Research on modeling and identification of machine tool joint dynamic characteristics

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Abstract
The dynamic characteristics of a joint affect the machine tool operations notably. In this paper, an improved approach is proposed to identify the dynamic stiffness of the joint and to construct a relevant dynamic model. The theoretical dynamic model assembled of two beams is built using the frequency response functions. The identification formulas are derived based on the mechanical equilibrium condition and the displacement compatibility condition to describe the relationship between the assembled structures and substructures. Then an inconsistent equation containing the identification relationship is developed. The equivalent value of the dynamic stiffness is extracted employing the least square method. In the identification process, a part of frequency response functions, which is difficult to be measured, is considered as an intermediate variable to avoid introducing errors. According to the results, the calculated dynamic responses of the assembly show better agreement with the experimental measured data in comparison with the results derived from a typical method, which validates the feasibility. And the results demonstrate higher accuracy of the proposed method.

Keywords: Dynamic stiffness, Joint, Joint identification, Modeling, Frequency response function

1. Introduction

Machine tools and mechanical structures are assembled by various components and usually consist of substructures or subsystems. The substructures are connected to each other by joints, which may result in the discontinuity of the mechanical structure in practice. The complex joint behaviors influence the properties of the whole machine significantly (Mehrpouya et al., 2013; Gao et al., 2015). Actually the dynamic stiffness of the assembled structure is highly dependent on the joint stiffness, which takes up 60-80% of the entire dynamic stiffness (Burdekin et al., 1979) and causes 40-60% deformations (Vafaei et al., 2002) generally. Furthermore, tiny relative displacements or angular displacements exist between two junction surfaces with the dynamic preload, which can store as well as can dissipate energy and show both elasticity and damping properties (Jalali, 2016). Thus, the modeling and identification of the joint characteristics is the key to predict the performance of the machine tools.

The joint characteristics can be hardly modeled accurately using a purely analytical approach because of its complexity and variability (Liu et al., 2015). Hence two typical methods employing the experimental data are developed to study the joint properties. The first one consists in the usage of the measured modal parameters, and the other one consists in the usage of the frequency response functions (FRFs). But for those structures containing closely coupled and heavily damped modes (Wang, 2012), it is difficult to obtain the accurate modal data which is necessary for the modal method. So, the FRFs method is more widely used in the subsequent researches.

The FRFs method for the joint identification was proposed by Okubo and Miyazaki (Okubo et al., 1984). On this basis a more advanced method was developed (Tsai et al., 1988). In this study, the substructure synthesis method with FRFs was proposed to identify the parameters of a build-up structure composed of two lapped beams. Based on the substructure synthesis method, Wang and Chuang (Wang et al., 2004) established a joint model with diagonal matrices and took the effect of noise into consideration. And Ren and Beards (Ren et al., 1995) proposed four basic
formulas to identify joint dynamic characteristics with FRFs, which is significant for the joint study. They constructed a relatively complete matrix, which considers mass, stiffness and damping, in order to describe the dynamic stiffness. For the best solution, they introduced a criterion to reduce the effect of measurement errors. However, in the above researches, the dimensions of the joint model were decreased in the modeling process, that means that only the translational stiffness was considered. And the coupling relationship among two substructures of a build-up structure was also ignored due to the difficulty of measurement. For a joint model considering only the translational stiffness, the identification result performed well in a low frequency range. But in a higher frequency range, the result was not satisfactory. Yang et al. (Yang et al., 2003) improved the method. They considered both the translational and rotational stiffness together in the modeling process, and used a coupling stiffness matrix to model the joint instead of a set of translational springs. But in their method, many inverse operations were introduced into the formulation and caused numerical errors. In order to avoid the errors and to improve the identification accuracy, Yang and Park (Yang et al., 2003) proposed a method using subset FRFs measurements. They applied partly measured and partly estimated FRFs to extract the joint parameters. This method solved the problem caused by unmeasurable FRFs, but, in the same time, the errors from estimation were introduced, and the accuracy of identification was affected. Li et al. (Li et al., 2012; Li et al., 2013) developed a new approach based on the Yang and Park’s method to identify the characteristics of the bolted joints, in which the FRFs and finite element model were applied together on the identification process. In the method, the DOFs of joint were completely considered, and the ill-conditioning problems caused by the contamination of measurement noise were solved. Based on four basic formulas (Ren et al., 1995), Čelič and Boltežar (Čelič et al., 2008; Čelič et al., 2009) also proposed an improved method which considered the effects of both translational and rotational degrees of freedom (DOFs). In their method, the DOFs were operated systematically, and the model dimensions were extended so that it was applicable for more complicated models. Furthermore, a new identification algorithm for the least-squares solution was used and the method feasibility was validated numerically and experimentally. Because of its simplification and practicality, their methods are used comprehensively in many joint identification researches. However, in these methods, the internal excitation force in the joint part was assumed as zero in the derivation process. That means the influence of the internal excitation force on the external nodes dynamic performance was ignored to avoid introducing some FRFs which are difficult to be measured. As a consequence, the errors caused by the assumption were introduced into the calculation, and the identification accuracy was decreased.

In this paper, an improved joint modeling and identification method is proposed. The method is based on the substructure synthesis method using FRFs and the FRFs coupling method. In contrast to the previous methods, in which the coupling relationship among substructures and the effect of the internal excitation force are ignored, both factors that will influence the identification accuracy, are considered in the identification process. That means the coupling relationship and the internal nodes excitations are expressed as a part of FRFs and are introduced into the joint identification formulas. To eliminate the errors caused by estimating, this part of FRFs, which is difficult to be measured, is treated as the intermediate variable instead of the estimated constant value and is offset in the identification with some special algorithms eventually.

To acquire the dynamic properties, a complete dynamic model is established, and an identification algorithm is developed based on the force equilibrium condition and displacement compatibility condition. Then the least square method was employed to solve the inconsistent equations and to transform the dynamic stiffness to an equivalent value. Based on the experimentally measured FRFs, the equivalent dynamic stiffness is obtained. With the equivalent value, a finite element model was established for predicting the response of the assembly. In comparison with the results calculated from another typical method, the predicted response data of the assembled structure using the proposed method shows better agreement with the experimentally measured data, which verified the feasibility and the identification accuracy of the proposed method.

2. Basic identification method and dynamical model

As a proof of principle, a simplified mechanical structure, which is a combination of different substructures and joints, is analyzed. The structure can be divided into three parts according to the substructure synthesis method: substructures I, joints J and assembled structures as shown in Fig. 1(a).
Fig. 1  The theoretical dynamic model of joint. The model is established during the internal and external force analysis for assembly system, substructure system and joint system.

Hence the relationship between the input and the output of the assembly can be presented as:

\[
\begin{bmatrix}
X_I^1 \\
X_I^2 \\
X_J^1 \\
X_J^2
\end{bmatrix} =
\begin{bmatrix}
H_{i1} H_{i1}^{11} H_{i1}^{12} H_{i1}^{12} \\
H_{i2} H_{i2}^{11} H_{i2}^{12} H_{i2}^{12} \\
H_{j1} H_{j1}^{21} H_{j1}^{22} H_{j1}^{22} \\
H_{j2} H_{j2}^{21} H_{j2}^{22} H_{j2}^{22}
\end{bmatrix}
\begin{bmatrix}
F_I^1 \\
F_I^2 \\
\tilde{F}_J^1 \\
\tilde{F}_J^2
\end{bmatrix},
\]  

(1)

where \(X_I\) and \(X_J\) are internal joint nodal displacement vectors of the substructure, and \(H\) is the FRF matrix. Superscript “1” and “2” denote the substructure of the assembled structure. \(F_I\) and \(\tilde{F}_J\) are the force vectors applied at the internal nodes and joint nodes of the assembled structure, respectively.

The input-output relationship of the substructure is expressed as:

\[
\begin{bmatrix}
X_I^1 \\
X_I^2 \\
X_J^1 \\
X_J^2
\end{bmatrix} =
\begin{bmatrix}
H_{i1} H_{i2} 0 0 \\
H_{i2} H_{i2} 0 0 \\
0 0 H_{j1} H_{j2} \\
0 0 H_{j2} H_{j2}
\end{bmatrix}
\begin{bmatrix}
F_I^1 \\
F_I^2 \\
\tilde{F}_J^1 \\
\tilde{F}_J^2
\end{bmatrix},
\]  

(2)

where \(F_I\) is the force vector applied on the joint nodes of the substructure, as shown in Fig. 1(b).

And for the joint dynamic stiffness, the following equation is applied:
\[
\begin{pmatrix}
\bar{F}_{ij} \\
\bar{F}_{ij}
\end{pmatrix} = D \begin{pmatrix}
X_{ij} \\
X_{ij}
\end{pmatrix},
\quad D = \begin{bmatrix}
D_{11} & D_{12} \\
D_{21} & D_{22}
\end{bmatrix},
\]  
(3)

\[
D = K + i\omega C,
\]  
(4)

where \( \bar{F}_{ij} \) denotes the force vector applied at the joint nodes of the joint structure. \( D \) is the joint dynamic stiffness matrix. \( K \) is the joint stiffness, and \( C \) represents the joint damping.

According to the mechanical equilibrium condition, it is obvious that

\[
\begin{pmatrix}
F_{ij} \\
F_{ij}
\end{pmatrix} + \begin{pmatrix}
\bar{F}_{ij} \\
\bar{F}_{ij}
\end{pmatrix} = \begin{pmatrix}
\bar{F}_{ij} \\
\bar{F}_{ij}
\end{pmatrix}.
\]  
(5)

Rearranging Eq. (2) leads to

\[
\begin{pmatrix}
X_{ij} \\
X_{ij}
\end{pmatrix} = \begin{bmatrix}
H_{y}^i & 0 \\
0 & H_{y}^i\end{bmatrix} \begin{pmatrix}
F_{ij} \\
F_{ij}
\end{pmatrix} + \begin{bmatrix}
H_{y}^i & 0 \\
0 & H_{y}^i\end{bmatrix} \begin{pmatrix}
F_{ij} \\
F_{ij}
\end{pmatrix},
\]  
(6)

\[
\begin{pmatrix}
X_{ij} \\
X_{ij}
\end{pmatrix} = \begin{bmatrix}
H_{y}^i & 0 \\
0 & H_{y}^i\end{bmatrix} \begin{pmatrix}
F_{ij} \\
F_{ij}
\end{pmatrix} + \begin{bmatrix}
H_{y}^i & 0 \\
0 & H_{y}^i\end{bmatrix} \begin{pmatrix}
F_{ij} \\
F_{ij}
\end{pmatrix}.
\]  
(7)

Substituting Eq. (5) and Eq. (7) into Eq. (3)-(4) yields

\[
\begin{pmatrix}
\bar{F}_{ij} \\
\bar{F}_{ij}
\end{pmatrix} = D \begin{bmatrix}
H_{y}^i & 0 \\
0 & H_{y}^i\end{bmatrix} \begin{pmatrix}
F_{ij} \\
F_{ij}
\end{pmatrix} + \begin{bmatrix}
I + D \begin{bmatrix}
H_{y}^i & 0 \\
0 & H_{y}^i\end{bmatrix}\end{bmatrix} \begin{pmatrix}
\bar{F}_{ij} \\
\bar{F}_{ij}
\end{pmatrix}.
\]  
(8)

Rearranging Eq. (1) leads to

\[
\begin{pmatrix}
X_{ij} \\
X_{ij}
\end{pmatrix} = \begin{bmatrix}
H_{y}^{1i} & H_{y}^{12} \\
H_{y}^{2i} & H_{y}^{22}\end{bmatrix} \begin{pmatrix}
F_{ij} \\
F_{ij}
\end{pmatrix} + \begin{bmatrix}
H_{y}^{1i} & H_{y}^{12} \\
H_{y}^{2i} & H_{y}^{22}\end{bmatrix} \begin{pmatrix}
\bar{F}_{ij} \\
\bar{F}_{ij}
\end{pmatrix}.
\]  
(9)

Substituting Eq. (8) into Eq. (9) yields

\[
\begin{pmatrix}
X_{ij} \\
X_{ij}
\end{pmatrix} = \begin{bmatrix}
H_{y}^{1i} & H_{y}^{12} \\
H_{y}^{2i} & H_{y}^{22}\end{bmatrix} \begin{pmatrix}
F_{ij} \\
F_{ij}
\end{pmatrix} + \begin{bmatrix}
H_{y}^{1i} & H_{y}^{12} \\
H_{y}^{2i} & H_{y}^{22}\end{bmatrix} \begin{pmatrix}
I + D \begin{bmatrix}
H_{y}^i & 0 \\
0 & H_{y}^i\end{bmatrix}\end{bmatrix} \begin{pmatrix}
\bar{F}_{ij} \\
\bar{F}_{ij}
\end{pmatrix}.
\]  
(10)
Comparing Eq. (6) with Eq. (10) yields

$$
\begin{bmatrix}
H^1_{ij} & H^2_{ij} \\
H^1_{ji} & H^2_{ji}
\end{bmatrix} +
\begin{bmatrix}
H^1_{ij} & H^1_{ij} \\
H^2_{ij} & H^2_{ij}
\end{bmatrix} D 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix} = 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix}.
$$

(11)

$$
\begin{bmatrix}
H^1_{ij} & H^2_{ij} \\
H^1_{ji} & H^2_{ji}
\end{bmatrix} (I + D 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix}) = 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix}.
$$

(12)

It should be noticed that the FRF matrix

$$
\begin{bmatrix}
H^1_{ij} & H^1_{ij} \\
H^2_{ij} & H^2_{ij}
\end{bmatrix} 
\begin{bmatrix}
H^1_{ij} & H^2_{ij} \\
H^1_{ji} & H^2_{ji}
\end{bmatrix}
$$

which includes the information of the internal excitation is difficult to be measured in practice. So, it is treated as an intermediate variable and is offset in the following procedures. This operation can avoid introducing the unpredictable errors caused by estimating, and improve the identification accuracy.

From Eq. (11) and Eq. (12), it is obvious that

$$
\left( \begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix} \right)^{-1} 
\begin{bmatrix}
H^1_{ij} & H^2_{ij} \\
H^1_{ji} & H^2_{ji}
\end{bmatrix} D 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix} = 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix} (I + D 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix})^{-1}.
$$

(13)

Pre-multiplying Eq. (13) by matrix

$$
\begin{bmatrix}
H^1_{ij} & H^1_{ij} \\
H^2_{ij} & H^2_{ij}
\end{bmatrix}^{-1}
\begin{bmatrix}
H^1_{ij} & H^2_{ij} \\
H^1_{ji} & H^2_{ji}
\end{bmatrix}
$$

yields

$$
D 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix}^{-1} = 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix}^{-1} \begin{bmatrix}
H^1_{ij} & H^1_{ij} \\
H^2_{ij} & H^2_{ij}
\end{bmatrix}^{-1} \begin{bmatrix}
H^1_{ij} & H^2_{ij} \\
H^1_{ji} & H^2_{ji}
\end{bmatrix}^{-1} (I + D 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix})^{-1}.
$$

(14)

Rearranging Eq. (14) leads to

$$
D 
\begin{bmatrix}
H^1_{ij} & H^2_{ij} \\
H^1_{ji} & H^2_{ji}
\end{bmatrix}^{-1} \begin{bmatrix}
H^1_{ij} & H^1_{ij} \\
H^2_{ij} & H^2_{ij}
\end{bmatrix}^{-1} \begin{bmatrix}
H^1_{ij} & H^2_{ij} \\
H^1_{ji} & H^2_{ji}
\end{bmatrix}^{-1} 
\begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix} \begin{bmatrix}
H^1_{ij} & 0 \\
0 & H^2_{ij}
\end{bmatrix}^{-1} = I.
$$

(15)

Thus, the joint dynamic stiffness identification function can be written as:

$$
D \cdot H_f = I.
$$

(16)
where \( H_j \) can be expressed as:

\[
H_j \equiv \begin{bmatrix}
H_{y_1} & 0 & 0 & 0 & 0 \\
0 & H_{y_2} & 0 & 0 & 0 \\
0 & 0 & H_{y_2} & H_{y_1}^{12} & 0 \\
0 & 0 & H_{y_1}^{12} & H_{y_2} & 0 \\
0 & 0 & 0 & 0 & H_{y_2}
\end{bmatrix}.
\] (17)

From Eq. (16) and Eq. (17) the dynamic stiffness matrix \( D \) can be identified. And the matrixes

\[
\begin{bmatrix}
H_{y_1}^{12} \\
H_{y_2}^{12} \\
H_{y_1}^{21} \\
H_{y_2}^{21}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
H_{y_1}^{12} \\
H_{y_2}^{12} \\
H_{y_1}^{21} \\
H_{y_2}^{21}
\end{bmatrix}
\]

that contain the coupling relationship among substructures in the joint system are also considered sufficiently in the identification, which can remedy the error caused by reducing dimensions using the substructure synthesis method. In the meanwhile, the effect caused by the internal excitation on the external nodes is also taken into consideration. And the FRF matrix

\[
\begin{bmatrix}
H_{y_1}^{12} \\
H_{y_2}^{12} \\
H_{y_1}^{21} \\
H_{y_2}^{21}
\end{bmatrix}
\]

is treated as an intermediate variable in our method which can be offset to avoid introducing errors caused by using the estimated FRF matrix.

3. Equivalent dynamic stiffness study

The dynamic stiffness \( D \) varies with different excitation frequencies which enhances the difficulty in the modeling process. In order to solve the problem, the equivalent methods are proposed to transform the dynamic result of \( D \) to a constant value that can facilitate the joint modeling and calculation.

3.1 Generalized inverse matrix (GIM) approach

A typical equivalent approach is to use the generalized inverse matrix. Expanding Eq. (16) to the range of all excitation frequencies leads to

\[
\bar{D}[H_j(\omega_1)\cdots H_j(\omega_n)] = [I\cdots I].
\] (18)

Pre-multiplying Eq. (18) by matrix \([H_j(\omega_1)\cdots H_j(\omega_n)]^\top\) yields

\[
\bar{D} = [I\cdots I][H_j(\omega_1)\cdots H_j(\omega_n)]^\top,
\]

\[
\bar{D} = \bar{K} + i\bar{C},
\] (19)

where \([\cdot]^\top\) is the generalized inverse matrix. \( \bar{D}, \bar{K} \) and \( \bar{C} \) represent the equivalent values of dynamic stiffness, stiffness and hysteretic damping, respectively.

However, the usage of generalized inverse matrix can lead to a result with a complicated asymmetric matrix, which does not exist in the actual physical field.
3.2 Derivation approach based on GIM

Based on the generalized inverse matrix, Ren and Beard (Ren et al., 1995) proposed a derivation algorithm, which can transform Eq. (16) to the \((N_r \times N_c)\) linearly independent equations system.

Rearranging Eq. (17) leads to

\[
E(\omega)\omega = g(\omega),
\]

\[
T_r = \begin{bmatrix} I & \frac{\omega^2}{\omega_0^2} I & I \end{bmatrix},
\]

\[
d(\omega) = T_r \omega.
\]

\[
x = \{k \quad \omega_0^2 m \quad c\}^T,
\]

\[
x = \left[ \sum_{r=1}^{n} \left( E(\omega_r) T_r \right)^T E(\omega_r) T_r \right]^{-1} \sum_{r=1}^{n} \left( E(\omega_r) T_r \right)^T g(\omega_r)
\]

where \(E(\omega)\) and \(g(\omega)\) are the coefficient matrices. \(d(\omega)\) is a frequency-dependent vector which elements are constructed from \(D \cdot T_r\) is a transformation matrix, and \(x\) is the vector of the solution composed of dynamic parameters.

Using this method, the decoupling between the stiffness and damping can be achieved. Although there will not be any complicated asymmetric matrix, it may lead to the matrix equations without solutions or a negative stiffness matrix.

3.3 Equivalent approach based on least square method

In this paper, an equivalent method based on the least square method was developed to transform the dynamic value. In the presented equivalent process, using the least square method can ensure the symmetry of the dynamic stiffness matrix in the transformation and avoid introducing a negative matrix or a matrix equation without solutions.

From Eq.(16), it is obvious that

\[
D(1,1)H_r(1,1) + D(1,2)H_r(2,1) = I(1,1),
\]

\[
D(1,1)H_r(1,2) + D(1,2)H_r(2,2) = I(1,2),
\]

\[
D(2,1)H_r(1,1) + D(2,2)H_r(2,1) = I(2,1),
\]

\[
D(2,1)H_r(1,2) + D(2,2)H_r(2,2) = I(2,2).
\]
Rearranging Eqs. (25)-(28) leads to

\[
H_j(1,1)D(1,1)+(H_j(1,1)+H_j(2,1))D(1,2)+D(2,2)H_j(2,1)
= I(1,1)+I(2,1)
\tag{29}
\]

\[
H_j(1,2)D(1,1)+(H_j(1,2)+H_j(2,2))D(1,2)+D(2,2)H_j(2,2)
= I(1,2)+I(2,2)
\tag{30}
\]

Rearranging Eq. (29) and Eq. (30) results in

\[
H_e \cdot \mathbf{D} = H_k.
\tag{31}
\]

where \(H_e = \begin{bmatrix} H_j(1,1) & H_j(1,1)+H_j(2,1) & H_j(1,2) & H_j(1,2)+H_j(2,2) & H_j(2,2) \end{bmatrix}\) and \(H_k = \begin{bmatrix} I(1,1)+I(2,1) & I(1,2)+I(2,2) \end{bmatrix}\) are the coefficient matrices.

\[
\mathbf{D} = \begin{bmatrix} D(1,1) \\ D(1,2) \\ D(2,2) \end{bmatrix}
\]

is the column vector consisted of the joint dynamic parameter.

Expanding Eq. (31) to the whole excitation frequencies range leads to

\[
\mathbf{\ddot{H}}_e \cdot \mathbf{D} = \mathbf{\ddot{H}}_k.
\tag{32}
\]

where \(\mathbf{\ddot{H}}_e\) can be written as:

\[
\mathbf{\ddot{H}}_e = \begin{bmatrix} H_e(\omega_1) \\ \vdots \\ H_e(\omega_n) \end{bmatrix}
\tag{33}
\]

And \(\mathbf{\ddot{H}}_k\) is expressed as:

\[
\mathbf{\ddot{H}}_k = \begin{bmatrix} H_k(\omega_1) \\ \vdots \\ H_k(\omega_n) \end{bmatrix}
\tag{34}
\]
So, it can be expressed as:

\[
\begin{bmatrix}
H_c(\omega) \\
\vdots \\
H_s(\omega)
\end{bmatrix} = \begin{bmatrix}
H_k(\omega) \\
\vdots \\
H_n(\omega)
\end{bmatrix} \bar{D}.
\]  

(35)

Equation (32) is the formulation for the whole frequency range. It can be expressed as Eq. (35), which is an inconsistent equation contained the identification information. Using the least square method, the equation is solved, and the equivalent dynamic stiffness \( \bar{D} \) is extracted, which can be presented as:

\[
\bar{D} = (\bar{H}_c^T(\omega) \bar{H}_c(\omega))^{-1} \bar{H}_c^T(\omega) \bar{H}_k(\omega).
\]  

(36)

Actually, there are still some other effective ways using the least square method (Gu et al., 2017) to transform the dynamic value not investigated in this paper.

4. Experimental setup

To obtain the FRFs data for modeling and identification of the joint, an experiment is carried out using LMS TEST LAB. The experiment schematic is shown in Fig. 2. The test rig consists of two parallel beams with the length of 350 mm and a rectangular cross section of 30 \( \times \) 10 mm\(^2\). Between the two beams there is no gasket in joint part. Normally, the gasket will not be considered as a substructure in joint dynamic properties identification process, because its effect on identification accuracy is very limited and can be omitted directly. In order to simulate an unconstrained status of the structure (namely the free-free boundary condition), two elastic strings are used to suspend the test assembly. The real-time responses are excited by impacting a force hammer on the input point. Then the data are acquired at the output point with an acceleration sensor to generate the FRFs for the identification. Test points are located at the central axis of the beams to minimize the effect of the torsional modal on the results. In order to avoid the noise signal interferences, every test point is impacted for 5 times to acquire the average data. It should be noticed that the directions of excitation and the impact should be kept constant to ensure that the coherence functions approach to 1.

(a) Impact test setup.
Before the test, a trial run with the substructures is proceeded to ensure an appropriate experimental setting and the accuracy of the results. Namely, a test point is chosen as the driving point in the trial test to confirm the system parameters such as bandwidth, trigger boundary, window function.

The effective bandwidth of the sampling frequency in the experiment is from 0 to 3200Hz. Considering the flexible status and dynamic nonlinearity of the joint exist in the practical situation, a torque of 60 Nm is applied on the screw in this experiment.

5. Identification of dynamic characteristics of joint

Before using the acquired FRFs data to describe the responses of the structures and to identify the stiffness, the FRFs in the accelerated form are converted into the displacement form to facilitate the identification.

The experimentally measured FRFs of two substructures and the assembly are shown in Fig. 3. For each structure there are three groups of FRFs data to describe the impacting response in different test points and in different excitation ways. The data $H_{11}$, $H_{22}$, $H_{33}$, $H_{44}$ describe the responses on the original point, and $H_{21}$, $H_{12}$, $H_{43}$, $H_{34}$ indicate responses in the cross-point. According to the results, there is a difference between the responses of two substructures and those of the assembly, as shown in Fig. 3(d). The reason is that the impacting response of the assembly contains the joint dynamic information, which can be identified based on the presented identification formulae using the measured FRFs data.

According to Eqs. (31)-(35), the least square solution of the inconsistent equation can be obtained. This least square solution is a column vector consisting of the equivalent dynamic stiffness matrices, so the equivalent dynamic stiffness can be expressed as:

$$
\vec{D} = \begin{bmatrix} 6.3631 & -4.9824 \\ -4.9824 & 6.3631 \end{bmatrix} \times 10^6 \text{ N/m}
$$
Fig. 3 The experimentally measured FRFs of substructures and assembly. The FRFs are acquired in the form of acceleration magnitude. They are transformed to displacement using MATLAB program to make the curve graphs more understandable.

6. Numerical simulation and validation

In order to validate the feasibility and identification accuracy of the proposed method, a numerical simulation is carried out to predict the responses of the assembly using the proposed method and another typical method developed by Čelič and Boltezar (Čelič et al., 2008). In the end the predicted results of two methods are compared with the experimentally measured data.

A dynamic finite element model composed of two independent substructures is built for simulating the practical experimental setup, as shown in Fig. 4(a). In order to construct a relationship between two substructures for simulating the joint, the element matrix 27 is employed to build a two-node-points connection containing damping and stiffness information. Element matrix 27 is an element of ANSYS APDL, which is defined by two node points and coefficient matrix. The stiffness, mass and damping data can be introduced as real constant in this element directly. It contains spring-damping element information, which has an excellent prediction performance in the high frequency range and a good stability in dealing with the coupling relationship. So, it is applied generally in joint modeling to simulate joint properties because of its excellent simulation performances. Then the calculated joint dynamic data $\mathbf{D}$ is introduced in this element. In simulation process, the boundary conditions of finite element model are completely the same as the real circumstances. Fig. 4(b) illustrates the modeling of joint using the element Matrix 27.
Fig. 4  The finite element model for bolted joint. The model is meshed by Hex Dominant Method in ANSYS. And there are 576 elements. In order to construct the Spring-Damping element, a gap of 1 mm is built between two substructures. The dynamic stiffness is introduced in the element to connect each other and simulate joint.

The relevant material coefficients of two beams are listed in Table 1.

|                      | Substructure A | Substructure B |
|----------------------|----------------|----------------|
| Material             | Steel          | Cast iron      |
| Young’s modulus (N/m²) | 2.10E11        | 1.3E11         |
| Poisson ratio        | 0.3            | 0.3            |
| Density (kg/m³)      | 7850           | 7350           |

In the numerical simulation, an excitation is applied on the assembly in the same way as in the experiment setting to predict the assembly response. Fig. 5 shows the response data of the assembly from the proposed method, Čelíc’s method and experiment. Only the cross-point FRFs data are presented, because they contain the complete dynamic information of joint and can reflect the effect of the joint on the combination accurately.

The detailed comparison results of two methods with the experimental values are listed in Table 2.
Fig. 5  Comparison of two predicted results with measured data. The result predicted by Čelič’s method is shown as short dash line. The value by proposed method is presented as normal dash line. And the experimental results is shown as full line.

Table 2  The resonant frequencies comparison results of proposed method and Čelič’s method with experimental values.

|                     | Experimental value (Hz) | Value by Čelič’s method (Hz) | Percentage error | Value by proposed method (Hz) | Percentage error |
|---------------------|-------------------------|------------------------------|------------------|-------------------------------|------------------|
| First order resonant frequency | 865                     | 756                          | 12.6%            | 881                           | 1.8%             |
| Second order resonant frequency | 1534                    | 1525                         | 0.8%             | 1589                          | 3.5%             |
| Anti-resonance point      | 2093                    | 2465                         | 17.8%            | 2275                          | 8.7%             |
| Third order resonant frequency | 2695                    | 3103                         | 22.0%            | 2790                          | 3.5%             |
| Average error            |                         |                              | 13.3%            |                               | 4.4%             |

As already analyzed, if the joint dynamic properties are accurately identified, the response of the assembly will be predicted accurately from the substructure FRFs and the identified joint parameters. From Figure 5, it can be seen that the predicted FRFs using both methods generally agree with the experimental value and reflect the resonant frequencies of the assembly approximately. However, for Čelič’s results, some clear discrepancies appear within the low frequency range of 500-1000Hz and within the high frequency range from 2000 to 3000Hz. The reason for the discrepancies is that the internal excitation force of the joint is assumed to zero in the identification process, which causes errors with the experimental value and decreases the prediction accuracy. In comparison with the results of Čelič’s method, the FRFs data predicted using the proposed method is closer to the experimentally measured FRFs. Although at the frequency of 2000Hz, there is an error in predicting the anti-resonance point between two results that is caused by the nonlinearity of the materials and the joint (Dong et al., 2016), most of the resonance frequencies are still predicted correctly with an average error of ±4.4%. And because of the consideration of the internal coupling relationship, the predicted results match the measured value very well within the low frequency range from 0-900Hz with 1.8% error. The comparison result demonstrates that the joint dynamic properties are identified accurately in this paper using the proposed method. The effectiveness and feasibility of the method are verified. And because of the better agreement and less error between the predicted value from the proposed method and the experimental value, the identification accuracy is also improved. Besides, the good agreement between the predicted results from spring-damping model and experimental value proved that the applied spring-damping element is able to simulate joint properties accurately in a certain frequency range using proposed method.

7. Conclusion

In this paper, an improved method is presented to identify the joint dynamic characteristics based on the substructure synthesis method and the FRFs method. A theoretical dynamic model of joint is established, and the identification formulae based on the force equilibrium condition and the displacement compatibility condition are derived. On the basis of the least square method, an equivalent method is proposed to transform the dynamic stiffness to a constant value. In order to obtain the FRFs of structures and to extract the dynamic stiffness of joint, an experiment is carried out. And a numerical simulation is carried out based on the finite element model to predict the assembly response. The predicted results from the proposed method and another typical identification method are compared with the experimental results. As a result, the predicted FRFs data using the proposed method agrees better with the experimentally measured values with an average error of ±4.4% in comparison with the data using Čelič’s method. The feasibility and effectiveness of the proposed method are verified and the improvement of the
identification accuracy is also validated.

In the presented dynamic identification algorithm, the internal coupling relationship in the joint was taken into account. And the effect caused by the internal excitation on the external nodes was also considered in the identification process. In the approach, the FRFs data of structures was applied on the external coordinates in the dynamic model instead of the joint coordinates. That means that some experimentally unmeasurable FRFs were taken as the intermediate variables instead of being ignored. And in the calculation procedure, the FRFs were offset to avoid introducing errors and to improve the identification accuracy. Furthermore, the least square method was used to solve the inconsistent equation containing the identification relationship in the equivalent process. It can also ensure the symmetry of the dynamic stiffness matrix and avoid the negative matrix.

It should be noted that, the inhomogeneity, anisotropism of materials and structural damping are not considered in this paper. In practical applications, such factors may also influence the joint dynamic properties identification and reduce the identification accuracy as well as the robustness, which will be further investigated in a future study.

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