Parameter identification in elasto-plasticity: distance between parameters and impact of measurement errors

Alexey V. Shutov1,2 | Anastasia A. Kaygorodtseva1,2

1 Lavrentyev Institute of Hydrodynamics, pr. Lavrentyva 15, 630090, Novosibirsk, Russia
2 Novosibirsk State University, ul. Pirogova 2, 630090, Novosibirsk, Russia

Correspondence
Alexey V. Shutov, Lavrentyev Institute of Hydrodynamics, pr. Lavrentyva 15, 630090, Novosibirsk, Russia.
Email: alexey.v.shutov@gmail.com

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A special aspect of parameter identification in finite-strain elasto-plasticity is considered. Namely, we analyze the impact of the measurement errors on the resulting set of material parameters. In order to define the sensitivity of parameters with respect to the measurement errors, a mechanics-based distance between two sets of parameters is introduced. Using this distance function, we assess the reliability of certain parameter identification procedures. The assessment involves introduction of artificial noise to the experimental data; the noise can be both correlated and uncorrelated. An analytical procedure to speed up Monte Carlo simulations is presented. As a result, a simple tool for estimating the robustness of parameter identification is obtained. The efficiency of the approach is illustrated using a model of finite-strain viscoplasticity, which accounts for combined isotropic and kinematic hardening. It is shown that dealing with correlated measurement errors, most stable identification results are obtained for non-diagonal weighting matrix. At the same time, there is a conflict between the stability and accuracy.

KEYWORDS
distance between parameters, finite strain, isotropic and kinematic hardening, measurement error, parameter identification, viscoplasticity

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1 INTRODUCTION

The classical approach to the identification of material parameters is based on the minimization of a certain error functional (target function), which reflects the deviation of simulation results from the available experimental data. [3] This procedure is rather general, but, unfortunately, does not provide any insight into the “quality” of the identified parameters. A practicing engineer might want to know, how reliable the obtained set of material parameters is. In order to overcome this problem, one can analyze the sensitivity of the material parameters with respect to the measurement errors.

A simple tool for assessing the quality of the identification procedure are correlation matrices: The identification procedure is considered reliable if the correlations between individual parameters are separated from ±1, see, for example, [8,9,18,34]. If, in contrast, the correlation between parameters $p_i$ and $p_j$ is close to ±1, then a small change in $p_i$ can be compensated by a suitable change in $p_j$, retaining the same simulation results. In such a situation, parameters $p_i$ and $p_j$ can not be identified in a robust way (cf. [13]). The covariance of identified material parameters provides a more detailed information on the quality of the identification procedure. In [8], a set of experiments is identified which leads to the smallest sensitivity (in terms of covariance) of the parameters to the experimental errors.
Obviously, the quality of the parameter identification depends on the completeness of the experimental data. A big body of information can be provided by experiments with an inhomogeneous stress-strain state. Therefore, in a number of publications (cf. [10,18,21,36]) the parameter identification is carried out using finite element method to model experiments with heterogeneous stress-strain distribution. For two-dimensional problems with measured displacement fields, the virtual field method can be used as well with the advantage that expensive FEM computations are not needed (cf. [1,6,7,24]).

In [31] the sensitivity of the material parameters with respect to measurement errors was minimized by an appropriate choice of the weighting coefficients. The main idea behind this procedure is that some experimental data may be more important than the others. Another aspect is that the accurately measured experimental data must be granted a larger weight than the less precise ones. Thus, [3] contains the following recommendation: If one needs to join two different target functions \( \Phi_1 \) and \( \Phi_2 \) pertaining to two different experiments in a single target function \( \Phi \), the following weights should be chosen: \( \Phi = \frac{1}{\sigma_1^2} \Phi_1 + \frac{1}{\sigma_2^2} \Phi_2 \), where \( \sigma_1^2 \) and \( \sigma_2^2 \) are the variances of the noise in both experiments. In a more general case, the weighting coefficients can be chosen in such a way as to ensure that the errors in residuals exhibit equal variance. For instance, having \( N \) experimental data \( Exp_1, Exp_2, \ldots, Exp_N \) with variances \( \sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2 \) and corresponding model predictions \( Mod_1, Mod_2, \ldots, Mod_N \) one may choose the error functional

\[
\Phi = \frac{1}{\sigma_1^2} (Exp_1 - Mod_1)^2 + \frac{1}{\sigma_2^2} (Exp_2 - Mod_2)^2 + \cdots + \frac{1}{\sigma_N^2} (Exp_N - Mod_N)^2.
\]

Note that this formula does not account for the correlation between measurement errors in different experiments. A further generalization of this formula will be discussed in Section 5.3.

In order to decide on which identification strategy is most insensitive to the measurement errors, the sensitivity must be measured in numbers. A straightforward approach, based on the sensitivity of individual material parameters \( p_i, i = 1, \ldots, n \), does not allow to grasp the collective behaviour of the parameter set \( \vec{p} = (p_1, \ldots, p_n)^T \). Thus, a metric in the space of material parameters is needed which allows one to measure a distance between two sets of material parameters. In the current paper, a mechanics-based metric in the space of material parameters is proposed. This metric is advantageous over the conventionally used Euclidean metric \((l_2\)-metric\)). In particular, the mechanics-based metric is invariant under re-parametrization of the material model. The metric is well defined even when different material parameters poses different dimensions. Loosely speaking, when working with the mechanics-based metric the impact of each parameter is proportional to its influence on the stress response.

The paper is organized as follows. In Section 2 a general procedure for finding material parameters is discussed, which is based on the minimization of a certain least-square error functional. In Section 3 we present a short overview of different stochastic models of noise used in reliability analysis and a simple solution for linearized model response is presented. In Section 4 the announced physics-based metric is defined. A series of demonstration problems of parameter identification is solved in Section 5. Finally, in Section 6 the main results are summarized and discussed.

A coordinate-free tensor formalism is used in the current paper (see [15,32]). Bold-faced symbols stand for the first- and second-rank tensors. The trace and determinant of a tensor \( \mathbf{A} \) are denoted by \( \text{tr}(\mathbf{A}) \), \( \det(\mathbf{A}) \); \( \mathbf{A}^{\text{T}} \) and \( \tilde{\mathbf{A}} \) stand for the deviatoric and unimodular parts of \( \mathbf{A} \), respectively.

## 2 | A GENERAL PROCEDURE FOR FINDING MATERIAL PARAMETERS

Assume that in a certain experimental program a set of \( N \) experimental observations is available. Here, the result of each observation is a certain real number. Denote by \( \mathbf{Exp} = (Exp_1, Exp_2, \ldots, Exp_N)^T \) the corresponding vector of experimental data. For a given physical model, the corresponding theoretical predictions are denoted by \( \mathbf{Mod} = (Mod_1, Mod_2, \ldots, Mod_N)^T \). In a standard setting, these theoretical values depend on \( n \) real-valued parameters \( p_1, p_2, \ldots, p_n \). We write for brevity \( \vec{p} = (p_1, p_2, \ldots, p_n)^T \). Obviously, the number of parameters should be smaller than the number of experimental results: \( n < N \).

**Remark 1.** In some applications it is reasonable to impose restrictions on the set of material parameters. Some of the restrictions represent algebraic equations of type \( g_k(\vec{p}) = 0, \ldots, g_k(\vec{p}) = 0 \). Restrictions of another type are given by inequalities \( h_l(\vec{p}) > 0, \ldots, h_l(\vec{p}) > 0 \). Here, \( k \) and \( l \) is the number of equality constraints and inequality constraints, respectively. Equality constraints are not consider in the current study; in some cases they can even be used to regularize the identification procedure by reducing the number of material parameters.\(^{[34]}\) As for inequality constraints, some authors suggest that a “good” identification procedure must satisfy these constraints in a natural way (cf. the discussion in [18]). Therefore, they are not introduced in the current setting as well.
Let $W$ be a given square $N \times N$ matrix; assume that it is symmetric and positive definite. Usually, the parameter identification is reduced to the following optimization (minimization) problem

$$\hat{p} = \text{argmin}(\Phi(\hat{p})), \quad \Phi(\hat{p}) = \|W\text{Resid}\|^2 = W\text{Resid}^TW\text{Resid}, \quad \text{Resid} = \text{Exp} - \text{Mod}. \quad (2)$$

Here, $\text{Resid}$ is the so-called residuum, being seen as a deviation of theoretical results from the experimental data. This optimization problem is equivalent to the minimization of the $l_2$-norm of a modified residuum $W\text{Resid}$:

$$\hat{p} = \text{argmin}(\Phi(\hat{p})), \quad \Phi(\hat{p}) = \|W\text{Resid}\|^2 = W\text{Resid}^TW\text{Resid}, \quad W\text{Resid} := W^{1/2}\text{Resid}. \quad (3)$$

In contrast to (2), problem (3) can be solved using standard procedures, like the well-established and highly reliable Levenberg-Marquardt method.[20] Some considerations regarding a “good” matrix $W$ will be presented in Section 5.3.

### 3 | INTRODUCTION OF NOISE TO EXPERIMENTAL STRESS-STRAIN CURVES

#### 3.1 | Types of noise

It is natural to assume that the available experimental data $\text{Exp} = (\text{Exp}_1, \text{Exp}_2, \ldots, \text{Exp}_N)^T$ are contaminated by measurement errors. In other words, in reality, noisy data are available. Usually one assumes that the error is additive:[3]

$$\text{Noisy data}_i = \text{Exp}_i + \text{Noise}_i. \quad (4)$$

In order to analyze the dependence of the parameter identification procedure on this measurement error, we need a stochastic model of $\text{Noise}_i$. The most simple model of noise is given by the assumption that the measurement errors are independent random variables with a normal distribution with a zero mean and standard deviation $\sigma$ (white noise, see Figure 1 (left))

$$\text{Noisy data}_i \in \mathcal{N}(\mu, \sigma^2), \quad \mu = 0. \quad (5)$$

The white-noise-model is a good choice in many practical situations due to the central limit theorem of the probability theory. The central limit theorem states that, if a large number of independent random variables is added, their normalized sum converges to a random variable with a normal distribution. Therefore, the white noise assumption is used in a number of studies to assess the stability of a certain identification procedure (cf. [1,7,31]).

Another stochastic description of noise is provided by the so-called autoregressive model (AR-model). Using the AR-model, it is possible to account for impact of previous measurement errors on the next measurement. For instance, assuming the Yule-Walker equations, the following scheme is obtained (see, for instance, [8,9,29])

$$\text{Noise}_i = a\text{Noise}_{i-1} + \epsilon_i, \quad a \in [0, 1), \quad (6)$$

where $\epsilon_i$ is a sequence of independent normally distributed random variables: $\epsilon_i \in \mathcal{N}(0, \sigma^2)$; $a$ is the autoregression parameter. According to (6), the errors are not independent random variables, but correlated in a certain way (see Figure 1 (middle)). Such a correlation can arise due to inertia effects in the testing equipment. For example, in experiments involving the occurrence of the Lüders bands, one may assume a noise caused by resonance in the load cell.[2]
Another stochastic model of noise which we call “two-source model” is as follows. We assume that two independent sources of noise are active, responsible for correlated and non-correlated errors, respectively. Thus we have

\[ \text{Noise}_i = \text{Noise}_{i\text{non-correlated}} + \text{Noise}_{i\text{correlated}}. \]  

(7)

Here \( \text{Noise}_{i\text{non-correlated}} \) corresponds to the previously mentioned white-noise. Thus, it is a sequence of independent normally distributed variables with the standard deviation \( \sigma_1 \):

\[ \text{Noise}_{i\text{non-correlated}} \in \mathcal{N}(0, \sigma_1^2). \]  

(8)

Dealing with experimental identification of stresses, the correlated error (8) can be caused by a wrong calibration of dynamometer or a wrong measurement of the sample cross-section (see Figure 1 (right)). The “two-source model” will be employed for Monte Carlo computations in Section 5.3.

In some studies, the sensitivity of the material parameters is analyzed assuming that the noise is uniformly distributed in a certain interval.\(^{[13]}\) This stochastic model is implemented not due to its plausibility, but rather due to its simplicity. Another stochastic model may consider additional noise caused by a play in the testing assembly.\(^{[31]}\) Note that this section refers solely to the stress-strain curves. Dealing with experimentally measured fields like the displacements obtained by digital image correlation, more sophisticated techniques are needed (cf. \([25]\)). In the more general context of reliability analysis, one may consider the Weibull or lognormal distributions of noise. Typically, this is done for positive strength parameters like elasticity modulus or yield stress of the material (cf. \([16]\)).

### 3.2 Fast optimization for linearized response

Let us consider the optimization problem (3) with the vector of experimental data \( \overline{\text{Exp}} \). For a given matrix \( \mathbf{W} \), the minimizing set of parameters \( \overline{p} \) and the Jacobian of the model response are as follows

\[ \overline{p} = \arg \min \Phi, \quad J = \left. \frac{\partial \text{Mod}(\overline{p})}{\partial \overline{p}} \right|_{\overline{p}}. \]

(9)

Along with this basic (unperturbed) optimization problem we consider a case where the measurements are contaminated by a small noise. In order to build an analytical procedure, the model response function \( \text{Mod}(\overline{p}) \) is linearized near \( \overline{p} \):

\[ \text{Mod}(\overline{p}) \approx \text{Mod}^{lin}(\overline{p}) = \text{Mod}(\overline{p}_s) + J(\overline{p} - \overline{p}_s). \]

(10)

The error functional, which corresponds to the optimization problem with noisy data, takes the form

\[ \Phi^{\text{noisy}}(\overline{p}) = \overline{\text{Resid}}^\dagger \mathbf{W} \overline{\text{Resid}}, \quad \overline{\text{Resid}} = \overline{\text{Exp}} + \text{Noise} - \text{Mod}^{lin} = \overline{\text{Exp}} + \text{Noise} - \text{Mod}(\overline{p}_s) - J(\overline{p} - \overline{p}_s). \]

(11)

This error functional can be re-written in the following form

\[ \Phi^{\text{noisy}}(\overline{p}) = \overline{\mathbf{W}} \overline{\text{Resid}}^\dagger \overline{\text{Resid}}, \quad \overline{\mathbf{W}} \overline{\text{Resid}} := \mathbf{W}^{1/2} \overline{\text{Resid}}. \]

(12)

We introduce the following abbreviation

\[ \overline{A} := \mathbf{W}^{1/2} \left( \overline{\text{Exp}} + \text{Noise} - \text{Mod}(\overline{p}_s) + J(\overline{p} - \overline{p}_s) \right). \]

(13)

Thus, the error functional is a quadratic function of \( \overline{p} \)

\[ \Phi^{\text{noisy}}(\overline{p}) = \left( \overline{A} - \mathbf{W}^{1/2} J \overline{p} \right)^\dagger \left( \overline{A} - \mathbf{W}^{1/2} J \overline{p} \right). \]

(14)

Its derivative with respect to \( \overline{p} \) is given by the linear function

\[ \frac{\partial \Phi^{\text{noisy}}(\overline{p})}{\partial \overline{p}} = -2 \left( \overline{A} - \mathbf{W}^{1/2} J \overline{p} \right)^\dagger \mathbf{W}^{1/2} J. \]

(15)
Since $\vec{p}$ is a local extremum of $\Phi_{\text{noisy}}(\vec{p})$, this derivative is equal to zero. After some rearrangements we arrive at the analytical solution

$$\vec{p} = (J^T \mathbf{W} J)^{-1} (\mathbf{W}^{1/2} J)^T \vec{A}. \quad (16)$$

Note that in the noise-free case (when Noise = 0), the original solution is restored, thus yielding $\vec{p} = \vec{p}_*$. 

4 HOW TO MEASURE A DISTANCE BETWEEN TWO SETS OF PARAMETERS

Having two sets of material parameters, $\vec{p}^{(1)}$ and $\vec{p}^{(2)}$, one needs to estimate the distance between them. Such a distance $\text{dist}(\vec{p}^{(1)}, \vec{p}^{(2)})$ should be small if the parameter sets are close to each other in a certain sense. In the following sections, this function will be used to estimate the dependence of material parameters on the experimental errors.

4.1 Euclidean norm

One of the simplest ways to measure a distance is to employ the Euclidean norm

$$\text{dist}^{\text{Euclidean}}(\vec{p}^{(1)}, \vec{p}^{(2)}) := \sqrt{\sum_{i=1}^{n} (p_i^{(1)} - p_i^{(2)})^2}. \quad (17)$$

Unfortunately, in most practical situations, this norm is physically absurd since the parameter set $\vec{p}$ encapsulates quantities with different physical dimensions. In order to resolve this issue, a non-dimensional Euclidean norm can be used (see, for instance, [31])

$$\text{dist}^{\text{Eucl. non-dimens.}}(\vec{p}^{(1)}, \vec{p}^{(2)}) := \sqrt{\sum_{i=1}^{n} \left( \frac{p_i^{(1)} - p_i^{(2)}}{p_i^*} \right)^2}, \quad (18)$$

where $p_i^*$ is a typical (characteristic) value of the parameter $p_i$. Obviously, there is a certain arbitrariness in the choice of $p_i^*$ and substantial difficulties will arise when $p_i^* = 0$. 

For a number of complex material models certain material parameters can not be identified with a suitable accuracy. At the same time, large variance in these bad trackable parameters does not have any substantial impact on the overall stress response. Thus, an essential drawback of the Euclidean norm is that all the parameters $p_i$ are treated by (18) in the same way, regardless of their importance for the physical problem under consideration.

Another aspect is as follows. The Euclidean metric is sensitive to a re-paramterization of the material model. More precisely, when the parameter set $\vec{p}$ is a one-to-one function of some new parameters $\vec{\rho}$ ($\vec{p} = \vec{p}(\vec{\rho})$), then $\text{dist}(\vec{p}(\vec{\rho}^{(1)}), \vec{p}(\vec{\rho}^{(2)})) \neq \text{dist}(\vec{\rho}^{(1)}, \vec{\rho}^{(2)})$. In order to resolve these problems, mechanical considerations are needed.

4.2 Mechanics-based metric in the space of material parameters

Dealing with finite-strain elasto-plasticity, it is reasonable to introduce a strain-controlled loading path at a certain material point. Let $T$ be the duration of the loading process and $\mathbf{F}(t)$ be the deformation gradient tensor given as a function of time $t \in [0, T]$. Assuming a simple material of Noll’s type, the local history of the Cauchy stress tensor $\mathbf{T}(t)$ is a unique function of the local deformation history $\mathbf{F}(t)$ and material parameters $\vec{p} = (p_1, p_2, \ldots, p_n)^T$:

$$\mathbf{T}(t, \vec{p}) = \mathbf{T} \left( \mathbf{F}(t'), \vec{p} \right), \quad \text{for all } t \in [0, T]. \quad (19)$$

1 For so-called “simple materials with initial conditions”, the stress history may depend on the initial values of the internal variables. The initial values can be included in the set of material parameter $\vec{p}$. 

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On the left-hand side of this relation, the dependence of the stress response on the deformation history \( \mathbf{F}(t) \) is omitted for brevity of notation. The deformation history \( \mathbf{F}(t) \) and the material model (19) uniquely define a mechanics-based distance between two sets of material parameters \( \bar{\mathbf{p}}^{(1)} \) and \( \bar{\mathbf{p}}^{(2)} \) as

\[
\text{dist}^\mathbf{F}(\bar{\mathbf{p}}^{(1)}, \bar{\mathbf{p}}^{(2)}) := \max_{t \in [0,T]} \| \mathbf{T}(t, \bar{\mathbf{p}}^{(1)}) - \mathbf{T}(t, \bar{\mathbf{p}}^{(2)}) \|.
\] (20)

In order to confirm that \( \text{dist}^\mathbf{F}(\bar{\mathbf{p}}^{(1)}, \bar{\mathbf{p}}^{(2)}) \) defines a metric on a certain set of material parameters, one needs to check the following conditions:

\[
\begin{align*}
(i) & : \quad \text{dist}^\mathbf{F}(\bar{\mathbf{p}}^{(1)}, \bar{\mathbf{p}}^{(2)}) \geq 0, \\
(ii) & : \quad \text{dist}^\mathbf{F}(\bar{\mathbf{p}}^{(1)}, \bar{\mathbf{p}}^{(2)}) = 0 \quad \text{if and only if} \quad \bar{\mathbf{p}}^{(1)} = \bar{\mathbf{p}}^{(2)}, \\
(iii) & : \quad \text{dist}^\mathbf{F}(\bar{\mathbf{p}}^{(1)}, \bar{\mathbf{p}}^{(2)}) = \text{dist}^\mathbf{F}(\bar{\mathbf{p}}^{(2)}, \bar{\mathbf{p}}^{(1)}), \\
(iv) & : \quad \text{dist}^\mathbf{F}(\bar{\mathbf{p}}^{(1)}, \bar{\mathbf{p}}^{(2)}) \leq \text{dist}^\mathbf{F}(\bar{\mathbf{p}}^{(1)}, \bar{\mathbf{p}}^{(3)}) + \text{dist}^\mathbf{F}(\bar{\mathbf{p}}^{(2)}, \bar{\mathbf{p}}^{(3)}).
\end{align*}
\]

Conditions \((i)\), \((ii)\), \((iii)\), and \((iv)\) are trivially satisfied. Condition \((ii)\) is satisfied only if the loading program \( \mathbf{F}(t) \) makes each material parameter “visible”. In the purely elastic range, two different sets of hardening parameters may produce the same stress response, thus yielding a zero distance between these sets of parameters. In order to avoid this undesired effect, the prescribed strains must be large enough. Concrete examples will be considered in Section 5.3. Due to its definition, the introduced metric \( \text{dist}^\mathbf{F} \) has a dimension of the mechanical stress.

Remark 2. Definition (20) is based on a local strain history to keep the computation of the metric as simple as possible. Obviously, some other application-related distances can be defined using a solution of a practical boundary value problem. On the other hand, as will be shown in Section 5.3, a concrete choice of the local loading history is not so important and different loading histories yield similar results (at least for the considered set of loading histories).

5 | ILLUSTRATION PROBLEM: MODEL WITH COMBINED ISOTROPIC-KINEMATIC HARDENING

5.1 | Experimental data for the steel 42CrMo4

For demonstration purposes we consider the parameter identification problem, based on the experimental data, reported in [35] for the steel 42CrMo4. During testing, thin-walled tubular specimens were subjected to non-monotonic torsion. The measured shear stresses (engineering stresses) are plotted versus the shear strain in Figure 2 (top left). As can be seen from the figure, the material exhibits a strong Bauschinger effect coupled to expansion of the elastic domain. In order to describe this type of stress response, finite-strain plasticity models with a combined isotropic-kinematic hardening are usually implemented. The presented measurement results will serve as a basis for the identification of material parameters. As discussed in [35], the initial (as-received) state can be idealized as isotropic. This observation is important for the identification of the initial state.

5.2 | Deterministic plasticity model of Shutov and Kreißig (2008)

The model of finite-strain viscoplasticity proposed by Shutov and Kreißig (cf. [33]) is formulated in a geometrically exact manner. The description of the nonlinear kinematics is based on the nested split of the deformation gradient tensor, originally proposed by Lion in [19]. Relations between stresses and elastic strains are of hyperelastic type (cf. [11]). This combination of constitutive assumptions is shown to have numerous advantages over competing alternatives.\[30\] The model accounts for nonlinear isotropic and kinematic hardening, it is objective and thermodynamically consistent, it is free from spurious shear stress oscillations under simple shear, and it is (weakly) invariant under isochoric changes of the reference configuration.\[30,37\] Some micromechanical arguments in favour of the nested multiplicative split are presented in [40].

The deformation at a material point is captured by the right Cauchy-Green tensor \( \mathbf{C} := \mathbf{F}^\mathsf{T} \mathbf{F} \). The current state of the material is described by internal variables of the right Cauchy-Green type: \( \mathbf{C} \) for the inelastic strains and \( \mathbf{C}^1, \mathbf{C}^2 \) for the inelastic strains of substructure. Tensors \( \mathbf{C}, \mathbf{C}^1, \mathbf{C}^1^1, \) and \( \mathbf{C}^2 \) are symmetric and positive definite. Additionally, two scalar-valued internal variables are employed: accumulated inelastic arc-length (Odqvist parameter) \( s \) and its dissipative part \( s_d \).
FIGURE 2  Experiment on non-monotonic torsion of thin-walled specimens made from steel 42CrMo4. Engineering stresses are depicted. Top left: experimental results from [35]. Top right: results of optimization with $W = \text{diag}(1, 1, \ldots, 1)$. Bottom left: results of optimization with $W_{ij} = \delta_{ij}/\text{Cov}_{ij}$. Bottom right: results of optimization with $W = \text{Cov}^{-1}$

By $\psi$ we denote the Helmholtz free energy per unit mass. Assume that it is decomposed into the following summands (cf. [35]):

$$\psi = \psi_{\text{el}}(CC^{-1}) + \psi_{\text{kin1}}(C_1C_{1i}^{-1}) + \psi_{\text{kin2}}(C_1C_{2i}^{-1}) + \psi_{\text{iso}}(s - s_d).$$

(21)

Here, $\psi_{\text{el}}(CC^{-1})$ captures the energy storage due to macroscopic elastic deformations; $\psi_{\text{kin1}}(C_1C_{1i}^{-1})$, $\psi_{\text{kin2}}(C_1C_{2i}^{-1})$ and $\psi_{\text{iso}}(s - s_d)$ are parts of the energy stored in defects of crystal lattice, they are related to kinematic and isotropic hardening. Important limitation of the approach is that the functions $\psi_{\text{el}}$, $\psi_{\text{kin1}}$, and $\psi_{\text{kin2}}$ are isotropic. For practical computations we use the following constitutive assumptions:

$$\rho_R\psi_{\text{el}}(A) = \frac{k}{2}(\ln \sqrt{\text{det}(A)})^2 + \frac{\mu}{2}(\text{tr}A - 3),$$

(22)

$$\rho_R\psi_{\text{kin1}}(A) = \frac{c_1}{4}(\text{tr}A - 3), \quad \rho_R\psi_{\text{kin2}}(A) = \frac{c_2}{4}(\text{tr}A - 3),$$

(23)

$$\rho_R\psi_{\text{iso}}(s_e) = \frac{\gamma}{2}(s_e)^2, \quad A : = (\text{det}(A))^{-1/3}A,$$

(24)

for any second-rank tensor $A$ and scalar $s_e$. Here, $k$, $\mu$, $c_1$, $c_2$, $\gamma$ are material parameters; $\rho_R$ denotes the mass density in the reference configuration. Employing the standard Coleman-Noll procedure, the second Piola-Kirchhoff stress $\tilde{T}$ is related to strains through

$$\tilde{T} = 2\rho_R \frac{\partial \psi_{\text{el}}(CC^{-1})}{\partial C} |_{C_1=\text{const}},$$

(25)

Recall that the mechanics-based metric operates with the Cauchy stress $T$, which is given by

$$T = \frac{1}{\text{det}(F)} F \tilde{T} F^T.$$

(26)

Two backstresses $\tilde{X}_1$ and $\tilde{X}_2$ as well as the overall backstress $\tilde{X}$ are used in the current paper to capture the translation of the yield surface in the stress space. These tensors operate on the reference configuration; they are computed through

$$\tilde{X}_1 = 2\rho_R \frac{\partial \psi_{\text{kin1}}(C_1C_{1i}^{-1})}{\partial C_1} |_{C_1=\text{const}}, \quad \tilde{X}_2 = 2\rho_R \frac{\partial \psi_{\text{kin2}}(C_1C_{2i}^{-1})}{\partial C_1} |_{C_2=\text{const}}, \quad \tilde{X} = \tilde{X}_1 + \tilde{X}_2.$$

(27)
A hardening variable \( R \in \mathbb{R} \), which is responsible for isotropic expansion of the yield surface, is related to scalar-valued internal variables:

\[
R = \rho_R \frac{\partial \psi_{\text{iso}}(s - s_0)}{\partial s} \bigg|_{s_0=\text{const}},
\]

(28)

For viscoplastic models, stress states beyond the elastic domain are possible. The corresponding viscous overstress \( f \) depends on the applied strain rate; it is defined by

\[
f := \mathcal{G} - \sqrt{\frac{2}{3}(K + R)}, \quad \mathcal{G} := \sqrt{\text{tr}((\mathbf{C}\dot{T} - \mathbf{C}_i\mathbf{X})^D)^2},
\]

(29)

where \( K \) is the initial quasi-static yield stress, \((\cdot)^D\) stands for the deviatoric part of a tensor, \( \mathcal{G} \) is the driving force of the viscoplastic flow. An inelastic multiplier \( \lambda_i \) is introduced which equals the norm of the inelastic strain rate; \( \lambda_i \) is computed employing the Perzyna law of viscoplasticity

\[
\lambda_i = \frac{1}{\eta} \left( \frac{f}{k_0} \right)^m, \quad \langle x \rangle := \max(x, 0).
\]

(30)

Here, \( \eta \) and \( m \) are the viscosity and the stress exponent, respectively; \( k_0 \) is set equal to 1 MPa in order to obtain a non-dimensional quantity in the bracket. The evolution of the internal variables is specified by the following constitutive equations

\[
\mathbf{C}_i = 2\frac{\lambda_i}{\mathcal{G}}(\mathbf{C}\dot{T} - \mathbf{C}_i\mathbf{X})^D \mathbf{C}_i, \quad i = 1, 2,
\]

(31)

\[
\dot{\mathbf{C}}_{1i} = 2\lambda_i \dot{x}_1 (\mathbf{C}_i \mathbf{X}_1)^D \mathbf{C}_{1i}, \quad \dot{\mathbf{C}}_{2i} = 2\lambda_i \dot{x}_2 (\mathbf{C}_i \mathbf{X}_2)^D \mathbf{C}_{2i},
\]

(32)

\[
\dot{s} = \sqrt{\frac{2}{3}} \mathcal{G}, \quad \dot{s}_d = \frac{\beta}{\gamma} \dot{s} R.
\]

(33)

Here, \( \dot{x}_1, \dot{x}_2 \) are parameters governing the saturation of the kinematic hardening; \( \beta \) is responsible for the saturation of the isotropic hardening; \((\cdot)^D\) is the material time derivative (differentiation with respect to the time \( t \) while the particle is held fixed). In the current study we assume that the initial state is isotropic, undeformed, and stress free. This yields the following initial conditions

\[
\mathbf{C}_i|_{t=0} = \mathbf{C}_{1i}|_{t=0} = \mathbf{C}_{2i}|_{t=0} = 1, \quad s|_{t=0} = s_d|_{t=0} = 0. \quad (34)
\]

Differential equations (31) and (32) describe an incompressible flow:

\[
\det(\mathbf{C}_i) = \det(\mathbf{C}_{1i}) = \det(\mathbf{C}_{2i}) = 1.
\]

(35)

Robust and efficient numerical procedures for the case where \( \psi_{\text{kin1}} \) and \( \psi_{\text{kin2}} \) are of neo-Hookean type are presented in [38]. The case where \( \psi_{\text{kin1}} \) and \( \psi_{\text{kin2}} \) are of Mooney-Rivlin type can be dealt with using an explicit update formula from [39]. The model is implemented into the nonlinear FEM-code MSC.MARC. Practical applications were analyzed using the model of Shutov and Kreißig in [27,28,37]. This model was derived later using rather general modelling principles in [4] and [17]. A similar model was proposed by Vladimirov et al. in [41].

The hardening parameters \( \gamma, \beta, c_1, c_2, \dot{x}_1, \) and \( \dot{x}_2 \) possess a clear mechanical interpretation. First, the isotropic hardening of Voce type is governed by \( \gamma \) and \( \beta \). The quantities \( \gamma / \beta \) and \( 1 / \beta \) provide the saturation value of the isotropic hardening and the rate of the saturation, respectively. Next, the nonlinear kinematic hardening of Armstrong-Frederick type depends on \( c_i, \dot{x}_i, i = 1, 2 \). In this model, \( 1 / \dot{x}_i \) and \( 1 / (c_i \dot{x}_i) \) are proportional to the saturation level of the backstress and the corresponding saturation rate.

Note that the material model summarized in this section is deterministic. The reader interested in stochastic constitutive models is referred to [14] and references cited therein. Solution strategies for problems with uncertainties in material properties and applied loads are discussed in [26].

5.3 Monte Carlo computations using noisy data

Some preliminary results regarding the identification of the material parameters for the steel 42CrMo4 were presented in [35]. Certain parameters which appear in the material model can be identified by general considerations. In particular, the elastic
TABLE 1  Pre-identified parameters for the steel 42CrMo4

| $k$ [MPa] | $\mu$ [MPa] | $\eta$ [s] | $m$ [-] | $K$ [MPa] |
|-----------|-------------|------------|--------|---------|
| 135 600   | 52000       | $5 \cdot 10^5$ | 2.26  | 335     |

TABLE 2  Identified material parameters of the steel 42CrMo4 using noise-free experimental data

| $\gamma_{e}$ [MPa] | $\beta_{e}$ [-] | $c_{1e}$ [MPa] | $c_{2e}$ [MPa] | $x_{1e}$ [1/MPa] | $x_{2e}$ [1/MPa] |
|-------------------|-----------------|----------------|----------------|------------------|------------------|
| $W = \text{Cov}^{-1}$ | 435.22         | 2.625          | 1 661.7        | 24 672           | 0.003810         | 0.004282         |
| $W = \text{diag}(1, 1, \ldots, 1)$ | 321.92         | 2.003          | 1 488.4        | 20 512           | 0.004087         | 0.004526         |
| $W_{ij} = \delta_{ij}/\text{Cov}_{ij}$ | 312.60         | 1.913          | 1 505.5        | 20 687           | 0.004089         | 0.004516         |

Constants $k$ and $\mu$ are extracted from the experimental data in the elastic domain. The viscosity parameters $\eta$ and $m$ can be obtained from a series of uniaxial tension tests with different loading rates (cf. [35]). The pre-identified material parameters are summarized in Table 1. The remaining material parameters describe the nonlinear isotropic and kinematic hardening. They are packed now into the vector $\vec{p} = (\gamma, \beta, c_1, c_2, x_1, x_2)^T$. Since the mechanisms of isotropic and kinematic hardening are active at the same time, the corresponding parameters must be identified simultaneously (cf. [5,34]).

In this subsection we demonstrate a procedure for numerical estimation of the sensitivity of the parameter cloud is expected to be less than 1%. Error estimation of the Monte Carlo method is somewhat larger for the strategy with $W = \text{Cov}^{-1}$, see Table 2. The corresponding simulation results are compared with the experimental data in Figure 2. As can be seen from the figure, a good correspondence between the simulation and experiment is observed for the strategies with $W = \text{diag}(1, 1, \ldots, 1)$ and $W_{ij} = \delta_{ij}/\text{Cov}_{ij}$. Both strategies provide similar results (see Table 2). On the other hand, the deviation from the experiment is somewhat larger for the strategy with $W = \text{Cov}^{-1}$.

For the Monte Carlo simulations a total number of $N_{\text{noise}} = 10000$ instances of noisy data were considered. Since the regular error estimation of the Monte Carlo method is $C/\sqrt{N_{\text{noise}}}$ with a certain constant $C$, the relative error in the computed size of the parameter cloud is expected to be less than 1%.

The stochastic parameters of the noise (7) are $\sigma_1 = 10$ MPa, $\sigma_2 = 5$ MPa. In order to speed up the Monte Carlo sampling, the model response is linearized near $\vec{p}$ according to (10). For $j = th$ instance of noise, the corresponding parameter set $\vec{p}_j$ is

\[ 1 \text{ These particular values of } \sigma_1 \text{ and } \sigma_2 \text{ are taken here for demonstration purposes. Note that for small noise the absolute values of } \sigma_1 \text{ and } \sigma_2 \text{ are not important in a search for the best optimization strategy. An important quantity is the ratio } \sigma_2/\sigma_1. \]
During the computation of the metric, the viscous part of the material model is deactivated by setting the viscosity to zero. For each of the normalized parameters is provided in Table 3. A very small variance indicates that the parameters are much more insensitive to the experimental noise than the other parameters. This is due to the fact that $\chi_1$ and $\chi_2$ are directly connected to the saturation levels of backstresses while backstress saturation is well defined by the available experimental data. On the other hand, the saturation of the isotropic hardening takes place at the initial stage of the loading process. Thus, the main body of the experimental data does not carry any information regarding the saturation of the isotropic hardening. Therefore, the hardening parameters $\gamma$ and $\beta$ are much more sensitive to the applied noise.

The size of the parameter cloud is then defined as the average distance between $\vec{p}_a$ and $\vec{p}^{(j)}$

$$\text{Size} = \frac{1}{N_{\text{noise}}} \sum_{j=1}^{N_{\text{noise}}} \text{dist}(\vec{p}_a, \vec{p}^{(j)}).$$

(38)

In order to define the distance between two sets of parameters, a suitable deformation history is needed. In this study we consider three different histories in the time interval $t \in [0, 4]$, $t$ is a non-dimensional loading parameter here. In the first history, strain-controlled incremental loading is borrowed from the real experimental program, see Figure 2 (top left). For the second and the third histories we set

$$F(t) = F'(t),$$

where $F'(t)$ is a linear interpolation between certain key-points $F_1, F_2, F_3,$ and $F_4$:

$$F'(t) := \begin{cases} 
(1-t)F_1 + (t)F_2 & \text{if } t \in [0, 1] \\
(2-t)F_2 + (t-1)F_3 & \text{if } t \in (1, 2) \\
(3-t)F_3 + (t-2)F_4 & \text{if } t \in (2, 3) \\
(4-t)F_4 + (t-3)F_1 & \text{if } t \in (3, 4)
\end{cases}.$$

For the 2nd history we employ the following key-points

$$F_1 := 1, \quad F_2 := 1.2 \, e_1 \otimes e_1 + (1.2)^{-1/2} (e_2 \otimes e_2 + e_3 \otimes e_3),$$

$$F_3 := 1, \quad F_4 := 1.2 \, e_2 \otimes e_2 + (1.2)^{-1/2} (e_1 \otimes e_1 + e_3 \otimes e_3).$$

and for the 3rd history we have

$$F_1 := 1, \quad F_2 := 1.2 \, e_1 \otimes e_1 + (1.2)^{-1/2} (e_2 \otimes e_2 + e_3 \otimes e_3),$$

$$F_3 := 1 + 0.2 \, e_1 \otimes e_2, \quad F_4 := 1.2 \, e_2 \otimes e_2 + (1.2)^{-1/2} (e_1 \otimes e_1 + e_3 \otimes e_3).$$

The used deformation histories are visualized in Figure 3.

The sizes of the parameter clouds are listed in Table 4. The results of the Monte Carlo computations indicate that the strategy with $W = \text{Cov}^{-1}$ yields parameters, which are most stable with respect to the considered noise. On the other hand, the strategies with $W = \text{diag}(1, 1, \ldots, 1)$ and $W_{ij} = \delta_{ij}/\text{Cov}_{ij}$ are almost equivalent to each other regarding stability of the parameter identification. Another important conclusion is that the choice between the considered deformation histories is not very important in defining the distance function (20). As is seen from Table 4, the sizes of the parameter clouds are somewhat smaller for the 1st deformation history. This is due to the fact that the 1st history is taken from the real experiment used to identify the parameters.

---

1 During the computation of the metric, the viscous part of the material model is deactivated by setting the viscosity to zero.
2nd deformation history

3rd deformation history

Figure 3 Homogeneous deformation histories used to define the mechanics-based metric

Table 4 Sizes of the parameter clouds in terms of the mechanics-based distance for the three parameter sets given in Table 2

|                | $W = \text{Cov}^{-1}$ | $W = \text{diag}(1,1,\ldots,1)$ | $W_{ij} = \delta_{ij}/\text{Cov}_{ij}$ |
|----------------|------------------------|-------------------------|----------------------------------|
| 1st history (from the experiment) | 5.139 MPa              | 7.263 MPa               | 7.215 MPa                        |
| 2nd deformation history           | 5.522 MPa              | 7.739 MPa               | 7.687 MPa                        |
| 3rd deformation history           | 5.562 MPa              | 7.745 MPa               | 7.682 MPa                        |

At the same time, for the 2nd and 3rd histories we are dealing with an extrapolation of the model response to other loading cases which may introduce additional errors.

6 DISCUSSION AND CONCLUSION

A simple mechanics-based definition of metric in the space of material parameters is introduced (see Equation (20)). In contrast to the formal use of the Euclidean norm, this metric accounts for the importance of each material parameter for the stress response. The metric is invariant under re-parametrization of the material model; it is not based on the artificial normalization of parameters. An important observation is that the specific choice of the deformation history among the three alternatives has only a minor impact on the results of computations (see Table 4).

A strain-controlled loading is implemented in (20) to define a distance between two sets of material parameters. This choice is reasonable for models of finite strain plasticity and viscoplasticity. Dealing with models of creep (including creep damage), a stress-controlled loading can be used instead.

A local strain history is considered to define the metric (see Equation (20)). This definition can be naturally generalized by considering a representative boundary value problem. The most reasonable results are expected when this boundary value problem would be close to a specific application.

For each instance of the noisy data, the corresponding parameters are identified using the linearized problem, where a closed-form solution is available. Therefore, the presented approach is computationally efficient. In a more general case of a large noise, the assumption (10) must be dropped and a straightforward solution of the optimization problem will be needed.

In case of a correlated noise (cf. stochastic model (7)), a good stability of the identified parameters can be achieved by using non-diagonal weighting matrix $W$. Unfortunately, there is a conflict between the accuracy in the description of the experimental data and the stability of the identified parameters with respect to the experimental errors: The strategy with $W = \text{Cov}^{-1}$, which provides most stable results, yields larger deviation of the computed stress response from the experimental data. Probably, while solving practical problems, a compromise between the stability and accuracy of the parameter identification needs to be found. This compromise should be based on the a-priory knowledge of the stochastic parameters of the experimental error.
The presented method of estimating the sensitivity of the material parameters with respect to the experimental errors can be useful in various situations. Basically, it can be employed to assess the quality of a certain identification procedure. When dealing with experimental data pertaining to different types of experiments, like tension-compression or non-monotonic torsion, a problem arises of how to combine these data in a single error functional. If a realistic model of stochastic noise for different experiments is available, the method can be used to define suitable weighting coefficients. Application of the mechanics-based metric to optimal experimental design problems (cf. [12]) is also promising.

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ORCID

Alexey V. Shutov https://orcid.org/0000-0002-5624-9732
Anastasia A. Kaygorodtseva https://orcid.org/0000-0002-6031-5018

REFERENCES

[1] S. Avril, M. Grédiac, F. Pierron, Sensitivity of the virtual field method to noisy data, Comput. Mech. 2004, 34, 439.
[2] S. Avril, F. Pierron, M. A. Sutton, J. Yan, Identification of viscoplastic parameters and characterization of Lüders behaviour using digital image correlation and the virtual fields method, Mech. Mater. 2008, 40, 729.
[3] J. V. Beck, K. J. Arnold, Parameter Estimation in Engineering and Science, John Wiley and Sons 2007.
[4] C. Bröcker, A. Matzenmiller, An enhanced concept of rheological models to represent nonlinear thermoviscoplasticity and its energy storage behavior, Contin. Mech. Thermodyn. 2013, 25, 749.
[5] G. B. Broggiato, F. Campana, L. Cortese, The Chaboche nonlinear kinematic hardening model: calibration methodology and validation, Meccanica 2008, 43, 115.
[6] M. Grédiac, N. Fournier, P.-A. Paris, Y. Surrel, Direct determination of elastic constants of anisotropic plates by modal analysis: experimental results, J. Sound Vib. 1998, 210, 643.
[7] M. Grédiac, F. Pierron, Applying the virtual fields method to the identification of elasto-plastic constitutive parameters, Int. J. Plast. 2004, 22, 602.
[8] T. Harth, S. Schwan, J. Lehn, F. G. Kollmann, Identification of material parameters for inelastic constitutive models: statistical analysis and design of experiments, Int. J. Plast. 2004, 20, 1403.
[9] T. Harth, J. Lehn, Identification of material parameters for inelastic constitutive models using stochastic methods, GAMM-Mitt. 2007, 30, 409.
[10] S. Hartmann, J. Gibmeier, B. Scholtes, Experiments and material parameter identification using finite elements. Uniaxial tests and validation using instrumented indentation tests, Exp. Mech. 2006, 46, 5.
[11] P. Haupt, Continuum Mechanics and Theory of Materials, Springer 2002.
[12] R. Herzog, F. Ospald, Parameter identification for short fiber-reinforced plastics using optimal experimental design, Int. J. Numer. Meth. Engrg. 2016, https://doi.org/10.1002/nme.5371.
[13] H. Johansson, K. Runesson, Calibration of a class of non-linear viscoelasticity models with sensitivity assessment based on duality, Int. J. Numer. Meth. Engrg. 2007, 69, 2513.
[14] P. Junker, J. Nagel, An analytical approach to modeling the stochastic behavior of visco-elastic materials, ZAMM 2018, 98, 1249.
[15] M. Itskov, Tensor Algebra and Tensor Analysis for Engineers. With Applications to Continuum Mechanics, Springer 2019.
[16] M. M. Kaminski, A generalized stochastic perturbation technique for plasticity problems, Comput. Mech. 2010, 45, 349.
[17] R. Kießling, R. Landgraf, R. Scherzer, J. Ihlemann, Introducing the concept of directly connected rheological elements by reviewing rheological models at large strains, Int. J. Solids Struct. 2016, 97-98, 650.
[18] R. Kreißig, U. Benedix, U. J. Görke, M. Lindner, Identification and estimation of constitutive parameters for material laws in elastoplasticity, GAMM-Mitt. 2007, 30, 458.
[19] A. Lion, Constitutive modelling in finite thermoviscoplasticity: a physical approach based on nonlinear rheological elements, Int. J. Plast. 2000, 16, 469.
[20] M. I. A. Lourakis, A brief description of the Levenberg-Marquardt algorithm implemented by levmar, Foundation of Research and Technology 2005, 4, 1.
[21] R. Mahnken, E. Stein, A unified approach for parameter identification of inelastic material models in the frame of the finite element method, Comput. Methods Appl. Mech. Eng. 1996, 136, 225.
[22] K. Naumenko, H. Altenbach, Modeling of Creep for Structural Analysis, Springer 2007.
[23] W. Noll, A new mathematical theory of simple materials, Arch. Ration. Mech. Anal. 1972, 48, 1.
[24] B. Rahmania, I. Villemure, M. Levesque, Regularized virtual fields method for mechanical properties identification of composite materials, Comput. Methods Appl. Mech. Engrg. 2014, 278, 543.
[25] B. V. Rosić, A. Kachcerová, J. Sýkora, O. Pajonk, A. Litvinenko, H. G. Matthies, Parameter identification in a probabilistic setting, *Engineering Structures* 2013, 50, 179.
[26] B. V. Rosić, H. G. Matthies, Variational theory and computations in stochastic plasticity, *Archives of Computational Methods in Engineering* 2015, 22, 457.
[27] R. Scherzer, C. B. Silbermann, J. Ihleman, FE-simulation of the Presta joining process for assembled camshafts — local widening of shafts through rolling, *IOP Conf. Series: Materials Science and Engineering* 2016, 118, 012039.
[28] R. Scherzer, C. B. Silbermann, R. Landgraf, J. Ihleman, FE-simulation of the Presta joining process for assembled camshafts — modelling of the joining process *IOP Conf. Series: Materials Science and Engineering* 2017, 181, 012030.
[29] T. Seibert, J. Lehn, S. Schwan, F. G. Kollmann, Identification of material parameters for inelastic constitutive models: stochastic simulations for the analysis of deviations, *Contin. Mech. Thermodyn.* 2000, 12, 95.
[30] A. V. Shutov, J. Ihleman, Analysis of some basic approaches to finite strain elasto-plasticity in view of reference change, *Int. J. Plast.* 2014, 63, 183.
[31] A. V. Shutov, A. A. Kaygorodtseva, N. S. Dranishnikov, Optimal error functional for parameter identification in anisotropic finite strain elasto-plasticity, *IOP Conf. Series: Journal of Physics: Conf. Series* 2017, 894, 012133.
[32] A. V. Shutov, R. Kreißig, Application of a coordinate-free tensor formalism to the numerical implementation of a material model, *Z. Angew. Math. Mech.*, 2008, 88, 888.
[33] A. V. Shutov, R. Kreißig, Finite strain viscoplasticity with nonlinear kinematic hardening: Phenomenological modeling and time integration, *Comput. Methods Appl. Mech. Eng.* 2008, 197, 2015.
[34] A. V. Shutov, R. Kreißig, Regularized strategies for material parameter identification in the context of finite strain plasticity, *Technische Mechanik* 2010, 30, 280.
[35] A. V. Shutov, C. Kuprin, J. Ihleman, M. F.-X. Wagner, C. Silbermann, Experimentelle Untersuchung und numerische Simulation des inkrementellen Umformverfahrens von Stahl 42CrMo4, *Mater. wiss. Werkst. tech.* 2010, 41, 765.
[36] A. V. Shutov, A. Yu, Larichkin, Finite strain transient creep of D16T alloy: identification and validation employing heterogeneous tests, *IOP Conf. Series: Journal of Physics: Conf. Series* 2017, 894, 012110.
[37] A. V. Shutov, S. Pfeiffer, J. Ihleman, On the simulation of multi-stage forming processes: invariance under change of the reference configuration, *Materials Science and Engineering Technology* 2012, 43, 617.
[38] A. V. Shutov, Efficient implicit integration for finite-strain viscoplasticity with a nested multiplicative split, *Comput. Methods Appl. Mech. Eng.* 2016, 306, 151.
[39] A. V. Shutov, Efficient time stepping for the multiplicative Maxwell fluid including the Mooney-Rivlin hyperelasticity, *Int. J. Numer. Methods Eng.* 2017, 113, 1851.
[40] A. V. Shutov, Fully coupled two-phase composite model for microstructure evolution during non-proportional severe plastic deformation, *Defect Diffus. Forum* 2018, 385, 234.
[41] I. Vladimirov, M. Pietryga, S. Reese, On the modelling of non-linear kinematic hardening at finite strains with application to springback — Comparison of time integration algorithms, *Internat. J. Numer. Methods Engrg.* 2008, 75, 1.
[42] Z. L. Zhang, A sensitivity analysis of material parameters for the Gurson constitutive model, *Fatigue Fract. Engng Mater. Struct* 1996, 19, 561.

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