Sequential Cooperative Energy and Time-Optimal Lane Change Maneuvers for Highway Traffic

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Abstract—We derive optimal control policies for a Connected Automated Vehicle (CAV) and cooperating neighboring CAVs to carry out a lane change maneuver consisting of a longitudinal phase where the CAV properly positions itself relative to the cooperating neighbors and a lateral phase where it safely changes lanes. In contrast to prior work on this problem, where the CAV “selfishly” seeks to minimize its maneuver time, we seek to ensure that the fast-lane traffic flow is minimally disrupted (through a properly defined metric) and that highway throughput is improved by optimally selecting the cooperating vehicles. We show that analytical solutions for the optimal trajectories can be derived and are guaranteed to satisfy safety constraints for all vehicles involved in the maneuver. When feasible solutions do not exist, we include a time relaxation method trading off a longer maneuver time with reduced disruption. Our analysis is also extended to multiple sequential maneuvers. Simulation results where the controllers are implemented show their effectiveness in terms of safety guarantees and up to 35% throughput improvement compared to maneuvers with no vehicle cooperation.

I. INTRODUCTION

Advances in transportation system technologies and the emergence of Connected Automated Vehicles (CAVs), also known as “autonomous vehicles”, have the potential to drastically improve a transportation network’s performance in terms of safety, comfort, congestion reduction and energy efficiency.

Recent developments focusing on autonomous car-following control are given in [1], [2], [3]. However, automating a lane change maneuver remains a challenging problem that has attracted increasing attention in recent years [4], [5], [6]. However, in these papers, only one vehicle can be controlled during the maneuver and no analytical solutions for the generated trajectories are provided.

The emergence of CAVs creates the opportunity for cooperation among vehicles traveling on multi-lane roads to carry out automated lane-change maneuvers [7], [8], [9]. In particular, when controlling a single vehicle and checking on the feasibility of a maneuver depending on the state of nearby traffic, as in [10], [11], the maneuver is often infeasible without the cooperation of other vehicles, especially under heavier traffic conditions. In contrast, a cooperative approach can allow multiple interacting vehicles to implement controllers enabling a larger set of feasible maneuvers. Aside from enhancing safety, this cooperative behavior can also improve the throughput, hence reducing the chance of congestion.

In our previous work, [12] we provided a time and energy optimal analytical solution for the maneuver shown in Fig. 1, in which the controlled vehicle C attempts to overtake an uncontrollable vehicle U by using the left lane to pass. A decentralized analytical solution is provided based on the cooperation and communication with two neighboring vehicles (vehicles 1 and 2) with the goal of minimizing the total maneuver time and subsequently determining trajectories that minimize the energy consumed by all three cooperating vehicles. This approach applies to a wider range of scenarios relative to those in [13], [8], [10].

However, by seeking to minimize C’s maneuver time, [12] adopts a vehicle-centric (selfish) viewpoint which ignores the effect of the maneuver on all remaining vehicles. As a result, since vehicle 2 typically decelerates to allow C to get ahead of it, this deceleration may cause a traffic flow slowdown in the left lane which can negatively impact throughput, especially in congested scenarios. Moreover, the analysis assumes that vehicles 1 and 2 are predetermined rather than being optimally selected among a set of possible cooperation candidates.

The contribution of this paper is the alleviation of the aforementioned limitations in such “selfish” maneuvers by adopting a “system-centric” optimality viewpoint. This provides a decentralized but socially optimal solution in the sense that our optimal controller design ensures that the resulting traffic throughput on the highway is improved by adding vehicles to the fast lane while (i) limiting the “disruption” that cooperation among multiple vehicles on the road can cause on the fast lane traffic flow, and (ii) determining an optimal pair of cooperating vehicles, which play the role of 1 and 2 in Fig. 1, within a set of feasible such candidates. Additionally, we focus on the realization of multiple...
sequential lane-changing maneuvers under the assumption of cooperation allowance by surrounding vehicles. As in [12], we decompose the maneuver into a longitudinal component followed by a lateral component. In the longitudinal part, our approach is based on first determining an optimal maneuver time for C subject to all safety and speed and acceleration constraints for vehicles C, 1, and 2 and such that C attains a desired final speed that matches that of the left lane traffic flow. We then solve a fixed terminal time decentralized optimal control problem for each of the two cooperating vehicles in which energy consumption is minimized. In the lateral phase, we solve a decentralized optimal control problem seeking to jointly minimize the time and energy consumed which is no different than the one presented in [12]. Our analysis also allows the determination of a vehicle pair that results in minimal acceleration/deceleration for this pair. This minimizes the possibility of excessive braking or deceleration of the rear vehicle in the pair (2 in Fig. 1), quantified through an appropriate “disruption metric”. Our results show that vehicles form natural platoons that dictate the free-flow speed of the fast lane in a two-lane highway.

The rest of the paper is organized as follows. Section II presents the formulation of the longitudinal lane-change maneuver problem. In Section III, a complete optimal control solution to coordinate multiple lane change maneuvers is obtained. Section IV provides simulation results for several representative examples and we conclude with Section VI.

II. Problem Formulation

In this section, we formulate the system-centric version of the cooperative maneuver setting presented in [12] which was exclusively vehicle-centric. As in [12], we decompose the maneuver into a longitudinal component followed by a lateral component. The former is significantly different, including the determination of a minimally disrupting cooperative pair (playing the role of 1 and 2 in Fig. 1). However, the latter is the same and we omit details that are given in [12].

Let C be the vehicle that initiates an automated maneuver. This can be manually triggered by the driver of C deciding to overtake vehicle U or automatically triggered by a given distance detected from an uncontrollable vehicle U ahead of C, as shown in Fig. 2. Assuming that all vehicles other than U are CAVs, we will henceforth refer to them as such.

Let S(t) be a set of vehicles on the left (fast) lane which are in the vicinity of C at time t and contains all candidate vehicles to cooperate with C in planning its lane-changing maneuver. For simplicity, once N members of S(t) are fixed, their indices are ordered \{1, \ldots, N\} starting with the CAV furthest ahead of C so that i + 1 denotes the CAV immediately following i (see Fig. 2). This set is limited by the communication range between C and other vehicles in its vicinity, but it may otherwise be selected based on other criteria as well. As shown in Fig. 2, we use the parameters L_r and L_f to define this set, where L_r is a given backward distance from the rear end of C and L_f is a forward distance from the front of vehicle U. Thus, letting x_i(t) denote the longitudinal position of vehicle i along its current lane with respect to a given origin O, we define S(t) to consist of fast lane CAVs as follows:

\[
S(t) := \{i \mid x_i(t) \leq x_U(t) + L_f, \\
x_i(t) \geq x_C(t) - L_r\}
\] (1)

It is now clear that any pair of vehicles in S(t) selected to cooperate with C at a time t is of the form \{i, i + 1\} with i, i + 1 \in S(t). An optimal pair, selected as described in the sequel, is, therefore, a subset of S(t) denoted by \(S^*(t) = \{i^*, i^* + 1\}\). If multiple maneuvers are to be executed, we index them by k = 1, 2, \ldots and write \(S_k(t)\) to represent the associated set corresponding to the specific CAV C that initiates the maneuver. We first limit ourselves to the simpler notation S(t).

For every vehicle \(i \in S(t)\) its dynamics take the form

\[
\dot{x}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t)
\] (2)

where, in addition to \(x_i(t)\), we define \(v_i(t)\) and \(u_i(t)\) to be vehicle \(i\)'s velocity and (controllable) acceleration respectively. Without loss of generality, we define the origin for CAV i involved in a maneuver to be the position \(x_C(t_0)\) of CAV C, where \(t_0\) denotes the time at which the maneuver starts. We will use \(t_f\) to denote the time when the longitudinal maneuver is complete. The control and speed are constrained as follows:

\[
u_{\min} \leq u_i(t) \leq u_{\max}, \quad \forall t \in [t_0, t_f]
\]

\[
v_{\min} \leq v_i(t) \leq v_{\max}, \quad \forall t \in [t_0, t_f]
\] (3)

where \(v_{\max} > 0\) and \(v_{\min} > 0\) denote the maximum and minimum speed allowed, usually determined by the rules of the highway. Similarly, \(u_{\max} > 0\) and \(u_{\min} < 0\) are i's maximum and minimum acceleration control.

Safety Constraints. Let \(d_i(v_i(t))\) be the speed-dependent safety distance of CAV i, defined as the minimum required distance between i and its immediately preceding vehicle:

\[
d_i(v_i(t)) = \varphi v_i(t) + \delta,
\] (4)

where \(\varphi\) is a constant value that denotes the reaction time (usually \(\varphi = 1.8\) s [14]), \(\delta\) is a constant, and \(d_i(v_i(t))\) is specified from the center of i to the center of its preceding vehicle. We can now define all safety constraints that must be satisfied during a lane-changing maneuver of C when cooperating with any two CAVs \(i, i + 1\):

\[
x_U(t) - x_C(t) \geq d_C(v_C(t)), \quad \forall t \in [t_0, t_f]
\] (5a)

\[
x_i(t) - x_{i+1}(t) \geq d_{i+1}(v_{i+1}(t)), \quad \forall t \in [t_0, t_f]
\] (5b)

\[
x_C(t_f) - x_{i+1}(t_f) \geq d_{i+1}(v_{i+1}(t_f)),
\] (5c)

\[
x_i(t_f) - x_C(t_f) \geq d_C(v_C(t_f))
\] (5d)
Traffic Disruption. We now seek to measure the extent to which a successful lane-changing maneuver may disrupt the left-lane traffic. Given any time \( t_f > t_0 \) and \( T = t_f - t_0 \), let \( x_i(t_f) \) be the terminal position of \( i \) as determined by some control policy \( u_i(t) \), \( t \in [t_0, t_f] \). We then define

\[
\Delta_i(T) = x_i(t_f) - \left[ x_i(t_0) + v_i(t_0)T \right] \tag{6}
\]

which specifies the difference between the actual terminal position of \( i \) under some control policy and its ideal terminal position obtained by maintaining a constant speed \( v_i(t_0) \). This is “ideal” in the sense that the vehicle’s uniform motion is undisrupted, hence also minimizing the energy consumption which would be due to any acceleration/deceleration. This motivates the definition of the following maneuver disruption metric \( D_{i,i+1}(T) \) applied to two cooperating CAVs \( i, i + 1 \):

\[
D_{i,i+1}(T) = \gamma \Delta_i^2(T) + (1 - \gamma) \Delta_{i+1}^2(T), \tag{7}
\]

where \( \Delta_i(T) \) defined in (6) is the disruption caused to \( i \) due to an acceleration/deceleration control \( u_i(t) \) applied to it relative to its undisrupted final position. The weight \( \gamma \in [0, 1] \) is included to potentially place more emphasis on one cooperating CAV over the other. Note that this quadratic disruption metric depends only on the total maneuver time \( T \) and the terminal positions \( x_i(t_f) \) and \( x_{i+1}(t_f) \) for CAVs \( i \) and \( i + 1 \). One can see that \( D_{i,i+1}(T) \) implicitly penalizes the time that CAV \( i + 1 \) (which normally decelerates to accommodate \( C \)) would take to accelerate back to the free flow speed. Further, if \( \Delta_{i+1}(T) \) is large and the vehicle following \( i + 1 \) is closely behind, then \( D_{i,i+1}(T) \) explicitly penalizes the deceleration of this vehicle and the time that it would take to accelerate back to the free flow speed. Clearly, if there are multiple maneuvers indexed by \( k = 1, 2, \ldots, \), we can seek to minimize an aggregate metric \( D_{\text{Total}} \) by adding individual disruptions \( D_{k,i,i+1}(T_k) \).

Optimization Problem. We consider two objectives for the longitudinal maneuver problem: first, we wish to minimize the maneuver time \( t_f \) experienced by CAV \( C \) and its cooperating vehicles; second, we wish to minimize the energy consumption of each of the three cooperating CAVs \( C, i \) and \( i + 1 \). At the same time, we must satisfy the safety constraints (5) and vehicle constraints (3). Finally, we must ensure that the disruption metric (7) does not exceed a given threshold \( D_{th} \).

The overall optimization problem is outlined next:

1. CAV \( C \) determines an optimal terminal time \( t_f^* \) for the maneuver and control \( \{u_C^*(t)\}, t \in [t_0, t_f^*] \) so as to minimize a given objective function \( J_C \) (to be defined in the sequel) subject to the vehicle dynamics (2) and constraints (5a), (5c), (5d) and (3). Moreover, its optimal terminal speed \( v_C(t_f^*) \) must be close to (or exactly match) a desired speed \( v_d \) that matches the fast-lane speed.

2. The solution \( t_f^* \) specifies \( S(t_f^*) \), the set from which an optimal pair \( (i^*, i^* + 1) \) of cooperating CAVs must be selected so as to minimize the disruption metric \( D_{i,i+1}(T^*) \) in (7), where \( T^* = t_f^* - t_0 \). Since \( D_{i,i+1}(T^*) \) depends on the values of \( x_i(t_f^*) \) and \( x_{i+1}(t_f^*) \) in (6), its minimization

![Fig. 3: Cooperative Maneuver Flow Diagram](image)

depends on the optimal trajectories selected by CAVs \( i \in S(t_f^*) \). This requires the determination of optimal controls \( \{u_i^*(t)\}, t \in [t_0, t_f^*] \) for all \( i \in S(t_f^*) \) minimizing a given objective function \( J_i \) (to be defined in the sequel) subject to the vehicle dynamics (2) and constraints (5b), (5c), (5d) and (3).

3. Finally, we determine an optimal pair \( (i^*, i^* + 1) \) which minimizes the disruption metric \( D_{i,i+1}(T^*) \) over all \( i \in S(t_f^*) \). In addition, this solution must satisfy the requirement \( D_{i,i^*,i^*+1}(T^*) \leq D_{th} \).

A solution to this problem, consisting of \( (i^*, i^* + 1) \) and \( \{u_C^*(t), u_{i^*}^*, u_{i^*+1}^*\}, t \in [t_0, t_f^*] \), may not exist. In the next section, we present a detailed solution approach with the following key elements: (i) We specify the objective functions \( J_C \) and \( J_i \) for any \( i \in S(t) \). (ii) We obtain the optimal cooperating pair \( (i^*, i^* + 1) \) without solving all (time-consuming) optimal control problems for \( \{u_i^*(t)\}, t \in [t_0, t_f^*] \). (iii) We include a time relaxation on \( t_f^* \) so that if a feasible solution does not exist, we seek one for a relaxed value \( t_f > t_f^* \). The relaxation process captures the trade-off between the “selfish” goal of \( C \) to minimize its maneuver time and the system-wide “social” goal of minimizing traffic flow disruptions while ensuring an increase in throughput by adding vehicles to the fast lane.

III. DECENTRALIZED OPTIMAL CONTROL SOLUTION

The overall description of the solution process for the optimal maneuver ensuring a minimal disruption that satisfies a given threshold \( D_{th} \) is given in Fig. 3.

**CAV C Optimal Trajectory:** Given a CAV \( C \) traveling behind an uncontrolled vehicle \( U \), a “start maneuver” request is sent to surrounding vehicles and the starting time \( t_0 \) is defined when \( x_U(t) - x_C(t) \leq d_{\text{start}} \), where \( d_{\text{start}} \) denotes the minimum distance at which CAV \( C \) decides to initiate a lane-changing maneuver. It is important to point out that at that instant the values of the optimal maneuver time and the entire optimal trajectory of \( C \) can be evaluated, which also enables planning the complete solution of the problem.

We now define the optimal control problem (OCP) for CAV \( C \) as follows:

\[
\min_{\{t_f, u_C(t)\}} \beta(t_f - t_0) + \int_{t_0}^{t_f} \frac{1}{2} u_C^2(t) dt \tag{8}
\]
\[
x_U(t) - x_C(t) \leq d_C(v_C(t)), \quad \forall t \in [t_0, t_f]
\]
\[
(v_C(t_f) - v_d)^2 \leq \delta_{tol}, \quad 0 \leq t_f \leq T_{th}
\]
where \( \beta = \alpha \max \left\{ \frac{u_{C_{\min}}^2}{2(1 - \alpha)}, \frac{u_{C_{\max}}^2}{2} \right\} \) and \( \alpha \in [0, 1] \) is an adjustable weight that penalizes travel time relative to the total energy cost for CAV \( C \). Note that by properly normalizing the energy cost term \( u_C^2(t) \), the integrand above is a convex combination of time and an energy metric. The last three constraints in (8) capture (i) the safe distance constraint (5a) between \( C \) and \( U \) (assuming that the position and speed of \( U \) can be sensed or estimated by \( C \)), (ii) the requirement that the terminal speed \( v_C(t_f) \) matches a desired speed \( v_d \) within a tolerance \( \delta_{tol} \geq 0 \) and (iii) \( T_{th} \) is specified as the maximum tolerable time to perform a lane-changing maneuver. In practice, if the last constraint cannot be met for a given \( T_{th} \), CAV \( C \) has the option of either relaxing this value or simply aborting the maneuver. Finally, note that problem (8) is solved given the initial position and speed of CAV \( C \).

The solution of this OCP can be analytically obtained through standard Hamiltonian analysis similar to OCPs formulated and solved in [12], thus, we omit the details. It is worth pointing out that depending on the weight \( \beta \) and the starting distance \( d_{start} \) of the maneuver, the form of the corresponding optimal trajectory can be either strictly accelerating or first decelerating followed by an accelerating component so that (5a) is satisfied regardless of the initial conditions of CAV \( C \).

Construction of Cooperative Set \( S(t_f) \): Once (8) is solved, the optimal terminal time \( t_f \) and terminal position \( x_C(t_f) \) are determined. Therefore, the cooperative set \( S(t_f) \) can be determined from (1) over different feasible positions \( x_i(t_f) \). In particular, we can compute optimal trajectories for each \( i \in S(t_0) \) on an OCP solved by \( i \) to minimize its energy and then use \( x_i(t_f) \) in \( S(t_f) \). However, we exploit the fact that the actual set \( S(t_f) \) of candidate CAVs is unaffected as long as the conditions \( x_i(t) \leq x_U(t) + L_f \) and \( x_i(t) \geq x_C(t) - L_r \) are satisfied. Rather than the time-consuming process of solving OCPs for all \( i \), we define \( S(t_f) \) as follows:

\[
S(t_f) := \left\{ i \mid x_i(t_0) + v_i(t_f - t_0) \leq x_U(t_f) + L_f, \quad x_i(t_0) + v_i(t_f - t_0) \geq x_C(t_f) - L_r \right\},
\]
where \( x_i(t_0) + v_i(t_f - t_0) \) is the position of \( i \) at time \( t_f \) under constant speed. Alternatively, we can use in (1) the value of \( x_i(t_f) \) under maximal acceleration to determine a set \( S_{max}(t_f) \) and maximal deceleration to determine a set \( S_{min}(t_f) \) and then define \( S(t_f) = S_{max}(t_f) \cup S_{min}(t_f) \). Also note that by adjusting the parameters \( L_f \) and \( L_r \), the size of this set can be adjusted to include as many candidate CAVs as desired, subject only to the constraint that any \( S(t) \) must include CAVs within the communication range of CAV \( C \) to allow full cooperation.

Optimal Cooperative Pair \((i^*, i^* + 1)\): The optimal cooperative pair \((i^*, i^* + 1)\) among all \( i \in S(t_f) \) is the one that minimizes the disruption metric in (7) by selecting terminal positions resulting in minimal disruption. These terminal positions \( x_i(t_f^*), x_{i+1}(t_f^*) \) are subject to the vehicle constraints specified in (3). As a result, a set of feasible terminal positions can be defined as shown in Fig. 2, Section III-A of [12]. Therefore, each \( x_i(t_f^*) \) must be constrained to this set which we denote by \( X_{i,f}^{feas}(t_f^*) \).

We can now formulate the following Quadratic Program (QP) whose solution provides the optimal terminal positions \( x_i(t_f^*), x_{i+1}(t_f^*) \) for any pair \( i, i + 1 \in S(t_f^*) \):

\[
\min_{x_i(t_f^*), x_{i+1}(t_f^*)} \gamma \Delta^2(t_f) + (1 - \gamma) \Delta^2_{i+1}(t_f)
\]
\[
\text{s.t. } x_i(t_f^*) - x_C(t_f^*) \geq d_C(v_C(t_f^*)), \quad x_C(t_f^*) - x_{i+1}(t_f^*) \geq \max \left\{ d_{i+1} \left( v_{i+1}(t_f^*) \right) \right\}, \quad x_{i+1}(t_f^*) - x_i(t_f^*) \geq \max \left\{ d_i \left( v_i(t_f^*) \right) \right\}, \quad x_i(t_f^*) \in X_{i,f}^{feas}(t_f^*), \quad x_{i+1}(t_f^*) \in X_{i+1,f}^{feas}(t_f^*)
\]
where the first three constraints are the safe distance requirements in (5) that \( i \) and \( i + 1 \) must satisfy. The values of the \( \max \{ \cdot \} \) terms in (10) are assumed to be given by prespecified maximum safe distances; for simplicity, \( \max \{ d_{i+1}(v_{i+1}(t_f^*)) \} = d_{i+1}(v_{i+1}(t_0)) \) and \( \max \{ d_i(v_i(t_f^*)) \} = d_i(v_i(t_0)) + u_{min}(t_f^* - t_0) \). Note that in the third constraint it is possible that \( i - 1 = 0 \), i.e., CAV \( i \) is the first in the set \( S(t_f^*) \) and there may not always exist a vehicle ahead of it, in which case we simply assign \( x_{i-1}(t_f^*) \) an arbitrarily large value (e.g., \( x_{i-1}(t_f^*) = x_i(t_f^*) + 1000m \)).

The solution of (10) for every pair \((i, i + 1)\) provides the optimal terminal positions \( x_i^*(t_f^*) \) and \( x_{i+1}^*(t_f^*) \) that minimize the disruption metric \( D_{i,i+1}(T) \). We shall denote this optimal value as \( D_{i,i+1}(T) \). We can now determine the optimal cooperative pair \((i^*, i^* + 1)\) from

\[
(i^*, i^* + 1) = \arg \min_{(i, i+1) \in S(t_f)} D_{i,i+1}(T)
\]
\[
\text{s.t. } D_{i,i+1}(T) \leq D_{th}
\]
where \( D_{th} \) is a prespecified maximum disruption level. This is a simple minimization problem comparing the values of \( D_{i,i+1}(T) \) obtained from (10) over a finite set consisting of pairs \((i, i + 1)\) that satisfy the disruption constraint above.

If no solution to (11) is found, it is still possible to derive a solution based on the analysis in [12], with no consideration of disruption. Alternatively, we may proceed with the time relaxation process described in Section III. However, we first complete the solution process by computing optimal trajectories for CAVs \((i^*, i^* + 1)\) if such a pair is identified.

Optimal Trajectories for CAVs \((i^*, i^* + 1)\): For any CAV \( i \), we define its objective function to be:

\[
J_i(u_i(t)) = \int_{t_0}^{t_f^*} \frac{1}{2} u_i^2(t) dt
\]
so that CAV \( i \) (where \( i = i^* \) or \( i = i^* + 1 \)) solves the following fixed terminal time and position OCP:

\[
\min_{\{u_i(t)\}} \int_{t_0}^{t_f^*} \frac{1}{2} u_i^2(t) dt
\]
where $x^*_i(t_f)$ is the optimal terminal position in (10) when $i = i^*$ or $i = i^* + 1$. This is an OCP of the same form as those solved in [12], thus, we omit the details of the solution. 

**Maneuver Time Relaxation:** As already mentioned, it is possible that no solution to problem (11) may be found. The most common reason is due to the fact that the optimal maneuver end time $t'_f$, determined by CAV $C$ at the first step of the solution approach, is too short to allow $C$ to reach a speed sufficiently close to $v_d$ and for cooperating CAVs to adjust their positions so as to satisfy the safety constraints in (5). In such cases, it is possible to perform a relaxation of $t'_f$ obtained through (8) by trading it off against the energy consumption due to the maneuver extension. Thus, the new terminal time is given as $t'_f = t_f \lambda_{i_f}$, where $\lambda_{i_f} > 1$ is a relaxation factor. Observe that this time modification changes the form of the OCP (8), since the terminal time is now fixed at $t'_f > t'_f$ and the solution will lead to a new terminal position $x_c(t'_f)$ for CAV $C$. The new OCP formulation is as follows:

$$
\min_{\{u(C(t))\}} J_C(u(C(t))) = \int_{t_0}^{t'_f} \frac{1}{2} u^2_C(t) dt \\
\text{s.t. (2), (3), and} \\
x_U(t) - x_C(t) \geq d_C(v_C(t)), \quad \forall t \in [t_0, t_f], \\
(v_C(t'_f) - v_d)^2 \leq \delta_{tol}
$$

This process may continue, as shown in Fig. 3, until a feasible solution is determined or the constraint $T \leq T_{th}$ is violated.

**Remark:** Despite time relaxation, problem (14) can still be infeasible if $d_{start}$ is small or if the constraint (5a) is active at $t_0$. Therefore, CAV $C$ can abort the maneuver and wait a specified time interval for the next opportunity window.

**Sequential Maneuvers:** Clearly, we can perform a series of individual maneuvers following Fig. 3 indexed by $k$ that minimize the aggregate metric $D_{total} = \sum_{k=1}^{N} D_{total}^{C_k}(T_k)$. For each maneuver $k$ we can compute a series of system-centric (social) optimal trajectories that start sequentially by defining a CAV $C$ as soon as maneuver $k-1$ has completed its corresponding lateral phase. Thus, the initial time for maneuver $k$ is upper bounded by the terminal time of maneuver $k-1$ ($t^b_k \geq t^f_{k-1}$). Note that CAV $C$ for maneuver $k-1$ can become a CAV candidate for the $k$th maneuver. It is also possible to **parallelize** a number of such maneuvers by allowing them to start simultaneously given a set of target vehicles (CAV $C$).

**IV. SIMULATION RESULTS**

In this section, we provide simulation results illustrating the time and energy-optimal controllers we have derived and comparing their performance against a baseline of non-cooperating vehicles. We test our algorithm using the traffic simulation software PTV Vissim. Our simulation setting consists of a straight two-lane highway segment with a 5000m and an allowable speed range of $v = [16, 33]$ m/s. We define a traffic flow of 6000 vehicles/hour. The incoming traffic is spawned with a desired speed of $v_d = 29$ m/s. Similarly, the inter-vehicle safe distance ($\delta$) is given by $\delta = 1.5 m$ and headway parameter $\phi$ drawn from a normal distribution $N(0.6, 0.04)$ s. In order to simulate congestion generation, we spawn an uncontrolled vehicle $U$, travelling on the right-lane (slow lane) with a constant speed $v_U = 16$ m/s throughout the simulation. The corresponding CAV $C$ is defined as vehicle $U$’s immediately following vehicle. For the maneuver start distance we select $d_{start}$ from $N(70, 10)$ m for every CAV $C$ initiating a maneuver. The control limits specified for every CAV are given by $u_{min} = -7$ m/s$^2$ and $u_{max} = 3.3$ m/s$^2$. It is assumed that all CAVs in the simulation share the same control bounds.

**CAV C Longitudinal Maneuver:** We apply the formulation proposed for CAV $C$ in (8) for different settings using a constant weight factor $\alpha = 0.4$, desired speed $v_d = 29$ m/s, and a relaxation constant $\delta_{tol} = 4 m^2/s^4$. Thus, we provide simulation results for different initial conditions pertaining only to vehicles $U$ and $C$ as described in Table I for Cases 1 and 2. Similarly, for Case 3 we show a sample trajectory generated from the relaxation of the optimal time proposed in (14). It can be seen for Cases 1 and 2 that the optimal terminal time solution $t'_f$ inversely varies with the starting distance $d_{start}$. Thus, for Case 1, when $d_{start} = 70$ m, the resulting maneuver time is $t'_f = 3.58 s$ and a control strategy of constant acceleration to reach $v_d$ is shown in Fig. 4a. Conversely, for Case 2, when $d_{start} = 14$ m, the resulting maneuver time is $t'_f = 13.03 s$ given that CAV $C$ needs to first undergo a deceleration segment followed by a segment with no acceleration to provide enough space to undergo a final acceleration segment allowing CAV $C$ to reach $v_d$ without violating the safety constraint (5a) as shown in Fig. 4b. Lastly, for Case 3, a relaxed terminal time constraint is provided with $t'_f = 14.65 s$. It can be seen from Fig. 4c that the resulting optimal trajectory of CAV $C$, similar to Case 2, is composed of a negative acceleration segment followed by constant acceleration.

| Case | $d_{start}$ (m) | $x_{C,k}^b$ (m) | $x_{C,k}^f$ (m) | $v_{C,k}^b$ (m/s) | $v_{C,k}^f$ (m/s) | $t_C$ (s) | $v_{C}(t_f)$ (m/s) |
|------|----------------|----------------|----------------|----------------|----------------|-----------|----------------|
| Case 1 | True | 30 | 382 | 16 | 222 | 25 | 3.38 |
| Case 2 | True | 14 | 913 | 16 | 188 | 25 | 14.65 |

**Sequencal Maneuvers:** We have also implemented a series of system-centric (social) optimal maneuvers. For this purpose, we discretize the start of the maneuvers as explained in Section III. Additionally, a study was performed to determine the optimal parameters that would lead to minimal energy, time, and disruption, along with throughput improvements. Table II summarizes the results of our throughput study comparing the throughput under no cooperation with the case of vehicle-centric (selfish) maneuvers as in [12] and with the system-centric maneuvers presented in this paper with different parameters. The throughput analysis is performed by counting the number of vehicles within a 120 s window that crosses a measurement point at 2000 m from the
candidate set that minimizes a disruption metric for the fast lane traffic flow to ensure it never exceeds a given threshold. Simulation results show the effectiveness of the proposed method with improvements of 35% in throughput over the no-cooperation case. Ongoing work aims to perform multiple maneuvers simultaneously while still minimizing traffic disruption. We are also working towards extending our analysis to a mixed traffic setting with both CAVs and human-driven vehicles.

**TABLE II: Throughput Simulation Study: Throughput results over 120 s analysis window**

| Description          | Relaxation | Count | Vehicle Flow [veh/hour] | Avg. Travel Time [s] | Avg Speed [m/s] |
|----------------------|------------|-------|------------------------|----------------------|-----------------|
| System-centric       | T          | 199   | 1500                   | 36.5                 | 37.5            |
| System-centric       | F          | 0.5   | 45                     | 77.8                 | 19.8            |
| Vehicle-centric      | T          | 150   | 1350                   | 78.1                 | 21.7            |
| Vehicle-centric      | F          | 0.5   | 45                     | 78.1                 | 19.8            |
| No Cooperation       | T          | 1110  | 38.4                   | 37                   | 19.7            |

**REFERENCES**

[1] D. Zhao, X. Huang, H. Peng, H. Lam, and D. J. LeBlanc, “Accelerated evaluation of automated vehicles in car-following maneuvers,” *IEEE Trans. on Intelligent Transportation Systems*, vol. 19, no. 3, pp. 733–744, 2018.

[2] M. Wang, W. Daamen, S. P. Hoogendoorn, and B. van Arem, “Cooperative car-following control: Distributed algorithm and impact on moving jam features,” *IEEE Trans. on Intelligent Transportation Systems*, vol. 17, no. 5, pp. 1459–1471, 2016.

[3] M. Wang, S. P. Hoogendoorn, W. Daamen, B. van Arem, and R. Happee, “Game theoretic approach for predictive lane-changing and car-following control,” *Transportation Research Part C: Emerging Technologies*, vol. 58, pp. 73–92, 2015.

[4] J. Nilsson, M. Brännström, E. Coelingh, and J. Fredriksson, “Longitudinal and lateral control for automated lane change maneuvers,” *Proc. of 2015 American Control Conf.*, pp. 1399–1404, 2015.

[5] C. Bax, P. Leroy, and M. P. Hagenzieker, “Road safety knowledge and policy: A historical institutional analysis of the Netherlands,” *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 25, pp. 127–136, 2014.

[6] S. He, J. Zeng, B. Zhang, and K. Sreenath, “Rule-based safety-critical control design using control barrier functions with application to autonomous lane change,” in *2021 American Control Conference (ACC)*. IEEE, 2021, pp. 178–185.

[7] H. N. Mahjoub, A. Tahmbris-Sarvestani, H. Kazemi, and Y. P. Fallah, “A learning-based framework for two-dimensional vehicle maneuver prediction over v2v networks,” *Proc. of 15th IEEE Intl. Conf. on Dependable, Autonomic and Secure Computing*, pp. 156–163, 2017.

[8] J. Nilsson, M. Brännström, E. Coelingh, and J. Fredriksson, “A dynamic automated lane change maneuver based on vehicle-to-vehicle communication,” *Transportation Research Part C: Emerging Technologies*, vol. 62, pp. 87–102, 2016.

[9] T. Li, J. Wu, C.-Y. Chan, M. Liu, C. Zhu, W. Lu, and K. Hu, “A cooperative lane change model for connected and automated vehicles,” *IEEE Access*, vol. 8.

[10] M. A. S. Kamal, M. Mukai, J. Murata, and T. Kawabe, “Model predictive control of vehicles on urban roads for improved fuel economy,” *IEEE Trans. on Control Systems Technology*, vol. 21, no. 3, pp. 831–841, 2013.

[11] A. Katriniok, J. P. Maschuw, F. Christen, L. Eckstein, and D. Abel, “Optimal vehicle dynamics control for combined longitudinal and lateral autonomous vehicle guidance,” *Proc. of 2013 Control Conf.*, pp. 974–979, 2013.

[12] R. Chen, C. G. Cassandras, A. Tahmarsi-Sarvestani, S. Saigusa, H. N. Mahjoub, and Y. K. Al-Nadawi, “Cooperative time and energy-optimal lane change maneuvers for connected automated vehicles,” *IEEE Transactions on Intelligent Transportation Systems*, 2020.

[13] J. Nilsson, M. Brännström, E. Coelingh, and J. Fredriksson, “Lane change maneuvers for automated vehicles,” *IEEE Trans. on Intelligent Transportation Systems*, vol. 18, no. 5, pp. 1087–1096, 2017.

[14] K. Vogel, “A comparison of headway and time to collision as safety indicators,” *Accident Analysis & Prevention*, vol. 35, no. 3, pp. 427–433, 2003.