Threshold suppression of $\Lambda$ spin-orbit splitting

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Abstract

New experimental data on medium to heavy single $\Lambda$ hypernuclei revealed a much larger spin-orbit splitting than observed in older measurements of light hypernuclei. Taking into account particle threshold effects and the density-dependence of in-medium coupling constants the apparent suppression of spin-orbit strength in light hypernuclei as well as the spin-orbit structure observed in medium to heavy nuclei are explained in a unified manner within the density dependent relativistic hadron field theory. It is concluded that the most valuable information on the $\Lambda$ spin-orbit dynamics in finite nuclei has to be extracted from medium to heavy mass nuclei.

Key words: hypernuclei, spin-orbit splitting
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1 Introduction

Guided by experiment, e.g. in $^{13}_\Lambda C$ [1] and $^{16}_\Lambda O$ [2] it is a long standing opinion that the spin-orbit splitting of the $\Lambda$ single particle levels in $\Lambda$ hypernuclei should be very small or almost zero. The most prominent theoretical explanation for this effect has been given through the quark-spin substructure of the $\Lambda$ leading to an additional $\Lambda-\omega$ tensor coupling that almost exactly cancels the more conventional scalar-vector spin-orbit force of the $\Lambda$ single particle states in finite systems [3–5].

A reanalysis of older $^{13}_\Lambda C^*$ [7] and $^{16}_\Lambda O^*$ [6] proton emitter emulsion data by Dalitz et al., however, lead already to a remarkably bigger splitting, i.e. $\approx 0.8$ MeV for $^{13}_\Lambda C$ and $1.56\pm 0.12$ MeV for $^{16}_\Lambda O$ though the statistics was not so good. New experiments at KEK, measuring high resolution $\Lambda$ single-particle spectra

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for medium and heavy hypernuclei, show a spin-orbit splitting for the $\Lambda$ levels in the range of 1–2 MeV [8]. This is quite sizeable but still smaller by a factor of about 2 than the splitting to be expected from the general quenching of $\Lambda$–N interactions.

In this letter we will show that the reduced splitting in medium and heavy nuclei can be attributed to the strong delocalization of the $\Lambda$ wave functions in the nucleus due to the shallow $\Lambda$ potential (sect. 3). In light nuclei with weakly bound $\Lambda$ states particle threshold effects are superimposed on the general delocalization. The very small splitting in light nuclei is due to these continuum threshold effects which lead to a squeezing of the single particle levels close to threshold and thereby almost completely dilutes generic spin-orbit potential effects in this mass region (sect. 4). According to our investigations, the extremely small splitting of the spin-orbit doublets in light $\Lambda$ hypernuclei as well as the reduced splitting of those in medium to heavy hypernuclei can be completely explained in the framework of the density dependent relativistic hadron (DDRH) field theory [9,10].

2 The DDRH field theory for $\Lambda$ hypernuclei

The DDRH field theory [10] is an effective density dependent lagrangian field theory of nucleons and mesons based on microscopic nucleon–nucleon interactions. The extension of DDRH to the strangenes sector was described in [9],
Table 1
Comparison of the measurements by Nagae et al. [8] on $^5\Lambda V$ with DDRH calculations.

\[
\begin{array}{|c|c|c|}
\hline
\text{State} & \text{Exp.} & \text{Theor.} \\
\hline
1s_{1/2} & -19.97\pm0.13 & -19.89 \\
\Delta E(1p) & 1.3\pm0.3 & 1.05 \\
E_{\text{centroid}}(1p) & - & 11.5 \\
\Delta E(1d) & 2.0\pm0.2 & 1.81 \\
E_{\text{centroid}}(1d) & - & 3.09 \\
\hline
\end{array}
\]

Where we refer to for theoretical details. Medium effects are included by mapping the ladder sum of Dirac-Brückner (DB) self-energies onto the vertices which are in our case functionals of the baryonic field operators. As input for the vertices DB calculations for the Bonn A potential have been used. Besides the standard kinetic and mass terms for baryons and mesons the DDRH lagrangian incorporates the interaction part:

\[
L_{\text{int}} = \Psi_F^\dagger \Gamma_\sigma (\Psi_F, \Psi_F) \Psi_F \sigma - \Psi_F^\dagger \Gamma_\omega (\Psi_F, \Psi_F) \gamma_\mu \Psi_F \omega^\mu \\
- \frac{1}{2} \Psi_F^\dagger \tilde{\Gamma}_\rho (\Psi_F, \Psi_F) \gamma_\mu \Psi_F \bar{\rho}^\mu - e \Psi_F^\dagger \tilde{Q} \gamma_\mu \Psi_F A^\mu
\]

Because of the functional dependence of the vertices on the field operators the baryonic equations of motion contain additional rearrangement self energies accounting for static polarization effects of the nuclear medium [9,10]. In figure 1 and table 1 we compare our theoretical results with the new experimental data by H. Hotchi and T. Nagae [8]. Our calculations have been performed with the numerical parameter set derived from Dirac-Brückner theory and free Λ-N scattering in [9]. The Λ single particle spectra of $^5\Lambda V$ and $^8\Lambda Y$ are reproduced almost perfectly. This indicates that:

1. The dynamics of the Λ is – in medium to heavy nuclei – almost completely governed by the nuclear mean field which is in line with the experiment [8] where shell structures even for deeply bound Λ-states are clearly resolved.
2. In view of this not much room seems to be left for QCD related phenomena like dissolving of the Λ in the nuclear medium or partial deconfinement of the strange quark, as conjectured e.g. in [11].
3. A Λ–ω tensor coupling as it arises e.g. from the quark meson coupling prescription and leads to an almost vanishing spin-orbit splitting (see e.g. [5]) is not supported by the analysis.

This underlines the sensibility of a hadronic description of in-medium hyperons and their interactions also around saturation density. It also stresses
Table 2

Localization coefficients $N_{\Lambda,n}(r_{rms})$ as defined in eq. (2) for $\Lambda$ and neutron states. The reduced values for $\Lambda$ states indicate the increased delocalization of $\Lambda$ wave functions.

the importance of $\Lambda$ hypernuclei in investigating the microscopic structure of baryonic interactions.

3 The spin-orbit potential and the delocalization of the $\Lambda$ wave function

As was indicated in [9] the reduced spin-orbit splitting in medium and heavy $\Lambda$ hypernuclei can be attributed to the delocalization of $\Lambda$ wave-functions. Since the spin-orbit energy is determined by the overlap integral of the single particle wave function and the spin-orbit potential – where the latter is well localized at the nuclear surface – a strong delocalization of the wave function leads to a sizeable reduction of the spin-orbit interaction energy.

A suitable measure for the reduced overlap of the $\Lambda$ wave function with the nuclear interior is to define a localization coefficient by

$$N_{\Lambda,n}(R) = N_o \int_0^R dr \, r^2 \, |F_{\Lambda,n}(r)|^2$$

(2)

describing the fraction of the probability density which is localized within the volume $\frac{4}{3} \pi R^3$. Here, $F_{\Lambda,n}$ is the upper component of the respective Dirac wave function and $N_o$ is chosen such that $N_{\Lambda,n}(R) \to 1$ for $R \to \infty$. In table 2 values $N_{\Lambda,n}(R)$ for $R = \sqrt{\langle r^2 \rangle}$ for $\Lambda$ and neutron orbitals in a number of nuclei are shown. The overlap of the $\Lambda$ wave functions with the bulk of the nuclear mass distribution is seen to be strongly reduced compared to corresponding neutron states. The delocalization increases rapidly with decreasing binding energy with a particular strong enhancement for $\Lambda$ states. The results show clearly that a sizeable part of the $\Lambda$ wave function lies “outside” the nucleus, a
much larger fraction than in the neutron case. A comparison with experiment, see figure 1 and table 1, shows that this mechanism is able to explain the fine structure of Λ single particle spectra.

4 Continuum threshold effects on the spin-orbit splitting

Experimentally observed single particle spectra in low mass Λ hypernuclei show a much more drastic reduction of the spin-orbit splitting. This led to the conjecture that the spin-orbit splitting in hypernuclei should be in general very small and could be reasonably explained by assuming that the spin of the Λ is carried solely by the strange quark.

The level spacing in light nuclei is, however, strongly affected by the continuum threshold which is in these small systems – especially for the rather weakly bound Λs – quasi omnipresent. As is known from neutron rich nuclei, e.g. [12–15], the single particle level spacing becomes compressed for states approaching the threshold.

Fig. 2. The figure shows the strong decrease of the spin orbit splitting for the 1p doublet in $^{16}_ΛO$ when the doublet’s centroid is pushed towards the continuum threshold. The dot marks the actual theoretical value for $^{16}_ΛO$, the data point is taken from [6].

The threshold compression effect is visible already in the Ca region but is especially important around and below $^{16}_ΛO$. The typical feature in these nuclei is that the centroid energy of a spin-orbit doublet is rather close to threshold, mainly because of the shallowness of the binding potential. Under such conditions the spectrum of valence levels becomes compressed inducing an apparent reduction of spin-orbit strength because the less bound j=ℓ-1/2 member of a spin-orbit doublet tries to avoid the crossing in the unbound region. This “hindrance” is related to the general phenomenon of “avoided level crossing” in
quantal systems because passing over into the continuum means that for example a $p_{1/2}$-level would have to cross the region of low energy s-wave continuum levels before re-appearing as a single particle resonance.

We illustrate these special effects for the 1p spin-orbit doublet in $^{16}_{\Lambda}O$. Approaching the continuum threshold is enforced by modifying artificially the $\Lambda$ potential depth by variation of the $\Lambda-\omega$ coupling constant. Figure 2 shows the dependence of the splitting strength of the 1p doublet on the centroid energy $E_{\text{centroid}} = \frac{E_{p3/2} + E_{p1/2}}{2}$. Even strongly bound states ($E_B < -6\text{MeV}$) are still considerably affected by the threshold. The effect only saturates if a state of another parity from the continuum is lowered into the bound region, in this example a $1d_{5/2}$ orbit. The same behavior is also found in the 1p-shell doublet in $^{13}_{\Lambda}C$, often referred to as the “standard hypernucleus”.

The continuum threshold effect reflects the peculiar dependence of matrix elements on wave function overlaps for weakly bound states. In such cases - as observed in recent studies of pure isospin dripline nuclei - generic properties of interactions are masked behind wave function effects. This has important consequences for extracting the desired information on the underlying interaction from data: The threshold effect leads to local variations in energy splittings not following the systematics derived from well bound systems. Obviously, this will strongly inhibit a model independent derivation of $\Lambda$ interaction strengths from measurements, especially in light nuclei. Around and beyond $A \sim 40$, however, the threshold effect rapidly vanishes. Since the new KEK data [8] for $^{51}V$ and $^{89}Y$ were obtained in mass region well outside the threshold regime they can be expected to provide suitable information on the generic $\Lambda$ spin-orbit strength, indicating a larger value than assumed before.

5 Conclusion

The larger than expected spin-orbit splitting observed in recent KEK experiments on medium mass nuclei are well reproduced by relativistic DDRH mean-field calculations. Universal parameter sets, derived previously from Dirac-Brueckner theory, were used. The results indicate that the medium dependence of hyperon-coupling constants is essential for the good description. The calculations explain consistently also the apparent suppression of spin-orbit strength for weakly bound $\Lambda$ orbitals in light nuclei by the compression of spectra close to the continuum threshold due to avoided level crossing. It is worthwhile to emphasize strongly the importance of high-resolution data, like those of ref. [8], for hypernuclear structure theory and the understanding of hyperon interactions.
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References

[1] M. May et al., Phys. Rev. Lett. **47** (1981) 1106.
[2] W. Brückner et al. [Heidelberg-Saclay-Strasbourg Collaboration], Phys. Lett. **B79** (1978) 157.
[3] J. Mares and B. K. Jennings, Phys. Rev. **C49** (1994) 2472.
[4] M. Chiapparini, A. O. Gattone and B. K. Jennings, Nucl. Phys. **A529** (1991) 589.
[5] K. Tsushima, K. Saito, J. Haidenbauer and A. W. Thomas, Nucl. Phys. **A630** (1998) 691 [nucl-th/9707022].
[6] R. H. Dalitz, D. H. Davis, D. N. Tovee and T. Motoba, Nucl. Phys. A **625** (1997) 71.
[7] R. H. Dalitz, D. H. Davis and D. N. Tovee, Nucl. Phys. A **450** (1986) 311C.
[8] H. Hotchi, Doctor Thesis, Dept. of Physics, University of Tokyo (2000), 200024003 KEK Report 2000-3, April 2000, H; T. Nagae, T. Nagae, Nucl. Phys. A **670** (2000) 269c.
[9] C. M. Keil, F. Hofmann and H. Lenske, Phys. Rev. **C61** (2000) 064309 [nucl-th/9911014].
[10] C. Fuchs, H. Lenske and H. H. Wolter, Phys. Rev. **C52** (1995) 3043 [nucl-th/9507044].
[11] H. J. Pirner, Phys. Lett. **B85** (1979) 190.
[12] D. Cortina-Gil, T. Baumann, H. Geissel, H. Lenske et al., EurPhys.J A (2000) (in press).
[13] H. Lenske, J.Phys. G: Nucl. Part. Phys. 24 (1998) 1429.
[14] T. Baumann, H. Geissel, H. Lenske et al., Phys.Lett. B 439 (1998) 256.
[15] W. Schwab, H. Geissel, H. Lenske et al. Z.Phys. A350 (1995) 283.