Student’s mistake in algebraic fraction: an analysis using AVAE categories

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Abstract. The role of algebra in mathematics is important because it related to other materials such as geometry, calculus, basic matrices, trigonometry, statistics, vectors, and other mathematical studies. However, students’ mistakes in solving algebraic fractions are often found due to the fact that students’ are lacking in understanding the algebra and arithmetic. Therefore, in this research, researchers interested in analyse some students’ mistake by AVAE categories in the topic of algebraic fraction. The samples were 30 junior high school students. The data were collected by testing of simplifying algebraic fraction. The data were used to analyse the students’ error using AVAE (Arithmetic, variables, algebraic expression, and equal sign) categories and eight of them were interviewed. The results showed that students have difficulty in understanding arithmetic and algebraic expressions.

1. Introduction
Algebra became a significant field in mathematics learned from elementary schools to universities. It studies concepts or principles of simplification and problem solving using certain symbols or letters[1]. The role of algebra in mathematics is important because it relates to other materials such as geometry, calculus, basic matrices, trigonometry, statistics, vectors, and other mathematical studies[2,3]. For example, algebraic problems can be solved geometrically by first visualizing functions or equations or otherwise problems that exist in geometry can be solved algebraically. Therefore, students inevitably should have a good understanding of the algebraic concept to be able to follow easily the learning in each field of mathematics that is related to that knowledge.

Algebra junior high school consists of operations (subtraction, addition, division, multiplication) of algebraic forms and algebraic fractions. In junior high school, algebra students still face difficulties in understanding algebra, students are still fixated on numerical answers even though in the algebra the focus is on derivative procedures, relationships and expressions in general[4]. The lacking comprehension of algebraic concepts will consequently cause students hard to solve algebraic problems.

The difficulties of students in algebra learning in some countries experience the same problems, especially in Indonesia. Jupri, Drijvers, and Van den Heuvel-Panhuizen [2] report the difficulties that occur on the students because students have difficulty in changing mathematical models to real situations and vice versa. Other difficulties of concern, the difficulty of students in understanding algebraic expressions, applying arithmetic operations in numerical and algebraic expressions, understanding the differences in the meaning of the sign “=”, and understanding variables[2,5,6].
In school area, the most crucial student difficulties of algebraic materials are found in algebraic fractions. Based on Makonye & Khanyile's study, it was reported that 98.6% of students do errors in simplifying algebraic fractions in the pre-test questions [7]. Further research to identify students’ mistakes and misconceptions in simplifying algebraic fractions, and to identify the extent material to which teacher teaching could overcome identified errors [3]. In this study, researchers interested to identifying students’ errors in simplifying algebraic fraction by AVAE (Arithmetic, Variable, Algebraic Expressions, and Equal Sign) categories in the topic on Algebraic Fraction.

there are five categories of student difficulties in Algebra learning, namely ARITH, VAR, AE, EQS and MATH. However, researchers only examines four aspects which related to student difficulties in conducting arithmetic operations (ARITH), understanding algebraic notation (VAR), algebraic expressions (AE) and identifying the difference of “=” (EQS) [2].

Difficulties in the ARITH include operations, rules, and properties. Operations were grouped into two subcategories: mistakes in carrying out operations on numbers, and mistakes in arithmetic operations on algebraic expressions. Then, for rules, students made mistakes in applying either numerical or algebraic expressions. Properties were three subcategories as follows: (i) misapplication of the commutative property in calculating a division in numerical expressions, (ii) misuse of the distributive property of a multiplication over an addition, and (iii) an improper use of the additive inverse property in solving problem. Difficulties in VAR contain variables as a varying quantity and variable as an unknown. In the AE, the difficulties were parsing obstacle, expecting answer obstacle, lacking of closure obstacle, and lacking of gestalt view. In the EQS, students wrote an error notation as a result of a combination of operations and ignored the equal sign and applied an incorrect simplification on algebraic expression [2].

2. Methods
This research is utilizing a qualitative method by case study approach due to the capability of method to capture a variety of qualitative information. The subjects were 30 students form at one of junior high school located in Bandung. The research used a set of test question to identify type of students’ error including developing all items and making some modifications. Nine items were defined as the final instrument to analyse competency with algebraic fractions. The test was conducted using a pencil and paper instrument in which calculators were not allowed. Students were encouraged to show all of their work. The test were designed to measure concept knowledge and computational fluency. The students finished the algebraic fractions test within a fifty-minute period. We analysed each test item. The test items were categorised and errors were analysed by AVAE (Arithmetic, Variable, Algebraic Expression and Equal Sign). We also interviewed the respondent. It was conducted if the students failed to answer correctly in their written test.

3. Result and Discussion
This section shares the main result of this research which deals with the following themes: what errors students make in solving algebra problems.

Based on the findings in the test, the researcher summarizes them in the Table 1. The Table 1 shows the percentage of student’s error for each category. Within each category there were several common errors. The Table 1 shows 64.73% students made mistakes in ARITH category, 1.5% student made mistakes in VAR category, 18.5% students made mistakes in AE category and 4.4% student made mistakes in EQS category. The following is a question-by-question analysis from the algebraic fraction test.
Table 1. Percentage of student’s errors

| Category | Subcategory | Student’s errors (30*9 = 270 cases) |
|----------|-------------|--------------------------------------|
| ARITH    | 1 (i)       | 19 (7.03%)                           |
|          | (ii)        | 60 (22.2%)                           |
|          | 2 (i)       | 74 (27.4%)                           |
|          | 3 (i)       | 0                                    |
|          | (ii)        | 22 (8.1%)                            |
|          | Total       | 175 (64.73%)                         |
| VAR      | 4 (i)       | 3 (1.1%)                             |
|          | (ii)        | 1 (0.4%)                             |
|          | Total       | 4 (1.5%)                             |
| AE       | 5 (i)       | 50 (18.5%)                           |
|          | 6 (i)       | 0                                    |
|          | 7 (i)       | 0                                    |
|          | 8 (i)       | 0                                    |
|          | Total       | 50 (18.5%)                           |
| EQS      | 9 (i)       | 3 (1.1%)                             |
|          | (ii)        | 9 (3.3%)                             |
|          | Total       | 12 (4.4%)                            |
| Total    |             |                                      |

3.1 The errors of ARITH

To illustrate the errors of ARITH, we selected three examples to check the subcategory of ARITH related to operation, rule and properties. Based on the result of tests and interviews some students answered \( \frac{5}{a} + \frac{20}{a} = \frac{25}{a^2} \) and \( \frac{25}{2a} \). From the answers, we observed that students made mistakes in determining the denominator. In fact, the fractions have like denominator, so they can add without rewriting. However, students add or multiply the denominator due to the lacking of understanding the concept of the rules of adding algebraic fractions.

The concept of the rules of adding algebraic fractions is no different from ordinary fractions which are algebraic fraction must attention to the denominator as same as fractions. So if like denominator exist, numerator will be operated only (see eq. 1).

\[
\frac{a}{cx} + \frac{b}{cx} = \frac{a+b}{cx}
\]  

However, students forget the concept and made errors in operating algebraic fractions. Algebra and arithmetic have a great deal in common. For example, to add or subtract arithmetic or algebraic fractions with unlike denominators, you must first find a common denominator.

Figure 1 shows that student understands how to operate two algebraic fraction term and then, find a common denominator, but how to find the lowest common denominator is troublesome. Consequently, the student made an error of ARITH when simplifying it. The student does not follow the rules of order arithmetical operation in algebraic fraction. Students should divide it with the same number. But, students don't do it. Based on the interviews, the student said she divide 75 and 90 then divide 72 by 6 (results of division 75 and 90) to get the answer.

Find the sum of \( \frac{5b}{6a} \) and \( \frac{12}{15b} \) = ⋯
The researcher selects two student answers in this problem as shown in Figure 2. Figure 2 shows that students do not fully understand the rules of division in algebraic fractions. In the first part, students directly divide the numerator by the numerator and the denominator by the denominator without following the rules in the division of algebraic fractions. In algebra, divided by multiplying by the reciprocal (eq. 2).

\[
\frac{a}{cx} \div \frac{b}{dy} = \frac{a}{cx} \times \frac{dy}{b}, \quad \frac{b}{dy} \text{ and } \frac{dy}{b} \text{ are reciprocals}
\]  

(2)

In the second part, students know the rule. But make an error in determining the denominator. Based on the results of interviews students assume that if algebraic fraction is like denominator then just operate without rewriting.

3.2. The error of variable (VAR)

Based on the result of written test in Figure 3, the students’ works show that there is an error in understanding the meaning of variables. The students ignored variables in solving the problem of algebraic fractions that should equate the denominator also apply to variables not just numbers. As Linchevski & Herscovics [8] research revealed that some studies show that students often secrete variables or symbols when performing an algebraic term operation as well as ignoring variables and only focusing on the summation operation. The error of algebraic expressions (AE).

\[
\frac{8}{5xy} - \frac{2}{2x^2} = \ldots
\]  

Figure 1. Work of student with errors of ARITH

Figure 2. Work of student with errors of ARITH

Figure 3. Work of student with errors of variables
Algebraic expressions (AE) consist of four categories of errors namely parsing obstacle, expected answer obstacle, lack of closure obstacle and lack of gestalt view. To illustrate how errors are made by students, we present example of the subcategories that have the most errors.

Malik wrote \[
\frac{12x}{5x+10} \div \frac{4}{5} = \frac{12x+4}{(5x+10)+5} = \frac{3x}{x+2}
\] is Malik's answer correct? Justified your answer.

The question has several possible errors, both errors in variables, arithmetic and algebraic expression. But, we concern about error of algebraic expression. From these results, it seems that students had only learned the algebra algorithmically without a conceptual understanding of algebraic expression. For instance, that students made mistake simplifying \(5x + 10\) into \(15x\) (parsin obstacle) and divide \(15x\) by \(5\) into \(3x\) (lack of closure obstacle).

3.3. The error of equal sign (EQS)

The students have trouble to solve this problem. In written test many students do not solve completely. We have confirmed by interviews they said, "The task is difficult because we are forgetting how to solve".

Solve this problem by looking for the value of \[\frac{2x+7}{6} - \frac{2x-9}{10} = 3\]. We present one of the students' answers.

The other situation the student gets the incorrect answer. Based on interviews' result, we findings the student assumes three is a value of the equation. Theirs assumes show that student gets misconception about the equal sign. So, the student think how to find the result of equation is 3.

4. Conclusion

The findings of this research showed that the most student’s error were ARITH(Arithmetic) and AE (algebraic expression) categories. In the ARITH category, students did not understand the concepts of fractions so that they felt difficult to solve the algebraic fraction problems. The students were confused about the concept of fraction in addition and subtraction which signed students did them as same as did fraction multiplication. Therefore, students have difficulty in solving algebraic fraction problems. Moreover, this case causes students experience many errors because students do not understand algebraic expressions (AE). The students are still needed to be improve on the concept of ordinary number operations. Hence, when they face an algebraic problem, the students do not determine on the differences algebra and numbers. Thus, it causes the students had mistaken in doing operation. Therefore, the findings of this research can be as a reference for teachers to find alternative ways to solve students’ problem in algebraic fraction.

5. References

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