Evolutionary Status of Late-Type Contact Binaries

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ABSTRACT

The old model of an unevolved, cool contact binary, in which the secondary component is strongly oversized due to energy transfer from the primary, and the whole system is out of thermal equilibrium, encounters serious problems.

I present a new scenario for evolution of contact binaries, which solves the problem of thermal nonequilibrium by assuming that contact binaries are past mass exchange with a mass ratio reversal. The scenario is divided into three phases. In Phase I loss of angular momentum (AM) due to magnetized wind of a detached binary is followed until the primary component fills its critical Roche lobe. In Phase II mass transfer takes place until mass ratio reversal. Arguments are given in favor of such a process in pre-contact binaries. In Phase III an approximate evolutionary path of the contact binary is followed until a possible coalescence. AM loss, evolutionary effects of the components and mass transfer to the primary are taken into account.

It is concluded that W UMa type binaries are old objects with secondaries in an advanced evolutionary stage, possibly with small helium cores. Both components fulfill the mass-radius relation for contact binaries while being in thermal equilibrium.

1. Introduction

W UMa type stars are binaries of spectral type F0–K5 which have both components surrounded by a common envelope lying between the inner and outer Lagrangian zero velocity equipotential surfaces (Mochnacki 1981).

Kuiper (1941) noted that contact binaries with unequal zero-age components should not exist because radii of the components must fulfill two, mutually contradictory conditions: one resulting from the mass-radius relation for zero-age stars and the other, relating sizes of the Roche lobes, identical in this case with stellar sizes, to stellar masses. The fact that contact binaries are nevertheless observed is known as “Kuiper paradox”. Lucy (1968) stressed that such a configuration can be either the result of subsequent evolution of zero-age detached systems or it is not in equilibrium. Following Kuiper (1941) he assumed the latter to be true and developed a theory of zero-age contact binaries with a common convective envelope. The theory explained the observed light curves of W UMa type binaries but encountered several problems when confronted with other observations of these stars (e.g., Mochnacki 1981, 1985, Rucinski 1993). On the other hand, several observational and theoretical facts point toward the advanced evolutionary status of W UMa type binaries. Numerical simulations
of a binary formation favor an early fragmentation of a protostellar cloud resulting in a formation of a detached binary (Boss 1993). Fission of a rapidly rotating protostar into two cores was never observed (Bonnel 2001). This result is in agreement with a lack of contact binaries among T Tau stars and among members of the youngest stellar clusters. Unless the exceptional orbital angular momentum loss (AML) takes place, the shortest period zero-age binaries with solar type components should have periods about 2 days or more (see the discussion by Stepien 1995). This is several times more than required for a contact configuration. However, stars of the lower main sequence (MS) lose angular momentum (AM) via a magnetized wind with efficiency increasing with increasing angular velocity (Mestel 1984, Kawaler 1988, Stepien 1991). Assuming synchronous rotation of the close binary components AML results in a decrease of orbital AM and shortening of the orbital period. Stars approach each other and ultimately form a contact system (Vilhu 1982, Mochnacki 1985). Closer consideration of this mechanism shows that it takes several Gyr for a detached binary with an initial period of the order of 2 days to lose enough AM and to form a contact system (Stepien 1995). For longer periods it takes correspondingly more time. Observations of W UMa type stars in old stellar clusters confirm this result. They are virtually unknown in young clusters but their frequency of occurrence rapidly increases for open clusters with age exceeding 4–5 Gyr (Kaluzny and Rucinski 1993, Rucinski 1998) and for globular clusters (Rucinski 2000). Note, however, that the AML time scale of $\approx 5–14$ Gyr is equal to the nuclear time scale for stars with masses $0.9–1.3 \, M_\odot$. Because of this coincidence, massive primaries can evolve up to, or even beyond terminal age MS (TAMS) by the time their shrinking Roche lobes reach the stellar surface. Filling up the Roche lobe by the more massive component results in mass exchange, a mass ratio reversal and the formation of a contact binary in which a secondary has a hydrogen depleted core.

A relatively high frequency of occurrence of W UMa type stars among MS dwarfs, particularly in the Galactic bulge i.e., among very old disk stars (Rucinski 1998, 2002, Szymański, Kubiak and Udalski 2001) indicates that the total life time of a contact configuration cannot be as short as a stellar thermal time scale of, say, 10 to 100 million years, but it should last several Gyr. This argument can be made more quantitative. Halbwachs et al. (2004) present the latest statistics concerning the properties of binary stars of spectral types F7–K. According to their results there are 56 binaries with orbital periods shorter than 10 years among 405 stars of the unbiased sample. This gives the frequency of 13.5%. Of these binaries two (possibly three) have periods shorter than 3 days – short enough to form a contact binary during their MS evolution. This gives a frequency of short period binaries equal to 0.005–0.007. Another estimate of this frequency can be obtained assuming that the observed binary distribution is described by the Duquennoy and Mayor (1991) formula. Integration of their formula over periods shorter than 3 days and then over periods shorter than 10 years gives a value of 0.02 for the ratio of both integrals. This multiplied by 13.5% gives 0.003 for the frequency of short period stars. Both methods give similar results suggesting that this frequency is in the range of 0.003–0.007. Fre-
quency of occurrence of W UMa stars was recently determined anew by Rucinski (2002). According to him it is equal to 0.002 for the local field stars and to 0.008 for the OGLE I sample. Approximate equality of both frequencies, i.e., of short period detached binaries and contact binaries indicates that the life time of a binary in the contact phase is of the same order as the life time in the detached phase. The latter time is equal to several Gyr as it was argued above.

To sum up the above arguments we see that a typical W UMa type binary enters the contact configuration when it is at least 4–5 Gyr old and spends several more Gyr in this phase. This conclusion agrees with the age of 8 Gyr obtained for the field W UMa stars from their space motions (Guinan and Bradstreet 1988). In the light of these results the basic assumption of Lucy (1968) that W UMa type stars are very young turns out to be incorrect.

It will be argued in the present paper that a typical W UMa star is a binary past mass exchange in the case A or early B (i.e., when the initially more massive component is close to, or immediately past TAMS). After mass ratio reversal the presently more massive component is a weakly evolved MS star whereas the less massive component is an evolutionary advanced star with hydrogen depleted in its center. Such a pair of stars can fulfill the mass-radius relation for contact binaries while being in thermal equilibrium. This solves the Kuiper paradox in a natural way.

The paper is organized as follows: in Section 2 observational data on radii and masses of components of W UMa stars are compared with evolutionary model calculations of single stars. Section 3 lists basic assumptions and equations used to describe the evolution of a binary. Next, the evolution of the orbital parameters of one binary with a moderate mass ratio and the total mass of 1.8 \( M_\odot \) is discussed. Three main phases are distinguished:

I – approach to contact,
II – mass exchange on a thermal time scale,
III – evolution of a contact binary until merging of the components.

The last Section contains the main conclusions of the paper.

2. Comparison of Observations with Evolutionary Models of Single Stars

The most complete, recent set of physical parameters of contact binaries, obtained in a uniform way, has been published by Maceroni and van’t Veer (1996). Their Table 3 contains absolute parameters of 78 binaries, which were calculated using the available at that time photometric and spectroscopic data. Due to lack of spectroscopic data for many stars, photometric mass ratios have been used which increases uncertainty of the listed parameters for these stars. Later, spectroscopic mass ratios have been determined for several such stars. In some cases their values agree well with the photometric values but in other cases, like AO Cam and LS Del they badly disagree. Such stars were ignored when plotting Fig. 1.
We restrict our attention to cool contact binaries with $M_{\text{tot}} \leq 2 \, M_\odot$, where $M_{\text{tot}}$ is the total mass of the binary (see Maceroni and van't Veer 1996). All binaries except AH Cnc, classified as W-type by Maceroni and van’t Veer, are included into our analysis but several massive A-type binaries are not. The data given by Maceroni and van’t Veer (1996) are plotted in Fig. 1. The primary components which we denote B-components (see Section 3) are marked with asterisks and the secondary, less massive A-components are marked with diamonds. Dotted lines give mass-radius relations for (from bottom to top): ZAMS stars, TAMS stars and evolved stars with small helium cores. They are based on models obtained by VandenBerg (1985, see also Rucinski 1992), Schaller et al. (1992) and Sienkiewicz.* Masses of the helium cores were selected in such a way that the resulting mass-radius relation for the evolved stars forms an upper bound to the observed values.

Inspection of Fig. 1 confirms the conclusion reached by Hilditch et al. (1988)

*At a request of the present author a set of evolutionary models of single, low mass stars was calculated by Dr. Ryszard Sienkiewicz using his recently updated evolution code. Models with masses 0.1–0.7 $M_\odot$ were evolved (at a constant mass) from ZAMS up to the formation of helium core of 0.08–0.15 of the solar mass (depending on the stellar mass).
that all primaries of the W-type binaries lie at, or close to ZAMS whereas their secondaries lie close to, or beyond TAMS. In case of A-type binaries with extreme mass ratios primaries approach TAMS in the mass-radius diagram although they lie closer to ZAMS in the mass-luminosity diagram (Yakut and Eggleton 2005), whereas secondaries lie significantly above TAMS, with radii bound from above by models with small helium cores. Such a situation when a less massive component of a binary is more advanced evolutionary, is known as an Algol paradox and is an indication that the system was subject to the mass exchange process resulting in a reversal of the mass ratio (Paczyński 1971). It is assumed here that the same process has taken place in most of the observed W UMa stars. As a result, the presently less massive components occupy the position at, and above TAMS (Fig. 1) because they are oversized due to their advanced evolutionary stage and not due to energy transfer from the more massive components.

3. Evolutionary Scenario – a Binary with Different Mass Components

A great variety of initial conditions of close binary stars, in particular of masses of individual components and orbital periods results in a plethora of their possible evolutionary scenarios (e.g., Eggleton 1996, Eggleton and Kiseleva-Eggleton 2002, Yakut and Eggleton 2005). It is beyond a scope of the present paper to determine precisely a range of the initial parameters of binaries leading to W UMa type stars. Instead, evolution of orbital parameters of one selected binary from ZAMS till merger of the components will be followed in detail and discussed (preliminary results are given by Stepień 2004).

The following main phases will be considered:

I) AML due to magnetized wind during the MS evolution and filling of the Roche lobe by the more massive A component,

II) fast mass exchange on a thermal time scale till reversal of the mass ratio and formation of a contact configuration,

III) slow evolution of the contact binary toward the extreme mass ratio and ultimate merging of the components.

3.1 Initial Configuration

As an initial configuration a detached ZAMS binary with masses $M_{A,1}=1.2 \ M_\odot$, $M_{B,1}=0.6 \ M_\odot$ is adopted. The mass of the larger component is close to the upper mass limit for stars possessing strong coronal emission associated with a significant mass loss via magnetized wind. The emission decreases rapidly for stars with still larger masses (Schmitt et al. 1985). The mass of the smaller component was assumed to be significantly lower than that of the larger component to stress the difference in evolutionary effects of both components on the resulting contact configuration and to obtain a “typical” W UMa system with
mass ratio close to 0.5 (Shu and Lubow 1981) after the mass exchange. Binaries with the initial mass ratio closer to one will be discussed later.

The initial mass ratio \( q_i = M_{A,i} / M_{B,i} = 2 \) (note that \( q \) is defined throughout the paper as the ratio of the mass of component A to the mass of component B) whereas the initial total mass of the binary is \( M_{\text{tot},i} = M_{A,i} + M_{B,i} = 1.8 \, M_{\odot} \).

The assumed value of the initial orbital period, \( P_{\text{orb},i} = 2 \) days, is equal to the short limit period of cool ZAMS binaries adopted by Stepień (1995). The radii of both components, semi-axis \( a \) of the orbit and the initial AM are given in Table 1.

It is assumed in the following that both components are active and lose AM via magnetized wind and that they rotate in a full synchronization with the orbital period. Their spin AM is neglected compared to orbital AM.

\[
\begin{align*}
\text{Table 1} & \\
\text{Evolutionary history of the discussed binary} \\
\begin{array}{cccccccccc}
\text{evolutionary} & \text{age} & M_A & M_B & q & R_A & R_B & P_{\text{orb}} & a & H_{\text{orb}} \\
\text{stage} & \text{Gyr} & M_\odot & M_\odot & & R_\odot & R_\odot & \text{days} & R_\odot & 10^{51} \text{g} \cdot \text{cm}^2/\text{s} \\
\text{ZAMS} & 0 & 1.2 & 0.6 & 2 & 1.1 & 0.49 & 2 & 8 & 9.2 \\
\text{End of Phase I} & 6 & 1.14 & 0.54 & 2.1 & 1.3 & 0.5 & 0.46 & 3 & 5.1 \\
\text{End of Phase II} & 6.1 & 0.55 & 1.08 & 0.51 & 0.8 & 1.02 & 0.32 & 2.3 & 4.3 \\
\text{Coalescence} & 11 & 0.1 & 1.48 & 0.07 & 0.4 & 1.3 & 0.46 & 2.2 & 1.1 \\
\end{array}
\end{align*}
\]

3.2 Phase I

Loss of AM momentum of a cool close binary via magnetized wind was discussed by Villu (1982) and, more recently, by Stepień (1995). The latter author derived and calibrated a semi-empirical formula for the orbital period variation of a close cool binary, based on the rotation evolution of solar type stars. The formula does not contain any free parameters.

\[
\frac{dP_{\text{orb}}}{dt} = -(2.6 \pm 1.3) \times 10^{-10} P_{\text{orb}}^{-1/3} e^{-0.2 P_{\text{orb}}} \quad (1)
\]

where \( P_{\text{orb}} \) is in days and time in years. For very short orbital periods the exponential factor is close to unity and it varies very little during the subsequent evolution of the orbital period of the discussed binary (in our case, from 2 days to about 0.3 of a day). It can therefore be left out. This simplifies significantly the following considerations. Note, that the 50% uncertainty of the numerical factor on RHS of Eq. (1) is much larger than the error resulting from such an approximation.

The third Kepler law can be expressed as

\[
P_{\text{orb}} = 0.1159 \sqrt{a^3/M_{\text{tot}}} \quad (2)
\]
where \( a \), the semi-axis, is in units of solar radius and \( P_{\text{orb}} \) again in days.

From these two formulas an equation for the time derivative of semi-axis \( a \) can be derived and integrated

\[
a = \sqrt{a_{\text{init}}^2 - 9 \times 10^{-9} t}
\]

where the initial value of semi-axis \( a_{\text{init}} \) is in solar radii and time \( t \) is in years. The uncertainty of the numerical coefficient in Eq. (3) is of the same order as the one in Eq. (1).

Effective radii of the Roche lobes of both components are approximated to better than 1\% by formulae (Eggleton 1983)

\[
\begin{align*}
    r_A &= 0.49q^{2/3} / 0.6q^{2/3} + \ln(1 + q^{1/3})^3, \\
    r_B &= 0.49q^{-2/3} / 0.6q^{-2/3} + \ln(1 + q^{-1/3})^3.
\end{align*}
\]

Our initial binary has semi-axis \( a_{\text{init}} \approx 8 \text{ R}_\odot \) (Table 1). It follows from Eq. (3) that after about 6 Gyr it decreases to \( a \approx 3 \text{ R}_\odot \). The size of the Roche lobe of star A is then \( r_A = 0.44a \approx 1.3 \text{ R}_\odot \). Approximately at the same time star A reaches TAMS and its radius increases to the same value of 1.3 \text{ R}_\odot so the star fills up its Roche lobe (Fig. 2 top). The size of the critical lobe of star B is at that time \( r_B = 0.32a = 0.96 \text{ R}_\odot \), \( i.e. \), it is nearly twice as large as the stellar radius which has practically not changed since ZAMS (Fig. 2 bottom).

The wind carries away not only AM but also mass. Observations show that the mass loss rate due to magnetized wind in the most active single stars is of the order of \( 10^{-11} \text{ M}_\odot/\text{year} \) (Wood et al. 2002). During 6 Gyr such a star loses \( \approx 0.06 \text{ M}_\odot \). Assuming that both components of the binary lose mass at the same rate, their masses become equal to 1.14 \text{ M}_\odot and 0.54 \text{ M}_\odot, respectively, at the end of Phase I.

The value of the orbital AM can be computed from the formula

\[
H_{\text{orb}}^2 = GM_{\text{tot}}^3 \frac{a q^2}{(1 + q)^2}.
\]

After 6 Gyr the orbital AM decreases to about 55\% of its initial value.

### 3.3 Phase II

When star A fills its Roche lobe mass transfer begins. The general paradigm says that star B will react to the mass flux by rapid swelling. After \( \approx 0.1 \text{ M}_\odot \) has been transferred, star B fills its Roche lobe and a contact binary is formed (Sarna and Fedorova 1989 and references therein). At this point, convective envelopes of both components merge with a single value of specific entropy (Lucy 1968). The upraised value of the entropy in the convective envelope of star B (compared to its value in the detached phase) keeps it oversized. Any additional mass transfer
is blocked but energy flows from star A to star B through the neck between the stars. The resulting configuration is out of thermal equilibrium and oscillates around the marginal contact (Lucy 1976, Flannery 1976). If both stars are unevolved, the faster nuclear evolution of star A results in net mass flow from star B and its ultimate swallowing by star A as it ascends the red giant branch. To sum up, after the initial, modest mass transfer from star A, star B swells irreversibly and it transfers all its mass to star A on a nuclear time scale of the accreting star, until merger occurs.

Is the paradigm correct? One of its basic assumptions, that contact binaries are unevolved objects, is certainly wrong. Observations and theoretical arguments show that W UMa stars are formed from initially detached binaries and that this process takes at least several Gyr. It means that primaries with masses close to, or exceeding 1 M⊙ have enough time to approach, or even to pass beyond TAMS. Unless the mass ratio is close to unity, both stars have therefore very different internal structures when mass transfer begins. The binary resembles a pre-Algol configuration. Classical Algols originate from binaries with more massive primaries, not possessing subphotospheric convective zones, and with higher orbital AM, compared to W UMa type stars. They emerge from a common envelope phase (assumed to be formed during a rapid mass transfer) with mass ratio reversed.

The assumption that convective envelopes of both components of a cool contact binary must lie on the same adiabatic curve, has been challenged by Shu et al. (1979) but their hypothesis of a contact discontinuity requires the energy flow from cooler to hotter medium. Recently, however, new models of energy flow between components of a cool contact binary have been developed in which the convective zone of a less massive component is separated from the common envelope lying above the critical Roche surface (Martin and Davey 1995, Kähler 2002a, 2002b, 2004). The models do not violate the second law of thermodynamics but they require another mechanism for adjusting the sizes of the components to the Roche geometry. While such models indicate only a possibility of solving the problem of cool contact binaries in an alternative way, the conclusive evidence for the existence of cool close binaries with mass ratio reversed came from observations. Recent, more accurate modeling of several contact, or nearly contact binaries resulted in detection of a significant number of Algols with periods shorter than 1 day (Rucinski 1993). Many of them have orbital periods of 0.5–0.6 of a day (Shaw 1994, Prihulla, Kreiner and Tremko 2003, Budding et al. 2004). An Algol with the shortest known period is W Crv with \( P_{\text{orb}} = 0.38 \) of a day (Rucinski and Lu 2000). It is clear that the paradigm does not apply to these systems and their existence proves that we are still far from satisfactory understanding of the mass transfer process in close binary stars (see also Shu and Lubow 1981). Note that all the very short period Algols will soon become contact binaries due to AML and evolutionary radius increase of the presently more massive components. It seems therefore fully justified to abandon the requirement of equal entropy convective zones and to assume that a typical W UMa type star has gone through mass transfer with a mass ratio reversal just as Algols do. The possibility of existence of contact binaries with
mass ratio reversed has already been invoked in the past by several authors discussing individual systems with special properties (Tapia and Whelan 1975, Kraicheva, Tutukov and Yungelson 1986, Sarna and Fedorova 1989, Eggleton 1996) but these systems were considered to be exceptions from the general rule stating that contact binaries have not reversed mass ratios. An opposite view is accepted in the present paper. It assumes that few systems are in the phase discussed e.g., by Sarna and Fedorova (1989) in which the net mass transfer proceeds from a more to a less massive component but the great majority of the cool contact, or near contact systems is past the mass ratio reversal. A similar view was expressed by Iben and Livio (1993) who suggest that secondaries of W UMa type binaries should be more advanced evolutionary than primaries.

The following scenario for Phase II is adopted. When star A overflows its Roche lobe it starts transferring mass to star B. Whatever the reaction of star B is, we assume that it can accept a significant portion of mass on a short (thermal in this case) time scale. Apart from not fully understood details of the evolution of the system in the common envelope, we assume that the mass transfer continues until the equilibrium radius of star A becomes smaller than its Roche lobe (a standard assumption when considering formation of Algols). This occurs after the mass ratio reversal. Fig. 3 shows the process as a function of $q$. The mass exchange begins when $q \approx 2.1$ and it ends when the equilibrium radius of star A does not exceed the Roche lobe. Fig. 3 does not describe an exact behavior of stellar radii during the mass exchange phase. The curves labeled “equil. rad. A” and “equil. rad. B” are used only to determine the beginning and the end of the mass exchange episode. They are equal to the TAMS (equil. rad. A) and ZAMS (equil. rad. B) radii of stars with masses varying during the exchange.

If AM is conserved, the equilibrium radius of star B is slightly smaller after the end of mass transfer than its Roche lobe (Fig. 3 top). As a result, a semi-detached binary of the type of a short-period Algol is formed, similar to W Crv (Rucinski and Lu 2000). Such systems will exist in a semi-detached state until the presently more massive star (star B in our case) fills its Roche lobe due to radius increase and/or AML, and a contact binary is formed. However, as modeling of mass exchange in Algols indicate, a significant percentage of AM is lost during the mass transfer in a common envelope phase, together with a small amount of mass (Sarna and De Greve 1996), e.g., for S Cnc the results suggest 5% mass loss and 43% AML. We can calculate how much AM our binary must lose during Phase II to emerge as a contact system. The result shows that the loss of 15% of AM is enough for the equilibrium radius of star B to become equal to the size of its Roche lobe and for the formation of a contact binary consisting of two stars in thermal equilibrium (Fig. 3, bottom). Such AML, accompanied by a mass loss of 3%, was adopted when plotting Fig. 2. The duration of Phase II was assumed to be equal to $10^8$ years which corresponds approximately to the thermal time scale.

At the end of Phase II the components of the binary have $R_A = 0.80 \, R_\odot$, $R_B = 1.02 \, R_\odot$ and $q \approx 0.51$. When plotted in the mass–radius diagram, both components land in the regions occupied by components of W-type contact
binaries (see the starting points of the heavy lines with arrows in Fig. 1).

### 3.4 Phase III

After the mass transfer on a thermal time scale has been completed the system becomes a typical W-type contact binary in a global thermal equilibrium. Large scale circulations in the common envelope carry energy from star B to star A, which results in equalization of surface temperatures of both stars (Martin and Davey 1995). Kähler (2004) developed recently a model of a cool contact system in thermal equilibrium. In his model the energy is transported from the hotter component to the outermost part of the envelope of the cooler component, separated from the bulk of the convective zone by a radiative layer. As a result, deep convective layers of both components have different entropies. In addition to circulations in the common envelope he also assumed the existence of circulations in a radiative interior of the less massive component, carrying a significant fraction of its nuclear energy flux. The value of this fraction is an additional free parameter of the problem. This removes the over-constraint in treatment of contact binaries with a common convective envelope and permits to obtain a unique solution in thermal equilibrium with the secondary component adequately oversized. He applied this model to unevolved, or only slightly evolved binaries. It would be interesting to extend his model to the binary discussed in the present paper.

The analysis of the evolutionary models of low mass stars obtained by Sienkiewicz shows that a helium core increases in these stars at an approximate rate of $0.01 \ M_\odot/Gyr$ hence building a small (less than $0.1 \ M_\odot$) helium core takes several Gyr. The mass–radius relations for such stars are shown in Fig. 4. Note that models were evolved at a constant mass (along the vertical lines in Fig. 4). The stellar radius increases with the increasing mass of the helium core at a constant stellar mass but it decreases with the decreasing stellar mass at a constant core mass.

Assuming that the AML rate during Phase III has its saturated value found for Phase I we obtain again a coincidence of both time scales governing the evolution in contact: nuclear time scale of the lower mass component and orbital AML time scale. As a result, it is possible to calculate approximate evolutionary tracks of both components in the mass–radius diagram (Fig. 4, see also Fig. 1) and the time variations of the orbit parameters (Fig. 2). After about 4 Gyr the mass of star A decreases to about $0.3 \ M_\odot$ whereas the mass of its helium core increases to $0.04 \ M_\odot$. Average net mass transfer from star A to B during this time is $6 \times 10^{-11} \ M_\odot/year$. This is hydrogen rich matter. Because star B converts in thermonuclear reactions about $10^{-11} \ M_\odot/year$ of hydrogen into helium, which is several times less, its degree of chemical non uniformity decreases and it stays close to ZAMS. When its mass increases beyond $1.3 \ M_\odot$, the convection zone gradually disappears, which is accompanied by a decrease of magnetic activity. The system evolves into A-type contact binary with the primary star unevolved. However, when the mass of star A becomes significantly less than the initial core mass with nuclear reactions, i.e., $0.25–0.30 \ M_\odot$, helium enriched
matter starts to flow to star B. At the same time its rate of conversion hydrogen to helium increases due to increased luminosity. Both these effects increase the chemical non uniformity of star B so it moves away from ZAMS. After another 1 Gyr the mass of star A decreases beyond 0.1 \( M_\odot \) and both components merge into a single, rapidly rotating star (Rasio 1995). This completes Phase III.

4. Discussion and Conclusions

4.1 Uncertainties

Because of a significant uncertainty of the AML rate, the time needed to reach the Roche lobe by a more massive component in Phase I is known with accuracy of only \( \approx 50\% \) (see Eq. 1). In our scenario it was assumed that the star fills its Roche lobe just reaching TAMS. With AML rate decreased to 50\% of the adopted value star A reaches TAMS while still being inside the Roche lobe and an additional 1 Gyr is needed to fill the lobe whereas for AML rate increased by 50\% star A reaches its critical lobe when it is close, but not yet exactly at TAMS. This is a minor modification of the further evolution of the discussed system.

The adopted mass loss rate is known not better than to a factor of 2 (Wood et al. 2002) but because it is low anyway, it has a very small influence on the considered time scale. Only for values several times higher than adopted, the mass loss will significantly modify the evolution of a close binary (Eggleton and Kiseleva-Eggleton 2002, Yakut and Eggleton 2005).

A good example of a detached binary in this phase is a very active system XY UMa with the orbital period of 0.48 of a day. According to Pribulla et al. (2001) the components have masses of 1.1 \( M_\odot \) and 0.66 \( M_\odot \), and radii of 1.16 \( R_\odot \) and 0.63 \( R_\odot \), respectively. The primary fills already 68\% of its Roche lobe whereas the secondary fills only 22\% of its Roche lobe. In about 1–2 Gyr the primary (i.e., star A in our notation) fills up its Roche lobe and mass transfer begins.

The beginning of mass transfer initiates the second phase. It has been usually accepted that after accreting about 0.1 \( M_\odot \) the less massive star swells, fills its Roche lobe and a contact binary is formed with a common convective envelope within which a strong energy flux flows through the neck, keeping the less massive star oversized (Lucy 1968, 1976, Flannery 1976, Webbink 1976, Sarna and Fedorova 1989). Recent observations of several short period Algols with periods down to 0.4 of a day contradict, however, this picture (Shaw 1994, Rucinski and Lu 2000, Pribulla et al. 2003). Such stars must have gone through mass exchange with the mass ratio reversal just like longer period ordinary Algols. We are still unable to model correctly the common envelope phase of mass exchange with all hydrodynamical effects in 3D geometry, energy balance, etc. taken properly into account. It is therefore adopted in the present paper that mass exchange, taking place in the discussed close binary, ends with mass ratio reversal. The variable V361 Lyr with orbital period of 0.31 of a day, recently discussed by Hilditch et al. (1997) seems to be now in the second phase. Accord-
ing to their analysis the more massive component (1.26 M\(_{\odot}\)) fills its Roche lobe and transfers mass at a rate of $10^{-7}$ M\(_{\odot}\)/yr to the secondary (0.87 M\(_{\odot}\)) which has a radius equal to only 50\% of its Roche lobe. A hot spot is visible at the location where the mass stream falls on the surface of the secondary. Using the results of Sarna and Fedorova (1989) the authors predict that mass transfer will last mere $\approx 10^6$ years and will end with swelling of the secondary, until it fills its Roche lobe. They stress that V361 Lyr is a very exceptional system caught in an extremely short lasting phase of evolution. We should note, however, that this interpretation requires even more exceptional situation. The secondary does not yet show any sign of expansion. It means that the star is either caught at the very beginning of the mass transfer phase when it has not yet had time to swell upon accreting mass or that the secondary can accommodate the accreted matter without substantial swelling.

In case of a binary with mass ratio distinctly different from one, as considered here, a conservative mass exchange results in a formation of the short-period Algol. The binary spends the first part of Phase III in a semi-detached configuration until a contact system is formed. For a binary with an initial mass ratio $q \approx 1.2$ a conservative mass exchange in Phase II takes place on a nuclear time-scale because any mass loss from star A results in shrinkage of its radius below the size of the Roche lobe. Further mass loss requires an evolutionary expansion of star A (Stępień 2006). The binary spends a noticeable time in a near contact phase, with a more massive component losing mass on a nuclear time scale. After mass ratio reversal an ordinary Algol is formed again. The duration of the Algol evolutionary stage can be shortened, or skipped altogether if enough AM is lost during Phase II. In any case a contact binary is ultimately formed with both components in thermal equilibrium. Later, it evolves towards an extreme mass ratio A-type binary. The common envelope, in which transport of energy takes place, is rather thin (a fraction of mass lying above the critical Roche surface) as discussed recently by Kühler (2004) and it is separated from the bulk of the convective zone of star A by a radiative layer. The separation results in different entropies of deep convective envelopes of both stars. The star W Crv, mentioned above, is in the Algol phase (Rucinski and Lu 2000) whereas $\epsilon$ CrA, an A-type contact binary with $q = 0.13$, is in a more advanced stage of the third phase. Stars like SX Crv ($q = 0.07$) or AW UMa ($q = 0.08$) are probably close to coalescence.

The rate of evolution of the binary in Phase III is determined by the AML rate and evolutionary effects of both components. These processes result in a net mass transfer from star A to star B until coalescence occurs. Unfortunately, as the following discussion shows, the total AML rate in Phase III is known with even greater uncertainty than that indicated by Eq. (1). The binary spends several (five in our case) Gyr in this phase. The time spent in Phase III may be sufficient for star B to complete its main sequence evolution. Its subsequent expansion will cause the common envelope to approach the outer critical Roche surface until a mass loss from the critical point L2 occurs. As the detailed calculations show, the specific AM of the matter flowing from L2 to infinity is 10–20 times larger than the specific orbital AM (Shu, Lubow and Anderson
It means that the mass loss from L2 is as efficient in removing AM from the binary as the magnetized wind. For a significant mass loss from L2 the total AML rate may be considerably increased over that resulting solely from the wind. On the other hand, AML rate via a magnetized wind may be substantially lower in contact binaries than in detached systems. Measured X-ray fluxes of W UMa type variables are lower by a factor of 4–5 compared to rapidly rotating single stars (Stepień, Schmitt and Voges 2001). If AML rate scales with the X-ray flux we expect the correspondingly lower AML rates in contact binaries. The unknown amount of AML from L2 as well as of AML rate via a wind increase substantially an uncertainty of the adopted AML rate in Phase III.

The luminosity of a binary in Phase III essentially does not differ from a model with TRO. The total luminosity is dominated by the nuclear luminosity of the more massive star which in both models is a MS object. Depending on the degree of hydrogen depletion in the core its luminosity changes approximately by a factor of two between ZAMS and TAMS (at a constant mass). The contribution of the less massive component is slightly larger in the present model than in the TRO model, because the star burns hydrogen in the shell during Phase III but the total luminosity is changed insignificantly.

Fig. 5 shows the orbital period variations of the discussed system through all three phases of binary evolution shown in Fig. 2. The shape of the plotted curve suggests a separation between the period distributions of detached and contact binaries. We expect the detached binaries with solar type components to have periods longer than about 0.5 of a day whereas orbital periods of the contact binaries should be concentrated around 0.3–0.4 of a day. Investigation of a large sample of contact binaries from OGLE I program shows indeed a strong peak in the period distribution of these stars at $P_{\text{orb}} = 0.35$ of a day (Rucinski 1998). A similar peak was obtained by Szymański et al. (2001). However, if AML rate is substantially reduced, the mass transfer from star A to B, forced by an evolutionary expansion of star A, will result in an increase of the orbital period when $q \to 0$. This should be visible as a systematic difference between periods of variables with moderate and very low values of $q$ (i.e., entering and leaving Phase III). Observations do not show this effect. The average orbital period of 11 binaries with $q < 0.15$, listed in the catalog by Pribulla et al. (2003), is equal to 0.4 of a day. Obviously, a self-regulating mechanism takes place: a decrease of AML rate, with the mass flux unchanged, results in an increase of the period, hence sizes of the Roche lobes, which, in turn, lowers the mass flux until the period increase is stopped. Lower AML rate should result in the lengthening of a duration of Phase III. For a significantly higher AML rate than adopted here (e.g., due to intense mass loss from L2) the duration of Phase III will be correspondingly shortened.

4.2 Period Variations

Many cool contact binaries exhibit period variations (e.g., Kreiner, Kim and Nha 2001, Qian 2003). While many $O-C$ curves are flat or show only quasi periodic
variations, parabolas have been fitted to others, suggesting systematic period increase/decrease. Because a typical time interval covered by the observations extends over only some tens of years, the resulting detection threshold for period variations is $\dot{P} \approx 10^{-7}$. Several stars show period variations of this order (Qian 2002), which gives the corresponding time scale equal roughly to the stellar thermal time scale. Some authors interpret these variations as resulting from mass transfer between the components. Such alternate mass transfer (visible as a respective period increase/decrease) has been predicted by the TRO theory (Lucy 1976, Flannery 1976) and the observations are interpreted in support of this model.

The recent, precise observations of a few hundred of W UMa type stars by the OGLE team, with a detection threshold for period variations of $10^{-8}$, showed that most of the observed stars reveal period variations of this order or less. Their distribution can be fitted with a Gaussian centered at zero (Kubiak, private communication). Variations exceeding substantially the threshold value are rare. Such a distribution is at odds with the TRO theory. If a large fraction of W UMa type stars were in the TRO phase, we would expect a two point distribution centered roughly at $\pm (10^{-7} - 10^{-8})$. The Gaussian distribution is expected for random period variations. Random variations can arise e.g., from fluctuations in mass transfer associated with the energy transfer between the components. To transfer the required amount of energy (of the order of $0.5 \, L_\odot$) from the hotter to cooler component in a convective way, of the order of $2 \times 10^{-5} \, M_\odot/\text{year}$ must be transferred (back and forth) through the neck between the stars. The rough estimate of energy transfer along the equipotential surface, i.e., at a constant pressure is

$$\Delta L = c_p \dot{M} \Delta T$$

where $\Delta L$ is the transferred luminosity, $c_p$ specific heat at the constant pressure, $\dot{M}$ mass flux and $\Delta T$ is the difference in temperature between the matter flowing from the hotter to the cooler component and returning back.

As a conservative estimate, $2 \times 10^8$ erg/g·K was adopted for the value of $c_p$. This is the largest value appearing in tables of convective envelopes given by Baker and Temesvary (1966). A rather high value of $10^3$ K was adopted for $\Delta T$. For lower $c_p$ and/or less cooling, the mass flux will be correspondingly higher (Webbink 1977 estimates this flux at $6 \times 10^{-4} \, M_\odot/\text{year}$). The typical observed period variations require the mass transfer of the order of $10^{-8} \, M_\odot/\text{year}$ or less, which is only $\approx 10^{-3} - 10^{-4}$ of the estimated mass flux. Such fluctuations can be expected, e.g., as a result of magnetic activity variations. Significant change of coverage of the stellar surface by dark spots perturbs the thermal structure of the stellar convection zone and it may result in a slight change of the stellar radius on a thermal time scale (Spruit 1982) modifying correspondingly the mass flux.
4.3 Summary of the Main Conclusions

An evolutionary scenario is presented for formation and evolution of cool contact binaries of W UMa type. W UMa type binaries are defined as binaries in which the initial masses of both components do not exceed the mass limit for the existence of a subphotospheric convection zone. It is assumed that detached, cool binaries with short orbital periods (about 2-3 days) and solar type primaries (i.e., with masses $\approx 0.9-1.3 M_\odot$) are progenitors of contact binaries. The assumption is in agreement with the current view about the origin of close binaries. The numerical models seem to exclude fission and they favor early fragmentation of the protostellar cloud as the way of close binary formation, with the result that ZAMS binaries are expected to have orbital periods not shorter than about 2 days. Observations of very young stars support this view.

The binaries lose angular momentum (AM) via magnetized wind from both components. As it turns out, the time scale for orbital evolution due to AML of such binaries is equal to several Gyr, or more, depending (primarily) on the length of the initial orbital period. Such time scales are the same as nuclear time scales for MS evolution of primaries. As a result of this coincidence, the primary is evolutionary advanced (i.e., it is close to, or beyond TAMS) when the orbit shrinks, due to AML, so that the critical Roche lobe approaches its surface. According to this scenario we should observe few, if any, young W UMa-type stars. Observations support this picture: the number of such stars is very low in stellar clusters with ages less than about 4-4.5 Gyr and it increases rapidly in older ones.

The Roche lobe overflow results in mass transfer to the secondary. It is assumed in the present scenario that the mass transfer continues (through the common envelope phase) until the mass ratio reversal. Such an exchange mass episode is similar to the one taking place in Algols and it is different from that adopted by Lucy (1976) and Flannery (1976) in case of W UMa stars. They assumed that specific entropy of the convective envelopes of both components takes the same value during the common envelope phase. As a result, net mass exchange stops after transferring a modest amount of matter and is replaced by energy transfer. The turbulent convective energy transport between the components keeps both convective zones on the same adiabatic curve but the resulting configuration of both stars is in global thermal nonequilibrium. Thermal Relaxation Oscillations are necessary for a long time scale existence of such binaries. However, the recent models of energy transfer between the components of a W UMa type binary indicate that large scale circulations, flowing in a relatively thin common envelope (above the inner critical Roche surface), can transport energy with a required efficiency (Martin and Davey 1995, Kähler 2002a, 2002b, 2004), hence the requirement of identical specific entropy in both convective layers is no longer needed. This favors the Algol-type mass transfer with mass ratio reversal. The existence of several Algols with periods close to 0.5 of a day supports the scenario. The system emerges as a contact binary, if enough AM is lost during the common envelope or as a short-period Algol which transforms, after losing excess AM, also in a contact binary.
The consecutive evolution in contact is governed by AML, assumed to be as efficient as in the detached phase, and the nuclear evolution of the present secondary which is the formerly more massive component. The secondary is hydrogen depleted in the center and it builds a small helium core. Time scales of both processes are again close to one another. AML keeps the orbit compact whereas nuclear evolution forces secular mass transfer from the secondary to the primary. The binary evolves towards an extreme mass ratio system, which, after several Gyr ends with a merging of both components.

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Fig. 2. Variations in time of the radius of star A (top) and star B (bottom) compared with the variations of the orbital semi-axis and size of the respective critical Roche lobe over all three phases of evolution discussed in the text. The curves are labeled correspondingly. Phase I (left-hand part of each figure) is separated from Phase III (right-hand part) by a narrow dip in variation of semi-axis, accompanied by rapid changes of radii of both stars, which then merge with sizes of the corresponding Roche lobes (Phase II).
Fig. 3. Variations of the critical Roche lobe sizes (dashed lines) during the fast mass exchange (Phase II). Mass exchange begins at $q \approx 2.1$ when the equilibrium radius of star A reaches the size of its Roche lobe and ends around $q \approx 0.5$ when it decreases below the lobe size. During the mass transfer the equilibrium radii of both stars (dash-dotted lines) do not describe their actual radii because stars develop a common envelope, very likely reaching beyond an outer critical surface. In a conservative case (top) the equilibrium radius of star B is at the end of mass transfer smaller than the size of its Roche lobe which results in a formation of a short-period Algol. Loss of $\approx 15\%$ of the orbital AM during the common envelope phase results in a formation of a contact binary (bottom).
Fig. 4. The evolution of the discussed binary during Phase II. Dotted lines show mass-radius relations for ZAMS and TAMS stars and dash-dotted lines show the same relations for stars with different helium cores in solar mass, as indicated. Heavy solid and broken lines are schematic evolutionary tracks of both components of the discussed binary, with arrows indicating the direction of evolution.

Fig. 5. Time behavior of the orbital period of the discussed binary through all three phases.