CKM angles from non-leptonic B decays using SU(3) flavour symmetry

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Abstract. We discuss the determination of the CKM angles $\gamma$ and $\alpha$ using recent data from non-leptonic B decays together with flavour symmetries. Penguin effects are controlled by means of the CP-averaged branching ratio $B_d \rightarrow \pi^\pm K^\mp$. The information from $A_{CP}(B_d \rightarrow J/\psi K_S)$ (two solutions for $\phi_d$), $R_b$ and $\gamma$ allow us to determine $\beta$, even in presence of New Physics not affecting $\Delta B = 1$ amplitudes. In this context we address the question of to what extent there is still space for New Physics.

PACS. 13.25Hw Hadronic decays of mesons – 11.30Er CP violation

1 Introduction

B physics is one of the most fertile testing grounds to check the CKM mechanism of CP violation in the SM \footnote{\cite{B} \footnote{\cite{M}}, but also to look for the first signals of New Physics \footnote{\cite{N}} in the pre-LHC era.

The huge effort at the experimental level at the B factories and future hadronic machines \footnote{\cite{E}} has produced, already, several impressive results. First, the measurement of $\sin \phi_d$ from the mixing induced CP asymmetry of the decay $B_d \rightarrow J/\psi K_S$. Second, the measurement of a series of non-leptonic B decays: $B_d \rightarrow \pi K$, $B_d \rightarrow \pi \pi$ and in the future hadronic machines $B_s \rightarrow K K$ will be also accessible.

These non-leptonic B decays play a fundamental role in the determination of the CKM angle $\gamma$. The main problem in analysing them is how to deal with hadronic matrix elements and how to control penguin contributions. Our approach \footnote{\cite{B} \footnote{\cite{M}} \footnote{\cite{N}} \footnote{\cite{E}}} extract the maximal possible information from data using flavour symmetries to try to reduce as much as possible the uncertainties associated to QCD hypothesis.

2 CKM angle $\gamma$ from non-leptonic decays: $B_d \rightarrow \pi \pi$, $B_d \rightarrow \pi K$ and $B_s \rightarrow K K$

We start writing down a general amplitude parametrization of $B_d \rightarrow \pi^+ \pi^-$ in the SM \footnote{\cite{B} \footnote{\cite{M}}:}

$$A(B^c_d \rightarrow \pi^+ \pi^-) = C \left( e^{i\gamma} - de^{i\theta} \right)$$

All the hadronic information is collected in

$$de^{i\theta} \equiv \frac{1}{R_b} \left( \frac{A_{\text{pen}}^u}{A_{\text{CC}}^u + A_{\text{pen}}^u} \right) \quad C \equiv \lambda^3 A R_b \left( A_{\text{CC}}^u + A_{\text{pen}}^u \right)$$

where $A_{\text{CC}}^u$ are current-current contributions and $A_{\text{pen}}^u$ are differences between penguin contributions with a quark $q = u, c$ and a quark top inside the loop.

This amplitude allow us to construct the corresponding CP asymmetries \footnote{\cite{B} \footnote{\cite{M}} \footnote{\cite{N}} \footnote{\cite{E}}:}

$$A_{\text{CP}}^{\text{dir}} = \text{func}(d, \theta, \gamma) \quad A_{\text{CP}}^{\text{mix}} = \text{func}(d, \theta, \gamma, \phi_d)$$

Following a similar procedure we can write down the amplitude for a closely related process:

$$A(B^0_s \rightarrow K^+ K^-) = \left( \frac{\lambda}{1 - \lambda^2/2} \right) C' \left[ e^{i\gamma} + \left( \frac{1 - \lambda^2/2}{\lambda} \right) d' e^{i\theta'} \right]$$

whose corresponding asymmetries will depend on \footnote{\cite{B} \footnote{\cite{M}} \footnote{\cite{N}} \footnote{\cite{E}}:}

$$A_{\text{CP}}^{\text{dir}} = \text{func}(d', \theta', \gamma) \quad A_{\text{CP}}^{\text{mix}} = \text{func}(d', \theta', \gamma, \phi_s)$$

The crucial point, here, is that the hadronic parameters $d'$, $\theta'$ and $C'$, has exactly the same functional dependence on the penguins that $d$, $\theta$ and $C$, except for the interchange of a $d$ quark by an $s$ quark.

As a consequence, both processes can be related via U-spin symmetry, reducing the total number of parameters to five: $\gamma$, $d$, $\theta$, $\phi_d$ and $\phi_s$. At this point, one must check the sensitivity of the results to the breaking of U-spin symmetry. This is explained in subsection \footnote{\cite{B} \footnote{\cite{M}} \footnote{\cite{N}} \footnote{\cite{E}}:}

Looking a bit more in detail, one finds that $d$ is indeed not a free parameter, but it can be constrained or substituted using an observable called $H$ \footnote{\cite{B} \footnote{\cite{M}} \footnote{\cite{N}} \footnote{\cite{E}}:}

$$H \equiv \frac{1}{e} \left| \frac{C}{C'} \right|^2 \left[ M_{B_d} \Phi(M_{K^+}, M_{K^-}) \tau_{B_d} \right] \left[ \frac{BR(B_d \rightarrow \pi^+ \pi^-)}{BR(B_s \rightarrow K^+ K^-)} \right]$$

This quantity requires the knowledge of $BR(B_s \rightarrow K^+ K^-)$, which is still not available. However, we can already now
evaluate $H$ by making contact with the B factories and substitute $B_s \to K^+K^-$ by $B_d \to \pi^\pm K^{\mp}$. These two processes differ by the spectator quark and certain exchange and penguin annihilation topologies that are expected to be small [8]. This leads to the following value for $H$ [9]:

$$H \approx \frac{1}{\epsilon} \left( \frac{f_K}{f_\pi} \right)^2 \left[ \frac{BR(B_d \to \pi^+\pi^-)}{BR(B_d \to \pi^\pm K^{\mp})} \right] = 7.5 \pm 0.9 \quad (1)$$

Due to the dependence of $H$ only on $\cos \theta \cos \gamma$ in the U-spin limit, we obtain immediately a constrained range for $d$: $0.2 \leq d \leq 1$. Also, using the exact expression for $H$ we can obtain $d$ as a function of $H$, $\theta$ and $\gamma$.

It is important to insist here that once the data on the branching ratio of $B_s \to KK$ will be available, the spectator quark hypothesis will not be necessary and only U-spin breaking effects will be important.

2.1 Prediction for CKM-angle $\gamma$

Let’s take as starting point the general expression [6]:

$$A_{\text{dir}}(B_d \to \pi^+\pi^-) = \mp \left[ \frac{\sqrt{4d^2 - (u + vd)^2} \sin \gamma}{(1 - u \cos \gamma) + (1 - v \cos \gamma)d^2} \right]$$

where $u, v, d = F_1(A_{\text{mix}}^f, H, \gamma, \phi_d(B_d \to J/\Psi K_S); \xi, \Delta \theta)$. The parameters $\xi, \Delta \theta$ will account for the U-spin breaking and are discussed in subsection 2.2.2.

Using present world average for $\sin \phi_d = 0.734 \pm 0.054$, one obtains two possible solutions for the weak mixing angle:

$$\phi_d = (47^{+13}_{-9})^\circ \vee (133^{+9}_{-2})^\circ.$$  

We will refer later on to these two solutions like scenario A and B, respectively.

Concerning experimental data, the situation is still uncertain, but improving. Present naive average of Belle and Babar data is [10]:

$$A_{\text{dir}}(B_d \to \pi^+\pi^-) = -0.38 \pm 0.16$$

$$A_{\text{mix}}^f(B_d \to \pi^+\pi^-) = +0.58 \pm 0.20$$

The intersection of the two experimental ranges of $A_{\text{dir}}^f$ and $A_{\text{mix}}^f$ allow us, using Eq. (2), to determine the range for $\gamma$. The first range, corresponding to take $\phi_d = 47^\circ$ is:

$$32^\circ \leq \gamma \leq 75^\circ$$  

For the second solution $\phi_d = 133^\circ$ one obtains:

$$105^\circ \leq \gamma \leq 148^\circ$$  

Both plots are symmetric (see [6,11]). This is a consequence of the symmetry $\phi_d \to 180^\circ - \phi_d$, $\gamma \to 180^\circ - \gamma$ that Eq. (2) exhibits. It is remarkable the stability of the range for $\gamma$ if we compared it with previous analysis [11].

2.2 Sensitivity to parameters $H$, $\xi$ and $\Delta \theta$

Here we will analyze the sensitivity of the determination of $\gamma$ on the variation of the different hadronic parameters.

2.2.1 $H$ and the spectator quark hypothesis

Let’s fix the solution $\phi_d = 47^\circ$ and take the experimental branching ratios of $B_d \to \pi\pi$ and $B_d \to \pi K$ to determine $H$. We vary $H$ inside its experimental range Eq. (1) at one, two and three sigmas to take into account the uncertainty associated to the spectator quark hypothesis. We find at one sigma a very mild influence in the determination of $\gamma$. The error induced in the range of $\gamma$ is about $\pm 2^\circ$.

For the very conservative range of up to three sigmas we find a maximal error of $6^\circ$. Moreover, if the experimental value of $H$ tends to increase the range for $\gamma$ tends to decrease, allowing for a narrower determination.

Finally, the uncertainty associated to $H$ will be drastically reduced once the $\text{BR}(B_s \to KK)$ is known and $H$ will be taken safely in a narrower range.

2.2.2 U-spin breaking: $\xi$ and $\Delta \theta$

U-spin breaking is the most important uncertainty. We will follow two different strategies to keep it under control:

a) Once the data from the CP asymmetries and branching ratio of $B_s \to KK$ will be available and $\phi_s$ will be measured from the CP-asymmetry of $B_s \to J/\Psi \phi$, we will be able to test directly from data U-spin breaking. Taking $\phi_d$ from $B_d \to J/\Psi K_S$ we have 4 observables (the CP asymmetries) and 3 unknowns ($d, \theta, \gamma$). Then, we can add $d'$ as another free parameter and data will tell us the amount of U-spin breaking.

b) Already now, we can define two quantities $\xi = d'/d$ and $\Delta \theta = \theta' - \theta$ that parametrizes the amount of U-spin breaking. In order to test the sensitivity of $\gamma$ to the variation of these parameters, we allow them to vary in a range. If we allow for a very large variation of $\xi$ between 0.8 and 1.2, the larger error in the determination of $\gamma$ is of $\pm 5^\circ$. Concerning $\Delta \theta$, its influence is negligibly small, a variation of 40$^\circ$ induces an error of at most 1 degree.

Other studies on U-spin breaking can be found in [12].

3 Determination of CKM angles $\alpha$ and $\beta$ in SM and with New Physics in the mixing

Next point is how to determine $\alpha$ and $\beta$ [11]. Here, in addition, we will also allow for Generic New Physics affecting the $B_d^0 - \bar{B_d}^0$ mixing, but not to the $\text{BR}(B, S) = 1$ decay amplitudes, i.e, this type of New Physics is consistent with the determination of $\gamma$ explained in the previous section. Our inputs are [9,13].
which implies the range for $\Delta M_d$ obtained from exclusive/inclusive transitions mediated by $b \to u\nu\ell$ and $b \to c\ell\tau$. Two important remarks are: a) This is an observable practically insensitive to New Physics, b) from $R_b^{\text{max}} = 0.46$ we can extract a robust maximum possible value for $\beta$: $|\beta|_{\text{max}} = 27^\circ$, respected by the two scenarios.

- $\phi_d$ obtained as discussed in previous sections.
- $\phi_d$ from $A^{\text{mix}}_b(B_d \to J/\psi K_S)$ is used as an input for the CP asymmetries of $B_d \to \pi\pi$, but NOT to determine $\beta$, since we assume that New Physics could be present. Also $\Delta M_d$ and $\Delta M_s/\Delta M_d$ are not used as inputs, due to their sensitivity to New Physics.

Using these inputs we obtain two possible determinations for $\alpha$, $\beta$ and $\gamma$, corresponding to the two possible values of $\phi_d$.

### 3.1 Scenario A: Compatible with SM

This scenario corresponds to the first solution $\phi_d = 47^\circ$, which implies the range for $\gamma$ given in Eq. (4). Together with $R_b$ we obtain the black region shown in Fig 1. It implies the following prediction for the CKM angles:

$$78^\circ \leq \alpha \leq 136^\circ \quad 13^\circ \leq \beta \leq 27^\circ \quad 32^\circ \leq \gamma \leq 75^\circ$$

and the error associated with $\xi \in [0.8,1.2]$ is $\Delta \alpha = \pm 4^\circ$, $\Delta \beta = \pm 1^\circ$ and $\Delta \gamma = \pm 5^\circ$. It is interesting to notice that this region is in good agreement with the usual CKM fits [14]. To illustrate it we have shown in Fig. 1 also the prediction from the SM interpretation of different observables: $\Delta M_d$, $\Delta M_s/\Delta M_d$, $\epsilon_K$ and $\phi_d^{\text{SM}} = 2\beta$.

### 3.2 Scenario B: New Physics

The second solution: $\phi_d = 133^\circ$ cannot be explained in the SM context and requires New Physics contributing to the mixing [9,13]. Models with New sources of Flavour mixing can account for this second solution with only two very general requirements [9]: a) The effective scale of New Physics is larger than the electroweak scale and b) the dimensional effective coupling ruling $\Delta B = 2$ processes can always be expressed as the square of two $\Delta B = 1$ effective couplings. Supersymmetry provides a perfect example, in particular, through the contribution of gluino mediated box diagrams with a mass insertion $\delta_{\text{in}}^B$ at [9].

In this case, $\gamma$ lies in the second quadrant Eq. (4) and $\beta$ is indeed smaller than in the previous scenario. The result is still consistent with the $\epsilon_K$ hyperbola. $\Delta M_{d,s}$ are not shown here, since they would be affected by New Physics. The black region obtained (see Fig 2) corresponds to the following prediction for the CKM angles:

$$22^\circ \leq \alpha \leq 60^\circ \quad 8^\circ \leq \beta \leq 22^\circ \quad 105^\circ \leq \gamma \leq 148^\circ$$

with same errors associated to $\xi$ as in Scenario A. It is interesting to remark that this second solution has also interesting implications for certain rare decays like $K^+ \to \pi^+\nu\bar{\nu}$ [15]. Using this second solution we find a better agreement with experiment than with the SM solution. Concerning $B_d \to \mu^+\mu^-$, we find also sizeable differences depending on the scenario used.

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