EXPERIMENTAL CONSTRAINTS ON $\gamma$-RAY PULSAR GAP MODELS AND
THE PULSAR GeV TO PULSAR WIND NEBULA TeV CONNECTION

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ABSTRACT

The pulsar emission mechanism in the gamma ray energy band is poorly understood. Currently, there are several models under discussion in the pulsar community. These models can be constrained by studying the collective properties of a sample of pulsars, which became possible with the large sample of gamma ray pulsars discovered by the Fermi Large Area Telescope. In this paper we develop a new experimental multi-wavelength technique to determine the beaming factor ($f_0$) dependence on spin-down luminosity of a set of GeV pulsars. This technique requires three input parameters: pulsar spin-down luminosity, pulsar phase-averaged GeV flux, and TeV or X-ray flux from the associated pulsar wind nebula (PWN). The analysis presented in this paper uses the PWN TeV flux measurements to study the correlation between $f_0$ and $\dot{E}$. The measured correlation has some features that favor the Outer Gap model over the Polar Cap, Slot Gap, and One Pole Caustic models for pulsar emission in the energy range of 0.1–100 GeV, but one must keep in mind that these simulated models failed to explain many of the most important pulsar population characteristics. A tight correlation between the pulsar GeV emission and PWN TeV emission was also observed, which suggests the possibility of a linear relationship between the two emission mechanisms. In this paper we also discuss a possible mechanism to explain this correlation.

Key words: gamma rays: general – pulsars: general – radiation mechanisms: non-thermal – stars: neutron

1. INTRODUCTION

Currently, there are several GeV pulsar models being discussed in the pulsar community. These models can be constrained either by studying individual pulsars in detail or alternatively using the collective properties of a sample of pulsars. The large sample of pulsars discovered by the Fermi Large Area Telescope (Fermi-LAT) provides a good place to study the collective properties of GeV pulsars.

The GeV luminosity as a function of other pulsar parameters is a fundamental quantity which models must predict. However, the potential utility of pulsar luminosity is limited by two factors: inability to measure the beaming factor ($f_0$) (also called the beam correlation factor) (Watters et al. 2009) and inaccurate distance measurements. The factor $f_0$ provides the correction to extrapolate the observed phase-averaged flux from the Earth line of sight to the full sky flux for a given beam shape. It is an essential factor needed to convert observed fluxes to luminosity:

$$L_\gamma = 4\pi d^2 f_0 F,$$

where $L_\gamma$ is the pulsar luminosity, $d$ is the distance to the pulsar from Earth and $F$ is the phase averaged flux measured at Earth. Since $f_0$ is a model dependent parameter, luminosity calculations are also model dependent. Therefore, one option to constrain GeV pulsar emission models is to use the collective properties of the luminosity distribution, but the uncertainty on distance measurements degrades the accuracy of the luminosity distribution. This issue can be resolved by studying the ratio of the flux from pulsars to that of their associated pulsar wind nebulae (PWNs), which yields a distance-independent parameter.

This paper uses the ratio between a pulsar’s GeV flux and the TeV flux from its associated PWN as the first application of this method. Using this ratio we obtain the dependence of $f_0$ on spin-down luminosity $\dot{E}$ for a sample of pulsars. This allows us to compare the experimentally measured dependence of $f_0$ on $\dot{E}$ with the theoretical expectation of four $\gamma$-ray pulsar gap models (Pierbattista et al. 2012).

2. THE SAMPLE OF PULSARS AND THEIR ASSOCIATED PWNs

Recently the Fermi-LAT produced its second pulsar catalog (Abdo et al. 2013) with 117 high-confidence $\gamma$-ray ($\geq 0.1$ GeV) pulsars. In addition, TeV $\gamma$-ray observatories, such as Milagro, VERITAS, and H.E.S.S., have measured TeV fluxes coming from PWNs. From a literature survey we found 14 GeV pulsars in the Fermi-LAT pulsar catalog for which the associated PWNs were also measured by TeV observatories.

Tables 1 and 2 summarize the properties of these 14 objects. This analysis uses the values of spin-down luminosity ($\dot{E}$), distance to the pulsar ($d$), and phase averaged flux in the energy range of 0.1–100 GeV ($G_{100}$) reported in the Fermi-LAT Second Pulsar Catalog.

2.1. Discussion of TeV PWN Measurements

The integrated energy fluxes around 35 TeV of the associated PWNs ($F_{\text{TeV}}$) are listed in column 3 of Table 2. All Milagro TeV measurements in this column are derived from Table 1 in Abdo et al. (2009a). Hereafter, we will refer to Abdo et al. (2009a) as the Milagro 0FGL search. In that publication the Milagro collaboration performed a targeted search for galactic sources in the Fermi Bright Source List, which is also known as 0FGL (Abdo et al. 2009b). The Milagro 0FGL search found TeV emission coinciding with 14 Fermi bright sources. Among these 14 sources, 9 have spatial associations with pulsars in the Fermi-LAT Second Pulsar Catalog. The Milagro 0FGL search paper reported the
### Table 1
Properties of a Sample of GeV Pulsars Cataloged in the Fermi-LAT Second Pulsar Catalog

| Pulsar Name | Association | $\log_{10} \left( E\left(10^{14} \text{ erg s}^{-1}\right) \right)$ | $G_{100}$ | Distance |
|-------------|-------------|-------------------------------------------------|----------|----------|
| J0007+7303  | CTA 1       | 35.65                                           | 40.1 ± 0.4 | 1.4 ± 0.3 |
| J0534+2200  | 0FGL J0534.6+2201\(^a\) | 38.64                                           | 129.3 ± 0.8 | 2.0 ± 0.5 |
| J0631+1036  | 0FGL J0631.8+1034\(^a\) | 35.24                                           | 4.7 ± 0.3   | 1.0 ± 0.2 |
| J0633+1746  | 0FGL J0634.0+1745\(^a\) | 34.52                                           | 423.3 ± 1.2 | 0.2 ± 0.1 |
| J0835–4510  | Vela        | 36.84                                           | 906 ± 2     | 0.29 ± 0.02 |
| J1420–6048  | K3 in Kookabura | 37.01                                           | 17.0 ± 1.4  | 5.6 ± 0.9 |
| J1509–5850  | MSH 15–52   | 35.71                                           | 12.7 ± 0.7  | 2.6 ± 0.5 |
| J1833–1034  | G21.5–0.9   | 35.73                                           | 5.9 ± 0.5   | 4.7 ± 0.4 |
| J1907+0602  | 0FGL J1907.6+0602\(^a\) | 36.45                                           | 25.4 ± 0.6  | 3.2 ± 0.3 |
| J1958+2846  | 0FGL J1958.1+2848\(^a\) | 35.53                                           | 9.1 ± 0.4   | <18.5    |
| J2021+3651  | 0FGL J2020.8+3649\(^a\) | 36.53                                           | 49.4 ± 0.8  | 10.0 ± 0.4 |
| J2021+4026  | 0FGL J2021.5+4026\(^a\) | 35.06                                           | 95.5 ± 0.9  | 1.5 ± 0.4 |
| J2032+4127  | 0FGL J2032.2+4122\(^a\) | 35.44                                           | 10.6 ± 0.6  | 3.7 ± 0.6 |
| J2229+6114  | 0FGL J2229.0+6114\(^a\) | 37.35                                           | 25.3 ± 0.4  | 0.8 ± 0.13 |
| ...         | Boomerang   | ...                                             | ...        | ...      |

Note. $G_{100}$ is the phase averaged flux of the pulsar GeV emission in the 0.1–100 GeV energy band.

### Table 2
TeV Flux of the Associated PWNs of a Sample of GeV Pulsars Cataloged in the Fermi-LAT Second Pulsar Catalog

| Pulsar Name | Association | $F_N \left(10^{-14} \text{ TeV} \text{ s}^{-1} \text{ cm}^{-2}\right)$ |
|-------------|-------------|-------------------------------------------------|
| J0007+7303  | CTA 1       | 1.4 ± 0.9 \(^a\)                                |
| J0534+2200  | 0FGL J0534.6+2201\(^a\) | 5.4 ± 0.3 \(^b\)                                |
| J0631+1036  | 0FGL J0631.8+1034\(^a\) | 1.5 ± 0.4 \(^a\)                                |
| J0633+1746  | 0FGL J0634.0+1745\(^a\) | 1.2 ± 0.4 \(^a\)                                |
| J0835–4510  | Vela        | 16.4 ± 9 \(^d\)                                 |
| J1420–6048  | K3 in Kookabura | 5.4 ± 1.3 \(^c\)                               |
| J1509–5850  | MSH 15–52   | 6.2 ± 0.9 \(^f\)                                |
| J1833–1034  | G21.5–0.9   | 0.99 ± 0.6 \(^e\)                               |
| J1907+0602  | 0FGL J1907.6+0602\(^a\) | 3.9 ± 0.5 \(^c\)                                |
| J1958+2846  | 0FGL J1958.1+2848\(^a\) | 1.1 ± 0.3 \(^b\)                                |
| J2021+3651  | 0FGL J2020.8+3649\(^a\) | 3.6 ± 0.3 \(^b\)                                |
| J2021+4026  | 0FGL J2021.5+4026\(^a\) | 1.2 ± 0.3 \(^b\)                                |
| J2032+4127  | 0FGL J2032.2+4122\(^a\) | 2.1 ± 0.3 \(^b\)                                |
| J2229+6114  | 0FGL J2229.0+6114\(^a\) | 2.3 ± 0.4 \(^b\)                                |
| ...         | Boomerang   | 1.3 ± 1.9 \(^h\)                                |

Notes. $F_N$ is the integrated energy flux of the PWN TeV emission in the 35.4–35.5 TeV energy band.

\(^a\) VERITAS measurement. Energy flux derived by extrapolating the SED. Reference: Aliu et al. (2013).

\(^b\) Milagro measurement. Reference: Abdo et al. (2009a).

\(^c\) H.E.S.S. measurement. Reference: Aharonian et al. (2006a).

\(^d\) H.E.S.S. measurement. Reference: Aharonian et al. (2006b).

\(^e\) H.E.S.S. measurement. Energy flux derived by extrapolating the SED. Reference: Aharonian et al. (2006c).

\(^f\) H.E.S.S. measurement. Reference: Aharonian et al. (2005).

\(^g\) H.E.S.S. measurement. Energy flux derived by extrapolating the SED. Reference: H.E.S.S. Collaboration et al. (2007).

\(^h\) VERITAS measurement. Energy flux derived by extrapolating the SED. Reference: Aliu et al. (2013).

The authors argue that the flux calculated at 35 TeV has the least dependence of the calculated flux on the true spectrum. Because Milagro does not report the full SED for these pulsars but only the differential photon flux at 35 TeV, we calculated the integrated energy flux over a 1 TeV band around 35 TeV with a SED of $E^{-2.6}$ and used the Milagro flux uncertainties.

The Milagro 0FGL search paper mentioned that TeV emission might come from the pulsar and/or from the associated PWN. However, it is very unlikely to get a significant contribution from the pulsar to the TeV flux measured by Milagro. The best example of this is the Crab pulsar, which is the brightest TeV object measured by Milagro. The VERITAS Collaboration (VERITAS Collaboration et al. 2011) observed pulsed $\gamma$-rays in the energy range of $\sim$100–$\sim$200 GeV. The measured energy spectrum is well described by a simple power law, without a cut-off, by:

$$\frac{dN}{dE} = \left(4.2 \pm 0.6 \text{ stat} \pm 2.4 \text{ syst}\right) \times 10^{-11} \left(\frac{E}{150 \text{ GeV}}\right)^{-3.8 \pm 0.5 \text{ stat} \pm 0.2 \text{ syst}} \text{ TeV}^{-1} \text{ cm}^{-2} \text{s}^{-1} \text{ mrad}^{-2} \text{ sr}^{-1} \text{ day}^{-1}$$

An extrapolation of this energy spectrum gives a differential photon flux of $4.2 \times 10^{-20}$ photons TeV$^{-1}$cm$^{-2}$s$^{-1}$ at 35 TeV. This is 0.03% of the TeV flux observed coincident with the Crab pulsar by Milagro. In addition, a theoretical model proposed in Aharonian et al. (2012) predicts a sharp cut-off below $\sim$500 GeV, so the extrapolated flux at 35 TeV from the pulsar might be even lower. These considerations lead us to conclude that the TeV emissions observed coincident with pulsars come predominantly from their associated PWNs.

We performed a literature search for PWNs measured by Air Cherenkov telescopes in the TeV band. We found H.E.S.S. SEDs for five other PWNs, which are also listed in Tables 1 and 2: Crab, Vela, K3 in Kookabura, MSH 15–52, and G 21.5–0.9. In order to be consistent with the Milagro measurements, we integrated the energy flux within a 1 TeV energy band around 35 TeV using the H.E.S.S. SEDs. The SEDs of the Crab,
3. METHOD

The ratio of the pulsar GeV luminosity \( L_P \) to the luminosity of the associated PWN \( L_N \) can be written in terms of the corresponding pulsar and PWN flux:

\[
\frac{L_P}{L_N} = \frac{4\pi d_P^2 f_0 G_{100}}{4\pi d_N^2 F_N} = f_0 \times \frac{G_{100}}{F_N}.
\]  

(3)

Taking their ratio cancels the distance but retains the beaming factor \( f_0 \), which can be written as:

\[
f_{0i} = \frac{L_P}{L_N} r
\]  

(4)

where

\[
r = \frac{F_N}{G_{100}},
\]  

(5)

which is the observed flux ratio. This relation is a mathematical identity which is valid for each individual pulsar and its associated PWN. This identity, however, cannot be used to derive \( f_{0i} \) for a given pulsar, because \( L_P \) is not measurable without \( f_{0i} \). However, we can extract information on the \( \dot{E} \) dependence of \( f_{0i} \) for a selected group of pulsars by using models that predict the \( \dot{E} \) dependence of \( L_P \) and \( L_N \).

Many high energy pulsar models (e.g., Harding 1981; Harding &Muslimov 2002, 2003; Muslimov & Harding 2003; Takata et al. 2010) predict a power law relationship between \( L_P \) and \( \dot{E} \):

\[
L_P = k_P \cdot \dot{E}^q
\]  

(6)

For a given pulsar, \( k_P \) is independent of \( \dot{E} \), but depends on other pulsar properties such as the angle between the direction of the magnetic dipole axis and the rotation axis. Both \( k_P \) and the power \( q \) are model-dependent. Later in the paper, we will discuss the implications of different choices of \( q \).

For PWNs Mattana et al. (2009) discussed the correlations between \( \dot{E} \) and PWN luminosity in TeV and X-ray energy bands. Using H.E.S.S. measurements they showed that PWN TeV luminosity is not correlated with \( \dot{E} \). This observation is consistent with the theoretical expectation that TeV photons are generated by the accumulated high-energy electrons in PWNs (Mattana et al. 2009). Therefore, for a given ensemble of GeV pulsars we can choose a characteristic PWN TeV luminosity \( k_N \), independent of \( \dot{E} \). Mattana et al. (2009) also showed that the X-ray luminosity versus \( \dot{E} \) distribution can be fitted by a power law model. Therefore we can generalize the X-ray luminosity versus \( \dot{E} \) distribution and TeV luminosity versus \( \dot{E} \) distribution as:

\[
L_N = k_N \cdot \dot{E}^m,
\]  

(7)

where \( m = 0 \) for the TeV luminosity versus \( \dot{E} \) distribution.

Both of these energy bands are good candidates for applying our method. The work presented in this paper uses PWN TeV luminosity. Another analysis with PWN X-ray luminosity is in progress.

When we combine the model expectations in Equation (6) for pulsars and Equation (7) for PWNs with Equation (4), we obtain \( f_{0i} \) for a specific pulsar \( i \):

\[
f_{0i} = \left( \frac{k_P}{k_N} \right) \cdot r_i \cdot \dot{E}_i^{(q-m)}.
\]  

(8)

In log–log space we can rewrite this equation as:

\[
\log f_{0i} = \log \left( \frac{k_P}{k_N} \right) + \log r_i + (q-m) \log (\dot{E}_i).
\]  

(9)

In this equation \( r_i \) and \( \dot{E}_i \) are measurable quantities, but the coefficients \( k_P \) and \( k_N \) are unknowns and vary pulsar to pulsar. In this paper we do not intend to measure the \( f_{0i} \) of individual pulsars. Instead we intend to obtain the \( f_{0i} \) dependence on \( \dot{E} \) for an ensemble of GeV pulsars using \( k_P \) and \( k_N \), where \( k_P \) and \( k_N \) are typical values of \( k_P \) and \( k_N \) appropriate for our ensemble of pulsars. We make this explicit by defining \( d_i \) as the difference between typical values and the pulsar-dependent constants:

\[
d_i = \log \left( \frac{k_P}{k_N} \right) - \log \left( \frac{\dot{k}_P}{\dot{k}_N} \right).
\]  

(10)

We can rewrite Equation (9) with the parameter \( d_i \) as,

\[
\log f_{0i} - d_i = \log \left( \frac{\dot{k}_P}{\dot{k}_N} \right) + \log r_i + (q-m) \log (\dot{E}_i).
\]  

(11)

Although \( d_i \) is not measurable for individual pulsars, we can use this expression to obtain the dependence of an estimate of

Vela, and MSH 15–52 PWNs were measured in the energy ranges of 440 GeV–40 TeV, 550 GeV–65 TeV, and 250 GeV–40 TeV, respectively. Therefore, their integrated energy flux around 35 TeV can be obtained without any extrapolation. However, the SEDs of K3 in Kookabura and G 21.5–0.9 PWNs were measured in the energy ranges of 200 GeV–25 TeV and 150 GeV–5 TeV, respectively. Therefore, their integrated energy fluxes obtained around 35 TeV by extrapolation of their SEDs which might be an overestimate if there is a cutoff below 35 TeV.

VERITAS has published SEDs of the Boomerang and CTA 1 PWNs. In both cases the SEDs of these sources were measured in the energy range of 1–15 TeV. Therefore, as for K3 and G 21.5–0.9, the integrated energy flux obtained around 35 TeV by extrapolating the SEDs might be an over estimate, if there is a cutoff before 35 TeV. For all H.E.S.S. and VERITAS measured PWNs, errors on the integrated flux are estimated by a standard Gaussian Monte Carlo propagation of the uncertainties of the SED fit, with the 16th percentile as the lower error bar and the 84th percentile as the upper.

There are two independent measurements for both Boomerang and the Crab. In each case, the measurements agree within experimental errors and differ by less than a factor of two. Both measurements for these PWNs are shown in the following plots but we use their weighted average when doing fits, which does not alter any of the conclusions in this analysis.
measurements contributes significantly to the luminosity error bars and that the extrapolated points ($\bar{E} = 35.7, 37.0, 37.3,$ and $37.5$) do not appear to be outliers. This distribution has a linear correlation coefficient of $0.09$. The small linear correlation coefficient suggests that $L_N$ is not correlated with $\bar{E}$ so we conclude that PWN TeV luminosity is not a function of $E$. Therefore, one has to expect zero slope for the best fit linear fit for the data points. The best fit linear fit for our data points has a slope of $0.03 \pm 0.06$, which is consistent with zero.

In summary, we argue that the observations are consistent with the theoretical expectation of no $\bar{E}$ dependence in the PWN TeV luminosity, which we discussed in Section 3. Therefore, we use the model value $m = 0$ in Equation (8). We fit a constant to the log $L_N$ data, yielding $31.6 \pm 0.05$, and use this value for the model parameter log $\bar{k}_N = 31.6$.

5. GEV PULSAR MEASUREMENTS

Next we return to the power law model for the pulsed GeV emission $L_P$. We proceed with the analysis on the basis of $q = \frac{3}{2}$, because there are several high energy pulsar models which predict a power law index near $\frac{1}{2}$ (e.g., Harding & Muslimov 2002, 2003; Muslimov & Harding 2003; Takata et al. 2010), and further, because in this paper we compare our results with a sample of pulsars simulated using $L_P = k_P \bar{E}^{\frac{3}{2}}$ as an underlying model assumption. Later we will discuss the effect of $q$ on the best fit parameters in Equation (16).

Finally we can select a reasonable value of $\bar{k}_P$ by using Figure 9 of Abdo et al. (2013) which has an illustrative line for a $L_P = k_P \bar{E}^{\frac{3}{2}}$ model with an additional constraint of $f_\Omega = 1$. Taking a suitable point from the line, log $\bar{E} = 39$ and log $(L_P/f_\Omega)$ = 36, we find log $\bar{k}_P = 16.5$.

6. ANALYSIS

As a summary of Sections 4 and 5, we proceed with our analysis using model parameters $q - m = \frac{1}{2}$ and log$_{10}$ ($\bar{k}_P/\bar{k}_N$) = $-15.1$. With these model parameters we can rewrite Equation (12) as follows:

$$\log_{10} \bar{k}_N = \log_{10} \left( \hat{k}_P \bar{E}_i^{\frac{1}{2}} \right) - 15.1$$

The correlation between log$_{10}$ $\bar{k}_N$ = $y_i$ and log$_{10}$ $\bar{E}_i$ = $x_i$ is shown in Figure 3. It appears that above log $\bar{E} \approx 35$, this distribution has a linear correlation with

$$\hat{y} = (-11.04 \pm 1.13) + (0.28 \pm 0.03) \cdot x_i,$$

where

$$y_i = \log \bar{k}_N = \log f_\Omega = d_i = \log \left( \frac{k_P}{k_N} \right) + \log \bar{n} + (q - m) \log \bar{E}_i,$$

$$x_i = \log \bar{E}_i.$$

Therefore, we can fit a phenomenological power law model for log $\bar{E} > 35$,

$$f_\Omega = a + b \cdot \log \bar{E},$$

Figure 1. PWN TeV luminosity vs. pulsar GeV luminosity normalized with respect to the beaming factor $(f_\Omega)$. Red squares are measured by Milagro. Red stars are measured by H.E.S.S. Green circles are measured by VERITAS. The two dotted lines have a slope of 1 but different arbitrary intercepts. The linear correlation coefficient is $R = 0.82$. The error bars are dominated by the distance measurement uncertainties.

$$f_{\Omega i} (\hat{f}_{\Omega i})$$ on $\bar{E}_i$, where

$$\log \hat{f}_{\Omega i} = \log f_{\Omega i} - d_i = \log \left( \frac{k_P}{k_N} \right) + \log \bar{n} + (q - m) \log \bar{E}_i.$$  

Thus $d_i$ is a correction factor between using typical values and the unknown pulsar-dependent values. We will estimate the magnitude of any such effects in Section 6.

This summarizes our method of extracting the $E$ dependence of $f_{\Omega i}$. We now proceed to discuss our choices for the constants $q$, $m$, $\bar{k}_P$ and $\bar{k}_N$ in more detail, and examine how well data supports these choices.

4. TEV PWN MEASUREMENTS

First we consider whether pulsar and TeV PWN luminosity exhibit sufficient correlation to make it worthwhile to work with their ratio. Figure 1 shows the correlation between $L_P/f_\Omega$ and $L_N$. The blue squares are PWNs measured by Milagro; the red stars by H.E.S.S., and the green circles by VERITAS. It appears that the PWNs measurements of the three experiments are generally consistent. Four of the TeV measurements ($L_N = 34.3, 35.0, 35.2, 35.8$) have been extrapolated from lower energy, but do not appear to be outliers to the general scatter of the distribution. The large error bars on this plot are due to the distance uncertainties. Also note that the TeV luminosity (y-axis) spans a larger range than the GeV (x-axis), making the error bars appear to be less important for TeV luminosities. The dotted lines (drawn to guide the reader’s eye) represent two lines of $L_N \propto \frac{L_x}{f_\Omega}$ with different arbitrary intercepts. Even though the error bars are large a reasonable correlation is obtained with a linear correlation coefficient of $R = 0.82$, which we judge to be sufficiently encouraging to proceed.

Next we examine the correlation between $L_N$ and $\bar{E}$. The $L_N$ versus $\bar{E}$ distribution for our PWN sample is shown in Figure 2. Again we note that the uncertainty on the distance
of removing $d_i$ from $\hat{f}_0$ (that is using the single pulsar-independent values of $k_p$ and $k_\Omega$). We see that although these deviations represent the information about specific pulsars compared to the overall model (trend), the deviations of individual pulsars from the trend are not so large as to invalidate the model extraction of the trend, as this scale ($\epsilon = 0.08$) is notably smaller than the variation of $\hat{y}$ across the range of $\dot{E}$.

7. DISCUSSION

7.1. Constraints on $\gamma$-ray Pulsar Gap Models

Recently, Pierbattista et al. (2012) studied four gamma ray pulsar acceleration models. They synthesized a pulsar population based on a radio emission model and four $\gamma$-ray pulsar gap models: Outer Gap (OG), Polar Cap (PC), Slot Gap (SG) and One Pole Caustic (OPC). Their model simulations of the correlation between $\dot{f}_0$ and $\dot{E}$ are shown in Figure 4. In all four model predictions, for pulsars with $\dot{E} > 10^{35}$ erg s$^{-1}$, $\dot{f}_0$ can be reasonably fit by a straight line in log–log space and the best fits give the following slopes: $m_{\text{PC}} = 0.05 \pm 0.23$, $m_{\text{SG}} = 0.003 \pm 0.004$, $m_{\text{OG}} = 0.12 \pm 0.01$ and $m_{\text{OPC}} = 0.26 \pm 0.007$.\(^3\) The slopes of the PC and SG models are consistent with zero, and the slope of the OPC is very small. The OG model is the only model that predicts a positive slope that is not consistent with zero. However, the small number of data points for the PC model have a large scatter and while the slope is consistent with zero, the uncertainty on the slope is much larger than for the other models.

We can now compare our data points from Figure 3 with the expectations from the four pulsar gap models. First we note some gross characteristics in comparing the simulations to our data points. Both the data and the models have $\log f_0$ near 0, and the scatter of individual simulated pulsars about their trend line is not grossly different from the scatter in the data, despite the fact that the data extraction used single individual values of $k_\Omega$ and $k_p$, while the simulation used the full information about individual simulated pulsars. This similarity is in agreement with the idea that the scatter due to $\epsilon$ is not so large as to lose all information about $f_0$. These simulated data points show a tighter distribution of the simulated radio-loud pulsars to the best fit line above $\log_{10}(\dot{E}) = 35$ while radio-quiet pulsars have a wider distribution. Especially in the OG model radio-quiet pulsars deviate to low $\dot{f}_0$ values for $\log_{10}(\dot{E}) < 35$. We note that the radio-quiet pulsars in our data also have a tighter distribution about the best fit line above $\log_{10}(\dot{E}) = 35$ and the radio-quiet pulsars with $\log_{10}(\dot{E}) < 35$ deviate to low $\dot{f}_0$ values. The authors of Pierbattista et al. (2012) also noted that the range of variation of their $f_0$ was less than that of their $L_p$ for the same range of $\dot{E}$, at least for the SG and OG models. We see a similar trend in our data.

The experimental $f_0$ versus $\dot{E}$ distribution has a non-zero slope of $b = 0.27 \pm 0.03$ for $\dot{E} > 10^{35}$ erg s$^{-1}$. This would tend to disfavor the PC, SG and OPC models, despite the OPC providing the best overall agreement with Fermi-LAT pulsars among the models considered. However, the slope in our data is over twice that expected by the OG model for $\dot{E} > 35$ erg s$^{-1}$. Below that value of $\dot{E}$, the expected correlation between $f_0$ and $\dot{E}$
A better sensitivity TeV survey such as HAWC promises to help our understanding of PWN visibility selection effects. Our estimates of \( f_{\Omega} \) may also be biased if there is an unexpected residual dependence of the PWN TeV luminosity on \( \dot{E} \) hidden in the TeV data scatter, or if there is hidden dependence on the selected TeV energy range. The extracted slope of the \( f_{\Omega} \) versus \( \dot{E} \) distribution linearly depends on any hidden slope in the PWN TeV luminosity versus \( \dot{E} \) distribution. The TeV energy range of a 1 TeV band around 35 TeV was chosen because of the Milagro data set. Ideally, we would prefer to do this study with a more uniform sample of PWN TeV energy fluxes obtained around the inverse Compton peak of each individual PWN. Using X-ray PWN luminosity instead of TeV luminosity would also offer complementary visibility selection effects, to allow assessment whether such selection effects are important.

This result also depends on the uncertainties of the theoretically predicted correlation between GeV pulsar luminosity and spin-down luminosity, \( L_p \propto E^{\delta} \), \( q = \frac{\gamma}{2} + \delta \). We made Figure 3 for \( L_p \propto E^{\gamma} \) (\( \delta = 0 \)), because it is natural in several pulsar models to have this relation (Abdo et al. 2010) and it is one of the underlying assumptions in Pierbattista et al. (2012). However, the slope of \( \log_{10}(f_{\Omega}) \) versus \( \log_{10}(\dot{E}) \) distribution depends linearly on \( \delta \), as seen in Equation (15). Therefore we can write that for \( \dot{E} > 10^{35} \) erg s\(^{-1}\) the slope of \( \log_{10}(f_{\Omega}) \) versus \( \log_{10}(\dot{E}) \) distribution is \( 0.28 \pm 0.03 \) stat. One can use this expression to determine the slope of \( \log_{10}(f_{\Omega}) \) versus \( \log_{10}(\dot{E}) \) distribution under different models. For example, Takata et al. (2010) discuss a

\[ \dot{E} \] for the OG model becomes more dispersed and the \( f_{\Omega} \) distribution for a given \( \dot{E} \) has a tail toward smaller \( f_{\Omega} \) values, especially for radio quiet pulsars. This feature is also consistent with the experimentally obtained \( f_{\Omega} \) versus \( \dot{E} \) distribution. The two radio quiet pulsars with \( \dot{E} \) below \( 10^{35} \) erg s\(^{-1}\) (PSR J0633+1746 and PSR J2021+4026) have smaller \( f_{\Omega} \) values compared to radio loud pulsars. By considering both features above and below \( \dot{E} \approx 10^{35} \) erg s\(^{-1}\), we can conclude that our data sample has some features which favor the OG model for pulsar emission in the energy range 0.1–100 GeV over the SG and OPC models, though even the OG model does not quantitatively match our measurements. However, we cannot reach any conclusions about the PC model.

The discrepancies with the models might be due to the systematic limitations or inadequacies of Pierbattista et al.’s (2012) simulations or biases in our data sample. As the authors of Pierbattista et al. (2012) mention, results of all four simulated models are lacking pulsars visible by Fermi-LAT with \( \dot{E} > 3 \times 10^{35} \) erg s\(^{-1}\) and characteristic age <100 kyr, and overpredict the number of low \( \dot{E} \) pulsars. Furthermore they also mention that the simulated OG model in particular fails to explain many of the most important pulsar population characteristics, including the distributions of period, characteristic age, \( L_p \) and \( \dot{E} \). This could certainly affect their \( f_{\Omega} \) versus \( \dot{E} \) distributions. Therefore, we cannot provide tight constraints using this synthetic pulsar population. However, a comparison of our results with an improved model simulation could provide tighter constraints.

Figure 4. Dependence of the beaming factor \( f_{\Omega} \) on spin-down luminosity \( \dot{E} \) for four models derived by Pierbattista et al. (2012) a sample of simulated pulsars. This plot was reproduced by M. Pierbattista with linear fits for pulsars above \( \dot{E} = 10^{35} \) erg s\(^{-1}\). For the original figure refer to Pierbattista et al. (2012). Red and green markers refer to the radio-loud and radio-quiet pulsars, respectively. Black lines refer to the best linear fits for the pulsars with \( \dot{E} > 10^{35} \) erg s\(^{-1}\). Slopes of the best fit lines are \( m_{\text{PC}} = 0.05 \pm 0.23 \), \( m_{\text{SG}} = 0.003 \pm 0.004 \), \( m_{\text{OG}} = 0.12 \pm 0.01 \) and \( m_{\text{OPC}} = 0.026 \pm 0.007 \).
high-energy emission from the outer gap that expects $L_P \propto \dot{E}^2$. Under this model the slope $\delta = 0.125$ and slope becomes $0.405 \pm 0.03_{\text{stat}}$. Another example is Muslimov & Harding (2003), which discuss a high-energy pulsar emission model that depends on the local magnetic field. In the low magnetic field scenario Muslimov & Harding (2003) expect $L_P \propto \dot{E}^2$, $\delta = -0.07$. Under this model the slope becomes $0.21 \pm 0.03_{\text{stat}}$.

7.2. Pulsar GeV Emission to PWN TeV Emission Connection

The correlation between the pulsar GeV emission and the PWN TeV emission shown in Figure 1 leads one to suspect a common underlying cause for the two emission mechanisms. One property relevant to both emissions is the electron–positron current of the pulsar wind ($I_{\text{wind}}$). The GeV energy flux from pulsars is thought to be directly related to the instantaneous value of $I_{\text{wind}}$, because the GeV pulsed emission from the magnetosphere is often thought to be produced by curvature emission by the most recently produced electron–positron population in the wind. Since the luminosity is roughly proportional to the population of electrons and positrons, we can write:

$$L_{\text{PSR GeV}} \propto I_{\text{wind}}.$$  \hspace{1cm} (18)

Inside PWNe, TeV photons are often thought to be produced by the up-scattering of ambient photons by the relativistic electrons and positrons, known as inverse Compton radiation. Therefore, $L_N$ should depend on the relativistic electron–positron population and the ambient photon population in the PWN. However, for the relativistic electrons and positrons that produce TeV photons by inverse Compton scattering the typical cooling time is larger than the lifetime of pulsars (Mattana et al. 2009). Therefore, the population of these electrons and positrons becomes proportional to the integral of $I_{\text{wind}}$ over the pulsar lifetime, instead of proportional to the instantaneous value of $I_{\text{wind}}$. However, we could suggest a proportionality between the ambient photon field density ($\rho_{\text{ph}}$) and $I_{\text{wind}}$. There are two different ambient photon fields which could be relevant to the production of TeV $\gamma$-rays: photons from synchrotron radiation and far-infrared photons (Atoyan & Aharonian 1996). The density of synchrotron radiation photons in the X-ray energy band is roughly proportional to the density of the freshly injected pulsar wind (Mattana et al. 2009). In addition far-infrared seed photons can be made by heating the pulsar wind, as described in section 2.2 of Arons (1996). Therefore the $\rho_{\text{ph}}$ may be roughly proportional to $I_{\text{wind}}$. If $\rho_{\text{ph}}$ is proportional to $L_N$, that would yield:

$$L_N \propto \rho_{\text{ph}} \propto I_{\text{wind}}.$$  \hspace{1cm} (19)

Hence,

$$L_N \propto L_{\text{PSR GeV}}.$$  \hspace{1cm} (20)

While these considerations are suggestive, a more detailed theoretical study is clearly needed to fully understand this correlation.

8. CONCLUSION

We have developed a new multi-wavelength technique to study the collective properties of the GeV pulsar beaming factor $f_{\Omega}$ with respect to the pulsar spin down luminosity $\dot{E}$. This technique uses a distance independent parameter, $\frac{R}{D}$, to obtain the correlation between $f_{\Omega}$ and $\dot{E}$. It allowed us to use pulsars with poorly known distance measurements to study $f_{\Omega}$. Using this technique we have experimentally obtained the $f_{\Omega}$ versus $\dot{E}$ dependence for pulsar emission in the $0.1$–$100$ GeV energy band. Under the model assumptions of an $\dot{E}^2$ dependence of GeV pulsed emission but no $\dot{E}$ dependence of TeV PWN emission, we find a dependence of $f_{\Omega}$ on $\dot{E}$. Our experimentally obtained correlation between $f_{\Omega}$ and $\dot{E}$ has some features which favor the theoretical $f_{\Omega}$ versus $\dot{E}$ distribution of the OG model obtained by Pierbattista et al. (2012). However, this specific comparison is limited by the modeling uncertainties of Pierbattista et al.’s (2012) simulated pulsar sample. Applying this same multi-wavelength method to X-ray data for PWNe is attractive since it may have a more precisely measurable $\dot{E}$ dependence than the present TeV data.

Pulsar GeV emission and PWN TeV emission are correlated, with a linear correlation coefficient of $R = 0.82$, although TeV PWN emission has no correlation to $\dot{E}$. This observed GeV to TeV correlation suggests the possibility of a linear relationship between $\gamma$-ray emission mechanisms in pulsars and TeV emission mechanisms in PWNe. However, it is not possible to explain this linear relationship using the electron–positron populations of curvature radiation inside the magnetosphere and synchrotron radiation in the PWN. An alternative possibility is a linear relationship between the ambient photon density ($\rho_{\text{ph}}$) in the PWN and the pulsar wind current ($I_{\text{wind}}$). A more detailed theoretical study will be needed to fully understand this correlation.

In the near future, TeV experiments under development such as HAWC, CTA, and Lhasso will have greater sensitivity than Milagro. The observed GeV to TeV luminosity correlation makes it likely that these observatories will detect PWNe associated with many more of the GeV pulsars Fermi has observed, leading to prospects of a higher-statistics and higher precision version of this analysis.

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