Probe Chiral Magnetic Effect with Signed Balance Function

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In this paper we propose a pair of observables as alternative ways to study the charge separation induced by Chiral Magnetic Effect (CME) in relativistic heavy ion collisions. They are, the out-of-plane to in-plane ratio of fluctuation of the difference between signed balance functions measured in pairs rest frame, and the ratio of it to similar measurement made in the laboratory frame. We have studied both observables with simulations including flow-related backgrounds, and for the first time we’ve pointed out and considered backgrounds that are related to resonance’s global spin alignment. The two observables have similar positive responses to signal, and opposite, limited responses to identifiable backgrounds arising from resonance flow and spin alignment. We have also tested our observables with two realistic models, namely, a multi-phase transport (AMPT) model and anomalous-viscous fluid dynamics (AVFD) model. The two observables, when cross examined, will provide useful insights in the study of CME-induced charge separation.

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I. INTRODUCTION

It has been pointed out that the hot and dense matter created in relativistic heavy-ion collisions may form metastable domains where the parity and time-reversal symmetries are locally violated [1], creating fluctuating, finite topological charges. In noncentral collisions, when such domains interplay with the ultra-strong magnetic fields produced by spectator protons [2], they can induce electric charge separation parallel to the system’s orbital angular momentum — the chiral magnetic effect (CME) [2–4].

In an event, charge separation along the angular momentum vector may be described by sine terms in the Fourier decomposition of the charged-particle azimuthal distribution

\[
\frac{dN_\pm}{d\phi} \propto 1 + 2v_1 \cos(\Delta \phi) + 2v_2 \cos(2\Delta \phi) + 2v_3 \cos(3\Delta \phi) + \cdots + 2a_\pm \sin(\Delta \phi) + \cdots,
\]

where \(\Delta \phi = \phi - \Psi_{RP}\) is particle’s azimuthal angle with respect to the reaction plane (\(\Psi_{RP}\)). The reaction plane is defined by the beam direction and the line connecting the centroids of two colliding nuclei at their closest approach (impact parameter \(b\)). \(v_1, v_2, v_3\) and \(a_\pm\) are coefficients accounting for the directed, elliptic and triangular flow [5], respectively. The \(a_\pm\) (\(a_+ = -a_-\)) parameter describes the charge separation effect. In a parity violating domain, net-positive and net-negative topological charges can be produced with equal likelihood, causing the sign of \(a_\pm\) to fluctuate from event to event depending on event’s net topological charge. This makes \(a_\pm\) not possible to be distinguished on an event-by-event basis. However one can instead study the effect of \(a_\pm\)’s fluctuation (\(a_1^2\)), where \(a_1 \equiv |a_+| \equiv |a_-|\), noting that \(\langle a_+ \rangle = \langle a_- \rangle = 0\).

To study the CME experimentally one has to look for the enhanced fluctuation of charge separation in the direction perpendicular to the reaction plane, relative to the fluctuation in the direction of reaction plane itself. This is the basis of all CME searches in heavy-ion collisions. Experimental searches for CME have been going on for a decade, with multiple methods [6–11] and carried out by experiments at both RHIC [7, 8, 12–14] and LHC [15, 16]. So far there is no conclusive evidence for the existence of CME in heavy ion collisions, see [3] for a progress review. The major challenge in CME searches is that backgrounds, in particular those related to elliptic flow of resonances, can produce similar enhancement in fluctuation in the direction perpendicular to the reaction plane [17–22]. To have the background under control, the STAR experiment at RHIC has collected collisions from isobaric collisions and the data analysis is on-going.

In this paper, we propose a pair of observables to study the CME effect. One of them is the out-of-plane to in-plane ratio of fluctuation of the difference between signed balance functions measured in particle pairs’ rest frame, and the other is the ratio of it to the similar measurement made in the laboratory frame. We will show that the two observables have positive responses to signal, but opposite, limited responses to identifiable backgrounds arising from resonance flow and global spin alignment. In following sections we will first describe the signed balance function and how we came up with our observables, followed by the discussion of toy model studies with various background scenarios, including flow related backgrounds and a background that is caused by resonance’s global spin alignment. The latter has not been considered previously. We will also present results from a realistic model with pure background, namely a multi-phase transport (AMPT) model [23], as well as from a model with both signal and background, namely the anomalous-viscous fluid dynamics (AVFD) model [24, 25]. At the end we will summarize.
II. SIGNED BALANCE FUNCTION

The balance function (BF), in its general form, describes the absolute separation of particles in phase space [26, 27]. At RHIC and LHC, the balance function in pseudorapidity, $B(\Delta\eta)$, which spans the absolute difference in pseudorapidity between two balancing particles, $\Delta\eta = |\eta_1 - \eta_2|$, is usually used to study the delayed hadronization in head-on collisions[27–31].

The signed balance function, previously proposed to study the magnetic field in heavy ion collisions[32], considers the signed difference instead of the absolute separation of particles in phase space. Before going further in details, let’s first introduce the coordinate system used in this paper. The $x$-axis is set by the direction of the impact parameter ($\hat{b}$) which is also the direction of the reaction plane. The $z$-axis represents the beam direction ($\hat{p}_{\text{beam}}$), and the $y$-axis ($\hat{y} = -\hat{b} \times \hat{p}_{\text{beam}}$) is perpendicular to the reaction plane. The magnetic field direction, as well as the global angular momentum vector, are pointing in ($-\hat{y}$) direction. With this setup, the charge separation due to CME is along the $y$-axis. This setup is the same as in [32]. Phase spaces that are relevant in this study are particles momentum in $x$ and $y$ direction, $p_x$ and $p_y$, respectively. We invoke two signed balance functions,

$$B_{\beta,\gamma}(S_y) = \frac{N_{\alpha\beta}(S_y) - N_{\alpha\gamma}(S_y)}{N_{\alpha\beta}},$$

$$B_{N,\gamma}(S_y) = \frac{N_{-\alpha\gamma}(S_y) - N_{-\alpha\gamma}(S_y)}{N_{-\alpha\gamma}},$$

Here $S_y$ for $N_{\alpha\beta}(S_y)$ is positive if particle $\alpha$ is leading particle $\beta$ ($p^\alpha_y > p^\beta_y$), and negative if vise versa. $N_{\alpha\beta}(S_y)$ denotes the number of positive-negative pairs with a sign of $S_y$ in an events. $N_{\alpha\beta}(S_y)$, $N_{\beta\gamma}(S_y)$ and $N_{\beta\gamma}(S_y)$ are defined in a similar way. $N_{\alpha\beta}(\pm)$ is the number of positive (negative) particles in an events. Similarly, $B_{\beta,\gamma}(S_x)$ and $B_{N,\gamma}(S_x)$ can also be defined.

To not to confuse $S_y(x)$ with the sign of charge, let’s label $S_y(x) = +1(−1)$ for $\alpha$ leading (tailing) $\beta$ in $N_{\alpha\beta}$ terms. With that, we calculate an event by event difference between $B_\beta$ and $B_N$ :

$$\delta B_y(\pm 1) = B_{\beta,\gamma}(\pm 1) - B_{N,\gamma}(\pm 1),$$

and

$$\Delta B_y = \delta B_y(1) - \delta B_y(-1).$$

Note that by definition $\delta B_y(1) = -\delta B_y(-1)$. A $\Delta B_x$ term can also be obtained similarly. When there is no CME effect, for a positive-negative particle pair, the probability of the positive particle leading the negative one equals the probability of tailing it. This means that $B_{\beta,\gamma}(x)$ and $B_{N,\gamma}(x)$ are in principle measuring the same quantity, and the distribution of $\Delta B_y(x)$ is only subject to statistical fluctuation (top row of Fig. 1). When there is CME effect, within an event the two probabilities become unbalanced, resulting more pairs with particles of one charge-type leading than tailing the other type. This makes for each event $B_{\beta,\gamma}$ and $B_{N,\gamma}$ to tend to be different from each other, and as a consequence, the distribution of $\Delta B_y$ has a broadened width (bottom row of Fig. 1). On the other hand, the distribution of $\Delta B_x$ is not broadened as there is no charge separation in $x$ direction. To cancel out the statistical fluctuation, one can calculate the ratio of the width of the distribution of $\Delta B_y$ to that of $\Delta B_x$, $r = \sigma_{\Delta B_y}/\sigma_{\Delta B_x}$. $r$ will be at unity for the case without CME, and greater than unity for the case with it. The strength of the CME will be positively correlated with $r$’s deviation from unity.

FIG. 1: (Color online) Cartoon illustration of positive (red) and negative (blue) particle directions (left plots) and $S_{\Delta p_y}$ distribution (middle plots) of an event, as well as the $\Delta B_y$ distribution over many events (right plots). The top row is for the case without CME, and the bottom one, with CME.

FIG. 2: (Color online) Cartoon illustration of a pair viewed in the laboratory frame (left) and pair’s rest frame (right).

The ratio $r$ can be calculated in the laboratory frame ($r_{\text{lab}}$) and pair’s rest frame ($r_{\text{rest}}$). We argue that the rest frame is the most appropriate frame to study charge separations. This can be understood in an intuitive way.
the clearest observation of two particles moving away from each other has to be, naturally, made by an observer who is at rest with the two-particle system under consideration. Note in the rest frame two particles are traveling back-to-back, and in this particular frame leading(tailing) simply means particle traveling in positive(negative) y direction – making it easy to be identified in the signed BF approach. Fig. 2 we give an example to illustrate this point. The cartoon on the left depicts a pair in the laboratory frame, and it is not counted as a case of charge separation by the signed BF approach, as both particles have same p_y. When the same pair is viewed in the rest frame (right cartoon), it is clearly a case of charge separation. Indeed by definition r_{rest} is always the most sensitive one when responding to real charge separation, however, it is not guaranteed so when responding to backgrounds – it may lag behind r_{lab}. It would be useful to calculate the ratio of the two,

$$R_B = \frac{r_{\text{rest}}}{r_{\text{lab}}}$$

where the subscript “B” stands for Balance Function. We will show below with simulations that while R_B responds positively to signal (like each of r_{rest} and r_{lab} themselves does), it may respond in the opposite direction (relative to r_{rest} and r_{lab}) to backgrounds. This information can be useful under certain scenarios in identifying charge separation induced by backgrounds. For example, if r_{rest} is above unity and R_B is below it (or vice versa), then it is an indication of background contribution. We will show that this is in particular true when the signal is weak.

For convenience, in this paper at a few places we will refer to either of the three ratios being above unity, which can be caused by CME and/or background, as apparent charge separation. The apparent charge separation is what is usually measured in experiments.

III. TOY MODEL SIMULATIONS

In this section, we present a series of toy model simulations for various signal and/or background scenarios. We start with simple cases followed by cases with relatively more realistic considerations. For all cases, a simulated event consists of 324 primordial charged pions (162 for each charge type), and 33 ρ resonances that each decays into a π^+ + π^- pair. This configuration gives a total multiplicity that matches the multiplicity within 2 units of rapidity for 30 – 40% central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [33], while maintaining the ratio in yield of ρ resonance to negative particles at ~17% [34]. The decay of $\rho \rightarrow \pi^+ + \pi^-$ is implemented with PYTHIA6 [35]. Primordial pions and ρ resonances are allowed to have their own v_2 and v_3, and in addition, primordial pions can have finite CME signal ($a_1 > 0$), and ρ resonances can have finite global spin alignment ($\rho_{00} \neq 1/3$) [36–41]. Unless otherwise specified, following [21], primordial pions are generated according to a Bose-Einstein distribution [33], $dN_{\pi^\pm}/dm_T^2 \propto (e^{m_T/T_{BE}} - 1)^{-1}$, where $m_T = \sqrt{p_T^2 + m_{\pi}^2}$ is the π^± rest mass, and $T_{BE}$ is set to be 212 MeV in order to have a $\langle p_T \rangle$ of 400 MeV [33]. ρ resonances are generated according to $dN_\rho/dm_T^2 \propto e^{-(m_T-m_{\rho})/T}/[T(m_\rho + T)]$, where T is set to be 317 MeV for having a $\langle p_T \rangle$ of 830 MeV [34], and $m_\rho$ is the rest mass of ρ-resonance. Note that the only available experimental data for ρ-resonance spectra at RHIC energies is measured for 40 – 80% central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [34], which does not match the 30 – 40% centrality mentioned above. However, for a qualitative study we don’t think this mismatch will affect our conclusion. In this paper we didn’t consider finite event plane resolution in simulations, but if needed it can be taken into account with well-established procedure [5]. In all simulations in this paper the reaction plane is assumed to be known exactly. The $dN/dy$($dN/d\eta$) distributions is taken to be flat in a range of [-1,1] for primordial pions (ρ resonances). By default ~1 million events are simulated for each data point in almost all figures of this section, except for Fig. 3 in which ~10 million events for each data point are simulated.

### A. Signal only

$\begin{array}{c}
\text{FIG. 3: (Color online) } r_{\text{rest}} \text{ and } r_{\text{lab}} \text{(top panel), as well as } R_B \text{ (bottom panel) for a simple-case simulation in which only } a_1 \text{ is introduced, no backgrounds.}
\end{array}$
In Fig. 3, we present a simulation with CME signal only, no backgrounds. In this simple case, \( r_{\text{rest}} \) and \( r_{\text{lab}} \) (top panel), as well as \( R_B \) (bottom panel) are consistent with unity when \( a_1 = 0 \), and increase with increasing \( a_1 \). The deviation from unity for \( R_B \) is about an order of magnitude smaller than that for \( r_{\text{rest}} \) and \( r_{\text{lab}} \), which is not a surprise as the additional sensitivity of \( r_{\text{rest}} \) over \( r_{\text{lab}} \) is a second order effect. Indeed as \( r_{\text{lab}} \) and \( r_{\text{rest}} \) are visually very close to each other, for clarity reason in the rest of the paper we choose to show only \( r_{\text{rest}} \) in figures.

B. Resonance \( v_2 \) as fixed value

![Figure 4](image-url)  
**FIG. 4:** (Color online) \( r_{\text{rest}} \) and \( R_B \) as a function of resonance \( v_2 \), for various transverse spectra. formulae for spectra are from [33] and [21], with temperatures for each are individually tuned to yield \( \langle p_T \rangle \) of 400 MeV and 830 MeV for pions and \( \rho \) resonances, respectively. Data points are shifted slightly in horizontal direction for clear view ( similar shift, when needed, has been applied for other plots in this paper ).

In the simulation presented in Fig. 4, we introduce elliptic flow for \( \rho \) resonances. Here all resonances are generated according to same \( v_2 \) regardless of their \( p_T \). Cases with \( p_T \) dependent \( v_2 \) will be considered later in the paper. We see that when there is no signal (\( a_1 = 0 \)) and \( v_2 \) of resonance is the only background, \( r_{\text{rest}} \) increases with increasing resonance \( v_2 \) (top panel). Note that when there is finite elliptic flow for resonances, in the rest frame although pairs that are originated from real resonance decays do not contribute to the apparent charge separation, pairs from random combinatorial background do. This will cause both \( r_{\text{rest}} \) and \( r_{\text{lab}} \) (not shown) to increase with increasing resonance \( v_2 \).

The bottom panel of Fig. 4 shows that \( R_B \), on the contrary, decreases with increasing resonance \( v_2 \) – an opposite trend than \( r_{\text{rest}} \). This is because that \( r_{\text{rest}} \) is not guaranteed to be more sensitive than \( r_{\text{lab}} \) when responding to backgrounds, and for this particular case, \( r_{\text{rest}} \) is less responsive to the increase of resonance \( v_2 \). This pattern of opposite trend has been observed for all spectra formulae that can practically describe data [33].

![Figure 5](image-url)  
**FIG. 5:** (Color online) \( r_{\text{rest}} \) and \( R_B \) as a function of resonance \( v_2 \), for various \( a_1 \) values.

In Fig. 5, we repeat a similar study with various \( a_1 \) introduced to primordial pions, with the spectra of primordial pions and resonances set to be the default setup as aforementioned at the beginning of section III. As expected, both \( r_{\text{rest}} \) and \( R_B \) increase with increasing \( a_1 \), on top of values induced by resonance \( v_2 \) alone.

C. Resonance \( \rho_{00} \)

It has been pointed that resonances with even spin can possess global spin alignment which tends to, in their rest frames, align two daughters either in the \( y \) direction (\( \rho_{00} > 1/3 \)), or in the \( x-z \) plane (\( \rho_{00} < 1/3 \) [36–41]. Considering the projection of many pairs onto the
transverse plane only, loosely speaking the global spin alignment acts like “elliptic flow” in the rest frame. For a reason similar to elliptic flow, the global spin alignment is also expected to cause an apparent charge separation. Such effect has not been discussed previously. In Fig. 6 is also expected to cause an apparent charge separation. For the case of no global effects introduced anywhere. For the case of no global spin alignment ($\rho_0 = 1/3$), $r_{\text{rest}}$ and $R_B$ are at unity as they should. When there is global spin alignment ($|\rho_0 - 1/3| > 0$), both ratios are not at unity anymore, and $r_{\text{rest}}$ and $R_B$ change in opposite directions when responding to the change of $\rho_0$. This pattern holds again for all transverse spectra shapes that we’ve considered.

In Fig. 7, we repeat similar studies with various $a_1$ introduced to primordial pions, with spectra of primordial pions and resonances set to be default ones as mentioned in the beginning of section III. As expected, both $r_{\text{rest}}$ and $R_B$ increase with increasing $a_1$, on top of values induced by resonance $\rho_0$ alone.

Note in this study we have chosen a wide $\rho_0$ range in order to clearly identify/demonstrate the pattern, which may have exaggerated the situation. Experimentally the only available $\rho_0$ measurement with decent statistical error has been made for $\phi$-meson for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV [42], and it is found to be less than 0.38 for $p_T > 1.2$ GeV/c. $\rho_0$ measurements for $K^{*0}$-meson suffers from large statistical error [42, 43]. So far there is no experimental guidance on $\rho_0$ for $\rho$-resonance, and our study calls for such measurements.

D. $p_T$ dependent $v_2$ and $v_3$ of primordial pions

In this subsection and subsections that followed, we study how our observables respond to realistic flow effects. We use the NCQ-inspired function [44] to introduce elliptic flow for primordial pions and $\rho$ resonances,

$$v_2/n = a/(1 + e^{-(m_T-m_0)/n-b}/c) - d,$$  \hspace{1cm} (7)

where $n = 2$ is the number of constituent quarks. By default parameters $a$, $b$, $c$ and $d$ take same values as in [21] for $30 - 40\%$ central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Unless otherwise specified, $v_3$ at any given $p_T$ is set to be $1/5$ of corresponding $v_2$ [45], for both primordial pions and $\rho$ resonances. No spin alignment is introduced for $\rho$ resonances. The study with realistic flow together with global spin alignment will be presented in a later subsection.

To change $v_2(p_T)$, we vary the parameter $a$ in Eq. (7). The effect of this variation on $v_3$ is illustrated in Fig. 8. In Fig. 9 we present $r_{\text{rest}}$ and $R_B$ as a function of $a$-parameter of primordial pions. Not that in this study $v_3$...
for primordial pions also changes with a-parameter, as \( v_3 \) at any given \( p_T \) has been set to be 1/5 of corresponding \( v_2 \). \( v_2(p_T) \) and \( v_3(p_T) \) for \( \rho \) resonances are introduced according to their aforementioned default configurations and are kept unchanged.

We see that when there is no CME induced charge separation \( (a_1 = 0) \), \( r_{\text{rest}} \) and \( R_B \) are at opposite sides of unity, and this is largely due to the presence of finite \( v_2 \) of \( \rho \)-resonance. When there is finite \( a_1 \), \( r_{\text{rest}} \) increases slightly with the a-parameter, and surprisingly \( R_B \) also increases with increasing a-parameter. This has to be caused by a combination of finite \( \rho \)-resonance \( v_2 \) and the change of \( v_2 \) of primordial pions. However, the reasoning for it at microscopic, dynamical level is not obvious for the moment and is a subject of future study.

In Fig. 10, we study the effect of \( v_3 \) alone on our observables by varying \( v_3(p_T) \) of primordial pions while keeping everything else unchanged. This is implemented by setting \( v_3 \) to be a fraction, which itself varies, of \( v_2 \) everywhere in \( p_T \), while keeping \( v_2(p_T) \) unchanged. Both \( r_{\text{rest}} \) and \( R_B \) are presented as function of ratio of \( v_3/v_2 \). As a reminder the case that is close to data is with \( v_3/v_2 = 0.2 \).

FIG. 8: (Color online) \( v_2(p_T) \) implemented in simulations, for \( \rho \) resonances (top panel) and primordial pions (bottom panel). Within each panel, a-parameter values in Eq. (7) are, from top to bottom, 0.275, 0.225, 0.175, 0.125 and 0.075, respectively. For each panel the curve with solid black line corresponds to the case with default value (0.125) of a-parameter taken from [21].

FIG. 9: (Color online) \( r_{\text{rest}} \) and \( R_B \) as a function of a-parameter of primordial pions in \( v_2(p_T) \) description, for various \( a_1 \) values.

We don’t see obvious dependence on \( v_3 \) change of primordial pions.

E. \( p_T \) dependent \( v_2 \) and \( v_3 \) of \( \rho \) resonances

In this subsection we repeat similar studies in the previous subsection, but instead of varying the flow of primordial pions, here we vary the flow of \( \rho \) resonances while keep flow of primordial pions unchanged at their default configuration. No spin alignment is introduced for \( \rho \)-resonance.

Fig. 11 shows \( r_{\text{rest}} \) and \( R_B \) as a function of a-parameter of \( \rho \) resonances. When there is no CME induced separation, \( r_{\text{rest}} \) and \( R_B \) deviate from unity in opposite directions. With a finite \( a_1 \), both observables increase on top of the values for the case of \( a_1 = 0 \), and the pattern that \( r_{\text{rest}} \) and \( R_B \) respond in opposite directions to the change of a-parameter can be seen for all \( a_1 \) values.

In Fig. 12, following a similar procedure in Fig. 10 we vary resonance \( v_3(p_T) \) while keep everything else unchanged. Like the case for \( v_3 \) of primordial pions (Fig. 10), there is no noticeable effect due to \( v_3 \) change of \( \rho \) resonances.
dependent flow effect, we again mentioned configuration in section III D. We see (Fig. 13) presence of flow effects.

observables are not at unity for $\rho$ primordial pions and $p$ acting together with resonance elliptic flow, low but instead of having no flow-effects, here we include background in CME-related analysis.

tion in reaction plane. Both effects will influence the fluctuation of primordial $p$ close to each other and preferentially close to the reaction plane, while high $p$ angle, while high $p$ tends to emit two daughters close to each other and preferentially more perpendicular to the reaction plane than

In a recent publication [22], it is pointed out that when acting together with resonance elliptic flow, low $p_T$ resonances have a tendency of emitting two daughters preferentially more perpendicular to the reaction plane than high $p_T$ resonances because of the large decay opening angle, while high $p_T$ resonances tend to emit two daughters close to each other and preferentially close to the reaction plane. Both effects will influence the fluctuation in $x$- and $y$-direction and should be considered as background in CME-related analysis.

To repeat a such study for our observables, we fixed $v_2$ of $\rho$ resonance to be 6% as in [22], and let all resonances have same $p_T$ for which the value itself can vary between simulations. Primordial pions are simulated again with realistic flow and spectra as aforementioned. In Fig. 14 we see that the $p_T$ change, over a range of 0.5 – 2 GeV/c, has an observable effect on our observables. We’d like to point out that, although we have a dedicated study for this effect, it is not an additional, independent effect on top of existing effects already presented in the paper. This effect has been taken into consideration automatically when taking a characteristic $v_2$ and transverse spectra in simulations. However, it would be an interesting study in terms of understanding how the choice of slope (which changes $\langle p_T \rangle$) of transverse spectra would affect our observables. To investigate into this, in Fig. 15 we simulate primordial pions and $\rho$ resonances according to their corresponding default characteristic flow and spectra as mentioned earlier, and calculate $r_{\text{rest}}$ and $R_B$ for a series of temperature of $\rho$-resonance spectra around its nominal value of 317 MeV. The study is repeated for various $a_1$ values. We see that $r_{\text{rest}}$ changes for merely $\sim 2\%$ relatively over a temperature span of 40% change.

**F. Resonance $\rho_{00}$ together with $p_T$ dependent $v_2$ & $v_3$ of primordial pions and $\rho$ resonances**

In this subsection we repeat the $\rho_{00}$ study in Fig. 7, but instead of having no flow-effects, here we include $p_T$ dependent flow effect, $v_2(p_T)$ and $v_3(p_T)$, for both primordial pions and $\rho$ resonances according to the aforementioned configuration in section III D. We see (Fig. 13) again $r_{\text{rest}}$ and $R_B$ change in opposite directions when responding to $\rho_{00}$ change. Unlike in Fig. 7, here both observables are not at unity for $\rho_{00} = 1/3$ due to the presence of flow effects.

**G. Resonance $p_T$**

In a recent publication [22], it is pointed out that when acting together with resonance elliptic flow, low $p_T$ resonances have a tendency of emitting two daughters preferentially more perpendicular to the reaction plane than high $p_T$ resonances because of the large decay opening angle, while high $p_T$ resonances tend to emit two daughters close to each other and preferentially close to the reaction plane. Both effects will influence the fluctuation in $x$- and $y$-direction and should be considered as background in CME-related analysis.
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In this section we present calculations of our observ-
ables based on two popular realistic models, namely
AMPT and AVFD model.

The AMPT model [23] uses the Heavy Ion Jet Interac-
tion Generator (HIJING [46, 47]) for generating the ini-
tial conditions, the Zhang’s Parton Cascade (ZPC [48])
for modeling the partonic scatterings, and A Relativis-
model for treating hadronic scatterings. The version (v2.25t4cu) we used is a version
with string melting, in which it treats the initial condition
as partons and uses a simple coalescence model to
describe hadronization. It is also a version with charge-
conservation being assured, which is particularly impor-
tant for CME related model-studies. For details, see [23].

The AVFD framework [24, 25] implements the anom-
alous transport current from CME into fluid dynamics
framework to simulate the evolution of fermion currents
on an event-by-event basis and to evaluate the result-
charge separation in QGP, on top of the neutral bulk background described by the VISH2+1 hydrody-
amic simulations [51] with Monte-Carlo Glauber initial
conditions, followed by a URQMD hadron cascade stage
[52, 53]. This new tool allows one to quantitatively and
systematically investigate the CME signal and account

FIG. 12: (Color online) $r_{\text{rest}}$ and $R_B$ as a function of $v_3(p_T)/v_2(p_T)$ ratio of $\rho$ resonances, for various $a_1$ values.

FIG. 13: (Color online) $r_{\text{rest}}$ and $R_B$ as a function of resonance $\rho_{00}$, for various $a_1$ values. Realistic flow
effects have been included for both primordial pions and $\rho$ resonances.

IV. AMPT AND AVFD MODEL

In this section we present calculations of our observ-
ables based on two popular realistic models, namely
AMPT and AVFD model.

Both AMPT and AVFD models are known to have
a good description of experimental data, including parti-
cle’s yield, spectra and flow. They can serve as good bas-
lines for apparent charge separation arising from pure
grounds. In addition, the CME feature implemented
in AVFD will allow us to study our observable’s response
to signal in a relatively realistic environment of back-
grounds. Fig. 16 shows $r_{\text{rest}}$ and $R_B$ as a function of centrality for AMPT and AVFD events. Each point in the figure is calculated with $\sim 2$ million model-events. To match
typical acceptance cuts used by the STAR collaboration,
only particles fall in $|\eta|<1$ and $0.2<p_T<2$ GeV/$c$
are considered in the analysis. For the two cases of no
CME (AMPT, and AVFD with $n_3/s = 0$), $r_{\text{rest}}$ values is
in between 1 and 1.005 depending on centrality, and is
smallest if compared to cases with CME. $r_{\text{rest}}$ increases
clearly with increasing $n_3/s$, indicating a very good sen-
sitivity to CME. Unfortunately our limited computing
resource cannot facilitate producing/analyzing a larger
data set of AVFD events in reasonable amount of time, and
our statistical error for $R_B$ is too large to demon-
strate its response to finite $n_3/s$. Indeed $R_B$ requires a
much larger statistics than $r_{\text{rest}}$ which may limit its usage
FIG. 14: (Color online) \(r_{\text{rest}}\) and \(R_B\) as a function of \(\rho\) resonance \(p_T\) as a fixed value.

FIG. 15: (Color online) \(r_{\text{rest}}\) and \(R_B\) as a function of temperature of the transverse spectra of \(\rho\)-resonance.

FIG. 16: (Color online) \(r_{\text{rest}}\) and \(R_B\) as a function of centrality, calculated for events from AMPT and AVFD models. The AMPT model has no built-in CME effect. In the AVFD model the CME is implemented by finite ratio of axial charge over entropy \((n_5/s)\), resulting finite average \(a_1\) (observed \(a_1\)) for all charged particles, including primordial ones and those from resonance decays.

in practice. That said, we have demonstrated that even \(r_{\text{rest}}\) itself alone is a sensitive CME probe. In general our proposed observables behave as expected for realistic models.

V. SUMMARY

We’ve proposed a pair of observables, \(r_{\text{rest}}\) and \(R_B\), as alternative ways to study the charge separation induced by CME in relativistic heavy ion collisions. We have studied both observables with toy model simulations, as well as two realistic models, namely, AMPT and AVFD. Our toy model study includes flow-related backgrounds, and for the first time, backgrounds that are related to the global spin alignment of resonances. We’ve shown that the two observables have similar positive responses to signal, and opposite, limited responses to identifiable backgrounds arising from resonance flow and global spin alignment. The opposite responses to backgrounds is in particular obvious when the signal is weak. This informa-
tion can be useful under certain scenarios in identifying charge separation induced by backgrounds. For example, putting aside the effect of global spin alignment which hopefully can be under control with a dedicated study in the future, if both \( r_{\text{est}} \) and \( R_B \) are above unity, then we have an evidence strongly supporting the existence of CME.

In practice the usage of \( R_B \) may be limited by its requirement of more statistics than \( r_{\text{est}} \), however, we have demonstrated that even \( r_{\text{est}} \) itself alone can serve as a sensitive CME probe. Like any other approach, our procedure do not provide a complete, clean solution under all possible scenarios. A quantitative statement on signal versus background has to rely on realistic simulations, and, better to be made with the help of additional, external information (such as information from isobaric collisions). That said, our observables do provide useful insights into the problem from a unique perspective.

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