Scale Dependent Dimension of Luminous Matter in the Universe

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(March 19, 2022)

Abstract

We present a geometrical model of the distribution of luminous matter in
the universe, derived from a very simple reaction-diffusion model of turbu-
 lent phenomena. The apparent dimension of luminous matter, \( D(l) \), depends
linearly on the logarithm of the scale \( l \) under which the universe is viewed:
\[
D(l) \sim 3 \log(l/l_0)/\log(\xi/l_0),
\]
where \( \xi \) is a correlation length. Comparison
with data from the SARS red-shift catalogue, and the LEDA database pro-
vides a good fit with a correlation length \( \xi \sim 300 \) Mpc. The geometrical
interpretation is clear: At small distances, the universe is zero-dimensional
and point-like. At distances of the order of 1 Mpc the dimension is unity,
indicating a filamentary, string-like structure; when viewed at larger scales
it gradually becomes 2-dimensional wall-like, and finally, at and beyond the
correlation length, it becomes uniform.

Uniformity of the background radiation requires that the universe must be homogeneous
at the largest scale; this is known as the cosmological principle. However, a decade ago,
Coleman and Pietronero [1] suggested that the universe, at length scales \( L \) up to a couple
of Mpc is fractal with fractal dimension, \( D \sim 1.2 \), based on a study of the CfA galaxy
catalogue. Subsequent studies seemed to confirm this picture: Guzzo et al. [2] found \( D = 1.2 \)
for \( L = 1 - 3 \) Mpc, increasing to \( D = 2.2 \) for \( L = 3 - 10 \) Mpc, from the Perseus-Pisces
catalogue. Martinez and Coles [3] found that the dimension gradually increases from 2.25 to
2.77 at length scales increasing from 1 – 50 Mpc. These empirical studies have recently been
reviewed by Wu et al [4]. Even though there is a general agreement about the existence of
fractal galactic structures at moderate scales, there is still intense debate whether or not the
universe is homogeneous at very large scales and, if so, how the transition to homogeneity
takes place [3] [5]. The value of the homogeneity scale and the matter distribution within
such scale have great cosmological consequences.

We propose that the distribution of luminous matter in the Universe can be described
by a new geometric scaling form that we discovered recently [7] in a different context. This
description leads to a reconciliation of observational data at various scales and a consistent
phenomenology of the crossover to homogeneity. A sharp transition to homogeneity at
300Mps is predicted.

The model is a simple non-equilibrium reaction-diffusion “forest-fire” model [8] [9], pro-
posed to capture the essential features of turbulent systems, where energy is injected at the largest scale, and dissipated at a small length scale. We found that in a range of length scales between these two limits the dimension of the luminous field (fire distribution) varies gradually from zero to three, and that the distribution becomes homogeneous beyond a correlation length which depends on the energy injection rate. The model operates near a dynamical “critical point”, with diverging correlation length.

As we will show below, analysis of galaxy maps indicates that the geometrical structure of luminous matter in the universe is very similar to that of the forest-fire model. Our alternative form provides a better fit to the data than conventional models. The underlying picture is one where luminous matter is being created and destroyed in an ongoing non-equilibrium dynamical process. This similarity is appealing in that it suggests that the universe shares the basic characteristic features of other dynamical systems, so perhaps the dynamics of the universe is not unique, but belongs to a more general universality class of non-equilibrium turbulent systems.

Usually, systems near equilibrium criticality are self-similar, or fractal, for length scales below the correlation length; hence fractal behavior can often be viewed as a consequence of criticality. However, the forest-fire model does not show simple power-law (fractal) scaling below the correlation length. Numerical studies show that the average amount of dissipation \( n(l) \), within a cube box of size \( l \) that contains dissipation, obeys

\[
\log(n) \sim \left( \frac{3 \log(l/l_0)}{2 \log(\xi/l_0)} \right) \log(l/l_0),
\]

where \( l_0 \sim 1 \) lattice spacing for the forest fire model. At the correlation length \( \xi \), there is a sharp cross-over to a homogeneous 3d structure. Thus, the correlation length is identical to the homogeneity length, as is usually the case in critical phenomena. However, the lack of self-similarity implies that one can derive the correlation length from observations in a range of much smaller length scales.

This equation can be interpreted in terms of an apparent fractal dimension for luminous matter that varies linearly with the logarithm of the length scale:

\[
D(l) = \frac{d \log(n)}{d \log(l)} \sim 3 \left( \frac{\log(l/l_0)}{\log(\xi/l_0)} \right).
\]

We suggest that the length scale dependent behaviour observed in this model may be sufficiently general that it is worthwhile to make a detailed comparison with real astronomical data. The underlying viewpoint is that the galactic dynamics is turbulent, with stellar objects interacting with one another in reaction-diffusion type processes through shock waves, super-novae explosions, galaxy mergers, etc. Apart from an overall amplitude, there are only two fitting parameters in our proposed galaxy distribution, namely the upper length scale \( \xi \), where the distribution becomes uniform, and the lower cutoff, \( l_0 \), where the distribution becomes point like.

In their seminal work, Sylos-Labini, Pietronero and coworkers analysed several database catalogues of galaxy maps. From the databases, they created volume-limited (VL) samples containing all galaxies exceeding a certain absolute luminosity within a given volume. Then they calculated the conditional density \( \Gamma^*(l) \), which is the average density of
galaxies within a sphere of size \( l \). This quantity corresponds to the density \( n(l) \) defined above, divided by the volume \( l^3 \). Thus, the resulting prediction for \( \Gamma^*(l) \) becomes

\[
\log (\Gamma^*(l)) \sim \left( \frac{3}{2} \frac{\log(l/l_0)}{\log(\xi/l_0)} - 3 \right) \log(l/l_0).
\]

Namely, on a log-log plot there is a pure quadratic dependence, rather than the linear dependence found for self-similar fractal structures.

We have fitted the above expression to the conditional densities extracted by Pietronero et al. from two widely different data bases with consistent results. The LEDA database is a heterogeneous compilation of data from the literature containing more than 200,000 galaxies. The Stromlo-APM red shift survey (SARS [12]) consists of 1797 galaxies. Figure 1 shows results from the fits, with two different cut-offs for the LEDA database. The labeling follows Sylos Labini et al., with the numbers representing the lower luminosity cut-offs. Obviously, there are larger fluctuation for the sparser, but perhaps higher quality, Stromlo-APM data set than for the LEDA database. The fits are very good in view of the fact that the only fitting parameters are the upper and lower length scales, \( \xi \) and \( l_0 \), respectively. In contrast to conventional critical phenomena, the correlation length enters the expression for length scales below the correlation length. We are therefore able to fit the correlation length to the data, despite the fact that no data is available at and beyond the projected correlation length.

The upper length scale is the one where the curves become flat, \( d = 3 \). The three fits yield very consistent values of this length scale, \( \xi = 260 \) Mpc from the LEDA16 data, \( \xi = 275 \) Mpc from the LEDA14 data, and \( \xi = 380 \) Mpc from the APM data.

The empirical logarithmic scale dependence of the dimension can be seen directly by re-plotting the data in figure 1: Figure 2 shows \( D(l) = 2 \times \log(\Gamma^*(l)/\Gamma(l_0))/\log(l/l_0) + 3 \). All data sets yield linear behavior. The correlation length is found by linear extrapolation to the point where \( D(l) \) assumes the value of 3. The dimensions derived from the intense galaxies, LEDA 16 and APM 18, are essentially identical, but the LEDA 14 data yield a somewhat steeper scale dependence. However, they all converge at essentially the same homogeneity length.

We predict a sharp crossover to uniformity, i. e. a sharp kink in the curve, at this length, which will be readily observable once data becomes available. Actually, there is a recent analysis based on ESO Slice Project galaxy redshift survey which indicates that the fractal dimension is close to 3 for the length scale greater than 300 Mpc [13]. Also, the intermediate data points are predicted to follow the straight lines in figure 2.

The lower cut-off, \( l_0 \), is the scale at which the slope of the curves in the figure assumes the value of -3. We find \( l_0 = 370 \) light-years, \( l_0 = 3700 \) light-years, and \( l_0 = 330 \) light years for the three samples, respectively. This scale is determined with less precision than the correlation length \( \xi \). It is not clear how well our scaling form applies to the analysis of the galaxy distribution at small length scales.

The geometry of the luminous set is not fractal when viewed over the entire range of scales, since there is no self-similarity for different scales. Nevertheless, the scale dependent dimension has a clear geometrical interpretation: At small distances, the universe is zero-dimensional and point-like. Indeed, energy dissipation takes place on individual point objects, like stars and galaxies. At distances of the order of 1 Mpc the dimension is unity,
indicating a filamentary, string-like structure; when viewed at larger scales it gradually becomes 2-dimensional wall-like, and finally at the correlation length, $\xi$, it becomes uniform.

It might be instructive to compare with more conventional interpretations of the large scale structure [14]. The conditional density can be related to a correlation function $g(r)$ through [11]

$$\Gamma^*(l) \sim \langle n \rangle (1 + g(l)),$$

where $\langle n \rangle$ is the mean density of galaxies. For instance, the field theory of de Vega et al. [15] yields an expression of this form. The correlation function is often assumed to be of the form $g(l) = (r_0/l)^\gamma$. Figure 1 also shows a fit to this expression, with $r_0 = 10$ Mpc and $\gamma = 1.3$. The fit is clearly inferior, flattening out at too small length scales. This is in accordance with the observations by Sylos Labini et al. that the value of fitted parameter $r_0$ depends heavily on the range of length scales used. At larger scales, the difference between the two fits is even more pronounced; when further data becomes available in the near future, one should be able to discriminate even better between the two pictures. In this traditional view, there is a smooth crossover to homogeneity when the amplitude, expressed in terms of $r_0$ reaches unity. In contrast, following our “critical phenomena” viewpoint, there is a sharp, possibly exponential, cutoff of the non-uniform part of the correlation function at the correlation length.

This has some important cosmological consequences. In the traditional formulation, one usually visualizes that the amplitude $r_0$ of the power-law fluctuations increases with time, starting from the time of the decoupling of radiation from hadronic matter, leading to an increase of the cross-over length to homogeneity. In our phenomenology, the correlation length $\xi$ is the only parameter, so it is this quantity which is increasing with time. The average density of galaxies in the universe is equal to the density within the correlation length, i.e. $\langle n \rangle = \Gamma^*(\xi) \sim 1/\xi^{3/2}$. Thus, once the correlation length has been determined, one knows the density of galaxies. Assuming that the entire density of hadronic matter scales as that of the luminous galaxies studied here, one might get an estimate of the mass of the universe. From the fit to LEDA 14 one gets that the density of galaxies in the entire universe with apparent magnitude greater than 14 is $\langle n \rangle = 2 \times 10^{-3}$ Mpc$^{-3}$. From the fit to the APS 18 data we find that the density of galaxies with apparent luminosity greater than 18 is $\langle n \rangle = 3 \times 10^{-4}$ Mpc$^{-3}$. The traditional fits give much larger values for the density of galaxies in the universe, depending on the range of length scales used in the fit [11].

In the forest fire model the energy flux (which determines the average density of fires) is an independent parameter, namely the growth rate of trees, whereas for the universe it is self-consistently determined by the dynamics. The forest fire model exhibits self-organised criticality [10], in the sense that the correlation length diverges as the tree growth rate $p \to 0$. All fire goes extinct as the correlation length reaches the system size. As the universe expands, the correlation length $\xi(t)$ increases faster than the size of the universe $R(t)$ and the universe become more and more inhomogeneous. One might speculate that as the correlation length reaches the size of the universe, all the luminous matter is extinguished, and we are left with a universe without luminous matter!
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Acknowledgment. We thank Maya Paczuski, Kim Sneppen, and Jakob Bak for helpful discussions and comments on the manuscript.

Figure Captions

Figure 1. Conditional average densities for various galaxy catalogues (arbitrary scale), as derived by Sylos Labini et al [11], compared with fits to equation 1, yielding \( \xi = 260 \) Mpc from the LEDA16 data, \( \xi = 275 \) Mpc from the LEDA14 data, and \( \xi = 380 \) Mpc from the APM data. The broken line is a conventional fit to equation 4 with \( \gamma = 1.3 \), \( r_0 = 10 \) Mpc.

Figure 2. Scale dependent dimension \( D(l) \) derived from the data points in figure 1 as explained in text. We conjecture that future data points follow the straight lines, and saturate sharply to \( D = 3 \) at the correlation length.
