Quantum mechanics as “space-time statistical mechanics”?

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(Dated: 8th October 2018)

Abstract

In this paper we discuss and analyse the idea of trying to see (non-relativistic) quantum mechanics as a “space-time statistical mechanics”, by using the classical statistical mechanical method on objective microscopic space-time configurations. It is argued that this could perhaps be accomplished by giving up the assumption that the objective “state” of a system is independent of a future measurement performed on the system. This idea is then applied in an example of quantum state estimation on a qubit system.

PACS numbers: 01.70.+w, 03.65.Ca, 03.65.Ta
Keywords: retrocausation, space-time, statistical mechanics, quantum state estimation

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I. INTRODUCTION

The intention of this article is to analyse some aspects of quantum mechanics in its standard form and try to see them from a space-time perspective. By doing so, perhaps one could get ideas for a new interpretation or a new conceptual foundation of quantum mechanics. Things that are hard to understand or seem strange in the standard form of quantum mechanics, could perhaps be understood in a better and more clear way if we see quantum mechanics from a space-time perspective. In this paper we are not trying or claiming to give a complete description or explanation of such things as what the elementary constituents of nature are, which laws govern their dynamics, etc. The basic idea presented in this paper is, in short, that there are objective configurations corresponding to a system between preparation and measurement and that these configurations are determined by both preparation and measurement observables. In sections II-IV background and motivation for this idea is presented. Section V introduces the idea of “space-time statistical mechanics” and in section VI this idea is applied in an example of quantum state estimation on a qubit system.

II. THE QUANTUM STATE

Let us consider a spin-1/2 system of an electron that has been prepared in the quantum state $|z+\rangle$. Formally, the quantum state before the measurement, in this case $|z+\rangle$, is independent of the choice of measurement observable. But, that the system and measurement apparatus can be treated independently in the theory, does not mean they are also independent in objective reality. (The concept of objective reality is something idealised and does not necessarily exist, but it is probably fair to say that to many physicists the final goal of science is to discover and describe the objective reality behind what we experience. Doing so, it would be important to separate theory from the concept of objective reality. There are
often many different models that are able to account for the same experimental
facts and data. Thus in the creation of a theory one has a freedom of choice when
it comes to designing the theory.)

Consider a classical two level system isolated from its environment. In classical
mechanics one assumes that the two levels are objective configurations of the
system and that the system always is in one or the other of those configurations.
If the exact configuration of the system is unknown, one could describe the system
as a statistical distribution \( \{ p, (1 - p) \} \) over the two configurations. A preparation
could then be a procedure that puts the system in such a distribution and the
preparation would in this sense completely determine the statistical “state” of
the system. But this is not how an isolated two level system, prepared (in the
quantum mechanical meaning of the word preparation) in some way, is described
in quantum mechanics. Instead it is mathematically described as a set of statistical
distributions, e.g.

\[
|z+\rangle = \{ p(z+)(\alpha+), p(z+)(\alpha-) \}, \{ p(z+)(\beta+), p(z+)(\beta-) \}, \ldots
\]

\[
= \{ |\langle \alpha+|z+\rangle|^2, |\langle \alpha-|z+\rangle|^2 \}, \{ |\langle \beta+|z+\rangle|^2, |\langle \beta-|z+\rangle|^2 \}, \ldots,\}
\]

(1)

where every distribution corresponds to an observable\(^1\) that can be measured,
\( \{ \alpha, \beta, \ldots \} \), and the probabilities for a specific ensemble are given by the usual rules
for calculating probabilities in quantum mechanics. There is of course nothing
peculiar or signs of any strange physics by describing the system in this way, as
this only lists every possible statistical distribution corresponding to a particular
experimental arrangement measuring some observable. But, strictly seen, in quan-
tum mechanics one only speaks about preparations and outcomes of measurements
performed on a system\(^1\). Quantum mechanics does not really speak about ob-
jective configurations of a system\(^2\) and one runs into difficulties when trying to

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\(^1\) For simplicity we do not concern ourselves with POVMs and consider only pure states.

\(^2\) However, this depends on the interpretation of quantum mechanics one considers.
Figure 1: A single-photon-source and a single-photon-detector.

replace the description of the system in [1] with only one statistical distribution over some objective configurations. There have been attempts to create microscopic models (hidden variables) behind QM, as for example the interpretation of QM by Bohm (a combined particle-wave model) [2,3]. These type of models often feels somewhat constructed and too fantastic to be true. A reasonable approach would be to first see if it is not possible to explain QM with more established concepts, for example the particle and space-time concepts without postulating such things as guiding waves or fields as in Bohm’s interpretation.

III. THE WAVE-PARTICLE DUALITY

The contents of this section are probably well known to the reader, but will be repeated for clarity. Assume we have a single photon source that can emit single photons on demand. When we press the button on the source, the single photon detector clicks (with some delay time of course, due to the photon traveling time). (See figure 1.)

If we place a 50/50 beamsplitter (BS) in the photon’s path, as in figure 2, there will be one and only one click in one of the detectors, and there is a 50% probability for either one of the detectors to click. If one of the detectors clicks, one knows with certainty that the other detector will not click. After a click in one of the detectors there are no more detectable traces of the photon anywhere.
The fact that only one of the detectors clicks is hard to explain if the photon is of purely “wave nature”. If the photon were of purely “wave nature”, one would expect the wave packet to split into two waves when passing through the BS and there would be no reason why only one of the detectors would click and the other one not. This seems to suggest that whatever the photon is, it is something localized in one and only one of the arms. That is, at the beamsplitter, the photon goes out in one and only one of the arms. In this sense the photon seems
rather to be of “particle nature”.

But what happens if we let the two photon paths meet again at another BS, as in figure 3? (We assume all the time that there is only one photon present, i.e. we only push the single-photon-source button once and do not push it again until the first photon is detected.) Since the photon seemed to be localized in only one of the arms and the BS are identical, if we repeat the experiment several times, we would in this case expect each of the detectors to click 50% of the time. But what actually happens when one performs the experiment is that one of the detectors always clicks and the other one never clicks! This is not what we would expect if the photon is of purely “particle nature”, i.e. localized in one and only one of the arms behind the first BS. But in the setup in figure 3 we could explain that one of the detectors always and the other one never clicks, if the photon is of “wave nature”. But we have already seen and argued in the setup with only one BS that a “wave nature” of the photon is not able to account for the behaviour with that setup. So the photon seems to be of neither purely wave nature nor purely particle nature.

But maybe the behaviour of the photon depends on the whole experimental setup in some way? To look into this question, consider again the setup in figure 3. But this time, while the photon is on its way to the detectors, we have the additional choice to detect the photon behind the first BS instead (see figure 4). The dotted line indicates that the photon is expected to be somewhere between the first BS and the place where two detectors will be placed if we choose to detect the photon behind the first BS.

What is the “physical reality” of the photon at the stage indicated by the dotted line? In other words, what and where is the photon at the stage indicated by the dotted line?

Depending on whether we choose to detect the photon behind the first or the
Figure 4: Two identical 50/50 BS and two pairs of single-photon-detectors that can be placed either behind the first BS or the second. The dotted line indicates that the photon is somewhere between the first BS and the place where two detectors will be placed if we choose to detect the photon behind the first BS. 

second BS, we would be inclined to draw different conclusions on what is the nature of the photon at this stage. But remember that we make the choice between detecting the photon behind the first or second BS only after the photon has been emitted by the source. Intuitively, one would expect that a supposed objective “state” or “objective configuration” of the photon would be completely determined by the source and the fact that we choose to detect the photon behind the first or second BS does not in any way affect or change its objective “state” at the stage indicated by the dotted line. If this is the case, again it seems as if the photon can neither be of purely particle nature nor purely wave nature. But what is it then?
In view of what have been said so far, let us consider the following possibility of explaining the nature and behaviour of photons. Since photons upon detection behave as though they were particles, maybe they are particles after all. If we assume that they are particles, what else do we have to change then to explain their un-particle-like behaviour? Let us go back to figure 4 again. One way in which we can uphold the idea that photons are particles, is if the path the photon takes depends on whether the detection is made behind the first or second BS! In other words, the path the photon takes depends on the whole experimental setup. This means for example that, which path the photon takes between the two BS:s depends not only on the source and the first BS, but also on whether the detectors are eventually placed behind the first or the second BS.

Let us generalise this idea: Maybe the reason it is difficult to describe the objective configuration of the system between preparation and measurement, is that it is not determined by the preparation alone, but by both preparation and measurement instead; i.e. knowing both what the prepared state and the choice of measurement observable is. For example, assume that the state was prepared in \(|z+\rangle\) and then measured in the \(x\)-direction. Then, in this situation we could assign the following statistical distribution as a representative of the objective statistical “state” of the system between preparation and measurement:

\[
\{p_{(z+)(x+)}, p_{(z+)(x-)}\} = \{|\langle x+|z+\rangle|^2, |\langle x-|z+\rangle|^2\}.
\] (2)

Here the two objective configurations of the system would be influenced by the choice of measurement observable. So, by giving up the idea that the preparation
alone should completely determine a “state” of the system, one has the possibility of describing the system with one statistical distribution over some objective configurations.

Assume that this is really how things are. But how could it possibly be like that!? How can a choice that is not yet made determine the objective state of the photon now!? Does not this imply retrocausation, i.e. future events being the cause of events in the past!?

Let us take the following view on the situation: In quantum mechanics the system is described by a set of statistical distributions, one for every possible measurement observable, which is called the quantum state of the system. The theory is thus mathematically constructed so that the description of the system is formally independent of the settings (that determine the measurement observable) of the measurement apparatus. This reflects the belief that a pre-measurement description of the system should be independent of the settings of the measurement apparatus and this belief is put into the formal structure of quantum mechanics. Viewed from an everyday or classical perspective this is a quite natural assumption or construction. But taking into consideration the strange behaviour of phenomena that quantum mechanics describes and predicts, it is not necessarily a natural assumption to make anymore. Also from a space-time perspective it is not necessarily a natural assumption to make, as will be discussed more in what follows.

At the beginning of the twentieth century Einstein revolutionized physics with his special theory of relativity [4]. The theory states that the speed of light is the highest signal velocity; this fact is often called “Einstein causality”. The concept of space-time was introduced and replaced the “Newtonian view” on
space and time as two separate entities. The concept of simultaneity no longer had any absolute meaning. The theory suggests that one should see the world as a four-dimensional space-time that is and not, as before, as a three-dimensional absolute space that changes with time. By doing so Einstein was later on able to understand gravity as curvature in this space-time \[5\]. But although we can accept and intellectually understand the concept of space-time, the human brain still perceives the world in a more Newtonian way. We feel “the passage of time” and we only experience “one moment at the time”. This is probably a reason why physicists still question whether the space-time should be considered as something real or just a useful concept, despite the success of the concept of space-time.

It is a common opinion (in view of Bell’s theorem \[6\]) that QM is non-local. But the concept of non-locality is related to the concept of simultaneity and things like action at a distance. In \[7\], p.61, Einstein writes:

There is no such thing as simultaneity of distant events; consequently there is also no such thing as immediate action at a distance in the sense of Newtonian mechanics.

There are several assumptions needed to prove Bell’s inequalities and from the fact that quantum mechanics does not satisfy Bell’s inequalities one cannot draw the conclusion that it is non-local (see e.g. \[8, 9\]). The assumptions needed to derive Bell’s inequalities all seem supported by common sense and dropping some of them seem to assume strange “non-local type correlations” among instrument settings and hidden variables. But as Morgan writes in his paper \[8\]:

The violation of Bell inequalities can be modelled by entirely local random fields, but leave an awkward question of how the nonlocal correlations might have
been established in the first place (that is, how did the “conspiracy” arise?).

So how could these type of correlations come about? What would be their origin? These type of correlations could perhaps find a more natural explanation when seen from a space-time perspective. In the space-time view the whole universe is, so to say, created “all at once”. The correlations would then be there from the beginning. The space-time would be created as a continuous whole, and correlations “forward” and “backward” in time would not be a conceptual problem when considering physical events. From a space-time perspective, a measurement “forward” in time would be similar to a preparation “backward” in time.\footnote{Similar ideas can be found in articles by O. Costa de Beauregard \cite{10,11} and references therein: “A measurement thus is a reversed preparation - a retroparation. Such is the basis of the zigzagging causality model of EPR correlations.” \cite{11}} Here we have at least some motivation for the idea that an objective pre-measurement “state” or configuration could depend not only on the preparation but also on the measurement apparatus settings.

In cases such as classical mechanics, special relativity and normally for the hidden variable theories of QM, the theories are often assumed deterministic in the following sense. An exhaustive description of all the variables on any time-slice of space and time, determines uniquely the variables for the whole space-time. When we say, in this paper, that the objective “state” of the system should not only depend on the preparation but also on the measurement apparatus settings, we do not exclude the possibility that the theory could be deterministic in the just mentioned sense. We are only saying that maybe the origin of the “non-local type correlations” could find a more natural explanation when seen from a space-time perspective.
Let us again consider the experimental arrangement where an apparatus prepares a “spin up” state in the z-direction and an apparatus will measure the state in some direction $\vec{n}$. The choice of $\vec{n}$ is determined by an observer, e.g. a human or a machine. In spirit with the space-time approach, assume now that the experimental arrangement including the system that chooses $\vec{n}$ is timelessly described and that the measurement observable is some $\vec{n}\gamma$. Then the state between preparation and measurement could as before be seen as a normal statistical mixture of two “objective configurations”:

$$\{p_{(z+)(\vec{n}\gamma+)}, p_{(z+)(\vec{n}\gamma-)}\} = \{|\langle \vec{n}\gamma+|z+\rangle|^2, |\langle \vec{n}\gamma-|z+\rangle|^2\},$$

with probabilities as calculated from the rules of QM. Consider as another example an arrangement that prepares the entangled Bell-state

$$|\psi\rangle \equiv \frac{1}{\sqrt{2}}(|z+\rangle|z+\rangle + |z-\rangle|z-\rangle)$$

and then measures the observables ($\vec{n}$, $\vec{n}'$) on the two (spatially separated) subsystems. If we assume that the (fixed) observables for this arrangement are ($\vec{n}\delta$, $\vec{n}\epsilon$), the state between preparation and measurement could instead be seen as a normal statistical mixture of four “objective configurations”:

$$\{p_{\psi(\vec{n}\delta+)(\vec{n}\epsilon+)}, p_{\psi(\vec{n}\delta+)(\vec{n}\epsilon-)}, p_{\psi(\vec{n}\delta-)(\vec{n}\epsilon+)}, p_{\psi(\vec{n}\delta-)(\vec{n}\epsilon-)}\} =$$

$$\{|\langle \vec{n}\delta+|\langle \vec{n}\epsilon+|\psi\rangle|^2, |\langle \vec{n}\delta+|\langle \vec{n}\epsilon-|\psi\rangle|^2, |\langle \vec{n}\delta-|\langle \vec{n}\epsilon+|\psi\rangle|^2, |\langle \vec{n}\delta-|\langle \vec{n}\epsilon-|\psi\rangle|^2\}. \quad (5)$$

V. “SPACE-TIME STATISTICAL MECHANICS”

The considerations made so far give a hint that quantum mechanics maybe could be recreated or interpreted as a “space-time statistical mechanics” (STSM). To explain what is meant by that consider the following: The settings on the preparation- and measurement apparatus are under the control of the observer.
Figure 5: A qubit is prepared in a state $|\tilde{n}+\rangle$ by the lower apparatus. The upper apparatus then measures the state in the $\tilde{n}'$ direction and the outcome is $\tilde{n}'+$.

The macroscopic parameters of this arrangement are $\tilde{n}+$ and $\tilde{n}'$.

He can e.g. choose what state he wants to prepare or what observable to measure. These settings we call macroscopic parameters. If these parameters are fixed, one gets a specific experimental arrangement (see figure 5). It could e.g. be the arrangement where a state is prepared as “spin-up” in the $z$-direction and measured in the $x$-direction. The macroscopic parameters would then be ’$z+$’ and ’$x$’. The “state” of the system between preparation and measurement corresponding to this arrangement is the statistical mixture of two objective configurations of the system:

$$\{p(z+)(x+), p(z+)(x-)\} = \{ |\langle x+|z+\rangle|^2, |\langle x-|z+\rangle|^2 \}, \quad (6)$$

again with probabilities calculated from the rules of QM. Which one of the two configurations is then realised and how should one describe the system? One could treat this arrangement as in classical statistical mechanics. We assume that the macroscopic space-time configuration of the arrangement is determined by
the macroscopic parameters. Further we assume that the microscopic space-time configuration of the arrangement is completely determined by some *microscopic parameters*. The idea is then that there are many different microscopic space-time configurations that give rise to a particular macroscopic space-time configuration. Further, the microscopic parameters, corresponding to the same macroscopic parameters, also determine which of the two configurations in (6) that is realized and thus the measurement outcome. Considering an ensemble of identical experimental arrangements, all with the same macroscopic parameters and where by arrangement we mean a preparation device together with a measurement apparatus (and a possible observer), one would have a representation of the statistical mixture in (6). The method presented here is analogous to the statistical mechanical method, which is the reason why we talk about “space-time statistical mechanics”. To be able to recreate quantum mechanics as a “space-time statistical mechanics”, one would probably need to find rules for how the microscopic space-time configurations are “drawn” and “selected”, and then find a systematic way of treating statistical ensembles over microscopic space-time configurations that gives rise to the probabilities that can be calculated from QM.

VI. THE STSM IDEA APPLIED IN QUANTUM STATE ESTIMATION

Let us apply the STSM idea in quantum state estimation. Consider *many* spin-1/2 systems each in some unknown state $|\theta_k, \phi_k\rangle$, where $\theta_k \in [0, \pi]$ and $\phi_k \in [0, 2\pi]$ for $k = 1, 2, 3, \ldots$, and assume that from many experiments we have observed that the mean value of the observable $\hat{\sigma}_z$ is $\bar{\sigma}_z$. To every $|\theta_k, \phi_k\rangle$ corresponds a vector $\bar{n}(\theta_k, \phi_k)$ on the Bloch sphere for which

$$
(\bar{n}(\theta_k, \phi_k) \cdot (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z))|\theta_k, \phi_k\rangle = |\theta_k, \phi_k\rangle.
$$

(7)
According to the STSM idea, the set $S$ of possible objective configurations of the system can be labeled by $(\bar{n}(\theta, \phi), \bar{n}(0,0), r)$ for short $(\theta, \phi, r)$, where $\bar{n}(0,0) \cdot (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z) = \hat{\sigma}_z$ and $r = \pm 1$. The systems are distributed over the configurations $(\theta, \phi, r)$ with the probability density $p(\theta, \phi, r)$.

How can we choose the most unbiased $p(\theta, \phi, r)$ which is compatible with the data $\bar{\sigma}_z$? – By means of the maximum-relative-entropy method. [12, 13, 14, 15]

But $p(\theta, \phi, r) = p(r|\theta, \phi) p(\theta, \phi)$ and since we want $p(r|\theta, \phi)$ to obey the rules of quantum mechanics, i.e. $p(+1|\theta, \phi) = \cos^2 (\frac{\theta}{2}) \equiv q(+1|\theta, \phi)$ and $p(-1|\theta, \phi) = \sin^2 (\frac{\theta}{2}) \equiv q(-1|\theta, \phi)$, we are not going to use the Lagrange-multiplier method with this non-linear constraint. Instead we apply the maximum-relative-entropy method on probability densities of the form $\tilde{p}(\theta, \phi, r) \equiv q(r|\theta, \phi) p(\theta, \phi)$, not varying $q(r|\theta, \phi)$ but only $p(\theta, \phi)$.

First, since the mean value comes from many experiments, we can assume that the preparation must be such that

$$\bar{\sigma}_z = \langle \hat{\sigma}_z \rangle = \sum_r \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r \cdot \tilde{p}(\theta, \phi, r) \sin \theta \, d\phi \, d\theta$$

$$= \sum_r \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r \cdot q(r|\theta, \phi) p(\theta, \phi) \sin \theta \, d\phi \, d\theta$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} [q(+1|\theta, \phi) - q(-1|\theta, \phi)] \, p(\theta, \phi) \sin \theta \, d\phi \, d\theta$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} [\cos^2 (\theta/2) - \sin^2 (\theta/2)] \, p(\theta, \phi) \sin \theta \, d\phi \, d\theta$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \cos \theta \, p(\theta, \phi) \sin \theta \, d\phi \, d\theta. \quad (8)$$

This constraint of course does not select a unique $\tilde{p}(\theta, \phi, r)$, but we can choose
the one among them which has maximum relative entropy

\[ H^{(3)}(\tilde{p}(\theta, \phi, r)) \equiv - \sum_r \int_0^{2\pi} \int_0^\pi \tilde{p}(\theta, \phi, r) \log \frac{\tilde{p}(\theta, \phi, r)}{\bar{m}(\theta, \phi, r)} \sin \theta \, d\phi \, d\theta \]

\[ = \int_0^{2\pi} \int_0^\pi p(\theta, \phi) \log \frac{p(\theta, \phi)}{m(\theta, \phi)} \sin \theta \, d\phi \, d\theta \equiv H^{(2)}(p(\theta, \phi)). \]

Here \( \bar{m}(\theta, \phi, r) \) is the prior probability density and \( \bar{m}(\theta, \phi, r) = q(r|\theta, \phi) m(\theta, \phi) \), since the prior probability distribution should also obey the rules of quantum mechanics. In the last step we have used that \( p(+1|\theta, \phi) + p(-1|\theta, \phi) = 1 \). We see from (9) that, to find the \( \tilde{p}(\theta, \phi, r) \) that maximizes \( H^{(3)}(\tilde{p}(\theta, \phi, r)) \), it is enough to find the \( p(\theta, \phi) \) that maximizes \( H^{(2)}(p(\theta, \phi)) \) and the sought probability density will then be given by \( \tilde{p}(\theta, \phi, r) = q(r|\theta, \phi) p(\theta, \phi) \). Let \( p'(\theta, \phi) \) be the probability density that maximizes \( H^{(2)}(p(\theta, \phi)) \) under the data constraint in (8), where the prior probability density \( m(\theta, \phi) \) is taken to be constant; \( m(\theta, \phi) = 1/(4\pi) \). The solution to the non-linear constraint problem is

\[ \tilde{p}'(\theta, \phi, r) = q(r|\theta, \phi) \frac{m(\theta, \phi)}{Z} e^{-\lambda \cos \theta} \]

where

\[ Z \equiv \int_0^{2\pi} \int_0^\pi m(\theta, \phi) e^{-\lambda \cos \theta} \sin \theta \, d\phi \, d\theta. \]

(The Lagrange multiplier \( \lambda \) is determined from the data constraint.) One can easily show that to this probability density corresponds the density matrix

\[ \hat{\rho} = \sum_r \int_0^{2\pi} \int_0^\pi \tilde{p}'(\theta, \phi, r) |\theta, \phi\rangle \langle \theta, \phi| \sin \theta \, d\phi \, d\theta = \frac{1}{2} (\hat{I} + \sigma_z \sigma_z) \].

Note that this is the same density matrix one would find with the “von Neumann entropy method” [12, 13], in which one instead seeks the density matrix that maximizes the von Neumann entropy, under the same data constraint \( \sigma_z = \langle \sigma_z \rangle \).
Acknowledgments

The author would like to thank Piero G. Luca Mana, Peter Morgan, Anders Karlsson and Gunnar Björk for advice, encouragement, and useful discussions.

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