Simulation and assessment of the nitrogen cycle in a constant-head, one pump (CHOP) aquaponics system

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Abstract. Aquaponics, or the co-culture of fish and plants, has been widely regarded as a solution to the growing food requirement of an increasingly urban landscape. However, difficulties in scaling up arise particularly for start-up growers due to the lack of reliable models to describe the nutrient balance in an aquaponics set-up. In this study, a comprehensive model for the products of the nitrogen cycle is presented in a constant-head one pump (CHOP) aquaponics assembly in order to have a picture of their concentrations in the system, thereby eliminating expensive trial and error adjustments. The growth rate and rate of waste generation of Nile Tilapia, \textit{Oreochromis niloticus} (Linnaeus, 1758) was mathematically represented alongside the growth rate and rate of nitrogen assimilation of lettuce (\textit{Lactuca sativa} L.). The evolution of nitrifying bacteria in the biofilter was also modelled. The condition of high recirculation rates was assumed to eliminate the spatial variation of concentration in small-volume modular tanks. The results of the simulation suggest that a linear propagation of both fish and plant through time will result to a deficiency of nutrients for the plants in the beginning of the fish growing cycle, and an excess of nitrates towards the end. To manage rising nitrate levels, the adoption of a staggered growing system was suggested instead of changing water. Overall, the developed model performed satisfactorily in providing a reference to the grower, and is hoped to be extended to other nutrients as well in the future.

1. Introduction

As wild fish stocks continue to decline due to overfishing, several ways are being developed to ease the pressure on wild stocks while continuing to feed a growing population. One such solution is aquaponics. Aquaponics is a combination of hydroponics and aquaculture, wherein the waste generated by a fish population in a closed culture supplies the nutrients essential for plant growth [1]. This configuration allows the grower to skip mineral fertilizers for the plants, as well as not having to worry about the detrimental effect of waste accumulation to the fish. This synergistic growing technique has been shown to both boost fish and plant production as well as minimize both water discharge and land use. For instance, fish stocking densities can be as high as 220 fishes/m$^3$ tank volume and plant density values can reach well over 60 plants/m$^2$ planting space [2], values that are dangerous and unsustainable if applied to conventional pond and field farming. Furthermore, the growing popularity of aquaponics is in part due to its encouraging profitability and minimal capital investment for small scale systems [3].

Regardless of the configuration used, the grower is primarily concerned on the health and growth of both fish and plant for they are the source of income. Several compounds are of outmost interest, particularly the transformations of nitrogen in the system. Fish excretes ammonia through their gills...
and feces, which in turn is oxidized by ammonia-oxidizing bacteria (AOB) to nitrite, and nitrite-oxidizing bacteria (NOB) to nitrate [4]. Plants are then left with the task of absorbing nitrate to prevent its accumulation in the system. Proper balance of organisms is crucial, as well as timing (introduction and harvest), to make sure that there would be no undesirable lack or excess of nutrients.

Several studies have attempted to model the nitrogen cycle in different aquaculture and aquaponics set-ups. A previous study provided a basic model for an aquarium without regard to fish growth and actual waste production as a function of fish age [5]. An explicit second-order Heun method was used for numerical simulation. The developed equations model the evolution of the components of the nitrogen cycle not only as a function of time but also including spatial variations, making the resultant model complex. For small scale-applications, the terms for spatial variations can be removed if enough circulation is maintained. Furthermore, the model does not incorporate the consumption of nitrate by plants. Another study modeled the nitrogen cycle along with other nutrients in a commercial INAPRO aquaponics system growing Nile tilapia and tomato [6]. Fourth order Runge-Kutta and Euler’s method were used to obtain numerical solutions to the presented equations. The developed equations are numerous and unwieldy, especially if one is modelling a small-scale system. The model for nutrient consumption was assumed to be a penalized version of growth, where the biomass accumulation of the plant would be stunted by a factor depending on the concentration of nutrients (N and P), rather than modelling the N-uptake of the plant directly through the roots. Also, the equations are suited for indoor greenhouse plant growing only. A model for a smaller scale system (PAFF box) growing Nile tilapia, lettuce, and basil, was also developed [7]. However, the developed model is especially suited for places in temperate regions and growing inside greenhouses, thus appearing to have a higher energy consumption due to water heating, indoor lighting, and temperature control.

The main objective of this study is to provide a mathematical basis for the design and simulation of basic nutrient dynamics in a constant-head one pump (CHOP) aquaponics system, with regard to bioenergetic growth models applied to Nile tilapia (Oreochromis niloticus) and lettuce (Lactuca sativa L.). This basic dynamic simulation will allow for manipulation of input variables to give the grower a starting point for the proper design and balance of a recirculating aquaculture system, eliminating the need for actual trial-and-error experimentation which is often costly and time-consuming. Furthermore, this model will allow for relevant sizing calculations and can generate process-based data, which if used together with market data, can be a useful tool in the successful start-up of an aquaponics venture.

2. Methodology

2.1. Definition of system

![CHOP aquaponics assembly](image)

**Figure 1.** CHOP aquaponics assembly (A: fish tank; B: mechanical filter; C: sump tank; D: water pump; E: biological filter (trickling filter); F: plant box; G: inlet for make-up water; H: air pump).
The general system layout for a CHOP configuration is shown in figure 1. Design parameters used for simulation are listed in table 1. CHOP was chosen as the starting point because it is modular and can be easily scaled up.

| Parameter                  | Value                  | Parameter                  | Value                  |
|----------------------------|------------------------|-----------------------------|------------------------|
| Fish tank volume, $V_{ft}$  | 1 m$^3$                | Feed rate, $f$              | $40 \, g \cdot (m^2 \cdot d)^{-1}$ |
| Sump tank volume, $V_{st}$  | 2 m$^3$                | Mean temperature, $T_m$     | 28°C                   |
| Plant bed volume, $V_{pb}$  | 1 m$^3$                | Mean radiation, $r_m$       | 4000 $Wh \cdot (m^2 \cdot d)^{-1}$ |
| Total system volume, $V_T$  | 4 m$^3$                | Air pump flow rate, $Q_{air}$ | 8 $L \cdot min^{-1}$   |
| Fish stock density, $F_D$   | 150 fishes $\cdot m^{-3}$ | Recirculation rate, $R$     | 200%                   |
| Planting density, $P_D$     | 60 plants $\cdot m^{-2}$ | Pumping rate, $Q_{water}$   | 8 $m^3 \cdot h^{-1}$   |

Several assumptions are used in the development of the model. For the base case scenario, it was assumed that there are no water changes, and that all water lost by evapotranspiration are replaced daily to keep the system volume constant. An initial average feeding rate was also included, but this value may change if the fish-to-plant ratio was also changed. The introduction of fishes and plants in the system are assumed to be continuous, i.e. the next batch is ready for transfer as soon as a batch was harvested. Mean daily temperature, mean daily radiation, air flow rate, water pumping rate, and recirculation rate are all assumed constant. Operation was also assumed continuous throughout the year without downtime.

2.2. Fish growth

Nile tilapia (Oreochromis niloticus) was chosen as a subject because of the availability of models developed to predict its growth over time. In this study, they will be assumed to be bought as fingerlings weighing 10 grams each and reared for 150 days to reach a marketable size of about 450 g per fish.

The bioenergetics growth model developed for pond-raised tilapia [8] was modified to account for higher levels of aeration and full access to artificial feed (1). The growth rate was modelled to be mainly dependent on controllable factors in a closed system such as feeding rate and temperature, as well as intrinsic characteristics of the fish species (feed assimilation and fasting catabolism).

$$\frac{dW_f}{dt} = b(1 - ae_0)SvqW_f(f_0 + m \ln t) - W_f^n k_{min} \exp[(T - T_{min})]$$  \hspace{1cm} (1)

The above equation models the absolute growth rate (g fresh weight/d) of a single fish. The solution of the differential equation will yield the weight of the fish ($W_f$) as a function of time ($t$). If it is assumed that all fish will grow at the same rate, then the solution can be multiplied by the number of fishes to obtain the total weight of fishes in the system. The term $(f_0 + m \ln t)$ is a fractional value that, if multiplied to the weight of the fish, will give the amount of feed to be given at that particular day. For accelerated growth, fishes are usually fed to satiety, and the amount to be fed greatly varies on the brand of commercial feed used. In this study, fishes are assumed to be initially fed 7% of their weight per day, and is logarithmically decreased until it reaches 2.5% of their weight per day as a finishing feed [4].

2.3. Plant growth

Leaf lettuce (Lactuca sativa L.) was chosen as the model plant for this study because its uptake of nitrate has been well documented. A reparametrized Gompertz growth model for lettuce [9] given in equation (2) was used without modification.

$$W_p = W_p^0 + (9.0977W_h - 17.254)(7.26 \times 10^{-5})^{exp(-0.05041t)}$$  \hspace{1cm} (2)

The recommended grow out period for lettuce is 30 days starting from transplanting. The
approximated fresh weight at harvest, $W_h$, was 200 g/plant. Seedlings are assumed to have an initial weight $W_p^0$ of 5 g each. Since the growth period for the plants is much less than that of the fish, multiple batches of plants will be used in the assumption that both plants and fish are present at any time within the 150-day grow-out period.

2.4. Growth of nitrifying bacteria

Two classes of bacteria are of interest, ammonia oxidizing bacteria (AOB) and nitrite oxidizing bacteria (NOB). Biomass growth for both classes are modelled using a modified Gompertz growth model with availability of space (biofilter surface area) and nutrients as the limiting factors. The equations for AOB and NOB (3) are identical in form, but differs in values of constants.

$$
\frac{d y_x}{dt} = \mu_x \exp \left\{ - \exp \left[ \frac{\mu_x e}{\ln (W_x^\text{max} / W_x^0)} (\lambda - t) + 1 \right] + \frac{\mu_x e}{\ln (W_x^\text{max} / W_x^0)} (\lambda - t) + 2 \right\}
$$

The subscript $x$ denotes either AOB or NOB. The variable $y$ is the natural logarithm of the dimensionless biomass weight, i.e., $y_{\text{AOB}} = \ln (W_{\text{AOB}} / W_{\text{AOB}}^0)$.

$$
\mu_{\text{AOB}} = \mu_{\text{AOB}}^{\text{max}} \left( \frac{C_{\text{TAN}}}{C_{\text{TAN}} + K_{\text{TAN}}} \right) \left( \frac{C_{\text{O}_2}}{C_{\text{O}_2} + K_{\text{O}_2}} \right)
$$

$$
\mu_{\text{NOB}} = \mu_{\text{NOB}}^{\text{max}} \left( \frac{C_{\text{NO}_2}}{C_{\text{NO}_2} + K_{\text{NO}_2}} \right) \left( \frac{C_{\text{O}_2}}{C_{\text{O}_2} + K_{\text{O}_2}} \right)
$$

2.5. Balances of chemical species

Upon assuming that the system is well mixed due to a high recirculation rate, the spatial variation of nutrients can be ignored. The general balance for any dissolved species can then be written as (7) where $x$ represents the chemical specie.

$$
\frac{dx}{dt} = r_x = (\frac{dx}{dt})_{\text{generation}} - (\frac{dx}{dt})_{\text{consumption}}
$$

The differential equations describing the behaviour of concentration versus time for ammonia, nitrite, nitrate, and oxygen are summarized in table 2.

| Substance | Equation |
|-----------|----------|
| TAN [10-12] | \( \frac{dC_{\text{TAN}}}{dt} = \frac{1}{V_T} \left[ q_{\text{N}} W_f (f_b + m \ln t) a_1 a_2 a_3 - 1.266 W_{\text{AOB}} \mu_{\text{AOB}} \left( \frac{1}{Y_{\text{AOB}}} + i_N \right) \right] \) |
| Nitrite [13] | \( \frac{dC_{\text{NO}_2}}{dt} = \frac{1.266}{V_T} \left( \frac{1}{Y_{\text{AOB}}} + i_N \right) \left( \frac{46}{18} W_{\text{AOB}} \mu_{\text{AOB}} - W_{\text{NOB}} \mu_{\text{NOB}} \right) \) |
| Nitrate [14] | \( \frac{dC_{\text{NO}_3}}{dt} = \frac{1}{V_T} \frac{1}{48} \left( \frac{1}{Y_{\text{NOB}}} + i_N \right) W_{\text{NOB}} \mu_{\text{NOB}} - l_{\text{max}} L N_p z \left( \frac{C_{\text{NO}_3}}{C_{\text{NO}_3} + K_{\text{NO}_3}} \right) \) |
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\[
\frac{dC_{O_2}}{dt} = K_N \left( \frac{P_{O_2}}{H} - C_{O_2} \right) - \frac{1}{V_f} \left[ 4.2528 \times 10^{-3} N_f W_f + 1.266 \left( \frac{1.5 - Y_{AOB}}{Y_{AOB}} - \frac{48}{14} \right) \mu_{AOB} W_{AOB} \right. \\
+ 1.266 \left( \frac{1.14 - Y_{NOB}}{Y_{NOB}} - \frac{48}{14} \right) \mu_{NOB} W_{NOB} \right] 
\]

(11)

2.6. Program initialization

All solutions to the ordinary differential equations presented were solved using Euler’s method with step size \( h = 0.0001 \) in Matlab. Additional constraints are placed for the concentrations of TAN, nitrite, nitrate, and dissolved oxygen. If during the iteration a negative value is obtained, the program automatically converts it to zero to signify that the consumption cannot be greater than generation. Thus, zero concentrations could mean that there is a deficit of the nutrient. Values of constants used and initial conditions are given in the Nomenclature section along with definitions and units of variables used.

3. Results and discussion

3.1. Plant and fish yield

The growth curve of tilapia is shown on figure 2. For an aquaponics system with 150 fishes being fed with 500 g of feed everyday on average, and assuming a fish feeding rate of 40 g feed per day per m² of planting area, 12.5 m² of planting area will be used. For a plant stocking density of 60 heads of lettuce per m² of planting area, 750 plants will be grown per cycle. The plants are grown in 30 days, so 5 cycles will be grown alongside the grow-out period of the fish. A total harvest of 66 kg tilapia and 846 kg of lettuce is projected at the end of 150 days. This does not include survival rates, which can average 97% per m³ for a well-circulated system [2]. The growth model for tilapia was validated against several experimental data and good agreement was observed between actual and predicted values because the model represents more or less the average growth curve of tilapia (figure 2). The model for lettuce was no longer validated because it was fitted from experimental data.

![Figure 2. O. niloticus (left) and L. sativa (right) simulated growth curves. Solid line denotes simulated data while points represent experimental values (○[17]; △[18]; □[19]).](image)

3.2. Concentration of key chemical species

If an aquaponics system is to be started with a new, clean biofilter, it would take some time before AOB and NOB can colonize the filter. The process can be accelerated by seeding the filter with a starter culture, as what was described in this study.
Ammonia starts to accumulate once the fish are introduced to the system. As such, the biofilter was immediately able to oxidize ammonia due to the introduced AOB culture. As seen in figure 3, ammonia rapidly accumulates until day 3, after which its concentration starts to decline until it stabilizes at about 25 days. The same can be said for nitrate. Their concentrations again started to increase beyond 100 days, which suggests that the waste generation of larger fish is putting more stress on the biofilter. If the fish are to be kept for longer periods, an upsizing of the filter would be recommended to increase the available surface area for bacterial growth. However, it should be noted that the critical value for total ammonia and nitrate varies between 1-2 mg/L and 3-5 mg/L, respectively. Either way, the simulation shows that safe levels are maintained throughout the grow-out period. For comparison, the simulated concentrations of ammonia and nitrite are within reasonable agreement with actual operational values [4]. It is worth noting that since ammonia is a weak electrolyte and it occurs at low concentrations, most of it is ionized to form the ammonium ion. The amount of unionized ammonia which is much more toxic to fish is negligibly small at the reported concentrations of total ammonia.

![Graph](image)

**Figure 3.** Concentration profiles of ammonia and nitrite (left) and nitrate (right). The solid and dashed lines in the nitrate concentration profile indicates with and without 1% daily water change, respectively.

The concentration of nitrate naturally fluctuates due the non-continuity of nitrate uptake by plants (figure 3). Divisions in the figure indicate separate growing periods for plants (30 days each). Young lettuce have a small ceiling of nitrate uptake due to their short roots. As the plants mature, their root systems branch out and are capable of absorbing more nitrate. However, tilapia can only tolerate nitrate concentrations up to 800 mg/L long term, or up to 1000 mg/L for short periods [17]. Elevated nitrate concentrations have adverse effects not only for fishes but also for plants grown hydroponically. Tilapia grown in an aquaculture system that are chronically exposed to elevated nitrate levels (1000 ppm) were shown to have significantly lower specific growth rates, higher feed conversion ratios, higher plasma nitrite and nitrate concentrations, lower blood haemoglobin, higher blood methemoglobin, and with more gill abnormalities than fishes exposed to lower nitrate concentrations (0-500 ppm) [20]. For plants, exposure to high nitrate concentrations could result to higher intracellular nitrate content and reduced growth. In a previous study, leafy vegetables exposed to high nitrate concentrations (450 and 600 ppm) resulted to significantly stunted growth, dramatic increase in intracellular nitrate content, and reduced nitrate reduction ability [21]. Both fishes and plants seem to be on their optimum growth and health of nitrates are kept below 500 ppm. In the current set-up, mortality rates can increase if nothing is done to bring nitrates concentration down. One common remedy to this problem is to perform daily partial water changes (DPWC). This minimizes
the accumulation of nitrates by continuously removing a small percentage of the total volume from the system. Shown in Figure 3 is the accumulation of nitrates if 1% of the total system volume is discarded and replaced daily. This results to a significant reduction in the maximum nitrate concentration, from about 1000 mg/L (no DPWC) to just above 400 mg/L.

Another possible solution is to install a bypass for an in-line denitrification systems to manage excess nitrates. This can be done by either chemical (inorganic) or biological denitrification (algae culture, duckweed culture, etc.).

The concentration of dissolved oxygen was observed to be relatively stable as long as aeration is uninterrupted. The simulation shows that the concentration of dissolved oxygen continually decreases as the fish grows, mainly due to their increase in oxygen requirement. Also, the oxygen consumption of AOB and NOB are significant. This is an important parameter to monitor because mortality rates of the fish will appreciably increase if the DO level falls below 5 ppm in higher stocking densities [4]. Also, lower DO levels will decrease the specific reaction rate of AOB and NOB, causing more ammonia and nitrite to accumulate in the system. These problems are circumvented by ensuring sufficient aeration. In this study, aeration is achieved by using air pumps to increase the water-air surface area and inducing turbulence to increase the mass transfer coefficient of oxygen in water.

The proposed model did not include the oxygen consumption of AOB and NOB lysate by metabolism of heterotrophic bacteria, oxygen consumption by plant respiration, and oxygen consumption of decaying organic matter. This implies that the simulated DO concentration profile can decrease during actual operation. Using an open trickling tower and ensuring rigorous solid waste removal can minimize the contribution of the aforementioned effects. The mentioned simplifications are recommended to be included in modifications of this model.

4. Conclusions
A dynamic simulation of the basic nitrogenous nutrients in a small-scale aquaponics system for growing both tilapia and lettuce was performed. For model simplicity and ease of use, spatial variations are neglected which is reasonable for small systems with high recirculation rates. Simulated results showed stable ammonia and nitrite concentrations, and manageable nitrate concentrations upon minimal daily partial water changes or staggered schedule of plant and fish culture. Dissolved oxygen was shown to be adequate at all times. Simulation results can be modified by changing critical parameters such as fish and plant stocking, system volume, average daily temperature, and configuration.

Appendices

| Symbol | Definition (units) | Value | Symbol | Definition (units) | Value |
|--------|-------------------|-------|--------|-------------------|-------|
| a      | Fraction of assimilated feed used for catabolism | 0.158 | $\mu_{\text{NOB}}$ | Specific reaction rate of NOB (1/d) | var. |
| $a_1$  | Mass fraction of nitrogen in fish feed | 0.0466 | $\mu_{\text{NOB}}^{\text{max}}$ | Maximum specific reaction rate of NOB (1/d) | 0.48 |
| $a_2$  | Fraction of ingested nitrogen released by fish | 0.639 | $n$ | Weight exponent for fasting catabolism | 0.81 |
| $a_3$  | Fraction of released nitrogen as ammonia in water | 0.825 | $\nu$ | Unionized ammonia factor | 1 |
| $b$    | Efficiency of feed assimilation | 0.946 | $P_{O_2}$ | Partial pressure of oxygen in atmosphere (atm) | 0.21 |
| $\delta$ | DO factor | 1 | $q$ | Feeding efficiency | 1 |
| $e_0$  | Coefficient of food energy reduction for catabolism | 0.327 | $R$ | Average solar radiation (Wh/m²·d) | 4000 |
| $f$    | Feeding rate (%body weight/d) | var. | $R_{GR}$ | Relative growth rate (g/g·d) | var. |
| $f_0$  | Initial feeding rate (%body weight/d) | 0.069 | $R_{SR}$ | Root to shoot ratio (cm/g) | var. |
| $H$    | Henry’s Law constant (mg/L·atm) | 36.88 | $T$ | Mean daily temperature (°C) | 28 |
\( H_{25} \) Henry’s Law constant at 25°C (mg/L·atm)
\( I_{max} \) Maximum nitrate uptake of plant (pmol/cm²-s)
\( i_N \) Nitrogen content of biomass (mg N/mg COD)
\( j \) Coefficient of catabolism change in \( T \) (1°C)
\( K_{TAN} \) Half-saturation constant for ammonia (mg/L)
\( K_{NO2} \) Half-saturation constant for nitrite (mg/L)
\( K_{NO3} \) Half-saturation constant for nitrate (mg/L)
\( K_{O2} \) Half-saturation constant for dissolved oxygen (mg/L)
\( K_{a} \) Overall liquid mass transfer coefficient (1/d)
\( K_{a20} \) Overall liquid mass transfer coefficient at 20°C (1/d)
\( k_{min} \) Coefficient of fasting catabolism at \( T_{min} \) (g/d)
\( L \) Total plant root length (cm)
\( \lambda \) Time lag (d)
\( m \) Feeding rate decrease factor
\( N_f \) Number of fishes
\( N_p \) Number of plants
\( \mu_{AOB} \) Specific reaction rate of AOB (1/d)
\( \mu_{AOB}^{max} \) Maximum specific reaction rate of AOB (1/d)
\( T_{min} \) Minimum livable temperature for fish (°C)
\( t \) Time (d)
\( W_{AOB} \) Biomass weight of AOB
\( W_{NOB} \) Biomass weight of NOB
\( W_f \) Weight of an individual fish (g)
\( W_p \) Weight of an individual plant (g)
\( Y_{AOB} \) True growth yield of AOB (mg COD/mg N)
\( Y_{NOB} \) True growth yield of NOB (mg COD/mg N)
\( \ln \) Natural logarithm of the ratio
\( \ln \) Conversion factor (mg·s/pmol·d)

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