The 3rd International Conference on Sustainable Energy Information Technology (SEIT 2013)

Influencing factors of methane/moist-air laminar boundary layer in micro-channel

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Abstract

Aiming at the flow characteristics of methane/moist-air laminar boundary layer in the millimeter micro channels, this paper solved the governing equation by Matlab programming and researched two kinds of micro channels' boundary layer properties under three different conditions. The two micro channels are micro plate and micro circular tube, the spacing between two plates and the diameter of the circular tube are both 1 mm, with the length both 40 mm. The influences of air/carbon mole ratio, water/carbon mole ratio, air mass flow and the wall friction on the velocity characteristic of boundary layer as well as the boundary layer thickness under different working conditions were analyzed. Preliminary get that the boundary layer thickness of micro channels is up to micron magnitude, which lays a foundation for the further study of the methane/moist-air combustion and methane/moist-air stagnation flow characteristics in micro channels.

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Key Words: Miro channels; the velocity boundary layer; laminar flow; stagnation flow

1. Introduction

With the rapid development of micro-electromechanical system (MEMS) technology, the influence of micro-devices on many areas has become increasingly evident. Nowadays, the miniaturization of device and the micro electro-mechanical system research have become important subjects and hot issues\textsuperscript{[1]}. Broadly speaking, micro reactor is a small reaction system which is created through precision processing and micro processing technology, the size of micro channel is no more than 1 mm in general. The smallest unit of micro flow system is called micro channel structure or micro-chamber. Owing to the decrease of
the scale, the ratio of surface area relative to volume will increase. The relative importance of various forces which affect the flow will change in the small scale system, therefore, the flow in micro channels is more complicated than in conventional sizes.

At present, the research fruits of scholars for the flow laws of microscale fluid in micro channels are still far from reaching the level of maturity. Scholars generally think that the phenomenon of fluid flow in micro channel has its unique mechanisms and laws. About the problem of boundary layer flow, Roslinda Nazar et al. [2] made a research on the unstable mixed convection boundary layer flow on the porous medium stagnation point area, and the simplified boundary layer partial differential equation was solved numerically with Keller-Box method; Saeed Dinarvand [3] analyzed stagnation flow of the rotary table and its velocity characteristics using Homotopy Analysis Method; Anuar Ishak et al. [4] studied the steady laminar flow boundary layer equations; Bachok et al. [5] researched the effects of homogeneous–heterogeneous reactions on the stagnation-point flow towards a stretching sheet; But they did not refer to the properties of stagnation flow.

It is considered that when the fluid flows through the object surface, the fluid speed would reduce to zero relative to the object due to some influencing factors such as friction, impact and viscous force, and the flow whose layer velocity is blocked to zero is called stagnation flow. The most common stagnation flow model is formed by the fluid impacting on wall vertically [6-8], the other model is formed by the fluid flowing along the wall, but the conception, formation mechanism and influencing factors of the latter model are almost new to the world. The purpose of this paper is to explore the stagnation flow characteristics through the laminar boundary layer characteristics in micro channels.

2. Physical and Mathematical Models

2.1. Physical Model

Aiming at the boundary layer characteristics of the millimeter magnitude microscale channels under different inlet mass flow conditions, we established the physical model of two parallel flat micro-channel of the spacing of 1 mm and a diameter of 1 mm microtubes, both of which are 40 mm in length (As shown in Fig. 1.). We assuming that the boundary conditions is adiabatic, and all of the wall surfaces are coated with catalyst. Using a laminar flow model under the no-slip condition, we get the analytic equations about the velocity and boundary layer thickness in the micro channels through the processing of analysis the governing equations, then using the Matlab language programming, we analyzed the methane/moist-air boundary layer flow characteristics and the main influencing factors in two micro channels, respectively.

( a ) ( b )

Fig.1. (a) schematic diagram of parallel plate micro-channel; (b) schematic diagram of tube micro-channel

2.2. Mathematical Model

1) The parallel plate microchannel

Because the characteristic geometric size of the microchannel is 1 mm, which is much larger than the molecular mean free path of the mixed gas, the reaction space, the mass inlet flow rate and flow velocity of the mixture is very small, therefore we neglected the volume force, the dissipation effect of the flow as well as the gas radiation in the calculation. In addition, there was no consideration about the microscale effect in the flow, we still used the N-S equations of continuous medium.
At first, we selected characteristic parameters $u_e$ and $L$, through converting the governing equations into dimensionless form, we get the simplified boundary layer equations by orders of magnitude:

$$
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0
$$

(1.1)

Definite solution of the boundary conditions for: $y = 0; u = 0, v = 0$ Then the stream function $\psi(x,y)$ and dimensionless variables $\eta = y/\sqrt{\frac{2\mu}{\rho}\frac{a}{U}}$, $x' = y/g(y)$, $F = \frac{\psi}{\eta g(y) U}$ velocity and the boundary layer thickness was obtained, respectively. Finally we obtained the plate velocity and boundary layer thickness using the Matlab language programming.

2) The tube microchannel

We derived the boundary layer equations of the tube flow with the momentum integral method. The momentum equation for incompressible fluid is as follows:

$$
\frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} \right) u_r + \frac{1}{r^2} \frac{d}{dr} \left( r^2 u_\theta \right) = \frac{1}{\rho} \frac{d}{dr} \left( \rho u_r \frac{d}{dr} \right)
$$

(2.1)

Where $u_r$ is the speed in the core area, the fluid is considered to be the ideal potential flow.

The total flow in the pipeline is the sum of flow in the core area and in the boundary layer, that is:

$$
Q = \pi R^2 v, \quad \pi (R - \delta) u_e = 2\pi (R - \delta) v
$$

(2.2)

Then assuming the boundary layer velocity distribution in the laminar flow obeys the secondary distribution: $u_x = a + by + cy^2$, after rearranging the above equation, the velocity equation is converted into:

$$
u_x = u_x \left( \frac{2y}{\delta} \frac{a}{\delta^2} \right)
$$

(2.3)

The boundary shear stress meets Newton internal friction law, namely

$$
\tau_v = \mu \frac{du_x}{dy} = \frac{2\mu u_x}{\delta} = 2\mu v [\delta (\delta^2 + \frac{2\delta}{3R} + \frac{\delta^2}{6R^2})]
$$

(2.4)

After plugging the above several formulas into the momentum integral equation, we get the dimensionless relation:

$$
\frac{dY}{dX} = \frac{20}{\text{Re} (18Y + 18Y^2)}
$$

(2.5)

Where $Y = d/R, X = x/D, \text{Re} = VD/\nu$ is the pipe Reynolds number.
3. Simulated Conditions

Water to carbon ratio and the mass flow rate are important factors affecting CH$_4$/H$_2$O catalytic reforming reaction in the micro cavity. To explore the boundary layer characteristics and influencing factors of methane/moist-air laminar flow in micro-channels, three different conditions of methane/moist-air in the air/carbon ratio, water/carbon ratio and inlet mass flow at room temperature and atmospheric pressure were analyzed. Three kinds of conditions of the physical parameters of the gas mixture are shown in the following tables:

Table 1. The physical parameters of the feed gas (methane / air) at 25 °C, 0.1 Mpa, $q_m=0.6$ g·h$^{-1}$

| air/carbon ratio $\alpha$ | $\rho_m$ [kg/m$^3$] | $\mu_m$ [kg/(m·s)] | $V_m$ [m$^3$/s] | plate Velocity [m/s] | Re | tube Velocity [m/s] | Re |
|--------------------------|---------------------|---------------------|---------------|----------------------|----|---------------------|----|
| 0.6                      | 0.975               | $2.08\times10^{-5}$ | $2.13\times10^{-5}$ | 0.17                 | 319| 0.22                | 413|
| 1.2                      | 1.06                | $2.01\times10^{-5}$ | $1.90\times10^{-5}$ | 0.16                 | 338| 0.2                | 464|
| 2.4                      | 1.14                | $1.95\times10^{-5}$ | $1.17\times10^{-5}$ | 0.15                 | 350| 0.19                | 444|
| 3.6                      | 1.18                | $1.92\times10^{-5}$ | $1.63\times10^{-5}$ | 0.14                 | 344| 0.18                | 442|

Table 2. The physical parameters of the feed gas (methane / air/water) at 300 °C, 0.1 Mpa, $q_m=1.2$ g·h$^{-1}$

| water/carbon ratio $\beta$ | $\rho_m$ [kg/m$^3$] | $\mu_m$ [kg/(m·s)] | $V_m$ [m$^3$/s] | plate Velocity [m/s] | Re | tube Velocity [m/s] | Re |
|----------------------------|---------------------|---------------------|---------------|----------------------|----|---------------------|----|
| 1.0                        | 0.494               | $2.50\times10^{-5}$ | $5.06\times10^{-5}$ | 0.67                 | 530| 0.86                | 680|
| 1.5                        | 0.472               | $2.45\times10^{-5}$ | $5.20\times10^{-5}$ | 0.71                 | 546| 0.90                | 692|
| 2.0                        | 0.453               | $2.37\times10^{-5}$ | $5.24\times10^{-5}$ | 0.74                 | 565| 0.94                | 718|
| 2.5                        | 0.434               | $2.29\times10^{-5}$ | $5.27\times10^{-5}$ | 0.77                 | 584| 0.98                | 744|

Table 3. The physical parameters of the feed gas (methane / air/water) at 300 °C, 0.1 Mpa, $\beta=2.0$

| inlet mass flow $q_m$ [g·h$^{-1}$] | $\rho_m$ [kg/m$^3$] | $\mu_m$ [kg/(m·s)] | $V_m$ [m$^3$/s] | plate Velocity [m/s] | Re | tube Velocity [m/s] | Re |
|------------------------------------|---------------------|---------------------|---------------|----------------------|----|---------------------|----|
| 0.6                                | 0.45                | $2.38\times10^{-5}$ | $5.29\times10^{-5}$ | 0.37                 | 279| 0.47                | 355|
| 0.9                                | 0.447               | $2.38\times10^{-5}$ | $5.32\times10^{-5}$ | 0.56                 | 421| 0.71                | 534|
| 1.2                                | 0.453               | $2.37\times10^{-5}$ | $5.24\times10^{-5}$ | 0.74                 | 565| 0.94                | 718|
| 1.5                                | 0.447               | $2.38\times10^{-5}$ | $5.32\times10^{-5}$ | 0.93                 | 699| 1.19                | 894|
4. Results and Discussion

4.1. The impact of air/carbon ratio

From the above table 1 we can see that the first condition is to pass the methane/air mixture into micro-channels with the inlet mass flow rate of 0.6g·h\(^{-1}\) under the ambient temperature and pressure. Through changing the air/carbon ratio of the mixed gas component, we obtain the following results:

First, Fig. 2. (a) shows that by changing the air/carbon molar ratio (from 0.6 to 3.6) of the mixed gas component, the velocity varying in the parallel plate micro-channel under different air/carbon ratio is almost the same, that is, from 0 to mainstream velocity \(U_e\) and then remains the constant \(U_e\). Wherein the horizontal mainstream velocity \(U_e\) reaches the maximum when air/carbon ratio is equal to 0.6, while with the increase of the air/carbon ratio, the achieved mainstream velocity \(U_e\) gradually decreases. And we achieve the corresponding mainstream velocity \(U_e\) fastest when air/carbon ratio is equal to 2.4. Fig. 2. (b) reveals that the trend of vertical velocity changes in the parallel plate micro-channel is basically the same with the horizontal velocity's. But the vertical mainstream velocity \(V_e\) is an order of magnitude smaller than the level of horizontal mainstream velocity \(U_e\). The Fig.2.(b) shows the maximum and the minimum mainstream velocity \(V_e\) appear at air/carbon ratio 0.6 and 2.4, respectively.

![Horizontal velocity profile](image1)

![Vertical velocity profile](image2)

![Frictional resistance coefficient profile](image3)

![Boundary layer thickness profile](image4)

**Fig.2. (a)** The profile of horizontal velocity in the parallel plate micro-channel; **(b)** The profile of vertical velocity in the parallel plate micro-channel; **(c)** The profile of frictional resistance coefficient in the parallel plate micro-channel; **(d)** The profile of boundary layer thickness in the parallel plate micro-channel.

Fig.2. (c) shows that the wall frictional resistance coefficient decreases with the increase of the microscale flat plate length, and (the frictional resistance coefficient) first decreases and then increase with the increase of the air/carbon ratio. The maximum and the minimum friction coefficient \(C_f\) (0.87, 0.03) appear at air/carbon ratio 0.6 and 2.4, respectively. Fig. 2. (d) reveals that the laminar boundary layer thickness in the parallel plate micro-channel increases gradually with the increase of the plate length. With the increase of air/carbon ratio, the boundary layer thickness first decreased and then increased. The maximum boundary layer thickness reaches micrometers. Wherein the boundary layer thickness reaches 1.5\(\mu\)m and 1.2\(\mu\)m when air/carbon ratio is 0.6 and 2.4, respectively.

4.2. The impact of water/carbon ratio
From the above table 2 we know that the second condition is to pass the methane/air /water mixture into micro-channels with the inlet mass flow rate of 1.2g·h⁻¹ at 300°C , 0.1MPa. Through changing the water/carbon ratio of the mixed gas component, we obtain the following results:

From Fig.3.(a~d) we can see that the the overall trend of velocity characteristics, friction coefficient and the boundary layer thickness change in the microscale flat channel is the same as the first condition’s. Mainstream velocity of the horizontal direction and the vertical direction within the boundary layer both increases gradually with the increase of water/carbon ratio(from 1.0 to 2.5),while Ve still an order of magnitude smaller than Ue. Fig.3.(c)shows the frictional resistance coefficient is basically unchanged with water/carbon ratio increased,decreases with the increase of plate length . The maximum static friction coefficient is about 0.92. Fig.3.(d)shows the boundary layer thickness variation: the boundary layer thickness in microscale plate channel increases with the increase of the plate length while decreases with the increase of water/carbon ratio. The maximum boundary layer thickness range is 1.16μm to 1.18μm under different water/carbon ratio conditions.

4.3. The impact of inlet mass flow

Table 3 shows that the third condition is to pass the methane/air /water mixture into micro-channels with the water/carbon ratio 2.0 at 300°C , 0.1MPa. Through changing the inlet mass flow of the mixture component, we get the following results:
Fig. 4. (a) The profile of horizontal velocity in the parallel plate micro-channel; (b) The profile of vertical velocity in the parallel plate micro-channel; (c) The profile of frictional resistance coefficient in the parallel plate micro-channel; (d) The profile of boundary layer thickness in the parallel plate micro-channel.

Fig. 4. (a, b) shows that, with the increase of inlet mass flow, the horizontal mainstream velocity $U_e$ increases gradually; the horizontal velocity $U_e$ is also increasing with the increase of water/carbon ratio. The vertical mainstream velocity $V_e$ also increases with the inlet flow and water/carbon ratio. Fig. 4. (c) shows that, wall friction coefficient is further reduced with the increase of inlet flow. The maximum static friction coefficient is about 0.63, which decreases with the increasing plate length. The influence of water/carbon ratio to the change of friction coefficient can be almost ignored. Fig. 4. (d) shows the boundary layer thickness in microscale plate channel increases with the increase of the plate length and increases with the increase of inlet flow. The maximum boundary layer thickness range is 0.8 $\mu$m to 1.6 $\mu$m under different inlet flow conditions.

4.4. Boundary layer thickness in the microscale tube channel

We get three corresponding boundary layer thickness distribution after passing the mixed gas into the micro tube channel under three different conditions. Table 1, 2, 3 conditions corresponding to the boundary layer thickness distribution of Fig. 5. (a, b, c).

Fig. 5. (a, b, c) The profile of boundary layer thickness in the tube micro-channel under three different conditions.

From Fig. 5. (a) we can see, the boundary layer thickness in the microscale tube increases with the tube length, first decreases and then increases with the increase of air/carbon ratio. The maximum boundary layer thickness is approximately 20 $\mu$m when air/carbon ratio is equal to 0.6. Fig. 5. (b) shows that the microscale tube boundary layer thickness is increased with the circular tube length and decreased with the increase of water/carbon ratio under the second condition. The maximum and the minimum boundary layer thickness appear at water/carbon ratio 1.0 and 2.5, respectively. Fig. 5. (c) shows boundary...
layer thickness increases gradually with the increase of micro tube length, while decreases with the inlet mass flow. The maximum boundary layer thickness is also 20μm in the third condition.

5. Conclusion

Aiming at two millimeter micro-channels, this paper researched the air carbon ratio under different temperature and intake flow and water carbon than the impact of the the micro-channel boundary layer characteristics, the main conclusions are as follows:

(1) The velocity varying trend is the same in the parallel plate micro-channel under three different conditions, that is, increase from rest to mainstream velocity Ue and Ve and then keep the constants. while Ve an order of magnitude smaller than Ue.

(2) The velocity of the mix gas in the parallel plate micro-channel is affected by air/carbon ratio, water/carbon ratio, temperature, inlet mass flow and other factors. Horizontal mainstream velocity Ue decreases with the increase of air/carbon ratio; Vertical mainstream velocity Ve first decreases and then increases with the increase of air/carbon ratio; Ue, Ve both increase with the increase of water/carbon ratio, temperature, inlet mass flow.

(3) The friction coefficient decreases with the increasing plate length in the parallel plate micro-channel. The maximum friction coefficients is 0.87, 0.63, 0.89, respectively, in three different conditions. The minimum friction coefficients is about 0.03. Frictional resistance coefficient first decrease and then increase with the increase of air/carbon ratio; The coefficient is basically unchanged with the change of water/carbon ratio and inlet flow.

(4) Boundary layer thickness both in parallel plate micro-channel and micro tube channel show a parabolic increase with the increasing plate length, First decreases then increases with the increase of air/carbon ratio, decreases with the increase of water/carbon ratio and inlet flow.

(5) We obtained the maximum boundary layer thickness of the parallel plate and tube micro-channel is micrometers, which provides evidence for the further study of stagnation thickness.

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