A sea surface-based drag model for large eddy simulation of wind-wave interaction

Aditya K. Aiyer  
Department of Mechanical and Aerospace Engineering,  
Princeton University, Princeton, NJ, 08544  
aaiyer@princeton.edu

Luc Deike  
Department of Mechanical and Aerospace Engineering  
Princeton University, Princeton, NJ, 08544  
1deike@princeton.edu

Michael E. Mueller  
Department of Mechanical and Aerospace Engineering  
Princeton University, Princeton, NJ, 08544  
muellerm@princeton.edu

December 14, 2021

Abstract

Monin-Obukhov similarity theory (MOST) is a well tested approach for specifying the fluxes when the roughness surfaces are homogeneous. For flow over waves (inhomogeneous surfaces), phase-averaged roughness length scales are often prescribed through models based on the wave characteristics and the wind speed. However, such approaches lack generalizability over different wave ages and steepnesses due to the reliance on model coefficients tuned to specific datasets. In this paper, a sea surface-based hydrodynamic drag model applicable to moving surfaces is developed to model the pressure-based surface drag felt by the wind due to the waves. The model is based on the surface gradient approach of Anderson and Meneveau [2010] applicable to stationary obstacles and extended here to the wind-wave problem. The wave drag model proposed specifies the hydrodynamic force based on the incoming momentum flux, peak wave phase speed, and the surface frontal area. The drag coefficient associated with the wind-wave momentum exchange is determined based on the wave steepness. The wave drag model is used to simulate turbulent airflow above a monochromatic wave train with different wave ages and wave steepnesses. The mean velocity profiles and model form stresses are validated with available lab-scale experimental data and show good agreement across a wide range of wave steepnesses and wave ages. The drag force is correlated with the wave surface gradient, and out-of-phase with the wave height distribution by a factor $\pi/2$ for the sinusoidal wave-train considered. The computational savings are significant over phase-resolved models with the effect of the waves on the mean velocity profiles well represented. These results demonstrate that the current approach is sufficiently general over a wide parameter space compared to wave phase-averaged models with a minimal increase in computational cost.

1 Introduction

The physics of wind-wave interactions involve an interplay between atmospheric turbulence and ocean waves and play an important role in the context of numerous geophysical and engineering applications. Wind-wave interactions have been a continuous topic of research spanning numerous experimental [Shemdin and Hsu, 1967, Grare et al., 2013, Buckley and Veron, 2017, Buckley et al., 2020], theoretical [Miles, 1993, Janssen, 1991, Belcher, 1999, Kudryavtsev and Makin, 2004], and computational [Sullivan et al., 2000, 2008, 2014, Yang and Shen, 2010, Yang et al., 2013, Cao et al., 2020, Cao and Shen, 2021] studies devoted to quantifying the momentum exchange between the waves and the airflow above. Many physical processes rely on accurately quantifying the heat, mass, and momentum exchanges that take place at the air-sea interface and are important for numerous practical applications such as weather and climate modeling, safe steering of naval vessels, and offshore wind energy harvesting.
Measurements of air pressure in the laboratory and field have been carried out to quantify the momentum exchange at the air-sea interface [Shemdin and Hsu, 1967, Snyder et al., 1981, Banner, 1990; Donelan et al., 2006, Pierson and Garcia, 2008, Grare et al., 2013]. The results and findings from these experiments have been summarized by Grare et al. [2013] where they analyzed the growth rate and associated form drag due to energy transfer between the wind and the waves for different experimental data. They found that the normalized wind energy input values showed good collapse as a function of wave steepness rather than the wave age that is typically used in the literature for the datasets analyzed. Detailed experimental measurements of the airflow structure and wind stress over the wave surface within the airflow’s viscous sublayer were carried out by Buckley and Veron [2017] and Buckley et al. [2020]. They found that the form drag dominated the total air-water momentum flux for high wave slopes while the viscous drag was important for low wave slopes. The high resolution and quality of the wind and wave profile measurements provide a useful database for comparison with models and simulations.

The component of the total momentum flux $\tau$ imparted by the wind to the wave field can be written as a sum of the pressure and tangential viscous stresses. Considering a sloped surface $\eta(x,t)$ varying as a function of horizontal coordinate $x$ and time $t$ representing the wave, the horizontal component of the stress can be written as:

$$\tau = p_s - \frac{\eta_x}{\sqrt{1+\eta_x^2}} + \frac{\tau_{\text{visc}}}{\sqrt{1+\eta_x^2}},$$

(1)

where $p_s$ is the surface pressure, $\tau_{\text{visc}}$ is the local viscous surface tangential shear stress, and $\eta_x = \partial\eta/\partial x$ is the local interface slope. A more detailed discussion of the various stress components is provided in the Appendix of Grare et al. [2013]. This wave coherent stress acts as a source of energy to the wave field. The total energy input to the wave field can be related to the total momentum flux at the surface $S_{in} = \tau c$, where $c$ is the wave phase speed. Knowledge of the dependence of the energy input on wave characteristics is important in modeling the wind-sea momentum flux. Two common parameterizations for the energy input have been developed by Jeffreys [1925] and Miles [1957]. The former takes the form:

$$S_{in} = \frac{1}{2} \rho_a s_{A/2} (ak)^2 c^3 \left[ \left( \frac{U_{A/2}}{c} - 1 \right) \left( \frac{U_{A/2}}{c} - 1 \right) \right],$$

(2)

where $\rho_a$ is the air density, $s_{A/2}$ is the sheltering coefficient evaluated at a height $z = \lambda/2$ above the water surface with $\lambda$ being the wavelength, $k = 2\pi/\lambda$ is the wavenumber, $a$ is the amplitude of the wave, and $U_{A/2}$ is the mean streamwise velocity at $z = \lambda/2$. The latter parameterization by Miles [1957] is given by

$$S_{in} = \frac{\beta}{\omega E} \rho_a \left( \frac{u_s}{c} \right)^2,$$

(3)

where $E$ is the wave energy density, $\omega$ is the angular frequency, $\rho_w$ is the water density, $u_s$ is the surface friction velocity, and $\beta$ is the normalized wave growth coefficient. The angular frequency and the wavenumber are related by the dispersion relation

$$\omega = \sqrt{gk}$$

where $g$ is the gravitational acceleration.

Computational simulations provide an alternative approach to these theoretical parameterizations. Direct Numerical Simulation (DNS) provides the highest fidelity description of the problem. However, these simulations have been restricted to low Reynolds number flows of air-flow over a wavy surface due to high computational costs associated with resolving the viscous layer for high Reynolds number [Sullivan et al., 2000, Lin et al., 2008, Yang and Shen, 2010, Druzhinin et al., 2012]. Large Eddy Simulation (LES) has been shown to yield high-fidelity results for flow over complex terrain resolving the large- and intermediate-scale turbulent motions and only requiring modeling of the unresolved turbulence effects. The effect of waves in LES of the marine atmospheric boundary layer (MABL) has been incorporated either using wave phase-resolved simulations [Sullivan et al., 2008, 2014, Yang et al., 2014, Sullivan et al., 2018, Hao et al., 2018] or wave phase-averaged models [Charnock, 1955, Donelan, 1990, Toba et al., 1990, Drennan et al., 2005]. The former approach relies on constructing a terrain-following grid and having high enough resolution to resolve the near wave dynamics. The approach has been used successfully to simulate boundary layer flow with the bottom boundary specified using either a sinusoidal wave train [Sullivan et al., 2008] or the full 3D wave spectrum [Sullivan et al., 2014, Yang et al., 2014]. Yang et al. [2014] developed a dynamic coupling procedure to include the effect of air flow pressure field on the wave motion as a kinematic boundary condition. This approach is quite computationally expensive and becomes infeasible for high Reynolds number atmospheric flows due to the large separation of scales ranging from a few meters to a kilometer for the atmospheric boundary layer.

In order to simulate high Reynolds number flows, a wall model is needed to specify the boundary fluxes [Piomelli and Balaras, 2002]. The phase-averaged approach models the total momentum flux $\tau$ by specifying a surface roughness for the waves and calculating the surface fluxes based on Monin-Obukhov similarity theory (MOST) [Moeng, 1984]:

$$\tau = \rho_a \left[ \frac{\kappa}{\log(z_r/z_0) - \psi_m(z/L)} \right]^2 u_r^2,$$

(4)
where $\kappa = 0.4$ is the Von-Karman constant, $z_0$ is the surface roughness, $u_r$ is the reference velocity at height $z_r$, and $\psi_m$ is an empirical function of the Obukhov length $L$, which accounts for stratification effects. In the MOST framework, molecular viscous stresses are neglected as they are small compared to the surface stress for high Reynolds number flows. Unlike stationary rough wall applications, where MOST theory is applicable, the sea surface roughness elements are all in motion with waves of different wavelengths propagating with different speeds according to the wave dispersion relation. To account for this, numerous models have been proposed to parameterize an equivalent effective roughness scale $z_0$. A widely used model is one proposed by Charnock [1955] where the surface roughness $z_0$ is calculated as

$$z_0 = \frac{\alpha_{ch} u_r^2}{g},$$  \hspace{1cm} (5)

where $\alpha_{ch} = 0.01 - 0.03$ is an empirical constant [Garratt, 1977]. The wide range of values for $\alpha_{ch}$ have been attributed to factors such as the wind speed, wave age, wave steepness, and water depth. Donelan [1990] Smith et al. [1992], [Taylor and Yelland, 2001], [Fairall et al., 2003], [Edson et al., 2013], [Jiménez and Dudhia, 2018]. Alternative formulations have been proposed based on the peak wave age $c_p/u_s$ [Donelan, 1990] Toba et al. [1990] Smith et al. [1992];

$$z_0 = A(u_s/c_p)^B,$$  \hspace{1cm} (6)

where $A$ and $B$ are empirical parameters that vary across datasets Deskos et al. [2021], Taylor and Yelland [2001] proposed a parameterization based on the characteristic wave steepness $H_s/\lambda_p$:

$$z_0/H_s = 1200(H_s/\lambda_p)^{3.4},$$  \hspace{1cm} (7)

where $H_s$ is the characteristic wave-height, $\lambda_p$ is the peak wavelength, and the factors 1200 and exponent 3.4 are determined using data consisting of waves with larger fetch (“old waves”). Due to the modeling uncertainties present in phase-averaged formulations, purely relying on MOST formulations developed for stationary surfaces may not be sufficiently general. There is a need to develop a model that would be applicable to a range of wave steepnesses and wave ages yet still retaining the computational efficiency of these wave phase-averaged parameterizations.

In the atmospheric boundary layer (ABL), the effect of forest canopies and vegetation has been represented using drag-based models termed canopy stress models Shaw and Schumann [1992], Su et al. [1998], Arthur et al. [2019]. The hydrodynamic drag force due to obstacles in the flow is modeled as a function of the obstacle frontal area and the incoming momentum flux. Browne et al. [2001] applied the canopy stress model to flow over stationary sinusoidal ridges. Anderson and Meneveau [2010] developed a surface-gradient drag (SGD) model where the obstacle roughness was vertically unresolved but resolved horizontally. Arthur et al. [2019] implemented the canopy stress model framework in the Weather Research and Forecasting (WRF) model and applied it to both resolved and unresolved roughness. The canopy stress approach provides a simple framework in which effects of obstacles and topographies could be included without additional computational complexity and without requiring empirical correlations for some effective roughness.

In the current work, a canopy stress approach is adapted to the marine atmospheric boundary layer to develop a model applicable to the wind-wave problem. The ease of implementation of this model along with the lower associated computational cost makes it an attractive alternative to more expensive wave phase-resolved terrain following approaches. The rest of the paper is organized as follows. The governing equations used for the LES are described in §2. The canopy stress approach provides a simple framework in which effects of obstacles and topographies could be included without additional computational complexity and without requiring empirical correlations for some effective roughness.

In §3, conclusions are drawn. The governing equations used for the LES are described in §2. The canopy stress-based wave drag model formulation is described in §3. The model validation and results are presented in §4. Conclusions are drawn in §5.

## 2 Computational Framework

The LES calculations were performed using NGA, which is a structured, finite difference, low Mach number flow solver Desjardins et al. [2008], MacArt and Mueller [2016]. The wind velocity field is described using the filtered Navier-Stokes equations in the incompressible limit:

$$\nabla \cdot \bar{\mathbf{u}} = 0,$$  \hspace{1cm} (8)

$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla \bar{\mathbf{u}} = -\frac{1}{\rho} \nabla \bar{\rho} + \nabla \cdot \bar{\boldsymbol{\sigma}} + \bar{\mathbf{F}}_d.$$  \hspace{1cm} (9)

The equations are discretized on a Cartesian grid $(x, y, z)$, where $x$ and $y$ are the streamwise and spanwise coordinates respectively, and $z$ is the vertical coordinate. In Equations (8) and (9), $\bar{\mathbf{u}} = (\bar{u}, \bar{v}, \bar{w})$ is the velocity vector with the tilde denoting variables filtered on the LES grid; $\bar{F}_d$ is the drag force applied in the streamwise and spanwise directions, representing the effects of the waves; and $\bar{\sigma}_{ij} = 2\nu \bar{S}_{ij} + \tau_{ij}^{4}$ is the total deviatoric stress, where $\nu$ is the molecular viscosity, $\bar{S}_{ij}$ is the resolved strainrate tensor and $\tau_{ij}^{4}$ is the subfilter stress (SFS) tensor. The SFS tensor is modeled...
using a Lilly-Smagorinsky type subfilter viscosity model $\tau_{wall}^l = 2\nu_T \hat{S}_{ij}$, where the subfilter viscosity is computed using the Anisotropic Minimum Dissipation (AMD) model [Rozema et al., 2015] [Abkar and Moin 2017]. The subfilter viscosity is given by

$$\nu_T = -\frac{(\hat{\partial}_i \hat{u}_i)(\hat{\partial}_j \hat{u}_j)}{(\hat{\partial}_i \hat{u}_m)(\hat{\partial}_j \hat{u}_m)},$$

where $\hat{\partial}_i = \sqrt{C_i} \delta_i \delta_j$ ($i = 1, 2, 3$) is the scaled gradient operator and $C_i = 1/3$ is the modified Poincare constant [Abkar and Moin 2017].

In LES of high Reynolds number flows, the viscous boundary layer at the bottom boundary at the waves is unresolved. A wall-layer model is used to impose the correct surface stress for the bottom boundary. The total roughness scale $\zeta_0$ is generally written as a sum of the smooth surface roughness $\zeta_{0,s}$ and a wave-based roughness scale $\zeta_{0,w}$, such that the total roughness $\zeta_0 = \zeta_{0,s} + \zeta_{0,w}$ [Yang et al., 2013]. The wave-based roughness scale is generally parameterized using phase-averaged roughness models, as discussed in the Introduction. However, in this work, the wave drag model will replace the wave-based roughness scale $\zeta_{0,w}$ to provide better generalizability and accuracy. For low wind speeds over smooth surfaces, the roughness scale Reynolds number, defined based on the friction velocity $u_\ast$ and the roughness scale $\zeta_{0,s}$, approaches a constant value $Re_{\zeta_0,s} = 0.11$ [Fairall et al., 1996]. The microscale surface roughness length scale $\zeta_{0,s}$ can then be calculated as:

$$\zeta_{0,s} = 0.11 \frac{\nu_w}{u_\ast}$$

To apply the wall-stress locally, the stress is enforced using an enhanced subfilter viscosity at the wall along with a Dirichlet boundary condition for the velocity components. The subfilter viscosity is set such that:

$$\nu_T = \frac{\tau_{wall}}{P} \Delta z / 2 \frac{\zeta}{\tilde{U}_{avg}} - \nu,$$

where $\tilde{U}_{avg} = \sqrt{\tilde{u}_r^2(x,y,t) + \tilde{v}_r^2(x,y,t)}$ is the magnitude of the tangential wind velocity relative to the wave surface. The relative surface velocities $\tilde{u}_r, \tilde{v}_r$ are calculated using the velocity at the first grid point and the wave surface orbital velocity [Yang et al., 2013] [Sullivan et al., 2014] [Yang et al., 2014]:

$$\tilde{u}_r = \tilde{u}(x,y,z_1,t) - \tilde{u}_s(x,y,t), \quad \tilde{v}_r = \tilde{v}(x,y,z_1,t) - \tilde{v}_s(x,y,t),$$

where $\tilde{u}_s, \tilde{v}_s$ are the wave orbital velocities. The velocities are filtered at twice the grid scale to calculate the wallstress and are denoted by ($\tilde{\cdots}$). This additional filtering reduces velocity fluctuations significantly and improves the local applicability of the logarithmic law [Bou-Zeid et al., 2005]. The wall stress is given by [Moeng, 1984]:

$$\tau_{wall} = \left[ \frac{\kappa \tilde{U}_{avg}}{\log \frac{\zeta z_1}{\zeta_{0,s}}} \right]^2,$$

where $\Delta z / 2$ is the first off-wall grid point, and $\tilde{\eta}(x,y,t)$ is the sea surface elevation filtered at the LES resolution $\Delta$.

3 Wave drag model

The proposed drag model for the waves is based on a canopy stress-type approach used for simulating flows over vegetation or forest canopies. The model is based on applying a hydrodynamic drag force proportional to the incoming momentum flux onto a frontal surface area [Brown et al., 2001]. This offers the advantage of incorporating the wave characteristics (surface topology and wave speed) into the formulation without relying on empirical parameterizations. Typically, for LES where the wave-air interface is unresolved, the effect of the waves is solely modeled by the subfilter stress. In this section, a model is developed to represent the pressure drag due to the waves and applied locally as a boundary force in the LES domain.

The surface gradient drag model of [Anderson and Meneveau, 2010] is extended to account for moving boundaries in order to model flow over waves. Figure 1 shows a sketch of the model setup. The waves are vertically unresolved and lie below the first LES grid cell. Consider the control volume encompassing the wave surface and the first grid point, moving with a characteristic phase velocity $c$. The incoming momentum flux associated with the flow over a surface with an area $A$ is given by $\int \rho \bar{u} \cdot dA$, where $\bar{u}_r = \bar{u} - c$ is the relative velocity between the incoming flow and the phase velocity and $\bar{u}$ are the velocities evaluated at $z = \Delta z / 2$. It is important to note that the phase velocity has been used in the
wave steepness, wave frontal area. The wave surface height is not resolved in the LES grid, and its effects are completely represented by wavelengths propagating with different wave speeds is incorporated in the formulation, with the force contribution from each wavelength calculated separately. The flow frontal area normal to the incoming flow can be calculated based on the surface gradient as \( \hat{n}_u \cdot \nabla \eta \Delta_x \Delta_z \), where \( \hat{n}_u = \hat{u}_x / U^\lambda \) and \( U^\lambda = \sqrt{(\hat{u} - c_x)^2 + (\hat{v} - c_y)^2} \) is the tangential velocity relative to the wave propagation speed. The momentum flux can then be written as \( M_{\text{wave}} = \rho \hat{u} U^\lambda \hat{n}_u \cdot \nabla \eta \Delta_x \Delta_z \). In the wind-wave problem, only a fraction of the incoming momentum flux is imparted as drag and quantified in the model through a drag coefficient \( C_D \). The wave drag forcing term is included in the momentum equation by dividing the momentum flux by the local grid volume. Additionally, the forcing is only applied if \( \hat{n} \cdot \nabla \eta > 0 \), that is, only on the "front" of the wave with respect to the flow. The wave drag model is then expressed as:

\[
F_{d,i} = -\frac{M_{\text{wave}}}{\Delta_x \Delta_y \Delta_z} = -C_D \rho u_i U^\lambda R \left( \frac{\hat{n}_{u,k}}{\eta_k} \frac{\partial \eta}{\partial x_k} \right) \quad \text{for } i = x, y. \tag{15}
\]

\( R(x) = 0.5(x + |x|) \) is the ramp function that ensures that the drag force is only applied when the flow is incident on the wave frontal area. The wave surface height is not resolved in the LES grid, and its effects are completely represented by Equation (15). However, the waves must be well resolved in the horizontal direction to ensure adequate representation of the wave topology.

The drag coefficient can be determined by considering the energy input from the wind to the waves as described in Equations (2) and (3). The form derived by Jeffreys [1925] gives a quadratic dependence of the energy input for both the wave steepness, \( a_k \), and the inverse wave age \( c / U_{1/2} \) (or equivalently \( c / u_* \)). The energy input from the model for a sinusoidal wave train can be calculated by substituting \( \eta = a \cos(k(x - ct)) \) in Equation (15):

\[
S_{in,LES} = F_d \Delta_x \Delta_z = C_D \rho (a_k) c^3 \frac{\hat{u}_{\text{ave}}^2}{c^2} \left( \frac{\hat{u}_{\text{ave}}^2}{c^2} - 1 \right). \tag{16}
\]

Note that the energy input is evaluated by integrating \( R(\cos(k(x - ct))) \) over one wavelength and assuming that the velocity at the first grid point is approximately constant, and any resulting prefactors are absorbed into \( C_D \). The model energy input varies linearly with the wave steepness and quadratically with wave age. Comparing Equations (2) and (3), the drag coefficient can be set proportional to the wave steepness, \( C_D = C_{\text{LES}} \cdot a_k \) to recover a scaling similar to the Jeffreys [1925] parameterization for the wave energy input. \( C_{\text{LES}} \) should be an order unity coefficient assuming that the velocity at the first grid point.

Peirson and Garcia [2008] analyzed the dependence of the sheltering coefficient to be a constant. Peirson and Garcia [2008] analyzed the dependence of the sheltering coefficient to be a constant. Peirson and Garcia [2008] analyzed the dependence of the sheltering coefficient to be a constant. Peirson and Garcia [2008] analyzed the dependence of the sheltering coefficient to be a constant. Peirson and Garcia [2008] analyzed the dependence of the sheltering coefficient to be a constant.
prefactor $C_{Belcher} = (β_f + β_t)/(2 + β_f (ak)^2)$, where $β_f$ and $β_t$ are contributions due to the form and tangential stress respectively. For values of $β_f = 12$ and $β_t = 20$ [Peirson and Garcia, 2008], the prefactor varies from $C_{Belcher} = 16$ (equal to that derived by [Plant, 1982]) at $ak = 0$ to $C_{Belcher} = 11$ at $ak = 0.27$. Incorporating a similar behavior into $C_{LES}$, the drag coefficient $C_D$ can be written as:

$$C_D = \frac{P}{1 + Q(ak)^2} ak,$$

where $P = 1.2$ and $Q = 6$ to be consistent with Belcher [1999]. The drag force applied in LES at the first grid point can now be written as

$$F_{d,i} = -\frac{P}{1 + Q(ak)^2} (ak) \frac{ρ}{Δz} u_i U^2 R \left( \hat{n}_{i,k} \frac{∂\bar{η}}{∂x_k} \right) \quad i = x, y.$$

The force acts only in the horizontal directions, to remain consistent with the Monin-Obukhov similarity theory implementation (Equation (13)). This assumption neglects the effects of a surface-normal velocity that could become important for swell conditions. The wave drag model in Equation (18) combined with the Monin-Obukhov similarity theory represents the effects of waves with surface elevation $\bar{η}(x,y)$.

It is important to note that Equation (18) is written for moderate wave phase speeds or “young-waves”. In the case of swell, where the wave age is much higher, momentum is transported from the waves to the wind. The efficiency of momentum transfer from the waves to wind would modify the drag coefficient used in the model and is left for future studies.

### 4 LES of wind over a sinusoidal wave-train

#### 4.1 Simulation setup

The wave surface is specified by $\bar{η}(x,t) = a \cos(k(x-ct))$, and the orbital velocities are defined using the deep water wave solutions:

$$u_x = aω \cos(k(x-ct)); \quad v_x = 0; \quad w_x = aω \sin(k(x-ct)).$$

The computational domain size in the horizontal directions is $L_x = L_y = 5λ$. The horizontal grid is discretized using $N_x = N_y = 128$ points, ensuring that the wave is horizontally well-resolved on the LES grid. The vertical extent $H$ and discretization $N_z$ are chosen to ensure that the wave lies below the cell center of the first grid point, $a = 0.95Δz/2$. Therefore, the wave is unresolved in the vertical direction, and its effects are solely specified by the wave drag model described in the previous section. Details of the grid and wave properties for the different configurations are provided in Table 1. Periodic boundary conditions are applied in the $x$ and $y$ directions, and the flow is driven with a constant external pressure-gradient. The pressure-gradient results in a friction velocity (or surface stress) $u^2_f = (\partial P/\partial x)H$. A free-slip boundary condition is used for the top of the domain. The simulations are run for a total 200 friction times $T_{eddy} = H/u_x$ with averaging carried out over the last 50 friction times.

#### 4.2 Description of Datasets

The model is validated using lab-scale experiments from Buckley and Veron [2016, 2017, 2019] and Buckley et al. [2020]. The validation datasets consist of a wide parameter space with wave steepnesses ranging from $ak = 0.06$ to $ak = 0.27$ and wave ages from $c/u_s = 1.4$ to $c/u_s = 7$. 

| Case | $ak$ | $c/u_s$ | $u_s$ | $N_z$ | $H/λ$ |
|------|------|---------|------|------|-------|
| Buckley et al. [2020] | 0.06 | 6.57 | 0.073 | 48 | 1 |
| | 0.12 | 3.91 | 0.167 | 28 | 1 |
| | 0.27 | 0.18 | 0.538 | 16 | 1 |
| Buckley and Veron [2016, 2017] | 0.27 | 1.4 | 0.672 | 24 | 2 |
| Different Wave Steepnesses | 0.1 | 4.2 | 0.2 | 16 | 1 |
| Different Wave Ages | 0.1 | 7.84 | 0.111 | 16 | 1 |

Table 1: Wave train simulation parameters.
Buckley et al. [2020] carried out two-dimensional high resolution measurements of the airflow above wind-generated waves. The experiment consisted of a tank specifically designed for the study of air-sea interactions where wind-waves were generated by a computer-controlled recirculating wind tunnel. The apparatus and data processing techniques are described in detail in Buckley and Veron [2017]. A total of four cases are presented combining data from Buckley and Veron [2017] and Buckley et al. [2020]. The wave characteristics of the cases considered are provided in Table 1.

4.3 Results and Discussions

4.3.1 Mean Velocity Statistics and Shear Stress

In this section, the emphasis is on the mean velocity profiles obtained using the wave drag model for different wave steepnesses and wave ages. In Figure 2, the mean velocity profiles are shown from the LES with the wave drag model, hereafter referred to as LES-WDM for the different wave steepnesses and compared to the experimental data. Additionally, for each case, the profiles obtained by using the Charnock model and the steepness-dependent roughness model are calculated using the logarithmic law for the velocity profiles with the roughness length calculated using Equations (5) and (7), respectively. Modeling the form drag using the LES-WDM provides remarkable improvement over the standard phase-averaged approaches (Eq. 5 & 7) across the full range of wave steepnesses and wave ages considered. As the wave steepness increases and the wave age decreases, the mean wind velocity measured by the experiments are reduced due to increasing resistance from the waves. The LES-WDM accurately captures this effect with slower velocities predicted for higher wave steepness and low wave age. Figure 2a shows the comparison for ak = 0.06. For small wave steepnesses the deviation from the log-law is small, with κ = 0.4. The effective roughness length scale predicted by the phase-averaged models of Charnock [1955] and Taylor and Yelland [2001] is too small, resulting in faster velocities for ak = 0.06, 0.12 and 0.27, but the roughness parameterization in Equation (7) does show good agreement for the ak = 0.2 case. The roughness length scale predicted by the Charnock model only depends on the friction velocity and has no explicit dependence on the wave steepness or wave age. The steepness dependent model only accounts for the wave steepness effect, neglecting the wave propagation speed. For small wave ages (Figures 2c and 2d), the steepness dependent model performs better. The results from the Charnock model could be improved by using a higher value of the parameter \( \alpha_{ch} \); however, this constant cannot be determined a priori. Note that the LES-WDM formulation is developed for coarse grids and the focus is on predicting the velocity profiles above the peak wave amplitude. Therefore, for the higher wave steepnesses only a few points lie in the range of the experimental data as the near-field of the wave is unresolved.

To highlight and isolate the effects of wave age and steepness on the airflow we consider two distinct sets of simulations. First, the wave steepness is fixed at ak = 0.1, and the wave age varies as c/u* = 4.2, c/u* = 7.84, and c/u* = 11.5. The second set of simulations has fixed wave age c/u* = 4.2 and wave steepnesses ak = 0.1, ak = 0.15, and ak = 0.2. Figure 3a shows that the normalized mean velocity is faster for higher wave ages. Based on Equation 18 increasing the wave phase speed reduces the wave form drag applied by the model. The wave offers smaller resistance to the wind, and the blocking effect on the airflow is diminished. In Figure 3b, the effect of only changing the wave steepness is highlighted. Larger steepness of the wave results in higher blockage thereby applying more drag on the airflow. Additionally, the reduction in the overall mean velocity profile is not uniform between ak = 0.1, ak = 0.15 and ak = 0.2 as the effect of the steepness on the wave drag is not linear. The averaged total shear stress consists of the averaged resolved Reynolds stress \( \langle u'w' \rangle \), where \( u' = u - \bar{u} \) is the fluctuating part of the resolved LES velocity obtained by subtracting the time averaged part of the velocity, and the mean subfilter shear stress \( \tau_{xz} \). The normalized resolved and subfilter shear stresses from the four validation cases are shown in Figure 4. For each case, the normalized resolved stress shown in Figure 4a increases linearly from the top boundary towards the surface and is the dominant contribution to the total stress. The mean normalized subfilter stress depicted in Figure 4b is negligible far from the surface and starts to increase rapidly below z/H = 0.1. The total normalized stress shown in Figure 4c is similar to a pressure gradient driven turbulent boundary layer, but the total normalized shear stress does not reach -1 at the surface. The additional stress at the wave surface is provided by the wave form drag model. At the surface, the total stress is the sum of the form drag imposed by the model and the subfilter stress. Figure 5 shows the partition of the total stress as a function of time for the different cases. The contribution of the normalized form stress and the subfilter stress (from MOST) sums up to the imposed non-dimensional pressure gradient for each case considered.

4.3.2 Model Form Drag

To understand the effects of the wave drag model in the LES, the spanwise-averaged velocity and the wave drag force applied at the first grid point are analyzed in Figure 6 for ak = 0.12. The form drag for phase-resolved simulations is obtained from the resolved pressure \( \partial \eta / \partial x \). In the current LES, as the waves are unresolved in the vertical direction,
the model form stress depends on the incoming momentum flux $M_{wave}$, and the drag coefficient for the wind-wave transfer, and is based on Equation (18):

$$ D_p = M_{wave}/(\Delta_x \Delta_y) = \frac{P}{1 + Q(ak)^2} \left( \frac{\rho \bar{u} U^2}{\Delta_z} \right), $$

(20)

The top panel depicts the spanwise averaged model form drag for two time instants. The drag force is correlated to the surface gradient of the wave height distribution and is out of phase with the wave surface (shown in the bottom panel) by a factor of $\pi/2$. The magnitude of the peaks of the drag force show temporal dependence due to the variability of the turbulent velocity at the first grid point (shown in the bottom panel). Additionally shown in the top panel is the time and space averaged value of the normalized form drag. There is considerable variation of the drag applied along the wave surface with respect to the mean. This effect is also observed in experiments and phase-resolved simulations [Hara and Sullivan, 2015; Sullivan et al, 2018; Husain et al., 2019; Buckley et al., 2020; Yousefi et al., 2020]. The bottom panel of Figure 6 depicts the spanwise average of the streamwise velocity at the wave surface (first grid point) for two distinct time steps. The wave height distribution is also depicted for reference. The velocity accelerates in the upstream side of the surface peak where correspondingly the force applied by the model (top panel of Figure 6) is zero and then decelerates as the flow encounters the next wave surface corresponding to a maximum in the applied force. Phase-averaged models purely apply the averaged drag and would not capture this variation in the velocity and drag.
The mean velocity is faster for higher wave ages and lower wave steepnesses. The transfer of momentum is from the waves to the wind a scenario common during conditions of swell, where fast moving waves (with $c/u_\ast > 25$) generated from distant systems transfer energy to the slower wind. Future studies would work towards developing a model for fast moving waves.

Figure 7a highlights the average normalized form drag defined in Equation 20 as a function of wave steepness. The LES-WDM shows good agreement with the experimental data from Banner [1990], Banner and Peirson [1998], and Peirson and Garcia [2008] and accurately predicts the change of form drag for different steepness values considered here. The form drag as a function of wave-steepness shows similar qualitative trend. The experimental scatter is in part due to the uncertainties in the measurements. Note that the form stress calculated here is the stress due to the wave drag model and different wave steepnesses $ak = 0.1, ak = 0.15$, and $ak = 0.2$ and wave age fixed at $c/u_\ast = 4.2$. The resolved stress peaks just above the surface for each case. The magnitude of the normalized subfilter surface stress is less than the imposed stress, $| \widetilde{\tau_{xz}} |_{wall} | < 1$ with the remaining contribution from the wave drag model. Note $H = \lambda$ except for $ak = 0.27$, where $H = 2\lambda$.

Figure 7b is shown in Figure 7b. The simulations are run at a fixed steepness, and the grid is set such that $\Delta z = 0.06, c/u_\ast = 6.57; a_k = 0.12, c/u_\ast = 3.91; a_k = 0.2, c/u_\ast = 1.8; a_k = 0.27, c/u_\ast = 1.4$. The resolved stress peaks just above the surface for each case. The magnitude of the normalized subfilter surface stress is less than the imposed stress, $| \widetilde{\tau_{xz}} |_{wall} | < 1$ with the remaining contribution from the wave drag model. Note $H = \lambda$ except for $ak = 0.27$, where $H = 2\lambda$.
Figure 5: Normalized form stress ($\Delta \tau_{F_{D,i}}$) and total stress ($\tau_i$) as a function of non-dimensional time for different wave steepnesses, and wave ages. The solid lines represent the form stress, and the corresponding dashed lines represent the total stress for $a_k = 0.2$, $c/\nu = 1.8$; $a_k = 0.12$, $c/\nu = 3.91$; and $a_k = 0.06$, $c/\nu = 6.57$. The contribution of the form stress increases as a function of wave steepness.

Figure 6: The solid lines depict the spanwise averaged normalized form drag (top panel) and normalized streamwise velocity (bottom panel) at the wave surface for $a_k = 0.12$ as a function of horizontal distance. The line colors correspond to two time instances. The corresponding scaled wave surface-gradient distribution is shown in the top panel using the dotted lines and the wave height distribution is shown in the bottom panel using the dashed lines. Additionally shown in the top panel is the time averaged normalized form drag. The velocity shows the characteristic acceleration on the leeward side of the wave with respect to the wave trough followed by deceleration. The wave drag force is correlated to the wave slope and has a phase difference of $\pi/2$ with respect to the wave height distribution.
Figure 7: Normalized form stress as a function of (a) wave steepness and (b) Wave age. In the left panel the symbols correspond to simulation results from the LES-WDM (current work), and LES from Husain et al. [2019], and experimental data from Banner [1990], Banner and Peirson [1998], Peirson and Garcia [2008], and Buckley et al. [2020]. In the left panel, apart from the LES-WDM validation cases, the form stress from the LES At $a_k = 0.1$ and $c/u_+ = 4.2, 7.84$ and 11.6 are shown. In the right panel, the LES-WDM at different wave ages $c/u_+ = 4.2, 7.84$ and 11.6 and $a_k = 0.1$ is compared to DNS data from Sullivan et al. [2000], Kihara et al. [2007], and Yang and Shen [2010]. The model shows good agreement with the existing experimental and numerical data.

4.3.3 Computational Cost Reduction

An estimate of the computational cost reduction can be made by considering the grid spacing used in phase-resolved LES with the current formulation. For instance the grid resolution used in the current LES-WDM approach is 10 times coarser in the streamwise and spanwise directions and more than a 100 times coarser than the grid spacing used in the wall normal direction of the phase-resolved LES of Cao and Shen [2021]. The coarse resolution used here allows the accurate prediction of the mean velocity profiles and wave form stresses with an $O(10^4)$ reduction in computational cost based on grid resolutions.

5 Summary and Conclusions

Characterizing the momentum transfer between wind and waves is critical in numerous atmospheric applications. Wave phase-resolving simulations through immersed boundary or terrain following approaches come with a high computational cost for simulating atmospheric flows. On the other extreme, wave phase-averaged approaches in the Monin-Obukhov similarity theory framework are not sufficiently generalizable to a broad range of conditions. In this work a wave-based drag model applicable to wall-modeled Large Eddy Simulations is proposed. The model quantifies the momentum transfer between the wind and the wave-field through a canopy stress approach. The surface-gradient based approach of Anderson and Meneveau [2010] is extended to moving boundaries, and the formulation adapted for the wind-wave problem. The drag coefficient is found to be proportional to the wave steepness to give the correct scaling for the form stress.

The model is applied and tested on various cases of turbulent boundary layer flow over idealized water waves of different wave steepnesses and wave ages. The predicted mean velocity profiles show good agreement with data from experiments. The form stress predicted by the model is in good agreement with existing experimental data and shows the correct trend with both wave age and wave steepness. For faster waves, the model is capable of predicting a negative form drag, where the momentum is transferred from the wave-field to the wind. For old waves, or swell conditions, different parameterizations for the drag-coefficient need to be developed and is left for future studies. The model showed generalizability over the parameter space studied when compared to phase-averaged roughness models and requires no tuning parameters or empirical constants. The inherent spatial and temporal variability associated with the the drag force imposed by the waves on the airflow is well represented.
The framework proposed here to incorporate wave effects can serve as a low-cost accurate LES framework to simulate airflow in the marine atmospheric boundary layer with $O(10^4)$ reduction in computational cost compared to phase-resolved approaches as it allows for much larger grid spacing and timesteps. The generalizability and ease of implementation of the wave drag model allows scalability to large domains including full-scale offshore wind farms.

Acknowledgements

The authors gratefully acknowledge financial support from the Princeton University Andlinger Center for Energy and the Environment and High Meadows Environmental Institute. The simulations presented in this article were performed on computational resources supported by the Princeton Institute for Computational Science and Engineering (PICSciE) and the Office of Information Technology’s High Performance Computing Center and Visualization Laboratory at Princeton University.

References

Mahdi Abkar and Parviz Moin. Large-eddy simulation of thermally stratified atmospheric boundary-layer flow using a minimum dissipation model. *Boundary-Layer Meteorology*, 165(3):405–419, 2017. doi: 10.1007/s10546-017-0288-4.

William Anderson and Charles Meneveau. A Large-Eddy Simulation Model for Boundary-Layer Flow Over Surfaces with Horizontally Resolved but Vertically Unresolved Roughness Elements. *Boundary-Layer Meteorology*, 137(3): 397–415, 2010. ISSN 00068314. doi: 10.1007/s10546-010-9537-5.

Robert S. Arthur, Jeffrey D. Mirocha, Katherine A. Lundquist, and Robert L. Street. Using a canopy model framework to improve large-eddy simulations of the neutral atmospheric boundary layer in the weather research and forecasting model. *Monthly Weather Review*, 147(1):31–52, 2019. ISSN 15200493. doi: 10.1175/MWR-D-18-0204.1.

M. L. Banner and W. L. Peirson. Tangential stress beneath wind-driven air–water interfaces. *Journal of Fluid Mechanics*, 364:115–145, 1998. doi: 10.1017/S0022112098001128.

Michael L. Banner. The influence of wave breaking on the surface pressure distribution in wind—wave interactions. *Journal of Fluid Mechanics*, 211:463–495, 1990. doi: 10.1017/S0022112090001653.

S.E. Belcher. Wave growth by non-separated sheltering. *European Journal of Mechanics - B/Fluids*, 18(3):447–462, 1999. ISSN 0997-7546. doi: 10.1016/S0997-7546(99)80041-7. Three-Dimensional Aspects of Air-Sea Interaction.

Elie Bou-Zeid, Charles Meneveau, and Marc Parlange. A scale-dependent Lagrangian dynamic model for large eddy simulation of complex turbulent flows. *Physics of Fluids*, 17(2):1–18, 2005. ISSN 10706631. doi: 10.1063/1.1839152.

A. R. Brown, J. M. Hobson, and N. Wood. Large-eddy simulation of neutral turbulent flow over rough sinusoidal ridges. *Boundary-Layer Meteorology*, 98(3):411–441, 2001. ISSN 00068314. doi: 10.1023/A:1018703209408.

M. P. Buckley and F. Veron. The turbulent airflow over wind generated surface waves. *European Journal of Mechanics - B/Fluids*, 73:132–143, 2019. ISSN 0997-7546. doi: https://doi.org/10.1016/j.euromechflu.2018.04.003. Breaking Waves.

M. P. Buckley, F. Veron, and K. Yousefi. Surface viscous stress over wind-driven waves with intermittent airflow separation. *Journal of Fluid Mechanics*, 905, 2020. ISSN 14697645. doi: 10.1017/jfm.2020.760.

Marc P. Buckley and Fabrice Veron. Structure of the airflow above surface waves. *Journal of Physical Oceanography*, 46(5):1377 – 1397, 2016. doi: 10.1175/JPO-D-15-0135.1.

Marc P. Buckley and Fabrice Veron. Airflow measurements at a wavy air–water interface using piv and lif. *Experiments in Fluids*, 58(11):161, 2017.

Tao Cao and Lian Shen. A numerical and theoretical study of wind over fast-propagating water waves. *Journal of Fluid Mechanics*, 919:A38, 2021. doi: 10.1017/jfm.2021.416.

Tao Cao, Bing Qing Deng, and Lian Shen. A simulation-based mechanistic study of turbulent wind blowing over opposing water waves. *Journal of Fluid Mechanics*, 2020. ISSN 14697645. doi: 10.1017/jfm.2020.591.

H. Charnock. Wind stress on a water surface. *Quarterly Journal of the Royal Meteorological Society*, 81(350):639–640, 1955. doi: https://doi.org/10.1002/qj.49708135027.

Olivier Desjardins, Guillaume Blanquart, Guillaume Balarac, and Heinz Pitsch. High order conservative finite difference scheme for variable density low mach number turbulent flows. *Journal of Computational Physics*, 227(15):7125–7159, 2008. ISSN 0021-9991. doi: https://doi.org/10.1016/j.jcp.2008.03.027.
Georgios Deskos, Joseph C. Y. Lee, Caroline Draxl, and Michael A. Sprague. Review of wind-wave coupling models for large-eddy simulation of the marine atmospheric boundary layer. *Journal of the Atmospheric Sciences*, pages 1–75, 2021. ISSN 0022-4928. doi: 10.1175/jas-d-21-0003.1.

M. A. Donelan. Air-sea interaction. *Ocean Engineering Science*, 9B:239–292, 1990.

Mark A. Donelan, Alexander V. Babanin, Ian R. Young, and Michael L. Banner. Wave-follower field measurements of the wind-input spectral function. Part II: Parameterization of the wind input. *Journal of Physical Oceanography*, 36(8):1672–1689, 2006. ISSN 00223670. doi: 10.1175/JPO2933.1.

William M. Drennan, Peter K. Taylor, and Margaret J. Yelland. Parameterizing the sea surface roughness. *Journal of Physical Oceanography*, 35(5):835–848, 2005. ISSN 00223670. doi: 10.1175/JPO2704.1.

O. A. Druzhinin, Y. I. Troitskaya, and S. S. Zilitinkevich. Direct numerical simulation of a turbulent wind over a wavy water surface. *Journal of Geophysical Research: Oceans*, 117(C11), 2012. doi: https://doi.org/10.1029/2011JC007789.

James B. Edson, Venkata Jampana, Robert A. Weller, Sebastien P. Bigorre, Albert J. Plueddemann, Christopher W. Fairall, Scott D. Miller, Larry Mahrt, Dean Vickers, and Hans Hersbach. On the exchange of momentum over the open ocean. *Journal of Physical Oceanography*, 43(8):1589–1610, 2013. ISSN 15200485. doi: 10.1175/JPO-D-12-0173.1.

C. W. Fairall, E. F. Bradley, D. P. Rogers, J. B. Edson, and G. S. Young. Bulk parameterization of air-sea fluxes for tropical ocean-global atmosphere coupled-ocean atmosphere response experiment. *Journal of Geophysical Research: Oceans*, 101(C2):3747–3764, 1996. doi: https://doi.org/10.1029/95JC03205.

C. W. Fairall, E. F. Bradley, J. E. Hare, A. A. Grachev, and J. B. Edson. Bulk Parameterization of Air Sea Fluxes: Updates and Verification for the COARE Algorithm. *Journal of Climate*, 16:571–591, February 2003.

J. R. Garratt. Review of drag coefficients over oceans and continents. *Monthly Weather Review*, 105(7):915 – 929, 1977. doi: 10.1175/1520-0493(1977)105<0915:RODCOO>2.0.CO;2.

Laurent Grare, William L. Peirson, Hubert Branger, James W. Walker, Jean Paul Giovanangeli, and Vladimir Makin. Growth and dissipation of wind-forced, deep-water waves. *Journal of Fluid Mechanics*, 722:5–50, 2013. ISSN 00221120. doi: 10.1017/jfm.2013.88.

Xuanting Hao, Tao Cao, Zixuan Yang, Tianyi Li, and Lian Shen. Simulation-based study of wind-wave interaction. *Procedia IUTAM*, 26:162–173, 2018.

Tetsu Hara and Peter P. Sullivan. Wave boundary layer turbulence over surface waves in a strongly forced condition. *Journal of Physical Oceanography*, 45(3):868–883, 2015. ISSN 15200485. doi: 10.1175/JPO-D-14-0116.1.

Nyla T. Husain, Tetsu Hara, Marc P. Buckley, Kianoosh Yousefi, Fabrice Veron, and Peter P. Sullivan. Boundary layer turbulence over surface waves in a strongly forced condition: LES and observation. *Journal of Physical Oceanography*, 49(8):1997–2015, 2019. ISSN 15200485. doi: 10.1175/JPO-D-19-0070.1.

P. A. E. M Janssen. Quasi-linear Theory of Wind-Wave Generation Applied to Wave Forecasting. *Journal of Physical Oceanography*, 21(11):1631–1642, 1991.

Harold Jeffreys. On the formation of water waves by wind. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 107(742):189–206, 1925. ISSN 09501207.

Pedro A. Jiménez and Jimy Dudhia. On the need to modify the sea surface roughness formulation over shallow waters. *Journal of Applied Meteorology and Climatology*, 57(5):1101–1110, 2018. ISSN 15588432. doi: 10.1175/JAMC-D-17-0137.1.

N. Kihara, H. Hanazaki, T. Mizuya, and H. Ueda. Relationship between airflow at the critical height and momentum transfer to the traveling waves. *Physics of Fluids*, 19(1):015102, 2007. doi: 10.1063/1.2409736. URL https://doi.org/10.1063/1.2409736.

V. N. Kudryavtsev and V. K. Makin. Impact of swell on the marine atmospheric boundary layer. *Journal of Physical Oceanography*, 34(4):934–949, 2004. ISSN 00223670. doi: 10.1175/1520-0485(2004)034<0934:IOSOTM>2.0.CO;2.

M. Y. Lin, C. H. Moeng, W. T. Tsai, P. P. Sullivan, and S. E. Belcher. Direct numerical simulation of wind-wave generation processes. *J. of Fluid Mech.*, 616:1–30, 2008. doi: 10.1017/S0022112008004060.

Jonathan F. MacArt and Michael E. Mueller. Semi-implicit iterative methods for low mach number turbulent reacting flows: Operator splitting versus approximate factorization. *Journal of Computational Physics*, 326:569–595, 2016. ISSN 0021-9991. doi: https://doi.org/10.1016/j.jcp.2016.09.016.

John Miles. Surface-Wave Generation Revisited. *Journal of Fluid Mechanics*, 256:427–441, 1993. ISSN 14697645. doi: 10.1017/S0022112093002836.
John W. Miles. On the generation of surface waves by shear flows. *Journal of Fluid Mechanics*, 3(2):185–204, 1957. doi: 10.1017/S0022112057000567.

C. Moeng. A Large-Eddy-Simulation Model for the Study of Planetary Boundary-Layer Turbulence. *Journal of Atmospheric Sciences*, 41(13):2052–2062, 1984.

William I. Peirson and Andrew W. Garcia. On the wind-induced growth of slow water waves of finite steepness. *Journal of Fluid Mechanics*, 608:243–274, 2008. ISSN 00221120. doi: 10.1017/S002211200800205X.

Ugo Piomelli and Elias Balaras. Wall-layer models for large-eddy simulations. *Annual Review of Fluid Mechanics*, 34(1):349–374, 2002. doi: 10.1146/annurev.fluid.34.082901.144919.

William J. Plant. A relationship between wind stress and wave slope. *Journal of Geophysical Research: Oceans*, 87(C3):1961–1967, 1982. doi: https://doi.org/10.1029/JC087iC03p01961.

Wybe Rozema, Hyun J. Bae, Parviz Moin, and Roel Verstappen. Minimum-dissipation models for large-eddy simulation. *Physics of Fluids*, 27(8):085107, 2015. doi: 10.1063/1.4928700.

Roger H. Shaw and Ulrich Schumann. Large-eddy simulation of turbulent flow above and within a forest. *Boundary-Layer Meteorology*, 61(1-2):47–64, 1992. ISSN 00068314. doi: 10.1007/BF02033994.

Peter P. Sullivan, James C. McWilliams, and Chin Hoh Moeng. Simulation of turbulent flow over idealized water waves. *Journal of Fluid Mechanics*, 102:47–85, 1981. doi: 10.1017/S002211209000486X.

Peter P. Sullivan, Michael L. Banner, Russel P. Morison, and William L. Peirson. Turbulent flow over steep and unsteady waves under strong wind forcing. *Journal of Physical Oceanography*, 48(1):3–27, 2018. ISSN 15200485. doi: 10.1175/JPO-D-17-0118.1.

Peter P. Sullivan, Michael L. Banner, Russel P. Morison, and William L. Peirson. Turbulent flow over steep and unsteady waves under strong wind forcing. *Journal of Physical Oceanography*, 48(1):3–27, 2018. ISSN 15200485. doi: 10.1175/JPO-D-17-0118.1.

Peter K. Taylor and Margaret J. Yelland. The dependence of sea surface roughness on the height and steepness of the waves. *Journal of Physical Oceanography*, 31(2):572 – 590, 2001. doi: 10.1175/1520-0485(2001)031<0572:TDOSSR>2.0.CO;2.

Y. Toba, N. Iida, H. Kawamura, N. Ebuchi, and I. Jones. Wave dependence of sea-surface wind stress. *Journal of Physical Oceanography*, 20:705–721, 1990.

Di Yang and Lian Shen. Direct-simulation-based study of turbulent flow over various waving boundaries. *Journal of Fluid Mechanics*, 650:131–180, 2010. ISSN 00221120. doi: 10.1017/S0022112009993557.

Di Yang, Charles Meneveau, and Lian Shen. Dynamic modelling of sea-surface roughness for large-eddy simulation of wind over ocean wavefield. *Journal of Fluid Mechanics*, 726:62–99, 2013. ISSN 00221120. doi: 10.1017/jfm.2013.215.

Di Yang, Charles Meneveau, and Lian Shen. Effect of downwind swells on offshore wind energy harvesting – A large-eddy simulation study. *Renewable Energy*, 70:11–23, 10 2014. ISSN 0960-1481. doi: 10.1016/J.RENENE.2014.03.069.

Kianoosh Yousefi, Fabrice Veron, and Marc P. Buckley. Momentum flux measurements in the airflow over wind-generated surface waves. *Journal of Fluid Mechanics*, 895:A15, 2020. doi: 10.1017/jfm.2020.276.