ABSTRACT

Recently, in a series of works by Liu and Li (L&L), they claimed that there exists a timing asynchrony of $-25.6\text{ ms}$ between the spacecraft attitude and radiometer output timestamps in the original raw WMAP time-ordered data (TOD). L&L reprocessed the WMAP data while the aforementioned timing asynchrony has been corrected, and they obtained an alternative CMB map in which the quadrupole dropped to nearly zero. In the present work, we try to see the implications to dark energy cosmology if L&L are right. While L&L claimed that there is a bug in the WMAP pipeline which leads to significantly different cosmological parameters, an interesting question naturally arises, namely, how robust is the current dark energy cosmology with respect to systematic errors and bugs? So, in this work, we adopt the alternative CMB data of L&L as a strawman to study the robustness of dark energy predictions.

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I. INTRODUCTION

Dark energy cosmology has been one of the most active fields in astronomy and physics, since the exciting discovery of current accelerated expansion of our universe [1]. The first evidence came from the observation of Type Ia supernovae (SNIa) in 1998 [2]. Five years later, cosmology entered the so-called “precision era” in 2003 when the first year Wilkinson Microwave Anisotropy Probe (WMAP) observations of the cosmic microwave background (CMB) had been released [3]. Up to now, the CMB observation is still a very powerful probe for cosmology, and the CMB data from WMAP mission provide the most important basis.

However, in the passed years, some unusual phenomena have been found from the CMB data released by WMAP team. Remarkably, among these unusual phenomena, it is claimed that there exists a preferred direction in the CMB temperature map (known as the “Axis of Evil” in the literature) [4]. In fact, there are two different approaches to deal with this problem. The first one is to admit that this phenomenon is an observational fact and try to explain it in the cosmological theories (see e.g. [5–7] and references therein). The second approach is to consider instead that this phenomenon might be an artifact due to the observational systematics. For example, in a series of works by Liu and Li (L&L) [8–10], they claimed that there exists a timing asynchrony of $-25.6$ ms between the spacecraft attitude and radiometer output timestamps in the original raw WMAP time-ordered data (TOD). If one fixed this problem, the most of CMB quadrupole component disappear. It is worth noting that recently their findings has been confirmed by several independent authors (see e.g. [11–13]). In [8], L&L reprocessed the WMAP data while the aforementioned timing asynchrony has been corrected, and they obtained an alternative CMB map in which the quadrupole dropped to nearly zero. In addition, using the alternative CMB data, they constrained the cosmological parameters of the standard flat ΛCDM model, and found that these cosmological parameters have been changed notably. For convenience, we reproduce their main results in Table I. It is easy to see that the fractional matter density $\Omega_m = \Omega_b + \Omega_{c,0}$ of L&L [8] is considerably larger than the one of WMAP [14, 15].

| Description                      | Symbol | WMAP [14, 15] | L&L [8] |
|----------------------------------|--------|--------------|--------|
| Hubble constant (km/s/Mpc)       | $H_0$  | $71.9^{+2.7}_{-2.5}$ | $71.0 \pm 2.7$ |
| Baryon density                   | $\Omega_b$ | $0.0441 \pm 0.0030$ | $0.052 \pm 0.0030$ |
| Cold dark matter density         | $\Omega_c$ | $0.214 \pm 0.027$ | $0.270 \pm 0.027$ |
| Dark energy density              | $\Omega_{\Lambda}$ | $0.742 \pm 0.030$ | $0.678 \pm 0.030$ |
| Fluc. Ampl. at 8$h^{-1}$ Mpc     | $\sigma_8$ | $0.796 \pm 0.036$ | $0.921 \pm 0.036$ |
| Scalar spectral index            | $n_s$  | $0.964^{+0.014}_{-0.015}$ | $0.957 \pm 0.015$ |
| Reionization optical depth       | $\tau$ | $0.087 \pm 0.017$ | $0.109 \pm 0.017$ |

TABLE I: The cosmological constraints on the standard flat ΛCDM model, reproduced from [8].

To our knowledge, there is still controversy about the findings of L&L in the community. In the present work, we try to be neutral as much as possible. Instead of discussing the detailed data process of WMAP mission (as in the works by L&L [8–10] or other authors [11–13]), in this work we would like to see the implications to dark energy cosmology if L&L are right. While L&L claimed that there is a bug in the WMAP pipeline which leads to significantly different cosmological parameters, an interesting question naturally arises, namely, how robust is the current dark energy cosmology with respect to systematic errors and bugs? So, in this work, we adopt the alternative CMB data of L&L as a strawman to study the robustness of dark energy predictions.

As is well known, using the full CMB data to perform a global fitting consumes a large amount of computation time and power. As a good alternative, one can instead use the shift parameter $R$ from the CMB data, which has been considered extensively in the literature. It is argued that the shift parameter $R$ is model-independent, and it contains the main information of the CMB data [16]. So, in Sec. [1] we firstly derive the corresponding shift parameter $R$ from the alternative CMB data of L&L [8].
In addition, we briefly introduce the other observational data, such as SNIa and the baryon acoustic oscillation (BAO), which are also used in the present work. In Sec. III we perform the data analysis by using the cosmological observations introduced in Sec. II. In particular, we discuss the tension between CMB and SNIa in Sec. III A; we study the age problem in dark energy models in Sec III B; and the cosmological constraints on dark energy models are considered in Sec. III C. Finally, the conclusion and discussions are given in Sec. IV.

II. OBSERVATIONAL DATA

A. Shift parameter $R$ from the alternative CMB data

As mentioned above, using the shift parameter $R$ from the CMB data is a good approach to fit the cosmological models. As is well known, the shift parameter $R$ of the CMB data is defined by [17] (see also [14–16])

$$R \equiv \Omega_{m0}^{1/2} \int_0^{z_*} \frac{dz}{E(z)},$$

where $\Omega_{m0}$ is the present fractional energy density of pressureless matter; $E \equiv H/H_0$ in which $H \equiv \dot{a}/a$ is the Hubble parameter; $a = (1 + z)^{-1}$ is the scale factor (we have set $a_0 = 1$; the subscript “0” indicates the present value of corresponding quantity; $z$ is the redshift); a dot denotes the derivative with respect to cosmic time $t$; the redshift of recombination $z_*$ is given by [18] (see also [14, 15])

$$z_* = 1048 \left[ 1 + 0.00124 \left( \Omega_{b0} h^2 \right)^{-0.738} \right] \left[ 1 + g_1 \left( \Omega_{m0} h^2 \right)^{g_2} \right],$$

where $\Omega_{m0} = \Omega_{b0} + \Omega_{c0}$, and $\Omega_{b0}$, $\Omega_{c0}$ are the present fractional energy densities of baryon and cold dark matter, respectively; $h$ is the Hubble constant $H_0$ in units of 100 km/s/Mpc; and

$$g_1 = \frac{0.0783 \left( \Omega_{b0} h^2 \right)^{-0.238}}{1 + 39.5 \left( \Omega_{b0} h^2 \right)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1 \left( \Omega_{b0} h^2 \right)^{1.81}}.$$

The shift parameter $R$ relates the angular diameter distance to the last scattering surface, the comoving size of the sound horizon at $z_*$, and the angular scale of the first acoustic peak in CMB power spectrum of temperature fluctuations [14, 16, 17].

In principle, one should obtain $z_*$ and $R$ from the full (alternative) CMB data. However, this is a hard work consuming a large amount of computation time and power. On the other hand, L&L have not published their full alternative CMB data in which the timing asynchrony of $\sim 25.6$ ms between the spacecraft attitude and radiometer output timestamps in the original raw WMAP time-ordered data (TOD) has been corrected. Therefore, in this work we use instead the Monte Carlo method [12]. We choose the standard flat $\Lambda$CDM model to be the fiducial model, since its cosmological parameters have been constrained by both WMAP [14, 15] and L&L [8] (see Table I). We generate the Gaussian distributions for $\Omega_{b0}$, $\Omega_{c0}$ and $h$ from their best-fit parameters and the corresponding 1$\sigma$ uncertainties given in Table I. Then, we randomly sample the parameters $\Omega_{b0}$, $\Omega_{c0}$ and $h$ from their corresponding Gaussian distribution for $N_{mc}$ times. For each $(\Omega_{b0}, \Omega_{c0}, h)$, we can obtain the corresponding $z_*$ and $R$ from Eqs. (2) and (1), respectively. Finally, we can determine the means and the corresponding 1$\sigma$ uncertainties for $z_*$ and $R$ from these $N_{mc}$ samples, respectively. Notice that in this work we have done $N_{mc} = 10^6$ samplings. At first, we check this method by calculating $z_*$ and $R$ from the WMAP data, and find that our results are well consistent with the ones given by WMAP team [14, 15]. Then, we turn to the alternative CMB data of L&L [8], and finally find that the corresponding redshift of recombination $z_*$ is given by

$$z_* = 1088.610 \pm 2.373,$$

and the shift parameter $R$ is given by

$$R = 1.761 \pm 0.014.$$
Obviously, the shift parameter $R$ from L&L’s alternative CMB data is fairly larger than the one of WMAP [14, 15]. It is argued that the shift parameter $R$ is model-independent, and it contains the main information of the CMB data [16]. Therefore, in the literature the shift parameter $R$ has been used extensively to constrain cosmological models. For the alternative CMB data of L&L [8], the corresponding $\chi^2$ from the shift parameter $R$ reads

$$\chi^2_{\text{CMB}} = \left( \frac{R - 1.761}{0.014} \right)^2.$$  (6)

B. SNIa and BAO data

In addition to the CMB data, we also consider the observations of SNIa and BAO. The data points of the latest 557 Union2 SNIa compiled in [20] are given in terms of the distance modulus $\mu_{\text{obs}}(z_i)$. On the other hand, the theoretical distance modulus is defined as

$$\mu_{\text{th}}(z_i) \equiv 5 \log_{10} D_L(z_i) + \mu_0,$$  (7)

where $\mu_0 \equiv 42.38 - 5 \log_{10} h$ and

$$D_L(z) = (1+z) \int_0^z \frac{dz}{E(z; p)},$$  (8)

in which $p$ denotes the model parameters. The corresponding $\chi^2$ from the 557 Union2 SNIa reads

$$\chi^2_{\mu}(p) = \sum_i \frac{|\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)|^2}{\sigma^2_{\mu_{\text{obs}}}(z_i)},$$  (9)

where $\sigma$ is the corresponding 1σ error. The parameter $\mu_0$ is a nuisance parameter but it is independent of the data points. One can perform an uniform marginalization over $\mu_0$. However, there is an alternative way. Following [21, 22, 28], the minimization with respect to $\mu_0$ can be made by expanding the $\chi^2_{\mu}$ of Eq. (9) with respect to $\mu_0$ as

$$\chi^2_{\mu}(p) = \tilde{A} - 2\mu_0 \tilde{B} + \mu_0^2 \tilde{C},$$  (10)

where

$$\tilde{A}(p) = \sum_i \frac{|\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, p)|^2}{\sigma^2_{\mu_{\text{obs}}}(z_i)},$$

$$\tilde{B}(p) = \sum_i \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0, p)}{\sigma^2_{\mu_{\text{obs}}}(z_i)},$$

$$\tilde{C} = \sum_i \frac{1}{\sigma^2_{\mu_{\text{obs}}}(z_i)}.$$  (11)

Eq. (10) has a minimum for $\mu_0 = \tilde{B}/\tilde{C}$ at

$$\tilde{\chi}^2_{\mu}(p) = \tilde{A}(p) - \frac{\tilde{B}(p)^2}{\tilde{C}}.$$  (11)

Since $\chi^2_{\mu, \text{min}} = \tilde{\chi}^2_{\mu, \text{min}}$ obviously, we can instead minimize $\tilde{\chi}^2_{\mu}$ which is independent of $\mu_0$. It is worth noting that the corresponding $h$ can be determined by $\mu_0 = \tilde{B}/\tilde{C}$ for the best-fit parameters.

Similar to the case of shift parameter $R$, one can use the the distance parameter $A$ of the measurement of the BAO peak in the distribution of SDSS luminous red galaxies [23], which is also model-independent and contains the main information of the observation of BAO. The distance parameter $A$ is defined by [23]

$$A \equiv \Omega_m^{1/2} E(z_b)^{-1/3} \left[ \frac{1}{z_b} \int_0^{z_b} \frac{dz}{E(z)} \right]^{2/3},$$  (12)
where \( z_b = 0.35 \). In [24], the value of \( A \) has been determined to be \( 0.469 (n_s/0.98)^{-0.35} \pm 0.017 \). Here the scalar spectral index \( n_s \) is taken to be 0.957 from L&L [8] (see also Table I). The corresponding \( \chi^2_{BAO} = (A - A_{\text{obs}})^2 / \sigma_A^2 \).

The best-fit model parameters are determined by minimizing the corresponding \( \chi^2 \). As in [25, 26, 76], the 68.3% confidence level (C.L.) is determined by \( \Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} \leq 1, 2 \) and 3, respectively, where \( n_p \) is the number of free model parameters. Similarly, the 95.4% C.L. is determined by \( \Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} \leq 4, 6.17 \) and 8.02 for \( n_p = 1, 2 \) and 3, respectively.

| Observation | \( \chi^2_{\text{min}} \) | \( \Omega_{m0} \) | \( w_0 \) | \( w_a \) |
|-------------|----------------|-------------|---------|---------|
| SNIa        | 541.430        | 0.420       | -0.863  | -5.490  |
| SNIa+A      | 542.636        | 0.278       | -1.007  | -0.105  |
| SNIa+R      | 542.523        | 0.295       | -1.021  | -0.314  |
| SNIa+A+R    | 542.936        | 0.288       | -0.969  | -0.529  |

TABLE II: The \( \chi^2_{\text{min}} \) and the best-fit values of \( \Omega_{m0} \), \( w_0 \) and \( w_a \) for the various observations.

FIG. 1: The 68.3% and 95.4% C.L. contours in the \( w_0 - w_a \) plane and the \( w_0 - \Omega_{m0} \) plane for the observations of SNIa alone (blue dashed lines) and SNIa+A+R (black solid lines). We also show the best-fit values for the observations of SNIa alone (blue box), SNIa+A (red star), SNIa+R (green triangle) and SNIa+A+R (black point).

III. DATA ANALYSIS

In this section, we perform the data analysis by using the cosmological observations mentioned in the previous section. In particular, we discuss the tension between CMB and SNIa in Sec. III A; we study the age problem in dark energy models in Sec. III B; and the cosmological constraints on dark energy models are considered in Sec. III C.

A. Tension between CMB and SNIa

There is a long history of the tension between CMB and SNIa. In [27, 28], the Gold04 SNIa dataset [29] has been shown to be in 2\( \sigma \) tension with the SNLS SNIa dataset [30] and the WMAP observations.
Although the Gold04 sample was updated to Gold06 [31], several years later, the tension still persisted. In [32], the tension and systematics in the Gold06 SNIa dataset has been investigated in great detail. Later, the number of SNIa has been significantly increased to 300 ~ 400 when the Union [33] and Constitution [34] SNIa datasets have been released. Subsequently, in [35] it is found that there is still significant tension between the WMAP CMB data and the Union/Constition SNIa datasets. The latest SNIa dataset is the Union2 compiled in [20], which is an updated version of Union. However, in [36, 37] it is found that the tension between the WMAP CMB data and the Union2 SNIa dataset still persisted.

Obviously, the tension comes from both sides: CMB and SNIa. In [32, 35–37], the authors mainly concentrated in SNIa datasets, and tried to find the outliers responsible for the tension. However, from the works by L&L [8–10], we become aware of the possible systematics in the CMB data. So, it is natural to see whether the alternative CMB data of L&L [8] can alleviate the tension.

In this subsection, we follow the discussions in [35], and consider the familiar Chevallier-Polarski-Linder (CPL) model [38], in which the equation-of-state parameter (EoS) of dark energy is parameterized as

\[ w(z) = w_0 + w_a(1 - a) = w_0 + w_a \frac{z}{1 + z}, \]

where \( w_0 \) and \( w_a \) are constants. As is well known, the corresponding \( E(z) \) is given by [19, 32, 41, 71]

\[ E(z) = \left[ \Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})(1 + z)^3(1 + w_0 + w_a) \exp \left( -\frac{3w_a z}{1 + z} \right) \right]^{1/2}. \]

Here, we consider the shift parameter \( R \) from the alternative CMB data of L&L [8], the latest Union2 SNIa dataset [20], and the distance parameter \( A \) [24] with \( n_s \) from L&L [8]. We fit the CPL model to the observations of SNIa alone, SNIa, SNIa+R, and SNIa+R, respectively. The best-fit values are presented in Table III. In Fig. 1 we also present the 68.3% and 95.4% C.L. contours in the \( w_0 - w_a \) plane and the \( w_0 - \Omega_{m0} \) plane. From Fig. 1 we find that there is still tension (beyond 2σ) between the alternative CMB data of L&L [8] and the latest Union2 SNIa dataset [20]. However, when we compare Fig. 1 and Table III of the present work with Figs. 1, 2 and Tables I, II of [32], it is easy to see that the tension has been alleviated to some extent in fact. The alleviation mainly comes from two sides: the best-fit \( \Omega_{m0} \) for L&L’s alternative CMB data is increased, whereas the best-fit \( \Omega_{m0} \) for the Union2 SNIa dataset is decreased. It is worth noting that the contribution to the alleviation from Union2 SNIa dataset is larger than the one from L&L’s alternative CMB data [8].

### B. Age problem

In history, the age problem played an important role in the cosmology for many times (see e.g. [42] for a brief review). The main idea is very simple: the universe cannot be younger than its constituents. For example, the matter-dominated Friedmann-Robertson-Walker (FRW) universe can be ruled out because its age is smaller than the ages inferred from old globular clusters. The age problem becomes even more serious when we consider the age of the universe at high redshift (rather than today, \( z = 0 \)). So far, there are some old high redshift objects (OHROs) been discovered. For instance, the 3.5 Gyr old galaxy LBDS 53W091 at redshift \( z = 1.55 \) [15], the 4.0 Gyr old galaxy LBDS 53W069 at redshift \( z = 1.43 \) [14], the 4.0 Gyr old radio galaxy 3C 65 at \( z = 1.175 \) [13], and the high redshift quasar B1422+231 at \( z = 3.62 \) whose best-fit age is 1.5 Gyr with a lower bound of 1.3 Gyr [10]. In addition, the old quasar APM 08279+5255 at \( z = 3.91 \) is also used extensively, whose age is estimated to be 2.0—3.0 Gyr [17, 18]. To assure the robustness of our analysis, we use the most conservative lower age estimate 2.0 Gyr for the old quasar APM 08279+5255 at \( z = 3.91 \) [17, 18], and the lower age estimate 1.3 Gyr for the high redshift quasar B1422+231 at \( z = 3.62 \) [46]. In the literature, there are many works on the age problem in the dark energy models, see e.g. [42, 49–55] and references therein.

In this subsection, we would like to consider the age problems in the flat ΛCDM model and the holographic dark energy (HDE) model, which have been discussed in the literature by using the earlier observational data (see e.g. [42, 50, 51, 54]). Of course, our main goal is to see whether the age problem can be alleviated with the alternative CMB data of L&L [8].
FIG. 2: The $\chi^2$ and likelihood $\mathcal{L} \propto e^{-\chi^2/2}$ as functions of $\Omega_{m0}$ for the $\Lambda$CDM model.

| Description       | $\Omega_{m0}$ | $S(3.91)$ | $S(1.43)$ | $S(1.55)$ | $S(1.75)$ | $S(3.62)$ |
|-------------------|---------------|-----------|-----------|-----------|-----------|-----------|
| best fit         | 0.288         | 0.799     | 1.120     | 1.195     | 1.310     | 1.345     |
| $1\sigma$ lower bound | 0.275         | 0.817     | 1.144     | 1.221     | 1.337     | 1.376     |
| $1\sigma$ upper bound | 0.302         | 0.781     | 1.097     | 1.170     | 1.283     | 1.315     |
| $2\sigma$ lower bound | 0.263         | 0.836     | 1.169     | 1.248     | 1.366     | 1.408     |
| $2\sigma$ upper bound | 0.316         | 0.763     | 1.074     | 1.145     | 1.257     | 1.286     |

TABLE III: The ratio $S(z) \equiv T_z(z)/T_{obj}$ at $z = 3.91, 1.43, 1.55, 1.75$ and $3.62$, for various model parameters $\Omega_{m0}$ (within $2\sigma$ uncertainty) of the $\Lambda$CDM model (the corresponding $h = 0.696$).

1. Age problem in the $\Lambda$CDM model

The age of our universe at redshift $z$ is given by [42, 50, 54]

$$t(z) = \int_0^\infty \frac{d\tilde{z}}{(1 + \tilde{z})H(\tilde{z})}.$$ (15)

It is convenient to introduce the so-called dimensionless age parameter [50, 54]

$$T_z(z) \equiv H_0 t(z) = \int_0^\infty \frac{d\tilde{z}}{(1 + \tilde{z})E(\tilde{z})}.$$ (16)

At any redshift, the age of our universe should be larger than, at least equal to, the age of the OHRO, namely

$$T_z(z) \geq T_{obj} \equiv H_0 t_{obj}, \quad \text{or equivalently,} \quad S(z) \equiv T_z(z)/T_{obj} \geq 1,$$ (17)

where $t_{obj}$ is the age of the OHRO. It is worth noting that from Eq. (16), $T_z(z)$ is independent of the Hubble constant $H_0$. On the other hand, from Eqs. (17), $T_{obj}$ is proportional to the Hubble constant $H_0$. The lower $H_0$, the smaller $T_{obj}$ is.
As is well known, for the flat ΛCDM model,

\[ E(z) = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}. \]  

(18)

It is easy to obtain the total \( \chi^2 \) and likelihood \( \mathcal{L} \propto e^{-\chi^2/2} \) as a function of the single model parameter \( \Omega_m \) for the ΛCDM model, by using the combined observations of the shift parameter \( R \) from the alternative CMB data of L&L [8], the latest Union2 SNIa dataset [20], and the distance parameter \( A \) [24] with \( n_s \) from L&L [8]. We present the corresponding \( \chi^2 \) and likelihood due to the influence of SNIa and BAO, it is still larger than the one from WMAP alone [14, 15] and the one from WMAP+SNIa+BAO [14, 15].

In Table III, we present the ratio \( S(z) \equiv T_z(z)/T_{obj} \) at \( z = 3.91, 1.43, 1.55, 1.175 \) and 3.62, for various model parameters \( \Omega_m \) within 2σ uncertainty of the ΛCDM model. Obviously, \( T_z(z) > T_{obj} \) holds at \( z = 1.43, 1.55, 1.175 \) and 3.62, whereas \( T_z(z) < T_{obj} \) at \( z = 3.91 \). So, the old quasar APM 08279+5255 at \( z = 3.91 \) cannot be accommodated (beyond 2σ). The age problem still exists in the ΛCDM model, even when the alternative CMB data of L&L [8] has been taken into account.

Let us have a closer observation. As mentioned above, the alternative CMB data of L&L [8] favor a larger \( \Omega_m \) (this point has also been mentioned by L&L themselves [8]). So, it is natural to see how the model parameter \( \Omega_m \) affects the age of our universe at redshift \( z \). In Fig. 3 we present the dimensionless age parameter \( T_z \) as a function of redshift \( z \) for various \( \Omega_m \) in the ΛCDM model (notice that from Eq. (16), \( T_z(z) \) is independent of the Hubble constant \( H_0 \)). It is easy to see that at any redshift \( z \), the larger \( \Omega_m \), the smaller age is. Therefore, the age problem becomes even worse in the ΛCDM model, since the alternative CMB data of L&L [8] favor a larger \( \Omega_m \).

2. Age problem in the HDE model

Previously, in [54], the age problem in the HDE model has been discussed in great detail. Here we would like to see whether the age problem can be alleviated with the alternative CMB data of L&L [8].
The HDE model \cite{56, 57} was proposed from the holographic principle \cite{58} in the string theory. HDE is now an interesting candidate of dark energy, which has been studied extensively in the literature. The energy density of HDE reads \cite{57} (see also e.g. \cite{54, 59, 60})

\[
\rho_\Lambda = 3c^2 m_p^2 L^{-2},
\]

where the numerical constant $3c^2$ is introduced for convenience, and $m_p \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass. In \cite{57}, $L$ has been chosen to be the future event horizon

\[
R_h = a \int^\infty_1 \frac{d\tilde{t}}{a} = a \int^\infty_a \frac{d\tilde{a}}{H\tilde{a}^2}.
\]

From Eqs. (11), (20), and the energy conservation equation $\dot{\rho}_\Lambda + 3H\rho_\Lambda (1 + w_\Lambda) = 0$, it is easy to find that (see e.g. \cite{54, 57, 59, 60})

\[
\frac{d\Omega_\Lambda}{dz} = -(1+z)^{-1} \Omega_\Lambda (1 - \Omega_\Lambda) \left(1 + \frac{2}{c} \sqrt{\Omega_\Lambda}\right),
\]

where $\Omega_\Lambda$ is the fractional energy density of HDE. From the Friedmann equation $H^2 = \left(\rho_m + \rho_\Lambda\right) / \left(3m_p^2\right)$, we have

\[
E(z) = \left[\frac{\Omega_{m0}(1+z)^3}{1 - \Omega_\Lambda(z)}\right]^{1/2}.
\]

There are 2 independent model parameters, namely $\Omega_{m0}$ and $c$. One can obtain $\Omega_\Lambda(z)$ by solving the differential equation (21) with the initial condition $\Omega_\Lambda(z = 0) = 1 - \Omega_{m0}$. Substituting $\Omega_\Lambda(z)$ into Eq. (22), we can find the corresponding $E(z)$ and then the total $\chi^2 = \chi^2_{m0} + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}}$. By minimizing the total $\chi^2$, we find the best-fit parameters $\Omega_{m0} = 0.298$ and $c = 0.658$, while $\chi^2_{\text{min}} = 546.628$. The corresponding $h = 0.703$ for the best fit. In Fig. 4, we present the corresponding 68.3% and 95.4% C.L. contours in the $\Omega_{m0} - c$ parameter space for the HDE model. Comparing with the results of \cite{59, 60}, it is easy to see that $\Omega_{m0}$ becomes larger, whereas $c$ becomes smaller. Note that $c < 1$ within 2$\sigma$ region, the EoS of HDE can be less than $-1$ \cite{61}.
In Table IV we present the ratio $S(z) \equiv T_z(z)/T_{obj}$ at $z = 3.91, 1.43, 1.55, 1.175$ and $3.62$, for various model parameters $(\Omega_{m0}, c)$ (within 2σ uncertainty) of the HDE model (the corresponding $h = 0.703$).

![Graph](image1)

**FIG. 5:** The dimensionless age parameter $T_z$ as a function of redshift $z$ for various $\Omega_{m0}$ and $c$ in the HDE model.

In Table IV we present the ratio $S(z) \equiv T_z(z)/T_{obj}$ at $z = 3.91, 1.43, 1.55, 1.175$ and $3.62$, for various model parameters $(\Omega_{m0}, c)$ (within 2σ uncertainty) of the HDE model. Obviously, $T_z(z) > T_{obj}$ holds at $z = 1.43, 1.55, 1.175$ and $3.62$, whereas $T_z(z) < T_{obj}$ at $z = 3.91$. So, the old quasar APM 08279+5255 at $z = 3.91$ cannot be accommodated (beyond 2σ). The age problem still exists in the HDE model, even when the alternative CMB data of L&L [8] has been taken into account. Further, comparing with the results in [54], it is easy to find that actually the age problem becomes even worse in the HDE model.

Again, we would like to see how the model parameters $\Omega_{m0}$ and $c$ affect the age of our universe at redshift $z$. In Fig. 5 we present the dimensionless age parameter $T_z$ as a function of redshift $z$ for various $\Omega_{m0}$ and $c$ in the HDE model (notice that from Eq. (16), $T_z(z)$ is independent of the Hubble constant $H_0$). It is easy to see that at any redshift $z$, for a fixed $c$, the larger $\Omega_{m0}$, the smaller age is; on the other hand, for a fixed $\Omega_{m0}$, the larger $c$, the smaller age is. However, from Fig. 5 we also find that the influence to the age from $\Omega_{m0}$ is significantly stronger than the one from $c$. As mentioned above, the alternative CMB data of L&L [8] favor a larger $\Omega_{m0}$ and a smaller $c$. Therefore, it is not surprising to find that the age problem becomes even worse in the HDE model.
C. Cosmological constraints on dark energy models

In this subsection, we consider the cosmological constraints on various dark energy models, by using the combined observations of the shift parameter $R$ from the alternative CMB data of L&L [8], the latest Union2 SNeIa dataset [20], and the distance parameter $A$ [24] with $n_s$ from L&L [8]. It is worth noting that the cosmological constraints on ΛCDM model, CPL model and HDE model have already been obtained in Secs. III A and III B. In the followings, we further consider the cosmological constraints on the other four dark energy models.
1. XCDM model

It is well known that in the spatially flat universe which contains pressureless matter and dark energy whose EoS is a constant $w_X$, the corresponding $E(z)$ is given by (see e.g. [59, 60])

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})(1+z)^{3(1+w_X)}}. \quad (23)$$

By minimizing the corresponding total $\chi^2 = \tilde{\chi}^2 \mu + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}}$, we find the best-fit parameters $\Omega_{m0} = 0.290$ and $w_X = -1.063$, while $\chi^2_{\text{min}} = 543.45$. The corresponding $h = 0.700$ for the best fit. In Fig. 6, we present the corresponding 68.3% and 95.4% C.L. contours in the $\Omega_{m0} - w_X$ parameter space for the XCDM model. Comparing with the corresponding results obtained in [59, 60], it is easy to find that $\Omega_{m0}$ becomes fairly larger, whereas $w_X$ becomes slightly smaller.

2. DGP model

One of the simplest modified gravity models is the well-known Dvali-Gabadadze-Porrati (DGP) braneworld model [62, 63], which entails altering the Einstein-Hilbert action by a term arising from large extra dimensions. For a list of references on the DGP model, see e.g. [64, 65] and references therein. As is well known, for the spatially flat DGP model (here we only consider the self-accelerating branch), $E(z)$ is given by [63–65]

$$E(z) = \sqrt{\Omega_{m0}(1+z)^3 + \Omega_{rc}} + \sqrt{\Omega_{rc}}, \quad (24)$$

where $\Omega_{rc}$ is a constant. It is easy to see that $E(z = 0) = 1$ requires

$$\Omega_{m0} = 1 - 2\sqrt{\Omega_{rc}}. \quad (25)$$

Therefore, the DGP model has only one independent model parameter $\Omega_{rc}$. Notice that $0 \leq \Omega_{rc} \leq 1/4$ is required by $0 \leq \Omega_{m0} \leq 1$. It is easy to obtain the total $\chi^2 = \chi^2_{\mu} + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}}$ as a function of the single model parameter $\Omega_{rc}$. In Fig. 7, we plot the corresponding $\chi^2$ and likelihood $L \propto e^{-\chi^2/2}$. The best fit has $\chi^2_{\text{min}} = 625.2$, whereas the best-fit parameter is $\Omega_{rc} = 0.122^{+0.005}_{-0.009}$ (with 1σ uncertainty) $+0.009$ (with 2σ uncertainty). The corresponding $h = 0.678$ for the best fit. From Eq. (25), $\Omega_{m0} = 0.302$ corresponds to the best-fit $\Omega_{rc}$. Comparing with the corresponding results obtained in [59, 60], it is easy to find that $\Omega_{rc}$ becomes fairly smaller, and hence $\Omega_{m0}$ becomes fairly larger.

3. New agegraphic dark energy model

The so-called new agegraphic dark energy (NADE) model was proposed in [66, 67], based on the Károlyházy uncertainty relation which arises from quantum mechanics together with general relativity. In fact, the NADE model is an upgraded version of the agegraphic dark energy (ADE) model [68–70]. In the NADE model, the dark energy density is given by [66, 67]

$$\rho_q = \frac{3n^2m_p^2}{\eta^2}, \quad (26)$$

where $n$ is a constant of order unity; $\eta$ is the conformal time which is defined by

$$\eta \equiv \int \frac{dt}{a} = \int \frac{da}{a^2H}. \quad (27)$$

From the Friedmann equation $H^2 = (\rho_m + \rho_q) / (3m_p^2)$, the energy conservation equation $\dot{\rho}_m + 3H\rho_m = 0$, Eqs. (26) and (27), it is easy to find that the equation of motion for $\Omega_q$, the fractional energy density of NADE, is given by [66, 67]

$$\frac{d\Omega_q}{dz} = -\Omega_q (1 - \Omega_q) \left[ 3(1+z)^{-1} - \frac{2}{n} \sqrt{\Omega_q} \right]. \quad (28)$$
FIG. 8: The $\chi^2$ and likelihood $\mathcal{L} \propto e^{-\chi^2/2}$ as functions of $n$ for the NADE model.

On the other hand, from the energy conservation equation $\dot{\rho}_q + 3H(\rho_q + \rho_p) = 0$, Eqs. (26) and (27), one can find that the EoS of NADE is given by [66, 67]

$$w_q \equiv \frac{p_q}{\rho_q} = -1 + \frac{2}{3n} \sqrt{\frac{\Omega_q}{a}}. \quad (29)$$

From Eqs. (26), (27) and (29), we have a very important result, namely, $\Omega_q(n^2a^2/4)$ in the matter-dominated epoch (we strongly refer to [66] for detailed arguments). Thanks to this special analytic feature $\Omega_q = n^2a^2/4$ in the matter-dominated epoch, NADE is a single-parameter model in practice. If $n$ is given, we can obtain $\Omega_q(z)$ from Eq. (28) with the initial condition $\Omega_q(z_{ini}) = n^2(1 + z_{ini})a$. Then, all other physical quantities, such as $\Omega_m(1 + z)$ and $w_q(z)$ in Eq. (29), can be obtained correspondingly. So, $\Omega_m = 1 - \Omega_q(z = 0)$ and $w_q = w_q(z = 0)$ are not independent model parameters. The only free model parameter is $n$ in the NADE model. From the Friedmann equation $H^2 = \frac{\rho_m + \rho_q}{3m_p^2}$, we have

$$E(z) = \left[ \frac{\Omega_m0(1 + z)^{3(1 - \Omega_q(z))}}{1 - \Omega_q(z)} \right]^{1/2}. \quad (30)$$

If the single model parameter $n$ is given, we can obtain $\Omega_q(z)$ from Eq. (28) with the initial condition $\Omega_q(z_{ini}) = n^2(1 + z_{ini})a$. Thus, we get $\Omega_{m0} = 1 - \Omega_q(z = 0)$. Then, $E(z)$ is on hand. Therefore, we can find the corresponding total $\chi^2 = \chi^2_{\text{S}} + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}}$. In Fig. 8 we plot the total $\chi^2$ and likelihood $\mathcal{L} \propto e^{-\chi^2/2}$ as functions of $n$. The best fit has $\chi^2_{\text{min}} = 595.001$, whereas the best-fit parameter is $n = 2.693_{-0.073}^{+0.072}$ (with 1σ uncertainty) and $\chi^2_{\text{min}} = 595.001$, whereas the best-fit parameter is $n = 2.693_{-0.073}^{+0.072}$ (with 2σ uncertainty). The corresponding $h = 0.681$ for the best fit. On the other hand, we find that $\Omega_{m0} = 0.299$, $\Omega_{q0} = 0.701$ and $w_{q0} = -0.793$ correspond to the best-fit $n$. Comparing with the corresponding results obtained in [51, 61, 71], we find that $n$ becomes fairly smaller, and correspondingly $\Omega_{m0}$ becomes fairly larger.

4. Ricci dark energy model

The so-called Ricci dark energy (RDE) model was proposed in [72], which can be regarded as a variant of the HDE model mentioned above, while its corresponding cut-off $L$ in Eq. (19) is chosen to be proportional
to the Ricci scalar curvature radius. In [72], there is no physical motivation to this proposal for \( L \) in fact. Later, in [73] it is found that the Jeans length \( R_{\text{CC}} \) which is determined by \( R_{\text{CC}}^2 = \dot{H} + 2H^2 \) gives the causal connection scale of perturbations in the flat universe. Since the Ricci scalar is also proportional to \( \dot{H} + 2H^2 \) in the flat universe, the physical motivation for RDE has been found in [73] actually. In the RDE model, the corresponding \( \rho_\Lambda \) is given by [72]

\[
\rho_\Lambda = 3\alpha \Omega_m^2 \left( \dot{H} + 2H^2 \right),
\]

where \( \alpha \) is a positive constant (when \( L \) is chosen to be \( R_{\text{CC}} \) in Eq. (19), one can see that \( \alpha = c^2 \) in fact).

Substituting Eq. (31) into Friedmann equation, it is easy to find that [59, 60, 72]

\[
E(z) = \left[ \frac{2\Omega_{m0}}{2 - \alpha} (1 + z)^3 + \left( 1 - \frac{2\Omega_{m0}}{2 - \alpha} \right) (1 + z)^{4-2/\alpha} \right]^{1/2}.
\]

There are two independent model parameters, namely \( \Omega_{m0} \) and \( \alpha \). By minimizing the corresponding total \( \chi^2 = \chi^2_\mu + \chi^2_{\text{CMB}} + \chi^2_{\text{BAO}} \), we find the best-fit parameters \( \Omega_{m0} = 0.355 \) and \( \alpha = 0.317 \), while \( \chi^2_{\text{min}} = 588.303 \). In Fig. 9 we present the corresponding 68.3\% and 95.4\% C.L. contours in the \( \Omega_{m0} - \alpha \) parameter space for the RDE model. Comparing with the corresponding results obtained in [59, 60], it is easy to find that \( \Omega_{m0} \) becomes fairly larger, whereas \( \alpha \) becomes significantly smaller.

5. Comparison of dark energy models

In Table V we summarize all the 7 dark energy models considered in this work. As mentioned above, it is common that L&L’s alternative CMB data [8] favor a larger \( \Omega_{m0} \) in all these dark energy models. Here we briefly consider the comparison of these models. Following [59, 60], we adopt three criterions used extensively in the literature, i.e., \( \chi^2_{\text{min}}/\text{dof} \), Bayesian Information Criterion (BIC) and Akaike Information Criterion (AIC). Note that the degree of freedom \( \text{dof} = N - k \), whereas \( N \) and \( k \) are the number of data points and the number of free model parameters, respectively. The BIC is defined by [74]

\[
\text{BIC} = -2 \ln \mathcal{L}_{\text{max}} + k \ln N,
\]

FIG. 9: The 68.3\% and 95.4\% C.L. contours in the \( \Omega_{m0} - \alpha \) parameter space for the RDE model. The best-fit parameters are also indicated by a black solid point.
TABLE V: Summarizing all the 7 dark energy models considered in this work.

| Model | ΛCDM | XCDM | CPL | DGP | NADE | HDE | RDE |
|-------|------|------|-----|-----|------|-----|-----|
| Ω_{m0} = 0.288 | Ω_{m0} = 0.290 | Ω_{m0} = 0.288 | Ω_c = 0.122 | n = 2.693 | Ω_{m0} = 0.298 | Ω_{m0} = 0.355 |
| w_X = −1.063 | w_0 = −0.969 | w_a = −0.529 |
| χ^2_{min} = 545.239 | 543.45 | 542.936 | 625.2 | 595.001 | 546.628 | 588.303 |
| k | 1 | 2 | 3 | 1 | 1 | 2 | 2 |
| χ^2_{min}/dof | 0.977 | 0.976 | 0.977 | 1.120 | 1.066 | 0.981 | 1.056 |
| ΔBIC | 0 | 4.537 | 10.349 | 79.961 | 49.762 | 7.715 | 49.390 |
| ΔAIC | 0 | 0.211 | 1.697 | 79.961 | 49.762 | 3.389 | 45.064 |
| Rank | 1 | 2 | 3 | 4 | 7 | 5 | 4 |

where \( \mathcal{L}_{max} \) is the maximum likelihood. In the Gaussian cases, \( \chi^2_{min} = -2 \ln \mathcal{L}_{max} \). So, the difference in BIC between two models is given by \( \Delta \text{BIC} = \Delta \chi^2_{min} + \Delta k \ln N \). The AIC is defined by \[ \text{AIC} = -2 \ln \mathcal{L}_{max} + 2k. \] (34)

The difference in AIC between two models is given by \( \Delta \text{AIC} = \Delta \chi^2_{min} + 2\Delta k \). In Table V, we present \( \chi^2_{min}/dof \), ΔBIC and ΔAIC for all the 7 models considered in this work. Notice that ΛCDM has been chosen to be the fiducial model when we calculate ΔBIC and ΔAIC. From Table V it is easy to see that the rank of models is coincident in all the 3 criterions (\( \chi^2_{min}/dof \), BIC and AIC). The ΛCDM model is still the best, whereas the DGP model is still the worst. Comparing with the corresponding results obtained in [59, 60], we find that the rank of these models has only slight change. In other words, the influence to the rank from L&L’s alternative CMB data [8] is fairly minor.

IV. CONCLUSION AND DISCUSSIONS

Recently, in a series of works by L&L [8–10], they claimed that there exists a timing asynchrony of \( −25.6 \text{ ms} \) between the spacecraft attitude and radiometer output timestamps in the original raw WMAP time-ordered data (TOD). In [8], L&L reprocessed the WMAP data while the aforementioned timing asynchrony has been corrected, and they obtained an alternative CMB map in which the quadrupole dropped to nearly zero. In the present work, we try to see the implications to dark energy cosmology if L&L are right. While L&L claimed that there is a bug in the WMAP pipeline which leads to significantly different cosmological parameters, an interesting question naturally arises, namely, how robust is the current dark energy cosmology with respect to systematic errors and bugs? So, in this work, we adopt the alternative CMB data of L&L as a strawman to study the robustness of dark energy predictions.

In this work, we found that L&L’s alternative CMB data [8] favor a larger \( \Omega_{m0} \) in all the dark energy models. As a result, we found that the tension between CMB and SNIa can be alleviated to some extent, since SNIa dataset usually favors a large \( \Omega_{m0} \). However, the age problem becomes even worse in the dark energy models, since a larger \( \Omega_{m0} \) usually leads to a smaller age of our universe at any redshift \( z \). On the other hand, we found that L&L’s alternative CMB data [8] do not significantly change the rank of dark energy models from the perspective of model comparison. Of course, it is a big advantage that the quadrupole dropped to nearly zero in the alternative CMB map of L&L (see [8–10]). Altogether, we consider that it is better to keep neutral to L&L’s findings so far. While WMAP has been critical in establishing a highly successful cosmology, it is most important that these results are verified by other independent experiments, such as Planck.

Finally, we stress that this work is a hypothetical study in fact, since the works of L&L [8–10] are highly controversial in the community. One should be aware of the risk that the present work is likely to be completely irrelevant if it turns out that the works of L&L are indeed flawed. Here are some
remarks to be taken serious (we thank the anonymous referee for pointing out these issues). Firstly, in fact there are many discussions on L&L’s claims in the community (see e.g. [77–79]), and many people (including both WMAP and Planck experts) believe that L&L are mistaken in their claims. Secondly, if the high-ℓ spectrum really is as discrepant as claimed by L&L, one could expect that many ground-based experiments would also have seen this effect. Thirdly, it is worth asking whether the timing offset was applied correctly in the actual map making but not in the calibration step. Up to now, to our knowledge, the discussions on L&L’s claims in the community are still unsettled, whereas L&L are still defending their claims persistently in more papers (see [80] for instance) and in many conferential talks around the world. We hope that this ongoing controversy could be finished with a firm and reliable conclusion in the near future.

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