Emergence of semi-localized Anderson modes in a disordered photonic crystal as a result of overlap probability

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Abstract. In this paper we study the effect of positional randomness on transmissional properties of a two dimensional photonic crystal as a function of a randomness parameter $\alpha$ ($\alpha = 0$ completely ordered, $\alpha = 1$ completely disordered). We use finite-difference time-domain (FDTD) method to solve the Maxwell’s equations in such a medium numerically. We consider two situations: first a $90^\circ$ bent photonic crystal wave-guide and second a centrally pulsed photonic crystal micro-cavity. We plot various figures for each case which characterize the effect of randomness quantitatively. More specifically, in the wave-guide situation, we show that the general shape of the normalized total output energy is a Gaussian function of randomness with wavelength-dependent width. For centrally pulsed PC, the output energy curves display extremum behavior both as a function of time as well as randomness. We explain these effects in terms of two distinct but simultaneous effects which emerge with increasing randomness, namely the creation of semi-localized modes and the shrinking (and eventual destruction) of the photonic band-gaps. Semi-localized (i.e. Anderson localized) modes are seen to arise as a synchronization of internal modes within a cluster of randomly positioned dielectric nano-particles. The general trend we observe shows a sharp change of behavior in the intermediate randomness regime (i.e. $\alpha \approx 0.5$) which we attribute to a similar behavior in the underlying overlap probability of nano-particles.

1 Introduction

Photonic crystals (PC), structures with a periodic refractive index distribution, are a subject of intense worldwide research [1–5]. Among various PC structures, two dimensional (2D) PCs based on a finite cluster of cylinders of infinite length have attracted most attention [1,2], since these structures are relatively easy to fabricate or simulate. PCs have potential for many technological applications. For example, in contrast with the traditional wave-guides in which guiding principle is based on total internal reflection, PC wave-guides have a fundamentally different mechanism [6]. These wave-guides can propagate the light in a path on the scale of wavelength without diffraction losses even at corners, because light leakage is forbidden for frequencies in the photonic band-gap [7].

The effect of disorder in 2D PCs with $90^\circ$ bent wave-guide was investigated by Langtry et al. [8] as well as Kwan et al. [9]. In the former, the sensitivity of wave-guiding to different degree of disorder, mostly in radius of cylinders, refractive index, and filling factor have been studied by highly accurate multipole method. In the latter, by using multiple-scattering method, it has been shown that the transmission spectrum of the wave-guide is more sensitive to the uniformity in position of first layer surrounding the wave-guide.

In the first part of this article, we study, using finite-difference time-domain (FDTD) method [10], the normalized output energy of a $90^\circ$ bent wave-guide in 2D PC consisting of a square array of long dielectric cylinders in the air background. By introducing a wide range of randomness in the position of all cylinders, we show that the output energy normalized to the ordered case have a Gaussian shape, when plotted with respect to randomness factor. This behavior is shown for any wavelength in the original PC band-gap.

In the second part of this article, we study the effect of positional randomness on the light energy output from a PC micro-cavity, where four cylinders are removed from the center of square array and a short Ricker wavelet pulse [11] is initiated in the center of the micro-cavity. We use standard FDTD method to study the time evolution of output energy for TM electromagnetic field in the above system. The normalized output energy curve as a function of time for different randomness factors starts out with a sharp minimum followed by a broad maximum. Previous investigations are mainly concerned with cavity mode frequencies and quality factor versus strength of randomness [12] as well as the variation of density-of-states with respect to reduced frequency [13]. Additionally, some researchers have undertaken theoretical studies.
of positional disorder effect on resonant mode distributions and resonance frequencies in 2D photonic crystal micro-cavity [14,15], and experimental activity accompanied by numerical simulation of spatial intensity distribution of localized modes in 2D open microwave cavity [16]. In the latter, for detection and simulation of localized modes, the random medium is excited by more than one thin antennas around the center of randomized PC. However, in the present work, we excite the medium by only one central Ricker wavelet pulse and then the system is left to its own in order to study how the energy of the pulse exists from boundaries of PC when it is subjected to the different strengths of randomness.

Using transfer matrix method, the first study of the effect of induced randomness in radius or refractive index of the cylinders has shown the appearance of propagation states in the original band-gap [17]. Band-gaps, although with some narrowing, were found to persist even for large amount of disorder. This phenomenon has been observed both theoretically [18–21] as well as experimentally [22,23], especially in the case of positional disorder in cylinders [18,24]. These investigations mainly concerned analysis of transmission through PC slabs. However, in the present work the source is placed in the center of our system and band-gap resistance to disorder is observed in the transmission spectrum of the output light. We show that the formation of non-central semi-localized modes, besides the light localization in the central micro-cavity, can explain dynamical behavior of normalized output energy for various randomness factors.

Here, the emergence of semi-localized (or Anderson localized) modes [25] in our randomized PC is due to synchronization of internal reflection modes in nano-particles which are situated near each other in a suitable yet random manner. Previously, similar ideas have been proposed in the context of coupled particle cavities [26,27] in order to interpret experimental results in powder random lasers [28,29]. However, the medium there is active and the particle sizes (4–20 μm) are much larger than emission wavelength. Here, on the other hand, we see such effects arise naturally as a result of randomness in a passive medium within clusters of nano-particles (≈100 nm). More recently, Anderson localization has become a controversial topic in disordered PC’s [25,30,31]. Here, we provide further evidence for existence of such modes in 2D PC’s. Similar localization phenomena has also been seen in [32] where randomness in size of cylinders has been considered.

This paper is arranged as follows: in Section 2 we briefly explain the set-up for our numerical simulations. Section 3 entails the main results of our study which includes numerical results as well as discussion of the relevant physical effects. We summarize our main results in Section 4.

2 Numerical simulation

In this paper we concentrate on a PC model which consist of a square lattice of cylinders with refractive index

![Fig. 1.](image)

3 with radius $0.3d$, where $d$ is the lattice constant of the crystal (Fig. 1a). The surrounding of cylinders is air with refractive index 1. This kind of crystal has a band-gap in wavelength region $3.0d < \lambda < 3.7d$ [33]. In order to introduce disorder in the position of cylinders, we consider displacement $\delta d$ for each cylinder, which is a random number between zero and a percentage of lattice constant $d$. We call this percentage randomness factor \( \alpha \). We consider the cylinders axes aligned in $z$ direction, so in the disordered case, like Figure 1b, each cylinder displaces from its ordered position, as $\delta d_x = u_x \times \alpha \times d$ in $x$ direction and as $\delta d_y = u_y \times \alpha \times d$ in $y$ direction; where $u_x$ and $u_y$ are random numbers with flat distribution between $-1$ and $+1$. We perform our simulation with $d = 180$ nm, so we choose visible light, $\lambda \sim 550$ nm, in the edge of band-gap, to study the effect of randomness better.

To simulate propagation of light, we use FDTD method, and for truncating computational space, we use CPML absorbing boundary condition [34].

Our model is symmetric along $z$ axis, so choosing TMz mode, we carry out the simulation in two dimensions. For each randomness factor $\alpha$, we repeat simulation with several different random arrangements (ensemble) of cylinders, and average over the various realizations of randomness. For first scenario, i.e. propagation of light by a wave-guide in PC (Fig. 1c), we enter a plane-wave to the wave-guide and calculate the output energy at the other end; while for the second scenario, i.e. propagation of a pulse from the center of PC, we use a Ricker wavelet pulse with central wavelength, $\lambda = 550$ nm, as initial pulse and calculate total output energy on all sides of crystal. In the second scenario, we remove four central cylinders to prevent the source from overlapping with randomly arranged neighboring cylinders and we therefore have a cavity in the center of the PC (Fig. 1b).

3 Numerical results and discussions

3.1 wave-guide PC

Figure 2 shows normalized output of guided energy in a 90° bent wave-guide PC. For each randomness factor, the output energy is averaged over 200 different random samples and normalized to the output energy of the ordered case ($\alpha = 0$). As we expect, output energy decreases with increasing disorder, however, we note the following observations: