In conjunction with recent numerical $\lambda \partial_0 A_0 + \nabla \cdot \vec{A} = 0$ “$\lambda$-gauge” results reported in a companion paper, we construct an $N \to \infty$ Wilson loop picture of $\lambda$-gaugefixing in which (I) the $\lambda$-gauge expectation value of a link chain $C$ is the weighted sum over Wilson loops made by joining to $C$ all selfavoiding chains $\tilde{C}$ closing $C$. (II) Weights $A_{\tilde{C}}$, containing all the $\lambda$-dependence, are given by the $\beta = 0$ $\lambda$-gauge expectation value of $\tilde{C}$. (III) $A_{\tilde{C}}$ equals path-products of coefficients from the trace expansion of the gaugefixing Boltzmann weight. From (II) and (III) we deduce formulas for $\beta = 0$ quark matrix elements. We find that $M_{q}^{(\lambda)}$ decreases with increasing $\lambda$; the quark propagator dispersion relation is not covariant when $\lambda \neq 1$; and $\Delta I = 1/2$ matching coefficients are $\lambda$-independent. These strong coupling features are qualitatively consistent with numerical $\beta = 5.7$ and 6.0 results briefly described here for comparison purposes but mainly presented in a companion paper.
I. MOTIVATION AND RESULTS

Traditionally gaugefixing is done only in weak coupling perturbation theory, where it is needed to define perturbative quark and gluon propagators. In lattice QCD, gaugefixing is unnecessary for computing gauge invariant correlation functions. However, since local gauge symmetry cannot break spontaneously the only way to see nonvanishing gauge variant correlation functions is by gaugefixing. Accordingly, lattice gaugefixing has drawn considerable attention in recent years. As described in our companion paper, gluon, quark and photon propagators, effective masses, and wavefunctions have been studied in special cases of “λ-gauges”

\[ \lambda \partial_0 A_0 + \nabla \cdot \vec{A} = 0. \]  

The numerical work has prompted analytical and computational studies of longitudinal and topological gaugefixing ambiguities and their effects on gluon, quark and photon correlation functions and operator product expansion coefficients determined from gauge covariant matching conditions.

Nonperturbative gaugefixing is a complex subject. Since quarks are confined \( M_q \) may (or may not) depend on gauge. For example, compare the exactly solvable Schwinger model in Coulomb gauge to covariant gauges parametrized by gauge parameter \( \xi \). (We were not able to solve the model in \( \lambda \)-gauges.) While the actual situation in dimension \( D = 4 \) QCD and lattice QCD may be arguably different, it is helpful to have a litmus test for discarding broad arguments (“Mass is gauge invariant in perturbation theory; hence quark mass is gauge invariant.”) which do not distinguish between the Schwinger model and other gauge theories. In the Schwinger model, quarks are confined but the photon is physical and has a mass from the \( U_A(1) \) anomaly. We define “effective mass” as the inverse correlation length of the zero momentum propagator. While the photon has a gauge invariant effective mass (equal to its physical mass), the effective quark mass varies with gauge parameter \( \xi \). Coulomb gauge—the unitary gauge for the Schwinger model—has no unphysical modes and the quark propagator is the physical amplitude for quark propagation. Due to dielectric breakdown of the vacuum, the quark propagator—the amplitude to have just one quark at \( x \neq 0 \) starting with an \( x = 0 \) quark—vanishes and the effective Coulomb gauge quark mass diverges. In covariant gauges, the presence of unphysical modes (or, alternatively, Gupta-
Bleuler physical-state conditions on the Hilbert space) ruins this physical interpretation of the quark propagator. Covariant gauge quark propagators are not amplitudes for quark propagation and do not vanish despite confinement.

As reported in our companion paper [3], effective quark and gluon masses $M_q$ and $M_G$ were evaluated on $\beta = 5.7$ and $6.0$ quenched $D = 4$, color $N = 3$ Wilson lattices in $\lambda$-gauges by matching gaugefixed $\vec{p} = 0$ propagators at large $t_E$ to the free particle ansatz

$$\lim_{t_E \to \infty} \sum_x e^{i\vec{p} \cdot \vec{x}} \langle V_x \psi_x \bar{\psi}_0 V_0^\dagger \rangle = Z_q^{(\lambda)} \left( \frac{M_q^{(\lambda)} + i\vec{p}}{2E_q^{(\lambda)}} \right) e^{-E_q^{(\lambda)}|t_E|},$$

where $E_q^{(\lambda)}(\vec{p} = 0) \equiv M_q^{(\lambda)}$ and $Z_q^{\text{free}} = 1$. The “$(\lambda)$” superscript anticipates $\lambda$-dependence, although sometimes we will omit it for brevity. The role of background gauge field gauge transformations $V_x$ will be explained shortly. The idea of monitoring chiral symmetry with quark masses goes back to the Gross-Neveu model [13], where the flavor $N_f \to \infty$ effective quark mass $M_q = M_c + \pi m_q + O(m_q^2)$ is a continuous, increasing function of bare mass $m_q$.

Similarly, matching the vacuum expectation value of the operator product expansion of $\psi_x \bar{\psi}_0$ in massless QCD to a free fermion propagator yields for $N = 3$, flavor $N_f = 3$ and Landau gauge [14]

$$\lim_{\mu^2 > -p^2 \to \infty} M_q(\mu, \mu) \sim \frac{4g^2(\mu_o)}{p^2} \left( \frac{g(\mu)}{g(\mu_o)} \right)^{8/9} \langle \langle \bar{\psi}_0 \psi_0 \rangle |_{\mu} \rangle.$$

Numerical fits in lattice QCD to $M_q^{(\lambda)} = b^{(\lambda)} M_\pi^2 + M_c^{(\lambda)}$, motivated by the CPTh relation $M_\pi^2 \propto m_q$ between pion and current quark mass [3], yield $b^{(1)} \sim 2.7(3) \times 10^{-4}/MeV$ and $M_c^{(1)} \sim 350(40) MeV$ at $\beta \sim 6.0$. At both $\beta = 5.7$ and $6.0$ effective quark and gluon masses decrease as $\lambda$ grows [3] so that, roughly, $M_q^{(2)}/M_q^{(1)} \sim 0.9$ and $M_q^{(25)}/M_q^{(1)} \sim 0.75$, all plus or minus $\sim 15\%$ jackknife errors.

In this paper, we put forth a $\lambda$-gauge $\beta = 0$, color $N \to \infty$ solution of lattice QCD with quenched Wilson fermions in an infinite volume lattice. Lattice gaugefixing is implemented by a lattice Fadeev-Popov method; we couple the links to a quenched Higgs gaugefixing field $\{V_x\}$ in the fundamental representation of $SU(N)$ [3]. Gluons do not propagate at $\beta = 0$, where link fields oscillate randomly. Let us focus momentarily on quark propagators. $\lambda$-gauge quark propagators are $V_x \psi_x$ “meson” propagators in this formulation—as written in (3). An expression for $V_x \psi_x$ meson propagators follows from hopping expanding the Higgs and quark fields, doing the $\beta = 0$ link integrals—which project out all but zero area Wilson

3
loops—and resumming the hopping expansions. However, as described in Section II C, we are unable to resum graphs where Higgs paths recur (Recurrence and other such notions are defined in Section II A.) since the hopping expansion weight of such paths differ from same-pathlength nonrecurrent graphs. To get around this we resort to a “trace orthogonality approximation,” which does not differentiate between recurrent and nonrecurrent Higgs paths. This approximation is tantamount to taking the $N \to \infty$ limit before resumming because, as we show in Section II C, there is no difference between recurrent and nonrecurrent paths in the brutally truncated $N \to \infty$ limit. Taking $N \to \infty$ before resummation is an approximation because the hopping expansion is (apparently) not absolutely convergent at infinite $N$.

In the trace orthogonality approximation, only selfavoiding quark paths contribute to the $\beta = 0$ quark propagator because the $\{V_x\}$, being quenched, dress only quark paths without internal loops. Hence nonselfavoiding quark paths are suppressed by infinite string tension at $\beta = 0$. However, since are unable to (re)sum over only selfavoiding quark paths, we make an additional approximation and sum over all (for technical reasons) nonbacktracking quark paths. When we do this we find, as shown in Section III, that the $\beta = 0$, $r = 0$, $N \to \infty$ zero momentum quark propagator pole $M_q^{(\lambda)}$ is analytic and linear in $m_q$ as $m_q \to 0$. Expanding

$$M_q^{(\lambda)} = M_c^{(\lambda)} + B^{(\lambda)} m_q + \mathcal{O}(m_q^2)$$ \hspace{1cm} (4a)

yields

$$M_c^{(\lambda)} = \frac{\sqrt{2D - 1}}{2(32D - 7)\lambda} \begin{cases} 11 + 14D - 25\lambda^2 & \lambda \leq \frac{1}{2}; \\ -25 + 200\lambda + 224(D - 1)\lambda^2 & \lambda \geq \frac{1}{2}; \end{cases}$$ \hspace{1cm} (4b)

$$B^{(\lambda)} = \begin{cases} \frac{499 - 1474D + 1600D^2 + (1376D - 751)\lambda^2}{2\lambda(32D - 7)^2} & \lambda \leq \frac{1}{2}; \\ \frac{-751 + 1376D + 6008\lambda - 11008DX + (-4032 - 1568D + 25600D^2)\lambda^2}{8(32D - 7)^2\lambda(4\lambda - 1)} & \lambda \geq \frac{1}{2}. \end{cases}$$ \hspace{1cm} (4c)

Note that the $\lambda$-dependence of $M_c^{(\lambda)}$ is qualitatively the same whether $D \to \infty$ (where the number of recurrences and selfintersections are negligible) or $D = 4$. This suggests that recurrences and selfintersections are not responsible for $\lambda$-dependence. Therefore we generally quote formulas for all $D$ when we might be safer with their $D \to \infty$ limits. In any
case, we do not find any qualitative difference between the $\lambda$-dependence of $M_q^{(\lambda)}$ at finite and infinite $D$ in our approximation scheme.
FIGURES

FIG. 1. λ-dependence comparison of numerical quark masses to the β = 0, N → ∞ Formula (4b) evaluated at D = 4. The numerical masses are rescaled so that at λ = 1 all data points are normalized to \( M_q^{(1)} \). Ref. [3] provides details of the numerical simulation.

As depicted in Fig. 1, Eq. (4b) mimics the λ-dependence of the β = 5.7 and 6.0 numerical data. Most of the change occurs between λ = 1 and λ = 2, and \( M_q^{(λ)} \) stabilizes to a nonzero value as \( λ \to \infty \). This qualitative agreement between strong coupling and numerical behavior helps give confidence that the numerical λ-dependence of \( M_q \) is not a finite volume artifact or due to details of how \( M_q \) is extracted in the numerical simulations.

For technical reasons we compute the quark propagator dispersion relation not at \( r = 0 \) but at \( r = 1 \), where it is

\[
E_q^{(λ)} = M_q^{(λ)} + \sum_{i=1}^{D-1} \overline{g}_i^2 \overrightarrow{p}_i^2 + \mathcal{O}(a^3) , \quad E_q^{(λ)} = ±ip_0 ,
\]

(5a)

\[
\overline{g}_i(λ) \equiv \begin{cases} 
\frac{3}{4λ} & λ \leq \frac{1}{2}; \\
\frac{3λ}{4λ-1} & λ \geq \frac{1}{2}.
\end{cases}
\]

(5b)

Since \( \overline{g}_i(λ \neq 1) \neq 1 \), the quark dispersion relation is not covariant and, hence, the propagator is not free particle-like except in Landau gauge. \( \overline{g}_i \) drops out if \( \overrightarrow{p} = 0 \), where the quark propagator is indistinguishable from the free particle propagator. (However \( M_q^{(λ)} \) remains λ-dependent.)

As described in Section IIIC, matching coefficients for \( \sigma d \) subtraction of \( ΔI = 1/2 \) Rule operators in the \( β = 0, N \to \infty \) limit assuming trace orthogonality are given by

\[
α_{O_±[Γ_1Γ_2]} = (δ_{Γ_1Γ_2,SS} + \frac{1}{2D/2N} f_{Γ_1Γ_2}^{SS} ) \langle ψ^0 ψ^0 \rangle
\]

(6)

where Fierz coefficients \( f_{Γ_1Γ_2}^{Γ_3Γ_4} \) are given in Eq. (87b). Since \( \langle ψ^0 ψ^0 \rangle \) is gauge invariant these β = 0 matching coefficients, which are directly related to physical continuum decay rates, are gauge invariant. At β = 6.0, \( α_{O_±[Γ_1Γ_2]} \) also seem to be λ-independent up to statistical errors [3].
II. LATTICE GAUGEFIXING

As exemplified by several models—notably QED\(_{3+1}\)—the thermodynamical limit of strong coupling lattice gauge theories may correspond to different field theories than the \(\beta \to \infty\) ones. In particular, proving gauge dependence of quark mass in the lattice Schwinger model would not reveal much about quark mass in the continuum Schwinger model since the lattice Schwinger model has a phase transition in \(\beta\) [15]. (This transition, if the critical point is unique, doesn’t ruin confinement since Schwinger model quarks are confined at both weak and strong coupling.) Analogously, the following \(\beta = 0, N \to \infty\) solution should be viewed as a toy model and not something necessarily related to QCD.

The \(\lambda\)-gauge lattice expectation value of a lattice operator \(\Theta\) is

\[
\langle \Theta \rangle \equiv \left[ \left[ [\Theta]_f\right]_v \right]_u
\]

where

\[
[\Theta]_\theta \equiv z_{\theta}^{-1} \int [d\theta] e^{-S_\theta} \Theta, \quad z_\theta \equiv \int [d\theta'] e^{-S_{\theta'}} , \quad \theta \in \{f, v, u\},
\]

\[
S_f \equiv \sum_{x,y} \psi_x \left[ \delta_{x,y} + K_B \sum_n (r - \hat{n}) U_{x,n} \delta_{y,x+\hat{n}} \right] \psi_y ,
\]

\[
S_v[\Omega] \equiv - \sum_{x,n} \frac{J_n}{2} \text{tr} \Omega_{x,n} , \quad n \in \pm \{0, \cdots, D - 1\},
\]

\[
J_{-n} \equiv J_n \equiv \lambda_n / \xi, \quad \lambda_\mu \equiv (\lambda, \bar{\lambda}, \cdots, \bar{\lambda}), \quad \Omega_{x,n} \equiv V_x U_{x,n} V_{x+\hat{n}}^\dagger ,
\]

\[
S_u[U] = S_u[\Omega] = \beta \text{ Re} \sum \text{tr} \square .
\]

The sum in (7f) ranges over all lattice plaquettes \(\text{tr} \square\). In (7c) quarks are quenched by choice. Consistency with the Fadeev-Popov method requires \(\{V_x\}\) to be quenched—inverse partition function \(z^{-1}_v = z^{-1}_v[U]\) plays the role of lattice Fadeev-Popov determinant. Since as described in Section [13] the hopping expansion turns \([\Theta]_f\) into a composite operator of links and transformations, in this Section we focus on the gaugefixed expectation value \(\langle V_{y\delta} V_{w\rho}^\dagger \rangle\) of continuous link chains \(y\delta\). The strategy is to integrate out \(\{V_x\}\) to get an expression for

\[
V_{\alpha\beta,ij}[U] \equiv \left[ (V_y)_{\alpha\beta} (V_w^\dagger)_{ij} \right]_v .
\]
This integration is simple enough to be practical because gauge symmetries relate \( \mathcal{V} \) to continuous link chains and \( z^{-1}_w \) suppresses all disconnected link chain loops. Then gaugefixing can be viewed as an operator insertion of \( \mathcal{V} \) into

\[
\langle V_y(y\dot{w})V_w^\dagger \rangle_{\alpha j} = \left[ [V_{yy}\dot{w}V_w^\dagger] \right]_{\alpha j} = \left[ (y\dot{w})_\beta i \mathcal{V}_{\alpha\beta;ij}[U] \right]_u,
\]

the usual gauge invariant lattice expectation value.

We will show that in the \( \xi \propto 1/N \) and \( N \rightarrow \infty \) limit

\[
[(V_y)_{\alpha\beta}(V_w^\dagger)_{ij}]_v = \delta_{\alpha j} \sum_{w\dot{y}} A_{w\dot{y}} (w\dot{y})_{i\beta} \quad (\text{as } N \rightarrow \infty),
\]

where the sum ranges over a complete set of selfavoiding continuous link chains \( w\dot{y}. \) Orthonormality of the \( w\dot{y} \) with respect to inner product

\[
\langle \Theta_1 | \Theta_2 \rangle \equiv \int [dU] \frac{1}{N} \text{tr}(\Theta_1^\dagger \Theta_2), \quad [dU] \equiv \prod_{x,\mu} dU_{x,\mu}
\]

identifies coefficients \( A_{w\dot{y}} \) with \( \beta = 0 \) \( \lambda \)-gauge expectation values:

\[
A_{w\dot{y}} = \frac{1}{N^2} \int [dU] \text{tr}(y\dot{w}[V_w^\dagger V_y]_v) = \frac{1}{N^2} \text{tr}(V_{yy}\dot{w}V_w^\dagger)_{\beta=0}.
\]

If \( y = w = x \), the only continuous selfavoiding chain is \( x\dot{x} = 1 \). Since \( V_x V_x^\dagger = 1 \), Eq. (12) implies \( A_{x\dot{x}} = 1/N \). Hence \( [(V_x)_{\alpha\beta}(V_x^\dagger)_{ij}]_v \) is independent of \( [U] \) and

\[
\langle (V_x)_{\alpha\beta}(V_x^\dagger)_{ij} \rangle = \langle (V_x)_{\alpha\beta}(V_x^\dagger)_{ij} \rangle_v = \frac{1}{N} \delta_{\alpha j} \delta_{\beta i} = \int dV \ V_{\alpha\beta}V_{ij}^\dagger.
\]

Eq. (13) implies gaugefixing does not affect closed link loops \( \bigcirc_x \) since

\[
\langle (V_x \bigcirc_x V_x^\dagger)_{\alpha j} \rangle = \frac{1}{N} \delta_{\alpha j} \left[ \text{tr}\bigcirc_x \right]_u = \langle (\bigcirc_x)_{\alpha j} \rangle.
\]

### A. Link Operator Definitions

A continuous link chain or randomwalk path may selfintersect (touch itself at right angles), recur (touch itself at 0° or 180°), or backtrack (immediate recurrence as in the sequence “\( \cdots U_{x,\mu} U_{x+\hat{\mu},-\mu} \cdots \)”). A selfavoiding link chain or randomwalk path does not selfintersect or recur.
A tree $T$ is a continuous random walk path which is nonbacktracking but may be self-intersecting and otherwise recurrent. A branch $B$ is a continuous, possibly self-intersecting and definitely recurring random walk loop enclosing a zero area minimal surface. A tip of $B$ is a backtracking subsegment of $B$. Every $B$ has at least one tip, perhaps more. Every random walk path is a sequence of trees and branches.

Continuous link chains extending from $y$ to $w$, not necessarily straight or self-avoiding, are denoted by

$$ y \star w \equiv U_{y,\mu} U_{y,\mu+\nu} \ldots U_{w,\delta,\delta} , \quad y \star w^\dagger = w \star y $$

where “…” denotes a continuous but not necessarily self-avoiding link chain. Selfavoiding link chains $y \tilde{\star} w$ are accented with “$\tilde{}$.” Examples of recurrent and self-intersecting link chains are $\square$, a continuous chain which traces out a unit box and recurs at one side, and $\chi$, which wraps around a unit box and selfintersects at one corner. Link chain loops $x \star x$ are also denoted by $\bigcirc$. The unit square chain is $\bigcirc \equiv \square$, whose trace is the plaquette. If $\bigcirc$ begins and ends at $x$, then it is denoted $\bigcirc_x$.

If the path traced out by $\bigcirc_x$ is a branch, then it is denoted $\bigcirc_x^B$. Since $U_{x,\mu} U_{x,\mu+\nu} = 1$}

$$ \bigcirc_x^B = 1 \quad \forall \bigcirc_x^B . $$

(16)

Factorizing link chains into a product of tree and branch subsegments yields

$$ y \star w = \bigcirc_y^B \bigcirc_x^B \bigcirc_x^B \cdots \bigcirc_z^B z \star w \bigcirc_w^B = y \star T_w . $$

(17)

Hence we will assume that $y \star w = y \star T_w$ unless specifically noted.

By $SU(N)$ identities such as Schur’s lemma,

$$ \int dU \ D_{ij}^{(\nu)} (U) \ D_{\alpha\beta}^{(\nu')} (U) = \frac{1}{\dim(\nu)} \delta^{\nu\nu'} \delta_{i\alpha} \delta_{j\beta} , $$

(18)

continuous link chains are orthonormal with respect to (11). However some disconnected chains mix with each other. For example, a nonorthonormal basis for gauge invariant link operators is

$$ \mathcal{B}_o \equiv \{1, \{\text{tr}\bigcirc\}, \{\text{tr}\bigcirc \text{tr}\bigcirc\}', \cdots\} \ 1 , \ \text{tr}1 = N $$

(19)

where $\{\bigcirc\}$ is the set of all nonbacktracking link loops in the lattice. It is possible to orthonormalize $\mathcal{B}_o$ by constructing irreducible combinations out of its link operators which mix.
B. Residual Gauge Symmetries and the Link Expansion

QCD gauge transformations are
\[ U_{x,\mu} \rightarrow Q_x U_{x,\mu} Q_x^\dagger_{x+\hat{\mu}}, \quad V_x \rightarrow V_x, \quad \psi_x \rightarrow Q_x \psi_x. \]  
(20)

While \( S^v \) breaks QCD gauge symmetry, \( S^u, S^f, \) and \( S^v \) are invariant under
\[ U_{x,\mu} \rightarrow R_x U_{x,\mu} R_x^\dagger, \quad V_x \rightarrow V_x R_x^\dagger, \quad \psi_x \rightarrow R_x \psi_x. \]  
(21)

We will refer to (20) as “\( Q \)” transformations and (21) as “\( R \)” transformations. \( R \) is a symmetry of \( [\Theta]_{\theta \in \{f,v,u\}} \) (and hence \( \langle \Theta \rangle \)) whereas \( Q \) is a symmetry of only \( [\Theta]_{\theta \in \{f,u\}} \). By Elitzur’s theorem [1] any \( R \)-variant operator such as \( V_x \) and \( U_{x,\mu} \) has zero \( \langle \Theta \rangle \) expectation value. \( R \)-invariant operators, including \( Q \)-variant ones like \( \Omega \), are not suppressed.

Global transformation “\( L_{XY} \)” where
\[ U_{x,\mu} \rightarrow U_{x,\mu}, \quad V_x \rightarrow L_{XY} V_x, \quad \psi_x \rightarrow \psi_x \]  
(22)
is a symmetry of \( [\Theta]_{\theta \in \{f,v,u\}} \). \( L_{XY} \) is equivalent to global color symmetry. The \( XY \) designation is because in the \( \beta \rightarrow \infty \) limit the system approaches the \( XY \) model, which has \( L_{XY} \) symmetry.

Under \( U_{x,\mu} \rightarrow U'_{x,\mu} = Q_x U_{x,\mu} Q_x^\dagger_{x+\hat{\mu}} \) partition function \( z_v[U] \) obeys
\[ z_v[U'] = \int [dV] e^{-S^v[\Omega']} = \int [d(VQ^\dagger)] e^{-S^v[M]} = z_v[U]. \]  
(23)

As a \( Q \) invariant functional of links \( z_v \), the inverse of the Fadeev-Popov determinant, is expandable in terms of gauge invariant link structures. Such structures are comprised of links joined together into closed networks with gauge invariant bonds \( \delta_{ij} \) and \( \epsilon_{i_1...i_N} \). Let us take \( N \mapsto \infty \) or \( U(1) \) and throw out the latter. Then following (19), with \( \circ \neq 1 \) and \( \circ' \neq 1 \),
\[ z_v[U] = (1 + \sum_{\circ} Z_v^\circ \text{tr} \circ + \sum_{\circ' \neq \circ} Z_v^{\circ\circ'} \text{tr} \circ \text{tr} \circ' + \cdots) Z_v^\circ, \]  
(24a)

\[ Z_v^\circ \equiv \int [dU] z_v[U] = Z_v^1 + \sum_{\circ \neq 1} Z_v^{\circ\circ'} \text{tr} \circ \text{tr} \circ' + \cdots \]  
(24b)
The reason for factorizing out $Z^o_v$ in (24a) and the reason it can be expanded in terms of zero area (colorless) link structures is because $Z^o_v$ is the colorless part of $z_v[U]$. Since it is colorless, $Z^o_v$ survives at $\beta = 0$.

If $W$ is in the fundamental representation and $\chi^{(\nu)}$ the character of irreducible representation $\nu$, the character expansion of $e^{z\text{Retr}W}$ is given by

$$b_{\nu}(z) = b_{\nu}(z) \equiv \int dW \chi^{(\nu)}(W) e^{\frac{z}{2} \text{tr}(W + W^\dagger)},$$

$$e^{\frac{z}{2} \text{tr}(W + W^\dagger)} = \sum_{\nu} b_{\nu}(z) \chi^{(\nu)}(W), \quad f_{\nu} \equiv \frac{b_{\nu}}{N b_1}. \tag{25a}$$

Since by (7d) $e^{-S_v[\Omega]} = \prod_{x,\mu} e^{J_{\mu}\text{Retr}_x \Omega_{x,\mu}}$,

$$e^{-S_v[\Omega]} = Z^o_v \sum_{\{\nu_{x,\mu}\}} \prod_{x,\mu} N f_{\nu_{x,\mu}}(J_{\mu}) \chi^{(\nu_{x,\mu})}(\Omega_{x,\mu}) \tag{25c}$$

where the reason for factoring out $Z^o_v$ will be apparent shortly and where

$$/\Theta/ \equiv \int dW \Theta e^{\frac{z}{2} \text{tr}(W + W^\dagger)} \int dW' e^{\frac{z}{2} \text{tr}(W' + W'^\dagger)}, \quad f_N(z) \delta_{ij} \equiv /W_{ij}/, \tag{25d}$$

$$\|f_N(z)\| \leq 1, \quad \lim_{z \to 0} f_N(z) = \frac{z}{2N}; \quad f_N(\infty) = 1 \quad \text{(finite } N). \tag{25e}$$

The first inequality in (25e) follows from $\|\text{tr}W\| \leq N$; the second from expanding $e^{z\text{Retr}W}$ in $z$; the third since the integrands are dominated by MaxRetr$W$ when $z \to \infty$. Integrating both sides of (24a) over $[dU]$, interchanging integration order $[dU][dV] \to [dV][dU]$ and changing variables $[dU]$ to $[d\Omega]$ yields

$$Z^o_v = \int [dU][dV] \prod_{x,\mu} e^{J_{\mu}\text{Retr} \Omega_{x,\mu}} = \prod_{x,\mu} b_1(J_{\mu}) \tag{26}$$

$L_{XY}$ symmetry implies

$$[(V_y)_{\alpha\beta}(V_w^\dagger)_{ij}]_v = \delta_{\alpha j} \ t_{i\beta} [U; w, y]. \tag{27}$$

Since $t[U; w, y]$ transforms under local $R$ like $[(V_y)_{\alpha\beta}(V_w^\dagger)_{ij}]_v$, at $y$ it transforms like $N$, at $w$ like $\nabla$, and the links away from $y$ and $w$ must be bonded gauge invariantly either by the $SU(N)$ identity tensor or the completely antisymmetric $\epsilon$ tensor.

To appreciate that (10) is not a selfevident result—it’s valid only in the $N \to \infty$ limit and orthogonal trace approximation to be described—consider the $U(1)$ case where
\[ U_{x,\mu} \equiv e^{i\phi_{x,\mu}}, \quad V_{x} \equiv e^{i\theta_{x}}, \quad S^{\nu} = - \sum_{x,\mu} J^{\mu} \cos(\theta_{x} - \theta_{x+\mu} + \phi_{x,\mu}). \] (28a)

The \( U(1) \) character expansion is given by
\[ e^{z \cos \theta} = I_{0}(z) \sum_{n=-\infty}^{\infty} t_{n}(z)e^{-in\theta}, \quad \chi^{(n)}(e^{-i\theta}) = e^{-in\theta}, \] (28b)
\[ I_{-n}(z) = I_{n}(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\theta \ e^{in\theta + z \cos(\theta)}, \quad t_{n} \equiv I_{n}/I_{0}, \] (28c)
\[ \varphi_{\nu}[\phi] = z_{\nu}/Z_{\nu}^{0} = \sum_{\{n_{x,\mu}\}} \prod_{x,\mu} t_{n_{x,\mu}}(J_{\mu}) e^{-i\nu_{x,\mu}\phi_{x,\mu}}, \] (28d)
\[ \{n_{x,\mu}\} \equiv \{\{n_{x,\mu}\}|\sum_{m} n_{x,m} = 0\}, \] (28e)
\[ [V_{y} V_{w}^{\dagger}]_{v} = \varphi_{w}^{-1}[\phi] \sum_{\{n_{x,\mu}\}} \prod_{x,\mu} t_{n_{x,\mu}}(J_{\mu}) e^{-i\nu_{x,\mu}\phi_{x,\mu}}, \] (28f)
\[ \{n_{x,\mu}\}^{''} \equiv \{\{n_{x,\mu}\}|\sum_{m} n_{x,m} = \delta_{x,y} - \delta_{x,w}\}. \] (28g)

The \( U(1) \) identity used to obtain (28d-28g) is
\[ \int dV_{x} \prod_{m} \chi^{(\nu_{x,m})(\Omega_{x,m})} = \delta(\sum_{m'} \nu_{x,m'}) \prod_{m} \chi^{(\nu_{x,m})}(U_{x,m} V_{x+m}^{\dagger}) \] (29)

where \( \chi^{(\nu_{-\mu})(\Omega_{x,\mu})} \equiv \chi^{(\nu_{-\mu})(\Omega_{x,\mu})^{\dagger}}. \)

Because \( U(1) \) character coefficients do not satisfy
\[ t_{n}(z) = (t_{1}(z))^{n}, \] (30)
link loops in the numerator of \([V_{y} V_{w}^{\dagger}]_{v}\) which touch open chains \( w \cdot \vec{y} \) are not cancelled by corresponding \( z_{v} \) loops and (14) does not apply to \( U(1) \). Formula (14) only applies to systems consistent with both (29) and (30).

The \( SU(N) \) version of (29) is complicated because there is more than one way of making a singlet out of \( \nu_{-D+1} \otimes \cdots \otimes \nu_{D-1} \). \( SU(\infty) \) traces, on the other hand, obey an analog of (29):
\[ \int dV_{x} \prod_{m=-D+1}^{D-1} \text{tr}^{n_{x,m}}(V_{x} O_{x,m}) \text{tr}^{l_{x,m}}(O_{x,m} V_{x}^{\dagger}) \] (31a)
\[ = \delta(\pi_{x,D-1} - \ell_{x,D-1}) \sum_{\sigma_{x} \in S_{x,D-1}} \prod_{j=0}^{\pi_{x,D-1}} \left[ \frac{\text{tr}(O_{x,M_{x}(j)} O_{x,M_{x}(\sigma(j))}^{\dagger})}{N} \right] \]
where

\[ O_{x,m} = \begin{cases} U_{x,\mu} V_{x+\hat{\mu}}^\dagger & m = \mu \geq 0; \\ V_{x-\hat{\mu},\nu}^\dagger V_{x-\hat{\nu},m} & m = -\nu \leq 0; \end{cases} \]  

(31b)

\[ n_{x,m} = \begin{cases} n_{x,\mu} & m = \mu; \\ l_{x-\hat{\nu},\nu} & m = -\nu; \end{cases} \]

\[ l_{x,m} = \begin{cases} l_{x,\mu} & m = \mu; \\ n_{x-\hat{\nu},\nu} & m = -\nu; \end{cases} \]  

(31c)

\[ n_{x,m}, l_{x,m} \equiv \sum_{m'=\pm D+1} n_{x,m'}, M_x = \begin{cases} 1 - D & 0 \leq j \leq n_{x,1-D}; \\ \vdots & \vdots \\ D - 1 & \pi_{x,D-2} < j \leq \pi_{x,D-1}. \end{cases} \]  

(31d)

\( S_p \) is the permutation group on \( p \) elements. Eq. (31a) says that the numerators and denominators of \( t[U; w, y] \) are comprised of chains made by joining links from the trace expansion of \( e^{-S_v} \) in all possible permutations consistent with \( \{n_{x,m}, l_{x,m}\} \) conservation at each site. In Section II C we will find that relevant coefficients of the link expansion satisfy (30) and that the \( SU(\infty) \) analog of (28d-28g) is consistent with Eq. (10).

**C. Trace Expansion and Orthogonality Approximation**

We will call the use of (31a) the “trace orthogonality approximation.” The reason its use is an approximation even in the \( N \to \infty \) limit is as follows. If \( 2 \leq N < \infty \), an exact group integral formula is

\[ \int dVV_{i_1i_1} V_{\alpha_1\beta_1} V_{i_2i_2} V_{\alpha_2\beta_2} = \frac{1}{N^2 - 1} \left[ \delta_{\alpha_1 \beta_1} \delta_{\alpha_2 \beta_2} - \frac{1}{N} \delta_{i_1i_2} \delta_{\alpha_1 \alpha_2} \delta_{\beta_1 \beta_2} + (\alpha_1 \leftrightarrow \alpha_2, \beta_1 \leftrightarrow \beta_2) \right]. \]  

(32)

Since the second and fourth terms in the RHS of (32) are suppressed by \( 1/N \) relative to the other terms, (32) agrees with (31a) in the \( N \to \infty \) limit. However, using (32) in place of (31a) to compute the leading \( O(J_\mu) \) hopping expansion term of \( A_\square \), the Eq. (10) coefficient where \( \square \) is a continuous five-link chain which traces out a unit square and recurs at one side, yields

\[ A_\square \approx \frac{1}{N} \left( \frac{J_\mu}{2N} \right)^5 [1 - 1 - 1] = -\frac{1}{N} \left( \frac{J_\mu}{2N} \right)^5 \neq 0. \]  

(33)
The first two terms of the middle expression come from the numerator of \( t[U; w, y] \); the third
from a denominator plaquette and a numerator link. Since \( J \propto N \) (as described below),
\( A_\square = \mathcal{O}(1/N) \) if we take \( N \to \infty \) after resummation of the trace expansion. On the other
hand, \( A_\square = 0 \) if one uses Eq. (31a), valid to leading \( \mathcal{O}(1/N) \), to integrate out the \([dV]\) (as
described below). The latter is tantamount to taking \( N \to \infty \) before resummation. The
discrepancy arises because the leading \( \mathcal{O}(1/N) \) contribution to \( A_\square \) comes from subleading
terms of (31a) which have been thrown out. We have not been able to improve the trace
orthogonality approximation to account properly for coefficients of recurrent chains.

The orthogonal trace approximation leads to the following for

\[
Z[L, K] = \int dW \ e^{\frac{1}{2} tr[LW + W^\dagger K]}, \quad Z[J, J] = b_1(J). \tag{34a}
\]

Expanding in \( L \) and \( K \), taking \( N \to \infty \), imposing

\[
\int dW \ \text{tr}^n(LW)\text{tr}^l(W^\dagger K) = n! \left( \frac{\text{tr}(LK)}{N} \right)^n \delta_{nl} \quad (N \to \infty) \tag{34b}
\]

and resumming gives

\[
\tilde{Z}[L, K] = \sum_{l=0}^{\infty} \sum_{n=0}^{2^n-l} 2^{-l} \int dW \ \frac{\text{tr}^n(LW)\text{tr}^{l-n}(W^\dagger K)}{n! (l-n)!} = e^{\frac{\text{tr}(LK)}{4N}}. \tag{34c}
\]

(34c) is a special case of (31a). We change notation from \( Z \mapsto \tilde{Z} \) to emphasize that the
interchange of integration and resummation leads to a discrepancy, described below, between
\( \tilde{Z} \) and the original \( Z \).

Using \( \tilde{Z} \) as a generating function yields (“/” is defined in (25d).)

\[
F^{(l,n)}_{ij_1j_2 \cdots ij_l}_{kl_1o_1 \cdots kl_no_n} \equiv /W^\dagger_{ij_1} \cdots W^\dagger_{ij_l} W_{kl_1o_1} \cdots W_{kn_o} /, \tag{35a}
\]

\[
2 \frac{\partial \log \tilde{Z}}{\partial L_{ji}} \bigg|_{K=J} = 2 \frac{\partial \log \tilde{Z}}{\partial K_{ji}} \bigg|_{L=J} = F_{ij}^{(1,1)} = F_{ij}^{(1,0)} = \delta_{ij} f_N(J), \tag{35b}
\]

\[
f_N(J) = \frac{J}{2N} \quad (J << 2N, \ N \to \infty). \tag{35c}
\]

Since following (34c)

\[
\frac{\partial^{n+1} \log \tilde{Z}}{\partial K_{ji_1} \cdots \partial K_{ji_l} \partial L_{o_1k_1} \cdots \partial L_{o_nk_n}} = 0 \quad \text{(if } n \text{ or } l > 1\text{)}, \tag{35d}
\]
\[ F_{i_1 j_1 \ldots i_n j_n}^{(l,n)} (J) - \prod_{p=1}^{n+l} \delta_{i_p j_p} f_N(J) = 0 \quad (35e) \]

by induction. Combining (35b) and (35e) yields
\[ \frac{1}{N_{n+l}} \frac{1}{\text{tr}^n W} \frac{\text{tr}^l W^+}{\text{tr}^l W^+} = f_n^{n+l}(J) \quad (n \geq 0, \ l \geq 0) . \quad (35f) \]

Eq. (35f) is our preliminary SU(\infty) analog of (30) which we will now improve.

Formula (35c) violates the inequality of (25e) if \( J > 2N \) because \( \tilde{Z} \) is not valid to leading \( \mathcal{O}(N) \). Rather, following (34b), it is only asymptotically valid as \( N \to \infty \). Since (34c) is analytic in \( \text{tr}(LK) \), the violation implies \( \tilde{Z} \neq Z \). This discrepancy arises from applying (34b) to (34c) on \( n \to \infty \) (including \( n \geq N \)) terms before resummation.

An exactly solvable limit is \( N \to \infty \) and \( J \propto N \). In this double limit, achieved by identifying the gauge parameter \( \xi \) with \( \xi \propto 1/N \) so that
\[ N \to \infty, \ \xi \to 0, \ \frac{J}{2N} \equiv \lambda , \quad (36) \]
the \( N \) dependence cancels out leaving a nontrivial \( \lambda \)-dependent solution. The exact generating function is [11]

\[ \lim_{N \to \infty} Z[2N \ell, 2N \kappa] \equiv \int dW \ e^{N \text{tr}(\ell W + \kappa W^+)} \equiv e^{N^2 w(\ell, \kappa)} , \quad (37a) \]
\[ w(\ell, \kappa) = \begin{cases} \ell \kappa & \sqrt{\ell \kappa} \leq \frac{1}{2} ; \\ 2\sqrt{\ell \kappa} - \frac{3}{4} - \frac{1}{4} \log(4\ell \kappa) & \sqrt{\ell \kappa} > \frac{1}{2} ; \end{cases} \quad (37b) \]
\[ \lim_{\ell \to 0} f_N(J) = \frac{\partial w}{\partial \ell} \bigg|_{\ell=\kappa=\lambda} = \begin{cases} \lambda & \lambda \leq \frac{1}{2} ; \\ 1 - \frac{1}{4\lambda} & \lambda \geq \frac{1}{2} ; \end{cases} \quad (37c) \]
\[ \frac{1}{N^2} /\text{tr}^2 W / - f_N(J) = \frac{1}{N^2} \frac{\partial^2 w}{\partial \alpha^2} = \mathcal{O} \left( \frac{1}{N^2} \right) . \quad (37d) \]

By induction
\[ \frac{1}{N_{n+l}} \frac{1}{\text{tr}^n W} \frac{\text{tr}^l W^+}{\text{tr}^l W^+} = \mathcal{O} \left( \frac{1}{N^2} \right) . \quad (37e) \]

Since the LHS terms in (37c) and (37e) are \( \mathcal{O}(1) \), property (35f) is valid when \( J \equiv 2N \lambda \) in the \( N \to \infty \) limit.
The link expansion of \( t[U; w, y] \) follows from trace expansion
\[
e^{z \text{Retr} \Omega} = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \tau(n, l; z) \frac{\text{tr}^n \Omega \text{tr}^l \Omega^\dagger}{n! \ l!}.
\]  
(38)

Taylor expansion of \( e^{z \text{Retr} \Omega} \), which defines the exponential operator, implies \( \tau(n, l; J) = (J/2)^{n+l} \). In fact, this identification is only asymptotically consistent with using (31a) in the limit \( J \ll 2N \) and \( N \to \infty \). As \( J \propto N \to \infty \), integral formula (31a) cannot be interchanged with resummation. If we insist on applying (31a) before resumming the trace expansion, \( \tau(n, l; J) \) must be redefined so that correlation functions evaluated using (31a) and (38) are consistent with correlation functions evaluated directly from \( Z \) of Eq. (37a).

The matching condition
\[
\sum_{n=0}^{\infty} \frac{\tau(n, n + k; J)}{n!} = b_1(J) N^k f^k_N(J) = \sum_{n=0}^{\infty} \frac{\tau(n + k, n; J)}{n!}
\]  
(39)

implies when \( N \to \infty \)
\[
\tau(n, l; J) = N^{l+n} f^l_n(J) \left\{ \begin{array}{ll}
1 & J \ll 2N; \\
\text{e}^{N^2[w(\lambda, \lambda) - \lambda^2]} & J = 2N\lambda.
\end{array} \right.
\]  
(40)

The \( SU(\infty) \) analog of (30),
\[
\tau(n, l + k; J) = \tau(n + k, l; J) = N^k f^k_N(J) \tau(n, l; J),
\]  
(41)

will be referred to as “factorization.”

Applying Eqs. (31a) and (38) leads to the \( SU(\infty) \) version of (28d-28g). The \([dU]\) integral of Eq. (12) for \( A_{\bar{w} \bar{y}} \) projects the the numerator and denominator of \( t[U; w, y] \) down to the colorless part except along \( \bar{w} \bar{y} \). At other sites the denominator contributions cancel the corresponding numerator contributions. Each link of \( \bar{w} \bar{y} \) coming from a (31a) integration costs a factor of \( 1/N \). Additionally there is an overall factor of \( \text{tr}(\bar{w} \bar{y}^\dagger) / N^2 = 1/N \). Hence

\[
A_{\bar{w} \bar{y}} = \frac{\wedge_{\{n,x,m\}} \tau(n_{x,m}, n_{x,m} + p_{x,m}, J_m)}{N \wedge_{\{l,y,o\}} \tau(l_y, l_y, J_o)} N^{p_{x,m}} = \frac{1}{N} \prod_{\{x,m\} \in \bar{w} \bar{y}} f_N(J_m)
\]  
(42a)

where
\[
\wedge_{\{n,x,m\}} \equiv \sum_{\{n_{x,m}\}} \prod_{\{x,m\}} \ , \quad p_{x,m} = \begin{cases} 1 & \text{if } \{x, m\} \subseteq \bar{w} \bar{y} \\
0 & \text{if } \{x, m\} \not\subseteq \bar{w} \bar{y}
\end{cases}
\]  
(42b)
Let $a(\Theta)$ be the coefficient of link chain $\Theta$ in the numerator or denominator of $t[U; w, y]$. Consider again $A_\Box$ where $\Box$ is a link chain which traces out a unit square and recurs at one side. The chain $\Box$ does not contribute to $A_\Box$ in the same manner that $w \bar{\cdot} y$ contributes to $A_{w \bar{\cdot} y}$ because, due to factorization, $a(\Box) = a(-) a(\Box)$ so that

$$
\frac{a(-) + a(\Box)}{1 + a(\Box)} = a(-) \left( \frac{1 + a(\Box)}{1 + a(\Box)} \right) = a(-) .
$$

(43) says that the $\Box$ contribution to $A_\Box$ has already been counted in $A_-$. This implies that $A_{\text{recurrent or selfintersecting chain}} = 0$.

As described in the beginning of this subsection (44), the trace orthogonality approximation result, is inconsistent with the leading $J_\mu$ expansion contribution. The problem is that the leading contribution to $A_\Box$ is not from the leading term of (32). In (33) the middle “$-1$” comes from the second term of (32)—a term which is neglected in (31a). Hence it does not appear in $A_\Box$ if (31a) is applied to do the $[dV]$ integrals.

**D. Wilson Loop Picture of Gaugefixing**

Eqs. (3), (10), and (42a) imply that as $N \to \infty$ within the trace orthogonality approximation

$$
\langle V_y(y \bar{\cdot} w) V_{w,\bar{\cdot}y}^\dagger \rangle_{\alpha j} = \delta_{\alpha j} \sum_{w \bar{\cdot} y} A_{w \bar{\cdot} y} (J) \text{ tr}[y \bar{\cdot} w, w \bar{\cdot} y]_u ,
$$

(45)

$$
A_{w \bar{\cdot} y} = \frac{1}{N^2} \text{ tr}(V_{yy} \bar{\cdot} w, V_{w,\bar{\cdot}y}^\dagger)_{\beta = 0} = \prod_{\{x,\mu\} \in y \bar{\cdot} w} f_N(J_\mu) ,
$$

(46)

where $f_N$ is given in (35c) if $J_\mu$ is finite and in (37c) if $\xi \to 1/N$. These formulas have the following interpretation:

- By Eq. (45) the gaugefixed expectation value of a link chain $y \bar{\cdot} w$ is the weighted sum over all Wilson loops made by joining to $y \bar{\cdot} w$ a selfavoiding link segment $w \bar{\cdot} y$. This holds at all $\beta$.

- By (46), weights $A_{w \bar{\cdot} y}$ are proportional to the $\beta = 0$ gaugefixed expectation value of $w \bar{\cdot} y$. *All the gauge dependence is in these weights.*
• $A_w$ is a path-product of $e^{-S^v}$ trace expansion coefficients. In the orthogonal trace approximation (34b), nonselfavoiding operators do not contribute to $[V_w^\dagger V_y]_v$ and the $\beta = 0$ expectation value of nonselfavoiding links vanish.

• Following Eq. (41) $\beta = 0$ $\lambda$-gauge expectation values satisfy factorization. If $P_{SA}$ is a selfavoiding path

$$\langle \prod_{\{y,n\} \in P_{SA}} \Omega_{y,n} \rangle|_{\beta = 0} = \prod_{\{y,n\} \in P_{SA}} \langle \Omega_{y,n} \rangle|_{\beta = 0}. \quad (47)$$

These results permit us to read off the Wilson line $\lambda$-gauge expectation value, which is proportional to the heavy quark propagator [3]. Asymptotically it can be parametrized as

$$\lim_{t_E \to \infty} \frac{1}{N} \text{tr} \{ V(t_E, \vec{y}) \hat{U}_{\vec{y}}(y) V(t_E, \vec{0}) \} = Z_H e^{-V_H^{(\lambda)}|t_E|}, \quad x \equiv (t_E, \vec{0}). \quad (48)$$

$V_H^{(\lambda)}$ can be interpreted as $V_H^{(\lambda)} = M^{(\lambda)}_{q,\text{heavy}} - m_q$. At $\beta = 0$ and $N \to \infty$,

$$Z_H = 1, \quad V_H^{(\lambda)} = -\log[f_N(J_0)]. \quad (49)$$

Hence $V_H^{(\lambda)}$ decreases with increasing $\lambda$.

III. HOPPING EXPANSION AND RESUMMATION

In the $N \to \infty$ hopping expansion of $[\Theta]_f$, the $\psi_x$ operator hops from site to site leaving behind a trail of

$$(r - \hat{\rho}) K_B U_{y,n}, \quad n \in \{\pm 0, \cdots, \pm(D-1)\} \quad (50)$$

factors to mark its randomwalk path $P$. Pauli Exclusion is irrelevant because quark quenching effectively requires quark paths to selfintersect indefinitely.\footnote{The author thanks C. Bernard for pointing this out.}
Therefore the quenched background field quark propagator is

\[
[\psi_x \bar{\psi}_0]_f = \sum_P \prod_{\{y,n\} \in P} (r - \phi_k) K_B U_{y,n}
\]

(51)

where \( \{y, n\} \) are the locations and directions along \( P \). (Assume implicit pathordering when appropriate.) Following Eq. (17), decompose the sum over \( P \) into a sum over trees and branches. The gaugefixed expectation value of branches is trivial by (16). \( \beta = 0 \) expectation values of nonselfavoiding trees, as discussed in Section II C, are suppressed in the orthogonal trace approximation. Thus following (16) Eq. (51) is equivalent to

\[
P_q \equiv \langle V_x \psi_x \bar{\psi}_0 V_0^\dagger \rangle = \sum_{\{T_{SA}\}} \prod_{\{y,n\} \in T_{SA}} f_N(J_n) \sum_{\{B_y\} | y \in T_{SA}} \prod_{\{y,n\} \in P} \left( r - \phi_k \right) K_B
\]

(52)

where \( P \) is the selfavoiding tree \( T_{SA} \) going through sites \( y \) with a branch \( B_y \) at each site.

As there is no technique for summing selfavoiding trees as specified in (52), we shall sum all trees \( T \). This approximation is justified if \( D > 4 \) by the fact that the quark mass pole is determined in the \( t \geq L \to \infty \) limit. In this limit the average number of selfintersections and recurrences [17]

\[
\bar{\sigma} = L \times \text{(path density)} \sim \frac{L^2}{(x^2)^{D/2}} = L^{2-D/2}
\]

(53)

vanishes if \( D > 4 \) for randomwalk paths. (Nonbacktracking randomwalks are presumably at least as selfavoiding.) This suggests that the number of nonselfavoiding paths, being negligible, may not affect the quark mass pole.

Whether selfavoidance affects the propagator pole or not is analogous to whether

\[
f(a, x) \equiv \sum_{L=0}^{\infty} a_L x^L, \quad \lim_{L \to \infty} a_L = a^L
\]

(54)

guarantees \( f(a, x) \) of the form \( f(a, x) \propto \frac{1}{1-ax} + \text{finite} \). A counterexample satisfying (54) is \( a_L = a^L + 1/L \) in which case a second pole exists at \( x = 1 \). Hence anomalous quark mass poles or mass shifts stemming from the inclusion of nonselfavoiding paths is a possibility which cannot be ruled out.

There are three effects we wish to investigate: (i) spontaneous chiral symmetry breaking, (ii) gauge dependence of the dispersion relation, and (iii) proximity of the effective quark propagator to free particle form. Since the source of (i) is certainly different from (ii) and (iii), we may pursue them separately. If the dispersion relation is gauge dependent
at \((r, \beta)\), there is no reason—barring a critical point—to believe this property would not persist to other \((r, \beta)\) values since \((r, \beta)\) are unrelated to gaugefixing. To study (ii) and (iii) we set \((r = 1, \beta = 0)\) where we can solve for the full quark propagator and its dispersion relation. In this limit, spontaneous symmetry breaking is absent. To study the combination of spontaneous symmetry breaking and gauge dependence, we set \((r = 0, \beta = 0)\) where we can solve for \(M_q^{(\lambda)}\).

### A. \(N \to \infty, r = 1, \beta = 0\) Quark Propagator

Since \((1 - \gamma_\mu)(1 + \gamma_\mu) = 0\), \(r = 1\) quarks cannot backtrack and the hopping expansion cannot generate any zero area Wilson loops. Hence at \(\beta = 0\)

\[
\langle \bar{\psi}_0 \psi_0 \rangle_{\text{cont}} = -2 CK_B, \quad C \equiv 2^{D/2}N, \quad K_B \equiv \frac{1}{2(m_q + D)},
\]

where \(\psi^\text{cont} = \sqrt{2K_B}\psi^\text{cont}\) and \(C\) is from the Dirac and color traces. The \(r = 1\) pion mass obeys \[18\]

\[
cosh(M_\pi) = 1 + \frac{1 - 4K_B^2(D + 1) + 16DK_B^4}{8K_B^2(1 - 2(D - 1)K_B^2)},
\]

which has \(K_c = 1/4\) at \(D = 4\).

Upon the approximation \(T_{SA} \to T\) in Eq. (52), the sum over trees is implemented by solving the recursion relation

\[
P_q(x) = -K_B \delta_{x,0} - K_R \sum_n f_n(J_n) \neq P_q(x - n).
\]

Fourier transform \(P_q(x) \propto \sum_p e^{ipx} \tilde{P}_q(p)\) produces

\[
\tilde{P}_q(p) = \frac{C}{g_0} \left[ \frac{m_R + \sum \mu g_\mu (1 - \cos p_\mu)}{[m_R + \sum \mu g_\mu (1 - \cos p_\mu)]^2 + \sum \mu g_\mu^2 \sin^2 p_\mu} \right],
\]

\[
g_\mu(\lambda) \equiv f_N(2N\lambda), \quad \overline{g}_\mu \equiv \frac{g_\mu}{g_0}, \quad m_R \equiv \frac{1}{2K_B g_0} - \sum_{\mu=0}^{D-1} \overline{g}_\mu.
\]

For \(i \in \{1, \cdots, D - 1\}\) \(\overline{g}_i\) is given in Eq. (54). \(M_q^{(\lambda)}\), the \(p = 0\) pole of \(\tilde{P}_q(p)\), is related to continuum pole \(m_R\) by

\[
sinh M_q^{(\lambda)} = m_R.
\]
In Landau gauge where $g_\mu = 1$, $P_q$ reduces to free particle form with $m_R = m_q/g_0$. Otherwise, $P_q$ obeys a noncovariant Dirac equation which in continuum language is

$$\left(i \sum_\mu \mathcal{J}_\mu \gamma_\mu \partial_\mu + M_\lambda^{(\lambda)}\right)P_q(x) = 0 . \quad (61)$$

Eq. (61) corresponds to dispersion relation (5a).

When $K \to K_c = 1/4$, $M_\lambda^{(\lambda)}$ is negative—for nonnegative values of $M_\lambda^{(\lambda)}$ there is no massless pion. This reflects the absence of chiral symmetry and spontaneous chiral symmetry breaking at $(r = 1, \beta = 0)$. Thus we turn to $(r = 0, \beta = 0)$ in order to examine the $\lambda$-dependence of $M_\lambda^{(\lambda)}$ in the presence of spontaneous chiral symmetry breaking.

**B. $N \to \infty, r = \beta = 0$ Quark Mass**

When $r = 0$, by virtue of $\gamma_n \gamma_{-n} = 1$, the Dirac matrix of each branch is 1 and the sum over branches in (62) is equivalent to renormalizing $K_B$ to \[ K_R \equiv K_B W(K_B) . \quad (62a) \]

$W(K_B)$, the weighted sum over all branch configurations at one site \[ 19,20 \], is deduced as follows. The number $I(L)$ of length $L \geq 1$ “irreducible” branches, branches with a single base stem, is

$$I(L) = (2D - 1) \sum_{p=0}^{L-1} \sum_{\{l_i\}} \prod_{i=1}^{p} \left[ \frac{2D - 1}{2D} I(l_i) \right] \quad (I(0) = 0), \quad (62b)$$

the sum over all arrangements of its irreducible subbranches. At any site

$$W(K_B) = 1 + \sum_{L=1}^{\infty} (-K_B^2)^L I(L) = \frac{1}{1 - w_I(K_B)} \quad (62c)$$

where

$$w_I(x) \equiv \sum_{L=1}^{\infty} x^L I(L) = \frac{x(2D - 1)}{1 - \frac{2D - 1}{2D} w_I(x)} . \quad (62d)$$

Definition (62d) factorizes the RHS of (62b) to give the RHS of (62c). Equating the RHS of (62d) to the LHS gives a polynomial whose solution yields

$$K_R = 1/(m_q + \sqrt{m_q^2 + (2D - 1)}). \quad (62e)$$
Eq. (62a) reduces (52) to

$$P_q(x) = \sum_{T_{SA}} \prod_{(n) \in T_{SA}} (-K_R) \hat{n} f_N(J_n) .$$  \hspace{1cm} (63)

In this approach gauge invariant ($r = 0, \beta = 0$) meson propagators are \[19,21\]

$$\langle \bar{\psi}_x \gamma_x \bar{\psi}_0 \gamma_0 \rangle = N \sum_T \text{tr} \left( \Gamma \left[ \prod_{(n) \in T} K_R \hat{n} \right] \Gamma \left[ \prod_{(m) \in T^1} K_R \hat{m} \right] \right)$$  \hspace{1cm} (64)

where $T$ may be nonselfavoiding since no gaugefixing is involved. Since trees are nonbacktracking—a backtrack makes a branch—its Fourier transform $D_m(p)$ obeys nonbacktracking randomwalk recursion relation

$$\text{tr}[D_m \Gamma] = C - K_R^2 \sum_n \text{tr}[\gamma_m \gamma_n (1 - \delta_{n,-m}) D_n e^{-ipm}]$$  \hspace{1cm} (65)

where $\Gamma = \gamma_5$ for the pion and $\gamma_3$ for the rho. The commutation of $\Gamma$ with the $\gamma_m$ splits

$$M_\rho^2 = 4 + M_\pi^2$$  \hspace{1cm} (66)

from

$$M_\pi^2 = \frac{4(D-1)}{\sqrt{2D-1}} m_q + \mathcal{O}(m^2) .$$  \hspace{1cm} (67)

While $M_\rho$ does not vanish as $m_q \to 0$, $M_\pi$ vanishes and the pion (actually $2^D$ pions) plays the role of Goldstone boson for chiral symmetry breaking. The same sort of arguments give \[20,21\]

$$\langle \bar{\psi}_0 \psi_0 \rangle = -C \frac{D \sqrt{2D - 1 + m_q^2} - (D - 1)m_q^2}{D^2 + m_q^2} , \quad C = 2^{D/2} N$$  \hspace{1cm} (68)

for the chiral order parameter, which is nonzero when $m_q \to 0$.

1. Nonbacktracking Approximation

Since including backtracking overcounts branches, already summed by $K_R$, we will enforce nonbacktracking but otherwise permit recurrence and selfintersection. The sum over nonbacktracking trees is implemented by

$$P_q(x)_n = -\frac{\langle \bar{\psi}_0 \psi_0 \rangle}{2DC} \delta_{x,0} - K_R f_N(J_n) \hat{n} \sum_m (1 - \delta_{m,-n}) P_q(x - n)_m$$  \hspace{1cm} (69)
where \( P_q(x) = \sum_n P_q(x)_n \). Subscript \( n \) indicates the direction from which site \( x \) was approached. The Fourier transform of Eq. (69) produces

\[
\tilde{P}_q(p)_n = -\frac{\langle \bar{\psi}_0 \psi_0 \rangle}{2DC} + \sum_m \mathcal{M}_{nm}(E, \vec{p}) \tilde{P}_q(p)_m ,
\]

(70)

\[
\mathcal{M}_{nm}[E, \vec{p}] = -K_R f_N(J_n) \hat{p} (1 - \delta_{m,-n}) e^{ipn} ,
\]

(71)

where \( \mathcal{M} \) is a \( 2D \times 2[D/2] \) matrix.

Effective quark mass \( M_q^{(\lambda)} \) obeys

\[
\text{det}(1 - \mathcal{M}[M_q^{(\lambda)}, \vec{0}]) = 0.
\]

(72)

Explicit solution by Mathematica\( \text{\textcopyright} \) at \( D = 2 \) and \( D = 4 \) reveals that, despite being a \( 2D \) polynomial in \( e^{-M_q^{(\lambda)} t} \), Eq. (72) has only two distinct solutions corresponding to \( \pm M_q^{(\lambda)} \). The positive energy solution reduces to

\[
\sinh M_q^{(\lambda)} = \frac{1 - (2D - 3)\bar{g}^2K_R^2 - g_0^2K_R^2 - (2D - 1)\bar{g}^2g_0^2K_R^4}{2(1 + \bar{g}^2K_R^2)g_0K_R^2}
\]

where \( \bar{g} \equiv g_i \) for \( i \in \{1, \cdots, D - 1\} \). In the absence of gaugefixing \( g_{\mu} \rightarrow 0 \) and \( M_q^{(\lambda)} \rightarrow \infty \).

In the absence of renormalization and perfect gaugefixing, \( K_R \rightarrow 1/(2m_q) \), \( g_{\mu} \rightarrow 1 \) and the free particle relation \( \sinh M_q^{(\lambda)} = m_q \) is recovered.

Following Eq. (4a), the chiral limit is extracted by replacing \( K_R \) with \( m_q \) according to Eq. (62c) and expanding about \( m_q = 0 \). If \( d \equiv 2D - 1 \),

\[
M_c^{(\lambda)} = \frac{1 + 4D(D - 1) - (3 + 4D(D - 2))\bar{g}^2 - d(g_0^2 + \bar{g}^2g_0^2)}{\sqrt{d(2d + 2\bar{g}^2)g_0}} ,
\]

(74a)

\[
B^{(\lambda)} = \frac{1 + 4D(D - 1) + d(2D\bar{g}^2 + g_0^2) - (d - 2)\bar{g}^4 + d(g_0^4 + (\bar{g}^2 + 3d - 1)\bar{g}^2g_0^2)}{2(d + \bar{g}^2)g_0} .
\]

(74b)

The choice \( \lambda_i \equiv \tilde{\lambda} = 1 \) (for \( i = 1, \cdots, D - 1 \)) yields (13) and (4d). While \( M_c^{(\lambda)} \) is continuous, its second derivative is discontinuous at \( \lambda = 1/2 \).

2. \( D \rightarrow \infty \text{ Limit} \)

In the \( D \rightarrow \infty \) limit, there is no difference between selfavoiding and random paths. In this limit, the leading \( O(1/D) \), \( r = 0 \) effective quark mass is straightforwardly expressible in terms of the chiral symmetry parameter:

23
\[ M_c^{(\lambda)} = \frac{7C}{16\langle \bar{\psi}_0 \psi_0 \rangle g_0}, \quad B^{(\lambda)} = \frac{25}{32g_0}. \] (75)

\[ 1/g_0(\lambda) \equiv 1/f_N(2N\lambda) \] is a continuous, monotonically decreasing function whose 2nd derivative is discontinuous at \( \lambda = 1/2 \).

**C. Matching Coefficients**

Call the weighted sum of (naive) lattice operators whose matrix elements reproduce matrix elements of a continuum QCD operator the “lattice representation” of said continuum operator and the weights “lattice matching coefficients.” Verifying the \( \Delta I = 1/2 \) Rule in lattice gauge theory is a longstanding unsolved problem because it has not been possible to determine all the lattice matching coefficients of the continuum electroweak Hamiltonian responsible for \( K \to \pi \) matrix elements, related to \( K^0 \to \pi^+\pi^- \) matrix elements by chiral perturbation theory. Specifically, we are interested in operators

\[
O_{\pm}^{\text{cont}}[LL] \equiv z_{\pm}^J J^\text{latt} + \sum_{\Gamma_1, \Gamma_2} z_{\pm}^{\Gamma_1 \Gamma_2} O_{\pm}^{\text{latt}}[\Gamma_1 \Gamma_2], \tag{76a}
\]

where

\[
O_{\pm}^{qq}[\Gamma_1 \Gamma_2] \equiv F_{\Gamma_1 \Gamma_2}^{qq} \pm H_{\Gamma_1 \Gamma_2}^{qq}, \tag{76b}
\]

\[
F_{\Gamma_1 \Gamma_2}^{qq} = \bar{s}_0 \Gamma_1 d_0 \cdot \bar{q}_0 \Gamma_2 q_0, \quad H_{\Gamma_1 \Gamma_2}^{qq} = \bar{s}_0 \Gamma_1 q_0 \cdot \bar{q}_0 \Gamma_2 d_0 \tag{76c}
\]

\[ \Gamma_1 \Gamma_2 \in \{ LL, SS, PP, TT, LR \}, \quad J(0) \equiv s(0)d(0). \tag{76d} \]

The RHS of (76a) is the combination of lattice operators required to reproduce \( K \to \pi \) matrix elements of continuum operator \( O_{\pm}^{\text{cont}}[LL] \). “Matching” coefficients \( z_{\pm}^{\Gamma_1 \Gamma_2} \) are determined by gauge invariant numerical methods. The problem at hand is determining \( z_{\pm}^J \). Proportional to \( (m_s + m_d)a^{-2} \), the \( z_{\pm}^J \) coefficients are beyond weak coupling perturbation theory or the usual lattice methods. As described in Ref. [3], we use lattice gaugefixing as a technical device to determine values of these matching coefficients. The idea is to nonperturbatively replicate what is done in WCPTh, that is, to make the lattice representations of the continuum operators reproduce an ansatz, motivated by flavor symmetry considerations, for continuum
quark correlation functions. This gauge covariant matching condition imposes constraints on the lattice matching coefficients sufficient to determine them. We find that matching coefficients numerically determined in this way are $\lambda$-independent (within jackknife errors)—as they must be if they ultimately contribute to the matrix elements of gauge invariant continuum operators.

The matching condition invoked in Ref. [3]

$$\langle s|O_{\pm}^{\text{cont}}[LL]|d \rangle = 0 , \tag{77}$$

the parity even quark-equivalent of the parity odd hadronic renormalization condition that $K \to \text{vac}$, implies that

$$z_{J}^{\pm} = - \sum_{\Gamma_1 \Gamma_2} \alpha_{O_{\pm}[\Gamma_1 \Gamma_2]} z_{\pm}^{\Gamma_1 \Gamma_2}, \tag{78a}$$

$$\langle s|O_{\pm}^{\text{latt}}[\Gamma_1 \Gamma_2]|d \rangle \equiv \alpha_{O_{\pm}[\Gamma_1 \Gamma_2]} \langle s|J^{\text{latt}}(0)|d \rangle. \tag{78b}$$

Since quarks are not part of the physical $S$-matrix, Eq. (78b) may give a value of $\alpha_{O_{\pm}[\Gamma_1 \Gamma_2]}$ different from its physical sector value. In the Schwinger model, [10] not all matching conditions can be satisfied. For the satisfiable ones, different gauge variant matching conditions (relations between different gauge variant correlation functions) may lead to different (or similar) values for matching coefficients. The differences stem from unphysical gauge variant modes due to gaugefixing ambiguities—gauge variant operators transform differently under residual gauge symmetries. Nonetheless matching coefficients are $\xi$ independent and, thus, plausibly gauge invariant. Matching with different gauge variant correlation functions correspond to adopting physically inequivalent definitions of the matching coefficients. Hence the use of quark matrix elements (as opposed to, for example, diquark matrix elements) must be justified in a physical way before proceeding—as done above Eq. (78a) for this $\Delta I = 1/2$ Rule example.

We have two purposes in this Section: (a) to understand how $\lambda$-independence of $\alpha_{O_{\pm}[\Gamma_1 \Gamma_2]}$ comes about; and (b) to derive a consequence of $\beta = 0$ factorization which can be compared to $\beta = 5.7$ numerical results. This comparison lends insight into why $\beta = 0$ formulas mimic so closely the $\beta = 5.7$ and 6.0 data.

Following (78b) we are interested in $\lambda$-gauge quark correlation functions of $O_{\pm}^{\text{latt}}[\Gamma_1 \Gamma_2]$ and $J^{\text{latt}}(0)$. Because of $\beta = 0$ factorization, quark propagators in the fermionic Wick
expansion of correlation functions do not interfere with each other. Hence the $\beta = 0$ correlation functions are given by replacing the background field quark propagators in their Wick expansion with the $\beta = 0$ value of $\langle V_x \bar{\psi}_x \psi_0 V_0^\dagger \rangle$.

Let $\{a, b\}$ be color and $\{\alpha, \beta\}$ be Dirac indices and define

$$Q^{ab}(t) \equiv \sum_x \langle (V\psi)^a_x (\bar{\psi}_0 V^\dagger)^b \rangle, \quad \chi^{ab}_{\alpha\beta} \equiv \langle (\psi_0^a)(\bar{\psi}_0^b) \rangle$$

where $Q \in \{D, S, C\}$ corresponding to $\psi \in \{d, s, c\}$. If $t_y = -t$, by (63)

$$\sum_y \langle V_0 \bar{\psi}_0 \psi_0 V_y^\dagger \rangle = \gamma^5 Q^\dagger(-t) \gamma^5.$$  (80)

By $R$ symmetry and (68),

$$\chi^{ab}_{\alpha\beta} = -\delta^{ab} \delta_{\alpha\beta} \langle \bar{\psi}_0 \psi_0 \rangle/(2D/2N).$$  (81)

In fact, a stronger statement can be made for $\chi$. Let “$\beta = 0$ graph” refer to any single hopping expansion graph of $[\psi_0 \bar{\psi}_0]^f$ which contributes nontrivially to $\chi$ at $\beta = 0$. Then, since whenever $\bigodot_x$ encloses zero area $\prod_{(y,n) \in \bigodot_x} U_{y,n} \neq 1$ and the $[dU]$ integral in (81) is trivial,

$$[[(\psi_0^a)(\bar{\psi}_0^b)]^f|_{\beta=0} \propto \delta^{ab} \delta_{\alpha\beta}.$$  (82)

This color diagonality of $[\psi_0 \bar{\psi}_0]^f$ permits factorization of correlation functions containing $[\psi_0 \bar{\psi}_0]^f$ “bubble” contractions at $\beta = 0$. While $[\psi_0 \bar{\psi}_0]^f$ graphs are not diagonal at $\beta > 0$, at $\beta = 5.7$

$$[[(\psi_0^a)(\bar{\psi}_0^b)]^f \propto \delta^{ab} \delta_{\alpha\beta} + 0.1\% \text{ fluctuations.}$$  (83)

On a typical $16b y 24$ gauge configuration with $\lambda = 1$, $K = .094$, and in Dirac space

$$[\psi_0 \bar{\psi}_0]^f = \frac{2K}{10^3} \begin{pmatrix}
(995,.1) & (0,.0) & (0,.0) & (1,.0) \\
(0,.0) & (995,.1) & (1,.0) & (0,.0) \\
(-.2,.0) & (-.1,.3) & (995,.1) & (0,.0) \\
(-.1,.3) & (.2,.0) & (0,.0) & (995,.1)
\end{pmatrix};$$  (84)

with $K = .166$,
\[
[\psi_0^\dagger \psi_0]_f = \frac{2K}{10^3} \begin{pmatrix}
(902., .9) & -(1.1, .6) & (−.4, 0) & (−.9, 2.1) \\
(2.8, .9) & (903., 4.2) & (−.9, 2.1) & (−1.9, 0) \\
(−2.7, 0) & (1.1, 4.1) & (902., −.9) & (2.8, −.9) \\
(−1.1, 4.1) & (1.7, 0) & (−1.1, .6) & (903., −4.2)
\end{pmatrix}.
\] (85)

In the color off-diagonal sector with \( K = .166 \),
\[
[\psi_0^\dagger \psi_0]_f = \frac{2K}{10^3} \begin{pmatrix}
(−8.0, 3.6) & (5.7, 2.1) & (11., 5.1) & (.04, 8.4) \\
(2.6, −2.3) & (−9.8, 7.4) & (−20., 8.4) & (−5.3, 8.2) \\
(−2.1, 7.0) & (6.5, 1.1) & (4.0, .00) & (7.3, 5.1) \\
(5.9, 7.6) & (1.4, 11.) & (3.9, −1.2) & (1.3, −13.)
\end{pmatrix}.
\] (86)

In general, (I) fluctuations grow as \( K \) increases; (II) in the color singlet sector, scalar components dominate; (III) in the color nonsinglet sector, the nonscalar fluctuations are greater than the scalar, sometimes by as much as an order of magnitude; (IV) these statements are valid for all \( \lambda \) and axial gauge configurations with typical variation between configurations of \( \sim 20\% \).

The Euclidean Dirac matrices we use obey Fierz relations
\[
\Gamma_1^{αβ} \Gamma_2^{γδ} = \sum_{Γ_3Γ_4} f_{Γ_1Γ_2}^{Γ_3Γ_4} \Gamma_3^βΓ_4^δ
\] (87a)
with
\[
f_{SS} = f_{PP} = \frac{1}{4}, \quad f_{VV} = −f_{AA} = 1, \quad f_{TT} = 3,
\] (87b)
\[
[\gamma^μ(1 ± γ^5)]_{ij}[\gamma^μ(1 ± γ^5)]_{kl} = −[\gamma^μ(1 ± γ^5)]_{ji}[\gamma^μ(1 ± γ^5)]_{kj}.
\] (87c)

If \( t_y = −t_x \equiv −t \), Wick expansion and Eqs. (86a), (81) and (87a–87c) imply
\[
\mathcal{J} ≡ \sum_{x, y} \langle V_x S_x J(0) \bar{d}_y V_y^\dagger \rangle = S(t) \gamma_5 D^\dagger(−t) \gamma^5,
\] (88)
\[
\sum_{x, y} \langle V_x S_x H_{Γ_1Γ_2}^{qq} \bar{d}_y V_y^† \rangle = \begin{cases}
\sim \text{tr}(\chi_{αβ}L) = 0 & \Gamma_1Γ_2 = LL; \\
−\frac{1}{2\gamma^2Nf} J_{Γ_1Γ_2} \mathcal{J}^{cd}(\psi_0\psi_0) & \text{otherwise};
\end{cases}
\] (89)
\[
\sum_{x, y} \langle V_x S_x F_{Γ_1Γ_2}^{qq} \bar{d}_y V_y^† \rangle = \begin{cases}
\mathcal{J}^{cd}(\psi_0\psi_0) & \Gamma_1Γ_2 = SS; \\
0 & \text{otherwise}.
\end{cases}
\] (90)
Comparing (89) and (90) to (88) yields

\[
\alpha_{F_{\Gamma_1\Gamma_2}} = \begin{cases} 
\langle \psi_0 \psi_0 \rangle_{\Gamma_1 \Gamma_2} & \Gamma_1 \Gamma_2 = SS; \\
0 & \text{otherwise}; 
\end{cases}
\] 

(91)

\[
\alpha_{H_{\Gamma_1\Gamma_2}} = \begin{cases} 
0 & LL; \\
-\frac{1}{2^{D/2N}} \langle \psi_0 \psi_0 \rangle f_{\Gamma_1 \Gamma_2}^{SS} & \text{otherwise.}
\end{cases}
\] 

(92)

Therefore, \(\alpha_{H_{\Gamma_1\Gamma_2}}\) is the dominant contribution to \(\alpha_{O_{\pm}[\Gamma_1\Gamma_2]}\) unless \(\Gamma_1 \Gamma_2 = SS\), in which case \(\alpha_{H_{SS}}\) is \(1/N\) suppressed relative to \(\alpha_{F_{SS}}\). By (87b) the \(\beta = 0\) ratios are

\[
\alpha_{H_{SS}} : \alpha_{H_{PP}} : \alpha_{H_{VV}} : \alpha_{H_{AA}} : \alpha_{H_{TT}} = 1 : 1 : 4 : -4 : 12 ,
\]

(93)

which as described in Ref. [3] are approximately numerically reproduced at \(\beta = 5.7\) and 6.0.

Furthermore, appropriate linear combinations of the \(\alpha_\Theta\) give Eq. (6). Since \(\langle \psi_0 \psi_0 \rangle\) is gauge invariant, \(\alpha_{O_{\pm}[\Gamma_1\Gamma_2]}\) is gauge invariant.

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