Nash Equilibria for Exchangeable Team against Team Games and their Mean Field Limit

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Abstract—We study stochastic mean-field games among finite number of teams each with large finite as well as infinite numbers of decision makers (DMs). We establish the existence of a Nash equilibrium (NE) and show that a NE exhibits exchangeability in the finite DM regime and symmetry in the infinite one. We establish the existence of a randomized NE that is exchangeable (not necessarily symmetric) among DMs within each team for a general class of exchangeable stochastic games. As the number of DMs within each team drives to infinity (that is for the mean-field games among teams), using a de Finetti representation theorem, we establish the existence of a randomized NE that is symmetric (i.e., identical) among DMs within each team and also independently randomized. Finally, we establish that a NE for a class of mean-field games among teams (which is symmetric) constitutes an approximate NE for the corresponding pre-limit game among teams with mean-field interaction and large but finite number of DMs.

I. INTRODUCTION

Stochastic teams entail a collection of DMs acting together to optimize a common cost function, but not necessarily sharing all the available information. At each time stage, each DM only has partial access to the global information which is defined as the information structure (IS) of the team [1]. If there is a pre-defined order according to which the DMs act, then the team is called a sequential team. For sequential teams, if each DM’s information depends only on primitive random variables, the team is static. If at least one DM’s information is affected by an action of another DM, the team is said to be dynamic. Teams as a generalization of classical single DM stochastic control problems have many application areas such as decentralized stochastic control [2], [3], networked control [2], [4], communication networks [4], cooperative systems [5], [6], large sensor networks [7], and electricity markets and smart grid design [8]. In this paper, we study a class of stochastic games where each player in the game entails a sequential team with finite as well as countably infinite number of DMs. A related set of results involves those on mean-field games and teams.

Mean-field games (MFGs) are limit models of symmetric non-zero-sum non-cooperative finite DM games with a mean-field (MF) interaction (see e.g., [9], [10]). For applications of MFGs in financial engineering, economics, and pricing in markets see [11]. The existence of a Nash equilibrium (NE) for MFGs has been established in several studies including [10], [12], [13], [14], [15]. In contrast to the setting of MFGs, in this paper, we study stochastic MFGs involving teams. Such problems exhibit additional nuances due to lack of convexity in team policies under decentralized information structures, see e.g., [16], [17], [18]. Such models have several applications as they combine MF game and team theory. A natural application area is in sensor networks where a large collection of decentralized sensors (acting as a large team) transmit their information to a fusion center over the network in the presence of a collection of strategic jammers. As another application, consider information sharing across a medium via interference channels involving a collection of the multi-terminal encoder and decoder pairs where each encoder-decoder pair can be viewed as a team whose information serves as noise for the other teams. Finally, another application is in the energy market where multiple distributed renewable energy stations (each can be viewed as a large team distributed in various locations) compete for optimal energy production and pricing policies.

Unlike games where NE is often of interest, the notion of optimality among each team is global optimality. In general, NE (in the language of teams, person-by-person optimal solutions) only arrive at local optima and not global optima. This gap is specially significant for stochastic teams with countably infinite number of DMs (or MF teams). Hence, the results on games may be inconclusive regarding global optimality for teams without uniqueness of NE (e.g., see [10], [19], [20], [12], [21]).

Related to MF teams, results on social optima control problems have primarily focused on the linear quadratic Gaussian model, where the centralized performance has been shown to be achieved asymptotically by decentralized controllers (see e.g., [22], [23], [24]). MF teams under decentralized IS have been studied in [25], [26], [27]. For such models, the existence and convergence of a globally optimal solution have been established in [25], [26], [27]. In this paper, we study a class of MFGs among finite number of teams with infinite DMs under the concept of NE which is globally optimal among DMs within each team but NE among teams. Related to our setting of games among large teams, a MF competition game has been studied among two large teams with centralized IS in [28]. In addition, for zero-sum games involving finite teams, the existence of a randomized saddle-point equilibrium has been established in [17]. In this paper, our focus is on exchangeable games among teams with both large and infinite number of DMs to establish the existence of a NE that is exchangeable for the
finite case and symmetric in the MF limit.

A. Contributions
1) For exchangeable games among teams with finite number of DMs, we establish the existence of a randomized NE, and show that it is exchangeable among DMs within each team (player). This NE might be asymmetric and also correlated among DMs within each team (player).
2) As the number of DMs within each team drives to infinity (for the MFG among teams), we establish the existence of a randomized NE, and show that it is symmetric (i.e., identical) and independently randomized.
3) Finally, we show that a NE for a MFG among teams (which is symmetric, independently randomized, and scalable with the number of DMs) constitutes an approximate NE for the corresponding game among teams with MF interaction and large but finite number of DMs.

Due to space constraints, the proofs of the results are not included here are available in the full paper [29].

II. PROBLEM FORMULATION
A. A generalized intrinsic model
Consider the class of stochastic games where DMs act in a pre-defined order (i.e., a sequential game [30]). Under the intrinsic model for sequential games (described in discrete time), any action applied at any given time is regarded as applied by a separate individual DM, who acts only once. However, depending on the desired equilibrium concepts, IS, and cost functions, it may be suitable to consider a collection of DMs as a single player acting as a team. In this setting, teams take part in a game. For this setting, Witsenhausen’s intrinsic model is inadequate as it views each DM individually, which does not capture a joint deviation of a collection of DMs acting as a team, required for our equilibrium concept. To formalize this class of games, we introduce a class of M-player games using the generalized intrinsic model in [31] building on Witsenhausen’s original one-shot DM based intrinsic model [1], where each player consists of a collection of (one-shot) DMs either with finite or infinite members. A formal description has the following components:

- Let \( M := \{1, 2, \ldots, M\} \) denote the set of players. For each \( i \in M \), let \( N_i := \{1, 2, \ldots, N_i\} \) denote the collection of DMs, \( D_{M_k} \) for \( k \in N_i \), acting as player \( i \) (PL\( ^i \)) (in other words, PL\( ^i \) encapsulates the collection of DMs indexed by \( N_i \)). We denote DM\( ^i_k \) of PL\( ^i \) by DM\( ^i_k \).
- There exists a collection of measurable spaces \((\Omega, \mathcal{F}), (U^i_k, \mathcal{U}^i_k), (\mathcal{Y}^i_k, \mathcal{U}^i_k)\), for \( k \in N_i \) and \( i \in M \), specifying the system’s distinguishable events, DMs’ control and observation spaces. The observation and action spaces are standard Borel spaces, and are described by the product spaces \( \prod_{k \in N_i} \mathcal{Y}^i_k \) and \( \prod_{k \in N_i} \mathcal{U}^i_k \), respectively for each PL\( ^i \).
- The \( \mathcal{Y}^i_k \)-valued observation variables are given by \( y^i_k = h^i_k(\omega, (u^p)_{(s,p) \in L^i_k}) \), where \( L^i_k \) denotes the set of all DMs acting before DM\( ^i_k \) (i.e., \( (s,p) \in L^i_k \) if DM\( ^i_k \) acts before DM\( ^i_k \)).

An admissible policy for each PL\( ^i \) is denoted by \( \gamma^i := \{y^i_k\}_{k \in N_i} \in \mathcal{Y}^i \) with \( u^i_k = \gamma^i_k(y^i_k) \) and the set of admissible policies for each PL\( ^i \) is described by the product space \( \mathcal{Y}^i := \prod_{k \in N_i} \mathcal{Y}^i_k \), where \( \Gamma^i_k \) is the set of all Borel measurable from \( \mathcal{Y}^i_k \) to \( \mathcal{U}^i_k \).
- There is a probability measure \( \mathbb{P} \) on \((\Omega, \mathcal{F})\), making it a probability space. Let \( \mathbb{E} \) denote the expectation with respect to \( \mathbb{P} \). The prior probability measures in general can be subjective (for each DM or PL\( ^i \)); however, in this paper, we will not discuss this, assuming that DMs have access to the common correct prior \( \mathbb{P} \).

We will also allow for randomized policies, where in addition to \( y^i_k \), each DM\( ^i_k \) has access to common and private randomization. This will be made precise in Section III-A.

B. Problems studied
We study stochastic games with each player involving finite (but large) number of DMs, or countably infinite number of DMs. For simplicity in our notation, we consider only two players in the game, denoted by PL\( ^1 \) and PL\( ^2 \) (i.e., \( M = \{1, 2\} \)). Our results remain valid for the finite player setting. We address the following questions:

1) Does there exist a NE for games with finite number of DMs? Is this NE exchangeable? We address these two questions in Theorem 1.
2) Does there exist a NE for games with countably infinite number of DMs? Does this NE admit symmetry properties? We address these two questions in Theorem 2.
3) Do Nash equilibria for games with countably infinite number of DMs constitute approximate Nash equilibria for the corresponding games with finite but large number of DMs? We address this question in Theorem 3.

The NE concept for such a class of games among teams should take into account the fact that DMs within players face a team problem with the desired global optimality notion for fixed policies of the other players. Hence, establishing the existence of a NE that exhibits exchangeability or symmetry requires further analysis that is not required for the classical stochastic games where each player is a singleton. This is because, each team must respond optimally, and not only person-by-person-optimally. Furthermore, to utilize a fixed point theorem, convexity and compactness of a set of policies (for each player, entailing all its DMs) are required. To address this difficulty, we endow a topology on the set of randomized policies which leads to the convexity and compactness of a set of exchangeable randomized policies. As the number of DMs goes to infinity, we use a de Finetti representation theorem and an argument used in [27] to establish symmetry and address the preceding questions.

Our focus is on exchangeable stochastic games, and hence, we suppose that action and observation spaces are identical through DMs of each player and are subsets of appropriate dimensional Euclidean spaces, i.e., \( \mathcal{U}^i_k = \mathcal{U}^i \subseteq \mathbb{R}^{n_i} \) and \( \mathcal{Y}^i_k = \mathcal{Y}^i \subseteq \mathbb{R}^{m_i} \) for all \( i \in M \), where \( n_i \) and \( m_i \) are positive integers. We first introduce a class of exchangeable static games with finite number of DMs and then a class of static MFGs.
C. Finite DM static game \( P_N \)

Consider a stochastic game with finite number of DMs, i.e., for each PL \( i \), \( N_i = \{ 1, \ldots, N_i \} \), \( \gamma^i_N = (\gamma^i_1, \ldots, \gamma^i_{N_i}) \) and \( \Gamma^i_N := \prod_{k=1}^{N_i} \Gamma_k^i \). Let the expected cost function for each PL \( i \) under a policy profile \( \gamma^{i,2} \) be given by

\[
J_N^i(\gamma^{i,2}) := \mathbb{E}^{i,2}_{\gamma^i_N} \left[ c(\omega_0, u^{i,1}_{N}) \right]
\]

for some Borel measurable cost function \( c^i : \Omega_0 \times \prod_{j=1}^{N_i} \mathcal{U}^j \to \mathbb{R}_+ \). Define \( \omega_0 \) as the \( \Omega_0 \)-valued, cost function relevant, exogenous random variable, taking values from a Borel space \( \Omega_0 \) with its Borel \( \sigma \)-field \( \mathcal{F}_0 \).

In the above, we used the notation \( \omega_0, u^{i,1}_{N}, u^{i,2}_{N} \) for \( \omega_0 \) as the \( \Omega_0 \)-valued, cost function relevant, exogenous random variable, taking values from a Borel space \( \Omega_0 \) with its Borel \( \sigma \)-field \( \mathcal{F}_0 \).

Re-write our game formulation and the equilibrium notion to incorporate randomized policies; see (6) and (7).

\[
\mu^N(B|\omega_0) = \prod_{i=1}^{N_i} \prod_{k=1}^{N_i} \int_{B^i_k} f^i_k(y^i_k, \omega_0, y^i_{-k}, \gamma^i) Q_k^i(dy^i_k), \tag{2}
\]

where \( y^i_{-k} = (y^i_{1:N_i}) \setminus \{ y^i_k \} \).

Under this assumption, via change of measure argument (see e.g., [32], [31]), the observations of each DMs are independent and also independent of \( \omega_0 \). This allows us to introduce a topology under which the space of randomized policies is Borel (see Section III-A). In addition, our main results (Theorems 1 and 2) impose the following assumptions.

Assumption 3:

(i) \( (y^i_k)_{k \in N_i} \) and \( (y^i_k)_{k \in N_i} \) are independent, conditioned on \( \omega_0 \);

(ii) For all \( i \in \mathcal{M} \), \( (y^i_k)_{k \in N_i} \) have an identical distribution, conditioned on \( \omega_0 \).

Assumption 4: For \( i \in \mathcal{M} \),

(i) \( \mathcal{U}^i \) is compact.

(ii) \( c^i(\omega_0, \cdot, \cdot, \cdot) \) in (1), is continuous for all \( \omega_0 \).

Under Assumption 3, we can rewrite (2) as follows:

\[
\tilde{\mu}^N(B|\omega_0) = \prod_{i=1}^{N_i} \prod_{k=1}^{N_i} \int_{B^i_k} \tilde{f}^i_k(y^i_k, \omega_0) Q^i(dy^i_k), \tag{3}
\]

where \( \tilde{f}^i : \mathcal{Y}^i \times \Omega_0 \to \mathbb{R}_+ \) and \( Q^i \) are identical for all DMs within player \( i \) for \( i = 1, 2 \).

In Section III, we establish the existence of a randomized NE for \( P_N \), and we show that this NE is symmetric (identical) among DMs within players. To this end, we first establish the existence of a randomized NE for \( P_N \) that is exchangeable, and then, we use this result to arrive at the existence result for \( P_N \). For our results in Section III, we let Assumptions 1 and 2 to hold. For exchangeable games with finite number of DMs, we consider Assumption 4, but we relax Assumption 3, where we allow DMs to have (finite)-exchangeable that can be correlated (see Assumption 5). As the number of DMs is driven to infinity, we let Assumptions 3 and 4 hold.

III. EXCHANGEABLE STATIC GAMES

In this section, we study exchangeable static stochastic games. We first introduce randomized policies with their suitable topology, and then, we establish the existence of a
NE for \( \mathcal{P}_N \) that is exchangeable, and a NE for \( \mathcal{P}_1 \) that is symmetric. Finally, we provide an approximation for NE for \( \mathcal{P}_N \) that is symmetric.

### A. Topology on control policies for static games

We now introduce randomized policies as Borel probability measures, equipped with a suitable topology. Following [33], [32], via a change of measure argument, Assumption 2 enables us to equivalently view observations of DMs of players independent and also independent of \( \omega_0 \). Under Assumption 2, we separate DMs’ policy spaces (both across the player and DMs) and equip them with a topology. Let \( \mathcal{R}_k \) be the set of all \( P \in \mathcal{P}(\mathcal{Y}^k \times \mathcal{Y}^0) \) such that \( P(B) = \int_B \Pi_k \{d\mu_k|y \} Q_k(y) \{d\nu_k|y \} \), \( B \in \mathcal{B}(\mathcal{Y}^k \times \mathcal{Y}^0) \), where \( \Pi_k \) is a stochastic kernel from \( \mathcal{Y}^0 \) to \( \mathcal{Y}^k \). \( \mathcal{R}_k \) is Borel measurable under the weak convergence topology [34]. We identify the set of relaxed policies for each DM \( \Gamma_k \), by \( \mathcal{R}_k \) with \( \gamma_k \), if and only if

\[
\gamma_k \Rightarrow (d\mu_k|y) Q_k(y) \Rightarrow (d\nu_k|y) Q_k(y) \quad \text{(4)}
\]

We equip \( \mathcal{P}(\mathcal{Y}^k \times \mathcal{Y}^0) \) with the w-s topology which is the coarsest topology under which the map \( \int \kappa(y_k, u_k) P(d\gamma_k) : \mathcal{P}(\mathcal{Y}^k \times \mathcal{Y}^0) \to \mathbb{R} \) is continuous for every measurable and bounded function \( \kappa \), which is continuous in \( u_k \) for every \( y_k \).

Unlike the weak convergence topology, \( \kappa \) need not be continuous in \( y_k \) (see [35]). Since the marginals on observations are fixed, under the w-s topology, the convergence coincides with that in the weak convergence topology, i.e., the convergence of probability measures is the weak convergence (see [35, Theorem 3.10]). Hence, we can view convergence in (4) in terms of w-s topology without any loss of generality.

1) Randomized policies for \( \mathcal{P}_N \): The above formulation of relaxed policies for \( \Gamma_N := \prod_{k=1}^N \Gamma_k \) allows us to introduce the set randomized policies \( L_N := \mathcal{P}(\Gamma_N) \) for each PL as a collection of Borel probability measures on \( \Gamma_N \), where Borel \( \sigma \)-field \( \mathcal{B}(\Gamma_N) \) is induced by the topology in (4). In the following, we introduce different sets of randomized policies by allowing common and individual randomness among DMs. Let \( L_{n,CO} \) be

\[
\begin{align*}
&\left\{ P_\pi \in L_N \right\} \forall A_k \in \mathcal{B}(\Gamma_k) : P_\pi(\gamma_k \in A_1, \ldots, \gamma_N \in A_N) \\
= &\int_{z^i \in [0,1]} \left[ \prod_{k=1}^{N_i} P_{\pi,k}(\gamma_k \in A_k[z^i]) \eta_k(dz^i) \right] \mathcal{P}([0,1]),
\end{align*}
\]

where \( \eta_k \) is the distribution of common randomness (independent from the intrinsic exogenous system random variables). In the above, for every fixed \( z^i \), \( P_{\pi,k} \in \mathcal{P}(\Gamma_k) \) corresponds to an independent randomized policy for each DM \( \Gamma_k \) \( (k \in N_i) \) and \( i \in M \). Since \( L_{n,CO} \) and \( L_{n,Ex} \) are equal (see [27, Theorem A.1]), \( L_{n,Ex} \) corresponds to randomized policies induced by individual and common randomness.

We now recall the definition of exchangeability.

**Definition 3:** Random variables \( x^{1:m} \) defined on a common probability space are \( m \)-exchangeable if for any permutation \( \sigma \) of the set \( \{1, \ldots, m\} \), \( \mathcal{L}(x^{\sigma(1):\sigma(m)}) = \mathcal{L}(x^{1:m}) \), where \( \mathcal{L}(\cdot) \) denotes the (joint) law of random variables.
and the symmetric ones by

\[ L_{PR,SYM}^i \triangleq \left\{ \pi \in \mathcal{L}^i \mid \forall \omega_k \in \mathcal{B}(\Gamma^i_k) : \pi(\omega_k) \in \mathcal{P}(\Gamma^i_k) \right\} \]

We note that \( L_{PR}^i \) and \( L_{PR,SYM}^i \) are not convex sets; however, \( L_{PR}^i \) contains the set of extreme points of the convex set \( L_{CO}^i \).

In the following, we first present two lemmas on convexity, compactness, and relationships between the preceding set of exchangeable randomized policies. We use the following lemmas, for our main results in Theorems 1 and 2.

**Lemma 1:** Consider \( L^{i}_{EX,N}, L^{i}_{CO,SYM,N}, L^{i}_{EX}, \) and \( L^{i}_{CO,SYM} \). Then, the following holds:

1. \( L^{i}_{CO,SYM} = L^{i}_{EX} \).
2. \( L^{i}_{CO,SYM,N} \subseteq L^{i}_{EX,N} \); however, in general \( L^{i}_{CO,SYM,N} \neq L^{i}_{EX,N} \).

**Lemma 2:** Let \( U^i \) be compact, then \( L^{i}_{EX,N} \) and \( L^{i}_{EX} \) are convex and compact.

### B. Existence of an exchangeable NE for \( \mathcal{P}_N \)

The expected costs \( J^i_{\pi,N}(P^1_\pi, P^2_\pi) \) for \( \mathcal{P}_N \) under \( (P^1_\pi, P^2_\pi) \in L^{i}_{N} \times L^{i}_{N} \) are given by

\[
J^i_{\pi,N}(P^1_\pi, P^2_\pi) := \limsup_{N \to \infty} \int L^i \cdot (dy) \cdot (dy) \cdot \mu^N \cdot (dy_0, dy) \cdot c^i_N(y, y_0, \omega_0),
\]

where \( \mu^N \) is the joint probability measure of \( \omega_0, y_1^{N_1}, y_2^{N_2} \), and \( c^i_N(y, y_0, \omega_0) \) is continuous and (uniformly) bounded for all \( \omega_0 \).

Since i.i.d. random variables are (in)exchangeable, \( \mathcal{P}_N \) implies Assumptions 5 (iii) and (iv). The following theorem establishes the existence of an exchangeable NE of \( \mathcal{P}_N \).

**Theorem 1:** Consider the game \( \mathcal{P}_N \) with a given IS. Let Assumptions 1, 2, and 5 hold. Then, there exists an exchangeable NE \( (P^1_{\pi}, P^2_{\pi}) \) for \( \mathcal{P}_N \) among all policies in \( L^{i}_{N} \times L^{i}_{N} \), i.e., \( (P^1_{\pi}, P^2_{\pi}) \in L^{i}_{EX,N} \times L^{i}_{EX,N} \).

Theorem 1 only guarantees the existence of an exchangeable NE which might not be symmetric since not all finite exchangeable random variables are i.i.d. (see Lemma 1). We cannot guarantee the existence of a symmetric NE since restricting to symmetric policies in finding the best response policies might be with a loss for the DMs within the player. This is because fixing the policies of other players to symmetric or exchangeable policies, DMs within a deviating player face a team problem that might not admit a symmetric optimal solution; e.g., see [27, Example 1].

### C. Existence of a symmetric NE for \( \mathcal{P}_\infty \)

The expected costs \( J^i_{\pi,N} \) for \( \mathcal{P}_N \) with the MF interaction under a randomized policy profile \( (P^1_\pi, P^2_\pi) \in L^{i}_{N} \times L^{i}_{N} \) can be defined similar to (6) with

\[
c_N^i(y, y_0, \omega_0) = \int L^{i} \cdot (dy) \cdot (dy) \cdot \mu^N \cdot (dy_0, dy) \cdot c_N^i(y_0, y, \omega_0).
\]

where \( \mu^N \) is the marginal of \( \omega_0, y_1^{N_1}, y_2^{N_2} \) to the first \( N_1 \) components, and \( \mu^N \) is the distribution of \( (\omega_0, y_1^{N_1}, y_2^{N_2}) \).

The following theorem establishes existence of an NE for \( \mathcal{P}_\infty \) that is symmetric and independent.

**Theorem 2:** Consider the game \( \mathcal{P}_\infty \) with a given IS. Under Assumptions 2, 3, and 4, there exists an independently randomized symmetric NE profile \( (P^1_{\pi}, P^2_{\pi}) \) for \( \mathcal{P}_\infty \) among all policies in \( L^{i}_{1} \times L^{i}_{2} \), i.e., \( (P^1_{\pi}, P^2_{\pi}) \in L^{i}_{PR,SYM} \times L^{i}_{PR,SYM} \).

In contrary to Theorem 1 where we established the existence of exchangeable randomized policies, in Theorem 2, we established the existence of a symmetric randomized NE with only independent randomness. The reason is because, the set of randomized policies with only independent randomness is not convex, and hence, the Kakutani-Fan-Glicksberg fixed point theorem does not apply. However in the limit as the number of DMs drives to infinity, thanks to the [27, Lemma 2 and Theorem 1], we consider a single DM as a representative DM for each team with countably infinite number of DMs whose apply a symmetric policy. This allows us to use the Kakutani-Fan-Glicksberg fixed point theorem on the corresponding set of randomized policies with independent randomization (for a single DM), which is convex and compact.

### D. Approximations of a symmetric NE for \( \mathcal{P}_N \)

By Theorem 1, there exists an exchangeable NE for \( \mathcal{P}_N \) with exchangeable costs. This NE is not necessarily symmetric, independently randomized, and varies by the number of DMs in the game. On the other hand, Theorem 2 established the existence of a NE for \( \mathcal{P}_\infty \) that is symmetric and independently randomized. We now address the following question: Does there exist a scalable (with respect to the number of DMs) approximate NE for \( \mathcal{P}_N \) with the MF interaction that is...
symmetric and independently randomized? We answer this question in affirmative by showing that a symmetric independently randomized NE for \( \mathcal{P}_\infty \) constitutes an approximate NE for \( \mathcal{P}_N \).

**Theorem 3:** Consider the games \( \mathcal{P}_N \) and \( \mathcal{P}_\infty \) with a given IS. Let Assumptions 2, 3, and 4 hold. Then, an independently symmetric randomized NE for \( \mathcal{P}_\infty \) constitutes an \( \epsilon_{N_1,2} \)-NE for \( \mathcal{P}_N \) among all policies in \( L_1 \times L_N \), where \( \epsilon_{N_1,2} \rightarrow 0 \) as \( N_1, N_2 \rightarrow \infty \).

### E. Exchangeable dynamic games

A class of exchangeable stochastic dynamic games among teams has also been studied in the full paper [29], where under technical conditions generalizing the change of measure argument over teams acting in multiple stages and further continuity of dynamics, it has been shown that results here can be generalized to the exchangeable dynamic games. Due to page limitations, we only have focused on static games.

### IV. Conclusion

In this paper, we have studied stochastic static MFGs among teams with finite and infinite numbers of DMs. We have established the existence of a NE for exchangeable games among teams with finite number of DMs, and have shown that this NE is exchangeable. For MFGs among teams with infinite number of DMS, we have established the existence of a NE for exchangeable games among teams with finite and infinite numbers of DMs. We have established the existence of an approximate NE that exhibits symmetry and is independently randomized for games among teams with MF interaction and finite number of decision makers, using a symmetric NE of the corresponding MFG that is independently randomized.

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