Design and Simulation of MEMS Devices using Interval Analysis

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Abstract. Modeling and simulation of MEMS devices are used to optimize the design, to improve the performance of the device, to reduce time to market, to minimize development time and cost by avoiding unnecessary design cycles and foundry runs. The major design objectives in any device design, is to meet the required functional parameters and the reliability of the device. The functional parameters depend on the geometry of the structure, material properties and process parameters. All model parameters act as input to optimize the functional parameters. The major difficulty the designer faces is the dimensions and properties used in the simulation of the MEMS devices can not be exactly followed during fabrication. In order to overcome this problem, the designer must test the device in simulation for bound of parameters involved in it. The paper demonstrates the use of interval methods to assess the electromechanical behaviour of micro electromechanical systems (MEMS) under the presence of manufacturing and process uncertainties. Interval method guides the design of pullin voltage analysis of fixed-fixed beam to achieve a robust and reliable design in a most efficient way. The methods are implemented numerically using Coventorware and analytically using Intlab.

Keywords: Uncertainty analysis, pullin voltage analysis, Interval Analysis, Intlab, Coventorware, Micro Electro Mechanical Systems
1. Introduction

Micro-electromechanical systems (MEMS) is a process technology used to create tiny integrated devices or systems that combine mechanical and electrical components. They are fabricated using integrated circuit (IC) batch processing techniques and can range in size from a few micrometers to millimeters; these devices (or systems) have the ability to sense, control, and actuate on the micro scale, and generate effects on the macro scale [6]. Electrostatic MEMS is a special branch under micromechanics with a wide range of application specific devices such as switches, micro-mirrors, micro-resonators, etc. Modelling and simulation of electrostatic MEMS devices play an important role in the design phase in predicting device characteristics. Electrostatic pull-in is a well-known sharp instability in the behavior of an elastically supported structure subjected to parallel plate electrostatic actuation [1].

Uncertainty analysis is a technique by which one can determine, with good approximation, whether a system will work within raw specification limits when the parameters vary between their limits [2]. The manufacturing of structural components is generally associated with manufacturing imperfections. In general, geometry of a component cannot be reproduced only within certain finite tolerances. If the influencing variables are uncertain, a direct consequence is that the response parameters are uncertain as well [3]. In the present work, interval analysis is implemented for pull-in voltage analysis of a micro fixed-fixed beam by using IntLab for analytical simulation and using Coventorware for numerical simulation, by considering uncertainty in geometric and material property simultaneously, and individually.

2. Pull-in analysis of fixed-fixed beam

A simple representation of an electrostatic pull-in device is shown in figure (1). It consists of two parallel conductive plates forming a capacitor with an effective overlap area $A_{\text{eff}}$ and separated by a gap spacing $d$. The bottom plate is fixed and the top plate is suspended by a spring with stiffness $K_{\text{eff}}$. By applying a dc voltage $V_{\text{DC}}$ across the plates, an electrostatic attractive force $F_{\text{el}}$ is induced which leads to a decrease of the gap spacing, thereby stretching the spring. This results in an increase of the spring force $F_s$, which counteracts the electrostatic force. Pull-in instability occurs as a result of the fact that the electrostatic force increases non-linearly with decreasing gap spacing, whereas the spring force is a linear function of the change in the gap spacing. In simple terms, the pull-in voltage $V_{\text{PI}}$ can be defined as the voltage at which the restoring spring force can no longer balance the attractive electrostatic force. In effect, the gap spacing is closed to zero at the onset of pull-in. In real MEMS pull-in structures, however, the situation is more complicated as the elastic member is not a simple lumped spring, but typically a continuous member.

$$d = d(V_{\text{DC}})$$

Figure 1. Lumped Pull-in system
The pull-in voltage can be easily derived based on the balance of forces and minimization of potential energy and is given by

\[ V_{\text{PI}} = \sqrt{\frac{8 \cdot K_{\text{eff}} \cdot d_0^3 \cdot \varepsilon_0 \cdot A_{\text{eff}}}{27}} \]

Where \( d_0 \) is the zero-voltage gap spacing and \( \varepsilon_0 \) is the permittivity of free space [1].

2.1 Design of fixed-fixed beam
A micro fixed-fixed beam of the dimensions given in the table 1 is designed in Coventorware. Pull-in voltage is analyzed using Cosolve solver for the actual dimension of the beam and the analysis gives the pull in voltage to be 38.4188V.

| Parameters                        | Nominal Value |
|-----------------------------------|---------------|
| Length of the beam (l), \( \mu m \) | 300           |
| Width of the beam (b), \( \mu m \)  | 50            |
| Thickness of the beam (h), \( \mu m \) | 3             |
| Zero voltage gap spacing (\( d_0 \)), \( \mu m \) | 1             |
| Young’s modulus (E), GPa         | 77            |
| Built-in stress (\( \sigma_0 \)), MPa | 100          |

The corresponding displacement vs. pullin voltage and 3-D view of the beam after the pull-in voltage is reached in the structure is shown in Figure2.

Figure 2. Pull in Analysis
3. Design using interval analysis

Interval Analysis is a technique used to estimate the bounds on various model outputs based on the bounds of the model inputs and parameters. In the interval method approach, uncertain parameters are assumed to be unknown but bounded and each of them has upper and lower limits without a probabilistic structure. Every uncertain parameter is described by an interval \([a, b]\) where \(a\) is the lower limit of the interval and \(b\) is the upper limit of the interval and \(a\) and \(b\) are real numbers. The number is known to lie between values but the exact value is unknown. Interval arithmetic is an elegant tool for practical work with inequalities, approximate numbers, error bounds, and more generally with certain convex and bounded sets.

Let \(x = [a, b]\) and \(y = [c, d]\) be two interval numbers, \(a\) and \(c\) are lower limits, \(b\) and \(d\) are upper limits and \(a, b, c, d\) are real.

1. Addition: \(x + y = [a, b] + [c, d] = [a + c, b + d]\)
2. Subtraction: \(x - y = [a, b] - [c, d] = [a - d, b - c]\)
3. Multiplication: \(xy = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)]\)
4. Division: \(1/x = [1/b, 1/a]\)

For carrying out uncertainty analysis of fixed-fixed beam, the pullin analysis was performed in Coventorware, with all the dimensional parameters and material properties 5% less than the nominal value and 5% more than the nominal values and the resulting pull in voltage is \([32.5, 45.3125]\) volts. On the similar line, considering a single dimensional parameter for example length, the pullin analysis was performed with \(\pm 5\%\) of the nominal value of the length and the same procedure was repeated for the width of the beam, thickness of the beam, gap spacing, Young’s modulus and built in stress, considering each parameter individually at a time, the results obtained are tabulated in the table 2.

| Parameters          | Parameter Interval | Numerical Simulation of Pullin voltage(v) |
|---------------------|--------------------|------------------------------------------|
| Length of the beam (l) \(\mu\text{m}\) | \([285, 315]\) | \([32.5, 41.25]\) |
| Width of the beam (b) \(\mu\text{m}\) | \([48.5, 51.5]\) | \([38.75, 38.75]\) |
| Thickness of the beam (h) \(\mu\text{m}\) | \([2.85, 3.15]\) | \([35.9375, 35.625]\) |
| gap spacing \(d_0\) \(\mu\text{m}\) | \([0.95, 1.05]\) | \([36.25, 41.25]\) |
| Young’s modulus (E) GPa | \([73.15, 80.85]\) | \([37.8125, 40]\) |
| Built-in stress \((\sigma_0)\) MPa | \([85, 115]\) | \([38.4375, 38.75]\) |

INTLAB is a tool for calculating interval arithmetic’s using MATLAB. Using this, pull-in voltage for the fixed-fixed beam model is calculated by inputting all inputs in interval. The pullin voltage with nominal values for all the parameters is 35.4188 volts. The pull-in voltage after the uncertainty of \(\pm 5\%\) applied to all the parameters like young’s modulus, stress, length, width, thickness and air gap is \([27.4047, \ 45.8610]\). Pullin voltage is also analyzed applying uncertainty individually for each of the parameter and the results are tabulated in the table 3.
Table 3. Results of pullin analysis with uncertainty using INTLAB

| Parameters                  | Parameter Interval | Analytical Simulation of Pullin voltage (v) |
|-----------------------------|--------------------|--------------------------------------------|
| Length of the beam (l) μm   | [285, 315]         | [32.5, 41.25]                              |
| Width of the beam (b) μm    | [48.5, 51.5]       | [38.75, 38.75]                             |
| Thickness of the beam (h) μm| [2.85, 3.15]       | [35.9375, 35.625]                          |
| gap spacing (d₀) μm         | [0.95, 1.05]       | [36.25, 41.25]                             |
| Young’s modulus (E) GPa     | [73.15, 80.85]     | [38.4375, 38.75]                           |
| Built-in stress (σ₀) MPa    | [85, 115]          | [37.8125, 40]                              |

4. Conclusion:
Interval methods provide a computationally efficient method for treating uncertainties in engineering calculation. In this work we have proposed and demonstrated the use of interval approach to analyze the effect of parameter uncertainty on pullin voltage analysis for a fixed-fixed beam. It was observed that the change in length and gap spacing tends to be the most influencing parameters, which needs to be tightly controlled.

The objective of the study is to show how FEA and interval based design can be used to simulate the effects of manufacturing tolerances on the behavior of the device. The interval methods can be used to quantify more reliable and robust design and analysis procedure for MEMS device. This provides guidance for necessary design changes in a most efficient way.

5. References
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