$B_K$ with dynamical overlap fermions

Norikazu Yamada (KEK/GUAS) for JLQCD Collaboration

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Collaboration with

- S. Hashimoto, T. Kaneko, H. Matsufuru, J. Noaki (KEK)
- S. Aoki (Univ. of Tsukuba)
- H. Fukaya (RIKEN)
- T. Onogi (YITP)
Introduction

• New physics could contribute to $K^0$-$\bar{K}^0$ mixing with visible size.
  - only through the loop diagram in the SM

• Lattice QCD can (must) determine $B_K$ with high precision.
### Parameters

| $\beta$ | $\beta$ of site \# of site Gauge dynamical and valence quarks $Q_{\text{top}}$ | $2.30$ (including two flavors of dynamical quarks) $16^3 \times 32$ Iwasaki + extra Wilson quarks + ghosts ($m_0 = 1.6$) overlap ($m_0 = 1.6$) $0, -2$ and $-4$ |
|---------|------------------------------------------------|--------------------------------------------------------------------------------------------------|
| $m_{\text{sea}}$ | $0.015$ | $0.025$ | $0.035$ | $0.050$ | $0.070$ | $0.100$ |
| $\# \text{ of traj.}$ | $10,400$ | $10,400$ | $10,000$ | $10,000$ | $10,000$ | $10,000$ |
| $Q = 0$ | | | | | | |
| $Q = -2$ | | $0.050$ | | | | |
| $Q = -4$ | | | $0.050$ | | | |
| $m_{\text{sea}}$ | | | | $5,000$ | | |
| $\# \text{ of traj.}$ | | | | | $5,000$ | |

$r_0 = 0.49 \text{ fm} \Rightarrow a = 0.1184(12)(11) \text{ fm} \ (1/a = 1.67(2)(2) \text{ GeV})$

$$(L/a)^3 \times (T/a) = 16^3 \times 32 \Rightarrow V \approx (1.9 \text{ fm})^3$$
## Parameters

| $\beta$  | $\#$ of site | Gauge  | dynamical and valence quarks | $Q_{\text{top}}$ | $m_s/6$ | $0, -\frac{m_s}{2}$ | $m_s$ |
|---------|--------------|--------|-----------------------------|------------------|---------|----------------|--------|
|         |              |        |                             |                  | $Q = 0$ |                  |        |
|         |              |        |                             |                  |        |                  |        |
| m$_{\text{sea}}$ | 0.015       | 0.025  | 0.035                       | 0.050            | 0.070   | 0.100           |        |
| $\#$ of traj. | 10,400       | 10,400 | 10,000                      | 10,000           | 10,000  | 10,000          |        |
| m$_{\text{sea}}$ |              |        |                             |                  | $Q = -2$ |                  |        |
| $\#$ of traj. |              |        |                             |                  |        |                  |        |
| m$_{\text{sea}}$ | 0.050        |        |                             |                  |        |                  |        |
| $\#$ of traj. | 5,000        |        |                             |                  |        |                  |        |
| m$_{\text{sea}}$ |              |        |                             |                  | $Q = -4$ |                  |        |
| $\#$ of traj. |              |        |                             |                  |        |                  |        |
| m$_{\text{sea}}$ | 0.050        |        |                             |                  |        |                  |        |
| $\#$ of traj. | 5,000        |        |                             |                  |        |                  |        |

$r_0 = 0.49$ fm $\Rightarrow a = 0.1184(12)(11)$ fm $\quad (1/a = 1.67(2)(2)$ GeV$)$

$(L/a)^3 \times (T/a) = 16^3 \times 32$ $\Rightarrow V \approx (1.9$ fm$)^3$
Calculation of 3-pt functions

Set a wall source at $t_1$ and $t_2$ and the four-quark operator at $t$.

- calculated with
  $(t_1, t_2) = (0, 16), (8, 24), (0, 24), (8, 0), (16, 8), (24, 16)$
  (some of them are equivalent by translation)
- averaged over equivalent ones.

⇒ two sets of 3-pt functions,
  set A with $|t_1 - t_2| = 24$
  set B with $|t_1 - t_2| = 16$

- 2-pt funcs. are calculated with 4 different $t_{source}$ and averaged after proper shift.
- Low-mode average is implemented for all valence propagators.
Fitting 3-pt functions

- Fit set A & set B simultaneously
- Combine the fit result with that of 2-pt funcs.

$B_P$
Two major contaminations

\[
C^{(3)}_{L_\mu L_\mu}(t_2, t, t_1) = \sum_{\vec{x}} \langle 0 | (A_4^w(t_2))^{\dagger} O_{L_\mu L_\mu}(t, \vec{x})(A_4^w(t_1))^{\dagger} | 0 \rangle
\]

relevant \quad = \quad C_0 \langle \bar{P}|O_{L_\mu L_\mu} |P\rangle e^{-m_P(t_2 - t_1)}
Two major contaminations

\[ C_{L_\mu L_\mu}^{(3)}(t_2, t, t_1) = \sum_{\vec{x}} \langle 0 | (A_4^\dagger(t_2)) | O_{L_\mu L_\mu}(t, \vec{x}) (A_4^\dagger(t_1)) | 0 \rangle \]

relevant \[\rightarrow\]

\[ = C_0 \langle \tilde{P} | O_{L_\mu L_\mu} | P \rangle e^{-m_P(t_2-t_1)} \]

+ \[C_1 \langle \tilde{P}' | O_{L_\mu L_\mu} | P \rangle e^{-(m_{P'}+m_P)t_2-t_1} \]

\[ \times \cosh \left( (m_{P'} - m_P) \left( t - \frac{t_2 + t_1}{2} \right) \right) \]
Two major contaminations

\[ C_{L_\mu L_\mu}^{(3)}(t_2, t, t_1) = \sum_{\vec{x}} \langle 0 | (A_4^w(t_2))^\dagger O_{L_\mu L_\mu}(t, \vec{x}) (A_4^w(t_1))^\dagger | 0 \rangle \]

relevant \[ = C_0 \langle \bar{P} | O_{L_\mu L_\mu} | P \rangle e^{-m_P(t_2-t_1)} \]

excited state \[ + C_1 \langle \bar{P}' | O_{L_\mu L_\mu} | P \rangle e^{-(m_{P'}+m_P)\frac{t_2-t_1}{2}} \]

\[ \times \cosh \left( (m_{P'}-m_P) \left( t - \frac{t_2 + t_1}{2} \right) \right) \]

wrapping contribution \[ + C_2 \langle 0 | O_{L_\mu L_\mu} | P, P \rangle e^{-m_P N_t - \Delta_P(t_2-t_1)/2} \]

\[ \times \cosh \left[ (2m_P + \Delta_P) \left( t - \frac{t_2 + t_1}{2} \right) \right], \]

\( (\Delta_P = E_{\text{total}} - 2 \times m_P) \)

- 3-pt functions are fit to this functional form to obtain \( B_P \).
- Fit range dependence was checked.
NPR with RI-MOM

Mixing of \((V-A)\otimes(V-A)\) with others

Chiral extrapolation of \(Z_{BK}^{\text{RGI}}\)

Preliminary result:

\[
Z_{BK}^{\text{RGI}} = 1.217(6) \\
Z_{BK}^{\text{MS}}(2\text{GeV}) = 0.862(4)
\]

In the following, focus on \(B_K^{\text{MS}}(2\text{ GeV})\).
Sea quark mass dependence

- No clear dependence is seen except in the right-bottom region.

\[ m_{ss}^2 \sim B_0(m_{\text{sea}} + m_{\text{sea}}) \]
Finite volume effect

NLO PQChPT analysis of finite volume effect:
[D. Becirevic and G. Villadoro, Phys. Rev. D 69, 054010 (2004)]

$$\frac{\Delta B_K}{B_K} = \frac{B_K(L) - B_K(\infty)}{B_K(\infty)},$$
$$r = \frac{m_{v1}}{m_s^{\text{phys}}}, \quad r_{\text{sea}} = \frac{m_{\text{sea}}}{m_s^{\text{phys}}}.$$}

Data with $m_{\text{val}} < m_{\text{sea}}$ could receive sizable effects.

Omit such data in the following analysis.

cf. G. Colangelo, S. Durr and C. Haefeli, Nucl. Phys. B 721, 136 (2005)
(NLO correction) $\approx$ (LO correction) for $m_\pi$
(NLO correction) $\approx 0.5 \times$ (LO correction) for $f_\pi$
Test with NLO PQChPT

- NLO PQChPT formula for mesons of degenerate quarks
  \[B_P = B_P^\chi \left[ 1 - \frac{6 m_P^2}{(4\pi f)^2} \ln \left( \frac{m_P^2}{\mu^2} \right) \right] + (b_1 - b_3) m_P^2 + b_2 m_{ss}^2,\]
  \[m_{ss}^2 \sim B_0(m_{sea} + m_{sea}).\]
  (4 free parameters)

- \(f\) controls the size of curvature.
- Fit the data with varying fit range
- Using only data consisting of degenerate quarks.
Test with NLO PQChPT

- While $\chi^2$/dof is reasonable for all fit ranges, $f$ largely depends on them.
- For two smallest ranges, $f$ is roughly consistent with $f = 111(4)(2)$ MeV obtained from the direct calculation.
Test with NLO PQChPT

- Fixing \( f = 111 \text{ MeV} \), only the two shortest fit ranges give acceptable \( \chi^2 \).

Our three or four lightest quarks are inside the NLO ChPT regime.
Determination of $B_K$

All data including non-degenerate quarks (but restricting data with $m_{\text{val}} \geq m_{\text{sea}}$) are fit to

$$B_{12} = B_{12}^\chi \left[1 - \frac{2}{(4\pi f)^2} \left\{ m_{ss}^2 + m_{11}^2 - \frac{3 m_{12}^4 + m_{11}^4}{2 m_{12}^2} + m_{12}^2 \left( \ln \left( \frac{m_{12}^2}{\mu^2} \right) + 2 \ln \left( \frac{m_{22}^2}{\mu^2} \right) \right) - \frac{1}{2} \left( \frac{m_{ss}^2 (m_{12}^2 + m_{11}^2)}{2 m_{12}^2} + \frac{m_{11}^2 (m_{ss}^2 - m_{11}^2)}{m_{12}^2 - m_{11}^2} \right) \ln \left( \frac{m_{22}^2}{m_{11}^2} \right) \right\} \right]$$

$$+ b_1 m_{12}^2 + b_3 m_{11}^2 \left( -2 + \frac{m_{11}^2}{m_{12}^2} \right) + b_2 m_{ss}^2 + c_1 m_{11}^2 m_{12}^2 + \frac{c_2 m_{12}^4}{1 + c_3 m_{12}^2 + c_4 m_{12}^4},$$

$$m_{ij}^2 \sim B_0 (m_{vi} + m_{vj}),$$

[ Golterman and Leung, PRD57(1998)5703] (except for $O(p^4)$ part)

To describe heavy region, “$O(p^4)$ terms” are added.
Determination of $B_K$ 

$B_K^{\text{MS}} (2 \text{ GeV}) = 0.532 - 0.535$ depending on the fit range.

From fit with 4 $m_{\text{sea}}$ data, 

Preliminary result:

$B_K^{\text{MS}} (2 \text{ GeV}) = 0.533(7)$

(Error is statistical only.)
Effect of fixing topology

Studied in R. Brower et al., PLB560(2003)64.

Since the explicit evaluation of the effect to $B_K$ is not available, we assume the correction to be

$$\sim \frac{m_P^2}{(4\pi f)^2} \frac{1}{\langle Q^2 \rangle} \left(1 - \frac{Q^2}{\langle Q^2 \rangle}\right)$$

For our lattice,

$$\langle Q^2 \rangle = \chi_t V_4 = m_q \Sigma V_4 / N_f \sim 10 \quad \text{at} \quad m_q = 0.050$$

the correction to the Q=0 result is expected to be 1.4 % at $m_q=0.05$, and the difference between Q=0 and -2 (-4) to be 0.6 % (2.2 %).

This has been checked numerically.
Effect of fixing topology

Results at $m_{\text{sea}}=0.05$ for data with degenerate quarks.

No clear tendency is seen at fully unquenched point as expected. Naive assumption gives a conservative estimate?

Strategy to study this effect is now available.

S. Aoki et al, arXiv:0707.0396 [hep-lat]
Summary

• $B_K$ was calculated with dynamical overlap.

• Our three or four lightest quarks appear to be inside the NLO ChPT regime.

• Systematic errors are now under investigation.
Fit range dependence

After taking into account the excited state, fit range dependence disappears.

Fit range:

$$[t_{\text{min}}, t_{\text{max}}] = [16 - dt, 24 + dt]$$ for set A,
$$[8 - dt, 8 + dt]$$ for set B
Gauge action

Iwasaki RG (\(\beta=2.30\))

- extra Wilson fermions with mass \(-m_0\)
to prevent \(\lambda_{Hw}=0\) from appearing (⇔ fixing topological charge)

- ghosts with twisted mass \(\mu\)
to suppress unwanted UV effects due to extra Wilson fermions

Boltzmann weight is changed to

\[
\det \left| \frac{H_w(-m_0)^2}{H_w(m_0)^2 + \mu^2} \right| \exp(-S_g)
\]

For details, JLQCD, PRD74 (2006)094505
Valence & Sea

Overlap fermion with Wilson kernel \[\text{[Neuberger, 1998]}\]

\[D_{ov} = \left( m_0 + \frac{m_q}{2} \right) + \left( m_0 - \frac{m_q}{2} \right) \gamma_5 \text{sgn} [H_w(-m_0)] \quad (m_0 = 1.6)\]

• 6 quark masses in \(m_s/6 < m_q < m_s\) \((0.34 < m_\pi/m_\rho < 0.67)\)
  lightest pion \(\Rightarrow m_\pi \approx 290\) MeV, \(m_\pi L \approx 2.8\)

• **Low-mode precondition** and **Low-mode average** are implemented for all valence propagators.