Dynamic chirality in the interacting boson model

S Brant¹ and D Tonev²,³

¹ Department of Physics, Faculty of Science, University of Zagreb, 10000 Zagreb, Croatia
² INFN, Laboratori Nazionali di Legnaro, 35020 Legnaro, Italy
³ Institute for Nuclear Research and Nuclear Energy, BAS, 1784 Sofia, Bulgaria

E-mail: brant@phy.hr

Abstract. The chiral interpretation of twin bands in odd-odd nuclei is investigated in the Interacting Boson Model framework. The analysis of the wave functions shows that the possibility for angular momenta of the valence proton, neutron and core to find themselves in the favorable, almost orthogonal geometry is present, but not dominant. Such behavior is found to be similar in nuclei where both the level energies and the electromagnetic decay properties display the chiral pattern, as well as in those where only the level energies of the corresponding levels in the twin bands are close together. The difference in the structure of the two types of chiral candidates nuclei can be attributed to different $\beta$ and $\gamma$ fluctuations, induced by the exchange boson-fermion interaction, i.e. by the antisymmetrization of odd fermions with the fermion structure of the bosons. In both cases the chirality is weak and dynamic. Among the nuclei that are candidates for chiral behavior, the best candidate is probably the odd-even $^{135}$Nd, where the $\beta$ and $\gamma$ fluctuations are strongly reduced and chirality, although dynamic in origin, is rather close to the static limit.

1. Introduction

The rotation of triaxial nuclei may give rise to pairs of identical $\Delta I = 1$ bands with the same parity in odd-odd nuclei - the chiral doublet bands [1, 2, 3]. The condition for the appearance of chiral doublet bands is that the proton and neutron Fermi levels are located in the lower part of the valence proton high-$j$ (particle-like) and in the upper part of the valence neutron high-$j$ (hole-like) subshells (or vice versa), and the core is triaxial. The angular momenta of the valence particles are aligned along the short and long axes of the triaxial core, while the angular momentum of the rotational core is aligned along the intermediate axis. The three angular momenta can be arranged to form two systems that differ by intrinsic static chirality, a left- and a right-handed system. When this chiral symmetry is broken in the body-fixed frame, the restoration of the symmetry in the laboratory frame results in the occurrence of degenerate doublet $\Delta I = 1$ bands. Chirality in odd-mass nuclei is a novel feature [4, 5]. The conditions for chirality in odd-mass nuclei can be more favorable than in odd-odd nuclei. They can be fulfilled for three-particle configurations of the type one-proton-two-neutrons or one-neutron-two-protons.

Several theoretical models have been applied in a number of articles, like Tilted Axis Cranking Model [1], Two Quasiparticle + Triaxial Rotor Model [1, 6], Core-Particle-Hole Coupling Model [7]. All these models have one assumption in common, they suppose a rigid triaxial core. On the contrary, all nuclei in which twin bands have been observed have another characteristics in common, they are in regions of masses where even-even nuclei are $\gamma$-soft, i.e. effectively...
triaxial but not rigid. It is evident that nuclei in these mass regions can not reach the full requirements needed for the existence of chirality, but they can approach them, or at least retain some fingerprints of chirality.

2. IBFFM calculations

The structure of twin bands in odd-odd nuclei was also described in the Interacting Boson Fermion-Fermion Model (IBFFM) [8, 9], in the version where there is no distinction between proton bosons and neutron bosons. The triaxial equilibrium deformation was generated by the cubic (three-body) term [10, 11] added to the standard IBM-1 core Hamiltonian. In the case of the $^{134}$Pr nucleus [12, 13, 14] the calculated IBFFM $B(E2)$ and $B(M1)$ values for twin bands based on the $\pi h_{11/2} \nu h_{11/2}$ configuration have been in excellent agreement with the experimental values, while the results of the Two Quasiparticle + Triaxial Rotor Model (TQPTR) calculations displayed sizable differences in respect to experimental data (Fig.1). In the IBFFM besides the orientation in space, the deformation of the core is the additional degree of freedom. Valence quasiparticles are coupled to all structures of the boson core that are present in the basis, limited by the total boson number. In the IBFFM calculation we have performed, the core was not rigid.

Due to the underlying symmetry, the pair of chiral twin bands should exhibit systematic properties. The yrast and the side bands should be nearly degenerate. In the angular momentum region where chirality sets in (which for the A~130 mass region corresponds to $I \geq 12$) the $B(E2)$ values of the electromagnetic transitions de-exciting analogue states of the chiral twin bands should be almost equal. Correspondingly the $B(M1)$ values should exhibit odd-even staggering, being for the $\pi h_{11/2} \nu h_{11/2}$ configuration much bigger for transitions de-exciting states with odd spins than for transitions de-exciting states with even spins. The $B(M1)$ values for $\Delta I = 1$ transitions connecting the side to the yrast band should have the odd-even staggering.
out of phase with respect to the $B(M1)$ staggering for transitions de-exciting states in the yrast and the side bands, i.e. for the $\pi h_{11/2} \nu h_{11/2}$ configuration $B(M1)$ values for transitions de-exciting states with even spins have to be much bigger than for the ones de-exciting states with odd spins. Neither of the conditions for the $B(E2)$ and $B(M1)$ values was found in $^{134}$Pr. This nucleus belongs to a class of odd-odd nuclei in which a pair of twin bands is close in excitation energy, but the electromagnetic decay properties do not show the chiral pattern. For shortness they will be denoted as case A nuclei.

To investigate the setting up of chirality in a certain nucleus, is crucial to determine the $B(E2)$ and $B(M1)$ values. Unfortunately, such measurements have been done only for few nuclei of interest: $^{134}$Pr, $^{132}$La and $^{128}$Cs. In the case of $^{128}$Cs the electromagnetic decay properties display the expected chiral pattern [15]. Odd-odd nuclei where the pair of twin bands is close in excitation energy and the electromagnetic decay properties display the chiral pattern, will be denoted as case B nuclei.

Odd-odd nuclei in the A~130 mass region can be classified as case A or case B nuclei. In all these nuclei the cores are $\gamma$-soft, their odd-proton odd-mass neighbors have also a similar structure and their odd-neutron odd-mass neighbors have a similar structure, too. IBFFM calculations have been performed for such nuclei, taking the same $\gamma$-soft core, same fermion occupation probabilities, same strengths of the boson-fermion interactions for the odd proton, and different strengths of the boson-fermion interactions for the odd neutron [16]. The main difference was in the strength of the exchange interaction for the odd neutron: in the case A it was strong and in the case B was weaker (but still rather strong). The results are presented in Fig.2.
Figure 3. Calculated distributions $\zeta (J_{\pi} j_{\nu})$ (panel (a)), $\psi (J_{\pi} R)$ (panel (d)) and $\xi (j_{\nu} R)$ (panel (g)) of the angles between $\vec{J}_{\pi}$, $\vec{j}_{\nu}$ and $\vec{R}$, and the $\sigma$ distribution (panel (j)) for the 39/2$_1$ state of the yrast band A in $^{135}$Nd. In panels (b), (e), (h) and (k) the corresponding distributions for the yrast $\pi h_{11/2} \nu h_{11/2}$ 15$_1$ state in $^{134}$Pr are presented. In panels (c), (f), (i) and (l) the corresponding distributions for the 39/2$_1$ state of the yrast band A in $^{137}$Nd are presented.

In the case A the composition of the yrast band, in terms of contributions from core states, shows that the yrast band is basically built on the ground-state band of the even-even core. With increasing spin the admixture of the $\gamma$-band of the core becomes more pronounced. The side band wave functions contain large components of the $\gamma$-band and with increasing spin, of higher-lying collective structures of the core, that near the band crossing become dominant. The analysis of the distributions $\zeta (J_{\pi} j_{\nu})$, $\psi (J_{\pi} R)$ and $\xi (j_{\nu} R)$ [16] of the angles between $\vec{J}_{\pi}$, $\vec{j}_{\nu}$ and $\vec{R}$, and the $\sigma$ distribution (details of the procedure can be found in Ref. [14]), found that the presence of configurations with the angular momenta of the proton, neutron and core in the for chirality favorable, almost orthogonal geometry, is substantial but far from being dominant (panels in the second column of Fig.3). The analysis of the distributions $\zeta (J_{\pi} j_{\nu})$, $\psi (J_{\pi} R)$ and $\xi (j_{\nu} R)$ for the case B has shown no significant differences in respect to the case A [16]. The presence of all chiral signatures in the case B is not the consequence of (for chirality) more favorable geometry.
The main, but decisive, difference is revealed in the analysis of distributions in the $\beta - \gamma$-plane [16] (details of the procedure and the definition of the deformation parameters $\beta$ and $\gamma$ can be found in Ref. [14]). The $\beta$ and $\gamma$ distributions for cases A and B are sizeably different. The two bands have significantly more similar deformations in the case B than in the case A (Fig. 4). The yrast bands in both cases are similar, with fluctuations on higher spins being somewhat smaller in the case B. The side band on medium and higher spins has far less shape fluctuations and a significant decrease of components with small $\beta$ in the case B. Different deformations and larger fluctuations in the case A reflect in $B(E2)$ values, that are much smaller in the side band than in the yrast band. In the case B, due to more similar deformations, the $B(E2)$ values for transitions between states in the side band are closer to the $B(E2)$ values in the yrast band. Smaller shape fluctuations and more similar deformations in the case B increase the probability that the two bands could be even and odd superpositions of separated left-handed and right-handed configurations, that reflects in the chiral-like behavior of $B(M1)$ values. The distribution of the core structure in the wave functions reveals the dynamical mechanism. In the case B the weaker, but still strong, boson-fermion interactions (particularly the exchange interaction) for the neutron (hole-like) quasiparticle are not strong enough to admix big components from higher lying core bands. The ground and the $\gamma$-band components are dominant in the states of the side band. All chiral signatures are present, but large shape fluctuations sizably reduce their magnitude in respect to the full chiral predictions. The $\gamma$-softness and shape fluctuations prevent the angular momenta of the proton, neutron and core to create chiral favorable geometry on the scale they would do if the core is triaxial. For nuclei where the boson-fermion interaction is stronger, other higher lying core structures admix into the states of the side (and partially into the yrast, too) band, increasing the shape fluctuations. This allows the admixture of more contributions from near axial shapes and consequently washes out the $B(M1)$ staggering. The only visible signature of dynamical chirality remains the vicinity of the excitation energies of the two bands.

However, the possible formation of chiral bands is not limited to odd-odd nuclei. Whenever three configurations in a nucleus have large angular momentum components along the three
principal axes, the conditions for chirality are present. In odd-mass nuclei this can be fulfilled for three-particle configurations of the type one-proton-two-neutrons or one-neutron-two-protons. In this case the two-particle configuration takes the role of the one-particle configuration (of the same type) in the odd-odd nucleus. The situation in odd-mass nuclei could be even more favorable for chirality, because the two-particle angular momentum is longer than the one-particle angular momentum in odd-odd nuclei. The first observation of a possible three-quasiparticle chiral structure was in $^{135}$Nd [4]. The negative-parity ground state band in $^{135}$Nd has the one-neutron-quasiparticle $\nu h_{11/2}$ hole-like structure. At spin $25/2$ it is crossed by a three-quasiparticle band with the structure interpreted by the $\nu h_{11/2}(\pi h_{11/2})^2$ configuration. The two-proton-quasiparticle configuration $(\pi h_{11/2})^2$ consists of $\pi h_{11/2}$ particle-like quasiprotons and is therefore particle-like. A twin band interpreted by the same $\nu h_{11/2}(\pi h_{11/2})^2$ configuration was observed as the yrare band rather close in excitation energy. In Ref. [4] these bands have been interpreted as a pair of chiral bands. Recently, $B(E2)$ and $B(M1)$ values deexciting states of the twin bands in $^{135}$Nd have been measured [5]. The corresponding $B(E2)$ and $B(M1)$ values have been found to be almost identical in both bands, giving a strong argument to interpret this pair of bands as chiral partner bands.

![Figure 5. IBFFM $\beta$ and $\gamma$ distributions for $^{135}$Nd (first and second column) and $^{137}$Nd (third and forth column): panels (a) and (b) for the $31/2_1$ and $31/2_2$ states, panels (c) and (d) for the $37/2_1$ and $37/2_2$ states and panels (e) and (f) for the $43/2_1$ and $43/2_2$ states.](image)

The properties of the three-quasiparticle bands in $^{135}$Nd have also been investigated [17] within the interacting boson fermion plus broken pairs framework (IBFBPM) [18]. The IBFBPM calculations have shown that both partner bands are basically built on the ground-state band of the even-even core. The admixture of the $\gamma$-band components of the core is small. This
behavior is different than in the IBFFM wave functions of odd-odd nuclei. The two-proton angular momentum is twice as long as the one-proton angular momentum in the neighboring odd-odd nuclei, providing a better possibility for the formation of chiral structures. The analysis of the distributions $\zeta(J_\pi j_\nu)$, $\psi(J_\pi R)$ and $\xi(j_\nu R)$ of the angles between $\vec{J_\pi}$, $\vec{j_\nu}$ and $\vec{R}$ and of the $\sigma$ distributions shows that in the states of both bands the presence of configurations with the angular momenta of the protons, neutron and core in the for chirality favorable, almost orthogonal geometry, is substantial. It is not so pronounced as if it were for a rigid triaxial core, but is far more pronounced than in the case of neighboring odd-odd nuclei (Fig.3). The partner bands show a structural change at medium spins that is due to shape fluctuations (Fig.5). For states with $I \leq 35/2$ shape fluctuations are of the order of those in the neighboring odd-odd nuclei [14, 16]. At spin $37/2$ shape fluctuations are reduced, and for higher spins, although different values of $\beta$ are present, the distribution shows one dominant peak. The structure changes from the geometry dominated by shape fluctuations into the geometry that is rather close to static chirality. This transition from the vibrational into a static chiral regime has recently been obtained in the calculations of TAC with RPA [5].

The structure of the $N = 77$ $^{137}$Nd nucleus looks very similar to $^{135}$Nd. But $^{137}$Nd is less deformed and closer to the close shell. The IBFFPM predictions for $^{137}$Nd [17] show that shape fluctuations in the yrare band are far more pronounced than in the yrast band. The fluctuations are not reduced for higher spins. On the contrary, in the yrare band the shape fluctuations are enhanced for higher spins and besides triaxial slightly prolate components, in the distribution oblate components appear (Fig.5). This leads to the conclusion that in $N = 77$ nuclei the structure of twin bands is more determined by shape fluctuations and prolate-oblate coexistence than by chirality.

3. Conclusions
The analysis of the wave functions of chiral candidates bands in the framework of the Interacting Boson Model, shows that the possibility for angular momenta of the valence proton, neutron and core to find themselves in the favorable, almost orthogonal geometry is present, but not dominant. The difference in the structure of chiral candidates nuclei can be attributed to different $\beta$ and $\gamma$ fluctuations. The coupling of the proton and neutron quasiparticles to the shape degrees of freedom is the dominant mechanism, leading to a weak chirality of dynamical origin. In some cases, in odd-even nuclei, this dynamic chirality can be close to static chirality.

References
[1] Frauendorf S and Meng J 1997 Nucl. Phys. A 617 131
[2] Dimitrov V I, Frauendorf S and Dönau F 2000 Phys. Rev. Lett. 84 5732
[3] Starosta K, Koike T, Chiara C J, Fossan D B and LaFosse D R 2001 Nucl. Phys. A 682 375c
[4] Zhu S et al 2003 Phys. Rev. Lett. 91 132501
[5] Mukhopadhyay S et al 2007 Phys. Rev. Lett. 99 172501
[6] Ragnarsson I and Semmes P 1988 Hyperfine Interact. 43 423
[7] Starosta K, Chiara C J, Fossan D B, Koike T, Kuo T T S, LaFosse D R, Rohoziński S G, Droste Ch, Morek T and Srebrny J 2002 Phys. Rev. C 65 044328
[8] Brant S, Paar V and Vretenar D 1984 Z. Phys. A 319 355
[9] Brant S and Paar V 1988 Z. Phys. A 329 151
[10] Heyde K, Van Isacker P, Waroquier M and Moreau J 1984 Phys. Rev. C 29 1420
[11] Casten R F, von Brentano P, Heyde K, Van Isacker P and Jolie J 1985 Nucl. Phys. A 439 289
[12] Brant S, Vretenar D and Ventura A 2004 Phys. Rev. C 69 017304
[13] Tonev D et al 2006 Phys. Rev. Lett. 96 052501
[14] Tonev D et al 2007 Phys. Rev. C 76 044313
[15] Grodner E et al 2006 Phys. Rev. Lett. 97 172501
[16] Brant S, Tonev D, de Angelis G and Ventura A 2008 Phys. Rev. C 78 034301
[17] Brant S and Petrache C M 2009 Phys. Rev. C 79 054326
[18] Vretenar D, Bonsignori G and Savoia M 1995 Z. Phys. A 351 289