Collective Dynamics of Intrinsic Josephson Junctions in HTSC

Yu.M. Shukrinov$^{1,2}$ and F. Mahfouzi$^3$

$^1$ Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow Region, 141980, Russia
$^2$ Laboratory of Theoretical Physics, Physical Technical Institute, Tajik Academy of Sciences, Dushanbe, 734063, Tajikistan
$^3$ Institute for Advanced Studies in Basic Sciences, P.O.Box 45195-1159, Zanjan, Iran

E-mail: shukrinv@theor.jinr.ru

Abstract. The dynamics of a stack of intrinsic Josephson junctions (IJJ) in the high-$T_c$ superconductors is theoretically investigated with both the quasineutrality breakdown effect and quasiparticle charge imbalance effect taken into account. The current-voltage characteristics (IVC) of IJJ are numerically calculated in the framework of capacitively coupled Josephson junctions model and charge imbalance model including set of differential equations for phase differences, kinetic equations and generalized Josephson relations. We obtain the branch structure in IVC and investigate it as a function of model parameters such as coupling constant, McCumber parameter and number of junctions in the stack. The dependence of branch slopes and branch endpoints on the coupling and disequilibrium parameters are found. We study the nonequilibrium effects created by current injection and show that the increase in the disequilibrium parameter changes essentially the character of IVC. The new features of the hysteresis behavior of IVC of IJJ are obtained.

1. Introduction

The phase dynamics in the IJJ have attracted a great interest because of rich and interesting physics from one side and perspective of applications from the other one [1, 2]. Different couplings between junctions determine a variety of current-voltage characteristics (IVC) observed in HTSC [3, 4, 5, 6, 7, 8]. Individual junctions area, its oxygen content and different coupling barrier layers might play an important role as well [9].

Two theoretical models are widely used to describe IJJ: capacitively coupled Josephson junctions (CCJJ) model [4, 10, 11, 12] and charge imbalance (CIB) model [6, 13, 14]. In CCJJ model a non-vanishing generalized scalar potential appears due to the breaking of charge neutrality, but in CIB model it is related to the quasiparticle charge imbalance as well. Probably both effects exist in HTSC because the thickness of superconducting layers is smaller than the Debye length and the characteristic length of disequilibrium relaxation.

In this paper we study the branch structure in IVC in framework of these two models.
2. Models and Results

In the CCJJ model the dynamics of the gauge-invariant phase difference $\varphi_{l,l+1}$ between superconducting layers $l$ and $l + 1$ is described by the equation

$$\frac{d^2\varphi_{l,l+1}}{d\tau^2} = (1 - \alpha \nabla^{(2)})(J/J_c - \sin(\varphi_{l,l+1})) - \beta \frac{d\varphi_{l,l+1}}{d\tau}$$

where $\alpha$ is the coupling parameter, $J$ and $J_c$ are the external dc current and the Josephson critical current, respectively. The derivative operator $\nabla^{(2)}$ is defined as

$$\nabla^{(2)} f_l = f_{l+1} + f_{l-1} - 2f_l.$$

To obtain the dimensionless form of the equation we have used $\tau = \omega_p t$, $\omega_p^2 = 2eJ_c/\hbar C$ and $\beta = 1/\omega_p RC$, where $R$ is the resistance and $C$ is the capacity of the junctions.

Fig.1a shows IVC for 11 junctions calculated by the equation (1) at $\alpha = 1$, $\beta = 0.1$ under restriction that patterns of distribution of phase rotating junctions are symmetric [5]. We used the boundary conditions due to the proximity effect with $\gamma = s/s_0 = s/s_N$, where $s_0$ and $s_N$ are the thickness of the first and last superconducting layers, respectively. In case of 11 junctions in the stack we have 45 different branches with a different slopes and different number of rotating junctions [15].

Fig.1b shows $\alpha$-dependence of the branch’s endpoints. Thin curves show the all possible cases of $\alpha$-dependence; thick curves demonstrate two examples of the broken dependence for the branches 6 and 28.

In the CIB model the dynamics of the gauge-invariant phase difference $\varphi_{l,l+1}$ between superconducting layers $l$ and $l + 1$ is described by the equation

$$\frac{d^2\varphi_{l,l+1}}{d\tau^2} = (1 - \alpha \nabla^{(2)})(J/J_c - \sin(\varphi_{l,l+1}) - \psi_l - \psi_{l+1} - \beta \frac{d\varphi_{l,l+1}}{d\tau}) - \frac{\dot{\psi}_l - \dot{\psi}_{l+1}}{\beta}$$

Figure 1. The total branch structure in the IVC of 11 IJJ (a) and $\alpha$-dependence of the branch’s endpoints (b). Thin curves in (b) show all possible cases of $\alpha$-dependence; thick curves demonstrate two examples of the broken dependence for the branches 6 and 28.
and kinetic equations

\[ \zeta \dot{\psi}_0 = \eta \gamma d \left( \frac{J}{J_c} - \beta \dot{\phi}_{0,1} + \psi_1 - \psi_0 \right) - \psi_0 \]
\[ \zeta \dot{\psi}_l = \eta \left( \beta (\dot{\phi}_{l-1,l} - \dot{\phi}_{l,l+1}) + \nabla^{(2)} \psi_l \right) - \psi_l \]
\[ \zeta \dot{\psi}_n = \eta \gamma d \left( \beta \dot{\phi}_{n,n+1} - \frac{J}{J_c} + \psi_{n-1} - \psi_n \right) - \psi_n \]  \hfill (3)

Here \( \psi_l \) is the charge imbalance potential on superconducting layers [6] and parameter \( b \) determines the injection of quasiparticles from boundaries. To obtain the voltage we use the generalized Josephson relation

\[ \frac{V_{l,l+1}}{\beta J_c R} = (1 - \alpha \nabla^{(2)})^{-1} (\dot{\psi}_{l,l+1} + \frac{\psi_l - \psi_{l+1}}{\beta}) \]  \hfill (4)

The Laplacian operator on the boundaries is defined as \( \nabla^{(2)} f_{0,1} = f_{1,2} - (1 + \gamma) f_{0,1}, \nabla^{(2)} f_{N-1,N} = f_{N-2,N-1} - (1 + \gamma) f_{N-1,N}. \)

We have solved numerically this system of equations using fourth order Runge-Kutta method in the presence of very small noise with maximum \( 10^{-10} \). The result of simulation of the total branch structure in IVC of 10 IJJ at \( \beta = 0.2 \) and \( \zeta = 0.1 \) is presented in Fig.2, which shows IVC at \( \eta = 0.3, \alpha = 0 \) (left picture) and at \( \eta = 0.1, \alpha = 1 \) (right picture). In first case we demonstrate

![Figure 2](image_url)  \hfill \textbf{Figure 2.} Charge imbalance branch structure of the IVC of 10 IJJ at \( \beta = 0.2 \)

the branching of IJJ without breaking of charge neutrality in superconducting layers (\( \alpha = 0 \)), only due to the charge imbalance effect. The new branches appear and the value of the current endpoints for branches depends a different way on disequilibrium parameter \( \eta \). There are 14 different \( \eta \)-dependencies for the endpoints, which are shown in Fig.3 (right). At \( \eta = 0, \alpha = 0 \) the branch structure of IVC disappears.

In second case (right picture in Fig.2) we demonstrate the branching of IVC of 10 IJJ simultaneously by breaking of charge neutrality and charge imbalance in the superconducting layers. As we can see, at these values of parameters \( \eta = 0.1, \alpha = 1 \) the branch structure is close to the equidistant one and just a small splitting of some branches exists. The splitting of branches with increase of disequilibrium parameter \( \eta \) for the states with one and five junctions in rotating state is shown in Fig.3 (left), where the \( \eta \)-dependence of the branch’s slope for branches
with one and five junctions in R-state presented. We define slope $n$ as $n = V/I$, where the dimensionless voltage $V = V/V_c$ and current $I = I/I_c$ are used. The five branches appear for the states with one rotating junction: lower curve corresponds to the states $R(1)$ and $R(10)$; the upper one corresponds to the states $R(5)$ and $R(6)$. There are 23 branches for the states with five rotating junctions: lower curve corresponds to the states $R(1,2,3,4,5)$ and $R(6,7,8,9,10)$; the upper one corresponds to the states $R(1,3,5,7,9)$ and $R(2,4,6,8,10)$.

In conclusion, the branch structure in IVC in CCJJ and CIB models is essentially different. Detail study of its dependence on model parameters is important for the comparison of theoretical consideration and experimental results and for application of IJJ as superconducting devices [16, 17].

**Acknowledgments**

Yu.M.S. gratefully acknowledges the financial support of INTAS grant No. 01-0617. We thank N.M.Plakida, Y.Sobouti, M.R.H.Khajehpour for useful discussion and cooperation.

**3. References**

[1] R. Kleiner, F. Steinmeyer, G. Kunkel and P. Muller, Phys. Rev. Lett. 68, 2394 (1992).
[2] G. Oya, N. Aoyama, A. Irie, S. Kishida, and H. Tokutaka, Jpn. J. Appl. Phys., 31, L829 (1992).
[3] L. N. Bulaevskii, D. Dominguez, M. Maley, A. Bishop, and B. Ivlev, Phys. Rev. B 53, 14601 (1996).
[4] T. Koyama and M. Tachiki, Phys. Rev. B 54, 16183 (1996).
[5] M. Machida, T. Koyama, and M. Tachiki, Phys. Rev. Lett. 83, 4816 (1999).
[6] D. A. Ryndyk, Phys. Rev. Lett. 80, 3376 (1998); D. A. Ryndyk, J. Keller, and C. Helm, J. Phys.: Condens. Matter 14, 815 (2002).
[7] Ch. Helm, Ch. Preis, F. Forsthofer, J. Keller, K. Schleenger, R. Kleiner, and P. Muller, Phys. Rev. Lett. 79, 737 (1997); Ch. Preis, Ch. Helm, K. Schmalzl, J. Keller, R. Kleiner, and P. Muller, Physica C 362, 51 (2001).
[8] Yu.M. Shukrinov, Kh. Nasrulaev, M. Sargolzaei, G. Oya, A. Irie, Supercond. Sci. Technol., 15, 178, (2002).
[9] P. Seidel, A. N. Grib, Yu. M. Shukrinov, J. Scherbel, U. Hubner, F. Schmid, Physica C362, 102, (2001).
[10] M. Machida, T. Koyama and M. Tachiki, Physica C300, 55 (1998).
[11] H. Matsumoto, S. Sakamoto, F. Wajima, T. Koyama, M. Machida, Phys. Rev. B 60, 3666 (1999).
[12] M. Machida, T. Koyama, Phys. Rev. B 70, 024523 (2004).
[13] S. Artemenko and A. Kobelkov, Phys. Rev. Lett. 78, 3551 (1997).
[14] D. A. Ryndyk, J. Keller, Phys. Rev. B 71, 054507 (2005).
[15] Yu. M. Shukrinov and F. Mahfouzi, submitted to Physica C.
[16] A. Barone, G. Paterno, Physics and Applications of the Josephson Effect, John Wiley and Sons, 1982.
[17] K. K. Likharev, Introduction to the Dynamics of Josephson Junctions, Nauka, 1985.