An effective model for entropy deposition in high-energy pp, pA, and AA collisions

J. Scott Moreland, Jonah E. Bernhard, and Steffen A. Bass
Department of Physics, Duke University, Durham, NC 27708-0305
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We introduce TRENTo, a new initial condition model for high-energy nuclear collisions based on eikonal entropy deposition via a “reduced thickness” function. The model simultaneously predicts the shapes of experimental proton-proton, proton-nucleus, and nucleus-nucleus multiplicity distributions, and generates nucleus-nucleus eccentricity harmonics consistent with experimental flow constraints. In addition, the model provides a possible resolution to the “knee” puzzle in ultra-central uranium-uranium collisions.

Over the last decade, the ultra-relativistic heavy-ion collision programs at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) have succeeded in producing and exploring a novel, highly excited phase of QCD matter dubbed the strongly-interacting Quark-Gluon Plasma (sQGP) [1-7]. A major goal of current research is the quantification of fundamental sQGP properties, typically accomplished by matching experimental measurements to computational models for the full spacetime evolution of heavy-ion collisions [8, 9]. While viscous relativistic fluid dynamics provides a stable, well-tested description of the thermalized sQGP medium [10-15], the initial state of the collision remains poorly constrained and constitutes the largest source of uncertainty in modern computational models [16, 17].

Initial condition models generate profiles of energy or entropy at the sQGP thermalization time to be evolved by fluid dynamics. This is accomplished by two general approaches: dynamical models, which explicitly simulate the initial state and pre-equilibrium evolution of the collision [18][21]; and simpler non-dynamical models, which neglect pre-equilibrium evolution and construct static profiles at the thermalization time.

The most successful dynamical model is IP-Glasma [18]. IP-Glasma uses weakly-coupled color-glass condensate (CGC) effective field theory [22][24] and classical Yang-Mills evolution to quantitatively describe the latest event-by-event data on higher-order flow harmonics and a variety of other observables [25]. More recently, strongly-coupled AdS/CFT holography was applied to create a dynamical simulation of central nuclear collisions at the LHC [19]. While these models are all based on approximations of QCD or related quantum field theories, they encounter limits in their application to the full range of collision systems [19][26], and their explicit treatment of pre-equilibrium dynamics comes at a significant computational cost.

Non-dynamical models generate initial conditions directly at the thermalization time by asserting an ansatz for entropy deposition. Although they cannot explain sQGP formation, they provide the necessary input for fluid dynamics and can constrain the outcome of ab-initio initial condition calculations. The most widely-used prescription is the two-component Monte Carlo Glauber model, which determines participating nucleons via optical overlap and deposits energy or entropy for each participant and binary nucleon-nucleon collision. Despite its simplicity, the Glauber model has qualitatively fit many experimental measurements [27] and inspired a number of similar models. Notably, a participant quark model was proposed to describe the transverse-energy distributions of proton-proton, deuteron-gold, and gold-gold collisions at RHIC without invoking binary collision scaling [28]. This may suggest that the two-component wounded nucleon and binary collision ansatz is merely a proxy for participant quark scaling in nucleus-nucleus collisions.

In this work we introduce TRENTo, a new initial condition model for high-energy proton-proton, proton-nucleus, and nucleus-nucleus collisions. It is an effective model, intended to generate realistic Monte Carlo initial entropy profiles without assuming specific physical mechanisms for entropy production, pre-equilibrium dynamics, or thermalization.

Suppose a pair of projectiles labeled $A$, $B$ collide along beam axis $z$. Each projectile is represented by the beam-integrated density of its nuclear matter, typically called a nuclear thickness:

$$T_{A,B}(x, y) = \int dz \rho_{A,B}(x, y, z).$$

The construction of the thickness functions will be addressed shortly; first, we postulate the following:

1. The eikonal approximation is valid: entropy is produced if $T_A$ and $T_B$ eikonally overlap.
2. There exists a scalar field $f(T_A, T_B)$ which converts projectile thicknesses into entropy deposition.

The function $f$ is proportional to the entropy created at mid-rapidity and at the hydrodynamic thermalization time:

$$f \propto dS/dy \big|_{r=\tau_0}.$$  

It should provide an effective description of early collision dynamics: it need not arise from a first-principles calculation, but it must obey basic physical constraints.

With this in mind, we introduce for $f$ the reduced thickness

$$f = T_R(p; T_A, T_B) = \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}.$$
so named because it takes two thicknesses \( T_A, T_B \) and "reduces" them to a third thickness, similar to a reduced mass. The dimensionless parameter \( p \) may take any real value in \((−\infty, \infty)\) and is to be determined by experiment. This functional form—known as the generalized mean—simplifies to arithmetic, geometric, and harmonic means for certain values of \( p \), i.e.

\[
T_R(p; T_A, T_B) = \begin{cases} 
\frac{T_A + T_B}{2} & p = 1 \text{ (arithmetic)}, \\
\sqrt{T_A T_B} & p = 0 \text{ (geometric)}, \\
\frac{2T_A T_B}{T_A + T_B} & p = -1 \text{ (harmonic)}.
\end{cases}
\] (4)

More generally, \( p \) quantifies the attenuation of entropy production in asymmetric \((T_A \neq T_B)\) regions of the collision, as demonstrated in FIG. 1. As \( p \) decreases, the degree of attenuation increases. This significantly impacts the behavior of the model, for instance \( p = 1 \) is precisely a participant nucleon model, while \(-1 \lesssim p \lesssim 0\) mimics saturation in CGC-based models.

The reduced thickness possesses several other key properties. It is

(a) continuous and monotonically increasing:

if \( T_A \leq T_A' \) and \( T_B \leq T_B' \), then \( T_R \leq T_R' \);

(b) symmetric in the projectile thicknesses:

\( T_R(p; T_A, T_B) = T_R(p; T_B, T_A) \);

(c) independent of \( p \) when the thicknesses are equal:

\( T_R(p; T, T) = T \); and

(d) vanishes when \( T_A \) and \( T_B \) vanish: \( T_R(p; 0, 0) = 0 \).

In fact, for \( p \leq 0 \), \( T_R \) vanishes if either \( T_A \) or \( T_B \) do.

On the other hand, positive values of \( p \) introduce small violations of the eikonal entropy production postulate, similar to a Glauber model.

Finally, the reduced thickness provides the basis of the model name: \( \text{TRENTo} \), for Thickness-Reduced Event-by-event Nuclear Topology.

We now detail the construction of the thickness functions \( T_{A,B}(x, y) \), which combined with the definition of the reduced thickness completes the specification of the model. The procedure is constructed from the ground up to handle a variety of collision systems; we begin with the simplest case.

Consider a collision of two protons \( A, B \) with impact parameter \( b \) along the \( x\)-direction and nuclear densities

\[
\rho_{A,B} = \rho(\text{proton})(x \pm b/2, y, z),
\] (5)

and assume that the integral \( \int dz \rho_{\text{proton}} \) either has a closed form or may be evaluated numerically, so that the proton thickness functions can be calculated. The protons collide with probability \[29\]

\[
P_{\text{coll}} = 1 - \exp \left[ -\sigma_{gg} \int dx \int dy \int dz \rho_{A} \int dz \rho_{B} \right],
\] (6)

where the integral in the exponential is the overlap integral of the proton thickness functions and \( \sigma_{gg} \) is an effective parton-parton cross-section tuned so that the total proton-proton cross-section equals the experimental inelastic nucleon-nucleon cross-section \( \sigma_{N,N} \).

\( P_{\text{coll}} \) is sampled once to determine if the protons collide; assuming they do, each is assigned a fluctuated thickness

\[
T_{A,B}(x, y) = \gamma_{A,B} \int dz \rho_{A,B}(x, y, z),
\] (7)

where \( \gamma_{A,B} \) are independent random numbers sampled from a gamma distribution with unit mean,

\[
P(\gamma; k) \propto \gamma^{k-1} e^{-\gamma/k},
\] (8)

with the shape parameter \( k > 0 \) to be fixed by experiment. The gamma distribution is chosen for its flexibility—it is exponential for \( k = 1 \) and becomes Gaussian for large \( k \)—and because it is the continuous analog of the negative binomial distribution, which has historically been used to fit proton-proton multiplicity fluctuations.

With the projectile thickness functions in hand, the reduced thickness is calculated to furnish the initial transverse entropy profile up to an overall normalization factor,

\[
dS/dy|_{\tau = \tau_0} \propto T_R(p; T_A, T_B).
\] (9)

Composite collision systems such as proton-nucleus and nucleus-nucleus are essentially treated as superpositions of proton-proton collisions. A set of nucleon positions is chosen for each projectile \( A, B \), typically by sampling an uncorrelated Woods-Saxon distribution or from more realistic correlated nuclear configurations when available [32]. The collision probability [6] is sampled for each pairwise interaction and those nucleons that collide with at least one partner are labeled "participants" while the rest are discarded. The fluctuated thickness function of nucleus \( A \) then reads

\[
T_A = \sum_i \gamma_i \int dz \rho_{\text{proton}}(x - x_i, y - y_i, z - z_i),
\] (10)
where $\gamma_i$ and $(x_i, y_i, z_i)$ are the random fluctuation factor and position, respectively, of participant $i$ in nucleus $A$. $T_B$ follows analogously.

This completes the construction of TRENTo. In summary, the model deposits entropy proportional to the reduced thickness function $F_\gamma$. The reduced thickness is defined as the generalized mean of fluctuated projectile thickness functions $\langle R_i \rangle$, where the projectile thicknesses consist of only participant nucleons, each independently fluctuated by a gamma random number $\gamma_i$.

We now demonstrate TRENTo’s ability to simultaneously describe a wide range of collision systems. Note that the reduced thickness parameter $p$, gamma fluctuation parameter $k$, and nucleon profile $\rho_{\text{proton}}$ are not rigorously constrained—to do so would require a systematic model-to-data comparison which is beyond the scope of this work. Therefore, the following results do not necessarily represent the best-fit of the model to data.

The experimentally observed charged-particle multiplicity $N_{\text{ch}}$ is to a very good approximation proportional to the total initial entropy $S$, and hence proportional to the integrated reduced thickness:

$$N_{\text{ch}} \propto \int dx \, dy \, T_R.$$  \hfill (11)

To compare with experiment, we generate a large ensemble of minimum-bias events, integrate their $T_R$ profiles, and rescale by an overall normalization constant.

The left panel of FIG. 2 shows the $N_{\text{ch}}$ distribution from $10^6$ proton-proton simulations using reduced thickness parameter $p = 0$ (geometric mean), gamma fluctuation parameter $k = 1.25$, and Gaussian beam-integrated proton density

$$\int dz \rho_{\text{proton}} = \frac{1}{2\pi B} \exp\left(-\frac{x^2 + y^2}{2B}\right)$$  \hfill (12)

with effective area $B = (0.6 \text{ fm})^2$. Proton-lead and lead-lead distributions (middle and right panels) use identical model parameters, except for the overall normalization factor, whose $\sim 20\%$ variation across collision systems is consistent with differences in experimental beam energy and kinematic cuts (annotated in the figure).

The $N_{\text{ch}}$ distributions compare favorably with the corresponding experimental measurements by ALICE [30, 31]. TRENTo simultaneously reproduces the shapes of all three distributions, similarly to how the participant quark model fits the transverse-energy distributions of several collision systems at RHIC [32].

Eccentricity harmonics $\varepsilon_n$ are calculated using the definition

$$\varepsilon_n e^{in\phi} = -\frac{\int dx \, dy \, r^n e^{in\phi} T_R}{\int dx \, dy \, r^n T_R}.$$  \hfill (13)

Figure 3 shows ellipticity $\varepsilon_2$ and triangularity $\varepsilon_3$ for lead-lead collisions as a function of centrality for reduced thickness $p = 1, 0, -1$. There is a clear trend of increasing eccentricity (particularly $\varepsilon_2$) with decreasing $p$. This may be understood by the attenuation property of the reduced thickness: as $p$ decreases, asymmetric regions of the collision produce less entropy, which accentuates the elliptical overlap shape in non-central collisions and enhances their eccentricity.

In addition, we perform the test proposed by [17], which uses flow data and hydrodynamic calculations to determine an experimentally allowed band for the ratio of root-mean-square eccentricities $\sqrt{\langle \varepsilon_2^2 \rangle}/\sqrt{\langle \varepsilon_3^2 \rangle}$ as a function of centrality. Among available initial condition models only IP-Glasma consistently falls within the allowed region. As shown in the right panel of FIG. 3, TRENTo yields excellent agreement with the allowed band for $p = 0$ (geometric mean).

As a final novel application, we test the performance of TRENTo in ultra-central uranium-uranium collisions, where typical Glauber models are notably inconsistent with experimental data. Unlike e.g. gold and lead, uranium nuclei have a highly deformed prolate spheroidal

![FIG. 2. Multiplicity distributions for proton-proton, proton-lead, and lead-lead collisions. The blue histograms are TRENTo results from $10^6$ minimum-bias events for each collision system, all with reduced thickness parameter $p = 0$ (geometric mean) and gamma fluctuation parameter $k = 1.25$. The normalization constants indicated in the legends are tuned to match the experimental distributions (points with error bars) from ALICE [30, 31].]
shape, so uranium-uranium collisions may achieve maximal overlap via two distinct orientations: “tip-tip”, in which the long axes of the spheroids are aligned with the beam axis and the overlap area is circular; or “side-side”, where the long axes are perpendicular to the beam axis and the overlap area is elliptical, as shown in FIG. 4. Hence side-side collisions will in general have larger initial-state ellipticity $\varepsilon_2$ and final-state elliptic flow $v_2$ than tip-tip.

In the two-component Glauber model, tip-tip collisions produce more binary nucleon-nucleon collisions than side-side, so tip-tip collisions have larger charged-particle multiplicity $N_{\text{ch}}$. Therefore, the most central uranium-uranium events are dominated by tip-tip collisions with maximal $N_{\text{ch}}$ and small $v_2$, while side-side collisions have a smaller $N_{\text{ch}}$ and somewhat larger $v_2$. This predicted drop in elliptic flow as a function of $N_{\text{ch}}$ is known as the “knee” [36].

Recent data by STAR on uranium-uranium collisions exhibits no evidence of a knee [35, 37], at odds with Glauber model predictions. It has been proposed that fluctuations could wash out the knee [38], but a recent flow analysis suggests that it would still be visible [39].

The reduced thickness ansatz [3] does not exhibit binary collision scaling and hence predicts a roughly constant ellipticity $\varepsilon_2$ in ultra-central uranium-uranium events, as shown in FIG. 5. The slope of $\varepsilon_2$ as a func-
tion of $N_{ch}$ is approximately equal for uranium-uranium and gold-gold, in contrast to the Glauber model which predicts a much steeper slope for uranium. Short of conducting a full hydrodynamic analysis, TRENTo appears to be more consistent with STAR data than the Glauber model, and behaves similarly to IP-Glasma \[37\].

In summary, we have developed TRENTo, a new initial condition model for high-energy nuclear collisions based on eikonal entropy deposition via a “reduced thickness” function. The model simultaneously fits proton-proton, proton-nucleus, and nucleus-nucleus multiplicity distributions along with lead-lead eccentricity harmonics, and provides a possible resolution to the “knee” puzzle in ultra-central uranium-uranium collisions.

TRENTo is an effective model. While it does not assume a particular physical mechanism for entropy production or include pre-equilibrium dynamics, it is computationally cheap, flexible by design, and mimics the characteristics of other initial condition models, e.g. its multiplicity distributions are similar to participant quark model predictions, and eccentricity harmonics resemble those of IP-Glasma.

As a next step, we will couple the initial condition generator to viscous relativistic fluid dynamics as part of a complete simulation of the spacetime evolution of relativistic heavy-ion collisions \[15\]. This will form the basis of a modern framework for extracting fundamental sQGP properties through systematic model-to-data comparison. Additionally, we plan to implement subnucleonic structure and generalize the model to three dimensions, enabling study of collective behavior in proton-proton and proton-nucleus collisions.

TRENTo will be publicly released in early 2015 and made available at \texttt{github.com/Duke-QCD/trento}. The community is encouraged to use, test, and contribute to the source code.

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