NEUTRINO Mass SPECTRUM AND MIXING
FROM NEUTRINO OSCILLATION DATA*

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ABSTRACT

Two schemes of mixing of four massive neutrinos with two close neutrino masses separated by a gap of $\Delta m^2 \sim 1 \text{ eV}^2$ can accommodate solar, atmospheric and LSND neutrino oscillation data. It is shown that long-baseline $\bar{\nu}_e \rightarrow \bar{\nu}_s$ and $\nu_\mu \rightarrow \nu_e$ transitions are strongly suppressed in these schemes. The scheme of mixing of three massive neutrinos with a mass hierarchy that can describe solar and atmospheric data is also discussed. It is shown that in this scheme the effective Majorana mass that characterizes the matrix element of neutrinoless double-beta decay is smaller than $\sim 10^{-2} \text{ eV}$.

1. Introduction

The strong evidence in favor of oscillations of atmospheric neutrinos obtained in the Super-Kamiokande experiment[1] made the problem of neutrino masses and mixing one of the central problems of the physics of elementary particles. The Super-Kamiokande evidence is the first important step in the investigation of the phenomenon of neutrino mixing proposed many years ago by B. Pontecorvo[2]. There is no doubt that an understanding of the physical origin of neutrino masses and mixing will require many new experiments.

The Super-Kamiokande data can be explained with $\nu_\mu \rightarrow \nu_\tau$ or $\nu_\mu \rightarrow \nu_s$ oscillations...
tions with
\[ 5 \times 10^{-4} \text{eV}^2 \lesssim \Delta m_{\text{atm}}^2 \lesssim 5 \times 10^{-3} \text{eV}^2 \] (1)
and a large mixing angle.

Indications in favor of neutrino mixing were obtained also in solar neutrino experiments. From the analysis of the existing data it follows that
\[ \Delta m_{\text{sun}}^2 \sim 10^{-5} \text{eV}^2 \text{ (MSW)} \quad \text{or} \quad \Delta m_{\text{sun}}^2 \sim 10^{-10} \text{eV}^2 \text{ (vac. osc.)} \] (2)

Finally, indications in favor of \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \) oscillations were obtained in the accelerator LSND experiment\(^4\). If all other data on the search for \( \nu_\mu \rightarrow \nu_e \) transitions in short-baseline (SBL) experiments are taken into account from the analysis of the data of this experiment it follows that
\[ 0.3 \text{eV}^2 \lesssim \Delta m_{\text{SBL}}^2 \lesssim 2.2 \text{eV}^2 . \] (3)

We will discuss here what conclusion about the neutrino mass spectrum and the elements of neutrino mixing matrix can be obtained from the results of all neutrino oscillation experiments. We will consider also some consequences for the future experiments that can be inferred from the model independent analysis of the existing data.

We will present in the beginning the general theoretical framework of neutrino mixing\(^5\).

2. Phenomenological theory of neutrino mixing

All existing data on the investigation of neutrino processes are perfectly described by the standard charged-current (CC) and neutral-current (NC) Lagrangians
\[ \mathcal{L}_I^{\text{CC}} = \frac{-g}{\sqrt{2}} \sum_{\ell = e, \mu, \tau} \bar{\nu}_\ell \gamma_\alpha \ell_L W^\alpha + \text{h.c.} , \]
\[ \mathcal{L}_I^{\text{NC}} = \frac{-g}{2 \cos \theta_W} \sum_{\ell = e, \mu, \tau} \bar{\nu}_\ell \gamma_\alpha \nu_\ell L Z^\alpha + \text{h.c.} . \]

The CC and NC interaction Lagrangians (4) and (5) conserve electron \( L_e \), muon \( L_\mu \) and tau \( L_\tau \) lepton numbers and CC interactions determine the notion of flavor neutrinos \( \nu_\ell \) and antineutrinos \( \bar{\nu}_\ell \) \( (\ell = e, \mu, \tau) \). There are no indications in favor of violation of lepton numbers in weak processes and from the existing experiments very strong bounds on the relative probabilities \( R \) of lepton number violating processes were obtained. For example, for \( \mu \rightarrow e \gamma, \mu \rightarrow 3e \) it was found:
\[ R_{\mu \rightarrow e \gamma} \leq 5 \times 10^{-11}, \quad R_{\mu \rightarrow 3e} \leq 10^{-12} . \] (6)

The neutrino mixing hypothesis\(^2\) is based on the assumption that neutrino masses are different from zero and the neutrino mass term does not conserve lepton numbers.
Only a Dirac mass term is allowed in the case of quarks. This is connected with the fact that quarks are charged particles. Massive neutrinos can be Dirac particles (if the total lepton number \( L = L_e + L_\mu + L_\tau \) is conserved) or Majorana particles (if neutrino mass term does not conserve any lepton number).

The Dirac neutrino mass term has the form

\[
\mathcal{L}^D = - \sum_{\alpha, \beta} \overline{\nu_{\alpha R}} M_{\alpha \beta}^D \nu_{\beta L} + \text{h.c.},
\]

where \( M^D \) is a complex \( 3 \times 3 \) non-diagonal matrix. It is obvious that the Dirac mass term conserves total lepton number. After the standard diagonalization, for the left-handed fields \( \nu_{\alpha L} \) we have

\[
\nu_{\alpha L} = \sum_{i=1}^3 U_{\alpha i} \nu_i L,
\]

where \( \nu_i \) is the field of the Dirac neutrino with mass \( m_i \) \( (i = 1, 2, 3) \) and \( U \) is the unitary mixing matrix. The Dirac mass term can be generated by the same standard Higgs mechanism with which the masses of quarks and leptons are generated.

The general Majorana mass term that does not conserve lepton numbers has the form

\[
\mathcal{L}^M = \mathcal{L}^M_L + \mathcal{L}^D + \mathcal{L}^M_R.
\]

with

\[
\mathcal{L}^M_L = - \frac{1}{2} \sum_{\alpha, \beta} (\nu_{\alpha L})^c M_{\alpha \beta}^L \nu_{\beta L} + \text{h.c.}.
\]

Here \( M^L \) is a complex \( 3 \times 3 \) symmetric matrix and \( (\nu_{\alpha L})^c \equiv C \nu_{\alpha L}^T \) (\( C \) is the charge conjugation matrix). The mass term \( \mathcal{L}^M_R \) can be obtained from Eq. (10) with the change \( L \rightarrow R \).

After the diagonalization of the mass term (10) we have

\[
\nu_{\alpha L} = \sum_{i=1}^n U_{\alpha i} \nu_i L, \quad (\nu_{\alpha R})^c = \sum_{i=1}^n U_{\alpha i} \nu_i L.
\]

where \( \nu_i = \nu_i^c \) is a Majorana field with mass \( m_i \).

The Majorana mass term (10) can be generated only in the framework of theories beyond the Standard Model. In this mass term three flavor left-handed fields \( \nu_{\alpha L} \) and three right-handed fields \( \nu_{\alpha R} \) enter. The number of massive Majorana particles is equal in this case to six. In general, if the number of right-handed fields that enter into the mass term is equal to \( n_R \), the number of massive Majorana fields is equal to \( n = 3 + n_R \).

Let us notice that in the case of a left-handed Majorana mass term

\[
\mathcal{L}^M = \mathcal{L}^M_L
\]
only flavor left-handed fields enter into Lagrangian. The number of Majorana neutrinos is equal in this case to three.

Two possible options are usually discussed in the Majorana case:

1. The *see-saw* option.

   If the lepton numbers are violated by the right-handed Majorana mass term at a mass scale much larger than the electroweak scale, the Majorana neutrino mass spectrum is composed by three light masses \( m_i \) \( (i = 1, 2, 3) \) and three very heavy masses \( M_i \) \( (i = 1, 2, 3) \) that characterize the scale of lepton number violation. The light neutrino masses are given in this case by the see-saw formula

   \[
   m_i \sim \frac{(m_F^i)^2}{M_i} \ll m_i^F \quad (i = 1, 2, 3),
   \]

   where \( m_F^i \) is the mass of the charged lepton or up-quark in the \( i \)th generation. The see-saw mechanism provides a plausible explanation for the smallness of neutrino masses with respect to the masses of all other fundamental fermions.

2. The *sterile neutrino* option.

   If all the Majorana masses in Eq.(9) are small, active neutrinos \( \nu_e, \nu_\mu, \nu_\tau \) can transfer into the sterile particles \( \nu_{as} \) that are quanta of the right-handed fields \( \nu_{aR} \). Notice that sterile neutrinos can appear in the framework of see-saw mechanism under some additional assumptions (“singular see-saw”\]).

   We will consider two possible scenarios:

   1. All three indications in favor of neutrino oscillations are confirmed.
   2. Only the solar and atmospheric neutrino indications in favor of neutrino mixing are confirmed.

3. Four massive neutrinos

   At least four massive neutrinos are needed\[^{[13]}\] in order to have three different scales of \( \Delta m^2 \). The three types of neutrino mass spectra that can accommodate the solar, atmospheric and LSND scales of \( \Delta m^2 \) are shown in Fig[1]. In all these mass spectra there are two groups of close masses separated by a gap of the order of 1 eV which gives \( \Delta m^2_{41} \equiv m_4^2 - m_1^2 \simeq \Delta m^2_{\text{LSND}} \sim 1 \text{ eV}^2 \).

   Only the largest mass-squared difference \( \Delta m^2_{41} \) is relevant for the oscillations in short-baseline (SBL) experiments and the SBL transition probabilities have the same
dependence on the parameter $\Delta m^2_{41} L/2p$ as the standard two-neutrino probabilities:

$$P_{\nu_\alpha \to \nu_\beta} = \frac{1}{2} A_{\alpha;\beta} \left( 1 - \cos \frac{\Delta m^2_{41} L}{2p} \right),$$  \hspace{1cm} (14)$$

$$P_{\nu_\alpha \to \nu_\alpha} = 1 - \frac{1}{2} B_{\alpha;\alpha} \left( 1 - \cos \frac{\Delta m^2_{41} L}{2p} \right).$$  \hspace{1cm} (15)$$

Here $L$ is the source-detector distance and $p$ is the neutrino momentum.

The oscillation amplitudes $A_{\alpha;\beta}$ and $B_{\alpha;\alpha}$ depend on the elements on the mixing matrix $U$ and on the form of the neutrino mass spectrum:

$$A_{\alpha;\beta} = 4 \left| \sum_i U_{\beta i} U^*_{\alpha i} \right|^2,$$  \hspace{1cm} (16)$$

$$B_{\alpha;\alpha} = 4 \left( \sum_i |U_{\alpha i}|^2 \right) \left( 1 - \sum_i |U_{\alpha i}|^2 \right),$$  \hspace{1cm} (17)$$

where the index $i$ runs over the indices of the first or (because of the unitarity of $U$) second group of neutrino masses.

The results of SBL reactor $\bar{\nu}_e$ and accelerator $\nu_\mu$ disappearance experiments in which no oscillations were found imply that $B_{\alpha;\alpha} \leq B^0_{\alpha;\alpha}$ for $\alpha = e, \mu$. The upper bounds $B^0_{\alpha;\alpha}$ for the amplitudes $B_{\alpha;\alpha}$ are given by the exclusion curves of SBL disappearance experiments and depend on the value of $\Delta m^2_{41}$. Using Eq. (17), these upper
bounds imply the following constraints for the quantities $\sum_i |U_{\alpha i}|^2$ ($\alpha = e, \mu$):

$$\sum_i |U_{\alpha i}|^2 \leq a^0_{\alpha} \quad \text{or} \quad \sum_i |U_{\alpha i}|^2 \geq 1 - a^0_{\alpha}, \quad (18)$$

where

$$a^0_{\alpha} = \frac{1}{2} \left(1 - \sqrt{1 - B^0_{\alpha\alpha}}\right). \quad (19)$$

The most stringent values of $a^0_e$ and $a^0_\mu$ are given by the results of the Bugey reactor experiment\cite{16} and the CDHS\cite{17} and CCFR\cite{18} accelerator experiments.

We have considered the range $10^{-1} \leq \Delta m^2_{41} \leq 10^3 \text{eV}^2$. In this range $a^0_e \lesssim 4 \times 10^{-2}$ and $a^0_\mu \lesssim 2 \times 10^{-1}$ for $\Delta m^2_{41} \gtrsim 0.3 \text{eV}^2$. Thus, from the results of disappearance experiments it follows that $\sum_i |U_{ei}|^2$ and $\sum_i |U_{\mu i}|^2$ can be either small or large (close to one).

From the four possibilities for the quantities $\sum_i |U_{ei}|^2$ and $\sum_i |U_{\mu i}|^2$ (small-small, small-large, large-small and large-large) for each neutrino mass spectrum in Fig.1 only one possibility is compatible with the results of solar and atmospheric neutrino experiments\cite{12,13}.

In the case of spectra I and II we have

$$|U_{ek}|^2 \leq a^0_e \quad \text{and} \quad |U_{\mu k}|^2 \leq a^0_\mu, \quad (20)$$

with $k = 4$ for the mass spectrum I and $k = 1$ for the mass spectrum II. In the case of spectrum IIIA we have

$$\sum_{i=1,2} |U_{ei}|^2 \leq a^0_e \quad \text{and} \quad \sum_{i=1,2} |U_{\mu i}|^2 \geq 1 - a^0_\mu, \quad (21)$$

whereas in the case of spectrum IIIB we have

$$\sum_{i=3,4} |U_{ei}|^2 \leq a^0_e \quad \text{and} \quad \sum_{i=3,4} |U_{\mu i}|^2 \geq 1 - a^0_\mu. \quad (22)$$

In the case of spectra I and II $\nu_\mu \rightarrow \nu_e$ transitions in SBL experiments are strongly suppressed. In fact the upper bound of $A_{e\mu}$ is given by

$$A_{e\mu} \leq 4 |U_{ek}|^2 |U_{\mu k}|^2 \leq 4 a^0_e a^0_\mu. \quad (23)$$

In Fig\,\ref{fig:fig1} the upper bound (23) is compared with the latest LSND-allowed region (90\% CL). Fig\,\ref{fig:fig1} shows that the spectra of type I and II (that include also the hierarchical spectrum) are disfavored by the result of the LSND experiment (they are compatible with the results of the LSND experiment only in the narrow region of $\Delta m^2_{41}$ around $0.2 - 0.3 \text{eV}^2$, where there is no information on $B_{\mu\mu}$). On the other hand, it is easy to show that spectra IIIA and IIIB are compatible with the results of the LSND experiment. Thus we come to the conclusion that from all possible spectra of four
massive neutrinos only spectra IIIA and IIIB are favored by the data of LSND and all other neutrino oscillation experiments.

We discuss now some consequences of the schemes with mass spectra IIIA and IIIB for future experiments. Let us consider in the framework of these two schemes the value of the effective mass \(m(\beta)\) measured in tritium \(\beta\)-decay experiments and the value of the effective Majorana mass \(|\langle m \rangle|\) measured in neutrinoless double-\(\beta\) decay experiments. In the schemes IIIA and IIIB we have respectively

\[
m(\beta) \simeq m_4, \quad |\langle m \rangle| \leq m_4, \tag{24}
\]

and

\[
m(\beta) \leq a_0^0 m_4 \ll m_4, \quad |\langle m \rangle| \leq a_0^0 m_4 \ll m_4. \tag{25}
\]

Therefore, if the scheme IIIA is realized in nature, there is a possibility to see the effects of the relatively large neutrino mass \(m_4 \simeq \sqrt{\Delta m_{41}^2}\) in future tritium \(\beta\)-decay experiments and in neutrinoless double-\(\beta\) decay experiments.

Let us consider now neutrino transitions in long-baseline (LBL) experiments. In the scheme IIIA the LBL transition probabilities are given by

\[
P_{\nu_{\alpha} \rightarrow \nu_{\beta}}^{\text{LBL}} = \sum_{k=1,2} U_{\alpha k}^* e^{-\frac{\Delta m_{21}^2 L}{2 E_{\nu}}} U_{\beta k}^2 + \sum_{j=3,4} U_{\alpha j}^* U_{\beta j}^2. \tag{26}
\]
The transition probabilities in the scheme IIIB can be obtained from (26) with the change $1 \leftrightarrow 3, 4$. The inequalities (21) and (22) imply rather strong constraints on the probabilities of $\bar{\nu}_e \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$ transitions in LBL experiments. Indeed, for the probability of $\bar{\nu}_e \rightarrow \bar{\nu}_e$ transitions we have

$$P^{\text{LBL}}_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \geq \left( \sum_{j=3,4} |U_{ej}|^2 \right)^2 \geq (1 - a_0^e)^2$$

(27)

in scheme IIIA and

$$P^{\text{LBL}}_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \geq \left( \sum_{k=1,2} |U_{ek}|^2 \right)^2 \geq (1 - a_0^e)^2$$

(28)

in scheme IIIB. Hence, in both schemes IIIA and IIIB $P^{\text{LBL}}_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ is close to one and we expect that the LBL transition probability of $\bar{\nu}_e$ into any other state is small. Indeed, in both schemes we have

$$1 - P^{\text{LBL}}_{\bar{\nu}_e \rightarrow \bar{\nu}_e} \leq a_0^e (2 - a_0^e).$$

(29)

This limit is shown by the solid line in Fig. 3. The exclusion line obtained in the CHOOZ experiment (dash-dotted line) and the final sensitivity of the CHOOZ experiment (dash-dot-dotted line) are also shown. It can be seen that for $\Delta m^2_{31} \lesssim 1 \text{eV}^2$ the upper bound (23) for $1 - P^{\text{LBL}}_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ is much smaller than the upper bound reached in CHOOZ experiment and also much smaller than the final sensitivity of the CHOOZ experiment.

4. Three massive neutrinos

If the results of the LSND experiment will not be confirmed by future experiments, the most plausible scheme is the one with mixing of three massive neutrinos and a mass hierarchy:

$$m_1 \ll m_2 \ll m_3.$$  

(30)

The investigations of neutrino oscillations does not allow to answer the fundamental question: are massive neutrinos Dirac or Majorana particles? Only investigations of neutrinoless double-$\beta$ decay could allow to answer this question. In the case of a three-neutrino mass hierarchy for the effective Majorana mass we have:

$$|\langle m \rangle| \simeq |U_{e3}|^2 \sqrt{\Delta m^2_{31}}.$$  

(31)

The results of reactor neutrino experiments imply an upper bound for $|U_{e3}|^2$: $|U_{e3}| \leq a_0^e$, with $a_0^e$ given in Eq. (19). Therefore the effective Majorana mass is bounded by

$$|\langle m \rangle| \lesssim a_0^e \sqrt{\Delta m^2_{31}}.$$  

(32)
The value of this upper bound as a function $\Delta m^2_{31}$ obtained from 90% CL exclusion plots of the Bugey and CHOOZ experiments is presented in Fig. 4 (the solid and dashed line, respectively). The region on the right of the thick straight solid line is forbidden by the unitarity bound $|\langle m \rangle| \leq \sqrt{\Delta m^2_{31}}$.

Also the results of the Super-Kamiokande atmospheric neutrino experiment imply an upper bound for $|U_{e3}|^2$. The shadowed region in Fig. 4 shows the region allowed by Super-Kamiokande results at 90% CL that we have obtained using the results of three-neutrino analysis performed by Yasuda.

Figure 5 shows that the results of the Super-Kamiokande and CHOOZ experiments imply that $|\langle m \rangle| \lesssim 10^{-2}$ eV.

The observation of neutrinoless double-$\beta$ decay with a probability that corresponds to a value of $|\langle m \rangle|$ significantly larger than $10^{-2}$ eV would mean that the masses of three neutrinos do not have a hierarchical pattern and/or exotic mechanisms (right-handed currents, supersymmetry with violation of R-parity, ...) are responsible for the process.

Let us notice that from the results of the Heidelberg-Moscow $^{76}$Ge experiment it follows that $|\langle m \rangle| \lesssim 0.5 - 1.5$ eV. The next generation of neutrinoless double-$\beta$ experiments will reach $|\langle m \rangle| \simeq 10^{-1}$ eV. Possibilities to reach $|\langle m \rangle| \simeq 10^{-2}$ eV are under discussion.

5. Conclusions

The neutrino mass spectrum and the structure of the neutrino mixing matrix depend crucially on the confirmation of the results of the LSND experiment. If this results will be confirmed we need (at least) four massive neutrinos with mass spectrum of type IIIA or IIIB (see Fig. 1). If the results of the LSND experiment will not be confirmed, a plausible scenario is the one with three massive neutrinos and a mass hierarchy. The investigation of the nature of massive neutrinos (Dirac or Majorana?) will require in this case to reach a sensitivity at the level of $10^{-2}$ eV in the search for neutrinoless double-$\beta$ decay.

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