Thermodynamics and statistical physics of quasiparticles within the quark-gluon plasma model

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We consider thermodynamic properties of a quark-gluon plasma related to quasiparticles having the internal structure. For this purpose, we employ a possible analogy between quantum chromodynamics and non-Abelian Proca-Dirac-Higgs theory. The influence of characteristic sizes of the quasiparticles on such thermodynamic properties of the quark-gluon plasma like the internal energy and pressure is studied. Sizes of the quasiparticles are taken into account in the spirit of the van der Waals equation but we take into consideration that the quasiparticles have different sizes, and the average value of these sizes depends on temperature. It is shown that this results in a change in the internal energy and pressure of the quark-gluon plasma. Also, we show that, when the temperature increases, the average value of characteristic sizes of the quasiparticles increases as well. This leads to the occurrence of a phase transition at the temperature at which the volume occupied by the quasiparticles is compared with the volume occupied by the plasma.

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I. INTRODUCTION

Quark-gluon plasma (QGP) can be defined as a high-temperature state of a non-Abelian SU(3) gauge field and quarks where hadrons are broken apart. The transition from a hadronic state to the QGP takes place at some critical temperature $T_c$. A full theoretical treatment of the QGP is still absent, and one might expect that such a description can be done only within the framework of the nonperturbative quantization of such strongly nonlinear system like quantum chromodynamics (QCD). For this reason, every approximate description clarifying any characteristic features of the QGP may be helpful.

In the present paper, we study the role of quasiparticles in the QGP. Lattice [1-5] and analytical investigations [6-10] indicate that the QGP contains various quasiparticles: monopoles, dyons, binary bound states [quark-quark (qq), quark-antiquark (qq̄), gluon-gluon (gg), quark-gluon (qg), etc.]. Quasiparticles are extended objects having their own energy and sizes. We wish to show here that: (a) their own energy causes a change in the internal energy of the QGP (within the approximation used in the present paper, the QGP is modeled as a gas consisting of quasiparticles); (b) nonzero sizes cause a change in the equation of state.

The item (b) can be easily explained: the volume where particles are in motion differs from the geometric volume $V$ where the QGP is located. This is entirely similar to that happens in real gases described by the van der Waals equation. In analogy to real gases, one can introduce a correction to the volume as follows: $V \to (V - V_q)$, where $V_q$ is the volume occupied by all quasiparticles. The significant difference from real gases is the fact that in our case $V_q$ becomes a function of temperature; this happens because when the temperature is changed, the internal structure (i.e., the distribution of fields creating a quasiparticle, as well as the energy and characteristic sizes) of the quasiparticles is also changed. It is evident that, within such an approach, the critical temperature $T_c$ occurs as a temperature at which the quasiparticles break apart and a transition to a state without quasiparticles takes place.

In Ref. [11], arguments in favour of that QCD can exhibit general similarities to non-Abelian Proca-Dirac-Higgs theory are given. In particular, it was shown there that within non-Abelian Proca-Dirac-Higgs theory there is a mass...
gap, at least for one frozen parameter. Since QCD should possess a mass gap, it is assumed that the mechanism of creation of such a gap in QCD may be the same. In non-Abelian Proca-Dirac-Higgs theory, the presence of the mass gap is related to the nonlinear Dirac equation. It is therefore assumed in Ref. [11] that in QCD the nonlinear Dirac equation occurs as a result of approximate description of the interaction between see quarks and gluons. This can happen as follows. In the Lagrangian, the interaction between quarks and gluons is described by the term $\hat{\psi} \lambda^B \hat{A}_\mu \hat{\psi}$, where $\hat{\psi} = \langle \hat{\psi} \rangle + \delta \hat{\psi}$, $\hat{A}_\mu = \langle \hat{A}_\mu \rangle + \delta \hat{A}_\mu$. Here $\langle \hat{\psi} \rangle$ and $\langle \hat{A}_\mu \rangle$ are valence quarks and gluons, and $\delta \hat{\psi}$ and $\delta \hat{A}_\mu$ are sea quarks and gluons; $\lambda^B$ is the Gell-Mann matrices. One can assume that the quantum average of the term $\langle \delta \hat{\psi} \lambda^B \delta \hat{A}_\mu \delta \hat{A}_\mu \delta \hat{\psi} \rangle$ will approximately look like $\langle \delta \hat{\psi} \lambda^B \delta \hat{A}_\mu \delta \hat{\psi} \rangle \approx \phi (\xi \xi)^2$, where the scalar field $\phi$ approximately describes the sea gluons and the spinor field $\xi$ approximately describes the sea quarks. Thus, in QCD, there can occur the nonlinear Dirac equation which approximately describes the interaction between sea quarks and gluons.

In order to study thermodynamic properties of the QGP, we employ here this analogy between the QGP in QCD and a plasma in non-Abelian Proca-Dirac-Higgs theory. In such theory, the simplest particlilelike solution is the solution containing only a spinor field, presumably describing a virtual pair of quarks – a spinball. In the QGP, such an object is a quasiparticle.

II. CONTROL PARAMETERS

In order to describe the properties of the QGP consisting of quasiparticles, one can use the control parameters known from the theory of strongly correlated systems: the coupling parameter $\Gamma$, the degeneracy parameter $\chi$, and the Brueckner parameter $r_s$ [12]. As applied to the QGP, they are

$$\Gamma = \frac{\overline{W}_q}{kT}, \quad \chi = \frac{n_q v_q}{\overline{V}_q}, \quad r_s = \frac{\overline{l}_q}{\overline{q}}. \quad (1)$$

Here $\overline{W}_q$ is the average value of the quasiparticle energy; $n_q$ is the concentration of quasiparticles; $\overline{V}_q$ is the average value of the volume of one quasiparticle; $\overline{l}_q$ is the average distance between the quasiparticles; $\overline{q}$ is the characteristic size of quasiparticles. It is evident that $\overline{l}_q \approx \overline{q}^{1/3}$.

III. STATISTICAL INTEGRAL

We consider a QGP model containing, apart from valence quarks and gluons, quasiparticles having finite sizes and the internal energy which is created by the field from which a quasiparticle is constructed. For example, this can be quasiparticles mentioned in Ref. [6]: monopoles, dyons, binary bound states [quark-antiquark ($qq$), quark-antiquark ($q\bar{q}$), gluon-gluon ($gg$), quark-gluon ($qg$), etc.]. Apparently, one can construct such quasiparticles only using nonperturbative QCD. Since at the present time we have no computational methods based on nonpertubative QCD, we will employ an assumption stated in Ref. [11], according to which non-Abelian Proca theory containing nonlinear spinor and Higgs fields possesses some properties similar to QCD. It was shown in Ref. [11] that in such a theory there is a mass gap, at least for one fixed parameter determining particlelike solutions within this theory. The presence of the mass gap is related to the nonlinear Dirac equation, as it was discovered in Refs. [13, 14]. It was supposed in Ref. [11] that the mass gap in QCD has the same nature, and the nonlinear Dirac equation occurs as a result of approximate description of the interaction between sea quarks and gluons. This means that non-Abelian Proca theory plus the nonlinear Dirac equation plus the Higgs field and nonperturbative QCD may have common properties.

We therefore consider here a plasma consisting of valence quarks and Proca gluons and containing also quasiparticles described by particlelike solutions from non-Abelian Proca-Dirac-Higgs theory. Our purpose is to show that the presence of the internal structure of the quasiparticles leads to a considerable change in such thermodynamic quantities like the internal energy and pressure.

The total energy of a nonrelativistic particle $W$ consists of two parts: the energy $W_q$ (associated with the energy related to the presence of the internal field structure) and the kinetic energy $W_k$. That is,

$$W = W_q + W_k \equiv W_q + \frac{e^2 m^2}{2W_q}. \quad (4)$$
where we have taken into account that the mass of a quasiparticle is \( m_q = W_q/c^2 \).

For simplicity, we will consider quasiparticles assuming that they do not interact with each other and with valence quarks and gluons containing in the QGP. The finiteness of quasiparticle sizes must be also taken into account. To do this, to a first approximation, we use the idea coming from the van der Waals equation: particles move not in the volume \( V \) but in a smaller volume \( (V - V_q) \), where \( V_q \) is the volume occupied by the quasiparticles. It is evident that the volume occupied by \( N \) quasiparticles is

\[
V_q = \sum_{i=1}^{N} v_i,
\]

where \( v_i \) is the characteristic volume occupied by the \( i \)-th quasiparticle.

Integration over coordinates in the statistical integral, by taking into account the sum (5), runs into great difficulty. Therefore, to simplify the problem, we estimate the sum (5) as follows:

\[
V_q \approx N\overline{v}_q = Vn_q\overline{v}_q = V\chi,
\]

where we have taken into account the definition (2) for the degeneracy parameter \( \chi \). This simplification permits us to write the statistical integral as the product of the corresponding integrals for every single particle,

\[
Z(T) \approx Z_{\text{(quarks + gluons)}} \left[ \int dV dp_z dp_y dp_z \rho(\gamma) d\gamma e^{-\frac{W_q(\gamma) + \frac{v^2}{2\rho(\gamma)} - \Delta}{kT}} \right]^N = Z_{\text{(quarks + gluons)}} Z_{\text{quasiparticles}}. \tag{7}
\]

Here \( Z_{\text{(quarks + gluons)}} \) is the statistical integral for valence quarks and gluons; \( Z_{\text{quasiparticles}} \) is the statistical integral associated with the presence of the internal structure of quasiparticles; \( \gamma \) is the set of parameters on which the energy of quasiparticles depends; \( T \) is the temperature; \( \rho(\gamma) \) is the density of states. The constant \( \Delta \) corresponds to the minimum energy of a quasiparticle (the mass gap) from which the energy will be reckoned (see below).

For simplicity, in the present paper, we consider only the case of noninteracting quasiparticles. Our main purpose is to study the effects related to the presence of particles having the internal structure: the internal energy and finite sizes. The presence of the internal energy is taken into account by the term \( W_q \) in Eq. (4). Then, using the approximation (6), the integration over the volume in (7) gives us \((V - V_q)\), and the statistical integral takes the form

\[
Z_{\text{quasiparticles}}(T) = Z_0 T^{3N/2} (V - V_q)^N \left[ \int W_q^{3/2}(\gamma) \rho(\gamma) e^{-\frac{W_q(\gamma) - \Delta}{kT}} d\gamma \right]^N = Z_0 T^{3N/2} (V_q)^N = Z_0 T^{3N/2} V^{N} (1 - \chi)^N (Z_q)^N. \tag{8}
\]

Here \( Z_q \) is the statistical integral per one quasiparticle and all dimensional constants are collected in the normalization constant \( Z_0 \). For simplicity, in performing numerical calculations in subsequent sections, we suppose that all properties of quasiparticles depend only on one parameter \( \gamma \) (i.e., we will consider the case where the set of parameters \( \gamma \) contains only one parameter). It is evident that characteristic sizes of the quasiparticles (and hence the volume) depend on temperature because the temperature changes, the average volume will also change: \( \overline{v}_q = \overline{v}_q(T) \).

The internal energy of the quasiparticles is defined as follows:

\[
U_{\text{quasiparticles}} = \frac{1}{Z_{\text{quasiparticles}}} \sum_{i=1}^{N} \left[ W_{q,i}(\gamma_i) + W_{k,i} - \Delta \right] e^{-\frac{W_{q,i}(\gamma_i) + W_{k,i} - \Delta}{kT}} \prod_{i=1}^{N} dV_i dp_i \rho(\gamma_i) d\gamma_i, \tag{9}
\]

where \( W_{q,i} \) and \( W_{k,i} \) correspond to energies of the \( i \)-th particle. Since

\[
\int e^{-\frac{W_q(\gamma) + W_k}{kT}} dV dp_z dp_y dp_z d\gamma \sim T^{3/2} (V - V_q) \int W_q^{3/2}(\gamma) \rho(\gamma) e^{-\frac{W_q(\gamma)}{kT}} d\gamma,
\]

the expression for the internal energy (9) takes the following form:

\[
\frac{U_{\text{quasiparticles}}}{N} \equiv U_{\text{quasiparticle}} = \frac{\int W_q^{3/2}(\gamma) e^{-\frac{W_q(\gamma)}{kT}} \rho(\gamma) d\gamma}{\int W_q^{3/2}(\gamma) e^{-\frac{W_q(\gamma)}{kT}} \rho(\gamma) d\gamma} - \Delta + \frac{c^2}{2} \frac{\int \frac{\rho^2}{W_q(\gamma)} e^{-\frac{W_q(\gamma) + W_k}{kT}} \rho(\gamma) dp_z dp_y dp_z d\gamma}{\int e^{-\frac{W_q(\gamma) + W_k}{kT}} \rho(\gamma) dp_z dp_y dp_z d\gamma} - \Delta + \frac{3}{2} kT. \tag{11}
\]
The gas pressure of one quasiparticle is defined as follows:

\[
\frac{p_{\text{quasiparticles}}}{N} = -\frac{1}{N} \frac{\partial F_{\text{quasiparticles}}}{\partial V} = \frac{kT}{V - V_q} = \frac{kT}{V (1 - \chi)} = \frac{kT}{V (1 - n_q v_q)}.
\]  

(12)

We emphasize that since the average volume of one quasiparticle depends on temperature, the equation of state \[12\] will differ strongly from the van der Waals equation. Also note that below we choose \( \rho(\gamma) = \text{const.} \)

IV. A QUALITATIVE ESTIMATE OF THE STATISTICAL INTEGRAL \( Z_q \) IN THE PRESENCE OF THE MASS GAP

In this section we estimate the statistical integral \( Z_q \) and quantities related to this integral. To do this, we assume that in the region where the parameters \( \gamma \) are changed, the energy \( W_q \) tends to infinity on the boundary of this volume, \((W_q)_{\partial V_q} \rightarrow \infty\), and the dependence of the energy on the parameters \( W_q(\gamma) \) has an inverted bell shape. This means that inside the volume \( V_q \) there is a point where the energy takes its minimum value \( \Delta \). This also means that there is some characteristic volume \( \Omega_0 \) outside which the energy \( W_q \) goes to zero sufficiently fast. If the minimum value of the energy is positive, i.e., \( \Delta > 0 \), one can say about the presence of a mass gap for such quasiparticles. Then one may estimate the statistical integral per one particle as follows:

\[
Z_{\text{quasiparticle}} \approx Z_0 T^{3/2} (V - V_q) W_q^{3/2} (\Delta) e^{-\frac{W_q(\Delta)}{\rho_0} \Omega_0(T)}.
\]

(13)

Here, we have taken into account that the size of the volume \( \Omega_0 \) in the parameter space \( \gamma \) depends on temperature: \( \Omega_0 = \Omega_0(T) \). Substituting the expression \[13\] in Eqs. \[11\] and \[12\], we have

\[
U_{\text{quasiparticle}} \approx \frac{3}{2} kT + W_q (\Delta),
\]

\[
p_{\text{quasiparticle}} = \frac{kT}{V - V_q} = \frac{kT}{V (1 - \chi)} = \frac{kT}{V (1 - n_q v_q)}.
\]

(14)

(15)

V. FERMION-PROCA GLUON PLASMA: THE SIMPLEST MODEL

In the above discussion, we have argued that the fermion-Proca gluon plasma may have common features with the QGP in QCD. Consistent with this, in this section we give a more detailed study of our model for such plasma.

The construction of statistical physics for a fermion-Proca gluon plasma is an extremely complicated problem since the corresponding particlelike solutions depend on a number of parameters. At the moment, it appears to be impossible to find the energy spectrum of these solutions for the whole set of the parameters. We therefore consider a simplified model for such a problem. To do this, we freeze the degrees of freedom related to a Proca field and consider only a spinor field described by the nonlinear Dirac equation. Our purpose will be to calculate the statistical integral for a gas consisting of spinballs: a fermion-Proca gluon plasma where the Proca field is frozen.

Since we freeze the degrees of freedom of the Proca field, a quasiparticle in the fermion-Proca gluon plasma is described by the nonlinear Dirac equation \[16\] (recall that, according to Ref. \[11\], it is assumed that this equation approximately describes the interaction between quarks and gluons) without the Proca Field \( A_\mu^a \) (cf. the equations of Ref. \[11\] where \( A_\mu^a = 0 \) and the scalar field is assumed to be constant),

\[
\frac{1}{2} \hbar \gamma^\mu \partial_\mu \psi + \Lambda \bar{\psi} \psi - m_f \psi = 0.
\]

(16)

Here \( m_f \) is the mass of the spinor field and \( \Lambda \) is a constant. Introducing \( l_0 = \frac{\hbar}{m_f c} \) and redefining \( \psi \sqrt{\frac{\Lambda}{m_f c}} \rightarrow \psi \), the Dirac equation \[16\] takes the following dimensionless form:

\[
[i\gamma^\mu \partial_\mu (\bar{\psi} \psi) - 1] \psi = 0.
\]

(17)

\textit{Ansatz} for the doublet of the spinor field is taken in the form \[11\]

\[
\psi^T = e^{-i\frac{\theta}{2}} \left\{ \begin{pmatrix} 0 \\ -u \end{pmatrix}, \begin{pmatrix} u \\ 0 \end{pmatrix}, \begin{pmatrix} i v \sin \theta e^{-i \varphi} \\ -iv \cos \theta \end{pmatrix}, \begin{pmatrix} -iv \cos \theta \\ -iv \sin \theta e^{i \varphi} \end{pmatrix} \right\}.
\]

(18)
where $E/\hbar$ is the spinor frequency. Substitution of (18) in (17) yields the following equations describing a spinball as a quasiparticle in the spinball plasma:

$$\ddot{v} + \ddot{v} = \ddot{u} \left( -1 + \tilde{E} + \frac{\tilde{a}^2 - \tilde{a}^2}{\bar{x}^2} \right),$$

$$\ddot{u} - \ddot{u} = \ddot{v} \left( -1 - \tilde{E} + \frac{\tilde{a}^2 - \tilde{a}^2}{\bar{x}^2} \right),$$

(19)

(20)

where we have introduced the following dimensionless quantities: $\bar{x} = r/l_0$, $\tilde{E} = (l_0/\hbar c)E$, $(\ddot{u}, \ddot{v}) = \sqrt{l_0} (u, v)$.

Solving Eqs. (19) and (20) for different values of the parameter $\tilde{E}$, we get the data given in Table I. Figs. 1 and 2 show the families of the corresponding solutions for the functions $\ddot{u}(\bar{x})$ and $\ddot{u}(\bar{x})$. The profiles of the dimensionless energy density are shown in Fig. 3. (Notice here that Figs. 1-5 are taken from Ref. [15].)

| $\tilde{E}$ | 0.1 | 0.2 | 0.4 | 0.6 | 0.8 | 0.9 | 0.99 | 0.999 | 0.9999 |
|------------|-----|-----|-----|-----|-----|-----|------|-------|--------|
| $\tilde{u}_1$ | 1.103640998 | 1.20242169 | 1.3371184 | 1.3896214 | 1.2745556 | 1.06477 | 0.4191646 | 0.9999 | 0.99999 |
| $\tilde{W}_q$ | 18585 | 2985.04 | 470.563 | 526.77 | 66.4478 | 49.285 | 74.2536 | 213.603 | 668.854 |
| $l_2$ | 10.7004 | 5.39175 | 2.61383 | 1.6336 | 1.04804 | 0.563584 | 0.000946652 | 0 | 0 |

TABLE I: Eigenvalues $\tilde{u}_1$ (the expansion coefficient in the vicinity of the center $\tilde{u}_1$) and the energy $\tilde{W}_q$ as functions of the parameter $\tilde{E}$.

Asymptotic behavior of the solutions shown in Figs. 1 and 2 is as follows:

$$\ddot{u}(\bar{x}) \approx \tilde{u}_\infty e^{-\bar{x}\sqrt{1-\tilde{E}^2}},$$

$$\ddot{v}(\bar{x}) \approx \tilde{v}_\infty e^{-\bar{x}\sqrt{1-\tilde{E}^2}},$$

(21)

(22)

where $\tilde{u}_\infty, \tilde{v}_\infty$ are some constants. These expressions permit us to determine the characteristic size of such particlelike solutions (quasiparticles) as $\tilde{E} \to 1$:

$$l_1 \approx \frac{l_0}{\sqrt{1-\tilde{E}^2}}.$$  

(23)

It is seen that when $\tilde{E} \to 1$ the characteristic size of the quasiparticle tends to infinity: $l_1 \to \infty$. From the analysis of Figs. 1, 2, and 3 it also follows that when $\tilde{E} \to 0$ the characteristic size of the quasiparticle $l_2 \to \infty$. But if in the first case by the characteristic size $l_1$ we mean the semi-width of the curves $\ddot{u}(\bar{x})$ or $\ddot{v}(\bar{x})$, in the second case by the characteristic size $l_2$ we mean the distance from the maximum of the curves $\ddot{u}(\bar{x})$ or $\ddot{v}(\bar{x})$ to the origin of coordinates. It is seen that the location of this maximum shifts to the right from the origin of coordinates as $\tilde{E} \to 0$. The dependencies of $l_1$ and $l_2$ on the energy $\tilde{E}$ are shown in Fig. 4. The numerical results indicate that when $\tilde{E} \to 0, 1$ the characteristic sizes $l_{1,2} \to \infty$.

The energy density of a quasiparticle in the spinball plasma is (cf. Ref. [15])

$$w_q = \frac{m_f c^5}{\hbar^3 \Lambda^{1/2}} \left[ \tilde{E} \ddot{u}^2 + \ddot{v}^2 + \frac{1}{2} \left( \ddot{u}^2 - \ddot{v}^2 \right)^2 \right] = \frac{m_f c^5}{\hbar^3 \Lambda^{1/2}} \tilde{w}_q,$$

(24)

where the dimensionless $\Lambda = \Lambda_v (m_f c l_0^3)$. The energy of the spinball is

$$W_q(E) = 4\pi \frac{m_f c^2}{\sqrt{\Lambda}} \int_0^\infty x^2 \tilde{w}_q(\tilde{E}) dx = 4\pi \frac{m_f c^2}{\sqrt{\Lambda}} \tilde{W}_q(\tilde{E}).$$

(25)

Here, we have introduced the dimensionless energy $\tilde{W}_q(\tilde{E})$ and emphasized that the spinball energy depends only on one parameter $\tilde{E}$.

Summarizing, one can say that the quasiparticle (spinball) has the following properties:

- The energy spectrum of the spinball is restricted from below, i.e., this spectrum possesses a mass gap discovered in Refs. [13, 14].
- The spinball energy depends on one parameter $\tilde{E} \in [0, 1]$, and this energy tends to infinity as $\tilde{E} \to 0, 1$. 

The characteristic sizes of the spinball also tend to infinity as $\bar{E} \to 0, 1$.

It was supposed in Ref. [11] that there is some relationship between non-Abelian Proca-Dirac-Higgs theory and QCD. This relationship consists in the fact that in Proca-Dirac-Higgs theory there is a mass gap, at least for a fixed value of one of the parameters ensuring the existence of particlelike solutions within such theory. The analysis indicates that the existence of the mass gap is related to the nonlinear Dirac equation: there is no mass gap in the absence of spinor fields. In turn, numerical lattice calculations indicate that there is a mass gap in QCD; therefore, it is argued in Ref. [11] that perhaps the presence of the mass gap in QCD is related to the fact that the interaction between sea quarks and sea gluons can be approximately described using the nonlinear Dirac equation. Also, this
permits one to assume that a plasma in the theory where quasiparticles are described by particlelike solutions found in Ref. [11] may serve as an approximation in describing the properties of the QGP in QCD.

In subsequent sections, we will study changes in statistical physics and thermodynamics of plasma within Proca-Dirac-Higgs theory induced only by spinballs. This is because, a study of a gas consisting of more complicated quasiparticles supported also by a non-Abelian Proca field is a much more difficult problem since the energy of such particlelike solutions depends on a larger number of parameters; this greatly complicates a numerical study of statistical physics and thermodynamics of such a gas.

It is evident that the spinball gas approximation is a rough one: some statistical and thermodynamic characteristics of such a plasma (Proca quark-gluon plasma) will be lost. In particular, this applies to the loss of a phase transition between hadronic matter and the Proca quark-gluon plasma. This happens because a spinball contains only a spinor field but not a gluon field. Therefore, when the temperature drops down, hadronic matter does not form.

In this case there is another phase transition: when the temperature rises, the size of a spinball increases. For some temperature $T_c$, this size becomes comparable to the distance between quasiparticles. Then the typical distance between spinballs is defined as follows:

$$l_q \sim (n_q)^{-1/3}. \quad (26)$$

It is evident that in this case it is already impossible to describe our system as a plasma consisting of quasiparticles. This means that there is a phase transition from a thermodynamic state (spinball plasma) to a state where such quasiparticles disappear.

We can estimate a characteristic size of the spinball $l_q$ as $\bar{E} \to 1$ using the formula (23). Unfortunately, there is no analytic formula to estimate the characteristic size as $\bar{E} \to 0$. Therefore, in Sec. V A, we will discuss a toy model where the main characteristics of the spinball energy spectrum and the dependence (23) are kept and where we introduce an ad hoc dependence of $l_q$ on the parameter defining the particlelike solution. Then, in Sec. V B we will carry out similar calculations for a spinball plasma when the dependence $l_q(\bar{E})$ is specified using the interpolation of the curves given in Fig. 5.

A. Toy model

To calculate the statistical integral [8], let us assume that the energy of a quasiparticle is defined as follows:

$$W_q(x) = \frac{\Delta}{1 - x^2}. \quad (27)$$

Here $\Delta$ is the minimum value of the energy (the mass gap) and the parameter $x$ is the parameter $\gamma$ from [8]. As $x \to 1$ and $x \to -1$, the characteristic size of such quasiparticles $l_q$ is defined as

$$\frac{l_q}{l_0} = \frac{a}{(1 - x)} + \frac{b}{(1 + x)}, \quad (28)$$

where $l_0$, $a$, and $b$ are some constants. We have chosen the terms appearing in the right-hand side of Eq. (28) so that they coincide qualitatively with the dependencies $l_1$ and $l_2$ shown in Fig. 5. In particular, the characteristic sizes of a quasiparticle diverge for $x \to \pm 1$.

In numerical calculations of the internal energy $U$, of the pressure $p$, and of the average value of the characteristic volume $v_q$ for one quasiparticle we use the formulas [see Eqs. (11) and (12)]

$$\bar{U}_{\text{quasiparticle}}(\bar{T}) \equiv \frac{\bar{U}_{\text{quasiparticles}}(\bar{T})}{N_q} = \frac{\int W_q^{5/2}(x) e^{-W_q(x)} dx}{\int W_q^{5/2}(x) e^{-W_q(x)} dx} - \hat{\Delta} + \frac{3}{2} \bar{T}, \quad (29)$$

$$\bar{\rho}_{\text{quasiparticles}}(\bar{T}) = \frac{\bar{\rho}}{N_q} = \frac{\bar{T}}{V - V_q} = \frac{\bar{T}}{V(1 - \chi)} = \frac{\bar{T}}{V(1 - n_qv_q)}, \quad (30)$$

$$\bar{v}_q(\bar{T}) \equiv \frac{\int (\frac{l_q}{v_0})^3 W_q^{3/2}(x)e^{-W_q(x)} dx}{Z_q}, \quad (31)$$

where we have introduced dimensionless $\bar{T} = T/T_0$, $(\bar{W}_q, \bar{\Delta}, \bar{U}) = (W_q, \Delta, U)/(kT_0)$, $\bar{\rho}_{\text{quasiparticles}} = [l_0^3/(kT_0)] \rho_{\text{quasiparticles}}$, $\bar{V} = V/l_0^3$. Here $T_0$ is some characteristic temperature, $v_0 = l_0^3$, and the partition function $Z_q$ is calculated according to the formula (8),

$$Z_q(\bar{T}) = \int W_q^{3/2}(x)e^{-W_q(x)} dx. \quad (32)$$
In the above integrals, the integration is taken over the range $-1 < x < 1$. The results of numerical calculations for the internal energy, the equation of state, and the average volume of quasiparticles are given in Figs. 6-8, respectively. Also, Fig. 6 shows the contribution to the entropy

$$S = k \ln Z + U/T$$

(33)

coming from the internal energy.

According to the equation of state (30) and Fig. 7, within the toy model under consideration, there is a phase transition when $n_q v_q \to 1$ and the pressure diverges. This means that the formula (29) for the internal energy can be used only up to the temperature $T \lessapprox T_c$, where $T_c$ is the critical temperature at which the phase transition occurs.

The main results for the toy model are as follows:

- At zero temperature, the average volume of quasiparticles is not zero, $\overline{V_q}(0) \neq 0$ (see Fig. 8). Apparently, this is due to the presence of the mass gap in the energy spectrum of the quasiparticles.

- In the presence of the mass gap $\Delta$ the internal energy must be calculated using the formula (9). Otherwise, $U(0) \neq 0$, and when one defines the entropy using the standard formula (33), the second term in this expression diverges.

- According to Eq. (26), there is some temperature $T_c$, defined from the expression

$$\overline{V_q}(T_c) \approx \frac{1}{n_q}$$

(34)
at which a phase transition takes place. Before the phase transition $T < T_c$ and the spinball plasma consists of quasiparticles (spinballs); after the phase transition the particlelike objects (spinballs) are destroyed and the matter has another equation of state.

### B. Spinball plasma

In this section we consider a more realistic model within which the QGP is modeled by a quark-Proca-gluon plasma, which, in turn, is simplified so that from all quasiparticles only spinballs remain. By the quark-Proca-gluon plasma we mean a plasma where quasiparticles are described by particlelike solutions obtained within non-Abelian Proca-Dirac-Higgs theory.

For convenience of performing numerical calculations, we rescale the integrand variables $W_q$ and $T$ in Eq. (8) as follows: $\tilde{W}_q = W_q/\Delta, \tilde{T} = T/\Delta$, where the dimensionless $\Delta$ and $\tilde{T}$ are defined after Eq. (31) and the dimensionless $\tilde{W}_q$ is taken from Eq. (26). Then the statistical integral $Z_q$ from (8) takes the form

$$Z_q \sim \tilde{\Delta}^{3/2} \int \tilde{W}_q^{3/2}(\gamma) e^{-\tilde{W}_q^{(\gamma)}} d\gamma.$$  

Table I shows the values of the energy $\tilde{W}_q$ for different values of the parameter $\tilde{E}$. Using these data, one can fit the dimensionless function $\tilde{W}_q(\tilde{E})$ in the form

$$\tilde{W}_q = \frac{W_0}{(1-x)} + \frac{W_1}{x^{\beta}}$$  

with the following magnitudes of the fitting parameters: $W_0 \approx 2.4, W_1 \approx 42.28, \alpha \approx 0.6, \beta \approx 2.64$. Here $x$ corresponds to the parameter $\tilde{E}$.

Because of the presence of the internal structure of quasiparticles, the equation of state is modified as follows:

$$p_q = -\frac{\partial F_q}{\partial V} = kT \frac{\partial \ln Z_q}{\partial V} = kN_q \frac{T}{V - V_q},$$  

where we have used the expression (12). Remembering that the volume $V_q$ can be estimated as $V_q \approx N_q \ell_q$, where $N_q$ is the quasiparticle number and $\ell_q(T)$ is the average volume of one quasiparticle that depends on temperature, we can rewrite Eq. (37) in the form

$$\frac{p_q}{n_q} = \frac{kT}{1 - n_q \ell_q(T)},$$  

where we have separated out the term depending on temperature. Notice that from the physical point of view (in the spirit of the van der Waals equation) the expression $(n_q \ell_q)^3 \frac{p_q}{n_q}$ is nothing but the fraction of the volume $V$ occupied by the quasiparticles. It is evident that when this fraction tends to unity a phase transition must occur. Fig. 9 shows the ration $p_q/n_q$ as compared with the equation of state of a perfect gas $p/n = kT$. [Note here that to calculate the average volume $\ell_q$ we have used Eq. (31) where the characteristic size $\ell_q$ is specified using the interpolation of the curves given in Fig. 5]. When $n_q \ell_q(T) \rightarrow 1$, it is seen that the equation of state of the spinball plasma differs considerably from that of a perfect gas.

Finally, according to Eq. (11), the contribution to the internal energy from one quasiparticle, related to the internal structure of the quasiparticle, is defined by the formula

$$\tilde{U}_{\text{quasiparticle}}(\tilde{T}) \equiv \frac{\tilde{U}_{\text{quasiparticles}}(\tilde{T})}{N_q} = \frac{\int \tilde{W}_q^{5/2}(\gamma) e^{-\tilde{W}_q^{(\gamma)}} d\gamma}{\int \tilde{W}_q^{3/2}(\gamma) e^{-\tilde{W}_q^{(\gamma)}} d\gamma} - \tilde{\Delta},$$  

where we have used the dimensionless variables given after Eq. (31). The behavior of this term is shown in Fig. 10.

It may also be noted that, according to the equation of state (38) and Fig. 9 in the spinball plasma, as in the case of the toy model considered in Sec. V A, there is a phase transition when $n_q \ell_q \rightarrow 1$ and the pressure $p_q \rightarrow \infty$. This means that the formula (39) can be applied to calculate the contribution to the internal energy only up to the temperature $T \lesssim T_c$, and the critical temperature $T_c$ can be defined by the relationship (34).
VI. CONCLUSIONS

In the present study, we have been continuing investigations within the framework of the model of Ref. [11] where a possible relationship between Proca-Dirac-Higgs theory and QCD is suggested. It is argued in Ref. [11] that the presence of a nonlinear spinor field results in the appearance of a mass gap in Proca-Dirac-Higgs theory. Also, it is assumed there that the nonlinear Dirac equation may occur as a result of approximate description of quantum nonperturbative effects in QCD in describing the interaction between sea quarks and gluons.

On the basis that there may be some correspondence between the quark-gluon plasma in QCD and the quark-Proca-gluon plasma in non-Abelian Proca-Dirac-Higgs theory, in the present paper, we have studied the influence that the internal structure of quasiparticles (i.e., their own energy and finite sizes) has on thermodynamics of the QGP. We have shown that this causes changes both in the internal energy and in the equation of state of the QGP. In doing so, we take into account the finiteness of sizes of the quasiparticles in the spirit of the van der Waals equation by means of the substraction of some volume from the total volume occupied by the QGP. But, in contrast to the case of real gases described by the van der Waals equation, quasiparticles in the QGP have different sizes depending on the own energy of a given quasiparticle. It is evident that when the temperature varies the average size of a quasiparticle will also change, resulting in the corresponding change in the internal energy and pressure of the QGP.

It appears that quasiparticles (spinballs) in such plasma also play another important role: when one supplies/removes heat, a part of it is used to change the own energy of the quasiparticles. When the plasma is heated, there is a temperature at which a phase transition takes place. This phase transition occurs because the average size of quasiparticles increases with temperature, and at some temperature the volume of all quasiparticles begins to approach the volume of the plasma. As a result, a description of the plasma at a microscopic level, using quasiparticles, becomes inapplicable. After the phase transition, quasiparticles disappear, and one has to deal with another microscopic description of the QGP.

In summary, the main results of the studies are as follows:

- We use a possible analogy between the QGP in QCD and the plasma in non-Abelian Proca-Dirac-Higgs theory proposed in Ref. [11].
- The expressions for the internal energy and pressure of the QGP containing quasiparticles have been obtained.
- As the simplest approximation for such a plasma, we have considered the case where quasiparticles are spinballs that are described by particlelike solutions within non-Abelian Proca-Dirac-Higgs theory containing only a nonlinear spinor field. One can assume that such a spinball may describe a pair of sea quarks.
- For the spinball plasma, we have numerically obtained the dependencies of the internal energy and pressure on temperature.
- Within this approximation, it is shown that there is a phase transition from a state with quasiparticles to a state where they are absent.

As a final remark, let us note that according to our assumption about a possible relationship between Proca-Dirac-Higgs theory and QCD, similar processes must take place both in the QGP and in QCD. This means that the QGP contains quasiparticles (perhaps they are analogues of Proca monopoles, Proca dyons, spinballs, etc.) which cause a
significant change in thermodynamics of the QGP. In particular, the presence of the phase transition has the result
that a gluon field supporting quasiparticles rearranges into some other structures. For instance, a transition from a
hadronic state to a plasma can perhaps be explained by the fact that in the hadronic state at least some part of the
gluon field is placed inside flux tubes between quarks. When the temperature increases, the cross section of such tubes
grows up to some temperature at which they will occupy almost the entire volume; this leads to the phase transition,
after which the gluon field is rearranged into quasiparticles.

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