STAR FORMATION INEFFICIENCY AND KENNICUTT-SCHMIDT LAWS IN EARLY-TYPE GALAXIES

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ABSTRACT

Star formation in disk galaxies is observed to follow the empirical Kennicutt-Schmidt law, a power-law relationship between the surface density of gas \( \Sigma_{\text{gas}} \) and the star formation rate per unit surface \( \Sigma_* \). In contrast to disk galaxies, early-type galaxies (ETGs) are typically associated with little to no star formation, and thus are usually termed “quiescent”. Recent observations, however, have noted the presence of massive gaseous cold disks in ETGs, raising the question as to why the conversion of gas into stars is truly inefficient. With the aid of the latest simulations, performed with our high-resolution hydrodynamic numerical code MACER, we reevaluate the traditional classification of ETGs as quiescent, dead galaxies. In fact, in the presence of moderate galaxy rotational support, circumnuclear gaseous disks of kpc size form following cooling episodes of the ISM, and are not destroyed by AGN feedback. The issue of star formation in such disks is therefore unavoidable. In MACER we follow star formation by considering two channels, that of 1) Toomre instability and 2) gas cooling/Jeans instability. We find that the resulting Kennicutt-Schmidt laws for the simulated ETGs reproduce the observed slope in disk galaxies, though with considerable scatter and lower normalization by a factor of \( \approx 2 \) or more for the highest mass galaxies. The Toomre instability is the main driver of the slope, while cooling/Jeans induced star formation dominates the central regions. Observational checks of our star formation predictions are thus essential for confirming the form of local star formation laws and reassessing star formation inefficiency in ETGs. The process we describe is similar to that of the star forming disks - of much lower luminosity - found in the central spheroidal regions of local spirals, such as the MW and M31.

Keywords: galaxies: elliptical and lenticular; galaxies: star formation

1. INTRODUCTION

Understanding how the physical properties of interstellar gas affect star formation is important for developing models of galactic evolution and possibly explaining the differences in star formation rate (SFR) across different galaxy types. A robust empirical correlation between the SFR and gas density in disk galaxies was first reported by [Kennicutt 1998, hereafter K98] and [Kennicutt 1989, hereafter K89] laws relating SFR and gas surface densities read \( \Sigma_* \propto \Sigma_{\text{gas}} \), with \( 1 < n < 3 \). Starting with Kennicutt’s compilation of H\(_\alpha\) measurements to trace star formation and HI and CO data to trace atomic and molecular gas, it has been found that both global (Kennicutt 1998) and resolved (Kennicutt 1989) laws relating SFR and gas surface densities read \( \Sigma_* \propto \Sigma_{\text{gas}} \), with \( 1 < n < 3 \), and \( n \approx 1.4 \) as the most accepted value. This empirical law, universally known as the Kennicutt-Schmidt law (hereafter KS) forms the critical basis for current theoretical and numerical work on common disk galaxies. Here and in the following, the global observables entail averaging the gas surface density and the star formation rate within some prescribed radius in the galactic disk, while the resolved values of \( \Sigma_{\text{gas}} \) and \( \Sigma_* \) are just angular averages over annuli of prescribed inner and outer radii \( R \) and \( R + \Delta R \).

Due to the importance of the subject, it is not surprising that a huge amount of observational, numerical, and theoretical work has been done to elucidate the physics behind the observed star formation rates. As the real phenomenon involves turbulent hydrodynamics, gravitational instabilities, radiative transport, magnetic fields, etc., in general a phenomenological approach is used (with considerable success) in the attempt to capture the basic physical principles driving star formation. In this framework, two main channels of star formation at work in relatively isolated galaxies (i.e., galaxies where gas rich merging events and other environmental effects such as near encounters, tidal stripping and harassment, can be neglected) can be easily identified. The first is related to local/global instabilities in rotating gaseous disks (hereafter the “Toomre instability”, see e.g. Binney & Tremaine 2008). The second is the also well known criterion based on the comparison between the cooling and the Jeans instability.
times (hereafter the “cooling/Jeans” channel). Both channels are almost certainly at work in star formation episodes hosted by rotating, massive cold gaseous disks. Regarding the Toomre instability channel, theory and simulations have suggested that in disk galaxies, gravitational instability criteria (Boissier et al. 2003; Kennicutt 1998) accurately replicate both the KS power-law (see Leroy et al. 2008 for qualification) and the cutoff threshold, while different disk thickness (Bacchini et al. 2019), turbulence (Shetty & Ostriker 2008), and shear (Davis et al. 2014) can contribute to the observed scatter in $n$. Additionally, efforts have also gone into examining modified KS relations with terms including orbital velocities and velocity dispersions of stars (Schaye 2004), with the aim of finding a universal relationship between gas and SFR densities in all types of star-forming galaxies (see Sun et al. 2023). Regarding the cooling/Jeans channel, it is obvious that in case of dense and cold gas, the consequences of the associated short time scales are inescapable, resulting in fast cooling, fragmentation, collapse of the ISM, and consequent star formation. For example, numerical simulations restricted to the cooling/Jeans channel (with no AGN feedback, and in absence of Toomre instability) have shown that the resulting star formation rates are in nice qualitative agreement with the observed slope of the KS empirical law (Negri et al. 2015).

Therefore, a first natural question is if the Toomre and the cooling/Jeans channels are cooperative, or complementary, in the process of star formation in rotating gaseous disks. A second, more observationally motivated question, concerns the systematic differences of star formation as a function of the morphological type of the host galaxy. In fact, in contrast to disk galaxies, ETGs are typically classified as “red and dead” stellar systems, with quenched star formation as a result of a post star-burst depletion of the molecular gas reservoir (Cappellari et al. 2011, Baron et al. 2022), and successive maintenance of galactic winds due to the cooperative effects of SNIIa, and recurrent AGN feedback events fueled by periodic cooling of the ISM produced by the galaxy aging stellar population (e.g., Ciotti et al. 1991 and Ciotti & Ostriker 1997; see also Kim & Pellegrini 2012). However, in sufficiently massive ETGs, the SNIIa and AGN feedback effects are not strong enough to maintain the galaxy devoid of the gas produced by the stars, and so in such systems some central star formation will occur. Our work (Ciotti et al. 2022, hereafter C22) and that of others leads to the conclusion that for massive ETGs ($M_\star \approx 10^{11}$), roughly $10^{11} M_\odot$ of gas is ejected by feedback processes (SNIIa and AGN feedback) and a remaining $10^{10} M_\odot$ falls to a central kpc disk in what was initially termed “cooling flow” (e.g., see Fabian 1994, Matthews & Brightent 2003) and references therein. There it can fall to the center to feed the AGN of be transformed locally into a (top heavy) population of new stars, the most massive of which will eject, by dynamical processes, some fraction of the gas from which they were born. In fact observations show that ETGs possess some reservoir of cold gas, with approximately 50 percent of massive ETGs (stellar mass $M_\star > 10^{10} M_\odot$) containing $10^{9} - 10^{10} M_\odot$ of cold gas in the form of atomic and molecular hydrogen (e.g., see Cappellari et al. 2011, Li et al. 2020); while the gas reservoirs for massive disk galaxies can reach $10^{10} - 10^{11} M_\odot$ (Zhou et al. 2021, Saintonge & Catinella 2022), validating the picture of overall quiescent ETGs, non-negligible star formation seems inevitable given the two channels above. Quite remarkably, it has been noted that ETGs - for given amount of cold gas available - lie on a significantly lower regime compared to starburst and disk galaxies on the global KS relations (Davis et al. 2014), implying that conversion from gas to stars in ETGs is more inefficient per unit mass. This inefficiency of star formation, despite non-negligible reservoirs of cold gas, has been identified as one of the most persistent problems in the field of star formation (Peng et al. 2015).

As numerical simulations have demonstrated (Negri et al. 2015, C22), ordered rotation enhances ISM instabilities and radiative cooling; because the cooling rate grows quadratically with density, the increasing ISM density triggers (as noted above) a cooling flow. As the catastrophically cooling ISM looses its thermal pressure, it accumulates onto a circumnuclear disk due to the angular momentum barrier. Though the gaseous disk masses of ETGs are typically one to two orders of magnitude less than what has been observed for spiral galaxies, disk instabilities and density-dependent cooling rates dictate that nonzero star formation is inevitable. ETGs, however, are noted to be significantly less efficient in turning this cold gas to stars in comparison to spiral galaxies. The reason why star formation is so inefficient in ETGs, however, is not well understood; possible explanations include the stabilizing influence of stellar population on gaseous disks (Kawata et al. 2007), low disk self-gravity and increased shear (Martig et al. 2013), and counter-rotation of stellar and gaseous populations (Osman & Bekki 2017). Confirmations of theoretical predictions via observations of star formation in ETGs are hampered by the lack of resolution of inner star-forming disks; recent far-infrared observations by Baron et al. (2022) have suggested that a potential resolution of this problem can be that much of the star formation is obscured.

In this paper we address the two problems mentioned above, taking advantage of the latest suite of simulations produced with the current version of our high-resolution hydrodynamical simulation code MACER, which includes numerical algorithms for the radiative cooling of the ISM, the formation of cold gaseous disks in the presence of ordered rotation, and star formation following simple feedback of various forms and robust input physics. We notice that the resolution of MACER (parsec scale in the inner regions) greatly exceeds that of most observations and typical cosmological simulations, enabling the simulation and analysis of star forming disks with half mass radius close to the galactic nucleus.

We are therefore in a good position to study the star formation inefficiency problem in ETGs and the form of local star formation laws. Applying MACER towards analyzing the formation of dense star-forming disks close to the galactic nucleus, we have concluded that the observed inefficiency in SFR can be partially explained due to an underestimation of total star formation and the rapid ejection of cold gas. While the degree of star formation is certainly less than that of disk galaxies, we note that the formation of cold, centrifugally supported equatorial disks due to the conservation of angular momentum in the ISM of rotating ETGs creates an environment conducive for star formation following cooling flows (see C22). Dense stellar disks embedded in these cold gaseous disks, if they exist, would lie close to the galactic center beyond the resolution of what most resolved observations have attempted, and would be vulnerable to destruction via late-time (dry) merging. Gas lost from evolving stars is considerable and can be estimated at $\approx 10\%$ of the mass of the remaining stars (Kim & Pellegrini 2012).

This paper is organized as follows: in Section 2, we provide a brief summary of the relevant numerical physics of MACER; in Section 3.1, we discuss the position of ETGs on the global KS law with respect to the cooling flow problem; and in Section 3.2-3, we discuss resolved KS law to examine the dynamics of local SF, followed by a summary of results.
In this paper we address some aspect of star formation in ETGs by a detailed analysis of the suite of numerical simulations presented in C22, obtained with the latest 2D version of our high-resolution code MACER (Massive AGN Controlled Ellipticals Resolved), built upon Athena++ (version 1.0.0, see Stone et al. 2020). With this code we solve the Eulerian hydrodynamical equations for the ISM of ETGs with a central supermassive black hole, while including the effects of circumgalactic (CGM) gaseous infall, dust grain evolution, stellar feedback, AGN activity, and the metallic evolution of the ISM and also providing an accurate treatment of stellar dynamics. At this stage, simulations are performed under the assumption of axisymmetry, and phenomena such as merging, and tidal interactions with other galaxies are not taken into account. For a detailed description of the input physics, and other technical aspects, see C22 and references therein.

For the models discussed here, the hydrodynamical equations are solved in spherical ($r, \theta$) coordinates, while allowing for rotation in the $\varphi$ direction. The outer boundary of the computational domain is fixed to $r = 250$ kpc from the galactic center on a logarithmic radial grid, while the inner boundary is set to either 2.5 pc or 25 pc, the latter of which is used for a (relatively) fast parameter-space exploration; we notice that the obtained resolution, also in the less resolved case, is significantly finer than the $\approx 150$ pc inner boundary of typical cosmological simulations and resolves the fiducial Bondi radius, allowing infall to the central BH to be appropriately computed.

The galaxy models used are the axisymmetric JJe dynamical models consisting of a moderately flat Jaffe ellipsoidal stellar distribution (corresponding to an E3 galaxy) embedded in a dark matter halo, resulting in a spherical Jaffe density profile (Ciotti et al. 2021, C22). The additional gravitational field produced by a quasi-isothermal DM halo is also considered, to model the effects of a group/cluster halo.

We analyze the star formation properties in three families of models, defined by their initial stellar mass, and fully described in C22 (see Table 1 therein). In the family of high mass galaxy models (HM) the initial stellar mass is $M_* = 7.8 \times 10^{11}$ $M_{\odot}$, the edge-on circularized effective radius is $< R_e > = 11.8$ kpc, and the stellar central velocity dispersion $\sigma_{e0} = 312$ km s$^{-1}$. In the second family of medium mass galaxy models (MM) the initial stellar mass is $M_* = 3.35 \times 10^{11}$ $M_{\odot}$, the edge-on circularized effective radius is $< R_e > = 7.04$ kpc, and the stellar central velocity dispersion $\sigma_{e0} = 265$ km s$^{-1}$. In the third family of low mass galaxy models (LM) the initial stellar mass is $M_* = 1.54 \times 10^{11}$ $M_{\odot}$, the edge-on circularized effective radius is $< R_e > = 4.57$ kpc, and the stellar central velocity dispersion $\sigma_{e0} = 223$ km s$^{-1}$. All the models are simulated over a time interval of $\Delta t = 12$ Gyr from $t_0 = 2$ Gyr, i.e., we start our simulations after the galaxy formation epoch.

In the presence of some galaxy rotation, the angular momentum associated with the gas injected into the ISM by the evolving stars (if not ejected by the galaxy as a galactic wind) leads to the unavoidable formation of a cold and rotationally supported gaseous disk in the galaxy equatorial plane, where star formation occurs (in the simulations, the gravity of the resulting stellar disk is also considered): of course, higher the rotation, bigger the gaseous disk produced by ISM cooling, with important consequences for star formation. It is therefore natural to explore how star formation depends on the galaxy rotational structure while maintaining all the other properties of the model fixed. Accordingly, among the models presented in C22, we focus our study on two different rotational realizations for each of the LM, MM, and HM galaxies. For each mass the two families are named constant-$k$ (and the models are indicated as HMe, MMe, and LMe), and exponential-$k$ (HMe, MMe, and LMe) for the following reasons. As is well known, the azimuthal velocity field of a rotating stellar system can be (and usually is) split in its ordered and dispersion components by adopting a generalised Satoh (1980) $k$-decomposition (see equation 11 in C22). In the constant-$k$ rotation case the Satoh parameter is constant over the galaxy body, with $k = 0$ corresponding to a non-rotating galaxy, with the flattening totally supported by tangential velocity dispersion, while $k = 1$ corresponds to a “fast” rotating galaxy (the isotropic rotator), with the flattening fully supported by ordered rotation. In our HMe, MMe, and LMe models we consider this latter possibility, i.e., they are fast spinning ETGs. In the exponential-$k$ rotation case we considered instead the spatially-dependent Satoh parameter

$$k(r) = e^{-r/<R_e>},$$

so that the galaxy rotational support (and angular momentum injection from the rotating stellar population) decreases significantly at large radii, while in the central regions stars rotate almost as fast as an isotropic rotator. Summarizing, in C22 and in the present study, angular momentum injection in the ISM is maximal in constant $k > 1$ models.

2.1. Star formation

Star formation is implemented in MACER as follows. A first, necessary condition to activate the star formation routines entails checking, grid by grid and at each time step, that the gas temperature is lower than $4 \times 10^4$ K. Under this condition, $\dot{\rho}_*,$ is computed by considering the two channels mentioned in the Introduction: the Toomre instability channel, and the cooling/Jeans channel. As we will see, in some circumstances the two channels are both active; in this case $\dot{\rho}_*$ is defined as the sum of the two rates (provided the total mass consumption rate in the considered time step does not exceed a prescribed fraction of the available mass in the considered grid).

The first channel of star formation is related to the Toomre instability (e.g., see Toomre 1964, Binney & Tremaine 2008). As is well known, the local stability of a rotating, self-gravitating disk can be understood in terms of the relative importance of temperature (or velocity dispersion) vs. disk gravitational field (i.e. surface density). Specifically, a self-gravitating gaseous disk is locally stable when

$$Q(R) \equiv \frac{c_s(R)\kappa(R)}{\pi G\Sigma_{gas}(R)} > 1,$$

Notice that changing rotational support at fixed structure affects some aspect of the ISM evolution, changing the amount of thermalization of the stellar ejecta due to stellar velocity dispersion and to the relative motion of stellar streaming rotational field and ISM velocity field. For an extensive discussion see Negri et al. (2014a) and Negri et al. (2014b); see also Kim & Pellegrini (2012).
where in our case $R$ is the distance from the galaxy center in the disk plane, $\Sigma_{\text{gas}}$ is the gas surface density of the disk, $c_s$ is the sound velocity, and $\kappa$ is the radial epicyclic frequency. As an effect of disk instability, angular momentum of the rotating gas is transferred outward due to a non-axisymmetric gravitational torques; correspondingly, mass is transferred inwards and is eventually accreted on the central SMBH. Unstable over-densities in the infalling material (in the present simulations, due to the imposed geometry, specifically gaseous rings) trigger bursty phases of star formation, which then decrease the surface density, increase $Q(R)$ above unity, and re-stabilize the ISM, in a sort of self-regulation (e.g., see Bertin & Lodato 1999). In Gan et al. (2019) the star formation rate $\dot{\rho}_{\text{star}}(R)$ is determined by bremsstrahlung emission, Compton heating and cooling, and recombination (Gan et al. 2019), with allowance for heating due to the central flaring AGN. The dynamical timescale is the minimum between the Jeans collapse timescale $\tau_{\text{Jeans}} = \sqrt{3\pi/32G\rho_{\text{gas}}}$ and the rotational timescale $\tau_{\text{rot}} = 2\pi R/\kappa_{\text{disk}}$. If the dynamical time dominates, $\tau_{\text{SF},C} \propto \rho_{\text{gas}}^{-1/2}$, whereas if the cooling timescale dominates, $\tau_{\text{SF},C} \propto \rho_{\text{gas}}^{3/2}$, with the resulting star formation rate proportional to either $\rho_{\text{gas}}^{3/2}$ or $\rho_{\text{gas}}^{1/2}$, respectively.

3. RESULTS

3.1. Gaseous and stellar disks from MACER simulations, and the global KS law

A preliminary discussion of the global properties of the gaseous rotating cold disks in ETGs (and of the associated stellar disks) produced by MACER simulations can be found in C22 (see Table 2 therein); here we consider several additional properties of the circumnuclear disks, providing all the information needed to address the focus of this paper. A complete summary of the results is contained in Table 1 and Table 2. In the Tables, the data are given for increasing galaxy mass, and for each mass in order of increasing amount of ordered rotation. In particular, in Table 1 we give the stellar and gaseous properties of the circumnuclear disks, both at the end of the simulations (corresponding to a simulation time of $\Delta t = 12$ Gyr, and to a final age of the galaxy of 14 Gyr), and time-averaged over $\Delta t$. In particular, time-averaged quantities are indicated with a bar over the corresponding symbol, so that the time-averaged values of the quantity $f(t)$ is given by

$$\bar{f} = \frac{1}{\Delta t} \int_{t_0}^{t_0 + \Delta t} f(t') \, dt',$$  

where, as assumed in MACER, at the initial time of simulation the age of the galaxy is $t_0 = 2$ Gyr. Therefore, for each of the six models, the columns in Table 1 give the total mass $\Delta M_*$ of new stars formed in the circumnuclear disk, the time-average $\overline{R}_{\text{M}_*}$ of the instantaneous half-mass radius of star formation $R_{\text{M}_*/2}(t)$, the total mass $M_{\text{M}_*}$ of the circumstellar stellar disk at the end of the simulation (where the difference $\Delta M_* - M_{\text{M}_*}$ is accounted for stellar evolution), and the final half-mass radius $R_{\text{M}_*/2}$ of the stellar disk. Then, for the properties of the circumnuclear gaseous cold (defined as in C22 as gas with $T \leq 5 \times 10^5$ K) disk, for each model we report the time-average $\overline{\Sigma}_{\text{gas}}$ of the gas mass in the circumnuclear disk, together with the time-averages $\overline{R}_{\text{gas}}$ and $\overline{R}_{\text{gas}}$ of the gaseous disk half-mass and truncation radii, respectively. The last two columns of Table 1 finally give the values at the end of simulation of the disk total gas mass $M_{\text{gas}}$, and the associated truncation radius $R_{\text{gas}}$.

In Table 2 additional information about gas content and star formation in the circumnuclear disks are reported to study the global properties of the simulated KS relations. Accordingly, for the six models we construct the equivalent surface star formation rate, defined from the quantities in Table 1 as

$$\overline{\Sigma}_* = \frac{\Delta M_*}{2\pi \overline{R}_{\text{M}_*/2}} \frac{\Delta t}{\Delta t},$$  

i.e., the time-independent and spatially uniform star formation rate of a fictitious starforming disk of half-mass radius $\overline{R}_{\text{M}_*/2}$, forming a total stellar mass $\Delta M_*$ over the time $\Delta t$. We then compute the time-average of the (spatially averaged) surface density of the disk within the instantaneous half-mass radius, given by

$$\overline{\Sigma}_{\text{gas}} = \frac{M_{\text{gas}}(t)}{2\pi \overline{R}_{\text{gas}/2}(t)},$$  

$^2$ A semi-analytical algorithm mimicking this process is also included in MACER, and is essential for producing SMBH accretion in presence of ordered rotation.
Star formation results for the 3 families of models in order of increasing galaxy stellar mass, with radially dependent (subscript “e”), and isotropic (subscript “c”) Satoh decomposition for the streaming rotational velocity of the stellar population; a bar over a quantity means time average over the simulation time $\Delta t = 12$ Gyr defined as in equation (5). The columns give: (1) total mass of new stars formed; (2) time-average of the instantaneous half-mass radius of star formation $R_{\text{hmass}}/2(t)$; (3) total mass of the circumnuclear stellar disk at the end of the simulation: the difference $\Delta M_* - M_{\text{dis}}$ is accounted for stellar evolution; (4) final half-mass radius of the stellar disk; (5) time-average of the cold ($T < 5 \times 10^4$ K) gas mass in the circumnuclear disk; (6) time-average of the half-mass radius of the cold gaseous disk; (7) time-average of the truncation radius of the cold gaseous disk; (8) final value of the cold gas mass in the circumnuclear disk; (9) final value of the truncation radius of the cold gaseous disk.

We finally evaluate the global, time-averaged star formation efficiency, naturally defined as

$$\epsilon_*= \frac{\dot{M}_*(t)}{\dot{M}_{\text{gas}}(t)} \approx \frac{\Delta M_*}{\dot{M}_{\text{gas}} \Delta t},$$

and, for comparison with the observed KS law, we also estimate the global, time-averaged star formation amplitude as

$$<A_* > \equiv \log_{10} < \dot{\Sigma}_* > - 1.4 \log_{10} < \dot{\Sigma}_{\text{gas}} >, \tag{9}$$

where, following the common approach. $< \dot{\Sigma}_* >$ is in units of $M_\odot$ kpc$^{-2}$ yr$^{-1}$, and $< \dot{\Sigma}_{\text{gas}} >$ in $M_\odot$ pc$^{-2}$.

Before focusing on the properties of the resolved KS laws for the six models, we list the main global results, beginning with Table 1. We immediately note a few robust trends. 1) The total mass of newly formed stars in the circumnuclear disk $\Delta M_*$ is in the range of $10^8 - 10^9 M_\odot$ and increases systematically with galaxy mass; at fixed galaxy mass, star formation is also larger for isotropic rotators (subscript “c” in Column 1). Both these trends are quite natural; in particular, the enhanced star formation in case of substantial galaxy rotation confirms previous studies (Negri et al. 2015, Gan et al. 2019) that angular momentum tends to increase ISM cooling and successive instabilities. 2) The present day mass $M_{\text{dis}}$ of the resulting stellar disks is obviously less than $\Delta M_*$, a consequence of stellar evolution: notice that the quite large difference between the two quantities (with a reduction factor slightly larger than $\simeq 50\%$) in all the models is due to the adopted top-heavy IMF in the simulations (see C22). 3) The present day half-mass radius $R_{\text{dis}}/2$ of the stellar disk is found, for all models, in the range $\simeq 100 - 300$ pc, being obviously larger for the three isotropic rotator models: notice how well $R_{\text{dis}}/2$ matches $R_{\text{hmass}}/2$ in Column 2, the time-averaged half-mass radius of star formation in the disk.

The properties of the stellar disks reflect nicely the properties of their gaseous component, the source of ISM material for star formation: some systematic differences are however important and should be noticed. 4) The present-day mass $M_{\text{gas}}$ of cold gas in the disk, in each of the three different families of models, is significantly larger (with values as high as $6 \times 10^{10} M_\odot$) in the isotropic rotators than in the corresponding low-rotation cases, where in these latter cases $M_{\text{gas}}$ is only slightly larger than $M_{\text{dis}}$. 5) A similar comment applies to the size of the gaseous component of the disks, as can be see by comparing Columns (4) and (7): in all models, the stellar disks are obviously restricted to the central regions of the parent gaseous disks, that can easily reach the kpc scale.

All the previous results reflect in the global quantities concerning star formation in the models, as reported in Table 2: in each of the three families of galaxy models, the averaged surface density star formation rate $< \dot{\Sigma}_*$ is higher in the low-rotation models, while the opposite happens for averaged cold gas surface density $< \dot{\Sigma}_{\text{gas}} >$. The corresponding global, time-averaged star formation efficiencies are in general of few percent: therefore, the efficiency with which cold gas is converted to stars in the numerical simulations is within the lower range predicted by [Kennicutt (1998)] for spiral galaxies. The last, but particularly important quantity, one of the focus points of the present paper, is the amplitude of the simulated KS laws. From the last column in Table 2, we found that $< A_* >$ is in the range $(-5.1, -3.4)$; generally, these amplitudes lie lower than, but not extremely so, the Kroupa IMF-corrected KS amplitude of measured for spirals/starbursts [Davis et al. (2014)]; lower amplitudes systematically correspond to models with low rotation of the stellar population. Therefore, a reduction of the rotational support in ETGs at fixed galaxy mass, appears to decrease the amplitude of the resulting KS law, and one is tempted to conclude that specific angular momentum of the stellar population could be the leading parameter determining the normalization coefficient of the KS law.
nicely aligned parallel to the heavy solid line showing the observed KS law tail of red points in the top-right panel, absent in the top-left panel. The second feature of the simulated points to be noticed is the large scatter (in both models) of amplitudes in equation (9) for the low mass models are reversed from those in Figure 4, suggesting that cold gas is over counted for the exponential case - a rapid decrease in rotation leaves much of the gas unable to produce stars.

Relevant parameters of the simulated KS law in the simulated ETGs reported in Table 1. The columns give: (1) the equivalent (time-independent and spatially uniform) surface star formation rate \( \dot{\Sigma}_* \) of a fictitious starforming disk of half-mass radius \( R_{\text{hf},/2} \), forming a total stellar mass \( \Delta M_* \) over the time \( \Delta t \), defined as in equation [6]; (2) time-average of the average surface density of cold gas in the disk \( \langle \Sigma_{\text{gas}} \rangle \), defined as in equation [7]; (3) time-average of the global star formation efficiency \( \epsilon_* \), defined as in equation [8]; (4) amplitude of the KS star formation law, defined as in equation [9].

### 3.2. Resolved KS Laws

Even though in principle star formation can happen everywhere over the galaxy model, the simulations revealed that all star formation occurs in rotationally supported, thin, cold disks in the galaxy equatorial plane. To investigate the radial dependence of the star formation rate \( \dot{\Sigma}_* \), as a function of the surface gas density \( \Sigma_{\text{gas}} \), i.e., the resolved KS law as produced by the MACER simulations, for each of the six models we compute the mass of cold \((T \leq 5 \times 10^5 \text{ K})\) ISM, and the star formation rate, over radial annuli \((R, R + \Delta R)\), and then we divide the results for \(2\pi R \Delta R\). This computation is repeated, for each annulus, with a sampling time of 1 Gyr over the time \( \Delta t = 12 \text{ Gyr} \) spanned by the simulations, so that for each of the analyzed models we construct a large set of points \( (\Sigma_{\text{gas}}, \Sigma_*) \), corresponding to different radii in the disk, and to different evolutionary times.

With the aid of Figure[1] we begin by illustrating the results for the two high-mass models HMe, in the low rotation case with the exponentially decreasing Saloh parameter \( k(r) \) (HMe, left panels), and for the fast rotating, isotropic rotator (HMc, right panels) with constant \( k = 1 \). In the top panels, all the points \( (\Sigma_{\text{gas}}, \dot{\Sigma}_*) \) are plotted with a color scheme mapping the distance from the galaxy center, with decreasing distance from red to blue. A few trends are apparent. The first is just a confirmation that the cold rotating disk extends more in the fast rotating isotropic HMe model than in the less rotating HMc model, as is obvious from the tail of red points in the top-right panel, absent in the top-left panel. The second important result is that the bulk of the points are nicely aligned parallel to the heavy solid line showing the observed KS law

\[
\dot{\Sigma}_* \approx 10^{-3.76} \Sigma_{\text{gas}}^{1.4}
\]

as given by Davis et al. (2014), where the units are the same as in Table 2, and the amplitude \( A_* = -3.76 \), similar to what found by other authors. For example Boquien et al. (2011) report observed amplitudes in the range \(-3.83 \lesssim A_* \lesssim -3.02 \) for spiral galaxies when surface densities/rates are computed in the same units as above. The third result is that our resolved KS laws, albeit parallel to the observed law, are clearly below it, with lower star formation rate at given surface density of the cold gas in the disk, i.e. in the simulations ETGs gaseous disks are predicted to be inefficient with respect to observed KS law observed in disk galaxies by almost an order of magnitude. Notice that from Table 2 this is reflected by the time-averaged, global value of \( \langle A_* \rangle \approx -4.1 \) for the low-rotating HMe model, but not for the isotropic rotator HMc (where \( A_* \approx -3.4 \)). The fourth feature of the simulated points to be noticed is the large scatter (in both models) of \( \Sigma_{\text{gas}} \) at approximately constant \( \Sigma_* \), both in the very central regions, and at the edge of the gaseous rotating disk. Physically, the scatter implies that at the very high (center) and very low (disk outskirts) star formation rates, the gas surface density is only weakly correlated with star formation, with significant differences in \( \Sigma_{\text{gas}} \) at fixed \( \Sigma_* \) at the center, and with almost uncorrelated \( \Sigma_{\text{gas}} \) and \( \dot{\Sigma}_* \) at large radii. More quantitatively, star formation rate becomes abruptly horizontal near the center, with \( \dot{\Sigma}_* \gtrsim 10^{9.5} M_\odot \text{pc}^{-2} \text{Gyr}^{-1} \). At larger radii (\( \approx 2.5 \text{ kpc} \)) we also see an abrupt threshold cutoff at gas densities less than \( 10^{-3} \text{ g cm}^{-2} \). K89 remarks that a similar behavior is a consistently observed phenomena for disk galaxies, and that this cutoff threshold is typically found to occur within the range of \( 10^{-3} - 10^{-4} \text{ g cm}^{-2} \), which is incidentally very similar to what we observe for our ETG model in Figure 1. That this feature is also present in our simulations is encouraging regarding the veracity of our star formation recipe and similarity of star-formation laws across different galaxy types. Following K89, we argue that the universal cause for this phenomena arises from stability considerations captured by the simple Toomre star formation channel: at lower densities, the gas is stable against large-scale perturbations, suppressing star formation. High resolution observations of ETGs are essential, therefore, for confirming or refuting the remaining similarity of star formation dependence on gas density between disk and ETGs found in our work.

All these trends are sufficiently interesting to merit a further scrutiny. In fact, we recall that in the MACER treatment of star formation, \( \dot{\Sigma}_* = \dot{\Sigma}_* \text{Q} + \dot{\Sigma}_* \text{C} \), different possibilities are considered, i.e., star formation can be produced by Toomre instability, by cooling/Jeans instabilities, or by a combination of the two, depending on the cold gas state (see Section 2). Therefore, a

\[ \text{Notice that the amplitude in equation } (10) \text{ is not the estimated time and space averaged amplitude } \langle A_* \rangle \text{ in equation } (9). \] In particular, the relative sizes of amplitudes in equation (9) for the low mass models are reversed from those in Figure 4, suggesting that cold gas is over counted for the exponential case - a rapid decrease in rotation leaves much of the gas unable to produce stars.
Resolved star formation rate of the high-mass (HM) models for the low-rotation case with exponentially declining Satoh $k(r)$ parameter with (HM)$_e$, left column, and for fast rotating $k = 1$ isotropic model (HM$_c$, right column). In the top panels local star formation rates $\dot{\Sigma}^*_\ast = \dot{\Sigma}^*_{Q} + \dot{\Sigma}^*_C$ (in units of $M_\odot$ kpc$^{-2}$ yr$^{-1}$) are given as a function of the local disk gas surface density $\Sigma_{\text{gas}}$ (in units of $M_\odot$ pc$^{-2}$), with a sampling time of 1 Gyr over the time span $\Delta t = 12$ Gyr, so the plots represent also the history of star formation in the model. For reference, the solid line represents the Davis et al. (2014) observed KS law in equation (10), of slope $1.4$ and amplitude $A^* = -3.76$. In the bottom panels red and blue points separate respectively the contribution of Toomre instability ($\dot{\Sigma}^*_{Q}$) and cooling/Jeans instabilities ($\dot{\Sigma}^*_C$) to the total star formation rate of each point in the top panels.

natural question is which, if either, of the two channels is dominant in different regions of the ($\Sigma_{\text{gas}}, \dot{\Sigma}^*_\ast$) planes. For this reason, in the bottom panels of Figure 1 we plot star formation resulting from different formation channels, indicating the star formation rate due to Toomre instability in red and the star formation rate due to cooling/Jeans instabilities in blue; all the points in the top panels corresponding to star formation with both channels active now split in pairs of red and blue points. From the bottom panels, we conclude that the bulk of the KS law obtained by MACER is due to Toomre instabilities, which are responsible for the large scatter of the points at the edge of the cold disk. Conversely, star formation in the very central regions is dominated by cooling/Jeans instabilities. Notice the almost vertical strip of blue points at high values of $\Sigma_{\text{gas}}$; these points are not missing in the top panels, they just represent a marginal star formation well below the Toomre instability channel, and they “disappear” when added to the red points relative to the same numerical cell and same sampling time. A complementary view of the relative importance of Toomre and cooling/Jeans instabilities, where the dependence of the main channels of star formation as a function of the distance from the center can be clearly seen, is given in Figure 2 nicely confirming the conclusions above.
Figure 2. Star formation channels for models HMe (left) and HMc (right), color coded to show the position inside the rotating cold gaseous disk in the equatorial plane of the models. From bottom panels it is apparent how cooling/Jeans instability star formation is mainly confined to the very central regions, and producing the highest values of star formation. Toomre instabilities instead occur over all the disk.

The dependence of the surface density profile on disk scale height, which rapidly evolves over the disk, accounts for the reason for the different relevance of the two channels in different locations of the gaseous disk. We identify the regime between $2.5 \times 10^{-1}$ and $2.5$ kpc as dominated by the Toomre channel, which produces a SFR identical to the KS power law. K89, and more recently Boissier et al. (2003), have both remarked that the Toomre instability criterion remarkably reproduces SFR power laws in spiral galaxies; despite the different gas properties and disk structure of ETGs, the form of local star formation is the same in our computations.

Figures 3 and 4 nicely confirm and extend the global picture illustrated above also to different galaxy masses, with simulated (resolved) star formation KS laws parallel to the observed one, characterized by lower amplitude $A_\nu$, and by quite significant scatter in the central regions and at the edge of the circumnuclear gaseous disks.

### 3.3. The volumetric KS law

As mentioned in §1, volumetric KS laws have also been proposed and investigated as alternatives to the standard KS law built from surface densities. Owing to the thinness of the stellar disks and difficulties of measuring volume densities in galaxies, surface density star formation laws are more frequently used. However, surface densities are frequently affected by projection effects and the flaring of the disk thickness; thus, volumetric density relations may offer more accurate insight into star formation laws
Figure 3. Resolved star formation rate of the medium-mass (MM) models, for the low-rotation case with exponentially declining Satoh $k(r)$ parameter with (MM_e, left column), and for fast rotating $k = 1$ isotropic model (MM_c, right column). In the top panels local star formation rates $\dot{\Sigma}_* = \dot{\Sigma}_* + \dot{\Sigma}_* C$ (in units of $M_\odot kpc^{-2} yr^{-1}$) are given as a function of the local disk gas surface density $\Sigma_{gas}$ (in units of $M_\odot pc^{-2}$), with a sampling time of 1 Gyr over the time span $\Delta t = 12$ Gyr, so the plots represent also the history of star formation in the model. For reference, the solid line represents the Davis et al. (2014) observed KS law in equation (10), of slope $1.4$ and amplitude $A_* = -3.76$. In the bottom panels red and blue points separate respectively the contribution of Toomre instability ($\dot{\Sigma}_* Q$) and cooling/Jeans instabilities ($\dot{\Sigma}_* C$) to the total star formation rate of each point in the top panels.

While also being more applicable for simulations. Due to these observational difficulties, however, it is thought that conversion between surface densities and volumetric densities is non-trivial (Bacchini et al. 2019, 2020). Our numerical simulations allow us to study volumetric power laws while avoiding observational 2D projection effects.

We study the radial behavior of volumetric star formation laws by averaging SFR and gas densities over radial annuli over 1 Gyr timesteps throughout the $\Delta t = 12$ Gyr of evolution. The two panels of Figure [S] contain plots of the fit (heavy solid line) of the obtained resolved volumetric KS law for the high mass models. We observe that the overall volumetric densities follow a remarkable correspondence with the surface density relationships in Figure [I] and that the fit is improved in the low radii cooling regime with less horizontal spread. Using Equation 4, a dynamical timescale on the order of $\rho_{gas}^{-1/2}$ explains the good $n \approx 1.5$ power law fit, consistent with some observational work (Sun et al. 2023); we conclude that in MACER simulations it is the longer dynamical time rather than the cooling time that dominates the star formation in this spatial region, favoring $n \approx 1.5$ rather than the occasionally observed $n \approx 2$ (Bacchini et al. 2020, heavy dashed lines). In any case, also the resolved volumetric KS
laws obtained from the simulations confirm a lower efficiency of star formation at given gas density in the circumnuclear disks of the simulated ETGs. While the volumetric KS relation corresponds with our physical prescription at small radii, the surface density KS relation matches less well, as noted in §3.2. This can be attributed to a rapidly flaring scale height, as seen in the left hand panel of Figure 5, resulting in projection effects; we note, however, that Toomre instability channel matches well in both cases. The threshold cutoff observed at ≈ 2.5 kpc once again is attributed to Toomre instability. Ultimately, we propose that surface-density SF laws, which are easier to measure, can be translated into volumetric-density SF laws, which describe the form of a local SF, given the close correspondence between the two. More importantly, we once again assert that the same empirical SF laws in disk galaxies can be applied towards ETGs. High-resolution observations of ETGs to confirm these power law fits will thus be beneficial for applications towards semi-analytical star formation recipes in simulations and for understanding the local analytic form of star formation laws.

4. SUMMARY AND CONCLUSIONS
Figure 5. Volumetric star formation law for the low-rotation, high-mass model HM\textsubscript{e} (left), and the high-mass, fast rotating (isotropic) model HM\textsubscript{c} (right), a figure analogous to the top panels in Figure 1. The total volumetric star formation rates \( \dot{\rho}^* \) are given as functions of local gas density \( \rho_{\text{gas}} \), with sampling time of 1 Gyr throughout the \( \Delta t = 12 \) Gyr time spanned by the simulation. The heavy solid line shows a fit to the \( \dot{\rho}^* \propto \rho_{\text{gas}}^{1.5} \) volumetric KS law, while the heavy dashed line is the observed relation derived for spirals and dwarf galaxies (equation 4.1 in Bacchini et al. 2020).

In this paper we present a detailed study of star formation laws in in ETGs by analysing the results of high resolution numerical simulations reported in C22 and performed with the latest version of our MACER code. For any axisymmetric ETG with some level of ordered rotation in their stellar populations, both numerical simulations and simple physical arguments have clearly established that the ISM cools and develops large-scale instabilities, inevitably leading to the formation of cold and rotating gaseous disks in the galaxy equatorial plane. This is confirmed by recent observations demonstrating the existence of massive gaseous disks in 50\% of ETGs (Young et al. 2011). Theoretically, such disks should be prone to star formation, raising the question as to why their observed star formation is so inefficient in comparison to disk galaxies (Negri et al. 2015).

Similar to what happens in rotating disk galaxy models, our simulations lead to the formation of rotating, cold, gaseous disks with kpc scale in the presence of galaxy rotation, gas cooling, and angular momentum. We implement star formation over this cold gas by considering two different channels: one based on Toomre instability, and another determined by the longer of the local cooling and dynamical (Jeans) times of the cold ISM. When the local values of density and temperature of the ISM are respectively larger and smaller than two prescribed threshold values, both channels are active, and the star formation rate is their sum. Remarkably, our star formation recipes result in a SFR-gas scaling law quite similar to the observed KS relation.

The Toomre instability star formation channel reproduces an \( n \approx 1.4 \) resolved KS star formation power law with a lower cutoff threshold, which is analogous to what has been observed in disk galaxies and can be attributed to gravitational stability considerations. The cooling channel also obeys a power-law density relationship, but deviates from the KS star formation law at higher densities closer to the center of the galaxy, which may be attributed to a rapidly flaring scale height. In preliminary 3D simulations more recently performed, smaller scale, nonaxisymmetric fluctuations tend to remove this small discrepancy. Volumetric star formation power laws are also explored in effort to remove artificial effects of disk flaring on surface density measurements, and we find a close correspondence between the volumetric and surface density forms of the KS power law. Globally, we find that our simulated ETGs lie approximately parallel but with a lower normalization to the same KS relation as disk galaxies.

The similarity of the observed star formation power-laws in disk galaxies with our simple numerical implementation in ETGs is remarkable, implying that these KS laws provide simple recipes that can be used in semi-analytical models of ETGs as well. The prediction of the existence of small star-forming disks in ETGs encourages high-resolution observations of galactic centers as well as further work in understanding their dynamical properties and exploring the form of local - rather than empirical - star formation laws to see if our predictions correspond with reality. Therefore, observational checks of our predictions are imperative for evaluating these conclusions and for determining the evolution of cool gas with respect to the star formation inefficiency problem. ETGs are not quiescent in our view and observations are need to check our SFR estimates: cooling flows events that occur in rotating ETGs (from the non ejected gas) should be observed as miniature versions of the standard observed disk star forming regions in normal spiral systems, and the central regions of the MW and M31.
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