NON DENTABLE SETS IN BANACH SPACES WITH SEPARABLE DUAL

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ABSTRACT. A non RNP Banach space $E$ is constructed such that $E^*$ is separable and RNP is equivalent to PCP on the subsets of $E$.

The problem of the equivalence of the Radon-Nikodym Property (RNP) and the Krein Milman Property (KMP) remains open for Banach spaces as well as for closed convex sets. A step forward has been made by Schachermayer's Theorem [S]. That result states that the two properties are equivalent on strongly regular sets. Rosenthal, [R], has shown that every non-RNP strongly regular closed convex set contains a non-dentable subset on which the norm and weak topologies coincide. In a previous paper ([A-D]) we proved that every non RNP closed convex contains a subset with a martigale coordination. Furthermore we established the $P\alpha\ell$-representation for several cases. The remaining open case in the equivalence of RNP and KMP is that of $B$-spaces or closed convex sets where RNP is equivalent to PCP in their subsets. Typical example for a such structure are the subsets of $L^1(0,1)$. H. Rosenthal raised the question if this could occur when the dual of the space is separable. W. James ([J2]) also posed a similar problem. The aim of the present paper is to give an example of a Banach space $E$ with separable dual failing RNP, and RNP is equivalent to PCP on its subsets. As consequence we get that $E$ does not contain $c_0(\mathbb{N})$ isomorphically and hence it does not embed into a Banach space with an unconditional skipped F.D.D. On the other hand $E$ semiembeds into a Banach space with an unconditional basis. The last property allows us to conclude that every closed convex non-RNP subset of $E$ contains a closed non-dentable set with a $P\alpha\ell$-representation. We recall that a closed set $K$ has a $P\alpha\ell$-representation if there is an affine, onto, one to one continuous map from the atomless probability measures on $[0,1]$ to the set $K$. In particular RNP is equivalent to KMP on the subsets of $E$. The space $E$ is realized by applying the Davis-Figiel-Johnson- Pelczynski factorization method to a convex symmetric set $W$ of a Banach space $E_{in}$ constructed in this paper. Finally as a consequence of the methods used in the proofs of the example we obtain that every separable $B$-space $X$ such that $X^{**}/X$ is isomorphic to $\ell^1(\Gamma)$ has RNP.

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We start with some definitions, notations and results necessary for our constructions.

A closed convex bounded set $K$ is said to be $\delta$-non dentable, $\delta > 0$, if every slice of $K$ has diameter greater than $\delta$. A closed convex set has RNP if it contains no $\delta$-non dentable set. A closed $K$ subset of a B-space has the P.C.P. if for every subset $L$ of $K$ and for all $\varepsilon > 0$ there exists a relatively weakly open neibhd of $L$ with diameter less than $\varepsilon$. It is well known that RNP implies P.C.P, but the converse fails [B-R].

In the sequel $D$ denotes the dyadic tree namely the set of all finite sequences of the for $a = \{0, \varepsilon_1, ..., \varepsilon_n\}$ with $\varepsilon_i = 0$ or 1. For $a$ in $D$ the length of $a$ is denoted by $|a|$. A natural order is induced on $D$, that is $a \prec \beta$ if the sequence $a$ is an initial segment of the sequence $\beta$. Two elements $a, \beta$ of $D$ are called incomparable if they are incomparable in the above defined order. We notice, for later use, that each $a$ in $D$ determines a unique basic clopen subset $V_a$ in Cantor’s group $\{0, 1\}^\mathbb{N}$ and $a, \beta$ are incomparable if $V_a \cap V_\beta = \emptyset$.

A basic ingredient in the definition of the space $E$ is Tsirelson’s norm as it is defined in [F-J]. We recall that the norm of this space satisfies the following implicit fixed point property.

For $x = \sum_{\kappa=1}^{m} \lambda_\kappa t_\kappa$

$$|| \sum_{K=1}^{m} \lambda_\kappa t_\kappa ||_T = \max\{ \max_\kappa |\lambda_\kappa|, \frac{1}{2} \sup_{j=1}^{n} \sum_{j=1}^{n} ||E_j x||_T \}$$

where the “sup” is taken over all choices

$$m < E_1 < E_2 < ... < E_n$$

$E_1, ..., E_n$ is an increasing sequence of intervals in the set of natural numbers and $E_j x$ is the natural projection of $x$ in the space generated by vectors of the basis $\{t_k : k \in E_j\}$ Tsirelson’s space is a reflexive Banach space with an unconditional basis not containing any $\ell^p$ for $1 < p < \infty$.

1.a The space $E_u$

The space $E_u$ will be defined to have an unconditional basis indexed by the dyadic tree $D$ and denoted by $(e_a)_{a \in D}$. For a sequence of reals $(\lambda_\alpha)_{\alpha \in D}$ which is eventually zero we define

$$|| \sum_{a \in D} \lambda_a e_a || = \sup\{ || \sum_{i=1}^{\ell} \lambda_{a_i} t_{k_i} ||_T : \{a_i\}_{i=1}^{\ell} \text{ are incomparable,}$$

$$|a_i| = \kappa_i, \kappa_1 < \kappa_2 < ... < \kappa_\ell \}.$$
1.1 Proposition. The dual of the space $E_u$ is separable.

Proof. The spare $E_u$ has an unconditional basis hence it is enough to show that $\ell^1$ does not embed into $E_u$ $[J_1]$.

Suppose, on the contrary, that $\ell^1$ embeds into $E_u$. Then, by standard arguments, we can find $\ell_1 < \ell_2 < ... < \ell_k < ...$ an increasing sequence of natural numbers and \{x_k\}_{k=1}^{\infty} a normalized sequence in $E_u$ equivalent to the usual basis of $\ell^1$, and

$$x_k = \sum_{\ell_k < |\alpha| < \ell_{k+1}} \lambda_\alpha e_\alpha$$

The definition of the norm of $E_u$ and elementary properties of Tsirelson’s norm show that

$$||\sum_{k=1}^{m} \mu_k x_k|| \leq ||\sum_{k=1}^{m} \mu_k t_{\ell_{k+1}}||_T$$

so \{t_{\ell_k}\}_{k=2}^{\infty} is equivalent to the basis $\ell^1$. This contradicts to the reflexivity of $T$. □

A consequence of the above Proposition is that the basis $(e_\alpha)_{\alpha \in D}$ is shrinking. Therefore every $x^{**}$ in $E_u^{**}$ has a unique representation as

$$x^{**} = w^* \lim_{n \to \infty} \sum_{|\alpha| \leq n} \lambda_\alpha e_\alpha := w^* - \sum_{\alpha \in D} \lambda_\alpha e_\alpha$$

and $\lambda_\alpha = < x^{**}, e_\alpha^* >$.

We define the support of $x^{**}$, denoted by $\text{supp } x^{**}$, to be the set

$$\{\alpha \in D : < x^{**}, e_\alpha^* > \neq 0\}.$$
and

\[ \| P_{[n, \infty)} (x^{**}) \| = \lim_{m \to \infty} \| P_{[n,m]} (x^{**}) \| \]

To establish the result it is enough to show that for \( \varepsilon > 0 \) there exists \( n(\varepsilon) \) such that for all \( m > n(\varepsilon) \)

\[ \| P_{[m, \infty)} (\sum_{i=1}^{k} x_i^{**}) \| \geq \frac{1}{2} \sum_{i=1}^{k} d(x_i^{**}, E_u) - \varepsilon. \]

Actually \( n(\varepsilon) = \max\{ \kappa, |a_1|, \ldots |a_k| \} \).

Choose any \( m > n(\varepsilon) \). Inductively we define \( \{ q_i, \ell_i \}_{i=1}^{\kappa} \) such that

\[ m < q_1 < \ell_1 < \ldots < q_k < \ell_k \]

and \( \| P_{[q_i, \ell_i]} (x_i^{**}) \| > d(x_i^{**}, E_u) - \frac{\varepsilon}{2^k} \).

For each \( 1 \leq i \leq k \) there is a set \( \{ \beta_j^i : 1 \leq j \leq s(i) \} \) of incomparable elements of \( D \) such that \( q_i \leq |\beta_j^i| \leq \ell_i \) and

\[ \| P_{[q_i, \ell_i]} (x_i^{**}) \| = \| \sum_{j=1}^{s(i)} \lambda^i_{\beta_j^i} t_{|\beta_j^i|} \|_T. \]

Notice that \( a_i < \beta_j^i \) for all \( j = 1 \ldots s(i) \).

Observe that \( \bigcup_{1 \leq i \leq k} \{ \beta_j^i : 1 \leq j \leq s(i) \} \) consists of pairwise incomparable elements. So

\[ \| P_{[m, \infty)} (\sum_{i=1}^{\kappa} x_i^{**}) \| \geq \| P_{[m, \ell_k]} (\sum_{i=1}^{\kappa} x_i^{**}) \| \geq \]

\[ \| \sum_{i=1}^{\kappa} \sum_{j=1}^{s(i)} \lambda^i_{\beta_j^i} t_{|\beta_j^i|} \|_T \geq \frac{1}{2} \sum_{i=1}^{\kappa} \| \sum_{j=1}^{s(i)} \lambda^i_{\beta_j^i} t_{|\beta_j^i|} \|_T \]

\[ = \frac{1}{2} \sum_{i=1}^{\kappa} \| P_{[q_i, \ell_i]} (x_i^{**}) \| \geq \frac{1}{2} \sum_{i=1}^{\kappa} d(x_i^{**}, E_u) - \varepsilon \quad \Box \]

Consider the following closed convex subset of the unit ball of \( E_u \)

\[ K = \{ x \in E_u : x = \sum_{n=0}^{\infty} \sum_{|a|=n} \lambda_a e_a, \lambda_0 = 1, \lambda_a \geq 0, \lambda_a = \lambda_{(a,0)} + \lambda_{(a,1)} \}. \]

It is easily verified that \( K \) is the closed convex hull of a \( \frac{1}{2} \)-tree \( (d_\alpha)_{\alpha \in D} \) where for every \( a \) in \( D \) \( d_\alpha \) is defined by the conditions \( e_{(\alpha)}^* (d_\alpha) = 1, e_{(\alpha,0)}^* (d_\alpha) = e_{(\alpha,1)}^* (d_\alpha) = \frac{1}{2} e_{(\alpha)}^* (d_\alpha) \) and \( d_\alpha \in K \).

We set \( W = co(K \cup -K) \) and we denote by \( \tilde{W} \) its \( w^* \) closure in \( E_u^{**} \). Notice that \( x^{**} \in \tilde{W} \) if \( |e_{(\alpha)}^* (x^{**})| \leq 1, e_{(\alpha,0)}^* (x^{**}) + e_{(\alpha,1)}^* (x^{**}) = e_{(\alpha)}^* (x^{**}) \) for all \( a \) in \( D \). Hence we could define a map

\[ T : M ((0, 1)^{\mathbb{N}}) \to \tilde{W} \]
with the rule
\[ T(\mu) = w^* - \sum_{\alpha \in D} \mu(V_\alpha)e_\alpha \]
where \( V_\alpha = \{ \gamma \in \{0, 1\}^\mathbb{N} : \gamma \uparrow |\alpha| = a \} \)
Clearly \( T \) is one to one and onto. Furthermore
\[ ||T(\mu)|| \leq \sup \{ \sum_{i=1}^k |\mu(V_{\alpha_i})| : \{\alpha_i\}_{i=1}^k \text{ incomparable} \} = ||\mu||. \]
Hence \( T \) is extended to a bounded linear operator from \( M(\{0, 1\}^\mathbb{N}) \) onto the linear span of \( \hat{W} \) denoted by \( <\hat{W}> \).

1.b The Space \( E \)

The space \( E \) is the result of the application of Davis-Figiel-Johnson-Pelczynski [D] factorization method to the set \( W \) defined above.

We give the precise definition and certain properties of the space \( E \).

\[ E = \{ y \in E_u : |||y||| = (\sum_{n=1}^{\infty} ||y||_n^2)^{\frac{1}{2}} < \infty \} \]
Here \( |||\cdot|||_n \) denotes the Minkowski’s gauge of the set \( 2^nW + \frac{1}{2^n}B_{E_u} \).

Let \( J : E \to E_u \) be the natural injection. We notice that \( J[B_E] \) contains the set \( W; \) hence \( E \) fails RNP.

The operator \( J \) satisfies the following properties.

P.1.: \( J^{**} : E^{**} \to E_u^{**} \) is one to one and \( J^{**}[E^{**}] \cap E_u = J[E] \).

As consequence of this property \( E^* \) is separable.

P.2.: \( J \) is a weak to weak homeomorphism on the bounded subsets of \( E \). This is a consequence of P.1 and it implies that \( J[L] \) is closed for all \( L \), closed convex bounded subsets of \( E \). In particular \( J \) is a semiembedding.

P.3.: Let \( L \) be a closed convex bounded subset of \( E \) failing RNP. Then \( J[L] \) is non RNP. If not, \( J[L] \) is an RNP set, hence for any \( L \)-valued operator \( S : L^1 \to E \) the operator \( JoS \) is representable by a function \( \varphi \) in \( L_{j[L]}^\infty \). Then the function \( \Psi = J^{-1}\varphi \) represents the operator \( S \) and \( K \) is RNP. ([B - R])

P.4.: If \( L \) is bounded subset of \( E \) and \( J[L] \) fails P.C.P. then \( L \) fails P.C.P.

Indeed, for \( \{y_n\}_{n=1}^{\infty} \) in \( L \) such that \( J(y_n) \to J(y) \) and \( ||J(y_n) - J(y)|| > \delta > 0 \) P.2. ensures that \( y_n \to y \) and also \( ||y_n - y|| > \frac{\delta}{||J||} \). Hence \( y \) is not a point of continuity.

P.5.: \( J^{**}[E^{**}] \subseteq <\hat{W}> \)

For this, notice that \( B_{E^{**}} \subseteq 2^n\hat{W} + \frac{1}{2^n}B_{E_u^{**}} \) hence
\[ J^{**}[B_{E^{**}}] \subseteq \cap_n(2^n\hat{W} + \frac{1}{2^n}B_{E_u^{**}}) \subseteq <\hat{W}>. \]

We proceed to the proof of the main property of the space \( E \).
1.3 Proposition. Let \( K \) be a closed, convex, bounded, non RNP subset of \( E \). Then \( K \) fails P.C.P.

Proof. Property 3, mentioned before, ensures that \( J[K] \) is non RNP closed subset of \( E \). Hence for some \( \delta > 0 \) there exists a convex closed \( L \) subset of \( J[K] \) which is \( \delta \)-nondentable. Our goal is to show that every weak neighd in \( L \) has diameter greater than \( \frac{\delta}{256} \). By a result due to Bourgain \([B]\) it is enough to show that for every \( S_1, S_2, \ldots, S_n \) slices of \( \tilde{L} \) there exists \( x^{**}_i \) in \( S_i \) \( i = 1, 2, \ldots, n \) such that for all \( (\lambda_i)_{i=1}^n \in \mathbb{R}_+^n \)

\[
d(\sum_{i=1}^n \lambda_i x^{**}_i, E_u) > \frac{\delta}{256}
\]

Given \( S_1, S_2, \ldots, S_n \) slices of \( \tilde{L} \). Using Lemma 2.7 from [R] we choose \( (x^{**}_{\xi,i})_{\xi \omega_1} \) an uncountable subset of \( S_i \) such that

\[
d(x^{**}_{\xi,i} - x^{**}_{\zeta,i}, E_u) > \frac{3\delta}{8} \text{ for } \xi \neq \zeta.
\]

Recall that \( \tilde{L} \) is a subset of \( J^{**}[E^{**}] \subset \overline{<W>} \) and that \( T[M\{0,1\}^N] \) is norm dense into \( \overline{<W>} \). Hence there are \( (\mu_{\xi,i})_{\xi \omega_1, i \leq n} \) such that

\[
||T\mu_{\xi,i} - x^{**}_{\xi,i}|| < \frac{\delta}{256}
\]

Also, it is known that \( M(\{0,1\}^N) = (\sum_{\gamma < 2^\omega} \oplus L^1(\lambda_\gamma))_1 \)

where \( \{\lambda_\gamma\}_{\gamma < 2^\omega} \) are pairwise singular probability measures on \( \{0,1\}^N \) and \( L^1(\lambda_\gamma) = L^1[0,1] \) or \( L^1(\lambda_\gamma) = \mathbb{R} \).

Therefore

\[
\mu_{\xi,i} = \sum_{\gamma < 2^\omega} \frac{d\mu_{\xi,i}}{d\lambda_\gamma}
\]

where the sum is taken in \( \ell_1 \)-norm.

Choose \( F_{\xi,i} \) finite subset of \( 2^\omega \) so that the measure \( \mu'_{\xi,i} = \sum_{\gamma \in F_{\xi,i}} \frac{d\mu_{\xi,i}}{d\lambda_\gamma} \) satisfies

\[
||T\mu'_{\xi,i} - x^{**}_{\xi,i}|| < \frac{\delta}{256}
\]  \hspace{1cm} (1)

In particular for \( \xi \neq \zeta \) we get

\[
d(T\mu'_{\xi,i} - T\mu'_{\zeta,i}, E_u) > \frac{\delta}{256}
\]  \hspace{1cm} (2)
Apply Erdős-Rado’s Lemma [C-N] to the family \( \{ F_\xi = \bigcup_{i=1}^{n} \xi_i, \xi < \omega_1 \} \) and find \( A \) uncountable, \( F \) finite such that for \( \xi \neq \zeta \in A \)

\[
F_\xi \cap F_\zeta = F.
\]

We set \( \lambda_F = \sum_{\gamma \in F} \lambda_\gamma \) and for \( \xi \in A \)

\[
\nu_{\xi} = \mu_{\xi} - \frac{d\mu_{\xi,i}}{d\lambda_F}
\]

**Claim:** For all \( i = 1, \ldots, n \) the set \( B_i = \{ \xi \in A : d(T\nu_{\xi,i}, E_u) \leq \frac{\delta}{16} \} \) is at most countable.

**Proof of the Claim.** Suppose that for some \( i \) the set \( B_i \) is uncountable. Then, since \( L^1(\lambda_F) \) is separable, there are \( \xi \neq \zeta \) in \( B_i \) such that

\[
|| \frac{d\mu_{\xi,i}}{d\lambda_F} - \frac{d\mu_{\zeta,i}}{d\lambda_F} || < \frac{\delta}{16}
\]

But then

\[
d(T\mu'_{\xi,i} - T\mu'_{\zeta,i}, E_u) < \frac{\delta}{4}
\]

which contradicts inequality (2) and this completes the proof of the claim.

Choose \( \xi_1 < \xi_2 < \ldots < \xi_n \) in \( A \) such that

\[
d(T\nu_{\xi,i}, E_u) > \frac{\delta}{16} \tag{3}
\]

In the rest of the proof we will denote \( (\xi_i, i) \) by \( \xi_i \).

Notice that the measures \( \nu_{\xi_1}, \ldots, \nu_{\xi_n}, \lambda_F \) are pairwise singular. Choose \( W_1, \ldots, W_n \) pairwise disjoint clopen subsets of \( \{0, 1\}^\mathbb{N} \) pairwise disjoint such that for \( i = 1, \ldots, m \)

\[
|| \nu_{\xi_i} |W_i^c| || < \frac{\delta}{128} \text{ and } || \frac{d\mu_{\xi_i}}{d\lambda_F} | \bigcup_{j=1}^{n} W_j || < \frac{\delta}{128} \tag{4}
\]

We are ready to prove the desired property. Indeed, for \( \lambda_i \geq 0, \sum_{i=1}^{n} \lambda_i = 1 \) we have

\[
d(\sum_{i=1}^{n} \lambda_i T\mu_{\xi}, E_u) \geq d(\sum_{i=1}^{n} \lambda_i T\mu_{\xi,i} \upharpoonright \bigcup_{j=1}^{n} W_j, E_u) \geq
\]

\[
d(\sum_{i=1}^{n} \lambda_i (T\nu_{\xi_i} \upharpoonright W_i), E_u) - \sum_{i=1}^{n} \lambda_i || T\nu_{\xi_i} \upharpoonright \bigcup_{j \neq i} W_j || -
\]

\[
\sum_{i=1}^{n} \lambda_i || \frac{d\mu_{\xi_i}}{d\lambda_i} \bigcup_{j=1}^{n} W_j ||
\]

\[
\sum_{i=1}^{n} \lambda_i || \frac{d\mu_{\xi_i}}{d\lambda_i} \bigcup_{j=1}^{n} W_j ||
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\]

\[
\sum_{i=1}^{n} \lambda_i || \frac{d\mu_{\xi_i}}{d\lambda_i} \bigcup_{j=1}^{n} W_j ||
\]
From Lemma 1.2 we get
\[ d(\sum_{i=1}^{n} \lambda_i (T \nu_{\xi_i} \upharpoonright W_i), E_u) \geq \frac{1}{2} \sum_{i=1}^{n} \lambda_i d(T \nu_{\xi_i} \upharpoonright W_i, E_u) \]
and from (3) and (4) we get
\[ d(\sum_{i=1}^{n} \lambda_i T \mu'_{\xi_i}, E_u) > \frac{3\delta}{4} - \delta \frac{64}{128} = \frac{\delta}{128} \]
Finally from (1) we have
\[ d(\sum_{i=1}^{n} \lambda_i x_{\xi_i}^*, E_u) > \frac{\delta}{256} \]
So L fails P.C.P., and P.4 ensures that \( J^{-1}(L) \) also fails this property. □

1.4 Remark The space E does not contain a subspace isomorphic to \( c_0(\mathbb{N}) \). This is because \( c_0(\mathbb{N}) \) contains a non RNP closed convex subset on which norm and weak topologies coincide. Therefore E does not embed into a space with an unconditional skipped block finite dimensional decomposition. The last follows from the fact that E fails P.C.P. and it does not contain \( c_0(\mathbb{N}) \). Finally E semiembeds into \( E_u \) a space with an unconditional basis.

1.5 Proposition. The properties RNP and KMP are equivalent on the subsets of E. Furthermore if K is closed convex non RNP subset of E then it contains a subset L with a Po\ell-representation.

Proof. As we mentioned before if K is closed convex bounded non RNP then \( J[K] \) carries the same properties and it is contained into \( E_u \) which has an unconditional basis. Therefore, there exists an L closed convex subset of \( J[K] \) with a Po\ell-representation [A-D]. Then \( J^{-1}[L] \) has the same property. □

We conclude with the following result.

1.6 Theorem. Suppose that X is a separable Banach space such that \( X^{**}/X \) is isomorphic to \( \ell^1(\Gamma) \). Then X has RNP.

Proof. Assume that X contains a \( \delta \)-non dentable subset K. Then the techniques developed in the proof of Proposition 1.3 shows that K is non strongly regular.

Actually every \( \sum_{i=1}^{u} \lambda_i S_i \) convex combination of slices will have diameter greater than \( \frac{\delta}{256} \). Hence by a result due to Bourgain [B], \( \ell^1 \) embeds into \( X^* \), and by Pelczynski’s Theorem [P] \( M[0,1] \) embeds into \( X^{**} \). But then there exists a sequence \( (x_n^{**})_{n \in \mathbb{N}} \) weakly convergent to zero and \( d(x_n^{**}, X) > \delta \). This contradicts the Schur property of \( \ell^1(\Gamma) \). □

1.7 Remark Odell in [O] has constructed a separable B-space X with \( X^{**}/X \cong \ell^1(2^\omega) \). From a theorem by Lindenstrauss [L] follows that every separable B space X and its dual \( X^* \) are of the form \( Z^{**}/Z \) for some separable Banach space \( Z \).
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References

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