Numerical simulation of ultrasonic testing reliability of civil aircraft considering the influence of the angle between the sound beam axis and the crack orientation

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Abstract. In order to quantify the influence of the angle between the axis of the sound beam and the crack orientation on the detection reliability during the ultrasonic testing, this paper uses COMSOL Multiphysics to simulate the numerical simulation of ultrasonic testing and explores the variation of the ultrasonic detection amplitude with the angle. The function of fitting the function is given, and then the numerical simulation method of ultrasonic detection reliability considering the influence of the angle between the beam axis and the crack orientation is proposed. According to the numerical simulation case of ultrasonic detection reliability, the following conclusions are obtained. When the detecting device detects a crack of less than 2.23 mm and the detection probability is greater than 90%, the angle between the axis of the sound beam emitted by the detecting device and the direction of the crack must be greater than 70°; If the detecting device detects a crack greater than 0.7 mm and the angle is greater than 86°, the probability of detecting the crack is greater than 61.35%.

1. Introduction
Ultrasonic testing is one of the important techniques in modern industrial in-situ non-destructive testing. It has a series of advantages such as high sensitivity, strong penetrability, good direction, fast detection speed, low cost, relatively simple equipment and harmless to the human body [1]. In the non-destructive testing process, not all defects existing in the object to be inspected can be accurately found in all cases. There are many factors affecting the reliability of the non-destructive testing technology, such as the performance of the equipment, the material of the test block, the internal structure, and the levels of inspectors [2]. These uncertainties have been dominated. For a detection system, what are the main influencing factors? What are the secondary factors? How to correct and compensate for the uncertainty caused by these factors? These problems should be studied and quantified one by one so that the test results meet the specified requirements. It then provides an effective basis for product designed, material selection and quality assessment.

At present, the reliability of non-destructive testing is the basis of non-destructive testing technology, and it is the focus of research and technical research by domestic and foreign scholars. The breakthrough of this technology will make the application prospect of non-destructive testing technology more broad. In order to quantify the ability of the detection system to detect cracks and determine the applicability of the detection scheme, "the probability of detection (POD)" can be...
introduced to characterize the reliability of ultrasonic detection [3], and the POD curve is used as a criterion for quantifying the capability evaluation of the non-destructive testing system.

2. Research content

Practical engineering experience is shown below. When the angle between the axis of the sound beam and the orientation of the crack (hereinafter referred to as "the angle") is 90°, the crack detection amplitude is often also the largest. When the angle is greater than or less than 90°, the crack detection amplitude will decrease. As the angle is furthering away from 90°, the crack detection amplitude will be smaller. When the angle is too large or too small, the probe may not receive crack echoes, making the crack undetectable. In this paper, the angle between the beam axis and the crack orientation is taken as the research object. By exploring the variation law of the angle and the detection amplitude, the function relationship is fitted and the variation amplitude of the detection amplitude under different angles is quantified. Applying the above results to the calculation of the POD curve, the POD curves at different angles and the crack size with a detection probability of 90% can be obtained. This provides a scientific approach to the research and evaluation of ultrasonic testing reliability.

3. Reliability theory of non-destructive testing

3.1. Theoretical overview of POD

Non-destructive testing reliability analysis is based on statistical and mathematical statistical methods to calculate and analyze the test results. It determines the functional relationship between POD and crack size, fits the POD(a) curves, and calculates the minimum crack size for a specific probability of detection [4]. The crack size under the condition of 95% confidence and 90% detection probability is often taken as an evaluation index for non-destructive testing reliability.

The common ultrasonic test result data is divided into two types: "Hit/Miss" type and "Signal Response" type [5]. Among them, "Signal Response" type data contains more information about cracks. It not only reflects that the signal response amplitude exceeds the detection threshold, but also quantifies the extent of exceeding the detection threshold and is ultimately reflected on the POD. For "Signal Response" type data, a represents the crack size, and \( \hat{a} \) represents the amplitude of the detection signal, and there is a log-linear relationship between the two and obeys a log-normal distribution [6], which is called "Log-Logistic" model.

3.2. POD calculation of Log-logistic model

The linear relationship between \( \ln \hat{a} \) and \( \ln a \) in the Log-logistic model is as follows:

\[
\ln \hat{a} = A + B \cdot \ln a + \delta
\]

where A and B are the intercept and the slope respectively. \( \delta \) is the error term and is generally considered to be \( \delta \sim N(0, \sigma_\delta^2) \). The POD(a) based on this linear relationship can be represented by a probability density function:

\[
POD(a) = \int_{\hat{a}_a}^{\infty} g(a)(\hat{a}) \, d\hat{a}
\]

where \( g(a)(\hat{a}) \) — probability density function; \( \hat{a}_a \) — detection threshold. When \( \hat{a} > \hat{a}_a \), the crack is detectable and POD(a) can be expressed as:

\[
POD(a) = \text{Prob}\left[ \delta > \ln \hat{a}_a \right] \cdot \left( A + B \times \ln(a) \right)
\]

Because of \( \delta \sim N(0, \sigma^2) \), the equation (3) can be expressed as:

\[
POD(a) = 1 - \Phi \left( \frac{\ln \hat{a}_a - (A + B \times \ln(a))}{\sigma_\delta} \right)
\]
where $\Phi(z)$ is the distribution function of the standard normal distribution. Using the characteristics of the standard normal distribution, the above equation can be simplified to:

$$\text{POD}(a) = \Phi \left( \ln \left( \frac{\hat{a}_{th}}{\hat{a}} \right) - A \frac{\ln a - A}{B} - \frac{\sigma}{\delta} \right)$$

(5)

The probability integral lower limit $\mu$ and the standard deviation $\sigma$ of the log size logarithm are:

$$\mu = \frac{\ln (\hat{a}_{th})}{B} - \frac{A}{B} \sigma = \frac{\delta}{B}$$

(6)

Therefore POD(a) can be written as follows:

$$\text{POD}(a) = \Phi \left( \frac{\ln (a) - \mu}{\sigma} \right)$$

(7)

Using equation (6), we can also estimate the lower confidence limit of the log-mean of the crack detection amplitude for a given confidence level, so that the curve can be plotted under the confidence, and the logarithm of the crack detection amplitude is given at a given confidence. The lower confidence limit for the mean is calculated as follows:

$$\ln a_{CL} = A + B \times \ln a - u \times \frac{\sigma}{\sqrt{n}} + \delta$$

(8)

Substituting equation (8) into equations (3) to (5), the lower limit of the probability integral of the log size logarithm (lna) at a given confidence is:

$$\mu_{CL} = \mu + u_{CL} \times \frac{\sigma}{\sqrt{n}}$$

(9)

Therefore, the POD(a) is written as follows under the given confidence:

$$\text{POD}(a)_{CL} = \Phi \left( \frac{\ln (a) - \mu_{CL}}{\sigma} \right)$$

(10)

4. Establishment and verification of ultrasonic testing simulation model

In order to study the influence of the angle between the beam axis and the crack orientation on the reliability of ultrasonic testing, this paper uses COMSOL Multiphysics to carry out ultrasonic testing simulation analysis. Ultrasonic detection amplitude which changed with the angle of inclusion was studied under different crack sizes.

This paper uses the two-dimensional axisymmetric model in COMSOL Multiphysics software to model. The numerical simulation verification model of ultrasonic testing is a crack-free cylinder with a diameter of 50 mm and a height of 20 mm. The material is selected as the built-in material Ti-6Al-4V, and the wave velocity is set at 6000 m/s. The model is shown in figure 1.

![Figure 1. Simulation model of crack-free cylinder.](image)
Set the probe diameter to 6mm, the frequency is 5MHz, and the simulation obtains 4 bottom echoes. The actual echo detection of the four echo waveforms given by Vamsi is shown in figure 2(a) [7], and the four echo waveforms obtained by the simulation are shown in figure 2(b). After the simulation is completed, the amplitude and propagation time of the bottom surface echo is extracted from four consecutive times, and the attenuation ratio of the adjacent amplitude (the ratio of the previous echo amplitude to the amplitude of the next echo) is compared with the experimental data of Vamsi. The result is as table 1.

![Actual ultrasonic detection waveform.](image1)

**Figure 2.** Echo waveforms obtained by actual detection and simulation.

**Table 1.** Comparison of actual ultrasonic testing and simulation results.

| Number | Echo amplitude | Echo time | Attenuation ratio |
|--------|----------------|-----------|------------------|
|        | Test value    | Simulation value | Test value    | Simulation value | Test value | Simulation value |
| 1      | 170           | 153       | 6.94            | 6.99           | \         | \                 |
| 2      | 59            | 71        | 13.92           | 13.8           | 2.86      | 2.16              |
| 3      | 36            | 42        | 20.88           | 20.6           | 1.66      | 1.69              |
| 4      | 21            | 26        | 27.33           | 27.4           | 1.69      | 1.62              |

It can be seen from the table that the error of the experimental values and the simulated values of the three eigenvalues are 4.45%, 1.81% and 4.14%, respectively. The error is small and the attenuation of the ultrasonic echo is consistent. In addition, the propagation time between consecutive backwall echoes of the A-scan signal obtained by the experimental and numerical simulations is substantially the same. Therefore, it can be considered that the software simulation model is reliable in ultrasonic propagation and attenuation, and can be further used to simulate the ultrasonic response of cracks.

Therefore, take the 2mm flat bottom hole crack as an example. A two-dimensional model was established for simulation research, in which the model length was 50 mm and the height was 30 mm. The flat bottom hole model was 2 mm long and 10 mm high. The two-dimensional model ultrasonic detection instantaneous sound field slice diagram and waveform diagram is shown in figure 3.

![Instantaneous sound field slice.](image2)

**Figure 3.** Ultrasonic testing of two-dimensional model flat-bottom hole crack.
5. Study on the influence of the angle on crack detection

In order to quantify the influence degree of the angle on the ultrasonic testing results, the 5mm flat bottom hole crack is taken as the research object, and an angle value is taken at intervals of 2° in the [70°, 90°] interval for simulation study. In the simulation study, the incident direction of the acoustic wave is kept unchanged, and the crack orientation is changed to change the angle. Taking the angle of 82° as an example, the instantaneous sound field slice diagram and the ultrasonic detection waveform diagram are shown in figure 4 and figure 5.

The simulation results with the angle [70°, 90°] are as follows:

| Parameter       | Simulation results |
|-----------------|--------------------|
| The angle/°     | 70  72  74  76  78  80  84  86  88  90 |
| Echo sound pressure /Pa | 25  30  36  40  51  64  90  130  178  218  240 |
| Reduction/%     | 89.66  87.31  84.98  83.24  78.72  73.43  62.24  45.87  25.59  8.97  0 |

The detection amplitude at 90° is regarded as 1. The simulation results under other angles are converted. Matlab is used to fit the simulation results in table 2, and then the function relationship is as follows:

$$K_\theta = \frac{0.176 \cdot \theta^2 - 4.2 \cdot \theta + 57.73}{\theta^2 - 3.68 \cdot \theta + 57.89}$$

(11)

where $\theta$ is the angle value at which the angle is offset by 90°, and $K_\theta$ is the proportional coefficient of the amplitude of the echo. According to formula (11), the variation of the ultrasonic detection echo amplitude of the crack with the angle is shown in figure 6:

It can be seen that after the angle changes, we can multiply the corresponding value by the crack detection amplitude at 90° to obtain the detection amplitude at the corresponding angle. Since the Log-logistic model is based on the log-linear relationship between the crack size and the detected amplitude, the POD is calculated. Therefore, when the detected amplitude is multiplied by a certain proportional coefficient and then the logarithm is obtained, there is still a log-linear relationship.
between the crack size and the crack size. That means the Log-logistic model can still be used to calculate the POD curve affected by the angle.

6. Reliability of ultrasonic testing considering the influence of angle

The simulation results of ultrasonic testing under different crack sizes are given. The corresponding detection amplitudes of cracks of different sizes at angles of 86°, 82°, 78°, 74°, and 70° were calculated using the contents of the fourth section, as shown in table 3.

Table 3. Ultrasonic simulation detection amplitude at different angles.

| The angle/° | Detecting amplitude |
|-------------|---------------------|
| 90          | 14.77 19.55 27.93 28.70 36.56 55.59 75.56 81.62 130.33 |
| 86          | 10.92 14.45 20.65 21.22 27.03 41.10 55.86 60.34 96.36 |
| 82          | 5.65  7.48 10.69 10.99 14.00 21.28 28.93 31.25 49.90 |
| 78          | 3.06  4.05  5.79  5.95  7.57 11.52 15.65 16.91 27.00 |
| 74          | 2.06  2.73  3.90  4.01  5.10  7.76 10.54 11.39 18.19 |
| 70          | 1.70  2.25  3.21  3.30  4.20  6.38  8.68  9.37  14.97 |

According to the crack detection amplitude under different angles, the POD calculation method of "Signal Response" type data is adopted. The Matlab is used to calculate and fit the POD of a certain crack size at a specific angle. The minimum crack size corresponding to the curve at different angles and the 90% probability of detection is shown in figure 7.

Figure 7. POD curves at different angles and their corresponding $a_{90}$ and $a_{90/95}$ crack sizes (The angles from left to right are 90°, 86°, 82°, 78°, 74°, and 70°, respectively).

Under the same detection conditions, when the ultrasonic detecting sound beam axis is perpendicular to the crack orientation, the detection reliability is the highest and the corresponding $a_{90}$ and $a_{90/95}$ detectable crack sizes are also the smallest. As the angle $\theta$ deviates from 90°, the crack detection amplitude gradually decreases, and the detection reliability decreases in turn. The corresponding $a_{90}$ and $a_{90/95}$ crack sizes also gradually increase. The minimum detectable minimum crack size is shown in table 4.

Table 4. Detectable minimum crack size at different angles.

| The angle/° | Detectable minimum crack size |
|-------------|-----------------------------|
|             | $a_{90}$/mm | $a_{90/95}$/mm |
| 90          | 0.63        | 0.70          |
| 86          | 0.75        | 0.84          |
| 82          | 1.10        | 1.23          |
| 78          | 1.57        | 1.77          |
| 74          | 1.98        | 2.23          |
| 70          | 2.23        | 2.50          |
7. Conclusions
In order to quantify the influence of the angle between the beam axis and the crack orientation on the
detection reliability, the crack detection probability (POD) under different conditions was obtained by
numerical simulation of in-situ ultrasonic inspection. The main conclusions are summarized as follows:

1) Considering the influence of the angle, the numerical simulation of COMSOL Multiphysics can
be used to obtain the ultrasonic detection amplitude under different angles. After substituting into the
Log-logistic model, the POD curve of the ultrasonic detection affected by the angle of inclusion can be
calculated. The minimum crack size can be calculated, which the detection device can detect with a
probability of 90%. This forms a numerical simulation method for the reliability of in-situ ultrasonic
testing.

2) As the angle is offset by 90°, the crack detection amplitude decreases rapidly. When the angle is
changed from 85° to 80°, the detection amplitude is changed from 64.17% to 26.67% of the highest
level, which is reduced by 37.5%.

3) In the case of a 90% probability of detection, when the angle is changed from 90° to 70°, the
minimum detectable crack size is reduced from 0.63 mm to 2.23 mm, which is 71.75% lowered. If the
detecting device detects a crack of less than 2.23 mm with a 90% detection probability, the angle
between the axis of the sound beam and the orientation of the crack must be greater than 70°. If the
detecting device detects a crack greater than 0.7 mm and the angle is greater than 86°, the probability
of detecting the crack is greater than 61.35%. If the detecting device detects a crack greater than 0.7
mm and the angle is greater than 86°, the probability of detecting the crack is greater than 61.35%.

8. Prospect
In this paper, the reliability of in-situ ultrasonic testing is carried out for the edge crack, and many
numerical simulations and calculations are carried out. However, there is still much room for
development.

1) The aircraft has many machine structures and many forms of damage. In addition to ultrasonic
testing, there are various non-destructive testing techniques such as eddy current testing, radiation
detection and magnetic particle testing. Therefore, various non-destructive testing reliability studies
can be carried out for different defects in other structures. In addition, the probability of detecting
defects in the structure can be more effectively improved, and the reliability of the aircraft structure
can be improved.

2) In this paper, the influence of two influencing factors on the reliability of in-situ ultrasonic
testing is studied in the form of two-dimensional curve. In order to analyze the influence of multi-
factors more directly, the three-dimensional form can be used to study the reliability of other structures.

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