A logical analysis of Stapp’s non-locality theorem

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Abstract

According to an argument proposed by Stapp, Quantum Mechanics violates the Locality Principle if the two hypotheses of Free Choices and No backward-in-time influence are assumed to hold, without the need of introducing hidden variables or criteria of reality. We develop an approach that endows the new hypotheses with a logico-mathematical formulation which allow us to perform an analysis of Stapp’s argument, based on ordinary, not counterfactual, logic. According to our results, the analyzed argument is not able to conclude that Quantum Mechanics violates the Locality Principle.

1 Introduction

In Physics the Principle of Locality can be expressed through the various conditions it implies. One particular locality condition is the following one.

(L) Locality Condition. In Nature, if $R_\alpha$ and $R_\beta$ are two space-time regions separated space-like, then operations completely confined in $R_\alpha$, whose realization depends on free choices made in $R_\alpha$, must have no influence in $R_\beta$.

An empirical theory is said to be a local theory if it is consistent with the principle of locality. Therefore, all predictions of a local theory must not contradict the locality condition (L).

Quantum Mechanics, as an empirical theory, establishes relationships between occurrences of physical events – including the occurrences of measurements’ outcomes – if the physical system is assigned a given state vector $|\psi\rangle$; the quantum theoretical predictions, all confirmed by the experimental observations so far performed, per se entail no violation of locality conditions. In fact, conflicts between Quantum Mechanics and locality arise only if further conditions, which do not belong to the genuine set of quantum postulates, are required to hold.
For instance, in order to show inconsistency between Quantum Mechanics and locality, Bell and his followers [1,2,3,4] had to introduce the existence of *hidden variables* as further condition, i.e. they needed to assign pre-existing values to observables which are not measured. The need for hidden variables is often based, in the literature, on the criterion or reality [5]:

(R) **Criterion of Reality.** If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

The strategy put into effect by these “classical” non-locality theorems for reaching inconsistency between Quantum Mechanics and locality consists in showing that under certain ideally realizable circumstances the value assignment entailed by the further conditions in agreement with quantum mechanics and with the locality principle is contradictory.

A different approach leading to the need for faster than light transfer of information was followed by Stapp [6]. He found unsatisfactory the alleged demonstrations of Bell and followers, just because they “rest explicitly or implicitly on the local-hidden-variable assumption that the values of the pertinent observables exist whether they are measured or not. That assumption conflicts with the orthodox quantum philosophy” [7]. Instead of assuming the further conditions entailing a value assignment to non-measured observables, he supplemented the standard quantum postulates with two new assumptions:

(NBITI) one assumption asserts that *once a measurement outcome has actually occurred in a region, no action in a space-like separated future region can change its value*;

(FC) the other assumption establishes that *given a concrete specimen of the physical system, the choice made in each region as to which experiment will be performed in that region is a localized free variable.*

Then he exploited a set of *quantum predictions*, we collectively refer to as (Q), for a particular physical setting [3]. This physical setting identifies four quantum observables $D^{(1)}$, $D^{(2)}$, $B^{(1)}$, $B^{(2)}$ and a state vector $|\psi\rangle$ such that, in particular, the operations for measuring $D^{(1)}$, $D^{(2)}$ are completely confined in a space-time region $\mathcal{R}_\alpha$ which is separated space-like from a region $\mathcal{R}_\beta$ where the operations for measuring $B^{(1)}$, $B^{(2)}$ are completely confined.

The conditions (NBITI), (FC), and (Q) constitute the hypotheses of Stapp’s theorem aiming to show a violation of the locality condition (L). The proof is hinged on a statement (SR), we make explicit later, having the status of a physical law within $\mathcal{R}_\beta$.

From the three hypotheses Stapp proved two propositions:

**Proposition 1.** If $D^{(2)}$ is measured in $\mathcal{R}_\alpha$ then (SR) holds in $\mathcal{R}_\beta$.

**Proposition 2.** If $D^{(1)}$ is measured in $\mathcal{R}_\alpha$ then (SR) does not hold in $\mathcal{R}_\beta$.

Therefore, following Stapp, the locality condition (L) is violated because the *validity of law (SR) in $\mathcal{R}_\beta$ turns out to depend on which measurement is freely chosen to perform in $\mathcal{R}_\alpha$.* This different non-locality proof is coherent with the starting motivation; indeed, no assignment of values to unmeasured observables is required.
Now, the conditions for the validity of the previously considered classical non-locality proofs [1]-[4] underwent deep analyses [8]-[10]; as a result, the possibility of recovering locality to quantum mechanics turns out to be open. For instance, in [8],[9] Adenier and Khrennikov show how a bias in the sample selection involved in Bell’s theorem allow for a local realistic explanation of the violation of Bell’s inequality. In [10] the theorems which adopt the criterion of reality as further condition, like the theorems of Bell and followers [1]-[3], have been analyzed. It has been shown that the theorems are valid if a wide interpretation of the criterion of reality is adopted. But they fail if the criterion is interpreted according to its strict meaning; therefore, these theorems can be re-interpreted as arguments against the wide interpretation of the criterion of reality rather than as proofs of non-locality.

The aim of the present work is to perform an analysis also of Stapp’s non locality theorem, to establish whether the possibility of recovering locality is open also in this case. The above explained profound difference between the logical mechanism of Stapp’s proof and that of Bell type theorems makes ineffective the methodologies used in [8]-[10]. Our method for the present case consists in endowing the theoretical apparatus with terms and relations which enable to express Stapp’s new concepts and their consequences as formal statements, so that Stapp’s argument can be reformulated into a language more suitable for a logical analysis. The results of such an analysis show that the proof of Proposition 2 drops into a logical pitfall within Stapp’s theoretical scheme developed by us. Therefore we cannot conclude that a violation of locality follows from Stapp’s argument.

In section 2 Stapp’s argument is described within a suitable theoretical apparatus. Once established the basic quantum formalism for describing the concepts at issue, in subsection 2.1 the hypotheses of the theorem are formulated. The logical structure of the argument is shown in subsection 2.2, where it is made clear that it yields the violation of locality if two specific statements, Proposition 1 but also Proposition 2 hold.

Section 3 is devoted to perform the analyses of the proofs of Proposition 1 and of Proposition 2 from the point of view of ordinary logic. To do this, in subsection 3.1 we express the consequences of the new hypotheses introduced by Stapp, namely (FC) and (NBITI), as formal statements within the theoretical apparatus developed for describing Stapp’s approach. Then the analyses of the proofs of Proposition 1 and Proposition 2 are respectively performed in subsection 3.2 and 3.3 on a logico-mathematical ground. While Proposition 1 turns out to be valid, the proof of Proposition 2 drops into an explicitly shown logical pitfall which invalidates the proof.

In section 4 we show how our work relates to the literature about the subject.

2 The theorem

Given a quantum state vector $|\psi\rangle$ of the Hilbert space $\mathcal{H}$ which describes the physical system, let $S(|\psi\rangle)$ be a support of $\psi$, i.e. a concrete set of specimens of the physical system whose quantum state is represented by $|\psi\rangle$. Let $D$ be any two-value observable, i.e. an observable having only two possible values denoted by $-1$ and $+1$, and hence represented by a self-adjoint operator $\hat{D}$ with purely discrete spectrum $\sigma(\hat{D}) = \{-1, +1\}$. 
Fixed any support $S(|\psi\rangle)$, every two-value observable $D$ identifies the following extensions in $S(|\psi\rangle)$:

- the set $D$ of specimens in $S(|\psi\rangle)$ which actually undergo a measurement of $D$;
- the set $D_+$ of specimens of $D$ for which the outcome $+1$ of $D$ has been obtained;
- the set $D_-$ of specimens of $D$ for which the outcome of $D$ is $-1$.

On the basis of the meaning of these concepts we can assume that the following statements hold (see [10], p.1268).

1. If $D$ is a two-value observable then for all $|\psi\rangle$ a support $S(|\psi\rangle)$ exists such that $D \neq \emptyset$;
2. $D_+ \cap D_- = \emptyset$ and $D_+ \cup D_- = D$;
3. If $\langle \psi | \hat{D} \psi \rangle \neq -1$ then $S(|\psi\rangle)$ exists such that $D_+ \neq \emptyset$, and if $\langle \psi | \hat{D} \psi \rangle \neq +1$, then $S(|\psi\rangle)$ exists such that $D_- \neq \emptyset$;
4. Two observables $D$ and $B$ are separated, written $D \bowtie B$, if their respective measurements require operations confined in space-like separated regions $R_\alpha$ and $R_\beta$.

According to standard quantum theory, two observables $D$ and $B$ can be measured together if and only if the corresponding operators commute with each other; therefore also the following statements hold for any pair of two-value observables $D$, $B$.

1. $[\hat{D}, \hat{B}] \neq 0$ implies $D \cap B = \emptyset$ for all $S(|\psi\rangle)$;
2. $[\hat{D}, \hat{B}] = 0$ implies $\forall \psi \exists S(|\psi\rangle)$ such that $D \cap B \neq \emptyset$.

Given a pair $D, B$ of two-value observables such that $[\hat{D}, \hat{B}] = 0$, we say that the correlation $D \rightarrow B$ holds in the quantum state $\psi$ if, whenever both $A$ and $B$ are actually measured, i.e. if $x \in D \cap B$, then $x \in D_+$ implies $x \in B_+$; so we have the following definition.

1. $D \rightarrow B$ if $x \in D_+$ implies $x \in B_+$, whenever $x \in D \cap B$.

This correlation admits the following mathematical characterization [11].

1. $D \rightarrow B$ if $\frac{1 + \hat{D}}{2} \frac{1 + \hat{B}}{2} \psi = \frac{1 + \hat{D}}{2} \psi$.

2.1 Hypotheses of the theorem

Now we explicitly establish the three premises of Stapp’s theorem.

(FC) Free Choices: “This premise asserts that the choice made in each region as to which experiment will be performed in that region can be treated as a localized free variable.” [6]

(NBITI) No backward in time influence: “This premise asserts that experimental outcomes that have already occurred in an earlier region [...] can be considered fixed and settled independently of which experiment will be chosen and performed later in a region spacelike separated from the first.” [6]

(Q) The third premise affirms the existence, as established by Hardy [3] according to quantum mechanics, of four observables $D^{(1)}, D^{(2)}, B^{(1)}, B^{(2)}$ and of a particular state vector $|\psi\rangle$, for a certain physical system, which satisfy the following conditions:
(q.i) $D^{(1)}$, $D^{(2)}$ are confined in a region $R_\alpha$ separated space-like from the region $R_\beta$ wherein the observables $B^{(1)}$ and $B^{(2)}$ are confined, with $R_\alpha$ lying in time earlier than $R_\beta$. Hence in particular $D^{(\cdot)} \succ B^{(\cdot)}, \ j,k \in \{1,2\}$.

(q.ii) $[\hat{D}^{(1)}, \hat{D}^{(2)}] \neq 0, \ [\hat{B}^{(1)}, \hat{B}^{(2)}] \neq 0; \ -1 \neq \langle \psi | \hat{D}^{(j)} | \psi \rangle \neq +1, \ -1 \neq \langle \psi | \hat{B}^{(j)} | \psi \rangle \neq +1$.

(q.iii) $[\hat{D}^{(j)}, \hat{B}^{(k)}] = 0, \ j,k \in \{1,2\}$ and in the state vector $|\psi \rangle$ the following chain of correlations holds.

\begin{itemize}
  \item a) $D^{(1)} \rightarrow B^{(1)}$
  \item b) $B^{(1)} \rightarrow D^{(2)}$
  \item c) $D^{(2)} \rightarrow B^{(2)}$
\end{itemize}

(q.iv) $S(|\psi \rangle)$ and $x_0 \in S(|\psi \rangle)$ exist such that $x_0 \in D^{(1)}_+ \cap B^{(2)}$.

In fact, this last condition is implied from the following non-equality satisfied by Hardy’s setting.

$$\left\langle \psi \left| \frac{1+\hat{D}^{(1)}}{2} - \frac{1-\hat{B}^{(2)}}{2} \right| \psi \right\rangle \neq 0. \quad (4)$$

Since the l.h.s. is nothing else but the quantum probability that a simultaneous measurement of $D^{(1)}$ and $B^{(2)}$ yields respective outcomes +1 and −1, the non-equality states that the correlation $D^{(1)} \rightarrow B^{(2)}$ does not hold. Therefore, by (3.i) it implies (q.iv).

### 2.2 Logical structure of the proof

The logical mechanism of the non-locality proof at issue is based on the following pivotal statement.

**Proposition 1.** If a measurement of $D^{(2)}$ is performed in region $R_\alpha$, then (SR) is valid. Equivalently, Statement (SR) holds in $R_\beta$ for all $x \in D^{(2)}$.

**Proposition 2.** If a measurement of $D^{(1)}$ is performed in region $R_\alpha$, then (SR) is not valid. Equivalently, Statement (SR) does not hold in $R_\beta$ for all $x \in D^{(1)}$.

If both these propositions hold, then the validity of statement (SR) in $R_\beta$ would depend on what is freely decided to measure in region $R_\alpha$, separated space-like from $R_\beta$; hence a violation of the locality condition (L) would happen.

In fact, Stapp gives his own proofs \cite{Stapp} that both Proposition 1 and Proposition 2 actually follow from the premises (FC), (NBITI) and (Q). Accordingly, the locality condition (L) should be violated if the three premises hold.
3 Logical analysis

In this section we shall examine, from a mere logical point of view, the proofs of Proposition 1 and Proposition 2 as drawn by Stapp. Let us begin by considering Proposition 1.

\textbf{Proposition 1.} \( x \in D^{(2)} \) implies (SR) holds for this \( x \).

\textbf{Stapp’s Proof.} “The concept of ‘instead’ [in (SR)] is given a unambiguous meaning by the combination of the premises of ‘free’ choice and ‘no backward in time influence’; the choice between \([B^{(2)}]\) and \([B^{(1)}]\) is to be treated, within the theory, as a free variable, and switching between \([B^{(2)}]\) and \([B^{(1)}]\) is required to leave any outcome in the earlier region \([R_0]\) undisturbed. But the statements \([q.iii.a] \) and \([h.iii.b]\) can be joined in tandem to give the result (SR)” [6].

The steps of this proof are carried out by appealing to their intuitiveness, rather than by means of the usual logico-mathematical methods; for instance, how the concept of instead can be given a unambiguous meaning, and which unambiguous meaning, is not explicitly shown. In this form the proof unfits for the aimed analysis. This lack is fulfilled in the following subsection; we shall endow the concept of ‘instead’ with a mathematical counterpart within the theoretical apparatus, in order to make explicit its role and formally verifiable the proofs.

3.1 The meaning of “instead”

In order to comply with the starting motivation of Stapp’s approach, the phrase

“\textit{if, instead, }\textit{B}^{(2)}\textit{ had been performed the outcome would have been }\textit{+1}”

in (SR) must entail no assignment of a pre-existing value +1 to \( B^{(2)} \), since \( B^{(2)} \) is a non-measured observable. Then the phrase must be interpreted as a \textit{prediction} about the outcome of a measurement of \( B^{(2)} \), which is valid also if the measurement of \( B^{(2)} \) is not performed. Of course, this kind of validity goes beyond the conceptual domain of standard Quantum Mechanics; but our task is just to establish whether it can be consistently introduced within Stapp’s theoretical framework where (NBITI) and (FC) add to standard Quantum Mechanics. Then we introduce the formal relation

\[ x \in \mathbb{B}_+^{(2)} \]

To indicate that the prediction in the phrase is \textit{valid} for a given specimen \( x \in \mathcal{S}(\ket{\psi}) \).

In general, given a two-value observable \( B \), by \( \mathbb{B}_+ \) (resp., \( \mathbb{B}_- \)) we shall denote a set of specimens \( x \in \mathcal{S}(\ket{\psi}) \) for which the prediction of the outcome +1 (resp., −1) for a measurement of \( B \) is valid, also if the measurement of \( B \) is not actually performed. The consistency of this new concept requires that the following statements hold.

\[ \mathbb{B}_+ \cap \mathbb{B}_- = \emptyset; \] (5.\textit{i})

\[ x \in \mathbb{B}_- \text{ implies } x \notin \mathbb{B}_+ \quad \text{and} \quad x \in \mathbb{B}_+ \text{ implies } x \notin \mathbb{B}_-. \] (5.\textit{ii})

As above noticed, statements such as \( x \in \mathbb{B}_+ \) or \( x \in \mathbb{B}_- \) do not make sense within standard Quantum Mechanics; our next step is to single out \textit{which conditions} make

\( x \in \mathbb{B}_+ \) or \( x \in \mathbb{B}_- \).
valid these kind of statements in a theory where the postulates of standard Quantum Mechanics are supplemented with the assumptions (FC) and (NBITI) of Stapp’s approach.

Let $D$ be a two-value observable confined in a region $\mathcal{R}_\alpha$ separated space-like from another region $\mathcal{R}_\beta$ where two other two-value observables $B$ and $F$ are confined, with $\mathcal{R}_\alpha$ located in time before $\mathcal{R}_\beta$; moreover, let the empirical implications $D \rightarrow B$ and $D \rightarrow F$ hold, with $[\hat{B}, \hat{F}] \neq 0$ and hence $B \cap F = \emptyset$ according to (2.iv). Let us suppose that in $\mathcal{R}_\alpha$ the observable $D$ is actually measured on a specimen $x \in S(|\psi\rangle)$ and that the outcome $+1$ is obtained; i.e., we are supposing that $x \in D_+$. Now, if later in $\mathcal{R}_\beta$ the observable $F$ is chosen to be measured on this $x$, i.e. if $x \in F$, then the prediction, made before measuring $F$, that the outcome will be $+1$ is valid according to standard Quantum Mechanics because the following conditions hold.

(C.1) The outcome of $D$ is not changed by the choice of measuring $F$, by (NBITI).
(C.2) Such an outcome is $+1$.
(C.3) $D \rightarrow F$ and $x \in D \cap F$, so that (3.i) applies.

But if we assume that also (FC) holds, then the choice of measuring $F$ or $B$ (or another observable) in $\mathcal{R}_\beta$ is free, so that at this stage, before choosing the measurement to perform in $\mathcal{R}_\beta$, $F$ and $B$ are on the same footing in our theory. Then the prediction that the outcome of a measurement of $B$ will be $+1$ must be equally valid with respect to the prediction for $F$, also if $B$ is not measured because $B \cap F = \emptyset$; hence in this case (FC) and (NBITI) imply that the relation $x \in B_+$ is valid. Thus we can conclude, in general, that the following statement hold.

$$(5.iii) \quad D \rightarrow B \quad \text{implies} \quad x \in D_+ \Rightarrow x \in B_+$$

if $D$ and $B$ are two-value observables respectively confined in space-like separated regions $\mathcal{R}_\alpha$ and $\mathcal{R}_\beta$, with $\mathcal{R}_\alpha$ located in time before $\mathcal{R}_\beta$.

Remark 3.1. It must be stressed that the stronger implication

$$(5.iv) \quad \text{If} \quad D \rightarrow B \quad \text{then} \quad x \in D_+ \Rightarrow x \in B_+$$

cannot be inferred under the same conditions for $D$ and $B$ by extending the argument which makes use of (NBITI) and (FC) for inferring (5.iii). Indeed, in (5.iv) no actual outcome for $D$ is available which enable us to invoke Quantum Mechanics for making a prediction of the outcome for $F$ or $B$. Moreover, the absence of the actual outcome forbids to apply (NBITI).

Hence, the supplementation of quantum mechanics with the assumptions (NBITI) and (FC) entails conditions (5.i)-(5.iii) which give the concept of “instead” an unambiguous consistent formulation. The new theoretical concepts make possible to re-formulate the crucial statement (SR) of Stapp’s argument in the following neologized simple form

$$(SR)_\nu \quad x \in B_+^{(1)} \quad \text{implies} \quad x \in B_+^{(2)}$$

$^1$The validity of conclusion (5.iii) seems to depend on the existence of an observable $F$ satisfying the conditions required by the argument. In fact, such an existence is ensured under quite general conditions. For instance, if such an observable $F$ does not exist in $\mathcal{R}_\beta$, it will exist in another region $\mathcal{R}_{\beta'}$ suitably shifted in space-time with respect to $\mathcal{R}_\beta$, so that the region $\mathcal{R}_\gamma = \mathcal{R}_\beta \cup \mathcal{R}_{\beta'}$ is still space-like separated from $\mathcal{R}_\alpha$ and in its future. Therefore our reasoning carried out with $\mathcal{R}_\gamma$ replacing $\mathcal{R}_\beta$ will lead to the conclusion (5.iii).
with evident gain in formal precision and clarity.

### 3.2 Proposition 1.

Now we can perform the analysis of Proposition 1’s proof.

**Proposition 1.** $x \in D(2) \Rightarrow (SR)_\nu$ holds in $R_\beta$.

**Logical expansion of the proof.**

(E.1) Let us suppose that the antecedent of Proposition 1 holds:

$$x \in D(2).$$

(E.2) Let us suppose that the antecedent of $(SR)_\nu$ holds too:

$$x \in B^{(1)}_\nu.$$  \hspace{1cm} (6.ii)

(E.3) Hence (6.i) and (6.ii) imply

$$x \in B^{(1)} \cap D(2).$$ \hspace{1cm} (6.iii)

(E.4) Then (q.iii.b), (6.ii) and (6.iii) imply

$$x \in D^{(2)}_\nu.$$ \hspace{1cm} (6.iv)

(E.5) (q.iii.c), (6.iv) and (5.iii) imply

$$x \in B^{(2)}_\nu.$$ \hspace{1cm} (6.v)

Since the validity of the steps from (E.3) to (E.5) is self-evident, in order that this re-worded proof be completely correct, it is sufficient to prove that specimens satisfying (6.i) and (6.ii) actually exist. Now, by (q.iii.b), (3.ii) and (q.ii) we have

$$\frac{1+\hat{B}^{(1)}_\nu}{2} \psi = \frac{1+\hat{B}^{(1)}_\nu}{2} \psi \neq 0.$$  \hspace{1cm} (5.iii)

Therefore $\langle \psi \mid \frac{1+\hat{B}^{(1)}_\nu}{2} + \frac{1+\hat{D}^{(2)}_\nu}{2} \psi \rangle \neq 0$. But this last is just the probability that a simultaneous measurement of $B^{(1)}$ and $D^{(2)}$ yields respective outcomes +1 and +1; being it non vanishing we have to conclude that specimens $x$ satisfying (6.i) and (6.ii) actually exist.

Thus our analysis does agree with Stapp’s conclusion that $(SR)$ holds if $D^{(2)}$ is measured in $R_\alpha$.

### 3.3 Proposition 2.

Now we submit Proposition 2 to our analysis.

**Proposition 2.** $x \in D^{(1)} \not\Rightarrow (SR)$ holds for this $x$ in $R_\beta$.

This Proposition is equivalent to the following statement.

$x_0 \in D^{(1)}$ exists such that the antecedent of $(SR)$ is true but the consequent is false

By making use of the neologized form $(SR)_\nu$, such a statement admits the following formulation.

$$\exists x_0 \in D^{(1)}, \quad x_0 \in B^{(1)}_\nu \quad \text{but} \quad x_0 \not\in B^{(2)}_\nu.$$ \hspace{1cm} (7)
Stapp’s Proof: “Quantum theory predicts that if \([D^{(1)}]\) is performed, then outcome \([+1]\) appears about half the time. Thus, if \([D^{(1)}]\) is chosen, then there are cases where \([x \in D^{(1)}_+]\) is true. But in a case where \([x \in D^{(1)}_+]\) is true, the prediction [(q.iii.a)] asserts that the premise of (SR) is true. But statement [(q.iv)], in conjunction with our two premises that give meaning to ‘instead’, implies that the conclusion of (SR) is not true: if \([B^{(2)}]\) is performed instead of \([B^{(1)}]\), the outcome is not necessarily \([+1]\), as it was in case \([D^{(2)}]\). So, there are cases where \([D^{(1)}]\) is true but (SR) is false.” [6]

Conclusion (7) is attained by Stapp through the following sequence of statements we translate from his proof.

\[(S.1) \text{A support } S(\psi) \text{ exists such that } D^{(1)}_+ \neq \emptyset. \]

\[(S.2) \ x \in D^{(1)}_+ \Rightarrow x \in B^{(1)}_. \]

\[(S.3) \text{The antecedent of (SR)}_
u \text{ holds } \forall x \in D^{(1)}_+. \]

\[(S.4) \exists x_0 \in D^{(1)}_+ \text{ such that } x_0 \in B^{(2)}_. \]

\[(S.5) \ x_0 \notin B^{(2)}_. \]

Let us now check the validity of each step.

Statement (S.1) holds by (2.iii) and (q.ii).

Statement (S.3) is implied from (S.1) and (S.2).

Statement (S.4) holds because of (q.iv).

Statement (S.5) holds because of (S.4) and (5.ii).

We see that all steps (S.1), (S.3), (S.4), (S.5) hold true according to a logical analysis.

But statement (S.2), is it valid? Statement (S.2) is nothing else but the translation into our language of the phrase “But in a case where \([x \in D^{(1)}_+]\) is true, the prediction [(q.iii.a)] asserts that the premise of (SR) is true” stated by Stapp in his proof. Hence, according to Stapp’s proof, (S.2) holds because of (q.iii.a) \([D^{(1)}] \rightarrow B^{(1)}\). But

\[x \in D^{(1)}_+ \Rightarrow x \in B^{(1)}_. \]

follows from \([D^{(1)}] \rightarrow B^{(1)}\) if \(x \in D^{(1)} \cap B^{(1)}\) holds too, because of (3.i). However, this last condition cannot hold for the specimen \(x_0\) considered in (S4), because it has been characterized by the two conditions \(x_0 \in D^{(1)}_+\) and \(x_0 \in B^{(2)}_.\). But if \(x_0 \in B^{(2)}_.\) holds then \(x_0 \in B^{(2)}\) obviously holds too, so that the premise of (SR), \(x_0 \in B^{(1)}_+\), cannot hold because \(B^{(1)}_+\) and \(B^{(2)}_+\) do not commute with each other and therefore \(B^{(1)}_+ \cap B^{(2)}_+ = \emptyset\), by (q.ii) and (2.iv).

Remark 3.2. In fact, a contradiction would arise if (5.iv) in remark 3.1 were valid. Indeed, the statement

\[x \in D^{(1)}_+ \Rightarrow x \in B^{(1)}_+ \]

(8)
can be deduced from the relation \([D^{(1)}] \rightarrow B^{(1)}\) stated by the prediction (q.iii.a) and by (5.iii). From (8), (q.iii.b) and (5.iv) we could deduce \(x \in D^{(2)}_+\); thereby \(x \in B^{(2)}_+\) would
follow, by (q.iii.c) and (5.iv). But \( x \in B^{(2)}_+ \) would imply \( x \notin B^{(2)} \), by (5.ii). Therefore, if (5.iv) were valid, the statement

\[
x \notin B^{(2)} \quad \text{for all} \quad x \in D^{(1)}_+
\]

would be true. This statement contradicts quantum mechanics prediction (q.iv). However, such a contradiction would be not a proof of Proposition 2, because it does not concern with the validity of \((SR)^\nu\). In other words, Proposition 2 is not even proved by assuming (5.iv). Rather, this contradiction would prove that Quantum Mechanics is not consistent with (NBITI) and (FC), if (5.iv) could be inferred from (NBITI) and (FC); but this is not the case, as stressed in remark 3.1.

4 Final comments

The object of the analysis performed in the present work is the final result of an investigation pursued by Stapp during many years [12]-[18] with the aim of proving inconsistency between Quantum Mechanics and locality without the need of attributing values to unmeasured observables. The earlier proposals [19]-[21] were explicitly based on counterfactual reasonings; they received severe criticisms [22],[23] which questioned the validity of the proof just on the ground of counterfactuals theory [24]. Stapp disputed [21],[25] these criticisms, but also expressed dissatisfaction with proofs based on counterfactual concepts: 

"[...] these theories, though useful in other ways, do not provide a completely adequate foundation for the study of the deep physical question of locality: basic physical conclusions should not rest on arbitrary conventions [which infiltrate counterfactuals theory]" [25]. Then he presented improvements of the proof, with the aim of making the argument valid without the need of rules of counterfactuals theory. [26],[27],[16],[17], until the final version [6] about which he states: “my 2004 proof, although retaining some of the trappings and language of counterfactual argumentation, is based on a substantially different foundation. The combination of my assumptions of ‘free choices’ and of ‘no backward-in-time influence’ amounts to the assumption that theories covered by my new work are to be compatible with the idea of ‘fixed past, open future’. This conceptualization circumvents, at the foundational level, the need for counterfactuals” [7].

However the debate has not reached a shared conclusion, because the criticisms about the counterfactual character of the proof continued [28]-[31]. On the other hand, Stapp always answered by essentially arguing that the new formulations of the proof do not make use of counterfactuals theory [18],[7].

The analysis performed in the present work differs from the previous criticisms. The key step of our work is realized in subsection 3.1. Here the theoretical apparatus of Quantum Mechanics has been endowed with the further statements and relations (5.i)-(5.iii) which formally express the new assumptions (FC) and (NBITI) introduced by Stapp. This is done without invoking counterfactual concepts. In particular, statement (5.iii) is the translation of the ‘fixed past, open future’ principle into an appropriate formulation within a coherent theoretical apparatus developed from Quantum Mechanics. Therefore our investigation is not based on a criticism of Stapp’s methods or on a rejection of his new concepts. Rather, we develop a coherent theoretical formulation from his conceptualization, where the proof can be formulated in logico mathematical
language and its validity verified by a purely logico-mathematical analysis which shows that according to our approach the proof is not valid.

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