An ordered ranking multi-attributive model for decision-making systems with attributes of control systems software

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Abstract. The authors present a compensation model for the general ranking of alternatives in their order of preference, considering the dependence of the attributes of software for spacecraft control systems. Among the discussed features is one that consists in the use of a set of target functions, which cannot be optimized simultaneously, due to their inherent incommensurability and target conflict.

1. Introduction

During the initial design stage of spacecraft control systems, it is necessary to decide on the structure of the system’s multi-version software by using the fuzzy programming method. This enables the software engineer to set the degree of ‘attribute preference’ and the ‘attainability percentage’ for the targets, when determining options for the formation of the multi-version software structure [1–3]. The result, as a rule, is a set of non-dominated solutions to the problem of forming multi-version software for control system. However, the selection of many non-dominated alternatives for solving multi-attribute problems is often unsatisfactory. The chief reason for this lies in the fact that with a sufficiently large initial set of options, the power of the set of non-dominated alternatives also increases, making it complicated to make a definite decision.

Thereby the selection of a multitude of non-dominated alternatives may only be considered only as the preliminary stage of the entire process. In order to make a decision, it is necessary to perform the next stage, which consists in reducing the resulting set and determining the best alternative.

A significant number of modules of the multi-version software of spacecraft control systems, their redundant versions, as well as limitations of real needs, such as the cost of the system’s development, implementation, and modification, oblige the designer to select the optimal composition of the multi-version software with consideration for a number of attributes [4–7].

Given the existence of a sufficiently large number of alternatives, their assessment is largely influenced by the traits of the individual behind the decision-making process; therefore, when selecting...
the best alternative from a number of proposed ones, it is necessary to consider a significant subjectivity factor in the assessments.

2. Compensatory multi-attributive general ranking model for optimal alternatives
In modern information technologies, decision-making is considered to be a set of decisions in conditions of certainty, which make it possible to choose unambiguous, consistent, correct decisions based on formalized control models for the objects and their environment. The formation of certain probabilistic decisions in conditions of uncertainty should also be considered as decision-making [8; 9]. Furthermore, the issue incorporates all aspects including the class of problems uncertainties, the final solution of which is performed outside the technology. In this case, the information is converted to a form that simplifies and facilitates decision-making by other methods.

It can be stated that, presently, various methods and approaches are used to support decision-making, which complement one other. Decision-making includes choosing a sequence of actions and its implementation. Decision support systems (DSS) are based on obtaining multivariate decisions using a variety of methods.

Decision support methodology includes a multitude of schemes and technologies, which can be implemented partially or completely using software and information systems.

The simplest decision-making model includes four main, cyclically repeating stages:

- Collection, analysis and transformation of data.
- Obtaining options for solutions (alternatives).
- Development of criteria for evaluating decisions.
- Selection of one of the options based on the selected criteria.

There are different conditions and situations for decision-making, and there are different levels of this process. At the top, is the conceptual level. It enables making generalized diagrams. Decision-making technology generally includes the following stages:

- Identification of a problem or task that needs to be addressed.
- Formation of the problem at the verbal level.
- Search for necessary information.
- Formalization of assignments.
- Data analysis and processing.
- Formation of sets of alternatives.
- Prediction.
- Assessment of decision-making results.

At the first stage, there exists a problem or task that requires a solution. As a rule, it is formed at the verbal (non-formalized) level of communication. Thereafter, it is necessary to carry out a search and collect data on the aspect of the given problem or task. Simultaneously, not only the data necessary for the solution, but also for the methods for solving such problems is gathered. Later, the problem is formalized or formed at the formal level [10].

At the information analysis and processing stage, complex processing is performed using an array of methods, including computer methods and expert analysis, direct calculation, heuristic methods, and optimization methods.

A possible change in conditions is provided by obtaining sets of solutions. Mutually exclusive solutions are called alternatives. Therefore, for completing and considering the changing conditions as a result of the analysis, the formation of alternatives is carried out, which is presented as an important stage in the decision-making scheme. The presentation of the generalized data is carried out in a convenient form.
The next stage (it is also one of the most important) is the stage of obtaining forecast estimates. At this stage, using the available solutions, data on the conditions and methods of obtaining a forecast and the forecasted estimates are obtained. This explains the choice of the forecasting method and the forecast verification method.

After assembling a data set, which consists of alternatives, data on the conditions dynamics, forecast estimates, assessment of forecast reliability, etc, complex processing is performed using an expert approach. This scheme works with different technologies and control levels.

The connection between the listed stages should be emphasized. They form a hierarchical sequence. If at one of the stages there is no possibility of its implementation, then the transition to the next is not carried out. Decision support means that a group of these methods is aimed not only at obtaining decisions, but also at preparing recommendations for the decision maker. Thereby decision support includes three groups of tasks:

1. Obtaining a set of solutions.
2. Preparing the criteria for evaluating the obtained solutions.
3. Choosing a solution from preexisting pools.

When forming decisions and evaluating them, there is a problem of correctly extracting the necessary data.

The use of modern technology and decision-making tools calls for the use of formalized data. Therefore, the effectiveness of the methods for obtaining solutions and decision support depends on the formalization of the problem. So, when decomposing a complex problem to the operational level and fully formalizing the conditions for solving the problem at this level, it is effective to use operations research methods to obtain solutions. If decomposition and complete formalization are impossible, methods of statistical evaluation, theory of fuzzy sets, etc. are used.

The choice of the best option for designing multi-version software from the entire set of possible implementations can be made using the compensation model of multi-attribute decision making, which allows performing a general ranking of alternatives based on the order of their preference for individual attributes and the interrelationship.

The developed model of decision-making on the composition of the multi-version software assumes the ranking of possible options for their formation in order of preference. Naturally, the first ranked option is the best. Only the order of preference is used as input allows avoiding scaling of attributes of a qualitative type [11, 12].

This model is based on the ranking of alternatives for individual attributes. On the basis of this 'private' ranking, the general order of preference of alternatives is determined, taking into account all the attributes and the relationship between them.

There is a linear assignment method that allows for a general ranking of alternatives [13]. According to it, the general rank is calculated as the sum of the ranks for individual attributes. In this case, the data on the relationship between the attributes is ignored:

$$ r_{i gen} = \sum_{j=1}^{n} r_{ij}, \ i = 1, m, $$

where m is the number of alternatives; n is number of attributes; r_{ij} is the rank of the i-th alternative by the j-th attribute; r is the number of ranks (r = m).

However, for the majority of decision-making tasks and, in particular, for choosing the option of forming the multi-version software, it is important to note this dependence [14]. Consequently, a multi-attribute compensation model was developed based on this method for the general ranking of alternatives in order of preference, considering the dependence of attributes.

The idea of compensation in this case is necessary for considering the dependence between attributes: a change in the value of one of them leads to a change in the values of any other attributes.

Let us define the matrix π as a square non-negative matrix m x m, whose elements π_{ik} represent the number (or frequency) of ranking of the alternative Ai by the r-th rank. Matrix π is formed on the basis of the ranking matrix of alternatives for individual attributes D:
\[ \pi_{ij} = \sum_{i=1}^{n} \mathcal{I}(D_{ij}^i) \cdot w_i; \ i = \overline{1,m}; \ j = \overline{1,r}, \]  

(2)

\[ \mathcal{I}(D_{ij}^i) = \begin{cases} 1, & \text{if } D_{ij}^i = i, \\ 0, & \text{if } D_{ij}^i \neq i, \end{cases} \]  

(3)

where \( \mathcal{I}(x) \) is the indicator function; \( w_i \) is the weighting factor of the \( i \)-th attribute.

With different weights, the elements of the matrix \( \pi \) represent the sum of the weights of the attributes of the corresponding rank. It is assumed that the weights are normalized.

Obviously, \( \pi_{ik} \) determines the contribution of the alternative \( A_i \) to the overall ranking. The larger the value of \( \pi_{ik} \), the fairer the assignment of the \( r \)-th rank to the alternative \( A_i \).

We define the permutation matrix \( Q \), as a square matrix \( m \times m \), whose elements \( Q_{ir} = 1 \), if the alternative \( A_i \) is assigned a common rank \( r \), and \( Q_{ir} = 0 \), otherwise. The objective function can be written as follows:

\[ \max Q_{ij} \sum_{i=1}^{m} \sum_{j=1}^{r} \pi_{ij} Q_{ij}, \]  

(4)

where

\[ \sum_{j=1}^{r} Q_{ij} = 1; \ i = \overline{1,m}, \]  

(5)

\[ \sum_{i=1}^{m} Q_{ij} = 1; \ j = \overline{1,r}. \]  

(6)

The above conditions mean that only one rank can be assigned to the alternative \( A_i \), and rank \( r \) can be assigned to only one alternative.

Let us denote the optimal permutation matrix representing the solution of the above linear programming problem as \( Q^* \). Then the optimal ordering can be achieved by multiplying the matrix \( A \) containing the numbers of alternatives by \( Q^* \).

Consider an example of a model application of the proposed multi-attribute compensation model of general ranking, when choosing one of the three alternatives. Let all attributes have increasing preference, that is, the greater the value of the attribute, the more preferable the alternative.

Let us assume that the ranks of these alternatives for three separate attributes correspond to those given in Table 1. Therefore, the first alternative has the first rank according to the first attribute, the first rank according to the second, and the second according to the third.

**Table 1.** Ranking alternatives by individual attributes.

| Attribute | 1 | 2 | 3 |
|-----------|---|---|---|
| rank      | 1 | A1| A1| A2|
|           | 2 | A2| A3| A1|
|           | 3 | A3| A2| A3|

The ranking by individual attributes can be represented as a matrix \( D \), the elements of which are the indices of the ranked alternatives:

\[ D = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 2 & 3 \end{bmatrix}. \]  

(7)

Based on this matrix, we can obtain matrix \( \pi \), whose elements represent the number of assignments to an alternative of each of the ranks. Therefore, the first alternative was assigned the first rank twice, the second rank – once, and the third – never, which corresponds to the value in the first row of the matrix:
\[
\pi = \begin{bmatrix}
2 & 1 & 0 \\
1 & 1 & 1 \\
0 & 1 & 2
\end{bmatrix}.
\] (8)

For the weighted coefficients \( w_1 = 0.2, w_2 = 0.4, w_3 = 0.4 \), the elements of the matrix \( \pi \) will change as follows:
\[
\pi = \begin{bmatrix}
0.2 + 0.4 & 0.4 & 0 \\
0.4 & 0.2 & 0.4 \\
0 & 0.4 & 0.2 + 0.4
\end{bmatrix} = \begin{bmatrix}
0.6 & 0.4 & 0 \\
0.4 & 0.2 & 0.4 \\
0 & 0.4 & 0.6
\end{bmatrix}.
\] (9)

The optimal permutation matrix \( Q^* \), which determines the overall rank of each of the alternatives, has the form:
\[
Q^* = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}.
\] (10)

It can be seen that the first alternative (the first column corresponds to it) has a common rank equal to one (the first row), the second alternative received the second rank and the third – the third. Based on the \( Q^* \) matrix, we get the following order of preferences:
\[
A_1 > A_2 > A_3.
\] (11)

The compensation model of multi-attribute decision making allows for a general ranking of alternatives in order of preference, based on ranking for individual attributes and the relationship between attributes. The advantage of this model lies in its unique practical application. Similar procedures are required from an expert when collecting data to set the order of preference for alternatives by attributes. The proposed approach allows avoiding the difficulties that arise when developing an attribute rating scale.

3. Conclusion
The proposed modified method of ordered preference can be used to select the optimal computing system from a number of available ones. The best option for the formation of multi-version system software is based on the implementation of multivariate decisions using various methods of multi-attribute decision making. The multi-attribute decision-making compensation model considers data on attributes interrelation; in this case, compensation involves taking into account the dependence between attributes, due to the fact that a change in the value of one of them results in the change in the values for any of the other attributes. The modified multi-attributive method of ordered preference through similarity with an ideal solution enables determining the best option for developing multi-version system software in a variety of alternatives.

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