Study of high-frequency edge of turbulent cascade on the surface of He-II.

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Abstract. We present the results of recent experimental study of capillary wave turbulence on the surface of superfluid $^4$He. Capillary waves on the charged liquid surface were excited by an external electric force and detected by means of an optical technique. It was observed for the first time that a local maximum in wave amplitudes at frequencies of the order of the viscous cut-off frequency can be formed in the case where the wave system is excited by a low-frequency periodical driving force. We suppose that the reason for this wave energy condensation could be in detuning of the nonlinearly generated waves from the resonant frequency of the cell of finite size, i.e. the mechanism considered earlier in the frozen turbulence theory. It was found also that the decrease of wave amplitudes within a viscous damping region observed in the case of excitation by a periodical force is much steeper than that observed in the case of noisy driving force.

1. Introduction
Turbulence in the system of waves on the liquid surface has been in focus of numerous investigations during the last decade [1]. Turbulence in the system of capillary waves (capillary turbulence) is of particular interest because this is a suitable model system for experimental study in laboratory conditions. Theory of capillary turbulence has been developed in 60s of the previous century [1, 2]. Our previous experiments [3, 4] have shown that the use of quantum fluids for investigations of capillary turbulence can provide additional advantages with respect to traditional techniques. In particular, low viscosity and high nonlinearity of capillary waves on the surface of liquid hydrogen have allowed one to observe and to study in great details steady-state and non-stationary Kolmogorov-Zakharov spectrum of capillary turbulence in a wide range of frequencies [3, 5]. Some new results devoted to evolution of the wave spectrum within the inertial range of frequencies (i.e. at frequencies higher than the driving frequency but much lower than the Kolmogorov viscous frequency) are considered in another paper in this issue [6].

In the present paper we focus on peculiarities in shape of the turbulent wave spectrum at high frequencies observed in experiments with He-II, a superfluid phase of liquid $^4$He. It is known [2] that the energy of turbulent capillary waves is transferred due to nonlinearity from low frequencies (where it is injected into the system) towards the high frequency spectral domain, where the energy flux is eventually absorbed by viscosity. We observed for the first time that under certain conditions the wave energy can “condensate” at frequencies of the order of the viscous cut-off
scale, thus producing a local *maximum* in turbulent wave distribution. We suppose that the main reason of the wave energy condensation can be in restrictions for the resonance conditions for three-wave processes due to finite sizes of an experimental cell. Thus our observations can be considered as a first experimental support of the theory of capillary turbulence in a finite geometry [8, 9, 10], see below. It is interesting to note that a similar mechanism was proposed recently for explanation of restrictions of the energy transfer in a system of interacting planetary vortices [11].

2. Experimental technique

In the reported experiment with superfluid $^4$He we used a technique [4] previously used in our studies of capillary turbulence on the surface of liquid hydrogen [5, 3, 12]. Liquid helium was condensed in the copper cup of inner diameter 30 mm and of depth 4 mm. The surface of liquid helium was charged with positive charges (snowballs) created by a radioactive plate inside the liquid. Constant DC voltage $U \approx 600$ V was applied between the cup and the copper plate (upper electrode) placed in vapor at a distance of 3 mm above the liquid surface. Surface waves were excited by applying a low-frequency AC voltage with amplitudes from 1 through 80 V to the cup, in addition to DC voltage. Oscillations of the surface were measured with help of a laser beam reflected from the surface. Time variations of the total power of the beam $P(t)$ were measured by a photodetector and were digitized by an analogue-to-digital converter. Spectrum of the surface oscillations $I_\omega$, as a function of wave frequency $\omega$, was computed via Fast Fourier Transform (FFT) of the measured signal $P(t)$, $I_\omega \sim |P_\omega|^2$. The measurements were done with superfluid $^4$He at temperature $T \approx 1.6$ K. For comparison, the temperature of the normal-to-superfluid state transition for liquid $^4$He at saturated vapor pressure is equal to $T_\lambda = 2.17$ K.

In the reported measurements two types of low-frequency driving force were used. In experiments of the first type the surface was driven by a sinusoidal force with frequency $\omega_p$ equal to a resonant frequency of the cell. In the measurements of the second type a driving by a noisy force was used. The spectral width of the noisy force was of the order of or less than its mean frequency, i.e. a situation of narrow-band pumping [1] was realized, see also [6]. In this case the electrical noise was produced by a digital noise generator.

![Figure 1](image.png)

*Figure 1.* Evolution of the turbulent spectrum with increasing the driving force amplitude. The driving frequency is equal to $\omega_p/2\pi = 120$ Hz. Angle amplitudes of the excited waves with frequency $\omega = \omega_p$ are equal to (a) 0.02 and (b) 0.05 rad. Solid lines correspond to power-law dependence $I_\omega \sim \omega^{-3.7}$. On Figure (b) a local maximum formed at $\omega/2\pi \approx 4$ kHz is clearly seen (highlighted by a red circle).
Figure 2. Wave spectrum observed when driving the surface by a sinusoidal force with frequency $\omega_p/2\pi = 34$ Hz. Formation of a maximum at the spectrum is clearly seen on the main plot (circled) and in the inset in an enlarged scale.

3. Results and Discussion

Figure 1 shows evolution of a turbulent spectrum of capillary waves on the surface of He-II with increasing amplitude of a periodical driving force. The driving frequency was equal to $\omega_p/2\pi = 120$ Hz. It is seen on the spectra that a fundamental peak is formed at a frequency $\omega = \omega_p$. Harmonics of the driving frequency generated due to nonlinearity form a wave cascade, as seen on Fig. 1. In the inertial range dependence of the peak amplitudes on frequency is described well by a power law function $I_\omega \sim \omega^{-m}$ with the index $m \approx 3.7$. This is in agreement with the weak turbulence theory [1, 2], which gives the value $m = 21/6$. It is clearly seen on Fig. 1a that the wave cascade is cut off at high frequencies $\omega/2\pi \approx 2$ kHz due to viscosity, in accordance with our previous observations [3, 4]. Level of the instrumental noise in these measurements was $\approx 0.3$ in the used dimensionless units.

It is evident from Fig.1b that a local maximum on the wave spectrum is formed near a viscous cut-off when the driving amplitude increases. Fig. 2 shows the wave spectrum with similar maximum (circled), which was observed in case where the surface waves were driven at $\omega_p/2\pi = 34$ Hz. Inset on Fig. 2 shows the maximum in an enlarged scale.

We should stress that formation of this maximum was only observed if the surface was driven by a periodic force (measurements of the first type). In measurements of the second type (noisy driving) the wave spectrum always decreased monotonically with a wave frequency rising. Turbulent distributions measured when driving by a periodic force with a relatively small amplitude (blue curve) and by a narrow-band noisy pumping (red curve) are shown on Figure 3 as typical examples of the results obtained. It is also seen on Figure 3 that within a dissipative frequency range (marked by vertical red dashed lines) the wave amplitudes in the turbulent spectrum generated by a harmonic force (blue curve) are decreased more rapidly than those in the spectrum generated by a noisy force. We can see, for example, that for harmonic drive the wave amplitudes decreased in about 6 orders over a relatively short frequency the range from 1000 Hz through 2500 Hz.

We can attribute formation of a local maximum at the turbulent spectrum to partial condensation of the wave energy at frequencies of the order of viscous cut-off frequency scale. One of the possible reasons for the condensation can be a “bottleneck phenomenon” for nonlinear wave interaction arising due to discreteness of the spectrum of resonant frequencies for capillary waves in a cylindrical cell, in accordance with theoretical predictions of the works [8, 9, 10]. The distance (in frequency scales) between two neighbouring $n$-th and $(n + 1)$-th resonances, $\Delta \omega_n \propto d\omega(k)/dk \propto n^{1/2}$, grows when the resonant number $n$ increases. It can be a reason why the detuning of the high-frequency harmonics from the resonant frequency of the cell should be only important for sufficiently high wave frequencies, i.e. at relatively high driving amplitudes. In the case of narrow-band drive the wave energy flux propagates to higher frequencies (where it is absorbed by viscosity) probably due to relatively higher density of states of the waves.
4. Conclusion

We report the first observation of energy condensation in a turbulent system of capillary wave on the surface of superfluid $^4$He. We found that a local maximum in the wave distribution in the high-frequency spectral domain can sometimes be formed if the surface is driven by a periodical low-frequency force. In the case where the waves were excited by a noisy drive no wave condensation was observed. We suppose that a reason of this peculiarity in the energy transfer is in the influence of the resonant conditions for capillary waves in a restricted geometry of experiment.

Acknowledgments

We acknowledge L. P. Mezhov-Deglin, V. E. Zakharov, E. A. Kuznetsov, S. Nazarenko, E. Kartashova and P. V. E. McClintock for fruitful discussions and thank V. N. Khlopinskii and A. V. Lokhov for experimental assistance. The work was supported by RFBR grants Nos. 06-02-17253 and 07-02-00728, by the Presidium of RAS in frames of programs "Quantum Macrophysics" and "Fundamental problems in nonlinear dynamics" and by Royal Society (UK).

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