The deformation of porous sandstones; are Byerlee friction and the critical state line equivalent?

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1. Introduction

Byerlee (1968) and later with more data (Byerlee, 1978) suggested that, to a useful approximation, the sliding friction of rock-on-rock is independent of rock type, characterized by a friction coefficient of 0.85 at small effective normal stresses and 0.6 at larger normal stresses. This relationship was supported by a compilation of experimental data. The generalization is commonly applied to describe such behaviour. Previous work showed that the yield surface is substantially independent of rock type when mean stress and differential stress are normalized by the grain crushing pressure, implying that the critical state line is rock type-independent and equivalent to the frictional sliding criterion. We test these hypotheses using previously published data for a range of porous sandstones augmented by new experimental results on Hollington and Berea sandstones deformed to large strains to define the critical state line over a wide range of pressures for each rock type. Results confirm the rock type-independence of the critical state line and show that it is nearly equivalent to frictional sliding. These relationships point to a simple procedure for estimating approximately the mechanical properties of sandstones based on petrographic characteristics.

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A B S T R A C T

Certain rock properties that depend on intergranular fracture and frictional sliding appear to be independent of rock type. This relationship is true for the rock-on-rock frictional sliding coefficient. The generalization has been widely applied to geomechanical modelling of upper crustal strength. Porous sandstones can be relatively weak and poorly cohesive, hence susceptible to deformation involving grain fragmentation and pore collapse. The critical state theory is commonly applied to describe such behaviour. Previous work showed that the yield surface is substantially independent of rock type when mean stress and differential stress are normalized by the grain crushing pressure, implying that the critical state line is rock type-independent and equivalent to the frictional sliding criterion. We test these hypotheses using previously published data for a range of porous sandstones augmented by new experimental results on Hollington and Berea sandstones deformed to large strains to define the critical state line over a wide range of pressures for each rock type. Results confirm the rock type-independence of the critical state line and show that it is nearly equivalent to frictional sliding. These relationships point to a simple procedure for estimating approximately the mechanical properties of sandstones based on petrographic characteristics.

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applied to describe deformation behaviour. Strength and mode of inelastic failure depends strongly on porosity and on the magnitude of effective mean stress. For small mean stresses, failure is rather like the failure of low porosity rocks in which, after initially elastic behaviour, microfracturing leads to dilatancy and the formation of a shear-oriented fault plane along which frictional sliding becomes localized following a differential stress drop. Lower porosity rocks display higher ultimate strength than higher-porosity rocks, and ultimate strength increases with increased mean stress. Additionally, finer grain size for a given porosity also leads to greater strength (Zhang et al., 1990).

At small mean pressures, failure leading to fault localization is usually described only in terms of the greatest and least principal stresses, $\sigma_1$ and $\sigma_3$, according to the linear failure criterion

$$\sigma_1 - \sigma_3 = a + b\sigma_3$$

in which $a$ and $b$ are material characteristics. $a$ is the unconfined compressive strength, and gradient $b$ is commonly about 3. Failure leading to fault formation can also be expressed in terms of the slope of the Mohr envelope to the stress circles for the combinations of greatest and least principal stresses at failure. Thus

$$\tau = \tau_0 + \mu\tan\psi$$

where $\tau_0$ is called the cohesive strength and $\psi$ is the angle of ‘internal’ friction, so that the internal friction coefficient is $\tan\psi$. The intercepts and slopes in these two alternative ways to describe failure are linked by

$$a = 2\tau_0/b\quad \text{and} \quad b = (1 + \sin\psi)/(1 - \sin\psi)$$

Unlike non-porous rocks, the more porous rocks can fail under purely hydrostatic stress, through the isotropic collapse of the pore space (e.g. Georgiannopoulos and Brown, 1978; Cundall and Strack, 1979; Elliott and Brown, 1985; Wong et al., 1997; Karner et al., 2003, 2005; Pettersen, 2007; Brzesowsky et al., 2011). The effective hydrostatic pressure, $P^*$, (total applied hydrostatic pressure minus pore fluid pressure, $P_f$) required for this failure mode is smaller for more porous rocks, and to some degree also for rocks of coarser grain sizes. Thus when loaded non-hydrostatically to failure under increasing amounts of effective mean stress, porous rocks display a transition from dilatant failure with fault localization, to compactive failure in which local intergranular shear stresses facilitate the onset of grain fragmentation that leads to pore collapse, until eventually $P^*$ is reached. Hence between the pressure at which the dilatancy to compaction transition occurs and the hydrostatic pressure $P^*$ at which isotropic pore collapse can occur, initial inelastic yielding occurs for progressively decreasing deviatoric stress (Fig. 1). In this case the yield stress is taken as the onset of inelastic permanent deformation, and is generally held to correspond to the onset of either permanent dilatation or compaction. This behaviour leads to the formation of a ‘capped’ yield surface on a $Q/P$ diagram, a plot of differential stress $Q = (\sigma_1 - \sigma_3)$ vs effective mean pressure $P = (\sigma_1 + 2\sigma_3 - 3P_f)/3$ for the case of uniaxial symmetric loading, where $\sigma_3$ is the total confining pressure. This regime of shear-enhanced compaction (Curran and Carroll, 1979), is where the non-hydrostatic part of the stress state assists the hydrostatic part in overcoming the resistance to pore collapse. Post-yield deformation is often mesoscopically ductile, which is defined as capacity for large distributed strain, but without specifying the deformation mechanism, but may involve the formation of a number of compaction bands or shear bands, particularly near the crest of the yield surface (e.g. Underhill and Woodcock, 1987; Wong, 1990; Wong et al., 1997; Issen and Rudnicki, 2000; Baud et al., 2000a, 2006; Schultz and Siddharthan, 2005; Fossen et al., 2007; Schultz et al., 2010; Das et al., 2011). Deformation does not become localized into a single shear fault because compaction causes local hardening, which leads to spreading of the deformation throughout the rock mass.

These failure characteristics of porous rocks have been extensively explored during several decades (e.g. Georgiannopoulos and Brown, 1978; Elliott and Brown, 1985; Wong et al., 1997; Baud et al., 2006; Wong and Baud, in press) and capped yield surfaces have been mapped out for several rock types (e.g. Baud et al., 2006). Different kinds of geometric descriptive yield surface models have been applied (e.g. DiMaggio and Sandler, 1971; Carroll, 1980) and compared (Baud et al., 2006) although the different geometries do not arise from fundamentally different physical models. These principles also apply to fluid saturated porous rocks at high temperatures, e.g. in the deformation of rocks rendered porous through dehydration reactions (Rutter et al., 2009) and in the flow of partially molten rocks (e.g. Rutter et al., 2006).

As porosity decreases, so the diameter of the yield surface arc expands, and the peak on the curve lies on a line that separates dilatant from compactive behaviour (Fig. 1), i.e. it is the locus of points corresponding to deformation at constant volume, where competing compactive and dilatant strains are equal. It usually corresponds to the separation between macroscopically brittle (localized faulting) and distributed (macroscopically ductile) deformation (Scott and Nielsen, 1991b; Rutter and Hadizadeh, 1991; Menéndez et al., 1996; Baud et al., 2006). This boundary is the critical state line. Muir Wood (1990), Schultz and Siddharthan (2005) and Schultz et al. (2010) argued that the critical state line
should be related to the friction coefficient determined from rock-on-rock sliding experiments because they both describe essentially similar processes of grain fracture and frictional sliding. Farmer (1983) explicitly interpreted the critical state line as equivalent to the Coulomb frictional strength of rocks, with the implication that if the frictional sliding behaviour of crustal rocks is largely independent of rock type, then so too should be the critical state line.

In this paper we set out to test the hypotheses that (a) the critical state line is approximately independent of rock type and (b) that the critical state line is equivalent to the frictional sliding line, by means of some re-analysis of previously published data together with new data on two silicic porous rock types.

2. Friction data of Byerlee (1978) in the Q/P coordinate frame

Conventionally, rock-on-rock friction data are presented as plots of shear stress versus effective normal stress across the sliding plane. Most experimental data were obtained from axisymmetric compression tests (‘triaxial’ tests with a hydrostatic confining pressure), with the shear and normal stresses computed from the greatest \((s_1)\) and least \((s_2 = s_3)\) principal stresses, and the orientation \(\theta\) of the fault plane with respect to the maximum principal stress direction. Other studies employed a true direct-shear testing configuration in which normal and shear stresses were obtained directly (e.g. Marone et al., 1990; Mair and Marone, 1999). For our present purpose of comparing friction and critical state data, sliding friction data must be recast in terms of principal stresses. This recasting requires the orientation of the sliding plane to be known for each pair of shear stress and normal stress values. This information is not always tabulated in publications, therefore where necessary we have used the Byerlee (1978) generalization to assign a most likely fault plane orientation \(\theta\), with respect to the maximum principal stress as follows:

\[
\theta = 45 - \left( \tan^{-1} \mu \right) / 2
\]

so that for normal stresses up to 200 MPa, where the average friction coefficient, \(\mu = 0.85, \theta = 24.8^\circ\), and for normal stresses above 200 MPa, where the average friction coefficient, \(\mu = 0.60, \theta = 29.5^\circ\).

Knowing shear stress \(\tau\), effective normal stress \(s_n\), and \(\theta\) from the geometry of the Mohr diagram, the mean stress \((s_1 + s_3)/2\) and the differential stress \((s_1 - s_3)\) required for sliding are obtained as

\[
(s_1 - s_3) = 2\tau/\sin 2\theta \quad \text{and} \quad (s_1 + s_3)/2 = s_n + \tau/\tan 2\theta \quad (1)
\]

Note that it is implicit that any dependence of the sliding criterion on the intermediate principal stress is ignored. Whilst previous studies have investigated the influence of the intermediate principal stress on failure of intact rock by fracturing (e.g. Mogi, 1967; Colmenares and Zoback, 2002), no corresponding studies exist for its influence on slip on a pre-existing fault surface.

For comparison with critical state data, the friction data in terms of principal stresses must be further recast in terms of differential stress \((Q = (s_1 - s_3))\) and effective mean stress \(P = (s_1 + 2s_3 - 3P_p)/3\), where \(P_p\) is the pore fluid pressure. Q as a measure of distortion-producing stress can be defined as equal to the maximum deviatoric principal stress, given by \((s_1 - (s_1 + s_2 + s_3))/3\), but because for axisymmetric loading this is equivalent to \(2(s_1 - s_3)/3\), it has become conventional to plot the differential stress rather than the deviatoric stress. For axisymmetric compression, \(P = (s_3 - P_p) + (s_1 - s_3)/3\), thus each increment of differential stress during loading increases the mean effective pressure by one third of the differential stress. This outcome means that in the axisymmetric shortening test the loading path on a Q/P plot rises from the abscissa with a slope of 3.

Setting \(M = (s_1 - s_3)(s_3 - P_p) + (s_1 - s_3)/3\) as the slope of the line representing the friction data in the Q/P coordinate frame and \(b = 1 - (s_1 - s_3)(s_3 - P_p)\) (the corresponding slope in the differential stress vs effective confining pressure coordinate frame) we can obtain \(M\) in terms of \(b\) as

\[
M = 3(b - 1)/(2 + b) \quad (2)
\]

This can be recast in terms of friction angle \(\psi = \tan^{-1} \mu\) using

\[
b = (1 + \sin \psi)/(1 - \sin \psi) \quad (3)
\]

Substituting for \(b\) in (2) and rearranging

\[
M = 6 \sin \psi/(3 - \sin \psi) \quad (4)
\]

This transformation was noted by Muir Wood (1990). Fig. 2 shows the compilation of friction data of Byerlee (1978) transformed into the Q/P coordinate frame in this way. Examples of the correspondence between sliding friction angle \(\psi\), coefficient of sliding friction \(\mu = \tan \psi\) and \(M\) are indicated as follows:

| \(\psi\) (degrees) | \(\mu\) | \(M = 6 \sin \psi/(3 - \sin \psi)\) |
|-------------------|--------|----------------------------------|
| 26.56             | 0.50   | 1.05                             |
| 30.96             | 0.60   | 1.24                             |
| 34.99             | 0.70   | 1.42                             |
| 38.65             | 0.80   | 1.58                             |
| 40.36             | 0.85   | 1.65                             |

As noted earlier, a large amount of newer friction data has been obtained for granular silicate rocks since Byerlee (1978), but it all falls within the confines of the dataset compiled by Byerlee. We will use data obtained from quartz sandstones as a basis for comparison with critical state data.

![Fig. 2. Compilation of friction data (after Byerlee, 1978) with a single best-fit line shown, compared to frictional sliding data for porous sandstone, all transformed into the Q/P coordinate frame.](image-url)
2.1. Porous sandstone friction

Frictional sliding data for porous sandstones and fault gouge were compiled from the following sources (porosities in parentheses):

Scott and Nielsen (1991a and 1991b), for Tennessee sandstone (6%); Colorado red sandstone (6.3%); Gold sandstone (13.9%); Berea sandstone (19.9%); DV (Dobbs Valley)-1 sandstone (20.9%); DV-2 sandstone (22.2%); DV-3 sandstone (27.6%).

Numelin et al. (2007), natural fault gouge.

Cuss et al. (2003), Tennessee sandstone (7%), Darley Dale sandstone (13%), Penrith sandstone (28%).

Mair and Marone (1999), synthetic fault gouge.

This study, Hollington sandstone (25%) and Berea sandstone (17.7%).

These frictional sliding data are plotted separately from the compilation of Byerlee (1978) on Fig. 2. All data are reasonably consistent with the Byerlee generalization, but within the sandstone group a tendency exists for friction in the more porous rocks to be slightly less than for less porous rocks.

3. New experiments on Hollington sandstone and Berea sandstone

3.1. Experimental details

In most previous literature (but see Cuss et al., 2003), post-yield deformation at large confining pressures was not usually taken to sufficiently high strains to reach the critical state condition, that is recognized when strain hardening and volumetric compaction ceases, and when deformation continues at constant differential stress, potentially with no further microstructural changes. Thus the critical state line could only be defined as the locus of points passing through peaks in the yield surface for rocks of different porosities.

New experimental data were therefore obtained from the porous Hollington and Berea sandstones, with the particular aim of testing to sufficiently high strains that the critical state line at large differential stresses is intersected by the stress path for axisymmetric compression. These data therefore complement the low strain data previously reported in the literature.

The Hollington red sandstone comes from a quarry on the outcrop of the Triassic Sherwood sandstone group in Staffordshire, England. The solid components (Fig. 3a) are dominated by 70% equant quartz sand grains with the remainder consisting of feldspar (17%), clays (8%), detrital mica (3%) and opaques (2%), including a thin coating of hematite around grains. A veneer of quartz cement around grains provides cohesion. Cores were cut normal to bedding, which is revealed by grain size and mineralogical banding (about 2 cm wide) with colour (hematite) contrasts. Total initial porosity is 25.6 ± 0.6% (by helium porosimetry), and grain size distribution mean is 230 ± 63 μm.

The Carboniferous Berea sandstone was cored in a single direction (normal to weakly-defined bedding laminae) from a block supplied by Dr D. Lockner (US Geological Survey). Total porosity is 17.7 ± 0.7% (by density measurement), grain size distribution mean is 170 ± 50 μm. This rock type was previously described and used extensively in programs of mechanical testing (e.g. Zhang et al., 1990; Wong et al., 1997; Baud et al., 2000b; Menéndez et al., 1996). The material used here is slightly less porous than blocks previously (21% porosity) used by Wong, Baud, Menéndez and co-workers (op. cit.), and hence is somewhat stronger. The solid components consist of 80% equant quartz grains, feldspar (7%), clay minerals (7%), with the remainder carbonate and opaque grains.

Axisymmetric shortening experiments were carried out on cylindrical specimens 15, 20 or 25 mm in diameter with a length: diameter ratio of 2.5:1, at total confining pressures ranging up to 400 MPa, using ReolubeDOS synthetic oil as a confining medium. The smaller diameter samples were used at the higher pressures to keep axial loads within the limits of the testing machine. Samples were jacketed either in polyolefin or heat-shrink tubes, which support negligibly small loads. Pore volumetry was used to measure porosity changes during deformation, with pore fluid pressure being maintained constant at 2, 10 or 16 MPa. To measure porosity changes during deformation requires that the pore pressure is kept constant, so that the pore volumometer measures the amount of fluid expelled from the rock during compaction, or the amount of fluid drawn in during dilatation. Such experiments are therefore performed under ‘drained’ conditions.

All experiments were carried out at room temperature at strain rates near $10^{-4}$ s$^{-1}$ for Hollington sandstone and $2 \times 10^{-5}$ s$^{-1}$ for the less permeable Berea sandstone. The effectiveness of drainage in fluid saturated tests was checked by observing the rapidity of response to small pore pressure steps. Hollington sandstone was tested both oven-dried (at 75 °C) and saturated with the same fluid as the confining medium. Berea sandstone specimens were tested water-saturated with a pore pressure of 2 or 10 MPa. Baud et al. (2000b) reported comparative data on the effect of water on the strength of Berea sandstone (amongst other porous sandstone types). Although a weakening effect on Berea sandstone occurs, it is small and little greater than the range of experimental error.

![Fig. 3.](image-url)
The evolution of specimen cross-sectional area with progressive deformation is required for calculation of differential stress. The increase in cross-sectional area with each increment of axial shortening strain was first calculated assuming constant volume deformation, then compensation was applied for the volume change obtained from the pore volumeter, which reduces the assumed amount of cross-sectional area increase when volume compaction occurs. Homogeneous deformation must be assumed for these calculations.

Measurements of differential stress at small (elastic) strains were made to an accuracy of ±1 MPa and hydraulic pressure measurements to ±0.5 MPa. The resolution of pore volumeter measurements was 0.5 mm³. Axial displacements were measured from outside the pressure vessel to a resolution of 0.01 mm, but after correcting for confining pressure and axial load-dependent axial apparatus distortion, the accuracy of strain measurements is estimated to be about ±0.2%. The effects of heterogeneous deformation of the sample and the correction required for cross-sectional area changes mean that differential stresses reported at high strains are probably accurate to only ±5%, but the largest contributor to scatter in experimental results is likely to be variability between different rock samples arising from bedding laminae heterogeneities.

3.2. Experimental results

Results are summarized in Table 1 and graphically in Figs. 4 and 5. Under steadily increasing effective hydrostatic pressure, both rocks displayed near-linear elastic compression of the pore space. Pore collapse of Hollington sandstone occurred at \( P^* = 190 \pm 15 \) MPa (12 tests). Fig. 3b shows the damage done at the grain scale to Hollington sandstone showing collapse of Hollington sandstone occurred at \( P^* \) was not attained for Berea sandstone within the range of the hydraulic pump used, but is expected to be at about 450 MPa.

For small effective confining pressures under axisymmetric loading, both rock types failed by localized faulting with stress drop. At higher effective confining pressures, faulting became suppressed, leading to a transition to failure by distributed ductile cataclastic flow. Yield (the onset of permanent, inelastic deformation, given with an estimated accuracy of about ±5% of the reported differential stress) was often recognized from the differential stress vs strain curve (Fig. 4), but it is most clearly expressed by deviation of the volumetric strain or porosity change curve from the hydrostat (Fig. 5). Yield stress values for Hollington sandstone show considerable scatter. It is suspected that this scatter is due to between-specimen variations owing to the marked lithological banding that characterizes this rock.

Wherever possible, deformation was continued to high strains, 16% shortening for Berea sandstone and 25–30% shortening for Hollington sandstone. Over much of the range of ductile flow, recovered specimens of Hollington sandstone showed some relative flow localization by the formation of compaction bands normal to the maximum compression direction and about 7 mm apart. The bands were still apparent even though the entire sample volume was heavily damaged, with loss of all cohesion, so that recovered samples could be completely crumbled under small pressure from the fingers.

Throughout the ductile flow regime, post-yield deformation was accompanied by volumetric compaction and strain hardening (Fig. 4) until eventually flow at constant volume was attained, which also corresponds to the attainment of steady flow at constant differential stress. At this condition, the tendency for continued compaction by pore collapse is balanced by dilatancy associated with frictional sliding and crack formation. Examination of deformed specimens from tests stopped at different amounts of strain after the onset of constant volume deformation showed that whilst constant volume flow was attained during distributed ductile flow, shortly afterwards some shear band formation would occur. Small stress drops sometimes seen during constant volume flow were linked by tie lines (stress path lines) of slope 3. Strain-hardening ductile flow occurs for yield points lying to the right of the peak on the yield curve, continuing for larger amounts of ductile strain for tests starting at the higher confining pressure up to \( P^* \). Although not normally done in previous studies of deformation of porous sandstones, we performed several tests in which the initial confining pressure applied was larger than \( P^* \). In these cases, some isotropic

| Specimen number | Total conf. press. (MPa) | Pore press. (MPa) | Differential stress (MPa) at first calculated assuming constant volume deformation | Fault angle (deg) |
|-----------------|-------------------------|------------------|----------------------------------|-----------------|
| (a) Hollington Sandstone (dry [pore pressure = 0] or oil wet, 25% porosity) |
| 107             | 0                       | 0                | 20                               | 0               |
| 105             | 36                      | 17               | 82                               | 66              |
| 51              | 40                      | 16               | 65                               | 45              |
| 16              | 43                      | 16               | 100                              | 78              |
| 8               | 46                      | 16               | 74                               | 65              |
| 104             | 56                      | 17               | 78                               | 82              |
| 7               | 78                      | 16               | 80                               | 80              |
| 21              | 80                      | 16               | 107                              | 90              |
| 103             | 96                      | 18               | 120                              | 113             |
| 40              | 103                     | 16               | 84                               | 105             |
| 19              | 106                     | 16               | 110                              | 83              |
| 29              | 123                     | 17               | 100                              | 150             |
| 23              | 110                     | 17               | 85                               | 145             |
| 37              | 124                     | 16               | 90                               | 80              |
| 24              | 130                     | 17               | 95                               | 202             |
| 80              | 135                     | 16               | 89                               | 172             |
| 33              | 137                     | 0                | 100                              | 304             |
| 6               | 152                     | 16               | 58                               | 203             |
| 41              | 169                     | 16               | 82                               |                 |
| 79              | 175                     | 16               | 65                               | 258             |
| 100             | 186                     | 17               | 35                               | 272             |
| 101             | 190                     | 16               | 60                               | 248             |
| 17              | 192                     | 16               | 45                               |                 |
| 18              | 195                     | 17               | 50                               |                 |
| 106             | 196                     | 16               | 40                               | 390             |
| 30              | 207                     | 0                | 40                               | 362             |
| 10              | 220                     | 16               | 38                               | 273             |
| 35              | 235                     | 0                | 65                               | 417             |
| 12              | 258                     | 16               | 27                               | 279             |
| 31              | 260                     | 0                | 80                               | 460             |
| 36              | 291                     | 0                | 85                               | 684             |
| 75              | 309                     | 22               | 60                               |                 |
| 76              | 316                     | 17               | 60                               |                 |
| Hydrostatic 206 | 16                      | mean value, ±15 MPa of 12 tests |

| (b) Berea sandstone (water wet, 17.7 % porosity) |
| 10              | 12                      | 2                | 110                              | 24              |
| 1              | 42                      | 2                | 202                              | 104             |
| 9              | 82                      | 2                | 240                              | 200             |
| 2              | 102                     | 2                | 245                              | 230             |
| 16             | 102                     | 2                | 272                              | 230             |
| 11             | 142                     | 2                | 322                              | 330             |
| 12             | 210                     | 10               | 278                              | 332             |
| 15             | 250                     | 10               | 250                              | 353             |
| 13             | 300                     | 10               | 170                              | 371             |
| 14             | 345                     | 10               | 320                              | 659             |
pore collapse occurred before the differential load was applied, permanently reducing porosity by up to 3% for the highest pressure tests. This behaviour resulted in a yield stress that increased with confining pressure applied in excess of \( P^* \), and very long post-yield strain-hardening paths that correspond to increased rates of strain hardening, rather than to greater strains when attaining constant stress/constant volume flow. The same pattern of behaviour of the stress/strain data was reported by Cuss et al. (2003) for Penrith and Darley Dale sandstones.

Fig. 7 summarizes in a similar way the Hollington sandstone data together with that for Berea sandstone from the present study and also data for Penrith sandstone and Darley Dale sandstone from the study of Cuss et al. (2003). This shows how the yield curve is expanded for rocks of lower porosity. It also shows how the constant flow stress/constant volume (critical state line) trend is, to a fair approximation, independent of rock type, whether measured as the constant volume flow stress or as the locus of the apices of successive yield curves for rocks of different porosities. The behaviours of Penrith and Hollington sandstones when loaded from initially applied confining pressures beyond \( P^* \) are also comparable.

4. Discussion of experimental results

4.1. Comparison of the critical state line with frictional sliding data

Fig. 8 compares differential stress \( Q \) with effective mean stress \( P \) at critical state, determined at the attainment of steady flow/
constant volume flow with $Q$ vs $P$ for frictional sliding in porous sandstones previously presented in Fig. 2. Also included are data on serpentinite (from Rutter et al., 2009) rendered porous (20%) by dehydration to olivine plus talc at 550 °C at low effective pressure, and then mechanically tested at 450 °C. The results show that the critical state concept applies equally to high temperature deformation that involves pore collapse and that the critical state line slope remains the same.

The frictional sliding slope for sandstones is 1.43, corresponding (via Eq. (4)) to a friction coefficient of 0.71. The critical state best-fit line slopes at 1.18 ± 0.05, corresponding to a friction coefficient of 0.58. Thus the data do not support the view that the two slopes are identical, as suggested by Farmer (1983), Muir Wood (1990), and Schultz and Siddharthan (2005). They are, however, closely similar, and the experimental errors implicit in the two datasets overlap within the data range. The differences in central tendency of the two groups of data may be attributable to the fact that, whilst both describe similar physical processes of grain fracture, rotation and frictional sliding, they do not describe similar degrees of fracture damage. Intensely localized frictional sliding in a fault zone involves much greater degree of damage than distributed cataclastic flow, perhaps involving several orders of magnitude difference in accommodated local strain. The particle-size distributions and their porosities are likely to be different at the same effective pressure, and it seems likely that these differences will be reflected in differences in mechanical behaviour. However, in the same way that sliding friction is to a useful approximation independent of rock type, the critical state line is likewise independent, even though the slopes are not identical. As we explore below, this similarity can form part of a useful basis for first-pass modelling of the mechanical behaviour of porous sandstones.

In making such generalizations, we recognize that force-fitting all critical state data into a single linear envelope can
obscure more subtle differences in real trends for individual rock types with respect to the general pattern of behaviour. Such differences can be crucial to distinguishing between different mathematical models of yield cap evolution, as explored by Baud et al. (2006).

4.2. Post-yield porosity evolution

As cataclastic flow proceeds with strain hardening in the post-yield part of the loading path, it is accompanied by progressive porosity reduction until the critical state condition is reached. Wong et al. (1992) and Wong and Baud (1999) presented data for Berea sandstone that shows how the attainment of a given porosity defines a succession of expanding yield loci, in the same way that rocks of initially lower porosities are characterized by initially larger yield curves. Rutter et al. (2009) showed similar behaviour for dehydrated serpentinite and the same can be demonstrated in the behaviour of Hollington sandstone. Loading paths starting from progressively higher effective pressures therefore arrive at the critical state line at higher differential stresses and hence at lower porosities and greater degrees of microstructural damage.

Fig. 9 shows a compilation that demonstrates how porosity at critical state decreases as effective confining pressure $P$ increases for the several different rock types referred to above, plus Gosford and Boise sandstones (Edmond and Paterson, 1972; Wong et al., 1997; Baud et al., 2000b). This is a projection of the critical state line onto the $P$ (MPa) vs porosity (%) plane and a hyperbolic function

$$P = (1/\phi^0 - 0.037)/0.000149$$

is shown fitted to these data. The curve can also be taken as the line separating localized faulting behaviour from distributed ductile flow (the brittle–ductile transition). Thus this graph shows how the effective confining pressure required for ductility varies with initial porosity. Rutter and Hadizadeh (1991), Scott and Nielsen (1991b), Wong et al. (1992), Menéndez et al. (1996), Wong et al. (1997) and Wong and Baud (1999) compiled data on the brittle–ductile transition for various sandstones in this way (Fig. 9). This curve, as an alternative way to plot critical state data, appears to be independent of rock type.

As suggested above, the critical state data may lie slightly below the frictional sliding data because, at the same effective mean pressure, the comminuted rock in a narrow fault zone is likely to be more compact, finer-grained and hence more microstructurally ‘evolved’. In the same way that porosity decreases systematically along the critical state line, a similar evolution might be expected for fault gouge derived from framework silicates. Some experimental studies have shown that gouge porosity evolves with progressive shearing, e.g. Marone et al. (1990). Several studies showed that compaction, as measured by reducing thickness of a synthetic granular gouge layer or fluid expulsion, accompanies progressive shearing (Marone et al., 1990; Mair and Marone, 1999; Crawford et al., 2008), tending to a constant porosity at steady-state sliding. Presently, insufficient data are available to evaluate whether porosity at the attainment of stable sliding varies systematically with effective mean pressure for a range of gouge types and over a sufficiently wide range of pressures.

4.3. Controls on the value of $P^*$

From a theoretical model based on pore collapse by Hertzian indentation cracking, Zhang et al. (1990) proposed that $P^*$, the effective hydrostatic pressure required for the onset of grain crushing, is determined by the product of initial porosity $\phi$ and grain radius $r$, such that a plot of log $P^*$ vs log ($\phi r$) is linear with a slope $-3/2$. This corresponds to the normal consolidation curve.

Fig. 10 shows such data compiled by Wong et al. (1997), augmented by our data for Hollington and Berea sandstones, together with that for Penrith and Tennessee sandstones (Cuss et al., 2003). The new data are consistent with the earlier compilation. A least-squares best-fit line

$$\log P^*(\text{MPa}) = 0.603 - 1.09 \log (\phi r(\text{mm}))$$

is shown, with a standard error of ±0.25 on log $P^*$, or about ±17% of the value of $P^*$.

Schultz et al. (2010) employed the relation $log P^* = -1.5 \log (\phi r)$ to estimate $P^*$ for Navajo sandstone. The slope in this equation is constrained by the Hertzian model and the line passes approximately through the centroid of the data (Fig. 10), but empirical Eq. (6) is a significantly better descriptor of the presently available data.

4.4. Normalization of yield surface data and prediction of sandstone properties

Wong et al. (1997) showed that, to a useful approximation, yield surface data lie on a single yield surface when normalized as $Q/P^*$ versus $P/P^*$. Cuss et al. (2003) found this correspondence also applied to Tennessee, Darley Dale and Penrith sandstones and, as Fig. 11 shows, it applies to Hollington sandstone and even dehydrated serpentinite. Given that the critical state line can be defined as passing though the apices of successive yield surfaces, the critical state line can be identified as independent of rock type. Wong et al. (1997) suggested that normalized yield surface data might be approximated by a single elliptical yield envelope given by

$$\frac{(P/P^* - 0.5)^2}{(Q/(0.6 P^*))^2} = 1$$

This equation corresponds to a linear critical state line on a $Q/P$ plot with a slope of $0.6/0.5 = 1.2$, which may be compared to the best-fit slope of 1.18 for the critical state line shown on Fig. 8 obtained from $Q/P$ data at the attainment of steady flow at constant volume. Geometric implications of this generalization (Fig. 12) are:
The value of effective mean pressure at critical state is always $0.5 P^*$, and the value of effective confining pressure for axisymmetric loading at the critical state at initial yield, equivalent to the brittle–ductile transition, is always $0.3 P^*$. Geometric similarity requires that if log $P^*$ varies linearly with log ($\phi r$), then log effective mean pressure $P_{\text{em}}$ at critical state must vary similarly with log ($\phi r$), on a parallel trend at lower pressure separated by log (2.0), i.e.

$$\log P_{\text{em}}(\text{MPa}) = 0.603 - 1.09 \log(\phi r(\text{mm})) - \log(2.0) \quad (8)$$

The available data are consistent with this interpretation to a degree of uncertainly comparable to that which applies to the log $P^* \propto \log (\phi r)$ relationship (Fig. 10).

Fig. 11 also shows that when confining pressure is increased beyond $P^*$ before applying differential stress, a new, rising yield curve becomes established that also is approximately independent of rock type after normalization, at least for Hollington and Penrith sandstones and for dehydrated serpentinite.

Using the logarithmic relation between $P^*$ and $\phi r$ given by Eq. (6), Eq. (7) can be recast in terms of $\phi r$. Thus potentially to a useful approximation, the yield and critical state characteristics of porous sandstones are determined entirely by the product of porosity with initial mean grain radius. Schultz et al. (2010) used this approach to estimate the strength of Navajo sandstone and conditions for the formation of shear and compaction bands. One would naturally expect that the nature and geometry of the cementing phase, the degree of maturity (mineralogy, sorting and grain shape) of a sediment and the form of the grain size distribution (not just mean grain size) would influence behaviour. These factors are presumably responsible for some or most of the observed scatter in the data.

Fig. 12 recasts Fig. 1 in such a way as to illustrate that the above generalizations provide a basis for making a first estimate of the mechanical behaviour any porous sandstone, and possibly of other porous rock types, starting from the petrographically determinable parameters of porosity and mean grain radius. This generalization may find application in making first estimates of the likely properties of reservoir rocks, the mismanagement of which can result in unwanted pore collapse and ground surface subsidence. This result applies particularly to axisymmetric loading, but can probably be generalized by recasting $Q$ in terms of deviatoric stress.

4.5. Drained versus undrained deformation

All experimental data described above was acquired under drained conditions, with the pore fluid pressure, where employed, maintained constant despite dilatancy or compaction. Deformation under undrained conditions can be of more relevance in nature, where pore pressure and hence effective pressure variations result

![Fig. 10. Plot of log $P^*$ vs log (porosity ($\phi$) x grain size ($r$)) from a compilation by Wong et al. (1997) (triangles) augmented with data from the present study (squares) and from Cuss et al. (2003), with upper best-fit (solid) line shown. Standard error in log $P^*$ is ±0.25 log units. Broken line shows the function used by Schultz et al. (2010) to estimate $P^*$, based on the Hertzian cracking model of Zhang et al. (1996).](image1)

![Fig. 11. Compilation of yield data for porous sandstones from Wong et al. (1997) augmented by data on Hollington and Berea sandstones (this study), dehydrated serpentinite (Rutter et al., 2009) and Penrith, Darley Dale and Tennessee sandstones (Cuss et al., 2003). Dashed curves show upper and lower limits to yield behaviour for $P/P^* < 1$ according to Wong et al. (1997). Data for Hollington and Penrith sandstones and dehydrated serpentinite extending to the region $P/P^* > 1$ are also shown to follow a similar trend. Porosity (%) shown in parentheses with each rock type.](image2)
from pore volume changes. The commonly cited $k_{c}$ loading path during burial of sandy rocks in a sedimentary basin is much less steep than the undrained experimental axisymmetric loading path of slope 3 commonly used in experiments. Clearly, a loading path slope of less than 1.2, the average critical state line slope, will never result in critical state being attained under drained conditions. However, under undrained or partially drained conditions, the stress path on a $Q/P$ plot can deviate according to whether dilatancy or compaction leads to change in the pore pressure. Thus compaction typically raises pore pressure in the post-yield regime, shifting the stress path leftwards until the critical state is attained. In the engineering and petroleum geomechanics literature, a great deal of attention is given to stress path evolution under undrained conditions because of its importance in reservoir management and the potential equivalence between frictional and critical state data is likely affected by different porosities and degrees of comminution arising from the greatly different strains attained.

5. Summary and conclusions

1) By means of a re-examination of previously published data together with the results of new experiments on Hollington sandstone and Berea sandstone, we have found that to a useful approximation the critical state line for porous sandstones is largely independent of rock type, characterized by a best-fit slope of $1.18 \pm 0.05$ in the $Q/P$ coordinate frame.

2) Previous work of Wong et al. (1997), and augmented by data from the present study, suggests that yield of porous sandstones lies on a single elliptical surface $[(P/P^*-0.5)/0.5]^2+ [Q/(0.6 P^*)]^2 = 1$ which differential and effective mean stress parameters $Q$ and $P$ are normalized by dividing by $P^*$, the effective hydrostatic stress required to initiate isotropic pore collapse. This relationship implies that they share a common critical state line with a slope of 1.2. The prolongation of the critical state line to higher pressures can also be determined by deforming a sandstone beyond the yield point until flow at constant differential stress and constant volume is attained. The critical state line defined in this way was also found to be approximately independent of rock type, with a slope of $1.18 \pm 0.05$ on a $Q/P$ plot. The critical state line also marks the transition between failure by dilatancy with the formation of a shear fault and accompanying stress drop (on the low pressure side) and more ductile, distributed cataclastic deformation involving compaction (on the higher pressure side).

3) The slope of the variation of shear stress with normal stress across a fault defines the friction coefficient $\mu$, which Byerlee (1978) found to be approximately independent of rock type to the extent that the generalization can be usefully employed in geomechanical modelling. When mapped into $Q/P$ space the slope for porous sandstones is 1.43, corresponding to $\mu = 0.71$. It is slightly higher than the critical state line slope (1.18) for a range of porous sandstones defined as above, with some indication that the slope is slightly greater for rocks of smaller initial porosity. Thus the potential equivalence between frictional and critical state data is likely affected by different porosities and degrees of comminution arising from the greatly different strains attained.

4) Above the yield surface on the ductile, compactant side of the critical state line, porosity decreases during post-yield strain-hardening flow until the critical state line is attained at a particular value of porosity. Porosity attained at a particular point along the critical state line also appears to be largely independent of rock type.

5) The value of $P^*$ for any given rock type is predicted moderately well from the value of the product porosity $\times$ grain radius (as first suggested by Zhang et al., 1990), using the empirical relationship $\log (P^* \text{ MPa}) = 0.625 \text{ log (porosity $\times$ grain-radius mm)}$. Thus it can be inferred that to a useful approximation, porosity $\times$ grain radius provides a basis for predicting the mechanical behaviour, in terms of strength and ductility, of porous sandstones in general and potentially of a wider range of porous rocks. It can be used in this way exactly as the Byerlee generalization has been applied to the description of fault behaviour in the Earth’s upper crust. This result may find application in estimating the likely properties of vesicular basalt) (e.g. Shimada, 1981; Heap et al., 2011; Zhu et al., 2011). Porous carbonate rocks behave mechanically in a comparable way to sandstones but can be more complex owing to potentially more complex pore/cement relationships and the capacity of calcite for a degree of intracrystalline plastic deformation even at low temperatures. We have therefore not attempted to analyze their behaviour together with silicic sandstones.

Fig. 12. Generalization of the behaviour of porous sandstones expressed as normalized differential stress plotted against normalized effective mean stress, with $P^*$ as given by Eq. (6) used as the normalization factor: We assume all sandstones’ yield stresses lie on the elliptical yield surface given by Eq. (7), with an axial ratio 12:1 and that all are represented by a single critical state line of slope 1.2. This implies that the brittle-ductile transition confining pressure for all sandstones is $0.3 P^*$ and the effective mean stress at this point is $0.5 P^*$. From the yield ellipse, for any stress path the yield stress may be estimated, and the $Q$ and $P$ values at which critical state is reached. Along the critical state line the porosity reduces with effective mean pressure $P_{\text{em}}$ according to Eq. (8), which can be used to estimate porosity decrease along any strain hardening stress path. The form of successive yield curves with reducing porosity is indicated, all normalized to the original $P^*$ value.

4.6. Other rock types

We have focused on the behaviour of porous sandstones because more data are available for them. However, a growing amount of experimental study is available for the mechanical behaviour of porous carbonate rocks (e.g. Baud et al., 2000a; Vajdova et al., 2004, 2010; Wong and Baud, in press; Zhu et al., 2010) and rocks of igneous origin (tuffs and other volcaniclastics,
reservoir sandstones from simply-acquired petrographical information.

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