Simplified chain inflation

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Abstract. We propose a simplified chain inflation model and calculate the primordial power spectra of the scalar and tensor fluctuations. The spectral index and the tensor–scalar ratio are respectively 0.972 and 0.089, which are consistent with present cosmological observations.

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Inflation models proposed by Guth in [1] not only explain the large scale homogeneity and isotropy of the universe, but also provide a natural mechanism for generating the observed magnitude of inhomogeneity (see [2,3] etc for a review). During the period of inflation, quantum fluctuations are generated within the Hubble horizon, and then stretch outside the horizon to become classical. In the subsequent deceleration phase after inflation these frozen fluctuations re-enter the horizon, and seed the matter and radiation density fluctuations observed in the universe.

In the last two decades many inflation models have been proposed. However, none of them can be naturally realized in a fundamental theory. Gravity governs the background evolution during inflation and quantum effects provide a natural mechanism for generating the primordial perturbations to seed the fluctuations for forming the large scale structure of the universe and the anisotropies in cosmic microwave background radiation which can be measured precisely. A quantum theory of gravity is needed before we can understand the full theory in the early universe. Nowadays string theory, as we know, is the only self-consistent quantum theory of gravity. String theory can only live in ten-dimensional spacetime. We need to compact ten-dimensional string theory to four dimensions. Many free parameters appear after compactifications, even though there is
no free dimensionless parameter in string theory in ten dimensions. Recent developments
for the flux compactifications [4, 5] suggest that a huge number of metastable string vacua
emerge in string theory. The whole space of such string vacua is called the string landscape.

Cosmological observation implies that the density perturbation is roughly $\delta \rho / \rho \sim 10^{-5}$ in our universe. A flat potential of the inflaton is called for in the inflation model.
On the other hand, a flat potential naturally brings in a large enough number of e-folds
during inflation to solve the problems of the hot big bang model. Usually we can expect
one or more small dimensionless parameters in the potential of the inflaton to be related
to the small density perturbation. One question we should ask is why there is such a
small parameter in a fundamental theory. Since the number of the metastable vacua in
the string landscape is so huge, it offers an opportunity to explain this small number.
Actually many inflation models in string theory, those of [6]–[10] and so on, have been
proposed in the last few years. But what the distinguishing phenomenon is for a string
landscape is still an open question. The authors in [11] suggested that the string landscape
has a model independent prediction which says that our universe should be spatially open.

Even though the shape of the string landscape has not been worked out, heuristically
we expect the metastable vacuum with large vacuum energy to have a very short lifetime.
In [12], Tye suggested that our universe would decay rapidly from a site with large vacuum
energy to a long-lived metastable vacuum with a small positive cosmological constant
through the resonance tunnelling. Thus it is possible to dynamically solve today’s dark
energy problem. However, inflation is still needed in the early universe. Fortunately, chain
inflation [13]–[15] becomes generic in this scenario. In this model, the universe tunnelled
rapidly through a series of metastable vacua with different vacuum energies. After many
tunnelling events, more than 60 e-folds are obtained and the problems in the hot big
bang are solved. In [16], the authors use a numerical method to calculate the amplitude
of the primordial power spectra. However, their simulation result implies a quite large
tensor–scalar ratio which is not consistent with cosmological observations\(^1\).

In this short note, we propose a simplified chain model and suggest a new method for
computing the density perturbations, spectral index and tensor–scalar ratio. Our results
produce a nice fit to the data.

In the chain inflation model, the universe begins in a metastable vacuum with a large
positive vacuum energy. The system could tunnel to the lower minima along a variety of
possible directions in the string landscape. However, many of the neighbouring minima
are inaccessible, since the probabilities for tunnelling into them are too small. In [18, 19],
the authors investigated a classical field theory in which there are two homogeneous
equilibrium states with different energy densities. In the quantum version of the theory,
the state of higher energy density becomes unstable through barrier penetration. For
simplicity, we only quantified the tunnelling probability by modelling a single tunnelling
event for a scalar field theory with potential

$$V(\phi) = \frac{1}{4} \lambda (\phi^2 - a^2)^2 + \frac{\sigma}{2a}(\phi - a), \hspace{1cm} (1)$$

\(^1\) In [16], the amplitude of the scalar power spectrum is given by $\Delta_k^2 \sim \beta / \alpha^2$. On the other hand, the power
spectrum for the tensor modes is roughly the same as that for the perturbation of the scalar field. The amplitude
of the tensor perturbations in [16] is $\Delta_T^2 \sim \beta$. Thus the tensor–scalar ratio takes the form $r = \Delta_T^2 / \Delta_k^2 \sim \alpha^2$.
However, their simulation result shows that $\alpha$ is much larger than 1, which contradicts WMAP3 [17] where
$r \leq 0.65 \text{ (95\% CL)}. \hspace{1cm}$
where $\sigma$ is the vacuum energy difference between two neighbouring metastable vacua. The tunnelling probability is roughly given by
\[ \Gamma \sim \exp \left( -\frac{27\pi^2 \kappa^4}{2 \sigma^3} \right), \] (2)
where $\kappa$ is the tension of the brane interpolating between the two vacua. In [20,21], Guth et al showed that the probability of a point remaining in a false de Sitter vacuum is roughly given by
\[ p(t) \sim e^{-\frac{4\pi}{3} \beta H t}, \] (3)
where the dimensionless parameter $\beta$ is
\[ \beta = \frac{\Gamma}{H^4}, \] (4)
and $H$ is the Hubble parameter in this vacuum. Thus the lifetime of the field in this metastable vacuum is estimated as
\[ \tau \simeq \frac{3}{4\pi H \beta}. \] (5)
A lower bound on the dimensionless parameter $\beta$ is obtained:
\[ \beta \geq \frac{9}{4\pi} \] (6)
in order for percolation and thermalization to be achieved. In this paper, we focus on the case with the lifetime of the metastable vacua much shorter than the Hubble time $H^{-1}$, or equivalently $\beta \gg 3/4\pi$.

In the simplified chain inflation we assume that $\sigma$ is always a constant. Since the inflation responsible for the observed modes of the density perturbations only lasts a few e-folds, the Hubble parameter $H$ can be taken as a constant during this period. Naively the vacuum energy goes like
\[ \rho_V \simeq \rho_{V,i} - \frac{\sigma}{\tau}, \] (7)
where $\rho_{V,i}$ is the initial vacuum energy. The vacuum energy drops $\sigma/(H\tau)$ per Hubble time $H^{-1}$. Here we also assume that $\sigma$ is much smaller than $\rho_V$ and then the series of tunnelling events can be regarded as a continuous evolution of a scalar field. The Hubble parameter is governed by the Friedmann equation
\[ H^2 = \frac{\rho_V}{3M_p^2} = \frac{\rho_{V,i} - (\sigma/\tau)t}{3M_p^2}, \] (8)
where $M_p$ is the reduced Planck scale. The number of e-folds before the end of inflation is related to the energy density $\rho_V$ by
\[ N_e = \int_t^{t_{\text{end}}} H(t') dt' \simeq \frac{2}{3\sqrt{3}M_p \sigma} \frac{\tau^{3/2}}{\rho_V^{3/2}}, \] (9)
Simplified chain inflation

or, equivalently,
\[ \rho_V \simeq \left( \frac{3\sqrt{3}}{2} \frac{M_p \sigma}{\tau} N_e \right)^{2/3}, \]
(10)
\[ H^2 \simeq \left( \frac{\sigma/\tau}{2M_p^2 N_e} \right)^{2/3}. \]
(11)

According to equation (8), the ‘slow-roll’ parameter \( \epsilon \) is given by
\[ \epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\sigma/\tau}{6M_p^2 H^3} = \frac{1}{3N_e}. \]
(12)

The meaning of \( \sigma/\tau \) is nothing but the change of the vacuum energy per time unit. Generically the tunnelling from one site to another site with a lower cosmological constant is accompanied by some radiation due to the bubble percolation, nucleation and collision. During inflation, radiation is inflated away. Suppose the change of the vacuum energy at the last step is large, which implies that the change of the vacuum energy per time unit is quite large and then the ‘slow-roll’ condition \( \epsilon \ll 1 \) is broken down. On the other hand, the loss of the vacuum energy is transferred to radiation and then the energy density is dominated by radiation. Now chain inflation has ended and reheating happened.

Now the amplitude of the primordial power spectra for the scalar and tensor perturbations can be expressed as respectively
\[ \Delta_R^2 = \frac{H^2/M_p^2}{8\pi^2\epsilon} = \frac{3}{2^{11/3}\pi^2} \left( \frac{\sigma/\tau}{M_p^5} \right)^{2/3} N_e^{5/3}, \]
(13)
\[ \Delta_T^2 = \frac{H^2/M_p^2}{\pi^2/2} = \frac{2^{1/3}}{\pi^2} \left( \frac{\sigma/\tau}{M_p^5} \right)^{2/3} N_e^{2/3}. \]
(14)

The spectral index and the tensor–scalar ratio are given by
\[ n_s = 1 + \frac{d \ln \Delta_R^2}{d \ln k} \simeq 1 - \frac{d \ln \Delta_T^2}{d N_e} = 1 - \frac{5}{3N_e}, \]
(15)
\[ r = \frac{\Delta_T^2}{\Delta_R^2} = \frac{16}{3N_e}. \]
(16)

For \( N_e = 60 \), the spectral index and the tensor ratio are respectively \( n_s = 0.972 \) and \( r = 0.089 \). The results for WMAP three-year data are presented in [17]. To properly compare our results with WMAP results, we show our results in figure 1. Our results are nicely compatible with WMAP at 95% confidence level.

Using the WMAP normalization \( \Delta_R^2 = 19.9^{+1.3}_{-1.8} \times 10^{-10} \), we find
\[ \frac{\sigma/\tau}{M_p^5} = 8.6 \times 10^{-16}. \]
(17)

The Hubble parameter and the energy density at \( N_e = 60 \) are respectively \( H \simeq 3 \times 10^{-5} M_p \) and \( \rho_V \simeq 2.7 \times 10^{-9} M_p^4 \). Requiring \( \sigma \ll \rho_V \) yields
\[ \frac{\tau}{t_p} \ll 3 \times 10^6, \quad \text{or} \quad \frac{\tau}{H^{-1}} \ll 90, \]
(18)
where $t_p = M_p^{-1}$ is the Planck time. Our requirement that the lifetime of the metastable vacua is much shorter than the Hubble time $H^{-1}$ is reasonable.

To summarize, a simplified chain inflation model is proposed. In this model, the vacuum energy drops only a little per step and the series of tunnellings just looks like the continuous evolution. We also estimate the density perturbations and we find that our model can fit the present data very well.

In the simplified chain inflation, we assume that the vacuum energy drops the same energy density $\sigma$ in each tunnelling event. However we do not expect it to really happen in a string landscape where the situation becomes much more complicated. It is even possible that in some steps along the chain the system tunnels to the vacua with slightly higher energy density. But we can take our semi-quantitative calculation as an averaged effect. Our results only depend on $\sigma/\tau$. Naively we suggest that $\sigma/\tau$ should be replaced by the average value $\langle \sigma/\tau \rangle$ in the string landscape. Generically there are a vast number of chains, but only a few of them can satisfy what we have observed. The amplitude of the scalar power spectra depends on the details of the chain. But the spectral index and the tensor–scalar ratio can be taken as quite insensitive predictions of the string landscape.

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