Precision measurements of the $pp \rightarrow \pi^+ pn$ and $pp \rightarrow \pi^+ d$ reactions: importance of long-range and tensor force effects

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Abstract

Inclusive measurements of pion production in proton–proton collisions in the forward direction were undertaken at 400 and 600 MeV at COSY using the Big Karl spectrograph. The high resolution in the $\pi^+$ momentum ensured that there was an unambiguous separation of the $pp \rightarrow \pi^+d/\pi^+pn$ channels. Using these and earlier data, the ratio of the production cross sections could be followed through the $\Delta$ region and compared with the predictions of final state interaction theory. Deviations are strongly influenced by long-range terms in the production operator and the tensor force in the final $pn$ system. These have been investigated in a realistic $pp \rightarrow \pi^+d/\pi^+pn$ calculation that includes $S \equiv D$ channel coupling between the final nucleons. A semi-quantitative understanding of the observed effects is achieved.

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Pion production in nucleon-nucleon collisions at intermediate energies involves a delicate interplay between the basic production mechanism and the strong interactions between the two or three particles in the final state. Information on this might be obtained by comparing the \( pp \rightarrow \pi^+d \) and \( pp \rightarrow \pi^+pn \) reactions. With this in mind, we measured simultaneously the two final states for forward-going pions at a proton beam energy of \( T_p = 951 \text{ MeV} \) by studying the inclusive \( pp \rightarrow \pi^+X \) reaction and achieving a mass resolution of \( \sigma_X = 97 \text{ keV}/c^2 \) in the region of the deuteron peak \[1\]. This is almost four times better than that of the previous best experiment \[2\] and means that events corresponding to the \( \pi^+d \) two-body final state could be unambiguously separated from those of the \( \pi^+pn \) continuum. Moreover, the high resolution was sufficient to show that the production of \( S \)-wave spin-singlet \{pn\}_s final states is negligible compared to the spin-triplet \{pn\}_t.

Simultaneous measurement of the \( \pi^+d \) and \( \pi^+pn \) final states allows one to evaluate the cross section ratio

\[
R_{pn/d} = \frac{d^2\sigma}{d\Omega dx}(pp \rightarrow \pi^+ \{pn\}_t) / \frac{d\sigma}{d\Omega}(pp \rightarrow \pi^+ d)
\]

with few systematic errors, being untroubled by questions of relative normalization of different experiments. Here \( x = \varepsilon/B \) is the excitation energy \( \varepsilon \) in the \( pn \) system in units of the deuteron binding energy \( B \).

If the coupling between the \( S \) and \( D \) states through the tensor force is neglected, then the Földy-Wilkin final state interaction (FSI) theorem \[3\] shows that

\[
R_{pn/d} \approx N \frac{p(x)}{p(-1)} \frac{\sqrt{x}}{2\pi(x+1)},
\]

where \( p(x) \) and \( p(-1) \) are the pion c.m. momenta for the \( \pi^+pn \) and \( \pi^+d \) channels, respectively. The normalization factor \( N \), which must be unity at the deuteron pole when \( x = -1 \), should differ little from this above threshold provided that the pion production operator is of short range \[3\]. However, a value of \( N \) close to two was required in order to fit the 951 MeV results \[1\].

The effects of the tensor force have been studied for pion production in \( pp \rightarrow \pi^+d \), where dramatic changes were found when the \( S \equiv D \) coupling was introduced \[4, 5\]. Since, on kinematic grounds, the \( D \)-state amplitudes might be expected to change sign between the deuteron bound state and the \( pn \) continuum, it was suggested \[1\] that the discrepancy in the value of \( N \) might be explained as being due to \( S-D \) interference that was not included in the
FSI theorem [3]. However, no detailed theoretical estimates had been made of the influence of the tensor force for the $\pi^+\{pn\}$ continuum final state and it is not clear whether it is this or the finite range of the production operator that is the crucial feature.

The 951 MeV data were taken above the peak associated with the production of the $\Delta$ resonance. Since the $S$–$D$ interference is predicted to change sign around the resonance [1, 4, 5], we have repeated the experiment at one energy below the $\Delta$ peak ($T_p \approx 400$ MeV) and at another close to it ($T_p \approx 600$ MeV).

The present experiment is a straightforward extension of that previously reported [1] and so only the salient points of the set-up will be described here. Since we are interested principally in the ratio of the cross sections for the $pp \rightarrow \pi^+d/\pi^+\{pn\}$ reactions as a function of the excitation energy $\varepsilon$ of the $pn$ system, it is crucial to measure this energy with high resolution for well identified pions. In order to optimize the momentum resolution of the system, the proton beam of the COSY synchrotron was electron cooled at injection, accelerated to the requested momentum, and then stochastically extracted. The resulting beam was focussed onto the center of a target cell that was only 2 mm thick, with windows made of 1 $\mu$m Mylar. The beam spot was less than 0.5 mm in diameter, with divergences of 1.1 and 1.3 mrad in directions perpendicular to the beam. These characteristics were much better than those achieved without beam cooling and, in particular, the background was considerably reduced.

Positive pions were detected near the forward direction using the Big Karl magnetic spectrograph [7]. Their positions and track directions in the focal plane were measured with two sets of multiwire drift chambers, each composed of six layers. The chambers were followed by scintillator hodoscopes that were used to measure the time of flight over a distance of 3.5 m, which led to the unambiguous $\pi^+$ identification. The set-up allowed the excitation energy $\varepsilon$ in the $pn$ system to be determined with high precision.

Count rates were converted to cross sections by normalizing the integrated deuteron peaks to the zero-degree $pp \rightarrow \pi^+d$ values, for which there are extensive data [8]. The results of these and our previous measurement [1] are shown in Fig. 1 as a function of $\varepsilon$. Although corrections for acceptance etc., are included, these are slowly varying for $\varepsilon < 20$ MeV. It is clear that there is an excellent separation between the $\pi^+\{pn\}$ and $\pi^+d$ channels, which leads to robust determinations of $R_{pn/d}$. There is also no sign of any $\{pn\}$ spin-singlet production, which would show up as a narrow peak for $\varepsilon \approx 1$ MeV. An analysis of the
isospin-related $pp \rightarrow \{pp\}, \pi^0$ reaction \[9\] shows that the cross section should be less than 0.1 $\mu$b/sr MeV.

FIG. 1: (Color online) Forward differential cross sections for the $pp \rightarrow \pi^+ X$ reaction for three different beam energies as functions of the $pn$ excitation energy $\varepsilon$. The left axis is for $X = d$ while the right are for the $X = pn$ continuum. The measurements are shown as histograms and the colors indicate different settings of the magnetic spectograph. The curves are the results of the Fältd-Wilkin model of Eq. (1), with the normalization factors $N$ being given in Table I. Also shown in Fig. 1 are the $pp \rightarrow \pi^+ \{pn\}$ predictions of the FSI theorem \[3\], with the normalization factors $N$ required to bring the calculations and data into agreement for $\varepsilon \approx 10$ MeV being given in Table I. The values of $N$ depend somewhat on the interval considered as well as on the background assumptions. Similar to Ref. \[2\] we subtracted a constant background extending from the deuteron peak to the continuum, which is 1.5, 4.8, and 0.66 $\mu$b/(sr MeV) and were included in the curves shown at 401, 601 and 951 MeV, respectively.

Our measurements can be supplemented by the results obtained at TRIUMF at 420 and 500 MeV \[2\]. Although the resolution in $\varepsilon$ was somewhat poorer than that achieved with Big Karl, it was sufficient to separate the $\pi^+ d$ and $\pi^+ pn$ final states. The data, which typically cover a range from 24$^\circ$ to 100$^\circ$ in laboratory angle, were transformed into the c.m. system
TABLE I: Normalization factor $N$ of Eq. (2) for various beam energies as determined by fitting data at $\varepsilon \approx 10$ MeV. The results of Ref. [2] required an extrapolation in angle to compare with the GEM results in the forward direction.

| $T_p$ (MeV) | Normalization factor $N$ | Reference |
|------------|--------------------------|-----------|
| 401        | 0.51 ± 0.06              | This work |
| 420        | 0.94 ± 0.07              | [2]       |
| 500        | 1.11 ± 0.07              | [2]       |
| 601        | 1.06 ± 0.04              | This work |
| 951        | 2.2 ± 0.1                | [1]       |

to allow values of $R_{pn/d}$ to be extracted, assuming that the two cross sections were to be evaluated for the same beam energy and production angle $\theta_x$. The normalization factors are also given in Table I, where some of the error arises from that in the angular extrapolation to the forward direction.

In order to study the results in more detail, the previous two-body calculations [1] have been extended to the full three-body final state. In the presence of the tensor force, this involves the nontrivial task of evaluating slowly converging overlap integrals of $S$ and $D$ final scattering wave functions with the initial $pp$ states. As in Ref. [10], these were performed by rotating the contour integration in the region outside the range of the $pn$ potential into the positive or negative imaginary directions, as required to ensure convergence. The numerical evaluations were carried out with the Reid soft core potential [11], which is quite adequate at low energies.

In addition to the direct production from distorted $NN$ states, $s$-wave pion rescattering was also taken into account. If half of the available energy in the $NN$ case is ascribed to the intermediate pion then this can place it on the energy shell for beam energies above the two-pion threshold [4, 12]. This therefore also leads to a long range mechanism and an integral which has to be handled in the same way as that for the direct production amplitude. Rescattering in the $p$ wave through the $\Delta(1232)$ isobar is accounted for by the $\Delta N$ admixtures generated in the initial states by the coupled channels method (with associated changes in the $NN$ force). As discussed for the case of a decaying $\Delta$ in Ref. [4],
all the final pion energy is assumed to be already in the intermediate pion, putting it on the energy shell. However, in this case the $\Delta N$ wave function is of finite range and this raises no convergence issues.

FIG. 2: (Color online) Same as Fig. 1 but for two- and three-body final state calculations. Solid curves include $S$-$D$ interference, which is omitted for the dashed curves.

In order to investigate in detail the influence of the $D$-state component, the full tensor coupling with mixed $S + D$ final state was first studied, with the results of the model calculations being compared with the data in Fig. 2. The tensor force was then switched off and the intermediate range $pn$ attraction multiplied by a factor of 1.3 to reproduce the correct deuteron binding energy. The results are plotted with a resolution determined by a Gaussian fit to the deuteron peak and a constant background included, as for Fig. 1. The calculations with the tensor force account well for the data in the continuum, though they overestimate the deuteron contribution at the highest beam energy. The calculations without $S$-$D$ interference also lie well above the data here, whereas the agreement with the data is very good for the two lower energies. The two types of calculations almost agree with one another at 400 and 600 MeV.

We now compare the results of the dynamical model with the Fäldt-Wilkin FSI theorem of Eq. (2) by evaluating the ratio $R_{pn/d}$ and hence the normalization factor $N$. The values
with the tensor force included are shown for a range of beam energies in Fig. 3. Below 650 MeV, the ratio is less than unity while for higher energies it is above, and this feature remains when the tensor force is omitted. It is remarkable that, within the dynamical model used here, the FSI theorem is also satisfied to a high accuracy for mutually consistent wave functions even when the tensor force is switched on. It is important to note that the beam energy dependence of the calculations reflects qualitatively the behavior of the data, which are also shown in Fig. 3 for $\varepsilon \approx 10\text{ MeV}$. Since this is very similar whether the tensor force is included or not, it suggests that the coupling between the $S$ and $D$ states is not primarily responsible for the energy dependence of $N$ and that this probably arises from the long range components in the production operator [4, 5].

In summary we have measured the forward cross section for the $pp \rightarrow \pi^+ X$ reaction at two energies, one corresponding to the center of the $\Delta$ resonance and one below. The high resolution achieved allowed a clear separation of the two channels, $X = d$ and $X = pn$. Comparison of the production of the two channels using the Fälldt-Wilkin theorem [3] shows the necessity for the introduction of normalization factors $N$. These factors together, with one derived earlier [1] and two obtained by extrapolating TRIUMF data [2] to zero degrees, follow a smooth dependence as function of the beam energy. The comparisons were made for $\varepsilon \approx 10\text{ MeV}$ so that some deviations might arise from $P$-wave $pn$ pairs. However, these contribute incoherently and would not change the picture qualitatively.

To check the suggestion that the energy dependence of $N$ stems from an interference

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FIG. 3: (Color online) Normalization factor $N$ of Eq. (2), evaluated in the present pion production model with the tensor force included, for the indicated energies $\varepsilon$ in the $pn$ system. Also shown are the values extracted from the experimental data at $\varepsilon \approx 10\text{ MeV}$, as presented in Table I.
between the $S$ and $D$ state in the continuum, model calculations were performed for both $\pi^+d$ and $\pi^+pn$ final states with and without the tensor force. For the lower bombarding energies both types of calculation reproduce the data in both channels. For the highest energy only that with tensor force accounts for the continuum part of the data, though the deuteron pole is still overestimated. The theoretical results of Fig. 3 give a semi-quantitative understanding of the energy dependence of the FSI normalization factor $N$ shown in Table 1. With or without the tensor force, the curves cross unity around $T_p \approx 650$ MeV, which corresponds to the maximum of the $pp \to \pi^+d$ excitation function and hence to the center of $\Delta(1232)$ production. The tensor force seems therefore not too be the dominant effect in the energy dependence of $N$.

Although the Fäldt-Wilkin theorem was originally proven only for $S$-waves, it was unexpected that the dynamical calculations with the tensor force also satisfy the theorem when extrapolated to the deuteron pole. However, even if the theorem is valid at the pole itself, when it is used as an approximation to the $pn/d$ ratio in the continuum it is seen that this can lead to errors, especially above the $\Delta$ resonance. These deviations, shown as a function of $\varepsilon$ in Fig. 3, may arise from the long range part of the pion production operator associated with the on-shell intermediate pions. This is supported by the fact that $N$ changes from below to above one at an energy which corresponds to the $\Delta$ excitation but also to the two pion threshold.

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