Sampling method with proportional probability to variable areas in stands of *Eucalyptus dunnii* Maiden in Brazil

Método de amostragem com probabilidade proporcional a áreas variáveis em povoamentos de *Eucalyptus dunnii* Maiden no Brasil

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Abstract

Many forest companies in Brazil started using plots of varying sizes in their continuous forest inventories (CFI), mainly to avoid the effects of marginal trees and to obtain more consistent estimates in the CFI. Consequently it was quite important to present a comparison between the results of applying a sampling method in which trees are selected with probability proportional to a variable average area with the method in which the trees are selected with probability proportional to a fixed area plot. In this study, the theory for a sampling method was developed, in which a selection of a group of trees in a plot is done with probability proportional to a variable average area of occupation (PPVA) per 60 trees in a plot. The data is composed by 41 permanent plots from a forest inventory of *Eucalyptus dunnii* Maiden in Santa Catarina, Brazil. In order to compare the PPVA method with the traditional fixed area method (PPA), the area was imaged with an unmanned aerial vehicle (UAV), allowing to obtain a full list of living trees (census). For this fraction, 7 plots for PPVA method and 7 for the PPA method were also measured, allowing the comparison of the total volume obtained in the census with PPVA and PPA results. The total volume of the plot obtained as a function of the census, when compared to the PPVA method, did not present significant differences at the 99% probability level, while the fixed area method was statistically different at the 95% probability level, resulting in an overestimation of 7.5% higher than that found in the census. The PPAV sampling method provides parameter estimates for characterization of the forest population at a lower cost; it becomes more effective than sampling with fixed area plots, because they are practical and operationally easy for delimitation in the field, providing appropriate estimates and more accurate average occupational area for each individual tree (m²) in the population.

Keywords: PPS sampling; Fixed area plot; Continuous forest inventory.

Resumo

Muitas empresas florestais no Brasil começaram a usar parcelas de tamanhos variados em seus inventários florestais contínuos (CFI), principalmente para evitar os efeitos das árvores marginais e obter estimativas mais consistentes. Consequentemente, foi muito importante apresentar uma comparação entre os resultados da aplicação de um método de amostragem em que as árvores são selecionadas com probabilidade proporcional a uma área média variável com o método em que as árvores são selecionadas com probabilidade proporcional a uma parcela de área fixa. Neste estudo, foi desenvolvida a teoria para um método de amostragem, no qual a seleção de um grupo de árvores em uma parcela é feita com probabilidade proporcional a uma área média variável (PPAV) de ocupação por 60 árvores na...
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**INTRODUCTION**

The continuous forest inventory in companies requires the definition of the method and the sampling process as a structure for obtaining data in a pre-defined area. Regarding the sampling method most used in continuous forest inventories in companies, the use of plots of fixed areas always resulted in border-line-trees or marginal trees, which results in undesirable variations in the estimators obtained per sampling unit. In this way, companies started to use sample units of variable area-sizes in order to avoid such occurrences. However, the forest managers did not consider this problem; that is, they continue to use an average area to obtain the inventory estimates. Based on this evidence, we decided to develop the appropriate methodology to obtain the correct estimators from sample units with probability proportional to average area, called the PPAV method.

We considered as a sampling method the theoretical probabilistic criterion that specifies how trees should be selected to participate in a sampling unit. Bitterlich (1948) proposed the first alternative sampling method to fix-area plot, named thereafter as probability proportional to size (PPS) sampling, in which trees are selected in a circular plot with a probability proportional to basal area, or to \( d_{10}^2 \). This sampling method brought new insights for the sampling theory applied to forest inventories and was named, mainly in the United States, as a point sampling. Some other PPS sampling methods followed, varying the size criterion for probability selection of trees, and changing the form of the plot from a circle to a line. The most important contributions are owing to Strand (1958), Cottam & Curtis (1956), and Prodan (1968).

Even though these methods have introduced interesting alternatives to reduce costs of data collection in forest inventories, they present some restrictions, such as bias on the sampling estimates, difficulties in getting dominant heights, and so on. In many forest companies around the world fixed area plots are still the most preferable method for sampling trees in forest inventories to avoid such restrictions.

In some companies in Brazil a variant of the fixed area method (FAM), whose sample units (SUs) are variable in their sizes, is adopted. For example, it will include six or more rows in the planting area with 10 plants per row i.e., approximately 60-hole planting for six rows. In these circumstances, the lines 1 and 6 of the plots will always have 10 trees each, but in lines 2 to 5 would not always have 10 trees each, because the spatial arrangement of trees in each planting row may vary, mainly owing to planting operational conditions and soil relief within the plot. Despite this, the companies have decided, for the sake of consistency of statistical sampling estimates, to completely avoid the occurrence of marginal trees because, in the fixed area sampling method the problem is dealt with in an approximate manner.

The objective of the present study was to provide the theoretical structure of the sampling method, in which individuals in a population are selected with a probability proportional to a variable average area of occupation (PPAV) by 60 trees in the plot and...
generate the estimators for density, basal area, and volume. Specifically, we applied this method simultaneously with the FAM to samples of a continuous forest inventory and compared these estimates with those obtained from the census.

MATERIAL AND METHODS

The data used for the present research were collected in an unthinned *Eucalyptus dunnii* stand, in the northern plateau region of Santa Catarina, from a continuous forest inventory carried out in 2015, in an area of 123.9 hectares, located in Três Barras (Latitude: 26° 8'52.94"S and Longitude: 50°20'59.30"W), Santa Catarina, Brazil (Figure 1). The climate in the region of this study is of the Cfb type, according to the Köppen description, with well-distributed rainfall throughout the year and an average temperature of the coldest month below 18°C. Originally, the local vegetation was formed by the Atlantic Forest biome and Mixed Ombrophilous Forest. The relief of the region is slightly undulating, and the soils are predominantly clayey, with good water retention capacity and mostly acidic (Empresa Brasileira de Pesquisa Agropecuária, 2006).

The FAM, in which trees are selected with a probability proportional to area (PPA) was compared with the variable area method, in which trees are selected with a probability proportional to the average area (PPAV) of 60 trees per plot.

Data description to elucidate the PPVA sampling method

A total of 41 permanent plots were measured, with dimensions of 6 lines per 10 plants in the planting line, yielding an average variable sample area of 629.01 m² (Table 1). This forest is approximately 6.2 years old, the trees are spaced at 3.0 m × 4.0 m, and the total area is 123.90 hectares.
Table 1: Data from compartments (an administrative unit inside the stand delimited by roads and/or streams) of *Eucalyptus dunnii* Maiden in Três Barras, north plateau of Santa Catarina, Brazil

| Results | Compartment Number Sampled | Age (years) | Number of Plots | Number of pits | Average Plot Areas per Compartment (m²) |
|---------|-----------------------------|-------------|-----------------|----------------|----------------------------------------|
| 01      | 6.1                         | 2           | 64              | 641.23         |
| 02      | 6.3                         | 2           | 62              | 621.27         |
| 03      | 6.3                         | 4           | 61              | 638.20         |
| 04      | 6.3                         | 2           | 62              | 618.11         |
| 05      | 6.2                         | 1           | 65              | 651.33         |
| 06      | 6.1                         | 6           | 63              | 639.00         |
| 07      | 6.2                         | 1           | 63              | 566.17         |
| 08      | 6.2                         | 3           | 62              | 646.03         |
| 09      | 6.3                         | 3           | 61              | 618.03         |
| 10      | 6.3                         | 2           | 64              | 634.52         |
| 11      | 6.3                         | 3           | 60              | 612.66         |
| 12      | 6.2                         | 5           | 61              | 631.70         |
| 13      | 6.1                         | 3           | 63              | 615.86         |
| 14      | 6.3                         | 1           | 61              | 690.80         |
| 15      | 6.2                         | 3           | 63              | 609.25         |
| Total   | 15                          | -           | 41              | 935            |
| Average |                             | 6.2         | 2.7             | 623            |

We used the calculations of mean, variance, standard deviation of the mean, and sampling error based on the complete random sampling design with a maximum admissible error of 10%, at different probability levels of 90, 95, and 99%, whose *t* tabulated values were: 1.684, 2.021, and 2.704, respectively.

**Data description for comparison between the variable area and fixed area methods**

Images from unmanned aerial vehicle were used to obtain the number of trees per hectare. The images were processed with orthorectification and georeferencing at a very high spatial resolution (25 cm), captured by means of the SenseFly eBee unmanned aerial vehicle (UAV), in *Eucalyptus dunnii* Maiden compartments, adjacent to the study area, aged 6.2 years (Figure 2).

**Figure 2:** Image Detail of High Spectral Resolution and Allocation of Crown Points in a Plantation of *Eucalyptus dunnii* Maiden trees, in Três Barras, Santa Catarina, Brazil.
The total count of individuals (census) was performed manually with the support of the ArcGis 10.2.2 software, where each living tree returned showing on its crown an attribute of item and coordinates of its surface (UTM - Universal Transverse Mercator Coordinates) and allowing to form a specific shapefile with all the trees of the compartment (Figure 3).

Figure 3: Spatial distribution of 5,952 trees, identified in the image of high spectral resolution from a 7.302-hectare stand of *Eucalyptus dunnii* Maiden, in Três Barras, Santa Catarina, Brazil.

The distribution of trees had an initial spacing of 4.5 m between lines and 3 m between trees in the planting row. However, we detected variations of ~20 cm in these distances, covering, therefore, the limits of 4.30 m < between lines < 4.70 m and 2.80 m < between plants in the row < 3.20 m. Such variations, observed in a reformation area of second rotation, are owing to steepness, regulation of the planting machines, planting mechanization activity, and the route deviations between the rows.

Subsequently, seven plots with a total area of 700 m² were randomly allocated in the stand. Other seven plots with a variable area, composed by 6 planting lines with 10 plants in each line, were allocated randomly (Figure 4A and Figure 4B), leading to a density of approximately one plot per hectare. The total number of trees within plots were counted in the same way as in the census previously described.
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**Figure 4:** Random distribution of 7 plots of fixed area (A) and 7 plots of variable area (B) in a stand of *Eucalyptus dunnii* Maiden, in Três Barras, Santa Catarina, Brazil. Where: A is the distribution of the sampled fixed area plots in the compartment; B is the distribution of the sampled variable area plots in the compartment.

### Theoretical development of the variable area method

This proposal comes after many reflections and evaluations on the *pps* sampling methods already known in the literature, as well as on the *FAM*. In many practical applications in forest inventories, it was detected that *FAM* presents undesirable practical circumstances, as mentioned before, mainly the occurrence of marginal trees and incorrect evaluation of the average coverage area (m²) of the trees in the population.

### Probability criteria for tree selection

The probability criterion for tree selection is proportional to its average occupation area in the plot. To ensure equal probability of selection of trees in the stand, we proceeded as follows:

a) Each plot was randomly selected, such that its left side was identified as one planting line, starting with one tree, and following up to the next ten subsequent trees. In the same way, another ten trees were taken in the sixth line selected on the right side of the previous one. Subsequently, the plot was closed on the limits of the intermediate lines, that is, from the 2nd to the 5th, in such a way that there were no marginal trees.

b) The distances between and along the lines resulting from the final fit were measured and formed a rectangle; the product of its side lengths resulted in the variable area of the plot, i.e., $A_i$.

c) Once the limits of the plot were closed, the trees in the 2nd to 5th lines were counted to obtain the variable number of individuals in the plot, that is, $n_i$. If $A_i$ had $C_i$ established planting holes, then $n_i \leq C_i$.

d) As can be noted, in each plot the selection of trees was conditioned to an average area $a_i$ associated to the tree condition $X_i$, such that we could establish the probability of its occurrence, which was obtained by relations (1) and (2).

$$ a_i = \frac{A_i}{n_i} \quad (1) $$

$$ p_i = \frac{a_i}{\bar{a}} \quad (2) $$
Where \( A \) is the area of the sampled population.

If \( A \) is one ha, then the inverse of the probability defined in (2) would be the estimator of the number of trees per hectare in each plot, i.e.,

\[
\frac{1}{p_i} = p^{-1} = \frac{A}{a_i} = \frac{A_{ni}}{A_i} = N_i
\]

(3)

It should be noted that this estimator is variable for each plot, typically a method of sampling with probability proportional to size (pps), which in this case is the area \( a_i \).

In this sampling procedure, in area \( A \), a dichotomous condition occurs, that is, the plot is represented by the average area \( a_i \), to which the trees that integrate it occur with equal probability and such a set will be considered equal to one or success and the other sets of trees \( (M-1) \) in the forest that do not qualify for the sampling will be considered equal to zero, or failure. Under these circumstances, we can obtain the mathematical expectation for the total number of trees per hectare \( N \), or \( E(\sum_{i=1}^{M} X_i) \):

\[
E\left(\sum_{i=1}^{M} X_i\right) = \sum_{i=1}^{M-1} [X_i = 0] p^{-1} + \sum_{i=1}^{1} [X_i = 1] p^{-1} = \sum_{i=1}^{1} \frac{A_{ni}}{A_i} = \frac{10,000 n_i}{A_i}
\]

Therefore:

\[
N_i = 10,000 \frac{n_i}{A_i}
\]

(4)

It is considered that, to obtain the estimators of basal area and volume per hectare, mathematical expectations are obtained in a similar manner.

The mathematical expectation of the basal area will be obtained by summing all possible sets of trees, from which their average transversal areas are taken, i.e.

\[
(\sum_{i=1}^{n_i} g_i) n_i^{-1} = \bar{g}_i
\]

(5)

If this is taken in the total forest area \( E(\sum_{i=1}^{M} \bar{g}_i X_i) \) will be obtained as follows:

\[
E\left(\sum_{i=1}^{M} \bar{g}_i X_i\right) = \sum_{i=1}^{M-1} [\bar{g}_i (X_i = 0)] p^{-1} + \sum_{i=1}^{1} [\bar{g}_i (X_i = 1)] p^{-1} = \sum_{i=1}^{1} \frac{A_{ni}}{A_i} = \frac{10,000 \bar{g}_i n_i}{A_i}
\]

\[
\bar{G}_i = 10,000 \frac{\bar{g}_i n_i}{A_i}
\]

(6)

The mathematical expectation of the volume will be obtained by the sum of all possible combinations of \( n_i \) trees, from which we take their individual volumes, i.e.

\[
(\sum_{i=1}^{n_i} v_i) n_i^{-1} = \bar{v}_i
\]

(7)

The mathematical expectation of the volume \( E(\sum_{i=1}^{M} \bar{v}_i X_i) \) will be obtained in (8):
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\[
E \left( \sum_{i=1}^{M} \bar{v}_i X_i \right) = \sum_{i=1}^{M-1} \left[ \bar{v}_i (X_i = 0) p^{-1} \right] + \sum_{i=1}^{1} \left[ \bar{v}_i (X_i = 1) p^{-1} \right]
\]

\[
E \left( \sum_{i=1}^{M} \bar{v}_i X_i \right) = \sum_{i=1}^{1} \bar{v}_i p^{-1} = \bar{v}_i p^{-1} = \frac{A \bar{v}_i n_i}{A_i} = \frac{10,000 \bar{v}_i n_i}{A_i}
\]

\[V_i = 10,000 \bar{v}_i n_i A_i^{-1}\] (8)

**RESULTS AND DISCUSSION**

**Variable area method**

The sampling method PPVA was applied to data from 41 plots and the results are presented in Table 2. The number of pits varied from 58 to 65, with an average of 62.49. The number of trees per plot varied from 30 to 62, with average 56.66.

The sum of the transversal areas of the trees per plot varied from 0.687 m² to 1.623 m², with average 1.312 m². The volume of the trees per plot varied between 8.000 m³ to 21.196 m³, with average of 15.597 m³ (Table 2).

Table 2 shows that even when dealing with a stand from the same annual production unit, silvicultural techniques, and initial spacing (3.5 m × 3.0 m), a high variability of the plot areas occurred, ranging from 555.84 to 690.80 m², with average of 629.00 m². It was also evident that there was a variation of the initial density of planting, even allowing to present inferences about the silvicultural consistency at the time of its implementation, a characteristic that would not be explicitly evident in the FAM.

The estimators of PPVA sampling are presented in Table 3. The sample estimators resulted in an average density of 902 trees·ha⁻¹, average basal area of 20.88 m²·ha⁻¹, and average volume of 248.167 m³·ha⁻¹.

**Table 2:** Results of the estimators obtained from 41 plots of *Eucalyptus dunnii* Maiden in Tres Barras, Santa Catarina, Brazil.

| Stand Number | Plot Number | Pits in the Plots | Trees per Plots \(n_i\) | Plot area \(A_i\) (m²) | Average area per tree \(A_i = A_i/n_i\) (m²) | Sum of the transversal areas (m²) | Sum of the trees volumes (m³) |
|--------------|-------------|------------------|-----------------------|----------------------|---------------------------------|-------------------------------|-----------------------------|
| 3            | 1           | 61               | 59                    | 658.5                | 11.2                            | 1.505                         | 19.312                      |
| 3            | 2           | 62               | 57                    | 584.0                | 10.2                            | 1.272                         | 13.929                      |
| 8            | 3           | 60               | 60                    | 634.8                | 10.6                            | 1.552                         | 21.044                      |
| 8            | 4           | 60               | 60                    | 651.2                | 10.9                            | 1.548                         | 21.196                      |
| 8            | 5           | 61               | 59                    | 607.4                | 10.3                            | 1.261                         | 16.596                      |
| 10           | 6           | 65               | 60                    | 637.7                | 10.6                            | 1.415                         | 17.062                      |
| 10           | 7           | 58               | 57                    | 598.5                | 10.5                            | 1.244                         | 15.306                      |
| 2            | 8           | 65               | 49                    | 598.5                | 12.2                            | 1.167                         | 13.422                      |
| 8            | 9           | 62               | 60                    | 659.4                | 11.0                            | 1.349                         | 15.693                      |
| 2            | 10          | 62               | 60                    | 684.0                | 11.4                            | 1.242                         | 12.059                      |
| 19           | 11          | 58               | 53                    | 597.2                | 11.3                            | 1.347                         | 15.451                      |
| 11           | 12          | 65               | 56                    | 651.3                | 11.6                            | 1.262                         | 13.276                      |
| 19           | 13          | 63               | 55                    | 621.1                | 11.3                            | 1.513                         | 20.524                      |
| 19           | 14          | 63               | 52                    | 635.8                | 12.2                            | 1.244                         | 14.652                      |
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Table 2: Results of the application of the PPAV method in 41 plots of *Eucalyptus dunnii* Maiden in Tres Barras, Santa Catarina, Brazil, for average of transversal area ($g_i$), average of volume ($v_i$), number of the trees per hectare ($N_i$), basal area ($G_i$) and volume per hectare ($V_i$).

| Stand Nº | Plot Nº | Pits in the Plots | Trees per Plots $n_i$ | Plot area $A_i$ (m$^2$) | Average area per tree $A_i = A_i/n_i$ (m$^2$) | Sum of the transversal areas (m$^2$) | Sum of the trees volumes (m$^3$) |
|----------|---------|-------------------|----------------------|--------------------------|------------------------------------------|-----------------------------------|----------------------------------|
| 3        | 1       | 0.0255            | 0.3273               | 896                      | 22.85                                    | 303.94                            | 393.53                           |
| 3        | 2       | 0.0223            | 0.2444               | 976                      | 21.78                                    | 312.20                            | 393.53                           |
| 3        | 3       | 0.0259            | 0.3507               | 945                      | 24.45                                    | 312.20                            | 339.53                           |
| 8        | 4       | 0.0258            | 0.3533               | 921                      | 23.76                                    | 306.60                            | 325.482                          |
| 8        | 5       | 0.0214            | 0.2813               | 971                      | 20.75                                    | 306.60                            | 273.220                          |
| 8        | 6       | 0.0236            | 0.2844               | 941                      | 22.20                                    | 306.60                            | 267.552                          |
| 10       | 7       | 0.0218            | 0.2685               | 952                      | 20.79                                    | 300.98                            | 255.738                          |
| 10       | 8       | 0.0238            | 0.2739               | 819                      | 19.50                                    | 300.98                            | 224.269                          |
| 10       | 9       | 0.0225            | 0.2616               | 910                      | 20.46                                    | 300.98                            | 237.999                          |
| 2        | 10      | 0.0207            | 0.2010               | 877                      | 18.16                                    | 20.79                            | 176.316                          |
| 8        | 11      | 0.0254            | 0.2915               | 887                      | 22.56                                    | 20.79                            | 258.708                          |
| 2        | 12      | 0.0225            | 0.2371               | 860                      | 19.37                                    | 20.79                            | 203.833                          |
| 19       | 13      | 0.0275            | 0.3732               | 886                      | 24.36                                    | 20.79                            | 330.460                          |
| 11       | 14      | 0.0239            | 0.2818               | 818                      | 19.57                                    | 20.79                            | 230.443                          |

Table 3: Results of the application of the PPAV method in 41 plots of *Eucalyptus dunnii* Maiden in Tres Barras, Santa Catarina, Brazil, for average of transversal area ($g_i$), average of volume ($v_i$), number of the trees per hectare ($N_i$), basal area ($G_i$) and volume per hectare ($V_i$).
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| Stand Nº | Plot Nº | \( \overline{g_i} \) (m²) | \( \overline{v_i} \) (m³) | \( N_i \) (ha) | \( G_i \) (m²) | \( V_i \) (ha) |
|----------|---------|-----------------|-----------------|-------------|-------------|-------------|
| 19       | 15      | 0.0215          | 0.2431          | 880         | 18.88       | 214.001     |
| 19       | 16      | 0.0262          | 0.3291          | 956         | 25.01       | 314.508     |
| 12       | 17      | 0.0189          | 0.1980          | 938         | 17.70       | 185.661     |
| 12       | 18      | 0.0228          | 0.2453          | 905         | 20.62       | 221.960     |
| 12       | 19      | 0.0245          | 0.3014          | 994         | 24.33       | 299.549     |
| 18       | 20      | 0.0226          | 0.2270          | 814         | 18.42       | 184.796     |
| 12       | 21      | 0.0227          | 0.2466          | 802         | 18.22       | 197.692     |
| 12       | 22      | 0.0233          | 0.2479          | 888         | 20.68       | 220.110     |
| 12       | 23      | 0.0223          | 0.2227          | 855         | 19.03       | 190.465     |
| 18       | 24      | 0.0236          | 0.3066          | 1030        | 24.28       | 315.765     |
| 18       | 25      | 0.0250          | 0.3168          | 690         | 17.28       | 218.579     |
| 34       | 26      | 0.0264          | 0.3457          | 996         | 26.29       | 344.234     |
| 34       | 27      | 0.0204          | 0.2427          | 1077        | 21.96       | 261.532     |
| 15       | 28      | 0.0278          | 0.3544          | 767         | 21.36       | 271.929     |
| 15       | 29      | 0.0224          | 0.2767          | 1014        | 22.73       | 280.701     |
| 34       | 30      | 0.0239          | 0.3167          | 870         | 20.81       | 275.449     |
| 15       | 31      | 0.0210          | 0.2297          | 916         | 19.24       | 210.537     |
| 33       | 32      | 0.0262          | 0.3574          | 828         | 21.69       | 296.001     |
| 31       | 33      | 0.0203          | 0.2470          | 995         | 20.16       | 245.799     |
| 32       | 34      | 0.0246          | 0.3057          | 926         | 22.80       | 282.939     |
| 32       | 35      | 0.0229          | 0.2667          | 497         | 11.36       | 132.411     |
| 21       | 36      | 0.0247          | 0.3311          | 954         | 23.53       | 315.768     |
| 21       | 37      | 0.0195          | 0.1881          | 870         | 16.96       | 163.667     |
| 31       | 38      | 0.0241          | 0.2800          | 900         | 21.65       | 251.928     |
| 31       | 39      | 0.0233          | 0.2708          | 922         | 21.50       | 249.706     |
| 32       | 40      | 0.0191          | 0.1939          | 1043        | 19.89       | 202.284     |
| 21       | 41      | 0.0188          | 0.1778          | 1010        | 18.97       | 179.578     |
| Total    |        | 0.0232          | 0.2756          | 36,996      | 855.93      | 10,174.864  |

The variations in the number of pits, number of plants and area of plots observed using the method with variable areas explain why the results of the sampling are closer to the census values, that is, with greater accuracy.

The statistical analysis by complete random sampling procedure, applied to 41 plots is presented in Table 4, and the results show that this sample size produced a standard error of less than the maximum admissible error value of 10%, whose values were 5.48, 6.57, and 8.80%, respectively at probability levels of 90, 95 and 99%, respectively.

### Table 4: Statistical analysis of the results with the application of PPVA method in 41 plots of *Eucalyptus dunnii* Maiden in Três Barras, Santa Catarina, Brazil.

| Population | Area (ha) | Volume (m³/ha⁻¹) | Variance (m³.ha⁻¹)² | Standard Deviation (m³.ha⁻¹) | Standard Error (m³.ha⁻¹) | Standard Error % |
|------------|-----------|------------------|---------------------|-----------------------------|-------------------------|------------------|
| Mean       | 123.90    | 248.167          | 2,716.764           | 52.123                      | 8.140                   | -                |
| Minimum    | -         | 132.411          | -                   | -                           | -                       | -                |
| Maximum    | -         | 344.234          | -                   | -                           | -                       | -                |
| Probability 90% | - | - | - | - | 13.593 | 5.48% |
| Probability 95% | - | - | - | - | 16.315 | 6.57% |
| Probability 99% | - | - | - | - | 21.832 | 8.80% |

The confidence intervals for the volume in m³·ha⁻¹ and for the total population of 123.90 ha, are presented in Table 5, from where it is also possible to verify the t values of the Student distribution and the sample sizes required to meet each probability level.
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Table 5: Confidence intervals for the volume estimators obtained with PPVA sampling method in 41 plots of *Eucalyptus dunnii* Maiden in Tres Barras, Santa Catarina, Brazil.

| Estimator       | t values | Required Sample Size | Confidence Intervals          |
|-----------------|----------|----------------------|-------------------------------|
| Mean (90%)      | 1.684    | 12                   | 234.57 ≤ X ≤ 261.76           |
| Mean (95%)      | 2.021    | 18                   | 231.85 ≤ X ≤ 264.48           |
| Mean (99%)      | 2.704    | 32                   | 226.34 ≤ X ≤ 270.00           |
| Total (90%)     | -        | -                    | 29,063.77 ≤ X ≤ 32,432.76     |
| Total (95%)     | -        | -                    | 28,726.48 ≤ X ≤ 32,769.40     |
| Total (99%)     | -        | -                    | 28,042.97 ≤ X ≤ 33,452.92     |

Comparison of the fixed area and variable area methods with the results of the census

A total of 5,952 live trees were identified and counted in the census with the high-resolution image. Also, the delimitation of the compartment polygon allowed to accurately extract the net area of planting, as well as the density of trees per hectare and for the total area sampled (Table 6). In addition, with the average volume (m$^3$) of the trees sampled in the plot, it was possible to estimate the volume per hectare and for the total area.

Table 6: Results of the census conducted in a stand of *Eucalyptus dunnii* Maiden in Três Barras, Santa Catarina, Brazil

| Nº Trees (ha) | Stand area (ha) | Nº Trees (total) | Volume (m$^3$ per tree) | Volume (m$^3$.ha$^{-1}$) | Total Volume (m$^3$) |
|---------------|-----------------|------------------|--------------------------|--------------------------|----------------------|
| 819.5         | 7.302           | 5,984            | 0.325                    | 266.3                    | 1,944.8              |

With the simulation of fixed area plots, the results shown in Table 7 were obtained, in which a total of 6,433 trees was estimated by the method, showing a total volume for the compartment of 2,090.600 m$^3$.

Table 7: Density and volume obtained with the FAP in a stand of *Eucalyptus dunnii* Maiden in Três Barras, Santa Catarina, Brazil

| Pits per Plot | Plot Area (m$^2$) | Nº of Trees (ha) | Stand Area (ha) | Nº of Trees (total) | Volume per ha (m$^3$.ha$^{-1}$) | Total Volume (m$^3$) |
|---------------|------------------|-----------------|-----------------|---------------------|--------------------------------|----------------------|
| 61            | 700              | 866.7           | 7.302           | 6,328               | 281.700                        | 2,056.700            |
| 63            | 700              | 900.0           | 7.302           | 6,572               | 292.500                        | 2,135.800            |
| 64            | 700              | 916.7           | 7.302           | 6,694               | 297.900                        | 2,175.400            |
| 61            | 700              | 883.3           | 7.302           | 6,450               | 287.100                        | 2,096.300            |
| 63            | 700              | 900.0           | 7.302           | 6,572               | 292.500                        | 2,135.800            |
| 62            | 700              | 883.3           | 7.302           | 6,450               | 287.100                        | 2,096.300            |
| 57            | 700              | 816.7           | 7.302           | 5,963               | 265.400                        | 1,938.100            |
| Average       | 700              | 881.0           | 7.302           | 6,433               | 286.300                        | 2,090.600            |
The simulation performed based on the variable area plot (PPAV) is presented in Table 8, in which a total of 6,034 trees and a volume of 1,961.100 m³ were estimated.

### Table 8: Density and volume obtained with PPAV in a stand of Eucalyptus dunnii Maiden in Três Barras, Santa Catarina, Brazil.

| Pits per Plot | Plot Area (m²) | Nº Trees (ha) | Plot Area (ha) | Nº Trees (total) | Volume per ha m³ha⁻¹ | Total Volume (m³) |
|---------------|---------------|---------------|---------------|-----------------|----------------------|------------------|
| 60            | 726.2         | 826.2         | 7.302         | 6,033           | 268.500              | 1,960.700        |
| 59            | 739.3         | 798.1         | 7.302         | 5,827           | 259.400              | 1,893.900        |
| 61            | 727.4         | 838.6         | 7.302         | 6,123           | 272.500              | 1,990.100        |
| 60            | 723.7         | 829.1         | 7.302         | 6,054           | 269.400              | 1,967.500        |
| 59            | 711.1         | 829.7         | 7.302         | 6,058           | 269.700              | 1,969.000        |
| 59            | 720.2         | 819.2         | 7.302         | 5,982           | 266.200              | 1,944.100        |
| 60            | 711.1         | 843.8         | 7.302         | 6,161           | 274.200              | 2,002.400        |
| Total         | 722.7         | 826.4         | 7.302         | 6,034           | 268.600              | 1,961.100        |

Comparative results of the analysis of variance between the census, FAM, and PPAV methods are presented in Table 9.

### Table 9: Density and volume obtained in variable area plots in a stand of Eucalyptus dunnii Maiden in Três Barras, Santa Catarina.

| Method | Nº of Trees (ha) | Compartiment Area (ha) | Nº of Trees (total) | Volume per ha m³.ha⁻¹ | Total Volume (m³) | Difference (%) |
|--------|------------------|------------------------|---------------------|-----------------------|-------------------|----------------|
| Census | 819.5            | 7.302                  | 5,984               | 266.3                 | 1,944.800        | -              |
| FAM    | 879.6            | 7.302                  | 6,423**             | 285.9**               | 2,087,400**      | 7.3%           |
| PPAV   | 826.4            | 6,034** ns             | 268.6** ns          | 1,961,100** ns        | 0.8%             |

* Significant at 95%; ** significant at 99%; ns not significant.

The estimates were highly accurate when the variable area plot (PPVA) was used to obtain the density and total volume of the evaluated compartment, with no significant differences at 95 and 99% probability levels. The FAM presents bias, overestimating in 7.3% the density and total volume of the evaluated compartment and presenting an outcome significantly different from that obtained in the census, at 99% probability level.

### Sampling method: Probability proportional to a variable area (PPAV)

Although the literature on Brazilian forestry does not specify or use the terminology of sampling with probability proportional to a variable area (PPAV), as a new sampling method, with estimators appropriately developed by mathematical expectation, it is well known that variable plot size has been used in practice for some time. Mello et al. (2009) applied two sampling methodologies: random sampling and geostatistics to evaluate the sampling behavior with fixed area plots and in-line plots (120 m by 2 planting lines, thus variable area) to obtain the volume in m³ of approximately 104 ha of Eucalyptus grandis plantation in the state of São Paulo, Brazil. The conversion of the estimates per hectare was made by taking the average area of the plots and finding, afterwards, the average conversion factor as is done for the FAM.
Londero et al. (2015) used data from plots with variable area to calibrate the 3-PG model for *Eucalyptus saligna* Smith in the region of Guaíba-RS, Brazil, composed of 6 lines with $n$ trees, totaling 60 holes planted evaluated in each plot in their study. They used variable area plots, but they have not used a *pps* sampling structure to estimate any quantitative variables for the forest stands, therefore they did not formulate a conversion area factor to obtain estimates per hectare.

Almeida et al. (2016) tested sampling with fixed area plots and with plots whose areas were defined by the fixed number of hole planting in a stand of eucalyptus in MG, Brazil. They tested the values obtained in these two plot sizes and found no statistically significant differences between them and concluded that these two sampling methods would result in equally consistent estimates per hectare. Therefore, they applied the conversion factor normally used for fixed area plots to obtain the estimates per hectare.

Moscovich et al. (1992) referred to Bitterlich, Strand, Prodan, and Quadrant sampling methods framing them as variable area methods in a comparative study carried out in an *Araucaria angustifolia* forest in Rio Grande do Sul, Brazil. In the international literature it is also common to find the mention of variable area when referring to Bitterlich and Prodan sampling methods (Husch et al., 2003; Hamilton & Bensted-Smith, 1989). In our research the concept of variable area plot refers to a consequent resulting plot size after selecting a fixed number of trees in the forest stand.

Batcheler & Craib (1985) tested the Prodan's variable area and *FAM* with different plot sizes to obtain estimates of the number of trees per hectare and the basal area with data from native forests of New Zealand, and concluded that variable and small area sampling is robust and suitable for ecological research of populations used in their studies. Once more, the concept of variable area in this study refers to Prodan's sampling method, certainly a *pps* sampling, in which the selection of trees to participate in the plot is applied to a set of trees, usually 6 trees, in which the probability is proportional to the radius of the 6th tree in a circle randomly selected in the stand.

Alijanpour et al. (2018) compared the performance of the *FAM* and the Bitterlich variable area method to assess quantitative and qualitative biodiversity in a sub-humid forest in northwestern Iran. The results indicated that the quantitative characteristics are significantly different between the two methods, whereas there were no significant differences in richness, biodiversity indices and qualitative characteristics. Again, the concept of variable area in this research refers to Bitterlich's sampling method, certainly a *pps* sampling, in which the selection of trees to participate in the plot is done with a probability proportional to the transversal area of each tree, or ($d^2$) in a circle randomly selected in the stand. Similarly, Fallah et al. (2000) compared two sampling methods with fixed area (10 acres) and variable area (Bitterlich) plots in oak forests in Persia, assessing the number of trees per hectare, basal area and time required to record data. The results showed that the 10-acre sampling method was more adequate for the forest inventory.

The most important influence on the estimates is the average occupation area ($m^2$) of the trees in the population, while the sampling with variable area can identify variations of spacing specific to each plot and, therefore, generate more consistent estimates to the compartment.

The variable area sampling method is already a reality in several forest companies in Brazil, among them the Eco-Brazil Forests in Tocantins, Investment groups in Maranhão, and F. Plywood factory S.A in Tres Barras, Santa Catarina, however they are still considering the estimates as if they were taken in fixed area plots.

Given the possible variability of existing gaps in the forestry companies, whether they are caused by slope, soil types, equipment, and other silvicultural practices that exert influence on tree coverage average area in the population, it is recommended to use the variable area sampling method to generate more efficient and accurate estimators in the referred areas.

The methodology with selection of trees proportional to a variable area (PPAV) could significantly mitigate the problem of marginal trees in the plots, correct possible systematic errors of spacing, and operationally facilitate the implementation and re-measurement of plots. As the variable area plots do not contain marginal trees, they provide better conditions.
Sampling method with proportional probability to variable areas in stands of *Eucalyptus dunnii* Maiden in Brazil

to obtain the CFI estimates, such as representativeness and stability of diametric, hypsometric, and volumetric distributions, as well as mortality and in-growth control, among others. The sample structure assures uniformity, or fixation of the same plot in different types of spacing, guaranteeing the estimates to maintain fairness of results despite the variation in spacing. This sample structure guarantees minimum representation of trees in the plot to generate their estimates with precision and statistical consistency. The most influential feature of PPAV sampling is the average occupancy area (m$^2$) of trees in the population, which can identify spatial variations in each plot and thus generates more consistent estimates to the stand.

**CONCLUSIONS**

PPAV sampling method provides the best parameter estimates for characterization of the forest population using variable area plots.

PPAV is more effective than sampling with fixed area plots, because its plot structure is easier to install in the field.

Using an average area of variable-size plots as if fixed-size plots have been used in the sampling is inappropriate.

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