Agegraphic Model based on the Generalized Uncertainty Principle

S. Davood Sadatian and A. Sabouri

Abstract Many models of dark energy have been proposed to describe the universe since the beginning of the Big Bang. In this study, we present a new model of agegraphic dark energy (NADE) based on the three generalized uncertainty principles $KMM$ (Kempf, Mangan, Mann), Nouicer and $GUP^*$ (higher orders generalized uncertainty principle). Using the obtained relations from three of types of $GUP$, in the form of three scenarios (Emergent, Intermediate, Logamediate), we consider three different eras of the universe evolution. Also we describe the evolution and expansion of the universe in each subsection. We will plot the obtained relations in these models for better comparison.

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Keywords Generalized Uncertainty Principle, Agegraphic Model, Expansion Universe.

1 Introduction

According to Type Ia supernova observations and observed data [1-12], we know the universe is expanding rapidly today. Extensive studies show that an energy with unknown nature and negative pressure that is called dark energy causes this accelerated expansion [13-15]. Dark energy is one of the most interesting topics in cosmology today, and various models have been proposed to describe it [16-18]. Only 4% of the universe is made of recognizable (known) matter and energy (baryonic), about 23% of it is dark matter, and about 73% is dark energy, which is now known to cause the universe to expand [19]. In this regards, many models of dark energy have been proposed to describe the universe, but so far no one have been able to explain the nature of dark energy itself. The simplest model for dark energy is the cosmological constant [20], but this model has a problem with quantum gravity as the vacuum value of some quantum fields [15]. In the $ΛCDM$ model, dark energy makes about 74% and dark matter about 22% of the universe are so effective on the critical density of the universe [21-23]. Spergel et al [24] and Seljak et al [25] combined the data from WMAP and the Supernova Legacy survey to show that can make a noticeable boundary on state equation of dark energy in a flat universe ($ω = −0.97^{+0.07} _{-0.09}$). However, some of the important models that have been proposed for dark energy are: quintessence [26], phantom [27], quantum [28], dynamical models [29], tachyon [30], chaplygin gas [31], Ricci [32], electromagnetic [33], $k$-essence [34], modified gravity [35] and etc. There are also two models for dark energy based on the holographic principle [36], one is the holographic dark energy model [37] and the other is the agegraphic dark energy model (ADE) [38]. A holographic model is based on the Bekenstein-Hawking energy boundary ($E_{BH} ≥ E_Λ$) with energy $E_{BH}$ for a universe-size black hole where $L^3 ρ_Λ ≤ m_P^2 L$ ($L$ cosmic length scale, $m_P$ Planck mass, $ρ_Λ$ vacuum energy density) [39]. Another model based on the karolyhazy relation [40,41], $δt = λ_P^2 t^2$, and the energy-time uncertainty, $ΔE ∼ t^{-1}$, in Minkowski space-time which results in $ρ_q ∼ ΔE ∼ m_P^2$ (to estimate the quantum energy density of Minkowski space-time metric fluctuations) [42]. Here, $ρ_q$ Energy density is called metric perturbations, which is used with the age of the universe $T = ∫_0^T dt$ as the energy density of the agegraphic model. While the vacuum energy density, $ρ_Λ$, is used as the energy density of the holographic model [43,44]. The $ADE$ model considers the relationship between Heisenberg uncertainty in quantum mechanics and gravitational effects in general relativity [45]. According to
In this model, dark energy is created from fluctuations in space-time and the field of matter[46]. In the ADE model, instead of the future event horizon, the age of the universe is used as a measure of length, and this solves the problem of causality in the holographic model[47]. In general relativity, space-time can be measured without limitation in measurement accuracy, but in quantum mechanics the Heisenberg uncertainty relationship creates measurement accuracy.

The original agegraphic model of dark energy (ADE) is defined as, \( \rho_D = \frac{3n^2m_p^2}{H^2} \), \((m_p = 8\pi G \simeq 10^8 Gev)\) which is Newton’s gravity constant, \( n^2 \) represents a constant, and \( T \) is the age of the universe \((T = \int dt = \int \frac{da}{aH})\), which \( t \) is cosmic time, \( a \) scale factor and \( H \) is a Hubble constant[46]. The basic problem with the ADE model is its contradiction with the matter-dominated universe, therefore, Wei and Cai [38] proposed a new ADE model to solve this problem. For the age of the universe, they presented \( T \) instead of \( \eta \), which was a conformal time scale, and the energy density changed to \( \rho_D = \frac{3n^2m_p^2}{\eta} \). But \( \eta \) changed as \( \eta = \int \frac{dt}{\dot{a}} = \int \frac{da}{aH} \), i.e. a coefficient \( \frac{1}{\eta} \) was added to the time scale. Wei and Cai also provided an important value for the stability of their model, which they called the squared speed of sound, \( v_s^2 \), and defined as \( v_s^2 = \frac{\delta}{\dot{\delta}} \). In order to develop the stability of the background, the sign of, \( v_s^2 \) was very important, and if \( v_s^2 < 0 \) in general relativity it created an instability disturbance. In this regard, many models studied such as: Myung [48] showed that the sign of \( v_s^2 \) is negative for the holographic model, its reason is because of future events horizon as negative IR cutoff. Kim et al [49] concluded that the sign of \( v_s^2 \) for the agegraphic model is also always negative and creates instability in the model. Pasqua et al [50] for the dark energy model based on the uncertainty principle showed that this model is unstable in the power-law form of the scale factor \( a(t) \). When the effects of quantum gravity become more important, the Heisenberg uncertainty principle is no longer estimated, and the generalized uncertainty principle (GUP), which is related to string theory, presents the Planck scale as the minimum length, and hence describes the evolution of the early universe [51,52]. In the new agegraphic model (NADE), two important parameters are the equation of state, \( \omega \), and the square speed of sound, \( v_s^2 \), which \( \omega \) determines the nature of the evolution of the universe and \( v_s^2 \) the stability of the evolution of the universe [49]. Rahul et al [20] investigated a new interactive agegraphic dark energy model based on GUP and examined the behavior and evolution of the universe in three scenarios: Emergent, Intermediate, and Logamediate. In recent years, extensive studies have been done on the new agegraphic model of dark energy to better understand the expansion and evolution of the universe like Kurmar and singh[53] who studied the evolution of the universe at the late time by the NADE model, and they showed that in this model, by transition the phase from the matter domination era in the early universe to an accelerated phase at the late time, can crossed from the phantom divided line. Also, Hosseinkhani has studied the thermodynamics of a new interactive agegraphic model [54].

In following, we want to present a new agegraphic dark energy model (NADE) in the framework of the three generalized uncertainty principles mentioned in the reference [56] (KMM,Novicerc,GUPs) and investigate the evolution and expansion of the universe in three Emergent, Intermediate, Logamediate scenarios. In this regard, in section two, we review and compare the original, new, and interactive agegraphic model, and then in the third section, we present the new agegraphic model based on the three GUPs mentioned. Finally, in the fourth section, we examine the proposed model in the form of three scenarios for the evolution and expansion of the universe and we summarize our results in the conclusion section.

2 A review on the agegraphic dark energy models

In this section, we review and compare the original, new and interactive agegraphic models of dark energy.

2.1 Original agegraphic dark energy (ADE)

As mentioned in the previous section, the basis of the ADE model is the relation (Karolyhazy) and the energy-time uncertainty in Minkowski space-time that we have:

\[ \delta t = \lambda t^\frac{1}{2} \]

\[ \Delta E \sim t^{-1} \]

Which is resulted from these two equations:

\[ \rho_q \sim \frac{\Delta E}{\delta t^3} \sim \frac{m_p^2}{t^2} \]
In relation (3) instead of time factor $t$ in this model, the age of the universe is replaced as follows:[57]

$$T = \int_0^a \frac{da}{aH}$$

(4)

where $a$ is the scale factor and $H \equiv \dot{a}/a$ the Hubble constant. Energy density in this model is defined as follows:[57]

$$\rho_q = \frac{3n^2m_p^2}{T^2}$$

(5)

where $m_p$ is Planck’s mass and $T$ is the age of the universe. In this model, instead of $T$, the age of the universe is set, and by doing so, the causality problem in the holographic model is solved and the coefficient $3n^2$ is to parameterize some uncertainties, such as the effects of space-time curves, various quantum fields in the universe, and etc [38]. The Friedmann equation in this model (considering a FRW flat universe containing agegraphic dark energy and pressureless matter) is defined as follows:

$$H^2 = \frac{1}{3m_p^2}(\rho_m + \rho_q)$$

(6)

Using the energy density relationship in this model, the fractional energy density for pressureless matter and agegraphic dark energy is equal to:[57]

$$\Omega_q = \frac{n^2}{H^2T^2}$$

(7)

$$\Omega_m = 1 - \Omega_q$$

(8)

Using relationships (5),(6),(7) the fractional energy density for pressureless matter and agegraphic dark energy is equal to:[57]

$$\dot{\Omega}_q = \Omega_q(1-\Omega_q)(3 - \frac{2\sqrt{\Omega_q}}{n})$$

(9)

Now using the energy conservation equation $\dot{\rho}_q + 3H(\rho_q + p_q) = 0$ and equations (5) and (7) the equation of parameter of state (EoS) of the agegraphic dark energy is obtained as:

$$\omega_q = -1 + \frac{2\sqrt{\Omega_q}}{3n}$$

(10)

The interesting point about relation (9) is that if $\Omega_q \to 0$ then $\omega_q \to -1$ at the early time while if $\Omega_q \to 1$ then $\omega_q \to -1 + \frac{2}{n}$ late time. The first is called the matter-dominated epoch, in which $\Omega_q \simeq 3\Omega_q$ and $\omega_q \simeq -1$, as a result $\Omega_q \propto a^3$. This means that with $\omega_q \simeq -1$ the agegraphic dark energy in the epoch of matter-dominated mimics a cosmic constant while the energy density of pressureless matter is considered as $\rho_m \propto a^{-3}$[58]. Therefore, there is an implied confusion in this agegraphic dark energy model[38]. In fact, in the matter-dominated epoch we have:

$$\Omega_q \ll 1 \Rightarrow \frac{2}{3} a \propto t^3 \propto a^3$$

$$\Rightarrow (according \ Eq.5) \rho_q \propto a^{-3}$$

On the other hand, because $\rho_m \propto a^{-3}$ has $\Omega_q \simeq const$ which is in conflict with the previous $\Omega_q \propto a^3$ and as a result the agegraphic dark energy does not dominate and this is unacceptable. Two solutions have been proposed to solve this confusion[38]:

Solution 1: Placing $T + \delta$ instead of $T$ ($\delta$ is a constant with time dimension) in results obtained in Equation (5) [58]:

$$\rho_q = \frac{3n^2m_p^2}{(T+\delta)^2}$$

(11)

In this solution, $T \ll \delta$ is at the early time, and as a result $\rho_q \simeq const$, which means that the agegraphic dark energy behaves like a cosmic constant. At the late time, $T \gg \delta$, in this state $\rho_q$ is approximately equal with Equation (5). And in the intermediate state, $T \sim \delta$, in this state the existence of $\delta$ cannot be ignored and as a result there will be no more tracking behavior. This solution overrules the basis of the relationship in this model (karolyhazy relation (1)) and becomes only a phenomenological model [38].

Solution 2: Because Equation (5) is derived from Minkowski space-time and on the other hand because space-time is very curved in the early time, so this equation has no validity in the early time and therefore $n \propto a(t)$ is variable [58]. In this solution, if we consider a critical $T_c$ in the early or middle period, the coefficient $n$ can be considered almost a constant in Equation (5), thus with this method, confusion can be eliminated, although the validity of the equation (5) remove in early time[38].

2.2 New agegraphic dark energy (NADE)

In this section, we review a new model of agegraphic which is a better model to solve the confusion mentioned above. According to the vacuum energy density of the holographic principle, $\rho_A = 3c^2m_p^2L^{-2}$. If we consider $L$ as $\frac{1}{m_p}$, the equation of state parameter (EoS) of the holographic dark energy is zero and therefore can not accelerate the expansion of the universe [44]. Now, if we consider $L$ as the particles horizon, in this case, because the parameter of the equation of state becomes
\( \omega_\Lambda > \frac{1}{3} \), this case can not also describe the acceleration of the expansion of the universe [59]. Finally, if \( L \) be the the future event horizon of the universe, in this case, the EoS holographic energy is equal to: [44,59-61]:

\[
\omega_\Lambda = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c}
\]  
(12)

Therefore, it is obvious that \( \omega_\Lambda < \frac{1}{3} \) and can be a necessary condition for accelerating the expansion of the universe. This model is called the holographic model, and because of the future event horizon, it shows the eternal acceleration of the universe but faces the causality problem. In the new dark energy agegraphic model (NADE), instead of \( T \) as the age of the universe, a conformal time \( \eta \) is chosen, defined as \( d\eta = \frac{dt}{a} \) (\( t \) is a cosmic time) to solve the causality problem [38]. Therefore, the energy density of the NADE model is presented as:

\[
\rho_q = \frac{3n^2m_p^2}{\eta^2}
\]  
(13)

\[
\eta = \int \frac{dt}{a} = \int \frac{da}{a^2H}
\]  
(14)

and for fractional energy density

\[
\Omega_q = \frac{n^2}{H^2\eta^2}
\]  
(15)

For a flat FRW universe containing new agegraphic dark energy and pressureless matter, using the energy conservation equation \( \dot{\rho}_m + 3H\rho_m = 0 \) and equations (6), (13) and (15) we have [38]:

\[
\frac{d\Omega_q}{da} = \frac{1}{a}\Omega_q(1 - \Omega_q)(3 - \frac{2\sqrt{\Omega_q}}{na})
\]  
(16)

Also, from the energy conservation equation

\[
\dot{\rho}_q + 3H(\rho_q + p_q) = 0
\]

and equations (13) and (15), the equation of state (EoS) can be obtained for the NADE model:

\[
\omega_q = -1 + \frac{2\sqrt{\Omega_q}}{3na}
\]  
(17)

In late time, if \( a \to \infty \) and \( \Omega_q \to 1 \) then \( \omega_q \to -1 \) and in early time if \( a \to 0 \) and \( \Omega_q \to 0 \) then \( \omega_q \) of relation (17) Not available. In the matter-dominated epoch we have:

\[
H^2 \propto \rho_m \propto a^{-3} \Rightarrow \sqrt{\rho_m} \propto \frac{dt}{a} \propto d\eta
\]

\( \Rightarrow \) (according Eq.13 and \( \rho_q \propto a^{-2} \)) \( \eta \propto a^{-1} \)

And from the energy conservation equation \( \dot{\rho}_q + 3H\rho_q(1 + \omega_q) = 0 \), the parameter of the state equation is equal to: \( \omega_q = -\frac{2}{3} \). According to \( \rho_m \propto a^{-3} \), it follows that \( \Omega_q \propto a^2 \) and with comparing \( \omega_q = -\frac{2}{3} \) and Equation (17):

\[
\Omega_q = \frac{n^2a^2}{4}
\]  
(18)

Thus the equation of motion in the matter-dominated epoch is obtained:

\[
\frac{d\Omega_q}{da} = \frac{1}{a}\Omega_q(3 - \frac{2\sqrt{\Omega_q}}{na})
\]  
(19)

Now consider a radiation-dominated universe, which contains NADE energy and background matter with the equation parameter \( \omega_m \) (for a particular case, \( \omega_m = 0 \) for pressureless matter and \( \omega_m = \frac{1}{3} \) for radiation). Again, using the energy conservation equation \( \dot{\rho}_m + 3H\rho_m(1 + \omega_m) = 0 \) and equations (6), (13) and (15), the equation of motion for \( \Omega_q \) is [38]:

\[
\frac{d\Omega_q}{da} = \frac{1}{a}\Omega_q(1 - \Omega_q)(3(1 + \omega_m) - \frac{2\sqrt{\Omega_q}}{na})
\]  
(20)

also in radiation-dominant epoch:

\[
H^2 \propto \rho_r \propto a^{-4} \Rightarrow \sqrt{\rho_r} \propto \frac{dt}{a} \propto d\eta
\]

\( \Rightarrow \) (according Eq.13 and \( \rho_q \propto a^{-2} \)) \( \eta \propto a \)

And from the energy conservation equation \( \dot{\rho}_q + 3H\rho_q(1 + \omega_q) = 0 \), the parameter of the equation of state is equal to: \( \omega_q = -\frac{1}{3} \) and according to \( \rho_r \propto a^{-4} \), it follows that \( \Omega_q \propto a^2 \) and with comparing \( \omega_q = -\frac{1}{3} \) and Equation (17):

\[
\Omega_q = \frac{n^2a^2}{4}
\]  
(21)

Thus the equation of motion in the radiation-dominated epoch is obtained [38]:

\[
\frac{d\Omega_q}{da} = \frac{1}{a}\Omega_q(4 - \frac{2\sqrt{\Omega_q}}{na})
\]  
(22)

In summary, in the matter-dominated epoch \( \omega_q = -\frac{2}{3} \) and \( \Omega_q = \frac{n^2a^2}{4} \) and in the radiation-dominant epoch, \( \omega_q = -\frac{1}{3} \) and \( \Omega_q = n^2a^2 \), which leads to domination of the new agegraphic dark energy. At the late time \( \omega_q \to -1 \) and \( a \to \infty \) that NADE mimics a cosmic constant. Note that both the matter-dominated epoch and the radiation-dominated epoch are be \( \Omega_q \ll 1 \) and \( a \ll 1 \) [38]. Therefore, the NADE model is very different from the ADE model and the evolutionary behavior of the NADE model is similar to the holographic dark.
energy, except that the NADE model does not have the causal problem [38]. In the NADE model, Equation (1) (Karolyhazy equation) naturally follows the entropy boundary of the holographic black hole [41].

2.3 Interactive agegraphic dark energy (IADE)

In this model, Wei [47] extended the NADE model by including the interaction between the new agegraphic dark energy and the background matter whose the equation of state parameter is $\omega_m = \text{const.}$ By including an interactive $Q$ expression, Wei obtained the following relationships between the new agegraphic dark energy and the background matter:

$$\dot{\rho}_q + 3H\rho_q(1 + \omega_q) = Q$$

(23)

$$\dot{\rho}_m + 3H\rho_m(1 + \omega_m) = -Q$$

(24)

The energy conservation equation in this case is $\dot{\rho}_{\text{tot}} + 3H(\rho_{\text{tot}} + p_{\text{tot}}) = 0$. Using equations (6), (13), (15), and (24) the equation of motion for $\Omega_q$ in this model is equal to:

$$\frac{d\Omega_q}{da} = \frac{1}{a} \Omega_q[(1-\Omega_q)(3(1+\omega_m) - \frac{2\sqrt{3\eta}}{na}) - \frac{Q}{3m^2_pH^3}]$$

(25)

Also from equations (13), (15) and (23) the equation of state parameter of the new agegraphic dark energy is obtained as [47, 62]

$$\omega_q = -1 + \frac{2\sqrt{\Omega_q}}{3na} - \frac{Q}{3H\rho_q}$$

(26)

According to Equation (17), $\omega_q > -1$ and cannot cross the phantom divided line, if $Q = 0$ (non-interaction mode) and return to the NADE model again. But in the case $Q \neq 0$ (interaction mode) according to Equation (25) if $Q < 0$ then $\omega_q > -1$ and still can not cross the phantom divided line, but if $Q > 0$ then $\omega_q > -1$ or $\omega_q < -1$ and therefore is likely to cross the phantom divided line and it has a phantom behavior. In general, the NADE model is the third single-parameter model after the $\Lambda$CDM model and the DGP braneworld model [63]. It is also found in [63] that the coincidence problem in the NADE model is naturally solvable and that this model is consistent with the cosmic observations of Type Ia supernovae, the cosmic microwave background, and the large scale structure. So the NADE model is a very good and logical model for describing dark energy in cosmology.

3 Investigation of NADE model based on three types of GUP ($KMM, Nouicer, GUP^*$)

In this section, we present a new Agegraphic dark energy model (NADE) based on the three types of generalized uncertainty principles mentioned in [56] ($KMM$, $Nouicer$ and $GUP^*$). This model based on $KMM$ GUP has done before, but based on $Nouicer$ and $GUP^*$ GUP is investigated for the first time in this paper.

3.1 NADE model based on $KMM$ GUP:

This part considers the KMM generalized uncertainty principle first proposed by Kim et al. [66] for the new agegraphic dark energy model (NADE):

$$\Delta E \Delta t \geq 1 + \beta(\Delta E)^2$$

(27)

In unit $c = h = k_B = 1$, $\beta$ here, like the Planck length, is a length scale $\beta \sim \frac{1}{m_p} \sim L_p$. With the solution of Eq.(27), we reach $\Delta E_G = \frac{1}{\Delta t} + \frac{\beta}{4\Delta t^2}$. (According to cosmological purpose such as $\Delta t \sim t$, so it can also be written as $\Delta E_G = \frac{1}{t} + \frac{\beta}{4t^2}$). The energy density obtained from GUP in this case is equal to [66]:

$$\rho_G = \frac{\Delta E_G}{\Delta t^2}$$

(28)

According to Karolyhazy time fluctuations, $\delta t = t_p^{\frac{2}{q}}t^{\frac{1}{q}}$ results $\beta = \frac{2}{n^2m_p^2}$, $t_p = \frac{4}{3n^2m_p^2}$ where $q$ and $n$ are two constant parameters and $m_p$ is Planck’s mass. Energy density is defined by the parameters $q$ and $n$ and Planck mass $m_p$ as follows:

$$\rho_G = \frac{3n^2m_p^2}{t^2} + \frac{3q^2}{t^4}$$

(29)

Now, according to the new agegraphic dark energy model, instead of the cosmic time parameter $t$, the age of the universe $T$, and then to solve the causality problem in this model. Now, instead of $T$, it is placed the conformal time $\eta$, which is defined as follows:

$$\eta = \int^a_0 \frac{da}{a^2H} = \int^t_{-\infty} \frac{dt}{aH}; \{t = \ln a; H = \frac{\dot{a}}{a}\}$$

(30)
Finally
\[ \rho_G = \frac{3n^2m_p^2}{\eta^2} + \frac{3q^2}{\eta^4} \] (31)

On the other hand, Friedman’s equations for a flat universe and also its continuity equations are:
\[ H^2 = \frac{1}{3m_p^2}(\rho_G + \rho_m); \] (32)
\[ \rho_G + 3H(\rho_G + P_G) = 0 \] (33)
\[ \rho_m + 3H\rho_m = 0; \] (34)
where \( \rho_G \) and \( P_G \) are the energy density and pressure obtained from GUP respectively and \( \rho_m \) are the density of cold dark matter (CDM) without pressure \( (P_m = 0) \).

Introducing the density parameters \( \Omega_i = \frac{\rho_i}{3H^2m_p^2} \) and the energy density equation obtained in this part and \( \Omega_G + \Omega_m = 1 \) we have:
\[ \Omega_G = \frac{n^2}{H^2\eta^2}(1 + \frac{q^2}{m_p^2n^2\eta^2}) \] (35)

According to Eq.34 and for the pressure expression we have:
\[ P_G = -\frac{1}{3} \frac{d\rho_G}{dt} - \rho_G \] (36)

that as consequence the parameter of state equation is as follows:
\[ \omega_G = \frac{P_G}{\rho_G} = -1 + \frac{2e^{-t}\sqrt{\Omega_G}}{3n}(\frac{m_p^2n^2\eta^2 + 2q^2}{m_p^2n^2\eta^2 + q^2}) \] (37)

Using the expression \( \Omega_G + \Omega_m = 1 \) and Eq.34 and Eq35, the evolution equation to define \( \omega_G \) and conformal time are described as follows [66]:
\[ \frac{d\Omega_G}{dt} = 3\omega_G\Omega_G(1 - \Omega_G) \] (38)
\[ \frac{d\eta}{dt} = \frac{1}{H_0}(\frac{He^{-t}}{H}) = \frac{1}{H_0}\sqrt{\frac{1 - \Omega_G}{\Omega_m}} \] (39)

where \( H_0 \) is the present Hubble parameter. According to the equation of the state parameter in this part, it is expected that dark energy will dominate in the present and the future. In the far past \( (t \to -\infty \) or \( a \to 0) \) the matter-dominated universe with \( \omega_G = -\frac{1}{3} \) and \( \Omega_G = n^2a^2 \) and the radiation-dominated universe with \( \omega_G = -\frac{1}{3} \) and \( \Omega_G = n^2a^2 \) is recovered [38]. According to [66], the evolution of the new agegraphic dark energy model for the critical parameter \( n \) and \( q = 1 \) with the initial conditions \( \Omega_G = 0.72 \) and \( \eta_0 = \frac{1}{H_0} \) has been investigated and shows that in the far past, the matter-dominated \( (\Omega_G \to 0 \) and \( \Omega_m \to 1) \) and in the far future, radiation-dominated \( (\Omega_G \to 1 \) and \( \Omega_m \to 0) \) will occur. In other words, in terms of critical \( n \) \( (n_c = 2.7999) \) we have:
\[ \{ n < n_c; \omega_G \to \infty \} \]
\[ \{ n = n_c; \omega_G \to -\frac{2}{3} \} \]
\[ \{ n > n_c; \omega_G \to -1 \} \] (40)

Because of the GUP represents the Planck scale \( (L = L_p) \) and is related to the Planck period \( (t = t_p = 10^{-43}s) \), it describes the reason of the cosmological constant problem well, and it does not change the dark energy-dominated universe in the present and the far future significantly [66].

3.2 NADE model based on Nouicer GUP

Nouicer’s principle of generalized uncertainty is about field theory in non-anticommutative superspace, which is defined in terms of energy-time as follows [56]:
\[ \Delta E \Delta t \geq e^{\beta E^2} \] (41)

So that in unit \( c = \hbar = k_B = 1 \) it has the following solution:
\[ \Delta E_G = \frac{\beta}{\Delta t^2ln\Delta t} \to (\Delta t \sim t) \frac{\beta}{t^2ln\Delta t} \] (42)

According to the relation of the energy density obtained from GUP, \( (\rho_G = \frac{\Delta E_G}{\Delta t}) \) and also \( \delta t = \frac{t_0^2}{t_p} \) and \( t_p = \frac{1}{3m_p^2} \) and \( \beta = \frac{q^2}{m_p^2} \), the energy density in this case, in terms of parameters \( q \) , \( n \) and Planck mass, \( m_p \), is defined as follows:
\[ \rho_G = \frac{9n^2q^2m_p^2}{t^2ln\eta} \] (43)

Now for investigating Nouicer GUP for the NADE model in Eq.43 instead of \( t \), we place cosmic age, \( T \), and then to solve the causality problem in this model instead of \( T \), we place the conformal time \( \eta \) which we will finally have:
\[ \rho_G = \frac{9n^2q^2m_p^2}{\eta^2ln\eta} \] (44)
According to Eq.32, Eq.33, Eq.34, and the derivative of the expression $\Omega_G + \Omega_m = 1$ and the density parameter $\Omega_i = \frac{\rho_i}{3H^2m_p^2}$ we have:

$$\Omega_G = \frac{3q^2n^2}{H^2\eta^3ln\eta}$$  \hspace{1cm}  (45)

Now using Eq.36 and Eq.44, the parameter of state equation for this model is obtained as follows:

$$\omega_G = \frac{P_G}{\rho_G} = \frac{1}{3ln\eta}$$  \hspace{1cm}  (46)

The equation of evolution related to Eq.46 is defined according to the derivative of the expression $\Omega_G + \Omega_m = 1$ and also Eq.34 and Eq.35 and the conformal time as follows:

$$\frac{d\Omega_G}{dt} = -3\Omega_G(1 + \omega_G)$$  \hspace{1cm}  (47)

As we see in Fig.1.a and Fig.1.b, the behavior of the equation of state parameter and the evolution equations for dark energy ($\Omega_G$) and cold dark matter ($\Omega_m$) are plotted in the NADE model based on the Nouicer GUP. In Fig.1.b, according to the Nouicer model, in the far past time, both matter and radiation are dominated and tend to infinity ($\Omega_G \rightarrow +\infty$ and $\Omega_m \rightarrow -\infty$) and have a repulsive effect from each other, which causes accelerated expansion of the universe. With the evolution of the universe and the passing of time, the effects of $\Omega_G$ and $\Omega_m$ converge and neutralize each other’s effect, which means that $\Omega_G$ and $\Omega_m$ have no effect on the evolution and expansion of the universe in the direction of moving towards the present. But, with the effect of $\omega_G$ and its decreasing behavior, the expansion of the universe increases with the evolution of the universe. The interesting note is that the higher the value of the parameter $n$, especially when $n > n_c$, the universe expands more rapidly. In the NADE model based on the Nouicer GUP because $\omega_G > -1$ therefore has a quintessence-like behavior.

3.3 NADE model based on $GUP^\ast$ (The higher order generalized uncertainty principle)

The GUP is perturbative, meaning that for smaller values the GUP parameter is set, and on the other hand, the maximum momentum defined in the general relativity is not set in GUP. To solve these problems, the higher order generalized uncertainty principle was defined [56]. The energy-time uncertainty relationship of $GUP^\ast$ is defined as follows:

$$\Delta E\Delta t \geq \frac{\hbar}{2(1 - \beta\Delta E^2)}$$  \hspace{1cm}  (48)

The solution of this equation is as follows (in unit $c = \hbar = k_B = 1$):

$$\Delta E_G = \frac{\hbar t}{2(t^2 - \beta)}$$  \hspace{1cm}  (49)

where $\beta$ is the GUP parameter and $\hbar$ is the reduced Planck constant,

$$\hbar = \frac{\hbar}{2\pi} = \frac{6.7 \times 10^{-34}}{2\pi} \approx 1.06633 \times 10^{-34} Js$$  \hspace{1cm}  (50)

According to the relation of the energy density obtained from GUP, ($\rho_G = \Delta E_G/\delta t^2$) and also $\delta t = t_p t^\ast$, and $t_p = \frac{1}{3n^2m_p^2}$ and $\beta = \frac{n^2m_p^2}{\eta}$, the energy density in this case, in terms of parameters $q$ and $n$ and Planck
mass, $m_p$ is defined as follows:

$$\rho_G = \frac{9n^6m_p^6\hbar}{2(t^2n^2m_p^2 - q^2)} \quad (51)$$

For investigating $GUP^*$ in the NADE model in Eq.43 instead of $t$, we place cosmic age, $T$, and then to solve the causality problem in this model instead of $T$, we place the conformal time $\eta$ which we will finally have:

$$\rho_G = \frac{9n^6m_p^6\hbar}{2(\eta^2m_p^2n^2 - q^2)} \quad (52)$$

According to Eq.32,Eq.33,Eq.34, and the derivative of the expression $\Omega_G + \Omega_m = 1$ and the density parameter $\Omega_i = \frac{m_p^2}{3H^2}$ we have:

$$\Omega_G = \frac{3hm_p^4n^6}{2H^2(\eta^2m_p^2n^2 - q^2)} \quad (53)$$

Now according to Eq.36 and Eq.52, the parameter of state equation for this model is obtained as follows:

$$\omega_G = -1 + \frac{2\eta^2}{3(\eta^2m_p^2n^2 - q^2)} \quad (54)$$

The equation of evolution related to Eq.46 is defined according to the derivative of the expression $\Omega_G + \Omega_m = 1$ and also Eq.34 and Eq.35 and the conformal time as follows:

$$\frac{d\Omega_G}{dt} = 3\Omega_G(\omega_G(\eta^2m_p^2n^2 - q^2) - q) \quad (55)$$

In Fig.2.a and Fig.2.b, the behavior of the state parameter equation and the evolution of dark energy $\Omega_G$ and cold dark matter $\Omega_m$ are plotted in the NADE model based on $GUP^*$, respectively. In Fig.2.a, in the far past time, $\omega_G$ has a decreasing behavior and because with the evolution of the universe, the value of the state parameter is $\omega_G = -1$ and then decreases rapidly ($\omega_G < -1$), so most of the time in the far past, It has a same cosmological constant behavior and then shows phantom-like behavior. And in the far future, $\omega_G$ has a decreasing and quintessence-like behavior ($\omega_G > -1$) and at the present time $\omega_G$ does not have a certain value. In Fig.2.b, in the far past time, tend to $\Omega_G \rightarrow -\infty$ and $\Omega_m \rightarrow +\infty$ that indicate in the far past, both matter and radiation is dominated and have a repulsive effect on each other that it causes the accelerated expansion of the universe. And in the far future time $\Omega_G$ and $\Omega_m$ converge to a certain constant value, and both in the far past and far future times all three $\omega_G, \Omega_G$ and $\Omega_m$ are dominated. In the Table.1, it summarizes the results obtained in this section.

| GUP     | The far past time | The present time | The far future time |
|---------|------------------|-----------------|-------------------|
| KMM     | Increasing the radiation effect with the universe evolution. | $\omega_G \rightarrow -\frac{1}{3}$ | $\Omega_G \rightarrow 1$ |
|         | $\Omega_m \rightarrow 0$ | $\Omega_G \rightarrow 0$ | $\Omega_G \rightarrow 1$ |
| Nouicer | Decreasing the matter and radiation effect and increasing the effect of state parameter equation. | $\omega_G \rightarrow -1$ | $\Omega_m \rightarrow 0$ |
|         | $\Omega_m \rightarrow 0$ | $\Omega_G \rightarrow 0$ | $\Omega_G \rightarrow 1$ |
| GUP*    | Lack of the matter and radiation and the parameter of state equation dominated. | $\omega_G \rightarrow -\infty$ | $\Omega_m \rightarrow 0$ |
|         | $\Omega_G \rightarrow -\infty$ | $\Omega_G \rightarrow 0$ | $\Omega_G \rightarrow 1$ |

Table 1 The evolution of the universe in three types of GUP(KMM, Nouicer and GUP*).
4 Investigation of Emergent, Intermediate and Logamediate scenarios in GUP types (KMM, Nouicer, GUP∗)

In this section, we examine the three scenarios mentioned for the universe in[67,68] in the types of GUPs mentioned in this paper, and check the evolution of the universe in each case. As respects the three scenarios mentioned for the NADE model are based on the KMM GUP in the various works previously reviewed (see:[20,46 and 66]), for this reason we regardless from re-describing it only is summarized in Table.2 at the end of this section, so in this section we will concentrate on two types of GUP (Nouice and GUP∗) with and without interaction.

4.1 Emergent scenario based on Nouice GUP

The scale factor in the emergent scenario is defined as follows[67]:

\[ a(t) = a_0(B + e^{At})^m; a_0 > 0, A > 0, B > 0 \]  (56)

Now if we place the above relation in the general expression of Eq.30, we will have:

\[ \eta = \frac{(1 + Be^{-At})^m(B + e^{At})^{-m}2F_1[m, m, 1 + m, -Be^{-At}]}{Aa_0m} \]  (57)

where \( 2F_1 \) is a hypergeometric function defined as

\[ 2F_1 = [a, b, c, z] = \sum_{i=0}^{\infty} \frac{a! b! c!}{i! i! i!} z^i \]

By placing Eq.57 in Eq.44, the energy density in this part is obtained as follows:

\[ \rho_{G1} = \frac{(-9A^3a_0^3m^3(1 + Be^{-At})^{-3m}(B + e^{At})^{-3m}n^2q^2m^2) \times \left(2F_1[m, m, 1 + m, -Be^{-At}^3]\right)^{-1}}{Aa_0m} \]  (58)

According to the conservation Eq.24 and Eq.56 in this part we have for dark matter density:

\[ \rho_{m1} = \rho_{m0}[a_0(B + e^{At})^m]^{-3(1+\omega_m+Q)} \]  (59)

where \( Q \) is the expression of interaction between dark energy of NADE model based on GUP with dark matter without pressure, which in all parts, two modes of interaction \( (Q \neq 0) \) and non-interaction \( (Q = 0) \) are considered. Using Eq.56 in this part, the Hubble parameter in terms of cosmic time is calculated as follows:

\[ H = \frac{\dot{a}}{a} = \frac{mAe^{At}}{B + e^{At}} \]  (60)

Using the conservation Eq.23 and Eq.60, in this part the dark energy pressure is calculated as follows:

\[ p_{G1} = \frac{-Q - \rho_{G1}}{3H} - \rho_{G1} \]  (61)

and using Eq.58, Eq.59, Eq.61, the equation of the total state parameter, \( \omega_{total} \), is obtained as follows:

\[ \omega_{total} = \frac{p_{G1}}{\rho_{G1} + \rho_{m1}} \]  (62)
The behavior of the $\omega_{\text{total}}$ state parameter equation, given in Eq.62, is plotted in Fig.3 in both the interaction and non-interaction modes under the emergent scenario based on the Nouicer GUP.

![Fig. 3](image)

**Fig. 3** Evolution of state parameter equation in terms of the cosmic time under the emergent scenario of the universe based on the Nouicer GUP. With values $n = 1.2$, $q = 1$, $m_p = 1$, $Q = 0.05$, $a_0 = 0.12$, $B = 2.3$, $A = 5.6$, $m = 2$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)

According to Fig.3, the emergent scenario based on the Nouicer GUP is only able to describe the evolution of the universe in the far past and present time, and cannot describe the far future time. In the mode of interaction with the evolution of the universe, the value of $\omega_{\text{total}}$ decreases, and this indicates the accelerated expansion of the universe in the far past, and also in the mode of non-interaction in this part, the universe expands with a constant rate in all of the past period. Because of the value of $\omega_{\text{total}} > -1$, the universe behaves a quintessence-like in the emergent scenario under the Nouicer GUP. In Fig.4.a and Fig.4.b, the parameters $\rho_{\text{total}}$ and $p$ are plotted in two modes of interaction and non-interaction, respectively.

In Fig.4.a, where $\rho_{\text{total}}$ values remain constant in both interaction and non-interaction modes, then its value decreases rapidly in a shorter time. And in Fig.4.b, the behavior of the $P$ parameter in both interaction and non-interaction modes is similar to the behavior of the parameter of state equation in Fig.3, and its reason is obviously due to Eq.62, and $\rho_{\text{total}}$ constant remaining. To examine the stability or instability of different dark energy models, a parameter called the speed square sound is expressed, which Kim and Wei first proposed for their new agegraphic model, which is defined as follows [49]:

$$\nu_s^2 = \frac{\dot{\rho}}{\rho} \quad (63)$$

In Fig.5, in both interaction and non-interaction modes, the sound speed square, $\nu_s^2$, are plotted under the Nouicer GUP emergent scenario of the universe, which behaves similarly to the parameter of state equation in this part. According to Fig.5, the behavior of $\nu_s^2$ with the evolution of the universe in the far past has a decreasing behavior and always remains at the non-negative level, which indicates the stability of the NADE model under the emerging scenario based on the Nouicer GUP.
4.2 Intermediate scenario based on Nouicer GUP

The scale factor in the emergent scenario is defined as follows [68]:

\[ a(t) = e^{\lambda t^\beta}; \{\lambda > 0, 0 < \beta < 1\} \]  

(64)

Now if we place the above relation in the general expression of Eq.30, we will have:

\[ \eta = -\frac{\lambda^2 \Gamma[\frac{1}{\beta}, \lambda t^\beta]}{\beta} \]  

(65)

where \( \Gamma[x,z] \) is an incomplete gamma function defined as \( \Gamma[x,z] = \int_x^\infty t^{z-1}e^{-t}dt \). With placing Eq.65 in Eq.44, we have:

\[ \rho_{G2} = -\frac{9n^2q^2m_n^2\beta^3\lambda^2}{\Gamma[\frac{1}{\beta}, \lambda t^\beta]'\ln(\frac{\lambda^2}{\beta})} \]  

(66)

According to the conservation Eq.24 under intermediate scenario we have:

\[ \rho_{m2} = \rho_{m0}(e^{\lambda \nu^2})^{-3(1+\omega_m+Q)} \]  

(67)

where \( Q \) is the expression of interaction between dark energy of NADE model based on GUP with dark matter without pressure, which in all parts, two modes of interaction \( (Q \neq 0) \) and non-interaction \( (Q = 0) \) are considered. The Hubble parameter according to Eq.64 in this part is:

\[ H = \frac{\dot{a}}{a} = \lambda \beta t^{-1+\beta} \]  

(68)

According to the conservation Eq.23 and Eq.68, in this part the dark energy pressure is calculated as follows:

\[ p_{G2} = -\frac{Q - \rho_{G2}}{3H} - \rho_{G2} \]  

(69)

The parameter of state equation and the square of the speed of sound in this part according to the Eq.66, Eq.67 and Eq.69 are respectively:

\[ \omega_{total} = \frac{p_{G2}}{\rho_{G2} + \rho_{m2}} \]  

(70)

\[ \nu_s^2 = \frac{p_{G2}}{\rho_{G2} + \rho_{m2}} \]  

(71)

Fig.6 shows the behavior of the \( \omega_{total} \) state parameter equation in Eq.70 given under the intermediate scenario based on the Nouicer GUP. According to Fig.6, the intermediate scenario, like the emergent scenario based on the Nouicer GUP, can only describe the evolution of the universe in the far past time, and can not describe the evolution of the universe in the present and the far future time. In this part, the mode of interaction and non-interaction have exactly the same behavior, in other words, interaction has no effect on the evolution behavior of the universe. As we see in Fig.6, with the evolution of the universe \( \omega_{total} \) has an increasing behavior, which means that in the far past, unlike the emergent scenario in the previous part, in this scenario, based on the Nouicer GUP, the universe does not have accelerated expansion but has decelerated behavior. Because of in this part is \( \omega_{total} > -1 \), so the universe has a quintessence-like behavior.

In Fig.7.a and Fig.7.b, the parameters \( \rho_{total} \) and \( P \) are plotted in terms of cosmic time, respectively. In these diagrams, the mode of interaction and non-interaction have exactly the same behavior, and with the evolution of the universe, the passing of time, the value of \( \rho_{total} \) increases and the value of \( P \) increases in the negative direction \( (t \to -\infty) \) in other words with the evolution of the universe, the value of negative \( P \) is decreased.

Fig.8 shows the sound speed square behavior, \( \nu_s^2 \), for the stability or instability of the model in this part. As shown in Fig.8, with the evolution of the universe and the passing of time, \( \nu_s^2 \) has an increasing behavior in this part and always remains at a positive level \( (\nu_s^2 > 0) \), which indicates the stability of the NADE model in the intermediate scenario based on Nouicer GUP.
4.3 Logamediate scenario based on Nouicer GUP

The scale factor in the emergent scenario is defined as follows [68]:

\[ a(t) = e^{\mu(\ln t)^\alpha}; \{\alpha > 1, \mu > 0\} \]  \hspace{1cm} (72)

Now if we place the above relation in the general expression of Equation 30, we will have:

\[ \eta = \int \frac{dt}{e^{\mu(\ln t)^2}} = \frac{e^{\frac{\mu}{\alpha} \sqrt{\pi} \text{erf} \left( -\frac{1+2\mu \ln t}{2\sqrt{\mu}} \right)}}{2\sqrt{\mu}} \]  \hspace{1cm} (73)

Due to the lack of a analytic solution of the above conformal time, and to solve this problem, we consider the above conformal time for the specific case \( \alpha = 2 \), which we will have:

\[ \eta = \int \frac{dt}{e^{\mu(\ln t)^2}} = \frac{e^{\frac{\mu}{2} \sqrt{\pi} \text{erf} \left( -\frac{1+2\mu \ln t}{2\sqrt{\mu}} \right)}}{2\sqrt{\mu}} \]  \hspace{1cm} (74)

where \( \text{erf} \) is an error function and is defined as \( \text{erf} = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} dt \). By placing Eq.74 in Eq.44, the energy density in this part is obtained as follows:

\[ \rho_G = \frac{72n^2q^2m_p^2\mu^2}{e^{\frac{\mu}{2} \sqrt{\pi} \text{erf} \left( -\frac{1+2\mu \ln t}{2\sqrt{\mu}} \right)} \left(3\ln \left(\frac{e^{\frac{\mu}{2} \sqrt{\pi} \text{erf} \left( -\frac{1+2\mu \ln t}{2\sqrt{\mu}} \right)}}{2\sqrt{\mu}}\right) \right)^3} \]  \hspace{1cm} (75)

According to the conservation Eq.24 and Eq.72 under the logamediate scenario we have:

\[ \rho_m = \rho_m \left( e^{\mu(\ln t)^\alpha} \right)^{-3(1+\omega_m+Q)} \]  \hspace{1cm} (76)

\[ \rho_m \]

where \( Q \) is an expression related to interaction. According to the scale factor equation in the logamediate scenario (Eq.72), the Hubble parameter in this part is calculated as follows:

\[ H = \frac{\dot{a}}{a} = \frac{\mu\alpha(\ln t)^{-1+\alpha}}{t} \]  \hspace{1cm} (77)

According to survival Eq.24 and Eq.77, in this part, the expression pressure is obtained as follows:

\[ p_G = \frac{-Q - \rho_G}{3H} - \rho_G \]  \hspace{1cm} (78)

The parameter of state equation and the square of the speed of sound in this part, as in the previous parts,
Fig. 8  Evolution of the sound speed squares in terms of the cosmic time to check the stability or instability of the dark energy model under the intermediate scenario of the universe based on Nouicer GUP. With values $n = 2$, $q = 1$, $m_{p} = 1$, $Q = 0.05$, $\lambda = 1.9$, $\beta = 0.5$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)

are as follows:

$$\omega_{total} = \frac{\rho_{G3}}{\rho_{G3} + \rho_{m3}}$$  \hspace{1cm} (79)$$

$$\nu_{s}^2 = \frac{\rho_{G3}}{\rho_{G3} + \rho_{m3}}$$  \hspace{1cm} (80)$$

Fig.9 shows the behavior of the state parameter equation in terms of the cosmic time under the logamediate scenario of the universe based on Nouicer GUP. With values $n = 1.0013$, $q = 1$, $m_{p} = 1$, $Q = 0.05$, $\alpha = 2$, $\mu = 1.4$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)

then remains constant, and after the value of $\omega_{total}$ is constant, value of $P$ changes from negative to positive level.

In Fig.11, to investigate stability and instability, the behavior of the square of the speed of sound, $\nu_{s}^2$, is plotted in terms of cosmic time. According to the diagram $\nu_{s}^2$, in this part the behavior of $\nu_{s}^2$ is divided into two step. In the first step, $\nu_{s}^2$ has neither increasing nor decreasing behavior (parabolic behavior) and is always at a positive level ($\nu_{s}^2 > 0$), which indicates the stability of the NADE model under the logamediate scenario based on the Nouicer GUP, but in the second step, $\nu_{s}^2$ has an increasing behavior and is always at a negative level ($\nu_{s}^2 < 0$), which indicates the instability of the NADE model under the logamediate scenario based on the Nouicer GUP.

4.4 Emergent scenario based on $GUP^*$

According to the scale factor defined in the emergent scenario (Eq.56), and its placement in the general expression of Eq.30 (equation of conformal time obtained in Eq.57) and also the placement of Eq.57 in Eq.52 (related to the energy density $GUP^*$ in the previous section) energy density in this part is obtained as follows:

$$\rho_{G3} = -\frac{9n^{2}m_{p}^{2}\hbar A_{q}m(1 + Be^{-At})^{-m}(B + e^{At})^{m}}{2n^{2}m_{p}^{2}F_{1}[m, m, 1 + m, -Be^{-At}] - q^{2}}$$  \hspace{1cm} (81)$$

"Fig. 10. a and Fig. 10. b, the parameters $\rho_{total}$ and $P$ are plotted in terms of cosmic time, respectively. In these diagrams, the two modes of interaction and non-interaction behave exactly the same. with the evolution of the universe and the passing of time, the value of $\rho_{total}$ increases rapidly and then remains constant and always remains at a negative level ($\rho_{total} < 0$). And in Fig. 10. b, the value of $P$ also increases rapidly and then remains constant, and after the value of $\omega_{total}$ is constant, value of $P$ changes from negative to positive level."

"In Fig. 9, the behavior of the state parameter equation in terms of the cosmic time under the logamediate scenario of the universe based on Nouicer GUP. With values $n = 1.0013$, $q = 1$, $m_{p} = 1$, $Q = 0.05$, $\alpha = 2$, $\mu = 1.4$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)"
Evolution of energy density parameter, $\rho_{\text{total}}$, in terms of the cosmic time under the logamediate scenario of the universe based on Nouicer GUP. With values $n = 1.0013$, $q = 1$, $m_p = 1$, $Q = 0.05$, $\alpha = 2$, $\mu = 1.4$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)

Using Eq.23 and the Hubble parameter defined in the emergent scenario (Eq.60), the new agegraphic dark energy pressure in this part is obtained as follows:

$$\rho_{G4} = \frac{-Q - \rho G4}{3H} - \rho G4$$  \hspace{1cm} (82)

According to Eq.81, Eq.82 and Eq.59, the state parameter equation, $\omega_{\text{total}}$, and the square of the speed of sound, $v_s^2$, are:

$$\omega_{\text{total}} = \frac{\rho G4}{\rho G4 + \rho m1}$$  \hspace{1cm} (83)

$$v_s^2 = \frac{p G4}{\rho G4 + \rho m1}$$  \hspace{1cm} (84)

In Fig.12, the equation of state parameter equation is plotted under the emergent scenario based on $GUP^*$. Unlike the emergent scenario in the Nouicer GUP, the emergent scenario based on $GUP^*$ describes the evolution of the universe in the far past, present, and the far future times. In fact, the behavior of $\omega_{\text{total}}$ in this part is the same as the behavior of $\omega_{\text{total}}$ in the emergent scenario based on Nouicer GUP, with the difference that the evolution of the universe in this part, in addition to the far past, is also generalized to the present and the far future. According to Fig.12, in the mode of interaction with the evolution of the universe, the value of $\omega_{\text{total}}$ decreases and in the present and the far future time, the value of $\omega_{\text{total}}$ remain constant, which means that the universe in the far past experienced a period of accelerated expansion and with the evolution of the universe in the present and the far future of the universe expand at a constant rate, and in non-interactive mode at all times, the universe does not have accelerated expansion. According to $\omega_{\text{total}} > -1$, so the universe has a quintessence-like behavior in this part.

In Fig.13.a and Fig.13.b, the behavior of the $\rho_{\text{total}}$ and $P$ parameters are plotted in terms of cosmic time. In these two mode diagrams, interaction and non-interaction behave the same exactly. With passing of time and evolution of the universe, the value of $\rho_{\text{total}}$ is constant and at a certain time in the far future, its value decreases rapidly, and the parameter $P$ is constant with the evolution of the universe and passing of time, at a
Evolution of state parameter equation in terms of the cosmic time under the emergent scenario of the universe based on $GUP^*$. With values $n = 1.2$, $q = 1$, $m_p = 1$, $Q = 0.05$, $a_0 = 0.12$, $B = 2.3$, $A = 5.6$, $m = 2$, $\hbar = 2$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)

certain time in the far future, its value increases rapidly.

In Fig.14, the sound speed squared behavior in both interaction and non-interaction modes is the same as the behavior of $\omega_{total}$, and $\nu_s^2$ has a decreasing behavior in this part and always remains positive level ($\nu_s^2 > 0$), which indicates the stability of the NADE model under emergent scenario based on $GUP^*$.

4.5 Intermediate scenario based on $GUP^*$

According to the scale factor defined in the intermediate scenario (Eq.64), and its placement in the general expression of Eq.30 (equation of conformal time obtained in Eq.65) and also the placement of Eq.65 in Eq.52 (related to the energy density $GUP^*$ in the previous section) energy density in this part is obtained as follows:

$$\rho_{G5} = \frac{-9n^6m_p^6\hbar\beta\lambda^{\frac{3}{2}}}{2(n^6m_p^6\Gamma[\frac{1}{2}, \lambda\beta] - q^2)}$$

Using Eq.23 and the Hubble parameter defined in the intermediate scenario (Eq.68), the new agegraphic dark energy pressure in this part is obtained as follows:

$$p_{G5} = \frac{Q - \rho_{G5}}{3H} - \rho_{G5}$$

According to Eq.85, Eq.86 and Eq.67, the parameter of state equation, $\omega_{total}$, and the square of the speed of sound, $\nu_s^2$, are:

$$\omega_{total} = \frac{p_{G5}}{\rho_{G5} + \rho_{m2}}$$

$$\nu_s^2 = \frac{p_{G5}}{\rho_{G5} + \rho_{m2}}$$

In Fig.15, the behavior of the state parameter equation in the intermediate scenario based on $GUP^*$ is plotted. The intermediate scenario based on $GUP^*$ is only able to describe the universe in the present and the far future time and is not able to describe the evolution of the universe in the far past time, and also in this part, interaction and non-interaction modes behave exactly the same. According to Fig.15, the evolution behavior of the universe is divided into two steps; In the first
Fig. 14  Evolution of the sound speed squares in terms of the cosmic time to check the stability or instability of the dark energy model under the emergent scenario of the universe based on GUP*. With values $n = 1.2$, $q = 1$, $m_p = 1$, $Q = 0.05$, $a_0 = 0.12$, $B = 2.3$, $A = 5.6$, $m = 2$, $h = 2$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)

step, with the evolution of the universe and the passing of time, the value of $\omega_{\text{total}}$ decreases, which means that the universe has an accelerated expansion, and in the second step, the value of $\omega_{\text{total}}$ decreases and then equals a constant value of $\omega_{\text{total}} = -1$, which means the universe in The second step experiences an accelerated expansion and then continues to expand at a constant rate. In the first step, because $\omega_{\text{total}} < -1$, the universe behaves the phantom-like, and in the second step, first because of $\omega_{\text{total}} > -1$, the universe behaves the quintessence-like, and then because of $\omega_{\text{total}} = -1$ behaves like the cosmological constant.

In Fig.16.a and Fig.16.b, the behavior of the parameters $\rho_{\text{total}}$ and $P$ are plotted in terms of cosmic time, and both interaction and non-interaction modes behave the same. With evolution of the universe and passing of time, the value of $\rho_{\text{total}}$ decreases in both steps. The value of the $P$ parameter increases rapidly in the first step and then decreases rapidly in the second step.

Fig.17 shows the sound speed squared behavior in terms of cosmic time. According to Eq.17, in the first step of the evolution of the universe in this part, $\nu_s^2$ has a parabolic behavior and is always at a negative level ($\nu_s^2 < 0$), which indicates the instability of the NADE model, and in the second step of evolution, $\nu_s^2$, has a decreasing behavior and always remains at a positive level, indicating the stability of the NADE model under the intermediate scenario based on GUP*.

4.6 Logamediate scenario based on GUP*

According to the scale factor defined in the intermediate scenario (Eq.72), and its placement in the general expression of Eq.30 (equation of conformal time obtained in Eq.74) and also the placement of Eq.74 in Eq.52 (related to the energy density GUP* in the previous section) energy density in this part is obtained as follows:

$$\rho_{G6} = \frac{9n^6m_p^6\hbar}{e^{\frac{1}{n}}\sqrt{\pi}n^2m_p^3erf\left(\frac{1+2\omega_{\text{total}}}{2\sqrt{n}}\right)} - q^2$$  \hspace{1cm} (89)

Using Eq.23 and the Hubble parameter defined in the logamediate scenario (Eq.77), the new agegraphic dark energy pressure in this part is obtained as follows:

$$p_{G6} = \frac{Q - \rho c_b}{3H} - \rho_{G6}$$  \hspace{1cm} (90)

According to Eq.89, Eq.90 and Eq.76, the parameter of state equation, $\omega_{\text{total}}$, and the square of the sound speed, $\nu_s^2$, are:

$$\omega_{G6} = \frac{p_{G6}}{\rho_{G6} + \rho_{m3}}$$  \hspace{1cm} (91)

$$\nu_s^2 = \frac{p c_b}{\rho_{G6} + \rho_{m3}}$$  \hspace{1cm} (92)

Fig.18 shows the behavior of the state parameter equation in terms of cosmic time under the logamediate
Evolution of energy density parameter, $\rho_{\text{total}}$, in terms of the cosmic time under the intermediate scenario of the universe based on GUP*. With values $n = 2$, $q = 1$, $m_p = 1$, $Q = 0.05$, $\lambda = 1.9$, $\beta = 0.5$, $\hbar = 2$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)

Evolution of pressure parameter $P$ in terms of the cosmic time under the intermediate scenario of the universe based on GUP*. With values $n = 2$, $q = 1$, $m_p = 1$, $Q = 0.05$, $\lambda = 1.9$, $\beta = 0.5$, $\hbar = 2$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)

The logamediate scenario based on GUP* is only able to describe the evolution of the universe in the present and the far future time, and is not able to describe the evolution of the universe in the far past time. Also, in this part, both interaction and non-interaction modes behave exactly the same. According to Fig.18, the evolution of the universe is divided into three steps; In the first step, the value of $\omega_{\text{total}}$ increases rapidly, which means that the universe has decelerated expansion. In the second step, $\omega_{\text{total}}$ has a parabolic behavior, which means that the universe will first have decelerated expansion and then accelerated expansion. And in the third step, the value of $\omega_{\text{total}}$ decreases rapidly, which means that the universe has an accelerated expansion. In the first step, first $\omega_{\text{total}} = -1$ and then its value increases ($\omega_{\text{total}} > -1$) and this means that in the first step, the universe will behave like a cosmological constant and then has a quintessence-like behavior. In the second step $\omega_{\text{total}} < -1$ and this means that the universe will have phantom-like behavior in this step. And in the third step, the universe will have the same behavior of the first step. Therefore, it will first have a quintessence-like behavior ($\omega_{\text{total}} > -1$) and then it will behave like a cosmological constant ($\omega_{\text{total}} = -1$).

In Fig.19a and Fig.19b, the behavior of the parameters $\rho_{\text{total}}$ and $P$ are plotted in terms of cosmic time, respectively. With evolution of the universe and passing of time, the value of $\rho_{\text{total}}$ decreases in all the steps, and the parameter $P$ decreases in the first and third steps, and in the second step it has a parabolic behavior. In these diagrams, the two modes of interaction and non-interaction have the same behavior.

In Fig.20, the sound speed squared behavior is plotted in terms of cosmic time. In the first step, $\nu_s^2$ has a parabolic behavior and in the second step, because of $\nu_s^2$ decreases with the evolution of the universe and tends towards $\nu_s^2 \to -\infty$, in the second step, the NADE model is unstable. In the third step, $\nu_s^2$ has a decreasing behavior. In the first and third steps of the evolution of the universe, in the major part of the diagram, $\nu_s^2$ is positive and in a minor part is negative, so by multiply from the positive and negative sign and the result of the negative answer ($\nu_s^2 < 0$), we conclude that $\nu_s^2$ is generally negative and this indicates the instability of
the NADE model under the logamediate scenario based on $GUP^*$. In Tables 2, 3 a summary of what is considered in Section 4 is summarized.

5 Conclusion

The model (NADE) is investigated based on three types of GUP defined in [56] (KMM, Nouicer and $GUP^*$) under three scenarios with different scale factors (Emergent, Intermediate and Logamediate). Due to the extensive work already done for the NADE model based on the KMM GUP [see 20, 46, 49, 50, 57, 66], we decided to describe this NADE model based on the Nouicer and $GUP^*$. In the second section, the types of dark energy models (ADE) are reviewed and also in the first subsection of the third section, the NADE model based on KMM GUP is reviewed. In the second subsection of the third section is defined the NADE model based on the Nouicer GUP. We concluded that in this subsection, with the evolution of the universe and the passing of time, the universe has an accelerated expansion, and the reason is to neutralize the effects of the equations of evolution of energy and dark matter ($\Omega_G$ and $\Omega_m$) and reduce the equation of state parameter ($\omega_G$). On the other hand, it was concluded that in the far past, in the NADE model based on Nouicer GUP, both matter and radiation are dominant, and with the evolution of the universe, their effect on expansion decreases, and instead the value of $\omega_G$ decreases and causes accelerated expansion of the universe. In the third subsection of the third section, the NADE model is defined based on $GUP^*$. In this subsection, radiation ($\Omega_G$), matter ($\Omega_m$) and state equation parameter ($\omega_G$) are all dominated in the far past and far future times, which means that in these times the universe has an acceleration expansion. In the present time, none of these parameters are dominated, which all of these results are summarized in Table 1.

In Section 4, the behavior and evolution of the universe in the NADE model were investigated based on three emergent, intermediate and logamediate scenarios based on KMM, Nouicer and $GUP^*$. In each of the scenarios and GUPs, the behavior of the parameter of state equation $\omega_{total}$, energy density, $\rho_{total}$, pressure, $P$, and sound speed squared, $\nu_s^2$ were investigated. According to [20, 46], the KMM

Fig. 18 Evolution of state parameter equation in terms of the cosmic time under the logamediate scenario of the universe based on $GUP^*$. With values $n = 1.0013$, $q = 1$, $m_p = 2$, $Q = 0.05$, $\alpha = 2$, $\mu = 1.4$, $h = 2$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)

Fig. 19 [a](Up). Evolution of energy density parameter, $\rho_{total}$, in terms of the cosmic time under the logamediate scenario of the universe based on $GUP^*$. [b](Down). Evolution of pressure parameter $P$ in terms of the cosmic time under the logamediate scenario of the universe based on $GUP^*$. With values $n = 1.0013$, $q = 1$, $m_p = 2$, $\alpha = 2$, $\mu = 1.4$, $h = 2$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)
GUP is done and we just reviewed it in Table 2. But the Nouicer GUP and $GUP^*$ show different behaviors than the KMM GUP of the universe. In the emergent scenarios based on Nouicer GUP and $GUP^*$, the universe shows quintessence-like behavior. In the intermediate scenario based on Nouicer GUP, the universe shows quintessence-like behavior and based on $GUP^*$ the universe shows quintessence-like, phantom-like and cosmological constant behavior. In the logamediate scenario based on Nouicer GUP the universe has the cosmological constant behavior in the present time and the quintessence-like in the far future time. In the logamediate scenario based on $GUP^*$, the behavior and evolution of the universe is limited to three steps, in the first and third steps of evolution, the universe has a constant cosmological and quintessence-like behavior, but in the second step of evolution, the universe shows phantom-like behavior. The NADE model based on KMM GUP is unstable under all scenarios but the NADE model based on Nouicer GUP is stable under emergent and intermediate scenarios, and under the logamediate scenario is stable in the first step of evolution and unstable in the second step of evolution. In $GUP^*$, under the emergent scenario, the NADE model is stable, and under the intermediate scenario, the model is unstable in the first step of evolution, also the model is stable in the second step of evolution, and the NADE model based on $GUP^*$ is unstable under the logamediate scenario. In general, the behavior and evolution of the universe in the types of GUPs and under all three mentioned scenarios in Tables 2, 3 are summarized.

Fig. 20 Evolution of the sound speed squares in terms of the cosmic time to check the stability or instability of the dark energy model under the logamediate scenario of the universe based on $GUP^*$. With values $n = 1.0013$, $q = 1$, $m_p = 2$, $Q = 0.05$, $\alpha = 2$, $\mu = 1.4$, $\hbar = 2$ (the values of the vertical axis are obtained according to the values of the mentioned parameters.)
Table 2  summary table of results.

| GUP   | Scenario     | $\omega_{total}$ | $\rho_{total}$ | P      | $\upsilon_s^2$ | Description                                      |
|-------|--------------|-------------------|-----------------|--------|-----------------|--------------------------------------------------|
| KMM   | Emergent     | Increasing behavior | $\omega_{total} < -1$ Phantom-like behavior | Upward ↑ | Downward ↓ | $\upsilon_s^2 < 0$ | The model is unstable. |
| KMM   | Intermediate | Increasing behavior | $\omega_{total} < -1$ Phantom-like behavior | Upward ↑ | Downward ↓ | $\upsilon_s^2 < 0$ | The model is unstable. |
| KMM   | Logamediate  | Increasing and decreasing behavior $\omega_{total} > -1$ quintessence-like behavior | Downward ↓ | Upward ↑ | $\upsilon_s^2 < 0$ | The model is unstable. |
| Nouice| Emergent     | The Description of the universe evolution in the far past. Decreasing behavior in the interaction mode and be constant in the non-interaction mode. $\omega_{total} > -1$ Quintessence-like behavior | Downward ↓ | Downward ↓ | $\upsilon_s^2 > 0$ | The model is stable. |
| Nouice| Intermediate | The Description of the universe evolution in the far past. Increasing behavior. $\omega_{total} > -1$ Quintessence-like behavior | Upward ↑ | Downward ↓ | $\upsilon_s^2 > 0$ | The model is stable. |
| Nouice| Logamediate  | The Description of the universe evolution in the present and the far future times. Decreasing behavior $\omega_{total} > -1$ For far future time and $\omega_{total} = -1$ For present time. Quintessence-like behavior for the far future time and cosmological constant behavior for the present time | Upward ↑ | Upward ↑ | First step $\upsilon_s^2 > 0$ and second step $\upsilon_s^2 < 0$ | In the first step, the model is stable and in the second step, the model is unstable. |
Table 3  summary table of results.

| GUP     | Scenario     | $\omega_{total}$ Description | $\rho_{total}$ | $P$ | $\upsilon^2_s$ | Description                                                                 |
|---------|--------------|--------------------------------|-----------------|-----|----------------|-----------------------------------------------------------------------------|
| $GUP^*$ | Emergent     | The Description of the universe evolution in all of the times. Decreasing behavior for the interaction mode and constant behavior for non-interaction mode $\omega_{total} > -1$ Quintessence-like behavior | Downward ↓      | Upward ↑     | $\upsilon^2_s > 0$ | the model is stable.                                                        |
| $GUP^*$ | Intermediate | The Description of the universe evolution in the present and the far future times. Decreasing behavior. $\omega_{total} \leq -1$ in the first step And $\omega_{total} \geq -1$ in the second step. The phantom-like and cosmological constant behavior in the first step And The quintessence-like and cosmological constant behavior in the second step | Downward ↓ for both steps | Upward↑ For the first step And Downward ↓ For the second step | First step $\upsilon^2_s < 0$ and second step $\upsilon^2_s > 0$ | In the first step the model is stable and in the second step the model is stable. |
| $GUP^*$ | Logamediate  | The Description of the universe evolution in the present and the far future times. Increasing behavior in the first step , parabolic behavior in the second step and decreasing behavior in the third step. $\omega_{total} \geq -1$ in the first and third steps and $\omega_{total} < -1$ In the second step. Quintessence-like and cosmological constant behavior in the first and third steps and Phantom-like behavior in the second step | Downward ↓      | Downward ↓ For the first and third steps of evolution. Parabolic ↓ For the second step of evolution | In generally and multiplying the sign of $\upsilon^2_s$ in all of step $\upsilon^2_s < 0$ | The model is unstable. |
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