CONTRASTIONS ON DARK ENERGY MODELS FROM RADIAL BARYON ACOUSTIC SCALE MEASUREMENTS

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ABSTRACT

We use the radial baryon acoustic oscillation (BAO) measurements of Gaztañaga et al. to constrain parameters of dark energy models. These constraints are comparable with constraints from other “nonradial” BAO data. The radial BAO data are consistent with the time-independent cosmological constant model but do not rule out time-varying dark energy. When we combine radial BAO and the Kowalski et al. Union Type Ia supernova data, we get very tight constraints on dark energy.

Key words: cosmological parameters – distance scale – large-scale structure of universe

1. INTRODUCTION

Recent Type Ia supernova (SNIa) data (Kowalski et al. 2008; Rubin et al. 2008; Sahni et al. 2008) confirm at high significance, that the cosmological expansion is currently accelerating. If we assume that general relativity is valid on cosmological length scales, this cosmological acceleration requires that the universe’s current energy density budget is dominated by far an approximately spatially uniform component—dark energy—with negative pressure \( p < -\rho / 3 \), where \( \rho \) is the dark energy density.

The most economic and the oldest form of dark energy is Einstein’s cosmological constant \( \Lambda \) (Peebles 1984), which is time independent and has an equation of state \( p_\Lambda = -\rho_\Lambda \). The time-independent cosmological constant model (ΛCDM) provides a reasonably good fit to most current cosmological data (see, e.g., Ratra & Vogeley 2008; Frieman et al. 2008), but despite this success a lot of alternative models of dark energy have been proposed over the years. One reason for this is that cosmological data cannot yet tightly constrain the various options currently under debate, although it is thought that in next decade a large amount of more precise new data will tightly constrain or measure departures from the now standard ΛCDM model (see, e.g., Podariu et al. 2001a; Wang 2007; Barnard et al. 2008; Tang et al. 2008). The second reason is that a cosmological constant is not straightforward to understand on a more fundamental level; in particular, it is difficult to accept as fundamental the needed new energy scale of a few meV.

In some scalar field dark energy models a nonlinear attractor solution ensures that this scale of a few meV is not fundamental, rather it follows from a much higher energy scale as the scalar field energy density decreases during the cosmological expansion (Peebles & Ratra 1988; Ratra & Peebles 1988). In this paper, we also consider this \( \Lambda \)CDM model in which the dark energy is modeled as a slowly rolling scalar field \( \phi \) with self-interaction potential energy density \( V(\phi) \propto \phi^{-\alpha} \), where \( \alpha > 0 \). A number of other models with time-varying dark energy have been proposed.5 We also consider the XCDM parametrization which is often used to describe time-varying dark energy models.5 In XCDM, dark energy is assumed to be a perfect fluid with effective equation of state \( p_x = \omega_x \rho_x \), where \( \omega_x \) is a number less than \(-1/3\).

Given a model, it is possible to compute quantities such as the Hubble parameter \( H(z) \) and the angular diameter distance \( d_A(z) \) as a function of redshift \( z \). Since these quantities are observable (or observational results depend on them), given perfect data it is possible in principle to compare observational results with theoretical predictions and determine which dark energy model provides a better fit to the data. There are a few difficulties however. Theoretical predictions not only depend on dark energy model parameters, they also can depend on a number of other cosmological parameters, such as the energy density of nonrelativistic matter or baryonic matter or radiation, the energy density perturbation spectral index, the total mass of neutrinos, the Hubble constant, etc. While some of these parameters are directly measurable, some of them have to be simultaneously determined from the data. This leads to degeneracies in cosmological parameter space.

The other issue is possible systematic errors in cosmological observations, which are very difficult to trace. It is therefore reassuring that the supernova data, indication of dark energy is confirmed by cosmic microwave background (CMB) anisotropy data. More precisely, assuming the CDM model of structure formation and assuming that dark energy does not evolve in time, CMB anisotropy data are consistent with negligible space curvature (see, e.g., Podariu et al. 2001b; Page et al. 2003; Doran et al. 2007; Dunkley et al. 2008; Komatsu et al. 2008), so in conjunction with low measured nonrelativistic matter density

4 Modifications of general relativity on cosmological length scales that might do away with dark energy have also been discussed (see, e.g., Wei & Zhang 2008; Tsujikawa & Tatekawa 2008; Capozziello et al. 2008; Sotiriou & Faraoni 2008; Bamba et al. 2008). In the models we consider in this paper, dark energy affects other fields only through gravity. For dark energy models with other couplings see La Vacca et al. (2009), Wu & Zhang (2008), Jesus et al. (2008), Antusch et al. (2008), Wu et al. (2008), and references therein. For other dark energy models see Zhang & Chen (2008), Alcaniz et al. (2009), Setare & Saridakis (2009), Basilakos & Plionis (2008), Shapiro & Solà (2008), and references therein.

5 The XCDM parametrization cannot describe time-varying \( \phi \)CDM dark energy at late times (see, e.g., Ratra 1991).
(see, e.g., Chen & Ratra 2003b) CMB anisotropy data also demand dark energy.

To convincingly remove degeneracies and understand and cancel the effect of unwanted systematic errors it’s necessary to have many independent cosmological tests. Other recently discussed cosmological probes include the angular size of radio sources and quasars as a function of redshift (see, e.g., Chen & Ratra 2003a; Podariu et al. 2003; Daly et al. 2007; Santos & Lima 2008), strong gravitational lensing (see, e.g., Lee & Ng 2007; Oguri et al. 2008; Zhang et al. 2009; Zhu & Sereno 2008), weak gravitational lensing (see, e.g., Doré et al. 2007; La Vacca & Colombo 2008; Hoekstra & Jain 2008; Schmidt 2008), measurements of the Hubble parameter as a function of redshift (see, e.g., Samushia & Ratra 2006; Lin et al. 2008; Dev et al. 2008; Fernandez-Martinez & Verde 2008), galaxy cluster gas mass fraction versus redshift data (see, e.g., Allen et al. 2004; Chen & Ratra 2004; Allen et al. 2008; Sen 2008), and the growth of large-scale structure (see, e.g., Basilakos et al. 2008; Waizmann & Bartelmann 2009; Mainini 2008; Abram et al. 2008).

A measurement of the large-scale structure, baryon acoustic oscillation (BAO) peak length scale provides another possible cosmological test. This was first measured a few years ago in the two-point correlation function of luminous red galaxies in the Sloan Digital Sky Survey (SDSS) catalog at $z = 0.35$ (Eisenstein et al. 2005, also see Hütsi 2006) and from 2dFGRS data (Cole et al. 2005) and then later in the joint SDSS and 2dF catalogs at $z = 0.24$ and $0.43$ (Percival et al. 2007a).  

One attractive feature of the BAO measurements is that they do not explicitly depend on the value of Hubble constant. These BAO measurements were used in the original papers to constrain parameters of the ΛCDM and XCDM models. See Ishida et al. (2008), Lazkoz et al. (2008), Santos & Jesus (2008), and Samushia & Ratra (2008b), and references therein, for recent discussions of BAO data constraints on these and other models.

The Percival et al. (2007a) measurement was made at only two redshifts and by itself does not provide a robust and reliable test of dark energy models, but in combination with other data, it does provide useful constraints on cosmological parameters. However, a number of surveys are planned in the next few years that will measure the BAO scale accurately and at a variety of redshifts up to $z = 1.2$. This upcoming BAO data, especially when combined with other data, will prove very useful in tightly constraining dark energy parameters (see, e.g., Wang et al. 2009).

Recently Gaztañaga et al. (2008a) argued that these measurements of the BAO scale were essentially measurements orthogonal to the line of sight and so statistically independent from a line-of-sight measurement of the BAO scale, even if the same galaxy catalog is used for both measurements. Gaztañaga et al. (2008a) used the SDSS data to compute the line of sight or radial BAO scale in redshift space for two ranges of redshift and Gaztañaga et al. (2008b) showed the resulting constraints on the spatially flat XCDM model. These constraints from the radial BAO scale data are quite similar to the constraints derived from earlier “nonradial” BAO scale measurements. In this paper, we extend the analysis to the ΛCDM and φCDM models of dark energy. The data are consistent with spatially flat ΛCDM. However, these current radial BAO measurements (like current nonradial BAO measurements) cannot tightly constrain time-varying dark energy, although the situation is anticipated to improve in the next few years.

We also derive constraints on these models from a combined analysis of the radial BAO data and the Kowalski et al. (2008) Union SNIa data. Since the radial BAO and SNIa data constraints are almost orthogonal, the constraints from the combined data are significantly tighter than those from either individual data sets. The combined constraints favor a close to spatially flat ΛCDM model (more so than the SNIa data), but do not yet completely rule out time-varying dark energy.

Our paper is organized as follows. In Section 2, we describe the radial BAO measurements we use. In Section 3, we explain how we derive constraints on different dark energy models from radial BAO and SNIa data. We present and discuss our results in Section 4.

2. RADIAL BARYON ACOUSTIC SCALE

In a spherically symmetric universe, the two-point correlation function is a function of two variables, $ξ = ξ(σ, π)$, where $σ$ is the separation along the line of sight and $π$ is the separation on the sky. It can also be expressed as a function of absolute separation $r = \sqrt{σ^2 + π^2}$ and the cosine of the angle between the line of sight and the direction of separation, $μ = π/r$. The correlation function can then be decomposed into multipole moments

$$ξ_l(r) = \int_{-1}^{+1} ξ(\mu)P_l(\mu)d\mu,$$

where $P_l$ is the $l^{th}$ order Legendre polynomial. Multipole moments of different orders can be related to each other if one has a complete theory of linear and nonlinear evolution. Although high multipoles that describe the “shape” of baryon acoustic oscillation imprints on the matter distribution are very difficult to measure in practice, theoretically they are independent of the monopole and could provide additional structure formation tests.

Initial work considered only the averaged over direction monopole part of the correlation function,

$$ξ_0(r) = \frac{1}{2} \int_{-1}^{+1} ξ(\mu)d\mu,$$

and found a BAO peak signal at a comoving distance of $r \approx 110h^{-1}$ Mpc ($h$ is the Hubble constant $H_0$ in units of $100$ km s$^{-1}$ Mpc$^{-1}$). This measurement was however mostly transverse to the line-of-sight direction $π$; the weight of separation along the line of sight contributes less than $1\%$ (Gaztañaga et al. 2008a). Consequently, it is fair to assume that the radial baryon acoustic peak scale measurement in the line-of-sight direction $ξ(σ)$ is statistically independent from that measured from $ξ(π)$, even if the same galaxy sample is used for both measurements.

Gaztañaga et al. (2008a) recently used SDSS data to measure the radial baryon acoustic scale in two redshift ranges $z \sim 0.15−0.30$ with radial BAO peak scale $Δz = 0.0407 ± 0.0014$ and $z \sim 0.40−0.47$ with $Δz = 0.0442 ± 0.0016$ (both one being...
standard deviation errors). Theoretically, the radial BAO peak scale is given by

\[ \Delta z = H(z) r_s(z_d)/c, \]  

(3)

where \( H(z) \) is the Hubble parameter at redshift \( z \), \( r_s(z_d) \) is the sound horizon size at the drag redshift \( z_d \), at which baryons were released from photons, and \( c \) is the speed of light. \( H(z) \) can be easily computed in a given cosmological model and depends on model parameters such as the nonrelativistic matter density and the time dependence of dark energy.

To compute \( r_s \), Gaztañaga et al. (2008b) use two different methods. One is to use the ratio \( I_l/I_s \) between the distance to the last-scattering surface and \( r_s \) measured by CMB anisotropy experiments and compute the sound horizon at photon decoupling from

\[ r_s(z) = \frac{\pi(1 + z_s)d_A(z_s)}{l_s}. \]  

(4)

Here \( z_s \) is the redshift at the photon decoupling and \( d_A \) is the angular diameter distance. Alternatively, one can use priors on the fractional energy density parameters of baryonic matter, \( \Omega_b \), nonrelativistic matter, \( \Omega_m \), and relativistic matter, \( \Omega_r \), from, e.g., CMB anisotropy measurements, and compute the sound horizon at the drag redshift from

\[ r_s(z_d) = \frac{c}{H_0 \sqrt{3 \Omega_m}} \times \int_{0}^{a(z_d)} \frac{da}{\sqrt{(a + 1.69\Omega_b/\Omega_m)(1 + \alpha 0.75\Omega_b/\Omega_m)}}. \]  

(5)

Both options have similar drawbacks. One has to assume priors on “nuisance” parameters like \( I_l \) or various energy densities. CMB anisotropy measurements themselves have measurement errors that must be accounted for, otherwise the errors on the estimates of dark energy model parameters of interest will be underestimated. Also, the best-fit values for nuisance parameters given by CMB anisotropy data are in general different for every cosmological model and also depend on model parameter values. To be fully consistent when using priors one would have to reanalyze CMB experiments for each cosmological model (and model parameter value) instead of using a single set of values for \( I_l \), \( \Omega_b \), \( \Omega_m \), and \( \Omega_r \).

At present, however, the BAO scale is measured only in two redshift ranges and does not provide very tight parameter constraints compared to other observational tests. Hence, as long as we are interested in preliminary constraints on dark energy from BAO scale measurements we may use the simplified approach of Gaztañaga et al. (2008b), keeping in mind that when more and better quality BAO scale measurements become available a more complete, careful, and time-consuming analysis will be warranted.

3. COMPUTATION

We compare the two measured radial BAO peak scales (Gaztañaga et al. 2008a) with the predictions of three dark energy models. The models we consider are standard ΛCDM, the XCDM parametrization of the dark energy’s equation of state, and the φCDM model with an inverse power-law potential energy density \( V(\phi) \propto \phi^{-\alpha} \). In these dark energy models, at late times, we can compute the redshift-dependent Hubble parameter from

\[ H(z) = H_0 \sqrt{\Omega_m(1 + z)^3 + \Omega_\Lambda} + (1 - \Omega_m - \Omega_\Lambda)(1 + z)^2 \]  

(ACDM),

\[ H(z) = H_0 \sqrt{\Omega_m(1 + z)^3 + \Omega_\Lambda} \]  

(XCDM),

\[ H(z) = H_0 \sqrt{\Omega_m(1 + z)^3 + \Omega_\Lambda(\alpha, z)} \]  

(φCDM).

(7)

(8)

In the φCDM model, the energy density of the scalar field has to be computed as a function of the redshift and \( \alpha \) parameter by numerically solving the equations of motion.⁸

In all three models, the background evolution is described by two parameters. One is the nonrelativistic matter fractional energy density parameter \( \Omega_m \) and the other one is a parameter \( p \) that characterizes the dark energy. For the ΛCDM model, \( p \) is the cosmological constant fractional energy density parameter \( \Omega_\Lambda \), in the XCDM parametrization it is the equation of state parameter \( \omega_\cdm \) and in the φCDM model it is \( \alpha \) which describes the steepness of the scalar field self-interaction potential. In this paper, we consider only spatially flat XCDM and φCDM models, while in the ΛCDM case spatial curvature is allowed to vary, with the space curvature fractional energy density parameter \( \Omega_k = 1 - \Omega_m - \Omega_\Lambda \). The angular diameter distance is determined in terms of the Hubble parameter through

\[ d_A(z) = \frac{1}{\sqrt{\Omega_k H_0(1 + z)}} \sin \left[ \sqrt{\Omega_k} H_0 \int_0^z \frac{dz'}{H(z')} \right] \]  

(Ωk > 0),

(9)

\[ d_A(z) = \frac{1}{1 + \Omega_\cdm} \int_0^z \frac{dz'}{H(z')} \]  

(Ωk = 0),

(10)

\[ d_A(z) = \frac{1}{\sqrt{-\Omega_k} H_0(1 + z)} \sinh \left[ \sqrt{-\Omega_k} H_0 \int_0^z \frac{dz'}{H(z')} \right] \]  

(Ωk < 0).  

(11)

We use both methods proposed in Gaztañaga et al. (2008b) to estimate the sound horizon at recombination. We compute theoretically predicted model values of the radial baryon acoustic scale in redshift space, \( \Delta z_{\text{obs}}(\Omega_m, p, \nu) \), at redshift \( z_d = 0.24 \) and \( z_s = 0.43 \), where by \( \nu \) we denote the “nuisance” parameters.

We assume that the measurements at \( z = 0.24 \) and 0.43 are independent and that the errors are Gaussianly distributed. To constrain model parameters we compute

\[ \chi^2(\Omega_m, p, \nu) = \sum_{i=1,2} \left( \Delta z_{\text{obs}, i}(\Omega_m, p, \nu) - \Delta z_{\text{obs}, i} \right)^2 / \sigma_{\Delta z_{\text{obs}, i}}^2, \]  

(12)

where the two observed values, \( \Delta z_{\text{obs}, i} \pm \sigma_{\Delta z_{\text{obs}, i}} \), are listed above Equation (3) and define a likelihood function

\[ L_{\text{BAO}}(\Omega_m, p, \nu) \propto \exp(-\chi^2(\Omega_m, p, \nu)/2). \]  

(13)

We integrate over nuisance parameters to get a two-dimensional likelihood for the cosmological parameters of interest \( L_{\text{BAO}}(\Omega_m, p) = \int L_{\text{BAO}}(\Omega_m, p, \nu) d\nu \). For the prior likelihood of nuisance parameters, we use Gaussian distribution functions with Wilkinson Microwave Anisotropy Probe (WMAP) five year recommended means and variances \( l_s = \)

⁸ For a discussion of the scalar field dynamics see, e.g., Podariu & Ratra (2000).
302 ± 0.87, \( z_s = 1090 \pm 1 \) (with 40% positive correlation between two measurements), \( \Omega_b h^2 = 0.0227 \pm 0.0066 \), \( \Omega_m h^2 = 0.133 \pm 0.0064 \), and \( \Omega_\Lambda = 2.45 \times 10^{-5} \) (Dunkley et al. 2008). For each dark energy model, we define the best-fit values of parameters as a pair \((\Omega_m, p)\) that maximizes the two-dimensional likelihood function \( L_{BAO}(\Omega_m, p) \). The 1\( \sigma \), 2\( \sigma \), and 3\( \sigma \) confidence level contours are defined as the sets of parameters \((\Omega_m, p)\) for which \( L_{BAO}(\Omega_m, p) \) is less than its maximum value by multiplicative factors of \( \exp(-2.30/2) \), \( \exp(-6.18/2) \), and \( \exp(-11.83/2) \), respectively. In Figures 1–3, we show two-dimensional constraints on cosmological parameters from radial BAO data for each of three dark energy models and two sets of priors.

Since radial BAO measurements alone, at the moment, cannot tightly constrain cosmological parameters we combined them with the Kowalski et al. (2008) Union SNIa data. In our derivation of the SNIa constraints we closely follow Kowalski et al. (2008). We define a likelihood function for cosmological parameters, \( L_{SNI}(\Omega_m, p, H_0) \), for each dark energy model. We marginalize over \( H_0 \) with a flat noninformative prior in the range \( 40 < H_0/(\text{km s}^{-1} \text{Mpc}^{-1}) < 100 \) to get a two-dimensional likelihood \( L_{SNI}(\Omega_m, p) \). The constraints on the \( \phi \)CDM model from the supernova data are shown in Figure 4.9 Since radial BAO and supernova measurements are independent, we multiply the individual likelihoods to get a joint likelihood function

\[
L(\Omega_m, p) = L_{BAO}(\Omega_m, p) L_{SNI}(\Omega_m, p).
\]

We show constraints derived from the joint likelihood function in Figures 5–7.

From the joint two-dimensional likelihood function we define the marginal one-dimensional likelihood functions for parameters \( \Omega_m \) and \( p \) (assuming uniform priors on both parameters in

For the constraints on ACMD and XCDM models from the supernova “Union” data set, see Figure 12 in Kowalski et al. (2008).
the ranges $\Omega_m \in (0.0, 1.0)$, $\Omega_\Lambda \in (0.0, 1.0)$, $\omega_x \in (-2.0, 0.0)$, and $\alpha \in (0.0, 5.0)$) through

$$\mathcal{L}(\Omega_m) = \int \mathcal{L}(\Omega_m, p) dp.$$  \hspace{1cm} (14)
Figure 8. One-dimensional likelihood functions for cosmological parameters for the three models derived using the joint likelihood from radial BAO and type Ia supernova data. Thick lines show results derived using the WMAP measured ratio \( l \), while thin lines are derived using the WMAP value for the sound horizon at recombination \( r_s \). In each of the six subplots total likelihoods are normalized to one.

### Table 1

| Model          | \( \Omega_m \) Range | \( \rho \) Range |
|----------------|-----------------------|------------------|
| \( \Lambda \)CDM (prior \( l \)) | 0.27 – 0.31 \( \Omega_r = 0.70 \) | 0.64 < \( \Omega_k \) < 0.75 |
| \( \Lambda \)CDM (prior \( r_s \)) | 0.24 – 0.32 \( \Omega_r = 0.68 \) | 0.62 < \( \Omega_k \) < 0.77 |
| XCDM (prior \( l \)) | 0.24 – 0.27 \( \omega_k = -0.91 \) | -1.00 < \( \omega_k \) < -0.88 |
| XCDM (prior \( r_s \)) | 0.25 – 0.28 \( \omega_k = -0.93 \) | -1.03 < \( \omega_k \) < -0.85 |
| \( \phi \)CDM (prior \( l \)) | 0.24 – 0.26 \( \alpha = 0.27 \) | 0.02 < \( \alpha < 0.54 \) |
| \( \phi \)CDM (prior \( r_s \)) | 0.25 – 0.27 \( \alpha = 0.20 \) | \( \alpha < 0.43 \) |

Note. 

* Best-fit values of cosmological parameters \( \Omega_m \) and \( \rho \), and 1σ confidence level intervals, from one-dimensional likelihood functions derived from a joint analysis of radial BAO and type Ia supernova data. Entries labeled as “prior \( l \)” are computed using the WMAP measured ratio \( l \) (and correspond to the thick lines in the figures). Entries labeled as “prior \( r_s \)” are computed using the WMAP measured value for the sound horizon at recombination \( r_s \) (and correspond to the thin lines in the figures).

\[
\mathcal{L}(p) = \int \mathcal{L}(\Omega_m, \rho) d\Omega_m.
\]

In Figure 8, we show one-dimensional likelihood functions for \( \Omega_m \) and \( \rho \) for the three dark energy models. From these one-dimensional likelihood functions, in all three models, we define the best-fit value of parameters as the values that maximize the likelihood, with highest posterior density 1σ confidence level intervals as the values of parameters for which \( \int_{\sigma \chi} dx \mathcal{L}(\chi) \int_{\sigma \chi} dx \mathcal{L}(\chi) = 0.68 \) and with \( \mathcal{L}(\chi) \) higher everywhere inside the interval than outside. The values of one-dimensional best-fit parameters and corresponding 1σ confidence level intervals are listed in Table 1.

### 4. RESULTS AND DISCUSSION

Figure 1 shows the Gaztaña et al. (2008a) radial BAO scale constraints on the \( \Lambda \)CDM model. When we use the WMAP measured value of the ratio \( l \) (thick lines) the model is constrained to be very close to the spatially flat case. When we use the WMAP measured value of the sound horizon at recombination \( r_s \) (thin lines) the constraints are weaker, the spatial curvature is not well constrained, and the nonrelativistic matter \( \Omega_m \) has to be less than 0.45 at about 3σ. These contours in Figure 1 are in reasonable accord with those shown in Figure 2 of Samushia & Ratra (2008b) which were derived using the “nonradial” BAO peak scales measured by Percival et al. (2007a) and Eisenstein et al. (2005). When the radial BAO data is combined with supernova data, the resulting constraints are significantly stronger, see Figure 5. The nonrelativistic matter density parameter is now constrained to be 0.15 < \( \Omega_m < 0.35 \) at about 3σ confidence level while the cosmological constant density parameter lies in the 0.45 < \( \Omega_k < 0.9 \) range. The best-fit values are close to the spatially flat model. The constraints computed from the joint one-dimensional likelihoods shown in Figures 8(a) and 8(b) are listed in Table 1. They are much more restrictive than those derived from the individual data sets, because those constrain combinations of dark energy parameters that are in some sense “orthogonal” in parameter space.

For the XCDM parametrization (for which we consider only spatially flat models) the confidence level contours derived from radial BAO measurements are broad and a range of \( \Omega_m \) and \( \omega_k \) values are acceptable. The constraints are shown in Figure 2. They are similar to the results shown in Gaztaña et al. (2008b, Figure 1). When we use the WMAP measured value of the ratio \( l \) (thick lines) the nonrelativistic matter \( \Omega_m \) has to be less than 0.4 at about 3σ. These confidence level contours are in reasonable accord with the ones shown in Samushia & Ratra (2008b, Figure 3) which were derived using the nonradial BAO data of Percival et al. (2007a) and Eisenstein et al. (2005). However, compared to the nonradial BAO scale measurements the radial BAO scale measurements better constrain \( \omega_k \) from below and tend to favor higher values of it. The joint constraints from radial BAO and supernova data are shown in Figure 6. The joint likelihood constrains the equation of state parameter to be \(-0.7 < \omega_k < -1.2\) at about 3σ.
confidence, while \( 0.2 < \Omega_m < 0.35 \). For XCDM, the joint constraints do not depend much on the method used for the analysis of the radial BAO data, unlike the \( \Lambda \)CDM and \( \phi \)CDM cases. One-dimensional joint likelihood functions for \( \Omega_m \) and \( \omega_\Lambda \) are shown in Figures 8(c) and 8(d) and the corresponding constraints are given in Table 1.

The spatially flat \( \phi \)CDM model confidence level contours are shown in Figure 3. Here, the radial BAO measurements constrain \( \Omega_m \) to be between 0.15 and 0.4 at about 3\( \sigma \), but the \( \alpha \) parameter is not constrained well and large values of \( \alpha \) (relatively rapidly evolving dark energy) are not ruled out, although the likelihood peaks at \( \alpha = 0 \). These results are similar to the ones derived in Samushia & Ratra (2008b, Figure 4) using the nonradial BAO peak scale measurements of Percival et al. (2007a) and Eisenstein et al. (2005). Figure 6 shows SNIa constraints on \( \phi \)CDM. From the Union data set alone, the nonrelativistic dark energy models and constraining cosmological parameters.

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REFERENCES

Aframand, R. R., Battista, R. C., Liberato, L., & Rosenfeld, R. 2009, Phys. Rev. D, 79, 023516
Alcaniz, J. S., Silva. R., Carvalho, F. C., & Zhu, Z.-H. 2009, Class. Quantum Gravity, 26, 105023
Allen, S. W., et al. 2004, MNRAS, 353, 457
Allen, S. W., et al. 2008, MNRAS, 383, 879
Antusch, S., Das, S., & Dutta, K. 2008, J. Cosmol. Part. Phys., JCAP08(2008)016
Bamba, K., Nojiri, S., & Odintsov, S. D. 2008, J. Cosmol. Part. Phys., JCAP08(2008)045
Barnard, M., et al. 2008, Phys. Rev. D, 78, 043528
Basilakos, S., Nessey, S., & Perivolaropoulos, L. 2008, MNRAS, 387, 1126
Basilakos, S., & Plionis, M. 2008, arXiv:0807.4590
Benitez, N., et al. 2009, ApJ, 691, 241
Blake, C., Collister, A., Bridle, S., & Lahav, O. 2007, MNRAS, 374, 1527
Blake, C., & Glazebrook, K. 2003, ApJ, 594, 665
Capozziello, S., Martin-Murono, P., & Rubano, C. 2008, Phys. Lett. B, 664, 12
Chae, K.-H., Chen, G., Ratra, B., & Lee, D.-W. 2004, ApJ, 607, L71
Chen, G., & Ratra, B. 2003a, ApJ, 582, 586
Chen, G., & Ratra, B. 2003b, PASP, 115, 1143
Chen, G., & Ratra, B. 2004, ApJ, 612, L1
Cole, S., et al. 2005, MNRAS, 362, 505
Daly, R. A., et al. 2009, ApJ, 691, 1058
Dev, A., Jain, D., & Lohiya, D. 2008, arXiv:0804.3491
Doran, M., Robbers, G., & Wetterich, C. 2007, Phys. Rev. D, 75, 023003
Doré, O., et al. 2007, arXiv:0712.1599
Dunkley, J., et al. 2009, ApJS, 180, 306
Eisenstein, D. J., et al. 2005, ApJ, 633, 560
Fernandez-Martinez, E., & Verde, L. 2008, J. Cosmol. Part. Phys., JCAP08(2008)023
Friedman, J. A., Turner, M. S., & Huterer, D. 2008, ARAA, 46, 385
Gaztañaga, E., Cabrè, A., & Hui, L. 2008a, arXiv:0807.3551
Gaztañaga, E., Miquel, R., & Sánchez, E. 2008b, arXiv:0808.1921
Glazebrook, K., et al. 2007, arXiv:astro-ph/0701876
Hoekstra, H., & Jain, B. 2008, Annu. Rev. Nucl. Part. Syst., 58, 99
Hu, W., & Haiman, Z. 2003, Phys. Rev. D, 68, 063004
Hütsi, G. 2006, A&A, 449, 891
Ishida, E. E. O., Reis, R. R. R., Toribo, A. V., & Waga, I. 2008, Astropart. Phys., 28, 547
Jesus, J. F., Santos, R. C., Alcaniz, J. S., & Lima, J. A. S. 2008, Phys. Rev. D, 78, 063514
Komatsu, E., et al. 2009, ApJS, 180, 330
Kowalski, M., et al. 2008, ApJ, 686, 749
La Vaia, G., & Colombo, L. P. L. 2008, J. Cosmol. Part. Phys., JCAP08(2008)007
La Vaia, G., Colombo, L. P. L., Vergani, L., & Bonometto, S. A. 2009, ApJ, 697, 1946
Laizooz, R., Nesseris, S., & Perivolaropoulos, L. 2008, J. Cosmol. Part. Phys., JCAP07(2008)012
Lee, S., & Ng, K.-W. 2007, Phys. Rev. D, 76, 043518
Lin, H., Zhang, T.-J., & Yuan, Q. 2008, arXiv:0804.3135
Linder, E. V. 2003, Phys. Rev. D, 68, 083504
Mainini, R. 2008, J. Cosmol. Part. Phys., JCAP07(2008)003
Oguri, M., et al. 2008, AJ, 135, 512
Padihampanah, N., et al. 2007, MNRAS, 378, 852
Page, L., et al. 2003, ApJS, 148, 233
Peebles, P. J. E. 1984, ApJ, 284, 439
Peebles, P. J. E., & Ratra, B. 1988, ApJ, 325, L17
Peebles, P. J. E., & Ratra, B. 2003, Rev. Mod. Phys., 75, 559
Percival, W. J., et al. 2007a, MNRAS, 381, 1053
Percival, W. J., et al. 2007b, ApJ, 657, 51
Podariu, S., Daly, R. A., Mory, M., & Ratra, B. 2003, ApJ, 584, 577
Podariu, S., Nugent, P., & Ratra, B. 2001a, ApJ, 553, 39
Podariu, S., & Ratra, B. 2000, ApJ, 532, 109
Podariu, S., Souradeep, T., Gott, J. R., Ratra, B., & Vogeley, M. S. 2001b, ApJ, 559, 9

These future BAO measurements should prove to be of great signifi-
