The measurable angular distribution of
\[ \Lambda_0^b \to \Lambda_c^+(\to \Lambda^0\pi^+)\tau^-(\to \pi^-\nu\bar{\nu}) \] decay

Quan-Yi Hu\(^1\), Xin-Qiang Li\(^2\), Ya-Dong Yang\(^2\) and Dong-Hui Zheng\(^2\)

\(^1\)School of Physics and Electrical Engineering, Anyang Normal University, Anyang, Henan 455000, China
\(^2\)Institute of Particle Physics and Key Laboratory of Quark and Lepton Physics (MOE), Central China Normal University, Wuhan, Hubei 430079, China

Abstract

In \( \Lambda_0^b \to \Lambda_c^+(\to \Lambda^0\pi^+)\tau^-(\to \pi^-\nu\bar{\nu}) \) decay, the solid angle of the final-state particle \( \tau^- \) cannot be determined precisely since the decay products of the \( \tau^- \) include an undetected \( \nu\bar{\nu} \). Therefore, the angular distribution of this decay cannot be measured. In this work, we construct a measurable angular distribution by considering the subsequent decay \( \tau^- \to \pi^-\nu\bar{\nu} \). The full cascade decay is \( \Lambda_0^b \to \Lambda_c^+(\to \Lambda^0\pi^+)\tau^-(\to \pi^-\nu\bar{\nu}) \). The three-momenta of the final-state particles \( \Lambda^0, \pi^+, \) and \( \pi^- \) can be measured. Considering all Lorentz structures of the new physics (NP) effective operators and an unpolarized initial \( \Lambda_b \) state, the five-fold differential angular distribution can be expressed in terms of ten angular observables \( \mathcal{K}_i(q^2, E_\pi) \). By integrating over some of the five kinematic parameters, we define a number of observables, such as the \( \Lambda_c \) spin polarization \( P_{\Lambda_c}(q^2) \) and the forward-backward asymmetry of \( \pi^- \) meson \( A_{FB}(q^2) \), both of which can be represented by the angular observables \( \hat{\mathcal{K}}_i(q^2) \). We provide numerical results for the entire set of the angular observables \( \hat{\mathcal{K}}_i(q^2) \) and \( \mathcal{K}_i \) both in the Standard Model and in the NP benchmark point, and find that the NP which can resolve the anomalies in \( \bar{B} \to D(\ast)\tau^-\nu\bar{\nu} \) decays has obvious effects on the angular observables \( \hat{\mathcal{K}}_i(q^2) \) and \( \mathcal{K}_i \) with \( i = 1c, 2ss, 2cc, 4sc, 4s \).
1 Introduction

The anomalous measurements [1–9] on $B \to D^{(*)}\tau^\pm\bar{\nu}_\tau$ decays indicate the existence of new physics (NP) that breaks the universality of lepton flavour in $b \to c\tau^\pm\bar{\nu}_\tau$ transition. At the typical energy scale $\mu \simeq m_b$, the $b$-hadron decays involving $b \to c\tau^\pm\bar{\nu}_\tau$ transition are governed by the following effective Hamiltonian:

$$\mathcal{H}_{\text{eff}} = \sqrt{2} G_F V_{cb} [g_V (\bar{c}\gamma^\mu b)(\bar{\tau}\gamma_\mu \nu_{\tau L}) + g_A (\bar{c}\gamma^\mu \gamma_5 b)(\bar{\tau}\gamma_\mu \nu_{\tau L})]$$

$$+ g_S (\bar{c}b)(\bar{\tau}\nu_{\tau L}) + g_P (\bar{c}\gamma_5 b)(\bar{\tau}\nu_{\tau L})$$

$$+ g_T (\bar{c}\sigma^{\mu\nu}(1 - \gamma_5)b)(\bar{\tau}\sigma_{\mu\nu}\nu_{\tau L}) + \text{H.c.},$$

(1.1)

where $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$. $\nu_{\tau L} = P_L \nu_\tau$ is the field of left-handed neutrino. The left-handed chirality projector $P_L = (1 - \gamma_5)/2$. In the Standard Model (SM), the Wilson coefficients satisfy $g_V = -g_A = 1$ and $g_S = g_P = g_T = 0$. To understand the anomalies in $B \to D^{(*)}\tau^\pm\bar{\nu}_\tau$ decays, a number of global analyses have been carried out [12–17], finding that some different combinations of Wilson coefficients can well explain these anomalies. In addition, a large number of studies have been done in some specific NP models, such as leptoquarks, $R$-parity violating supersymmetric models, charged Higgses, and charged vector bosons; see for instance Refs. [18–39]. In these NP scenarios, the $\Lambda^0_b \to \Lambda^{+}_c \tau^\mp\bar{\nu}_\tau$ decay, which is also governed by the $b \to c\tau^\pm\bar{\nu}_\tau$ transition, will receive contributions from the NP.

The baryonic decay $\Lambda^0_b \to \Lambda^{+}_c \tau^\mp\bar{\nu}_\tau$ could be useful to confirm possible Lorentz structures of the NP effective operators and to distinguish the specific NP models. Due to the spin-half nature of $\Lambda_c$ and $\Lambda^0_b$ baryons, all the effective operators in Eq. (1.1) can affect $\Lambda^0_b \to \Lambda^{+}_c \tau^\mp\bar{\nu}_\tau$ decay. But for the mesonic counterparts, the operators $(\bar{c}\gamma^\mu \gamma_5 b)(\bar{\tau}\gamma_\mu \nu_{\tau L})$ and $(\bar{c}\gamma_5 b)(\bar{\tau}\nu_{\tau L})$ cannot affect the $B \to D$ processes, and operator $(\bar{c}b)(\bar{\tau}\nu_{\tau L})$ cannot affect the $B \to D^*$ processes. The large production cross section of $\Lambda^0_b$ on the LHC and the clear $\Lambda^0_b \to \Lambda^{+}_c \tau^\mp\bar{\nu}_\tau$ transition form factors [40–44] make $\Lambda^0_b \to \Lambda^{+}_c \tau^\mp\bar{\nu}_\tau$ a good candidate to complement the $B \to D^{(*)}\tau^\pm\bar{\nu}_\tau$ decays. In the previous studies of the NP contributions in $\Lambda^0_b \to \Lambda^{+}_c \tau^\mp\bar{\nu}_\tau$ decay [13, 45–50], especially in some studies considering the angular distribution of the cascade decay $\Lambda^0_b \to \Lambda^{+}_c \to \Lambda^{0}\pi^\mp\tau^\pm\bar{\nu}_\tau$ [51–54], the information of the polar and azimuthal angles ($\theta_\tau, \phi_\tau$) of the final-state particle $\tau^\pm$ may

---

$^1$In this work, we only consider left-handed neutrinos. The effective Hamiltonian containing right-handed neutrinos can be found in Refs. [10, 11]. It can be derived from the identity $\sigma^{\mu\nu} \gamma_5 = -\frac{i}{2} \epsilon^{\mu\nu\alpha\beta} \sigma_{\alpha\beta}$ that the operator $(\bar{c}\sigma^{\mu\nu}(1 + \gamma_5)b)(\bar{\tau}\sigma_{\mu\nu}\nu_{\tau L})$ is absent. We use the convention $\epsilon_{0123} = -\epsilon^{0123} = 1$. 

2
be used. However, as pointed out in Ref. [55], the polar and azimuthal angles \( (\theta_\tau, \phi_\tau) \) cannot be determined precisely since the decay products of the \( \tau^- \) include an undetected \( \nu_\tau \). Therefore, in this work, we construct a measurable angular distribution by considering the subsequent decay \( \tau^- \rightarrow \pi^- \nu_\tau \). The full cascade decay is \( \Lambda_0^b \rightarrow \Lambda_0^c (\rightarrow \Lambda^0 \pi^+ \tau^-(\rightarrow \pi^- \nu_\tau) \bar{\nu}_\tau) \), which includes two undetected final-state particles \( \nu_\tau \) and \( \bar{\nu}_\tau \), as well as three visible final-state particles whose three momenta can be measured: \( \Lambda^0, \pi^+, \) and \( \pi^- \).

Our paper is organized as follows. In Section 2, we define the independent transverse amplitudes and give the analytical results of the measurable angular distribution of the five-body decay \( \Lambda_0^b \rightarrow \Lambda_0^c (\rightarrow \Lambda^0 \pi^+ \tau^-(\rightarrow \pi^- \nu_\tau) \bar{\nu}_\tau) \), with an unpolarized \( \Lambda_0^b \). Discussions of the integrated observables are included in Section 3. The numerical analyses and results are shown in Section 4. Our conclusions are finally made in Section 5. In the Appendix A, we present the detailed calculation procedures and some conventions.

2 Analytical results

In this section, we directly list the analytical results of the angular distribution. The detailed calculation procedures are presented in the Appendix A.

2.1 Transverse amplitudes

In order to get the compact form of the analytical results, we adopt the helicity-based definition of the \( \Lambda_b \rightarrow \Lambda_c \) form factors [56], which are given in Ref. [41]. The matrix elements of vector and axial vector currents can be expressed by six helicity form factors \( F_+^+, F_-^+, F_0^+, G_+^+, G_-^+, G_0^+ \). Using the Ward-like identity for the \( \Lambda_b \rightarrow \Lambda_c \) matrix elements

\[
\langle \Lambda_c | \bar{c} (\gamma_5) b | \Lambda_b \rangle = \frac{q_\mu}{m_b \mp m_c} \langle \Lambda_c | \bar{c} (\gamma_\mu) \bar{b} | \Lambda_b \rangle,
\]

the matrix elements of scalar and pseudoscalar currents can be written in terms of \( F_0^+ \) and \( G_0^+ \), respectively. In the absence of the tensor operator, we can define six independent transverse amplitudes as follows

\[
A_{\perp t} = A_{\perp t}^{SP} + \frac{m_\tau}{\sqrt{q^2}} A_{\perp t}^{VA},
\]

\[
A_{\parallel t} = A_{\parallel t}^{SP} + \frac{m_\tau}{\sqrt{q^2}} A_{\parallel t}^{VA},
\]
\[ A_{\perp 1} = -2 F_{\perp} \sqrt{Q} - g V, \]  
(2.4)  
\[ A_{\parallel 1} = -2 G_{\perp} \sqrt{Q} + g A, \]  
(2.5)  
\[ A_{\perp 0} = F_{\perp} \frac{m_{\Lambda_b} - m_{\Lambda_c}}{\sqrt{Q}} g V, \]  
(2.6)  
\[ A_{\parallel 0} = G_{\perp} \frac{m_{\Lambda_b} - m_{\Lambda_c}}{\sqrt{Q}} g A. \]  
(2.7)

Here, \( \perp \) and \( \parallel \) stand for the different transverse states. The subscript \( t \) represents time-like \( \tau^- \bar{\nu}_\tau \) state; the subscripts 1 and 0 denote the magnitude of the \( z \)-component of the \( \tau^- \bar{\nu}_\tau \) angular momentum in the vector \( \tau^- \bar{\nu}_\tau \) state. \( Q_{\pm} \equiv (m_{\Lambda_b} \pm m_{\Lambda_c})^2 - q^2 \). The time-like transverse amplitudes \( A_{SP}^{\perp t} \), \( A_{SP}^{\parallel t} \), \( A_{VA}^{\perp t} \), and \( A_{VA}^{\parallel t} \) are respectively defined as

\[ A_{SP}^{\perp t} = F_0 \frac{m_{\Lambda_b} - m_{\Lambda_c}}{\sqrt{Q}} g S, \quad A_{SP}^{\parallel t} = -G_0 \frac{m_{\Lambda_b} + m_{\Lambda_c}}{m_b + m_c} g P, \]  
(2.8)  
\[ A_{VA}^{\perp t} = F_0 \frac{m_{\Lambda_b} - m_{\Lambda_c}}{\sqrt{Q}} g V, \quad A_{VA}^{\parallel t} = G_0 \frac{m_{\Lambda_b} + m_{\Lambda_c}}{\sqrt{Q}} g A. \]  
(2.9)

The matrix elements of the tensor currents can be expressed by four helicity form factors \( h_+, h_\perp, \tilde{h}_+, \) and \( \tilde{h}_\perp \), and we need to define four additional independent transverse amplitudes as follows

\[ A_{T}^{\perp 1} = 4 h_{\perp} \sqrt{Q} \frac{m_{\Lambda_b} + m_{\Lambda_c}}{\sqrt{q^2}} g T, \]  
(2.10)  
\[ A_{T}^{\parallel 1} = 4 \tilde{h}_{\perp} \sqrt{Q} \frac{m_{\Lambda_b} - m_{\Lambda_c}}{\sqrt{q^2}} g T, \]  
(2.11)  
\[ A_{T}^{\perp 0} = -2 h_{\perp} \sqrt{Q} - g T, \]  
(2.12)  
\[ A_{T}^{\parallel 0} = -2 \tilde{h}_{\perp} \sqrt{Q} + g T. \]  
(2.13)

The superscript \( T \) indicates that an amplitude arises only when there are tensor operators.

### 2.2 Angular distribution

The measurable angular distribution of the five-body \( \Lambda^0_b \rightarrow \Lambda_c^+ \rightarrow \Lambda^0 \pi^+ \rightarrow \pi^- \rightarrow \tau^- \bar{\nu}_\tau \) decay, with an unpolarized \( \Lambda^0_b \), is described by the \( \tau^- \bar{\nu}_\tau \) invariant mass squared \( q^2 \); the helicity angle of \( \Lambda^0_b \) baryon in the \( \Lambda_c^+ \) rest frame, \( \theta_{\Lambda_c} \); as well as the energy, the polar angle, and the azimuthal angle of \( \pi^- \) in the \( \tau^- \bar{\nu}_\tau \) center-of-mass frame, \( E_\pi, \theta_\pi, \) and \( \phi_\pi \). For more details, we refer to
Table 1: Contributions of the NP Wilson coefficients to the various transverse amplitudes.

| Transverse Amplitudes | Couplings |
|-----------------------|-----------|
| \(A_{\perp t}\), \(A_{\perp 0}\) | \(g_V\) |
| \(A_{\| t}\), \(A_{\| 0}\) | \(g_A\) |
| \(A_{\perp t}\) | \(g_S\) |
| \(A_{\perp t}\) | \(g_P\) |
| \(A_{\perp t}\) | \(g_V, g_S\) |
| \(A_{\perp t}\) | \(g_A, g_P\) |
| \(A_{\| t}\), \(A_{\| 0}\), \(A_{\perp 0}\), \(A_{\| 0}\) | \(g_T\) |

Figure 1: Definition of the angles in the unpolarized \(\Lambda^0\) \(\to\Lambda^+\)(\(\to\Lambda^0\pi^+\))\(\pi^-(\to\pi^-\nu_\tau)\bar{\nu}_\tau\) decay. Figure 1 and the Appendix A. The five-fold differential decay rate can then be written as

\[
\frac{d^5\Gamma}{dq^2dE_\pi d\cos \theta_\pi d\phi_\pi d\cos \theta_\Lambda} = \frac{G_F^2 |V_{cb}|^2 |p_{\Lambda_c}|^2 (q^2)^{3/2}m^2}{256\pi^4m_{\Lambda_b}^2 (m^2 - m_{\pi}^2)^2} B(\tau \to \pi^-\nu_\tau) B(\Lambda_c \to \Lambda\pi^+) \\
\times K(q^2, E_\pi, \cos \theta_\Lambda, \cos \theta_\pi, \phi_\pi),
\]

where \(|p_{\Lambda_c}| = \sqrt{Q^+Q^-/(2m_{\Lambda_c})}\) is the magnitude of the \(\Lambda_c\) three-momentum in the \(\Lambda_b\) rest frame. By rearranging \(N^{iS}_i |A_i|^2, N^{R}_{i,j} \text{Re}[A_i A_j^\ast], \text{and } N^{I}_{i,j} \text{Im}[A_i A_j^\ast]\) pieces of Eq. (A.51), which are listed in Table 3, 4, and 5, the angular distribution \(K\) can be expressed as a set of trigonometric functions as follows

\[
K(q^2, E_\pi, \cos \theta_\Lambda, \cos \theta_\pi, \phi_\pi) = \sum_{i=1}^{10} \mathcal{K}_i(q^2, E_\pi) \Omega_i(\cos \theta_\Lambda, \cos \theta_\pi, \phi_\pi) \\
\equiv (K_{1ss} \sin^2 \theta_\pi + K_{1cc} \cos^2 \theta_\pi + K_{4c} \cos \theta_\pi)
\]
\begin{align*}
+ (K_{2ss} \sin^2 \theta_\pi + K_{2cc} \cos^2 \theta_\pi + K_{2c} \cos \theta_\pi) \cos \theta_\Lambda \\
+ (K_{3sc} \sin \theta_\pi \cos \theta_\pi + K_{3s} \sin \theta_\pi) \sin \theta_\Lambda \sin \phi_\pi \\
+ (K_{4sc} \sin \theta_\pi \cos \theta_\pi + K_{4s} \sin \theta_\pi) \sin \theta_\Lambda \cos \phi_\pi,
\end{align*}

(2.15)

where the ten angular observables \( \mathcal{K}_i(q^2, E_\pi) \) can be completely expressed by the transverse amplitudes, the dimensionless factors (see Eqs. (A.53)–(A.64)), and the asymmetry parameter \( \alpha_{A_c} \) (see Eq. (A.38)) as follows

\begin{align*}
\mathcal{K}_{1ss} &= \left[ S_1 |A_{\perp_1}|^2 + (S_1 - S_3) |A_{\perp_3}|^2 + (S_1 + S_3) |A_{\perp_0}|^2 \\
&+ (S_1^T - S_3^T) |A_{\perp_1}^T|^2 + (S_1^T + S_3^T) |A_{\perp_0}^T|^2 + (\perp\leftrightarrow\parallel) \right] \\
&+ \text{Re} \left[ (R_1 - R_3) A_{\perp_1} A_{\perp_1}^* + (R_1 + R_3) A_{\perp_0} A_{\perp_0}^* + (\perp\leftrightarrow\parallel) \right], \\
\mathcal{K}_{1cc} &= \left[ S_1 |A_{\perp_1}|^2 + (S_1 + S_3) |A_{\perp_1}|^2 + (S_1 - 3S_3) |A_{\perp_0}|^2 \\
&+ (S_1^T + S_3^T) |A_{\perp_1}^T|^2 + (S_1^T - 3S_3^T) |A_{\perp_0}^T|^2 + (\perp\leftrightarrow\parallel) \right] \\
&+ \text{Re} \left[ (R_1 + R_3) A_{\perp_1} A_{\perp_1}^* + (R_1 - 3R_3) A_{\perp_0} A_{\perp_0}^* + (\perp\leftrightarrow\parallel) \right], \\
\mathcal{K}_{1c} &= 2 \text{Re} \left[ S_2 A_{\perp_1} A_{\perp_1}^T + S_2^T A_{\perp_1}^T A_{\perp_1}^T \right] \\
&+ \text{Re} \left[ R_2 A_{\perp_1} A_{\perp_1}^T - \sqrt{2} R_t A_{\perp_1} A_{\perp_0}^* - \sqrt{2} R_t^T A_{\perp_1} A_{\perp_0}^* + (\perp\leftrightarrow\parallel) \right], \\
\mathcal{K}_{2ss} &= 2 \alpha_{A_c} \left[ S_1 A_{\perp_1} A_{\perp_1}^* + (S_1 - S_3) A_{\perp_1} A_{\perp_0}^* + (S_1 + S_3) A_{\perp_0} A_{\perp_0}^* \\
&+ (S_1^T - S_3^T) A_{\perp_1}^T A_{\perp_1}^T + (S_1^T + S_3^T) A_{\perp_0}^T A_{\perp_0}^T \right] \\
&+ \alpha_{A_c} \left[ (R_1 + R_3) A_{\perp_1} A_{\perp_0} A_{\perp_0}^* + (R_1 - 3R_3) A_{\perp_0} A_{\perp_0} A_{\perp_0}^* + (\perp\leftrightarrow\parallel) \right], \\
\mathcal{K}_{2cc} &= 2 \alpha_{A_c} \left[ S_1 A_{\perp_1} A_{\perp_1}^* + (S_1 + S_3) A_{\perp_1} A_{\perp_0}^* + (S_1 - 3S_3) A_{\perp_0} A_{\perp_0}^* \\
&+ (S_1^T + S_3^T) A_{\perp_1}^T A_{\perp_1}^T + (S_1^T - 3S_3^T) A_{\perp_0}^T A_{\perp_0}^T \right] \\
&+ \alpha_{A_c} \left[ (R_1 + R_3) A_{\perp_1} A_{\perp_0} A_{\perp_0}^* + (R_1 - 3R_3) A_{\perp_0} A_{\perp_0} A_{\perp_0}^* + (\perp\leftrightarrow\parallel) \right], \\
\mathcal{K}_{2c} &= \alpha_{A_c} \left[ S_2 |A_{\perp_1}|^2 + S_2^T |A_{\perp_1}^T|^2 + (\perp\leftrightarrow\parallel) \right] \\
&+ \alpha_{A_c} \left[ R_2 A_{\perp_1} A_{\perp_1}^T - \sqrt{2} R_t A_{\perp_1} A_{\perp_0}^* - \sqrt{2} R_t^T A_{\perp_1} A_{\perp_0}^* + (\perp\leftrightarrow\parallel) \right], \\
\mathcal{K}_{3sc} &= 2 \sqrt{2} \alpha_{A_c} \left[ 2S_3 A_{\perp_1} A_{\perp_0}^* + 2S_3^T A_{\perp_1}^T A_{\perp_1}^T \right]
\end{align*}
\[+ R_3 A_{\perp_1} A^T_{\perp_0} - R_3 A_{\perp_0} A^T_{\perp_1} - (\perp \leftrightarrow \|) \]

\[K_{3s} = - \frac{\alpha_s}{\sqrt{2}} \text{Im} \left[ \sqrt{2} R_t A_{\perp_1} A^*_{\perp_1} + \sqrt{2} R_t A_{\perp_1} A^T_{\perp_1} + 2 S_2 A_{\perp_1} A^*_{\perp_0} + R_2 A_{\perp_1} A^T_{\perp_0} + R_2 A_{\perp_0} A^T_{\perp_1} + 2 S_2 A_{\perp_1} A^*_{\perp_0} - (\perp \leftrightarrow \|) \right], \tag{2.22}

\[K_{4sc} = 2 \sqrt{2} \alpha_s \text{Re} \left[ R_3 A_{\perp_0} A^T_{\perp_1} - 2 S_3 A_{\perp_1} A^*_{\perp_0} - R_3 A_{\perp_1} A^T_{\perp_0} - 2 S_2 A_{\perp_1} A^*_{\perp_0} - (\perp \leftrightarrow \|) \right], \tag{2.23}

\[K_{4s} = \frac{\alpha_s}{\sqrt{2}} \text{Im} \left[ \sqrt{2} R_t A_{\perp_1} A^*_{\perp_1} + \sqrt{2} R_t A_{\perp_1} A^T_{\perp_1} + 2 S_2 A_{\perp_1} A^*_{\perp_0} + R_2 A_{\perp_1} A^T_{\perp_0} + R_2 A_{\perp_0} A^T_{\perp_1} + 2 S_2 A_{\perp_1} A^*_{\perp_0} - (\perp \leftrightarrow \|) \right]. \tag{2.24}

The time-like pieces of our results can be completely formulated by transverse amplitudes \(A_{\perp_1}\) and \(A_{\parallel_1}\), without using \(A^{SP}_{\perp_1}\), \(A^{SP}_{\parallel_1}\), \(A^{VA}_{\perp_1}\), and/or \(A^{VA}_{\parallel_1}\). This is consistent with the Ward-like relation (2.1). We can obtain the differential decay rate \(d\Gamma/dq^2\) as a function of \(q^2\) by integrating over \(E_\pi, \cos \theta_A, \cos \theta_\pi\), and \(\phi_\pi\). Apart from the factors \(B(\tau \to \pi^-\nu_\tau)\) and \(B(\Lambda_c \to \Lambda\pi^+)\), we find that our results of \(d\Gamma/dq^2\) (see Eq. (3.5)) are complete agreement with those in Ref. [41], which discusses the \(\Lambda_b \to \Lambda_c\bar{\tau}\bar{\nu}_\tau\) decay in the presence of all dimension-six operators.

In the SM, \(g_V = -g_A = 1\) and \(g_S = g_P = g_T = 0\), the angular observables \(K_{3sc}\) and \(K_{3s}\) are vanishing. Therefore, a non-vanishing \(K_{3sc}\) or \(K_{3s}\) indicates that there is NP effect, which induces a complex contribution to the amplitude.

- Suppose that the angular distribution is found to contain the component \(\sin \theta_\pi \cos \theta_\pi \sin \theta_A \sin \phi_\pi\).

This indicates that at least one of \(\text{Im}[A_{\perp_1} A^T_{\perp_1}]\), \(\text{Im}[A_{\perp_0} A^T_{\perp_1}]\), \(\text{Im}[A_{\parallel_1} A^T_{\parallel_1}]\), and/or \(\text{Im}[A_{\parallel_0} A^T_{\parallel_1}]\) is nonzero, which implies that \(g_T \neq 0\), and that the \(g_T\) has a different phase than \(g_V\) or \(g_A\).

- Suppose that the angular distribution is found to contain the component \(\sin \theta_\pi \sin \theta_A \sin \phi_\pi\).

This indicates that the imaginary part of at least one of \(g_S g_V^\dagger\), \(g_P g_A^\dagger\), \(g_V g_A^\dagger\), \(g_S g_T^\dagger\), \(g_P g_T^\dagger\), \(g_V g_T^\dagger\), and \(g_A g_T^\dagger\) is not equal to zero.
3 Observables

The five-fold differential decay rate of $\Lambda_b^0 \rightarrow \Lambda^+_c (\rightarrow \Lambda^0 \pi^+) \tau^- (\rightarrow \pi^- \nu_\tau) \bar{\nu}_\tau$ decay depends on five measurable kinematic parameters $q^2$, $E_\pi$, $\theta_\Lambda$, $\theta_\pi$, and $\phi_\pi$, and a complete experimental analysis may be limited by statistics. By integrating some kinematic parameters, abundant observables can be constructed.

3.1 Hadron-side observables

By integrating over the lepton-side kinematic parameters $E_\pi$, $\theta_\pi$, and $\phi_\pi$, we can obtain the two-fold differential decay rate as follows

$$\frac{d^2\Gamma}{dq^2 d \cos \theta_\Lambda} = \frac{1}{2} \frac{d\Gamma}{dq^2} \left[ 1 + \alpha_{\Lambda_c} P_{\Lambda_c}(q^2) \cos \theta_\Lambda \right]. \quad (3.1)$$

Here, $P_{\Lambda_c}(q^2)$ represents the $\Lambda_c$ spin polarization, which is defined as

$$P_{\Lambda_c}(q^2) \equiv \frac{d\Gamma^+_{\Lambda_c=1/2}/dq^2 - d\Gamma^-_{\Lambda_c=-1/2}/dq^2}{d\Gamma^+_{\Lambda_c=1/2}/dq^2 + d\Gamma^-_{\Lambda_c=-1/2}/dq^2}. \quad (3.2)$$

The differential decay rates for the polarized intermediate state $\Lambda_c$ baryon are given by

$$\frac{d\Gamma_{\Lambda_c=\pm 1/2}}{dq^2} = N \left( A_0^+ + A_1^+ \kappa_\tau + A_2^+ \kappa_\tau^2 \right), \quad (3.3)$$

where $A_0^+ \equiv \frac{3}{2} |A_{+1} \pm A_{||}|^2 + |A_{\perp 1} \pm A_{||}|^2 + |A_{\perp 0} \pm A_{||}|^2$

$$+ 2 |A^T_{\perp 1} \pm A^T_{||}|^2 + 2 |A^T_{\perp 0} \pm A^T_{||}|^2,$$

$$A_1^+ \equiv -6 \text{Re} \left[ (A_{+1} \pm A_{||}) (A^T_{+1} \pm A^T_{||})^* (A_{\perp 0} \pm A_{||}) (A^T_{\perp 0} \pm A^T_{||})^* \right],$$

$$A_2^+ \equiv \frac{1}{2} |A_{+1} \pm A_{||}|^2 + \frac{1}{2} |A_{\perp 0} \pm A_{||}|^2 + 4 |A^T_{+1} \pm A^T_{||}|^2 + 4 |A^T_{\perp 0} \pm A^T_{||}|^2,$$

where the dimensionless parameter $\kappa_\tau \equiv m_\tau/\sqrt{q^2}$, and the factor

$$N \equiv \frac{G_F^2 |V_{cb}|^2 |p_{\Lambda_c}| q^2}{384 \pi^3 m_{\Lambda_c}^2} \left( 1 - \kappa_\tau^2 \right)^2 B(\tau \rightarrow \pi^- \nu_\tau) B(\Lambda_c \rightarrow \Lambda \pi^+). \quad (3.4)$$

Further integrating over the variable $\theta_\Lambda$, we can obtain the following differential decay rate depending only on $q^2$,

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma^+_{\Lambda_c=1/2}}{dq^2} + \frac{d\Gamma^-_{\Lambda_c=-1/2}}{dq^2}.$$
\[ \mathcal{N} \left[ (A_0^+ + A_0^-) + (A_1^+ + A_1^-) \kappa_\tau + (A_2^+ + A_2^-) \kappa_\tau^2 \right]. \]  

(3.5)

Our \(d\Gamma/dq^2\) (apart from \(\mathcal{B}(\tau \to \pi^- \nu_\tau)\mathcal{B}(\Lambda_c \to \Lambda\pi^+)\)) is consistent with that in Ref. [41], which has also been checked by Refs. [13, 53]. Since we have integrated over all the lepton-side kinematic parameters, the observables constructed above are not affected by \(\tau\) decay dynamics, so they are also applicable to light leptons \(\ell = \mu, e\) (Necessary replacement \(m_\tau \to m_\ell\) and removal of factor \(\mathcal{B}(\tau \to \pi^- \nu_\tau)\) are required). The universality of lepton flavor can be tested by comparing the predicted values of observables \(d\Gamma/dq^2\) or \(P_{\Lambda_c}(q^2)\) of \(\tau\) and \(\ell\).

### 3.2 Lepton-side observables

By integrating over the hadron-side kinematic parameters \(\theta_\Lambda\), and one or two lepton-side kinematic parameters, we can construct a variety of observables. These observables depend on at least one kinematic parameter of \(\pi^-\), so they only exist in \(\tau\) channels, and specifically for the \(\tau \to \pi^- \nu_\tau\) decay.

The differential decay rates for which \(E_\pi\) has not been integrated over can be expressed simply as the angular observables \(K_i\). To reduce the uncertainty of theoretical predictions, we use \(d^3\Gamma/(dq^2dE_\pi)\) to normalize them.

\[
\frac{d^3\Gamma}{dq^2dE_\pi d\cos \theta_\pi} = \frac{3}{2} \frac{d^2\Gamma}{dq^2dE_\pi} \frac{\kappa_1 ss \sin^2 \theta_\pi + \kappa_1 cc \cos^2 \theta_\pi + \kappa_1 c \cos \theta_\pi}{2\kappa_1 ss + \kappa_1 cc},
\]

(3.6)

\[
\frac{d^3\Gamma}{dq^2dE_\pi d\phi_\pi} = \frac{1}{2\pi} \frac{d^2\Gamma}{dq^2dE_\pi} \left(1 + \frac{3\pi^2}{16} \kappa_3 s \sin \phi_\pi + \kappa_4 s \cos \phi_\pi \right),
\]

(3.7)

with

\[
\frac{d^2\Gamma}{dq^2dE_\pi} = \frac{4\kappa_\tau^2}{d^2q^2 \sqrt{(\kappa_\tau^2 - \kappa_\tau^2)^2} (1 - \kappa_\tau^2)^2 \left( (A_0^+ + A_0^-) + (A_1^+ + A_1^-) \kappa_\tau + (A_2^+ + A_2^-) \kappa_\tau^2 \right)}.
\]

(3.8)

The forward-backward asymmetry of the \(\pi^-\) meson can be obtained by the difference between the integrals of the Eq. (3.6) on the interval \([0, \pi/2]\) and \([\pi/2, \pi]\). We can define the following asymmetry \(A_{FB}(q^2, E_\pi)\) as a function of \(q^2\) and \(E_\pi\),

\[
A_{FB}(q^2, E_\pi) = \frac{\int_0^1 dq^2 dq^2 d\cos \theta_\pi dq^2 d\cos \theta_\pi dq^2 d\cos \theta_\pi - \int_{-1}^0 dq^2 dq^2 d\cos \theta_\pi dq^2 d\cos \theta_\pi dq^2 d\cos \theta_\pi}{\frac{d^2\Gamma}{dq^2dE_\pi}} \frac{3 \kappa_1 c}{2 \kappa_1 ss + \kappa_1 cc}.
\]

(3.9)
The difference between the integrals of the Eq. (3.7) on the interval \([0, \pi]\) and \([\pi, 2\pi]\) can isolate the angular observable \(K_{3s}\), which is nonzero only if the NP induces a complex contribution to the amplitude.

Further integrating over the \(\pi^-\) energy \(E_\pi\) in Eqs. (3.6) and (3.7), one can obtain the two-fold differential decay rates \(d^2\Gamma/(dq^2d\cos \theta_\pi)\) and \(d^2\Gamma/(dq^2d\phi_\pi)\). Similarly, we can use them to construct asymmetry observables that do not depend on the variable \(E_\pi\). For example, the forward-backward asymmetry of the \(\pi^-\) meson as a function of \(q^2\) can be defined as

\[
A_{FB}(q^2) = \frac{\int_0^1 dq^2 \frac{d^2\Gamma}{dq^2d\cos \theta_\pi} d\cos \theta_\pi - \int_{-1}^0 dq^2 \frac{d^2\Gamma}{dq^2d\cos \theta_\pi} d\cos \theta_\pi}{\int dq^2 \frac{d^2\Gamma}{dq^2d\cos \theta_\pi} d\cos \theta_\pi}. \tag{3.10}
\]

This result can also be obtained by integrating over \(E_\pi\) separately in the numerator and denominator in Eq. (3.9).

### 3.3 The angular observables \(\hat{K}_i(q^2)\) and \(\hat{K}_i\)

Starting from five-fold differential decay rate (2.14), integrating over the variable \(E_\pi\), and after proper normalization, we can obtain the following angular function

\[
\hat{K}(q^2, \cos \theta_\Lambda, \cos \theta_\pi, \phi_\pi) = \frac{\int dq^2 dE_\pi d\cos \theta_\Lambda d\cos \theta_\pi dE_\pi}{\int dq^2 dE_\pi dE_\pi dq^2 dE_\pi} \int dq^2 dE_\pi dE_\pi dq^2 dE_\pi \int d\cos \theta_\Lambda d\cos \theta_\pi d\phi_\pi dE_\pi
\]

\[
= \frac{3}{8\pi} \left( \sum_{i=1}^{10} \hat{K}_i(q^2) \Omega_i(\cos \theta_\Lambda, \cos \theta_\pi, \phi_\pi) \right), \tag{3.11}
\]

with the angular observables \(\hat{K}_i(q^2)\) given by

\[
\hat{K}_i(q^2) = \frac{\int K_i(q^2, E_\pi) dE_\pi}{\int (2K_{1ss} + K_{1cc}) dE_\pi}. \tag{3.12}
\]

Comparing Eqs. (3.9), (3.10) and (3.12), we can easily get the \(A_{FB}(q^2) = \frac{3}{2} \hat{K}_{1c}(q^2)\). The spin polarization of \(\Lambda_c\) baryon satisfies \(\alpha_{\Lambda_c} P_{\Lambda_c}(q^2) = 2\hat{K}_{2ss}(q^2) + \hat{K}_{2cc}(q^2)\).

If the variables \(E_\pi\) and \(q^2\) in Eq. (2.14) are simultaneously integrated over, we can obtain the following angular distribution

\[
\hat{K}(\cos \theta_\Lambda, \cos \theta_\pi, \phi_\pi) = \frac{\int dq^2 dE_\pi d\cos \theta_\Lambda d\cos \theta_\pi dE_\pi dq^2}{\int dq^2 dE_\pi dE_\pi dq^2 dE_\pi} \int dq^2 dE_\pi dE_\pi dq^2 \int dq^2 dE_\pi dE_\pi dq^2 \int d\cos \theta_\Lambda d\cos \theta_\pi d\phi_\pi dE_\pi
\]

\[
= \frac{3}{8\pi} \left( \sum_{i=1}^{10} \hat{K}_i \Omega_i(\cos \theta_\Lambda, \cos \theta_\pi, \phi_\pi) \right), \tag{3.13}
\]
Table 2: Predictions for the entire set of angular observables in the SM and in the NP benchmark point. The asymmetry parameter $\alpha_{\Lambda_c}$ is factored out in $\hat{K}_i$ with $i = 2ss$, $2cc$, $2c$, $4sc$, and $4s$. The observable $\hat{K}_{1cc}$ can be obtained as $1 - 2\hat{K}_{1ss}$, and $\hat{K}_{3sc} = \hat{K}_{3s} = 0$.

| observable          | SM       | NP benchmark point |
|---------------------|----------|--------------------|
| $\hat{K}_{1ss}$     | 0.323 ± 0.001 | 0.323 ± 0.001     |
| $\hat{K}_{1c}$      | 0.224 ± 0.005 | 0.243 ± 0.004     |
| $\hat{K}_{2ss}/\alpha_{\Lambda_c}$ | $-0.240 ± 0.004$ | $-0.258 ± 0.004$ |
| $\hat{K}_{2cc}/\alpha_{\Lambda_c}$ | $-0.279 ± 0.005$ | $-0.295 ± 0.004$ |
| $\hat{K}_{2c}/\alpha_{\Lambda_c}$ | $-0.274 ± 0.003$ | $-0.277 ± 0.003$ |
| $\hat{K}_{4sc}/\alpha_{\Lambda_c}$ | $-0.024 ± 0.002$ | $-0.019 ± 0.002$ |
| $\hat{K}_{4s}/\alpha_{\Lambda_c}$ | $-0.149 ± 0.004$ | $-0.122 ± 0.005$ |

Our choice of the normalization in Eq. (3.12) and (3.14) make the first two angular observables exactly satisfy the relationships, $2\hat{K}_{1ss}(q^2) + \hat{K}_{1cc}(q^2) = 1$ and $2\hat{K}_{1ss} + \hat{K}_{1cc} = 1$.

4 Numerical results

The model-independent analyses to study the NP effects in $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ decays have been completed in the previous literatures [12–17]. In order to illustrate our results numerically, we select a best-fit point which is labelled “Min 1b” in Ref. [15] as the NP benchmark point:

$$g_V = -g_A = 1.14, \quad g_S = 0, \quad g_P = 0.18, \quad g_T = 0.02.$$  \hspace{1cm} (4.1)

This point does not provide new CP violating phases, so the angular observables $\mathcal{K}_{3sc}$ and $\mathcal{K}_{3s}$ are still missing. Next, we should discuss the remaining angular observables $\mathcal{K}_i$, including the functions $\hat{K}_i(q^2)$ and the numbers $\hat{K}_i$, in the SM and in the NP respectively. For angular observables $\mathcal{K}_i$ with $i = 2ss$, $2cc$, $2c$, $4sc$, and $4s$, we factor out the asymmetry parameter $\alpha_{\Lambda_c} = -0.82 ± 0.09$ [53], because it can bring great uncertainty to these observables, thus interfering with the emergence of the NP effects.
Figure 2: The angular observables $\hat{K}_i(q^2)$ as a function of $q^2$, predicted both within the SM (central values: blue solid curves, uncertainties: blue bands) and in the NP benchmark point (central values: purple solid curves, uncertainties: purple bands). The asymmetry parameter $\alpha_{\Lambda_c}$ is factored out in $\hat{K}_i(q^2)$ with $i = 2ss, 2cc, 2c, 4sc$, and $4s$. The observable $\hat{K}_{1cc}(q^2)$ can be obtained as $1 - 2\hat{K}_{1ss}(q^2)$, and $\hat{K}_{3sc}(q^2) = \hat{K}_{3s}(q^2) = 0$.

In our numerical analysis, we use the $\Lambda_b \to \Lambda_c$ form factors computed in lattice QCD including all the types of Lorentz structures of the NP effective operators [40, 41]. The results
of the angular observables $\hat{K}_i(q^2)$ as a function of $q^2$ are shown in Figure 2. Since the asymmetry parameter $\alpha_{\Lambda_c}$ has been factored out, the uncertainties of the observables mainly come from the $\Lambda_b \to \Lambda_c$ form factors. Benefiting from the correlation between the uncertainties of the $\Lambda_b \to \Lambda_c$ form factors and from the cancellations through normalization to the decay rate, the predicted angular observables $\hat{K}_i(q^2)$ have small uncertainties. This allows us to discuss the NP effects in them. We find that the NP which can resolve the anomalies in $\bar{B} \to D(\ast)\tau^-\bar{\nu}_\tau$ decays has obvious effects on the angular observables $\hat{K}_{1c}(q^2)$, $\hat{K}_{2ss}(q^2)$, $\hat{K}_{2cc}(q^2)$, $\hat{K}_{4sc}(q^2)$, and $\hat{K}_{4s}(q^2)$, but hardly affects the $\hat{K}_{1ss}(q^2)$ and $\hat{K}_{2c}(q^2)$ and, of course, $\hat{K}_{1cc}(q^2)$, $\hat{K}_{3sc}(q^2)$ and $\hat{K}_{3s}(q^2)$. The values of the corresponding angular observables $\hat{K}_i$ are provided in Table 2.

5 Conclusions

Inspired by the anomalies in $\bar{B} \to D(\ast)\tau^-\bar{\nu}_\tau$ decays, many works have been done to explore possible NP patterns in $b \to c\tau^-\bar{\nu}_\tau$ transition by studying the baryonic counterparts, that is, the $\Lambda_0^b \to \Lambda_+^c\tau^-\bar{\nu}_\tau$ decay or the cascade decay $\Lambda_0^b \to \Lambda_+^c(\to \Lambda^0\pi^+)\tau^-\bar{\nu}_\tau$. Comparing with $\bar{B} \to D(\ast)\tau^-\bar{\nu}_\tau$ decays, the baryonic counterparts could be useful to confirm more possible Lorentz structures of the NP effective operators. However, the angular distribution of them cannot be measured since the solid angle of the final-state particle $\tau^-$ cannot be determined precisely. Therefore, in this work, we further consider the subsequent decay $\tau^- \to \pi^-\nu_\tau$ to construct a measurable angular distribution. The full process is $\Lambda_0^b \to \Lambda_+^c(\to \Lambda^0\pi^+)\tau^-\to \pi^-\nu_\tau$, which includes three visible final-state particles $\Lambda^0$, $\pi^+$, and $\pi^-$ whose three momenta can be measured.

For an unpolarized initial $\Lambda_b$ state, the five-fold differential angular distribution including all Lorentz structures of the NP effective operators can be expressed in terms of ten angular observables $K_i(q^2, E_\pi)$, which can be completely expressed by ten independent transverse amplitudes, the asymmetry parameter $\alpha_{\Lambda_c}$ related to $\Lambda_+^c \to \Lambda^0\pi^+$ decay, and some dimensionless factors given in Eqs. (A.53)–(A.64). Our results are consistent with the Ward-like relation, and when the transverse states $\perp$ and $\parallel$ are exchanged, they have good symmetry or asymmetry. We also find that our results of $d\Gamma/dq^2$, which can be obtained by integrating over the kinematic parameters $E_\pi$, $\cos\theta_\Lambda$, $\cos\theta_\pi$, and $\phi_\pi$, are complete agreement with those in Ref. [41]. Based on these, we believe that our results are correct.

If the angular distribution is found to contain the nonzero component $\sin\theta_\pi\cos\theta_\pi\sin\theta_\Lambda\sin\phi_\pi$,
this will be an unquestionable sign of the NP, indicating that the tensor operator must exist and that the corresponding Wilson coefficient $g_T$ has a different weak phase than $g_V$ or $g_A$.

We obtain a number of observables by integrating over some of the five kinematic parameters. On the hadron side, there are the $\Lambda_c$ spin polarization $P_{\Lambda_c}(q^2)$ and certainly the differential decay rate $d\Gamma/dq^2$. Since all the lepton-side kinematic parameters have been integrated over, these observables are not affected by $\tau^{-}$ decay dynamics, so their expressions are applicable to light leptons $\ell = \mu, e$ (Necessary replacement $m_\tau \rightarrow m_\ell$ and removal of factor $B(\tau \rightarrow \pi^-\nu_\tau)$ are required). On the lepton side, there are the three-fold differential angular distributions $d^3\Gamma/(dq^2dE_\pi d\cos \theta_\pi)$ and $d^3\Gamma/(dq^2dE_\pi d\phi_\pi)$, and the two-fold differential decay rate $d^2\Gamma/(dq^2dE_\pi)$, as well as the $\pi^-$ meson forward-backward asymmetry $A_{FB}(q^2)$. These observables depend on at least one kinematic parameter of $\pi^-$, so they only exist in $\tau$ channels, and specifically for the $\tau \rightarrow \pi^-\nu_\tau$ decay. The $P_{\Lambda_c}(q^2)$ and $A_{FB}(q^2)$ can be represented by the angular observables $\hat{K}_i(q^2)$.

Using the $\Lambda_b \rightarrow \Lambda_c$ form factors computed in lattice QCD including all the types of Lorentz structures of the NP effective operators, we predict the entire set of the angular observables $\hat{K}_i(q^2)$ and $\hat{K}_i$ both in the Standard Model and in the NP benchmark point, which is a best-fit solution labelled “Min 1b” in Ref. [15]. We find that the NP which can resolve the anomalies in $\bar{B} \rightarrow D^{(*)}\tau^-\bar{\nu}_\tau$ decays has obvious effects on the angular observables $\hat{K}_i(q^2)$ and $\hat{K}_i$ with $i = 1c, 2ss, 2cc, 4sc$, and $4s$.

Acknowledgements

This work is supported by the National Natural Science Foundation of China under Grant Nos. 11947083, 12075097, 11675061 and 11775092.

A The detailed calculation of the measurable angular distribution

The differential decay rate of the unpolarized $\Lambda_b^0 \rightarrow \Lambda_c^+(\rightarrow \Lambda^0\pi^+)\tau^-\rightarrow \pi^-\nu_\tau\bar{\nu}_\tau$ decay can be written as

$$d\Gamma = \frac{1}{2m_{\Lambda_b}} |\mathcal{M}|^2 d\Pi_5(p_{\Lambda_b}; p_{\pi^-}, p_\nu, p_{\bar{\nu}}, p_\Lambda, p_{\pi^+}),$$

(A.1)
where the squared matrix element is

$$|M|^2 = \sum_{\lambda} \frac{1}{2} \sum_{\lambda_{\Lambda b}} |M^\lambda_{\lambda_{\Lambda b}}|^2,$$

$$M^\lambda_{\lambda_{\Lambda b}} = \sum_{\lambda_{\Lambda c}, \lambda_{\tau}} \frac{M^\lambda_{\lambda_{\Lambda c} \lambda_{\tau}}(\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau) M^\lambda_{\lambda_{\Lambda b}}(\Lambda_c \to \Lambda\pi^+) M^\lambda_{\lambda_{\Lambda c}}(\tau \to \pi^- \nu_\tau)}{(p^2_{\Lambda c} - m^2_{\Lambda c} + i m_{\Lambda c} \Gamma_{\Lambda c}) (p^2_\tau - m^2_\tau + i m_\tau \Gamma_\tau)},$$

(A.2)

as well as the five-body phase space$^2$ is

$$d\Pi_5(p_{\Lambda b}; p_{\pi^+}, p_\nu, p_{\bar{\nu}}, p_\Lambda, p_{\pi^+}) = \frac{dq^2 dp^2_{\tau} dp^2_\Lambda}{(2\pi)^3} d\Pi_2(p_{\Lambda b}; q, p_{\Lambda c})$$

$$\times d\Pi_2(q; p_\tau, p_\bar{\nu}) d\Pi_2(p_{\tau^-}, p_{\nu^-}) d\Pi_2(p_{\bar{\nu}}, p_\Lambda, p_{\pi^+}).$$

(A.3)

The $\lambda_x$ stands for the helicity of the particle $x$. We drop the helicity indices $\lambda_{\pi\pm}$ as they are null and fix $\lambda_{\nu^+} (\lambda_{\nu^-})$ to $\frac{1}{2} (-\frac{1}{2})$.

Using the effective Hamiltonian given in Eq. (1.1), one can express the helicity amplitude of $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ decay as

$$M^\lambda_{\lambda_{\Lambda b}}(\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau) = \sqrt{2} G_F V_{cb} \left( H^\lambda_{\lambda_{\Lambda b}} \lambda_{\Lambda c} \lambda_{\tau} + \sum_{\lambda} \eta_\lambda H^\lambda_{\lambda_{\Lambda b}} \lambda_{\Lambda c} \lambda_{\tau} + \sum_{\lambda, \lambda'} \eta_\lambda \eta_{\lambda'} H^\lambda_{\lambda_{\Lambda b}} \lambda_{\Lambda c} \lambda_{\tau} \lambda_{\tau'} \right).$$

(A.4)

Here $\lambda^{(t)} = t, \pm 1, 0$ indicates the helicity of the virtual vector boson $W^*$. The number of the helicity indexes depends on the Lorentz structure of the effective operator. The factor $\eta$ that appears here is due to the use of the completeness relation (Eq. (A.19)) of the polarization vectors of the virtual vector boson. The hadronic and leptonic helicity amplitudes are respectively defined as

$$H^\lambda_{\lambda_{\Lambda b}} \equiv \langle \Lambda_c(\lambda_{\Lambda c}) | g_S(\bar{c}b) + g_P(\bar{c}\gamma_\mu b) | \Lambda_b(\lambda_{\Lambda b}) \rangle,$$

(A.5)

$$H^\lambda_{\lambda_{\Lambda b}} \equiv \epsilon^{\mu*}(\lambda) \langle \Lambda_c(\lambda_{\Lambda c}) | g_V(\bar{c}\gamma_\mu b) + g_A(\bar{c}\gamma_\mu \gamma_5 b) | \Lambda_b(\lambda_{\Lambda b}) \rangle,$$

(A.6)

$$H^\lambda_{\lambda_{\Lambda b}} \equiv g_T \epsilon^{\mu*}(\lambda) \epsilon^{\nu*}(\lambda') \langle \Lambda_c(\lambda_{\Lambda c}) | \bar{c}\sigma_{\mu\nu}(1 - \gamma_5) b | \Lambda_b(\lambda_{\Lambda b}) \rangle,$$

(A.7)

and

$$L_{\lambda\tau} \equiv \langle \tau^- (\lambda_{\tau}) \bar{\nu} | \bar{p}_{L\nu} | 0 \rangle,$$

(A.8)

$^2$In this work, the $n$-body phase space is most generally defined as

$$d\Pi_n(P; p_i) = \left( \prod_i \frac{d^3 p_i}{(2\pi)^3 2E_i} \right) (2\pi)^4 \delta^{(4)} \left( P - \sum_i p_i \right).$$

15
\[ L_{\lambda}^{\mu} = \epsilon^\mu(\lambda) \langle \tau^- (\lambda_\tau) \bar{\nu} | \bar{\tau} \gamma_\mu P_L \nu | 0 \rangle, \tag{A.9} \]

\[ L_{\lambda,\lambda'}^{\mu} = (-i) \epsilon^\mu(\lambda) \epsilon^{\mu'}(\lambda') \langle \tau^- (\lambda_\tau) \bar{\nu} | \bar{\tau} \sigma_{\mu\nu} P_L \nu | 0 \rangle, \tag{A.10} \]

where \( \epsilon^\mu(\lambda) \) is the polarization vector of the virtual vector boson with helicity of \( \lambda \).

Using the narrow width (\( \Gamma_y \ll m_y \)) approximation

\[ \frac{1}{(p_y^2 - m_y^2)^2 + m_y^2 \Gamma_y^2} = \frac{\pi}{m_y \Gamma_y} \delta(p_y^2 - m_y^2), \quad (y = \Lambda_c, \tau) \tag{A.11} \]

in Eq. (A.1) and by integrating over the \( dp_y^2 dp_{\Lambda_c}^2 \), one can obtain two on-shell relations \( p_{\Lambda_c}^2 = m_{\Lambda_c}^2 \) and \( p_\tau^2 = m_\tau^2 \), as well as

\[ d\Gamma = \frac{dq^2}{2^5 \pi m_{\Lambda_b} m_\tau \Gamma_{\Lambda_c} \Gamma_{\Lambda_c}} d\Pi_2(p_{\Lambda_b}; q, p_{\Lambda_c})d\Pi_2(q; p_\tau, p_\nu)d\Pi_2(p_\tau; p_{\pi^-}, p_\nu)d\Pi_2(p_{\Lambda_c}; p_\Lambda, p_{\pi^+}) \]

\[ \times \sum_{\lambda,\lambda_0} \sum_{\lambda_c,\lambda_\tau} | \mathcal{M}_{\lambda_0}^{\lambda_{\Lambda_b}}(\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau) \mathcal{M}_{\lambda_c}^{\lambda_{\Lambda_c}}(\Lambda_c \to \Lambda \pi^+) \mathcal{M}_{\lambda_\tau}(\tau \to \pi^- \nu_\tau) |^2. \tag{A.12} \]

Since each individual two-body phase space or helicity amplitude is Lorentz invariant in Eqs. (A.12) and (A.4), one can finish each part of \( d\Gamma \) in different reference frames. In this work, we should consider three measurable reference frames — the \( \Lambda_b \) rest frame, the \( \Lambda_c \) rest frame and the \( \tau^- \bar{\nu}_\tau \) center-of-mass frame.

### A.1 In the \( \Lambda_b \) rest frame

In this frame, we calculate the hadronic helicity amplitudes \( H \) and the two-body phase space \( d\Pi_2(p_{\Lambda_b}; q, p_{\Lambda_c}) \). We choose the three-momentum of the \( \Lambda_c \) baryon to point to the \( +z \) direction and the three-momentum of the virtual vector boson \( W^* \) to point to the \( -z \) direction, see Figure 1. The momenta of \( \Lambda_b, \Lambda_c \), and \( W^* \) are respectively given by

\[ p_{\Lambda_b}^\mu = (m_{\Lambda_b}, 0, 0, 0), \quad p_{\Lambda_c}^\mu = (E_{\Lambda_c}, 0, 0, |p_{\Lambda_c}|), \quad q^\mu = (q_0, 0, 0, -|q|). \tag{A.13} \]

The spinors of \( \Lambda_b \) and \( \Lambda_c \) are then given by [57, 58]

\[ u_{\Lambda_b} \left( \frac{1}{2} \right) = \left( \sqrt{2m_{\Lambda_b}}, 0, 0, 0 \right)^T, \quad u_{\Lambda_b} \left( -\frac{1}{2} \right) = \left( 0, \sqrt{2m_{\Lambda_b}}, 0, 0 \right)^T, \tag{A.14} \]

\[ u_{\Lambda_c} \left( \frac{1}{2} \right) = \left( \beta_{\Lambda_c}^+, 0, \beta_{\Lambda_c}^-, 0 \right)^T, \quad u_{\Lambda_c} \left( -\frac{1}{2} \right) = \left( 0, \beta_{\Lambda_c}^+, 0, -\beta_{\Lambda_c}^- \right)^T. \tag{A.15} \]
with $\beta^\pm_x \equiv \sqrt{E_x} \pm m_x$. In this frame, the polarization vectors of the virtual vector boson $W^*$ can be written as [57, 58]

$$e^\mu(t) = q^\mu / \sqrt{q^2}, \quad (A.16)$$
corresponding to $J_W = 0, \lambda_W = 0$, and

$$e^\mu(\pm 1) = (0, \pm 1, -i, 0)/\sqrt{2}, \quad (A.17)$$
corresponding to $J_W = 1, \lambda_W = \pm 1, 0$. The well-known completeness relation can be expressed as

$$g^{\mu\nu} = \sum_{\lambda \in \{t, \pm 1, 0\}} e^\mu(\lambda)e^{\nu*}(\lambda)\eta_\lambda, \quad (A.19)$$

with $\eta_t = 1$ and $\eta_{\pm 1, 0} = -1$.

By integrating over the two-body phase space, we can get

$$\int d\Pi_2(p_\lambda; q, p_\lambda) = |p_\lambda| / 4\pi m_\lambda, \quad (A.20)$$
as well as $|p_\lambda| = |q| = \lambda^{1/2}(m^2_\lambda, m^2_\Lambda, q^2)/(2m_\Lambda)$, $E_\lambda = (m^2_\lambda + m^2_\Lambda - q^2)/(2m_\Lambda)$, and $q_0 = (m^2_\Lambda - m^2_\Lambda + q^2)/(2m_\Lambda)$. The Källén function $\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, and $\lambda(m^2_\Lambda, m^2_\Lambda, q^2) = Q_+Q_-$. The nonzero hadronic helicity amplitudes $H$ can be expressed by the transverse amplitudes as follows

$$
\begin{align*}
H^{1/2}_{1/2} &= (A^{SP}_{1\bot} + A^{SP}_{1||}) / \sqrt{2}, & H^{-1/2}_{-1/2} &= (A^{SP}_{1\bot} - A^{SP}_{1||}) / \sqrt{2}, \\
H^{1/2,1}_{1/2} &= (A^{VA}_{1\bot} + A^{VA}_{1||}) / \sqrt{2}, & H^{-1/2,1}_{-1/2} &= (A^{VA}_{1\bot} - A^{VA}_{1||}) / \sqrt{2}, \\
H^{1/2,0}_{1/2} &= (A_{1\bot} + A_{1||}) / \sqrt{2}, & H^{-1/2,0}_{-1/2} &= (A_{1\bot} - A_{1||}) / \sqrt{2}, \\
H^{-1/2,-1,1}_{1/2} &= H^{-1/2,0,0,-1}_{1/2} = \frac{A^{T}_{1\bot} - A^{T}_{1||}}{2\sqrt{2}}, & H^{1/2,1,0}_{1/2} &= H^{-1/2,0,0,1}_{1/2} = \frac{A^{T}_{1\bot} + A^{T}_{1||}}{2\sqrt{2}}, \\
H^{-1/2,2,1,0}_{1/2} &= H^{1/2,1,0}_{1/2} = \frac{A^{T}_{1\bot} + A^{T}_{1||}}{2\sqrt{2}}, & H^{-1/2,2,1,-1}_{1/2} &= H^{1/2,1,0}_{1/2} = \frac{A^{T}_{1\bot} - A^{T}_{1||}}{2\sqrt{2}},
\end{align*}
$$

(A.21) - (A.26)

together with the other eight non-vanishing tensor-type helicity amplitudes related to the above ones by

$$H^{\lambda}_{\lambda a} = -H^{\lambda}_{\lambda a}. \quad (A.27)$$

17
A.2 In the $\Lambda_c$ rest frame

In this frame, we calculate the helicity amplitude $\mathcal{M}^{\Lambda}_{\Lambda_c}(\Lambda_c \rightarrow \Lambda \pi^+)$ and the two-body phase space $d\Pi_2(p_{\Lambda_c}; p_{\Lambda}, p_{\pi^+})$. The momentum of $\Lambda_c$ and $\Lambda$ are respectively given by

$$\tilde{p}_{\Lambda_c}^\mu = (m_{\Lambda_c}, 0, 0, 0), \quad p_{\Lambda}^\mu = (E_{\Lambda}, |p_{\Lambda}| \sin \theta_{\Lambda}, 0, |p_{\Lambda}| \cos \theta_{\Lambda}). \quad (A.28)$$

The “~” here and the following are only used to distinguish the representations of the same kinematic quantity in different reference frames. The spinors of $\Lambda_c$ and $\Lambda$ are given by [57, 58]

$$\tilde{u}_{\Lambda_c} \left( \frac{1}{2} \right) = \left( \sqrt{2m_{\Lambda_c}}, 0, 0, 0 \right)^T, \quad \tilde{u}_{\Lambda_c} \left( -\frac{1}{2} \right) = \left( 0, \sqrt{2m_{\Lambda_c}}, 0, 0 \right)^T, \quad (A.29)$$

$$u_\Lambda \left( \frac{1}{2} \right) = \left( \beta_\Lambda^+, \cos \frac{\theta_\Lambda}{2}, \beta_\Lambda^- \sin \frac{\theta_\Lambda}{2}, \beta_\Lambda^- \cos \frac{\theta_\Lambda}{2}, \beta_\Lambda^+ \sin \frac{\theta_\Lambda}{2}, \beta_\Lambda^+ \cos \frac{\theta_\Lambda}{2} \right)^T, \quad (A.30)$$

$$u_\Lambda \left( -\frac{1}{2} \right) = \left( -\beta_\Lambda^+ \sin \frac{\theta_\Lambda}{2}, \beta_\Lambda^- \cos \frac{\theta_\Lambda}{2}, \beta_\Lambda^- \sin \frac{\theta_\Lambda}{2}, -\beta_\Lambda^- \cos \frac{\theta_\Lambda}{2}, -\beta_\Lambda^+ \sin \frac{\theta_\Lambda}{2}, -\beta_\Lambda^+ \cos \frac{\theta_\Lambda}{2} \right)^T, \quad (A.31)$$

By integrating over the $\delta^{(4)}$ term and the azimuthal angle $\phi_\Lambda$ in two-body phase space, one can get

$$d\Pi_2(p_{\Lambda_c}; p_{\Lambda}, p_{\pi^+}) = \frac{1}{8\pi m_{\Lambda_c}} |p_{\Lambda}| d\cos \theta_{\Lambda}, \quad (A.32)$$

as well as $|p_{\Lambda}| = \lambda^{1/2}(m_{\Lambda_c}^2, m_\Lambda^2, m_{\pi}^2)/(2m_{\Lambda_c})$ and $E_\Lambda = (m_{\Lambda_c}^2 + m_\Lambda^2 - m_{\pi}^2)/(2m_{\Lambda_c})$.

The helicity amplitude

$$\mathcal{M}^{\Lambda}_{\Lambda_c}(\Lambda_c \rightarrow \Lambda \pi^+) = i\tilde{u}_\Lambda(\lambda_\Lambda)(A + B\gamma_5)u_{\Lambda_c}(\lambda_{\Lambda_c}),$$

$$= i\lambda_{\Lambda_c}^\dagger(\lambda_\Lambda)(S + P\sigma \cdot \tilde{p}_\Lambda)\lambda_{\Lambda_c}(\lambda_{\Lambda_c}). \quad (A.33)$$

Where $S = \sqrt{2m_{\Lambda_c}}\beta_\Lambda^+ A$ stand for the parity-violating s-wave amplitude and $P = -\sqrt{2m_{\Lambda_c}}\beta_\Lambda^- B$ stand for the parity-conserving p-wave amplitude. $\sigma = (\sigma^1, \sigma^2, \sigma^3)$ is a vector composed of Pauli matrices. $\tilde{p}_\Lambda$ is the unit vector along the direction of $\Lambda$ baryon. The four helicity amplitudes are

$$\mathcal{M}^{1/2}_{1/2}(\Lambda_c \rightarrow \Lambda \pi^+) = i(S + P) \cos \frac{\theta_\Lambda}{2}, \quad \mathcal{M}^{1/2}_{-1/2}(\Lambda_c \rightarrow \Lambda \pi^+) = i(S + P) \sin \frac{\theta_\Lambda}{2}, \quad (A.34)$$

$$\mathcal{M}^{-1/2}_{1/2}(\Lambda_c \rightarrow \Lambda \pi^+) = -i(S - P) \sin \frac{\theta_\Lambda}{2}, \quad \mathcal{M}^{-1/2}_{-1/2}(\Lambda_c \rightarrow \Lambda \pi^+) = i(S - P) \cos \frac{\theta_\Lambda}{2}. \quad (A.35)$$

The two helicity amplitudes in Eq. (A.34) correspond to $\lambda_\Lambda = \frac{1}{2}$. By using them, one can obtain that the decay rate of $\Lambda_c \rightarrow \Lambda \pi^+$ with $\lambda_\Lambda = \frac{1}{2}$ is

$$\Gamma^{\lambda_\Lambda = 1/2}(\Lambda_c \rightarrow \Lambda \pi^+) = \frac{|p_{\Lambda}|}{16\pi m_{\Lambda_c}^2} |S + P|^2. \quad (A.36)$$
In the same way, one can obtain the decay rate

\[ \Gamma_{\lambda\Lambda} = \frac{|p_{\lambda}|}{16\pi m_{\Lambda_{c}}^2} |S - P|^2, \]  

by using the two helicity amplitudes in Eq. (A.35). The angular asymmetry parameter \( \alpha_{\Lambda_{c}} \) is defined as

\[ \alpha_{\Lambda_{c}} \equiv \frac{2\Re(S^* P) |S|^2 + |P|^2}{\Gamma_{\lambda\Lambda}}, \]  

and one can immediately get the relations

\[ \Gamma_{\lambda\Lambda} = \frac{1}{2} \Gamma_{\lambda\Lambda} = \frac{1}{2}, \]  

\[ \Gamma_{\lambda\Lambda} = \frac{1}{2} \Gamma_{\lambda\Lambda} = \frac{1}{2} = \frac{1}{2}(1 - \alpha_{\Lambda_{c}}). \]  

### A.3 In the \( \tau^-\bar{\nu}_\tau \) center-of-mass frame

In this frame, we calculate the leptonic helicity amplitudes \( L \) and the helicity amplitudes \( M_{\lambda\tau}(\tau \rightarrow \pi^{-}\nu_\tau) \), as well as the two-body phase spaces \( d\Pi_2(q; p_\tau, p_\nu) \) and \( d\Pi_2(p_\tau; p_{\pi^-}, p_\nu) \). The momentum of \( \pi^- \) is defined as \( p_{\pi^-} = (E_{\pi^-}, |p_{\pi^-}| \hat{p}_{\pi^-}) \) with

\[ \hat{p}_{\pi^-} = (\sin \theta_{\pi^-} \cos \phi_{\pi^-}, \sin \theta_{\pi^-} \sin \phi_{\pi^-}, \cos \theta_{\pi^-}), \tag{A.40} \]

is the unit vector along the direction of \( \pi^- \). In this frame, the polarization vectors of the virtual vector boson \( W^* \) are changed to

\[ \tilde{\epsilon}^\mu(t) = (1, 0, 0, 0), \quad \tilde{\epsilon}^\mu(\pm 1) = (0, \pm 1, -1, 0)/\sqrt{2}, \quad \tilde{\epsilon}^\mu(0) = (0, 0, 0, -1). \tag{A.41} \]

The helicity amplitudes \( M_{\lambda\tau}(\tau \rightarrow \pi^{-}\nu_\tau) \) can be written as

\[ M_{\lambda\tau}(\tau \rightarrow \pi^{-}\nu_\tau) = i\sqrt{2}G_F V_{ud}^* \bar{f}_\tau \bar{u}_{\nu_\tau} \gamma^\mu P_L v_\tau(\lambda_\tau), \tag{A.42} \]

and one can obtain that the decay rate is

\[ \Gamma(\tau \rightarrow \pi^{-}\nu_\tau) = \frac{G_F^2 |V_{ud}|^2 f_\tau^2 (m_\tau^2 - m_\pi^2)^2}{16\pi m_{\pi^-}}. \tag{A.43} \]

Using the relation \( \sum_{\lambda_\tau} u_\tau(\lambda_\tau) \bar{u}_\tau(\lambda_\tau) = \bar{\nu}_\tau + m_\tau \), we link the \( \bar{u}_{\nu_\tau} \gamma_\mu P_L v_\tau(\lambda_\tau) \) in Eq. (A.42) with the leptonic helicity amplitudes \( L \), and we can obtain the new leptonic helicity amplitudes as follows

\[ L = m_\tau p_\pi^\mu \bar{u}_{\nu_\tau} \gamma_\mu P_L v_\tau, \tag{A.44} \]

\[ L_\lambda = m_\tau^2 \epsilon^\mu(\lambda) \bar{u}_{\nu_\tau} \gamma_\mu P_L v_\tau, \tag{A.45} \]
\[ L_{\lambda'} = m_\tau \left[ p_\pi \cdot \epsilon(\lambda') \epsilon^\mu(\lambda') - p_\pi \cdot \epsilon(\lambda) \epsilon^\mu(\lambda) + i\epsilon^{\rho\alpha\mu\nu} p_\pi \rho \epsilon_\alpha(\lambda) \epsilon_\nu(\lambda') \right] \bar{u}_\nu \gamma_\mu P_L v_\rho. \] (A.46)

Next, we deduce the phase spaces \( d\Pi_2(q; p_\tau, p_\nu) \) and \( d\Pi_2(p_\pi; p_\pi^-, p_\nu) \) simultaneously in the \( \tau^-\bar{\nu}_\tau \) center-of-mass frame [55].

\[
d\Pi_2(q; p_\tau, p_\nu) d\Pi_2(p_\pi; p_\pi^-, p_\nu) = \int_{\delta^{(8)}} \frac{d^3p_\tau}{(2\pi)^3 2E_\tau} \left( \frac{d^3p_\nu}{(2\pi)^3 2E_\nu} \right) \left( \frac{d^3p_\pi}{(2\pi)^3 2E_\pi} \right) \left( \frac{d^3p_\pi^-}{(2\pi)^3 2E_\pi^=} \right)
\]

\[
= \int_{\delta^{(2)}} \frac{1}{2^8\pi^4} \frac{d^3p_\tau}{E_\tau} \delta \left( \sqrt{q^2} - E_\tau - |p_\tau| \right) \left( \frac{d^3p_\pi}{E_\pi} \right) \left( \frac{d^3p_\pi}{|p_\tau - p_\pi|} \right) \left( \frac{d^3p_\pi^-}{E_\pi} \right) \delta \left( E_\tau - E_\pi - |p_\tau - p_\pi^-| \right). \quad \text{(A.47)}
\]

The momentum-conservation relations \( p_\nu = -p_\tau \) and \( p_\nu = p_\pi - p_\pi^- \) hold. Since three-momentum \( p_\pi \) can be measured experimentally, the remaining two \( \delta \) functions will be used to integrate over the two variables in \( d^3p_\tau \). We define the solid angle of \( \tau^- \) relative to the direction of \( \pi^- \) instead of the \( z \)-axis as \( (\theta_{\pi\tau}, \phi_{\pi\tau}) \). Next, we will see that the magnitude \( |p_\tau| \) and the \( \pi^- - \tau^- \) opening angle \( \theta_{\pi\tau} \) can be determined theoretically.

Using formula \( \delta(g(t)) = \sum_i \delta(t - t_i) / |g'(t_i)| \) where \( g(t_i) = 0 \) and \( g'(t_i) \neq 0 \) to deduce the remaining two \( \delta \) functions, one has

\[
d\Pi_2(q; p_\tau, p_\nu) d\Pi_2(p_\pi; p_\pi^-, p_\nu) = \frac{1}{2^8\pi^4} \frac{1}{\sqrt{q^2}} d\phi_{\pi\tau} dE_\pi d\cos \theta_{\pi\tau} d\phi_{\pi\tau}, \quad \text{(A.48)}
\]

as well as

\[
|p_\tau| = \frac{q^2 - m_\tau^2}{2\sqrt{q^2}}, \quad \cos \theta_{\pi\tau} = \frac{2E_\pi E_\tau - m_\pi^2 - m_\tau^2}{2 |p_\tau| |p_\pi|}. \quad \text{(A.49)}
\]

Accordingly, the variables \( q^2 \) and \( E_\pi \) can take

\[
m_\pi^2 \leq q^2 \leq (m_{\Lambda_\pi} - m_{\Lambda_\tau})^2, \quad \frac{m_\tau^2 m_\pi^2 q^2}{2m_\pi^2 \sqrt{q^2}} \leq E_\pi \leq \frac{m_\tau^2 + q^2}{2\sqrt{q^2}}. \quad \text{(A.50)}
\]

So far, all of the pieces of Eq. (A.12) have been completed.

\[ \cos \theta_{\pi\tau} = \cos \theta_\pi \cos \theta_\tau + \sin \theta_\pi \sin \theta_\tau \cos(\phi_\pi - \phi_\tau), \]

\[ \cos \phi_{\pi\tau} = \frac{\sin \theta_\tau \sin(\phi_\pi - \phi_\tau)}{\sqrt{\sin^2 \theta_\tau \sin^2(\phi_\pi - \phi_\tau) + (\cos \theta_\tau \sin \phi_\pi \sin \phi_\tau \cos \phi_\tau \cos(\phi_\pi - \phi_\tau))^2}}, \]

\[ \sin \phi_{\pi\tau} = \frac{-\cos \theta_\tau \sin \phi_\pi + \cos \theta_\tau \sin \phi_\tau \cos(\phi_\pi - \phi_\tau)}{\sqrt{\sin^2 \theta_\tau \sin^2(\phi_\pi - \phi_\tau) + (\cos \theta_\tau \sin \phi_\pi \sin \phi_\tau \cos \phi_\tau \cos(\phi_\pi - \phi_\tau))^2}}. \]

\[ \]

20
The dimensionless factors in Table 3, 4, and 5 are given by

\[
S_t = \frac{N^S}{|A_t|^2} = \kappa_{\pi}^2 - \kappa_{\pi}^4 - \kappa_{\pi}^2,
\]

\[
S_1 = \left( \frac{\kappa_{\pi}}{8 (\omega_\pi - \kappa_{\pi}^2)} \right) \left[ \kappa_{\pi}^2 (\omega_\pi^2 - 4\omega_\pi + 6) \right] + (2\omega_\pi - \kappa_{\pi}^2) (2\omega_\pi + \omega_\pi - 1) \kappa_{\pi}^2 - 3\kappa_{\pi}^4 + 6 (1 - 2\omega_\pi) \omega_\pi^2,
\]

\[
S_2 = \frac{\kappa_{\pi}^2 (\omega_\pi^2 - 2\omega_\pi + 1) (\omega_\pi - \kappa_{\pi}^2)}{\sqrt{\omega_\pi^2 - \kappa_{\pi}^2}},
\]

Table 3: The enumeration of \(N^S|A_i|^2\) pieces of Eq. (A.51).

**A.4 The five-fold differential decay rate**

The five-fold differential decay rate is

\[
\frac{d^5\Gamma}{dq^2dE_\gamma d\cos \theta_\pi d\phi d\cos \theta_{\Lambda}} = \frac{G_F^2 |V_{cb}|^2 |p_{A_{\Lambda}}| (q^2)^{3/2} m^2}{2^{8} \pi^4 m_{\Lambda}^2 (m_\pi^2 - m_\gamma^2)^2} B(\tau \to \pi^- \nu_\tau) B(A_c \to \Lambda\pi) \times \sum_{i,j} \left( N_i^S |A_i|^2 + N_{i,j}^R \text{Re}[A_i A_j^*] + N_{i,j}^I \text{Im}[A_i A_j^*] \right),
\]

where the terms \(N_i^S |A_i|^2\), \(N_{i,j}^R \text{Re}[A_i A_j^*]\), and \(N_{i,j}^I \text{Im}[A_i A_j^*]\) are respectively listed in Table 3, 4, and 5.

To make the expressions more compact, we define the following dimensionless parameters

\[
\kappa_{\pi} \equiv \frac{m_\pi}{\sqrt{q^2}}, \quad \kappa_{\pi}^4 \equiv \frac{m_\pi^4}{\sqrt{q^2}}, \quad \omega_\pi \equiv \frac{E_\pi}{\sqrt{q^2}}.
\]

The dimensionless factors in Table 3, 4, and 5 are given by

\[
S_t = \frac{N^S}{|A_t|^2} = \kappa_{\pi}^2 - \kappa_{\pi}^4 - \kappa_{\pi}^2,
\]

\[
S_1 = \left( \frac{\kappa_{\pi}}{8 (\omega_\pi - \kappa_{\pi}^2)} \right) \left[ \kappa_{\pi}^2 (\omega_\pi^2 - 4\omega_\pi + 6) \right] + (2\omega_\pi - \kappa_{\pi}^2) (2\omega_\pi + \omega_\pi - 1) \kappa_{\pi}^2 - 3\kappa_{\pi}^4 + 6 (1 - 2\omega_\pi) \omega_\pi^2,
\]

\[
S_2 = \frac{\kappa_{\pi}^2 (\omega_\pi^2 - 2\omega_\pi + 1) (\omega_\pi - \kappa_{\pi}^2)}{\sqrt{\omega_\pi^2 - \kappa_{\pi}^2}},
\]
| Transverse Amplitudes | $N_i^R \text{Re}[A_i A_i^*]$ |
|------------------------|-------------------------------|
| $\text{Re}[A_{11}, A_{11}^*]$ | $2\alpha_{\Lambda} S_2 \cos \theta_\Lambda$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $\alpha_{\Lambda} R_1 \sin \theta_\Lambda \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} R_1 \cos \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} R_1 \cos \theta_\Lambda \cos \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $\alpha_{\Lambda} R_1^2 \sin \theta_{\phi} \sin \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} R_1^2 \cos \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} R_1^2 \cos \theta_\Lambda \cos \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\alpha_{\Lambda} R_1 \sin \theta_\Lambda \sin \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} R_1 \cos \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\alpha_{\Lambda} R_1^2 \cos \theta_\Lambda \cos \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} R_1^2 \cos \theta_\Lambda \cos \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\alpha_{\Lambda} R_1^2 \sin \theta_{\phi} \sin \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} R_1^2 \cos \theta_\Lambda \cos \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $2 S_2 \cos \theta_{\phi} + 2 \alpha_{\Lambda} \cos \theta_{\phi} (S_1 + S_3 \cos 2 \theta_{\phi})$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $\sqrt{2} \alpha_{\Lambda} S_2 \sin \theta_{\phi} \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-2 \sqrt{2} \alpha_{\Lambda} S_3 \sin \theta_\Lambda \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $R_1 + \alpha_{\Lambda} R_2 \cos \theta_\Lambda \cos \theta_{\phi} + R_3 \cos 2 \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $R_2 \cos \theta_{\phi} + \alpha_{\Lambda} \cos \theta_{\phi} (R_1 + R_3 \cos 2 \theta_{\phi})$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $\sqrt{2} \alpha_{\Lambda} R_3 \sin \theta_{\phi} \sin 2 \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $(\alpha_{\Lambda} R_2 / \sqrt{2}) \sin \theta_\Lambda \sin \theta_{\phi} \sin \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} R_3 \sin \theta_\Lambda \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $2 \sqrt{2} \alpha_{\Lambda} S_3 \sin \theta_\Lambda \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} S_2 \sin \theta_\Lambda \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $R_2 \cos \theta_{\phi} + \alpha_{\Lambda} \cos \theta_{\phi} (R_1 + R_3 \cos 2 \theta_{\phi})$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $R_1 + \alpha_{\Lambda} R_2 \cos \theta_\Lambda \cos \theta_{\phi} + R_3 \cos 2 \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $\sqrt{2} \alpha_{\Lambda} R_3 \sin \theta_{\phi} \sin 2 \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} R_3 \sin \theta_{\phi} \sin 2 \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $(\alpha_{\Lambda} R_2 / \sqrt{2}) \sin \theta_\Lambda \sin \theta_{\phi} \sin \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} R_3 \sin \theta_{\phi} \sin 2 \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $2 \sqrt{2} \alpha_{\Lambda} S_3 \sin \theta_\Lambda \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} S_2 \sin \theta_\Lambda \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $R_1 - R_3 - 2 R_3 \cos 2 \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $\alpha_{\Lambda} \cos \theta_{\phi} (R_1 - R_3 - 2 R_3 \cos 2 \theta_{\phi})$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} R_3 \sin \theta_{\phi} \sin 2 \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\alpha_{\Lambda} R_2 / \sqrt{2} \sin \theta_\Lambda \sin \theta_{\phi} \sin \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $\alpha_{\Lambda} \cos \theta_{\phi} (R_1 - R_3 - 2 R_3 \cos 2 \theta_{\phi})$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $R_1 - R_3 - 2 R_3 \cos 2 \theta_{\phi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $2 S_2 \cos \theta_{\phi} + 2 \alpha_{\Lambda} \cos \theta_{\phi} (S_1 + S_3 \cos 2 \theta_{\phi})$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $\sqrt{2} \alpha_{\Lambda} S_2^2 \sin \theta_{\phi} \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-2 \sqrt{2} \alpha_{\Lambda} S_3^2 \sin \theta_{\phi} \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $2 \sqrt{2} \alpha_{\Lambda} S_3^2 \sin \theta_{\phi} \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $-\sqrt{2} \alpha_{\Lambda} S_2^2 \sin \theta_{\phi} \sin \theta_{\phi} \cos \phi_{\pi}$ |
| $\text{Re}[A_{11}, A_{11}^*]$ | $2 \alpha_{\Lambda} \cos \theta_{\phi} (S_1^2 - S_3^2 - 2 S_2^2 \cos 2 \theta_{\phi})$ |

Table 4: The enumeration of $N_i^R \text{Re}[A_i A_i^*]$ pieces of Eq. (A.51).

22
| Transverse Amplitudes | \( N^I \) |
|-----------------------|----------------|
| \( \text{Im}[A_{l1}A_{11}^*] \) | \(-\alpha_{c}R_t \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{l1}A_{11}^*] \) | \(-\alpha_{c}R_t^{T} \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{12}^*] \) | \(\alpha_{c}R_t^{T} \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{12}^*] \) | \(\alpha_{c}R_t^{T} \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{l1}A_{10}^*] \) | \(2\alpha_{c}S_3 \sin \theta_{\Lambda} \sin 2\theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{l1}A_{10}^*] \) | \(-\sqrt{2}\alpha_{c}S_2 \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{l1}A_{10}^*] \) | \(\sqrt{2}\alpha_{c}R_3 \sin \theta_{\Lambda} \sin 2\theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{l1}A_{10}^*] \) | \(-\alpha_{c}R_2/\sqrt{2} \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(\sqrt{2}\alpha_{c}S_2 \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(-2\alpha_{c}S_3 \sin \theta_{\Lambda} \sin 2\theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(\alpha_{c}R_2/\sqrt{2} \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(-\sqrt{2}\alpha_{c}R_3 \sin \theta_{\Lambda} \sin 2\theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(-\alpha_{c}R_2/\sqrt{2} \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(\alpha_{c}R_2/\sqrt{2} \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(\sqrt{2}\alpha_{c}R_3 \sin \theta_{\Lambda} \sin 2\theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(2\sqrt{2}\alpha_{c}S_3^{T} \sin \theta_{\Lambda} \sin 2\theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(-\sqrt{2}\alpha_{c}S_2^{T} \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(\sqrt{2}\alpha_{c}S_2^{T} \sin \theta_{\Lambda} \sin \theta_{\pi} \sin \phi_{\pi} \) |
| \( \text{Im}[A_{||1}A_{11}^*] \) | \(-\sqrt{2}\alpha_{c}S_3^{T} \sin \theta_{\Lambda} \sin 2\theta_{\pi} \sin \phi_{\pi} \) |

Table 5: The enumeration of \( N^I_{ij} \), \( \text{Im}[A_iA_j^*] \) pieces of Eq. \((A.51)\).
\[ S_3 = \frac{\kappa^2}{8 (\omega^2 - \kappa^2)} \left[ \kappa^4 (2 \omega^2 - \kappa^2 + 4 \omega^2 - 2 \omega) + (\omega - 2 \omega^2) (2 \omega^2 - 6 \omega^3 + 3) \kappa^2 - \kappa^4 + 2 (1 - 2 \omega^2) \omega^2 \right] , \]  
\[ (A.56) \]

\[ S^T_1 = \frac{1}{2 (\omega^2 - \kappa^2)} \left\{ \kappa^4 (2 \omega^2 \kappa^2 + 5 \kappa^4 + 2 \omega^2 - 3) + 4 \omega^2 \kappa^2 [(3 \omega^2 - 1) \kappa^2 - \omega] \right\} + \kappa^2 \left[ (-6 \omega^2 + 10 \omega^3 + 3) \kappa^2 + 2 (3 - 2 \omega^2) \omega \kappa^2 + 2 \omega^2] - \kappa^6 \right\} , \]  
\[ (A.57) \]

\[ S^T_2 = \frac{4 \kappa^2 (2 \omega^2 - \omega^2 + 1) (\omega^2 - \omega \kappa^2)}{\sqrt{\omega^2 - \kappa^2}} , \]  
\[ (A.58) \]

\[ S^T_3 = \frac{1}{2 (\omega^2 - \kappa^2)} \left\{ \kappa^4 (-6 \omega^2 \kappa^2 + \kappa^2 - 6 \omega^2 + 1) - 4 \omega^2 \kappa^2 [(\omega^2 - 1) \kappa^2 + \omega] \right\} + \kappa^2 \left[ (2 \omega^2 - 2 \omega^2 - \omega^2 + 1) \kappa^4 + 2 \omega^2 (6 \omega^2 - 1) \kappa^2 + 2 \omega^2] + 3 \kappa^6 \right\} , \]  
\[ (A.59) \]

and

\[ R_1 = \sqrt{2} (\omega^2 - 1) \kappa^2 (2 \omega^2 \kappa^2 - \kappa^4 - \kappa^2) \]  
\[ (A.60) \]

\[ R^T_1 = \frac{2 \sqrt{2} \left[ \kappa^2 (2 \omega^2 \kappa^2 + \kappa^4 - \omega) - \omega \kappa^2 (\kappa^2 - 2 \omega^2) + \kappa^4 \right]}{\sqrt{\omega^2 - \kappa^2}} , \]  
\[ (A.61) \]

\[ R_1 = \frac{\kappa^4}{2 (\omega^2 - \kappa^2)} \left\{ \kappa^2 [(\omega^2 + 2) \kappa^2 + (4 \omega^2 + 8 \omega^2 - 6) \kappa^2 - 4 \omega^2 + \omega] \right\} + \kappa^2 \left[ (-6 \omega^2 + \omega^2 + 2) + \omega \kappa^2 [(1 - 4 \omega^2) \kappa^2 - 4 (\omega^2 - 1) \omega] \right\} , \]  
\[ (A.62) \]

\[ R_2 = \frac{2 \kappa^2 (\kappa^4 - \kappa^2) (\kappa^2 - 2 \omega^2 + 1)}{\sqrt{\omega^2 - \kappa^2}} , \]  
\[ (A.63) \]

\[ R_3 = \frac{\kappa^4}{2 (\omega^2 - \kappa^2)} \left\{ \kappa^2 [(3 \omega^2 - 2) \kappa^4 + (-4 \omega^2 + 8 \omega^2 - 2) \kappa^2 + (3 - 4 \omega^2) \omega] \right\} + \kappa^4 \left[ (-2 \omega^2 + 3 \omega^2 - 2) + \omega \kappa^2 [(3 - 4 \omega^2) \kappa^2 + 4 (\omega^2 - 1) \omega] \right\} . \]  
\[ (A.64) \]

References

[1] BABar collaboration, Evidence for an excess of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$ decays, Phys. Rev. Lett. 109 (2012) 101802 [1205.5442].

[2] BABar collaboration, Measurement of an Excess of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$ Decays and Implications for Charged Higgs Bosons, Phys. Rev. D 88 (2013) 072012 [1303.0571].
[3] LHCb collaboration, Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \to D^{(*)}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \to D^{(*)}\mu^-\bar{\nu}_\mu)$, Phys. Rev. Lett. 115 (2015) 111803 [1506.08614].

[4] Belle collaboration, Measurement of the branching ratio of $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ relative to $\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell$ decays with hadronic tagging at Belle, Phys. Rev. D 92 (2015) 072014 [1507.03233].

[5] Belle collaboration, Measurement of the $\tau$ lepton polarization and $R(D^*)$ in the decay $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$, Phys. Rev. Lett. 118 (2017) 211801 [1612.00529].

[6] LHCb collaboration, Measurement of the ratio of the $B^0 \to D^{*-}\tau^+\nu_\tau$ and $B^0 \to D^{*-}\mu^+\nu_\mu$ branching fractions using three-prong $\tau$-lepton decays, Phys. Rev. Lett. 120 (2018) 171802 [1708.08856].

[7] Belle collaboration, Measurement of the $\tau$ lepton polarization and $R(D^*)$ in the decay $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$ with one-prong hadronic $\tau$ decays at Belle, Phys. Rev. D 97 (2018) 012004 [1709.00129].

[8] LHCb collaboration, Test of Lepton Flavor Universality by the measurement of the $B^0 \to D^{*-}\tau^+\nu_\tau$ branching fraction using three-prong $\tau$ decays, Phys. Rev. D 97 (2018) 072013 [1711.02505].

[9] Belle collaboration, Measurement of $R(D)$ and $R(D^*)$ with a semileptonic tagging method, Phys. Rev. Lett. 124 (2020) 161803 [1910.05864].

[10] Z. Ligeti, M. Papucci and D.J. Robinson, New Physics in the Visible Final States of $B \to D^{(*)}\tau\nu$, JHEP 01 (2017) 083 [1610.02045].

[11] R. Mandal, C. Murgui, A. Peñuelas and A. Pich, The role of right-handed neutrinos in $b \to c\tau\bar{\nu}$ anomalies, JHEP 08 (2020) 022 [2004.06726].

[12] A.K. Alok, D. Kumar, J. Kumar, S. Kumbhakar and S.U. Sankar, New physics solutions for $R_D$ and $R_{D^*}$, JHEP 09 (2018) 152 [1710.04127].

[13] Q.-Y. Hu, X.-Q. Li and Y.-D. Yang, $b \to c\tau\nu$ transitions in the standard model effective field theory, Eur. Phys. J. C 79 (2019) 264 [1810.04939].
[14] A.K. Alok, D. Kumar, S. Kumbhakar and S. Uma Sankar, Solutions to $R_D$-$R_{D^*}$ in light of Belle 2019 data, *Nucl. Phys. B* 953 (2020) 114957 [1903.10486].

[15] C. Murgui, A. Peñuelas, M. Jung and A. Pich, Global fit to $b \to c\tau\nu$ transitions, *JHEP* 09 (2019) 103 [1904.09311].

[16] K. Cheung, Z.-R. Huang, H.-D. Li, C.-D. Lü, Y.-N. Mao and R.-Y. Tang, Revisit to the $b \to c\tau\nu$ transition: in and beyond the SM, 2002.07272.

[17] S. Kumbhakar, Signatures of complex new physics in $b \to c\tau\bar{\nu}$ transitions, 2007.08132.

[18] Y. Sakaki, M. Tanaka, A. Tayduganov and R. Watanabe, Testing leptoquark models in $\bar{B} \to D^{(*)}\tau\bar{\nu}$, *Phys. Rev. D* 88 (2013) 094012 [1309.0301].

[19] M. Bauer and M. Neubert, Minimal Leptoquark Explanation for the $R_{D^{(*)}}$, $R_K$, and $(g-2)_g$ Anomalies, *Phys. Rev. Lett.* 116 (2016) 141802 [1511.01900].

[20] S. Fajfer and N. Košnik, Vector leptoquark resolution of $R_K$ and $R_{D^{(*)}}$ puzzles, *Phys. Lett. B* 755 (2016) 270 [1511.06024].

[21] S. Iguro, T. Kitahara, Y. Omura, R. Watanabe and K. Yamamoto, $D^*$ polarization vs. $R_{D^{(*)}}$ anomalies in the leptoquark models, *JHEP* 02 (2019) 194 [1811.08899].

[22] X.-Q. Li, Y.-D. Yang and X. Zhang, Revisiting the one leptoquark solution to the $R(D^{(*)})$ anomalies and its phenomenological implications, *JHEP* 08 (2016) 054 [1605.09308].

[23] D. Bečirević, I. Doršner, S. Fajfer, N. Košnik, D.A. Faroughy and O. Sumensari, Scalar leptoquarks from grand unified theories to accommodate the $B$-physics anomalies, *Phys. Rev. D* 98 (2018) 055003 [1806.05689].

[24] A. Angelescu, D. Bečirević, D. Faroughy and O. Sumensari, Closing the window on single leptoquark solutions to the $B$-physics anomalies, *JHEP* 10 (2018) 183 [1808.08179].

[25] A. Crivellin, C. Greub and A. Kokulu, Explaining $B \to D\tau\nu$, $B \to D^{*}\tau\nu$ and $B \to \tau\nu$ in a 2HDM of type III, *Phys. Rev. D* 86 (2012) 054014 [1206.2634].

[26] A. Celis, M. Jung, X.-Q. Li and A. Pich, Sensitivity to charged scalars in $B \to D^{(*)}\tau\nu_\tau$ and $B \to \tau\nu_\tau$ decays, *JHEP* 01 (2013) 054 [1210.8443].
[27] N. Deshpande and X.-G. He, *Consequences of R-parity violating interactions for anomalies in $\bar{B} \to D^{(*)}\tau\bar{\nu}$ and $b \to s\mu^+\mu^-$*, *Eur. Phys. J. C* **77** (2017) 134 [1608.04817].

[28] W. Altmannshofer, P. Bhupal Dev and A. Soni, *$R_{D^{(*)}}$ anomaly: A possible hint for natural supersymmetry with R-parity violation*, *Phys. Rev. D* **96** (2017) 095010 [1704.06659].

[29] Q.-Y. Hu, X.-Q. Li, Y. Muramatsu and Y.-D. Yang, *$R$-parity violating solutions to the $R_{D^{(*)}}$ anomaly and their GUT-scale unifications*, *Phys. Rev. D* **99** (2019) 015008 [1808.01419].

[30] Q.-Y. Hu, Y.-D. Yang and M.-D. Zheng, *Revisiting the $B$-physics anomalies in $R$-parity violating MSSM*, *Eur. Phys. J. C* **80** (2020) 365 [2002.09875].

[31] P. Ko, Y. Omura and C. Yu, *$B \to D^{(*)}\tau\nu$ and $B \to \tau\nu$ in chiral $U(1)'$ models with flavored multi Higgs doublets*, *JHEP* **03** (2013) 151 [1212.4607].

[32] A. Celis, M. Jung, X.-Q. Li and A. Pich, *Scalar contributions to $b \to c(u)\tau\nu$ transitions*, *Phys. Lett. B* **771** (2017) 168 [1612.07757].

[33] S.M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, *Non-abelian gauge extensions for $B$-decay anomalies*, *Phys. Lett. B* **760** (2016) 214 [1604.03088].

[34] A. Datta, M. Duraisamy and D. Ghosh, *Diagnosing New Physics in $b \to c\tau\nu_\tau$ decays in the light of the recent BaBar result*, *Phys. Rev. D* **86** (2012) 034027 [1206.3760].

[35] W. Altmannshofer, P.B. Dev, A. Soni and Y. Sui, *Addressing $R_{D^{(*)}}$, $R_{K^{(*)}}$, muon $g-2$ and ANITA anomalies in a minimal R-parity violating supersymmetric framework*, *Phys. Rev. D* **102** (2020) 015031 [2002.12910].

[36] M. Freytsis, Z. Ligeti and J.T. Ruderman, *Flavor models for $\bar{B} \to D^{(*)}\tau\bar{\nu}$*, *Phys. Rev. D* **92** (2015) 054018 [1506.08896].

[37] A. Greljo, D.J. Robinson, B. Shakya and J. Zupan, *$R(D^{(*)})$ from $W'$ and right-handed neutrinos*, *JHEP* **09** (2018) 169 [1804.04642].

[38] A. Azatov, D. Bardhan, D. Ghosh, F. Sgarlata and E. Venturini, *Anatomy of $b \to c\tau\nu$ anomalies*, *JHEP* **11** (2018) 187 [1805.03209].
[39] S. Bansal, R.M. Capdevilla and C. Kolda, Constraining the minimal flavor violating leptoquark explanation of the $R_{D^{(*)}}$ anomaly, Phys. Rev. D 99 (2019) 035047 [1810.11588].

[40] W. Detmold, C. Lehner and S. Meinel, $\Lambda_{b} \rightarrow p\ell^{-}\bar{\nu}_{\ell}$ and $\Lambda_{b} \rightarrow \Lambda_{c}\ell^{-}\bar{\nu}_{\ell}$ form factors from lattice QCD with relativistic heavy quarks, Phys. Rev. D 92 (2015) 034503 [1503.01421].

[41] A. Datta, S. Kamali, S. Meinel and A. Rashed, Phenomenology of $\Lambda_{b} \rightarrow \Lambda_{c}\tau\bar{\nu}_{\tau}$ using lattice QCD calculations, JHEP 08 (2017) 131 [1702.02243].

[42] LHCb collaboration, Measurement of the shape of the $\Lambda_{b}^{0} \rightarrow \Lambda_{c}^{+}\mu^{-}\bar{\nu}_{\mu}$ differential decay rate, Phys. Rev. D 96 (2017) 112005 [1709.01920].

[43] F.U. Bernlochner, Z. Ligeti, D.J. Robinson and W.L. Sutcliffe, New predictions for $\Lambda_{b} \rightarrow \Lambda_{c}$ semileptonic decays and tests of heavy quark symmetry, Phys. Rev. Lett. 121 (2018) 202001 [1808.09464].

[44] F.U. Bernlochner, Z. Ligeti, D.J. Robinson and W.L. Sutcliffe, Precise predictions for $\Lambda_{b} \rightarrow \Lambda_{c}$ semileptonic decays, Phys. Rev. D 99 (2019) 055008 [1812.07593].

[45] R. Dutta, $\Lambda_{b} \rightarrow (\Lambda_{c}, p)\tau\nu$ decays within standard model and beyond, Phys. Rev. D 93 (2016) 054003 [1512.04034].

[46] X.-Q. Li, Y.-D. Yang and X. Zhang, $\Lambda_{b} \rightarrow \Lambda_{c}\tau\bar{\nu}_{\tau}$ decay in scalar and vector leptoquark scenarios, JHEP 02 (2017) 068 [1611.01635].

[47] E. Di Salvo, F. Fontanelli and Z. Ajaltouni, Detailed Study of the Decay $\Lambda_{b} \rightarrow \Lambda_{c}\tau\bar{\nu}_{\tau}$, Int. J. Mod. Phys. A 33 (2018) 1850169 [1804.05592].

[48] A. Ray, S. Sahoo and R. Mohanta, Probing new physics in semileptonic $\Lambda_{b}$ decays, Phys. Rev. D 99 (2019) 015015 [1812.08314].

[49] N. Penalva, E. Hernández and J. Nieves, Further tests of lepton flavour universality from the charged lepton energy distribution in $b \rightarrow c$ semileptonic decays: The case of $\Lambda_{b} \rightarrow \Lambda_{c}\ell\bar{\nu}_{\ell}$, Phys. Rev. D 100 (2019) 113007 [1908.02328].

[50] X.-L. Mu, Y. Li, Z.-T. Zou and B. Zhu, Investigation of effects of new physics in $\Lambda_{b} \rightarrow \Lambda_{c}\tau\bar{\nu}_{\tau}$ decay, Phys. Rev. D 100 (2019) 113004 [1909.10769].
[51] T. Gutsche, M.A. Ivanov, J.G. Körner, V.E. Lyubovitskij, P. Santorelli and N. Habyl, Semileptonic decay $\Lambda_b \rightarrow \Lambda_c + \tau^- + \bar{\nu}_\tau$ in the covariant confined quark model, Phys. Rev. D 91 (2015) 074001 [1502.04864].

[52] S. Shivashankara, W. Wu and A. Datta, $\Lambda_b \rightarrow \Lambda_c \tau \bar{\nu}_\tau$ Decay in the Standard Model and with New Physics, Phys. Rev. D 91 (2015) 115003 [1502.07230].

[53] P. Böer, A. Kokulu, J.-N. Toelstede and D. van Dyk, Angular Analysis of $\Lambda_b \rightarrow \Lambda_c (\rightarrow \Lambda \pi) \ell \bar{\nu}$, JHEP 12 (2019) 082 [1907.12554].

[54] M. Ferrillo, A. Mathad, P. Owen and N. Serra, Probing effects of new physics in $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ decays, JHEP 12 (2019) 148 [1909.04608].

[55] B. Bhattacharya, A. Datta, S. Kamali and D. London, A measurable angular distribution for $\bar{B} \rightarrow D^* \tau^- \bar{\nu}_\tau$ decays, JHEP 07 (2020) 194 [2005.03032].

[56] T. Feldmann and M.W. Yip, Form factors for $\Lambda_b \rightarrow \Lambda$ transitions in the soft-collinear effective theory, Phys. Rev. D 85 (2012) 014035 [1111.1844].

[57] P. Auvil and J. Brehm, Wave Functions for Particles of Higher Spin, Phys. Rev. 145 (1966) 1152.

[58] H.E. Haber, Spin formalism and applications to new physics searches, in 21st Annual SLAC Summer Institute on Particle Physics: Spin Structure in High-energy Processes (School: 26 Jul - 3 Aug, Topical Conference: 4-6 Aug) (SSI 93), pp. 231–272, 4, 1994 [hep-ph/9405376].