(\pi \to K) = G_B [\bar{d}\gamma_\mu (1 - \gamma_5) u][\bar{s}\gamma^\mu (1 - \gamma_5) u],
\]

or

\[
M(\pi \to K) = G_B [\bar{d}Q_\mu u][\bar{s}Q^\mu u],
\tag{21}
\]

where $G_B$ is Fermi of $B$ boson constant which is connected with Fermi constant $G_W$ of $W$ by the following relation

\[
G_B = G_F \sin \theta, \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2},
\]

and $Q_\mu = \gamma_\mu (1 - \gamma_5)$.

The mass Lagrangian $L$ for this diagram in the framework of standard approach is [7]

\[
L = M(\pi \to K).
\tag{22}
\]

Then the mass differences in squared form which response for $\pi \to K$ and $K \to \pi$ transitions is

\[
m_1^2 - m_2^2 = <\pi \mid L \mid K> + <K \mid L \mid \pi>
\tag{23}
\]

(we suppose that $K$ meson is on the mass shell of $\pi$ meson). Therefore

\[
m_1^2 - m_2^2 \simeq 2m_\pi \Delta m_{12}
\tag{24}\]
\[ \Delta m_{12} = \frac{1}{2m_\pi}[<\pi \mid L \mid K> + <K \mid L \mid \pi>] \]  

(25)

Now we compute mass difference. For this goal we use the following expressions:

\[ <0 \mid \bar{d}Q_\mu u \mid \pi> = \phi_\pi f_\pi p_\mu, \]
\[ <0 \mid \bar{s}Q^{\mu}u \mid K> = \phi_K f_K p^{\mu}, \]  

(26)

where \( \phi_\pi, \phi_K, f_\pi, f_K \), correspondingly, are the wave functions and the constant decays of \( \pi \) and \( K \) mesons, \( p_\mu \) is momentum of \( \pi \) meson.

It is necessary to remark that the following relation for constant decays on mass shells will be:

\[ f_\pi(m_\pi) = f_K(m_\pi), \]  

(27)

Then from equation (23) using equations (26), (27) we obtain the following expression:

\[ \Delta m^2 = m_1^2 - m_2^2 = f_\pi^2 m_\pi^2 G_B. \]  

(28)

or (see Eq. (4))

\[ m_{\pi K} = \Delta m_{12} = f_\pi^2 m_\pi G_B. \]  

(29)

### 5 Probability of \( \pi \xrightarrow{B} K \) Virtual Oscillations with Account of \( \pi \) Decays

If at \( t = 0 \) we have the flow \( N(\pi, 0) \) of \( \pi \) mesons, then at \( t \neq 0 \) this flow will decrease since \( \pi \) mesons decay and then we have the following flow \( N(\pi, t) \) of \( \pi \) mesons:

\[ N(\pi, t) = \exp(-\frac{t}{\tau_0})N(\pi, 0), \]  

(30)

where \( \tau_0 = \tau_0 \frac{E_\pi}{m_\pi}. \)

The expression for the flow \( N(\pi \to K, t) \), i.e. probability of \( \pi \) to \( K \) meson transitions at time \( t \), has the following form:

\[ N(\pi \to K, t) = N(\pi, t)P(\pi \to K, L) \]  

(31)
where
\[ P(\pi \to K, L) = \sin^2 \left( \pi \frac{L}{L_{\text{osc}}} \right), \]
\[ L_{\text{osc}} = \frac{2.48 \rho_\pi (\text{MeV})}{|m_1^2 - m_2^2| (\text{eV}^2)^m}. \]
and
\[ m_1^2 - m_2^2 = f^2 \pi m_\pi G_B. \]

The expression for probability of \( \pi \to K \) oscillations \( P(\pi \to K, t) \), in the approach where the phase volume is taken into account, has the following form [4,5]:

\[ N(\pi \to K, t) = N(\pi, t) \sin^2 \left[ \frac{\pi t}{\tau(\pi \to B \to K)} \right] = \]
\[ = N(\pi, 0) \exp \left( - \frac{t}{\tau_0} \right) \sin^2 \left[ \frac{\pi t}{\tau_0} \frac{m_\pi^4}{m_B^4} \right] \left( \frac{m_u}{m_d} \right)^2. \] \hspace{1cm} (32)

Probability of \( \pi \to K \) real oscillations \( P(\pi \to K, t) \) in the case of real oscillations is described by the following expression (see Eq. (8)) [4,5]:

\[ P(\pi \to K, t) = \sin^2 2\beta \sin^2 \left[ \pi t \frac{m_K^2}{2p} \right], \]
where
\[ \sin^2 2\beta \simeq \frac{4m_{\pi K}^2}{(m_\pi - m_K)^2} \simeq 0. \]

The kinematics of \( K \) meson production processes in quasielastic processes is given in work [4].

6 Conclusion

The elements of the theory of vacuum oscillations and the model of dynamical expansion of the theory of weak interactions works at the tree level, i.e. the model of dynamical analogy of Cabibbo-Kobayashi-Maskawa matrices and its further development, were given. It was shown that the quarks and massive vector bosons must be structural and these
structural particles (subparticles) must interact to generate quark and vector boson masses. In this case the problem of singularity cancellations does not arise in this model. It was also shown that, for self consistency of the theory, the weak decays of $K$ mesons must go through massive vector boson $B$ but not $W$ boson.

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