Parameterized and exact algorithms for class domination coloring. (English) [Zbl 1444.68147]

Summary: A class domination coloring (also called as cd-coloring) of a graph is a proper coloring such that for every color class, there is a vertex that dominates it. The minimum number of colors required for a cd-coloring of the graph $G$, denoted by $\chi_{cd}(G)$, is called the class domination chromatic number (cd-chromatic number) of $G$. In this work, we consider two problems associated with the cd-coloring of a graph in the context of exact exponential-time algorithms and parameterized complexity. (1) Given a graph $G$ of $n$ vertices, find its cd-chromatic number. (2) Given a graph $G$ and integers $k$ and $q$, can we delete at most $k$ vertices such that the cd-chromatic number of the resulting graph is at most $q$? For the first problem, we give an exact algorithm with running time $O(2^n n^4 \log n)$. Also, we show that the problem is FPT with respect to the number of colors $q$ as the parameter on chordal graphs. On graphs of girth at least 5, we show that the problem also admits a kernel with $O(q^3)$ vertices. For the second (deletion) problem, we show NP-hardness for each $q \geq 2$. Further, on split graphs, we show that the problem is NP-hard if $q$ is a part of the input and FPT with respect to $k$ and $q$. As recognizing graphs with cd-chromatic number at most $q$ is NP-hard in general for $q \geq 4$, the deletion problem is unlikely to be FPT when parameterized by the size of deletion set on general graphs. We show fixed parameter tractability for $q \in \{2, 3\}$ using the known algorithms for finding a vertex cover and an odd cycle transversal as subroutines.

For the entire collection see [Zbl 1355.68020].

MSC:

68R10 Graph theory (including graph drawing) in computer science
05C15 Coloring of graphs and hypergraphs
05C85 Graph algorithms (graph-theoretic aspects)
68Q17 Computational difficulty of problems (lower bounds, completeness, difficulty of approximation, etc.)
68Q25 Analysis of algorithms and problem complexity
68Q27 Parameterized complexity, tractability and kernelization

Full Text: DOI arXiv

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