Turbulent Free-Space Optical Communication Utilizing Degree of Polarization

NICHOLAS J. SAVINO1,†, RAVI K. SARIPALLI1,2, JACOB M. LEAMER1, WENLEI ZHANG1, DENYS I. BONDAR1,∗, AND RYAN T. GLASSER1

1Department of Physics and Engineering Physics, Tulane University, New Orleans, Louisiana 70118, USA
2SLAC National Accelerator Laboratory, Menlo Park, California 94305, USA
†nsavino@tulane.edu
∗dbondar@tulane.edu

Abstract: Free-space optical (FSO) communication is often subject to atmospheric turbulence, which can result in loss of data and introduce security threats. Degree of polarization (DOP), a classical property of propagating light, is shown to be preserved when passed through experimentally generated turbulence. We also find that the state of polarization is preserved. Due to this resistance to turbulence, which does not require adaptive optics or post processing, we suggest that DOP could pose a valuable degree-of-freedom for multiplexed FSO communication and basis dependent encryption schemes. Experimental and simulation results are compared.

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1. Introduction

Free-space optical (FSO) communication is a widely used method of sending data quickly over long distances. It allows for wide bandwidths, licence free spectra, and high bit rates [1–5]. Polarization transformers have been shown to be able to manipulate the state of polarization (SOP) and DOP [6], in ways we believe are suitable for FSO communication [7]. The manipulation of polarization has been shown to encode and transmit information securely [8]. Light, when treated classically with Maxwell’s equations, is represented such that the electric field is a plane wave where

\[
\begin{align*}
\vec{E}_x(t) &= \hat{i}E_{0x}(t) \cos \left[ (\hat{k}z - \hat{\omega}t) + \epsilon_x(t) \right], \\
\vec{E}_y(t) &= \hat{j}E_{0y}(t) \cos \left[ (\hat{k}z - \hat{\omega}t) + \epsilon_y(t) \right],
\end{align*}
\]

(1)

and

\[
\vec{E}(t) = \vec{E}_x(t) + \vec{E}_y(t).
\]

(2)

Degree of polarization (DOP) is a property of light that can be calculated from the Stokes parameters. The Stokes parameters \(s_0, s_1, s_2, s_3\) are defined by the electric field such that

\[
\begin{align*}
s_0 &= \langle E_{0x}^2 \rangle_T + \langle E_{0y}^2 \rangle_T, & s_1 &= \langle E_{0x}^2 \rangle_T - \langle E_{0y}^2 \rangle_T, \\
s_2 &= \langle 2E_{0x}E_{0y} \cos \epsilon \rangle_T, & s_3 &= \langle 2E_{0x}E_{0y} \sin \epsilon \rangle_T,
\end{align*}
\]

(3)

where DOP is defined as

\[
DOP = \frac{\sqrt{s_1^2 + s_2^2 + s_3^2}}{s_0}.
\]

(4)

Atmospheric turbulence poses a significant threat for FSO communication [1]. Attenuation in signal created by turbulent fluctuations can result in increased bit-error rate and data security...
concerns. Work is constantly being done to improve FSO such that it can become resistant to turbulence [9], for which many techniques require cumbersome adaptive optics, advanced algorithms, and/or machine learning [10–14]. We propose the use of the use of DOP as a viable degree-of-freedom in FSO that is subject to atmospheric turbulence. It has been shown that near DOP = 100% it is preserved in the presence of turbulence [15,16] and its use may lead to improvements in FSO [17,18]. Methods such as coherent binary polarization shift keying suggest promise in improving FSO in turbulence [19]. Here we show that an arbitrary DOP is preserved when passed through turbulent media. We measure the DOP of several partially polarized SOP that is passed through lab generated turbulence and compare them with those of the input state.

Turbulence in Earth’s atmosphere is created by fluctuating air currents and manifests itself in fluctuating index of refraction of the air [20–22]. These fluctuations lead to scattering and absorption of light passing through the air, which in turn, leads to fluctuating attenuation of a transmitted signal [23]. Atmospheric turbulence can be modeled by several different distributions including, gamma-gamma, log-normal, and negative exponent distributions. In this work, we focus on log-normal turbulence, which is an accepted model for weak atmospheric turbulence [24]. Log-normal turbulence is modeled by the intensity distribution

\[
 f(I) = \frac{1}{2I\sqrt{2\pi}\sigma^2_I} \exp\left(-\frac{(\ln \frac{I}{I_0} + \frac{\sigma^2_I}{2})^2}{2\sigma^2_I}\right),
\]

where \(I_0\) is the mean irradiance and \(\sigma^2_I\) is the scintillation index [23,24]. The scintillation index can also be calculated directly from the measured intensities where

\[
 \sigma^2_I = \frac{<I^2> - <I>^2}{<I>^2}.
\]

2. Experiment

The experimental setup is shown in Fig. 1. The desired partially polarized states are generated by a Mach-Zehnder interferometer, where a the polarizing beamsplitter generates beams of horizontal and vertical polarizations, which are then recombined on a 50:50 non-polarizing beamsplitter (BS), such that they are side by side, with the edges of the beams are not overlapping [25]. The DOP of these states is controlled by adjusting relative intensity of each arm of the interferometer. This is done by attenuating one arm by tuning a the variable neutral-density filter. In order to change the basis from horizontal/vertical to an arbitrary basis, the light is passed through a quarter-wave plate (QWP), half-wave plate (HWP), and another QWP.

When the tank is placed as shown in Fig. 1, such that the light propagates through the bubbles, we refer the the measured state as the ‘output state.’ The tank can be removed from the setup in order to determine the SOP and DOP without turbulence, referred to as ‘input states.’

2.1. Turbulence Generation

Air bubbles in water have been shown to scatter and change the behavior of light passing through the interfaces [26,27]. We generate turbulence like effects in-lab, forgoing the need for long distance atmospheric testing. Intensity fluctuations are generated by passing the beam of light through water in a glass fish tank with submerged bubblers. When the light passes through the columns of bubbles, we find the generated intensity fluctuations are consistent with that of log-normal atmospheric turbulence [24,28]. Bubbles in water have previously been studied as a means of creating to atmospheric turbulence like effects [28].

Atmospheric turbulence that is modeled by a log-normal intensity distribution can be classified by the scintillation index [24]. Over the course of the experiment, between all intensity projection
measurements, we observe an average scintillation index of $5 \pm 1$, calculated with Equ. 6. A scintillation index less than 1 is within the accepted range for that of weak atmospheric turbulence [29,30]. The turbulence appears to randomly fluctuate throughout the experiment. We believe the fluctuations in levels of turbulence are due to the bubbler system shifting as bubbles are produced throughout the experiment.

2.2. Polarization State Tomography

Tomography measurements on the polarization matrix can be conducted to obtain the Stokes parameters of the input and output states using a polarimeter. However, we use an intensity detection scheme that only require a QWP, linear polarizer, and photodetector to obtain polarization projection measurements [31,32]. Four intensity measurements are performed: $I(0^\circ, 0^\circ)$, $I(0^\circ, 90^\circ)$, $I(0^\circ, 45^\circ)$, and $I(45^\circ, 45^\circ)$. $I(\psi, \phi)$ is the intensity measured by the photodetector with the fast axis of the QWP (in the detection scheme) at angle $\psi$, and the axis of transmission of the linear polarizer at angle $\phi$ (both with respect to the horizontal axis). With these intensity measurements, we can calculate the Stokes parameters [32]:

$$s_0 = I(0^\circ, 0^\circ) + I(0^\circ, 90^\circ),$$
$$s_1 = I(0^\circ, 0^\circ) - I(0^\circ, 90^\circ),$$
$$s_2 = 2I(45^\circ, 45^\circ) - s_0,$$
$$s_3 = 2I(0^\circ, 45^\circ) - s_0.$$  \hspace{1cm} (7)

From this, we obtain the DOP of the states using Eq. 4. We examine 120000 intensity measurements over 24 seconds for each polarization projection.

It is important to recognize that our results rely on many measurements being recorded, then time averaged, to show DOP is preserved. The setup in Fig. 1 only allows us to take the intensity measurements independently at different times. Due to the irreproducibility and chaotic nature of turbulence, enough data points to recover the entire turbulence distribution must be taken.
3. Simulation

We use the COMSOL Ray Optics [33] package to simulate the experimental setup shown in Fig. 1. The code for running these simulations can be found on Github [34]. In these calculations, we consider light propagating in the x-direction through water that contains bubbles. Because the speed of the velocity of the bubbles in the experiment is negligible compared to the speed of light, we choose to have stationary bubbles in the simulation, which simplifies the model. This means that our simulation results are snapshots of the experimental setup. We perform simulations for light that is initially vertically polarized and for light with an arbitrary initial state of polarization. In either case, we consider five initial values for the DOP and run 1000 simulations for each DOP. In every simulation, the number, size, and position of the bubbles along the x-direction are determined by sampling a uniform distribution. The position of the bubbles along the y and z directions is determined using a normal distribution to decrease the number of bubbles that are placed outside of the path of the rays. The seed for the random number generation is different for every simulation. The COMSOL Ray Optics package keeps track of the Stoke’s parameters of each ray throughout the simulation, so calculating the initial and final DOP for a given simulation can be done by calculating the average $s_1$, $s_2$, $s_3$, and $s_0$ of the rays at the beginning and end of the simulation and using Eq. (4).

Additionally, we simulate the propagation of light through atmospheric turbulence for an arbitrary state of polarization using the COMSOL Ray Optics software [33]. Here we consider three values for the initial DOP and perform 1000 simulations for each DOP. In every simulation, atmospheric turbulence is modeled as fluctuations in the index of refraction $n(\vec{r})$. These fluctuations can be generated using the power spectral density of $n(\vec{r})$, which is given by the Von Kármán spectrum [35]

\[
\phi_n(\vec{k}) \sim \frac{0.033 C_n^2}{(\vec{k}^2 + \kappa_0^2)^{11/6}} e^{-\vec{k}^2/\kappa_m^2},
\]

where $\vec{k}$ is the spatial frequency, $C_n^2$ is the structure constant, and $\kappa_0$ and $\kappa_m$ are the frequencies corresponding to the inner and outer scales of the turbulence respectively. The kernel $G(\vec{r})$ that defines the statistics of the fluctuations in position space can then be calculated as [36, 37]

\[
G(\vec{r}) = (F_{\vec{k} \rightarrow \vec{r}})^{-1} [\sqrt{\phi_n(\vec{k})}],
\]

where $r$ is the position and $F_{\vec{k} \rightarrow \vec{r}}^{-1}$ denotes the inverse Fourier transform. By convolving $G(\vec{r})$ with random uniform noise, fluctuations in $n(\vec{r})$ with the desired statistics can be generated [36, 37] and provided to COMSOL for the simulations. As before, the code for generating these fluctuations and performing the simulations can be found on Github [34].

4. Results

We obtain experimental results for five different partially polarized input states with varying DOP. We take the average of 120000 data points for each intensity measurement. The Stokes parameter $s_1$ is varied. In Fig. 2, we plot the values of the input and output DOP, derived from the Stokes parameters. The experimental results are compared with the results from the simulation with stationary bubbles. The individual experimental Stokes parameters are shown in Tab. 1. The diagonal black line in Fig. 2 represents equal input and output DOP. However, we note that there is significant intensity attenuation of the beam as it propagates through the bubbles and water, which is to be expected. We see a reduction in average intensity by a factor of about 20.

In Fig. 3, we show the location of the input and output states in the Poincare sphere representation of normalized in three-dimensional Stokes space. We see the corresponding input and output states lie very close to each other on the Poincare sphere.

While we measure all Stokes parameters, a full tomography of the polarization matrix is not necessarily required for communication purposes, given appropriate encoding and decoding.
Fig. 2. Measured DOP of the output states through underwater turbulence compared to the measured DOP of the input states for different partially polarized states. The output DOP with respect to the input DOP. Input and output DOP are equal at the black line ($y = x$). Experimentally obtained data is shown in red and simulation results are shown in blue. The error bars represent 99% confidence interval.

| state | $s_1/s_0$ | $s_2/s_0$ | $s_3/s_0$ | DOP               |
|-------|-----------|-----------|-----------|-------------------|
|       | input     | output    | input     | output            | input     | output            |
| 0     | 0.0857    | 0.140     | -0.0435   | 0.00870          | -0.0412   | -0.0478          |
| 1     | 0.357     | 0.389     | -0.0457   | -0.0676          | -0.0321   | -0.0699          |
| 2     | 0.528     | 0.554     | -0.0378   | -0.0794          | -0.0244   | -0.104           |
| 3     | 0.739     | 0.714     | -0.053    | -0.0302          | -0.0227   | -0.0576          |
| 4     | 0.993     | 0.923     | -0.0567   | -0.0997          | -0.0151   | -0.0832          |

Table 1. Measured mean normalized Stokes parameters of the input and output partially polarized states on the horizontal/vertical axis through experimental underwater turbulence.

protocols are chosen. If we use the input states shown in Fig. 3, only measurement along the $s_1$ Stokes parameter is necessary to determine the encoded value. Thus, the measurement can be done in a more robust manner, with only two intensity measurements, $I(0°, 0°)$ and $I(0°, 90°)$. Due to our choice of basis, where $s_2$ and $s_3$ are approximately 0, we can recover the DOP with only $s_0$ and $s_1$. This measurement can be generalized to all directions on the Poincare sphere by exploiting its symmetry, as there is no preferred direction on the Poincare sphere.

We can choose some arbitrary basis to generate arbitrary states. The individual Stokes
parameters, when measuring states along an arbitrary basis are shown in Tab. 2. Thus, our results suggest that SOP is also preserved. The input and output DOPs, along with the locations of each state on the Poincare sphere are shown in Fig. 4

Table 2. Measured mean normalized Stokes parameters of the input and output arbitrary partially polarized states through experimental underwater turbulence.

Furthermore, we verify that SOP and DOP are preserved when we simulate light passing through atmospheric turbulence, as described in Sec. 3. The results from the atmospheric turbulence simulations are shown in Fig. 5. We choose the same arbitrary basis as in experiment and the stationary bubbles simulation for consistency.
(a) The output DOP with respect to the input DOP for an arbitrary basis. Input and output DOP are equal at the black line ($y = x$). Experimentally obtained data is shown in red and stationary bubble simulation results are shown in blue. The error bars represent 99% confidence interval. We see DOP is preserved for arbitrary states.

(b) Arbitrary input and output states in the Poincare sphere representation. Experimental input states are orange triangles, experimental output states are green diamonds, stationary bubbles simulation input states are red squares, stationary bubbles simulation output states are blue circles. On the Poincare sphere: D = diagonal, A = anti-diagonal, L = left, R = right, V = vertical, H = horizontal.

Fig. 4. Measured DOP of the output states through underwater turbulence compared to the measured DOP of the input states for different arbitrary partially polarized states. Experimental and stationary bubbles simulation results are compared.

(a) The output DOP with respect to the input DOP for an arbitrary basis. Input and output DOP are equal at the black line ($y = x$). Atmospheric turbulence simulation results are shown in blue. The error bars represent 99% confidence interval. We see DOP is preserved for arbitrary states.

(b) Arbitrary input and output states in the Poincare sphere representation. Atmospheric turbulence simulation input states are red squares, atmospheric turbulence simulation output states are blue circles. On the Poincare sphere: D = diagonal, A = anti-diagonal, L = left, R = right, V = vertical, H = horizontal.

Fig. 5. Measured DOP of the output states through simulated atmospheric turbulence compared to the measured DOP of the input states for different arbitrary partially polarized states.
5. Discussion

Due to the nature of the experiment, the level of turbulence fluctuates slightly between data sets, while remaining in or near the regime of atmospheric turbulence. Changing levels of turbulence does not appear to alter DOP preservation. We do note that the DOP is not exactly preserved, as the input states are slightly outside of confidence interval for most measured output states. We suspect this is caused by error in experimental state generation and measurement, and is not a direct result of the turbulence. A more sensitive state generation and detection setup can reduce this error.

We can significantly decrease the time scales needed to obtain results if we collect multiple intensity measurements at the same time. Measurements can be made anywhere on the entire Poincare sphere if $I(0°,0°), I(0°,90°), I(0°,45°)$, and $I(45°,45°)$ are measured simultaneously. A polarimeter that uses a rotating wave-plate/polarizer scheme, where the intensity is continuously measured as a function of time as a the wave-plate/polarizer spin can result in near instantaneous detection. The measurements would need to be taken at a faster rate than the fluctuations of the turbulence. Some time averaging may still be necessary unless the entire state observes uniform turbulence across the entire beam profile.

DOP and SOP show promise as a method of encoding information in FSO communication that is preserved when passed through turbulence. The sender and receiver only need passive optics to encode and decode the respective data; no active or adaptive optics are necessary. Turbulence resilience without the need of adaptive optics could be of particularly beneficial for advancements in ground-to-satellite communication, as bypassing adaptive optics can result in significant weight savings [38]. We believe the methods presented here can work in tandem with other correction and security measures to continue to improve upon FSO communication. Due to the nature of our experiment, we believe these methods may also hold promise in improving underwater optical communication [39].

Multiplexing along different Stokes parameters is made possible when arbitrary SOP is generated. As discussed in Sec. 2, to encode onto an arbitrary polarization state, the light can be passed through a QWP, followed by a HWP, and then another QWP. Rotating these three components allows for any point within the Poincare sphere. Our results have potential to improve already existing polarization dependant communication schemes [7,40–42]. Encryption schemes similar to quantum key distribution [40] can also be performed, where the sender and receiver share a key which determines which bases to measure. Since DOP is a projection onto the three-dimensional Poincare sphere, it can potentially be used as a turbulence resistant communication mode, in which information is carried in both the base and value. As the Poincare sphere is analogous to the Bloch sphere, applications can potentially be implemented in the field of quantum information.

Furthermore, we show the intensity fluctuations generated with our setup are consistent with models used to describe true atmospheric turbulence on the macro scale [24]. Results obtained in experiment and simulation agree with one another. More work will need to be done to verify that changing turbulence properties do not alter the DOP and SOP.

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**Data availability.** Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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