Distribution Center Location Model Based on Gauss-Kruger Projection and Gravity Method

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Abstract—The latitude and longitude coordinates are converted into Gaussian plane coordinates through the forward calculation of the Gauss-Kruger projection, and then the location is selected through the center of gravity method, and then the location result is converted into the latitude and longitude coordinates through the reverse calculation of the Gauss-Kruger projection. This method was then applied to the location of distribution centers among the four cities in Zhejiang Province, and reasonable results were obtained.

1. INTRODUCTION

As an important part of the modern logistics system, the distribution center is at the center of the logistics system and is the dense area of the goods flow in the logistics system. Therefore, reasonable planning or not has a significant impact on reducing the overall freight of the logistics system and enhancing the quality of logistics. Scientific distribution center location selection needs to consider the actual situation of the logistics system. The most basic distribution center location problem is the single-point location problem, that is, only one distribution center is set up for distribution of multiple demand points. The distribution center location method used in the early days was the center of gravity method. With the advancement of mathematics, computer science, and social science, scholars have also tried to use a fuzzy approach, Robust Optimization, TOPSIS methodology, heuristic algorithms and other methods for distribution center location selection.

As a classic location method, the center of gravity method still has strong practical significance. The center of gravity method mainly studies the influence of the distance between logistics nodes and freight on the total cost. It determines the location of distribution center by solving the center of gravity of logistics system. Scholars of the Nitto Research Institute improved the method of solving the center of gravity method, constructed a differential equation based on the partial derivative of the total freight, and obtained the best distribution center through an iterative calculation. XiaoChun Lu et al. pointed out that due to the vector operation in the center of gravity formula, it does not correspond to the actual location problem conditions, and demonstrated the reliability of the differential method. Yan Zhang et al. combined the Gravity Method and Fuzzy-AHP Model to expand the center of gravity method, not only considering the freight, but also comprehensively considering the natural conditions, social environment, and economic factors that affect the site selection, making the results closer to reality. MaoSheng Yang et al. improved the objective function of the logistics system, and comprehensively...
considered the location problem of the center of gravity method in the case of variable operating costs and fixed cost reduction. JiLi Kong et al. combined the Hierarchical Clustering and Gravity Method, so that the center of gravity method can be applicable to the multi-node location problem.

In the worldwide distribution center location problem, the center of gravity method is usually used for location selection based on the latitude and longitude. However, the center of gravity method uses the Euclidean distance to calculate the distance between two points in the logistics system. The Euclidean distance is the plane distance, the coordinate system is a spherical coordinate system, and there is an error in obtaining the location of the distribution center directly through the latitude and longitude coordinates. In order to eliminate the error, it is necessary to convert the latitude and longitude coordinates to plane coordinates before using the center of gravity method.

Commonly used methods of latitude and longitude to plane coordinate conversion mainly includes Gaussian projection, UTM projection, Mercator projection, etc. Gaussian projection is generally used in surveying and mapping projects in China. After converting the latitude and longitude coordinate system into a Gaussian plane coordinate system, use the center of gravity method for location selection, which can solve the error problem caused by the difference between the spherical coordinate system and the plane coordinate system.

The method of converting the spherical coordinate system into a plane coordinate system will greatly reduce the additional transportation cost caused by the location error caused by inaccurate calculation in the location of large-scale logistics facilities. In addition, the idea of the projection method will make it possible to transfer algorithms on the plane coordinate system to the spherical coordinate system, so it has greater practical significance.

2. MATERIALS AND METHODS

There is \( n \) demand point in a certain area, the demand of each point is \( W_j (j = 1, 2, \ldots, n) \), the longitude and latitude coordinates are \( (B_j, L_j) (j = 1, 2, \ldots, n) \), and the freight rate from the new distribution center to the demand point is \( C_j (j = 1, 2, \ldots, n) \).

2.1. Forward calculation of the Gauss-Kruger projection.

Convert the longitude and latitude coordinates of the demand points into the Gaussian plane coordinates \( (x_j, y_j) (j = 1, 2, \ldots, n) \) using the Gauss-Kruger projection.

The conversion formula is

\[
\begin{align*}
x_j &= X + \frac{\lambda_j N_j}{2} \sin \varphi_j \cos \varphi_j \\
&\quad + \frac{\lambda_j N_j}{24} \sin \varphi_j \cos^3 \varphi_j (5 - \tan^2 \varphi_j) \\
&\quad + \frac{9 \epsilon^2 \cos^2 \varphi_j + 4 \epsilon^4 \cos^4 \varphi_j}{720} \\
&\quad + \frac{\lambda_j N_j}{720} \sin \varphi_j \cos^5 \varphi_j (61 - 58 \tan^2 \varphi_j + \tan^4 \varphi_j) \quad (1)
\end{align*}
\]

\[
\begin{align*}
y_j &= \lambda_j N_j \cos \varphi_j + \frac{\lambda_j^3 N_j}{6} \cos^3 \varphi_j (1 - \tan^2 \varphi_j) \\
&\quad + \frac{\lambda_j^3 N_j}{120} \cos^5 \varphi_j (5 - 18 \tan^2 \varphi_j) \\
&\quad + \tan^4 \varphi_j + 14 \epsilon^2 \cos^2 \varphi_j - 58 \epsilon^2 \cos^2 \varphi_j \tan^2 \varphi_j \quad (2)
\end{align*}
\]

Among them:
\( x_j \) Ordinate in Gaussian coordinate system of the jth demand point.

\( y_j \) Abscissa in Gaussian coordinate system of the jth demand point.

\( \lambda_j \) The difference between the longitude of the jth demand point and the central longitude in radian, and

\[
\lambda_j = (L_j - L^0) \frac{\pi}{180^\circ}
\]  

\( L^0 \) The central longitude.

\( L_j \) Longitude of the jth coordinate.

\( \phi_j \) The difference between the latitude and the equator of the jth demand point in radian, and

\[
\phi_j = \frac{\pi B_j}{180^\circ}
\]  

\( N_j \) The Radius of Curvature in Prime Vertical, and

\[
N_j = a(1 - e^2 \sin^2 \phi_j)^{\frac{1}{2}}
\]  

\( a \) Semimajor axis of ellipsoid.

\( b \) Minor axis of ellipsoid.

\( f \) Flattening of ellipsoid, and

\[
f = \frac{a-b}{a}
\]  

\( e \) First eccentricity, and

\[
e = \frac{\sqrt{a^2 - b^2}}{a}
\]  

\( e' \) Second eccentricity, and

\[
e' = \frac{\sqrt{a^2 - b^2}}{b}
\]  

\( X \) Meridian arc length of Prime Vertical, and

\[
X = a_0 \phi_j - \frac{a_2}{2} \sin 2\phi_j + \frac{a_4}{4} \sin 4\phi_j - \frac{a_6}{6} \sin 6\phi_j + \frac{a_8}{8} \sin 8\phi_j
\]  

The calculation formula of \( a_0, a_2, a_4, a_6, a_8 \) is

\[
a_0 = m_0 + \frac{m_2}{2} + \frac{3}{8} m_4 + \frac{5}{16} m_6 + \frac{35}{128} m_8
\]  

\[
a_2 = m_2 + \frac{m_4}{2} + \frac{15}{32} m_6 + \frac{7}{16} m_8
\]  

\[
a_4 = \frac{m_4}{8} + \frac{3}{16} m_6 + \frac{7}{32} m_8
\]  

\[
a_6 = \frac{m_6}{16}
\]  

\[
a_8 = \frac{m_8}{128}
\]  

\( m_0, m_2, m_4, m_6, m_8 \) are calculated according to the formula (15)-(19)
2.2. Site selection using the Gravity Method.

In part A, the Gaussian plane coordinates $(x_j, y_j) (j = 1, 2, \ldots, n)$ of each demand point are obtained. If the coordinates of the new distribution center are $(x, y)$, the total transport cost function can be expressed as (20)

$$H = \sum_{j=1}^{n} C W_j D_j$$  \hspace{1cm} (20)

The location of the new distribution center should minimize the total transportation cost of the logistics system, namely

$$\text{Min } H = \sum_{j=1}^{n} C W_j D_j$$  \hspace{1cm} (21)

The $H$ is the total transportation cost and the Euclidean distance from the new distribution center to each demand point $(x_j, y_j) (j = 1, 2, \ldots, n)$ is calculated as (22)

$$D_j = \sqrt{(x - x_j)^2 + (y - y_j)^2}$$  \hspace{1cm} (22)

By using the least square method to find the extreme value of the total transportation cost function, the distribution center coordinate which makes the total transportation cost $H$ minimum can be obtained, and the coordinate $(x, y)$ satisfies

$$\left\{ \begin{array}{l}
\frac{\partial H}{\partial x} = \sum_{j=1}^{n} \frac{C W_j (x - x_j)}{D_j} = 0 \\
\frac{\partial H}{\partial y} = \sum_{j=1}^{n} \frac{C W_j (y - y_j)}{D_j} = 0
\end{array} \right.$$  \hspace{1cm} (23)

According to formulas (23) and (24), (25) and (26) can be obtained.

$$\left\{ \begin{array}{l}
x' = \frac{\sum_{j=1}^{n} C W_j x_j / D_j}{\sum_{j=1}^{n} C W_j / D_j} \\
y' = \frac{\sum_{j=1}^{n} C W_j y_j / D_j}{\sum_{j=1}^{n} C W_j / D_j}
\end{array} \right.$$  \hspace{1cm} (25, 26)

That is

\[
\begin{align*}
m_0 &= a(1-e^2) \\
m_2 &= \frac{3}{2}e^2m_0 \\
m_4 &= 5e^2m_2 \\
m_6 &= \frac{7}{6}e^2m_4 \\
m_8 &= \frac{9}{8}e^2m_6
\end{align*}
\]  \hspace{1cm} (15-19)
Calculate \( x, y \) iteratively by formulas (27) and (28), and let \( \varepsilon_1 \) be the iteration accuracy, the algorithm steps are shown in the Fig 1.

2.3. Reverse calculation of the Gauss-Kruger projection.
On the basis of obtaining the Gaussian plane coordinates \((x^*, y^*)\) of the optimal location of the distribution center in part B, the optimal location coordinates of the distribution center are transformed into latitude and longitude coordinates \((B^*, L^*)\) by reverse calculation of the Gauss-Kruger projection.

The reverse calculation formula of Gauss-Kruger projection is as follows

\[
\left\{ \begin{array}{l}
\phi^* = \phi_j - \frac{y^* \tan \phi_j}{2M_j N_j} + \frac{y^* \tan \phi_j}{24M_j N_j^3} (5 + 3 \tan^2 \phi_j) \\
+ e^2 \cos^2 \phi_j - 9e^2 \cos^2 \phi_j \tan^2 \phi_j \\
- \frac{y^* \tan \phi_j}{720M_j N_j^5} (61 + 90 \tan^2 \phi_j + 45 \tan^4 \phi_j)
\end{array} \right. \tag{29}
\]

\[
\lambda^* = \frac{y}{N_j \cos \phi_j} - \frac{y^3}{6N_j^3 \cos \phi_j} (1 + 2 \tan^2 \phi_j)
+ e^2 \cos^2 \phi_j + \frac{y^3}{120N_j^5 \cos \phi_j} (5 + 28 \tan^2 \phi_j)
+ 24 \tan^4 \phi_j + 6e^2 \cos^2 \phi_j + 8e^2 \cos^2 \phi_j \tan^2 \phi_j \tag{30}
\]

Among them:

\( \phi^* \) The difference in radian between the latitude of the site selection result and the latitude of the equator, and
\[ \phi^* = \frac{\pi B^*}{180^\circ} \]  \hspace{1cm} (31)

\( B^* \) Latitude of site selection result.

\[ \lambda^* = (L' - L_0) \cdot \frac{\pi}{180^\circ} \]  \hspace{1cm} (32)

\( \lambda^* \) The difference in radian between the longitude of the site selection result and the central meridian, and

\[ L^* \] Longitude of site selection result.

\[ N_f = a(1 - e^2 \sin^2 \phi_f)^{\frac{1}{2}} \]  \hspace{1cm} (33)

\[ M_f = a(1 - e^2)(1 - e^2 \sin^2 \phi_f)^{\frac{3}{2}} \]  \hspace{1cm} (34)

\( \phi_f \) The latitude corresponding to the arc length of meridian at that time of \( x = X \).

According to the meridian arc length formula (9) iterative calculation, and let \( \varepsilon_2 \) be the iteration accuracy, the iterative calculation process of \( \phi_f \) is shown in Fig. 2.

Fig 2. Parameter iteration process.

3. Results & Discussion

A company in Zhejiang Province needs to build a new distribution center to deliver goods to Hangzhou, Jiaxing, Huzhou and Shaoxing.

This problem is solved by using the Gauss-Kruger projection and the center of gravity method:

The longitude and latitude data of four cities are available from the coordinate picker of AMAP developer platform. Assuming that road transportation is utilized, the transportation rate is all 0.35 RMB/Ton-kilometers. The aggregate social retail sales data of 2018 are obtained by querying the 2019 Zhejiang Statistical Yearbook to estimate the demand for each city.

Using the CGCS2000 coordinate system, the length of the long axis is 6378140 m, the Flattening of ellipsoid is 1/298.257222101, the iterative accuracy of the center of gravity method is \( 6.10^{-8} \), and the iterative accuracy of Gauss-Kruger projection is \( \pi 10^{-3} / 648 \). The calculation process is realized by writing C++ program on the Xcode11.7. The coordinates of each node and the total social retail sales are shown in the table 1.

| City names | Longitude coordinates | Latitude coordinates | Vertical coordinates (km) | Transverse coordinates (km) | Total social retail sales in 2018 (¥ 100 million) |
|------------|-----------------------|---------------------|---------------------------|-----------------------------|---------------------------------------------|
|            |                       |                     |                           |                             |                                             |
The final location result is 3347.397454 km in ordinate and 20.206909 km in abscissa. Convert it to latitude and longitude coordinates as (120.209947, 30.245853).

The result is shown in the fig. 3, where the blue dots represent the demand points and the red dots represent the distribution center.

![Fig 3. Site selection results](image)

### 4. conclusions

The location of the distribution center in the logistics system plays an important role in reducing logistics costs. This paper combines the Gauss-Kruger projection and the center of gravity method to select the location of the logistics center, and comprehensively considers the difference between the plane coordinate system and the spherical coordinate system, which is more reasonable than using the longitude and latitude coordinates directly. However, the center of gravity method only considers the straight-line distance between two points instead of the actual distance, and does not consider other factors such as the construction cost of the logistics center. In addition, the location of the site selection result does not necessarily have the conditions for the construction of a logistics center. Therefore, the site selection results can only be used as a reference, and other factors should be comprehensively considered for site selection decisions.

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