Decay of flux vacua to nothing

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Abstract

We construct instanton solutions describing the decay of flux compactifications of a 6d gauge theory by generalizing the Kaluza-Klein bubble of nothing. The surface of the bubble is described by a smooth magnetically charged solitonic brane whose asymptotic flux is precisely that responsible for stabilizing the 4d compactification. We describe several instances of bubble geometries for the various vacua occurring in a 6d Einstein-Maxwell theory namely, $AdS_4 \times S^2$, $\mathbb{R}^{1,3} \times S^2$, and $dS_4 \times S^2$. Unlike conventional solutions, the bubbles of nothing introduced here occur where a two-sphere compactification manifold homogeneously degenerates.
I. INTRODUCTION

The necessity of extra dimensions has strong theoretical backing in the context of string theory, but stabilizing the shape and size moduli of the compactification manifold has historically been one of the most challenging obstacles for realistic model building. Field theories with higher rank fluxes wound on internal cycles were proposed long ago as a remedy to this problem [1–4]. Similar mechanisms have been incorporated in string theory compactifications [5–7] which suggest the existence of a tremendous multitude of stable and metastable vacua, the so-called string landscape [8]. The interplay between these solutions and eternal inflation [9, 10] opens the possibility for transitions between the various flux-vacua [11–15]. Furthermore, it has been found that there exist more exotic classes of transitions which change the effective dimensionality of spacetime [13, 16–19]. Although work on these transitions is in its early stages, already it appears they may have interesting theoretical [20] and observational [21–24] consequences.

In this paper we generalize a new decay channel that has recently been shown to exist in axionic flux compactifications [25]. This instability renders vacua susceptible to decay via the nucleation of a generalized bubble of nothing [26], one that is charged with respect to the flux which induces the spontaneous compactification (See also [14, 27] for a discussion of related ideas).

This paper is organized as follows. In section II we discuss the 6d Einstein-Maxwell landscape. In section III we embed the Maxwell theory in the simplest non-abelian gauge theory, yielding the Einstein-Yang-Mills-Higgs model of SU(2). In section IV we describe new instanton configurations in detail and provide explicit numerical examples within a family of solutions. Finally, we conclude in section V.

II. THE EINSTEIN-MAXWELL LANDSCAPE IN 6d

The Einstein-Maxwell theory in 6d [3] is a remarkably simple model which nevertheless enjoys many important features of more realistic flux compactifications of string theory [7]. The action is given by

\[ S = \int d^6 x \sqrt{-g} \left( \frac{1}{2k^2} R - \frac{1}{4} F_{MN} F^{MN} - \Lambda \right), \]
where our conventions are as follows. Six dimensional indices are indicated with capital latin letters, \( M,N = 0 \ldots 5 \). The 6\( d \) reduced Planck mass is written \( M(6) = 1/\sqrt{\kappa} \), and \( \Lambda \) is the six dimensional cosmological constant, which we will assume to be non-negative.

This model was explored in detail in \[13, 19, 20\], where it was shown to possess distinct families of flux compactifications: a magnetic sector with geometry \((A)dS_4 \times S^2\) or \(\mathbb{R}^{1,3} \times S^2\), an electric sector with spacetime \(AdS_2 \times S^4\), and a higher dimensional vacuum with no flux, \(dS_6\). Several possible transitions between these sectors were discussed in \[13–15, 18, 19\], which suggest the existence of a complex multi-dimensional landscape even in this simple model.

Here we study a new decay channel for the 4\( d \) flux vacua, the nucleation of a bubble of nothing \[26\]. The portion of this landscape under consideration is the magnetic sector (four large dimensions), which we will now review.

The equations of motion obtained from the action in Eq. (1) are

\[
R_{MN} - \frac{1}{2} g_{MN} R = \kappa^2 T_{MN},
\]

\[
\frac{1}{\sqrt{-g}} \partial_M \left( \sqrt{-g} F^{MN} \right) = 0,
\]

with energy-momentum tensor

\[
T_{MN} = g^{LP} F_{ML} F_{NP} - \frac{1}{4} g_{MN} F^2 - g_{MN} \Lambda.
\]

In the magnetic sector, the metric takes the form

\[
ds^2 = g_{MN} dx^M dx^N = g_{\mu\nu} dx^\mu dx^\nu + C^2 d\Omega_2^2,
\]

where \( g_{\mu\nu} \) describes a four dimensional maximally symmetric space,\(^1\) and the compactification manifold is a 2-sphere of radius \( C \).

The field strength in this sector is given by the monopole-type configuration \[3\],

\[
F_{\theta\phi} = -F_{\phi\theta} = \frac{n}{2e} \sin \theta,
\]

which respects the chosen isometries of the metric, and saturates the Dirac quantization condition \( \int_{S^2} F = 2\pi n/e \), where \( n \in \mathbb{Z} \) and \( e \) is the quantum of electric charge. With this

\(^1\) The 4\( d \) part of the metric has Ricci scalar \( R^{(4)} = 12H^2 \), where \( H^2 \) may be negative.
ansatz, the electromagnetic equations of motion are automatically satisfied, and the Einstein equations lead to the relations for $H$ and $C$

$$3H^2 + \frac{1}{C^2} = \kappa^2 \left( \frac{n^2}{8e^2C^4} + \Lambda \right),$$  \hspace{1cm} (7)

$$6H^2 = \kappa^2 \left( \Lambda - \frac{n^2}{8e^2C^4} \right).$$  \hspace{1cm} (8)

This can be solved in terms of the parameters of the 6d theory and the magnetic flux number $n$, yielding the solutions

$$C^2 = \frac{1}{\kappa^2\Lambda} \left( 1 \mp \sqrt{1 - \frac{3n^2}{4n_0^2}} \right),$$

$$H^2 = \frac{2\kappa^2\Lambda}{9} \left[ 1 - \frac{2n_0^2}{3n^2} \left( 1 \pm \sqrt{1 - \frac{3n^2}{4n_0^2}} \right) \right],$$  \hspace{1cm} (9)

where we have defined

$$n_0^2 = \frac{2e^2}{\kappa^4\Lambda}.\hspace{1cm} (10)$$

The twofold existence of solutions when $\Lambda > 0$ can be understood by looking at Fig. 1, the 4d effective potential for the radion, which governs the size of the extra dimensions. Following [13], we generalize the six dimensional metric ansatz to

$$ds^2 = g_{MN} dx^M dx^N = e^{-\psi(x)/M_P} g^{(4)}_{\mu\nu} dx^\mu dx^\nu + e^{\psi(x)/M_P} C_0^2 d\Omega^2_2,$$  \hspace{1cm} (11)

with $C_0 = 1/\sqrt{2\kappa^2\Lambda}$. Together with the monopole-type configuration for the Maxwell field, this ansatz allows integration of the full 6d action over the internal manifold, yielding a 4d effective theory with low energy action

$$S = \int d^4x \sqrt{-g^{(4)}} \left( \frac{1}{2} M_P^2 R^{(4)} - \frac{1}{2} \partial_\mu \psi \partial^\mu \psi - V(\psi) \right),$$  \hspace{1cm} (12)

with the potential for the canonical radion $\psi$ given by

$$V(\psi) = \frac{4\pi}{\kappa^2} \left( \frac{n^2}{2n_0^2} e^{-3\psi/M_P} - e^{-2\psi/M_P} + \frac{1}{2} e^{-\psi/M_P} \right).$$  \hspace{1cm} (13)

The 4d Planck mass $M_P$ is dependent on the volume of the compactification manifold via $M_P^2 = 4\pi C^2/\kappa^2$. In Fig. 1, we plot the effective potential $V(\psi)$ for three choices of flux number $n$. We can immediately see that at most one of the two solutions shown in Eqs. (9) can be stable, while the other, once perturbed, will roll to either the stable solution or
decompactification. Henceforth, we consider only the stable solutions to the equations of motion.²

Transitions between flux vacua are mediated by instantons constructed [13] from the magnetically charged black 2-branes known to exist in the spectrum of the theory [29,30]. On the other hand, it was recently suggested in [25] that there should be a special transition that would decrease the flux number of the compactification to zero. It is clear that if such a transition occurs, there will be no obstacle for the internal geometry to collapse and create a large coordinate region of volume measure zero, a bubble of nothing [26]. Furthermore, the surface of this bubble must act as a source for the magnetic flux present in the asymptotic region of the compactification, and so we generalize the bubble of nothing to include this charge. Recently, two of us have demonstrated the existence of charged bubbles of nothing in a simple axionic flux compactification. In that fully backreacting 5d solution, the surface of the bubble is a de Sitter vortex charged with respect to the axion [25]. In this paper, we generalize the instability to the more realistic landscape of the 6d Einstein-Maxwell theory. One clear candidate for the bubble of nothing is a generalization of the well-known

² Perturbative stability of flux compactifications has been discussed previously in [28].
codimension three Dirac monopole. We require a solution where the extra-dimensional spacetime is smooth everywhere, in particular in the region where the 2-sphere degenerates to zero size. This is difficult to achieve in the Einstein-Maxwell model, since it seems to inevitably lead to a singularity at the location of the monopole. We solve this problem in a natural way by introducing new degrees of freedom which resolve the singularity: by embedding the model in a non-abelian gauge theory which is known to possess smooth magnetically charged solitons of codimension three, the Yang-Mills-Higgs model [31, 32], which we will now review.

III. THE EINSTEIN-YANG-MILLS-HIGGS LANDSCAPE

One can imagine an embedding of the Einstein-Maxwell theory presented in the previous section into more complicated models which include new degrees of freedom only in the UV, and so would not distort the landscape of 4d flux vacua computed previously. Here we realize this with a specific Einstein-Yang-Mills SU(2) model with an adjoint Higgs breaking the gauge symmetry to U(1), which we identify with the Maxwell field described above. This is one of the first flux compactification models described in the literature [1], and as we will see, it is well suited to our goal of finding a UV completion of the Einstein-Maxwell flux vacuum instability known as a bubble of nothing.

The model is defined by the action

$$S = \int d^6x \sqrt{-g} \left( \frac{1}{2\kappa^2} R - \frac{1}{4} F^a_{MN} F^{aMN} - \frac{1}{2} D_M \Phi^a D^M \Phi^a - V(\Phi) - \Lambda \right),$$

(14)

with

$$V(\Phi) = \frac{\lambda}{4} (\Phi^a \Phi^a - \eta^2)^2,$$

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + e\epsilon^{abc} A^b_M A^c_N,$$

$$D_M \Phi^a = \partial_M \Phi^a + e\epsilon^{abc} A^b_M \Phi^c.$$

(15)

Varying the action with respect to the fields yields the equations of motion

$$R_{AB} - \frac{1}{2} g_{AB} R = \kappa^2 T_{AB},$$

(16)

$$\frac{1}{\sqrt{-g}} D_M \left( \sqrt{-g} D^M \Phi \right)^a = \lambda \Phi^a \left( \Phi^b \Phi^b - \eta^2 \right)^2,$$

(17)

$$\frac{1}{\sqrt{-g}} D_N \left( \sqrt{-g} F^{MN} \right)^a = e\epsilon^{abc} \left( D^M \Phi^b \right) \Phi^c,$$

(18)
where the energy-momentum tensor is given by

\[ T_{AB} = D_A \Phi^a D_B \Phi^a + F_{AM}^a F_{B}^{aM} + g_{AB} \mathcal{L}, \]  

(19)

with

\[ \mathcal{L} = -\frac{1}{2} D_A \Phi^a D^A \Phi^a - \frac{1}{4} F_{MN}^a F^{aMN} - V(\Phi) - \Lambda. \]  

(20)

A. Compactification solutions

In Cremmer et al. [1] it was shown that the preceeding equations lead to a spontaneous compactification of the 6d spacetime after turning on a monopole-type flux in the spontaneously broken gauge theory, similar to what was presented in the abelian case of the previous section. Cremmer et al. restricted themselves to the flat 4d spacetime \( \mathbb{R}^{1,3} \times S^2 \), and although their work discussed only the \( n = 1 \) flux compactification, they managed to find several types of solutions. Here we generalize such compactifications to arbitrary integer \( n \) flux vacua by choosing a matter field ansatz

\[ \Phi^a = \eta \, p_c (\sin \theta \cos n \phi, \sin \theta \sin n \phi, \cos \theta), \]
\[ A^\mu_a = A^a_r = 0, \]
\[ A^a_\theta = \left( 1 - \frac{w_c}{e} \right) (\sin n \phi, -\cos n \phi, 0), \]
\[ A^a_\phi = \frac{n (1 - w_c)}{e} \sin \theta (\cos \theta \cos n \phi, \cos \theta \sin n \phi, -\sin \theta), \]

(21)

with \( n \in \mathbb{Z} \). The suitability of this ansatz can be motivated by computing the topological charge for this configuration [33] via

\[ \frac{1}{4\pi} \int d\theta d\phi |\Phi|^{-3} \epsilon_{abc} \Phi^a \partial_\theta \Phi^b \partial_\phi \Phi^c = n, \]

(22)

where \( |\Phi| = \sqrt{\Phi^a \Phi^a} \).

Interestingly, for \( n > 1 \) the equations of motion constrain the possible values of the constants in the ansatz Eq. (21) to be, \( p_c = 1 \) and \( w_c = 0 \). The covariant derivative for the scalar triplet then vanishes, and the energy momentum tensor induced by this configuration

\[ \text{In Appendix A we discuss in detail some of the peculiar properties of this type of compactification which are special to } n = 1. \]
is precisely that found in the abelian flux vacua. This can be understood by looking at the
form of the electromagnetic tensor \[ F_{MN} = \frac{\Phi^a}{|\Phi|} F^a_{MN} + \frac{1}{e|\Phi|^3} e^{abc} \Phi^a D_M \Phi^b D_N \Phi^c, \] (23)
which in this case becomes
\[ F_{\theta\phi} = \frac{n}{e} \sin \theta, \] (24)
and the equations of motion reduce to
\[ 3H^2 + \frac{1}{C^2} = \kappa^2 \left( \frac{n^2}{2e^2C^4} + \Lambda \right), \] (25)
\[ 6H^2 = \kappa^2 \left( \Lambda - \frac{n^2}{2e^2C^4} \right). \] (26)

Note that there is a small discrepancy in the definition of the coupling constant \( e \) with respect to the abelian case. The charge \( e \) here is twice the value of the same symbol appearing in the Maxwell theory. This is reconciled with saturation of Dirac’s charge quantization condition by noting that the smallest-charged particle in the non-abelian theory would be an \( SU(2) \) doublet, whose charge is equal to \( e/2 \) using the present non-abelian convention for \( e \). With this dictionary, the \( SU(2) \) theory is indistinguishable from the abelian theory in the IR.

We turn now to discussion of the non-perturbative decay of flux vacua by the generalized bubble of nothing, and so only consider the perturbatively stable solutions of the equations of motion. Following the arguments presented in the Einstein-Maxwell theory, these are specified by the two length scales
\[ C^2 = \frac{1}{\kappa^2\Lambda} \left( 1 - \sqrt{1 - \frac{3\kappa^4\Lambda n^2}{2e^2}} \right), \]
\[ H^2 = \frac{2\kappa^2\Lambda}{9} \left[ 1 - \frac{e^2}{3\Lambda\kappa^4n^2} \left( 1 + \sqrt{1 - \frac{3\kappa^4\Lambda n^2}{2e^2}} \right) \right]. \] (27)
The landscape of vacua is identical to the pure electromagnetic case, in particular we see that the theory has 4d compactifications \( AdS_4 \times S^2, \mathbb{R}^{1,3} \times S^2 \), and \( dS_4 \times S^2 \).

IV. BUBBLE OF NOTHING SOLUTIONS

Bubbles of nothing in a simple toy flux compactification were discussed in [25], where they were identified as solitonic defects whose intrinsic worldvolume is a codimension-two de Sitter space.
We are interested in finding similar objects in a higher dimensional spacetime where the compactification manifold is a 2-sphere. This leads us to the metric ansatz

\[ ds^2 = B^2(r)(-dt^2 + \cosh^2 t \, d\Omega^2_2) + dr^2 + C^2(r)(d\theta^2 + \sin^2 \theta \, d\phi^2) . \]

(28)

We are searching for solutions that describe the decay of flux compactifications to a bubble of nothing, i.e., solutions where the extra-dimensional space wound with magnetic flux degenerates to a point at some value of \( r \), which we gauge fix to \( r = 0 \). This implies the existence of a magnetic source at the degeneration loci. We satisfy this requirement by placing a solitonic magnetic brane centered at \( r = 0 \), making use of our UV completion of the low energy Einstein-Maxwell theory. An appropriate ansatz in this case is therefore the hedgehog configuration,

\[ \Phi^a = \eta \, p(r)(\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta) , \]

\[ A_\mu^a = A_\rho^a = 0 , \]

\[ A_\theta^a = \frac{1 - w(r)}{e}(\sin \varphi, - \cos \varphi, 0) , \]

\[ A_\varphi^a = \frac{1 - w(r)}{e} \sin \theta(\cos \theta \cos \varphi, \cos \theta \sin \varphi, - \sin \theta) . \]

(29)

For simplicity we are considering only KK-spherically symmetric solutions. This demands that we restrict to the case with \( n = \pm 1 \), since higher winding solutions are incompatible with spherical symmetry [34, 35]. (We do not expect any conceptual difficulty in finding higher \( n \) solutions of reduced symmetry, but they will be more challenging to construct numerically.)

Using the above ansatz, the equations of motion for the matter fields in Eqs. (17-18) become

\[ p'' + \left( 3 \frac{B'}{B} + 2 \frac{C'}{C} \right) p' - \frac{2w^2p}{C^2} - \lambda \eta^2 p(p^2 - 1) = 0 \]

(30)

and

\[ w'' + 3 \frac{B'}{B} w' + \frac{w(1 - w^2)}{C^2} - e^2 \eta^2 p^2 w = 0 . \]

(31)

The Einstein equations are

\[ G^0_0 = - \frac{1}{B^2} - \frac{1}{C^2} + \left( \frac{B'}{B} \right)^2 + 4 \frac{B'C'}{BC} + \left( \frac{C'}{C} \right)^2 + 2 \frac{B''}{B} + 2 \frac{C''}{C} = \kappa^2 T^0_0 , \]

\[ G^r_r = - \frac{3}{B^2} - \frac{1}{C^2} + 3 \left( \frac{B'}{B} \right)^2 + 6 \frac{B'C'}{BC} + \left( \frac{C'}{C} \right)^2 = \kappa^2 T^r_r , \]

\[ G^\theta_\theta = - \frac{3}{B^2} + 3 \left( \frac{B'}{B} \right)^2 + 3 \frac{B'C'}{BC} + 3 \frac{B''}{B} + \frac{C''}{C} = \kappa^2 T^\theta_\theta , \]

(32)
FIG. 2: Left: An \( n = 1 \) flux compactification, with hue representing wound flux. The \( S^2 \) compactification manifold is shown as an \( S^1 \), along with one of the four large dimensions. Right: A bubble of nothing occurs when the compactification manifold degenerates. The radion \( C(r) \) is plotted as a function of radial distance \( r \) along the large dimension. Saturation is given by the scalar field magnitude \( p(r) \), which along with \( C(r) \), vanishes at the core of the soliton.

with energy-momentum tensor specified by

\[
T^0_0 = -\left[ \eta^2 \left( \frac{p'^2}{2} + \frac{p^2 w^2}{C^2} \right) + \frac{1}{e^2 C^2} \left( w'^2 + \frac{(1 - w^2)^2}{2C^2} \right) + \frac{\lambda \eta^4}{4} (p^2 - 1)^2 + \Lambda \right],
\]

\[
T^r_r = \eta^2 \left( \frac{p'^2}{2} - \frac{p^2 w^2}{C^2} \right) + \frac{1}{e^2 C^2} \left( w'^2 - \frac{(1 - w^2)^2}{2C^2} \right) - \frac{\lambda \eta^4}{4} (p^2 - 1)^2 - \Lambda, \tag{33}
\]

\[
T^\theta_\theta = -\eta^2 \frac{p'^2}{2} + \frac{(1 - w^2)^2}{2e^2 C^4} - \frac{\lambda \eta^4}{4} (p^2 - 1)^2 - \Lambda.
\]

Below we will separately study the three different asymptotic 4\( d \) effective geometries, \( AdS_4 \), \( \mathbb{R}^{1,3} \), and \( dS_4 \). Notice that the asymptotic geometry is specified once one fixes the values of \( \Lambda \) and \( e \). Nevertheless, one may find qualitatively different solutions depending on the values of the other two fundamental parameters, \( \eta \) and \( \lambda \). Having explored the form of the solutions in this two dimensional parameter space, we will comment below on the different behaviors that one may encounter.

All the solutions we present in this paper have a magnetically charged soliton at \( r = 0 \), which because of the 2 + 1 dimensional de Sitter invariance of its world-volume, can be called inflating. Inflating braneworld solutions with similar asymptotic behavior to those presented here have been previously discussed in a different context in [36].

One can show that the most general smooth solution describing the soliton core has
expansion about \( r = 0 \) given by

\[
\begin{align*}
p(r) &= p_1 r + \cdots, \\
w(r) &= 1 + w_2 r^2 + \cdots, \\
B(r) &= B_0 + B_2 r^2 + \cdots, \\
C(r) &= r + C_3 r^3 + \cdots.
\end{align*}
\]

(34)

Using the equations of motion we can write all coefficients, \( B_2, C_3, \) etc., in terms of three locally undetermined constants, \( B_0, p_1, \) and \( w_2 \). We give the explicit form of these expansions in Appendix B. In the following sections we use the numerical technique known as the multiple shooting method to demonstrate the existence of a solution and determine the values of the coefficients \( B_0, p_1, \) and \( w_2 \), such that the matter and metric fields approach the asymptotic form of the appropriate flux compactification. We review the multiple shooting method in Appendix F.

A. The decay of \( \text{AdS}_4 \times S^2 \) vacua

The first compactification we consider is to \( \text{AdS}_4 \times S^2 \), which occurs for all values of \( n \) in a landscape with \( \Lambda \leq 0 \), as well as for \( n < e^2/(2\kappa^4 \Lambda) \), regardless of \( \Lambda \). Bubbles of nothing were studied \([25]\) in a much simpler landscape whose vacua are all of this type. The minimal case is \( \Lambda = 0 \), i.e., Freund-Rubin \([2]\), which we begin with here. In order to construct the bubble of nothing for this case, we impose boundary conditions compatible with the asymptotic compactification geometry. Within our \( SO(1,3) \times SO(3) \) invariant metric ansatz Eq. (28), the asymptotically \( \text{AdS}_4 \times S^2 \) solution is

\[
\begin{align*}
p(r) &\to 1, \\
w(r) &\to 0, \\
C(r) &\to C_\infty, \\
B'(r)/B(r) &\to |H|,
\end{align*}
\]

(35)
as \( r \to \infty \). The values of \( |H| \) and \( C_\infty \) for the \( n = 1, \Lambda = 0 \) case can be seen in Eq. (27) to be

\[
C_\infty = \sqrt{\frac{3\kappa^2}{4e^2}}, \quad |H| = \sqrt{\frac{4e^2}{27\kappa^2}}.
\]

(36)

The full solution, shown in Fig. (3), interpolates between the near-core expansion given by Eq. (34) and the asymptotic solution Eq. (35). The AdS bubble of nothing geometry is
FIG. 3: A bubble of nothing in an AdS compactification. The core of the monopole (surface of the bubble) is at \( r = 0 \), where the \( S^2 \) degenerates. The “warp factor” \( B(r) \) is nonzero at the core, and grows exponentially toward the AdS boundary, where all fields approach their vacuum values. Throughout, we use reduced Planck units (\( \kappa = 1 \)).

Illustrated in Fig. (4), which may be seen as the Euclidean solution, or as the spatial solution at the moment of nucleation. After nucleation the bubble expands exponentially eventually reaching the conformal boundary of the 4\( d \) anti-deSitter space. This can be seen in Fig. (5) where we show the 4\( d \) conformal diagram of the bubble of nothing geometry for this Ad\( S_4 \) compactification.\(^4\)

As a six dimensional geometry, one can interpret the behavior of the warp factor \( B(r) \) as indicating the presence of a throat-like region in our spacetime. The gravitational potential due to the warp factor reveals the (in this case) attractive nature of the bubble geometry (\( B(r) \) is decreasing toward the bubble). Gravitationally attractive throats appear in the context of warped compactifications [5]. A gapped warped throat (e.g., Klebanov-Strassler) in global coordinates may even be thought of as a bubble of nothing geometry, albeit with cylindrical rather than de Sitter isometry, and lacking a negative mode.

\(^4\) Note that all the conformal diagrams in this paper describe the 4\( d \) part of the geometry in Eq. (28). Every interior point represents a large \( S^2 \) from the 4\( d \) part of the spacetime times the small \( S^2 \) compactification manifold.
FIG. 4: Illustration of a bubble of nothing in an AdS compactification. The vertical position represents the radion $C(r)$, plotted as a function of the warp factor $B(r)$, represented here by radial position. The Euclidean $SO(4)$ or spatial $SO(3)$ symmetry is manifest in the rotational symmetry of the illustration. Hue represents the wound electromagnetic flux (see Fig. 2), and saturation is proportional to the scalar field magnitude, $p(r)$. The thick black ring represents the position of the defect.

FIG. 5: Conformal diagrams of the AdS geometries. Left: The de Sitter slicing of $AdS_4$ covers the shaded region. Right: The bubble of nothing geometry only exists in the shaded region outside the bubble wall denoted by the thick black line.
In our solutions, the bubble wall represents the smooth termination of this throat. We can use the gravitational properties of the throat as a proxy for the effective 4\textit{d} tension of the bubble. Since a bubble of nothing accelerates toward an outside observer, the throat is attractive, and the apparent 4\textit{d} tension of the domain wall is negative \cite{37,38}. One can see this by calculating the effective tension one would have to place in a 4\textit{d} spacetime to orbifold two identical copies of the shaded region in Fig. (5).

**B. The decay of $\mathbb{R}^{1,3} \times S^2$ spacetimes**

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure_6.png}
\caption{A bubble of nothing in a Minkowski compactification}
\end{figure}

We may uplift the effective 4\textit{d} cosmological constant to zero for the $n = 1$ vacua by raising the 6\textit{d} cosmological constant to

$$\Lambda = \frac{e^2}{2\kappa^4}.$$  

The asymptotic solution is then given by

$$\begin{align*}
p(r) &\rightarrow 1, & w(r) &\rightarrow 0, \\
C(r) &\rightarrow C_\infty = \frac{\kappa}{\epsilon}, & B'(r) &\rightarrow 1,
\end{align*}$$

(37)
FIG. 7: A bubble of nothing in a Minkowski compactification. The vertical position represents the radion $C(r)$, plotted as a function of the warp factor $B(r)$, whose minimum occurs at the core of the defect (thick dark ring)

as $r \to \infty$. A numerical example of the bubble of nothing geometry in this vacuum is shown in Fig. (6).

As mentioned before, we have introduced new parameters $\lambda$ and $\eta$ into our model which affect local properties of the bubble wall, but which are independent from the asymptotic solution. In Figs. (8 - 9) we give an example of such variation by finding a new type of bubble solution that is qualitatively different in the near tip region.

There, the warp factor $B(r)$ displays a punt shape, like a wine bottle achieving its minimum slightly away from the core. We can understand the existence of this family of solutions demonstrating the different form of the warp factor as a competition between the two contributions to the 4d gravitational properties in this region, one coming from the bubble of nothing itself, and the other from the magnetic 2-brane located at the surface of the bubble.

C. The decay of $dS_4 \times S^2$ spacetimes

Bubbles of nothing in de Sitter space are complicated by the existence of a cosmological horizon\(^5\). The bubble geometry in this case has an exterior region of finite radius, $0 < r < r_h$,

\(^5\) For a critique of exponential decay of de Sitter vacua, see \[39\].
FIG. 8: A punted bubble of nothing in a Minkowski compactification. The minimum of the warp factor $B(r)$ does not occur at the core.

FIG. 9: Illustration of a punted bubble of nothing in Minkowski space. The vertical separation represents the radion $C(r)$, plotted as a function of the warp factor $B(r)$, whose minimum occurs away from the core of the defect (thick dark ring).
where \( r_h \) is defined by \( B(r_h) = 0 \). This is denoted by the horizon which bounds region I in Fig. (10). Expanded about the horizon at \( r = r_h \), the solution takes the form

\[
\begin{align*}
p(r) &= p_h + p_2(r - r_h)^2 + \cdots , \\
w(r) &= w_h + w_2(r - r_h)^2 + \cdots , \\
B(r) &= (r - r_h) - B_3(r - r_h)^3 + \cdots , \\
C(r) &= C_h - C_2(r - r_h)^2 + \cdots ,
\end{align*}
\]

(38)

where \( p_2, w_2, B_3, C_2, \) etc. can be found in terms of the three field values at the horizon, \( p_h, w_h, \) and \( C_h \). We relegate the more complete expressions for these expansions to Appendix C.

Unlike the asymptotically flat or AdS bubbles of nothing, there is no topological distinction between a bubble of nothing in \( dS_4 \times S^2 \) and other physical solutions, including \( dS_6 \). Intuitively, a bubble of nothing should be a boost-invariant solution with a degenerating extra-dimensional fiber, which within our ansatz is a zero for the function \( C(r) \). The broadness of these criteria becomes apparent when one considers the anisotropic slicing of \( dS_6 \), whose metric is given by \( B(r) = \cos r, C(r) = \sin r \):

\[
ds^2 = \cos^2(r)(-dt^2 + \cosh^2 t \, d\Omega_2^2) + dr^2 + \sin^2 r \, d\Omega_2^2 .
\]

(39)

Remarkably, this appears to be a bubble of nothing\(^6\). In this case, any observer is on the core of the bubble at \( r = 0 \) and sees a cosmological horizon at \( r = \pi/2 \), with topology given by an \( S^2 \times S^2 \) fibration of \( S^4 \).

As this example demonstrates, we must adopt a more restrictive definition if we demand the bubble of nothing describe a decay channel for flux compactifications. We will therefore look not only for boost-invariant solutions with a smooth core region but also solutions with an asymptotic region which approaches a 4d flux vacuum. This requires us to determine the behavior of the solutions beyond the cosmological horizon, in what we denote by region II of Fig. (10). One can do this by analytically continuing the metric ansatz across this horizon via the substitution \( r \to it \) and \( t \to \chi + i\pi/2 \) in Eq. (28), yielding

\[
ds^2 = -dt^2 + B^2(t)d\mathcal{H}_3^2 + C^2(t)d\Omega_2^2 ,
\]

(40)

where \( d\mathcal{H}_3^2 \) is the unit metric on three dimensional hyperbolic space,

\[
d\mathcal{H}_3^2 = d\chi^2 + \sinh^2 \chi d\Omega_2^2 .
\]

(41)

\(^6\) A 5d version of this interpretation appears in [37].
Matter fields in this region are given by

\[ \Phi^a(t) = \eta p(t)(\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta), \]
\[ A_{\mu}^a(t) = A_{r}^a(t) = 0, \]
\[ A_{\theta}^a(t) = \frac{1 - w(t)}{e}(\sin \varphi, -\cos \varphi, 0), \]
\[ A_{\varphi}^a(t) = \frac{1 - w(t)}{e} \sin \theta(\cos \theta \cos \varphi, \cos \theta \sin \varphi, -\sin \theta). \] (42)

The general expansion of the fields about the light-cone \( t = 0 \) yields

\[ p(t) = p_h - p_2t^2 + \cdots, \]
\[ w(t) = w_h - w_2t^2 \cdots, \]
\[ B(t) = t + B_3t^3 + \cdots, \]
\[ C(t) = C_h + C_2t^2 + \cdots, \] (43)

where the three undetermined coefficients \( p_h, w_h, \) and \( C_h \) are trivially related to the field values across the horizon \( r = r_h \).

Using the time-continued equations of motion shown in Appendix [D], we numerically integrate the solution forward in time, taking as initial conditions the values of the fields at the horizon separating the future region (II) from the spacelike region (I).
For certain parameter values, one can find solutions for the bubble of nothing asymptotic to $dS_4 \times S^2$. Shown in Fig. (11-12) is a numerical solution in region I, where the behavior $B(r) \to 0$ signals the appearance of a cosmological horizon, and the remaining functions behave as described in Eq. (38). Following the procedure outlined above, one can find that indeed this solution relaxes to the appropriate $dS_4 \times S^2$ compactification. We show in Fig. (13) the posterior evolution of the fields beyond the horizon in region II.

There is however, a different class of solution one can find in this future directed region. There are solutions which lead to runaway behavior for the radion $C(t)$. This is manifestly different from a compactification. In fact, it is not difficult to see that at late times this geometry asymptotes to six dimensional de Sitter space written in an anisotropic gauge. The interpretation of these types of solutions is not as a bubble of nothing in a flux compactification, but as an instanton describing the creation of smooth magnetically charged 2-branes in $dS_6$ [19]. We give an example of such type of solutions in Appendix E.

An interesting feature of the bubble of nothing in de Sitter compactifications is the symmetry between the ‘excised’ region and the undisturbed region, as can be seen in Figs. (10) and (12). In fact, within the family of solutions are those where the excised region is far larger than the undisturbed region. In the framework of this paper, this should be inter-

\[ \eta = 2.75 \]
\[ \lambda = 0.01 \]
\[ \Lambda = 0.65 \]
\[ \epsilon = 1.0 \]
FIG. 12: A bubble of nothing in $dS_4 \times S^2$. This illustrates either the Euclidean solution, with cosmological horizon antipodal to the hole, or the global spatial solution at the moment of nucleation. Interpreted as the spontaneous collapse of a super-horizon sized region to nothing. Another interpretation of this solution is that of an instanton describing the spontaneous creation of an open flux compactification. The ambiguity between these interpretations disappears when considering the analogous solutions in flat and AdS compactifications. We therefore refer to these solutions as *bubbles from nothing* [40].

V. CONCLUSIONS

We have demonstrated a new instability of flux compactifications, and a new topology for the bubble of nothing. Like the original bubble of nothing of the Kaluza–Klein vacuum, the bubbles we present are smooth gravitational instantons asymptotic to a compactification geometry. The principal new ingredients are

- The bubble surface is charged with respect to the flux employed to stabilize the compactification.

- The solutions describe the smooth degeneration of an $S^2$, rather than the previously known $S^1$ cases.
\[ \eta = 2.75 \quad \lambda = 0.01 \quad \Lambda = 0.65 \quad e = 1.0 \]

\[ \frac{\sinh^{-1} HB(t)}{H} \]

\[ \frac{C(t)}{C_{\infty}} - 1 \]

\[ w(t) \]

\[ 1 - p(t) \]

FIG. 13: The future-directed evolution of the fields in region I of Fig. (10). The solution asymptotes to the 4d flux vacuum as \( t \to \infty \).

- The bubbles have a variety of 4d effective tensions, which can be negative or positive.
- The solutions preserve the isometry of the compactification manifold only for flux number \( n = \pm 1 \).
- The instability may occur for perturbatively stable flat, AdS, or dS compactifications, although it does not exist for all parameter values \( \eta, \lambda \).

A consequence of the topology of the bubbles of nothing presented here is that spin structure cannot play a role in excluding the instability; every configuration considered here satisfies \( \pi_1(M_6) = 0 \).

Families of solutions exist for anti de Sitter, Minkowski, as well as de Sitter compactifications, although the taxonomy of a bubble of nothing in \( dS_4 \times S^2 \) is complicated by the lack of a topological distinction from other solutions which are physically distinct (e.g., defects...
in $dS_6$ and the *bubble from nothing*). Roughly speaking, a bubble of nothing should be gravitationally attractive over a large range of distances, meaning an observer must accelerate away from the bubble in order to avoid collision. In this case, the effective 4d tension of the boundary of spacetime is negative. Because of the natural de Sitter slicing, the throat picture of a bubble of nothing is reminiscent of the dS/dS correspondence [41].

On the other hand, spin structure does not allow one to project out the bubble of nothing instability in this more general case. It would therefore be interesting to find what mechanism forbids the decay of supersymmetric flux vacua to nothing.

By increasing the monopole parameters $\eta$ and $\lambda$, one can preserve the long range attractive nature of the solution despite a short range gravitational repulsion, as shown in Fig. (9).

A more drastic solution, to be discussed in a future publication [40], describes a purely repulsive boundary, which we have referred to as a “*bubble from nothing*.” Although this solution is topologically equivalent to a bubble of nothing in de Sitter compactifications, the corresponding interpretation (certainly in the flat and AdS case) is distinct from that of a bubble of nothing.

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Appendix A: Compactification Solutions for $n = 1$

For compactifications of unit flux number, Eqs. (16), (17), and (18) reduce to

\[
3H^2 + \frac{1}{C^2} = \kappa^2 \left( \frac{\eta^2 p^2 w^2}{C^2} + \frac{(1 - w^2)^2}{2e^2 C^4} + \frac{\lambda \eta^4}{4} (p^2 - 1)^2 + \Lambda \right),
\]

\[
6H^2 = \kappa^2 \left( - \frac{(1 - w)^2}{2e^2 C^2} + \frac{\lambda \eta^4}{4} (p^2 - 1)^2 + \Lambda \right),
\]

\[
0 = \frac{2pw^2}{C^2} + \lambda \eta^2 p (p^2 - 1),
\]

\[
0 = \frac{w(1 - w^2)}{C^2} - e^2 \eta^2 p^2 w. \tag{A1}
\]

The solution to these equations is not unique. They can be categorized by their stability, as below.

1. **Stable Compactification Solutions**

Stable solutions exist when the fields relax to their vacuum values, $p = 1$, $w = 0$, yielding

\[
H^2 = \frac{2\kappa^2 \Lambda}{9} - \frac{2e^2}{27\kappa^2} \left( 1 + \sqrt{1 - \frac{3\kappa^4 \Lambda}{2e^2}} \right),
\]

\[
C^2 = \frac{1}{\kappa^2 \Lambda} \left( 1 - \sqrt{1 - \frac{3\kappa^4 \Lambda}{2e^2}} \right). \tag{A2}
\]

One can see that this is a stable configuration by looking at the 4d effective action about this solution. (See the main part of the text for a discussion on this point.) These are the most interesting solutions for our purpose, although one can find several other solutions which are unstable.

2. **Unstable Compactification Solutions**

For completeness, we construct unstable configurations which exist in the $n = 1$ case.\(^7\) Our numerical solutions approach only stable configurations, as should be the case for purely non-perturbative instabilities.

\(^7\) The first type of unstable solutions presented here may also occur for $n > 1$, although none of the subsequent examples generalize in this way.
a. Compactifications with $p = 1$ and $w = 0$

Here the matter fields have relaxed to their respective vacua, but $C$ is sitting at an unstable equilibrium for the size of the compactification manifold. In other words, these solutions are the straightforward generalization of the Nariai compactification solutions. The solutions take the form

\begin{align*}
H^2 &= \frac{2\kappa^2 \Lambda}{9} - \frac{2e^2}{27\kappa^2} \left( 1 - \sqrt{1 - \frac{3\kappa^4 \Lambda}{2e^2}} \right), \\
C^2 &= \frac{1}{\kappa^2 \Lambda} \left( 1 + \sqrt{1 - \frac{3\kappa^4 \Lambda}{2e^2}} \right),
\end{align*}

(A3)

b. Compactification with $p = 0$ and $w = 1$

These configurations are clearly unstable since vanishing $p$ implies that the scalar triplet sits at the top of its potential. Here,

\begin{align*}
H^2 &= \frac{\kappa^2}{24} (\lambda \eta^4 + 4\Lambda), \\
C^2 &= \frac{8}{\kappa^2 (\lambda \eta^4 + 4\Lambda)},
\end{align*}

(A4)

c. Non-zero constant $p$-$w$ Solutions

This is a solution where both $p$ and $w$ are non-zero constants different from their vacuum values. We have checked numerically that this type of solution is unstable to decompactification, as was indicated in [42].

To simplify the notation, we define the quantity

\[ \alpha = \sqrt{(8e^2 + 4\lambda(\eta^2 \kappa^2 - 1))^2 - 24\kappa^2(e^2(\lambda \eta^4 + 4\Lambda) - 2\lambda \Lambda)}, \]

(A5)
allowing the solutions to be written

\[ p = \sqrt{\frac{-4e^2 + 2\lambda + \lambda \eta^2 \kappa^2 \pm \frac{1}{2}\alpha}{(3\lambda - 6e^2)}}, \]

\[ w = \sqrt{\frac{\lambda \left(43^2 \eta^2 + 2\kappa^2 \lambda \Lambda + e^2(\lambda \eta^4 \kappa^2 - 2\lambda \eta^2 - 4\kappa^2 \Lambda)\right) \pm \frac{1}{2}e^2 \eta^2 \alpha}{\kappa^2(2e^2 - \lambda) \left(e^2(\lambda \eta^4 + 4\Lambda) - 2\lambda \Lambda\right)}} , \]

\[ H^2 = \frac{\kappa^2 \left(2e^2 - \lambda\right) \left(2e^2 - \lambda + \eta^2 \kappa^2 \lambda \pm \frac{1}{2}\alpha\right)}{9(2e^2 - \lambda) \left(4e^2 - 2\lambda + 2\eta^2 \kappa^2 \lambda \pm \frac{1}{2}\alpha\right)} , \]

\[ C^2 = \frac{6\kappa^2}{4e^2 - 2\lambda + 2\lambda \eta^2 \kappa^2 \pm \frac{1}{2}\alpha} . \]  

\[ (A6) \]

**Appendix B: Expansion about the soliton core**

The expansion of the equations of motion takes the following form for the most general smooth solution describing the core of the magnetically charged inflating 2-brane. In the ansatz of Eqs. \([28][29] \),

\[ p(r) = p_1 r + \cdots , \]

\[ w(r) = 1 + w_2 r^2 + \cdots , \]

\[ B(r) = B_0 + \frac{B_1}{12} \left( 1 + \frac{3(2e^2 + B_0^2 e^2 \eta^2 \kappa^2 p_1^2 + 8B_0^2 w_2^2 \kappa^2)}{2B_0^2 e^2} \left( \frac{\kappa^2}{4} \left( 6\eta^2 p_1^2 + \eta^4 \lambda + \frac{24w_2^2}{e^2} + 4\Lambda \right) \right) \right) r^2 + \cdots , \]

\[ C(r) = r - \frac{(2e^2 + B_0^2 e^2 \eta^2 \kappa^2 + 8B_0^2 w_2^2 \kappa^2)}{12B_0^2 e^2} r^3 + \cdots . \]

This expansion depends on three locally undetermined constants, \( B_0, p_1, \) and \( w_2 \), in terms of which all subsequent terms in the near-core expansions may be specified.
Appendix C: Expansion about the cosmological horizon

Some of the solutions we obtain possess a cosmological horizon, defined by $B(r_h) = 0$. The general form for a smooth expansions about the horizon $r = r_h$ is

$$p(r) = p_h + p_2^h(r - r_h)^2 + \cdots,$$
$$w(r) = w_h + w_2^h(r - r_h)^2 + \cdots,$$
$$B(r) = (r - r_h) - B_3^h(r - r_h)^3 + \cdots,$$
$$C(r) = C_h - C_2^h(r - r_h)^2 + \cdots,
$$

where the coefficients of this expansion are given by

$$p_2^h = p_h \left[ 2 w_h^2 - \lambda \eta^2 C_h^2 (1 - p_h) \right] / 8 C_h^2,$$
$$w_2^h = w_h \left[ w_h (w_h + e^2 \eta p_h^2 C_h^2) - 1 \right] / 8 C_h^2,$$
$$B_3^h = \frac{1}{288 C_h^4 e^2} \left\{ 10 (w_h^2 - 1)^2 \kappa^2 + 8 e^2 C_h^2 (\eta^2 p_h^2 w_h^2 \kappa^2 - 1) - \kappa^4 \lambda (p_h^2 - 1) + 4 \Lambda \right\},$$
$$C_2^h = \frac{1}{48 C_h^3 e^2} \left\{ 2 \kappa^2 (w_h^2 - 1)^2 + 4 e^2 C_h^2 (\kappa^2 w_h^2 \eta^2 p_h^2 - 1) + \kappa^4 \lambda (p_h^2 - 1)^2 + 4 \Lambda \right\}
+ \frac{1}{4 e^2 C_h^2} \left[ 10 \kappa^2 (w_h^2 - 1)^2 + 8 e^2 C_h^2 (\eta^2 p_h^2 w_h^2 \kappa^2 - 1) - \kappa^4 \lambda (p_h^2 - 1)^2 + 4 \Lambda \right]\}.
$$

Much like the near-core expansion, the general form near the cosmological horizon is written in terms of three constants, in this case $C_h$, $p_h$, and $w_h$.

Appendix D: Time-dependent Equations

Following the ansatz of Eq. (40), the time-dependent Einstein equations are

$$C_0^\theta = \frac{3}{B(t)^2} - \frac{1}{C(t)^2} - 3 \left( \frac{B'(t)}{B(t)} \right)^2 - 6 \frac{B'(t) C'(t)}{B(t) C(t)} - \left( \frac{C'(t)}{C(t)} \right)^2 = \kappa^2 T_0^\theta,$$
$$C_r^r = \frac{1}{B(t)^2} - \frac{1}{C(t)^2} - \left( \frac{B'(t)}{B(t)} \right)^2 - 4 \frac{B'(t) C'(t)}{B(t) C(t)} - \left( \frac{C'(t)}{C(t)} \right)^2 - 2 \frac{B''(t)}{B(t)} - 2 \frac{C''(t)}{C(t)} = \kappa^2 T_r^r,$$
$$C_\theta^\theta = \frac{3}{B(t)^2} - 3 \left( \frac{B'(t)}{B(t)} \right)^2 - 3 \frac{B'(t) C'(t)}{B(t) C(t)} - \frac{3}{B(t)} - \frac{C''(t)}{C(t)} = \kappa^2 T_\theta^\theta,$$

(D1)
where energy-momentum tensor on the right hand side of these equations is given by

\[ T_{00} = -\left[ \eta^2 \left( \frac{p'(t)^2}{2} + \frac{p(t)^2w(t)^2}{C(t)^2} \right) + \frac{1}{e^2C(t)^2} \left( w'(t)^2 - \frac{(1 - w(t)^2)^2}{2C(t)^2} \right) + \frac{\lambda \eta^4}{4} (p(t)^2 - 1)^2 + \Lambda \right], \]

\[ T_{rr} = \eta^2 \left( \frac{p'(t)^2}{2} - \frac{p(t)^2w(t)^2}{C(t)^2} \right) + \frac{1}{e^2C(t)^2} \left( w'(t)^2 - \frac{(1 - w(t)^2)^2}{2C(t)^2} \right) - \frac{\lambda \eta^4}{4} (p(t)^2 - 1)^2 - \Lambda, \]

\[ T_{\theta\theta} = \eta^2 \left( \frac{p'(t)^2}{2} + \frac{(1 - w(t)^2)^2}{2e^2C(t)^4} \right) - \frac{\lambda \eta^4}{4} (p(t)^2 - 1)^2 - \Lambda. \] (D2)

Eqs.(17-18) become

\[ p''(t) + \left( 3 \frac{B'(t)}{B(t)} + \frac{2C''(t)}{C(t)} \right) p'(t) + \frac{2w(t)^2p(t)}{C(t)^2} + \lambda \eta^2 p(t)(p(t)^2 - 1) = 0 \] (D3)

and

\[ w''(t) + 3 \frac{B'(t)}{B(t)} w'(t) - \frac{w(t)(1 - w(t)^2)}{C(t)^2} + e^2 \eta^2 p(t)^2 w(t) = 0. \] (D4)

**Appendix E: Smooth Brane solution in \( dS_6 \)**

As we explained in the main body of the text, our metric and matter ansatz allows for the description of solutions that should probably not be considered bubbles of nothing in the sense that we use it in the present context. They are nevertheless worth mentioning, since they are interesting geometries in their own right. Perhaps the most important example of this is the type of solutions which describe the nucleation of smooth magnetically charged de Sitter branes in \( dS_6 \). Similar instanton solutions have been recently discussed in the literature [19] using black branes. The difference between these two type of instantons is the existence of a horizon on the black brane solutions that is not present in the smooth cores that we study here. The possibility of a smooth solitonic brane is again due to the fact that we have extended our model to include additional degrees of freedom that resolve the singularities of the Dirac monopole solution.

By choosing appropriate values for the parameters in the theory one can find solutions of this type. We show in Fig. (14) the instanton solution within region I of the spacetime [see Fig. (10)]. The solution in this region is rather similar to the bubble of nothing solutions obtained in the main body of the text. However, once we continue across the lightcone into region II, things fall apart. [see Fig. (15)]. In particular the “radion” \( C(t) \) grows without bound, signaling the decompactification of spacetime. This shows that one cannot consider
FIG. 14: The near-core region of what appears to be a bubble of nothing, but which does not in fact asymptote to a 4d region.

this solution to be relevant for a compactified spacetime since its global structure is markedly different from anything four dimensional. On the other hand, we can see that the form of the metric rapidly approaches that of Eq. (39), the anisotropic ansatz for $dS_6$. This validates the interpretation of this instanton as the quantum mechanical creation of a $2 + 1$ dimensional solitonic de Sitter brane in a $dS_6$ bulk.

Appendix F: Numerical Techniques

The defining field configuration of a bubble of nothing is the existence of a smooth core where the extradimensional fiber degenerates. In the case at hand, this requirement leaves three unknown initial field values for a seventh order ordinary differential system. These three parameters must be determined by evolving the fields between the core and a sufficiently asymptotic region, where the boundary conditions are known.

The three unknown parameters can be obtained by treating them as “shooting parameters.” This means a guess is made, followed by the numerical evolution of the resulting solution toward the asymptotic boundary conditions. If the numerical evolution fails to asymptote to the boundary conditions, the shooting parameters are appropriately adjusted,
\[ \eta = 1.5 \quad \lambda = 0.25 \quad \Lambda = 0.57 \quad e = 1.0 \]

FIG. 15: The decompactification of the 4\(d\) geometry surrounding the faux bubble of nothing geometry found in Fig. (14).

and the procedure is repeated. However this method is rendered intractable by the exponentially growing modes of the differential equations. Because we would like to evolve the fields across distances much longer than the Compton wavelength of the most massive of the four fields, the behavior is extremely sensitive to the initial conditions. This would require one to maintain tremendous numerical precision throughout the evolution of the solution. The inefficiency arises because one must cancel the coefficient of the three growing modes to far more numerical precision than the eventual solution warrants.

A solution to this problem is easily implemented using the so-called multiple shooting method \[\text{[43]}\]. The integration interval is divided into many subintervals of length less than the Compton wavelengths of the fields. Since each interval is small, the numerical evolution depends approximately linearly on the initial conditions. The many intervals are then pieced together by demanding that each function be continuous and differentiable across the
boundaries of the subintervals. This is achieved using Newton’s method. By extrapolating the mismatch between all neighboring subintervals as a function of the shooting parameters and multiple boundary conditions, an optimal guess for the improvement of the shooting parameters can be made. In this sense, the solution is found by a combination of shooting and relaxing of the fields between subintervals. A non-linear problem in few dimensions is traded for a linear problem in many dimensions. The approximately linear nature of the problem is important for Newton’s method to work.

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