Features of propagation of slow deformation perturbations in geomedia with faults

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Abstract. Our work is performed in the framework of the mathematical model suggested by P.V. Makarov to describe the joint generation and propagation of ordinary stress waves (propagating with the sound speed) and slow deformation waves of the inelastic nature in loaded elastoplastic media. Structural models of block media with weakened regions (faults or deformation sites) are constructed for test calculations on generation and propagation of slow deformation waves in nonlinear dissipative media. The features of propagation of deformation waves are investigated for different fault orientations and implementations of the algorithm of cellular automata for transmitting slow perturbation. The zigzag and diamond nature of propagating fronts of slow deformation disturbances is revealed.

1. Introduction

Practical and particularly important problems of catastrophic damage of various objects, including the problems of earthquakes prediction, as well as the forecast of dangerous dynamic phenomena in rock massif with depletion are associated with the solution of essential tasks for establishing mechanisms for the formation of a site of break-down. They are also related to the solution of the problem of migration of deformation activity and seismic activations, which together form a complex scientific problem of the development of the deformation process that ends in catastrophic destruction [1].

As known, perturbations in solids media can be transmitted by stress waves with sound speeds or plastic (Kolsky) waves with markedly lower speeds but of about the same order. The speeds of tectonic flows, which are determined by geological and GPS data, are no greater than several centimeters per year, i.e., 14–15 orders of magnitude lower than those of sound, and characterize the creep [2]. These motions cannot be expounded as waves. Slow deformation processes have been subjects of study in the physics of plasticity, e.g. Lüders fronts are well understood as propagating autowaves. Speeds of these perturbations have an intermediate value of about 2×10⁻⁵ m/s. In the Earth Sciences over the past 40 years, the concept of deformation waves of the Earth or slow motions the speed of which is 5–6 orders of magnitude less than the velocity of sound and 7–8 orders of magnitude greater than the typical of tectonic flows has been developed and widely discussed [3].

We are talking about perturbations of stress-strain behavior in geomedium, which are interpreted as a wave and propagate from the source of the perturbation (these are tectonic plate boundaries and faults of different scales as a rule). In any multiscale solid—plastic metal, brittle rock, quasi-brittle geological material, etc., slow deformation perturbations can occur. Their speeds are completely controlled by one
factor—the rate of generation of defects and/or damage in these media under dynamic action applied [3].

Note, that there are experimentally identified deformation waves [4] which propagate from fault to fault and which were interpreted from the very beginning as a reflection of autowave processes. At this special attention is paid to the formulation of a phenomenological model of the processes based solely on the observed parameters. In [4], the presence of spatial-temporal migration of deformation processes in fault zones is reliably established. At the same time, it seems that a wave of excitation of abnormal deformation processes extend over the whole space, moreover, fault zones acting as excitable elements. The propagation speeds of deformation waves are in the range from the first kilometers to tens of kilometers per year, depending on the nature of the propagation of these waves. The main difference in the pattern of propagation of these zones is that in the case of strain excitation transfer from fault to fault (“inter-fault” wave), the speed value is in the range of 20 to 30 km/year or more. If the excitation of processes occurs within limits of a single fault zone (“intra-fault” wave), the speed changes from 10 to 4 km/year or less. The observed processes have the character of the “token passing” of anomalous activity from one geodynamically activated object to another. Summarizing these results, we can assume that the spatial-temporal migration of anomalies of modern movements of the earth's surface in fault zones is a consequence of autowave deformation processes in a geodynamically active, excitable geological medium, which is an open system.

So, such slow deformation processes in a media with faults zones need additional investigation by different methods and approaches. One of the helpful methods for studying complex phenomena is numerical modeling and simulation.

The present work aimed to study the features of generation and propagation of slow deformation perturbations in an elastoplastic medium with a fault by the methods of numerical modeling.

2. Mathematical model of strain perturbations
A model proposed by the authors [3] for describing slow deformation perturbations is based on the general fundamental concept of loaded solids:

- All loaded solids, including geomedia, are hierarchically organized multiscale nonlinear dynamic systems. In such a multi-scale media, it is possible to generate deformation perturbations of various physical nature, including slow waves of plastic deformation—Lüders fronts in metallic samples, deformation perturbations, as well as waves of damage and/or destruction in quasi-brittle and brittle materials and geomedia caused by perturbations of stress-strain behavior. The multiscale hierarchical organization of a solid allows the generation of slow wave trains of different scales from microscales of elementary inelastic (plastic) events to scales of global tectonic flows and large tectonic plates, and due to deformation self-similarity on different scales, they can be described by a unified mathematical model.

- The scales, the range of action and velocities of the forming deformation fronts are completely determined by the amount of energy delivered to the medium, the level of generated stress, and the rates of kinetic processes of generation of inelastic (plastic) deformations and/or damage in the loaded medium.

The mathematical implementation of these propositions includes the basic compute core expressed by equations (1)–(6).

\[
\begin{align*}
\frac{dp}{dt} + \rho \text{div} \vec{v} & = 0; \quad \rho \frac{d\vec{v}}{dt} = \frac{\partial \sigma_{ij}}{\partial x_j} + \rho \vec{F}_i; \quad \frac{d\vec{e}}{dt} = \frac{1}{\rho} \sigma_{ij} \dot{\varepsilon}^T_{ij} - q_{ii}; \\
\sigma_{ij} & = -P \delta_{ij} + S_{ij}; \quad P = \frac{1}{3} \sigma_{ii}; \quad \dot{\varepsilon}^T_{ij} = \frac{1}{2} \left( \frac{\partial \sigma_{ij}}{\partial x_j} + \frac{\partial \sigma_{ij}}{\partial x_i} \right) = \dot{\varepsilon}^p_{ij} + \dot{\varepsilon}^p_{ij}; \\
\dot{\sigma}_{ij} & = \lambda (\dot{\theta}^t - \dot{\theta}^p) \delta_{ij} + 2\mu (\dot{\varepsilon}^p_{ij} - \dot{\varepsilon}^p_{ij}); \quad \dot{\theta}^t = \dot{\varepsilon}^T_{ii}; \\
\dot{\varepsilon}^p_{ij} & = \dot{\lambda} \frac{\partial \sigma_{ij}}{\partial \varepsilon_{ij}}, \text{if } f(\sigma_{ij}) \geq 0; \text{otherwise } \dot{\varepsilon}^p_{ij} = 0
\end{align*}
\]

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\[ g(\sigma_{ij}) = \frac{\lambda}{3} I_1(\sigma_{ij}) + f_2^{1/2}(\sigma_{ij}) + \text{const}; \quad f(\sigma_{ij}) = \frac{\alpha}{3} I_1(\sigma_{ij}) + f_2^{1/2}(\sigma_{ij}) - Y; \]  

\[ I_1(\sigma_{ij}) = \sigma_{ii}; \quad f_2(\sigma_{ij}) = \frac{1}{2} S_{ij} S_{ij}. \]  

Here equations (1) express conservation laws, \( \rho \) is the density, \( \mathbf{u}_i \) are the components of the velocity vector, \( \sigma_{ij} \) are the components of the stress tensor, \( F_i \) are the components of the vector of the bulk forces, \( E \) is the internal energy of a unit of initial volume, \( q_i \) are the components of the heat flux vector, the dot above the symbol means the material time derivative. Equations (2)–(3) are the constitutive evolutionary equations of the first group, \( P \) is the pressure, \( \delta_{ij} \) is the Kronecker delta, \( \dot{\epsilon}^f_{ij} \) and \( \dot{\epsilon}^P_{ij} \) are the elastic and plastic components of the total strain rate \( \epsilon^f_{ij} \), respectively. The constitutive evolutionary equations of the second group define the plastic strain rate using the flow rule (4) and functions of the yield surface and plastic potential (5). The following notation is also used: \( \lambda \) and \( \mu \) are the Lame coefficients, \( S_{ij} \) are the components of the deviatoric stress tensor, \( I_1(\sigma_{ij}) \) is the first stress invariant, \( f_2(\sigma_{ij}) \) is the second invariant of the deviatoric stress tensor, \( f(\sigma_{ij}) \) is the yield function, \( g(\sigma_{ij}) \) is the plastic potential function, \( \Lambda \) is the dilatancy coefficient, \( \alpha \) is the internal friction coefficient.

Combined with the method of cellular automata, these equations allow to implement in calculations a cooperative coordinated response of a medium to loading [5-7] and to describe slow deformation fronts in inelastic nature in nonlinear elastic-plastic media [1, 3].

The complete system of equations of continuum mechanics was solved using the finite difference method [8]. According to the algorithm of cellular automata, fault zones—narrow elongated areas inclined to the axis of the load application—were defined as the areas where plastic deformation can be generated in our simulations here.

The behavior of a region of the medium with a fault in the conditions of uniaxial compression along the vertical axis using different implementations of the algorithm of cellular automata for transmitting slow perturbation was studied. Specifically, two variants of the neighborhood were used in two-dimensional models of cellular automata: von Neumann and Moore. In the von Neumann neighborhood (Figure 1, a), the neighbors of a given cell are cells that have a common edge with the given cell. Thus, each cell has exactly four neighbors. In the Moore neighborhood (Figure 1, b), two cells are adjacent if they have either a common edge or a common vertex. That is, there are eight cells in the local neighborhood of each cell (apart from itself).

![Figure 1. Examples of the neighborhood type: (a) von Neumann; (b) Moore.](image-url)

3. Results and discussion

Figure 2 shows chronograms of the distributions of effective inelastic strains that reflect the propagation of deformation perturbation fronts in nonlinear media with different implementation options for cellular automata algorithms and with a fault inclined at an angle of 45 degrees. The fronts of deformation perturbations depicted in the figure have different shapes. All the fronts have two main advances in mutually perpendicular directions, and the vertical advance is always greater than the horizontal one. This is due to the formation of several angular fronts, which edges are inclined at approximately 45°. Calculations with the Moore neighborhood lead to the formation of one large triangular front from each edge of the fault, they absorb other small fronts. In the case of the von Neumann neighborhood, the number of triangular fronts is greater and they are different in size. Sometimes larger fronts absorb...
smaller ones. The wave fronts in both versions of the neighborhoods have zigzag shape, but it is more pronounced in the case of the von Neumann neighborhood.

![Wave fronts in both versions of the neighborhoods](image)

**Figure 2.** Propagation of deformation perturbations in the medium when implementing cellular automata with (a–c) von Neumann and (d–f) Moore neighborhoods.

Figure 3 shows chronograms of the effective inelastic strain distributions in the cases of the von Neumann and Moore neighborhoods realizations when the faults are inclined at an angle of 0 degrees. Wave fronts originate symmetrically in two places at both ends of the fault, then propagate towards each other and at some point absorb each other due to the interaction of waves (Figure 3, c, f). The front of slow perturbations has a diamond shape.

![Wave fronts in a medium with a fault inclined at an angle of 0 degrees](image)

**Figure 3.** Propagation of deformation perturbations in the medium with a fault inclined at an angle of 0 degrees when implementing cellular automata with (a–c) von Neumann and (d–f) Moore neighborhoods.

The results of simulations in a medium with a fault inclined at an angle of 90 degrees are depicted in Figure 4. One can see that the fronts of slow perturbations also originate symmetrically at the ends of
the fault and move towards each other. But in this case, there is no such strong absorption of wave fronts. The shapes of the fronts of slow perturbations have the same diamond shape.

![Figure 4. Propagation of deformation perturbations in the medium with a fault inclined at an angle of 90 degrees when implementing cellular automata with (a–c) von Neumann and (d–f) Moore neighborhoods.](image)

4. Conclusion

Thus, it is shown that the propagating fronts of slow deformation perturbations in a nonlinear medium with faults inclined at different angles have different shapes owing to the action of the applied load, the maximum shear stresses, and the number of cells involved in the model of cellular automata. As a result of the studies of the origin and propagation of slow deformation perturbations in a medium with a fault tilted at an angle of 45 degrees, the zigzag nature of wave fronts is revealed in both versions of the neighborhoods. However, the propagated fronts of slow deformation perturbations in a nonlinear medium when the faults are tilted at an angle of 0 and 90 degrees have a diamond shape. The amount of zigzags is greater in the case of the von Neumann neighborhood. The speed of the fronts in the case of the Moore neighborhood is higher than in the case of the von Neumann neighborhood. Simulations were made using our own computer program.

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