Constituent quark masses obtained from hadron masses with contributions of Fermi-Breit and Glozman-Riska hyperfine interactions

V. Borka Jovanović, 1 S. R. Ignjatović, 2 D. Borka, 1 and P. Jovanović 3

1 Laboratory of Physics (010), Vinča Institute of Nuclear Sciences, University of Belgrade, P.O. Box 522, 11001 Belgrade, Serbia
2 Department of Physics, Faculty of Science, University of Banja Luka, Mladen Stojanovića 2, 78000 Banja Luka, Bosnia and Herzegovina
3 Astronomical Observatory, Volgina 7, 11060 Belgrade, Serbia

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The quark model predicts that the masses of the mesons and baryons are given by the sum of the constituent masses of the quarks and the hyperfine splitting. As described in [1], the term with hyperfine splitting emanates from a spin-spin interaction produced by one gluon exchange, which also depends on the masses of the quarks (for mesons this term contains a mass sum of a quark pair and for baryons it contains a sum over all the possible quark pairs).

In Refs. [4] the baryons were studied in a quark model with color hyperfine interactions. Some parameters of meson and baryon spectroscopy, including some predictions of the quark masses, can be found in [2]. In Ref. [2], from the meson and baryon relations, the authors fitted both the mass differences and mass ratios with a single set of quark masses. In [2] estimations of masses were made via magnetic moments of baryons and via the vector meson masses. In Refs. [2, 3] it was shown that in the constituent quark model, the Feynman-Hellmann theorem and semiempirical mass formulas can be applied to give useful information about the masses of mesons and baryons.

In this paper we use a schematic study (two quark interaction) to calculate meson and baryon theoretical masses from which the constituent quark masses can be obtained. This paper is organized in the following way: in Section II we give complete method how to calculate theoretical masses, in Section III we explain fitting procedure and our main results, discussion and conclusions are given in Section IV.

I. INTRODUCTION

Using the colored version of the Fermi-Breit hyperfine interaction (FB HFI) [10, 11] and the Glozman-Riska hyperfine interaction (GR HFI) [14, 15], we present estimations of the theoretical meson and baryon masses. Strong FB HFI Hamiltonian [13] has the following form:

$$ H_{FB} = C \sum_{i<j} \frac{\bar{q}_i \sigma \sigma \cdot q_j}{m_i m_j} (\lambda_i^F \lambda_j^F), $$

and it has explicit color and spin exchange dependence and implicit (by way of quark masses) flavor dependence. Here $\sigma_i$ are the Pauli spin matrices, $\lambda_i^F$ are the color Gell-Mann matrices and $C$ is a constant. FB contribution to hadron masses is given by: $m_{\nu,FB} = |\nu\rangle \langle H_{FB} |\chi\rangle |\nu\rangle$, where $\chi$ denotes the spin wave function and $\nu$—the flavor wave function. Strong GR Hamiltonian [14] is of the form:

$$ H_{GR} = -C_\chi \sum_{i<j} (-1)^{\alpha_{ij}} (\lambda_i^F \lambda_j^F) \left( \frac{\bar{q}_i \sigma \sigma \cdot q_j}{m_i, m_j} \right), $$

$$ = \left\{ \begin{array}{ll}
-1, & q\bar{q} \\
+1, & q\bar{q} or \bar{q}\bar{q} \end{array} \right\}, $$

where $\lambda_i^F$ are Gell-Mann matrices for flavor SU(3), $\sigma_i$ are the Pauli spin matrices and $C_\chi$ is a constant. We employ this schematic flavor-spin interaction between quarks and antiquarks which leads to Glozman-Riska HFI contribution to hadron masses: $m_{\nu,GR} = |\nu\rangle \langle H_{GR} |\chi\rangle |\nu\rangle$, where $m_i$ are the constituent quark effective masses: $m_i = m \neq m_s$ and $\nu$—flavor wave functions.

For mesons, there are two flavor SU(3) multiplets according to product: $3 \otimes 3 = 1 + 8$, i.e. one singlet and one octet. For baryons, according to: $3 \otimes 3 \otimes 3 = 1 + 8_{MS} + 8_{MA} + 10$, there is one singlet, two octets (one mixed symmetric (MS) and one mixed antisymmetric (MA)) and one decuplet. Young diagrams for these SU(3)$_F$ multiplets, as well as the weight diagrams, can be found in textbooks (see e.g. [17, 18]). Regarding baryons, in case of octet members (as they have the MS and MA...
part of the flavor wave function), the contribution of HFI is calculated as sum of the symmetric and antisymmetric part, so we start from the following expression: 
$|\nu \uparrow\rangle = \frac{1}{\sqrt{2}} (|\nu_{MS} + \nu_{MA}\rangle$.

III. FITTING PROCEDURE

For our calculations we used multidimensional least-square fit of masses, using subroutine "lfit" from Numerical Recipes in FORTRAN [19], modified according to the instructions in the last paragraph of §15.4 of [19]. We fitted simultaneously all parameters: $m_u$, $m_s$, $m_c$ and the constant $C$ in such a way that we minimized $\chi^2$ between the measured and theoretical masses. Equations for meson and baryon theoretical masses (see §IV) are at first linearized by expansion in Taylor's series up to the first order, and in that way we obtained the corresponding system of linear equations for differences between experimental and theoretical masses in which the unknown variables are corrections of the parameters. These corrections, as well as their corresponding uncertainties, are determined using linear least square method, and then the parameters are accounted for the values of these obtained corrections. With these new (corrected) parameters the previous procedure is repeated until the fit converges, that is while the $\chi^2$ value between the experimental and theoretical masses decreases. In that way, after several iterations, we obtain final values for the parameters and their uncertainties. The uncertainties are estimated during fitting procedure as square roots of the corresponding diagonal elements of covariance matrix, according to eq. (15.4.15) of Ref. [19].

For every analyzed system of equations a fast convergence is obtained, even in the case when starting values of parameters differ very much from their final values, which favors the correctness of our theoretical model as well as our fitting method.

IV. RESULTS AND DISCUSSION

We calculated FB and GR contributions to meson and baryon masses. The mass formulas of mesons and baryons with FB HFI included are given in eqs. (3) - (4), while their mass formulas with GR HFI included can be found in [15] (eqs. (6)-(10) for mesons and eqs. (11)-(13) for baryons).

Here we study the hadron masses using FB and GR HFI in schematic approximation (two-particle interaction). The masses of constituent quarks $m_u (= m_d)$, $m_s$, $m_c$ and the constants $C^m$ and $C^b$ are calculated from least-square fit of the theoretical equations for meson and baryon masses with HFI included. The corresponding experimental masses are taken from "Particle Data Group" site: http://pdg.lbl.gov [20].

We give the equations for meson masses with FB interaction included for the following mesons: light pseudoscalar mesons $\pi$, $K$, light vector mesons $\rho$, $K^*$, $\omega$, $\phi$, charmed mesons $D$, $D^*$, strange charmed mesons $D_S$, $D_S^*$ and double charmed mesons $\eta_c$, $J/\psi$ (eqs. (3)). In these equations, the constant for mesons is denoted by $C^m$. We did not calculate $\eta$ and $\eta'$ contribution because of their mixing and they cannot be described within such a model. Because of that mixing (due to the same quantum numbers), their flavor wave functions are given only in a first approximation [21] and therefore calculations are not sufficiently precise. The mixing of the states also changes the properties and shifts masses from the theoretical predictions.

$$
\begin{align*}
    m_\pi &= 2m_u - 3C^m \frac{m_c}{m_u} ; m_K = m_u + m_s - 3C^m \\
    m_\rho &= 2m_u + \frac{m_s}{m_c} ; m_{K^*} = m_u + m_s + \frac{m_c}{m_s} \\
    m_\omega &= 2m_u + \frac{m_s}{m_c} ; m_\phi = 2m_u + C^m m_u m_c \\
    m_D &= m_u + m_c - \frac{3C^m}{2m_u} ; m_{D^*} = m_u + m_c + \frac{C^m}{2m_u} \\
    m_{D_s} &= m_s + m_c - \frac{3C^m}{m_u} ; m_{D_s^*} = m_s + m_c + \frac{C^m}{m_u} \\
    m_{\eta_c} &= 2m_c - \frac{3C^m}{m_u} ; m_{J/\psi} = 2m_c + \frac{C^m}{m_u}.
\end{align*}
$$

(3)

We also present the theoretical mass equations with FB HFI included for the following baryons: light baryon octet $N$, $\Sigma$, $\Xi$, $\Lambda$, light baryon decuplet $\Delta$, $\Sigma^*$, $\Xi^*$, $\Omega$ and heavy baryons $\Sigma_c$, $\Xi_c$, $\Lambda_c$, $\Sigma_c^*$, $\Omega_c$ (eqs. (4)). Constant for baryons is denoted by $C^b$.

$$
\begin{align*}
    m_N &= 3m_u - 3C^b \frac{1}{2m_u} \\
    m_\Sigma &= 2m_u + m_s + 2C^b (1 - \frac{m_u}{m_s}) \\
    m_\Xi &= m_u + 2m_s + 2C^b \frac{1}{m_s} \left( \frac{1}{4} + \frac{m_u}{m_s} \right) \\
    m_\Lambda &= 2m_u + m_s - 3C^b \frac{1}{2m_u} \\
    m_\Delta &= 3m_u + 3C^b \frac{1}{2m_u} \\
    m_{\Sigma^*} &= 2m_u + m_s + C^b \frac{1}{m_u} \left( \frac{1}{2} + \frac{m_u}{m_s} \right) \\
    m_{\Xi^*} &= m_u + 2m_s + C^b \frac{1}{m_u} \left( \frac{1}{2} + \frac{m_u}{m_s} \right) \\
    m_{\Omega} &= 3m_u + 3C^b \frac{1}{2m_u} \\
    m_{\Sigma_c} &= 2m_u + m_c + 2C^b \frac{1}{m_c} \left( \frac{1}{4} - \frac{m_u}{m_c} \right) \\
    m_{\Xi_c} &= m_u + 2m_c + 2C^b \frac{1}{m_c} \left( \frac{1}{4} - \frac{m_u}{m_c} \right) \\
    m_{\Lambda_c} &= 2m_u + m_c - 3C^b \frac{1}{2m_u} \\
    m_{\Sigma_{cc}} &= 2m_u + m_c + C^b \frac{1}{m_u} \left( \frac{1}{2} + \frac{m_u}{m_c} \right) \\
    m_{\Omega_c} &= 2m_u + m_c + C^b \frac{1}{m_u} \left( \frac{1}{2} + \frac{m_u}{m_c} \right). \quad (4)
\end{align*}
$$
TABLE I: Constituent quark masses obtained from the meson fits. For each combination of mesons, the results are obtained from their masses with FB HFI included (upper rows) and with GR HFI (lower rows). Note that constant $C^m$ differs for the two HFIs.

| Fit No. | Mesons | $m_u = m_d$ | $m_s$ | $m_c$ | $C^m$ ($\times 10^5$MeV$^3$) | $\chi^2$ ($\times 10^6$) |
|---------|--------|-------------|--------|--------|----------------------------|--------------------------|
| 1       | $\pi, K, \rho, K^*, \omega, \varphi, D, D^*$ | 318.12 ± 0.11 | 477.29 ± 0.21 | 1591.42 ± 0.21 | 1.5977 ± 0.0017 | 0.095336 |
|         | $D^*, D_S, D_S^*, \eta_c, J/\psi$ | 304.55 ± 0.11 | 534.33 ± 0.19 | 1576.94 ± 0.21 | 2.1331 ± 0.2346 | 0.304832 |
| 2       | $\pi, K, \rho, K^*, \omega, \varphi$ | 308.05 ± 0.12 | 484.91 ± 0.24 | - | 1.5043 ± 0.0017 | 0.000411 |
|         | | 292.35 ± 0.12 | 552.80 ± 0.22 | - | 2.0284 ± 0.0023 | 0.143289 |
| 3       | $D, D^*, D_S, D_S^*, \eta_c, J/\psi$ | 454.75 ± 0.47 | 547.48 ± 0.49 | 1524.36 ± 0.35 | 2.6936 ± 0.0095 | 0.003475 |

TABLE II: The same as Table I, but for baryon fits. Note that for heavy baryons fit did not converge and constitutive quark masses could not be obtained.

| Fit No. | Baryons | $m_u = m_d$ | $m_s$ | $m_c$ | $C^b$ ($\times 10^7$MeV$^3$) | $\chi^2$ ($\times 10^6$) |
|---------|---------|-------------|--------|--------|----------------------------|--------------------------|
| 1       | $N, \Sigma, \Xi, \Lambda, \Delta, \Sigma^*, \Xi^*$, $\Omega, \Sigma_c, \Lambda_c, \Sigma_c^*, \Omega_c$ | 381.53 ± 0.09 | 537.26 ± 0.18 | 1368.15 ± 0.27 | 1.3454 ± 0.0021 | 0.153491 |
|         |         | 541.04 ± 0.18 | 655.00 ± 0.22 | 1358.62 ± 0.25 | 2.4875 ± 0.0047 | 0.125597 |
| 2       | $N, \Sigma, \Xi, \Lambda, \Delta, \Sigma^*, \Xi^*$, $\Omega$ | 362.52 ± 0.10 | 538.68 ± 0.20 | - | 1.2789 ± 0.0020 | 0.001165 |
|         |         | 500.42 ± 0.20 | 621.20 ± 0.25 | - | 1.0951 ± 0.0043 | 0.002179 |

TABLE III: Constituent quark masses obtained from hadron fits when hadrons with two $c$-quarks are excluded. The upper rows correspond to FB HFI, and the lower rows to GR HFI.

| Hadrons | $m_u = m_d$ | $m_s$ | $m_c$ | $C$ ($\times 10^5$MeV$^3$) | $\chi^2$ ($\times 10^6$) |
|---------|-------------|--------|--------|----------------------------|--------------------------|
| Mesons | $\pi, K, \rho, K^*, \omega, \varphi, D, D^*$ | 314.75 ± 0.12 | 466.80 ± 0.21 | 1627.31 ± 0.25 | 1.5546 ± 0.0017 | 0.033123 |
|         | $D^*, D_S, D_S^*, \Lambda_c, \Sigma_c, \Omega_c$ | 300.88 ± 0.11 | 526.65 ± 0.19 | 1605.62 ± 0.26 | 2.0660 ± 0.0023 | 0.263221 |
| Baryons | $N, \Sigma, \Xi, \Lambda, \Delta, \Sigma^*, \Xi^*$, $\Omega, \Sigma_c, \Lambda_c, \Sigma_c^*, \Omega_c$ | 365.69 ± 0.09 | 530.08 ± 0.18 | 1700.17 ± 0.37 | 1.2513 ± 0.0019 | 0.015451 |

TABLE IV: Predicted masses (in MeV) of constituent quarks in different phenomenological models.

| Mass | Ref. [5] | Ref. [8] | Ref. [1] | Ref. [22] | Ref. [6] | Ref. [7] | Ref. [15] |
|------|---------|---------|---------|---------|---------|---------|---------|
| $m_u = m_d$ | 220 | 300 | 310 | 290 | 360 | 337.5 | 311 |
| $m_s$ | 419 | 475 | 483 | 460 | 540 | 486 | 487 |
| $m_c$ | 1628 | 1640 | - | 1650 | 1710 | 1550 | 1592 |

For each set of equations, the minimized $\chi^2$ values for masses are calculated by the formula: $\chi^2 = \sum_{i=1}^{N} \frac{(T_i - E_i)^2}{\sigma_i^2}$, where $T_i$ is the model prediction for the hadron mass, $E_i$ the experimental hadron mass and $\sigma_i$ the uncertainty of the mass.

The values of constituent quark masses and their uncertainties (in MeV) are given in Tables I and II. It is noticeable that the fits are satisfactory for both HFIs and in cases of all mesons, light mesons and light baryons. They show that the constituent mass is only slightly modified by the dynamics of confinement. The constituent masses differ for the quarks in mesons and those in baryons. This is not unexpected since they are confined in quite different systems. Also, as noted in [8] there are no theoretical reasons why the masses determined from the baryon fits should coincide exactly with those determined from the meson fits because these quark masses are constitutive masses, i.e. effective ones. Roncaglia et al. [8, 9]
also showed that constraints (inequalities) for mass differences are stronger for baryons than for mesons, and therefore requirement for the same set of quark masses for both baryons and mesons would result in significantly poorer fits \[8, 9\].

In the case of heavy mesons, the fit resulted in somewhat increased values for \(u\) and \(s\) quarks, and in the case of heavy baryons fit even did not converge. Therefore, our results suggest that both of the analyzed HFIs can be used in a first approximation for modeling the term with hyperfine splitting in the quark model only for light mesons and light baryons. That is both HFIs well describe systems which contain quarks from SU(3)\(\text{F}\), i.e. light quarks: \(u, d, s\). In order to verify this conclusion, we also show the results for the case when hadrons with two \(c\)-quarks are excluded (see Table \[\text{III}\]): meson fit without \(\eta\) and \(J/\psi\), and baryon fit without \(\Xi_{cc}\). From these two fits we can calculate mass differences \(m_c - m_u\) and \(m_c - m_s\) which may be then compared with the corresponding inequalities given by Roncaglia et al. \[8, 9\], obtained from the Feynman-Hellmann theorem. We can see from Table \[\text{III}\] that in this case the constituent quark masses obtained for FB HFI satisfy all of the inequalities given in \[8, 9\], contrary to Tables \[\text{I}\] and \[\text{II}\] where FB HFI breaks some of these inequalities. Also, from Table \[\text{III}\] one can see that GR HFI violates one or more of the inequalities, but to a lesser extent than in Tables \[\text{I}\] and \[\text{II}\].

Even more, in this case obtained quark masses are more realistic than those from Tables \[\text{I}\] and \[\text{II}\]. From each set of equations (except for heavy baryons) we calculated masses of constituent quarks and constants \(C^m\) and \(C^b\), and we got slightly different values, but all of them are from expected mass ranges of constituent quarks. As one can see from Table \[\text{IV}\] our predictions for constituent quark masses are in the range of masses obtained using different phenomenological models. When comparing these two HFIs, it is interesting to note that the values of obtained quark masses are similar although one interaction is color-spin, and the other one is flavor-spin. As it can be seen from Tables \[\text{I}\] and \[\text{II}\] two different HFIs result with masses of constituent quarks which are more similar than those obtained by the same HFIs applied to different groups of hadrons (e.g. light mesons, heavy mesons, light baryons, ...). For example, results for \(m_u (= m_d)\) and \(m_c\) for FB HFI applied on light and heavy mesons are more different than results for \(m_u (= m_d)\) and \(m_s\) for FB and GR HFIs applied on light mesons.

When comparing FB, the color-spin interaction, with GR HFI, which is flavor-spin interaction, it is interesting that we obtained similar results. We can conclude that both HFIs describe masses of light mesons and baryons very well. It shows that both HFIs well describe systems which contains quarks from SU(3)\(\text{F}\). In case of heavy baryons the fits for both HFIs did not converge, which might indicate that quark model which includes these two HFIs is not satisfactory approximation for heavy baryons. It might also indicate that FB and GR HFI are not the complete effective two-quark interactions and therefore they cannot be successfully applied to quark systems out of SU(3) group. If we do not take into account hadrons with two \(c\)-quarks the FB HFI becomes a good approximation, even for hadrons having one heavy \(c\)-quark. We also show that the constituent quark masses are very sensitive to the system in which they are confined, and their values differ less or more in different systems (for example heavy mesons and light mesons). We obtained that least-square fit is more accurate for FB than for GR interaction in case of light mesons, but in case of heavy mesons and light baryons accuracy of the fit is similar.

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