Neutrino oscillations in the presence of the crust magnetization

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Abstract

It is noted that the crustal magnetic spectrum exhibits the signal from the partly correlated domain dipoles on the space-scale up to approximately 500 km. This suggests the nonzero correlation among the dynamical variables of the ferromagnetic magnetization phenomenon on the small domain scale inside the earth’s crust also. Therefore the influence of the mean of the zero component of the polarization on the CP matter-induced violation indices is discussed.

Keywords: neutrino oscillation

1. Introduction: Magnetization of the lithosphere

The nonzero magnetization of the geological structures of the crust of the earth was reported some time ago in the geophysical publications [1][2][3]. Hence the neutrino oscillation phenomenon in the presence of the magnetization of the earth’s lithosphere is expected from the partly correlated domain dipoles on the intermediate space-scale (up to approximately 500 km) of the coherently magnetized geological structures [4]. It is the signal of the (anti) parallel correlations of the spins of the ferromagnetic (mainly iron’s) domains on the far longer scale than the exchange energy is able to explain, which in turn suggests the initial condition for the analysis i.e. that they were formed as such.

2. The effective $\nu S M$ Hamiltonian and $\nu S M$ transition rate

The low-energy effective Standard Model (e$\nu S M$) potential Hamiltonian for the Dirac neutrino ($D$) charged current interaction with the electrons of the medium has the following form:

\[
(H^0_D)_{ij} = \sqrt{2} G_F N_C (\alpha_{L})_{ij} \left( \frac{\nu^e_i}{E_e} - \frac{m_e \nu^{\mu}_{\mu i}}{E_e} \right) n_{\nu j},
\]

where the first and second term originate in the vector ($V$) and axial-vector ($A V$) currents, respectively [5][6]. Indexes $i, j = 1, 2, 3$ are for three massive neutrinos $\nu_i$ and index $e$ stands for the background electrons. The quantities $N_C$, $E_e$, and $m_e$ are the electron density, energy and mass, respectively. The space component $\mathcal{H}$ of the four-vector $n^\mu = (1, \mathcal{H})$ points to the direction of the neutrino momentum. \((\alpha_{L})_{ij} = |U^L_{\nu^e i}|^2 U^L_{\nu^e j}\) is the $\nu S M$ coupling [7], where $U^L$ is the unitary neutrino mixing matrix in the charged ($C$) current interaction, and the $V - A V$ factor $\epsilon^L_{\nu^e}$ is the global $\nu S M$ coupling constant.

The mechanical four-momentum $\mu^e \equiv (\mu^0, \vec{\mu}_e)$ is defined as $\mu^e = p^e - eA^e$, where $A^e$ is the electromagnetic four-potential acting on the background electron. The electron polarization four-vector $\vec{\mu}^e$ is equal to:

\[
\vec{\mu}^e = \left[ \vec{\mu} \cdot \vec{\lambda}_e, \vec{\lambda}_e + \vec{\mu} (\vec{\lambda}_e \cdot \vec{\lambda}_e) \right] / m_e (m_e + E_e) \equiv (\vec{s}^e, \vec{z}^e),
\]

where $\vec{\lambda}_e = \chi_e^1 \sigma_1 \chi_e^2$ ($\chi_e^1 \chi_e^2 = 1$) is the electron’s polarization and $\chi_e^1$ is its two component spinor. The quantities $\langle \vec{s}^e \rangle$ and $\langle \vec{z}^e \rangle$ are the thermodynamical means. Finally, the $\nu S M$ meets both the relation \((H^0_D)_{ij} = 0\) and \((H^0_D)_{ii} = -(H^0_D)_{ji}\) in the case of the Dirac antineutrino ($\bar{D}$).

2.1. The thermodynamical means

With the isotropy assumption of the electron momentum [9], $\langle \vec{s}^e \rangle \approx 0$, we obtain $\langle \vec{z}^e \rangle \equiv (\langle \vec{z}^e \rangle, \langle \vec{s}^e \rangle) \approx (1, \vec{0})$ for the $V$ term in Eq. (1). From Eq. (2) we see that for the $A V$ term in Eq. (1) the crust unpolarized matter condition $\langle \vec{s}^e \rangle \approx 0$ leads to $\langle \vec{z}^e \rangle \approx (\vec{0}, \vec{0})$. As besides the initial condition mentioned in Section 1, the crust impact on the correlations seen in the magnetic power spectrum around the globe has mainly the ferromagnetic origin [1] hence for the temperatures in the crust [8] the magnetic moments inside each of the ferromagnetic domains are overwhelmingly parallel arranged [9]. Due to the potential $A^e$ inside one domain there appears the nonzero mean correlation between polarization $\vec{\lambda}_e$ and mechanical momentum $\vec{\mu}_e$ inside each of the ferromagnetic domains, having the same sign for every one of them. It results in the nonzero mean value of the
zero polarization component $\langle s^0 \rangle \neq 0$ along the whole experimental baseline $L$ (which stands in the fundamental opposition to the earth’s crust mean space component behavior $\langle s \rangle \approx 0$). The full solid-state analysis should follow. To test the impact of the described phenomenon on the neutrino oscillation in the crust the natural baselines of $L \leq 874$ km for the current experiments could be used. But the specially builded plants with the short but totally ferromagnetic baselines are thinkable also.

2.2. The $N_e$ magnetization form of the potential Hamiltonians and transition rate formula

The above considerations lead to the Dirac neutrino and antineutrino $\nu$SM Hamiltonians:

$$\begin{align*}
(H^{0D}_-)_i & = \sqrt{2} G_F N_i (a L i L)(1 - N_e) , \\
(H^{0\overline{T}}_+)_j & = -(H^{0D}_-)_j ,
\end{align*}$$

where

$$N_e \equiv \left( \frac{\vec{g}_e \cdot \vec{A}_e}{E_e} \right).$$

Here $N_e$ is the only nonzero term connected with the magnetization of the background electrons. The $\nu$SM effective Hamiltonians $H$ for the neutrino or antineutrino are, in the helicity - mass base $|\lambda, i\rangle$, $i = 1, 2, 3$, the $3 \times 3$ dimensional matrices:

$$H^D = M + H^{0D}_- , \quad H^{0\overline{T}} = M + H^{0D}_+ , \quad M = \text{diag}(E^0_1, E^0_2, E^0_3).$$

$M$ is the vacuum mass term, $E^0_i = E_\nu + \omega^0_i$, and $E_\nu$ is the energy for the massless neutrino [7]. In the $\nu$SM and for the homogenous medium and (in practise) relativistic neutrinos, the $\alpha$ to $\beta$ flavor oscillation probability $P_{\nu_\alpha \rightarrow \nu_\beta}(L)$ factors out in the differential transition rate formula [6]:

$$\frac{d\sigma_{\beta \mu}}{dQ_{\beta \mu}} = \int_D A^\nu_{\beta \mu} \sum_{i, \ell} U^\nu_{\beta i} U^\nu_{\mu i} U^\ell_{\beta \ell} U^\ell_{\mu \ell} e^{(i\Delta m^{eff}_\beta^2 / 2E_\ell)} .$$

$$\equiv \frac{d\sigma_{\beta \mu}}{dQ_{\beta \mu}} (m_\ell = 0) \times P_{\nu_\alpha \rightarrow \nu_\beta}(L) ,$$

where $f_D$ is the kinematical factor, $A^\nu_{\beta \mu}$ is the function of the energies and momenta of the particles in the detection process, and $\Delta m^{eff}_\beta^2$ is the neutrinos effective square mass difference in the medium calculated with Eqs. (3) - (5).

3. The numerical results. Advancing steps in the analysis

The variety of neutrino oscillation observables could be used for the purpose of the vacuum oscillation parameters estimation. As the experiments are performed on the earth hence the dependance of these observables on the crust magnetization has to be well understood. The simplest one is the oscillation probability $P_{\nu_\alpha \rightarrow \nu_\beta}(L)$ plotted on Figure 1 up to the limit baseline $L = 874$ km (taken as the approximate maximum value of the neutrino path in the earth’s crust). Then the transition rate given by Eq. (6) for the number of events in the detector follows. Yet, in the full oscillation data analyzes the number of the events is useful for the preliminary analyzes only. What matters is the functional dependance of the observable on the probability oscillation in the particular phenomena also. The steps from the observable unsensitive to the $N_e$ magnetization to the sensitive ones are as follows:

1. The observable unsensitive to the $N_e$ magnetization of the earth’s crust is $R_{\mu/e}$, the ratio-of-ratios for the muon vs. electron atmospheric neutrinos [7]. Its weak dependance on $N_e$ is connected with the general fact that the linear dependance of the transition rates on $H^{0D}$ and $H^{0\overline{T}}$ cancels out (due to opposite signs for $D$ and $\overline{D}$ in Eq. (3)). Hence $R_{\mu/e}$ would be perfect for the vacuum parameters estimation. Unfortunately the experimental errors for $R_{\mu/e}$ are bigger than 3%.

2. The observables dependent on the $N_e$ magnetization:
   (a) The up-down asymmetry $A^\nu_{\mu/e-down}$ in the atmospheric neutrinos experiments [7] seems to be unprofitable for the decision about the solitary earth’s crust $N_e$ dependance. Therefore the following paper is going to be devoted to the all-directions analyzes on $N_e$ dependance in the “through-earth” up-down asymmetry.
   (b) The CP matter-induced violation $A^{CP}_{\mu/e}$ is sensitive to $N_e$ (see Section 3.1).
   (c) The $N_e$-CP matter-induced asymmetry defined below is the observable (very) sensitive to the $N_e$ magnetization. The accelerator neutrino could be taken into account also.

![Figure 1: Panel: (a) The (noticeable) change of $P_{\nu_\mu \rightarrow \nu_e}$ with $N_e$. (b) The relative change of $\Delta P_{\mu/e \rightarrow \nu_\beta} = 100\% \left( P_{\nu_\beta \rightarrow e \mu} - P_{\nu_\beta \rightarrow e \mu} \right) / P_{\nu_\beta \rightarrow e \mu}$, with $N_e$ is even more visible.](image)
3.1. The observable sensitive to magnetization: The CP matter-induced violation

Even when the fundamental Lagrangian is CP symmetric (the mixing matrix $U^{\ell}$ phase $\delta = 0$), we could observe the matter-induced violation of the CP symmetry expressed in the (non-vanishing) difference [21]:

$$A_{\alpha \beta}^{CP} = P_{\alpha \beta}^{\nu} - P_{\alpha \beta}^{\bar{\nu}} ; \quad \alpha, \beta = e, \mu, \tau .$$

(7)

The plots on Figure 2 show, for relatively low neutrino energy ($E_\nu = 2 \text{ GeV}$) and $L = 500$ or 874 km, the noticeable change of $A_{\mu \bar{\nu}}^{CP}$ with $N_e$. Hence with $A_{\mu \bar{\nu}}^{CP}$ we have finally obtained the observable sensitive to the earth’s crust $N_e$ magnetization, although, because of the experiment sensitivity to $\Delta m^2$, with the increase of $E_\nu$ the $N_e$ dependence is decreasing.

![Figure 2: The change of $A_{\mu \bar{\nu}}^{CP}$ with $N_e$ for $\beta = e, \mu, \tau$ (from top to bottom) and for the neutrino energy $E_\nu = 2 \text{ GeV}$ and $L = 500$ or 874 km.](image)

3.2. The $N_e$-CP matter-induced asymmetry. Even the accelerator neutrinos matter

The $N_e$-CP matter-induced asymmetry could be defined as follows:

$$A_{\alpha \beta}^{N_e-CP} = \sigma_{\nu_\alpha \rightarrow \nu_\beta} - \sigma_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta} = \sigma_{\nu_\alpha N \rightarrow \nu_\beta} P_{\nu_\alpha \rightarrow \nu_\beta} - \sigma_{\nu_\alpha N \rightarrow \bar{\nu}_\beta} P_{\nu_\alpha \rightarrow \bar{\nu}_\beta} ,$$

(8)

where the initial flavors are $\alpha = e, \mu$ or $\tau$ and index $\beta = e, \mu$ or $\tau$ is for the events in the detector. The second equality in Eq.(8) is valid for the sSM relativistic case (see Eq.(5)), where the index $N$ in the total cross section $\sigma_{\nu_\alpha} \nu_e$ and $\sigma_{\nu_\alpha} N$ signifies the nucleon. Let us notice that even if $A_{\alpha \beta}^{CP} = 0$ we have $A_{\alpha \beta}^{N_e-CP} \neq 0$. We could define also the ratio:

$$RA_{\alpha \beta}^{N_e-CP} = 100% \frac{A_{\alpha \beta}^{N_e-CP} (N_e)}{A_{\alpha \beta}^{N_e-CP} (N_e = 0)} .$$

(9)

Let us make some predictions e.g. for the K2K experiment. Here for $L = 250$ km and $E_\nu = 1.3 \text{ GeV}$ the neutrino beam was the almost pure mu neutrino one. Hence to calculate $A_{\mu \bar{\nu}}^{N_e-CP}$ the experiment should be rebuild for the possibility of the production of the $\bar{\nu}_\mu$ beam also. Then with $N_e = 0.23$ we obtain $RA_{\mu \bar{\nu}}^{N_e-CP} \approx -20\%$ which validates the conclusion that with $A_{\mu \bar{\nu}}^{N_e-CP}$ the correction of the experimental values of $\sigma_{\nu_\alpha \nu_\beta}$ for the neutrino-nucleon inelastic scattering would be possible also.

4. The conclusions

The impact of the zero component of the mean polarization $\langle \vec{P}_e \rangle$ from ferromagnetic domains of the earth’s crust on the basic neutrino oscillation observables has been discussed. Two of them, i.e. the CP matter-induced violations, $A_{\mu \bar{\nu}}^{CP}$ and $A_{\mu \bar{\nu}}^{N_e-CP}$, exhibit the greatest influence of the $N_e \equiv \langle \vec{P}_e \cdot \vec{\lambda}_e / E_\nu \rangle$ background media magnetization. It follows that under the precise knowledge of the $N_e$ magnetization along the baseline $L$ (natural or artificially designed) and for relatively low neutrino energies (2 - 10 GeV or even for the reactor’s ones) we could improve both the estimation of the vacuum neutrino oscillation parameters [22] and the neutrino cross sections also. Finally, as for the purpose of the geomagnetic analysis of the huge Minto block [3] in Canada [10] the samples from the rocks were taken every 10 km, hence the direct measurements of the crust magnetization for one e.g. 295 km long neutrino T2K baseline should not be the too difficult task as well.

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1For information on $\sigma_{\nu_\alpha \nu_\beta}$ and $\sigma_{\nu_\alpha \bar{\nu}_\beta}$ see: J.A. Formaggio, G.P. Zeller, From eV to EeV: Neutrino cross sections across energy scales, Rev.Mod.Phys. 84 (2012) 1307-1341, arXiv:1305.7513 [hep-ex].

2Or that to improve the estimation of the vacuum neutrino oscillation parameters, the precise knowledge of $N_e$ along the baseline $L$ is necessary.

3 Its northern part is located approximately 2400 km from Fermilab.
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