Single-particle potential in a chiral approach to nuclear matter including short range NN-terms

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Abstract

We extend a recent chiral approach to nuclear matter of Lutz et al. [Phys. Lett. B474 (2000) 7] by calculating the underlying (complex-valued) single-particle potential $U(p, k_f) + i W(p, k_f)$. The potential for a nucleon at the bottom of the Fermi-sea, $U(0, k_{f0}) = -20.0\,\text{MeV}$, comes out as much too weakly attractive in this approach. Even more seriously, the total single-particle energy does not rise monotonically with the nucleon momentum $p$, implying a negative effective nucleon mass at the Fermi-surface. Also, the imaginary single-particle potential, $W(0, k_{f0}) = 51.1\,\text{MeV}$, is too large. More realistic single-particle properties together with a good nuclear matter equation of state can be obtained if the short range contributions of non-pionic origin are treated in mean-field approximation (i.e. if they are not further iterated with $1\pi$-exchange). We also consider the equation of state of pure neutron matter $E_n(k_n)$ and the asymmetry energy $A(k_f)$ in that approach. The downward bending of these quantities above nuclear matter saturation density seems to be a generic feature of perturbative chiral pion-nucleon dynamics.

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1 Introduction

The present status of the nuclear matter problem is that a quantitatively successful description can be achieved, using advanced many-body techniques [1], in a non-relativistic framework when invoking an adjustable three-body force. Alternative relativistic mean-field approaches, including non-linear terms with adjustable parameters or explicitly density-dependent point couplings, are also widely used for the calculation of nuclear matter properties and finite nuclei [2].

In recent years a novel approach to the nuclear matter problem based on effective field theory (in particular chiral perturbation theory) has emerged [3, 4, 5]. The key element there is a separation of long- and short-distance dynamics and an ordering scheme in powers of small momenta. At nuclear matter saturation density the Fermi-momentum $k_{f0}$ and the pion mass $m_\pi$ are comparable scales ($k_{f0} \simeq 2m_\pi$), and therefore pions must be included as explicit degrees of freedom in the description of the nuclear many-body dynamics. The contributions to the energy

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per particle of isospin-symmetric nuclear matter $\bar{E}(k_f)$ originating from chiral pion-nucleon dynamics have been calculated up to three-loop order in refs.\cite{3,4}. Both calculations include the $1\pi$-exchange Fock-diagram and the iterated $1\pi$-exchange Hartree- and Fock-diagrams. In ref.\cite{3} irreducible $2\pi$-exchange is also taken into account and a momentum cut-off $\Lambda$ is used to regularize the few divergent parts associated with chiral $2\pi$-exchange. The resulting cut-off dependent contribution to $\bar{E}(k_f)$ is completely equivalent to that of a zero-range NN-contact interaction (see eq. (15) in ref.\cite{3}). At that point the work of Lutz et al. \cite{3} deviates and it follows a different strategy. Two zero-range NN-contact interactions (acting in $^3S_1$ and $^1S_0$ NN-states) proportional to the parameters $g_0 + g_A^2/4$ and $g_1 + g_A^2/4$ are introduced (see eq. (4) in ref.\cite{3}). The components proportional to $g_A^2/4$ cancel the zero-range contribution generated by the $1\pi$-exchange Fock-diagram. The other components proportional to $g_0$ and $g_1$ are understood to subsume all non-perturbative short-range NN-dynamics relevant at densities around nuclear matter saturation density $\rho_0$. In order to be consistent with this interpretation the NN-contact vertices proportional to $g_{0,1}$ are allowed to occur only in first order. Furthermore, according to ref.\cite{3} pions can be treated perturbatively (at least) in the $^1S_0$ partial-wave of NN-scattering if the zero-range pieces they generate are removed order by order. Therefore, the NN-contact vertex proportional to $g_A^2/4$ occurs also in higher orders (see Fig. 1 in ref.\cite{3} which includes diagrams with “filled circle” and ”open circle” vertices).

Despite their differences in the treatment of the effective short-range NN-dynamics both approaches\cite{3,4} are able to reproduce correctly the empirical nuclear matter properties (saturation density $\rho_0$, binding energy per particle $-\bar{E}(k_{f0})$ and compressibility $\bar{K}$) by adjusting only one parameter, either the coupling $g_0 + g_1 \simeq 3.23$ or the cut-off $\Lambda \simeq 0.65$ GeV. Note that in dimensional regularization all diagrams evaluated in ref.\cite{3} are finite. In the chiral approach of the Munich group\cite{4,3} the asymmetry energy $A(k_f)$, the energy per particle of pure neutron matter $\bar{E}_n(k_n)$ as well as the (complex) single-particle potential $U(p,k_f) + i W(p,k_f)$ below the Fermi-surface ($p \leq k_f$) have been calculated. Good results (in particular for the asymmetry energy, $A(k_{f0}) = 33.8$ MeV, and the depth of the single-particle potential, $U(0,k_{f0}) = -53.2$ MeV) have been obtained with the single cut-off scale $\Lambda \simeq 0.65$ GeV adjusted to the binding energy per particle $-\bar{E}(k_{f0}) = 15.3$ MeV. Moreover, when extended to finite temperatures \cite{3} this approach reproduces the liquid-gas phase transition of isospin-symmetric nuclear, however, with a too high value of the critical temperature $T_c = 25.5$ MeV.

It is the purpose of this work to investigate in the approach of Lutz et al.\cite{3} the single-particle potential $U(p,k_f) + i W(p,k_f)$ in isospin-symmetric nuclear matter as well as the neutron matter equation of state $\bar{E}_n(k_n)$ and the asymmetry energy $A(k_f)$. One of our major conclusions will be that any strong short-range NN-dynamics of non-pionic origin should be kept at the mean-field level. It should not be further iterated with pion-exchange in contrast to the prescription of power-counting rules.

## 2 Nuclear matter equation of state

Let us first reconsider the equation of state of isospin-symmetric nuclear matter as it follows from the calculation of ref.\cite{3}. Even though all contributions to the energy per particle $\bar{E}(k_f)$ have been given explicitly in ref.\cite{3} we prefer to write down again the extra terms generated by the NN-contact interactions proportional to $g_{0,1} + g_A^2/4$ (using a more compact notation). The first diagram in Fig. 1 gives rise to a contribution to energy per particle of the form:

$$\bar{E}(k_f) = \frac{(\gamma + 1) g_A^2 k_f^3}{(4\pi f_\pi)^2},$$  

(1)
Figure 1: Additional in-medium diagrams generated by the NN-contact interactions introduced in ref.[3]. The two NN-contact interactions proportional to $\gamma + 1$ and $\gamma_n + 1$ are symbolized by the filled square vertex. The last diagram is to be understood such that quadratic terms (such as $\gamma^2$, $\gamma \gamma_n$ and $\gamma_n^2$) are omitted.

where we have introduced (for notational convenience) the coefficient $\gamma$ by the relation $(\gamma + 1)g^2_A/2 = g_0 + g_1 + g^2_A/2$. In the second and third diagram in Fig. 1 the contact-interaction proportional to $\gamma + 1$ is iterated with $1\pi$-exchange or with itself (dropping the $\gamma^2$-contribution). Putting a medium-insertion\footnote{This is a technical notation for the difference between the in-medium and vacuum nucleon propagator. For further details, see section 2 in ref.[1].} at each of two nucleon propagators with equal orientation one gets:

$$\bar{E}(k_f) = \frac{3(\gamma + 1)g^4_A M m^4_\pi}{5(8\pi)^3 f^4_\pi} \left[ 11u - \frac{1}{2u} - (10 + 8u^2) \arctan 2u + \left( \frac{1}{8u^3} + \frac{5}{2u} \right) \ln(1 + 4u^2) \right], \quad (2)$$

with the abbreviation $u = k_f/m_\pi$. One observes that eq.(2) receives no contribution from the third diagram in Fig. 1 since $\int_0^\infty dl$ is set to zero in dimensional regularization. The second and third diagram in Fig. 1 with three medium-insertions give rise to the following contribution to the energy per particle:

$$\bar{E}(k_f) = \frac{9g^4_A M m^4_\pi}{(4\pi f_\pi)^4 u^3} \int_0^u dx \int_0^1 dy \left[ 2uxy + (u^2 - x^2 y^2) H \right] \left[ \frac{\gamma + 1}{2} \ln(1 + s^2) - \frac{s^2}{4} \right], \quad (3)$$

with the auxiliary functions $H = \ln(u + xy) - \ln(u - xy)$ and $s = xy + \sqrt{u^2 - x^2 + x^2 y^2}$. In the chiral limit $m_\pi = 0$ only the contribution coming from the last term, $-s^2/4$, in the second square bracket survives. The corresponding double integral $\int_0^u dx \int_0^1 dy \ldots$ has the value $2u^7(\ln 4 - 11)/105$. The expansion of the energy per particle up to order $O(k^4_f)$ is completed by adding to the terms eqs.(1,2,3) the contributions from the (relativistically improved) kinetic energy, from 1$\pi$-exchange and from iterated 1$\pi$-exchange written down in eqs.(5-11) of ref.[1]. In case of the 1$\pi$-exchange contribution (eq.(6) in ref.[1]) we neglect of course the small relativistic $1/M^2$-correction of order $O(k^6_f)$.

Now, we have to fix parameters. The pion decay constant $f_\pi = 92.4$ MeV and the nucleon mass $M = 939$ MeV are well-known. As in ref.[1] we choose the value $g_A = 1.3$. This corresponds via the Goldberger-Treiman relation to a $\pi NN$-coupling constant of $g_{\pi NN} = g_A M/f_\pi = 13.2$ which agrees with present empirical determinations of $g_{\pi NN}$ from $\pi N$-dispersion relation analyses \footnote{This predicted equilibrium density $\rho_0 = 0.138$ fm$^{-3}$ (corresponding to a Fermi-momentum of}. We set $m_\pi = 135$ MeV (the neutral pion mass) since this is closest to the expected value of the pion mass in the absence of isospin-breaking and electromagnetic effects.

The dashed line in Fig. 2 shows the equation of state of isospin-symmetric nuclear matter in the approach of ref.[3] using the abovementioned input parameters. The coefficient $\gamma = 4.086$ has been adjusted such that the minimum of the saturation curve lies at $\bar{E}(k_{f_0}) = -15.3$ MeV \footnote{This predicted equilibrium density $\rho_0 = 0.138$ fm$^{-3}$ (corresponding to a Fermi-momentum of}. The predicted equilibrium density $\rho_0 = 0.138$ fm$^{-3}$ (corresponding to a Fermi-momentum of
$k_f_0 = 250.1 \text{ MeV}$) is somewhat too low. The same holds for the nuclear matter compressibility $K = k_f^2 \bar{E}''(k_f) = 202 \text{ MeV}$. Of course, if we use the input parameters of ref. [3] ($f_\pi = 93 \text{ MeV}$, $g_A = 1.26$, $m_\pi = 140 \text{ MeV}$ and $g_0 + g_1 = 3.23$ corresponding to $\gamma = 4.07$) we exactly reproduce the numerical results of that work. We emphasize that the different treatment of the two components of the NN-contact interaction is essential in order to get (realistic) nuclear binding and saturation in the framework of ref. [3]. If both components were treated on equal footing in first order (technically this is realized by deleting the contribution coming from the third diagram in Fig. 1) the energy per particle $\bar{E}(k_f)$ would not even develop a minimum.

3 Real single-particle potential

Next, we turn to the real part of the single-particle potential $U(p, k_f)$ below the Fermi-surface ($p \leq k_f$) in the framework of ref. [3]. As outlined in ref. [3] the contributions to $U(p, k_f)$ can be classified as two-body and three-body potentials. From the first diagram in Fig. 1 one gets a contribution to the two-body potential of the form:

$$U_2(p, k_f) = -\frac{2(\gamma + 1)g_A^2k_f^3}{(4\pi f_\pi)^2},$$  \hspace{1cm} (4)
Figure 3: The lower curve shows the real part of the single-particle potential $U(p, k_f)$ at saturation density $k_f = 250.1 \text{ MeV}$ in the approach of Lutz et al. [3]. The upper curve includes in addition the relativistically improved kinetic energy $T_{\text{kin}}(p) = p^2/2M - p^4/8M^3$.

which is just twice its contribution to the energy per particle (see eq.(1)). From the second diagram in Fig. 1 one derives a contribution to the two-body potential of the form:

$$U_2(p, k_f) = \frac{(\gamma + 1)g_A^4Mm^4}{(4\pi f^4)} \left\{ u + \frac{1}{4x}(x^3 - 3x - 3u^2x - 2u^3) \arctan(u + x) \\
+ \frac{1}{4x}(x^3 - 3x - 3u^2x + 2u^3) \arctan(u - x) \\
+ \frac{1}{8x}(1 + 3u^2 - 3x^2) \ln \frac{1 + (u + x)^2}{1 + (u - x)^2} \right\}, \tag{5}$$

with the abbreviation $x = p/m$. The second and third diagram in Fig. 1 give each rise to three different contributions to the three-body potential. Altogether, they read:

$$U_3(p, k_f) = \frac{3g_A^4Mm^2}{(4\pi f^4)} \int_{-1}^{1} dy \left\{ 2uxy + (u^2 - x^2y^2)H \left[ \frac{\gamma + 1}{2} \ln(1 + s^2) - \frac{s^2}{4} \right] \\
+ \int_{-xy}^{s-xy} d\xi \left[ 2u\xi + (u^2 - \xi^2) \ln \frac{u + \xi}{u - \xi} \right] \frac{(2\gamma + 1)(xy + \xi) - (xy + \xi)^3}{2[1 + (xy + \xi)^2]} \\
+ \int_{0}^{u} d\xi \frac{\xi^2}{x} \left[ (\gamma + 1) \ln(1 + \sigma^2) - \frac{\sigma^2}{2} \right] \ln \left| \frac{x + \xi y}{x - \xi y} \right| \right\}, \tag{6}$$

with the auxiliary function $\sigma = \xi y + \sqrt{u^2 - \xi^2 + \xi^2y^2}$. The real single-particle potential $U(p, k_f)$ is completed by adding to the terms eqs.(4,5,6) the contributions from $1\pi$-exchange and iterated $1\pi$-exchange written down in eqs.(8-13) of ref.[3]. Again, the (higher order) relativistic $1/M^2$-correction to $1\pi$-exchange (see eq.(8) in ref.[3]) is neglected for reasons of consistency.
Figure 4: The lower curve shows the real part of the single-particle potential $U(p, k_{f0})$ at saturation density $k_{f0} = 270.3$ MeV in a mean-field treatment of the NN-contact interaction.

The lower curve in Fig. 3 shows the momentum dependence of the real single-particle potential $U(p, k_{f0})$ at saturation density $k_{f0} = 250.1$ MeV as it arises in the framework of Lutz et al. $^3$. The predicted potential depth for a nucleon at the bottom of the Fermi-sea is only $U(0, k_{f0}) = -20.0$ MeV. In magnitude this is much smaller than the typical depth $U_0 \approx -53$ MeV of the empirical optical model potential $^9$ or the nuclear shell model potential $^{10}$. The upper curve in Fig. 3 includes the (relativistically improved) single-nucleon kinetic energy $T_{\text{kin}}(p) = p^2/2M - p^4/8M^3$. As required by the Hugenholtz-van-Hove theorem $^{12}$ this curve ends at the Fermi-surface $p = k_{f0}$ with the value $\bar{E}(k_{f0}) = -15.3$ MeV. A further important check is provided by the sum rule for the two- and three-body potentials $U_{2,3}(p, k_f)$ written down in eq.(5) of ref.$^5$. It holds with very high numerical accuracy in the present calculation.

The overly strong momentum dependence of $U(p, k_{f0})$ comes from the second and third diagram in Fig. 1 in which the NN-contact interaction proportional to the large coefficient $\gamma + 1$ is further iterated. We propose to drop these three-loop diagrams and to keep the NN-contact interaction (of unspecified dynamical origin) at the mean-field level. The resulting equation of state obtained by leaving out the contributions eqs.(2,3) and adjusting $\gamma = 6.198$ is shown by the full line in Fig. 2. The predicted saturation density is now $\rho_0 = 0.174$ fm$^{-3}$ (corresponding to a Fermi-momentum of $k_{f0} = 270.3$ MeV) and the nuclear matter compressibility has the
value $K = 253 \text{ MeV}$. Note that the scheme of ref.\cite{3} modified by a mean-field treatment of the NN-contact interaction becomes equivalent to the truncation at fourth order in small momenta of our previous work \cite{1,2} after the identification of parameters, $\gamma + 1 = 10g_A^2\Lambda M/(4\pi f)^2$, with $\Lambda$ denoting the cut-off scale.

The lower full curve in Fig. 4 shows the momentum dependence of the real single-particle potential at saturation density $k_{f0} = 270.3 \text{ MeV}$ which results in a mean-field approximation of the NN-contact interaction (by leaving out the contributions eqs.(5,6)). The predicted potential depth $U(0,k_{f0}) = -54.8 \text{ MeV}$ is in good agreement with that of optical model \cite{10} or nuclear shell model potentials \cite{11}. Most importantly, the total single-particle energy $T_{\text{kin}}(p) + U(p,k_{f0})$ (upper curve) grows now monotonically with the nucleon momentum $p$, as it should. The up- and downward bending of the lower full curve in Fig. 4 is however still too strong. The negative slope of $U(p,k_{f0})$ at the Fermi-surface $p = k_{f0}$ leads to a too large effective nucleon mass $M^*(k_{f0}) \approx 2.9 M$ which reflects itself in a too high critical temperature $T_c \approx 25 \text{ MeV}$ of the liquid-gas phase transition \cite{7}. More elaborate calculation of nuclear matter in effective (chiral) field theory are necessary in order to cure this problem of the too large effective nucleon mass $M^*(k_{f0})$.

4 Imaginary single-particle potential

In this section, we discuss the imaginary part of the single-particle potential $W(p,k_f)$ for $p \leq k_f$ as it arises in the scheme of Lutz et al.\cite{3}. This quantity determines the half-width of nucleon-hole states in the Fermi-sea. As outlined in ref.\cite{3} the contributions to $W(p,k_f)$ can be classified as two-body, three-body and four-body terms. From the second and third diagrams in Fig. 1 one derives a two-body term of the form:

$$W_2(p,k_f) = \frac{g_A^4 M m^4}{(8\pi)^3 f^4} \left\{ u^2 x^2 + \frac{3u^4}{2} - \frac{x^4}{10} + (\gamma + 1) \left[ 4 + 14u^2 - \frac{22x^2}{3} \right. \right.$$

$$+ \frac{2}{x} (3x^2 - 3u^2 - 1) \left[ \arctan(u + x) - \arctan(u - x) \right]$$

$$\left. + \frac{1}{x} (x^3 - 3x - 3u^2 x - 2u^3) \ln[1 + (u + x)^2] \right\}. \quad (7)$$

The associated three-body term reads:

$$W_3(p,k_f) = \frac{3\pi g_A^4 M m^4}{(4\pi f^4)} \int_{-1}^{1} dy \left\{ (\gamma + 1) \left[ 2xy(s - \arctan s) - \frac{s^2}{2} \right. \right.$$

$$\left. + \left( \frac{1}{2} + u^2 - x^2 y^2 \right) \ln(1 + s^2) \right] + s^2 \left( \frac{s^2}{8} - \frac{u^2}{2} - \frac{sxy}{3} + \frac{x^2 y^2}{2} \right)$$

$$\left. + \int_{0}^{u} dx \frac{\xi^2}{x^2} \theta(x - \xi | y) \left[ (\gamma + 1) \ln(1 + \sigma^2) - \frac{\sigma^2}{2} \right] \right\}, \quad (8)$$

and the four-body term is given by the expression:

$$W_4(p,k_f) = \frac{3\pi g_A^4 M m^4}{(4\pi f^4)} \left\{ (\gamma + 1) \left[ \frac{4x^2}{3} - 1 + \ln(1 + 4x^2) + \left( \frac{1}{2x} - 2x \right) \arctan 2x \right] \right.$$

$$\left. - \frac{4x^4}{15} + \int_{-1}^{1} dy \left[ s^2 \left( \frac{u^2}{2} - \frac{s^2}{4} + \frac{2sxy}{3} - \frac{x^2 y^2}{2} \right) \right. \right.$$

$$\left. + \left( \frac{1}{2x} - 2x \right) \arctan 2x \right\}. \quad (9)$$
The additional contributions from the iterated $1\pi$-exchange Hartree- and Fock-diagram are collected in eqs.(20-25) of ref.[5]. The total imaginary single-particle potential evaluated at zero nucleon momentum ($p = 0$) can even be written as a closed form expression:

\[
W(0, k_f) = \frac{3\pi g_A^4 M m^4}{(4\pi f_\pi)^4} \left\{ \frac{u^4}{2} + (\gamma - 2)u^2 - \frac{2u^2}{1 + u^2} + \frac{\pi^2}{12} + \text{Li}_2(-1 - u^2) \right. \\
+ \left[ 4 - \gamma + \ln(2 + u^2) - \frac{1}{2} \ln(1 + u^2) \right] \ln(1 + u^2) \right\},
\]

(10)

where $\text{Li}_2(-a^{-1}) = \int_0^1 d\zeta (\zeta + a)^{-1} \ln \zeta$ denotes the conventional dilogarithmic function.

The dashed line in Fig. 5 shows the momentum dependence of the imaginary single-particle potential $W(p, k_{f0})$ at saturation density $k_{f0} = 250.1$ MeV as it arises in the approach of ref.[3]. The predicted value $W(0, k_{f0}) = 51.1$ MeV lies outside the range $20 - 40$ MeV obtained in calculations based on (semi)-realistic NN-forces [13, 14]. The full line in Fig. 5 corresponds to a mean-field approximation of the NN-contact interaction. Up to a slight change in the equilibrium Fermi-momentum $k_{f0} = 270.3$ MeV the full curve in Fig. 5 agrees with the one shown in Fig. 4 of ref.[5]. The considerably reduced value $W(0, k_{f0}) = 28.4$ MeV indicates the large contribution of the iterated diagrams in Fig. 1 to the imaginary single-particle potential $W(p, k_f)$. Note that both curves in Fig. 5 vanish quadratically near the Fermi-surface as required by Luttinger’s theorem [15]. As further check on our calculation we verified the zero sum rule: $\int_0^{k_f} dp p^2 [6W_2(p, k_f) + 4W_3(p, k_f) + 3W_4(p, k_f)] = 0$, for the two-, three- and four-body components $W_{2,3,4}(p, k_f)$ written in eqs.(7,8,9).
5 Neutron matter

In this section we discuss the equation of state of pure neutron matter. In the scheme of Lutz et al. [3] the energy per particle of pure neutron matter $\bar{E}_n(k_n)$ depends exclusively on the coefficient $g_1$ parameterizing the short-range NN-interaction in the channel with total isospin $I = 1$. There is no need to write down explicitly the contributions of the diagrams in Fig. 1 to $\bar{E}_n(k_n)$. These expressions are easily obtained from eqs.(1,2,3) by replacing $k_f$ by the neutron Fermi-momentum $k_n$, by replacing the coefficient $\gamma$ by a new one $\gamma_n$, and by multiplying the formulas with a relative isospin factor $1/3$. The relation $(\gamma_n + 1)g_A^2/4 = g_1 + g_A^2/4$ defines this new coefficient $\gamma_n$. The additional contributions to $\bar{E}_n(k_n)$ from the kinetic energy, $1\pi$-exchange and iterated $1\pi$-exchange are written down in eqs.(32-37) of ref.[4] (neglecting again the relativistic $1/M^2$-correction to $1\pi$-exchange).

The dashed line in Fig. 6 shows the energy per particle of pure neutron matter $\bar{E}_n(k_n)$ versus the neutron density $\rho_n = k_n^3/3\pi^2$ as it arises in the approach of ref. [3]. The coefficient $\gamma_n = 0.055$ has been adjusted to the empirical value of the asymmetry energy $A(k_{f_0} = 250.1\,\text{MeV}) = 33.2\,\text{MeV}$ (see next section). The downward bending of the dashed curve in Fig. 6 above $\rho_n > 0.15\,\text{fm}^{-3}$ is even stronger than in our previous work [4] (see Fig. 8 therein). This property can be understood by taking the chiral limit ($m_\pi \to 0$) of the calculated neutron matter equation of state and considering the coefficient $\beta_n$ in front of the term $k_n^4/M^3$. In the approach of Lutz et al. [3] one has:

$$\beta_n = -\frac{1}{70}\left(\frac{g_{\pi N}}{4\pi}\right)^4 \left(4\pi^2 + 17 + 16\ln 2\right) - \frac{3}{56} = -1.23,$$

which is 2.2 times the negative value of $\beta_n$ found in ref.[4]. The full line in Fig. 6 shows the
equation of state of pure neutron matter obtained in mean-field approximation of the \(nn\)-contact interaction proportional to \(\gamma_n + 1\) after adjusting \(\gamma_n = 0.788\) to the empirical value of the asymmetry energy \(A(k_{f0} = 270.3 \text{ MeV}) = 33.2 \text{ MeV}\). The downward bending of the full curve in Fig.6 is weaker and it sets in at somewhat higher densities \(\rho_n > 0.2 \text{ fm}^{-3}\). The dotted line in Fig.6 stems from the many-body calculation of the Urbana group \[16\]. This curve should be considered as a representative of the host of existing realistic neutron matter calculations which scatter around it. The systematic deviations observed in Fig.6 indicate that the neutron matter equation of state of ref.\[4\] cannot be improved by a different treatment of the short-range NN-dynamics alone. The downward bending of \(\bar{E_n}(\rho_n)\) above \(\rho_n > 0.2 \text{ fm}^{-3}\) seems to be a generic feature of perturbative chiral \(\pi N\)-dynamics truncated at three-loop order.

### 6  Asymmetry energy

Finally, we turn to the density dependent asymmetry energy \(A(k_f)\) in the approach of ref.\[3\]. The asymmetry energy is generally defined by the expansion of the energy per particle of isospin-asymmetric nuclear matter (described by different proton and neutron Fermi momenta \(k_{p,n} = k_f(1 \mp \delta)^{1/3}\) around the symmetry line: \(\bar{E_{as}}(k_p, k_n) = \bar{E}(k_f) + \delta^2 A(k_f) + \mathcal{O}(\delta^4)\). Evaluation of the first diagram in Fig.1 leads to the following contribution to the asymmetry energy:

\[
A(k_f) = \frac{g_A^2 k_f^3}{3(4\pi f_\pi)^2} (3\gamma - 2\gamma_n + 1),
\]

with the coefficients \(\gamma = 2(g_0 + g_1)/g_A^2\) and \(\gamma_n = 4g_1/g_A^2\) in the notation of ref.\[3\]. Putting a medium-insertion at each of two nucleon propagators with equal orientation one gets from the second and third diagram in Fig.1:

\[
A(k_f) = \frac{g_A^4 M m_n^4}{3(8\pi)^3 f_\pi^3} \left\{ 2(\gamma + 1)u + 8(2\gamma - \gamma_n + 1)u^2 \arctan 2u \\
+ \left[(2\gamma_n - 6\gamma - 4)u - \frac{\gamma + 1}{2u} \right] \ln(1 + 4u^2) \right\}.
\]

The same diagrams with three medium-insertions give rise to the following contribution to the asymmetry energy:

\[
A(k_f) = \frac{g_A^4 M m_n^4}{4\pi f_\pi^4 u^3} \int_0^u dx x^2 \int_{-1}^1 dy \left\{ 2uxy + (u^2 - x^2 y^2)H \right\} \left( 4ss' - \frac{2}{3}s' s^2 - \frac{2}{3}ss'' - \frac{7}{2}s^2 \right) \\
+ (\gamma + 1) \left\{ \frac{uxy(11u^2 - 15x^2 y^2)}{3(u^2 - x^2 y^2)} + \frac{1}{2}(u^2 - 5x^2 y^2)H \right\} \ln(1 + s^2) - \frac{4u^2 s^2 H}{3(1 + s^2)} \\
+ \frac{2uxy + (u^2 - x^2 y^2)H}{6(1 + s^2)^2} \left[ 8s(1 + s^2)(3s + s'' - 5s') + (1 - s^2)(3s^2 - 8ss' + 8s'^2) \right] \\
+ 2u^2(\gamma_n + 1) \left[ \frac{2uxy \ln(1 + s^2)}{3(u^2 - x^2 y^2)} + \left( \ln(1 + s^2) + \frac{2s^2}{3(1 + s^2)} \right) H \right] \right\},
\]

with \(s' = u \partial s/\partial u\) and \(s'' = u^2 \partial^2 s/\partial u^2\) denoting partial derivatives. In the chiral limit \(m_\pi = 0\) only the terms in the first line of eq.(14) survive. The corresponding double integral \(\int_0^u dx x^2 \int_{-1}^1 dy \ldots\) has the value \(4u^7(\ln 4 - 1)/15\). The asymmetry energy is completed by adding to the terms eqs.(12,13,14) the contributions from the kinetic energy, (static) 1\(\pi\)-exchange and iterated 1\(\pi\)-exchange written down in eqs.(20-26) of ref.\[4\].
Figure 7: The asymmetry energy $A(k_f)$ versus the nucleon density $\rho = 2k_f^3/3\pi^2$. The dashed line corresponds to the approach of Lutz et al. [3] and the full line shows the result obtained in mean-field approximation of the NN-contact interactions. The parameter $\gamma_n$ is in each case adjusted to the (empirical) value $A(k_{f0}) = 33.2\text{ MeV}$ [9].

The dashed line in Fig. 7 shows the density dependence of the asymmetry energy $A(k_f)$ in the approach of Lutz et al. [3] with the coefficient $\gamma_n = 0.055$ adjusted (at fixed $\gamma = 4.086$) to the empirical value $A(k_{f0}) = 250.1\text{ MeV} = 33.2\text{ MeV}$ [9]. The full line in Fig. 7 corresponds to the result obtained in mean-field approximation of the NN-contact interaction by dropping the contributions eqs. (13,14). In that case the empirical value $A(k_{f0}) = 270.3\text{ MeV} = 33.2\text{ MeV}$ [9] is reproduced by tuning (at fixed $\gamma = 6.198$) the coefficient $\gamma_n$ to the value $\gamma_n = 0.788$. Both curves in Fig. 7 behave rather similarly. In each case the asymmetry $A(k_f)$ reaches it maximum close to the respective saturation density $\rho_0$ and then it starts to bend downward. Since the same (unusual) feature has also been observed in ref.[4] it seems to be generic for perturbative chiral $\pi N$-dynamics truncated at three-loop order.

7 Concluding remarks

In this work we continued and extended the chiral approach to nuclear matter of Lutz et al. [3] by calculating the underlying single-particle potential. The potential for a nucleon at the bottom of the Fermi-sea $U(0, k_{f0}) = -20.0\text{ MeV}$ is not deep enough. Most seriously, the total single-particle energy $T_{\text{kin}}(p) + U(p, k_{f0})$ does not grow monotonically with the nucleon momentum $p$. The thereof implied negative effective nucleon mass at the Fermi-surface $M^*(k_{f0}) \simeq -3.5M$ and the negative density of states will ruin the behavior of nuclear matter at finite temperatures. The half-width of nucleon-holes at the bottom of the Fermi-sea $W(0, k_{f0}) = 51.1\text{ MeV}$ comes also out too large in that approach. A good nuclear matter equation of state and better (but still not yet optimal) single-particle properties can be obtained if
the NN-contact interaction (proportional to the coefficient $g_0 + g_1 + g_2^a/2$) is kept at the mean-field level and not further iterated. The energy per particle of pure neutron matter $\bar{E}_n(k_n)$ and the asymmetry energy $A(k_f)$ depend on a second parameter $g_1$ in the scheme of ref. [3]. Their density dependence is similar to the results of the one-parameter calculation in ref. [4]. The downward bending of $\bar{E}_n(k_n)$ and $A(k_f)$ above saturation density $\rho_0$ (less pronounced if the NN-contact interaction is kept at mean-field level) seems to be generic for perturbative chiral $\pi N$-dynamics. More elaborate calculations of nuclear matter in effective (chiral) field theory which fulfill all (semi)-empirical constraints are necessary.

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