Partition Function on Not-flat Brane

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Abstract

We show that a partition function on the not-flat D1-brane can be written in the same form as that on the flat one in $\alpha'$-order. In this case the information of the curvature of the brane configuration is included in tachyon beta function.
1 Introduction and Summary

On a flat D-brane in the bosonic string theory, Born-Infeld action can be obtained as the partition function of the two-dimensional nonlinear sigma model action on an open string world sheet. It is calculated by the path-integral. Then, the factor like Born-Infeld action appears in the scattering amplitudes calculated by the path-integral formalism [1][2]. In these papers, the scattering amplitudes of closed string states with a D0, 1-brane are studied. It is shown that the Born-Infeld-like factor on the arbitrary configuration of a D-brane can be written as that in the case of a flat brane by renormalizing the embedding function \( f^\mu \).

Here we introduce the open string tachyon and calculate the partition function on the arbitrary configuration of a D-brane. It is shown that the partition function can be expressed as the same factor as that on a flat brane up to an \( \alpha'^2 \) order term. In such a case, however, the tachyon beta-function is shifted from that in the flat case.

In this letter we introduce a open string tachyon field and try to cancel a contribution from the curvature of the brane configuration. Following the method of [2], we setup tools needed for calculation in Sect. 2. In Sect. 3 the path-integral is performed and then we obtain the partition function and renormalize a tachyon field.

2 Setup - Geodesic Normal Coordinate Expansion

We deal with a D1-brane in flat 26-dimensional space-time of the bosonic string theory. The bosonic open string world-sheet action with a gauge field \( A_\mu \) and an open string tachyon field \( T \) on the boundary is given by [3]

\[
S = \frac{1}{4\pi \alpha'} \int_\Sigma d^2 \bar{\sigma} \partial_\bar{a} X^\mu \partial_\bar{a} X'^\nu \eta_{\mu\nu} + i \int_{\partial \Sigma} d\theta A_\mu(X) \partial_\theta X^\mu + \frac{1}{\epsilon} \int_{\partial \Sigma} d\theta T(X). \tag{2.1}
\]

In (2.1) we take the world sheet to be a unit disk \( \Sigma \) and represent its boundary by \( \partial \Sigma \), \( \theta \) is the one-dimensional coordinate on \( \partial \Sigma \), the subscript \( \bar{a} \) of \( \partial_\bar{a} X^\nu \) means the coordinate on the world sheet \( \bar{\sigma} \), \( X^\mu \) are the coordinates of the target 26-dimensional space, the metric of the target space is taken as \( \eta_{\mu\nu} = \text{diag}(-1, 1, \cdots 1) \), and \( \epsilon \) stands for the short distance cutoff parameter.
Following [1] and [2], the partition function for the brane configuration \( f^\mu(X(\theta)) \) is evaluated as
\[
Z = c \int DX^\mu Dt(\theta) D\sigma(\theta) \delta(X^\mu(\theta) - f^\mu(t(\theta), \sigma(\theta))) \exp(-S),
\]
where \( t \) and \( \sigma \) are the coordinates on the D1-brane and \( c \) is an overall factor independent of \( A \) and \( T \).

Using the method in [4], we reduce the two-dimensional action to the one-dimensional action on boundary of the world-sheet \( \Sigma \):
\[
S = \frac{1}{4\pi\alpha'} X^\mu \cdot N^{-1}_{\mu\nu} X^\nu + i \int_{\partial\Sigma} d\theta A_\mu \cdot \partial_\theta X^\mu + \int_{\partial\Sigma} d\theta \frac{T}{\epsilon}.
\]
(2.3)
The Neumann function \( N \) and its inverse are given by
\[
N^{\mu\nu} = \eta^{\mu\nu} \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} e^{-k\epsilon} \cos k(\theta - \theta')
\]
(2.4)
\[
N^{-1}_{\mu\nu} = \eta_{\mu\nu} \frac{1}{\pi} \sum_{k=1}^{\infty} e^{-k\epsilon} k \cos k(\theta - \theta').
\]
(2.5)
The dot in (2.3) means the following operation;
\[
A \cdot B(\theta, \theta') = \int_0^{2\pi} d\phi A(\theta, \phi) B(\phi, \theta').
\]
(2.6)

In order to calculate the partition function (2.2), we expand \( f \) in terms of the geodesic normal coordinates \( \zeta^a \) [1] [2] [3]:
\[
f^\mu(t(\theta), \sigma(\theta)) = f^\mu(t, \sigma) + \partial_a f^\mu(t, \sigma) \zeta^a(\theta) + \frac{1}{2} K^\mu_{ab}(t, \sigma) \zeta^a(\theta) \zeta^b(\theta) + \frac{1}{3!} K^\mu_{abc}(t, \sigma) \zeta^a(\theta) \zeta^b(\theta) \zeta^c(\theta) + \cdots,
\]
(2.7)
where
\[
h_{ab} = \partial_a f^\mu \partial_b f^\nu \eta_{\mu\nu},
\]
(2.8)
\[
h^{ab} = (h^{-1}),
\]
(2.9)
\[
K^\mu_{ab} = P^{\mu\nu} \partial_a \partial_b f^\nu = \partial_a \partial_b f^\mu - \partial_c f^\mu \Gamma^c_{ab}
\]
(2.10)
\[
\eta^{\mu\nu} = h^{\mu\nu} + P^{\mu\nu},
\]
(2.11)
\[
h^{\mu\nu} = \partial_a f^\mu h^{ab} \partial_b f^\nu.
\]
(2.12)

* It is imposed that \( \partial F \) and \( \partial \partial F \) can be ignored.
\( \nabla \) denotes the covariant derivative for the subscripts a and b, \( h_{ab} \) the induced metric on the D1-brane, \( \Gamma^c_{ab} \) the Christoffel symbol for the induced metric \( h_{ab} \), \( K^\mu_{ab} \) the extrinsic curvature, and \( P^{\mu\nu} \) and \( h^{\mu\nu} \) stand for projectors to orthogonal and tangent directions to the D1-brane, respectively.

Following the background field method, we separate \( X(\theta) \) into the zero-mode \( x \) and the fluctuation \( \xi(\theta) \),

\[
X^\mu(\theta) = x^\mu + \xi^\mu(\theta)
\]

with the condition

\[
\oint_{\partial \Sigma} d\theta \xi^\mu(\theta) = 0.
\]

(2.13)

Then the \( \delta \)-function splits into factors corresponding to the different modes;

\[
\delta(X^\mu(\theta) - f^\mu(t, \sigma), \sigma(\theta))) = \delta(x^\mu + \xi^\mu(\theta) - f^\mu(t, \sigma) - \partial_a f^\mu(t, \sigma) \zeta^a(\theta) - \cdots)
\]

\[
= \delta(x^\mu - f^\mu(t, \sigma)) \delta(\xi^\mu(\theta) - \partial_a f^\mu(t, \sigma) \zeta^a(\theta) - \cdots)
\]

\[
= \int D\nu_\mu \exp \left[ i \int d\theta \nu_\mu(x^\mu - f^\mu(t, \sigma)) \right] \cdot
\]

\[
\int D\nu_\mu \exp \left[ i \int d\theta \nu_\mu(\xi^\mu(\theta) - \partial_a f^\mu(t, \sigma) \zeta^a(\theta) - \cdots) \right].
\]

Note that the fluctuation of \( X^\mu \) comes from the change of the place where the open string is connected to the D1-brane, but not from the change of the form of the function \( f \), that is, the change of the form of the D1-brane.

On the other hand, we transform the fields \( \xi^\mu \) to \( \rho^a \) for which the determinant of the metric is \(-1\). \( ^\dagger \)

\[
\xi^\mu(\sigma^\bar{\alpha}) = \hat{e}^\mu_A \rho^A(\sigma^\bar{\alpha}),
\]

\[
x^A = \hat{e}^A_\mu x^\mu,
\]

\[
\hat{e}_{a}^\mu = N^b_a \partial_b f^\mu(t, \sigma), \quad a, b = 0, 1,
\]

\[
\hat{e}_{\alpha}^\mu = N^\beta_\alpha \hat{e}^\mu_\beta,
\]

\[
\hat{e}_{a}^\mu \hat{e}_{\mu a} = 0,
\]

\[
\hat{e}_{a}^\mu \hat{e}_{\mu \beta} = \delta_{a\beta}.
\]

\( ^\dagger \) We can take \( \rho^a \) to be the orthonormal coordinates \( ^\ddagger \), but we do not so here for simplicity.
where $A$, $a$ and $\alpha$ denote all space-time directions, tangent and orthogonal directions to the D1-brane, respectively. Normalization factors are

$$
N_B^A = \text{diag}(N_0, N_1, \cdots, 1),
$$

$$
N_0 = \frac{1}{\sqrt{-h_{00}}}, \quad N_1 = \sqrt{\frac{h_{00}}{h}}, \quad h = \det(h_{ab}).
$$

In the same time new tensors are defined as

$$
\bar{h}_{AB} = 2\pi\alpha' \hat{e}_A^\mu \hat{e}_B^\nu = \text{diag}(\bar{h}_{ab}, \delta_{\alpha\beta}),
$$

$$
\bar{h}_{ab} = \hat{e}_a^\mu \hat{e}_b^\nu = \left[
\begin{array}{cc}
N_0^2 h_{00} & N_0 N_1 h_{01} \\
N_0 N_1 h_{01} & N_1^2 h_{11}
\end{array}
\right],
$$

$$
\det(\bar{h}_{ab}) = -1,
$$

$$
\bar{F}_{AB} = 2\pi\alpha' \hat{e}_A^\mu \hat{e}_B^\nu F_{\mu\nu}, \quad \bar{F}_{ab} \neq 0,
$$

$$
\bar{u}_{AB} = 2\pi\alpha' \hat{e}_A^\mu \hat{e}_B^\nu \partial_{\mu} \partial_{\nu} T, \quad \bar{u}_{ab} \neq 0,
$$

$$
\hat{\nu}_A = \hat{e}^B_A \nu_B, \quad \hat{\nu}_{0A} = \hat{e}^B_A \nu_{0B}.
$$

In the following discussion the metrics in (2.4) and (2.5) change from $\eta$ to $\bar{h}$; The Neumann functions with the new metric are denoted by $N^{AB}$ and $N^{-1}_{AB}$. It is supposed that $A_\mu$ is nonzero only for the component tangent to the D1-brane, and $A$ and $T$ depend only on the coordinates tangent to the D1-brane. In this case $\bar{F}_{\alpha A}$ and $\bar{u}_{\alpha A}$ vanish and then $\bar{u}$ can be written in the covariant form

$$
\bar{u}_{ab} = 2\pi\alpha' N_a^c N_b^d J^\mu \partial_{\mu} J^\nu \partial_{\nu} T = 2\pi\alpha' N_a^c N_b^d \nabla_c \nabla_d T.
$$

By use of them, the action is rewritten in terms of $\rho$:

$$
S = \frac{1}{4\pi\alpha'} \left[ \rho^A \cdot N^{-1}_{AB} \cdot \rho^B + i \int_{\partial \Sigma} d\theta \bar{F}_{AB}(x^A) \rho^A \partial_{\theta} \rho^B + \frac{1}{\epsilon} \int_{\partial \Sigma} \bar{u}_{AB}(x^A) \rho^A \rho^B + \cdots \right],
$$

$$
+ \frac{1}{\epsilon} \int_{\partial \Sigma} d\theta a(x^A),
$$

where $\cdots$ means higher derivative terms of $A_\mu$ and $T$, and we ignore them here. In this action, regarding the terms from the tachyon field, $a$ and $\bar{u}$, as perturbation, the conditions imposed on $\rho^a$ and $\rho^\alpha$ are

$$
N^{-1}_{ab} \cdot \rho^b(\theta) + i \bar{F}_{ab} \partial_{\theta} \rho^b(\theta) = 0
$$

$$
N^{-1}_{\alpha\beta} \cdot \rho^\beta(\theta) = 0.
$$

\footnote{If $\bar{u}$ is included in the Green function, the Partition function looks like that in \cite{8}.}
On the other hand, the conditions from the $\delta$-function are given by

\[
\rho^a(\theta) = (N^{-1})^a_b \left[ \zeta^b(\theta) - \frac{1}{3!} K^\mu_{cd} K^\nu_{\mu lm} h^{bc} \zeta^d(\theta) \zeta^l(\theta) \zeta^m(\theta) + \cdots \right], \tag{2.23}
\]

\[
\rho^\alpha(\theta) = \hat{e}_{\alpha\mu} \left[ \frac{1}{2} \partial_\alpha \partial_\mu f^\mu \zeta^a(\theta) \zeta^b(\theta) + \frac{1}{3!} \left\{ \partial_\alpha \partial_\beta \partial_\gamma f^\mu - 3 \Gamma^d_{ab} \partial_\alpha \partial_\beta \partial_\gamma f^\mu \right\} \zeta^a(\theta) \zeta^b(\theta) \zeta^c(\theta) + \cdots \right]. \tag{2.24}
\]

From (2.22) we can put $\rho^a$ as zero.

The Green function

Let think about the Green function for (2.22). The Green function satisfies the following condition;

\[
\mathcal{N}^{-1}_{AB} \cdot \tilde{M}^{BC} + i \tilde{F}_{AB} \partial_B \tilde{M}^{BC} = \delta_A^C \delta(\theta - \theta'). \tag{2.25}
\]

A solution of this equation is [5]

\[
\tilde{M}^{AB}(\theta, \theta') = \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{1}{n} e^{-\epsilon n} \left[ G^{-1AB} \cos n(\theta - \theta') + i \Theta^{AB} \sin n(\theta - \theta') \right], \tag{2.26}
\]

where $G^{-1}$ and $\Theta$ are respectively symmetric and antisymmetric tensors defined as

\[
\left( \frac{1}{h - F} \right)^{AB} = G^{-1AB} + \Theta^{AB}. \tag{2.27}
\]

The Green function (2.26) has the short distance divergence such that

\[
\tilde{M}^{AB}(\theta, \theta) = \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{1}{n} e^{-\epsilon n} G^{-1AB} = -\frac{1}{\pi} G^{-1AB} \ln \epsilon + \mathcal{O}(\ln(1 - \epsilon)). \tag{2.28}
\]

The first term in (2.28) diverges as $\epsilon$ closes to zero.

3 Path Integral

Let carry out the calculations of the path integrals. At first, we integrate with respect to $\rho^a$. Since the integration of $\rho^a$ give only a factor independent of $A$, $T$ and those derivatives, we do not deal with it.
The part of the exponent of the partition function including $\rho^a$ is

\[
I_{\rho^a} = \int D\rho^a \exp \left\{ -\frac{1}{4\pi\alpha'}[\rho^a \cdot \mathcal{N}_{ab}^{-1} \cdot \rho^b + i\bar{F}_{ab}\rho^a \cdot \partial_b \rho^b + 2\pi\alpha' \partial_\rho(F_{\mu\nu})\epsilon_\mu^a \epsilon_\nu^b \rho^a \cdot \rho^b \partial_b x^c \\
+ \frac{1}{\epsilon} \bar{u}_{ab}\rho^a \cdot \rho^b + \mathcal{O}(\rho^3)] + i\hat{\nu}_a \cdot \rho^a \right\}. \tag{2.29}
\]

We treat $\bar{u}$ and $\partial F$ as perturbation. Integrating with respect to $\rho^a$, we have

\[
I_{\rho^a} \propto [\text{Det}'(\mathcal{N}_{ab}^{-1} + i\bar{F}_{ab}\partial_b)]^{-1}[1 - i\pi\alpha' \int d\theta\partial_\nu(F_{\rho\mu})\epsilon_\rho^a \epsilon_\mu^b \hat{M}^{ab}(\theta, \theta)\partial_b x^c \\
- \frac{1}{2\epsilon} \int d\theta\text{tr}[\bar{u}\hat{M}(\theta, \theta)] \exp[-\pi\alpha' \hat{\nu}_a \cdot \hat{M}^{ab} \cdot \hat{\nu}_b]. \tag{2.30}
\]

Det is defined as the determinant with respect to modes of the fields and Det' stands for also the determinant excluding the contributions from zero-modes. det and tr are defined as that with subscripts $a$ and $b$.

Let integrate with respect to $\hat{\nu}_a$. The part of $Z$ including $\hat{\nu}_a$ is

\[
I_{\hat{\nu}_a} = \int D\hat{\nu}_a \exp[-\pi\alpha' \hat{\nu}_a \cdot \hat{M}^{ab} \cdot \hat{\nu}_b - i \int d\theta\hat{\nu}_a J^a], \tag{2.31}
\]

where

\[
J^a = (N^{-1})^a_b \left( \zeta^b - \frac{1}{3!} K^\mu_{ab} K_{\mu c d} h^{e a} \zeta^b \zeta^c \zeta^d + \mathcal{O}(\zeta^4) \right). \tag{2.32}
\]

Then we obtain

\[
I_{\hat{\nu}_a} \propto [\text{Det}'(\hat{M}^{ab})]^{-1} \exp \left\{ -\frac{1}{4\pi\alpha'} J^a \cdot \hat{M}^{-1} \cdot J^b \right\}. \tag{2.33}
\]

The factor $[\text{Det}'(\hat{M}^{ab})]^{-1}$ is canceled by the factor $[\text{Det}'(\mathcal{N}_{ab}^{-1} + i\bar{F}_{ab}\partial_b)]^{-1}$.

Finally we integrate over $\zeta^a$. In order to identify the derivative expansion with the $\alpha'$ expansion, we rescale $\zeta$ as follows:

\[
\zeta^a = N_b^a \sqrt{\alpha'} \bar{\zeta}^b, \quad D\zeta^a = \frac{1}{\alpha'} \sqrt{-h} D\bar{\zeta}^a. \tag{2.34}
\]

The exponent including $\bar{\zeta}^a$ is obtained as

\[
- \frac{1}{4\pi} \int d\theta d\theta' \left[ \bar{\zeta}^a(\theta) D_{ab}(\theta, \theta') \bar{\zeta}^b(\theta') \\
- \frac{\alpha'}{3} N_b^a N_c^a (N^{-1})^a_c K_{\mu c d} h^{e a \mu} (\hat{M}^{-1}(\theta, \theta'))_{e a} \bar{\zeta}^b(\theta) \bar{\zeta}^c(\theta) \bar{\zeta}^d(\theta) \bar{\zeta}^a(\theta') \right] \\
+ \mathcal{O}(\alpha'^{\frac{7}{2}}). \tag{2.35}
\]
where

\begin{equation}
D_{ab}(\theta, \theta') = (\bar{M}^{-1}(\theta, \theta'))_{ab} + i2\pi\alpha' N^a_{a'} N^b_{b'} \hat{\nu}^a(\theta) \hat{\epsilon}_{a\mu} \partial_{\mu} \bar{f}^a \delta(\theta - \theta').
\end{equation}

(2.36)

Since \(\hat{\nu}^a\) does not have the zero mode and \(\bar{M}^{ab}(\theta, \theta)\) is independent of \(\theta\), \(\int d\theta \hat{\nu}^a(\theta) \bar{M}^{ab}(\theta, \theta)\) vanishes. Then, \(\text{Det}'(D) = \text{Det}'(\bar{M}^{-1}) + O(\alpha'^2)\).

By integration with respect to \(\zeta\), regarding \(\alpha'\) higher order terms as perturbation, the partition function becomes

\begin{equation}
Z \propto \int DxDtD\sigma \delta(x^\mu - f^\mu(t, \sigma)) \sqrt{-\bar{h}} \left[ \text{Det}'(\bar{M}^{-1}) \right]^{-\frac{1}{2}} \exp \left( \frac{-1}{\epsilon} \int d\theta T(x) \right) \{ 1 - i\pi\alpha' \int d\theta \partial_{\mu}(F_{\mu\nu}) \hat{\epsilon}^\nu_{a} \hat{\epsilon}^\rho_{b} \hat{\epsilon}^\mu_{c} \bar{M}^{ab}(\theta, \theta) \partial_{\theta} x^c \\
- \frac{1}{2\epsilon} \int d\theta \left[ \bar{u}_{ab} \bar{M}^{ba}(\theta, \theta) \right. \\
+ \frac{\alpha'}{3} \pi \left[ 2N^c_{b} N^{c^\prime}_{c} K^\mu_{\alpha'\beta'} K_{\mu'\nu'\alpha''} h^{\alpha''\nu'} M^{bc}(\theta, \theta) + N^c_{c} N^d_{d} K^\nu_{\alpha'\beta'} K_{\mu'\nu'\alpha''} h^{\alpha''\nu'} M^{cd}(\theta, \theta) \right] \\
+ O(\alpha'^3) \} \},
\end{equation}

(2.37)

where

\begin{equation}
[\text{Det}'(\bar{M}^{-1})]^{-\frac{1}{2}} = [\text{Det}'(\bar{M})]^{\frac{1}{2}} = \sqrt{\det(\bar{h} + \bar{F})}.
\end{equation}

(2.38)

In the calculation of (2.38) we used the zeta-function regularization.

It is shown that the \(\alpha'\) reading terms include divergences and the partition function is the same form as that on the flat D-brane when these divergent terms are canceled.

**\(\beta\) function of tachyon**

We have shown that the \(\alpha'\) leading terms include divergences. Since the term proportional to \(\partial(F)\) vanish when \(F\) is constant, the divergence included in the second term in (2.37) is ignored. We renormalize the tachyon field to cancel other two terms in (2.37). The renormalized tachyon denoted by \(T_R\) is defined in the \(\alpha'\) order as follows:

\begin{equation}
T(x) = \epsilon [T_R + \text{tr}[\bar{u}G^{-1}] \ln \epsilon + \frac{\alpha'}{3} (2N^c_{b} N^{c^\prime}_{c} K^\mu_{\alpha'\beta'} K_{\mu'\nu'\alpha''} h^{\alpha''\nu'} G^{-1bc} + N^c_{c} N^d_{d} K^\nu_{\alpha'\beta'} K_{\mu'\nu'\alpha''} h^{\alpha''\nu'} G^{-1cd} ) \ln \epsilon].
\end{equation}

(2.39)
Substituting (2.19) to (2.39), the beta-function of a tachyon becomes

\[
\beta_T = -\left[ T_R + 2\pi\alpha' N^a_b N^b_c G^{-1ab} \nabla_a' \nabla_b' T_R \right. \\
+ \left. \frac{\alpha'}{3} (2N^b_c K^\mu_{\alpha'^c} K_{\mu'\alpha'^d} h^d'a' G^{-1bc} + N^c_d K^\mu_{\alpha'^c} K_{\mu'\alpha'^d} h^{a'b'} G^{-1cd}) \right]. 
\]

(2.40)

The \( \beta_T \) is different from that of the case of a flat brane by an inhomogeneous term. Because of the term the tachyon satisfies the equation different from that of the flat brane under the condition that the beta function vanishes. We see that the geometrical information of the brane configuration is included in the tachyon.

The effective action \( S \) including the tachyon is related to the partition function \( Z \). Coefficients of the divergent terms in (2.37), the tachyon kinetic term and the term including the extrinsic curvatures, are scheme dependent and we can determine these coefficients from the claim that \( \delta S / \delta T = 0 \) is equivalent to \( \beta_T = 0 \). In general this calculation is complicated. Exceptionally in the case where the last term in (2.40), the normalization factor \( N^a_b \), the induced metric \( \bar{h}_{ab} \) and the field strength \( \bar{F}_{ab} \) are constant (for example when the D1-brane forms a tube), we can determine these coefficients as in the flat case $^\S$.

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$^\S$ Alternately we define the new field

\[
\tilde{T} = T + \frac{\alpha'}{3} (2N^b_c K^\mu_{\alpha'^c} K_{\mu'\alpha'^d} h^d'a' G^{-1bc} + N^c_d K^\mu_{\alpha'^c} K_{\mu'\alpha'^d} h^{a'b'} G^{-1cd}). 
\]

(2.41)

Then \( \beta_T = 0 \) means that \( \tilde{T} \) satisfies the ordinary equation of motion of the tachyon on a flat brane and we can obtain the effective action as that on the flat brane.
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