POLAR CAP MODEL FOR PULSAR HIGH-ENERGY EMISSION

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Abstract

The study of physical processes associated with particle acceleration in the open field line region above the polar cap (PC) of an isolated neutron star (NS) plays a fundamental role in our understanding and interpretation of high-energy emission from pulsars. The systematic study of particle acceleration and formation of electron-positron pair fronts above the PCs of NSs was initiated two decades ago. The detailed analysis of these processes is now possible with the development of pair cascade codes that enables us to calculate the spectra and pulse profiles of high-energy emission from pulsars. The calculation of pair formation and $\gamma$-ray production is being improved to include new results on the PC physics. We briefly outline the current status of the PC model for pulsar high-energy emission, focusing on some of our most recent results on the theoretical modeling of the PC acceleration and $\gamma$-ray emission.

1. Introduction

The small but growing subpopulation of $\gamma$-ray active radio pulsars raises our hopes in advancing the theoretical models of pulsar emission. In this paper we concentrate on the picture in which both the particle acceleration and $\gamma$-ray emission occur in the inner magnetosphere of a neutron star (NS) above the magnetic pole. The corresponding models, known as polar cap (PC) models, favor relatively small obliquities (single pole emission), and they crucially depend on curvature radiation (CR) and on the QED process of magnetic pair creation. The prototype of the PC models has been suggested by Ruderman & Sutherland (1975). Some outstanding problems and progress in this field were addressed clearly by Curtis Michel, Jonathan Arons, and other speakers at the conference. In this written report we will schematically describe the relevant results of our
most recent work that are being prepared for publication in a more complete form. The bottom line of this presentation is that the PC model for pulsar high-energy emission is generally a viable approach, but it needs to be revised and improved in a number of important theoretical aspects.

2. Observation Summary

Let us briefly summarize the main observational facts that need to be explained by any model of $\gamma$-ray emission from pulsars (see Thompson et al. 1997 for a review). First, most $\gamma$-ray light curves show two peaks (separated by 0.4-0.5 in phase) with bridge emission. These peaks are in phase with the similar hard X-ray peaks. Second, in the high-energy band (few to tens GeV) of the spectra of $\gamma$-ray pulsars there seem to be spectral breaks or cutoffs (the exception is PSR 1509-58 for which this occurs between 4 and 30 MeV). Third, there is a systematic variation of spectral hardness through the $\gamma$-ray pulse profile. In Geminga this hardness variation seems to be more complex or even behave in the opposite sense than in other pulsars for which such a study has been performed. And fourth, albeit more ambiguous, is that the hardness of the phase-averaged spectra tend to increase with the pulsar spin-down age. In addition, for some $\gamma$-ray pulsars (e.g. Geminga, Vela, and PSR 0656+14) the soft component of the X-ray spectrum has a broad profile centered between the $\gamma$-ray peaks. Finally, in all $\gamma$-ray pulsars excluding Crab, there is a phase offset between the corresponding peaks at different wavelengths, from radio to gamma.

3. Basics of the PC Model and Main Input Physics

Several types of pulsar models have studied particle (electron/positron) acceleration due to charge deficits at different locations in the neutron star magnetosphere. Polar cap (PC) models consider the formation of a parallel electric field, $E_\parallel \equiv E \cdot B / B$, in the open field region near the magnetic poles, while outer gap models consider acceleration in the outer magnetosphere, near the null charge surface (see Mestel 1998 for the most recent and comprehensive review of pulsar electrodynamics).

The initial calculations of electron-positron pair formation fronts (PFFs) assumed that the primary electrons began accelerating at the NS surface and that curvature radiation (CR) was the only mechanism for providing pair-producing photons (Arons 1983). In recent years, it has become clear that inverse-Compton scattering (ICS) of soft thermal X-ray photons from the hot NS surface by the primary electrons is also an important mechanism above the PC. As well as being a significant energy loss (Xia et al. 1985, Daugherty & Harding 1989, Sturner
1995) and radiation (Sturner & Dermer 1994) mechanism, ICS can also provide photons capable of producing pairs. The standard models of PC acceleration thus need substantial revision.

We use the electric field due to inertial frame dragging above the NS surface, first calculated by Muslimov & Tsygan (1990, 1992; hereafter MT90, 92). The regime under which the generation of this field occurs is actually the same as implied in the electric field computations by Arons & Scharlemann (1979; hereafter AS79), assuming space-charge limited flow in flat space. An electric field must be present above the NS surface because as charges flow along open magnetic field lines, the corotation, or Goldreich-Julian charge density $\rho_{GJ}$, cannot be maintained. Even though the actual charge density $\rho = \rho_{GJ}$ and therefore $E_{\parallel} = 0$ at the surface, the curvature of the field lines causes the area of the open field region, through which the particles flow, to increase faster with height than $\rho_{GJ}$, and a charge deficit grows. Thus, the $E_{\parallel}$ cannot be shorted-out and grows with altitude up to about one stellar radius above the surface, and then it gradually declines. General relativity causes a significant modification (MT90, 92) of this induced electric field, through the effect of dragging of inertial frames, a consequence of the distortion of spacetime by a rotating gravitating body. Thus the Goldreich-Julian charge density, which is the charge density required to make magnetospheric particles drift in corotation with the star, will differ from that in flat space. This charge difference enhances $E_{\parallel}$ over what it would be in flat space, by a factor of 50 - 100 for a typical 1 s pulsar. The frame-dragging contribution (see e.g. first term in Eqn [1]) to $E_{\parallel}$ depends on $\cos \chi$, where $\chi$ is the pulsar obliquity angle. Particle acceleration may therefore occur throughout the entire open field line region, with the relative contribution of the frame dragging component to $E_{\parallel}$ being strongest for pulsars with small obliquities. In our modeling we properly incorporate the effect of the screening of $E_{\parallel}$ at some height above the stellar surface by creation of electron-positron pairs in the strong magnetic field. For illustration we now present an explicit formula for the case where the screening of $E_{\parallel}$ occurs at altitude of order of the PC size,

$$E_{\parallel} \simeq -\frac{3}{2} \left(\frac{\Omega R}{c}\right) \frac{B_0}{1 - \varepsilon} \left(1 - \frac{z}{z_0}\right) z \left[\kappa \cos \chi + \frac{1}{2} \theta_0 \xi H(1) \delta(1) \sin \chi \cos \phi\right], \quad (1)$$

where $\Omega$ is the pulsar rotation frequency, $B_0$ is the surface value of the magnetic field strength at the magnetic pole, $\varepsilon = r_\text{g}/R$, $r_\text{g}$ is the gravitational radius of the NS, $z$ is the altitude scaled by the stellar radius $R$, $z_0$ is the altitude at which the electric field is fully screened out, $\kappa = \varepsilon I/M R^2$, and $I$, and $M$ are the moment of inertia and mass of the NS, respectively, $\xi \equiv \theta/\theta_0$ is the magnetic colatitude scaled by the half-opening angle of the PC, $\phi$ is the azimuthal angle, and the functions $H$ and $\delta$ are the correction factors due to the strong gravity, with $H(1) \delta(1) \simeq 1$. 
Fig. 1. Left: $E_{||}$, Lorentz factor, $\gamma$, and energy loss-rates ($\dot{\gamma}$) due to non-resonant/resonant ICS and CR as functions of the acceleration lengths. Right: The same as in Left, but for the CR-controlled PFFs. Here $H_0$ is the height of lower PFF (start of acceleration) in units of $R$.

4. Modeling of the PFFs and $\gamma$-ray Emission

Another effect which has never been included in PC acceleration models is the formation of a lower PFF due to the positrons that are turned around and accelerated downward from the electron PFF. Although the number of positrons which are accelerated downward is small compared to the primary current and even to the charge deficit at the upper PFF, the multiplicity of the downward cascades is quite large. Thus, the amount of charge produced by only a small number of downward moving positrons may be sufficient to establish a second PFF. Although downward going cascades have been discussed in previous papers (AS79), their effect on the acceleration of primaries has not been investigated.

In our most recent study (Harding & Muslimov 1998b; hereafter HM98b) we investigate the acceleration of primary electrons and secondary (downward-moving) positrons above a pulsar PC, assuming space-charge limited flow (free emission) of particles from the NS surface. Both electrons and positrons suffer energy loss and emit photons from CR and ICS. In Fig. 1 we summarize some of our calculations for the electrons to illustrate the self-consistent solutions of the PFFs controlled by ICS and CR. We compute the location of both electron and positron PFFs due to one-photon pair production as a function of magnetic colatitude and height of the lower PFF. When the electrons are assumed to accelerate from the NS surface, we find that ICS photons produce the PFFs, in agreement with the results of Zhang & Qiao (1996). However, we also find that there is substantial
Fig. 2. Profiles of the maximum electron Lorentz factor and the total acceleration length of the CR-produced upper pair front (see Fig. 1, Right panel) across the PC. The magnetic field strength is in units of $B_{cr} \approx 4.4 \cdot 10^{13}$ G.

asymmetry between the scattering of upward-going electrons and downward-going positrons by the same thermal X-ray photons: the electrons scatter these photons at angles less than $\pi/2$, while the positrons scatter the photons head-on. The photons scattered by positrons of the same energy are therefore more energetic and produce pairs in a shorter distance. These pairs may poison $E_{\parallel}$ up to some altitude above the surface. We demonstrate that (see Fig. 1) the double PFFs controlled by CR tend to set up at much higher altitudes than those controlled by ICS. This fact is essential for reproducing the widely separated double-peaked $\gamma$-ray (and also hard X-ray) profiles observed in $\gamma$-ray pulsars. Also, we suggest that stable, double PFFs can form only when CR photons produce them; i.e. at a height where electron CR losses overtake ICS losses (see Fig. 1, Right panel).

One interesting result of our computations is that the accelerated voltage limited by CR-controlled PFFs is only a function of magnetic colatitude (i.e. geometry of the open field lines), ranging between $\sim 10^7$ and $3 \cdot 10^7 mc^2$, and is insensitive to pulsar parameters such as period and surface magnetic field and even to the height of the acceleration. In Fig. 2 we present the profiles of the electron upper PFF controlled by CR (i.e. in the regime of electron acceleration where the CR losses dominate the ICS losses) and the corresponding maximum Lorentz factor as functions of coordinate $\xi$ (see the notations following Eqn [1]). Figure 2 illustrates that the electron maximum Lorentz factor (total voltage drop) decreases toward the PC rim, $\xi = 1$, because of decrease in $E_{\parallel}$, while the height of the PFF increases at the magnetic axis, because of significant increase in the pair-production
attenuation length. The stable location of the lower PFF depends primarily of surface magnetic field, and PC size and temperature, ranging between 0.5 and 1.0 stellar radius, but is insensitive to period.

5. On the PC Heating and Surface X-ray Emission

Some fraction of positrons produced e.g. in the first cascades may return toward the PC surface (an accurate determination of the fraction of backflowing positrons requires a self-consistent calculation of the screening of $E_\parallel$ by the cascade pairs and the modification of the pair spatial distribution by the screened $E_\parallel$). These backflowing positrons slightly suppress the voltage all the way down to the bottom of the polar magnetic flux tube, so that the electric field $E_\parallel$ (and also potential $\Phi$) vanishes at the height $H_0$ above the stellar surface rather than at the actual stellar surface. This occurs because the flux of backflowing positrons is equivalent to the corresponding enhancement of the total electron current and therefore of the maximum number of electrons per second to be ejected into the acceleration region. Thus, to satisfy the zero-electric field boundary condition the ejection radius should fix itself at the larger value corresponding to the higher rate of supply of Goldreich-Julian charge. It is interesting that, just by using very general reasoning, we can derive an upper limit on the fraction of returning positrons.

The maximum possible total power put into the backflowing positrons can be estimated as (cf. Muslimov & Harding 1997; hereafter MH97, Eqns [76]-[78]; and see HM98b):

$$\{L_{e^+}\}_{\text{max}} \approx \lambda L_{sd},$$  \hspace{1cm} (2)

where

$$\lambda \approx (3/4)x^3\left(\kappa^2 x^2 \cos^2 \chi + (3/16)H^2 \phi^2 \sin^2 \chi \right),$$  \hspace{1cm} (3)

and $L_{sd}$ is the spin-down luminosity of a pulsar, $x = 1/(1 + H_0/R)$, $H_0$ is the altitude of the lower (positron) PFF (see Fig. 1). For relatively small obliquities and typical pulsar spin periods of 0.1 - 1 s the first term in Eqn (3) dominates, and we get

$$\{L_{e^+}\}_{\text{max}} \approx (3/4)\kappa^2 x^6 L_{sd}.\tag{4}$$

For the 1.4 $M_\odot$ NS and for a broad range of realistic equations of state of dense matter (see e.g. Ravenhall & Pethick 1994 for the calculations of the NS moment of inertia for various equations of state) $I/(MR^2) \approx (0.2 - 0.25)(1 - r_g/R)^{-1}$. Thus, for the NS of 1.4 $M_\odot$ and 8-10 km radius (which is consistent with the realistic stellar models) we can estimate $\kappa \approx 0.15 - 0.27$. Given $x \approx 0.5 - 1$, Eqn (4) yields $\lambda \approx (3 \cdot 10^{-4} - 2 \cdot 10^{-2})(\kappa/0.15)^2$. This estimate combined with
the estimated X-ray luminosities (that imply isotropic emission) of pulsars (see Becker & Trümper 1997) may have rather interesting implications.

For example, if the X-ray fluxes in some of these pulsars are dominated by the photons from the heated (e.g. by the backflowing positrons) PC, then this may indicate that these pulsars operate in an oscillatory regime (see Sturrock 1971, and also HM98b). Alternatively, we should note that according to the estimates by Becker & Trümper the pulsed X-ray luminosities (for the case of isotropic emission), $L_{\text{x}}^p$, for e.g. Crab, Vela, Geminga, and PSR 0656+14 are, respectively, $1.6 \cdot 10^{-3}$, $0.7 \cdot 10^{-5}$, $1.3 \cdot 10^{-4}$, and $3.7 \cdot 10^{-3} L_{\text{sd}}$. If we assume that at least for Vela, Geminga, and PSR 0656+14 (see e.g. HM98a for the modeling of the soft X-ray and $\gamma$-ray emission for these pulsars) the X-ray emission is beamed into a solid angle of $\sim 1$ steradian, then for these pulsars we can estimate that $\lambda_{\text{x anis}}^p \equiv L_{\text{x anis}}^p / L_{\text{sd}} \sim 6 \cdot 10^{-7}$, $10^{-5}$, and $3 \cdot 10^{-4}$, respectively. The corresponding PC temperature for these pulsars can be estimated as $T_{\text{pc}} \sim (0.6 - 1) \cdot 10^6 (10^5 \lambda_{\text{x anis}}^p)^{1/4} (B_0/4 \cdot 10^{12} \text{ G})^{1/2} (R/8 \text{ km})^{3/4} (P/0.1 \text{ s})^{-3/4} \text{ K}$. Thus, the estimated value of $\lambda_{\text{x anis}}^p \sim (3 \cdot 10^{-5} - 10^{-2})$ may indicate that the backflowing positrons precipitate onto the effective area smaller than that of the standard PC (e.g. the returning positrons focus toward the magnetic axis, HM98b), and/or that the fraction of the returning positrons is well below the maximum possible one. Then the latter would support the quasi-steady state (MH97) rather than the oscillatory (Sturrock 1971) regime of pulsar operation.

6. Conclusions

In conclusion we emphasize that the PC model is capable of explaining and reproducing (see Harding 1996, Daugherty & Harding 1996) the main observational facts mentioned in § 2: the widely separated double-peaked profiles (also in hard X-rays) with bridge component, very steep high-energy spectral cutoffs (due to magnetic pair production), and systematic soft-hard-soft hardness variation in the pulse (due to the softening of emerging spectra by cascades).

The novel developments we would like to introduce here are: 1) the double PFF controlled by CR, a self-limiting buildup where the electrons are accelerated from the lower front and a small fraction of positrons returning from the upper front to produce the lower front; 2) the establishment of the PFFs and pair cascades at higher altitudes; and 3) the revised treatment of a feedback between the inflow of cascade particles into the acceleration region and accelerating electric field, including the PC heating by returning positrons.

Although the PC model has had success in accounting for a number of main observational features of $\gamma$-ray pulsars, there remain some puzzling observational
data and exceptions (e.g., Crab: alignment of γ-ray pulses with those at other wavebands; Geminga: complex behaviour of spectral hardness through the pulse; Geminga, Vela, and PSR 0656+14: soft pulsed X-ray emission; etc.). Given the advancement in understanding of PC acceleration discussed above, we will be able to model characteristics of pulsar high-energy emission from PC pair cascades in more detail and ultimately address these unsolved problems.

7. References

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