Research Article

Identification of Dynamic Parameters for Robots with Elastic Joints

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This paper presents a novel method for identifying dynamic parameters of robot manipulators with elastic joints. A procedure based on the Lagrangian formulation of the dynamic model is proposed. Each term is inspected to search for a linear relationship with the dynamic parameters, thus enabling the linearization of robot dynamic model. Hence, the torque vector is expressed as the product of a regressor matrix, suitably defined by the vector of dynamic parameters. A parametric identification based on a least-squares technique is applied to determine dynamic parameters of robots with elastic joints. The correctness of the proposed procedure has been tested in simulation on two robotic structures with elastic joints of different complexity, that is, a 2-degree-of-freedom (dof) and a 6-dof manipulator, controlled with a PD control in the joint space.

1. Introduction

Robot dynamics describes the relationship between motion and forces by means of a number of parameters, including link and motor masses, inertias, and friction. Dynamic parameters can be grouped into the following three main categories, based on the contribution they provide to generate motor torques [1]:

(i) unidentifiable parameters: if a variation of the parameter does not modify the robot dynamic behavior;
(ii) linearly independent parameters: if a variation of the parameter modifies the robot dynamic behavior in a way that is not repeatable by varying another parameter (or a set of other parameters);
(iii) linearly dependent parameters: if a variation of the parameter modifies the robot dynamic behavior in a way that is repeatable by varying another parameter (or a set of other parameters).

The knowledge of robot dynamic parameters is helpful for simulation purposes [2], for control purposes [3, 4], and for mechatronic design purposes (i.e., to optimize robot design by studying the interaction between the robot and its environment [5]). Notwithstanding, a proper estimation is often unavailable. Hence, the need emerges of resorting to methods for estimating them; for clarity, a schematic classification of these methods is provided in Figure 1, with a brief description.

This paper is focused on one specific branch of the tree in Figure 1, that is, the Parameters identification procedures belonging to the category of offline estimation.

They consist of “exciting” the dynamics of a manipulator making it move along optimized trajectories through proper torque commands [1, 6]. These methods are generally used to identify parameters of open kinematic chains, generally neglecting coupled dynamics and transmission elasticity. They are grounded on three possible approaches. The first approach exploits the Newton-Euler formulation (N.-E.) of the dynamic model [1, 7, 8]. The second approach is based
Figure 1: Methods for dynamic parameters identification (readapted from [26]).

on the Lagrangian formulation of the dynamic model [9, 10]. The third one exploits again energetic considerations [11, 12], as in the second approach based on the Lagrangian formulation. However, it evaluates the energy of each configuration without relating the external torques to the Lagrangian of the system. In [13] a comparative analysis of the computational burden of the Lagrangian formulation with respect to the energetic formulation is provided.

All the aforementioned approaches are formulated for robots with rigid joints and links and cannot be easily extended to the case of robots with mechanical elasticity because of the different dynamic model. On the other hand, when existing, elastic phenomena cannot be neglected because of the degradation of robot performance they may cause [3, 14, 15].

It is worth noticing that only a few works cope with the identification of the elasticity in single joints due to the transmission systems and, to our knowledge, no work does exist on the identification of the complete dynamics of robots with mechanical elasticity. For instance, in [16] a method to estimate one actuator elasticity is proposed, but motor inertia is supposed to be known. The work in [17] presents the parameter identification of one elastic joint based on a nonlinear model, and at least four series of experiments are needed to apply the proposed approach.

This paper intends to propose a novel procedure for identifying the dynamic parameters of robots with elastic joints. It is grounded on the Lagrange formulation of the dynamic model of the manipulator, thus accounting for robot elastic energy in addition to kinetic and gravitational energy contributions. Open kinematic chains and absence of coupled dynamics are supposed. Furthermore, joint elasticity is assumed to be concentrated in the transmission system between the motor and the joint. This entails that 14 dynamic parameters should be estimated for each joint. They are link mass, three coordinates of the center of mass, six components of the inertia tensor, motor inertia, static and viscous friction coefficients (assuming the Coulomb friction model for static friction), and transmission elasticity. Special attention is paid to the extraction of the regressor matrix, in order to linearize robot dynamic model and facilitate parametric identification. Also a method is proposed to identify the category to which a parameter belongs (i.e., unidentifiable, linearly dependent, or else independent).

The paper is structured as follows. Section 2 introduces the notation that is adopted in this paper. In Section 3, linearization of robot dynamic model with respect to dynamic parameters is presented. The regressor matrix relating external and actuation torques with dynamic parameters is extracted in Section 4. Parameter identification is carried out in Section 5; it also proposes a method to discriminate the category to which a parameter belongs. Finally, the proposed procedure is validated on two simulated robots (a planar and a 6-dof robot) in Section 6. Section 7 summarizes the main achievements of this work, presenting conclusions and future works.

2. Notation

In this paper, the following notation is used. All the parameters are defined in the robot base frame, that is, “0” frame, if not specified by other apexes. Regarding kinematic parameters we have the following:

(i) \( \theta_i \): angular position of motor \( i \), divided by reduction gear \( k_{m_i} \) of transmission \( i \);
(ii) \( q_i \): angular position of link \( i \), which is different from \( \theta_i \) due to transmission elasticity;
(iii) \( p_i, p_i, R_i, \omega_i \): position, velocity, rotation matrix, and angular velocity of reference frame \( i \) on link \( i \);
(iv) \( z_{m_i} \): rotation axis of rotor of motor \( i \), expressed in frame \( i - 1 \).

For defining robot dynamic parameters, it is useful to introduce the concept of “body.” Body \( i \) is the system composed of link \( i \) and the stator of the motor rigidly
connected to it. Thus, the following notation applies to dynamic parameters:

(i) $m_i$: mass of body $i$;
(ii) $c_i$: product of mass and center of mass of body $i$, represented in frame $i$;
(iii) $I_i$: inertia tensor of body $i$ represented in frame $i$ and relatively to its frame origin;
(iv) $\gamma_i$: inertia of rotor of motor $i$;
(v) $K_i$: elasticity of transmission $i$;
(vi) $f_s$, $f_v$: static and viscous friction coefficients of motor $i$. Coulomb friction model is assumed. Friction is supposed to be concentrated in motors, as proposed in the literature \[18,19\]. In other words, it is assumed that all the components of link $i$ prone to friction rotate with angular velocity $\theta_i$.

3. Dynamic Model of Robot Manipulators with Elastic Joints

The following assumptions are provided:

(1) The robot has an open kinematic chain of rigid bodies, driven by electrical actuators through elastic joints undergoing small deformations in the domain of linear elasticity.
(2) Rotors of motors are rigid bodies with uniform density around their rotation axes.
(3) Stator of motor $i$ is rigidly connected to joint $i - 1$.

Robot kinetic energy $T$, gravitational potential energy $U_g$, and elastic potential energy $U_e$ can be computed as follows:

$$T = \sum_{i=1}^{n} \left( \frac{1}{2} m_i \ddot{\mathbf{p}}_i + \mathbf{p}_i \mathbf{T} \omega_i \times (\mathbf{R} \mathbf{c}_i) + \frac{1}{2} \omega_i^T I_i \omega_i + k_m \dot{\theta}_i \gamma_i + \dot{\mathbf{z}}_m^T \mathbf{R} \mathbf{c}_i + \frac{1}{2} k^2 \dot{\theta}_i^2 \gamma_i \right),$$

$$U_g = -\sum_{i=1}^{n} \left( m_i \mathbf{g}_i^T \mathbf{p}_i + \mathbf{g}_0^T \mathbf{R} \mathbf{c}_i \right),$$

$$U_e = \sum_{i=1}^{n} K_i (\theta_i - \dot{\theta}_i)^2,$$

where $n$ is the number of robot joints and $\mathbf{g}_0$ represents the gravity vector. For the analytical formulation of $T$, refer to [20] and Appendix A; for $U_g$, refer to [20]; for $U_e$, refer to [21].

By applying the Lagrangian formulation, dynamic model of robots with elastic joints can be expressed as [3, 21]

$$\mathbf{\tau} = B(q) \ddot{q} + C(q, \dot{q}) \dot{q} + f_s \dot{q} + f_v + \text{sign}(\dot{q})$$

$$+ \mathbf{g}(q) + K_e \ddot{q},$$

where $\mathbf{\tau} = [(\mathbf{J}(q)^T \mathbf{h}_e) \mathbf{u}]^T \in \mathbb{R}^{(2n \times 1)}$ is the external torque vector, $\mathbf{J}(q) \in \mathbb{R}^{(6 \times n)}$ is the robot Jacobian matrix, $\mathbf{h}_e \in \mathbb{R}^{(6 \times 1)}$ is the generalized vector of external forces, and $\mathbf{u} \in \mathbb{R}^{(n \times 1)}$ is the actuation torque vector; $\ddot{q} = [\ddot{q}^T \ddot{\theta}_e]^T \in \mathbb{R}^{(2n \times 1)}$ is the generalized joint vector, and

(1) $B(q) \in \mathbb{R}^{(2n \times 2n)}$ is the inertia matrix, related to total robot kinetic energy $T$ as follows:

$$T = \frac{1}{2} \ddot{q}^T B(q) \ddot{q};$$

(2) matrix $C(q, \dot{q}) \in \mathbb{R}^{(2n \times 2n)}$ is the centrifugal and Coriolis torque matrix, related to elements of $B(q)$ as

$$C(q, \dot{q}) = \sum_{k=1}^{n} \eta_k \mathbf{H}^k(q) - \text{asym} \left( \begin{bmatrix} \ddot{q}^T \mathbf{H}^k(q) \\ \ddot{q}^T \mathbf{H}^n(q) \\ \text{zeros}(n, 2n) \end{bmatrix} \right),$$

being operator $\text{asym}(\mathbf{M})$ defined as $\text{asym}(\mathbf{M}) = (\mathbf{M} - \mathbf{M}^T)/2$ and $\mathbf{H}^k(q) = \partial \mathbf{B}(q)/\partial q_k$; $\text{zeros}(n, 2n)$ represents the $(n \times 2n)$ null matrix. The relationship between matrix $C(q, \dot{q})$ and matrices $\mathbf{H}^k(q), k = 1 \ldots n$, is shown in Appendix B;

(3) the generalized static and viscous friction matrices $f_s$ and $f_v$ are given by

$$f_s = \begin{bmatrix} \text{zeros}(n, n) & \text{zeros}(n, n) \\ \text{zeros}(n, n) & \text{diag} \{f_{s_1}, \ldots, f_{s_n}\} \end{bmatrix},$$

$$f_v = \begin{bmatrix} \text{zeros}(n, n) & \text{zeros}(n, n) \\ \text{zeros}(n, n) & \text{diag} \{f_{v_1}, \ldots, f_{v_n}\} \end{bmatrix};$$

(4) $\mathbf{g}(q) = \partial U_g/\partial q$ is the gravity vector, where $U_g$ is the gravitational potential energy;

(5) $K_e \ddot{q} = \partial U_e/\partial q$ is the contribution of elasticity, being $K_e$ expressed as [21]

$$K_e = \begin{bmatrix} \mathbf{K} & -\mathbf{K} \\ -\mathbf{K} & \mathbf{K} \end{bmatrix},$$

where $U_e$ is the elastic potential energy and $\mathbf{K} = \text{diag}(K_1, \ldots, K_n)$ is the joints elasticity matrix.

As shown in Appendix A, by exploiting the relationships between linear and angular velocity with joint velocity, inertia matrix $B(q)$ can be written as follows:

$$B(q) = \begin{bmatrix} B_m(q) + B_z(q) + B_t(q) & B_d(q) \\ B_d(q)^T & B_Y \end{bmatrix},$$
where
\[
B_m(q) = \sum_{i=1}^{n} m_i J_P^{(i)} T_{P}^{(i)},
\]
\[
B_c(q) = \sum_{i=1}^{n} \left( J_P^{(i)} (R_c)_{[x]} J_P^{(i)} - J_P^{(i)} (R_c)_{[x]} J_O^{(i)} \right),
\]
\[
B_t(q) = \sum_{i=1}^{n} J_O^{(i)} R_i R_i^T J_O^{(i)},
\]
\[
B_d(q) = \left[ Y_1 k_{m_1} J_O^{(1)} R_1 z_{m_1} \cdots Y_n k_{m_n} J_O^{(n)} R_n z_{m_n} \right],
\]
\[
B_T = \text{diag} \left\{ Y_1 k_{m_1}^2, \ldots, Y_n k_{m_n}^2 \right\}.
\]

(8)

Symbol \(\cdot_{[x]}\) denotes the skew-symmetric matrix corresponding to the cross product. Also note that \(\cdot_{[x]} = -\cdot_{[x]}\), \(J_P^{(i)}\) and \(J_O^{(i)}\) are the position and orientation Jacobians of body \(i\).

It is worth observing that submatrix \(B_m(q)\) is linear with body masses \(m_i\), \(B_c(q)\) is linear with products of body masses and body centers of mass \(c_i\), \(B_t(q)\) is linear with body inertia tensors \(I_i\), and submatrices \(B_d(q)\) and \(B_T\) are linear with rotor inertias \(Y_i\).

Once matrix \(B(q)\) is defined, matrices \(H^{(k)}(q)\) can be easily evaluated as
\[
H^{(k)}(q) = \begin{bmatrix} H_m^{(k)}(q) + H_c^{(k)}(q) + H_t^{(k)}(q) + H_d^{(k)}(q) \\ H_d^{T}(q) \end{bmatrix}^{T} \begin{bmatrix} \Pi_T \\ \Pi_Y \\ \Pi_f \\ \Pi_K \end{bmatrix},
\]
where
\[
H_m^{(k)}(q) = \sum_{i=1}^{n} m_i \left( A_{P,(k)}^{(i)} \right)_P^{T} + \left( J_O^{(i)} (R_c)_{[x]} \right) J_P^{(i)}
\]
and \(A_{P,(k)}^{(i)} = \partial f_P^{(i)} / \partial q_k\), \(A_{O,(k)}^{(i)} = \partial f_O^{(i)} / \partial \dot{q}_k\), and \((J_O^{(i)})_P R_i = \partial R_i / \partial \dot{q}_k\).

Finally, coefficient \(j\) of gravity vector \(g(q)\) can be easily shown to be equal to
\[
g(q) = \frac{\partial \tau_j}{\partial q_j} = \begin{bmatrix} g(q)_{m,j} + g(q)_{c,j} \\ 0 \end{bmatrix},
\]
where \(g(q)_{m,j} = -\sum_{i=1}^{n} m_i J_O^{(i)} (R_i c_i)\) depends only on masses, while \(g(q)_{c,j} = -\sum_{i=1}^{n} J_O^{(i)} (R_i c_i)\) depends only on products of masses and centers of gravity.

4. Linearization of Robot Dynamic Model

Formulation of robot dynamics as in Section 3 makes each term depend only on one dynamic parameter. This has the consequent advantage of simplifying the linearization of the dynamic model and the evaluation of regressor matrix \(Y(q, \dot{q}, \ddot{q})\), which relates the measured generalized external torques \(\tau\) to the robot dynamic parameters. Hence, robot dynamic model in (2) can be linearized as
\[
\tau = \left( Y_x (q, \dot{q}) + Y_C (q, \ddot{q}) + Y_{oth} (q, \dot{q}, \ddot{q}) \right) \Pi = Y (q, \dot{q}, \ddot{q}) \Pi,
\]
where \(Y_x(q, \ddot{q})\) is the regressor of the inertia matrix, \(Y_C(q, \dddot{q})\) is the regressor of the centrifugal and Coriolis torque matrix, \(Y_{oth}(q, \dot{q}, \ddot{q})\) is the regressor of all the remaining terms in (2), and
\[
\Pi = \begin{bmatrix} \Pi_m^{T} & \Pi_c^{T} & \Pi_T^{T} & \Pi_Y^{T} & \Pi_f^{T} & \Pi_K^{T} \end{bmatrix}^{T}
\]
is the parameter vector composed of the subvectors of mass parameters \(\Pi_m = [m_1 \cdots m_n]^{T}\), product of mass and center of mass parameters \(\Pi_c = [c_1^T \cdots c_n^T]^{T}\), body inertia parameters \(\Pi_T = [I_1^T \cdots I_n^T]^{T}\), rotor inertia parameters \(\Pi_Y = [Y_1 \cdots Y_n]^{T}\), static friction coefficient parameters \(\Pi_f = [f_{s,1} \cdots f_{s,n}]^{T}\), viscous friction coefficient parameters \(\Pi_{f,v} = [f_{v,1} \cdots f_{v,n}]^{T}\), and elasticity parameters \(\Pi_K = [K_1 \cdots K_n]^{T}\). Body inertia \(I_i^T\) is expressed as \(I_i^T = [I_{xx,i} I_{yy,i} I_{zz,i} I_{xy,i} I_{xz,i} I_{yz,i}]^{T}\), where the first three parameters represent the inertia moments and the last three parameters represent the inertia products.

In the following the expression of the regressor matrix related to each term of the dynamic model is presented.
First, product $B(q)\ddot{q}$ is considered. It can be rewritten as

$$B(q)\ddot{q} = \left[ Y_{B,m}(q, \dot{q}) \Pi_m + \sum_{i=1}^{n} Y_{B,i}(q, \dot{q}) \Pi_i + Y_{R,i}(q, \ddot{q}) \Pi_Y \right].$$  \hspace{1cm} (15) 

The regressor matrices in (15) can be obtained from (8) as

$$Y_{B,m}(q, \dot{q}) = \left[ Y_{B,m,1} \cdots Y_{B,m,n} \right] \in \mathbb{R}^{n \times n}$$ \hspace{1cm} (16) 

being $Y_{B,m,i} = J_{P}^{(i)} J_{O}^{(i)} q \in \mathbb{R}^{n \times 1}$

$$Y_{B,i}(q, \dot{q}) = \left[ Y_{B,i,1} \cdots Y_{B,i,n} \right] \in \mathbb{R}^{n \times 3n}$$ \hspace{1cm} (17) 

being $Y_{B,i,j} = \left( J_{P}^{(i)} J_{O}^{(j)} \right)_{\{x\}} R_i \in \mathbb{R}^{n \times 3}$

$$Y_{R,i}(q, \ddot{q}) = \left[ Y_{R,i,1} \cdots Y_{R,i,n} \right] \in \mathbb{R}^{n \times n}$$ \hspace{1cm} (18) 

being $Y_{R,i,j} = \theta_k m_j \left( J_{O}^{(i-1)} \right)^{T} R_{i-1} z_{m_j} \in \mathbb{R}^{n \times 1}$

From (16)–(20) it follows that $B_n q = Y_{B,n} \Pi_n$, $B_i q = Y_{B,i} \Pi_i$, $B_i \ddot{q} = Y_{R,i} \Pi_Y$, and $B_i \dot{q} + B_i \ddot{q} = Y_{B,i} \Pi_Y$. Consequently, product $B(q)\ddot{q}$ can be expressed as

$$Y_B(q, \ddot{q}) \Pi,$$ where

$$Y_B(q, \ddot{q}) = \left[ \begin{array}{cccc} Y_{B,m}(q, \dot{q}) & Y_{B,i}(q, \dot{q}) & Y_{R,i}(q, \ddot{q}) & \text{zeros (n, 3n)} \\ \text{zeros (n, n)} & \text{zeros (n, 3n)} & \text{zeros (n, 6n)} & \text{zeros (n, 3n)} \end{array} \right] \in \mathbb{R}^{(2n \times 14n)}.$$  \hspace{1cm} (21)

Analogously, product $C(q, \dot{q})\ddot{q}$ can be written as (see Appendix B)

$$C(q, \dot{q})\ddot{q} = Y_C(q, \ddot{q}) \Pi,$$

where

$$Y_C(q, \ddot{q}) = \left[ \begin{array}{cccc} Y_{H,i}(q, \dot{q}) & Y_{H,i}(q, \dot{q}) & Y_{H,i}(q, \ddot{q}) & \text{zeros (n, 3n)} \\ \text{zeros (n, n)} & \text{zeros (n, 3n)} & \text{zeros (n, 6n)} & \text{zeros (n, 3n)} \end{array} \right] \in \mathbb{R}^{(2n \times 14n)},$$  \hspace{1cm} (23) 

where

$$Y_{H,i}(q, \dot{q}) = \left[ Y_{H,i,1} \cdots Y_{H,i,n} \right] \in \mathbb{R}^{n \times n}$$ \hspace{1cm} (24) 

being $Y_{H,i,j} = A_{P,i,j}^{(i-1)} J_{P}^{(i)} q + J_{P}^{(i)} A_{P,i,j}^{(i)} q \in \mathbb{R}^{n \times 1}$

$$Y_{H,i}(q, \ddot{q}) = \left[ Y_{H,i,1} \cdots Y_{H,i,n} \right] \in \mathbb{R}^{n \times 3n}$$ \hspace{1cm} (25) 

being $Y_{H,i,j} = J_{O}^{(i)} \left( J_{O}^{(i)} \right)_{\{x\}} R_i \in \mathbb{R}^{n \times 3}$. Where $Y_{H,i}(q, \ddot{q})$ is defined so that $H^{(i)}(q, \ddot{q}) = Y_{H,i}(q, \ddot{q}) \Pi$. Similarly to matrix $Y_B(q, \ddot{q})$, matrix $Y_{H,i}(q, \ddot{q})$ is given by

$$\left( \begin{array}{c} \hat{q}^{T} Y_{H,i}^{(i)}(q, \ddot{q}) \\ \vdots \\ \hat{q}^{T} Y_{H}^{(i)}(q, \ddot{q}) \\ \text{zeros (n, 2n)} \end{array} \right) \Pi,$$

where $Y_{H,i}(q, \ddot{q})$ is defined so that $H^{(i)}(q, \ddot{q}) = Y_{H,i}(q, \ddot{q}) \Pi$. Similarly to matrix $Y_B(q, \ddot{q})$, matrix $Y_{H,i}(q, \ddot{q})$ is given by

$$\left( \begin{array}{c} \hat{q}^{T} Y_{H,i}^{(i)}(q, \ddot{q}) \\ \vdots \\ \hat{q}^{T} Y_{H}^{(i)}(q, \ddot{q}) \\ \text{zeros (n, 2n)} \end{array} \right) \Pi,$$

where $Y_{H,i}(q, \ddot{q})$ is defined so that $H^{(i)}(q, \ddot{q}) = Y_{H,i}(q, \ddot{q}) \Pi$. Similarly to matrix $Y_B(q, \ddot{q})$, matrix $Y_{H,i}(q, \ddot{q})$ is given by

$$\left( \begin{array}{c} \hat{q}^{T} Y_{H,i}^{(i)}(q, \ddot{q}) \\ \vdots \\ \hat{q}^{T} Y_{H}^{(i)}(q, \ddot{q}) \\ \text{zeros (n, 2n)} \end{array} \right) \Pi,$$

where $Y_{H,i}(q, \ddot{q})$ is defined so that $H^{(i)}(q, \ddot{q}) = Y_{H,i}(q, \ddot{q}) \Pi$. Similarly to matrix $Y_B(q, \ddot{q})$, matrix $Y_{H,i}(q, \ddot{q})$ is given by

$$\left( \begin{array}{c} \hat{q}^{T} Y_{H,i}^{(i)}(q, \ddot{q}) \\ \vdots \\ \hat{q}^{T} Y_{H}^{(i)}(q, \ddot{q}) \\ \text{zeros (n, 2n)} \end{array} \right) \Pi.$$
As for the rigid case [7, 10], it exploits the property of linearity with motion variables \( \dot{q} \) and \( q \) of matrix \( R \) and the parameter \( \theta \) of matrix \( K \), which can be written as:

\[
Y_{g_m}(q) = \begin{bmatrix} -S_0^T j_{i,1}^{(1)} & \cdots & -S_0^T j_{i,1}^{(n)} \\ \vdots & \ddots & \vdots \\ -S_0^T j_{i,n}^{(1)} & \cdots & -S_0^T j_{i,n}^{(n)} \end{bmatrix} \in \mathbb{R}^{n \times n},
\]

\[
Y_{g_s}(q) = \begin{bmatrix} -S_0^T (j_{i,1}^{(1)})_{x|_r} & \cdots & -S_0^T (j_{i,n}^{(1)})_{x|_r} \\ \vdots & \ddots & \vdots \\ -S_0^T (j_{i,1}^{(n)})_{x|_r} & \cdots & -S_0^T (j_{i,n}^{(n)})_{x|_r} \end{bmatrix} \in \mathbb{R}^{n \times 3n}.
\]

On the other hand, \( Y_{f_s} \), \( Y_{f_f} \), and \( Y_K \) are diagonal matrices defined as:

\[
Y_{f_s}(\dot{\theta}) = \text{diag}\{\text{sign}(\dot{\theta}_1), \ldots, \text{sign}(\dot{\theta}_n)\} \in \mathbb{R}^{n \times n},
\]

\[
Y_{f_f}(\dot{\theta}) = \text{diag}\{\dot{\theta}_1, \ldots, \dot{\theta}_n\} \in \mathbb{R}^{n \times n},
\]

\[
Y_K(\ddot{q}) = \text{diag}\{\ddot{q}_1 - \ddot{\theta}_1, \ldots, \ddot{q}_n - \ddot{\theta}_n\} \in \mathbb{R}^{n \times n}.
\]

### 5. Parametric Identification Procedure

A parametric identification procedure for robots with elastic joins is proposed to identify vector of dynamic parameters \( \Pi \). As for the rigid case, it exploits the property of linearity of robot dynamic model with respect to the vector of dynamic parameters \( \Pi \). The parametric identification procedure requires recording manipulator actuation torques \( u \) and motion variables \( \ddot{q}, \dot{q}, \text{ and } q \), while moving the robot in the free space along suitable trajectories. For each measurement \( k \), with \( k = 1, \ldots, w \), matrix \( W_k = Y(\ddot{q}_k, \dot{q}_k, q_k) \in \mathbb{R}^{2w \times 14n} \)

\[
Y_{H_{g_m}}(q, \ddot{q}) = \text{diag}\{Y_{H_{g_m}}^{(1)}(q, \ddot{q}), \ldots, Y_{H_{g_m}}^{(n)}(q, \ddot{q})\} \in \mathbb{R}^{n \times n},
\]

\[
Y_{H_{g_s}}(q, \ddot{q}) = \text{diag}\{Y_{H_{g_s}}^{(1)}(q, \ddot{q}), \ldots, Y_{H_{g_s}}^{(n)}(q, \ddot{q})\} \in \mathbb{R}^{n \times 3n},
\]

\[
Y_{H_{g_s}}(q, \ddot{q}) = \text{diag}\{Y_{H_{g_s}}^{(1)}(q, \ddot{q}), \ldots, Y_{H_{g_s}}^{(n)}(q, \ddot{q})\} \in \mathbb{R}^{n \times 3n}.
\]
(3) If column $i$ of matrix $W$ is not null and cannot be obtained as a linear combination of other columns, then parameter $\pi_i$ is independently identifiable.

In order to evaluate whether parameters are independently identifiable or not, Gauss-Jordan elimination procedure can be applied to evaluate a base matrix $L$ for matrix $W$, that is, a set of linearly independent columns of matrix $W$. Then, matrix $K$ can be introduced, composed of not-null columns of matrix $W$ which are not columns of matrix $L$. Thus, each column $K_i$ of matrix $K$ can be written as a linear combination of columns $L_j$ of matrix $L$ as follows:

$$K_i = a_{i,1}L_1 + \cdots + a_{i,m}L_m = LA_i$$

where $m$ is the number of vectors composing the base of $W$.

Eventually, by grouping vector $\Pi$ into two subvectors $\Pi_L$ and $\Pi_K$, the following relations can be obtained:

$$W\Pi = [L \ K] \begin{bmatrix} \Pi_L \\ \Pi_K \end{bmatrix}$$

and, due to relation (39),

$$W\Pi = L[I_{m \times m}] A \begin{bmatrix} \Pi_L \\ \Pi_K \end{bmatrix} = L\Pi^*,$$  

where $I_{m \times m}$ is the $(m \times m)$ identity matrix, $A = [A_1 \ \cdots \ A_{4 \times m}]$ is the matrix composed of columns $A_i$, and $\Pi^* = \Pi_L + A\Pi_K$ is the vector of parameters to be identified. Thus, parameter $\Pi_{LK}$ is independently identifiable if row $j$ of matrix $A$ is a null vector; otherwise, parameter $\Pi_{Lj}$ is identifiable only in linear combination with some parameters of vector $\Pi_K$. Parameters of vector $\Pi_K$ are identifiable only in linear combination.

Further, vector $\Pi^*$ can be computed through a least-squares approach as

$$\Pi^* = (L^T L)^{-1} L^T \Pi.$$

6. Validation and Results

Simulation tests have been performed to validate the proposed identification procedure of dynamic parameters on robots with elastic joints. To this purpose, two robotic structures of different complexity have been modeled. The former is a planar manipulator with 2 dofs and elastic joints (see Figure 2). The latter is a 6-dof manipulator modeled as a PUMA 560 with additional elastic joints.

The planar manipulator moves in the vertical plane; hence, gravitational acceleration $g$ is directed along axis $-y_0$ with respect to the base reference system ($g_0 = [0 \ -9.81 \ 0]^T$ m/s²).

Kinematic and dynamic parameters of the planar robot are as follows: $a_1 = a_2 = 0.5$ m ($a_i$: link length), $l_1 = l_2 = 0.25$ m ($l_i$: center of mass distance; see Figure 2), $m_1 = 20$ kg, $m_2 = 10$ kg ($m_i$: link mass), $m_{m1} = m_{m2} = 1$ kg ($m_{m_i}$: motor mass), $k_{m_1} = k_{m_2} = 0.1$ (kg/m/s: transmission reduction gear), $K_1 = 3000$ N·m/rad, $K_2 = 1800$ N·m/rad ($K_i$: transmission elasticity), $I_{m1} = 0.4667$ kg·m², $I_{m2} = 0.2333$ kg·m² ($I_{m_i}$: link inertia around rotation axis), $Y_1 = 21.18$ kg·m², and $Y_2 = 12.10$ kg·m² ($Y_i$: motor inertia). It is supposed that friction torque $F_i$ acts on joint $i$. It accounts for the Stribeck effect and is modeled as follows [22]:

$$F_i = f_{i,1} \dot{\theta}_i + f_{i,2} \text{sign}(\dot{\theta}_i) - f_{i,3} \text{sign}(\dot{\theta}_i)e^{-|\dot{\theta}_i|/f_{i,4}}$$

$$- f_{i,5} \text{sign}(\dot{\theta}_i)e^{-|\dot{\theta}_i|/f_{i,6}}$$

where

$$f_{i,1} = 0.1434 \quad f_{i,2} = 0.1391$$
$$f_{i,3} = 0.2499 \quad f_{i,4} = 0.2451$$
$$f_{i,5} = 0.0537 \quad f_{i,6} = 0.1073$$
$$f_{i,1} = 0.2332 \quad f_{i,2} = 0.3576$$
$$f_{i,3} = 0.2499 \quad f_{i,4} = 0.2451$$

In accordance with model in (5), only the static and viscous components of the friction torque are taken into account, thus neglecting the nonlinear effects as shown, for example, in Figure 3 for the Coulomb friction. In the identification procedure, static and viscous components will be identified (as shown in (32) and (33)), while the nonlinear effects will be regarded as external disturbances.

Figure 2: 2-dof planar robot with elastic joints. Gravity vector is directed along $-y_0$ axis.
The computed body parameters are given by

\[ m_1 = 21 \text{ kg} \quad m_2 = 10 \text{ kg} \]

\[ c_1 = [-5 \quad 0 \quad 0]^T \text{ kg} \cdot \text{m} \]

\[ c_2 = [-2.5 \quad 0 \quad 0]^T \text{ kg} \cdot \text{m} \]

\[ I_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1.25 & 0 \\ 0 & 0 & 13.8167 \end{bmatrix} \text{ kg} \cdot \text{m}^2 \]  

\[ I_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.625 & 0 \\ 0 & 0 & 0.8583 \end{bmatrix} \text{ kg} \cdot \text{m}^2. \]  

The PD control proposed in [3] has been used to control robot motion in the free space. Excitation trajectories based on fifth order B-splines, as in [23], have been used as reference trajectory for each joint.

Joint angular positions have been sampled at 1 kHz and quantized to simulate a real scenario, by assuming to have a high resolution encoder (1 250 000 counts/round). Data have been filtered through a Butterworth IIR low-pass filter of fifth-order and cut-off frequency of 20 Hz. Motor and joint angular velocities and accelerations have been computed through a first-order numerical differentiation. Additionally, the effect of sensor noise on the performance of the proposed identification procedure has been evaluated. To this purpose, a Gaussian noise with zero mean value has been added to motor and joint position, velocity, and acceleration and the error percentage between real and estimated dynamic parameters has been measured for five different levels of signal-to-noise ratio (i.e., \( \text{SNR} = 20, \text{SNR} = 70, \text{SNR} = 80, \text{SNR} = 100, \) and \( \text{SNR} = 200 \)), in addition to absence of noise. Motion variables have been obtained by means of the robot forward dynamics also accounting for the friction model in (43). In Figure 4 motor torques applied to the simulated manipulator are shown, with the corresponding motion in the joint space. As shown in Figure 5, the maximum difference between link and motor position is 0.39° and 0.10° for first and second joint, respectively.

A total of 8900 measurements have been performed, thus constructing observation matrix \( \mathbf{W} \) and torque vector \( \mathbf{T} \) in (36). In order to reject nonlinear friction effects, measurements where joint velocity was lower than a threshold value of 0.5 rad/s have been discarded (a total of 1334 measurements have been discarded). A systematic approach to define this threshold value is presented in [24]. By inspecting matrix \( \mathbf{W} \), vector \( \mathbf{Π^*} \) of 14 identifiable parameters (or linear combination of parameters) has been extracted by means of relation (41), and matrix \( \mathbf{L} \) has been computed. The obtained values of parameters in \( \mathbf{Π^*} \) in absence of measurement noise are reported in Table 1, where \( c_i = [c_{x,i} \ c_{y,i} \ c_{z,i}]^T \) (in SI units).

As one can easily observe in Table 1, the error between the obtained estimated parameters and the real ones is very low. The normalised mean error is 0.103; it becomes 0.00862
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Joint position (rad)

Joint 2 position (rad)

\[ q_1 \]

\[ \theta_1 \]

\[ q_2 \]

\[ \theta_2 \]

Figure 5: Zoom on the main differences between motor and link positions for the 2-dof planar manipulator.

Figure 6: Actuation torques for the 6-dof manipulator have been generated with the PD control in [3]; corresponding robot motion in the joint space is shown.

Table 1: Results of the identification procedure.

| Parameter | Real value | Estimated value |
|-----------|------------|-----------------|
| \( \pi_1^* = K_1 \) | 3000.00 N·m/rad | 3000.79 N·m/rad |
| \( \pi_2^* = K_2 \) | 1800.00 N·m/rad | 1799.79 N·m/rad |
| \( \pi_4^* = m_1 - 4 \cdot I_{zz,1} + 4 \cdot I_{zz,2} \) | -30.8333 kg | -30.8352 kg |
| \( \pi_5^* = m_2 - 4 \cdot I_{zz,2} \) | 65667 kg | 65670 kg |
| \( \pi_6^* = c_{x,1} + 2 \cdot I_{zz,1} \) | 22.6333 kg·m | 22.6245 kg·m |
| \( \pi_7^* = c_{x,2} + 2 \cdot I_{zz,2} \) | -0.7833 kg·m | -0.7835 kg·m |
| \( \pi_8^* = y_{e,1} \) | 0.0000 kg·m | -1.04 \times 10^{-5} kg·m |
| \( \pi_9^* = y_{e,2} \) | 0.0000 kg·m | 3.48 \times 10^{-6} kg·m |
| \( \pi_{10}^* = Y_1 \) | 21.1800 kg·m^2 | 21.1446 kg·m^2 |
| \( \pi_{11}^* = f_{s,1} \) | 0.3302 N·m | 0.1016 N·m |
| \( \pi_{12}^* = f_{s,2} \) | 0.3576 N·m | 0.1249 N·m |
| \( \pi_{13}^* = f_{V,1} \) | 0.1434 N·m·s/rad | 0.1288 N·m·s/rad |
| \( \pi_{14}^* = f_{V,2} \) | 0.1391 N·m·s/rad | 0.1389 N·m·s/rad |

Table 2: Variation of the estimation error with signal-to-noise ratio.

| Signal-to-noise ratio | Normalised mean error |
|-----------------------|-----------------------|
| SN = 20               | 3.84 \times 10^{-3}   |
| SN = 70               | 44.50                 |
| SN = 80               | 4.73                  |
| SN = 100              | 0.157                 |
| SN = 200              | 0.111                 |

One can observe that for high values of signal-to-noise ratio, up to SN = 80, the error still remains small and comparable to the absence of noise. For lower values of signal-to-noise ratio, the error becomes very high.

As aforementioned, the proposed identification procedure of dynamic parameters has also been applied to a 6-dof robot manipulator with elastic joints. It has been modeled as a PUMA 560 with elastic joints. Robot dynamic parameters (i.e., 0.862 per cent as percentage error) without the static friction, thus pointing out that probably the neglected non-linear terms of the adopted model may cause an increase of the error. Furthermore, Table 2 reports the normalised mean error between the estimated parameters and the real ones in the case of noisy measurements of position, velocity, and acceleration, for five different levels of signal-to-noise ratio (i.e., SN = 20, SN = 70, SN = 80, SN = 100, and SN = 200).
Figure 7: Zoom on the main differences between motor and link positions for the 6-dof manipulator.
Velocity Composition. If points $p_i$ and $p_j$ are rigidly connected, their velocities are related as

$$\dot{p}_i = \dot{p}_j + \omega_i \times (p_i - p_j),$$

(A.1)

where $\omega_i$ is the angular velocity.

Steiner’s Theorem for Tensors. Steiner’s theorem relates inertia tensor $a T^b_i$; it expresses the inertia tensor of body $c$ relatively to point $b$ and defined in reference frame $a$. Symbol $I_c$ indicates the inertia tensor when $a = b = c$.

Moreover, the following relations hold.

Table 3: Real and estimated parameters related to joint elasticity and masses for the 6-dof manipulator.

| $\pi_1$ | $\pi_2$ | $\pi_3$ | $\pi_4$ | $\pi_5$ | $\pi_6$ | $\pi_7$ | $\pi_8$ | $\pi_9$ | $\pi_{10}$ | $\pi_{11}$ | $\pi_{12}$ |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----------|-----------|-----------|
| Real   | 2000   | 2000   | 2000   | 2000   | 2000   | 0.10   | 17.4   | 4.80   | 0.82      | 0.34      | 0.09      |
| Estimated | 1999.3 | 1999.8 | 1999.6 | 1999.9 | 1999.3 | 0.0985 | 17.41  | 4.7988 | 0.819     | 0.339     | 0.0889    |

Table 4: Real and estimated parameters related to the centers of mass for the 6-dof manipulator.

| $\pi_{13}$ | $\pi_{14}$ | $\pi_{15}$ | $\pi_{16}$ | $\pi_{17}$ | $\pi_{18}$ | $\pi_{19}$ | $\pi_{20}$ | $\pi_{21}$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Real      | 2000      | 0         | 0         | -6.3301   | 0.1044    | 3.9585    | -0.0974   | -0.0677   | 0.336     |
| Estimated | -0.25E-4  | -0.22E-4  | -0.15E-5  | -6.33087  | 0.10425   | 3.95835   | -0.09769  | -0.0679   | 0.33587   |
| $\pi_{22}$ | $\pi_{23}$ | $\pi_{24}$ | $\pi_{25}$ | $\pi_{26}$ | $\pi_{27}$ | $\pi_{28}$ | $\pi_{29}$ | $\pi_{30}$ |
| Real      | 0.0156    | 0         | 0         | 0         | 0         | 0         | 0         | 0.0029    |
| Estimated | -0.00015  | 0.015     | -0.12E-4  | -0.21E-4  | -0.15E-4  | -0.25E-4  | -0.18E-4  | 0.0026    |

where $p_i$ is the position of the center of mass of link $i$, and $0 T^i_0$ is its inertia tensor.

Indicating with $p_i$ the position of the center of mass of body $i$ and exploiting relations (A.1) and (A.2), kinetic energy can be expressed as

$$T_i = \frac{1}{2}m_i p_i^T \dot{p}_i + \frac{1}{2} \omega_i^T 0 T^i_0 \omega_i. \tag{A.6}$$

Energy of motor $m+1$, rigidly connected to link $i$, is given by

$$T_{m+1} = \frac{1}{2}m_{m+1} p_{m+1}^T \dot{p}_{m+1} + \frac{1}{2} \omega_{m+1}^T 0 m_{m+1} \omega_{m+1}. \tag{A.7}$$

For computing motor kinetic energy, it should be considered that rotor of motor $m+1$ rotates with angular velocity

$$\omega_{m+1} = \omega_i + k_{m+1} z_{m+1} \dot{\theta}_{j+1}. \tag{A.8}$$

and that rotor inertia tensor of motor $m+1$ relatively to its center of mass and defined in a reference frame having its $z_{m+1}$ axis parallel to the rotation axis is given by

$$m_{m+1} 0 m_{m+1} = \begin{bmatrix} I_{p_{m+1}} & 0 & 0 \\ 0 & I_{p_{m+1}} & 0 \\ 0 & 0 & Y_{j+1} \end{bmatrix}, \tag{A.9}$$

being the rotor regarded as a rigid body rotating around axis $z_{m+1}$.

Further, according to (A.3)

$$0 m_{m+1} 0 z_{m+1} = (R_{m+1}^m m_{m+1} z_{m+1} R_{m+1}^T) \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

$$= R_{m+1}^m m_{m+1} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

$$= R_{m+1}^m \begin{bmatrix} 0 & 0 & Y_{j+1} \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 0 & Y_{j+1} \end{bmatrix}^T$$

$$= Y_{j+1} z_{m+1} = Y_{j+1} R_{m+1} z_{m+1}, \tag{A.10}$$
and, due to symmetry of inertia tensor,
\[
0^{T}z_{m_{i+1}} = \begin{pmatrix} 0 \ y_{m_{i+1}} \ 0^{T}z_{m_{i+1}} \end{pmatrix}^{T} = \begin{pmatrix} Y_{i+1} \ R_{i} \ z_{m_{i+1}} \end{pmatrix}^{T} = Y_{i+1} z_{m_{i+1}}^{T} R_{i}^{T}. \tag{A.11}
\]

Thus, according to (A.1), (A.2), (A.10), and (A.11), kinetic energy of motor \( m + 1 \) is expressed as
\[
T_{m_{i+1}} = \frac{1}{2} m_{m_{i+1}} P_{C_{i}} P_{C_{i}}^{T} + m_{m_{i+1}} P_{C_{i}}^{T} \left[ \omega_{i+1} \right] k_{m_{i+1}} \hat{\theta}_{i+1} z_{m_{i+1}}^{T} R_{i}^{T} \omega_{i} + \frac{1}{2} Y_{i+1} k_{m_{i+1}}^{2} \hat{\theta}_{i+1}^{2}.
\tag{A.12}
\]

Thus, considering that

(i) total inertia of body \( i \) can be expressed as \(0^{T}I_{C_{i}} = 0^{T}I_{i} + 0^{T}C_{i}\),

(ii) total mass of body \( i \) can be expressed as \( m_{i} = m_{0} + m_{m_{i}} \),

(iii) center of mass of body \( i \) related to centers of mass of link \( i \) and motor \( i + 1 \) as \( m_{m_{i}} P_{C_{i}} = m_{0} P_{C_{i}} + m_{m_{i}} P_{m_{i+1}} \),

(iv) position of center of mass of body \( i \) can be expressed in the \( i \)th reference frame as \( P_{C_{i}} = R_{i}^{T} P_{C_{i}} + p_{i} \)

(v) total inertia of body \( i \) can be expressed in the \( i \)th reference frame, through (A.3) as \(0^{T}I_{i} = R_{i}^{T}I_{i}^{T}R_{i}^{T}\),

then the robot total kinetic energy in (A.4) can be expressed as
\[
T = \sum_{i=1}^{n} \left( \frac{1}{2} m_{i} \dot{p}_{i}^{T} \dot{p}_{i} + \dot{p}_{i}^{T} \omega_{i} \right) R_{i} \dot{c}_{i} + \frac{1}{2} \omega_{i}^{T} R_{i} I_{i}^{T} R_{i}^{T} \omega_{i} + Y_{i} k_{m_{i}}^{2} \dot{\theta}_{i}^{2} \right) \tag{A.13}
\]

where \( I_{i} = I_{i}^{j} \) and \( c_{i} = m_{i} P_{C_{i}} \) and body parameters \( m_{i}, c_{i} \), and \( I_{i} \) are related to link and motor parameters as follows
\[
m_{i} = m_{i} + m_{m_{i}},
\]
\[
c_{i} = c_{i} + m_{0} P_{C_{i}} + m_{m_{i}} P_{m_{i+1}},
\]
\[
I_{i} = I_{i}^{j} = I_{i}^{j} + I_{m_{i+1}}^{j} = I_{m_{i+1}}^{j} + m_{m_{i}} \left[ \begin{pmatrix} \dot{p}_{m_{i+1}}^{T} / |x| \ |p_{m_{i+1}}| / |x| \end{pmatrix}^{T} \begin{pmatrix} \dot{p}_{m_{i+1}} / |x| \ |p_{m_{i+1}}| / |x| \end{pmatrix} \right]
\tag{A.14}
\]

A.1. Formulation of Matrix \( B(q) \). Kinetic energy expressed as in (A.13) can be decomposed into five terms as \( T = T_{m} + T_{e} + T_{r} + T_{Y1} + T_{Y2} \), where \( T_{m} \) linearly depends on masses, \( T_{e} \) linearly depends on products of masses and centers of gravity, \( T_{r} \) linearly depends on inertia tensors, and \( T_{Y1} \) and \( T_{Y2} \) linearly depend on rotor inertias.
Correspondingly, four blocks of inertia matrix $B(q)$ can be defined.

**Dependence on Masses.** $T_m$ can be written as

$$
T_m = \sum_{i=1}^{n} \frac{1}{2} \dot{m}_i \dot{p}_i \dot{p}_i \left( \frac{i^{(i)}}{p} \right) \dot{q}
$$

$$
= \frac{1}{2} \dot{q}^T \left( \sum_{i=1}^{n} \dot{m}_i \dot{p}_i \dot{p}_i \left( \frac{i^{(i)}}{p} \right) \right) \dot{q}.
$$

(A.15)

**Dependence on Products of Masses and Centers of Gravity.** $T_e$ can be written as

$$
T_e = \sum_{i=1}^{n} p_i^T \omega_i R_i c_i = \sum_{i=1}^{n} p_i^T \left( \omega_i \times (R_i c_i) \right)
$$

$$
= -\frac{1}{2} \dot{q}^T \sum_{i=1}^{n} \left( \frac{i^{(i)}}{p} \right) \left( (R_i c_i) \times \dot{j}^{(i)} q \right)
$$

$$
= \frac{1}{2} \dot{q}^T \sum_{i=1}^{n} \left( \frac{i^{(i)}}{p} \right) \left( (R_i c_i) \times \dot{j}^{(i)} q \right) \dot{q}
$$

$$
= \frac{1}{2} \dot{q}^T \left( \sum_{i=1}^{n} \dot{m}_i \dot{p}_i \dot{p}_i \left( \frac{i^{(i)}}{p} \right) \right) \dot{q}.
$$

(A.16)

**Dependence on Inertia Tensors.** One can write

$$
T_i = \sum_{i=1}^{n} \frac{1}{2} \omega_i R_i R_i^T \omega_i = \sum_{i=1}^{n} \frac{1}{2} \dot{q}^T \left( \frac{i^{(i)}}{p} \right) R_i R_i^T \dot{j}^{(i)} q
$$

$$
= \frac{1}{2} \dot{q}^T \left( \sum_{i=1}^{n} \dot{m}_i \dot{p}_i \dot{p}_i \left( \frac{i^{(i)}}{p} \right) \right) \dot{q}.
$$

(A.17)

**Dependence on Rotor Inertias.** In (A.13) two terms can be identified, the first one is linearly varying and the latter one is quadratically varying with reduction gear $k_{m_i}$. In particular, the first term is given by

$$
T_{Y1} = \sum_{i=1}^{n} \frac{1}{2} \dot{Y}_i k_{m_i}^2 \dot{z}_{m_i} \dot{z}_{m_i}^T \dot{w}_{i-1}
$$

$$
= \sum_{i=1}^{n} \left( \frac{1}{2} \dot{Y}_i k_{m_i}^2 \dot{z}_{m_i} \dot{z}_{m_i}^T \dot{w}_{i-1} \right)
$$

$$
= \frac{1}{2} \dot{q}^T \left[ \frac{1}{2} \dot{Y}_i k_{m_i}^2 \dot{z}_{m_i} \dot{z}_{m_i}^T \dot{w}_{i-1} \right] \dot{q}
$$

$$
= \frac{1}{2} \dot{q}^T \left[ \frac{1}{2} \dot{Y}_i k_{m_i}^2 \dot{z}_{m_i} \dot{z}_{m_i}^T \dot{w}_{i-1} \right] \dot{q}
$$

$$
= \frac{1}{2} \dot{q}^T \left[ \frac{1}{2} \dot{Y}_i k_{m_i}^2 \dot{z}_{m_i} \dot{z}_{m_i}^T \dot{w}_{i-1} \right] \dot{q}.
$$

(A.18)

The latter one can be written as

$$
T_{Y2} = \frac{1}{2} \dot{Y}_i k_{m_i}^2 \dot{z}_{m_i} \dot{z}_{m_i}^T \dot{w}_{i-1}
$$

$$
= \frac{1}{2} \dot{q}^T \left[ \frac{1}{2} \dot{Y}_i k_{m_i}^2 \dot{z}_{m_i} \dot{z}_{m_i}^T \dot{w}_{i-1} \right] \dot{q}.
$$

(A.19)

Thus, (A.15), (A.16), (A.17), (A.18), and (A.19) allow expressing total kinetic energy in (A.13) as

$$
T = \frac{1}{2} \dot{q}^T B_m \dot{q} + \frac{1}{2} \dot{q}^T B_r \dot{q} + \frac{1}{2} \dot{q}^T B_d \dot{q}
$$

$$
+ \frac{1}{2} \dot{q}^T B_d \dot{q} + \frac{1}{2} \dot{q}^T B_d \dot{q} + \frac{1}{2} \dot{q}^T B_d \dot{q}
$$

$$
= \frac{1}{2} \dot{q}^T B (\dot{q}, \dot{q}).
$$

(A.20)

where inertia matrix $B(q)$ is given by

$$
B (q) = \left[ B_m(q) + B_r(q) + B_d(q) \right]^T B_r(q) B_d(q) B_d(q).
$$

(A.21)

Note that matrix $B(q)$ depends only on $q$.

**B. Robot Centrifugal and Coriolis Torque Matrix**

Elements $c_{ij}$ of matrix $C(q, \dot{q})$ are related to elements of $B(q)$ as

$$
c_{ij} = \sum_{k=1}^{n} c_{ijk} \ddot{q}_k, \quad c_{ijk} = \frac{1}{2} \left( \frac{\partial b_{ij}}{\partial q_k} - \frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ij}}{\partial \dot{q}_k} \right),
$$

being $c_{ijk}$ the Christoffel symbols.

Thus, three matrices $C'(q, \dot{q}), C''(q, \dot{q})$, and $C'''(q, \dot{q})$ can be introduced, so that

$$
C'(q, \dot{q}) = C'(q, \dot{q}) + C''(q, \dot{q}) + C'''(q, \dot{q}).
$$

(B.2)

Coefficients of matrices $C', C'', C'''$ are given by

$$
c_{ij}' = \frac{1}{2} \sum_{k=1}^{n} \frac{\partial b_{ij}}{\partial \dot{q}_k},
$$

(B.3)

$$
c_{ij}'' = -\frac{1}{2} \sum_{k=1}^{n} \frac{\partial b_{ij}}{\partial \dot{q}_k},
$$

(B.4)

$$
c_{ij}''' = \frac{1}{2} \sum_{k=1}^{n} \frac{\partial b_{ij}}{\partial \dot{q}_k}.
$$

(B.5)

Moreover, $n$ matrices $H^{(k)}$ can be defined as

$$
H^{(k)} = \frac{\partial}{\partial \dot{q}_k} B(q) = \frac{\partial}{\partial \dot{q}_k} B(q) \quad k = 1 \ldots n,
$$

(B.6)

being $B(q)$ dependent only on $q$.

It can be easily observed from (B.3) that

$$
c_{ij}' = \frac{1}{2} \sum_{k=1}^{n} h_{ij}'^{(k)} \dot{q}_k = \frac{1}{2} \sum_{k=1}^{n} h_{ij}'^{(k)} \dot{q}_k.
$$

(B.7)
and, consequently,

\[ C'(q, \dot{q}) = \frac{1}{2} \sum_{k=1}^{n} \partial \phi_k H^{(k)}. \]  

(B.8)

From (B.4), exploiting the symmetry property of matrix B(q) (i.e., \( b_{jk} = b_{kj} \)), one can write

\[ e''_{ij} = -\frac{1}{2} \sum_{k=1}^{2n} \partial b_{kj} \dot{q}_k = -\frac{1}{2} \sum_{k=1}^{2n} h_{kj}\dot{q}_k. \]  

(B.9)

Hence,

\[ C''(q, \dot{q}) = -\frac{1}{2} \begin{bmatrix} \dot{q}^T H^{(1)} \\ \vdots \\ \dot{q}^T H^{(2n)} \end{bmatrix} = -\frac{1}{2} \begin{bmatrix} \dot{q}^T H^{(1)} \\ \vdots \\ \dot{q}^T H^{(2n)} \end{bmatrix} \]  

(B.10)

Eventually, from (B.5), one can write

\[ C'''(q, \ddot{q}) = \frac{1}{2} \begin{bmatrix} H^{(1)y} q \cdots H^{(2n)y} q \end{bmatrix} \]  

(B.11)

\[ = \frac{1}{2} \begin{bmatrix} H^{(1)y} q \cdots H^{(2n)y} q \text{ zeros } (2n, n) \end{bmatrix} \]  

(B.12)

Thus, (B.2) yields

\[ C(q, \ddot{q}) = C'(q, \dot{q}) + C''(q, \dot{q}) - C''(q, \dot{q})^T \]  

(B.13)

Given regressor matrix \( Y_{H^{(k)}} \), (B.8) and (B.10) allow writing

\[ C'(q, \dot{q}) \ddot{q} = \frac{1}{2} \sum_{k=1}^{n} \left( \dot{q}_k \left( H^{(k)} \dot{q}_k \right) \right) \]  

(B.15)

and, from (B.12),

\[ C'''(q, \ddot{q}) = \frac{1}{2} \left[ Y_{H^{(1)}} \Pi \cdots Y_{H^{(n)}} \Pi \text{ zeros } (2n, n) \right] \dot{q}. \]  

(B.17)

Element \( i \) of vector \( C'''(q, \ddot{q}) \ddot{q} \) is given by

\[ \left( C'''(q, \ddot{q}) \ddot{q} \right)_i = \sum_{j=1}^{2n} e'''_{ij} \ddot{q}_j = \sum_{j=1}^{2n} \left( \frac{1}{2} \sum_{k=1}^{n} y_{ik} \pi_k \right) \ddot{q}_j \]  

(B.18)

where \( y_{jk}^{(i)} \) is the element \((ik)\) of matrix \( Y_{H^{(i)}} \), and \( y_{ij}^{(i)} \) is the row \((i)\) of matrix \( Y_{H^{(i)}} \). Hence, vector \( C'''(q, \ddot{q}) \ddot{q} \) is given by

\[ C'''(q, \ddot{q}) \ddot{q} = \frac{1}{2} \sum_{j=1}^{2n} \left( \pi_j \right) \Pi = \frac{1}{2} \sum_{j=1}^{n} \left( \ddot{q}_j Y_{H^{(i)}} \Pi \right) \]  

(B.19)

which is equal to vector \( C'(q, \dot{q}) \dot{q} \) as defined in (B.15).

Consequently, regressor \( Y_C(q, \dot{q}, \ddot{q}) \) can be defined as

\[ Y_C(q, \dot{q}, \ddot{q}) = \sum_{k=1}^{n} \left( \dot{q}_k Y_{H^{(k)}} \right) - \frac{1}{2} \begin{bmatrix} \ddot{q}^T Y_{H^{(1)}} \\ \vdots \\ \ddot{q}^T Y_{H^{(n)}} \end{bmatrix} \]  

(B.20)

### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

### Authors’ Contribution

Loredana Zollo and Edoardo Lopez contributed equally to this paper.
Endnotes

1. In (18) and (26) operator \((a)_{[0]}\) has been introduced, which generates a matrix \((3 \times 6)\) from a \((3 \times 1)\) vector \(a\). It is defined as

\[
(a)_{[0]} = \begin{bmatrix}
a_1 & 0 & 0 & a_2 & a_3 & 0 \\
0 & a_2 & a_3 & 0 & a_1 & 0 \\
0 & 0 & a_3 & a_1 & a_2 & 0
\end{bmatrix}.
\]

Given a \((3 \times 3)\) symmetric matrix \(S\) of elements \(s_{ij}\), operator \((a)_{[0]}\) allows expressing product \(Sa\) as

\[
Sa = (a)_{[0]} [s_{11} \ s_{22} \ s_{33} \ s_{12} \ s_{13} \ s_{23}]^T.
\]

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