Axion dark matter from topological defects

Ken’ichi Saikawa (DESY)

Based on
T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi, PRD85, 105020 (2012) [1202.5851]
T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi, JCAP01, 001 (2013) [1207.3166]
M. Kawasaki, KS, T. Sekiguchi, PRD91, 065014 (2015) [1412.0789]
A. Ringwald, KS, PRD93, 085031 (2016) [1512.06436]
QCD axion as dark matter candidate

- Motivated by Pecccei-Quinn mechanism as a solution of the strong CP problem

- Spontaneous breaking of continuous Pecccei-Quinn symmetry at
  \[ T \approx F_a \approx 10^{8-11}\text{GeV} \]
  “axion decay constant”

- Nambu-Goldstone theorem
  \[ \rightarrow \] emergence of the (massless) particle \equiv \text{axion}

- Axion has a small mass (QCD effect)
  \[ \rightarrow \] pseudo-Nambu-Goldstone boson

- \[ m_a \sim \frac{\Lambda_{\text{QCD}}^2}{F_a} \sim 6 \times 10^{-5}\text{eV} \left( \frac{10^{11}\text{GeV}}{F_a} \right) \]

- \( \Lambda_{\text{QCD}} \approx O(100)\text{MeV} \)

- Tiny coupling with matter + non-thermal production
  \[ \rightarrow \] good candidate of cold dark matter
Axions in the inflationary universe: two scenarios

- Pre-inflationary PQ symmetry breaking
  - Severe isocurvature constraints
  - Tuning of the initial field value ("anthropic window")

- Post-inflationary PQ symmetry breaking ← this talk
  - Formation of topological defects

Hamann, Hannestad, Raffelt and Wong (2009)
How axions are produced?

If PQ symmetry is broken after inflation, there are three contributions:

1. Re-alignment mechanism
2. Radiation from strings
3. Collapse of string-wall systems

- Total abundance is sum of all these contributions
- All these effects have to be quantitatively taken into account
Re-alignment mechanism

- Axion field starts to oscillate at
  \[ m_a(T_{osc}) \approx 3H(T_{osc}) \]
- Temperature dependence of axion mass is important
  \[ m_a(T)F_a = \sqrt{\chi(T)} \]
- Recently, the lattice calculations of \( \chi \) in full QCD became available

\[ \frac{m_a}{m_a^0} \quad \text{and} \quad \frac{3H}{m_a^0} \]

\[ \chi \text{[fm}^{-4}] \quad T[\text{MeV}] \]

\[ \frac{m_a}{m_a^0} \quad 3H/m_a^0 \]

\[ \frac{\theta_a}{\text{[GeV}^{-1}]^2} \quad N_a \]

Wantz and Shellard (2009)

\( m_a < 3H \)
axion is frozen

\( m_a \approx 3H \)
axion starts rolling,
turns into pressureless matter.

Borsanyi et al. (2016)
Axionic string and axionic domain wall

Pecccei-Quinn field (complex scalar field)
\[ \Phi = |\Phi| e^{ia(x)/\eta} \]
\[ a(x) : \text{axion field} \]

\[ F_a = \eta / N_{\text{DW}} \]

String formation \( T \lesssim F_a \)
Spontaneous breaking of \( U(1)_{\text{PQ}} \)
\[ V(\Phi) = \frac{\lambda}{4} (|\Phi|^2 - \eta^2)^2 \]

Domain wall formation \( T \lesssim 1\text{GeV} \)
QCD effect
\[ V(\Phi) = \frac{\lambda}{4} (|\Phi|^2 - \eta^2)^2 + m_a^2 \eta^2 (1 - \cos(a/\eta)) \]

Field space

Coordinate space

Strings attached by domain walls
Domain wall problem

- Domain wall number \( N_{DW} \)
- \( N_{DW} \) degenerate vacua

\[
V(a) = \frac{m_a^2 \eta^2}{N_{DW}^2} \left( 1 - \cos(N_{DW} \frac{a}{\eta}) \right)
\]

\( N_{DW} \): integer determined by QCD anomaly

- If \( N_{DW} = 1 \), string-wall systems are unstable
  - Collapse soon after the formation

- If \( N_{DW} > 1 \), string-wall systems are stable
  - Coming to overclose the universe

Zel'dovich, Kobzarev and Okun (1975)

- We may avoid this problem by introducing an explicit symmetry breaking term
  - Sikivie (1982)
  - (walls become unstable)

\[
V(\Phi) = \frac{m_a^2 \eta^2}{N_{DW}^2} \left( 1 - \cos(N_{DW} \frac{a}{\eta}) \right) - \Xi \eta^3 (\Phi e^{-i\delta} + h.c.)
\]

\( \Delta V \sim \Xi \eta^4 \)
### Domain wall problem

- **Domain wall number** $N_{DW}$
  - $N_{DW}$ degenerate vacua
  
  \[
  V(a) = \frac{m_a^2 \eta^2}{N_{DW}^2} (1 - \cos(N_{DW} a/\eta))
  \]
  
  $N_{DW}$: integer determined by QCD anomaly

- **If** $N_{DW} = 1$, string-wall systems are **unstable**
  - Collapse soon after the formation

- **If** $N_{DW} > 1$, string-wall systems are **stable**
  - Coming to overclose the universe

  Zel'dovich, Kobzarev and Okun (1975)

- **We may avoid this problem by introducing an explicit symmetry breaking term** Sikivie (1982)
  (walls become unstable)

  \[
  V(\Phi) = \frac{m_a^2 \eta^2}{N_{DW}^2} (1 - \cos(N_{DW} a/\eta)) - \Xi \eta^3 (\Phi e^{-i\delta} + h.c.)
  \]

- $\Delta V \sim \Xi \eta^4$
Production of axions in the early universe

(post-inflationary PQ symmetry breaking scenario)

Inflation

PQ symmetry breaking
- Formation of strings
- Axion acquires a mass
- Formation of domain walls

QCD phase transition
- Axion acquires a mass
- Formation of domain walls

$T \lesssim F_a \simeq 10^{8-11} \text{ GeV}$

$T \lesssim 1 \text{ GeV}$

Collapse of string-wall systems

Annihilation of domain walls before they overclose the universe

$N_{DW} = 1$

$N_{DW} > 1$

soon after formation

String-wall networks exist for a long time
Production of axions in the early universe
(post-inflationary PQ symmetry breaking scenario)

- Inflation
  \[ T \lesssim F_a \approx 10^{8-11} \text{ GeV} \]

- \( \text{PQ symmetry breaking} \)
  - Formation of strings
  - Axion acquires a mass
  - Formation of domain walls

- QCD phase transition
  \[ T \lesssim 1 \text{ GeV} \]

  \( V(a) \)

  \( N_{DW} = 1 \quad \text{soon after formation} \)

  \( N_{DW} > 1 \quad \text{String-wall networks exist for a long time} \)

- Collapse of string-wall systems
- Annihilation of domain walls before they overclose the universe

\( \Omega_{a,\text{real}} \)

(i) Coherent oscillation
(re-alignment mechanism)
Production of axions in the early universe

(post-inflationary PQ symmetry breaking scenario)

Inflation

PQ symmetry breaking
- Formation of strings

QCD phase transition
- Axion acquires a mass
- Formation of domain walls

\[ T \lesssim F_a \approx 10^{8-11} \text{ GeV} \]

\[ T \lesssim 1 \text{ GeV} \]

(i) Coherent oscillation (re-alignment mechanism)
\[ \Omega_{a,\text{string}} \]

(ii) Radiation from strings
\[ \Omega_{a,\text{real}} \]

String-wall networks exist for a long time

Collapse of string-wall systems

Annihilation of domain walls before they overclose the universe
Production of axions in the early universe
(post-inflationary PQ symmetry breaking scenario)

**Inflation**
- Formation of strings

**PQ symmetry breaking**
- Axion acquires a mass
- Formation of domain walls

**QCD phase transition**
- Axion acquires a mass
- Formation of domain walls

- Coherent oscillation (re-alignment mechanism)
- Radiation from strings
- Wall decay

**Collapse of string-wall systems**
- Soon after formation

**Annihilation of domain walls**
- Before they overclose the universe

- \( N_{DW} = 1 \)
- \( N_{DW} > 1 \)

\( T \lesssim F_a \approx 10^{8-11} \text{ GeV} \)

\( T \lesssim 1 \text{ GeV} \)
Numerical simulation: $N_{DW} = 1$

Hiramatsu, Kawasaki, KS and Sekiguchi (2012)

- Solve the classical field equations on lattice
- Number of grids in simulation box: $N^3 = 512^3$
Spectrum of radiated axions

Contribution to relic abundance

\[
\frac{\langle \omega_a \rangle}{m_a} (t_{\text{decay}}) = 3.23 \pm 0.18
\]

\[
\rho_a (t_{\text{today}}) = m_a \frac{\rho_a (t_{\text{decay}})}{\langle \omega_a \rangle} \left( \frac{R(t_{\text{decay}})}{R(t_{\text{today}})} \right)^3
\]

\[
\rho_a (t_{\text{decay}}) \approx \rho_{\text{defects}}(t_{\text{decay}})
\]

Hiramatsu, Kawasaki, KS and Sekiguchi (2012)
Kawasaki, KS and Sekiguchi (2015)
Axion dark matter abundance ($N_{DW} = 1$)

- **Re-alignment mechanism**  
  Borsanyi et al. (2016), Ballesteros, Redondo, Ringwald and Tamarit (2016)

  \[ \Omega_{a,\text{real}} h^2 \approx (3.8 \pm 0.6) \times 10^{-3} \left( \frac{F_a}{10^{10} \text{ GeV}} \right)^{1.165} \]

- **Production from string-wall systems**

  \[ \Omega_{a,\text{string-wall}} h^2 \approx 1.2^{+0.9}_{-0.7} \times 10^{-2} \left( \frac{F_a}{10^{10} \text{ GeV}} \right)^{1.165} \]

- **Total axion abundance**

  \[ \Omega_{a,tot} h^2 \approx 1.6^{+1.0}_{-0.7} \times 10^{-2} \left( \frac{F_a}{10^{10} \text{ GeV}} \right)^{1.165} \]

\[ \Omega_{a,tot} \leq \Omega_{\text{CDM}} \]
\[ \Omega_{\text{CDM}} h^2 \approx 0.12 \]

- **Large uncertainty comes from estimation of the string density in numerical simulations**

\[ F_a \lesssim (3.8-9.9) \times 10^{10} \text{ GeV} \]
\[ m_a \gtrsim (0.6-1.5) \times 10^{-4} \text{ eV} \]
Models with $N_{DW} > 1$

- Domain walls are long-lived and decay due to the explicit symmetry breaking term
  \[ \Delta V = -\Xi \eta^3 (\Phi e^{-i\delta} + \text{h.c.}) \]

- The contribution from long-lived domain walls leads to the possibility that axions explain CDM at lower $F_a$ or larger $m_a$
  \[ \Omega_{a,\text{wall}} h^2 \approx (0.09 \ 0.17) \times \left( \frac{\Xi}{10^{-52}} \right)^{-1/2} \left( \frac{F_a}{10^9 \text{GeV}} \right)^{-1/2} \quad (\text{for} \ N_{DW} = 6) \]

- Several constraints on the explicit symmetry breaking parameter $\Xi$

  ➤ Poster presentation
  “Axion dark matter in the post-inflationary Peccei-Quinn symmetry breaking scenario”
Search for axion dark matter

Search space in photon coupling $g_{a\gamma} \sim \alpha/(2\pi F_a)$ vs. mass $m_a$

Mass ranges predicted in the post-inflationary PQ symmetry breaking scenario can be probed by various future experimental studies.

Ringwald and KS (2016)
Summary

- The scenario where PQ symmetry is broken after inflation is investigated
- Radiation from string-wall systems gives additional contribution to the CDM abundance
- Axion can be dominant component of dark matter if

\[ F_a \simeq (3.8-9.9) \times 10^{10} \text{ GeV} \]
\[ m_a \simeq (0.6-1.5) \times 10^{-4} \text{ eV} \]  
for \( N_{DW} = 1 \)

\[ F_a \simeq \mathcal{O}(10^8-10^{10}) \text{ GeV} \]
\[ m_a \simeq \mathcal{O}(10^{-4}-10^{-2}) \text{ eV} \]  
for \( N_{DW} > 1 \)

- Mass ranges can be probed in the future experiments
Backup slides
Astrophysical and cosmological constraints

- Astrophysical observations give lower (upper) bounds on $F_a (m_a)$
- Dark matter abundance gives upper (lower) bounds on $F_a (m_a)$ [and also a lower (upper) bound for DFSZ models]
- DFSZ models can explain CDM abundance at lower $F_a (m_a)$ due to the additional contribution from long-lived string-wall systems
• Open Fabry-Perot resonator and a series of current-carrying wire planes

• Searches for axion like particles in the 68.2-76.5μeV mass range were demonstrated

• Potentially searches in the mass range 40-400μeV in the future
\[ N_{DW} > 1: \text{long-lived domain walls} \]

- Domain walls are long-lived and decay due to the bias term
  \[ V_{\text{bias}}(\Phi) = -\Xi \eta^3 (\Phi e^{-i\delta} + \text{h.c.}) \]

- For small bias
  Long-lived domain walls emit a lot of axions which might exceed the observed matter density
  \[ \text{Cosmology} \rightarrow \text{large bias is favored} \]

- For large bias
  Bias term shifts the minimum of the potential and might spoil the original Peccei-Quinn solution to the strong CP problem
  \[ \bar{\theta} = \frac{2\Xi N_{DW}^3 F_a^2 \sin \delta}{m_a^2 + 2\Xi N_{DW}^2 F_a^2 \cos \delta} < 7 \times 10^{-12} \]
  \[ \delta : \text{phase of bias term} \]
  \[ \text{CP} \rightarrow \text{small bias is favored} \]

- Consistent parameters?
Constraints

- **Axion density** \( \Omega_{a,\text{mis}} + \Omega_{a,\text{string}} + \Omega_{a,\text{dec}} \leq \Omega_{\text{CDM}} \)
- **Neutron electric dipole moment (NEDM)** \( \bar{\theta} < 0.7 \times 10^{-11} \)
- **Astrophysical constraint (SN1987A)** \( F_a > 4 \times 10^8 \text{GeV} \)
Constraints

- Axion density \( \Omega_{a,\text{mis}} + \Omega_{a,\text{string}} + \Omega_{a,\text{dec}} \leq \Omega_{\text{CDM}} \)

- Neutron electric dipole moment (NEDM) \( \bar{\theta} < 0.7 \times 10^{-11} \)

- Astrophysical constraint (SN1987A) \( F_a > 4 \times 10^8 \text{GeV} \)
Numerical simulation

• Discretize the spatial coordinate

\[
\vec{x} \rightarrow (i, j, k)
\]

\[
i, j, k = 0, 1, \ldots, N - 1
\]

\[
\Phi(\vec{x}) \rightarrow \Phi_{i,j,k}
\]

\[
\nabla^2 \Phi(\vec{x}) \rightarrow (\nabla^2 \Phi)_{i,j,k}
\]

\[
= \frac{1}{12(\Delta x)^2} \left[ 16(\Phi_{i+1,j,k} + \Phi_{i-1,j,k} + \Phi_{i,j+1,k} + \Phi_{i,j-1,k} + \Phi_{i,j,k+1} + \Phi_{i,j,k-1}) \\
- (\Phi_{i+2,j,k} + \Phi_{i-2,j,k} + \Phi_{i,j+2,k} + \Phi_{i,j-2,k} + \Phi_{i,j,k+2} + \Phi_{i,j,k-2}) - 90\Phi_{i,j,k} \right]
\]

• Solve the classical EOM for complex scalar \( \Phi = \phi_1 + i\phi_2 \) on lattice

\[
\ddot{\phi}_i + 3H \dot{\phi}_i - \frac{\nabla^2}{\mathcal{R}^2(t)} \phi_i = - \frac{\partial V}{\partial \phi_i} \quad i = 1, 2
\]

• Number of grids in simulation box : \( N^3 = 512^3 \)
Numerical simulations: $N_{DW} = 1$

- Solve the classical EOM for complex scalar $\Phi = \phi_1 + i\phi_2$ on 3D lattice

$$
\ddot{\phi}_1 + 3H\dot{\phi}_1 - \frac{\nabla^2}{a^2}\phi_1 = -\lambda\phi_1(|\Phi|^2 - \eta^2) - \frac{\lambda}{3}T^2\phi_1 + m_a^2\eta
$$

$$
\ddot{\phi}_2 + 3H\dot{\phi}_2 - \frac{\nabla^2}{a^2}\phi_2 = -\lambda\phi_2(|\Phi|^2 - \eta^2) - \frac{\lambda}{3}T^2\phi_2
$$

- Include temperature dependence of axion mass \cite{Wantz&Shellard,PRD82,123508(2010)}

$$
\frac{m_a(T)^2}{F_a^2} = c_T\kappa^{n+4}\left(\frac{T}{F_a}\right)^{-n}
$$

$\kappa \equiv \Lambda_{QCD}/F_a = \Lambda_{QCD}/\eta$

| $c_T$  | 6.26 |
|-------|------|
| $c_0$ | 1.0  |
| $\lambda$ | 1.0 |
| $\kappa$ | 0.2-0.4 |

- Number of grids in simulation box: $N^3 = 512^3$

- (Comoving) Box size: $L = 20$ (\ $\Delta x = L/N \approx 0.039$)

- Numerical computation is carried out in SR16000 at the Yukawa Institute Computer Facility
Numerical simulations: $N_{DW} > 1$

- Solve the classical EOM for complex scalar field
  
  $\Phi = \phi_1 + i\phi_2$

  $\ddot{\phi}_1 + 3H\dot{\phi}_1 - \frac{\nabla^2}{a^2(t)}\phi_1 = -\lambda\phi_1(\phi_1^2 + \phi_2^2 - \eta^2) - \frac{\partial V_a}{\partial \phi_1}$

  $\ddot{\phi}_2 + 3H\dot{\phi}_2 - \frac{\nabla^2}{a^2(t)}\phi_2 = -\lambda\phi_2(\phi_1^2 + \phi_2^2 - \eta^2) - \frac{\partial V_a}{\partial \phi_2}$

  $\frac{\partial V_a}{\partial \phi_1} = -\frac{m^2n}{N_{DW}^2} (\cos \theta \cos N_{DW}\theta + N_{DW} \sin \theta \sin N_{DW}\theta)$

  $\frac{\partial V_a}{\partial \phi_2} = -\frac{m^2n}{N_{DW}^2} (\sin \theta \cos N_{DW}\theta - N_{DW} \cos \theta \sin N_{DW}\theta)$

- Parameters

|      |     |
|------|-----|
| $\lambda$ | 0.1 |
| $m/\eta$  | 0.1 |
| $N_{DW}$  | 2 - 6 |

- Number of grids in simulation box: $N^3 = 512^3$

- (Comoving) Box size: $L = 80$  ($\Delta x = L/N \simeq 0.156$)

- Note: we do not include the bias term ($\Xi$ term)

  - evolution of the stable networks
Numerical simulations: $N_{DW} > 1$

- $8192^2, 16384^2, 32768^2$ (2D) $\rightarrow$ decay time of domain walls

- $512^3$ (3D) $\rightarrow$ spectrum of radiated axions

Hiramatsu, Kawasaki, KS and Sekiguchi (2012)
Kawasaki, KS and Sekiguchi (2015)
Initial Conditions ($N_{DW} = 1$)

- Treat $\phi_1$ and $\phi_2$ as two independent real scalar fields with correlation function in the finite temperature

$$\langle \phi_i(k) \phi_i(k') \rangle = \frac{n_k}{E_k} (2\pi)^3 \delta^3(k + k') \quad i = 1, 2$$

$$\langle \dot{\phi}_i(k) \dot{\phi}_i(k') \rangle = E_k n_k (2\pi)^3 \delta^3(k + k')$$

- No correlation in the $k$ space

  - Generate $\phi_i(k)$ as Gaussian with

$$\langle |\phi(k)|^2 \rangle = \frac{n_k}{E_k} V_b \quad \langle |\dot{\phi}(k)|^2 \rangle = E_k n_k V_b$$

$$\langle \phi(k) \rangle = \langle \dot{\phi}(k) \rangle = 0$$

  - Fourier transform to obtain $\phi_i(x)$ and $\dot{\phi}_i(x)$

$$V_b \simeq (2\pi)^3 \delta^3(0)$$

: volume of the simulation box
Initial Conditions ($N_{DW} > 1$)

- Treat $\phi_1$ and $\phi_2$ as two independent real scalar fields with correlation function

\[
\langle \phi_i(\mathbf{k})\phi_i(\mathbf{k}') \rangle = \frac{1}{2k} (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \quad \phi = \phi_1 + i\phi_2
\]

\[
\langle \dot{\phi}_i(\mathbf{k})\dot{\phi}_i(\mathbf{k}') \rangle = \frac{k}{2} (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \quad (i = 1, 2)
\]

- No correlation in the $k$ space

- Generate $\phi_i(\mathbf{k})$ as Gaussian with

\[
\langle |\phi(\mathbf{k})|^2 \rangle = \frac{k}{2} V_b \quad \langle |\phi(\mathbf{k})|^2 \rangle = \frac{1}{2k} V_b
\]

\[
\langle \phi(\mathbf{k}) \rangle = \langle \dot{\phi}(\mathbf{k}) \rangle = 0
\]

\[
V_b \simeq (2\pi)^3 \delta^{(3)}(0)
\]

: volume of the simulation box

- Fourier transform to obtain $\phi_i(\mathbf{x})$ and $\dot{\phi}_i(\mathbf{x})$
Map of the phase of PQ field

- **green:** true vacuum, $\theta = 0$
- **blue:** $\theta = -\pi$
- **red:** $\theta = \pi$
- **domain wall:** width $\sim m_a^{-1}$
- **string:** width $\sim F_a^{-1}$
Identification of defects

String exists if $\Delta \theta > \pi$
Evolution of string-wall systems

- After the production, strings obey scaling solution

\[ \rho_{\text{string}} = \xi \frac{\mu}{t^2} \]

“O(1) strings in a horizon volume”

\[ \mu = \pi \eta^2 \ln \left( \sqrt{\lambda \eta t} / \sqrt{\xi} \right) \] : energy per length

- Walls also obey scaling solution

\[ \rho_{\text{wall}} = A \frac{\sigma}{t} \]

\[ \sigma \sim m_a F_a^2 \] : wall tension

- Scaling parameters

\[ \xi, A \sim O(1) \]

contain relatively large uncertainties
Evolution of long-lived domain walls

- Walls obey scaling solution if $\Xi = 0$: $\rho_{\text{wall}} = A \frac{\sigma}{t}$
- **Decay time** (estimated from the condition $\Xi \eta^4 \gtrsim A \sigma / t$)

\[
t_{\text{dec}} = C_d \frac{A \sigma}{\Xi \eta^4 [1 - \cos(2\pi N / N_{DW})]}
\]

- $C_d$ is determined from numerical simulation

\[
\left. \frac{A / V(\Xi)}{A / V(\Xi = 0)} \right|_{t_{\text{dec}}} = 0.1
\]

![Graph showing evolution of domain walls](image)

$C_d \simeq 2 - 5$
Area parameter

• Area parameters increase for large $N_{DW}$

\[ \rho_{wall} = A \frac{\sigma_{wall}}{t} \]

| $N_{DW}$ | $A(\tau_f)$ ($N = 8192, \tau_f = 160$) |
|---------|-----------------|
| 2       | 0.694 ± 0.113  |
| 3       | 1.10 ± 0.20    |
| 4       | 1.41 ± 0.13    |
| 5       | 1.84 ± 0.17    |
| 6       | 2.24 ± 0.21    |

• Slightly increase with time?

\[ A(\tau) = A_{form} \left( \frac{\tau}{\tau_{form}} \right)^{2(1-p)} \]

$p = 0.92–0.93$

• It is not clear whether this slight increase continues in later times, so we consider both two cases, “exact scaling” ($p=1$) and “deviation from scaling” ($p<1$)
String-wall contribution to CDM abundance

- On the mean energy $\langle \omega_a \rangle$ of axions radiated from string-wall systems

**Case A**

$\langle \omega_a \rangle \sim m_a$

Nagasawa and Kawasaki (1994)

- Radiated axion is mildly relativistic
- Contribution for DM abundance can be large

**Case B**

$\langle \omega_a \rangle \sim m_a \log\left(\frac{F_a}{m_a}\right)$

Chang, Hagmann and Sikivie (1999)

- Spectrum is hard
  \[dE/dk \sim 1/k\]
- Contribution for DM abundance is subdominant

\[\rho_a(t_{\text{today}}) = m_a n_a(t_{\text{today}}) = m_a \frac{\rho_a(t_{\text{decay}})}{\langle \omega_a \rangle} \left(\frac{R(t_{\text{decay}})}{R(t_{\text{today}})}\right)^3\]

$R(t)$: scale factor of the universe

- This controversy can be resolved by field theoretic lattice simulation of defect networks
Radiation of axions

- Compute power spectrum by using data of scalar field $\Phi(t, x)$ obtained by simulations

$$\frac{1}{2} \langle \dot{a}(t, k)^* \dot{a}(t, k') \rangle = \frac{(2\pi)^3}{k'^2} \delta^{(3)}(k - k') P(k, t)$$

$$\dot{a}(t, k) = \int d^3x e^{ik \cdot x} \dot{a}(t, x) \quad \dot{a}(t, x) = \text{Im} \left[ \frac{\dot{\Phi}}{\Phi} (t, x) \right]$$

- We overestimate the energy of axions if we include data on the defects

$$\dot{a}(t, x)$$

$$= \dot{a}_{\text{free}}(t, x) + \text{(contamination from defects)}$$

[Diagram showing radiated axions and higher energy]
Masking analysis

\( \alpha(x) \): contains contamination from defects

\( \alpha_{\text{free}}(x) \): use masked data only

\[
\frac{1}{2} \langle \hat{a}_{\text{free}}(k) \hat{a}_{\text{free}}(k') \rangle = \frac{(2\pi)^3}{k^2} \delta^{(3)}(k - k') P(k)
\]

Hiramatsu, Kawasaki, Sekiguchi, Yamaguchi and Yokoyama (2011)
Computation of the power spectrum (1)

- The moving defects can contaminate the spectrum
  \[ \dot{a}(t, \mathbf{x}) = \dot{a}_{\text{free}}(t, \mathbf{x}) + \text{(contamination from strings)} \]

- Introduce window function
  \[ W(\mathbf{x}) = \begin{cases} 
  0 & \text{(near strings)} \\
  1 & \text{(elsewhere)}
\end{cases} \]

- Masked axion field
  \[ \tilde{a}(\mathbf{x}) \equiv W(\mathbf{x}) \dot{a}(\mathbf{x}) = W(\mathbf{x}) \dot{a}_{\text{free}}(\mathbf{x}) \]

- We can compute the masked power spectrum
  \[ \tilde{P}(k) \equiv \frac{k^2}{V} \int \frac{d\Omega_k}{4\pi} \frac{1}{2} |\tilde{a}(k)|^2 \]

  This is different from the true power spectrum \( \langle \tilde{P}(k) \rangle \neq P_{\text{free}}(k) \)
Computation of the power spectrum (2)

- The true power spectrum is given by
  \[
  \frac{1}{2} \langle \hat{a}_{\text{free}}(t, \mathbf{k})^* \hat{a}_{\text{free}}(t, \mathbf{k}') \rangle = \frac{(2\pi)^3}{k^2} \delta^{(3)}(\mathbf{k} - \mathbf{k}') P_{\text{free}}(k, t)
  \]

- Define PPSE of \( P_{\text{free}}(k) \)

  \[
  P_{\text{PPSE}}(k) \equiv \frac{k^2}{V} \int \frac{dk'}{2\pi^2} M^{-1}(k, k') \tilde{P}(k')
  \]

  with a window weight matrix

  \[
  M(k, k') \equiv \frac{1}{V^2} \int \frac{d\Omega_k}{4\pi} \frac{d\Omega_{k'}}{4\pi} |W(\mathbf{k} - \mathbf{k'})|^2
  \]

  \[
  \int \frac{k''^2 dk''}{2\pi^2} M^{-1}(k, k') M(k', k'') = \frac{2\pi^2}{k^2} \delta(k - k'')
  \]

- It can be shown that \( \langle P_{\text{PPSE}}(k) \rangle = P_{\text{free}}(k) \)

Hiramatsu, Kawasaki, Sekiguchi, Yamaguchi & Yokoyama (2011)
Procedure to estimate the power spectrum

beginning to collapse

$t_i$ initial time of the simulation

$t_1$

string identification

masked map

evaluate $P(k, t_1)$

completion of collapse

$t_d \lesssim t_2$

string identification (if it exists)

masked map

evaluate $P(k, t_d)$

shift $P(k, t_1)$ into $t = t_d$

and subtract it from $P(k, t_d)$

$t_f$ final time of the simulation
Subtraction of pre-existing radiations

- Compute spectrum at two different times $t_1$ and $t_2$
- Subtract contributions radiated before $t_1$  

$t_1$: formation time of walls

$$\Delta P(k, t_2) = P(k, t_2) - P(k, t_1) \frac{\omega_a(k, t_2)}{\omega_a(k, t_1)} \left( \frac{R(t_1)}{R(t_2)} \right)^3$$

$$\omega_a(k, t) = \sqrt{m_a^2 + k^2/R^2(t)}$$
Averaged axion energy

- Dependence on $\kappa = \frac{\Lambda_{QCD}}{F_a}$

\[ \tilde{\epsilon}_w = \frac{\omega_a}{m_a} (t_{\text{decay}}) = 3.23 \pm 0.18 \]