ABSTRACT  An original thermal model of a single room structure is developed by using the tensorial network-based Kron’s method. The modelling principle is using the equivalent RC-network of wall, door and air constituting the house. For a better understanding, the temperature propagation was assumed only in a 1-D horizontal direction. The problem geometrization is defined in function of rectangular approximation meshing. After the determination of the equivalent thermal resistor and thermal capacitor, the innovative thermal circuit representing the room is elaborated. The methodology of the Kron’s formalism, implicitly described with the different action steps is introduced. The thermal room Kron’s method is implemented from the branch to mesh spaces before the expression of the problem metric. The thermal transfer functions (TTFs) at three cases of indoor points, situated near, middle and far of the door are established from the Kron’s problem metric. The feasibility of the room thermal Kron’s TTF model is validated with SPICE TTF simulations in both frequency and time domains. The thermal cut-off frequencies are verified with very good correlation between the established TTF model and simulation. An excellent prediction of transient responses with unit-step and arbitrary waveform temperature signals with a minimal and maximal amplitude of about 20°C and 40°C is proposed.

INDEX TERMS  Thermal modelling, room model, zonal model, RC-network, 1-D propagation, Kron’s method, frequency domain analysis, transient response, thermal transfer function (TTF).

I. INTRODUCTION
The modern urban city design engineering should overcome the energy and thermal comfort performances [1], [2]. The building engineers are wondering constantly about the ideal room temperature for the comfort living [3].

To face up this challenge, an efficient investigation must be conducted on the dependence between the indoor and outdoor temperature in function of the climate environment [4]. Diverse solutions were deployed against the building and home comfort performances [5]–[8]. Innovative smart home technology was introduced in order to adapt the indoor and outdoor temperatures [5]. Material and architectural solutions based on the insulating layer location, distribution and orientations were developed [6],[7]. Seasonal passive design strategies were also introduced [8]. Whatever the envisaged solutions, an efficient predictive approach is undeniably helpful for the building engineers during the design phases.

Emphatically, thanks to the tremendous boom of the computer-aided design, simulation tools [9]–[14] were developed. The trends of the simulation were aimed to the multiple objectives as the numerical method order reduction [10], building energy efficiency [11], distributed computation technique [12], novel approach to overcome legacy code limitations [13], [14] and the sensitivity of peak-load reduction computing tool in the function of environmental
systems [28], [29]. Moreover, the Kron’s method was also introduced in the 1930s for the treatment of electrical machine structures [27]. This unfamiliar method is initiated in [31]. Despite, research work performed on the Kron’s method modelling of electronic and electrical circuits [27]–[30], the method is not at all familiar to the thermal research engineer. For this reason, an original thermal network modelling of room indoor temperatures is explored in this work. The developed modeling approach will provide an overview on how we can manage building thermal flows in a general manner according to the Kron’s formalism. The knowledge about the thermal performance is useful to design and control heating and cooling systems in the building environment. Moreover, as the Kron’s method guarantees a literally fast computation time, the developed model can be exploited in the future to predict quickly the performances of building in function of materials and geometry. But a simple case of study should be performed to illustrate the developed model understandability and feasibility.

The present paper is organized in four main sections described as follows:

- Section II is dedicated to the room structure problem formulation. The 3-D representation of the room will be described. Then, the basic way to determine the equivalent building block elements as the thermal resistors and thermal capacitors will be introduced.

- Section III is focused on the main action about the Kron’s method modelling of the building thermal network. First, the classical thermal circuit will be defined. Then, the thermal structure equivalent graph topology will be explored. The branch and mesh space analyses will be drawn in order to determine the TTF at the different meshing indoor test points.

- Section IV introduces the validation of the thermal Kron’s model. First, the proof-of-concept (POC) of a room structure will be parametrized and then, the comparisons between the calculated and simulated TTF will be discussed. The performance of the developed method will be pointed out with respect to the existing thermal simulators [32].

- Then, Section V is the final conclusion.

II. STATEMENT OF THE ROOM THERMAL MODEL PROBLEM

The present section formulates the thermal room problem under investigation. The problem statement must begin with the consideration of the room structure 3-D description. After that, we will introduce the elaboration of the equivalent thermal resistor and thermal capacitor constituting each element of the room.

A. 3-D DESCRIPTION OF THE ROOM UNDER INVESTIGATION

In the present study, we consider a computer server room which is located in the basement of the building. The location
is well-isolated over the three sides, except the front face, as illustrated in Fig. 1(a). In fact, the main goal of the present modelling is to evaluate the different internal temperatures. In mid-term, according to the requirement about the comfort temperature for the future building standard, the proposed model may serve to choose the optimal place, where we can install the fan and optimize its control, under fast computation.

- The outdoor side is assumed separated by the wall side with the door.
- The environment humidity, wind effect and indoor shadow are neglected.
- And the other three sides of the room are assumed to be adiabatic as explained in Fig. 1(b). In other words, these faces are assumed to be in the ideal case of well isolated from the other adjacent rooms.

**B. POSITIONNING OF THE INDOOR NODES FOR THE UNKNOWN TEMPERATURES**

Before the elaboration of the equivalent thermal network of the room, it would be necessary to define the key points where the temperature needs to be assessed. To do this, different nodes are taken as reference test points in the room. Fig. 2 highlights the positioning configuration of the indoor test nodes. The meshing strategy can be determined based on the inhomogeneity of the facing wall and the indoor nodes positioning according to the distance with the door. Subsequently, our main interest in the present study can be formulated by the determination of:

- Near test plane which corresponds to the temperature near the door or in the internal face of the wall,
- Middle test plane which designates the temperature in the medium line of the room,
- And far test plane is the temperature at the nodes in the opposite side of the door and back of the room.

After this geometrical analysis, let us describe the systemic approach of the room thermal modelling.

**C. SYSTEM APPROACH OF THE THERMAL MODELLING PROBLEM**

To model the indoor temperature of the room, we will proceed with systemic approach as explored in [20]–[27]. The present subsection introduces the representative thermal model of the room structure.

1) **RECALL ON THE SYSTEMIC BLACK BOX MODEL**

Fig. 3 represents the general diagram of the thermal black box for treating the thermal problem. It acts as a two-port
The following thermal resistor and capacitor are defined under the thermal propagation direction indicated in Fig. 4. By denoting the bulk material thermal conductivity, \( \lambda \), the associated thermal resistor is analytically defined by [33]:

\[
R = \frac{s_3}{\lambda s_1 s_2}.
\]

(4)

By denoting the bulk material specific heat, \( h \), and mass density, \( \rho \), the thermal capacitor is analytically defined by [33]:

\[
C = \rho h s_1 s_2 s_3.
\]

(5)

Knowing these \( R \) and \( C \) parameters, the RC thermal networks can be drawn in function of the structure as the case of the room scenario given in Fig. 2.

3) RC-NETWORK EQUIVALENT TO THE ELEMENTARY STRUCTURE

The common way to treat such a structure depends on the fundamental network of each elementary block constituting the system. The building block of the thermal RC network modelling is shown in Fig. 5(a). This illustrative elementary bulk structure consists of outdoor air-wall material-indoor air by supposing that the temperature is propagating from outdoor to indoor in 1-D direction. The reference nodes, \( M_{\text{out}} \) and \( M_{\text{in}} \) enable to define the equivalent network. This structure can be modelled as the content of the black box introduced in Fig. 3(a) with its thermal resistance, \( R \), and thermal capacitor, \( C \), [20]–[27] interconnected as depicted in Fig. 5(b).

Based on this introductive definition, the Kron’s modelling of our room structure will be elaborated in the following section.

III. DEVELOPMENT OF THE ORIGINAL KRON’S MODEL OF THE ROOM THERMAL RESPONSES

The present section develops the description of the classical equivalent thermal network. Through the equivalent graph topology, the thermal Kron’s model will be implemented. Then, the TTFs at the different indoor nodes will be established. Before this analytical development, it is worth to introduce in understandable way the methodology of the unfamiliar Kron’s method in the following subsection.


A. METHODOLOGY OF THE KRON’S FORMALISM APPLICATION

The unfamiliar Kron’s formalism [36] is a particularly efficient approach to model the engineering problem represented as networked systems. However, extra-effort needs to be paid for the non-specialist building design engineer to be familiar to the method. Following this motivation, our research work serves pedagogically to make this power formalism to be open to any engineers.

First and foremost, so far, it is interesting to remind that the Kron’s formalism is outstandingly well adapted for electromagnetic circuits. Meanwhile, it contributes indirectly to different applications as social, mechanical, thermal, chemical, etc., uses.

Then, in the practical engineering point of view, the Kron’s formalism consists in modelling of any physical problem via the network approach. The analytical theory is the tensorial mathematization of the elementary objects as 1- or 2-rank tensors often named simplex. The system complex can be represented by the simplex interaction with tensorial operations.

To summarize the TAN methodology to solve typical thermal problem as the case of our room shown in Figs. 1, we propose the workflow of Fig. 6 [25].

To explain more clearly, the application of Kron’s formalism, we can consider the network shown in Fig. 7. The practical methodology to treat this type of graph is explained in the following items:

- The Kron’s method should start for the problem formulation which corresponds to the room structure initiated in Figs. 1 for the present study.
- Afterwards, the problem must be geometrized in function of the targeted solution.
- Then, with the basic laws’ physics, we can transform the structure into an electrical thermal circuit.
- In the next step of the modeling, the thermal circuit must be translated as a Kron’s graph topology. For the better understanding we can look over the two-mesh graph shown in Fig. 6.
- The topological parameter of the graph must be tabulated. For the example, of the case of Fig. 6, the Kron’s universe is made of three branches \( \{B_1, B_2, B_3\} \). Each branch has an impedance represented by the components denoted \((A, B, C)\). Under the topological parametrization, we can see that this graph is a single network, \( R \), having two nodes, \( \{N_1, N_2\} \). In consequence, the number of meshes (M) required to analyze the system is given by this Euler-Poincaré topological invariant formula [31]:

\[
M = B - N + R. \tag{6}
\]

- From this graph, the systemic analysis can be performed first in the branch space and then, in the mesh space.
- Based on the graph topology, the last phase of the Kron’s method can be the mathematization. The most general way of this mathematization should be the tensorial approach in the adequate space (branch, node, summit, mesh …).

B. ELABORATION OF THE THERMAL EQUIVALENT CIRCUIT

The meshing geometrization of the room structure will serve as the initial step of the Kron’s modelling. However, for the
basic understanding, it would be essential to establish the classical thermal circuit based on the RC-network.

1) NODED GEOMETRICAL DESCRIPTION OF THE ROOM STRUCTURE

For a better understanding of the Kron’s modelling, we propose to start with the geometrization by considering the meshing related to the door and the wall widths. Accordingly, Fig. 8(a) presents the top view of the room. The external temperature source is naturally induced in the outdoor environment. The time-dependent instantaneous temperature is represented by $T_{\text{source}}(t)$. This source is connected to the three nodes, $N_0$, which are connected to the outside of the room wall and door. Then, the test points corresponding to the temperatures near the source are denoted by $N_1$, $N_2$ and $N_3$. The medium nodes for the middle plane are indicated by $M_1$, $M_2$ and $M_3$. Then, the far nodes are referenced by $F_1$, $F_2$ and $F_3$.

The structure meshing is configured in Fig. 8(b) by taking into account the wall length, $d_w$, and door width, $d_d$. The wall and door thickness are assumed to be equal to $d$. It can be seen that the axis $(N_2M_2F_2)$ constitutes a symmetrical axis of the overall structure. It means that the lateral thermal responses referring to the nodes, $N_1$ and $N_3$, $M_1$ and $M_3$, and $F_1$ and $F_3$, are identical.

From an analytical point of view, the unknown parameters of this thermal modelling problem can be expressed by the components of the following vector:

$$
T_{\text{unknowns}} = \begin{bmatrix}
T(N_1) = T(N_3) \\
T(N_2) \\
T(M_1) = T(M_3) \\
T(M_2) \\
T(F_1) = T(F_3) \\
T(F_2)
\end{bmatrix}.
$$

Under such formulation, the thermal problem under study can be redrawn as depicted in Fig. 9 by the determination of the multi-port TTF. In brief, the overall room structure under study can be assumed as 7-port system with single input and six outputs.

2) IDENTIFICATION OF NETWORK LUMPED R AND C ELEMENTS

Before the thermal circuit design, the specification of each elementary components between the nodes, $N_m$, $M_m$ and $F_m$. ($m = 0, 1, 2, 3$) of the structure shown in Fig. 3 is necessary. Table 1 summarizes these components for three constituting materials, wall, door and indoor air. Each component is

| Material         | Connecting nodes                                                                 | Nature     | Parameter |
|------------------|----------------------------------------------------------------------------------|------------|-----------|
| Wall             | $N_0N_1$ and $N_0N_3$                                                          | Resistor   | $R_w$     |
| Door             | $N_2N_1$                                                                        | Resistor   | $R_d$     |
| Door             | $N_2N_3$                                                                        | Capacitor  | $C_d$     |
| Indoor air       | $N_0N_1$, $N_0N_3$ (with $m=1,2,3$) and $N_0N_{m+1}$, $M_0M_{m+1}$, $F_0F_{m+1}$ (with $m=1,2$) | Capacitor  | $C_1, C_2, C_3$ |
determined by formulas (4) and (5). Let notice that, we have chosen to stay on its components for the sake of simplicity and to avoid unnecessary computational efforts.

3) CIRCUIT DESIGN OF THE ROOM THERMAL NETWORK
The thermal equivalent network is established from the RC-model introduced in Fig. 5(b) applied to the wall, door and meshed indoor air. The circuit design, drawn in Fig. 10, can be understood with the positions of referential nodes, N<sub>m</sub>, M<sub>m</sub> and F<sub>m</sub> (m = 0, 1, 2, 3). It is noteworthy that the circuit is built under the reference temperature, T<sub>a</sub>, implicitly the potential ground node. The thermal source, T<sub>source</sub>, is connected to N<sub>0</sub>. In the circuit problem, the main unknown parameters are the temperature at the other nodes as indicated in vector (6).

We would like to emphasize herein about the equivalent circuit construction. The design of the circuit approach depicted in Fig. 10 is related to the hypothesis of the temperature source propagation in 1-D in the near zone. The effect of the lateral RC network can be neglected with the considered size of geometrical meshing. In the far zone, we also adopted the adiabatic hypothesis which enables to avoid the temperature change in the far zone corners. In difference to the near and far zone, the middle one is essentially constituted by the air. Therefore, we assumed that in the middle area, the propagation of the temperature from nodes M<sub>1</sub> and M<sub>3</sub> to the lateral walls can be represented by network R<sub>3</sub>C<sub>3</sub>.

C. IMPLEMENTATION OF THERMAL KRON’S MODELLING
The implementation of the Kron’s model is performed via graph topology and the tensorial approach. The next paragraphs develop the detailed expression of the room thermal model.

1) KRON’S GRAPH TOPOLOGY
Fig. 11 represents the Kron’s equivalent graph topology of the thermal network introduced in Fig. 10. The circuit design is implemented with the thermal capacitors connected to the ground ambient temperature in virtual 3-D view. This thermal circuit universe is built with the interconnections of branches. In this circuit, each branch is constituted by a thermal resistor or a thermal capacitor. The circuit can also be seen in another space based on the mesh representation. The rigorous Kron’s method analysis can be performed with the topological parameters addressed in Table 2.

| Designation | Branch | Node | Mesh | Port |
|-------------|--------|------|------|------|
| Parameter   | B      | N    | M    | P    |
| Value       | 18     | 8    | 11   | 7    |

Knowing these parameters, the tensorial branch analysis with the Kron’s method will be proposed in the next paragraph.
2) BRANCH SPACE ANALYSIS

The branch space variable can be referred with the positive integer index, \( b = 1, 2, \ldots, B \). As covariable, the branch temperature of the graph introduced in Fig. 11 can be expressed as:

\[
T_b = \begin{bmatrix} T_{source} & 0 & \ldots & 0 \end{bmatrix}.
\tag{8}
\]

It acts as a diagonal matrix because the circuit does not present any interbranch coupling phenomenon. Each submatrix corresponds to the constituting block of our room structure as the wall, door and indoor air which represented by the branch thermal impedances, respectively:

\[
[Z_{wall}] = \begin{bmatrix} R_w & 0 & 0 & 0 \\ 0 & Z_w & 0 & 0 \\ 0 & 0 & R_w & 0 \\ 0 & 0 & 0 & Z_w \end{bmatrix}
\tag{11}
\]

\[
[Z_{door}] = \begin{bmatrix} R_d \\ 0 \\ Z_d \\ 0 \end{bmatrix}
\tag{12}
\]

and the \((11 \times 11)\)-dimension matrix, \([Z_{air}]\) expressed in equation (13), as shown at the bottom of this page. The thermal capacitive impedance elements are defined by:

\[
Z_{N_1} = Z_{N_3} = Z_w = \frac{1}{C_{ws}}
\tag{14}
\]

\[
Z_{N_2} = Z_d = \frac{1}{C_ds}
\tag{15}
\]

\[
Z_{M_1} = Z_{M_3} = Z_{13}
\tag{16}
\]
be performed with the connectivity matrix denoted \( Z_{M} \), \( Z_{F} \) and \( Z_{I} \). The branch to space conversion of the thermal variables must be handled with the positive integer index, \( m \), \( n \), \( b \), \( m \), \( n \), \( b \), \( m \), \( n \), \( b \), respectively. In this notation, the ternary elements, “-1”, “0” and “1” corresponds to the coefficients of the branch flux in function of mesh fluxes with respect to the second Kirchoff circuit node’s law.

It can be demonstrated that the mesh temperature covariable is expressed as:

\[
W_m = \begin{bmatrix} T_{source} & 0 & \ldots & 0 \end{bmatrix}.
\]

We point out that according to the tensor algebra, \( \Xi_m^b \) represents the matrix transpose of \( \Xi_m^b \). Substantially, the mesh impedance of our thermal graph can be obtained from the relationship:

\[
\Omega_{mn} = \Xi_m^b b \Xi_n^b.
\]

It is noteworthy that the subscript, \( n \), plays the same role as \( m \). Therefore, it can be defined as a positive integer index, \( n = 1, 2, \ldots, M \).

4) EXPRESSIONS OF TTF VECTORS

Based on the previously expressed mesh impedance, the metric of the thermal problem can be formulated by:

\[
W_m = \Omega_{mn} Q^n.
\]

Substituting the mesh power flux given in (23) into the previous metric, it yields the following tensorial solution of the posed problem:

\[
W_m = \Omega_{mn} \Xi_m^b b P^b.
\]

Therefore, the branch power flux vector is:

\[
P^b = \Xi_n^b Y_{nm} W_m
\]

where we expressed the mesh admittance as:

\[
Y_{nm} = \Omega_{mn}^{-1}.
\]

3) MESH SPACE ANALYSIS

The mesh space variable can be referred with the positive integer index, \( m = 1, 2, \ldots, M \). Based on the Kron’s method, the branch to space conversion of the thermal variables must be performed with the connectivity matrix denoted \( \Xi_m^b \). This matrix enables to realize the following transform of the branch temperature, \( T_b \), and power flux, \( P^b \), into the mesh temperature, \( W_m \), and power flux, \( Q^m \), written as:

\[
W_m = \Xi_m^b T_b
\]

\[
Q^m = \Xi_m^b P^b.
\]

The global connectivity matrix of the graph shown in Fig. 8 in function constituting block (door, wall, indoor air) ones is expressed as:

\[
\Xi_m^b = \begin{bmatrix} \Xi_{wall}^b \\ \Xi_{door}^b \\ \Xi_{air}^b \end{bmatrix}.
\]

The connectivity sub-matrices representing the walls, door and the indoor air are denoted, \( \Xi_{wall}^b \), \( \Xi_{door}^b \) and \( \Xi_{air}^b \), respectively. The expressions of these sub-matrices are given in equations (25), (26) and (27), as shown at the bottom of this page, respectively. In this notation, the ternary elements, “-1”, “0” and “1” corresponds to the coefficients of the branch flux in function of mesh fluxes with respect to the second Kirchoff circuit node’s law.

\[
[\Xi_{wall}] = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}
\]

\[
[\Xi_{door}] = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}
\]

\[
[\Xi_{air}] = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}
\]

\[
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\]

\[
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}
\]

\[
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\]
It means that the TTFs associated to the temperature vector given in equation (7) can be determined by:

\[
\begin{align*}
TTF_{N_1}(s) &= TTF_{N_2}(s) = \frac{Z_w(s)P^5(s)}{T_{source}(s)} \\
TTF_{N_2}(s) &= \frac{Z_w(s)P^5(s)}{T_{source}(s)} \\
TTF_{M_1}(s) &= TTF_{M_2}(s) = \frac{Z_1(s)P^{12}(s)}{T_{source}(s)} \\
TTF_{F_1}(s) &= TTF_{F_3}(s) = \frac{Z_2(s)P^{13}(s)}{T_{source}(s)} \\
TTF_{F_2}(s) &= \frac{Z_2(s)P^{13}(s)}{T_{source}(s)}.
\end{align*}
\] (34)

To verify the effectiveness of the developed thermal Kron’s method, these tensorial analytical expressions were programmed in Matlab environment. The investigated results will be described in the next section.

IV. VALIDATION OF RESULTS

To verify the feasibility of the thermal Kron’s model, the present section introduces the room structure proof of concept (POC). The validations are based on frequency and time domain analyses. Comparisons between calculated results and simulations are explored in this section. The model was implemented as a Matlab routine program. The simulations are carried out in the environment of electronic circuit simulator commercial tool ADS® from Keysight Technologies®. Then, the discussed results will be examined in the next subsections.

A. POC DESCRIPTION

Before the exploration of the simulated results, it is worth to describe the thermal structure POC. As aforementioned, the present study is assumed under the hypothesis cited in Section II.A and the door is in a closed state.

1) DESCRIPTION OF THE POC PHYSICAL PARAMETERS

The overall geometrical sizes of the thermal structure were introduced earlier in Fig. 1. In the present case of study, we assume that the wall and wood are made in unfired clay bricks [34] and wood across the grain [35]. Table 3 indicates the physical parameters of each material constituting these elements.

Based on these specifications, the \( R \) and \( C \) thermal lumped components constituting the branch of the POC circuit were determined by means of equations (4) and (5), respectively.

We underline that the thermal resistor and thermal capacitors depend on the terminal nodes of meshing element. Subsequently, the thermal resistors were calculated as follows:

\[
R_w = \frac{d}{\lambda_w d_w L_x}, \quad R_d = \frac{d}{\lambda_d d_d L_x}.
\] (35) (36)

To verify the effectiveness of the developed thermal Kron’s method, these tensorial analytical expressions were programmed in Matlab environment. The investigated results will be described in the next section.

### Table 3. Physical parameters of the thermal circuit POC representing the room structure.

| Structure | Designation | Parameters | Values |
|-----------|-------------|------------|--------|
| Wall      | Geometrical sizes with propagation \( N_1 \rightarrow N_2 \) | \( s_1 = d_a \) | 2 m |
|           | \( s_2 = L_y \) | \( s_3 = d \) | 2.3 m |
|           | \( s_4 = d \) | 0.104 m |
|           | Thermal conductivity \( \lambda_w \) | 0.9 W/mK |
|           | Specific heat \( h_a \) | 545 J/kgK |
|           | Mass density \( \rho_w \) | 1788 kg/m³ |
| Door      | Geometrical sizes with propagation \( N_1 \rightarrow N_2 \) | \( s_1 = d_a \) | 2 m |
|           | \( s_2 = L_y \) | \( s_3 = d \) | 2.3 m |
|           | \( s_4 = d \) | 0.05 m |
|           | Thermal conductivity \( \lambda_d \) | 0.2 W/mK |
|           | Specific heat \( h_a \) | 1400 J/kgK |
|           | Mass density \( \rho_d \) | 800 kg/m³ |
|           | Thermal conductivity \( \lambda_d \) | 0.026 W/mK |
|           | Specific heat \( h_a \) | 1007 J/kgK |
|           | Mass density \( \rho_d \) | 1.16 kg/m³ |

The associated thermal capacitors are given by:

\[
C_w = \rho_w h_a d_w L_x d \\
C_d = \rho_d h_a d_d L_x d \\
C_1 = \frac{1}{2} \rho_a h_a d_w L_x d \\
C_2 = \frac{1}{2} \rho_a h_a d_L L_x \\
C_3 = \frac{1}{4} \rho_a h_a L_x L_d d_w.
\] (37) (38) (39) (40)
2) DESCRIPTION OF THE SIMULATED THERMAL CIRCUIT DESIGN

The POC schematic of the thermal circuit design is displayed in Fig. 12. This circuit was drawn in the SPICE environment, usually exploited to the electrical and electronic circuit designs. The temperature excitation source is represented by the voltage source connected to the node, $N_0$. The unknown indoor temperatures correspond to the voltages at the other nodes indicated by Near$_{1,2}$, Mid$_{1,2}$ and Far$_{1,2}$ of the POC circuit. During the simulations, the voltages at these nodes can be assessed and compared with the excitation one in order to determine the TTFs.

The comparisons between calculated and simulated frequency domain results will be examined in the next subsection.

B. FREQUENCY DOMAIN RESULTS

The present case of the frequency domain is rarely elaborated in the area of thermal building engineering. One of the original points of Kron’s method is the possibility to perform such a study and understanding the behavior of the thermal structure constituting the room. The following results of the frequency domain are based on Matlab computation of the Kron’s model compared to simulations with the SPICE AC analyses.

1) TTF MAGNITUDES

To visualize the significant behaviors of the TTFs, two ranges of frequencies were considered. In the first case, the frequency domain was investigated in the frequency band up to $f_{\text{max}} = 10 \, \mu\text{Hz}$. In the other case, the analysis is performed up to $f_{\text{max}} = 100 \, \mu\text{Hz}$. Linear and semi-logarithmic plots of magnitudes were presented for a better understanding of the thermal responses at the different test points.

Figs. 13 and Figs. 14 present the comparisons between the calculated and simulated TTFs at the near, mid and far positions. The simulations are plotted in solid lines and the Kron’s method calculated ones are shown in dotted lines. As expected, the Kron’s model is in very good agreement with the SPICE simulation. It is noteworthy that as seen in Fig. 13(a) and in Fig. 14(a), in the near side of the wall and door, the TTF is close to the unity. More generally, at extremely low frequencies below 100 nHz, all the TTFs can be assumed as equal to unity. However, in the mid and far TTF responses plotted in Figs. 13(b) and 14(b), and in Figs. 13(c) and 14(c), respectively, we have a typical
low-frequency thermal behavior. The mid TTF attenuates more than 20 dB at higher frequencies more than 5 µHz as depicted in Fig. 14(b). However, the far TTF attenuates more than 20 dB only 2 µHz. Then, at extremely high frequencies, the attenuation increases with slope 20 dB/decade.

The magnitude responses in linear and semi-logarithmic plots respecting the wider frequency analyses up to \(f_{\text{max}} = 100 \, \mu\text{Hz}\) are exposed in Figs. 15 and in Figs. 16, respectively. It can be understood from Fig. 15(a) that the temperature thorough the door is much higher than the temperature through the lateral walls. Along the symmetrical axis, the attenuation remains lower than 10 dB up to 100 µHz. However, the wall TTF has reached 20 dB attenuation only at about 60 µHz. Then, as seen in Figs. 15(b) and in Figs. 16(b), the mid TTF attenuation reaches 40 dB at 30 µHz. Then, strong attenuation of about 50 dB is realized with the far TTF beyond 20 µHz.

2) TTF PHASE RESPONSES

These AC analyses guarantee that the fast variation of the temperature cannot propagate through the considered wall and door structures. In the worst case of thermal shielding, the propagation can be limited in the near zone of the source.

The phase responses of the TTFs were explored only in linear plots because the phase information at lower frequencies is almost trivial and near zero. However, the two cases of the frequency limits, \(f_{\text{max}} = 10 \, \mu\text{Hz}\) to \(f_{\text{max}} = 100 \, \mu\text{Hz}\) were also considered for the present AC analyses.

Figs. 17 and Figs. 18 present the comparisons between the calculated and simulated TTF phases. Once again, it can be underlined that the Kron’s calculated results are in very good correlation with the simulations. For all cases of TTFs, the phases are decreasing with the frequency. This behavior corresponds to the phase responses of typical linear passive systems as low-pass filters. The phase shifts of the near zone in the lateral indoor part are limited to 20° below 10 µHz. However, the symmetrical axis is almost two times higher. As depicted in Fig. 17(b) and in Fig. 17(c), the mid and far TTF phase shifts exceed 100° beyond 7 µHz and 1 µHz, respectively.

The increase of the TTF phase shifts for all positions is confirmed by Figs. 18. For further validation of the previous simulations, transient analyses were also carried out. The next subsections describe the obtained results.

C. TIME DOMAIN RESULTS

The present thermal time domain analyses are performed based on the consideration of unit step and arbitrary waveform excitation temperatures. The reference ambient temperature was fixed to \(T_a = 20^\circ\text{C}\) for the time domain investigation. Only simulation results are exposed in this case of analysis. For the better understanding about the transient behavior of indoor temperature in the different zones...
FIGURE 15. Linear plot of simulated and calculated TTF magnitudes in (a), near, (b) middle and (c) far test nodes up to $f_{max} = 100 \, \mu$Hz.

FIGURE 16. Semi-logarithmic plot of simulated and calculated TTF magnitudes in (a), near, (b) middle and (c) far test nodes up to $f_{max} = 100 \, \mu$Hz.

FIGURE 17. Comparisons of simulated and calculated TTF phases in (a), near, (b) middle and (c) far test nodes up to $f_{max} = 10 \, \mu$Hz.

FIGURE 18. Comparisons of simulated and calculated TTF phases in (a), near, (b) middle and (c) far test nodes up to $f_{max} = 100 \, \mu$Hz.

1) UNIT-STEP EXCITATION TIME-DOMAIN RESULTS

In this case, the excitation signal corresponds to a unit-step temperature with $T_{min} = T_a$ and $T_{max} = 40^\circ$C.

(near, mid and far), the present time-domain analyses were performed under two different time windows with $t_{max} = 5$ hours and also, $t_{max} = 500$ hours.
Figs. 19 present the transient analysis with $t_{\text{max}} = 5$ hours. The sampling time was fixed equal to 30 seconds. As depicted in Fig. 19(a), the symmetrical axis and lateral near zone temperature reaches the half amplitude equal to 30°C, at about 0.5 hours and 2.5 hours, respectively. However, as illustrated in Fig. 19(b) and in Fig. 19(c), the responses at the mid and far zones are negligible, the temperatures are almost kept to $T_a$. It points out that the airflow pattern during was neglected in the present case of study.

To visualize the mid and far zone responses, the unit step transient results up to $t_{\text{max}} = 500$ hours are introduced in Figs. 20. It can be emphasized that the mid zone temperature reaches the half amplitude only after 70 hours as seen in Fig. 20(b). Fig. 20(c) highlights that the half amplitude can be reached only after 120 hours. This long-time range time domain analyses explain that the temperature in the far zone of the source are almost same for $F_1$ and $F_2$.

2) ARBITRARY WAVEFORM EXCITATION TIME-DOMAIN RESULTS

The present time domain analysis is carried out by considering an arbitrary waveform temperature. The same as the previous unit step analyses, the two different time ranges were considered. This time sampling was chosen similarly to the unit step response analysis.

Figs. 21 show the simulated transient results for $t_{\text{max}} = 5$ hours. We emphasize that in the considered time range, as shown in Fig. 21(a), the temperature variation is considerably smoothed in the near zone. The indoor temperature does not exceed the amplitude 40°C. However, as illustrated in Fig. 21(b) and in Fig. 21(c), the mid and far
zone temperatures are remained unchanged as predicted in the unit step time response of the previous paragraph.

The transient responses with $t_{\text{max}} = 500$ hours are displayed in Figs. 22. The near zone responses of Figs. 22(a) highlights that the transfer function is close to unity because the signals are extremely slow. However, the mid and far zone transient responses of Fig. 22(b) and Fig. 22(c) highlight that the temperature responses are smoothed. Only half of the maximal amplitudes are reached. With the explored test case of Figs. 2, the minimal, maximal and mean resultant of the temperatures at each node is recapitulated in Table 5.

### TABLE 5. Minimal, maximal and mean values of each node temperature (in °C) corresponding to $t_{\text{max}} = 500$ hours.

| Node | Min. | Max. | Mean |
|------|------|------|------|
| N1   | 20   | 39.9 | 25.9 |
| N2   | 20   | 37.7 | 25.9 |
| Difference | 0 | 0.2 | 0 |
| M1   | 20   | 28.8 | 24.9 |
| M2   | 20   | 29   | 24.9 |
| Difference | 0 | -0.2 | 0 |
| F1   | 20   | 27.8 | 24.4 |
| F2   | 20   | 27.9 | 24.4 |
| Difference | 0 | -0.1 | 0 |

### D. ADVANTAGES AND DRAWBACKS OF THE THERMAL KRON’S MODEL

The same as all modelling methods, the developed Kron’s thermal method presents certain advantages and drawbacks.

The main benefits of the Kron’s tensorial approach compared to the thermal building 3-D computational methods are:

- Inexistence of pre-processing time to design,
- Easiness of implementation as a graph topology,
- Possibility to consider different parameters which can be assumed as tensorial variables,
- Feasibility of analyses in both frequency and time domains,
- And very fast computation time.

However, the weaknesses of the Kron’s method are:

- Necessitate a good back ground in TAN approach which is not always familiar to the building engineers,
- And the dependence to the approximation of R and C elements.

Behind the present feasibility study, further investigation must be conducted in the future for more complex building in different seasons of various rural and urban applications.

### V. CONCLUSION

An original modelling method of temperature propagation through building room is investigated. The model is based on the Kron’s method applied to RC thermal network. After the classical thermal circuit introduction, the Kron’s graph is elaborated. Then, the thermal circuit was analyzed in the branch and mesh spaces. Then, the thermal metric of the circuit is compactly expressed in tensorial notation. The main solution of the thermal problem is formulated with the TTF at different indoor test points identified as temperature nodes.

The feasibility of the thermal Kron’s modelling was verified by SPICE simulation of a single door room. As expected, a good correlation between the frequency responses of the TTF at near, middle and far positions of the door was obtained.

From practical point of view, the developed Kron’s model is dedicated to the building design engineers who need to evaluate quickly the thermal performances of a room or more complex building. It presents the following benefits. The model enables:

- To compare the effect of the materials and the building geometry.
- To manage the building thermal flows in a general manner.
- The perform the feasibility study which is useful to design and control heating and cooling systems in the built environment.

Similar to all pioneer research work on preliminary understandability and feasibility, the present study on unfamiliar modelling method is validated with intentionally simple proof-of-concept. Meanwhile, in the near future, a significant research work should be made for the application exploitation to real cases of building. Under this perspective, further work will be made to develop an operational tool based on the Kron’s model.
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