The effect of non-Gaussianity on error predictions for the Epoch of Reionization (EoR) 21-cm power spectrum

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ABSTRACT

The EoR 21-cm signal is expected to become increasingly non-Gaussian as reionization proceeds. We have used semi-numerical simulations to study how this affects the error predictions for the EoR 21-cm power spectrum. We expect SNR = \sqrt{N_k} for a Gaussian random field where N_k is the number of Fourier modes in each k bin. We find that the effect of non-Gaussianity on the SNR does not depend on k. Non-Gaussianity is important at high SNR where it imposes an upper limit [SNR]_f. It is not possible to achieve SNR > [SNR]_f even if N_k is increased. The value of [SNR]_f falls as reionization proceeds, dropping from \sim 500 at \tilde{x}_H = 0.8 - 0.9 to \sim 10 at \tilde{x}_H = 0.15. For SNR < [SNR]_f we find SNR = \sqrt{N_k}/A with A \sim 1.5 - 2.5, roughly consistent with the Gaussian prediction. We present a fitting formula for the SNR as a function of N_k, with two parameters A and [SNR]_f, that have to be determined using simulations. Our results are relevant for predicting the sensitivity of different instruments to measure the EoR 21-cm power spectrum, which till date have been largely based on the Gaussian assumption.

Key words: methods: statistical, cosmology: dark ages, reionization, first stars, cosmology: diffuse radiation

1 INTRODUCTION

Observations of the redshifted 21-cm signal from neutral hydrogen (H I) are a very promising probe of the Epoch of Reionization (EoR), and there is a considerable observational effort underway to detect the EoR 21-cm power spectrum e.g. GMRT1 (Paciga et al., 2013), LOFAR2 (Yatawatta et al., 2013; van Haarlem et al., 2013), MWA3 (Tingay et al., 2013; Bowman et al., 2013), and PAPER4 (Parsons et al., 2014; Jacobs et al., 2014). Observing the EoR 21-cm signal is one of the key scientific goals of the future telescope SKA5 (Mellema et al., 2013). It is important to have quantitative predictions of both, the expected EoR 21-cm power spectrum and the sensitivity of the different instruments to measure the expected signal.

On the theoretical and computational front, a considerable amount of effort has been devoted to simulate the expected EoR 21-cm signal (e.g. Gnedin 2000; Zahn et al. 2005; Mellema et al. 2006; Trac & Cen 2007; Thomas et al. 2009; Battaglia et al. 2013). There also have been several works to quantify the sensitivity to the EoR signal for different instruments (e.g. Morales 2005; McQuinn et al., 2006; Beardsley et al. 2013, Jensen et al. (2013) and Pober et al. 2014) have recently made quantitative predictions for detecting the EoR 21-cm power spectrum with the MWA, LOFAR, SKA and PAPER respectively.

The sensitivity of any instrument to the EoR 21-cm power spectrum is constrained by the errors, a part of which arises from the system noise of the instrument and another component which is inherent to the signal that is being detected (cosmic variance). It is commonly assumed, as in all the sensitivity estimates mentioned earlier, that the system noise and the EoR 21-cm signal are both independent Gaussian random variables. This is a reasonably good assumption at large scales in the early stages of reionization when the H I is expected to trace the dark matter. Ionized bubbles, however, introduce non-Gaussianity (Bharadwaj & Pandey, 2005) and the 21-cm signal is expected to become highly non-Gaussian as the reionization proceeds. This transition in the 21-cm signal is clearly visible in Figure 1.

In this Letter we use semi-numerical simulations of the EoR 21-cm signal to study the effect of non-Gaussianities on the error estimates for the 21-cm power spectrum. Not only is this important...
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for correctly predicting the sensitivity of the different instruments, it is also important for correctly interpreting the observation once an actual detection has been made. The entire analysis here focuses on the errors which are intrinsic to the 21-cm signal, and we do not consider the system noise corresponding to any particular instrument.

Throughout the Letter, we have used the Planck+WP best fit values of cosmological parameters \( \Omega_m = 0.3183, \Omega_{\Lambda 0} = 0.6817, \Omega_k = 0.022032, h = 0.6704, \sigma_8 = 0.8347, \) and \( n_s = 0.9619 \) (Planck Collaboration et al., 2013).

2 SIMULATING THE 21-CM MAPS

The evolution of the mass averaged neutral fraction \( \bar{x}_{HI}(z) \) during EoR is largely unconstrained. Instead of choosing a particular model for \( \bar{x}_{HI}(z) \), we have fixed the redshift \( z = 8 \) and considered different values of \( \bar{x}_{HI} \) at an interval of 0.1 in the range \( 1.0 \geq \bar{x}_{HI} \geq 0.3 \) in addition to \( \bar{x}_{HI} = 0.15 \). For each value of \( \bar{x}_{HI} \) we have simulated 21 statistically independent realizations of the 21-cm map which were used to estimate the mean \( P_b(k) \) and the rms. fluctuation \( \delta P_b(k) \) of the 21-cm power spectrum. We have used these to study how \( P_b(k) \) and particularly \( \delta P_b(k) \) evolve as reionization proceeds i.e. \( \bar{x}_{HI} \) decreases.

The simulations are based on three main steps: (1.) determine the dark matter distribution at the desired redshift, (2.) identify the collapsed halos (3.) generate the reionization map using an excursion set formalism (Furlanetto et al., 2004) under the assumption that the collapsed halos host the ionizing sources and the hydrogen exactly traces the dark matter.

We have used a particle-mesh N-body code to simulate the \( z = 8 \) dark matter distribution in a \((150.08 \, \text{Mpc})^3 \) comoving volume with a \( 2144^3 \) grid using 1072^3 dark matter particles. The standard Friends-of-Friends (FoF) algorithm was used to identify collapsed dark matter halos from the output of the N-body simulation. We have used a fixed linking length 0.2 times the mean inter-particle separation, and require a halo to have at least 10 particles which corresponds to a minimum halo mass of \( 7.3 \times 10^5 h^{-1} M_\odot \).

We have assumed that the number of ionizing photons from a collapsed halo is proportional to its mass. It is possible to achieve different values of \( \bar{x}_{HI} \) by appropriately choosing this proportionality factor. The ionizing photon field was used to construct the hydrogen ionization fraction and the \( H^I \) brightness temperature using the homogeneous recombination scheme of Choudhury et al. (2009). Following Majumdar, Bharadwaj & Choudhury (2013), the simulated \( H^I \) distribution was mapped to redshift space to generate the 21-cm maps. The steps outlined in this paragraph used a low resolution grid 8 times coarser than the N-body simulations.

Figure 1 shows a section through one of the simulated three dimensional 21-cm maps with \( \bar{x}_{HI} = 1 \) and 0.5 in the left and right panels respectively. The brightness temperature \( T_b(x) \) is to a good approximation a Gaussian random field for \( \bar{x}_{HI} = 1 \). The homogeneous recombination scheme implemented here predicts an “inside-out” reionization where the high density regions are ionized first and the low density regions later. The image at \( \bar{x}_{HI} = 0.5 \) is dominated by several ionized bubbles which preferentially mask out the high density regions, the low density regions are left untouched. We expect the statistics of \( T_b(x) \), or equivalently \( \bar{T}_b(k) \) its Fourier transform, to show considerable deviations from the original Gaussian distribution. The induced non-Gaussianity will reflect in the sizes and distribution of the ionized bubbles and we expect the non-Gaussianity to increase as reionization proceeds.

3 RESULTS

Figure 2 shows the brightness temperature fluctuation \( \Delta_b^2(k) = k^2 P_b(k)/2\pi^2 \) as a function of \( k \) for different values of \( \bar{x}_{HI} \). The average power spectrum \( P_b(k) \) and the \( 1 - \sigma \) errors \( \delta P_b(k) \) were calculated using 21 independent realizations of the simulation, and the \( k \) range has been divided into 10 equally spaced logarithmic bins. Note the change in \( \Delta_b^2(k) \) as reionization proceeds. At \( \bar{x}_{HI} = 0.5 \), the non-Gaussian Poisson noise of the discrete ionized regions makes a considerable contribution to \( \Delta_b^2(k) \) at length-scales that are larger than the typical bubble radius. The ionized regions percolate at smaller \( \bar{x}_{HI} \) where the Poisson noise of the surviving discrete \( H^I \) regions makes a considerable contribution to \( \Delta_b^2(k) \). While these effects have an imprint on the predicted \( \Delta_b^2(k) \), the power spectrum does not capture the fact the predicted signal is non-Gaussian. The error estimates for the power spectrum, however, are affected by the non-Gaussianity of the 21-cm signal.

We expect the signal to noise ratio to follow \( \text{SNR} = P_b(k)/\delta P_b(k) = \sqrt{N_k} \) if the 21-cm signal is a Gaussian random field, \( N_k \) here is the number of Fourier modes in each \( k \) bin. We have tested the Gaussian assumption by plotting the simulated SNR as a function of \( \sqrt{N_k} \) in Figure 3 where the 45° dashed line shows...
we have also considered 20 and 40 equally spaced logarithmic bins (Figure 4). The relation between $N_k$ and $k$ changes (i.e., the value of $C$ changes) if we change the number of bins, however we find that curves showing the SNR as a function of $N_k$ do not change. We therefore conclude that the effect of the non-Gaussianity on the SNR (or equivalently $\delta P(k)/P(k)$) does not depend on $k$, it depends only on $x_{HI}$ and $N_k$.

We find that the function

$$\text{SNR} = \frac{\sqrt{N_k}}{A} \left[ 1 + \frac{N_k}{(A^{[\text{SNR}]})^2} \right]^{-0.5}$$

(1)

depends on $x_{HI}$ and is independent of $k$. The parameter $A$ quantifies the deviation from the Gaussian prediction in the low SNR regime ($\text{SNR} \ll [\text{SNR}]$) where we have $\text{SNR} = \sqrt{N_k}/A$. We find that the value of $A$ increases from $A \sim 1.2$ at $x_{HI} = 0.15$ to $A \sim 2.5$ at $x_{HI} = 0.9$. Surprisingly, in this regime the SNR approaches the Gaussian prediction as the reionization proceeds. In contrast, the value of $[\text{SNR}]$ decreases by a factor of $\sim 50$ as the $x_{HI}$ falls from 0.9 to 0.15. The deviations from the Gaussian predictions seen at large SNR increase as reionization proceeds.

4 DISCUSSION AND CONCLUSIONS

We may think of the EoR 21-cm signal as a combination of two components, one a Gaussian random field and another a non-Gaussian component from the discrete ionized bubbles. The picture is slightly changed as the reionization proceeds and the ionized regions percolate. The non-Gaussian component then arises from the discrete HI clumps. The Gaussian components in the different Fourier modes $T_{ij}(k)$ are independent, the non-Gaussian components however are all correlated. The contribution to $\delta P(k)/P(k)$ from the Gaussian component comes down as $1/\sqrt{N_k}$, whereas the non-Gaussian contribution remains fixed even if $N_k$ is increased. The Gaussian assumption gives a reasonable description at low SNR, the non-Gaussian contribution however sets an upper limit $[\text{SNR}]$. It is not possible to increase the SNR beyond this by combining the signal from more Fourier modes. The non-Gaussianity
increases as reionization proceeds, and $[\text{SNR}]_l$ falls from $\sim 500$ at $\bar{x}_{\text{HI}} = 0.8 - 0.9$ to $\sim 10$ at $\bar{x}_{\text{HI}} = 0.15$.

We have used a simple model of reionization, and held $z = 8$ fixed. The predictions will be different if effects like inhomogeneous recombination are included, and the evolution of $\bar{x}_{\text{HI}}$ with $z$ is taken into account. The present work highlights the fact that non-Gaussian effects could have an important effect on the error predictions for the EoR 21-cm power spectrum.

In conclusion, the effect of the non-Gaussianity on the error estimates (cosmic variance) does not depend on the value of $k$. It is adequate to use the Gaussian assumption at low SNR. The non-Gaussian effects are important at high signal to noise ratio ($\text{SNR} \sim [\text{SNR}]_l$). The limiting signal to noise ratio $[\text{SNR}]_l$ varies with $\bar{x}_{\text{HI}}$ as shown in Figure 5.

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