Study of the neutron skin thickness of $^{208}$Pb in mean field models

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Abstract. We study whether the neutron skin thickness $\Delta r_{np}$ of $^{208}$Pb originates from the bulk or from the surface of the neutron and proton density distributions in mean field models. We find that the size of the bulk contribution to $\Delta r_{np}$ of $^{208}$Pb strongly depends on the slope of the nuclear symmetry energy, while the surface contribution does not. We note that most mean field models predict a neutron density for $^{208}$Pb between the halo and skin type limits. We investigate the dependence of parity-violating electron scattering at the kinematics of the PREX experiment on the shape of the nucleon densities predicted by the mean field models for $^{208}$Pb. We find an approximate formula for the parity-violating asymmetry in terms of the central radius and the surface diffuseness of the nucleon densities of $^{208}$Pb in these models.

1. Introduction

The neutron skin thickness of nuclei is defined as the difference between the root mean square radius of neutrons and protons,

$$\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}. \quad (1)$$

The extraction of neutron radii and neutron skins from the experiment is in general dependent on the shape of the neutron distribution used in the analysis [1, 2, 3, 4]. The data typically do not indicate unambiguously if the difference between the peripheral neutron and proton densities is caused by an extended bulk radius of the neutron density, by a modification of the width of the surface, or by some combination of both effects. In particular, the neutron skin $\Delta r_{np}$ of $^{208}$Pb is nowadays attracting significant interest in both experiment and theory since it has a close relation with the density dependence of the nuclear symmetry energy and with the equation of state of neutron-rich matter [5], which have a large impact in diverse problems of nuclear physics and astrophysics where neutron-rich matter is involved [5, 6, 7].

2. Method

The analysis of bulk and surface contributions to the neutron skin thickness of a nucleus requires proper definitions of these quantities based on nuclear density distributions [8]. Using the
Figure 1. Linear correlation of $\Delta r_{np}$ of $^{208}$Pb with the density slope of the nuclear symmetry energy $L$. The dependence on $L$ of the bulk and surface contributions defined in Eqs. (2) and (3) is also displayed.

standard definitions of the equivalent sharp radius $R$ and surface width parameter $b$ of a nucleon density profile, we have shown in Refs. [8, 4] that one can write $\Delta r_{np} = \Delta r_{np}^{\text{bulk}} + \Delta r_{np}^{\text{surf}}$, where

$$\Delta r_{np}^{\text{bulk}} = \sqrt{\frac{3}{5}} (R_n - R_p)$$

(2)

and

$$\Delta r_{np}^{\text{surf}} = \sqrt{\frac{3}{5}} \left( \frac{b_n^2}{R_n} - \frac{b_p^2}{R_p} \right).$$

(3)

We recall that $R_q$ stands for the radius of a uniform sharp distribution whose density equals the bulk value of the actual density and that has the same number of particles. Therefore, one has a natural splitting of $\Delta r_{np}$ in terms of a bulk part (2) independent of surface properties, and a part (3) of surface origin [8, 4].

3. Results

3.1. Model predictions for bulk and surface contributions

Nonrelativistic (NRMF) and relativistic (RMF) mean field models often differ in their predictions for properties of asymmetric nuclear matter. A common example is the value predicted for the density slope $L$ of the nuclear symmetry energy at saturation density, which may show large discrepancies in the MF interactions. The $L$ parameter is defined as

$$L = 3\rho_0 \frac{\partial c_{\text{sym}}(\rho)}{\partial \rho} \bigg|_{\rho_0}$$

(4)

where $c_{\text{sym}}(\rho)$ is the symmetry energy and $\rho_0$ the nuclear saturation density.

In Figure 1 we show the linear correlation of $\Delta r_{np}$ of $^{208}$Pb with the parameter $L$ and demonstrate that it mainly arises from the bulk part of $\Delta r_{np}$ within a large and representative set of mean field models of very different nature. Relativistic (RMF) models: G2, NLC, NL-SH, TM1, NL-RA1, NL3, NL3*, NL-Z, NL1, DD-ME2 and FSUGold; and nonrelativistic (NRMF) models: HFB-8, MSk7, D1S, SGII, D1N, Sk-T6, HFB-17, SLy4, SkM*, SkSM*, SkMP, Ska,
Figure 2. Parity violating asymmetry (DWBA) in $^{208}$Pb for 1 GeV electrons at 5° scattering angle. MF results and those obtained with the parametrized formula given in the text.

Sk-Rs and Sk-T4. The original references to the different interactions can be found in [9, 10] for the Skyrme models, [11] for the Gogny models, and [12, 13, 14, 15, 16] and Ref. [19] in [12] for the RMF models. From this figure, one sees that whereas the bulk contribution to the neutron skin thickness of $^{208}$Pb changes largely among the different mean-field interactions, the surface contribution remains confined to a narrow band of values. The shape of the neutron density is more uncertain than the proton density in $^{208}$Pb, and even if the neutron rms radius is determined (e.g. in PREX [17]), it can correspond to different shapes of the neutron density. As discussed in [4], from the study of the two-parameter Fermi functions fitted to the self-consistent densities of the MF models, we find that for $^{208}$Pb the central radii $C_n$ vary within the windows $C_n \approx 6.7 - 6.85$ fm in NRMF and $6.8 - 7$ fm in RMF, $C_p \approx 6.65 - 6.7$ fm in NRMF and $6.7 - 6.77$ fm in RMF, and that the diffuseness parameters $a_q$ vary within the windows $a_n \approx 0.53 - 0.55$ fm in NRMF and $0.55 - 0.59$ fm in RMF and $a_p \approx 0.43 - 0.47$ fm. From these results, we can conclude that most of the MF models predict a neutron distribution for $^{208}$Pb between the halo-type limit (where $C_n - C_p = 0$) and the skin-type limit (where $a_n - a_p = 0$). The halo-type is supported by models with a very soft symmetry energy. Models with a stiffer symmetry energy (larger $L$ values) have $C_n - C_p$ more and more different from zero. However, a pure skin-type is not found in any mean-field model as $a_n - a_p$ is always non-vanishing.

3.2. Parity violating electron scattering at the kinematics of the PREX experiment

Parity-violating electron scattering (PVES) probes the neutron density in a nucleus since the $Z^0$ boson couples mainly to neutrons [17]. The PREX experiment at JLab [18] aims to determine the neutron rms radius of $^{208}$Pb by PVES. We have investigated the dependence of PVES on the shape of the neutron and proton densities of $^{208}$Pb within the MF models. To this end, we
compute the parity-violating asymmetry

\[ A_{LR} \equiv \left( \frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega} \right) \left/ \left( \frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega} \right) \right. \]  

(5)

at the PREX kinematics [18]. We obtain the differential cross sections \( d\sigma_+/d\Omega \) for right and left-handed electrons by performing the exact phase shift analysis (DBWA) of the Dirac equation [19] for incident electrons moving in the potentials \( V_\pm(r) = V_{\text{Coulomb}}(r) \pm V_{\text{weak}}(r) \) [4]. See Ref. [4] for the details of the calculations. We display in Fig. 2 the results for \( A_{LR} \) as a function of \( C_n - C_p \) and \( a_n - a_p \) of the two parameter Fermi distributions fitted to the \(^{208}\text{Pb}\) MF densities. We have found that the results can be reasonably parametrized by the formula \( 10^7 A_{LR} \approx \alpha + \beta (C_n - C_p) + \gamma (a_n - a_p) \) with \( \alpha = 7.33 \), \( \beta = -2.45 \text{ fm}^{-1} \) and \( \gamma = -3.62 \text{ fm}^{-1} \), which is depicted by the crosses in the figure. Recently, in [20] we have analyzed systematically the correlations of \( A_{LR} \) with the neutron skin thickness of \(^{208}\text{Pb}\) and with the slope of the symmetry energy in the nuclear MF models.

4. Conclusions

We have found that the known linear correlation of \( \Delta r_{np} \) of \(^{208}\text{Pb}\) with the density derivative of the nuclear symmetry energy arises from the bulk part of \( \Delta r_{np} \). This implies that an experimental determination of \( R_n \) of \(^{208}\text{Pb}\) could be as useful for constraining \( L \) as a determination of \( \langle r^2 \rangle_n^{1/2} \). MF models can accomodate the halo-type distribution in \(^{208}\text{Pb}\) if the symmetry energy is very soft but do not support a purely skin-type distribution. We find a simple parametrization of \( A_{LR} \) in terms of \( C_n - C_p \) and \( a_n - a_p \) that would provide a new correlation between the central radius and the surface diffuseness of the distribution of neutrons in \(^{208}\text{Pb}\) assuming the proton density known from experiment.

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