Dark states in electromagnetically induced transparency controlled by a microwave field

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Abstract
The rigorous dark-state conditions, i.e. two-photon resonance, for a $\Lambda$-type electromagnetically induced transparency (EIT) are extended with the system controlled by a microwave field. The extended dark states are extremely sensitive to the phase of the microwave field in a narrow interval. However, the dark state is no longer required for EIT in this scheme and EIT without dark states is present.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
An electromagnetically induced transparency has drawn tremendous attention during the last two decades [1, 2]. The transparency together with dramatic dispersion has wide applications in light velocity control [3, 4], nonlinear optics [5–9] and quantum information [10–15]. Dark states, as the key property of EIT, are of great importance in quantum control of atom and optical quanta [10, 11]. The existence of dark states is significant to an EIT-based medium, especially for optical processing and quantum manipulations such as coherent population trapping (CPT), coherent population transfer [12] and photon storage [10].

Microwaves coupling the ground states can manipulate the characteristics of the EIT medium [16–22]. For a closed system, the population trapping is very sensitive to the relative phase of the electromagnetic fields [16]. Microwave interaction has been applied to excite the Raman trapped state and to influence the CPT in a $\Lambda$ system [18]. Fast and slow light phenomena in the EIT medium controlled by microwave have been theoretically predicted [20, 21]. Constructive and destructive interference in the presence of a microwave field have been experimentally realized based on a V-type system in solid ($\text{Pr}^{3+}\text{YAlO}_3$) [19] and recently in $\Lambda$-type atom vapor [22]. A recent study predicts that the microwave field can even provide additional control to the light storage system [23].

In this paper, we investigate a $\Lambda$-type system controlled by a microwave field coupling ground states as in [18, 20, 22]. Other than the early works which emphasized on absorption and dispersion properties of the medium, we focused on the behaviour of dark states under this configuration and its limitations. In this scheme, two-photon detuning condition has been extended and the phase sensitivity of the dark states is studied. With significant ground states dephasing the dark states are proved to be vanished, while the transparency still exists provided that the intensity and phase of the microwave field are chosen appropriately. That is to say, EIT can be established in severe dephasing medium without dark states.

2. Rigorous dark-state condition
Consider a closed three-level $\Lambda$-type system shown in figure 1. Basically, it is a $\Lambda$-type EIT system with probe and pumping optical fields coupling the ground states [1] and [2] to the excited state [3]. A controlling microwave field with relative phase $\phi$ to the optical fields is applied between the two ground states. Under the rotating frame, the Hamiltonian is
\[ H = \hbar \Delta_1 |1\rangle \langle 1| + \hbar \Delta_2 |2\rangle \langle 2| - \hbar (g |1\rangle \langle 3| + G |2\rangle \langle 3| + \Omega \epsilon^{\phi} |1\rangle \langle 2| + \text{h.c.}), \]

where \( g \), \( G \) and \( \Omega \) are the Rabi frequencies of the probe light, pumping light and microwave field, respectively. \( \Delta_1 \) and \( \Delta_2 \) are the optical detunings of the probe and pumping, and the microwave field is assumed to be resonant for simplicity.

The eigenstates of the system are

\[ |\psi_i\rangle = -e^{i\phi}[g^2 + \lambda_i, \Delta_2 - \lambda_i, i]|1\rangle + (e^{i\phi} g G - \Omega \lambda_i)|2\rangle + [G \Omega + e^{i\phi} g (\Delta_2 - \lambda_i)]|3\rangle, \]

where \( \lambda_i = \lambda_{1,2,3} \) are eigenvalues of the Hamiltonian and satisfy

\[ \lambda^2 - \lambda^2 (g^2 + G^2 + \Omega^2) + 2gG \cos \phi + (g^2 - \lambda^2) \Delta_2 + (G^2 - \lambda^2) \Delta_1 + \lambda \Delta_1 \Delta_2 = 0. \]

A dark state requires elimination of the excited state \( |3\rangle \), which means \( G^2 + \epsilon^{\phi} g (\Delta_2 - \lambda_i) = 0 \). Since the eigenvalue of the Hamiltonian is always real, \( e^{i\phi} \) needs to be real and so \( \phi = 0 \) or \( \pi \).

(a) If \( \phi = 0 \), one has \( G \Omega + g (\Delta_2 - \lambda_i) = 0 \). Substituting it into (3) yields

\[ \Delta_1 - \Delta_2 = -\frac{\Omega}{g} (g^2 - G^2), \]

where \( \Delta_1 - \Delta_2 \) is called the two-photon detuning. This is the dark-state condition in our scheme. In the absence of the microwave field, \( \Omega = 0 \), this condition can be reduced into two-photon resonance \( \Delta_1 = \Delta_2 \) [1, 24]. Dressed states under this condition are

\[ |\lambda^0_{\phi}\rangle : G|1\rangle - g|2\rangle, \quad |\lambda^0_+\rangle : g|1\rangle + G|2\rangle + \lambda_+|3\rangle, \]

where \( \lambda^0_{\phi, \pm} \) are the eigenvalues with \( \phi = 0 \).

(b) If \( \phi = \pi \), following the same process, the dark-state condition is derived as

\[ \Delta_1 - \Delta_2 = -\frac{\Omega}{g} (g^2 - G^2), \]

with eigenvalues and eigenstates

\[ |\lambda^\pi_{\phi}\rangle : G|1\rangle - g|2\rangle, \quad |\lambda^\pi_+\rangle : g|1\rangle + G|2\rangle + \lambda_+|3\rangle, \]

where

\[ \lambda^\pi_0 = \frac{g \Omega}{G} + \Delta_1, \]

\[ \lambda^\pi_\pm = \frac{G \Omega + g \Delta_1 \pm \sqrt{4 (g^2 + G^2) g^2 + (G \Omega - g \Delta_1)^2}}{2g}. \]

For both the above cases without microwave, the corresponding states reduce to the same dark state as in EIT under the same two-photon resonance condition. With a microwave field, however, the two-photon resonance condition can be extended. A direct result shows that the two-photon detuning can vary in a wide range by only changing the intensity of the microwave field.

Dark states in our scheme are the result of the coherence among the two optical fields and the microwave field. Without the microwave field, the dark state, as in EIT, is \( |D\rangle = (G|1\rangle - g|2\rangle)/\sqrt{g^2 + G^2} \). In comparison, the dressed states of the microwave transitions are \( (\epsilon^{\phi} |1\rangle \pm |2\rangle)/\sqrt{2} \). If no coherence happens [18], the direct transition of the dark state to the microwave dressed states requires \( \phi = 0 \) or \( \pi \), \( g = G \) and \( \Delta_1 = \Delta_2 \), which is only a special case of conditions (4) and (6). However, it is the coherent interaction that promotes the two-photon resonance condition into the condition that the two-photon detuning is controllable. For the states dressed by the microwave field, the dark state has changed the population distribution of the components in the dressed states, which is not possible in the absence of the optical fields. The coherent process benefits both the optical EIT dark states and the microwave fields.

3. Dampings included: the density operator approach

It is well known that the most important property of dark state is the elimination of the excited state, and hence the spontaneous emission can be ignored. Since the dephasing between ground states is usually negligible for atom vapor, dark states could remain very stable in such a medium. However, in some cases when the ground states’ dephasing is significant, the dark-state condition has to be justified and the influence of the dephasing has to be considered. For this reason the density operator approach is required. For the Hamiltonian shown in (1), the matrix equations read

\[ \rho_{11} = 2\gamma_1 \rho_{33} + (ig \rho_{31} + i \rho_{21} \Omega e^{\phi} + \text{h.c.}), \]

\[ \rho_{22} = 2\gamma_1 \rho_{33} + (ig \rho_{32} - i \rho_{21} \Omega e^{\phi} + \text{h.c.}), \]

\[ \rho_{21} = 2 |(\Delta_1 - \Delta_2) - \kappa| \rho_{21} + ig \rho_{23} + i G \rho_{31} + 2 \rho_{21} e^{-i\phi} (\rho_{11} - \rho_{22}), \]

\[ \rho_{31} = 2 |(\Delta_1 - \Delta_2) - \kappa| \rho_{31} - i \rho_{21} \Omega e^{-i\phi} + i G \rho_{21} + ig (\rho_{11} - \rho_{33}), \]

\[ \rho_{32} = 2 |(\Delta_1 - \Delta_2) - \kappa| \rho_{32} + 2 i \rho_{11} \Omega e^{\phi} + ig \rho_{12} + ig (\rho_{22} - \rho_{33}), \]

where \( \gamma_{1,2} \) are the radiation damping rates from the excited states to \( |1\rangle \) and \( |2\rangle \), respectively, and \( \kappa \) is the dephasing rate between ground states.
The dark-state condition requires \( \rho_{13} = 0 \). In the resonance case \( \Delta_1 = \Delta_2 = 0 \), the dark-state condition is reduced to
\[
\cos^2 \phi = 1 + \frac{2 \gamma}{g^2 G^2(4\gamma + \kappa)} \left[ 2g^2 G^2 \gamma k^2 + 4\kappa \Omega^2 \gamma^2 \right] \times \left[ (g^2 + G^2) + (g^2 - G^2)^2 \right] \left[ \kappa(g^2 + G^2) + 2\gamma \Omega^2 \right].
\]
(8)

Only when \( g = G \) and \( \kappa = 0 \), can \( \phi \) be real with \( \cos \phi = \pm 1 \). This agrees exactly with the dark-state conditions (4) and (6) in the resonance case. For the non-resonant case, the dark-state condition is the same as (4) and (6) with \( \kappa = 0 \) (see appendix). Figure 2 shows that with different \( \phi \), the excited state population changes from a high occupation to zero. It is obvious that the existence of the dephasing term will inevitably make \( \cos \phi > 1 \) whether the microwave field is applied or not, so that the dark states can never be achieved.

Around \( \phi = 0, \pi \), the dark state is very sensitive to the shift of \( \phi \). In the resonance case neglecting ground state dephasing, the excited state population can be solved as
\[
\rho_{13} = \frac{1}{3} \left( 1 - \frac{A}{A + 6g^2 \Omega^2 \sin^2 \phi} \right),
\]
(9)
where \( A = 2(g^2 - \Omega^2)^2 + g^2 \Omega^2 + 8\gamma \Omega^2 > 0 \) and \( G = g \) for dark states requirement. It is obvious that when \( \phi = \pi/2, 3\pi/2 \), \( \rho_{13} \) reaches the maximum while \( \phi = 0, \pi \) for the minimum, which is the dark state with \( \rho_{13} = 0 \).

So full width at half maximum of the population versus \( \phi \), as illustrated in figure 3, is
\[
\delta_\phi = 2 \sin^{-1} \left( \frac{1}{6} + \frac{4\gamma^2}{3g^2 \Omega^2} + \frac{(g^2 - \Omega^2)^2}{3g^4 \Omega^2} \right).
\]
(10)

Thus, for strong field coupling the atom states, the excited state population will be very sensitive to the phase of the microwave field and so does the dark state. In contrast, the population is extremely insensitive to the phase of the microwave field if (4) or (6) is not satisfied. This phase dependence effect is similar to the phase sensitivity in [16]. For real applications, the relative phase \( \phi \) can be controlled by a microwave phase shifter or an optical delay line as suggested in [22]. According to experimental setups, the control can be simplified. For example, in the experimental setup in [22], the relative phase can be changed simply by moving the cell and microwave cavity along the propagation direction of the optical fields.

4. EIT without dark states

Dark state plays a very important role in EIT, and total transparency can be achieved only in dark state. Up to now only \( \Lambda \)-type atom can satisfy this condition [2]. The other three-level systems can exhibit EIT transparency window but could not generate dark state [2, 25], so that the transparency windows are always accompanied with absorption. In our scheme, as mentioned above, the existence of the ground states dephasing limits the applications of the dark states irrespective of that the microwave field exists or not. But with the help of the microwave field, the total transparency can be achieved even with an occupied excited state. A previous study has already given an example [20].

For \( G \gg g \) as EIT requires, let \( g = \zeta G \) where \( \zeta \ll 1 \). The imaginary part of \( \rho_{31} \), which represent the absorption of the medium, can be solved from the density matrix equations in the resonance case as
\[
\text{Im}[\rho_{31}] = \frac{\Omega \zeta \sin \phi + \kappa \zeta^2 + O(\zeta^3)}{2g + O(\zeta^2)} \approx \frac{\Omega \zeta \sin \phi + \kappa \zeta^2}{2g}.
\]
(11)
Hence, if \( \sin \phi = -\kappa \zeta / \Omega = -\kappa g / \Omega G \), transparency occurs (\( \text{Im}[\rho_{31}] = 0 \)). Obviously this condition is far from the dark-state condition and the excited state is occupied.

Figure 4 shows the excited state population and absorption/dispersion properties. EIT does occur without dark states. This is because the presence of the microwave field can generate gain rather than absorption. It is clear in (11) that \( \Omega \sin \phi \zeta \) is the contribution from the microwave field (gain if \( \sin \phi < 0 \) and \( \kappa \zeta^2 \) is the absorption caused by dephasing. Thus, the transparency without dark states can be understood as the result of the balance between the absorption (due to dephasing) and gain (due to the microwave field). As in figure 5, with different Rabi frequencies.

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**Figure 2.** Dark-state condition. \( g = G = \Omega = \gamma, \gamma_1 = \gamma_2 = \gamma, \Delta_1 = 0, \kappa = 0 \). The excited state population changes from a high occupation (\( \phi = \pi/2 \), dashed blue line) to zero in dark state \( \phi = 0 \) (solid red line). (Colour online.)

**Figure 3.** Phase sensitivity of excited state population. \( g = G = \Omega = 20 \gamma, \gamma_1 = \gamma_2 = \gamma, \Delta_1 = \Delta_2 = 0, \kappa = 0, \rho_{31} \) is extremely sensitive to the phase around the dark-state condition and not sensitive to the phase in any other conditions.
of the microwave field the medium represents absorption, transparency and gain. Figure 6 shows the advantage of the microwave controlled EIT. Without ground states dephasing, the medium is transparent to the probe. If the ground states dephasing is large enough, however, the transparency will disappear. Including the microwave field with proper phase, the transparency is regained due to the balance between the microwave caused gain and the dephasing caused absorption.

Figure 6. Comparison of the absorption profile with or without the microwave field. $g = 0.05\gamma, G = \gamma, \Omega = 0.05\gamma, \gamma_1 = \gamma_2 = \gamma, \Delta_2 = 0, \kappa = \gamma$ and $\phi = \pi/2$. (1) EIT with $\kappa = 0$ and $\Omega = 0$ (red dashed line); (2) EIT with $\kappa \gamma$ and $\Omega = 0$ (blue dot dashed line), the transparency window is almost closed; (3) EIT with $\kappa = \gamma$ and $\Omega = \pi/2$ (black solid line), the transparency window is regained with the presence of the microwave field. (Colour online.)

Based on this method a good EIT signal can be achieved even in the high dephasing medium.

5. Conclusion

In conclusion, we study a $\Lambda$-type EIT medium controlled by a microwave field coupling the ground states. From the Hamiltonian in the rotating frame the dark-state conditions are derived. Dark states under these conditions are the direct consequence of the coherence among the optical fields and microwave field. This kind of dark state is very sensitive to the relative phase between the microwave field and the optical fields. However, with significant ground states dephasing, the dark states can never be achieved, which is similar to EIT without the microwave field. Even without dark states, owing to the presence of the microwave field, a gain mechanism is introduced and the balance of the gain and the loss from the dephasing can be balanced so that transparency can be regained without dark states. Thus, the microwave field provides more possible methods for the quantum control of the inner states of the atom structure, enhances EIT in a high dephasing medium and benefits other EIT-based phenomena. One should note that, other than in atom vapors [22], recent studies in circuit QED exhibit the EIT phenomena in microwave frequencies [26, 27], which may provide a much more convenient way to realize this kind of configuration.

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Appendix

The full solution of the density matrix equation is very complicated and not necessary to be given here. If $\rho_{33} = 0$, the equation gives the requirement for $\phi$ as

$$\cos \phi = A \pm \sqrt{B},$$

where

$$A = \frac{2\gamma}{g G \Omega(4\gamma + \kappa)^2}[(2\gamma g^2 + \kappa \Omega^2 - G^2(2\gamma + \kappa))\Delta_1 + \text{[-}2(2\gamma + \kappa)g^2 + \kappa \Omega^2 + 2G^2\gamma]\Delta_2],$$

$$B = \frac{g^2 G^2(4\gamma + \kappa)^2 \Omega^2}{g^2 G^2(4\gamma + \kappa)^2 \Omega^2} + \frac{2\gamma[\kappa \Omega^2 + 2(g^2 + G^2)\gamma]}{g^2 G^2(4\gamma + \kappa)^2 \Omega^2} \{g^2[2\gamma(\Delta_1 - \Delta_2) - \kappa \Delta_2]^{\gamma} + G^2[\kappa \Delta_1 + 2\gamma(\Delta_1 - \Delta_2)]^{\gamma} + 2\gamma\kappa \Omega^2(\Delta_1 + \Delta_2)^{\gamma}\}.$$ 

So $B > 0$. Since

$$B - (1 - A)^2 = \frac{4\gamma^2}{g^2 G^2(4\gamma + \kappa)^2 \Omega^2}(g^2 - G^2)\Omega$$

$$+ gG(\Delta_1 - \Delta_2)^2 + \frac{2\gamma\kappa}{g^2 G^2(4\gamma + \kappa)^2 \Omega^2}[2g^2 \gamma G^2 + 4(g^2 + G^2)\gamma \Omega^2 + (g G^2 - \Omega^2) - G G \Delta_1]^{\gamma}$$

$$+ [G^2 - \Omega^2] - g G (\Delta_1 - \Delta_2)^2],$$

it is clear that $B - (1 - A)^2 \geq 0$ and so $A + \sqrt{B} \geq 1$ and $A - \sqrt{B} \leq 1; \phi$ the equality holds when $(g^2 - G^2)\Omega + gG(\Delta_1 - \Delta_2) = 0$ and $\kappa = 0$. Following the same step, one has $B - (1 + A)^2 \geq 0$ and so $A - \sqrt{B} \leq -1$ and $A + \sqrt{B} \geq -1$; the equality holds when $(g^2 - G^2)\Omega - gG(\Delta_1 - \Delta_2) = 0$ and $\kappa = 0$. The combination gives

$$A + \sqrt{B} \geq 1, \quad \phi = 0, \kappa = 0, $$

$$\rightarrow (g^2 - G^2)\Omega + gG(\Delta_1 - \Delta_2) = 0;$$

$$A - \sqrt{B} \leq -1, \quad \phi = \pi, \kappa = 0,$$

$$\rightarrow (g^2 - G^2)\Omega - gG(\Delta_1 - \Delta_2) = 0,$$

which is exactly the same as (4) and (6).

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