Abstract

We examine the dielectron invariant mass spectrum and the variation of the polarization asymmetry as a function of that mass in the event generator of the five-dimensional Bethe-Heitler differential cross section of the conversion of linearly polarized photons that we have implemented recently as a Geant4 Physics Model. We compare the results obtained with simulated samples to analytical expressions.

Studies of \( b \rightarrow s\gamma \) decays at LHC and Belle II experiments could provide hints of physics beyond the standard model in flavor-changing neutral current processes; issues have been reported in the simulation of the properties of the background induced by genuine \( \gamma \)-ray conversions in their detectors. Our results demonstrate that using the five-dimensional model would help solve these issues.

keywords: gamma rays, pair conversion, polarization, radiative decays, dielectron, Geant4

1 Motivation

The standard model of elementary particles and of their interactions (SM) predicts that \( b \rightarrow s\gamma \) decays produce photons that are left-handed to a high accuracy [1]. Photon polarimetry offers therefore a unique sensitivity to beyond-the-standard-model (BSM) processes that provide right-handed couplings.

Among the various methods that have been used or that are being developed to achieve the measurement, the analysis can involve the angular distribution of the final state including a prompt conversion of the photon to an \( \ell^+\ell^- \) pair at the \( B \) decay vertex ("short distance lepton pairs"), or the "nuclear" conversion of a real photon in the material of the detector far away from the vertex ("long distance lepton pairs"), [2,3]. In the later case, the system formed by the two vector particles (\( K^* \) and \( \gamma \)) stay in a coherent state until decay and conversion.

In both cases (short distance, discussion of eq. (2.15) of [3]; long distance, [4]) most of the sensitivity lies at low values of the lepton pair invariant mass, \( \mu \), therefore the analysis is performed as a function of \( \mu \) and light-lepton (electron-positron) final states are used. In that context, understanding the \( \mu \) spectrum and polarization asymmetry of the conversion of background photons in the detector can be critical to mastering the systematics; the examination of event samples obtained by simulation with the Geant4 toolkit [5] showed [7,6] a departure from the QED prediction [4]. The physics models that were existing in Geant4 at that time were sampling probability density functions (pdf) that are products of one-dimensional Erzätze, with a focus on the distributions of the one-track polar angle and on the energy share between the electron and the positron, but with an inconsistent behavior for other variables [8]. In particular the polarized model was found to be flawed [8].

In this paper we examine the dielectron invariant mass spectrum and the variation of the polarization asymmetry as a function of that mass in the event generator of the five-dimensional Bethe-Heitler differential cross section of the conversion of linearly polarized photons. We compare the results obtained with simulated samples to analytical expressions.
2 Dielectron invariant mass spectrum

We examine the $\mu$ spectra provided by the exact, that is, five-dimensional, event generator of the Bethe-Heitler conversion process that we have recently implemented in Geant4 [9, 10, 11, 12], and we compare it with the Bethe-Heitler analytical spectrum obtained by Borsellino [4]. Note that in contrast with Bethe-Heitler [13], the algorithm used here does not assume that the energy carried away by the recoiling target be negligible (compare the differential element of eq. (20) of [13] to that of eqs. (1)-(3) of [9]).

The $\mu$ spectrum shows a maximum at very low mass ($\mu < 2 \text{MeV}/c^2$) and then decreases as $1/\mu^3$ [4], so using the low-mass approximation expression from [4] is legitimate ($\mu \ll E$, where $E$ is the energy of the incident photon). Indeed for the nuclear conversion of 40 GeV photons on silicon, the fraction of the spectrum above $\mu = 1 \text{GeV}/c^2$ is found to be as small as $2 \times 10^{-5}$ in the simulated samples.

Figure 1 shows the comparison of the spectra obtained with the 5D algorithm of [9] to the low-mass approximation analytical expression [4], for which the screening of the nuclear field by the atomic electrons is described with a form factor (eq. (10) of [4] or eq. (3.4) of [6]). The simulated spectrum and the analytical expression are found to agree with each other, except for the lowest invariant masses. Borsato [6] had noticed a large discrepancy between the fractions of events having an invariant mass $\mu > 10 \text{MeV}/c^2$ predicted by their Geant4 simulation (3.2%) and computed from [4] (5.4%); we obtain a much better agreement with the 5D generator, with a fraction found to be $(5.57 \pm 0.07)\%$.

Figure 1: Spectrum of the $e^+e^-$ invariant mass, minus its minimal value of $2m$, for nuclear conversions of 40 GeV photons on silicon (grey points: Monte Carlo sample; black curve: analytical expression from [4]). The bin sizes have been set so that the numbers of events per bin are identical. The vertical “error bar” for each bin is located at the barycenter of the events that enter that bin.

Variations on the nature of the target (C, Si, Ge) or on the photon beam energy (40 GeV, 40 TeV) induced very little variation of the spectra, be it analytical or from simulated data (plots not shown).

3 Polarization Asymmetry

For gamma rays converting to an $e^+e^-$ pair, the measurement of the fraction and of the angle of the linear polarization of is based on the analysis of the distribution of an azimuthal angle, $\phi$, describing the orientation
of the final state \[14\]

\[
d\sigma/d\phi \propto (1 + A \times P \cos(2\phi)), \tag{1}
\]

- \(A\), the polarization asymmetry, depends on the energy, \(E\), of the photon and varies from \(\pi/4\) at threshold \[15\] (when the masses of the particles of the pair are much smaller than that of the target, compare Fig. 3 of \[11\] to Fig. 5 of \[9\]) to 1/7 at very high-energy \[16\]. Over most of the MeV-GeV energy range which is presently accessible to experimentalists, \(A\) is close to 0.15 – 0.20 (see e.g. Fig. 3 of \[15\]).

- \(P\) is the linear polarization fraction of the photon beam.

Using the Weizsacker-Williams approximation, Wick has obtained \[17\] an expression for the polarized double-differential cross section, \(d\sigma/d\phi\), as a function of \(|\beta|\), where \(|\beta|\) and \(\theta_\ell\) are, in the pair center-of-mass frame, the velocity and the polar angle of either of the leptons.

As the pair invariant mass, \(\mu\), is related to \(|\beta|\), \(\mu = 2m/\sqrt{1 - \beta^2}\), we can obtain the variation of the polarization asymmetry as a function of the pair invariant mass \(\mu\) \[6\]. Wick’s expression integrates \[17\] to

\[
d\sigma/d\phi \propto (1 + \frac{1}{3} \cos^2 \phi), \tag{2}
\]

that is

\[
d\sigma/d\phi \propto (1 + \frac{1}{7} \cos 2\phi); \tag{3}
\]

that is \(A = 1/7\), which is the high-energy limit of the Boldyshev-Peresunko high-energy asymptotic expression obtained \[16\] from the Bethe-Heitler differential cross section

\[
A \approx \frac{4}{9} \log \left(\frac{2E}{mc^2}\right) - \frac{20}{28} \log \left(\frac{2E}{mc^2}\right) - \frac{218}{27}. \tag{4}
\]

Integrating Wick’s expression for \(d\sigma/\sin\theta_\ell d\theta_\ell d\phi\) on \(\theta_\ell\), we obtain the polarization asymmetry as a function of \(|\beta|\)

\[
A = \frac{\beta(\beta^2 - 1)}{\beta(\beta^2 - 2)} + \frac{(1 - \beta^4)}{(3 - \beta^4)} \text{arctanh} \beta. \tag{5}
\]

We compute the polarization asymmetry on simulated samples using the moment’s method \[18, 15\], the azimuthal angle of the event being defined as the bisectrix of the azimuthal angles of the electron and of the positron \[15\]. For 40 GeV photons converting on point-like charges we obtain \(A = (16.22 \pm 0.14)\%\), which is marginally compatible with the Boldyshev-Peresunko \[16\] value of \(A = 15.8\%\), while for conversions on silicon we obtain \(A = (16.70 \pm 0.14)\%\). These values are significantly higher than Wick’s high-energy limit of \(A = 1/7 \approx 14.3\%\).

Figure 2 shows the variation of the polarization asymmetry as a function of dielectron invariant mass for several targets. On most of the mass range, say above \(2m + 0.5\) MeV/c\(^2\), the shape of the polarization asymmetry for simulated data is well described by eq. \[5\], with some underestimation though, as we observed above on the whole sample. At lower masses, simulated spectra are affected by the \(q^2\) suppression due to screening in the case of conversions on full atom (with their electron cloud), where \(q\) is the momentum of the recoiling target, and eq. \[5\] obtained from Wick’s model fails to represent the data.

Examination of the variation of the polarization asymmetry as a function of dielectron invariant mass for several incident photon energies (Fig. 3) confirms the interpretation of Wick’s model as a high-energy approximation.
Figure 2: Variation of the polarization asymmetry of the nuclear conversions of 40 GeV photons, as a function of the dielectron invariant mass minus its minimal value of $2m$, for simulated samples (conversions on germanium (light grey), on silicon (grey) and on point-like charged targets (black)), compared to the analytical expression (eq. 5) obtained from Wick’s differential cross section [17]. The bin sizes have been set so that the numbers of events per bin are identical. The vertical “error bar” for each bin is located at the barycenter of the events that enter that bin.

4 Conclusion

We have obtained the spectra of the dielectron invariant mass, $\mu$, and the variation with $\mu$ of the polarization asymmetry, $A$, of gamma-ray conversions to $e^+e^-$ pairs in the algorithm [9] used in the five-dimensional generator of the Bethe-Heitler differential cross section. We have partly integrated Wick’s expression of the differential cross section to obtain the polarization asymmetry as a function of $\mu$ (eq. 5). At large masses, the results on simulated data agree with Borsellino’s expression (spectrum) and with eq. (5) (polarization asymmetry). At smaller masses though, close to $\mu = 2m$, Borsellino’s and Wick’s expressions are found to fail to describe the simulated data.

Apart from the very small invariant mass range of $\mu < 1\text{ MeV}/c^2$ that is irrelevant for the analyses performed in [7, 6] and for which form-factor effects affect the spectra and the polarization asymmetry, simulated results are found to be in agreement with past published analytical expressions.

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Figure 3: Variation of the polarization asymmetry of nuclear conversions on silicon as a function of the dielectron invariant mass minus its minimal value of $2m$, for simulated samples with various energies (40 TeV (light grey), 40 GeV (grey) and 40 MeV (black)), compared to the analytical expression (eq. 5) obtained from Wick’s differential cross section [17]. The bin sizes have been set so that the numbers of events per bin are identical. The vertical “error bar” for each bin is located at the barycenter of the events that enter that bin.

References

[1] D. Atwood, M. Gronau and A. Soni, “Mixing induced CP asymmetries in radiative B decays in and beyond the standard model,” Phys. Rev. Lett. 79 (1997) 185

[2] F. Bishara and D. J. Robinson, “Probing the photon polarization in $B \to K^*\gamma$ with conversion,” JHEP 1509 (2015) 013

[3] Y. Grossman and D. Pirjol, “Extracting and using photon polarization information in radiative B decays,” JHEP 0006, 029 (2000)

[4] A. Borsellino, “Momentum Transfer and Angle of Divergence of Pairs Produced by Photons,” Phys. Rev. 89 (1953) 1023.

[5] J. Allison et al. “Recent developments in Geant4,” Nucl. Instrum. Meth. A 835 (2016) 186

[6] M. Borsato, “Study of the $B^0 \to K^{*0}e^+e^-$ decay with the LHCb detector and development of a novel concept of PID detector: the Focusing DIRC,” CERN-THESIS-2015-219

[7] R. Aaij et al. [LHCb Collaboration], “Angular analysis of the $B^0 \to K^{*0}e^+e^-$ decay in the low-q$^2$ region,” JHEP 1504 (2015) 064

[8] P. Gros et al. [HARPO Collaboration], “γ-ray telescopes using conversions to $e^+e^-$ pairs: event generators, angular resolution and polarimetry,” Astropart. Phys. 88 (2017) 60

[9] D. Bernard, “A 5D, polarised, Bethe-Heitler event generator for $\gamma \to e^+e^-$ conversion,” Nucl. Instrum. Meth. A 899 (2018) 85

[10] I. Semeniouk and D. Bernard, “C++ implementation of Bethe-Heitler, 5D, polarized, $\gamma\to e^+e^-$ pair conversion event generator,” Nucl. Instrum. Meth. A 936 (2019) 290
[11] D. Bernard, “A 5D, polarised, Bethe-Heitler event generator for $\gamma \rightarrow \mu^+\mu^-$ conversion,” arXiv:1910.12501

[12] V. Ivanchenko et al. [Geant4 Collaboration], “Progress of Geant4 electromagnetic physics developments and applications,” EPJ Web Conf. 214 (2019) 02046.

[13] H. Bethe and W. Heitler, “On the Stopping of Fast Particles and on the Creation of Positive Electrons”, Proceedings of the Royal Society of London A, 146 (1934) 83.

[14] T. H. Berlin and L. Madansky, “On the Detection of gamma-Ray Polarization by Pair Production”, Phys. Rev. 78 (1950) 623.

[15] P. Gros and D. Bernard, “$\gamma$-ray polarimetry with conversions to $e^+e^-$ pairs: polarization asymmetry and the way to measure it,” Astropart. Phys. 88 (2017) 30.

[16] V. F. Boldyshev and Y. P. Peresunko, “Electron-positron pair photoproduction on electrons and analysis of photon beam polarization,” Yad. Fiz. 14 (1971) 1027.

[17] “Detection of gamma-Ray Polarization by Pair Production”, G. C. Wick, Phys. Rev. 81, 467 - 468 (1951).

[18] D. Bernard, “Polarimetry of cosmic gamma-ray sources above $e^+e^-$ pair creation threshold,” Nucl. Instrum. Meth. A 729, 765 (2013).