Coherent structure dynamics and identification during the multistage transitions of polymeric turbulent channel flow

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Abstract. Drag reduction induced by polymer additives in wall-bounded turbulence has been studied for decades. A small dosage of polymer additives can drastically reduce the energy dissipation in turbulent flows and alter the flow structures at the same time. As the polymer-induced fluid elasticity increases, drag reduction goes through several stages of transition with drastically different flow statistics. While much attention in the area of polymer-turbulence interactions has been focused on the onset and the asymptotic stage of maximum drag reduction, the transition between the two intermediate stages – low-extent drag reduction (LDR) and high-extent drag reduction (HDR) – likely reflects a qualitative change in the underlying vortex dynamics according to our recent study [1]. In particular, we proposed that polymers start to suppress the lift-up and bursting of vortices at HDR, leading to the localization of turbulent structures. To test our hypothesis, a statistically robust conditional sampling algorithm, based on Jenong and Hussain [2]'s work, was adopted in this study. The comparison of conditional eddies between the Newtonian and the highly elastic turbulence shows that (i) the lifting “strength” of vortices is suppressed by polymers as reflected by the decreasing lifting angle of the conditional eddy and (ii) the curvature of vortices is also eliminated as the orientation of the head of the conditional eddy changes. In summary, the results of conditional sampling support our hypothesis of polymer-turbulence interactions during the LDR-HDR transition.

1. Introduction

It is widely known that adding a small amount of polymers into Newtonian turbulence will significantly modify the flow statistics and structures. As a result, the friction factor of the flow is considerably reduced by polymers [3, 4]. In certain flow setups, such as the flow in a straight channel, polymer-induced drag reduction can reach up to 80%. This phenomenology is highly valuable in the development of new flow control strategies for enhancing the transportation efficiency of fluids. One example of the application of polymer additive drag reduction is the Trans-Alaska Pipeline system which saves the pump power by injecting polymers in to the pipe [5]. However, although the polymer additives drag reduction has been intensively studied in the past 60 years, the complex mechanisms behind them are still not fully understood.

In polymeric turbulence, several flow stages occur sequentially as the elasticity increases: the onset of drag reduction (ODR), low-extent drag reduction (LDR), high-extent drag reduction (HDR) and maximum drag reduction (MDR). Before ODR, the effect of polymers on the flows is indiscernible and the mean flow is statistically indistinguishable from those of Newtonian turbulence. Further raising the elasticity leads to the enhancement of drag reduction which
eventually converges to an universal upper bound, i.e. MDR. Between the ODR and MDR, recent experimental and numerical studies suggest the existent of two intermediate stages: LDR and HDR [4, 6, 7]. Out recent study indicated that these two stages are distinguished by the regions where flow statistics are affected by polymers: at the LDR stage polymer effects are constrained to the buffer layer while they extend to the whole wall layer at the HDR stage [1].

Same as many other drag reduction strategies, polymer additives drag reduction is the result of the disruption of the turbulence generation cycle. A widely accepted explanation of the polymer-turbulence interaction is that polymers can damp the intensity of the near-wall streamwise vortices in turbulent flows [3]. Dubief et.al. [8] found that the polymer forces applied to the near vortices tend to damp the ejection and sweeping process. Meanwhile, Ptasinski et.al. [9] suggested that polymers could absorb and redistribute the turbulence kinetic energy since the polymer work in the shear stress balance is negative. The mechanism of polymer damping vortices is in good agreement with observations from DNS simulations and experiments [3] and is sufficient to explain the occurrence of drag reduction at ODR. However, this mechanism is not sufficient to explain the drastic changes of flow statistics and coherent structures at the LDR-HDR transition.

The qualitatively different behaviors between these two intermediate stages suggest the existent of another mode of polymer-turbulence interaction that starts during the LDR-HDR transition. In addition to the indiscriminate suppression of vortex intensity, several recent studies observed additional effects of polymers on the coherent structures. Yarin et.al. [10] studied the thin vortex filaments and found that the generation of horseshoe and hairpin vortexes in the near-wall region is prevented by polymers in the high elasticity regime. Moreover, Biancofiore et.al. [11] observes that the critical amplitude of perturbation to trigger a sustained turbulence increases with elasticity at the high elasticity regime, but is constant at the low elasticity regime. This phenomenon is related to the considerable suppression of the lifting strength of near-wall streaks in high elasticity regime. In addition, our recent study [12] on the laminar-turbulence transition of polymeric flows found that polymers can stabilize the primary streak-vortex structures and suppress the bursting of vortexes. These studies all relate the polymer-turbulence interaction at the HDR stage to the lifting process of coherent structures and its following bursting event.

In our recent study [1], a systematic study on the statistical and dynamic changes of turbulence during the LDR-HDR transition was done and a new mechanism which links the LDR-HDR transition to the modification of coherent structures was proposed. In our hypothesis, the lifting and breakdown of vortices and streaks are suppressed by polymers after the LDR-HDR transition. As a result, the bursting process is weakened which prevents the transportation of energy from the buffer layer to the log-law layer and leads to the decreasing of turbulent intensity in the log-law layer. This mechanism is consistent with all known observations of the HDR stage, e.g. turbulence localization and the changing flow statistics in the log-law layer. However, direct evidences are still needed. The conditional sampling approach allows us to understand the statistical properties of vortices and offer an accessible way to study the polymer-turbulence interaction.

The conditional sampling method was initially used in experimental studies of turbulence to obtain quantitative information of a turbulent flow; the readers are referred to Antonia’s comprehensive review [13] for more details. In essence, the conditional sampling method is used to obtain the best estimation of certain targeted events. In early experimental studies, the quadrant (based on Q2 and Q4 events) [14] and the Variable Interval Time Average (VITA; based on the large variances of the streamwise velocity) [15] schemes gained considerable attention. But these methods are originally not appropriate to reveal the spatial details of the flow field [16]. Later, the development of numerical simulation allows researchers to have fully 3D representation of turbulence and then spatial conditional sampling techniques were proposed. In most of these
spatial conditional sampling techniques, three steps are involved: (i) applying a predetermined event identification criterion to the flow field of turbulence, (ii) extract individual coherent structures from the target field, and (iii) determining the reference points for alignment and implementing zone average.

For step (i), the velocity-gradient-tensor-based algorithms are one of the most widely adopted local criteria (i.e. applied to individual points) in the area of vortex identification [17]. The simplest criterion in this category is vorticity. However, the vorticity is not able to distinguish the difference between the pure shear and the real swirling motion [2, 17]. Then, Hunt et.al.[18] introduced the \( Q \) quantity

\[
Q \equiv \frac{1}{2}(||\Omega|| - ||S||),
\]

(1)
to describe the swirling motion, i.e. the so-called \( Q \)-criterion. The strain-rate tensor \( S \) and the vorticity tensor \( \Omega \) in equation (1) are the symmetric and antisymmetric parts of the velocity-gradient tensor \( \nabla v \), respectively. In the current study, The \( Q \)-criterion is adopted to identify the vortex structures in polymeric turbulence.

Step (ii) is of particular importance to obtain representative structures in the flows. For this step, a common approach in the literature takes advantage of the spatial/temporal separation of structures. For instance, applications based on the VITA and quadrant schemes [19, 20] employ the spatial continuity to recognize individual coherent structures. For those algorithms, a careful selection of a cutoff threshold is needed to decrease the potential influence of structure percolation. On the other hand, Hussain et.al. [21] chose the local extrema of vorticity as the sampling events. In their approach, the local extrema are picked without predefining the cutoff threshold which could effectively avoid the percolation issue. Interestingly, Jeong et.al. [2] extended Hussain et.al. [21]’s 2D local extreme conditional sampling to the 3D spatial fields by computing the local extrema of the \( \lambda_2 \) eigenvalue at each streamwise plane. This method is designed to capture the centre line of streamwise vortices and is robust to the cutoff threshold.

As for Step (iii), the geometry center is normally adopted as the reference point for alignment [2, 19, 21, 22]. However, a smearing problem may raise due to the shape variation of vortex structures. To deal with the smearing issue, a filter is usually set up to discard structures that differ greatly from the target events [16, 19]. Furthermore, Jeong et.al. [2] shifted the reference point according to cross-correlation between realizations.

In polymeric turbulence, the near-wall vortices are elongated and weakened by polymers [3], meanwhile, the long streamwise vortices dominate the near-wall flow field [23]. Therefore, we adopt the Jeong et.al. [2]’s method which is highly efficient in capturing the streamwise vortices. In this study, the original Jeong et al.’s method is adapted to the polymeric case and conditional vortex structures therein are sampled. Comparison is then made with the Newtonian turbulence. The results offer new insight into the coherent structure modification by polymers during the LDR-HDR transition.

2. Methodology

2.1. Direct numerical simulation

In this study, Direct Numerical Simulation (DNS) is adopted to simulate the polymeric turbulence in a plane Poiseuille geometry with a fixed pressure drop. The flow geometry is shown in figure 1, where \( x, y \) and \( z \) denote the streamwise, wall-normal and spanwise directions, respectively. The periodic boundary condition is applied to \( x \)- and \( z \)-directions while the no-slip boundary condition is used in the \( y \)-direction. The governing equations are

\[
\frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla p + \frac{1}{Re} \nabla^2 v + \frac{2(1-\beta)}{ReWi} (\nabla \cdot \tau_p),
\]

(2)

\[
\nabla \cdot v = 0,
\]

(3)
Figure 1. The conceptual plot of the flow geometry

\[
\frac{\alpha}{1 - \frac{tr(\alpha)}{b}} + \frac{Wi}{2} \left( \frac{\partial \alpha}{\partial t} + \mathbf{v} \cdot \nabla \alpha - \mathbf{\alpha} \cdot \nabla \mathbf{v} - (\mathbf{\alpha} \cdot \nabla \mathbf{v})^T \right) = \frac{b\delta}{b+2}, \quad (4)
\]

\[
\tau_p = \frac{b+5}{b} \left( \frac{\alpha}{1 - \frac{tr(\alpha)}{b}} - \left( 1 - \frac{2}{b+2} \right) \delta \right). \quad (5)
\]

In the above equations, the Reynolds number is defined as \( Re \equiv \rho U_{CL} l / \eta \) where \( \rho \), \( U_{CL} \), \( l \), and \( \eta \) are the fluid density, laminar center-line velocity, half-channel height, and fluid viscosity, respectively; under this definition, the friction Reynolds number \( Re_\tau = \sqrt{2Re} \). The Weissenberg Wi number, defined as \( Wi \equiv 2\lambda U/l \) (\( \lambda \) is the polymer relaxation time), measures the level of elasticity. \( \beta \equiv \eta_s/\eta \) is the ratio between the solvent and solution viscosities. The FENE-P constitutive equations (4)-(5) \([24]\) is adopted to determine the polymer stress tensor \( \tau_p \) in equation (2), where \( \alpha \) is the polymer conformation tensor and the maximum extensibility parameter \( b \) constrains the square length of the polymer chains \( tr(\alpha) \).

A series of Wi under \( Re = 3711 \) (i.e., \( Re_\tau = 84.85 \)) is investigated. The rheological parameters \( \beta \) and \( b \) are fixed to 0.97 and 5000, respectively. The streamwise and spanwise periods are \( L_x^+ \times L_z^+ = 4000 \times 800 \), where the superscript “+” indicates quantities in turbulent inner scales. A Fourier-Chebyshev-Fourier pseudo-spectral scheme is applied for spatial discretization. Meanwhile, a third-order semi-implicit backward-differentiation-Adams-Bashforth scheme\([25]\) is used to integrate equations in time. In addition, an artificial diffusion term \( 1/(ScRe) \nabla^2 \alpha \) with the Schmidt number \( Sc = 0.5 \) is introduced in equation (4) to achieve better numerical stability.

2.2. Conditional sampling

In this study, vortexes are identified by adopting the \( Q \)-criterion (equation (1)) \([17]\). \( Q = 0 \) indicates a pure shear flow while a large negative and positive \( Q \) correspond to regions dominated by extensional and rotational flows, respectively. The vortex structures are chosen by satisfying \( Q > Q_{rms} \) in the 3D instantaneous flow field; \( Q_{rms} \) is the root-mean-square of \( Q \).

The conditional sampling algorithm based on Jeong et.al.\([16]\)’s method involves several steps: (1) detect vortex region which satisfies \( Q > 0.7Q_{rms} \) in the 3D instantaneous flow field; (2) calculate the local maxima of the accepted regions at each y-z plane – the local maxima are regarded as the x-centreponts of streamwise vortices; (3) group the local maxima to individual vortexes by adopting a cone detection method, connecting these points form the centrelines of vortices; (4) categorize vortexes into two categories according to the senses of rotation (clockwise/anticlockwise), the sense is determined by the sign of streamwise vorticity at the vortex axis; (5) realizations are discarded if they do not satisfy: (a) the vortex streamwise length \( l_x^+ \geq 50 \), and (b) the average height of the vortex axis \( h^+_y \leq 50 \); (6) the x-centrepoint
Figure 2. Schematic diagram of the cone algorithm method

located at $y^+ = 50$ is chosen as the reference point for alignment and realizations with the same sense of rotation are aligned and averaged to obtain the conditional eddy.

In order to extract individual vortices, the cone detective method is involved. In this method, two x-centrepoints are considered to belong to the same vortex when they satisfy: (a) the two points locate at two adjacent y-z planes; (b) the downstream point is the closest local maximum on its plane to the upstream point; and (c) the distance between the downstream point and the projection of the upstream point is smaller than a threshold, i.e., the downstream point locates within a confining cone extending from the upstream point. A schematic plot of the cone detective method is shown in figure 2. Dots in this figure are the x-centrepoints while the dot dash lines indicate the wall of a vortex. Centrepoints are grouped by the detective method if they fall into the cone (the triangles in figure 2). However, note that this method is inefficient to detect vortices with a high deviation of vortex line from the streamwise direction, e.g. vortex with a strong lifting tendency as shown in the figure.

In Jeong et.al.’s original method [16], the geometry center of vortex axis is chosen to be the reference point for realization alignment. However, since the lengths of streamwise vortexes are different, fixing the reference point to the geometry center will lead to the misalignment of heads and tails of vortices, as shown in figure 3a, and cause the smearing problem. This is worse for high Wi cases where the vortices have a broader range of size and shapes. Instead, we move the reference point for realization alignment to the x-centrepoint at $y^+ = 50$. The new reference point ensures the precise alignment of the heads of vortices (figure 3b). Benefiting from this modification, some important dynamics of vortices, such as the lifting process, can be accurately captured. In addition, the choice of the reference point relaxes the strict constraint of vortex length in step (5) of the conditional sampling algorithm, as the variation of vortex length now has less effects on the conditional eddies (especially in the head region) compared with the original method.

3. Results and discussion

The presence of multiple stages in the polymeric turbulence suggests the existence of multiple types of polymer-turbulence interactions at different stages. In this study, we focus on the change of interaction involved in the LDR-HDR transition. As has been discussed in the literature[3, 4], the statistical quantities of turbulence in LDR and HDR stages exhibit qualitatively different
behaviors. Figure 4 shows the mean velocity profiles of Newtonian, LDR (Wi = 16), HDR (Wi = 48) and MDR (Wi = 80) at Re$\tau$ = 86.15. Starting at the ODR (Wi ≤ 10), the velocity profiles lift up as Wi increases and reach an upper bound at the MDR stage. The identification between LDR and HDR depends on the influenced zones of polymers. For the LDR case (Wi = 16), the effect of polymers is constrained in the buffer layer ($y^{+}$ ≈ 5 ∼ 30): the velocity profile rises in buffer layer but stays parallel to the Newtonian profile outside the buffer layer region ($y^{+}$ > 30). By contrast, the slopes of the mean velocity profiles at HDR and MDR stages differ from the Newtonian profile in the whole channel.

The systematic analysis of the statistical and dynamic changes during the LDR-HDR transition is the focus of a separate study of ours [1]. Therein, we hypothesized that at the LDR-HDR transition polymers start to suppress the bursting of turbulence by preventing vortices from lifting up. The typical vortex structures extracted from the instantaneous flow field before and after the LDR-HDR transition are presented in figure 5. Before the LDR-HDR transition, vortices tend to aggregate into vortex packets. A hairpin vortex is observed at the downstream end of this vortex packet. As the head of the hairpin vortex lifts up, the vortex eventually breaks
down after a period of time. In addition to the hairpin, other type of vortices (e.g. the pseudo-streamwise vortices) also have a high tendency of lifting. In fact, the lift-up and bursting of vortices play an important role in the instability-based vortex regeneration cycle [26, 27]. However, in the high elasticity turbulence (HDR and MDR), the lift-up strength of vortexes is weakened and the hairpin vortexes are eliminated by polymers, as shown in figure 5c. Instead, longer and smoother streamwise vortices with weaker lifting strength dominate the near-wall coherent structures. Note that these streamwise vortices usually organize to a ultra-long (the streamwise length $l^+_x \geq 400$) streamwise vortex string in which the head of a upstream vortex overlaps the tail of an adjacent downstream vortex. The vortex string is similar to those vortexes generated by the parent-offspring vortex regeneration cycle[28]. Eventually, as the strength of vortices exceeds a certain threshold, the vortex string suddenly bursts and forms a group of vortexes in the adjacent region (figure 5d).

The modification of the vortex organization pattern is due to the change of vortex regeneration mechanism. In our hypothesis, since the instability-based mechanism is interrupted by polymers in HDR and MDR stage due to the prevention of lifting and bursting of vortices, the other mechanism – the parent-offspring vortex mechanism, becomes exposed.

To understand the statistical characteristics of vortices at different stages, the aforementioned conditional sampling algorithm, improved based on ref. [16], is implemented. Figure 6 shows the probability density functions (PDF) of streamwise lengths of vortexes at different $y^+$. The PDF distribution of the polymeric flow (Wi = 96) covers a wider range from 0 to over 800 wall units while the Newtonian turbulence only expands to 500 wall units. Also, the peak of the PDF distribution adheres to the wall ($y^+ = 30$) in the Newtonian case while it approaches the center of the channel ($y^+ = 50$) in the polymeric case. To alleviate the potential smearing problem (figure 3), a minimum streamwise length cutoff of vortexes is set to filter vortexes with a short streamwise length. In Jeong et al’s conditional sampling of the Newtonian flow, the cutoff ($l^+_x = 200$) is larger than the average length of vortexes. As a consequence, the relatively large cutoff biases the sample by eliminating many qualified structures, which limits the representativeness of the conditional eddies. Despite the long cutoff, the issue of uneven vortex length still apparently caused smearing in their conditional eddy. The new criterion for selecting the reference point (the x-centrepoint located at $y^+ = 50$) in our improved algorithm significantly alleviates the smearing issue with a much smaller cutoff ($l^+_x = 50$). As a consequence, more realizations are included in constructing the conditional eddy which improves the representativeness of the
conditional eddy.

Figure 6. Probability density functions of vortex streamwise lengths: the color flood is for the \( Wi=96 \) case and lines are for the Newtonian case.

Based on their streamwise length, vortices can be grouped into two categories: major vortices \((l_x \geq l_{\text{cutoff}})\) and fragments \((l_x \leq l_{\text{cutoff}})\). On the other hand (as reviewed above), the vortex regeneration mechanisms in wall-bounded flow can be categorized into two types [29]: the parent-offspring and instability-based mechanisms. Noting that the parent-offspring mechanism depends on the direct contact between the parent and the offspring vortices while the steak instability does not, major vortices may thus be categorized according to their generation mechanisms based on their spatial proximity to other vortices. Pseudo-streamwise vortices overlapping with another vortex with the opposite sense of rotation are categorised to the parent-offspring vortex class and other vortices are categorised to the instability vortex class. By employing this classification, we are able to investigate the changing vortex regeneration mechanism at different stages. The average number of vortex centrepoints \( N_c \) of these two classes for different \( Wi \) is plotted in figure 7. Since the threshold to determine fragments is arbitrary, a number of thresholds are tested and the trend of the profiles is robust to the threshold. In figure 7, the cutoff threshold is 150.

The critical \( Wi \) of the PO-LDR, LDR-HDR and HDR-MDR transitions in figure 7 are 10, 24 and 80, respectively. The number of centrepoints in the instability class starts to decrease at \( Wi = 20 \) which is close to the LDR-HDR transition, suggesting a strong relation between the LDR-HDR transition and the suppression of the instability mechanism. The parent-offspring vortices, however, does not decrease until well after \( Wi = 32 \). In other words, the parent-offspring mechanism is more persistent to the strong polymer effect at HDR. On the other hand, fragments continuously decrease after the onset of drag reduction as polymers suppress the overall intensity of vortices. These observations are in good agreement with our proposed polymer-turbulence interaction during the LDR-HDR transition. However, readers should note that figure 7 is an imprecise estimation of vortex number at different classes. For example, the poor capability of current vortex tracking method in tracking curved vortexes will overestimate the number of vortices in the fragment class. To overcome this issue, the detective algorithm needs to be improved to accommodate higher vortex curvatures, which will be the focus of our future work.

The conditional eddies are obtained in cases before (Newtonian) and after (\( Wi = 96 \)) the LDR-HDR transition and are presented in figure 8. As a reminder, vortex realizations are
Figure 8. (a) The top and (b) side views of the conditional eddies of the Newtonian (red) and Wi = 96 (blue) turbulence.

separated into two classes according to the sign of streamwise vorticity before zone averaging. In figure 8, only the counter-clockwise conditional eddy are shown since the clockwise conditional eddy is its mirror image.

Figure 8(a) shows the top view of the conditional eddies. In general, the length of the conditional eddy at Wi = 96 is longer than that of the Newtonian case, which is consistent with observations in instantaneous flow field images. Comparing the two cases, the tilting angles are similar, but the head of the Newtonian eddy bends further to the positive z direction, which is attributed to the prevalence of highly curved vortexes, e.g. hairpins. In figure 8(b), the lifting angle of the conditional eddy before the transition (Newtonian) is larger than that after the transition (Wi = 96), which agrees with our previous discussion of polymers suppressing the lifting process of vortices. Also, in both cases, an additional iso-surface with opposite streamwise vorticity appears under the head of the main body. Its existence indicates a high probability of another vortex with the opposite sense of rotation showing up under the head of the upstream vortex. Note that in the Newtonian case, the additional structure appears irregular; only in the high-Wi case does the additional eddy become more coherent. This change is possibly a reflection of the increasing importance of the parent-offspring mechanism.

4. Conclusions
Despite recent efforts in the literature, a complete picture of the polymer-turbulence interaction is still missing. Especially, the interaction responsible for the LDR-HDR transition remains a puzzle. A hypothesis was put forward in our recent study on the LDR-HDR transition[1], in which polymers suppress vortex regeneration through streak instability by preventing the lift-up and bursting of vortexes. In this study, a conditional sampling algorithm improved from that of ref. [16] is employed to compare the vortex dynamics before and after the LDR-HDR transition.

Observations of the conditional eddies obtained support our hypothesis. The lifting angle of the Newtonian eddy is considerably larger than that of the Wi = 96 case. Also, the different shapes of the vortex head suggest that highly curved vortexes (e.g., hairpins) existing in the Newtonian and LDR cases are suppressed by polymers at the HDR and MDR stages.

The near-wall vortices are also divided into three classes: one is fragments and the other two are major vortices generated by the streak-instability and parent-offspring mechanisms. The number/size of vortices generated by the streak-instability mechanism drops during the LDR-HDR transition (Wi = 20) while those in the parent-offspring class remain frequent until well
beyond $Wi = 32$. This result also agrees well with our hypothesis of the modification of vortex regeneration mechanism.

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