Neutrino spin-flavor oscillations in rapidly varying external fields

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Abstract

We study the neutrino oscillations in presence of rapidly varying external fields. The effective Hamiltonian describing the evolution of the system of the neutrinos is derived. We examine the resonance conditions in the case of general rapidly varying external fields. The neutrino spin-flavor oscillations in the combination of the constant transversal and twisting magnetic fields are considered. We also discuss the applications of the developed method to the neutrino oscillations in the magnetic fields of the Sun.

For the first time the idea of neutrino oscillations was put forward in Ref. [1]. Since then many experimental and theoretical studies of neutrino oscillations have been carried out. One of the most intriguing puzzles in neutrino physics is the solar neutrino deficit. Nowadays it is experimentally established (see, e.g., Ref. [2]) that the disappearance of solar electron neutrinos can be explained by the LMA-MSW solution [3, 4]. The process of neutrino oscillations is likely to be the most reliable explanation of the solar neutrino deficit. There are several theoretical models of the solar neutrino oscillations. The best solution of the solar neutrino problem is the LMA solution. However other scenarios such as spin-flavor precession (see Refs. [5–12]) are also considered. Spin-flavor precession requires a neutrino to possess a (transitional) magnetic moment (see Refs. [5, 6, 13]).

Recently the resonant neutrino spin-flavor oscillations were studied in Ref. [14] where an attempt was made to reproduce the data of the major solar neutrino experiments using the peak profiles of the solar magnetic field. In order to explain the time modulation of the observed solar neutrino flux, which is correlated with the solar magnetic activity, solar neutrinos were assumed to be converted into sterile neutrinos. The most favorable mass square difference was $\sim 10^{-8}$ eV$^2$. Different approach to the solar neutrino problem was made in Ref. [15]. Supposing that neutrino spin-flavor precession played a subdominant

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role in comparison with flavor oscillations the combined action of these two mechanisms was examined. It was also possible to impose the constraints on the characteristics of a neutrino, namely to restrict its magnetic moment, and the strength of the solar magnetic field. The solar electron neutrino transitions into non-electron antineutrino were studied in Ref. [16]. On the basis of the recent SNO data the indication of the neutrino Majorana magnetic moment was obtained in this paper.

In order to make any reliable predictions of the solar neutrino fluxes one should also take into account the realistic density and magnetic field profiles. A global analysis of the spin-flavor precession solution to the solar neutrino problem was given in Ref. [17]. Two flavor oscillations scenario in the optimized self-consistent magneto-hydrodynamical magnetic field profile was adopted.

Recent helioseismological data provide new information of the solar neutrino fluxes and the profile of the solar magnetic field. A careful analysis of the predicted solar neutrino fluxes was performed in Ref. [18]. The influence of the external magnetic field on the matter density profile and, hence, the neutrino oscillations was also discussed.

We studied neutrino oscillations in the electromagnetic fields of various configurations in our previous works. First it is necessary to mention that the Lorentz invariant formalism for the description of neutrino spin-flavor oscillations was elaborated in Ref. [19]. In Ref. [20] we worked out the method for the investigation of the neutrino evolution equation solution near the resonance point. This technique is of great importance when the evolution equation cannot be solved analytically. The parametric resonance in neutrino oscillations in periodically varying electromagnetic fields was elaborated in Ref. [21]. The major feature of these our papers was the use of the perturbation theory in solving the neutrino evolution equation. It should be noted that the perturbation theory was also used in Ref. [22] where the parametric resonance in neutrino oscillations in matter with periodically varying density was studied. In Ref. [23] we proposed the Lorentz invariant quasi-classical approach for the description of the neutrino spin oscillations in arbitrary non-derivative external fields.

In this paper we study the neutrino oscillations in presence of general rapidly varying fields. It should be noted that influence of the rapidly varying fields on mechanical oscillations was investigated in Ref. [24]. It is also well known that there are certain analogies between mechanical and neutrino oscillations. We start from the neutrino evolution equation with the Hamiltonian which accounts for the neutrino interaction with rapidly varying fields. Note that we do not fix the explicit form of the Hamiltonian. Then we derive the new effective Hamiltonian governing the time evolution of the averaged neutrino wave function. Therefore the obtained new Hamiltonian allows one to study the neutrino conversion in presence of arbitrary rapidly varying external fields. We examine the resonance conditions in the case of general rapidly varying external fields. Our result is also beyond the perturbation theory because no assumptions about the smallness of the rapidly varying fields are made. On the basis of the elaborated technique we consider the neutrino spin-flavor precession in the combination of the constant transversal and twisting magnetic fields. Some of the possible applications to the solar neutrino problem are also discussed.

Let us consider the evolution of the two neutrinos $\nu = (\nu_1, \nu_2)$, which can belong
to different flavors and helicity states. The evolution of the system is described by the Schrödinger type equation,

\[ i \frac{d\nu}{dt} = H\nu, \quad (1) \]

where the Hamiltonian \( H \) involves the external fields, e.g., electromagnetic field, interaction with matter etc., and the characteristics of the neutrino. Here we do not specify the explicit form of the Hamiltonian but just suppose that it is decomposed into two terms,

\[ H = H_0 + \mathcal{H}, \quad \mathcal{H}(t + \tau) = \mathcal{H}(t). \quad (2) \]

The first term in Eq. (2), \( H_0 \), corresponds to the neutrino interaction with constant or slowly varying external fields. In presence of only this term the solution of Eq. (1) can be easily found. The solution is known to be periodical with the typical frequency \( \Omega_0 \sim 1/L_{\text{eff}} \), where \( L_{\text{eff}} \) is the oscillations length. The second term in Eq. (2), \( \mathcal{H}(t) \), corresponds to rapidly varying external fields. The frequency \( \omega = 2\pi/\tau \) should be much greater than \( \Omega_0 \): \( \omega \gg \Omega_0 \). Note that we will not make any assumptions about the strength of the varying external fields.

We will seek the solution of the Eqs. (1) and (2) in the form (see also Ref. [24]),

\[ \nu(t) = \nu_0(t) + \xi(t). \quad (3) \]

In Eq. (3) the function \( \xi(t) \) is the small rapidly oscillating one. We will use the mean value of the function \( f(t) \) which is denoted as,

\[ \bar{f}(t) = \frac{1}{\tau} \int_t^{t+\tau} f(t') \, dt'. \]

Thus \( \nu_0(t) \) is the slowly varying function during the time \( \tau \), and the mean value of the function \( \xi(t) \) is zero.

Substituting Eq. (3) in Eq. (1) we obtain

\[ i\dot{\nu}_0 + i\dot{\xi} = H_0\nu_0 + \mathcal{H}\nu_0 + \mathcal{H}\xi + H_0\xi. \quad (4) \]

Despite the function \( \xi \) is small, its derivative can be, in principle, not small. Therefore equating the rapidly oscillating terms in left and right-handed sides of Eq. (4) we receive the equation for the function \( \xi \)

\[ i \frac{d\xi}{dt} = \mathcal{H}\nu_0, \quad (5) \]

Other rapidly oscillating terms in Eq. (4) contain small factor \( \xi \) and hence neglected in Eq. (5). Equation (5) can easily integrated and we find its solution

\[ \xi(t) = -i \left( \int \mathcal{H}(t) \, dt \right) \nu_0(t). \quad (6) \]

Here we take into account that variations of the function \( \nu_0 \) are small.
Averaging slowly varying terms in Eq. (4) over the period $\tau$ and allowing for 
\[ \dot{\nu}_0 = \dot{\nu}_0, \quad \dot{H}_0\nu_0 = H_0\nu_0, \quad \dot{H}_0\xi = H_0\xi = 0, \]
we obtain the equation for the function $\nu_0$
\[ i\frac{d\nu_0}{dt} = H_0\nu_0 + \overline{\mathcal{H}\xi}. \]
With help of Eq. (6) this equation can transformed into more common form
\[ i\frac{d\nu_0}{dt} = H_{\text{eff}}\nu_0, \quad (7) \]
where
\[ H_{\text{eff}} = H_0 - i\hat{\mathcal{H}}(\int \mathcal{H} dt). \quad (8) \]

In derivation of Eqs. (7) and (8) we did not make any assumptions about smallness of the interaction described by the Hamiltonian $\mathcal{H}$. Thus our result is beyond the perturbation theory used in Refs. [20–22].

The Hamiltonian $H$ can always be represented in the form,
\[ H = (\sigma h), \]
where $h$ is an arbitrary 3D-vector. Using this representation we discuss two cases
\[ \overleftrightarrow{H}_0 = (\sigma_\perp h_\perp), \mathcal{H} = \sigma_3 h_3, \]
\[ \overleftrightarrow{\mathcal{H}} = (\sigma_\perp h_\perp), H_0 = \sigma_3 h_3. \]
Here we introduced 2D-vectors $\sigma_\perp = (\sigma_1, \sigma_2)$ and $h_\perp = (h_1, h_2)$.

In the former case the application of the developed in the present paper technique leads to no interesting consequences. Indeed, using Eq. (8) we find that
\[ H_{\text{eff}} = (\sigma_\perp h_\perp) - i\hat{\mathcal{I}}h_3(\int h_3 dt), \]
where $\hat{\mathcal{I}}$ is the unit matrix. It is well known that terms in the Hamiltonian proportional to the unit matrix can be omitted. For example, if we study the neutrino oscillations in the matter with rapidly varying density, $n(t)$, it will cause no essential effect on the oscillations process. However, if one accounts for all components of the vector $h$ in $\mathcal{H}$, i.e. $\mathcal{H} = (\sigma h)$, it could result in some interesting effects. This case requires special careful examination.

Now we consider the latter case. First we introduce two new auxiliary 2D-vectors,
\[ a = h_\perp, \text{ and, } b = \int a \, dt. \]
Again using Eq. (8) we obtain that
\[ H_{\text{eff}} = \sigma_3 h_3 - i\hat{\mathcal{I}}(ab) + \sigma_3(a_1b_2 - a_2b_1) \to \sigma_3 \left[ h_3 + (a_1b_2 - a_2b_1) \right]. \]
Therefore the off-diagonal rapidly varying terms can shift the resonance point. It was shown in Ref. [23] that both axial-vector and tensor and pseudotensor interactions can cause the neutrino spin precession. Thus the rapidly varying external fields of the mentioned above types, e.g., electromagnetic fields, interaction with moving and polarized matter etc., will nontrivially affect the neutrino oscillation process. We study below one the possible examples.

In the following let us discuss the neutrino evolution in matter under the influence of a combination of the two types of magnetic fields

☞ Constant transversal magnetic field $B_0$,

☞ Twisting magnetic field $B(r)$.

Note that if magnetic field is constant in time we may consider only its transversal component with respect to neutrino velocity. These magnetic field configurations were studied separately in the majority of works devoted to the neutrino spin-flavor oscillations. For instance, only constant transversal component of a magnetic field was taken into account in Refs. [5–9], and the effect of only twisting magnetic field on the neutrino oscillations was considered in Refs. [10–12].

In our case the Hamiltonians $H_0$ and $\mathcal{H}$ have the form

$$H_0 = \begin{pmatrix} V/2 & \mu B_0 \\ \mu B_0 & -V/2 \end{pmatrix},$$

(9)

and

$$\mathcal{H} = \begin{pmatrix} 0 & \mu B e^{-i\omega t} \\ \mu B e^{i\omega t} & 0 \end{pmatrix}.$$  

(10)

where $\mu$ is the neutrino magnetic moment. In Eq. (9) we introduced the quantity,

$$\frac{V}{2} = \frac{\Delta m^2}{4E} \Theta - \frac{G_F}{\sqrt{2}} n_{\text{eff}},$$

where $\Theta$ is the function of the vacuum mixing angle $\theta_{\text{vac}}$ (the explicit form of $\Theta$ for various transitions of the $\nu_{iL} \leftrightarrow \nu_{jR}$ type can be found in Ref. [12]), $\Delta m^2$ is the difference of the neutrino mass squared, $E$ is the neutrino energy, $n_{\text{eff}}$ is the effective matter density, $\omega$ is the frequency of the transversal magnetic field variation, $G_F$ is the Fermi constant.

Using Eqs. (8)-(10) it is possible to derive the expression for the $H_{\text{eff}}$,

$$H_{\text{eff}} = \begin{pmatrix} V/2 - (\mu B)^2/\omega & \mu B_0 \\ \mu B_0 & -V/2 + (\mu B)^2/\omega \end{pmatrix}.$$  

(11)

Since $B_0$ and $B$ do not depend on time in Eq. (11), we can solve Eq. (7) for the case of the effective Hamiltonian given in Eq. (11). The transition probability is expressed in the following way,

$$P(t) = A \sin^2 \left( \frac{\pi t}{L} \right),$$

(12)
where
\[
A = \frac{(\mu B_0)^2}{[V/2 - (\mu B)^2/\omega]^2 + (\mu B_0)^2},
\]
and
\[
\frac{\pi}{L} = \Omega = \sqrt{[V/2 - (\mu B)^2/\omega]^2 + (\mu B_0)^2}.
\]

Thus the validity of the developed method (the frequency of the twisting magnetic field, \(\omega\), should be much greater than the characteristic “frequency” of a system in the absence of this field) can expressed as the constraint,
\[
\omega \gg \Omega_0 = \sqrt{(V/2)^2 + (\mu B_0)^2}.
\]

Now let us discuss the resonance conditions in neutrino oscillations in rapidly varying magnetic fields. From Eqs. (12) and (13) it follows that if the condition is satisfied,
\[
\frac{V}{2} \simeq \frac{(\mu B)^2}{\omega},
\]
then \(A \simeq 1\) and the transition probability can achieve great values. This phenomenon is analogous to resonance amplification of spin-flavor oscillations. However, in our case the resonance is attained not due to the zero value of the parameter \(V\).

We again emphasize that the strength of the twisting magnetic field \(B\) can be not small in the proposed technique. The only assumption made consists in the great value of \(\omega\). Therefore we can discuss two cases:

☞ \(B_0 \gg B\),
☞ \(B \gg B_0\).

In the former case the value of \(A\) in Eq. (13) is great independently of \(B\). This case corresponds to the usual spin-flavor precession in constant transversal magnetic field. However, in the latter case the value of \(A_0 = A(B = 0)\) is much less than unity. Hence the transition probability at the absence of the additional twisting magnetic field is small. In this case we can choose the parameters so [see Eq. (14)] that the amplitude of the transition probability is great. It is this situation which is of interest because the essential neutrino spin-flavor conversion is attained not in a strong constant magnetic field (which is very difficult to generate), but in the small perturbations of a magnetic field (which can be of sufficient strength). Such perturbations can have twisting structure. From Eqs. (13)-(14) it is possible to derive the restriction imposed on the \(B\) and \(B_0\),
\[
\left(\frac{B}{B_0}\right)^2 \gg 1.
\]

Now we consider the possible application of the developed technique to the spin-flavor neutrino oscillations in the magnetic fields of the Sun. The solar magnetic field is unlikely
to be only either constant transversal or twisting. Therefore we can apply the elaborated in this paper method to the description of the solar neutrino conversion. The propagation of the neutrino flux in the combination of constant transversal and twisting solar magnetic fields is schematically depicted in Fig. 1. We consider one of the possible channels of neutrino oscillations, namely $\nu_{eL} \leftrightarrow \nu_{\mu R}$ conversion. First we should estimate the parameter $V$ in Eq. (9). In our case the function $\Theta$ and the effective matter density have the form (see, e.g., Ref. [12]),

$$\Theta = \frac{1 + \cos 2\theta_{\text{vac}}}{2}, \quad n_{\text{eff}} = \left( n_e - \frac{1}{2} n_n \right).$$

Let us discuss the neutrino with the following properties: $\Delta m^2 \approx 10^{-5}$ eV$^2$, $\theta_{\text{vac}} \approx \pi/4$, which are not excluded by the modern experimental results (see, for instance, Ref. [25]). We take the neutrino energy $E \approx 10$ MeV, which corresponds to the $^8$B solar neutrinos. Matter is supposed to consist mainly of the hydrogen, i.e. $n_n \approx 0$, $n_e = n_p$ (as a consequence of the system electroneutrality) and have the density $d \approx 1.4 \text{ g} \cdot \text{cm}^{-3}$, which is close to the mean density of the solar matter. For these parameters we obtain that $V/2 \approx 10^{-14}$ eV.

Now let us evaluate the strength of the magnetic fields, $B_0$ and $B$, necessary for the 10% conversion of the initial $\nu_{eL}$ beam. We suppose that neutrinos conversion occurs along the distance $D = t \approx R_\odot/10 \approx 3.5 \times 10^{14}$ eV$^{-1}$, where $R_\odot$ is the solar radius. We also suppose that neutrinos have the transitional magnetic moment $\mu = 10^{-10} \mu_B$. Setting $P(t) = 0.1$ in Eq. (12) we obtain $B_0 \approx 18.5$ kG. From the condition (13) we can derive the strength of the twisting magnetic field: $B \approx 3.2 B_0 \approx 59.2$ kG. The resonance condition (14) is satisfied if $\omega \approx 1.1 \times 10^{-13}$ eV. On the other hand $\pi/D \approx 9.0 \times 10^{-15}$ eV. Thus several (about 10) periods of the magnetic field variation are along the neutrino trajectory.

In conclusion we note that in this paper we have examined the neutrino oscillations in

![Figure 1: Neutrino flux propagation in the magnetic fields of the Sun.](image)
general rapidly varying fields. We have started with the usual Schrödinger type evolution equation. The Hamiltonian involved both slowly and rapidly varying in time terms. Instead of solving the evolution equation directly we have derived the new effective Hamiltonian which described the evolution of the averaged neutrino wave function. This new effective Hamiltonian enabled one to study the neutrino oscillations in arbitrary rapidly varying external fields. We have examined the resonance conditions in the case of general rapidly varying external fields. It has been revealed that matter with rapidly varying density caused no effect to neutrino oscillations. We have demonstrated that rapidly varying axial-vector (e.g., interaction with moving or polarized matter), tensor and pseudotensor external fields could result in nontrivial effects in neutrino oscillations. It is worth mentioning that, in contrast to Refs. [20, 21], the strength of rapidly varying fields have not been limited. Therefore the elaborated method was beyond the perturbation theory since we have not carried out any expansions over the strength of the external fields. On the basis of the derived new effective Hamiltonian we have considered the neutrino spin-flavor oscillations in the combination of the constant transversal and twisting magnetic fields. It has been shown that described mechanism of the neutrino oscillations could be important in the neutrino spin-flavor conversion in the magnetic fields of the Sun.

References

[1] B. Pontecorvo, “Inverse beta processes and nonconservation of lepton charge,” JETP, vol. 7, p. 172, 1958.

[2] S. N. Ahmed et al., “Measurement of the total active $^8$B solar neutrino flux at the Sudbury Neutrino Observatory with enhanced neutral current sensitivity,” Phys. Rev. Lett., vol. 92, p. 181301, 2004, nucl-ex/0309004.

[3] L. Wolfenstein, “Neutrino oscillations in matter,” Phys. Rev. D, vol. 17, pp. 2369–2374, 1978.

[4] S. P. Mikheev and A. Yu. Smirnov, “Resonance enhancement of oscillations in matter and solar neutrino spectroscopy,” Sov. J. Nucl. Phys., vol. 42, pp. 913–917, 1985.

[5] K. Fujikawa and R. E. Shrock, “Magnetic moment of a massive neutrino and neutrino-spin rotation,” Phys. Rev. Lett., vol. 45, pp. 963–966, 1980.

[6] J. Schechter and J. W. F. Valle, “Majorana neutrinos and magnetic fields,” Phys. Rev. D, vol. 24, no. 7, pp. 1883–1889, 1981.

[7] M. B. Voloshin, M. I. Vysotskii, and L. B. Okun’, “Electromagnetic properties of neutrino and possible semianual variation cycle of the solar neutrino flux,” Sov. J. Nucl. Phys., vol. 44, p. 440, 1986.

[8] C.-S. Lim and W. J. Marciano, “Resonant spin-flavor precession of solar and supernova neutrinos,” Phys. Rev. D, vol. 37, pp. 1368–1373, 1988.
[9] E. Akhmedov, “Resonant amplification of neutrino spin rotation in matter and the solar-neutrino problem,” *Phys. Lett. B*, vol. 213, pp. 64–68, 1988.

[10] A. Y. Smirnov, “The geometrical phase in neutrino spin precession and the solar neutrino problem,” *Phys. Lett B*, vol. 260, pp. 161–164, 1991.

[11] E. K. Akhmedov, S. T. Petcov, and A. Y. Smirnov, “Neutrinos with mixing in twisting magnetic fields,” *Phys. Rev. D*, vol. 48, pp. 2167–2181, 1993.

[12] G. G. Likhachev and A. I. Studenikin, “Neutrino oscillations in magnetic fields of the Sun, supernovae and neutron stars,” *JETP*, vol. 81, p. 419, 1995.

[13] M. Dvornikov and A. Studenikin, “Electric charge and magnetic moment of a massive neutrino,” *Phys. Rev. D*, vol. 69, no. 7, p. 073001, 2004, hep-ph/0305206.

[14] B. C. Chauhan and J. Pulido, “LMA and sterile neutrinos: a case for resonance spin flavour precession?,” *J. High Energy Phys.*, vol. 0406, p. 008, 2004, hep-ph/0402194.

[15] E. K. Akhmedov and J. Pulido, “Solar neutrino oscillations and bounds on neutrino magnetic moment and solar magnetic field,” *Phys. Lett. B*, vol. 553, pp. 7–17, 2003, hep-ph/0209192.

[16] S. K. Kang and C. S. Kim, “Implications of SNO and BOREXINO results on neutrino oscillations and Majorana magnetic moment,” *Phys. Lett. B*, vol. 584, pp. 98–102, 2004, hep-ph/0403059.

[17] J. Barranco, O. Miranda, T. Rashba, V. Semikoz, and J. Valle, “Confronting spin flavor solutions of the solar neutrino problem with current and future solar neutrino data,” *Phys. Rev. D*, vol. 66, p. 093009, 2002, hep-ph/0207326.

[18] S. Couvidat, S. Turck-Chieze, and A. G. Kosovichev, “New solar seismic models and the neutrino puzzle,” 2002, astro-ph/0203107.

[19] A. Egorov, A. Lobanov, and A. Studenikin, “Neutrino oscillations in electromagnetic fields,” *Phys. Lett. B*, vol. 491, pp. 137–142, 2000, hep-ph/9910476.

[20] M. S. Dvornikov and A. I. Studenikin, “Neutrino oscillations in the field of a linearly polarized electromagnetic wave,” *Phys. At. Nucl.*, vol. 64, no. 9, pp. 1624–1627, 2001.

[21] M. S. Dvornikov and A. I. Studenikin, “Parametric resonance in neutrino oscillations in periodically varying electromagnetic fields,” *Phys. At. Nucl.*, vol. 67, no. 4, pp. 719–725, 2004.

[22] P. M. Fishbane and S. G. Gasiorowicz, “Equations for neutrino propagation in matter,” *Phys. Rev. D*, vol. 64, p. 113017, 2001, hep-ph/0012230.

[23] M. Dvornikov and A. Studenikin, “Neutrino spin evolution in presence of general external fields,” *J. High Energy Phys.*, vol. 09, p. 016, 2002, hep-ph/0202113.
[24] P. L. Kapitza, “Pendulum with vibrating suspension,” *Usp. Fiz. Nauk*, vol. 44, no. 1, pp. 7–20, 1951.

[25] K. Eguchi *et al.*, “First results from KamLAND: Evidence for reactor anti-neutrino disappearance,” *Phys. Rev. Lett.*, vol. 90, p. 021802, 2003, hep-ex/0212021.