Vector leptoquark mass limits and branching ratios of $K^0_L, B^0, B_s \to l_i^+ l_j^-$ decays with account of fermion mixing in leptoquark currents

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Abstract

The contributions of the vector leptoquarks of Pati-Salam type to the branching ratios of $K^0_L, B^0, B_s \to l l'$ decays are calculated with account of the fermion mixing in the leptoquark currents of the general type. Using the general parametrizations of the mixing matrices the lower vector leptoquark mass limit $m_V > 86$ TeV is found from the current experimental data on these decays. The branching ratios of the decays $B^0, B_s \to l l'$ predicted at $m_V = 86$ TeV are calculated. These branching ratios for the decays $B^0 \to \mu^+ \mu^-$, $e\mu$, and $B_s \to e^+ e^-$ are by order of $2 \div 4$ less than their current experimental limits. For the decays $B_s \to e\tau, \mu\tau$ these branching ratios are of order $10^{-10}$ and $10^{-9}$ respectively. The predicted branching ratios will be useful in the current and future experimental searches for these decays.

Keywords: Beyond the SM; four-color symmetry; Pati–Salam; leptoquarks; B physics; rare decays.

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The search for a new physics beyond the Standard Model (SM) is one of the aims of the current experiments at the LHC. There is a lot of new physics scenarios (such as supersymmetry, left-right symmetry, two Higgs model, extended dimension models, etc.) which are now under experimental searches at the LHC.

One of the possible variants of new physics can be induced by the well known Pati-Salam idea on the possible four color symmetry between quarks and leptons of the vector-like type regarding leptons as the quarks of the fourth color in frame of the gauge group $G_{PS} = SU_V(4) \times SU_L(2) \times SU_R(2)$ [1]. This group has three gauge coupling constants related to the strong electromagnetic and weak coupling constants and can be regarded as an intermediate stage in the symmetry breaking of the GUT group $SO(10)$ [2–4] embedded in the more large group $E_6$ [5,6]. The four color symmetry of the vectorlike type immediately predicts new gauge particles - the vector leptoquarks $V_{\alpha}, \alpha = 1, 2, 3$ which belong to the multiplet $(15, 1, 1)$ of the group $G_{PS}$ and form the color triplet $(3,1)_{2/3}$ of the SM group $G_{SM} = SU_c(3) \times SU_L(2) \times U(1)$.

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In general case the leptoquarks as the vector or scalar particles carrying both the baryon and lepton numbers appear in many models and can led to varied new physics effects, the comprehensive review of physics effects generated by leptoquarks can be found in [7]. In the last years the vector $(3,1)_{2/3}$ leptoquarks are used to explain the known anomalies in the semileptonic $B$ meson decays with simultaneous satisfying the other experimental data. Using the conventional Pati-Salam vector leptoquarks this goal is achieved by the appropriate choice of the vector leptoquark couplings to fermions [8], by account of the fermion mixing in leptoquark currents of the special form [9], in frame of new gauge leptoquark model extending the $SU(4)$ four color group by the additional factor $SU(3)'$ [10], by extending the fermion sector of the Pati-Salam model [11], in a three-site gauge model using the Pati-Salam gauge group for each fermion generation [12]. Another approach to the explanations of the $B$-decay anomalies is attempted in the model with the composite leptoquarks which is based on the Pati-Salam $SU(4)$ group in the context of a new strongly interacting sector [13][14]. In this approach the vector leptoquarks couple primarily to the fermions of the third generation with their couplings to the the fermions of the first two generations being suppressed.

The lower mass limits for the vector leptoquarks from their direct searches are of about or less 1 TeV. The essentially more stringent lower mass limits are resulting from the rare decays of pseudoscalar mesons. The most stringent of them are resulting from the $K^0_l \rightarrow e^+\mu^+$ decay and with neglect of fermion mixing in leptoquark currents are of order of 2 000 TeV [15–19].

It should be noted however that the fermion mixing in leptoquark currents is quite natural. It is as natural as the fermion mixings in the week currents which are described by the well known matrices $V_{CKM}$ and $U_{PMNS}$ in the quark and lepton sectors respectively. Indeed, the mass eigenstates of left- and right-handed quarks and leptons $Q_{paa}^{L,R}$, $l_{ia}^{L,R}$ can enter to interactions with gauge and scalar fields in general case through the superpositions

$$Q_{paa}^{L,R} = \sum_q (A_{Q_a}^{L,R})_{pq} Q_{qaa}^{L,R}, \quad l_{ia}^{L,R} = \sum_j (A_{l_a}^{L,R})_{ij} l_{ja}^{L,R},$$

where $A_{Q_a}^{L,R}$ and $A_{l_a}^{L,R}$ are unitary matrices describing the fermion mixing and diagonalizing the mass matrices of quarks and leptons, $p, q, i, j = 1, 2, 3, ...$ are the quark and lepton generation indexes, $a = 1, 2$ and $\alpha = 1, 2, 3$ are the $SU_L(2)$ and $SU_c(3)$ indexes, $Q_{q1} \equiv u_q = (u, c, t)$, $Q_{q2} \equiv d_q = (d, s, b)$ are the up and down quarks, $l_{j1} \equiv \nu_j$ are the mass eigenstates of neutrinos and $l_{j2} \equiv l_j = (e^-, \mu^-, \tau^-)$ are the charged leptons. In the weak interaction the matrices $A_{Q_a}^{L,R}$ and $A_{l_a}^{L,R}$ form the CKM and PMNS matrices as $C_Q = (A_{Q_1})^+ A_{Q_2} = V_{CKM}$, $C_l = (A_{l_1})^+ A_{l_2} = U_{PMNS}^+$. In analogous way in the interaction of quarks and leptons with leptoquarks the matrices $A_{Q_a}^{L,R}$ and $A_{l_a}^{L,R}$ led to specific matrices

$$K_a^{L,R} = (A_{Q_a}^{L,R})^+ A_{l_a}^{L,R}$$

of the fermion mixing in leptoquark currents. The fermion mixing in leptoquark currents can essentially lower the mass limits on leptoquark masses. The current experimental data on the decays

$$K^0_L, B^0, B_s \rightarrow l_i^+ l_j^-$$

give now the possibility to obtain new lower mass limits for the leptoquarks with account of the fermion mixing in leptoquark currents.

In this paper the contributions of the vector leptoquarks of Pati-Salam type to the decays [3] are calculated and analysed with account of the fermion mixing in leptoquark
factors depending on the mixing matrices $K$ standard way, the factors $R^\dagger$ currents, $m$ the branching ratios $Br^\dagger$ interaction (4) of vector leptoquarks with quarks and leptons is not purely vectorlike. 

with account the gluonic corrections to the pseudoscalar quark $\bar{K}$ of the $SU_V(4)$ symmetry breaking, $P_{L,R} = (1 \pm \gamma_5)/2$ are the left and right operators of fermions and $K_{L,R}^2$ are the mixing matrices (2) for down fermions. It should be noted that in the case of the chiral ($K_L^2 \neq K_R^2$) mixing the interaction (4) of vector leptoquarks with quarks and leptons is not purely vectorlike.

Denoting the sums of the branching ratios of charge conjugated final states as

$$ Br_V(P \to e^+\mu^-) + Br_V(P \to \mu^+e^-) \equiv Br_V(P \to e\mu), $$ (5)

$$ Br_V(P \to e^+\tau^-) + Br_V(P \to \tau^+e^-) \equiv Br_V(P \to e\tau), $$ (6)

$$ Br_V(P \to \mu^+\tau^-) + Br_V(P \to \tau^+\mu^-) \equiv Br_V(P \to \mu\tau), $$ (7)

the branching ratios $Br_V(P \to ll')$ of the decays of pseudoscalar mesons $P = (K_L^0, B^0, B_s)$ into lepton-antilepton pairs $ll' = e^+e^-, \mu^+\mu^-, e\mu, e\tau, \mu\tau, \tau^+\tau^-$ induced by the vector leptoquarks $V$ in the case of neglecting the electron and muon masses

$$ m_e, m_\mu \ll m_\tau, m_{K^0}, m_{B^0}, m_{B_s} $$ (8)

can be written as

$$ Br_V(P \to ll') = B_P \beta_{P,ll'}^2 \quad \text{for} \quad ll' = e^+e^-, \mu^+\mu^-, e\mu, $$ (9)

$$ Br_V(P \to l\tau) = B_P (1 - m_\tau^2/m_P^2)^2 \beta_{P,\tau l}^2 \quad \text{for} \quad l\tau = e\tau, \mu\tau, $$ (10)

$$ Br_V(P \to \tau^+\tau^-) = B_P \sqrt{1 - 4m_\tau^2/m_P^2} \left[ \beta_{P,\tau^+\tau^-}^2 - (m_\tau^2/m_P^2) \right] [k_{P,33}^L - k_{P,33}^R]^2, $$ (11)

where

$$ B_P = \frac{m_P \pi \alpha_{em}^2(M_c) f_P^2 \bar{m}_P^2 (R_P^\dagger)^2}{2 m_P^4 \Gamma_{P}^{tot}} $$ (12)

are the typical branching ratios of these decays and $\beta_{P,ll'}^2, |k_{P,33}^L - k_{P,33}^R|^2$ are the mixing factors depending on the mixing matrices $K_L^2, K_R^2$. The entering into (12) form factors $f_P$ parametrize the matrix elements of the axial and pseudoscalar quark currents in the standard way, the factors $R_P^\dagger$ account the gluonic corrections to the pseudoscalar quark current, $\bar{m}_P = m_P^2/(m_{d_P} + m_{d_P})$, $m_P, \Gamma_{P}^{tot}$ are the mass and total width of $P$ meson and $m_{d_P}, m_{d_P}$ are the masses of its valency quarks, $m_V$ is the mass of the vector leptoquark.

With account of the definitions (5)–(7) the mixing factors $\beta_{P,ll'}^2$ have the form

$$ \beta_{P,e^+e^-}^2 = \beta_{P,11}^2, \quad \beta_{P,\mu^+\mu^-}^2 = \beta_{P,22}^2, \quad \beta_{P,\tau^+\tau^-}^2 = \beta_{P,33}^2, $$ (13)

$$ \beta_{P,e\mu}^2 = \beta_{P,12}^2 + \beta_{P,21}^2, \quad \beta_{P,e\tau}^2 = \beta_{P,13}^2 + \beta_{P,31}^2, \quad \beta_{P,\mu\tau}^2 = \beta_{P,23}^2 + \beta_{P,32}^2, $$ (14)

where the mixing factors $\beta_{P,ij}^2$ are related to the matrix elements $(K_L^2)_{pi}, (K_R^2)_{qj}$ of the mixing matrices $K_L^2, K_R^2$. 

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The mixing matrices $K^L_2,$ $K^R_2$ are the unitary $3 \times 3$ matrices and in general case each of them can be parametrized by three angles and six phases as

$$K^L_2 = e^{i\phi^L_0} \begin{pmatrix} c_{12}c_{13}e^{i\phi_1} & s_{12}c_{13}e^{i\phi_2} & s_{13}e^{i\phi_3} \\ (-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta})e^{i\phi_{21}} & (c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta})e^{i\phi_{22}} & s_{23}c_{13}e^{i\phi_{23}} \\ (s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta})e^{i\phi_{21}} & (c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta})e^{i\phi_{22}} & c_{23}c_{13}e^{i\phi_{23}} \end{pmatrix} L,R,$$ (15)

where

$$\varphi^L_{21} = (\varphi_1 + \varepsilon)L,R, \quad \varphi^L_{22} = (\varphi_2 + \varepsilon)L,R, \quad \varphi^L_{23} = (\varphi_3 + \delta + \varepsilon)L,R,$$

$$\varphi^L_{31} = -(\varphi_2 + \varphi_3 + \delta + \varepsilon)L,R, \quad \varphi^L_{32} = -(\varphi_1 + \varphi_3 + \delta + \varepsilon)L,R, \quad \varphi^L_{33} = -(\varphi_1 + \varphi_2 + \varepsilon)L,R,$$

$$s_{ij}^L = \sin \theta^L_{ij}, \quad c_{ij}^L = \cos \theta^L_{ij}, \quad \theta^L_{12}, \theta^L_{23}, \theta^L_{13} \text{ and } \delta^L_{LR}, \varepsilon^L_{LR}, \varphi^L_{0L}, \varphi^L_{1L}, \varphi^L_{2L}, \varphi^L_{3L} \text{ are the arbitrary angles and phases of the matrices } K^L_2.$$ Keeping in mind that the phases of the quark and lepton states are fixed by the standard choice of the matrices $V_{CKM}$ and $U_{PMNS}$ for the matrices $K^L_2$ we use the general form (15)-(17).

With account of (15)-(17) the factors $\beta^2_{P_{ij}}, |k^L_{P_{33}} - k^R_{P_{33}}|^2$ can be expressed in terms of mixing angles and phases of the matrices $K^L_2.$ For $P = K^0_L$ the expressions $\beta^2_{K^0_{L}^{ij}}$ have the form

$$\beta^2_{K^0_{L}^{11}} = \left( |(s_{12}^L c_{23}^L e^{i\phi^L_1} + c_{12}^L s_{23}^L e^{i(\delta^L + \epsilon^L)}) c_{12}^R c_{13}^R + (s_{12}^R c_{23}^R e^{i\phi^R_1} + c_{12}^R s_{23}^R e^{i(\delta^R + \epsilon^R)}) c_{12}^L c_{13}^L |^2 + L \leftrightarrow R \right) / 4,$$ (18)

$$\beta^2_{K^0_{L}^{22}} = \left( |(c_{12}^L c_{23}^L e^{i\phi^L_2} - s_{12}^L s_{23}^L e^{i(\delta^L + \epsilon^L)}) c_{12}^R c_{13}^R + (c_{12}^R c_{23}^R e^{i\phi^R_2} - s_{12}^R s_{23}^R e^{i(\delta^R + \epsilon^R)}) c_{12}^L c_{13}^L |^2 + L \leftrightarrow R \right) / 4,$$ (19)

$$\beta^2_{K^0_{L}^{12}} = \beta^2_{K^0_{L}^{21}} = \left( |(s_{12}^L c_{23}^L e^{i\phi^L_1} + c_{12}^L s_{23}^L e^{i(\delta^L + \epsilon^L)}) s_{13}^R c_{13}^R + (-c_{12}^R s_{23}^R e^{i\phi^R_2} + s_{12}^R s_{23}^R e^{i(\delta^R + \epsilon^R)}) c_{12}^L c_{13}^L |^2 + L \leftrightarrow R \right) / 4.$$ (20)

The mixing factors $\beta^2_{P_{ij}}, |k^L_{P_{33}} - k^R_{P_{33}}|^2$ for $P = (B^0, B_s)$ are more complicated and can be presented in the form

$$\beta^2_{B^0_{i1}} = \left( |(s_{12}^L s_{23}^L - c_{12}^L c_{23}^L e^{i\phi^L_1}) e^{i(\delta^L + \epsilon^L)} c_{13}^R |^2 + (c_{12}^R c_{13}^R)^2 + L \leftrightarrow R \right) / 2,$$ (21)

$$\beta^2_{B^0_{i2}} = \left( |(c_{12}^L s_{23}^L + s_{12}^L c_{23}^L e^{i\phi^L_2}) e^{i(\delta^L + \epsilon^L)} c_{13}^R |^2 + (s_{12}^R c_{13}^R)^2 + L \leftrightarrow R \right) / 2,$$ (22)

$$\beta^2_{B^0_{i3}} = \left( |(s_{12}^L s_{23}^L - c_{12}^L c_{23}^L e^{i\phi^L_1}) c_{13}^R |^2 + (s_{12}^R c_{13}^R)^2 + L \leftrightarrow R \right) / 2,$$ (23)

$$\beta^2_{B^0_{i4}} = \left( |(c_{12}^L s_{23}^L + s_{12}^L c_{23}^L e^{i\phi^L_2}) c_{13}^R |^2 + (c_{12}^R c_{13}^R)^2 + L \leftrightarrow R \right) / 2,$$ (24)

$$\beta^2_{B^0_{i5}} = \left( |(c_{23}^L s_{13}^R - c_{23}^L c_{13}^L e^{i(\delta^L - \epsilon^L)} m_{\tau} / (2m_{B^0} R_{B^0}^V) |^2 (c_{12}^R c_{13}^R)^2 + L \leftrightarrow R \right) / 2,$$ (25)

$$\beta^2_{B^0_{i6}} = \left( |(c_{23}^L c_{13}^R - c_{23}^L c_{13}^L e^{i(\delta^L - \epsilon^L)} m_{\tau} / (2m_{B^0} R_{B^0}^V) |^2 (s_{12}^R c_{13}^R)^2 + L \leftrightarrow R \right) / 2,$$ (26)

$$\beta^2_{B^0_{i7}} = \left( |(s_{23}^L s_{13}^L e^{i(\delta^L - \epsilon^L)} m_{\tau} / (2m_{B^0} R_{B^0}^V) |^2 (s_{12}^L c_{23}^L e^{i\phi^L_1})^2 + L \leftrightarrow R \right) / 2,$$ (27)

$$\beta^2_{B^0_{i8}} = \left( |(s_{23}^L s_{13}^L e^{i(\delta^L - \epsilon^L)} m_{\tau} / (2m_{B^0} R_{B^0}^V) |^2 (c_{12}^L c_{23}^L e^{i\phi^L_2})^2 + L \leftrightarrow R \right) / 2,$$ (28)

$$\beta^2_{B^0_{i9}} = \left( |(c_{23}^L s_{13}^L e^{i(\delta^L - \epsilon^L)} m_{\tau} / (2m_{B^0} R_{B^0}^V) |^2 (c_{12}^L c_{23}^L e^{i\phi^L_1})^2 + L \leftrightarrow R \right) / 2,$$ (29)

$$|k^L_{B^0_{i33}} - k^R_{B^0_{i33}}|^2 = \left| c_{23}^L c_{13}^L - c_{23}^L c_{13}^L e^{i(\delta^L - \epsilon^L)} m_{\tau} / (2m_{B^0} R_{B^0}^V) \right|^2 + L \leftrightarrow R \right) / 2,$$ (30)

and

$$\beta^2_{B_{i1}} = \left( |(s_{12}^L s_{23}^L - c_{12}^L c_{23}^L e^{i\phi^L_1}) e^{i(\delta^L + \epsilon^L)} c_{13}^R |^2 + (c_{12}^R c_{13}^R)^2 + L \leftrightarrow R \right) / 2,$$ (31)
\[ \beta^2_{B,22} = (|C_{23}^{L}|^2 s_{23}^L + C_{23}^{L} C_{23}^{R} s_{23}^L e^{i\delta L})^2 \left| e^{R} R_{23} s_{12}^R - s_{12}^R R_{23} e^{-i\delta R}\right|^2 + L \leftrightarrow R \), \]
\[ \beta^2_{B,32} = (|C_{23}^{L}|^2 s_{23}^L - C_{23}^{L} C_{23}^{R} s_{23}^L e^{i\delta L})^2 \left| e^{R} R_{23} s_{12}^R + s_{12}^R R_{23} e^{-i\delta R}\right|^2 + L \leftrightarrow R \),
\[ \beta^2_{B,23} = (|C_{23}^{L}|^2 s_{23}^L - C_{23}^{L} C_{23}^{R} s_{23}^L e^{i\delta L})^2 \left| e^{R} R_{23} s_{12}^R + s_{12}^R R_{23} e^{-i\delta R}\right|^2 + L \leftrightarrow R \),
\[ \beta^2_{B,33} = (|C_{23}^{L}|^2 s_{23}^L - C_{23}^{L} C_{23}^{R} s_{23}^L e^{i\delta L})^2 \left| e^{R} R_{23} s_{12}^R + s_{12}^R R_{23} e^{-i\delta R}\right|^2 + L \leftrightarrow R \).

where

\[ \chi_1^{L,R} = \phi_0^{L,R} - \phi_1^{L,R} - \phi_2^{L,R}, \quad \chi_2^{L,R} = \phi_0^{L,R} + \phi_3^{L,R}. \]  

The mixing factors (13), (14), (18)–(20) describe in the general form the effect of the fermion mixing in leptoquark currents in the case (8) on the branching ratios \( Br_V (P \to ll') \) of the decays of pseudoscalar mesons \( P = (K_0^0, B^0, B_s) \) into lepton-antilepton pairs. These mixing factors can be used for the analysis of the branching ratios (39)–(41) in dependence on the mixing angles and phases of the mixing matrices (15).

As seen from the (25)–(30), (35)–(40) the mixing factors \( \beta_{P,\tau}^2, \beta_{P,\mu}^2, \beta_{P,\tau+\tau^-}^2, \beta_{P,\tau^-}^2, \beta_{P,\mu}^2, |k_{P,33}^L - k_{P,33}^R|^2 \) for \( P = (B^0, B_s) \) depend on the phases \( \phi_0^{L,R}, \phi_1^{L,R}, \phi_2^{L,R}, \phi_3^{L,R} \) through their combinations (41). With fixed values of mixing angles \( \theta_{12}^{L,R}, \theta_{23}^{L,R}, \theta_{13}^{L,R} \) and phases \( \delta^{L,R} \) these mixing factors can be minimized over phases (41) by the conditions

\[ \chi_1^L - \epsilon^L - \chi_1^R + \epsilon^R = 0, \quad \chi_2^L - \chi_2^R = 0, \quad \delta^L + \epsilon^L - \delta^R - \epsilon^R = 0. \]

The most stringent lower mass limits for vector leptoquark are resulting from the experimental data on the branching ratios \( Br_V (K_{L}^0 \to ll') \). As a result for the more small masses \( m_V \) the mixing factors \( \beta_{K_{L}^0, ll'}^2 \) must be very small (close to zero) and can be assumed in the further analysis to be equal to zero

\[ \beta_{K_{L}^0, ll'}^2 = 0. \]  

for \( ll' = e^+e^-, \mu^+\mu^-, e\mu \), where \( \beta_{K_{L}^0, ll'}^2 \) are given by the relations (13), (14) and (18)–(20).

There are two solutions of the equations (43):

solution A: \( \theta_{23}^L = \theta_{23}^R = \pi/2, \quad \theta_{13}^L = \theta_{13}^R = \theta_{13} \), \( \delta^L + \epsilon^L + \delta^R + \epsilon^R = \pi \)  

and

solution B: \( \theta_{13}^L = \theta_{13}^R = \pi/2. \)
In both cases (44) and (45) the mixing factors $\beta_{P,33}^2$ and $|k_{P,33}^L-k_{P,33}^R|^2$ for $P = (B^0, B_s)$ are equal to zero, which gives that in these cases $Br_V(B^0 \to \tau^+\tau^-) = Br_V(B_s \to \tau^+\tau^-) = 0$.

In the case (42), (44) we obtain from (21)–(40) the nonzero mixing factors (13), (14) in the form

$$\beta_{P,0,e^+e^-}^2 = \beta_{P,0,\mu^+\mu^-}^2 = c_{13}^2((s_{12}^L r_{12}^R)^2 + (s_{12}^R c_{12}^L)^2)/2, \quad (46)$$

$$\beta_{P,0,\mu\tau}^2 = c_{13}^2((s_{12}^L s_{12}^R)^2 + (c_{12}^L c_{12}^R)^2), \quad (47)$$

$$\beta_{P,0,\mu^+\mu^-}^2 = (1 - m_\tau/(2m_{P,0}^V_R))s_{13}^2((s_{12}^L)^2 + (s_{12}^R)^2)/2, \quad (48)$$

$$\beta_{P,0,\mu^+\mu^-}^2 = (1 - m_\tau/(2m_{P,0}^V_R))s_{13}^2((c_{12}^L)^2 + (c_{12}^R)^2)/2, \quad (49)$$

and

$$\beta_{B_s,e^+e^-}^2 = \beta_{B_s,\mu^+\mu^-}^2 = s_{13}^2((s_{12}^L r_{12}^R)^2 + (s_{12}^R c_{12}^L)^2)/2, \quad (50)$$

$$\beta_{B_s,\mu\tau}^2 = s_{13}^2((s_{12}^L s_{12}^R)^2 + (c_{12}^L c_{12}^R)^2), \quad (51)$$

$$\beta_{B_s,\mu^+\mu^-}^2 = (1 - m_\tau/(2m_{B_s}^V_R))s_{13}^2((s_{12}^L)^2 + (s_{12}^R)^2)/2, \quad (52)$$

$$\beta_{B_s,\mu^+\mu^-}^2 = (1 - m_\tau/(2m_{B_s}^V_R))s_{13}^2((c_{12}^L)^2 + (c_{12}^R)^2)/2. \quad (53)$$

As seen from (46)–(53) the mixing factors $\beta_{P,0,\mu^+\mu^-}^2$, $\beta_{B_s,\mu^+\mu^-}^2$ in the case (42), (44) depend in general case on three mixing angles $\theta_{13}$, $\theta_{12}$, $\theta_{21}$. In the case (42), (44) the mixing factors (13), (14) depend on two effective mixing angles $\theta^L$, $\theta^R$ with $\sin \theta^{L,R} = |c_{23}c_{13} - s_{23}s_{13}e^{i\delta_{12}|L,R}$ and can be obtained from (46)–(53) by the substitutions $\theta_{12} \to \pi/2$, $\theta_{12} \to \pi/2$. The branching ratios (9)–(11) with the factors $\beta_{P,0,\mu^+\mu^-}^2$ depending on the mixing angles have been numerically analysed with account of experimental data on the decays (3). We vary the mixing angles $\theta_{13}$, $\theta_{12}$, $\theta_{21}$ in the case (42), (44) and the mixing angles $\theta^L$, $\theta^R$ in the case (42), (44) to find the minimal vector leptoquark mass $m_V$ satisfying these data. In the numerical analysis we use the data on the masses $m_{l_i}$, $m_{d_i}$ of leptons and quarks, the data on the masses $m_P$, life times $\tau_P$ ($\tau_P \to \Gamma_P^{tot}$) and the form factors $f_P$

$$f_{K^0} = f_K = 155.72 \text{ MeV}, \quad f_{B^0} = 190.9 \text{ MeV}, \quad f_{B_s} = 227.2 \text{ MeV}$$

of mesons $P = (K_L^0, B^0, B_s)$ from ref. [20]. The experimental data on the branching ratios $Br(P \to \ell\ell')^{exp}$ are also taken from the ref. [20] except the branching ratios of the decays $B^0, B_s \to \mu^+\mu^-$, $\tau^+\tau^-$ for which we use the current data

$$Br(B^0 \to \mu^+\mu^-)^{exp} < 3.4 \cdot 10^{-10} \quad [21], \quad (54)$$

$$Br(B^0 \to \tau^+\tau^-)^{exp} < 2.1 \cdot 10^{-3} \quad [22], \quad (55)$$

$$Br(B_s \to \mu^+\mu^-) = (3.0 \pm 0.6_{-0.3}^{+0.2}) \cdot 10^{-9} \quad [21], \quad (56)$$

$$Br(B_s \to \tau^+\tau^-)^{exp} < 6.8 \cdot 10^{-3} \quad [22] \quad (57)$$

of refs. [21][22] and the branching ratios of the decays $B^0, B_s \to e\mu$ for which we use the recent data

$$Br(B^0 \to e\mu)^{exp} < 1.0 \cdot 10^{-9} \quad [23], \quad (58)$$

$$Br(B_s \to e\mu)^{exp} < 5.4 \cdot 10^{-9} \quad [23] \quad (59)$$

of the ref. [23]. For the mass scale $M_c$ of the $SU_V(4)$ symmetry breaking we choose the value $M_c = 100 \text{ TeV}$. 

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Using (46)–(53) for the case (44) we have found the lower vector leptoquark mass limit

\[ m_V > 86 \, TeV \]  

(60)

in the case of the chiral mixing and the mass limit \( m_V > 87.4 \, TeV \) in the case of the vectorlike \( (\theta_{12}^L = \theta_{12}^R \equiv \theta_{12}) \) one. The case (45) with the chiral and vectorlike \( (\bar{\theta}^L = \bar{\theta}^R) \) mixings gives the mass limits \( m_V > 89.3 \, TeV \) and \( m_V > 90.3 \, TeV \) respectively. As seen, the mass limit (60) is the lowerest one.

The effect of the fermion mixing on the decays of type (3) has been also considered in [21] but for the particular choice of the mixing matrices corresponding to the case (44) with the additional restrictions \( \theta_{12}^L = \theta_{12}^R = \pi/2 \). Besides, in comparing the theoretical results for \( B^0, B_s \) mesons with the experimental data only the first terms in the branching ratios (5)–(7) and in the mixing factors (14) have been taken into account, which contradicts the usual treatment of the experimental branching ratios as the sums over charge conjugated final states. By these two reasons the lower vector leptoquark mass limit \( m_V > 38 \, TeV \) of ref. [21] seems questionable (for example instead of this value the case (44) with \( \theta_{12}^L = \theta_{12}^R = \pi/2 \) gives in fact the mass limit \( m_V > 101.3 \, TeV \)).

The mass limit (60) is resulting from the current experimental data on the branching ratios \( Br(P \rightarrow l l')^{\exp} \) of the ref. [20] and (54)–(59) with account of the fermion mixing in leptoquark currents of the general form. As seen, it is essentially lower than that of order of 2,000 \( TeV \) obtained with neglect of the fermion mixing [15] and noticeably exceeds the mass limit of order of 1 \( TeV \) resulting from the current direct searches for the vector leptoquarks. It is worthy to note that the mass limit (60) can be further lowered by account of the possible destructive interference of the contributions from the vector leptoquarks with those from the scalar leptoquarks which are also predicted in the scalar sector of the models with the four color symmetry. The possibility of such interference in the minimal model with the four color symmetry based on the gauge group \( G_{MQLS} = SU_V(4) \times SU_L(2) \times U_R(1) \) \([25, 27]\) in the case of neglect of the fermion mixing \( (K_2^L = K_2^R = I) \) has been demonstrated in [18, 19].

We have calculated the branching ratios \( Br_V(P \rightarrow l l') \) predicted by the vector leptoquarks with the lower allowed mass \( m_V = 86 \, TeV \). This value of the vector leptoquark mass is ensured by the appropriate values of the mixing angles \( \theta_{13}, \theta_{12}^L, \theta_{12}^R \) from the region

\[ \theta_{13} = 1.183 - 1.187, \quad \theta_{12}^L(\theta_{12}^R) = 0.00(0.81) - 0.763(\pi/2) \]  

(61)

with the branching ratios \( Br_V(P \rightarrow l l') \) being invariant under exchange \( \theta_{12}^L \leftrightarrow \theta_{12}^R \) in accordance with (46)–(53). The analysis shows that the variations of the branching ratios under variations of the mixing angles \( \theta_{13}, \theta_{12}^L, \theta_{12}^R \) within the region (61) for the decays \( B^0, B_s \rightarrow e^+e^-, \mu^+\mu^-, e\mu \) do not exceed 3\% and the branching ratios of the decays \( B^0 \rightarrow e\tau, \mu\tau \) and \( B_s \rightarrow e\tau, \mu\tau \) are of order of \( 10^{-9} \) and \( 10^{-10} - 10^{-9} \) respectively. For definiteness in the second column of the Table 1 we present the branching ratios \( Br_V(P \rightarrow l l') \) for \( m_V = 86 \, TeV \) at \( \theta_{13} = 1.183, \theta_{12}^L(\theta_{12}^R) = 0.00(0.81) \). In the third column of the Table 1 we present for comparison the current experimental data on the decays under consideration.

As seen from the Table 1 the branching ratios predicted by the vector leptoquarks with \( m_V = 86 \, TeV \) for the decays \( B^0, B_s \rightarrow \mu^+\mu^-, e\mu \) are close to (or compatible with) the corresponding experimental data. It means that these decays are now most suitable for search for the vector leptoquarks and for setting the new more stringent limits on their mass. The decays \( B^0, B_s \rightarrow \mu^+\mu^-, e\mu \) as the perspective ones for obtaining the new mass limit for the vector leptoquarks have been pointed out in ref. [28] at the data of ref. [20] and (54)–(57), this conclusion is now confirmed at the data (58)–(59) with improving the vector leptoquark mass limit from \( m_V > 78 \, TeV \) of ref. [28] to the current mass limit (60).
Table 1: Branching ratios $\text{Br}_V(P \rightarrow ll')$ predicted by the vector leptoquarks with the lower mass $m_V = 86\text{ TeV}$ with account of fermion mixing at $(\theta_{13})\theta_{12}^L(\theta_{12}^R) = (1.183)0.00(0.81)$

| $\text{Br}(P \rightarrow ll')$ | $\text{Br}_V(P \rightarrow ll')$ | $\text{Br}(P \rightarrow ll')^{\text{exp}}$ |
|-------------------------------|----------------------------------|------------------------------------------|
| $\text{Br}(K_L^0 \rightarrow e^+e^-)$ | 0 | $(9^{+6}_{-4}) \cdot 10^{-12}$ |
| $\text{Br}(K_L^0 \rightarrow \mu^+\mu^-)$ | 0 | $(6.84 \pm 0.11) \cdot 10^{-9}$ |
| $\text{Br}(K_L^0 \rightarrow e\mu)$ | 0 | $< 4.7 \cdot 10^{-12}$ |
| $\text{Br}(B^0 \rightarrow e^+e^-)$ | $3.39 \cdot 10^{-10}$ | $< 8.3 \cdot 10^{-8}$ |
| $\text{Br}(B^0 \rightarrow \mu^+\mu^-)$ | $3.39 \cdot 10^{-10}$ | $< 3.4 \cdot 10^{-10}$ |
| $\text{Br}(B^0 \rightarrow e\mu)$ | $6.1 \cdot 10^{-10}$ | $< 1.0 \cdot 10^{-9}$ |
| $\text{Br}(B^0 \rightarrow e\tau)$ | $1.6 \cdot 10^{-9}$ | $< 2.8 \cdot 10^{-5}$ |
| $\text{Br}(B^0 \rightarrow \mu\tau)$ | $4.4 \cdot 10^{-9}$ | $< 2.2 \cdot 10^{-5}$ |
| $\text{Br}(B^0 \rightarrow \tau^+\tau^-)$ | 0 | $< 2.1 \cdot 10^{-3}$ |
| $\text{Br}(B_s \rightarrow e^+e^-)$ | $3.0 \cdot 10^{-9}$ | $< 2.8 \cdot 10^{-7}$ |
| $\text{Br}(B_s \rightarrow \mu^+\mu^-)$ | $3.0 \cdot 10^{-9}$ | $(3.0 \pm 0.6^{+0.3}_{-0.2}) \cdot 10^{-9}$ |
| $\text{Br}(B_s \rightarrow e\mu)$ | $5.35 \cdot 10^{-9}$ | $< 5.4 \cdot 10^{-9}$ |
| $\text{Br}(B_s \rightarrow e\tau)$ | $3.9 \cdot 10^{-10}$ | $-$ |
| $\text{Br}(B_s \rightarrow \mu\tau)$ | $1.1 \cdot 10^{-9}$ | $-$ |
| $\text{Br}(B_s \rightarrow \tau^+\tau^-)$ | 0 | $< 6.8 \cdot 10^{-3}$ |

As concerns the decays $B^0 \rightarrow e^+e^-, e\tau, \mu\tau,$ and $B_s \rightarrow e^+e^-$ the current experimental limits on these decays essentially (by order of $2 \div 4$) exceed the corresponding expected contributions from the vector leptoquarks and the search for these decays will need the correspondingly high statistics.

The experimental data on the decays $B_s \rightarrow e\tau, \mu\tau$ by now are absent and the vector leptoquarks with mass $m_V = 86\text{ TeV}$ predict for these decays the branching ratios of order of $10^{-10}$ and $10^{-9}$ respectively.

The contributions of the vector leptoquarks to branching ratios of the decays $B^0, B_s \rightarrow \tau^+\tau^-$ are very small (under assumptions (43) they are equal to zero) and the search for these decays seems to be difficult.

The numerical predictions of the branching ratios of the decays $B^0, B_s \rightarrow ll'$ presented in the Table 1 will be useful in the current and future experimental searches for these decays.

In conclusion we resume the results of the work.

The contributions of the vector leptoquarks of Pati-Salam type to the branching ratios of $K_L^0, B^0, B_s \rightarrow ll'$ decays are calculated and analysed with account of the fermion mixing in the leptoquark currents of the general type. The general parametrizations of the mixing matrices in the leptoquark currents are proposed and the mixing factors in the branching ratios as the general functions on mixing angles and phases are found.

With account of the fermion mixing in leptoquark currents of the general form the lower vector leptoquark mass limit $m_V > 86\text{ TeV}$ is found from the current experimental data on the branching ratios of these decays.

The numerical predictions of the branching ratios of the decays $B^0, B_s \rightarrow ll'$ at the
lower vector leptoquark mass $m_V = 86$ TeV are obtained in comparison with the current experimental data. It is shown that for the decays $B^0, B_s \to \mu^+\mu^-, e\mu$ the predicted branching ratios are close to the experimental data and these decays are most promising for the search for the vector leptoquarks and for setting the new more stringent vector leptoquark mass limits whereas the analogous predictions for the decays $B^0 \to e^+e^-, e\tau, \mu\tau$, and $B_s \to e^+e^-$ are essentially (by order of $2 \div 4$) less than their current experimental limits and the search for these decays will need the more high statistics.

For the decays $B_s \to e\tau, \mu\tau$ the experimental data by now are absent, the expected branching ratios for these decays can be of order or less than $10^{-10}$ and $10^{-9}$ respectively.

The numerical predictions of the branching ratios of the decays $B^0, B_s \to ll'$ at the current lower vector leptoquark mass $m_V = 86$ TeV will be usefull in the current and future experimental searches for these decays.

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