A Froggatt-Nielsen flavor model for neutrino physics

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Superheavy neutrinos can, via the seesaw model, provide a mechanism for lepton number violation. If they are combined with flavor violation as characterized by the Froggatt-Nielsen mechanism, then the phenomenology for the neutrinos in oscillation experiments, neutrinoless double beta decay, and other experiments can be described by a relatively few number of parameters. We describe the low-energy neutrino mass matrix and show that the results are consistent with currently available data.

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I. INTRODUCTION

An amusing property of the neutrino masses and mixing is that two mixing angles are large. An early expectation was that instead the mixing between flavors would mirror the small mixing observed for quarks and charge leptons. However, neutrino masses may not be Dirac masses like their quark and charged lepton counterparts. They can obtain their small masses by virtue of a seesaw mechanism giving light neutrinos with Majorana masses, so perhaps there can be different opportunities for generating large mixing angles.

One large mixing angle accounts for the solar neutrino oscillations, whereas the other is involved in the atmospheric neutrino data. Analysis of the flavor symmetry in the charged lepton and quark sectors of the Standard Model tries to account for the hierarchy of masses and the smallness of the mixing angles observed. For example the Froggatt-Nielsen mechanism introduces a horizontal symmetry and a small parameter, which characterizes the breaking of the symmetry, can be used to accommodate the pattern of masses and mixings observed there. For this reason the Froggatt-Nielsen mechanism is usually considered as an avenue in flavor models for explaining small mixing angles and large mass hierarchies, a pattern not observed in the neutrino sector. However, for neutrino physics a combination of the seesaw mechanism with the Froggatt-Nielsen mechanism gives rise to new possibilities for accounting for the neutrino data.

II. DEFINITION OF THE MODEL

In this paper we extend an earlier model[1] which is based on a symmetry

$$G_{SM} \times U(1)_H \times U(1)_L,$$

where $G_{SM}$ is the Standard Model gauge group, $U(1)_H$ is the usual horizontal (or flavor) symmetry of Froggatt-Nielsen[2] models, and $U(1)_L$ is lepton number. Scalar fields will be used to break the two $U(1)$ symmetries. The familiar Froggatt-Nielsen scalar field $S_H$ breaks the horizontal symmetry. Some other scalar fields which break the horizontal $U(1)_H$ will also carry lepton number. Neutrino anarchy[3] will be used to make the second and third generations of light neutrinos with large mixing. This approach was used in Ref. [1] which made the then popular but now obsolete prediction of a vanishing value for the $U_{e3}$ matrix element (i.e. a vanishing third mixing angle) of the light neutrino mixing matrix. This model prediction occurred because only two generations of superheavy neutrinos were needed to accommodate the neutrino data existing at that time. Now that a nonvanishing $U_{e3}$ is required by the neutrino experimental data, the model can be extended to include three (or more) generations of heavy neutrino to accommodate this measurement.

In Froggatt-Nielsen models there is a small parameter which gives rise to hierarchies in the charged lepton and quark sectors. In a fully anarchical three generational model the horizontal charges would be assigned to be the same for each of the three generations. Then the challenge is to understand small quantities in neutrino physics, like the ratio of $\Delta m^2$ observed in solar and atmospheric neutrino oscillations as well as the smallness of the third mixing angle. In the model described in this paper, on the other hand, the horizontal symmetry is again broken using a small Froggatt-Nielsen parameter, but this small parameter is also used to suppress the small quantities mentioned above. An agreement with the experimental neutrino oscillation data[4] can be obtained in a very economical way.

A scalar field $S_{HL}$ which carries charge $-1$ under $U(1)_H$ (which generates the mass and mixing angle hierarchies in the charged lepton and quark sectors of the Standard Model when the Froggatt-Nielsen scalar field $S_H$ receives a vev). However, the new field $S_{HL}$ also violates lepton number upon spontaneous symmetry breaking of $U(1)_L$. All light neutrino Majorana masses must reflect this symmetry breaking. In Ref. [1] this was called “combining” the lepton number violation with the flavor symmetry violation, and the resulting model exhibits a meshing of the seesaw and Froggatt-Nielsen mechanisms. It has been noted before that there are interesting and more subtle possibilities for the Froggatt-Nielsen mechanism in the neutrino sector which may involve a

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A mechanism to obtain the large observables in neutrino physics is an $L_e - L_\mu - L_\tau$ symmetry, which can be implemented in a seesaw model and be perturbatively broken by the Froggatt-Nielsen mechanism. Before including these small effects, the light neutrino mass matrix in the flavor basis has the form

$$m_{\ell i} = m \begin{pmatrix} 0 & \sin \theta & \cos \theta \\ \sin \theta & 0 & 0 \\ \cos \theta & 0 & 0 \end{pmatrix}. \quad (2)$$

This matrix has mass eigenvalues 0, $-m$, and $+m$, so it is of the inverted hierarchy type. It is diagonalized by a unitary matrix $U_{\nu}$ which contains a maximal mixing angle which is responsible for atmospheric neutrino oscillations. i.e.

$$\sin^2 2\theta_A = \sin^2 2\theta \, ,$$
$$\sin^2 2\theta_\odot = 1. \quad (3)$$

The second and third generations are assigned the same charges under the symmetries (neutrino anarchy). The first generation is assigned a different $U(1)_H$ charge, and the breaking of this symmetry via the Froggatt-Nielsen mechanism, with interactions consistent with Eq. (1), are needed to fill in the zeros in this matrix. Therefore the spontaneous breaking of the $U(1)_H$ symmetry determines the size and structure of the perturbations.

The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix receives contributions from the nonalignment of the flavors and masses in the charged lepton sector as well as the neutrino mass matrix. The matrix is defined as

$$U_{\text{PMNS}} = U_L^1 U_\nu \, , \quad (4)$$

where $U_L$ is the matrix that diagonalizes the charged lepton sector. Since their exists a large hierarchy in the charged lepton ($e, \mu, \tau$) masses, Froggatt-Nielsen models should give small mixing angles in the charged lepton sector, and the matrix $U_L$ is approximately the identity. Therefore the mixing angles arising from $U_\nu$ to provide the dominant contributions to the mixing angles observed in the experiments in $U_{\text{PMNS}}$, and we henceforth restrict our attention to the diagonalization of this matrix.

In Ref. [1] we introduced a full three-generation model but with only two generations of superheavy neutrinos. In this paper we expand the set of superheavy neutrinos to include a full set of three generations. If this is done assuming the second and third generations of heavy neutrinos have the same horizontal charges (anarchy), then the model naturally accounts for all the available neutrino data: large mixing angles for solar and atmospheric neutrino oscillations as well as a nonzero value for $U_{e3}$ and the possibility of CP violation in the lepton sector of the Standard model. The model has an inverted mass hierarchy imposed by the $L_e - L_\mu - L_\tau$ symmetry, exactly one massless neutrino, and the possibility of an observable signal in neutrinoless double beta decay experiments.

The scales in the model are:

1) the electroweak breaking scale determined by the vevs of Higgs doublets, $\langle \phi_a \rangle$;

2) $M_{HL} \equiv \langle S_{HL} \rangle \sim \langle S_{HL} \rangle$, the lepton number breaking scale;

3) $M_H \equiv \langle S_H \rangle \sim \langle S_H \rangle$, the horizontal symmetry breaking scale;

4) $M_F$, the mass scale of Froggatt-Nielsen vector-like quarks and leptons.

The Froggatt-Nielsen parameter

$$\lambda_H \equiv \frac{\langle S_H \rangle}{M_F} = \frac{M_H}{M_{HL}}. \quad (5)$$

can be used to generate the mass and mixing angle hierarchies in the quark and charged lepton sectors of the Standard Model. It is small and is often associated with the value of Cabibbo angle ($\sim 0.2$) since it is used to generate the mass and mixing angle hierarchies in the quark and charged lepton sectors of the Standard Model (as mentioned above, to achieve the hierarchy in masses and mixing needed to account for the data, usually large powers of the parameter is needed). The lepton number breaking introduces another parameter

$$\lambda_L^2 \equiv \frac{\langle S_H \rangle^2}{\langle S_{HL} \rangle^2} \equiv \frac{M_H}{M_{HL}}. \quad (6)$$

This parameter, being the ratio of two vevs breaking the horizontal symmetry, may or may not be small. In the case where $\lambda_L$ is small, the neutrinos may display a mass hierarchy which is significantly different from that suggested by their Froggatt-Nielsen $U(1)_H$ alone. Alternatively, as in the model presented in this paper, there can be additional symmetry considerations that constrain the model predictions. Our results will apply whether $\lambda_L \sim 1$, $\lambda_L \sim \lambda_H$, or even smaller.

The fields of the model and their associated $U(1)_H$ charges

$$L_0, L_{i+1}^{(k)}, N^i_{+1}, N^{-i}_{-1}, N^i_0, N^{-i}_0 \, , \quad (7)$$

where $k = 1, 2$ to account for the three generations of lepton doublets $L$ in the Standard Model. The fields $N$ are the Standard Model singlets. The fields $L$ and $N$ give rise to the usual Dirac mass terms. Vector-like couplings can arise between $N$ and $\bar{N}$ while lepton violating couplings arise between two $N$ fields or two $\bar{N}$ fields. The indices $i, j$ indicate the possibility of multiple pairs of each horizontal charge. Any number of additional pairs of superheavy neutrinos of each type will produce a model with the properties of the minimal viable model, $i = 1$ and $j = 1, 2$. In Ref. [1] only one pair of each type ($i = 1$ and $j = 1$) was considered which produces the most predictive and economical description of the neutrino data (at the time). It turns out introducing a second pair $N^{(2)}_0, \bar{N}^{(2)}_0$ is sufficient to produce a nonzero prediction
for $U_{e3}$ entry in the PMNS matrix. Furthermore, this additional pair of leptons makes the field content of the model consistent over the three generations.

The fermion fields have lepton number assignments as given in Table II. The $U(1)_L$ symmetry ensures lepton number $L = L_e + L_\mu + L_\tau$ conservation before symmetry breaking from vevs of the scalar fields. With $k = 1, 2$ we have the three Standard Model generations $L_{+1}, L_0^{(1)}$, $L_0^{(2)}$. In particular the charges of the heavy neutrinos $N_{+1}$ and $N_{0}^{(k)}$ have the same charges as the light fields, $L_{+1}$ and $L_0^{(k)}$ respectively. The $\bar{N}$ fields have the opposite charges of their $N$ partners in all cases. For this reason, it is natural to suppose that for this case below, but the more generic equations can be obtained by simple and obvious generalizations.

The rows and columns are ordered as given in Eq. (7). The $L_e - L_\mu - L_\tau$ symmetry, respected by the $S_{HL}$ and $\bar{S}_{HL}$ fields, can be understood from the charges assigned to the fields in Eq. (7). These fields acquire vevs that violate overall lepton number $L$, but do not violate $L_e - L_\mu - L_\tau$. The fields $S_H$ and $S'_{HL}$ also respect the $U(1)_L$ symmetry, but their interactions violate $L_e - L_\mu - L_\tau$ conservation. The small Froggatt-Nielsen parameter $\lambda_H$, which arises from the breaking of the $U(1)_H$ symmetry and accounts for the hierarchies in the charged lepton and quark sectors of the Standard Model, arises from the vev of the $S_H$ field. It sets the scale here for the level of breaking of the $L_e - L_\mu - L_\tau$ symmetry.

The collection of heavy neutrino fields is economical, with one pair of heavy lepton fields $N$ and $\bar{N}$ for each flavor generation. The lepton number violating scalar fields $S_{HL}$ and $\bar{S}_{HL}$ carry the horizontal symmetry charge. Upon receiving vacuum expectation values (vevs), they introduce a scale $M_{HL} = \langle S_{HL} \rangle \sim \langle \bar{S}_{HL} \rangle$ which then via the seesaw mechanism sets the scale for $m$ in Eq. (2).

Even after the spontaneous breaking of the $U(1)_L$ symmetry, the (accidental) $L_e - L_\mu - L_\tau$ symmetry of the $S_{HL}$ and $\bar{S}_{HL}$ fields interactions prevent contributions to the zero entries in Eq. (2). A third scale is associated with vevs of the fields $S_H$ and $S'_{HL}$ and is small, $O(\lambda_H)$, compared to $M_F$. It is this last scale which accounts for the $L_e - L_\mu - L_\tau$ violation. The $S_H$ and $S'_{HL}$ fields are singlets under $U(1)_L$, so their vevs cannot by themselves contribute to the zero entries in Eq. (2). The $S_H$ and $S'_{HL}$ fields, the first of which is envisioned to play a role in the Froggatt-Nielsen mechanism in the quark sector, are assigned zero charges under $L_e - L_\mu - L_\tau$.

First consider the situation where the horizontal symmetry breaking is temporarily turned off (i.e. $\lambda_H = 0$). With the charges of the fields chosen as in Eq. (7), the following symmetric mass matrix results:

$$\begin{pmatrix}
\phi_u & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & M_F & \phi_u & \phi_u \\
0 & 0 & 0 & S_{HL} & 0 & S_{HL} \\
0 & 0 & 0 & S'_{HL} & M_F & M_F \\
0 & 0 & 0 & \bar{S}_{HL} & 0 & \bar{S}_{HL} \\
0 & 0 & 0 & \bar{S}_{HL} & M_F & M_F \\
0 & 0 & 0 & \bar{S}_{HL} & M_F & M_F \\
0 & 0 & 0 & \bar{S}_{HL} & M_F & M_F \\
\end{pmatrix}.$$ (8)

The rows and columns are ordered as given in Eq. (7). The entries $S_{HL}$ and $\bar{S}_{HL}$ represent the vevs of the fields. As in all types of Froggatt-Nielsen models there is understood to be an undetermined order one coefficient in front of every nonvanishing entry in this matrix since the symmetries only enforce that certain elements are zero but does by itself enforce any relationships between nonzero elements apart from an overall scale. Some of the entries are related because the matrix is symmetric, but other entries which appear identical are only equivalent up to these coefficients (for example, $M_F$’s in the 4 – 5 entry and in the 7 – 8 entry). We imagine the scale $M_{HL}$ is comparable to the scale $M_F$ (so that $\lambda_L \sim \lambda_H$), whereas $M_H = \langle S_H \rangle \sim \langle S_{HL} \rangle$ is smaller (and ignored here temporarily). The effect of assigning a nonzero $U(1)_H$ charge to the fields giving rise to lepton number violation, $S_{HL}$ and $\bar{S}_{HL}$, is to move the contributions off the diagonal, and as a consequence generate the pseudo-Dirac structure in the effective light neutrino mass matrix.

The mass matrix in Eq. (8) gives rise to the light neutrino mass matrix $U_\nu$ in Eq. (2) upon integrating out the heavy sector. This mass matrix has one maximal angle, one angle $\theta$ which is given in terms of (undetermined) order one coefficients in the full mass matrix in Eq. (8). Because the source of the light neutrino masses arises from a limited number of interactions, we can define $\tan \theta_1$ to be the ratio of the 3 – 7 and the 2 – 7 entries in the matrix in Eq. (5). Likewise define $\tan \theta_2$ to be the ratio of the 3 – 9 and the 2 – 9 entries. These two contributions add with in general differing weights, $m_1$ and
$m_2$, to give the overall mixing angle $\theta$:

$$
 m \sin \theta = m_1 \sin \theta_1 + m_2 \sin \theta_2 \\
 m \cos \theta = m_1 \cos \theta_1 + m_2 \cos \theta_2
$$

(9)

The mass parameters $m_1$ and $m_2$ have complicated but completely known expressions in terms of the nonzero entries in Eq. (8). If the second pair of superheavy neutrino fields $N_0^{(2)}, \bar{N}_0^{(2)}$ is absent as in the model in Ref. [1], then there is necessarily an alignment in angles $\theta = \theta_1$ (there is no angle $\theta_2$ in that case) which eventually extends to the perturbative contributions to the light neutrino mass matrix (see Eqs. (13) and (14) below). The overall scale $m$ is generated as a seesaw-type mass, and the undetermined ratio of order one coefficients has been expressed as the angle $\theta$ in Eq. (9). These terms break the lepton number symmetry but do not violate the $L_e - L_\mu - L_\tau$ symmetry.

Now we consider the small perturbations coming from the vevs of the fields $S_H$ and $S_{H}^{\prime}$ whose interactions violate $L_e - L_\mu - L_\tau$ and whose vevs at the same time as breaking the horizontal symmetry. Since these the Froggatt-Nielsen parameters are assumed to be small to explain the hierarchies in the fermion masses and mixings, they simultaneously account for the smallness of the perturbations to Eq. (2). In fact, with these perturbations the experimental data, which requires a solar angles not maximal and a mass splitting between the two nonzero light neutrino masses (for the solar neutrino $\Delta m^2$), can be accommodated as a natural consequence of the Froggatt-Nielsen breaking of the horizontal symmetry.

First the effects of the field $S_H$ arise from the coupling $S_H N_{i+1}^{-1} \bar{N}_0^{(j)}$ which conserves the horizontal charge $U(1)_H$. Upon spontaneous symmetry breaking the scalar field receives a vev and gives contributions in elements of the mass matrix that were previously zero, $j$ these will contribute only to the $1-1$ entry of the light neutrino mass matrix. These contributions must necessarily involve the lepton-number breaking vev of the $S_{H}$ field as well as a Froggatt-Nielsen suppression of order $P$.

In addition to this perturbation, there is a field $S_{H}^{\prime}$ with $U(1)_H$ charge $+1$, then it provides a coupling $S_{H}^{\prime} N_0^{(j)} \bar{N}_{-1}$. Since this interaction has charge $L_e - L_\mu - L_\tau = -2$, it will produce perturbations in the $2 - 3$ subblock in the light neutrino mass matrix. After $S_{H}^{\prime}$ receives a vev, it introduces contributions to certain entries in Eq. (10) that were previously zero.
Here the perturbation $P' = \mathcal{O}(\lambda_H)$ is naturally small by the Froggatt-Nielsen mechanism. The effective light neutrino mass matrix in Eq. (2) becomes,

$$m_{\ell 3} = m \begin{pmatrix} z \sin \theta & \cos \theta \\ y & d \\ \cos \theta & d & x \end{pmatrix}, \quad (13)$$

where the small quantities $x, y, d$ are of order the perturbation, $P'$. The relationship between the four terms in the $2 - 3$ subblock of this matrix is enforced by the restricted nature of the couplings of the lepton doublets $L$ to the heavy neutrino fields $N$ and $\bar{N}$. Furthermore the parameters are related to each other so that the $3 \times 3$ matrix $m_{\ell 3}$ has a vanishing determinant and there is one massless neutrino. This condition is enforced by the symmetries of the model. The form in Eq. (13) yields a light neutrino mass matrix that can be phenomenologically acceptable for describing all the data from atmospheric and solar neutrino oscillations. In fact the quantities $x, y, d$ have the form

$$x = z'_{11} \cos \theta_1 + z'_{22} \cos \theta_2 + z'_{12} \cos \theta_1 \cos \theta_2$$
$$y = z'_{11} \sin \theta_1 + z'_{22} \sin \theta_2 + z'_{12} \sin \theta_1 \sin \theta_2$$
$$d = \frac{1}{2} z'_{12} \sin \theta_2 \cos \theta_2$$

$$+ \frac{1}{2} z'_{12} \sin \theta_1 \cos \theta_2 + \sin \theta_2 \cos \theta_1). \quad (14)$$

The quantities $z'_{ij}$ are of order $P'$ in the perturbation. The formulas in Eq. (13) have an obvious generalization to the case where more (than two) superheavy fields $(N_0^{(j)}, \bar{N}_0^{(j)}, j = 1, 2, 3, \ldots)$ are present. On the other hand, when only one such superheavy field is present, $\theta_1 \equiv \theta$ and the terms involving $\theta_2$ are absent. Then the perturbations can be seen to align necessarily with the atmospheric mixing angle $\theta$. This yields an unacceptable value of $U_{\ell 3} = 0$ (see Eqn. (13) below).

The more constrained model presented in Ref. [1] has sometimes been categorized as an example of the Simple Real Scaling (SRS) ansatz [8, 10]. However, that constrained model falls into the SRS class of theories by accident (i.e. the SRS form is not required by the symmetries). As a consequence of being an SRS model, it indeed exhibits an unacceptable vanishing third neutrino mixing angle. The generalization to the model described in this paper eliminates this phenomenological prediction (while keeping a massless neutrino), and puts the new model outside the SRS class [11, 19].

III. PHYSICAL PREDICTIONS OF THE MODEL

This mass matrix in Eq. (13), involving perturbations $x, y, z, d$ collectively of order $\delta$, can be related to the physical observables [8]. The atmospheric neutrino oscillation parameters are

$$\sin^2 2\theta_A = \sin^2 2\theta + \mathcal{O}(\delta^2),$$
$$\Delta m^2_A = -m^2 + 2\Delta m^2_{\odot} + \mathcal{O}(\delta^2). \quad (15)$$

The solar neutrino parameters are given by

$$\sin^2 2\theta_{\odot} = 1 - \left( \frac{\Delta m^2_{\odot}}{4\Delta m^2_A} - z \right)^2 + \mathcal{O} \,
$$

$$R = \frac{\Delta m^2_{\odot}}{\Delta m^2_A} = 2(z + \bar{y} \cdot \bar{x}) + \mathcal{O}(\delta^2). \quad (16)$$

These formulas give the leading perturbations away from leading order ones in Eq. (3). Finally one has

$$U_{\ell 3} = \vec{A} \cdot (\vec{v} \times \vec{x}) + \mathcal{O}(\delta^3). \quad (17)$$

where $\vec{v} = (\cos^2 \theta, \sin^2 \theta, \sqrt{2} \sin \theta \cos \theta)$, $\vec{x} = (x, y, \sqrt{2}d)$ and $\vec{A} = \frac{1}{\sqrt{2}}(1, 1, 0)$. Full unapproximated expressions can be obtained by using the results of Ref. [20].

From Eq. (13) we have

$$\bar{v} \cdot \vec{x} = z'_{11} \cos^2 \alpha_1 + z'_{22} \cos^2 \alpha_2 + z'_{12} \cos \alpha_1 \cos \alpha_2$$
$$U_{\ell 3} = \frac{1}{\sqrt{2}} z'_{12} \sin \theta_2 \cos \theta_2$$

$$+ \frac{1}{\sqrt{2}} z'_{12} \sin (\alpha_1 + \alpha_2). \quad (18)$$

where $\alpha_i = \theta - \theta_i$ are the angles measuring the nonalignment of the contributions from each superheavy neutrino pair with the total mixing angle $\theta$ between the second and third generations in the light neutrino mass matrix (Eq. (13)). It is straightforward to find parameter values which produce observables in acceptable agreement with experiment. Again there is an obvious generalization to
Eqs. [9] and [13] when extra superheavy neutrinos are included, but the overall mass pattern remains the same.

The light neutrino mass matrix describes one massless neutrino, so it automatically satisfies the condition \( \det M_\nu = 0 \) [21], [22]. This condition has been exploited in Froggatt-Nielsen models previously, as an supplementary constraint on the parameters [23] of a model with a normal mass hierarchy.

Dirac phases can be easily included in the model. However, the symmetries do not constrain these phases, so no unique prediction for CP violation in neutrino oscillations can be made, beyond the expected overall scale associated with the product of the three nonzero mixing angles.

IV. SUMMARY AND CONCLUSIONS

We have presented a model where lepton number violation arises from the vev of a scalar field which also carries a horizontal symmetry charge. The large mixing angles observed in neutrino oscillation experiments arise from an \( L_e - L_\mu - L_\tau \) symmetry together with Froggatt-Nielsen neutrino anarchy in the second and third generations. Froggatt-Nielsen symmetry breaking then accounts for the smaller observables. The resulting light neutrino physics has an inverted hierarchy, two large and one small mixing angle, and one exactly massless neutrino. There is a nonzero expectation for neutrinoless double beta decay (the mass parameter measured in those experiments is \( m_2 \)) as is familiar in inverted hierarchy models. The form of the light neutrino mass matrix, determined at a high energy scale where the spontaneous symmetry breaking occurs, and its phenomenology are unaffected under renormalization group evolution. Increasing the number of heavy neutrino fields beyond the minimum number of three generations yields the same overall phenomenology. However, the case of three generations with two having exactly the same charge assignments is sufficient to provide consistency with the experimental neutrino data.

One can take the point of view that a full three-generational anarchy in Froggatt-Nielsen charges is compatible with the neutrino data, and the small quantities are just accounted for by small parameters and/or cancelations. This is certainly reasonable since the hierarchy in the neutrino observables is quite mild. On the other hand, the model presented here offers a reason for their smallness in parallel with the quark and charge lepton hierarchies, namely the same Froggatt-Nielsen mechanism.

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