The magnetic properties and electronic structure of (K,Tl)$_y$Fe$_{1-x}$Se$_2$ is studied using first-principles calculations. The ground state is checkerboard antiferromagnetically coupled blocks of the minimal Fe$_4$ squares, with a large block spin moment $\sim 11.2\mu_B$. The magnetic interactions could be modelled with a simple spin model involving both the inter- and intra-block, as well as the n.n. and n.n.n. couplings. The calculations also suggest a metallic ground state except for $y = 0.8$ where a band gap $\sim 400 - 550$ meV opens, showing an antiferromagnetic insulator ground state for (K,Tl)$_{0.8}$Fe$_{0.2}$Se$_2$. The electronic structure of the metallic (K,Tl)$_y$Fe$_{1-x}$Se$_2$ is highly 3-dimensional with unique Fermi surface structure and topology. These features indicate that the Fe-vacancy ordering is crucial to the physical properties of (K,Tl)$_y$Fe$_{1-x}$Se$_2$.

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Superconductivity (SC) with moderate high transition temperatures $T_c$ has been observed in a broad family of the iron-based materials. They are typically represented by the 1111-type LaFeAsO, the 122-type BaFe$_2$As$_2$, the 111-type LiFeAs, and the 11-type FeSe. The parent compounds of these materials show a universal strip-like (collinear) antiferromagnetic (SDW) order except for the 11-type iron chalcogenides where the magnetic order is bicollinear. The magnetic properties are closely related to a common two-dimensional Fe-atom square lattice and the electronic structures are featured by the cylinder-like hole and electron pockets around the $\Gamma$ and $M$ points respectively, with relatively weak dispersions along the c-axis. By electron or hole doping the Fermi surfaces evolve smoothly in accordance with rigid band shift and the SC instability is enhanced once the magnetic order is suppressed.

Recently, a new family of the 122-type FeSe compounds K$_y$Fe$_2$Se$_2$ and (Ti,K)$_y$Fe$_{2-x}$Se$_2$ have been found to exhibit SC with transition temperatures $T_c \sim 30K$. Moreover, the iron deficient compound (Ti,K)$_y$Fe$_{2-x}$Se$_2$ shows two remarkable features: (i) The SC (appears for $x \sim 0.12 - 0.3$, $y \sim 1$) is in proximity to an insulating phase (for relatively larger $x$, or $y \sim 0.8$) [16-18], and (ii) the Fe-vacancies may exhibit some ordered superstructures [16]. Early Mössbauer experiment for TiFe$_{2-x}$Se$_2$ [19] and recent transmission electron microscopy on KFe$_{2-x}$Se$_2$ (for $x \sim 0.4 - 0.5$) [20] provide clear evidence for the tetragonal and orthorhombic superstructures in the FeSe layer. Previous first-principles calculations suggested that the Fe-vacancy orthorhombic superstructure could be stabilized with an stripe-like (collinear) AFM ground state in (Ti,K)Fe$_{1.5}$Se$_2$ [21,22]. The insulating behavior with an activation gap $\sim 60$ meV in transport observed for $x \sim 0.5$ or around [16] could be attributed to a moderate large short-ranged Fe-3d electron correlation, manifesting a possible Mott insulator driven by kinetic energy reduction due to the ordered Fe-vacancies [22]. The Mott-transition can be indeed realized by a relatively smaller $U_c$ in a two-orbital model with vacancy orderings [23,24].

So far it is yet unclear about the precise location of the critical Fe-deficiency $x_c$ (for $y = 1$) where the metal-insulator transition takes place. One should notice that in experiments the real Fe-content is sample dependent and may deviate from the nominal ones, and possibly (in addition to introduce electrons to the FeSe layer), the role of TI atoms is to stabilize the Fe-vacancy orderings while the role of alkaline atoms (K, Rb, Cs) is to achieve higher Fe-content. Among all the iron deficient compounds, (K,Tl)$_y$Fe$_{1-x}$Se$_2$ ($x = 0.4$) is of special interest due to its closer proximity to the transition point $x_c$ and the peculiar ordering pattern of the Fe-vacancies which can be stabilized in tetragonal crystalline superstructure [16,20]. This is the simplest vacancy superstructure with the highest symmetry since all iron atoms are 3-coordinated equivalently. Thus it is especially interesting to understand the electronic and magnetic structures of (K,Tl)$_y$Fe$_{1-x}$Se$_2$ with the tetragonal Fe-vacancy superstructure.

In this paper, we performed extensive study on TiFe$_{1.6}$Se$_2$, using the first-principles simulations. We have also performed calculations on KFe$_{1.6}$Se$_2$ for the magnetic structure as well as the density of states (DOS), which agrees well with TiFe$_{1.6}$Se$_2$ results. We found that tuning $y$ only leads to the rigid band shift for a specific Fe-deficiency $x$. Thus our results are valid for the mixture system (K,Tl)$_y$Fe$_{1.6}$Se$_2$. In particular, we used the Vienna Ab-initio Simulation Package (VASP) [25,26], which employs the plane-wave basis set and the projected augmented wave (PAW) method [27]. A body-centered orthorhombic primitive cell (FIG. 1(a)) was used throughout the calculation unless otherwise specified. A 360 eV energy cut-off and a $4 \times 4 \times 4 \Gamma$-centered k-grid were chosen to ensure the convergence of the total energy to 1 meV/cell. All the geometries were optimized until the forces on each atom smaller than 0.01 eV/Å and the total pressure smaller than 0.5 kBar. For the DOS calculations, a much finer k-grid of $16 \times 16 \times 16$ and the tetrahedra method were used.

Similar to (K,Tl)$_{1.5}$Se$_2$, (K,Tl)$_y$Fe$_{1.6}$Se$_2$ has three different stacking patterns. Here we focus on the in-plane magnetic structure and consider the $AA$-stacking only. As in...
FIG. 1: [1(a)] The geometry and primitive cell from top-view. The shaded (blue) region indicates the fundamental block with the four Fe-atoms at the corners. The area encircled by the dashed line for the AFM2 configuration (the ground state) is $74.75\, \text{Å}^2$. The periodic boundary condition for such blocks extended over the whole lattice is imposed. [1(b)] The proposed magnetic couplings. $(J_1, J_2)$ and $(J'_1, J'_2)$ represent the intra-block and inter-block (n.n., n.n.n.) couplings, respectively. [1(c)] to [1(j)] Various magnetic configurations. The red/blue atoms indicate the Fe-atoms with positive/negative total magnetic moment, respectively. For the AFM1 configuration, two primitive cells consist a magnetic unit cell. In all figures, we show only the Fe atoms to enhance the visibility.

dicated in $(\text{K,Tl})\text{Fe}_{1.6}\text{Se}_2$, the stacking ordering contributes only a negligible secondary correction to the total energy unless it changes the symmetry of the crystal lattice[22]. We also consider the anti-ferromagnetic (AFM) inter-layer coupling while its magnitude is negligible owing to the large inter-layer distances. As a check, we performed test calculations on one of our spin configurations. It turns out that inter-layer magnetic coupling contribution is $<1\, \text{meV/Fe}$ for TlFe$_{1.6}$Se$_2$.

To begin with, we first study the ground state magnetism by considering 8 possible in-plane AFM configurations [FIG. 1(c) to 1(j)], as well as the non-magnetic (NM) and ferromagnetic (FM) orderings. We list the relaxed geometry parameters as well as their relative energies in TABLE I. The AFM0 configuration could be regarded as the checker-board AFM; whereas the AFM1 and AFM6 configurations are the bi-collinear and zig-zag collinear orderings respectively. Our calculations suggest a ground state of the AFM2 type, whose configuration energy is $433\, \text{meV/cell}$ (or $54\, \text{meV/Fe}$) lower than the second lowest (AFM6) configuration. Due to the symmetry of Fe-vacancies, all Fe sites are equivalent in $(\text{K,Tl})\text{Fe}_{1.6}\text{Se}_2$, forming perfect square Fe blocks (indicated by the blue units in FIG. 1(b)) intercalated by Fe-vacancies. The AFM2 configuration can thus be regarded as checker-board antiferromagnetically coupled blocks of parallel aligned spins. Furthermore, due to the structure distortion induced by the Fe-vacancies, the magnetic couplings could be classified

| $a\,(\text{Å})$ | $c\,(\text{Å})$ | $m_{FV}\,(\mu_B)$ | $E_A\,(\text{meV/Fe})$ |
|----------------|----------------|-----------------|-----------------|
| NM 3.8649(3.8649) | 13.1812 | 0 | 0 |
| FM 3.7903(3.7904) | 14.7087 | 2.8 | -62 |
| AFM0 3.9026(3.9026) | 13.6949 | 2.3 | -131 |
| AFM1 3.8494(3.8182) | 14.1779 | 2.7 | -183 |
| AFM2 3.8667(3.8668) | 14.2420 | 2.8 | -254 |
| AFM3 3.8043(3.8517) | 14.1884 | 2.7 | -199 |
| AFM4 3.8892(3.8892) | 13.9058 | 2.5 | -175 |
| AFM5 3.8882(3.8778) | 13.9053 | 2.6 | -180 |
| AFM6 3.7645(3.8440) | 14.3045 | 2.8 | -200 |
| AFM7 3.8994(3.8385) | 14.0930 | 2.6 | -183 |
Where, \( n \) denotes the block index, \( \delta \) is short for the nearest neighbouring block to block \( n \), \( \alpha \) is the site-index which goes from 1 to 4, \( \alpha_3 \) selects the site connecting to the nearest neighbouring block \( \delta \); \( J_1 \) and \( J_1' \) \((J_2 \) and \( J_2' \)) indicate the n.n. (the n.n.n.) couplings of intra- and inter-block, respectively. If we further use the approximation that only the \( S^z \) component is involved (Ising model with \( S^z \) being the same in all configurations at each Fe sites), we could fit the energetics of the 8 AFM configurations using the least squares method to obtain \( J = 2JS^2 \). The resulting intra-block couplings \( J_1 \) and \( J_2 \) are -86 meV and -9 meV, respectively; while the inter-block couplings \( J_1' \) and \( J_2' \) are -29 meV and 38 meV, respectively, with fitting correlation \( \sigma = 97.04\% \). The AFM inter-block n.n.n. interaction \( J_2' \) dominates the inter-block interactions and both intra-block interactions are FM, thus a block-type checkerboard AFM configuration is favored.

We then examine the electronic structure of \((\text{K,TL})_y$$Fe_{1.0}\text{Se}_2$$. We show the band structures of the AFM2 and NM states for \( y=1.0 \) in FIG. 2(b) and 2(a), respectively. The band structure of the AFM2 state suggests a metallic nature, with a band gap \( \sim 400 \) meV for \( \text{TiFe}_{1.0}\text{Se}_2 \) (or \( 550 \) meV for \( \text{KFe}_{1.0}\text{Se}_2 \)) for the states 16 meV (or 39 meV ) below \( E_F \). Interestingly, the DOS from the top of the band gap to \( E_F \) integrates to exactly 0.2 electron per \((\text{K,TL})_y$$Fe_{1.0}\text{Se}_2 \) \((y=1.0) \) formula, suggesting that the material would become an insulator if \((\text{K,TL}) \) content is decreased by 20\%. As a check, we have further performed calculations for \( \text{K}_{0.8}$$Fe_{1.0}\text{Se}_2 \), assuming two types of K-vacancy orderings. Then a band gap \( \sim 600 \) meV shows up at \( E_F \) for either cases. The result implies that the K-vacancies shift the chemical potential but their orderings do not change the band structures.

Another interesting feature of the \((\text{K,TL})_y$$Fe_{1.0}\text{Se}_2 \) electronic structure is that in the NM state the band energies disperse significantly along \( k_z \)-axis, manifesting its highly 3-dimensional characteristics. This feature is fundamentally different from the \( \text{KFe}_{2}\text{Se}_2 \) compound and all other iron pnictides. Of course, both the NM and AFM DOS indicate that the Fe-3d orbitals dominate the states near \( E_F \), similar to all iron-based superconductors.
Finally, we reconstruct the Fermi surfaces (FIG. 3) of the metallic \((\mathrm{K,Tl})_y\mathrm{Fe}_{1.6}\mathrm{Se}_2\) \((y=1.0)\) by fitting the band structure using the maximally localized wannier functions \(\text{(MLWFs)}\)\cite{29,30}. The Fermi surface of both NM and block-spin AFM \((\mathrm{K,Tl})_y\mathrm{Fe}_{1.6}\mathrm{Se}_2\) are highly 3-dimensional, although their specific shape is quite different from each other. It is worthwhile to notice that due to the Fe-vacancy superstructure the first Brillouin zone \((\text{BZ})\) is not the same as the one in \((\mathrm{K,Tl})\mathrm{Fe}_2\mathrm{Se}_2\), and thus the M point in FIG. 3 is not \((\pi/a, \pi/a)\). Nevertheless, the Fermi surface topology of the present \((\mathrm{K,Tl})_y\mathrm{Fe}_{1.6}\mathrm{Se}_2\) compound is quite unique compared to either \(\mathrm{KFe}_2\mathrm{Se}_2\)\cite{28} or all other iron pnictides. It strongly indicates that the formation of Fe-vacancy superstructures is crucial to the electronic structures of the \((\mathrm{K,Tl})_y\mathrm{Fe}_{2-x}\mathrm{Se}_2\) compounds. Unlike the change in \(y\), the topological change of the Fermi surface across \(x \sim 0.4\) indicates that the electronic and magnetic structures for different Fe-vacancy ordered materials can not be approached by a rigid shift of the chemical potential.

It is worth noting that while the Fe-vacancy ordering is crucial to the block spin magnetic pattern and the finite gap for \((\mathrm{K,Tl})_x\mathrm{Fe}_{1.6}\mathrm{Se}_2\), the strong disorder of the Fe-vacancies may destroy the magnetic ordering leading to the metallic ground state. In the vacancy disordered state, the first Brillouin Zone would remain the same as \((\mathrm{K,Tl})\mathrm{Fe}_2\mathrm{Se}_2\) and thus the rigid-band model would remain effective. The whole electronic structure in that case could then be approximated with a hole-doped \((\mathrm{K,Tl})_y\mathrm{Fe}_{2-x}\mathrm{Se}_2\)\cite{28}. Actually, it has been suggested that randomly distributed Fe-vacancies in the doped antiferromagnetic (or Mott) insulator may lead to a spin-singlet \(s\)-wave superconductor \cite{24}.

In conclusion, we have performed first-principles calculations on \((\mathrm{K,Tl})_y\mathrm{Fe}_{1.6}\mathrm{Se}_2\). A block-type checkerboard antiferromagnetic ground state was identified and the AFM inter-block n.n.n. coupling interaction dominates. Our calculations suggest a metallic ground state for \(y = 1\) with a 400-550 meV band gap which appears slightly below the Fermi level and an insulating ground state for \((\mathrm{K,Tl})_0.3\mathrm{Fe}_{1.6}\mathrm{Se}_2\). The experimentally observed insulating behavior may be due to both the 20% \((\mathrm{K,Tl})\) deficiency and the iron vacancy superstructure. Furthermore, the electronic structures of the metallic states show a significant 3-dimensional feature with a unique Fermi surface topology, indicating that the formation of Fe-vacancy superstructure is crucial to the physical properties of \((\mathrm{K,Tl})_y\mathrm{Fe}_{2-x}\mathrm{Se}_2\).

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**Note added**: Recently, we become aware of a paper by W. Bao et al.\cite{51} on neutron diffraction experiment for \(\mathrm{K}_0.8\mathrm{Fe}_{1.6}\mathrm{Se}_2\). Our result is in agreement with the reported magnetic ordering pattern.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Fermi surfaces of \(\mathrm{TlFe}_{1.6}\mathrm{Se}_2\) reconstructed using MLWFs at (a,c): non-magnetic (NM) state and (b,d): block-spin antiferromagnetic state; (c,d): Cross section of the Fermi surface of \(\mathrm{TlFe}_{1.6}\mathrm{Se}_2\) the metallic \((\mathrm{K,Tl})\) spin AFM Fermi surfaces are 3D-like.}
\end{figure}

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