HUBBLE PARAMETER MEASUREMENT CONSTRAINTS ON THE COSMOLOGICAL DECELERATION–ACCELERATION TRANSITION REDSHIFT

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ABSTRACT

We compile a list of 28 independent measurements of the Hubble parameter between redshifts 0.07 ≤ z ≤ 2.3 and use this to place constraints on model parameters of constant and time-evolving dark energy cosmologies. These H(z) measurements by themselves require a currently accelerating cosmological expansion at about, or better than, 3σ confidence. The mean and standard deviation of the six best-fit model deceleration–acceleration transition redshifts (for the three cosmological models and two Hubble constant priors we consider) are z_{da} = 0.74 ± 0.05, in good agreement with the recent Busca et al. determination of z_{da} = 0.82 ± 0.08 based on 11 H(z) measurements between redshifts 0.2 ≤ z ≤ 2.3, almost entirely from baryon-acoustic-oscillation-like data.

Key words: cosmological parameters – cosmology: observations – dark energy

Online-only material: color figure

1. INTRODUCTION

In the standard picture of cosmology, dark energy powers the current accelerating cosmological expansion but played a less significant role in the past when non-relativistic (cold dark and baryonic) matter dominated and powered the then-decelerating cosmological expansion. It is of some interest to determine the redshift of the deceleration–acceleration transition predicted to exist in dark energy cosmological models. There have been a number of attempts to do so; see, e.g., Lu et al. (2011a), Giostri et al. (2012), Lima et al. (2012), and references therein. However, until very recently, this has not been possible because there has not been much high-quality data at high enough redshift (i.e., for z above the transition redshift in standard dark energy cosmological models).

The recent Busca et al. (2012) detection of the baryon acoustic oscillation (BAO) peak at z = 2.3 in the Lyα forest has dramatically changed the situation by allowing for a high-precision measurement of the Hubble parameter H(z) at z = 2.3, well in the matter-dominated epoch of the standard dark energy cosmological model. Busca et al. (2012) use this and 10 other H(z) measurements, largely based on BAO-like data, and the Riess et al. (2011) Hubble Space Telescope determination of the Hubble constant, in the context of the standard ΛCDM cosmological model, to estimate a deceleration–acceleration transition redshift of z_{da} = 0.82 ± 0.08.

In this Letter, we extend the analysis of Busca et al. (2012). We first compile a list of 28 independent H(z) measurements. We then use these 28 measurements to constrain cosmological parameters in three different dark energy models and establish that the models are a good fit to the data and that the data provide tight constraints on the model parameters. Finally, we use the models to estimate the redshift of the deceleration–acceleration transition. Busca et al. (2012) have 1 measurement (of 11) above their estimated z_{da} = 0.82, while we have 9 of 28 above this (and 10 of 28 above our estimated redshift z_{da} = 0.74). Granted, the Busca et al. (2012) z = 2.3 measurement carries great weight because of the small, 3.6%, uncertainty, but 9 of our 10 high-redshift measurements from Simon et al. (2005), Stern et al. (2010), and Moresco et al. (2012) include three 11%, 13%, and 14% measurements from Moresco et al. (2012) and three 10% measurements from Simon et al. (2005), all six of which carry significant weight.

Dark energy, most simply thought of as a negative pressure substance, dominates the current cosmological energy budget. In this Letter, we consider three dark energy models.

The first one is the “standard” spatially flat ΛCDM cosmological model (Peebles 1984). In this model, a little over 70% of the current energy budget is dark energy (Einstein’s cosmological constant Λ), non-relativistic cold dark matter (CDM) being the next largest contributor (a little over 20%), followed by non-relativistic baryonic matter (about 5%). In the ΛCDM model, the dark energy density is constant in time and does not vary in space. ΛCDM has a number of well-known puzzling features (see, e.g., Peebles & Ratra 2003).

These puzzles could be eased if the dark energy density is a slowly decreasing function of time (Ratra & Peebles 1988). In this Letter, we consider a slowly evolving dark energy scalar field model as well as a time-varying dark energy parameterization.

In ΛCDM, time-independent dark energy density is modeled as a spatially homogeneous fluid with equation of state p_X = -w_X X, where p_X and ρ_X are the fluid pressure and energy density. Much use has been made of a parameterization of slowly decreasing dark energy density known as ΧCDM where dark energy is modeled as a spatially homogeneous fluid with equation of state p_X = -w_X X ρ_X. The equation of state parameter w_X < -1/3 is independent of time and p_X and ρ_X are the pressure and energy density of the X-fluid. When w_X = -1 the ΧCDM parameterization reduces to the complete and consistent ΛCDM model. For any other value of w_X < -1/3 the ΧCDM...
parameterization is incomplete as it cannot describe spatial inhomogeneities (see, e.g., Ratra 1991; Podariu & Ratra 2000).

For computational simplicity, in the XCDM case we assume a spatially flat cosmological model. The φCDM model is the simplest, consistent, and complete model of slowly decreasing dark energy density (Ratra & Peebles 1988). Here dark energy is modeled as a scalar field, φ, with a gradually decreasing (in φ) potential energy density V(φ). In this Letter, we assume an inverse power-law potential energy density V(φ) ∝ φ−α, where α is a nonnegative constant (Peebles & Ratra 1988). When α = 0 the φCDM model reduces to the corresponding CDM model.

Many different data sets have been used to derive constraints on the three cosmological models we consider here.4 Of interest to us here are measurements of the Hubble parameter as a function of redshift (e.g., Jimenez et al. 2003; Samushia & Ratra 2006; Samushia et al. 2007; Sen & Scherrer 2008; Chen & Ratra 2011b; Duan et al. 2011; Aviles et al. 2012; Seikel et al. 2012). Table 1 lists 28 H(z) measurements.5 We only include independent measurements of H(z), listing only the most recent references.

![Table 1](image)

| z  | H(z) (km s⁻¹ Mpc⁻¹) | σ_H (km s⁻¹ Mpc⁻¹) | Reference |
|----|---------------------|---------------------|-----------|
| 0.070 | 69 | 19.6 | 5 |
| 0.100 | 69 | 12 | 1 |
| 0.120 | 68.6 | 26.2 | 5 |
| 0.170 | 83 | 8 | 1 |
| 0.179 | 75 | 4 | 3 |
| 0.199 | 75 | 5 | 3 |
| 0.200 | 72.9 | 29.6 | 5 |
| 0.270 | 77 | 14 | 1 |
| 0.280 | 88.8 | 36.6 | 5 |
| 0.350 | 76.3 | 5.6 | 7 |
| 0.352 | 83 | 14 | 3 |
| 0.400 | 95 | 17 | 1 |
| 0.440 | 82.6 | 7.6 | 6 |
| 0.480 | 97 | 62 | 2 |
| 0.593 | 104 | 13 | 3 |
| 0.600 | 87.9 | 6.1 | 6 |
| 0.606 | 92 | 8 | 3 |
| 0.730 | 97.3 | 7.0 | 6 |
| 0.781 | 105 | 12 | 3 |
| 0.875 | 125 | 17 | 3 |
| 0.880 | 90 | 40 | 2 |
| 0.900 | 117 | 23 | 1 |
| 1.037 | 154 | 20 | 3 |
| 1.300 | 168 | 17 | 1 |
| 1.430 | 177 | 18 | 1 |
| 1.530 | 140 | 14 | 1 |
| 1.750 | 202 | 40 | 1 |
| 2.300 | 224 | 8 | 4 |

References. (1) Simon et al. 2005; (2) Stern et al. 2010; (3) Moresco et al. 2012; (4) Busca et al. 2012; (5) Zhang et al. 2012; (6) Blake et al. 2012; (7) Chuang & Wang 2012b.

Figure 1. Solid (dot-dashed) lines show 1σ, 2σ, and 3σ constraint contours for the XCDM model from the H(z) data given in Table 1 for the prior H₀ ± σ_H₀ = 68 ± 2.8 km s⁻¹ Mpc⁻¹ [H₀ ± σ_H₀ = 73.8 ± 2.4 km s⁻¹ Mpc⁻¹]. The filled (empty) circle best-fit point is at (Ω_m, Ω_Λ) = (0.29, 0.72) (0.32, 0.91) with χ²_min = 18.24 [19.30]. The dashed diagonal line corresponds to spatially flat models, the dotted line demarcates zero-acceleration models, and the area in the upper left-hand corner is the region for which there is no big bang. The 2σ intervals from the one-dimensional marginalized probability distributions are 0.15 < Ω_m < 0.42, 0.35 < Ω_Λ < 1.02 [0.20 < Ω_m < 0.44, 0.62 < Ω_Λ < 1.14].

![Figure 1](image)

Figure 2. Solid (dot-dashed) lines show 1σ, 2σ, and 3σ constraint contours for the XCDM model from the H(z) data given in Table 1 for the prior H₀ ± σ_H₀ = 68 ± 2.8 km s⁻¹ Mpc⁻¹ [H₀ ± σ_H₀ = 73.8 ± 2.4 km s⁻¹ Mpc⁻¹]. The filled (empty) circle best-fit point is at (Ω_m, Ω_Λ) = (0.29, −1.04) [0.26, −1.30) with χ²_min = 18.18 [18.53]. The dashed horizontal line at ω_X = −1 corresponds to spatially flat XCDM models and the curved dotted line demarcates zero-acceleration models. The 2σ intervals from the one-dimensional marginalized probability distributions are 0.23 < Ω_m < 0.35, −1.51 < ω_X < −0.64 [0.22 < Ω_m < 0.31, −1.78 < ω_X < −0.92].

![Figure 2](image)

result from analyses of a given data set. The values in Table 1 have been determined using a number of different techniques; for details, see the papers listed in the table caption. Table 1 is the largest set of independent H(z) measurements considered to date.
Figure 3. Solid (dot-dashed) lines show 1σ, 2σ, and 3σ constraint contours for the φCDM model from the \(H(z)\) data given in Table 1 for the prior \(H_0 \pm 3\sigma_{H_0} = 68 \pm 2.8 \text{ km s}^{-1} \text{ Mpc}^{-1}\) \([H_0 \pm 3\sigma_{H_0} = 73.8 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}]\). The filled (empty) circle best-fit point is at \((\Omega_{\text{m}0}, \alpha) = (0.29, 0)\) \((0.25, 0)\) with \(\chi^2_{\text{min}} = 18.24\) \([20.64]\). The horizontal axis at \(\alpha = 0\) corresponds to spatially flat ΛCDM models and the curved dotted line demarcates zero-acceleration models. The one-dimensional marginalized probability distributions are \(0.17 \leq \Omega_{\text{m}0} \leq 0.34\), \(\alpha \leq 2.2\) \([0.16 \leq \Omega_{\text{m}0} \leq 0.34, \alpha \leq 0.7]\).

We first use these data to derive constraints on cosmological parameters of the three models described above. The constraints derived here are compatible with cosmological parameter constraints determined by other techniques. These constraints are more restrictive than those derived by Farooq & Ratra (2012) using the previous largest set of \(H(z)\) measurements as well as those derived from the recent SNIa data compilation of Suzuki et al. (2012). The \(H(z)\) data considered here require accelerated cosmological expansion at the current epoch at about or more than 3σ confidence.

Our Letter is organized as follows. In the next section we present constraints from the \(H(z)\) data on cosmological parameters of the three models we consider, establish that the three models are very consistent with the \(H(z)\) data, and use the models to estimate the redshift of the cosmological deceleration–acceleration transition. We conclude in Section 3.

2. CONSTRAINTS FROM THE \(H(z)\) DATA

Following Farooq et al. (2013), we use the 28 independent \(H(z)\) data points listed in Table 1 to constrain cosmological model parameters. The observational data consist of measurements of the Hubble parameter \(H_{\text{obs}}(z)\) at redshifts \(z_i\), with the corresponding one standard deviation uncertainties \(\sigma_i\). To constrain cosmological parameters \(p\) of the models of interest, we build the posterior likelihood function \(\mathcal{L}_H(p)\) that depends only on \(p\) by integrating the product of \(\exp\left(-\chi^2_H/2\right)\) and the \(H_0\) prior likelihood function \(\exp\left(-(H_0 - \bar{H}_0)^2/(2\sigma^2_{H_0})\right)\), as in Equation (18) of Farooq et al. (2013). We marginalize over the nuisance parameter \(H_0\) using two different Gaussian priors with \(H_0 \pm \sigma_{H_0} = 68 \pm 2.8 \text{ km s}^{-1} \text{ Mpc}^{-1}\) (Chen et al. 2003; Chen & Ratra 2011a) and with \(H_0 \pm \sigma_{H_0} = 73.8 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}\) (Riess et al. 2011). As discussed there, the Hubble constant measurement uncertainty can significantly affect cosmological parameter estimation (for a recent example see, e.g., Calabrese et al. 2012). We determine the parameter values that maximize the likelihood function and find 1σ, 2σ, and 3σ constraint contours by integrating the likelihood function, starting from the maximum and including 68.27%, 95.45%, and 99.73% of the probability.

Figures 1–3 show the constraints from the \(H(z)\) data for the three dark energy models we consider and for the two different \(H_0\) priors. In all six cases the \(H(z)\) data of Table 1 require accelerated cosmological expansion at the current epoch, at or better than 3σ confidence. The previous largest \(H(z)\) data set used, that in Farooq & Ratra (2012), required this accelerated expansion at or better than 2σ confidence. Comparing Figures 1–3 here to Figures 1–3 of Farooq & Ratra (2012), we see that in the XCDM and φCDM cases the \(H(z)\) data we use in this Letter significantly tighten the constraints on \(w_X\) and \(\alpha\), but...
do not much affect the $\Omega_{m0}$ constraints. However, in the $\Lambda CDM$ case the $H(z)$ data used here tighten constraints on both $\Omega_{\Lambda}$ and $\Omega_{m0}$. We found that as we increase the value of the nuisance parameter $H_0$ the best-fit point for $\Lambda CDM$ moves from the spatially flat case to the closed case, and for XCDM the best-fit point moves almost orthogonally to the flat $\Lambda CDM$ line, toward more negative values of $\omega_X$.

As indicated by the $\chi^2_{\text{min}}$ values listed in the captions of Figures 1–3, all six best-fit models are very consistent with the $H(z)$ data listed in Table 1. It is straightforward to compute the cosmological deceleration–acceleration transition redshift in these cases. They are 0.706 [0.785], 0.695 [0.718], and 0.698 [0.817] for the $\Lambda CDM$, XCDM, and $\phi CDM$ models with prior $H_0 \pm \sigma_{H_0} = 68 \pm 2.8 \text{ km s}^{-1}\text{ Mpc}^{-1} \{H_0 \pm \sigma_{H_0} = 73.8 \pm 2.4 \text{ km s}^{-1}\text{ Mpc}^{-1}\}$. The mean and standard deviation give $z_{\text{da}} = 0.74 \pm 0.05$, in good agreement with the recent Busca et al. (2012) determination of $z_{\text{da}} = 0.82 \pm 0.08$ based on less data, possibly not all independent. Figure 4 shows $H(z)/(1+z)$ data from Table 1 and the six best-fit model predictions as a function of redshift. The deceleration–acceleration transition is not impossible to discern in the data.

From Figure 4 one sees that there are only six data points for $z > 1$ but 22 data points for $z < 1$. The larger errors of some of the $z < 1$ data, as compared to those of the $z > 1$ measurements, are likely responsible for the excellent reduced $\chi^2$ values of the best-fit models.

3. CONCLUSION

In summary, we have extended the analysis of Busca et al. (2012) to a larger independent set of 28 $H(z)$ measurements and determined the cosmological deceleration–acceleration transition redshift $z_{\text{da}} = 0.74 \pm 0.05$. These $H(z)$ data are well described by all six best-fit models and provide tight constraints on the model parameters. The $H(z)$ data require accelerated cosmological expansion at the current epoch, and are consistent with the decelerated cosmological expansion at earlier times predicted and required in standard dark energy models. While the standard spatially flat $\Lambda CDM$ model is very consistent with the $H(z)$ data, current $H(z)$ data are not able to rule out slowly evolving dark energy. More, and better quality, data are needed to better discriminate between constant and slowly evolving dark energy density; these data are likely to soon be in hand.

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