An Experimental Study of Adaptive Control for Evolutionary Algorithms

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Abstract

The balance of exploration versus exploitation (EvE) is a key issue on evolutionary computation. In this paper we will investigate how an adaptive controller aimed to perform Operator Selection can be used to dynamically manage the EvE balance required by the search, showing that the search strategies determined by this control paradigm lead to an improvement of solution quality found by the evolutionary algorithm.

Keywords: Algorithms, Design Experimentation, Measurement, Performance

1. Introduction

During the past decades, Evolutionary Algorithms (EAs) [Holland, 1975; Goldberg, 1989; Eiben and Smith, 2003] have been successfully applied to many optimization problems. From a high level point of view, EAs manage a set of potential solutions of a problem – a population of individuals according to the evolutionary metaphor. The population is progressively modified by variation operators in order to converge to an optimal solution with regards
to a fitness function, which evaluates the quality of the individuals. Two well-known concepts are commonly used to describe the behavior of an EA: exploitation – which reflects the ability of the algorithm to converge to an optimum – and exploration – which insures that the algorithm is able to visit sufficiently sparse areas of the search space. The balance between exploration and exploitation (referred to as EvE) is widely recognized as a key issue of the overall search performance. This balance often relies on the adjustment of several parameters (e.g., size of the population and application rates of the different operators).

Significant progress has been achieved in parameter setting (Lobo et al., 2007). Following the taxonomy proposed by Eiben et al. (1999), tuning techniques adjust the parameters of the algorithm before the run while control techniques modify the behavior of the algorithm during the search process. Efficient tuning methods are now available using statistical tools such as racing techniques (Birattari et al., 2002) or meta-algorithms that explore the parameters’ space (e.g., ParamILS (Hutter et al., 2007) or Revac (Nannen et al., 2008)). Control techniques have also been proposed in order to provide adaptive or self-adaptive EAs (Eiben et al., 2007).

In this paper, we focus on Adaptive Operator Selection (AOS) methods (Maturana et al., 2012) from the control point of view: the operator selection problem consists in selecting, out of a set of available operators, which one should be applied at a given iteration of the evolutionary process. The aim of AOS is to control the EvE balance in order to improve search efficiency. Nevertheless, in most of the related works (see section 2), the control of the EvE balance has been only partially investigated. Most of the approaches focus on exploitation and use the quality of the population as a unique criterion to guide the search (Thierens, 2007; Gong et al., 2010; Fialho et al., 2008).

Furthermore, there are few works that use several criteria to assess the utility of the operators (Maturana and Saubion, 2008), but in these works the EvE balance is kept fixed. Since it has been shown that an efficient algorithm requires different parameter values during the search for achieving better results (Linhares and Yanasse, 2010), the EvE balance should be dynamically controlled.

The purpose of our work is twofold. Firstly, we investigate the management of more dynamic control strategies. The framework proposed by
Maturana et al. (2010) is used to implement a generic controller. This controller must thus identify the suitable operators at each step of search in order to achieve the required EvE balance, which may change dynamically according to a given control strategy. Then we want to assess the impact of dynamic control on the performance of the algorithm. Our experimental methodology is organized as follows:

1. **Assessing the operators management:**
   - by assessing whether the controller is able to identify the required operators in presence of non-efficient operators, i.e., in presence of noisy operators;
   - by checking whether the controller is able to manage a policy in which the desired EvE balance is modified along the search.

2. **Evaluating the solving performances:**
   - by checking whether the controlled EA is able to solve problems efficiently with regards to existing algorithms.

In this paper we recall the main literature on the topic on section 2 before describing the controller in section 3. Then, we introduce the experimental setting in section 4 before discussing results obtained through the experimental phase: section 5 focuses on the management of the operators, and solving performance is investigated in section 6.

2. **Related Works**

Parameter setting (Lobo et al., 2007; Eiben and Smit, 2012) is an important challenge for building efficient and robust EAs. As mentioned in the introduction, using an EA requires us to define its basic structural components and to set the values of its behavioral parameters. The components may be considered as structural parameters of the algorithm. Therefore, parameter setting in EA addresses two general classes of parameters: *structural* and *behavioral* (alternatively, the terms *numerical* and *symbolic* parameters

\footnote{In this paper, we call controller, the complete architecture that allows us to perform adaptive operator selection.}
are used (Smit and Eiben, 2009). Concerning structural parameters, automated tuning techniques (Hoos, 2012) can be used as tools for selecting the initial configuration of the algorithm. The configuration and the discovery of new heuristics from building blocks is also addressed by the concept of hyper-heuristics (Burke et al., 2010). We may also mention self-adaptive operators that mainly consists in encoding directly the parameters of the operator in the individuals. This approach also allows the algorithm to dynamically manage the EvE balance and has been successfully applied for solving combinatorial and continuous optimization problems [Zhang and Sanderson (2009); Tang et al. (2014); Qin et al. (2009); Tang and Wang (2013)]. Note that an adaptive management of the operators, which dynamically adds and discards operators during the search, has been proposed by Maturana et al. (2010).

In this paper, we focus on behavioral parameters and we limit our investigation to the Adaptive Operator Selection. AOS can be seen as the choice of the best policy for selecting the operators during the search and different methods have been proposed to this goal.

Let us consider n operators: the probability of selecting operator $op_i$ at time $t$ is $s_i(t)$. In a static setting, the probability of selecting $op_i$ (for each $i$) is fixed over time (i.e., $s_i(t) = s_i(t')$, for any $t$ and $t' \in [1, t_{max}]$), and can be determined by an automated tuning process. Contrary to a static tuning of the operator application rates, adaptive operator selection consists in selecting the next operator to apply at time $t+1$ by adapting the selection probability during the search according to the performance of the operators. Let us consider an estimated utility $u_i(t)$ of operator $op_i$ at time $t$. This utility of the operators has to be re-evaluated at each time, classically using a formula $u_i(t+1) = (1-\alpha)u_i(t) + \alpha r_i$ where $r_i$ is the reward associated to the application of operator $op_i$ (immediate performance) and $\alpha$ is a coefficient that control the balance between past and immediate performance, as done in classic reinforcement learning techniques (Sutton and Barto, 1998). Note that $\alpha$ can be set to $\frac{1}{t+1}$ in order to compute the mean value. A classic selection mechanism is the probability matching selection rule (PM) and can be formulated as:

$$s_i(t+1) = p_{\text{min}} + (1 - n \times p_{\text{min}}) \frac{u_i(t+1)}{\sum_{k=1}^{n} u_k(t+1)}.$$  \hspace{1cm} (1)

where a non negative $p_{\text{min}}$ insures a non zero selection probability for all operators (Goldberg, 1990; Lobo and Goldberg, 1997).
Thierens (Thierens, 2005, 2007) has explored a winner-take-all strategy for AOS, based on the quality (or fitness) of the population:

\[
\begin{align*}
   s_{i^*}(t+1) &= s_{i^*}(t) + \beta (p_{\text{max}} - s_{i^*}(t)) \\
   s_i(t+1) &= s_i(t) + \beta (p_{\text{min}} - s_i(t))
\end{align*}
\]

where \( i^* = \text{argmax}\{u_i(t), i = 1..n\} \), \( p_{\text{max}} = 1 - (n-1)p_{\text{min}} \) and \( \beta \) is a parameter to adjust balance of this winner-take-all strategy.

Alternatively, AOS can also be considered as a multi-armed bandit problem, which is a classic reinforcement learning problem (Sutton and Barto, 1998). The initial multi-armed bandit problem was introduced in the context of the experiment design by (Robbins, 1952). It was formulated as the maximization of the total gain of a gambler who could make \( n \) tosses with two coins \( A \) and \( B \) with a gain of 1 for each head but nothing for tails. The biases of the coins are unknown. This problem is known as the Two-armed Bandit and has been extended to multi-armed bandit by Rodman (Rodman, 1978). Later, Auer (Auer, 2002) has proposed to use this problem to manage the compromise between exploration and exploitation in optimization algorithms. The MAB (Multi-Armed Bandit) algorithms that uses an UCB (Upper Confidence Bound) in order to approximate the expected benefit of an operator \( op_i \) at time \( t \) have been firstly extended to AOS by Da Costa et al. (2008): the operator that maximizes \( Mab_i(t) \) in the following formula is selected:

\[
Mab_i(t) = u_i(t) + C \sqrt{\frac{\log \sum_{j \in 1..n} n_j(t)}{n_i(t)}},
\]

where \( r_i(t) \) is the reward of operator \( op_i \) at time \( t \), \( n_i(t) \) is the number of times operator \( op_i \) has been applied so far, and \( C \) is the scaling factor used to properly balance rewards and application frequency. In the initial multi-armed bandit problem, the expected gain of each possible action is supposed to be fixed over time. Therefore, in Da Costa et al. (2008), the authors propose to use a Page-Hinkley test in order to detect a change of the behavior of the operators and thus to reset \( r_i(t) \) and \( n_i(t) \). In Fialho et al. (2010b), an improved technique has been proposed for comparing the respective performance of the operators.

Note that the equation \( Mab_i(t) \) uses \( n_i(t) \) as a way to avoid forgetting less favorable operators, supposing that all operators were included from the start of the search. Indeed, if one of them were introduced to the eligible set in
the middle of the search, it would be necessary to apply the operator several times to catch up with respect to the other ones. This would imply a waste of time and an eventual degradation of the search if the new operator would not be suited to the current search requirements. In order to deal with this situation, a variation of the AOS was proposed in (Maturana et al., 2010) that considers idle time instead of the number of times an operator has been applied.

Focusing on the performance measures, Whitacre et al. (2006) consider extreme values over a few applications of the operators, based on the idea that highly beneficial but rare events might be more beneficial than regular but smaller improvements.

Most works rely on quality as the only criterion used for control. Nevertheless, EA literature has constantly been concerned with maintaining the diversity of the population in order to avoid premature convergence (McKay, 2000). Therefore, Maturana and Saubion (2008) have proposed another AOS method, which manages simultaneously the mean quality and the diversity of the population: these two criteria are clearly related to the exploitation and the exploration of the search process. The impact of an operator is thus recorded in two sliding time windows and used to select the next operator according to a given search trajectory, which is defined in this two-dimensional performance space.

Maturana et al. (2009) have evaluated several combinations of various control components using ideas from Fialho et al. (2008), Da Costa et al. (2008), and Maturana and Saubion (2008). These works have investigated different methods for rewarding operators according to their performances, and different operator selection techniques for choosing the most suitable operator according to its past rewards.

In all these works the balance between these criteria, which can be seen as an abstraction of the exploration-exploitation balance, is set according to a fixed and predefined search policy. In this paper, instead, we want to explore alternate possibilities offered by this powerful AOS framework in order to provide a more dynamic management of the algorithm’s behavior with regards to this balance.

3. A Generic Controller for Selecting Variation Operators

This section describes a generic controller for Adaptive Operator Selection (AOS) in evolutionary algorithms. In order to assess the generality of our
controller, we consider a generic EA that may include several operators. This controller is connected to the algorithm by a simple I/O interface:

- the EA sends the controller the identifier of last applied operator identifier and its associated performance values;
- the controller tells the EA which operator should be applied next.

AOS relies on performance criteria which are computed and received from the EA. These criteria are meaningful measures of the utility of the applied operator over the search. In order to keep an independence from the EA, the criteria are calculated by the latter, and sent to the AOS. The specific criteria set considered in this work is the one used in Maturana and Saubion (2008), where two performance criteria are used to reflect the EvE balance: the mean quality (fitness) of the population and the diversity (entropy) of the population. The choice of the mean quality is rather straightforward; the choice of the entropy need some justification. For instance, we could have used the fitness diversity or the edit distance (Rivest et al., 1992) instead, but they appeared too much correlated with the fitness (Burke et al., 2004). Preliminary experiments have shown that entropy shows a negligible correlation with fitness when the controller aims to favor diversity. Hence, entropy provides us with a clear information on the phenotypic distribution of the population. Please notice that each time we mention the values of the criteria, we are interested in their variation rather than in their absolute current values.

The controller mechanism is divided into four basic stages: Aggregation Criteria Computation, Reward Computation, Credit Assignment, and Operator Selection. These stages define a chain of modules, which are presented in figure 1. Each module has its own inputs, outputs and parameters. The parameters of each module are highlighted into boxes at the right of each module.

**Aggregated Criteria Computation.** This module records the impact of the successive applications of an operator during the search. This impact corresponds to the variation of the value of the above mentioned criteria. In order to deal with the long-term behavior of the operators, the values are recorded in a sliding window of size Twin. A sliding window $W_{ij}$ is associated to each pair $(op_i, j)$ of operator $op_i$ and criterion $j$. The impact is then computed as the result of a function $Fwin$ applied on the
window for each criterion. $F_{win}$ can be instantiated to max if one aims at detecting outliers, or mean if one wants to smooth the behavior of the operator. The input of this module are the identifier of last applied operator ($op_i$) and the observed variation of the $k$ criteria values ($v_1 \ldots v_k$); the output – sent to the Reward Computation module – is thus a vector $[op_i, F_{win}(W_{i1}, Twin), \ldots, F_{win}(W_{ik}, Twin)]$. Note that at the end of ACC, only one (aggregated) scalar value is issued for each couple (operator, criteria).
**Reward Computation.** Once the behavior of each operator is computed, we are interested in assessing comparatively the available operators. This comparative measure is denoted as reward. In this work we will use the Compass method \cite{Maturana2008}, that defines a search angle $\theta \in [0, \frac{\pi}{2}]$ in the 2-dimensional space defined in the $\Delta\text{Diversity}/\Delta\text{Quality}$ space, as shown on figure 2. Each operator is thus represented in this two-dimensional space according to its previous aggregated impact, and associated to a vector $opdir_i$.

![Figure 2: Compass Reward Computation.](image)

A search policy is thus fully defined by the value of $\theta$: $\theta = 0$ corresponds to a policy in which the diversity is fostered and the quality is neglected; $\theta = \frac{\pi}{2}$ corresponds to a policy in which the quality is fostered and the diversity is neglected. The reward is computed as the scalar product between the vector defined by $\theta$ and $opdir_i$.

In Compass, the angle $\theta$ stands for the variable $SDir$ in the reward computation module. However, it must be noted that other measures may be used to establish the search policy.\footnote{The term *reward* is usually used in AOS methods and refers to the benefit provided by the application of a particular action.}

The vector $[op_i, F\text{win}(W_{i1}, Twin), \ldots, F\text{win}(W_{ik}, Twin)]$ is the input of this module. Only one (aggregated) scalar value is determined for each couple (operator, criteria). The output of this module is the reward of the operator $op_i$, corresponding to its impact according to the criteria expressed as a single value.

\footnote{For instance, \cite{Veerapen2012} proposes a method to vary the preference between two criteria in local search: quality and distance from the search trajectory. A parameter $\alpha$ controls which of these two criteria must be preferred in a Pareto-based comparison among them. In this case, $SDir$ could be mapped to the $\alpha$ parameter.}

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The rewards obtained by an operator will be closely related to the state of the search. Figure 3 shows the rewards obtained by an exploration operator into a context of a search strategy that encourages exploration during the first 5,000 iterations and exploration during the remaining 5,000. Notice how this operator is better rewarded when its behavior is coherent with the policy defined by the strategy during the first half of the search (see section 4.2 for more details).

Figure 3: Operator 0011: rewards

Example 1. Figure 4 shows the impact on the two criteria observed on two runs of a EA capable of using just a single variation operator. Figure 4 (a) refers to the application of an intensification oriented operator, labelled as 1111; Figure 4 (b) refers to the run based on the application of a diversification oriented operator, labelled as 6011 (both will be defined in section 4.2). Each plot shows the number of iterations performed by the EA on the x-axis and the percentage difference for the observed criterion (i.e., $\Delta Q$ on figure 4 (a) and $\Delta D$ on figure 4 (b)).

We can remark that the operator 6011 has an impact over quality that is often bigger in magnitude than 1111. Even though this impact is often negative, leading to worsen the solution quality (figure 4 (a)). Accordingly, 1111 has an unstable impact on diversity (figure 4 (b)). Hence, when defining the experimental setting, the user has to keep on mind that the different criteria are often intertwined, and that choosing to favor a criteria does not mean that the other will be aimed to be constant during all the search process.
Credit Assignment. Credit is defined as a measure that characterizes the reward obtained by an operator recently. In order to capture this typical reward profile, the rewards assigned to operators are stored in a sliding window of size $T'\text{win}$. The Credit Assignment module works in the same way as the aggregated criteria computation method: it computes an aggregated credit for each operator, stored in a time window of size $T'\text{win}$ using a specific function $F'\text{win}$. These credits are computed over a given period of time $T'\text{win}$ using a specific function $F'\text{win}$ which aggregates thus the successive rewards obtained by the operator. These values, that represent the operator’s credit w.r.t. to their performances, represent the output of this component, and they are sent to the Operator Selection component. Previous studies (Maturana et al., 2009) have shown that $T'\text{win}$ and $F'\text{win}$ have significantly less impact on the behavior of the controller than $T\text{win}$ and $F\text{win}$. Therefore, to reduce the combinatorial complexity of our analysis we will not address the issue of instantiating these two parameters and we will set them to the values used in that work.

Operator Selection. Once the credits have been computed for each operator, AOS must select one of them to recommend its application to the EA on the next iteration. This module determines the next operator to be applied by the EA, according to the credits (which are the input of the module). The operator selection is performed by means of a Method which has to be defined by the user/developer. In this paper, after having performed prelim-
inar experiences, we use the probability matching selection rule (see Section 2). This selection method is used with the mean function for $F_{\text{win}}$. We do not address the comparisons between methods, but such comparisons can be found in (Maturana et al., 2009; Fialho et al., 2010a; Maturana et al., 2010; di Tollo et al., 2011). PM reduces the number of parameters in the selection method and has shown good results on the problem we want to use for benchmarking (SAT problem).

4. Experimental Setting

This section describes the experimental setting used to explore the behavior of AOS. The EA is detailed in subsection 4.1, the operators in subsection 4.2 and the benchmarks in subsection 4.3.

4.1. Basic Structure of the Evolutionary Algorithm

Our purpose is to investigate how our controller influence the search process. To this aim we have chosen to tackle the satisfiability problem (SAT) (Biere et al., 2009) for two main reasons. On the one hand, many different problems can be encoded into SAT formalism, which provides different search landscapes and instances’ structures for experiments. On the other hand, the EA we use is based on GASAT (Lardeux et al., 2006), that includes several variation operators whose performances are known according to previous studies (Maturana et al., 2010). The selection process consists of a classic tournament over two randomly chosen individuals and the insertion process replaces the oldest individual of the population. The algorithm applies one operator at each step producing one individual from two parents.

The combination of the Evolutionary Algorithm and the controller is sketched in figure 5.

Population size has been set to 30. Since we want to observe the long term effects of the controller, the number of generations is set to 100,000 as default value. Concerning computation time, we stress out that, given the size of the individuals kept fixed, the execution time is constant for each operator application, so the computation effort will be assessed by the number of crossovers performed during the search. The EA and the controller have been coded in C++ and are available upon request. Experiments have been run on a 280-core, 792 GFlop computer cluster.
4.2. Operators definition

The operator to be applied is selected by the controller from a set of 20 variation operators (out of more than 300 operators defined by Maturana et al. (2010)). These operators are specific to the SAT problem and can be defined by a combination of four basic features:

1. selection of clauses that are false in both parents;
2. action on each of the false clauses;
3. selection of clauses that are true in both parents;
4. action on each of the true clauses.

An operator can be represented by a quadruplet $f_1, f_2, f_3, f_4$, where $f_i$ is the value of feature $i$ according to table 1. A variation operator is a function $S \times S \rightarrow S$, where $S$ is the set of all possible individuals (i.e., the search space).

All variables that remain unassigned in the resulting individual are valued using a classic uniform process (Sywerda, 1989). In our experiments, we have selected the following operators, grouped according to their expected effect (Lardeux et al., 2006; Maturana et al., 2010):

- **exploration**: 0011, 0035, 0015, 4455, 6011;
- **exploitation**: 1111, 1122, 5011, 3332, 1134, 0022, 2352, 4454, 1224, 0013;

Figure 5: Combination of EA and AOS
The following basic example highlights how variation operators may be used to get better individuals from a fitness point of view.

**Example 2.** Let us consider a small SAT instance with three Boolean variables $a, b$ and $c$, and three clauses \( c_1 \equiv a \lor \neg b \lor \neg c \), \( c_2 \equiv \neg a \lor b \) and \( c_3 \equiv \neg a \lor c \). The purpose of a SAT solver is to find a satisfying assignment, for instance \( \{a \leftarrow 1, b \leftarrow 1, c \leftarrow 1\} \), where true and false are classically denoted as 1 and 0. In our EA, an individual (that represents a Boolean assignment) is a triple \( (v_a, v_b, v_c) \), whose values represent the Boolean values assigned to $a$, $b$ and $c$. The fitness of an individual corresponds to the number of true clauses. The fitness of \( (111) \) is thus 3. The operators are applied on two individuals in order to produce a new one.

Let us consider the two assignments \( (110) \) and \( (100) \) as input for an operator. \( (110) \) satisfies \( c_1 \) and \( c_2 \) but not \( c_3 \) and its fitness is 2; \( (100) \), whose fitness is 1, satisfies only \( c_1 \). Therefore, \( c_3 \) is false for both assignments. If we consider the operator \( 1111 \), it will select clause \( c_3 \) as common false clause and change variable $a$ to 0 (since for \( (110) \) it leads to \( (010) \) and for \( (100) \) it leads to \( (000) \), both with fitness 3). The resulting individual is obtained by setting $a$ to 0, and finally, by completing uniformly: $c$ is set to 0, having the same value in \( (110) \) and \( (100) \); $b$ can be set either to 1 or to 0. In both cases, we get an individual with a fitness value 3 (either \( (010) \) or \( (000) \)), which improves the quality of the population.

Notice that in this example we have considered a classic notion of fitness function, which has to be maximised. However, given that SAT problem is often treated as a minimization problem (minimize the number of false clauses), from here on we will use the term fitness and quality in terms of a minimization problem.

### 4.3. Instances

In order to assess the general purpose of the controller, different representative SAT instances have been selected from the following problems categories:

- Random 3-SAT instances (Cook and Mitchell, 1997);
- Random k-SAT instances sampled from the phase transition region (Cheeseman et al., 1991);
• 3 Bit Colorable flat graphs (Hogg, 1996);

• Subgraph Isomorphism Problems (Anton and Olson, 2009);

• Hard Handmade instances (Chatalic and Simon, 2000).

For more details, we forward the interested reader to the SAT competition’s website http://www.satcompetition.org/. The main instance’s features are reported in Table 2. For each experiment on the same instance, the same initial population is used.

5. Operators Management

The goal of the controller is to manage the trade-off between exploitation and exploration. Therefore, the operators design and choice are of the utmost importance, since some operators lead to the concentration of the population into specific areas of the search space (exploitation), while other operators may be more related to exploration, depending on the current search state.

In this section we study some relevant features of our controller. Operators management in presence of null operators is discussed in section 5.1, while the definition of search strategies is presented in section 5.2.

5.1. Experiments with null operators

We use the term null operator to identify operators that take two individuals as input, and outputs one of them, thus having no effect over the population if used jointly with an appropriate insertion process (they replace an individual by itself, and therefore the variation of the criteria is 0).

We have carried out experiments using a set of operators containing an exploration-oriented operator (6011), an exploitation-oriented one (1111) and 18 null operators (identified by the tuples 70** in figures 6 and 7). Our purpose is to check whether the controller discriminates amongst the proposed operators according to the desired level of exploitation-exploration. In the following pictures, we show in top part the frequency of application of all operators (labelled on the x-axis). The remaining three parts show, respectively, the variation of entropy, the variation of the $\theta$ parameter (labelled as angle) and the fitness evolution of all individuals over time (steps are labelled in the x-axis).
The controller is expected to identify the null operators. In the same time, the controller must apply the non-null operator that fits the required behavior (defined by $\theta$). Notice that null operators are not significantly used, and the proportion of application on non-null operators produced the expected effect on the search.

Figure 6: Experiment with null operators, different fixed $\theta$ values. Instance 3bits.

By defining a sequence of changes of policy throughout the search, we can define a search strategy (see section 5.2). This is done by varying the value of $SDir$ in the reward computation module, i.e., the angle $\theta$. Figure 7 shows the application frequency and the behavior (in terms of entropy and quality) when alternating between extreme angles.

Figure 7: Experiment with null operator, changing the angle. Instance Simon.
We can notice that the controller also succeeds in detecting the suitable operators according to the different required search direction, and relegate null operators to a second place.

5.2. Search strategies

As stated in the introduction, we are interested in considering dynamic policies during the search. This defines either a predefined or a dynamic change between policies that allow us to guide the search according to a previously defined or a reactive schedule, respectively. In this work we explore the following simple strategies that guide the search by changing the value of the angle $\theta$:

- **INCREASE**: To split the execution time into several epochs and to increase the angle value in equally distributed levels in $[0, \frac{\pi}{2}]$.

- **DECREASE**: To split the execution time into several epochs and to decrease the angle value in equally distributed levels in $[0, \frac{\pi}{2}]$.

- **ALWAYSMOVING**: To split the execution time into several epochs and to alternate the angle value between 0 and $\frac{\pi}{2}$ (as shown in the previous section).

- **REACTIVEMOVING**: Similar to **ALWAYSMOVING** but setting $\theta$ to $\frac{\pi}{2}$ when the entropy value is less than 0.9 and to 0 when the quality has not increased for 200 consecutive iterations.

In order to show how AOS orient the search by changing the angle, figure 8 presents the variation of the population’s mean quality and diversity when the value of angle $\theta$ changes in the range $[0, \frac{\pi}{2}]$ for two different strategies, using the operators listed in section 4.2.

We remark that the controller succeeds in determining, for each epoch corresponding to a given $\theta$ value, which operator has to be used in order to foster the given policy (operator 6011 for the exploration epochs and 1111 for the exploitation epochs).

6. Solving Performance

In this section we study the effect of the controller in terms of improvement of the solutions obtained by the EA. In section 6.1 we show that the
introduction of the controller leads to solutions whose quality is comparable (when not better) with regards to other selection methods. In section 6.2 we study the behavior of the diverse dynamic strategies presented in section 5.2. In section 6.3 we discuss results obtained by adding a tabu search mechanism to the GA+controller, in order to escape from local optima and to get better performances.

6.1. Controller vs. Tuning Methods

We start our analysis by comparing our combination EA+controller with two other solving approaches:

- an EA that uses a uniform random selection of the operators introduced in section 5

- an EA whose operator application rates have been optimally tuned by using ParamILS (see section 2). The operator selection is achieved according to a roulette-wheel mechanism whose operators’ application probability are known \textit{a priori}.

\footnote{As for ParamILS implementation, we have defined a discrete set of values for the 20 parameters, consisting of 11 equi-distanced possible values in the range [0 1]. We have used the Focused-ILS variant, setting the cut-off time to 70, since for at least 75% of the instances, GASAT completes within 70 seconds. The overall time budget allocated for the whole process has been set to 20000 seconds \cite{Hoos2012}.}
The controller is first used with fixed search policies $\theta = \frac{\pi}{4}$, $\theta = \frac{\pi}{2}$. Note that a fixed policy does not mean that the application rates of the operators are fixed but rather than they are adapted dynamically in order to maintain the desired fixed trade-off between quality and diversity.

Results are presented in table 3, where the quality of the best solution found over 30 runs is reported for the different strategies labelled on columns. The best solution amongst strategies are boldfaced.

We remark that the $\theta = \frac{\pi}{4}$ controller provides better results than the random selection. As for ParamILS, we remark that it shows good performances, especially when tackling Random k-SAT instances. Anyhow, when tackling these instances, ParamILS results are not significantly different from the fixed angle’s ones. On other instance instead, the controller (fixed angles or Alwaysmoving) performs always better than ParamILS.

We also remark that just focusing on quality ($\theta = \frac{\pi}{2}$) represents actually a good stand-alone criterion for some instances, but nevertheless fails to reach good solutions for many instances.

Indeed, the choice of the suitable operators with regards to a given compromise between criteria has to be coupled by a strategy that determines how much time has to be spent in achieving this given compromise. If the population recent history indicates that no further improvements can be reached with regards to this compromise, keeping on having the same controller setting can result in a waste of computational time, which could be more effectively used otherwise. The fixed policies leads to results that are not satisfactory since assignments are hardly found for some instances and we have to turn to more dynamic control strategies.

### 6.2. Experiments with dynamic strategies

In order to improve results obtained in section 6.1, we are interested in using the strategies described in section 5.2. In particular, we will use the dynamic strategy labelled as $\text{REACTIVEMOVING}$, in which $\theta$ values switches between 0 and $\frac{\pi}{2}$ according to the state of the search.

In order to assess the performance of these strategies, we use a fixed angle policy ($\theta = \frac{\pi}{2}$) as baseline and a steady-state GA \cite{Lardeux2006} that uses the optimized operator CC (1111 w.r.t. our operator taxonomy). Note that this crossover has been optimized using time consuming experiments.

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5 Column ID represents the instance number, see table 2.
on several SAT instances. Table 4 shows results obtained by the diverse strategies labelled on columns.

The policies that perform “blind” $\theta$ variations (INCREASE, DECREASE and AM) can offer better results since they provide the mechanism to escape from the current search local optimum, but improvements are hindered by the inefficient use of the computation time. The ReactiveMoving instead, offers the best results, given its capability to adapt to the search scenario.

We want to remark that the ReactiveMoving strategy offers results which are comparable to the ones obtained by the CC-based algorithms. Please notice that the CC-based algorithm has been tuned by means of time-consuming experience, whilst ReactiveMoving do not require preliminary experiments.

6.3. Memetic Algorithms and different operators set

In memetic algorithms (Moscato, 1989), the solution generated by variation operators - typically crossover or recombination operators - are refined by a local search algorithm.

The integration of a Tabu Search (Glover and Laguna, 1999) mechanism in an EA for the SAT problem has been proposed in (Lardeux et al., 2006), showing that this combination leads to improvement of the initial performances. This memetic algorithm is sketched in figure 9. We have conducted experiments by adding the Tabu mechanism to the following strategies:

- Increasing;
- ReactiveMoving;
- Fixed Angle ($\theta = \frac{\pi}{2}$);

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6 We have also tried to implement a dynamic version of Increase and Decrease, but in these approaches we have faced the problem to implement the idle mechanism w.r.t. intermediate angle values: it is not clear when the value of $\theta$ has to be changed when its value is different from 0 or $\frac{\pi}{2}$. This investigation is left for further works.
• One operator only, no control and no strategy (as for section 6.2, the operator is the CC (1111), i.e., the best performing exploitation operator).

We have compared them with a stand-alone Tabu Search and with the steady-state GA based on the CC operator, which is still nowadays a reference EA for SAT. The results of this investigation are summarised in table 5.

By comparing the results with those outlined in table 4, we can observe that by adding a simple Tabu Search, the performance of the controlled GA is better than the non-Tabu controlled version, no matter the strategy used. Furthermore, we can state that the combination Controller + Tabu offers results which are comparable (and even better, see instance 31) to the CC based GASAT + Tabu. The comparison amongst GA+control+Tabu strategies and Tabu only shows that the single-steady Tabu Search does not offer satisfactory results over a broad set of instances (unif*, simon, 3bit and F500): over these instances, a simple comparison with table 4 shows that even a non-tabu ReactiveMoving performs better. This allow us to state that the good performances of Tabu + Control are not due just to the Tabu mechanism, as it could be argued: the adaptive operation selection provide the Tabu search with an efficient way to escape from local optima, with the advantage to be general w.r.t. the instance at hand.

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7Length of the Tabu list has been set to = 10 percent of the number of variables. The process ends when 1 000 000 iterations have been performed.
In order to check the robustness of our findings we have defined 20 different sets of operators, each containing an exploitation-oriented operator, an exploration-oriented operator, and 18 randomly chosen ones out of the 300s operators derived from the table. For each operator set, we have rerun experiments using the policies defined above. For the 20 sets, we have remarked that the Tabu+Control policies perform better than Tabu-only and policies that do not use Tabu improvement. Adding the Tabu algorithm helps in improving the results of the control policies. We can anyhow remark that non-Tabu policies also provide robust results with regards to the different sets of operators.

Additionally we have run a pairwise Wilcoxon test on the best solutions found by the different policies for each of the 30 rounds found over all instances, in order to verify the Tabu skill to allow the controlled GA escaping from local optima. All possible pairwise combinations amongst

- \( \text{TABU} + \text{INCREASING} \),
- \( \text{TABU} + \text{REACTIVE MOVING} \),
- \( \text{TABU} + \text{CC} \),

have a p-value greater than 0.05, leading us to accept the H-hypothesis that the distribution from which they are drawn are equivalent. Conversely, the tiny p-value found by a pairwise comparisons about each of the aforementioned strategies and TABU ONLY lead us to confirm that TABU is to be used as a feature to add to a controlled GA instead of using a stand-alone strategy.

7. Conclusions

In this paper we have investigated the control ability of adaptive control techniques for EAs. The control consists in achieving a dynamic adaptation of the algorithm with respect to a given search policy that is defined according to high level criteria, i.e., the quality and the diversity of the population. We have considered various control strategies, in order to handle more dynamic scenarios.

This work has addressed some important aspects related to the automatic control of EAs, namely:
1. The ability to identify and select suitable operators for achieving a given search strategy;

2. The ability to maintain a given search policy by automatically adjusting the EAs’ parameters, by means of selecting the operator to apply at each step of the search process;

3. The ability to solve problems and to perform better than non-controlled EAs.

Results show that dynamic strategies are better than fixed search policies, in terms of solution quality and operators management. Furthermore, the dynamic version allows the EA to better allocate computational time and is more robust w.r.t. the setting of the controller.

The contribution of this paper is thus focused on providing deep insights for users willing to use EAs for solving specific problems. In this context, adaptive control can be used for two complementary purposes:

- Controlling a basic EA in which classic or less known operators have been included without having any knowledge about parameters setting. In particular, in presence of many parameters (as in our study, where we consider 20 operators), it is virtually impossible to forecast the impact of the application of these operators during the search, while it would be more intuitive to think in terms of search policy, managing a higher level criterion.

- Improving the design of EAs for expert users, for which adaptive control can be used to study the behavior of customised operators according to various search scenarios. We have shown that a good controller may achieve good results using “average” operators compared to the best performing stand-alone ones, whose design normally requires the execution of costly and time-consuming experiences.

Further work will be devoted to autonomously modify the operator set during the execution time, and to devise new criteria to define the desired behavior.

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Table 1: Combinatorial definition of crossover operators

For each iteration $i$:

1. **Selection of clauses that are false in both parents**
   - 0. do nothing
   - 1. select them in chronological order
   - 2. choose randomly one
   - 3. choose randomly one in the set of smallest clauses
   - 4. choose randomly one in the set of biggest clauses
   - 5. Fleurant Ferland (Fleurent and Ferland, 1996)
   - 6. flip variables which are identical in both parents

   **Action on each of the false clauses**
   - 0. do nothing
   - 1. flip the variable that maximizes the number of true clauses and minimizes the number of false clauses
   - 2. same as previous one, but the flip is not performed when the corresponding child’s clause is already verified to be true
   - 3. flip all the variables
   - 4. flip the literal which appears less often in the others clauses

2. **Selection of clauses that are true in both parents**
   - 1. do nothing
   - 2. select them in chronological order
   - 3. choose randomly one
   - 4. choose randomly one in the set of smallest clauses
   - 5. choose randomly one in the set of biggest clauses

   **Action on each of the true clauses**
   - 1. do nothing
   - 2. set to true the variable that whose flip minimizes the number of false clauses
   - 3. set all the literals to true
   - 4. set to true the literal whose negation appears less often in the other clauses
   - 5. set all the literals to false
| ID | Instance name | variables | clauses |
|----|---------------|-----------|---------|
| **Random 3-SAT** | | | |
| 1  | F500          | 500       | 2150    |
| **Random k-SAT instances** | | | |
| 2  | unif-k7-r89-v65-c5785-S1481196126 | 65 | 5785 |
| 3  | unif-k7-r89-v65-c5785-S1678989107 | 65 | 5785 |
| 4  | unif-k7-r89-v65-c5785-S2099893633 | 65 | 5785 |
| 5  | unif-k7-r89-v65-c5785-S316555917 | 65 | 5785 |
| 6  | unif-k7-r89-v65-c5785-S461794864 | 65 | 5785 |
| 7  | unif-k7-r89-v75-c6675-S1299158672 | 75 | 6675 |
| 8  | unif-k7-r89-v75-c6675-S1534329206 | 75 | 6675 |
| 9  | unif-k7-r89-v75-c6675-S1572638390 | 75 | 6675 |
| 10 | unif-k7-r89-v75-c6675-S1785258608 | 75 | 6675 |
| **3 Bit Colorable** | | | |
| 11 | flat50-293    | 150       | 545     |
| 12 | flat50-297    | 150       | 545     |
| 13 | flat50-298    | 150       | 545     |
| 14 | flat50-299    | 150       | 545     |
| 15 | flat50-3      | 150       | 545     |
| 16 | flat50-30     | 150       | 545     |
| **Subgraph Isomorphism Problems** | | | |
| 17 | new-difficult-20 | 360   | 15466  |
| 18 | new-difficult-21 | 399   | 18184  |
| 19 | new-difficult-22 | 440   | 22276  |
| 20 | new-difficult-23 | 483   | 25396  |
| 21 | new-difficult-24 | 528   | 30728  |
| 22 | new-difficult-26 | 624   | 38944  |
| 23 | new-difficult-28 | 728   | 48442  |
| 24 | satsgi-n23himBHm26 | 598   | 14076  |
| 25 | satsgi-n23himBHm27 | 621   | 14927  |
| 26 | satsgi-n25himBHm27 | 675   | 16900  |
| 27 | satsgi-n25himBHm29 | 725   | 18875  |
| 28 | satsgi-n28himBHm30 | 840   | 23548  |
| 29 | sgi-difficult4  | 483    | 15156  |
| **Hard handmade** | | | |
| 30 | sgi-difficult7 | 728    | 28986  |
| 31 | Simon         | 2424   | 14812  |
| 32 | 3bit          | 8432   | 31310  |
Table 3: Best solution fitness for Controller ($\theta = \frac{\pi}{4}$ and $\theta = \frac{\pi}{2}$), Random Selection and ParamILS.

| ID  | $\theta = \frac{\pi}{4}$ | $\theta = \frac{\pi}{2}$ | Random | ParamILS |
|-----|--------------------------|--------------------------|--------|----------|
|     | Min | Std | Min | Std | Min | Std | Min | Std |
| 1   | 5   | 13.09 | 1   | 11.7 | 53  | 6.72 | 59  | 5.06 |
|     | Random 3-SAT                  | Random k-SAT instances            | 3 Bit Colorable | Subgraph Isomorphism Problems | Hard handmade |
| 2   | 12  | 16.41 | 12  | 1.35 | 11  | 1.58 | 14  | 1.56 |
| 3   | 15  | 16.62 | 12  | 1.69 | 14  | 1.99 | 13  | 1.55 |
| 4   | 14  | 16.65 | 2   | 3.19 | 13  | 1.47 | 9   | 2.57 |
| 5   | 12  | 15.96 | 7   | 2.56 | 12  | 1.79 | 12  | 1.70 |
| 6   | 15  | 15.31 | 2   | 3.23 | 15  | 1.71 | 12  | 1.90 |
| 7   | 17  | 20.68 | 15  | 1.80 | 17  | 1.66 | 14  | 2.47 |
| 8   | 19  | 19.41 | 2   | 3.80 | 16  | 1.72 | 15  | 1.87 |
| 9   | 17  | 19.82 | 5   | 3.39 | 17  | 2.08 | 17  | 2.09 |
| 10  | 14  | 20.27 | 4   | 2.43 | 17  | 2.04 | 17  | 1.36 |
| 11  | 1   | 1.27  | 0   | 2.04 | 13  | 1.90 | 9   | 1.60 |
| 12  | 13  | 1.96  | 0   | 1.23 | 12  | 1.62 | 11  | 0.75 |
| 13  | 13  | 2.03  | 0   | 2.16 | 11  | 1.61 | 10  | 1.47 |
| 14  | 3   | 1.17  | 0   | 1.65 | 12  | 1.58 | 9   | 1.26 |
| 15  | 14  | 1.68  | 0   | 2.17 | 12  | 1.65 | 8   | 1.90 |
| 16  | 4   | 9.72  | 0   | 2.61 | 11  | 0.66 | 0   | 1.18 |
| 17  | 3   | 11.35 | 4   | 4.33 | 11  | 0.86 | 9   | 1.92 |
| 18  | 3   | 15.79 | 4   | 3.38 | 9   | 0.84 | 8   | 1.04 |
| 19  | 5   | 2.37  | 5   | 1.71 | 9   | 1.08 | 10  | 0.70 |
| 20  | 3   | 3.68  | 5   | 1.85 | 11  | 0.76 | 11  | 1.00 |
| 21  | 3   | 4.31  | 3   | 2.06 | 13  | 0.88 | 11  | 2.72 |
| 22  | 13  | 5.44  | 5   | 3.02 | 14  | 0.99 | 13  | 1.25 |
| 23  | 4   | 4.89  | 8   | 3.42 | 3   | 0.77 | 3   | 1.58 |
| 24  | 0   | 5.65  | 0   | 3.49 | 2   | 1.02 | 5   | 0.98 |
| 25  | 0   | 6.34  | 0   | 3.17 | 6   | 0.99 | 2   | 2.07 |
| 26  | 0   | 5.65  | 0   | 3.96 | 6   | 1.44 | 7   | 1.27 |
| 27  | 0   | 7.96  | 0   | 3.94 | 5   | 1.30 | 5   | 1.47 |
| 28  | 0   | 6.72  | 0   | 5.10 | 5   | 1.12 | 7   | 1.02 |
| 29  | 0   | 8.65  | 0   | 3.96 | 7   | 1.58 | 6   | 1.43 |
| 30  | 1   | 10    | 0   | 5.69 | 8   | 4.96 | 6   | 1.25 |
| 31  | 19  | 27.26 | 20  | 15.39 | 70  | 33.19 | 59  | 7.79 |
| 32  | 68  | 100.48| 21  | 65.53 | 157 | 76.37 | 138 | 18.66 |
Table 4: Best and standard deviation of fitness for several Controller settings.

| ID | π     | INC | DEC | AM | RM | CC ONLY |
|----|-------|-----|-----|----|----|---------|
|    | Min   | Std | Min | Std | Min | Std | Min | Std | Min | Std | Min | Std |
| Random 3-SAT |       |     |     |     |     |     |     |     |     |     |     |     |
| 1  | 53    | 6.72| 5   | 13.09| 1  | 11.70| 5   | 6.18| 2   | 11.24| 7   | 5.43 |
| Random k-SAT instances |       |     |     |     |     |     |     |     |     |     |     |     |
| 2  | 12    | 1.35| 0   | 1.41 | 1  | 1.15 | 6   | 1.09| 1   | 2.84 | 1   | 1.15 |
| 3  | 12    | 1.69| 2   | 0.66 | 1  | 0.57 | 11  | 1.35| 1   | 1.55 | 1   | 0.9  |
| 4  | 2     | 3.19| 0   | 0.61 | 0  | 0.55 | 4   | 1.19| 0   | 4.4  | 1   | 1.11 |
| 5  | 7     | 2.56| 0   | 1.08 | 0  | 1.3  | 12  | 3.18| 0   | 1.76 | 0   | 1.44 |
| 6  | 2     | 3.23| 1   | 0.001 | 1  | 0.1  | 12  | 2.02| 2   | 1.48 | 1   | 1.1  |
| 7  | 15    | 1.80| 1   | 1.45 | 2  | 1.1  | 1   | 1.64| 0   | 0.58 | 2   | 1.02 |
| 8  | 2     | 3.80| 1   | 0.001 | 1  | 0.1  | 17  | 0.88| 0   | 1.75 | 2   | 0.95 |
| 9  | 5     | 3.39| 1   | 0.65 | 1  | 0.81 | 3   | 1.31| 0   | 4.53 | 1   | 1.05 |
| 10 | 4     | 2.43| 0   | 0.3  | 0  | 0.24 | 17  | 1.96| 1   | 4.1  | 1   | 1.03 |
| 3 Bit Colorable |       |     |     |     |     |     |     |     |     |     |     |     |
| 11 | 0     | 2.04| 2   | 0.70 | 1  | 0.67 | 4   | 1.52| 1   | 4.4  | 2   | 1.53 |
| 12 | 0     | 1.23| 1   | 0.63 | 1  | 0.51 | 5   | 4.09| 1   | 1.62 | 2   | 1.15 |
| 13 | 0     | 2.16| 0   | 0.81 | 1  | 0.8  | 4   | 21.09| 1  | 1.71 | 1   | 1.7  |
| 14 | 0     | 1.65| 1   | 0.84 | 1  | 0.53 | 4   | 1.42| 1   | 1.35 | 1   | 1.63 |
| 15 | 0     | 2.17| 1   | 0.001 | 1  | 0.001 | 7  | 2.10| 0   | 4.7  | 1   | 1.47 |
| 16 | 0     | 2.61| 2   | 0.001 | 1  | 0.001 | 11  | 1.43| 0   | 2.4  | 1   | 1.78 |
| Subgraph Isomorphism Problems |       |     |     |     |     |     |     |     |     |     |     |     |
| 17 | 4     | 4.33| 2   | 0.001 | 3  | 0.001 | 12  | 1.27| 0   | 0    | 0   | 0.38 |
| 18 | 4     | 3.38| 3   | 1.25 | 2  | 1.33 | 16  | 1.27| 0   | 0    | 0   | 0.54 |
| 19 | 5     | 1.71| 2   | 0.47 | 3  | 0.48 | 13  | 0.84| 0   | 0    | 0   | 0.9  |
| 20 | 5     | 1.85| 2   | 0.3  | 2  | 0.001 | 13  | 1.55| 0   | 0    | 0   | 0    |
| 21 | 3     | 2.06| 2   | 0.001 | 3  | 0.001 | 3   | 1.52| 0   | 0    | 0   | 0.25 |
| 22 | 5     | 3.02| 2   | 0.001 | 3  | 0.001 | 16  | 4.11| 0   | 0    | 0   | 0.34 |
| 23 | 8     | 3.42| 3   | 0.71 | 3  | 0.65 | 3   | 0.84| 0   | 0    | 0   | 0.04 |
| 24 | 0     | 3.49| 0   | 0.44 | 0  | 0.53 | 6   | 0.92| 0   | 0    | 0   | 0    |
| 25 | 0     | 3.17| 0   | 0.47 | 0  | 0.57 | 8   | 1.51| 0   | 0    | 0   | 0    |
| 26 | 0     | 3.96| 0   | 0.4  | 0  | 0.39 | 9   | 1.90| 0   | 0    | 0   | 0    |
| 27 | 0     | 3.94| 0   | 0.85 | 0  | 1.28 | 9   | 1.31| 0   | 0    | 0   | 0.45 |
| 28 | 0     | 5.10| 0   | 0.3  | 0  | 0.63 | 17  | 3.85| 0   | 0    | 0   | 0.6  |
| 29 | 0     | 3.96| 0   | 0.001 | 0  | 0.001 | 9   | 1.64| 0   | 0.18 | 0   | 0.54 |
| 30 | 0     | 5.69| 0   | 0.97 | 0  | 0.90 | 3   | 0.74| 0   | 0.37 | 0   | 0.3  |
| Hard handmade |       |     |     |     |     |     |     |     |     |     |     |     |
| 31 | 20    | 15.39| 9   | 12.26| 10  | 19.36| 96  | 19.07| 17  | 9.4  | 21  | 4.48 |
| 32 | 21    | 65.53| 53  | 0.64 | 51  | 0.54 | 12  | 2.00 | 9   | 243.22| 15  | 7.68 |
Table 5: Best and standard deviation of fitness for several Controller settings.

| ID | TABU + INC | TABU + RM | TABU + θ = \frac{π}{2} | TABU + CC | TABU ONLY |
|----|------------|----------|------------------------|----------|-----------|
|    | Min        | Std      | Min                     | Std      | Min       | Std       | Min       | Std       |
| 1  | 0          | 0.5      | 0.5                     | 0        | 0.84      | 0         | 0.75      | 9         | 1.17      |
| 2  | 0          | 0.4      | 0.42                    | 0        | 0.42      | 0         | 0.77      | 5         | 0.79      |
| 3  | 0          | 0.67     | 0.34                    | 0        | 0.58      | 0         | 0.77      | 5         | 0.99      |
| 4  | 0          | 0.37     | 0                       | 0        | 0.51      | 0         | 0.47      | 5         | 0.87      |
| 5  | 0          | 0.25     | 0.39                    | 0        | 0.4       | 0         | 0.67      | 5         | 0.89      |
| 6  | 0          | 0.37     | 0.38                    | 0        | 0.38      | 0         | 0.68      | 4         | 0.78      |
| 7  | 0          | 0.34     | 0.25                    | 0        | 0.34      | 0         | 0.26      | 6         | 0.99      |
| 8  | 0          | 0.41     | 0.44                    | 0        | 0.36      | 0         | 1.2       | 5         | 0.7       |
| 9  | 0          | 0.18     | 0.21                    | 0        | 0.18      | 0         | 0.54      | 5         | 0.7       |
| 10 | 0          | 0.31     | 0.41                    | 0        | 0.31      | 1         | 0.55      | 6         | 0.73      |

Random k-SAT instances

|    | 11 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 12 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 13 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 14 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 15 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 16 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

3 Bit Colorable

|    | 17 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 18 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 19 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 20 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 21 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 22 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 23 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 24 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 25 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 26 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 27 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 28 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 29 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|    | 30 | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

Subgraph Isomorphism Problems

|    | 31 | 12  | 3.47 | 15 | 2.61 | 16 | 2.98 | 21 | 3.02 | 22 | 1.64 |
|    | 32 | 3   | 1.5  | 4  | 1.2  | 3  | 1.41 | 4  | 1.27 | 15 | 2.84 |