Intuitionistic fuzzy decision support based on EDAS and grey relational degree for historic bridges reconstruction priority

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Abstract
Bridge management includes all actions in the life cycle of the bridge, to ensure its safety, stability, and functionality. Numerous problems have been identified that are primarily related to the organization of planning and the role of decision-making in the reconstruction of the historic pedestrian bridges. The planning process for the reconstruction of these bridges is crucial due to increased traffic load, poor condition, or damage to bridges. Some of these bridges are part of the cultural heritage, while some are unfairly neglected. The motivation for this research arose from the need to establish the priority for the reconstruction of historic pedestrian bridges to achieve their safety, stability, functionality, and cultural preservation. For this reason, a new decision support model based on intuitionistic fuzzy group decision-making to the multi-criteria analysis is created. The model combines multi-criteria method Evaluation Based on Distance from Average Solution and grey relational degree (GRD) with intuitionistic fuzzy theory. Three relevant decision groups of experts are formed, with the knowledge and expertise in the area of research problematic, establishing criteria for the evaluation. A new approach to the consistency of criteria weights is proposed. The intuitionistic fuzzy likelihood function is developed for the aggregation of bridge evaluations. Furthermore, GRD values are calculated to determine the reconstruction priority ranking of bridge for each decision group. The final ranking is defined by integrating Integer Linear Programming (ILP) and Ant Colony Optimization (ACO), determining spatial-functional, time, and financial constraints.

Keywords Decision support model · Intuitionistic fuzzy theory · EDAS · Grey relational degree · Historic pedestrian bridges · Reconstruction priority

1 Introduction
The challenging task for engineers and conservatives is the protection of the buildings under the cultural heritage. Usually, these buildings are neglected and no proper managerial activities are provided for them. Among these buildings are historic bridges that require major or extensive remediation activities to ensure their safety, stability, and functionality. Considering that most of them are exposed to a long period of negligence and to environmental and anthropological impacts, their elements need to be preserved from degradation, and for this reason, adequate conservation and precedence to their values must be specified. Most of these bridges are derelict and no proper maintenance is conducted. As modern civilization requires more intensive traffic flow, it is important to make these bridges more functional and adaptable to new requirements, to extend their life cycle, and preserve their historical importance. Due to all mentioned, adequate materials and technologies are needed during reconstruction or maintenance to keep their authenticity. Planning their recovery demands qualitative and thorough decisions among engineers, conservatives, and project managers, which is not an easy mission. Before the implementation of any reconstruction activity on these bridges, all regulations

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of conservation and local spatial development plans need to be taken into account. Attention should also be given to the fact, that historic bridges are assets that need not only conservative protection but also the preservation of their socio-economic features. Which bridge has priority in reconstruction is a decision that demands objective and thoughtful judgments of various aspects of an interdisciplinary nature. Using a decision support model is an effective tool which can systematic and synthetically describe fundamental issues when it comes to resolving complex types of problems in the decision-making process. Objective and impartial decision-making is often compound and challenging in most construction projects, and it requires support from various experts from different engineering fields. Additional difficulties arise during the establishment of a corresponding set of criteria as one of the parts of the decision support model (Radziszewskaz Zielina and Sladowski, 2017). In spite of restrictions such are technical, environmental, financial, or socio-economic, with the decision support principles, it can be easy for managers to define the most appropriate activity of reconstruction for each bridge. Since planning and decision-making are the main components of construction management, the planning of recovery of historic pedestrian bridges is important due to the management quality. Assessment of bridge safety and functionality is multi-criterion decision-making (MCDM) task, that deals with the assessment of criteria such as safety, functionality, cultural preservation, environmental impact, stability, etc. Therefore, the MCDM approach can be used for bridge safety assessment. The demanding nature of the observed problem motivated the authors to examine the possibility of forming a unique decision support (DS) model that would upgrade group decision-making to the reconstruction planning of historic pedestrian bridges. Group decision-making is used in tasks of selection of the best alternative from the observed set of alternatives or the priority ranking of feasible alternatives. In both tasks, group decision-making is implemented via the attitudes and opinions of various decision-makers (Wang et al. 2020). To enable such a group decision making, a new DS model is created that integrates an intuitionistic fuzzy theory with multi-criteria method Evaluation of the Distance from the Average Solution (EDAS) and grey relational degree (GRD). The EDAS method (Keshavarz Ghorabaee et al. 2015) uses the average solution for evaluating the alternatives. It is based on measuring a positive distance from average (PDA) and a negative distance from average (NDA) when evaluating alternatives. The alternative with a higher value of PDA and smaller NDA is selected as the best one. It is useful for problems with multiple conflicting criteria whose generation and evaluation are often uncertain and vagueness. The GRD is defined by Grey Relational Analysis (GRA). It is introduced by Deng (1989) as a multi-criteria procedure to evaluate the uncertain relations between the alternatives using criteria. It is a process of grey knowledge, which deals with the indistinct and vagueness information on different parameters and diverse environments. It requires a simple and straightforward assessment with a reasonable amount of interpretative data. The theory of GRA has been widely applied in some uncertain problems such as decision-making, pattern recognition, and alike, particularly under the discrete data and fuzzy information (Stanjukić et al. 2017). It is used to determine the vicinity to the ideal solution and the remoteness of the anti-ideal solution, respectively. The process of the proposed DS model is illustrated through decision-making to the historic pedestrian bridge remediation as a socially sensitive and very ill structure problem. Transparent decisions with the lowest predicted losses are crucial for managers (Rashidi, Lemass, and Gibson 2010). Objective and impartial decision-making is a difficult task, as in each industry so it is in construction engineering and cultural heritage management. It is a complex and time-consuming process due to the comprehensive construction project’s character, and it must be supported by experts from different areas of the construction industry. Most of the difficulties occur in the definition of a suitable set of criteria needed in the group decision-making process (Li et al. 2009). The major contributions and novelties of the research are as follows:

(i) A new decision support model under intuitionistic fuzzy theory is proposed to achieve qualitative group decision-making to solve the outranking problem of reconstruction planning of historic pedestrian bridges. Therefore, membership, non-membership, and hesitation degrees are included.

(ii) The proposed algorithm contains an extended EDAS method based on GRD, integrated into the intuitionistic fuzzy numbers.

(iii) A set of criteria is established for the alternatives assessment according to safety, stability, functionality, etc. Thereby, fractional programming and intuitionistic fuzzy weighted averaging operator are used to calculate criteria weights, and thus, a consistency determination is proposed.

(iv) Final common ranking of the bridges is obtained by combining Integer Linear Programming and Ant Colony Optimization, using financial, time, and spatial-functional constraints.

The research has five sections. In section two the preliminaries of intuitionistic sets and the proposed methodology are presented. In section three, a numerical example of the decision support model for the historic pedestrian bridges built till the end of the Austro-Hungarian
Monarchy in Split-Dalmatian County, Croatia, is explained. In section four, the discussion is provided with sensitivity analysis and comparison with other methodologies, and the last section gives concluding remarks.

1.1 Literature review

Problems of multi-criteria decision-making are crucial matters in comprehensive engineering, management, and economics due to the complex and uncertain decision-making and disparate human thinking ability and knowledge capacity (Boran and Akay 2014). The fuzzy set theory was developed by Zadeh (1965) to cope with uncertain information. It has been applied in various areas, such as multi-criteria decision making (Liu et al. 2012), uncertainty definition of linguistic variables (Liu and Yu 2014), SIS epidemic model analysis, and treatment control function (Adak and Jana 2021), bridge condition assessment (Dabous 2008), managing disaster relief under earthquake concerns (Mohammadi et al., 2020), optimization of electrical discharge machine (Fazlollahtabar and Gholizadeh 2019), solving the multi-objective problem of manufacturing system scheduling (Huang and Süer 2015), parametrization in the problems with vague data (Nawaz and Akram 2021), network systems connectivity (Poulik, Das and Ghorai 2021), optimization of robotic flexible assembly cells scheduling using Taguchi method combined with fuzzy logic and multi-objective analysis (Abd, Abhary and Marian 2016), the definition of phytoplankton–zooplankton model using triangular fuzzy number (Meng and Wu 2020). The fuzziness arises from vagueness, uncertainty, and incomplete information which is often present in project managers’ problems (Yang and Lin 2013). The fuzzy set concept is generalized by Atanassov (1986), and Atanassov and Gargov (1989) who provided the intuitionistic fuzzy sets (IFSs) concept. It is a quite effective way to deal with uncertainty and obscurity. Lately, more and more attention was given to IFS in different research fields. Atanassov (2017) gave a comparison of IFSs with Type-1 fuzzy sets and the possibility for the transformation of some concepts from IFSs theory to T1-IFS theory. Chen, Cheng, and Chio (2016) proposed a new fuzzy multi-criteria group decision-making method using intuitionistic fuzzy sets and a methodology of evidential reasoning. Atanassov, Mavrov, and Atanassova (2014) developed a new model of intuitionistic fuzzy multi-criteria decision-making called inter-criteria decision making. Wei et al. (2011), Wei (2010), and Hou (2010) investigated the intuitionistic fuzzy multi-attribute decision-making problems with the unknown information about attribute weights. Xu (2008) focused on the dynamic intuitionistic fuzzy multi-attribute decision-making problems, proposing new intuitionistic fuzzy aggregation operators. Xia and Xu (2012) developed two methods, based on entropy and cross-entropy to determine criteria weights, and proposed intuitionistic fuzzy entropy and cross-entropy measures. Lin, Yuan, and Xia (2007) proposed an approach for fuzzy multi-criteria decision-making under an intuitionistic fuzzy environment, with satisfiability and non-satisfiability degree of each alternative regarding a set of criteria represented by intuitionistic fuzzy sets. Furthermore, the intuitionistic fuzzy theory was also used in sustainable energy decision-making problems (Gumus, Kucukvar, and Tatari 2016), pattern recognition (Cheng and Chang 2015; Vlachos and Sergiagis 2007), biparametric similarity measure (Boran and Akay 2014), image fusion (Balasubramanian and Ananthi 2011) personal and supplier selection (Zhang and Liu 2019; Boran et al. 2009), application of similarity in group decision-making (Szmidt and Kacprzyk, 2004), measuring the amount of knowledge (Szmidt and Kacprzyk 2014), partner selection and evaluation systems of product development (Büyüközkan and Güleryüz 2016), etc. All of the abovementioned research proposed models and methods for the specific case study and problematic using IFS theory to gain certainty and accuracy in various multi-criteria and demanding tasks.

In a multi-criteria decision-making process combined under intuitionistic fuzzy theory, the criteria weights and alternative evaluations are formed into the intuitionistic fuzzy information, and sometimes their determination can be difficult due to the lack of time, experience, knowledge, or data in the problem domain (Kahraman et al. 2017). The traditional multi-criteria methods such as EDAS might fail in dealing with the intuitionistic fuzzy multicriteria decision-making problems. For this reason, fuzzy set theory and intuitionistic fuzzy set theory are integrated into the EDAS method. Kahraman et al. (2017) applied intuitionistic fuzzy EDAS to solid waste disposal, Keshavarz Ghorabaee (2016) used extended EDAS for fuzzy MCDM to supplier selection, Li, Wang and Wang (2018) incorporated power aggregation operators with the EDAS method combined with linguistic neutrosophic theory to solve fuzzy multi-criteria group decision-making problems, Ilieva, Yankova and KliSareva-Belcheva (2018) presented a new model of fuzzy EDAS modifications of distance measure to improve decision analysis. Ye, Zhan and Xu (2021) used tight fuzzy rough sets to obtain intuitionistic fuzzy data from uncertain data and developed a new multi-attribute decision-making methodology using PROMETHEE II and EDAS method under an intuitionistic fuzzy environment. Mi and Liao (2019) developed a hesitant fuzzy best–worst method to define criteria weights and integrated them in the normalized form into the EDAS method. Keshavarz Ghorabaee et al. (2017) proposed a model for evaluation of supplier and order assignment considering environmental criteria using interval type-2
fuzzy sets and the EDAS method. Ju et al. (2020) developed a new approach based on the EDAS method for the selection of the optimal healthcare alternatives for waste disposal. Thereat, alternatives assessments and criteria weights are determined by experts using multi-granular linguistic distribution assessments. Karatop et al. (2021) used Fuzzy Analytical Hierarchy Process, Fuzzy Failure Mode and Effect Analysis and EDAS method to define optimal decisions for the renewable energy sector in Turkey, also applying risk analysis. Peng, Dai, and Yuan (2017) proposed three approaches for the interval-valued fuzzy soft decision-making problems by multi-attributive Border Approximation Area Comparison and EDAS to solve some unreasonable conditions and promote the development of decision-making methods. Wei, Wei and Guo (2021) in their research extended the EDAS method to solve multiple attribute group decision-making under the probabilistic linguistic term sets, while Ren et al. (2021) extended and used the EDAS technique under the four-branch fuzzy theory to handle the multi-criteria decision-making problems. Menekse and Camgöz Akdag (2022) proposed a hybrid multi-criteria group decision-making model by fusing the EDAS method with the analytic hierarchy process (AHP) using spherical fuzzy sets to solve the problem of selecting a videoconferencing tool for distance education. On the other hand, the grey relational analysis (GRA) method (Deng, 1982) is based on the analysis of different relationships between the discrete data sets and decision-making in multi-criteria situations (Xu, 2008). Rani et al. (2021) used a technique of the GRA method under an intuitionistic fuzzy environment to explore and obtain an ideal cellular mobile telephone service provider in the Madhya Pradesh region, India. Zhang et al. (2013) used intuitionistic fuzzy GRA for the multi-criteria decision-making problems in which the criteria value took the form of an intuitionistic trapezoidal fuzzy number, where the criteria weight was unknown. Zhang et al. (2014) presented a multi-attribute decision-making approach with interval-valued intuitionistic fuzzy evaluation measures for each alternative using GRA. Dey, Pramanik, and Giri (2015) developed multi-criteria group decision-making for weaver selection in Khadi institutions in an intuitionistic fuzzy environment based on GRA. Wei et al. (2011) created a model of intuitionistic fuzzy GRA for problem research on multi-attribute decision-making with intuitionistic fuzzy information in which the average weighted information is completely unknown. The above-given literature used EDAS or GRA method in a simple fuzzy or intuitionistic fuzzy form, but none of them combined these two methods and used them in intuitionistic fuzzy group decision-making. Unlike some of the presented studies with unknown criteria or attribute weights, in this research weights are given by each decision-maker and aggregated group weights are calculated at the beginning of the proposed model, and then further used to gain GRD for each alternative.

The advantage of the newly developed model regarding presented studies in the literature is that it is applied to the more complex and ill-structure problems, with multiple criteria and alternatives which proves that the research is more adaptive to higher demand problems. It is evident from the research in the literature review that EDAS based on GRD has not been used to solve the multi-criteria problem in integration with the intuitionistic fuzzy theory, especially in the field of construction engineering and management. Therefore, it is a significant research contribution and an important topic to create a new decision-making approach applying EDAS and GRD under an intuitionistic fuzzy environment in decision-making to rank and obtain the most indispensable historic bridge for the remediation. Such a decision-making approach will avoid the deficiency of the traditional multi-criteria decision-making procedure and form a new concept of intuitionistic fuzzy decision information. Fusing EDAS with GRD authors found it very useful for this and similar problems because both methods can deal with complex multi-criteria issues. On the one hand, the EDAS method calculates the distance of each alternative from the average solution, it is more efficient and accessible. The problems of construction engineering and management, with various conflicting criteria, multiple alternatives, multiple decision groups with different professions, and within each group there are multiple stakeholders with different styles, opinions and, experiences, tend to effective and practical solutions that are less time-consuming. On the other hand, GRD belongs to grey system theory and is suitable when it comes to problems with compound interrelationships among multiple factors and variables, and it solves MCDM problems by combining introduced criteria values considered for every alternative into the single value. This way, alternatives with multiple conflicting criteria can be easily compared using GRD. Besides, for the proposed group decision-making problem, the authors decided to apply an intuitionistic fuzzy theory because complex construction management problems are often based on imprecise and uncertain or incomplete information. For this reason, to achieve a tractable and robust solution, it is inevitable to reach for the methodology that will utilize the minimum tolerance for imprecision and uncertainty in achieving solutions to ill-structured decision-making problems. Intuitionistic fuzzy
Intuitionistic fuzzy decision support based on EDAS and grey relational degree for historic bridges remediation group decision-making.

2 Materials and methods

In the following section, some basic facts and procedures related to intuitionistic fuzzy sets (IFSs) are given. Firstly, the IFSs and intuitionistic fuzzy weighted averaging operators are defined.

2.1 Preliminaries of intuitionistic fuzzy sets

IFS theory is introduced by Atanassov (1986) and is an extension of the classical Fuzzy Set Theory which deals with uncertainty and duality. Intuitionistic fuzzy set \( \alpha \) in a finite set \( X \) can be written as:

\[
\alpha = \{ (x, \mu_\alpha(x), \gamma_\alpha(x)) | x \in X \}
\]

where \( \mu_\alpha(x) : X \rightarrow [0, 1] \), \( \gamma_\alpha(x) : X \rightarrow [0, 1] \) are membership function and non-membership function, respectively, of element \( x \) in \( X \) of set \( \alpha \), such that:

\[
0 \leq \mu_\alpha(x) + \gamma_\alpha(x) \leq 1
\]

The third parameter of IFS is \( \pi_\alpha(x) \), known as the intuitionistic fuzzy index or hesitation degree that denotes the hesitation of \( x \) to \( \alpha \):

\[
\pi_\alpha(x) = 1 - \mu_\alpha(x) - \gamma_\alpha(x)
\]

And it is obvious that for every \( x \in X \):

\[
0 \leq \pi_\alpha(x) \leq 1
\]

If the \( \pi_\alpha(x) \) is small, then \( x \) is more certain. But if \( \pi_\alpha(x) \) is great, then \( x \) is more uncertain.

When \( \mu_\alpha(x) = 1 - \gamma_\alpha(x) \) for all elements of the universe, the ordinary fuzzy set concept is regained (Wei et al. 2021).

Definition 1 (Liao and Xu, 2014) Let \( \alpha = (\mu_\alpha(x), \gamma_\alpha(x), \pi_\alpha(x)) \) be an intuitionistic fuzzy set (IFS), where \( \mu_\alpha(x) \in [0, 1] \), \( \gamma_\alpha(x) \in [0, 1] \), and \( \pi_\alpha(x) = 1 - \mu_\alpha(x) - \gamma_\alpha(x) \).

Definition 2 (Gumus, Kucukvar and Tatari, 2016) Let \( \alpha = (\mu_\alpha, \gamma_\alpha, \pi_\alpha) \) and \( \beta = (\mu_\beta, \gamma_\beta, \pi_\beta) \) be two IFSs of the set \( X \), then the following statement are defined as follows:

\[
\alpha \cdot \beta = (\mu_\alpha \cdot \mu_\beta, \gamma_\alpha + \gamma_\beta - \gamma_\alpha \cdot \gamma_\beta, 1 - \mu_\alpha \cdot \mu_\beta - \gamma_\alpha + \gamma_\alpha \cdot \gamma_\beta)
\]

\[
(\mu_\alpha)^\lambda, 1 - (1 - \mu_\alpha)^\lambda, (1 - \mu_\beta)^\lambda - (\mu_\beta)^\lambda, \lambda > 0
\]

\[
(\gamma_\alpha)^\lambda, 1 - (1 - \gamma_\alpha)^\lambda, (1 - \gamma_\beta)^\lambda - (\gamma_\beta)^\lambda, \lambda > 0
\]

2.2 Proposed DS model

In this study, an integrated DS model for group multi-criteria decision-making problems of historic bridge remediation planning is proposed. It is formed using intuitionistic fuzzy theory fused with the EDAS method and GRD to obtain bridge ranking by each decision group (DG) and common ranking of bridges obtained by ILP-ACO simulation. The proposed model consists of 17 steps. The steps of the newly developed DS model are given below and presented in detail in Figs. 1 and 2, where Fig. 1 shows the ranking of bridges by each DG, while Fig. 2 presents the final priority ranking of bridges. The pseudo-code of the proposed DS model is given in Algorithm 1.

Step 1. Definition of alternatives and criteria.

To start any MCDM procedure, it is necessary to determine the optimal alternative within the analysed set of alternatives \( A = \{A_1, A_2, \ldots, A_m\} \) evaluated by set of criteria \( C = \{C_1, C_2, \ldots, C_n\} \). Groups of experts \( G = \{G_1, G_2, \ldots, G_g\} \) consisting of experts/decision makers \( E = \{E_1, E_2, \ldots, E_p\} \) have been formed to obtain the optimal alternative(s).

Step 2. Calculate DMs’ weights.

Definition 3 (Boran et al. 2009) There are \( p \) decision makers (DM) in decision group (DG). The weights of DMs are given as linguistic values expressed by IFNs. If \( A_t = (\mu_t, \gamma_t, \pi_t) \) is an IFN of the \( t \)th DM’s rating, then the weight of the \( t \)th DM can be obtained as:

\[
\lambda_t = \frac{\left( \sum_{i=1}^{p} \frac{\mu_t \cdot \pi_t}{\mu_t + \gamma_t} \right)}{\sum_{i=1}^{p} \frac{\mu_t \cdot \pi_t}{\mu_t + \gamma_t}}
\]
where \( \sum_{i=1}^{p} \lambda_i = 1 \).

**Step 3.** Determine criteria weights using an intuitionistic fuzzy judgment matrix.

All criteria may not necessarily be of equal importance, and for this reason, a set of ratings needs to be determined that will be used for criteria weights. Suppose the DM provides intuitionistic fuzzy judgments for a pairwise comparison matrix.

**Definition 4.** Let \( A = (a_{ij})_{n \times n} \) be a judgment matrix where \( a_{ij} = (\mu_{ij}, \gamma_{ij}, \pi_{ij}) \) satisfies given condition, i, j = 1, 2, ..., n, and \( \mu_{ij} \) denotes the preference degree of the criterion \( x_i \) over the criterion \( x_j \) given by DM, \( \gamma_{ij} \) denotes the preference degree of the criterion \( x_j \) over the criterion \( x_i \) by DM, \( \pi_{ij} \) denotes uncertainty degree.

\[
\tilde{P}_{ij} = \left( \tilde{P}_{ij}^\mu, \tilde{P}_{ij}^\gamma \right) = \left\{ \begin{array}{ll}
(0.5, 0.5) & \text{if } i = j \\
\left( \frac{2\mu_{ij}}{\mu_{ij} - \gamma_{ij} + \mu_{ij} - \gamma_{ij} + 2}, \frac{2\mu_{ij}}{\mu_{ij} - \gamma_{ij} + \mu_{ij} - \gamma_{ij} + 2} \right) & \text{if } i \neq j
\end{array} \right.
\]

**Step 4.** Create an intuitionistic fuzzy preference relation based on multiplicative consistency.

**Definition 5.** Let \( \tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, ..., \tilde{\omega}_n)^T = \left( (\tilde{\omega}_1^\mu, \tilde{\omega}_1^\gamma), (\tilde{\omega}_2^\mu, \tilde{\omega}_2^\gamma), ..., (\tilde{\omega}_n^\mu, \tilde{\omega}_n^\gamma) \right)^T \) be an intuitionistic fuzzy priority weight vector of the IFPR \( \tilde{R} = (\tilde{r}_{ij})_{n \times n} \), where \( \tilde{\omega}_i = (\tilde{\omega}_i^\mu, \tilde{\omega}_i^\gamma) \) is a multiplicative consistent IFPR is established as follows:

\[
\tilde{r}_{ij} = (\mu_{ij}, \gamma_{ij}) = \left\{ \begin{array}{ll}
(0.5, 0.5) & \text{if } i = j \\
\left( \frac{2\mu_{ij}}{\mu_{ij} - \gamma_{ij} + \mu_{ij} - \gamma_{ij} + 2}, \frac{2\mu_{ij}}{\mu_{ij} - \gamma_{ij} + \mu_{ij} - \gamma_{ij} + 2} \right) & \text{if } i \neq j
\end{array} \right.
\]

\[
(\tilde{\omega}_i^\mu, \tilde{\omega}_i^\gamma) \text{ for all } i = 1, 2, ..., n.
\]

An intuitionistic fuzzy preference relation \( \tilde{R} = (\tilde{r}_{ij})_{n \times n} \) is called multiplicative consistent if the following multiplicative transitivity is satisfied:

\[
r_{ij} \cdot r_{jk} \cdot r_{ki} = r_{ik} \cdot r_{kj} \cdot r_{ji}, \quad i, j, k = 1, 2, ..., n
\]

Multiplicative consistent IFPR is established as follows:

where \( \omega_i^\mu, \omega_i^\gamma \in [0, 1], \omega_i^\mu + \omega_i^\gamma \leq 1, \sum_{j=1, j \neq i}^n \omega_j^\mu \leq \omega_i^\mu, \text{ and } \omega_i^\mu + n - 2 \geq \sum_{j=1, j \neq i}^n \omega_j^\gamma, \text{ for all } i = 1, 2, ..., n.

If there is a normalized intuitionistic fuzzy weight vector \( \tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, ..., \tilde{\omega}_n)^T \), for IFPR \( \tilde{R} = (\tilde{r}_{ij})_{n \times n} \) and \( \tilde{r}_{ij} = (\mu_{ij}, \gamma_{ij}) \), defined as:

\[
\tilde{r}_{ij} = (\mu_{ij}, \gamma_{ij}) = \left\{ \begin{array}{ll}
(0.5, 0.5) & \text{if } i = j \\
\left( \frac{2\mu_{ij}}{\mu_{ij} - \gamma_{ij} + \mu_{ij} - \gamma_{ij} + 2}, \frac{2\mu_{ij}}{\mu_{ij} - \gamma_{ij} + \mu_{ij} - \gamma_{ij} + 2} \right) & \text{if } i \neq j
\end{array} \right.
\]

\[
(\tilde{\omega}_i^\mu, \tilde{\omega}_i^\gamma) \text{ for all } i = 1, 2, ..., n.
\]

Assignment according to the intuitionistic fuzzy value (IFV), and it satisfies \( \omega_i^\mu, \omega_i^\gamma \in [0, 1] \) and \( \omega_i^\mu + \omega_i^\gamma \leq 1 \). The membership and non-membership degree of the criteria \( x_i \) as an intuitionistic fuzzy importance are \( \tilde{\omega}_i^\mu \) and \( \tilde{\omega}_i^\gamma \), respectively. An intuitionistic fuzzy weight vector \( \tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, ..., \tilde{\omega}_n)^T \) with \( \tilde{\omega}_i = (\tilde{\omega}_i^\mu, \tilde{\omega}_i^\gamma), \omega_i^\mu, \omega_i^\gamma \in [0, 1] \) and \( \omega_i^\mu + \omega_i^\gamma \leq 1 \) for \( (i = 1, 2, ..., n) \) is normalized if it satisfies given condition (Wang 2013; Liao and Xu, 2014):

\[
\sum_{j=1, j \neq i}^n \omega_j^\mu \leq \omega_i^\mu, \omega_i^\mu + n - 2 \geq \sum_{j=1, j \neq i}^n \omega_j^\gamma, \text{ for all } i = 1, 2, ..., n.
\]

(9)

(10)
In decision making is often difficult to obtain multiplicative consistency, and for that reason, it is expected that the deviation between given IFPR and its associated multiplicative consistent IFPR be as small as possible. Introducing the deviation variables $e_{ij}$ and $n_{ij}$, and smaller the absolute deviations are $|e_{ij}|$ and $|n_{ij}|$, respectively, the more exact result. Using fractional programming, the intuitionistic fuzzy weights can be obtained as:

$$\text{Min } Z = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left( e_{ij}^+ + e_{ij}^- + \tilde{e}_{ij}^+ + \tilde{e}_{ij}^- \right)$$ (13)

Fig. 1 DS model: priority ranking of bridges by each DG (steps 1–14)
Fig. 2 DS model: final priority ranking of bridges (steps 15–17)

\[
\begin{align*}
\left\{ \begin{array}{l}
2\omega_i^{(\mu)} - \omega_i^{(\gamma)} + \omega_j^{(\mu)} - \omega_j^{(\gamma)} + 2 = \mu_{ij} - \epsilon_{ij}^+ + \epsilon_{ij}^- = 0, \quad i = 1, 2, \ldots, n - 1; j = i + 1, \ldots, n \\
2\omega_i^{(\mu)} - \omega_i^{(\gamma)} + \omega_j^{(\mu)} - \omega_j^{(\gamma)} + 2 = \mu_{ij} - \epsilon_{ij}^+ + \epsilon_{ij}^- = 0, \quad i = 1, 2, \ldots, n - 1; j = i + 1, \ldots, n
\end{array} \right.
\end{align*}
\]

s.t. \( \omega_i^{(\mu)} , \omega_i^{(\gamma)} \in [0, 1] , \omega_i^{(\mu)} + \omega_i^{(\gamma)} \leq 1, \quad i = 1, 2, \ldots, n - 1 \)

\[
\sum_{j=1, j\neq i}^{n} \omega_j^{(\gamma)} \leq \omega_i^{(\gamma)},
\]

\[
\omega_i^{(\mu)} + n - 1 \geq \sum_{j=1, j\neq i}^{n} \omega_j^{(\gamma)}, \quad i = 1, 2, \ldots, n - 1
\]

\[
\epsilon_{ij}^+ \geq 0, \epsilon_{ij}^- \geq 0, \epsilon_{ij} \geq 0, \xi_{ij} \geq 0, \epsilon_{ij}^+ \epsilon_{ij}^- = 0, \epsilon_{ij}^+ \xi_{ij} = 0, \quad i = 1, 2, \ldots, n - 1; j = i + 1
\]
The IFPR $\tilde{R} = (\tilde{r}_{ij})_{n \times n}$ is multiplicative consistent if and only if $Z = 0$, and $Z$ is the optimal value of the objective function. Since such excellent multiplicative consistent IFPR is hard for the experts to establish, a consistency measure $C_{\tilde{R}}$ must be obtained (with the definition of consistency threshold $\delta$) to measure the degree of consistency for the IFPR $\tilde{R}$. It is very important to include the uncertainty degree when calculating the distance between two IFSs.

**Step 5. Determination of consistency.**

**Definition 6** Let $A$ and $B$ be two IFSs, then the distance measure between $A$ and $B$ is defined as $d(A,B)$, which satisfies the following properties:

(i) $0 \leq d(A,B) \leq 1$;
(ii) $d(A,B) = 0$ iff $A = B$;
(iii) $d(A,B) = d(B,A)$.

Szmidt and Kacprzyk (2000) proposed the following normalized distance measures between $A$ and $B$:

The normalized Hamming distance is given as follows:

$$d(A,B) = \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A - \mu_B| + |\bar{\gamma}_A - \bar{\gamma}_B| + |\bar{\pi}_A - \bar{\pi}_B|)$$  \hspace{1cm} (14)

The normalized Euclidean distance:

$$d(A,B) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (|\mu_A - \mu_B|^2 + |\bar{\gamma}_A - \bar{\gamma}_B|^2 + |\bar{\pi}_A - \bar{\pi}_B|^2)}$$  \hspace{1cm} (15)

Extending (14) and (15) into generalized IF normalized distance and using generalized mean operator by Dyckhoff and Pedrycz (1984) as follows:

$$M(A_1, \ldots, A_n) = \left( \frac{1}{n} \sum_{i=1}^{n} d_i \right)^{1/\lambda}$$  \hspace{1cm} (16)

(14) and (15) can be written as:

$$d(A,B) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( (|\mu_A - \mu_B|^2 + |\bar{\gamma}_A - \bar{\gamma}_B|^2 + |\bar{\pi}_A - \bar{\pi}_B|^2) \right)}$$  \hspace{1cm} (17)

Using Hausdorff distance in generalized IF normalized form as:

$$d(A,B) = \sqrt[3]{\frac{\max\left(\max\left(\frac{\sum_{i=1}^{n} \max(|\mu_A - \mu_B|^2, |\bar{\gamma}_A - \bar{\gamma}_B|^2, |\bar{\pi}_A - \bar{\pi}_B|^2)}{3}\right)\right)}{n}}$$  \hspace{1cm} (18)

If $d(A,B)$ satisfies the following properties:

(i) $0 \leq d(A,B) \leq 1$;
(ii) $d(A,B) = 0$ iff $A = B$;
(iii) $d(A,B) = d(B,A)$;
(iv) If $A \subseteq B \subseteq CA, B, C \in IFs(x)$ then $d(A,C) \geq d(A,B)$ and $d(A,C) \geq d(B,C)$, then $d(A,B)$ is a distance measure between IFSs $A$ and $B$.

Observing (17) and (18), a new generalized IF normalized distance between two IFSs can be proposed as follows:

$$d(A,B) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( (|\mu_A - \mu_B|^2 + |\bar{\gamma}_A - \bar{\gamma}_B|^2 + |\bar{\pi}_A - \bar{\pi}_B|^2) \right)}$$  \hspace{1cm} (19)
\[
\begin{align*}
&\left|\mu_A - \mu_C\right|^2 + \left|\gamma_A - \gamma_C\right|^2 + \left|\pi_A - \pi_C\right|^2 \quad \text{\(R\)} \\
&\frac{6}{\max\left(\left|\mu_A - \mu_C\right|^2, \left|\gamma_A - \gamma_C\right|^2, \left|\pi_A - \pi_C\right|^2\right)} \quad \text{\(R\)} \\
&\geq \left|\mu_A - \mu_B\right|^2 + \left|\gamma_A - \gamma_B\right|^2 + \left|\pi_A - \pi_B\right|^2 \quad \text{\(R\)} \\
&\frac{3}{\max\left(\left|\mu_A - \mu_B\right|^2, \left|\gamma_A - \gamma_B\right|^2, \left|\pi_A - \pi_B\right|^2\right)} \quad \text{\(R\)}
\end{align*}
\]

Which proves property 4. So it is clear that \(d(A, B)\) is the distance measure between IFSs \(A\) and \(B\).

Assume that \(R = (r_{ij})_{n \times n}\) is an IFPR with \(r_{ij} = (\mu_{ij}, \gamma_{ij}, \pi_{ij})\), \(i, j = 1, 2, \ldots, n\), then \(R\) is an acceptable multiplicative consistent IFPR if:

\[
d(R, R^*) \leq \delta
\]

where \(d(R, R^*)\) is the distance measure between given \(R\) and its corresponding multiplicative consistent IFPR \(R^*\), and \(\delta\) is the consistency threshold. As there is a need to calculate the differences only over the upper triangular elements, the denominator is used as \((n - 1)(n - 4)\). Finally, the distance between \(R\) and \(R^*\) can be written as:

\[
d(R, R^*) = \sqrt{\frac{1}{(n - 1)(n - 4)} \sum_{j=1}^{n} \left(\frac{\left|\mu_A - \mu_B\right|^2 + \left|\gamma_A - \gamma_B\right|^2 + \left|\pi_A - \pi_B\right|^2}{6} \right) + \frac{\max\left(\left|\mu_A - \mu_B\right|^2, \left|\gamma_A - \gamma_B\right|^2, \left|\pi_A - \pi_B\right|^2\right)}{3}}
\]

where \(\lambda\) is the DM’s weight given by (8). Then the consistency measure \(C_R\) can be calculated as:

\[
C_R = 1 - d(R, R^*) \geq \delta
\]

If the determined consistency threshold \(\delta\) is not achieved, then the IFPRs have no acceptable consistency, and DMS are asked to go back to step 3 and define new criteria weights. The consistency is taken as \(\delta = 0.90\).

Step 6. Determine aggregated IF criteria weights.

Criteria importance is denoted by \(\omega\) and opinions of all DMS in each DG about each criterion need to be gathered together after the criteria weight consistency is established. As the IFWGA operator highlights the individual influence, and for that reason is more sensitive to \(w_j\) in IFS (j = 1, 2, ..., n), it is proposed for calculating the weights of criteria.

Definition 7 (Xu, 2007a) If \(\omega_j^{(i)} = (\mu_j^{(i)}, \gamma_j^{(i)}, \pi_j^{(i)})\) is an intuitionistic fuzzy number given to criterion \(x_j\) by the \(th\) DM, then the aggregated criteria weights, using IFWGA operator, can be defined as follows:

\[
\omega_j^{(a)} = \text{IFWGA}_{\lambda}\left(\omega_j^{(1)}, \omega_j^{(2)}, \ldots, \omega_j^{(p)}\right) = \left(\omega_j^{(1)}\right)^{\lambda_1}\left(\omega_j^{(2)}\right)^{\lambda_2}\ldots\left(\omega_j^{(p)}\right)^{\lambda_p}
\]

\[
= \left(\prod_{j=1}^{p}\left(1 - \gamma_j^{(t)}\right)^{\lambda_t}\right) - \left(\prod_{j=1}^{p}\left(1 - \gamma_j^{(t)}\right)^{\lambda_t}\right)
\]

where \(\omega = (\omega_1, \omega_2, \ldots, \omega_j)\) and \(\omega_j = (\mu_j, \gamma_j, \pi_j)\), \(j = 1, 2, \ldots, n\), \(\mu_j = \prod_{j=1}^{p}\left(1 - \gamma_j^{(t)}\right)^{\lambda_t}\), \(\gamma_j = 1 - \prod_{j=1}^{p}\left(1 - \gamma_j^{(t)}\right)^{\lambda_t}\), \(\pi_j = \prod_{j=1}^{p}\left(1 - \gamma_j^{(t)}\right)^{\lambda_t}\), and \(\lambda\) is obtained by (8).

Step 7. Calculate high accuracy function.

If \(\mu_i, \gamma_i \in [0, 1]\) and \(\mu_i + \gamma_i \leq 1\), i, j, ..., n, then \(A\) is called intuitionistic fuzzy judgement matrix. If \(\pi_{ij} = 0, 1, \ldots, n\), then the intuitionistic fuzzy judgement matrix \(A\) can be divided into two complement judgement matrices formally \(A_1 = (\mu_{ij})_{n \times n}\) and \(A_2 = (\gamma_{ij})_{n \times n}\), where \(\mu_{ij}, \gamma_{ij} \in [0, 1]\), \(\mu_i + \mu_j = 1\), \(\gamma_i + \gamma_j = 1\), \(\mu_i = \gamma_i = 0.5\), i, j, ..., n.

Definition 8 (Ma and Hu, 1997) Let \(A = (a_{ij})_{n \times n}\) be an antisymmetric matrix, if \(a_{ij} = -a_{ji}, i, j, \ldots, n\), and \(A = (a_{ij})_{n \times n}\) be a transfer matrix, if \(a_{ij} = -a_{ji}, a_{ij} = a_{ik} + a_{kj}, i, j, \ldots, n\), and \(b(a_{ij})\) is called score function of \(a_{ij}\).

Definition 9 (Cao, Wu, and Liang, 2015) Let \(A = (a_{ij})_{n \times n}\) be an intuitionistic fuzzy judgement matrix, then \(B = (b_{ij})_{n \times n}\) is called the score judgement matrix, where \(b_{ij} = b(a_{ij}) = \mu_{ij} - \gamma_{ij}, i, j, \ldots, n, b(a_{ij})\) is called score function of \(a_{ij}\).

Definition 10 (Cao, Wu, and Liang, 2015) Let \(A = (a_{ij})_{n \times n}\) be an intuitionistic fuzzy judgement matrix and \(B = (b_{ij})_{n \times n}\) its score judgement matrix, then \(B = (b_{ij})_{n \times n}\) is an antisymmetric matrix, and if \(b_{ij} = b(a_{ij}) = \mu_{ij} - \gamma_{ij}\),
\[ b_{ij} = b(a_{ij}) = \mu_j - \gamma_j, \quad \text{then} \quad b_{ij} = \mu_j - \gamma_j = \gamma_i - \mu_i = -\gamma_j \text{ for all } i, j = 1, 2, \ldots, n. \]

**Definition 11** (Zhang and Xu, 2012) Let \( A = (\mu_A, \gamma_A, \pi_A) \) be an intuitionistic fuzzy number, a similarity function \( S(A) \) of intuitionistic fuzzy value can be presented as follows:

\[
S(A) = \frac{1 - \mu_A}{1 + \pi_A}
\]

(24)

And the accuracy function is:

\[
H(A) = \mu_A + \gamma_A (25)
\]

For two intuitionistic fuzzy numbers \( A = (\mu_A, \gamma_A, \pi_A) \) and \( B = (\mu_B, \gamma_B, \pi_B) \) there is:

(i) If \( S(A) > S(B) \), then \( A > B \).
(ii) If \( S(A) = S(B) \), then \( A = B \).
(iii) If \( H(A) > H(B) \), then \( A > B \).
(iv) If \( H(A) = H(B) \), then \( A = B \); in this case score of \( A \) is not better than \( B \). The next, the high accuracy function is defined to evaluate the degree of accuracy of IFS.

From the definition of accuracy function (25), \( H(A) \) can be also demonstrated as:

\[
H(A) = \mu_A + \gamma_A = 1 - \pi
\]

The value of \( \pi \) denotes a measure of hesitation. The larger \( \pi \) the higher hesitation degree of the DM’s evaluation. Consequently, the lower the value of \( H(A) \), the less is the degree of accuracy of the IFS of \( A \). Furthermore, if the accuracy function is the same for two IFSs, then one set is no better than the other which does not satisfy DM’s requirements. For this reason, two functions, score \( S(A) \) and accuracy \( H(A) \) are added to obtain a function of high accuracy, which can give a better measure of the accuracy degree of DM’s evaluation. So, using these two functions, the high accuracy to which the alternative \( A_i \) is better than \( B_j \) can be measured as follows (Lin, Yuan, and Xia, 2006):

\[
F(A) = S(A) + \frac{1 - H(A)}{2} = \frac{1 - \mu_A}{1 + \pi_A} + \frac{\pi_A}{2}
\]

(26)

where \( F(A) \in [0, 1] \). The larger the value of \( F(A) \), the better is alternative \( A_i \) over \( B_j \).

**Step 8.** Evaluate the alternatives: aggregated IF decision matrix.

Assume that \( D^{(k)} = a_{ij}^{(k)} \) is an intuitionistic fuzzy decision matrix of each DM, and \( \lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_p\} \) is the each DM’s importance within each group, where \( \sum_{i=1}^{p} \lambda_i = 1, \lambda_i \in [0,1] \). Evaluation of bridges by each DG’s member must be gathered so the aggregated intuitionistic fuzzy decision matrix can be design. Let suppose that the evaluations of DMs are log-normally distributed. The aggregated evaluations of each DG are expressed as the common evaluations per logarithmic size interval with the DM’s importance \( \lambda_i \in [0,1] \), and can be formulated with the log-normal distribution as:

\[
P(d|\delta, \sigma^2) = \frac{1}{d^2\sigma\sqrt{2\pi}} e^{-\left(\frac{(\ln d - \delta)^2}{2\sigma^2}\right) }
\]

(27)

where \( d_i = (\mu_i, \gamma_i / \pi_i) \) is the evaluation of each bridge by each criterion, \( i = 1, 2, \ldots, m, j = 0, 1, \ldots, n \), and \( \lambda_i \in [0,1] \) is the DM’s importance. To calculate likelihood of aggregated evaluations, likelihood function is used. The function of the lognormal distribution for a \( d_i j_s \ (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n) \) is derived by taking the product of the probability densities of the individual \( d_i j_s \):

\[
L(\delta, \sigma^2|d_i^1, \ldots, d_i^m) = \prod_{i=1}^{m} \frac{1}{d_i^m \sigma\sqrt{2\pi}} e^{-\left(\frac{(\ln d_i - \delta)^2}{2\sigma^2}\right) j}
\]

\( = 1, 2, \ldots, n, t = 1, 2, \ldots p \) (28)

The log-likelihood function of the log-normal for the series of \( d_{ij} \ s \ (i = 1, 2, \ldots, m, j = 1,2,\ldots, n) \) is then derived by taking the natural log of the likelihood function:

\[
\ln L(\delta, \sigma^2|d_i^1, \ldots, d_i^m) = \ln \left( \prod_{i=1}^{m} \frac{1}{d_i^m \sigma\sqrt{2\pi}} e^{-\left(\frac{(\ln d_i - \delta)^2}{2\sigma^2}\right) j} \right)
\]

\[
= \ln 1 - \lambda_i \ln d_i - \ln\sigma\sqrt{2\pi} + \left( -\frac{1}{2} \frac{(\ln d_i - \delta)^2}{\sigma} \right) + \ldots + \ln 1 - \lambda_m \ln d_m - \ln\sigma\sqrt{2\pi} + \left( -\frac{1}{2} \frac{(\ln d_m - \delta)^2}{\sigma} \right) \quad t = 1, 2, \ldots p
\]

(29)

After taking derivate with respect to \( \delta \), the expression is derived as:

\[
\frac{\partial}{\partial \delta} L(\delta, \sigma^2|d_i^1, \ldots, d_i^m) = \left( \frac{-1}{\sigma^2} (\lambda_1 \ln d_1 - \delta) \right) + \ldots + \left( \frac{-1}{\sigma^2} (\lambda_m \ln d_m - \delta) \right) - \frac{1}{\sigma^2} \sum_{i=1}^{m} (\lambda_i \ln d_i) - m\delta)
\]

(30)

Setting the derivate equal to zero, such as:
\[
\frac{\partial}{\partial \delta} L \left( \delta, \sigma^2 | d_1^*, \ldots, d_m^* \right) = -\frac{1}{\sigma^2} \left( \sum_{i=1}^{m} (\lambda_i \text{Ind}_{ij}) - m\delta \right) = 0
\]

\[
m\delta = \frac{\sum_{i=1}^{m} (\lambda_i \text{Ind}_{ij})}{\sigma^2}
\]

\[
\delta = \frac{\sum_{i=1}^{m} (\lambda_i \text{Ind}_{ij})}{m}, j = 1, 2, \ldots, nt = 1, 2, \ldots p
\]

Using expression (7), \( \lambda \ln d \) is defined as:

\[
\lambda \text{Ind} = \lambda \ln(1 - \mu) - \ln(1 - \mu) \cdot (1 - \lambda) \ln y
\]

Then, IF likelihood of aggregated evaluations is defined as follows:

\[
\delta_{ij} = \frac{1}{m} \sum_{i=1}^{m} \left( 1 - (1 - \mu)^{(1-\lambda)} \right) \frac{1}{m} \sum_{i=1}^{m} \left( 1 - (1 - \lambda) \right)
\]

where \( \mu_{ij} \) and \( \gamma_{ij} \) are membership and non-membership degree, respectively and \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \) and \( t = 1, 2, \ldots, p \).

Then, the aggregated IF decision matrix is established as:

\[
D_{ij}^{(g)} = \begin{bmatrix}
(\mu_{11}(d_1), \gamma_{11}(d_1), \pi_{11}(d_1)) & (\mu_{21}(d_2), \gamma_{21}(d_2), \pi_{21}(d_2)) & \cdots & (\mu_{m1}(d_1), \gamma_{m1}(d_1), \pi_{m1}(d_1)) \\
(\mu_{12}(d_1), \gamma_{12}(d_1), \pi_{12}(d_1)) & (\mu_{22}(d_2), \gamma_{22}(d_2), \pi_{22}(d_2)) & \cdots & (\mu_{m2}(d_2), \gamma_{m2}(d_2), \pi_{m2}(d_2)) \\
\vdots & \vdots & \ddots & \vdots \\
(\mu_{1n}(d_1), \gamma_{1n}(d_1), \pi_{1n}(d_1)) & (\mu_{2n}(d_2), \gamma_{2n}(d_2), \pi_{2n}(d_2)) & \cdots & (\mu_{mn}(d_n), \gamma_{mn}(d_n), \pi_{mn}(d_n))
\end{bmatrix}
\]

For beneficial criteria:

\[
PDA_j = \frac{\max_0 \left( F(D_{ij}^{(g)}) - F(\tilde{A}V_{ij}^{(g)}) \right)}{F(\tilde{A}V_{ij}^{(g)})}, g = 1, 2, \ldots, 1
\]

For non-beneficial criteria: \( NDA_j \)

\[
NDA_j = \frac{\max_0 \left( F(\tilde{A}V_{ij}^{(g)}) - F(D_{ij}^{(g)}) \right)}{F(\tilde{A}V_{ij}^{(g)})}, g = 1, 2, \ldots, 1
\]

Step 11. Calculate NWPDA and NWNDA.

After the IFPDA and IFNDA are calculated, the high accuracy function of alternatives assessments is determined as explained in Step 7. Then, NWPDA and NWNDA are defined as follows:

\[
\text{NWPDA}_j = \frac{\sum_{i=1}^{n} F(w_{ij}^{(g)}) \cdot PDA_j}{\max_j \left( F(w_{ij}^{(g)}) \cdot PDA_j \right)}, g = 1, 2, \ldots, 1
\]

Step 9. Calculate IF average alternative.

The IF average alternative of each DG is calculated as follows:

\[
AV_{ij}^{(g)} = \frac{\sum_{i=1}^{m} D_{ij}^{(g)}}{n} = \frac{1}{n} \sum_{i=1}^{m} \mu_{ij}^{(g)} \cdot \frac{1}{n} \sum_{i=1}^{m} \gamma_{ij}^{(g)} \cdot \frac{1}{n} \sum_{i=1}^{m} \pi_{ij}^{(g)}
\]

where \( j = 1, 2, \ldots, n \) and \( g = 1, 2, \ldots, l \).

Step 10. Calculate PDA and NDA.

Hereby, the matrices of positive distance from average (PDA), and negative distance from average (NDA) are calculated using high accuracy values, calculated by Eq. (26), of intuitionistic fuzzy aggregated decision matrix and intuitionistic fuzzy average solution, according to the type of criteria as follows:

\[
\text{NWNDA}_j = 1 - \frac{\sum_{i=1}^{n} F(w_{ij}^{(g)}) \cdot NDA_j}{\max_j \left( F(w_{ij}^{(g)}) \cdot NDA_j \right)}, g = 1, 2, \ldots, 1
\]

Step 12. Calculate reference determination.

Hereby, the grey relational analysis starts. Reference determination is defined by assessing the ideal solution (IS) and anti-ideal (AIS) solution. The IS is introduced as:

\[
\chi_{ij}^{+} = \begin{cases} 
\max_{1 \leq i \leq m} \text{NWPDA}_{ij} & \text{benefit,} \\
\min_{1 \leq i \leq m} \text{NWPDA}_{ij} & \text{non - benefit,}
\end{cases}
\]

\[
g = 1, 2, \ldots, l
\]

Similarly, AIS is depicted as follows:
Intuitionistic fuzzy decision support based on EDAS and grey relational degree for historic

\[ y_{ij}^+(g) = \begin{cases} \min_{1 \leq i \leq m} \frac{NWPDA_{ij}}{\text{benefit}}, \\ \max_{1 \leq i \leq m} \frac{NWPDA_{ij}}{\text{non-benefit}} \end{cases}, \quad i = 1, 2, \ldots, l \]  

(41)

**Step 13.** Calculate grey relational coefficient.

Using square of difference, grey relational coefficient of each alternative from the IS is calculated as:

\[
\tau_{ij}^+ = \frac{\min_{1 \leq i \leq m} \min_{1 \leq j \leq n} \left( \sqrt{\frac{(NWPDA(x_j) - y_{ij}^+(g))^2}{NWPDA(x_j) - y_{ij}^+(g)^2}} \right) + \rho \max_{1 \leq i \leq m} \max_{1 \leq j \leq n} \left( \sqrt{\frac{(NWPDA(x_j) - y_{ij}^+(g))^2}{NWPDA(x_j) - y_{ij}^+(g)^2}} \right)}
\]

\[
\tau_{ij}^- = \frac{\min_{1 \leq i \leq m} \min_{1 \leq j \leq n} \left( \sqrt{\frac{(NWNDA(x_j) - y_{ij}^-(g))^2}{NWNDA(x_j) - y_{ij}^-(g)^2}} \right) + \rho \max_{1 \leq i \leq m} \max_{1 \leq j \leq n} \left( \sqrt{\frac{(NWNDA(x_j) - y_{ij}^-(g))^2}{NWNDA(x_j) - y_{ij}^-(g)^2}} \right)}
\]

(42)

(43)

In (42) and (43), \( \rho \) is identification coefficient and it can be adjusted so the difference between series of \( \tau_{ij}^+ \) and \( \tau_{ij}^- \) to \( y_{ij}^+(g) \) and \( y_{ij}^-(g) \), respectively, can be achieved. It has a range of \( \rho \in [0, 1] \), and the smaller its value is, the larger is GRC. Usually, \( \rho = 0.5 \) is applied because it offers moderate distinguishing effect and stability (Lin, Lu, and Lewis, 2007). Changing the value of \( \rho \) will only change the magnitude of the relational coefficient but it does not affect the rank of the GRD (Chiang, Tsai, and Wang, 2002).

**Step 14.** Determine GRD: ranking by each DG.

In this step, the ranking of each DG is provided calculating the GRD as shown below:

\[ NWPDA_{ij} = \frac{\sum_{s} x_{i} \sim a_{i}}{\sum_{s} x_{i} \sim b_{i}}, \quad s.t. \sum_{s} x_{i} \leq c_{i}, \quad x_{i} \in \{0, 1\}, \quad i = 1, 2, \ldots, n \]

where \( x_{i} \) is GRD obtained in previous step, \( \sim \) holds for =, < or >, \( a_{i} \) and \( b_{i} \) and \( c_{i} \) are spatial-functional, time

conventionally, otherwise, common ranking is determined using numerical simulation by Integer Linear Programming based on Ant Colony Optimization (ILP-ACO) as the authors proposed. Taking into the account strategy of SDC, remediation is planned to be implemented through one investment cycle (four years). Firstly, the goal function and constraints are determined for each DG as follows:

\[ \max \sum_{i=1}^{n} \tau_{i} x_{i} \]

\[
\begin{align*}
\sum_{s} x_{i} & \sim a_{i} \\
\sum_{s} x_{i} & \sim b_{i} \\
\sum_{s} f x_{i} & \leq c_{i} \\
x_{i} & \in \{0, 1\}, \quad i = 1, 2, \ldots, n
\end{align*}
\]

(45)
and financial constraints, respectively. \( m \) are months needed for the remediation of each bridge, and \( f \) is an estimated budget for remediation of each bridge. \( x_i \) denote bridges. For each year of the planned investment cycle for the remediation, the set of bridges for each DG is defined.

**Step 16. Definition of the common set of bridges.**

The common set of bridges is determined using ACO (Dorigo and Stützle, 2004) simulation as follows:

\[
\Delta \theta^k_{ij} = \begin{cases} 
1, & \text{\( k^{th} \) DG path on the bridge} \ i, j \\
0, & \text{otherwise}
\end{cases}
\]

(46)

where \( \Delta \theta \) is the value of how many times the path from bridge \( i \), to bridge \( j \) is repeated, where \( k \) is the \( k^{th} \) DG. If the path does not lead to a bridge, it is equal to 0, otherwise, it is equal to \( \frac{1}{L_k} \). \( L_k \) is the level of path by the \( k^{th} \) DG. The amount of total length of each path is calculated by summing all lengths between bridges, as given:

\[
\theta^k_{ij} = \sum_{k=1}^{m} \Delta \theta^k_{ij}
\]

(47)

where \( m \) is the total number of DGs.

The selected path needs to be the shortest one. To choose the right path the probability is calculated as follows:

\[
P_{ij} = \frac{(\theta_{ij})^a (\varphi_{ij})^b}{\sum (\theta_{ij})^a (\varphi_{ij})^b}
\]

(48)

where \( \varphi_{ij} = \frac{1}{L_{ij}} \), and \( a, b = 1 \).

**Step 17. Final common ranking of the bridges.**

The common final ranking is defined by the set of bridges determined for each year of the investment circle.

---

**Input:** Dataset features (historic and new data)  
Selection of bridges: \( A = \{A_1, A_2, ..., A_m\} \)  

**Output:** Priority ranking by each DG; Final priority ranking

| Step | Description |
|------|-------------|
| 1 | Problem discussion; Data collection |
| 2 | Generate DMs: \( E = \{E_1, E_2, ..., E_p\} \); Divide into DGs: \( G = \{G_1, G_2, ..., G_n\} \); Calculate DM’s weights by (8) |
| 3 | Examine bridges |
| 4 | Define relevant criteria; Divide criteria into beneficial and non-beneficial |
| 5 | Form IF judgement matrices using LVs and IFNs |
| 6 | Calculate IF weights; Define objective function and constraints by (13) |
| 7 | Define the IFPR matrix |
| 8 | Check the consistency by (22) |
| 9 | If \( C_k \geq \delta \) then adopt IFPR matrix |
| 10 | else |
| 11 | go back to step 5 |
| 12 | Aggregate weights: IFWGA by (23) |
| 13 | Calculate high accuracy function by (26) |
| 14 | Evaluate alternatives by criteria; Create decision matrices |
| 15 | Calculate matrices by (33) |
| 16 | EDAS: Average alternative; PDA and NDA; NWPDA and NWNDAD |
| 17 | GRA: Reference determination; grey relational coefficient (beneficial, non-beneficial); GRD |
| 18 | Priority ranking by each DG |
| 19 | If \( \{DG \text{ accepts ranking}\} \) then go to a goal function |
| 20 | else |
| 21 | go back to step 1 |
| 22 | repeat |
| 23 | \( \{0,1\} \) LP: form a goal function and constraints by (45) |
| 24 | Select first set of bridges by each DG; Initialize Ant paths; Overlap the paths |
| 25 | Define the first bridge for rehabilitation; start of paths |
| 26 | Calculate the shortest path; Define the probability of the path |
| 27 | Final priority ranking of bridges for the first year of the investment cycle |
| 28 | until all bridges are ranked in each year of the investment cycle |
| 29 | Final priority ranking of the bridges |

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3 Numerical illustration of DS model

The list of historic bridges built during the end of the Austro-Hungarian Monarchy in the area of SDC is established by (Rogulj et al., 2019). Among them are pedestrian bridges, evaluated in this study. Conservative department in Split (Ministry of Culture), Croatia is managing pedestrian bridges that belong to the cultural heritage of SDC. According to the extensive and detailed inspection made by structural and conservative experts, these bridges need to be subjected to construction and architectural activity, such as remediation, to sustain their load capacity, durability, and functionality. Three DGs of experts are formed to assess the alternatives: (DG1) civil engineers and architects, (DG2) conservatives, and (DG3) economists. Nine alternatives are generated and the most indispensable for the remediation will be the one with the highest calculated GRD value. These alternatives are (A1) Kastilac, Kastel Gomilica; (A2) Ricevica, Kastel Stafilic; (A3) The bridge next to the Castel Vitturi, Kastel Luksic; (A4) The bridge next to the Cosic mill, Grab; (A5) The bridge next to the Samardic mill, Grab; (A6) Small bridge over river Grab, Grab; (A7) Gorucica, Sinj; (A8) Vrboska, Hvar; (A9) Franjo Josip, Brac. Also, the same three interactive DGs are asked to establish criteria. Generated criteria are given with their beneficial/non-beneficial determination as follows:

(C1) Safety and stability (non-beneficial) – represents safety that meets the standards (Eurocodes) for bridges, mechanical resistance and stability, and seismic safety.

(C2) Complexity of reconstruction (non-beneficial) – represents the complexity of project design, static calculation, the complexity of reconstruction design, and capacity of the whole structure.

(C3) Load (beneficial) – represents the capacity of the people, and for some bridges a load of vehicles they can bear.

(C4) Cost (non-beneficial) – it is related to the reduction of the project and reconstruction cost, from project design to the reconstruction activities and disposal of the waste material.

(C5) Functionality (non-beneficial) – it is related to the traffic quality and forming of a suitable structural reply to the inflicted deformations throughout the bridge functioning phase.

(C6) Environmental impact (non-beneficial) – it is linked to the protection of the environment and surrounding landscape from the reconstruction activities and the impact of the used materials.

(C7) Reconstruction duration (non-beneficial) – presents the predicted time for the reconstruction, from assembling required mechanical equipment to the finishing works and waste material disposal which needs to be minimized.

(C8) Preservation of cultural heritage (beneficial) – it is related to the preservation of the original appearance and aesthetics of the bridge, all within the laws and regulations of the Ministry of Culture.

Three DGs are formed and they have seven, four, and three members, respectively. The first decision group (DG1) contains seven, the second decision group (DG2) has four, and the third (DG3) has three decision-makers. To each member of each group, importance is given according to his/her years of experience in the field of historical civil and structural construction and conservative management. In Table 1 detailed list of linguistic values (LVs) for DMs’ importance is presented and their transformation to IFNs (the same LVs and IFNs are used for the criteria weights).

To obtain the importance of the DMs, (8) is used. The calculated values of the importance of $\lambda$ for each member of each group are given in Table 2.

Furthermore, each DM defined criteria weights using LVs from Table 1, and thus, intuitionistic fuzzy judgment matrices are formed. Using a model of fractional programming defined by Eq. (13), the priority vectors are defined. Then, the IFPR matrix with intuitionistic fuzzy criteria weights is calculated for each DM and the consistency is checked. Matrices which were not consistent...
(CR < 0.90), decision-makers were asked to redefine criteria weights. After the consistency is achieved for all DM’s criteria weights, IFWGA is used to calculate aggregated matrix for each DG. In Table 3, the priority vector and consistency ratio are given for each DM.

Furthermore, high accuracy function \( F \) values are calculated by Eq. (26) to obtain defuzzyfied criteria weights. These defuzzyfied criteria weights are then normalized as presented in Table 4. It is obvious that the stability and functionality of the bridge are the most important for all groups.

### Table 3 Priority vector for each DM and consistency ratio.

|     | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 | CR |
|-----|----|----|----|----|----|----|----|----|----|
| DG1 | DM1 | \( \mu \) | 0.63 | 0.49 | 0.57 | 0.24 | 0.16 | 0.42 | 0.26 | 0.31 | 0.90 |
|     | \( \gamma \) | 0.16 | 0.51 | 0.28 | 0.62 | 0.73 | 0.56 | 0.73 | 0.69 |    |    |
| DM2 | \( \mu \) | 0.72 | 0.23 | 0.62 | 0.06 | 0.03 | 0.31 | 0.09 | 0.13 | 0.92 |    |
|     | \( \gamma \) | 0.12 | 0.77 | 0.25 | 0.92 | 0.96 | 0.61 | 0.90 | 0.86 |    |    |
| DM3 | \( \mu \) | 0.58 | 0.36 | 0.54 | 0.17 | 0.14 | 0.27 | 0.18 | 0.18 | 0.92 |    |
|     | \( \gamma \) | 0.13 | 0.64 | 0.22 | 0.68 | 0.66 | 0.71 | 0.81 | 0.70 |    |    |
| DM4 | \( \mu \) | 0.76 | 0.09 | 0.18 | 0.02 | 0.00 | 0.09 | 0.03 | 0.04 | 0.90 |    |
|     | \( \gamma \) | 0.07 | 0.91 | 0.77 | 0.60 | 0.99 | 0.88 | 0.97 | 0.96 |    |    |
| DM5 | \( \mu \) | 0.76 | 0.26 | 0.52 | 0.09 | 0.07 | 0.26 | 0.13 | 0.17 | 0.94 |    |
|     | \( \gamma \) | 0.11 | 0.74 | 0.31 | 0.85 | 0.85 | 0.73 | 0.78 | 0.82 |    |    |
| DM6 | \( \mu \) | 0.67 | 0.35 | 0.59 | 0.17 | 0.13 | 0.34 | 0.17 | 0.34 | 0.95 |    |
|     | \( \gamma \) | 0.13 | 0.65 | 0.33 | 0.75 | 0.71 | 0.64 | 0.71 | 0.64 |    |    |
| DM7 | \( \mu \) | 0.75 | 0.44 | 0.40 | 0.18 | 0.22 | 0.38 | 0.21 | 0.19 | 0.93 |    |
|     | \( \gamma \) | 0.13 | 0.56 | 0.51 | 0.72 | 0.64 | 0.55 | 0.68 | 0.74 |    |    |

### Table 4 Normalized defuzzyfied values of criteria weights

|     | DG1 | DG2 | DG3 |
|-----|-----|-----|-----|
| C1  | 0.19 | 0.19 | 0.18 |
| C2  | 0.16 | 0.18 | 0.18 |
| C3  | 0.17 | 0.19 | 0.12 |
| C4  | 0.04 | 0.07 | 0.13 |
| C5  | 0.10 | 0.10 | 0.15 |
| C6  | 0.16 | 0.09 | 0.13 |
| C7  | 0.08 | 0.07 | 0.08 |
| C8  | 0.11 | 0.09 | 0.04 |

### Table 5 LVs and IFNs for the alternative assessment

| LVs | IFNs |
|-----|------|
| Extreme poor (EP)/Extreme low (EL) | (0.05, 0.95, 0.00) |
| Very poor (VP)/Very low (VL) | (0.15, 0.80, 0.05) |
| Poor (P)/Low (L) | (0.25, 0.65, 0.10) |
| Medium poor (MP)/Medium low (ML) | (0.35, 0.55, 0.10) |
| Medium (M)/Fair (F) | (0.40, 0.45, 0.15) |
| Medium good (MG)/Medium high (MH) | (0.55, 0.35, 0.10) |
| Good (G)/High (H) | (0.65, 0.25, 0.10) |
| Very good (VG)/Very high (VH) | (0.80, 0.15, 0.05) |
| Extreme good (EG)/Extreme high (EH) | (0.95, 0.05, 0.00) |
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All alternatives are evaluated by each DM. The evaluations are given using LVs and their belonging IFNs as presented in Table 5. The assessments of alternatives by each DM are aggregated into the DG’s common opinion. For each alternative, membership, non-membership, and hesitation degree are calculated. These degrees present how much each alternative belongs to the highest level of assessment EG/EH by each criterion. Meaning, that the alternative with the lowest membership degree, the highest non-membership degree, and minimum hesitation, the least is near to the EG/EH level, and this alternative is considered the most necessary when it comes to the remediation of the bridges. This statement is especially important for the criterion C1 and C3, where for a bridge near EG/EH level, remediation is not necessary. Criterion C2 “binds” to criterion C1, meaning, that bridge with low safety and stability often demands more complex remediation, hence, bridges with higher complexity of remediation have higher priority for the remediation. The same can be said for criterion C3. Bridges with a higher load can cause higher stress and thus greater damage. For that reason, bridges that have membership degrees near EG/EH level according to the criterion C3, are considered more important for the remediation. In this case, bridges do not suffer high dynamical loads, because there are mostly situated in the inner part of SDC where the pedestrian load is very low. Furthermore, with criteria C4-C7, the situation is a little bit different. Namely, DMs preferred more bridges with lower financial expenses and time duration of reconstruction. Hence, bridges that are closer to the EP/EL level by criteria C4-C7 are preferable. Criterion C8 presents the preservation of cultural heritage, therefore, a bridge that is closer to the EG/EH, by this criterion is considered more necessary for the remediation. Although all bridges should be equally important by this criterion, the current safety and stability of a bridge greatly influence the final decision of the DMs. From this statement, it is obvious that the importance of criterion C1 has the most significant role in defining the final ranking. According to the aggregated assessments of bridges, it is clear that the A1 bridge has the lowest membership by criterion C1 for all three DGs. The highest membership degree according to C1 has A2, A4, and A5 by DG1, A3 and A4 by DG2, and A3, A5, A6, and A7 by DG3. Although these bridges have the highest values for C1, their safety and stability are still not satisfying and reconstruction is needed but it is not necessary as it is for A1. Furthermore, Figs. 3–8 present the membership and non-membership degrees of each alternative by each DG. Also, the average alternative (AA) is presented which is further used for the calculation of NWPDa and NWNDA of each DG. NWPDa and NWNDA are shown in Figs. 9 and 10, respectively.

Furthermore, the GRCs for beneficial and non-beneficial criteria are calculated, as well as GRD for each group. The GRD values and final ranking by each DG are presented in

### Table 6: Spatial-functional constraints:

| Zone  | DG1     | DG2     | DG3     |
|-------|---------|---------|---------|
| Zone 1| $x_1 + x_2 + x_3 \geq 2$ | $x_1 + x_2 + x_3 \geq 1$ | $x_1 + x_2 + x_3 \geq 1$ |
| Zone 2| $x_1 + x_2 + x_3 \geq 1$ | $x_3 + x_4 + x_5 \geq 1$ | $x_3 + x_4 + x_5 \geq 1$ |
| Zone 3| $x_7 \leq 1$ | $x_7 \leq 1$ | $x_7 \geq 0$ |
| Zone 4| $x_8 + x_9 = 1$ | $x_8 + x_9 = 1$ | $x_8 + x_9 = 1$ |
GRD values and constraints as explained in (45) and Fig. 2. ILP is used to define a set of bridges for remediation for it is necessary to determine the final common ranking. The considered more prioritized for the remediation.

Spatial-functional constraints, given in Table 6, are defined according to the spatial distribution of bridges, and by this analogy A1, A2, and A3 are in the first zone (town of Kastela), A4, A5, and A6 are in the second zone (town of Grab), A7 in the third zone (town of Sinj), and A8 and A9 are in the fourth zone (islands Brac and Hvar). Time constraints are defined by the duration of each bridge remediation within each year of the investment cycle, and the financial constraints are determined by the cost of remediation in the monetary units. Time and financial constraints for simplicity are taken as equal for each DG.

Goal function for each DG:

DG1:max\{0.633x_1 + 0.424x_2 + 0.392x_3 + 0.424x_4 + 0.424x_5 + 0.615x_6 + 0.421x_7 + 0.370x_8 + 0.357x_9\}

DG2:max\{0.723x_1 + 0.469x_2 + 0.558x_3 + 0.558x_4 + 0.480x_5 + 0.476x_8 + 0.480x_7 + 0.478x_8 + 0.262x_9\}

DG3:max\{0.636x_1 + 0.388x_2 + 0.500x_3 + 0.488x_4 + 0.541x_5 + 0.541x_6 + 0.500x_7 + 0.314x_8 + 0.488x_9\}

Time constraint: \(8x_1 + 2x_2 + 3x_3 + 7x_4 + 6x_5 + 3x_6 + 3x_7 + 4x_8 + 7x_9 \leq 12\)

Financial constraint: \(30x_1 + 5x_2 + 5x_3 + 20x_4 + 10x_5 + 5x_6 + 5x_7 + 15x_8 + 20x_9 \leq 30\)

For the first set of bridges, only A1 is chosen to be reconstructed in the first year as this bridge is the most deteriorated and is a priority by each DG. The set of bridges for remediation in the second year of the investment cycle is obtained as follows: for DG1, A2, A3, A6, and A8, and for DG2 and DG3 is A2, A6, and A8. These sets are simulated by ACO to provide a common ranking of bridges for the first investment year.

Bridges are positioned in polygons, as presented in Fig. 12, with one polygon for each group. Connections inside polygons (red, green, and blue) are paths. These paths represent the ranking of the first set, starting from the bridge with the highest GRD and ending with the smallest. Between each bridge, the length of the path is measured.

The length of each path is calculated as follows:

\(L_{DG1} = 23.12 \rightarrow \Delta \theta_{DG1}^{ij} = \frac{1}{23.12} = 0.043; \quad L_{DG2} = 16.28 \rightarrow \Delta \theta_{DG2}^{ij} = \frac{1}{16.28} = 0.061; \quad L_{DG3} = 18.51 \rightarrow \Delta \theta_{DG3}^{ij} = \frac{1}{18.51} = 0.054.\)

The overlap of all paths shows the number of pheromones on each length of each path, as presented in Fig. 13.

Calculating the probability, the direction of each path is determined from the common source, A6, which has the highest GRD for DG1 and DG3. From Fig. 12 it is obvious that the path leads from A6 to A2 with the probability \(P_{(A6 \rightarrow A2)} = 82.98\%\). Following the length between bridges, the ranking for the second set is defined as \(A6 > A2 > A8 > A3\). The same procedure is repeated for the third and fourth sets with the exclusion of the bridges from previous sets. The final common ranking of bridges within each year is obtained as follows:
1st year: A1; 2nd year: A6 > A2 > A8 > A3; 3rd year: A4 > A7; 4th year: A5; next investment cycle, 1st year: A9.

### 4 Discussion

#### 4.1 Sensitivity analysis-stability intervals

To examine how modifying the weights of criteria affect the final ranking of historic pedestrian bridges, the stability intervals are used to calculate the upper and lower values of initial criteria weights. The values of the high accuracy function of criteria weights calculated by (26) are used as initial to determine lower and upper weights of interval. Within these intervals, the obtained final ranking of each DG remains unchanged, which confirms the stability of the given weights. In Table 7, intervals of criteria weights for each group are presented.

Different sets of criteria weights are applied to the defined case study to check the stability of obtained rankings. In this case, only the final ranking of each group is checked on sensitivity, assuming that it also affects the final common ranking. These sets of criteria weights are defined for eight criteria. Weights $w_i$ are used to express the relative importance of diverse criteria. The stability intervals of criteria weights are demonstrated by Mareschal (1988), who proposed a process for sensitivity analysis that introduces stability intervals. The weight values of one criterion inside the stability interval do not change the results from the preliminary set of weights. Using initial weights $w_i$ for each criterion, the weight stability interval $[w_i^L, w_i^U]$ is determined. The ranking of solutions obtained with initial weights will be modified if the weight value is out of the stability interval. The weight for the $i$th criterion may increase or decrease from its initial value $w_i$, thus the changed weight can be described as $w_i' = \alpha \cdot w_i$. To have the changed weights normalized, meaning $\sum_{k=1}^{n} w_k' = 1$, other

### Table 7 Stability intervals of criteria weights for each DG

| DG1 | DG2 | DG3 |
|-----|-----|-----|
| $w_L$ | $w_U$ | $w_L$ | $w_U$ | $w_L$ | $w_U$ |
| C1  | 0.19 | 0.18 | 0.21 | 0.19 | 0.17 | 0.21 |
| C2  | 0.16 | 0.15 | 0.17 | 0.18 | 0.17 | 0.21 |
| C3  | 0.04 | 0.04 | 0.08 | 0.07 | 0.07 | 0.13 |
| C4  | 0.10 | 0.05 | 0.16 | 0.10 | 0.20 | 0.15 |
| C6  | 0.16 | 0.14 | 0.19 | 0.09 | 0.13 | 0.11 |
| C7  | 0.08 | 0.08 | 0.09 | 0.07 | 0.08 | 0.07 |
| C8  | 0.11 | 0.11 | 0.22 | 0.09 | 0.18 | 0.04 |

### Table 8 Comparison with IF TOPSIS and IF VIKOR

| DG1 | DG2 | DG3 |
|-----|-----|-----|
| $\alpha$ | $Q$ | Rank |
| A1 | 0.547 | 1 | 1 |
| A2 | 0.527 | 3 | 3 |
| A3 | 0.525 | 1 | 1 |
| A4 | 0.522 | 4 | 4 |
| A5 | 0.508 | 5 | 5 |
| A6 | 0.526 | 2 | 2 |
| A7 | 0.527 | 6 | 6 |
| A8 | 0.487 | 6 | 6 |
| A9 | 0.362 | 9 | 9 |

| $\alpha$ | $Q$ | Rank |
| A1 | 0.312 | 1 | 1 |
| A2 | 0.457 | 3 | 3 |
| A3 | 0.593 | 1 | 1 |
| A4 | 0.563 | 4 | 4 |
| A5 | 0.588 | 5 | 5 |
| A6 | 0.326 | 2 | 2 |
| A7 | 0.718 | 8 | 8 |
| A8 | 0.684 | 6 | 6 |
| A9 | 0.362 | 9 | 9 |

rs = 0.933, rs = 0.950, rs = 0.967, rs = 0.983
weights are changed, keeping initially defined ratios: 
\[ w'_k = zw_k, \quad k \neq i, k = 1, \ldots, n. \]

The function \( \tau(x) \) is obtained from the expression 
\[ \tau(x) = \frac{1}{1 \sum_{k \neq i} w_k} = 1 \]

in the following form 
\[ \tau = \frac{1}{1 \sum_{k \neq i} w_k}. \]

The parameter \( x \) can vary in the interval \( 0 \leq x \leq \frac{1}{w_i} \). Applying the EDAS method with different values of \( x \), the interval \( x_1 \leq x \leq x_2 \), can be achieved with initial weights, and this interval is a stability interval. The weight stability interval can be defined as 
\[ w'_i \leq w_i \leq w'_i, \quad \text{where} \quad w'_i = x_1 w_i \quad \text{and} \quad w'_i = x_2 w_i. \]

The weight stability intervals are defined for each criterion, \( i = 1, \ldots, n \), with the values of the initial weights.

For each DG, small deviations of stability intervals of the criteria weights are determined. Values of weights within these intervals keep obtained final rankings of each DG the same as provided by initial weights.

### 4.2 Comparison with other MCDM methods

Hereby, the comparison of ranking results obtained by each DG with other MCDM methods under the intuitionistic fuzzy environment such are TOPSIS (Memari et al., 2019) and VIKOR (Zeng, Chen, and Kuo 2019) is presented. Table 8 shows the results of priority ranking of the bridges for each group by the proposed method, IF TOPSIS and IF VIKOR. Also, the values of R and Q are given for better insight. For the IF TOPSIS method, the higher R value is, the more prior bridge is for the reconstruction, and for the IF VIKOR method, the lower the Q value is, the higher is bridges positioned in the rank. The correlation of the rankings between the proposed methodology and each of the two methods is calculated by Spearman’s rank correlation coefficient. Spearman’s rank correlation coefficient, \( r_s \), is a statistical approach used to define the relationship strength between two different variables. To have a higher correlation between the proposed methodology and each method, \( r_s \) needs to be greater than 0.8. The \( r_s \) of the proposed methodology is higher than 0.9 for both methods in all three groups. The highest correlation is shown to be for the IF TOPSIS method for DG3, which is 0.983. It is obvious that the ranking of the proposed methodology is highly correlated with the two MCDM methods. Hence, the proposed method is very efficient.

Despite the high correlation, some shortcomings in the two methods are present. This primarily refers to the solutions on which each method is based. The IF TOPSIS and IF VIKOR are calculated by taking into account ideal solutions, while IF EDAS GRD observes both ideal solutions and average solutions. This way, the number of reference points is increased thus allowing an accurate and

| Related MCDM studies | Fuzzy Type | MCDM method | Integrated method | GDM | Final common ranking |
|----------------------|------------|-------------|------------------|-----|----------------------|
| Wei et al. (2011)    | Intuitionistic | GRA      | –                | No  | –                    |
| Zhang, Jin and Lin (2013) | Intuitionistic | GRA      | Entropy          | No  | –                    |
| Zhang et al. (2014)  | Interval-valued intuitionistic | GRA      | –                | No  | –                    |
| Dey, Pramanik and Giri (2015) | Intuitionistic | GRA      | –                | Yes | IFWAA                |
| Keshavarz Ghorabaee et al. (2016) | Fuzzy | EDAS      | –                | No  | –                    |
| Kahraman et al. (2017) | Interval-valued intuitionistic | EDAS      | –                | No  | –                    |
| Peng, Dai and Yuan (2017) | Interval-valued | MABAC     | –                | No  | –                    |
| EDAS                 | –          | EDAS       | –                | No  | –                    |
| Keshavarz Ghorabaee et al. (2017) | Interval type-2 | EDAS      | –                | No  | –                    |
| Ilieva, Yankova and Klisareva-Belcheva (2018) | Fuzzy | EDAS       | –                | No  | –                    |
| Li, Wang and Wang (2018) | Neutrosophic | EDAS      | –                | Yes | LNPWA                |
| Mia and Liao (2019)   | Hesitant   | BW         | EDAS             | No  | –                    |
| Ju et al. (2020)      | –          | EDAS       | –                | No  | –                    |
| Karatop et al. (2021) | Fuzzy      | AHP FMEA   | EDAS             | No  | –                    |
| Ye, Zhan and Xu (2021) | Intuitionistic | PROMETHEE II EDAS | –      | No  | –                    |
| Proposed DS model     | Intuitionistic | EDAS      | GRA (Grey relational degree) | Yes | ILP-ACO               |

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From Table 9, most of the studies used intuitionist fuzzy efficiency in comparison with other studies. As can be seen in the novelty of the proposed study and to indicate its greater method, group/individual decision-making, and obtained a final common ranking by all groups or by decision-makers. All mentioned features of studies were important to define the approach for consistency is defined with its properties and proofs, and the threshold was determined to be greater than 0.90. Also, integration of IF likelihood and intuitionistic fuzzy property is used to define the approach for alternatives assessments aggregation. The proposed methodology, as it was mentioned before, used both ideal solutions and average solution in regard to related studies that only used ideal solutions or an average ones. Finally, to summarize all mentioned, the proposed intuitionistic DS model base on the EDAS and grey relational degree with the integration of ILP-ACO has proven to be novel and better in comparison with the related studies.

4.3 Advantages and disadvantages

The proposed model is unique, it improves a group decision-making process, and it gives free expression of a DM’s attitude that moves the boundaries of the previous decision-making by civil engineers, architects, conservatives, project managers, users, etc., on this issue. The advantages of the proposed intuitionistic fuzzy DS model:

(i) It can be applied to any type of decision-making with a high level of problem complexity. The model has all components needed for the qualitative expert’s judgments and final decisions. For this case study, where historic pedestrian bridges are analyzed, eight criteria are determined to evaluate and rank bridges, these criteria are specific because they can be applied to other bridges and objects, and their number can be upgraded if needed.

(ii) It integrates and hybridizes the strengths of two known and practical methods, EDAS and GRA, using both ideal solutions and average solution as references.

(iii) It uses fractional programming to gain criteria weights and further calculates consistency by a newly developed model for consistency determination.
(iv) It proposes not only an integration of the EDAS method and GRD for the group decision-making problems under an intuitionistic fuzzy environment but also a systematic and sustained methodology with technological and managerial advances. It integrates various constraints defined by DMs using linear programming to achieve the main goal in the planned financial cycle and uses optimization techniques enabling DMs to achieve their goals in time and money-limiting situations.

(v) The approach is straightforward, it successfully reflects real-life problematics with inherent uncertainties and dualities by using IFS theory, and can be applied to various different, more complex problems with a greater amount of criteria, alternatives, DMs and DGs with different attitudes and strategies. It can effectively produce a final decision when there are many groups whose rankings need to be united.

(vi) The proposed methodology is well structured and can be easily implemented in software.

Although the methodology has many advantages, there are some disadvantages. It seems too long and it might be too complicated for some beginners. It demands the involvement of too many decision-makers, thus, too many interviews and discussions. Also, the process of alternatives evaluation is too long. For this reason, in future research directions, the focus needs to be put on developing an intelligent expert system that includes all the information about objects and can gain relevant outcomes by decreasing the number of conversations with experts. Furthermore, a model that will shorten the assessment of alternatives but still give substantial results will be proposed.

5 Conclusion

A new hybrid DS model is created to improve reconstruction planning of historic pedestrian bridges, using group decision making with multi-criteria analysis under the intuitionistic fuzzy environment. It is a delicate and complex problem, especially for the civil engineers, architects, and conservative experts as managers of these bridges. To define the reconstruction priority ranking of historic pedestrian bridges, many factors such as safety, stability, functionality, environmental impact, cultural preservation, etc., needed to be included in the decision-making process. In the proposed model, three DGs are formed according to their area of expertise and relevant knowledge. Members of each DG are asked to create a list of relevant criteria according to which safety and stability, the complexity of reconstruction, load, cost, functionality, environmental impact, reconstruction duration and preservation of cultural heritage will be used to evaluate bridges for the reconstruction priority. Criteria weights are determined by judgment IF matrices, then the priority vector is calculated using the IFPR matrix. For each IF matrix, the consistency is determined using the newly defined model for a consistency determination. Historic bridges are evaluated using defined LVs and IFNs. An aggregated decision matrix is calculated for each DG by the IF likelihood function. The EDAS method is fused with GRD, making a new multi-criteria approach for the systems with limited, uncertain, and incomplete information, under an intuitionistic fuzzy environment. The combination of GRD and EDAS has proved that it can assist in solving sensitive problems and making adequate decisions that in real-life problems often require expert support. Values of GRD were calculated for each DG. These values determined the position of each bridge in the ranking for each DG. Finally, to obtain a final common ranking, a model of [0, 1] ILP-ACO is used. Sensitivity analysis is conducted to gain intervals of weights of criteria within which the same rankings are obtained. The proposed DS model is compared with other MCDM methods such as IF TOPSIS and IF VIKOR where according to Spearman’s rank correlation coefficient, the correlation is higher than 0.9 for both methods.

According to the abovementioned, the newly developed model has proven to be suitable for the considered motive, and because of its flexible character, it could be the starting input for future research on the reconstruction planning of the historic objects. The study contains a quite amount of relevant and qualitative data and information, and although there is no study that deals with the herein described problem, and uses the proposed methodology, the study gives an extensive literature review on the used methods. The utilized MCDM methods are robust, giving distinct results valuable for the group decision-making. Finally, this research offers guidelines for experts, managers, and policymakers, and supplies decision-makers with valuable support for making reliable and straightforward decisions.

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