Fault tolerant quantum key distribution based on quantum dense coding with collective noise

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We present two robust quantum key distribution protocols against two kinds of collective noise, following some ideas in quantum dense coding. Three-qubit entangled states are used as quantum information carriers, two of which forming the logical qubit which is invariant with a special type of collective noise. The information is encoded on logical qubits with four unitary operations, which can be read out faithfully with Bell-state analysis on two physical qubits and a single-photon measurement on the other physical qubit, not three-photon joint measurements. Two bits of information are exchanged faithfully and securely by transmitting two physical qubits through a noisy channel. When the losses in the noisy channel is low, these protocols can be used to transmit a secret message directly in principle.

Keywords: Quantum key distribution; fault tolerant; quantum dense coding; collective noise.

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I. INTRODUCTION

Quantum dense coding and quantum teleportation are two typical examples in quantum information exploiting striking nonclassical properties of entangled states to perform otherwise impossible tasks. Quantum dense coding, which was first proposed in 1992 \cite{1}, enables the communication of two bits of classical information with the transmission of a qubit. This feature makes it have a good application in secure quantum communication \cite{2,3}. Quantum dense coding was demonstrated by using photon pairs entangled in polarization in 1996 \cite{4} and subsequently was investigated experimentally by using some other quantum systems, such as continuous variables \cite{5}, nuclear magnetic resonance \cite{6} and atomic qubits \cite{7}. The theoretical researches have been broadened to high-dimension quantum states \cite{2}, more than two entangled qubits \cite{8} and hyperentangled quantum states \cite{9}.

In quantum dense coding, the Bell-state analysis (BSA) is one of the key important steps. Although the four Bell states are orthogonal to each other and is deemed to be discriminated deterministically in principle, BSA is always a technical difficulty, which yielding low efficiencies and low discrimination fidelities. Using only the polarization degree of freedom of photons, three Bell states can at best be discriminated with linear optics, which reduces the attainable channel capacity of quantum dense coding from 2 bits to $\log_23 \approx 1.585$ bits \cite{4}.

Recently, cross-Kerr nonlinearity was used to implement two-qubit controlled-NOT gate \cite{10}, whose character can also be used to solve the BSA problem. Furthermore, hyperentanglement (i.e., entanglement in more than one degree of freedom (DOF)) enables a full BSA in polarization DOF with linear optics \cite{9,11}. In particular, Schuck et al. \cite{12} proposed a complete deterministic linear-optics BSA by employing the intrinsic time-energy correlation of photon pairs generated with high temporal definition in 2006. If we can control the time of arrival of these two photons at beam splitter (BS), i.e., make them arrive at the BS simultaneously, these four Bell states can be discriminated faithfully in principle and the experiment gives its success probability between 81%-89%.

Although there are many kinds of physical systems which can be selected by quantum communication, photon is the most popular one, relying on its good feature in preparation and manipulation. However, during the transmission, the polarization DOF of photons is incident to be influenced by the thermal fluctuation, vibration and the imperfection of the fiber, which we call them noise in total. At present, we always suppose the noise in a quantum channel is a collective one \cite{13}. That is, the fluctuation of noise is slow in time. If several photons transmit through the noisy channel simultaneously or they are close to each other, the alternation arising from the noise on each qubit is identical. With this kind of noise, several methods have been proposed to cancel or reduce the noise effect, such as entanglement purification \cite{14}, quantum error correct code (QECC) \cite{13}, single-photon error rejection \cite{16} and decoherence-free subspace (DFS) \cite{17,18}. QECC encodes one logical bit into several physical qubits according to the kind of noise, and the user measures the stabilizer codes to detect the errors and then correct them. Compared with QECC, single-
photon error rejection schemes require less quantum resource, although they succeed probabilistically. Entanglement purification is a method to distill a maximally entangled states from some less-entanglement states by sacrificing several samples affected by the noise. The influence of noise can only be reduced in entanglement purification and infinite steps are needed to get a perfect maximally entangled state. DFS can be composed of several qubits which suffer from the same noise and compensate the effect of noise, and then it posses the invariability against the noise. This feature can be used to encode messages in quantum communication over a noisy channel. Boileau et al. \[18\] proposed a quantum key distribution (QKD) scheme with a collective random unitary noise by using linear combinations of two singlet states $|\psi^-, x\rangle$.

In this paper, we present two fault-tolerant quantum key distribution schemes against two different kinds of collective noise, the collective-dephasing noise and the collective-rotation one, following some ideas in quantum dense coding. Three-qubit entangled states are utilized as quantum information carriers. Each logical qubit transmitted is composed of two physical qubits and is invariant during the transmission. The message is encoded on each logical qubit transmitted with one of four unitary operations and can be read out with a Bell-state analysis and a single-photon measurement on each three-qubit quantum system, not a three-qubit joint measurement (i.e., a Greenberger-Horne-Zeilinger (GHZ) state measurement). Two bits of information are exchanged faithfully and securely by transmitting two physical qubits through a noisy channel. When the losses in the noisy channel is low, our quantum cryptography schemes can, in principle, be used to transmit a secret message directly.

II. FAULT TOLERANT QUANTUM KEY DISTRIBUTION BASED ON QUANTUM DENSE CODING WITH COLLECTIVE NOISE

A. review of quantum cryptography based on quantum dense coding in an ideal condition

We review the basic principle of quantum cryptography based on quantum dense coding in an ideal condition first \[2, 3\]. Suppose the receiver Alice wants to share a set of key with the sender Bob. She prepares a sequence of EPR (Einstein-Podolsky-Rosen) pairs: $(00)\pm(11)$ and $(01)\pm(10)$ with each $e\pm$ randomly in one of four Bell states:

$$|\phi^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{AB},$$

$$|\psi^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{AB},$$

where the subscripts $A$ and $B$ represent two entangled photons, $|0\rangle$ and $|1\rangle$ are the two eigenvectors of $\sigma_z$. Alice picks up photon $B$ in each pair to form the sequence $S_B$ and sends it to Bob who performs one of four unitary operations on each photon in the sequence $S_B$ randomly. These four operations are

$$U_{00} = U_I = I = |0\rangle\langle 0| + |1\rangle\langle 1|,$$

$$U_{01} = U_\sigma = \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|,$$

$$U_{10} = U_\sigma = \sigma_x = |1\rangle\langle 0| + |0\rangle\langle 1|,$$

$$U_{11} = U_y = -i\sigma_y = |1\rangle\langle 0| - |0\rangle\langle 1|. \quad (3)$$

The subscripts $00, 01, 10$ and $11$ are the two bits of information represented by the corresponding operations. $\sigma_x, \sigma_y, \sigma_z$ are Pauli operators. These four unitary operations will change the Bell states from one to another.

$$|\Phi^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0_L\rangle_B \pm |1\rangle_A|1_L\rangle_B),$$

$$|\Psi^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|1_L\rangle_B \pm |1\rangle_A|0_L\rangle_B). \quad (4)$$

Here the subscript $L$ denotes the logical qubit, which can be made of several physical qubits experiencing the same noise and then compensating the noise effect due to the peculiar relationship between them. The logical qubits are invariant with the noise and are constructed according to the form of noise.

It is well known that quantum dense coding can be used to improve the capacity of secure quantum communication \[2, 3\]. The security of quantum communication is its most important character, compared with classical communication. If quantum dense coding cannot be used for secure quantum communication, it will lose the significance as a classical signal is far easier to be prepared and robust against the channel noise. For instance, in a classical way, Alice and Bob can communicate easily with the two states $|0\rangle$ and $|1\rangle$ against a collective-dephasing noise shown in Eq. (3). Also they can communicate by using the two states $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ to avoid the effect of the collective-rotation noise shown in Eq. (4). However, a secure quantum communication resorts to at least two nonorthogonal states \[19, 22\]. That is, it requires that the superposition of two eigenvectors is also free to the collective noise. The two classical states $\{|0\rangle, |1\rangle\}$ are immune to a collective-dephasing noise, but their superposition is not. So do the two states $\{|+\rangle, |-\rangle\}$ to a
collective-rotation noise. We will introduce two methods to construct a logical qubit against two kinds of collective noise, which will retain the security of quantum cryptography simultaneously. We discuss them in detail below.

B. Quantum cryptography based on quantum dense coding against a collective-dephasing noise

A collective-dephasing noise can be described as

\[ U_{\text{dp}}|0\rangle = |0\rangle, \quad U_{\text{dp}}|1\rangle = e^{i\phi}|1\rangle, \]

where \( \phi \) is the noise parameter and it fluctuates with time. In general, a logical qubit encoded into two physical qubits with antiparallel parity is immune to this kind of noise as these two logical qubits acquire the same phase factor \( e^{i\phi} \)

\[ |0_L\rangle = |01\rangle, \quad |1_L\rangle = |10\rangle. \]

In this way, the four entangled logical states used for quantum cryptography based on quantum dense coding against a collective-dephasing noise can be made up of four three-qubit entangled states as

\[ |\Phi^\pm_{\text{dp}}\rangle_{AB_1B_2} = \frac{1}{\sqrt{2}}(|00\rangle_A|01\rangle_{B_1B_2} \pm |11\rangle_A|10\rangle_{B_1B_2}), \]

\[ |\Psi^\pm_{\text{dp}}\rangle_{AB_1B_2} = \frac{1}{\sqrt{2}}(|00\rangle_A|10\rangle_{B_1B_2} \pm |11\rangle_A|01\rangle_{B_1B_2}). \]

(7)

The logical qubit \( B \) is composed of two physical qubits \( B_1 \) and \( B_2 \).

Our quantum cryptography scheme based on quantum dense coding against a collective-dephasing noise works as follows.

(1) Similar to Refs. [2, 24], Alice prepares a sequence of quantum systems in the entangled state \( |\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB} = \frac{1}{\sqrt{2}}(|00\rangle_A|01\rangle_{B_1B_2} + |11\rangle_A|10\rangle_{B_1B_2}) \). Alice divides the quantum systems into two photon sequences, \( S_A \) and \( S_B \). \( S_A \) is composed of all the logical qubits \( A \) in the quantum systems and \( S_B \) is made up of the logical qubits \( B \), which is composed of two physical qubits \( B_1 \) and \( B_2 \).

(2) Alice sends the logical qubit sequence \( S_B \) to Bob and keeps the sequence \( S_A \).

(3) After receiving the sequence \( S_B \), Bob chooses randomly a subset of logical qubits as the samples for error rate analysis and the other logical qubits make up of the message sequence \( S_{BM} \) for generating the private key. Bob measures each logical qubit in the samples by choosing randomly one of the two bases \( \sigma^B_0 \equiv \sigma^{B_1} \otimes \sigma^{B_2} \) and \( \sigma^B_1 \equiv \delta^{B_1} \otimes \delta^{B_2} \). Also he tells Alice which logical qubits are chosen as samples. Under these two bases, the original entangled state can be written as

\[ |\Phi^+_{\text{dp}}\rangle_{AB_1B_2} = \frac{1}{\sqrt{2}}(|00\rangle_A|01\rangle_{B_1B_2} + |11\rangle_A|10\rangle_{B_1B_2}) \]

\[ = \frac{1}{2}([|+\rangle_A(|+\rangle + |-\rangle)_{B_1B_2}
\quad + |\rangle_A(|-\rangle + |+\rangle)_{B_1B_2}] \].

(8)

That is, the outcomes obtained by Alice and Bob are correlated if they choose the same bases for their logical qubits. However, the vicious action done by Eve will destroy this correlation and be detected, same as those in Refs. [22, 28]. Alice and Bob can exploit the correlation to check the security of the transmission of the logical qubits run from Alice to Bob.

(4) For the message sequence \( S_{BM} \), Bob operates each logical qubit with one of the four unitary operations, \{\( \Omega_{00}, \Omega_{01}, \Omega_{10}, \Omega_{11} \)\} which is made up of two unitary operations on two physical qubits, i.e.,

\[ \Omega_{00} = \Omega_I = I_1 \otimes I_2, \]

\[ \Omega_{01} = \Omega_z = U_{z1} \otimes I_2, \]

\[ \Omega_{10} = \Omega_x = U_{x1} \otimes U_{x2}, \]

\[ \Omega_{11} = \Omega_y = U_{y1} \otimes U_{x2}. \]

(9)

The subscripts of \( \Omega \) denotes the code \{00, 01, 10, 11\}, and the subscripts 1 and 2 represent the unitary operations performing on the photons \( B_1 \) and \( B_2 \), respectively. The relation between the initial state, final state and the operator is shown in Table I. These four operations will not drive the states prepared by Alice out of the subspace \( S = \{ |\Phi^+_{\text{dp}}\rangle_{AB_1B_2}, |\Psi^+_{\text{dp}}\rangle_{AB_1B_2} \} \). In other words, the states operated by Bob are also antinoise when they are transmitted back to Alice through a collective-dephasing noise channel. After the unitary operation, Bob sends the sequence \( S_{BM} \) back to Alice.

| \( |\Phi^+_{\text{dp}}\rangle_I \) | \( |\Phi^+_{\text{dp}}\rangle_f \) | \( |\Psi^+_{\text{dp}}\rangle_I \) | \( |\Psi^+_{\text{dp}}\rangle_f \) |
|---|---|---|---|
| \( \Omega_{00} \) | \( \Omega_{01} \) | \( \Omega_{10} \) | \( \Omega_{11} \) |
| \( \Omega_{01} \) | \( \Omega_{00} \) | \( \Omega_{11} \) | \( \Omega_{10} \) |
| \( \Omega_{10} \) | \( \Omega_{11} \) | \( \Omega_{00} \) | \( \Omega_{01} \) |
| \( \Omega_{11} \) | \( \Omega_{10} \) | \( \Omega_{01} \) | \( \Omega_{00} \) |

(5) Alice picks up each entangled quantum system from the two photon sequences \( S_{AM} \) and \( S_{BM} \). Here \( S_{AM} \) is made up of the remaining logical qubits in \( S_A \) after the first sampling for analyzing error rate of the first transmission from Alice to Bob. Alice performs a Bell-state measurement on the two photons \( AB_1 \) and a single-photon measurement on the photon \( B_2 \) with \( X = \sigma_x \) basis for distinguishing the four GHZ states \( |\Phi^+_{\text{dp}}\rangle_{AB_1B_2} \) and \( |\Psi^+_{\text{dp}}\rangle_{AB_1B_2} \). These four states to be discriminated can be written as

\[ |\Phi^+_{\text{dp}}\rangle_{AB_1B_2} = \frac{1}{\sqrt{2}}(|\Phi^+\rangle_{AB_1} \otimes |+\rangle_{B_2} - |\Phi^-\rangle_{AB_1} \otimes |\rangle_{B_2}), \]

\[ |\Psi^+_{\text{dp}}\rangle_{AB_1B_2} = \frac{1}{\sqrt{2}}(|\Psi^+\rangle_{AB_1} \otimes |+\rangle_{B_2} + |\Psi^-\rangle_{AB_1} \otimes |\rangle_{B_2}). \]

(10)
Here \( |\pm\rangle = (1/\sqrt{2})(|0\rangle \pm |1\rangle) \) are two eigenvectors of Pauli operator \( \sigma_x \). From this expression, one can see that with these two measurement outcomes Alice can distinguish these four three-photon GHZ states deterministically and read out the operations done by Bob on the logical qubits \( B \) in principle.

(6) For checking the security of the second transmission from Bob to Alice, Alice chooses randomly a subset of the outcomes of the measurements on three-qubit states operated by Bob as the samples for error rate analysis, similar to Ref. 29. She requires Bob tell her his operations on these samples and then analyzes the security of the second transmission by herself. If the error rate is very low, she tells Bob that their quantum communication is secure in principle; otherwise, they will discard their outcomes and repeat their quantum communication from the beginning.

(7) If their quantum communication is secure, Alice and Bob exploit error correction and privacy amplification techniques to distill a private key, same as Ref. 23.

As the logical qubits are always in the maximal mixture of \( |0_B\rangle \) and \( |1_B\rangle \) during the transmission, the intercept-resending attack strategy by Eve will not work. The secret message are encoded on logical qubits with four unitary operations, which may be eavesdropped by inserting spy photons \( 29 \), in particular by inserting delay photons as the invisible photons with a different wavelength compared to the legitimate photons can be filtered out by a special filter \( 29 \) before Bob performs his operations. In order to guarantee the security, Bob has to select parts of samples to check the multiphoton rate as the attack with spy photons will increase the number of photons in each quantum signal. In detail, Bob can choose another subset of logical qubits \( B \) and analyzes its multiphoton rate with photon number splitters (PNSs). That is, for each logical qubit \( B \), Bob uses two PNSs to split each physical qubit \( (B_1 \text{ and } B_2) \) and then measures the number of the photons in each legitimate physical qubit. If the multiphoton rate is unreasonably high, Bob terminates the transmission and they repeat the communication from the beginning, same as Ref. 29.

Certainly, the technique of photon number splitters is not mature in the application of quantum communication. At present, Bob can use beam splitters instead of PNSs to complete the task of analyzing the multiphoton rate, same as Ref. 29.

Without Trojan horse attack with spy photons, the transmission of the sequence \( S_B \) from Alice to Bob is completely same as that in Bennett-Brassard-Mermin 1992 (BBM92) quantum key distribution (QKD) protocol \( 22 \) which has been proven secure \( 30 \), \( 31 \). Thus, there is no problem in the security of the transmission of the sequence \( S_B \) from Alice to Bob in principle. On the other hand, the reduced matrix of each logical qubit \( B \) from Bob to Alice is \( \rho_B = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \), which means that Eve cannot get useful information if she only eavesdrops the transmission from Bob to Alice in principle. That is to say, our quantum cryptography based on fault tolerant quantum dense coding against a collective-dephasing noise is secure in principle.

C. Quantum cryptography based on quantum dense coding against a collective-dephasing noise

Another kind of noise model called a collective-rotation noise operates as

\[
U_r|0\rangle = \cos \theta|0\rangle + \sin \theta|1\rangle, \\
U_r|1\rangle = -\sin \theta|0\rangle + \cos \theta|1\rangle. \tag{11}
\]

The parameter \( \theta \) depends on the noise and fluctuates with time. With such a type of collective noise, \( |\phi^\pm\rangle \) and \( |\psi^\pm\rangle \) are invariant. Logical qubits can be chosen as

\[
|0_L\rangle = |\phi^+\rangle, \quad |1_L\rangle = |\psi^-\rangle. \tag{12}
\]

Then the states for quantum cryptography based on quantum dense coding can be chosen as

\[
|\Phi^\pm_{AB} \rangle_{AB12} = \frac{1}{\sqrt{2}} (|0\rangle_A|\phi^+\rangle_B12 \pm |1\rangle_A|\psi^-\rangle_B12), \\
|\Psi^\pm_{AB} \rangle_{AB12} = \frac{1}{\sqrt{2}} (|0\rangle_A|\psi^-\rangle_B12 \pm |1\rangle_A|\phi^+\rangle_B12). \tag{13}
\]

Also, Alice divides the quantum systems into two photon sequences, \( S_A \) and \( S_B \). She sends the logical qubit sequence \( S_B \) to Bob and keeps the sequence \( S_A \). After receiving the sequence \( S_B \), Bob chooses randomly a subset of logical qubits as the samples for error rate analysis and the other logical qubits make up of the message sequence \( S_{BM} \) for generating the private key. Bob measures each logical qubit in the samples by choosing randomly one of the two bases \( \sigma_z^B \equiv \sigma_z^B \otimes \sigma_y^B \) and \( \sigma_y^B \equiv \sigma_x^B \otimes \sigma_y^B \). Also he tells Alice which logical qubits are chosen as samples. Under these two bases, the original entangled state can be written as

\[
|\Phi^\pm_{AB} \rangle = \frac{1}{2} (|0\rangle_A(|00\rangle + |11\rangle)_B12 \\
+ |1\rangle_A(|01\rangle - |10\rangle)_B12) \\
= \frac{1}{\sqrt{2}} (|y\rangle_A|y\rangle_B12 - |y\rangle_A|y\rangle_B12) \tag{14}
\]

That is, the outcomes obtained by Alice and Bob are correlated if they choose the same bases for their logical qubits. For the message sequence \( S_{BM} \), Bob operates each logical qubit with one of the four unitary operations \( \{\Theta_{00}, \Theta_{01}, \Theta_{10}, \Theta_{11}\} \) which can be written as

\[
\Theta_{00} = \Theta_1 = I_1 \otimes I_2, \\
\Theta_{01} = \Theta_z = U_{21} \otimes U_{x2}, \\
\Theta_{10} = \Theta_x = U_{x1} \otimes U_{x2}, \\
\Theta_{11} = \Theta_y = I_1 \otimes U_{y2}. \tag{15}
\]
With these four operations, Bob can manipulate the total state of the quantum system $AB_1B_2$ and then send back the two photons $B_1B_2$ to Alice. The relation between the initial state, final state and the operation is shown in Table II. After the operation, Bob sends the sequence $S_{BM}$ back to Alice.

| $|\Phi^+\rangle_f$ | $|\Phi^-\rangle_f$ | $|\Psi^+\rangle_f$ | $|\Psi^-\rangle_f$ |
|---------------|---------------|---------------|---------------|
| $|\Theta_{00}\rangle$ | $|\Theta_{01}\rangle$ | $|\Theta_{10}\rangle$ | $|\Theta_{11}\rangle$ |

As these four entangled states in Eq. (13) are orthogonal, they can be discriminated faithfully in principle. With the measurement results and the initial states, Alice can deduce the unitary operations performed by Bob. Different from the case with a collective-dephasing noise, Alice first performs a Hadamard operation on $B_1$ and then takes a Bell-state measurement on $A$ and $B_1$ and a single-photon measurement on the photon $B_2$. The effect of the Hadamard operation can be written as

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),$$

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle).$$

Under the Hadamard operation, the four states in Eq. (13) are transformed into

$$|\Phi^\pm\rangle_{AB_1B_2} \rightarrow \frac{1}{\sqrt{2}}(|\phi^\pm\rangle_{AB_1} \otimes |+\rangle_{B_2} - |\psi^\mp\rangle_{AB_1} \otimes |\rangle_{B_2}),$$

$$|\Psi^\pm\rangle_{AB_1B_2} \rightarrow \frac{1}{\sqrt{2}}(|\psi^\pm\rangle_{AB_1} \otimes |+\rangle_{B_2} - |\phi^\mp\rangle_{AB_1} \otimes |\rangle_{B_2}).$$

(17)

The combinations of a Bell-state measurement and an $X$ basis single-photon measurement are different for these four states. In this way, Alice can read out the information about the operations done by Bob and they can share two bits of information by means of transmitting two physical qubits in a collective-dephasing noise.

As for the security of this quantum cryptography protocol, it can be made as the same as the case with a collective-dephasing noise.

### III. DISCUSSION AND SUMMARY

In quantum communication with a noisy channel, the two parties need introduce some other physical qubits to prevent a logical qubit from noise or decrease the success probability of faithful transmission of qubits. When quantum dense coding is used for quantum cryptography in a noisy channel, three-photon quantum systems are the optimal ones as the quantum dense coding based on the measurements on wavepackets is not easy to be implemented at present. If these two quantum dense coding schemes are not used for secure quantum communication, they are not optimal as the effect of the two kinds of collective noise can be avoid with two orthogonal single-photon states and each qubit can carry one bit of information. However, only two orthogonal single-photon states cannot ensure the security of quantum cryptography.

In summary, we have presented two fault-tolerant quantum key distribution protocols against two different kinds of collective noise, following some ideas in quantum dense coding. Each logical qubit, which is invariant when it is transmitted in a collective-noise channel, is made up of two physical qubits. The information is encoded on logical qubits with four unitary operations, which will not destroy the antinoise feather of the quantum systems. Alice can read out Bob’s message with a Bell-state analysis and a single-photon measurement on each three-qubit quantum system, not three-qubit joint measurements (i.e., GHZ-state measurements). Two bits of information are exchanged faithfully and securely by transmitting two physical qubits through a noisy channel. When the losses in the noisy channel is low, our quantum cryptography schemes can, in principle, be used to transmit a secret message directly.

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