Performance Analysis of Cognitive Radio Networks With Burst Dynamics

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This work was supported in part by the National Natural Science Foundation of China under Grant 61663024, in part by the Erasmus+ Programme under Grant 573879-EPP-1-2016-1-FR-EPPKA2-CBHE-JP, and in part by the Hongliu First Class Discipline Development Project of Lanzhou University of Technology, China.

\textbf{ABSTRACT} The impact of the traffic characteristics of secondary users (SUs) on the performance of cognitive radio networks (CRNs) should be understood for designing operation rules. This paper focuses on multi-type burst services and network congestion problem in CRNs and evaluates the QoS of SUs based on multiple cross-layer considerations. A two-state Markov-modulate Bernoulli process (MMBP-2) is adopted to model packet flows of SUs with different burst degrees in CRNs. We propose a two-dimensional discrete queuing model to consider spectrum access, burstiness of traffic, network congestion, channel environment, user activity and finite buffer. We construct an iterative algorithm to compute the steady-state distribution of the proposed queuing model and determine the performance metrics like the throughput, the average delay, the average queue length and the total packet loss probability. Numerical analysis evaluates the QoS of SUs under different burst environments.

\textbf{INDEX TERMS} Cognitive radio networks, burst traffic, network congestion, queuing model, two-dimensional Markov chain, performance evaluation.

I. INTRODUCTION

\textbf{A. MOTIVATION}

At present, the operation of the traditional wireless communication systems is based on the static licensed allocation of the radio spectrum. Research shows that the utilization of the licensed spectrum is low because licensed (primary) users are not always active on the allocated bands \cite{1}. For the improvement of spectrum utilization \cite{2}, cognitive radio (CR) technology allows unlicensed (secondary) users to detect idle periods of the licensed spectrum and transmit data during periods when primary users (PUs) are not active \cite{3}, \cite{4}. Sensing ability and reconfigurable ability are the necessary features of CR technology. Some licensed share access (LSA) concepts have been proposed to cope with unpredictable environments \cite{5}–\cite{7}. The performance evaluation of cognitive radio networks (CRNs) where the complex interaction between various factors (the burstiness of traffic, unpredictable environment, etc.) happens could help design operation rules.

\textbf{B. RELATED WORK}

According to whether the PUs allow the SUs to coexist with them, CR is divided into three modes: interweave, overlay and underlay \cite{8}. The research of this paper is mainly based on the overlay mode. In this mode, the SUs can excavate the spectrum hole of the PUs and communicate on the basis of guaranteeing the transmission performance of the PUs. In the overlay mode, the SUs satisfy their own services through cooperates with the PUs, in the form of renting the spectrum that the PUs do not often use or relaying the communication between the PUs. Chavez-Santiago \textit{et al.} \cite{9} proposed an ultra-wideband radio technology where the secondary systems can utilize the specific cognitive radio licensed bands to ensure reliable transmission. Researchers \cite{5}–\cite{7} investigated sharing rules where PUs cannot preempt the services of SUs during a predefined time interval, which protects the transmission of SUs.
Queuing model can describe the network model and analyze the system performance [10]. Therefore, it is an effective method to establish a queuing model to evaluate the performance of the secondary system in CRNs. Based on the cross-layer consideration, [11]–[14] investigated the access mode of the secondary system with the use of the queuing theory. Jian et al. [12] set up a discrete-time finite-state Markov chain to explore the impact of sensing errors, medium-access control (MAC) scheme, adaptive modulation and coding (AMC), automatic repeat-request (ARQ) and buffer size on the quality of service (QoS) of SUs. Simulation results [12] showed the influence of system settings on the QoS metrics of SUs. Furthermore, Zhang et al. [13] considered cross-layer factors such as path loss, ARQ and buffer capacity to establish the M/M/1/K queuing model to analyze the delay of SUs. Their results [13] indicated that the buffer capacity of SUs and the tolerable number of ARQ play an essential role in average delay and packet loss probability. Queuing models were established in [15], [16] to characterize the spectrum access mode of multi-priority SUs traffics, so as to evaluate the QoS. Yang et al. [17] established an objective function to obtain the optimal spectrum access strategy based on the interference quantity on PUs and the spectrum leasing cost. They applied a queuing model to evaluate the throughput of the secondary system. However, the above works did not consider the problem of multi-type burst services, especially the modeling and performance evaluation of SUs traffic with different burst degrees. Consequently, this motivates the investigation of our study, as observed in Table 1, which summarizes aspects considered by the literary works and shows what are new in this paper compared to the current results. This paper considers the physical layer and data link layer mechanism, traffic of multi-type burst, congestion control based on the random early detection (RED) [18] algorithm to evaluate the QoS of SUs. In addition, we also take the channel model at the physical layer into consideration.

By considering the listed parameters, multi-type burst services are modeled and analyzed. Since data traffic bursts at any time scale, there exists a certain correlation between traffics with different burst degrees. Autocorrelation is used to describe burstiness. The two-state Markov-modulate Bernoulli process (MMBP-2) model differentiates an input source into two states, then the correlation between them can be established with the Markov chain [19].

| Ref. | Cross-layer Consideration | Spectrum Sensing | Channel Fading | Multi-type SUs Traffic | Burst Traffic | Finite Buffer | Queuing Model | Focused Parameters | QoS Evaluation Indexes of SUs |
|------|---------------------------|------------------|----------------|------------------------|--------------|---------------|---------------|-------------------|--------------------------------|
| 2012 [11] | Yes | Yes | Yes | No | No | Yes | No | ARQ / AMC / Finite buffer size / Sensing errors / Rayleigh fading | Spectral efficiency / Power |
| 2013 [12] | Yes | Yes | Yes | No | No | Yes | Yes | MAC protocol / ARQ / AMC / Finite buffer size / Sensing errors / Block fading | Average queue length / Packet dropping rate / Packet collision rate |
| 2016 [13] | Yes | Yes | Yes | No | No | Yes | Yes | ARQ / Finite buffer size / Sensing errors / Nakagami-m fading | Average delay / Packet loss rate |
| 2017 [14] | Yes | Yes | No | No | No | No | Yes | ARQ / Sensing errors | Average delay / Throughput |
| 2016 [15] | Yes | No | Yes | Yes | No | No | Yes | Priority of SUs / Infinite buffer / Gaussian noise / Priority of SUs | Average waiting time / Reneging probability / System idle probability |
| 2018 [16] | No | No | No | Yes | No | No | Yes | Interference quantity on PUs / Spectrum leasing cost | Average waiting time / Number of users |
| 2019 [17] | No | Yes | No | No | No | No | Yes | Throughput / blocking rate / Dropping rate / Interference quantity on the PUs | Throughput / Average queue length / Average delay / Packet loss rate |
| Proposed | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Multi-type SUs traffic / Burst traffic / Congestion control / Finite buffer size / Nakagami-m fading | Average throughput / Average queue length / Average delay / Packet loss rate |

C. NOVELTY AND CONTRIBUTIONS

In this paper, through studying the spectrum access mode of multi-type SUs services with different burst degrees, we consider multiple cross-layer factors that affect spectrum access including channel environment, user activity, multi-type burst services, finite buffer and network congestion. The above factors are integrated into a two-dimensional probability space and a two-dimensional stochastic process is proposed to describe the system. The main contributions of this paper are as follows:

1) A model integrates multi-factors and cross-layer considerations. This paper integrates multiple affecting factors of spectrum access, such as multi-type burst services, network congestion, channel environment,
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user activity and finite buffer. The traffics generated by SUs belong to one of two types, and the MMBP-2 process models their bursty nature. The proposed model also considers the congestion control performed by the dual-threshold RED algorithm. To our best knowledge, no such model is available so far.

2) Construct a two-dimensional Markov chain. A two-dimensional Markov chain is constructed to integrate the above mentioned affecting factors into a two-dimensional probability space. Then, a mathematical model is used to solve the performance evaluation problem.

3) Design of iterative algorithm: An iterative algorithm is presented to compute the steady-state probabilities of the two-dimensional Markov chain and the performance metrics (throughput, average delay, average queue length and total packet loss probability).

4) Congestion control management approaches (no congestion control, dynamic random early detection, and random early detection) are compared. The impact of channel fading on the system performance is investigated.

II. QUEUING MODEL

A. ASSUMPTIONS

We consider a CRN system that operates in discrete-time slots. A clustered time-slot CRN model with a finite number of PUs and SUs is illustrated in Figure 1. The information transmitted through CR may be of multiple types, including data, image, audio and video. The SUs are heterogeneous, independent and identically distributed (i.i.d). Each cluster has one base station for PUs (PU_BS) and one for SUs (SU_BS).

The direction of data transmission considered in this paper is the uplink from SU_BS to PU_BS. According to the past channel observation, the probability of a channel idle in the next slot is estimated. The scheduling of the SUs is performed by the SU_BS based on the spectrum estimation results. At each time slot, the cognitive phase and the transmission phase may happen. If PU does not occupy the channel at the beginning of a specific slot, the PU will not use the channel throughout the slot.

In this paper, we make further assumptions as well:

1) Control node can perform orderly scheduling for multi-type SUs traffics, and then the collision between SUs can be avoided.

2) PUs and SUs apply the overlay mode of a predefined resource sharing rule where the PUs cannot preempt the services of the SUs within a predefined time interval [5].

3) The SU packets enter the channel according to the First Come First Service (FCFS) principle.

As shown in Figure 2, a buffer with finite capacity (K) is set for SUs at SU_BS. Let N_PU and N_SU be the number of PUs and SUs, respectively.

In CRNs, SUs may transmit packets carrying either data/image or audio/video. These traffics are mostly burst and usually has a certain correlation between traffics with different burst degrees. In data link layer, this paper adopts MMBP-2 as the input source model to characterize the queuing dynamic of traffics with different burst degrees. MMBP-2 consists of two states to differentiate traffics with different burst degrees. Markov chain can be used to link the transitions between the states to depict the correlation between the different type of packets. The data/image service has a short frame length and high delay tolerance, while the audio/video service has a low tolerance for delay. Therefore, traffics of a specific bursty degree in the data link layer are described by the discrete-time, Markov-modulated Bernoulli process (MMBP-2) with two states (S_1 and S_2), where S_1 represent the state of a particular SU when it transmits an audio/video packet while S_2 corresponds to the SU’s state when it sends a data/image traffic.

Since burst traffics likely cause congestion, a dual-threshold queue management scheme named random early detection (RED) is adopted for the SUs queuing in the buffer, where the congestion of secondary network is controlled by actively dropping a small part of data packets.

SU packets leaving the buffer will enter the channel and begin the service. A SU can access any idle channel, and the sum of the probabilities of accessing all the idle channels is 1. A channel corresponds to a server in the queuing model. In what follows, we only describe the internal user activity
of one of the channels, so the established queuing model has a single server. Assume that the Nakagami fading model describes the characteristic of a channel in the physical layer. Not that the probability that the specific data packet of SUs is successful received on the channel is affected by the propagation link, the activity of PUs and the cognitive ability of SUs. Therefore, the channel fading and the user activity jointly determine the successful reception probability, \(\sigma_{SU}\), of SUs.

If the arrival of the SU traffic is in state \(S_1\) (\(S_2\)) in the current slot, the probability of remaining in \(S_1\) (\(S_2\)) in the next slot is \(p (q)\), and the probability of transferring to state \(S_2\) (\(S_1\)) is \(1 − p (1 − q)\). The process obeys the Bernoulli distribution and the rate of sending a packet in states \(S_1\) and \(S_2\) are \(\alpha_1\) and \(\beta_1\), respectively.

The MMBP-2 process is characterized by the transition probability matrix \(P\) and the arrival rate matrix \(\Lambda_1\) as:

\[
P = \begin{bmatrix}
p & 1-p \\
1-q & q
\end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix}
\alpha_1 & 0 \\
0 & \beta_1
\end{bmatrix}
\]  

(1)

The MMBP-2 process has the following characteristics [19].

1) The average arrival rate:

\[
\rho = (q\alpha_1 + \beta_1)/(\beta + q)
\]  

(2)

2) The squared coefficient of variation of the interarrival time, \(c^2\), describes the burst degree of SUs traffic as:

\[
c^2 = \frac{2\rho[(\beta + q)^2 + (q\alpha_1 + \beta_1)(p + q - 1)]}{(\beta + q)[q\alpha_1 + \beta_1 + \alpha_1\beta_1(p + q - 1)]} - 1
\]  

(3)

3) The first-order autocorrelation coefficient of the interarrival time, \(\phi(1)\), and the \(x\)-th order auto-correlation coefficient of the number of SU’s arrivals based on MMBP-2, \(\Phi(x)\), which describes the correlation between the arrivals of SUs traffic as:

\[
\phi(1) = \frac{\alpha_1\beta_1(\alpha_1 - 1)^2\beta_1[p + q - 1]^2}{c^2(p + q)^2[q\alpha_1 + \beta_1 + \alpha_1\beta_1(p + q - 1)]^2}
\]  

(4)

\[
\Phi(x) = \frac{\beta_1[p + q - 1]^x}{(q\alpha_1 + \beta_1)[q(1 - \alpha_1) + \beta(1 - \beta_1)]}
\]  

(5)

A buffer with the finite capacity, \(K\), is set for SUs at each terminal in \(SU_{BS}\). There are \(SU_N\) secondary users. We assume that \(SU_{BS}\) applies a control mechanism with \(L_1\) and \(L_2\) thresholds, \(0 < L_1 < L_2 < K\), (see Figure 3).

Upon the arrival of a packet,

1) if the queue length is less than \(L_1\), the packet is stored in the buffer;
2) if the queue length is between \(L_1\) and \(L_2\), and the arrival process is in state \(S_1\), the packet is dropped with probability \(1 - \alpha_2\);
3) if the queue length is between \(L_1\) and \(L_2\), and the arrival process is in state \(S_2\), the packet is dropped with probability \(1 - \beta_2\);
4) if the queue length exceeds \(L_2\), and the arrival process is in state , it is dropped with probability \(1 - \alpha_3\);
5) if the queue length exceeds \(L_2\), and the arrival process is in state \(S_2\), it is dropped with probability \(1 - \beta_3\).

Note that \(\alpha_3 < \alpha_1, \beta_3 < \beta_1\), the values of \(\alpha_m\) and \(\beta_m (m = 1, 2, 3)\) are set as:

\[
\alpha_m = \begin{cases}
\alpha_1, & 0 \leq j < L_1 \\
\alpha_1 + (\alpha_3 - \alpha_1)(j - L_1 + 1)/L_2 - L_1 + 1, & L_1 \leq j < L_2 \\
\alpha_3, & L_1 \leq j \leq K;
\end{cases}
\]  

(6)

\[
\beta_m = \begin{cases}
\beta_1, & 0 \leq j < L_1 \\
\beta_1 + (\beta_3 - \beta_1)(j - L_1 + 1)/L_2 - L_1 + 1, & L_1 \leq j < L_2 \\
\beta_3, & L_1 \leq j \leq K
\end{cases}
\]  

(6)

where \(j, (0 \leq j \leq K, j \in Z)\) is the current length of the SUs queue.

Let \(\sigma_{SU}\) denotes the probability that the data packet of SUs is successful received on channel. Assume that radio propagation link between any pair of nodes generated in the transmission process in the transmission process is affected by the independent stationary Nakagami flat-fading channel, and the envelope of the received signal is \(r_{SU}\). SUs can always perform correct spectrum sensing. Thus, the probability that the SU packet is successfully received when only SU exists in channel, \(P_{[SU_{succ}|SU]}\), can be obtained:

\[
P_{[SU_{succ}|SU]} = \Pr \left[ |r_{SU}(t)|^2 > \frac{C_{SU}R_{SU}}{P_{SU}} \right]
\]  

(6)
where $P_f$ is the false alarm probability. Since the probability of $L_{PU} = 0$ is 1, then we obtained:

$$\sigma_{SU} = P_{SU \text{succ}}(SU) \cdot (1 - P_f)$$  \hspace{1cm} (9)

In addition, [13] also discusses the probability that the SUs being received successfully with sensing errors. Since the focus of this study is to evaluate the service performance of secondary system without considering the collision between packets, the situation of sensing errors will not repeat here.

### B. QUEUING ANALYSIS

Let $j$ represents the current queue length of SUs in the buffer. A two-dimensional Markov chain $\{s, j\}, s = 1, 2; j = 0, 1, 2, \ldots, L_1 - 1, L_1, \ldots, L_2, L_2 + 1, \ldots, K$ is used to describe the system. The transition diagram of the Markov chain is depicted in Figure 4.

The probability that the queue length is $j$ at the current slot, and it will not change at the next slot is $P_{s,j \rightarrow s,j}^R$. Let $P_{s,j \rightarrow s,j+1}^A$ denote the probability that the current queue length is $j$ and it changes to $j+1$ at the next slot. Let $P_{s,j \rightarrow s,j-1}^D$ be the probability that the current queue length is $j$ and it goes to $j-1$ at the next slot. The rise in the queue length indicates that a data packet enters the SU queue, while the decrease is due to the departure of a packet after completing the service.

The probability that the sending rate is $\alpha_m$ and the queue length is $j$ at the current slot while the sending rate changes to $\beta_m$ and the queue length changes to $j-1$ at the next slot is $P_{1,j \rightarrow j-1}^D$. The probability that the sending rate is $\beta_m$ and the queue length is $j$ at the current slot, then the sending rate changes to $\alpha_m$ and the queue length is $j-1$ at the next slot is $P_{2,j \rightarrow j-1}^D$. $P_{1,j \rightarrow j-1}^D$ or $P_{2,j \rightarrow j-1}^D$ represents the probability that the sending rate is $\alpha_m(\beta_m)$, the queue length is $j$ and the queue length changes to $j-1$ at the next slot but the sending rate is not change. Thus, the transition probabilities of two-dimensional Markov chain which consist of $P_{s,j \rightarrow s,j-1}^D, P_{s,j \rightarrow s,j+1}^A$ and $P_{s,j \rightarrow s,j}^R$ are as follows:

1) The probability, $P_{s,j \rightarrow s,j-1}^D, (1 \leq j \leq K)$, of a packet departure at the next slot is:

$$P_{1,j \rightarrow j-1}^D = \begin{cases} (1 - \alpha_1) & 1 \leq j \leq L_1 \\ (1 - \alpha_2) & L_1 < j \leq L_2 \\ (1 - \alpha_3) & L_2 < j \leq K \\ \sigma_{SU}(1 - \alpha_1)(1 - \beta_1) & 1 \leq j \leq L_1 \\ \sigma_{SU}(1 - \alpha_2)(1 - \beta_2) & L_1 < j \leq L_2 \\ \sigma_{SU}(1 - \alpha_3)(1 - \beta_3) & L_2 < j \leq K \\ \end{cases}$$

2) The probability, $P_{1,j \rightarrow s,j+1}^A, (0 \leq j < K)$, of a packet arrival at the next slot is:

$$P_{1,j \rightarrow 1,j+1} = \begin{cases} \alpha_1 & j = 0 \\ \alpha_2(1 - \sigma_{SU}) & 1 \leq j < L_1 \\ \alpha_3(1 - \sigma_{SU}) & L_1 \leq j < L_2 \\ \alpha_4(1 - \sigma_{SU}) & L_2 \leq j < K \\ \end{cases}$$

3) The probability, $P_{s,j \rightarrow s,j}^R, (0 \leq j \leq K)$, that the packet still in its original queue length state at the next slot is:

$$P_{1,j \rightarrow 2,j} = \begin{cases} (1 - \alpha_1)(1 - \beta_1) & j = 0 \\ \sigma_{SU}(1 - \alpha_1)(1 - \beta_1) & 1 \leq j < L_1 \\ \sigma_{SU}(1 - \alpha_2)(1 - \beta_2) & L_1 \leq j < L_2 \\ \sigma_{SU}(1 - \alpha_3)(1 - \beta_3) & L_2 \leq j < K \\ \end{cases}$$

$$P_{2,j \rightarrow 1,j} = \begin{cases} (1 - \beta_1) & j = 0 \\ \beta_2(1 - \sigma_{SU}) & 1 \leq j < L_1 \\ \beta_3(1 - \sigma_{SU}) & L_1 \leq j < L_2 \\ \beta_3(1 - \sigma_{SU}) & L_2 \leq j < K \\ \end{cases}$$

$$P_{2,j \rightarrow 2,j} = \begin{cases} (1 - \beta_1) & j = 0 \\ \beta_2(1 - \sigma_{SU}) & 1 \leq j < L_1 \\ \beta_3(1 - \sigma_{SU}) & L_1 \leq j < L_2 \\ \beta_3(1 - \sigma_{SU}) & L_2 \leq j < K \\ \end{cases}$$
Note that there are two possible scenarios when the queue length remains unchanged in the last case above: i) either the number of arrival and departure packets cancels each other out, or ii) there is no arrival and departure packet in the SU queue. When the first scenario occurs, a packet has been served successfully. Otherwise, the packet is not served successfully. In particular, when the SU queue is empty, no packet can be served. When it is full, the probability that the packet being successfully served is 1.

Let $\pi(s, j)$ be the steady-state probability of state $(s, j)$, $(0 \leq j \leq K$, $s = 1$ or $s = 2)$. Then the balance equations of the two-dimensional Markov chain are as follows (13), as shown at the bottom of the next page.

Let $\pi(j) = \pi(1, j) + \pi(2, j)$ represents the steady-state probability of the queue length $j$, $(j = 0, 1, 2, \ldots, K)$. Therefore, equation (13) can be transformed into the following form:

$$\pi Q = \pi$$ and $\pi e = 1$, where $e = [1, 1, 1, \ldots, 1]_{(K+1) \times 1}$

(15)

where $\pi = [\pi_0, \pi_1, \pi_2, \ldots, \pi_K]$ is the steady-state distribution of the system model.

C. CALCULATION OF THE STEADY-STATE DISTRIBUTION

Following [20], an iterative algorithm is constructed to solve (14) for steady-state distribution. To ease the comprehension, we present steps for $L_1 = 4$, $L_2 = 11$, $K = 15$. In this case, matrix $Q$ can be expressed as $Q$, as shown at the bottom of the page 110634.

Note that $Q$ is a matrix of size $32 \times 32$ and composed of block matrices of size $2 \times 2$.

The idea is to reduce the block matrix with size $16 \times 16$ to a one-block matrix with size $1 \times 1$ in the forward reduction phase. After getting the steady-state solution of one-block matrix (named boundary vector), the solution is expanded layer by layer to obtain the steady-state solution of matrix $Q$ in the backward expansion phase. The layer here is the row of $Q$, and the number of the layers corresponds to the current queue length of the SU queue, $j$, in the buffer. The details of our algorithm are as follows:

1) FORWARD REDUCTION PHASE

Firstly, permute the rows and columns of the blocks of layer 0-15 of matrix $Q$ in the following order:

$$
\begin{pmatrix}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 1 & 3 & 5 & 7 & 9 & 11 & 13 & 15
\end{pmatrix}
$$

(16)

That is, the blocks of $Q$ in layer 1 goes to the place of layer 2, layer 2 goes to the place of layer 4, etc. As a result,
\[
\begin{align*}
\pi(1, 0) &= P^R_{1,0 \rightarrow 1,0} \pi(1, 0) + P^R_{2,0 \rightarrow 1,0} \pi(2, 0) + P^D_{2,1 \rightarrow 1,0} \pi(2, 1) + P^D_{1,1 \rightarrow 1,0} \pi(1, 1) \\
\pi(1, 1) &= P^R_{1,1 \rightarrow 1,1} \pi(1, 1) + P^R_{2,1 \rightarrow 1,1} \pi(2, 1) + P^A_{1,0 \rightarrow 1,1} \pi(1, 0) + P^A_{2,0 \rightarrow 1,1} \pi(2, 0) \\
&\quad + P^D_{2,2 \rightarrow 1,1} \pi(2, 2) + P^D_{1,2 \rightarrow 1,1} \pi(1, 2) \\
&\quad \ldots \\
\pi(1, K-1) &= P^R_{1,K-1 \rightarrow 1,K-1} \pi(1, K-1) + P^R_{2,K-1 \rightarrow 1,K-1} \pi(2, K-1) \\
&\quad + P^A_{1,K-2 \rightarrow 1,K-1} \pi(1, K-2) + P^A_{2,K-2 \rightarrow 1,K-1} \pi(2, K-2) \\
&\quad + P^D_{2,K-1 \rightarrow 1,K-1} \pi(2, K) + P^D_{1,K-1 \rightarrow 1,K-1} \pi(1, K) \\
\pi(1, K) &= P^R_{1,K-1 \rightarrow 1,K} \pi(1, K) + P^R_{2,K-1 \rightarrow 1,K} \pi(2, K) + P^A_{1,K-1 \rightarrow 1,K} \pi(1, K-1) \\
&\quad + P^A_{2,K-1 \rightarrow 1,K} \pi(2, K-1) \\
\pi(2, 0) &= P^R_{2,0 \rightarrow 2,0} \pi(2, 0) + P^R_{1,0 \rightarrow 2,0} \pi(1, 0) + P^D_{2,1 \rightarrow 2,0} \pi(2, 1) + P^D_{1,1 \rightarrow 2,0} \pi(1, 1) \\
\pi(2, 1) &= P^R_{2,1 \rightarrow 2,1} \pi(2, 1) + P^R_{1,1 \rightarrow 2,1} \pi(1, 1) + P^A_{2,0 \rightarrow 2,1} \pi(2, 0) + P^A_{1,0 \rightarrow 2,1} \pi(1, 0) \\
&\quad + P^D_{2,2 \rightarrow 2,1} \pi(2, 2) + P^D_{1,2 \rightarrow 2,1} \pi(1, 2) \\
&\quad \ldots \\
\pi(2, K-1) &= P^R_{2,K-1 \rightarrow 2,K-1} \pi(2, K-1) + P^R_{1,K-1 \rightarrow 2,K-1} \pi(1, K-1) \\
&\quad + P^A_{2,K-2 \rightarrow 2,K-1} \pi(2, K-2) + P^A_{1,K-2 \rightarrow 2,K-1} \pi(1, K-2) \\
&\quad + P^D_{2,K-1 \rightarrow 2,K-1} \pi(2, K) + P^D_{1,K-1 \rightarrow 2,K-1} \pi(1, K) \\
\pi(2, K) &= P^R_{2,K \rightarrow 2,K} \pi(2, K) + P^R_{1,K \rightarrow 2,K} \pi(1, K) + P^A_{1,K-1 \rightarrow 2,K} \pi(1, K-1) \\
&\quad + P^A_{2,K-1 \rightarrow 2,K} \pi(2, K-1).
\end{align*}
\]

\[
Q_{2(K+1) \times 2(K+1)} =
\begin{pmatrix}
E'_1 & F'_1 \\
D_1 & E_1 & F_1 \\
\vdots & \vdots & \vdots & \vdots \\
D_1 & E_1 & F_1 \\
D_2 & E_2 & F_2 \\
D_3 & E_3 & F_3 \\
\vdots & \vdots & \vdots & \vdots \\
D_3 & E_3 & F_3 \\
K-1 & K & K
\end{pmatrix}
\]

\[
F_1 = \begin{pmatrix}
\alpha_1 p \\
\beta_1(1-q) \\
\alpha_1(1-p) \\
\beta_1 q
\end{pmatrix} = \Lambda_1 P;
\]

\[
E'_1 = \begin{pmatrix}
(1-\alpha_1) p \\
(1-\alpha_1)(1-p) \\
(1-\alpha_1)(1-p) \\
1-\beta_1 q
\end{pmatrix} = (I-\Lambda_1) P;
\]

\[
D_m = \begin{pmatrix}
\sigma_{SU}(1-\alpha_m) p \\
\sigma_{SU}(1-\alpha_m)(1-q) \\
\sigma_{SU}(1-\alpha_m)(1-p) \\
\sigma_{SU}(1-\beta_m) q
\end{pmatrix} = \sigma_{SU}(I-\Lambda_m) P;
\]

\[
E_m = \begin{pmatrix}
[\alpha_m \sigma_{SU} + (1-\alpha_m)(1-\sigma_{SU})] p \\
[\beta_m \sigma_{SU} + (1-\beta_m)(1-\sigma_{SU})](1-q) \\
[\beta_m \sigma_{SU} + (1-\beta_m)(1-\sigma_{SU})](1-q) \\
[\beta_m \sigma_{SU} + (1-\beta_m)(1-\sigma_{SU})] q
\end{pmatrix} = \Lambda_m \sigma_{SU} P + (I-\Lambda_m)(1-\sigma_{SU}) P;
\]

\[
F_m = \begin{pmatrix}
(1-\sigma_{SU}) \alpha_m p \\
(1-\sigma_{SU}) \alpha_m(1-p) \\
(1-\sigma_{SU}) \beta_m q \\
(1-\sigma_{SU}) \beta_m(1-q)
\end{pmatrix} = (1-\sigma_{SU}) \Lambda_m P;
\]

\[
E'_3 = \begin{pmatrix}
[\alpha_3 + (1-\alpha_3)(1-\sigma_{SU})] p \\
[\alpha_3 + (1-\alpha_3)(1-\sigma_{SU})](1-q) \\
[\beta_3 + (1-\beta_3)(1-\sigma_{SU})] q \\
[\beta_3 + (1-\beta_3)(1-\sigma_{SU})](1-q)
\end{pmatrix} = \Lambda_3 P + (I-\Lambda_3)(1-\sigma_{SU}) P.
\]
the original order [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] is updated to [0, 2, 4, 6, 8, 10, 12, 1, 3, 5, 7, 9, 11, 13, 15], and we obtained matrix $Q_0$.

Next, divide $Q_0$ into two groups (group $S$ and group $T$) with four parts according to the layer. Reduce $Q_0$ to half the size of the original block matrix according to $Q_1 = Q_{1S}^{(0)} + Q_{1S}^{(0)}(-Q_{1S}^{(0)})^{-1}Q_{1S}^{(0)}$. The resulting submatrix $Q_1$ inherits the matrix form of $Q_0$. The reduced $Q_1$ is a new parent matrix $Q_0$, as shown at the bottom of the page.

Then, permute the layer of the new parent matrix as shown in Figure 5. Repeat the above reduction steps 1-2 until the
original block matrix \( Q_0 \) is reduced to a one-block matrix \( Q_4 \): 
\[ Q_0 \rightarrow Q_1 \rightarrow Q_2 \rightarrow Q_3 \rightarrow Q_4; \]

2) BOUNDARY VECTOR OBTAINING PHASE
Obtain the boundary solution vector \( \pi^{(4)} \) according to \( \pi^{(4)} Q_4 = \pi \);

3) BACKWARD EXPANSION PHASE
The solution vector of \( Q_3 \) can be obtained from \( \pi^{(3)} = \pi^{(4)} [-Q_0^{-1} Q_3^{(3)} - I] \), where \( I \) is the identity matrix. After finite iterations \( \pi^{(4)} \rightarrow \pi^{(3)} \rightarrow \pi^{(2)} \rightarrow \pi^{(1)} \rightarrow \pi^{(0)} \), \( \pi^{(0)} = \pi^{(1)} [-Q_0^{-1} Q_3^{(0)} - I] \) can be obtained.

Following the reverse order in Figure 5 as:
\[
\begin{pmatrix}
0 & 2 & 4 & 6 & 8 & 10 & 12 & 14 & 1 & 3 & 5 & 7 & 9 & 11 & 13 & 15 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15
\end{pmatrix},
\]

the blocks of \( \pi^{(0)} \) in layer 2 return back to the place of layer 1, layer 4 return back to the place of layer 2. The order of the steady-state \( \pi^{(0)} \), \([0, 2, 4, 6, 8, 10, 12, 14, 1, 3, 5, 7, 9, 11, 13, 15] \) is updated to \([0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] \). Finally, the matrix \( \pi \) after layer updating is the steady-state distribution of matrix \( Q \) with size 1×32.

Note that a computational method in [21] can also be applied as well to determine the steady-state distribution.

D. PERFORMANCE MEASURES
Based on the stationary distribution, the following system performance metrics can be determined.

1) The average queue length, \( E(L) \), is:
\[
E(L) = \sum_{j=0}^{K} j[\pi(1, j)+\pi(2, j)]
\] (16)

2) The average throughput, \( E(S) \), is the number of packets successfully transmitted by the secondary system per unit time (packets/slot) as:
\[
E(S) = P_{SU}(1-P_f)[1-\pi(1, 0)-\pi(2, 0)]
\] (17)

TABLE 2. Arrival rate of different traffics.

| Index | \( \alpha_i \) | \( \beta_i \) | \( \rho \) | \( c^2 \) | \( \phi \) | \( \Phi \) |
|-------|---------------|--------------|----------|---------|--------|--------|
| STRONG | 1 0.55 0.0004 | 0.5 50 0.1 | 0.1 |
| 2 0.60 0.0007 | 0.5 100 0.2 | 0.2 |
| 3 0.65 0.0009 | 0.5 150 0.3 | 0.3 |
| 4 0.70 0.0011 | 0.5 200 0.4 | 0.4 |
| 5 0.75 0.0013 | 0.5 250 0.5 | 0.5 |
| 6 0.80 0.0015 | 0.5 300 0.6 | 0.6 |
| 7 0.85 0.0017 | 0.5 350 0.7 | 0.7 |
| 8 0.90 0.0018 | 0.5 400 0.8 | 0.8 |
| WEAK | 1 0.55 0.0004 | 0.5 50 0.1 | 0.1 |

3) Average delay, \( E(W) \), is determined by the Little’s Law:
\[
E(W) = \frac{E(L)}{E(S)} = \frac{\sum_{j=0}^{K} j[\pi(1, j)+\pi(2, j)]}{\sigma_{SU}[1-\pi(1, 0)-\pi(2, 0)]}
\] (18)

4) Packet loss probability of the secondary system, \( D_{pl} \), is:
\[
D_{pl} = [\pi(1, K)+\pi(2, K)][1-P_{SU}(1-P_f)]
\] (19)

III. NUMERICAL RESULTS
In this section, three study cases are carried out with parameters as \( R_{SU} = 5, m_{SU} = 1, P_{SU} = 20, C_{SU} = 6, P_f = 0.05, K = 15 \).

A. IMPACT OF CONGESTION CONTROL ON SECONDARY SYSTEM PERFORMANCE
We compare three congestion control schemes: RED, no congestion control (NCC) and dynamic random early detection (DRED). The difference between RED and DRED is the number of thresholds. RED adopts dual-threshold control (\( L_1 = 3, L_2 = 12 \) are used for the study), while DRED uses a single threshold (\( L = 12 \) for the study).

Table 2 contains the parameters (\( \alpha_1, \beta_1, \rho, c^2, \phi, \Phi \)) of the arrival process for the numerical study. The index represents the bursty degree of the arrival process. The higher the index is, the heavier the traffic places the load on the system.
In Figure 6, we plot the average delay and the packet loss probability versus the index for three congestion control schemes. It can be observed that when the index value increases, the value of $E(W)$ and $D_{pl}$ will also increase. Since RED and DRED drop a small number of packets to prevent the queue overflow. Therefore, the system with NCC has lower $D_{pl}$ than with RED or DRED. However, as can be seen from Figure 6, the secondary network has higher $E(W)$. As we know, $E(L)$ is in proportion to $E(W)$, that is, the QoS of SUs with NCC environment is obviously low.

Although the packet loss probability of RED and DRED is higher, the difference value does not exceed 0.1 compared with the NCC environment, which is still acceptable. When the index value is between 4 and 6, the packet loss probability and the average delay increase significantly with the RED/DRED mechanism. When it exceeds 6, the growth slope slows down. That is, congestion mechanisms effectively cope with the bursty traffic to improve the performance of the secondary system. Furthermore, RED achieves a lower delay and packet loss probability than DRED.

**B. IMPACT OF CHANNEL FADING ON SECONDARY SYSTEM PERFORMANCE**

This section investigates the impact of the Nakagami fading channel. Figure 7 plots the average queue length and the packet loss probability versus the index and the parameter, $m_{SU}$, of the Nakagami fading channel. Note that the smaller $m_{SU}$ represents the severe fading of the propagation link, the worse the channel state is.

From Figure 7, we can observe that the average queue length and the packet loss probability are more significant in the network environment with more bursty traffic. Moreover, the severe fading leads to higher $E(L)$ and $D_{pl}$.

Results show that the channel fading in physical layer has a great influence on secondary system. In CRNs, the transmission quality of SUs is indeed affected by the channel environment, and the fading parameter focused on this paper is necessary for the evaluation of secondary network performance.

**C. PERFORMANCE ANALYSIS OF SECONDARY SYSTEM**

This section explores the overall performance of secondary system in network environments with different burst degrees by adjusting dual-threshold in queue management. Let us fix $L_1 = 3$, and adjust $L_2$, we discuss how to control and optimize system parameters to improve the overall performance. Threshold difference $L_2 - L_1$ represents the smooth interval of congestion control. Figures 8 and 9 show the comparison of the average delay and packet loss probability versus the setting of $(\rho, c^2, \phi, \Phi)$ and $L_2 - L_1$. The value of the statistics $(\rho, c^2, \phi, \Phi)$ are also presented in Table 2.

As observed from Figure 8, the increase of $L_2 - L_1$ decreases the average delay due to the traffic with a high
bursty degree. However, the average delay decrease comes with a high packet loss probability (see Figure 9). It is worth mentioning that the increasing packet loss probability is still within tolerance. With the increase of $L_2-L_1$, RED has a more chance to alleviate the instantaneous congestion before the queue length grows too large. From Figure 9, it can be observed that the proper setting of $L_2-L_1$ helps the stable operation of the system.

Figures 10 and 11 show the average throughput and average queue length versus $L_2-L_1$ and burst degree. As shown in Figure 10, under the influence of channel environment and burst traffic, the results show that the system throughput fluctuates around 0.324. Compared with the model established in the general environment, the throughput in burst environment is relatively low. This is because when burst traffic arrives, the system may violate the normal service rules due to the late service. Furthermore, the proposed model can maintain the stable throughput under the network environment with serious burstiness.

In addition, Figure 11 shows that the average queue length and the average delay have similar trends. It is not difficult to find in Little’s law that the average queue length varies directly as the average delay. Thus, the stable throughput makes similar graph trends of the queue length and delay.

Secondary network performance shows a downward trend when the more serious burst occurs. However, properly increasing the smooth interval of RED can control the deterioration of system performance by dropping little packets in advance. The access model with MMBP-2 as the input source established in this paper can maintain the stability of the QoS of SUs in complex environments. Moreover, in CRNs with multi-type services, different types of traffics have different QoS requirements. Real-time audio/video services are more sensitive to delay, while non-real-time services have lower delay requirements. The model parameters can be set according to actual transmission environment.

IV. CONCLUSION

We have presented a queuing model to evaluate the impact of bursty traffic in CRNs. SUs traffic sending audio/video services and data/image services can be modeled as the MMBP-2 process. Our model considers the RED congestion control, Nakagami fading channel. We have also constructed an iterative algorithm to obtain the steady-state solution and the performance metrics such as throughput, average queue length, queuing delay and packet loss probability. Extensive numerical results have been performed to investigate the impact of various parameters on the system. We have shown that RED with the dual-threshold could alleviate the deterioration of bursty traffic to improve network QoS. The research parameters in this paper provides a novel idea for performance analysis of CRNs with burstiness.

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