RADIATIVE TRANSFER EFFECTS IN He I EMISSION LINES

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ABSTRACT

We consider the effect of optical depth of the 2 3S level on the nebular recombination spectrum of He I for a spherically symmetric nebula with no systematic velocity gradients. These calculations, using many improvements in atomic data, can be used in place of the earlier calculations of Robbins. We give representative Case B line fluxes for UV, optical, and IR emission lines over a range of physical conditions: \( T = 5000-20,000 \) K, \( n_e = 1-10^8 \) cm\(^{-3} \), and \( \tau_{2345} = 0-100 \). A FORTRAN program for calculating emissivities for all lines arising from quantum levels with \( \ell \leq 10 \) is also available from the authors. We present a special set of fitting formulae for the physical conditions relevant to low-metallicity extragalactic H\( \alpha \) regions: \( T = 12,000-20,000 \) K, \( n_e = 1-300 \) cm\(^{-3} \), and \( \tau_{2345} < 2.0 \). For this range of physical conditions, the Case B line fluxes of the bright optical lines 4471 \( \AA \), 5876 \( \AA \), and 6678 \( \AA \) are changed less than 1%, in agreement with previous studies. However, the 7065 \( \AA \) corrections are much smaller than those calculated by Izotov & Thuan based on the earlier calculations by Robbins. This means that the 7065 \( \AA \) line is a better density diagnostic than previously thought. Two corrections to the fitting functions calculated in our previous work are also given.

Subject headings: atomic data — cosmological parameters — cosmology: observations — ISM: abundances — ISM: general

1. INTRODUCTION

In a previous paper (Benjamin, Skillman, & Smits 1999, hereafter Paper I), we presented a calculation of the nebular recombination line spectrum of He I which combined the atomic data and recombination cascade of Smits (1991, 1996) with the collision strengths data of Sawey & Berrington (1993). The resulting Case B line fluxes were tabulated, fitting functions were provided, and estimates of uncertainties in atomic data on the helium abundance estimates were also given.

An additional uncertainty in estimating helium abundances is the effect of radiative transfer. By assuming Case B, we have assumed that most nebulae have a large optical depth in transitions arising from the 1 1S level. However, a significant population of electrons may also build up in the metastable 2 3S level. Absorption and reprocessing of photons from the 2 3S level to upper levels will alter the recombination cascade and resulting helium abundance estimates.

The last full treatment of the recombination cascade combined with optical depth was provided by Robbins (1968), and this work remains the classic reference for most researchers. Additional work on this problem has been done by Almog & Netzer (1989), Proga, Mikolajewska, & Kenyon (1994), and Sasselov & Goldwirth (1995). However, these papers, while they considered the radiative transfer problem, used models of the helium atomic physics with an accuracy less than the 1% level necessary for meaningful cosmological constraints. As a result, while they were useful in demonstrating the relative effect of radiative transfer, the absolute effect has not been addressed since the improvements in the atomic data. We address this problem here.

This work is of particular interest to the derivation of the primordial helium abundance. Accuracy in the atomic physics of better than 1% is required to produce abundance estimates that provide meaningful constraints on models of Big Bang nucleosynthesis. Currently, observational studies are claiming accuracies on order of less than 1% (e.g., \( Y_p = 0.2452 \pm 0.0015 \), Izotov et al. 1999; \( Y_p = 0.2345 \pm 0.0026 \), Peimbert et al. 2000). In Paper I, we performed a Monte Carlo exercise to estimate the uncertainty in the theoretical emissivities due to uncertainties in the input atomic data and obtained an estimate of 1.3%-1.5%. Recently, Olive & Skillman (2001) have demonstrated that minimization routines used to solve for physical conditions and He abundances simultaneously can result in erroneous solutions due to the degeneracies of the dependencies of the helium emission lines on the various physical conditions (i.e., temperature, density, optical depth, and underlying absorption). Obviously, using accurate atomic physics in these calculations reduces the systematic uncertainties in the abundance calculations.

2. CALCULATIONS

For this work, we consider the radiative transfer of a spherically symmetric nebula with no systematic expansion. We assume that the recombination is in the Case B limit, so that the only radiative transfer effects are those that arise from the metastable 2 3S level. The assumption of no systematic expansion is justified by detailed studies of the velocity fields of giant extragalactic H\( \alpha \) regions (e.g., Yang et al. 1996, and references therein), which show them to be...
dominated by turbulent, and not organized, motions. We parameterize our calculations in terms of the line center optical depth of the 3889 Å line, \( \tau_{3889} = n(2^3S) \sigma_{3889} R_s \), where \( n(2^3S) \) is the density of atoms in the metastable state.\(^1\) The cross section at line center is given by

\[
\sigma = (6.73 \times 10^{-31} \text{cm}^{-2}) \, A \lambda_\lambda^2 \left( \frac{T_d}{n_{\text{He}}} + \xi^2 \right)^{-1/2},
\]

where \( A(\text{s}^{-1}) \) is the spontaneous transition probability; \( \lambda_\lambda \) is the wavelength in Angstroms; the gas temperature is \( T_d = T/(10^4 \text{ K}) \); \( n_{\text{He}} = 4.0 \), and the “turbulent” velocity, \( \xi \), is typically 40 km s\(^{-1}\).

For a nebula of Stromgren radius, \( R_s = (30 \text{ pc}) \), \( Q_{3889} \), the optical depth is

\[
\tau = 9.68 \times 10^{-14} Q_{3889}^{1/3} n_{\text{H}}^{1/3} p_{-7} A_6 T_d^{-1/2} \lambda_\lambda^{3},
\]

where \( Q_{3889} = Q_{3889}/(10^{48} \text{ cm}^{-3}) \) is the luminosity of hydrogen ionizing photons; \( n_{\text{H}}(\text{cm}^{-3}) \) is the hydrogen particle density, \( A_6 = A/(10^6 \text{ s}^{-1}) \) is the spontaneous transition probability, and \( p_{-7} = [n(2^3S)/n_{\text{He}+}] / 10^{-7} \). A helium abundance of 10\(^\%\) by number is assumed. Turbulent velocities lower this optical depth and can be accounted for by replacing \( T_d \) with \( T_d + n_{\text{He}} \xi^2 / 166 \), where \( \xi \) is in km s\(^{-1}\). For low densities, the population in the \( 2^3S \) level is in the range of \( 1 \). For fixed \( T \) and \( p \), the variation of optical depth with ionizing luminosity and particle density is shown in Figure 1.

For a spherically symmetric nebula, the mean probability of escape of a photon with a frequency characterized by

\[
x = (\nu - \nu_0)/\Delta \nu \text{ is given by (Cox & Mathews 1969)}
\]

\[
p(\tau_x) = \frac{3}{4 \pi^2} \left[ 1 - \frac{1}{2 \tau_x^2} + \left( \frac{1}{\tau_x} + \frac{1}{2 \tau_x^2} \right) e^{-2 \tau_x} \right],
\]

where \( \nu_0 \) is the line center frequency and \( \Delta \nu = (\nu_0/\epsilon)(2kT/n_{\text{He}} + \xi^2)^{1/2} \). Integrating this escape probability over the entire line profile yields a total escape probability of \( \epsilon(\tau) = 1/\pi^{1/2} \int_0^{\infty} p(\tau_x) e^{-x^2} dx \). As noted in Osterbrock (1989), this can be approximated to within 10\(^\%\) with the function \( \epsilon(\tau) = 1.72/(1.72 + \tau) \). To get a more exact result, we have evaluated this integral numerically, with a precision of better than 0.01\(^\%\).

The calculation of statistical equilibrium given in equation (4) of Paper I is then simply modified by replacing the spontaneous decay rates \( A \) with \( \epsilon(\tau) A \), yielding a revised set of level populations and emissivities. In the calculations here, we used a model atom with individual levels up to \( n = 20 \); otherwise, the calculations are the same as in Paper I. The relative optical depths of lines in the \( n^3P-2^3S \) series are given in Table 1.

### Table 1

| \( n \) | \( \lambda \) (Å) | \( \tau \) | \( n \) | \( \lambda \) (Å) | \( \tau \) |
|---|---|---|---|---|---|
| 2... | 10833 | 2328.35 | 12... | 2654 | 0.86 |
| 3... | 3890 | 100.00 | 13... | 2646 | 0.66 |
| 4... | 3189 | 32.77 | 14... | 2639 | 0.51 |
| 5... | 2946 | 14.68 | 15... | 2634 | 0.42 |
| 6... | 2830 | 7.88 | 16... | 2630 | 0.34 |
| 7... | 2765 | 4.74 | 17... | 2627 | 0.30 |
| 8... | 2724 | 3.08 | 18... | 2624 | 0.23 |
| 9... | 2697 | 2.12 | 19... | 2621 | 0.21 |
| 10... | 2678 | 1.53 | 20... | 2619 | 0.18 |
| 11... | 2664 | 1.10 | ... | ... | ... |

*Normalized to \( \tau_{3889} = 100 \).

\(^{1}\) Note that in the text we refer to emission lines using their wavelength in air. However, the tables use the vacuum wavelengths derived from the energy levels described in Paper I.

3. RESULTS

3.1. Tables, Fitting Formulae, and Comparisons

We have calculated the emissivity, \( j = j(n, T, \tau_{3889}) \), of recombination lines for the same temperature and density grid as in Paper I, but for optical depths up to \( \tau_{3889} = 100 \). The ratio of the line emissivity for optical depth \( \tau_{3889} \) to the emissivity for zero optical depth is given by optical depth factor \( f_{\text{line}}(\tau_{3889}) = j(n, T, \tau_{3889}) / j(n, T, 0) \). Figure 2 shows how the optical depth factor of several lines varies as a function of optical depth for a case with \( n_e = 10^2 \text{ cm}^{-3} \) and \( T = 10,000 \text{ K} \) and \( 20,000 \text{ K} \). Although we include the collisional coupling between the triplet and singlet levels, only the triplet lines show noticeable differences. Even for densities as high as \( n_e = 10^8 \text{ cm}^{-3} \), the effect of optical depth in the \( 2^3S \) level on the singlet lines is small—less than 0.4\(^\%\) in the worst case.

Table 2 shows all optical and IR lines that change by more than 5\(^\%\) over the range \( \tau_{3889} = 0-100 \) for the case with \( n_e = 10^2 \text{ cm}^{-3} \) and \( T = 10,000 \text{ K} \). This provides a guide to which lines are most affected by optical depth. From Figure 2, note the strong dependence of the 7065 Å line on both \( \tau_{3889} \) and temperature. The decrease in \( f_{7065}(\tau_{3889}) \) with

![Fig. 1.—Dependence of optical depth in 3889 Å line center as a function of ionizing luminosity of source, \( Q_{3889} \), and hydrogen particle density, \( n_{\text{He}} \). This is assuming a fixed helium abundance of 10\(^\%\) (by number), a temperature of 10\(^4\) K, no turbulent velocity, and \( n(2^3S)/n_{\text{He}+} = 10^{-7} \). Using a typical turbulent velocity of \( \xi = 40 \text{ km s}^{-1} \) would reduce the optical depth by a factor of 6.2. The scaling of optical depth with these quantities is given in eqs. (1) and (2).](image-url)
increasing temperature (at constant density) arises due to a decreasing population in the $2^3S$ level caused by the increasing collisional excitations out of that level. We also provide polynomial fit coefficients in Table 3 for these lines that approximate the calculated results to within 10%, and better than 1%, for a few selected optical lines at a single temperature and density.

Unfortunately, the optical depth factor is sometimes a complex function of density, temperature, and $\tau_{3889}$, so no simple functional form was found that could provide a good (i.e., better than 10%) fit for $f_{\text{line}}(\tau_{3889})$ over our full three-dimensional parameter space. This difficulty is illustrated by Figure 3, which shows $f_{\text{3865}}$ at $\tau_{3889} = 100$. As a result, we have written a program which does a linear interpolation on our grid of results to estimate the emissivities. This program may be obtained from the authors.2

In Figure 4, we have compared the results of our new calculations with the results of Robbins’s (1968) for a density of $n_e = 100$ cm$^{-3}$. Robbins’ results are shown as filled symbols, ours are shown as open symbols, and a solid line shows the result of the fitting formulae in Table 3. This figure shows differences between the relative line strengths for different temperatures, particularly for $\lambda 7065$. This demonstrates again that the radiative transfer effects are dependent on other factors beside $\tau_{3889}$. As a result, the fitting formulae given in Table 3 should be used with caution, since they are only for a single choice of temperature and density.

Second, although Robbins only reported results on a very coarse grid, it is instructive to note the difference in $f_{3889}$ as a function of $\tau_{3889}$. Our calculated $\lambda 3889$ line strengths do not decrease as strongly with increasing $\tau_{3889}$. Since $\lambda 3889$ is essentially a resonance line, most $\lambda 3889$ absorptions are followed directly by a $\lambda 3889$ emission, with the result that the line strength falls off relatively slowly with increasing $\tau_{3889}$ (less than 50% at $\tau_{3889} = 10$). The absolute value of $f_{3889}$ as a function of $\tau_{3889}$, therefore, depends on the formalism used in the calculation. Robbins (1968) used an “exact redistribution” function from Unno (1952), while here we use complete redistribution. In the complete redistribution case, the absorbed wavelength and re-emitted wavelength are completely unrelated. In addition, Robbins (1968) used a numerical integration of the equations of radiative transfer, while our result is (almost) analytical and therefore has much higher numerical accuracy. Thus, we believe our calculations should be the more accurate.

A significant advantage to using an escape probability formalism as opposed to calculating the “exact” solution as in Robbins (1968) is that it allows one to quickly change the characteristics of the radiative transfer. Since these effects depend upon the exact geometry of the system, which will always be poorly known, our ability to correct line fluxes for modifications due to self-absorption will always be limited. In particular, density inhomogeneities, ionization inhomogeneities, large-scale velocity structure, and planar versus spherical geometry can all affect the resulting radiative transfer to differing degrees. Our calculations also do not include the effects of dust absorption or the effects of the hydrogen emission line at 3889.1 Å.

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2 Program and input data file may be obtained from the Web site http://wisp.physics.wisc.edu/~benjamin.
In lieu of calculating a large number of grids of differing geometries and density/velocity structure, we have attempted to estimate the robustness of our results using two alternate forms of the escape probability corresponding to differing geometries and scattering assumptions. For the case with $T = 10^4$ K and $n_e = 10^{-2} \text{ cm}^{-3}$, we have recalculated the recombination cascades using the simple form $\epsilon = [1 - \exp(-\tau)]/\tau$ for a plane-parallel geometry (Rybicki 1984; Almog & Netzer 1989). The second form comes from integrating the local escape probability given by equation (I.2) in Avrett & Hummer (1965). We find that for the restricted range of optical depth of interest for primordial helium abundance studies, the difference in all the bright optical lines is less than 1%. For the extended range to $\tau_{3889} = 100$, all the triplet lines that are not part of the $2^3S-^3P$ series are modified by 7% at the most. Lines in the $2^3S-^3P$ series are affected much more. Like other lines, they are modified indirectly by the change in the $2^3S$ level population (which is a small fractional change in the line flux), but also directly by the reprocessing of photons in this series.

Another important effect for radiative transfer is the presence or absence of velocity gradients. Our treatment implicitly assumes that the expansion rate of a nebula is significantly less than the intrinsic Doppler width of the line, i.e., $w = v_{\text{exp}}/v_{\text{Dopp}} \ll 1$, where $v_{\text{Dopp}} = (166T_4/m_He + \zeta^2)^{1/2}$ as above. For extragalactic H II regions, we are generally justified in assuming that $w \approx 0$. However, using the results of Robbins (1968), one can estimate the effects of expansion velocity for an uniform density sphere. We find that the effects of expansion can be approximated by replacing $\tau_{3889}$ with $\tau_{3889, \text{eff}} \approx \tau_{3889}/(1 + 0.4w)$. This approximation is good to within 5% up to $\tau_{3889} = 30$ and 20% for $\tau_{3889} = 70$.

### 3.2. Relevance to Primordial Helium Determinations

Part of the motivation for this work was to estimate accurate radiative transfer corrections in determinations of
helium abundances in low-metallicity extragalactic H II regions. Thus, as in Paper I, we have calculated optical depth correction factors over a limited grid of physical parameters: \( T = 12,000 - 20,000 \) K, \( n_e = 1 - 300 \) cm\(^{-3}\), and \( \tau_{3889} < 2.0 \). The upper bound on the optical depth is chosen for two reasons. First, most observations of extragalactic H II regions find optical depths less than 2.0 (e.g., Izotov & Thuan 1998). This is due to the large (\( \geq 25 \text{ km s}^{-1} \)) velocity dispersion typical of these objects (e.g., Melnick, Terlevich, & Mølles 1987, and references therein). Second, if an object requires a large optical depth correction to derive the helium abundance, it is probably an inherently uncertain abundance calculation and may not therefore be useful in determining the primordial helium abundance.

The upper bound on density follows from the same principle; i.e., as the correction for collisional excitation becomes large, the uncertainty in that correction must also grow, rendering the abundance determination less certain. While the density range covers the entire range of densities considered for NGC 346 by Peimbert et al. (2000) and almost all of the targets studied by Izotov & Thuan (1998), Peimbert, Peimbert, & Luridiana (2001) have derived much higher densities for several of the targets in the Izotov & Thuan (1998) sample. If it proves necessary to consider a grid with a larger density range, it is possible to produce one with the provided programs.

We find that the optical depth correction factors for the bright optical lines 4471 Å and 5876 Å are less than 1% in this limited range. It is the corrections to 7065 Å and 3889 Å that are of most concern. In Table 4, we give fitting functions that allow one to estimate the helium abundance given an observed line ratio of \( \tau_{\text{line}} = I_{\text{line}}/I_{\text{H}} \). The helium abundance by number for a given line is given by

\[
\nu_{\text{line}} = r_{\text{line}} \frac{F_{\text{line}}(n_e, t)}{f_{\text{line}}(n_e, t, \tau_{3889})}.
\]

The function \( F_{\text{line}}(n_e, t) = A_T B_0 + B_1 n_e \) is taken from Paper I, while the optical depth function, \( f_{\text{line}}(n_e, t, \tau_{3889}) = 1 + (\tau/2) [a + (b_0 + b_1 n_e + b_2 n_e^2)] \), is calculated here for the limited range \( \tau \leq 2.0 \). For the 7065 Å line, however, it is necessary to use a different functional form. The H\( \beta \) emissivity is fitted by \( \epsilon_{\text{H} \beta}(n_e, t) = 4\pi \epsilon_{\text{H} \beta}/n_e \tau_{3889} = 12.450 \times 10^{-26} \times 0.917 \) with a maximum error of 0.3% over our limited parameter space. The 7065 emissivity at \( \tau = 0 \) is fit by \( \epsilon_{\text{H} \beta}(n_e, t) = 4\pi \epsilon_{\text{H} \beta}/n_e \tau_{3889} = C + (D_0 + D_1 n_e + D_2 n_e^2) \), where \( (C, D_0, D_1, D_2) = (3.494, -0.793, 1.30 \times 10^{-3}, -6.96 \times 10^{-7}) \times 10^{-26} \), and the optical depth function \( f_{\text{7065}}(n_e, t, \tau_{3889}) \) is the same as above with parameters \( (a, b_0, b_1, b_2) = (0.359, -3.46 \times 10^{-2}, -1.84 \times 10^{-4}, 3.039 \times 10^{-7}) \). Then the helium abundance by number using the 7065 line is given by

\[
\nu_{\text{7065}} = r_{\text{7065}} \frac{\epsilon_{\text{H} \beta}(n_e, t)}{\epsilon_{\text{H} \beta}(n_e, t)/f_{\text{7065}}(n_e, t, \tau_{3889})}.
\]

These functions fit \( \nu_{\text{7065}} \) within 2.2% over our limited range.

Figure 5 shows the same functions as in Figure 4 but only for the low values of \( \tau_{3889} \) relevant to estimating the primordial helium abundance. We include the fits to the optical depth correction factors contained in Table 4 and refer to them as “BSS limited fit.” A striking feature in Figure 5 is the large deviation of the Izotov & Thuan fitting formula from our newly calculated values for the 7065 Å line. Since the Izotov & Thuan fit is based on the very coarse grid of
Limited Range Fitting Formulae For Helium Abundance Including Optical Depth Effects

| Line (Å) | A   | B₀   | B₁   | a   | b₀   | b₁   | b₂   | Max. Error (%) |
|----------|-----|------|------|-----|------|------|------|----------------|
| 3889     | 0.904 | -0.173 | -5.4 × 10⁻⁴ | -1.06 × 10⁻¹ | 5.14 × 10⁻⁵ | -4.20 × 10⁻⁷ | 1.97 × 10⁻¹⁰ | 0.8           |
| 4026     | 4.297 | 0.090 | -6.3 × 10⁻⁶ | 1.43 × 10⁻³ | 4.05 × 10⁻⁴ | 3.63 × 10⁻⁸ | ...           | 0.2           |
| 4387     | 16.255 | 0.109 | -3.7 × 10⁻⁶ | ...   | ...   | ...   | ...           | 0.2           |
| 4471     | 2.010 | 0.127 | -4.1 × 10⁻⁴ | 2.74 × 10⁻³ | 8.81 × 10⁻⁴ | -1.21 × 10⁻⁶ | ...           | 1.0           |
| 4686     | 0.0816 | 0.145 | ...     | ...   | ...   | ...   | ...           | 0.3           |
| 4714     | 19.555 | -0.461 | -13.5 × 10⁻⁴ | 5.77 × 10⁻² | -7.12 × 10⁻³ | -3.10 × 10⁻⁵ | 3.99 × 10⁻⁸ | 1.9           |
| 4922     | 7.523 | 0.150 | -1.9 × 10⁻⁴ | ...   | ...   | ...   | ...           | 0.6           |
| 5876     | 0.735 | 0.230 | -6.3 × 10⁻⁴ | 4.70 × 10⁻³ | 2.23 × 10⁻³ | -2.51 × 10⁻⁶ | ...           | 1.2           |
| 6678     | 2.580 | 0.249 | -2.0 × 10⁻⁴ | ...   | ...   | ...   | ...           | 0.3           |
| 7065     | See text | ...     | ...     | ...   | ...   | ...   | ...           | ...           |
| 7281     | 13.835 | -0.307 | -7.0 × 10⁻⁴ | ...   | ...   | ...   | ...           | 2.0           |

Note.—Helium number abundance is given by $y = (I_{\text{line}}/I_{\text{He}})F(n_e, T)/f(n_e, T, \tau_{3889})$, where $I_{\text{line}}/I_{\text{He}}$ is the observed line ratio, $F(n_e, T) = 4\pi n_e^2 k_n$ (from Paper I), and the optical depth correction factor is given by $f(n_e, T, \tau_{3889}) = 1 + (\tau_{3889}/2)[a + (b_0 n_e + b_1 n_e^2)\tau]$. These functions apply only for cases in the range $n_e = 1\quad 10^3$ cm⁻³ and $T = 1.2\quad 2.0 \times 10^4$ K. The maximum uncertainty over this range is noted.

Evidence of this can be seen by considering the predictions of the Izotov & Thuan functions at $\tau_{3889} = 0.5$. Figure 5 shows that at $\tau_{3889} = 0.5$, the 3889 Å line has decreased by roughly 3%, while the Izotov & Thuan fit for the 7065 Å line has increased by roughly 25%. At 20,000 K, the ratio of the emissivities for the $\lambda 3889$ and $\lambda 7065$ lines is 3.85, which, after correcting for energy per photon, means that there are roughly 2.1 $\lambda 3889$ photons emitted for each $\lambda 7065$ photon. Since the increase in the 7065 Å line with increasing $\tau_{3889}$ comes from the conversion of $\lambda 3889$ photons, the 7065 Å line should not increase faster than 2.1 times the 3889 Å line decreases. Thus, in the range of interest, the Izotov & Thuan fit has overestimated the radiative transfer effects on the 7065 Å line by roughly a factor of 4 at $\tau_{3889} = 0.5$, and higher at lower values of $\tau_{3889}$.

This difference has important consequences. Olive & Skillman (2001) explored various possibilities for systematic errors that could result by using minimization routines to solve for physical conditions and He abundances simultaneously. Since the 7065 Å line plays the main role as the density diagnostic in these minimization routines, overestimating the 7065 Å dependence on $\tau_{3889}$ leads to errors in the density estimates (which directly affect the helium abundance determinations). The good news is that the 7065 Å line dependence on $\tau_{3889}$ is much less than previously thought, so that using the 7065 Å as a density diagnostic will increase in diagnostic power. That is, the degeneracies between the 3889 Å and 7065 Å lines will be decreased. This is important since the 3889 Å line is subject to systematic observational uncertainties, most notably separation from the 3889 H I line.

The error estimation work of Olive & Skillman (2001) will need to be revisited with the newly calculated fitting formulae. We recommend that all future helium abundance calculations from minimization routines take advantage of the new fitting formulae presented in Table 4. The consequence of our results on the determination of the primordial helium values given by Robbins, it does not accurately represent the results at low $\tau_{3889}$, where it is actually being applied. The parabolic form of the fitting function is not appropriate to the physics, as $f$ increases nearly linearly with $\tau_{3889}$ for low values of $\tau_{3889}$ and turns over only at relatively high values of $\tau_{3889}$.

Fig. 5.—A plot similar to that shown in Fig. 4, zooming in on the range of $\tau_{3889}$ from 0 to 3. Note that the y-axis ranges are different from those of Fig. 4, and, again, change from panel to panel. The striking feature in this figure is the large difference between the new calculated values for $\lambda 7065$ and the interpolation fit to the Robbins (1968) calculations derived by Izotov & Thuan (1998). For low values of $\tau_{3889}$, the new calculations show that the amplification of the $\lambda 7065$ line is much lower than would have been inferred from the Izotov & Thuan (1998) fit. This implies that the $\lambda 7065$ line is a better density diagnostic than previously thought. As in Fig. 4, the differences in the results for the two different temperatures for $\lambda 3889$ are small compared to the y-axis range, and thus, the circular and square symbols are overplotted.
abundance is not yet clear. Only about one-third of the targets in the sample of Izotov & Thuan (1998) showed evidence for detectable radiative transfer effects. Moreover, since the main effect comes through the densities calculated from the 7065 Å line, the effect is likely to be small in most cases. The total effect will need to be calculated on a case-by-case basis. However, the sense of the correction will be that higher 7065 Å line strengths will be interpreted as being due to higher densities, not higher values of $3889$. As can be seen using the helium abundance fitting functions in Table 9 of Paper I, this will result in a lower helium abundance for objects for which radiative transfer corrections have been applied.

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APPENDIX

CORRECTIONS TO PAPER I

Two corrections to the fitting formulae provided in Paper I should be noted. First, the fitting formula for primordial helium abundance in Table 9 for the 6678 line should be given by $f = 2.58e^{0.249-n_{10^4}}$. Second, the values of fluxes in Tables 5–7 for Brackett γ emissivities were in error. The values of the emissivities in units of $10^{-27}$ ergs s$^{-1}$ cm$^{-3}$ for $T = (5000, 10,000, 20,000)$ K should be $(7.25, 3.44, 1.54), (7.20, 3.41, 1.53)$, and $(6.99, 3.33, 1.51)$ for $n_e = 10^2, 10^4, 10^6$ cm$^{-3}$. The corresponding fitting functions (good to within 8%) are $3.39 \times 10^{-27}T^{-1.116}$, $3.37 \times 10^{-27}T^{-1.116}$, and $3.29 \times 10^{-27}T^{-1.104}$.

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