This chapter presents the background and motivation for why this book came about. Needless to say, this book has two key words besides “sociophysics,” and these are “evolutionary game” and “mathematical epidemiology.” Game theory is a contemporary mathematical concept founded in the middle of the twentieth century by the milestone work of John von Neumann and Oskar Morgenstern; “Theory of Games and Economic Behavior” (Princeton University Press, 1944). Following their work, John Nash played an important role in applying the theory to various domains, including economics, information science, and biology; for his efforts, he was awarded the Nobel Prize in Economics in 1994 due to his establishment of the
concept of Nash Equilibrium.\footnote{Any standard textbooks on game theory will carefully explain the concept of Nash Equilibrium. For example, see Tanimoto (2014, 2015, 2019).} Briefly, game theory can be described as a mathematical template for modeling human decision-making processes. If one intends to address the time-evolutionary aspect of a dynamical system, this is called evolutionary game theory (EGT), as distinct from classic game theory, which focuses on a static situation in which several game players with several strategies mutually interact to maximize their benefits (called a “payoff”) at a certain moment. After the concept of complex science emerged in the 1990s and computational resources surged, EGT was combined with multi-agent simulation (MAS) to open up a new horizon through which it is possible to approach many complex problems related to human social systems that contain physical systems as subordinates. Such problems had previously been considered unsolvable because human behavior was seen as too stochastic to appropriately predict the dynamics of decision-making. A disease spreading in our society is a good example. Epidemiology—meaning the study of how an epidemic spreads on a human network—can be said to be sufficiently predictable because we know that it obeys a simple physics: the principle of diffusion phenomena. But the behavior of each individual is so varied, so prone to stochastic deviation, and so significantly influenced by information from the media that it is hard to predict how a disease will really spread through a complex human social system. In fact, whether one commits to pre-emptive vaccination or not is deeply related to the costs of the illness and the vaccination, and is also significantly influenced by the extent of the current outbreak as it stochastically evolves in time. This is quite a difficult task: but this book aims to address it in the following chapters.

To begin with, this chapter presents the holistic background that has motivated the author to produce the present book. We also introduce the concepts of complex human–physics systems, mathematical epidemiology, and evolutionary game theory.

1.1 Modeling of a Social–Complex System: A Human–Physics System

Most fields in science and engineering are primarily concerned with physical systems. Science specializes in understanding the physical phenomena of a natural, or even an artificial system, and building a mathematical model to describe the dynamics and underlying mechanisms of the phenomena. If that particular phenomenon brings profitable outputs like resources and energy to human society, engineering picks up where science leaves off so that the output can be maximized by means of careful design and optimized parameterization of the controlling variables. No matter what differences there are between systems, the most important issue in
such an approach is how the focal physical phenomenon can be modeled to ensure dynamical prediction and optimization.

Let us start with a simple and familiar example. We imagine the design process of an engine, as shown in Fig. 1.1. Under the given constraint(s), we seek an optimal design for the whole vehicle system, which requires the best-performing engine, as well as the best-streamlined shape of a body. To optimize the engine performance considering the multi-objective situation, in practice, we consider not only maximizing the fuel efficiency, but also minimizing the exhaust of SOx, NOx, and PM. Numerous engineers have made great efforts towards this end since the nineteenth century, and these efforts continue so as to control what happens in each cylinder of the engine, which is basically a problem of complex physics. At one time, this was very difficult; but now, the foundations for this research have been revealed by several sophisticated sciences, including combustion engineering, heat-transfer theory, thermodynamics, and fluid dynamics. All of this basically lies within the sphere of physics, which is more tangible and controllable than a fully stochastic system like human psychology, even if there are some noise effects resulting from the uncertainty of mechanical design and from material, fuel, and other considerations. This is because most of the phenomena involved in an internal-combustion engine can be traced by a set of laws of physics–mathematical equations.

Fig. 1.1 Designing an engine, which can be described fully in terms of physics. Breaking it down into pieces, we expect that a cylinder-level optimization is likely to result in an optimal design for the vehicle as a whole.
Here, we may notice that one crucially important element has been neglected in the discussion above: the human. If human commitment is not considered, you might miss out on the substance of reality. In this case, the subject is a human; that is a driver. By committing, as a driver, to a vehicle system, a human is able to draw a benefit. A driver may intend to maximize the performance of their engine through their control of the gas pedal; but in general, most drivers usually aim not to maximize their fuel efficiency nor to minimize their environmental impact, but to maximize their own benefit by minimizing the travel time to their destination. And this becomes much more complicated when there are many vehicles, with many drivers, each in turn driven by their respective decision-making processes. A society collecting many vehicles, say agents, might suffer a conflict between mutually competing agents individually seeking each local maximization, which consequently triggers a traffic jam. A jam inevitably leaves all agents with a meager level of benefit (see Fig. 1.2). Here, we must note that predicting human decision making is quite a difficult subject, as humans do not obey a set of deterministic equations like those of an engine. Rather, their behavior must be guessed through the empirical findings of experimental psychology with a fully stochastic mathematical framework.
Therefore, a precise discussion and prediction at the level of each cylinder, engine, or vehicle no longer works. We must account for the existence of a driver, and also consider a multiple-driver situation; this can be called a “society,” since the performance of an engine is totally controlled by its driver; and the driver’s control is fully dependent upon social context—how other drivers are driving and whether it results in a jam. Thus, we must model not only an independent physical system, but also simultaneously model the effect due to interference by the human who controls the system, in the context of a society that we can regard as an ensemble of individuals. All of these components—physics, human, and society—must be integrated into the same modeling template with a common spatiotemporal structure. This is the concept of a human–physics system from the standpoint of social physics (sociophysics).

We once introduced the concept of a human–environmental–social system, where the sphere of the “environment” implies an assembly of many physical sub-systems. For example, an urban climate system contains an urban block-scale model and a building-physics model as its subordinates while it is connected with a regional meso-scale climate model as a supra-structure (see Fig. 1.3). The human–physics system is a more generalized concept than the human–environmental–social

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2Tanimoto (2015, 2019).
system; the three are, respectively, driven by different principles, as schematically illustrated in Fig. 1.4. The physical, human, and social systems together compose a kind of a layer structure. They are mutually connected by relations between the action–reaction chain. If individuals try to extract much more benefit from a physical system, or if agents take extreme action towards it, the physics system may harshly react to them, depending on the inherent potential capacity that the physical system has. Thus, we must see the three subsystems as an integrated system, and must build up a certain comprehensive model. This is the most important kernel of a human–physics system.

As you might imagine, the dynamical system in which an epidemic spreads and individuals protect themselves based on own decisions is one of the most appropriate applications of the concept of a human–physics system, and is the subject of the present book.

1.2 How the Spread of an Infectious Disease Can be Modeled?—Mathematical Epidemiology

The counterpart to the “physics model” in the previous section in the context of an epidemic is provided by “mathematical epidemiology.” A communicable disease mediated by either bacteria or a virus spreads through human contact; typically, the frequency with which a focal agent engages in physical human contact is represented by their degree on the physical network with which they are involved. Here, “degree” means the number of links the agent has; or, say, the number of immediate neighbors around them. Thus, an agent with a higher degree would be exposed to a
higher risk of infection. Such agents are called “hubs.” Whether the focal agent would be infected when they have at least a single infectious neighbor depends on a certain stochastic process. The so-called transmission rate, usually denoted by $\beta$ [day$^{-1}$ person$^{-1}$], which indicates how frequently an infectious agent is communicated to a healthy agent, would differ from one agent to another depending on their specific situation. But speaking approximately, the spread of a disease on a social network connecting all agents in a society can be mathematically formulated by the physics of diffusion phenomena. Such physics are basically represented by a simple linear partial differential equation, $\frac{\partial C}{\partial t} = \kappa \frac{\partial^2 C}{\partial x^2}$, where $C$ is the concentration of the transferred object, $\kappa$ is so-called diffusion coefficient for the object, and $t$ and $x$, respectively, indicate time and space. If you have already introduced an underlying network into your model to represent physical contacts, the evolution of a physical system in time can be treated in a much easier way. The state of an agent is plural. Presuming what is known as SIR dynamics—one of the most general mathematical epidemiology models, which will be deliberately introduced in Chap. 3—we can note that there are three distinct subpopulations: Susceptible (S), meaning those who are healthy yet vulnerable to infection, Infected (I), and Recovered (R), meaning those who may never be infected again due to acquired immunity. The main question to be answered is whether or not an S agent gets infected (i.e., whether S transfers to I) if this S agent has an I agent in their neighborhood. The answer primarily depends upon the transmission rate $\beta$. Such a binary-state transfer from S to I is ubiquitously observed on a certain underlying network in many natural systems. A good analogy is that of a forest fire (see Fig. 1.5), which has been used as a template for percolation theory. Percolation theory is the mathematical model used to describe stochastic diffusion processes to emulate a “fire” (or an “epidemic” in the current discussion) spreading on a network. Inspired by Fig. 1.5, we can draw another schematic explanation; Fig. 1.6, which explains that mathematical epidemiology works as a model for describing the physical dynamics composing a human–physics system, thereby quantifying how an epidemic spreads on a human complex network. A more detailed mathematical framework is presented to readers in Chap. 3.

1.3 How Human Behavior Can be Modeled?—Evolutionary Game Theory

In this section, we should discuss the remaining component in a human–physics system: human commitment. Needless to say, a human is not like a tree in the woods in the case of a forest fire (Fig. 1.5). Trees are passive, and cannot take actions to protect themselves; but humans can.

3Not “infected” but “infectious”. Those two terms have different meanings, as discussed on Chap. 3.
Let us suppose a situation of exposure to the risk of an epidemic breakout. In fact, we have gained practical experience of this in the wake of the worldwide COVID-19 pandemic. Each individual has a different social background (nationality, income, gender, job, religion, etc.) and thus a different decision-making mechanism; however, they have several provisions available for avoiding infection, as schematically shown in Fig. 1.7.

Committing to pre-emptive vaccination has been thought effective against such epidemics as influenza; and a late vaccination can be another alternative to a pre-emptive one, by which an individual commits to a vaccination in the middle of a season, which might be thought more beneficial (information-advantageous, precisely speaking) than pre-emptive vaccination for someone who seeks to free ride on herd immunity (i.e., doing nothing despite others vaccinating). Because such an egocentric person may avoid infection without paying any cost for vaccination due to the shield of herd immunity, which can be regarded as a kind of public good.

As other alternatives, gargling and wearing a mask may be worthy of consideration. In fact, COVID-19 shows that these strategies are not insignificant at all although it is said that some American people disapprove. Such “intermediate defense measures” backed by new lifestyle habits are thought meaningful, since

![Forest fire modeled by percolation theory. The fire spreads to healthy trees. The front region consists a “wave” of burning, which can be likened to the I state of the SIR model. Sites of burned ruins can be likened to the R state, although in this case, they do not recover.](image-url)
the required cost is quite small, even if the expected effect might be smaller than that of medical provisions like vaccination.

Another new lifestyle habit we have learnt from COVID-19 is so-called social distancing. Central and local governments, as well as public-health authorities, have addressed this at length, because maintaining sufficient physical distance from others can directly reduce the transmission rate; $\beta$. A recommendation of “keeping social distance” encourages people to refrain from taking public transportation during rush hour, for an example, which may result in the emergence of a new lifestyle in our society.

Fig. 1.6  The role that the mathematical epidemiology playing to predict a disease spreading on a human complex network. Agents are connected by both physical network, actual physical contacts and virtual network, emulating a social relation mutually sharing information. Epidemic spreads on the physical network. If an $S$ agent who has an infectious neighbor, depending on transmission rate, $\beta$, he may be infected. If he spreads out some useful information about the disease to his neighbors, it would be helpful for them, since those neighbors are able to prepare the disease front comes to their site.
Several measures have been taken by governments and public-health authorities to suppress the spread of COVID-19 that have annoyed individuals and devastated the economy. The biggest of these is the so-called lock-down, whereby most activities in an area are forcefully subdued; thus, people must stay home basically all of the time, which certainly decreases transmission, but also brings down economic indicators. If the spread of COVID-19 could have been successfully controlled in the early stage of breakout in China, such fierce provisions all over the world may have been avoided. Perhaps we missed a chance to cope with the virus.

The key point is that the extent of each alternative intervention used to control the spread of a disease depends on each individual’s decision at a certain time and in a certain social context; however, its effect is shared with all people in a society. A situation of “expecting individual dedication but the fruits are given to all” carries with it a social dilemma. This is why we must dovetail the concept of evolutionary game theory with the physical process of the spread of disease.
Game theory provides us with a quite powerful quantifying framework for modeling human decision-making processes, including whether or not each strategy should be taken if an individual was to refer to its expected benefits. Evolutionary game theory adds the concept of dynamics to classic game theory. Chapter 2 presents some of the mathematical fundamentals of evolutionary game theory.

For last several years, there has been a wave of epidemiological studies in which mathematical epidemiology is coupled with the evolutionary game theory. The challenge of modeling such a human–physics system is highlighted in this book. Such studies have been concerned with what is known as the “vaccination game,” which is mathematically explained in Chap. 3. In the chapters following Chap. 3, we will deliver several applications considering various provisions besides vaccination, which we name the “intervention game”; this is a more advanced and resilient concept than the vaccination game.

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