Dynamics in the Ising field theory after a quantum quench

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Abstract. We study the real-time dynamics of the order parameter $\langle \sigma(t) \rangle$ in the Ising field theory after a quench in the fermion mass, which corresponds to a quench in the transverse field of the corresponding transverse field Ising chain. We focus on quenches within the ordered phase. The long-time behaviour is obtained analytically by a resummation of the leading divergent terms in a form-factor expansion for $\langle \sigma(t) \rangle$. Our main result is the development of a method for treating divergences associated with working directly in the field theory limit. We recover the scaling limit of the corresponding result by Calabrese et al (2011 Phys. Rev. Lett. 106 227203), which was obtained for the lattice model. Our formalism generalizes to integrable quantum quenches in other integrable models.

Keywords: form factors, integrable quantum field theory, spin chains, ladders and planes (theory), exact results

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1. Introduction

Recent experimental advances have made it possible to study the non-equilibrium dynamics of trapped cold atomic gases [1,2]. A key feature of these systems is that they are only weakly coupled to their environments, which makes it possible to study non-equilibrium dynamics in essentially isolated systems. This has led to an intense theoretical effort to address fundamental questions [3] regarding the non-equilibrium dynamics of many-body quantum systems.

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One issue of particular interest concerns the role played by dimensionality and conservation laws. As shown by the ‘quantum Newton’s cradle’ experiments of Kinoshita et al [2], quasi-one-dimensional condensates exhibit behaviour that is dramatically different from two- and three-dimensional ones. In particular, it was observed that the late-time behaviour cannot be described in terms of an effective temperature: the systems does not ‘thermalize’ [4]. One possible explanation [2] for this behaviour is that the experimental system is close to being integrable. This has engendered a vigorous research effort aimed at clarifying the role played by quantum integrability in the late time and stationary state behaviour in non-equilibrium dynamics [5]–[10].

A simple and attractive way of inducing non-equilibrium evolution is by means of a quantum quench. One prepares a system in the ground state of a given Hamiltonian \( H(h_0) \), where \( h_0 \) is an experimentally tunable parameter such as a magnetic field or an interaction strength. At time \( t = 0 \) the parameter \( h_0 \) is then changed instantaneously from \( h_0 \) to \( h \), and at subsequent times the system evolves according to the quantum dynamics induced by the new Hamiltonian \( H(h) \). One of the main models studied in the context of quantum quenches has been the transverse field Ising chain [10]–[12]. This is on the one hand because the model has a free fermion representation which makes analytical progress possible. On the other hand the model is the simplest paradigm of a quantum phase transition and therefore is an ideal testing ground for questions relating to non-equilibrium evolution in the vicinity of quantum critical points. The stationary and late-time behaviour of correlation functions in the transverse field Ising chain after a quantum quench has recently been determined analytically by Calabrese et al [10].

Analysis of quantum quenches in interacting integrable models is difficult [13] and remains a largely open challenge, although important progress has been made by combining numerical and integrable model techniques [8]. A special role is played by integrable quenches in integrable quantum field theories. These are characterized as follows. As shown by Calabrese and Cardy [6] the quench problem can be mapped to an equivalent theory defined in a strip geometry. The initial state plays the role of a boundary condition, and for an integrable quench this boundary condition does not spoil the integrability of the theory. Hence, for these special initial states one can use methods of integrable quantum field theory [14,15] with boundaries [16] to analyse the time evolution of observables. An important step in this direction was taken by Fioretto and Mussardo [9], who considered the stationary state behaviour of one-point functions in integrable quenches and in particular in the Ising field theory. A serious complication that arises in the field theory limit is that singularities associated with kinematical poles appear. This problem is particularly acute for two-point correlators and is closely related to the one encountered when calculating finite-temperature dynamical correlation functions in integrable models [17]–[19]. To date two general ways of dealing with these singularities have been developed. The first [17]–[19] is to use a finite-volume regularization for matrix elements [20], while the second is a subtraction scheme that works directly in the infinite volume [17]. The aim of the present work is to apply these methods to the problem of quench dynamics in the ordered phase of the Ising field theory. Regulating the theory in a finite volume reduces all calculations to a particular limit of the analysis for the lattice Ising model [10,21] and we therefore do not report any details here. We focus on the infinite-volume regularization proposed in [17] and apply it to the quench problem at late, finite times. This requires a significant generalization of the regularization procedure,
which constitutes the main result reported here. In forthcoming work we will apply this method to a quench in the sine–Gordon model.

The outline of this paper is as follows. In section 2 we introduce the Ising field theory as the scaling limit of the transverse field Ising chain. In section 3 we discuss quenches in the fermion mass of the field theory, which corresponds to a quench in the transverse field of the related Ising chain. In section 4 we develop a new method to calculate the time evolution of correlation functions in integrable field theories, which constitutes the main result of our work. In section 5 we apply this method to the one-point function of the order parameter field, which relaxes exponentially to zero as shown in equations (46) and (47). In section 6 we discuss the relation of quenches in the fermion mass to the extrapolation time regularization introduced in [9] and we conclude in section 7. Technical details of the derivations have been moved to the appendices.

2. Quantum Ising chain

We start with the transverse field Ising chain

$$H_{\text{lat}} = -J \sum_i (\sigma_i^x \sigma_{i+1}^x + h \sigma_i^x).$$  \hspace{1cm} (1)

Here $\sigma^x$ and $\sigma^z$ are the Pauli matrices and $J$ is the exchange energy. The Hamiltonian (1) is invariant under the $\mathbb{Z}_2$-transformation $\sigma_i^x \to \sigma_i^x, \sigma_i^z \to -\sigma_i^z$. For $h < 1$ this symmetry is broken, the order parameter field $\sigma_i^z$ takes a non-zero expectation value, and the ground state is two-fold degenerate. On the other hand, for $h > 1$ the system possesses a unique ground state and the expectation value of the order parameter field vanishes. The two phases are separated by a quantum critical point at $h = 1$. At small deviations from criticality, $|h - 1| \ll 1$, one can pass to the scaling limit [22] ($a_0$ is the lattice spacing)

$$J \to \infty, \quad h \to 1, \quad a_0 \to 0,$$  \hspace{1cm} (2)

while keeping fixed both the gap $M$ and the velocity $v$

$$2J|1 - h| = M, \quad 2Ja_0 = v.$$  \hspace{1cm} (3)

The order parameter in the scaling limit must be defined as

$$\sigma(x) \propto (1 - h^2)^{-1/8} \sigma_i^z,$$  \hspace{1cm} (4)

where $x = na_0$. It is customary to choose the normalization of the field $\sigma(x)$ such that

$$\lim_{x \to 0} \langle 0|\sigma(x)\sigma(0)|0 \rangle = \frac{1}{|x|^{1/4}},$$  \hspace{1cm} (5)

which implies

$$\sigma_i^z \to 2^{1/24} e^{1/8} \mathcal{A}^{-3/2} a_0^{1/8} \sigma(x),$$  \hspace{1cm} (6)

with Glaisher’s constant

$$\mathcal{A} = 1.282 427 129 100 62 \ldots.$$  \hspace{1cm} (7)
The Hamiltonian in the scaling limit reads
\[
H = \int_{-\infty}^{\infty} \frac{dx}{2\pi} \left[ \frac{i\psi}{2} (\psi \partial_x \psi - \bar{\psi} \partial_x \bar{\psi}) - iM \bar{\psi} \psi \right],
\]
where \(\psi\) and \(\bar{\psi}\) are the two components of a Majorana fermion. The model (8) is conformally invariant at the critical point \(M = 0\) (see for example [23]). In the ordered phase, which we will consider throughout this paper, the mass is positive.

An important notion is the mutual semi-locality of operators [14, 15, 24, 25]. This is most easily established by defining complex coordinates \(z = \tau + iz, \bar{z} = \tau - iz\) and then considering the operator product \(O_1(\tau, x) O_2(0, 0) = O_1(z, \bar{z}) O_2(0, 0)\). If we take \(O_1\) counterclockwise around \(O_2\) in the plane, i.e. we perform the analytic continuation \(z \rightarrow e^{2\pi i z}, \bar{z} \rightarrow e^{-2\pi i \bar{z}}\), the operators \(O_1\) and \(O_2\) are said to be mutually semi-local if
\[
O_1(e^{2\pi i z}, e^{-2\pi i \bar{z}}) O_2(0, 0) = l_{O_1 O_2} O_1(z, \bar{z}) O_2(0, 0).
\]
The phase \(l_{O_1 O_2}\) is called the semi-locality factor. The two fields are mutually local if \(l_{O_1 O_2} = 1\). Semi-locality is the mildest form of non-locality, in general the right-hand side of (9) may be more complicated. The mutual semi-locality factor of the spin and disorder operators can be extracted from their operator product expansion [23, 24]
\[
\sigma(z, \bar{z}) \mu(0, 0) \sim \frac{1}{\sqrt{2} |z|^{1/4}} [e^{i\pi/4} \sqrt{z} \psi(0) + e^{-i\pi/4} \sqrt{\bar{z}} \bar{\psi}(0)].
\]
This implies that when taking \(\sigma\) once around \(\mu\) one obtains an extra minus sign, i.e. \(l_{\sigma \mu} = -1\). In the same way one finds \(l_{\psi \mu} = l_{\bar{\psi} \mu} = l_{\psi \sigma} = l_{\bar{\psi} \sigma} = -1\). On the other hand, the disorder field \(\mu\) is local with respect to itself.

We use the disorder field \(\mu\) as the fundamental field creating the excitations. This implies that the fundamental excitations are viewed as bosons. We denote the corresponding annihilation and creation operators by \(A(\theta)\) and \(A^\dagger(\theta)\) respectively. They fulfill the Faddeev–Zamolodchikov algebra [26]
\[
\begin{align*}
A(\theta_1)A(\theta_2) &= SA(\theta_2)A(\theta_1), & A^\dagger(\theta_1)A^\dagger(\theta_2) &= SA^\dagger(\theta_2)A^\dagger(\theta_1), \\
A(\theta_1)A^\dagger(\theta_2) &= 2\pi \delta(\theta_1 - \theta_2) + SA^\dagger(\theta_2)A(\theta_1),
\end{align*}
\]
with the scattering matrix \(S = -1\). The basis of scattering states can now be constructed by
\[
|\theta_1, \ldots, \theta_n\rangle = A^\dagger(\theta_1) \cdots A^\dagger(\theta_n)|0\rangle,
\]
where the vacuum state \(|0\rangle\) is defined by \(A(\theta)|0\rangle = 0\). The energy and momentum of the scattering states are expressed in terms of the rapidities \(\theta_i\) as
\[
E = M \sum_{i=1}^{n} \cosh \theta_i, \quad P = \frac{M}{v} \sum_{i=1}^{n} \sinh \theta_i.
\]

In this paper we study the one-point function of the order parameter field \(\sigma\). The relevant matrix elements (form factors) in the ordered phase are given by [24, 25, 27]
\[
f(\theta_1, \ldots, \theta_{2n}) = \langle 0 | \sigma | \theta_1, \ldots, \theta_{2n} \rangle = i^n \tilde{\sigma} \prod_{i,j=1}^{2n} \tanh \frac{\theta_i - \theta_j}{2},
\]
where
\[
\tilde{\sigma} = \langle 0 | \sigma | 0 \rangle = 2^{1/12} e^{-1/8} A^{3/2} \left( \frac{M}{v} \right)^{1/8}.
\]
3. Quench in the fermion mass

We now consider a sudden change of the transverse field in (1) at time \( t = 0 \) from \( h_0 \) to \( h \). This quench has been studied previously by several authors [9, 11, 12]. Most importantly, in [10, 21] the time evolutions of both the one-point and two-point functions of the order parameter after a quench were determined analytically. One of the methods developed in [10, 21] is a form-factor approach for the lattice model. In the following we consider the time evolution of the one-point function directly in the scaling limit (2), (3). As we have mentioned before, our key objective is to generalize the form-factor approach to quantum field theories in order to analyse integrable quenches in interacting systems such as the sine–Gordon model. However, a second interesting issue is related to commutativity of limits: \textit{a priori} it is unknown whether a quench in the scaling limit is the same as the scaling limit of a quench. We will come back to this question in sections 5.6 and 7. In the following we resolve this question for the particular case of the one-point function of the order parameter in the ferromagnetic phase of the Ising model.

In the field theory (8) the quench in the transverse field corresponds to a quench in the fermion mass, i.e. at time \( t = 0 \) we switch from \( M_0 \) to \( M \). The time evolution for \( t > 0 \) is governed by (8), while the initial state can be expressed in terms of the eigenstates of (8) as [12, 13]

\[
|\Psi_0\rangle = \exp \left( \int_0^\infty \frac{d\xi}{2\pi} K_q(\xi) A^\dagger(-\xi) A^\dagger(\xi) \right) |0\rangle, \tag{16}
\]

where

\[
K_q(\xi) = i \tan \left[ \frac{1}{2} \arctan(\sinh \xi) - \frac{1}{2} \arctan \left( \frac{M}{M_0} \sinh \xi \right) \right] \equiv i \hat{K}_q(\xi). \tag{17}
\]

We note that the quench matrix satisfies \( K_q(\xi) = SK_q(-\xi) = -K_q(-\xi) \) and that \( \hat{K}_q(\xi) \in \mathbb{R} \) for \( \xi \in \mathbb{R} \). Furthermore, for any finite initial mass, \( M_0 < \infty \), the integral

\[
\int_0^\infty \frac{d\xi}{2\pi} |K_q(\xi)|^2 \tag{18}
\]

is convergent. We note the similarity of the initial state (16) with the boundary state [16] introduced in the context of integrable field theories with boundaries, which can be used to study the physical properties of systems with defects or impurities [28, 29]. Starting from the initial state (16) we calculate the time evolution of the one-point function of the order parameter field,

\[
\langle \sigma(t) \rangle \equiv \frac{\langle \Psi_0 | \sigma(t) | \Psi_0 \rangle}{\langle \Psi_0 | \Psi_0 \rangle}, \quad \sigma(t) = e^{iHt} \sigma e^{-iHt}. \tag{19}
\]

4. Method

The strategy to calculate the one-point function (19) after the quench is as follows. (i) We formally expand [9, 10, 18] the numerator and denominator in powers of the quench matrix \( K_q \). (ii) We evaluate each term in these expansions using a combined approach based...
on a regularization of the appearing form factors following Smirnov [14] as well as the \(\kappa\) regularization recently introduced in the study of dynamical correlation functions at finite temperatures [17]. (iii) In the resulting expression the singularities in the numerator and denominator, which are due to the infinite volume of the model (8), cancel each other. In particular, we show by explicit calculation up to \(\mathcal{O}(K_q^4)\) that this procedure yields well-defined results which agree with a finite-volume regularization. (iv) Finally the resulting series has to be resummed [10] in order to obtain a well-defined long-time limit. The calculation of two-point functions follows the same lines, although the explicit expressions become considerably more complicated.

4.1. Formal expansion

The first step in the calculation of (19) is the formal expansion of both the numerator and the denominator in powers of the quench matrix \(K_q\). This expansion yields for the numerator

\[
\langle \Psi_0\vert \sigma(t)\vert \Psi_0 \rangle = \sum_{m,n=0}^{\infty} \int_0^\infty \frac{d\xi'_1 \cdots d\xi'_m d\xi_1 \cdots d\xi_n}{m!(2\pi)^m n!(2\pi)^n} \times \prod_{i=1}^{m} K_q(\xi'_i)^* \prod_{j=1}^{n} K_q(\xi_j) e^{2Mt\sum_i \cosh \xi'_i} e^{-2Mt\sum_j \cosh \xi_j} \times \langle \xi'_1, -\xi'_1, \ldots, \xi'_m, -\xi'_m | \sigma | -\xi_n, \xi_n, \ldots, -\xi_1, \xi_1 \rangle \tag{20}
\]

\[
\equiv \sum_{m,n=0}^{\infty} C_{2m,2n}(t). \tag{21}
\]

Note that the indices \(2m\) and \(2n\) correspond to the numbers of particles originating from the left and right initial states respectively. Similarly, the expansion of the denominator reads

\[
\langle \Psi_0\vert \Psi_0 \rangle = 1 + \sum_{n=1}^{\infty} \int_0^\infty \frac{d\xi'_1 \cdots d\xi'_n d\xi_1 \cdots d\xi_n}{n!(2\pi)^n n!(2\pi)^n} \times \prod_{i=1}^{n} K_q(\xi'_i)^* K_q(\xi_i) \langle \xi'_1, -\xi'_1, \ldots, \xi'_n, -\xi'_n | -\xi_n, \xi_n, \ldots, -\xi_1, \xi_1 \rangle \tag{22}
\]

\[
\equiv \sum_{n=0}^{\infty} Z_{2n}. \tag{23}
\]

The normalization in (19) can therefore be formally expanded in the following way:

\[
\frac{1}{\langle \Psi_0\vert \Psi_0 \rangle} = 1 - Z_2 + Z_2^2 - Z_4 + \ldots. \tag{24}
\]

We note that (24) is merely used for defining linked clusters, i.e. identifying the parts of the numerator in (19) that diverge in the infinite volume.
4.2. Regularization procedure

The matrix elements in the terms $C_{2m,2n}(t)$ and $Z_{2n}$ possess kinematical poles whenever $\xi_i = 0$ and therefore have to be regularized. Following Smirnov [14] we proceed as follows. Let $A$ denote a set of one-particle excitations and $A_1$ and $A_2$ a partition of $A$. The scattering matrix arising from the commutations necessary to rewrite $|A\rangle$ as $|A_2A_1\rangle$ is denoted by $S_{AA_1}$, i.e. $|A\rangle = S_{AA_1}|A_2A_1\rangle = S_{AA_2}|A_1A_2\rangle$. If $A$ and $B$ denote two sets of one-particle excitations, the form factors of $\sigma$ one-particle excitations, the form factors of $\sigma$ present by virtue of the semi-locality of the spin operator with respect to the fundamental field and is given by

$$
\langle A|\sigma|B\rangle = \sum_{A=A_1\cup A_2, B=B_1\cup B_2} d(B_2) S_{AA_1} S_{B_1B} \langle A_2|B_2\rangle \langle A_1 + i0|\sigma|B_1\rangle,
$$

(25)

where the sum is over all possible ways to break the sets $A = A_1 \cup A_2$ and $B = B_1 \cup B_2$ into subsets. The scalar products $\langle A_2|B_2\rangle$ as well as the corresponding terms in the $Z_{2n}$ are easily evaluated using the Faddeev–Zamolodchikov algebra (11). The factor $d(A)$ is present by virtue of the semi-locality of the spin operator with respect to the fundamental field and is given by

$$
d(A) = (-1)^{n(A)},
$$

(26)

where $n(A)$ denotes the number of elements in $A$. Using (25) the poles in the form factors have been shifted away from the real axis. The terms $\langle A_2|B_2\rangle$ correspond to the disconnected pieces of the form factors. As all rapidities in the remaining matrix elements are distinct, they can be evaluated using the crossing relation

$$
\langle \theta'_1 + i0, \ldots, \theta'_m + i0|\sigma|\theta_1, \ldots, \theta_n \rangle = f(\theta'_1 + i\pi + i\eta_1, \ldots, \theta'_m + i\pi + i\eta_m, \theta_1, \ldots, \theta_n),
$$

(27)

where $\eta_i \to 0^+$. We note that one can also shift the rapidities in the set $A_1$ to the lower half plane, which results [28] in different scattering and phase factors in (25) but leaves the final result unchanged.

It is clear that the right-hand side of (25) may still contain divergences due to the intertwining of particles with rapidities $\xi_i$ and $-\xi_i$ in the initial state (16). These divergences are a consequence of working in the infinite volume and have to be cancelled against similar divergences originating from the norm of the initial state (23). In order to exhibit these cancellations we need to identify these divergences explicitly. To this end we use the $\kappa$-regularization scheme recently introduced in the study of finite-temperature correlation functions [17]. For each pair of rapidities $\{-\xi_i, \xi_i\}$ in the ket states we introduce an auxiliary real parameter $\kappa_i$ to shift the rapidities away from the singularities. The resulting expressions have to be understood as generalized functions of the auxiliary variables $\kappa_i$. In order to exhibit the cancellations of terms in the Lehmann representation of (19) that diverge in the infinite volume we define a smooth function $P(\kappa)$ which is strongly peaked around $\kappa = 0$ and satisfies

$$
P(0) = L, \quad \int d\kappa P(\kappa) = 1.
$$

(28)

Here $L$ can be thought of as the length of the system in the finite-volume regularization (see appendix F). One possible choice is $P(\kappa) = L e^{-\pi L^2 \kappa^2}$. Using this regularization
scheme the first non-trivial term in the expansion (23) reads

\[ Z_2 = \int d\kappa P(\kappa) \int_0^\infty \frac{d\xi'd\xi}{(2\pi)^2} K_q(\xi')^* K_q(\xi) \langle \xi', -\xi' | -\xi + \kappa, \xi + \kappa \rangle \]

\[ = \int d\kappa P(\kappa) \delta(-2\kappa) \int_0^\max\{0,-\kappa\} d\xi K_q(\xi + \kappa)^* K_q(\xi) \]

\[ - \int d\kappa P(\kappa) \delta(-2\kappa) \int_0^\max\{0,-\kappa\} d\xi K_q(-\xi - \kappa)^* K_q(\xi) \]

\[ = \frac{L}{2} \int_0^\infty d\xi |K_q(\xi)|^2. \]

The infinite-volume divergence is now clearly exhibited and (31) facilitates comparison with the finite-volume regularization (see appendix F). Further examples for the application of (25) and the \(\kappa\)-regularization are presented in appendices A–E.

### 4.3. Cancellation of singularities

Using the \(\kappa\)-regularization as described in section 4.2 all terms in the expansions (21) and (23) are finite but may contain terms \(\propto L^k\), or equivalently \(\propto \delta(-2\kappa_i)\), that diverge in the infinite-volume limit. However, when we consider the one-point function (19) given by their quotient and expand again in powers of \(K_q\),

\[ \langle \sigma(t) \rangle = \frac{\sum_{m,n=0}^\infty C_{2m,2n}(t)}{\sum_{n=0}^\infty Z_{2n}} \equiv \sum_{m,n=0}^\infty D_{2m,2n}(t), \]

all terms \(\propto L^k\) with \(k \geq 1\) cancel each other and the remaining functions \(D_{2m,2n}(t)\) are finite in the infinite-volume limit \(L \to \infty\). This can be thought of as a linked-cluster expansion and is analogous to the finite-temperature case [17].

### 4.4. Resummation

Performing the steps outlined in the preceding sections we obtain the expansion (32) for which the infinite-volume limit can safely be performed. After taking this limit we can study the long-time behaviour of the one-point function. Doing so we observe that the leading contribution to the term \(D_{2m,2n}(t)\) will grow as \(\propto t^\alpha\) with the power \(\alpha\) depending on the numbers of particles \(2m\) and \(2n\). As we will show below, however, these divergences can be resummed leading to a well-defined long-time behaviour of the one-point function, which we present in section 5.6.

### 5. Results

In this section we present the results for the leading terms in the expansion (32). We consider terms up to \(O(K_q^4)\) and concentrate on the dominant contributions in the long-time limit. Technical details of the derivation are presented in appendices A–D. In section 5.6 we present the final result for the long-time behaviour of the one-point function (19) after the resummation of the leading contributions in the series (32).
5.1. Terms in $\mathcal{O}(K_q^0)$ and $\mathcal{O}(K_q)$

The terms up to linear order in $K_q$ do not contain form factors possessing both incoming and outgoing particles. Thus there exist no kinematical poles, a regularization following the procedure discussed in section 4.2 is not necessary and we straightforwardly obtain

$$D_{00} = C_{00} = \langle 0|\sigma|0 \rangle = \bar{\sigma}$$

as well as

$$D_{20}(t) + D_{02}(t) = C_{20}(t) + C_{02}(t) = \bar{\sigma} \int_0^\infty \frac{d\xi}{2\pi} \hat{K}_q(\xi) \tanh \xi [e^{2Mt\cosh \xi} + e^{-2Mt\cosh \xi}].$$

Here the real function $\hat{K}_q(\xi)$ was defined in (17). The long-time behaviour of this term is obtained by a stationary phase approximation,

$$D_{20}(t) + D_{02}(t) = -\frac{\bar{\sigma}}{8\sqrt{\pi}} \left( 1 - \frac{M}{M_0} \right) \frac{\cos(2Mt - \pi/4)}{(Mt)^{3/2}}, \quad Mt \gg 1. \quad (35)$$

5.2. Terms in $\mathcal{O}(K_q^2)$

In this order there exist three terms. The first two originate from $C_{20}(t)$ and $C_{04}(t)$, which do not possess kinematical poles. In the long-time limit we obtain $D_{20}(t) + D_{04}(t) \sim \cos(4Mt)/(Mt)^{5}$ which constitutes a sub-leading correction to (35).

In contrast $C_{22}$ contains a form factor possessing both incoming and outgoing particles and hence kinematical poles appear. Performing the calculation as outlined in sections 4.2 and 4.3 we obtain (see appendix B for details of the derivation)

$$D_{22}(t) = C_{22}(t) - Z_2 \bar{\sigma} = -\bar{\sigma} \Gamma t + D_{22}'(t), \quad (36)$$

$$\Gamma = \frac{2M}{\pi} \int_0^\infty d\xi |K_q(\xi)|^2 \sinh \xi, \quad (37)$$

$$D_{22}'(t) = \bar{\sigma} \text{Re} \int_0^\infty d\xi' \int_0^\infty \frac{d\xi}{2\pi} \int_0^\infty \frac{d\xi}{2\pi} \hat{K}_q(\xi') \hat{K}_q(\xi) \tanh \xi' \tanh \xi \times \coth^2 \frac{\xi' - \xi}{2} \coth^2 \frac{\xi' + \xi}{2} e^{2Mt(\cosh \xi' - \cosh \xi)}. \quad (38)$$

Here the contour of integration $\gamma_-$ lies in the lower half plane and can be explicitly parametrized by $(0 < \phi_0 \leq \pi/4)$

$$\gamma_-(s) = \begin{cases} -is, & 0 \leq s \leq \phi_0, \\ (s - \phi_0) - i\phi_0, & \phi_0 \leq s < \infty. \end{cases} \quad (39)$$

Clearly the first term in (36) dominates the long-time behaviour. It can be thought of as the second term in the expansion of $\bar{\sigma} e^{-t\Gamma}$ in powers of $K_q$, see section 5.6. The late-time behaviour of the second contribution (38) is dominated by the region $\xi' \approx 0$ and $\xi = -is$ with $s \approx 0$. Expanding the integrand and changing to polar coordinates then gives

$$D_{22}'(t) \approx -\frac{\bar{\sigma}}{32\pi} \frac{(1 - M/M_0)^2}{Mt}, \quad Mt \gg 1. \quad (40)$$

We note that the behaviour of (36) is in agreement with the finite-volume regularization presented in appendix F.
5.3. Terms in $O(K^3_q)$

In this order the terms containing kinematical poles are $C_{42}(t)$ and $C_{24}(t)$. The calculation presented in appendix C yields

$$D_{42}(t) = D_{24}(t)^* = C_{42}(t) - Z_2 C_{20}(t) = -\Gamma t D_{20}(t) + \cdots,$$

where the dots represent sub-leading terms that fall off at least as $\sim 1/(Mt)$ in the long-time limit. Again we find a term showing an explicit linear-time dependence, which can be viewed as the second term in the expansion of $D_{20}(t) e^{-\Gamma t}$.

5.4. Terms in $O(K^4_q)$

The calculation of $D_{44}(t)$ requires the introduction of two independent auxiliary parameters $\kappa_1$ and $\kappa_2$ (see appendix D). The result is given by

$$D_{44}(t) = C_{44}(t) - Z_2 C_{22}(t) + (Z_2^2 - Z_4) \bar{\sigma} = \frac{\bar{\sigma}}{2} (\Gamma t)^2 - \Gamma t D'_{22}(t) + D'_{44} + \cdots,$$

$$D'_{44} = \bar{\sigma} \Re e \int_0^\infty \frac{d\xi_1}{2\pi} |K_q(\xi_1)|^2 \int_{\gamma_-} \frac{d\xi_2}{2\pi} \tilde{K}_q(\xi_2)^2$$
$$\times \left( \cot h^2 \frac{\xi_1 + \xi_2}{2} - \tanh^2 \frac{\xi_1 + \xi_2}{2} - \cot h^2 \frac{\xi_1 - \xi_2}{2} + \tanh^2 \frac{\xi_1 - \xi_2}{2} \right),$$

where the dots again represent sub-leading terms that fall off at least as $\sim 1/(Mt)$ in the long-time limit. The first term in (42) can be viewed as the third term in the expansion of $\bar{\sigma} e^{-\Gamma t}$, while the second corresponds to the second term in the expansion of $D'_{22}(t) e^{-\Gamma t}$. On the other hand $D'_{44}$ is independent of time and represents a correction of order $K^4_q$ to $D_{00} = \bar{\sigma}$. We further note that $D_{44}(t)$ does not contain terms that are linear in $t$ (as $D'_{22}(t) \propto 1/(Mt)$).

5.5. Leading-time dependence of higher-order terms

Finally, we argue in appendix E that the leading terms in the long-time behaviour of $D_{2m,2m}(t)$ and $D_{2m+2,2m}(t)$ are given by

$$D_{2m,2m}(t) = \frac{\bar{\sigma}}{m!} (-\Gamma t)^m + \cdots,$$

$$D_{2m+2,2m}(t) = D_{2m,2m+2}(t)^* = \frac{1}{m!} (-\Gamma t)^m D_{20}(t) + \cdots,$$

where the dots represent terms that grow at most as $\propto t^{m-1}$ for large times.

5.6. Resummation and long-time behaviour

As we have shown in the previous sections, the expansion (32) contains terms that grow at long times as powers of $t$. Hence, in order to obtain a well-defined result in the long-time limit we have to resum these divergences. From the results presented in the previous subsections we deduce the leading long-time behaviour $(Mt \gg 1)$ of the one-point function
of $\sigma$ after a mass quench,
\[
\langle \sigma(t) \rangle = \bar{\sigma} \left[ 1 + \frac{\alpha}{Mt} - \frac{1 - M/M_0 \cos(2Mt - \pi/4)}{8\sqrt{\pi} (Mt)^{3/2}} + \cdots \right] e^{-\Gamma t} + \cdots,
\]
where the dots represent sub-leading corrections to the prefactor as well as terms that decay faster than $e^{-\Gamma t}$, respectively. The relaxation rate is given by
\[
\Gamma(M, M_0) = \frac{2M}{\pi} \int_0^\infty d\xi |K_q(\xi)|^2 \sinh \xi + O(K_q^6).
\]
We have determined the relaxation rate up to order $K_q^6$. The fact that there is no contribution in $O(K_q^4)$ follows from the absence of terms linear in $t$ in $D_{44}(t)$. This finding is in complete agreement with the corresponding result for the lattice model [10]. Consistent calculation of the relaxation rate in $O(K_q^6)$ would require the derivation of the contributions to $D_{66}(t)$ that grow linearly in time.

The $1$ and the $1/(Mt)^{3/2}$ term in the prefactor of the exponential in (46) have been established by considering particular contributions to all orders and showing that they exponentiate (see (44) and (45)). A detailed discussion of this point for the lattice Ising chain is given in [21]. On the other hand, the $\sim 1/(Mt)$ contribution in the prefactor is a conjecture based on our results for the leading contributions (in the expansion in powers of the quench matrix) in the Lehmann representation of the one-point function. From (40) and (42) we deduce
\[
\alpha = -\frac{(1 - M/M_0)^2}{32\pi} + O(K_q^3).
\]
Finally, we stress that our results agree with the scaling limit of a quench in the transverse field of the Ising chain in the ordered phase [10, 21].

6. Extrapolation time

The $K$-matrix for fixed boundary conditions in the Ising field theory is given by [16]
\[
K_{\text{fixed}}(\xi) = i \tanh \frac{\xi}{2}.
\]
We note that this is obtained as a limit of the quench $K$-matrix (17),
\[
K_{\text{fixed}}(\xi) = \lim_{M_0 \to \infty} K_q(\xi).
\]
In the quench problem a finite value of $M_0$ is required to render rapidity integrals convergent at large energies. In particular, the decay rate $\Gamma$ depends on $M_0$ and diverges in the limit $M_0 \to \infty$. For quenches in interacting integrable quantum field theories it is currently not known how to express a given initial state in terms of eigenstates of the post-quench Hamiltonian [13]. An exception is initial states that correspond to integrable boundary conditions. In order to use this information in the context of quantum quenches, a prescription for how to ‘regularize’ the corresponding $K$-matrices at large rapidities is required. Fioretto and Mussardo introduced an ‘extrapolation time’ $\tau_0$ by the replacement [9]
\[
K_{\text{fixed}}(\xi) \to i \tanh \frac{\xi}{2} e^{-2M_0 \cosh \xi} \equiv K_{\tau_0}(\xi).
\]
Using this regularized $K$-matrix to perform our calculations results in a decay rate

$$\Gamma_{\tau_0}(M) = \frac{2M}{\pi} \int_0^{\infty} d\xi |K_{\tau_0}(\xi)|^2 \sinh \xi + \mathcal{O}(K_{\tau_0}^6).$$

By comparing (52) and (47) and requiring the decay rates to be equal, $\Gamma_{\tau_0}(M) = \Gamma(M, M_0)$, it is possible to relate the extrapolation time $\tau_0$ to the initial mass $M_0$.

7. Conclusions

In this work we have considered the time evolution of the order parameter after a quantum quench of the mass in the Ising field theory. We have focused on a quench within the ordered phase. We found exponential decay of the order parameter to zero (46). Our results agree with the scaling limit of a quantum quench performed in the ordered phase of the transverse field Ising chain [10, 21]. Our main achievement is of technical nature: we have shown how to carry out calculations in the field theory limit. Here, unlike for the lattice model, additional divergences occur that need to be regulated appropriately. We have shown how to use techniques developed recently in the study of integrable quantum field theories at finite temperatures to overcome this problem. Our method generalizes to interacting integrable quantum field theories such as the sine–Gordon and $O(N)$ non-linear sigma models. This opens the door for analysing quantum quenches in these theories, at least for particular classes of initial states related to integrable boundary conditions (‘integrable quenches’) [9]. Work on the sine–Gordon model is under way.

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Appendix A. Calculation of $\langle \Psi_0 | \Psi_0 \rangle$

In this appendix we evaluate the leading terms in the expansion (23) of the norm of the initial state $|\Psi_0\rangle$. Obviously one has $Z_0 = 1$, while $Z_2$ was already calculated in section 4.2 with the result

$$Z_2 = \frac{L}{2} \int_0^{\infty} d\xi |K_{\xi}(\xi)|^2 \equiv \delta(-2\kappa) \int_0^{\infty} d\xi |K_{\xi}(\xi)|^2.$$

In the last step we have reintroduced the auxiliary variable $\kappa$. We will explicitly retain the auxiliary variables $\kappa_i$ throughout the appendices, but keep in mind that all expressions have to be understood as generalized functions of $\kappa_i$ as discussed in section 4.2. As all expressions are multiplied by the strongly peaked functions $P(\kappa_i)$ we can drop all terms $\propto \kappa_i^n$ with $n \geq 1$. In contrast, all irregular terms $\propto \delta(\kappa_i)$ as well as all divergent terms $\propto \kappa_i^n$ with $n \leq -1$ have to cancel when considering the expansion (32). It is the purpose of these appendices to show by explicit evaluation up to $\mathcal{O}(K_i^4)$ that these terms indeed cancel each other and that the remaining terms $\propto \kappa_i^0$ yield the results for $D_{2m,2n}(t)$ presented in section 5.

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The next term $Z_4$ requires the introduction of two auxiliary variables $\kappa_1$ and $\kappa_2$. Starting from (22) we have to regularize the overlap element

$$
\langle \xi_1', -\xi_1', \xi_2', -\xi_2' | -\xi_2, \xi_2, -\xi_1, \xi_1 \rangle 
\equiv \langle \xi_1', -\xi_1', \xi_2', -\xi_2' | -\xi_2 + \kappa_2, \xi_2 + \kappa_2, -\xi_1 + \kappa_1, \xi_1 + \kappa_1 \rangle 
$$
(A.2)

$$
= (2\pi)^4 \delta(\xi_1' - \xi_1 - \kappa_1) \delta(-\xi_1' + \xi_1 - \kappa_1) \delta(\kappa_2 - \xi_2 - \kappa_2) \delta(-\xi_2 + \xi_2 - \kappa_2) 
- \delta(\xi_1' - \xi_1 - \kappa_1) \delta(-\kappa_1 + \xi_2 - \kappa_2) \delta(\kappa_2 - \xi_2) \delta\left(-\xi_2 + \xi_2 - \kappa_2\right) 
- \delta(\xi_1' - \xi_2 - \kappa_2) \delta(-\kappa_2 + \xi_1 - \kappa_1) \delta(-\xi_1 + \xi_1 - \kappa_1) \delta\left(-\xi_1' + \xi_1 - \kappa_1\right) 
+ \delta(\xi_1' - \xi_2 - \kappa_2) \delta(-\kappa_2 + \xi_1 - \kappa_1) \delta(-\xi_1 + \xi_1 - \kappa_1) \delta\left(-\xi_1' + \xi_1 - \kappa_1\right) 
+ \ldots]
$$
(A.3)

Here the dots represent 12 further combinations of $\delta$-functions which lead, in analogy to (30), to terms containing integrals restricted to intervals like $0 < \xi_i < \kappa_i$. These terms in turn yield contributions of the $\kappa^n$ with $n \geq 1$ which vanish when performing the $\kappa$-integrations. Hence we have not written these terms in (A.3). Now straightforward evaluation of the four terms yields

$$
Z_4 = \frac{1}{2} \delta(-2\kappa_1) \delta(-2\kappa_2) \left( \int_0^\infty d\xi |K_q(\xi)|^2 \right)^2 
- \frac{1}{2} \delta(-2\kappa_1 - 2\kappa_2) \int_0^\infty d\xi K_q(\xi + \kappa_1)^* K_q(\xi - \kappa_1)^* K_q(\xi)^2.
$$
(A.4)

### Appendix B. Calculation of $D_{22}$

The first term in the expansion (21) which involves kinematical poles is $C_{22}(t)$, which after shifting the rapidities in the ket by $\kappa$ reads

$$
C_{22}(t) = \int_0^\infty d\xi' d\xi \frac{d\xi'}{(2\pi)^2} |K_q(\xi')|^* K_q(\xi) \langle \xi', -\xi'|\sigma|-\xi + \kappa, \xi, \kappa + \xi \rangle e^{2Mit(cosh\xi' - cosh \xi)}. \tag{B.1}
$$

We decompose the form factor into its connected and disconnected pieces using (25),

$$
\langle \xi', -\xi'|\sigma|-\xi + \kappa, \xi, \kappa + \xi \rangle = (2\pi)^2 \bar{\sigma} \left[ \delta(\xi' - \xi - \kappa) \delta(-\xi' + \xi - \kappa) - 2\pi \delta(\xi' - \xi - \kappa) f(-\xi' + i\pi + i\eta, -\xi + \kappa) 
- 2\pi \delta(-\xi' + \xi - \kappa) f(\xi' + i\pi + i\eta, \xi + \kappa) 
+ 2\pi \delta(\xi' + \xi - \kappa) f(-\xi' + i\pi + i\eta, \xi + \kappa) 
+ 2\pi \delta(-\xi' + \xi - \kappa) f(\xi' + i\pi + i\eta, -\xi + \kappa) 
+ f(\xi' + i\pi + i\eta_1, -\xi' + i\pi + i\eta_2, -\xi + \kappa, \xi + \kappa) \right], \tag{B.2}
$$

with $\eta, \eta_1 \rightarrow 0^+$. Insertion of (B.2) into (B.1) yields three different types of terms, which we denote by $C_{22}^0$, $C_{22}^1(t)$, and $C_{22}^2(t)$ respectively.

The first line simply gives

$$
C_{22}^0 = \bar{\sigma} \delta(-2\kappa) \int_0^\infty d\xi |K_q(\xi)|^2.
$$
(B.3)
The second term is obtained from the second to fifth lines in (B.2), which yield
\[
C_{22}^1(t) = i\tilde{\sigma} \coth \frac{2\kappa - i\eta}{2} \int_{-\infty}^{\infty} \frac{d\xi}{2\pi} K_q(\xi + \kappa) K_q(\xi) e^{2Mit[\cosh(\xi + \kappa) - \cosh \xi]}.
\] (B.4)

Hereby we have already omitted terms of the form
\[
-i\tilde{\sigma} \coth \frac{2\kappa - i\eta}{2} \int_{0}^{\infty} \frac{d\xi}{2\pi} K_q(\kappa - \xi) K_q(\xi) e^{2Mit[\cosh(\kappa - \xi) - \cosh \xi]} \propto \kappa^2
\] (B.5)

which vanish in the \(\kappa\)-regularization scheme due to
\[
\int d\kappa P(\kappa) \kappa^n \rightarrow 0 \quad \text{for} \quad L \rightarrow \infty, \quad n \geq 1.
\]

We can further analyse (B.4) by expanding the integrand up to \(O(\kappa)\),
\[
K_q(\xi + \kappa) K_q(\xi) e^{2Mit[\cosh(\xi + \kappa) - \cosh \xi]} = |K_q(\xi)|^2 + \kappa \dot{K}_q(\xi) \frac{d\dot{K}_q(\xi)}{d\xi} + 2Mit\kappa |K_q(\xi)|^2 \sinh \xi + O(\kappa^2).
\] (B.6)

The contributions from the second and third terms vanish as they are antisymmetric under \(\xi \rightarrow -\xi\), thus we arrive at
\[
C_{22}^1(t) = 2i\tilde{\sigma} \coth \frac{2\kappa - i\eta}{2} \int_{0}^{\infty} \frac{d\xi}{2\pi} |K_q(\xi)|^2.
\] (B.7)

Finally, the sixth line yields
\[
C_{22}^2(t) = \tilde{\sigma} \int_{0}^{\infty} \frac{d\xi'}{(2\pi)^2} K_q(\xi')^* K_q(\xi) \tanh \xi' \tanh \xi e^{2Mit[\cosh \xi' - \cosh \xi]}
\times \coth \frac{\xi' + \xi - \kappa + i\eta_1}{2} \coth \frac{\xi' - \xi - \kappa + i\eta_1}{2}
\times \coth \frac{\xi' - \xi + \kappa - i\eta_2}{2} \coth \frac{\xi' + \xi + \kappa - i\eta_2}{2}.
\] (B.8)

In order to isolate the singularities we may shift the \(\xi'\)-contour to the upper half plane or the \(\xi\)-contour to the lower half plane. Doing so we pick up contributions from the poles at \(\xi' = \xi - \kappa + i\eta_2\), \(\xi' = -\xi - \kappa + i\eta_2\) and \(\xi = \xi' + \kappa - i\eta_2\), \(\xi = -\xi' + \kappa - i\eta_1\), respectively, and we obtain
\[
C_{22}^2(t) = \tilde{\sigma} \Re e \int_{0}^{\infty} \frac{d\xi'}{2\pi} \int_{\gamma_-} \frac{d\xi'}{2\pi} \hat{K}_q(\xi') \hat{K}_q(\xi) \tanh \xi' \tanh \xi
\times \coth^2 \frac{\xi' - \xi}{2} \coth^2 \frac{\xi' + \xi}{2} e^{2Mit[\cosh \xi' - \cosh \xi]}
\times \{ \Theta(\xi - \kappa) \hat{K}_q(\xi - \kappa) e^{2Mit[\cosh(\xi - \kappa)]}
+ \Theta(-\xi - \kappa) \hat{K}_q(-\xi - \kappa) e^{2Mit[\cosh(\xi + \kappa)]} \}
\] (B.9)

\[
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\] (B.10)
stress that the first term exactly cancels the product ξ
ξ
(iii) evaluate the terms by shifting the contours of integration for
follow the same steps as above: (i) shift the rapidities in the ket by the auxiliary variable
half plane, and (iv) expand the result in κ
κ
where
Z
Now the first term in (B.12) cancels
C
We restrict ourselves to
C
where the path γ
−
lies in the lower half plane and was explicitly defined in (39).
Expanding (B.10) and (B.11) in κ yields
−2iσ coth \( \frac{2\kappa - i(\eta_1 + \eta_2)}{2} \) \( \int_0^\infty \frac{d\xi}{2\pi} \hat{K}_q(\xi) e^{2M\Gamma t \cosh \xi} \)
× \{ Θ(κ + κ) \hat{K}_q(κ + κ) e^{-2M\Gamma t \cosh (κ + κ)}
− Θ(−κ + κ) \hat{K}_q(−κ + κ) e^{-2M\Gamma t \cosh (−κ + κ)} \}, \hspace{1cm} (B.11)
where we have used
\[ \int_0^\infty d\xi \hat{K}_q(\xi) \frac{d\hat{K}_q}{d\xi}(\xi) = \left. \frac{1}{2} \hat{K}_q(\xi)^2 \right|_0^\infty = 0. \] \hspace{1cm} (B.13)
Now the first term in (B.12) cancels \( C_{22}^1(t) \) and we arrive at the final result
\[ C_{22}(t) = \bar{\sigma} \delta(-2\kappa) \int_0^\infty d\xi |K_q(\xi)|^2 - \frac{2M\bar{\sigma} t}{\pi} \int_0^\infty d\xi |K_q(\xi)|^2 \sinh \xi + D'_{22}(t) \]
\[ = \bar{\sigma} Z_2 - \bar{\sigma} \Gamma t + D'_{22}(t), \] \hspace{1cm} (B.14)
where \( Z_2, \Gamma \) and \( D'_{22}(t) \) are defined by (A.1), (37), and (B.9) or (38) respectively. We stress that the first term exactly cancels the product \( Z_2 C_{00} = \bar{\sigma} Z_2 \), while the second term equals the second term in the expansion of \( \bar{\sigma} e^{-\Gamma t} \).

Appendix C. Calculation of \( D_{42} \) and \( D_{24} \)

We restrict ourselves to \( C_{42}(t) \) as \( C_{24}(t) = C_{42}(t)^* \). For the calculation of \( C_{42}(t) \) we follow the same steps as above: (i) shift the rapidities in the ket by the auxiliary variable \(-\xi, \xi \rightarrow -\xi + \kappa, \xi + \kappa\), (ii) analytically continue the resulting form factor using (25), (iii) evaluate the terms by shifting the contours of integration for \( \xi'_1 \) and \( \xi'_2 \) to the upper half plane, and (iv) expand the result in \( \kappa \) up to \( \mathcal{O}(\kappa^0) \). In particular one can show that all ill-defined terms \( \propto 1/\kappa \) (see (B.7) for a similar term in \( \mathcal{O}(K_q^2) \)) cancel each other. After straightforward calculation we obtain
\[ C_{42}(t) = \delta(-2\kappa) C_{20}(t) \int_0^\infty d\xi |K_q(\xi)|^2 - \Gamma t C_{20}(t) \]
\[ + \bar{\sigma} \int_0^\infty \frac{d\xi}{2\pi} \hat{K}_q(\xi)^3 \tanh \xi e^{2M\Gamma t \cosh \xi} \]
\[ - 2i\bar{\sigma} \int_{\gamma_+} \frac{d\xi'}{2\pi} \int_0^\infty \frac{d\xi}{2\pi} \hat{K}_q(\xi') |K_q(\xi)|^2 \tanh \xi' \]
\[ \times \left( \coth \frac{\xi' - \xi}{2} - \tanh \frac{\xi' - \xi}{2} - \coth \frac{\xi' + \xi}{2} + \tanh \frac{\xi' + \xi}{2} \right) e^{2M\Gamma t \cosh \xi'} \] \hspace{1cm} (C.2)
The final result is

\[\mathcal{F}(t) = \bar{\gamma} + \frac{\bar{\gamma}}{2} \int_{t_o}^{t} \frac{d\xi_1}{2\pi} \int_{t_o}^{t} \frac{d\xi_2}{2\pi} \mathcal{K}_q(\xi_1) \tanh \xi \prod_{i=1}^{2} \hat{\mathcal{K}}_q(\xi'_i) \tanh \xi'_i \]

\[\times \prod_{i=1}^{2} \coth^2 \frac{\xi'_i - \xi}{2} \coth^2 \frac{\xi'_i + \xi}{2} \tanh^2 \frac{\xi'_1 - \xi'_2}{2} \tanh^2 \frac{\xi'_1 + \xi'_2}{2} \mathcal{K}_q(\xi_1') \tanh \xi_1' \]

\[\times \frac{e^{2\Gamma t}}{\left(\coth(\xi_1' + \coth(\xi_2' - \coth(\xi))\right)}, \quad (C.3)\]

where the path \(\gamma_+\) lies in the upper half plane and is explicitly defined by \((0 < \phi_0 \leq \pi/4)\)

\[\gamma_+(s) = \begin{cases} \text{i}s, & 0 \leq s \leq \phi_0, \\ (s - \phi_0) + i\phi_0, & \phi_0 \leq s < \infty. \end{cases} \quad (C.4)\]

The first term in \((C.1)\) cancels against the product of \(Z_2 C_20(t)\), while the second term corresponds to the second term in the expansion of \(C_20(t) e^{-\Gamma t}\). All other terms constitute sub-leading corrections to \((46)\).

**Appendix D. Calculation of \(D_{44}\)**

The calculation follows the same steps as outlined above. The main difference is that we have to introduce two auxiliary variables, i.e. the form factor in \(C_{44}(t)\) becomes

\[\langle \xi'_1, -\xi'_1, \xi'_2, -\xi'_2|\sigma|-\xi_2, \xi_2, -\xi_1, \xi_1 \rangle \]

\[\rightarrow \langle \xi'_1, -\xi'_1, \xi'_2, -\xi'_2|\sigma|-\xi_2 + \kappa_2, \xi_2 + \kappa_2, -\xi_1 + \kappa_1, \xi_1 + \kappa_1 \rangle, \quad (D.1)\]

which is analytically continued using \((25)\). After a tedious but straightforward evaluation of the resulting terms up to \(O(\kappa_1^4, \kappa_0^2)\) one can explicitly show that all ill-defined terms

\[\propto \coth \frac{2\kappa_1 - i0}{2}, \quad \propto \coth \frac{2\kappa_2 - i0}{2}, \quad \propto \coth \frac{2\kappa_1 + 2\kappa_2 - i0}{2} \quad (D.2)\]

cancel each other, where we have employed for example

\[\int d\kappa_1 d\kappa_2 \kappa_1^{m} \kappa_2^{n} \coth \frac{2\kappa_2 - i0}{2} P(\kappa_1) P(\kappa_2) \propto L^{1-m-n} \rightarrow 0 \quad \text{for } m + n \geq 2. \quad (D.3)\]

The final result is

\[C_{44}(t) = \bar{\gamma} Z_1 + \frac{1}{2} \left(\delta(-2\kappa_1) + \delta(-2\kappa_2)\right) \left(D_{22}'(t) - \bar{\gamma} \Gamma t\right) \int_0^{\infty} d\xi |\mathcal{K}_q(\xi)|^2 \]

\[+ \frac{\bar{\gamma}}{2} \left(\Gamma t\right)^2 - \Gamma t D_{22}'(t) \quad (D.4)\]

\[+ \bar{\gamma} \Re \int_0^{\infty} d\xi_1 |\mathcal{K}_q(\xi_1)|^2 \int_{\gamma_-} d\xi_2 |\mathcal{K}_q(\xi_2)|^2 \]

\[\times \left(\coth^2 \frac{\xi_1 + \xi_2}{2} - \tanh^2 \frac{\xi_1 + \xi_2}{2} - \coth^2 \frac{\xi_1 - \xi_2}{2} + \tanh^2 \frac{\xi_1 - \xi_2}{2}\right) \quad (D.5)\]
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\[ + 2\sigma \Re \int_0^\infty \frac{d\xi'}{2\pi} \int_{\gamma_-} \frac{d\xi}{2\pi} \hat{K}_q(\xi')^3 \hat{K}_q(\xi) \tanh \xi' \tanh \xi \]
\[ \times \coth^2 \frac{\xi' - \xi}{2} \coth^2 \frac{\xi' + \xi}{2} e^{2Mt(\cosh \xi' - \cosh \xi)} \quad (D.6) \]
\[ + 2\sigma \int_{\gamma_+} \frac{d\xi'}{2\pi} \int_0^\infty \frac{d\xi_1}{2\pi} \int_{\gamma_-} \frac{d\xi_2}{2\pi} \hat{K}_q(\xi') \hat{K}_q(\xi_1)^2 \hat{K}_q(\xi_2) \]
\[ \times \tanh \xi' \tanh \xi_2 \coth^2 \frac{\xi' - \xi_2}{2} \coth^2 \frac{\xi' + \xi_2}{2} \]
\[ \times \left[ \tanh \frac{\xi' - \xi_1}{2} - \tanh \frac{\xi' + \xi_1}{2} - \coth \frac{\xi' - \xi_1}{2} + \coth \frac{\xi' + \xi_1}{2} + \tanh \frac{\xi_1 - \xi_2}{2} - \tanh \frac{\xi_1 + \xi_2}{2} - \coth \frac{\xi_1 - \xi_2}{2} + \coth \frac{\xi_1 + \xi_2}{2} \right] \]
\[ \times e^{2Mt(\cosh \xi' - \cosh \xi_2)} \quad (D.7) \]
\[ + \frac{\bar{\sigma}}{4} \int_{\gamma_+} \frac{d\xi'_1}{(2\pi)^2} \int_0^\infty \frac{d\xi_1}{(2\pi)^2} \int_{\gamma_-} \frac{d\xi_2}{(2\pi)^2} \prod_{i=1}^2 \hat{K}_q(\xi'_i) \hat{K}_q(\xi_i) \tanh \xi'_i \tanh \xi_i \]
\[ \times \prod_{i=1}^2 \coth^2 \frac{\xi'_i - \xi_j}{2} \coth^2 \frac{\xi'_i + \xi_j}{2} \tanh^2 \frac{\xi_1 - \xi_2}{2} \tanh^2 \frac{\xi_1 + \xi_2}{2} \]
\[ \times \tanh^2 \frac{\xi'_1 - \xi'_2}{2} \tanh^2 \frac{\xi'_1 + \xi'_2}{2} e^{2Mt\sum_i(\cosh \xi'_i - \cosh \xi_i)} \], \quad (D.8)\]

where \( Z_4, \Gamma \) and \( D'_{22}(t) \) are defined in (A.4), (37) and (B.9) respectively. The paths \( \gamma_{\pm} \) are defined in (39) and (C.4). The leading contributions are given by the first line (D.4), the second line (D.5) yields the time-independent term \( D'_4 \), and (D.6)–(D.8) constitute sub-leading corrections that fall off at least as \( \sim 1/(Mt) \) in the long-time limit.

Now \( D_{44}(t) \) is obtained by (see (B.14))

\[ D_{44}(t) = C_{44}(t) - Z_2 C_{22}(t) + (Z_2^2 - Z_4)\bar{\sigma} \quad (D.9) \]
\[ = \left( \delta(-2\kappa) - \frac{1}{2}\delta(-2\kappa_1) - \frac{1}{2}\delta(-2\kappa_2) \right) \left( \bar{\sigma} \Gamma t - D'_{22}(t) \right) \int_0^\infty d\xi |K_q(\xi)|^2 \quad (D.10) \]
\[ + \frac{\bar{\sigma}}{2}(\Gamma t)^2 - \Gamma t D'_{44}(t) + D'_{44} + \cdots, \quad (D.11) \]

where the dots represent the sub-leading terms (D.6)–(D.8). Here (D.10) vanishes due to

\[ \delta(-2\kappa) - \frac{1}{2}\delta(-2\kappa_1) - \frac{1}{2}\delta(-2\kappa_2) \equiv \int d\kappa \delta(-2\kappa) P(\kappa) \]
\[ - \frac{1}{2} \int d\kappa_1 d\kappa_2 [\delta(-2\kappa_1) + \delta(-2\kappa_2)] P(\kappa_1) P(\kappa_2) \quad (D.12) \]
\[ = \frac{L}{2} - \frac{L}{4} \int d\kappa_2 P(\kappa_2) = \frac{L}{4} \int d\kappa_1 P(\kappa_1) = 0, \quad (D.13) \]

and we arrive at (42).

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Appendix E. Leading-time dependence of $D_{2m,2m}$ and $D_{2m+2,2m}$

The leading behaviour of $D_{2m,2m}(t)$ in the long-time limit is obtained from $C_{2m,2m}(t)$ given in (21) by (i) introducing the auxiliary variables $\kappa_i$, $i = 1, \ldots, m$, (ii) regularizing the form factor according to (25) while keeping only the connected piece (i.e. the term with $A_2 = B_2 = 0$), (iii) evaluating the connected form factor (14), and (iv) shifting the $\xi_i$-contours to the upper half plane and keeping only the contributions from the poles at $\xi_i = \xi_j - \kappa_j + i0$. The result after these steps reads

$$\bar{\sigma} \frac{(-2i)^m}{m!} \prod_{i=1}^{m} \coth \frac{2\kappa_i - i0}{2} \int_0^\infty \frac{d\xi_1 \cdots d\xi_m}{(2\pi)^m} \prod_{i=1}^{m} K_q(\xi_i - \kappa_i)^* K_q(\xi_i) e^{2M(t)(\cosh(\xi_i - \kappa_i) - \cosh \xi_i)} \times \prod_{i<j}^{m} \frac{\tanh \frac{\xi_i - \xi_j + \kappa_i - \kappa_j}{2}}{\tanh \frac{\kappa_i - \kappa_j}{2}} \times \frac{\coth \frac{\xi_i - \xi_j - \kappa_i - \kappa_j + i0}{2}}{\coth \frac{\xi_i - \kappa_j}{2}}.$$  

(E.1)

Now expanding in the $\kappa_i$s we obtain

$$\bar{\sigma} \frac{(-4Mt)^m}{m!} \int_0^\infty \frac{d\xi_1 \cdots d\xi_m}{(2\pi)^m} \prod_{i=1}^{m} |K_q(\xi_i)|^2 \sinh \xi_i + \cdots = \bar{\sigma} \frac{1}{m!} \left( -2 \frac{M}{\pi} \right) \int_0^\infty \frac{d\xi}{|K_q(\xi)|^2} \sinh \xi \right)^m t^m + \cdots,$$

(E.2)

where the dots represent terms that grow at most as $\propto t^{m-1}$ for large times. As the disconnected pieces of $C_{2m,2m}(t)$ also grow at most as $\propto t^{m-1}$ we deduce that the leading-time dependence of $D_{2m,2m}(t)$ is given by (44). Following the same line of argument for $C_{2m+2,2m}(t)$ we arrive at (45).

Appendix F. Finite-size regularization

In our calculation in the infinite-volume system we have regularized the kinematical poles in the form factors using a combination of the analytic continuation (25) together with the $\kappa$-regularization. An alternative procedure to regularize the kinematical poles is to study [10, 18, 21] the system with a finite length $L$. In this case the Hilbert space divides itself into two sectors: the Neveu–Schwarz (NS) sector corresponding to antiperiodic boundary conditions and the Ramond (R) sector corresponding to periodic ones. The rapidities in these sectors are quantized according to

$$\text{NS: } ML \sinh \xi_p = 2\pi p, \quad p \in \mathbb{Z} + \frac{1}{2},$$

(F.1)

$$\text{R: } ML \sinh \theta_q = 2\pi q, \quad q \in \mathbb{Z}.$$  

(F.2)

The spin operator connects the two sectors, its form factors in the finite system read [30]

$$\langle \sigma | q_1, \ldots, q_m \rangle = S(L) \prod_{i=1}^{m} g(\xi_p) \prod_{j=1}^{n} g(\theta_q) F_{mn}(\xi_{p1}, \ldots, \xi_{pn}, |\theta_{q1}, \ldots, \theta_{qn}|).$$  

(F.3)

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Figure F.1. $D_{22}^{L}(t)$ (black line, left axis) and $D_{22}^{L}(t) + \Gamma L t$ (red line, right axis) for $\tau_0 = 0.1$, $L = 50$ and $N = 300$. For comparison we show $D_{22}(t)$ (black stars) and $D^{'}_{22}(t)$ (red stars) as defined in (36) and (38) respectively. We observe excellent agreement between the finite-volume regularization and the infinite-volume results.

where the function $F_{mn}$ is the infinite-volume form factor (see (14) and (27))

$$F_{mn}(\xi_1, \ldots, \xi_m|\theta_1, \ldots, \theta_n) = i^{\lfloor (m+n)/2 \rfloor} \bar{\sigma} \prod_{i,j=1}^{m \atop i<j} \tanh \frac{\xi_i - \xi_j}{2} \prod_{i,j=1}^{n \atop i<j} \tanh \frac{\theta_i - \theta_j}{2} \prod_{i=1}^{m} \prod_{j=1}^{n} \coth \frac{\xi_i - \theta_j}{2},$$

the symbol $\lfloor \rfloor$ denotes the floor function, and the constant as well as the leg factors are

$$S(L) = 1 + O(e^{-L}), \quad g(\theta) = \tilde{g}(\theta) = \frac{1}{\sqrt{ML \cosh \theta}} + O(e^{-L}).$$

We stress that due to the quantization of the rapidities the singularities in the form factor (F.4) are regularized.

As we consider quenches in the ordered phase which breaks the $\mathbb{Z}_2$ invariance the initial state in a system of length $L$ has the form [10,21]

$$|\Psi_0\rangle_L = \frac{1}{\sqrt{2}} [|\Psi_0\rangle_{NS} + |\Psi_0\rangle_R],$$

where

$$|\Psi_0\rangle_{NS/R} = \exp \left( \sum_{k \in NS/R \atop k > 0} K_q(\xi_k) A^\dagger(-\xi_k) A^\dagger(\xi_k) \right) |0\rangle_{NS/R},$$

and $|0\rangle_{NS/R}$ denotes the vacuum state in the corresponding sector. In the regularization introduced by Fioretto and Mussardo (see section 6) the quench matrix $K_q(\xi_k)$ has to be replaced by $K_{\tau_0}(\xi_k)$, which we will do in the numerical evaluations presented in figures F.1 and F.2. The time evolution starting from the initial state (F.6) now reads

$$\langle \sigma(t) \rangle = \frac{2}{NS} \frac{\langle \sigma(t)|\Psi_0\rangle_R}{\langle \Psi_0|\Psi_0\rangle_{NS} + R \langle \Psi_0|\Psi_0\rangle_R} = \sum_{m,n=0}^{\infty} D_{2m,2n}^{L}(t).$$
Figure F.2. $D_{44}^L(t)$ (black line, left axis) and $D_{44}'^L(t) \equiv D_{44}^L(t) - (\Gamma^L t)^2/2 + \Gamma^L t (D_{44}^L(t) + \Gamma^L t)$ (red line, right axis) for $\tau_0 = 0.1$, $L = 50$ and $N = 300$. For comparison we show $D_{44}(t)$ (black stars) and $D_{44}'(t)$ (red stars) as defined in (42) and (43) respectively. Note that in contrast to $D_{44}'$ the finite-volume term $D_{44}'(t)$ depends on the time as it also includes corrections which fall off as powers of $Mt$ (these corrections are incorporated in the dots in (42)). We observe excellent agreement between the finite-volume regularization and the infinite-volume results.

Straightforward calculation of the norms gives

$$Z_{NS/R}^L = \sum_{n=0}^{\infty} Z_{NS/R,2n}^L,$$  \hspace{1cm} (F.9)

$$Z_{NS/R,2}^L = \sum_{k,l \in NS/R, k,l > 0} \frac{|K_q(\xi_k)K_q(\xi_l)|}{Z_{NS/R}^L},$$ \hspace{1cm} \sum_{k \in NS/R, k > 0} = \sum_{|k|,|l| = \frac{1}{2}} |K_q(\xi_k)|^2,$$  \hspace{1cm} (F.10)

$$Z_{NS/R,4}^L = \left( \sum_{k \in NS/R, k > 0} |K_q(\xi_k)|^2 \right)^2 - \frac{1}{2} \sum_{k \in NS/R, F.11}

We note the analogy to the infinite-volume results (A.1) and (A.4) respectively. The first non-trivial term in the expansion (F.8) reads

$$D_{44}^L(t) = C_{22}^L(t) - \frac{\sigma}{2} \left( Z_{NS,2}^L + Z_{R,2}^L \right),$$  \hspace{1cm} (F.12)

$$C_{22}^L(t) = \sum_{p \in NS, p > 0} \sum_{q \in R, q > 0} K_q(\xi_p)^* K_q(\xi_q) \frac{\sigma}{\tau_0} N \left( \frac{\sigma}{\tau_0} \right) \frac{e^{2Mt(cosh(\xi_p) + cosh(\xi_q))}}{2}$$  \hspace{1cm} (F.13)

$$= \frac{\sigma}{(ML)^2} \sum_{p \in NS, p > 0} \sum_{q \in R, q > 0} K_q(\xi_p)^* K_q(\xi_q) \frac{\tanh(\xi_p) \tanh(\xi_q)}{\cosh(\xi_p) \cosh(\xi_q)}$$  \hspace{1cm} (F.14)

$$\times \coth^2 \frac{\xi_p - \xi_q}{2} \coth^2 \frac{\xi_p + \xi_q}{2} e^{2Mt(cosh(\xi_p) + cosh(\xi_q))}.$$
In the same way a straightforward calculation yields

\[
D_{44}^{L}(t) = \frac{\bar{\sigma}}{4(ML)^{2}} \sum_{p_{1},p_{2} \in NS} \sum_{q_{1},q_{2} \in R} K_{q}(\xi_{p_{1}})^{*} K_{q}(\xi_{p_{2}})^{*} K_{q}(\xi_{q_{1}}) K_{q}(\xi_{q_{2}}) \prod_{i=1}^{2} \tanh \frac{\xi_{p_{i}}}{\cosh \xi_{p_{i}}} \tanh \frac{\xi_{q_{i}}}{\cosh \xi_{q_{i}}}
\]

\[
\times \tanh^{2} \frac{\xi_{p_{1}} + \xi_{p_{2}}}{2} \tanh^{2} \frac{\xi_{p_{1}} - \xi_{p_{2}}}{2} \tanh^{2} \frac{\xi_{q_{1}} + \xi_{q_{2}}}{2} \tanh^{2} \frac{\xi_{q_{1}} - \xi_{q_{2}}}{2}
\]

\[
\times \prod_{i,j=1}^{2} \frac{\coth^{2} \frac{\xi_{p_{i}} + \xi_{q_{j}}}{2} - \frac{2}{\coth^{2} \frac{\xi_{p_{i}} - \xi_{q_{j}}}{2}}}{\cosh^{2} \frac{\xi_{p_{i}} - \xi_{q_{j}}}{2} e^{2\mu t} \sum_{i}(\cosh \xi_{p_{i}} - \cosh \xi_{q_{j}})}
\]

\[
- \frac{1}{2} C_{22}^{L}(t) (Z_{NS,2}^{L} + Z_{R,2}^{L}) + \frac{\bar{\sigma}}{2} Z_{NS,2}^{L} Z_{R,2}^{L} + \frac{\bar{\sigma}}{4} (Z_{NS,4}^{L} + Z_{R,4}^{L}). \tag{F.15}
\]

Finally, the relaxation rate (47) up to \(O(K_{q}^{2})\) is given by

\[
\Gamma^{L} = -\frac{2M}{L} \sum_{p \in NS} \left| K_{q}(\xi_{p}) \right|^{2} \tanh \xi_{p} - \frac{2M}{L} \sum_{q \in R} \left| K_{q}(\xi_{q}) \right|^{2} \tanh \xi_{q}. \tag{F.16}
\]

We have evaluated (F.14) and (F.15) numerically using the replacement \(K_{q} \rightarrow K_{m}\) for several values of \(\tau_{0}, L, \) and \(N,\) where \(N\) denotes the UV cutoff for the momentum numbers \(p\) and \(q.\) The results are shown in figures F.1 and F.2. We observe excellent agreement with the results obtained in the infinite volume.

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