Critical Exponents and Critical Amplitude Ratio of The Scalar Model from Finite-temperature Field Theory

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Abstract

The critical exponents and the critical amplitude ratio of the scalar model are determined using finite-temperature field theory with auxiliary mass. A new numerical method is developed to solve an evolution equation. The results are discussed in comparison with values obtained from the other methods.

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1 introduction

Phase transition is an important phenomenon in particle physics, cosmology, and condensed matter physics. Quark Gluon Plasma should be present at the heavy ion collision and will give us a lot of valuable information on particle physics [1]. The investigation into the chiral phase transition suggest that a number of flavor may be bounded above [2, 3]. In cosmology, the electro-weak phase transition should be first order for electro-weak baryogenesis [4, 5] and is investigated attentively [6, 7, 8, 9, 10, 11, 12, 13, 14]. Needless to say, a variety of phase transitions are observed and investigated precisely in condensed matter physics.

Field theoretical approach is essential in order to investigate these phase transitions: finite-temperature/chemical-potential field theory [1, 15], perturbative and non-perturbative renormalization group [16, 17, 18, 19, 20, 21, 22], field theory on lattice [23], and so on. Temperature can be naturally introduced by statistical principle using finite-temperature field theory. Not all the phase transitions, however, can be investigated by it; the perturbation theory, which is the most powerful method at zero temperature, often breaks down around the critical temperature because of many interactions in thermal bath [24, 25]. Indeed the perturbation theory fails, when it is applied to either a second-order or a weakly first-order phase transition.

Drummond et.al. [26] proposed a new method using an auxiliary mass in order to avoid this difficulty. We utilized their idea and developed a new method to calculate the effective potential. We, then, investigated the phase transition of the scalar model using the auxiliary-mass method and showed it is second order correctly [27]. It is a great advance in finite-temperature field theory, because the phase transition in the scalar model is indicated to be first order incorrectly by the perturbation theory with daisy resummation [7, 28]. We note that the method was able to reproduce the result with super-daisy approximation [29].

Since the equation we must solve in the auxiliary-mass method is a non-linear partial differential equation for the effective potential, it can not be solved analytically and must be solved by a numerical method. It is, however, difficult to solve partial differential equations numerically because of the
numerical instability \[30\]. What is worse, the non-linearity of the equation prevents us from using the methods established in case of a linear equation. We could not, therefore, make mesh size arbitrary small; The investigation in Ref.\[27\] was not accurate quantitatively. In the present paper, we use an improved numerical method given in the appendix, which do not suffer from the instability, and get accurate universal quantities. Unlike the rough values in \[27\], they are beyond the values obtained from Landau approximation.

The present paper is organized as follows. In the next section we review the auxiliary-mass method developed in \[27\]. In section 3 the effective potential is shown as temperature varies. We, then, focus on the behaviour of it around the critical temperature and calculate the universal quantities. These values are compared with values obtained from the other methods. Summary and discussion is presented in section 4. In appendix we devote to explain the numerical method we used.

2 Review of the auxiliary-mass method

We review the method to calculate an effective potential at temperature where the perturbation theory is not reliable \[27\]. We consider $\lambda \phi^4$ theory which is defined by the Lagrangian density

$$L_E = -\frac{1}{2} \left( \frac{\partial \phi}{\partial \tau} \right)^2 - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4 + J\phi + c.t., \quad (1)$$

where $J$ is an external source function. If $m^2$ is negative, the scalar field $\phi$ develops the non-vanishing field expectation value at $T = 0$. First, the effective potential is calculated with a positive mass squared $M^2$ which is as large as the temperature $T^2$. This selection of the mass permit us to use the perturbation theory without failure, because the loop expansion parameter there is $\lambda T/M \sim \lambda \[7, 31, 32\]$, which is small when the coupling constant $\lambda$ is small. Using the perturbation theory, the effective potential is calculated as follows,

$$V = \frac{1}{2} M^2 \phi^2 + \frac{\lambda}{4!} \phi^4 + \frac{T}{2\pi^2} \int_0^\infty dr r^2 \log \left[ 1 - \exp \left( -\frac{1}{T} \sqrt{r^2 + M^2 + \frac{\lambda}{2} \phi^2} \right) \right].$$
Here, only the one loop thermal correction is left and the quantum correction is neglected, because it should be negligible when the coupling constant $\lambda$ is sufficiently small.

We, then, extrapolate the effective potential (2) to the negative mass squared $m^2 = -\mu^2$ using the following evolution equation,

$$\frac{\partial V}{\partial m^2} = \frac{1}{2}\bar{\phi}^2 + \frac{1}{2\pi i} \int_{-\infty+\epsilon}^{+\infty+\epsilon} dp_0 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{-p_0^2 + p^2 + m^2 + \frac{1}{2}\bar{\phi}^2 + \Pi e^{\beta p_0} - 1}$$

(3)

where $\bar{\phi}$ is an expectation value of the field and $\Pi = \Pi(p^2, -p_0^2, \bar{\phi}, m^2, \tau)$ is a full self energy. The thermal correction is left and the quantum correction is neglected here, too. Of course, $\Pi$ can not be calculated exactly; we need an appropriate approximation in order to calculate the effective potential from (2). Because the effective potential is a generating function of n-point functions with zero external momentum, neglect of momentum dependence in $\Pi$ allows us to replace as follows,

$$m^2 + \frac{\lambda}{2}\bar{\phi}^2 + \Pi(0, 0, \bar{\phi}, m^2, \tau) \rightarrow \frac{\partial^2 V}{\partial \bar{\phi}^2}. \quad (4)$$

The evolution equation (3) can be converted to partial differential equation using this replacement as follows,

$$\frac{\partial V}{\partial m^2} = \frac{1}{2}\bar{\phi}^2 + \frac{1}{4\pi^2} \int_0^\infty dr r^2 \sqrt{r^2 + \frac{\partial^2 V}{\partial \bar{\phi}^2}} \exp \left( \frac{1}{T} \sqrt{r^2 + \frac{\partial^2 V}{\partial \bar{\phi}^2}} \right) - 1$$

(5)

The effective potential can be calculated by solving the partial differential equation (3) with the initial condition (2). The effective potential has an imaginary part below the critical temperature and an analytic continuation is done so that this imaginary part is negative [27]. Since the evolution equation (3) is complicated non-linear partial differential equation, it can be solved only by numerical methods.

1 Hereafter we use unit $\mu = 1$. All dimensionful quantities are measured in the unit.
Figure 1: Real part of the effective potential ($\lambda = 1$). The values of the origin are set to zero. A stable point comes to be zero smoothly as temperature increases.

3 Results

We calculate the effective potential numerically using the method in the Appendix. The real part of the effective potential as temperature varies is shown in fig.1. A stable field expectation value $\bar{\phi}_c$, where the effective potential has its minimum, comes to be zero smoothly as temperature increases. This indicates that second-order phase transition takes place in this model correctly [33, 34]. The imaginary part of the effective potential below the critical temperature is shown in fig.2. One can observe a magnitude of it increases as a field expectation value decreases; this illustrate that a state with smaller field expectation value is less stable below the critical temperature. The critical temperature as a function of the coupling constant $\lambda$ is shown in fig.3. This shows a similar behaviour to the leading result obtained in Ref.[35], but has a slight difference ($\sim 2\%$). In the remaining of this section we determine some critical exponents: $\beta, \delta, \gamma^{+/-}$,and $\alpha$. The amplitude ratio $\chi^+/\chi^-$ is also determined. The results are summarized in table.1.

First, we observe the stable point $\bar{\phi}_c$ carefully. Figure.4 shows $\bar{\phi}_c$ as a function of temperature. It decreases monotonically and vanish smoothly as temperature increases. We, then, focus on its behaviour near the critical tem-
Figure 2: Imaginary part of the effective potential ($\lambda = 1$). The magnitude, which shows the instability of the state, increases as a field expectation value decreases.

Figure 3: Phase diagram of $\lambda \phi^4$ theory. Second-order phase transition is observed on the boundary. The dots represent values calculated using auxiliary-mass method. The dotted line represents the leading result of perturbation theory [35].
Figure 4: Stable field expectation value as a function of temperature ($\lambda = 1$). It decreases monotonically and vanishes smoothly as temperature increases.

Temperature $T_c$ and determine $\beta$, which relate a magnetization to temperature near $T_c$. This is defined as follows,

$$\phi_c \propto (-\tau)^\beta \quad (\tau \sim 0, T < T_c)$$

(6)

where $\tau = (T - T_c)/T_c$. We plot $\log(\phi_c)$ against $\log(-\tau)$ in fig.5; we fit the data to linear function and draw it in fig.5. We determine $\beta$ from the gradient of it. We find $\beta = 0.385$.

Next, we determine the exponent $\delta$ which is defined as follows,

$$\bar{\phi} \propto J^{1/\delta} = \left(\frac{\partial V}{\partial \bar{\phi}}\right)^{1/\delta} \quad (T = T_c).$$

(7)

One can derive the following relation from this,

$$V \propto \bar{\phi}^{\delta+1} \quad (T = T_c).$$

(8)

We show the effective potential at $T_c$ in fig.6. We plot $\log(V)$ against $\log(\bar{\phi})$ in fig.7; we fit the data to linear function and draw it in fig.7. We determine $\delta$ from the gradient of it. The result is $\delta = 4.0$.
Figure 5: Plot of $\log(\bar{\phi}) - \log(-\tau)$ ($\lambda = 1$). The data points are fit to linear function. Using its gradient, $\beta$ is determined.

Figure 6: Effective potential at the critical temperature ($\lambda = 1$).
Figure 7: Plot of $\log(V) - \log(\bar{\phi})$ ($\lambda = 1$). The data points are fit to linear function. Using its gradient, $\delta$ is determined.

Then, we determine $\gamma^{+/-}$ and $\chi^{+}/\chi^{-}$. They are defined as follows through the susceptibility,

$$\chi \equiv \frac{\partial \bar{\phi}}{\partial J} \bigg|_{J=0} \sim \chi_{+} \tau^{-\gamma^{+}} \quad (\tau \sim 0, T > T_c), \quad (9)$$

$$\chi \equiv \frac{\partial \bar{\phi}}{\partial J} \bigg|_{J=0} \sim \chi_{-} \tau^{-\gamma^{-}} \quad (\tau \sim 0, T < T_c). \quad (10)$$

To calculate $\chi$, we relate $\chi$ to the curvature using the following identity derived from the definition of the effective potential,

$$\left. \frac{\partial \bar{\phi}}{\partial J} \right|_{J=0} = \left. \left( \frac{\partial^2 V}{\partial \bar{\phi}^2} \right)^{-1} \right|_{\bar{\phi} = \phi_c}. \quad (11)$$

We show $\left( \frac{\partial^2 V}{\partial \phi^2} \right)^{-1}|_{\bar{\phi} = \phi_c}$ as a function of temperature in fig. 8. We also plot $\log(\bar{\phi})$ against $\log(|\tau|)$ in fig. 3, we fit the data to linear functions and draw them in fig. 4. We determine $\gamma^{+/-}$ from the gradient of it and $\chi^{+}/\chi^{-}$ from the intercepts. We find $\gamma \equiv \gamma^{+} = \gamma^{-} = 1.37$, $\chi^{+}/\chi^{-} = 3.4$.
Figure 8: Curvature at minimum point $\frac{\partial^2 V}{\partial \phi^2}$ as temperature varies ($\lambda = 1$).

Figure 9: Plots of $\log(\frac{\partial^2 V}{\partial \phi^2}) - \log(|\tau|)$ ($\lambda = 1$). The data points are fit to linear functions. Using their gradients, $\gamma^+/\gamma^-$ and $\chi^+/\chi^-$ are determined.
Finally, we pay attention to the second derivative of the effective potential with respect to temperature, which is proportional to the specific heat $C$. The exponent $\alpha$ is defined as follows:\footnote{Though the amplitude ratio of the specific heat can also be defined, it is no determined because of the numerical reason.}

$$C \propto \frac{\partial^2 V}{\partial \tau^2} \propto \tau^{-\alpha} \quad (\tau \sim 0).$$

This derivative is shown in fig.10 as a function of temperature. We focus on its behaviour around $T_c$ in fig.11 and observe that it blows up there. One of the critical exponent $\alpha$ is determined using this. The result is $\alpha = 0.12$.

The results are summarized in table.1 and compared with results obtained by various methods. Discussion is presented in the next section.
Figure 11: Specific heat $C$ as a function of $\tau$ around the critical temperature. One can observe that it blows up around the critical temperature. One of the critical exponent $\alpha$ is determined from this.

4 Summary and discussion

The critical exponents and the amplitude ratio were determined using the auxiliary-mass method developed in ref. [27] by the improved numerical method in the appendix. The results are summarized in table 1. We found that $\lambda \phi^4$ theory shows second-order phase transition as it should be. Though the critical exponents calculated here do not satisfy the scaling relations, they satisfy inequalities of critical exponents. For example, the inequalities given by Griffiths [38],

$$\gamma^- \geq \beta(\delta - 1), \quad (13)$$

$$\gamma^+(\delta + 1) \geq (2 - \alpha)(\delta - 1) \quad (14)$$

are satisfied. In the following we compare our result with other's.

First, the results are compared with the values obtained by the perturbative finite-temperature field theory with daisy resummation. Since first-order phase transition is indicated at one-loop order [7, 28], the critical exponents can not be determined by the perturbation theory. At two-loop order, second-order phase transition is observed and the critical exponents are same as those
Table 1: Critical exponents and critical amplitude obtained from various methods. Since first-order phase transition is indicated, the critical exponents cannot be determined using finite-temperature field theory (F-T) within one-loop order. We note that there are many non-perturbative methods based on the renormalization group (R-G) idea which we do not refer here. The central values of them are shown. Values in parenthesis are determined using scaling relations.

| Method          | $\gamma$ | $\nu$ | $\beta$ | $\alpha$ | $\delta$ | $\eta$ | $\chi^+ / \chi^-$ |
|-----------------|----------|-------|---------|----------|----------|--------|------------------|
| F-T perturbation theory 1-loop | 1.37 | 0.385 | 0.12 | 4.0 | 3.4 |
| F-T perturbation theory 2-loop | 1.0 | 0.5 | 0.5 | 0.0 | 3.0 | 0.0 | 2.0 |
| R-G perturbation theory | 1.24 | 0.630 | 0.325 | 0.11 | 4.82 | 0.317 | 4.82 |
| R-G perturbation theory fixed dim. | 1.24 | 0.631 | 0.327 | 0.11 | 4.79 | 0.349 | 4.70 |
| R-G non-perturbative sharp cut off [19] | (1.38) | 0.690 | (0.345) | (-0.07) | (5.0) | 0.0 | * |
| R-G non-perturbative smooth cut off [17] $\partial^1$ | (1.32) | 0.660 | (0.33) | (0.02) | (5.0) | 0.00 | * |
| R-G non-perturbative smooth cut off [17] $\partial^2$ | (1.20) | 0.618 | (0.327) | (0.146) | (4.67) | (0.054) | * |
| R-G lattice Monte Carlo [37] | 1.24 | 0.629 | 0.324 | 0.113 | 4.83 | 0.027 |
| Landau approximation binary fluids | 1.236 | 0.625 | 0.325 | 0.112 | 4.3 |
| Landau approximation liquid-vapor | 1.24 | 0.625 | 0.316 | 0.107 | 5.0 |
| Landau approximation antiferromagnets | 1.25 | 0.64 | 0.328 | 0.112 | 4.9 |

by the Landau approximation \(^3\). In comparison with these values, the results obtained in the present paper are considerably good.

Second, they are compared with the values obtained by renormalization group and by lattice simulation, which agree with greatly. In comparison with these accurate values, our results are not very good. These errors are probably caused by the replacement \(^3\). Since this replacement is based on the neglect of momentum dependence in $\Pi$, we have to take into account the momentum dependence in order to improve our results \(^3\).

\(^3\) We used the two-loop order effective potential calculated in \([\text{ref.}]\). We determined the critical exponents from this both numerically and analytically.
As mentioned in Sec.1, the finite-temperature field theory is optimum theory in order to investigate phase transitions; it is based on statistical principle and can deal with both first-order and second-order phase transition. The perturbation theory, however, often breaks down and it prevent us from using the finite-temperature field theory. The -mass method enables the finite-temperature field theory to be used in various situations.

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A Numerical method

The numerical method, which we use to solve (5), is explained in this appendix. The partial differential equation (5) is written as follows,

\[ \frac{\partial V}{\partial m^2} = \frac{1}{2} \phi^2 + f(\frac{\partial^2 V}{\partial \phi^2}). \]  

(15)

Here, \( f(x) \) is the integral in (5). First, we make a lattice shown in fig.12. The partial differential equation (5) is, then, differenced as follows [30],

\[
\frac{V_{i,j+1} - V_{i,j}}{\Delta m^2} = \frac{1}{2} \phi_i^2 + f\left( \alpha \left( \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{(\Delta \phi)^2} \right) + (1 - \alpha) \left( \frac{V_{i,j} - 2V_{i,j-1} + V_{i,j-2}}{(\Delta \phi)^2} \right) \right). \]  

(16)

The parameter \( \alpha \) decides where the laplacian \( \frac{\partial^2 V}{\partial \phi^2} \) is evaluated. If \( \alpha = 0 \) is selected, the laplacian is evaluated at (a) in fig.12. The method with this selection is called the explicit method, which we used in [27]. This method is simple, because \( V_{x,j+1} \) is determined only by substituting \( V_{x,j} \) into the right hand side. It, however, suffers from a numerical instability, when smaller mesh \( \Delta \phi \) is chosen [30]; therefore, we could not make mesh small in [27]. If \( \alpha = 1 \) is selected, the laplacian is evaluated at (b) in fig.12. The method with this selection is called the implicit method, which does not suffer from the numerical instability at least if \( f(x) \) is a linear function [30] — as far as we know, when \( f(x) \) is not linear function like our case, not many things are known —. If \( \alpha = 1/2 \) is selected, the laplacian is evaluated at (c) in fig.12.
The method with this selection is called the Crank-Nicholson method, which also does not suffer from the numerical instability at least if \( f(x) \) is a linear function. What is more, the result converges more rapidly with decreasing \( \Delta m^2 \) using this method \cite{30}. Both the implicit and the Crank-Nicholson method, however, requires us to solve coupled non-linear equation (16); this prevents us from using established method in the case \( f(x) \propto x \).

We developed two methods in order to overcome this difficulty. First method is based on the Taylor expansion of \( f(x) \). The equation (16) is rewritten as follows,

\[
\frac{V_{i,j+1} - V_{i,j}}{\Delta m^2} = \frac{1}{2} \phi_t^2 + f \left( \frac{V_{i+1,j+1} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right) + \alpha \left( \frac{V_{i+1,j+1} - 2V_{i,j+1} + V_{i-1,j+1}}{(\Delta \phi)^2} - \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right).
\]

(17)

Since the quantity in the parenthesis behind \( \alpha \) is the variation of the laplacian per one step, it is small if \( \Delta m^2 \) is sufficiently small. We, then, expand \( f(x) \) around \( \frac{V_{i+1,j+1} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \).

\[
\frac{V_{i,j+1} - V_{i,j}}{\Delta m^2} = \frac{1}{2} \phi_t^2 + f \left( \frac{V_{i+1,j+1} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right).
\]

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\[ + \alpha \left( \frac{V_{i+1,j+1} - 2V_{i,j+1} + V_{i-1,j+1}}{(\Delta \phi)^2} - \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right) \]
\[ \times f' \left( \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right) + \text{higher order terms} \tag{18} \]

This coupled equation is linear with respect to \( V_{x,j+1} \) and can be solved easily [30].

The second method is based on an iteration. In order to solve equation (16), we iterate as follows until a solution is found,

\[ \frac{V_{i,j+1}^{n+1} - V_{i,j}}{\Delta m^2} = \frac{1}{2} \frac{\phi_i^2}{\phi_i^2} + f \left( \alpha \left( \frac{V_{i+1,j+1}^{n+1} - 2V_{i,j+1}^n + V_{i-1,j+1}^n}{(\Delta \phi)^2} \right) \right. \]
\[ \left. + (1 - \alpha) \left( \frac{V_{i+1,j}^{n+1} - 2V_{i,j}^n + V_{i-1,j}^n}{(\Delta \phi)^2} \right) \right) \tag{19} \]

Here, \( n \) is the number of the iteration. Note that we can not replace \( V_{i,j+1}^n \) with \( V_{i,j+1}^{n+1} \) unlike the Gauss-Seidel method, which is a powerful method if \( f(x) \) is a linear function [30]. Next, relaxation method is used in order to improve the convergence [30]. Since this procedure is identical with the linear case, we only write down the iteration equation without an explanation.

\[ \frac{V_{i,j}^{n+1} - V_{i,j}}{\Delta m^2} = \omega \left( \frac{1}{2} \phi_i^2 + f \left( \alpha \left( \frac{V_{i+1,j+1}^{n+1} - 2V_{i,j+1}^n + V_{i-1,j+1}^n}{(\Delta \phi)^2} \right) \right. \right. \]
\[ \left. \left. + (1 - \alpha) \left( \frac{V_{i+1,j}^{n+1} - 2V_{i,j}^n + V_{i-1,j}^n}{(\Delta \phi)^2} \right) \right) \right) \]
\[ +(1 - \omega)V_{i,j}^n. \tag{20} \]

Here, the relaxation parameter \( \omega \) is determined only by experience. The results by the two methods agree greatly. In the present paper, the latter method is used in order to determine the universal quantities.

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