Closed system approach to open systems: tunnelling decay of interacting cold bosons in an optical lattice

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Abstract
A Bose–Hubbard Hamiltonian, modelling cold bosons in an optical lattice, is used to simulate the dynamics of interacting open quantum systems as subsystems of a larger, closed system, avoiding complications like the introduction of baths, complex absorbing potentials or absorbing boundaries. The numerically exact unitary dynamics is compared with the effective descriptions of the subsystems based on non-Hermitian Hamiltonians or Lindblad master equations. The validity of popular models with constant decay rates is explicitly analysed for decaying single and double wells. For simplicity, we concentrate on simulations with very small particle numbers (mostly $N = 2, 3$) and neglect the interaction in parts of the system where this is appropriate. We furthermore present a discrete lattice version of the Siegert approximation method for calculating decay rates.

(Some figures may appear in colour only in the online journal)

1. Introduction

The behaviour of interacting quantum particles in open systems is of fundamental interest. Such systems have been studied experimentally and theoretically in different contexts, including electronic transport in semiconductors and nanostructures [1] and cavity QED [2].

Cold atom experiments [3–6] permit a new way to study interacting quantum particles in open systems and thus a way to test different theoretical approaches for describing these systems, such as effective non-Hermitian Hamiltonians or master equations. Benefits of the cold atom approach include the possibility of creating various trap geometries, tunability of the interaction between the particles and the absence of defects and impurities. In particular, one can realize simple setups where open systems can be studied as smaller subsystems of a larger, closed system, the dynamics of which is governed by familiar Hermitian Hamiltonians leading to unitary time evolution. In this paper, this concept is implemented theoretically for the particular situation of the tunnelling decay of cold bosons within an optical lattice, modelled by the Bose–Hubbard Hamiltonian.

In recent years, the decay dynamics of trapped cold bosons was considered in various contexts. The nonexponential tunnelling decay of Bose–Einstein condensates was analysed in the Gross–Pitaevskii mean-field approximation for different trap geometries [7–12]. Few-boson tunnelling was considered in [13] by means of a numerically exact method yielding deviations from the mean-field behaviour for small particle numbers. Furthermore, the decay of bosons in optical lattices was studied using effective non-Hermitian Hamiltonians [14–16], Lindblad master equations and the Bogoliubov backreaction approximation [17, 18]. In these works, phenomenological models with localized constant decay rates are used. While the latter appear appropriate for describing decay mechanisms such as e.g. particle loss due to a focused laser beam or an electron beam (the latter was implemented successfully in a recent experiment [19]), this does not have to be the case for tunnelling decay considering the nonexponential behaviour found in some of the studies mentioned above.

Here, we analyse the tunnelling decay of cold bosons in a lattice within the framework of the one-dimensional
Figure 1. Bose–Hubbard lattice where two sites (filled circles) forming a double well with the internal tunnelling coefficient \( J \) are weakly coupled (coefficient \( \omega \ll \Omega \)) to a long chain of sites (empty circles) with the internal tunnelling coefficient \( \Omega \).

Bose–Hubbard model

\[
\dot{H} = \sum_j \left( \epsilon_j a_j^\dagger a_j - \frac{J_j}{2} (a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j) + \frac{U_j}{2} (a_j^\dagger)^2 a_j^2 \right),
\]

where \( a_j \) is the annihilation operator of a particle in the lattice site, \( j \) and \( \epsilon_j \) are on-site single-particle energies. The interaction parameters \( U_j \) depend on the shapes of the local ground states in the respective lattice sites, and the local tunnelling rates \( J_j \) on the overlap of the ground states in the respective adjacent sites [20].

For a high total number of bosons \( N \), the operators \( a_j \) can be replaced by the complex numbers \( \sqrt{N} c_j \) representing their respective coherent state expectation values (see e.g. [21]), which leads to the mean-field Hamiltonian

\[
H = \sum_j \left( \epsilon_j |c_j|^2 - \frac{J_j}{2} N (c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) + \frac{U_j}{2} N(N-1) |c_j|^4 \right).
\]

The dynamics of the on-site amplitudes \( c_j \) is then given by \( i\dot{c}_j = \frac{\partial H}{\partial c_j^\dagger} \), which yields the system of coupled discrete nonlinear Schrödinger equations or Gross–Pitaevskii equations

\[
i\dot{c}_j = \epsilon_j c_j - \frac{J_j}{2} c_{j+1} - \frac{J_{j-1}}{2} c_{j-1} + U_j (N-1) |c_j|^2 c_j
\]

in scaled units with \( \hbar = 1 \) that we will use throughout this paper. In the following, the dynamics obtained from the numerically exact solutions of equations (1) and (3) is compared with the results of effective theoretical models, including non-Hermitian Hamiltonians and master equations, describing small subsystems that are weakly coupled to the rest of the lattice. This is illustrated in figure 1 for a double-well subsystem. For simplicity, we will neglect the inter-particle interaction in parts of the system, and the Bose–Hubbard simulations (equation (1)) are mostly performed with very few particles (\( N = 2, 3 \) only).

The main objectives of this paper can be summed up as follows.

- An experimentally realizable system, namely cold bosons in a lattice, is used to study the non-Hermitian quantum dynamics of interacting particles under clean, well-controlled conditions.
- Effective descriptions of open systems, such as e.g. non-Hermitian Hamiltonians or Lindblad master equations, are compared with numerically exact calculations within genuinely closed systems without any additional approximations such as complex absorbing potentials, absorbing boundaries or the introduction of particle baths.
- The validity of popular phenomenological models with constant decay rates is explicitly tested for a concrete mechanism, namely tunnelling decay within a Bose–Hubbard lattice.
- The full Bose–Hubbard dynamics is compared with the mean-field approximation.
- The relative technical simplicity of our approach makes sure that the results are neither obscured nor compromised by mathematical or numerical subtleties.
- Methodically, the discrete lattice version of the Siegert approximation method is presented as a technically simple alternative to decay rate calculations based on Green’s functions or Fermi’s golden rule.

In section 2, we consider single-well tunnelling, i.e. tunnelling out of one site coupled to a long chain; double-well tunnelling will be analysed in section 3.

2. Single-well tunnelling

We consider a situation where one site, let us say site 0, is coupled weakly to a long chain of sites, i.e. we choose the tunnelling coefficients \( J_j = \Omega, j > 0 \), and \( J_0 = \omega \ll \Omega \). For the sake of simplicity, we further choose \( \epsilon_0 = \epsilon, U_0 = U \) and \( \epsilon_j = 0, U_j = 0 \) for \( j > 0 \), such that there is no interaction within the long chain.

In the following, we will find an approximation to the decay coefficient for tunnelling from the single site into the chain by means of the Siegert approximation method [22, 10, 23], which we adopt for use in a discrete lattice. In the case of an open, decaying system, the stationary Schrödinger equation does not really describe a time-independent state but rather a quasistationary decay process. Usually, the decay is accounted for by the imaginary part of the chemical potential \( \mu \), which can be interpreted as a decay rate. The Siegert approximation method is based on neglecting this imaginary part and using instead the continuity equation in order to determine the quasistationary decay rate. A more detailed introduction to the Siegert approximation method can be found in [23].

By means of the usual ansatz \( c_j(t) = c_j \exp(-i\mu t) \) with the chemical potential \( \mu \), equation (3) leads to the quasistationary nonlinear Schrödinger equations

\[
\mu c_0 = -\frac{\omega}{2} c_1 + \epsilon c_0 + U(N-1) |c_0|^2 c_0
\]

\[
\mu c_1 = -\frac{\omega}{2} c_0 - \frac{\Omega}{2} c_2
\]

\[
\mu c_j = -\frac{\Omega}{2} (c_{j-1} + c_{j+1}), \quad j > 1.
\]

For \( j \geq 1 \), we make an outgoing wave (Siegert) ansatz \( c_j \sim A \exp(-i\mu t) \) (i.e. no backreflection at \( +\infty \)) in analogy with outgoing plane waves in continuous space. Thus, equation (6) leads to the dispersion relation \( \mu(k) = -\Omega \cos(k) \). In order to obtain the current in the region \( j > 1 \), we consider \( \partial_j |c_j|^2 = c_j^\dagger \dot{c}_j + c_j \dot{c}_j^\dagger \) (see e.g. [24]). From (6), we obtain

\[
|c_j|^2 = -\frac{\Omega}{2} (c_{j+1}^\dagger c_j - c_j^\dagger c_{j+1} + c_{j-1}^\dagger c_j - c_j^\dagger c_{j-1}) = -\mathcal{J}_{j,j+1} + \mathcal{J}_{j,j-1},
\]

where we have identified the currents \( \mathcal{J}_{j,j+1} = i\Omega (c_{j+1}^\dagger c_j - c_j^\dagger c_{j+1})/2 \) and \( \mathcal{J}_{j,j-1} = i\Omega (c_{j-1}^\dagger c_j - c_j^\dagger c_{j-1})/2 \). Going from site \( j \) to sites \( j+1 \) and \( j-1 \), respectively. Inserting
and the dispersion relation leads to

\[ \omega_c^2 = \omega \sqrt{1 - \frac{\mu^2}{\Omega^2}}. \]

Due to \( \omega \ll \Omega \), the term \(-\omega c_0^2/2 = -\omega^2/2\Omega c_0 \exp(\imath k)\) in (4) can be neglected so that the chemical potential is approximately given by \( \mu \approx \epsilon + U(N-1)|c_0|^2 \). The occupation of the single site thus satisfies the differential equation

\[ |c_0|^2 = \frac{\omega^2}{\Omega} \sqrt{1 - \frac{(\epsilon + U(N-1)|c_0|^2)^2}{\Omega^2}} |c_0|^2. \]

Alternatively, this result can be obtained by means of Green’s functions (see e.g. [25]) or Fermi’s golden rule [26]. Equation (9) implies that due to the dispersion relation in the lattice, there is no decay for \( |\epsilon + U(N-1)|c_0|^2| \geq |\Omega| \) which can be confirmed numerically.

Now we turn to the full Bose–Hubbard model. We attempt an effective description by means of a master equation for the probabilities \( P_M \) that the single-site subsystem is occupied by \( M \) particles, \( 0 \leq M \leq N \) [27]. We apply the sequential tunnelling approximation, i.e. we neglect the possibility of simultaneous decay of two or more particles, concentrating on single-particle processes only [27]. As in the mean-field limit, the transition rates can be obtained using the Siegert approximation method. Instead of the mean-field system (4)–(6), we consider the Heisenberg equations \( \dot{\hat{a}}_j = [\hat{a}_j, \hat{H}] \) derived from the Hamiltonian (1). Since there is no interaction in the chain, the current can be obtained in complete analogy with the mean-field case. The continuity equation still holds so that the tunnelling rate per particle is still given by (8). The single-site system goes from an \( M \)-particle state to an \((M-1)\)-particle state when a particle tunnels into the chain. At site 0, the Heisenberg equation reads

\[ \dot{\hat{a}}_0 = -\frac{\omega}{2} \hat{a}_1 + \epsilon_0 \hat{a}_0 + U \hat{a}_0^2 \hat{a}_0. \]

The chemical potential for going from state \( |M\rangle \) to state \(|M-1\rangle\) is approximately determined by its stationary version

\[ \mu_M |M-1\rangle = \langle M-1 | \hat{H} | M \rangle \approx \langle M-1 | \hat{a}_0^d \hat{a}_0 | M \rangle, \]

where we have neglected the small first term due to \( \omega \ll \Omega \) in analogy with the mean-field case. Thus, the chemical potential reads \( \mu_M \approx \epsilon + U(M-1) \). The corresponding tunnelling rates per particle are given by

\[ \Gamma_M \approx \frac{\omega^2}{\Omega} \sqrt{1 - \frac{(\epsilon + U(M-1))^2}{\Omega^2}} \]

for states with \( M \) particles. This leads to the rate equation model

\[ \dot{P}_N(t) = -N \Gamma_N P_N(t), \quad \dot{P}_0(t) = \Gamma_1 P_1(t), \]

\[ \dot{P}_M(t) = (M+1) \Gamma_{M+1} P_{M+1}(t) - M \Gamma_M P_M(t), \]

\[ 1 \leq M \leq N - 1 \]

for the relative occupations \( P_M \) of the \( M \)-particle states. The total occupation of the first site is then given by

\[ n_0(t) = \sum_{M=0}^{N} M P_M(t). \]

The system can be integrated analytically:

\[ P_N(t) = \exp(-N \Gamma_N t) P_N(0), \quad P_0(t) = \Gamma_1 \int_0^t P_1(\tau) d\tau, \]

\[ P_M(t) = \int_0^t \exp(-M \Gamma_M(t-\tau)) (M+1) \Gamma_{M+1} P_{M+1}(\tau) d\tau, \]

\[ 1 \leq M \leq N - 1. \]

Now we compare the approximate descriptions obtained above with numerically exact integrations of the full mean-field system and the full Bose–Hubbard system for a finite lattice. Both in the mean-field and the Bose–Hubbard case, the simulations are performed by directly integrating the corresponding Schrödinger equations in time using a classical Runge–Kutta scheme. In the Bose–Hubbard case, the Hamiltonian is set up using techniques familiar from the method of exact diagonalization (see e.g. [28] for a recent review).

We start with all particles in the first site. Decay without backreflection can be observed for short times when there is no backreflection at the end of the finite chain. The simulation time \( T_s \) in a chain of length \( L \) is thus determined by \( (\Omega/2)T_s \approx L \), \( T_s \approx 2L/\Omega \) with the phase velocity \( \Omega/2 \). The results are shown in figure 2. In accordance with the results obtained in [13] for single-well tunnelling in a different (non-lattice) setup, there is a clear difference between few-particle tunnelling and mean-field tunnelling. In both cases, the numerically exact results (solid lines) are quite well described by the respective approximations discussed above (dashed lines). For \( N = 2 \) particles, there are some deviations, which might occur due to pair tunnelling, which has been neglected in the rate equation model. For both \( N = 3 \) particles and the mean-field, there is a much better correspondence between the approximations and the exact results, which might be due to the diminished relative importance of pair tunnelling. For comparison, the right panel also shows the rate equation prediction for \( N = 15 \) particles (dash–dotted line) which almost coincides with the mean-field results, which indicates that the mean-field limit is approached with relatively few particles.

3. Double-well tunnelling

Now we consider an open double well consisting of two sites 0 and 1 weakly coupled to a long chain of sites. The quasistationary states can again be obtained by means of the Siegert approximation method. The nonlinear Schrödinger equations describing our system in the mean-field limit now read

\[ \mu c_0 \approx \frac{J}{2} c_1 + \epsilon_0 c_0 + U(N-1)|c_0|^2 c_0 \]

\[ \mu c_1 \approx -\frac{J}{2} c_2 - \frac{J}{2} c_0 + \epsilon_1 c_1 + U(N-1)|c_1|^2 c_1 \]

\[ \mu c_2 \approx -\frac{J}{2} c_1 - \frac{J}{2} c_3 \]
\[ \mu c_j = -\frac{\mu}{\Omega} (c_{j-1} + c_{j+1}), \quad j > 2. \]  
\hspace{2cm} (21)

The outgoing wave solution, dispersion relation and current in the chain \((j \geq 2)\) are still given by \(c_j = A \exp(ik_j)\), \(\mu(k) = -\Omega \cos(k)\) and \(\mathcal{J} = -\Omega |A|^2 / \sqrt{1 - (\mu/\Omega)^2}\), respectively. Analogous to the single-site case, the amplitude \(A\) is obtained from equation (20) as \(A = c_{1 \omega/\Omega}\). The continuity equation \(|c_0|^2 + |c_1|^2 = \mathcal{J}\) for the first two sites then yields the double-well decay rate

\[ \Gamma_d = -\frac{J}{|c_0|^2 + |c_1|^2} \equiv \frac{|c_1|^2}{|c_0|^2 + |c_1|^2} \frac{\omega^2}{\Omega^2} \sqrt{1 - \frac{\mu^2}{\Omega^2}}, \]  
\hspace{2cm} (22)

where the chemical potential \(\mu\) is approximately determined by equations (18) and (19) and the coupling to the chain is neglected, i.e. \(\omega\) is set to zero. Often, phenomenological models with constant decay rates are used to describe open systems. For our single-site model with tunnelling decay from the previous section, this is justified for \(\mu \ll \Omega\) when \(\Gamma \approx \frac{\omega^2}{\Omega^2} = \gamma\). Physically, this means that the internal dynamics of the subsystem is slow compared to the transport velocity in the chain. In this limit, one can try to model our decaying double well by simply adding an imaginary constant \(-i\gamma/2\) to the on-site energy of the second well, which leads to the model

\[ i\dot{c}_0 = -\frac{J}{2} c_1 + \epsilon_0 c_0 + U(N-1) c_0^* c_0 \]  
\hspace{2cm} (23)

\[ i\dot{c}_1 = -\frac{J}{2} c_0 + \left( \epsilon_1 - \frac{\gamma}{2} \right) c_1 + U(N-1) c_1^* c_1, \]  
\hspace{2cm} (24)

considered in a number of recent works (see e.g. [17, 18, 15]). It is interesting to compare the predictions of this model for the decay rates of stationary states with the Siegert approximation result (22). To this end, we treat the term \(-i\gamma c_1/2\) as a small perturbation. The unperturbed potential and amplitudes \(c_0\) and \(c_1\) are then determined by the stationary states of the system (23)–(24) with \(\gamma\) set to zero, obtained in the usual way with the ansatz \(c_j(t) = \exp(-i\xi t) c_j, j = 0, 1\). The first-order perturbation theory correction is straightforwardly obtained as

\[ \Delta \mu = \frac{1}{|c_0|^2 + |c_1|^2} \begin{pmatrix} c_0^* & c_1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & -i\gamma/2 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \end{pmatrix} \]  
\hspace{2cm} (25)

This purely imaginary correction corresponds to a decay rate

\[ \Gamma'_d = -2\text{Im}(\Delta \mu) = \frac{|c_1|^2}{|c_0|^2 + |c_1|^2} \gamma = \frac{|c_1|^2}{|c_0|^2 + |c_1|^2} \frac{\omega^2}{\Omega}, \]  
\hspace{2cm} (26)

which is equal to (22) in the limit \(\mu \ll \Omega\). Therefore, both descriptions are compatible in this limit. This result can be straightforwardly generalized to any finite number of sites coupled to a chain.

The model given by (23) and (24) with \(\gamma = 0\) is a well-studied system whose eigenvectors and corresponding stability properties can be obtained analytically [29]. In the particularly simple symmetric case with \(\epsilon_0 = 0 = \epsilon_1\), the eigenvalues and eigenvectors are given by

\[ \mu = -J\xi/2 + U(N-1)n1^2/(1 + \eta^2), \]  
\hspace{2cm} (27)

with

\[ \eta \in \{\pm 1\}, \quad |U(N-1)n_1| \leq |J| \]  
\hspace{2cm} (28)

\[ \eta \in \{\pm 1\}, \quad -U(N-1)n_1/J \pm \sqrt{U^2(N-1)^2n_1^2/J^2 - 1}, \]
\[ |U(N-1)n_1| \geq |J|, \]  
\hspace{2cm} (29)

where \(n = |c_0|^2 + |c_1|^2\) is the norm of the eigenvector and \(\eta = c_0/c_1\) is real and nonzero. For a repulsive interaction \(U > 0\), the anti-symmetric solution with \(\eta = -1\) becomes dynamically unstable for \(|U(N-1)n_1| \geq |J|\). The two solutions that only exist for sufficiently strong interactions are strongly localized in either the left or the right site, thus breaking the symmetry of the system. Figure 3 shows the time propagation of two of the eigenstates of the symmetric double well due to the full mean-field dynamics of the whole system including the chain (solid lines) and according to the model (23), (24) (dashed-dotted). The two approaches coincide well for both the symmetric ground state (upper panels) and the eigenstate localized in the right well (lower panels). For the symmetric ground state, we find an almost quasisatationary decay behaviour since both the occupation \(|c_1(t)|^2\) of the second lattice site and the total occupation \(n(t) = |c_0(t)|^2 + |c_1(t)|^2\) of the double well show almost pure decay with only a small oscillation. Consequently, the total occupation \(n(t)\) is well described by a decay with the rate \(\Gamma'_{d}\) (dashed lines). For
the symmetry breaking state, we observe a quasistationary decay until the effective interaction \( U(N - 1) n(t) \) drops below the threshold value \( |J| \) for the existence of symmetry breaking eigenstates, and the site occupations start to oscillate. In spite of this fact, the total occupation \( n(t) \) is still reasonably well described by a decay with the rate \( \Gamma_d' \) (dashed line). This behaviour is in agreement with the previous studies on open double-well systems containing detailed discussions of the dynamics of the (quasi-) eigenstates [17, 18, 15, 11]. Figure 4 shows the time propagation for the initial conditions \( c_0(0) = 0, c_1(0) = 1 \), i.e. all particles are initially in the right well. For a moderate value \( U(N - 1) = 0.8 \) of the interaction (upper panels), the oscillatory behaviour of the full system (solid lines) is well captured by the non-Hermitian two-mode model (23), (24) (dash–dotted lines). The lower panels display the dynamics for a higher value \( U(N - 1) = 1.1 \) of the interaction which lies above the threshold \( U(N - 1) = 2|J| = 1 \) for running phase self-Trapping (see e.g. [29–34]), where interactions prevent a total population transfer between the wells in the corresponding Hermitian double-well system. Such an oscillation with a small amplitude can also be observed here for short times. However, after the first oscillation, the effective interaction \( U(N - 1)n(t) \) drops below the threshold value \( 2|J| = 1 \) so that oscillations with larger amplitudes are possible again. Before the threshold is reached, the non-Hermitian two-mode model (dash–dotted lines) provides an excellent description of the dynamics of the full system (solid lines); afterwards deviations occur but the qualitative behaviour is still correctly described.

Now, we again turn to the dynamics of the full Bose–Hubbard system and compare it with an effective two-mode description of our open double well, namely the Lindblad master equation [35]

\[
\dot{\hat{\rho}} = -i[H_0, \hat{\rho}] - \frac{1}{2} \gamma \left( \hat{a}_1^\dagger \hat{a}_1 \hat{\rho} + \hat{\rho} \hat{a}_1^\dagger \hat{a}_1 - 2\hat{a}_1 \hat{a}_1^\dagger \hat{\rho} \right) \tag{30}
\]

for the density matrix \( \rho \), where the second term describes constant decay from site 1 with rate \( \gamma \), whereas the first term provides the Hermitian part of the time evolution with the two-site Bose–Hubbard Hamiltonian

\[
\hat{H}_0 = -(J/2) (\hat{a}_0^\dagger \hat{a}_0 + \hat{a}_1^\dagger \hat{a}_1) + (U/2) (\hat{a}_0^2 \hat{a}_0^\dagger + \hat{a}_1^2 \hat{a}_1^\dagger) \tag{31}
\]

The time-dependent expectation values of an operator \( \hat{O} \) are then given by the trace \( \langle \hat{O}(t) \rangle = \text{Tr}(\rho(t)\hat{O}) \). Recently, it was shown [17, 18] that the Lindblad master equation (30) reduces to the nonlinear non-Hermitian model (23), (24) in the mean-field limit. For a small number of particles, equation (30) can be integrated directly; for higher particle numbers not considered here, it can be solved using Monte Carlo methods [35]. Figure 5 displays the relative occupation of the second lattice site \( N_1(t)/N \) for initial conditions, where only the second well is occupied at \( t = 0 \). Both for \( N = 2 \) particles (upper panels) and for \( N = 3 \) particles, the full Bose–Hubbard dynamics (solid lines) is well reproduced by the Lindblad master equation (dash–dotted lines). Compared to the mean-field case, the oscillatory dynamics of the system appears less regular due to
to the occurrence of various frequencies corresponding to transitions between different eigenstates of the many-(or here rather few-) particle system, an effect which, if regarded from a mean-field point of view, is also referred to as quantum fluctuations in this context [31, 30, 34].

4. Conclusion

In this paper, an experimentally realizable system, namely cold bosons in an optical lattice modelled by a Bose–Hubbard Hamiltonian, was used to theoretically investigate the dynamics of open interacting many-particle systems within a closed system setup. Open single- and double-well systems were simulated as one and two sites, respectively, weakly coupled to a long but finite Bose–Hubbard chain avoiding additional approximations due to absorbing boundaries, complex absorbing potentials or the introduction of baths.

Even for single-site tunnelling, deviations of the mean-field dynamics from the full Bose–Hubbard dynamics were found in accordance with [15]. The nonexponential decay behaviour and in particular the differences between Bose–Hubbard and mean-field were demonstrated to depend on the dispersion relation in the chain. Both in the mean-field and many-particle case, the dynamics was well described by rate equation models derived using a discrete lattice version of the Siegert approximation method.

It was shown that a description of tunnelling decay by means of constant local decay terms is justified if the chemical potential of the considered subsystem is small compared to the tunnelling coefficient in the lattice. In the latter limit, the dynamics of an open double well was analysed. For the latter, it was found that full mean-field and Bose–Hubbard dynamics is well described by a non-Hermitian nonlinear Schrödinger equation and a Lindblad master equation, respectively, with a constant decay term.

In summary, decaying interacting quantum systems were analysed for a concrete physical situation by means of a closed system approach, explicitly confirming the validity of popular effective theoretical descriptions that are usually applied on a phenomenological basis.

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