Charge expulsion, Spin Meissner effect, and charge inhomogeneity in superconductors

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Superconductivity occurs in systems that have a lot of negative charge: the highly negatively charged $(\text{CuO}_2)^{\text{\text{-}}}^\text{\text{-}}$ planes in the cuprates, negatively charged $(\text{FeAs})^-\text{\text{-}}$ planes in the iron arsenides, and negatively charged $B^-$ planes in magnesium diboride. And, in the nearly filled (with negative electrons) bands of almost all superconductors, as evidenced by their positive Hall coefficient in the normal state. No explanation for this charge asymmetry is provided by the conventional theory of superconductivity, within which the sign of electric charge plays no role. Instead, the sign of the charge carriers plays a key role in the theory of hole superconductivity, according to which metals become superconducting because they are driven to expel negative charge (electrons) from their interior. This is why NIS tunneling spectra are asymmetric, with larger current for negatively biased samples. The theory also offers a compelling explanation of the Meissner effect: as electrons are expelled towards the surface in the presence of a magnetic field, the Lorentz force imparts them with azimuthal velocity, thus generating the surface Meissner current that screens the interior magnetic field. In type II superconductors, the Lorentz force acting on expelled electrons that don’t reach the surface gives rise to the azimuthal velocity of the vortex currents. In the absence of applied magnetic field, expelled electrons still acquire azimuthal velocity, due to the spin-orbit interaction, in opposite direction for spin-up and spin-down electrons: the “Spin Meissner effect”. This results in a macroscopic spin current flowing near the surface of superconductors in the absence of applied fields, of magnitude equal to the critical charge current (in appropriate units). Charge expulsion also gives rise to an interior outward-pointing electric field and to excess negative charge near the surface. In strongly type II superconductors this physics should give rise to charge inhomogeneity and spin currents throughout the interior of the superconductor, to large sensitivity to (non-magnetic) disorder and to a strong tendency to phase separation.

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1. INTRODUCTION

In a perfectly conducting fluid, magnetic field lines move with the fluid (Alfven’s theorem[1]). Thus it is natural to infer that the expulsion of magnetic field lines from the interior of a metal making a transition to the superconducting state (Meissner effect[2]) is likely to be associated with radially outward motion of electric charge[3]. However, the conventional theory of superconductivity predicts no radial motion of charge in the transition to superconductivity[4]. Rather, BCS-London theory postulates that a spontaneous azimuthal charge motion near the surface is generated (Meissner current) to compensate the magnetic field in the interior, without however explaining what is the driving force for such a motion nor how angular momentum is conserved[5].

Instead, the theory of hole superconductivity[6] predicts that superconductors expel negative charge from their interior towards the surface as they enter the superconducting state[7, 8] to lower their kinetic energy[9] associated with quantum confinement[10], whether or not an external magnetic field is present. In the presence of an external magnetic field, the radial charge motion will ‘drag’ the magnetic field lines with it as in a classical plasma[1]: the magnetic Lorentz force on a radially moving charge acts in the azimuthal direction, and the deflected electron motion generates a magnetic field in direction opposite to the applied one. Thus, the theory offers a ‘dynamical’ explanation of the Meissner effect[11]. In the absence of applied magnetic field, the radial outflow of charge gives rise to a spontaneous spin current[12], predicted to exist in the ground state of all superconductors[13].

The hypothesis that the transition to superconductivity is associated with expulsion of negative charge is supported by the observation that high temperature superconductivity appears to be favored in materials with substructures that have excess negative charge, namely the $(\text{CuO}_2)^{-}$ planes in the cuprates, the $(\text{FeAs})^{-}$ planes in the iron arsenides, and the $B^{-}$ planes in magnesium diboride. The observation that in high $T_c$ materials normal-insulator-superconductor tunneling spectra are asymmetric, with larger current for a negatively biased sample[14, 15], is further evidence that superconductors have a tendency to expel negative charge. Finally, superconducting materials almost always exhibit a positive Hall coefficient in the normal state[16], which indicates electronic bands almost full with negative electrons.

Superconductivity arises in our theory when the Fermi level is close to the top of a band[17], and $T_c$ is enhanced when the ions are negatively charged[18]. Electron-hole asymmetric electronic polaron models describe the physics of pair formation[19], which in the low energy sector reduce to a Hubbard model with correlated hopping[20]. The pair (bipolaron) is lighter than the single polaron in these models[21] because the hopping amplitude increases with increasing local hole occupation...
due to electron-hole asymmetry\cite{22}, and this effect promotes pairing of hole carriers\cite{24}. The models are derived from basic atomic physics considerations of wide generality\cite{22}, and the theory is proposed to apply to all superconducting materials\cite{22}.

II. THE TWO ROUTES TO THE MEISSNER EFFECT

The fact that the Meissner effect is unexplained by the conventional theory is not generally recognized\cite{11}. The canonical momentum of an electron with superfluid velocity $\vec{v}_s$ is

$$\vec{p} = m_e \vec{v}_s + e \vec{A}$$

with $\vec{A}$ the magnetic vector potential. In the BCS ground state the expectation value $< \vec{p} >$ is 0, hence the superfluid velocity is given by

$$\vec{v}_s = -\frac{e}{m_e c} \vec{A} = \frac{e\lambda_L}{m_e c} \vec{B} \times \hat{n}$$

The second equality in Eq. (2) applies to a cylindrical geometry, where $\hat{n}$ is the outward pointing normal of the lateral surface of the cylinder and $\vec{B}$ is the magnetic field along the axis of the cylinder. The London penetration depth $\lambda_L$ is given by\cite{4}

$$\frac{1}{\lambda_L^2} = \frac{4\pi n_s e^2}{m_e c^2}$$

where $n_s$ is the superfluid density.

Eq. (2) embodies the Meissner effect\cite{4}. However the BCS ‘explanation’ just outlined does not explain how the electrons are driven to acquire this velocity starting from a normal state where the average velocity is zero in the presence of a static magnetic field, nor how the mechanical angular momentum of the carriers of the Meissner current is compensated\cite{11}.

A. Meissner current from orbit expansion

Consider an electron that moves radially outward from the axis of a cylinder in the presence of a magnetic field $\vec{B}$ parallel to the cylinder. The equation of motion is

$$m_e \frac{d\vec{v}}{dt} = \frac{e}{c} \vec{v} \times \vec{B} + \vec{F}_r$$

where the first term is the magnetic Lorentz force and the second term is a radial force arising from “quantum pressure” that drives the electron outward\cite{10}. From Eq. (4),

$$\vec{r} \times \frac{d\vec{v}}{dt} = \frac{e}{m_e c} \vec{r} \times (\vec{v} \times \vec{B})$$

where $\vec{r}$ is in the plane perpendicular to the axis of the cylinder. Hence $\vec{r} \cdot \vec{B} = 0$ and $\vec{r} \times (\vec{v} \times \vec{B}) = -(\vec{r} \cdot \vec{v}) \vec{B}$, and

$$\frac{d}{dt}(\vec{r} \times \vec{v}) = -\frac{e}{m_e c} (\vec{r} \cdot \vec{v}) \vec{B} = -\frac{e}{2m_e c} \left( \frac{d}{dt} \vec{r} \right)^2 \vec{B}$$

so that $\vec{r} \times \vec{v} = -(e/2m_e c)r^2 \vec{B}$, and the acquired azimuthal velocity in moving out a distance $r$ is

$$v_\phi = -\frac{e}{2m_e c} r B$$

Thus, to acquire the azimuthal speed Eq. (2) needed for the Meissner current requires the action of the Lorentz force over a radially outgoing motion to radius $r = 2\lambda_L$.

B. Meissner current from Faraday induction

For an electron orbiting in a circular orbit of radius $r$, as an external magnetic field perpendicular to the orbit is applied, an azimuthal electric field $E = (r/2c)\partial B/\partial t$ is generated by Faraday’s law, and the velocity of the electron changes as

$$\frac{dv}{dt} = \frac{eE}{m_e c} = \frac{er}{2m_e c} \frac{\partial B}{\partial t}$$

so that for a magnetic field increasing from 0 to $B$ the extra velocity acquired is

$$\Delta v = \frac{er}{2m_e c} B$$

which reduces to Eq. (2) if and only if the orbit has radius $r = 2\lambda_L$. Hence, the hallmark property of superconductors, that the same Meissner current Eq. (2) results when a magnetic field is applied to an already superconducting metal or when a normal metal becomes superconducting in a pre-existent magnetic field, can be understood from the assumption that superconducting electrons reside in mesoscopic orbits of radius $2\lambda_L$. A parallel reasoning leads to the development of a ground state spin current in the absence of applied fields\cite{13} (Spin Meissner effect) as we discuss in the following section.

III. THE TWO ROUTES TO THE SPIN MEISSNER EFFECT

The superconducting condensate carries a charge density $en_s$. Thus, in a charge-neutral system the superfluid moves in a compensating background of positive charge density $\rho = |e|n_s$. The interaction of the moving magnetic moments of the electrons with the positive background leads to a universal spin Meissner current with speed of magnitude\cite{13}

$$v_\sigma^0 = \frac{\hbar}{4m_e \lambda_L}$$

as we will shows in what follows, which parallels the discussion in the previous section.
A. Spin Meissner current from orbit expansion

Consider a magnetic moment $\vec{\mu}$ along the $z$ direction that moves radially outward with velocity $\vec{v}$. It is equivalent to an electric dipole moment $\vec{p}$

$$\vec{p} = \frac{\vec{v}}{c} \times \vec{\mu}$$  \hspace{1cm} (11)

In the presence of the radial electric field of the cylinder

$$\vec{E} = 2\pi \rho \vec{r} = 2\pi |e| n_s \vec{r}$$  \hspace{1cm} (12)

the electric dipole experiences a torque

$$\vec{\tau} = \vec{p} \times \vec{E} = (\frac{\vec{v}}{c} \times \vec{\mu}) \times \vec{E} = -2\pi |e| n_s \vec{r} \times (\frac{\vec{v}}{c} \times \vec{\mu})$$  \hspace{1cm} (13)

which causes a change in its angular momentum

$$\frac{d\vec{L}}{dt} = m_e \frac{d}{dt}(\vec{r} \times \vec{v}) = \vec{\tau}$$  \hspace{1cm} (14)

Hence

$$\vec{r} \times \frac{d\vec{v}}{dt} = \frac{2\pi en_s}{m_e} \vec{r} \times (\frac{\vec{v}}{c} \times \vec{\mu})$$  \hspace{1cm} (15)

Eq. (15) is identical to Eq. (5) if we define the ‘effective’ magnetic field

$$\vec{B}_\sigma = 2\pi n_s \vec{\mu}$$  \hspace{1cm} (16)

and hence leads to the azimuthal velocity Eq. (7) with $B_\sigma$ replacing $B$

$$v_\phi = -\frac{\pi en_s}{m_e} r \mu_B$$  \hspace{1cm} (17)

with $\mu_B = |e| \hbar / 2m_e c$ the Bohr magneton, so that

$$v_\phi = \frac{\pi n_s e^2 \hbar r}{2m_e^2 c^2} = \frac{\hbar r}{8m_e \lambda^2_L}$$  \hspace{1cm} (18)

where we have used Eq. (3) for the second equality in Eq. (18). The two electrons in a Cooper pair have opposite spin and orbit in opposite directions. The orbital angular momentum of each electron is

$$l = m_e r v_\phi = \frac{\hbar r^2}{8m_e \lambda^2_L}$$  \hspace{1cm} (19)

For $r = 2\lambda_L$, the azimuthal velocity Eq. (18) reduces to Eq. (10) and the orbital angular momentum is

$$l = \frac{\hbar}{2}.$$  \hspace{1cm} (20)

For any other value of $r$, the orbital angular momentum Eq. (19) is not $\hbar/2$.

B. Spin Meissner current from Maxwell induction

The same result for the azimuthal velocity Eq. (10) is obtained through a reasoning paralleling the second route to the Meissner current (Sect. IIB).

Consider a magnetic moment $\vec{\mu}$ in an orbit of radius $r$, with $\vec{\mu}$ oriented perpendicular to the plane of the orbit. Assume a radial electric field grows from 0 to a final value $\vec{E}$. According to Ampere-Maxwell’s law a magnetic field is induced by the varying electric field, satisfying

$$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t}$$  \hspace{1cm} (21)

which exerts an azimuthal force on the magnetic moment

$$\vec{F} = m_e \frac{d\vec{v}}{dt} = \nabla (\vec{\mu} \cdot \vec{B})$$  \hspace{1cm} (22)

We have

$$\nabla (\vec{\mu} \cdot \vec{B}) = (\vec{\mu} \cdot \nabla) \vec{B} + \vec{\mu} \times (\nabla \times \vec{B})$$  \hspace{1cm} (23)

In the geometry under consideration the first term in Eq. (23) is half the second term and points in opposite direction, so that

$$\vec{F} = m_e \frac{d\vec{v}}{dt} = \frac{1}{2} \vec{\mu} \times (\nabla \times \vec{B}) = \frac{1}{2c} \frac{\partial}{\partial t} (\vec{\mu} \times \vec{E})$$  \hspace{1cm} (24)

and the azimuthal velocity acquired is

$$\vec{v}_\phi = \frac{1}{2m_e c} \vec{\mu} \times \vec{E}$$  \hspace{1cm} (25)

and for the electric field given by Eq. (12)

$$\vec{v}_\phi = \frac{\pi |e| n_s}{m_e c} \vec{\mu} \times \vec{r}$$  \hspace{1cm} (26)

or

$$v_\phi = \frac{\pi n_e e^2 \hbar r}{2m_e^2 c^2} = \frac{\hbar r}{8m_e \lambda^2_L}$$  \hspace{1cm} (27)

in agreement with Eq. (18). Thus, just like for the Meissner effect, the same spin-current azimuthal speed Eq. (10) is obtained for a magnetic moment moving radially outward a distance $2\lambda_L$ in the presence of a radial electric field Eq. (12) as for a magnetic moment orbiting at radius $2\lambda_L$ that is subject to a time-dependent radial electric field that grows from zero to its final value Eq. (12).

Note that the finding that the orbital angular momentum of the electron in the Cooper pair is $\hbar/2$ (Eq. (20)) was not “built in”. Rather, it was derived (through two equivalent routes) from the hypothesis that the size of the orbit is $2\lambda_L$, which in turn was inferred from the existence of the Meissner effect, together with the reasonable assumption that the outgoing electron magnetic moment interacts with a positive background of equal charge density as the charge of the superfluid ($\rho = |e| n_s$).
The magnitude of the magnetic field that will stop the spin current velocity of one of the spin orientations (the one that is parallel to the applied \( \vec{B} \)) satisfies, according to Eqs. (2) and (10)

\[
v^s_\sigma = \frac{\hbar}{4m_e\lambda_L} - \frac{e\lambda_L}{m_e c} B_s
\]

hence it is given by

\[
B_s = -\frac{\hbar c}{4e\lambda_L^2} = \frac{\Phi_0}{4\pi\lambda_L^2}
\]  

(29)

with \( \Phi_0 = \hbar c/2|e| \) the flux quantum. Eq. (29) is essentially the lower critical field of a type II superconductor, \( H_{c1} \), that will drive the system normal \( [2] \), and it coincides with \( B_s \), Eq. (16). The flux of the “stopping field” \( B_s \) through the area of the orbit of radius \( 2\lambda_L \) is precisely the flux quantum \( \Phi_0 \).

IV. NEGATIVE CHARGE EXPULSION

We have shown in the previous sections that expansion of the electronic orbits from a microscopic dimension to a mesoscopic radius \( 2\lambda_L \) describes the Meissner effect and predicts the Spin Meissner effect \( [13] \). This expansion also gives rise to expulsion of negative charge from the interior of the superconductor towards the surface, as we discuss in what follows.

In the normal state, electronic orbits carry zero orbital angular momentum on average, however each electron has an intrinsic (spin) angular momentum \( \hbar/2 \). We can think of the spinning electron as a charge \( e \) orbiting at speed \( c \) in an orbit of radius given by the “quantum electron radius” \( r_q = h/(2m_e c) \). We have seen that as the orbit expands to radius \( 2\lambda_L \), the orbital angular momentum acquired is also \( \hbar/2 \) (Eq. 20). Thus the electron orbiting at radius \( 2\lambda_L \) with orbital angular momentum \( \hbar/2 \) can be regarded as a magnified image of the spinning electron, with “magnification factor” \( 2\lambda_L/r_q \). Hence it is natural to conclude that the charge \( e \) will be correspondingly reduced by the factor \( r_q/(2\lambda_L) \), so that the expelled negative charge density is

\[
\rho_- = en_s \frac{r_q}{2\lambda_L} = en_s \frac{v^0_\sigma}{c} .
\]  

(30)

Eq. (30) implies that the spin current can be equivalently regarded as being carried by charge densities \( en_s/2 \) orbiting at speed \( \pm v^0_\sigma \) or by charge densities \( \rho_-/2 \) orbiting at speed \( \pm c \) (the same charge density orbits in each direction in both cases in the absence of applied magnetic field). A similar result holds for the charge current as we discuss below.

Indeed it can be shown \( [27] \) that the requirement that the theory be relativistically covariant leads to the conclusion that a negative charge density \( \rho_- \) of magnitude given by Eq. (30) exists within a London penetration depth of the surface of superconductors. This negative charge was expelled from the interior of the superconductor in the transition to superconductivity \( [26] \), resulting in an interior positive charge density

\[
\rho_0 = -\frac{2\lambda_L}{R} \rho _- .
\]  

(31)

for a cylinder of radius \( R \). The electric field generated by this internal positive charge density increases linearly with \( r \), the distance to the cylinder axis, and reaches a maximum value

\[
E_m = 2\pi \rho_0 R = \frac{hc}{4\pi\lambda_L}
\]  

(32)

within a London penetration depth of the surface. Note that Eq. (32) is the same (in cgs units) as the “stopping” magnetic field Eq. (29) as well as the effective spin-orbit field Eq. (16).

We can also understand the result Eq. (32) from the following heuristic argument. The expelled charge density \( \rho_- \) is related to \( E_m \) by

\[
\rho_- = \frac{E_m}{4\pi\lambda_L}
\]  

(33)

due to charge neutrality. The Meissner current in an applied magnetic field \( B \) has magnitude

\[
j = n_s |e| v_s = \frac{c}{4\pi\lambda_L} B = \frac{\rho_- |e| B}{E_m}
\]  

(34)

Eq. (34) can be interpreted as the current created by the excess negative charge \( \rho_- \) moving at speed

\[
v_{\rho_-} = \frac{B}{E_m}
\]  

(35)

and suggests that superconductivity will be destroyed when \( v_{\rho_-} \) reaches the speed of light. This will occur for \( B = E_m \), thus the value of the magnetic field that stops the spin current and destroys superconductivity, Eq. (29), yields the value of the electric field near the surface \( E_m \) Eq. (32).

From Eqs. (3), (10) and (32) it follows that the electrostatic energy density due to the electric field \( E_m \) equals the kinetic energy density of the spin current

\[
\frac{1}{2} m_e (v^0_\sigma)^2 n_s = \frac{E_m^2}{8\pi}.
\]  

(36)

The same relation exists, as is well known \( [4] \), between the kinetic energy of the Meissner current and the magnetic energy density

\[
\frac{1}{2} m_e v_s^2 n_s = \frac{B^2}{8\pi} .
\]  

(37)

as can be seen from Eqs. (2) and (3).
V. ELECTRODYNAMICS OF CHARGE AND SPIN

The foregoing considerations lead to the following four-dimensional equation in the charge sector\[25\]

\[ J - J_0 = -\frac{c}{4\pi \lambda_L^2} (A - A_0) \]  

(38)

with the current four-vector given by

\[ J = (\vec{J}(r, t), ic\rho(r, t)) \]  

(39)

with \( \vec{J} \) the charge current and \( \rho \) the charge density, and the vector-potential four-vector given by

\[ A = (\vec{A}(r, t), i\phi(r, t)) \]  

(40)

with \( \vec{A} \) the magnetic vector potential and \( \phi \) the electric potential, related by the Lorenz gauge condition \( \text{Div}A = 0 \), with \( \text{Div} \equiv (\nabla, \partial/\partial t)ict \). The quantities with subindex 0 are

\[ J_0 = (0, ic\rho_0) \]  

(41a)

\[ A_0 = (0, i\phi_0(r)) \]  

(41b)

with \( \nabla^2\phi_0 = -4\pi\rho_0 \) and \( \rho_0 \) determined by Eqs. (30)-(32). The spatial part of Eq. (38) is the ordinary London equation. From the fourth component of Eq. (38) and Maxwell’s equations it follows that there exists an electrostatic field in the interior of superconductors that satisfies

\[ \nabla^2(\vec{E} - \vec{E}_0) = \frac{1}{\lambda_L}(\vec{E} - \vec{E}_0) \]  

(42)

with \( \vec{E}_0 \) the electrostatic field generated by the uniform charge density \( \rho_0 \).

The charge current four-vector Eq. (39) is composed of the sum of spin current four-vectors

\[ J = J_\uparrow + J_\downarrow \]  

(43)

and the spin current four-vectors satisfy\[25\]

\[ J_\sigma - J_{\sigma 0} = -\frac{c}{8\pi \lambda_L^2} (A_\sigma - A_{\sigma 0}) \]  

(44)

with

\[ J_\sigma = (\vec{J}_\sigma, ic\rho_\sigma) \]  

(45a)

\[ A_\sigma = (\vec{A}_\sigma, i\phi_\sigma) \]  

(45b)

\( \vec{J}_\sigma = e(n_\sigma/2)\vec{v}_\sigma \) is the component of the current of spin \( \sigma \) and \( \rho_\sigma \) is the charge density with spin \( \sigma \). The spin potentials are given by\[25\]

\[ \vec{A}_\sigma = \lambda_L \vec{\sigma} \times \vec{E}(r, t) + \vec{A}(r, t) \]  

(46a)

\[ \phi_\sigma(r, t) = -\lambda_L \vec{\sigma} \cdot \vec{B}(r, t) + \phi(r, t) \]  

(46b)

Finally, the quantities with subindex 0 are

\[ J_{\sigma 0} = (\vec{J}_{\sigma 0}(r), ic\rho_{\sigma 0}) \]  

(47a)

\[ \vec{J}_{\sigma 0}(r) = -\frac{c\rho_0}{2} \vec{\sigma} \times \hat{r} \]  

(47b)

\[ \rho_{\sigma 0} = \frac{\rho_0}{2} \]  

(47c)

and

\[ A_{\sigma 0} = (\vec{A}_{\sigma 0}(r), i\phi_{\sigma 0}(r)) \]  

(48a)

\[ \vec{A}_{\sigma 0}(r) = \lambda_L \vec{\sigma} \times \vec{E}_0(r) \]  

(48b)

\[ \phi_{\sigma 0}(r) = \phi_0(r) \]  

(48c)

These equations predict the existence of a spontaneous spin current flowing within a London penetration depth of the surface of the superconductor, with carrier densities (\( n_\sigma/2 \)) and opposite spin flowing in each direction with speed Eq. (10), and a spontaneous electric field throughout the interior of the superconductor, of maximum value given by Eq. (32)\[25\].

VI. ENERGETIC CONSIDERATIONS

The kinetic energy of a pair of electrons of opposite spin due to the spin current is \( \epsilon_p = m_e(v_\sigma^0)^2 \). When the applied magnetic field approaches \( B_\sigma \) (Eq. (29)), one of the members of the pair doubles its speed and the other one comes to a stop, hence the kinetic energy of the pair doubles, at which point the pair breaks up\[13\]. We conclude from this argument that the condensation energy of the pair is \( m_e(v_\sigma^0)^2 \) and hence that the condensation energy per electron is

\[ \epsilon_c = \frac{1}{2} m_e(v_\sigma^0)^2 \]  

(49)

which equals the electrostatic energy cost per electron due to the internal electric field, Eq. (36). This implies that each electron lowers its energy in entering the condensate by

\[ \nu \equiv 2\epsilon_c = \frac{\hbar^2 q_0^2}{4m_e} \]  

(50)

with \( q_0 = 1/2\lambda_L \). The system gives back half of this gain right away in the electrostatic energy cost Eq. (36), and the other half when the applied magnetic field destroys superconductivity.

What is the physical origin of this energy lowering? Note that \( \nu \) can be written as

\[ \nu = \frac{1}{2} \mu_B B_\sigma = \frac{|\vec{S}|^2 (v_\sigma^0 \times \vec{E})}{2m_e c^2} \]  

(51)
with $S = \hbar/2$ and $\vec{E}$ the radial electric field Eq. (12) at radius $r = 2\lambda_L$, normal to $\vec{v}_0^\parallel$. The second form of Eq. (51) is the usual spin-orbit energy including the correction for Thomas precession\cite{28}. From this we conclude that the condensation energy of the superconductor originates in the spin-orbit energy lowering arising from the interaction of the spin current of the condensate (of charge density $en_s$ and velocity Eq. (10)) with the compensating positive background charge density $|et_n|$. It is also interesting to note that $\nu$ determines the fraction of the superfluid charge density ($en_s$) that is expelled (as suggested in \cite{33}), through the relations

$$\rho_- = en_s(\frac{\nu}{eE_m\lambda L}) = en_s(\frac{\nu}{m_e c^2})^{1/2}$$  \hspace{1cm} (52)$$

where $eE_m\lambda L$ is the electrostatic energy difference between an electron at the center and at radius $2\lambda_L$ of a cylinder with charge density $|\rho_-|$. (The second expression in Eq. (52) was also found in ref.\cite{26} through different arguments). $\nu$ is also related to this electrostatic energy difference through

$$\nu = \frac{(eE_m\lambda L)^2}{m_e c^2}. \hspace{1cm} (53)$$

The parameter $\nu$ represents a change in the chemical potential between normal and superconducting states\cite{7},\cite{8}, which according to the theory of hole superconductivity is related to the slope of the energy-dependent gap function $\Delta_k$ by\cite{14},\cite{29},\cite{30}

$$\nu = \frac{1}{2} \frac{\partial}{\partial k} (\Delta_k)^2$$  \hspace{1cm} (54)$$

From Eqs. (50), (54) and (10) and using a free-electron dispersion relation $\epsilon_k = \hbar^2 k^2/2m_e$ we find that the energy-dependent gap function is given by

$$\Delta_k = \frac{\hbar^2 q_0 k}{2m_e} = \sqrt{2\nu} \epsilon_k. \hspace{1cm} (55)$$

Both the parameter $\nu$ and the gap at the Fermi energy $\Delta_{k_F}$ can be expressed in terms of the slope of the gap function at the Fermi energy, $m$, that determines the tunneling asymmetry\cite{14}:

$$m = \frac{\partial \Delta_k}{\partial \epsilon_k} \bigg|_{\epsilon = \epsilon_F} = \frac{q_0}{2k_F}. \hspace{1cm} (56a)$$

$$\Delta_{k_F} = 2m \epsilon_F \hspace{1cm} (56b)$$

$$\nu = 2m^2 \epsilon_F = m \Delta_{k_F} \hspace{1cm} (56c)$$

which illustrates that a sloped gap function is a necessary condition for superconductivity\cite{14},\cite{30},\cite{31}.

The quasiparticle energy is given within BCS theory by

$$E_k^* = (\epsilon_k - \mu)^2 + \Delta_k^2 = (\epsilon_k - \mu + \nu)^2 + \Delta_0^2 \hspace{1cm} (57)$$

with the minimum quasiparticle gap $\Delta_0 = \sqrt{2\nu - \nu^2}$. For the second equality in Eq. (57) we used Eq. (55). Eq. (57) shows that indeed $\nu$ is the change in the chemical potential in going from the normal to the superconducting state, as anticipated in the “correlated hopping” model of hole superconductivity\cite{7},\cite{14},\cite{29}.

Furthermore note that a spin current with speed Eq. (10) can be represented by the energy-wavevector relation\cite{32}

$$\epsilon_{k\sigma} = \epsilon_{-k\,-\sigma} = \frac{\hbar^2}{2m_e}(\vec{k} - \vec{q}_0)^2 \hspace{1cm} (58)$$

with $\sigma = \pm 1$, where $\vec{q}_0$ is a vector in the direction of the spin current flow of magnitude $q_0 = 1/2\lambda_L$ (since $h^{-1}(\partial\epsilon_{k\sigma}/\partial k - \partial\epsilon_{-k\,-\sigma}/\partial k)/2 = hq_0/(2m_e) = \vec{q}_0^2$). Eqs. (58) and (55) then imply that

$$\Delta_k = \frac{\epsilon_k^\parallel - \epsilon_{-k\,-\sigma}}{2} = \frac{\epsilon_k^\parallel - \epsilon_k}{2} \hspace{1cm} (59)$$

for $\vec{k} \parallel \vec{q}_0$. In other words, the superconducting energy gap is due to ‘spin splitting’\cite{33}. Note also that the ‘depairing’ speed which will cause the spin current to stop and the pairs to break is given by Eq. (10), which can be written as

$$\nu_s^0 = \frac{\Delta_{k_F}}{\hbar k_F} \hspace{1cm} (60)$$

in terms of the gap Eq. (55) at $k = k_F$. Eq. (60) for the critical speed is identical to what is obtained in conventional BCS theory\cite{4} even though the energy gap expression Eq. (55) was obtained through an entirely independent argument. Fig. 1 shows schematically the spin-split bands in momentum space and the associated charge configuration and spin current in real space.
vortex core has diameter or orbits of up and down spin electrons in a Cooper pair. A Cooper pair if and only if be enclosed by the orbits of both members of the same broken well before the charge speed reaches Eq. (10), a Cooper pair[4]. In extreme type I materials, pairs which is also the average distance between members of q
\[\text{of the orbits is } -\] geometrically, as shown in Fig. 2: Schematic depiction of a Cooper pair in a type I (a) and type II (b) superconductor. The vertical arrows denote the direction of the electron magnetic moment, and the horizontal arrows the orbiting direction. The radius of the orbit of each electron is \(2\lambda_L\), and the distance between the centers of the orbits is \(\xi\). In type II materials ((b)) with \(\xi < 2\lambda_L\) a normal vortex core of diameter \(\xi\) can be enclosed by both orbits of the same Cooper pair.

Finally, note that the condensation energy per electron Eq. (49) can be written, using Eq. (55) and the free-electron dispersion relation, as
\[\epsilon_c = \frac{\Delta_{\text{gap}}^2}{4\epsilon_F}\] (61)
Eq. (61) is consistent with the BCS expression for the condensation energy per unit volume[4]
\[\delta U = \frac{1}{2}N(0)\Delta^2\] (62)
with \(N(0)\) the density of states per spin at the Fermi energy, if we take \(N(0) = n_s/2\epsilon_F\) appropriate to a two-dimensional free-electron system.

Van der Marel[34] and Khomskii[35] have pointed out that quite generally in conventional BCS theory a shift in the chemical potential is predicted upon entering the superconducting state, of magnitude \(\Delta^2/4\epsilon_F\). In our case the shift in the chemical potential, \(\nu\), is twice as large (Eqs. (50) and (61)), and it is directly related to the slope of the gap function and to the existence of a spin current and of negative charge expulsion.

VII. TYPE I VERSUS TYPE II MATERIALS AND CHARGE INHOMOGENEITY

The above energetic considerations apply close to the crossover between type I and type II behavior, where \(H_c \sim H_{c1}\) and \(\lambda_L \sim \xi\), where \(\xi\) is the coherence length which is also the average distance between members of a Cooper pair[4]. In extreme type I materials, pairs are broken well before the charge speed reaches Eq. (10), \(q_0 \sim 1/\xi \ll 1/2\lambda_L\) in Eq. (55) and \(E_m = H_c\) rather than Eq. (32). We can understand the crossover between type I and type II behavior geometrically, as shown in Fig. 2. \(\xi\) is the distance between the centers of the \(2\lambda_L\) orbits of up and down spin electrons in a Cooper pair. A vortex core has diameter \(\xi\), and this normal region can be enclosed by the orbits of both members of the same Cooper pair if and only if \(\xi < 2\lambda_L\).

Why does the vortex core have to be enclosed by the orbits of both members of the same Cooper pair? The phase change for each electron in going around a loop enclosing a vortex is
\[\hbar\Delta\theta = \oint m_e\vec{v} \cdot d\vec{l} + \frac{e}{c}\phi_B\] (63)
where \(\phi_B\) is the enclosed magnetic flux. For each member of the Cooper pair this phase change is \(\pi\), corresponding to its angular momentum \(\hbar/2\), and thus \(\phi_B = (\hbar/2)c/e = \phi_0\) (assuming the integration loop is through a path where \(\vec{v} = 0\)), thus providing a new rationale for the factor of 2 in the flux quantum \(\phi_0\). If only one member of the Cooper pair were to enclose the vortex the wave function for the pair would not be single-valued.

The superconductor expels negative charge towards the surface and towards any interior normal regions, thus in the flux phase there will be excess negative charge in and around the vortex cores. Furthermore our model has a large sensitivity to disorder arising from the slope of the gap function, as discussed in [36]. In the presence of local potential variations due to impurities, vacancies, etc, the gap can be sharply reduced or vanish altogether giving rise to normal regions. There will be excess negative charge in and around those normal regions expelled from the superconducting regions, and a spin current will circulate around the interior normal regions as shown.

FIG. 2: Schematic depiction of a Cooper pair in a type I (a) and type II (b) superconductor. The vertical arrows denote the direction of the electron magnetic moment, and the horizontal arrows the orbiting direction. The radius of the orbit of each electron is \(2\lambda_L\), and the distance between the centers of the orbits is \(\xi\). In type II materials ((b)) with \(\xi < 2\lambda_L\) a normal vortex core of diameter \(\xi\) can be enclosed by both orbits of the same Cooper pair.

FIG. 3: Schematic depiction of a superconductor with strong disorder in the absence of applied magnetic field. Defects, grain boundaries, vacancies, etc. will result in patches of normal regions (hatched areas) surrounded by spin currents (dashed lines, with arrows pointing in the direction of flow of electrons with magnetic moment pointing out of the paper) and excess negative charge density (gray areas). The figure also shows the excess negative charge and spin current near the surface. If the system is cooled in the presence of a magnetic field, magnetic flux will be trapped in the hatched regions and a charge current will flow around those regions together with the depicted spin currents. The smallest normal regions have diameter of a coherence length.
schematically in Fig. 2. An applied magnetic field will concentrate in these normal or weakly superconducting regions and the spin current around them will acquire a charge current component.

Finally, the model has a strong tendency to phase separation in the regime where the carrier (hole) concentration is small [37]. This arises from the bandwidth dependence on the hole concentration [38]: the bandwidth increases with increasing hole concentration, favoring segregation into hole-rich and hole-poor regions. This tendency is also enhanced in the presence of disorder.

VIII. SUMMARY AND DISCUSSION

The theory discussed here proposes that superconductivity arises from the fundamental charge asymmetry of matter [39]. It was motivated by the discovery of high $T_c$ cuprates [20] but it was clear from the outset that if valid it would apply to all superconducting materials [40]. As the theory progressed it became increasingly apparent that it led to a radical departure from the conventional BCS theory [8, 21]. Later it became clear that even London electrodynamics had to be modified [27]. Finally it became clear that spin-orbit coupling plays a key role [13, 25]. The fact that at the end of this road one is led, unexpectedly, to a far more compelling (in our view) explanation [11] of the most fundamental property of superconductors, the Meissner effect, than the conventional theory proposes, is in our view a strong argument for its validity. Ultimately of course confirmation or refutation of the theory discussed here will come from experiment.

As the theory advanced we have gained increased understanding of what we knew we didn’t know, however we have also become aware of issues that previously had been ‘unknown unknowns’ [41]. Perhaps the most important of these issues, that remains to be understood, is the proper inclusion of the key role of the Dirac sea [42].

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