NONLINEAR PARKER INSTABILITY WITH THE EFFECT OF COSMIC-RAY DIFFUSION

T. KUWABARA
Graduate Institute of Astronomy, National Central University, 38 Wu-chuan Li, Chung-li, Tao-yuan 32054, Taiwan

K. NAKAMURA
Matsue National College of Technology, Nishi-ikoma-chou, Matsue-city, Shimane 690-8518, Japan

AND

C. M. KO
Department of Physics, Institute of Astronomy and Center for Complex Systems, National Central University, Chung-Li, Tao-yuan 32054, Taiwan

Received 2003 December 15; accepted 2004 February 16

ABSTRACT

We present the results of linear analysis and two-dimensional local magnetohydrodynamic (MHD) simulations of the Parker instability, including the effects of cosmic rays (CRs), in magnetized gas disks (galactic disks). As an unperturbed state for both the linear analysis and the MHD simulations, we adopted an equilibrium model of a magnetized, two-temperature, layered disk with constant gravitational acceleration parallel to the normal of the disk. The disk comprises a thermal gas, CRs, and a magnetic field perpendicular to the gravitational acceleration. CR diffusion along the magnetic field is considered; cross-field-line diffusion is supposed to be small and is ignored. We investigate two cases in our simulations. In the mechanical perturbation case, we add a velocity perturbation parallel to the magnetic field lines, while in the explosive perturbation case, we add CR energy into the sphere in which the CRs are injected. Linear analysis shows that the growth rate of the Parker instability becomes smaller if the coupling between the CRs and the gas is stronger (i.e., if the CR diffusion coefficient is smaller). Our MHD simulations of the mechanical perturbation confirm this result. We show that the falling matter is impeded by the CR pressure gradient; this causes a decrease in the growth rate. In the explosive perturbation case, the growth of the magnetic loop is faster when the coupling is stronger in the early stage. However, in the later stage the behavior of the growth rate becomes similar to the mechanical perturbation case.

Subject headings: cosmic rays — instabilities — ISM: magnetic fields — MHD

1. INTRODUCTION

It has been suggested that magnetic fields may play important roles for active phenomena in space, for example, in astrophysical jets (e.g., Shibata & Uchida 1985; Matsumoto et al. 1996), solar activity (e.g., Priest 1982; Yokoyama & Shibata 2001), and active galaxies (e.g., Kuwabara et al. 2000). With such active phenomena, if a gas layer is supported by horizontal magnetic fields against gravity, then the Parker instability may appear and can play an important role. Magnetic fields are also thought to play an important role in accretion disks (e.g., Stella & Rosner 1984; Kato & Horiuchi 1986) and galactic disks. For example, in galactic disks, interstellar matter (ISM) is aggregated and grows to giant cloud complexes in the spiral arms of galaxies via the Parker instability (e.g., Mouschovias 1974; Mouschovias et al. 1974; Elmegreen 1982a, 1982b; Elmegreen & Elmegreen 1986). On the other hand, cosmic rays (CRs) may also play an essential role in the dynamics of the ISM, since it is recognized that the energy density of CRs is of the same order as that of the magnetic field and turbulent gas motions (Parker 1969; Ginzburg & Ptuskin 1976; Ferrière 2001). The importance of the effects of CRs has been acknowledged, and a discussion concerning CRs was also given in the original work on the Parker instability (Parker 1966). For studying the dynamics of CRs, there are several approaches. The particle-particle approach treats the plasma and the CRs as particles; the fluid-particle approach treats the plasma as a fluid and the CRs as particles; and the fluid-fluid approach treats the plasma and the CRs as fluids. The hydrodynamic approach to the Parker instability and Parker-Jeans instability without CRs has been done by nonlinear calculation (e.g., Matsumoto et al. 1988; Shibata et al. 1989; Chou et al. 2000; Franco et al. 2002). On the other hand, in spite of the suggestions of many astrophysical applications, there are very few papers on the evolution of the Parker instability with the effects of CRs (Hanasz 1997; Hanasz & Lesch 2000). Hanasz & Lesch (2000) carried out calculations on the Parker instability induced by CR injection from a supernova under the thin flux tube approximation and suggested that the instability grows on shorter timescales, with the values of the diffusion coefficient decreasing. As the diffusion coefficient decreases, the diffusion speed of the CRs decreases. Since the region where the CR energy is injected keeps it for a long time, the dynamics is dominated by the interactions near the injection region.

In this paper we present the results of a linear analysis and MHD simulation of the Parker instability with the effects of CRs, starting from an equilibrium, two-temperature, layered disk. We adopt the hydrodynamic approach for CR propagation (Drury & Völk 1981; Ko 1992). The paper is organized as follows: In § 2 we present our physical assumptions and the equilibrium model. A linear stability analysis of the system is given in § 3, and the numerical results are described in § 4. Section 5 provides a summary and discussion.
2. MODELS

2.1. Assumptions and Basic Equations

We investigate the Parker instability with the effect of CRs in galactic disks. The basic equations are the MHD equations combined with the CR energy equation:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0,
\]

\[
\frac{\partial (\rho V)}{\partial t} + \nabla \cdot \left[ \rho VV + \left( P + P_c + \frac{B^2}{8\pi} \right) I - BB \right] - \rho g + 2\rho \Omega \times V = 0,
\]

\[
\frac{\partial P_c}{\partial t} + \nabla \cdot \left[ P_c V + \frac{B^2}{8\pi} I - \rho \nabla E - \rho \Omega \times B \right] = 0,
\]

\[
\frac{\partial}{\partial t} \left( \frac{P_g}{\gamma_g - 1} + \frac{1}{2} \rho V^2 + \frac{B^2}{8\pi} \right) + \nabla \cdot \left[ \frac{\gamma_g}{\gamma_g - 1} P_g V + \frac{c}{4\pi} E \times B \right] + V \cdot (\nabla P_c - \rho g) = 0,
\]

\[
\frac{\partial}{\partial t} \left( \frac{P_c}{\gamma_c - 1} \right) + \nabla \cdot \left( \frac{\gamma_c}{\gamma_c - 1} P_c V - V \cdot \nabla P_c \right) - \nabla \cdot \left[ \kappa \nabla \cdot \left( \frac{P_c}{\gamma_c - 1} \right) \right] = 0,
\]

where \( P_g \) and \( P_c \) are the gas pressure and the CR pressure, \( I \) is the unit tensor, \( \gamma_g \) and \( \gamma_c \) are the adiabatic indexes for the gas and the CRs, \( \kappa \) is the CR diffusion coefficient along the magnetic field, \( \mathbf{b} \) is the unit vector of the magnetic field, \( \Omega \) is the rotational angular frequency, and the other symbols have their usual meanings. In this model, self-gravity is ignored. The centrifugal force is assumed to be balanced by other forces (e.g., radial gravitational force of the galaxy). For simplicity, we ignore cross-field-line diffusion of the CRs. As a matter of fact, the ratio of the perpendicular to the parallel diffusion coefficient is quite small, for instance, 0.02–0.04 (Giacalone & Jokipii 1999; Ryu et al. 2003). We normalize these equations by the physical quantities related to the equilibrium model described in § 2.2.

The units of density and velocity are the density \( \rho_0 \) and sound speed \( C_{s0} \) at the midplane of the galactic disk in the equilibrium model. The unit of length is the scale height without the magnetic field and CRs, \( H_0 = C_{s0}^2/\gamma_g g_0 \), and the unit of time is the sound crossing time over 1 scale height, \( H_0/C_{s0} \). The two-dimensional calculation is carried out in the Cartesian coordinate system \((x, z)\), where we adopt the approximation \( \dot{x} = \dot{\phi} \) and \( \dot{z} = \dot{\psi} \) in the cylindrical coordinate system \((r, \phi, z)\) of the galactic disk, as did Mineshige et al. (1993) (see Fig. 1). Moreover, the calculation is carried out only in the region over the midplane of the galactic disk.

2.2. Equilibrium Model

We adopted the two-temperature, layered disk equilibrium model (Shibata et al. 1989) as the initial condition:

\[
T(z) = T_0 + (T_{\text{halo}} - T_0) \frac{1}{2} \left[ \tanh \left( \frac{z - z_{\text{halo}}}{w_{\text{leg}}} \right) + 1 \right],
\]

where the disk temperature is \( T_0 = 10^4 \) K, the halo temperature is \( T_{\text{halo}} = 25 \times 10^4 \) K, the height of the disk-halo interface is \( z_{\text{halo}} = 900 \) pc, and the width of the transition layer is \( w_{\text{leg}} = 30 \) pc. The magnetic fields are horizontal initially. The density, gas pressure, and CR pressure distributions are derived from the equation

\[
\frac{d}{dz} \left[ P_g + P_c + \frac{B^2(z)}{8\pi} \right] + \rho g_z = 0;
\]

subsequently, the total gas pressure scale height at \( z = 0 \) (midplane of the galactic disk) is \( H = (1 + \alpha + \beta)C_{s0}^2/\gamma_g g_0 \).
where \( \alpha, \beta, \) and \( g_c (>0) \) are the initial ratio of the magnetic pressure to the gas pressure, the initial ratio of the CR pressure to the gas pressure, and the gravitational acceleration, respectively. In this paper, we only consider constant \( \gamma_g, \gamma_c, \) and \( g_c. \) In the following simulations, we pick \( \gamma_g = 1.05 \) and \( \gamma_c = 4/3 \) and set \( \alpha = 1 \) and \( \beta = 1 \) initially. The system is initially homogeneous in the \( \xi \)-direction. For normalization, we take our units as follows: the unit of length is \( H_0 = C_{o,0}/(\gamma_0 g_{\text{GR}}) \) = 50 pc, the unit of density is \( \rho_0 = 1.6 \times 10^{-24} \text{ g cm}^{-3}, \) the unit of velocity is \( C_{o,0} = 10 \text{ km s}^{-1}, \) and the unit of time is \( H_0/C_{o,0} \approx 5 \text{ Myr}, \) where the subscript 0 denotes the value at the midplane of the galactic disk \( (z = 0). \)

### 3. LINEAR STABILITY ANALYSIS

#### 3.1. Linearized Equations

We perform standard linear stability analysis on the set of equations (1)--(5). Since the unperturbed state depends on \( z \) only, the perturbed quantities are assumed to have the form

\[
\xi = (\delta \rho, i \delta V_x, i \delta V_y, \delta V_z, \delta P_g, \delta P_e, \delta B_x, \delta B_y, -i \delta B_z),
\]

where \( \sigma \) is the growth rate and \( k_x \) and \( k_y \) are the wavenumbers in the \( x \) - and \( y \) -directions, respectively. For simplicity, instead of linearizing the energy equation, we assume an isothermal perturbation for the gas,

\[
\delta P_g = C^2_s \delta \rho,
\]

where \( C_s \) is the isothermal sound velocity.

Although there are nine perturbed quantities, it turns out that there are seven algebraic relations among them, if we assume no cross--field-line diffusion of CRs. First of all, the induction equation gives \((\delta B_x, \delta B_y, \delta B_z)\) in terms of \((\delta \rho, \delta V_x, \delta V_y, \delta V_z)\). The \( x \) - and \( y \) -momentum equations then give \((\delta V_x, \delta V_y)\) in terms of \((\delta \rho, \delta V_z, \delta P_g)\), where \( \delta P_t = \delta P_g + \delta P_e + B_x \delta B_y \). The nice consequence of no cross--field-line diffusion is that \( \delta P_e \) can be written in terms of \((\delta \rho, \delta V_z)\) (Note that the diffusion coefficient is also perturbed, because it depends on the direction of the magnetic field.) Thus, \( \rho \) can be written in terms of \((\delta V_z, \delta P_t)\). After some algebra, the continuity equation and the \( z \) -momentum equation become

\[
\frac{d}{dz} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = A_{11} \begin{bmatrix} A_{12} \\ A_{21} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix},
\]

where

\[
y_1 = \rho \delta V_z,
\]

\[
y_2 = \sigma \delta P_t = \sigma (\delta \rho + \delta P_e + \frac{B_x \delta B_y}{4\pi}),
\]

\[
K = \frac{\kappa k^2_s}{\sigma}, \quad \Sigma^2 = \sigma^2 + V^2_A k^2_s, \quad \Gamma = \frac{2\omega \Omega}{\Sigma}, \quad C^2_s = \frac{\gamma \rho c^2}{\rho},
\]

\[
D = 1 + \frac{C^2_s}{1 + K} + V^2_A \left[ \frac{\sigma^2}{\Sigma^2} + \frac{V^2_A k^2_s \Gamma^2}{\Sigma^2(1 + \Gamma^2)} \right],
\]

\[
A_{11} = \left[ 1 - \frac{\sigma^2}{\Sigma^2} - \frac{V^2_A k^2_s}{\Sigma^2(1 + \Gamma^2)} \left( \Gamma^2 + \frac{k_x}{k_{sx}} \right) \right] \frac{d \ln \rho}{dz}
\]

\[
\times \frac{1}{D} \left[ \frac{\sigma^2}{\Sigma^2} + \frac{V^2_A k^2_s}{\Sigma^2(1 + \Gamma^2)} \left( \Gamma^2 + \frac{k_x}{k_{sx}} \right) \right]
\]

\[
\times \left( g_x + \left[ 1 + \frac{C^2_s}{1 + K} + V^2_A \left[ \frac{\sigma^2}{\Sigma^2} + \frac{V^2_A k^2_s \Gamma^2}{\Sigma^2(1 + \Gamma^2)} \right] \right] \frac{d \ln \rho}{dz} \right),
\]

\[
A_{12} = \left( -\frac{k_x^2 + k_y^2}{\Sigma^2(1 + \Gamma^2)} - \frac{1}{D} \left[ \frac{\sigma^2}{\Sigma^2} + \frac{V^2_A k^2_s}{\Sigma^2(1 + \Gamma^2)} \left( \Gamma^2 + \frac{k_x}{k_{sx}} \right) \right] \right)
\]

\[
\times \left[ \frac{\sigma^2}{\Sigma^2} + \frac{V^2_A k^2_s}{\Sigma^2(1 + \Gamma^2)} \left( \Gamma^2 - \frac{k_y}{k_{sy}} \right) \right],
\]

\[
A_{21} = -\frac{\sigma^2}{\Sigma^2} + \frac{g_x}{D} \left[ 1 + \frac{C^2_s}{1 + K} + V^2_A \left[ \frac{\sigma^2}{\Sigma^2} + \frac{V^2_A k^2_s \Gamma^2}{\Sigma^2(1 + \Gamma^2)} \right] \right] \frac{d \ln \rho}{dz},
\]

\[
A_{22} = -\frac{g_x}{D} \left[ \frac{\sigma^2}{\Sigma^2} + \frac{V^2_A k^2_s \Gamma^2}{\Sigma^2(1 + \Gamma^2)} \right] \frac{d \ln \rho}{dz}.
\]

We solve equation (10) to find the eigenmodes with given boundary values. Consequently, the problem converges to a boundary value problem, and the growth rate of the perturbation is obtained as an eigenvalue.

#### 3.2. Boundary Conditions

We assume that the disk is symmetric with respect to the midplane \( (z = 0) \). Under this assumption, the sign of \( \delta V_z \) inverts beyond the midplane; on the other hand, the sign of \( \delta P_t \) should not change. Hence, the perturbed values of \( y_1 \) and \( y_2 \) should be antisymmetric and symmetric about \( z = 0 \), respectively. On the other hand, the matrix \( A \) in equation (10) nearly equals a constant in the region \( z \gg H \), and the WKB solution is applicable. Then, the asymptotic solutions, under the condition that \( y_1 \) and \( y_2 \) should vanish at large \( z \), are written as follows (e.g., Horiuchi et al. 1988):

\[
y_1 = \exp(i \lambda z),
\]

\[
(\lambda - A_{11}) y_1 = A_{12} y_2,
\]

where

\[
\lambda = \frac{1}{2} \left[ A_{11} + A_{22} - \sqrt{(A_{11} - A_{22})^2 + 4 A_{12} A_{21}} \right].
\]

We solve the set of the two linear differential equations (eq. [10]) by the shooting method. We integrate equation (10) from the outer boundary, by using the condition given by equations (19) and (20), to the inner boundary \( (z = 0) \) to obtain a trial value of \( \sigma \). This \( \sigma \) is regarded as an eigenvalue when the value of \( y_1 \) is small enough at \( z = 0 \).
3.3. Result of Linear Stability Analysis

In this analysis, we take the value of the CR diffusion coefficient $k$, the ratio of the CR pressure to the gas pressure $\beta$, and the rotational angular frequency $\Omega$ as parameters. The value of $k$ is estimated as $\sim 3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$ (Berezinskii et al. 1990; Ptuskin 2001; Ryu et al. 2003). In our units, $3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$ corresponds to 200. We thus take $k = 200$, $\beta = 1$, and $\Omega = 0$ as a fiducial case, and the ratio of the magnetic pressure to the gas pressure is taken as $\alpha = 1$ (initially).

Figure 2 (left) shows the dispersion relations for different $k$-values, where $\sigma$ is the growth rate and $k_x$ is the wavenumber along the direction of the magnetic field. In this calculation we set $\beta = 1$ and $\Omega = 0$. The growth rate becomes smaller as $k$ decreases from the fiducial value, $k = 200$. This result matches well the one given by Ryu et al. (2003). The maximum growth rate $\sigma_{\text{max}}$ for different $k$ occurs at roughly the same $k_x$. Figure 2 (right) shows the dependence of the maximum growth rate $\sigma_{\text{max}}$ on $k$ for different $k$.

Figure 3 (left) shows the dispersion relations for different $\beta$-values, where $\sigma$ is the growth rate and $k_x$ is the wavenumber along the direction of the magnetic field. In this calculation we set $k = 200$. As $\beta$ increases, the growth rate of long-wavelength perturbations decreases rapidly, and $k_x$ becomes larger. Figure 3 (right) shows the dependence of $\sigma_{\text{max}}$ on $\beta$ for different $\beta$.

4. TWO-DIMENSIONAL MHD SIMULATION

4.1. Numerical Procedure and Boundary Conditions

We solve the set of two-dimensional, nonlinear, time-dependent, compressible ideal MHD equations, supplemented with the CR energy equation (eqs. [1]–[5]), in Cartesian coordinates. We use the modified Lax-Wendroff scheme with artificial viscosity for the MHD part and the biconjugate gradients stabilized (BiCGstab) method for the diffusion part of the CR energy equation in the same manner as described in Yokoyama & Shibata (2001). The MHD code using the Lax-Wendroff scheme was originally developed by Shibata (1983).
Fig. 4.—Left: Dispersion relation for the Parker instability with the effect of CRs at different $\Omega$, where $\Omega$ is the rotational angular velocity of the disk. Right: Dependence of the maximum growth rate $\sigma_{\text{max}}$ on $\beta$ for different $\Omega$.

Fig. 5.—Time evolution of the CR pressure distribution, magnetic field lines, and velocity vectors in the models with $\kappa_\parallel = 200$ (top), 40 (middle), and 10 (bottom). The gray scale, the white curves, and the white vectors show the CR pressure distribution, magnetic field lines, and velocity vectors, respectively. The units of length and time are 50 pc and $10^6$ yr.
and has been extended by Matsumoto et al. (1996) and Hayashi et al. (1996). To test the part of the code relating to diffusion, we did a test run on a simple CR diffusion problem (see the Appendix).

We calculate only the region over the midpoint of the galactic disk in the \(x-z\) plane, as shown in Figure 1. The \(x\)-direction corresponds to the azimuthal direction, and the \(z\)-direction is in the direction of the rotational axis of the galactic disk. The partial derivative \(\partial / \partial y\) is neglected. We use the units in \(\S\) 2.2 for our simulations, and the equilibrium model in \(\S\) 2.2 is used as the initial equilibrium background. We then apply two types of perturbation, mechanical perturbations and explosive perturbations. In the case of mechanical perturbations, we add small velocity perturbations of the form

\[
v_x = 0.05 C_{s,0} \sin \left( \frac{2\pi x}{\lambda} \right)
\]

within the finite rectangular region of \(0 < x < \lambda / 2\) and \(4H_0 < z < 8H_0\), where \(\lambda = 20H_0\) is the horizontal wavelength of the small velocity perturbation, which is nearly equal to the most unstable wavelength derived from the linear analysis in the \(k_\parallel = 200\) case. The size of the simulation box is \((x_{\text{max}} \times z_{\text{max}}) = (80H_0 \times 187H_0)\), the number of grid points is \((N_x, N_z) = (101, 401)\), and the grid size is \(\Delta x = 0.8H_0\) and \(\Delta z = 0.15H_0\) when \(0 \leq z \leq 25.0H_0\) and increases with \(z\) otherwise. We assume a symmetric boundary for \(x = 0\) and \(z = 0\) and a free boundary for \(x = x_{\text{max}}\) and \(z = z_{\text{max}}\). On the other hand, in the explosive perturbation case, the CR energy (\(\sim 10^{50}\) ergs) is put into a cylindrical region placed at \((x, z) = (0, 6H_0)\) with volume \(V_{\text{exp}} = \pi r_{\text{exp}}^2 H_{\text{exp}}\), where \(r_{\text{exp}} = 25\) pc = 0.5\(H_0\) and \(y_{\text{exp}} = 50\) pc = \(H_0\). The size of the simulation box is \((x_{\text{max}} \times z_{\text{max}}) = (90H_0 \times 187H_0)\), the number of grid points is \((N_x, N_z) = (301, 401)\), and the grid size is \(\Delta x = \Delta z = 0.15H_0\) when \(0 \leq x \leq 35H_0\) and \(0 \leq z \leq 25H_0\) and increases with \(x\) and \(z\) otherwise. We assume symmetric boundaries for \(x = 0\) and \(z = 0\) and a free boundary for \(x = x_{\text{max}}\) and \(z = z_{\text{max}}\).

4.2. Numerical Results for the Mechanical Perturbation Case

In this subsection we show the results for the mechanical perturbation case. In order to examine the effect of the CR diffusion coefficient on the instability, we consider three different diffusion coefficients: \(k_\parallel = 10, 40, 200\). As can be seen from Figure 2, \(k_\parallel = 10, 40, 200\) correspond to a small, medium, and high growth rate, although the difference between the growth rates corresponding to \(k_\parallel = 40\) and 200 is small.

Figure 5 shows the time evolution of the CR pressure distribution, the magnetic field lines, and the velocity vectors for the \(k_\parallel = 200, 40,\) and 10 models. The gray-scale contour shows the CR pressure distribution, white curves show the magnetic field lines, and white vectors show the velocity vectors, where the white arrow at the upper right corner shows the reference velocity vector (\(5C_{s,0}\)). The middle column shows the results of the three models (\(k_\parallel = 200, 40,\) and 10) at the end of their linear growth (i.e., \(t = 28, 30,\) and 36, respectively). These times are decided from Figure 6, which shows the time evolution of \(V_x\) in each model. The initial perturbations grow to form the characteristic loop-like structures. In the linear phase, the form of the magnetic loop is the same in each model. In contrast, in the nonlinear phase, its form can be very different. In the \(k_\parallel = 200\) model, the shape of the magnetic loop is a beautiful omega shape. In the \(k_\parallel = 40\) model, the omega shape is distorted in the outer region, and a double-loop forms in the inner region. In the \(k_\parallel = 10\) model, the major feature is the double loop.

In order to compare the results of the MHD simulation and the linear analysis, we examine the temporal variation of \(V_x\) at a particular point. Figure 6 shows the time evolution of \(V_x\) at \((x, z) = (5.6, 7.95)\).

The solid curves show the growth rate of \(V_x\) (normalized to the sound velocity \(C_{s,0}\)) in each model: \(k_\parallel = 200, 40,\) and 10. The dashed lines, \(L1\) for \(k_\parallel = 200, L2\) for \(k_\parallel = 40,\) and \(L3\) for \(k_\parallel = 10\), show the growth rate obtained from the linear analysis. The dash-dotted line shows the initial Alfvén speed. The inclination obtained from the linear analysis agrees well with that obtained from the MHD simulation. The speed reaches the Alfvén speed and is saturated in the \(k_\parallel = 200\) model. On the other hand, the speed is saturated below the Alfvén speed in the \(k_\parallel = 40\) and 10 models, and the saturated speed decreases as \(k_\parallel\) decreases.

In Figure 7 the three panels in the first row show the CR pressure distribution (gray-scale contour), velocity vectors (white arrows), and a magnetic field line (white curve) at the end of the linear growth phase for the three models: \(k_\parallel = 200\) at \(t = 28t_0\) (Fig. 7, left panels), \(k_\parallel = 40\) at \(t = 30t_0\) (Fig. 7, middle panels), and \(k_\parallel = 10\) at \(t = 36t_0\) (Fig. 7, right panels). The arrow at the upper right corner shows a reference velocity vector equal to 5 times a unit velocity vector. As the value of \(k_\parallel\) decreases, the expansion speed of the magnetic loop becomes slower. The three panels in the second row show the CR pressure values along the magnetic field line depicted in the first row. In the \(k_\parallel = 200\) model, the CR pressure distribution in the magnetic loop (\(x < 15\)) is nearly uniform. As \(k_\parallel\) decreases, the CR pressure profile develops some structures. In addition, as \(k_\parallel\) decreases, the CR pressure decreases at the top of the loop (\(x \approx 0\)) but increases at the footpoint of the loop. The horizontal axis “\(L\)” is the distance along the magnetic field depicted in the first row, where \(L = 64.5\) (\(k_\parallel = 200\), \(61.1\) (\(k_\parallel = 40\), and \(57.8\) (\(k_\parallel = 10\)) at \(x = 50\). The three panels in the third row show the density distributions, \(\log_{10}(\rho/\rho_0)\), along the magnetic field line depicted in the first row. The distributions are very similar in each model, but we can recognize the difference of the distribution in the magnetic loop part. In the \(k_\parallel = 200\) model, the density in the loop is...
Fig. 7.—CR pressure, density, absolute velocity, and speed along the magnetic field line depicted in the first row at the end of linear growth in the models with $\kappa_\parallel = 200$ (left panels), 40 (middle panels), and 10 (right panels), where $L$ is the distance along the magnetic field line.
In each model, \( V_z \) becomes maximum at the position of the magnetic loop near the end of the linear growth, \( t \approx 29 \) for \( \kappa_{||} = 200 \), \( t \approx 31 \) for \( \kappa_{||} = 40 \), and \( t \approx 36 \) for \( \kappa_{||} = 10 \). Subsequently, \( V_z \) becomes large in the halo region, because the halo is pushed upward by the growing magnetic loops. The growth speed of the magnetic loop finally falls, and the growth is impeded similarly to the case without the effect of CRs (Kato et al. 1998, p. 475).

4.3. Numerical Results for the Explosive Perturbation Case

In this subsection we show the results for the explosive perturbation case. In this case we set a high CR pressure region at \((x, z) = (0, 6)\) with radius \(0.5H_0\) as our initial perturbation.

Figure 9 shows the time evolution of the CR pressure distribution \((\text{gray-scale contour})\), the magnetic fields \((\text{white curves})\), and the velocity vectors \((\text{white arrows})\) for the models \(\kappa_{||} = 10\) (Fig. 9, top) and 80 (Fig. 9, bottom). It is recognized that the growth speed of the magnetic loop in the \(\kappa_{||} = 10\) model is faster than that of the \(\kappa_{||} = 80\) model at \(t = 12\), contrary to the result for the mechanical perturbation case. Subsequently, at \(t = 24\), the growth of the magnetic loop in the \(\kappa_{||} = 80\) model overtakes the \(\kappa_{||} = 10\) model. The expansion speed of the magnetic loop becomes very slow in the \(\kappa_{||} = 10\) model, while it is still very fast in the \(\kappa_{||} = 80\) model.

In Figure 10 the two panels in the first row show the initial CR pressure distribution \((\text{gray-scale contour})\) and an initial magnetic field line \((\text{white line})\) in the models \(\kappa_{||} = 10\) (left panels) and 80 (right panels). The two panels in the second row show the time evolution of the CR pressure distribution along the magnetic field line depicted in the first row. The field line threads through the explosion region. The dotted line shows the initial distribution of the CR pressure, which is highly localized near \(x = 0\). In the \(\kappa_{||} = 10\) model, the high CR pressure region is localized for a relatively long time, because the diffusion speed is slow. The initially localized high CR pressure pushes the matter in the \(x\)-direction along the magnetic field lines rather effectively in the case of strong coupling (i.e., small diffusion coefficient). Thus, the density drops rather rapidly, while the CR pressure decreases slowly in the initial phase \((t < 2.5)\). When the magnetic loop penetrates into the low CR pressure region \((t > 2.5)\), the CR pressure inside the loop diminishes faster, partly because the magnetic tube has expanded and partly because the CR is carried by the downward flow of the matter. As matter accumulates at the footpoint of the magnetic loop, the magnetic tube becomes thinner, and the CR pressure builds up, because of the small \(\kappa_{||}\). When a significant CR pressure gradient has been built up against the infall of matter, the growth of the instability slows down. This can be confirmed by the small differences of the density and the CR pressure distributions between \(t = 20\) and 24. In the \(\kappa_{||} = 80\) model, diffusion is more important. The high CR pressure region disappears as CRs diffuse along the magnetic field. At \(t = 15\) the CR pressure is rather uniform throughout the whole region. The CR pressure gradient between the top of the loop and the footpoint is significant only after \(t = 24\), and the growth of the instability will not slow down until then. The two panels in the third row show the time evolution of the density distribution along the magnetic field line depicted in the first row. The field line threads through the explosion region. In the \(\kappa_{||} = 10\) model, matter is drained rapidly by the CR pressure gradient, and a large drop in density occurs near the top of the magnetic loop in a short time (at \(t = 2.5)\). As time proceeds,
the matter accumulates at the footpoint of the loop, where a high density region is formed. The draining rate of matter near the top of the loop reduces. In the $\kappa_{\parallel} = 80$ model, the density near the top of the magnetic loop decreases very slowly, until about $t = 15$. After that, the draining rate accelerates, and the density near the top of the loop becomes smaller than that of the $\kappa_{\parallel} = 10$ model at $t = 24$. The two panels in the last row show the CR pressure distribution, the velocity vectors, and the magnetic field line at $t = 24$. We should point out that, in order to emphasize the CR pressure distribution near the depicted magnetic field line, the gray scale used in Figure 10 is different from that used in Figure 9.

5. SUMMARY AND DISCUSSION

Using linear analysis and a time-dependent nonlinear calculation, we studied the Parker instability (or magnetic buoyancy instability) with the effect of CRs. Several works on linear analysis of the Parker instability with the effect of CRs have been published (e.g., Hanasz & Lesch 1997; Ryu et al. 2003). In Hanasz (1997) the CR energy equation (including diffusion) was not solved. In Ryu et al. (2003) the effect of rotation was not included, and only two cases of CR pressure were described (one was without CR, $\beta = 0$, and the other was with equal unperturbed CR and gas pressures, $\beta = 1$). Since Ryu et al. (2003) showed that the effect of cross–field-line diffusion of CRs is negligible in the context of the ISM, we neglected the effect of cross–field-line diffusion in our analysis for simplicity.

In the linear analysis, the growth rate becomes larger as the CR diffusion coefficient $\kappa_{\parallel}$ along the field line increases, and is saturated at large $\kappa_{\parallel}$. This result is consistent with the result by Ryu et al. (2003). The growth rate also becomes larger when the initial ratio $\beta$ of the CR pressure to the gas pressure increases, and is saturated at large $\beta$. This is consistent with Ryu et al. (2003), except for some slight differences. In our results, the maximum growth rate of the normal Parker case ($\beta = 0$) is almost half of the fiducial case ($\beta = 1$), and the critical wavenumber, over which the instability is stabilized, of the normal Parker case is about 0.7 times that of the fiducial case. In Ryu et al. (2003) the maximum growth rate of the normal Parker case is less than half of the $\beta = 1$ case, and the critical wavenumber of the normal Parker case is about half of the $\beta = 1$ case. The differences perhaps come from how the normalization was taken. In fact, we succeeded in producing their results by taking the same scale height under the same equilibrium condition. The scale height of the disk, $H = (1 + \alpha + \beta)C_{\perp,0}^{2}/(\gamma_{g}g_{d})$, changes with the values of $\alpha$ and $\beta$, and it takes the value $H = 2C_{\perp,0}^{2}/(\gamma_{g}g_{d})$ in the normal Parker case ($\alpha = 1, \beta = 0$), and $H = 3C_{\perp,0}^{2}/(\gamma_{g}g_{d})$ in the fiducial case ($\alpha = \beta = 1$). We allowed the change of the scale height because we preferred not to change the gravitational acceleration. This is the reason why we took the unit of length as
The effect of the rotation stabilizing the Parker instability is similar to that of the case without CRs. The $k_{x,\max}$ increases as $\Omega$ increases. Our result for the effect of rotation is consistent with that by Hanasz & Lesch (1997), except for the difference in the region of small wavenumber. The growth rate with rotation is small near $k_x = 0$. It increases linearly with the wavenumber in Hanasz & Lesch (1997). However, in our result the growth rate increases faster than linearly, and this is also observed in the normal Parker case (see Kato et al. 1998, p. 475).

In the MHD simulation, we showed that the growth rate of the instability becomes smaller when the diffusion coefficient $\kappa||$ becomes smaller, which agrees well with the result of the linear analysis. At the end of the linear growth, the morphology of the magnetic loop developed from the initial perturbation is more or less the same in the three models ($\kappa|| = 200, 40, \text{and} 10$) studied in the mechanical perturbation case. However, in the nonlinear stage, the magnetic loop in different models develops into different morphologies. From the distribution of CR pressure, density, and velocity along a magnetic field line at the end of linear growth, we found several characteristics. In the case of small diffusion coefficient (e.g., $\kappa|| = 10$, i.e., the coupling between the CRs and the gas is strong), the CR pressure distribution is rather non-uniform. CRs tend to accumulate near the footpoint of the magnetic loop, and the CR pressure gradient force toward the top of the loop becomes larger. The falling motion of matter is then impeded by the CR pressure gradient force, and the growth rate of the Parker instability decreases. In the case of a large diffusion coefficient (e.g., $\kappa|| = 200, 40$), the falling speed of matter along the magnetic field line exceeds the speed of sound, and a shock is formed near the footpoint of the

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**Fig. 10.** Time evolution of the CR pressure distribution, and the density distribution along a magnetic field line initially threading the region where the CR energy was injected. *Left panels*: $\kappa|| = 10$. *Right panels*: $\kappa|| = 80$. 

$H_0 = C^{2}_{\Omega,0} / (\gamma g_2)$. The effect of the rotation stabilizing the Parker instability is similar to that of the case without CRs. The $k_{x,\max}$ increases as $\Omega$ increases. Our result for the effect of rotation is consistent with that by Hanasz & Lesch (1997), except for the difference in the region of small wavenumber. The growth rate with rotation is small near $k_x = 0$. It increases linearly with the wavenumber in Hanasz & Lesch (1997). However, in our result the growth rate increases faster than linearly, and this is also observed in the normal Parker case (see Kato et al. 1998, p. 475).
magnetic loop. Moreover, the CR pressure distribution along the magnetic field line in the cases of large diffusion coefficient (e.g., $\kappa_\parallel = 200$) reminds us of the profile of CR pressure in CR-modified shocks (e.g., Drury & Völk 1981; Ko et al. 1997). The linear growth rate in the simulations agrees well with that in the linear analysis. We also found that the speed along the disk is saturated at the initial Alfvén speed. This result agrees with that in the normal Parker instability (i.e., without CRs; Matsumoto et al. 1988). The unperturbed state has the scale height $H = (C_s^2 + \beta C_s^2 + v_A^2)/2g_z$. When the Parker instability takes place, the gas falls down along the magnetic field lines. The CR pressure tends to distribute uniformly along the magnetic field line (at least in the case of large diffusion coefficient), and its contribution to the scale height disappears. Thus, the scale height along the magnetic field lines settles to $H_0 = C_s^2/g_z$ at later times. The released gravitational energy in the form of kinetic energy per unit mass is estimated as $V_A^2/2$. Hence, we obtain the same results as the normal Parker case, even when the effect of CRs is included.

The explosive perturbation case has been studied by Hanasz & Lesch (2001). They stated that the smaller the diffusion coefficient, the larger the growth rate of the instability. This trend is the opposite of what we found from linear analysis and simulation in the mechanical perturbation case. We thus computed the explosive perturbation case for a longer time. Our result showed that the growth rate is larger in the smaller diffusion coefficient model only in the early stage. The growth rate becomes smaller when compared to that of the large diffusion coefficient model in the later stage. The growth of instability is suspended by the CR pressure gradient force interfering with the falling motion of the matter in the small-$\kappa_\parallel$ model, while the magnetic loop can grow up to larger scales in the large-$\kappa_\parallel$ model.

Numerical computations were carried out on the VPP5000 at the National Astronomical Observatory, Japan. T. K. and C. M. K. are supported in part by the National Science Council, Taiwan, Republic of China, under the grants NSC-91-2112-M-008-006, NSC-90-2112-M-008-020, and NSC-91-2112-M-008-050.

APPENDIX

TEST CALCULATION

In this appendix we show the result of a simple CR diffusion problem to test the diffusion part of our numerical code. We solved the following diffusion equation (i.e., CR energy eq. [4] with $V = 0$):

$$\frac{\partial E_c}{\partial t} = \nabla \cdot (\kappa \mathbf{b} \cdot \nabla E_c),$$

where $E_c$ is the CR energy and $\kappa_\parallel$ is the diffusion coefficient along the magnetic field. We considered a uniform magnetic field with an $x$-component only, i.e., $\mathbf{B} = (B_x, 0)$. The test calculation itself is two-dimensional in the $x$-$z$ plane, but the content is the same as a one-dimensional calculation in the $x$-direction, because we just considered constant $\kappa_\parallel$, and the initial condition depended on $x$ only. We took the same initial condition as that used in Hanasz & Lesch (2003):

$$E_{c,0} = A \exp \left(-\frac{x^2}{w_0^2}\right),$$

where $x = B_x t$. Fig. 11.—Test result of a simple CR diffusion problem.
where $w_0 = 50^{1/2}$ is the initial half-width of the Gaussian profile, $x_p$ is the distance from the central point of the calculation region, and $A = 10$ is the value at $x_p = 0$. The number of grid points used in this calculation is $(N_x, N_z) = (400, 100)$, and $k_{||} = 100$. Figure 11 shows the initial distribution and the distribution at $t = 9.4$. In Figure 11 (right) ($t = 9.4$), the solid curve shows the analytical solution, and the squares show the numerical solution. The numerical solution completely matches the analytical solution.

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