DUALITY IN ASYMMETRIC QUANTUM OPTICAL RAMSEY INTERFEROMETERS

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A formalism have been recently derived [J. Martinez-Linares and D. Harmin, quantum-ph/0306057] allowing one to separate different sources of which-way information contributing to the total distinguishability $D$ of the ways in a two-way interferometer. Here we apply the formalism to a Quantum Optical Ramsey Interferometer where both sources, the a-priori predictability of the ways $P$ and the quantum "Quality" $Q$ of the which-way detector, stems from the same physical interaction. We show that the formalism is able to separate both sources of which-way information. Moreover, it is shown that $Q$ succeeds in quantifying the amount of quantum which-way information stored in the which-way detector even in cases where $D$ does not.

1 Introduction

The duality principle is at the core of the fundamentals of Quantum Mechanics since its foundations 1. In the last decade, new results have been found stressing the role of quantum correlations in the building of quantum which-way information (WWI) contributing to the distinguishability of the ways. In fact, quantum correlations can explain the disappearance of fringes even in situations where the usual Heisenberg relations cannot 2. Englert 3 quantifies the total distinguishability of the ways that can be potentially available to the experimenter by a parameter $D$ and connects it to the fringe visibility $V$ measured at the output port of a two-ways interferometer. Both quantities are related by the inequality

$$D^2 + V^2 \leq 1,$$

stating to which degree both distinguishability (particle like information) and visibility (wave-like information) are compatible. Equation (1) can therefore be interpreted as an expression of duality.

However, two different sources of WWI are represented in $D$. One is the Predictability $P$ of the ways, i.e., the a-priori WWI that the experimenter has about the ways. It stems from the preparation of the beam splitter (BS)
and the initial state of the two-level system (Quanton). The second source is purely quantum-mechanical, stemming from the quantum correlations established between the Quanton and the WWD, leading to the storage of WWI in the final state of the which-way detector (WWD). In recent work, from now on cited as [I], a measure of the latter contribution has been founded. This measure has been named the ”Quality” $Q$ of the WWD, since it tells us how ”good” the detector is, i.e., it quantifies the WWD’s ability to quantum correlate its final state with the Quanton’s alternatives. Both particle-like information measures, $P$ and $Q$, are related to the fringe visibility $V$ through the expression

$$\left(1 - P^2\right) Q^2 + P^2 + V^2 \leq 1.$$  \hspace{1cm} (2)

The above equation is also an expression of duality, since it relates fringe degradation to the availability of different sources of WWI. In the case we have only one source of WWI, i.e., $P = 0$ (or $Q = 0$), we obtain $D = Q$ (or $D = P$), and Eq. (2) devolves into Eq. (1).

Asymmetric interferometers are not uncommon. A number of proposed experiments are essentially asymmetric. For instance the Einstein recoiling slit in a Young double-slit interferometer, the Quantum optical Ramsey Interferometer (QORI) outlined by Englert \cite{Englert}, and the experiment by Haroche group \cite{Haroche}, in which beam splitting is performed by the quantized cavity-mode of a high finesse resonator. In all these cases, both BS and WWD are provided by the same physical interaction. The asymmetry on such devices is directly coupled to the ability of the WWD to get entangled with the Quanton. In [I], we applied the formalism to a quantum logic gate, the Symmetric Quanton Detecton System (SQDS), in which BS and WWD were physically independent. Now, we apply in this paper the formalism to a QORI. We will show that the formalism is able to separate both contributions $Q$ and $P$ even though they are both provided the same physical source.

2 The Quantum Optical Ramsey interferometer

In this section we specialize to the case of the quantum optical Ramsey interferometer described by Englert \textit{et al}. A schematic two-way interferometer is depicted in Fig. 1(a). The beam splitter (BS) distributes the input states between the 2 ways, which become entangled with the state of the quantum WWD. The phase shifter (PS) induces a state dependent shift $\pm \phi/2$. The beam merger (BM) recombines the contributions into the final state of the quantum. Measurements of the output build a fringe pattern versus variation of $\phi$.

In the case of a QORI, we consider a two-level atom to be the Quanton and a high finesse resonator to act jointly as a which-way detector (WWD) and a beam splitter (BS) [see Fig. 1(b)]. Before entering the resonator, the
Figure 1. (a) Schematic two-way interferometer setup. (b) Quantum Optical Ramsey interferometer. The ±-ways are realized by electronic states \( |a\rangle, |b\rangle \) of a 2-level Rydberg atom. The atomic transition is resonant with a cavity mode of a high finesse resonator acting as both BS and WWD. The PS is realized by a dc electric field, which Stark shifts the atomic levels. The BM is provided by a classical microwave resonant field performing a \( \pi/2 \) pulse on the state of the atom. The final output is measured in M by means of a field ionization state-selective technique (FISST).

atom is prepared in the upper level \( |a\rangle \). The atom interacts with the resonator, adding a photon to its quantized cavity mode if a resonant transition to the lower level \( |b\rangle \) occurs. Due to the high finesse of the resonator, the cavity field can keep track of the way taken by the atom since it can store the energy quantum liberated in the atomic transition. Thus, the same interaction both splits the beam and makes the two “ways” distinguishable. Next is the turn of the phase shifter (PS)—in the guise, for example, of an external electrostatic dc field applied at the central stage of the interferometer. The differential Stark shift between upper and lower levels induces a relative phase \( \phi \) that can be controlled externally upon variation of the strength of the applied potential. Finally, a classical microwave field at the port of the interferometer supplies the beam merger (BM), effecting a \( \pi/2 \) pulse after resonant interaction with the atom. The final state of the atom after crossing the interferometer is measured by means of state-selective field ionization techniques at M in Fig. 1(b).

By varying the phase \( \varphi \) in successive repetitions of the experiment, a fringe pattern can be built up in the detected probability for the atom to wind up in one state or the other. It is worth stressing that the atomic center-of-mass wavefunction is irrelevant for this setup, since it remains essentially unaffected by the quantum optical Ramsey field crossed by the atom. Therefore, the two paths refer exclusively to the internal electronic states of the atom, not to the actual trajectory of their center of mass.
As commented in the introduction, in this system both the BS and the WWD are provided by the same physical interaction. Thus, different preparations of the cavity field in the resonator would lead to different predictabilities and, in turn, to different degrees of quantum entanglement with the atoms. Our task is to quantify the two contributions to the distinguishability: the one stemming from quantum correlations established after the interaction between atom and detector, and the other associated with the asymmetry of the ways resulting from this operation. Both contributions are related to the fringe visibility at the output port of the interferometer by the duality expression

\[(1 - \mathcal{P}^2) Q^2 + \mathcal{P}^2 + V^2 = 1, \quad (3)\]

which invokes Eq. (2) for a system prepared initially in a pure state.\(^5\)

Phase-sensitive micromaser schemes have been extensively studied in the literature.\(^9\) The validity of (1) when applied to a QORI has been demonstrated by Englert\(^14\). The analysis of duality for the QORI is very similar to that presented in [I], once we remove the unitary constraint on the operators \(U^\pm\) in Eq. (5) of [I]. As a matter of fact, choosing for simplicity the initial pure Quanton state Bloch vector \(s_Q^{(0)} = (0, 0, 1)\), it is easy to check that Eq. (9) in [I] does give the fringe visibility contrast factor for the QORI, once the nonunitary replacements

\[U_+ \rightarrow \sqrt{2} C^\dagger, \]
\[U_- \rightarrow \sqrt{2} S a, \quad (4)\]

have been taken. Here we have abbreviated the Jaynes-Cummings operators.\(^15\)

\[S = \frac{\sin \left(\Omega \tau \sqrt{a a^\dagger + \lambda^2}\right)}{\sqrt{a a^\dagger + \lambda^2}} = S^\dagger, \]
\[C = \cos \left(\Omega \tau \sqrt{a a^\dagger + \lambda^2}\right) + i \lambda S, \quad (5)\]

where \(a, a^\dagger\) are the standard photon operators of the cavity mode, \(\tau\) the interaction time (i.e., the time of flight of the atom through the resonator), and \(\lambda\) is a detuning parameter normalized to twice the Rabi frequency \(\Omega\) of the atomic transition. In addition, Eq. (17) in [I] is still valid once we recall that the origin of the asymmetry in the predictability is the BS action, and not the initial polarization of the Quanton as was the case in Eqs. (13)–(14) in [I]. The computation of the relevant quantities of the formalism is now straightforward. In fact, it suffices to compute

\[w_+ = \langle C^\dagger C \rangle_0, \]
\[w_- = \langle S^2 a a^\dagger \rangle_0, \]
\[V = 2 \langle S a C \rangle_0, \quad (6)\]
where \( w_\pm \) are the probabilities that the atom takes the upper or lower way after the BS, and the averages are taken over the initial state of the cavity-mode.

The observation of an interference pattern in the output \( \mathbf{M} \), depends on the initial field state \( \rho_D^{(0)} \) prepared in the resonator. If it is unable to acquire which-way information, quantum interference is observable, due to the indistinguishability of the paths leading to the same final state. On the other hand, if the state of the detector becomes perfectly correlated with the particular path chosen by the atom (for instance when prepared initially in a Fock state), then no fringes are observable as implied by duality. In order to study the transition between these limits, we consider the cavity mode prepared in a pure coherent state with mean photon number \( \bar{n}_0 \geq 0 \) up to large values of \( \bar{n}_0 \) (e.g., 100). The numerical results for \( \mathcal{P}^2, Q^2, \) and \( V^2 \), calculated according to Eqs. (6) and (3) in this paper, are plotted in Fig. 2.

We also compare the results with the Englerts distinguishability satisfying

\[
\mathcal{D}^2 = (1 - \mathcal{P}^2) Q^2 + \mathcal{P}^2 = 1 - V^2, \tag{7}
\]

as shown in Eq. (31) of [I] for the case of pure state preparation.

The symmetrical case is illustrated in Fig. 2(a), where \( Q^2 = \mathcal{D}^2 \) (superposed thick lines) and \( V^2 \) (thin line) are plotted at resonance (\( \lambda = 0 \)). The values of the vacuum Rabi phase \( \Omega \tau \) have been optimized for each \( \bar{n}_0 \) in order to assure symmetrical operation of the BS (i.e., \( \mathcal{P} = 0 \)). The plot illustrates the loss of coherence induced by the acquisition of WWI. In the limit of high intensity of the cavity field, we have \( s_Q^2 = V^2 \to 1 \) and \( Q \to 0 \). This behavior can be easily understood, since the pure-state condition and the perfect fringe visibility that has been reached in this limit are incompatible with any storage of which-way information in the detector (see Eq. (41) in [I] and below). On the other hand, total loss of coherence is achieved for \( \bar{n}_0 = 0 \) (initial vacuum Fock state). In this case, the detector and the atom evolve into the maximally entangled state

\[
\frac{1}{\sqrt{2}} \left\{ |0\rangle_D |a\rangle_Q + i |1\rangle_D |b\rangle_Q \right\}. \tag{8}
\]

Perfect correlations are established between atom and detector, leading to maximum which-way information storage in the cavity field. Notice that the state (8), after tracing over the detector degrees of freedom, describes a totally unpolarized ensemble, with \( \rho_Q = \frac{1}{2} I \) and \( s_Q^2 = 0 \). We can say that all the information about the polarization of the atom has been transferred to the detector.

The difference between \( Q \) and \( \mathcal{D} \) becomes apparent in the case \( \mathcal{P} \neq 0 \) [see Eq. (7)]. Actually, \( Q \) can quantify the amount of which-way information that can be contained in the detector even in cases where \( \mathcal{D} \) cannot. This is illustrated in Fig. 2(b), where the same quantities as in Fig. 2(a) have been plotted for \( \mathcal{P} = 1 \). As can be seen in this figure, \( \mathcal{D} \) is tied to the predictability by
Figure 2. $Q^2$ (thick solid line), $D^2$ (thick dot-dashed line) and $V^2$ (thin solid line) for the quantum optical Ramsey interferometer. The cavity mode of the resonator is prepared in a coherent state with mean photon number $\bar{n}_0$. (a) Interferometer under resonant ($\lambda = 0$) and symmetrical ($P = 0$) operation, showing $Q^2 = D^2 = 1 - V^2$ ($Q$ and $D$ coincide, thick lines). The $P = 0$ condition is achieved by optimizing the value of $\Omega \tau$ to assure symmetrical operation of the BS for each $\bar{n}_0$. (b) Interferometer under resonant ($\lambda = 0$) and maximally asymmetrical operation ($P = 1$), so $V = 0$. In this case, we have optimized the value of $\Omega \tau$ so that $P = 1$ is achieved for each $\bar{n}_0$. Here $Q$ and $D$ are clearly different: $D \geq P = 1$ saturates $D$ to the value of the predictability. On the other hand, $Q$ succeeds in quantifying the quantum properties of the detector, even in this case of extreme predictability.

the inequality $D \geq P = \frac{1}{3}$ no matter the degree of which-way information stored in the state of the detector. This WWI is however still present in $Q$, which keeps invariant with respect to Fig. 3(a). This property is brought about by the special structure of the left hand side of relation (2), which cancels the contribution of $Q$ ($P$) in the case $P$ ($Q$) becomes maximum.

Notice that for $\bar{n}_0 = 0$ we have both $Q = 1$ and $P = 1$. Full WWI is stored in the quantum state of the cavity-mode of the resonator even in this extreme asymmetric case $P = 1$, that can be associated to an interferometer with a single way situation ($w_+ = 1$ or $w_- = 1$). This fact can be understood since the final state of the system (empty cavity $|0\rangle_D|a\rangle_Q$ or cavity with one photon $|1\rangle_D|b\rangle_Q$) continues to be perfectly correlated with the ways taken by
the Quanton (upper or lower way).

3 Summary

We consider in this paper a Quantum Optical Ramsey Interferometer (QORI), for which the Jaynes-Cummings interaction between an atom and a quantized mode of a cavity field supplies both the beam splitting and the WWD. We have applied our formalism to the QORI in order to elucidate the interplay between $Q$, $P$, and $D$ resulting from this coupling. The value of $Q$ can differ substantially from the actual value of $D$ in this essentially asymmetric interferometer, since the latter also involves the a-priori which-way information stemming from the imbalanced beam splitting involved in the interaction. We have shown that the formalism is able to separate both contributions. In fact, it is shown that the parameter $Q$ characterizes solely the quantum properties of the which-way detector. Moreover, it is shown that $Q$ quantifies the amount of which-way information that can be stored in the detector even in cases where $D$ cannot, e.g., when the latter is saturated by the a-priori source of WWI represented by $P$.

The observation of this effect is experimentally feasible. As a matter of fact, the visibility in Fig. 1(a) have been actually measured experimentally in the strong-coupling limit, $\Omega \tau \sim 1$, by Bertet et al\[9\] for cavity field mean photon number ranging between 0 and 14.

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