Determination of stiffness parameters of reinforced concrete structures using the decomposition method for calculating their survivability

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Abstract. A method for determining the generalized stiffness of reinforced concrete structural systems of monolithic buildings using the decomposition method is presented. The secondary calculated scheme for determining dynamic additional loading in the frame of a monolithic building is adopted in the form of a substructure isolated from the frame of a building in the zone of possible local failure under a special action. For such a system, generalized stiffnesses are defined, depending on the angular stiffness of the support restraints and the stiffness of the overlying floors (adjacent structural elements). Analytical dependencies and a method for determining the overall stiffness of reinforced concrete substructures with specified boundary conditions are proposed. The influence of boundary conditions on internal force factors of the substructure is investigated. Recommendations are given for improving the survivability of the considered structural systems of monolithic buildings, as well as suggestions for their protection from progressive collapse.

1. Introduction

In scientific research of scientists from many countries, research is conducted aimed at protecting buildings and structures from the process of collapse [1–5]. One of the directions of these researches is to study the redistribution of force flows in physically and structurally nonlinear systems when removing one of the load-bearing elements. When studying such processes, it is important to assess the stress state in the remaining elements of the structural system, taking into account the dynamic nature of the process. Analysis of publications shows that numerical studies of dynamic processes [6-8] cause a number of computational difficulties. This includes a significant amount of hard-to-see information, a colossal time spent on the calculation, the impossibility of qualitative analysis of the studied processes, etc. In this regard, it is proposed to use the decomposition method and level computational models for such an analysis [9]. At the same time, an important task becomes the analysis of factors affecting the overall stiffness of the substructure allocated from the structural system and the development of practical recommendations for designing the protection of buildings and structures against progressive collapse on this basis.

Studies of the process of the dynamic response of a substructure during the structural reconstruction of a structural system have been described in [10–13]. Here we consider the idealized
calculated diagram of the substructure with hinged or fixed supports. While in real conditions, reinforced concrete substructures as part of the entire structural system rest on supports with a specific angular stiffness or are not clamped absolutely rigidly (Figure 1). Therefore, this work aims to develop a method for determining the stiffness of a reinforced concrete substructure, taking into account the influence of the overlying floors and the stiffness of support anchors on the dynamic response of the investigated substructure.

![Figure 1](image)

**Figure 1.** Local failure zone (a) and selected substructure (b) and calculated scheme (c)

2. Method for calculating the total stiffness of a substructure by the force method

Following [10-13] for the considered substructure of a continuous two-span beam, the equation of motion with one degree of freedom can be written in the form:

\[
 u(t) = (u_n)_0 \left[ \frac{t}{t_r} - \frac{1 - 2\zeta^2}{t_\omega_D} e^{-\zeta t_\omega_D t} \sin t_\omega_D t - \frac{2\zeta}{t_\omega_n} \left(1 - e^{-\zeta t_\omega_n t} \cos t_\omega_n t\right) \right] \text{ if } 0 \leq t \leq t_r ;
\]

\[
 u(t) = (u_n)_0 \left[ 1 + \frac{2\zeta}{t_\omega_D} \left[ e^{-\zeta t_\omega_D t} \cos t_\omega_D t - e^{-\zeta t_\omega_n (t-t_r)} \cos t_\omega_n (t-t_r) \right] \right] - \frac{1 - 2\zeta^2}{t_\omega_D} \left[ e^{-\zeta t_\omega_D t} \sin t_\omega_D t - e^{-\zeta t_\omega_n (t-t_r)} \sin t_\omega_n (t-t_r) \right] \text{ if } t > t_r ,
\]

where \((u_n)_0\) - the static deflection of the substructure; \(\omega_D\) - the frequency of natural vibrations of the system, taking into account energy dissipation; \(\zeta\) - the damping coefficient; \(t_r\) - column failure time.

Time of dynamic additional loading of the substructure and taking into account the time of column failure:

\[
 t_d = t_r + \frac{T}{2},
\]

where \(T\) is the period of oscillation of the substructure.

Putting (2) in (1) we obtain an analytical expression for the maximum dynamic deflection of the substructure, taking into account the time of column failure without taking into account energy dissipation:

\[
 u(t)_{max} = (u_n)_0 \left[ 1 + \frac{2}{\omega_n t_r} \sin \frac{\omega_D t_r}{2} \right].
\]

It is not difficult to see that \(u(t)_{max}\) approaches two times the static deflection of the substructure at \(t_r \to 0\).
where $P_0$ is the external load on the substructure.

From formula (4) it follows that the maximum dynamic deflection after the sudden removal of the column is inversely proportional to the value of the total stiffness of the substructure $K$, that is, an increase in the survivability of the structural system can be provided by an increase in the total stiffness of the substructure.

Consider the factors that affect the value of the overall stiffness of the substructure $K$ in relation to a two-span continuous beam with different boundary conditions (figure 2a).

\[ u(t)_{\text{max}} = \lim_{t \to 0} \left[ (u_0) _0 \left( 1 + \frac{2}{\omega t} \sin \frac{\omega t}{2} \right) \right] = 2(u_0) _0 = \frac{2P_0}{K}, \]  

(4)

Figure 2. Calculated (a) and equivalent (b) substructure scheme: c, d, e, f – the plot of the bending moment from the action of unit forces and the action of external loads, respectively

The degree of static indeterminacy of a substructure is equal to 3, that is, the substructure is three times statically indeterminate. We form the main statically determinate system, discarding the three extra connections and construct an equivalent system, replacing the action of discarded connections with yet unknown reactions of connections $X_1$, $X_2$ and $X_3$ (figure 2b), where $X_1$, $X_2$ - bending moments in elastic - compliance nodes, and $X_3$ - the reaction in the spring simulating the influence of the overlying floors. Unknown bond reactions of the connections $X_1$, $X_2$ and $X_3$ are determined from canonical equations by the force method:

\[
\begin{align*}
\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \Delta_{1P_b} &= -X_1 / C_1; \\
\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \Delta_{2P_b} &= -X_2 / C_2; \\
\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \Delta_{3P_b} &= -X_3 / C_3,
\end{align*}
\]

(5)

where $C_1 = X_1 / \varphi_1, C_2 = X_2 / \varphi_2$ is the angular stiffness of compliance nodes, and $K_3 = X_3 / (u_0)_0$ - is the linear stiffness of the spring that models the influence of the overlying floors of the building frame.
The compliance coefficients are calculated using the well-known formulas of structural mechanics:

\[
\delta_{11} = \delta_{22} = l/3B_{red}, \quad \delta_{12} = \delta_{21} = l/6B_{red}, \quad \delta_{13} = \delta_{31} = l^2/16B_{red}, \quad \delta_{23} = l^2/48B_{red},
\]

\[
\Delta_{1p} = \Delta_{2p} = -P_0l^2/16B_{red}, \quad \Delta_{3p} = -P_0l^3/48B_{red},
\]

where \(B_{red}\) is the reduced flexural stiffness of the reinforced concrete substructure section taking into account cracks at the first stage of static-dynamic loading.

Putting these coefficients in (5), we obtain a system of equations:

\[
\begin{align*}
\left(\frac{l}{3B_{red}} + \frac{1}{C_1}\right)X_1 + \frac{l}{6B_{red}}X_2 + \frac{l^2}{16B_{red}}X_3 \bigg(1 - \frac{P_0l^2}{16B_{red}}\bigg) &= 0 \\
\frac{l}{6B_{red}}X_1 + \left(\frac{l}{3B_{red}} + \frac{1}{C_2}\right)X_2 + \frac{l^2}{16B_{red}}X_3 \bigg(1 - \frac{P_0l^2}{16B_{red}}\bigg) &= 0 \\
\frac{l^2}{16B_{red}}X_1 + \frac{l^2}{16B_{red}}X_2 + \left(\frac{l^3}{48B_{red}} + \frac{1}{K_3}\right)X_3 \bigg(1 - \frac{P_0l^3}{48B_{red}}\bigg) &= 0
\end{align*}
\]

(6)

For real reinforced concrete structures of the building frame, it can be assumed that \(C_1 = C_2\). Then solving system (6), we find:

\[
X_1 = X_2 = \frac{24B_{red} \cdot P_0C_1 \cdot l^2}{384B_{red}^2 + 192B_{red} \cdot C_1 \cdot l + K_3(8B_{red} + C_1 \cdot l)l^3}
\]

(7)

\[
X_3 = \frac{K_3(8B_{red} + C_1 \cdot l)l^3P_0}{384B_{red}^2 + 192B_{red} \cdot C_1 \cdot l + K_3(8B_{red} + C_1 \cdot l)l^3}
\]

(8)

Then, with the instantaneous removal of the middle support in the substructure (see figure 1, b) and the application of external load \(P_0\) the static deflection at the central point (the maximum static deflection of the substructure) will be:

\[
(u_{d})_0 = \frac{X_3}{K_3} = \frac{(8B_{red} + C_1 \cdot l)l^3P_0}{384B_{red}^2 + 192B_{red} \cdot C_1 \cdot l + K_3(8B_{red} + C_1 \cdot l)l^3}
\]

(9)

The overall stiffness of a reinforced concrete substructure can be determined from the ratio:

\[
K = \frac{P_0}{(u_{d})_0} = \frac{384B_{red}^2 + 192B_{red} \cdot C_1 \cdot l}{(8B_{red} + C_1 \cdot l)l^3} + K_3
\]

(10)

From the analysis of formula (10), it follows that the overall stiffness of the substructure depends on the span length \(l\), the reduced flexural stiffness \(B_{red}\), the stiffness of compliance support nodes \(C_i\) and the stiffness of the overlying floors of the building frame \(K_j\).

3. Results and discussion

As an example, consider a substructure in the form of a two-span continuous beam, made of heavy concrete of class B40 with a cross-section of 50x100 mm. The choice as an example of such a substructure is due to the fact that the substructure was isolated from the experimentally tested design of a two-span three-story reinforced concrete frame [9,13-15] and it is possible to compare experimental and theoretical results directly. The beam is reinforced symmetrically in the height of the cross-section in the compressed and tensioned zones with two rods with a diameter of 8 mm of class A500. The span of the substructure \(l\) is 1000 mm. The value of the reduced flexural stiffness of the reinforced concrete substructure section, taking into account the formation of cracks at the first stage of static-dynamic loading when applying the static part of the load, was calculated using the refined method [16] and amounted to 49.2 KNm^3. Then the calculation of the stiffness by formula (10) can be represented in the form of graphs describing the effect of the stiffness of compliance nodes \(C_i\) and overlying floors of the building frame \(K_j\) on the overall stiffness of the substructure.
Figure 3. Change in the overall stiffness of reinforced concrete substructure $K$ depending on the stiffness of compliance support nodes $C_1$ and overlying floors $K_3$:

a. 3D graphics, b. 2D graphics at $K_3=0$; $K_3=2B_{red}$ and $K_3=4B_{red}$ respectively

The analysis of the above graphs allows us to note the following: When $K_3=0$ and $C_1 \to 0$, we obtain a well-known stiffness formula for a beam with hinged supports $K = 48B/l^3$. At $K_3=0$ and $C_1 \to +\infty$, we obtain the well-known stiffness formula for a beam with fixed supports $K = \lim_{C_1 \to +\infty} \frac{384B^2 + 192B \cdot C_1 \cdot l}{(8B + C_1 \cdot l)^3} = \frac{192B}{l^3}$. Accordingly, with an increase $K_3$, the overall stiffness of the investigated reinforced concrete substructure increases.

This analysis allows us to make some recommendations to improve the survivability of reinforced concrete monolithic frames of multi-storey buildings to protect them from progressive collapse under special actions.

**Recommendations for protecting reinforced concrete monolithic structures of building frames from progressive collapse.**

The obtained theoretical calculated dependencies make it possible to formulate some recommendations for protecting the considered class of reinforced concrete structures from progressive collapse.

1. Increasing the reduced flexural stiffness ($B_{red}$) of the crossbar structures above the first floor by increasing the cross-section height of the crossbar.

2. The use of double working reinforcement in the structures of the floor disc above the first floor, which ensuring the perception of redistributed forces in the zones of removal of columns on the first floor.

3. Increasing the stiffness of the support nodes ($C_1$) of substructures by increasing cross-sections in the zones of the conjugation of columns and girders (figure 4c), and also by ensuring the continuity of all elements in the precast-monolithic floor disc by installing additional reinforcement, including pre-stressed reinforcement in the zones of alternating forces under special action (figure 4b).

4. Increasing the stiffness of the overlying floors $K_3$ and the entire structural system of the building frame due to the arrangement of outrigger floors and other stiffness elements.

5. Such as: reducing the own weight of building by using high-strength, lightweight reinforced concrete and fiber-reinforced concrete [17], setting contour and internal horizontal ties in the plane of floor discs, vertical ties in the direction of columns for the entire height of the building, and taking into
account the static work of the building frame when filling self-supporting partitions and walls made of small-piece materials.

a, b,

Figure 4. Schemes of constructive solutions for protecting the reinforced concrete frame from progressive collapse: a - due to increasing the stiffness of the support nodes; b - due to the continuity of the floor disc

4. Conclusions
A method and an algorithm for determining the generalized stiffness of substructures isolated from reinforced concrete frames of multi-storey buildings by the decomposition method are proposed, and secondary calculated schemes are constructed for them depending on the angular stiffness of the support restraints. The influence of boundary conditions on the internal force factors of the substructure during the sudden removal of the first-floor column in the considered building frame is established.

The obtained analytical dependencies for determining the stiffness of substructures and the given calculation recommendations can be used in the design of monolithic and precast reinforced concrete frames of multi-storey buildings in terms of their protection from progressive collapse under special actions.

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Acknowledgments
The reported study was funded by RFBR, project number 19-38-90111.