Axion perturbation spectra in string cosmologies

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Abstract

We discuss the semi-classical perturbation spectra produced in the massless fields of the low energy string action in a pre big bang type scenario. Axion fields may possess an almost scale-invariant \((\Delta n \approx 0)\) spectrum on large scales dependent upon the evolution of the dilaton and moduli fields to which they are coupled. As an example we calculate the spectra for three axion fields present in a truncated type IIB model and show that they are related with at least one of the fields having a scale-invariant or red \((\Delta n < 0)\) perturbation spectrum. In the simplest pre big bang scenario this may be inconsistent with the observed isotropy of the microwave background. More generally the relations between the perturbation spectra in low energy string cosmologies should reflect the symmetries of the theory.

Superstring theory is at present the best candidate for a theory uniting gravity with the other fundamental forces. If it is the correct description of our physical world, it must have important consequences for models of the very early universe. Much theoretical work is currently devoted to building models of cosmological inflation driven by slow-rolling, self-interacting scalar fields in the context of supergravity models. However, there are problems inherent in such an approach due to the large masses for the scalar fields that are generally introduced by supergravity terms \([1]\). A radically different cosmological scenario has been proposed by Gasperini and Veneziano based on the low-energy vacuum solutions derived from the generic superstring effective action \([2]\). Inflation in this pre big bang model is driven by the kinetic energy of the fast-rolling dilaton field rather than any interaction potential. A number of new problems appear in such a scenario, most notably the graceful exit problem \([3]\). There are also concerns about the specific initial conditions required \([4]\).

The key test of all these models of the very early universe is the spectrum of perturbations that they predict. In conventional slow-roll inflation the only perturbation spectra usually generated are the gravitational wave background and perturbations in a single quasi-massless inflaton field. This naturally produces a nearly scale-invariant spectrum of adiabatic density perturbations. By contrast there are potentially many massless scalar fields in a pre big bang string cosmology which each produce their own spectrum of perturbations. The fast-rolling dilaton and moduli fields can only yield a steep blue spectrum \([5]\). However, it was recently realised that axion fields, which are always present in the low-energy effective action \([6]\), may have significantly different spectral slopes due to their explicit coupling to the dilaton and moduli fields \([7, 8]\). For instance, the pseudo-scalar axion field, \(\sigma_1\), whose gradient is dual to the Neveu/Schwarz-Neveu/Schwarz (NS-NS) three-form field strength in four-dimensions \([9]\), can have a spectral tilt in the range from \(3 - 2\sqrt{3} \approx -0.46 \leq \Delta n_1 \leq 3\), depending upon the evolution of the dilaton and moduli fields \([7, 8]\). In highly symmetrical cases the spectrum becomes scale-invariant, \(\Delta n_1 = 0\) \([10]\). Durrer et al. \([11]\) have noted that such a spectrum may provide a novel scenario for structure formation induced by seed perturbations. The massless axion field can yield an almost scale-invariant
spectrum of density perturbations at horizon crossing whose amplitude is fixed by the string coupling constant at the end of the pre big bang era, \( \delta \rho / \rho \approx e^{\varphi} \approx 10^{-2} \). Hence a slightly “blue” spectrum, with \( \Delta n_t > 0 \), may be consistent with observations of the microwave background anisotropies. More detailed analyses are required to examine whether this model of structure formation can compete with the successful model based around conventional inflation \([12]\), but this example emphasises the potential challenge to the standard picture raised by the pre big bang scenario as well as the importance of perturbation spectra in testing these ideas.

Thus far, calculations of axion perturbations have been restricted to simple axion-dilaton systems \([\ref{7}, \ref{8}, \ref{10}, \ref{13}, \ref{14}]\). In this paper we investigate another important feature of the pre big bang model – the importance of perturbation spectra in testing these ideas.

The type IIB superstring contains a dilaton and a graviton and a two-form potential in the NS–NS sector of the theory, together with a second two–form potential and an axion field, \( \sigma_3 \), in the Ramond–Ramond (RR) sector. (There is also a four–form in the RR sector, but this can be consistently set to zero). A 4-D action may be derived by compactifying the 10-D spacetime on a 6-D Ricci-flat internal space so that

\[
ds_{10}^2 = e^{\varphi(x)} g_{\mu\nu}(x) dx^\mu dx^\nu + e^{\eta(x)}/\sqrt{\sigma} g_{ab} dX^a dX^b,
\]

where \( \varphi \) is the 4-D dilaton and \( y \) describes the volume of the internal space and is the only modulus field considered. We have included the conformal factor \( e^\varphi \) in our definition of the 4-D external metric \( g_{\mu\nu} \) in order to work in the 4-D Einstein frame, where the dilaton field is minimally coupled to gravity. In four dimensions the three-form field strengths from the NS-NS and RR sectors are dual to the gradients of two pseudo-scalar axion fields \( \sigma_1 \) and \( \sigma_2 \). The third axion field is the RR axion already present in the 10-D theory. In this dual formulation it can be shown that the equations of motion for the fields follow from an effective action \([13]\):

\[
S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ \hat{R} - \frac{1}{2} \left( \nabla \varphi \right)^2 - \frac{1}{2} \left( \nabla y \right)^2 \right. \\
- \frac{1}{2} e^{\sqrt{\sigma} y + \varphi} \left( \nabla \sigma_3 \right)^2 - \frac{1}{2} e^{\sqrt{\sigma} y + \varphi} \left( \nabla \sigma_2 \right)^2 - \frac{1}{2} e^{2\varphi} \left( \nabla \sigma_1 - \sigma_3 \nabla \sigma_2 \right)^2 \left. \right].
\]

where \( \kappa^2 = 8\pi/m_\text{Pl}^2 \). This describes a non-linear sigma model in Einstein gravity where the scalar fields parametrise an SL(3,R)/SO(3) coset \([15]\). The global symmetries of the action include the SL(2,Z) ‘S-duality’ of the original 10-D action \([16]\) and a \( Z_2 \) ‘T-duality’ corresponding to invariance under \( y \to -y \) \([17]\).

We assume that the external four dimensional spacetime is described by a flat Friedmann-Robertson-Walker (FRW) metric with the line element

\[
d\bar{s}^2 = \bar{a}^2(\eta) \left\{ -d\bar{t}^2 + \delta_{ij} dx^i dx^j \right\},
\]

where \( \bar{t} \) is the conformally invariant time coordinate and \( \bar{a}(\eta) \) is the scale factor. FRW solutions with non-zero spatial curvature can also be found \([14], [16]\). The familiar field equations of general relativity apply in the 4-D Einstein frame and the combined stress-energy tensor for homogeneous massless fields reduces to that for a perfect fluid with a maximally stiff equation of state \([20], [21]\), i.e., with pressure equal to energy density. This leads to the simple solution for the scale factor

\[
\bar{a} = \bar{a}_*|\eta|^{1/2}.
\]
The dilaton-moduli-vacuum solutions are monotonic power-law solutions

\[ e^\phi = e^{\phi_* |\eta| \sqrt{3} \cos \xi}, \]
\[ e^y = e^{y_* |\eta| \sqrt{3} \sin \xi}, \]

where the integration constant \( \xi \) determines the relative rate of change of the effective dilaton and internal volume respectively. If stable compactification has occurred and the volume of the internal space is fixed, we have \( \sin \xi = 0 \).

In order to understand the perturbation spectra produced in different fields it is revealing to look at conformally related metrics, \( g_{\mu\nu} \to \Omega^2 g_{\mu\nu} \). If the conformal factor \( \Omega^2 \) is itself homogeneous, the transformed metric remains a FRW metric but with scale factor \( a \to \Omega a \) and proper time \( t \to \int \Omega dt \).

A finite proper time interval in one frame does not necessarily coincide with a finite proper time in another frame and, in particular, we shall see that what seems to be a singular evolution in one frame may appear to be non-singular in another frame.

In the original string frame the scale factor evolves as

\[ a \equiv e^{\phi/2} = a_* |\eta|^{(1+\sqrt{3} \cos \xi)/2}, \]

and there is an accelerated expansion in this frame for \( \cos \xi < -1/\sqrt{3} \) if \( \eta < 0 \). In the Einstein frame we see that \( \eta \to 0^- \) always corresponds to a collapsing universe with \( \tilde{a} \to 0 \). However, for any value of \( \xi \) this fulfils one definition of inflation, namely, that the comoving Hubble length \( (|\tilde{a}/\tilde{a}'|)^{-1} = |\tilde{a}/\tilde{a}''| = 2|\eta| \) decreases with time \( [22] \). A given comoving scale that starts arbitrarily far within the Hubble scale in either conformal frame at \( \eta \to -\infty \) inevitably becomes larger than the Hubble scale in that frame as \( \eta \to 0^- \). This allows one to produce perturbations in the fields on scales much larger than the present Hubble scale from quantum fluctuations in flat-spacetime at earlier times.

Because the dilaton and moduli are both minimally coupled to the Einstein metric, the field equations for their linearised scalar perturbations are given by \([3, 23, 24] \)

\[ \delta \phi'' + 2\tilde{h}\delta \phi' + k^2 \delta \phi = 0 \]
\[ \delta y'' + 2\tilde{h}\delta y' + k^2 \delta y = 0 \]

where a prime denotes differentiation with respect to conformal time, the comoving Hubble rate in the Einstein frame is given by \( \tilde{h} \equiv \tilde{a}'/\tilde{a} \) and \( k \) is the comoving wavenumber. (Perturbations in the gravitational field obey a similar field equation \([23]\)). The singular evolution of the metric as \( \eta \to 0^- \) implies that their perturbation spectra grow dramatically on shorter wavelengths that leave the Hubble radius close to the singularity. This leads to steep blue spectra with spectral tilt \( \Delta n = 3 \) \([4]\) which leaves effectively no perturbations in these fields on large (astronomically observable) scales in our present universe.

On the other hand, axion fields’ kinetic terms retain a non-minimal coupling to the dilaton or moduli fields in the Einstein frame. This non-minimal coupling can be eliminated by a conformal transformation to an alternative conformally related metric, which we will refer to as the corresponding axion frame. For the NS-NS axion this is given by \([7]\)

\[ \bar{g}_{(1)\mu\nu} = e^{2\phi} \tilde{g}_{\mu\nu}, \]

and hence

\[ \bar{a}_1 \equiv e^{\phi} \tilde{a}. \]

The NS-NS axion field is a minimally coupled massless scalar field in this frame and thus axionic particles follow null geodesics with respect to this metric.

More generally, for the three axion fields in the truncated type IIB action given in Eq. (3) we can define

\[ \bar{a}_i^2 = \Omega_i^2 \tilde{a}^2, \]
where the conformal factor

$$\Omega_i^2 = \begin{cases} 
e^{2\varphi} & \text{for } \sigma_1 \\ \ne^{\varphi - \sqrt{3}y} & \text{for } \sigma_2 \\ \ne^{\varphi + \sqrt{3}y} & \text{for } \sigma_3 \end{cases}$$ \hspace{1cm} (12)$$

reflects the differing couplings of the axion fields to the dilaton and moduli in the Lagrangian.

Although conformally related to the string and Einstein frames, the metric “seen” by the axions may behave very differently. In terms of conformal time, the axionic scale factors for the dilaton-moduli-vacuum solutions given by Eqs. (5) and (7) evolve as

$$\bar{a}_i = \bar{a}_i |\eta|^{r_i+(1/2)}$$ \hspace{1cm} (13)

where

$$r_i = \begin{cases} \sqrt{3} \cos \xi & \text{for } \sigma_1 \\ \sqrt{3} \cos(\xi + \pi/3) & \text{for } \sigma_2 \\ \sqrt{3} \cos(\xi - \pi/3) & \text{for } \sigma_3 \end{cases}$$ \hspace{1cm} (14)

The proper time in the axion frame is given by

$$\bar{t}_i \equiv \int \bar{a}_i d\eta \sim |\eta|^{r_i+(3/2)}$$ \hspace{1cm} (15)

so it takes an infinite proper time to reach $\eta = 0$ for $r_i \leq -3/2$ and the scalar curvature for the axion metric, $\bar{R}_i \sim \bar{t}_i^{-2}$, vanishes as $\eta \to 0$. However, these same dilaton-moduli-vacuum solutions then reach $\eta \to \pm\infty$ in a finite proper time where $\bar{R}_i$ diverges. Because the conformal factor diverges as $|\eta| \to 0$ it stretches out the curvature singularity in the string metric into a non-singular evolution in the axion frame. But since $\Omega^2 = e^{\varphi} \to 0$ as $\eta \to \pm\infty$ the non-singular asymptotic behaviour in this limit in the Einstein or string frames gets compressed into a curvature singularity in the axion frame.

In terms of the proper time in the axion frame we have

$$\bar{a}_i = \bar{a}_i |\eta|^{-r_i/2} \left(1 + 2r_i / (3 + 2r_i)\right)$$ \hspace{1cm} (16)

For $r_i < -3/2$ we have conventional power-law inflation (not pole-inflation) with $\ln \bar{a}_i \sim \bar{p}_i \ln \bar{t}_i$, where $\bar{p}_i = 1 + [2/(-2r_i - 3)] > 1$. This has important consequences for the tilt of the power spectrum of semi-classical perturbations in the axion field produced on large scales.

The field equations for the linearised scalar perturbations in the axion fields are \[ \delta \sigma_i'' + 2\bar{h}_i \delta \sigma_i' + k^2 \delta \sigma_i = 0 \] (17) where the comoving Hubble rate in the axion frame for each field is given by

$$\bar{h}_i \equiv \frac{\bar{a}'_i}{\bar{a}_i} = \frac{\bar{a}'_i}{\bar{a}_i} + \frac{\Omega'_i}{\Omega_i}.$$ \hspace{1cm} (18)

The canonically normalised axion field perturbations are given by \[ v_i \equiv \frac{1}{\sqrt{2k}} \bar{a}_i \delta \sigma_i \] \hspace{1cm} (19)

and the equation of motion given in Eq. (17) can be re-written in terms of $v_i$ as

$$v_i'' + \left( k^2 - \frac{\bar{a}''_i}{\bar{a}_i} \right) v_i = 0.$$ \hspace{1cm} (20)
In the terminology of Ref. [14], the pump field $S$ for the perturbations in each axion field is given by the square of the scale factor in the corresponding conformal frame, $S_i = \bar{a}_i^2$. After inserting the power-law solution for the axion frame scale factor given in Eq. (13), we find that these equations give the general solutions

$$v_i = |k\eta|^{1/2} \left[ v_+ H^{(1)}_{\mu_i}(|k\eta|) + v_- H^{(2)}_{\mu_i}(|k\eta|) \right], \quad (21)$$

where $H^{(j)}_{\mu_i}$ are Hankel functions of order $\mu_i = |r_i|$.

For pre big bang solutions, i.e., $\eta < 0$, we can normalise modes on small scales at early times when all the modes are far inside the Hubble scale, $k \gg |\eta|^{-1}$. They can be assumed to be in the flat-spacetime vacuum\footnote{This is the power spectrum for a massless scalar field during power-law inflation which approaches the famous result $P_{\delta x} = 1$.}.

Allowing only positive frequency modes in the flat-spacetime vacuum state at early times requires that

$$v_i \to e^{-ik\eta}/\sqrt{2k} \quad (22)$$

as $k\eta \to -\infty$\footnote{It is interesting to note that in conventional inflation we have to assume that this result for a quantum field in a classical background holds at the Planck scale. Here, however, the normalisation is done in the zero-curvature limit in the infinite past.}.

This gives

$$v_+ = e^{i(2\mu_i+1)\pi/4} \frac{\sqrt{\pi}}{2\sqrt{k}} H^{(1)}_{\mu_i}(-k\eta), \quad v_- = 0 \quad (23)$$

and hence we have

$$\delta\sigma_i = \kappa \sqrt{\frac{\pi}{2k}} e^{i(2\mu_i+1)\pi/4} \frac{\sqrt{-k\eta}}{\bar{a}} H^{(1)}_{\mu_i}(-k\eta). \quad (24)$$

Just as in conventional inflation, this produces perturbations on scales far outside the horizon, $k \ll |\eta|^{-1}$, at late times, $\eta \to 0^-$. The power spectrum for perturbations is conventionally denoted by

$$P_{\delta x} \equiv \frac{k^3}{2\pi^2} |\delta x|^2, \quad (25)$$

and thus for modes far outside the horizon ($-k\eta \to 0$) we have\footnote{When $r = 0$ the dilaton remains constant and the axion frame and Einstein frame coincide, up to a constant factor. Thus, the axion spectra behave in the same way as those of the dilaton and moduli fields. The late time evolution in this case is logarithmic with respect to $-k\eta$\footnote{The Hubble rate in the axion frame can be written as $H_i = (1 + 2r_i)\Omega_i^{-1} \bar{H}$, where the conformal factor $\Omega_i^2$ is given in Eq. (12) and $\bar{H}$ is the expansion parameter in the Einstein frame.}}

$$P_{\delta\sigma_i} = 2\kappa^2 \left( \frac{C(\mu_i)}{2\pi} \right)^2 \frac{k^2}{\bar{a}^2} (-k\eta)^{1-2\mu_i}, \quad (26)$$

where the numerical coefficient

$$C(\mu_i) = \frac{2^{\mu_i} \Gamma(\mu_i)}{2^{\mu_i+1} \Gamma(3/2)} \quad (27)$$

approaches unity for $\mu_i = 3/2$.

The expression for the axion power spectrum can be written in terms of the field perturbation when each mode crosses outside the horizon ($k\eta_c = -1$):

$$P_{\delta\sigma_i}|_c = 2\kappa^2 \left[ \frac{C(\mu_i)}{r_i + (1/2)} \right]^2 \left( \frac{\bar{H}_i}{2\pi} \right)^2, \quad (28)$$

where $\bar{H}_i|_c$ is the Hubble rate in the axion frame\footnote{The factor $2\kappa^2$ arises due to our dimensionless definition of $\sigma_i$.} when $k\eta_c = -1$. This is the power spectrum for a massless scalar field during power-law inflation which approaches the famous result $P_{\delta x} = 1$. 

$2\kappa^2(\tilde{H}/2\pi)^2$ as $r_i \to -3/2$, this critical case arising when the expansion in the axion frame becomes exponential.

The amplitude of the power spectra at the end of the pre big bang phase can be written as

$$P_{\delta\sigma|s} = 2\kappa^2 \left[ \frac{C(\mu_i)}{r_i + (1/2)} \right]^2 \left( \frac{\tilde{H}_i}{2\pi} \right)^2 \left( \frac{k}{k_s} \right)^{3-2\mu_i}, \quad (29)$$

where $k_s$ is the comoving wavenumber of the scale just leaving the Hubble radius at the end of the pre big bang phase, $k_s\eta_s = -1$. The subsequent evolution of these perturbations may depend upon the nature of the exit from the pre big bang ($\eta < 0$) to the post big bang ($\eta > 0$). The simplest assumption is that the modes remain frozen-in on large scales ($|k\eta| \ll 1$) in which case the massless axion perturbations contribute an energy density

$$\dot{\rho}_i \sim \Omega_i^2 \frac{k^2}{a^2} P_{\delta\sigma|s} = C^2(\mu_i) \frac{k^3}{a^2} \left( \frac{\tilde{H}}{2\pi} \right)^2 \left( \frac{k}{k_s} \right)^{3-2\mu_i}, \quad (30)$$

in the Einstein frame. We note that although the amplitude of the perturbations in each axion field depends upon the conformal factor $\Omega_i^2$, the effective energy density in the Einstein frame for perturbations with $k \sim k_s$ of a massless axion field are independent of $\Omega_i^2$. Thus, the amplitude of density perturbations on larger scales depends only upon the spectral tilt. Durrer et al. have noted that a spectrum for a massless axion, slightly tilted towards smaller scales, may be consistent with the observed amplitude of anisotropies in the cosmic microwave background with $\Delta T/T \sim (\dot{\rho}_i)_{k=\text{crit}} \sim \kappa^2 \tilde{H}_i^2 (k/k_s)^{3-2\mu_i}$ for $\kappa^2 \tilde{H}_i^2 \sim e^\sigma \sim 10^{-2}$ [11].

The spectral tilt of the perturbation spectra is given by

$$\Delta n_i \equiv \frac{d \ln P_{\delta\sigma|s}}{d \ln k} \quad (31)$$

The spectral tilt for each of the fields follows from Eq. [20], and are shown in Figure 1. They take the values

$$\Delta n_i = 3 - 2\mu_i = 3 - 2\sqrt{3} \cos(\xi - \xi_i) \quad (32)$$

where

$$\xi_i = \begin{cases} 0 & \text{for } \sigma_1 \\ -\pi/3 & \text{for } \sigma_2 \\ \pi/3 & \text{for } \sigma_3 \end{cases} \quad (33)$$

The tilts depend crucially upon the value of $\mu_i$. The spectrum becomes scale-invariant spectrum in the limit $\mu_i \to 3/2$. The lowest possible value of the spectral index for any of the axion fields is $3 - 2\sqrt{3} \simeq -0.46$. Requiring conventional power-law inflation, rather than pole inflation, in the axion frame, guarantees a negatively tilted spectrum ($\Delta n_i < 0$) [11].

All three spectral indices for the axion fields in the truncated type IIB model which we have considered are determined by the single integration constant $\xi$. In particular, we find that one of the axion fields always has a red spectrum ($\Delta n_i < 0$) while the other two spectra are blue ($\Delta n_i > 0$), except in the critical case $|\cos \xi| = \sqrt{3}/2$, where two of the spectra are scale-invariant and only one is blue. This provides an example of the important phenomenological role that the RR sector of string theory can play in cosmological solutions [28].

More generally, the axion perturbation spectra can have different spectral indices, but in a given string model there is a specific relationship between them. This follows as a direct consequence of the symmetries of the effective action. These symmetries relate the coupling parameters between the

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5 This simple assumption was verified in the specific scenarios investigated in Ref. [18].

6 Note that although the power spectrum for axion perturbations diverges on large scales for $\Delta n_i < 0$, the energy density for modes outside the horizon is proportional to $k^2 P_{\delta\sigma|s}$, and this remains finite.
Figure 1: Spectral tilts $\Delta n_i$ for three axion fields’ perturbation spectra in truncated type IIB action as a function of integration constant $\xi$ in pre big bang solutions. The solid line corresponds to $\Delta n_1$, the dotted line to $\Delta n_2$ and the dashed line to $\Delta n_3$.

Various fields and are manifested in the spectra. Such perturbation spectra could provide distinctive signatures of the early evolution of our universe. The analysis presented above should be applicable to a wide class of non-linear sigma models coupled to gravity. In such models, the couplings between the massless scalar fields are specified by the functional form of the target space metric. These couplings determine the appropriate conformal factors analogous to those in Eq. (12) that leave the fields minimally coupled and it is the evolution of these couplings that directly determine the scale dependence of the perturbation spectra.

Because large symmetry groups which include SL(3,R) are ubiquitous in supergravity theories obtained from compactification of higher dimensional theories [29], our result raises a serious challenge for the pre big bang scenario. We have shown that at least one massless axion field in an SL(3,R) non-linear sigma model will have a negatively tilted spectrum. In the scenario considered by Durrer et al. [11], the amplitude of density perturbations at horizon crossing is determined by the string scale at the end of the pre big bang era, $\delta \rho/\rho \sim e^{\phi}$. Only a positively tilted spectrum can be consistent with the usual string scale, $e^{\phi} \sim 10^{-2}$ if density perturbations on larger scales are to remain compatible with the isotropy of the microwave background sky. This assumes that the axion remains massless. One would naïvely expect that the introduction of a mass for the axion field would only make matters worse. One possible way out, would be for the axion to develop a periodic potential in which case the axion might contribute a large fraction of the dark matter in our universe, but the large field fluctuations might lead to only small density fluctuations [30].

Acknowledgements

We are grateful to Andrew Liddle for useful discussions, and to Gabriele Veneziano for drawing our attention to the problems posed by the negatively tilted axion spectra. EJC and JEL are supported by the Particle Physics and Astronomy Research Council (PPARC), UK.

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