Can the $\pi^+\chi_{c1}$ resonance structures be $D^*\bar{D}^*$ and $D_1\bar{D}$ molecules?

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We use QCD sum rules to study the recently observed resonance-like structures in the $\pi^+\chi_{c1}$ mass distribution, $Z_1^+(4050)$ and $Z_2^+(4250)$, considered as $D^{*+}\bar{D}^{*0}$ and $D_1^*\bar{D}^*$ molecules with the quantum number $J^P = 0^+$ and $J^P = 1^-$ respectively. We consider the contributions of condensates up to dimension eight and work at leading order in $\alpha_s$. We obtain $m_{D^{*}\bar{D}^{*}} = (4.15 \pm 0.12)$ GeV, around 100 MeV above the $D^*D^*$ threshold, and $m_{D_1\bar{D}} = (4.19 \pm 0.22)$ GeV, around 100 MeV below the $D_1\bar{D}$ threshold. We conclude that the $D^{*+}\bar{D}^{*0}$ state is probably a virtual state that is not related with the $Z_1^+(4050)$ resonance-like structure. In the case of the $D_1\bar{D}$ molecular state, considering the errors, its mass is consistent with both $Z_1^+(4050)$ and $Z_2^+(4250)$ resonance-like structures. Therefore, we conclude that no definite conclusion can be drawn for this state from the present analysis.

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The recent discovery of several missing states and a number of unexpected charmonium like resonances in B-factories has revitalized the interest in the espectroscopy of the charmonium states. There is growing evidence that at least some of these new states are non conventional $c\bar{c}$ states, such as mesonic molecules, tetraquarks, and/or hybrid mesons. Among these new mesons, some have their masses very close to the meson-meson threshold like the $X(3872)$ \cite{1} and the $Z^+(4430)$ \cite{2}. Of special importance is the appearance of the $Z^+(4430)$, observed in the $\pi^+\psi'$ mass spectrum produced in the $B^0 \rightarrow K^-\pi^+\psi'$ decays. Being a charged state it can not be described as ordinary $c\bar{c}$ meson. Its nature is completely open, but an intriguing possibility is the interpretation as tetraquark state or molecular state \cite{3,4,5}.

The $Z^+(4430)$ observation motivated studies of other $B^0 \rightarrow K^-\pi^+(c\bar{c})$ decays. In particular, the Belle Collaboration has recently reported the observation of two resonance-like structures in the $\pi^+\chi_{c1}$ mass distribution \cite{6}. The significance of each of the $\pi^+\chi_{c1}$ structures exceeds 5$\sigma$ and, if they are interpreted as meson states, their minimal quark content must be $c\bar{c}u\bar{d}$. They were called $Z_1^+(4050)$ and $Z_2^+(4250)$, and their masses and widths are $M_1 = (4051 \pm 14^{+20}_{-31})$ MeV, $\Gamma_1 = 82^{+21}_{-17} - 22$ MeV, $M_2 = (4248^{+44+180}_{-29-35})$ MeV, $\Gamma_2 = 177^{+54}_{-39+61}$ MeV.

There are already theoretical interpretations for these structures as tetraquark states with $J^P = 1^-$ and as molecular $D^*\bar{D}^*$ state with $J^P = 0^+$ \cite{8}. In this work, due to the closeness of the $Z_1^+(4050)$ and $Z_2^+(4250)$ masses to the $D^*(2010)\bar{D}^*(2010)$ and $D_1(2420)\bar{D}(1865)$ thresholds respectively, we use the QCD sum rules (QCDSR) \cite{9,10,11,12}, to study the two-point functions of the $D^*\bar{D}^*$ molecule with $J^P = 0^+$, and the $D_1\bar{D}$ molecule with $J^P = 1^-$, to see if they can be interpreted as the new observed resonances structures $Z_1^+(4050)$ and $Z_2^+(4250)$ respectively. Since they were observed in the $\pi^+\chi_{c1}$ channel, the only quantum numbers that are known about them are $I^G = 1^-$. In previous calculations, the QCDSR approach was used to study the $X(3872)$ considered as a diquark-antidiquark state \cite{12} and as a $D^*\bar{D}$ molecular state \cite{13}, the $Z^+(4430)$ meson, considered as a $D^*\bar{D}_1$ molecular state \cite{5} and as tetraquark states \cite{14}, and the $Y$ mesons considered as molecular and tetraquark states \cite{15}. In some cases a very good agreement with the experimental mass was obtained. The QCDSR approach was also used to study the existence of a $D_s\bar{D}^*$ molecule with $J^P = 1^+$, that would decay into $J/\psi K^*\rightarrow J/\psi K\pi$ and, therefore, could be easily reconstructed \cite{13}.

Considering the $Z_1^+(4050)$ resonance structure as a $D^{*+}\bar{D}^{*0}$ molecule with $I^GJ^P = 1^-0^+$, a possible current describing such state is given by:

\begin{equation}
    j = (\bar{d}_a\gamma_\mu c_a) (\bar{c}_b\gamma^\mu u_b) ,
\end{equation}
where $a$ and $b$ are color indices.

The sum rule is constructed from the two-point correlation function:

$$
\Pi(q) = i \int d^4 x \, e^{iq.x} \langle 0 | T[j(x)j^\dagger(0)] | 0 \rangle.
$$

(2)

On the OPE side, we work at leading order in $\alpha_s$ in the operators and consider the contributions from condensates up to dimension eight. The correlation function in the OPE side can be written as a dispersion relation:

$$
\Pi^{\text{OPE}}(q^2) = \int_{4m^2}^{\infty} ds \frac{\rho^{\text{OPE}}(s)}{s - q^2},
$$

(3)

where $\rho^{\text{OPE}}(s)$ is given by the imaginary part of the correlation function: $\pi\rho^{\text{OPE}}(s) = \text{Im}[\Pi^{\text{OPE}}(s)]$.

In the phenomenological side, we write a dispersion relation to the correlation function in Eq. (2):

$$
\Pi^{\text{phen}}(q^2) = \int ds \frac{\rho^{\text{phen}}(s)}{s - q^2} + \cdots,
$$

(4)

where $\rho^{\text{phen}}(s)$ is the spectral density and the dots represent subtraction terms. The spectral density is described, as usual, as a single sharp pole representing the lowest resonance plus a smooth continuum representing higher mass states:

$$
\rho^{\text{phen}}(s) = \lambda^2 \delta(s - m_{D^*D^*}^2) + \rho^{\text{cont}}(s),
$$

(5)

where $\lambda$ gives the coupling of the current to the scalar meson $D^*D^*$:

$$
\langle 0 | j(D^*D^*)| \rangle = \lambda.
$$

(6)

For simplicity, it is assumed that the continuum contribution to the spectral density, $\rho^{\text{cont}}(s)$ in Eq. (5), vanishes below a certain continuum threshold $s_0$. Above this threshold, it is assumed to be given by the result obtained with the OPE. Therefore, one uses the ansatz

$$
\rho^{\text{cont}}(s) = \rho^{\text{OPE}}(s) \Theta(s - s_0),
$$

(7)

After making a Borel transform to both sides of the sum rule, and transferring the continuum contribution to the OPE side, the sum rules for the scalar meson $Z^+_1$, considered as a scalar $D^*D^*$ molecule, up to dimension-eight condensates, using factorization hypothesis, can be written as:

$$
\lambda^2 e^{-m_{D^*D^*}^2/M^2} = \int_{4m^2}^{s_0} ds \, e^{-s/M^2} \rho^{\text{OPE}}(s) + \Pi^{\text{mix}(\bar{q}q)}(M^2),
$$

(8)

where

$$
\rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{(\bar{q}q)}(s) + \rho^{(G^2)}(s) + \rho^{\text{mix}}(s) + \rho^{(\bar{q}q)^2}(s),
$$

(9)

with

$$
\rho^{\text{pert}}(s) = \frac{3}{2^9 \pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta^3} [ (\alpha + \beta)m_c^2 - \alpha \beta s ]^4,
$$

$$
\rho^{(\bar{q}q)}(s) = -\frac{3m_c^2 \langle \bar{q}q \rangle}{2^6 \pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\text{min}}}^{1-\alpha} \frac{d\beta}{\beta} [ (\alpha + \beta)m_c^2 - \alpha \beta s ]^2,
$$

$$
\rho^{(G^2)}(s) = \frac{m_c^2 \langle G^2 \rangle}{2^6 \pi^6} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\text{min}}}^{1-\alpha} d\beta (1 - \alpha - \beta) [ (\alpha + \beta)m_c^2 - \alpha \beta s ]^2,
$$

$$
\rho^{\text{mix}}(s) = -\frac{3m_c^2 m_0^2 \langle \bar{q}q \rangle}{2^6 \pi^4} \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} \frac{d\alpha}{\alpha} [ m_c^2 - \alpha (1 - \alpha) s ],
$$

$$
\rho^{(\bar{q}q)^2}(s) = \frac{m_c^2 \langle \bar{q}q \rangle^2}{4\pi^2} \sqrt{1 - 4m_c^2/s},
$$

(10)
where the integration limits are given by \( \alpha_{\text{min}} = (1 - \sqrt{1 - 4m_c^2/s})/2 \), \( \alpha_{\text{max}} = (1 + \sqrt{1 - 4m_c^2/s})/2 \), \( \beta_{\text{min}} = am_c^2/(s\alpha - m_c^2) \), and we have used \( \langle \bar{q}qGq \rangle = m_0^2\langle \bar{q}q \rangle \). We have neglected the contribution of the dimension-six condensate \( \langle g^3G^3 \rangle \), since it is assumed to be suppressed by the loop factor \( 1/16\pi^2 \). For completeness we have also included a part of the dimension-8 condensate contributions

\[
\Pi^{\text{mix} (\bar{q}q)}(M^2) = -\frac{m_c^2 m_0^2 \langle \bar{q}q \rangle^2}{8\pi^2} \int_0^1 d\alpha \frac{-m_0^2}{\alpha(1-\alpha)} \left[ 1 + \frac{m_c^2}{\alpha(1-\alpha)M^2} \right].
\]

One should note that a complete evaluation of the dimension-8 condensate contributions require more involved analysis including a nontrivial choice of the factorization assumption basis \cite{17}, which is beyond the scope of this calculation.

For a consistent comparison with the results obtained for the other molecular states using the QCDSR approach, we have considered here the same values used for the quark masses and condensates as in refs. \cite{5,13,15,18}: \( m_c(m_c) = (1.23 \pm 0.05) \text{ GeV} \), \( \langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3 \text{ GeV}^3 \), \( m_0^2 = 0.8 \text{ GeV}^2 \), \( \langle g^2G^2 \rangle = 0.88 \text{ GeV}^4 \), where \( g = \sqrt{4\pi\alpha_s} \).

To determine the Borel window, we analyse the OPE convergence and the pole contribution: the minimum value of the Borel mass is fixed by considering the convergence of the OPE, and the maximum value of the Borel mass is determined by imposing that the pole contribution must be bigger than the continuum contribution. To fix the continuum threshold range we extract the mass from the sum rule, for a given \( s_0 \), and accept such value of \( s_0 \) if the obtained mass is in the range 0.4 GeV to 0.6 GeV smaller than \( \sqrt{s_0} \). Using these criteria, we evaluate the sum rules in the Borel range \( 2.0 \leq M^2 \leq 3.5 \text{ GeV}^2 \), and in the \( s_0 \) range \( 4.5 \leq \sqrt{s_0} \leq 4.7 \text{ GeV} \).

From Fig. 1 we see that for \( M^2 \geq 2.5 \text{ GeV}^2 \) the contribution of the dimension-8 condensate is less than 20% of the sum of the other contributions. Using this fact as a criterion to establish a reasonable OPE convergence, we fix the lower value of \( M^2 \) in the sum rule window as \( M^2_{\text{min}} = 2.5 \text{ GeV}^2 \).

The comparison between pole and continuum contributions for \( \sqrt{s_0} = 4.6 \text{ GeV} \) is shown in Fig. 2. From this figure we see that the pole contribution is bigger than the continuum for \( M^2 \leq 2.9 \text{ GeV}^2 \). The maximum value of \( M^2 \) for which this constraint is satisfied depends on the value of \( s_0 \). The same analysis for the other values of the continuum threshold gives \( M^2 \leq 2.75 \text{ GeV}^2 \) for \( \sqrt{s_0} = 4.5 \text{ GeV} \) and \( M^2 \leq 3.1 \text{ GeV}^2 \) for \( \sqrt{s_0} = 4.7 \text{ GeV} \). In our numerical analysis, we shall then consider the range of \( M^2 \) values from 2.5 GeV$^2$ until the one allowed by the pole dominance criterion given above.

To extract the mass \( m_{D^*D^*} \), we take the derivative of Eq. (8) with respect to \( 1/M^2 \), and divide the result by Eq. (8).
In Fig. 2 we show the relative pole contribution (the pole contribution divided by the total, pole plus continuum, contribution) and the solid line shows the relative continuum contribution for $\sqrt{s_0} = 4.6$ GeV.

In Fig. 3 we show the $D^*D^*$ meson mass as a function of the sum rule parameter ($M^2$) for $\sqrt{s_0} = 4.4$ GeV (long-dashed line), $\sqrt{s_0} = 4.5$ GeV (dotted line), $\sqrt{s_0} = 4.6$ GeV (solid line) and $\sqrt{s_0} = 4.7$ GeV (dot-dashed line). The crosses indicate the upper and lower limits in the Borel region.

Using the Borel window, for each value of $s_0$, to evaluate the mass of the $D^*D^*$ meson and then varying the value of the continuum threshold in the range $4.5 \leq \sqrt{s_0} \leq 4.7$ GeV, we get $m_{D^*D^*} = (4.15 \pm 0.05)$ GeV.

Because of the complex spectrum of the exotic states, sometimes lower continuum threshold values are favorable in order to completely eliminate the continuum above the resonance state. Therefore, in Fig. 3 we also include the result for the $D^*D^*$ meson mass for $\sqrt{s_0} = 4.4$ GeV. We see that we get a very narrow Borel window, and for values of the continuum threshold smaller than 4.4 GeV there is no allowed Borel window. Considering then the continuum threshold in the range $4.4 \leq \sqrt{s_0} \leq 4.7$ GeV, we get $m_{D^*D^*} = (4.13 \pm 0.07)$ GeV.

To check the dependence of our results with the value of the charm quark mass, we fix $\sqrt{s_0} = 4.6$ GeV.
and vary the charm quark mass in the range $m_c = (1.23 \pm 0.05)$ GeV. Using $2.5 \leq M^2 \leq 2.9$ GeV we get $m_{D^*D^*} = (4.15 \pm 0.07)$ GeV.

Up to now we have taken the values of the quark-gluon mixed condensate and the gluon condensate without allowing any uncertainties. While from Fig. 1 we can see that a change in the gluon condensate value has little effect in our results, this is not the case for the quark-gluon mixed condensate. Allowing $m_0^2$ to vary in the range $m_0^2 = (0.8 \pm 0.1)$ GeV$^2$ and fixing $\sqrt{s_0} = 4.6$ GeV we get $m_{D^*D^*} = (4.15 \pm 0.06)$ GeV. Finally, assuming a possible violation of the factorization hypothesis, one should multiply $\langle \bar{q}q \rangle^2$ in Eqs. (10) and (11) by a factor $K$. Using $K = 2$, which means a violation of the factorization hypothesis by a factor 2, and $\sqrt{s_0} = 4.6$ GeV we get $m_{D^*D^*} = (4.12 \pm 0.02)$ GeV. Therefore, taking into account the uncertainties in the QCD parameters as discussed above we arrive at

$$m_{D^*D^*} = (4.13 \pm 0.07^{+0.09 +0.08 +0.01}_{-0.05 -0.04 -0.03}) \text{ GeV},$$

where the first, second, third and forth errors come from the uncertainties in $s_0$, $m_c$, $m_0^2$ and the factorization hypothesis respectively. Adding the errors in quadrature we finally arrive at

$$m_{D^*D^*} = (4.15 \pm 0.12) \text{ GeV},$$

where the central value is around 130 MeV above the $D^*D^*(4020)$ threshold, indicating the existence of repulsive interactions between the two $D^*$ mesons. Strong interactions effects might lead to repulsive interactions that could result in a virtual state above the threshold. Therefore, this structure may or may not indicate a resonance. However, considering the errors, it is compatible with the observed $Z_{1}^{+}(4050)$ resonance mass.

One can also deduce, from Eq. (8) the parameter $\lambda$ defined in Eq. (6). We get:

$$\lambda = (4.20^{+0.68 +0.46 +0.45 +0.19}_{-0.88 -0.30 -0.19 +0.03}) \times 10^{-2} \text{ GeV}^5,$$

where the first, second, third and forth errors come from the uncertainties in $s_0$, $m_c$, $m_0^2$ and the factorization hypothesis respectively. Adding the errors in quadrature we finally arrive at $\lambda = (4.20 \pm 0.96) \times 10^{-2} \text{ GeV}^5$.

Since to obtain the mass we have taken the derivative of the sum rule in Eq. (8), it is important also to check if the convergence of the OPE and the pole contribution dominance are also satisfied for the derivative sum rule.

From Fig. 4 we see that the OPE convergence is even better as from Fig. 1. Therefore, it is correct to fix the lower value of $M^2$ from the convergence of the original sum rule in Eq. (8). Regarding the pole contribution, we show in Table I the values of $M^2$ for which the pole contribution is 50% of the total contribution, for each value of $\sqrt{s_0}$.

![Fig. 4: Same as Fig. 1 for the derivative of Eq. (8).](image-url)
The two invariant functions, Π₁ and Π₀, appearing in Eq. (17), have respectively the quantum numbers of the spin 1 and 0 mesons. Therefore, we choose to work with the Lorentz structure \( g_{\mu\nu} \), since it gets contributions only from the \( 1^- \) state. The sum rule for the meson \( Z^+_2 \), considered as a vector \( D_1 D \) molecule, in the Lorentz structure \( g_{\mu\nu} \) can also be given by Eq. (8) with:

\[
\begin{align*}
\rho_{\text{pert}}(s) &= \frac{3}{2^7\pi^6} \int\limits_0^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int\limits_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)(1+\alpha+\beta) \left[ (\alpha+\beta)m_c^2 - \alpha\beta s \right]^4, \\
\rho^{(q\bar{q})}(s) &= \frac{3m_c(m_c^2)}{2^7\pi^4} \int\limits_0^{\alpha_{\max}} \frac{d\alpha}{\alpha} \int\limits_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} (1-\alpha-\beta) \left[ (\alpha+\beta)m_c^2 - \alpha\beta s \right]^2, \\
\rho^{(G^2)}(s) &= \frac{(g^2G_s^2)}{2^{11}\pi^6} \int\limits_0^{\alpha_{\max}} \frac{d\alpha}{\alpha} \int\limits_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} \left[ (\alpha+\beta)m_c^2 - \alpha\beta s \right] \left[ m_c^2(1-(\alpha+\beta)^2) - \frac{1}{\alpha} \right], \\
\rho^{(q\bar{q})^2}(s) &= \frac{3m_c^2(m_c^2)}{2^8\pi^4} \int\limits_0^{\alpha_{\max}} \frac{d\alpha}{\alpha} \int\limits_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} (2\alpha+3\beta)(1-(\alpha+\beta)m_c^2 - \alpha\beta s), \\
\rho^{m_{\bar{q}q}^2}(s) &= -\frac{m_c^2(m_c^2)}{2^4\pi^4} \sqrt{1-4m_c^2/s}, \\
\Pi^{\text{mix}(q\bar{q})}(M^2) &= -\frac{m_c^2m_0(m_c^2)}{2^5\pi^2} \int_0^1 \frac{d\alpha}{1-\alpha} e^{-\alpha M^2} \frac{1}{\alpha M^2} \left[ \alpha - \frac{m_c^2}{\alpha M^2} \right].
\end{align*}
\]

In this case, from Fig. 5 we see that we obtain a reasonable OPE convergence for \( M^2 \geq 2.4 \text{ GeV}^2 \). Therefore, we fix the lower value of \( M^2 \) in the sum rule window as \( M^2_{\text{min}} = 2.4 \text{ GeV}^2 \). The OPE convergence obtained from the derivative sum rule is better than the OPE convergence from the original sum rule. From the derivative sum rule we get a good OPE convergence for \( M^2 \geq 2.2 \text{ GeV}^2 \).

The upper limits for \( M^2 \) for each value of \( \sqrt{s_0} \) are given in Table II, for the original sum rule and for the derivative sum rule.
Table II: Upper limits in the Borel window for $D_1 D$ molecule with $J^P = 1^-$.  

| $\sqrt{s_0}$ (GeV) | $M_{\text{max}}^2$ (GeV$^2$) (sum rule in Eq.(S)) | $M_{\text{max}}^2$ (GeV$^2$) (derivative sum rule) |
|---------------------|-------------------------------------|-------------------------------------|
| 4.5                 | 2.56                                | 2.25                                |
| 4.6                 | 2.73                                | 2.42                                |
| 4.7                 | 2.90                                | 2.58                                |

Again we see that the upper limits in the Borel window imposed by the derivative sum rule are smaller than the ones obtained with the original sum rule, and this would restrict the range of values allowed for the continuum threshold. However, we will allow a small violation in the 50% pole contribution criterion in the derivative sum rule, and we will work in the Borel window allowed by the original sum rule.

In the case of the $D_1 D$ molecule we get a worse Borel stability than for the $D^* D^*$, in the allowed sum rule window, as a function of $M^2$, as can be seen by Fig. 6.

In Fig. 6 we show the $D_1 D$ meson mass as a function of the sum rule parameter ($M^2$) for $\sqrt{s_0} = 4.5$ GeV (dotted line), $\sqrt{s_0} = 4.6$ GeV (solid line) and $\sqrt{s_0} = 4.7$ GeV (dot-dashed line). The crosses indicate the upper and lower limits in the Borel region.
function of $M^2$.

Using the value of the continuum threshold in the range $4.5 \leq \sqrt{s_0} \leq 4.7$ GeV, and varying $m_c$, $m_0^2$ and $K$ as discussed above, we get

$$m_{D_1D} = (4.10^{+0.12}_{-0.08} +0.05 -0.05 +0.01 -0.09 +0.28) \text{ GeV},$$

(19)

where the first, second, third and forth errors come from the uncertainties in $s_0$, $m_c$, $m_0^2$ and the factorization hypothesis respectively. Adding the errors in quadrature we finally arrive at

$$m_{D_1D} = (4.19 \pm 0.22) \text{ GeV},$$

(20)

where the central value is around 100 MeV below the $D_1D(285)$ threshold, and around 60 MeV smaller than the mass of the $Z_2^+(4250)$ resonance structure. Therefore, in this case, there is an attractive interaction between the mesons $D_1$ and $D$ which can lead to the molecular state discussed above. Considering the uncertainties in Eq. (20), and the width of the $Z_2^+(4250)$ resonance structure: $\Gamma_2 = 177^{+54}_{-39} - 61$ MeV, it seems to us that it is possible to describe this structure as a $D_1D$ molecular state with $I^G J^P = 1^- 1^-$ quantum numbers. However, considering the uncertainties, the result in Eq. (20) is also compatible with the observed $Z_1^+(4050)$ resonance mass. Therefore, no definite conclusion can be drawn for this state from the present analysis.

For the value of the parameter $\lambda$ defined in Eq. (6) we get:

$$\lambda_{Z_2} = (1.60^{+0.40}_{-0.30} +0.30 -0.30 +0.66 -0.19 -0.10) \times 10^{-2} \text{ GeV}^5,$$

(21)

where the first, second, third and forth errors come from the uncertainties in $s_0$, $m_c$, $m_0^2$ and the factorization hypothesis respectively. Adding the errors in quadrature we finally arrive at $\lambda_{Z_2} = (1.88 \pm 0.77) \times 10^{-2} \text{ GeV}^5$.

In conclusion, we have presented a QCD analysis of the two-point function for possible $D^* D^*$ and $D_1D$ molecular states with $I^G J^P = 1^- 0^+$ and $I^G J^P = 1^- 1^-$ respectively. For the $D^* D^*$ molecule with $I^G J^P = 1^- 0^+$ we got a mass around 120 MeV above the $D^* D^*$ threshold, and around 160 MeV above the observed $Z_2^+(4050)$ mass. In the case of the $D_1D$ molecule with $I^G J^P = 1^- 1^-$ we got a mass around 100 MeV below the $D_1D$ threshold, and around 60 MeV smaller than the observed $Z_2^+(4250)$ mass. Therefore, their conclusion is that it is possible to describe the $Z_2^+(4250)$ resonance structure as a $D_1D$ molecular state with $I^G J^P = 1^- 1^-$ quantum numbers.

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