The role of stellar collisions for the formation of massive stars

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ABSTRACT

We use direct N-body simulations of gas embedded star clusters to study the importance of stellar collisions for the formation and mass accretion history of high-mass stars. Our clusters start in virial equilibrium as a mix of gas and protostars. Protostars then accrete matter using different mass accretion rates and the amount of gas is reduced in the same way as the mass of stars increases. During the simulations we check for stellar collisions and we investigate the role of these collisions for the build-up of high-mass stars and the formation of runaway stars.

We find that a significant number of collisions only occur in clusters with initial half-mass radii \( r_h \leq 0.1 \text{ pc} \). After emerging from their parental gas clouds, such clusters end up too compact compared to observed young, massive open clusters. In addition, collisions lead mainly to the formation of a single runaway star instead of the formation of many high-mass stars with a broad mass spectrum. We therefore conclude that massive stars form mainly by gas accretion, with stellar collisions only playing a minor role if any at all. Collisions of stars in the pre-main-sequence phase might, however, contribute to the formation of the most massive stars in the densest star clusters and possibly to the formation of intermediate-mass black holes with masses up to a few \( 100 \, M_\odot \).

Key words: stars: formation – galaxies: star clusters: general.

1 INTRODUCTION

An important unanswered question in modern astrophysics concerns the mechanism(s) by which stars and star clusters form. This is especially true in the case of massive stars, which are challenging objects to observe since they are rare, evolve on short time-scales, and are often highly extincted. Various theories have been proposed concerning the formation process of massive stars, like monolithic collapse of molecular cloud cores (McKee & Tan 2003), competitive gas accretion in small clusters (Larson 1978; Bonnell et al. 1997a) driven by turbulent fragmentation (Klessen & Burkert 2000; Klessen, Heitsch & Mac Low 2000; Klessen 2001) and stellar collisions and mergers in dense clusters (Bonnell, Bate & Zinnecker 1998; Clarke & Bonnell 2008). For further discussions, see Mac Low & Klessen (2004), McKee & Ostriker (2007), and Zinnecker & Yorke (2007).

Calculations of the collapse of protostellar clouds show that the collapse proceeds non-homologously and in such a way that the central regions rapidly form a protostellar core while the outer regions still continue to fall inward (Bodenheimer & Sweigart 1968; Larson 1969; Shu 1977). The formation of low- and intermediate-mass stars \( (M \leq 10 \, M_\odot) \) can readily be explained this way (Palla & Stahler 1993); however, this models fails for high-mass stars since for stars more massive than about \( 20 \, M_\odot \) the Kelvin–Helmholtz time-scale is larger than their formation time, meaning that these stars start core hydrogen burning while they are still accreting. Once the resulting radiation pressure from the stellar core exceeds the gravitational attraction, further accretion of gas is halted, limiting stellar masses to \( 20 \, M_\odot \) at least for spherically symmetric accretion (Kahn 1974; Wolfire & Cassinelli 1987). The effects of non-ionizing as well ionizing radiation, however, might overcome if accretion happens through an accretion disc which shields the gas from radiation (Yorke & Sonnhalter 2002; Krumholz et al. 2009; Peters et al. 2010a,b,c).

Based on the fact that massive stars are mainly found in the central regions of rich star clusters (Hillenbrand 1997; Hillenbrand & Hartmann 1998), Bonnell et al. (1998) presented a model for high-mass star formation in which high-mass stars form through gas accretion and repeated mergers in the centre of a star cluster. In their calculations, the clusters contracted while stars were accreting until the central density became high enough for stellar collisions to occur. Once a massive star has formed, the remaining cluster gas is removed and the cluster expands again. Clarke & Bonnell (2008) found furthermore that relaxation effects between protostars prevent low-mass clusters from reaching high-enough densities needed for...
collisions, in good agreement with the fact that high-mass stars are only found in massive clusters (Weidner & Kroupa 2006).

Direct simulations of the role of stellar collisions for star formation are still lacking. In the present paper we present N-body simulations of the early evolution of protoclusters. Our clusters start as a mix of gas and protostars and we follow the cluster evolution through the accretion and gas expulsion phases. Our paper is organized as follows. In Section 2 we describe the details of our simulations. In Section 3 we present our results and in Section 4 we draw our conclusions.

2 MODELS

All simulations were made with the graphics-card enabled version of the collisional N-body code NBODY6 (Aarseth 1999) on NVIDIA graphic processing units (GPU). NBODY6 uses a Hermite scheme to integrate the motion of stars, and uses an Ahmad-Cohen type neighbour scheme (Ahmad & Cohen 1973) for the calculation of forces from the nearest neighbours. It also uses KS (Kustanheimo & Stiefel 1965) and Chain regularization (Mikkola & Aarseth 1990) to deal with close encounters between stars.

In our simulations, the star clusters were assumed to be initially a mix of gas and accreting protostars. The gas was not simulated directly, instead its influence on the stars was modelled as a modification to the equation of motion of stars. We assumed that the star formation efficiency does not depend on the position inside the cluster, so gas and stars followed the same density distribution initially, which was given by a Plummer model. The influence of the external gas was calculated at each regular time step of NBODY6. Since NBODY6 uses a Hermite scheme to integrate the motion of stars, the correction terms due to the external gas on the acceleration and its first derivative were evaluated at each regular step. More details on the way the external gas was implemented can be found in Baumgardt & Kroupa (2007).

The overall star formation efficiency was set to be 30 per cent, i.e. 70 per cent of the initial gas was not converted into stars, which agrees with typical star formation efficiencies found for embedded clusters in the Milky Way (Lada & Lada 2003). We assumed that the removal of gas not converted into stars sets in after \( t_g = 0.2 \) Myr or, if the formation time-scale is longer, after the most massive stars have formed. After this time, the fraction of gas was assumed to decrease exponentially on a time-scale of \( \tau_M = 0.5 \) Myr, i.e.

\[
M_{\text{gas}}(t) = M_{\text{gas,init}} e^{-\left(t-t_g\right)/\tau_M}.
\]  

(1)

The time-scale for \( \tau_M \) was chosen such that our clusters are gas free after a few Myr, in agreement with observations which show that most embedded star clusters have ages of less than a few Myr (Lada & Lada 2003). All simulations were run for 10 Myr to allow a comparison of our simulations with observations of gas embedded star clusters and young open clusters.

All protostars started with a mass of \( 0.1 M_\odot \) initially. We assumed that all stars accrete with the same accretion rate. Most simulations were done with an accretion rate of \( M = 10^{-4} M_\odot \text{yr}^{-1} \), but we also made simulations with accretion rates of \( M = 10^{-3} \) and \( 10^{-3} M_\odot \text{yr}^{-1} \) to test the influence of the assumed accretion rate on our results. It is possible that the accretion rate on to massive stars is higher than on to low-mass stars, as e.g. indicated by infall motions and other accretion tracers (e.g. Fuller, Williams & Sridharan 2005; Keto & Wood 2006). Our simulations should therefore be regarded as first steps to address the question of stellar collisions during star formation. Due to the constant accretion rate for all stars, low-mass stars reach their final mass first, in agreement with observations of young open clusters in the Milky Way, which show that high-mass stars are still accreting after low-mass stars have completed their accretion (e.g. Herbig 1992; DeGioia-Eastwood et al. 2001; Kumar, Keto & Clerkin 2006). The mass of the gas was reduced in the same way as the stars gained mass such that the total cluster mass was constant before the onset of gas removal.

Accretion on to the protostars was stopped once the mass of each star reached a pre-specified final mass. The mass function of these final masses was given by a Kroupa IMF (Kroupa 2001) between lower and upper mass limits of \( m_{\text{low}} = 0.1 M_\odot \) and \( m_{\text{up}} = 15 M_\odot \). Due to the low upper mass limit in our runs, any star with mass \( m > 15 M_\odot \) cannot form directly but has to form via collisions between lower mass stars.

The radius evolution of the stars in our simulations was split into three different phases, the accretion phase, the cooling phase and the stellar evolution phase after stars have reached the main sequence. During the accretion phase protostars grow in mass until they have reached their final mass. Following Hosokawa & Omukai (2009), we assume that in the accretion phase protostellar radii depend on the actual mass of a star and its accretion rate, but not on the final mass. In our simulations, the radius of a star in the accretion phase is given by

\[
\frac{R_\ast}{R_\odot} = 421 \left( \frac{M}{M_\odot} \right)^{0.375} \left( \frac{\dot{M}}{M_\odot \text{yr}^{-1}} \right)^{0.33}.
\]  

(2)

In the contraction phase stars contract towards the zero-age main sequence (ZAMS) after having reached their final mass. During the contraction phase, we assume that the radius of a star depends on its mass and the time which has passed since the star has reached its final mass. To roughly match the results of the detailed calculations by Bernasconi & Maeder (1996), we assumed that for stars more massive than \( 7 M_\odot \), the stellar radius decreases exponentially towards the ZAMS such that

\[
R_\ast = R_{\text{ZAMS}} + \left( R_{\text{max}} - R_{\text{ZAMS}} \right) \exp\left(-\frac{(t - t_{\text{max}})}{t_{\text{cool}}}\right).
\]  

(3)

Here \( R_{\text{ZAMS}} \) are the ZAMS radii of stars which we have taken from Tout et al. (1996), assuming solar metallicity. \( R_{\text{max}} \) is the maximum radius reached during the accretion phase according to equation (2), \( t_{\text{max}} \) is the time when this radius is reached, and \( t_{\text{cool}} \) is the cooling time-scale which is given by

\[
\log \frac{t_{\text{cool}}}{\text{yr}} = 11.57 - 9.85 \times \log \frac{M}{M_\odot} + 2.86 \left( \log \frac{M}{M_\odot} \right)^2.
\]  

(4)

After \( 5 t_{\text{cool}} \), we assumed that stars start their main-sequence evolution, which was modelled according to the fitting formulae of Hurley, Pols & Tout (2000). For stars less massive than \( 7 M_\odot \), the Bernasconi & Maeder (1996) tracks differ significantly from an exponential decrease, so we assumed a linear decrease of the stellar radius with time according to

\[
\log \frac{R_\ast}{R_\odot} = \log R_{\text{ZAMS}} + \left( \log R_{\text{max}} - \log R_{\text{ZAMS}} \right) \times \frac{\log(t - (t_{\text{max}}))/\log(t_2 - \log t_1)}{\log(t_2 - \log t_1)},
\]  

for times \( t_1 < t < t_2 \), where the constants \( t_1 \) and \( t_2 \) were given by

\[
t_1 = 4.677 \times 10^{-3} \left( \frac{M}{M_\odot} \right)^{-2/3} \text{Myr}
\]  

(5)

\[
t_2 = 23.4 \left( \frac{M}{M_\odot} \right)^{-4/3} \text{Myr}.
\]  

(6)

For times \( t < t_1 \), we assumed \( R_\ast = R_{\text{max}} \), while for times \( t > t_2 \) stars were assumed to undergo standard stellar evolution. Fig. 1 shows...
the time evolution of stellar radii for two different masses according to Bernasconi & Maeder (1996) and our simple fitting formulae. We adjusted the accretion rate such that our models start at the same initial radius as the models of Bernasconi & Maeder (1996). Our fitting formulae are within 20 per cent of the Bernasconi & Maeder models for most of the time.

Stars were merged if their separation became smaller than the sum of their radii and the mass of the merger product was set equal to the sum of the masses of the stars, i.e. we assumed no disruption of stars and no mass-loss during a collision. Merged stars were assumed to be immediately back on their pre-main sequence or main-sequence track, so we did not allow for an increased radius of a star due to energy increase as a result of a collision. This is justified since thermal adjustment of massive collision products happens on time-scales of $10^3$–$10^4$ yr (Suzuki et al. 2007), which is much smaller than the typical time for collisions except in the most compact clusters. We also did not allow for tidal interactions during close flybys or collisions which do not lead to mergers between stars. Incorporation of these effects is difficult since they depend on the internal structure of stars and would need to be followed by detailed simulations of the pre-MS evolution and the collisions.

3 RESULTS

Observations of open clusters in the Milky Way show that the formation of O stars, i.e. stars with masses larger than $\sim 25 M_\odot$ is happening in clusters down to about 1000 stars (Weidner, Kroupa & Bonnell 2010), so in our simulations we followed the evolution of star clusters containing between $10^3$ and $10^4$ stars. Simulations with less than $10^3$ stars were repeated 10 times and the results were averaged to reduce the statistical noise of low-$N$ simulations. The initial half-mass radii of the clusters were varied between 0.033 and 1.0 pc. Embedded star clusters usually have radii between 0.3 and 1 pc (Lada & Lada 2003), so our simulations more than cover the range of half-mass radii found for embedded star clusters. For each mass and radius, we simulated two kinds of clusters, mass-segregated and mass-unsegregated models. In the unsegregated models, the stellar number density follows a Plummer profile and there is no correlation between the final mass of a star and its position in the cluster. The gain in mass by the stars is balanced by the mass-loss from the gas such that the enclosed mass within any radius does not change. Hence, the Lagrangian radii of unsegregated clusters remain almost constant in time throughout the accretion phase, except for possible relaxation effects.

In the segregated models, the stellar number density profile still follows a Plummer profile initially; however, we correlate the final mass of each star with its total energy in the cluster such that the most massive stars have the lowest energies and will form mainly in the core of the cluster. In the mass-segregated models, the mass profile therefore changes during the accretion phase. Due to the high number of massive stars in the core, the core mass increases and its size decreases as it tries to remain in virial equilibrium. The mass-segregated simulations mimic the behaviour of an embedded star cluster where the core mass increases due to gas which cools and falls into the centre. They are also similar to the star clusters discussed by Bonnell, Bate & Zinnecker (1997b), which also form a very compact core of massive stars. Table 1 summarizes our suite of simulations. It shows the number of runs done for any particular model, the number of stars, the assumed mass accretion rate, the type of run (Std: Unsegregated cluster, Segr: Mass-segregated cluster, Bin: Simulation with binaries), the initial half-mass radius, the number of collisions, and the maximum stellar mass and cluster half-mass radius at $T = 10$ Myr.

3.1 Unsegregated clusters

Fig. 2 depicts the evolution of Lagrangian radii in the unsegregated clusters. The evolution of Lagrangian radii is determined by three competing processes, two-body relaxation which decreases the core radii before core collapse and increases all radii after core collapse, as well as gas expulsion and mass-loss by stellar evolution which increase the cluster radii. The relative importance of the different processes depends on the initial half-mass radius and number of cluster stars. In the most compact clusters with $r_h = 0.033$ pc, the relaxation time is short enough that they undergo core collapse before gas expulsion sets in. They later expand due to gas expulsion and mass-loss from massive stars in the core and can expand by up to a factor of 10 within 10 Myr. Interestingly, the final radius reached after 10 Myr of evolution is the same for clusters starting with $r_h = 0.033$ and 0.10 pc. Hence the initial radius cannot be determined from the one after 10 Myr for very compact clusters. In more extended clusters, the initial relaxation time is too long to allow significant mass segregation before gas expulsion sets in, so these clusters become mass segregated and go into core collapse only on a much longer time-scale of several Myr. This is most clearly visible for the $N = 10^4$ stars, $r_h = 1$ pc cluster which has the longest relaxation time of all clusters studied and for which the core is contracting after $T = 5$ Myr. More compact clusters and clusters with lower particle numbers have developed some degree of mass segregation so that the expansion due to mass-loss outweighs contraction due to two-body relaxation and the clusters always expand.

Fig. 3 depicts the evolution of projected half-light radius of bright stars in the unsegregated clusters. We have calculated the Lagrangian radii from the positions of all stars with masses larger than $3 M_\odot$ that have not yet turned into compact remnants (mainly black holes given the short calculation times of our runs). The mass limit of $3 M_\odot$ was chosen to allow a comparison with observations since in distant star clusters stars fainter than this often cannot be observed and contribute only a small fraction of the total cluster light. Filled triangles show the age and half-light radii of open clusters from Portegies Zwart, McMillan & Gieles (2010), which are clusters in the mass range $10^4 M_\odot < M_c < 10^5 M_\odot$. Filled circles show the location of additional open clusters with masses in the
Table 1. Details of the performed N-body runs.

| $N_{\text{Run}}$ | $N_{\text{Star}}$ | $M_{\text{Acc}}$ (M$_\odot$ yr$^{-1}$) | Type | $r_{\text{ini}}$ (pc) | $\langle N_{\text{Coll}} \rangle$ | $M_{\text{Max}}$ (M$_\odot$) | $r_{\text{hpfin}}$ (pc) | $N_{\text{Run}}$ | $N_{\text{Star}}$ | $M_{\text{Acc}}$ (M$_\odot$ yr$^{-1}$) | Type | $r_{\text{ini}}$ (pc) | $\langle N_{\text{Coll}} \rangle$ | $M_{\text{Max}}$ (M$_\odot$) | $r_{\text{hpfin}}$ (pc) |
|-----------------|------------------|-----------------------------|------|----------------------|-------------------|------------------|-----------------|-----------------|-----------------|-----------------------------|------|----------------------|-------------------|------------------|-----------------|-----------------|
| 10              | 1000             | $10^{-4}$                   | Std  | 0.033               | 2.1               | 17.4             | 0.25            | 10              | 1000             | $10^{-4}$                   | Segr  | 0.033               | 9.7               | 19.0             | 2.36            |
| 10              | 3000             | $10^{-4}$                   | Std  | 0.033               | 5.4               | 25.4             | 0.20            | 10              | 3000             | $10^{-4}$                   | Segr  | 0.033               | 43.5              | 49.6             | 0.92            |
| 1               | 10000            | $10^{-4}$                   | Std  | 0.10                | 41.0              | 139.9            | 0.22            | 1               | 10000            | $10^{-4}$                   | Segr  | 0.10                | 2.9               | 27.6             | 0.31            |
| 10              | 1000             | $10^{-4}$                   | Std  | 0.10                | 0.3               | 12.3             | 0.23            | 10              | 1000             | $10^{-4}$                   | Segr  | 0.10                | 10.9              | 31.0             | 0.24            |
| 1               | 10000            | $10^{-4}$                   | Std  | 0.10                | 1.4               | 20.5             | 0.25            | 1               | 10000            | $10^{-4}$                   | Segr  | 0.10                | 54.0              | 272.1            | 0.23            |
| 10              | 1000             | $10^{-4}$                   | Std  | 0.33                | 0.1               | 13.1             | 0.47            | 10              | 1000             | $10^{-4}$                   | Segr  | 0.33                | 0.3               | 14.6             | 0.28            |
| 10              | 3000             | $10^{-4}$                   | Std  | 0.33                | 0.1               | 13.5             | 0.51            | 10              | 3000             | $10^{-4}$                   | Segr  | 0.33                | 2.7               | 27.4             | 0.28            |
| 1               | 10000            | $10^{-4}$                   | Std  | 0.33                | 0.0               | 14.1             | 0.62            | 1               | 10000            | $10^{-4}$                   | Segr  | 0.33                | 11.0              | 27.8             | 0.20            |
| 10              | 1000             | $10^{-4}$                   | Std  | 1.00                | 0.0               | 12.9             | 4.26            | 10              | 1000             | $10^{-4}$                   | Segr  | 1.00                | 0.0               | 13.1             | 0.32            |
| 10              | 3000             | $10^{-4}$                   | Std  | 1.00                | 0.0               | 13.3             | 4.17            | 10              | 3000             | $10^{-4}$                   | Segr  | 1.00                | 0.6               | 17.8             | 0.30            |
| 1               | 10000            | $10^{-4}$                   | Std  | 1.00                | 0.0               | 14.6             | 2.11            | 1               | 10000            | $10^{-4}$                   | Segr  | 1.00                | 1.0               | 25.6             | 0.22            |
| 10              | 1000             | $10^{-4}$                   | Bin  | 0.033               | 11.7              | 12.0             | 0.50            | 10              | 1000             | $10^{-3}$                   | Std   | 0.033               | 1.8               | 17.1             | 0.51            |
| 10              | 3000             | $10^{-4}$                   | Bin  | 0.033               | 42.0              | 25.4             | 0.31            | 10              | 3000             | $10^{-3}$                   | Std   | 0.033               | 6.6               | 32.4             | 0.24            |
| 10              | 1000             | $10^{-4}$                   | Bin  | 0.10                | 1.5               | 12.8             | 0.41            | 10              | 1000             | $10^{-3}$                   | Std   | 0.033               | 31.0              | 113.3            | 0.19            |
| 10              | 3000             | $10^{-4}$                   | Bin  | 0.10                | 14.4              | 18.7             | 0.32            | 10              | 3000             | $10^{-5}$                   | Std   | 0.033               | 3.3               | 20.7             | 0.55            |
| 10              | 1000             | $10^{-4}$                   | Bin  | 0.33                | 0.1               | 11.8             | 0.72            | 10              | 3000             | $10^{-5}$                   | Std   | 0.033               | 11.6              | 22.4             | 0.32            |
| 10              | 3000             | $10^{-4}$                   | Bin  | 0.33                | 0.7               | 13.2             | 0.25            | 10              | 3000             | $10^{-5}$                   | Std   | 0.033               | 68.0              | 285.6            | 0.29            |

Figure 2. Evolution of Lagrangian radii for unsegregated clusters as a function of time. Shown are half-mass radii (upper curves) and 10 per cent Lagrangian radii (lower curves). Clusters expand due to gas expulsion and mass-loss from massive stars. The most concentrated models reach core-collapse before gas expulsion sets in and expand up to a factor of 10 within the first 10 Myr due to strong mass-loss from the core and binary heating (upper left panel). Less concentrated clusters expand by a smaller amount within the first 10 Myr.

The range $10^3 M_\odot < M_C < 10^4 M_\odot$ and which contain massive stars according to Weidner et al. (2010). The parameters of the clusters shown can be found in Table 2.

It can be seen that Lagrangian radii of bright stars also mostly expand, except for clusters starting with half-mass radii $r_h = 0.33$ pc for which mass segregation causes a slight decrease at later times. The most compact clusters all reach radii of about $r_{hp} = 0.2$ pc at $T = 10$ Myr. This radius therefore represents a minimum radius for star clusters and there are indeed no observed clusters smaller than it. Simulations starting with half-mass radii $r_h \leq 0.1$ pc lead to radii significantly smaller than those of observed clusters. For
the unsegregated clusters discussed here the best match with observations is achieved for models starting with half-mass radii in the range $0.33 \, \text{pc} < r_h < 1 \, \text{pc}$.

We next discuss the number of stellar collisions in the unsegregated clusters. According to Portegies Zwart & McMillan (2002), the inspiral time of a star of mass $m > (m)$ is given by

$$T_{\text{INS}} = 3.3 \frac{(m)}{m} T_{\text{th}}$$

$$= 0.4 \frac{\sqrt{M_{\odot} r_h^{3/2}}}{\sqrt{G m} \ln 0.02 N}$$

where $T_{\text{th}}$ is the half-mass relaxation time of the cluster which we have taken from Spitzer (1987) and where $(m)$ is the mean stellar mass. Significant number of collisions are only to be expected after massive stars have spiraled into the cluster core, because low-mass stars have small radii and therefore smaller cross-sections for collisions. Fig. 4 depicts the number of stellar collisions in the unsegregated cluster models. In clusters less concentrated than $r_h \approx 0.1 \, \text{pc}$ hardly any collisions occur. The reason is the low stellar density of these models in combination with the large inspiral times of the massive stars. In clusters with half-mass radii larger than $r_h = 0.1 \, \text{pc}$, the inspiral times are larger than a few $0.1 \, \text{Myr}$, meaning that massive stars do not have time to accumulate in the centre while still in the accretion phase. Instead, they reach the centre only after massive stars have reached their final mass, in the pre-MS phase while roughly half of the collisions happen after $1 \, \text{Myr}$, i.e. by the time the cluster has already become gas free and collisions would possibly be observable through their collision products or flashes of bright emission.

### 3.2 Mass-segregated clusters

We next discuss the evolution of the mass-segregated clusters. In the segregated clusters energy and final mass of stars were correlated such that protostellar cores which will become the highest mass stars have the lowest energies. The high-mass stars therefore form in the cluster centre and do not have to spiral into the centre by dynamical friction. Furthermore, the cluster core contracts during the accretion process due to mass increase, further facilitating stellar collisions. Table 1 and Fig. 5 show the number of collisions in the segregated models. It can be seen that the number of collisions which occur in the runs are now about a factor of $10$ higher than in the unsegregated models. In particular in the most concentrated models starting with $r_h = 0.033 \, \text{pc}$, the number of collisions is now sufficiently high to build up a main sequence of high-mass stars. A closer look at the data also shows that in the mass-segregated models the collisions happen at earlier times on average, for the star cluster with $N = 10^{4}$ and $r_h = 0.033 \, \text{pc}$ cluster, for example, roughly half of all collisions happen in the pre-MS stage. We note that these results are not in contradiction to results obtained by Ardi, Baumgardt & Mineshige (2008), who found that primordial mass segregation does not lead to a significant increase in the number of stellar collisions, since in our simulations the increase in the number of collisions comes to a large part from an increase in the central density as a result of the mass accretion of the heavy stars. In contrast, Ardi et al. started their simulations with all stars being on the main sequence and the same mass density for segregated and unsegregated clusters, meaning that

| Cluster name | Age (Myr) | log Mass $(M_{\odot})$ | Radius (pc) | Ref |
|--------------|-----------|------------------------|-------------|-----|
| Arches       | 2.0       | 4.3                    | 0.4         | 1   |
| NGC 3603     | 2.0       | 4.1                    | 0.7         | 1   |
| Quintuplet   | 4.0       | 4.0                    | 2.0         | 1   |
| Westerlund 1 | 3.5       | 4.5                    | 1.0         | 1   |
| Trumpler 14  | 0.4       | 3.6                    | 0.5         | 1,2 |
| ONC          | 1.0       | 3.3                    | 1.0         | 3,4 |
| NGC 637      | 4.0       | 3.2                    | 0.3         | 3,5 |
| NGC 2244     | 1.9       | 3.8                    | 4.4         | 3,6 |
| NGC 6231     | 1.0       | 3.7                    | 1.8         | 3,7 |
| NGC 6530     | 2.3       | 3.0                    | 3.2         | 3,6 |
| Westerlund 2 | 1.5       | 3.9                    | 1.0         | 3,8 |
| DBS 2003     | 3.5       | 3.8                    | 0.2         | 3,9 |

Note: 1: Portegies Zwart et al. (2010), 2: Sana et al. (2010), 3: Weidner et al. (2010), 4: Hillenbrand & Hartmann (1998), 5: Yadav et al. (2008), 6: Chen, de Grijs & Zhao (1997), 7: Raboud (1999), 8: Ascaso et al. (2007) and 9: Borissova et al. (2008).
the stellar number density in the mass-segregated clusters was much lower than in the unsegregated models.

Fig. 6 depicts the evolution of the projected radius containing 50 per cent of bright stars as a function of time. Due to the initial mass segregation, all models have significantly smaller half-light light radii than the non-segregated models of the previous section. The final half-light radii are also significantly smaller than in the previous section, despite some expansion due to gas expulsion, stellar evolution, and dynamical heating by binary stars. The projected half-light radii of bright stars are for all models also well below the measured radii of young open clusters. It is therefore unlikely that any of the mass-segregated models represents the starting condition of any observed Milky Way cluster. Starting with even larger cluster radii would be a way to get a better fit to observed cluster radii; however, this would again reduce the number of collisions to too small levels. Strong cluster expansion on a time-scale of a few $10^6$ yr after the most massive stars have formed would be needed to bring the cluster radii into agreement with observed cluster radii. This may be achieved through a very small star formation efficiency of the order of 10 per cent or less, which however is much smaller than observed star formation efficiencies (Lada & Lada 2003). It therefore seems that primordial mass segregation and a shrinking cluster core are not a way to get a sufficiently high number of collisions for realistic clusters.

An additional problem for the hypothesis that high-mass stars form by stellar collisions is illustrated in Fig. 7. It shows the final mass function of stars in the four mass-segregated models which have the largest number of collisions. All models show a sharp drop in the number of stars at $m = 15 \, M_\odot$, meaning that collisions were not able to build a sufficient number of stars with $m > 15 \, M_\odot$. Instead, the main effect of collisions was the formation of a single massive star in each cluster, inspection of the runs shows that typically more than 90 per cent of all collisions involve the same star. This behaviour is similar to what Portegies Zwart & McMillan (2002) found for collisions between main-sequence stars in dense star clusters. The most massive star has a large radius and also attracts other stars via gravitational focusing. As a result it has by far the highest collision rate of all stars in the core and once it has grown beyond a certain critical mass, all further collisions happen nearly exclusively with this star. Instead of building many massive stars in the mass range $15 \, M_\odot < m < 100 \, M_\odot$, there is runaway growth of a single star only. This star can reach masses with more than $100 \, M_\odot$ in some of our runs. Although these runs are probably too concentrated to lead to realistic star clusters, runaway collisions might be possible in more massive star clusters with larger radii. Collisions will also be more common if stars more massive than the mass limit of 15 $M_\odot$ chosen here can form directly through gas accretion since higher mass stars have larger cross-sections. Runaway collisions between massive stars during the pre-MS phase could therefore lead to the formation of very massive stars in the densest star clusters, like the massive WN5-6 stars with masses up to 320 $M_\odot$ recently found in several massive star clusters by Crowther et al. (2010). They might also contribute to the formation of intermediate-mass black holes in dense star clusters (Portegies Zwart et al. 2004). These results are similar to results recently obtained by Moeckel & Clarke (2011), who also found that collisions between massive protostars lead to the formation of one or two extremely massive objects instead of a smooth filling of the top end of the mass spectrum.

3.3 Influence of the mass accretion rate and binary stars

Fig. 8 depicts the influence of the mass accretion rate on the number of collisions occurring in a cluster within the first 10 Myr. All clusters start from a half-light radius of $r_h = 0.033 \, pc$ and are...
unsegregated. According to equation (2), stars accreting with a larger rate are more extended, which will lead to a higher number of collisions. However, larger accretion rates also lead to shorter pre-main-sequence times which decreases the number of collisions. Fig. 8 shows that both effects nearly cancel each other. The accretion rate changes only by a factor of two despite the fact that the accretion rate varies between $10^{-5}$ and $10^{-3} M_{\odot} \text{yr}^{-1}$. This is too small to significantly change the results of the previous paragraphs.

Another possible effect which was not considered so far is the influence of binary stars. It is well known that most massive stars reside in binary or multiple systems (García & Mermilliod 2001; Lada 2006; Sana et al. 2007). Stellar binaries have a larger cross-section for collisions than single stars, which could enhance the number of collisions. We therefore performed a number of simulations containing primordial binaries. We assumed an initial binary fraction of 50 per cent in these runs and that all high-mass stars reside in binaries. Binary masses were chosen such that stars were ordered according to their mass and the components of each binary were drawn from consecutive stars in this list, i.e. the first binary contained the two most massive stars. Hence, the stellar binary fraction is 100 per cent for the high-mass stars, while all low-mass stars are single. Although being a strong simplification, our adopted binary distribution reflects the drop in binary fraction towards late spectral types seen for stars in the Galactic disc (Lada 2006).

The semimajor axis of the binaries were chosen randomly in log $r$ between a minimum radius three times as large as the sum of the radii reached at the end of the accretion phase and a maximum radius which was set equal to 100 au. For simplicity, we assumed in our runs that the semimajor axis of each binary remains constant during the accretion phase and increased the stellar velocities of the components to avoid a shrinkage of the binary.

Fig. 7 shows the resulting number of collisions in runs with binary stars. The number of collisions is large in very compact clusters with $r_h = 0.33$ pc. The projected half-light radii of these clusters after 10 Myr are similar to those of clusters without binaries and still smaller than about 0.3 pc (see Table 1). Such clusters therefore still end up too compact compared to observed open clusters. In more extended clusters the number of collisions is still not high enough to allow the build-up of a complete main sequence of massive stars. It is therefore likely that even in the presence of binaries, stellar collisions do not play a significant role for the formation of massive stars. A definite answer to this question can, however, only be made if a wider range of binary distributions is explored.

4 CONCLUSIONS

We have performed N-body simulations of the pre-main-sequence evolution of stars in stellar clusters, taking account gas accretion, primordial gas expulsion and collisions between stars. Our simulations show that it is very unlikely that all high-mass stars with masses $m > 20 M_{\odot}$ form from the collisions of lower mass stars. The reason for this is twofold. First, the necessary number of collisions between massive stars only occurs for central densities around $10^3 M_{\odot} \text{pc}^{-3}$, implying initial half-mass radii $r_h < 0.1$ pc for clusters of a few thousand stars. Such clusters remain highly concentrated within the first 10 Myr of their evolution, despite expansion due to gas expulsion and stellar evolution mass-loss, and therefore lead to clusters which are significantly more concentrated than known open clusters with O and B type stars which generally have radii around 1 pc. Our simulations show that the observed radii of young open clusters imply initial radii in the range 0.2–1 pc for most of them.

Secondly, even if a sufficient number of collisions occurs, this will normally lead to the formation of single runaway stars with
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Figure 9. Number of stellar collisions in runs containing 50 per cent binaries. Binaries increase the number of collisions by a factor of 5 to 10 compared to the corresponding single star runs. However, for clusters with initial half-mass radii \( r_h > 0.1 \) pc, this is still not large enough to create a sufficient number of high-mass stars.

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