Multiperiod Energy Market as Adjustable Robust Optimization: Intertemporal Pricing of Dispatchable Generators, Storage Batteries, and Uncertain Renewable Resources

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Abstract

In this paper, we present modeling and analysis of day-ahead multiperiod energy markets in which each competitive player or aggregator aims at making the highest profit by managing a complex mixture of different energy resources, such as conventional generators, storage batteries, and uncertain renewable resources. First, we develop a multiperiod energy market model in terms of an adjustable robust convex program. This market modeling is novel in the sense that the prosumption cost function of each aggregator, which evaluates the cost or utility to realize an amount of multiperiod energy prosumption, is a multivariable function resulting from a "parameterized" max-min program, in which the variable of the prosumption cost function is involved as a continuous parameter and the variable of dispatchable resources is involved as an adjustable variable for energy balance. This formulation enables to reasonably evaluate a reward for intertemporal dispatchability enhancement and a penalty for renewable energy uncertainty in a unified way. In addition, it enables to enforce a market regulation in which every aggregator is responsible for absorbing his renewable energy uncertainty by managing his own dispatchable energy resources. Second, in view of social economy as well as personal economy, we conduct a numerical analysis on the premise of several photovoltaic penetration levels. In this numerical analysis, we demonstrate that renewable generators do not always have priority of energy supply higher than conventional generators due to their uncertainty and limited dispatchability, meaning that the merit order of conventional and renewable generators can reverse. Furthermore, we analyze long-term evolution of competitive energy markets demonstrating that there can be found a social equilibrium of battery penetration levels, at which maximum personal profit with respect to battery system enhancement is attained.

Key words: Multiperiod energy markets, Adjustable robust optimization, Distributed energy resources, Uncertainty of renewable energy resources, Convex analysis.

1 Introduction

1.1 Research Background

The development of a smart grid has been recognized as one of the key issues in addressing environmental and social concerns, such as the sustainability of energy resources and the efficiency of energy management [1, 2]. In future power systems operation, it is crucial to appropriately manage a complex mixture of multiple types of energy resources, such as conventional generators, energy storage systems and devices, controllable and shiftable loads, and uncertain renewable resources, towards the realization of economically efficient supply-demand balance of energy.

The penetration of energy storage systems, such as electric vehicles and home energy storage systems, is generally supposed to be spatially distributed due to the limitation of installation capability [3]. Photovoltaic generator installation is also supposed to be spatially distributed, especially in Japan [4], for effective use of roof top spaces. Even though the impact of such individual materials and components on the grid may be tiny, the aggregation of them has high potential to serve for supply-demand balance. Thus an aggregator, a manager of accessible energy resources, can be a strong stakeholder in electricity markets, the modeling and analysis of which are the main subject of this paper.

1.2 Literature Review

The existing modeling and analysis of electricity markets are considerably wide-ranging in terms of objectives, settings, and concepts. For better understanding of our subject and focus, we first consider classifying existing works into the following (not necessarily mutually exclusive) categories.

1.2.1 Day-ahead and Real-time

The main difference between day-ahead and real-time markets can be explained in terms of "offline scheduling" and "online operation." In a day-ahead market, each market
player transacts their future operation schedule of energy resources, which mainly prescribes the amounts of energy production and consumption in consideration of ancillary service for stable supply [5–9]. On the other hand, each market player in a real-time market is supposed to transact their capability of energy resources to compensate discrepancies between operation schedules and real-time operation results [10–12]. Such discrepancies are possibly caused by several factors, e.g., load prediction errors, the difference of time resolutions in scheduling and real operation, and unexpected fluctuation of renewable resources. In general, profit-maximizing strategies in day-ahead and real-time markets are mutually correlated because of the limitation of energy resources, leading to dependency of day-ahead and real-time prices. Optimization models considering such a market correlation are analyzed in [8,13].

1.2.2 Spatial and Temporal

The notion of market transaction can be extended to space and time; see, e.g., [7,8] for models of spatiotemporal pricing. In particular, spatial pricing in electricity markets is called locational marginal pricing or nodal pricing [14–17], which can evaluate an impact of spatial constraints in a power network, such as the voltage capacity and thermal capacity of transmission lines. Depending on the levels of transmission line congestion, an independent system operator (ISO) determines spatially distributed energy prices based on a marginal social cost for stable power supply.

Energy prices can also have a temporal correlation [5,6,8,9]. This aspect deserves attention especially in recent day-ahead markets. This is because offline scheduling is significant in considering intertemporal optimization of the mixture of different types of energy resources. As mentioned in [1], an electricity market that can explicitly involve such intertemporal optimization is indispensable for making use of the power shiftability of batteries or flexible loads, which will work as key energy resources for future smart grid operation.

1.2.3 Open-loop and Closed-loop

The behavior of market players can be modeled as an optimization problem in which a personal profit maximization is performed in consideration of market prices. This modeling can be “open-loop,” meaning that a market price (or its prediction) is assumed to be given independently of the decision of a market player under consideration, i.e., a price-taker setting [5,6,9,13]. Such an open-loop model is valid, as long as the situations of most other players, e.g., the constitution of energy resources and the strategy in profit maximization, is stationary. However, in general, market prices are determined as assembly of the decisions of all market players. Such a market model is “closed-loop,” meaning that the player decision is mutually correlated with the resultant price [7,8,17,18]. This type of closed-loop modeling is crucial to discuss long-term evolution of competitive markets in which energy resource constitution is non-stationary.

1.2.4 Convex and Non-convex

Microeconomics is grounded in convex analysis. As thoroughly discussed in [16], an optimal energy price can be calculated as a marginal social cost of economic dispatch problems, which are often formulated as convex programs. This pricing method builds on the convexity of social costs, implying that a non-convex cost, such as the start-up cost of generators, cannot be reflected in the resultant marginal price; see, e.g., [5,7,12] for models with such a unit commitment problem. A theoretically grounded approach to dealing with such non-convexity is approximate convexification of social costs, called convex hull pricing. This method is shown to minimize the amount of particular side-payments caused by convexification. However, it generally yields another complicated issue on revenue adequacy, i.e., a financial issue on revenue allocation, which is often affected by a subjective decision of ISO; see [19] for details.

1.2.5 Bottom-up and Top-down

Last, but not least, the difference of market modeling concepts is explained in terms of “bottom-up” and “top-down.” A bottom-up approach is an engineering approach that is based on a stacking-up process of detailed elements. Existing works can mostly be classified into this category, developing complex integrated market models. Though such a model can serve for elaborate empirical studies, it is often cumbersome to find out key drivers and structures due to a high degree of parametric and structural freedom. A top-down approach is a systems theoretic approach (or a reverse engineering approach) that is based on a breaking-down process starting from an “appropriate level of abstraction.” Examples include [16,19], which are based on supply-demand models without distributed energy resources. A major advantage of this approach is to provide a simple but principled framework to discuss a universal law in electricity markets. Synthesis of both bottom-up and top-down approaches would be significant to develop a principled market framework.

1.3 Focus and Contribution of Present Paper

With the categorization in Section 1.2, we model and analyze an electricity market that is day-ahead, temporal, closed-loop, and convex, which we call a multiperiod energy market. In Section 3, based on a top-down approach, we first develop a mathematical foundation for modeling and analysis of the multiperiod energy market, where each competitive aggregator aims at making the highest profit by managing a complex mixture of different energy resources. The theoretical contributions in this paper are summarized as follows.

• We formulate a novel prosumption cost function of each aggregator as a multivariable function that results from a parameterized max-min program, i.e., the collection of an infinite number of max-min programs.

• We prove the convexity of such a prosumption cost function, in which the amount of multiperiod energy prosumption is involved as a non-adjustable variable while the variable of dispatchable resources is involved as an adjustable variable for energy balance.

It will be found that the resultant market clearing problem, which is formulated as a social cost minimization problem composed of a family of the parameterized max-min programs, is given as an “adjustable” robust convex program [20], which is also referred to as a two-stage robust convex program [21]. To the best of the authors’ knowledge, such an idea of describing a closed-loop energy market model based on an adjustable robust convex program has
not been reported in the literature, while scheduling problems or open-loop (price taker) market models with uncertain resources, such as robust unit commitment problems, have been discussed in terms of two-stage robust or stochastic optimization over recent years; see [22–25] and references therein.

The proposed market modeling enables to evaluate not only a reward for enhancement of intertemporal dispatchability owing to dispachable energy resources, but also a penalty for uncertainty due to renewable energy resources. Furthermore, it enables to enforce a “market regulation” in which each aggregator is responsible for absorbing the uncertainty of his renewable power generation so that his dispachable prosumption schedule can be regularly transacted in the day-ahead energy market. These properties give a clear distinction from the existing market models reviewed above.

In Section 4, on the premise of several photovoltaic penetration levels, we conduct a numerical analysis in view of social economy as well as personal economy. The contribution there is to give the following insights.

- Renewable generators do not always have priority of energy supply higher than conventional generators due to their uncertainty and limited dispatchability.
- There is a social equilibrium of battery penetration levels, at which maximum personal profit with respect to battery system enhancement is attained.

The first insight is relevant to the “merit order” of conventional versus renewable generators. The second is relevant to long-term evolution of competitive energy markets; see Section 2.2 for more details of motivation to study. This paper builds on preliminary versions [26, 27]. Compared with them, this paper newly gives the formulation of uncertain renewable resources, which enables the analysis of battery penetration levels.

### 2 Preliminaries

#### 2.1 Multiperiod Energy Market as Convex Optimization

In this subsection, to make the following discussion self-consistent, we overview the relation between a day-ahead energy market and an optimization problem from a top-level view of abstraction. For clarity, we first consider a small example where three market players, called aggregators, participate in a day-ahead market. In particular, the schedule of the total energy amount in the forenoon and that in the afternoon are supposed to be transacted, meaning that the whole day schedule transaction is divided into two energy markets at the AM and PM spots.

The objective of ISO is to find a suitable set of transaction energy amounts among the aggregators and a set of clearing prices assigned to the transaction energy amounts. For example, suppose that a market result is determined as in Table 1, where positive and negative energy amounts correspond to production and consumption, respectively. In this example, the transaction energy amounts can be written as

\[
x_1^* = \begin{pmatrix} 150 \\ 100 \end{pmatrix}, \quad x_2^* = \begin{pmatrix} -250 \\ -50 \end{pmatrix}, \quad x_3^* = \begin{pmatrix} 100 \\ -50 \end{pmatrix}
\]

and the clearing price can be written as

\[
\lambda^* = \begin{pmatrix} 10 \\ 5 \end{pmatrix}.
\]

Note that all these vectors, which are the decision variables of ISO, are two-dimensional vectors, the dimension of which corresponds to the number of time spots. Furthermore, the transaction energy amounts are balanced, i.e.,

\[
x_1^* + x_2^* + x_3^* = 0.
\]

This represents the balance of production and consumption (prosumption) energy amounts at every time spot.

Let \(A\) denote the label set of aggregators and let \(T\) denote the label set of time spots on the day of interest. In this paper, the event of finding a set of balanced prosumption profiles and a clearing price profile, denoted by \((x_\alpha^*, \lambda_\alpha)_{\alpha \in A}\) and \(\lambda^*\), is referred to as market clearing; A specific example in which

\[
A = \{1, 2, 3\}, \quad T = \{\text{AM, PM}\}
\]

is given above. With this terminology, we review a general market clearing problem in terms of intertemporal optimization. To this end, we introduce the notion of profit, which is a personal objective function of aggregators. Under a clearing price profile \(\lambda^*\), the resultant profit of the \(\alpha\)th aggregator is written as

\[
J_\alpha(x_\alpha^*; \lambda^*) = \langle \lambda^*, x_\alpha^* \rangle - F_\alpha(x_\alpha^*)
\]

where the inner product \(\langle \lambda^*, x_\alpha^* \rangle\) corresponds to the income obtained from the multiperiod energy market and \(F_\alpha\) denotes the cost function to realize a prosumption profile. Note that the income may be negative, meaning that the inner product term can be regarded as income or outgo depending on its sign. For instance,

\[
\langle \lambda^*, x_1^* \rangle = 2000, \quad \langle \lambda^*, x_2^* \rangle = -2750, \quad \langle \lambda^*, x_3^* \rangle = 750
\]

in the example above.

The social profit, which is a social objective function of ISO, is given as

\[
\sum_{\alpha \in A} J_\alpha(x_\alpha^*; \lambda^*) = \langle \lambda^*, \sum_{\alpha \in A} x_\alpha^* \rangle - \sum_{\alpha \in A} F_\alpha(x_\alpha^*). \tag{2}
\]

Note that the social income (the inner product) is zero as long as the balance of prosumption profiles is attained. From
this observation, we see that the social profit maximization problem is equivalent to the social cost minimization problem
\[
\min_{(x_\alpha)_{\alpha \in \mathcal{A}}} \sum_{\alpha \in \mathcal{A}} F_\alpha(x_\alpha) \text{ s.t. } \sum_{\alpha \in \mathcal{A}} x_\alpha = 0. \tag{3}
\]
It is clear that an optimal primal variable \((x^*_\alpha)_{\alpha \in \mathcal{A}}\) corresponds to the transacted prosumption profiles. Furthermore, an optimal dual variable \(\lambda^*\), i.e., the Lagrange multiplier associated with the prosumption balance constraint, corresponds to the clearing price profile. Thus the socially optimal market clearing problem is equivalent to the social cost minimization problem (3). In addition, the socially optimal market results can attain the profit maximization of each aggregator; see Proposition 3 for details.

2.2 Key Questions

In this subsection, we list several key questions to be discussed in this paper. The first question is:

Q1: How to model the cost function of prosumption profiles in a reasonable manner?

In the discussion of Section 2.1 above, we implicitly suppose that the prosumption cost function \(F_\alpha\) in (3) is a given (convex) function, which must be “multivariable,” i.e.,
\[
F_\alpha : \mathbb{R}^{|T|} \to \mathbb{R}.
\]
However, in practice, it is not always easy to identify such an intertemporally correlated cost function. In addition, aggregators are generally “prosumers,” meaning that the prosumption profile \(x_\alpha\) should be a complex mixture of different types of energy resources. A prosumption cost function should reflect idiosyncratic properties and structures of each prosumer. An answer to this question will be given in Section 3.

The second question is:

Q2: Is it reasonable to say in the multiperiod energy market that uncertain renewable resources have priority higher than dispatchable generators in terms of merit order?

It is often mentioned that renewable generators have priority higher than conventional generators because the marginal cost of renewable resources is zero (or close to zero). However, this conclusion is possibly premature in the multiperiod energy market. This is because the marginal cost function to realize a prosumption profile \(x_\alpha\) is not only “multivariable” but also “vector-valued,” i.e.,
\[
\partial F_\alpha : \mathbb{R}^{|T|} \to \mathbb{R}^{|T|} \tag{4}
\]
which is called the subdifferential of \(F_\alpha\) shown to be a monotone mapping [28]. This multi-dimensionality comes from the intertemporal correlation of prosumption cost functions. In addition, renewable and conventional generators should be distinguished in terms of the difference of realizable prosumption profiles, i.e., the difference of intertemporal dispatchability. Thus, it is not trivial to determine the “merit order” of different energy resources.

Furthermore, the uncertainty of renewable power generation must be considered in discussing its marginal cost. Note that, though the day-ahead prediction of renewable power generation necessarily involves some degree of uncertainty, the prosumption profile \(x^*_\alpha\) should be “deterministic” so that it can be regularly transacted in the day-ahead market. This implies that each aggregator should be responsible for absorbing the uncertainty of his renewable power generation. In general, large uncertainty can lead to conservative management of dispatchable energy resources; thereby reducing social profit as well as personal profit of each aggregator.

One prospective way to regulating the uncertain fluctuation of renewable power generation would be a smart use of energy storage systems. In fact, its potential has been attracting much attention towards effective integration of dispatchable and renewable power generation. However, the existing works have not yet been paid much attention on the question:

Q3: What level of energy storage penetration is economically reasonable from the viewpoints of not only social profit but also personal profit?

In general, large penetration of energy storage systems serves for improving social profit because it has a potential to reduce the waste of renewable power curtailments and to regulate load profiles by peak load shifting. However, from the viewpoint of aggregator’s “personal” profit, it is not clear what degree of increased profit deserves to make an investment in the enhancement of energy storage systems. We will discuss Q2 and Q3 through the numerical analysis in Section 4.

3 Mathematical Foundation

3.1 Aggregator Model

In this subsection, we provide a model of aggregators as incorporating their idiosyncrasy. This is a braking-down process to elaborate on the abstracted market model in (3). For simplicity of notation, we do not make a distinction among the aggregators in this model description, i.e., we drop the subscript \(\alpha\) as long as there is no chance of confusion.

The prosumption energy of an aggregator at the \(t\)th time spot can be described as
\[
x_t = g_t - l_t + p_t - q_t + \eta^{\text{out}} t - \frac{1}{\eta^{\text{in}}} \delta_t^\text{in}, \quad t \in \mathcal{T} \tag{5}
\]
where \(x_t \in \mathbb{R}\) denotes the resultant prosumption energy to the grid, \(g_t \in \mathbb{R}_+\) denotes the power generation of dispatchable generators, \(l_t \in \mathbb{R}_+\) denotes the load, \(p_t \in \mathbb{R}_+\) and \(q_t \in \mathbb{R}_+\) denote the power generation and power curtailment of renewable generators, and \(\delta^{\text{in}}_t \in \mathbb{R}_+\) and \(\delta^{\text{out}}_t \in \mathbb{R}_+\) denote the battery charge and discharge energy. The positive constants \(\eta^{\text{in}}\) and \(\eta^{\text{out}}\) denote the charge and discharge efficiency, respectively, each of which takes a value in \((0, 1]\).

Note that the sign of \(x_t\) is positive for outflow direction to the grid.

We denote the stacked vector of a symbol, called a profile, by that without the subscript \(t\). For example, the prosumption profile \(x = (x_t)_{t \in \mathcal{T}}\) represents the sequence of prosumption amounts, which is \(|\mathcal{T}|\)-dimensional. Throughout the paper, the load profile \(l\) is supposed to be a given constant vector, i.e., the prediction of the load profile is supposed to perform without a prediction error. On the other hand, the renewable power generation profile \(p\) is supposed to be an uncertain variable that can vary within a scenario set, denoted by \(\mathcal{P}\).
In particular,

\[ p \in \mathcal{P}, \quad \mathcal{P} := \{p^{(1)}, \ldots, p^{(m)}\}, \tag{6} \]

where \( m \) different scenarios are considered in the day-ahead prediction. For simplicity of discussion, we here suppose that \( \mathcal{P} \) is composed of a finite number of renewable scenarios, though the following discussion can be extended also to the case of continuous scenario sets.

The dispatchable power generation profile \( g \), the renewable power curtailment profile \( q \), and the battery charge and discharge power profiles \( \delta^{\text{in}} \) and \( \delta^{\text{out}} \) are decision variables that can be controlled by the aggregator. To realize a desired prosumption profile \( x \), the aggregator aims at controlling \( g, q \), and \( \delta := (\delta^{\text{in}}, \delta^{\text{out}}) \) in compliance with some physical constraints. In particular, the bounds for dispatchable power generation and the limitation of inverter and battery capacities are represented by given domains \( \mathcal{G} \) and \( \mathcal{D} \), i.e.,

\[ g \in \mathcal{G}, \quad \delta \in \mathcal{D}. \tag{7} \]

For example, these can represent a constraint \( q \leq g \leq \gamma \), where \( g \) and \( \gamma \) are given constant vectors representing the lower and upper bounds for generator outputs. The renewable power curtailment profile must not be greater than the the renewable power generation profile, i.e., \( 0 \leq q \leq p \) for each scenario \( p \in \mathcal{P} \). In this paper, this is written by

\[ q \in \mathcal{Q}(p), \quad p \in \mathcal{P}. \tag{8} \]

Note that the domain \( \mathcal{Q}(p) \) is dependent on which renewable power generation profile \( p \) arises from the scenario set \( \mathcal{P} \).

### 3.2 Characterization of Realizable Prosumption Profiles

We consider giving a mathematical description of intertemporal dispatchability of aggregator energy prosumption. With respect to each prosumption profile \( x \) under a renewable scenario \( p \in \mathcal{P} \), we denote the feasible domain of \( g, \delta \), and \( q \), which are called the dispatchable variables, by

\[ \mathcal{F}(x; p) := \{(g, \delta, q) \in \mathcal{G} \times \mathcal{D} \times \mathcal{Q}(p) : (5) \text{ holds}\}. \tag{9} \]

Note that \( x \) and \( p \) are involved as “parameters” in the constraint of the dispatchable variables. More specifically, a feasible domain \( \mathcal{F}(x; p) \) of the stacked version \( (g, \delta, q) \) of the dispatchable variables is determined as a standard set if the parameters \( x \) and \( p \) are fixed as constant vectors.

Based on this parameterized domain, we define the following set of realizable prosumption profiles.

**Proposition 1** Suppose that \( \mathcal{G} \) and \( \mathcal{D} \) are convex. Then the set of realizable prosumption profiles defined by

\[ \mathcal{X} := \{x \in \mathbb{R}^{|T|} : \mathcal{F}(x; p) \neq \emptyset, \forall p \in \mathcal{P}\} \tag{10} \]

is convex.

For explanation, let us consider the meaning of “\( 0 \in \mathcal{X} \).” This means that the prosumption profile \( x = 0 \), i.e., the supply-demand balance inside the aggregator, is surely realizable for “any” renewable scenario \( p \) arising from \( \mathcal{P} \). This can be seen as follows. Suppose that the \( i \)th renewable scenario \( p^{(i)} \) arises. Then, the condition of \( \mathcal{F}(0; p^{(i)}) \neq \emptyset \) means that there exists at least one combination of the dispatchable variables such that the supply-demand balance is attained in the \( i \)th renewable scenario, i.e.,

\[ \exists g \in \mathcal{G}, \quad \delta \in \mathcal{D}, \quad q \in \mathcal{Q}(p^{(i)}) \quad \text{s.t.} \quad 0 = g - 1 + p^{(i)} - q + \eta \delta^{\text{out}} - \frac{1}{\eta} \delta^{\text{in}}. \]

Therefore, \( \mathcal{F}(0; p) \neq \emptyset \) for all \( p \in \mathcal{P} \) means that, for each of all renewable scenarios, there exists at least one combination of the dispatchable variables such that the supply-demand balance inside the aggregator, i.e., \( x = 0 \), is attained. Note that \( (g, \delta, q) \) is an “adjustable” variable, meaning that it can vary such that the energy balance in (5) is attained for every scenario \( p \in \mathcal{P} \) under a prosumption profile \( x \in \mathcal{X} \).

As seen here, \( \mathcal{X} \) can be understood as the collection of all possible prosumption profiles that can be surely realizable for all renewable scenarios. In the rest of this paper, we suppose that each aggregator transacts only a prosumption profile \( x \) involved in \( \mathcal{X} \). This corresponds to a “market regulation” in which every aggregator is responsible for absorbing the uncertainty of his renewable power generation by managing his own dispatchable energy resources. Proposition 1 shows that this market regulation of uncertainty absorption ensures the convexity of \( \mathcal{X} \), crucial for marginal cost pricing.

**Remark 1** The idiosyncrasies of \( \mathcal{X} \) can be understood as the individual characteristics of intertemporal dispatchability that can be produced from energy resource management. For better understanding, let us consider a simple aggregator just having a conventional generator, i.e., \( x = g \). Then it is easy to see that \( \mathcal{X} = \mathcal{G} \). On the other hand, for an aggregator just having a renewable energy resource, i.e., \( x = p - q \), we see that

\[ \mathcal{X} = \{x \in \mathbb{R}^{|T|} : 0 \leq x \leq p_{\text{min}}\}, \quad p_{\text{min}} := \min \mathcal{P} \tag{11} \]

where the minimum of \( \mathcal{P} \) is taken in the element-wise sense. Note that \( p_{\text{min}} \) represents the lower envelope of all possible renewable scenarios. This means that the uncertainty is removed just by curtailment; The larger the uncertainty is, the smaller the domain \( \mathcal{X} \) is. Such a waste of energy curtailment can be reduced, e.g., if the aggregator has a battery. This battery introduction can actually enlarge the domain of \( \mathcal{X} \). In this sense, the “size” of \( \mathcal{X} \) can be understood as a level of intertemporal dispatchability (or flexibility) of the corresponding aggregator under the market regulation of uncertainty absorption.

### 3.3 Deduction of Prosumption Cost Function

We consider assigning a cost to each realizable prosumption profile \( x \in \mathcal{X} \). This gives an answer to Q1 in Section 2.2. To this end, we introduce the cost functions of dispatchable power generation and battery charge and discharge as

\[ G : \mathcal{G} \to \mathbb{R}, \quad D : \mathcal{D} \to \mathbb{R}, \tag{12} \]

which represent practically accessible cost functions, such as the fuel cost of generators and the deterioration cost of batteries. With this notation, we can deduce a convex prosumption cost function as follows.

**Proposition 2** Suppose that the generation cost function \( G \) and the battery cost function \( D \) are convex over convex
domains \( G \) and \( D \). Then the prosumption cost function
\[
F(x) := \max_{p \in P} \min_{(g, \delta, q) \in F(x; p)} \left\{ G(g) + D(\delta) \right\}
\]  
(13)
is convex over the convex domain \( \mathcal{X} \).

**NOTE** One may think that the convexity of \( F \) is not surprising because of assuming the convexity of \( G \) and \( D \). However, we remark that the presumption cost function is defined based on a “parameterized” max-min program in which the variable \( x \) is involved as a continuous parameter in the equality constraint (5). In fact, the max-min program in the right-hand side of (13) can be reduced to a set of scenario-wise convex programs if the parameter \( x \) is “fixed.” In other words, \( F \) is defined as the collection of an infinite number of scenario-wise convex programs. The significance of Proposition 2 is to show, by virtue of convex analysis theory, that such a complicated function is convex with respect to the vector-valued parameter \( x \). To the best of the authors’ knowledge, such an idea of formulating the cost function (or, at the same time, utility function) based on a parameterized max-min program has not been reported in the literature of multiperiod energy market modeling.

The meaning of the prosumption cost function can be explained as follows. For clarity, let us first discuss the value of \( F(0) \), i.e., the cost to realize the prosumption profile \( x = 0 \). Suppose that the \( i \)th renewable scenario \( p^{(i)} \) arises. Then, \( F(0; p^{(i)}) \), defined as in (9), represents the set of all possible combinations of \( g \in G, \delta \in D, \) and \( q \in Q(p^{(i)}) \) such that the supply-demand balance is attained inside the aggregator. Note that there can be an infinite number of combinations of the dispatchable variables satisfying this supply-demand balance. Thus, we can use this remaining degree of freedom to minimize the sum of the generation and battery costs, as in the minimization part of (13).

Let us denote the minimum value by \( F'(0; p^{(i)}) \), for which \( F' \) is defined as
\[
F'(x; p) := \min_{(g, \delta, q) \in F(x; p)} \left\{ G(g) + D(\delta) \right\}.
\]  
(14)
Note that both \( x \) and \( p \) in (14) are involved as parameters in the constraint of the minimization problem. This means that \( (g, \delta, q) \) is an adjustable variable, i.e., the minimizer \((g^*, \delta^*, q^*)\) is a function dependent on \( p \in P \) and \( x \in \mathcal{X} \).

The value of \( F'(0; p^{(i)}) \) corresponds to the minimum cost to attain the supply-demand balance under the specific renewable scenario \( p^{(i)} \). Finally, the value of \( F(0) \) is defined as the worst cost among all possible renewable scenarios, i.e.,
\[
F(0) = \max_{i \in \{1, \ldots, m\}} F'(0; p^{(i)}).
\]
As seen here, to find the value of \( F(0) \), or, more generally, to find the value of \( F(x) \) for a “fixed” parameter \( x \), is a single max-min program. Note that this max-min program is different from a robust optimization problem [29] defined as a “min-max” program for minimizing the worst-case cost. It should be further noted that the worst-case renewable scenario \( p^{(i)} \in P \) is a function dependent on \( x \in \mathcal{X} \). This means that the worst-case renewable scenario is not unique in general, and it depends on how much amounts of energy are bought or sold at the respective time spots.

The continuous function \( F(x) \) can be calculated as the collection of all minimum costs to realize respective prosumption profiles, provided that each \( x \) is an element of \( \mathcal{X} \). Note that the closed form of \( F \) cannot be written down in general; It encapsulates complex information of different energy resources, such as the uncertainty of renewable power generation and the physical constraints of generators and batteries.

**Remark 2** In Proposition 2, the worst case of renewable scenarios is considered. This can be replaced with the expectation with respect to renewable scenarios as
\[
F(x) := \sum_{i=1}^{m} \pi^{(i)} \min_{(g, \delta, q) \in F(x; p^{(i)})} \left\{ G(g) + D(\delta) \right\}
\]  
(15)
where \( \pi^{(i)} \) denotes the realization probability of the renewable scenario \( p^{(i)} \). This is also shown to be convex. Note that the variable \( x \) is involved here as a parameter of the expected cost minimization problem in the right-hand side of (15). Even though this \( F \) is defined in terms of expectation, every prosumption profile \( x \in \mathcal{X} \) is surely realizable for all possible renewable scenarios \( p \in P \), because \( \mathcal{X} \) is defined as the robust feasible domain in (10).

**Remark 3** We give a remark on the cost or penalty of renewable power curtailment. In the above formulation, we do not explicitly evaluate the cost of \( q \). However, this does not mean that the renewable power curtailment results in no loss of profit. For clarity, we consider an aggregator just having a renewable resource, i.e., \( x = p - q \). Then,
\[
F(x) = 0, \quad \forall x \in \mathcal{X}
\]
with \( \mathcal{X} \) given as in (11). This means that every \( x \) involved in \( \mathcal{X} \) is equally realizable without causing any prosumption cost. Consider the personal profit maximization with \( J \) defined as in (1). Whenever the resultant clearing price profile is positive, i.e., \( \lambda^* > 0 \), the profit-maximizing prosumption profile is found as \( x^* = p_{\text{min}} \). This shows that the least renewable power curtailment is naturally obtained as the optimal decision following the principle of profit maximization.

### 3.4 Multiperiod Energy Market Model

Because each prosumption cost function is convex as shown in Proposition 2, the socially optimal market clearing problem (3) is found to be an adjustable robust convex program. In particular, (3) is a first-stage problem to find the optimal non-adjustable variable \( (x_\alpha^*)_{\alpha \in A} \), while the family of
\[
F_\alpha(x_{\alpha}) = \max_{p_{\alpha} \in P_\alpha} \min_{(g_\alpha, \delta_\alpha, q_\alpha) \in F_\alpha(x_{\alpha}, p_{\alpha})} \left\{ G_\alpha(g_{\alpha}) + D_\alpha(\delta_{\alpha}) \right\}
\]
for all aggregators is the family of second-stage problems to find the optimal adjustable variables \((g_\alpha^*, \delta_\alpha^*, q_\alpha^*)_{\alpha \in A}\) such that the uncertainty of renewable resources is absorbed inside individual aggregators on their own responsibility.

Based on the convexity of the market model, we can see a clear relation between microeconomics and convex analysis.
To see this, we consider the Lagrangian dual problem
\[
    \max_{\lambda} \sum_{\alpha \in \mathcal{A}} x_\alpha \min \left\{ F_\alpha(x_\alpha) - \langle \lambda, x_\alpha \rangle \right\},
\]
which is equivalent to (3) owing to the strong duality. As shown in the following proposition, the clearing price profile, denoted by \( \lambda^* \), can achieve the maximal personal profit of every aggregator under the prosumption balance constraint.

**Proposition 3** Consider the Lagrangian dual problem (16). With the profit function \( J_\alpha \) defined as in (1), let
\[
    x_\alpha(\lambda) := \{ x_\alpha \in \mathcal{X}_\alpha : J_\alpha(x_\alpha; \lambda) \geq J_\alpha(x'_\alpha; \lambda), \forall x'_\alpha \in \mathcal{X}_\alpha \},
\]
be the set of all prosumption profiles that attain the maximum profit under a price profile \( \lambda \). Then, \( x_\alpha^* \) and \( \lambda^* \) are primal and dual solutions to (16) if and only if
\[
    x_\alpha^* \in x_\alpha(\lambda^*), \quad \forall \alpha \in \mathcal{A}, \quad \text{and} \quad \sum_{\alpha \in \mathcal{A}} x_\alpha^* = 0. \tag{18}
\]

**NOTE** We remark the significance of the proposed market model. One may think that an economic dispatch problem with uncertain renewable resources can be formulated as a robust optimization problem, where all available resources in the grid can be involved as the decision variables. However, in such a centralized formulation, an owner of renewable resources can be a “free rider,” who does not have responsibility for uncertainty management. In particular, the uncertainty of renewable resources can be covered by dispatchable resources owned by some other aggregators. In general, it is not easy to determine a reasonable price or side payment for using such resources of others, because a unique price invariant for renewable scenarios is difficult to determine. In contrast, our market model imposes self-responsibility to uncertainty management on each aggregator. This creates synergy between renewable and storage resources. Each aggregator has an incentive to own a storage resource in conjunction with a renewable resource because reducing a waste of energy curtailment due to uncertainty makes his own profit higher.

### 4 Numerical Analysis

#### 4.1 Simulation Setup

We consider a multiperiod energy market composed of 11 aggregators in which prosumption energy amounts are transacted at 24 time spots. The specification of each aggregator is given as follows. The first to tenth aggregators are supposed to be distributed energy resource (DER) aggregators each of whose prosumption profile is described as
\[
    x_\alpha = -l_\alpha + p_\alpha - q_\alpha + g_{\alpha}^\text{out} \delta_{\alpha}^\text{out} - \frac{1}{\eta_\alpha} g_{\alpha}^\text{in} \delta_{\alpha}^\text{in}, \quad \alpha \in \mathcal{A}_D
\]
where \( \mathcal{A}_D \) denotes the label set of the DER aggregators. The load profile of each DER aggregator is given as a typical profile of consumers such as residences, commercial facilities, and factories. For example, the first DER aggregator’s load profile is plotted by the black line with circles in Fig. 1(a1), which is the same as that in Fig. 1(b1). This load profile corresponds to the aggregate of 4.6 million residences in terms of consumption energy amounts. The aggregate of all DER aggregators’ load profiles, which amounts to 1035.1[GWh], is plotted by the dotted line of the upper envelope of Fig. 2(a1), which is identical to those in Figs. 2(a2) and (a3).

For renewable power generation, we consider two penetration levels of photovoltaic (PV) power generation. This is based on the premise of the feed-in tariff scheme in Japan. For example, the PV scenarios of the the first DER aggregator are plotted by the thin solid lines in Figs. 1(a1) and (b1), which correspond to low and high PV penetration levels, respectively. In this numerical analysis, ten scenarios are considered for each DER aggregator, i.e., \( m = 10 \) in (6). For the high PV penetration level, the aggregate average of all PV scenario sets subtracted from the aggregate of all load profiles, i.e.,
\[
    \sum_{\alpha \in \mathcal{A}_D} \left\{ l_\alpha - \frac{1}{m} \sum_{i=1}^{m} p_\alpha^{(i)} \right\},
\]
is plotted by the green dotted line in the middle of Fig. 2(a1), identical to those in Figs. 2(a2) and (a3).

In the following, we analyze market results as varying the penetration levels of batteries. The battery penetration level is defined as follows. The aggregate of all loads, i.e., \( \sum_{\alpha \in \mathcal{A}_D} l_\alpha \), can be regarded as 57 million residential loads in terms of consumption energy. Based on this, we say that the battery penetration level is \( r \% \) if each of \( r \% \) of 57 million residences has a battery with \( \pm 7[kW] \) inverter capacity and \( 14[kWh] \) battery capacity. Its charge and discharge efficiencies are supposed to be 0.95.

The battery cost function \( D_\alpha \) is given as follows. Let \( s_0^\alpha \) denote the initial amount of stored energy, i.e., the state of charge (SOC), which is defined as the deviation from a neutral value. It is reasonable to suppose that a higher level of the final SOC is more preferable than a lower level, and vice versa. To take into account this aspect, we suppose that each aggregator assesses the value of the final SOC by
\[
    D_\alpha(\delta_\alpha) = -d(s_\alpha^\text{fin}(\delta_\alpha)), \quad s_\alpha^\text{fin}(\delta_\alpha) := s_0^\alpha + \mathbf{1}^T (\delta_\alpha^\text{in} - \delta_\alpha^\text{out})
\]
where \( d : \mathbb{R} \rightarrow \mathbb{R} \) is a concave function given as
\[
    d(s) := \begin{cases} 
        a_4(s - \overline{\sigma}) + a_3 \overline{\sigma}, & \overline{\sigma} \leq s, \\
        a_3 s, & 0 \leq s < \overline{\sigma}, \\
        a_2 s, & \underline{s} \leq s < 0, \\
        a_1(s - \underline{s}) + a_2 \underline{s}, & s \leq \underline{s}.
    \end{cases}
\]
In the simulation, we set \( s_0 = 0 \) for all \( \alpha \in \mathcal{A}_D \), \( a_1 = 11 \), \( a_2 = 8 \), \( a_3 = 4 \), and \( a_4 = 1 \). The values of \( s \) and \( \overline{\sigma} \) are set to represent 37.5% and 62.5% of the full SOC.

The 11th aggregator is supposed to be a generation aggregator whose prosumption profile is \( x_{11} = g_{11} \). Supposing that this aggregator operates 13 types of conventional generators, we denote the fuel cost and the output limit of the ith generator by \( c^i[\text{JPY/kWh}] \) and \( g_i \), respectively. Then, the generation cost function is given as
\[
    G_{11}(g_{11}) = \min_{(g^i)_{i \in I}} \sum_{i \in I} c_i^i \mathbf{1}^T g_i
\]
s.t. \( g_{11} = \sum_{i \in I} g_i^i \) and \( g_i \in G_i, \forall i \in I \).
where $I$ denotes the label set of the generators. The fuel cost and the output limit of each generator are listed in Table 2. In this table, “Duration” corresponds to the period of output regulation. For example, the output of “Coal (A)” is regulated every 12 hours, i.e., its output is fixed during the day and night time. In the simulation, we construct every prosumption cost function $F_\alpha$ based on the definition of (15) with the uniform probability distribution $\pi(i) = 1/m$.

### 4.2 Simulation Results

Varying battery penetration levels, we plot the resultant social costs in Fig. 3(1), where the red and blue lines correspond to low and high PV penetration levels, respectively. From this figure, we see that the social cost decreases as the battery penetration level increases. This is a natural consequence because the realizable prosumption profile set $X_\alpha$ in (10), the direct product of which corresponds to the feasible domain of the social cost minimization problem (3), enlarges as the battery penetration level increases. Next, at 20% battery penetration level, we plot the resultant profiles of the first DER aggregator in Fig. 1, where (a1)-(a3) correspond to the low PV penetration level and (b1)-(b3) correspond to the high PV penetration level. In particular, the resultant prosumption profiles (with opposite signs) are plotted by the blue solid lines with squares and the resultant PV curtailment profiles subtracted from the corresponding PV scenarios are plotted by the solid lines with asterisks in Figs. 1(a1) and (b1), the resultant battery charge and dis-

![Diagram](image1)

**Fig. 1.** Profiles of the first DER aggregator at 20% battery penetration level. (a1-a3) Low PV penetration level. (b1-b3) High PV penetration level. (a1,b1) Prosumption (sign opposite), Load, PV scenarios, PV curtailments subtracted from PV scenarios. (a2,b2) Battery charge and discharge. (a3,b3) SOC.

![Diagram](image2)

**Fig. 2.** (a1-a3) Overviews of prosumption balance at high PV penetration level. (b1-b3) Clearing price profiles. (a1,b1) 0% battery penetration level. (a2,b2) 20% battery penetration level. (a3,b3) 50% battery penetration level.

### Table 2

| Type     | Max output [MW] | Cost [JPY/kWh] | Duration [h] |
|----------|-----------------|----------------|--------------|
| Oil (A)  | 4200            | 12.39          | 1            |
| Oil (B)  | 5000            | 11.31          | 1            |
| Oil (C)  | 4500            | 11.48          | 1            |
| LNG (A)  | 3000            | 5.48           | 1            |
| LNG (B)  | 4200            | 5.40           | 1            |
| LNG (C)  | 2000            | 6.02           | 1            |
| LNGCC (A)| 7500            | 2.48           | 1            |
| LNGCC (B)| 5000            | 4.07           | 1            |
| LNGCC (C)| 8000            | 1.91           | 6            |
| Coal (A) | 5000            | 4.23           | 12           |
| Coal (B) | 5600            | 3.58           | 12           |
| Coal (C) | 8500            | 1.95           | 12           |
| Coal (D) | 5000            | 4.88           | 12           |

![Diagram](image3)
charge profiles are plotted in Figs. 1(a2) and (b2), and the resultant SOC profiles are plotted in Figs. 1(a3) and (b3). From these figures, we see that the uncertainty of each set of PV scenarios is absorbed by appropriate PV curtailment and battery management.

At the high PV penetration level, we depict the overviews of prosumption balance in Figs. 2(a1)-(a3), which correspond to 0%, 20%, and 50% battery penetration levels, respectively. Furthermore, we plot the corresponding clearing price profiles in Figs. 2(b1)-(b3), showing that the increased battery penetration level results in price leveling-off. The black thick line in each of Figs. 2(a1)-(a3) represents the resultant prosumption profile \( x_{11} \), which is equal to \(- \sum_{\alpha \in A} x_{\alpha} \). The dark red area corresponds to the aggregate energy amount of PV curtailment that is given as the aggregate average of the resultant PV curtailment profiles of all DER aggregators. The blue lines in Figs. 2(a2) and (a3) represent the aggregate average of the resultant battery charge and discharge profiles. From these figures, we see that the average amount of PV curtailment properly decreases, i.e., the amount of thermal power generation decreases, as the battery penetration level increases. This implies that PV power generation with an appropriate amount of batteries has priority of energy supply higher than thermal generators while that without batteries does not always have such priority due to its uncertainty, giving an answer to Q2 in Section 2.2 with regard to merit order of different energy resources.

Finally, from the viewpoint of personal profit, we discuss a social equilibrium of battery penetration levels, giving an answer to Q3. We plot the values of of the first DER aggregator’s profit, i.e., \( J_1(\lambda^*; \lambda^*) \), in Fig. 3(2) where the red and blue lines correspond to the low and high PV penetration levels. From this figure, we see that the increasing rate of the personal profit is relatively high at lower battery penetration levels, while it decreases at higher battery penetration levels. This profit rate saturation can be explained by the fact that the profit from battery arbitrage as well as the profit from PV curtailment reduction decrease as the battery penetration level increases. This trend can be confirmed from Fig. 2.

Note that the enhancement of battery systems generally requires an additional cost for battery purchase. To take into account this, we calculate the actual net profit as in Fig. 3(3) where we estimate the battery purchase cost for one-day use considering its durability: The battery price is supposed to be 10,000 [JPY/kWh] and the durability to be 25 years. From this figure, we see that there is an equilibrium of the battery penetration level, highlighted by the diamond mark, that attains the maximum of the net profit for each PV penetration level. It should be emphasized that this equilibrium can be realized as the assembly of long-term decisions of competitive market players who pursue just personal profit maximization, not social profit maximization.

5 Concluding Remarks

In this paper, we presented modeling and analysis of day-ahead multiperiod energy markets. We modeled the multiperiod energy market as an adjustable robust convex program, in which each competitive player is responsible for absorbing the uncertainty of renewable power generation. Furthermore, by numerical analysis, we discussed the merit order of different energy resources and the existence of a social equilibrium of battery penetration levels, at which the maximum personal profit with respect to battery system enhancement is attained. Consideration of non-convexity in unit commitment problems and spatial constraints of transmission lines would be an interesting direction of future research.

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A Proof of Proposition 1

We see that \( \mathcal{X} \) can be written as

\[
\mathcal{X} = \bigcap_{p \in P} \mathcal{X}'(p), \quad \mathcal{X}'(p) := \{ x \in \mathbb{R}^{|T|} : F(x; p) \neq \emptyset \}.
\]

Note that \( \mathcal{X}'(p) \) is convex because it is an affine mapping of the convex domain \( G \times D \times Q(p) \). Thus, \( \mathcal{X} \) is also convex because it is an intersection of convex domains.

B Proof of Proposition 2

We see that \( x \) in (5) is given as an affine mapping of \( (g, \delta, q) \). As shown in Theorem 5.7 of [28], the image of a convex function under an affine mapping is a convex function. This implies that \( F' \) defined as in (14) is convex with respect to \( x \) for each \( p \in P \). In addition, the pointwise supremum of an arbitrary collection of convex functions is convex, as shown in Theorem 5.5 of [28]. Thus, the convexity of \( F \) is proven for any set \( P \), which may be discrete or continuous.

C Proof of Proposition 3

The convex conjugate of \( F_\alpha \) is defined as

\[
F_\alpha^*(\lambda) := \sup_{x_\alpha} \{ \langle \lambda, x_\alpha \rangle - F_\alpha(x) \},
\]

which is called the Legendre-Fenchel transformation [28]. For the maximization in (16), its first-order optimality condition yields the generalized equation

\[
0 \in \sum_{\alpha \in A} \partial F_\alpha(x^\star).
\]  

(C.1)

To prove the equivalence between (18) and (C.1), it suffices to show the identity of

\[
x_\alpha(\lambda) = \partial F_\alpha^*(\lambda).
\]  

(C.2)

Consider some \( x^\star_\alpha \in x_\alpha(\lambda) \), i.e., a maximizer of \( J_\alpha \) under a given \( \lambda \). Then, its optimality condition yields \( \lambda \in \partial F_\alpha(x^\star_\alpha) \). Because \( \partial F_\alpha^* \) is shown to be the inverse mapping of \( \partial F_\alpha \); see Theorem 23.5 of [28], it is equivalent to \( x^\star_\alpha \in \partial F_\alpha^*(\lambda) \). Hence, (C.2) is proven.