Study on the Mechanism of the Variable-Speed Rotor Affecting Rotor Aerodynamic Performance

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Abstract: The variable-speed rotor has proven to be a promising means to improve helicopter performance. Previous investigations on the aeromechanics of the variable-speed rotor are scarce. This work studies the mechanism of the variable-speed rotor affecting the rotor aerodynamic performance by means of dimensionless parameter analysis and reveals the various effect mechanisms under the various flight speeds and take-off weights. The current work shows that, during hover, the variable-speed rotor improves the rotor aerodynamic performance by reducing the blade dynamic pressure, while the non-uniform thrust distribution is attributed to the reduced rotor speed suppresses the performance improvement. In slow forward flight, the blade dynamic pressure reduction improves the rotor aerodynamic performance. In cruise and fast forward flight, both the blade dynamic pressure and advancing blade tip compression loss reductions improve the rotor aerodynamic performance. The study also shows that in forward flight, the rotor loading is smaller, and the effect of reducing the advancing blade tip compression loss through the variable-speed rotor is greater.

Keywords: variable-speed rotor; rotor performance improvement; nonlinear aerodynamics; dimensionless parameter analysis

1. Introduction

The variable-speed rotor, also called the optimum speed rotor or slowed speed rotor, adjusts the helicopter or rotorcraft rotor speed along with variation in the flight condition to maintain a relatively high aerodynamic efficiency. For example, a helicopter may reduce its rotor speed under the condition of low payload or cruise flight speed, and a tilt-rotor may reduce its rotor speed when transferring to fixed-wing flight mode from helicopter mode.

In recent years, the variable-speed rotor has proven to be a promising way to reduce rotor-required power, extend the flight speed, endurance and range, and the reduce rotor noise [1–4]. Furthermore, research from the ACARE (Advisory Council for Aeronautics Research) has shown that there is a potential benefit of reducing exhaust pollutants by reducing the required power [5]. Karem first proposed the Optimum Speed Rotor (OSR) concept in 1974, ran the OSR project, and finally, developed the hummingbird unmanned helicopter A160/A160T, which achieved a long-endurance flight test for about 20 h [4]. The Advancing Blade Concept (ABC) program implemented the variable-speed rotor in a coaxial helicopter (XH-59A), reducing the rotor speed by 30%, making high speed flights possible [6,7]. The tilt-rotor Bell XV-3 changes the rotor speed from 532 rpm in helicopter hover mode to 324 rpm in fixed-wing cruise flight mode, which balances the hover figure of merit (FM) and cruise efficiency [8]. Steiner, Gandhi and Yoshizaki [9] and Steiner [10] simulated the effect of the rotor speed on the required rotor power, shaft torque, maximum flight speed and ceiling and showed that the greatest improvement was achieved in cruise forward flight. The study of Khoshlahjeh showed that the variable-speed rotor...
can improve performance more significantly under conditions of low altitude and small take-off weight than with a high altitude and large take-off weight [11]. Misté, Benini and Garavello et al. analyzed the variable-speed rotor with a turboshaft engine and optimized the rotor speed based on the minimal required power and minimal fuel consumption. The results showed that in hover and fast flight, these two optimizations lead to different optimal rotor speeds, and the one based on the required power could not reduce the entire vehicle fuel consumption due to the deterioration in engine efficiency [12]. Then, a continuously variable transmission concept was proposed to solve this problem [13,14]. Han, Pastrikakis and Barakos compared the CFD (computational fluid dynamics) method and dynamic inflow modeling in a variable-speed rotor study, which revealed a high correlation between these two methods. Their results indicated that the shaft torque could increase along with an increase in power, leading to increased tail rotor thrust [15]. Han and Barakos studied the variable-speed in the tail rotor by setting three different tail rotor speed schedules, which achieved a 1.4% reduction in total helicopter power in cruise flight when the tail rotor speed and main rotor speed were independent and were both optimal. However, the results showed that this was ineffective for reducing the amount of power in high speed flight [16].

Many researchers have discussed the benefits of the variable-speed rotor on helicopter performance; however, previous investigations on the aeromechanics of the variable-speed rotor are scarce. Xu, Han and Li analyzed the aeromechanics of the variable-speed rotor and tested it on a 2.5 m scale rotor platform. The results showed that the airfoil AoA (angle of attack) and lift-to-drag ratio increased by optimizing the rotor speed; thus, the hover FM was improved by 32% and the cruise flight power was decreased by 22% [17]. Bowen-Davies and Chopra studied the mechanisms of the variable-speed rotor affecting the rotor’s required power, mainly the levels of profile power in slow forward flight and fast forward flight [18,19]. The study showed that in slow forward flight, blade dynamic pressure reduction is the main cause of reduction in the profile power and the total required power of the variable-speed rotor. In fast forward flight, the advancing blade compression loss and blade dynamic pressure reductions both decrease the profile power and the total required power. The hover case, which is dominated by the induced power, has not yet been discussed. Furthermore, considering that the variable-speed rotor is mostly applied to a long-endurance unmanned helicopter, whose weight could largely decrease in flight, the effects of rotor loading should be investigated when discussing the aeromechanics of the variable-speed rotor.

This current work mainly focuses on studying the mechanisms of the variable-speed rotor affecting the rotor aerodynamic performance and contributes to revealing the various effect mechanisms under the conditions of various flight speeds, including hover, and rotor loadings. A dimensionless parameter analysis is applied in this current work. The prescribed wake method with BVI (blade vortex interaction) correction is applied to predict the rotor inflow to capture the non-uniform distribution of rotor inflow which is of significance when predicting the induced power. The Leishman–Beddoes dynamic stall algorithm [20–22] is applied to model the airfoil nonlinear aerodynamic behaviors for a variable-speed rotor. The rigid flap assumption is applied in the current work which limits the study of the variable-speed rotor dynamics, for example, the resonance issue and blade root loading analysis. A helicopter simulation code developed in-house is applied in the current work.

2. Modeling

The rotor vortex theory assumes that rotor wake could be treated as a potential flow field, so the induced velocity is computed by the Biot–Savart Law. As Figure 1a shows, the current work segments start as straight line elements. The induced velocity on interest point $P$ by a certain element is

\[
dv_i = \frac{\Gamma}{4\pi} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \frac{r_1 \times r_2}{r_1 r_2 + r_1 \cdot r_2}
\]

where $r_1$ and $r_2$ are vectors from the straight line end-points to $P$. 

The vortex element position, always within the rotor disc, is expressed as
\[
z_v = -\mu z \Delta \psi - \lambda_{i0} \left( 1 - 2\mu x y + E \left( \frac{8}{15\pi} + \cos \psi_t + \frac{\mu_x (\psi_t - \psi_b)}{2r} - |y|^3 \right) \right) \Delta \psi. \tag{2}
\]

The vortex element position, always beyond the rotor disc, is expressed as
\[
z_v = -\mu z \Delta \psi - 2\lambda_{i0} \left( 1 - 2\mu x y + E \left( \frac{8}{15\pi} - |y|^3 \right) \right) \Delta \psi. \tag{3}
\]

The position of a vortex element, convecting from the inside to the outside of the disc, is expressed as
\[
z_v = -\mu z \Delta \psi - \frac{2\lambda_{i0} x_e}{\mu_x} \left( 1 - 2\mu x y + E \left( \frac{8}{15\pi} - |y|^3 \right) \right) \Delta \psi. \tag{4}
\]
where \(x_e\) is the x-axis position when the vortex element convects outside the disc, and \(y\) is the vortex element y-axis position. Variable \(E\) equals \(\frac{1}{2} \tan^{-1} \left( \frac{\mu_x}{\mu_z + \lambda_b} \right)\).

The vortex core model is introduced to eliminate the singularity of Equation (6):
\[
r_c = \sqrt{r_0^2 + 4\alpha \nu \Delta t} \tag{5}
\]
where \(r_0\) is the initial core radius, and is chosen to be 5% of the chord in the current work. The Lamb–Oseen constant, \(\alpha\), equals 1.25643.

The induced velocity on \(P\) is computed by the core-modification Bivot–Savart Law:
\[
dv_i = \frac{\Gamma h^2}{4\pi (r_1^4 + h^4)^{0.5}} \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \frac{r_1 r_2 - r_1 \times r_2}{r_1 r_2 + r_1 \cdot r_2}. \tag{6}
\]

In forward flight, upwash can appear from the advancing blade to the retreating blade. Some interactions may be caused, as shown in Figure 1b, between the wake trailing from the front blade and the following blade because of small convection, the so-called blade–vortex interaction (BVI). The lift-surface correction \([23]\) is applied in the current work to consider this phenomenon.

The lift coefficient by lift-surface modeling is
\[
\frac{L_{ls}}{\rho V^2} = \frac{r \sin \Lambda}{(r \sin \Lambda)^2 + (h + c_0)^2} - \frac{a'_o}{\frac{b_0 x_0}{x_0^2 + 4 (r \sin \Lambda)^2 (h + c_0)^2}} - \sum_{k=1}^{2} a_k \left( \frac{d}{dr \sin \Lambda} \right)^{2k} \frac{b_0 x_0}{(r \sin \Lambda + b_1)^2 + (h + c_k)^2 \cos b_2} \frac{b_0 x_0}{(r \sin \Lambda + b_1)^2 + (h + c_k)^2 \cos b_2}. \tag{7}
\]
The lift coefficient by lift-line modeling is

\[
\frac{L_{ll}}{\rho v^2 \Gamma} = \frac{r \sin \Lambda}{(r \sin \Lambda)^2 + (h + c_0)^2} \quad \text{and} \quad \frac{r \sin \Lambda}{(r \sin \Lambda)^2 + (h + c_1)^2}. \quad (8)
\]

The parameters in Equations (7) and (8) are expressed as functions of the collision angle, \( \Lambda \) [23].

When the distance, \( d \), between a certain vortex element and the computing point, \( P \), is smaller than a predefined value, 10 times the chord in the current work, the vortex element-induced velocity is computed as

\[
d\tilde{v}_i = d\hat{v}_i \min \left( \left| \frac{L_{ls}}{L_{ll}} \right| , 1 \right). \quad (9)
\]

Both the blade motion and the airfoil freestream vary periodically along with the rotor revolution. With an increase in AoA, flow separation could appear on the airfoil surface. Under the effects of periodic motion and freestream, the airfoil aerodynamic forces and moment behaviors are unsteady. With a further increase in AoA, the airfoil can experience dynamic stall which also affects the aerodynamic forces and moments. The unsteady forces and moments are related to the airfoil motion, freestream, AoA and their rates of change. When lowering the rotor speed, the advance ratio increases the blade flap and feathering equivalently, and the AoA range increases; therefore, simulation of the unsteady behavior is of great significance when discussing the variable-speed rotor aerodynamics.

The current work applies the Leishman–Beddoes algorithm to simulate the process of airfoil dynamic stall. The model separates the unsteady dynamic stall process into three sub-processes—attached flow, separation flow, and deep dynamic stall, and introduces some one-step delays based on airfoil steady and dynamic experiments to simulate the aerodynamics via the indicial response method and Duhamel’s integral.

The attached flow is the base for simulating the unsteady aerodynamic process. The Leishman–Beddoes algorithm applies two parts—initial impulsive loading, \( C_{IN}^I \), and circulatory loading, \( C_{CN}^C \), tending towards a steady state—to simulate this process:

\[
C_{IN}^I (k) = C_{IN}^C (k) + C_{IN}^I (k). \quad (10)
\]

The circulatory loading caused by variation in AoA is expressed as

\[
C_{CN}^C (k) = C_N (Ma) \alpha (k) - X (k) - Y (k) = C_N (Ma) \alpha_E (k) \quad (11)
\]

where \( X \) and \( Y \) are called deficiency functions and

\[
\begin{align*}
X (k) &= X (k - 1) e^{-b_1 \beta^2 \Delta s} + A_1 \Delta \alpha_E e^{-b_1 \beta^2 \Delta s/2} \\
Y (k) &= Y (k - 1) e^{-b_2 \beta^2 \Delta s} + A_2 \Delta \alpha_E e^{-b_2 \beta^2 \Delta s/2}
\end{align*}
\]

where \( A_1 = 0.3, b_1 = 0.14, A_2 = 0.7, b_2 = 0.53 \). The term, \( C_N (Ma) \), is the slope of the airfoil normal force to AoA, obtained through an airfoil steady experiment. The terms \( \alpha \) and \( \alpha_E \) mean AoA and equivalent AoA, respectively, at three-quarters of the chord. The variation \( \beta \) expresses the Prandtl–Glauert compression factor and the non-dimensional time constant, \( \Delta s = \frac{2V \Delta t}{c} \).

The impulsive loading caused by the variation in AoA, \( \Delta \alpha \), and the pitch rate, \( \Delta \eta = \frac{\Delta \theta c}{V} \), can be expressed as

\[
C_{IN}^I (k) = C_{INa}^I (k) + C_{IN\eta}^I (k) \quad (13)
\]
where the part by $\Delta \alpha$ is
\[
\begin{align*}
C_{Ia}(k) &= \frac{4T_a}{Ma} \left( \frac{\Delta \alpha_k}{\Delta t} - D_1(k) \right) \\
D_1(k) &= D_1(k-1) e^{\frac{-\Delta \alpha}{T_a}} + \left( \frac{\Delta \alpha_k - \Delta \alpha_{k-1}}{\Delta t} \right) e^{\frac{T_a}{\Delta \alpha}} \tag{14}
\end{align*}
\]
and the part by $\Delta q$ is
\[
\begin{align*}
C_{Inq}(k) &= \frac{T_q}{Ma} \left( \frac{\Delta q_k}{\Delta t} - D_2(k) \right) \\
D_2(k) &= D_2(k-1) e^{\frac{-\Delta q}{T_q}} + \left( \frac{\Delta q_k - \Delta q_{k-1}}{\Delta t} \right) e^{\frac{T_q}{\Delta \alpha}} \tag{15}
\end{align*}
\]

The time constants can be expressed as
\[
\begin{align*}
T_a &= \frac{0.75}{1 - Ma + \pi \beta^2 Ma^2 (A_1 b_1 + A_2 b_2) a} \frac{c}{c} \\
T_q &= \frac{0.75}{1 - Ma + 2\pi \beta^2 Ma^2 (A_1 b_1 + A_2 b_2) a} \frac{c}{c} \tag{16}
\end{align*}
\]

As the AoA becomes relatively large, flow separation may appear in the leading edge. Under this unsteady condition, there is so-called dynamic stall delay. This means that the leading edge pressure distribution occurs later than normal force variation, and the normal force variation is delayed after the AoA variation. The Leishman–Beddoes algorithm introduces a one-step delay to consider this phenomenon and uses a substitutive normal force, $C_{N}^{C}$, to trigger leading edge flow separation.

\[
\begin{align*}
C_{N}^{C}(k) &= C_{N}^{p}(k) - D_{p}(k) \\
D_{p}(k) &= D_{p}(k-1) e^{\frac{-\Delta \alpha}{T_p}} + \left( C_{N}^{p}(k) - C_{N}^{p}(k-1) \right) e^{\frac{T_p}{\Delta \alpha}} \tag{17}
\end{align*}
\]

where $T_p$ is a function of the Mach number, whose value is obtained from the airfoil data.

The leading edge flow separation affects the boundary layer and trailing edge separation, which causes circulatory loading loss and nonlinear variation in the aerodynamic force and moment. The algorithm considers this when computing the circulatory loading by the Kirchhoff plate flow solution. So, the unsteady normal force during trailing edge separation is

\[
C_{N}^{t}(k) = C_{N}^{C}(k) + C_{N}^{l}(k) \tag{18}
\]

where $C_{N}^{C}$ represents the normal force after the Kirchhoff flow correction.

During modeling, the point when the boundary layer starts to flow in reverse is treated as the trailing edge separation point, $f''$; thus, the corrected normal force can be expressed as

\[
C_{N}^{C}(k) = C_{N} (Ma) \left( 1 + \frac{\sqrt{f''(k)}}{2} \right)^2 (\alpha_E (k) - \alpha_0) \tag{19}
\]

Due to the dynamic stall delay, the algorithm introduces a one-step delay when computing the separation point:

\[
\begin{align*}
f''(k) &= f'(k) - D_f(k) \\
D_f(k) &= D_f(k-1) e^{\frac{-\Delta \alpha}{T_f}} + \left( f'(k) - f'(k-1) \right) e^{\frac{T_f}{\Delta \alpha}} \tag{20}
\end{align*}
\]
where the separation point before delay correction is expressed by a piecewise function of AoA:

\[
\begin{align*}
    f' &= 1 - 0.3e^{(\alpha_{\text{eff}} - \alpha_1)s_1^{-1}} & \alpha_{\text{eff}} &\geq \alpha_1 \\
    f' &= 0.04 + 0.66e^{(\alpha_1 - \alpha_{\text{eff}})s_2^{-1}} & \alpha_{\text{eff}} &< \alpha_1
\end{align*}
\]  

(21)

where the term \( \alpha_1 \) represents steady stall AoA, and \( s_1 \) and \( s_2 \) are steady stall parameters, obtained through an airfoil steady experiment and data fitting via Equations (19) and (21).

The effective AoA \( \alpha_{\text{eff}} \) is computed by the substitutive normal force, \( C'_{\text{N}} \), mentioned previously:

\[
\alpha_{\text{eff}} = \frac{C'_{\text{N}}(k)}{C_{\text{N}}(Ma)} \tag{22}
\]

The deep dynamic stall process characterizes the formation, convection and detachment of the leading edge vortex. The leading edge vortex contributes to the airfoil normal force with a vortex lift:

\[
C_{\text{V}}(k) = C_{\text{N}}^C(k) \left( 1 - \frac{1 + \sqrt{f''(k)}}{2} \right)^2. \tag{23}
\]

In an unsteady flow, the decay and accumulation of the leading edge vortex exist simultaneously:

\[
C_{\text{V}}(k) = C_{\text{V}}^C(k) - \left. C_{\text{V}}^C \right|_{k-1} e^{-\frac{\Delta S}{T_{\text{v}}}} + \left. C_{\text{V}}^C \right|_{k} e^{-\frac{\Delta S}{T_{\text{v}}}}. \tag{24}
\]

When the critical condition, \( C'_{\text{N}} > C_{\text{N}I} \), is satisfied, the leading edge vortex forms and convects downstream. During this process, Equation (24) is applied. When it comes to the trailing edge, the accumulation effect stops. The critical normal force, \( C_{\text{N}I} \), is obtained through an airfoil experiment. During the deep dynamic stall process, the leading edge vortex may form and detach continuously. Its frequency is determined by \( T_{s} = \frac{2(1-f)}{S_{\text{tr}}} \), where the Strouhal number, \( S_{\text{tr}} \), equals 0.2. If the endurance of the current vortex, \( t_{\text{v}} \), is larger than \( T_{s} \), Equations (23) and (24) are implemented again.

From the above, the normal force can be expressed as

\[
C_{\text{N}}^T = C_{\text{N}}^f + C_{\text{N}}^c \tag{25}
\]

and the chord-wise force can be expressed as

\[
\begin{align*}
    C_{\text{C}}^T(k) &= \eta C_{\text{N}}(Ma) \alpha_E \sin \alpha_E(k) \sqrt{f''(k)} & C_{\text{N}}(k) &\leq C_{\text{N}I} \\
    C_{\text{C}}^r(k) &= \eta C_{\text{N}}(Ma) \alpha_E \sin \alpha_E(k) \sqrt{f''(k)} \Psi & C_{\text{N}}(k) &> C_{\text{N}I}
\end{align*}
\]  

(26)

where \( \eta \) is a recovery factor and is often set to 0.95. The term \( \Psi = f^{D}(C'_{\text{N}} - C_{\text{NI}}) \) ensures a smooth chord-wise force with respect to the variation in AoA:

\[
\begin{align*}
    C_L &= C_{\text{N}}^T \cos \alpha + C_{\text{C}}^T \sin \alpha \\
    C_D &= C_{d0} + C_{\text{N}}^T \sin \alpha - C_{\text{C}}^T \cos \alpha
\end{align*}
\]  

(27)

In a series of ramp-up dynamic experiments by the University of Glasgow, Sheng, Galbraith and Coton found that the original Leishman–Beddoes algorithm predicts the dynamic stall vortex formation ahead of the actual situation when the Mach number is 0.12 [24,25]. Then, Sheng et al. proposed a new dynamic stall criterion in the original Leishman–Beddoes framework in a low Mach number situation (\( Ma < 0.3 \)). In the following study, the new criterion was found to be applicable in an oscillation freestream [26].
In the Sheng correction, there is a linear relationship between the reduced pitch rate, \( r = \frac{\dot{\alpha}}{2V} \), and the dynamic stall in AoA, \( \alpha_{ds} \):

\[
\alpha_{ds} = \alpha_{d0} + Ta r \quad (r \geq r_0).
\]  

(28)

The critical stall onset angle is defined as

\[
\begin{align*}
\alpha_{cr} &= \alpha_{d0} & r \geq r_0 \\
\alpha_{cr} &= \alpha_{ds} + \frac{(\alpha_{d0} - \alpha_{ss}) r}{r_0} & r < r_0
\end{align*}
\]  

(29)

where the steady stall angle, \( \alpha_{ss} \), and variables, \( \alpha_{d0} \), \( Ta \) and \( r_0 \), are obtained via an airfoil experiment.

The dynamic stall angle is derived by a one-step delay:

\[
\Delta\alpha' = (\alpha(s) - \alpha(s - \Delta s)) \left( 1 - e^{-\frac{s}{T_\alpha}} \right)
\]  

(30)

where the variable \( s \) expresses the non-dimensional time constant. In a low Mach number situation, the critical condition, \( \alpha' > \alpha_{cr} \), replaces \( C'_N > C_{NI} \) to be a new dynamic stall onset criterion.

Accordingly, the Sheng correction implements an equivalent angle shift in the fitting function of the trailing edge separation point, Equation (21):

\[
\begin{align*}
f' &= 1 - 0.3e^{(\alpha'-\alpha_1-\Delta\alpha_1)s_1^{-1}} & \alpha' \geq \alpha_1 \\
f' &= 0.04 + 0.66e^{(\alpha_1+\Delta\alpha_1-\alpha')s_2^{-1}} & \alpha' < \alpha_1
\end{align*}
\]  

(31)

where the shift angle matches

\[
\begin{align*}
\Delta\alpha_1 &= \alpha_{d0} - \alpha_{ss} & r \geq r_0 \\
\Delta\alpha_1 &= \frac{\alpha_{d0} - \alpha_{ss}}{r_0} & r < r_0.
\end{align*}
\]  

(32)

In the current work, the rigid flap assumption was applied, and a Jacobian method for trimming was implemented. It is known that the Jacobian matrix is used to determine the direction of updating feathering controls. We applied the C81 data and the linear inflow model to accelerate the matrix computation, and then used the dynamic stall algorithm and wake method to judge whether it was trimmed, maintaining trimming computation accuracy.

3. Validation

Figure 2 shows the NACA0012 airfoil dynamic stall validation under two freestream conditions: one represents no obvious dynamic stall and the other represents obvious deep dynamic stall, where the experimental data and Leishman solutions were derived from Reference [21]. In Figure 2a, the maximum AoA is not beyond the dynamic stall angle, so there is no leading edge vortex. Furthermore, the unsteady behavior is mainly a combination of the attached flow and trailing edge separation flow. When the AoA becomes larger, the deep dynamic stall appears as shown in Figure 2b. We can see that it differs greatly from Figure 2a. Due to the dynamic delay, the normal force increases further after AoA becomes larger than the static stall angle. Then, the boundary layer reversal point moves towards the leading edge and the vortex forms at Point 1. In this case, the vortex convects to the trailing edge at Point 2 before reaching the maximum AoA (Point 3); then, the \( C_n \) declines because of the vortex detachment. As the AoA decreases, reattached flow starts at Point 4. Compared with the experimental data and Leishman results, the present solutions have good correlations, and the modeling method is validated.

Figure 3 shows a \( C_n \) validation of the NACA23012 airfoil dynamic stall in a low Mach number situation. Compared with the experimental data and Sheng results [26], the present solutions have
good correlations. This validates that the Sheng correction effectively predicts the capture of the leading edge vortex and its subsequent normal force variations in a low Mach number freestream. The modeling parameters for NACA23012 in the current work are listed in Table 1 and were derived by fitting the C81 table data from Reference [27].

**Figure 2.** NACA0012 dynamic stall validation: (1) the leading edge vortex forms; (2) the vortex convects to the trailing edge; (3) the vortex decays in the wake; (4) the reattached flow starts; (5) the attached flow starts.

**Figure 3.** NACA23012 dynamic stall validation.

**Table 1.** NACA23012 static stall parameters.

| Ma  | 0.1   | 0.2   | 0.3   | 0.4   | 0.5   | 0.6   | 0.7   | 0.8   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|
| $C_N$ (deg$^{-1}$) | 0.1052 | 0.1068 | 0.1084 | 0.1134 | 0.1206 | 0.1312 | 0.1455 | 0.1742 |
| $\alpha_0$ (deg) | -1.058 | -1.054 | -1.05 | -1.062 | -1.101 | -1.102 | -1.214 | -1.197 |
| $\alpha_1$ (deg) | 14 | 13.16 | 13 | 12.92 | 11.23 | 7.638 | 4 | 0.7618 |
| $\psi_1$ (deg) | 0.3008 | 0.04658 | 0.2099 | 0.815 | 1.029 | 1.023 | 0.02218 | 0.6004 |
| $\psi_2$ (deg) | 1.322 | 2.293 | 2.109 | 2.298 | 4.842 | 6.595 | 6.33 | 2.05 |
| $C_{d0}$ | 0.0106 | 0.0106 | 0.0106 | 0.0102 | 0.0102 | 0.0103 | 0.0103 | 0.0287 |
| $C_{NI}$ | 1.471 | 1.494 | 1.517 | 1.364 | 1.113 | 0.9106 | 0.6207 | 0.24 |
Thanks to adequate public data, the current work used the Bo 105 helicopter as a base to implement the variable-speed rotor aerodynamic performance study; its rotor airfoil is NACA23012 and whose tail rotor airfoil is NACA0012. The key parameters for dynamic flight modeling of the Bo 105 helicopter were derived from Reference [28] and Reference [29]. Figure 4 shows a flight trim validation of the Bo 105 at sea level, compared with the flight test and Padfield results based on linear inflow modeling [29]. The take-off weight was 2200 kg. It can be seen that the collective feathering decreased at first and then increased as the flight speed increased. At a medium flight speed, the required rotor power was relatively low, as was the collective feathering. The present trim plot shows that in hover and low flight speed, there was a good correlation with the flight test, while in faster flight modes, the present collective feathering result was underestimated. During fast flight, the fuselage parasite power consumption dominates the rotor required power and increases with the cubic flight speed. Therefore, the deviation may be caused by an underestimated fuselage drag in fast flight; to simulate this, the current work applied the Padfield’s fitting function. The lateral cyclic feathering correlated better with the flight test, owing to the wake method.

4. Results and Discussion

It is clear that the blade dynamic pressure varies with the square of rotor speed, thus directly affecting the rotor required power. At the same time, the blade aerodynamic conditions, such as the airfoil AoA and Mach number, change with variation in the rotor speed which also affects the aerodynamic characteristics, such as the airfoil lift coefficient and the drag coefficient. The current work applied a dimensionless parameter analysis. This kind of analysis method could separate the variable-speed rotor performance from the dynamic pressure effects and focus on the aerodynamic characteristics. Based on a previous study, it is known that the rotor speed affects the induced power and profile power more significantly [30], which is proportional to the cubic rotor speed:

\[
P_i \sim \omega^3 \int_0^{2\pi} \int_0^1 \lambda_i dC_T
\]

\[
P_o \sim \omega^3 \int_0^{2\pi} \int_0^1 \left( \mu_0 dC_{H_0} + \frac{1}{R} dC_{Q_0} \right)
\]

where \(\lambda_i\) and \(\mu_0\) are, respectively, the induced velocity ratio and the advance ratio at the HP (hub plane) coordinate. The increased backward thrust force derived from the airfoil drag and the shaft torque derived from the airfoil drag are defined as \(dC_T\), \(dC_{H_0}\) and \(dC_{Q_0}\), respectively.
Therefore, we can use the remaining terms

\[ \eta_i = \int_0^{2\pi} \int_0^1 \lambda_i dC_T \]  
(35)

\[ \eta_o = \int_0^{2\pi} \int_0^1 \left( \mu_h dC_{H_o} + \frac{1}{R} dC_{Q_o} \right) \]  
(36)

to represent the variable-speed rotor characteristics. It is noteworthy that the values of variables \(\eta_i\) and \(\eta_o\) equal the corresponding power coefficients.

The sum of the induced power and profile power is defined as the direct power loss (DPL), which represents the part of total required power directly affected by the rotor speed. Figure 5 depicts the relationship between the DPL and the rotor speed for different flight speeds under the condition of given take-off weights. The four classic flight speeds are selected, representing hover, slow flight, cruise, and fast flight, respectively. Furthermore, the cubic rotor speed curve is also given. The variable-speed rotor aerodynamic characteristics are indicated by comparing the DPL variation with the cubic curve. When the rotor speed changes from a high value to a low value, if the DPL variation trend is slower than the cubic curve, this indicates that the dynamic pressure reduction is the main reason why a lower rotor speed could decrease the required power. If the DPL variation trend is faster, this indicates that the rotor aerodynamic performance is improved. That is, \(\eta_o\) and \(\eta_i\) are reduced. In this situation, both the advanced aerodynamic performance and the reduced dynamic pressure contribute to the required rotor power reduction. It can be seen from Figure 5 that, in hover and slow flight, the DPL variation trend is slower than the cubic curve, and the DPL variation with rotor speed is slight, which means that, primarily, the blade dynamic pressure reduction improves the rotor aerodynamic performance. During cruise and fast flight modes, the DPL variation trend is faster than the cubic curve, implying that both the aerodynamic performance improvement and dynamic pressure reduction play roles.

A dimensionless parameter analysis is applied to separate the variable-speed rotor performance from the dynamic pressure. The conditions of slow flight are listed in Table 2. It is noticed that the three cases have different true flight parameters, but the same advance ratio and blade loading coefficient.
Table 2. Variable-speed rotor low flight speed cases.

| Take-Off Weight | Advance Ratio | Blade Loading | Rotor Speed Ratio | Flight Speed |
|-----------------|---------------|---------------|-------------------|--------------|
| 1600 kg         | 0.15          | 0.074         | 0.9               | 29 m/s       |
| 1900 kg         | 0.15          | 0.074         | 0.98              | 32 m/s       |
| 2200 kg         | 0.15          | 0.074         | 1.06              | 35 m/s       |

Figures 6–8 respectively depict the time history of airfoil AoA, lift coefficient, $C_L$, and drag coefficient, $C_D$, with respect to the azimuth, for the above three kinds of slow flight cases. It can be observed that they have indiscernible aerodynamic characteristics, although with different rotor speeds, that is to say different required powers. The indiscernible aerodynamic characteristics imply that the variable-speed rotor is within a linear aerodynamic region and reduces its required power through a reduction in dynamic pressure.

A group of high flight speed cases is implemented. The take-off weights, rotor speed computation points and blade loadings remain the same as in low flight speed cases, and the flight speeds and advance ratios are doubled. The dimensionless parameters and true flight parameters are listed in Table 3.

![Figure 6](image1)

**Figure 6.** $\mu = 0.15$, AoA (angle of attack) with rotor speed.

![Figure 7](image2)

**Figure 7.** $\mu = 0.15$, lift coefficient, $C_L$, with rotor speed.
(a) $W_0 = 1600 \text{ kg, } K_\alpha = 0.9$  (b) $W_0 = 1900 \text{ kg, } K_\alpha = 0.98$  (c) $W_0 = 2200 \text{ kg, } K_\alpha = 1.06$

Figure 8. $\mu = 0.15$, drag coefficient, $C_D$, with rotor speed.

Table 3. Variable-speed rotor in fast flight speed cases.

| Take-Off Weight | Advance Ratio | Blade Loading | Rotor Speed Ratio | Flight Speed |
|-----------------|---------------|---------------|-------------------|--------------|
| 1600 kg         | 0.3           | 0.074         | 0.9               | 59 m/s       |
| 1900 kg         | 0.3           | 0.074         | 0.98              | 64 m/s       |
| 2200 kg         | 0.3           | 0.074         | 1.06              | 69 m/s       |

Figures 9–11 respectively depict the time history of airfoil AoA, the lift coefficient, $C_L$, and the drag coefficient, $C_D$, with respect to the azimuth, for the three kinds of high flight speed cases. It is found that the prominent drag coefficient appears at the tip of the advancing blade when the rotor speed is high. As the rotor speed decreases, the drag coefficient at the tip of the advancing blade decreases significantly. This implies that during a fast flight, the variable-speed rotor reduces the blade tip compression loss and improves the rotor aerodynamic performance by lowering its rotor speed. That is to say, the dynamic pressure and advancing blade tip compression reductions both contribute to a reduction in the required power for the variable-speed rotor. It is noticed that the three kinds of fast flight cases provided the same advance ratio and blade loading; however, significantly different aerodynamic characteristics were observed. The difference indicates that the variable-speed rotor works within a non-linear aerodynamic region, the non-linearity of which comes from the compression of the advancing blade tip and the dynamic stall of the retreating blade. Figure 10 shows that the region of higher lift coefficient $C_L$ becomes slightly larger as the rotor speed decreases, which means the potential onset of an airfoil dynamic stall. Although the rotor speed is high enough and the blade loading is relatively low, it is not enough to observe the obvious airfoil dynamic stall.

(a) $W_0 = 1600 \text{ kg, } K_\alpha = 0.9$  (b) $W_0 = 1900 \text{ kg, } K_\alpha = 0.98$  (c) $W_0 = 2200 \text{ kg, } K_\alpha = 1.06$

Figure 9. $\mu = 0.3$, AoA with rotor speed.
In hover and slow flight modes, the induced power dominates in the total required rotor power. In Figure 5, the value of the DPL during hover is not sensitive to variation in the rotor speed, which means an increasing $\eta_i$ occurs as rotor speed decreases. From Equation (33), it is known that the induced power is affected by both the value and distribution of the induced inflow and the thrust. If it is assumed that the induced inflow and thrust are uniform, the result given by Equation (33) matches the actuator disc theory.

Figure 12 shows the time history of the airfoil coefficient during hover. Considering that, during hover, the value of the airfoil lift coefficients varies as the rotor speed changes, a relative value, $\Delta C_L$, is given by removing the average to allow the effect of distribution on the aerodynamic performance for a variable-speed rotor to be observed. When the rotor speed is low, it is found that the lift coefficients in the advancing blade are high and those in the retreating blade are small, causing a more significant non-uniform distribution than when the rotor speed is high. Therefore, during hover, although a reduced-speed rotor can greatly decrease the dynamic pressure, the rotor required power is not affected. The reason for this non-uniformity in hover is that the rotor shaft torque increases as the rotor speed decreases, so an increasing tail thrust is required to balance in yaw; this is followed by a larger lateral tilt of TPP (tip-path plane) to provide enough rotor side force to balance the new tail thrust. In the current study, Steiner mentioned that the rotor required power could not be reduced by lowering its rotor speed during hover based on the unpublished research of the Pennsylvania State University; however, no further explanation has been given yet [10].
Figure 12. During hover, lift coefficient, $C_L$, with rotor speed.

The DPL variations with respect to rotor speeds were rearranged for different take-off weights under the given flight speeds, as shown in Figure 13. By comparing the trends in DPL variation with the rotor speed cubic curve, the take-off weight effect on the variable-speed rotor aerodynamic performance can be indicated. It can be seen in Figure 13b that the DPL variation becomes close to the cubic curve when the take-off weight is small, which means that in this situation, both the rotor aerodynamic performance improvement and dynamic pressure reduction contribute to the reduction in the required power. In forward flight, the lower the take-off weight, the more significant the DPL variation within a certain range of rotor speed is. Its trend is faster than the cubic curve, and it slows down near the rotor speed lower boundary. The results of Figure 13 imply that the rotor loading is lower; the effect of an improvement in aerodynamic performance on the reduction in the required power is greater.

Figure 13. Variation in DPL with rotor speed for different flight speeds.

A dimensionless parameter analysis was conducted for a high loading situation, whose parameters are listed in Table 4. The high loading cases had the same advance ratio and take-off weights as the fast flights above. Figure 14 shows a comparison of the airfoil drag coefficient, $C_D$, between light loading and heavy loading. When rotor loading is low, it can be seen that the lower rotor speed can reduce the advancing blade tip compression loss, without obvious onset of dynamic stall in the retreating blade, which results in lower profile power. In contrast, when the rotor loading is high, the retreating blade airfoil dynamic stall is remarkable, with a larger region of high $C_D$. The change in the advancing...
blade tip compression loss is not sensitive. The result suppresses the contribution of dynamic pressure reduction to the required power.

Near the reversal flow region, there is an obvious dynamic stall phenomenon because of the large AoA and the large change rate of AoA. Bowen-Davies and Chopra [31], Lind [32] and Lind and Jones [33–36] found through a series of simulation and experimental studies on the large advance ratio flight, that Leishman–Beddoes dynamic stall modeling can give a reasonable prediction of the reversal flow within the regular advance ratio ($\mu < 0.4$). However, the modeling is not capable of giving a reasonable prediction when $\mu > 0.7$. The maximum advance ratio in the current work was 0.4.

**Table 4. Variable-speed rotor high loading case.**

| Take-Off Weight | Advance Ratio | Blade Loading | Rotor Speed Ratio | Flight Speed  |
|-----------------|---------------|---------------|-------------------|--------------|
| 1600 kg         | 0.3           | 0.125         | 0.69              | 88 m/s       |
| 1900 kg         | 0.3           | 0.125         | 0.75              | 96 m/s       |
| 2200 kg         | 0.3           | 0.125         | 0.82              | 104 m/s      |

**Figure 14. Drag coefficient, $C_D$, with rotor speed for different rotor loadings.**

5. Conclusions

The current work applied a dimensionless parameter analysis to analyze and reveal the mechanisms of a variable-speed rotor affecting the rotor aerodynamic performance under the conditions of various flight speeds and take-off weights. The analysis concluded the following:

1. During hover and slow flight modes, the variable-speed rotor reduces the required power by reducing the blade dynamic pressure, whereas the non-uniform thrust distribution caused by a lower rotor speed suppresses improvements in aerodynamic performance.
2. During cruise and fast forward flight modes, the blade dynamic pressure and advancing blade tip compression loss reductions both improve the rotor aerodynamic performance.
3. During forward flight, the lower the rotor loading is, the greater the effect on reducing required power is. This effect is due to a reduction in the advancing blade tip compression loss in variable-speed rotors.
Author Contributions: M.Z., J.X., N.G. and Z.X. conceived the research and contributed to the literature search; J.X. conducted the simulation and analyzed the results; J.X. and Z.X. wrote and revised the paper.

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Abbreviations
AoA attack of angle
CFD computational fluid dynamics
DPL direct power loss
HP hub plane
TPP tip-path plane

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