Investigation of three lobes roots blower with special ejector

S.V. Vizgalov, G.N. Chekushkin, and M.V. Volkov

Kazan National Research Technological University, 68, K. Marx Str., Kazan, Russian Federation

E-mail: sv_kstu@rambler.ru

Abstract. A volumetric Roots type blower with spur three or two lobes rotors has a high performance at a small unit size and is used mainly at the pressure ratio in the range of 1.4 – 1.6. A significant limitation of this parameter is due to the nature of the compressor working process, in particular a rapid, almost instantaneous increase in the pressure in the working chamber because of back flow. This leads to a decrease in performance and efficiency, and increase of the compressor noise level. One of the ways to overcome this drawback is the use of bypassing of part of compressed and cooled gas at injection through the ejector into the working chamber that is simultaneously disconnected from both discharge and suction. Due to this, the pressure increase process becomes more gradual as related to the rotor angle of rotation, and additional suction of portions of gas from the suction line by ejection effect becomes possible, which increases the efficiency and performance of the compressor. Experimental data confirm the efficacy of this method. Research of the Roots blower combined action with an ejector attachments based on the developed mathematical model has been presented.

1. Introduction

The Roots blower belongs to positive displacement machines and has a feature of gas compression workflow associated with the working chamber volume stability transferred from the gas suction side to the gas discharge side of gas and great influence of gas backflow. As a result of this the compression process is almost isochoric, which leads to a drastic reduction in performance, volumetric and adiabatic efficiencies, pressure pulsation and noise level increase with increasing rotors speed and pressure in the discharge line. There are several well-known methods, which were adapted to the Roots blower with three or more lobes rotors. In keeping with those methods the high pressure gas is bypassed to the working chamber with the aim of the gas pulsation decrease in the discharge line. This method is not based on physical chamber volume reduction, but it helps to reduce the gas discharge pressure pulsation. It is implemented in the simplest way by executing the channels [1] on the internal surface of the housing and by gas bypassing without cooling it. Other studies provide compressed gas cooling in an integrated heat exchanger before the gas is bypassed through external piping connected to the compressor housing [2, 3]. In this case, the blower can be operated at higher pressure ratio of up to 1.8.

The more appropriate method is bypassing the compressed gas into the working camber of the blower through the ejector. This design of Roots blower with a discharge heat exchanger for cooling
gas and two ejectors is shown in Fig. 1. A part of gas in the amount of $m_{\text{byp}}$ is taken off through the pipelines 5 from the flow of gas compressed and cooled in the heat exchanger 4 and fed into the active ejector nozzle 6 wherein the expansion and acceleration of the gas flow and an additional fresh low pressure gas portion leak-in through the passive nozzle 8 into the mixing chamber 7 in the amount of $m_{\text{add}}$ take place. The total flow is supplied through the diffuser 9 and the rotary union 10 via the additional slit ports 3 located on the housing into the working cavity. The ports 3 are arranged along the entire length of the housing such that they are overlapped by the rotor when connecting the working cavity with the discharge port. Thus, we achieve both additional suction of fresh gas through the ejector and a more gradual increase in pressure within the cavity during the transfer thereof from suction to discharge during rotate of rotor. Adiabatic and volumetric efficiency of this supercharger layout depends on several factors: the location of the slit ports and the ejector nozzle geometric dimensions, depending on the pressure ratio of the supercharger.

The main objective of the study is to determine the effectiveness of this blower layout when operated at different discharge pressure conditions and a fixed position slit ports located on the housing blower. Besides, since the pressure ratio in the supercharger does not exceed 1.9 and the working gas is air, the ejector operating condition is subcritical and the nozzle has a conical shape with a decreasing cross section.

![Figure 1: Roots blower with ejectors](image-url)
2. Mathematical model

The simulated Roots blower has three lobes rotors, with a cycloid-circular rotor profile, theoretical capacity of 4,85 m$^3$/min, the rotor diameter of 113.7 mm, length of 150 mm, center distance between rotors is 75 mm. The working gas is air.

Design scheme of the supercharger is shown in Fig. 2. There are simultaneously 5 chambers in the machine at different stages of the working process designated as:

I – gas suction process into the working cavity of expanding volume V=f(φ), where the rotor rotation angle is φ=-γ_inlet - π/3 - γ_inlet (Fig. 2);

II, III – transfer of the constant volume cavity to the discharge side V=const; the cavity transfer angle is Δφ=π/3 + γ_inlet + γ_out;

IV – reverse gas leakage process (back flow) that starts at an angle of φ=2π/3 + γ_inlet + γ_out and ends at an angle of φ_{bf_end} provided pressure equalization in the adjacent cavities IV and V;

V – process discharge of gas from the decreasing volume cavity from the IV and V cavities unification movement until the rotation angle of φ=π/3 + φ_{bf_end}.

Figure 2: Roots blower design diagram

The dependence of the volume of each of the cavities on the angle of rotor rotation was determined by numerical integration, and are not given here.

At any position of the rotors determined by the angle φ, the cavities are displaced relative to each other to an angle of π/3. Gas leakage through the gaps occur between the chambers. Also, the chambers intercommunicate for some time with the inlet port - chamber I, gas discharge port - chamber V, as well as through the special slit ports with an ejector attachment - chambers II and III, through which the additional leakage of both fresh gas mass of m_{add} and discharge line bypassed flow mass of m_{byp} into the chamber takes place; the chamber IV is connected to the discharge line through the port a-b, the size of which is determined by the normal drawn from point ‘a’ onto the rotor profile; reverse gas leakage occurs through this port; this flow rate is determined by the pressure difference.
upstream and downstream of the port. Location of additional ports on the cylindrical part of the housing for ejector connection is symmetric and determined by the angle of $\gamma_{\text{eject}}$. The condition of absence of gas ejection through these ports is overlapping thereof by the rotor blade apex before the sharp rise in pressure in the reverse flow process; thus, the angle of $\gamma_{\text{eject}} \leq \pi/3 + \gamma_{\text{out}}$. The length of the port with respect to the angular coordinate is given by the value of $\Delta \gamma_{\text{eject}}$.

Simulated blower has the following design parameters:

\begin{align*}
\gamma_{\text{inlet}} &= 18,093 \, \text{deg}, \quad \gamma_{\text{out}} = 26,058 \, \text{deg}; \\
\gamma_{\text{eject}} &= 85 \, \text{deg}; \\
\Delta \gamma_{\text{eject}} &= 4.5 \, \text{deg}.
\end{align*}

The mathematical model of the Roots type compressor was developed on the basis of energy equilibrium equations in the working chamber and the ideal gas law. It takes into account the chamber volume change according to the rotor rotation angle $V=f(\varphi)$, the gas leak-in during the suction period, the gas mass transfer through the clearances between the adjacent chambers having various pressures, the gas heat exchange with the chamber walls, and the gas back-streaming into the working chamber at the discharge side. In this case the gas thermodynamic parameters in the volume of the working chamber are homogeneous at any period of time. The pressure and temperature gradients are zero in the chamber.

The equations of the model in the form of differential pressure and temperature dependences on the rotor rotation angle $\varphi$ are presented in the research works by authors Ibraev, Vizgalov [4, 5, 6]:

\begin{align*}
\frac{dp}{d\varphi} &= \frac{k-1}{k} \left( \frac{\omega \cdot dQ}{d\varphi} + \sum m_{i}^* \cdot h_{i}^* - \sum m_{i}^* \cdot h - \frac{k}{k-1} \cdot \omega \cdot \frac{dV}{d\varphi} \right) \\
\frac{dT}{d\varphi} &= \frac{(k-1) \cdot T}{P \cdot \omega} \left( \frac{\omega \cdot dQ}{d\varphi} + \frac{k-1}{k} \cdot \left( \sum m_{i}^* - \sum m_{i}^* \right) \cdot h + \sum m_{i}^* \cdot h_{i}^* - \sum m_{i}^* \cdot h - \frac{p}{k} \cdot \frac{dV}{d\varphi} \right)
\end{align*}

(1)

where $p, T$ are pressure and temperature of gas in the working chamber respectively, $V$- volume of chamber; $dQ$ is the elementary heat flux on the working chamber; $k$ is the adiabatic index ($k=1.4$ for air); $\omega$ is the rotor angular velocity, rad$\cdot$s$^{-1}$; $m$ is the mass flow through the ports and gaps. Indices: “inf” relates to the gas inflows into the chamber, “lks” relates to the leaks of gas from the chamber. The gas enthalpy in this case can be defined as for the ideal gas $h=C_{p} \cdot T$, where $C_{p}$ is the isobaric heat capacity of gas.

The elementary heat flux in the equation (1), taking into account the convective heat transfer between the gas and the walls of the chamber was determined by the equation

\begin{equation}
\frac{dQ}{dT} = \omega \cdot \frac{dQ}{d\varphi} = \bar{\alpha} (\varphi) \cdot f_{w} \cdot (T_{w} - T),
\end{equation}

(2)

where the heat transfer coefficient $\bar{\alpha}$ averaged over the surface of the walls $f_w$ was determined according to [7], where the experimental data were summarized by the equation of Nusselt for the periods of suction, transfer of the isolated chamber, and discharge; $T_{w}$ – temperature of wall (experimental).

Gas flow through the narrow clearances between the rotors and the housing compressor, between profiles of rotors were determined by the method proposed S.E. Zakharenko [8] by the following equation:

\begin{equation}
m = \sqrt{\frac{\rho_{1} \cdot \rho_{2} \cdot \ell \cdot \delta \cdot \left( \frac{\rho_{1}}{\rho_{2}} \right)^{\frac{\gamma}{\gamma - 1}}}{\ln \left( \frac{\rho_{1}}{\rho_{2}} \right) + \xi + \lambda \cdot \Delta}}, \quad \text{kg/s}
\end{equation}

(3)

where $\ell$, $\delta$ - clearance sizes, the front length and the minimum height of clearance; $\xi$ - coefficient of local resistances at the inlet and the outlet of the clearance due to a sudden contraction and expansion;
\( \lambda \) - friction coefficient of the gas in the clearance as a function of the Reynolds number, \( \Delta \) - shape factor of the clearance, were determined by work [8].

The blower operating area with respect to the pressure ratio value of \( \Pi = \frac{p_{\text{disch}}}{p_{\text{suc}}} \) is that the expansion ratio in the ejector nozzle is below the critical pressure ratio by using part of the compressed and cooled gas as the motive stream

\[
\frac{p}{p_{\text{disch}}} \leq \left( \frac{2}{k + 1} \right)^{\frac{1}{k-1}} = 0.529 ,
\]

where \( p \) is air pressure in the suction line, \( p = p_{\text{suc}} = 98,066 \text{ kPa} \); \( p_{\text{disch}} \) - pressure in discharge line (the maximum value is 187,0 kPa).

\[
\Delta p_{\text{chamb}} = p(\varphi) - p_{\text{suc}} - \text{is the difference of working chamber pressure and suction pressure; } \psi_1, \psi_2, \psi_3, \psi_4 \text{ are the velocity coefficients, recommended values based on the experimental results are } \psi_1=0.95, \psi_2=0.975, \psi_3=0.9, \psi_4=0.925 [9]; \text{ the parameter } \Pi_{a.p} \text{ is a gas-dynamic pressure function and is defined as}
\]

\[
\Pi_{a.p} = \frac{p_{\text{suc}}}{p_{\text{disch}}} = \frac{1}{\Pi} .
\]

The gas-dynamic velocity function of

\[
\text{Figure 3: Ejector design schematic}
\]
\[
\lambda_{a,p} = \frac{k+1}{k-1} \left(1 - \Pi_{a,p}^{k-1} \right),
\]
where \(\lambda = w_{\text{isent}}/w_{\text{sound}}\) is determined by the value \(\Pi_{a,p}\); the specific volume ratio is equal to

\[
v_a = \frac{P_{\text{disch}}}{p(\varphi)} T_a, \quad v_a = \frac{P_{\text{disch}}}{p_{\text{suc}} T_a};
\]

mixture temperature in the ejector is

\[
T_m = \frac{T_a + u \cdot T_{\text{suc}}}{1 + u};
\]

temperature of the motive stream fed to the nozzle is \(T_a = T_{\text{suc}} + \Delta T\); \(\Delta T\) is undercooling of gas in the heat exchanger.

The parameter in the equation (5) is defined as

\[
n = \frac{f_3}{f_3^*},
\]

where the gas-dynamic function \(q_{a,p} (q = \rho w_{\text{isent}}/\rho \omega)\) is defined as

\[
q_{a,p} = \left(\frac{k+1}{2}\right)^{1-k} \lambda_{a,p} \left(1 - \frac{\lambda_{a,p}^2 (k-1)}{k+1}\right)^{1/2};
\]

The parameter equal to the optimal ratio of the ejector mixing chamber area to the nozzle area is defined by the following equations

\[
\left\{ \begin{array}{l}
\frac{f_3}{f_3^*} = \frac{\sqrt{b^2 - 4a \cdot c}}{2a} - \frac{b}{2a}, \\
\end{array} \right.
\]

where coefficients are

\[
a = \psi_1 \cdot \psi_2 \cdot q_{a,p},
\]

\[
b = -\left[ \psi_1 \psi_2 + 2 \varepsilon_{a,p} \left( \frac{1}{\psi_3} - 0.5 \frac{v_m}{v_a} (1+u)^2 - (\psi_2 \psi_4 - 0.5 \frac{v_m}{v_a} u^2) \right) \right],
\]

\[
c = \frac{2 \varepsilon_{a,p}}{q_{a,p}} \left( \frac{1}{\psi_3} - 0.5 \frac{v_m}{v_a} (1+u)^2 \right)^{1/2},
\]

\[
\varepsilon_{a,p} = \left( \frac{1 - k-1}{k+1} \lambda_{a,p}^2 \right)^{1/2},
\]

where \(\varepsilon_{a,p}\) is the gas relative density \(\varepsilon = \rho_{\text{isent}}/\rho^n\).

The discharge bypass active flow consumption was defined as

\[
m_{\text{byps}} = \frac{k \cdot \Pi_s}{a_u} \cdot p_{\text{disch}} \cdot q_{a,p} F_{s1},
\]

where \(F_{s1}\) is the nozzle exit area \((d_{s1} \text{ is the determining size}), a_u \text{ is the motive flow critical speed in the nozzle defined as}

\[
a_u = \sqrt{\frac{2k}{k+1} RT_a}, \quad \Pi_s = \left( \frac{2}{k+1} \right)^{k/2},
\]

6
where $R$ is a gas constant.

Consumption of the additional passive nozzle inflow

$$m_{\text{add}} = u \cdot m_{\text{byp}},$$  \hspace{1cm} (17)

Thus, having the dependence

$$\frac{\Delta p_{\text{chamb}}}{p_{\text{suc}}} = f(u, d_1)$$  \hspace{1cm} (18)

and knowing the current pressure in the chamber $p(\varphi)$, the ejection coefficient value and the flow $m_{\text{add}}$ are defined.

The system of equations of the mathematical model was solved numerically using the method of Runge-Kutta-Feldberg [10]. Diagrams of pressure and temperature according to the angle of the rotor rotation resulted from solving the equations of mathematical model. Indicated power and indicated adiabatic efficiency of the blower were determined by integrating the curve $p = f(\varphi)$

$$N_{\text{ind}} = 6 \cdot \frac{\varphi}{2\pi} \int_{0}^{p(\varphi)} \varphi d\varphi, \quad \eta_{\text{ind}} = \frac{N_{\text{ind}}}{N_{\text{ad}}},$$  \hspace{1cm} (19), (20)

where $N_{\text{ad}}$ is the adiabatic compressor power.

Volumetric efficiency was determined as:

$$\eta_v = \frac{\pi \cdot (\sum m_{\text{disch}}(\varphi) - m_{\text{byp}})}{3\omega \cdot m_{\text{cham}}},$$  \hspace{1cm} (21)

where $\sum m_{\text{disch}}(\varphi)$ is the total value of the mass flow rate of gas through the outlet port to discharge line in during one revolution of the rotor, $m_{\text{cham}}$ is the mass of gas in isolated working chamber separated from the suction chamber at the angle $\varphi = (60^\circ - \gamma_{\text{inlet}})$.

3. Result and conclusions

Calculations of integral characteristics of Roots blowers was based on integration of mathematical model differential equations and finally are presented as dependence of volumetric efficiency $\eta_v$, adiabatic efficiency $\eta_{\text{ad}}$ and temperature increasing on pressure ratio $\Pi$ (The range of pressure ratio of 1.25 to 1.87). The points of gas injection on the housing (one of each side of the housing) of the Root's blower (angle $\gamma_{\text{eject}}$) and geometric parameters of the ejector were fixed during the all calculations (ejector's nozzle outlet diameter $d_1=5$ mm).

Calculated pressure-shaft angle and temperature-shaft angle diagrams of Roots blower were obtained using the developed mathematical model. The calculation results in the case of a pressure ratio of 1.6 and a rotating speed of 3000 rpm are presented in Figure 4. They include the Roots blower characteristics without the ejector (solid line), with the ejector and without bypass cooling (dashed line) and with the ejector and bypass cooling (dash-dotted line). In other working conditions the calculated characteristic curves have similar evolutions. The specific zones depending on the shaft angle and according to the schematic, which is presented in Figure 2, were denoted as I-V. There were two variants using the ejector: with and without cooling of the motive stream. In the case of the motive stream cooling its temperature was assumed 5 degree Celsius above the suction gas temperature (undercooling $\Delta T=5K$). It can be seen from the graph that in the case of using ejector the gas pressure starts increasing before the disconnection of the working chamber from the blower suction port (shaft angle of 20.8 deg.). It occurs due to the stream starting flowing through the ejector. For the following transporting process (zones II and III) there is the significant increase in the gas pressure, but at the same time the stream of the induced air falls rapidly. The rates of the temperature curves changes are similar to the rates of the pressure curves changes. However, influence of the motive stream cooling
on the temperature curves is more obvious. On the whole it should be noted that the motive stream cooling has a marginally impact on the pressure and temperature curves.

\[
\begin{align*}
\text{Figure 4: Influence of ejector and cooling bypass on } & P-\phi \text{ and } T-\phi \text{ diagrams (} \Pi = 1.6) \\
\text{Obtained integral parameters are presented in Fig. 5 (a, b), 6 where the curve 1 shows the characteristics of Root’s blower without ejector. In this case it should be noted that the volumetric efficiency and adiabatic efficiency decrease smoothly, but at the same time the discharge temperature increases significant. The volumetric efficiency has a parabolic behavior. The adiabatic efficiency has a quasi-linear behavior. The results of calculation of Root’s blower characteristics with ejector and}
\end{align*}
\]
without a cooling heat exchanger are presented by the curve 2. In this case the discharge temperature are reduced, which can be explained by an injection of low temperature gas from suction. At the same time the $\eta_v$ and $\eta_{ad}$ are increased and their decreasing rate depending on pressure ratio are reduced. It should be noted that the $\eta_v$ depending on pressure ratio has been increasing since $\Pi \approx 1.75$. In this case ejectors parameters are approaching to optimal and, hence, mass flow of injected gas from a suction are increased.

![Figure 5: Roots blower efficiency at pressure ratio](image)

(a) $\eta_v$ vs $\Pi$ (b) $\eta_{ad}$ vs $\Pi$

1 - Roots blower without ejector, 2 - with ejector and without cooling, 3 - with cooling $\Delta T=20K$, 4 - with cooling $\Delta T=5K$

![Figure 6: The temperature difference $\Delta T_{\text{blow}}=T_{\text{disch}}-T_{\text{suc}}$ at pressure ratio](image)

$\Delta T_{\text{blow}}$ vs $\Pi$

1 - Roots blower without ejector, 2 - with ejector and without cooling, 3 - with cooling $\Delta T=20K$, 4 - with cooling $\Delta T=5K$

Influences of motive stream temperature are presented by curves 3 and 4. The temperatures of motive stream have been taken as a function of suction temperature $T_s=T_{\text{suc}}+\Delta T$, where $\Delta T$ has been
taken 20K for the curve 3 and 5K for the curve 4 respectively. In this case the $\eta$ are increased by 0,5% on low pressure ratio side and by 1,65% on pressure ratio of $\Pi=1,8$ respectively. At the same time the increasing of $\eta_{ad}$ is absolutely insignificantly. It can be explained by increasing density of motive stream’s gas. Hence, there are no reasons to significant cooling motive stream.

The presented investigation shows advisability of using ejector for the Root blowers. But this investigation should be continued to determine the optimal ejector parameters depending on construction of Root’s blower and its nominal mode.

References
[1] US Pat. 4,215,977 Pulse-Free blower, 1980.
[2] PST/US88/04066 Hi-ratio reciprocation gas compressor, 1989.
[3] US Pat. 5,439,358 Recirculating rotary gas compressor, 1995.
[4] Ibraev A M, Vizgalov S V, Khisameev I G. 2013 Analysis of the basic geometrical parameters influence on the efficiency of the Roots-type compressor on the basis of thermodynamic processes simulation. 8th Int. Conf. on Compressors and their Systems (London: City University) 739.
[5] Vizgalov, S V, 2004, Vliyanie vnutrennego okhlazhdeniya na effektivnost rabocheho protsessa shesterenchatogo kompressora (Influence of internal cooling to efficiency of working process of Root's compressor(in Russian)). Dissertation, Kazan national research technology university. 212p.
[6] Ibraev, A.M., 1987. Povyshenie effektivnosti raboty rotornykh nagnetatelei vneshnego szhatiya na osnove analiza vliyaniya geometricheskikh parametrov na ikh kharakteristiki (Higher efficiency of rotary blower by analysis of geometry parameters to them characteristics (in Russian)). Dissertation, St.Petersburg State Polytechnical University. 208 p.
[7] Sharapov I I, Saifetdinov A G, Ibraev A M, Khamidullin M S, Khisameev I G 2013 Investigation of heat exchange in the working chamber of rotary compressor. 8th Int. Conf. on Compressors and their Systems (London: City University) 227.
[8] Zakharenko S. E. K voprosu o protechkakh gaza cherez shcheli (On the question of gas leakages through the gaps (in Russian)) Trudy LPI. - №2. - M.-L., Mashgiz, 1953. - 144-160 p.
[9] Sokolov, E., Zinger, N.M. Struinie apparati (Jet device (in Russian)), 1989. Moscow.: Energoatomizdat, 1989. – 352 p.
[10] Forsythe G., Malcolm, M., Moler C., 1977. Computer methods for mathematical computations. Prentice-Hall, Inc., Englewood cliffs, 1977.