gl(4|4) current algebra: free field realization and screening currents

Wen-Li Yang \textsuperscript{a,\textbullet}, Yao-Zhong Zhang \textsuperscript{b,c} and Xin Liu \textsuperscript{b}

\textsuperscript{a} Institute of Modern Physics, Northwest University, Xian 710069, P.R. China
\textsuperscript{b} Department of Mathematics, University of Queensland, Brisbane, QLD 4072, Australia
\textsuperscript{c} Physikalisches Institut, Universität Bonn, D-53115 Bonn, Germany

Abstract

The gl(4|4) current algebra at general level $k$ is investigated. Its free field representation and corresponding energy-momentum tensor are constructed. Seven screening currents of the first kind are also presented.

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1 Introduction

Two-dimensional non-linear sigma models with supermanifold target space naturally appear in the quantization of superstring theory on the AdS-type backgrounds. It was shown that the sigma model on the supergroup $PSU(1,1|2)$ can be used for quantizing the superstring theory on the $AdS_3 \times S^3$ background with Ramond-Ramond (RR) flux [1, 2]. It has been believed [3, 2] that the sigma model on $PSU(2,2|4)$ is related to the string theory on the $AdS_5 \times S^5$ background and that the understanding of the $PSU(2,2|4)$ sigma model (or its generalization, $GL(4|4)$-WZNW model) would shed important lights on the quantization of the string theory on the $AdS_5 \times S^5$ background. The study of the non-linear sigma model with supermanifold target spaces has also believed to be relevant for disordered systems and the integer quantum Hall transition [4, 5, 6, 7, 8].

In most circumstances, models of interest are believed to be more complicated than the WZNW models on supergroups. However, even the WZNW models on supergroups are far from being understood [9]. This is largely due to technical reasons (such as indecomposability of the operator product expansion (OPE), appearance of logarithms in correlation functions and continuous modular transformations of the irreducible characters [10]), combined with the lack of “physical intuition”.

Free field realization [11] has been proved to be a powerful method in the study the conformal field theory (CFT) such as WZNW models. In this letter, motivated by the applications both to superstring theory and condensed matter physics, we investigate the gl$(4|4)$ current algebra associated with the $GL(4|4)$ WZNW model at general level $k$.

2 gl$(4|4)$ current algebra

Let us start with some basic notation of the gl$(4|4)$ current algebra, i.e. the gl$(4|4)$ affine superalgebra [12, 13]. Let $\{E_{i,j} \mid i,j = 1, \ldots, 8\}$ be the generators of the finite dimensional superalgebra gl$(4|4)$. The generators obey the (anti-)commutation relations:

$$[E_{i,j}, E_{k,l}] = \delta_{j,k}E_{i,l} - (-1)^{(i+j)(k+l)}\delta_{i,l}E_{k,j}. \quad (2.1)$$

Here and throughout, we adopt the convention: $[a, b] = ab - (-1)^{|a||b|}ba$. The $\mathbb{Z}_2$ grading of the generators is

$$[E_{i,j}] = [i] + [j].$$
where \([1] = \ldots = [4] = 0, [5] = \ldots = [8] = 1\). Due to the non-simplicity of \(gl(4|4)\), besides the usually quadratic Casimir element

\[
C_1 = \sum_{i,j=1}^{8} (-1)^{|j|} E_{i,j} E_{j,i},
\]

(2.2)

there exists another independent quadratic Casimir element

\[
C_2 = \sum_{i,j=1}^{8} E_{i,i} E_{j,j} = \left( \sum_{i=1}^{8} E_{i,i} \right)^2.
\]

(2.3)

These two Casimir elements are useful in the following for the construction of the energy-momentum tensor.

Let \(V\) be a \(\mathbb{Z}_2\)-graded (4+4)-dimensional linear space with the orthonormal basis \(\{\langle i \rangle, i = 1, \ldots, 8\}\). The \(\mathbb{Z}_2\)-grading is chosen as: \([1] = \ldots = [4] = 0, [5] = \ldots = [8] = 1\). In fact \(V\) spans the fundamental representation of \(gl(4|4)\), namely, the generators of the algebra can be realized on \(V\) by

\[E_{i,j} = e_{i,j}, \quad i, j = 1, \ldots, 8,\]

where \(e_{i,j}\) is the matrix with entry 1 at the \(i\)-th row and \(j\)-th column, and zero elsewhere. We remark that the Casimir element \(C_1\) vanishes on \(V\), while \(C_2\) acts as a multiple of identity on \(V\). From the fundamental representation of \(gl(4|4)\), one can define the consistent, supersymmetric and invariant inner product [12, 13] as follows,

\[(E_{i,j}, E_{k,l}) = \text{str} \left( e_{i,j} e_{k,l} \right).
\]

Here \(\text{str}\) denotes the supertrace, i.e., \(\text{str}(a) = \sum_i (-1)^{|i|} a_{i,i}\).

The \(gl(4|4)\) current algebra is generated by the currents \(J_{i,j}(z)\) which are associated with the generators \(E_{i,j}\) of \(gl(4|4)\). The current algebra at a general level \(k\) obeys the following OPEs [11],

\[
J_{i,j}(z) J_{l,m}(w) = \frac{k (E_{i,j}, E_{l,m})}{(z-w)^2} + \frac{1}{(z-w)} \left( \delta_{j,l} J_{i,m}(w) - (-1)^{|i|+|j|} \delta_{i,m} J_{l,j}(w) \right).
\]

(2.5)

### 3 Free field realization

Free field realization of the \(gl(4|4)\) currents, in principle, can be obtained by a general method outlined in [14, 15, 16, 17, 18], where differential realizations of the corresponding finite
dimensional Lie (super) algebras play a key role. However, their constructions become very complicated for higher-rank algebras \[16,18,19,20\]. We have found a way to obtain explicit expressions of the differential realizations of \(gl(m|n)\). In our approach, the construction becomes much simpler. In this letter we will restrict our attention mainly to \(gl(4|4)\), as this already illustrates the main features. Generalizations to other algebras such as \(gl(m|n)\), more details and proofs, will be given elsewhere \[21\].

Let us introduce 12 bosonic coordinates \(\{x_{i,j}, x_{4+i,4+j}\ 1 \leq i < j \leq 4\}\) with the \(\mathbb{Z}_2\)-grading: \([x_{i,j}] = 0\), and 16 fermionic coordinates \(\{\theta_{i,4+j}\ i,j = 1, \ldots, 4\}\) with the \(\mathbb{Z}_2\)-grading: \([\theta_{i,4+j}] = 1\). These coordinates satisfy the following (anti-)commutation relations:

\[ [x_{i,j}, x_{k,l}] = 0, \ [\partial_{x_{i,j}}, x_{k,l}] = \delta_{ik}\delta_{jl}, \]
\[ [\theta_{i,4+j}, \theta_{k,4+l}] = 0, \ [\partial_{\theta_{i,4+j}}, \theta_{k,4+l}] = \delta_{ik}\delta_{jl}, \]

and the other commutation relations are vanishing. Then the generators corresponding to the simple roots in the standard (distinguished) basis \[13\] can be realized by the following differential operators:

\[
E_{j,j+1} = \sum_{k \leq j-1} x_{k,j} \partial_{x_{k,j+1}} + \partial_{x_{j,j+1}}, \quad 1 \leq j \leq 3, \quad (3.1)
\]
\[
E_{4,5} = \sum_{k \leq 3} x_{k,4} \partial_{\theta_{k,5}} + \partial_{\theta_{k,5}}, \quad (3.2)
\]
\[
E_{4+j,5+j} = \sum_{k \leq 4} \theta_{k,4+j} \partial_{\theta_{k,5+j}} + \sum_{k \leq j-1} x_{4+k,4+j} \partial_{x_{4+k,5+j}} + \partial_{x_{4+j,5+j}}, \quad 1 \leq j \leq 3, \quad (3.3)
\]
\[
E_{j,j} = \sum_{k \leq j-1} x_{k,j} \partial_{x_{k,j}} - \sum_{j+1 \leq k \leq 4} x_{j,k} \partial_{x_{j,k}} - \sum_{k \leq 4} \theta_{j,4+k} \partial_{\theta_{j,4+k}} + \lambda_j, \quad 1 \leq j \leq 3, \quad (3.4)
\]
\[
E_{4,4} = \sum_{k \leq 3} x_{k,4} \partial_{x_{4,k}} - \sum_{k \leq 4} \theta_{4+k} \partial_{\theta_{4+k}} + \lambda_4, \quad (3.5)
\]
\[
E_{4+j,4+j} = \sum_{k \leq 4} \theta_{4+k} \partial_{\theta_{4+k}} + \sum_{k \leq j-1} x_{4+k,4+j} \partial_{x_{4+k,4+j}}
- \sum_{j+1 \leq k \leq 4} x_{4+j,4+k} \partial_{x_{4+j,4+k}} + \lambda_{4+j}, \quad 1 \leq j \leq 4, \quad (3.6)
\]
\[
E_{j+1,j} = \sum_{k \leq j-1} x_{k,j+1} \partial_{x_{k,j}} - \sum_{j+2 \leq k \leq 4} x_{j,k} \partial_{x_{j+1,k}} - \sum_{k \leq 4} \theta_{j,4+k} \partial_{\theta_{j,4+k}}
- \sum_{j+1 \leq k \leq 4} x_{j,4+k} \partial_{x_{j+1,4+k}}
+ \sum_{j+2 \leq k \leq 4} x_{j+1,k} \partial_{x_{j+1,k}} + \sum_{k \leq 4} \theta_{j+1,4+k} \partial_{\theta_{j+1,4+k}}
\]
\[ E_{5,4} = \sum_{k \leq 3} \theta_{k,5} \partial_{x_k,4} + \sum_{2 \leq k \leq 4} \theta_{4,4+k} \partial_{x_{5,4+k}} - \theta_{4,5} \left( \sum_{2 \leq k \leq 4} (\theta_{4,4+k} \partial_{\theta_{4,4+k}} + x_{5,4+k} \partial_{x_{5,4+k}}) \right) + \theta_{4,5} (\lambda_4 + \lambda_5). \] (3.7)

\[ E_{5+j,4+j} = \sum_{k \leq 4} \theta_{k,5+j} \partial_{\theta_{k,4+j}} + \sum_{k \leq j-1} x_{4+k,5+j} \partial_{x_{4+k,4+j}} - \sum_{j+2 \leq k \leq 4} x_{4+j,4+k} \partial_{x_{5+j,4+k}} - x_{4+j,5+j} \left( \sum_{j+1 \leq k \leq 4} x_{4+j,4+k} \partial_{x_{4+j,4+k}} - \sum_{j+2 \leq k \leq 4} x_{5+j,4+k} \partial_{x_{5+j,4+k}} \right) + x_{4+j,5+j} (\lambda_{4+j} - \lambda_{5+j}), \quad 1 \leq j \leq 3, \] (3.8)

\[ E_{5+j,4+j} = \sum_{k \leq 4} \theta_{k,5+j} \partial_{\theta_{k,4+j}} + \sum_{k \leq j-1} x_{4+k,5+j} \partial_{x_{4+k,4+j}} - \sum_{j+2 \leq k \leq 4} x_{4+j,4+k} \partial_{x_{5+j,4+k}} - x_{4+j,5+j} \left( \sum_{j+1 \leq k \leq 4} x_{4+j,4+k} \partial_{x_{4+j,4+k}} - \sum_{j+2 \leq k \leq 4} x_{5+j,4+k} \partial_{x_{5+j,4+k}} \right) + x_{4+j,5+j} (\lambda_{4+j} - \lambda_{5+j}), \quad 1 \leq j \leq 3, \] (3.9)

where \( \{\lambda_j | j = 1, \ldots, 8\} \) are c-numbers which label the lowest weight vector of \( gl(4|4) \). The other generators associated with the non-simple roots can be constructed through the simple ones by the commutation relations,

\[ E_{i,j} = [E_{i,k}, E_{k,j}], \quad 1 \leq i < k < j \leq 8 \text{ and } 2 \leq j - i, \] (3.10)

\[ E_{j,i} = [E_{j,k}, E_{k,i}], \quad 1 \leq i < k < j \leq 8 \text{ and } 2 \leq j - i. \] (3.11)

One can prove that the differential realization (3.1)-(3.9) of \( gl(4|4) \) satisfies the commutation relations (2.1).

With the help of the differential realization we can obtain the free field realization (Wakimoto construction) of the \( gl(4|4) \) current algebra in terms of 12 bosonic \( \beta-\gamma \) pairs ((\( \beta_{i,j}, \gamma_{i,j} \)) and (\( \beta_{i,j}^\dagger, \gamma_{i,j} \)), for \( 1 \leq i < j \leq 4 \), 16 fermionic \( b-c \) pairs ((\( \psi_{i,j}^\dagger, \psi_{i,j} \)), for \( 1 \leq i, j \leq 4 \) and 8 free scalar fields (\( \phi_i \), for \( i = 1, \ldots, 8 \)). The free fields obey the following OPEs:

\[ \beta_{i,j}(z) \gamma_{k,l}(w) = -\gamma_{k,l}(z) \beta_{i,j}(w) = \frac{\delta_{ik}\delta_{jl}}{(z-w)}, \quad 1 \leq i < j \leq 4 \text{ and } 1 \leq k < l \leq 4, \] (3.12)

\[ \tilde{\beta}_{i,j}(z) \gamma_{k,l}(w) = -\gamma_{k,l}(z) \tilde{\beta}_{i,j}(w) = \frac{\delta_{ik}\delta_{jl}}{(z-w)}, \quad 1 \leq i < j \leq 4 \text{ and } 1 \leq k < l \leq 4, \] (3.13)

\[ \psi_{i,j}(z)\psi_{k,l}^\dagger(w) = \psi_{k,l}^\dagger(z)\psi_{i,j}(w) = \frac{\delta_{ik}\delta_{jl}}{(z-w)}, \quad 1 \leq i, j \leq 4 \text{ and } 1 \leq k, l \leq 4, \] (3.14)

\[ \phi_i(z)\phi_j(w) = (-1)^{|i|} \delta_{ij} \ln(z-w), \quad 1 \leq i, j \leq 8, \] (3.15)

and the other OPEs are trivial.

The free field realization of the \( gl(4|4) \) current algebra (2.5) is obtained by the following
in the differential realization (3.11)-(3.9) of \(gl(4|4)\) and a subsequent addition of anomalous terms linear in \(\partial\psi^\dagger(z)\), \(\partial\gamma(z)\) and \(\partial\bar{\gamma}(z)\) in the expressions of the currents. Here we present the realization of the currents associated with the simple roots,

\[
\begin{align*}
J_{j,j+1}(z) &= \sum_{l\leq j-1} \gamma_{l,j}(z)\beta_{l,j+1}(z) + \beta_{j,j+1}(z), \quad 1 \leq j \leq 3, \\
J_{4,5}(z) &= \sum_{l\leq 3} \gamma_{4,l}(z)\psi_{l,1}(z) + \psi_{4,1}(z), \\
J_{4+j,5+j}(z) &= \sum_{l\leq 4} \psi^\dagger_{l,j}(z)\psi_{l,j+1}(z) + \sum_{l\leq j-1} \bar{\gamma}_{l,j}(z)\bar{\beta}_{l,j+1}(z) + \bar{\beta}_{j,j+1}(z), \quad 1 \leq j \leq 3, \\
J_{j,j}(z) &= \sum_{l\leq j-1} \gamma_{l,j}(z)\beta_{l,j}(z) - \sum_{j+1 \leq l \leq 4} \gamma_{j,l}(z)\beta_{j,l}(z) - \sum_{l\leq 4} \psi^\dagger_{j,l}(z)\psi_{j,l}(z) \\
&\quad + \sqrt{k}\partial\phi_j(z) - \frac{1}{2\sqrt{k}} \sum_{l=1}^8 \partial\phi_l(z), \quad 1 \leq j \leq 3, \\
J_{4,4}(z) &= \sum_{l\leq 3} \gamma_{4,l}(z)\beta_{4,l}(z) - \sum_{l\leq 4} \psi^\dagger_{4,l}(z)\psi_{4,l}(z) + \sqrt{k}\partial\phi_4(z) \\
&\quad - \frac{1}{2\sqrt{k}} \sum_{l=1}^8 \partial\phi_l(z), \\
J_{4+j,4+j}(z) &= \sum_{l\leq 4} \psi^\dagger_{l,j}(z)\psi_{l,j}(z) + \sum_{l\leq j-1} \bar{\gamma}_{l,j}(z)\bar{\beta}_{l,j}(z) - \sum_{j+1 \leq l \leq 4} \bar{\gamma}_{j,l}(z)\bar{\beta}_{j,l}(z) \\
&\quad + \sqrt{k}\partial\phi_{4+j}(z) + \frac{1}{2\sqrt{k}} \sum_{l=1}^8 \partial\phi_l(z), \quad 1 \leq j \leq 4, \\
J_{j+1,j}(z) &= \sum_{l\leq j-1} \gamma_{l,j+1}(z)\beta_{l,j}(z) - \sum_{j+2 \leq l \leq 4} \gamma_{j,l}(z)\beta_{j+1,l}(z) - \sum_{l\leq 4} \psi^\dagger_{j,l}(z)\psi_{j+1,l}(z) \\
&\quad - \gamma_{j,j+1}(z) \left( \sum_{j+1 \leq l \leq 4} \gamma_{j,l}(z)\beta_{j,l}(z) + \sum_{l\leq 4} \psi^\dagger_{j,l}(z)\psi_{j,l}(z) \right) \\
&\quad + \gamma_{j,j+1}(z) \left( \sum_{j+2 \leq l \leq 4} \gamma_{j+1,l}(z)\beta_{j+1,l}(z) + \sum_{l\leq 4} \psi^\dagger_{j+1,l}(z)\psi_{j+1,l}(z) \right)
\end{align*}
\]

\(\beta_{i,j}(z)\) in the expressions of the currents. Here we present the realization of the currents associated with the simple roots,
In order to apply the free field realization of the \( gl(4|4) \) currents to compute conformal blocks of the associated WZNW-conformal field theory, we need to calculate the energy-momentum tensor of the associated CFT. The Sugawara tensor corresponding to the quadratic Casimir \( C_1 \) is given by

\[
T_1(z) = \frac{1}{2k} \sum_{i,j=1}^{8} (-1)^{|j|} : J_{i,j}(z) J_{j,i}(z) :
\]

\[
= \frac{1}{2} \sum_{l=1}^{8} (-1)^{|l|} \partial \phi_l(z) \partial \phi_l(z) + \frac{1}{2k} \frac{1}{2} \partial^2 \left( \sum_{i=1}^{4} (2i - 1) \left( \phi_i(z) + \phi_{9-i}(z) \right) \right)
\]

\[
+ \sum_{i=1}^{4} \left( \partial \gamma_{i,j}(z) \beta_{i,j}(z) + \partial \tilde{\gamma}_{i,j}(z) \tilde{\beta}_{i,j}(z) \right) + \sum_{i,j=1}^{4} \partial \psi^\dagger_{i,j}(z) \psi_{i,j}(z)
\]

\[
- \frac{1}{2k} \partial \bar{\phi}(z) \partial \bar{\phi}(z),
\] (4.1)

Here normal ordering of the free field expressions is implied. The free field realization for other currents associated with the non-simple roots can be obtained from the OPEs of the simple ones. It is straightforward to check that the above free field realization of the currents satisfy the OPEs of the \( gl(4|4) \) current algebra given in the last section.

### 4 Energy-momentum tensor

In order to apply the free field realization of the \( gl(4|4) \) currents to compute conformal blocks of the associated WZNW-conformal field theory, we need to calculate the energy-momentum tensor of the associated CFT. The Sugawara tensor corresponding to the quadratic Casimir \( C_1 \) is given by

\[
J_{5.4}(z) = \sum_{l=3}^{4} \psi^\dagger_{l,4}(z) \beta_{l,4}(z) + \sum_{2 \leq l \leq 4} \psi^\dagger_{4,l}(z) \tilde{\beta}_{4,l}(z)
\]

\[
- \psi^\dagger_{4,1}(z) \left( \sum_{2 \leq l \leq 4} \left( \psi^\dagger_{4,l}(z) \psi_{4,l}(z) + \tilde{\gamma}_{1,l}(z) \tilde{\beta}_{1,l}(z) \right) \right)
\]

\[
+ \sqrt{k} \psi^\dagger_{4,1}(z) \left( \partial \phi_4(z) + \partial \phi_5(z) \right) + (k + 3) \partial \psi^\dagger_{4,1}(z),
\] (3.23)

\[
J_{5+j,4+j}(z) = \sum_{l=4}^{4} \psi^\dagger_{l,j+1}(z) \psi_{l,j}(z) + \sum_{l=4}^{4} \tilde{\gamma}_{l,j+1}(z) \tilde{\beta}_{l,j}(z) - \sum_{j+2 \leq l \leq 4} \tilde{\gamma}_{j,l}(z) \tilde{\beta}_{j+1,l}(z)
\]

\[
- \tilde{\gamma}_{j,j+1}(z) \left( \sum_{j+1 \leq l \leq 4} \tilde{\gamma}_{j,l}(z) \tilde{\beta}_{j,l}(z) - \sum_{j+2 \leq l \leq 4} \tilde{\gamma}_{j+1,l}(z) \tilde{\beta}_{j+1,l}(z) \right)
\]

\[
+ \sqrt{k} \tilde{\gamma}_{j,j+1}(z) \left( \partial \phi_{4+j}(z) - \partial \phi_{5+j}(z) \right) - (k + 5 - j) \partial \tilde{\gamma}_{j,j+1}(z),
\] (3.24)
where \( \bar{\phi}(z) = \sum_{l=1}^{8} \phi_l(z) \) and the normal ordering of the free field expressions is implicit. On the other hand, the Sugawara tensor corresponding to the quadratic Casimir \( C_2 \) is given by

\[
T_2(z) = \frac{1}{2k} \sum_{i,j=1}^{8} \phi_i(z) \phi_j(z) := \frac{1}{2} : \partial \bar{\phi}(z) \partial \phi(z) : .
\] (4.2)

In order that all currents \( J_{i,j}(z) \) are primary fields with conformal dimensional one, we define the energy-momentum tensor \( T(z) \) as follow:

\[
T(z) = T_1(z) + \frac{1}{k} T_2(z)
= \frac{1}{2} \sum_{i=1}^{8} (-1)^{i+1} \partial \phi_i(z) \partial \phi_i(z) + \frac{1}{2\sqrt{k}} \partial^2 \left( \sum_{i=1}^{4} (2i - 1) (\phi_i(z) + \phi_{9-i}(z)) \right)
+ \sum_{1 \leq i < j} \left( \partial \gamma_{i,j}(z) \beta_{i,j}(z) + \partial \gamma_{i,j}(z) \beta_{i,j}(z) \right) + \sum_{i,j=1}^{4} \partial \psi_{i,j}^\dagger(z) \psi_{i,j}(z),
\] (4.3)

where the normal ordering of the free field expressions is implicit. We find

\[
T(z)T(w) = \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{(z-w)},
\] (4.4)

with a center charge \( c = 0 \). Moreover, it is easy to check that

\[
T(z)J_{i,j}(w) = \frac{J_{i,j}(w)}{(z-w)^2} + \frac{\partial J_{i,j}(w)}{(z-w)}, \quad 1 \leq i, j \leq 8.
\] (4.5)

Therefore, \( T(z) \) is the energy-momentum tensor of the \( gl(4|4) \) current algebra.

### 5 Screening currents

An important object in applying the free field realization to the computation of correlation functions of the associated CFT is screening currents. A screening current is a primary field with conformal dimension one and has the property that the singular part of the OPE with the affine currents is a total derivative. These properties ensure that integrated screening currents (screening charges) may be inserted into correlators while the conformal or affine Ward identities remain intact. This in turn makes them very useful in computation of the correlation functions \[22, 23\]. For the present case, we find seven screening currents

\[
S_j(z) = \left( \sum_{j+2 \leq l \leq 4} \gamma_{j+1,l}(z) \beta_{j,l}(z) + \sum_{l=1}^{4} \psi_{j+1,l}^\dagger(z) \psi_{j,l}(z) + \beta_{j,j+1}(z) \right) \bar{s}_j(z),
\]
\[1 \leq j \leq 3,\]
\[S_4(z) = \left( \sum_{2 \leq l} \bar{\gamma}_{1,l}(z) \psi_{4,l}(z) + \psi_{4,1}(z) \right) \tilde{s}_4(z),\]  
(5.1)
\[S_{4+j}(z) = \left( \sum_{j+2 \leq l \leq 4} \bar{\gamma}_{j+1,l}(z) \bar{\beta}_{j,l}(z) + \bar{\beta}_{j,j+1}(z) \right) \tilde{s}_{4+j}(z), \quad 1 \leq j \leq 3,\]  
(5.2)
where
\[\tilde{s}_j(z) = e^{-\frac{1}{\sqrt{k}}(\phi_j(z)-\phi_{j+1}(z))}, \quad \tilde{s}_4(z) = e^{-\frac{1}{\sqrt{k}}(\phi_4(z)+\phi_5(z))}, \quad \tilde{s}_{4+j}(z) = e^{\frac{1}{\sqrt{k}}(\phi_{4+j}(z)-\phi_{5+j}(z))}.\]  
(5.4)

The normal ordering of the free field expressions is implicit in the above equations. The nontrivial OPEs of the screening currents with the energy-momentum tensor and the \( gl(4|4) \) currents (3.16)-(3.24) are
\[T(z)S_j(w) = \frac{S_j(w)}{(z-w)^2} + \frac{\partial S_j(w)}{(z-w)} = \partial_w \left\{ \frac{S_j(w)}{(z-w)} \right\}, \quad 1 \leq j \leq 7,\]  
(5.5)
\[J_{i+1,i}(z)S_j(w) = (-1)^{[i]+[i+1]} \delta_{ij} \partial_w \left\{ \frac{k \tilde{s}_j(w)}{(z-w)} \right\}, \quad 1 \leq i, j \leq 7.\]  
(5.6)

The screening currents obtained here are associated with the simple roots and correspond to the first kind screening currents [24]. Moreover, \( S_4(z) \) is fermionic screening current and the others are all bosonic ones.

\section{6 Discussions}

We have studied the \( gl(4|4) \) current algebra at general level \( k \). We have constructed its Wakimoto free field realization (3.16)-(3.24) and the corresponding energy-momentum tensor (4.3). We have also found seven screening currents, (5.1)-(5.4), of the first kind.

To fully take the advantage of the CFT method, one needs to construct its primary fields. It is well-known that there exist two types of representations for the underlying finite dimensional superalgebra \( gl(4|4) \): typical and atypical representations. Atypical representations have no counterpart in the bosonic algebra setting and the understanding of such representations is still very much incomplete. Although the construction of the primary fields associated with typical representations are similar to the bosonic algebra cases, it is a highly non-trivial task to construct the primary fields associated with atypical representations [20].
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Note added: We became aware that free field realizations of $sl(N|N)$ (or $gl(N|N)$) current algebra were investigated previously in [25, 26]. There, the $sl(N|N)_k$ (or $gl(N|N)_k$) currents were expressed in terms of the $sl(N)_{k-N}$ and $sl(N)_{-k-N}$ currents and some $b$-$c$ pairs. As part of the results of our paper, we give the explicit expressions of $gl(4|4)_k$ currents in terms of free fields, by using a different method.

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