On The 5D Extra-Force according to Basini-Capozziello-Ponce De Leon Formalism and the Experimental Research of Extra Dimensions On-Board International Space Station (ISS) using Laser Beams.

Fernando Loup *
Residencia de Estudiantes Universitas Lisboa Portugal
April 8, 2009

Abstract

We analyze the possibility of Experimental Research of Extra Dimensions On-Board International Space Station (ISS) by using a Satellite carrying a Laser device (optical Laser) on the other side of Earth Orbit targeted towards ISS. The Sun will be between the Satellite and the ISS so the Laser will pass the neighborhoods of the Sun at a distance $R$ in order to reach ISS. The Laser beam will be Gravitationally Bent according to Classical General Relativity and the Extra Terms predicted by Kar-Sinha in the Gravitational Bending Of Light due to the presence of Extra Dimensions can perhaps be measured with precision equipment. By computing the Gravitational Bending according to Einstein we know the exact position where the Laser will reach the target on-board ISS. However if the Laser arrives at ISS with a Bending different than the one predicted by Einstein and if this difference is equal to the Extra Terms predicted by Kar-Sinha then this experience would proof that we live in a Universe of more than 4 Dimensions. We demonstrate in this work that ISS have the needed precision to detect these Extra Terms (see eq 137 in this work). Such experience would resemble the measures of the Gravitational Bending Of Light by Sir Arthur Stanley Eddington in the Sun Eclipse of 1919 that helped to proof the correctness of General Relativity although in ISS case would have more degrees of accuracy because we would be free from the interference of Earth Atmosphere.

1 Introduction

Much has been said about the so-called Extra Dimensional nature of the Universe. It was first proposed by Theodore Kaluza and Oskar Klein in 1918 in an attempt to unify Gravity and Electromagnetism. However the physical nature of the Extra Dimension was not well defined in a clear way. They used a so-called Compactification Mechanism to explain why we cannot see the 5D Extra Dimension but this mechanism was not clearly understood. Later on and with more advanced scientific knowledge other authors appeared with the same idea under the exotic concept of the so-called BraneWorld. In the BraneWorld concept our visible Universe is a 3 + 1 Dimensional sheet of Spacetime: a Brane involved by a Spacetime of Higher Dimensional nature. This idea came mainly from Strings Theory where Gravitational Forces are being represented by an Elementary Particle called Graviton while other interactions are represented by other sets of Elementary Particles: Electromagnetic Interaction is being represented by an Elementary Particle.

* spacetimeshortcut@yahoo.com

1 see [21] for an excellent account on Kaluza-Klein History
Particle called Photon. According to Strings Theory Gravitons are Closed Loops and can leave easily our 3 + 1 Dimensional Spacetime and escape into the Extra Dimensions while Photons are Open Strings and are "trapped" in our 3 + 1 Spacetime. To resume: In 1918 Kaluza-Klein tried to unify Gravity and Electromagnetism and Einstein among other scientists were interested in the same thing. But however inside the framework of the so-called Strings Theory how can a Closed Loop be unified with a Open String ??? How can an Interaction that with some degrees of freedom is allowed to probe the Extra Dimensional Spacetime be unified with Interactions confined to our 3 + 1 Spacetime ??? A puzzle to solve. So the so-called Strings Theory is trying to unify Gravity with Electromagnetism and other Interactions but the framework is not completed or not well understood. On the other hand the Compactification Mechanism in the original Kaluza-Klein theory explains why we cannot see beyond the 3 + 1 Spacetime because the Extra Dimensions are Compactified or Curled Up but it does not explains why we have 3 + 1 Uncurled or Uncompactified Dimensions while the remaining Extra Dimensions are Curled and what generates this Compactification Mechanism in the first place ??? We adopt in this work the so-called Basini-Capozziello Ponce De Leon formalism coupled to the formalisms of Mashoon-Wesson-Liu and Overduin-Wesson in which Extra Dimensions are not compactified but opened like the 3 + 1 Spacetime Dimensions we can see. There are small differences between these formalisms but Basini-Capozziello Ponce De Leon admits a non-null $^5 R_{AB}$ Ricci Tensor while the others make the Ricci Tensor $^5 R_{AB} = 0^2$ but essentially these formalisms are mathematically equivalent. In the Basini-Capozziello Ponce de Leon the ordinary Spacetime of 3 + 1 Dimensions is embedded into a large Higher Dimensional Spacetime, however in a flat or Minkowsly Spacetime the Spacetime Curvature eg Ricci and Einstein Tensors of the Higher Dimensional Spacetime reduces to the same Ricci and Einstein Tensors of a 3 + 1 Spacetime. This explains without Compactification Mechanisms why we cannot see beyond the 3 + 1 Spacetime, our everyday Spacetime is essentially Minkowskian or flat and a 5D Ricci Tensor reduces to a Ricci Tensor of a 3 + 1 Spacetime. On the other hand in this formalism all masses, electric charges and spins of all the Elementary Particles seen in 4D are function of a 5D rest-mass coupled with Spacetime Geometry. We can observe in 4D Spacetime a multitude of Elementary Particles with different masses, electric charges or spins but according to the Basini-Capozziello Ponce De Leon formalism eg the 5D to 4D Dimensional Reduction all these different Elementary Particles with all these 4D rest-masses, electric charges or spins are as a matter of fact a small group of 5D Elementary Particles with a 5D rest-mass and is the geometry of the 5D Spacetime coupled with the Dimensional Reduction that generates these apparent differences. Hence two particles with the same 5D rest-mass $M_5$ can be seen in 4D with two different rest-masses $m_0$ making ourselves think that the particles are different but the difference is apparent and is generated by the Dimensional Reduction from 5D to 4D. Look to the set of equations below: We will explore in this work these equations with details but two particles with the same 5D rest mass $M_5$ can be seen in the 4D with two different rest-masses $m_0$ if the Dimensional Reduction from 5D to 4D or the Spacetime Geometric Coupling $\sqrt{1 - \Phi^2(\frac{du}{ds})^2}$ is different for each particle. All the particles in the Table of Elementary Particles given below with non-zero rest-mass $m_0$ seen in 4D can as a matter of fact have the same rest-mass $M_5$ in 5D and the Dimensional Reduction term $\sqrt{1 - \Phi^2(\frac{du}{ds})^2}$ generates the apparent different 4D rest-masses. This is very attractive from the point of view of a Unified Physics theory. There exists a small set of particles in 5D and all the huge number of Elementary Particles in 4D is a geometric projection from the 5D Spacetime into a 4D one.  

---

2 see [21] pg 31 after eq 48 and see [2] eq 20 [11] eq 21 and [20] eq 8. We prefer to assume that exists matter in the 5D due to the last section of [20] about the particle Z.

3 we consider our Spacetime as a Schwarzschild Spacetime however at a large distance from the Gravitational Source it reduces to a Minkowsky SR Spacetime due to a large $R$ and the ratio $\frac{\Phi}{R}$ tends to zero.

4 extracted from the Formulary Of Physics by J.C.A. Wevers available on Internet.
eq 20,[11] eq 21 and [20] eq 8)([2] eq 14,[20] eq 1 and eq 2).

\[ m_0 = \frac{M_5}{\sqrt{1 - \Phi^2 \left(\frac{dy}{ds}\right)^2}} \]  

(1)

\[ dS^2 = g_{\mu \nu} dx^\mu dx^\nu - \Phi^2 dy^2 \]  

(2)

\[ dS^2 = ds^2 - \Phi^2 dy^2 \]  

(3)

| Particle | spin (\(h\)) | B | L | T | T3 | S | C | B* | charge (e) | \(m_0\) (MeV) | antipart. |
|----------|-------------|---|---|---|----|---|---|----|-------------|-------------|-----------|
| u        | 1/2 1/3 0 1/2 1/2 0 0 0 | +2/3 | 5 | \(\uparrow\) |
| d        | 1/2 1/3 0 1/2 −1/2 0 0 0 | −1/3 | 9 | \(\uparrow\) |
| s        | 1/2 1/3 0 0 0 −1 0 0 | −1/3 | 175 | \(\uparrow\) |
| c        | 1/2 1/3 0 0 0 0 1 0 | +2/3 | 1350 | \(\uparrow\) |
| b        | 1/2 1/3 0 0 0 0 0 −1 | −1/3 | 4500 | \(\uparrow\) |
| t        | 1/2 1/3 0 0 0 0 0 0 | +2/3 | 173000 | \(\uparrow\) |
| e^-      | 1/2 0 1 0 0 0 0 0 | −1 | 0.511 | e^+ |
| \(\mu^-\) | 1/2 0 1 0 0 0 0 0 | −1 | 105.658 | \(\mu^+\) |
| \(\tau^-\) | 1/2 0 1 0 0 0 0 0 | −1 | 1777.1 | \(\tau^+\) |
| \(\nu_e\) | 1/2 0 1 0 0 0 0 0 | 0 | 0(?) | \(\bar{\nu}_e\) |
| \(\nu_\mu\) | 1/2 0 1 0 0 0 0 0 | 0 | 0(?) | \(\bar{\nu}_\mu\) |
| \(\nu_\tau\) | 1/2 0 1 0 0 0 0 0 | 0 | 0(?) | \(\bar{\nu}_\tau\) |
| \(\gamma\) | 1 0 0 0 0 0 0 0 | 0 | 0 | \(\gamma\) |
| gluon    | 1 0 0 0 0 0 0 0 | 0 | 0 | \(\gamma\) |
| W^+      | 1 0 0 0 0 0 0 0 | +1 | 80220 | W^- |
| Z        | 1 0 0 0 0 0 0 0 | 0 | 91187 | Z |
| graviton | 2 0 0 0 0 0 0 0 | 0 | 0 | graviton |

We employ in this work the Basini-Capozziello Ponce De Leon Formalism to demonstrate that while in flat or Minkowsky Spacetime the curvature in 5D reduces to a one in 4D due to the Dimensional Reduction suffered by the Ricci and Einstein Tensors and we cannot tell if we live in a 5D or in a 4D Universe due to the absence of Strong Gravitational Fields but in an environment of Strong Gravity the 5D Ricci and Einstein Tensors cannot be reduced to similar 4D ones and the Curvature of a 5D Spacetime is different than the one of a 4D because the 5D extra terms in the Ricci and Einstein Tensors have the terms of the Strong Gravitational Field and cannot be reduced to 4D.Perhaps the study of the conditions of extreme Gravitational Fields in large Black Holes will tell if we live in a 5D Universe or in a 4D one.We also demonstrate that the International Space Station (ISS) can perhaps be used to study the Experimental Detection Of Extra Dimensions using the Gravitational Bending Of Light of the Sun or the similar for large Black Holes.Higher Dimensional Spacetimes affects the Gravitational Bending Of Light adding Extra Terms as predicted by Kar-Sinha(see abstact of [3]).(see also pg 73 in [20]).International Space Station is intended to be a laboratory designated to test the forefront theories of Physics and ISS can provide a better environment for Physical experiences without the interference of Earth secondary effects that will disturb careful measures specially for gravity-related experiments.(see pg 602-603 for the advantage of the free-fall conditions for experiments in [8]) We argue that the Gravitational Bending Of Light first measured by Sir Arthur Stanley Eddington in a Sun Eclipse in 1919 is widely known as the episode that made Einstein
famous but had more than 30 percent of error margin. In order to find out if the Extra Dimension exists (or not) we need to measure the factor $C^2$ of Kar-Sinha coupled to a Higher Dimensional definition of $m_0$ with an accurate precision of more than $\Delta \omega < 2.8 \times 10^{-4}$. (see pg 1783 in [3]). ISS can use the orbit of the Moon to create a "artificial eclipse"\(^5\) to get precise measurements of the Gravitational Bending Of Light and according to Kar-Sinha detect the existence of the 5D Extra Dimension although in this work we will propose a better idea. ISS will be used to probe the foundations of General Relativity and Gravitational Bending of Light certainly will figure out in the experiences (see pg 626 in [6]). ISS Gravitational shifts are capable to detect measures of $\frac{\Delta \omega}{\omega}$ with Expected Uncertainly of $12 \times 10^{-6}$ (see pg 629 Table I in [6]) smaller than the one predicted by Kar-Sinha for the Extra Dimensions although the ISS shifts are red-shifts and not Bending of Light similar precision can be achieved. Also red-shifts are due to a time delay in signals and the 5D can also affect this measure as proposed by Kar-Sinha.(see pg 1782 in [3]). The goal of ISS is to achieve a Gravitational Shift Precision of $2.4 \times 10^{-7}$ (see pg 631 Table II in [6]) by far more than enough to detect the existence of the 5D Extra Dimension. We believe that Gravitational Bending Of Light can clarify the question if we live in a Higher Dimensional Universe or not. A small deviation in a photon path different than the one predicted by Einstein can solve the quest for Higher Dimensional Spacetimes. We propose here the use of a Satellite with a Laser beam in the other side of the Earth Orbit targeted towards ISS. The Laser would pass the neighborhoods of the Sun at a distance $R$ in order to reach ISS and would be Gravitational Bent according to General Relativity and affected by the Kar-Sinha Extra Terms due to the presence of the Higher Dimensional Spacetime. Hence the Gravitational Bending Of Light can be measured with precision equipment of ISS. We demonstrate in this work that ISS have the needed precision to detect these Extra Terms (see eq 137 in this work). By computing the Gravitational Bending according to Einstein we know where the photons would reach ISS. However if the photons arrives at ISS with a Bending angle different than the one predicted by Einstein and if this difference is equal to the Kar-Sinha Extra Terms then we would have a proof that we live in a Universe of more than 4 Dimensions. We also examine Gravitational Red Shifts affected by the presence of the Extra Dimension. This experience made on-board ISS would have the same impact of the Sir Arthur Stanley Eddington measures of Gravitational Bending Of Light in the Sun Eclipse of 1919 and if the result is "positive" then the International Space Station ISS would change forever our way to see the Universe.

\(^5\)we agree that idea is weird but is better than to wait for a Sun Eclipse in the proper conditions
The Basini-Capozziello Ponce De Leon Formalism and resemblances with Mashoon-Wesson-Liu and Overduin-Wesson Formalisms

Basini-Capozziello Ponce de Leon argues that our 3 + 1 Dimensional Spacetime we can see is a Dimensional Reduction from a larger 5D one and according to a given Spacetime Geometry we can see(or not) the 5D Extra Dimension. This is also advocated in the almost similar Formalisms of Mashoon-Wesson-Liu and Overduin-Wesson. A 5D Spacetime metric is defined as([1] eq 32,[5] eq 18,[20] eq 62 and [9] pg 556 Section 2) and contains all the 3 + 1 Spacetime Dimensions of our observable Universe plus the 5D Extra Dimension. Then \(A, B, 0, 1, 2, 3\) are the Dimensions of the 4D Spacetime and 4 is the script of the 5D Extra Dimension(see [5] pg 2225 after eq 18 and again [9] pg 556).

\[
dS^2 = g_{AB} dx^A dx^B
\]

(4)

Note that this equation is common not only to Basini-Capozziello-Ponce De Leon but also to Mashoon-Wesson-Liu and Overduin-Wesson Formalisms. These formalisms advocates the Dimensional Reduction from 5D to 4D and we need to separate in this Spacetime Metric both the 3 + 1 Components of our visible Universe and the components of the 5D Extra Dimension. The resulting equation would then be([1] eq 56,[2] eq 12 and 14,[5] eq 42,[9] eq 32 and 33 without vector potential,[11] eq 10, [12] pg 308,[20] eq 109 and [13] pg 1346)

\[
dS^2 = g_{AB} dx^A dx^B = g_{\alpha\beta} dx^\alpha dx^\beta - \Phi^2 dy^2
\]

(5)

Note that when the Warp Field\(^10\) \(\Phi = 1\) the Spacetime Metric becomes:

\[
dS^2 = g_{AB} dx^A dx^B = g_{\alpha\beta} dx^\alpha dx^\beta - dy^2
\]

(6)

Writing the \(5 R_{\alpha\beta}\) Ricci Tensor and the \(5 R\) Ricci Scalar according to Basini-Capozziello using these equations:\((\alpha, \beta = 0, 1, 2, 3)([1] eq 58, [5] eq 44 and [20] eq 111. See also [21] eq 48 for \(5 R_{\alpha\beta}\))\(^11\)

\[
5 R_{\alpha\beta} = R_{\alpha\beta} - \frac{\Phi_{,\alpha,\beta}}{\Phi} - \frac{1}{2\Phi^2} \left( \Phi_{,\gamma\alpha,\beta,\lambda} - g_{\alpha\beta,44} + g^{\lambda\mu} g_{\alpha\lambda,4\beta\mu,4} - \frac{g^{\mu\nu} g_{\mu\nu,4\alpha\beta,4}}{2} \right)
\]

(7)

\[
5 R = R - \frac{\Phi_{,\alpha,\beta}}{\Phi} - \frac{1}{2\Phi^2} \left( \Phi_{,\gamma\alpha,\beta,\lambda} - g_{\alpha\beta,44} + g^{\lambda\mu} g_{\alpha\lambda,4\beta\mu,4} - \frac{g^{\mu\nu} g_{\mu\nu,4\alpha\beta,4}}{2} \right)
\]

(8)

Simplifying for diagonalized metrics we should expect for:

\[
5 R_{\alpha\beta} = R_{\alpha\beta} - \frac{\Phi_{,\alpha,\beta}}{\Phi} - \frac{1}{2\Phi^2} \left( \Phi_{,\gamma\alpha,\beta,\lambda} - g_{\alpha\beta,44} + g^{\mu\nu} g_{\mu\nu,4\alpha\beta,4} \right)
\]

(9)

\[
5 R = R - \frac{\Phi_{,\alpha,\beta}}{\Phi} - \frac{1}{2\Phi^2} \left( \Phi_{,\gamma\alpha,\beta,\lambda} - g_{\alpha\beta,44} + g^{\mu\nu} g_{\mu\nu,4\alpha\beta,4} \right)
\]

(10)

---

\(^6\)We will skip a tedious definition and concentrate on the Dimensional Reduction. A unfamiliar reader must study first [1] pg 122 Section 2.2 to pg 127,[5] pg 2225 Section 3 to pg 2229 and [20] pg 1434 Section 4 to pg 1441 see also [21] pg 29 Section 6 to pg 31

\(^7\)[12] with spacelike signature

\(^8\)see [13] pg 1341 the Campbell-Magaard Theorem

\(^9\)see [1] eq 57,[5] eq 43 and [21] eq 47.

\(^10\)the term "Warp" appears in pg 1340 in [2]

\(^11\)Working with diagonalized metrics the terms \(\alpha, \lambda, \mu, \beta\) and \(\nu\) are all equal
Note that the term \(4\Box \Phi = \nabla_\alpha \Phi^\alpha = g^{\alpha\beta}(\Phi_\alpha)_\beta = g^{\alpha\beta}[\Phi_\alpha - \Gamma^K_\alpha \Phi^K] \) corresponds to the D'Alembertian in 4D\(^{12}\). We can write for the Ricci Scalar the following expression:

\[
5R = R - 4\Box \Phi - g^{\alpha\beta} \left( \frac{\partial_\alpha \Phi \partial_\beta \Phi}{\Phi} - \partial_4 g_{\alpha\beta} \right) - g_{\alpha\beta} + \frac{g^\mu\nu g_{\mu\nu} g_{\alpha\beta}}{2} \tag{11}
\]

If according to Basini-Capozziello the terms \(g_{\alpha\beta}\) have no dependence with respect to the Extra Coordinate \(y\) after the Reduction from 5D to 4D then all the derivatives with respect to \(y\) vanish and we are left out with the following expression for the Ricci Scalar: ([1] eq 59, [5] eq 45 and [20] eq 116)). We will analyze this in details when studying the 5D to 4D Dimensional Reduction.

\[
5R = R - 4\Box \Phi \tag{12}
\]

Writing the remaining Ricci Tensors we should expect for ([21] eq 48)\(^{14}\):

\[
\hat{R}_{\alpha\beta} = \hat{R}_{\alpha\beta} - \frac{\partial_\beta (\partial_\alpha \Phi)}{\Phi} - \frac{1}{2\Phi^2} \left( \frac{\partial_\alpha \Phi \partial_\beta \Phi}{\Phi} - \partial_4 g_{\alpha\beta} \right) + g^{\gamma\delta} \partial_4 g_{\gamma\alpha} \partial_4 g_{\delta\beta} - \frac{g^{\gamma\delta} \partial_4 g_{\gamma\alpha} \partial_4 g_{\delta\beta}}{2},
\]

\[
\hat{R}_{\alpha4} = \frac{g^{4\gamma}}{4} (\partial_4 g_{\beta\gamma} \partial_\gamma g_{\alpha4} - \partial_\gamma g_{44} \partial_4 g_{\alpha\beta} + \frac{\partial_4 g^{\beta\gamma} \partial_4 g_{\gamma\alpha}}{2}) + \frac{g^{4\gamma}}{2} \partial_4 (\partial_\gamma g_{\beta\gamma}) + \frac{g^{\beta\gamma}}{2} \partial_4 (\partial_\gamma g_{4\beta}) + \frac{g^{\beta\gamma} g^{4\epsilon}}{4} \partial_4 g_{\gamma\beta} \partial_4 g_{4\epsilon} + \partial_4 \Phi \frac{\partial_\beta \Phi}{\Phi} - \frac{g^{\alpha\beta}}{2} \partial_4 g_{\alpha\beta} \partial_4 g_{\beta\beta} + \frac{g^{\alpha\beta} \partial_4 g_{\alpha\beta} \partial_4 g_{\beta\beta}}{4}, \tag{13}
\]

where “\(\Box\)” is defined as usual (in four dimensions) by \(\Box \Phi \equiv g^{\alpha\beta} \nabla_\beta (\partial_\alpha \Phi)\).

Note that the Overduin-Wesson definition is exactly equal to the one presented by Basini-Capozziello-Ponce De Leon. Both Formalisms are equivalent except that Basini-Capozziello-Ponce De Leon admits a \(5R_{\alpha\beta}\) not null.

Working with diagonalized Spacetime Metrics of signature (+, -, -, -)\(^{15}\) the Ricci Tensors would be written as:

\[
\hat{R}_{\alpha\alpha} = \hat{R}_{\alpha\alpha} - \frac{\partial_\alpha (\partial_\alpha \Phi)}{\Phi} - \frac{1}{2\Phi^2} \left( \frac{\partial_\alpha \Phi \partial_\alpha \Phi}{\Phi} - \partial_4 g_{\alpha\alpha} \right) + g^{\alpha\alpha} \partial_4 g_{\alpha\alpha} \partial_4 g_{\alpha\alpha} - \frac{g^{\alpha\alpha} \partial_4 g_{\alpha\alpha} \partial_4 g_{\alpha\alpha}}{2},
\]

\(^{12}\) see pg 129 in [1] and pg 2230 in [5]
\(^{13}\) see also pg 311 in [12]
\(^{14}\) adapted from the arXiv.org LaTeX file of [21] eq 48. Note the difference between the first term \(\hat{R}_{\alpha\beta}\) between [21] eq 48 ,[1] eq 58,[5] eq 44 and [20] eq 111
\(^{15}\) \(\alpha = \beta = \gamma = \delta = \epsilon\)
\[ \hat{R}_{a4} = \frac{g^{44}g^{\alpha\alpha}}{4}(\partial_4g_{\alpha\alpha} \partial_\alpha g_{44} - \partial_4g_{\alpha4} \partial_\alpha g_{4\alpha}) + \frac{\partial_4g^{\alpha\alpha} \partial_4g_{\alpha\alpha}}{2} \\
+ \frac{g^{\alpha\alpha} \partial_4(\partial_4g_{\alpha\alpha})}{2} - \frac{\partial_4g^{\alpha\alpha} \partial_4g_{\alpha\alpha}}{2} - \frac{g^{\alpha\alpha} \partial_4(\partial_4g_{\alpha\alpha})}{2} \\
+ \frac{g^{\alpha\alpha}g^{\alpha\alpha} \partial_4(\partial_4g_{\alpha\alpha})}{4} + \frac{g^{\alpha\alpha} \partial_4(\partial_4g_{\alpha\alpha})}{4} , \]

\[ \hat{R}_{44} = \Phi \Box \Phi - \frac{\partial_4g^{\alpha\alpha} \partial_4g_{\alpha\alpha}}{2} - \frac{g^{\alpha\alpha} \partial_4(\partial_4g_{\alpha\alpha})}{2} \\
+ \frac{\partial_4\Phi g^{\alpha\alpha} \partial_4g_{\alpha\alpha}}{2} - \frac{g^{\alpha\alpha}g^{\alpha\alpha} \partial_4(\partial_4g_{\alpha\alpha})}{4} , \quad (14) \]

Compare the first of the Ricci Tensors above with [20] eq 113. Note that the Mashoon-Wesson-Liu Formalism is exactly equal to the Basini-Capozziello-Ponce De Leon and Overduin-Wesson Formalisms. Look to the equations [9] eq 32 and 33 without vector potential. Compare with

\[ dS^2 = g_{\alpha\beta}dx^\alpha dx^\beta - \Phi^2dy^2 \quad (15) \]

One can see that we already presented this equation proving without shadows of doubt that the three formalisms are equivalent. Mashoon-Wesson-Liu in [9] pg 557 makes \( g_{44} = -\Phi^2 \). They also makes \( g_{44} = -1 \) (see pg 558) giving the equation below:

\[ dS^2 = g_{\alpha\beta}dx^\alpha dx^\beta - dy^2 \quad (16) \]

We already presented this equation: is the 5D Spacetime Geometry without the Warp Field.

One thing advocated by Mashoon-Wesson-Liu and Basini-Capozziello Ponce De Leon is the fact that the 5D Extra Dimension generate a 5D Extra Force that can be detected in 4D. (see [9] abstract and pgs 556,562 look to eq 24, pg 563 look to eq 31 and the comment below this equation, pg 565 eq 38,39 and the comments on the de-acceleration, pg 566 definition of \( \beta \), pg 567 look to the comment of a small force but detectable). (see also [2] abstract and pgs 1336,1337,1341 eq 20, pg 1342 eq 25, pg 1343 eq 30). We will now prove that the 5D Extra Force in both formalisms is equivalent.

If we have a Spacetime Geometry defined as:

\[ dS^2 = g_{\alpha\beta}dx^\alpha dx^\beta - \Phi^2dy^2 \quad (17) \]

\[ dS^2 = g_{\alpha\beta}dx^\alpha dx^\beta - [\phi(t,x)\chi(y)]^2dy^2 \quad (18) \]

where we defined the Warp Field \( \Phi \) according to Basini-Capozziello ([1] eq 76, [5] eq 70 and [20] eq 132 ) we have two choices:

- **\( M_5 \) the 5D Mass is not zero and we have matter in the 5D Extra Dimension according to one of the Ponce De Leon Options making also \( ^5R_{\alpha\beta} \) the Ricci Tensor in 5D not null.

- **\( M_5 \) The 5D Mass is zero and we have no matter in the 5D Extra Dimension according to another of the Ponce De Leon Options making also \( ^5R_{\alpha\beta} \) the Ricci Tensor in 5D null.

Overduin-Wesson and Mashoon-Wesson-Liu formalisms agree with the second option of Ponce De Leon. (see [21] pg 31 after eq 48 and [9] pg 557 eq 2)

According to Ponce De Leon in option 1 if we have a rest-mass in 5D \( M_5 \) this rest-mass will be seen in 4D as a rest-mass \( m_0 \) as follows ([2] eq 20,[11] eq 21 and [20] eq 8):
\[ m_0 = \frac{M_5}{\sqrt{1 - \Phi^2(d\phi/ds)^2}} \]  \hspace{1cm} (19)

\[ m_0 = \frac{M_5}{\sqrt{1 - [\phi(t, x)\chi(y)]^2(d\phi/ds)^2}} \]  \hspace{1cm} (20)

We have Quantum Chromodynamics for Quarks and a Quantum Electrodynamics for Leptons like Electron but as a matter of fact two particles with the same rest-mass in 5D M_5 can appear in our 4D Spacetime with different rest masses \( m_0 \) making one appear as a Quark and the other as a Lepton depending on the Dimensional Reduction from 5D to 4D or the Spacetime Coupling \( \sqrt{1 - \Phi^2(d\phi/ds)^2}, \sqrt{1 - [\phi(t, x)\chi(y)]^2(d\phi/ds)^2} \) although in 5D both particles are the same.

This is a very interesting perspective of Modern Physics. Why Quantum Electrodynamics and Quantum Chromodynamics in 4D while as a matter of fact in 5D both are the same???. Look again to the table below:\(^{16}\):

| Particle | spin (ℏ) B | L | T | T_3 | S | C | B^* | charge (e) | \( m_0 \) (MeV) | antipart. |
|-----------|-----------|---|---|-----|---|---|-----|-----------|-------------|----------|
| u         | 1/2 1/3 0 | 1/2 | 1/2 | 0 | 0 | 0 | +2/3 | 5         |            | \( \bar{u} \) |
| d         | 1/2 1/3 0 | 1/2 | -1/2 | 0 | 0 | 0 | -1/3 | 9         |            | \( \bar{d} \) |
| s         | 1/2 1/3 0 | 0 | 0 | 0 | -1 | 0 | 0 | 175       |            | \( \bar{s} \) |
| c         | 1/2 1/3 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1350      | 4500       | \( \bar{c} \) |
| b         | 1/2 1/3 0 | 0 | 0 | 0 | 0 | 0 | -1 | 173000    |            | \( \bar{b} \) |
| t         | 1/2 1/3 0 | 0 | 0 | 0 | 0 | 0 | 0 | 173000    | 1350       | \( \bar{t} \) |
| e^-       | 1/2 0 1 | 0 | 0 | 0 | 0 | 0 | -1 | 0.511     | e^+        |
| \( \mu^- \) | 1/2 0 1 | 0 | 0 | 0 | 0 | 0 | -1 | 105.658   | \( \mu^+ \) |
| \( \tau^- \) | 1/2 0 1 | 0 | 0 | 0 | 0 | 0 | -1 | 1777.1    | \( \tau^+ \) |
| \( \nu_e \) | 1/2 0 1 | 0 | 0 | 0 | 0 | 0 | 0 | \( \nu_e \) |
| \( \nu_{\mu} \) | 1/2 0 1 | 0 | 0 | 0 | 0 | 0 | 0 | \( \nu_{\mu} \) |
| \( \nu_{\tau} \) | 1/2 0 1 | 0 | 0 | 0 | 0 | 0 | 0 | \( \nu_{\tau} \) |
| γ         | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | γ         |
| gluon     | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | gluon     |
| W^+       | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 80220     | W^-        |
| Z         | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 91187     | Z          |
| graviton  | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | graviton  |

The Extra Force generated by the 5D seen in 4D for a massive 5D particle \( M_5 \) seen in 4D as \( m_0 \) according to Ponce De Leon is defined as follows([2] eq 25 and [20] eq 15):

\[ \frac{1}{m_0} \frac{dm_0}{ds} = -\frac{1}{2} u^u u^v \frac{\partial g_{uv}}{\partial y} dy + \Phi u^u \frac{\partial \Phi}{\partial x^u}(dy/ds)^2 \]  \hspace{1cm} (21)

We have here two choices:

- The Warp Field \( \Phi = [\phi(t, x)\chi(y)] \) ([1] eq 76,[5] eq 70 and [20] eq 132 ) is not null and we have a Warp Field coupled to the 5D Extra Dimension.

\(^{16}\)extracted from the Formulary Of Physics by J.C.A. Wevers available on Internet
The Warp Field $\Phi = 1$ and we have no Warp Field at all.

For a 5D Extra Dimension coupled with a Warp Field according to Basini-Capozziello([1] eq 76,[5] eq 70 and [20] eq 132 ) the Extra Force is given by:

\[
\frac{1}{m_0} \frac{dm_0}{ds} = -\frac{1}{2} u^u u^v \partial g_{uv} \frac{dy}{dy} - \phi(t, x) \chi(y) u^u \frac{\partial \phi(t, x) \chi(y)}{\partial x^u} \left( \frac{dy}{ds} \right)^2
\]

(22)

\[
\frac{1}{m_0} \frac{dm_0}{ds} = -\frac{1}{2} u^u u^v \partial g_{uv} \frac{dy}{dy} - \phi(t, x) \chi(y) u^u \frac{\partial \phi(t, x) \chi(y)}{\partial x^u} \left( \frac{dy}{ds} \right)^2
\]

(23)

If we have no Warp Field at all $\Phi = 1$ the equation is simply:

\[
\frac{1}{m_0} \frac{dm_0}{ds} = -\frac{1}{2} u^u u^v \partial g_{uv} \frac{dy}{dy}
\]

(24)

Note that this equation is exactly equal to the 5D Extra Force equation as defined by Mashoon-Wesson-Liu([9] eq 24 and 38 because they used $g_{44} = -\Phi^2 = -1$ see [9] pg 558).Of course we expected this result because Basini-Capozziello Ponce De Leon and Mashoon-Wesson-Liu formalisms are equivalent.

For the case of a null 5D rest-mass $M_5$ the option 2 of Ponce De Leon the equation of the 5D Extra Force seen in 4D is given by([2] eq 30 and [20] eq 19):

\[
\frac{1}{m_0} \frac{dm_0}{ds} = \frac{1}{2\Phi} \frac{\partial g_{uv}}{\partial y} u^u u^v - \frac{u^u}{\Phi} \frac{\partial \Phi}{\partial x^u}
\]

(25)

\[
\frac{1}{m_0} \frac{dm_0}{ds} = \frac{1}{2\phi(t, x) \chi(y)} \frac{\partial g_{uv}}{\partial y} u^u u^v - \frac{u^u}{\phi(t, x)} \frac{\partial \phi(t, x)}{\partial x^u}
\]

(26)

If we have a no Warp Field at all the equation becomes:

\[
\frac{1}{m_0} \frac{dm_0}{ds} = \frac{1}{2} \frac{\partial g_{uv}}{\partial y} u^u u^v
\]

(27)

This is equal to ([9] eq 24 and 38 with $dy = ds$ a Null-Like 5D Spacetime Geometry)

According to the following Spacetime Geometry as defined by Basini-Capozziello Ponce De Leon,Mashoon-Wesson-Liu and Overduin-Wesson formalisms([1] eq 56,[2] eq 12 and 14,[5] eq 42 and [20] eq 1)

\[
dS^2 = g_{uv} dx^u dx^v - \Phi^2 dy^2
\]

(28)

\[
dS^2 = ds^2 - \Phi^2 dy^2
\]

(29)

\[
ds^2 = g_{uv} dx^u dx^v
\]

(30)

We have three different types of Spacetime Geometries:

- **Timelike 5D Geometry**

  \[
dS^2 > 0 \rightarrow ds^2 - \Phi^2 dy^2 > 0 \rightarrow ds^2 > \Phi^2 dy^2 \rightarrow \frac{1}{\Phi^2} > \left( \frac{dy}{ds} \right)^2 \rightarrow Timelike5D
\]

(31)
• Null-Like 5D Geometry

\[ dS^2 = 0 \rightarrow ds^2 - \Phi^2 dy^2 = 0 \rightarrow ds^2 = \Phi^2 dy^2 \rightarrow \frac{1}{\Phi^2} = \left(\frac{dy}{ds}\right)^2 \rightarrow \text{Nulllike5D} \]  

(32)

• Spacelike 5D Geometry

\[ dS^2 < 0 \rightarrow ds^2 - \Phi^2 dy^2 < 0 \rightarrow ds^2 < \Phi^2 dy^2 \rightarrow \frac{1}{\Phi^2} < \left(\frac{dy}{ds}\right)^2 \rightarrow \text{Spacelike5D} \]  

(33)

Note that for a Null-Like 5D Geometry the equation of the 4D rest-mass \( m_0 \) in function of the 5D rest-mass \( M_5 \) is not valid. ([2] eq 20, [11] eq 21 and [20] eq 8).

\[ m_0 = \frac{M_5}{\sqrt{1 - \Phi^2 \left(\frac{dy}{ds}\right)^2}} \]  

(34)

Hence we suppose that for a Null-Like 5D Geodesics the Extra Dimension have no mass at all or all matter in the 5D Extra Dimension obeys Timelike 5D Geometries.

Then we can say that the Basini-Capozziello Ponce De Leon 5D formalism is for Timelike 5D Geometries because they admit a non-null 5D rest-mass while the formalisms of Mashoon-Wesson-Liu and Overduin-Wesson are valid for a Null-Like 5D Geometry where we have a \( M_5 = 0 \) a null 5D Ricci Tensor or a flat 5D Spacetime.

\[ \frac{1}{\Phi^2} > \left(\frac{dy}{ds}\right)^2 \rightarrow 1 > \Phi^2 \left(\frac{dy}{ds}\right)^2 \]  

(35)

\[ \frac{1}{\Phi^2} = \left(\frac{dy}{ds}\right)^2 \rightarrow 1 = \Phi^2 \left(\frac{dy}{ds}\right)^2 \]  

(36)

\[ \frac{1}{\Phi^2} < \left(\frac{dy}{ds}\right)^2 \rightarrow 1 < \Phi^2 \left(\frac{dy}{ds}\right)^2 \]  

(37)

Note that a small Warp Field \( 0 < \Phi^2 < 1 \) will generate a large \( \frac{1}{\Phi^2} \) ideal for a 5D Timelike Geodesics.

Although we can have a Null 5D rest-mass \( M_5 \) the Warp Field in the 5D Extra Dimension can still account for the generation of rest-masses in 4D.

See these Ponce De Leon Equations for the 4D rest-mass \( m_0 \) ([2] eq 27 and 28, [20] eq 16,17 and 18)

\[ m_0 = \pm \Phi \frac{dy}{d\lambda} \]  

(38)

\[ d\lambda = \frac{1}{m_0} ds \]  

(39)

Combining eqs 45 and 46 we can clearly see that: \( ^{17} \)

\[ \frac{dy}{ds} = \frac{1}{\Phi} \]  

(40)

\( ^{17} \) see pg 1343 in [2] but the result is obvious from [20] eq 10
The 5D Extra Force seen in 4D for massless particles in 5D is given by: ([2] eq 30, [20] eq 19)\(^{18,19}\)

\[
\frac{1}{m_0} \frac{dm_0}{ds} = 1 \frac{\partial g_{uv}}{2 \Phi} u^u u^v - \frac{u^u}{\Phi} \frac{\partial \Phi}{\partial x^u}
\]  \(41\)

This equation although for massless 5D particles have many resemblance with its similar for massive 5D particles as pointed out by Ponce De Leon and can easily be obtained combining eqs 15 and 18 of [20] (see pg 1343 in [2]).

According to the Table of Elementary Particles already presented in this work (two times and we think its enough) Photons or Gravitons have a 4D rest-mass \(m_0 = 0\) corresponding to a 5D Null-Like Spacetime Geometry or in hence a stationary particle a particle that is at the rest in the 5D Spacetime, a particle with a \(m_0 = \Phi \frac{dy}{d\lambda} \rightarrow m_0 = 0 \rightarrow \frac{dy}{d\lambda} = 0 \rightarrow \frac{dy}{ds} = 0\).

But of course we can have a 5D rest-mass \(M_5 = 0\) giving a non-null 4D rest-mass \(m_0 \neq 0\) even with a Warp Field \(\Phi = 1\) if \(\frac{dy}{d\lambda} \neq 0\) according to the following equations although we believe that non-null rest-masses \(m_0\) in 4D comes from non-null rest-masses \(M_5\) in 5D (see sections 8 and 9 about particle Z in [20]):

\[
m_0 = \Phi \frac{dy}{d\lambda}
\]  \(42\)

\[
d\lambda = \frac{1}{m_0} ds
\]  \(43\)

\[
\frac{dy}{ds} = 1
\]  \(44\)

\[
\frac{1}{m_0} \frac{dm_0}{ds} = 1 \frac{\partial g_{uv}}{2 \Phi} u^u u^v
\]  \(45\)

Note that if the Warp Field \(\Phi = 1\) with \(\frac{dy}{d\lambda} = 1\) and \(dS^2 = 0\) the equation of the 5D Extra Force for a massless particle in 5D \(M_5 = 0\) becomes equivalent to [9] eq 24 proving that the Ponce De Leon equations are equivalent to the Mashoon-Wesson-Liu ones.

\[
\frac{1}{m_0} \frac{dm_0}{ds} = 1 \frac{\partial g_{uv}}{2 \Phi} u^u u^v
\]  \(46\)

\(^{18}\)note that like for its analogous 5D massive counterpart the Warp Field function only of the Extra Coordinate makes the second term vanish (examine eqs 50 and 52 in [1])

\(^{19}\)compare this equation with [9] eq 24 and look for the + signal in this equation while [9] eq 24 only have the - sign
3 Dimensional Reduction from $5D$ to $4D$ according to Basini-Capozziello Ponce De Leon, Mashoon-Wesson-Liu and Overduin-Wesson. Possible Experimental Detection of Extra Dimensions in Strong Gravitational Fields or On-Board the International Space Station (ISS) using the Gravitational Bending Of Light in Extra Dimensions.

The most important thing to keep in mind when we study models of BraneWorlds or Extra Dimensions is to explain why we cannot "see" directly the presence of the Extra Dimension although we can "feel" its effects in the $4D$ everyday Physics. We avoid here the models with compactification or "curling-up" of the Extra Dimension because these models don’t explain why we have $3 + 1$ Large Dimensions while the remaining ones are small and "unseen" Extra Dimensions and also these models don’t explain what generates the "Compactification" or "Curling" mechanism in the first place. Also some of these models develop "Unphysical" features. An excellent account of the difference between compactified and uncompactified models of Extra Dimensions is given by [21](see pgs 2 to 31). We prefer to adopt the fact that like the $3 + 1$ ordinary Large Spacetime Dimensions the Extra Dimensions are Large and uncompactified but due to a Dimensional Reduction from $5D$ to $4D$ we cannot "see" these Extra Dimensions although we can "feel" some of its effects. (see abs and pg 123 of [1] when Basini-Capozziello mentions the fact that we cannot perceive Time as the fourth Dimension and hence we cannot perceive the Spacelike Nature of the $5D$ Extra Dimension). (see also pg 1424 and pg 1434 beginning of section 4 in [20]). (see also abs pg 2218 and 2219 of [5]. Note the comment on Dimensional reduction and a $4D$ Spacetime embedded into a larger $5D$ one). We will now demonstrate how the Dimensional Reduction from $5D$ to $4D$ work and why in ordinary conditions we cannot "see" the $5D$ Extra Dimensions but we can "feel" some of its effects. Also we will see that changing the Spacetime Geometry and the shape of the Warp Field the $5D$ Extra Dimension will become visible. (Dimensional Reductions from $5D$ to $4D$ appears also in pg 2040 of [4]). We know that in ordinary $3 + 1$ Spacetime the curvature of the Einstein Tensor is negligible and Spacetime can be considered as Minkowskian or flat where Special Relativity holds. A Minkowskian $5D$ Spacetime with a Warp Field can be given by (see eq 325 in [20]):

$$dS^2 = dt^2 - dX^2 - \Phi^2 dy^2$$  \hspace{1cm} (47)$$

The Warp Field considered here have small values between 0 and 1 nearly close to 0 and we recover the ordinary Special Relativity Ansatz. A Minkowskian $5D$ Spacetime with no Warp Field at all would be given by (see eq 326 in [20]):

$$dS^2 = dt^2 - dX^2 - dy^2$$  \hspace{1cm} (48)$$

The Ricci Tensors and Scalars for the Basini-Capozziello $5D$ Spacetime Formalism and Ansatz given by $dS^2 = g_{\mu\nu} dx^\mu dx^\nu - \Phi^2 dy^2$ are shown below: (see pg 128 eq 58 in [1], pg 2230 eq 44 in [5] and pg 1442 eqs 111 to 115 in [20])

$$5R_{\alpha\beta} = R_{\alpha\beta} - \frac{\Phi_{,a;b}}{\Phi} - \frac{1}{2\Phi^2}(\frac{\Phi_{,a}g_{\alpha\beta,4} + g_{\mu\nu}g_{\mu\nu,4a\beta,4}}{2} - g_{\alpha\beta,4}g_{,44} + g_{\alpha\beta,4}g_{,44})$$  \hspace{1cm} (49)$$

$$5R = R - \frac{\Phi_{,a;b}}{\Phi} g^{\alpha\beta} - \frac{1}{2\Phi^2} g^{\alpha\beta}(\frac{\Phi_{,a}g_{\alpha\beta,4} + g_{\mu\nu}g_{\mu\nu,4a\beta,4}}{2} - g_{\alpha\beta,4}g_{,44} + g_{\alpha\beta,4}g_{,44})$$  \hspace{1cm} (50)$$
$$5R = R - 4\Box \Phi \varphi - \frac{1}{2\Phi^2} g^{\alpha\beta}(\frac{\Phi A g_{\alpha\beta,4}}{\Phi} - g_{\alpha\beta,4} + \frac{g^{\mu\nu} g_{\mu\nu,4} A g_{\alpha\beta,4}}{2})$$  \hspace{1cm} (51)$$

For a 5D Spacetime Metric without Warp Field defined as $dS^2 = g_{\mu\nu} dx^\mu dx^\nu - dy^2$ the Ricci Tensor and Scalar would then be (see eqs 330 and 331 pg 1477 in [20]):

$$5R_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} (-g_{\alpha\beta,4} + \frac{g^{\mu\nu} g_{\mu\nu,4} A g_{\alpha\beta,4}}{2})$$  \hspace{1cm} (52)$$

$$5R = R - \frac{1}{2} g^{\alpha\beta} (-g_{\alpha\beta,4} + \frac{g^{\mu\nu} g_{\mu\nu,4} A g_{\alpha\beta,4}}{2})$$  \hspace{1cm} (53)$$

But remember that a Minkowskian 5D Spacetime in which Special Relativity holds have all the 3 + 1 Spacetime Metric Tensor Components defined by $g_{\mu\nu} = (+1, -1, -1, -1)$ (see pg 1476 and 1477 in [20]) and the derivatives of the Metric Tensor vanishes and hence we are left with the following results (see eqs 332 and 333 in [20]):

$$5R_{\alpha\beta} = R_{\alpha\beta} \hspace{1cm} (54)$$

$$5R = R \hspace{1cm} (55)$$

From the results above in a flat Minkowsky 5D Spacetime the Ricci Tensor in 5D is equal to its counterpart in 4D and since the Spacetime is flat then both are equal to zero. Then its impossible to tell if we live in a 4D Spacetime or in a larger 5D Extra Dimensional one (see pg 1477 after eq 333 in [20]). If the Geometry of a flat 5D Extra Dimensional Spacetime is equivalent to the Geometry of a 3 + 1 Spacetime we cannot distinguish if we live in a 4D or a 5D Universe. This is one of the most important things in the Dimensional Reduction from 5D to 4D as proposed by Basini-Capozziello. The 5D Extra Dimension is Large and Uncompactified but the physical reality we see is a Dimensional Reduction from 5D to 4D because we live in a nearly flat Minkowsky Spacetime\(^{20}\) where Special Relativity holds and the 5D Ricci Tensor is equal to the 3 + 1 counterpart. No Compactification mechanisms needed. The 5D Extra Dimension have a real physical meaning (see pg 2226 in [5] and pg 127 in [1]). (see also pg 2230 in [5] the part of the reduction of the Ricci Tensor from 5D to 4D eqs 44 and 45, if the Warp Field $\Phi = 0$ both 5D and 4D Ricci Tensors from eq 45 are equal. the same can be seen in pg 128 to 129 eqs 58 to 59 in [1], see also pg 1442 eqs 115 to 116 in [20]). If in a flat 5D Minkowsky Spacetime we cannot ”see” the Extra Dimension then we have three choices in order to tell if we live in a 5D Extra Dimensional Spacetime or a 3 + 1 Ordinary Dimensional one. The choices are:

- Making the Warp Field $\Phi \neq 0$ in order to generate a difference between the 5D Extra Dimensional Ricci Tensor and the 3 + 1 Spacetime counterpart according to eq 45 in [5], eq 59 in [1] and eq 116 in [20]. This difference can tell the difference between a 5D Universe and a 3 + 1 one. (see pg 1477 in [20])

- Making the 3 + 1 Spacetime Metric Tensor components be a function of the 5D Extra Dimension in order to do not vanish the derivatives of the Metric Tensor with respect to the Extra Dimension generating a difference between the 5D Ricci Tensor and the 3 + 1 counterpart according to eqs 330 and 331 pg 1477 in [20]. A Strong Gravitational Field of a Large Maartens-Clarkson 5D Schwarzschild

---

\(^{20}\)The System Earth-Sun have a weak Gravitational Field so around Earth the Spacetime is considered flat
Black String have the Spacetime Metric Tensor Components defined in function of the $4D$ rest-mass $M$ but the $4D$ rest-mass is function of the $5D$ Extra Dimensional Spacetime Geometry according to eq 20 in [2].

- Making both conditions above hold true

We will examine all of the items above in this section. We live in a region of Spacetime where the Warp Field $\Phi = 1$ then we cannot see the $5D$ Extra Dimension. Or we can live in a region of Spacetime where the Warp Field $\Phi = 0$ and this cancels out the term $\Phi^2 dy^2$ in the $5D$ Spacetime Ansatz making the Extra Dimension invisible. Or perhaps we can live in a region of spacetime where $0 \leq \Phi \leq 1$ but near to 0 or 1 so its very difficult to detect the presence of the $5D$ Extra Dimension although we can ”feel” some of its effects. Considering now a Warp Field $\Phi \neq 1$ the Minkowsky $5D$ Spacetime Ansatz would still have the terms of the $3 + 1$ Spacetime Metric Tensor given by $g_{\mu \nu} = (+1, -1, -1, -1)$. Hence the $5D$ Spacetime Ansatz would then be:

$$dS^2 = dt^2 - dX^2 - \Phi^2 dy^2 \quad (56)$$

The derivatives of the $3 + 1$ components of the Spacetime Metric Tensor vanishes but note that the $5D$ component do not vanish. The Ricci Tensor and Scalar would be given by the following expressions (see eqs 335, 336 and 337 in [20]):

$$5R_{\alpha \beta} = R_{\alpha \beta} - \frac{\Phi_{,a \cdot b}}{\Phi} \quad (57)$$

$$5R = R - \frac{\Phi_{,a \cdot b} g_{\alpha \beta}}{\Phi} \quad (58)$$

$$5R = R - \frac{4 \Box \Phi}{\Phi} \quad (59)$$

Note that now the scenario is different: while with the Warp Field $\Phi = 1$ the $5D$ Ricci Tensor is equal to its $3 + 1$ counterpart and we cannot tell if we live in a $5D$ or in a $3 + 1$ Universe but when the Warp Field $\Phi \neq 1$ there exists a difference between the Ricci Tensor in $5D$ and the $3 + 1$ one. The Geometrical Properties of Spacetime of the $5D$ Spacetime are now different than the $3 + 1$ equivalent one and this makes the $5D$ Extra Dimension visible. (see also pg 1478 after eq 337 in [20])

According to Basini-Capozziello the Warp Field can be decomposed in two parts: one in $3 + 1$ ordinary Spacetime and another in the $5D$ Extra Dimension given by the following equation: ([1] eq 76, [5] eq 70 and [20] eq 132 and 338)

$$\Phi = \phi(t, x) \chi(y) \quad (60)$$

Note that when we compute the covariant derivative of the Warp Field with respect to the $3 + 1$ Spacetime the terms of the $5D$ Extra Dimension are cancelled out and we are left with derivatives of the $3 + 1$ components of the Warp Field (see eq 339 in [20])

$$\Phi_{,a \cdot b} = \chi(y) [\phi_{,a \cdot b}] = \chi(y) [\phi_{,a}] - \Gamma^K_{\beta \alpha} \phi_K \rightarrow \frac{\Phi_{,a \cdot b}}{\Phi} = \frac{\chi(y) [\phi_{,a \cdot b}]}{\phi(t, x) \chi(y)} = \frac{[\phi_{,a \cdot b}]}{\phi(t, x)} = \frac{[\phi_{,a}] - \Gamma^K_{\beta \alpha} \phi_K}{\phi(t, x)} \quad (61)$$

The result shown below is very important. It demonstrates that only the $3 + 1$ component of the Warp Field $\phi(t, x)$ fortunately the component that lies in "our side of the wall" and its derivatives with (again
fortunately) respect to our 3 + 1 Spacetime coordinates can make the 5D Ricci Tensor be different than its 3 + 1 counterpart and since we are considering in this case a flat Minkowsky Spacetime the 4D Ricci Tensor reduces to zero and this means to say that $^5R_{\alpha\beta} = -\frac{\phi_{\alpha;b}}{\phi}$ or better $^5R_{\alpha\beta} = \frac{[(\phi_{\alpha})_{\beta} - \Gamma^K_{\beta\alpha}\phi_K]}{\phi(t,x)}$ 

$$^5R_{\alpha\beta} = R_{\alpha\beta} - \frac{\phi_{\alpha;b}}{\phi}$$ \hspace{1cm} (62) 

$$^5R_{\alpha\beta} = R_{\alpha\beta} - \frac{[(\phi_{\alpha})_{\beta} - \Gamma^K_{\beta\alpha}\phi_K]}{\phi(t,x)}$$ \hspace{1cm} (63) 

Note that if the 4D Ricci Tensor vanishes due to a flat Minkowsky Spacetime and we are left with derivatives of the 3 + 1 Spacetime components of the Warp Field with respect to (again fortunately for the second time) 3 + 1 Spacetime coordinates and we are left with a result in which the 5D Ricci Tensor and our capability to detect the existence of the 5D Extra Dimension depends on the shape of the 3 + 1 component of the Warp Field. If we can detect the derivatives of the Warp Field we can detect the existence of the 5D.

The other way to make the 5D Extra Dimension visible is to make the derivatives of the 3+1 Spacetime Metric Tensor components $g_{\mu\nu} = (g_{00}, g_{11}, g_{22}, g_{33})$ non-null with respect to the 5D Extra Dimension.

$$dS^2 = dt^2 - g_{\mu\nu}d(X^\mu)^2 - \Phi^2 dy^2$$ \hspace{1cm} (64) 

For our special case of diagonalized metric:

$$dS^2 = dt^2 - g_{\mu\nu}d(X^\mu)^2 - \Phi^2 dy^2$$ \hspace{1cm} (65) 

Considering the 3 + 1 Spacetime Metric Tensor Components $g_{00}$ and $g_{11}$ (see eqs 353 and 354 in [20]).

$$^5R_{00} = R_{00} - \Phi_{0,0} - \frac{1}{2\Phi^2}(\Phi_{,4}g_{00,4} - \frac{g_{00}^0g_{00,4}004}{2})$$ \hspace{1cm} (66) 

$$^5R_{11} = R_{11} - \Phi_{1,1} - \frac{1}{2\Phi^2}(\Phi_{,4}g_{11,4} - \frac{g_{11}^1g_{11,4}114}{2})$$ \hspace{1cm} (67) 

Now we can see that if the derivatives of $g_{00}$ and $g_{11}$ do not vanish with respect to the Extra Coordinate then the terms $\Phi_{0,0} - \frac{1}{2\Phi^2}(\Phi_{,4}g_{00,4} - \frac{g_{00}^0g_{00,4}004}{2})$ and $\Phi_{1,1} - \frac{1}{2\Phi^2}(\Phi_{,4}g_{11,4} - \frac{g_{11}^1g_{11,4}114}{2})$ will generates a difference between the 5D Ricci Tensor and its 3 + 1 Ordinary Spacetime Dimensional counterpart. Remember also that $g_{00}$ and $g_{11}$ can be defined as the Spacetime Metric Tensor Components of the Maartens-Clarkson 5D Schwarzschild Black String centered on a large Black Hole for example in which $M$ is the 4D rest-mass of the Black Hole but $M$ can be defined in function of the 5D Extra Dimensional rest-mass $M_5$ and also defined in function of the 5D Spacetime Geometry according to Ponce De Leon eq 20 in [2]. This can make the 5D Extra Dimension becomes visible. Writing the Maartens-Clarkson 5D Schwarzschild Cosmic Black String as follows:([7] eq 1,[20] eq 380):

$$dS^2 = [(1 - \frac{2GM}{R})dt^2 - \frac{dR^2}{(1 - \frac{2GM}{R})} - R^2d\eta^2] - \Phi dy^2$$ \hspace{1cm} (68) 

Where the Spacetime Metric Tensor Components of the Black String are given by:$g_{00} = (1 - \frac{2GM}{R})$ and $g_{11} = -(1 - \frac{2GM}{R})^{-1}$. The derivatives with respect to the Extra Coordinate are then$^{21}$:

$^{21}$only time and radial components are considered here.
\[
\frac{\partial g_{00}}{\partial y} = \frac{\partial (1 - 2G M)}{\partial y} = -2G \frac{\partial M}{\partial y} = -2G \left[ \frac{\partial M}{\partial y} \times R^{-1} + \frac{\partial R^{-1}}{\partial y} \times M \right]
\] (69)

We know that the 4D rest-mass \( M \) of the Maartens-Clarkson 5D Schwarzschild Cosmic Black String can be defined in function of the Ponce De Leon 5D rest-mass \( M_5 \) eq 20 in [2]. The final result would then be:

\[
\frac{\partial g_{00}}{\partial y} = -2G \frac{1}{R} \frac{M_5}{\sqrt{1 - \Phi^2 (\frac{dy}{ds})^2}} \left[ \Phi^2 \frac{dy}{ds} \frac{\partial \Phi}{\partial y} + (\frac{dy}{ds})^2 \Phi \frac{\partial^2 \Phi}{\partial y^2} - \frac{1}{R^2} \left( \frac{\partial R}{\partial y} \right) \frac{M_5}{\sqrt{1 - \Phi^2 (\frac{dy}{ds})^2}} \right] (70)
\]

\[
\frac{\partial g_{11}}{\partial y} = \frac{\partial g_{00}}{\partial y} g_{00} = -2G \frac{1}{R} \frac{M_5}{\sqrt{1 - \Phi^2 (\frac{dy}{ds})^2}} \left[ \Phi^2 \frac{dy}{ds} \frac{\partial \Phi}{\partial y} + (\frac{dy}{ds})^2 \Phi \frac{\partial^2 \Phi}{\partial y^2} - \frac{1}{R^2} \left( \frac{\partial R}{\partial y} \right) \frac{M_5}{\sqrt{1 - \Phi^2 (\frac{dy}{ds})^2}} \right] (71)
\]

Note that in a Strong Gravitational Field these derivatives will have high values and this will make the 5D Ricci Tensor be highly different than its 3 + 1 counterpart making the 5D Extra Dimension be visible but far away from the center of the Black String \( M_5 \) \( R \simeq 0 \) and the derivatives will vanish due to the Weak Gravitational Field however the term corresponding to the Warp Field will remain as shown below:

\[
5R_{00} = R_{00} - \frac{\Phi^{0;0}}{\Phi} (72)
\]

\[
5R_{11} = R_{11} - \frac{\Phi^{1;1}}{\Phi} (73)
\]

Then in a Weak or Null Gravitational Field\(^{22}\)is the Warp Field that can make the 5D Ricci Tensor be different than its 3 + 1 counterpart and can tell if we live in a 5D Extra Dimensional Spacetime or in an ordinary 3 + 1 one. Remember that at faraway distances from Gravitational Field the Spacetime is flat or Minkowskian and the 3 + 1 Ricci Tensor is zero or nearly zero. Then we could rewrite the two equations above as follows:

\[
5R_{00} = -\frac{\Phi^{0;0}}{\Phi} (74)
\]

\[
5R_{11} = -\frac{\Phi^{1;1}}{\Phi} (75)
\]

We already know that when computing derivatives of the Warp Field with respect to 3 + 1 Coordinates the 5D Extra Dimensional terms are cancelled out and we will get these results:

\[
5R_{00} = -\frac{[\phi_{0} - \Gamma_{0}^{K} \phi_{K}]}{\phi(t,x)} (76)
\]

\[
5R_{11} = -\frac{[\phi_{1} - \Gamma_{1}^{K} \phi_{K}]}{\phi(t,x)} (77)
\]

Writing the Ricci Tensors with the derivatives of the 3 + 1 Spacetime Metric Tensor Components of the Warp Field explicitly written we have:

\(^{22}\)eg Earth-Sun System or a Spaceship far away from a Black Hole
\[ 5R_{00} = -\frac{\partial^2 \phi(t,x)}{\partial t^2} + \Gamma_K \frac{\partial \phi(t,x)}{\partial x^K} \phi(t,x) \] (78)

\[ 5R_{11} = -\frac{\partial^2 \phi(t,x)}{\partial R^2} + \Gamma_K \frac{\partial \phi(t,x)}{\partial x^K} \phi(t,x) \] (79)

\[ \Gamma^{K} \frac{\partial \phi(t,x)}{\partial x^K} = \Gamma^{00} \frac{\partial \phi(t,x)}{\partial t} + \Gamma^{11} \frac{\partial \phi(t,x)}{\partial R} \phi(t,x) \] (80)

\[ \Gamma^{K} \frac{\partial \phi(t,x)}{\partial x^K} = \Gamma^{00} \frac{\partial \phi(t,x)}{\partial t} + \Gamma^{11} \frac{\partial \phi(t,x)}{\partial R} \phi(t,x) \] (81)

We still don’t know the shape of the Warp Field but remember that the $3 + 1$ component of the Warp Field can be coupled to Gravity as defined by Basini-Capozziello in [1] pg 119 and [5] pg 2235.

Considering only valid Christoffel Symbols we have

\[ \Gamma^{K} \frac{\partial \phi(t,x)}{\partial x^K} = \Gamma^{00} \frac{\partial \phi(t,x)}{\partial t} + \Gamma^{11} \frac{\partial \phi(t,x)}{\partial R} \phi(t,x) \] (82)

\[ \Gamma^{K} \frac{\partial \phi(t,x)}{\partial x^K} = \Gamma^{00} \frac{\partial \phi(t,x)}{\partial t} + \Gamma^{11} \frac{\partial \phi(t,x)}{\partial R} \phi(t,x) \] (83)

These expressions are valid for Strong or Weak Gravitational Fields. But we are considering here Weak Gravitational Fields where the Gravitational Force almost vanishes and the derivatives of the Spacetime Metric Tensor Components of the Maartens-Clarkson $5D$ Schwarzschild Cosmic Black String vanishes due to the term $\frac{M_5}{R}$ \(^{24}\) and the final expression for the $5D$ Ricci Tensors can be given by:

\[ 5R_{00} = -\frac{\partial^2 \phi(t,x)}{\partial t^2} \phi(t,x) \] (84)

\[ 5R_{11} = -\frac{\partial^2 \phi(t,x)}{\partial R^2} \phi(t,x) \] (85)

The $5D$ Ricci Scalar would be given by:

\[ 5R = 5R_{00} + 5R_{11} = -\frac{\partial^2 \phi(t,x)}{\partial t^2} \phi(t,x) + \frac{\partial^2 \phi(t,x)}{\partial R^2} \phi(t,x) = -\frac{1}{\phi(t,x)} 4 \Box \phi(t,x)^2 \] (86)

Remarkably we can extract the Ricci Scalar from the $5D$ to $4D$ Dimensional Reduction of Basini-Capozziello.

If the Warp Field Coupled to Gravity is defined by Basini-Capozziello then this can be regarded as a final proof that the $5D$ Extra Dimension really exists.

Considering now the case of the $5D$ Spacetime Metric with no Warp Field at all $\Phi = 1$ the difference between the $5D$ and the $4D$ Ricci Tensors will depend on the derivatives of the Spacetime Metric Tensor Components with respect to the $5D$ Extra Dimension that will vanish far away from the center of the Maartens Clarkson $5D$ Schwarzschild Cosmic Black String making the $5D$ be invisible but in the regions

\(^{24}\) diagonalized metrics

\(^{24}\) making $g_{00} = 1$ and $g_{11} = 1$
of intense Gravitational Field the 5D Ricci Tensor will be different than its 4D counterpart making the 5D Extra Dimension becomes visible

\[ 5R_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}( -g_{\alpha\beta,44} + \frac{g^{\mu\nu}g_{\mu\nu,4}g_{\alpha\beta,4}}{2}) \]  
(87)

\[ 5R = R - \frac{1}{2}g^{\alpha\beta}( -g_{\alpha\beta,44} + \frac{g^{\mu\nu}g_{\mu\nu,4}g_{\alpha\beta,4}}{2}) \]  
(88)

Writing the Ricci Tensor for the time and radial components we have (see eqs 343 and 344 pg 1479 in [20]):

\[ 5R_{00} = R_{00} - \frac{1}{2}( -g_{00,44} + \frac{g^{00}g_{00,4}g_{00,4}}{2}) \]  
(89)

\[ 5R_{11} = R_{11} - \frac{1}{2}( -g_{11,44} + \frac{g^{11}g_{11,4}g_{11,4}}{2}) \]  
(90)

We know that the derivatives of the Spacetime Metric Tensor Components will vanish making the 5D Ricci Tensor equal to its 3 + 1 counterpart and the 5D Extra Dimension will become invisible. But in the neighborhoods of the Black String center Gravity becomes so high that the Extra Terms will make the 5D Ricci Tensor be different than its 3 + 1 counterpart. Another way to measure the presence of the 5D Extra Dimension is to measure how the Extra Dimension affects the Gravitational Bending of Light in the vicinity of the Black String according to Kar-Sinha (see abstract of [3]). In one of our works (20 pg 1495 section 8) we proposed the use of the International Space Station ISS\(^{25,26}\) to measure the Kar-Sinha Gravitational Bending of Light of the Sun to find out if it can be affected by the presence of the 5D Extra Dimension. (see also pg 1467 before eq 290 in [20]) While the Sun have a “weak” Gravitational Field a Black String is a Black Hole in 5D and in the vicinity of the Black String perhaps the Gravitational Bending Of Light affected by the presence of the 5D Extra Dimension according to Kar-Sinha would be better noticeable (ISS could still be used in this fashion to measure Gravitational Bending Of Light affected by the presence of Extra Dimensions by observing accretion disks of Black Holes free from the disturbances of Earth Atmosphere. We propose here to use beams of neutrons or photons to measure the Extra Terms On-Board ISS ). Writing the Kar-Sinha Gravitational Bending Of Light affected by the presence of the 5D Extra Dimension in the neighborhoods of a Black String or in the neighborhoods

\(^{25}\) more on General Relativity and ISS in [6], [8], [16], [17] and [18]

\(^{26}\) ISS will also appear in the next section
of the Sun to be measured On-Board ISS with a non-null Warp Field as follows (see pg 1467 eq 288 to 291 and pg 1468 eq 295 to 296 in [20]) (see also pg 1781 eq 18 in [3])

\[ \Delta \omega = \frac{2GM}{c^2R} (2 + |\Phi \frac{dy}{cdt}|^2) \]  
(93)

\[ \Delta \omega = \frac{2GM}{c^2R} (2 + |\phi(t, x)\chi(y) \frac{dy}{cdt}|^2) \]  
(94)

\[ \Delta \omega = \frac{2G}{c^2R} \frac{M_5}{\sqrt{1 - \Phi^2(\frac{dy}{dt})^2}} (2 + |\Phi \frac{dy}{cdt}|^2) \]  
(95)

\[ \Delta \omega = \frac{2G}{c^2R} \frac{M_5}{\sqrt{1 - \phi(t, x)^2\chi(y)^2(\frac{dy}{dt})^2}} (2 + |\phi(t, x)\chi(y) \frac{dy}{cdt}|^2) \]  
(96)

The same expression for a null Warp Field would be given by:

\[ \Delta \omega = \frac{2GM}{c^2R} (2 + \left| \frac{dy}{cdt} \right|^2) \]  
(97)

\[ \Delta \omega = \frac{2G}{c^2R} \frac{M_5}{\sqrt{1 - \left( \frac{dy}{dt} \right)^2}} (2 + \left| \frac{dy}{cdt} \right|^2) \]  
(98)

In the above equations \( M_5 \) and \( M \) are the 5D and 4D rest-masses of the Sun or the Black Hole and \( R \) the distance between the accretion disk and the Black Hole or the distance between the photon beam and the Sun. We know that the Warp Field must have values between 0 and 1 so the shift in the Gravitational Bending Of Light must be very small making the value of the expression in 5D be close to its 4D counterpart. The presence of the Gravitational Constant \( G = 6.67 \times 10^{-11} \text{ Newton timesm}^2/\text{kg}^2 \) divided by the square of the Light Speed would make the things even worst. This is the reason why we need a Black String of large rest mass \( M \) or \( M_5 \) to make the shift noticeable. Perhaps in the Sun we would never be able to measure the shift. Note also the comment on [21] pg 71 that the derivative \( \frac{dy}{dt} \) analogous to our \( \frac{dy}{ds} \) is null for photons and we know from Ponce De Leon that the 5D Spacetime Metric \( ds^5 = ds^2 - \Phi^2 dy^2 \) is null for photons making \( ds^2 = \Phi^2 dy^2 \) \( M_5 = 0 \) and \( m_0 = 0 \). Remember that in 4D SR \( ds^2 = 0 \) for photons and \( \frac{dy}{ds} = 0 \) making the shift in 5D be equal to its 4D counterpart. Kar-Sinha mentions in pg 1783 [3] the fact that if the photon propagates in 5D the value of \( \frac{dy}{ds} < 2.8 \times 10^{-4} \) and the shift \( \Delta \omega \) affected by the 5D Extra Dimension must lie between the error margins of the observed values of pgs 39 to 41 in [19]. Remember also that the Spacetime at a distance \( R \) from the Sun where the photon passes by in order to be Bent or Deflected is described by the 5D Black String centered on the Sun according to [7] eq 1,[20] eq 380. Although \( \frac{dy}{dt} \) is zero for photons it may be not for the Sun Mass \( M \) and then the Extra Terms in the Gravitational Bending Of Light for photons may still be measurable anyway. The Gravitational Bending

---

\(^{27}\) equations written without Warp Factors and with the Gravitational Constant

\(^{28}\) see also eqs 156 to 158 pg 70 section 8.7 in [21].see also in the same reference the comment on the velocity along the 5D Extra Dimension in pg 71 after eq 159 \( \frac{dv}{dt} \) similar to our \( \frac{dy}{dt} \).see also between page 70 and 71 the comment that the shift is physically measurable. we will examine photon paths in the 5D Maartens-Clarkson Schwarzschild Cosmic Black String in this section also but we will use the Ponce De Leon point of view of pg 1343 after eq 30 in [2].look to the Ponce De Leon comment of genuine manifestation of the 5D Extra Dimension before section 4

\(^{29}\) this reference contains one of the best explanations for the Gravitational Bending Of Light Geometry and describes even the 30 percent margin of error in the 1919 measurements
can be observed for other particles with a non-null $M_5$ and a non-null $m_0$ and perhaps the study of the motion of high-speed relativistic particles from accretion disks of large Black Holes can tell the difference between the $5D$ $\Delta \omega$ and its $4D$ counterpart. For a non-null $\frac{dy}{ds}$ particle the Gravitational Bending formulas could be given by:

\[ \Delta \omega = \frac{2GM}{c^2 R} (2 + \frac{\Phi}{ds c dt} dy ds c dt)^2 \] (99)

\[ \Delta \omega = \frac{2GM}{c^2 R} (2 + [\phi(t, x)\chi(y)\frac{dy ds c dt}{ds c dt}])^2 \] (100)

\[ \Delta \omega = \frac{2G}{c^2 R} \frac{M_5}{\sqrt{1 - \Phi^2(\frac{dy}{ds}^2)}} (2 + \frac{\Phi}{ds c dt} dy ds c dt)^2 \] (101)

\[ \Delta \omega = \frac{2G}{c^2 R} \frac{M_5}{\sqrt{1 - \phi(t, x)^2 \chi(y)^2(\frac{dy}{ds}^2)}} (2 + \frac{\phi(t, x)\chi(y)}{ds c dt} dy ds c dt)^2 \] (102)

The same for a Warp Field $\Phi = 1$

\[ \Delta \omega = \frac{2GM}{c^2 R} (2 + \frac{dy ds c dt}{ds c dt})^2 \] (103)

\[ \Delta \omega = \frac{2G}{c^2 R} \frac{M_5}{\sqrt{1 - \frac{dy}{ds}^2}} (2 + \frac{dy ds c dt}{ds c dt})^2 \] (104)

Note that a relativistic beam of neutrons would not suffer the deflection by electromagnetic fields and could be used to measure the Extra Terms in the Gravitational Bending of Light due to the presence of the Higher Dimensional Spacetime but a beam of photons is more easy to be obtained. Consider a Satellite carrying a small Laser device in the other side of the Earth Orbit targeting the beam towards a target in ISS. The beam in order to reach ISS must pass at a distance $R$ from the Sun. The Extra Terms in $\Delta \omega$ could perhaps be measured in Outer Space On-Board International Space Station ISS free of the interference of the Earth Atmosphere. Computing the Classical Bending of Light as (see pg 1781 eq 18 in [3]):

\[ \Delta \omega_{\text{Classic}} = \frac{4GM}{c^2 R} \] (105)

If we observe these deviations of the Bending Angle

\[ \Delta \omega_{\text{ExtraTerms}} = \frac{2GM}{c^2 R} [\Phi \frac{dy}{ds c dt}]^2 \] (106)

\[ \Delta \omega_{\text{ExtraTerms}} = \frac{2GM}{c^2 R} [\frac{dy}{ds c dt}]^2 \] (107)

different than the original Classical value then we can demonstrate that we live in a $5D$ Higher Dimensional Spacetime.

The Laser beam would be affected by the Sun Mass $M$ while passing at a distance $R$ from the Sun but would reach ISS. Computing the Classical Einstein Bending we know where the Laser will reach the target. If the observed Bending is equal to the Classical Einstein then this would mean that there are no Extra Dimensions in the Universe. But if the deviated photon arrives at ISS with an angle different than
the one predicted by Einstein and if this difference in the angle matches the Extra-Terms predicted by Kar-Sinha then ISS would proof that we live in a Universe of more than 4 Dimensions. Then this experiment on-board ISS would have the same degree of importance of the measures of the Gravitational Bending of Light by Sir Arthur Stanley Eddington in the Sun Eclipse of 1919

Computing the Classical Einstein Bending Of Light for a Laser beam passing the Sun at distance \( R = 150,000 km \)\(^{30}\) we would have:

- **Mass of the Sun (in 4D):**
  \[
  M = 1.9891 \times 10^{30} \text{ kg}
  \]

- **Newton Gravitational Constant (in 4D):**
  \[
  G = 6.67 \times 10^{-11} \text{ Newton} \times m^2/kg^2
  \]

There exists a common factor between the Classical Bending Of Light in 4D and the Kar-Sinha Extra Terms given by:

\[
\Delta \omega_{\text{CommonFactor}} = \frac{2GM}{c^2R}
\]

The common factor would be given by:

\[
\Delta \omega_{\text{CommonFactor}} = 2 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{30} / (1.5 \times 10^8 \times 9 \times 10^{16}) = 1,877481481481 \times 10^{-5}
\]

\[
2 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{30} = 25,346 \times 10^{19}
\]

\[
1.5 \times 10^8 \times 9 \times 10^{16} = 13,5 \times 10^{24}
\]

\[
\frac{25,346}{13,5} = 1,877481481481
\]

The Classical Einstein Bending Of Light would be given by:

\[
\Delta \omega_{\text{Classic}} = 4 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{30} / (1.5 \times 10^8 \times 9 \times 10^{16}) = 3,754962962962 \times 10^{-5}
\]

\[
4 \times 6.67 \times 10^{-11} \times 1.9 \times 10^{30} = 50,692 \times 10^{19}
\]

\[
\frac{50,692}{13,5} = 3,754962962962
\]

We already outlined in the Introduction Section the fact that the goal of ISS is to achieve a Gravitational Shift Precision of \( 2.4 \times 10^{-7} \) (see pg 631 Table II in [6]) and the fact that ISS Gravitational shifts are capable to detect measures of \( \Delta \omega \) with Expected Uncertainty of \( 12 \times 10^{-6} \) (see pg 629 Table I in [6])

---

\(^{30}\)Remember that the Sun lies between ISS and the Satellite placed in the other side of the Earth Orbit. The laser must pass in the neighbourhoods of the Sun.
The difference between Kar-Sinha and Classical Gravitational Bending of Light is given by:

\[
\Delta \omega_{\text{KarSinha}} - \Delta \omega_{\text{Classic}} = \frac{2GM}{c^2R} (2 + [\Phi \frac{dy}{cdt}]^2) - \frac{4GM}{c^2R} \tag{118}
\]

\[
\Delta \omega_{\text{KarSinha}} - \Delta \omega_{\text{Classic}} = \frac{4GM}{c^2R} + \frac{2GM}{c^2R} [\Phi \frac{dy}{cdt}]^2 - \frac{4GM}{c^2R} \tag{119}
\]

\[
\Delta \omega_{\text{KarSinha}} - \Delta \omega_{\text{Classic}} = \frac{2GM}{c^2R} [\Phi \frac{dy}{cdt}]^2 \tag{120}
\]

\[
\Delta \omega_{\text{KarSinha}} - \Delta \omega_{\text{Classic}} = \Delta \omega_{\text{CommonFactor}} [\Phi \frac{dy}{cdt}]^2 \tag{121}
\]

\[
\Delta \omega_{\text{KarSinha}} - \Delta \omega_{\text{Classic}} = 1,877481481481 \times 10^{-5} [\Phi \frac{dy}{cdt}]^2 \tag{122}
\]

\[
\Delta \omega_{\text{KarSinha}} - \Delta \omega_{\text{Classic}} = \Delta \omega_{\text{ExtraTerms}} \tag{123}
\]

And this difference is equal to the Common Factor multiplied by the Kar-Sinha Extra Terms due to the presence of the Higher Dimensional Spacetime.

The Kar-Sinha additional terms depend on the derivative of the 5D Extra Coordinate or the value of the Warp Field \(\Phi\) due to the Extra Dimensional factors \([\Phi \frac{dy}{cdt}]^2\). We know that for a Timelike 5D geodesics according to Ponce De Leon we have \(0 < \Phi < 1\) and \(0 < [\Phi \frac{dy}{ds}]^2 < 1\) (see eq3 pg 1426 in [20] without the Warp Factors \(\Omega = 1\)) and for photons we would have a Null-Like Geodesics (see eq4 pg 1426 in [20] without the Warp Factors \(\Omega = 1\)) giving \(0 < [\Phi \frac{dy}{ds}]^2 = 1\). Assuming also a small derivative of the Extra Coordinate with respect to time \(\frac{dy}{cdt}\) then we would have very small values for the Extra Dimensional Term \([\Phi \frac{dy}{cdt}]^2\) at least for a Timelike Geodesics.

Examining now the case for photons in a Null-Like 5D Geodesics in the neighborhoods of the Sun as a Maartens-Clarkson Schwarzschild Black String:\[31\]

\[
dS^2 = 0 \longrightarrow ds^2 - \Phi^2 dy^2 = 0 \longrightarrow ds^2 = \Phi^2 dy^2 \longrightarrow \frac{1}{\Phi^2} = (\frac{dy}{ds})^2 \longrightarrow 1 = \Phi^2 (\frac{dy}{ds})^2 \tag{124}
\]

\[
1 = \Phi^2 (\frac{dy}{ds})^2 \longrightarrow 1 = \Phi^2 (\frac{dy}{cdt})^2 (\frac{cdt}{ds})^2 \longrightarrow \Phi^2 (\frac{dy}{cdt})^2 = (\frac{ds}{cdt})^2 \tag{125}
\]

\[
\Delta \omega_{\text{ExtraTerms}} = 1,877481481481 \times 10^{-5} [\Phi \frac{dy}{cdt}]^2 = 1,877481481481 \times 10^{-5} (\frac{ds}{cdt})^2 \tag{126}
\]

\[
dS^2 = [(1 - \frac{2GM}{c^2R})(cdt)^2 - \frac{dR^2}{(1 - \frac{2GM}{c^2R})} - R^2 d\eta^2] - \Phi dy^2 \tag{127}
\]

\[
ds^2 = [(1 - \frac{2GM}{c^2R})(cdt)^2 - \frac{dR^2}{(1 - \frac{2GM}{c^2R})} - R^2 d\eta^2] \tag{128}
\]

\[31\] We already outlined the fact that Kar-Sinha mentions in pg 1783 [3] the fact that if the photon propagates in 5D the value of \(\frac{dy}{ds}\) is less than \(2 \times 10^{-4}\) and the shift \(\Delta \omega\) affected by the 5D Extra Dimension must lie between the error margins of the observed values of pgs 39 to 41 in [19].
\[
\frac{ds^2}{(cdt)^2} = \left[ 1 - \frac{2GM}{c^2R} \right] - \frac{1}{1 - \frac{2GM}{c^2R}} \left( \frac{dR^2}{(cdt)^2} - R^2 \frac{d\eta^2}{(cdt)^2} \right)
\] (129)

\[
\frac{ds^2}{(cdt)^2} = \left[ 1 - \Delta\omega_{\text{CommonFactor}} \right] - \frac{1}{1 - \Delta\omega_{\text{CommonFactor}}} \left( \frac{dR^2}{(cdt)^2} - R^2 \frac{d\eta^2}{(cdt)^2} \right)
\] (130)

\[
1 - \frac{2GM}{c^2R} = 1 - \Delta\omega_{\text{CommonFactor}} = 1 - 1,877,481,481 \times 10^{-5} = 0.9998,122,518,518,519
\] (131)

\[
\frac{ds^2}{(cdt)^2} = \left[ 1 - \Delta\omega_{\text{CommonFactor}} \right] - \frac{1}{c^2} \left( \frac{1}{1 - \Delta\omega_{\text{CommonFactor}}} \frac{dR^2}{(dt)^2} - R^2 \frac{d\eta^2}{(dt)^2} \right)
\] (132)

Note that the term \(\frac{1}{c^2}\) will make the second right term neglectable due to the factor \(10^{-16}\).

\[
\frac{ds^2}{(cdt)^2} = \left[ 1 - \Delta\omega_{\text{CommonFactor}} \right] = 0.9998,122,518,518,519 = 9,998,122,518,518,519 \times 10^{-1}
\] (133)

\[
\Delta\omega_{\text{KarSinha}} - \Delta\omega_{\text{Classic}} = \Delta\omega_{\text{ExtraTerms}} = 1,877,481,481 \times 10^{-5} \left[ \Phi \frac{dy}{cdt} \right]^2
\] (134)

\[
\Delta\omega_{\text{KarSinha}} - \Delta\omega_{\text{Classic}} = \Delta\omega_{\text{ExtraTerms}} = 1,877,481,481 \times 10^{-5} \left( \frac{ds}{cdt} \right)^2
\] (135)

\[
\Delta\omega_{\text{KarSinha}} - \Delta\omega_{\text{Classic}} = \Delta\omega_{\text{ExtraTerms}} = 1,877,481,481 \times 10^{-5} \times 9,998,122,518,518,519 \times 10^{-1}
\] (136)

\[
\Delta\omega_{\text{KarSinha}} - \Delta\omega_{\text{Classic}} = \Delta\omega_{\text{ExtraTerms}} = 18,771,289,878,096,695,094,540,466,39 \times 10^{-6}
\] (137)

The result above is the most important of this work and means to say that the Kar-Sinha Extra Terms in the Gravitational Bending of Light due to presence of the Extra Dimensions are in the range of the detection capability of the International Space Station ISS enclosing the Gravitational Shift Precision of \(2.4 \times 10^{-7}\) (see pg 631 Table II in [6]) and the Gravitational Shifts of \(\Delta\omega\) with Expected Uncertainly of \(12 \times 10^{-6}\) (see pg 629 Table I in [6]). If this Extra Term is detected with "positive" results then the International Space Station ISS can demonstrate for the first time that our Universe have more than 4D Dimensions making the Physics of Extra Dimensions an Experimental Branch of Modern Physics.
4 Experimental Detection of Extra Dimensions using Gravitational Red-Shifts On-Board the International Space Station ISS

From the abstract of [6] we know that Gravitational Red Shifts are also considered to experiments on the International Space Station ISS. We already outlined before the Gravitational Shift precision of ISS. We also know from Kar-Sinha that Extra Dimensions affects the Gravitational Red Shift (see pg 1782 in [3]) generating Extra Terms in a way similar to the ones for the Gravitational Bending Of Light. We will propose in this Section a way to detect these Extra Terms On-Board ISS as a second proof that Extra Dimensions exists (or not).

There are two Classical expressions for the Gravitational Red-Shift $\Delta \lambda$ (The wavelength displacement in Spectral Lines due to Gravity as seen by a far away observer in free space). One approximate and one exact. The approximate expression is given by:

$$\Delta \lambda_{\text{approximate}} = \Delta \lambda_{\text{Classical}} = \frac{GM}{c^2 R}$$  \hspace{1cm} (138)

And the exact one by:

$$\Delta \lambda_{\text{exact}} = \frac{1}{\sqrt{1 - \frac{2GM}{c^2 R}}} - 1$$ \hspace{1cm} (139)

These expressions considering the Kar-Sinha Extra Terms due to the presence of the Extra Dimensions would be given by:

$$\Delta \lambda_{\text{approximate}_{ks}} = \frac{GM}{c^2 R} (1 + [\Phi \frac{dy}{ds} \frac{ds}{dt}]^2)$$  \hspace{1cm} (140)

$$\Delta \lambda_{\text{exact}_{ks}} = \frac{1}{\sqrt{1 - \frac{2GM}{c^2 R}} (2 + [\Phi \frac{dy}{ds} \frac{ds}{dt}]^2)} - 1$$ \hspace{1cm} (141)

$$\Delta \lambda_{\text{approximate}_{ks}} = \frac{GM}{c^2 R} (1 + [\Phi \frac{dy}{cdt}]^2)$$ \hspace{1cm} (142)

$$\Delta \lambda_{\text{exact}_{ks}} = \frac{1}{\sqrt{1 - \frac{2GM}{c^2 R}} (2 + [\Phi \frac{dy}{cdt}]^2)} - 1$$ \hspace{1cm} (143)

The approximate expression is more than enough to illustrate our point of view.

$$\Delta \lambda_{\text{KarSinha}} = \frac{GM}{c^2 R} (1 + [\Phi \frac{dy}{cdt}]^2)$$  \hspace{1cm} (144)

$$\Delta \lambda_{\text{KarSinha}} = \Delta \lambda_{\text{Classical}} (1 + [\Phi \frac{dy}{cdt}]^2) = \Delta \lambda_{\text{Classical}} + \Delta \lambda_{\text{Classical}} [\Phi \frac{dy}{cdt}]^2$$ \hspace{1cm} (145)

$$\Delta \lambda_{\text{KarSinha}} - \Delta \lambda_{\text{Classical}} = \Delta \lambda_{\text{Classical}} [\Phi \frac{dy}{cdt}]^2 = \Delta \lambda_{\text{ExtraTerms}}$$ \hspace{1cm} (146)

Our idea is to send a second Satellite with another Laser beam to the Venus Orbit but directed towards the ISS. The Satellite would send the Laser beam to ISS with a certain blue wavelength but when arriving
at ISS due to the difference of Sun Gravitational Fields between Earth and Venus the beam would be Red-Shifted and the Kar-Sinha Extra Terms due to the presence of Extra Dimensions could be detected.

In Venus the Gravitational Red Shift would be given by:

\[ \Delta \lambda_{KarSinhaVenus} = \frac{GM}{c^2 D_{Venus}} (1 + \left[ \Phi \frac{dy}{cdt} \right]_{Venus}^2 ) \] (147)

From the Previous Section we know that

\[ \Delta \lambda_{KarSinhaVenus} = \frac{GM}{c^2 D_{Venus}} (1 + \left[ \frac{ds}{cdt} \right]_{Venus}^2 ) \] (148)

With \( D_{Venus} \) being the Sun Radius \( R \) plus the distance \( d_{Venus} \) from Sun to Venus

On Earth the Gravitational Red Shift would be given by:

\[ \Delta \lambda_{KarSinhaEarth} = \frac{GM}{c^2 D_{Earth}} (1 + \left[ \Phi \frac{dy}{cdt} \right]_{Earth}^2 ) \] (149)

\[ \Delta \lambda_{KarSinhaEarth} = \frac{GM}{c^2 D_{Earth}} (1 + \left[ \frac{ds}{cdt} \right]_{Earth}^2 ) \] (150)

With \( D_{Earth} \) being the Sun Radius \( R \) plus the distance \( d_{Earth} \) from Sun to Earth

Since in Venus the Laser is still Blue-Shifted when sent towards ISS the Red Shift detected by ISS would be generated in the Venus-Earth trip.

\[ \frac{GM}{c^2 D_{Earth}} - \frac{GM}{c^2 D_{Venus}} = \frac{GM}{c^2} \left( \frac{1}{D_{Earth}} - \frac{1}{D_{Venus}} \right) \approx \frac{GM}{c^2} \left[ \frac{1}{d_{Earth}} - \frac{1}{d_{Venus}} \right] \] (151)

The Kar-Sinha Gravitational Red Shift in Extra Dimensions epresions from the Venus Earth trip would be given by the following expressions

\[ \Delta \lambda_{KarSinhaEarthVenus} = \frac{GM}{c^2} \left[ \frac{1}{d_{Earth}} - \frac{1}{d_{Venus}} \right] (1 + \left[ \Phi \frac{dy}{cdt} \right]^2 ) \] (152)

\[ \Delta \lambda_{KarSinhaEarthVenus} = \frac{GM}{c^2} \left[ \frac{1}{d_{Earth}} - \frac{1}{d_{Venus}} \right] (1 + \left[ \frac{ds}{cdt} \right]^2 ) \] (153)

And the Extra Terms due to the presence of the Extra Dimensions would be given by:

\[ \Delta \lambda_{ExtraTermsEarthVenus} = \frac{GM}{c^2} \left[ \frac{1}{d_{Earth}} - \frac{1}{d_{Venus}} \right] \left[ \Phi \frac{dy}{cdt} \right]^2 \] (154)

\[ \Delta \lambda_{ExtraTermsEarthVenus} = \frac{GM}{c^2} \left[ \frac{1}{d_{Earth}} - \frac{1}{d_{Venus}} \right] \left[ \frac{ds}{cdt} \right]^2 \] (155)

From the previous Section we know that ISS can measure these Extra Terms if the Extra Dimensions exists.
5 Conclusion-Physics of Extra Dimensions as an Experimental Branch of Physics for the first time

Our approach to the study of Extra Dimensions was centered on the Basini-Capozziello-Ponce de Leon Formalism. While other formalisms of Extra Dimensions uses $3 + 1$ uncompactified ordinary spacetime dimensions while the Extra Dimensions are compactified bringing the question of why $3 + 1$ large ordinary dimensions and the rest of the Extra Dimensions "curled-up" over themselves and what causes or generates the "compactification mechanism"????. In the Basini-Capozziello-Ponce de Leon Formalism the Extra Dimensions are large the same size of the $3 + 1$ ordinary dimensions avoiding the need of "exotic" compactification mechanisms but we cannot "see" these dimensions in normal conditions due to the reasons presented in this work. Also it can explain the multitude of particles seen in $4D$ as Dimensional Reductions from a small group of particles in $5D$ allowing perhaps the "unification" of Physics from the point of view of the Extra Dimensional Spacetime. This is very attractive from the point of view of a Unified Physics theory. There exists a small set of particles in $5D$ and all the huge number of Elementary Particles in $4D$ is a geometric projection from the $5D$ Spacetime into a $4D$ one([2] eq 20,[11] eq 21 and [20] eq 8).

\[ m_0 = \frac{M_5}{\sqrt{1 - \Phi^2(\frac{\partial^2}{\partial s^2})^2}} \]  
(156)

| Particle | spin ($h$) | B | L | T | T$_3$ | S | C | B* | charge ($e$) | $m_0$ (MeV) | antipart. |
|----------|------------|---|---|---|------|---|---|----|------------|------------|-----------|
| u        | 1/2 1/3 0 1/2 1/2 0 0 0 | +2/3 | 5 |    |      |    |    |    |            |            |           |
| d        | 1/2 1/3 0 1/2 -1/2 0 0 0 | -1/3 | 175 |   |      |    |    |    |            |            |           |
| s        | 1/2 1/3 0 0 0 -1 0 0 | -1/3 | 1350 | 9 |      |    |    |    |            |            |           |
| c        | 1/2 1/3 0 0 0 0 1 0 | +2/3 | 3500 | 5 |      |    |    |    |            |            |           |
| b        | 1/2 1/3 0 0 0 0 0 -1 | -1/3 | 173000 | 40 |      |    |    |    |            |            |           |
| t        | 1/2 1/3 0 0 0 0 0 0 | +2/3 | 173000 | 8 |      |    |    |    |            |            |           |
| e$^-$    | 1/2 0 1 0 0 0 0 0 | -1 | 0.511 | e$^+$  |    |      |    |    |    |            |            |           |
| $\mu^-$ | 1/2 0 1 0 0 0 0 0 | -1 | 105.658 | $\mu^+$ |    |      |    |    |    |            |            |           |
| $\tau^-$| 1/2 0 1 0 0 0 0 0 | -1 | 1777.1 | $\tau^+$ |    |      |    |    |    |            |            |           |
| $\nu_e$ | 1/2 0 1 0 0 0 0 0 | 0 | 0(?) | $\bar{\nu}_e$ |    |      |    |    |    |            |            |           |
| $\nu_\mu$| 1/2 0 1 0 0 0 0 0 | 0 | 0(?) | $\bar{\nu}_\mu$ |    |      |    |    |    |            |            |           |
| $\nu_\tau$| 1/2 0 1 0 0 0 0 0 | 0 | 0(?) | $\bar{\nu}_\tau$ |    |      |    |    |    |            |            |           |
| $\gamma$ | 1 0 0 0 0 0 0 0 | 0 | 0 | $\gamma$ |     |      |    |    |    |            |            |           |
| gluon    | 1 0 0 0 0 0 0 0 | 0 | 0 | gluon |    |      |    |    |    |            |            |           |
| W$^+$    | 1 0 0 0 0 0 0 0 | +1 | 80220 | W$^-$  |    |      |    |    |    |            |            |           |
| Z        | 1 0 0 0 0 0 0 0 | 0 | 91187 | Z |    |      |    |    |    |            |            |           |
| graviton | 2 0 0 0 0 0 0 0 | 0 | 0 | graviton |    |      |    |    |    |            |            |           |

Look to the Elementary Particles Table above: we have a multitude of Quarks, Leptons, Muons and Heavy particles with apparently different rest-masses $m_0$ in $4D$ but these particles can have the same rest-mass $M_5$ in the $5D$ Extra Dimension and the differences are being generated by the Dimensional Reduction from $5D$ to $4D$ according to the Basini-Capozziello-Ponce De Leon formalism. This can bring new perspectives.

\footnote{We know that we are repeating the table for the third time but the table coupled with the Ponce De Leon equation illustrates the beauty of this point of view}
for the desired dream of the Unification of Physics. We proposed here the use of a Satellite\textsuperscript{33} with a Laser device placed in the other side of Earth Orbit with the Sun between the Satellite and ISS. The satellite will send the Laser beam towards ISS and the beam must pass in the neighborhoods of the Sun in order to reach ISS. The beam will be Gravitationally Bent according to Einstein and if Extra Dimensions exist then the Extra Terms in the Gravitational Bending Of Light affected by the presence of the Extra Dimensions predicted by Kar-Sinha will appear. We demonstrated here that ISS have the needed precision to spot the Extra Terms predicted by Kar-Sinha and ISS could answer for the first time the question if the Universe have or not more than 4 Dimensions predicted by many Physics Theories but never seen before. Such an experiment would have the same degree of importance of the measures of Gravitational Bending Of Light made by Sir Arthur Stanley Eddington in the Sun Eclipse of 1919 that proved valid the Einstein General Theory Of Relativity. The implications of a “positive” result would be enormous making the Physics of Extra Dimensions an Experimental Branch of Physics for the first time\textsuperscript{34, 35}. All the theories of Physics Unification that predicts the existence of Extra Dimensions would be regarded as valid Physical Descriptions of Nature and not only mere Mathematical Models adjusted to “normalize consistently” some calculations and this would pose major modifications in Particle Physics since it could be proved that the multitude of apparent different particles we see in 4 Dimensions are Dimensional Reductions or Dimensional Projections from a small group of perhaps the same particles in 5 Dimensions according to the Basini-Capozziello Ponce De Leon formalism. If this experiment is performed with “positive” results then the International Space Station ISS could change drastically and forever our way to understand the Universe.

Young Jedi Knight Padawan: Stay Away From The Dark Side And May The Force Be With You\textsuperscript{36}

\textsuperscript{33}Such a Satellite could perfectly be christened as “Eddington” as a Homage to Sir Arthur Stanley Eddington

\textsuperscript{34}An excellent account of the progresses in this area is given by the link below:

\texttt{http://arXiv.org/find/\texttt{gr-qc}}\texttt{/physics/1/abs : +AND +dimensions + AND + experimental + extra/0/1/0/all/0/1}

\textsuperscript{35}Slightly modified from Frank Oz as Master Yoda in the George Lucas movie Star Wars Episode II The Attack Of The Clones

27
6 European Space Agency Satellite GAIA

After the publication acceptance of this work by NOVA Scientific Publishers in the book Space Exploration Research ISBN 978-1-60692-264-4 as Chapter IX we discovered the Satellite GAIA of European Space Agency. GAIA was originally scheduled to be launched in 2009 by a French rocket Ariane but it is now scheduled to be launched in 2011 by a French-Russian rocket Soyuz-Frigat. GAIA is able to measure the Gravitational Bending Of Light by amounts of $5 \times 10^{-7}$ (pg 4 in [22]) by far more than enough to spot our shift of $18 \times 10^{-6}$. The European Space Agency Satellite GAIA can detect if we live in a Higher Dimensional Universe.
7 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible." - Arthur C. Clarke

- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them." - Albert Einstein

---

37 special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C. Clarke

38 "Ideas And Opinions" Einstein compilation, ISBN 0 – 517 – 88440 – 2, on page 226. "Principles of Research" ([Ideas and Opinions], pp.224-227), described as "Address delivered in celebration of Max Planck’s sixtieth birthday (1918) before the Physical Society in Berlin"

39 appears also in the Eric Baird book Relativity in Curved Spacetime ISBN 978 – 0 – 9557068 – 0 – 6
8 Acknowledgements

We would like to express the most profound and sincere gratitude towards Doctor Frank Columbus Editor-Chief of NOVA Scientific Publishers United States of America for the invitation to write a paper for his presentation conference meeting entitled "Space Stations: Crew, Experiments and Missions." This arXiv.org paper is our answer to the invitation. We also would like to Acknowledge Professor Doctor Martin Tajmar of University Of Viena, Austria – ESA (European Space Agency) – ESTEC-SV (European Space Technology and Engineering Center - Space Vehicles Division) and Seibesdorf Austria Aerospace Corporation GmBH-ASPS (Advanced Space Propulsion Systems), for his kindness and goodwill for being our arXiv.org sponsor and to Acknowledge Paulo Alexandre Santos and Dorabella Martins da Silva Santos from University of Aveiro Portugal for the access to the scientific publication General Relativity and Gravitation(GRG). In closing we would also like to Acknowledge the Administrators and Moderators of arXiv.org at the Cornell University, United States of America for their agreement in accepting this document.
9 Remarks

The bulk of the bibliographic sources used in our research came from the refereed scientific publication General Relativity and Gravitation (GRG) from Springer-Verlag GmBH (formerly Kluver/Plenum Academic Publishing Corp)(ISSN:0001-7701 paper)(ISSN:1572-9532 electronic) under the auspices of the International Comitee on General Relativity and Gravitation and quoted by Deutsche Zentralblatt Math of EMS(European Mathematical Society). The Volume 36 Issue 03 March 2004 under the title:"Fundamental Physics on the ISS" was totally dedicated to test experimentally General Relativity,Quantum Gravity,Extra Dimensions and other physics theories in Outer Space on-board International Space Station(ISS) under the auspices of ESA(European Space Agency) and NASA(National Aeronautics and Space Administration). All the mention to pages of the references in the main text and in the footnotes of this work are for GRG and Liv Rev Rel references originally from the published version since we have access to GRG and Liv Rev Rel although we provide the number of the arXiv.org available GRG and Liv Rev Rel papers but for PhysRpt the page numbers are originally from the arXiv.org version since we cannot access this journal and sometimes exists differences in page numbers between the arXiv.org version and the published version due to different editorial styles preferred by scientific journals.\textsuperscript{40} We choose to adopt in our research mainly refereed published papers from these publications not only due to their prestige and reputation among the scientific community but also because we are advocating new points of view in this work but based on the solid ground of certifiable and credible references.

\textsuperscript{40}readers that can access GRG can compare for example gr-qc/0310078 with [2] or gr-qc/0603106 with [20]
10 Legacy

This work is dedicated to the memory of the British Astronomer Sir Arthur Stanley Eddington that measured for the first time the Gravitational Bending Of Light in the Sun Eclipse of 1919 proving valid the Einstein General Theory Of Relativity. This work is also dedicated with a feeling of gratitude to all the people of NASA(National Aeronautics and Space Administration),ESA(European Space Agency),to all the people of the Space Agencies of Canada,Russia,Japna,China,Brazil,Argentina,Mexico,Israel and India all of these people with major contributions and Manned Space Missions or responsible and involved in one way or another with the project of the International Space Station ISS
References

[1] Basini G. and Capozziello S. (2005). Gen Rel Grav 37 115.
[2] Ponce De Leon J. (2004). Gen Rel Grav 36 1335, gr-qc/0310078.
[3] Kar S. and Sinha M. (2003). Gen Rel Grav 35 1775.
[4] Loup F. Santos P. and Santos D. (2003). Gen Rel Grav 35 2035.
[5] Basini G. and Capozziello S. (2003). Gen Rel Grav 35 2217.
[6] C. Lammerzahl; G. Ahlers N. Ashby, M. Barmatz, P. L. Biermann, H. Dittus, V. Dohm, R. Duncan, K. Gibble, J. Lipa, N. Lockerbie, N. Mulders and C. Salomon. (2004). Gen Rel Grav 36 615
[7] Clarkson C. and Maartens R. (2005). Gen Rel Grav 37 1681, astro-ph/0505277
[8] Dittus H. (2004). Gen Rel Grav 36 601
[9] Mashhoon B., Wesson P. and Liu H. (1998). Gen Rel Grav 30 555
[10] Loup F. Santos P. and Santos D. (2003). Gen Rel Grav 35 1849
[11] Ponce De Leon J. (2003). Gen Rel Grav 35 1365, gr-qc/0207108
[12] Wesson P. (2003). Gen Rel Grav 35 307, gr-qc/0302092
[13] Seahra S. and Wesson P. (2005). Gen Rel Grav 37 1339
[14] Seahra S. and Wesson P. (2001). Gen Rel Grav 33 1731, gr-qc/0105041
[15] Billyard A. and Sajko W. (2001). Gen Rel Grav 33 1929, gr-qc/0105074
[16] Paik H., Moody M. and Strayer D. (2004). Gen Rel Grav 36 523
[17] Dittus H., Lammerzahl C. and Selig H. (2004). Gen Rel Grav 36 571
[18] Walz J and Hansch T. (2004). Gen Rel Grav 36 561
[19] Will C. M., (2006). Liv Rev Rel,(9) lrr-2006-3, gr-qc/0103036
[20] Loup F (2006). Gen Rel Grav 38 1423, gr-qc/0603106
[21] Overduin J.M. and Wesson P. (1997). Phys.Rept. 283 303-380, gr-qc/9805018
[22] Perryman M.A.C, Pace.O,(2000), ESA Bulletin 103