Statistical Analysis of the Metropolitan Seoul Subway System: Network Structure and Passenger Flows

Keumsook Lee\textsuperscript{a,b}, Woo-Sung Jung\textsuperscript{c} Jong Soo Park\textsuperscript{d} M. Y. Choi\textsuperscript{c,e,*}

\textsuperscript{a}Department of Geography, Sungshin Women's University, Seoul 136-742, Republic of Korea
\textsuperscript{b}Center for Transportation Studies, Boston University, Boston, MA 02215, USA
\textsuperscript{c}Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA
\textsuperscript{d}School of Computer Science and Engineering, Sungshin Women’s University, Seoul 136-742, Republic of Korea
\textsuperscript{e}Department of Physics and Astronomy, Seoul National University, Seoul 151-747, Republic of Korea

Abstract

The Metropolitan Seoul Subway system, consisting of 380 stations, provides the major transportation mode in the metropolitan Seoul area. Focusing on the network structure, we analyze statistical properties and topological consequences of the subway system. We further study the passenger flows on the system, and find that the flow weight distribution exhibits a power-law behavior. In addition, the degree distribution of the spanning tree of the flows also follows a power law.

Key words: passenger flow, transportation, subway, power law

PACS: 89.75.Hc, 89.40.Bb, 89.65.Lm

Complex networks have been an active research topic in the physics community since models for complex networks were announced \cite{12}. Subsequently, numerous real networks observed in biological and social systems as well as physical ones have been studied \cite{3,4,5,6,7,8,9,10,11,12,13,14,15}. Those studied also include transportation systems such as airline networks, subway networks,

* Permanent address: Department of Physics and Astronomy, Seoul National University, Republic of Korea
and highway systems. For example, studies of world-wide airport networks as well as Indian and Chinese airport networks have disclosed small-world behaviors and truncated power-law distributions \[6,7,8,9,10\]. For the subway systems in Boston and Vienna, various network properties, such as the clustering coefficient and network size, have been reported \[11,12\]. Further, statistical properties of the Polish public transport network have been examined \[13\] and the Korean highway system has been analyzed with respect to the gravity model \[14\].

In this manuscript, we consider the Metropolitan Seoul Subway (MSS) system, which consists of \( N = 380 \) stations, and serves as the major public transportation mode in the greater Seoul area, Republic of Korea. The system in an earlier phase was analyzed with regard to the accessibility measurement \[16\]. In the present phase, the maximum distance between a pair of stations in the system is 126 km while the minimum value is 238 m. When we construct the subway network with \( N \) nodes, each corresponding to a station, the number of links connecting two nearest nodes turns out to be 424. The characteristic path length \( L \) is defined in terms of the network distance \( n_{ij} \), which represents the shortest path length between nodes \( i \) and \( j \). Usually, the clustering coefficient \( C \) also provides an important measure for a complex network. However, since a few nodes of the subway system have only one nearest neighbor, the clustering coefficient \( C \) is not well defined. Accordingly, we define the clustering coefficient \( C^* \) of the subway system, excluding the nodes which have one neighbor. The eccentricity of node \( i \) corresponds to the greatest distance between \( i \) and other node. The radius \( R \) and the diameter \( D \) of a network are then defined to be the minimum eccentricity and the maximum eccentricity, respectively, among all nodes. We further define the efficiency according to

\[
\epsilon \equiv \frac{1}{N(N-1)} \sum_{i \neq j} \frac{1}{n_{ij}}. \tag{1}
\]

In the ideal case for the efficiency \( \epsilon \), all nodes are connected to each other, so that the network of \( N \) nodes has \( N(N-1)/2 \) links. Normalizing the efficiency to that in the ideal case, we obtain the network efficiency \( E \), which takes a value between zero and unity: \( 0 \leq E \leq 1 \).

We carry out measurement of the above quantities for the MSS network and show the results in Table \[1\]. Note that unlike, e.g., an airline network, it is not conceivable for a subway network to have all-to-all connections between stations. The number of links in a real network of \( N \) stations should be far less than \( N(N-1)/2 \), thus leading to the small value of the network efficiency \( E \). On the other hand, it is desirable to build the network in such a way that the physical distance measured along the links between a pair of stations is as short as possible, compared with the actual distance (along the straight line) between the two. Accordingly, it is more appropriate to use the physical
distance $d_{ij}$ rather than the network distance $n_{ij}$ in measuring the network efficiency $E$. Whereas the value of $E$ in the network distance represents the efficiency with respect to the ideal but unrealistic network (with all-to-all connections), the value in the physical distance measures the efficiency with respect to the optimal network in reality. In terms of the physical distance, the characteristic path length, diameter, and radius are given by $L = 27.9$ km, $D = 139$ km, and $R = 69.8$ km, respectively.

Figure 1 exhibits the distribution of the shortest path lengths, (a) in terms of the network distance (or path length) and (b) in terms of the physical distance. Namely, the horizontal axis of Fig. 1(a) represents the number of links between a pair of nodes (stations) while that of (b) the physical distance between them.

We next investigate the passenger flows on the system. The MSS network operates a smart card system which keeps track of the travel information of every passenger. Here we analyze the passenger flows on a single day, based on the transaction data of the smart card on 24 June, 2005. The total number of transactions or passenger flows in the MSS network was as many as 4,909,316 on that specific day.

The weight $w_{ij}$ of a link between stations $i$ and $j$ is taken to be the sum of passenger flows in both directions on the link, i.e., $i \rightarrow j$ and $j \rightarrow i$. The strength $s_i$ of station $i$ is then defined to be the sum: $s_i = \sum_{j=1}^{N} w_{ij}$. Displayed in Fig. 2 are the obtained distributions of (a) weights and (b) strengths for the MSS network. It is observed that the weight distribution $P(w)$ apparently exhibits power-law behavior with an exponent around 0.56, albeit restricted due to the finite system size; in contrast, the strength distribution follows a log-normal function with a peak at $s \approx 4 \times 10^4$. Note that in the subway network the weight of a link connecting two stations represents the passenger flow between them and the strength of a station corresponds to the number of passengers arriving at and departing from that station. Accordingly, while passenger flows do not have a characteristic size, numbers of passengers at single stations do have. In a metropolis, most facilities are located near stations, so that each station is naturally abundant in passengers, the number of which reflects the capacity of facilities located near the station. The fact that a majority of stations are used by a similar number of passengers, corresponding to the peak of the strength distribution, thus indicates that with residential and commercial facilities taken into account, places near most stations are already developed fully to accommodate dense and compact location of facilities. The peak of the strength distribution thus gives a measure for the characteristic capacity of the facilities near a station in the fully urbanized Seoul.

Spanning trees are widely used to analyze a complex network [17]. Of particular interest is the minimum spanning tree, which is constructed with weights
not larger than those of other possible spanning trees. In the subway system, on the other hand, the link which has a larger passenger flow than others is important, demanding to consider the maximum spanning tree. Figure 3 shows the maximum spanning tree of passenger flows in the MSS system. We then compute the degree distribution $P(k)$ of the maximum spanning tree, which is shown in Fig. 4. It is observed to be consistent with a power-law behavior: $P(k) \sim k^{-\gamma}$ with exponent $\gamma \approx 1.7$, obtained from the least-square fit.

To summarize, we have analyzed the Metropolitan Seoul Subway system consisting of 380 stations, and obtained various network measurements including the path length, clustering coefficient, diameter, and radius as well as the efficiency of the network. The path length, diameter, and radius have also been computed in terms of the physical distance between stations. We have further investigated the passenger flows in the system, and constructed the maximum spanning tree of the flows. It is found that the weight distribution displays a power-law behavior whereas the strength distribution follows a log-normal one. Also revealed is the power-law behavior of the degree distribution of the spanning tree. The detailed analysis and implications are left for further study.

**Acknowledgements**

This work was supported by the Korea Research Foundation through Grant No. KRF-2006-B00022 and by the Sungshin Women’s University Research Grant of 2008.

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|                | network distance | physical distance |
|----------------|------------------|-------------------|
| $N$            | 380              | ⍺                 |
| $L$            | 20.0             | 27.9 km           |
| $n_{\text{max}}$ | 62               | 139 km            |
| $C^*$          | $6.41 \times 10^{-3}$ | ⍺                 |
| $D$            | 62               | 139 km            |
| $R$            | 31               | 69.8 km           |
| $E$            | $7.86 \times 10^{-2}$ | 0.747             |

Table 1
Statistical properties of the Metropolitan Seoul Subway network.
Fig. 1. Probability distribution of the shortest path length in terms of (a) the network distance $n$ and (b) the physical distance $d$ between stations on semi-log scales.
Fig. 2. Probability distribution of (a) weight $w$ on the log-log scale and (b) strength $s$ on the semi-log scale. Lines are guides to the eye. In (a) the line has the slope $\gamma = 0.56$ whereas the line in (b) represents a log-normal distribution.
Fig. 3. Maximum spanning tree of passenger flows, consisting 380 stations in the Metropolitan Seoul Subway system.
Fig. 4. Probability distribution of the degree $k$ of the maximum spanning tree in Fig. 3 on the log-log scale. The slope of the line, serving as a guide to the eye, is given by $-1.7$. 