Interaction of high-speed plasma jet with a pulse of powerful microwave radiation

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Abstract. The interaction of high-speed plasma jet created by a discharge in an ablative capillary with powerful pulse of microwave radiation (W≈600 kW, λ=2.3 cm, τ=8 μs) is studied. A significant influence of microwave radiation pulse on the plasma jet flow pattern, connected with the development of instability similar to the instability of the free shear flows, is found. Evolution of instability depends on the initial level of perturbation and the plasma flow velocity. The typical for gas jet flows "classical" evolution scenario of instability, including the steps of perturbation amplification, the formation of large-scale vortex structures, their nonlinear interaction and the development of turbulence is realized only at high intensities of the initial perturbation and plasma velocity close to the threshold of the laminar-turbulent transition. In the case of low-speed plasma jets the perturbation amplification leads, eventually, to the interruption of the flow without obvious signs of turbulence. The scenario of instability attenuation is realized at low levels of initial perturbation and generally is common both for low-speed and for high-speed jets, and includes the perturbation zone extension with its simultaneous drift downstream. The drift velocity of the perturbation is comparable to the plasma velocity in the peripheral zone of the jet, which indicates the shear nature of the instability. A significant influence of the plasma jet’s condition on the spatial position of the microwave pulse energy release domain is found.

1. Introduction

The interaction of a microwave discharge with gas–plasma flows is one of the important problems of magneto plasma aerodynamics. The ability to control parameters, configuration and spatial localization of energy release domain by changing the parameters of microwave radiation and its focusing system is doubtless advantage of such a discharge. The efficiency of the incident flow parameters control convincingly demonstrated in the experiments in which the microwave discharge ignition in the vicinity of the streamlined body led to a significant deformation of the attached shock wave and decreasing the drag force [1]. Later, combined discharges were studied (microwave combined with a spark one), by means of which the microwave breakdown threshold was substantially reduced and the localization of the discharge in a desired spatial domain was provided. A laser or a pulsed high-voltage spark was used for the microwave discharge initiation in this case [2].

A promising and simple method of initiating the combined discharge is the use of a thin plasma jet created by a pulsed capillary discharge [3,4]. In general, such a plasma jet can also be used for effective control of the parameters of supersonic flow [5,6] with simultaneous initiation and significant decrease in microwave breakdown threshold in the demanded range. Moreover, high-power microwave radiation can influence the gasdynamic and plasma properties of the jet itself (in particular, its stability), which is of independent methodological and scientific interest for studying the dynamics of jet flows [7–9] under an external remote action. Therefore, the main objective of our experiments was to comprehend the variety of the phenomena arising upon the interaction of a plasma jet with...
powerful microwave radiation, which, in our opinion, is of undoubted interest for various applications. This is a continuation of research initiated in [10].

2. Experimental Facility

The object of our studies was the plasma jet outflowing into the air atmosphere created by a pulsed discharge in a capillary with ablative wall (figure 1(a)). The capillary is made from hydrocarbon polymer – polymethyl methacrylate. Its initial diameter is d=1 mm and the depth is h=4 mm. The anode (copper, graphite) is tightly mounted to the capillary inlet and the cathode is installed near the capillary outlet. The capacitive storage connected in series through an inductor is used for discharge power supply. Discharge power algorithm corresponds approximately to a half-wave of sine. The detailed description of discharge gap design, the experimental setup and methods of research can be found elsewhere [3,4].

We studied two plasma jet modes in the experiments - subsonic and supersonic - with substantially different flow velocities, jet lifetime, and power of the discharge pulse. Typical values of these parameters are following: velocity of the jet’s contact boundary (top front of the jet) - \( v_{\text{sub}} \approx 30 \text{ m/s} \) and \( v_{\text{sup}} \approx 250 \text{ m/s} \), velocity of plasma in paraxial zone of the jet - \( v_{\text{pl,sub}} \approx 600-3000 \text{ m/s} \) and \( v_{\text{pl,sup}} \approx 4000-10000 \text{ m/s} \), duration of discharge pulse - \( t_{\text{sub}} \approx 9 \text{ ms} \) and \( t_{\text{sup}} \approx 1 \text{ ms} \), peak discharge power (the maximum value of the instantaneous power achieved to the middle of the discharge pulse) - \( P_{\text{sub}} \approx 10 \text{ kW} \) and \( P_{\text{sup}} \approx 90 \text{ kW} \), - for subsonic and supersonic jet, respectively. The main criterion for selecting the parameters of the discharge pulse was obtaining a laminar flow mode in the capillary outlet. This mode allows creating thin jets with maximal length reaching 10-20 cm to the middle of the discharge pulse. At the same time, a laminar flow regime can be provided in a restricted range of discharge pulse parameters. One of the important parameters is a critical peak power, which excess is accompanied by the development of turbulence and as a consequence by the dramatic reduction in the jet’s length. The critical value of the peak power is \( P \approx 100 \text{ kW} \) for considered here discharge gap. This value is slightly higher than the peak power of a discharge pulse used for creation the supersonic jet. For a subsonic jet the peak power of a discharge pulse is substantially lower (for order of magnitude) than the critical value.

Study of interaction of the plasma jet with the microwave radiation was carried out in the metallic chamber (figure 1(b)). The diameter of the chamber is 0.7 m, its length - 1 m. Input of the microwave radiation is carried out through a waveguide mounted on an end flange of the chamber. Pulse magnetron is used as microwave radiation source with the following parameters: power \( W \approx 600 \text{ kW} \), wavelength \( \lambda = 2.3 \text{ cm} \), pulse duration \( \tau = 8 \mu \text{s} \). The parabolic mirror, which is mounted on the opposite side of the chamber, is used to focusing the microwave radiation. The electric field intensity in the focal spot is about \( E \approx 3 \text{ kV/cm} \), which is substantially below the threshold of self-ignition of the microwave discharge at atmospheric pressure. A capillary gap installed along the chamber axis in the vicinity of the focal spot region. By adjusting the spatial position of the discharger, the axis of the plasma jet is aligned with the main axis of the focusing system. To align the middle part of the jet with the focal plane of microwave radiation and to prevent the initiation of a microwave discharge on metal parts of the capillary gap, the last one is placed at a certain distance from the focal spot. The distance between the center of the focal spot and the capillary outlet was taken equal to 6 cm and did not change during the experiments.

The interaction of microwave radiation with a plasma jet was studied at different delays of a microwave pulse from the ignition of the capillary discharge. Temporal evolution of the plasma jet was recorded by a Motion Pro N3 high-speed video camera (exposure time of 1 \( \mu \text{s} \), frame rate of 10 kHz). PCO Sensicam double-frame camera (minimum exposure time of 100 ns) was used for detailed study of the initial stage of the interaction, that permit to synchronize the frame relatively the onset of a microwave pulse and the instant of ignition of the capillary discharge. This allowed us to follow the dynamics of the flow pattern during the microwave pulse as well as before its onset and after its termination.
3. Results
Switching on a microwave pulse leads to perturbation of the plasma jet, which dramatically changes the subsequent flow pattern. Numerous shots have shown that the instant of the microwave pulse is accompanied by a disturbance of the initial section of the jet, in which vicinity the main portion of the microwave radiation, perhaps, is absorbed. The analysis of numerous jets images obtained at the instant the microwave pulse, allowed us to determine the spatial position of energy release domain, the center of which is located at a distance of 1-3 cm from the capillary outlet. Spatial position of the energy release domain depends on the condition plasma outflowing from the capillary, whose parameters vary continuously during the discharge pulse. So the higher the instantaneous discharge power - the farther is the position of energy release domain relatively capillary outlet. Microwave pulse has a greater impact on subsonic jets. For such jets, switching on a microwave pulse leads to deformation of the initial section (along the length of about 10–15 mm), the core of which takes a form close to sinusoidal and at the periphery of which the formation of ordered vortex structures is observed (figure 2). The time from switching on a microwave pulse to the beginning of noticeable deformation does not exceed 2 μs and, to the end of the microwave pulse (approximately 1–2 μs before its termination), the amplitude of deformation and the sizes of the vortex structures reach the maximum. The main features of this flow pattern persist for relatively long time: during the remaining
part of the microwave pulse (approximately 2 μs) and for 20–30 μs after its termination. After this time, the drift of the perturbation downstream, caused by the income of new portions of plasma from the capillary, becomes noticeable. During the drift the amplitude of deformation of the initial section smoothly decreases and the full restoration of its initial form is reached approximately 50 μs after the termination of the microwave pulse. The total duration of the initial stage of the evolution of the subsonic jet perturbation does not exceed 60 μs and decreases with increasing plasma velocity. The duration of the initial stage for supersonic jets does not exceed 10 μs. The initial stage occupies a relatively short interval of time lasting less than a few percent of the total duration of the disturbance evolution that characterizes it as fast stage.

The subsequent evolution of the perturbation (the slow stage) develops by two main scenarios corresponding to its amplification or attenuation. Implementation of a particular scenario of the slow stage is determined by the intensity of initial perturbation, dependent upon the microwave power density in the absorption domain and its spatial position relative to jet, which essentially depend on the plasma parameters in the jet vicinity different for subsonic and supersonic regimes [11]. According to this criterion the perturbations is conveniently divided into strong and weak. The initial stage of the first scenario (strong perturbations) is characterized by the further increase in the transverse sizes and transformation of the perturbed section, the shape of which deviates more and more from sinusoidal (see frame 2 and inset in figure 3(a)). The transformation also affects the vortex structures, the sizes of
which decrease from the initial ones, apparently due to their nonlinear interaction and further fragmentation [7,8,12,13]. This scenario terminates with cardinal restructuring of the flow, whose resulting pattern depends on the flow velocity. For subsonic jets, the amplification of the perturbation, as a rule, terminates with a disruption of the initial flow and the formation at the place of disruption of a new contact boundary separating the sections of the jet to both sides of the disruption (figure 3(a)). Herewith, the section located after disruption slows down and relaxes over a time about $t_{rel} \approx 0.5-1$ ms. The velocity of newly formed contact boundary at the initial moment takes the value of $v_{sub, f} \approx 150-200$ m/s. Subsequently, in process of passage the pre-heated and slowly cooling section of relaxing plasma, its velocity decreases, but remains a 2-3 times higher until it reaches the cold gas area, where the jet’s front velocity decreases to its usual value - $v_{sub, f} < 30$ m/s [14].

Figure 3. The slow stage of the evolution of strong perturbation of (a) a subsonic and (b) a supersonic plasma jet under the action of a microwave radiation pulse: (a) $t_d = 2$ ms, (b) $t_d = 200$ μs, 1 – low-scale vortices; frame rate of 10 kHz, and the arrow $F_0$ indicates the position of the focal plane.
The duration of the slow stage of the evolution of a strong perturbation decreases with increasing velocity of the plasma jet. For supersonic jets it does not exceed 100 μs. The transience of stage under these conditions does not allow us to trace the dynamics of vortex structures in detail in contrast with subsonic jets. Termination of slow stage of evolution of strong disturbance is characterized by formation of clearly expressed turbulent section, which is drifted downstream under the action of plasma incoming from the capillary (figure 3(b)). It should be noted that, for subsonic jets, no explicit transition to small-scale turbulence is observed.

The second scenario is realized at weak level of initial perturbation. In this case the extension of the disturbed section along the jet axis in both directions on the background of its drift downstream takes place. This scenario is realized as for subsonic (figure 4(a)) and for supersonic (figure 4(b)) jets.

**Figure 4.** The evolution of weak perturbation of (a) a subsonic and (b) a supersonic plasma jet under the action of a microwave radiation pulse: (a) \( t_d = 4 \) ms, \( f = 10 \) kHz, (b) \( t_d = 150 \) μs, \( f = 60 \) kHz; the arrows indicate the position of upstream and downstream fronts of perturbation.

The drift velocity of the perturbation decreases downstream and changes in a range of \( v_d = 100-250 \) m/s for subsonic and of \( v_d = 400-600 \) m/s for supersonic jets (figure 4 and figure 5). As can be seen, the
drift velocity of the perturbation is comparable to the plasma velocity in the jet’s peripheral zone only, which clearly indicates the nature of the resulting instability as the instability of the shear layer. Decrease of the perturbation drift velocity downstream (figure 5), caused by mixing in the shear layer, just confirms this conclusion.

The expansion velocity of the perturbation zone is much smaller than the drift velocity, and changes in a range of $v_{\text{exp}} \approx 20$-60 m/s for subsonic and of $v_{\text{exp}} \approx 80$-280 m/s for supersonic jets (figure 5). At the same time, the expansion velocity of perturbation when it moves downstream changes in different ways: it decreases for subsonic and increases for supersonic jets (figure 5). As can be seen from figure 4(b), increasing the expansion velocity of perturbation of the supersonic jet is caused by an increase of the downstream front velocity, while the upstream front velocity decreases. The reasons for such a distinction are not well understood. It can be assumed that the different character of upstream and downstream front velocities and their inconstancy are caused by inhomogeneity of the plasma parameters (first of all, concentrations of the components) in the mixing layer, which thickness increases downstream the faster, the higher the flow velocity. Increase of the thickness of the mixing layer, in particular, causes the decrease in plasma density in a peripheral zone where the heavy elements are localized predominantly because high radial temperature gradients [4]. So, the plasma density in the peripheral zone along the high-speed jets can change significantly that may be assumed as a possible reason for different character of upstream and downstream fronts velocities. A detailed analysis of possible reasons for discovered feature is beyond the scope of this paper and requires a comprehensive research.

![Figure 5. The drift velocity $v_d$ and expansion velocity $v_{\text{exp}}$ of the perturbation zone along the axis of subsonic and supersonic jets.](image)

4. Discussion

Thus, the plasma jet a reaction to the exposure of microwave radiation pulse contains specific features and stages of the evolution of the instability of the free shear flows, to which, in particular, the jet flows and boundary layers are related. It is known [7,8,12,13,15,16], that such flows, if the inflection point in velocity profile is present, are always non-viscous unstable because Rayleigh instability, and initially unstable perturbations increase exponentially downstream. The starting point of this process is the primary Kelvin–Helmholtz instability, which leads to successive deconvolution of shear layers into vortex structures [17,18], the configuration of which is determined by the dominant spatial mode of instability - symmetrical (varicose) or asymmetrical (sinusoidal or helical - depending on the flow geometry) [19–22]. Subsequently, such vortex structures are interact with each other and combined [23], and appearing on this background the secondary instabilities can lead to their destruction and transition to turbulence [24–26].
All of the above properties of instability and the stages of its evolution in varying degrees are present in our case. However, there are some specificities. Firstly, the evolution of instability depends on the initial level of perturbation. In particular, the scenario finishing with the development of turbulence is realized at a high intensity of the initial perturbation and a rather high flow velocity, close to the threshold for the laminar–turbulent transition $Re-Re_{cr}$. Taking into account the data $Re_{cr} \approx 100–600$ for gas jets [27–29], we obtain the critical flow velocity $v_{cr} \approx 3–12$ km/s corresponding to our experimental conditions (hydrogen as the working gas, $T=10000$ K, $p=1$ bar, $d=1$ mm). This velocity is reached in paraxial zone of supersonic jets, in experiments with which the turbulent section is formed. Herein, no transition to turbulence is observed in subsonic jets and the excess of a certain threshold intensity of initial perturbation eventually leads to the disruption of the flow. At a low intensity of perturbation its amplification does not take place neither in subsonic nor in supersonic jets. Moreover, the dynamics of a weak perturbation is mainly determined by the parameters of the peripheral zone, significantly different from the parameters of a high-temperature jet core [4,14].

Secondly, the action of a microwave pulse always leads to the development of the asymmetric (helical) mode of instability, which, as a rule, dominates in jet flows with the Poiseuille velocity profile and in conditions of an asymmetric external action [22,30–32]. Although we admit the formation of a velocity profile close to the Poiseuille profile on the capillary outlet, which is quite possible due to the significantly nonuniform radial profiles of temperature and concentrations of plasma components [4], we suppose that the main reason for the excitation of the asymmetric mode is the asymmetric position of the energy release domain with respect to the jet axis. This fact is caused, primarily, by the asymmetry of the discharge current relative jet axis and, as a consequence, by the asymmetric profile of the electron density in the jet vicinity. Additional impact on the spatial position and the parameters of the energy release domain can provide electromagnetic excitation of the plasma jet as a single-wire long line, which was the subject of our previous studies [33].

An important result, in our opinion, is the experimentally observed dependence of the spatial position of energy release domain (its displacement by 4-5 cm from the focus toward the capillary) on the parameters of the plasma jet. The dependence of the spatial position of this domain from instant value of discharge power is experimentally established, according to which the distance between the energy release domain and the capillary outlet increases with instant power. However, its position never reaches the focus, even if the length of the plasma jet in the moment switching on the microwave pulse is significantly overlap this distance.

This result indicates that the position of energy release domain is determined by optimal conditions for absorption the microwave power, i.e. $q=\sigma E^2 \approx \max$ (where $\sigma(n_e, \omega)$, $E(n_e, \omega)$ – conductivity and electric field intensity, respectively, $n_e$ – electron density, $\omega=2\pi c/\lambda$ – the angular frequency of the electromagnetic wave). According to our estimations, taking into account the dependence of the field penetration depth from the electron density $\delta \approx \frac{c}{\omega_p} \sqrt{\frac{2}{\omega}}$, these conditions can be achieved closer to the plasma jet boundary, where the conductivity increases up to $\sigma=\sigma_0 \frac{\omega_p^2}{\nu}\approx 200$ S/m ($n_e=10^{15}$ cm$^{-3}$, $\delta=0.4$ mm, $T=6000$ K) if to compare with its value $\sigma=0.04$ S/m in the cutoff region ($n_e=n_{cr}=2 \cdot 10^{12}$ cm$^{-3}$, $\delta=2$ cm, $T=1000$ K). Here $\omega_p=\sqrt{\frac{n_e e^2}{m_0}}$ – Landmuir frequency, $\nu$ – elastic collision frequency of electrons with atoms and molecules (for air $\nu=1.7 \cdot 10^{13}$ s$^{-1}$ [34]), $n_0=p/k_B T$ – concentration of atoms, p – pressure, $T$ – temperature, $k_B=1.38 \cdot 10^{-23}$ J/K – Boltzmann constant, m, e, $\epsilon_0$ – elementary mass, charge, dielectric constant of vacuum, respectively. The absorbed power density and overpressure in this limiting case take the values $q\approx 2 \cdot 10^{13}$ W/m$^3$ and $p_{over}=q \cdot \tau<1.5$ kbar, respectively (for $E=3$ kW/cm, $\tau=8$ $\mu$s). The actual value of overpressure is obviously significantly below, firstly, because unaccounted dependence of the electric field intensity from plasma conductivity, and, secondly, due to the expansion of the energy release domain immediately after the start of heating. According to observations, the overpressure can reach maximum approximately by the end of the microwave pulse.
(6-8 microsecond after the switch on), that is well traces by the dynamics of the initial stage of the disturbance (figure 2).

5. Conclusions

Thus, the obtained results show the high efficiency of the impact of a powerful pulse of microwave radiation on the gas dynamic of a high-speed plasma jet. As shown by results of research, the mechanism of action is associated with the formation of a local region of overpressure due to the absorption of microwave radiation by the plasma in the jet's periphery. The perturbation of gasdynamic flow leads to the development of the instability, which contains specific features and stages of evolution the instability of free shear flows. However, compared with the gas jet flows, the picture of the plasma jet instability has a number of differences. In particular, the evolution of instability depends on the initial level of perturbation and the plasma velocity. Herewith, "classical" evolution scenario of instability, including the steps of perturbation's amplification, the formation of large-scale vortex structures, their nonlinear interaction and the development of turbulence is realized only at high intensities of the initial perturbation and the plasma velocity close to the threshold of the laminar-turbulent transition $Re_{cr}$. In the case of low-speed jets the perturbation's amplification leads, eventually, to the interruption of the flow without obvious signs of turbulence and to formation the new contact boundary at the place of disruption.

Scenario of instability evolution at low levels of initial perturbation in general is common for both low speed and high speed jets, and includes an extension of the perturbed section with its simultaneous drift downstream. The drift velocity of the perturbation is comparable to the plasma velocity in the jet's peripheral zone, indicating the shear nature of the instability. The observed features of the evolution of supersonic plasma jet instability associated with the downstream acceleration of the perturbation's front, indirectly confirms this conclusion. We believe that the probable reason of the observed features can be variation in plasma density in the peripheral zone of the jet due to expansion of the mixing layer downstream. This issue, obviously, requires comprehensive studies.

The presence of the plasma jet has a significant influence on the position of energy release domain. Founded interconnection between the spatial localization of the microwave discharge with the parameters of the plasma jet makes possible a new approach to formulating the problem of control of the spatial position and parameters of the energy release domain, which is important for many applications, in particular, magnetoplasma aerodynamics.

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