Future probes of the origin of CP violation

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Abstract. We review what has been learned about CP violation in the K system. It is natural to hypothesize that the observed CP-violating effects are caused by the Standard Model weak interaction. We describe the stringent future test of this hypothesis via experiments on the B system. Then, we see how new physics beyond the Standard Model could be revealed by this test.

1. What do we already know?

Before discussing the future probes of the origin of CP violation, let us briefly recall some of the things we already know.

All laboratory CP-violating effects seen so far occur in the neutral K system. We know that the neutral K mass eigenstates, $K_{\text{Short}}(K_S)$ and $K_{\text{Long}}(K_L)$, are not CP eigenstates, as they would be if the world were CP invariant. Rather, they are CP admixtures given by

$$
|K_S\rangle = |K_1\rangle + \epsilon |K_2\rangle
$$

$$
|K_L\rangle = |K_2\rangle + \epsilon |K_1\rangle.
$$

Here, $K_1$ and $K_2$ are, respectively, the CP-even and CP-odd eigenstates of CP, and $|\epsilon| = O(10^{-3})$. We know also that both $K_L$ and $K_S$ can decay to the CP-even final state $\pi^+\pi^-$. The ratio of their decay amplitudes, $\eta_{+-} \equiv \langle \pi^+\pi^- | T | K_L \rangle / \langle \pi^+\pi^- | T | K_S \rangle$, has the measured value

$$
\eta_{+-} = [(2.285 \pm 0.019) \times 10^{-3}] e^{i(43.56 \pm 0.56)\circ}.
$$

The CP-violating decay $K_L \to \pi^+\pi^-$ can, in principle, result from two effects. First, it can occur because, as described by Eqs. (1.1), $K_L$ has a small $K_1$ component, which can decay to $\pi^+\pi^-$ without violating CP. This type of CP violation, which results from CP-noninvariance of the $K^0 - \bar{K}^0$ mixing amplitudes which make $K_S$ and $K_L$ what they are, is referred to as “indirect CP violation.” In addition, it may be that even the $K_2$ component of $K_L$ can decay to $\pi^+\pi^-$. This type of CP violation, in which a decay amplitude itself violates CP, is called “direct CP violation.”

The CP-violating decay of $K_L$ to the CP-even final state $\pi^0\pi^0$ has also been seen, and it has been found that $\eta_{00} \equiv \langle \pi^0\pi^0 | T | K_L \rangle / \langle \pi^0\pi^0 | T | K_S \rangle$ has the value

$$
\eta_{00} = [(2.275 \pm 0.019) \times 10^{-3}] e^{i(43.5 \pm 1.0)\circ}.
$$

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If direct CP violation may be neglected compared with the indirect kind in \( K \to \pi \pi \) (that is, if we may take \( \langle \pi \pi | T | K_2 \rangle \approx 0 \)), then it follows from Eqs. (1.1) that
\[
\eta_{+-} = \eta_{00} = \epsilon. \tag{1.4}
\]
Comparing Eqs. (1.2) and (1.3), we see that, indeed, \( \eta_{+-} \) and \( \eta_{00} \) are at least very close to being equal. In addition, assuming CPT, one predicts that
\[
\arg(\epsilon) = (43.46 \pm 0.08)^\circ. \tag{1.5}
\]
This predicted phase of \( \epsilon \) is in excellent agreement with the measured phases of \( \eta_{+-} \) and \( \eta_{00} \). Thus, we have several pieces of evidence that Eq. (1.4) holds; that is, several indicators that direct CP violation \( \ll \) indirect CP violation in \( K \to \pi \pi \).

A further fact that we know is that the CP-violating asymmetry
\[
\delta_{\pi \ell \nu} \equiv \frac{\Gamma(K_L \to \pi^- \ell^- \nu) - \Gamma(K_L \to \pi^+ \ell^+ \bar{\nu})}{\Gamma(K_L \to \pi^- \ell^- \nu) + \Gamma(K_L \to \pi^+ \ell^+ \bar{\nu})} \tag{1.6}
\]
has the value [3]
\[
\delta_{\pi \ell \nu} = (3.27 \pm 0.12) \times 10^{-3}. \tag{1.7}
\]
Now, assuming the validity of the \( \Delta S = \Delta Q \) rule, \( K_L \to \pi^- \ell^- \nu \) comes only from the \( K^0 \) component of the \( K_L \), and \( K_L \to \pi^+ \ell^+ \bar{\nu} \) comes only from the \( \bar{K}^0 \) component. If we assume also that there is no significant direct CP violation in \( K \to \pi \ell \nu \), then
\[
|\langle \pi^- \ell^- \nu | T | K^0 \rangle| = |\langle \pi^+ \ell^+ \bar{\nu} | T | \bar{K}^0 \rangle|. \tag{1.8}
\]
It is then easily shown from Eq. (1.1) for \( K_L \) and from the familiar expressions for \( K_1 \) and \( K_2 \) in terms of \( K^0 \) and \( \bar{K}^0 \) that, for small \( \epsilon \),
\[
\delta_{\pi \ell \nu} = 2 \Re(\epsilon). \tag{1.9}
\]
Now, if direct CP violation is indeed negligible in \( K \to \pi \pi \), so that \( \epsilon \approx \eta_{+-} \) [cf. Eq. (1.4)], then Eq. (1.9) implies that
\[
\delta_{\pi \ell \nu} = 2 \Re(\eta_{+-}). \tag{1.10}
\]
From the experimental value (1.2) for \( \eta_{+-} \), we find that
\[
2 \Re(\eta_{+-}) = (3.31 \pm 0.04) \times 10^{-3}, \tag{1.11}
\]
in excellent agreement with the measured value (1.7) of \( \delta_{\pi \ell \nu} \). Thus, Eq. (1.10) does hold within errors, providing evidence that direct CP violation is indeed very small in \( K \to \pi \ell \nu \), and adding to the evidence that it is also very small in \( K \to \pi \pi \).

2. What is the origin of CP violation?

All laboratory CP-violating effects observed to date have been seen in kaon decays. Kaon decays are due to the weak interaction. Thus, the most obvious candidate for the source of CP violation is the weak interaction itself.

The weak interaction, as described by the Standard Model (SM), can produce CP violation only through complex phases in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix
\[
V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{2.1}
\]
Complex phases in \( V \) could not have any physical consequences (such as CP violation) if there were only two quark generations. Furthermore, in \( K \) decay and mixing, the leading SM processes (e.g., the process
s→u+π+d for K decay) do not involve the t or b quarks. Hence, speaking approximately, these processes “do not know” that the third quark generation exists. Thus, again speaking approximately, K decay and mixing cannot violate CP through these leading SM processes, but only through SM processes with smaller amplitudes. As a result, if complex phases in the SM quark mixing matrix V are the source of CP violation, we expect this violation to be small in K decays, as observed.

In the SM, we also expect that direct CP violation ≪ indirect CP violation, both in neutral K→πℓν and in neutral K→ππ, as observed. However, the relative smallness of direct CP violation in these decays does not point uniquely to the SM. For example, if CP violation arises, not from the SM weak interaction, but from a “superweak” interaction that affects K decay, then once again we expect that direct CP violation ≪ indirect CP violation in K→πℓν and K→ππ.

While the SM expectation for K→ππ is that Eq. (1.4) should hold approximately, it would take an accident for (|η−|² − |η0|²)/(|η−|² + |η0|²) to be much smaller than 10⁻⁴. Thus, vigorous efforts are being made at Fermilab, CERN, and Frascati to detect and measure a nonvanishing, if small, difference between η− and η0.

The SM picture of CP violation is compatible, not only with all existing information from the kaon system, but also with the bounds on the electric dipole moments of various elementary particles. To be sure, it appears that CP violation coming from CKM phases which produce their effects through the physics of the SM cannot account for the baryon asymmetry of the universe. Thus, this asymmetry may be pointing to physics beyond the SM. However, it is thought that this asymmetry developed when the universe was still at a temperature at or above MW. For CP violation at energies well below MW, CKM phases acting within the SM remain a very plausible explanation. The hypothesis that these phases, acting in this way, are indeed the origin of low-energy CP violation will be tested cleanly and in detail during the next ten to fifteen years. The test will be carried out mostly through experiments on the B system, but will also entail some important experiments on the K system. Physics beyond the SM could be revealed through failure of the SM of CP violation to pass the test posed by all these experiments.

3. Testing the SM of CP violation in B decays

3.1. What is there to measure?

In B decays, some of the anticipated CP-violating effects can cleanly probe the phases of various products of CKM elements, thereby testing whether these complex phases are indeed behind CP violation. There are only four independent phases of CKM products, which may be taken to be the quantities

\[
\alpha \equiv \arg \left( -\frac{V_{td}V_{tb}^*}{V_{ub}V_{ub}^*} \right), \quad (3.1)
\]

\[
\beta \equiv \arg \left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{td}^*} \right), \quad (3.2)
\]

\[
\chi \equiv \arg \left( -\frac{V_{cs}V_{cb}^*}{V_{ls}V_{ls}^*} \right), \quad (3.3)
\]

\[
\chi' \equiv \arg \left( -\frac{V_{us}V_{ub}^*}{V_{cs}V_{cd}^*} \right). \quad (3.4)
\]

The phases α and β may be pictured as two of the angles in the “db unitarity triangle”, which expresses pictorially the SM unitarity constraint that the d and b columns of the CKM matrix must be orthogonal:

\[
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad (3.5)
\]

This triangle is shown in Fig. 1. In a similar way, χ is an angle in the sb unitarity triangle, which expresses the orthogonality of the s and b CKM columns, and χ’ is an angle in the ds triangle, which expresses the orthogonality of the d and s columns. The angles α and β may both be quite large, but, given what we already know about the CKM elements, χ is at most a few percent of a radian, and χ’ at most a few milliradians.
The CP asymmetry in a given B decay mode will probe the phase of some phase-convention-independent product of CKM elements. One can show [8] that if $\varphi$ is the phase of such a product, then, mod $\pi$,

$$\varphi = n_\alpha \alpha + n_\beta \beta + n_\chi \chi + n_\chi' \chi', \quad (3.6)$$

where $n_\alpha$, $n_\beta$, $n_\chi$, and $n_\chi'$ are integers. For the $B$ decay mode studied in a typical CP experiment, each of these integers is 0, ±1, or ±2, so that the relation between $\varphi$ and the underlying angles $\alpha$, $\beta$, $\chi$, and $\chi'$ is almost trivial. Thus, the experiments that will study CP violation in the $B$ system may be thought of as, among other things, measurements of the angles $\alpha$, $\beta$, $\chi$, and $\chi'$. Interestingly, once these four angles are determined, the entire CKM matrix, including the magnitude and phase of each of its elements, follows from them. [8] Thus, in principle, CP violation in the $B$ system probes the entire content of the CKM matrix. However, it is the independent phases $\alpha$, $\beta$, $\chi$, and $\chi'$ of CKM products which are the probed quantities which are simply related to what will be observed.

As we shall see shortly, when the CP asymmetry in some $B$ decay mode probes a CKM phase $\varphi$ which is one of the small angles $\chi$ or $\chi'$, the CP asymmetry itself is small and, consequently, hard to measure. Indeed, when the phase probed is $\chi'$, the CP asymmetry is only $O(10^{-3})$, and experiments to study such a small asymmetry may never be practical in the $B$ system. However, experiments to measure the (hopefully) large angles $\alpha$, $\beta$, and $\gamma \equiv \pi - \alpha - \beta$ in the $db$ triangle are being very actively developed, and experiments to determine the $O(10^{-2}$ radians) angle $\chi$ are being contemplated as well.

The program to test the SM of CP violation via experiments on $B$ decays may be summarized as follows:

1. Measure the four independent phases $\alpha$, $\beta$, $\chi$, and $\chi'$ of CKM products. If the smallest of these, $\chi'$, is beyond reach, at least measure $\alpha$, $\beta$, and $\chi$. Focus first on $\alpha$ and $\beta$, since these phases may both be large.

2. To see whether the SM provides a consistent picture of CP-violating phenomena or leads to inconsistencies which point to physics beyond the SM, overconstrain the system as much as possible. To do so—

(a) Measure, if possible, CP asymmetries in different decay modes which, if the SM of CP violation is correct, all yield the same phase angle ($\beta$, for example). See whether these asymmetries actually yield the same numerical result.

(b) Measure independently the angles $\alpha$, $\beta$, and $\gamma$ in the $db$ triangle, and see whether these angles actually add up to $\pi$.

(c) Measure the lengths of the sides of the $db$ triangle (via experiments on non-CP-violating effects such as decay rates and neutral B mixing). See whether the interior angles implied by the measured lengths agree with those inferred directly from CP-violating asymmetries.
3.2. How will the phases of CKM products be cleanly measured?

The techniques through which CKM phase information can be extracted from B decays have been extensively discussed in the literature. [9] Here, we shall only recall some highlights.

With some notable exceptions, the B decays that can yield clean CKM phase information are of the neutral B mesons, \( B_d = (\bar{b}d) \) and \( B_s = (\bar{b}s) \). Each of these mesons mixes significantly with its antiparticle. In the SM, the \( B_q - B_{\bar{q}} \) mixing is due largely to the box diagram in Fig. 2. This diagram obviously imparts to the mixing amplitude \( A(B_q \rightarrow B_{\bar{q}}) \) the CKM phase \[ \text{arg}_{\text{CKM}} A(B_q \rightarrow B_{\bar{q}}) = 2 \text{arg} (V_{tq}V_{tb}）。 \] (3.7)

Owing to mixing, there are two ways in which a neutral B that is initially a pure \( B_q \) can decay to some final state \( f \): The \( B_q \) can decay directly to \( f \). Or, it can convert via \( B_q - B_{\bar{q}} \) mixing into a \( B_{\bar{q}} \), and then the \( B_{\bar{q}} \) decays to \( f \). The amplitude for the direct decay, \( A(B_q \rightarrow f) \), interferes with that for the mixing-induced decay, \( A(B_q \rightarrow B_{\bar{q}}) A(B_{\bar{q}} \rightarrow f) \). Suppose that in each of the amplitudes \( A(B_q \rightarrow f) \), \( A(B_q \rightarrow B_{\bar{q}}) \), and \( A(B_{\bar{q}} \rightarrow f) \), some one Feynman diagram, which of course is proportional to some product of CKM elements, dominates. Then the interference between the direct-decay and mixing-induced paths from the initial \( B_q \) to the final state \( f \) probes the single CKM phase \( \phi_{qf} \) which is the relative CKM phase of the two interfering paths. That is,

\[ \phi_{qf} = \text{arg}_{\text{CKM}} \left[ \frac{A(B_q \rightarrow f)}{A(B_q \rightarrow B_{\bar{q}}) A(B_{\bar{q}} \rightarrow f)} \right]. \] (3.8)

A particularly simple situation arises when \( f \) is a CP eigenstate. Suppose a B meson is at proper time \( \tau = 0 \) a pure \( B_q \) (pure \( B_{\bar{q}} \)). Let the rate for this B to decay to a CP eigenstate \( f \) at proper time \( \tau \) be denoted by \( \Gamma_{qf}(\tau) \) \([\Gamma_{qf}(\tau)]\). One finds that, if each of the amplitudes appearing in Eq. (3.8) is dominated by one Feynman diagram, then \( \Gamma_{qf}(\tau) \) and \( \Gamma_{qf}(\tau) \) are given by [9]

\[ \Gamma_{qf}(\tau) = e^{-\Gamma_q \tau} \left[ 1 + \frac{\eta_f \sin \phi_{qf} \sin(\Delta M_q \tau)}{2} \right]. \] (3.9)

Here, \( \Gamma_q \) is the width of the two mass eigenstates of the \( B_q - B_{\bar{q}} \) system (we neglect the expected \(~20\% width difference of the two \( B_q \) mass eigenstates). The quantity \( \Delta M_q \) is the positive mass difference between the two \( B_q \) mass eigenstates, and \( \eta_f \) is the CP parity of \( f \).

Since \( \Gamma_{qf}(\tau) \) and \( \Gamma_{qf}(\tau) \) are the rates for two CP-mirror-image processes, the asymmetry between these rates,

\[ \mathcal{A}_{qf}(\tau) = \frac{\Gamma_{qf}(\tau) - \Gamma_{qf}(\tau)}{\Gamma_{qf}(\tau) + \Gamma_{qf}(\tau)} = \eta_f \sin \phi_{qf} \sin(\Delta M_q \tau), \] (3.10)

is a violation of CP. Note that, assuming \( \eta_f \) and \( \Delta M_q \) to be known, a measurement of \( \mathcal{A}_{qf}(\tau) \) would yield a clean determination of \( \sin \phi_{qf} \).
When each of the amplitudes in Eq. (3.8) is dominated by one Feynman diagram, proportional to some product of CKM elements, then \( \varphi_{qf} \) is obviously the phase of some product of CKM elements. Thus, the CP asymmetry (3.10) cleanly probes the phase of a CKM product. Note from Eqs. (3.8)–(3.10) that, within the SM, it is only the CKM phase, and not the magnitude, of the mixing amplitude \( A(B_q \to B_q') \) to which CP-violating asymmetries are sensitive. More generally, if \( A(B_q \to B_q') \) should contain a contribution from beyond the SM, this contribution would affect CP-violating asymmetries only if it modified the \( \alpha \). CP-violating asymmetries are sensitive. More generally, if \( \varphi \) product of CKM elements, then

\[
A(B_q \to B_q') \gamma = \text{one of the small angles} \chi \text{ or } \chi', \ A_q \text{ is correspondingly small}.
\]

How does one identify the \( B \) decay modes for which \( \varphi_{qf} \) is a particular CKM phase angle of interest, such as \( \alpha \) or \( \beta \)? This question is most easily answered if we make Wolfenstein’s approximation \([12]\) to the CKM matrix. In this approximation, in a common phase convention, the only CKM elements which depart significantly from being real are \( V_{ub} \) and \( V_{td} \). Then, from Eqs. (3.1)–(3.4) and the constraint that \( \gamma = \pi - \alpha - \beta \),

\[
\alpha \equiv \pi + \arg (V_{td}) + \arg (V_{ub}), \tag{3.11}
\]

\[
\beta \equiv - \arg (V_{td}), \tag{3.12}
\]

\[
\gamma \equiv - \arg (V_{ub}), \tag{3.13}
\]

\[
\chi \equiv 0, \tag{3.14}
\]

\[
\chi' \equiv 0. \tag{3.15}
\]

Thus, from Eq. (3.7),

\[
\arg_{CKM} A(B_d \to \bar{B}_d) \cong -2 \beta, \tag{3.16}
\]

while

\[
\arg_{CKM} A(B_s \to \bar{B}_s) \cong 0. \tag{3.17}
\]

From these relations and Eq. (3.8) we see that if, for example, we would like \( \varphi_{qf} \) to be \( \sim \beta \), we may choose a \( B_d \) decay mode (where mixing involves \( \beta \)) in which the CKM elements appearing in the decay amplitudes \( A(B_q \to f) \) and \( A(B_q' \to f) \) are \( \sim \) real (so that there are no further CKM phases). An example, \( B_d \to D^+ D^- \), where \( \varphi_{qf} = 2 \beta \), is shown in Table 1. Similarly, if we wish \( \varphi_{qf} \) to be \( \sim \alpha \), we may choose a \( B_d \) decay mode in which the decay amplitudes involve \( V_{ub} \). An example, \( B_d \to \pi^+ \pi^- \), where \( \varphi_{qf} = -2 \alpha \), is given in Table 1. If we wish \( \varphi_{qf} \) to be \( \sim \gamma \), we may select a \( B_d \) decay mode (where mixing introduces no phase) in which the decay amplitudes involve \( V_{ub} \). An example \([13]\), \( B_s \to D_s^+ K^- \), where \( \varphi_{qf} = \gamma \), is shown in Table 1. In this example, the final state is not a CP eigenstate, and the decay rate is not described by Eq. (3.8). However, the CKM phase probed is still the relative CKM phase of the direct-decay and the mixing-induced paths to the final state, as described by Eq. (3.8).

To consider the small angle \( \chi \), we must go beyond Wolfenstein’s approximation. Table 1 lists a decay mode, \( B_s \to \psi \phi \), which probes \( \chi \). Indeed, from Eqs. (3.3), (3.8), and (3.7), and the final column of Table 1, we find that in \( B_s \to \psi \phi \), \( \varphi_{qf} = -2 \chi \).

### 3.3. New Wrinkles

We would like to mention several interesting recent developments concerning the test of the SM of CP violation in \( B \) decays.

First, data from the CLEO collaboration at Cornell suggest that in \( (\bar{B}_d \to \pi^+ \pi^-) \), the decay mode where information on the angle \( \alpha \) will almost certainly first be sought, the assumption that one diagram dominates \( A(B_d \to \pi^+ \pi^-) \), while other contributions to this decay amplitude may be safely neglected, is invalid. The relevant data are the branching ratio \([14]\)

\[
BR(B_d \to K^+ \pi^-) = (1.5^{+0.52}_{-0.42}) \times 10^{-5}, \tag{3.18}
\]

and the (90\%CL) bound \([14]\)

\[
BR(B_d \to \pi^+ \pi^-) < 1.5 \times 10^{-5}. \tag{3.19}
\]
Table 1. Illustrations of decay modes that probe $\beta$, $\alpha$, $\gamma$, and $\chi$. In the second column is shown the diagram which dominates $A(B_q\rightarrow f)$, and in the third the one which dominates $A(\overline{B}_q\rightarrow f)$. A wavy line in any of these diagrams denotes a $W$ boson. In the final column is given the CKM factor to which $A(B_q\rightarrow f)/A(\overline{B}_q\rightarrow f)$ is proportional.

| Decay mode | $A(B_q\rightarrow f)$ | $A(\overline{B}_q\rightarrow f)$ | Decay CKM factor |
|------------|-----------------------|-----------------------------------|-----------------|
| $B_d\rightarrow D^+D^-$ | ![Diagram](image1) | ![Diagram](image2) | $V_{cs}V_{cd}$/$V_{cb}V_{cd}$ |
| $B_d\rightarrow \pi^+\pi^-$ | ![Diagram](image3) | ![Diagram](image4) | $V_{ub}V_{ud}$/$V_{ub}V_{ud}^*$ |
| $B_s\rightarrow D_s^+K^-$ | ![Diagram](image5) | ![Diagram](image6) | $V_{us}V_{cs}$/$V_{ub}V_{us}^*$ |
| $B_s\rightarrow \psi\phi$ | ![Diagram](image7) | ![Diagram](image8) | $V_{cs}V_{cs}$/$V_{cb}V_{cs}^*$ |
Now, the decay amplitudes for \( B_d \to K^+\pi^- \) and \( B_d \to \pi^+\pi^- \) are expected to receive contributions from the diagrams in Fig. 3. For each decay mode, there is a tree diagram, labelled T in Fig. 3 and a “penguin” diagram, labelled P. In \( B_d \to \pi^+\pi^- \), the tree diagram is expected to dominate. The major difference between the tree diagram for \( B_d \to \pi^+\pi^- \), \( T_{\pi\pi} \), and the one for \( B_d \to K^+\pi^- \), \( T_{K\pi} \), is that \( V_{ud} \) in the former is replaced by \( V_{ts} \) in the latter. Thus,

\[
|T_{K\pi}| \approx \lambda |T_{\pi\pi}|, \quad (3.20)
\]

with \( \lambda \equiv V_{us}/V_{td} = 0.22 \). Similarly, the major difference between the penguin diagram for \( B_d \to \pi^+\pi^- \), \( P_{\pi\pi} \), and the one for \( B_d \to K^+\pi^- \), \( P_{K\pi} \), is that \( V_{td} \) in the former is replaced by \( V_{ts} \) in the latter. Thus, estimating that \( |V_{td}/V_{ts}| \approx \lambda \), we have

\[
|P_{K\pi}| \approx \frac{1}{\lambda} |P_{\pi\pi}|. \quad (3.21)
\]

The experimental results Eq. (3.18) and (3.19) suggest that \( BR(B_d \to \pi^+\pi^-) \leq BR(B_d \to K^+\pi^-) \). In view of relations (3.20) and (3.21), this in turn suggests that \( P_{\pi\pi} \) cannot be entirely negligible compared to \( T_{\pi\pi} \). Indeed, if we use Eqs. (3.20) and (3.21), but allow for various possible values of the relative phase of \( P_{\pi\pi} \) and \( T_{\pi\pi} \), and various possible values of that of \( P_{K\pi} \) and \( T_{K\pi} \), we find that

\[
\frac{|P_{\pi\pi}|}{T_{\pi\pi}} \approx \begin{cases} 0.1 - 0.4 & \text{if } BR(B_d \to \pi^+\pi^-) = BR(B_d \to K^+\pi^-) \\ 0.2 - 0.6 & \text{if } BR(B_d \to \pi^+\pi^-) = \frac{4}{3} BR(B_d \to K^+\pi^-) \\ 0.3 - 0.9 & \text{if } BR(B_d \to \pi^+\pi^-) = \frac{2}{3} BR(B_d \to K^+\pi^-) \end{cases} \quad (3.22)
\]

Clearly, one cannot safely assume that the penguin diagram may be neglected relative to the tree diagram in \( B_d \to \pi^+\pi^- \).

Fortunately, a technique was developed long ago to deal with a possibly non-negligible penguin contribution to \( B_d \to \pi^+\pi^- \). This technique exploits the fact that the penguin and tree diagrams for \( B \to \pi \pi \) have different isospin properties, and requires the experimental study of several isospin-related \( B \to \pi \pi \) decays.

A second recent development we would like to mention is the suggestion that experimental study of “cascade mixing” could yield the sign of \( \cos 2\beta \), and thereby eliminate the anticipated discrete ambiguities in the angles \( \alpha, \beta \), and \( \gamma \).

To test the SM of CP violation, one would like to determine the angles in the unitarity triangles, especially those in the \( db \) triangle of Fig. 1. However, as Eq. (3.10) illustrates, CP asymmetries determine only
trigonometric functions of these angles, leaving the angles themselves discretely ambiguous. As the time when the B-system CP experiments will be done approaches, means for eliminating these discrete ambiguities are being developed.

Most likely, the first CKM phase quantities to be determined will be sin 2β (from $\overline{B}_d \to \psi K_S$), sin 2α (from $\overline{B} \to \pi \pi$), and cos 2γ (perhaps from $B^+ \to D K^+$, as discussed shortly). If we assume that α, β, and γ are the three angles in a triangle, then a knowledge of sin 2α, sin 2β, and cos 2γ will fix these three angles completely, except for a two-fold ambiguity. That ambiguity would be resolved if one could determine either Sign (cos 2α) or Sign (cos 2β).

The sign of cos 2α might be found through analysis of the three-body decays $\overline{B}_d \to \pi^+ \pi^- \pi^0$. The sign of cos 2β could be found by studying the decay chain

$$\overline{B}_d \to \psi + K \to \psi + (\pi \ell \nu).$$

(3.23)

In this chain, neutral $B$ mixing before the primary decay is followed by neutral $K$ mixing after it. We refer to this as “cascade mixing”. This decay chain is sensitive to both sin 2β and cos 2β, even though $\overline{B}_d \to \psi + K_S$ depends only on sin 2β. To see why, let us consider Fig. 4, which shows the paths through which the decay chain (3.23) can occur. In Fig. 4, the mass eigenstates of the $B_d - \overline{B}_d$ system are labelled $B_H$ ($B_{\text{Heavy}}$) and $B_L$ ($B_{\text{Light}}$). As Fig. 4 indicates, a $B$ which starts out as a pure $\overline{B}_d$ or a pure $B_d$ has both a $B_H$ and a $B_L$ component. Either of these components can decay to either $\psi + K_L$ or $\psi + K_S$. Subsequently, either the $K_L$ or $K_S$ can decay to $\pi \ell \nu$, and the amplitudes for these decays are very comparable. Thus, there are four paths from the initial $\overline{B}_d$ to the final state, $\psi + (\pi \ell \nu)$. The amplitudes for all of these paths depend on the same CKM phase angle, β, but they depend on β in different ways. In the limit where CP is conserved (so that β and all other CKM phases vanish), all the intermediate states in Fig. 4, $B_H$, $B_L$, $\psi K_L$, and $\psi K_S$, are CP eigenstates. In particular, $CP(B_H) = CP(\psi K_S) = -1$, while $CP(B_L) = CP(\psi K_L) = +1$. Thus, the decays $B_H \to \psi K_L$ and $B_L \to \psi K_S$, represented by dashed lines in Fig. 4, connect states which in the CP-conserving limit are of opposite CP parity. Therefore, the amplitudes for these decays must vanish in this limit, and one finds that, in particular, they are proportional to sin β. By contrast, the decays $B_H \to \psi K_S$ and $B_L \to \psi K_L$, represented by solid lines in Fig. 4, connect states which in the CP-conserving limit are of the same CP parity. Thus, the amplitudes for these decays are expected to survive in this limit, and one finds that they are proportional to cos β. Now, from Fig. 4, we see that the decays $\overline{B}_d \to \psi K_S$ in $\overline{B}_d$ involve only two paths, one through $B_H \to \psi K_S$ and one through $B_L \to \psi K_S$. It is the interference between these two paths that leads to the CP asymmetry in $\overline{B}_d \to \psi K_S$. Since $A(B_H \to \psi K_S) \propto \cos \beta$, while $A(B_L \to \psi K_S) \propto \sin \beta$, the interference between them is proportional to $\cos \beta \sin \beta$, or to sin 2β. This is why $\overline{B}_d \to \psi K_S$ probes only sin 2β. In contrast, in the decay chain (3.23), there are the four paths shown in Fig. 4, and all of them interfere. Since $A(B_H \to \psi K_S)$ and $A(B_L \to \psi K_L)$ are both $\propto \cos \beta$, the interference between them is $\propto \cos^2 \beta$. Similarly, the interference between $A(B_H \to \psi K_L)$ and $A(B_L \to \psi K_S)$ is $\propto \sin^2 \beta$. Obviously, a suitable linear combination of $\cos^2 \beta$ and $\sin^2 \beta$ will yield cos 2β. This is why $\overline{B}_d \to \psi + K -> \psi + (\pi \ell \nu)$ can determine cos 2β.

The event rate for $\overline{B}_d \to \psi + K -> \psi + (\pi \ell \nu)$ is such that, hopefully, this mode could be used to extract $\text{Sign} (\cos 2\beta)$ at a hadron facility, although the extraction may not be feasible at an $e^+e^-$ $B$ factory.
Due to the smallness of the mixing amplitude, small effects from PBSM have a chance to be visible.
Table 2. The phase quantities actually measured by popular decay modes in the presence of PBSM.

| Process                  | Measures in the SM | Actually measures when PBSM is present |
|--------------------------|--------------------|----------------------------------------|
| $\overline{B}_d \to \pi^+\pi^-$ | $\sin 2\alpha$     | $\sin[2(\alpha + \theta_d)]$           |
| $\overline{B}_d \to \psi K_S$   | $\sin 2\beta$     | $\sin[2(\beta - \theta_d)]$           |
| $B^\pm \to DK^{\pm} \to (f_1,2)K^{\pm}$ | $\cos 2\gamma$     | $\cos 2\gamma$                        |
| $\overline{B}_s \to \psi \phi$   | $\sin 2\chi$      | $\sin[2(\chi + \theta_s)]$            |
| $\overline{B}_s \to D_s^\pm K^{\mp}$ | $\cos 2\gamma$     | $\cos[2(\gamma - 2\theta_s)]$         |

As already mentioned in Sec. 3.2, a contribution to the mixing amplitude $A(B_q \to \overline{B}_q)$ from PBSM can affect CP violation only if it changes the phase of $A(B_q \to \overline{B}_q)$. Now, there are combinations of measurements that could reveal that the weak (i.e., non-strong-interaction) phase of $A(B_q \to \overline{B}_q)$ does not have its SM value, given by Eq. (3.7). However, other combinations of measurements would devilishly hide this fact. Let us see why this is so.

Suppose that, while the $B$ decay amplitudes are unaffected by PBSM, the mixing amplitude $A(B_q \to \overline{B}_q)$ contains a contribution from PBSM which changes its weak phase to the SM value plus an offset $2\theta_q$:

$$\arg A(B_q \to \overline{B}_q) = 2 \arg (V_{tq}V_{d}\ast) + 2\theta_q; \quad q = d,s. \quad (4.2)$$

The weak phases probed by several popular $B$ decay modes are then modified as described in Table 2. Now, one important test of the SM of CP violation will be to see whether the angles $\alpha$, $\beta$, $\gamma$ extracted from $B$ decays satisfy the constraint

$$\alpha + \beta + \gamma = \pi, \quad (4.3)$$

as they must if they are actually the CKM phase angles in the $db$ unitarity triangle of Fig. 1. Suppose the values of $\alpha$, $\beta$, $\gamma$, and $\chi$ inferred from $B$ experiments are $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\chi}$, respectively. If $\alpha$, $\beta$, and $\gamma$ are extracted from the first three processes listed in Table 2 then as this Table shows, $\tilde{\alpha} = \alpha + \theta_d$, $\tilde{\beta} = \beta - \theta_d$, and $\tilde{\gamma} = \gamma$. Thus, while the measured angles $\tilde{\alpha}$ and $\tilde{\beta}$ are not the true CKM phase angles $\alpha$ and $\beta$, the measured angles nevertheless satisfy

$$\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma} = \pi, \quad (4.4)$$

thereby concealing the presence of PBSM.

One way to overcome this insensitivity to PBSM is to add a measurement of $\tilde{\chi} = \chi + \theta_s$ via the fourth process in Table 2. To an accuracy of a few per cent, the true angles $\alpha$, $\beta$, $\gamma$, and $\chi$ satisfy

$$\frac{\sin \alpha \sin \chi}{\sin \beta \sin \gamma} = \left| \frac{V_{ts}}{V_{td}} \right|^2. \quad (4.5)$$

The right-hand side of this relation is just the square of the Cabibbo angle, and is very accurately known. Now, for measured angles $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\chi}$ which are not the true angles $\alpha$, $\beta$, $\gamma$, and $\chi$, the relation (4.5) will in general fail, even if $\tilde{\alpha} + \tilde{\beta} = \alpha + \beta$ and $\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma} = \pi$. Thus, a nonvanishing $\theta_d$ and/or $\theta_s$ from PBSM would be revealed.

Another way to try to uncover evidence of PBSM in $B - \overline{B}$ mixing is to measure $\gamma$, not in the decay $B^\pm \to DK^{\pm}$, which does not involve $B - \overline{B}$ mixing, but in $\overline{B}_s \to D_s^\pm K^{\mp}$, which does. As indicated in Table 3, the latter decay would yield for $\gamma$ a measured value $\tilde{\gamma} = \gamma - 2\theta_s$. Combining this $\tilde{\gamma}$ with the $\tilde{\alpha}$ from $B_d \to \pi^+\pi^-$ and the $\tilde{\beta}$ from $\overline{B}_d \to \psi K_S$ would give

$$\tilde{\alpha} + \tilde{\beta} + \tilde{\gamma} = \pi - 2\theta_s. \quad (4.6)$$
A nonvanishing \( \theta_s \), if present, would thereby be revealed.

If \( \gamma \) is measured both in \( B^\pm \to D K^\pm \) and in \( B_s \to D_s^+ K^- \), and a nonvanishing \( \theta_s \) is present, different values will be obtained from the two measurements, uncovering the \( \theta_s \). This illustrates the virtue of making "redundant" measurements.\[26, 27\]

5. Summary

The most obvious candidate for the source of CP violation is the SM weak interaction. If this interaction is indeed the source, then CP violation comes from complex phase factors in the CKM matrix. The hypothesis that these phase factors are the origin of CP violation will be stringently tested in future experiments, mostly in the \( B \) system. Physics beyond the SM could be revealed by failures of this test. To seek such new physics in CP violation and related phenomena, we should overconstrain the system as much as possible, being careful not to restrict the measurements we make to those which could hide the new physics.

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The angle $\alpha$ is defined by Eq. (3.1), $\beta$ by Eq. (3.2) and $\gamma$ by $\gamma \equiv \arg \left(-V_{ud}V_{ub}^{*}/V_{cd}V_{cb}^{*}\right)$, or equivalently by $\gamma \equiv \pi - \alpha - \beta$. Thus, the true $\alpha$, $\beta$, and $\gamma$ always satisfy the constraint (4.3), even if unitarity is violated and the $d\bar{b}$ unitarity triangle does not close.

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If the tiny branching ratio for $K_L \to \pi^0\nu\bar{\nu}$ can be measured, we would gain additional clean CKM information which would help us significantly to test the SM of CP violation. See Buchalla G et al Ref. [7]

In the talk on which this paper is based, we included a discussion of the possible effect of supersymmetry on CP violation in the $B$ system. This effect can range from dramatic to invisible, depending on the particular version of supersymmetry which is involved. A nice treatment of this topic may be found in Grossman Y, Nir Y and Rattazzi R hep-ph/9701231.