COMPOSITE-TO-METAL TUBULAR LAP JOINT UNDER TORSIONAL LOADING

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During the damage evolution of bonded joints, different modes of failure can be detected. These modes include pure modes (Mode I, Mode II, Mode III) or their combinations (Mixed-mode) according to the type and the direction of loading. Some scientifically validated test methods are commonly used to obtain the fracture toughness by loading under pure Mode I, Mode II and their Mixed-mode. Some test methods were established to obtain the fracture toughness by loading under pure Mode III, however further research is required. The aim of this work is to clarify the mode ratio of a tubular lap joint under torsional loading and then to propose a method to determine the cohesive and damage parameters of this joint.

The presumed mode of failure is Mode III, or Mixed-mode with a predominance of Mode III. Emphasis is placed on the widespread applicability of the obtained results for similar specimens loaded under similar modes.

KEYWORDS
Bonded joint, composite-to-metal, finite element, Scotch-Weld DP490, damage modelling, identification

1 INTRODUCTION
Composite materials are used for machine tools, the automotive industry, and aviation, where designs mostly consist of multiple parts and materials. This places high demands on the inter-part connections. There are several ways to create metal-to-composite connections. These are namely riveted joints, bolted joints, interference fit joints, taped riveted joints, integrated joints and bonded joints [Lašová 2013]. For riveted and bolted joints, the connecting elements are placed in the holes in the connected parts. These holes cause a concentration of the stress and significantly degrade the mechanical properties of composites. Interference fit joints are less suitable for metal-to-composite connections because of the differences in thermal expansion, which can lead to an increase of the pressure or release of the interference fit joint [Bernardin 2016]. Therefore, composite-to-metal bonded joints seem to be a more appropriate option.

The structure of the bonded joint is described in Fig 1, where (1) are adherends (bonded parts), (2) and (6) are adhesive zones, (3) and (5) are transitional cohesive zones, and (4) is the cohesive zone.

2 BONDED JOINT MODELLING OF TUBULAR LAP JOINT
The mechanical properties of the bonded joints (strength, stiffness) can be determined analytically. The finite element method (FEM) should be selected for predicting the mechanical properties of bonded joints for complex surfaces and it is also a good solution for describing damage initiation and damage evolution (until complete failure occurs). There are many possibilities for modelling the adhesive layer and the adherends, differing in the level of idealisation and the size of the elements in the FEM model [Benzegagh 1996, Camanho 2002]. Damage initiation can be evaluated using strength criteria, or by principles of fracture mechanics which also contain damage evolution. Energy principles based on fracture mechanics are used for thin adhesive layers, where the macroscopic properties (Young’s modulus, Poisson’s ratio) of the adhesive do not correspond to the real behaviour of the bonded joint [Simulia 2013]. This description is based on research [Griffith 1920, Irwin 1957, Inglis 1913], whose aim was to clarify elliptical crack propagation in structures. One option for predicting damage initiation and damage evolution is a cohesive model corresponding with Griffith’s energy principles [Griffith 1920]. Cohesive and damage parameters describe the process of failure in a cohesive model. These parameters are not usually provided by the adhesive manufacturer and strongly depend on the geometry and the type of loading of the bonded joint. During the damage evolution of the bonded joints, different modes (mechanisms) of failure can be detected. These are namely the pure modes (Mode I, Mode II, Mode III) or their combination (Mixed-mode) and are shown in Fig. 2.

Figure 2. The failure modes
Each mode is described by its own cohesive and damage parameters. These are namely the critical strain energy release rate (SERR), which can be obtained for Mode I and II from experimental measurement according to ASTM [ASTM D5528-13 2014, ASTM D7905M-14 2013]. Several test methods were proposed to determine the critical energy release rate for Mode III [de Morais 2008, Liao 1996], for example, the Split Cantilever Beam test (SCB) [Davidson 2010], the Edge Crack Torsion test (ECT) [Ratcliffe 2004] or the Six Point Bending Plate test (SPBP) [Mehrabadi 2013]. Most of these contain a significant proportion of modes other than pure Mode III. There is no standardized testing method for testing components loaded by pure Mode III or by Mixed-mode containing Mode III. The authors of this research are focused in the long term on machine tool design and are also currently working on composite-to-metal drive shafts. Therefore, a tubular lap joint under torsional loading (TLTL) seemed to be a suitable choice. According to some sources [de Morais 2008, Liao 1996, Davidson 2010, Ratcliffe 2004, Mehrabadi 2013] these joints include a combination of modes (Mixed-mode), according to others [Li 2009] only pure Mode III is included. The aim of this work is to clarify the mode ratio of TLTL (pure Mode III or Mixed-mode with predominance of Mode III) and then propose a method to determine the
The cohesive and damage parameters of this joint. The results should be useful for the subsequent design of the standardized method of joints under pure Mode III.

2.1 ANALYTICAL CALCULATION

The least time-consuming calculation method for predicting the TLTL is an analytical calculation [Gay 2003]. This method simplifies the real stress behaviour $\tau_r$ (frictional shear stress component) to the average value $\tau_a$. Fig. 3 shows the typical distribution of the $\tau_r$ in the TLTL in the X direction ($L$ is the length of the bonded joint) [Aimmanee 2018, Kaya 1999, Chen 1992].

![Figure 3. The typical stress distribution ($\tau$) and the idealised stress distribution ($\tau_a$) in TLTL](image)

Fig. 4 shows all the parameters used for the analytical calculation according to Equation 1, which are described in Tab. 1.

$$\tau_{average} = \frac{M_t}{2\pi r^2 L} \leq 0.2 \cdot \tau_{rupture},$$ (1)

| Parameter | Description |
|-----------|-------------|
| $r$       | Inner tube radius |
| $\tau_{rupture}$ | Permissible shear stress |
| $L$       | Length of the bonded surface |
| $M_t$     | Applied torque |

**Table 1. All parameters used for analytical calculation**

2.2 STRENGTH CRITERIA EVALUATED BY FEM

There are many possibilities for modelling the bonded joint, differing in the level of idealisation of the adherends and adhesive using FEM.

![Figure 4. Sketch of TLTL analytical calculation](image)

The adhesive layer can be idealized with 1D elements with defined or infinite stiffness or by 3D elements. This approach is described in Fig. 5. The maximum shear stress value is determined from 1D, or 3D elements in relation to the permissible shear stress (given by the manufacturer of the adhesive). The other possibility for bonded joint idealisation is Tahmasebi’s model [Tahmasebi 1999], which combines the 3D solid elements with the 1D spring and 1D rigid elements. This model is described in Fig. 6 and Fig. 7.

![Figure 6. Tahmasebi’s model - 1D and 3D elements](image)

The maximum shear stress can be determined only in the middle of the adhesive layer (from the 1D spring elements), which is the major disadvantage of this model.

2.3 VIRTUAL CRACK CLOSURE TECHNIQUE

The virtual crack closure technique (VCCT) calculates the SERR, respecting that the energy needed to separate two surfaces is equal to the energy to close them [Gliszczynski 2019, Vigroux 2009, Krueger 2015]. This unique technique calculates iteratively the crack growth based on fracture mechanics using the nonlinear analysis with the help of convergence and stabilization algorithms. This method is applicable for bonded joints loaded by an arbitrary mode of failure (I, II, III or Mixed-mode), but the major disadvantage is the considerable time and computing demands.

2.4 EXTENDED FEM

The eXtended Finite Element Method (XFEM) also known as partition of unity method (PUM) or generalized FEM (GFEM), is a powerful and effective calculation technique for modelling failure problems without the requirement of remeshing. This method extends the conventional FEM and allows independent mesh crack modelling (see Fig. 8) [Belytschko 2009].

![Figure 5. 3D elements (left) and 1D spring elements (right)](image)

![Figure 6. Tahmasebi’s model - 1D spring elements and 1D rigid elements](image)

![Figure 7. Tahmasebi’s model - 1D spring elements and 1D rigid elements](image)

![Figure 8. Crack modelling independent on mesh](image)
The main field of application of this method is modelling of fracture, dislocations, grain boundaries, or phases interfaces.

2.5 COHESIVE ZONE MODEL

Nowadays, most FEM solvers use de-cohesion elements in different forms. For our specific purpose, we use cohesive surface based contact [Benzegagha 1996, Camanho 2002] instead of the cohesive elements, which are now the two basic methods of cohesive modelling. These elements are based on the cohesive zone modelling approach (CZM) [Barenblatt 1962, Dugdale 1960]. This approach describes the bonded joint from the initial load until complete failure using the specific criteria. The cohesive behaviour of the joint is characterized by the cohesive stiffness criterion \(k_n, k_s, k_t\), where \(n, s, t\) are directions of the coordinate system. The damage initiation criterion and damage evolution law were used to simulate the loading and softening process of the bonded joints. The damage initiation criterion refers to the degradation of the cohesive stiffness of the layer of the adhesive. In our case, it was described by the quadratic stress, see Eq. 2:

\[
\left(\frac{t_n}{t_n^0}\right)^2 + \left(\frac{t_s}{t_s^0}\right)^2 + \left(\frac{t_t}{t_t^0}\right)^2 = 1
\]

The denominator represents the maximum contact stress defined by the user \(t_n^0, t_s^0, t_t^0\) and the nominator represents traction stress. The \(t_n\) in Maxwell’s bracket describes the different behaviour of the adhesive layer in positive and negative directions. The damage initiation (B point position) can be described (in three directions) by Eq. 3:

\[
t = \begin{bmatrix} t_n \\ t_s \\ t_t \end{bmatrix} = \begin{bmatrix} k_n & 0 & 0 \\ 0 & k_s & 0 \\ 0 & 0 & k_t \end{bmatrix} \begin{bmatrix} \delta_n \\ \delta_s \\ \delta_t \end{bmatrix} = K \delta
\]

The pure modes are expected to be orthogonal in the loaded part. This should be reflected in the FE model, where the directions of Mode I-III should correspond with the directions of \(n, s, t\) in the FEM analysis. The crack tip orientation should coincide with the direction of Mode III. This assumption is valid only in the case of direct crack propagation. The critical SERR \(G_{Nc}, G_{Sc}, G_{Tc}\) describes the state of complete degradation of cohesive stiffness and is represented by the area under AB and BC curves in fig. 9.

The failure of the joint according to Mode I is performed using the Double Cantilever Beam (DCB) test method, the failure according to Mode II is performed using the End Notched Flexure (ENF) test method. The critical value of the \(G_{Nc}, G_{Sc}, G_{Tc}\) can be determined from experimental measurements according to the ASTM methods [ASTM D5528-13 2014, ASTM D7905M-14 2013] using Eq. 4 and Eq. 5 [Bernardin 2019, Ducept 2000, Zemčík 2008].

\[
G_{IC EXP} = \frac{3F_c \delta_c}{2b(a+\Delta)}
\]

\[
G_{IIc EXP} = \frac{9a^2 F_c \delta_c}{2b(2L^4+3a^4)}
\]

where \(F_c\) is the critical force related to the damage initiation, \(\delta_c\) is the corresponding opening displacement, \(b\) is the width of the specimen, \(a\) is the initial opening length and \(L\) is the displacement between the position of the load and one support. As stated in the introduction, the determination of critical value \(G_{IC}\) for Mode III \(G_{III}\) is not possible according to any standard method. Therefore, it is a task of this research to find one. Because the mode ratio in the tubular lap joint was unknown, the model was solved as loaded under Mixed-mode, defined by the 3D Power Law criterion [Reeder 2006] described in Eq. 6.

\[
\left(\frac{G_{Ic}}{G_{IC EXP}}\right)^\alpha + \left(\frac{G_{IIc}}{G_{IIc EXP}}\right)^\alpha + \left(\frac{G_{III}}{G_{III EXP}}\right)^\alpha = 1,
\]

where \(G\) is SERR, indexes I, II, III refer to the mode of failure, index \(c\) describes the critical value, index \(\alpha\) is a dimensionless coefficient.

3 EXPERIMENT

Since there is no standard method for testing bonded joints loaded under pure Mode III, the shape of the specimen had to be designed as well as the whole testing method. The TLTL was chosen based on the motivation of the authors and previous research [Bernardin 2013]. The specimens consisted of a composite tube (Pos.2) and two steel rollers (Pos.1 and Pos.3). The dimensions and arrangement of all the parts are shown in Fig. 10 and Fig. 11.

Figure 10. Sketch of specimen

Figure 11. Sketches of specimen

The lengths of the bonded surfaces on both metal parts were different (15 mm and 23 mm), because of the controlled failure of the bonded joint with the shorter bonded surface. The influence of the various adhesive thicknesses was examined in depth. The strength of the bonded joint is not influenced by the change of the adhesive layer in the range of thickness from 0.05 mm to 0.4 mm [Sedláček 2005]. A detailed view of the adhesive layer is shown in Fig. 12, where \(L\) represents the length, \(T\) represents the thickness of the adhesive layer. The 0.3 mm value of thickness was chosen according to the adhesive manufacturer [3M 1996]. It corresponds with the previously recommended range.
Mechanical properties of the composite tube are shown in Tab. 2, where \( V_m \) is the matrix volume ratio, \( V_f \) is the fibre volume ratio, \( E \) is the Young’s modulus and the 1, 2, 3 indexes refer to the material directions, \( v \) is Poisson’s ratio, and \( G \) is the shear modulus.

| Layer number | Thickness [mm] | Fibre Orientation [*] |
|--------------|----------------|-----------------------|
| 1            | 0.346          | 43.09                 |
| 2            | 0.341          | -43.98                |
| 3            | 0.968          | 0.00                  |
| 4            | 0.324          | 47.1                  |
| 5            | 0.320          | -47.82                |
| 6            | 1.284          | 0.00                  |
| 7            | 0.176          | 87.38                 |
| 8            | 0.349          | 87.41                 |
| 9            | 0.349          | 43.09                 |

The universal testing machine FU-O and the clamped specimen are shown in Fig. 13, and the measuring devices (motors, sensors, etc.) are labelled. All results obtained during measuring were processed using the TestControl software. The load was defined by angular velocity \( \omega = 1 \, \text{°/min} \) and the torque and the rotation were measured. The cohesive bonded failure (only in the adhesive layer) is expected during testing, but there was the concern that several external factors may affect the measuring results. These are namely the imperfect manual preparation of the bonded joint (variable circumferential thickness, air in the adhesive), imperfect manufacturing of the specimens (small roughness of the turned specimens), pull-up of the composite fibres, and the adhesive bonding failure (on the interface between the adhesive and adjacent part).

### 4 Finite Element Analysis

The model used for quasi-static FEM analysis was created in commercial software Abaqus/CAE 6.14. All parts were uniformly meshed using the eight-node linear brick elements (C3D8R). The partition cell feature was used for making the mesh more precise on the contact surfaces. The material and the boundary conditions were assigned to all three parts in the FEM model. The layer of the adhesive was idealised using surface-to-surface cohesive contact with defined mechanical properties.

The cohesive surface based contact was applied on all coincident node pairs (N1-N4) in the layer of the adhesive with defined thickness 0.3 mm (respecting the real thickness of the adhesive). The meshed FEM model with specific boundary conditions (BC1, BC2) and a definition of the coordinate system (X, Y, Z) is shown in Fig. 15.

### 5 Identification

The mechanical properties were identified using the gradient-based method optimisation in optiSlang software. \( k_n, k_t, t_n, t_t, G_{nc}, G_{tc}, \alpha \) were set as the input parameters. The initial values, lower and upper bounds are shown in Tab. 4.
The ruptured specimens are shown in Fig. 6. The pull-up of the fibre was not found for these specimens during testing, only cohesive failure in the layer of the adhesive. The relationship between torque and rotation is shown in Fig. 17 for all eight specimens with appropriate mechanism of failure.

The results show the difference of the maximum torque value (about 25%) and the different stiffness of each specimen. After reaching the maximum torque value the degradation of the bonded joint occurs and the values of the torque vary by up to 70%. This significant difference is caused by the different sizes of the active bonded surfaces. This, together with friction and broken glue (which still remains inside the joint), influences the value of the transmitted torque. The torque-rotation curve highlighted in Fig. 7 was considered to be a representative curve for the following optimisation. This curve was obtained by arithmetic average of all results. All the necessary parameters were found using the optimisation method and are shown in Tab. 6.

Table 6. Cohesive, damage parameters and objective function

| Parameters | Representative sample |
|------------|-----------------------|
| $k_w$ [GPa/m] | 120 |
| $k_x$ [GPa/m] | 130 |
| $k_t$ [GPa/m] | 350 |
| $f_c$ [MPa] | 2.3 |
| $f_x$ [MPa] | 20.8 |
| $f_t$ [MPa] | 14 |
| $G_{ic}$ [J/m²] | 141 |
| $G_{xc}$ [J/m²] | 3850 |
| $G_{tc}$ [J/m²] | 2850 |
| $\alpha$ | 1 |
| $r_g$ | 0.903 |

The parameters in the $n$ and $s$ directions were identified in previous research [Bernardin 2019] according to ASTM [ASTM D5528-13 2014, ASTM D7905M-14 2013]. Tab. 6 describes the cohesive and damage parameters of the representative specimen. The difference between torque and rotation of the experimentally obtained curves and the curves from FEM analysis is shown in Fig. 18.
Figure 18. Relationship between torque and rotation of experimentally obtained curves and curves from FEM analysis

The participation of the failure mode $i_a$ is based on 3D Power Law criterion and it is described by Eq. 8.

$$i_a = \frac{\sigma_a}{\sigma_{c}} \cdot 100 \% \quad (8)$$

where $a$ can be replaced by the specific mode (I, II or III). The participation of Mode I, II and III was determined using the results of FEM analysis and it is shown in Tab. 7.

| Participation of mode [%] | $i_I$ - Mode I | $i_{II}$ - Mode II | $i_{III}$ - Mode III |
|---------------------------|----------------|-------------------|---------------------|
|                           | $5 \cdot 10^{-5}$ | $3 \cdot 10^{-4}$ | 99.99964 |

Table 7. The participation of failure modes in bonded joint

The predominance of Mode III was confirmed by the FEM results (the parameter $i_{III}$ reaches almost 100%). According to the crack tip propagation detected in FEM, the parameters $k_{II}$, $t_{II}$, $G_{II}$ correspond to $k_{III}$, $t_{III}$, $G_{III}$ and have the biggest influence on the results. This corresponds to the real crack tip orientation and to the theory of the orthogonality of the pure mode orientation. The non-standardized samples do not have the same diameter along the length and contain sharp corners, notches, etc. It was also found that the stress distribution varies using different calculation methods (VCCT, CZM, Analytical calculation, XFEM) [Adams 1977, Aimmamee 2018, Kaya 1999, Chen 1992]. These are the main reasons that the shear stress propagation along the joint length (see Fig. 19) roughly, but not exactly, corresponds with the typical stress distribution in TLTL in general (see Fig. 3).

The following figures show the particular stress components in dependence on the Z direction (in plane shown in Fig. 15). Fig. 20 and Fig. 21 show the normal stresses $\sigma_1$ and $\sigma_3$. Fig. 22 and Fig. 23 show the shear stresses in the XY and XZ planes $\tau_\alpha$ and $\tau_\gamma$. All stresses were determined on the shorter side of the bonded surface, see ZSection (Fig. 15).

Figure 19. The stress distribution ($\tau_\alpha$) and the idealised stress distribution ($\tau_\alpha$) in examined TLTL obtained by FEM in X direction

Figure 20. The normal stress $\sigma_1$ in dependence on the Z direction obtained by FEM

Figure 21. The normal stress $\sigma_3$ in dependence on the Z direction obtained by FEM

Figure 22. The shear stress in the XY plane $\tau_\alpha$ in dependence on the Z direction obtained by FEM
Figure 23. The shear stress in the XZ plane τZ in dependence on the Z direction obtained by FEM

The CZM primarily evaluates the SERR values and not the stress values. Therefore, the previous stress comparison is for information only.

7 CONCLUSIONS

The mode ratio of TLTL was clarified using FEM analysis. Modes I and II were negligible, participation of Mode III was almost 100%. Therefore, this work confirmed the suitability of the proposed method for identifying the cohesive and damage parameters of these joints related to Mode III. These parameters are suitable for a tubular lap joint under torsional loading, with similar surface roughness (Ra = 3.2 μm), for arbitrary adherend stiffness, arbitrary length and adhesive layer in a range from 0.05 mm to 0.4 mm, and for arbitrary material of adherends [Bernardin 2019, Sediščak 2005]. The parameters obtained for Mode I, II, and III can be used to predict the behaviour of any bonded joints loaded by pure modes (I, II, III) or by Mixed-mode loading. However, the biggest disadvantage of this method is the large dispersion of the experimental results. Therefore, this method does not yet seem to be a suitable option as a Mode III standard method. The statistical method was not used to evaluate the results due to the small number of samples. Performing the experimental measurement with a higher number of specimens (hundreds) and the use of statistical methods is needed to increase the accuracy of the results.

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