Inclusions Behavior Analysis during Levitation Melting of Steel in Cold Crucible for Application to Cleanliness Assessment

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Fusion of metals in a cold crucible is known as a clean melting method without contamination by refractory. For this reason, it is commonly applied to the fusion of high melting point metals or reactive materials. As the usage of the clean melting function, this technique is also applied to the cleanliness assessment method as an advanced technique of electron beam melting method.

In this paper, oxide inclusions behavior in a cold crucible levitation melting of steel is discussed through fundamental experiments and numerical simulations in order to clarify the inclusions behavior during the fusion. Experiments showed a rapid removal of oxides from the bulk molten metal to the sample surface without remarkable change in shape and size distribution of contained oxides. Numerical analyses revealed the dominant factors of this phenomenon, and explained the reason why the size distribution of oxides does not change during the fusion.

KEY WORDS: levitation melting; inclusion; continuous casting; numerical simulation; quality assessment.

1. Introduction

Levitation melting in a cold crucible is well known as a clean melting method, which uses a water-cooled copper crucible whose inner horizontal section decreases in the downward direction to increase the electromagnetic force to sustain the gravitational force and levitate a molten metal melted by induction heating. This technique has been commonly used for melting of metals such as titanium and zirconium, which have high melting points and very reactive characteristics with refractory during a fusion. The application of this technique to the refining of metals is also tried for the dephosphorization of stainless steel by using a very reactive flux.

These researches are the applications for materials processing, however, the levitation melting apparatus is also the effective method for fundamental researches such as the analysis of deoxidation behavior and reaction equilibrium. It has been known that the cold crucible levitation melting increases the cleanliness in the melt, moreover, the removed inclusions are accumulated to the surface of the sample. From this aspect, the application for cleanliness assessment like an electron beam (EB) method was proposed based on the characteristics of cold crucible technique.

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Application to the cleanliness assessment is related to the circumstance that in metals industry there is a demand to find a suitable cleanliness assessment method of metal, which represents the products quality. Actually, various techniques for cleanliness assessment of steel have been developed and used for the determination of operating conditions during continuous casting. Table 1 shows the methods generally used for the quality assessment of steel with their characteristics including advantages and disadvantages. Each method has many advantages, however, the establishment of more rapid, more informative, more accurate and more representative method is still strongly demanded, because the earlier the quality determination is, the lower the production cost becomes through the change in operating conditions and the suitable treatment of semi-products. The cold crucible method is a kind of extraction method of inclusions from steel by the fusion. The above mentioned electron beam (EB) method is this kind of method, whose advantage is its rapid treatment. However, the change in morphology or in composition through the high heat flux of the electron beam cannot be ignored, especially in the case of inclusions whose origin is the slag in a ladle which has a relatively lower melting point. Another disadvantage is the amount of sample, which is normally some gram for one sample. This demands the increase in number of samples to increase the representativeness. In the aforementioned research, possibility to use a cold crucible technique for the quality assessment is proposed through experiments with

| Method | Treatment Time | Volume | Information |
|--------|----------------|--------|-------------|
| TIOE   | 10min          | 2g     | N0           |
| MS     | same days      | 200m   | Ye            |
| EB     | 1hr            | 2g     | mainlyAl2O3   |
| Slime  | 1month         | 100g/1kg| Yes (SEM)    |
| FZS    | real time      | 10g    | Yes           |
| UST    | same hours     | 1gram  | Yes (metal)  |

(TIOE: Total oxygen, MS: Microscope, EB: Electron beam, Slime: Electrolytic extraction, FZS: Electric sensing zone, UST: Ultrasonic testing.)

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aluminum-killed steel, in which the capability of the technique for aluminum oxides evaluation through the scanning electron microscopy (SEM) observation is presented. However, it takes much time to observe the dispersed inclusions on the round surface of samples. Consequently, for the establishment of cold crucible method, the establishment of post-processing of the extracted inclusions has been desired. On behalf of this demand, the authors have proposed the assessment technique by combining cold crucible, X-ray fluorescent method and surface electrolytic extraction method.

On the fundamentals of levitation melting in a cold crucible from the theoretical aspects, they have been evaluated through various modeling researches based on electromagnetism and fluid dynamics. Electromagnetic modeling was developed by using integral equation method, which enabled the prediction of electromagnetic force distribution and electrical characteristics such as the inductance needed for electrical design. Free surface shape analysis of a levitated metal can be predicted by using a functional method with minimizing the total energy. Once the Lorentz force field is obtained, fluid flow in the melt can be calculated by solving Navier–Stokes equations. These analyses revealed the fundamental phenomena in the cold crucible levitation melting.

In this paper, fundamental phenomena including the behavior of inclusions in levitation melting by using a cold crucible are discussed by using results of experiments and numerical simulations in order to clarify the characteristics of this method and to determine the control parameters.

2. Experimental Study on Oxide Particles Behavior during Levitation Melting

Fundamental research was conducted by using a cold crucible technique, which enables induction heating and levitation melting in a water-cooled copper crucible. Figure 1 shows a schematic view of the experimental apparatus and Table 2 shows its principal specs. The number of segments of the crucible is 16 and dimensions of the crucible are 34 mm in inner diameter and 40 mm in depth. Radio frequency alternative current (AC) field of 300 kHz is used for fusion taking account of the free surface stability, which is generally more stable with higher frequency. Steel samples containing about 60 ppm aluminum oxides including some slag type inclusions taken at the inlet region of the tundish in a continuous casting machine are melted, whose grade and dimensions are shown in Table 3. The atmosphere used for fusions is pure argon. After the sample is melted, power supply is kept by changing the holding time of fusion as from 1 to 10 min, at the temperature of about 20 K as superheat measured by a two-color pyrometer at the top of the sample. Figure 2 shows the relation between DC power input by the generator and the temperature measured by the two-color pyrometer at the top of the charge. On the state of levitation melting, a soft contact between the charge and the crucible is chosen so as to obtain a stable fusion, because excess power supply levitates the metal, however, it causes repeated attaching and detaching behavior of metals to the crucible, which causes a strong disturbance on the surface of charge and seemed to be harmful for the reproducibility of inclusion extraction and evaluation. The upper limit of the input power is also known by the saturation of superheat versus power input as shown in Fig. 2. This soft contact makes a solidified region (skull) formed just after the fusion of sample as schematically shown in Fig. 3. However, the volume of this region can be evaluated as less than 5% of total volume of the charge, and the effect of this area for the extraction amount of inclusions to the surface area can be negligible. To discuss the inclusion behavior, flow velocity at the surface of melt was measured by adding small iron tips at the top of the melt under video camera observation, which showed that the order of electromagnetically driven flow is about 0.2 m/s. By the evaluation of inclusion removal ratio, it was found that the holding time more than 5 min enables about 80% of oxide inclusions to go out to the surface as shown in Fig. 4. Figure 5 shows the concentration of aluminum oxide in the sample evaluated.
by repeating electrolytic extraction of inclusions. It can be observed that oxides are condensed to the surface by a fusion in a cold crucible. The change in size distribution of inclusions obtained by the electrolytic extraction method is shown in Fig. 6, which does not show a remarkable change in size distribution except for the ones whose diameters are in the order of 1 μm. On the slag type inclusion, which is originated to the slag flow out phenomena at the last stage of pouring of steel from the ladle to the tundish, even if its melting point is in the same level as the one of steel, the spherical shape does not be deformed during the fusion as shown in Fig. 7.

3. Numerical Study on Inclusions Behavior

3.1. MHD Analysis

To investigate this phenomenon, magnetohydrodynamic (MHD) analyses have been conducted. First, electromagnetic field is obtained by using the integral equation method.9,10) As the frequency of electromagnetic field is very high, the skin depth is very small which is evaluated as about 1.2 mm for a molten steel charge and 0.3 mm for the copper crucible. It is obtained from the relation of \( \delta = \sqrt{1/\mu_0 \sigma f} \) by using the values of electrical conductivity \( \sigma = 0.7 \times 10^6 \) S/m for steel and \( \sigma = 1 \times 10^7 \) S/m for copper, frequency \( f = 300 \) kHz and the permeability of vacuum \( \mu_0 = 4 \pi \times 10^{-7} \) A/m. Then the electromagnetic field can be assumed to exist only in the vicinity of the surface of electrically conductive materials such as segments of the copper crucible and the charge. The boundary is divided into surface elements and the induced current is calculated from the boundary integral equation, which is obtained by Biot–Savart’s law. The calculation is performed by the assumption of axisymmetric system and by introducing quasi-three dimensional modeling of the cold crucible system.10) Once the induced electric current field is obtained, then the induced magnetic field can be calculated by integrating the magnetic field generated by the given electric currents in the coils and that generated by induced electric currents. After obtaining the electric current and the magnetic filed, Lorentz force distribution can be calculated by the relation of \( F = J \times B \). In the aforementioned calculation, the force is
assumed to exist on the surface of charge by the hypothesis of very thin skin depth. Therefore, the body force inside the charge is obtained by assuming that the force normal to the surface is proportional to \( \exp\left(-\frac{2n}{\delta}\right) \), where \( n \) shows the distance from the surface of a charge perpendicular and internal direction to the surface. The surface value of Lorentz force is to be calculated by \( B^2/(2\mu_0) \). This assumption is acceptable because the skin depth is less than 10% of radii of the charge surface. By this assumption, the extrapolation of surface electromagnetic pressure can be performed to the inner direction of the charge. These analyses methods are summarized in the Appendix. It should be noted that in the following calculation, the bottom of the charge is a singular point concerning the electromagnetic force theoretically, however, the value of the force in the vicinity of this point was extrapolated because the diameter of the hole at the bottom of crucible is set to be small (about 1 mm). Figure 8 shows the calculated melt surface shape including free boundary. The larger the power input, the levitation state is improved, however the surface deformation becomes also remarkable. The condition of middle surface line marked by a thicker one is chosen for the following numerical calculations, because this condition was also found to be suitable by the experiment. Figure 9(a) shows the nodal system for the fluid flow simulation of a charge and (b) shows the contour diagram of Lorentz force distribution in the aforementioned case. In Fig. 9, remarkable skin effect, concentration of Lorentz force to the surface of the charge, is observed, which becomes the force to levitate the charge.

Once the boundary shape and the external force field are determined, the fluid flow field can be calculated by solving the Navier–Stokes equations by introducing obtained Lorentz force as the external volume force. Here, the small magnetic Reynolds number enables to make the calculation one way, that is, it is not necessary to solve Maxwell equations and Navier–Stokes equations simultaneously.

The equation of continuity and Navier–Stokes equation are shown as follows.

\[
\nabla \cdot \mathbf{u} = 0. \quad \text{(1)}
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p + \nabla \cdot (\nu \nabla \mathbf{u}) + \mathbf{g} + \frac{f}{\rho} \quad \text{... (2)}
\]

where, \( \mathbf{u} \): velocity of fluid, \( t \): time, \( p \): pressure, \( \rho \): density of fluid, \( \mathbf{g} \): acceleration of gravity, \( \nu \): kinematic viscosity of fluid and \( f \): external volume force.

In the calculation, standard \( k-\epsilon \) model was used as a turbulence model. The boundary condition of the flow is a free slip condition at the free surface and standard wall function at the interface between melt and skull caused by the soft contact between the melt and the crucible. Figure 10(a) shows the calculated flow field and Fig. 10(b) shows the distribution of the turbulence energy dissipation. One vortex tube is observed in this system and the order of flow velocity is about 0.2 m/s, which corresponds to the aforementioned observed value in the experiment. The formation of one vortex tube not two vortices tubes notwithstanding the existence of peak value of Lorentz force along the surface of molten metal is due to the fact that the flow pattern is decided by the electromagnetic force gradient at the free meniscus and the lower vortex is suppressed by the moderately weaken penetration of Lorentz force through the skull.

Fig. 8. Change in calculated surface shape of metal with power input.

Fig. 9. Nodal system and calculated Lorentz force distribution. (a) Nodal system, (b) Lorentz force distribution.

Fig. 10. Calculated flow field and turbulent energy dissipation distribution. (a) Flow field, (b) turbulent energy dissipation.
### 3.2. Analysis of Inclusion Behavior

The particle behavior was calculated by solving the equation of motion of the inclusion by Lagrangian type method.\(^1,1^1\) The forces counted in the simulation are the drag force, the buoyancy force, the acceleration by virtual mass, the force by pressure gradient and the electromagnetic reaction force,\(^2,2^5\) which acts because the inclusion is non-conductive material.

Equation of motion for inclusions is as follows.

\[
\frac{d\mathbf{u}_p}{dt} = \frac{18\mu C_D \text{Re}}{24\rho_p d_p^2} (\mathbf{u} - \mathbf{u}_p) + \left( \frac{\rho}{\rho_p} - 1 \right) g \\
+ \frac{\rho}{2\rho_p} \frac{d}{dt} (\mathbf{u} - \mathbf{u}_p) + \frac{\rho}{\rho_p} \frac{d\mathbf{u}}{dt} - f_r \\
\]

where,

\[
\text{Re} = \rho d_p |\mathbf{u} - \mathbf{u}_p|/\mu \tag{4}
\]

where, \(\rho_p\): density of inclusions, \(\mathbf{u}_p\): velocity of inclusions, \(g\): acceleration of gravity, \(d_p\): diameter of inclusions, \(C_D\): drag coefficient for solid sphere particle, \(\text{Re}\): Local Reynolds number, \(\mu\): molecular viscosity of fluid and \(f_r\): reaction force. The effect of turbulence on the particle movement is neglected in the trajectory calculation because of the not so high Reynolds number of the system.

On the effect of electromagnetic reaction force and the agglomeration of the inclusions, they are discussed later. Figure 11 shows the example of trajectories of inclusions whose diameters are 10 \(\mu m\) and 100 \(\mu m\). By the simulation, it is shown that the larger the inclusion size, the faster the removal from the bulk occurs. The numerical simulation revealed that the maximum removal \(i.e\). residence times for the inclusions, whose diameters are 1, 10 and 100 \(\mu m\), are approximately 300, 30 and 0.3 respectively. In the case of the particle of 1 \(\mu m\) in diameter, the residence time is more than the holding time of fusion, so that the residence time is chosen as the holding time (300 s). The factors, which are to be discussed, are the effect of electromagnetic reaction force and of agglomeration. The former is the reaction of Lorentz force to the metal and is described as follows.

\[
f_r = \frac{3}{4} f \tag{5}
\]

This relation is obtained by neglecting the electrical conductivity of inclusions.\(^1,1^3\)

The latter can be treated by the Saffman–Turner’s equation described as follows\(^1,1^4\) by neglecting the effect of Stokes collision term and Brownian term, which are small compared to the turbulent collision term.

\[
N_g = 1.3/8 \cdot \alpha_g (a_i + a_j)^{1/2} \left( \frac{\varepsilon \rho_p}{\mu} \right)^{1/2} n_i n_j \tag{6}
\]

where, the symbols denote that \(a_i, a_j\): inclusion diameters, \(N_g\): collision frequency of inclusions in unit volume and time, \(n_i, n_j\): number of inclusions in a unit volume, \(\alpha_g\): agglomeration efficiency, \(\varepsilon\): turbulent energy dissipation rate, \(\mu\): viscosity of fluid, \(\rho\): density of fluid, respectively. In order to evaluate the order of the agglomeration possibility, the agglomeration efficiency \(\alpha_g\) is defined simply as the constant value of 1.

The effect of electromagnetic reaction force is summarized in Fig. 12. The reaction force becomes larger than the buoyancy force only in the vicinity of sample surface \(i.e\). within the distance of about 2 mm from surface because of the very small skin depth in this system. Therefore once the particle enters into this electromagnetic skin, the particles rapidly moved to the surface. This phenomenon is more evident for the larger particles, which can be confirmed by the Fig. 11. This force also acts for the prevention of re-entrainment of the extracted inclusions on the surface.

On the other hand, the possibility of inclusions agglomeration during their residence time in the melt is listed in Table 4 in which the symbols denote that \(N_i\): collision frequency of inclusions in the sample during the residence time of inclusion, \(n_i\): number of inclusions in the sample, respectively. This is obtained by using the size distribution of inclusions in the sample and the residence times obtained from the numerical simulation of trajectory calculation. The size distribution of inclusions is measured by microscope for small inclusions whose diameters are less than 20 \(\mu m\) and by electrolytic extraction method for large inclusions whose diameters reach to some hundred micron meter. The table shows that the inclusions whose diameters are some micron meters can be agglomerated once in their residence time, however, very scarcely for larger inclusions.
whose diameters are more than 10 μm. This last result is useful for the application of this technique to the quality assessment, because larger inclusions are more harmful for the products.

4. Conclusions

Inclusions removal behavior during fusion in a cold crucible was examined through fundamental experiments and numerical analyses. The inclusions whose diameters are more than some micron meter are rapidly removed with scarce change in composition and in size. This fact was proved to be the effect of rapid fusion and of electromagnetically driven flow in the melt, which is only enough to enhance the agglomeration of inclusions of 1 μm in diameter. From this fundamental knowledge, a new technique of cleanliness assessment of inclusions is confirmed to be effective by combining fusion in a cold crucible for levitation melting with a X-ray fluorescent evaluation method, which has more advanced characteristics such as rapid, representative and informative ones than the previous methods.

Nomenclature

\[ A, A \]: Vector potential and its component (T \cdot m)
\[ a_x, a_y \]: Inclusion diameters (m)
\[ B_x \]: Radial component of magnetic flux density (T)
\[ B_y \]: Axial component of magnetic flux density (T)
\[ C_D \]: Drag coefficient (−)
\[ d_y \]: Diameter of secondary phase (m)
\[ E_{1y}, E_{2y} \]: Elliptic integral of first and second kind (−)
\[ F, F_y \]: Lorentz force vector and its components (N/m³)
\[ F(z) \]: Function of radii of boundary curve for surface tension (−/m)
\[ f \]: Lorentz force (N/m³)
\[ f_y \]: Electromagnetic reaction force (N/m³)
\[ g \]: Acceleration of gravity (m/s²)
\[ I \]: Coil current (A)
\[ i \]: Square root of −1 (−)
\[ j, j_0 \]: Induced electric current density (A/m²)
\[ N_{1y} \]: Collision frequency of inclusions in unit volume and time (−/m³ s)
\[ N_{2y} \]: Collision frequency of inclusions in the sample during residence times (−)
\[ n \]: Unit vector perpendicular to the boundary surface (−)
\[ n_x, n_y \]: Local coordinate normal to the boundary surface (m)
\[ n_x, n_y \]: Number of inclusions in a unit volume (−/m³)
\[ n_x', n_y' \]: Number of inclusions in the sample (−)
\[ p \]: Pressure (Pa)
\[ s \]: Symbol implying boundary surface

| \[a_x, a_y (\mu m)\] | \[n_x/n_y\] | Maximum Residue Time (s) | Collision frequency during residence time \(N_{2y} (-/sample)\) | Ratio \(N_{2y}/n_y\) | Agglomeration possibility |
|----------------------|------------|------------------------|------------------|-----------------|------------------|
| 1                    | 2.1 × 10²  | 300                    | 5.2 × 10⁵         | 2.4             | 1 μ m = 2 μ m    |
| 10                   | 1.7 × 10⁶  | 30                     | 3.3 × 10⁵         | 0.2             | Rare             |
| 100                  | 1.4 × 10⁷  | 0.3                    | 2.3 × 10⁴         | 1.6 × 10⁴       | Impossibly       |

\( t \): Time (s)
\( \text{Re} \): Local Reynolds number (−)
\( r \): Radial coordinate (m)
\( r_0 \): Radial coordinate of primary induced current position (m)
\( u \): Velocity of molten metal (m/s)
\( z \): Axial coordinate (m)
\( z_0 \): Axial coordinate of primary induced current position (m)
\( \alpha_t \): Agglomeration efficiency (−)
\( \gamma \): Surface tension coefficient (N/m)
\( \delta \): Skin depth (m)
\( \varepsilon \): Turbulent energy dissipation rate (m²/s³)
\( \varepsilon_0 \): Coefficient of displacement (−)
\( \lambda \): Lagrangian constant (N/m³)
\( \mu \): Viscosity (Pa s)
\( \mu_0 \): Magnetic permeability of vacuum (H/m)
\( \phi \): Scalar potential (V)
\( \rho \): Density of fluid (kg/m³)
\( \rho_0 \): Density of inclusion (kg/m³)
\( \tau \): Displacement vector perpendicular to the boundary (m)
\( \omega \): Angular frequency (rad/s)

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Appendix

The Eq. (A1) is derived from Maxwell’s equations and Ohm’s law by introducing the scalar and vector potential
neglecting the effect of motion of the liquid metal.

\[ j = -\sigma(\nabla \phi + i\omega A) \] ........................(A1)

where, the symbols denote that \( j \): induced electric current, \( \sigma \): electrical conductivity, \( i \): square root of \(-1\), \( \omega \): angular frequency, \( A \): vector potential, respectively. In this equation, the time-dependence of the induced field generated by the imposed sinusoidal electric current is taken account of as the form of \( \exp(i\omega t) \) and the separation of variable is already performed. As the vector potential generated by the induced electric current vector is expressed as the product of the function of distance, the induced electric current density and the scalar potential. Owing to the high electrical conductivity and high frequency of imposed field, the exponential decrease of the penetration of magnetic field in the conductive materials is obtained as Eq. (A2), so that the integration normal to the surface is separately possible.

Finally the singular boundary integral equation of induced electric current is obtained. If the horizontality of the induced electric current is assumed, the electric current is taken for a circular one so that the complex vector potential is expressed as follows simply in the cylindrical coordinate system.

\[ j = j_0 \exp \left\{ -\frac{(1+i)n}{\delta} \right\} \] ........................(A2)

\[ A = \frac{\mu_0}{\pi(1+i)} \int_S j(S)f_1(r_0,z_0,r,z)dS \]

\[ + \frac{\mu_0}{\pi} \sum_{\text{Col}} I \frac{\partial f_1}{\partial z} \] .................................(A3)

\[ f_1(r_0,z_0,r,z) = \sqrt{\frac{r_0}{mr}} \left\{ 1 - \frac{m}{2(m-1)} \right\} E_2(m) - E_1(m) \] ..............................(A4)

\[ m = \frac{4rr_0}{(r+r_0)^2+(z_0-z)^2} \] ...........................(A5)

where, the symbols denote that \( j_0 \): surface value of induced current, \( n \): distance normal to the surface, \( S \): symbol indicating surfaces, \( r_0, z_0, r, z \): radial and axial coordinates of induced current, \( E_1, E_2 \): elliptic integral of first and second kind and \( I \): coil current, respectively.

In the three dimensional analysis, the expression of the vector potential becomes more complex, hence the scalar potential must be also taken account of. By the discretization of the equation, the simultaneous equations are obtained. Here, the number of the discretized parameters becomes normally very large. In such a case, the application of the Kirchhoff’s law is very useful. This law can reduce the number of the parameters. The magnetic field can be obtained by using calculated induced electric current. The following relations are in case of two-dimensional calculation. Similar relations can be also obtained in three-dimensional case.

\[ B_r = \frac{\mu_0}{\pi(1+i)} \int_S j \frac{\partial f_1}{\partial z} dS + \frac{\mu_0}{\pi} \sum_{\text{Col}} I \frac{\partial f_1}{\partial z} \] ...........................(A6)

\[ B_z = \frac{\mu_0}{\pi(1+i)} \int_S \frac{\partial f_1}{\partial r} dS + \frac{\mu_0}{\pi} \sum_{\text{Col}} I \frac{\partial f_1}{\partial r} \] ...........................(A7)

where, \( B_r, B_z \) denote radial and axial component of magnetic flux density, respectively.

The direct free boundary problem in this case is defined as to determine the profile of the free surface of the charge when the other parameters are given. Here, the profile of the surface is deformed iteratively in decreasing the total energy of the system. The principle of the deformation is introduced by the functional method and the distribution of the displacement vector \( \tau \) is shown as follows.

\[ \tau = -\varepsilon_0 \left\{ \frac{B_r^2}{2\mu_0} + \rho g z + F(z) + \lambda \right\} \mathbf{n} \] ...........................(A8)

where \( \varepsilon_0 \) is the coefficient of displacement, \( \lambda \) is the Lagrange’s constant the value of which is decided by the condition of volume conservation, \( \gamma \) is the surface tension of the melt, \( F(z) \) is the function of radii for the surface energy and \( \mathbf{n} \) indicates the unit vector normal to the free surface of the liquid metal.