Rare decays $B \to X_d l^+ l^-$ and $B \to K^{(*)} l^+ l^-$ in SM and beyond

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After a brief review of the inclusive and exclusive rare semileptonic decays of $B$ mesons in the standard model (SM), an overview of recent theoretical developments in this field is given. New physics effects on various observables such as branching ratio, backward-forward asymmetry, polarization of lepton and CP violation in the decays are analyzed in models beyond SM (supersymmetric models and two Higgs doublet models).

1. INTRODUCTION

There are no flavor changing neutral currents (FCNC) at tree-level in the standard model (SM). FCNC appear at loop-levels and consequently offer a good place to test quantum effects of the fundamental quantum field theory on which SM based. Furthermore, they are very small at one loop-level due to the unitarity of Cabibbo-Kobayashi-Maskawa (CKM) matrix. In models beyond SM new particles may appear in the loop and have significant contributions to flavor changing transitions. Therefore, FCNC interactions give an ideal place to search for new physics. Any positive observation of FCNC couplings deviated from that in SM would unambiguously signal the presence of new physics. Searching for FCNC is clearly one of important goals of the next generation of high energy colliders [1].

In this talk I shall review the recent developments of rare decays $B \to X_d l^+ l^-$ and $B \to K^{(*)} l^+ l^-$, one kind of FCNC processes, in SM and beyond (SUSY and 2HDM) with emphasis on the latter. In 2HDM or SUSY the couplings of neutral Higgs bosons (NHBs) to down-type quarks or leptons are proportional to $m_t^2/m_w^2$ $\tan\beta$ which leads to significant effects on observables if $\tan\beta$ is large. So we shall pay particular attention to the large $\tan\beta$ case in the talk. I am sorry to say that some interesting works are not addressed due to the limited length of the paper.

2. INCLUSIVE DECAYS $B \to X_d l^+ l^-$

2.1. In SM

The effective Hamiltonian relevant to $b \to s$ transition in SM is

$$\mathcal{H}_{\text{eff}} = -4\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i(\mu),$$

(1)

Where operators $O_i$ and Wilson coefficients $C_i$ in leading logarithmic approximation (LLA) can be found in ref. [2]. The above Hamiltonian leads to the following free quark decay amplitude

$$\mathcal{M}(b \to s \ell^+ \ell^-) = \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} C_9 \left[ \bar{s} \gamma_\mu L b \right] \left[ \bar{\ell} \gamma^\mu \ell \right]$$

$$+ C_{10} \left[ \bar{s} \gamma_\mu L b \right] \left[ \bar{\ell} \gamma^\mu \gamma_5 \ell \right]$$

$$- 2 \bar{m}_b C_7 \left[ \bar{s} i \gamma_{\mu\nu} \frac{q^\nu}{s} R b \right] \left[ \bar{\ell} \gamma^\mu \ell \right],$$

(2)

where $L/R \equiv (1 \mp \gamma_5)/2$, $s = q^2$, $q = p_+ + p_-$, $\bar{s} = s/m_b^2$, $\bar{m}_b = m_b/m_B$. We put $m_s/m_b = 0$, but keep the leptons massive. In Eq. $C_9$ can be found in, e.g., ref. [3].

The dilepton invariant mass spectrum (IMS) including power corrections in the HQET approach in $B \to X_d \ell^+ \ell^-$ decays can be written as:

$$\frac{d\mathcal{B}}{d\bar{S}} = \frac{d\mathcal{B}^0}{d\bar{S}} + \frac{d\mathcal{B}^{1/m_b^2}}{d\bar{S}} + \frac{d\mathcal{B}^{1/q^2}}{d\bar{S}},$$

(3)

where the first term corresponds to the free quark decay $b \to s l^+ l^-$, the second term accounts for the $\mathcal{O}(1/m_b^2)$ power corrections[4], and the last term accounts for the non-perturbative interaction of a virtual $u\bar{u}$- and $c\bar{c}$-quark loop with soft
From Eq. (3), we obtain IMS for $b \to s\ell^+\ell^-$

$$\frac{d\Gamma(B \to X_s\ell^+\ell^-)}{d\hat{s}} = B_{sl} \frac{\alpha^2}{4\pi^2 f(m_c)\kappa(m_c)} \times (1 - \hat{s})^2(1 - \frac{4\hat{s}^2}{\hat{s}})^{1/2} \frac{|V_{tb}V_{ts}^*|^2}{|V_{cb}|^2} D(\hat{s}) \ ,$$

$$D(\hat{s}) = |C_{9}^{\epsilon f f}|^2(1 + \frac{2\hat{s}^2}{\hat{s}})(1 + 2\hat{s})$$

$$+ 4|C_{7}^{\epsilon f f}|^2(1 + \frac{2\hat{s}^2}{\hat{s}})(1 + \frac{2}{\hat{s}})$$

$$+ |C_{10}|^2[(1 + 2\hat{s}) + \frac{2\hat{s}^2}{\hat{s}}(1 - 4\hat{s})]$$

$$+ 12 \text{Re}(C_{7}^{\epsilon f f} C_{9}^{\epsilon f f} s^* C_{10}) (1 + \frac{2\hat{s}^2}{\hat{s}})$$ \ (4)

where $t = m_t/m_b$.

The forward-backward asymmetry (FBA) for $b \to s\ell^+\ell^-$ is given by

$$A(\hat{s}) = \frac{\int_0^1 dz \frac{d\Gamma(B \to X_s\ell^+\ell^-)}{d\hat{s}^2}}{\int_0^1 dz \frac{d\Gamma(B \to X_s\ell^+\ell^-)}{d\hat{s}}} = \frac{E(\hat{s})}{D(\hat{s})}, \ (5)$$

$$E(\hat{s}) = \text{Re}(C_{9}^{\epsilon f f} C_{10}^* + 2C_{7}^{\epsilon f f} C_{10}^*), \ (6)$$

where $z = \cos \theta$ and $\theta$ is the angle between the momentum of $B$ and that of $\ell^+$ in the center of mass frame of the dileptons $\ell^+\ell^-$. The longitudinal, transverse and normal polarizations of lepton (LP) are given by

$$P_L = (1 - \frac{4\hat{s}^2}{\hat{s}})^{1/2} \frac{D_L(\hat{s})}{D(\hat{s})},$$

$$P_N = \frac{3\pi}{4\hat{s}^{1/2}}(1 - \frac{4\hat{s}^2}{\hat{s}})^{1/2} \frac{D_N(\hat{s})}{D(\hat{s})},$$

$$P_T = \frac{3\pi t}{2\hat{s}^{1/2}} \frac{D_T(\hat{s})}{D(\hat{s})}, \ (7)$$

where

$$D_L(\hat{s}) = \text{Re} \left(2(1 + 2\hat{s})C_{9}^{\epsilon f f} C_{10}^* + 12C_{7}^{\epsilon f f} C_{10}^* \right),$$

$$D_N(\hat{s}) = \text{Im} \left(4tC_{10}^* C_{7}^{\epsilon f f} + 2t\hat{s} C_{9}^{\epsilon f f} C_{10}^* \right),$$

$$D_T(\hat{s}) = \text{Re} \left(-2C_{7}^{\epsilon f f} C_{10}^* + 4C_{9}^{\epsilon f f} C_{7}^{\epsilon f f} + \frac{4}{\hat{s}} |C_{7}^{\epsilon f f}|^2 \right)$$

$$- C_{9}^{\epsilon f f} C_{10}^* + \hat{s}|C_{9}^{\epsilon f f}|^2 \right) .$$ \ (8)

In LLA one obtains the following numerical results

$$Br(B \to X_s \mu^+ \mu^-) = 6.7 \times 10^{-6},$$

$$Br(B \to X_s \tau^+ \tau^-) = 2.5 \times 10^{-7}. \ (9)$$

Apart from the CKM-parametric dependence (estimate ±13%), the theoretical uncertainty on the branching ratio comes mainly from the scale dependence. The present experimental bound is

$$B(B \to X_s \ell^+ \ell^-) < 4.2 \times 10^{-5} \ (at \ 90\% \ C.L.)$$

Comparing the bound with Eq. (5), it follows that there is a room for new physics.

For the lepton polarization, one has $\langle P_L \rangle_r = -0.37$, $\langle P_T \rangle_r = -0.63$, $\langle P_N \rangle_r = 0.03 (0.02)$ for $\kappa_V = 2.35$. Assuming a total of $5 \times 10^9$ $B \to X_s \tau^+ \tau^-$ events, one can expect to observe $\sim 100$ identified $B \to X_s \tau^+ \tau^-$ events, permitting a test of the predicted polarization ($P_L = -0.37$, and ($P_T = -0.63$) with good accuracy.

### 2.2. Beyond SM

We limit to discuss the model II 2HDM and SUSY (MSSM, mSUGRA, and string theory and M-theory inspired models) in the talk. In these models one should include the contributions from exchanging NHBs in the large $\tan \beta$ case. Instead of Eq. (6), the effective Hamiltonian describing $B \to X_s \ell^+ \ell^-$ now becomes

$$H_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

$$+ \sum_{i=1}^{10} C_{Q_i}(\mu) Q_i(\mu), \ (10)$$

where $O_i$ is the same as that in Eq. (6) and $Q_i$’s come from exchanging neutral Higgs bosons and are defined by

$$Q_1 = \frac{e^2}{16\pi^2} (s^2 \bar{b}^\prime_R)(\bar{\tau} \tau),$$

$$Q_2 = \frac{e^2}{16\pi^2} (s^2 \bar{b}^\prime_R)(\tau \gamma_5 \tau),$$

$$Q_3 = \frac{g^2}{16\pi^2} (s^2 \bar{b}^\prime_R)(\sum_q \bar{q}_L q_R).$$
Wilson coefficients in models beyond SM have been given and are listed in the following. There are five different sets of contributions:

1) Model II 2HDM
   
   \( C_i = \frac{g^2}{16\pi^2} \left( s_L^2 b_R^2 \right) \left( \sum_q q_R^i q_L^j \right) \) (9a)
   
   \( C_5 = \frac{g^2}{16\pi^2} \left( s_L^2 b_R^2 \right) \left( \sum_q q_L^i q_R^j \right) \) (9b)
   
   \( C_6 = \frac{g^2}{16\pi^2} \left( s_L^2 b_R^2 \right) \left( \sum_q q_R^i q_L^j \right) \) (9c)
   
   \( C_7 = \frac{g^2}{16\pi^2} \left( s_L^2 \sigma_{\mu\nu} b_R^2 \right) \left( \sum_q q_R^i q_L^j \right) \) (9d)
   
   \( C_8 = \frac{g^2}{16\pi^2} \left( s_L^2 \sigma_{\mu\nu} b_R^2 \right) \left( \sum_q q_L^i q_R^j \right) \) (9e)
   
   \( C_9 = \frac{g^2}{16\pi^2} \left( s_L^2 \sigma_{\mu\nu} b_R^2 \right) \left( \sum_q q_R^i q_L^j \right) \) (9f)
   
   \( C_{10} = \frac{g^2}{16\pi^2} \left( s_L^2 \sigma_{\mu\nu} b_R^2 \right) \left( \sum_q q_R^i q_L^j \right) \) (11)

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      \( C_5 = \frac{g^2}{16\pi^2} \left( s_L^2 b_R^2 \right) \left( \sum_q q_L^i q_R^j \right) \) (9b)
      
      \( C_6 = \frac{g^2}{16\pi^2} \left( s_L^2 b_R^2 \right) \left( \sum_q q_R^i q_L^j \right) \) (9c)
      
      \( C_7 = \frac{g^2}{16\pi^2} \left( s_L^2 \sigma_{\mu\nu} b_R^2 \right) \left( \sum_q q_R^i q_L^j \right) \) (9d)
      
      \( C_8 = \frac{g^2}{16\pi^2} \left( s_L^2 \sigma_{\mu\nu} b_R^2 \right) \left( \sum_q q_L^i q_R^j \right) \) (9e)
      
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      \( C_{10} = \frac{g^2}{16\pi^2} \left( s_L^2 \sigma_{\mu\nu} b_R^2 \right) \left( \sum_q q_R^i q_L^j \right) \) (11)

   2) SUSY
      
      There are five different sets of contributions:

      a) the SM contribution with exchange of \( W^- \) and up-quarks; b) the charged Higgs contribution with \( H^- \) and up-quarks; c) the chargino contribution with \( \tilde{\chi}^- \) and up-squarks \( \tilde{u} \); d) the gluino contribution with \( \tilde{g} \) and down-squarks \( \tilde{d} \); and finally e) the neutralino contribution with \( \tilde{\chi}^0 \) and down-squarks.

   As pointed out in ref. [11, d), e)](9,10) can be negligible in most of region of the parameter space. \( C_i \) i=9,10 can be found in refs. [11,12,10] and \( C_{Q_1} \) in refs. [13,14,10]. It has been shown that \( C_{Q_4} \) i=1,2 is proportional to \( \frac{m_3 m_3 \tan^3 \beta}{m_2^2} \) in some regions of the parameter space, which make the contributions from NHBs significant for \( l=\mu, \tau \).

   It is well known that \( b \to s\gamma \) puts a very stringent constraint on the parameter space of various models. SUSY contributions can interfere either constructively or destructively [13], which is determined by the sign of \( \mu \). Therefore, one has to consider the constraint from \( B \to X_s \gamma \) in numerical calculations of observables for \( B \to X_s l^+ l^- \).

   The branching ratio (Br) of \( B \to X_s l^+ l^- \)

   \[ 2.0 \times 10^{-4} < B(B \to X_s \gamma) < 4.5 \times 10^{-4} \]  (13)

   translates into the constraint on \( C_{7eff} \) as

   \[ 0.249 \leq |C_{7eff,LLA}(\mu = 4.8 \text{ GeV})| \leq 0.374 \]  (14)

   in LLA and \( 0.28 \leq |C_{7eff}(m_B)| \leq 0.41 \) in NLO.

   The constraint leads to the correlation between \( C_{7eff} \) and \( C_{Q_1} \) in SUSY.

   The numerical results can be summarized as follows.

   A. In model II 2HDM for small \( \tan\beta \) the deviation of Br from SM is very small and the deviation for \( l=\tau \) is significant when \( \tan\beta \) is larger than 25 [14].

   B. In MSSM and mSUGRA

   For \( b \to s\mu^+ \mu^- \), there is no significant deviation from SM when \( \tan\beta \) is small (say, 2) and IMS is enhanced by a factor of 50% compared to SM in some region of the parameter space when \( \tan\beta \) is large (say, 30) [13]. For \( b \to s\mu^+ \mu^- \) and the large \( \tan\beta \) case, IMS can be enhanced by a factor of 100% and FBA is significantly different from SM in some region of the parameter space due to the contributions of NHBs [14].

   C. In string theory and M-theory inspired models
For $b \to s^+\tau^-$, IMS can be enhanced by a factor of 200% even 400% compared to SM in some large $\tan\beta$ regions of the parameter space in some models because of NHB contributions [13, 18].

3. EXCLUSIVE DECAYS $B \to (K, K^*)\ell^+\ell^-$

For exclusive semileptonic decays of $B$, to make theoretical predictions, additional knowledge of decay form factors is needed, which is related with the calculation of hadronic transition matrix elements. Hadronic transition matrix elements depend on the non-perturbative properties of QCD, and can only be reliably calculated by using a nonperturbative method. The form factors for $B$ decay into $K^{(*)}$ have been computed with different methods such as quark models [13], SVZ QCD sum rules [20], light cone sum rules (LCSRs) [21].

IMS, FBA and LP have also been computed in SM [22].

In the SM we obtain the following non-resonant branching ratios, denoted by $B_{nr}, (\ell = e, \mu)$ [23].

$$B_{nr}(B \to K\ell^+\ell^-) = 5.7 \times 10^{-7},$$
$$\Delta B_{nr} = (^{+27}_{-15}, ^{+1.7}_{-1}, ^{+1}_{-0.2}, ^{+2}_{-0.2})\%,$$
$$B_{nr}(B \to K\tau^+\tau^-) = 1.3 \times 10^{-7},$$
$$\Delta B_{nr} = (^{+22}_{-18}, ^{+0.4}_{-0.2}, ^{+3}_{-0.3}, ^{+1}_{-0.1})\%,$$

$$B_{nr}(B \to K^e^+e^-) = 2.3 \times 10^{-6},$$
$$\Delta B_{nr} = (^{+29}_{-17}, ^{+4}_{-1}, ^{+3}_{-0.2}, ^{+3}_{-0.3})\%,$$

$$B_{nr}(B \to K^\mu^+\mu^-) = 1.9 \times 10^{-6},$$
$$\Delta B_{nr} = (^{+26}_{-17}, ^{+0.4}_{-0.2}, ^{+0.7}_{-0.3}, ^{+1}_{-0.1})\%,$$

$$B_{nr}(B \to K^*\ell^+\ell^-) = 1.9 \times 10^{-7},$$
$$\Delta B_{nr} = (^{+4}_{-0.3}, ^{+13}_{-11}, ^{+0.6}_{-0.3}, ^{+0.3}_{-0.3})\%. \quad (15)$$

The first error in the $\Delta B_{nr}$ consists of hadronic uncertainties from the form factors. The other four errors given in the parentheses are due to the variations of $m_t$, $\mu$, $m_{bt,pc}$ and $\alpha_s(m_Z)$, in order of appearance. In addition, there is an error of $\pm 2.5\%$ from the lifetimes $\tau_B$. The largest parametric errors are from the uncertainties of the scale $\mu$ and the top quark mass, $m_t$.

IMS, FBA and LP have also been computed in extensions of SM [23, 25]. Here we only pay attention to FBA because some qualitative features to discriminate new physics from SM can be seen from the observable. For $B \to K^*\ell^+\ell^-$ decays FBA reads as follows [23]

$$\frac{dA_{FB}}{ds} = \frac{G_F^2 \alpha^2 m_B^5}{2\pi s} \left| V_{ts}^* V_{tb} \right| ^2 \tilde{s\bar{u}(s)^2} \left[ C_{10} \left( Re(C_{9}^{\text{eff}})VA_1 + \frac{\hat{m}_b}{\hat{s}} C_{7}^{\text{eff}} VT_2 (1 - \hat{m}_K^*) + A_1 T_1 (1 + \hat{m}_K^*) \right) \right], \quad (16)$$

where $A_i, T_i$ and $V$ are relevant form factors. The position of the zero $\hat{s}_0$ of FBA is given by

$$Re(C_{9}^{\text{eff}}(\hat{s}_0)) = \frac{\hat{m}_b}{\hat{s}_0} C_{7}^{\text{eff}} \frac{T_2(\hat{s}_0)}{A_1(\hat{s}_0)} (1 - \hat{m}_K^*) + \frac{T_1(\hat{s}_0)}{V(\hat{s}_0)} (1 + \hat{m}_K^*), \quad (17)$$

which depends on the value of $\hat{m}_b$, the ratio of the effective coefficients $C_{7}^{\text{eff}}/Re(C_{9}^{\text{eff}}(\hat{s}_0))$, and the ratio of the form factors shown above. In the Large Energy Effective Theory [20] one has a particularly simple form for the equation determining $\hat{s}_0$, namely [23]

$$Re(C_{9}^{\text{eff}}(\hat{s}_0)) = -\frac{\hat{m}_b}{\hat{s}_0} C_{7}^{\text{eff}} \frac{1 - \hat{s}_0}{1 + \hat{m}_K^* - \hat{s}_0}, \quad (18)$$

$$s_0 = 2.88^{+0.44}_{-0.38}\text{GeV}^2 \quad (19)$$

in the SM.

$$\frac{dA_{FB}}{ds}(B \to K^*\ell^+\ell^-)$$

is proportional to $C_{10}$ and has a characteristic zero under the condition

$$\text{sign}(C_{7}^{\text{eff}}Re(C_{9}^{\text{eff}})) = -1, \quad (20)$$

The condition in Eq. (20) provides a discrimination between the SM and models having new physics.

For $B \to K \ell^+\ell^-$ in SUSY is given by [24]

$$\frac{dA_{FB}^K}{ds} = -2\hat{m}_t \tilde{u}(s) Re(S_1 A^{"}) D^{-K}, \quad (21)$$

where

$$D^K = (|A|^2 + |C|^2)(\lambda - \frac{\tilde{u}(s)^2}{3})$$

$$+ |S_1|^2 (s - 4\hat{m}_t^2) + |C|^2 4\hat{m}_t^2 (2 + 2\hat{m}_K^* - s) + Re(C^* D^{"}) 8\hat{m}_t (1 - \hat{m}_K^2) + |D|^2 4\hat{m}_t^2 \tilde{s}, \quad (22)$$
\[
A'(s) = C_9^{e\ell}\bar{s} f_+(\bar{s}) + \frac{2\bar{m}_b}{1 + \bar{m}_K} C_7^{e\ell} f_T(\bar{s}),
\]
\[
S_1(\bar{s}) = \frac{1 - \bar{m}_K^2}{(\bar{m}_b - \bar{m}_s)} CQ_1 f_0(\bar{s}),
\]
\[
\hat{u}(\bar{s}) = \sqrt{\lambda(1 - \frac{\bar{m}_t^2}{\bar{s}})}
\]
\[
\lambda = 1 + \bar{m}_K^2 + \bar{s}^2 - 2\bar{s} - 2\bar{m}_K^2(1 + \bar{s})
\]

Note that the variable \( \hat{u} \) corresponds to \( \theta \) through the relation \( \hat{u} = -\hat{u}(\bar{s}) \cos \theta \). It is evident from Eq. (21, 23) that FBA for \( B \to K \ell^+ \ell^- \) vanishes if there are no contributions of NHBs and the contributions of NHBs can be large enough to be observed only in SUSY and/or 2HDM with large \( \tan \beta \) for \( l=\mu, \tau \), a non-zero FBA for \( B \to K \ell^+ \ell^- \) (\( l=\mu, \tau \)) would signal the existence of new physics.

4. CP VIOLATION

In SUSY phases of soft breaking parameters provide new sources of CP violation. The cancellation mechanism recently found makes the phases can be relatively large and experimental electric dipole moment (EDM) bounds for electron and neutron satisfied\([27, 29]\).

The direct CP asymmetries in decay rate and backward-forward asymmetry for \( B \to X_s l^+ l^- \) and \( \bar{B} \to X_s l^+ l^- \) are defined as

\[
A_{CP}^1(\bar{s}) = \frac{d\Gamma / d\bar{s} - d\Gamma / d\bar{s}}{d\Gamma / d\bar{s} + d\Gamma / d\bar{s}} = \frac{D(\bar{s}) - \bar{D}(\bar{s})}{D(\bar{s}) + \bar{D}(\bar{s})},
\]
\[
A_{CP}^2(\bar{s}) = \frac{A(\bar{s}) - \bar{A}(\bar{s})}{A(\bar{s}) + \bar{A}(\bar{s})}
\]
\[
A(\bar{s}) = 3\sqrt{\frac{1 - \frac{4\bar{t}^2}{\bar{s}}}{\bar{D}(\bar{s})}}.
\]

In SUSY and the model II 2HDM\([28, 17]\)

\[
D(\bar{s}) = 4|C_7^{e\ell}|^2\left(1 + \frac{2\bar{t}^2}{\bar{s}}\right)\left(1 + \frac{4\bar{t}^2}{\bar{s}}\right) + |C_9^{e\ell}|^2\left(2\bar{s} + 1\right)\left(1 + \frac{2\bar{t}^2}{\bar{s}}\right) + |C_{10}|^2\left[1 + 2\bar{s} + (1 - 4\bar{s})\frac{2\bar{t}^2}{\bar{s}}\right]
\]
\[
+ 12Re(C_9^{e\ell} C_7^{e\ell}\bar{s}(1 + \frac{2\bar{t}^2}{\bar{s}})) + \frac{3}{2}|C_{10}|^2\left(1 - \frac{4\bar{t}^2}{\bar{s}}\right)\bar{s} + \frac{3}{2}|C_{10}|^2\bar{s} + 6Re(C_{10} C_{Q_2} t),
\]
\[
E(\bar{s}) = Re(C_9^{e\ell} C_7^{e\ell}\bar{s} + 2C_7^{e\ell} C_{10}\bar{s} + C_9^{e\ell} C_{Q_2} t + 2C_7^{e\ell} C_{Q_1} t),
\]
\[
P_N = \frac{3\pi}{4} \sqrt{\frac{1 - \frac{4\bar{t}^2}{\bar{s}}}{\bar{s}^2}} Im\left[2C_7^{e\ell} f_C \bar{s} t + 4C_{10} C_7^{e\ell} t + C_7^{e\ell} f_C C_{Q_1} + 2C_7^{e\ell} f_C C_{Q_1} + C_{Q_2} \right]/D(\bar{s})
\]

The formulas in SM can be obtained from the above formulas by taking \( C_{Q_i} = 0 \) and \( C_i \) the value in SM. The numerical results are: \( A_{CP}^1(\bar{s}) \) is about 0.1% in SM\([27]\) and mSUGRA\([28]\) and can reach 1% in SUGRA with non-universal gaugino masses and large \( \tan \beta \). \( A_{CP}^2(\bar{s}) \) is about 0.1% in mSUGRA and 1% in SUGRA with non-universal gaugino masses respectively in large \( \tan \beta \) case for \( l=e, \mu \). It can reach even 50% for \( l=\tau \) in SUGRA with non-universal gaugino masses when \( \tan \beta \) is large\([29]\). \( P_N \) is negligible for \( l=e \) in SM and SUSY due to smallness of electron mass. \( P_N \) is still negligible for \( l=\mu \) and about 1% for \( l=\tau \) in SM\([27]\). It can reach 0.5% for \( l=\mu \) and 5% for \( l=\tau \) respectively in mSUGRA\([28]\) and 6% for \( l=\mu \) and 15% for \( l=\tau \) respectively in SUGRA with non-universal gaugino masses when \( \tan \beta \) is large.

5. CONCLUSIONS

The following conclusions can be drawn from the above discussions.

For the inclusive decay \( B \to X_s l^+ l^- \), the Br in SM is smaller than experimental bound, which implies there is a room for new physics. The deviation of the Br from SM is not significant for small \( \tan \beta \) and can be significant in some large \( \tan \beta \) region of parameter space in SUSY. For \( l=\mu, \tau \), much significant deviations can be reached in some large \( \tan \beta \) regions of parameter space in SUSY due to the NHB contributions.
And the FBA and LP for \( l = \mu, \tau \) are also significantly different from SM in the regions in SUSY. For the exclusive decays, \( \text{Br}(B \to K^{(*)} l^+ l^-) \) in SM is also smaller than the data. The zero of FBA for \( B \to K^* \ell^+ \ell^- \) provides a discrimination between SM and new models beyond SM. FBA of \( B \to K \ell^+ \ell^- \) for \( l = \mu, \tau \) does not vanish in some large \( \tan \beta \) regions of parameter space in SUSY and can be observed for \( l = \tau \) in B factories if nature chooses large \( \tan \beta \) and low sparticle mass spectrum. Finally, CP violation in \( B \to X_s \ell^+ \ell^- \) in SUGRA with non universal gaugino masses and large \( \tan \beta \) can be as large as be observed in B factories in the future.

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