Abstract—We consider asynchronous CDMA systems in no-fading environments and focus on a certain user. This certain user is called a desired user in this paper. In such a situation, an optimal sequence, maximum Signal-to-Interference plus Noise Ratio (SINR) and the maximum capacity for a desired user are derived with other spreading sequences being given and fixed. In addition, the optimal sequence and SINR for a desired user are written in terms of the minimum eigenvalue and the corresponding eigenvector of a matrix, respectively. Since it is not straightforward to obtain an explicit form of the maximum SINR, we evaluate SINR and obtain the lower and upper bounds of the maximum SINR. From these bounds, the maximum SINR may get larger as the quantities written in terms of quadratic forms of other spreading sequences decreases.

Index Terms—Asynchronous CDMA systems, Spreading sequence, Signal-to-Interference Noise Ratio, Capacity, Rayleigh quotient

I. INTRODUCTION

To investigate channel capacity is a significant task since channel capacity is the maximum achievable rate [1]. If the rate is smaller than given capacity, then there is a code whose maximum error converges to zero as the length of code words goes to infinity [2] [3]. Thus, if large channel capacity is achieved, then information can be sent at a high rate. These results have been proven in [4]. Furthermore, in a general channel, capacity has been obtained in [5] [6]. With a practical scheme, channel capacity has been obtained in [7]. Further investigations of capacity are expected to contribute to improvement in communication systems.

In Orthogonal Frequency Division Multiplexing (OFDM) systems, channel capacity with a non-linear amplifier has been obtained in [8]. In Multiple Input-Multiple Output (MIMO) systems, capacity has been investigated [9]. By contrast, in some situations, capacity with MIMO systems has not been obtained.

In Code Division Multiple Access (CDMA) systems, capacity has also been investigated. One of representative characteristics of CDMA systems is to use spreading sequences to communicate each other. Therefore, capacity may depend on spreading sequences. Further, it is known that capacity increases as Signal-to-Interference plus Noise Ratio (SINR) increases in practical schemes [7]. There are many works to obtain spreading sequence which achieve large SINR.

CDMA systems are divided into three kinds of systems. In synchronized CDMA systems, it is known that the Welch bound equality (WBE) sequences achieve the maximal capacity [10]. In chip synchronized CDMA systems with given and fixed delays, an algorithm to obtain sequences which achieves nearly maximum SINR has been suggested [11]. However, channel capacity in asynchronous CDMA systems have not been obtained. In asynchronous CDMA systems, many spreading sequences have been suggested to obtain large SINR. For more details, we refer the reader to [12] - [17]. Asynchronous CDMA systems have been investigated in [18] [19] [20]. Since it is known that correlations play important roles in CDMA systems, correlations of sequences have been investigated and bound of correlations have been obtained [21] [22]. Thus, sequences which achieve the equalities of such bounds have been obtained [23] [24] [25].

In this paper, we show the optimal sequence for a desired user in a sense of SINR with no fading environments. Further, we show that the maximum SINR and an optimal sequence are written in terms of the minimum eigenvalue and the corresponding eigenvector, respectively. Since we derive the maximum SINR, the maximum capacity is derived under an approximation. Although we clarify the maximum SINR, it does not seem to be straightforward to obtain its closed form. Thus, we evaluate the maximum SINR and derive lower and upper bounds of SINR. From these bounds, it turns out that maximum SINR gets larger as the quantities written in terms of quadratic forms of other spreading sequences decreases. Note that the maximum capacity of all the users has not been obtained and that the problem to obtain the capacity of all the users is still remaining.

II. SYSTEM DESCRIPTION

In this section, we show a model of asynchronous CDMA systems. This model has been investigated in [20] [26] [27]. We make the following assumptions.

1) a modulation scheme is Binary Shift Phase Keying (BPSK)
2) there is no fading effect.
3) the spreading sequences for the other users are given and fixed. Only a spreading sequence for a certain user is regarded as a variable.
4) Gaussian channel noise is given.
5) interference noise follows Gaussian.
6) interference noise is independent of Gaussian channel noise.

The assumptions 1, 2 and 3 are often made and CDMA systems in no-fading effect have been investigated [20] [11]. The assumptions 4 and 6 are usually made to analyze communication systems [20] [28]. The assumption 5 has been made in [20]. Further, in analysis of Signal-to-Noise Ratio (SNR), this assumption is often made since Gaussian noise is the worst kind of additive noise in the view of capacity [8] [3]. Thus, we consider the worst case in the view of capacity in asynchronous CDMA systems.

Let N be the length of spreading sequences and N is common for all the users. From the assumption 1, a data signal of the user k, b_k(t), is written as

\[ b_k(t) = \sum_{n=-\infty}^{\infty} b_{k,n} p_T(t - nT), \]  
(1)

where \( b_{k,n} \in \{-1, 1\} \) is the n-th component of symbol which the user k sends, \( T \) is the duration of one symbol and \( p_T(t) \) is the rectangular pulse written as

\[ p_T(t) = \begin{cases} 
1 & 0 \leq t < T \\
0 & \text{otherwise}
\end{cases}. \]

Then, the code waveform of the user k, \( s_k(t) \), is written as

\[ s_k(t) = \sum_{n=-\infty}^{\infty} s_{k,n} p_{T_c}(t - nT_c), \]  
(2)

where \( s_{k,n} \) is the n-th component of the spreading sequence of the user k and \( T_c \) is the width of each chip such that \( NT_c = T \). We assume the sequence \( (s_{k,n}) \) is periodic, that is, \( s_{k,n} = s_{k,n+N} \). Moreover, we assume the power normalization condition

\[ \sum_{n=-\infty}^{\infty} |s_{k,n}|^2 = N. \]  
(3)

This condition is often used [22] [21]. With the above signals, the transmitted signal of the user k, \( \zeta_k(t) \), is written as

\[ \zeta_k(t) = \sqrt{2P} \text{Re}[s_k(t)b_k(t) \exp(j\omega_c t + j\theta_k)], \]  
(4)

where \( P \) is the common signal power to all the users, \( j \) is the unit imaginary number, \( \omega_c \) is the common carrier frequency to all the users and \( \theta_k \) is the phase of the user k. Note that the signal \( \zeta_k(t) \) is called a Radio Frequency (RF) signal.

We assume that there are \( K \) users and that all the users are not synchronized. Then, the received signal \( r(t) \) is written as

\[ r(t) = \sum_{k=1}^{K} \zeta_k(t - \tau_k) + n(t), \]  
(5)

where \( \tau_k \) is the delay time of the user k and \( n(t) \) is additive white Gaussian noise (AWGN).

To analyze SINR for a certain user, we focus on the user \( i \) and the user \( i \) is called desired user. If the user \( i \) is a desired user and the received signal \( r(t) \) is the input to a correlation receiver matched to \( \zeta_i(t) \), then the corresponding output \( Z_i \) is written as

\[ Z_i = \int_0^T r(t) \text{Re}[s_i(t - \tau_i) \exp(j\omega_c t + j\psi_i)] dt, \]  
(6)

Without loss of generality, we assume \( \tau_i = 0 \) and \( \theta_i = 0 \). With a low-pass filter, we can ignore double frequency terms, and rewrite Eq. (6) as

\[ Z_i = \frac{1}{2} \sum_{k=1}^{K} \int_0^T \sqrt{2P} \text{Re}[s_k(t)b_k(t)\bar{s_i}(t) \exp(j\psi_k)] dt \]  
(7)

where \( \bar{z} \) is the complex conjugate of \( z \),

\[ \bar{s_i}(t) = \sum_{n=-\infty}^{\infty} s_{i,n} p_{T_c}(t - nT_c). \]  
(8)

Then, the output value \( Z_i \) is divided into there signals, the desired signal \( D_i \), the interference signal \( I_i \) and the AWGN signal \( N_i \), which are written as

\[ D_i = \sqrt{P} \int_0^T b_i(t) dt \]  

\[ I_i = \sqrt{P} \sum_{k \neq i} \text{Re}[\tilde{I}_{i,k}] \]  
(10)

\[ N_i = \int_0^T n(t) \text{Re}[s_i(t) \exp(j\omega_c t)] dt \]

where

\[ \tilde{I}_{i,k} = \int_0^T \mu_{i,k}(\tau; t) \exp(j\psi_k) dt. \]

Thus, the output \( Z_i \) is rewritten as

\[ Z_i = D_i + F_i + I_i + N_i. \]  
(11)

Since \( \text{E}\{I_i\} = \text{E}\{N_i\} = 0 \) and \( \text{E}\{D_i\} = T \sqrt{P}/2 \), we have \( \text{E}\{Z_i\} = T \sqrt{P}/2 \), where \( \text{E}\{X\} \) is the average of \( X \). Then, SINR of the user \( i \) is defined as

\[ \text{SINR}_i = \frac{PT^2/2}{\text{Var}\{I_i\} + \text{Var}\{N_i\}}, \]  
(12)

where \( \text{Var}\{X\} \) is the variance of \( X \). In particular, it is known in [20] [26] that the variance of \( N_i \) is written as

\[ \text{Var}\{N_i\} = \frac{1}{4} N_0 T. \]  
(13)
if \( n(t) \) has a two-sided spectral density denoted as \( \frac{1}{2}N_0 \).
In [27], the formula of SINR has been proposed as

\[
\text{SINR}(s_i)_i = \left\{ \frac{1}{6N^2} \sum_{k=1}^{K} \sum_{m=1}^{N} S_{m,k}^{i} + \frac{N_0}{2P_T} \right\}^{-1/2},
\]

(14)

where

\[
S_{m,k}^{i} = (s_i^* Q_m s_i) (s_k^* Q_m s_k) + (s_i^* \hat{Q}_m s_i) (s_k^* \hat{Q}_m s_k).
\]

(15)

In this paper, attention is drawn to this formula. The symbols in Eq. (12) are explained as follows. First, \( s_k \) is the vector written as

\[
s_k = (s_{k,1}, s_{k,2}, \ldots, s_{k,N})^T,
\]

(16)

the matrices \( Q_m \) and \( \hat{Q}_m \) are given by

\[
Q_m = V^* C_m V, \quad \hat{Q}_m = \hat{V}^* \hat{C}_m \hat{V},
\]

(17)

\( V \) and \( \hat{V} \) are unitary matrices whose \((m,n)\)-th component is written as

\[
V_{m,n} = \frac{1}{\sqrt{N}} \exp\left(-2\pi j \frac{mn}{N}\right),
\]

\( \hat{V}_{m,n} = \frac{1}{\sqrt{N}} \exp\left(-2\pi j n \frac{m}{N} + \frac{1}{2N}\right),
\]

(18)

and \( C_m \) and \( \hat{C}_m \) are diagonal matrices whose \((m,m)\)-th elements are given by

\[
(C_m)_{m,m} = \sqrt{1 + \frac{1}{2} \cos \left(2\pi \frac{m}{N}\right)},
\]

\[
(\hat{C}_m)_{m,m} = \sqrt{1 + \frac{1}{2} \cos \left(2\pi \frac{m}{N} + \frac{1}{2N}\right)},
\]

(19)

and the other elements are zero. In the above equations, \( x^* \) and \( z^* \) denote the transpose of \( x \) and the conjugate transpose of \( z \), respectively. Note that the matrices \( Q_m \) and \( \hat{Q}_m \) are positive semidefinite matrices since \( Q_m \) and \( \hat{Q}_m \) are Gram matrices. It is obvious that Eq. (14) depends on the vector \( s_i \).

### III. Optimal Sequence and SINR for Desired User in No Fading

In this section, we derive an optimal spreading sequence in no fading situation for the user \( i \). Since the optimal spreading sequence is derived, the maximum SINR and the maximum capacity for the user \( i \) are obtained.

In the previous section, we have made six assumptions. These are also assumed to be made in this section. From the assumption 1, 2, 3 and 6, which has been defined in Section II, when the spreading sequence \( s_i \) is given, SINR for the user \( i \) is written as Eq. (14). From assumption 3, Eq. (14) depends only on the \( s_i \) since the other spreading sequences \( s_k \) are fixed for \( k \neq i \). Therefore, to maximize SINR, we consider the following optimization problem

\[
(P_i) \quad \min \sum_{k=1}^{K} \sum_{m=1}^{N} S_{m,k}^{i},
\]

(20)

subject to \( ||s_i||^2 = N \).

It is clear that maximum SINR is obtained from the above optimization problem. In what follows, the problem \( (P_i) \) is rewritten in another form.

To analyze the optimization problem, we define the following matrix \( \Sigma \)

\[
\Sigma = \sum_{k=1}^{K} \sum_{m=1}^{N} (s_k^* Q_m s_k) Q_m + (s_k^* \hat{Q}_m s_k) \hat{Q}_m.
\]

(21)

The matrix \( \Sigma \) is constant since \( s_k \) is given and fixed for \( k \neq i \) under assumption 3. Further, the matrix \( \Sigma \) is positive semidefinite since the quantities \((s_k^* Q_m s_k)\) and \((s_k^* \hat{Q}_m s_k)\) are non-negative, and the matrices \( Q_m \) and \( \hat{Q}_m \) are positive semidefinite.

With the matrix \( \Sigma \), the optimization problem \( (P_i) \) is written as

\[
(P_i) \quad \min s_i^* \Sigma s_i
\]

(22)

subject to \( ||s_i||^2 = N \).

where \( ||z|| \) is the Euclidean norm of \( z \). Further, the above problem is equivalent to the following one

\[
(P_i) \quad \min \frac{s_i^* \Sigma s_i}{||s_i||^2/N}
\]

(23)

subject to \( ||s_i||^2 = N \).

Let the vector \( u_i \) be \( u_i = \frac{1}{\sqrt{N}} s_i \). Then, the problem \( (P_i) \) is rewritten as

\[
(P_i) \quad \min \frac{N \cdot u_i^* \Sigma u_i}{||u_i||^2}
\]

(24)

subject to \( ||u_i||^2 = 1 \).

It is obvious that the value of the objective function is invariant under the action \( u_i \mapsto cu_i \), where \( c \in \mathbb{C} \) is a non-zero scalar. This result implies that if we obtain a non-zero solution \( \hat{u}' \) which minimizes the objective function of \( (P_i) \), then we can obtain the feasible optimal solution \( \hat{u} \) as \( \hat{u} = \hat{u}'/||\hat{u}'|| \). Thus, we consider the following problem

\[
(P_i') \quad \min_{u_i \neq 0} \frac{N \cdot u_i^* \Sigma u_i}{||u_i||^2}
\]

(25)

This is the Rayleigh quotient of \( N \Sigma \) [29]. It is known that the optimal value coincides with the product of \( N \) and the minimum eigenvalue of \( \Sigma \), \( \lambda_{\min} \geq 0 \) and that the global minimizer of the problem \( (P_i) \) is the eigenvector corresponding to \( \lambda_{\min} \). Let \( u \) be such a minimizer. When the minimizer \( u \) is normalized as \( ||u|| = 1 \), the optimal spreading sequence for the user \( i \), \( s_i^* \) is written as

\[
s_i^* = \sqrt{N} \hat{u}.
\]

(26)

Then, the maximum SINR is written as

\[
\text{SINR}_i^* = \text{SINR}(s_i^*)_i = \left\{ \frac{\lambda_{\min}}{6N} + \frac{N_0}{2P_T} \right\}^{-1/2}.
\]

(27)

Further, it is known that the channel capacity is written in terms of Signal-to-Noise Ratio (SNR) if an input is continuous and channel noise is Gaussian [11] [30]. The sum of
interference noise and channel noise follows Gaussian since the sum of the two independent Gaussian variables follow Gaussian under assumptions 5 and 6 [31]. Even in a case where noise follows Gaussian, it is known that the channel capacity with a practical scheme is complicated [3] [7]. In [2], the channel capacity with BPSK scheme is close to one with a continuous channel in low SNR. Thus, we approximate the maximum channel capacity of the user \( i \) by one with a continuous channel. Under this approximation, from Eq. (27), the maximum channel capacity for the user \( i, C_i^* \), is approximated as

\[
C_i^* \approx \frac{1}{2} \log \left[ 1 + \frac{\lambda_{\text{min}}}{6N} + \frac{N_0}{2PT} \right].
\]  

(28)

As seen in the above discussions, the maximum SINR and the maximum channel capacity depend on the minimum eigenvalue of the matrix \( \Sigma \), and these maximums are achieved with the eigenvector corresponding to the minimum eigenvalue.

IV. ESTIMATING MAXIMUM SINR

We have obtained the maximum SINR in asynchronous CDMA systems for a desired user. It is obvious that the minimum eigenvalue \( \lambda_{\text{min}} \) depends on other spreading sequences \( s_k \ (k \neq i) \) since the matrix \( \Sigma \) depends on \( s_k \ (k \neq i) \). Thus, to analyze the maximum SINR, it is necessary to obtain the explicit form of \( \lambda_{\text{min}} \). However, it is not straightforward to obtain the explicit form \( \lambda_{\text{min}} \). In this section, we derive the lower and upper bounds of the maximum SINR. From these bounds, we can estimate the maximum SINR and the main factor related to SINR.

As seen in Eq. (21), the matrix \( \Sigma \) consists of two kinds of the matrices, \( Q_m \) and \( \hat{Q}_m \). From Eq. (17), the eigenvalues of the matrices \( Q_m \) and \( \hat{Q}_m \) are represented as the matrices \( C_m \) and \( \hat{C}_m \), respectively. Further, the matrices \( C_m \) and \( \hat{C}_m \) have at most one non-zero component at the \( (m, m) \)-th entry. Therefore, the matrix \( \Sigma \) is written as

\[
\Sigma = V^* \Lambda V + \hat{V}^* \hat{\Lambda} \hat{V},
\]  

(29)

where \( \Lambda \) and \( \hat{\Lambda} \) are diagonal matrices whose \( m \)-th diagonal components, \( \lambda_m \) and \( \hat{\lambda}_m \) are written as

\[
\lambda_m = \sqrt{1 + \frac{1}{2} \cos \left( 2 \pi \frac{m}{N} \right) \sum_{k=1}^{K} (s_k^* Q_m s_k)}
\]

\[
\hat{\lambda}_m = \sqrt{1 + \frac{1}{2} \cos \left( 2 \pi \left( \frac{m}{N} + \frac{1}{2N} \right) \right) \sum_{k=1}^{K} (s_k^* \hat{Q}_m s_k)}.
\]  

(30)

Since the matrices \( V \) and \( \hat{V} \) are unitary, the quantities \( \lambda_m \) and \( \hat{\lambda}_m \) are the eigenvalues of the matrices \( V^* \Lambda V \) and \( \hat{V}^* \hat{\Lambda} \hat{V} \), respectively. Note that the quantities \( \lambda_m \) and \( \hat{\lambda}_m \) depend on the spreading sequences \( s_k \) for \( k \neq i \).

With the above eigenvalues, the bounds of the maximum SINR for the user \( i \) are derived. First, we derive the upper bound. As seen in Eq. (27), the maximum SINR is written with the minimum eigenvalue of \( \Sigma \), \( \lambda_{\text{min}} \). Since \( \lambda_{\text{min}} \) is the minimizer of the Rayleigh quotient of \( \Sigma \), the following relations are obtained

\[
\lambda_{\text{min}} = \min_{u \neq 0} \frac{u^* \Sigma u}{\|u\|^2} = \min_{u \neq 0} \frac{u^* \left( V^* \Lambda V + \hat{V}^* \hat{\Lambda} \hat{V} \right) u}{\|u\|^2}
\]

\[
\geq \min_{u_{1} \neq 0} \frac{u_{1}^* V^* \Lambda V u_{1}}{\|u_{1}\|^2} + \min_{u_{2} \neq 0} \frac{u_{2}^* \hat{V}^* \hat{\Lambda} \hat{V} u_{2}}{\|u_{2}\|^2}
\]

\[
= \min_{m} \lambda_m + \min_{m} \hat{\lambda}_m.
\]  

(31)

In the above discussions, we have used Eq. (29). Then, the upper bound of the maximum SINR is written as

\[
\left\{ \frac{1}{6N} (\min_{m} \lambda_m + \min_{m} \hat{\lambda}_m) + \frac{N_0}{2PT} \right\}^{-1/2} \geq \text{SINR}_i^*.
\]  

(32)

On the other hand, to derive the lower bound of the maximum SINR, we use the following theorem [32].

**Theorem (Weyl).** Let \( A \) and \( B \) be the \( n \times n \) Hermitian matrices whose eigenvalues are written as \( \alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n \) and \( \beta_1 \geq \beta_2 \geq \cdots \geq \beta_n \), respectively. Further, we define the Hermitian matrix \( C = A + B \) whose eigenvalues are written as \( \gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_n \). Then, the following relation holds for \( k + l - 1 \leq n \)

\[
\gamma_{k+l-1} \leq \alpha_k + \beta_l.
\]  

(33)

The eigenvalues of sums of Hermitian matrices have been investigated in [33] [34] [35]. From the above theorem, there is a following inequality

\[
\lambda_{\text{min}} \leq \min \left\{ \min_m \lambda_m + \max_m \hat{\lambda}_m, \max_m \lambda_m + \min_m \hat{\lambda}_m \right\}.
\]  

(34)

The above result is obtained when we set \( k = n \) and \( l = 1 \) in the theorem. With Eq. (34), a lower bound of the maximum SINR is written as

\[
\left\{ \frac{1}{6N} \gamma + \frac{N_0}{2PT} \right\}^{-1/2} \leq \text{SINR}_i^*.
\]  

(35)

where

\[
\gamma = \min \left\{ \min_m \lambda_m + \max_m \hat{\lambda}_m, \max_m \lambda_m + \min_m \hat{\lambda}_m \right\}.
\]  

(36)

From Eqs. (31) and (34), we observe that the maximum SINR is related to the quantities \( \lambda_m \) and \( \hat{\lambda}_m \). These results imply that the maximum SINR is improved if the quantities \( \lambda_m \) and \( \hat{\lambda}_m \) are reduced. Therefore, if the spreading sequences \( s_k \) for \( k \neq i \) are designed to achieve lower \( \lambda_m \) and \( \hat{\lambda}_m \) for \( m = 1, 2, \ldots, N \), then higher SINR is obtained with the optimal sequence for the user \( i, s_i^* \).

V. CONCLUSION

In this paper, we have derived the optimal spreading sequence for the user \( i \), which achieves maximum SINR and maximum capacity under an approximation. It has turned out that the maximum SINR is written as the minimum eigenvalue
of the matrix $\Sigma$ and that the optimal spreading sequence is obtained as a corresponding eigenvector. Further, we have derived the lower and upper bounds of maximum SINR. From these bounds, the maximum SINR will get larger as the quantities $\lambda_{\max}$ and $\lambda_{\min}$ get smaller.

From our results, an iterative procedure to obtain spreading sequences achieving high SNR can be obtained. One of remained issues is to investigate the performance of the optimal spreading sequences and the properties of the sequences.

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