Electroweak baryogenesis induced by a scalar field

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Abstract

A cosmological pseudoscalar field coupled to hypercharge topological number density can exponentially amplify hyperelectric and hypermagnetic fields while coherently rolling or oscillating, leading to the formation of a time-dependent condensate of topological number density. The topological condensate can be converted, under certain conditions, into baryons in sufficient quantity to explain the observed baryon asymmetry in the universe. The amplified hypermagnetic field can perhaps sufficiently strengthen the electroweak phase transition, and by doing so, save any pre-existing baryon number asymmetry from extinction.

Preprint Number: BGU-PH-98/07
To generate baryon asymmetry from a state in which quark and antiquarks have equal abundances, the three celebrated Sakharov conditions have to be satisfied. In the scenario we propose, the Sakharov conditions are satisfied during an epoch in which a cosmological pseudo-scalar field coupled to hypercharge topological number density coherently rolls or oscillates. The scalar field motion creates a time dependent hypercharge topological number condensate which violates fermion number conservation through the abelian anomalous coupling \[ \lambda \bar{Y}Y \], and establishes the required departure from equilibrium, and its interactions violate C and CP symmetries. Pseudoscalar fields with the proposed axion-like coupling appear in several possible extensions of the Standard Model. They typically have only perturbative derivative interactions and therefore vanishing potential at high temperatures, and acquire a potential at lower temperatures through non-perturbative interactions. Their potentials take the generic form \[ V(\phi) = V_0^4 V(\phi/f) \], where \[ V_0 \] is a bounded periodic function characterized by a scale \( f \) ("Peccei-Quinn" scale), which could be as high as the Planck scale and a much smaller mass \( m \approx V_0^2/f \), which could be as low as a fraction of an eV, or as high as \( 10^{12} \) GeV. A particularly interesting mass range is the TeV range, expected to appear if potential generation is associated with supersymmetry breaking.

The fundamental role of hypermagnetic and hyperelectric fields in the context of electroweak (EW) baryogenesis has been recognized recently in several investigations. It was observed that: i) a topological number condensate can be released at the EW transition in the form of leptons and baryons, and ii) strong enough hypermagnetic fields could make the EW transition strongly first order even for those large values of the Higgs mass that have not been ruled out by LEP II experiment. Scalars with axion-like coupling to hypercharge fields were previously considered. Amplification of ordinary electromagnetic (EM) fields by such scalar fields was discussed, and their possible use for baryogenesis in [5]. We will assume that the universe is homogeneous and isotropic, and can be described by a conformally flat metric \( ds^2 = a^2(\eta)(d\eta^2 - dx_1^2 - dx_2^2 - dx_3^2) \) where \( a(\eta) \) is the scale factor of the universe, and \( \eta \) is conformal time related to cosmic time \( t \) as \( a(\eta)d\eta = dt \). In addition to the standard model fields we will consider a time-dependent pseudoscalar field \( \phi(\eta) \) with coupling \( \frac{1}{4}\phi Y\bar{Y} \) to the \( U(1)_Y \) hypercharge field strength and a potential \( V(\phi) = V_0^4 V(\phi/f) \). The coupling constant \( \lambda \) (which we will take as positive) has units of mass\(^{-1} \). For QCD axions, \( M \sim f \), but in general, it is not always the case, and we will therefore take \( M \) to be a free parameter, and in particular allow \( M < f \). We will assume that the universe is radiation dominated at some early time \( \eta = 0 \) at \( T \gtrsim 100 \) GeV before the scalar field becomes relevant.

Maxwell’s equations describing the hyper EM fields (we will drop the hyper from now on for brevity) in the resistive approximation of the EW plasma, coupled to the heavy pseudoscalar are the following

\[
\begin{align*}
(i) \quad & \nabla \cdot \vec{E} = 0 \quad (ii) \quad \nabla \cdot \vec{B} = 0 \\
(iii) \quad & \vec{J} = \sigma \vec{E} \quad (iv) \quad \frac{\partial \vec{B}}{\partial \eta} = -\nabla \times \vec{E} \\
(v) \quad & \frac{\partial \vec{E}}{\partial \eta} = \nabla \times \vec{B} - \lambda \frac{d\phi}{d\eta} \vec{B} - \vec{J}.
\end{align*}
\]

We have rescaled the electric and magnetic fields \( \vec{E} = a^2(\eta) \vec{E}, \vec{B} = a^2(\eta) \vec{B} \), and the physical conductivity \( \sigma = a(\eta)\sigma_c \). In the EW plasma \( \sigma_c \sim 10T \). The fields \( \vec{E}, \vec{B} \) are the flat...
space EM fields. We have assumed for simplicity vanishing bulk velocity \( \bar{v} \) of the plasma and vanishing chemical potentials for all species. Our results can be easily generalized to include non-zero bulk velocity or right electron chemical potential. The equation for the pseudoscalar \( \phi \) is the following

\[
\frac{d^2 \phi}{d\eta^2} + 2aH \frac{d\phi}{d\eta} + a^2 \frac{dV(\phi)}{d\phi} = \lambda a^2 \vec{E} \cdot \vec{B},
\]  

(2)

where \( H = \frac{1}{a} \frac{da}{d\eta} \) is the Hubble parameter. We will neglect the backreaction of the electromagnetic fields on the scalar field since it is irrelevant for most of the physics we would like to explore. The cosmic friction term \( 2aH \frac{d\phi}{d\eta} \) will not be relevant for most of our discussion. We therefore solve eq.(2) with vanishing r.h.s., and substitute the resulting amplification factor \( \lambda \) which can happen only if \( \lambda \) in many models of supergravity, and a value is obtained if the typical scale for scalar field motion is the Planck scale, as happens in many models of supergravity, and \( \lambda \leq 1/10^{10} \text{GeV} \).

We therefore solve eq.(2) with vanishing r.h.s., and substitute the resulting \( \phi(\eta) \) into eq.(2).

We are interested in solutions of the form \( \vec{E}(\vec{x}, \eta) = \int d^3 \vec{k} e^{-i\vec{k} \cdot \vec{x}} \hat{c}_k(\eta), \vec{B}(\vec{x}, \eta) = \int d^3 \vec{k} e^{-i\vec{k} \cdot \vec{x}} \hat{b}_k \beta_k(\eta) \), for which the electric and magnetic modes are parallel to each other. We find that the Fourier modes \( \hat{c}_k \) and \( \hat{b}_k \) are related, \( \hat{b}_k^\pm = \hat{b}_k^\mp (\hat{e}_1 \pm i\hat{e}_2) \), \( k\hat{c}_k^\pm \hat{e}_\mp(\eta) = \pm \hat{b}_k^\pm e^{i\beta_k(\eta)} \), where \( \hat{e}_1, \hat{e}_2 \) are unit vectors in the plane perpendicular to \( \vec{k} \) such that \( (\hat{e}_1, \hat{e}_2, \hat{k}) \) is a right-handed system. This type of solution has \( J_k \sim e^{i\hat{b}_k/\sigma}, \nabla \times \hat{B}_k \sim \hat{B}_k, \) and \( \nabla \times \vec{E}_k \sim \vec{E}_k \). The function \( \beta_k(\eta) \) obeys the following equation,

\[
\frac{\partial^2 \beta_k}{\partial \eta^2} \pm \sigma \frac{\partial \beta_k}{\partial \eta} + \left( k^2 \pm \lambda \frac{d\phi}{d\eta} \right) \beta_k(\eta) = 0.
\]  

(3)

Before plunging into a detailed discussion of the solutions of eq.(3), it is useful to consider the simple case of a constant \( \frac{d\phi}{d\eta} \). Then the solutions are simply

\[
\beta^\pm(\eta) = \beta_0^\pm e^{\omega^\pm \eta} + \beta_0^\pm e^{\omega^\mp \eta},
\]

where \( \omega_1,2 \) are the two roots of the quadratic equation \( \omega^2 + \sigma \omega + \left( k^2 \pm \lambda \frac{d\phi}{d\eta} \right) = 0, \omega_1,2 = \frac{1}{2} \left[ -\sigma \pm \sqrt{\sigma^2 - 4 \left( k^2 \pm \lambda \frac{d\phi}{d\eta} \right) } \right] \).

Now the qualitative behaviour of solutions is clear. If one of the eigenvalues is positive, which can happen only if \( \lambda | \frac{d\phi}{d\eta} | > k \), EM fields can grow exponentially. Otherwise fields are either oscillating or damped, as in ordinary magnetohydrodynamics. To obtain significant amplification, coherent scalar field velocities \( \frac{d\phi}{d\eta} \) over a duration are necessary, larger velocities leading to larger amplification. We estimate the wavenumber \( k \) of maximal amplification by looking for the maximum of \( \omega \) as a function of \( k \), \( k_{\text{max}} = \frac{1}{2} \lambda \frac{d\phi}{d\eta} \).

Another interesting approximate solution relevant for rolling fields can be obtained for \( k \ll T \). In this case, eq.(3) can be approximated by a first order equation

\[
\sigma \frac{\partial \beta}{\partial \eta} + \left( k^2 \pm \lambda \frac{d\phi}{d\eta} \right) \beta(\eta) = 0,
\]

which can be solved exactly, \( \beta = \beta_0 e^{-k^2 2 \eta} + \lambda \Delta \phi k/\sigma \), where \( \Delta \phi(\eta) = |\phi(\eta) - \phi(0)| \). The amplified mode is determined by the sign of \( \phi(\eta) - \phi(0) \). The amplification factor \( A^\pm(k, \eta) = \beta^\pm(\eta)/\beta^\pm(0) \) is maximal for \( k_{\text{max}} \eta = \frac{1}{2} \lambda \Delta \phi, A^\pm(k_{\text{max}}, \eta) = e^{\frac{1}{2}(\lambda \Delta \phi)^2 \frac{1}{8}} \). Looking at \( \eta \sim \eta_{\text{EW}} \) we obtain \( \frac{1}{\eta_{\text{EW}} \sigma} \sim 10^{-16} \), and therefore to obtain amplification \( \lambda \Delta \phi \gtrsim 10^8 \). For specific models an upper bound on \( \lambda \Delta \phi \) may appear, narrowing the range of allowed parameter space. A value of \( \lambda \Delta \phi \gtrsim 10^8 \) is not unnatural, for example, such a value is obtained if the typical scale for scalar field motion is the Planck scale, as happens in many models of supergravity, and \( \lambda \leq 1/10^{10} \text{GeV} \).

\[
\lambda \leq 1/10^{10} \text{GeV}.
\]
FIG. 1. Amplification of EM fields. The function $\beta$, exponentially scaled, is shown for the parameters $\Lambda = 55$, $k/m = 20$, $m = 6\, TeV$ and $\sigma = 40\, T$ at $T = 1\, TeV$.

If the scalar field is oscillating, the field’s velocity changes sign periodically and both modes can be amplified, each during a different part of the cycle. Each mode is amplified during one part of the cycle and damped during the other part of the cycle. Net amplification results when amplification overcomes damping, which occurs roughly when $\lambda d\phi/d\eta k \gtrsim \sigma^2$ (recall that $\sigma \sim 10^T$). Total amplification is exponential in the number of cycles.

We have studied numerically the solutions of eq.(3) for the case of an oscillating field for different parameters and situations. The concrete potential that we used for the numerical analysis is $V(\phi) = \frac{1}{2}m^2\phi^2$, assuming that the initial amplitude of the field is of order $f$ with $\Lambda \equiv f/M > 1$, $f/m > 1$. We find results in agreement with the previous qualitative discussion. Amplification occurs for a limited range of Fourier modes, peaked around $k/m \sim \Lambda$. The modes of the EM fields are oscillating with (sometimes complicated) periodic time dependence and an exponentially growing amplitude. In Fig. 1 we show an example of the time dependence of amplified EM fields for a specific mode. For the range of parameters in which fields are amplified, the amount of amplification per cycle for each of the two modes is very well approximated by the same constant $\Gamma(k/m, \Lambda, \sigma)$. A good approximate estimate for the average amplification after $N$ cycles is therefore $A^{\pm}(k, \eta) = N^{\pm,k}e^{N\Gamma}$, where $N^{\pm,k}$ represents the transient influence of the initial conditions of EM and scalar fields.

The scalar field is a very efficient amplifier of EM fields. For example, to obtain an amplification of $10^{12}$ for $\Lambda = 55$, $k/m = 20$, $m = 6\, TeV$ and $\sigma = 40\, T$, for oscillations occurring at a temperature of $1\, TeV$, we need just one cycle! In Fig. 2 we have presented an example of the dependence of the amplification per cycle factor $\Gamma$, on different parameters.
A detailed discussion of the end of oscillations or rolling is beyond the scope of the current investigation. However, we do know that once the oscillations or rolling stop, the fields are no longer amplified and obey a diffusion equation \[4\]. Modes with wave number below the diffusion value \(k < k_\sigma T \sim 10^{-8} T\), where \(\frac{k^2}{\eta_{EW}} = 1\), remain almost constant until the EW transition, their amplitude goes down as \(T^2\), and energy density as \(T^4\), maintaining a constant ratio with the environment radiation. Modes with \(k/T > k_\sigma\) decay quickly, washing out the results of amplification. We have seen that the range of amplified momenta for oscillating fields is not too different than \(T\), therefore scalar field oscillations have to occur just before, or during the EW transition. In that case, the amplified fields do not have enough time to be damped by diffusion. If the field is rolling, momenta \(k \ll T\) can be amplified, and therefore the rolling can end sometime before the EW transition.

To obtain the average magnetic energy density in amplified fields we have to average over the magnetic and scalar fields initial conditions. The averaging has different reasons: \(i\) statistical fluctuations inside the horizon of magnetic field initial conditions due to thermal and quantum fluctuations, and \(ii\) variations over causally disconnected regions. We will assume translation and rotation invariance of initial conditions, \(\langle B^\pm(\vec{k}, \eta)B^\pm(\vec{\ell}, \eta)^*\rangle_{\eta=0} = \delta^3(\vec{k} - \vec{\ell}) \delta_+ f^\pm(k)\). Using the magnetic power spectrum \(P^\pm_B(k) = 4\pi k^3 f^\pm(k)\), the average
initial energy density $\rho_B(0)$, given by $\rho_B(0) = \frac{1}{8\pi} < B^2 >|_{\eta=0}$, can be expressed as $\rho_B(0) = \frac{1}{8\pi} \int_0^\infty d\ln k \left( P_B^+(k) + P_B^-(k) \right)$.

The two modes of the magnetic field are amplified by amplification factors $A^\pm(k, \eta)$, which depend on the parameters of the model as discussed previously. The amplification factors depend, in addition, on the initial conditions of the magnetic field and on the initial conditions of the scalar field. For example, if the velocity of the scalar field $d\phi/d\eta$ is constant, the sign of the velocity determines that only $A^+$ or $A^-$ will be non-vanishing.

The magnetic energy density in amplified fields is given by

$$\rho_B = \frac{1}{8\pi} \int d\ln k \left\{ P_B^-(k) \left| A^-(k, \eta) \right|^2 + P_B^+(k) \left| A^+(k, \eta) \right|^2 \right\}. \quad (4)$$

The Chern-Simons number density is given by $\Delta n_{CS} = \frac{y_R^2 g^2}{16\pi^2} \int d\eta (E \cdot B)_\eta$, where $y_R = -2$ is the hypercharge of the right electron, and $g'$ is the hypercharge gauge coupling. Since in our case $E_k^\pm(\eta) = \pm \frac{1}{2} \partial_\eta B^\pm(k, \eta)$, then $(E^\pm(k, \eta)B^\pm(\bar{k}, \bar{\eta}))^* = \pm \frac{1}{2} \partial_\eta (B^\pm(k, \eta) B^\pm(\bar{k}, \bar{\eta}))^*$. Neglecting the initial $n_{CS}$ density,

$$n_{CS} = \frac{y_R^2 g'^2}{32\pi^2} \int d\ln k \frac{1}{k} \left\{ P_B^-(k) \left| A^-(k, \eta) \right|^2 - P_B^+(k) \left| A^+(k, \eta) \right|^2 \right\}. \quad (5)$$

Let us define two $k$-dependent asymmetry parameters, $\gamma_{AS}^B = \frac{P_B^+(k) - P_B^-(k)}{P_B^+(k) + P_B^-(k)}$, and $\gamma_{AS}^g = \frac{|A^-(k, \eta)|^2 - |A^+(k, \eta)|^2}{|A^-(k, \eta)|^2 + |A^+(k, \eta)|^2}$. Both parameters vary from 0 - no asymmetry, to ±1 - maximum asymmetry. Using the asymmetry parameters we may relate $n_{CS}$ to $\rho_B$,

$$n_{CS} = \frac{y_R^2 g'^2}{4\pi} \int d\ln k \frac{1}{k} \rho_B(k) \frac{\gamma_{AS}^B + \gamma_{AS}^g}{1 + \gamma_{AS}^B \gamma_{AS}^g}. \quad (6)$$

Further, we can compute the fractional energy density in coherent magnetic field configurations $\Omega_B(k) = \rho_B(k)/\rho_c$, where $\rho_c$ is the critical energy density, and the Chern-Simons fractional number density $n_{CS}/s$, where $s$ is the entropy density, and compare them. If the universe is radiation dominated $\rho_c = \frac{s^2}{30} g_* T^4$ and $s = \frac{2\pi^2}{45} g_* T^3$, therefore

$$\frac{n_{CS}}{s} = \frac{y_R^2 g'^2}{4\pi} \frac{3g_*}{4g_*} \int d\ln k \frac{T}{k} \Omega_B(k) \frac{\gamma_{AS}^B + \gamma_{AS}^g}{1 + \gamma_{AS}^B \gamma_{AS}^g}. \quad (7)$$

If the amplification factors $A^\pm_k$ are, as we have seen, sharp functions of $k$, peaked at $k_{max}$ and have width in $k$ of approximately $k_{max}$, then

$$\frac{n_{CS}}{s} \simeq 0.01 \frac{T}{k_{max}} \Omega_B(k_{max}) \frac{\gamma_{AS}^B + \gamma_{AS}^g}{1 + \gamma_{AS}^B \gamma_{AS}^g}(k_{max}), \quad (8)$$

where we have used numerical values for the prefactor coefficients.

According to Giovannini and Shaposhnikov [3], this Chern-Simons number will be released in the form of fermions which will not be erased if the EW transition is strongly first
order \(8\), and will generate a baryon asymmetry, \(\frac{n_B}{s} = -\frac{3}{2} \frac{n_{CS}}{s}\). An equal lepton number would also be generated by the same mechanism so that \(B - L\) is conserved. Note that the fact that baryon number asymmetry is generated at \(k_{max} \neq 0\) does not mean that baryon density is actually inhomogeneous on this short length scale \(L_{max} \sim 1/k_{max}\). Comoving neutron diffusion distance at the beginning of nucleosynthesis is much longer than \(L_{max}\) \([9,10]\), so that by that time inhomogeneities would have been erased by free streaming \([9]\).

If \(T/k_{max}\) is not too different than unity, as we have seen for the case of oscillating field, and \(\gamma_{B\text{AS}}^B\) and \(\gamma_{\phi\text{AS}}^\phi\) are small, it is possible to obtain \(\frac{n_B}{s} \sim 10^{-10}\) and have strong magnetic fields \(\Omega_B \sim 1\) present during the EW transition. If \(T/k_{max}\) is large and \(\gamma_{\phi\text{AS}}^\phi\) is order unity as we have seen in the rolling case, it is not possible to have strong magnetic fields without producing too many baryons.

The existence of hypermagnetic fields during the EW transition can influence the nature of the EW transition. For example, a homogeneous hypermagnetic field adds a pressure term to the symmetric phase which could lower the transition temperature and strengthen the EW transition. This well known effect in conductor-superconductor phase transitions \([11]\) was discussed in several investigations \([2,3]\), with seemingly inconclusive results, for the moment.

Our conclusions are therefore that

1. Depending on the ratio \(T/k_{max}\) and on the asymmetry parameters, it is possible to create the desired ratio \(\frac{n_{CS}}{s} \sim 10^{-10}\), and therefore to generate the observed baryon asymmetry in the universe.
2. Some asymmetry in the initial conditions of either \(B\) or \(\phi\) is required. The asymmetry can be very small and can be induced at a much higher scale. Possible sources for the initial asymmetry are temperature dependent potential that traps \(\phi\) at a preferred position, asymmetry in quantum fluctuations, etc. Large asymmetry in \(\phi\) can be expected and appears naturally.

ACKNOWLEDGMENTS

This work is supported in part by the Israel Science Foundation administered by the Israel Academy of Sciences and Humanities. D.O. is supported in part by the Ministry of Education and Science of Spain.
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