Universe Expansion Lensing II - Photometric and spectroscopic empirical evidences

Juan De Vicente-Albendea

Centro de Investigaciones Energéticas, Medioambientales y Tecnológicas (CIEMAT), Avda. Complutense 40, E-28040, Madrid, Spain
e-mail: juan.vicente@ciemat.es

Received; accepted

ABSTRACT

The foundations of the universe expansion were formulated close to a century ago through the General Relativity theory and the FLRW (Friedmann-Lemaître-Robertson-Walker) metric. Since early thirties, the Tolman’s surface brightness test was proposed to differentiate between static and expanding universes. According to Tolman’s model, the surface brightness of astronomical sources in an expanding universe should decrease with the redshift $z$ as $\mu \propto (1 + z)^{-n}$, while the luminosity-angular distances relation goes as $d_L = d_A(1 + z)^n$, with cosmic index $n_c = 2$. Recently, a new paradigm—Expansion Lensing (EL)—was presented. EL demonstrates the inverse square law within the FLRW geometry, predicting a different behaviour of surface-brightness given by $\mu \propto (1 + z)^{-2}$ and a cosmic index $n_c = 1$, i.e., $d_L = d_A(1 + z)$. In this paper, a method denominated cosmic vector inference is developed to experimentally determine the cosmic index $n_c$. The method has been applied to the SDSS DR15 galaxy sample obtaining $n_c = 1.0$, the value expected for the expansion lensing paradigm. On the other hand, the surface brightness $\mu(z)$ has been computed from spectra of two galaxy samples (SDSS and VIPERS) with the goal to constraint the galaxy evolution associated to Tolman’s and EL models. The results provide additional arguments supporting expansion lensing over Tolman’s model.

Key words. Cosmology: theory – Cosmology: observations – Galaxies: distances and redshifts – cosmological parameters – dark matter – dark energy

1. Introduction

The last hundred years have provided a great impulse towards the knowledge of the history and fate of the Universe. The field equations of general relativity formulated by Einstein (1915) on one hand, the Friedmann (1922), Lemaître (1927, 1931), Robertson (1933) and Walker (1937) (FLRW) model on the other, along with several cosmological distances defined from luminosity and angles subtended by objects, constitute the basis of the standard model. The matching between theoretical predictions and observational data provides information about the different components of the Universe (i.e., curvature, radiation, matter and dark energy).

Since the discovery of the universe expansion by Hubble (1929), many astronomical surveys have been performed to determine the evolution of the Universe. Different test have been proposed to confront expansion against other theories as the tired light (Zwicky 1929). The most conclusive test is the time dilution of Type Ia supernovae light curves that was suggested by Wilson (1939) and verified by Leibundgut et al. (1996) and Goldhaber et al. (2001). Another relevant test was proposed by Tolman in 1934 which predicts a surface brightness dropping as $\sim (1 + z)^{-4}$ with redshift $z$ for an expanding universe. The values obtained for the exponent $n$ in different data analysis (Hoyle & Sandage 1956), Sandage (1961), Petrosian (1976), Meier (1976), Sandage & Perelmuter (1991), Phare et al. (1996), Lubin & Sandage (2001) and Sandage (2010) differ from the predicted value of $n = -4$ and thus it is assumed the existence of a non negligible galaxy evolution effect.

Recently, Expansion Lensing (EL) was presented (De Vicente-Albendea 2020). EL predicts a new luminosity-angular distances relation $D_L = D_A(1 + z)$ unlike $D_L = D_A(1 + z)^2$ assumed by the standard model. In the same sense, the Tolman’s surface brightness-redshift relation changes from $\mu \sim (1 + z)^{-4}$ to $\mu \sim (1 + z)^{-2}$.

In this paper, a method denominated Cosmic Vector Inference (CVI) is developed to determine experimentally the luminosity-angular distance relation. The method has been applied to 1.2 million galaxies from SDSS DR15 sample (Aguado et al. 2019). On the other hand, the mean surface brightness of galaxy samples from SDSS and VIPERS (Guzzo et al. 2014) surveys have been computed from spectra. This amount allows one to evaluate the galaxy evolution on Tolman’s model against the expansion lensing paradigm. Relevant conclusions can be extracted from this comparison.

The rest of the paper is organized as follow. Section 2 introduces some basic elements of the standard model of cosmology and define the cosmic index that relates luminosity and angular distances. In Section 3 the Expansion Lensing paradigm is described. Section 4 develops the method Cosmic Vector Inference and applies it to determine de cosmic index. Section 5 describes the surface brightness measurement from the spectra, and analyse possible galaxy evolution according to the values predicted by the Tolman’s model and Expansion lensing.
2. Standard Model of cosmology: cosmic index $n_c = 2$

The Standard Model of cosmology compiles the current knowledge related to the beginning, evolution and fate of the Universe. The model is based on the Friedmann-Lemaître-Robertson-Walker (FLRW) metric Eq. [1] which is a solution of Einstein’s field equation of General Relativity describing a homogeneous and isotropic expanding universe,

$$ -c^2 dt^2 = -c^2 dt^2 + a(t)^2 \left[ \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right] $$

where $k$ describes the curvature and $a(t)$ is the scale factor responsible for the universe expansion.

Along with the main equations of cosmology there are several distances defined to link the theory with the observational data. Let us to reproduce here a brief summary of the distances and its relation with cosmological models described by relative densities $\Omega_M, \Omega_r, \Omega_\Lambda$ for matter, radiation, cosmological constant and curvature respectively (Hogg (1999)).

Let $E(z)$ be the function defined as:

$$ E(z) = \sqrt{\Omega_k(1+z)^2 + \Omega_M + \Omega_r} $$

The Line of sight Comoving Distance $D_C$ is defined by

$$ D_C(\Omega_k) = D_H \int_0^z \frac{dz'}{E(z')} $$

where $\Omega_k$ remarks the dependence from relative densities and where

$$ D_H = c/H_0 = 3000h^{-1} \text{Mpc} $$

is the Hubble distance.

The Transverse Comoving Distance $D_M$ is defined by

$$ D_M = \begin{cases} 
D_H \frac{\sinh[\sqrt{\Omega_k}D_C(\Omega_k)]}{\sqrt{\Omega_k}} & \text{for } \Omega_k > 0 \\
D_C(\Omega_k) & \text{for } \Omega_k = 0 \\
D_H \frac{\sin[\sqrt{\Omega_k}D_C(\Omega_k)]}{\sqrt{\Omega_k}} & \text{for } \Omega_k < 0 
\end{cases} $$

On the other hand, the Luminosity Distance defines the relation between the bolometric flux energy $f$ received at earth from an object to its bolometric luminosity $L$ by means of

$$ f = \frac{L}{4\pi D_L^2} $$

being

$$ D_L = D_M(1+z) $$

(standard model)

Eq. [7] provides the link between a measurable amount $D_L$ and the densities of the components of the Universe through $D_M(\Omega_M, \Omega_r, \Omega_\Lambda)$.

The Angular Diameter Distance $D_A$ is defined as the ratio between the size of the object $S$ and its angular size $\theta$

$$ D_A = \frac{S}{\theta} $$

The Angular Diameter Distance is related to the transverse comoving distance by

$$ D_M = D_A(1+z) $$

and taking into account Eq. [7] we have

$$ D_L = D_A(1+z)^2 $$

(standard model)

For reasons stated below, let us to denominate the $(1+z)$ exponent of this luminosity-angular relation as cosmic index $n_c$, being $n_c = 2$ for the current standard model. Finally the surface brightness ($\mu$) is given by

$$ \mu = l_s(1+z)^{-4} $$

(standard model)

where $l_s$ is the luminosity of the source per area unit and time unit. Note that $\mu$ only depends on the redshift when $l_s$ is constant.

Finding $l_s$ in Eq. [11] one has

$$ l_s = \mu(1+z)^4 $$

(standard model)

3. Expansion Lensing: cosmic index $n_c = 1$

Expansion Lensing is a new paradigm for the luminosity-angular distances relation. It is common in many fields of physics the euclidean inverse-square law, e.g., the flux decreases as the inverse of square of the distance between the source and the observer. Although the static euclidean geometry is not applicable to describe an expanding universe, De Vicente-Albendea (2020) demonstrates that the inverse-square law is still applicable within the FLRW geometry. Then, the relation between the different cosmological distances can be rewritten within the expansion lensing paradigm as

$$ D_L = D_M $$

(expansion lensing)

$$ D_L = D_A(1+z) $$

(expansion lensing)

being therefore the cosmic index $n_c = 1$. The flux can be expressed as
\[ f = \frac{L}{4\pi D_L^2} = \frac{L}{4\pi D_m^2} \]  

(15)

In the same way, expansion lensing also affects to surface brightness (\(\mu\)) that is transformed from Eq. [11] to

\[ \mu = l_S(1+z)^{-2} \quad \text{(expansion lensing)} \]  

(16)

where \(l_S\) is the luminosity of the source per area and time units. Finding \(l_S\) in Eq. [16] one obtains

\[ l_S = \mu(1+z)^2 \quad \text{(expansion lensing)} \]  

(17)

### 4. Empirical determination of cosmic index:

\(n_c = 1.0 \pm 0.05\)

In this section we develop Cosmic Vector Inference, a direct method to measure the cosmic index in the luminosity-angular distances relation. The luminosity distance can be expressed as

\[ d_L = \sqrt{\frac{L}{4\pi f_L}} \]  

(18)

and since the angular distance corresponds to the luminosity distance at emission, it can be expressed also as

\[ d_A = \sqrt{\frac{L}{4\pi f_A}} \]  

(19)

where \(f_A\) corresponds to the flux that would be measured in a static universe. Dividing both expressions we have

\[ \frac{d_L}{d_A} = \sqrt{\frac{f_A}{f_L}} \]  

(20)

On the other hand, the luminosity-angular distances relation is given by

\[ (1+z)^{n_c} = \frac{d_L}{d_A} \]  

(21)

where \(n_c = 2\) for the standard model and \(n_c = 1\) for expansion lensing. Substituting Eq. [20] in Eq. [21] we have

\[ (1+z)^{n_c} = \left(\frac{f_A}{f_L}\right)^{1/2} \]  

(22)

Taking base10 logarithm in both sides of the equation we have

\[ n_c \log(1+z) = \frac{1}{2} \log\left(\frac{f_A}{f_L}\right) \]  

(23)

Let us to consider a new class of magnitudes, natural magnitudes, defined as minus base10 logarithm of flux, dropping the usual meaningless 2.5 factor. In this scope, we can define:

- Luminosity magnitude as \(m_L = -\log f_L\)
- Angular magnitude as \(m_A = -\log f_A\)
- Cosmic magnitude as \(m_c = m_L - m_A\)
- Natural magnitudes e.g., \(m = (u/2.5, g/2.5, r/2.5, i/2.5, z/2.5, 1)\)

where \(ugriz\) correspond to common SDSS magnitudes and the last component 1 as been added for convenience.

In this way we have

\[ 2n_c \log(1+z) = m_L - m_A \]  

(24)

or

\[ 2n_c \log(1+z) = m_c \]  

(25)

defining the cosmic magnitude as \(m_c = m_L - m_A\).

On the other hand, a tentative expression can be considered to relate cosmic magnitude \(m_c\) and measured magnitudes \(m\). Thus, we assume there exists a vector \(V_c\) on magnitude space, denominated cosmic vector, where the projection of \(m\) produces the cosmic magnitude \(m_c\) for each galaxy.

\[ m_c = m \cdot V_c \]  

(26)

or substituting [25] in [26] we have

\[ 2n_c \log(1+z) = m \cdot V_{n_c} \]  

(27)

where \(V_{n_c}\) denote the value of \(V_c\) for \(n_c\).

To assess the validity of such assumption, we need a galaxy sample with spectroscopic redshift and properly measured photometric magnitudes in several bands. A regression can be applied on this sample to determine \(V_{n_c}\) for different values of the cosmic index \(n_c\). The success of \(n_c\) determination requires two conditions:

1. High correlation between both sizes of Eq. [27] to trust on the results. Note that \(n_c\) is solely a multiplicative factor on left size of Eq. [27] thus it is expected the same correlation for all values of \(n_c\).
2. Then, assuming this high correlation is achieved, what is the true value of \(n_c\)? The true value of \(n_c\) is the one that produces a cosmic vector \(V_c\) that meets

\[ \|V_{n_c}\| = 1 \]  

(28)

since it gives the real value of the cosmic magnitude, i.e., the true projection of measured magnitudes on a normalized cosmic vector.

In order to apply Cosmic Vector Inference to compute the cosmic index by Eq. [27] we resort to SDSS DR15 that provides simultaneously spectrum and photometric measurements for...
5. Surface Brightness: Tolman’s model vs Expansion Lensing

Few years after the discovery of the universe expansion, Tolman proposed the surface brightness ($\mu$) test to differentiate between static and expanding universes. According to this prediction, the surface brightness should decrease as Eq. [11] for an expanding universe. On the contrary, the expansion lensing paradigm predicts a new surface brightness relation in an expanding universe following Eq. [16]. In this section, the surface brightness of different galaxy samples is computed. The comparison of the measured surface brightness $\mu$ with the Tolman’s model and expansion lensing predictions provides constraints for galaxy evolution within each model. The feasibility of such galaxy evolution can be then analysed.

5.1. Measuring the surface brightness from spectra

Let $f_o(\lambda_o)$ be the observed spectrum (i.e. the flux density measured in erg/cm$^2$/s/$\lambda$) of a galaxy. Since the spectrum is taken with a constant fiber aperture, it can be converted to surface brightness ($\mu(\lambda_o)$) by dividing by the aperture. In what follows, let us to equate $f_o(\lambda_o)$ to $\mu(\lambda_0)$ by introducing (in the spectrum units) a constant $\beta$ representing the aperture normalization to arcsec$^{-2}$. On the other hand, let $z$ be the measured galaxy redshift. The surface brightness within the observed band $b_0$ would be given by

$$\mu_o = \int_{b_0} f_o(\lambda_o)d\lambda_o$$

(32)

To compare the surface brightness of galaxies with different redshifts, it is convenient to blue-shift by an amount $(1+z)$ the observed spectrum $f_o(\lambda_o)$ to a common rest-frame emission band $b_r$. To avoid distorting the data analyses with this transformation, the surface brightness $\mu$ represented in the common rest-frame $\mu_r$ should be equal to the measured one in the observation band $\mu_o$. Therefore, performing a change of variable on Eq. [32] we have

$$\lambda_o = (1 + z)\lambda_r$$

(33)

d$\lambda_o = (1 + z)d\lambda_r$

(34)

$$\mu_o = \int_{b_r} f_r(\lambda_r)d\lambda_o = \int_{b_r} (1 + z)f_o(\lambda_o)d\lambda_r = \int_{b_r} f_r(\lambda_r)d\lambda_r = \mu$$

(35)

and then

$$f_r(\lambda_r) = (1 + z)f_o(\lambda_o)$$

(36)

Therefore, Eq. [33] and Eq. [36] allow one to obtain the spectra in the common rest-frame band.

To prevent the surface brightness-redshift relationship $\mu(z)$ from the effects of galaxy evolution, one needs to find a source with constant luminosity along a large redshift period (i.e. a standard candle). The best known standard candle are supernovae Type Ia since they provide a very uniform luminosity. Though they have been used extensively in the last decades, the technique

Fig. 1: Cosmic redshift reconstruction along the cosmic index $n_c$

about 1.2 million of luminous galaxies. To ensure a uniform treatment for all galaxies, we select De Vaucouleurs magnitude (deVMag) which achieves accurate measurement of the flux of the bulge, the most luminous part of the galaxies.

We have applied Eq. [27] to this sample obtaining high correlation ($c = 0.91$) between both sides of the equation for all values of $n_c$. Thus, the first condition to properly measure $n_c$ is met. Regarding the second condition, alternatively to verifying Eq. [28] to determine the true value of $n_c$, we can inversely reconstruct the value of the redshift $z_c$. To properly evaluate $m_{n_c}$ projection we first normalize $V_{n_c}$

$$v_{n_c} = \frac{V_{n_c}}{||V_{n_c}||}$$

(29)

and then projects $m$ over $v_{n_c}$ to obtain $m_{n_c}$

$$m_{n_c} = m \cdot v_{n_c}$$

(30)

Finding $z$ in Equation [25]

$$z_c = 10^{\frac{m_{n_c}}{c}} - 1$$

(31)

Fig. 1 shows the results in the determination of the cosmic index $n_c$. We can see the cosmic redshift reconstruction for $n_c = 2$ (standard model) and $n_c = 1$ (expansion lensing). In Fig. 2 the redshift reconstruction has been extended along several $n_c$ values obtaining the true value for $n_c = 1$, which corresponds to the expansion lensing paradigm.

Fig. 2: Cosmic redshift reconstruction along the cosmic index $n_c$. 

Article number, page 4 of 8
is complex and there are some complications in their measurements associated to the eventuality and standardization issues as is related in Riess et al. (1998) and Perlmutter et al. (1999).

In the same sense, Luminous Red Galaxies (LRGs) constitute a very uniform and homogeneous set of galaxies that provides high luminosity up to redshift of cosmological interest. Though LRGs are not recognized standard candles, the study of the surface brightness on this sample allows one to constrain and analyze the galaxy evolution within the Tolman’s model and expansion lensing paradigm.

5.2. Surface brightness on SDSS

For luminosity studies on SDSS sample, we are interested on galaxies composed uniquely by bulge —mostly LRGs— since they represent a very luminous and homogeneous sample composed of very stable stars and hence foreseeable low luminosity mean evolution. Thus, it was selected a subcatalog by setting the selection parameter $\text{fracDeV} = 1$, which account for exclusive de Vaucouleurs profile and hence bulge-shape galaxies (~ 127,000 galaxies).

5.2.1. Surface brightness-redshift relation

Let us to apply Eq. 33 and Eq. 36 to obtain the rest-frame spectrum on bulge-SDSS sample. After shifting the sample to rest frame, all galaxy spectra become aligned (Fig. 4(a)). Such alignment can be better visualized by spectra normalization in wider wavelength bins and averaging in redshift bins (Fig. 4(b)). The surface brightness $\mu$ can be obtained by integrating the spectra in a common rest-frame emission band. The SDSS spectrum of galaxies was taken between (3650 − 10400)Å. The secure integration interval should not be larger than $\lambda_{\text{max}}/(1 + z_{\text{max}})$ to ensure that all rest-frame spectra have valid measured data. Thus, it has been selected a conservative wavelength integration interval $b_r = (3940 − 5200)\AA$ that meets the above restriction up to $z_{\text{max}} = 1$ and includes some characteristic LRG features as the 4000Å break and the absorption lines between (5160 − 5200)Å corresponding to low evolution stars. Then, the surface brightness $\mu$ in this band can be obtained by

$$\mu = \int_{b_r(z)} f_r(\lambda_r)d\lambda_r = \int_{b_r} f_r(\lambda_r)d\lambda_r$$

then

$$\mu = \int_{3940}^{5200} f_r(\lambda_r)d\lambda_r$$

Averaging the surface brightness $\mu$ of the different bulge-galaxies in redshift bins one obtains $\mu(z)$ (Fig. 5(a)). In this plot, it was also represented the surface brightness prediction by Tolman’s model (Eq. 11) and by expansion lensing paradigm (Eq. 16), assuming in both cases a constant value of $l_S$. The difference between a prediction and $\mu(z)$ would correspond to $l_S$ variation and hence galaxy evolution. The shadow along the line corresponds to galaxy dispersion. There is a notable divergent behaviour of $\mu$ for $z < 0.47$ and $z > 0.47$. Let one to focus by now on $z > 0.47$. In this case, $\mu(z)$ is very close to expansion lensing prediction and far from Tolman’s model one. As we show below in Section 5.2.2, the closeness to expansion lensing prediction indicates low passive evolution of bulge-SDSS sample as expected due to the low start formation rate.

5.2.2. bulge-SDSS dry mergers

Nevertheless, low passive evolution of bulge-SDSS sample in the emission wavelength band studied (3960 − 5200)Å does not explain the large break observed in Fig. 5(a) for $z < 0.47$. Thus, we need an explanation different from spectral evolution for this break. Let us to hypothesize an explanation. Fig. 5(b) shows the luminosity per surface unit $l_S$ for expansion lensing (Eq. 17) vs the number of galaxies per redshift bin $N(z)$. It can be appreciated that increments in the luminosity slope corresponds to drops in the number of galaxies $N(z)$. Although $N(z)$ depends on many factors including the spectroscopic selection function, it seems probable that the increase in luminosity slope of the bulge-SDSS sample be due to dry mergers (i.e., gas-poor galaxies merging with low star formation but significant stellar mass growth (Bell et al. 2006)). More clues about dry merging are given below.

Fig. 4: (a) Rest frame flux density (left) (b) Rest frame flux density resampled in wavelength and averaged by redshifts bins (right)
and then grows for $z < 0.45$. While the final growth could be explained by dry merging, the parallel decay of the luminosity in all wavelengths discard the galaxy spectral evolution, pointing to some unknown factor of cosmological origin, i.e., the fault of the Tolman’s model to explain the observations.

5.3. Surface brightness on VIPERS

The VIMOS Public Extragalactic Redshift Survey (VIPERS) was conceived to study the large-scale distribution and evolution of galaxies at $0.5 < z < 1.2$. In this paper we focus on W1 field of VIPERS PDR-2 (Scodeggio et al. (2018)) that provides spectrum and redshift measurements for about $\sim 60,000$ galaxies to $iAB < 22.5$.

5.3.1. $\mu(z)$-redshift relation

The spectra were measured at the band $b = (5500 − 9500)\,\AA$. Since our selected rest-frame band is $b_0 = (3960 − 5200)\,\AA$, the minimum and maximum redshifts with valid data at this band are

$$z_{\text{min}} = \frac{5500}{3960} - 1 = 0.39$$

and

$$z_{\text{max}} = \frac{9500}{5200} - 1 = 0.83$$

Fig. 7 shows the surface brightness $\mu$ of the VIPERS sample as a function of redshift. Note that within small fluctuations due to possible mergers and residual spectral evolution, $\mu(z)$ follows the prediction of expansion lensing for $z > 0.6$. On the contrary, as occurs with SDSS samples, VIPERS $\mu(z)$ transits far from surface brightness Tolman’s model prediction.

6. Conclusions

Early after the discovery of the universe expansion, Tolman proposed a surface brightness test as a mean to differentiate an expanding from a non-expanding universe. The test predicts the relation $\mu \sim (1 + z)^{-3}$ for an expanding universe. Recently, expansion lensing—a novel cosmological paradigm— was presented providing a new assessment of the flux received from cosmological sources. Thus, expansion lensing predicts surface brightness given by $\mu \sim (1 + z)^{-3}$ and a luminosity-angular distances relation given by $d_L = d_D (1 + z)^{3}$, with cosmic index $n_c = 1$ rather than the value $n_c = 2$ established by the standard model.

In this paper, empirical evidences of the reality of the expansion lensing paradigm are presented. On one hand, the method Cosmic Vector Inference is developed to determine the cosmic index. The method has been applied to the public SDSS DR15 catalog obtaining a value of $n_c = 1.0 \pm 0.05$. On the other hand, the surface brightness-redshift relation has been derived and analysed from DR15 SDSS and PDR-2 VIPERS spectroscopic data releases. The results also provide arguments favoring expansion lensing over Tolman’s model.

Based on these results, a deep revision of methods involving luminosity as cosmological probes have to be performed under the new Expansion Lensing look. The Hubble constant and the density components of the Universe (i.e. dark matter, dark energy) have to be reassessed on such luminosity probes.
Fig. 6: Luminosity per surface unit $l_\Sigma$ for bulge-SDSS sample. (a) Expansion Lensing: the behaviour of $l_\Sigma$ can be explained by minor galaxy spectral evolution from high to low redshift and by dry mergers at the lowest redshift bin, increasing substantially the luminosity but maintaining the spectrum shape as should be expected by mergers of the same galaxy type. (b) Tolman’s model: the luminosity drop simultaneously in all wavelengths from $z=0.85$ to $z=0.45$. This behaviour seems unfeasible by common galaxy evolution mechanisms, pointing to some unknown factor of cosmological origin, i.e. the fault of the Tolman’s model to explain the observations.

Fig. 7: Tolman’s model vs expansion lensing: Surface Brightness ($\mu$) in common rest-frame emission wavelength band (3960 – 5200)Å for VIPERS sample. Note how $\mu(z)$ evolves close to expansion lensing prediction for $z > 0.6$.

Acknowledgements. Funding support for this work was provided by the Autonomous Community of Madrid through the project TEC2SPACE-CM (S2018/NMT-4291). This paper uses data from public SDSS DR-15. Funding for the Sloan Digital Sky Survey IV has been provided by the Alfred P. Sloan Foundation, the U.S. Department of Energy Office of Science, and the Participating Institutions. SDSS-IV acknowledges support and resources from the Center for High-Performance Computing at the University of Utah. The SDSS web site is www.sdss.org. SDSS-IV is managed by the Astrophysical Research Consortium for the Participating Institutions of the SDSS Collaboration including the Brazilian Participation Group, the Carnegie Institution for Science, Carnegie Mellon University, the Chilean Participation Group, the French Participation Group, Harvard-Smithsonian Center for Astrophysics, Instituto de Astrofísica de Canarias, The Johns Hopkins University, Kavli Institute for the Physics and Mathematics of the Universe (IPMU) / University of Tokyo, the Korean Participation Group, Lawrence Berkeley National Laboratory, Leibniz Institut für Astrophysik Potsdam (AIP), Max-Planck-Institut für Astronomie (MPIA Heidelberg), Max-Planck-Institut für Extraterrestrische Physik (MPE), National Astronomical Observatories of China, New Mexico State University, New York University, University of Notre Dame, Observatório Nacional / MCTI, The Ohio State University, Pennsylvania State University, Shanghai Astronomical Observatory, United Kingdom Participation Group, Universidad Nacional Autónoma de México, University of Arizona, University of Colorado Boulder, University of Oxford, University of Portsmouth, University of Utah, University of Virginia, University of Washington, University of Wisconsin, Vanderbilt University, and Yale University. This paper uses data from the VIMOS Public Extragalactic Redshift Survey (VIPERS). VIPERS has been performed using the ESO Very Large Telescope, under the “Large Programme” 182.A-0886. The participating institutions and funding agencies are listed at http://www.vipers.unaaf.it.

References

Aguado D. S., Ahumada R., Almeida A., Anderson S. F., Andrews B. H., Anguiano B., Ortiz E. A., Aragón-Salamanca A., Argudo-Fernández M., Aubert M., et al., 2019, The Astrophysical Journal Supplement Series, 240, 23
Bell E. F., Naab T., McIntosh D. H., Somerville R. S., Caldwell J. A., Barden M., Wolf C., Rix H.-W., Beckwith S. V., Borch A., et al., 2006, The Astrophysical Journal, 640, 241
Blanton M. R., Bershady M. A., Abolfathi B., Albareti F. D., Prieto C. A., Almeida A., Alonso-García J., Anders F., Anderson S. F., Andrews B. H., et al., 2017, The Astronomical Journal, 154, 28
Blumenthal G. R., Faber S., Primack J. R., Rees M. J., 1984, Nature, 311, 517
Blumenthal G. R., Pagels H., Primack J. R., 1982, Nature, 299, 37
Bond J. R., Szalay A. S., Turner M. S., 1982, Physical Review Letters, 48, 1636
De Vicente-Albendea J., 2020, arXiv preprint astro-ph/2003.05307
Ellis R., Fabbro S., Fruchter A., et al., 2001, The Astrophysical Journal, 558, 2017
Hall, M., et al., 2019, arXiv preprint astro-ph/0905116
Hoye F., Sandage A., 1956, Publications of the Astronomical Society of the Pacific, 68, 301
Hubble E., 1929, Proceedings of the National Academy of Sciences, 15, 168
Leibundgut B., Schommer R., Phillips M., Riess A., Schmidt B., Spyromilio J., Walsh J., Sundysz N., Hamuy M., Maza J., et al., 1996, The Astrophysical Journal Letters, 466, L21
Lemaître G., 1931, Monthly Notices of the Royal Astronomical Society, 91, 483
Lubin L. M., Sandage A., 2001, The Astronomical Journal, 122, 1084
Meier D. L., 1976, The Astrophysical Journal, 207, 343
Pahre M. A., Djorgovski S., De Carollo R., 1996, The Astrophysical Journal Letters, 456, L79
Peebles P., 1982

Article number, page 7 of 8
Perlmutter S., Aldering G., Goldhaber G., Knop R., Nugent P., Castro P., Deustua S., Fabbro S., Goobar A., Groom D., et al., 1999, The Astrophysical Journal, 517, 565

Petrosian V., 1976, The Astrophysical Journal, 209, L1

Riess A. G., Filippenko A. V., Challis P., Clocchiatti A., Diercks A., Garnavich P. M., Gilliland R. L., Hogan C. J., Jha S., Kirshner R. P., et al., 1998, The Astronomical Journal, 116, 1009

Robertson H. P., 1933, Reviews of modern Physics, 5, 62

Sandage A., 1961, The Astrophysical Journal, 133, 355

Sandage A., 2010, The Astronomical Journal, 139, 728

Sandage A., Perlmutter J.-M., 1991, The Astrophysical Journal, 370, 455

Scodeggio M., Guzzo L., Garilli B., Granett B., Bolzonella M., de La Torre S., Abbas U., Adami C., Arnouts S., Bottini D., et al., 2018, Astronomy & Astrophysics, 609, A84

Walker A. G., 1937, Proceedings of the London Mathematical Society, 2, 90

Wilson O., 1939, The Astrophysical Journal, 90, 634

Zwicky F., 1929, Proceedings of the National Academy of Sciences, 15, 773