Lorentz-symmetry violation and dynamical flavour oscillations

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Abstract

We show how a mass mixing matrix can be generated dynamically, for two massless fermion flavours coupled to a Lorentz invariance violating (LIV) gauge field. The LIV features play the role of a regulator for the gap equations, and the non-analytic dependence of the dynamical masses, as functions of the gauge coupling, allows to consider the limit where the LIV gauge field eventually decouples from the fermions. Lorentz invariance is then recovered, to describe the oscillation between two free fermion flavours, and we check that the finite dynamical masses are the only effects of the original LIV theory.

1 Introduction

The generation of quark, lepton and vector boson masses, as described in the Standard Model due to their coupling with the Higgs boson, seems to have been confirmed by the latest experimental results at the Large Hadron Collider [1], with the discovery of a Higgs-like (scalar) particle. However, the origin of neutrino masses is still not well established, although the seesaw mechanism seems the most elegant and simple for such a purpose [2].

The possibility, therefore, of generating neutrino masses dynamically is still at play. A scenario has been proposed in [3], in which flavour oscillations can arise dynamically from the flavour-mixing interaction of two massless bare fermions with an Abelian gauge field, which has a Lorentz-Invariance-Violating (LIV) propagator. Lorentz symmetry violation is achieved by higher order space derivatives, which are suppressed by a large mass scale $M$. This mass scale allows the dynamical generation of fermion masses, as was shown in [4] with the Schwinger-Dyson approach. Another role of this mass scale is to lead to a finite gap equation, and therefore to regulate the model.
Moreover, LIV U(1) gauge models of the form suggested in [4] have been shown to arise in the low-energy limit of some consistent quantum gravity theories [5], for instance when the U(1) gauge theory is embedded in a stringy space time foam model, with the foamy structures being provided by (point-like) D-brane space-time defects (“D-particles”). In such microscopic models, the gauge field is one of the physical excitations on brane world universes interacting with the D-particles. It was observed in [5] that the LIV Lagrangian of [4] can be obtained from a Born-Infeld-type Lagrangian of the U(1) gauge field in the D-particle background, upon an expansion in derivatives. Lorentz Violation arises locally in such models as a result of the recoil of the D-particle defects during their interaction with open strings representing the U(1) excitations. Finally, such a microscopic model has been used for the study of decoherence in neutrino oscillations [6].

An important point is the following structure of the fermion dynamical mass [4]

\[ m_{\text{dyn}} \simeq M \exp(-a/e^2), \]  

(1)

where \( a \) is a positive constant and \( e \) is the gauge coupling. From the expression (1), one can see that it is possible to take the simultaneous limits \( M \to \infty \) and \( e \to 0 \), in such a way that the dynamical mass (1) remains finite, corresponding to a physical fermion mass. In the previous limit, the non-physical gauge field decouples from the theory, and we also note that the gauge dependence of the dynamical mass is avoided.

An essential feature of the mechanism described here is the following. Although LIV features are suppressed by the large mass scale \( M \), so that the corresponding effects are negligible at the classical level, quantum corrections completely change this picture, and lead to finite effects. In our present study, the finite effect is the dynamical generation of fermion masses, which is present even after setting the LIV-suppressing mass scale \( M \) to infinity and the coupling \( e \) to 0. Also, after this limit is taken, relativistic dispersion relations for fermions are recovered, such that the dynamical masses are the only finite effect from the original LIV model.

2 Model

The LIV Lagrangian considered in [3] is

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}(1 - \frac{\Delta}{M^2}) F^{\mu\nu} + \bar{\Psi}(i\partial - \tau A)\Psi, \]  

(2)
where $F_{\mu\nu}$ is the Abelian field strength for the gauge field $A^\mu$ and $\Delta = -\partial_i\partial^i$ is the Laplacian. The mass scale $M$ suppresses the LIV derivative operator $\Delta$, and can be thought of as the Plank mass, but which will eventually be set to infinity. $\Psi$ is a massless fermion doublet $\Psi = (\psi_1, \psi_2)$ and the flavour mixing matrix $\tau$ features the gauge couplings $(e_1, e_2, \epsilon)$ as

$$\tau = \begin{pmatrix} e_1 & -i\epsilon \\ i\epsilon & e_2 \end{pmatrix} = \frac{e_1 + e_2}{2}1 + \frac{e_1 - e_2}{2}\sigma_3 + \epsilon\sigma_2 ,$$

(3)

where $\sigma_i$ are the usual Pauli matrices and $1$ is the $2 \times 2$ identity matrix. The fermions $\psi_1$ and $\psi_2$ in eq.(2) are Dirac, but the structure of the gap equations which is discussed below remains the same in the case of Majorana fermions, hence the corresponding dynamical masses are independent of the nature of fermions.

In order to study the possibility of generating masses dynamically, we assume the dressed fermion mass matrix

$$M = \begin{pmatrix} m_1 & \mu \\ \mu & m_2 \end{pmatrix} = \frac{m_1 + m_2}{2}1 + \frac{m_1 - m_2}{2}\sigma_3 + \mu\sigma_1 ,$$

(4)

with eigenvalues $\lambda_{m \pm} = (m_1 + m_2)/2 \pm \sqrt{(m_1 - m_2)^2 + 4\mu^2}/2$. We allow for the presence of $M$ in the gap equations, obtained from the self-consistent Schwinger-Dyson equation for the fermion propagator, which has the usual structure and is not modified by the LIV term in the Lagrangian (2). If we neglect corrections to the wave functions, the vertices and the gauge propagator, the Schwinger-Dyson equation reads for our model

$$G^{-1} - S^{-1} = \int_p D_{\mu\nu} \tau^\mu G \tau^\nu ,$$

(5)

where $S, G, D_{\mu\nu}$ are the bare fermion propagator, the dressed fermion propagator and the gauge propagator respectively, given by (we denote $p_\mu = (\omega, \vec{p})$)

$$S = \frac{i\phi/p^2}{1 + \vec{p}^2/M^2} ,$$

$$G = \frac{\omega^2 + p(m_1 + m_2) + m_1 m_2 - \mu^2}{(p^2 - m_1^2)(p^2 - m_2^2) - 2\mu^2(p^2 + m_1 m_2) + \mu^4} \times \left[ \left( \frac{\phi - m_1 + m_2}{2} \right) 1 + \frac{m_1 - m_2}{2}\sigma_3 + \mu\sigma_1 \right] ,$$

$$D_{\mu\nu} = \frac{i}{1 + \vec{p}^2/M^2} \left( \frac{\eta_{\mu\nu}}{\omega^2 - \vec{p}^2} + \frac{P_{\mu\nu}}{(\omega^2 - \vec{p}^2)^2} \right) ,$$
where $\zeta$ is a gauge fixing parameter. The loop integral $[5]$ is finite as a consequence of the LIV term $\vec{p}^2/M^2$ in the denominator of the gauge propagator.

3 Solutions

The Schwinger-Dyson equation $[5]$ leads to 4 self-consistent gap equations, which must be satisfied by three unknowns $m_1, m_2, \mu$. This is possible provided constraints are satisfied, and several possibilities are derived in [3], which are summarized here:

(a) $\{m_1 = m_2 = 0, \mu \neq 0 \text{ and } e_1e_2 > e^2\}$ or $\{m_1 = -m_2 \neq 0 \text{ and } e_1 = e_2 > \epsilon\}$: Mixing but no oscillation because the mass eigenvalues are opposite. In both cases we have

$$m^2 + \mu^2 = M^2 \exp \left( \frac{-16\pi^2}{(4 + \zeta)(e_1e_2 - e^2)} \right), \quad (7)$$

where $m = |m_1| = |m_2|$, and only the sum $m^2 + \mu^2$ is constrained;

(b) $\{m_1 = m_2 \neq 0, \mu^2 = m_1m_2 \text{ and } e_1 = e_2, \epsilon = 0\}$: Mixing and flavour oscillations. In this situation, the mass matrix has eigenvalues

$$\lambda_+ = 2m = M \exp \left( -\frac{8\pi^2}{(4 + \zeta)e^2} \right), \quad \lambda_- = 0, \quad (8)$$

where $m = m_1 = m_2$ and $e = e_1 = e_2$. The mixing angle is $\pm \pi/4$, depending on the sign of $\mu$. Note that this mixing angle corresponds to $\theta_{23}$;

(c) $\{\mu = 0 \text{ and } \epsilon = 0\}$: No mixing. In this case, the two eigenvalues of the mass matrix are

$$m_i = M \exp \left( \frac{-8\pi^2}{(4 + \zeta)e_i^2} \right), \quad i = 1, 2, \quad (9)$$

so the mass matrix is diagonal in flavour space, with masses $m_i$.

The latter case is relevant to Majorana mass eigenstates. Indeed, one defines the Majorana fields $\nu_k$ in terms of the left-handed Dirac fields $\nu_k^L$ as $\nu_k = \nu_k^L + (\nu_k^L)^C$, such that the Lagrangian for free Majorana fields reads

$$\mathcal{L} = \frac{1}{2} \sum_k (\bar{\nu}_k i\gamma^5 \nu_k - m_k \nu_k^C \bar{\nu}_k) \quad (10)$$

where the extra overall factor $1/2$ compared to the Dirac case does not play a role in our discussion, as it can be absorbed in a redefinition of masses.
4 Discussion

One can consider the coupling of a doublet of (mass eigenstate) Majorana fields to the regulator \( U(1) \) gauge field \( A_\mu \) in the case (c). The fact that a Majorana field contains both chiralities allows for a straightforward extension of the Dirac case discussed in previous sections to the current situation. In this way, we are able to generate dynamically different mass eigenvalues \( M \) for the two species, without mixing, as implied by the corresponding solution. This is a consistent way of discussing the dynamical appearance of a Majorana mass for left-handed neutrinos of the standard model. The non-trivial mixing of flavour eigenstates is then obtained by coupling neutrinos to the physical \( SU(2)_L \) gauge fields of the standard model. Thus it is because Majorana neutrinos are mass eigenstates that the solution without mixing is relevant to the scenario with Majorana neutrinos.

In order to recover Lorentz invariance in the different solutions found, we finally take the simultaneous limits

\[
M \to \infty \quad \text{and} \quad e_1, e_2, \epsilon \to 0,
\]

in such a way that the dynamical masses are finite, and we denote the corresponding “renormalized” mass matrix by \( M_R \). This mass matrix can be fixed by experimental data, which in the interesting cases (b) and (c) leads to the couplings \( e_1, e_2 \) as functions of \( M \), for the limit (11) to be defined unambiguously. This procedure is independent of the gauge parameter \( \zeta \), and the resulting fermion mass is set to any desired value. In this limit, the gauge field decouples from fermions, and the only finite effect from Lorentz violation in the original model is the presence of finite dynamical masses for fermions. Indeed, the fermion dispersion relations are relativistic in the limit (11), since the fermion self-energy is then

\[
\Sigma(\omega, \vec{p}) \to -\frac{1}{4}(\omega \gamma^0 - \vec{p} \cdot \vec{\gamma})1 - M_R,
\]

such that time and space derivatives are dressed with the same (non-perturbative) corrections.

We also note that, because of flavour mixing interactions of fermions with the vector field, that latter may become massive \( 7 \), but this feature does not occur here in the relevant cases (b) or (c), where \( \epsilon = 0 \).

Finally, the case (b) might seem restrictive for Dirac fermions, since the mixing angle is necessarily equal to \( \pm \pi/4 \). This is a result of the high
symmetry of the $2 \times 2$ mass matrix, and the extension to the three-flavour case is planned, in order to allow for more flexibility and phenomenological studies.

References

[1] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716 (2012) 1 [arXiv:1207.7214 [hep-ex]]; S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716 (2012) 30 [arXiv:1207.7235 [hep-ex]].

[2] For a review see: R. N. Mohapatra, S. Antusch, K. S. Babu, G. Barenboim, M. -C. Chen, A. de Gouvea, P. de Holanda and B. Dutta et al., Rept. Prog. Phys. 70 (2007) 1757 [hep-ph/0510213] and references therein.

[3] J. Alexandre, J. Leite and N. E. Mavromatos, Phys. Rev. D 87 (2013) 125029 [arXiv:1304.7706 [hep-ph]].

[4] J. Alexandre, arXiv:1009.5834 [hep-ph]; J. Alexandre and A. Vergou, Phys. Rev. D 83 (2011) 125008 [arXiv:1103.2701 [hep-th]].

[5] N. E. Mavromatos, Phys. Rev. D 83 (2011) 025018 [arXiv:1011.3528 [hep-ph]].

[6] J. Alexandre, K. Farakos, N. E. Mavromatos and P. Pasipoularides, Phys. Rev. D 77 (2008) 105001 [arXiv:0712.1779 [hep-ph]].

[7] R. Jackiw, K. Johnson and , Phys. Rev. D 8 (1973) 2386; J. M. Cornwall, R. E. Norton and , Phys. Rev. D 8 (1973) 3338.