We evaluate the four-closed-string scattering amplitude by using the string path integral in the proper-time gauge. Identifying the Fock space representation of the four-closed-string-vertex, we obtain a field theoretic expression of the closed string scattering amplitudes. In the zero-slope limit, the four-closed-string scattering amplitude is shown to reduce to the four-graviton-scattering amplitude of the Einstein gravity. Given that the four-closed-string interaction is generated perturbatively by the three-closed-string interaction, the quantum theory of gravity may be described consistently as a low energy limit of the closed string field theory which contains a cubic closed string interaction only.

PACS numbers: 04.60.-m, 11.25.Db, 11.25.-w
Keywords: quantum gravity, graviton, scattering amplitude, string path integral

I. INTRODUCTION

Construction of a finite consistent quantum theory of gravity [1] [2] is one of the most outstanding problems in theoretical physics which turned theorists towards the string theory. Because the spin-two massless particle, corresponding to graviton is included in the spectrum of a free closed string, closed string field theory may be the right candidate for a unifying framework for the finite quantum theory of gravity. However, despite decades of efforts, construction of a consistent closed string field theory has never been accomplished.

In a recent work [3], we evaluated the three-closed string scattering amplitude by choosing the proper-time gauge [4] [5] for the string path integral [6] which depicts the three-closed string scattering. In the proper-time gauge, the string path integral can be written as integrals over the proper-times in a way similar to the Schwinger’s proper time representation of Feynman integrals of quantum field theory. For this reason, it becomes feasible in the proper-time gauge to identify the field theoretical expressions of the string path integrals which depict multiple string scatterings. If we evaluate the string path integral on a cylindrical surface by choosing the proper-time gauge, we may get the free field action of closed string [4]. The Fock space representation of three-closed string vertex may be obtained by recasting the string path integral on the string worldsheet called the pants diagram into the corresponding field theoretical expression [3]. The three-closed string scattering amplitude is found to be factorized entirely into those of the three-open string scattering amplitudes. It implies that the KLT [8] relations of the first quantized string theory may be extended to the second quantized string theory.

In this Letter, we study the four-closed string scattering amplitude by extending the previous work on the three-closed string scattering amplitude. The Green’s function on the worldsheet is explicitly constructed by mapping the worldsheet onto the complex plane and the Fock space representation of the four-closed-string vertex is obtained by rewriting the string path integral in the proper-time gauge in terms of the oscillatory basis. If a four-graviton state is chosen as the external four-closed string state, the Fock space representation of the four-closed-string vertex yields the four-graviton scattering amplitude. In the zero-slope limit, the resultant four-graviton scattering amplitude is found to be in perfect agreement with that of the Einstein gravity [9] [11].

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II. STRING PATH INTEGRAL FOR FOUR-CLOSED STRING SCATTERING

The four-closed string scattering amplitude is given by the string path integral of Polyakov on the string worldsheet with four external closed strings \[12\]

\[
\mathcal{W}_4 = g^2 \int D[X] D[h] \exp \left( i S + i \int_{\partial M} \sum_{i=1}^4 P^{(c)} \cdot X^{(c)} \, d\sigma \right),
\]

\[
S = -\frac{1}{4\pi} \int_M d\tau d\sigma \sqrt{-h} \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta} \eta_{\mu\nu}, \quad \mu, \nu = 0, \ldots, d - 1
\]

where \(\sigma^1 = \tau, \sigma^2 = \sigma\). Here \(d = 26\) for the bosonic string and \(d = 10\) for the open super-string. In terms of normal modes the string coordinate variable, \(X(\tau, \sigma) = X_L(\tau + \sigma) + X_R(\tau - \sigma)\) may be expanded as

\[
X_L(\tau, \sigma) = x_L + \sqrt{\frac{1}{2}} p_L(\tau + \sigma) + i \sqrt{\frac{1}{2}} \sum_{n \neq 0} n \omega_n e^{-in(\tau + \sigma)},
\]

\[
X_R(\tau, \sigma) = x_R + \sqrt{\frac{1}{2}} p_R(\tau - \sigma) + i \sqrt{\frac{1}{2}} \sum_{n \neq 0} n \tilde{\omega}_n e^{-in(\tau - \sigma)}.
\]

Fig.1 depicts the worldsheet of four-closed string scattering on which we introduce local coordinate patches. The two-dimensional worldsheet metric may be written in terms of the lapse and shift functions as follows

\[
\sqrt{-h} (h^\alpha\beta) = \frac{1}{N_1} \left( \frac{-1}{N_2^2} (N_1)^2 - (N_2)^2 \right).
\]

In the proper-time gauge the worldsheet metric on a local patch is fixed by two constants \(n_1\) and \(n_2\) which correspond to zero modes of the lapse and shift functions respectively. The proper time \(s\) on a local patch is given as \(s = n_1 \Delta \tau\) where \(\Delta \tau\) is the interval of the local patch along \(\tau\) direction.

![Fig. 1: Local coordinate patches and the Schwarz-Christoffel mapping from the string worldsheet onto the complex plane](image)

Making use of the reparametrization invariance of the Polyakov string path integral, we may fix the length parameters as \(\alpha_1 = \alpha_2 = 1\) and \(\alpha_3 = \alpha_4 = -1\). In order to evaluate the string path integral we map the worldsheet of four-closed string scattering onto the complex plane. If we choose the Koba-Nielsen variables as \(Z_1 = 0, Z_2 = Z, Z_3 = 1, Z_4 = \infty\), the Schwarz-Christoffel (SC) transformation from the worldsheet onto the complex plane is given by

\[
\rho = \ln z + \ln(z - Z) - \ln(z - 1) + i\pi.
\]

Two interaction points where two closed strings join together to form one closed string or one closed string splits into two closed strings are mapped to two points \((\tau_i, \sigma_i), i = 1, 2\) on the complex plane which are determined by \(\frac{\partial \rho}{\partial \tau} = 0\):

\[
\tau_1 = 2 \text{Re} \ln \left( 1 - \sqrt{1 - Z} \right), \quad \sigma_1 = 2 \text{Im} \ln \left( 1 - \sqrt{1 - Z} \right) + \pi,
\]

\[
\tau_2 = 2 \text{Re} \ln \left( 1 + \sqrt{1 - Z} \right), \quad \sigma_2 = 2 \text{Im} \ln \left( 1 + \sqrt{1 - Z} \right) + \pi.
\]
Then, it follows that the SC mapping from the individual local coordinate patch with local coordinate, \( \zeta_r = \xi_r + i \eta_r \), \( r = 1, 2, 3, 4 \) onto the complex plane may be written explicitly as

\[
\begin{align*}
\exp(-\zeta_1) &= \exp(-\tau_1 + i \sigma_3) \frac{(z - 1)}{z(z - Z)}, \\
\exp(-\zeta_2) &= -\exp(-\tau_1 + i \sigma_3) \frac{(z - 1)}{z(z - Z)}, \tag{6a} \\
\exp(-\zeta_3) &= \exp(-\tau_2 - i \sigma_2) \frac{z(z - Z)}{z - 1}, \\
\exp(-\zeta_4) &= -\exp(-\tau_2 - i \sigma_2) \frac{z(z - Z)}{z - 1}. \tag{6b}
\end{align*}
\]

### III. FOUR-CLOSED STRING SCATTERING AMPLITUDE

Integrating out \( X^\mu \) in Eq.(1a), we may write the four-closed string scattering amplitude may be written in terms of the momentum variables of four external strings, \( P(r), r = 1, 2, 3, 4 \) as follows

\[
\mathcal{W}_4 = g^2 \langle P | \exp \left\{ \sum_{r=1}^{4} \frac{\xi_r}{2} \left( (p_L^r)^2 + (p_R^r)^2 \right) + \frac{1}{4} \sum_{r,s,n,m} \tilde{C}^{rs}_{nm} e^{in|\xi_r + m|\xi_s} P_n(r) \cdot P_m(s) \right\} | 0 \rangle
\]

where \( |P\rangle \) is the momentum eigenstate and \( |0\rangle \) is the vacuum state in the oscillatory basis. Here \( \tilde{C}^{rs}_{nm} \) is the Fourier component of the Green’s function on the string worldsheet, analogous to the Neumann function for open string

\[
\ln |z_r - z'_s| = -\delta_{rs} \left\{ \sum_{r=1}^{4} \frac{e^{-n\Delta}}{2n} \left( e^{i\xi_r^2 - \eta_r^2} + e^{-i\xi_r^2 - \eta_r^2} \right) - \max(\xi, \xi') \right\} + \sum_{n,m} \tilde{C}^{rs}_{nm} e^{in|\xi_r + m|\xi_s} e^{im\eta_r} e^{im\eta_s}
\]

where \( \Delta = |\xi_r - \xi'_s| \). The four-closed string vertex \( |V_4\rangle \) may be identified by

\[
\mathcal{W}_4 = \langle P | \exp \left( \sum_{r=1}^{4} \xi_r L_0^{(r)} \right) | V_4 \rangle.
\]

The Fock space representation of the four-closed string vertex is given by

\[
| V_4 \rangle = e^{-\sum_{r=1}^{4} \frac{\xi_r}{2}} e^{\sum_{r=1}^{4} \int_{Z_r}^{Z_s} \left\{ \tilde{C}^{rs}_{00} \alpha_n^{(r)} \alpha_m^{(s)} + \tilde{C}^{rs}_{n-m} \alpha_n^{(r)} \alpha_m^{(s)} + \tilde{C}^{rs}_{n0} \alpha_n^{(r)} + \tilde{C}^{rs}_{n0} \alpha_n^{(s)} \right\}} | 0 \rangle.
\]

Explicit forms of the Neumann functions for closed string, \( \tilde{C}^{rs}_{nm} \), are given by

\[
\begin{align*}
\tilde{C}^{rr}_{00} &= \ln |Z_r - Z_s|, \quad r \neq s, \tag{11a} \\
\tilde{C}^{rr}_{00} &= -\sum_{i \neq r} \frac{\alpha_i}{\alpha_r} \ln |Z_r - Z_i| + \frac{1}{\alpha_r} r_0^{(r)}, \tag{11b} \\
\tilde{C}^{rs}_{n0} &= \tilde{C}^{rs}_{-n0} = \frac{1}{2n} \int_{Z_r}^{Z_s} \frac{dz}{2\pi i} \frac{1}{z - Z_{1/2}} e^{-n\zeta_r(z)}, \quad n \geq 1, \tag{11c} \\
\tilde{C}^{rs}_{nm} &= \tilde{C}^{rs}_{n-m} = \frac{1}{2nm} \int_{Z_r}^{Z_s} \frac{dz}{2\pi i} \int_{Z_r}^{Z_s} \frac{dz'}{2\pi i} \frac{1}{(z - z')^2} e^{-n\zeta_r(z) - m\zeta_s(z')}, \quad n, m \geq 1, \tag{11d} \\
\tilde{C}^{rs}_{n+m} &= \tilde{C}^{rs}_{-n-m} = 0, \quad n, m \geq 1. \tag{11e}
\end{align*}
\]

Introducing the \( SL(2, C) \) invariant measure for the Koba-Nielsen variables, we may write the four-closed scattering amplitude as follows:

\[
\mathcal{A}_4 = g^2 \int \prod_{r=1}^{4} \frac{dZ_r^2 |Z_a-Z_b| |Z_b-Z_c| |Z_c-Z_a|^2}{d^2Z_a d^2Z_b d^2Z_c} \langle \Psi_4 | V_4 \rangle
\]

where \( |\Psi_4\rangle \) denotes the external four-closed string state.
IV. FOUR-GRAVITON SCATTERING AMPLITUDE

The massless closed string state, $h_{\mu\nu} \alpha_{-1}^{(\mu} \tilde{\alpha}_{-1}^{\nu)} |0\rangle$ may be decomposed into graviton, anti-symmetric tensor, and scalar states. Choosing the symmetric traceless part of $h_{\mu\nu}$, we may write the external four-graviton state as

$$|\Psi_{[4]}\rangle = \prod_{r=1}^{4} \left\{ h_{\mu\nu} (p^{(r)}) \alpha_{-1}^{(\mu} \tilde{\alpha}_{-1}^{\nu)} \right\} |0\rangle. \quad (13)$$

The graviton field $h_{\mu\nu} (p^{(r)})$ is subject to the covariant gauge condition: $h_{\mu\nu} (p^{(r)}) p_{\mu}^{(r)} = h_{\mu\nu} (p^{(r)}) p_{\nu}^{(r)} = 0$. The four-graviton scattering amplitude follows from Eq. (12) and Eq. (13):

$$\mathcal{A}_{[4G]} = \frac{g^2 c_{[4]}}{4} \int \prod_{r<s} |Z_r - Z_s|^2 \frac{d^2 Z}{Z^*} e^{-2 \sum_{i=1}^{4} \tilde{c}_i^{(r)s}}$$

$$\langle 0 | \left\{ \prod_{i=1}^{4} h_{\mu\nu} (p^{(i)}) a_1^{(i)\mu} a_1^{(i)\nu} \right\} \left\{ \left( \sum_{r,s} C_{11}^{rs} a_1^{(r)\dagger} \cdot a_1^{(s)} \right)^{2} + \left( \sum_{r,s} \tilde{C}_{11}^{rs} a_1^{(r)\dagger} \cdot a_1^{(s)} \right) \left( \sum_{r,s} \tilde{C}_{10}^{rs} a_1^{(r)\dagger} \cdot p^{(s)} \right)^{2} \right\} |0\rangle \quad (14)$$

where $c_{[4]}$ is a normalization constant to be fixed later.

Some algebra brings us to the following expression for the four-graviton scattering amplitude:

$$\mathcal{A}_{[4G]} = g^2 c_{[4]} \int d^2 Z |Z|^{-\frac{1}{2}} [1 - Z]^{-\frac{1}{4}} h_{\mu_1 \nu_1} h_{\mu_2 \nu_2} h_{\mu_3 \nu_3} h_{\mu_4 \nu_4}$$

$$\left\{ \frac{1}{Z^* + (1 - Z^*)} \left( \sum_{r,s} C_{11}^{rs} a_1^{(r)\dagger} \cdot a_1^{(s)} \right)^{2} + \left( \sum_{r,s} \tilde{C}_{11}^{rs} a_1^{(r)\dagger} \cdot a_1^{(s)} \right) \left( \sum_{r,s} \tilde{C}_{10}^{rs} a_1^{(r)\dagger} \cdot p^{(s)} \right)^{2} \right\}$$

$$\left\{ Z^* \Rightarrow Z, \mu \Rightarrow \nu \right\} \quad (15)$$

where $s, t, u$ denote the Mandelstam variables defined as

$$s = - \left( p^{(1)} + p^{(2)} \right)^2, \quad t = - \left( p^{(2)} + p^{(3)} \right)^2, \quad u = - \left( p^{(1)} + p^{(3)} \right)^2. \quad (16)$$

For four-graviton scattering, $s + t + u = 0$. Because the integrand, if expanded, may contain as many as $27^2 = 729$ terms, it may be a laborious task to calculate the four-graviton scattering amplitude explicitly. Fortunately, thanks to the useful formula found by Kawai, Lewellen, and Tye [5], the integrand does not need to be expanded. We note that terms contained in the integrand take the following form

$$I = \int d^2 Z \prod_{r<s} |Z_r - Z_s|^2 \frac{p^{(r)} \cdot p^{(s)}}{Z^*} (Z^*)^{-n} (1 - Z^*)^{-p} Z^{-m} (1 - Z)^{-q} \quad (17)$$
where \( n, p, m, q \) are integers. \( I \) can be factorized into two integrals with two independent real variables, \( \eta \) and \( \xi \):

\[
I = \sin \left( \frac{\pi t}{8} \right) I_1(n, p) I_2(m, q), \tag{18a}
\]

\[
I_1(n, p) = \int_1^{\infty} d\eta |\eta|^{-\frac{3}{2}} |1 - \eta|^{-\frac{3}{2}} \eta^{-n} (1 - \eta)^{-p} = (-1)^p \frac{\Gamma \left( -\frac{3}{2} + n + p - 1 \right)}{\Gamma \left( \frac{5}{2} + n \right)}, \tag{18b}
\]

\[
I_2(m, q) = \int_0^1 d\xi |\xi|^{-\frac{3}{2}} |1 - \xi|^{-\frac{3}{2}} \xi^{-m} (1 - \xi)^{-q} = \frac{\Gamma \left( -\frac{3}{2} - m + 1 \right) \Gamma \left( -\frac{3}{2} - q + 1 \right)}{\Gamma \left( \frac{5}{2} - m - q + 2 \right)}. \tag{18c}
\]

Making use of Eq. (17) and Eqs. (18a, 18b, 18c), we find

\[
\mathcal{A}_{[4G]} = g^2 c_{[4]} h_{\mu_1 \nu_1} h_{\mu_2 \nu_2} h_{\mu_3 \nu_3} h_{\mu_4 \nu_4} \sin \left( \frac{\pi t}{8} \right) \left\{ I_1(2, 0) \eta^{\mu_1 \mu_2} p^{\mu_3 \mu_4} + I_1(0, 0) \eta^{\mu_1 \mu_2} p^{\mu_3 \mu_4} + I_1(0, 2) \eta^{\mu_1 \mu_2} p^{\mu_3 \mu_4} - \frac{1}{4} \eta^{\mu_1 \mu_2} \left( I_1(0, 1) p^{(1) \mu_3} p^{(2) \mu_4} + I_1(1, 1) p^{(1) \mu_3} p^{(2) \mu_4} + I_1(1, 1) p^{(4) \mu_3} p^{(3) \mu_4} + I_1(2, 1) p^{(4) \mu_3} p^{(3) \mu_4} \right) + \frac{1}{4} \eta^{\mu_1 \mu_2} \left( I_1(0, 1) p^{(1) \mu_3} p^{(2) \mu_4} + I_1(1, 1) p^{(1) \mu_3} p^{(2) \mu_4} + I_1(-1, 1) p^{(4) \mu_3} p^{(3) \mu_4} + I_1(0, 1) p^{(4) \mu_3} p^{(3) \mu_4} \right) - \frac{1}{4} \eta^{\mu_1 \mu_2} \left( I_1(1, 0) p^{(4) \mu_3} p^{(1) \mu_4} + I_1(1, 1) p^{(4) \mu_3} p^{(1) \mu_4} + I_1(1, 1) p^{(4) \mu_3} p^{(1) \mu_4} + I_1(1, 1) p^{(4) \mu_3} p^{(1) \mu_4} \right) - \frac{1}{4} \eta^{\mu_1 \mu_2} \left( I_1(-1, 1) p^{(1) \mu_3} p^{(4) \mu_3} + I_1(0, 1) p^{(1) \mu_3} p^{(4) \mu_3} + I_1(0, 1) p^{(2) \mu_3} p^{(4) \mu_3} + I_1(1, 1) p^{(2) \mu_3} p^{(4) \mu_3} \right) - \frac{1}{4} \eta^{\mu_1 \mu_2} \left( I_1(0, 1) p^{(3) \mu_1} p^{(4) \mu_2} + I_1(1, 1) p^{(3) \mu_1} p^{(4) \mu_2} + I_1(1, 1) p^{(3) \mu_1} p^{(4) \mu_2} + I_1(1, 1) p^{(3) \mu_1} p^{(4) \mu_2} \right) \right\} \rightarrow I_2(n, p, \mu \Rightarrow \nu) \right\}, \tag{19}
\]

To compare the obtained graviton scattering amplitude Eq. (19) with that of the Einstein gravity, we need to take the zero-slope limit. Using the momentum conservation \( \sum p^{(\mu)} = 0 \), the covariant gauge condition, and the on-shell condition, we find the four-graviton scattering amplitude in the zero-slope limit:

\[
\mathcal{A}_{[4G]} = 4 \kappa^2 h_{\mu_1 \nu_1} h_{\mu_2 \nu_2} h_{\mu_3 \nu_3} h_{\mu_4 \nu_4} \frac{1}{8 t s u} K_{\mu_1 \mu_2 \nu_3 \nu_4} K_{\nu_1 \nu_2 \mu_3 \mu_4}, \tag{20a}
\]

\[
K_{\mu_1 \mu_2 \nu_3 \nu_4} = -\frac{tu}{4} \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} - \frac{st}{4} \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} - \frac{us}{4} \eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} + \frac{\eta^{\mu_1 \mu_2}}{2} \left( p^{(1) \mu_3} p^{(2) \mu_4} + p^{(2) \mu_3} p^{(1) \mu_4} \right) + \frac{\eta^{\mu_1 \mu_3}}{2} \left( p^{(1) \mu_2} p^{(3) \mu_4} + p^{(3) \mu_2} p^{(1) \mu_4} \right) + \frac{\eta^{\mu_1 \mu_4}}{2} \left( p^{(2) \mu_1} p^{(3) \mu_3} + p^{(3) \mu_1} p^{(2) \mu_3} \right) + \frac{\eta^{\mu_2 \mu_3}}{2} \left( p^{(2) \mu_1} p^{(4) \mu_3} + p^{(4) \mu_1} p^{(2) \mu_3} \right) + \frac{\eta^{\mu_2 \mu_4}}{2} \left( p^{(2) \mu_1} p^{(4) \mu_2} + p^{(4) \mu_1} p^{(2) \mu_2} \right) + \frac{\eta^{\mu_3 \mu_4}}{2} \left( p^{(3) \mu_1} p^{(4) \mu_2} + p^{(4) \mu_1} p^{(3) \mu_2} \right). \tag{20b}
\]
where $\kappa = g \sqrt{\frac{c_4}{2} \pi / 2} = \sqrt{32 \pi G_{10}}$ and $K^{\mu_1 \mu_2 \mu_3 \mu_4}$ is the kinematic factor for four-gauge particle scattering in open string theory, which is totally symmetric in the four external particle lines [10]. The four-graviton scattering amplitude which results from the field theoretical expression of the four-string scattering is found to be in perfect agreement with that of the Einstein gravity in the zero-slope limit. It is noteworthy that the four-graviton scattering amplitude $A_{4G}$ given by Eq. (19) is valid for the full range of energy scale, thus it may be useful to study the four-graviton scattering amplitude in the high energy region.

V. CONCLUSIONS

We have shown that the four-graviton scattering amplitude of the Einstein gravity arises as a zero-slope limit of the four-closed string scattering amplitude by explicitly evaluating the corresponding string path integral in the covariant proper-time gauge. In the proper-time gauge, the Polyakov string path integral can be put into the form similar to the Schwinger’s proper-time representation of Feynman integral of quantum field theory so that we may identify the second quantized expressions of string theory. This paper is an extension of our recent works [5, 13]: We studied the string path integral for open string on $Dp$-branes and found that the scattering amplitudes of open string correctly reduce to those of the non-Abelian Yang-Mills gauge in the zero-slope limit. Along the same lines, we recast the string path integral which describes the three-closed string scattering, into the second quantized form and obtained the Fock space representation of the three closed string vertex [3]. The Neumann functions for closed string are found to be the same as those of open string in the case of three-closed-string scattering and consequently three-closed-string scattering amplitudes are factorized into those of three-open-string amplitudes. It may imply that the KLT relations in the first quantized theory may be fully extended to the second quantized string theory.

In this work we calculate the four-closed string scattering amplitude by using the string path integral in the proper-time gauge. According to the general studies of the spin two particle called graviton [14–19], all consistent theories of graviton must reduce to the Einstein gravity in the low energy limit. This general argument certainly applies to closed string theory, containing the massless spin-two particle state. In work [3] prequel to this, we obtained the Fock space representation of the three-closed-string vertex, with which a closed string field theory with the cubic interaction may be constructed. It was shown that the three-closed-string amplitude defined by the string path integral yields the three-graviton scattering amplitude of the Einstein gravity exactly. Here we further extend it to show that the four-graviton scattering amplitude is deduced in the zero-slope from the four-closed string scattering amplitude defined by the string path integral. Because the four-closed-string interaction may be generated perturbatively by the three-closed-string interaction as in the case of open string theory, the Einstein gravity may be consistently described as a low energy limit of the closed string field theory with the cubic interaction. This work may also be useful to study the Bern-Carrasco-Johansson duality [20] which relates the scattering amplitudes of non-Abelian gauge particles with those of gravitons within the framework of string theory.

Acknowledgments

This work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2017R1D1A1A02017805). The author thanks J. C. Lee and Y. Yang at NCTU (Taiwan) for useful discussions.

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