THE ROLE OF MATHEMATICS IN THE PALESTINIAN ECONOMY: ESTIMATING THE SPEED OF THE SWALLOWING OF PALESTINIAN LANDS BY THE ISRAELI SETTLEMENTS

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Abstract

The purpose of this paper is to demonstrate the importance of mathematics in the Palestinian economy by deriving a dynamic model for the speed—the rate of change—of the expansion for the Israeli settlements and thus the “swallowing” rate of Palestinian lands. Partial Differential Equations were used to derive a theoretical model for the “swallowing” rate of Palestinian lands by Israeli settlements, which in turn result in tremendous negative effects on both the agricultural and industrial sector of the Palestinian economy. The developed dynamic model explains the process of which the Israeli settlements expands and the variables affecting the “swallowing” rate. The data was plotted and a mathematical equation was fitted for the growth rate of the settlements. This will give the policymakers an approximation of how fast the settlements are expanding and may aid them in their strategic planning of Palestinian development.

Keywords: Swallowing of Palestinian lands, Israeli Settlements, Gompertz Model, Palestinian Economy, Partial Differential Equation
1. Introduction

Since one of the most threatening challenges facing the existence of the Palestinian people is the expansion of the Israeli settlements, this is due to the continuous increase in the number of Israeli settlements living in the West Bank. The purpose of this paper is to develop a dynamic model that estimates the speed – the rate of change – of the expansion of Israeli settlements. This paper demonstrates the importance of mathematics in the Palestinian economy through the derivation of a dynamic model that is causing the “swallowing” of Palestinian lands. This is a crucial issue as the spread of the Israeli settlements are an indication to the end of the Palestinian dream in having an independent Palestinian state. This paper will aid in creating a theoretical framework in understanding the process by which the settlements are expanding. More mathematical literature should be geared towards this issue.

The Israeli settlements are a major part of the Israeli occupation, where the restrictions that are imposed by the Israeli authorities – in order to secure the settlements – are used as a tool to have political, geographical, and economical control over the Palestinian resources. By doing so, the Israeli occupation is transformed from a military occupation to colonization and thus removing the Palestinians from their homes. From the first day, the occupation had the intention of controlling the natural resources of the nation. This is achieved by spreading its settlements in East Jerusalem, the Jordan Valley, and north of the Dead Sea as part of a plan in order to control the water resources, mining resources, and tourist sites. The Israeli strategy had been to increase the number of settlements and expand those existing. The Israeli authorities had also worked on creating a network of roads that connect settlements together and with the other side of the Green Line. In addition, the Israeli implemented a strategy of building industrial sites in the West Bank that serve the settlements, encouraging tourism in the settlements, and building institutions in the settlements that serve both the settlers and Israeli citizens outside of the West Bank. All this has been done on the expense of Palestinian lands.

The various economic indicators demonstrate the burden that the settlements have on the Israeli economy. In addition to all that, the settlements contribute to the change of demographics for the advantage of the Israelis in the West Bank (Atrash, 2014). The settlements affect negatively the following Palestinian economic sectors:

1. The agricultural sector is affected directly from the existence of the settlements, where the amount of land that can be used by Palestinians for agriculture is continuously decreasing. In addition, the total control of the water resources by the Israelis in the Palestinian territories due to the existence of the settlements affects the agricultural sector negatively.

2. The manufacturing sector is limited by the expansion of the settlements, where the settlements are occupying lands that can be used to build industrial zones. In addition,
settlements are built in locations that are strategic for manufacturing sites. For example, settlements are built along the Green Line. If the manufacturing site is built there, it will be closer to both Israeli and Palestinian markets. It will also be closer to seaports.

3. The increase in the number of settlers in the West Bank is colonizing more land that is affecting the expansion of the Palestinian cities. Thus, strategic planners have to constantly re-organize the patterns of expansion of Palestinian cities. In addition, it is driving the price of land in the Palestinian territories to rise. Thus making it more expensive for Palestinians to own or rent property.

(Atrash, 2014) Table 1 shows the number of settlers living in the Israeli settlements in the West Bank including Jerusalem by region.

Table 1: Number of Settlers in the West Bank Settlements by Year and Region, 1986-2016

| Year | West Bank (excluding Jerusalem) | Jerusalem | West Bank |
|------|--------------------------------|-----------|-----------|
| 1986 | 60766                          | 117550    | 190953    |
| 1987 | 67483                          | 123061    | 202885    |
| 1988 | 73403                          | 132460    | 221348    |
| 1989 | 79842                          | 137331    | 238060    |
| 1990 | 88888                          | 140872    | 252545    |
| 1991 | 100729                         | 146436    | 268756    |
| 1992 | 111673                         | 152219    | 285791    |
| 1993 | 122320                         | 156724    | 300159    |
| 1994 | 133572                         | 159684    | 316658    |
| 1995 | 140235                         | 158929    | 326053    |
| 1996 | 153974                         | 162842    | 341929    |
| 1997 | 167124                         | 170400    | 361150    |
| 1998 | 179087                         | 173986    | 379099    |
| 1999 | 190750                         | 175987    | 391049    |
| 2000 | 205113                         | 178437    | 405149    |
| 2001 | 215062                         | 181425    | 421738    |
| 2002 | 226712                         | 184944    | 437681    |
| 2003 | 240313                         | 187573    | 452622    |
| 2004 | 252737                         | 190534    | 470013    |
| 2005 | 265049                         | 193485    | 487618    |
| 2006 | 279479                         | 197071    | 496032    |
| 2007 | 294133                         | 196803    | 510904    |
| 2008 | 298661                         | 200545    | 529319    |
| 2009 | 314101                         | 205088    | 548438    |
| 2010 | 328774                         | 205746    | 565317    |
| 2011 | 343350                         | 209912    | 583907    |
| 2012 | 359571                         | 214362    | 602311    |
| 2013 | 373995                         | 218297    | 619285    |
| 2014 | 387949                         | 222325    | 636452    |
The graph and table above show a continuous increase in the number of Israeli settlers over time. In 1988, the total number of settlers was 190,953 with 117,550 settled in Jerusalem and 73,403 settled in the rest of the West Bank. In 1994, the Palestinian Authority took control of Jericho; the total number of settlers was 285,791 with 152,219 in Jerusalem and 133,572 in the rest of the West Bank. Thus, the number of settlers increased by approximately 29 percent in Jerusalem and approximately 82 percent in the rest of the West Bank since 1988. Meanwhile, in 2016 the total number of settlers reached 636,452 with 222,325 in Jerusalem and 414,127 in the rest of the West Bank. Thus, the number of settlers increased by approximately 43 percent in Jerusalem and approximately 210 percent in the rest of the West Bank since the year 1994. Clearly, time is not a constraint to the Israeli government; on the contrary, it is to their benefit. This shows the true intentions of the Israeli government towards giving up the West Bank despite of the signing of the Oslo Agreement with the Palestinians. The above data asserts that the Israeli government invested sufficiently in time to change the demographics on the ground to their advantage in the West Bank and create a new reality. Therefore, the settlements are a major obstacle to the two state solutions.

The monotonically increasing function shown in the graph clearly reflects the continuous increase in the number of Israeli settlers. Nonetheless, in this paper we are interested in the time and space dimension. The above table does not show the amount of land occupied by the Israeli settlers. However, the continuous increase in the number of Israeli settlers in the West Bank causes an increase in the demand for housing and associated services –such as schools, medical centers, shopping centers, universities, etc. - in the settlements. This
continuous increase results in an upward pressure on the price of land in the settlements and adds extra pressures to expand the borders of the settlement. This will have further implications in our analysis in creating the theoretical framework for estimating the amount of “swallowing” of Palestinian lands.

Let us take a closer look at some spatial facts concerning the settlements. The total area of the West Bank is 5664.5 km² including East Jerusalem, where the settlements occupy 2.7% of the total area of the West Bank. In addition, the settlements have an “area of jurisdiction” that is larger compared to the settlement built up area within the fence. The area of jurisdiction of settlements exceeds 9.3% of the area of the West Bank. Moreover, the settlements are connected with a well-developed road network that takes 2.3% of area to connect settlements together and connect them to their center of life in Israeli (Samara, 2019).

The table in Annex I shows the number of settlers living in each of the settlements, the initial area occupied by the settlement when it was first established, and the area of each settlement in 2016.

Table 2 shows only the settlements not including the illegal outposts. The table indicates that the largest settlement in terms of number of settlers is Modi’ih Ilit with 66,847 and the smallest is Niran with 91. Nonetheless, these two settlements are not the largest and smallest in terms of area. The largest settlement in terms of area is Bet Arye with 7814 m² and the smallest is Rotem with 55 m². From looking at the table above, we also notice that most of the settlements were established in the eighties.

2. Literature Review

In the 1970s, a new generation of mathematical urban theories manifested. These theories formed the new urban economics. This approach was successful in explaining certain economic and geographical phenomena in urban systems. In addition, it was also able to explain the behavior of households and firms in urban systems. Nonetheless, the problem was that most of the models in this approach required perfect competition. So, an assumption was made that both households and firms had perfect information about housing and land markets (Mills & Mackinnon, 1973).

In reality, urban development processes are dominated by imperfect information and disequilibrium. Thus, relaxing the assumption of perfect competition leads us to urban development models of geographical diffusion. These models describe dynamic urban development pattern formations by PDEs in which temporal and spatial economic development processes can be dealt with simultaneously. These models are aimed at trying to explain how urban patterns might be far away from the equilibrium determined by the new urban economics (Zhang, 1988).

Zhang (1988) developed a dynamic model for urbanization in the regional system containing the Central Business District, the urban, and the rural areas. The paper had described “eating up” of rural areas by the expanding the organization. Partial differential equations were used to represent...
the spatial and temporal land prices of the urban and rural areas. It’s also asserted the movement of the boundary of the urban areas. Three differential equations were used to demonstrate the movement of the boundary, and the deviation from the equilibrium due to the inflationary forces. It was concluded by construction of analytic solution for the fixed rural land price.

Kaashoek and Paelinck (1994) potentialized the spatial variables to generalize the partial differential equation in order to be used in spatial economics. This process was carried out for one dimensional wave equation. Linear, exponential, and tanner potentials were investigated. The result of the analysis had led to the utilization of the exponential equation in producing meaningful analytical solutions for spatial economic patterns. This paper has also discussed the problems of stability and strangeness of the dynamical solutions.

Kaashoek and Paelinck (2001) used potentialized wave and diffusion equations to emphasis the relationship between the resulting process and potentializing parameters. Empirical relevance to spatial economics was mentioned and discussed.

Redding and Rossi –Hansberg (2017) described what was achieved as a result of the utilization of quantitative models in spatial economic in this decade.

Marquez, Lasarte, and Lufin (2019) proposed a measurement for the procedure for the role of geographical position in economic inequality. In addition, they aimed to determine an approach for the decomposition of global inequality into both within and between countries components in order to assess which of these components is a result of neighborhood factors. This was applied to European countries. Based on the inequality analysis certain policies were recommended.

Vallone (2019) used geo-computation in spatial economic analysis. He constructed and applied a new set of algorithms and functions in the R programming language to deal with spatial economic data.

Portilla, Maza, and Villaverde (2019) examined the effects of inward FDI on economic growth –focusing on the headquarters effect- across the Spanish regions over the time period of 1996 to 2013. The paper had concluded that FDI does not always register where it is effectively made, but instead in the region in which the firm’s headquarters is located. This is achieved by estimating a panel spatial spillovers results: FDI does foster economic growth, only when the headquarters effect is properly addressing the question do spatial spillovers arise.

The above literature had asserted how spatial economics had developed over time. Now let us consider how mathematics was used in the Palestinian economy.

Paul de Boer and Marco Messaglia (2006) utilized mathematics and statistics to estimate the income elasticities and their use in a CGE model for Palestine. They used the Linear Expenditure Systems (LES) to estimate the consumption block for the computable general equilibrium model (CGE). To represent reality – where non-straight Engel curves, inferior commodities, elastic demand, and gross substitution exist – the use of the Indirect Addilog System (IAS) was
suggested. Thus, the income elasticities of the IAS were estimated from the 1998 Palestinian Expenditure and Consumption Survey (PECS). Calculating the income elasticities using the IAS model helped in trying to explain the consumption behavior.

Samarah (2016) studied the relationship between Gross Disposable Income (GDI) and consumption, and the consumption function for Palestine was estimated. Three econometric models were constructed to estimate the consumption function and calculus was used to derive the Marginal Propensity to Consume (MPC) in both the short and long run.

The key determinants of poverty status a household since the implementation of the economic reform program in Palestine were also identified using mathematical and statistical techniques. Through the utilization of the Logistic Regression Model it was found that the chance of a household to fall into poverty increases due to the unemployed adults, the large number of children below 18 years old, and the large dependency ratio (Elnamourty & Safi, 2012).

Mathematics was also able to analyze the impact of the size of domestic working labor force, Real Gross Domestic Capital Formation, real domestic exports and imports of goods and services, as well as political instability on the Real Gross Domestic Product (RGDP) in Palestine. The Cobb-Douglas function was used to formulize the functional relationship between the explanatory variables and the RGDP. The study had indicated the urgent need for increasing the level of investment into the Palestinian economy (Abu-Eideh, 2014).

Samarah (2018) had proved that there is a functional relationship between governance and economic growth in Palestine using real analysis. Thus, economic growth is a function of governance.

This paper will use partial differential equations to derive a dynamic model to better understand the rate of change in which the Israeli settlements are expanding in the West Bank. Thus, this paper will add a more regress and theoretical approach to the already limited existing literature.

3. Spatial Economics

In recent years, Partial Differential Equations (PDEs) had played an important role in economics. An example is the large literature on the design of optimal dynamic contracts and policies (Farhi & Werning, 2013). Labor economics, was another area in which PDEs were used in modeling the labor markets (Alvarez & Shimer, 2011) and reviewed by (Lentz & Mortensen, 2010).

A system of nonlinear PDEs was also used to model heterogeneous agents sharing a common mathematical structure over time. Thus, PDEs were able to present a continuous time formulation of the simplest model to study the effect of various policies and institutions on the inequality of income and wealth distributions (Achdou, Lasry, Lions, & Moll, 2014).

To understand the growth rate of the Israeli settlements, we will discuss how spatial economics based on a single space dimension x and time t can model reality. PDEs had long represented space-time dimensions. Thus, we will use PDEs to model the expansion
patterns of the Israeli settlements. Let us look at some examples of how PDEs can be used in spatial economics to describe the pattern of evolution of cities over space-time dimension.

Considering only one space variable x and time t, a PDE for some function $f(x,t)$ is a relation of the form:

$$g(x, t; f; f_x, f_t; f_{xx}, f_{xt}, f_{tt}; ...) = 0$$

In the above equation $g$ – in general- is a given function of both the independent variables x and t; of the unknown function $f$; and a finite number of its partial derivatives.

Nonetheless, in order to apply PDE to human spatial behavior we need two fundamental adoptions. The first is that the interaction at a distance should be taken into consideration explicitly. Thus, Potentialized Partial Differential Equations (PPDE) is considered. Thus, applying to the classical PDE the idea of “potential function”. The second adoption is the interpretations of PPDE in the presence of a bifurcation parameters relating to the openness of the spaces studied (Kaashoek & Paelinck, 2001, p. 464). A parameter is a numerical or other measurable factor forming on of a set that defines a system or sets the conditions of its operations. A Bifurcation parameter is most commonly used in the mathematical study of dynamic systems. A bifurcation occurs when a small smooth change made to the parameter values (the bifurcation parameters) of a system causes a sudden quantitative or topological change in its behavior.

Let us now consider the classical wave equation

$$\ddot{f}(x,t) = \alpha^2 f''(x, t)$$

This equation is an expression of local interaction. However, in spatial economics locality is rather the exception. Thus, in order to express the spatial interaction the wave equation should be generalized as follows:

$$\ddot{f}(x,t) = \alpha^2 \int_{-l}^{+l} w(x, \xi) f''(\xi, t) d\xi \quad (1)$$

Where $w(x, \xi)$ is called a “spatial discount function”, it represents the potential of a convolution with some variable over the line [-1, +1]. Thus, equation 1 is potentialized equation. Using the separation variable technique, we can rewrite the above equation as:

$$u(x)v(t) = \alpha^2 v(t) \int_{-l}^{+l} w(x, \xi) u''(\xi) d\xi$$

(Kaashoek & Paelinck, 1994, p. 585). Implying that

$$f(x, t) \triangleq u(x)v(t)$$

Let us now take a linear spatial discount function over the closed interval [-1, 1],

$$w(x, \xi) = 1 + \frac{1}{2}(\xi - x), \quad \xi < x.$$  

$$= 1 + \frac{1}{2}(x - \xi), \quad \xi \geq x.$$  

The potential function is

$$p(x) = \int_{-1}^{x} u''(\xi)[1 + \frac{1}{2}(\xi - x)] d\xi$$

$$= \int_{x}^{+1} u''(\xi)[1 + \frac{1}{2}(x - \xi)] d\xi$$
Integrating the potential with a factor of 1 will give us the following

\[ p_1(x) = [u'(1) - u'(-1)] \]

Integrating with a factor of x gives

\[ p_2(x) = -xu'(x) + \frac{1}{2}[u'(-1) - u'(1)]x \]

Finally, in

\[ \frac{1}{2} \left( \int_{-1}^{x} u''(\xi) \xi d\xi - \int_{x}^{+1} u''(\xi) \xi d\xi \right) \]

The indefinite integral is

\[ u'(\xi)\xi - u(\xi) \]

By integration by parts and collecting the terms we get

\[ p(x) = c + ax - u(x) \]

With

\[ c = \frac{1}{2} [u'(1) - u'(-1) + u(1) + u(-1)] \]

\[ a = \frac{1}{2} [u'(1) - u'(-1)] \]

The linear potential implies a straight line passing through the origin and rotating with time (Kaashoek & Paelinck, 1994, p. 587).

Let us now take an exponential spatial discount function

\[ w(x, \xi) = \frac{1}{\gamma} \exp\left(-\frac{|x - \xi|}{\gamma}\right) \]

In the above equation, \( \gamma \) is a scaling variable and \( \gamma > 0 \). Then the potential function is

\[ p(x) = \frac{1}{\gamma} \left\{ \int_{-1}^{x} u''(\xi) \exp\left[ \frac{x - \xi}{\gamma} \right] d\xi + \int_{x}^{+1} u''(\xi) \exp\left[ \frac{x - \xi}{\gamma} \right] d\xi \right\} \]

Refer to Kaashoek and Paelinck (1994) for the solution. “The resulting behavior is not unlike representing the occupational density of a city and its evolution over time” (Kaashoek & Paelinck, 1994, p. 588).

4. The Theoretical Framework for the Expansion of Settlements

In this section, we will use spatial economics to describe the speed of the growth of the Israeli settlements inside the West Bank and Jerusalem over time. We will develop a theoretical model that will describe the process of the border movement of the settlements. Our theoretical model will identify the factors that affect the movement of the borders. Thus, the theoretical model will explain the process of how the settlements expand and provide the theoretical framework for the evolution of the Israeli settlements.

A vital aspect of spatial economics in general is that in principle all relevant variables are spatially interrelated. The choice of space-and-time specifications is manifested naturally in a theoretical analysis. PDE contains these specifications, they allow the demonstrations of movement simultaneously over space –whatever is the dimension- and time. When solving such equations, the ideal solution is a function that contains the values of the considered variables in the analysis for every point in space and time.
We will start our analysis by developing a dynamic model for a representative Israeli settlement. We will then generalize the model to include all the settlements.

The total land for the region is constant—the total area of the West Bank including Jerusalem. The center of the system is in the center of the Israeli settlement and the system is symmetric. The Israeli settlement is located in the center and the Palestinian areas are surrounding the settlement. The Israeli settlement is causing the “swallowing of the Palestinian lands”. The dynamics of the system are spatial and temporal movements of the following variables:

- \( I(x,t) \) is the land prices in the Israeli settlement’ areas at \((x,t)\)
- \( P(x,t) \) is the land prices at the Palestinian areas at \((x,t)\)
- \( B(t) \) is the distance from the center of the Israeli settlement to the boundaries that separate the Palestinians from the Israeli areas.

Where \( P, B, \) and \( I \) belong to the set of real number.\(^1\)

The equilibrium land prices for both Israel and Palestine are determined using the supply and demand for land in the Israeli and Palestinian markets respectively over time and space dimensions. In other words, equilibrium land prices are determined by the forces of supply and demand for the two distinct markets. However, the actual land prices are not only dependent on the classical economic theory of supply and demand. Actual prices are also dependent on political factors such as the majority in the Knesset, the tradeoff between political turmoil and regional stability adopted by policy makers in the region, and political pressures from the outside. Thus, it is important for us to consider the difference between the actual land prices and the equilibrium land prices for both Israeli and Palestine.

Here \( I \) and \( P \) represent the deviations of the actual land prices from the long-run equilibrium land prices. Let us express the long-run equilibrium determined on the basis of the new urban economics as \((i^*(x), p^*(x))\) which is independent of time, and the actual regional pattern by \((i(x,t), p(x,t))\), then we will have the following

\[
I(x,t) = i(x,t) - i^*(x)
\]
\[
P(x,t) = p(x,t) - p^*(x)
\]

Thus, when we talk about land prices, we mean the deviations of the actual land prices from the equilibrium.

Assume that a positive excess supply is associated by positive prices and vice versa. In addition, there are other exogenous forces that will cause the deviation in the Palestinian prices of land from their equilibrium. In this study we are interested in the speed of the movement of the boundaries. Since we assume that the Israeli settlements should not exist so over the long-run the Israeli settlements would not exist and the entire West Bank including Jerusalem should be free from settlements. However, as mentioned before we are only interested and focused on the situation at the boundaries.
We assume that the speed of the movement of the boundary $-dB(t)/dt$ is negatively proportional to the gradient of the Palestinian land prices at the boundary. That is when the Palestinian land prices decrease the Israeli settlement boundaries will be tempted to expand at a fast rate. Meanwhile, if the prices are high the boundary will move at a slow rate. Usually, in the Palestinian areas the high prices of the Palestinian lands are associated with a high density of the Palestinians and as a result the boundaries of the settlement will not be tempted to move forward. In other words, the boundary will move at a very slow pace. The movement of the Israeli settlement boundary is positively proportional to the Israeli land prices in the settlement. A high land price at the settlement is usually associated with a high demand to live in that settlement and as a result, the boundary is pressured to expand.

Consider the location $B(t)-\delta x$ in the Israeli settlements where $\delta x$ is positive and sufficiently small. The gradient of the Israeli land prices near the boundary is represented as follows

$$\{I(B(t)-0,t)-I(B(t)-\delta x,t)/\delta x$$

The above equation represents the Israeli settlement price per mile between the boundary and its neighborhood. The value is usually negative, so we consider the absolute value. The larger the absolute value the more easily the boundary is moved.

Let us now consider the Palestinian land prices near the boundary represented by $B(t)+\delta y$, where $\delta y$ is positive and sufficiently small. The gradient of the Palestinian land prices near the boundary is

$$\{P(B(t)+\delta y,t)-P(B(t)+0,t)/\delta y$$

The speed of the movement of the boundary is given by

$$\frac{dB(t)}{dt} = h[I(B(t)-\delta x,t)/\delta x-kP{B(t)+\delta y,t)-P(B(t)+0,t)/\delta y$$

In the above equation, $h$ and $k$ are non-negative parameters. We will now allow for $\delta x$ and $\delta y$ to go to 0, then we can rewrite the above equation as:

$$\frac{dB(t)}{dt} = \{h \frac{\partial I}{\partial x} - k \frac{\partial P}{\partial x}\} x=B(t)$$

This equation describes the movement of the boundary. This is known as the Stefan condition (Zhang, 1988, p. 335). The above equation represents the expansion of the representative Israeli settlement and thus we obtain the “swallowing” rate of the Palestinian lands by the representative Israeli settlement. Looking at the above equation, it is clear that the difference in land prices in the Israeli settlements and the Palestinian territories affects the expansion rate of the border. Given that the land prices in the Israeli settlement are higher than the land prices in the Palestinian territories, the greater is the difference the faster is the expansion rate of the border. Thus, the land prices along the two sides of Israeli settlement border are crucial to the rate of expansion.

Since the difference in prices is not the only factor that causes the expansion of the settlement, we will add the variable $u(x,t)$ to represent the additional rate of expansion due to the political decision of the Israeli government that is not captured by land prices. The Israeli government’s strategy is
geopolitical, where its sole propose is to create a major in balanced favoring the Israeli population in the West Bank. Hence, we will have the following equation

\[
\frac{dB(t)}{dt} = \{h \frac{\partial l}{\partial x} - k \frac{\partial p}{\partial x}\} + u(x, t) \quad x=B(t),
\]

where \( u \) belong to the set of real numbers.

Here \( u \) is an exogenous variable, i.e. it is determined by factors outside the model. Nonetheless, there is not only one settlement in the occupied Palestinian territories. Therefore, we need to expand our model to include the \( n \) number of Israeli settlements and thus estimate the aggregate expansion rate for all the Israeli settlements as given below:

\[
\sum_{i=1}^{n} \frac{dB_i(t)}{dt} \quad \text{Where } n \text{ is a positive integer greater than } 0
\]

The above equation describes the rate of expansion of the borders for all the Israeli settlements in the West Bank including Jerusalem. Clearly, the equation will result in a finite number and the rate of expansion is limited to the actual total area of the West Bank that is under the control of the Israeli government. Only in these areas can the settlements grow. The areas controlled by the Palestinian government –known as area A- are protected against the expansion of the Israeli settlements.

Notably, the rate of expansion for each settlement is different depending on the difference between the Israeli and Palestinian land prices along the borders of the settlement. Unlike normal economic transactions, the leakage of Palestinian land to the Israeli government’s ownership is a shady deal. The Palestinians are usually subjected to identity theft Israeli institutions and the transaction is carried out on their behalf. Nonetheless, if this does not work other ways are adapted by the Israeli government in order to carry out the transaction. Finally, if all fails the Israeli authorities uses force and confiscate the land. In all cases, Israelis are willing to compensate the Palestinians if they agree to sell land. The price they offer is usually dependent on the Palestinian price of land. The higher the Palestinian price the higher is the Israeli price and the harder it is for the Israelis to give an offer. At the end of the day, the Israelis best interest is to show that they bought the land rather than taking it by force.

5. Data and Methodology

The data for the initial area and the area in 2016 for each of the settlement was taken from the Wafa organization website. Meanwhile, the establishment date and number of settlers inhabiting the settlement was taken from the B’tselem website.

With the aim of trying to estimate a functional equation for the expansion rate of the Israeli settlements, we will calculate the cumulative sum for the initial areas of the settlements. The cumulative sums are then plotted against time \( t \). We will then fit a model to estimate the “swallowing” rate of the Palestinian lands.

6. Results

The Excel program was used to analyze the data. Where the results of the analysis are given below.
Table 3: Total Growth. Years Passed, Growth Rate and Cumulative Sum for each of the Israeli Settlements

| Data settlement (1) | Date of Establishment | Initial Area (m²) | Area in 2016 (m²) | Number of Settlers in 2016 | Total Growth | Year passed growth rate | Cumulative sum |
|---------------------|-----------------------|-------------------|-------------------|-----------------------------|--------------|-------------------------|----------------|
| Kfar Etzion         | 1967                  | 258               | 567               | 1099                        | 1.197674419  | 49                      | 1.61991889 258 |
| Mehola              | 1968                  | 241               | 288               | 517                         | 0.195020747  | 48                      | 0.37186376 499 |
| Qalya               | 1968                  | 271               | 771               | 386                         | 1.84501845   | 48                      | 2.2021674 770 |
| Argaman             | 1968                  | 109               | 353               | 131                         | 2.23853211   | 48                      | 2.4783807 879 |
| Rosh Tzurim         | 1969                  | 247               | 463               | 934                         | 0.874493927  | 47                      | 1.3458672 1126 |
| Alon Shvut          | 1970                  | 492               | 463               | 3180                        | -0.058943089 | 46                      | -0.1319817 1618 |
| Mevo Horon          | 1970                  | 603               | 524               | 2566                        | -0.131011609 | 46                      | -0.3048074 2221 |
| Ma’ale Efrayim      | 1970                  | 359               | 489               | 1209                        | 0.362116992  | 46                      | 0.67408812 2580 |
| Yitav               | 1970                  | 117               | 322               | 321                         | 1.752136752  | 46                      | 2.2252176 2697 |
| Gilgal              | 1970                  | 181               | 727               | 178                         | 3.016574586  | 46                      | 3.0681924 2878 |
| Massu’a             | 1970                  | 162               | 692               | 162                         | 3.271604938  | 46                      | 3.20684493 3040 |
| Mizpe Shalem        | 1971                  | 72                | 108               | 54                          | 0.5           | 45                      | 0.9051051 3112 |
| Hamra               | 1971                  | 123               | 261               | 124                         | 1.1219512    | 45                      | 1.68591165 3235 |
| Kiryat Arba         | 1972                  | 466               | 787               | 2727                        | 0.688841202  | 44                      | 1.19812666 3701 |
| Har Gilo            | 1972                  | 224               | 507               | 1570                        | 1.263392857  | 44                      | 1.87385157 3925 |
| Beka’ot             | 1972                  | 115               | 344               | 187                         | 1.991304348  | 44                      | 2.52151462 4040 |
| Gittit              | 1973                  |                  | 430               | #DIV/0!                     | 43           | #DIV/0!                 | #DIV/0!       |
| Mechora             | 1973                  | 103               | 171               | 142                         | 0.660194175  | 43                      | 1.18589422 4143 |
| Ma’ale Adumim       | 1975                  | 3291              | 7010              | 37670                       | 1.130051656  | 41                      | 1.86137059 7434 |
| Ofra                | 1975                  |                  | 3605              | #DIV/0!                     | 41           | #DIV/0!                 | #DIV/0!       |
| Elazar              | 1975                  | 265               | 349               | 2568                        | 0.316981132  | 41                      | 0.67382615 7699 |
| Petza’el            | 1975                  | 319               | 897               | 257                         | 1.81192226   | 41                      | 2.55368316 8018 |
| Netiv Hagedud       | 1976                  | 234               | 1425              | 190                         | 5.08974359   | 40                      | 4.62006251 8252 |
| Ro’i                | 1976                  | 106               | 164               | 165                         | 0.547169811  | 40                      | 1.09704219 8358 |
| Beit El             | 1977                  | 688               | 557               | 6115                        | -0.190406977 | 39                      | -0.540135 9046 |
| Kedumim             | 1977                  |                  | 4323              | #DIV/0!                     | 39           | #DIV/0!                 | #DIV/0!       |
| Elkanah             | 1977                  | 867               | 1757              | 3898                        | 1.026528258  | 39                      | 1.82758711 9913 |
| Tekoa               | 1977                  | 527               | 844               | 3633                        | 0.601518027  | 39                      | 1.21488965 10440 |
| Kochav Hashchar     | 1977                  | 301               | 1033              | 1985                        | 2.431893688  | 39                      | 3.2123429 10741 |
| Neve Halamish       | 1977                  | 297               | 2510              | 1328                        | 7.451178451  | 39                      | 5.62509426 11038 |
| Bet Horon           | 1977                  | 235               | 199               | 1240                        | -0.153191489 | 39                      | -0.4254531 11273 |
| Shvei Shomron       | 1977                  | 276               | 391               | 897                         | 0.416666667  | 39                      | 0.89709407 11549 |
| Location      | Year  | Population | Employment | Density | Crime Rate | Population Density |
|---------------|-------|------------|------------|---------|------------|--------------------|
| Sal’it        | 1977  | 443        | 691        | 818     | 0.559819413 | 39                 |
| Rimmonim      | 1977  | 625        | #DIV/0!    | #DIV/0! | #DIV/0!    | #DIV/0!            |
| Migdal Oz     | 1977  | 95         | 464        | 605     | 3.884210526 | 39                 |
| Rehan         | 1977  | 86         | 298        | 224     | 2.465116279 | 39                 |
| Almog         | 1977  | 107        | 397        | 239     | 2.710280374 | 39                 |
| Niran         | 1977  | 107        | 397        | 91      | 2.710280374 | 39                 |
| Ari’el        | 1978  | 2378       | 4729       | 19220   | 0.988645921 | 38                 |
| Karnei Shomron| 1978  | 341        | 631        | 7102    | 0.850439883 | 38                 |
| Mitzpe Yeriho | 1978  | 331        | 747        | 2319    | 1.256797583 | 38                 |
| Kfar Tapuah   | 1978  | 181        | 396        | 1071    | 1.187845304 | 38                 |
| Mevo Dotan    | 1978  | 96         | 768        | 386     | 7          | 38                 |
| Tomer         | 1978  | 218        | 366        | 262     | 0.678899083 | 38                 |
| Kfar Adumim   | 1979  | 412        | 693        | 4271    | 0.682038835 | 37                 |
| Shilo         | 1979  | 387        | 3247       | 3727    | 7.390180879 | 37                 |
| Elon Moreh    | 1979  | 381        | 1047       | 1861    | 1.748031496 | 37                 |
| Shademot Mehola| 1979 | 164       | 363        | 608     | 1.213416343 | 37                 |
| Efrat         | 1980  | 992        | 1792       | 8658    | 0.806451613 | 36                 |
| Givon Hahadasha| 1980 | 1135      | #DIV/0!    | #DIV/0! | #DIV/0!    | #DIV/0!            |
| Ma’ale Shomeron| 1980 | 1037      | #DIV/0!    | #DIV/0! | #DIV/0!    | #DIV/0!            |
| Vered Yeriho  | 1980  | 252        | #DIV/0!    | #DIV/0! | #DIV/0!    | #DIV/0!            |
| Hemdat        | 1980  | 75         | 202        | 230     | 1.693333333 | 36                 |
| Beit Ha’arava | 1980  | 75         | 163        | 183     | 1.173333333 | 36                 |
| Yafit         | 1980  | 173        | 297        | 139     | 0.716763006 | 36                 |
| Bet Arye      | 1981  | 837        | 7814       | 4842    | 8.335722828 | 35                 |
| Psagot        | 1981  | 1847       | #DIV/0!    | #DIV/0! | #DIV/0!    | #DIV/0!            |
| Yaqir         | 1981  | 3016       | 574        | 1901    | -0.809681698 | 35                 |
| Barkan        | 1981  | 411        | 563        | 1798    | 0.369829684 | 35                 |
| Nili          | 1981  | 321        | 1296       | 1552    | 3.037383178 | 35                 |
| Ma’ale Mikhmas| 1981  | 221        | 1507       | 1323    | 5.819004525 | 35                 |
| Hinnanit      | 1981  | 163        | 698        | 1164    | 3.282208589 | 35                 |
| Ateret        | 1981  | 160        | 3393       | 875     | 20.20625    | 35                 |
| Shaqed        | 1981  | 229        | 463        | 864     | 1.021834061 | 35                 |
| Einav         | 1981  | 183        | 466        | 749     | 1.546448087 | 35                 |
| Mattityahu    | 1981  | 149        | 755        | 772     | 4.067114094 | 35                 |
| Ma’on         | 1981  | 167        | 443        | 539     | 1.652694611 | 35                 |
| Carmel        | 1981  | 182        | 347        | 605     | 0.906593407 | 35                 |
| Ma’ale Amos   | 1981  | 89         | 310        | 390     | 2.483146067 | 35                 |
| Neve Daniel   | 1982  | 268        | 457        | 2278    | 0.705223881 | 34                 |
| Nokdim        | 1982  | 231        | 336        | 2052    | 0.454545455 | 34                 |
| Settlement      | Year | Population | Area (km²) | Population Density | GDP per Capita (US$) |
|-----------------|------|------------|------------|--------------------|----------------------|
| Alei Zahav      | 1982| 339        | 608        | 1643               | 0.793510324          |
| Almon           | 1982| 194        | 689        | 1329               | 2.551546392          |
| Eshkolot        | 1982| 92         | 139        | 515                | 0.510869565          |
| Pnei Hever      | 1982| 98         | 522        | 548                | 4.326530612          |
| Telem           | 1982| 95         | 451        | 362                | 3.747368421          |
| Hermesh         | 1982| 96         | 445        | 223                | 3.635416667          |
| Naama           | 1982| 119        | 280        | 116                | 1.352941176          |
| Giv’at Ze’ev    | 1983| 1257       | 2741       | 16865              | 1.180588703          |
| Sha’arei Tikva  | 1983| 5811       |            |                    | #DIV/0!              |
| Immanuel        | 1983| 301        | 740        | 3309               | 1.458471761          |
| Har Bracha      | 1983| 238        | 350        | 2339               | 0.470588235          |
| Yitzhar         | 1983| 200        | 1248       | 1468               | 5.24                 |
| Dolev           | 1983| 280        | 1186       | 1331               | 3.235714286          |
| Susiya          | 1983| 268        | 460        | 1115               | 0.71641791           |
| Otziel          | 1983| 301        | 740        | 976                | 1.458471761          |
| Kiryat Netafim  | 1983| 141        | 339        | 910                | 1.404255319          |
| Ma’ale Levona   | 1983| 161        | 503        | 826                | 2.124223602          |
| Tene            | 1983| 185        | 390        | 768                | 1.108108108          |
| Asefar          | 1983| 86         | 201        | 688                | 1.337209302          |
| Mezadot Yehuda  | 1983| 208        | 429        | 466                | 1.0625               |
| Migdalim        | 1983| 66         | 136        | 305                | 1.060606061          |
| Maskiyyot       | 1983| 45         | 413        | 253                | 8.177777778          |
| Rotem           | 1983| 44         | 55         | 196                | 0.25                 |
| Avenat          | 1983| 43         | 89         | 193                | 1.069767442          |
| Adam(Geva Binyamin) | 1984| 457        | 1324       | 5278               | 1.897155361          |
| Eli             | 1984| 591        | 2167       | 4233               | 2.666666667          |
| Pedu’el         | 1984| 1682       |            |                    | #DIV/0!              |
| Itamar          | 1984| 182        | 677        | 1151               | 2.71978022           |
| Carmei Tzur     | 1984| 135        | 397        | 1047               | 1.940740741          |
| Nahaliel        | 1984| 90         | 1585       | 639                | 16.611111111         |
| Beit Hagai      | 1984| 142        | 869        | 573                | 5.11971831           |
| Adora           | 1984| 182        | 372        | 421                | 1.043956044          |
| Beitar Illit    | 1985| 2208       | 3632       | 51636              | 0.644927536          |
| Oranit          | 1985| 8652       |            |                    | #DIV/0!              |
| Kochav Ya’akov  | 1985| 564        | 2311       | 7394               | 3.09751773           |
| Hasmoneam       | 1985| 752        | 501        | 2826               | -0.333776596         |
| Etz Efrayim     | 1985| 233        | 344        | 2022               | 0.47639485           |
| Qedar           | 1985| 241        | 365        | 1555               | 0.514522822          |
| Shim’a          | 1985| 113        | 172        | 592                | 0.522123894          |

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| Settlement      | Year | Cumulative Sum | a    | b    | c   | Coefficient | 16 | Coefficient | 28 | Coefficient | 0.28 |
|-----------------|------|----------------|------|------|------|-------------|----|-------------|----|-------------|------|
| Har Adar        | 1986 | 1257           | 2741 | 3980 | 1.180588703 | 30 | 2.63270891 | 37222 |
| Nofim           | 1987 | 215            | 632  | 690  | 1.939534884  | 29 | 3.7880945 | 37437 |
| Na’ale          | 1988 | 314            | 1105 | 1661 | 2.51910828   | 28 | 4.59609024 | 37751 |
| Talmon          | 1989 | 3879           | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | 37751 |
| Zufin(Zufim)    | 1989 | 2087           | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | 37751 |
| Bat Ayin        | 1989 | 291            | 417  | 1307 | 0.432989691  | 27 | 1.34137213 | 38042 |
| Avne Hefetz     | 1990 | 269            | 789  | 1759 | 1.933085502  | 26 | 4.22550974 | 38311 |
| Revava          | 1991 | 240            | 605  | 2181 | 1.520833333  | 25 | 3.76759835 | 38551 |
| Rechelim        | 1991 | 67             | 217  | 668  | 2.23880597   | 25 | 4.81305926 | 38618 |
| Alfei Menashe   | 1993 |                |      | 7780 | #DIV/0!      | 23 | #DIV/0!     | 38618 |
| Modi’in Ilit    | 1996 | 2103           | 4549 | 66847| 1.163100333  | 20 | 3.93308853 | 40721 |
| Kafr Haoranim(Menora) | 1998 | 337            | 1329 | 2678 | 2.943620178  | 18 | 7.92083136 | 41058 |
| Bruchin         | 1999 |                | 818  | #DIV/0! | #DIV/0! | #DIV/0! | 41058 |
| Sansana         | 1999 | 74             | 188  | 377  | 1.540540541  | 17 | 5.63776011 | 41132 |
| Negohot         | 1999 | 53             | 227  | 289  | 3.283018868  | 17 | 8.93357685 | 41185 |
| Gilad Farm      | 2002 |                | #DIV/0! | #DIV/0! | #DIV/0! | #DIV/0! | 41185 |

Table 4: Cumulative Sum for the Settlements and the Gompertz Model Coefficients

| Year | Cumulative sum | a  | b  | c  |
|------|----------------|----|----|----|
| 1967 | 258            | 0  |    | 1.11E-11 |
| 1968 | 879            | 1  |    | 1.03E-08 |
| 1969 | 1126           | 2  |    | 1.81E-06 |
| 1970 | 3040           | 3  |    | 9E-05   |
| 1971 | 3235           | 4  |    | 0.001724 |
| 1972 | 4040           | 5  |    | 0.016049 |
| 1973 | 4143           | 6  |    | 0.08665  |
| 1975 | 8018           | 8  |    | 0.812004 |
| 1976 | 8358           | 9  |    | 1.681556 |
| 1977 | 12387          | 10 |    | 2.915108 |
| 1978 | 15932          | 11 |    | 4.418187 |
| 1979 | 17276          | 12 |    | 6.049656 |
| 1980 | 18591          | 13 |    | 7.671584 |
| 1981 | 24719          | 14 |    | 9.1801 |
| 1982 | 26251          | 15 |    | 10.51405 |
| 1983 | 30075          | 16 |    | 11.64938 |
| 1984 | 31854          | 17 |    | 12.5881 |
| 1985 | 35965          | 18 |    | 13.34743 |

Gompertz model: \( f(t) = a \times (\exp(-b \times \exp(-ct))) \)
7. Discussion

Looking at the above results we notice that the growth rate for each settlement vary from -0.81 to 9.38. However, the “Swallowing” rate of the Palestinian lands by the expansion of the Israeli settlements follows the Gompertz function. This is a time series mathematical model that uses a log exponential approximation to describe the growth rate. This rate follows an “S” shape where it is slowest at the beginning and at the end of a given time period.

Looking at the graph in table 4, we notice that there was a slow growth rate in the initial areas occupied by the Israeli settlements from the year 1967 to 1973. In 1967, the Israeli forces had succeeded in occupying the West Bank including East Jerusalem. Immediately after the occupation, the Israeli government quickly began “swallowing” East Jerusalem
by building settlements. Nonetheless, the rest of the West Bank was not invested in. From 1973 to 1984, the Israeli government moved forward with its plans to “swallow” the rest of the West Bank. This is reflected in our graph by the increase in the growth rate where the slope –rate of change- of the graph is increasing at an increasing rate. Finally, from the year 1984 to 2002, the growth rate in the number of settlements had started to increase at a decreasing rate. Thus, the Israeli government had continued with its “swallowing” by the building of new settlements but at a much slower pace.

Did the story end here? Unfortunately, not; we now move to our theoretical dynamic model to explain the rate of expansion of each of the existing settlements shown in table 3 under total growth and growth rate. In table 3, we calculated both the total growth and the growth rate for each settlement. However, the lack of data on the prices of land along, the borders of the settlements makes it hard to predict the expansion rate for each settlement.

8. Conclusion

This paper developed a dynamic theoretical model to explain the process by which the existing Israeli settlements are expanding and thus developing an equation to explain the “swallowing” rate of the Palestinian lands. It described a pattern in which the Israeli settlements are far away from their equilibrium – where the West Bank including Jerusalem should have zero settlements. The paper also derived an empirical mathematical time series model that can somewhat estimate the rate of growth of Israeli settlements, which will in turn provide policy makers with a better prediction of the “swallowing” of Palestinian lands.

The theoretical dynamic model outlined by this paper described the expansion of Israeli settlements. The resulting model had asserted that the rate of expansion of the settlements is dependent on the difference in land prices between the Israeli settlements and Palestinian territories and the benchmark set by the Israeli government.

So, are the Palestinians fully aware of the threats from the settlements? The answer is “yes”, however the Israeli government had succeeded in creating Palestinian dependency on the Israeli economy (Samarah and Rahman, 2017). This is turn would deter the efforts of Palestinians from slowing down the ‘swallowing’ of their lands.

What can the Palestinians do to slow down the “swallowing”? One way to prevent the expansion of the settlements by the Palestinian government is to constantly inflate Palestinian land prices along the borders of the settlements. The increase in prices minimizes the price difference between Palestinian and Israeli land. This can be done by providing both financial and logistical support for Palestinians who either live or own land along the borders. In addition, it will relieve Palestinians of the potential Israeli pressure inserted to sell their lands; and finally, it will limit the indirect ways that Israelis use to trick Palestinians into selling their land.

Over the long run as the Palestinian population grows, the demand curve for Palestinian land shifts to the right – given the
constant supply of land, which is represented by a vertical line – the equilibrium land price, will increase. Thus, it is important for the Palestinian government to provide all the necessary aid in order to strengthen the steadfastness or Palestinians on their lands.

The ambition of this paper is to open the way for more empirical studies to be conducted based on our theoretical framework and the empirical model. In addition, understanding the patterns of the increase in Israeli settlers and the pace of the settlement expansion will aid policymakers in attempting to limit their growth. Furthermore, policymakers can include the rate of expansion of settlements in their strategic plans and better plan Palestinian urban and rural development. Finally, with such predictions, policymakers can also try to minimize the negative effects of settlements as much as possible.

Annex I

Table 2: The Name of the Settlement, the Date of Establishment, the Initial Area Occupied by the Settlement when Established, and the Area Occupied in 2016.

| Name of Settlement         | Date of Establishment | Initial Area (m²) | Area in 2016 (m²) | Number of Settlers in 2016 |
|----------------------------|-----------------------|-------------------|-------------------|---------------------------|
| 1  Modi’in Illit            | 1996                  | 2103              | 4549              | 66847                     |
| 2  Beitar Illit             | 1985                  | 2208              | 3632              | 51636                     |
| 3  Ma’ale Adumim           | 1975                  | 3291              | 7010              | 37670                     |
| 4  Ari’el                  | 1978                  | 2378              | 4729              | 19220                     |
| 5  Giv’at Ze’ev            | 1983                  | 1257              | 2741              | 16865                     |
| 6  Oranit                  | 1985                  |                   |                   | 8652                      |
| 7  Efrat                   | 1980                  | 992               | 1792              | 8658                      |
| 8  Alfei Menashe           | 1993                  |                   |                   | 7780                      |
| 9  Kochav Ya’akov          | 1985                  | 564               | 2311              | 7394                      |
| 10 Kiryat Arba             | 1972                  | 466               | 787               | 7272                      |
| 11 Karnei Shomron          | 1978                  | 341               | 631               | 7102                      |
| 12 Beit El                 | 1977                  | 688               | 557               | 6115                      |
| 13 Sha’arei Tikva          | 1983                  |                   |                   | 5811                      |
| 14 Adam (Geva Binyamin)    | 1984                  | 457               | 1324              | 5278                      |
| 15 Bet Arye                | 1981                  | 837               | 7814              | 4842                      |
| 16 Kedumim                 | 1977                  |                   |                   | 4323                      |
| 17 Kfar Adumim             | 1979                  | 412               | 693               | 4271                      |
| 18 Eli                     | 1984                  | 591               | 2167              | 4233                      |
| 19 Elkana                  | 1977                  | 867               | 1757              | 3898                      |
| 20 Har Adar                | 1986                  | 1257              | 2741              | 3980                      |
| 21 Talmon                  | 1989                  |                   |                   | 3879                      |
| 22 Tekoa                   | 1977                  | 527               | 844               | 3633                      |
| 23 Shilo                   | 1979                  | 387               | 3247              | 3727                      |
| 24 Immanueld               | 1983                  | 301               | 740               | 3309                      |
| 25 Alon Shvut              | 1970                  | 492               | 463               | 3180                      |
| 26 Ofra                    | 1975                  |                   |                   | 3605                      |
| No. | City                  | Year | Population | Area | Peak Elevation |
|-----|-----------------------|------|------------|------|----------------|
| 27  | Hasmoneam             | 1985 | 752        | 501  | 2826           |
| 28  | Kfar Haoranim (Menora)| 1998 | 337        | 1329 | 2678           |
| 29  | Elazar                | 1975 | 265        | 349  | 2568           |
| 30  | Mevo Horon            | 1970 | 603        | 524  | 2566           |
| 31  | Mitzpe Yeriho         | 1978 | 331        | 747  | 2319           |
| 32  | Neve Daniel           | 1982 | 268        | 457  | 2278           |
| 33  | Har Bracha            | 1983 | 238        | 350  | 2339           |
| 34  | Zufin (Zufim)         | 1989 |            |      | 2087           |
| 35  | Revava                | 1991 | 240        | 605  | 2181           |
| 36  | Kfar Hashachar        | 1977 | 231        | 336  | 2052           |
| 37  | Nokdim                | 1982 |            |      | 1847           |
| 39  | Yaqir                 | 1981 | 3016       | 574  | 1901           |
| 40  | Etz Efrayim           | 1985 | 233        | 344  | 2022           |
| 41  | Elon Moreh            | 1979 | 381        | 1047 | 1861           |
| 42  | Barkan                | 1981 | 411        | 563  | 1798           |
| 43  | Avne Hefetz           | 1990 | 269        | 789  | 1759           |
| 44  | Pedu’el               | 1984 |            |      | 1682           |
| 45  | Na’ale                | 1988 | 314        | 1105 | 1661           |
| 46  | Har Gilo              | 1972 | 224        | 507  | 1570           |
| 47  | Qedar                 | 1985 | 241        | 365  | 1555           |
| 48  | Nili                  | 1981 | 321        | 1296 | 1552           |
| 49  | Yitzhar               | 1983 | 200        | 1248 | 1468           |
| 50  | Ma’ale Mikhmas        | 1981 | 221        | 1507 | 1323           |
| 51  | Itamar                | 1984 | 182        | 677  | 1151           |
| 52  | Dolev                 | 1983 | 280        | 1186 | 1331           |
| 53  | Alei Zahav            | 1982 | 339        | 608  | 1643           |
| 54  | Almon                 | 1982 | 194        | 689  | 1329           |
| 55  | Neve Halamish         | 1977 | 297        | 2510 | 1328           |
| 56  | Bet Horon             | 1977 | 235        | 199  | 1240           |
| 57  | Bat Ayin              | 1989 | 291        | 417  | 1307           |
| 58  | Ma’ale Efrayim        | 1970 | 359        | 489  | 1209           |
| 59  | Givon Nahalasha       | 1980 |            |      | 1135           |
| 60  | Hinnanit              | 1981 | 163        | 698  | 1164           |
| 61  | Kfar Etzion           | 1967 | 258        | 567  | 1099           |
| 62  | Susiya                | 1983 | 268        | 460  | 1115           |
| 63  | Carmei Tzur           | 1984 | 135        | 397  | 1047           |
| 67  | Otniel                | 1983 | 301        | 740  | 976            |
| 68  | Ateret                | 1981 | 160        | 3393 | 875            |
| 69  | Shvei Shamron         | 1977 | 256        | 391  | 897            |
| 70  | Kiryat Netafim        | 1983 | 141        | 339  | 910            |
| 71  | Shaqed                | 1981 | 229        | 463  | 864            |
| 72  | Ma’ale Levona         | 1983 | 161        | 503  | 826            |
| 73  | Tene                  | 1983 | 185        | 390  | 768            |
| 74  | Bruchin               | 1999 |            |      | 818            |
### The Role of Mathematics in the Palestinian Economy: Estimating the Speed of the Swallowing of Palestinian Lands by the Israeli Settlements

| Settlement       | Year | Number |
|------------------|------|--------|
| Einav            | 1981 | 183    |
| Mattityahu       | 1981 | 149    |
| Sal‘it           | 1977 | 443    |
| Nahaliel         | 1984 | 90     |
| Nofim            | 1987 | 215    |
| Rimmonim         | 1977 |        |
| Beit Hagai       | 1984 | 142    |
| Asefar           | 1983 | 86     |
| Rechelim         | 1991 | 67     |
| Shademot Mehola  | 1979 | 164    |
| Shim’a           | 1985 | 113    |
| Ma’on            | 1981 | 167    |
| Eshkolot         | 1982 | 92     |
| Mehola           | 1968 | 241    |
| Mezadot Yehuda   | 1983 | 208    |
| Migdal Oz        | 1977 | 95     |
| Pnei Hever       | 1982 | 98     |
| Carmel           | 1981 | 182    |
| Adora            | 1984 | 182    |
| Qalya            | 1968 | 271    |
| Ma’ale Amos      | 1981 | 89     |
| Gittit           | 1973 |        |
| Mevo Dotan       | 1978 | 96     |
| Telem            | 1982 | 95     |
| Sansana          | 1999 | 74     |
| Negohot          | 1999 | 53     |
| Yitav            | 1970 | 117    |
| Migdalim         | 1983 | 66     |
| Petza’el         | 1975 | 319    |
| Tomer            | 1978 | 218    |
| Vered Yeriho     | 1980 | 75     |
| Gilad Farm       | 2002 |        |
| Hemdat           | 1980 | 75     |
| Hermesh          | 1982 | 96     |
| Rehan            | 1977 | 86     |
| Almog            | 1977 | 107    |
| Maskiyyot        | 1983 | 45     |
| Beka’ot          | 1972 | 115    |
| Netiv Hagedud    | 1976 | 234    |
| Mizpe Shalem     | 1971 | 72     |
| Gilgal           | 1970 | 181    |
| Rotem            | 1983 | 44     |
| Ro’i             | 1976 | 106    |
| Beit Ha’arava    | 1980 | 75     |
| Yafit            | 1980 | 173    |
| Massu’a          | 1970 | 162    |
| Mechora          | 1973 | 103    |
| Argaman          | 1968 | 109    |
| Avenat           | 1983 | 43     |

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| No. | Village | Date of Establishment | Initial Area | Area in 2016 |
|-----|---------|-----------------------|--------------|--------------|
| 124 | Hamra   | 1971                  | 123          | 261          | 124          |
| 125 | Naama   | 1982                  | 119          | 280          | 116          |
| 126 | Niran   | 1977                  | 107          | 397          | 91           |

Source: The names of the settlements, date of establishment, and number of settlers were taken from B’t Selem. The initial area and area in 2016 were taken from Wafa organization.
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