Polarization of Inclusive $\Lambda_c$’s in a Hybrid Model

Gary R. Goldstein

aDepartment of Physics, Tufts University
Medford, MA 02155

A hybrid model is presented for hyperon polarization that is based on perturbative QCD subprocesses and the recombination of polarized quarks with scalar diquarks. The updated hybrid model is applied to $p+p \rightarrow \Lambda+X$ and successfully reproduces the detailed kinematic dependence shown by the data. The hybrid model is extended to include pion beams and polarized $\Lambda_c$’s. The resulting polarization is found to be in fair agreement with recent experiments. Predictions for the polarization dependence on $x_F$ and $p_T$ is given.

1. INTRODUCTION

Inclusively produced strange hyperons can have sizeable polarization \cite{1} over a wide range of energies. Evidence now indicates that charmed hyperons also have sizeable polarization \cite{2,3,4}. Many theoretical models have been proposed to explain various aspects of hyperon polarization data with varying success \cite{5,6,7,8}. All try to explain the large negative $\Lambda$ polarization. Because the hyperon data is in the region of high CM energy but relatively small transverse momentum ($p_T \sim 1$ GeV/c), soft QCD effects should play a major role in any theoretical explanation. Several years ago Dharmaratna and Goldstein developed a hybrid model for $\Lambda$ polarization in inclusive reactions \cite{9}. The model involves hard scattering at the parton level, gluon fusion and light quark pair annihilation, to produce a polarized heavy quark which then undergoes a soft recombination that, in turn, enhances the polarization of the hyperon. This scheme provided an explanation for the characteristic kinematic dependences of the polarization in $p+p \rightarrow \Lambda+X$. The use of perturbative QCD to produce the initial polarization for strange quarks, with their low current or constituent quark mass (compared to $\Lambda_{QCD}$) made the application of perturbation theory marginal, however.

In the heavy quark realm the perturbative contribution is more reliable. Given these circumstances, I have modified the original hybrid model to apply to heavy flavor baryons produced inclusively from either proton or pion beams. The results are encouraging, as the following will show (see ref. \cite{7} for a more complete treatment).

2. HYBRID MODEL

All of the models for $\Lambda$ polarization begin with the observation that $Q$-flavor hyperons of the type $\Lambda_Q \sim [ud]Q$ have their polarization carried primarily by the $Q$; the $[ud]$ must be a color anti-triplet isospin 0 spin scalar diquark (to the extent that gluons + $L +$ sea contributions can be ignored). How does the $Q$ itself get polarized in a production process? Consider parton + parton $\rightarrow Q \bar{Q}$. At tree level in QCD, there can be no single quark polarization for these two-body subprocesses, all diagrams being relatively real. This can be seen when the polarization is written in terms of helicity amplitudes $f_{a,b,c,d}$ for particles $A+B \rightarrow C+D$ as

$$P_Q \propto \sum_{a,b,d} f^*_{a,b,c,d} f_{a,b,c',d} (\sigma \cdot \hat{n})_{c,c'}$$
$$\propto \text{Im} \sum_{a,b,c,d} f_{a,b,c,d} f^*_{a,b,c,d},$$

(1)

where $\hat{n}$ is the normal to the scattering plane. Hence there has to be a phase difference and a flip–non-flip interference. In QCD with zero quark masses there are only non-flip vertices; helicity flip requires non-zero quark masses. And a relative imaginary part only arises beyond tree level \cite{10}. So the hybrid model incorporates the order $\alpha_s^2$ QCD perturbative calculation of interference between tree level and the large number of one loop diagrams to produce massive heavy quark polarization. (Only the imaginary parts of the one loop diagrams were needed, so the Cutkosky rules were used to simplify the calculation. For the lengthy results see ref. \cite{10,11} as well as an independent calculation in ref. \cite{12}). This gives rise to significant polarization \cite{11}, proportional to $\alpha_s(Q^2)$ and to complicated functions of the constituent quark mass. The
scale here is \( Q^2 \sim m_{QCD}^2 \). The results are illustrated in fig. 4 for the \( g + g \rightarrow Q + \bar{Q} \) case, with CM energy 26 GeV and outgoing quark flavors \( Q = d, s, c, b, \). The symmetry requires \( \mathcal{P}(\pi - \theta) = -\mathcal{P}(\theta) \), so backward \( Q \) has \( \mathcal{P} < 0 \). The magnitude of \( \mathcal{P} \) reaches \( \sim 6\% \) for the b-quark. It is clear that the polarization increases roughly as the quark mass. Similar results are obtained for \( q + \bar{q} \rightarrow Q + \bar{Q} \).

Figure 1. Polarization for the QCD subprocess of gluon fusion to quark pairs. The curves are for d, s, c, b quarks.

The cross sections for polarized \( Q \)-quarks (polarized normal to the production plane) must then be convoluted with the relevant structure functions for the hadronic beam and target. The inclusive cross section for hadron + hadron \( \rightarrow Q(\uparrow \text{ or } \downarrow) + X \) is obtained thereby. For protons on protons gluon fusion is the more significant subprocess.

The hadronization process, by which the polarized \( Q \) recombines with a [ud] diquark system to form a \( \Lambda_Q \), is crucial for understanding the subsequent hadron polarization. The backward moving, negatively polarized heavy quark must be accelerated to recombine with a fast moving diquark (resulting from remnants of the \( pp \) or \( p\bar{p} \) collision) to form the hadron with particular \( x_F \) while preserving the quark’s \( p_T \) value. Letting \( x_Q \) be the Feynman \( x \) for the heavy quark, the simple form, a linear mapping of the \( Q \) kinematic region,

\[
x_F = a + bx_Q
\]

is used for the recombination. Naively, if the \( Q \) has 1/3 of the final hyperon momentum (in its infinite momentum frame) and the diquark carries 2/3 of that momentum, then \( a = 2/3 \) and \( b = 1 \). The values actually used, \( a = 0.86 \) and \( b = 0.70 \), were chosen to fit the \( pp \rightarrow A + X \) data (that existed in 1990) at one \( x_F \) value. These parameters in eqn. 3 are not far from the naive expectation.

This recombination prescription is similar to the semi-classical dynamical mechanism used in the “Thomas precession” model of hyperon polarization [5], which posits that the s-quark needs to be accelerated by a confining potential or via a “flux tube” [6] at an angle to its initial momentum in order to join with the diquark to form the hyperon. The skewed acceleration gives rise to a spin precession for the s-quark. With the precession rate, \( \omega_T = (\gamma - 1)v \times a/v^2 \times p_Q \times A\mathbf{p}_T \sim -\hat{n} \), an energy shift \( -S \cdot \omega_T \propto +S \cdot \hat{n} \) occurs. Hence negative values of \( \langle S \cdot \hat{n} \rangle \) are energetically favored. In the Hybrid Model the \( Q \) has acquired negative polarization already from the hard subprocess before it is accelerated in the hadronic recombination process. That “seed” polarization gets enhanced by a multiplicative factor \( A \simeq 2\pi \) which simulates the Thomas precession. The Hybrid Model combines hard perturbative QCD with this simple model for non-perturbative recombination.

In summary, the hyperon polarization is given as

\[
\mathcal{P}_{\Lambda_Q}(x_F, p_T) = A \cdot \mathcal{P}_{Q}(x_Q(x_F), p_T)
\]

for each reaction \( g(x_1) + g(x_2) \) or \( q(x_1) + \bar{q}(x_2) \rightarrow Q \bar{Q} \), with the mapping function \( x_Q(x_F) \) obtained by inverting eqn. 2. From eqn. 3 the subprocess polarized cross sections,

\[
\frac{d^2 \sigma(\uparrow \text{ and } \downarrow)}{dx_Q dp_T} = \frac{d^2 \sigma(\uparrow \text{ and } \downarrow)}{dx_Q dp_T}
\]

for partons \( (i, j) \) at \( (x_1, x_2) \) can be obtained. These cross sections are convoluted with the gluon, quark and antiquark structure functions for the proton and pion [4], \( \hat{g}^{p,\pi}(x), \hat{q}^{p,\pi}(x), \hat{\bar{q}}^{p,\pi}(x) \), or generically \( f_i^{p,\pi}(x) \) leading to

\[
\frac{d^2 \sigma(\uparrow \text{ and } \downarrow)}{dx_Q dp_T} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_i^{p,\pi}(x_1) f_j^{\pi,\bar{q}}(x_2) \frac{d^2 \sigma(\uparrow \text{ and } \downarrow)}{dx_Q dp_T}.
\]

Next the recombination formula, eqn. 3 is applied to obtain the corresponding \( \Lambda_Q \) polarized...
cross section at $x_F = a + bx_Q$ and $p_T$. The polarization is obtained via

$$P_{\Lambda_Q}(x_F, p_T) = \frac{d^2\sigma(\uparrow) - d^2\sigma(\downarrow)}{d^2\sigma(\uparrow) + d^2\sigma(\downarrow)},$$

(6)

in an obvious notation.

Note that the linear form of eqn. 2 maps the $Q$-quark Feynman $x$ region $[-1, (1 - a)/b]$ into the $x_F$ region $[(a - b), +1]$ for the $\Lambda_Q$. The $p + p \to Q$ differential cross section, $d^2\sigma/dx_Qdp_T$ is mapped correspondingly into the $p + p \to \Lambda_Q$ cross section $d^2\sigma/dx_Fdp_T$. The measured cross sections for the latter are known to fall with positive $x_F$ and to fall precipitously with $p_T$, roughly as

$$(1 - x_F)\alpha e^{-\beta p_T^2},$$

(7)

overall [3], where $\alpha$ and $\beta$ are greater than 1.0 (for $\pi + p \to \Lambda + X$, $\alpha, \beta \approx 3.0$). However, the directly computed lowest order $p + p \rightarrow q$-quark cross section grows with $x_Q$ in the region (-1.0) and it falls more gradually with $p_T$ than the exponential in eqn. 2. Hence the more complete recombination scheme would have to temper the $x_F$ dependence and narrow the $p_T$ distribution. This will not affect the polarization calculation, though, since the individual up or down polarized cross sections will be altered in the same way. For a more thorough calculation this should be done, and work is underway on this point. The polarization results are the focus of this work.

3. COMPARISON WITH DATA AND PREDICTIONS

Applied to strange $\Lambda$ production, the hybrid model reproduces the detailed $(x_F, p_T)$ dependence of the data, with very slow energy dependence [13], as fig. 2 shows.

Note that an estimated 20 to 30% of the $\Lambda$'s come from $\Sigma^0 \to \gamma\Lambda$ [13], so the parameter $A$ in eqn. 2 is increased to 7.9. The agreement of the hybrid model with the wide range of data is excellent.

It is worth remarking that recently extensive data have been collected on $\Lambda$ polarization in many exclusive reactions [17], for which a simple form, $P = (-0.443 \pm 0.037)x_Fp_T$, approximates all the polarization data at $p_{lab} = 27.5$ GeV/c. That form provides lower bracketing values for the inclusive polarization, as fig. 2 indicates. In the hybrid model all the final states other than the $\Lambda$ arise from the hadronization of the $s$-quark and the remains of the incoming baryons. Therefore, in the hybrid model it would be anticipated that as the beam energy increases and/or more final states are included in the determination of the $\Lambda$ polarization, more complicated final states will be accompanied by much lower polarization as $p_T$ increases beyond 1 GeV/c.

In turning to $\Lambda_c$ production, there is a straightforward scaling up that occurs in the $P$ for $g + g$ and $q + \bar{q} \to c \uparrow + \bar{c}$. The seed polarization increases by $\sim 3$. The recombination with a fixed force/mass should have the same Thomas factor, but the overall recombination could scale as $M_{hadron}$, so a factor of $M_{\Lambda_c}/M_{\Lambda} \sim 2$ could apply. The scaled polarization in the reaction $\pi + p \to \Lambda_c \uparrow + X$ is obtained from the convolution of eqn. 2 with the $\pi$ structure functions for the beam [13]. The predicted kinematic dependences for $P(x_F, p_T)$ are shown in fig. 3 (without the hadron mass enhancement). Integrating over $x_F$ from -0.2 to +0.6 allows the comparison with the data of E791 [8, 18], as fig. 3 shows. The lower curve has taken the additional factor of 2 that could apply to the scaling of the recombination. The higher curve does not have that factor and gives a poorer fit, albeit not far from the large uncertainties in the data points.

4. CONCLUSION

In conclusion, these results are encouraging for the hybrid model. The Thomas enhanced gluon fusion model has been modified to include quark-
anti-quark annihilation, which should be more prominent for heavy baryon polarization in pion induced reactions, like the above $\pi^- + p \rightarrow \Lambda_c + X$. Experimental data can be analyzed into $x_F$ as well as $p_T$ bins, so the predictions from the hybrid model can be checked in detail. It is important to realize that the results for the $\Lambda_c$ were obtained without changing the parameters of the model that had been applied to the strange hyperons. Aside from the possible enhancement in $A$, everything else was simply scaled up by quark mass. This gives further credence to the results herein.

The somewhat ad hoc prescription for the recombination is being studied further in order to accommodate both the polarization and the cross section behavior of eqn. 7, with the kinematic variables $x_F$ and $p_T$. The overall factor $A$ may have some dependence on those variables as well, given that the semi-classical Thomas precession may have such dependence. Furthermore, an investigation of other hyperon production reactions, involving $\Sigma, \Sigma_c$, and $\Xi$, for example, is underway. Will $\bar{p} + p \rightarrow \Lambda + X$ carry significant, near energy independent polarization at collider energies? Can photoproduction of $\Lambda$ produce large polarizations also? These can be answered within the hybrid model.

The related strange meson asymmetries in $p + p \rightarrow K$ or $\pi$ or $\Lambda$ will be investigated in future work as well.

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