We study a cosmology by considering entropic issues where the emphasis is put on the change of sign of the inhomogeneous term associated to an energy non-conservation equation. We make a bare/effective description of the equation of state of the cosmic fluid through a bare(\(\omega\))/effective(\(\omega_{eff}\)) conservation equation. In the bare case, where we have a non conserved equation of state for the cosmic fluid, we describe the change in the sign of the inhomogeneous term at different times of the cosmic evolution. We show that by a redefinition of the adiabatic \(\omega\)-parameter we can recover the usual scheme for the cosmic evolution. In the effective case we show also that if the evolution is driven by dust or cosmological constant, the universe evolves on the thermal equilibrium. Additionally, by incorporating a quantum correction only cosmological constant can drive an evolution on thermal equilibrium. The future singularity present in the case when we do not incorporate this correction is avoided if we do it.
I. INTRODUCTION

From the works of E. Verlinde [1] and D. A. Easson et al [2], entropic gravity is visualized as an alternative to the standard (classical) theory of gravity described by the Einstein theory. In the first formalism the space is generated from the thermodynamics on a holographic screen. Here the information is the main ingredient for deriving gravity and the input is the holographic principle; the information is encoded at the boundary. The second formalism is based on the incorporation of surface terms in the gravitational action but gravity is still a fundamental theory. Both schemes drive to modifications in the usual Friedmann equations of the standard Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmology and so we can have another sand where we can inspect. Whether or not these approaches can give a new scope about gravity is a controversial issue [3].

This letter is organized as follows. In Section II we discuss an effective scheme ($\omega_{eff}$) for the equation of state (EoS) of the cosmic fluid through a redefinition of the adiabatic-$\omega$ parameter. In Section III we describe the sign change in the inhomogeneous term (in the bare EoS for the cosmic fluid) associated to the interchange of energy between the bulk and the boundary and we discuss also the absence of future singularities when a quantum correction is incorporated in the field equations. In Section IV we summarize the discussed aspects.

We will use the units $8\pi G = c = 1$.

II. $\Theta_{eff}$-FORMULATION.

We discuss the model [4] which is described by the following modified Friedmann equations

$$3H^2 = \rho + 3\alpha H^2 + 3\beta \dot{H},$$

$$\ddot{a}/a = \dot{H} + H^2 = -\frac{1}{6} (3\Theta - 2) \rho + \alpha H^2 + \beta \dot{H},$$

where the additional terms, $\alpha H^2$ and $\beta \dot{H}$ are originated on surface terms in the gravitational action and $\{\alpha, \beta\}$ are two constant parameters associated to surface curvature where $\alpha = 3/2\pi$ and $\beta = 3/4\pi$ and like a generalization this parameters can be bounded by $\alpha < 1$
and $0 \leq \beta \lesssim 3/4\pi$ in agreement to the observational data [2]. The $\Theta$-quantity is $\Theta = 1 + \omega$ where $\omega = p/\rho$ being $p$ the pressure and $\rho$ the energy density.

We will study two approaches in order to manipulate (1-2): the first is based on making a description of the model through an effective EoS ($\Theta_{\text{eff}}$). The second approach is based on a non conserved equation for $\rho$. So, we write the scheme (1-2), denoted as effective scheme, in the form

$$3H^2 = \frac{1}{1 - \alpha} \left(1 - \frac{3}{2} \beta \Theta \right) \rho,$$

(3)

$$\frac{\ddot{a}}{a} = -qH^2 = - \left(\frac{3}{2} \Theta_{\text{eff}} - 1\right) H^2,$$

(4)

where $\Theta_{\text{eff}}$ is defined through

$$\Theta_{\text{eff}} = (1 - \alpha) \left(1 - \frac{3}{2} \beta \Theta \right)^{-1} \Theta,$$

(5)

and by using (4) the deceleration parameter $q$ is given by

$$1 + q = \frac{3}{2} \Theta_{\text{eff}},$$

(6)

and the conservation equation becomes

$$\dot{\rho} + 3\Theta_{\text{eff}} H \rho = 0.$$ 

(7)

In the present effective scheme, this equation appears like an usual energy conservation equation and so, the scheme (1-2) can be saw as one standard FLRW scheme in the following sense, from (3-4) we can obtain the solution for the Hubble parameter

$$H(t) = H_0 \left[1 + \frac{3}{2} \Theta_{\text{eff}} H_0 (t - t_0)\right]^{-1},$$

(8)

and this solution it is a one standard given that $\Theta_{\text{eff}} > 0 \implies$ dark matter-evolution or quintessence-evolution and $\Theta_{\text{eff}} < 0 \implies$ phantom-evolution. If we consider early inflation, $\Theta \approx 0$, from (3-4) we obtain $H \approx \text{const.} \rightarrow \rho \approx \text{const.}$ and the acceleration is
\[
\frac{\ddot{a}}{a} = \frac{1}{3} \left( \frac{1}{1-\alpha} \right) \rho = \text{const.} > 0 \Rightarrow \alpha < 1,
\]  
(9)
so that if we look (3) we obtain \( \beta \Theta < 2/3 \) and this constraint for \( \beta \) will be improved later.

Particular cases about bare-effective description \( \Theta (\omega) - \Theta_{eff} (\omega_{eff}) \) are:

- cosmological constant

\[
\Theta = 0 (\omega = -1) \implies \Theta_{eff} = 0 (\omega_{eff} = -1),
\]  
(10)
and both descriptions, bare and effective, coincide,

- stiff matter

\[
\Theta = 2 (\omega = 1) \implies \Theta_{eff} = 2 (1 - \alpha) (1 - 3\beta)^{-1} (\omega_{eff} = -1 + 2 (1 - \alpha) (1 - 3\beta)^{-1}),
\]  
(11)

- dark matter

\[
\Theta = 1 (\omega = 0) \implies \Theta_{eff} = (1 - \alpha) \left( 1 - \frac{3}{2} \beta \right)^{-1} (\omega_{eff} = -1 + (1 - \alpha) \left( 1 - \frac{3}{2} \beta \right)^{-1}),
\]  
(12)

- phantom dark energy

\[
\Theta < 0 (\omega < -1) \implies \Theta_{eff} < 0 (\omega_{eff} < -1),
\]  
(13)
and in this case we are also coincidence in both schemes.

By using (6) we can fit the q-parameter in the following form

\[
\Theta_{eff} = 1 \ (q = 1/2) \text{ and } \Theta = 1 \implies \alpha = \frac{3}{2} \beta,
\]  
(14)
so that for dark matter we have \( \omega_{eff} = \omega = 0 \). Thus, we can write

\[
\Theta_{eff} = (1 - \alpha) (1 - \alpha \Theta)^{-1} \Theta,
\]  
(15)
and we have one parameter ($\alpha$) for fixing. Now, by considering $\Theta = 2 (\omega = 1)$, i.e. stiff matter, we have

$$\Theta_{eff} = 2(1 - \alpha)(1 - 2\alpha)^{-1} \rightarrow \omega_{eff} = -1 + \frac{1 - \alpha}{1/2 - \alpha}.$$  \hspace{1cm} (16)

and $1/2 < \alpha < 1 \implies \omega_{eff} < -1$, i.e., something like effective phantom stiff matter! We solve this problem by doing $\alpha < 1/2$, and in this case we have a quintessence-evolution driven for stiff matter. If $0 < \alpha << 1/2$ we have $\omega_{eff} \approx 1$. We note that $\alpha < 1/2 \implies \beta < 1/3$ (see (14)). So, the present effective scheme result to be on standard... and nothing new under the sun!, that is, we obtain the same as the usual FLRW-scheme by changing $\Theta \rightarrow \Theta_{eff}$.

III. $\Gamma$-FORMULATION.

We come back to (7) in order to obtain

$$\dot{\rho} + 3\Theta H\rho = \Gamma,$$  \hspace{1cm} (17)

where $\Gamma$ is the amount of energy non-conservation which is given by

$$\Gamma = 3(\Theta - \Theta_{eff}) H\rho = 3\alpha \Theta \left(\frac{1 - \Theta}{1 - \alpha \Theta}\right) H\rho,$$  \hspace{1cm} (18)

where we have used (15). This quantity, $\Gamma$, represents the interchange of energy between the bulk (the universe) and the boundary (Hubble horizon). At early times, $\Theta \approx 0 \iff H \approx const.$ and $\rho \approx const.$ (exponential inflation), so that we have $\Gamma \approx 0$ and the same occurs if we have dark matter ($\Theta = 1$). In the case of stiff matter ($\Theta = 2$) or radiation ($\Theta = 4/3$) we obtain $\Gamma < 0$ and the same occurs at late times if we have the possibility of phantom dark energy ($\Theta < 0$), and we note that $\Gamma > 0$ if we are in the quintessence-zone ($0 < \Theta < 1$). In the case of cosmological constant ($\Theta = 0$) we have $\Gamma = 0$.

By using the eqs. (15), (8) and (1), the expression (18) can be written in the equivalent forms

$$\Gamma (\Theta, \alpha; t) = 9H_0^2\alpha (1 - \alpha) \left(\frac{1 - \Theta}{1 - \alpha \Theta}\right)^2 \left[1 + \frac{3(1 - \alpha)\Theta}{2(1 - \alpha \Theta)}H_0(t - t_0)\right]^{-3},$$  \hspace{1cm} (19)
\[
\frac{8}{3} H_0^3 \frac{\alpha}{(1-\alpha)^2} \frac{(1-\Theta)(1-\alpha \Theta)}{\Theta^2} \left[ \frac{2(1-\alpha \Theta)}{3(1-\alpha) \Theta} + H_0 (t - t_0) \right]^{-3},
\]
(20)

\[
= -\frac{8}{3} \frac{\alpha}{(1-\alpha)^2} \frac{(1-\Theta)(1-\alpha \Theta)}{\Theta^2} (t_s - t)^{-3},
\]
(21)

where

\[
t_s = t_0 - \frac{2}{3 \Theta_{eff}} H_0^{-1} = t_0 - \frac{2(1-\alpha \Theta)}{3(1-\alpha) \Theta} H_0^{-1},
\]
(22)

and we can see that \( \Gamma (1, \alpha; t) = \Gamma (0, \alpha; t) = \Gamma (\alpha^{-1}, \alpha; t) = 0 \) and we have a future singularity for \( \Theta < 0 \) (this singularity will be removed by the inclusion of quantum corrections on (1-2), see later) and a past singularity if \( \Theta > \alpha^{-1} \). If \( 0 < \alpha < 1/2 \implies \Theta > 2 \), i.e., \( \omega > 1 \) (super stiff matter), but the observational data discards this past singularity. So,

\[
\Gamma (\Theta > 1, \alpha; t) < 0 \iff \omega > 0 \quad (\text{beyond dust zone}),
\]

\[
\Gamma (1, \alpha; t) = 0 \iff \omega = 0 \quad (\text{dust})
\]

\[
\Gamma (0 < \Theta < 1, \alpha; t) > 0 \iff -1 < \omega < 0 \quad (\text{quintessence zone}),
\]

\[
\Gamma (0, \alpha; t) = 0 \iff \omega = -1 \quad (\text{cosmological constant})
\]

\[
\Gamma (\Theta < 0, \alpha; t) < 0 \iff \omega < -1 \quad (\text{phantom zone}),
\]
(23)

and the sign of \( \Gamma \) it’s not always the same during the cosmic evolution. So, the evolution is developed out the thermal equilibrium at exception when we have dust or cosmological constant.

We come back now to (4). By using (18) we write the following expression for the acceleration

\[
\frac{\ddot{a}}{a} = \frac{3}{2} H^2 \left( \frac{2}{3} + \frac{\Gamma}{3H\rho - \Theta} \right),
\]
(24)

and we note that

\[
\Theta = \frac{2}{3} \iff \omega = -\frac{1}{3} \implies \frac{\ddot{a}}{a} = \left( \frac{H}{2\rho} \right) \Gamma \quad \text{and} \quad \Gamma = \left( \frac{\alpha}{3/2 - \alpha} \right) H\rho > 0.
\]
(25)

In standard cosmology, \( \Theta = 2/3 \iff \omega = -1/3 \) correspond to a curvature fluid (string gas) and in that case we have \( \ddot{a} = 0 \). Nevertheless, in the present case we have \( \ddot{a} > 0 \).
What happen today? From WMAP 5-7 [5]:

\[ 1 + \omega(0) = \Theta(0) \approx -0.10 \pm 0.14 \] (for \( \omega \)-time independent) so that we define \( \Theta^+(0) = 0.04 \) and \( \Theta^-(0) = -0.24 \). Thus, in accord to [18]

\[ \Gamma(0) > 0 \leftrightarrow \Theta(0) = \Theta^+(0) \quad \text{and} \quad \Gamma(0) < 0 \leftrightarrow \Theta(0) = \Theta^-(0), \quad (26) \]

and \( \Gamma(0) = 0 \) if the evolution is one standard driven by the cosmological constant, and in this case we do not notice today the ”presence” of \( \Gamma \) (the same happened when \( \Theta = 1 \), evolution driven by dust). So, we can say that \( \Gamma(0) \approx 0 \) and today the cosmic fluid is one conserved (delicate thermal equilibrium between the bulk and the boundary today?). We note that in the quintessence zone \( \Theta = 1 (\omega = 0) \) and \( \Theta = 0 (\omega = -1) \) we have energy transference from the boundary to the bulk (\( \Gamma > 0 \)). If the future is driven for phantom dark energy, then we have \( \Gamma < 0 \) plus a singularity and in this case we will have energy going from the bulk to the boundary and the temperature of the boundary (\( T_H = H/2\pi \)) will increase and we ask, what will happen when the singularity is nearby?, super hot boundary (\( T_H \rightarrow \infty \)) and bulk frozen (\( T \rightarrow 0 \)? We inspect now the phantom zone (\( \Theta < 0 \rightarrow \Gamma < 0 \)).

The Hubble temperature is given by \( T_H(t) = H(t)/2\pi \) and we do the following exercise (in order to have a feeling): by using [8] with \( \Theta_{eff} < 0 \) (phantom zone) we obtain

\[ H(a) = H_0 \left( \frac{a}{a_0} \right)^3 |\Theta_{eff}|^{1/2}, \quad (27) \]

so that the boundary temperature is

\[ T_H(a) = T_H(0) \left( \frac{a}{a_0} \right)^3 |\Theta_{eff}|^{1/2}. \quad (28) \]

For radiation (\( \Theta = 4/3 \) and \( \Theta_{eff} = (4/3) (1 - \alpha) / (1 - 4\alpha/3) \)) we have

\[ T_r(a) = T_r(0) \left( \frac{a}{a_0} \right)^{-(1-\alpha)/(1-4\alpha/3)}, \quad (29) \]

and we do now \( T_H(\bar{a}) = T_r(\bar{a}) \) (equilibrium) so that

\[ \bar{a} = a_0 \left( \frac{T_r(0)}{T_H(0)} \right)^{1/(\Delta+3|\Theta_{eff}|^{1/2})}, \quad (30) \]
where \( \Delta = (1 - \alpha) / (1 - 4\alpha / 3) \). Today \( T_r (0) \sim 3K \) and \( T_H (0) \sim 0 (10^{-30}) K \) and in this case we have thermal equilibrium at \( \bar{a} \sim 10^{12} a_0 \). We note that \( T_r (\bar{a}) \sim 10^{-12} T_r (0) \) and \( T_H (\bar{a}) \sim 10^{18} T_H (0) \sim 10^{-12} K \). The time \( (t_{eq}) \) at which we obtain this thermal equilibrium is given by

\[
t_{eq} - t_0 = (t_s - t_0) \left(1 - \frac{T_H (0)}{T_{CMB} (0)}\right)^{1/\Omega},
\]

where \( \Omega = 1 + 2/3 |\Theta_{eff}| \). So, we have \( t_{eq} \sim t_s \), and we have thermal equilibrium very near to the singularity.

Now, by adding the quantum corrections \( 3\gamma H^4 \) in (1) and \( \gamma H^4 \) in (2) \([4,6]\) it is straightforward to obtain

\[
3H^2 = \frac{1}{1 - \alpha} \left(1 - \frac{3}{2}\beta\Theta\right) \rho + 3\gamma \frac{\gamma}{1 - \alpha} H^4,
\]

and

\[
\dot{H} + H^2 = -\left(\frac{3}{2}\tilde{\Theta} - 1\right) H^2
\]

where

\[
\tilde{\Theta} = \Theta_{eff} \left(1 - \frac{\gamma}{1 - \alpha} H^2\right),
\]

and \( \Theta_{eff} \) is given by \([5]\). So, the new effective scheme becomes

\[
\dot{\rho} + 3\tilde{\Theta} H \rho = 0,
\]

and the new bare scheme is

\[
\dot{\rho} + 3\Theta H \rho = \tilde{\Gamma},
\]

where
\[
\tilde{\Gamma} = 3\Theta \left[ 1 - (1 - \alpha) \left( 1 - \frac{3}{2}\beta\Theta \right)^{-1} \left( 1 - \frac{\gamma}{1 - \alpha}H^2 \right) \right] H\rho. \tag{37}
\]

If we consider \( \alpha = (3/2)\beta \) we obtain

\[
\tilde{\Gamma} = 3\alpha\Theta \left( \frac{1 - \Theta}{1 - \alpha\Theta} \right) \left[ 1 + \frac{\gamma}{\alpha} \frac{H^2}{1 - \Theta} \right] H\rho \tag{38}
\]

and at difference of \( \Gamma (\Theta; t) \) (23) we can see that only \( \Theta = 0 (\omega = -1) \) implies \( \tilde{\Gamma} = 0 \). So, when the evolution is driven by dust we have not thermal equilibrium if we consider the quantum correction.

Now, from Eqs. (32,34) it is easy to obtain the following implicit solution for the Hubble parameter

\[
\frac{3}{2} \Theta_{\text{eff}} (t - t_0) = \frac{1}{H} - \frac{1}{H_0} + \frac{1}{2\delta} \ln \left( \frac{H - \delta}{H_0 - \delta} \right) \left( \frac{H_0 + \delta}{H + \delta} \right), \tag{39}
\]

where we are defined

\[
\delta = \sqrt{\frac{(1 - \alpha)}{\gamma}}. \tag{40}
\]

If we consider \( \Theta_{\text{eff}} < 0 \), Eq. (40) can be written in the form

\[
\frac{1}{H} - \frac{3}{2}\Theta_{\text{eff}} (t_s - t_0) = \frac{1}{2\delta} \ln \left( \frac{1 + \delta/H}{1} \right) \left( \frac{1 - \delta/H_0}{1 + \delta/H_0} \right), \tag{41}
\]

where \( t_s \) is given in (22) by doing \( \Theta_{\text{eff}} = -|\Theta_{\text{eff}}| \), and it is easy to verify when \( t = t_s \) there is not future singularity for \( H \) (in fact, when \( t = t_s \) Eq. (41) is satisfied only for \( H = H_s < H_0 \)). Finally, in accord to Eq. (32) we can visualize the auto-accelerated solution \( \rho = 0 \Rightarrow H = \sqrt{(1 - \alpha)/\gamma} \) and in this case \( \tilde{\Gamma} = 0 \) too and under phantom evolution we have \( \tilde{\Gamma} \neq 0 \).

IV. FINAL REMARKS

We have studied the entropic model given in (1,2) by doing a bare/effective description of the equation of state for the cosmic fluid. The effective description have showed to be an
usual, that is, only by redefining the parameter of the equation of state we find a standard FLRW cosmology. In the bare case, we find sign changes in the term which accounts the state of thermal equilibrium, and only when the evolution is driven by dust or cosmological constant we have thermal equilibrium. We have showed that during a phantom evolution it is possible to reach the thermal equilibrium between the bulk (radiation) and the boundary (Hubble horizon) in the nearby of the singularity. Finally, by adding a quantum correction to the modified Friedmann ‘equations only cosmological constant can drive the universe on thermal equilibrium and the future singularity, which it is present in absence of the quantum corrections, is avoided. So, under the scope of the entropic cosmology is it possible to have a phantom-free evolution.

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