Towards a possible solution for the coincidence problem: $f(G)$ gravity as background

Prabir Rudra

Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711 103, India.
Department of Mathematics, Pailan College of Management and Technology, Bengal Pailan Park, Kolkata-700 104, India.

Abstract

In this article we address the well-known cosmic coincidence problem in the framework of the $f(G)$ gravity. In order to achieve this, an interaction between dark energy and dark matter is considered. A set-up is designed and a constraint equation is obtained which generates the $f(G)$ models that do not suffer from the coincidence problem. Due to the absence of a universally accepted interaction term introduced by a fundamental theory, the study is conducted over three different forms of logically chosen interaction terms. To illustrate the set-up three widely known models of $f(G)$ gravity are taken into consideration and the problem is studied under the designed set-up. The study reveals that the popular $f(G)$ gravity models does not approve of a satisfactory solution of the long standing coincidence problem, thus proving to be a major setback for them as successful models of universe. Finally, two non-conventional models of $f(G)$ gravity have been proposed and studied in the framework of the designed set-up. It is seen that a complete solution of the coincidence problem is achieved for these models. The study also reveals that the b-interaction term is much more preferable compared to the other interactions, due to its greater compliance with the recent observational data.

Keywords: Dark energy, Dark matter, Modified gravity, coincidence, interaction.

Pacs. No.: 95.36.+x, 95.35.+d

1 Introduction

Recent observational evidences from Ia supernovae, CMBR via WMAP, galaxy redshift surveys via SDSS indicated that the universe is going through an accelerated expansion of late [1, 2, 3, 4, 5]. With this discovery the incompatibility of general relativity (GR) as a self sufficient theory of gravity came into light. Since no possible explanation of this phenomenon could be attributed in the framework of Einstein’s GR, a proper modification of the theory was required that will successfully incorporate the late cosmic acceleration. As the quest began, two different approaches regarding this modification came into existence.

According to the first approach, cosmic acceleration can be phenomenally attributed to the presence of a mysterious negative energy component popularly known as dark energy (DE) [6]. Here we modify the right hand side of the Einstein’s equation, i.e. in the matter sector of the universe. Latest observational data shows that the contribution of DE to the energy sector of the universe is $\Omega_d = 0.7$. With the passage of time, extensive search saw various candidates for DE appear in the scene. Some of the popular ones worth mentioning are Chaplygin gas models [7, 8], Quintessence Scalar field [9], Phantom energy field [10], etc. A basic feature of these models is that, they violate the strong energy condition i.e., $\rho + 3p < 0$, thus producing the observed cosmic acceleration. Recent reviews on DE can be found in [11, 12].

A different section of cosmologists resorted to an alternative approach for explaining the expansion. This concept is based on the modification of the gravity sector of GR, thus giving birth to modified gravity theories.
A universe associated with a tiny cosmological constant, i.e. the \( \Lambda \)CDM model served as a prototype for this approach. It was seen that the model could satisfactorily explain the recent cosmic acceleration and passed a few solar system tests as well. But with detailed diagnosis it was revealed that the model was paralyzed with a few cosmological problems. Out of these, two major problems that crippled the model till date are the Fine tuning problem (FTP) and the Cosmic Coincidence problem (CCP). The FTP refers to the large discrepancy between the observed values and the theoretically predicted values of cosmological parameters. Numerous attempts to solve this problem can be found in the literature. Among them, the most impressive attempt was undertaken by Weinberg in [13]. Although the approaches for the solutions are different, yet, almost all of them are basically based on the fact that the cosmological constant may not assume an extremely small static value at all times during the evolution of the universe (as predicted by GR), but its nature should be rather dynamical [14]. These drawbacks reduced the effectiveness of the model, as well as its acceptability, and hence alternative modifications of gravity was sought for. Some of the popular models of modified gravity that came into existence in recent times are loop quantum gravity [15, 16], Brane gravity [17, 18, 19], \( f(R) \) gravity [20 21 22], \( f(T) \) gravity [23 24 25 20], etc. Reviews on extended gravity theories can be found in [27, 28].

In this work we will consider \( f(G) \) model as the theory of gravity [34, 35]. Over the years, several modifications to GR have been achieved, by generalizing the Einstein-Hilbert Lagrangian used in GR. \( F(R) \) and \( F(T) \) gravities are common examples of such modifications. Gauss-Bonnet (GB) modification to GR is another way of modifying the Einstein gravity that has gained popularity over the past few years, because it is considered as a low energy limit of string theory. In this modification, one generally adds quadratic terms, specifically the GB terms, which involve second order curvature invariants. But as it turns out to be, the GB term becomes trivial in 4-dimension, and hence, it is used from another form of modified gravity, namely, the modified GB gravity [34]. Here an arbitrary function of the GB term, \( f(G) \) is added to the Einstein-Hilbert Lagrangian, to bring about the modification.

In [36] Bisabr studied cosmological coincidence problem in the background of \( f(R) \) gravity. Motivated by Bisabr’s work, we dedicate the present assignment to the study of the coincidence problem in \( f(G) \) gravity. The paper is organized as follows: Basic equations of \( f(G) \) gravity are furnished in section 2. In section 3, we discuss the coincidence problem. The set-up for the present study is discussed in section 4. We illustrate the designed set-up by a few examples in section 5, and finally the paper ends with a short conclusion in section 6.

## 2 Basic equations of \( f(G) \) gravity

The 4-dimensional action in \( f(G) \) gravity is given by,

\[
S = \frac{1}{\kappa^2} \int \sqrt{-g} \left[ \frac{R}{2} + f(G) \right] d^4x + S_m \tag{1}
\]

where \( R \) is the Ricci scalar curvature, \( f(G) \) is a generic function of the Gauss-Bonnet topological invariant \( G \), \( \kappa^2 = 8\pi G \) and \( S_m \) is the matter action.

Varying the above action with respect to the metric one can obtain the field equation as,

\[
G_{\mu\nu} + 8 \left[ R_{\mu\rho\sigma\nu} + R_{\mu\nu}g_{\rho\sigma} + \frac{1}{2} (g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}) R - R_{\rho\sigma}g_{\mu\nu} - R_{\mu\nu}g_{\rho\sigma} + R_{\mu\sigma}g_{\nu\rho} \right] \nabla^\rho \nabla^\sigma F'(G) + (G F'(G) - F) g_{\mu\nu} = T_{\mu\nu}^m \tag{2}
\]

where \( G_{\mu\nu} \) is the Einstein tensor, \( T_{\mu\nu}^m \) is the energy momentum tensor of matter. Here we consider \( \kappa^2 = 8\pi G = 1 \) and prime denotes ordinary derivative with respect to \( G \). For spatially flat Robertson-Walker metric

\[
d s^2 = -dt^2 + a(t)^2 dx^2 \tag{3}
\]

we have

\[
R = 6 \left( \dot{H} + 2H^2 \right), \quad G = 24H^2 \left( \dot{H} + H^2 \right) \tag{4}
\]

where \( H \) is the Hubble parameter and dot denotes the time derivative. Considering the universe to be filled with pressureless dark matter under the assumption of a flat universe the modified Friedmann equations for \( f(G) \) gravity are,

\[
3H^2 = GF'(G) - F - 24H^2 \dot{F}'(G) + \rho_m \tag{5}
\]

\[
-2\dot{H} = -8H^3 \ddot{F}'(G) + 16H \dot{H} \dot{F}'(G) + 8H^2 \ddot{F}'(G) + \rho_m \tag{6}
\]
The energy conservation equations are given by,

\[ \dot{\rho}_m + 3H\rho_m = Q \]  \hspace{1cm} (7)
\[ \dot{\rho}_G + 3H(1 + \omega_G)\rho_G = -Q \] \hspace{1cm} (8)

Here \( \omega_G = \frac{p_G}{\rho_G} \) is the EoS parameter of the energy sector and \( Q \) is the interaction between the matter and the energy sector of the universe. The EoS parameter is given by,

\[ \omega_G = -1 + \left( \frac{8H^2\ddot{G} + 16H\dot{H}\dot{G} - 8H^3\dot{G}}{G'F - F - 24H^3GF''} \right) \] \hspace{1cm} (9)

3 The Coincidence problem

The cosmic coincidence problem has been a serious issue in recent times regarding various dark energy models. Recent cosmological observations have indicated that the densities of the matter sector and the DE sector of the universe are almost identical in late times. It is known that the matter and the energy component of the universe have evolved independently from different mass scales in the early universe, then how come they reconcile to identical mass scales in the late universe! This is major problem having its roots in the very formation of the theory. Almost all the DE models known till date more or less suffer from this phenomenon.

Various attempts to solve the coincidence problem can be found in literature. Among them the most impressive are the ones which use the concept of a suitable interaction between the matter and the dark energy components of the universe, as given in the conservation equations (7) and (8). In this approach, it is considered that the two sectors of the universe have not evolved independently from different mass scales. Instead they evolve together, interacting with each other, allowing a mutual flow of matter and energy between the two components. Due to this exchange, the densities of the two components coincide in the present universe. Although the concept seems to be a really promising one, yet a problem persists. There is no universally accepted interaction term, introduced by a fundamental theory known till date.

It is known that both dark energy and dark matter are not universally accepted facts, but concepts which are still at the speculation level. Due to this unknown nature of both dark energy and dark matter, it is not possible to derive an expression for the interaction term \( Q \) from the first principles. Such a situation, demands us to use our logical reasoning and propose various expressions for \( Q \) that will be reasonably acceptable. The late time dominating nature of dark energy indicates that \( Q \) must be considered a small and positive value. On the other hand a large negative value of interaction will make the universe dark energy dominated from the early times, thus leaving no scope for the condensation of galaxies. So the most logical choice for interaction should contain a product of energy density and the hubble parameter, because it is not only physically but also dimensionally justified. So \( Q = Q(H\rho_m, H\rho_{de}) \), where \( \rho_{de} \) is the dark energy density. Since here we are not planning to add any dark energy by hand, so the effective density resulting from the extra terms of the modified GB gravity, \( \rho_G \) will replace \( \rho_{de} \). This leads us to three basic forms of interactions as given below [42]:

\[ \begin{align*}
\text{b - model : } Q &= 3bH\rho_m \\
\text{\eta - model : } Q &= 3\eta H\rho_G \\
\text{\Gamma - model : } Q &= 3\Gamma H(\rho_m + \rho_G),
\end{align*} \hspace{1cm} (10) \]

where \( b, \eta \) and \( \Gamma \) are the coupling parameters of the respective interaction models.

It is worth mentioning that due to its simplicity the most widely used interaction model is the \( b \)-model and is available widely in literature [41, 42, 43, 44].

4 The set-up

In this note we address the coincidence problem in \( f(G) \) gravity. \( f(G) \) gravity has evolved over the past decade as a candidate for modified gravity theory. From the literature it is known that \( f(G) \) gravity is itself self competent in producing the late cosmic acceleration without resorting to any forms of dark energy. Therefore in order to keep it simple and reasonable, we do not consider any separate dark energy components in the present study. The extra terms of the modified GB gravity provides the exotic nature and is considered as the dark energy. We consider the ratio of the densities of matter and dark energy as, \( r \equiv \rho_m/\rho_G \). Our aim is to devise a set-up that will aim towards a possible solution to the coincidence problem. We also want to set up a filtering process
that will separate the favorable \( f(G) \) models, that produce a stationary value of the ratio of the component densities, \( r \) from the unfavorable ones that do not. The time evolution of \( r \) is as follows,

\[
\dot{r} = \frac{\dot{\rho}_m}{\rho_G} - r \frac{\dot{\rho}_G}{\rho_G} 
\]  

(11)

Using eqns. (7), (8) and (11), we obtain

\[
\dot{r} = 3H\omega_G + \frac{Q}{\rho_G} (1 + r) 
\]  

(12)

Using the \( b \)-interaction given in eqn.(10), we get the expression for \( \dot{r} \) as,

\[
\dot{r} = 3Hr (b + br + \omega_G) 
\]

(13)

where \( \omega_G \) is given by eqn.(9). Now in order to comply with observations, it is required that universe should approach a stationary stage, where either \( r \) becomes a constant or evolves slower than the scale factor. In order to satisfy this \( \dot{r} = 0 \) in the present epoch, it leads to the following equation,

\[
g_1(f, H, r_s, q) = 0
\]

(14)

where

\[
g_1(f, H, r_s, q) = 3Hr_s \left[ b + \frac{\dot{H}}{H^2} + q + br_s + \frac{1}{(-f[G] + Gf'[G] - 576H^3 (\dot{H}HH + 2HH (HH + H^3))) f''[G]) \right] 
\]

\[
\left( \left(384\dot{HH} \left(\dot{HH}H^2 + 2\dot{H} \left(\dot{HH} + H^3\right)\right) - 192H^3 \left(\ddot{HH}H^2 + 2\ddot{H} \left(\ddot{HH} + H^3\right)\right) + 192H^2 \left(\dddot{HH}H^2 + 2\dddot{H} \left(\dddot{HH} + H^3\right)\right) + 6HH ^2 + 2\dddot{H} \left(3\dddot{HH} + H^3\right)\right) f''[G] + 4608H^2 \left(\dddot{HH}H^2 + 2\dddot{H} \left(\dddot{HH} + H^3\right)\right)^2 f'''[G] \right] 
\]

(15)

and \( r_s \) is the value of \( r \) when it takes a stationary value.

Using the \( \eta \)-interaction given in eqn.(10), we get the expression for \( \dot{r} \) as,

\[
\dot{r} = 3H [\eta + r (\eta + \omega_G)] 
\]

(16)

where \( \omega_G \) is given by eqn.(9). In order to satisfy this \( \dot{r} = 0 \) in the present epoch, it leads to the following equation,

\[
g_2(f, H, r_s, q) = 0
\]

(17)

where

\[
g_2(f, H, r_s, q) = 3H \left[ \eta + r_s \left( \frac{\dot{H}}{H^2} + q + \eta + \frac{1}{(-f[G] + Gf'[G] - 576H^3 (\dot{H}HH + 2HH (HH + H^3))) f''[G]) \right] 
\]

\[
\left( \left(384\dot{HH} \left(\dot{HH}H^2 + 2\dot{H} \left(\dot{HH} + H^3\right)\right) - 192H^3 \left(\ddot{HH}H^2 + 2\ddot{H} \left(\ddot{HH} + H^3\right)\right) + 192H^2 \left(\dddot{HH}H^2 + 2\dddot{H} \left(\dddot{HH} + H^3\right)\right) + 6HH ^2 + 2\dddot{H} \left(3\dddot{HH} + H^3\right)\right) f''[G] + 4608H^2 \left(\dddot{HH}H^2 + 2\dddot{H} \left(\dddot{HH} + H^3\right)\right)^2 f'''[G] \right) \right] 
\]

(18)

Using the \( \Gamma \)-interaction given in eqn.(10), we get the expression for \( \dot{r} \) as,

\[
\dot{r} = 3H \left[ \gamma [\gamma^2 + r (2\gamma + \omega_G) + \gamma] \right] 
\]

(19)

where \( \omega_G \) is given by eqn.(9). In this case, in order to satisfy \( \dot{r} = 0 \) in the present epoch, it leads to the following equation,

\[
g_3(f, H, r_s, q) = 0
\]

(20)
where

\[ g_3(f, H, r_s, q) = 3H \left[ \Gamma + r_s^2 \Gamma + r_s \left( \frac{\dot{H}}{H^2} + q + 2\Gamma + \frac{1}{-f(G) + GF'[G] - 576H^3 \left( \dot{H}H^2 + 2H \left( \dot{H}^2 + 2H^3 \right) \right) F''[G]} \right) \times \right. \]

\[ \left. \left( 384\dot{H}H \left( \dot{H}H^2 + 2\dot{H} \left( \dot{H}H + 2H^3 \right) \right) - 192H^3 \left( \dot{H}H^2 + 2\dot{H} \left( \dot{H}H + 2H^3 \right) \right) + 192H^2 \left( \ddot{H}H^2 + 2\ddot{H} \left( \dot{H} + 6H^2 \right) \right) \right) + 2\dot{H} \left( 3\dot{H}H + 2H^3 \right) \right) F''[G] + 4608H^2 \left( \ddot{H}H^2 + 2\ddot{H} \left( \dot{H}H + 2H^3 \right) \right)^2 F''[G] \right) \right] \] (21)

In our analysis we will consider \( H_0, r_0 \) and \( q_0 \) as the present day values of \( H, r \) and \( q \) respectively. As far as \( q \) is concerned, we start from the best fit parameterization obtained directly from observational data. Here we use a two parameter reconstruction function for \( q(z) \) [45, 46]

\[ q(z) = \frac{1}{2} + \frac{q_1 z + q_2}{(1 + z)^2} \] (22)

On fitting this model to Gold data set, we get \( q_1 = 1.47^{+1.89}_{-1.82} \) and \( q_2 = -1.46 \pm 0.43 \) [45]. We consider \( z_0 = 0.25 \) and using these values in eqn. (22), we get \( q_0 \approx -0.2 \). From recent observations, we obtain \( r_0 = \frac{\rho_m(z_0)}{\rho_T(z_0)} \approx \frac{1}{4} \) [47, 48, 49]. The present value of Hubble parameter, \( H_0 \) is taken as 72, in accordance with the latest observational data.

5 Illustration

We consider the scale-factor, \( a \) as a power-law form of time, \( t \) as given below,

\[ a = a_0 t^n \] (23)

In order to illustrate the above set-up we consider three different \( f(G) \) gravity models found widely in literature and test them for the coincidence phenomenon. The three models used are [50, 51, 52]:

**Model1:**

\[ F(G) = \alpha G^{m_1} + \beta G \ln G \] (24)

where \( \alpha, \beta \) and \( m_1 \) are constants, whose values depend on the cosmographic parameters [52].

**Model2:**

\[ F(G) = \frac{\alpha_1 G^{m_2} + b_1}{\alpha_2 G^{m_2} + b_2} \] (25)

where \( \alpha_1, \alpha_2, m_2, b_1 \) and \( b_2 \) are constants.

**Model3:**

\[ F(G) = a_3 G^{m_3} \left( 1 + b_3 G^{m_4} \right) \] (26)

where \( a_3, m_3, m_4 \) and \( b_3 \) are constants.

Using the model 1, i.e., (24) and eqn. (23) in eqn. (15), we get the following expression for the dynamical quantity \( g_1 \),

\[ g_1^{model1} = \frac{1}{t} \left[ b - \frac{1}{n} + q + br + \left( -24m_1 \left( \frac{n^3}{t^2} + \frac{n^4}{t^4} \right)^{m_1} \alpha - \frac{1}{t^5} 576n^3 \left( \frac{2n^3}{t^5} - \frac{2n}{t^3} + \frac{2n^3}{t^3} \right) \right) \right] \]
Similarly expressions for not include all of them in the manuscript. As it can be seen from above that the expressions are really lengthy, so we do
exponential model, which gives very interesting results when used in the designed set-up.

Fig 1: The plot of $g_1(f_0, H_0, r_{s0}, q_0)$ against time $t$ for model1 (red), model2 (blue) and model3 (green) using $b$ interaction. The other parameters are considered as $q = -0.2, r = 3/7, \alpha = 1, \beta = 4, n = 10, m_1 = 0.2, b = 1.5, a_1 = -1, b_1 = -1, a_2 = 2, b_2 = 0.5, m_2 = 1.5, a_3 = -1, b_3 = 0, m_3 = 1.5, m_4 = 0.$

\[
\left(24^{-2+m_1}(-1 + m_1)_{m_1} \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right)^{-2+m_1} \alpha + \frac{\beta}{24 \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right)} - 24 \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right) \beta \log \left[24 \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right)\right]\right] \\
+ 24 \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right) \left[24^{-1+m_1}m_1 \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right)^{-1+m_1} \alpha + \beta + \beta \log \left[24 \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right)\right]\right]^{-1} \times \\
\left(\frac{1}{t^2} - 4608n^2 \left(\frac{2n^3}{t^5} - \frac{2n \left(-\frac{n^2}{t^2} + \frac{n^3}{t^2}\right)}{t^2}\right)^2 \left(24^{-3+m_1}(-2 + m_1)(-1 + m_1)m_1 \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right)^{-3+m_1} \alpha - \frac{\beta}{576 \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right)}\right) \right. \\
+ \left.- \frac{384n^2 \left(\frac{2n^3}{t^5} - \frac{2n \left(-\frac{n^2}{t^2} + \frac{n^3}{t^2}\right)}{t^2} \right)}{t^3} - 192n^3 \left(\frac{2n^3}{t^5} - \frac{2n \left(-\frac{n^2}{t^2} + \frac{n^3}{t^2}\right)}{t^2}\right) \right) + 192n^2 \left(-\frac{6n^3}{t^6} + \frac{2n^2 \left(-\frac{n^2}{t^2} + \frac{n^3}{t^2}\right)}{t^2} + \frac{4n \left(-\frac{n^2}{t^2} + \frac{n^3}{t^2}\right)}{t^2}\right) \right) \\
\left(24^{-2+m_1}(-1 + m_1)m_1 \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right)^{-2+m_1} \alpha + \frac{\beta}{24 \left(-\frac{n^3}{t^4} + \frac{n^4}{t^4}\right)}\right) \right] \quad (27)
\]

Similarly expressions for $g_1$ is obtained for the other two $f(G)$ models. Expressions for $g_2$ and $g_3$ are also found for all the three gravity models. As it can be seen from above that the expressions are really lengthy, so we do not include all of them in the manuscript.

We have generated plots for $g_1, g_2$ and $g_3$ against cosmic time, $t$ for each of the three models in the figures 1, 2 and 3, for all the three forms of interactions, $b, \eta$ and $\Gamma$. Particular numerical values for the involved parameters have been considered which are in accordance with the recent observational data [50, 51, 52]. Moreover in figs 4 and 5, we have illustrated two non-conventional models of $f(G)$ gravity i.e., the logarithmic model and the exponential model, which gives very interesting results when used in the designed set-up.
Fig 2: The plot of $g_2(f_0, H_0, r_{s0}, q_0)$ against time $t$ for model1 (red), model2 (blue) and model3 (green) using $\eta$ interaction. The other parameters are considered as $q = -0.2, r = 3/7, \alpha = 1, \beta = 4, n = 10, m_1 = 0.2, \eta = 1.5, a_1 = -1, b_1 = -1, a_2 = 2, b_2 = 0.5, m_2 = 1.5, a_3 = -1, b_3 = 0, m_3 = 1.5, m_4 = 0.$

Fig 3: The plot of $g_3(f_0, H_0, r_{s0}, q_0)$ against time $t$ for model1 (red), model2 (blue) and model3 (green) using $\Gamma$ interaction. The other parameters are considered as $q = -0.2, r = 3/7, \alpha = 1, \beta = 4, n = 10, m_1 = 0.2, \Gamma = 5 \times 10^{10}, a_1 = -1, b_1 = -1, a_2 = 2, b_2 = 0.5, m_2 = 1.5, a_3 = -1, b_3 = 0, m_3 = 1.5, m_4 = 0.$
Variation of $g$ against $t$ for the model $f(G) = \sigma \log(G) + \tau$. The other parameters are considered as $q = -0.2, r = 3/7, \sigma = 5, \tau = 0.4, n = 3, b = 1.5, \eta = 1.5, \Gamma = 1.5$.

Fig 4:

Variation of $g$ against $t$ for the model $f(G) = \lambda e^{\delta G}$. The other parameters are considered as $q = -0.2, r = 3/7, \lambda = 2, \delta = 0.4, n = 3, b = 1.5, \eta = 1.5, \Gamma = 1.5$.

Fig 5:
6 Discussion and conclusion

From the figures it is evident that the $g$ vs $t$ curves become asymptotic near the time axis, when the cosmic time corresponds to the age of the universe, i.e. $14 \times 10^9$ years. As a result of this, $g$ never reaches the zero level. Hence $\dot{r} \neq 0$, makes the realization of a stationary phase extremely difficult. The asymptotic nature of the curves are indicative of the fact that as time evolves the trajectories move closer and closer to the time axis. Therefore for the given models, the coincidence problem is substantially alleviated with the evolution of time, but never ever gets solved. This is truly a set back for the models which are known to satisfy most of the solar system tests.

But from the set-up that we have designed in this assignment, we can generate as well as filter various models of $f(G)$ gravity which are completely free from the coincidence problem. Two such models have been illustrated in the figs.4 and 5. In fig.4, we have generated the plot of $g$ vs $t$, for the logarithmic model ($f(G) = \sigma \log(G) + \tau$) for all the three interaction terms. It can be seen from the plot that $g$ reaches the zero level for all the interaction terms at around $t = 8$. Particularly for the b-interaction, the stationary scenario is realized for $t > 11$. In fig.5, a similar plot has been generated for the exponential model ($f(G) = \lambda \exp(\delta G)$). From the plot, it is evident that a stationary scenario is achieved at around $t = 6$ for all the interactions. Particularly for the b-interaction a continuous stationary scenario is realized for $t \geq 7$. So from the above discussion it is quite clear that for the logarithmic and the exponential models a complete solution for the coincidence problem can be achieved following the set-up that we have designed in the present assignment. Looking at the plots 4 and 5, it must also be mentioned that the b-interaction term is much more preferable compared to the other interactions, since it helps us to realize a continuous stationary phase between dark energy and dark matter after a certain point of time in the cosmological time-line, thus complying with the recent observational data.

Acknowledgement:

The author sincerely acknowledges the anonymous referee for his or her constructive comments which helped the author to improve the quality of the manuscript.

References

[1] Perlmutter, S. et al. :- [Supernova Cosmology Project Collaboration] ApJ 517 565(1999)
[2] Spergel, D. N. et al. :- [WMAP Collaboration] Astron. J. Suppl 148 175(2003)
[3] Bennett, C. L. et al. :- Astrophys. J. Suppl. 148 1 (2003)
[4] Tegmark, M. et al. :- Phys. Rev. D 69 103501 (2004)
[5] Allen, S. W. et al. :- Mon. Not. Roy. Astron. Soc. 353 457 (2004)
[6] Riess, A. G. et al. :- [Supernova Search Team Collaboration] ApJ 607 665(2004)
[7] Kamenshchik, A., Moschella, U. and Pasquier, V. :- Phys. Lett. B 511 265(2001).
[8] Gorini, V., Kamenshchik, A. and Moschella, U. :- Phys. Rev. D 67 063509(2003).
[9] Ratra, B., Peebles, P. J. E. :- Phys. Rev. D 37 3406 (1988)
[10] Caldwell, R. R. :- Phys. Lett. B 545 23 (2002)
[11] Joyce, A., Jain, B., Khoury, J., Trodden, M. :- arxiv:1407.0059v1 (2014)
[12] Bamba, K., Capozziello, S., Nojiri, S., Odintsov, S. D. :- arxiv:1205.3421v3 (2012)
[13] Weinberg, S. :- Rev. Mod. Phys. 61 1 (1989)
[14] Bisabr, Y. :- Gen. Rel. Grav. 42 1211 (2010)
[15] Rovelli, C. :- liv. Rev. Rel. 11(1998)
[16] Ashtekar, A., Lewandowski, J. :- Class. Quantum. Grav. 21R53(2004)
[17] Brax, P. et. al :- Rep. Prog.Phys. 67 2183 (2004)
