Energy-efficient Scheduling for Multistage Manufacturing System with Parallel Machines

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Abstract. Increasing energy is prompting academicians to pay attention to energy efficient scheduling. This study establishes a mathematical model for multistage manufacturing system with parallel machines to minimize total energy consumption. A memetic algorithm based on differential evolution (MABDE) is developed to solve the model effectively. Two rules are incorporated into the proposed algorithm to narrow down the search space and improve search efficiency. Computational results demonstrate the effectiveness of the proposed model and algorithm.

1. Introduction

For the manufacturers, introducing power-efficient equipment [1] or designing embodied energy framework can be practical method for decreasing carbon footprints and energy consumption [2]. But compared to the aforementioned method, energy-efficient scheduling (EES) requires less capital investment [3].

In this paper, EES is incorporated into multistage manufacturing system (MMS) with parallel machines aiming at minimizing the total energy consumption. MMS with parallel machines, also known as hybrid flow shop scheduling problem (HFSP), is a generalization of the traditional flow shop scheduling problem (FFSP). Regarding FFSP with EES, Fang et al. [3] proposed a mathematical model for FSP with peak power load and verified the model by applying a case study. Luo et al. [4] presented an ant colony optimization for HFSP considering electricity consumption cost. Lin et al. [5] presented an integrated model for processing parameter optimization and HFSP, and adopted a teaching-learning-based optimization (TLBO) for minimizing make span and carbon footprint. Ding et al. [6] devised a modified multi-objective iterated greedy algorithm for a permutation flow shop scheduling with carbon emission. Lei et al. [7] developed a novel TLBO algorithm for HFSP taking into account the scheduling, machine assignment, and speed selection.

However, the above-mentioned literature did not consider the energy consumption of processing different types of products and that when machines are idle, which are the very issue covered in this study. Moreover, to solve this problem, a MABDE is proposed. MABDE incorporates into the rule of first available equipment (RFAE) and minimal energy consumption equipment (RMECE) to reduce the search space and improve search efficiency.

2. Problem Description
The problem can be depicted as follows. There are \( N \) jobs that need to be processed at \( M \) stages sequentially from stage 1 to stage \( M \). The \( m \)th stage consists of \( M_m \) parallel machines, and \( M_m \geq 1 \) at each stage. Each job \( n \) consists of a series operation \( O_{mn} \), \( m=1,2,\ldots,M \). The actualization of \( O_{mn} \) needs one machine out of the machine set \( M_m \) at stage \( M \). \( L_m=\{l_m\} \) is a random processing sequence of all jobs at the \( m \)th stage, where \( l_m \) denotes that the job’s processing order is \( l_m \) at stage \( m \). \( B_{nmj} \) is the processing time of \( O_{mn} \) on machine \( j \). \( B_{nmj} \) is the start time of \( O_{mn} \) on machine \( j \) at the stage \( m \), while \( F_{nmj} \) is the completion time of \( O_{mn} \) on the machine \( j \) at stage \( m \). \( EC_{nmj} \) denotes the unit energy consumption of \( O_{mn} \) on machine \( j \) at stage \( m \), and \( EL_{nmj} \) represents the unit energy consumption in unit time when machine \( j \) is idle at stage \( m \).

To formulate the model, the following assumptions are considered: The process is not interrupted; Each machine can process only one job at a time; and each job can be processed on only one machine at a time. Each job is independent of each other and can be processed at initial time; The processing time of each job on each machine is known; Transportation times of job between stages and unloading time of job on machine are neglected; The failure of machines is not considered throughout the production process, and the buffer capacity between adjacent stages is unlimited.

3. Mathematical Model

To formulate the problem clearly, two binary variables are introduced. \( x_{slm}=1 \) denotes that \( O_{mn} \) is processed in the \( l \)th order at stage \( m;0 \) otherwise. \( y_{nmj}=1 \) denotes that \( O_{mn} \) is processed on machine \( j \) at stage \( m;0 \) otherwise. The mathematical model can be formulated as follows.

\[
\text{Objective } \min \ EA = ECA + EIA
\]

s.t.

\[
\sum_{l=1}^{N} x_{nlm} = 1, \ n = 1,2,\ldots,N
\]

(2)

\[
\sum_{n=1}^{N} x_{nlm} = 1, \ l_m = 1,2,\ldots,N
\]

(3)

\[
\sum_{j=1}^{m} y_{nmj} = 1, \ n = 1,2,\ldots,N; \ m = 1,2,\ldots,M
\]

(4)

\[
F_{nmj} = B_{nmj} + P_{nmj}, n = 1,2,\ldots,N; \ m = 1,2,\ldots,M; \ j = 1,2,\ldots,M_m
\]

(5)

\[
F_{nmj} \leq B_{(d_{nm}+1)j}, n = 1,2,\ldots,N; \ m = 1,2,\ldots,M-1; j = 1,2,\ldots,M_m; j = 1,2,\ldots,M_{m+1}
\]

(6)

\[
\sum_{s=1}^{N} x_{slm} B_{sij} \leq \sum_{s=1}^{N} x_{slm} B_{sij} / l_m = 1,2,\ldots,N-1; j = 1,2,\ldots,M_m
\]

(7)

\[
\sum_{s=1}^{N} x_{slm} y_{nmj} F_{nmj} \leq \sum_{s=1}^{N} x_{slm} y_{nmj} B_{nmj} + L \left( 1 - \sum_{s=1}^{N} x_{slm} y_{nmj} \right),
\]

(8)

\[
I_{l_m}, l_m = 1,2,\ldots,N; l_m \leq l_{m'}, m = 1,2,\ldots,M; j, j' = 1,2,\ldots,M_m
\]

\[
ECA = \sum_{s=1}^{N} \sum_{m=1}^{M_m} \sum_{j=1}^{M_m} y_{nmj} EC_{nmj}, n = 1,2,\ldots,N; m = 1,2,\ldots,M; j = 1,2,\ldots,M_m
\]

(9)

\[
EIA = \sum_{s=1}^{N} \sum_{m=1}^{M_m} \sum_{j=1}^{M_m} \left( F_{sj} - \sum_{n=1}^{N} y_{nmj} B_{nmj} \right) \times EL_{nmj}, n = 1,2,\ldots,N; m = 1,2,\ldots,M; j = 1,2,\ldots,M_m
\]

(10)

\[
x_{slm} = 0,1, n,l_m = 1,2,\ldots,N; x_{nmj} = 0,1, n = 1,2,\ldots,N; m = 1,2,\ldots,M; j = 1,2,\ldots,M_m
\]

(11)

The objective function (1) minimizes the energy consumption. Constraint (2) indicates that the job \( n \) is processed once at the \( m \)-th stage. Constraint (3) represents that there is only one job at the \( l_m \)-th workstation at the \( m \)-th stage. Constraint (4) ensures that the job \( n \) is processed by only one machine at the \( m \)-th stage. Constraint (5) denotes the completion time of job \( n \) at the \( j \)-th machine at the \( m \)-th stage. Constraint (6) involves job-sequencing at the same machine. Constraint (7) indicates that the jobs scheduled in front takes precedence over the later jobs to begin processing at the first stage. Constraint
guarantees that when the jobs are scheduled on a machine in the same stage, the later job must start after the completion of former job, where L is a large number. Constraint (9) indicates the total energy consumption when all the jobs are processed in whole stages. Constraint (10) represents the total energy consumption when the machines are idle in the process of production. Constraint (11) defines the domain of decision variables.

4. MABDE Algorithm

In this paper, differential evolution is applied to the mutation of the memetic algorithm and greedy algorithm is used for local search, thus solving the problem of hybrid flow shop energy scheduling problem.

4.1. Coding and Decoding

Integer coding is used for the problem, and a chromosome represents a solution. For instance, there are five jobs to be processed in the system, a chromosome \{2,5,1,3,4\} represents that the processing order is job2→job5→job1→job3→job4 in the first stage. The first come first serve (FCFS) rule is applied in the following stage. Additionally, the rules of combining the available machine with the minimum energy consumption is applied as follows.

- **RFAE.** When a job selects a machine at a certain stage, the machine that can start the next job earliest is preferred.
- **RMECE.** When multiple machines satisfy the RFAE, the machine with minimal energy consumption is selected.

4.2. Population Initialization

Let PS be the population size, the initial population is randomly generated for two accounts: one is to avoid precocity, the other is to spread the initial solution into the whole solution space to the maximum extent.

4.3. Evaluation Fitness

For the \(p\)-th chromosome \(Chrom_{p}\), a boundary construction method is applied to convert objective function value \(EA\) to fitness shown as follows.

\[
Fit(f(p)) = \begin{cases} 
1 - EA(Chrom_{p}) & \text{if } EA(Chrom_{p}) \leq I, \\
0 & \text{otherwise}
\end{cases}
\]

where I is a sufficiently positive value.

4.4. Mutation Mechanism

An energy differential evolution (EDE) combined with the traditional insertion mutation is adopted to keep the excellent meme fragments as far as possible in the mutation process. The EDE can be defined as follows.

\[
\Delta_{EA}(k) = EA(posi_{k}, posi_{k-1}, \ldots, posi_{1}) - EA(posi_{k}, posi_{k-1}, \ldots, posi_{k-1}), 2 \leq k \leq N
\]

\[
posi_{mu} = \text{arg max} \{\Delta_{EA}(k)\}, 2 \leq k \leq N
\]

where \(posi_{k}\) indicates the \(k\)-th meme of chromosome \(Chrom_{p}\), and \(posi_{mu}\) denotes meme location in which the mutation occurs. The \(posi_{mu}\) is defined as the bottleneck location of energy consumption.

4.5. Crossover Mechanism

Due to the job sequence-based coding, a generalized order crossover (GOX) is adopted to ensure that the chromosomes are in the solution space after crossover. The specific steps are as follows.

Step 1: two paternal chromosomes \(Chrom_{p1}\) and \(Chrom_{p2}\) are randomly selected from population.

Step 2: a meme fragment with length \(CL\) is selected from \(Chrom_{p1}\) and placed it to the first \(CL\) location of offspring chromosome \(Chrom_{p1}\).
Step 3: another meme fragment that is different from that from step 2 is selected from \( \text{Chrome}_{p_2} \), and fill it successively into the location of offspring chromosome \( \text{Chrome}_{p_1} \) from \( CL+1 \) to \( N \) to generate the offspring chromosome \( \text{Chrome}_{p_1} \).

The generation of another offspring chromosome \( \text{Chrome}_{p_2} \) is the same as \( \text{Chrome}_{p_1} \).

### 4.6. Selection Mechanism

The roulette-wheel method is adopted to select the chromosome. It uses a probability distribution for selection in which the selection probability of a given schedule is proportional to its fitness.

### 4.7. Framework of the Proposed MABDE

Based on the previous illustration, the overall framework of the proposed MABDE is provided by Figure 1.

![Figure 1. The overall framework of the proposed MABDE](image)

### 5. Experimental results and analysis

To evaluate the performance of the proposed MABDE algorithm, this section provides an experimental evaluation of its performances and a comparative study with differential evolution (DE) and genetic algorithm (GA), considering significant benchmarking instances of different dimensions. All the algorithms are coded in MATLAB (R2014a) and run on a PC with 2.6 GHz and 4 GB of RAM memory, and Windows 10.

#### 5.1. Experimental setup

Two different scale instances are generated randomly. A small-scale instance is shown in Table 1. For the medium instance, 12 jobs \( (N=12) \) need to be processed in four stages \( (M=4) \). The alternative machines in each stage is \( M_1=3, M_2=3, M_3=2, M_4=2 \). \( \forall n \in N, m \in M \). \( P_{n mj} \) is generated as \( \lceil \text{rnd}^{uni}(45,50) \rceil \), where \( \text{rnd}^{uni}(45,50) \) is a uniform distribution between 45 and 50, and \( \lceil x \rceil \) aims to convert \( x \) into the nearest integer. Additionally, \( P_{n 2 j}=[\text{rnd}^{uni}(30,35)], P_{n 3 j}=[\text{rnd}^{uni}(30,40)], P_{n 4 j}=[\text{rnd}^{uni}(25,30)]; EC_{n 1 j}=[\text{rnd}^{uni}(80,100)], EC_{n 2 j}=[\text{rnd}^{uni}(70,95)], EC_{n 3 j}=[\text{rnd}^{uni}(70,90)], EC_{n 4 j}=[\text{rnd}^{uni}(65,90)], EI_{m j}=[\text{rnd}^{uni}(1,4)] \).

For small-scale instance, the population size is set to 20, maximum iteration is 100, mutation probability is 0.001, and crossover probability is 0.7. Meanwhile, the population size is set to 30, maximum iteration is 200, mutation probability is 0.001, and crossover probability is 0.7 for the medium instances.

#### 5.2. Effectiveness Analysis for MABDE

Regarding the stochastic nature of the proposed approach, 10 runs are realized for the resolution of the small-scale instance. Then an evaluating indicator \( (BR_{2/1}) \) is adopted to measure the experimental
results of algorithms. The indicator is calculated as \( BR_{2/1} = (AVG_1 - AVG_2) / AVG_2 \times 100\% \), where \( BR_{2/1} \) indicates the superior ratio of algorithm 2 to algorithm 1, and \( AVG_1 \) and \( AVG_2 \) indicate the average value for ten runs of algorithm 1 and algorithm 2, respectively. Table 2 summarize the computational results. Table 2 shows that the resulting solution of MABDE outperforms DE and GA, where the \( BR \) is up to 3.86% and 12.62% to DE and GA, respectively.

Table 1. A small-scale instance

| Job(n) | Stage 1 | Stage 2 | Stage 3 |
|--------|---------|---------|---------|
|        | \( P_{n11}(EC_{n11}) \) | \( P_{n12}(EC_{n12}) \) | \( P_{n21}(EC_{n21}) \) | \( P_{n22}(EC_{n22}) \) | \( P_{n31}(EC_{n31}) \) | \( P_{n32}(EC_{n32}) \) |
| 1      | 2 (11)  | 2 (10)  | 4 (7)   | 5 (9)   | 2 (11)  | 3 (9)   |
| 2      | 4 (14)  | 5 (9)   | 3 (9)   | 4 (15)  | 5 (13)  | 4 (11)  |
| 3      | 6 (11)  | 5 (11)  | 4 (9)   | 2 (10)  | 2 (11)  | 5 (12)  |
| 4      | 4 (4)   | 3 (11)  | 6 (15)  | 5 (11)  | 5 (14)  | 8 (11)  |
| \( EI_{mj} \) | 1       | 2       | 1       | 2       | 1       | 3       |

Table 2. Computational results of MABDE, DE and GA for the small-scale instance.

| Instance | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | BR   |
|----------|---|---|---|---|---|---|---|---|---|----|------|
| MABDE    | 183| 180| 184| 183| 182| 181| 180| 180| 181| 180| 0    |
| DE       | 192| 190| 189| 189| 188| 190| 188| 186| 187| 185| 3.86%|
| GA       | 207| 204| 208| 204| 205| 206| 201| 201| 204| 203| 12.62%|

5.3. Convergence Performance Analysis

In order to verify the convergence performance of MABDE, the three algorithms are executed 10 times independently for the medium-scale instance respectively, and the convergence curve is obtained as shown in Figure 2. Obviously, the proposed MABDE can search the satisfactory solution faster than DE and GA, which confirms that MABDE algorithm has outstanding search performance.

Figure 2. Convergence curve for algorithms.

Figure 3. Effect analysis of energy saving strategy

5.4. Energy Saving Analysis

To validate the energy consumption strategy, 9 groups of comprehensive-scale instances are generated randomly. In these instances, 3 groups of jobs\( (N=6,12,16) \) are processed in 3 stages \( (M=2,4,6) \). Alternative machines in each stage is \( \forall n \in N, m \in M \), \( P_{nmj} = \lceil \text{rand}(8,15) \rceil \), \( EC_{nmj} = \lceil \text{rand}(30,40) \rceil \), \( EI_{mj} = \lceil \text{rand}(3,8) \rceil \). Following parameters are applied in the three algorithms: \( PS=30 \), maximum iteration is set to 200, mutation probability is 0.001, and crossover probability is 0.7.
In order to carry on the effective analysis to the proposed energy saving strategy, two types of objective function values are achieved, namely minimum energy consumption and minimum makespan. Moreover, two indicators are adopted as follows.

$$R_{EA} = \frac{EA[G_{\text{EA}}] - EA[G_{\text{EA}}^G]}{EA[G_{\text{EA}}]} \times 100\%$$  \hspace{1cm} (15)

$$R_{\text{Makespan}} = \frac{\text{Makespan}[G_{\text{EA}}] - \text{Makespan}[G_{\text{Makespan}}]}{\text{Makespan}[G_{\text{Makespan}}]} \times 100\%$$  \hspace{1cm} (16)

where $R_{EA}$ is the energy consumption reduction rate, while $R_{\text{Makespan}}$ is the increase rate of makespan. $EA[G_{\text{EA}}]$ indicates the energy consumption with minimizing objective function of $EA$, while $EA[G_{\text{Makespan}}]$ denotes the energy consumption with minimizing objective function of makespan. Likewise, the meaning of other symbols in Eq. (16) can be known. Each instance is executed 10 times independently for the three algorithms respectively. Figure 3 shows the results of the energy saving strategy.

In terms of energy consumption, $R_{EA}$ has always between 34% and 35%. And with the increase of the problem scale, the reduction of energy consumption shows an uptrend. From the angle of processing time, $R_{\text{Makespan}}$ remains between 5.5% and 6.5% on the small-scale instances. With the increase in the scale of the problem, $R_{\text{Makespan}}$ has a slight increase trend. But the maximum value of $R_{\text{Makespan}}$ is 10.5% for all instances.

6. Conclusions

This work presents a mathematical model for the multistage manufacturing system scheduling problem with parallel machines for minimizing total energy consumption. To solve this problem, a memetic algorithm based on differential evolution is proposed. The computational results on different scale instances verify the efficiency and effectiveness of the proposed algorithm.

7. References

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