ABSTRACT

Cryptocurrencies typically aim at preserving the privacy of their users. Different cryptocurrencies preserve privacy at various levels, some of them requiring users to rely on strategies to raise the privacy level to their needs. Among those strategies, we focus on two of them: merge avoidance and mixing services. Such strategies may be adopted on top of virtually any blockchain-based cryptocurrency. In this paper, we show that whereas optimal merge avoidance leads to an NP-hard optimization problem, incentive-compatible mixing services are subject to a certain class of impossibility results. Together, our results contribute to the body of work on fundamental limits of privacy mechanisms in blockchain-based cryptocurrencies.

1. INTRODUCTION

Privacy is one of the desired properties by most users issuing online transactions. Privacy concerns have motivated the development of novel cryptocurrencies, and rankings of cryptocurrencies with respect to privacy. Monero and Zcash are highly ranked in that respect, whereas Bitcoin and Ethereum are known to suffer from weaknesses [1,12].

Although the history of transactions in blockchain systems is publicly available to anyone through the blockchain itself, the owners of the addresses involved in the transactions in the blockchain are in principle unknown. However, a number of techniques have been devised and employed to de-anonymize addresses. They range from basic heuristics, such as clustering the input addresses of transactions [11], to some more advanced techniques such as applying machine learning to infer the owners of addresses [3]. Merge avoidance and mixing services are two of the solutions to mitigate the effectiveness of such de-anonymization techniques [5,10,14].

Users in Bitcoin store their coins in wallets, and exchange funds through transactions, with each transaction comprising its inputs (funds sources) and outputs (funds recipients). A privacy issue exists in exploring the regularity of certain types of transactions to infer personal information.

If Alice, an employee of a coffee shop, is paid her salary in Bitcoin, one can infer Alice’s salary by noticing the existence of weekly or monthly transactions with the coffee shop’s wallet as input and Alice’s wallet as output. To avoid such an issue, one strategy consists in Alice creating multiple wallets to receive her salary.

Prior art. There has been a recent surge in interest on...
the privacy aspects of cryptocurrencies \cite{12,13}. Merge avoid-
ance was first proposed as an alternative to services such as 
CoinJoin \cite{6}. Among the tools that leverage merges to break
anonymity, Bitiodine is a notable example \cite{14}. Mixing ser-
vice has its roots in networks such as the Tor project \cite{2},
which aims to anonymize users in the Internet. The liter-
ature on mixing services has been rapidly growing \cite{8,15},
with measurements and results that are complementary to
those reported in this work.

Contributions. Our contributions are twofold.

Optimal merge avoidance is NP-hard: we show
that optimally allocating resources for merge avoidance
is NP-hard. The result involves a reduction from the partition
problem, and motivates heuristics for merge avoidance.

An impossibility result on incentive-compatible mix-
ing services: we prove that no incentive-compatible strat-
getic for resource allocation in mixing services can at the
same time account for strategic relays and strategic targets.
Resource allocation for incentive-compatible mixing services
must relax assumptions, e.g., pertaining to constant amount
of currency in the system, to be feasible.

2. MERGE AVOIDANCE

Merge avoidance in blockchain systems was first proposed
by Mike Hearn \cite{5}, one of the Bitcoin core developers,
by the end of 2013. It basically consists of splitting trans-
actions among multiple addresses, so as to avoid the easy
identification that two addresses are owned by the same
user. Our goal is to measure the impact and effectiveness
of merge avoidance, noting that the split of transactions in-
volves costs.

We model the Bitcoin transaction network as a graph com-
prising nodes of two types:

Value node: each transaction has a list of input value
nodes and generates a list of output value nodes. Each input
and output must correspond to an address that holds some
amount of currency (e.g., Bitcoins or BTC).

Transaction node: this node characterizes the action of
transferring value from input to output nodes. Each transac-
tion corresponds to a transaction identifier in the blockchain.

Directed edges are created between a transaction node and
each value node involved in this transaction, with directions
from input value nodes to the transaction node and from
the transaction node to the output nodes; see Figure 1(a).

The BTC graph spawns from its genesis block. Every new
block generates at least one new value node, corresponding
to the coinbase transaction, with a given reward. Value
nodes are set as input to transaction nodes, which output
one or more new value nodes. It is well known that roughly
80\% of the transactions in the BTC network output 2 value
nodes, one corresponding to the destination and the other
respective to a change to the transaction issuer.

One of the most natural strategies to cluster addresses is
to merge the input addresses for a transaction and assume
that they all belong to the same owner. That is because
it is likely that the owner of some set of addresses will use
the accumulated values together to pay for some goods or
service, and output the remainder to one change address.
Now, suppose that one learns the owner of one of those
addresses, e.g., through side channels, like online forums or
social networking, by social engineering, or by leveraging the

fact that the address belongs to a famous retailer, who issues
transactions at known values. Then, one can easily learn
the owner of additional addresses belonging to that user,
following the activity of his addresses and, subsequently, also
de-anonymize other elements in the network.

The goal of merge avoidance \cite{5} is to mitigate this weak-
ness by avoiding the use of multiple input addresses into
a transaction node, effectively modifying the transaction
graph. To that aim, if one intends to merge \( N \) addresses to
pay a single destination address, one alternative is to create
\( N \) separate transactions and output them to novel destina-
tion addresses belonging to the same user. The main cost
involved in this scheme relates to the extra fees incurred in
the issuing of such transactions.

Hardness of Optimal Merge Avoidance

We begin by considering a single transaction, illustrated in
Figure 1. The circles in Figure 1 represent value nodes and
the boxes represent transaction nodes, and the edge values
are the amount of BTC transferred. Figure 1(a) shows the
original graph, before transaction splitting for merge avoid-
ance, and Figure 1(b) shows one possible outcome of merge
avoidance — the owner of the addresses \( D1 \) and \( D2 \) is the
same user. To simplify presentation, we are not considering
any fees in this simplified view of the problem. Then, the
sum of the values from the input equals the sum of values
in the output and we work under such assumption.

A user builds a transaction, and before submitting it to
the blockchain the transaction goes through merge avoid-
ance. The output of merge avoidance is a new set of trans-
actions, whose outputs fulfill the originally requested out-
put values. The user then submits those transactions to the
miners, to add them into the blockchain.

Definition 2.1 (Multi-target merge avoidance). An original transaction has \( \ell \) input and \( r \) output value
nodes, with integer values \( s_i \) and \( t_j \), \( 1 \leq i \leq \ell \) and \( 1 \leq j \leq r \),
respectively. A set of modified values comprises integer ele-
ments \( m_{i,j} \) such that

\[
s_i = \sum_{j=1}^{r} m_{i,j}, \quad t_j = \sum_{i=1}^{\ell} m_{i,j}
\]

While solving the merge avoidance problem, the number of
transactions in the modified graph corresponds to the number of
strictly positive values in the set of modified values. The aim
is to minimize such number, subject to (1). Note that \( m_{i,j} \)
is the amount of coins routed from \( s_i \) to \( t_j \) under merge avoidance. Note also that we make no as-
sumptions about the semantics of the output values, e.g.,
regarding what is an effective transfer and what is a change.

The complexity of finding the minimum set of merge avoid-
ance transactions is established by the following result.

Theorem 2.1. Multi-target merge avoidance is NP-Hard.

Let \( \ell \) and \( r \) be the number of value nodes in the left and
right side of the transaction. A lower bound on the number
of transactions in the modified graph is \( \ell \). An upper bound
is given by \( \ell \cdot r \), which occurs when we need to transfer value
from each of the nodes in the left to each of the nodes in
right. In that case, the original graph is transformed into
a complete bipartite graph with \( \ell \cdot r \) edges, which in turn
yields a modified graph with \( \ell \cdot r \) transactions.
Given a set of ℓ value nodes with positive integer values \( s_i \), and one target output value \( v \), we want to determine the set \( K \), contained in the power set of \( \{1, \ldots, \ell\} \), with smallest cardinality, that meets the output, \( \min_{K \subseteq 2^{\{1,\ldots,\ell\}}} |K| \text{ such that } \sum_{i \in K} s_i \geq v. \) \( (2) \)

The smallest set corresponds to the optimal way of creating a merge avoidance set of transactions, minimizing the number of transactions while fulfilling the requested output value.

Under the above problem formulation, merge avoidance can be solved in polynomial time using a greedy algorithm. The algorithm first orders the input nodes of a transaction, in decreasing order based on their values. Then, it selects the first nodes whose sum of values is sufficient to resolve the considered transaction.

3. MIXING SERVICES

Another strategy to increase privacy in a blockchain system relies on the deployment of mixing services. These services attempt to conceal the identity of nodes in blockchain records by routing these records through a sequence of proxy nodes, successively replacing the identity of each node with that of its successor. This can be thought of as a “shuffling” of node identities, which does not remove the identity of the original node but rather hides it in a steganographic fashion. While this idea is not novel in itself, tracing back its origins to Web anonymity systems such as Tor \(^2\), it has become particularly appealing for cryptocurrency systems, as their records regard objects (quantities of some cryptocurrency) with direct monetary value. Mixing services usually operate under a rewarding scheme in which the intermediate nodes involved in the concealment process charge the node interested in having its own identity concealed; these transactions are usually managed externally through e.g. a system of credits, though cryptocurrency systems can establish that these payments are performed within the system itself, which simplifies their implementation.

In designing such mixing services, it is important not to assume that every node in the blockchain system is trustworthy, and therefore to design the service such that it is robust to attacks from its own nodes. In particular, we consider here edge insertion attacks. In such attacks, nodes falsely claim that additional nodes were involved in the process in order to receive the corresponding rewards on their behalf. This kind of attack is rather simple to prevent through the deployment of a centralized authority responsible for validating the service performed and verifying node identities, but becomes a challenge if both these tasks are designed to be done in a distributed fashion.

Next, we will work with the following definition for an edge insertion attack and associated terminology.

**Definition 3.3.** In an identity concealment process, an applicant node is a node which requests that its identity in a blockchain record be concealed, and a concealer node is any of the nodes involved in concealing the identity of the applicant node. A concealment route is a sequence of nodes \( \mathcal{R} := (r_0, r_1, \ldots, r_{n-1}, r_n) \) with \( r_0 \) being the applicant node and all others being concealer nodes. The length of this route is given by the number \( n \) of concealer nodes. The reward received by a concealer node \( r_i \) is denoted by \( R_i \), and the total cost paid by the applicant node is denoted by \( C \).

**Definition 3.4.** An edge insertion attack, performed by an attacking node \( r_i \) (\( 0 \leq i \leq n \)), occurs when \( r_i \) forges a set of Sybil nodes \( S \), replaces \( \mathcal{R} \) by a bogus concealment route \( \mathcal{R}' = (r_0, \ldots, r_{i-1}, s_1, \ldots, s_j, r_i, \ldots, r_n) \) := \( (r'_0, \ldots, r'_{n+j}) \), with \( j \in \mathbb{N} \) and \( s_1, \ldots, s_j \in S \), and acquires rewards directed to itself and all nodes in \( S \).

This definition of concealment route only covers mixing by routing funds through a long sequence of nodes. Mixing services usually apply additional strategies involving e.g. splitting and re-merging funds; nevertheless, any impossibility results that apply to concealment routes extend to general mixing services, whose tools include concealment routes.

**Impossibility Result**

In designing a distributed system which prevents edge insertion attacks, one can leverage the freedom in determining the values of the rewards to be paid by the applicant node. One possibility is to diminish the value of the individual reward for longer concealment routes. This way, even if malicious nodes are able to acquire more reward quotas, their smaller value would not be enough to make up for the reward they were originally entitled to.

However, it is important to note that not only concealing nodes can attempt to perform edge insertion attacks. Rather, the applicant node can also engage in such attempts. In this case, however, the attacker is not attempting to hoard more reward quotas than he was entitled to, but to obtain a certain amount of rewards in order to inadequately recover a portion of the original cost.

The following theorem states that, under mild assumptions on the reward scheme, it is impossible to simultaneously prevent edge insertion attacks from concealment and applicant nodes.
**Theorem 3.2.** Consider a reward scheme for an identity concealment process. Assume it follows the guidelines below:

**Zero-sum** The applicant node is charged for exactly the total reward paid to concealing nodes, i.e., the total amount of credits in the system is constant over time;

**Length-dependency** The amount of credits the applicant node is charged for is exclusively a function of the length of the concealment route;

**Uniformity** Rewards paid to concealing nodes are uniform, i.e., all concealing nodes receive equal rewards.

Then, this scheme cannot prevent, simultaneously, edge insertion attacks performed by the applicant node and by concealment nodes.

Proof. Consider an identity concealment process with a concealment route $R$ of length $l$. Denote by $C(l)$ the total cost charged from the applicant node, and by $R(l)$ the reward received by a concealer node. Since rewards are uniform, $R(l) = C(l)/l$ is equal for all concealer nodes.

Now, denote by $CR(l, k)$ the net reward received by an attacking concealer node, when adding $k \geq 0$ Sybil nodes to $R$. If $k = 0$, there is no attack and $CR(l, 0) = R(l)$. More generally, when an attack is performed, each concealing node receives a reward equal to $R(l + k)$ (since the bogus route has length $l + k$), but the attacker collects both its own reward and the rewards of every Sybil node, for a total reward of $CR(l, k) = (k + 1) \cdot R(l + k)$.

Analogously, denote by $AC(l, k)$ the net cost obtained by an attacking applicant node, when adding $k \geq 0$ Sybil nodes to $R$. If $k = 0$, there is no attack and $AC(l, 0) = C(l)$. More generally, when an attack is performed, the applicant node is charged a total cost of $C(l + k)$ due to the bogus route, but cashes back the share of this cost corresponding to the Sybil nodes, thus effectively paying a total of $AC(l, k) = C(l + k) - k \cdot R(l + k) = l \cdot R(l + k)$.

To prevent both kinds of attack simultaneously, the reward scheme must assure that the most beneficial scenario for an attacker occurs when there is no attack, i.e., $k = 0$. Under the scheme assumptions, this amounts to ensuring that the concealer reward is maximized when $k = 0$:

$$CR(l, k) \leq CR(l, 0) \quad l, k \in \mathbb{N},$$

and the applicant cost is minimized when $k = 0$:

$$AC(l, k) \geq AC(l, 0) \quad l, k \in \mathbb{N}.$$

Note that, for any $k > 0$, the first condition implies that $R(l + k) \leq R(l)/(k + 1)$, while the second implies that $R(l + k) \geq l \cdot R(l)/(k + 1)$. Both inequalities can only be simultaneously satisfied if $R(l)/(k + 1) \geq R(l)$, which is absurd.

Note that this impossibility results holds for a reward scheme satisfying rather loose conditions. From the perspective of this analysis, attempts at effectively preventing edge insertion attacks fall in two categories.

**New design relaxing constraints.** One option is to design a reward scheme that relaxes one or more constraints required by Theorem 3.2. In using such approach, one must be mindful of the consequences of the chosen design. For instance, the zero-sum condition can be relaxed by allowing credits to be added or removed from the system depending on the length of the concealment route, but the management of the total credit in the system would likely require some kind of centralized control to prevent the economic collapse of the system, which is certainly not desired in blockchain systems. The length-dependency condition can be relaxed by allowing the reward function to vary over time, but in addition to the possible necessity of exerting this variation in a centralized fashion, the effective result of this strategy would only be to prevent edge insertion attack of only one kind or the other at any given time instant.

**New design with additional security measures.** One alternative is to combine the reward scheme with additional security measures. In that case, the mechanism may prevent edge insertion attacks by the concealer nodes via a reward scheme, requiring the last concealer node to validate the concealment route in order to prevent edge insertion attacks by the applicant node.

Discussion. Although the mechanics of how fees are charged and paid to mixer nodes are out of the scope of this work, a fundamental assumption in our model is that both kinds of sybil attacks are feasible. Route insertion attacks, i.e., adding nodes to $R$, may be difficult in certain practical settings if the token owner needs to choose a route a priori and specify how much it is willing to pay in fees to the mixer. In onion routing, for instance, one chooses a route ahead of time, using source routing, and any deviations would result in a router not being able to decrypt the message.

4. REFERENCES

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