NONEXTENSIVE THERMOS TATISTICAL APPROACH TO
THE THERMOLUMINESCENCE DECAY

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Abstract

In this study, thermoluminescence decay is investigated within Tsallis thermostatistics(TT). We believe that this is the first attempt to handle thermoluminescence decay process within TT.

Keywords: Thermoluminescence, Tsallis thermostatistics.

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I. INTRODUCTION

The luminescence intensity of a pre-irradiated phosphor is recorded as a function of the rising temperature. The resulting curve between the luminescence intensity and temperature, known as the glow curve, first increases and then, after reaching a maximum, decreases. If \( n \) represents the occupied trap density at time \( t \), then the following equation gives the rate of thermal release of electron at temperature \( T \):

\[
\frac{dn}{dt} = -ns \exp(-E/k_B T).
\]

(1)

If the phosphor is heated at a constant rate of increase of temperature, i.e. \( dT/dt = \text{const} \tan t = r \), the above equation can be rewritten as

\[
\frac{dn}{n} = -\frac{s}{r} \exp(-E/k_B T) \exp(-s \int_{T_i}^{T} \exp(-E/k_B T')dT').
\]

(2)

If assuming initially at the start of heating, \( T = T_i, n = n_0 \), then the integration of Eq.(2) results in

\[
n = n_0 \exp \left(-\frac{s}{r} \int_{T_i}^{T} \exp(-E/k_B T')dT' \right).
\]

(3)

The thermoluminescence intensity \( I \) as a function of temperature is given as:

\[
I = -c \left( \frac{dn}{dt} \right) = cn_0 s \left[ \exp(-E/k_B T) \right] \left[ \exp \left(-\frac{s}{r} \int_{T_i}^{T} \exp(-E/k_B T')dT' \right) \right].
\]

(4)

II. TSALLIS THERMOSTATISTICS

Boltzmann-Gibbs statistics is used to study the systems having the following conditions:

(i) the spatial range of the microscopic interactions are short-ranged,
(ii) the time range of the microscopic memory is short-ranged,
(iii) the system evolves in a Euclidean-like space-time.
These kinds of systems are said to be extensive. If a system does not obey these restrictions, Boltzmann-Gibbs statistics seems to be inappropriate and a non-extensive formalism must be used. Tsallis thermostatistics (TT) is one of these formalisms.

TT has been applied to some concepts of thermostatistics [1] and also achieved in solving some physical systems, where BG statistics is known to fail: stellar polytropes [2], Levy-like anomalous diffusions [3-7], two-dimensional Euler turbulence [8], solar neutrino problem [9], velocity distributions of galaxy clusters [10], fully developed turbulence [11-14], electron-positron and other high-energy collisions [15-19], anomalous diffusion of Hydra viridissima [20] and nematic liquid crystals [21].

Since the paper by Tsallis [22], there has been a growing tendency to nonextensive statistical formalism. It has been shown that this formalism is useful, because it provides a suitable theoretical tool to explain some of the experimental situations, where standard thermostatistics seems to fail, due to the presence of long-range interactions, or long-range memory effects, or multi-fractal space-time constraints.

TT has been applied to various concepts of thermostatistics and achieved in solving some physical systems, where Boltzmann-Gibbs statistics is known to fail. Recently, Oliveira et al. [23] have pursued the idea, based on novel experimental and theoretical results, that manganites are magnetically non-extensive objects. This property appears in systems where long-range interactions and/or fractality exist, and such features have been invoked in recent models of manganites, as well as in the interpretation of experimental results. Therefore we could expect that this kind of materials could be examined within a non-extensive thermostatistics, one of which is called Tsallis thermostatistics (TT) and as you can see in our paper, TT has been applied to various concepts of thermostatistics and achieved in solving some physical systems, where Boltzmann-Gibbs statistics is known to fail. On the other hand, we are still working on application of TT on thermoluminescence process, so another studies on this subject are forthcoming, experimentally and theoretically.

TT considers three possible choices for the form of a nonextensive expectation value.
These choices have been studied in [24] and applied to two systems; the classical harmonic oscillator and the quantum harmonic oscillator. In that study, Tsallis et al. studied three different alternatives for the internal energy constraint. The first choice is the conventional one,

\[ \sum_{i=1}^{W} p_i \varepsilon_i = U_q^{(1)}. \] (5)

The second choice is given by

\[ \sum_{i=1}^{W} p_i^q \varepsilon_i = U_q^{(2)}. \] (6)

and regarded as the canonical one. Both of these choices have been applied to many different systems in the last years [25]. However both of them have undesirable difficulties. The third choice for the internal energy constraint is

\[ \frac{\sum_{i=1}^{W} p_i^q \varepsilon_i}{\sum_{i=1}^{W} p_i^q} = U_q^{(3)}. \] (7)

This choice is commonly considered to study physical systems because it is the most appropriate one, and is denoted as the Tsallis-Mendes-Plastino (TMP) choice. \( q \) index is called the entropic index and comes from the entropy definition,

\[ S_q = k \frac{1 - \sum_{i=1}^{W} p_i^q}{q - 1} \] (8)

where \( k \) is a constant, \( \sum_i p_i = 1 \) is the probability of the system in the \( i \) microstate, \( W \) is the total number of configurations. In the limit \( q \to 1 \), the entropy reduces to the well-known Boltzmann-Gibbs (Shannon) entropy.

The optimization of \( S_q \) leads to

\[ p_i^{(3)} \left( 1 - (1 - q)\beta (\varepsilon_i - U_q^{(3)}) / \sum_{j=1}^{W} (p_j^{(3)})^q \right)^{\frac{1}{q - 1}} \]

with

\[ Z_q^{(3)} = \sum_{i=1}^{W} \left[ 1 - (1 - q)\beta (\varepsilon_i - U_q^{(3)}) / \sum_{j=1}^{W} (p_j^{(3)})^q \right]^{\frac{1}{q - 1}}. \] (10)
This equation is an implicit one for the probabilities $p_i$. Therefore the \textit{normalized $q$-expectation value} of an observable is defined as

$$A_q = \frac{\sum_{i=1}^{W} p_i^q A_i}{\sum_{i=1}^{W} p_i^q} = \langle A_i \rangle_q$$

(11)

where $A$ denotes any observable quantity which commutes with the Hamiltonian. This expectation value recovers the conventional expectation one when $q = 1$. As mentioned above, Eq. (11) is an implicit one and in order to solve this equation, Tsallis et al. suggest two different approaches, "iterative procedure" and "$\beta \rightarrow \beta'$" transformation.

The luminescence intensity of a pre-irradiated phosphor is recorded as a function of the rising temperature. The resulting curve between the luminescence intensity and temperature, known as the glow curve, first increases and then, after reaching a maximum, decreases. If $n$ represents the occupied trap density at time $t$, then the following equation gives the rate of thermal release of electron at temperature $T$ within Tsallis thermostatics:

$$\frac{dn}{dt} = -ns(1 - (1 - q)E/kT)^{\frac{1}{1-q}}.$$  

(12)

It is important to note that the rate is expressed as a power law rather than exponential one. If the phosphor is heated at a constant rate of increase of temperature, i.e. $dT/dt = \text{const}$, then $t = r$, the above equation can be rewritten as

$$\frac{dn}{n} = -\frac{s}{r}(1 - (1 - q)E/kT)^{\frac{1}{1-q}}.$$  

(13)

We assume that initially at the start of heating, $T = T_i, n = n_0$. Then the integration of Eq. (13) results in

$$n = n_0 \left(1 + (1 - q) \left(\frac{-2 \int_{T_i}^{T} (1 - (1 - q)E/kT')^{\frac{1}{1-q}} dT' \left(\frac{1}{1-q} - 1\right)}{1 - q}\right)\right)^{\frac{1}{1-q}}.$$  

(14)

The thermoluminescence intensity $I$ as a function of temperature is given as:

$$I = -c \left(\frac{dn}{dt}\right) = cn_0s \left[(1 - (1 - q)E/kT)^{\frac{1}{1-q}}\right]$$

$$\left(1 + (1 - q) \left(\frac{-2 \int_{T_i}^{T} (1 - (1 - q)E/kT')^{\frac{1}{1-q}} dT' \left(\frac{1}{1-q} - 1\right)}{1 - q}\right)\right)^{\frac{1}{1-q}}.$$  

(15)
Figure shows the computed glow curves corresponding to Eq.(15). The parameters in the Eq.(15) are: $q = 5 \, K.s^{-1}$, electron trap depth $E = 0.8 \, eV$ and $s = 5 \times 10^{11} \, s^{-1}$. In the figure, it is shown the effect of the nonextensivity. In other words, if the rate of thermal release of electrons at temperature $T$ is expressed as a power law relation, then the glow intensity vs temperature behaves as shown in the figure. An interesting point is that in all the curves corresponding to different values of $q$ entropic index, the intensity approximately has the same value at $T \simeq 390K$.

III. RESULTS AND DISCUSSION

Presumably there is some normalization at 390 K in order to compare with $q = 1$ case. The peak sequence moves smoothly with $q$ value, this is in much the same way as caused by changing thermoluminescence kinetics if one varies the relative number of trapping and recombination sites and includes secondary, or back reactions. Whilst the data presented here are of a preliminary nature, it is sufficient to give encouragement for further developments.
REFERENCES

1. S. Abe, Y. Okamoto, Nonextensive statistical mechanics and its applications, Series Lectures Notes in Physics, Berlin: Springer-Verlag, 2001.

2. A.R. Plastino, A. Plastino, Phys. Lett. A 174 (1993) 384.

3. P.A. Alemany, D.H. Zanette, Phys. Rev. E 49 (1994) R956.

4. C. Tsallis, S.V.F. Levy, A.M.C. Souza, R. Maynard, Phys. Rev. Lett. 75 (1995) 3589.

5. D.H. Zanette, P.A. Alemany, Phys. Rev. Lett. 75 (1995) 366.

6. M.O. Careres, C.E. Budde, Phys. Rev. Lett. 77 (1996) 2589.

7. D.H. Zanette, P.A. Alemany, Phys. Rev. Lett. 77 (1996) 2590.

8. B.M. Boghosian, Phys. Rev. E 53 (1996) 4754.

9. G. Kaniadakis, A. Lavagno, P. Quarati, Phys. Lett. B 369 (1996) 308.

10. A. Lavagno, G. Kaniadakis, M.R. Monteiro, P. Quarati, C. Tsallis, Astrophys. Lett. Commun. 35 (1998) 449.

11. T. Arimitsu, N. Arimitsu, Phys. Rev. E 61 (2000) 3237.

12. T. Arimitsu, N. Arimitsu, J. Phys. A 33 (2000) L235.

13. C. Beck, Physic A 277 (2000) 115.

14. C. Beck, G.S. Lewis, H.L. Swinney, Phys. Rev. E 63 (2001) 035303.

15. D.B. Walton, J. Rafelski, Phys. Rev. Lett. 84 (2000) 31.

16. G. Wilk, Z. Wlodarczyk, Nucl. Phys. B (Proc. Suppl.) 75A (1999) 191.

17. G. Wilk, Z. Wlodarczyk, Phys. Rev. Lett. 84 (2000) 2770.

18. M.L.D. Ion, D.B. Ion, Phys. Lett. B 482 (2000) 57.

19. I. Bediaga, E.M.F. Curado, J. Miranda, Physica A 286 (2000) 156.

20. C. Beck, Physica A 286 (2000) 164.

21. A. Upadhyaya, J.-P. Rieu, J.A. Glazier, Y. Swada, Physica A 293 (2001) 549.

22. C. Tsallis, J. Stat. Phys. 52 (1988) 479.

23 M.S. Reis, J.P. Aráujo, V.S. Amaral, E.K. Lenzi and I.S. Oliveira, Phys. Rev. B 66 (2002) 134417.
24. C. Tsallis, R.S. Mendes, A.R. Plastino, Physica A 261 (1998) 534.

25. see http://tsallis.cat.cbpf.br for updated bibliography.

FIGURE CAPTIONS

Figure. The computed glow curves corresponding to Eq.(15) for various values of \( q \).