Analytical and experimental research of nonlinear resonant gas oscillations in a cube

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Abstract. The gas pressure in a cubic resonator is studied experimentally and numerically at the first resonant frequency. Lagrange’s approach is used to analytical study of problem. Comparison of the pressure oscillations obtained experimentally and numerically at different amplitudes of piston oscillation is carried out.

Introduction. In the operation of many modern technological equipment, oscillations in limited volumes of various shapes are observed [1-2]. The effect of the wave field on gas in such volumes leads to the formation of high-intensity flows, which can be useful for intensifying the mixing, sedimentation and heat exchange of the media [3-5]. An important practical application is a method for reducing the noise level in resonant-type hardware devices [6]. The flow velocity increasing with the help of such processes in channels and air ducts of rectangular cross section can increase the efficiency of aerosol capturing, which plays an important role in the filtration of harmful emissions [7, 8]. Revealing the features of oscillations in resonators is of great practical interest, since such knowledge will make it possible to reasonably approach the problems of designing industrial devices. The aim of this work is a theoretical and experimental research of nonlinear gas oscillations in a cube at a resonant frequency by piston amplitude variation.

1. Experimental study
Experimental studies were carried out on the installation in figure 1 [9]. The main element is vibration generator 1, brand ES-1-150 from Dongling Vibration. The vibration generator set in motion a flat piston 2 with a diameter of $2R = 0.1$ m. Oscillations were excited in a cube 3 with sides $L = 0.4$ m and $H = 0.4$ m. The gas pressure was measured by a piezoelectric sensor 4, model 8530C-15, from Bruel & Kjaer, located in the lower boundary of the cube. The signal from the sensor was recorded on a DSO 3062A digital oscilloscope (Agilent Technologies).

2. Analytical study
The Lagrange approach was used to analytical study the gas dynamics in a closed cubic resonator. This approach worked well in previous work [10], where it showed good agreement with experimental and numerical methods.
In this paper 1D-resonator \( 0 < y < H + l \cos \omega t \) is considered with piston oscillating on the bottom boundary with amplitude \( l \) and angular frequency \( \omega \) \((H – resonators height)\). The previously obtained the nonlinear wave equation [11] for viscous polytropic gas has the form

\[
y_{yt} = \frac{c_0^2}{\rho_0} y_{yy} + \frac{\mu}{\rho_0} \left( \frac{\lambda + 2}{3} \right) \left( \frac{y_{\eta\eta}}{y_{\eta}} \right),
\]

where \( y \) is the Eulerian coordinate of gas particle at time \( t \), \( \eta \) is the Lagrangian coordinate, \( \rho_0 \) is initial gas density, \( c_0 \) is undisturbed sound speed, \( \lambda \) is Lame parameter, \( \mu \) is dynamic viscosity.

The boundary conditions are:
\[
y(0, t) = 0, \quad y(H, t) = H + l \cos \omega t,
\]
initial conditions are:
\[
y(\eta, 0) = \eta, \quad y(\eta, 0) = 0.
\]

3. Results and discussion

Figure 2 depicts oscillograms obtained analytically and experimentally for gas oscillations in time at different excitation amplitudes at the first resonance frequency \( \nu_1 = 432 \) Hz. As can be seen from the presented data, gas oscillations are harmonic and continuous. An insignificant deformation of the pressure waveforms in the gas compression region is observed, which is evident at the maximum considered amplitudes. This deformation is characteristic of the considered resonant frequency.

| \( l \), mm | 0.015 | 0.025 | 0.035 | 0.045 | 0.05 | 0.055 | 0.065 |
|------------|-------|-------|-------|-------|------|-------|-------|
| Lagrange approach | | | | | | | |
| Experiment | | | | | | | |

Figure 2. Oscillograms of gas oscillations in a cube at a resonant frequency \( \nu_1 = 432 \) Hz at various excitation amplitudes.
These plots are in good agreement: the curves have the same shape and deviation from the undisturbed state. It can be seen that the amplitude of the oscillations increases with an increase in the amplitude of the piston.

The results of the dependence of the amplitude of the gas pressure on the intensity of excitation at the resonant frequency of oscillations were obtained (Fig. 3). The experimental data were approximated by a power function of the form, where for theoretical results $a = 0.42$, $b = 0.63$, and for relative experimental results $a = 0.18$, $b = 0.3$. As one can see, the gas oscillations at the measurement point at the bottom boundary of the cube occurred in a shock-free wave mode. A slightly different character of the curves is observed, which may be due to a number of reasons, such as: error of equipment and measurements, nonlinearity of the process, inaccuracy of the theoretical approach, etc.

![Figure 3. Gas pressure versus excitation amplitude at resonance frequency $\nu_1 = 432$ Hz. Solid lines – power-law approximation.](image)

As noted earlier, the amplitude of pressure oscillations increases with increasing piston amplitude. The graphs are in satisfactory agreement: they are approximately linear and they have general growth trend.

4. Conclusion.

Nonlinear resonant oscillations of gas in a cube are investigated. For analytical studies, the proven Lagrange approach was used. A number of experiments have been carried out during which graphs of pressure oscillations near the piston have been obtained. A good agreement is obtained for the analytical and experimental curves of gas pressure oscillations, which have a periodic smooth character with a weakly expressed nonlinearity. The agreement of the experimental resonance frequency and that obtained analytically was found. Satisfactory agreement of the curves nature is observed for the dependence of the gas pressure amplitude on the excitation intensity. It was found that with an increase in the piston amplitude, the pressure deviations from the undisturbed state increase: both for the experimental and analytical cases.

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