EXPLORING THE COSMIC REIONIZATION EPOCH IN FREQUENCY SPACE: AN IMPROVED APPROACH TO REMOVE THE FOREGROUND IN 21 cm TOMOGRAPHY

JINGYING WANG1, HAIGUANG XU1, TAO AN2, JUNHUA GU3, XUEYING GUO4, WEITIAN LI1, YU WANG2, CHENGZE LIU1, OLIVIER MARTINEAU-HUYNH3,4, AND XIANG-PING WU3

1 Department of Physics, Shanghai Jiao Tong University, 800 Dongchuan Road, Minhang, Shanghai 200240, China; hxgu@sjtu.edu.cn, zishi@sjtu.edu.cn
2 Shanghai Astronomical Observatory, Chinese Academy of Sciences, Nandan Road 80, Shanghai 200030, China
3 National Astronomical Observatories, Chinese Academy of Sciences, 20A Datun Road, Beijing 100012, China
4 Laboratoire de Physique Nucléaire et de Physique des Hautes Energies, CNRS/IN2P3 and Université Pierre et Marie Curie, Tour 12-22, 4 place Jussieu, F-75005 Paris, France

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ABSTRACT

With the intent of correctly restoring the redshifted 21 cm signals emitted by neutral hydrogen during the cosmic reionization processes, we re-examine the separation approaches based on the quadratic polynomial fitting technique in frequency space in order to investigate whether they work satisfactorily with complex foreground by quantitatively evaluating the quality of restored 21 cm signals in terms of sample statistics. We construct the foreground model to characterize both spatial and spectral substructures of the real sky, and use it to simulate the observed radio spectra. By comparing between different separation approaches through statistical analysis of restored 21 cm spectra and corresponding power spectra, as well as their constraints on the mean halo bias b and average ionization fraction x_e of the reionization processes, at z = 8 and the noise level of 60 mK we find that although the complex foreground can be well approximated with quadratic polynomial expansion, a significant part of the Mpc-scale components of the 21 cm signals (75% for 6 h^{-1} Mpc scales and 34% for 1 h^{-1} Mpc scales) is lost because it tends to be misidentified as part of the foreground when the single-narrow-segment separation approach is applied. The best restoration of the 21 cm signals and the tightest determination of b and x_e can be obtained with the three-narrow-segment fitting technique as proposed in this paper. Similar results can be obtained at other redshifts.

Key words: cosmology: theory – dark ages, reionization, first stars – early universe – methods: data analysis – methods: statistical – radio lines: general

Online-only material: color figures

1. INTRODUCTION

In the late stage of the dark ages of the universe, neutral hydrogen in the intergalactic medium began to be reionized by UV and soft X-ray photons (Baek et al. 2010) emitted by the first-generation sources, such as stars and/or quasars (see Morales & Wyithe 2010 for a recent review). This period, which started sometime from about 150 Myr to 500 Myr after the big bang, is known as the epoch of reionization (EoR), and is one of the few most important milestones in the evolution histories of both our universe and galaxies. The study of the line emission of neutral hydrogen at 21 cm (21 cm signals hereafter) coming from the surroundings of ionizing sources in EoR provides us with a novel opportunity to reveal the properties of the first-generation stars and/or quasars, as well as to constrain cosmological models (e.g., Iliev et al. 2002; Furlanetto et al. 2004a; Santos & Cooray 2006; Morales & Wyithe 2010). EoR is expected to be a major center of interest within the next one or two decades for the facilities working in the low-frequency radio band (≲1 GHz), such as the 21 Centimeter Array (21CMA)5, the Low Frequency Array (LOFAR)6, the Murchison Wide-field Array (MWA)7, and the Square Kilometer Array (SKA)8.

One of the serious challenges in observing the 21 cm signals from EoR is that they are extremely weak (∼10 mK). Having been redshifted to the low-frequency radio band (50–200 MHz, for 6 ≤ z ≤ 27; Morales & Wyithe 2010; Field 1959a, 1959b; Scott & Rees 1990; Madau et al. 1997), the 21 cm signals are drowned in the luminous foreground, which consists of our Galaxy, extragalactic discrete sources (e.g., radio galaxies and active galactic nuclei, i.e., AGNs), galaxy clusters, and other relatively minor contaminating sources. Theoretical and numerical efforts have been made in the past few years to probe how to correctly restore the redshifted 21 cm signals from this overwhelmingly luminous foreground, e.g., extraction of the 21 cm signals in either the angular power spectrum space (e.g., Di Matteo et al. 2002, 2004; Santos et al. 2005; McQuinn et al. 2006) or the frequency space (e.g., Wang et al. 2006; Gleser et al. 2006; Jelíč et al. 2008; Bowman et al. 2009; Harker et al. 2009b; Liu et al. 2009; Petrovic & Oh 2011), and both have been made on the a priori assumption that the integral foreground emission can be well described as a smooth continuum.

For the separation works in the frequency space, the total spectrum is usually fitted with a smooth function (e.g., an n-order polynomial model log T_{\text{fore}}(\nu) = a_0 + \sum_{i=1}^{N_{\text{poly}}} a_i \log \nu, where i = 1, 2, ..., N_{\text{poly}}) to describe the integral foreground brightness temperature T_{\text{fore}}(\nu). However, the first thing one should keep in mind is that the foreground models adopted by previous works are more or less relatively simple, e.g., extragalactic discrete sources are treated as point sources, showing none of the substructures, such as lobes and jets, that are often observed in nearby FRI and FRII radio galaxies, and/or possessing simple, featureless power-law radio spectra. Meanwhile, galaxy clusters, which contribute to the foreground at a non-negligible level compared with AGNs, are not taken into account or treated as simple power-law spectrum sources.
Revisiting the separation approach with more complex and more realistic foreground models is necessary. Second, a variety of theoretical and numerical works (e.g., Furlanetto et al. 2004b; Wyithe & Loeb 2004; Furlanetto & Oh 2005; Petrovic & Oh 2011) have already indicated that in the late phases of cosmic reionization, the typical sizes of ionized regions are of the order of a few tens of comoving Mpc, so that the 21 cm signals possess slowly varying components at ~1 MHz scales in frequency space due to a specific distribution of H II bubble sizes. For example, Mesinger & Furlanetto (2007) predicted that at \( z = 8 \), the diameter distribution of the H II bubble is peaked at \( D_{\text{bubble}}^{\text{peak}} |_{z=8} \approx 16 \) Mpc (corresponding to a frequency span of \( \Delta \nu_{\text{bubble}} \approx 0.9 \) MHz), along with a significant tail out to \( D_{\text{bubble}}^{\text{max}} |_{z=8} \approx 60 \) Mpc (\( \Delta \nu_{\text{bubble}} \approx 3.4 \) MHz). Apparently, the non-negligible, slowly varying 21 cm components contributed by these giant bubbles should have been entangled with the foreground in frequency space, and most of them are bound to be abandoned in foreground subtraction, especially when a single-narrow-frequency segment (e.g., \( \Delta \nu_{\text{segment}} \approx 2 \) MHz as used in Wang et al. 2006) is adopted in the quadratic polynomial fittings. In fact, there is a significant power loss on Mpc scales, as will be shown in Section 3.2.1.

In order to address the problems raised above, we attempt to re-examine and improve the polynomial fitting algorithm by applying a more complex foreground emission model that contains detailed emulations of both spatial and spectral features of the real sources (see Wang et al. 2010 for details, which followed Snellen et al. 2000, Giardino et al. 2002, Finkbeiner 2003, and Wilman et al. 2008 and references therein; W2010 hereafter), meanwhile employing a multi-narrow-frequency segment quadratic polynomial fitting technique to remove the foreground emission components in frequency space. We find that our new algorithm can reasonably correct the systematic bias introduced by the single-narrow-segment algorithm, which may lead to a reduction of the 21 cm signal power by about 75% for \( \gtrsim 6 h^{-1} \) Mpc scales and 34% for \( \gtrsim 1 h^{-1} \) Mpc scales, and therefore a significant underestimate of both the H II bubble size and growth rate. We also prove that at \( z = 0.5 \), our algorithm is efficient and reliable in restoring mass distribution on \( \gtrsim 30 h^{-1} \) Mpc scales (b contrast about 19% power will be lost with the single-segment approach), and thus it can be applied to extract the 21 cm signals emitted at intermediate redshifts to investigate the baryon acoustic oscillations (BAOs), which exhibit a typical scale of \( \approx 150 \) comoving Mpc (Morales & Wyithe 2010; Ansari et al. 2012).

Parallel to the polynomial fitting technique, a series of works have recently been published in an attempt to develop non-parametric techniques and apply them in observing the 21 cm signals, which do not assume a specific form for the contaminating foregrounds. Using the Wp smoothing method, Harker et al. (2009a, 2010) preferentially consider foreground models with as fewer inflection points as possible, which when applied to simulated LOFAR-EoR data, compare very favorably with parametric methods. FastICA, as presented in Chapman et al. (2012), accurately recovers the 21 cm power spectra by considering the statistically independent components of the foregrounds. With Generalized Morphological Component Analysis, Chapman et al. (2012) not only recover the power spectra to high accuracy, but also recover the simulated 21 cm signal maps exceedingly well using wavelet decomposition. A more detailed discussion of the non-parametric techniques can be found in the works listed above, and will not be presented here because it is beyond the scope of this paper.

Throughout this work, we adopt \( h = 0.71 \), \( \Omega_M = 0.27 \), \( \Omega_{\Lambda} = 0.73 \), and \( \Omega_b = 0.044 \). Unless otherwise stated, all errors are quoted at 68% confidence level.

2. SIMULATION OF LOW-FREQUENCY RADIO SPECTRA

2.1. 21 cm Signals of Neutral Hydrogen from EoR

According to Wang et al. (2006) and references therein, the three-dimensional (3D) power spectrum of the 21 cm signals from EoR at redshift \( z \) should take the form of

\[
P_{3D,21 \text{ cm}}(k, z) = \left(0.016 \text{ K}^2 \right)^\text{2} \frac{1}{h^2} \left( \frac{\Omega_b h^2}{10 \Omega_M} \right)^2 \left(1 + z\frac{0.3}{1.5}\right) \times \left(1 - x_e(z)\right)^2 + b(z) e^{-k^2 R^2(z)} x_e(z) \right) P_{3D, \text{ matter}}(k, z).
\]

where \( P_{3D, \text{ matter}}(k, z) \) is the 3D matter power spectrum at redshift \( z \), \( x_e(z) \) is the average ionization fraction, \( b(z) \) is the mean halo bias (the mean ratio of the mass overdensities of halo to field weighted by different halos), and \( R(z) = 100 \left(1 - x_e(z)\right)^{-1/3} \) kpc is the mean radius of the ionized patches in H II regions. To calculate \( P_{3D,21 \text{ cm}}(k, z) \), we determine \( b(z) \) by adopting the parameters used in the fiducial reionization model of Santos et al. (2003, 2005). At any given redshift \( z_0 \), we consider a redshift segment defined between \( z_0 \) and \( z_0 + \Delta z_{\text{segment}} \) (corresponding to the reference frequency \( v_0 \) by \( v_0 - \Delta \nu_{\text{segment}} < v \leq v_0 \) for the redshifted 21 cm signals), where \( \Delta z_{\text{segment}} \ll 1 \) so that \( P_{3D,21 \text{ cm}}(k, z_0) \) is roughly uniform and isotropic within the segment \( \Delta z_{\text{segment}} \) or \( \Delta z_{\text{segment}} \) (i.e., the impact of evolution effects can be ignored across \( \Delta z_{\text{segment}} \)). In this narrow redshift range, we are allowed to convert the derived \( P_{3D,21 \text{ cm}}(k, z_0) \) into the one-dimensional (1D) power spectrum using the integration formula

\[
P_{1D,21 \text{ cm}}(k) = \int_0^{\infty} P_{3D,21 \text{ cm}}(k', z_0) k' dk'.
\]

(Peacock 1999). The 1D power spectra are shown in Figure 1 for some typical redshifts. The spectrum (i.e., line-of-sight distribution) of the redshifted 21 cm signals in the segment \( v_0 - \Delta \nu_{\text{segment}} < v \leq v_0 \) is then calculated through the inverse Fourier transform

\[
T_{21 \text{ cm}}(x_n) = \frac{\sqrt{2\pi}}{L} \sum_{q=0}^{N-1} \left[ A_q (P_{1D,21 \text{ cm}}(k, z_0)) \cos \left( \frac{2\pi q}{L} x_n \right) + B_q (P_{1D,21 \text{ cm}}(k, z_0)) \sin \left( \frac{2\pi q}{L} x_n \right) \right],
\]

where \( L \) is the comoving space scale corresponding to \( \Delta z_{\text{segment}} \), the terms \( A_q (P_{1D,21 \text{ cm}}(k, z_0)) \) and \( B_q (P_{1D,21 \text{ cm}}(k, z_0)) \) are independent and random parameters sharing the same Gaussian distribution \( N(0, \sqrt{P_{1D,21 \text{ cm}}(k, z_0)/2}) \). \( k = 2\pi q/L \) is the wave number, \( x_n (n = 1, \ldots, N) \) is the line-of-sight location related to frequency \( v_0 \) under linear approximation in frequency space, and \( N \) is the number of frequency channels for the segment chosen to be large enough so that most of the information of \( P_{1D,21 \text{ cm}}(k, z) \) is included within the range \( 2\pi L/ \leq k \leq 2\pi N/L \). We adopt a segment width of \( \Delta \nu_{\text{segment}} = 2 \text{ MHz} \) and \( N = 50 \); therefore, the frequency resolution (i.e., the width of each frequency channel) is \( d\nu = \Delta \nu_{\text{segment}} / N = 40 \text{ kHz} \), which is a sufficiently high frequency resolution in order to reconstruct the local power spectrum for the give \( \Delta z_{\text{segment}} \) at \( z \).
Table 1
Foreground Emission Components at 150 MHz (Spatial Resolution = 12 arcmin²)

| Component                                | Mean Value (K) | rms (K) |
|------------------------------------------|----------------|---------|
| Galaxy synchrotron emission              | 237            | 24      |
| Galaxy free–free emission                | 0.877          | 0.205   |
| Galaxy clusters                          | 0.084          | 2.06    |
| Extragalactic discrete sources           | 88             | 7937    |
| Extragalactic discrete sources (under $S_{\text{cut}}$) | 23             | 29      |
| Total foreground                         | 326            | 7937    |
| Total foreground (with discrete sources under $S_{\text{cut}}$) | 261            | 38      |

2.2. Foreground Emission Components and Instrumental Noise

2.2.1. Foreground Components Simulation

The foreground emission components that contaminate the 21 cm signals have been continuously discussed and simulated in detail in many works (e.g., Jelić et al. 2008, 2010; Gleser et al. 2008; Bowman et al. 2009). In our previous work (W2010), we fully took into account the effects of random variations of model parameters in the ranges allowed by the observations in Monte Carlo simulations to model the emissions from our Galaxy, galaxy clusters, and discrete sources (i.e., star-forming galaxies, radio-quiet AGNs, and radio-loud AGNs), and constructed the 50–200 MHz radio sky maps with a frequency resolution of 40 kHz, following the works of Snellen et al. (2000), Giardino et al. (2002), Finkbeiner (2003), Wilman et al. (2008), and reference therein. All of the simulated sky maps are plotted in a circular field of view (FOV) with a radius of 5° centered at the north celestial pole, as is currently being observed with the 21CMA array, and the obtained images are pixelated with grids of 1024 × 1024 pixels, each image pixel covering approximately a 0.6 × 0.6 patch. We adopt the same model to create the contaminating foreground emission components used in this paper. Compared with Wang et al. (2006), the size of each sky pixel is fixed to 12 arcmin², which is no smaller than the area covered by the point-spread function of any operating or upcoming facility designed to detect EoR signals. We note that the foreground simulations in Wang et al. (2006) did not include discrete sources brighter than 0.1 mJy at 150 MHz, because the authors believed that all of them could be identified and safely removed before the signal separation step. In our work, however, we assume a conservative flux cut of 10 mJy at 150 MHz, since technically it is more realistic with current instrumentations (Liu et al. 2009; Pindor et al. 2011). The brightness temperatures of the simulated foreground at 150 MHz are listed in Table 1.

2.2.2. Instrument and Noise

The 21CMA array is a low-frequency radio interferometer array constructed in a remote area of Xinjiang, China. The array consists of 81 telescopes that are distributed in a “T” shape with the north–south and east–west baselines being 6 km and 4 km, respectively (Figure 2), and each telescope is an assembly of 127 logarithmic periodic antennas. The 21CMA array is designed to continuously observe the same patch of sky around NCP (i.e., 24 hr a day). The integration $uv$ coverage of 24 hr is shown
The thermal noise of 21CMA can be expressed in terms of brightness temperature as

$$\sigma_{\text{noise}} = \frac{T_{\text{sys}} \lambda^2}{A_{\text{eff}} \Omega / n(n-1) \tau d\nu}$$

(e.g., Thompson et al. 2001; W2010), where $T_{\text{sys}}$ is the system temperature, $\lambda$ is the wavelength, $A_{\text{eff}}$ is the effective area of a single radio telescope, $\Omega$ is the beam solid angle, $\tau$ is the effective integral time, and $n$ is the number of single radio telescopes. For 21CMA, we have $T_{\text{sys}} = 300$ K, $A_{\text{eff}} = 218$ m$^2$, and $n = 81$ (Table 2), thus

$$\sigma_{\text{noise}} = 425 \text{ mK} \left(\frac{\lambda}{2 \text{ m}}\right)^2 \left(\frac{1 \text{ MHz}}{d\nu}\right)^{1/2} \left(\frac{30 \text{ days}}{\tau}\right)^{1/2} \left(\frac{1 \times 10^{-7} \text{ sr}}{\Omega}\right).$$

Assuming a typical 21CMA survey of $d\nu = 40$ kHz, $\tau = 1$ yr, and $\Omega = 1 \times 10^{-6} \text{ sr}$ (i.e., 12 arcmin$^2$), the noise is $\sigma_{\text{noise}}|_{\nu=157.8 \text{ MHz}} = 60 \text{ mK}$ at $\lambda = 1.9$ m (corresponding to $\nu = 157.8 \text{ MHz}$), where the 21 cm signals emitted at $z = 8$ are expected to appear. Under the same observation conditions, the noise at any frequency is given as $\sigma_{\text{noise}}(\nu) = 60 \text{ mK}(\nu/157.8 \text{ MHz})^{-2}$. Following McQuinn et al. (2006), Chapman et al. (2012), and Zaroubi et al. (2012), the noise at each frequency $\nu$ is simulated as a Gaussian random field in the 21CMA $uv$ plane, and then transformed to the image plane to obtain the noise $T_{\text{noise}}(\nu)$, whose root-mean-square (rms hereafter) value is normalized to $\sigma_{\text{noise}}(\nu)$.

### 3. SIGNAL SEPARATION

#### 3.1. Single-segment Fits to Pure Complex Foreground

To extract the foreground components, $n$-order polynomial models are usually applied in the frequency space, e.g., Wang et al. (2006) performed a logarithmic form

$$\log T_{\text{fore}}'(\nu) = a_0 + a_1 \log \nu + a_2 \log^2 \nu$$

(6)

to approximate the integral foreground in a single segment with a width of $\Delta \nu_{\text{segment}} \approx 2 \text{ MHz}$ for $z_0 = 8$, where $a_i (i = 0, 1, 2)$ are the parameters that are all obtained by applying the least-squares fitting method. To determine if this approximation is appropriate when a complex foreground is introduced, and if it can be applied to a wider frequency segment of a width $\sim 10 \text{ MHz}$, as will be discussed in Sections 3.2 and 4.1, we fit the pure simulated integral foreground in the logarithmic frequency space using the linear polynomial (log $T_{\text{fore}}'(\nu) = a_0 + a_1 \log \nu$), the quadratic polynomial (Equation (6)), and the cubic polynomial (log $T_{\text{fore}}'(\nu) = a_0 + a_1 \log \nu + a_2 \log^2 \nu + a_3 \log^3 \nu$), respectively. Each of the three polynomial fittings is applied to 1000 randomly selected sky pixels on the simulated foreground map with different segment widths (i.e., $\Delta \nu_{\text{segment}} = 2i \text{ MHz}$ for $z_0 = 8$, where $i = 1, 2, \ldots, 40$) with a central frequency.
located at $v_0 = 1420.4$ MHz/(1 + $z_0$). As shown in Figure 4, the intensity residuals of linear polynomial fittings can reach up to $\sim 10$ mK in many cases, which is comparable with the intensity of the 21 cm signals, whereas the residuals of quadratic polynomial fittings ($\lesssim 1$ mK) are small enough to be negligible when compared to the intensities of both the instrumental noises and the 21 cm signals. Since the quadratic polynomial fitting is good enough to approximate the complex foreground for a large range of $\Delta^{\text{segment}}$, we decide that the cubic one with even smaller residuals is unnecessary in our work.

In Figure 4, it is also shown that the residuals, although statistically small enough to make the quadratic polynomial fittings acceptable when $\Delta^{\text{segment}} < 100$ MHz, increase as the segment broadens. We ascribe this to the fact that our foreground model (for details see Section 2.2.1 and W2010) is complex, and thus a broader segment means that the more complicated behaviors of the foreground have been involved in causing larger uncertainties. We note that the clustering of the foreground model (for details see Section 2.2.1 and W2010) does not lead to larger uncertainties. We note that the clustering of the foreground has been taken into account in our sky simulation, does not lead to causing larger uncertainties. We note that the clustering of the foreground model (for details see Section 2.2.1 and W2010) does not lead to larger uncertainties.

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As a first step, we apply the separation approach on a single segment with a narrow width of $\Delta^{\text{segment}} = 2$ MHz (the channel width $\Delta v = \Delta^{\text{segment}}/N = 2$ MHz/50 = 40 kHz) in order to examine if the information on the fluctuations of the 21 cm signals on Mpc scales is lost when our complex foreground model is applied. The segment width of 2 MHz is adopted to guarantee that the power spectrum of the 21 cm signals is roughly uniform within the corresponding redshift segment ($\Delta^{\text{segment}} \gtrsim 0.1$) and $P_{1D,21\ cm}(z)$ can be measured accurately.

We fit the total (foreground + noise + 21 cm signals) brightness–temperature spectrum $T_{\text{tot}}(v)$ with Equation (6) to determine the best-fit foreground $T_{\text{fore}}(v)$ and derive the residuals (raw spectrum hereafter) $T'_{21\ cm}(v) \equiv T_{\text{tot}}(v) - T_{\text{fore}}(v)$, which contain the 21 cm signals and a dominating noise component (i.e., $T'_{21\ cm}(v) = T_{21\ cm}(v) + T_{\text{noise}}(v)$). Then, we transform the raw spectrum into the corresponding power spectrum. By subtracting the theoretically predicted noise component from the averaged power spectrum for a sufficiently large number of sky pixels, we may obtain a mean restored power spectrum for the 21 cm signals (e.g., Harker et al. 2010; Chapman et al. 2012). When comparing the results with the input 1D power spectra created in the simulations (Section 2.1; Figure 1), we find that when 10,000 sky pixels (12 arcmin$^2$ for each) that cover about 1/3 FOV of the 21CMA (about 33 deg$^2$) are in use, the power spectrum of the 21 cm signals can be well recovered on small scales of $\lesssim 3$ h$^{-1}$ Mpc, whereas on larger scales the deviation is huge (reduced $\chi^2 = 5196$; Figure 5(a)). We measure the relative Mpc-scale power restoration in terms of the ratio of restored to simulated powers as $w_{k<1h/Mpc} = (P_{21\ cm}^1/P_{21\ cm}^0)_{k<1h/Mpc} = \left(\int_{2\pi/L} P_{21\ cm}^1(k)dk/\int_{2\pi/L} P_{21\ cm}^0(k)dk\right)_{k<1h/Mpc}$, where $P_{21\ cm}^0(k) = \int_{2\pi/L} P_{1D,21\ cm}^0(k)dk$ is the Mpc-scale power of the input 21 cm signals. We find that the mean value is

3.2. Separation at Noise Level of 60 mK

The ideal low noise level as applied in some previous works (e.g., Wang et al. 2006) might not be achieved until the next generation facilities have been built. For currently operating facilities, such as 21CMA and LOFAR, the noise will dominate the 21 cm signals from the EoR, so that direct restoration of the 21 cm signals is precluded (e.g., McQuinn et al. 2006; Gleser et al. 2008; Bowman et al. 2009; Harker et al. 2009a; Chapman et al. 2012). To study this case, here we assume a noise level of $\sigma_{\text{noise}}|_{z=8} = 60$ mK, which is achievable with a one-year observation of 21CMA (Section 2.2.2), and investigate how to restore the 21 cm signals.

3.2.1. Separation in Single-narrow-frequency Segment

As a first step, we apply the separation approach on a single segment with a narrow width of $\Delta^{\text{segment}} = 2$ MHz (the channel width $\Delta v = \Delta^{\text{segment}}/N = 2$ MHz/50 = 40 kHz) in order to examine if the information on the fluctuations of the 21 cm signals on Mpc scales is lost when our complex foreground model is applied. The segment width of 2 MHz is adopted to guarantee that the power spectrum of the 21 cm signals is roughly uniform within the corresponding redshift segment ($\Delta^{\text{segment}} \gtrsim 0.1$) and $P_{1D,21\ cm}(z)$ can be measured accurately.

We fit the total (foreground + noise + 21 cm signals) brightness–temperature spectrum $T_{\text{tot}}(v)$ with Equation (6) to determine the best-fit foreground $T_{\text{fore}}(v)$ and derive the residuals (raw spectrum hereafter) $T'_{21\ cm}(v) \equiv T_{\text{tot}}(v) - T_{\text{fore}}(v)$, which contain the 21 cm signals and a dominating noise component (i.e., $T'_{21\ cm}(v) = T_{21\ cm}(v) + T_{\text{noise}}(v)$). Then, we transform the raw spectrum into the corresponding power spectrum. By subtracting the theoretically predicted noise component from the averaged power spectrum for a sufficiently large number of sky pixels, we may obtain a mean restored power spectrum for the 21 cm signals (e.g., Harker et al. 2010; Chapman et al. 2012). When comparing the results with the input 1D power spectra created in the simulations (Section 2.1; Figure 1), we find that when 10,000 sky pixels (12 arcmin$^2$ for each) that cover about 1/3 FOV of the 21CMA (about 33 deg$^2$) are in use, the power spectrum of the 21 cm signals can be well recovered on small scales of $\lesssim 3$ h$^{-1}$ Mpc, whereas on larger scales the deviation is huge (reduced $\chi^2 = 5196$; Figure 5(a)). We measure the relative Mpc-scale power restoration in terms of the ratio of restored to simulated powers as $w_{k<1h/Mpc} = (P_{21\ cm}^1/P_{21\ cm}^0)_{k<1h/Mpc} = \left(\int_{2\pi/L} P_{21\ cm}^1(k)dk/\int_{2\pi/L} P_{21\ cm}^0(k)dk\right)_{k<1h/Mpc}$.
simply replacing our foreground with that of Wang et al. (2006) caused by the complex foreground employed in this work by single-narrow-segment quadratic polynomial fittings is applied. 2011), a part of the 21 cm signals is bound to be abandoned (Section 3.1), we propose to apply a multi-narrow-frequency ranges much wider than the largest scales of the 21 cm power spectra in each case are also plotted in the lower panels. (A color version of this figure is available in the online journal.)

\[
\tilde{w}|_{k<1 h/\text{Mpc}} = 0.25 \pm 0.04 \text{ for the total 10,000 cases, which indicates that the use of the single-narrow-segment quadratic polynomial fitting technique may cause an average power loss on Mpc scales (i.e., } \tilde{w}_{\text{loss}}|_{k<1 h/\text{Mpc}} = 1 - \tilde{w}|_{k<1 h/\text{Mpc}} \text{) of about 75% in the redshifted 21 cm signals. Since the existence of such Mpc-scale structures is expected to be quite common in the late phases of EoR (e.g., Furlanetto et al. 2004b; Petrovic & Oh 2011), a part of the 21 cm signals is bound to be abandoned along with the foreground if a separation algorithm based on single-narrow-segment quadratic polynomial fittings is applied. We exclude the possibility that such a Mpc-scale bias is caused by the complex foreground employed in this work by simply replacing our foreground with that of Wang et al. (2006) and obtaining similar results. An enlargement of } \Delta v_{\text{segment}} \text{ to the order of 10 MHz has been suggested in some previous works (e.g., Harker et al. 2009a; Petrovic & Oh 2011) to mitigate the Mpc-scale bias. However, as will be discussed in Sections 3.2.3 and 4, we do not suggest this method because it will introduce too many auxiliary frequency channels in the calculation, a relatively larger bias of } \tilde{w}|_{k<1 h/\text{Mpc}} \text{, and a relatively poorer determination of the physical conditions of EoR when compared with the three-narrow-segment separation approach.}

3.2.2. Separation in Three-Narrow-Frequency Segment

In the following separations, we attempt to introduce more segments and frequency channels into the spectral fittings as auxiliaries to determine the foreground more accurately. Inspired by the fact that the complex foreground can be well approximated with quadratic polynomial expansion in frequency ranges much wider than the largest scales of the 21 cm signals (Section 3.1), we propose to apply a multi-narrow-segment (hereafter multi-segment) quadratic polynomial fitting technique as an attempt to solve the problems raised in Section 3.2.1. The frequency resolution and the number of channels in each segment are } d\nu = 40 \text{ kHz and } N = 50 \text{, respectively, and the span of the segments is of the order of } 10 \text{ MHz. We carry out the foreground fittings by using the same quadratic polynomial as formulated in Equation (6) to approximate the low-frequency foreground simultaneously in three narrow segments, which are separated far away enough to guarantee that the 21 cm signals in different segments come from different ionized bubbles. Taking the redshift } z_0 = 8 \text{ (i.e., reference frequency } v_0 = 1420.4 \text{ MHz/(1 + } z_0) = 157.8 \text{ MHz) as an example, we define the three segments in such a way that each of them possesses the same bandwidth of } \Delta v_{\text{segment}} \text{ = 2 MHz as in Section 3.2.1, and they are separated by a gap measured either in redshift (i.e., a segment combination of } z_{\text{segment}} = \{z_0, z_1, z_2\}; \text{ here, } z_0 \text{ is the reference point for segment one, and } z_1 = z_0 + \Delta z_{\text{gap}} \text{ and } z_2 = z_0 - \Delta z_{\text{gap}} \text{ for segments two and three, where } \Delta z_{\text{gap}} = 0.5, 1.0, 1.5 \text{, respectively) or in frequency (i.e., a segment combination of } v_{\text{segment}} = \{v_0, v_1, v_2\}; \text{ here, } v_0 \text{ is the reference point for segment one, and } v_1 = v_0 + \Delta v_{\text{gap}} \text{ and } v_2 = v_0 - \Delta v_{\text{gap}} \text{ for segments two and three, where } \Delta v_{\text{gap}} = 10, 20, 30 \text{ MHz, respectively). We use the same method as described in Section 3.2.1 to restore the 21 cm signals and evaluate the quality of the results in } v_0 - 2 \text{ MHz } < v \leq v_0 \text{ using the reduced } \chi^2. \text{ In general, the results are significantly better than those of the single narrow segment and there exists a very weak dependence of separation quality on the segment gap size. Taking the segment combination of } z_{\text{segment}} = \{8, 9, 7\}, \text{ for example, the power spectrum of the 21 cm signals can be recovered very well with the three-narrow-segment approach (reduced } \chi^2 = 1.137 \text{ and } \tilde{w}|_{k<1 h/\text{Mpc}} \text{ = 1.00 } \pm 0.05; \text{ Figure 5(c)}, \text{ which is significantly better than that obtained with the single-narrow-segment separation } (\chi^2 = 5196 \text{ and } \tilde{w}|_{k<1 h/\text{Mpc}} \text{ = 0.25 } \pm 0.04), \text{ especially on large scales. It is found that the use of more than three segments is redundant. It does not help to improve the separation quality but lowers the efficiency of calculation.}
3.2.3. Single-segment Separations with a Larger $\Delta v_{\text{segment}}$

In order to reduce the systematic bias introduced by possible missubtraction of the Mpc-scale components in redshifted 21 cm signals in single-narrow-segment quadratic polynomial fittings, Petrovic & Oh (2011) qualitatively mentioned that a larger segment width should be adopted, especially in restoring the 21 cm signals in late reionization stages when the H ii bubble size becomes sufficiently large. A similar problem was also briefly mentioned by Wang et al. (2006), who suggested an enlargement of $\Delta v_{\text{segment}}$ by a factor of 10. To investigate if this is feasible, we repeat the single-segment separation calculations described in Section 3.2.1 for another 10000 rounds by increasing the segment width to $\Delta v_{\text{segment}} = 20$ MHz ($N = \Delta v_{\text{segment}}/dv = 500$) and examining the changes in the results. The goodness of fittings is much better than that of fittings with single-narrow-segment fittings (Table 3), which supports the ideas of Petrovic & Oh (2011) and Wang et al. (2006). However, we note that in single-wide-segment fittings, the goodness of both the reduced $\chi^2 = 1.365$ and $\bar{w}|_{k<1h/\text{Mpc}} = 0.97 \pm 0.05$ (Figure 5) are a bit worse when compared with those of our three-narrow-segment fittings (reduced $\chi^2 = 1.137$ and $\bar{w}|_{k<1h/\text{Mpc}} = 1.00 \pm 0.05$). In addition, as will be shown in Section 4.2, the single-wide-segment quadratic polynomial separation technique cannot yield the best constraints on the physical conditions of EoR.

We have also tried to enlarge $dv$ from 40 kHz to 100 kHz, which would reduce the required observing time to achieve $\sigma_{\text{noise}}|z=8| = 60$ mK. The results are shown in Figures 5(d)–(f), in which we can find that our three-narrow-segment method still works well, whereas the information for smaller scales would be lost due to the sampling reduction of frequency.

4. DISCUSSION

4.1. Further Comparison Between Different Approaches

The differences between the results obtained with single-segment and multi-segment separation approaches can be explained as follows. In the late stage of cosmic reionization (since $z \lesssim 9$ depending on models; e.g., Morales & Wyithe 2010 for a recent review; Santos et al. 2008), most of the H ii bubbles have been well developed. For example, by simulating the reionization process using the Lagrangian perturbation theory, Mesinger & Furlanetto (2007) predicted that at $z = 8$, the bubble size $D_{\text{bubble}}$ (corresponding to a frequency span of $\Delta v_{\text{bubble}}$) should range from $D_{\text{bubble}}|z=8| \sim 1$ Mpc ($\Delta v_{\text{bubble}}|z=8| \sim 0.05$ MHz) to $D_{\text{bubble}}|z=8| \sim 60$ Mpc ($\Delta v_{\text{bubble}}|z=8| \sim 3.4$ MHz) with a clear peak at $D_{\text{bubble}}|z=8| \sim 16$ Mpc ($\Delta v_{\text{bubble}}|z=8| \sim 0.9$ MHz) in number distribution. This means that the 21 cm signals can possess slowly varying components in frequency space at $z \gtrsim 1$ MHz scales. In single-narrow-segment (e.g., $\Delta v_{\text{segment}} = 2$ MHz) separations, the H ii bubble size $\Delta v_{\text{bubble}}$ is comparable to or even larger than the chosen $\Delta v_{\text{segment}}$. This makes it inevitable that a significant part of the Mpc-scale 21 cm components are treated as a part of the foreground, which is smooth on $\sim 100$ MHz scales, and then missubtracted (Figure 6). In single-segment separations with a larger $\Delta v_{\text{segment}}$, as the segment width increases to $\Delta v_{\text{segment}} \gg \Delta v_{\text{bubble}}$, the contributions of more than one H ii bubble located along line of sight are involved in the spectra to be fitted. Since the bubble and foreground components vary on quite different scales within the segment, the problem of missubtraction can be significantly mitigated, except that the power restorations for Mpc-scale components are not fully constrained (Table 3), mostly due to the appearance of giant bubbles that contribute smooth components on tens of Mpc scales. In three-narrow-segment separation fittings with narrow $\Delta v_{\text{segment}}$, the Mpc-scale 21 cm components in different segments come from different bubbles, i.e., they do not correlate with each other, whereas the foreground components in the three segments do. Therefore, the foreground and 21 cm signals are best determined, respectively.

This can also be clearly seen when the noise is sufficiently low (e.g., $\sigma_{\text{noise}}|z=8| = 1$ mK), although the realistic condition (e.g., $\sigma_{\text{noise}}|z=8| = 60$ mK) follows the same rule. Taking $\sigma_{\text{noise}}|z=8| = 1$ mK as an example, we repeat the separation using single-narrow-segment, three-narrow-segment, and single-wide-segment separation approaches with 1000 rounds, respectively. We evaluate the result of each round in a quantitative way by calculating the relative root-mean-square error as follows, $r_{\text{fit}} \equiv \sqrt{\sum (T_{\text{fit}}(\nu) - T_{\text{data}}(\nu))^2/[T_{\text{fit}}(\nu)]_{\text{MAX}}^2}$. With the single-narrow-segment approach, only 25% of the 1000 cases can yield acceptable ($r_{\text{fit}} \leq 0.1$; Figure 7 and see an example in Figure 8) results. We find that failed ($r_{\text{fit}} > 0.1$; Figure 7 and see an example in Figure 9) restorations tend to occur when the 21 cm signals possess significant large-scale ($k \lesssim 1h$ Mpc$^{-1}$) components along line of sight, which contribute a non-negligible part to the simulated 21 cm spectra (Figure 10). The best restoration of the 21 cm signals can be obtained with the three-narrow-segment fitting technique (Figures 7 and 9; Table 4), and in single-wide-segment fittings the scatter of power restoration on $\gtrsim 6h^{-1}$ Mpc scales.
Figure 6. Comparison between three separation techniques discussed in Sections 3 and 4. Upper panel: sketch diagram for the spatial distribution of ionized bubbles along line of sight at $z_0 = 8$. Lower panel: corresponding spectrum of the simulated 21 cm signals (black solid), along with the best fits (red solid: single narrow segment; green solid: single wide segment; blue star: three narrow segment) of $T_{\text{fore}} - T_{\text{true}}$, which presents the Mpc-scale components of 21 cm signals that have been entangled with the foreground. Clearly, the best-fit spectrum of the single narrow segment is significantly biased by the Mpc-scale components of the bubble, and best-fit spectrum of the single wide segment is biased by the tens of Mpc-scale components of the giant bubbles. The black dotted line marks $T_{\text{true}} - T_{\text{true}} = 0$, which is the most ideal result for the restoration of the 21 cm signals.

Figure 7. (a) Fractional distributions of the relative root-mean-square error $r_{\text{fit}}$ for the single-narrow-segment (black solid), the single-wide-segment (red square), the two-narrow-segment with $z$ gap (magenta solid; upper left to lower right are $\Delta z_{\text{gap}} = 1.5, 1.0, \ldots, -1.5$, respectively), the two-narrow-segment with $v$ gap (blue dash; upper left to lower right are $\Delta v_{\text{gap}} = -30, -20, \ldots, 30$ MHz, respectively), the three-narrow-segment with $z$ gap (red solid; upper left to lower right is $\Delta z_{\text{gap}} = 1.5, 1.0, 0.5$, respectively), and the three-narrow-segment with $v$ gap (green dash; upper left to lower right are $\Delta v_{\text{gap}} = 30, 20, 10$ MHz, respectively) separation approaches, each of which is calculated based on 1000 rounds of simulation and separation with $\sigma_{\text{noise}}|_{z=8} = 1$ mK (Table 4; Section 4.1). (b) Average power spectra of input and restored 21 cm signals and their 68% dispersion limits. The restorations are performed with the single-narrow-segment (blue) and three-narrow-segment (magenta) quadratic polynomial fitting techniques based on 1000 rounds of simulation and separation for each.

Figure 8. One case for successful separation with the single-narrow-segment quadratic polynomial fitting technique, for which the segment width is chosen to be 2 MHz, $\sigma_{\text{noise}}|_{z=8} = 1$ mK, and our complex foreground model is applied (Section 4.1). (a) Spectra of input 21 cm signals $T_{21\text{cm}}$, restored 21 cm signals plus noise $T_{21\text{cm}} + T_{\text{noise}}$, and the difference between $T_{21\text{cm}}$ and $T_{21\text{cm}} + T_{\text{noise}}$ are shown in black, blue, and green, respectively. (b) Corresponding power spectra for (a). The power spectrum of the complex foreground (cyan) is also plotted for comparison.
is larger when compared with our three-narrow-segment fittings (Table 4). We also show the result of the two-narrow-segment approach, which is similar to three-narrow-segment approach but has one less segment. In general, the results of the two-narrow-segment approach are better than those of the single-narrow-segment separations, but the goodness of separation is more or less sensitive to the choice of $\Delta z_{\text{gap}}$ or $\Delta \nu_{\text{gap}}$ (Figure 7).

We have also applied above separation approaches to higher redshifts of $z_0 = 9, 10, 12,$ and $14$, which correspond to reference frequencies of $\nu_0 = 142.0$ MHz, $129.1$ MHz, $109.3$ MHz, and $94.7$ MHz for redshifted 21 cm signals, respectively (Table 5). At these redshifts, where the cosmic reionization model adopted in this work (Santos et al. 2003, 2005; Section 2.1) predicts that the H$\text{II}$ bubbles are growing in their earlier stage and are less extended, the separation quality of our three-narrow-segment approach is still the best, although statistically the separation quality of the single-wide-segment approach is almost equally good.

### 4.2. Constraints on EoR Physics

In fiducial reionization models such as the one adopted in this work (Equation (1)), the power of the redshifted 21 cm signals is mainly determined by the mean halo bias $b$ and the average ionization fraction $x_e$ at a given redshift (Figure 11), whereas today both $b$ and $x_e$ are poorly constrained and suffer large uncertainties among different theoretical reionization models (Santos et al. 2003). In future observations, well restored 21 cm powers will greatly help us to derive $b$ and $x_e$, and then deduce H$\text{II}$ bubble size and growth rate. Since powers integrated in different scale ranges follow different contour lines on the $x_e$–$b$ diagram (e.g., $w|_{k<1h/Mpc}$ and $w|_{k<5h/Mpc}$ in Figure 11), it is feasible to constrain $b$ and $x_e$ simultaneously using the overlapping area of any two of these contours (Figure 12), the accuracy of which depends on how accurately we can restore the corresponding 21 cm powers. In Figure 12 and Table 3, we clearly see that for an ideal noise condition ($1$ mK), the tightest constraints ($b = 3.50_{-0.10}^{+0.25}$ and $x_e = 0.84_{-0.07}^{+0.05}$) are
obtained using the powers restored with our three-narrow-segment separation approach, when compared with the input model values \((b = 3.50\) and \(x_\star = 0.84;\) Santos et al. 2003). When using the single-wide-segment approach, the obtained

\[ b = 3.50^{+10.5}_{-0.20} \text{ and } x_\star = 0.83^{+0.13}_{-0.01} \]

are also unbiased, but their errors are larger by about one order of magnitude when compared with three-narrow-segment approach. When using the single-narrow-segment approach, both of the constraints

\[ r_{\text{fit}} > 0.1 \]

When the fits of single-narrow-segment separation are not acceptable, there exists a clear correlation between \(r_{\text{fit}}\) and \(P_{21\text{~cm}}(k<1h/\text{Mpc})\), i.e., \(\log P_{21\text{~cm}}(k<1h/\text{Mpc}) = 0.89\log r_{\text{fit}} - 1.95\) (solid), as can be derived with the maximum likelihood method.

Table 4

| Gap Width | \(r_{\text{fit}}\) | \(|\hat{w}|_{k<1h/\text{Mpc}}\) |
|-----------|------------------|-------------------------------|
| Single narrow segment (\(\Delta v_{\text{segment}} = 2\text{~MHz}\)) | ... | 0.153 ± 0.070 | 0.64 ± 0.24 |
| Single wide segment (\(\Delta v_{\text{segment}} = 20\text{~MHz}\)) | ... | 0.055 ± 0.034 | 0.98 ± 0.12 |
| Two narrow segment (\(\Delta v_{\text{segment}} = 2\text{~MHz}\)) | \(\Delta z_{\text{gap}}\) | 0.116 ± 0.083 | 1.09 ± 0.51 |
| | \(-1.0\) | 0.099 ± 0.072 | 1.03 ± 0.35 |
| | \(-0.5\) | 0.082 ± 0.054 | 0.97 ± 0.23 |
| | 0.5 | 0.062 ± 0.037 | 0.92 ± 0.16 |
| | 1.0 | 0.056 ± 0.034 | 0.91 ± 0.14 |
| | 1.5 | 0.050 ± 0.028 | 0.92 ± 0.13 |
| | \(\Delta v_{\text{gap}}\) (MHz) | -30 | 0.048 ± 0.026 | 0.91 ± 0.12 |
| | | -20 | 0.052 ± 0.030 | 0.91 ± 0.13 |
| | | -10 | 0.062 ± 0.037 | 0.91 ± 0.16 |
| | | 0.084 ± 0.059 | 0.96 ± 0.22 |
| | | 20 | 0.098 ± 0.069 | 1.02 ± 0.31 |
| | | 30 | 0.116 ± 0.086 | 1.13 ± 0.84 |
| Three-narrow-segment (\(\Delta v_{\text{segment}} = 2\text{~MHz}\)) | \(\Delta z_{\text{gap}}\) | 0.028 ± 0.010 | 1.00 ± 0.02 |
| | | 1.0 | 0.024 ± 0.006 | 1.00 ± 0.02 |
| | | 1.5 | 0.023 ± 0.005 | 1.00 ± 0.02 |
| | \(\Delta v_{\text{gap}}\) (MHz) | 10 | 0.027 ± 0.009 | 1.00 ± 0.02 |
| | | 20 | 0.023 ± 0.006 | 1.00 ± 0.02 |
| | | 30 | 0.022 ± 0.005 | 1.00 ± 0.02 |

Note. \(^a\) We run 1000 rounds of simulation and separation for each separation approach. \(\bar{r}_{\text{fit}}\) is the mean relative root-mean-square error for the restoration of the 21 cm signals, and \(\hat{w}|_{k<1h/\text{Mpc}}\) is the mean power restoration on \(\gtrsim 6 h^{-1}\) Mpc scales. Errors indicate the 68% confidence limits.

Figure 10. Relative root-mean-square error \(r_{\text{fit}}\) for the single-narrow-segment separation at \(z_0 = 8\) and the power of Mpc-scale components of the simulated 21 cm signals (Section 4.1). When the fits of single-narrow-segment separation are not acceptable, there exists a clear correlation between \(r_{\text{fit}}\) and \(P_{21\text{~cm}}(k<1h/\text{Mpc})\), i.e., \(\log P_{21\text{~cm}}(k<1h/\text{Mpc}) = 0.89\log r_{\text{fit}} - 1.95\) (solid), as can be derived with the maximum likelihood method.
(2.85 < b < 14 and 0.14 < xe < 0.98) are very poor. For a realistic noise condition (60 mK), the three-narrow-segment separation approach still works well (b = 3.55 +0.15 −0.15 and xe = 0.83 +0.02 −0.10; Figure 13 and Table 3), whereas the single-wide-segment approach introduces larger errors into the result (b = 3.75 +0.65 −0.35 and xe = 0.76 +0.12 −0.26). In such a sense, we conclude that the three-narrow-segment quadratic polynomial fitting technique is the best approach to restore the 21 cm signals.

4.3. Detection of BAO Signals at Intermediate Redshifts with Three-segment Separation Approach

We have also studied whether or not our three-segment separation technique can be applied to intermediate frequency bands to detect the characteristic BAO signals (Ansari et al. 2012), which can be used as a sensitive probe to investigate the nature of dark matter and to constrain cosmological models (e.g., Bharadwaj et al. 2009 for a review; Visbal et al. 2009; Bagla et al. 2010). We apply the single-narrow-segment, single-wide-segment, and three-narrow-segment approaches to the redshift range of 0.35–0.65, which corresponds to 861 MHz–1052 MHz in detection. The power spectrum of the brightness temperature of the 21 cm signals is assumed to follow that of the matter density field

\[ P_{3D,21 \text{cm}}(k, z) = 
\frac{T_{21 \text{cm}}^2(z) b^2(z)}{P_{3D,\text{matter}}(k, z)} \]  

(7)

(Seo et al. 2010), where \( T_{21 \text{cm}}(z) \) is the brightness temperature of the redshifted 21 cm signals averaged at \( z \), and \( b(z) \) is the mean bias. According to Barkana & Loeb (2007), Pritchard & Loeb (2008), and Chang et al. (2008), \( b(z) \approx 1 \) after the cosmic reionization, and \( T_{21 \text{cm}}(z) \) is estimated to be

\[ T_{21 \text{cm}}(z) = \frac{188 x_{\text{HI}}(z) \Omega_{\text{H}_0}}{\sqrt{\Omega_M(1+z)^3 + 1}} \Omega_M \]  

(8)

where \( x_{\text{HI}}(z) \) is the neutral hydrogen fraction at \( z \) and \( \Omega_{\text{H}_0} \) is the ratio of the total hydrogen mass density to the critical density at \( z = 0 \). In literature (e.g., Zwaan et al. 2005; Prochaska et al. 2005; Rao et al. 2006), a conservative estimate, \( x_{\text{HI}}(z) \Omega_{\text{H}_0} = 0.0037 \), is often assumed for the neutral hydrogen fraction at intermediate redshifts.

As a simple test, we carry out both single-segment and three-segment separations (\( \Delta v_{\text{gap}} = 0.15, 0.20, 0.25 \) or \( \Delta v_{\text{gap}} = 100, 150, 200 \text{ MHz} \)) at \( z_0 = 0.5 \) with a channel width \( dv = 0.1 \text{ MHz} \), a channel number N = 500 (i.e., a segment width \( \Delta v_{\text{segment}} = N dv = 50 \text{ MHz} \)), and \( \sigma_{\text{noise}} = 0.5 = 20 \text{ mK} \). As shown in Figure 14 and Table 6, the three-segment separation approach (e.g., \( \bar{r}_{\text{fit}} = 0.016 + 0.004 \) for \( z_{\text{segment}} = (0.5, 0.7, 0.3) \)) yields better results than the single-segment separation approach \( \bar{r}_{\text{fit}} = 0.036 + 0.015 \). The power loss in the reconstruction for the \( \geq 30 h^{-1} \text{ Mpc} \) scale structures is \( \bar{w}_{\text{loss}}[k<0.2h/\text{Mpc}] = 1 - \bar{w}[k<0.2h/\text{Mpc}] \approx 0.02 \) with the three-segment separation approach.
Figure 12. $x_e$ and $b$ determined by overlapping the $w'|k<1h/\text{Mpc}$ and $w'|k<5h/\text{Mpc}$ contours shown in Figures 11(a) and (b) on the $x_e$--$b$ map. The input model parameters ($x_e = 0.84$, $b = 3.50$) are shown as a black diamond, meanwhile the best restored parameters are shown with a red cross. Allowed parameter ranges (68% level) are marked in yellow (Table 3).

(A color version of this figure is available in the online journal.)

Figure 13. Same as Figure 12, but for $\sigma_{\text{noise}}|z=8|=60$ mK.

(A color version of this figure is available in the online journal.)

Figure 14. Same as Figure 7, but for $z_0 = 0.5$ (Table 6; Section 4.3).

(A color version of this figure is available in the online journal.)

| Gap Width | $f_{\text{fit}}$ | $w'|k<0.2h/\text{Mpc}$ |
|-----------|-----------------|---------------------|
| Single segment | 0.036 ± 0.015 | 0.81 ± 0.16 |
| Three segment | 0.15 | 0.016 ± 0.004 | 1.00 ± 0.02 |
|             | 0.20 | 0.016 ± 0.004 | 1.00 ± 0.02 |
|             | 0.25 | 0.016 ± 0.003 | 1.00 ± 0.02 |
| 3-segment with gap | 0.15 | 0.016 ± 0.004 | 1.00 ± 0.02 |
| 3-segment with gap | 0.20 | 0.016 ± 0.004 | 1.00 ± 0.02 |
| 3-segment with gap | 0.25 | 0.016 ± 0.003 | 1.00 ± 0.02 |
| $\Delta v_{\text{gap}}$ (MHz) | 100 | 0.016 ± 0.004 | 1.00 ± 0.02 |
|             | 150 | 0.015 ± 0.003 | 1.00 ± 0.02 |
|             | 200 | 0.015 ± 0.003 | 1.00 ± 0.02 |

Note. $a$ Parameters are defined in the same way as in Table 4, whereas the wavenumber range is confined to $k < 0.2h \text{ Mpc}^{-1}$ to calculate relative power restoration.
approach and $\bar{w}_{\text{loss}} |k < 0.25/\text{Mpc} = 0.19 \pm 0.16$ with the single-segment separation approach. Therefore, we conclude that the three-segment separation approach can successfully restore the 21 cm signals from intermediate redshifts better than the single-segment approach, showing its potential to be employed in detecting the BAO signals that typically possess characteristic $\lesssim 150$ comoving Mpc scales. More details on the simulation of the BAO signals and its separation from the foreground will be presented in the next paper, since it is beyond the scope of this work.

5. SUMMARY

In this work, we construct a complex foreground model and compare between different separation approaches by carrying out statistical analysis of the restored 21 cm spectra and corresponding power spectra, as well as their constraints on mean halo bias $b$ and average ionization fraction $x_e$. At $z = 8$, the best restoration of 21 cm signals and the tightest determination of $b$ and $x_e$ can be obtained with the three-narrow-segment quadratic polynomial fitting technique proposed in this work. We illustrate that the new technique also works well at redshifts higher than eight and is a potentially useful tool to probe the BAO at intermediate redshifts.

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