Winding optimization of composite frame by dry fiber rovings

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Abstract
Light-weight fibers reinforced polymer (FRP) composite frames are essential parts of vehicles body in the aerospace and automotive industries. Composite frames are often designed in complex curved 3D geometry through the dry winding process. The winding process of homogeneously wound-up layers of fibers without overlapping and gaps is the main challenge in the fabrication of frames with consistent thickness and acceptable quality. In this study, an industrial robot and winding head are set with a novel optimum process to wind the dry fiber with the specified angles on the frame, to fabricate it with minimum overlapping and local commulation of fibers, yet without gaps. Mathematical models and algorithms are developed to determine the optimal number of simultaneously winds rovings of fibers in a given layer. In addition, this study addresses the optimum dry winding of curved parts of frames that form a torus geometry. It is shown that the combination of layers of rovings wound successively on the frame at angles of 45°, 90° (i.e. the rovings are laid along with the frame), and −45°, is the most used variant of winding

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that provides the composite frame with higher strength. Results indicated that an optimal selection of the number and width of the rovings minimizes the overlap of the wound rovings, which saves up to 20% of the utilized fibers. The derived theory is verified on practical tests and experiments, which confirms the development of new suitable procedures to improve the fabrication of FRP composite frames.

Keywords
Robot winding, composite frame, mathematical model, helix, torus, roving, winding angle

Introduction
In the past decades, composite materials are increasingly replacing classic materials (such as wood, and metal) for their physical properties, energy-saving, and economic benefits.1–3 In general, composites are recognized with excellent mechanical properties, such as low weight, resistance to weathering, resistance to corrosion even in aggressive environments, long lifespan, etc., 3–5 A fiber-reinforced polymer (FRP) composite frame is a structure with a circular or rectangular cross-section that is made in close or open form, and that primarily is subjected to tensile, compressive, torsional, and bending loads.6,7 Fiber-reinforced polymer composite frames are used in the aerospace industry (e.g. as fuselage reinforcements, attaching windows to the fuselage, reinforcing the helicopter cabin),1,8,9 automotive applications (e.g. as car chassis, cab and door reinforcement),8 and also in shipbuilding. Engineering, energy generation (e.g. as wind turbine blade reinforcement), agriculture,10,11 pipeline transportation (e.g. oil, liquids, and gas),1,12 and sports equipment, are other use of FRP composite frames.6

The most commonly used production processes of FRP composite frames are braiding technology and filament winding. Braiding enables high adhesion of fibers to the frame surface, even in the case of a geometrically complicated frame shape,13,14 which enables the minimal risk of composite cracking during its loading. For special applications, it is possible to combine materials in partial directions (for example, carbon fibers/aramid fibers), or it is possible to exchange standard fibers to support synthetic yarn in one direction (unidirectional braiding).15–20 However, braiding technology is not applicable for the production of a composite closed frame with fiber reinforcement (technically difficult to implement).

The winding technology is used to manufacture frames with complex geometries, such as spherical and cylindrical vessels, tubes of various diameters, and a variety of convex and concave shapes.21–27 The main advantage of filament winding is the reinforcement content can be 60%–75%, to 80%, obtaining excellent mechanical characteristics. The second advantage is that the stiffness and strength of the composite are possibly enhanced in chosen directions by modifying the winding angle.

Some research has been done on the winding and optimization of composite frames. Duan et al.28 proposed and verified a two-stage optimization scheme for the composite frame design. They performed tensile, bending, and torsional stiffness calculations of
composite beams with a circular cross-section, that were expressed as explicit integral functions of the fiber winding angles. Numerical results showed that the proposed two-stage optimization eliminates the initial design dependence on fiber winding angle optimization; which helps to find a better design. A multistage design was developed by Yan et al. and Duan et al., which optimizes the winding angle, material, and laminate stiffness.

Similar issues are addressed in the book “Composite filament winding” where by combining the object geometry, roving parameters, strength, and feasibility to determine the optimal geometry for winding an object. Modeling of material properties depending on the physical-mechanical properties of the pressure vessel through the numerical simulation and derivation of the spiral winding angle was described by Zeng et al. Guo et al. developed a method for determining optimal beam winding trajectories. The method links structural design requirements with optimal structural properties including fiber orientation and volume reinforcement ratios. The resulting solution could be used directly to program the wrapping process. Similar problems have been addressed by Prado and Bodea.

Some studies have focused on establishing the relationship between fiber arrangement, winding geometry, material, and mechanical properties of the composite. Supian et al. investigated the effect of the orientation of the hybrid winding of a composite tube on energy absorption and failure modes. Differences in response to dynamic loads for FRP with different types of reinforcement (metal, woven fabric, winding reinforcement) were studied by Gowid et al. Li et al. built a model to predict the local change in fiber angle due to forming at 220°C. They used tubes with thermoplastic winding (CF/PA6 combination) for the experiment, while they compared the prediction of the winding angles resulting from the forming process with the real winding angles obtained by an automated precision measurement system.

Many researches have been done on the dry winding of composite frame, while some also studied the optimization process, however such optimization lack in focusing on the fiber arrangement to minimize the overlapping and accommodation of fibers without gaps. Therefore, this motivates the authors of this work to investigate specifically a novel approach of optimization procedure that involved an industrial robot and winding head to purposely arrange the dry fiber on a frame structure without overlapping and gaps. The research is also extended to a degree of considering complex structures such as torus.

In this regard, the optimized winding process is described in the following section Material and winding optimization. The mathematical model of the winding is briefly described in Subsection Fiber winding, the geometrical representation of wound rovings by helixes is shown in Subsection General rules of winding optimization, and the determination of the optimal number of rovings used and their width is solved by Subsections Selection of the optimal number of fiber rovings and width of roving and Winding a curved part of the frame focuses on the problem of determining the optimal number of rovings and their width when wound in a curved part of the frame. The results of the practical tests and experiments are presented in Section (Results and discussion).
Material and winding optimization

This section focuses on the winding method and process. Winding heads and industrial robots are used during the winding of rovings on the non-bearing frame with a circular cross-section.\textsuperscript{39,40} The winding head contains three revolving rings with coils for fiber rovings (see Figure 2(a)). Rovings lead from the coils to the feeder-winding ring, which is a circle in the center of the winding head. A small ring diameter concentrates the rovings to one area, and rovings are placed/winded on the frame’s core. The winding head is fixed in the robot’s working space, while the non-bearing frame (that is usually made of polyurethane) is attached to the end of the working arm (end-effector) of the industrial robot (see Figure 2(b)). Based on the determined trajectory of the robot, the frame passes through the winding head; three fiber layers are created simultaneously. The trajectory of robot arm movement depends on the shape of the frame.

Figure 1 shows a schematic of the development of the composite frame. The content of the paper focuses on the highlighted portion of the diagram.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{Overview diagram of the composite frame development process.}
\end{figure}
Fiber winding

During the winding process, based on the right determined trajectory of the robot, the frame structure is gradually passed through the winding head. Three layers of fibers with different directions/angles (often 45°, 90°, –45°) are placed on the surface of the frame (Figure 2(a)). A detailed procedure for calculating and determining the off-line trajectory

![Figure 2](image)

**Figure 2.** Winding head with three coil rings and frame (a), and connection of frame to the robotic arm and robot cooperation with winding head during the winding process (b).

![Figure 3](image)

**Figure 3.** Connection of closed frame to the robot-end-effector before starting the winding process (a), and example of the 3D shaped frame (b).
of the robot is described in the literature.\textsuperscript{8} Both frame types (open or closed) can be wound by rovings using this technique (see Figures 2 and 3(a)).

\textbf{Note 1.} Roving is a longitudinal fiber system that enables single filaments to be arranged in parallel, without twists. Fiber rovings (from carbon, glass, basalt, or aramid fibers) are used to produce areal and 3D composite reinforcement. The thickness of the fiber rovings depends on the number of filaments and the width of the roving. The available thicknesses of the rovings are; 3K–0.14 mm, 12K–0.18 mm, 24K–0.22 mm, and 48K–0.5 mm. This study uses roving 24K with a layer thickness between 0.21 – 0.24 mm through the frame winding process.

Performing the correct winding (i.e. correct winding angle and homogeneity of winding) of individual fiber layers is an essential prerequisite for producing composite with the required physical and mechanical properties.\textsuperscript{41} Adherence to the correct winding of fibers rovings in each layer of reinforcement is conditioned by the perpendicular passage of the frame through the plane \( \rho \) of winding of the fibers (see Figure 4 - plane \( \rho \) 1 of the fiber winding corresponding to coil ring \( k \) 1 of the head when winding the first layer of fibers). At the same time, it is necessary that intersection points of frame axis \( o \) with plane \( \rho \) 1 and axis \( s \) of winding head with plane \( \rho \) 1 have the smallest possible distance (see Figure 4 - in this case, the points of intersections of the plane are identical – point \( M \) 1). These conditions are described in detail in previous works by the same authors.\textsuperscript{1,9}

To achieve the correct fiber angle in the winding of a given layer at a constant speed of frame passage through the winding head, it is necessary to determine the correct angular speed of rotation of the ring with coils. This issue is solved in detail in previous work.\textsuperscript{1} The calculation of distance \( h \) of winding fibers on the frame from the coil ring at the required angle is also solved in detail in\textsuperscript{1} (see Figure 4 – distance \( h \) 1 of winding plane.

\includegraphics{figure4.png}

\textbf{Figure 4.} Model of the winding process that shows the formation of the first fiber layer on the composite frame; the frame goes through a winding head (Figure 2(a)) represented by three coil rings, and three layers of fiber are created. The schema of the first fiber layer formation is shown in this figure.
ρ1 from coil ring k1). At the same time, the article deals with the winding of 3D frames, which consist of several parts with different radii.

In the case of a geometrically complicated shape of a frame (see Figure 3(b)), it is difficult to determine a suitable robot trajectory. Optimization of the robot trajectory for such cases using a differential evolution algorithm is described in detail in previous works.\textsuperscript{1,9}

After winding the fibers, the frame is impregnated with resin and then thermally cured using, for example, (resin transfer molding) RTM technology.\textsuperscript{42}

**General rules of winding optimization**

The wound fiber roving can be represented by a helix formed on the surface of the frame. The roving has the shape of a rectangle in a cross-section. The height of the roving does not need to be considered for our purposes. The $u$-axis passes through the center of the roving (see Figure 5). The width of the roving is denoted by $m$, and the overlap of two consecutive wound rovings is denoted by $\delta$. Then it applies to the distance $d$ of two adjacent axes

$$d = m - \delta. \quad (1)$$

The roving is wound on the frame; the $u$-axis of roving forms a helix on the frame envelope (see Figure 6(a) and (b)), detail can be found in.\textsuperscript{43} Right-hand helix $h_R$ is formed

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{The winding part is represented by three rovings (red, blue, and green) in the plane. Axes $u_1$, $u_2$, and $u_3$ pass through the centers of rovings 1, 2, and 3 of width $m$. The overlap of rovings $\delta$ is formed by winding two consecutive rovings.}
\end{figure}

\textsuperscript{7} Mlýnek et al.
on the surface of the frame at a positive angle of winding (denoted by +), with a negative angle of winding (denoted by –) is formed left-handed helix \( h_L \).

The following sections focus only on the winding in a positive direction and the creation of a right-handed helix. The following relations derived for a positive winding angle will be completely analogous even for the case of a negative winding angle. Figure 6(a), shows one turn of helix \( h_R \) (the turn of roving). Helix \( h_R \) is defined by axis \( o \) (axis of wrapped frame), radius \( r \) (radius of frame), and pitch \( v \) (height of one helix turn, measured parallel to axis \( o \) of helix). The value of \( v \) is equal to the distance of points \( A \) and \( B \) in Figure 6(a). A characteristic triangle defines helix angle \( \alpha \) (see Figure 6(c), the length of the hypotenuse of the right triangle is equal to the length of helix \( h_R \) in one turn, \( \tan \alpha = \frac{v}{2\pi r} \)). This angle \( \alpha \) is also formed by the tangent \( t \) at each point \( P \) of the helix with its orthogonal projection \( t_1 \) (see Figure 6(a)).

Thus, the defined angle \( \alpha \) is named the angle of winding of the roving. Angle \( \beta \) is defined as

\[
\beta = \frac{\pi}{2} - \alpha. \tag{2}
\]

**Note 2.** Textile and composite specialists often use \( \beta \) angle (defined in equation (2)) to mark the winding angle.

In 3D Euclidean orthogonal coordinate system, the parametric equation of helix \( h_R \) in the homogenous form can be expressed as (see 43)

\[
h_R(t) = (x(t), y(t), z(t), 1) = (r \cos t, r \sin t, v_0 t, 1), \tag{3}
\]

where \( v_0 \) is reduced pitch of helix (length of translation during rotation of helix by one radian along \( o \) axis, \( v_0 = \frac{v}{2\pi} \), \( t \in <0, \infty> \)). Point \( A = (r, 0, 0, 1)^T \) is the initial point of right-handed helix \( h_R \) (see Figure 6(a)). One turn of \( h_R \) is defined by relation (3) for \( t \in <0, 2\pi> \).
Note 3. Parametric expression of left-handed helix $h_L$ (Figure 6(b)) can be expressed in the form $h_L(t) = (r \cos t, - r \sin t, v_0 t, 1), t \in <0, \infty$) The winding of the roving at a negative angle is represented by a left-handed helix.

The equation of the right-handed helix $h_R$ can also be expressed as the rotation of the point $A_1 = (r, 0, 0, 1)^T$ around axis $z$ (in our case $o \equiv z$, see Figure 6(a)), and its translation in a positive direction of axis $z$

$$h_R(t) = (x(t), y(t), z(t))^T = \begin{pmatrix} \cos t & -\sin t & 0 & 0 \\ \sin t & \cos t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & v_0 t \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r \\ \sin t \\ \cos t \\ 0 \end{pmatrix}.$$  

Now a helix $h_2_R$ that has the same parameters as $h_1_R$, and only $A_2$ is its initial point is considered. Point $A_2$ also lies on a circle with center $S_1$ and radius $r$, angle $A_1S_1A_2 = \lambda$ (see Figure 7(a)). Then point $A_2$ has coordinates $A_2 = (r \cos \lambda, r \sin \lambda, 0, 1)^T$ and helix $h_2_R(t)$ can be expressed in the form of

$$h_2_R(t) = (x(t), y(t), z(t), 1)^T = \begin{pmatrix} \cos t & -\sin t & 0 & 0 \\ \sin t & \cos t & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & v_0 t \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r \cos \lambda \\ r \sin \lambda \\ r \sin(t + \lambda) \\ v_0 t \end{pmatrix}.$$  

Note that relations $r(\cos t \cos \lambda - \sin t \sin \lambda) = r \cos(t + \lambda)$ and $r(\cos t \sin \lambda + \sin t \cos \lambda) = r \sin(t + \lambda)$ (see 44) are used in the last equality.

One turn (i.e. $t \in <0, 2\pi>$) of helices $h_1_R$ and $h_2_R$ is displayed in Figure 8.

Initial point $A_2$ of helix $h_2$ is rotated about the $z$-axis by angle $\lambda$ with respect to the initial point $A_1$ of the helix $h_1$ (see Figures 8 and 9). Attention will now turn to determine the distance of helices $h_1_R$ and $h_2_R$. The unfolding of the cylindrical surface with one turn of the helices $h_1_R$ and $h_2_R$ into the plane is shown in Figure 9.
Figure 7. Initial points of right-handed helices. Two adjacent initial points are rotated by angle $\lambda = \pi/2$ (a), and $\lambda = \pi/6$ (b).

Figure 8. Graph of one turn of helices $h_{1R}$ and $h_{2R}$. 
Figure 9 shows that the relationship $\sin \alpha = \frac{d}{\lambda r}$ is valid, where $\alpha$ is the helix angle, and $d$ is the distance of two adjacent helices. From there it follows the relationship

$$d = \lambda r \sin \alpha.$$  \hfill (4)

Relation (4) implies that the distance, $d$ depends on the size $\lambda$ of rotation, radius $r$ of helices $h_{1R}$ and $h_{2R}$, and the size of helix angle $\alpha$. Function $\sin$ is increasing on interval $(0, \pi/2)$. This means that with increasing angle $\alpha$ (required angle of winding of the fiber rovings), the distance $d$ will also increase.

The winding of one roving at an angle $\pm \pi/4$ ($\pm 45^\circ$), the second at an angle of $-\pi/4$ ($-45^\circ$) utilizing the winding head and an industrial robot are shown in Figure 10(a). The ring of the winding head with ten spools wound on fiber rovings shows in Figure 10(b).

Selection of the optimal number of fiber rovings and width of roving

The ideal winding of rovings is such that the individual fiber layers are homogeneous. The wound rovings are placed side by side, follow each other smoothly, the entire surface of the frame is covered with fibers, and there is no overlap of adjacent rovings. This ideal winding is only possible with a straight frame. However, achieving this condition is difficult if the frame is geometrically more rugged (2D or 3D configuration). To achieve a qualified and acceptable composite frame winding process, it is essential to ensure the full coverage of the frame surface with minimal $\delta$ overlap of adjacent rovings, see Figure 5.

As mentioned in the introduction of this section, axes $u_1$, and $u_2$ of two adjacent fiber rovings (see Figure 5) create two adjacent helices $h_{1R}$ and $h_{2R}$, during the winding process. The distance of these helices is given by relation (4). Equation (1) defines the relationship between the width $m$ of the rovings and distance $d$ of the two wound rovings.

Now the focus is on determining the optimal number $n$ of wound rovings and the width $m$ of roving. A straight frame with radius $r$ and helix angle $\alpha$ (see Figure 6(a)) is considered. The number of used rovings increases with the frame radius $r$ and helix angle $\alpha$ (winding angle). In general, narrower rovings are more appropriate than wider rovings to achieve a homogeneous winding of fibers onto the frame.
Following the relations (1) and (4), the width $m$ of used fiber rovings is given by the relation

$$m = d + \delta = \lambda \cdot r \sin \alpha + \delta. \quad (5)$$

The size of frame radius $r$ and the required $\alpha$ winding angle for the specified fiber layer is fixed. Equation (5) allows two basic procedures:

1. Determining the number $n$ of rovings used in the winding process of one layer of fibers (i.e. the number of bobbins used on the coil ring) for the specified $m$ width of roving.

2. Calculation of optimal roving width $m$ for specified number $n$ of rovings used during the winding process.

**Determining the optimal number of rovings.** It is assumed that the width $m$ of fiber roving, radius $r$ of the frame, and winding angle $\alpha$ is specified. The optimal number $n$ of rovings for winding the layer of fibers is calculated. The rotation angle $\lambda$ between two adjacent windings (see Figures 7 and 8) of rovings is expressed from relation (5). Rotation angle $\lambda$ about the z-axis is expressed by the relation (5) and zero overlaps of adjacent rovings (i.e. $\delta = 0$) is assumed. Then it applies $m = \lambda \cdot r \sin \alpha$ and so $\lambda = \frac{m}{r \sin \alpha}$. The optimal number $n$ of strands for winding a given layer is specified by the relation $n = \frac{2 \pi}{\lambda}$. Substituting $\lambda$ into the last relationship results in

![Figure 10](image-url)

(a) Fiber rovings on the surface of the frame at angles $+\frac{\pi}{4}$ ($+45^\circ$) and $-\frac{\pi}{4}$ ($-45^\circ$), and (b) ten spools with rovings as feeding devices on the ring of the winding head.
The entire surface of the frame is covered with wound rovings and at the same time, there are minimal overlaps of the rovings for the number \( n \) of rovings given by relation (6).

Note 4. The ceiling \( \lceil x \rceil \) of a real number, \( x \) is defined as \( \lceil x \rceil = \min \{ p \in Z; p \geq x \} \), where \( Z \) denotes a set of integers.

An actual angle of rotation between the two following rovings is denoted as \( \tilde{\lambda} \) and

\[
\tilde{\lambda} = \frac{2\pi}{n},
\]

(7)

where \( n \) is given by relation (6). Then actual overlap \( \tilde{\delta} \) is equal to

\[
\tilde{\delta} = m - \tilde{\lambda} r \sin \alpha.
\]

(8)

Values of optimal number \( n \) of used rovings, corresponding to the rotation angle \( \tilde{\lambda} \) between the two following rovings, and the corresponding overlap \( \tilde{\delta} \) are gradually given by relations (4), (6), (7), and (8) for specified width \( m \) of roving, radius \( r \) and winding angle \( \alpha \).

**Calculation of optimal roving width.** It is assumed that rotation angle \( \lambda \) between two adjacent rovings (and thus the number \( n \) of roving used), radius \( r \) of the frame, and winding angle \( \alpha \) are entered. Then, the optimal width \( m \) of roving is calculated. Overlap \( \delta \) is equal to zero in the case of optimal width \( m \) of roving. Then it implies from relation (5)

\[
m = \lambda r \sin \alpha.
\]

(9)

The optimal width \( \tilde{m} \) of the rovings will be chosen so that \( \tilde{m} \) is the smallest width of roving supplied by the manufacturer, which also meets the condition \( \tilde{m} \geq m \), where \( m \) is given by relation (9).

If the number of rovings \( n \) used for winding is known, then the angle \( \lambda \) of rotation for the neighboring two wound rovings is equal to the value

\[
\lambda = \frac{2\pi}{n}.
\]

(10)

The optimal \( m \) width of fiber roving for a specified number \( n \) of rovings, frame radius \( r \), and winding angle \( \alpha \) can be calculated using relations (9) and (10).

**Winding a curved part of the frame**

Winding a curved part of a frame with a circular cross-section is the most challenging task of winding technology. In general, the frame could be a 3D curved structure. However, the curved part of the frame is very often shaped only in 2D and forms part of a torus (see Figure 2(b), Figure 11(a), detail is provided in the literature ).43 Attention is given to a
more detailed study of winding such a part of the frame. The torus is defined by its center $S$, radius $R$ of the torus axis $\alpha$, and radius $r$ of a circular cross-section of the torus, as shown in Figure 11(b).

The outer part $s_1$ of the torus surface is larger than the inner part $s_2$ of the torus surface (see Figure 11(b)). This disproportion relates to the difficulty of making a homogeneous fiber layer. Figure 12 shows the winding of the roving on the surface of the torus at an angle of $-\frac{\pi}{6}$ ($-30^\circ$) and $-\frac{\pi}{6}$ ($-60^\circ$). The distance of the corresponding boundary points after one turn of roving is greater on the outer circumference of the torus than on the inner circumference. Said distance increases with growing winding angle $\alpha$.

Now, the attention is on the size of the circumference of circle $p_1$ and circle $p_2$ of the torus in Figure 11(b). Circle $p_1$ creates the outer circumference and circle $p_2$ inner circumference of the torus. Circumference size $O(p_1)$ of the circle, $p_1$ is equal to $O(p_1) = 2\pi(R + r)$ and circumference size $O(p_2)$ of the circle, $p_2$ is equal to $O(p_2) = 2\pi(R - r)$, where $R > r$. It is seen that $O(p_1) > O(p_2)$, then it applies to the proportion

$$\frac{O(p_2)}{O(p_1)} = \frac{2\pi(R - r)}{2\pi(R + r)} = \frac{2\pi R - 2\pi r}{2\pi R + 2\pi r} = \frac{2\pi r}{2\pi R + 2\pi r} - \frac{4\pi r}{2\pi R + 2\pi r} = 1 + \frac{2r}{R + r}.$$  \hspace{1cm} (11)

It follows from relation (11), that the greater value of $R$ than the value of $r$, the closer is the proportion $\frac{O(p_2)}{O(p_1)}$ to 1, and thereby better the curved part of the frame can be wound.

**Optimal number of rovings for winding of curved frame part.** As it is stated in the introduction, curved parts of the frames most often form parts of tori. Therefore, we will solve the problem of winding the frame in the shape of a torus. As already mentioned in subsection *fiber winding*, it is necessary for right winding the frame passes perpendicularly to the

![Figure 11](image1.png)  \hspace{1cm} ![Figure 11](image2.png)

**Figure 11.** Frame curvature often copies the shape of a torus (a). Floor plan of a torus - description of torus parameters (b).
winding plane of fiber rovings (see Figures 4 and 2(b), detail is described in the literature). It was assumed torus is connected to the robot-end-effector, rotates around its center \( S \) and goes through the winding head (and therefore through rotated coil ring \( k_1 \) which forms the first layer of wound fibers, similar to Figure 4) and orthogonally goes through the winding plane \( \rho_1 \) (i.e. its axis \( o \) orthogonal to \( \rho_1 \), see Figure 11(b)). It is assumed a constant circumferential velocity of the torus. The circumferential speed of axis \( o \) is denoted as \( u_{\text{torus}} \). The required winding angle of the fiber rovings is denoted as \( \alpha \). The circumferential velocity \( u_{\text{torus}} \) of torus axis, \( o \) is given by the relation

\[
u_{\text{torus}} = R \omega_{\text{torus}}, \tag{12}\]

where \( \omega_{\text{torus}} \) is the angular velocity of the torus. The angular velocity of rotated coil ring \( k_1 \) (see Figure 4) is denoted as \( \omega_{cr} \) and its circumferential velocity as \( u_{cr} \). Then it is true

\[
u_{cr} = L \omega_{cr}, \tag{13}\]

where \( L \) is the radius of coil ring \( k_1 \) (see Figure 4). Recall that \( v \) indicates the pitch of the helix formed by the roving when wound on the general frame at \( \alpha \) angle (see Section General rules of winding optimization). Then relation \( v = 2\pi r \tan \alpha \) (see Figure 6(c)) is used and can be written

\[
u_{cr} = \frac{2\pi L}{v} = \frac{2\pi L}{2\pi r \tan \alpha} = \frac{L}{r \tan \alpha}, \tag{14}\]

where \( \alpha \in (0, \pi/2) \).

It follows from relations (12), (13), and (14) \( \frac{\omega_{cr} L}{\omega_{\text{torus}} R} = \frac{L}{r \tan \alpha} \), that implies \( \tan \alpha = \frac{\omega_{\text{torus}} R}{\omega_{cr} r} \).

It follows from here
The required $\omega_{cr}$ angular velocity of rotated coil ring $k_1$ for the rovings to be wound at the specified angle $\alpha$ is determined from relation (15) to the assuming a constant angular velocity $\omega_{torus}$ of the torus. The calculated $\omega_{cr}$ angular velocity is determined with respect to the axis $o$ of the torus and its peripheral speed $u_{cr}$. But the peripheral speed of the outer circuit of the torus (circle $p_1$, see Figure 11(b)) is equal to the value $(R + r)$ $\omega_{torus}$ and peripheral speed of the internal circuit (circle $p_2$) is equal to $(R - r)$ $\omega_{torus}$. Then real winding angle $\alpha_{outer}$ of the outer circuit is equal to (see relation (15) on the right)

$$\alpha_{outer} = \arctg \left( \frac{(R + r) \omega_{torus}}{r \omega_{cr}} \right)$$

and real winding angle $\alpha_{internal}$ of internal circuit is equal to

$$\alpha_{internal} = \arctg \left( \frac{(R - r) \omega_{torus}}{r \omega_{cr}} \right).$$

The outer circumference $p_1$ (see Figure 11(b)) of the torus has a higher speed circumferential than internal circumference $p_2$. It is seen from relations (16) and (17) that winding angle $\alpha_{outer}$ on outer circumference $p_1$ of the torus is greater than the winding angle $\alpha_{internal}$ on internal circumference $p_1$ (because arctg is increasing function).

The possibilities of using derived relationships to achieve optimized winding of rovings on the frame are shown in the following section.

**Results and discussion**

In this section, the derived mathematical relations and conclusions for specific input parameters and their application to practical examples are applied.

**Calculation of optimal number of rovings**

Table 1 contains the calculated optimal number of rovings $n$ used for winding: specific roving width $m$, frame radius $r$, and prescribed winding angle $\alpha$. The overlap $\delta$ of neighboring rovings are simultaneously determined in each row of Table 1. Relations (6), (7), and (8) are used to calculate the values $n$ and $\delta$ in Table 1. Only a small number $n$ of rovings for optimal winding is needed for a smaller frame radius $r$, a smaller winding angle $\alpha$, and a larger roving width $m$ (see relation (6)). There may even be a case where it is optimal to wind only one roving (see Table 1).

Based on the values given in each row of Table 1, the percentage $p$ value of the area of the roving overlaps with respect to the total wound area can be calculated by a simple relation

$$p = \frac{\delta \cdot (n - 1)}{n \cdot m} \cdot 100.$$
For example, for the fourth row of the table from the bottom ($m = 15$($\text{mm}$), $r = 30$($\text{mm}$), $\alpha = 5^\circ$, $n = 2$), $p = 22.6330\%$; for the last row ($m = 15$($\text{mm}$), $r = 30$($\text{mm}$), $\alpha = 60^\circ$, $n = 11$), $p = 1.0145\%$. Using relation (18), appropriate (with minimal $p$) and inappropriate choices of input parameters can be determined.

**Table 1.** The specified optimal number $n$ of rovings and the corresponding overlap $\delta$ of adjacent rovings for the specified input parameters: rovings width $m$, frame radius $r$, and winding angle $\alpha$.

| Width of roving ($m$) ($\text{mm}$) | Radius ($r$) ($\text{mm}$) | Winding angle ($\alpha$) ($^\circ$) | (rad) | $(2\pi r \sin \alpha)/m$ | Number of rovings ($n$) | Overlap ($\delta$) ($\text{mm}$) |
|-------------------------------------|-----------------------------|--------------------------------------|-------|--------------------------|-------------------------|---------------------------|
| 5                                   | 10                          | 5                                    | 0.0815 | 1.0952                  | 2                       | 2.2634                    |
|                                    | 30                          | 0.5235                              | 6.2789 | 0.5143                  | 3                       | 1.3510                    |
|                                    | 45                          | 0.7853                              | 8.8811 | 0.0660                  | 3                       | 1.692                     |
|                                    | 60                          | 1.0471                              | 10.8772| 0.0560                  | 3                       | 0.0660                    |
| 20                                  | 5                           | 0.0815                              | 2.0480 | 1.0286                  | 3                       | 4.5266                    |
|                                    | 30                          | 0.5235                              | 18.8367| 0.0421                  | 4                       | 0.0660                    |
|                                    | 45                          | 0.7853                              | 26.6433| 0.0660                  | 1                       | 0.0558                    |
|                                    | 60                          | 1.0471                              | 32.6316| 0.0558                  | 3                       | 0.0558                    |
| 10                                  | 10                          | 5                                    | 0.0815 | 0.5120                  | 1                       | 4.5266                    |
|                                    | 30                          | 0.5235                              | 3.1394 | 2.1500                  | 4                       | 2.1500                    |
|                                    | 45                          | 0.7853                              | 4.4405 | 1.1187                  | 5                       | 1.1187                    |
|                                    | 60                          | 1.0471                              | 5.4386 | 0.9358                  | 6                       | 1.1187                    |
| 20                                  | 5                           | 0.0815                              | 1.0240 | 1.0286                  | 2                       | 4.5266                    |
|                                    | 30                          | 0.5235                              | 6.27895| 1.0286                  | 3                       | 4.5266                    |
|                                    | 45                          | 0.7853                              | 8.88115| 0.1319                  | 5                       | 0.0660                    |
|                                    | 60                          | 1.0471                              | 10.8772| 0.1116                  | 5                       | 1.0286                    |
| 30                                  | 5                           | 0.0815                              | 1.5360 | 1.7899                  | 2                       | 1.7899                    |
|                                    | 30                          | 0.5235                              | 9.41835| 5.8000                  | 3                       | 5.8000                    |
|                                    | 45                          | 0.7853                              | 13.3216| 0.4843                  | 6                       | 0.0660                    |
|                                    | 60                          | 1.0471                              | 16.3158| 0.4024                  | 8                       | 0.0660                    |
| 15                                  | 10                          | 5                                    | 0.0815 | 0.3413                  | 2                       | 9.5266                    |
|                                    | 30                          | 0.5235                              | 2.0929 | 4.5533                  | 3                       | 4.5533                    |
|                                    | 45                          | 0.7853                              | 2.9603 | 1.0139                  | 5                       | 1.0139                    |
|                                    | 60                          | 1.0471                              | 3.6257 | 3.5965                  | 8                       | 3.5965                    |
| 20                                  | 5                           | 0.0815                              | 0.6826 | 4.0532                  | 1                       | 4.0532                    |
|                                    | 30                          | 0.5235                              | 4.1859 | 2.4400                  | 3                       | 2.4400                    |
|                                    | 45                          | 0.7853                              | 5.9207 | 0.1979                  | 5                       | 0.1979                    |
|                                    | 60                          | 1.0471                              | 7.2514 | 1.4034                  | 8                       | 1.4034                    |
| 30                                  | 5                           | 0.0815                              | 1.024  | 6.7899                  | 2                       | 6.7899                    |
|                                    | 30                          | 0.5235                              | 6.2789 | 1.5428                  | 7                       | 1.5428                    |
|                                    | 45                          | 0.7853                              | 8.8811 | 0.1979                  | 9                       | 0.1979                    |
|                                    | 60                          | 1.0471                              | 10.8772| 0.1674                  | 11                      | 0.1674                    |
Example – winding with the optimal number of rovings

In this practical example, the goal is to wind carbon rovings on a straight core (straight frame). The winding parameters are; radius of frame $r = 16$ (mm), roving winding angle $\alpha = \frac{-\pi}{6}$ ($-30^\circ$) and roving width $m = 9$ (mm). Determining the optimal number of $n$ rovings used in winding is our task and coverage of the entire surface of the frame with minimal overlaps of consecutive rovings. Covering the entire surface of the frame with minimal overlaps of adjacent rovings will ensure a homogeneous winding of a given layer of fibers.

The longitudinal central axis of the roving forms a left-handed helix $h_L$ (see Figure 6(b)), pitch $v$ (height of one helix turn, measured parallel to axis $o$ of helix; see Figure 6(c)) is equal to $v = 2\pi r \tan \alpha = 58.04$ (mm). Based on the use of relation (6), the optimal number $n$ of used rovings is $n = 6$. Then the overlap $\delta$ of two adjacent rovings is equal to $\delta = 0.62$ (mm) based on the use of relations (6), (7), and (8). Figure 13 shows the winding of a layer of fibers on a straight core in the length of one roving turn. One (a), three (b), five (c), and six (d) rovings for winding the layer of fibers on the frame are successively used. The winding is performed with gaps in the partial figures with one, three, and five rovings. It can be seen that the sufficient optimal number $n$ of rovings is 6 (see Figure 13(d), the wound layer is without gaps).

![Figure 13. Presentation of the winding of rovings to a straight core. One (a), three (b), five (c), and six (d) rovings are wound on the core according to helix parameters (The individual pictures present this procedure).](image)

Calculation of optimal width of roving

The optimal $m$ width of fiber roving for a specified number $n$ of rovings, frame radius $r$, and winding angle $\alpha$ can be calculated using relations (9) and (10). The optimal values of the roving width $m$ are given in Table 2.
In practice, it is necessary to choose the smallest integer greater than width $m$ or equal to $m$ of the rovings supplied by the roving manufacturer at the specified number of rovings used (i.e. the number of used coils with rovings).

**Note 5.** Carbon rovings from Toho Tenax producer was used in the fabrication process. Carbon roving 24K has a width of 9(mm). Twenty-four thousand (24K) carbon filaments about a diameter 7(μm) create a rectangle cross-section of roving. Rovings with a different number of filaments are available on the market; for example, 12K or 6K. In proportion to the number of fibrils, the width of the roving also differs 12K = width 5(mm), 6K = width 2(mm).

The derived mathematical models in Section *Material and winding optimization*, enable the determination of the number $n$ of roving and their width $m$ for winding a compact layer of fibers rovings.

**Example – winding curved frame – part of the torus**

Let us consider the input values for winding rovings on a torus as listed in Table 3. Note that the second radius $r$ of the torus corresponds to the radius of the frame.

Firstly, $\omega_{cr}$ is determined (angular speed of the rotated coil ring) using relation (15)

$$\omega_{cr} = \frac{R \omega_{torus}}{r \tan} \approx \frac{0.2 \cdot 0.5}{0.02 \cdot 1} = \frac{0.1}{0.02} = 5 \text{ (rad/s)}.$$ 

| Input parameter                              | Designation | Value of parameter |
|----------------------------------------------|-------------|--------------------|
| The angular speed of torus                   | $\omega_{torus}$ | 0.5(rad/s)        |
| 1st radius of the torus                      | $R$         | 0.2(m)             |
| 2nd radius of torus                          | $r$         | 0.02(m)            |
| Winding angle                                | $\alpha$    | $\pi/4$(rad) (45°) |
| Number of used rovings to winding            | $n$         | 8                  |

Table 2. The calculated optimal roving width $m$ for specified winding input parameters.

| Radius ($r$) (mm) | Number of rovings ($n$) | Winding angle ($\alpha$) (°) | Winding angle ($\alpha$) (rad) | Optimal width of roving ($m$) (mm) |
|-------------------|-------------------------|-------------------------------|-------------------------------|------------------------------------|
| 10                | 4                       | 30                            | 0.5235                        | 7.8539                             |
| 12                | 45                      | 0.7853                        | 3.7023                        |
| 16                | 60                      | 1.0471                        | 3.4007                        |
| 20                | 10                      | 30                            | 0.5235                        | 6.2831                             |
| 16                | 45                      | 0.7853                        | 5.5535                        |
| 20                | 60                      | 1.0471                        | 5.4413                        |

Table 3. Input values for winding rovings on a frame with torus geometry.
Therefore, fiber rovings will be wound on the torus at angle $\alpha = \pi/4$ (rad) = 45° with respect to the axis $o$ of the torus at $\omega_{cr} = 5$ (rad/s) and $\omega_{torus} = 0.5$ (rad/s).

Rovings on the outer circumference of the torus (circle $p_1$, see Figure 11(b)) will be wound at an angle given by relation (16)

$$
\alpha_{outer} = \arctg \left( \frac{(R + r)\omega_{torus}}{r\omega_{cr}} \right) = \arctg \left( \frac{(0,2 + 0,02).0,5}{0,02.5} \right) = \arctg(1,1) = 0,8329 (\text{rad})
$$

$$
= 47,7260^\circ.
$$

At the same time, the rovings on the internal circumference of the torus (circle $p_2$) will be wound at an angle given by the relation (17)

$$
\alpha_{internal} = \arctg \left( \frac{(R - r)\omega_{torus}}{r\omega_{cr}} \right) = \arctg \left( \frac{(0,2 - 0,02).0,5}{0,02.5} \right) = \arctg(0,9) = 0,7328 (\text{rad})
$$

$$
= 41,9870^\circ.
$$

It could be seen that the rovings are wound at a greater angle on the outer circumference than on the inner circumference of the torus. As a result, there occurs often insufficient fiber rovings coverage of the surface on the outer circumference of the torus and overlapping of the fiber rovings on the inner circumference of the torus.

As already mentioned in the previous Section Material and winding optimization, the covering of the entire surface of the torus by rovings is an important condition for ensuring a sufficiently high-quality layer of reinforcing fibers rovings. The following, very important condition, is to minimize overlaps of the wound rovings.

Since eight rovings are used for winding (and thus also eight spools located on the circumference of the specified coil ring), two adjacent rovings on the frame are rotated by angle $\lambda = \pi/4$ (rad) = 45° (see relation (10) and Figures 7 and 8). The distance $d$ of the central axes of two adjacent wound strands of fibers (see Figure 5) is equal to (see relation (4))

$$
d = \lambda \cdot \sin \alpha = \frac{\pi}{4} \cdot 0,02 \cdot \sin \left( \frac{\pi}{4} \right) = 0,0111072 (\text{m}) = 1,11072 (\text{cm}).
$$

This distance $d$ corresponds to winding angle $\alpha$ with respect to the $o$-axis of the torus. Distance $d_{outer}$ of the central axes of two adjacent rovings on the outer circumference of the torus (see Figure 11(b), circle $p_1$) is equal to

$$
d_{outer} = \lambda \cdot \sin \alpha_{outer} = \frac{\pi}{4} \cdot 0,02 \cdot \sin(0,8329) = 0,011622949 \ (\text{m}) = 1,162249 \ (\text{cm}).
$$

Analogously, distance $d_{internal}$ of the central axes of two adjacent rovings for $\alpha_{internal}$ winding angle (on the internal circumference of a torus, circle $p_2$) is equal to

$$
d_{internal} = \lambda \cdot \sin \alpha_{internal} = \frac{\pi}{4} \cdot 0,02 \cdot \sin(0,7328) = 0,010508 \ (\text{m}) = 1,0508 \ (\text{cm}).
At width \( m \) of the roving equal to \( m = d_{\text{outer}} = 1,1622949 \text{(cm)} \), adjacent rovings of fibers will follow each other on the outer circumference of the torus, then overlap \( \delta = m - d_{\text{internal}} = 0,1114949 \text{(cm)} \approx 1,11 \text{(mm)} \) (see relation (1)) will be on the internal circumference.

The selection of roving in which the width \( \bar{m} \) is the smallest value greater than or equal to \( m = d_{\text{outer}} (\bar{m} \geq m = d_{\text{outer}}) \) from the rovings offered widths by the suppliers, seems to be a suitable approach for the winding process. In this case, the wound rovings on the outer circumference of the frame are free of gaps and overlaps and the overlaps on the inner circumference of the frame are minimized.

The following Figure 14 shows the winding of a torus-shaped frame successively with one, two, four, and six rovings at an angle of \(-45^\circ\). In this case, the width of the roving \( m = d_{\text{outer}} = 9 \text{(mm)} \). When six rovings are used, the frame surface is fully covered, the outer circumference is free of gaps and overlaps, and the inner circumference contains minimized overlaps.

**Winding angles on the outer and inner circumference of the torus**

Table 4 presented the same input parameters as in the previous Example 3.4, only \( \alpha \) angle will be changed. The quantities \( \omega_{\text{cr}}, d, \alpha_{\text{outer}}, d_{\text{outer}}, \alpha_{\text{internal}}, \) and \( d_{\text{internal}} \) are calculated in Table 4 by successively using relations (15), (16), (4) and (17) for each entered \( \alpha \).

Table 4 clearly shows that as the \( \alpha \) winding angle increases (relative to axis \( o \) of the torus, see Figure 11(b)), values of \( \alpha_{\text{outer}}, \alpha_{\text{internal}}, d, d_{\text{outer}}, \) and \( d_{\text{internal}} \) increase. At the same time, it is true \( \alpha_{\text{internal}} < \alpha < \alpha_{\text{outer}} \) and \( d_{\text{internal}} < d < d_{\text{outer}} \) for specific \( \alpha \). Thus, in general, the roving is wound at a greater angle on the outer circumference than on the inner circumference. For this reason, uncovered portions (gaps) of the outer periphery of the frame and overlaps of the rovings on the inner periphery of the frame often occur during the winding of the roving layer. The value of \( \omega_{\text{cr}} \) ((angular speed of rotating coil ring) decreases for increasing values of \( \alpha \) (relation (15)). As mentioned before angular speed \( \omega_{\text{torus}} \) of the torus is constant. As can be seen from the table, the difference \( \alpha_{\text{outer}} - \alpha_{\text{internal}} \) (see relations (16) and (17)) between the winding angle of roving on the outer circumference and on the inner circumference increases with the approximate raise in \( \alpha \) to angle \( \frac{\pi}{4} \) \((45^\circ)\). Then, the value of difference \( \alpha_{\text{outer}} - \alpha_{\text{internal}} \) decreases for increasing angle \( \alpha \) and \( \alpha > \frac{\pi}{4} \) \((45^\circ)\) due to a more gradual increase in the values of \( \arctan \) function.

**Winding angles on the circumference of the torus depending on the frame radius**

The same input values are considered as in Example 3.4 and the constant specified winding angle \( \alpha = \frac{\pi}{4} \) \((45^\circ)\). Only the radius \( r \) of the torus (see Figure 11(b)) will be changed. Table 5 contains calculated values \( \alpha_{\text{outer}}, \alpha_{\text{internal}}, d, d_{\text{outer}} \) and \( d_{\text{internal}} \) for
specification radii $r$. Table 5 demonstrates that with increasing value of radius $r$, differences $\alpha_{\text{outer}} - \alpha_{\text{internal}}$ and $d_{\text{outer}} - d_{\text{internal}}$ are also increasing.

Figure 14. Presentation of the winding of a torus-shaped frame successively with one (a), two (b), four (c), and six (d) with 9 mm wide rovings.

Table 4. Values of winding angles on the outer and inner circumference of the torus and the distance between the central axes of two adjacent wound rovings depend on the entered winding angle $\alpha$.

| $\alpha$ (°) | $\omega_{\text{cr}}$ (rad/s) | $d$ (mm) | $\alpha_{\text{outer}}$ (°) | $d_{\text{outer}}$ (mm) | $\alpha_{\text{internal}}$ (°) | $d_{\text{internal}}$ (mm) |
|-------------|-----------------|--------|-----------------|-----------------|-----------------|-----------------|
| 10          | 0.17453         | 28.3607| 2.725           | 10.9767         | 0.1915          | 2.991           | 9.0173          | 0.1573          | 2.462           |
| 30          | 0.52395         | 8.6602 | 7.854           | 32.4091         | 0.5656          | 8.418           | 27.4481         | 0.4790          | 7.240           |
| 45          | 0.78539         | 5.0000 | 11.101          | 47.7260         | 0.8329          | 11.622          | 41.9870         | 0.7328          | 10.508          |
| 60          | 1.04719         | 2.8868 | 13.596          | 62.3067         | 1.0874          | 13.908          | 57.3197         | 1.0004          | 13.221          |
| 80          | 1.39626         | 0.8816 | 15.461          | 80.8932         | 1.4118          | 15.509          | 78.9151         | 1.3773          | 15.414          |
**Conclusion**

This study focuses on the issue of achieving high-quality production of FRP composite frames. The winding process covers several layers of fiber rovings that are wound onto an open or closed non-load bearing frame. Ensuring the homogeneity and desired winding angle of each layer is a prerequisite for producing a quality composite frame. It is aimed to introduce the procedure to determine the optimal number of spools placed on a coil ring (i.e. the number of wound fiber rovings), and the width of the roving, which depend on the winding angle and radius of the frame. The number of rovings increases with the growth of the winding angle. It is shown that through the optimal determination of the number of rovings used along with rovings width, the possibility of material savings of up to 20% could be achieved. On the other hand, fiber winding optimization of curved frames, which often forms part of the torus, was developed and discussed. The optimized winding of rovings of such a curved frame is determined based on the geometric parameters of the torus. It is shown that, for a larger radius of the torus central-axis compared to the radius of the frame, a better-homogenized winding could be achieved. The winding parameters are chosen such that no gaps are created during winding on the outer circumference of the torus and at the same time roving overlaps are minimized on the inner circumference of the torus. A few experimental tests are also demonstrated in which the results of the test calculations are close to the experiment data.

In this study, the quality of winding is solved mainly from a geometric point of view. Ensuring the correct winding geometry is a necessary prerequisite for producing a quality fiber reinforcement for the composite frame. The procedures outlined in the article make it possible to determine the necessary parameters for the specific task of rovings winding onto the frame and thus avoid unsuitable set of parameters of the winding process.

Future research activities will focus on detailed testing of the physical performance of the composite frame under various static and fatigue loads. At the same time, the research will be extended to the generalization of the findings presented in this article to composite frames with a cross-section in the shape of an ellipse, triangle, rectangle, trapezoid, etc.

**Declaration of Conflicting Interests**

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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**Table 5.** Values of winding angles on the outer and inner circumference of the torus and the distance between the central axes of two adjacent wound rovings depend on the specified radius \( r \) of the torus.

| \( r \) (mm) | \( d \) (mm) | \( \alpha_{outer} \) (°) | \( d_{outer} \) (mm) | \( \alpha_{internal} \) (°) | \( d_{internal} \) (mm) |
|-------------|--------------|-----------------|-------------------|-----------------|-----------------|
| 20          | 11.107       | 47.7263         | 11.622            | 41.9872         | 10.508          |
| 50          | 27.768       | 51.3402         | 30.664            | 36.8699         | 23.561          |
| 100         | 55.536       | 56.3099         | 65.349            | 26.5551         | 35.111          |
| 150         | 83.304       | 60.2554         | 102.287           | 14.0362         | 29.452          |
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References

1. Mlýnek J, Rahimian Koloor SS and Martinec T. Fabrication of high-quality straight-line polymer composite frame with different radius parts using fiber winding process. Polymers 2021; 13(4): 497.
2. Zhang R, Li Z, Sun Q, et al. Design and characterization of the carbon fiber tube reinforced polymer composite for full ocean depth submersibles. Compos Sci Technol 2022; 217: 109074.
3. Abdi B, Koloor SSR, Abdullah MR, et al. Effect of strain-rate on flexural behavior of composite sandwich panel. In: Applied mechanics and materials. Trans Tech Publ, 2012.
4. Gay D. Composite materials: design and applications. 3rd ed. Taylor & Francis, 2014.
5. Koloor SSR, Karimzadeh A, Abdullah MR, et al. Linear-nonlinear stiffness responses of carbon fiber-reinforced polymer composite materials and structures: a numerical study. Polymers 2021; 13(3): 344.
6. Koloor S, Abdul-Latif A and Tamin MN. Mechanics of composite delamination under flexural loading. In: Key engineering materials. Trans Tech Publ, 2011.
7. Kashyzadeh KR, Koloor SSR, Omidi Bidgoli M, et al. An optimum fatigue design of polymer composite compressed natural gas tank using hybrid finite element-response surface methods. Polymers 2021; 13(4): 483.
8. Martinec T, Mlýnek J and Petrů M, Calculation of the robot trajectory for the optimum directional orientation of fibre placement in the manufacture of composite profile frames. Robot Comput-Integr Manuf 2015; 35: 42–54.
9. Mlýnek J, Petru M, Martinec T, et al. Fabrication of high-quality polymer composite frame by a new method of fiber winding process. Polymers 2020; 12(5): 1037.
10. Mlýnek J, Petrů M, Martinec T, et al. Design of composite frames used in agricultural machinery. In: Proceedings of the 7th TAE, Prague, Czech Republic, 17–20 September 2019.
11. Bahrami A, Soltani N, Pech-Canul M, et al. Development of metal-matrix composites from industrial/agricultural waste materials and their derivatives. Crit Rev Environ Sci Technol 2016; 46(2): 143–208.
12. Fu L, Zhang S, Cao G, et al. Strength design of tubular textile composites for pipeline rehabilitation under internal pressure. *Int J Press Ves Pip* 2022; 195: 104572.

13. Eschler E, Miadowitz T, Zaremba S, et al. Design optimization of a braided roof frame reinforcement by process-integrated local customization of component properties. *Appl Compos Mater* 2020; 27: 1–17.

14. Bodea S, Zechmeister C, Dambrosio N, et al. Robotic coreless filament winding for hyperboloid tubular composite components in construction. *Automat Constr* 2021; 126: 103649.

15. Kyosev Y. *Braiding technology for textiles: principles, design and processes*. Elsevier Science, 2014.

16. Arold B, Gessler A, Metzner C, et al. Braiding processes for composites manufacture. In: *Advances in composites manufacturing and process design*. Elsevier, 2015, 3–26.

17. Thomas GP. *Braided fibers - manufacturing, benefits and applications*. AZoM, 2021.

18. Bilisik K, Karaduman NS and Bilisik NE. Applications of braided structures in transportation. In: *Braided structures and composites: production properties mechanics and technical applications*. Milton Park: Taylor & Francis, 2015.

19. Emonts C, Grigat N, Merkord F, et al. Innovation in 3D braiding technology and its applications. *Textiles* 2021; 1(2): 185–205.

20. Yuksekkaya ME and Adanur S. Analysis of polymeric braided tubular structures intended for medical applications. *Textile Res J* 2009; 79(2): 99–109.

21. Rojas EV, Chapelle D, Perreux D, et al. Unified approach of filament winding applied to complex shape mandrels. *Compos Struct* 2014; 116: 805–813.

22. Skinner ML. Trends, advances and innovations in filament winding. *Reinf Plast* 2006; 50(2): 28–33.

23. Peters ST. *Composite filament winding*. ASM International, 2011.

24. Srivastava S and Hoda S. A brief theory on latest trend of filament winding machine. *Int J Adv Eng Res Sci* 2016; 3(4): 258859.

25. Sofi T, Neunkirchen S, Schleidewski R, et al. Path calculation, technology and opportunities in dry fiber winding: a review. *Adv Manuf: Polym Compos Sci* 2018; 4(3): 57–72.

26. Laval C. CADWIND 2006–20 years of filament winding experience. *Reinf Plast* 2006; 50(2): 34–37.

27. Lossie M and Van Brussel H. Design principles in filament winding. *Compos Manuf* 1994; 5(1): 5–13.

28. Duan Z, Yan J, Lee I, et al. A two-step optimization scheme based on equivalent stiffness parameters for forcing convexity of fiber winding angle in composite frames. *Struct Multidiscip Optim* 2019; 59(6): 2111–2129.

29. Yan J, Duan Z, Lund E, et al. Concurrent multi-scale design optimization of composite frame structures using the Heaviside penalization of discrete material model. *Acta Mechanica Sinica* 2016; 32(3): 430–441.

30. Duan Z, Jung Y, Yan J, et al. Reliability-based multi-scale design optimization of composite frames considering structural compliance and manufacturing constraints. *Struct Multidiscip Optim* 2020; 61(6): 2401–2421.

31. Koussios S. Integral design for filament winding—materials, winding patterns, and roving dimensions for optimal pressure vessels. *Composite Filament Winding. Composites* 2011; 3: 19–34.
32. Zeng W, Hu W, Liu H, et al. Finite element analysis of glass fiber winding molding of HDPE pressure vessel. *J Phys: Conf Ser* 2021; 1965.
33. Guo Y, Péreza MG, Serhat G, et al. A design methodology for fiber layup optimization of filament wound structural components. *Structures* 2022; 38.
34. Prado M. Computational design of fiber composite tower structures, 2020.
35. Saba AM, Khan AH, Akhtar MN, et al. Strength and flexural behavior of steel fiber and silica fume incorporated self-compacting concrete. *J Mater Res Technol* 2021; 12: 1380–1390.
36. Supian A, Sapuan S, Zuhri M, et al. Effect of winding orientation on energy absorption and failure modes of filament wound kenaf/glass fibre reinforced epoxy hybrid composite tubes under intermediate-velocity impact (IVI) load. *J Mater Res Technol* 2021; 10: 1–14.
37. Gowid S, Mahdi E, Youssef SS, et al. Experimental investigation of the dynamic characteristics of wrapped and wound fiber and metal/fiber reinforced composite pipes. *Compos Struct* 2021; 276: 114569.
38. Li Z, Zhu H, Du C, et al. Experimental study on cracking behavior of steel fiber-reinforced concrete beams with BFRP bars under repeated loading. *Compos Struct* 2021; 267: 113878.
39. Quanjin M, Rejab MRM, Idris MS, et al. Design and optimize of 3-axis filament winding machine. *IOP Conf Ser: Mater Sci Eng* 2017; 257.
40. Quanjin M, Rejab RM, Idris MS, et al. Filament winding technique: SWOT analysis and applied favorable factors. *SCIREA J Mech Eng* 2019; 3(1): 1–25.
41. Zhao L, Mantell SC, Cohen D, et al. Finite element modeling of the filament winding process. *Compos Struct* 2001; 52(3–4): 499–510.
42. Biron M. *Thermoplastics and thermoplastic composites*. Elsevier Science, 2012.
43. Shifrin T. *Differential geometry: a first course in curves and surfaces*. University of Georgia, 2015.
44. Watson A. *Textbook of Geometry*. NY RESEARCH Press, 2020.