Lie on the Fly: Strategic Voting in an Iterative Preference Elicitation Process

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Published online: 20 September 2019
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Abstract
A voting center is in charge of collecting and aggregating voter preferences. In an iterative process, the center sends comparison queries to voters, requesting them to submit their preference between two items. Voters might discuss the candidates among themselves, figuring out during the elicitation process which candidates stand a chance of winning and which do not. Consequently, strategic voters might attempt to manipulate by deviating from their true preferences and instead submit a different response in order to attempt to maximize their profit. We provide a practical algorithm for strategic voters which computes the best manipulative vote and maximizes the voter’s selfish outcome when such a vote exists. We also provide a careful voting center which is aware of the possible manipulations and avoids manipulative queries when possible. In an empirical study on four real world domains, we show that in practice manipulation occurs in a low percentage of settings and has a low impact on the final outcome. The careful voting center reduces manipulation even further, thus allowing for a non-distorted group decision process to take place. We thus provide a core technology study of a voting process that can be adopted in opinion or information aggregation systems and in crowdsourcing applications, e.g., peer grading in massive open online courses.

Keywords Iterative voting · Preference elicitation · Group decisions · Crowdsourcing

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1 Introduction

Voting procedures are used for combining voters’ individual preferences over a set of alternatives, enabling them to reach a joint decision. However, sometimes the full set of preferences is unavailable. Take, for example, a recruiting committee that convenes to decide on the appropriate candidate to fill a position. Ideally, each committee member is required to rank all applicants; then a joint decision is reached based on all opinions [see e.g. Chen et al. (2013)]. However, as their time is limited, each committee member is reluctant to describe and disclose a complete list of ranked preferences [see e.g. the discussion in Xia and Conitzer (2011)]. As another example, consider peer grading in massive open online courses (MOOCs). Since students are not professional educators, they are not trained to provide grades in absolute terms. Rather, students provide comparative information by answering some binary comparative queries [see e.g. Capuano et al. (2017)]. Even when the voter is acquainted with all of the candidates, it is easier to answer relative comparison queries than to rank all of the alternatives (Balakrishnan and Chopra 2012). Furthermore, voters are more accurate when making relative comparisons than (a) ranking all items (Miller 1956); and than (b) presenting precise numerical values (Damart et al. 2007).

To minimize the degree of open preference formulation, a preference elicitation technique can be applied. One such technique is iterative relative comparisons, i.e., querying the voters for their preference between two candidates. It has been theoretically shown that not all preferences are needed for reaching a joint decision when full information about voter preferences is available, and that the same joint decision can be reached with partial information (Konczak and Lang 2005). Consequently some practical algorithms for eliciting a minimal set of user preferences using relative comparisons are available, showing that in practice the required information can be cut by more than 50% (Lu and Boutilier 2011; Naamani-Dery et al. 2015a).

These algorithms assume that voters are sincere in their response and that intermediate results are not available to the voters. However these assumptions might not hold in real world scenarios. As a toy example, consider three candidates: \(c_1, c_2, c_3\). After a few iterations of voting is it apparent that either \(c_2\) or \(c_3\) will be elected (since \(c_1\) received zero votes and almost all of the voters have been queried). Now, assume that voter \(v_1\) prefers \(c_1\) over \(c_2\). However, given that \(c_1\) stands no chance of winning, \(v_1\) may choose to vote strategically and state that he prefers \(c_2\) over \(c_1\) in order to reduce \(c_3\)’s chance of winning. To the best of our knowledge, iterative preference elicitation has not been studied with strategic behavior nor has the issue of intermediate results been addressed. Specifically we assume that:

- **Intermediate results are available**—it is possible that the voters learn the intermediate results either directly, e.g., if the intermediate results are published during the process; or indirectly, e.g., the voters discuss their preferences and reveal how they responded to the relative comparison queries they received; or through an information leak, e.g., someone tapped in on the voting center in charge of aggregating the voters.

- **Strategic behavior is possible**—voters may adopt a strategic behavior in order to manipulate the outcome of the election towards their favorite candidates (Farquhar-
This behavior has been observed in online applications [see e.g. Zou et al. (2015)].

In this paper, we set out to study strategic behavior in incremental voting processes. We follow Lu and Boutilier (2011) and Naamani-Dery et al. (2015a) and assume that the voting center proceeds in rounds; in each round the voting center selects one relative comparison query, i.e., one voter to query for her preference between two candidates. Deviating from previous research, we assume that at the end of each round, the voting center directly exposes the intermediate results, i.e., the candidates that stand a chance to win the elections. Thus the voters may attempt to manipulate by sending the voting center an insincere response in order to promote or avoid certain candidates according to their truthful preferences. Note that voters cannot erase their past responses. In fact, the center never queries when it can infer an answer, as such a query would be wasted in terms of eliciting new information. Essentially, the center maintains a closure by inference of all obtained preferences. This means that the center would immediately recognize query responses inconsistent with previously obtained preferences. As with other knowledge bases, such inconsistencies can be resolved (Brewka 1991; Brewka et al. 1997), the simplest resolution being to invalidate the voter or even election entirely. As a result, the very process of finding an effective strategic vote becomes a non-trivial problem.

**Contributions:** There are two main challenges in a model that allows strategic voting in iterative preference elicitation. One challenge is from the voting center’s point of view, the second challenge is from the voter’s point of view. The voting center’s main challenge is to try and avoid manipulations by presenting only “safe” relative comparison queries to the voters, i.e., queries in which the voters must answer truthfully since they are not able to compute a strategic vote. Nevertheless, each voter’s challenge is to maximize her own selfish outcome, as is done in the voting literature since the classical works of Gibbard (1973) and Satterthwaite (1975). This is achieved if and when the voters are able to compute a strategic vote. For the first challenge, we present a **careful voting center** (Sect. 4.2) which tries to prevent manipulation. The voting center will not present queries over candidates that can be manipulated by a strategic voter, unless it has no other choice. However, this is only a conclusion to our development of a wide range of possible manipulations. More specifically, for the second challenge, we provide a **manipulation algorithm for voters** (Sect. 4.1) in an iterative voting setting. We present a dominance-based heuristic in order for voters to submit strategic responses that will maximize their outcome and will not contradict their previous statements. We prove the soundness and completeness of the manipulation algorithm, meaning, if a manipulative response (a preference over queried candidates) exists then a manipulative voter will submit it to the voting center. By addressing these challenges we provide the foundations for a realistic group decision support system.

An earlier and shorter version of this work was published in the proceedings of IJCAI-2015 (Naamani-Dery et al. 2015b). In this paper we expand both the theoretical and empirical parts of the research. Beyond providing detailed explanations of the manipulation algorithm for voters in an iterative voting setting, we formalize the constraints in which a voter is able to manipulate the voting. Also, we add a proof to the soundness and completeness of the manipulation algorithm. An additional important
contribution is in the wide set of experiments added to this version. We added three real-world data sets in order to prove that our conclusions are domain independent. The new experiments analyze interesting aspects of the manipulation algorithms and examine its impact in practice.

This paper is structured as follows. In Sect. 2 we survey the current work on preference elicitation with partial information and the current work on iterative voting processes. In Sect. 3 we lay out the model preliminaries before we present a detailed algorithm in Sect. 4. In Sect. 5 we set forth the empirical study carried out to answer our research questions. Finally, we provide some conclusions in Sect. 6.

2 Related Work

In this paper we fuse together two distinct subjects of interest: a) multi-query (iterated) preference elicitation and b) iterative voting processes. The distinction between the two is seemingly irreconcilable. On one hand, preference elicitation assumes truthful but communication limited voters. Iterative voting processes, on the other hand, focus explicitly on manipulative, deceitful voter nature, but make no limiting assumption on the amount of communication required to cheat. Nonetheless, as we will show, the two can co-exist, yielding a more realistic picture of time-extended voting processes with limited communication. To make our intent clearer, we provide an overview of the two subjects, and highlight our innovation against their background.

2.1 Preference Elicitation with Sincere Voters

Traditionally, preference elicitation is performed via multi-stage processes. At each stage the voting center selects one voter and queries for a portion of her preferences. It is assumed that voters respond sincerely, do not know each other’s preferences, and are unaware of any intermediate results. The latter, although not completely foolproof, provides one of the better insurances that voters will not attempt strategic manipulation of the outcome (Reijngoud and Endriss 2012). Under these conditions, the voting center can concentrate on minimizing the number of queries and the amount of information it requires from a voter.

The communication complexity of preference elicitation for various voting protocols has been analyzed, and upper and lower bounds are available (Conitzer and Sandholm 2005). In general, for most voting protocols, in the worst case voters should send their entire set of preferences. Other theoretical bounds for the computation of necessary winners have been previously addressed (Betzler et al. 2009; Pini et al. 2007; Walsh 2007). At the end of each iteration, it is possible to compute which candidates may still have a chance of winning and which will certainly win. These sets of candidates are known as the set of possible winners and a set of necessary winners respectively (Konczak and Lang 2005). This has been done in various settings, for example, in tournaments (Aziz et al. 2015). We adopt this approach and compute the set of possible winners at each stage of the voting process.
A candidate winning set is defined as the set of queries needed in order to determine whether a candidate is a necessary winner. For rules other than the plurality voting, computing this set is NP-hard (Ding and Lin 2013). Following this theorem, heuristics for preference elicitation have been suggested, with the goal of finding the necessary winner using a minimal number of queries. One such heuristic operates under the assumption that each voter holds a predefined decreasing order of the preferences.

A practical elicitation process that follows these assumptions is proposed for the Borda voting protocol using the minmax regret concept. The output is a definite winner or an approximate winner (Lu and Boutilier 2011). Another practical elicitation framework for the Range and Borda voting rules introduces two heuristics for choosing which voter to query regarding which candidates. One heuristic is based on the information gained by the query and the other heuristic tries to maximize the Expected Score (ES) (Naamani-Dery et al. 2014b, 2015a, 2016). In this paper we assume that a naive voting center, i.e. a voting center that does not attempt to block strategic behavior, will query the voters according to the ES heuristic.

2.2 Iterative Voting Processes and Strategic Depth

Like preference elicitation, iterative voting is also performed via multi-stage processes. At each stage a voter is selected to examine the current election outcome, and is granted the possibility to alter her ballot, after which the election outcome is re-evaluated.

A priori it is unclear whether an iterative voting process will ever stabilize either in strategy (where no voter wishes to change her ballot) or in outcome (where the election outcome no longer changes, even though ballots may). Both converging and cycling voting processes have been demonstrated (Branzei et al. 2013; Kukushkin 2011; Lev and Rosenschein 2012; Meir et al. 2010; Obraztsova et al. 2014; Reyhani and Wilson 2012). Encouraged by these results, researchers proceeded to study stable points of iterative voting processes (e.g. Rabinovich et al. 2015); investigated voting dynamics, i.e. families of iterative voting strategies, to provide convergence guarantees (Grandi et al. 2013; Lev and Rosenschein 2016; Meir et al. 2014; Obraztsova et al. 2015b; Reijngoud and Endriss 2012); and have even expanded the model to include the ability of candidates, rather than voters, to behave strategically (Brill and Conitzer 2015; Obraztsova et al. 2015a; Polukarov et al. 2015). Now, it is necessary to separate features of voting dynamics that support convergence, and actual strategic behavior that satisfies these features. Furthermore, even if a convergent strategy is found, can we ensure that it is computationally feasible? This question is not trivial. Although almost all voting rules can be manipulated (Gibbard 1973; Satterthwaite 1975), it may be difficult to calculate such a manipulation (Walsh 2011). In fact, in incomplete voting scenarios, both a manipulating strategy and an estimate of the election outcome can be computationally hard (Pini et al. 2007; Walsh 2007). This computational difficulty is of particular importance for iterative voting, as the information available to a voter at any given stage is incomplete.

One of the major breakthroughs in this direction came from considering a softer form of manipulation by strategic voting: one that will not worsen, but just may improve, the election outcome (Endriss et al. 2016; Meir et al. 2014; Reijngoud and
Endriss (2012). In a sense, this is a safe manipulation. It considers all possible situations where the given piece of information holds; and then chooses a ballot augmentation that would improve the outcome in some of those possible scenarios, but damage none. Termed a locally dominant strategy, this behavior is myopic in its original design. However, it is possible to push this idea a bit further, and allow each voter to act non-myopically, assuming even the unlikely case that others will act in her favor (Bannikova et al. 2016; Obraztsova et al. 2016). Of course this latter, optimistic form of behavior may result in a suboptimal stable point of the iterative voting process.

2.3 The Fusion

Against this background, our paper innovates the following fusion:

- **Preference Elicitation with Insincere Voters** Dropping the assumption that voters are always sincere, the elicitation process has to change dramatically as well. In particular, unlike any previous elicitation heuristic, we introduce dual purpose selection of preference queries. First, as was originally intended, these queries are designed to calculate the election winner as quickly as possible. Second, referring to the core principle of eliciting true preference, we design these queries to be manipulation resistant.

- **Iterative Voting with Partial Preferences** Following the common iterative voting assumption, we assume that intermediate results are available to the voters. However, differently from the common procedure, we investigate a situation where a voter is limited in her communication, and may only answer a given query.

- **Strategic Voting in an Iterative Preference Elicitation Process** We show, however, that in spite of the limited voter-center communication, voters may still manipulate the outcome. To achieve this, we rely on a variant of locally dominant principle of manipulation, where we introduce additional guidance in selecting the manipulative vote. Specifically, we require minimization of preference distortion applied to achieve the manipulation effect. Perhaps counter-intuitively, satisfying this additional guidance requirement leads to a polynomial time computable set of manipulations.

Another point of reference for our research, one that also may lead to a variety of applications, is that of crowdsourcing (Yuen et al. 2011). Both crowdsourcing and iterative voting deal with opinion aggregation. The classical difference is that in crowdsourcing voters derive utility from participation (e.g., get payed), whereas in voting scenarios voters derive utility from the aggregation outcome. However, more recent studies on crowdsourcing examine team formations with strategic voters as well (Wang et al. 2017; Jiang et al. 2017). In these cases, the crowdsourcing system attempts to address the concern that strategic voters may influence and skew the outcome away from the truth. Which makes these systems reminiscent (distantly as it may) to a voting center that attempts to eliminate manipulation. Strategyproof peer selection mechanisms have emerged [e.g. Aziz et al. (2016)], however they are not iterative, making a rather impractical assumption that voters consider the whole set of candidates at once.

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To conclude, the voting process in our paper is a combination of preference elicitation and iterative voting. We borrow from each to describe a more realistic voting interaction. In more detail, the voter is presented with the current election outcome (similar to iterative voting), and is requested to respond to one relative comparison query (similar to some preference elicitation heuristics). Furthermore, we consider strategic voting behavior, where the voter deviates from her true preferences and submits a response that maximizes her utility. Such interaction is an explicit meld of our two subjects of interest, where each portion necessarily adopts some features of the other. To appreciate this recall that preference elicitation typically assumes sincere voters (which we do not), and iterative voting processes commonly assume the ability to communicate a change of the entire preference order (which we assume to be limited). The fusion is further underlined by our introducing center’s awareness of the manipulative nature of voters. In other words, while the main goal of the center remains to determine the winning candidate with a minimal number of queries, the center must now also ensure that outcome reflects true voter preferences, without distortion. To this end, we design a voting center capable of identifying the opportunities of strategic manipulation by voters. This allows the center to balance the need to extract preference information and the need to suppress manipulations. The latter, in our design, takes the form of avoiding certain manipulation-prone queries, potentially stagnating some voters until no other recourse is possible, but query them. Notice that the center and voter population are now integral parts of the same system—their respective views of the problem are now linked, and all participants are treated as proactive.

3 The Model

Our model consists of: a set of candidates \( C = \{c_1, \ldots, c_m\} \), a set voters \( V = \{v_1, \ldots, v_n\} \), and a voting center. The opinion of a voter \( i \) about various candidates is expressed by a preference order (i.e., a ranking) \( P_{i}^{true} \) over the set of candidates, i.e., \( P_{i}^{true} = [c_{i_1}, \ldots, c_{i_m}] \) where \( c_{i_1} \) is \( v_i \)'s most preferred candidate. We denote the relative preference of candidate \( c_j \) over \( c_k \) by \( c_j \succ_i c_k \), omitting the subscript where the voter is obvious, or, equivalently by \( P_{i}^{true}(c_j, c_k) \). The space of all possible preference orders is denoted \( \mathcal{L}(C) \).

The voting center seeks to implement a voting rule, i.e. a mapping \( \mathcal{F} : \mathcal{L}(C)^n \to C \), by which preferences of voters are aggregated and mapped to a single candidate, termed the winner. Since iterative preference elicitation with strategic behavior is a novel idea, we chose to begin with the Borda voting rule (de Borda 1781).

The voters’ preferences \( P = (P_1, \ldots, P_n) \) are private, and unknown to either other voters or the voting center (and even a distribution of preferences within the voter population is unknown). Therefore, the winner calculation is replaced by an iterative, approximate elicitation process. Specifically, at every step of this iterative process a voter-item-item query is generated, that describes a question addressed to a single voter regarding her relative preference among some pair of candidates. Our model does not assume any specific query selection protocol and several possibilities for selecting the voter-item-item queries exist, however we do assume that the center would like to minimize the number of queries that it makes. For specific query selec-
tion protocols for this setting see Lu and Boutilier (2011) or Naamani-Dery et al. (2015a).

The voting center sends a voter-item-item query \((v_i, c_j, c_k)\), to which voter \(v_i\) can respond either \(c_j \succ c_k\), declaring that she prefers \(c_j\) to \(c_k\), or respond \(c_k \succ c_j\) to express the inverse preference. The voting center collects all query responses into a collection of partial preferences \(Q = (Q_1, \ldots, Q_n) \in Q^n\), where \(Q_i\) is the collection of pairwise preference expressions from voter \(v_i\). At each stage, \(Q\) can be used to estimate the range of possible candidate scores and an estimate of a Borda winner (Konczak and Lang 2005). This is done by considering whether a complete joint preference order \(P\) exists that could extend the partial information contained in \(CL(Q)\), i.e. all expressed preference comparisons and all preference comparisons inferred by transitivity 1.

Definition 1 (Possible and Necessary Winners) [adapted from Konczak and Lang (2005)] Let \(P_i \models Q_i\) denote the fact that \(\forall (c_j, c_k) \in CL(Q_i)\) holds \(P_i(c_j, c_k)\), and let \(Ext(Q) = \{P \in L(C^n) | \forall v_i \in V P_i \models Q_i\}\), that is we look at all possible complete joint preference profiles that are consistent with the responses recorded by the voting center. Then:

- \(c \in C\) is a necessary winner for \(Q\) if and only if \(\forall P \in Ext(Q)\) holds \(F(P) = c\);
- \(c \in C\) is a possible winner for \(Q\) if there exists a \(P \in Ext(Q)\) so that \(F(P) = c\).

The Possible Winners for \(Q\), \(PW = PW(Q) \subseteq C\), is a set that consists of all possible winners for a given set of voter preference responses to queries.

Notice that a similar, and somewhat inverse, process is also possible: we can begin from a set of Possible Winners and ask whether a complete joint preference order \(P\) is consistent with it. We will later employ this inverse reasoning in defining domination among possible voter strategies in Sect. 3.1.

The set of possible winners contains all candidates. Informally, the set of possible winners can be seen as all candidates that still have a chance to be declared as the winners. If a candidate is omitted from the set of the possible winners, it cannot be re-added later. The voting center calculates the set of possible winners (PW) after each query response. The voting center sends queries to the voters until the set of possible winners contains only one candidate. That candidate is the necessary winner.

To drive the voters to disclose their preferences, the voting center, before querying a voter, provides her with the current set of possible winners, PW, similarly to (Conitzer et al. 2011; Reijngoud and Endriss 2012). As voters are selfish, they are driven to answer a query as this is only manner in which they can effect a change in the possible winners in their favor.

This means that a voter’s response to a query is not necessarily dictated by their true preferences \(P_i^{true}\). Rather, a strategic voter may choose to answer a query using an alternative preference order \((P_i \neq P_i^{true})\) that forces the voting center to reshape the set of possible winners to be more beneficial to the queried voter. In the following Sect. 3.1 we discuss how such alternative orders compare to each other via the concept of local dominance, and in Sect. 4 we contribute a computational procedure to implement this strategic choice.

1 Notice that \(CL(Q)\) is a composition of \(CL(Q_i)\), i.e. transitive closures of individual voter response sets.
The voting center calculates the set of possible winners based on the received and the inferred preferences by transitivity. The voting center can and will detect logical inconsistency in query answers, if they do not conform (extend) to some complete preference order in $\mathcal{L}(C)$. Such inconsistency is implicitly assumed to be punishable, and undesirable by voters and the voting center alike. Thus, voters cannot just calculate momentary beneficial responses to queries, since they will be instantly caught. Therefore, (successful) strategic voters keep track of their preference orders ($P_i$).

The interaction between the voting center and voters is summarised by the following loop:

- At the beginning, each voter holds her true set of preferences $P_i = P_i^{true}$;
- As long as the Necessary Winner has not been identified:
  1. The voting center computes a set of the Possible Winners $PW$.
  2. The center selects a voter-item-item query, $\langle v_i(c_j, c_k) \rangle$;
  3. The voter $v_i$ is provided with the current set of Possible Winners $PW$;
  4. The voter decides whether or not to change her profile from her current profile $P_i$ to a new strategic profile $P'_i$;
  5. The voter responds with either $c_j \succ c_k$ or $c_k \succ c_j$ according to her updated profile $P'_i$;
  6. The center updates the incomplete profile ($Q$) by incorporating $v_i$’s answer and applying transitive closure.

Returning to the toy example presented in the Introduction, let us instantiate one step of this loop to clarify the process. Consider three candidates: $c_1$, $c_2$, $c_3$ and a voter $v_1$ with the preferences: $c_1 \succ c_2 \succ c_3$. The voting center computes the set of possible winners (step 1). Based on this set, the center selects the query $\langle v_1(c_1, c_2) \rangle$. Now, voter $v_1$ is asked to state her preference between $c_1$ and $c_2$ (step 2). The voter is also informed that the possible winners $PW$ are $c_2$ and $c_3$ (step 3). Since $c_1$, the candidate most preferred by $v_1$, is not in $PW$, $v_1$ chooses to vote strategically. In order to keep track of all of her strategic moves, the voter updates her profile: $P'_1 = c_2 \succ c_1 \succ c_3$ (step 4). The voter submits a response: $c_2 \succ c_1$ (step 5). The partial profile now known to the center is: $Q_1 = c_2 \succ c_1$ (step 6).

### 3.1 Locally Dominant Manipulation

In order to guarantee consistency of their answers, each voter maintains a current preference profile $P_i$ that she uses to answer queries from the voting center. A voter may change this profile, as long as it remains consistent with previous answers, if she deems such a change beneficial for the current and, possibly, future query answering.

As many preference orders are possible, to capture the benefit of adopting one particular preference order over another we employ the concept of local dominance manipulation model (Conitzer et al. 2011; Meir et al. 2014; Reijngoud and Endriss 2012). We formally instantiate local dominance below.

First, notice that voters cannot contradict their answers to previous queries. Thus, the voters ability to adopt additional manipulative changes to their current preference order are limited. Specifically, whatever preference order voter $v_i \in V$ decides to
adopt when answering a query $(v_i, c_k, c_j)$, this order will have to be a member of the set $P_i = \{ P \in L(C) | P \models Q_i \}$. Second, the improvement that all voters seek is that of a better final outcome declared by the voting center. Thus, a voter will change her current profile from $P_i$ to $P'_i$ if she recognizes that the change (and query answers that it engenders) possibly entails a better final outcome, i.e. an outcome that ranks higher with respect to her truthful preference. However, the voter would also want to guarantee that there is no possibility that the change would result in her being worse off when the final outcome is produced. Let us define these possibilities more formally.

**Definition 2 (Possible World and Outcome)** Let $PW$ be a set of Possible Winners. A joint preference profile, $P_{-i}$, of all agents in $V \setminus i$ is termed a possible world (from the perspective of $v_i \in V$), if it is consistent with the set $PW$. More formally, $P_{-i}$ is a possible world if exists $Q_{-i}$ so that $P_{-i} = Ext(Q_{-i})$ and $PW(Q_{-i}, Q_i) = PW$. That is, the use of joint preference profile $P_{-i}$, combined with voter $v_i$’s responses, could have led the voting center to generate $PW$. In turn, a possible outcome (of $P_i$), is the candidate that would be declared the winner by the voting center if it had full access to $P_i$ and some possible world $P_{-i}$.

Notice that $v_i$ has no access to $P_{true}^{-i}$ or the actual $Q_{-i}$, so the concept of possible worlds and outcomes is speculative. However, this speculation does allow $v_i$ to build a strategic argument, if she implicitly assumes all other voters to be persistently truthful and that they use $P_{true}^{-i}$ to answer all queries. This is because a possible world $P_{-i}$ may generate different possible outcomes for two different preference orders, $P_i$ and $P'_i$, of agent $i$. Thus, the preference order $P_i$, that voter $v_i$ uses to respond to the voting center queries, can now be used as a strategic manipulation means. More specifically, we can compare the relative benefit of the set of possible outcomes of $P_i$ vs the set of possible outcomes of $P'_i$. With that in mind, Local Dominance is a particular form of safe choice of $P_i$ with respect to its effect on the set of possible outcomes.

**Definition 3 (Local Dominance)** A preference order $P'_i$ is a local dominant over preference order $P_i$ if, in at least one possible world, the possible outcome of $P'_i$ is ranked higher than the possible outcome of $P_i$, and in none of the possible worlds the possible outcome of $P'_i$ is ranked lower than the possible outcome of $P_i$.

When queried, the voter is requested to submit her preference between two candidates only. However, in order to manipulate, more than a single change in current preferences $P_i$ might be needed. A voter would want to ensure that such a continual distortion does not accumulate into a grotesque misrepresentation of her original preferences $P_{true}^i$. Thus, from all of the possible changes in $P_i$, the voter will seek a change that requires the minimal number of swaps. I.e., the voter actively minimises the change in her current preferences at every step. Formally, to compare two current preference profiles $P_i$ and $P'_i$, we use the swap distance Bredereck et al. (2016), Kendall (1938) defined for two linear orders $P$ and $P'$. The distance counts the number of candidate pairs that are ordered differently by two ballots or linear orders.

Summarising these limitations on a manipulative change in current profile, a voter will change her profile to $P'_i \in P_i$ only under the following voting manipulation (VM) conditions:
Condition-1 The new preference profile $P_i'$ is a local dominant profile over $P_i$.

Condition-2 The new preference profile $P_i'$ has the minimal swap distance out of all possible (consistent with previous $i$’s responses) profiles.

For notational convenience, we will denote the set of all preference profiles that satisfy Condition-2 by $\mu(P_i)$. Formally, $\mu(P_i) = \arg\min_{P_i \models Q_i \cup \{(c_k, c_j)\}} d_{swap}(P_i, P)$, where $P_i \models Q_i$ denotes the fact that the preference order $P$ is consistent with the partial order of reported preferences $Q_i$. In turn, $d_{swap}(P_i, P)$ denotes the swap distance between two preference profiles.

We now define the scenarios where manipulation can be performed. For a given set of Possible Winners $PW = \{pw_1, \ldots, pw_l\}$ we define $PW_i$ as the ordered vector of possible winners for voter $v_i$: $PW_i = \{pw_{i1}, \ldots, pw_{il}\}$, where the order is w.r.t $P_i$. In particular, for any $1 \leq j < k \leq l$, we will have $P_i(pw_{ij}, pw_{ik})$, i.e. voter $i$ will prefer $pw_{ij}$ to $pw_{ik}$. We will omit the superscript where the agent is clear from the content, and simply write $PW_i = \{pw_1, \ldots, pw_l\}$.

To set the conditions for local dominance, we use the following set of “Common Givens”, a set of w.l.o.g. assumptions that we will use throughout the remainder of the paper:

- The current preference profile of $v_i$, $P_i$;
- The query is: $\langle v_i(c_j, c_k) \rangle$, and according to $P_i$: $c_j > c_k$;
- The ordered (w.r.t. $P_i$) vector of Possible Winners of $v_i$, $PW_i$;
- The closure of the set of current query responses, $Q_i$;
- Neither $Q_i(c_j, c_k)$ nor $Q_i(c_k, c_j)$ hold, i.e. there is no committed order among the query’s candidates.

Now, to describe our algorithms and their theoretical features, we will use an interval-like notation for subsets of candidates. The order within the interval will be that of a preference profile, $P_i$, or the set of previously stated preferences, $Q_i$. In particular, $[c ; P_i ; c']$ will denote a set of candidates in preference profile $P_i$ between two candidates, $c > c'$, inclusive of the two candidates themselves. At the same time, $(c ; P_i ; c')$ will denote the same set, but excluding the candidate $c$. Finally, we will naturally use $\infty$ in this notation, so that, e.g., $[c ; Q_i ; -\infty)$ will denote all candidates that have been reported to be below $c$ and $c$ itself. To maintain the common left-to-right descending order notation, interval boundary points will also appear in descending order. Notice that, in this respect, $+/ - \infty$ are used consistently. We will also allow preference orders to be imposed on (or limited to) an interval. Formally, $P \downarrow_{[c ; P' ; c']}$ will denote a complete order of elements of $[c ; P' ; c']$ consistent with the preference order $P$.

To instantiate the notation of preference intervals, and preference projections on such intervals, consider the following example. Let $P$ be a preference order over 6(six) candidates so that $c_2 > c_1 > c_3 > c_5 > c_4 > c_6$, and $P' = c_4 > c_6 > c_1 > c_5 > c_3 > c_2$. Then $[c_1, P, c_4] = \{c_1, c_3, c_4, c_5\}$, while $[c_1, P, c_4] = \{c_1, c_3, c_5\}$, and $[\infty ; P' ; c_1] = \{c_1, c_4, c_6\}$. Finally, the preference order $P'' = P \downarrow_{[\infty ; P' ; c_1]}$ will impose the order inherited from $P$ on $[\infty ; P' ; c_1] = \{c_1, c_4, c_6\}$. In particular, according to $P''$ would hold $c_1 > c_4 > c_6$.

In the following section, we provide our solution to the model. I.e., an efficient computational procedure to find an augmentation to the current preference profile $P_i$.
that satisfies all the three aforementioned properties: a) the order must be coherent with all answers given; b) the order must be based on a dominant answer to the current query; c) the order must allow as much future flexibility as possible, via minimising the momentary swap-distance distortion.

4 Interactive Local Dominance

Our model adapts the concept of Local Dominance to interactive voting scenarios, where voter preferences are only partially known at any given point in time. In this section we provide both the algorithmic and theoretical treatment of the Interactive Local Dominance (ILD) concept.

We begin by stating a key feature of ILD, and describe the algorithm to solve it for the particulars of our model. We then proceed with a detailed theoretical analysis of ILD and prove the correctness of our algorithmic solution.

4.1 Computing an Interactive Local Dominance Response

The following Theorem states that a manipulative response to a query has to maintain the same order of possible winners. Furthermore, for at least one pair of consecutive possible winners the distance between them grows. In turn, the Corollary states that these ordering and distance properties can be consistently traced from the truthful profile throughout all responses of a voter. We defer the proofs of these statements to Sect. 6, where we will present the encompassing and rigorous theoretical treatment of the guided locally-dominant manipulation.

**Theorem 1** Let us assume w.l.o.g. that the enumeration order of possible winners in \( PW_i \) is aligned with their order of appearance in \( P_i \), that is \( pw_\alpha > pw_\beta \) according to \( P_i \) if and only if \( a < b \). A preference profile \( P'_i \) is a local dominant profile over \( P_i \) if and only if the following holds:

- \( pw_\alpha > pw_{\alpha+1} \) for all \( \alpha \in [1, \ldots, l-1] \) w.r.t. \( P'_i \), i.e. the order of possible winners does not change;
- \( \left| \left[ pw_\alpha ; P'_i ; pw_{\alpha+1} \right] \right| \geq \left| \left[ pw_\alpha ; P_i ; pw_{\alpha+1} \right] \right| \) for all \( \alpha \in [1, \ldots, l-1] \), i.e. none of the intervals between two consecutive possible winners decreases;
- \( \exists \alpha \in [1, \ldots, l-1] \) so that \( \left| \left[ pw_\alpha ; P'_i ; pw_{\alpha+1} \right] \right| \geq \left| \left[ pw_\alpha ; P ; pw_{\alpha+1} \right] \right| \), i.e. at least one interval between two consecutive possible winner will grow.

**Corollary 1** Let \( \tau > t \), and \( P'_i, P^\tau_i \) are the preference profiles of voter \( v_i \) at times \( t \) and \( \tau \) respectively. Then the set of possible winners \( PW \) at time \( \tau \) will be ordered in the same way by \( P^\tau_i \) (the truthful preference of \( v_i \)), \( P'_i \) and \( P^\tau_i \). Furthermore, the size of each segment between consecutive possible winners in \( PW \) will monotonically grow from \( P^\tau_i \) to \( P'_i \) to \( P^\tau_i \), and the total size of these segments will grow strictly monotonically.

Given Theorem 1, it is easy to confirm whether any of the two given preference profiles, \( P, P' \), locally dominates the other. If we are able to compute the set \( \mu(P_i) \),
Require:

“Common Givens” w.l.o.g. assumptions

For query \((c_j, c_k)\) holds \(P_i(c_j, c_k)\)

1. Set \(d^{abs} \leftarrow \infty\), \(d^{loc} \leftarrow \infty\), \(P^{loc} \leftarrow P_i\)

2. if \(c_j, c_k\) do not satisfy Lemma 1 then

3. return \(P_i\)

4. end if

5. Set \(Z \leftarrow [c_j ; P_i ; c_k]\)

6. for \(z \in Z\) increasing w.r.t \(P_i\) do

7. \(X_{good} \leftarrow (\infty ; P_i ; z) \setminus [c_j ; Q_i ; \ominus \infty]\)

8. \(X_{bad} \leftarrow (\infty ; P_i ; z) \cap [c_j ; Q_i ; \ominus \infty]\)

9. \(Y_{good} \leftarrow (\ominus \infty ; P_i ; c_k) \setminus (\infty ; Q_i ; c_k)\)

10. \(Y_{bad} \leftarrow (\ominus \infty ; P_i ; c_k) \cap (\infty ; Q_i ; c_k)\)

11. Order \(X_{good}, X_{bad}, Y_{good}\) and \(Y_{bad}\) by \(P_i\)

12. Denote \(P_i'\) the preference order \((X_{good}, Y_{bad}, c_k, c_j, X_{bad}, Y_{good})\)

13. if \(d < d^{abs}\) then

14. \(d^{abs} = d\)

15. end if

16. if \(P_i'\) is LD and \(d < d^{loc}\) then

17. \(d^{loc} = d\)

18. \(P^{loc} = P_i'\)

19. end if

20. end for

21. if \(d^{abs} < d^{loc}\) then

22. return \(P_i\)

23. else

24. return \(P^{loc}\)

25. end if

Fig. 1 Voter manipulation function: meta-algorithm

then testing its elements would result in finding a feasible manipulation for a voter. Algorithm 1 (Fig. 1) scans the set of all preference profiles that may belong to \(\mu(P_i)\), thus composing \(\mu(P_i)\), and confirming that at least one of them locally dominates \(P_i\). The algorithm finds a feasible manipulation, if one exists.

More formally, Algorithm 1 operates under the “Common Givens” w.l.o.g. assumptions on a query \((c_j, c_k)\), and returns a manipulative preference order, if one exists to satisfy Condition-1 and Condition-2. Otherwise, Algorithm 1 keeps the preference order unchanged. Hence, either a new preference order \(P_i'\) is returned, where \(P_i'(c_k, c_j)\) holds, or the current preference order \(P_i\) is kept, where \(P_i(c_j, c_k)\). In the former case of the pair the response \(c_k \succ c_j\) will be added to \(Q_i\), while in the latter—the query response \(c_j \succ c_k\) will be used. Proof that the algorithm’s correctness and that it can run in polynomial time can be found in the appendix.

4.2 Careful Voting Center: Securing Against Interactive Local Dominance

Our analysis of ILD was directed to find as many safe (in local dominance sense) manipulation opportunities as possible for a voter to adopt. This, however, does not mean that all queries would prompt a manipulation. In fact, Theorem 1 places a clear
limitation on the space of manipulable queries. Since the order of possible winners can
not be altered by a locally-dominant manipulative change in the current preference
profile, all queries regarding relative preference among two possible winners will
follow $P_t$. Thus the queried voter will keep her current preference order unchanged.

**Corollary 2** Let us assume that voter $v_i$ at time $t$ has $P_t^i$ as its current preference
order, and let $PW$ be the set of possible winners calculated by the voting center at
time $t$. Then, for all $c_k, c_j \in PW$, $v_i$ will respond $(c_k, c_j)$ to a voter-item-item query
$(v_i, c_k, c_j)$ if $P_t^i(c_k, c_j)$, and $(c_j, c_k)$ if $P_t^i(c_j, c_k)$. In particular, after answering the
query $P_t^{i+1} = P_t^i$, i.e. the current preference profile will not change.

This opens the possibility to secure, at least in part, against manipulative voters
affecting the voting center’s calculation of the true Borda winner. We can simply
avoid, as much as possible, using queries that may prompt a manipulation attempt by
the queried voter.

**Definition 4** Let us term a query $(v_i, c_k, c_j)$, where both $c_k, c_j \in PW$, a safe query.
A voting center that directs to voters only safe queries, unless no safe query exists, is
termed a careful voting center.

### 5 Experimental Validation

We know that manipulations are not summarily avoidable, therefore our experiments
do not serve as a manipulation feasibility study. Rather, the experiments provide an
important statistical insight, not addressing the question of “whether”, but “how fre-
quently” manipulations are possible; and, what’s more important, how frequently do
they actually lead to altered election outcome. In order to evaluate the manipulation
impact on the preference elicitation process, we compared manipulative voters to
truthful voters in a careful and in a Naïve voting center setting. A careful voting center
selects only queries which are not manipulable, when such queries exist. If no such
queries exist (since they have been previously used), the voting center stops being
careful. A Naïve voting center does not consider which queries are manipulable and
which are not.

Algorithms that perform preference elicitation in iterations can be found in Lu and
Boutilier (2011, 2013), Naamani-Dery et al. (2015a, 2014b). In this paper we use
the Expected Score (ES) algorithm found in Naamani-Dery et al. (2015a). The ES
algorithm selects a voter-item-item pair where one of the items is the item with the
current maximum score. This algorithm is publicly available whereas some others are
used commercially and cannot be tampered with. As a baseline we used an algorithm
which randomly chooses the next query (denoted as RANDOM). Each algorithm (ES
and RANDOM) was studied in three states:

1. The voters always answer truthfully (ES+T, RANDOM+T).
2. The voters attempt to manipulate (ES+M, RANDOM+M).
3. Manipulative voters with a careful voting center (Careful-ES+M, Careful-
RANDOM+M).
Experiments were performed on four real world data sets: the Sushi data set (5000 preference rankings over 10 candidates) (Kamishima et al. 2005), the T-shirt data set (30 preference rankings over 11 candidates), the Courses-2003 data set (146 preference rankings over 8 candidates) and the Courses-2004 data set (153 preference rankings over 7 candidates). The three latter data sets were taken from the Preflib library (Mattei and Walsh 2013). The data sets were used to generate responses to elicitation queries, assuming a Borda voting rule. A random set of voter preference profiles ($P$) was sampled with return out of each data set. For each experiment setting, 20 sets of random profiles were evaluated. For each set of profiles, the experiment was conducted 40 times. Thus we reach an amount of 800 experiments for each experiment setting.

The amount of candidates was set to the maximal amount in each data set. The amount of profiles $P$ was first varied on a range of 10, 20, ..., 100 voters. Since we suspected that fewer voters may lead to more fragile situations, we also examined scenarios with only 4, 5, ..., 20 voters.

In order to conclude which algorithm performs best over multiple data sets, we followed a robust non-parametric procedure proposed by García et al. (2010). We first used the Friedman Aligned Ranks test in order to reject the null hypothesis that all heuristics perform the same. This was followed by the Bonferroni-Dunn test to find whether one of the heuristics performs significantly better than other heuristics.

5.1 Measures

The evaluation focused on the effect of the manipulations on the iterative process. We examined three dependent variables:

1. Manipulation rate in a careful and a regular voting center - When a voter has an opportunity to manipulate, she will. We therefore first set out to check how often do the voters actually have an opportunity to manipulate, and whether a careful voting center can reduce the manipulation rate.
2. Manipulation impact on the final result - Even when manipulations occur during the iterative process, it does not mean they impact on the final result, i.e., the chosen winner. We consider a manipulation process successful only if the chosen winner is different than the winner when no manipulations occur.
3. Manipulations impact on the number of iterations - How manipulations impact the process length, measured as the number of iterations.

5.2 Results

In the following sections, we examine each of the measures in turn.

5.2.1 Manipulation Rate in a Careful and a Regular Voting Center

Across all four data sets, the ratio of queries that have actually been manipulated is very small: the average manipulation ratio in all experiments is 0.003 for the Sushi, T-shirt and Courses-2003 data sets, and 0.005 for the Courses-2004 data set. The
highest result was received for the Courses-2004 data set, with the Random+M algorithm, and 4 voters: 0.0114 of the queries were manipulated. To illustrate, in Fig. 2 we present the results obtained from the four data sets. The x-axis presents the number of voters, and the y-axis the ratio of the queries manipulated (i.e., the number of manipulated queries divided by the number of total queries). The behavior is similar in all data sets. Unanimously across all data sets, the Friedman test detected a significant difference (with a p-value of at most 0.05) between the four manipulating algorithms: ES+M, RANDOM+M, Careful-ES+M and Careful-RANDOM+M. The Bonferroni-Dunn post-hoc test reveals that Careful-RANDOM+M outperforms the other algorithms, and that ES+M and Careful-ES+M outperform RANDOM+M. Namely, less manipulations occur with ES+M and Careful-ES+M algorithms than in the RANDOM+M algorithm, and even less manipulations occur with the Careful-RANDOM+M algorithm.

At first thought, one might expect Careful-ES+M to exhibit a lower manipulation rate than Careful-RANDOM+M, since RANDOM selects queries at Random. However, the manipulation rate is the lowest for Careful-RANDOM+M, regardless of the number of voters. This can be explained by the query selection process. Careful-RANDOM randomly selects a query out of all the safe queries available (i.e., queries that are “safe” cannot be manipulated). ES algorithms first create a pool of possible queries that are “ES compatible” (according to the ES algorithm). The regular ES algorithm selects one of these queries at Random. The Careful-ES algorithm has an additional selection requirement—a query that is both ES compatible and safe. If none is found, it will select an ES query that is not safe and is thus exposed to manipulation more often. The set of ES compatible and safe is a subset of the safe queries and thus there are less opportunities for ES to send safe queries.
5.2.2 Manipulation Impact on the Final Result

In some cases, there is zero impact on the final result, meaning that although manipulations might occur, they do not alter the final result, i.e., the winning candidate remains the same. The most severe impact on the final result occurs while using the RANDOM+M algorithm, and even then the proportion of the cases where the final result is changed remains quite low with proportions of: 0.09, 0.11, 0.13 and 0.16 for the Sushi, Courses-2003, Courses-2004 and T-shirt data sets respectively. Figure 3 presents the average proportion of the impact on the final result (i.e., the number of cases where the manipulation changed the final result, out of all cases). Unanimously across all data sets, the Friedman test detected a significant difference (with a p-value of at most 0.05) between the four manipulating algorithms: ES+M, RANDOM+M, Careful-ES+M and Careful-RANDOM+M. The Bonferroni-Dunn post-hoc test confirmed that Careful-RANDOM+M outperforms RANDOM+M. Namely, using the Careful-Random+M algorithm, the final result is less likely to deviate from the result obtained when all voters vote sincerely. This conclusion aligns with the previous section—less manipulations occur with the Careful-RANDOM+M, therefore the outcome is less subject to change.

5.2.3 Manipulations Impact on the Number of Iterations

Figure 4 illustrates the average percentage of the data set queried until a necessary winner is found, for a changing number of voters. Unanimously across all data sets, the Friedman test detected a significant difference (with a p-value of at most 0.05) between the 6 algorithms. The Bonferroni-Dunn post-hoc test confirmed that ES outperforms all the RANDOM algorithm variations (RANDOM+T, RANDOM+M,
Careful-RANDOM+M). Namely, the ES algorithm needs less queries in order to detect a necessary winner. ES+M outperforms all RANDOM algorithm variations except in the Courses-04 data set where there is no significant difference between ES+M and RANDOM+T Careful-ES+M outperforms all RANDOM algorithm variations except in the Sushi data set where there is no significant difference between Careful-ES+M and RANDOM+T and in the T-shirt data set where there is no significant difference between Careful-ES+M and Careful-RANDOM+M. There is no significant difference between ES+M and Careful-ES+M. However, there is a significant difference in favor of ES: when no manipulations occur, the result is reached faster. A possible explanation is that manipulative responses cause candidates that would have been removed from the possible winners set in a regular iterative voting to still be considered as possible thus the set of possible winners decreases more slowly.

For the RANDOM algorithms the trend is the opposite: in all data sets but Sushi, Careful-RANDOM+M performs better than the other RANDOM variations. For the Sushi data set, RANDOM+M performs better than the other RANDOM variations. The results indicate that the number of iterations can be reduced when the queries are chosen in a non-Random method—either using the ES algorithm, or using manipulations on the RANDOM algorithms. When some thought is put into choosing the queries, either via a careful voting center or via a non-random algorithm, less queries are used. However, when the query selection process is not random (e.g. using the ES algorithm) the algorithm itself results in a lower number of iterations, and any additions to the algorithm, such as manipulations, or a careful voting center, only hamper the process.

We can conclude that in practice, manipulations do not pose a big threat on an incremental iterative voting process when the voters submit one preference at a time,
since the manipulation rate is low and they rarely modify the outcome. A careful voting center can reduce the manipulation rate, but the tradeoff is that in turn, there is an increase in the amount of queries needed in order to end the iterative process and find a winner. We can therefore state that publishing the list of Possible Winners during a voting process is quite safe from a manipulation perspective.

6 Conclusions

In this work, we have developed and studied a novel combination of two iterative processes found in social choice: iterative preference elicitation using a voting center and the manipulative modification of preferences by voters in Iterative Voting. Traditionally, the design of the former intends to reduce the amount of query requests the voting center sends in order to obtain election outcomes, but assumes voters will reveal their true preference. The latter, on the other hand, presumes that voters may misreport their preferences and vote strategically. We illustrated how a voter may attempt to manipulate a voting center. We provided a set of algorithms to detect and exploit manipulation opportunities that would drive the voting center to declare an election outcome that is more beneficial to the manipulating voter. Our manipulation detection algorithms allow us to build a careful voting center that avoids manipulable queries.

We present two novel contributions: (1) a careful voting center and (2) a manipulation algorithm for voters. Both of these are tailored specifically for the Borda voting rule. Since iterative preference elicitation with strategic behavior is a novel idea, we chose to begin with the Borda voting rule. It is possible to devise a careful center and a manipulation algorithm for other voting rules. To do so, some changes are required:

1. The voting center computes the set of possible winners according to the Borda protocol. However, the possible winners can be computed according to other protocols (Aziz et al. 2015; Konczak and Lang 2005).

2. We suggest two ways for the voting center to select a voter-item-item query. The first method is to randomly select a query. This will work with any voting rule. The second method is to use the ES algorithm Naamani-Dery et al. (2015a) which is tailored specifically for the Borda voting rule. To the best of our knowledge, such methods currently exist for the Borda and Range voting rules only (Lu and Boutilier 2011; Naamani-Dery et al. 2014a). We leave to future work the development of intelligent selection method of methods for other rules.

3. The local dominant manipulation (Sect. 3.1) and the interactive local dominance (Sect. 4) are specifically tailored for Borda. Adjustment to scoring rules is relatively easy and methods in the same spirit can be developed for any other voting rule. We leave this for prospective future work.

In this paper we focused on finding a single winner, and on investigating how manipulating the preferences will influence that winner. Thus, our model is befitting for strict preferences of individuals over candidates. An interesting question is how the model can be expanded to accommodate weak preferences. Our model holds in a scenario where the individuals have ties in the preferences over the candidates, but there is still a single winner. However, this is not a very realistic scenario, since in most
cases, ties in the preferences lead to ties in the winning candidates, so that there is a set of possible winners instead of a single winner. This may be an interesting model to investigate in the future.

Our experiments show that a careful voting center is effective. Specifically, we show that: (a) Voters do not have many manipulation opportunities to begin with. Even when we assume that a voter will manipulate whenever she has the opportunity, the ratio of queries that have actually been manipulated is very small. The careful voting center reduces the manipulations and the manipulation rate is the lowest for Careful-RANDOM+M. (b) There is a very low impact of manipulations on the final result. Even on the rare occasions that manipulations do occur, they usually do not alter the final result. Again, the careful voting center assists, and the most severe impact on the final result occurs while using the RANDOM+M algorithm. (c) A careful voting center can reduce the manipulation rate, but the tradeoff is that there is an increase in the amount of queries needed in order to end the iterative process and find a winner. Another tradeoff is that advanced elicitation schemes such as the one found in Naamani-Dery et al. (2015a) can reduce the iteration process, but in turn are more prone to manipulations than a Random selection scheme.

We would like to underscore that iterative voting processes with preference elicitation are not an artificial construction. Rather, they occur naturally, as demonstrated by the Introduction section examples: a hiring committee process, and a peer grading system. The latter, in fact, suggests that iterative voting processes with preference elicitation are a consequence of crowdsourcing solutions in general. Of course, modern crowdsourcing acknowledges the fact that task solvers have selfish motives, and seek to tap into them [see e.g. Dasgupta and Ghosh (2013); Roughgarden and Schrijvers (2017)]. However, to the best of our knowledge, a direct study of explicit strategic behavior towards manipulating the crowdsourcing outcome has yet to be attempted. Which makes our contribution that more significant. Our results show that a careful design of the voting procedure can greatly diminish the effect of strategic behavior, which can, for example, be directly translated into MOOC grading stability. The link between crowdsourcing applications and voting procedures is much richer than this one application, and we fully intend to pursue further developments of our approach in this area.

Appendix

Proof: Theorem 1

Let us assume that for some \( l \) holds \( pw_{l+1} > pw_l \). Consider a partial joint profile, \( \mathcal{R}_{-i} \), where \( \text{score}(pw_l, \mathcal{R}_{-i}) = \text{score}(pw_{l+1}, \mathcal{R}_{-i}) = \eta \) and all other voters have a score of at most \( \eta - m \). \( \mathcal{R}_{-i} \) is a possible (partial) joint profile given the set \( PW_i \). Now, if the voter \( i \) submits \( P_i \), then \( pw_l \) will become the winner. If \( P'_i \) is submitted, then \( pw_{l+1} \) will win the elections. This contradicts the definition of dominance: \( P_i' \) does not dominate \( P_i \).

Let us now assume that for some \( l \) holds \( ||pw_l; P_i' : pw_{l+1}|| < ||pw_l; P_i : pw_{l+1}|| \). Similar to the previous case, construct a possible (partial) joint profile \( \mathcal{R}_{-i} \) so that
that satisfies Condition-1 and Condition-2, if such a preference profile exists.

As before, if \( P_i \) is submitted by the voter \( i \), then \( pw_i \) wins the elections, and if \( P_i' \) is submitted, then \( pw_{w_i} \) wins. Again, this contradicts \( P_i' \) dominating \( P_i \).

Lastly, assuming that \( \| [pw_i : P_i : pw_{l+1}] \| \geq \| [pw_i : P_i : pw_{l+1}] \| \) for all \( l \in [1, \ldots, k-1] \) holds, but there is no \( l \in [1, \ldots, k-1] \) so that \( \| [pw_i : P_i' \wedge \text{Condition-2}, if such a preference profile exists.

Interactive Local Dominance Response Analysis

Theorem 1 has given a higher level structure to the set of possible manipulations in our model. It has allowed the reader to build intuition, comprehend our algorithm construction and understand their application examples. Now, it is possible to provide the detailed theoretical treatment, and in this section we will provide complete definitions supporting the algorithmic design. Proofs of all the theorems and

**Theorem 2** Assume that "Common Givens" w.l.o.g. conditions hold. Algorithm 1 works in polynomial time in the number of voters and candidates, and finds a \( P_i' \) that satisfies Condition-1 and Condition-2, if such a preference profile exists.

Before we prove Theorem 2, i.e. the correctness of Algorithm 1, we provide a set of lemmas that are needed for supporting the proof. All lemmas adopt the "Common Givens" w.l.o.g. assumptions mentioned in Sect. 4.1. Let us examine voter \( v_i \)'s profile. The preferred order of possible winners according to \( v_i \) is: \( P_i = [pw_1, pw_2, \ldots, pw_k] \). When \( v_i \) is queried for her preference between \( c_j \) and \( c_k \), her response is: \( c_j > c_k \). We would like to build a new profile \( P_i' \) where voter \( v_i \)'s response to the same query is: \( c_k > c_j \). We need \( P_i' \) to satisfy conditions Condition-1 and Condition-2, i.e. \( P_i' \) should be a local dominant profile over \( P_i \) and also have the minimal swap distance to \( P_i \) out of all possible profiles.

The only way to create a profile \( P_i' \), that is local dominant and has a minimal swap distance, is if in profile \( P_i \):

- \( c_j \) is above \( pw_1 \) and \( c_k \) is below \( pw_1 \): \( P_i : \cdots > c_j > \cdots > pw_1 > \cdots > c_k > \cdots \)
- \( c_j \) is between \( pw_1 \) and \( pw_1 \) and \( c_k \) is below \( pw_1 \): \( P_i : \cdots > pw_1 > \cdots > c_j > \cdots > pw_1 > \cdots > c_k > \cdots \)
- \( c_j \) is above \( pw_1 \) and \( c_k \) is between \( pw_1 \) and \( pw_1 \): \( P_i : \cdots > c_j > \cdots > pw_1 > \cdots > c_k > \cdots > pw_1 > \cdots \)

As an example of the latter case, if \( P_i = [c_j, \ldots, pw_1, \ldots, c_k, \ldots pw_1] \) then switching between \( c_j \) and \( c_k \) by adding \( c_j \) to the sequence that is below \( pw_1 \) and above \( pw_1 \: P_i' = \{pw_1, \ldots, c_k, c_j, \ldots, pw_1\} \) results in a profile \( P_i' \) that is a local dominant with a minimal swap distance, i.e. satisfies both conditions Condition-1 and Condition-2.
Lemma 1 Assume that there is $P_i'$ that satisfies Condition-1 and Condition-2. Then either of the following holds:

- below $pw_1$ and above $pw_l$: $P_i : \cdots > pw_1 > \cdots > c_j > \cdots > c_k > \cdots > pw_l > \cdots$
- below $pw_l$: $P_i : \cdots > pw_1 > \cdots > pw_l > \cdots > c_j > \cdots > c_k > \cdots$
- above $pw_1$: $P_i : \cdots > c_j > \cdots > c_k > \cdots > pw_1 > \cdots > pw_l > \cdots$

Then for $P_i'$ to be a local dominant profile over $P_i$, the total distance between $pw_1$ and $pw_l$ should increase with respect to the total distance between $pw_1$ and $pw_l$ in profile $P_i$. Therefore, in these cases one must not only switch between $c_j$ and $c_k$ but must also insert at least one candidate between $pw_1$ and $pw_l$ so that the total distance is increased. However, inserting a candidate between $pw_1$ and $pw_l$ results in a profile $P_i'$ that is local dominant but does not have a minimal swap distance.

Formally, the above descriptions can be expressed as:

**Lemma 1** Assume that there is $P_i' \neq P_i$ that satisfies Condition-1 and Condition-2. Then either of the following holds:

- $P_i(c_j, pw_1)$ and $P_i(pw_l, c_k)$;
- $P_i(c_j, pw_1)$ and $c_k \in [pw_1; P_i; pw_l]$;
- $P_i(pw_1, c_k)$ and $c_j \in [pw_1; P_i; pw_l]$.

**Proof: Lemma 1**

Let us assume the contrary, i.e. that, in addition to Condition-1 and Condition-2, either of the following holds:

- $c_j, c_k \in [pw_1; P_i; pw_l]$;
- $c_j, c_k \in (+\infty; P_i; pw_l]$;
- $c_j, c_k \in [pw_1; P_i; -\infty)$

Because Condition-1 holds for $P_i'$, i.e. $P_i'$ locally dominates $P_i$, it follows from Theorem 1 that

$$[[pw_1; P_i; pw_l]] < [[pw_1; P_i'; pw_l]].$$

Hence, there is a candidate $c \in C$ so that either $P_i(c, pw_1)$ and $P_i'(pw_1, c)$, or $P_i(pw_1, c)$ and $P_i'(c, pw_l)$. Due to the symmetry of these two cases, let us assume without loss of generality that the former case holds, i.e. $P_i(c, pw_1)$ and $P_i'(pw_1, c)$. Let us assume that $c$ is the highest candidate for which this condition holds with respect to $P_i'$. Formally:

$$\forall \hat{c} \neq c \quad P_i(\hat{c}, pw_1) \wedge P_i'(pw_1, \hat{c}) \Rightarrow P_i'(c, \hat{c})$$  \hspace{1cm} (1)$$

Let $c'$ be the candidate immediately above $c$ w.r.t $P_i'$, i.e. $P_i'(c', c)$ and the segment $(c' ; P_i' ; c) = \emptyset$. Let us show that the candidate pair $(c', c) \neq (c_k, c_j)$, in each of the contrary sub-cases:

- If $c_j, c_k \in [pw_1; P_i; pw_l]$, then $c \neq c_j$ and $c \neq c_k$ since $c \in (+\infty; P_i; pw_1)$.
- If \( c_j, c_k \in (+\infty; P_i; pw_1) \), then \( c \neq c_j \), otherwise we obtain contradiction to the Eq. 1, because \( P'_i(c_k, c_j) \).
- If \( c_j, c_k \in [p w_1; P_i; -\infty) \), then \( c \neq c_j \) and \( c \neq c_k \) since \( c \in (+\infty; P_i; pw_1) \).

Furthermore, \( c \) and \( c' \) are such that \( P_i(c, c') \). Otherwise we again obtain contradiction to Eq. 1, since by the choice of \( c \) and \( c' \) holds that \( P_i(c, pw_1) \) and \( |(pw_1; P_i; c')| \geq 1 \) (i.e. \( c' \) is either \( pw_1 \) or below it).

Let us then consider \( P'_i'' \) obtained from \( P'_i \) by swapping \( c \) and \( c' \). It is easy to see that \( d_{swap}(P_i, P'_i'') \leq d_{swap}(P_i, P'_i), \) yet \( P'_i'' \models CL(Q_i \cup (c_k, c_j)) \). This contradicts the assumption that Condition-2 holds for \( P'_i \).

As before, let us assume that in \( P_i, c_j \succ c_k \). In \( P'_i \) the order of these two candidates is switched so that \( c_k \succ c_j \). Let us denote the set of all profiles that have a minimal swap distance from \( P_i \) as \( \mu(P_i) \). In order for \( P'_i \in \mu(P_i) \), i.e., in order for \( P'_i \) to have a minimal swap distance from \( P_i, c_k \) and \( c_j \) need to be ordered directly one after the other, with no other candidates separating them. Formally:

**Lemma 2** Let \((c_j, c_k)\) be the query, and let there be \( c \) so that \( P'_i(c_k, c) \) and \( P'_i(c, c_j) \), i.e. \((c_k; P'_i; c_j) \neq \emptyset\), then \( P'_i \notin \mu(P_i) \).

**Proof:** **Lemma 2**

Let us have a closer look at the closed interval \([c_k; P'_i; c_j]\). There is a pair of candidates \((c, c') \in [c_k; P'_i; c_j]\), so that \( P_i(c', c) \) and \((c; P'_i; c') = \emptyset \). Because \((c_k; P'_i; c_j) \neq \emptyset \), it holds that \((c, c') \neq (c_k, c_j) \). Let \( P'_i'' \) be a preference order obtained from \( P'_i \) by swapping \( c \) and \( c' \). It is easy to see that \( P'_i'' \models Q_i \) and \( d_{swap}(P_i, P'_i'') \leq d_{swap}(P_i, P'_i). \) i.e. \( P'_i \notin \mu(P_i) \).

Besides the proximity of \( c_j, c_k \), we can also show that certain sets of elements remain in their original order. In particular, the following lemma shows that two subsets of elements, those with the closest consistent \( P'_i \in \mu(P_i) \) places either above \( c_k \) or below \( c_j \), inherit their relative order from \( P_i \).

**Lemma 3** Let \( P'_i \in \mu(P_i) \), then the following two equations hold

\[
P'_i \downarrow_{(c_k; P'_i; c)} = P_i \downarrow_{(\infty; P'_i; c_k)} \quad (2)
\]

\[
P'_i \downarrow_{(c_j; P'_i; -\infty)} = P_i \downarrow_{(c_j; P'_i; -\infty)} \quad (3)
\]

**Proof:** **Lemma 3**

Let us assume that the Eq. 2 does not hold. Then, there are two candidates, \( c, c' \) so that \((c'; P'_i; c) = \emptyset,\) \( P'_i(c', c) \) and \( P_i(c, c') \). Furthermore, it holds that \( P_i(c, c_k) \). Let us define a new preference order \( P'_i'' \) obtained from \( P'_i \) by swapping \( c \) and \( c' \). Then \( d_{swap}(P_i, P'_i'') < d_{swap}(P_i, P'_i) \) and \( P'_i'' \models CL(Q_i \cup ((c_k, c_j))) \), i.e. \( P'_i \notin \mu(P_i) \), contradicting the lemma’s premise.

We obtain the same kind of contradiction by assuming that Eq. 3 does not hold. Hence the Lemma’s conclusion: both Eqs. 2 and 3 must hold.

\[\square\]
Furthermore, if we consider two candidates that the original preference order $P_i$ places outside the span between $c_j$ and $c_k$, then they demarcate an upper and a lower candidate intervals that maintain both their order and composition in $P_i'$.

Lemma 4 Let $P_i' \in \mu(P_i)$, and let $c'_j, c'_k \in C$ so that $P_i(c'_j, c_j)$ and $P_i(c_k, c'_k)$. Then the following equations hold

\[ (\infty; P_i; c'_j] = (\infty; P_i'; c'_j]\quad (4) \]
\[ [c'_k; P_i; -\infty) = [c'_k; P_i'; -\infty). \quad (5) \]

Proof: Lemma 4

Let us assume that the Eq. 4 does not hold, in spite of the lemma’s premise being true. That is, there exists a candidate $c'_j$ so that $P_i(c'_j, c_j)$ and $(\infty; P_i; c'_j) \neq (\infty; P_i'; c'_j)$.

Three possible sub-cases exist in this context:

1. $\exists c \in C$ s.t. $P_i(c'_j, c) \land P_i'(c, c'_j)$
2. $\exists c \in C$ s.t. $P_i(c, c'_j) \land P_i'(c'_j, c)$
3. Neither of the above holds.

If $\exists c \in C$ s.t. $P_i(c'_j, c) \land P_i'(c, c'_j)$, then it is easy to see that a pair of candidates $(c, c')$ exists so that $(c; P_i'; c') = \emptyset, P_i'(c, c')$, and either $P_i(c', c'_j)$ or $c = c'_j$. Let us then obtain a preference order $P_i''$ from $P_i'$ by swapping $c$ and $c'$. It holds that $d_{swap}(P_i, P_i'') < d_{swap}(P_i, P_i')$ and, in addition, $P_i' \models CL(Q_i \cup \{(c_k, c_j)\})$. Hence, we contradict the lemma’s premise that $P_i' \in \mu(P_i)$.

The sub-case where it holds that $\exists c \in C$ s.t. $P_i(c, c'_j) \land P_i'(c'_j, c)$ is similar to the above.

Let us now investigate the third sub-case. It occurs if there is no element that has switched from being above (below) $c'_j$ in $P_i$ to being below (above) $c'_j$ in $P_i'$. In particular the following two sets are equal (as sets):

\[ B = \{c \in C | P_i(c, c'_j)\} = \{c \in C | P_i'(c, c'_j)\} \]

If $(\infty; P_i; c'_j) = \emptyset$, then the assumption of Eq. 4 not being true can not hold. If, however, $(\infty; P_i; c'_j) \neq \emptyset$, then $P_i \downarrow_B \neq P_i' \downarrow_B$. That is, there is a pair of candidates $c, c' \in B$ so that $(c'; P_i'; c) = \emptyset, P_i(c, c') and P_i'(c', c)$. Defining an alternative order $P_i''$ obtained from $P_i'$ by swapping $c$ and $c'$, we once again obtain a contradiction to the premise $P_i' \in \mu(P_i)$.

We conclude that Eq. 4 must hold. Symmetric proof establishes Eq. 5. \qed

Now, as Lemma 2 showed, $c_j$ and $c_k$ are placed next to each other, when changing the preference order from $P_i$ to $P_i'$. However, to achieve this some other elements may need to be separated. The following lemma shows that this does not occur without need. That is, if two elements were placed next to each other in $P_i$, but not in $P_i'$, then they were separated to accommodate the placement of $c_k$ and $c_j$ between them.
Lemma 5 Let \( P'_i \in \mu(P_i) \), and let \( a, b \in C \) be two candidates so that \( P_i \downarrow_{[a,b]} = P'_i \downarrow_{[a,b]} \), \( (a ; P_i ; b) = \emptyset \), and \( (a ; P'_i ; b) \neq \emptyset \). Then \( a \in (\infty ; P'_i ; c_k] \) and \( b \in [c_j ; P'_i ; -\infty) \).

Proof: Lemma 5

Let us assume that the Lemma’s conclusion does not hold. In particular this would mean that \( c_j, c_k \notin (a ; P'_i ; b) \). On the other hand, \((a ; P'_i ; b) \neq \emptyset\), so there is a candidate \( c \in (a ; P'_i ; b) \). Because \( a \) and \( b \) are next to each other in the preference ordering \( P_i \), i.e., \((a ; P_i ; b) = \emptyset\), it holds that either \( P_i(c, a) \) or \( P_i(b, c) \). Which, in turn, implies that \( P_i \downarrow_{[a;P'_i:b]} \neq P'_i \downarrow_{[a;P'_i:b]} \). Therefore, there is \( c' \in [a;P'_i:b] \) so that \( c' \neq c \) and \( (c'; P'_i ; c) = \emptyset \), i.e., \( c \) and \( c' \) are next to each other in the ordering \( P'_i \). Furthermore, it must hold that these two elements were switched between \( P_i \) and \( P'_i \), that is \( P_i(c, c') \) and \( P'_i(c', c) \). Let us define a new preference order \( P''_i \) by swapping \( c \) and \( c' \) in \( P'_i \). It would hold that \( d_{\text{swap}}(P''_i, P_i) \leq d_{\text{swap}}(P'_i, P_i) \), while \( P''_i \models CL(Q_i \cup \{c_k, c_j\}) \), thus contradicting the premise that \( P'_i \in \mu(P_i) \).

One final observation that we will need to prove Theorem 2 has to do with the general change in the relative position of elements committed by \( Q_i \) to a particular order w.r.t \( c_k \) or \( c_j \). Lemma 6 shows that among all elements above (correspondingly, below) \( c_k \) (correspondingly, \( c_j \)) only those committed to be ordered after \( c_j \) (correspondingly, before \( c_k \)) will change their relative position when moving from preference order \( P_i \) to \( P'_i \). All other elements will maintain their order.

Lemma 6 Let \( P'_i \in \mu(P_i) \). Let \( c \) be some candidate so that \( P'_i(c, c_k) \) and \( (c, c_k) \notin Q_i \). Then the following equality holds:

\[
(\infty ; P'_i ; c] = (\infty ; P_i ; c] \setminus [c_j ; Q_i ; -\infty). 
\]

Symmetrically, let \( c \) be some candidate so that \( P'_i(c_j, c) \) and \( (c_j, c) \notin Q_i \). Then:

\[
[c ; P'_i ; -\infty) = [c ; P_i ; -\infty) \setminus (\infty ; Q_i ; c_k].
\]

Proof: Lemma 6

First, notice that if \( c \notin (c_j ; P_i ; c_k) \), then the lemma is a direct conclusion if Lemma 4. In more detail, if \( P_i(c, c_j) \), then only the premise of the first equation holds. Furthermore, since \( P_i \models Q_i, (\infty ; P_i ; c] \cap [c_j ; Q_i ; -\infty) = \emptyset \). Thus, the lemma’s conclusion requires that \((\infty ; P'_i ; c] = (\infty ; P_i ; c] \setminus [c_j ; Q_i ; -\infty) \), which holds due to Lemma 4. Symmetrically, if \( P_i(c_k, c) \), then the premise of the second equation is true, and the conclusion similarly holds according to Lemma 4. Therefore, in the remainder of this proof, we will assume that \( c \in (c_j ; P_i ; c_k) \).

Now, let us assume that there is in fact a candidate \( c \in C \) that satisfies the first premise of the lemma, but violates its conclusion. Denote by \( X \) the following set:

\[
X = \{c \in C | P'_i(c, c_k), (c, c_k) \notin Q_i, (+\infty ; P'_i ; c] \neq (+\infty ; P_i ; c] \setminus [c_j ; Q_i ; -\infty) \}
\]
Let $x$ denote the least preferred candidate of $X$ w.r.t the preference order $P_i'$, i.e., for any $x \neq c \in X$ holds that $P_i'(c, x)$.

From Lemma 3 we know that $P_i' \downarrow_{(+\infty : P_i' : x]} = P_i \downarrow_{(+\infty : P_i' : x]}$. Which also means that $(+\infty : P_i' : x] \subseteq (+\infty : P_i : x]$. Furthermore, since $P_i' \models CL\left(Q_i \cup \{(c, j)\}\right)$, for all $c \in [c_j ; Q_i] ; -\infty)$ holds that $P_i'(c_j, c)$ or $c = c_j$. Since $P_i'(x, c)$, by the transitivity of $P_i'$ it is also true that $P_i'(x, c)$ for all $c \in [c_j ; Q_i] ; -\infty)$. Hence, we obtain $(+\infty : P_i' : x] \cap [c_j ; Q_i] ; -\infty) = \emptyset$. In addition, since we have assumed that the lemma’s conclusion does not hold, we obtain the following strong subsumption:

$$(+\infty : P_i' : x] \subseteq (+\infty : P_i : x] \setminus [c_j ; Q_i] ; -\infty) \quad (6)$$

This means, in particular, that there is a candidate $y \in C$ so that $P_i(y, x)$, $P_i'(x, y)$, and $(c_j, y) \notin Q_i$. Taking into account Lemma 3, $P_i$ and $P_i'$ have the following overall structures:

- $P_i : \ldots > c_j > \ldots > y > \ldots > x > \ldots > c_k > \ldots$
- $P_i' : \ldots > x > \ldots > c_k > c_j > \ldots > y > \ldots$

Let us denote $A$ the number of candidates between $x$ and $c_k$ with respect to $P_i'$, i.e. $A = |(x ; P_i' ; c_k]|$, and, correspondingly $B = |[c_j ; P_i' ; y]|$.

Consider now alternative preference orderings $R$ and $R'$, obtained from $P_i'$ by either moving $x$ below $y$ or, alternatively, moving $y$ just below $x$. That is, $P$ and $P'$ have the following structures:

- $R : \ldots > c_k > c_j > \ldots > y > x > \ldots$
- $R' : \ldots > y > x > \ldots > c_k > c_j > \ldots$

Furthermore, $P_i' \downarrow_{C \setminus \{x, y\}} = R \downarrow_{C \setminus \{x, y\}} = R' \downarrow_{C \setminus \{x, y\}}$. Let us now denote by $D = d_{swap}(P_i, P_i')$, and consider $d_{swap}(P_i, R)$ and $d_{swap}(P_i, R')$.

It holds that $P_i \downarrow_{[c_j ; P_i' ; c_k]} = R \downarrow_{[c_j ; P_i' ; c_k]}$, while $P_i'(x, c)$ for any $c \in [c_j ; P_i' ; y)$, hence $P$ is closer to $P_i$ by $B$ element swaps. At the same time $P_i \downarrow_{(x ; P_i' ; c_k]} = P_i' \downarrow_{(x ; P_i' ; c_k]}$, yet $R(c, x)$ for all $c \in (x ; P_i' ; c_k]$. Similarly the order of $x$ and $y$ is also “restored”, i.e. it holds that $R(y, x)$, $P_i(y, x)$, and $P_i'(x, y)$. As a result we have $d_{swap}(P_i, P) = D - B + A - 1$. Similarly $d_{swap}(P_i, R') = D - A + B - 1$. Since either $-A + B - 1 < 0$ or $-B + A - 1 < 0$, we have that either $R$ or $R'$ is closer to $P_i$ than $P_i'$. Because no pair of candidates $x, y, c_j, c_k$ is restricted by $Q_i$, we also have that both $R \models CL\left(Q_i \cup \{(c_k, c_j)\}\right)$ and $R' \models CL\left(Q_i \cup \{(c_k, c_j)\}\right)$, therefore violating the assumption of $P_i' \in \mu(P_i)$.

These lemmas are the setting for the proof of Theorem 2—the correctness of our algorithm.

**Proof: Theorem 2**

First, let us do away with the question of computational complexity of the Algorithm 1, as the simpler portion of the algorithm’s analysis.

Prior to the main loop of the algorithm, a preliminary feasibility of manipulation is run in Line 2, based on Lemma 1. The Lemma includes a finite number of membership
checks, each of which runs in time linear in the number of candidates. It does, however, presume that the set of possible winners can be obtained efficiently. Since we use the definition of the PW set from (Konczak and Lang 2005, Lu and Boutilier 2013, Naamani-Dery et al. 2014), it can be found efficiently in the number of voters and candidates. Therefore, the overall preliminary check of Line 2 is polynomial in both voter and candidate set sizes.

Once the pre-check is complete the main loop of the algorithm is repeated for every candidate in the worst case. If we show that each loop is polynomial in the size of the problem as well, the overall algorithm’s complexity will be obtained.

Lines 7-10 operate on ordered subsets of the candidate set, and each such operation takes at most \(2|C|\) basic steps to complete. The arguments of these operations are also polynomial-time constructed. One, \(P_i\), is given explicitly as input, and taking a sub-interval of it is linear in \(|C|\). The other is obtained, e.g., by a spanning tree traversal of \(Q_i\). However, \(Q_i\) set is at most quadratic in the number of candidates. Hence, the calculates of sets \(X_{good}, X_{bad}, Y_{good}\) and \(Y_{bad}\) take time, polynomial in the candidate set size. Line 11 is a sanity check, since the aforementioned subsets of \(C\) were obtained from an ordered sequence of candidates, hence linear in \(|C|\). Similarly, construction of the new preference order \(P_i'\) takes linear time. Calculating the distance \(d_{swap}(P_i, P_i')\) (line 13) takes at most \(|C|^2\) steps, as it is equivalent to running the bubble-sort algorithm. The last non-trivial step, line 17, depends on how efficiently we can confirm the Local Dominance property of \(P_i'\) with respect to \(P_i\). This confirmation, however, can be performed by using the three conditions of Theorem 1. As we have already accounted for the calculation of the PW set, each condition of the Theorem takes time linear in \(|C|\). We conclude that the main loop runs at most in \(O(|C|^3)\). Hence, the overall run time of the algorithm is polynomial in the sizes of \(V\) and \(C\) sets.

Now, given that we know that the Algorithm 1 operates in polynomial time, let us prove that it operates correctly.

Let \(P_i' \in \mu(P_i)\), and let us analyze its structure.

Let us assume for the moment, that there are two elements \(a_1, b_2 \in C\) that satisfy the conditions of Lemma 6, i.e., \(P_i'(a_1, c_k), P_i'(c_j, b_2)\) and \((a_1, c_k) \notin Q_i, (c_j, b_2) \notin Q_i\). Furthermore, let us assume that \(a_1\) is the minimum \((b_2\) is the maximum\) such element with respect to \(P_i'\). Combining this assumption with Lemma 2, \(P_i'\) can be broken down into the following structure \(P_i' = (F_j, G_k, c_j, G_j, F_k)\), where the intervals \(F_j, F_k, G_j, G_k\) are characterized as follows:

- \(F_j = (\infty ; P_i' ; a_1]\)
- \(F_k = [b_2 ; P_i' ; -\infty)\)
- \(G_k \subset (\infty ; Q_i ; c_k)\)
- \(G_j \subset (c_j ; Q_i ; -\infty)\)

Let us now denote \(a_2, b_1 \in C\), so that \(P_i(a_1, a_2), P_i(b_1, b_2)\) and \((a_1 ; P_i ; a_2) = \emptyset, (b_1 ; P_i ; b_2) = \emptyset\). Such \(a_2\) and \(b_1\) exist, since \(P_i(a_1, c_k)\) and \(P_i(c_j, b_2)\) due to Lemma 3. Furthermore, it also entails that \(P_i(a_2, c_k)\) and \(P_i(b_1, c_j)\).

The following three subcases are possible.

- Either \((a_1 ; P_i' ; a_2) \neq \emptyset\) or \((b_1 ; P_i' ; b_2) \neq \emptyset\).
Sub-Case A $a_2 = b_2$

Sub-Case B $P'_i(a_2, b_2)$

Sub-Case C Both $(a_1 ; P_i' ; a_2) = \emptyset$ and $(b_1 ; P_i' ; b_2) = \emptyset$.

W.l.g., let us first assume that $(a_1 ; P_i' ; a_2) \neq \emptyset$. Then, according to Lemma 5, $P'_i(c_j, a_2)$. Combining this with Lemma 3, we obtain that $P_i(c_j, a_2)$. Thus, we also conclude that $a_2 \in [c_j ; P_i ; c_k]$.

Sub-Case A If in addition, $a_2 = b_2$, then the following holds according to Lemma 6 and setting $z = a_2$ in Algorithm 1:

$$F_j = (\infty ; P_i' ; a_1] = (\infty ; P_i ; a_1]\{c_j ; Q_i ; -\infty) = X_{good}$$

$$F_k = [a_2 ; P_i' ; -\infty) = [a_2 ; P_i ; -\infty]\{\infty ; Q_i ; c_k] = Y_{good}$$

By its definition $G_j \subset (c_j ; Q_i ; -\infty)$. Furthermore, $G_j \subset [c_j ; P_i' ; a_2]$. Thus, by Lemma 3, $G_j \subset [c_j ; P_i ; a_2]$. Since $a_2 \notin G_j$, we conclude that $G_j \subset (\infty ; P_i ; a_2)$. Hence, by setting $z = a_2$ in Algorithm 1, we have:

$$G_j = (\infty ; P_i ; a_2) \cap (c_j ; Q_i ; -\infty) = X_{bad}$$

Similarly, $G_k \subset (\infty ; Q_i ; c_k)$ by definition. Furthermore, $G_k \subset [a_1 ; P_i' ; c_k]$. Thus, by Lemma 3, $G_k \subset [a_1 ; P_i ; c_k]$. Since $a_1 \notin G_k$ and $(a_1 ; P_i ; a_2) = \emptyset$, we can conclude that $G_k \subset [a_2 ; P_i ; -\infty)$. Letting $z = a_2$ in Algorithm 1, we have:

$$G_k = \{z ; P_i ; -\infty) \cap (\infty ; Q_i ; c_k] = Y_{bad}$$

Thus we have $P'_i = (X_{good}, Y_{bad}, c_k, c_j, X_{bad}, Y_{good})$ during a run of the Algorithm 1, where $z = a_2$. That is, this sub-case of $P'_i \in \mu(P_i)$ will be recovered by the Algorithm 1.

Sub-Case B Let us now consider the situation where, rather than $a_2 = b_2$, we have $P'_i(a_2, b_2)$. Similar to the case where $a_2 = b_2$, we will have that $a_2 \in [c_j ; P_i ; c_k]$ and that $X_{good} = F_j, Y_{bad} = G_k$ when Algorithm 1 constructs a hypothetical manipulative preference profile with $z = a_2$. It remains to show that $X_{bad}$ and $Y_{good}$ combine into the segment $(c_j ; P_i' ; -\infty) = (G_j, F_k)$, and then conclude that, even if $P'_i(a_2, b_2)$, the preference profile $P'_i$ will be discovered by Algorithm 1 for $z = a_2$. To this end, let us have a closer look at segments $[a_2 ; P_i' ; b_2]$ and $[a_2 ; P_i ; b_2]$.

Let there be $x \in [a_2 ; P_i ; b_2]\{a_2 ; P_i' ; b_2]$. If $P'_i(c_j, x)$, then, according to Lemma 3, we obtain a contradiction that $x \in [a_2 ; P_i' ; b_2]$. Hence $P'_i(x, c_k)$. On the other hand, it must be that $P_i(a_1, x)$, since $P_i(a_1, a_2)$ and $x \in [a_2 ; P_i ; b_2]$. Hence, $x \in G_k \subset (\infty ; Q_i ; c_k)$. Notice also that, due to Lemma 3, we have $[a_2 ; P_i' ; b_2]\{a_2 ; P_i ; b_2] = \emptyset$. As a result, $[a_2 ; P_i' ; -\infty) = [a_2 ; P_i ; -\infty] \{\infty ; Q_i ; c_k] = Y_{bad}$, where $Y_{bad}$ is computed for $z = a_2$.
Finally, notice that it is impossible to have $P'_i(b_2, a_2)$, and that the reasoning is symmetric for the case where $(b_1; P'_i; b_2) \neq \emptyset$. Hence, if either $(a_1 ; P'_i ; a_2) \neq \emptyset$ or $(b_1 ; P'_i ; b_2) \neq \emptyset$, then $P'_i$ is discovered by Algorithm 1.

**Sub-Case C** Let us now have a closer look at a $P'_i$ where $(a_1 ; P'_i ; a_2) = (b_1 ; P'_i ; b_2) = \emptyset$.

Denote $d_1, d_2$ a pair of candidates that satisfy conditions\(^2\) of Lemma 6, and, in addition, that $P_i(a_1, d_1)$ and $(a_1; P_i; d_1)$ is minimal.

Then the reasoning of Sub-Case B above can be repeated, replacing $b_2$ by $d_2$ in its arguments. We conclude that $P'_i$ with $(a_1 ; P'_i ; a_2) = \emptyset$ and $(b_1 ; P'_i ; b_2) = \emptyset$ will also be discovered by Algorithm 1. In other worlds Algorithm 1 will discover all elements of $\mu(P_i)$. As the algorithm selects a locally dominant order $P'_i$ among all those that it finds, the final outcome will satisfy both condition Condition-1 and Condition-2. \(\square\)

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\(^{2}\) Notice that such a pair always exists.
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