THE FULL CURVATURE EFFECT EXPECTED IN EARLY X-RAY AFTERGLOW EMISSION FROM GAMMA-RAY BURSTS

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ABSTRACT

We explore the influence of the full curvature effect on the flux of the early X-ray afterglows of gamma-ray bursts (GRBs) in cases where the spectrum of the intrinsic emission is a power law. We find that the well-known $t^{-2+\beta}$ curve appears only when the intrinsic emission is extremely brief or the emission arises from exponential cooling. The timescale of this curve is independent of the Lorentz factor. The resulting light curve exhibits two phases if the intrinsic emission has a power-law spectrum and a temporal power-law profile of infinite duration. The first phase is a rapid decay in which the light curve is well described by the $t^{-2+\beta}$ curve. The second phase is a shallow decay in which the power-law index of the light curve is obviously smaller than in the first phase. The start of the shallow phase is strictly constrained by, and can in turn set a lower limit on, the radius of the fireball. In the case of power-law emission that lasts for only a limited time, there will be a third phase after the $t^{-2+\beta}$ curve and the shallow decay phase, which is much steeper than the shallow phase. As a sample application, we fit the *Swift* XRT data for GRB 050219A with our model and show that the curvature effect alone can roughly account for this burst. Although the fit parameters cannot be uniquely determined, because of the various choices in the fitting, a lower limit on the fireball radius of this burst can be estimated, which is $\sim 10^{14}$ cm.

Subject headings: gamma rays: bursts — gamma rays: theory — relativity

1. INTRODUCTION

The canonical gamma-ray burst (GRB) X-ray afterglow light curve, containing five components after the prompt emission phase, is a great finding from the *Swift* satellite (Chincarini et al. 2005; Nousek et al. 2006; O’Brien et al. 2006; Zhang et al. 2006; Zhang 2007). The first of the five components is the so-called steep decay phase, which generally extends to $\sim 10^2$–$10^3$ s with a temporal decay slope of typically $-3$ or much steeper (Vaughan et al. 2006; Cusumano et al. 2006; O’Brien et al. 2006).

A hint in this phase suggesting that we are seeing high-latitude emission from the fireball’s surface is that it is typically smoothly connected to the prompt emission phase (Tagliaferri et al. 2005; Barthelmy et al. 2005; Liang et al. 2006). Generally, the steep decay phase has been interpreted as being a consequence of the so-called curvature effect (Fenimore et al. 1996; Kumar & Panaitescu 2000; Dermer 2004; Dyks et al. 2005; Butler & Kocevski 2007a; Liang et al. 2006; Panaitescu et al. 2006; Zhang et al. 2006, 2007), which is a combination of the time delay and the Doppler shifting of the intrinsic spectrum, as well as other relevant factors, due to the expansion of the fireball (see Qin et al. 2006 for a detailed explanation). This effect has recently been intensively studied in the prompt gamma-ray phase in terms of the profile of the full light curve of pulses, the spectral lags, the power-law relation between pulse width and energy, and the evolution of the hardness ratio (Sari & Piran 1997; Qin 2002; Ryde & Petrosian 2002; Kocevski et al. 2003; Qin & Lu 2005; Shen et al. 2005; Lu et al. 2006; Peng et al. 2006; Qin et al. 2004, 2005, 2006).

As early as a decade ago, Fenimore et al. (1996) found that, as a result of the curvature effect, the light curve arising from the emission of an infinitely thin shell will be a power law of the observer’s time when the rest-frame photon number spectrum is a power law and the emission occurs within an infinitesimal time interval. In this case, the two power-law indices are related by $\alpha = 2 + \beta$, where $\alpha$ is the light-curve index and $\beta$ is the spectral index. Concerning the X-ray afterglow emission, Kumar & Panaitescu (2000) also found that, because of the curvature effect, the light curve of a shock-heated fireball shell radiating a power-law spectrum within the observational band (i.e., the X-ray band in early afterglow observations) is a power law of time as well and that the relation $\alpha = 2 + \beta$ holds in this situation.

As revealed by Figure 7 of Nousek et al. (2006), the relation $\alpha = 2 + \beta$ is in rough agreement with the data on the steep decay phase of some *Swift* bursts. However, this figure also shows that the real relationship between the two indices for some bursts significantly deviates from the $\alpha = 2 + \beta$ line. This might be due to an erroneous setting of time zero, which should be reset to the time when the central engine restarts (see Liang et al. 2006). In addition, subtracting the contribution of the underlying afterglow by more or less can lead to other values of the temporal index $\alpha$ (for a detailed explanation, see Zhang 2007).

The derivation of the relation $\alpha = 2 + \beta$ in previous papers has been based on the leading terms of the curvature effect. Does this relation still hold (or, in what situations will it hold) when the full curvature effect is considered? This question motivates our investigation here. The structure of the paper is as follows: In $\S$ 2, we present a general analysis of the full curvature effect for the case in which the intrinsic emission is a power law. In $\S$ 3, we discuss the light curves of power-law emission associated with several typical intrinsic temporal profiles. Presented in $\S$ 4 is an example of the application of our model. Conclusions are presented in the last section.
2. FIREBALL LIGHT CURVES FROM INTRINSIC EMISSION WITH A POWER-LAW SPECTRUM

An observation of the emission from an expanding fireball will be influenced by the time delays from different areas of the fireball’s surface, the variations of intensity due to the growth of the fireball radius, the variation of the time contraction factor, and shifting of the intrinsic spectrum associated with the angle to the line of sight. Taking all these factors into account, one may come to a full description of the so-called curvature effect (see also Qin et al. 2006 for a detailed explanation). Consider a constantly expanding fireball shell emitting within a proper-time interval $t_{0\min} \leq t \leq t_{0\max}$ and over an angle $\theta_{\min} \leq \theta \leq \theta_{\max}$, where $\theta$ is the angle to the line of sight. Assume that the energy range of the emission is unlimited. Following the same approach adopted in Qin (2002) and Qin et al. (2004), one can verify that the flux density expected to be measured by a distant observer at laboratory time $t_{\text{ob}}$ is

$$f_{\nu}(t_{\text{ob}}) = \frac{2\pi c^2 \int_{t_{0\min}}^{t_{0\max}} t_{\text{ob}}(t_{0}, t_{\nu}) (t_{0} - t_{0,c}) (t_{0} - t_{\nu}) \Gamma + D/c - (t_{\text{ob}} - t_{\nu}) \left[R_{c}/c + (t_{0} - t_{0,c}) \Gamma v/c \right]^2 dt_{0}}{D^2 \Gamma^2 \left[R_{c}/c - D/c - (t_{\text{ob}} - t_{\nu}) \left(v/c \right) \right]^2},$$

(1)

where $t_{0\min}$ and $t_{0\max}$ are determined by

$$t_{0\min} = \max \left\{ t_{0\min}, \frac{t_{\text{ob}} - t_{c} - D/c + (R_{c}/c) \cos \theta_{\max}}{1 - (v/c) \cos \theta_{\max}} \Gamma + t_{c} \right\},$$

(2)

$$t_{0\max} = \min \left\{ t_{0\max}, \frac{t_{\text{ob}} - t_{c} - D/c + (R_{c}/c) \cos \theta_{\min}}{1 - (v/c) \cos \theta_{\min}} \Gamma + t_{c} \right\},$$

(3)

and $\nu_{0}$ and $t_{0}$ are related by

$$\nu_{0} = \frac{R_{c}/c - [D/c - (t_{\text{ob}} - t_{c})] \left(v/c \right)}{R_{c}/c + (t_{0} - t_{0,c}) \Gamma v/c} \Gamma \nu.$$  

(4)

The observation time is confined to the range

$$\left[ 1 - (v/c) \cos \theta_{\min} \right] \left[ (t_{0\min} - t_{0,c}) \Gamma + t_{c} \right] + \left[ t_{c}(v/c) - R_{c}/c \right] \cos \theta_{\min} + D/c \leq t_{\text{ob}} \leq \left[ 1 - (v/c) \cos \theta_{\max} \right] \left[ (t_{0\max} - t_{0,c}) \Gamma + t_{c} \right] + \left[ t_{c}(v/c) - R_{c}/c \right] \cos \theta_{\max} + D/c.$$  

(5)

Outside this interval, no photons from the source are detectable by the observer.

A power-law spectrum is commonly observed in early X-ray afterglows, especially during the steep decay phase (e.g., Vaughan et al. 2006; Cusumano et al. 2006; O’Brien et al. 2006). In this paper, we focus only on the case of intrinsic emission that has a power-law spectrum, which is expected in the case of synchrotron emission produced by shocks and has generally been assumed in previous investigations (e.g., Fenimore et al. 1996; Sari et al. 1998; Kumar & Panaitescu 2000). Let the intensity of the intrinsic emission be $I_{\nu}(t_{0}, \nu_{0}) = I_{0}(t_{0}) \nu^{-\beta}$ (Kumar & Panaitescu 2000). One may find from equation (1) that

$$f_{\nu}(t_{\text{ob}}) = \frac{2\pi c^2 \nu^{-\beta} \int_{t_{0\min}}^{t_{0\max}} I_{\nu}(t_{0}) \left[R_{c}/c + (t_{0} - t_{0,c}) \Gamma v/c \right]^{2+\beta} \left[(t_{0} - t_{0,c}) \Gamma + D/c - (t_{\text{ob}} - t_{c}) \right] dt_{0}}{D^2 \left(\Gamma v/c \right)^{2+\beta} \left[t_{\text{ob}} - t_{c} + R_{c}/v - D/c \right]^{2+\beta}},$$

(6)

where equation (4) has been employed. Assigning

$$t \equiv t_{\text{ob}} - t_{c} + R_{c}/v - D/c,$$

(7)

one arrives at

$$f_{\nu}(t) = \frac{2\pi c^2 \nu^{-\beta}}{D^2 \left(\Gamma v/c \right)^{2+\beta}} \int_{t_{0\min}}^{t_{0\max}} I_{\nu}(t_{0}) \left[R_{c}/c + (t_{0} - t_{0,c}) \Gamma v/c \right]^{2+\beta} \left[(t_{0} - t_{0,c}) \Gamma + R_{c}/v - t \right] dt_{0},$$

(8)

with

$$t_{0\min} = \max \left\{ t_{0\min}, \frac{t - R_{c}/v + (R_{c}/c) \cos \theta_{\max}}{1 - (v/c) \cos \theta_{\max}} \Gamma + t_{c} \right\},$$

$$t_{0\max} = \min \left\{ t_{0\max}, \frac{t - R_{c}/v + (R_{c}/c) \cos \theta_{\min}}{1 - (v/c) \cos \theta_{\min}} \Gamma + t_{c} \right\},$$

(9)

$$\nu_{0} = \frac{R_{c}/v + (t_{0} - t_{0,c}) \Gamma}{R_{c}/v + (t_{0} - t_{0,c}) \Gamma} \nu,$$

(10)

and

$$\left[ 1 - (v/c) \cos \theta_{\min} \right] \left[ (t_{0\min} - t_{0,c}) \Gamma + t_{c} \right] + \left[ t_{c}(v/c) - R_{c}/c \right] \cos \theta_{\min} + R_{c}/v - t \leq t \leq \left[ 1 - (v/c) \cos \theta_{\max} \right] \left[ (t_{0\max} - t_{0,c}) \Gamma + t_{c} \right] + \left[ t_{c}(v/c) - R_{c}/c \right] \cos \theta_{\max} + R_{c}/v - t.$$  

(11)
The meaning of $t$ as defined by equation (7) can be seen by employing equation (8) of Qin et al. (2004) (where the quantity $t$ is now written as $t_{0,\text{min}}$). According to the latter equation, emission from a fireball radius $R_c = 0$ (which occurs at $t_0 = t_c$) corresponds to $t = 0$, and emission from an angle of $\theta = 0$ at any $R_c$ (occurring at $t_0 = t_c$) gives rise to $t = (R_c/c)(1 - \beta) \simeq (R_c/c)/2\Gamma^2$. The quantity $(R_c/c)/2\Gamma^2$ is nothing but the travel time of the fireball’s surface from the point of explosion to $R_c$, contracted by a factor of $1/(2\Gamma^2)$ since the area $\theta = 0$ moves toward the observer with Lorentz factor $\Gamma$. Thus, $t = 0$ is the moment when photons emitted at $R_c = 0$ reach the observer. Even for $t_0 = t_c$, one would have $t > 0$ if $R_c > 0$. The emission time $t_0 = t_c$ does not mean that photons are radiated at $R_c = 0$. Instead, it means that the photons are emitted from the surface of a fireball of radius $R_c$, which is measured at $t_c$ (see Appendix A of Qin 2002).

Note that if the range of the power law is limited, this will constrain the integration limits $t_{0,\text{min}}$ and $t_{0,\text{max}}$, which when then differ from equations (2) and (3) or equation (9) (see Qin 2002). In the following, we adopt Kumar & Panaitescu’s (2000) assumption: the intrinsic emission is a strict power law within the energy range corresponding to the observed energy channel. Thus, equations (2) and (3) (or eq. [9]) are applicable.

According to equation (9), $\int_{t_{0,\text{min}}}^{t_{0,\text{max}}} I_d(t_0)[R_c/c + (t_0 - t_{0,c})\Gamma/v/c]^{2+\beta}((t_0 - t_{0,c})\Gamma + R_c/v - t)dt_0$ is a function only of $t$. Let

$$h(t) \equiv \int_{t_{0,\text{min}}}^{t_{0,\text{max}}} I_d(t_0)[R_c/c + (t_0 - t_{0,c})\Gamma/v/c]^{2+\beta}((t_0 - t_{0,c})\Gamma + R_c/v - t)dt_0.$$  

Equation (8) can then be written as

$$f_\nu(t) = \frac{2\pi c^2}{D^2(\Gamma v/c)^{2+\beta}} h(t)^{(2+\beta)} \nu^{-\beta}.$$  

This shows that in the case that the power-law relation $I_d(t_0) = I_d(0)\nu_0^{-\beta}$ holds for the intrinsic radiation intensity within the energy range corresponding to the observed energy channel (due to Doppler shifting), a power-law spectrum will also obtain within the observed channel, and its index will be exactly the same as that of the intrinsic spectrum.

Taking the factor $h(t)$ as a constant, equation (13) leads to

$$f_\nu(t) \propto t^{-(2+\beta)} \nu^{-\beta}.$$  

This is the well-known flux density associated with the curvature effect, which reveals the relation between the temporal and spectral power-law indices: $\alpha = 2 + \beta$, where $\alpha$ is the temporal index [e.g., when assuming $f_\nu(t) \propto t^{-\alpha}$] (see Fenimore et al. 1996; Kumar & Panaitescu 2000).

3. TIME FACTORS OTHER THAN A POWER LAW

Let us consider intrinsic emission that is a $\delta$-function of time. In this situation, effects arising from the duration of real intrinsic emission are omitted, and therefore those coming only from the expansion of the fireball’s surface will be clearly seen.

Without loss of generality, we assume that $I_d(t_0) = I_d(0)(t_0 - t_{0,c})$ and take $t_{0,\text{min}} = 0$ and $t_{0,\text{max}} = \pi/2$ (this corresponds to the half of the fireball’s surface facing us and is adopted throughout this paper). One then finds from equation (11) that

$$R_c/v - R_c/c \leq t \leq R_c/v.$$  

Within this observation time, the integral equation (12) becomes

$$h(t) = I_d(R_c/c)^{2+\beta}(R_c/v - t).$$  

Therefore,

$$f_\nu(t) = \frac{2\pi c^2 I_d(R_c/c)^{2+\beta} \left(\frac{R_c}{v} - t\right)^{-(2+\beta)} \nu^{-\beta}}{D^2}.$$  

When $t$ is outside the time range given by equation (15), $h(t) = 0$ and thus $f_\nu(t) = 0$.

Based on Qin (2002) and Qin et al. (2004), one can check that the term $[(t_0 - t_{0,c})\Gamma + D/c - (t_{0b} - t_c)]$ in equation (1), or the term $[(t_0 - t_{0,c})\Gamma + R_c/v - t]$ in equation (12), comes from the projection of the infinitesimal fireball surface area within the angle concerned, say, $\theta$, to the distant observer, that is, $\cos \theta$. This term becomes $R_c/v - t$ for $\delta$-function temporal radiation under the new time definition in equation (7). Longer observation times correspond to larger line-of-sight angles, and thus the cos $\theta$ term becomes smaller.

Note that when one considers only a very small cone pointing toward the observer, this term can be ignored, since it varies very mildly within a range of angles close to $\theta = 0$. However, what we are discussing here is the steep decay phase of an early X-ray afterglow, which has generally been assumed to arise from high-latitude emission. In this situation, the variation of this term will be significant.

According to equation (10), another noticeable term, $[R_c/c + (t_0 - t_{0,c})\Gamma/v/c]$ in equations (1) and (12), reflects a shift in frequency. Equation (10) suggests that the flux observed at frequency $\nu$ and time $t$ is contributed by rest-frame photons of frequency $\nu_0$ emitted at a proper time $t_0$ (note that the flux will also be contributed by rest-frame photons of other frequencies $\nu_0'$ emitted at other proper times $t_0'$ so long as they satisfy eq. [10], and that the value of the flux is determined by all these possible photons). The quantity $[R_c/c + (t_0 - t_{0,c})\Gamma/v/c]$ represents the shift in frequency when the observation time $t$ is fixed. This term is independent of the observation time, but because of its
coupling with \((t_0 - t_{0,c})\Gamma\) in the term \([t_0 - t_{0,c})\Gamma + D/c - (t_{thb} - t_c)\), it may also affect the light curve. Similarly, the term \(I_0(t_0)\) might also play a role due to its coupling with \((t_0 - t_{0,c})\Gamma\) in the term \([\frac{t_0 - t_{0,c})\Gamma + D/c - (t_{thb} - t_c)}{C0}\).

The last factor affecting the light-curve profiles is the range of integration in equation (12). This might differ from time to time, since the surface area of the fireball sending photons to the observer, which are observed at time \(t\), may change over time. As shown in equation (9), both \(t_{0,min}\) and \(t_{0,max}\) are determined by the observation time \(t\). A time-dependent integration range in equation (12) probably could make the decay phase of the light curve deviate from a strict power law.

In the following, we show how these time factors affect the decay phase of light curves that arise from intrinsic emission with a power-law spectrum.

### 3.1. Delta-Function Temporal Profile

We first consider the case in which the temporal profile of the intrinsic emission is a \(\delta\)-function of time. The light curve that arises from this emission is \(I_v,\delta[1 - t/(R_v/v)]^{-\Gamma - (2+\beta)}\), according to equation (17). We take \(I_v,\delta = 1\) and \(\beta = 1\) to plot the curves. We consider fireballs with \(R_v/v = \frac{1}{2} \times 10^2\) s and \(R_v/v = \frac{1}{4} \times 10^3\) s, which correspond to typical radii of \(R_e \approx 10^{15}\) cm and \(R_e \approx 10^{13}\) cm, respectively (see Ryde & Petrosian 2002).

Shown in Figure 1 are the \(I_v,\delta[1 - t/(R_v/v)]^{-\Gamma - (2+\beta)}\) and \(I_v,\delta t^{-1-2+\beta}\) light curves associated with these two radii. We find that in the case of very brief intrinsic emission, although the \(R_v/v - t\) term in equation (17) plays a role in the decay phase, the temporal curve closely follows a power law over most of the phase. Following the power-law curve is a tail that drops off speedily because of the effect of the \(R_v/v - t\) term. A remarkable feature revealed by this figure is that the power-law decay phase is solely determined by, and very sensitive to, the radius of the fireball and that the power-law range itself can tell us how large the fireball radius is. For example, a power-law range found to extend to 100 s calls for a fireball larger than \(10^{13}\) cm, and one being found to extend to 10,000 s implies a radius over \(10^{15}\) cm. This conclusion is surprisingly independent of the Lorentz factor, and it holds only when the intrinsic emission is extremely short, so that its temporal profile can be treated as a \(\delta\)-function. As illustrated by Figure 1, a strict \(t^{-1+\beta}\) curve followed by a speedily falling off tail is a feature of extremely short intrinsic emission. When this feature is observed, one can estimate the fireball radius merely from the timescale of the power-law decay phase, so long as the spectrum is a power law and the relation \(\alpha = 2 + \beta\) holds.

### 3.2. Exponential Temporal Profile

Second, let us consider intrinsic emission with a temporal profile that is an exponential of time and check whether the resulting light curve differs from that due to \(\delta\)-function emission. We ignore the contribution from the rise phase of the emission of shocks (this corresponds to the situation in which the rise time is extremely short). The intrinsic, decaying light curve is assumed to have the form \(I_0(t_0) = I_0 \exp \left[-(t_0 - t_{0,c})/\sigma_d\right]\) for \(t_0 > t_{0,c}\). Equation (13) now becomes

\[
f_v(t) = I_v h_v(t) t^{-\Gamma - (2+\beta)} v^{-\beta} \tag{18}
\]

with

\[
h_v(t) = \int_{t_{0,min}}^{t_{0,max}} \frac{[1 + (t_0 - t_{0,c})\Gamma v/R_e]^{2+\beta} [t_0 - t_{0,c})\Gamma v/R_v + 1 - t v/R_v] d\sigma_d}{\exp \left\{\frac{(t_0 - t_{0,c})/\sigma_d}{c}\right\}} \quad \text{for} \quad t_0 > t_{0,c}, \tag{19}
\]

\[
t_{0,min} = \max \left\{ t_{0,c}, \frac{t - R_c/v}{\Gamma} + t_{0,c} \right\}, \quad t_{0,max} = \frac{t - R_v/v + R_c/c}{(1 - v/c)\Gamma} + t_{0,c}, \tag{20}
\]

where \(I_v\) is a constant and the observation time \(t\) is confined by

\[
R_c/v - R_v/c \leq t. \tag{21}
\]
Without loss of generality, we take \( t_0 = 0, c = 0 \). Equations (19)–(20) then become:

\[
\begin{align*}
he(t) &= \int_{t_{0,\text{min}}}^{t_{0,\text{max}}} \left( 1 + t_0 \Gamma v/R_c \right)^{2+\beta} (t_0 \Gamma v/R_c + 1 - tv/R_c) dt_0 \quad (t_0 > 0), \\
\tilde{t}_{0,\text{min}} &= \max \left\{ 0, \frac{t - R_c/v}{\Gamma} \right\}, \quad \tilde{t}_{0,\text{max}} = \frac{t - R_c/v + R_c/c}{(1 - v/c)\Gamma}.
\end{align*}
\]

Here we take \( L_0 = 1 \) and \( \beta = 1 \) and adopt \( R_c/v = \frac{1}{2} \times 10^5 \) s to plot the light curves. For the Lorentz factor, we consider \( \Gamma = 10 \) and \( \Gamma = 100 \). Shown in Figure 2 are the light curves plotted for different values of the width of the exponential function, \( \sigma_d (0.1, 1, 10, \text{and } 100 \text{ s}) \). The light curves are quite similar to those arising from the \( \delta \)-function intrinsic emission (see Fig. 1, where the feature of a curve followed by a speedily falling off tail is observed). Because of the contribution of the exponential decay in the intrinsic emission, the time range of the light curves is slightly larger than in the case of \( \delta \)-function intrinsic emission (this can be observed when the width is large enough; see Fig. 2, bottom right). This is understandable, since beyond \( \sigma_d \) the emission of the exponential function dies away rapidly and therefore its contribution can be ignored.

In the case that both the width of the exponential function and the Lorentz factor of the fireball are large, the resulting light curves obviously will deviate from those arising from \( \delta \)-function emission in the domain of the falling-off tail, where the slope of the tail of the former becomes obviously mild (see Fig. 2, bottom right). Aside from this, no other characteristics distinguish between the two kinds of light curve.

### 3.3. Power-Law Temporal Profile

Third, we check whether an observed light curve arising from emission with a power-law spectrum has something to do with the intrinsic decay behavior when the decay curve is a power law of time. Here we also ignore the contribution from the rise phase of the emission from shocks and then consider only emission with an intrinsic power-law decay (this will occur when the cooling follows a power law).

#### 3.3.1. Infinite Power-Law Decay Time

Assuming that the power-law decay time is infinite, the intrinsic decaying light curve is taken to be \( I_0(t_0) = \left[ (t_0 - t_{0,0})/(t_{0,0} - t_{0,c}) \right]^{-\alpha_0} \) for \( t_0 > t_{0,0} \). Here we take \( I_0(t_0) \) as a constant denoting the time when the power-law decay emission begins. In this case, equation (13) becomes

\[
f_{\nu}(t) = I_0 h_{\nu}(t) t^{-(2+\beta)\nu^{-\beta}}
\]
with

$$h_p(t) = \int_{t_0,\min}^{t_0,\max} \left[ 1 + \frac{t - t_0}{t_0 - t_0, c} \right]^{\Gamma v/R_c} d\tilde{t}_0$$

$$\tilde{t}_0,\min = \max \left\{ t_0,0, \frac{t - R_c/v}{\Gamma} + t_0, c \right\}, \quad \tilde{t}_0,\max = \frac{t - R_c/v + R_c/c}{(1 - v/c)\Gamma} + t_0, c,$$

where $I_P$ is a constant and the observation time $t$ is limited to

$$(t_0,0 - t_0, c)(1 - v/c)\Gamma - R_c/c + R_c/v \leq t.$$

Without loss of generality, we take $t_0, c = 0$. Equations (25)–(27) then become

$$h_p(t) = \int_{t_0,\min}^{t_0,\max} \left( 1 + \frac{t_0}{R_c} \right)^{\Gamma v - \beta} \left( \frac{t_0}{t_0,0} \right)^{-\alpha_0} dt_0 \quad (t_0 > t_0,0),$$

$$\tilde{t}_0,\min = \frac{t - R_c/v + R_c/c}{(1 - v/c)\Gamma}, \quad \left( 1 - \frac{v}{c} \right) \Gamma t_0,0 - \frac{R_c}{c} + \frac{R_c}{v} \leq t.$$

Here we take $I_Pv^{-\beta} = 1$ and $\beta = 1$ to plot the light curves. For the Lorentz factor, we adopt $\Gamma = 10$ and $\Gamma = 100$. We consider two typical values of the fireball radius, $R_c = 10^{15}$ cm and $R_c = 10^{13}$ cm. For the intrinsic temporal power-law index $\alpha_0$, we take values of 2, 2.5, 3, 4, and 5, and for the times when the power-law decay emission begins, we take $t_0,0 = 0.01, 0.1, 1$ s.

The corresponding light curves are displayed in Figure 3. Because of the contribution from $h_p(t)$, some new features are observed. There exist two kinds of light curve: (1) a $t^{-(2+\beta)}$-curve followed by a shallow decay curve with an index obviously smaller than $2 + \beta$ (type I) and (2) a $t^{-(2+\beta)}$-curve followed by a very steep decay phase (shown as a “cutoff” curve) and then a shallow decay curve with index smaller than $2 + \beta$ (type II). The type II curves tend to appear in cases where the intrinsic temporal power-law index is large, the Lorentz factor is small, and the onset of the intrinsic temporal power law is early (compare the left panels of Figs. 3a, 3b, and 3c, or the right panels of these figures). The very steep decay curve appears very close to the time marked by that in the light curve arising from $\delta$-function emission (Fig. 3, gray lines) (in fact, relative to the latter, the former shifts to slightly larger times). This means that the position in time of the very steep decay phase of the type II light curves is mainly determined by the radius of the fireball, which can serve as an indicator of the latter (see also the discussion in the two previous subsections). For the light curves of type I, the start of the shallow decay phase can appear from very early on to around 300 s for a fireball with radius $R_c = 10^{13}$ cm, depending on the intrinsic temporal power-law index $\alpha_0$, the Lorentz factor $\Gamma$, and the onset time $t_0,0$ of the intrinsic temporal power law (see Figs. 3a–3c, left). The smaller the values of $\alpha_0$, $\Gamma$, and $t_0,0$, the larger the timescale of the start of the shallow decay phase. For a fireball with radius $R_c = 10^{15}$ cm, the conclusions drawn from the type I light curves remain the same, except that the maximum of the start time of the shallow decay phase can be as high as 3000 s. For both types I and II, the slope of the shallow decay curve increases with increasing $\alpha_0$.

As revealed by the bottom left panel of Figure 3c, as a special case of type I, some light curves appear to be single power laws, with indices significantly smaller than $2 + \beta$. These are in fact the shallow decay phase of the corresponding light curves. The onset of this phase shifts to much smaller timescales as a result of the larger values of the Lorentz factor and the onset time $t_0,0$ of the intrinsic temporal power-law index for a given value of the fireball radius.

### 3.3.2. Limited Power-Law Decay Time

One might note that there is no upper limit on the intrinsic power-law decay emission considered above. Let us set such a limit and then check to see whether it can give rise to other noticeable features in the observed light curves. The intrinsic decaying light curve is assumed to be $I_0(t_0) = \left[ (t_0 - t_0,0)/(t_0,0 - t_0, c) \right]^{-\alpha_0}$ for $t_0,0 < t_0 < t_0,\max$. Also, we take $t_0, c = 0$. In this situation equations (24) and (28) hold, while equation (29) is replaced by

$$t_0,\max = \min \left\{ t_0,\max, \frac{t - R_c/v + R_c/c}{(1 - v/c)\Gamma} \right\}, \quad \left( 1 - \frac{v}{c} \right) \Gamma t_0,0 - \frac{R_c}{c} + \frac{R_c}{v} \leq t \leq t_0,\max \Gamma + \frac{R_c}{v}.$$

The parameters adopted in producing Figure 3 are also adopted here to create the light curves. Among the cases studied in Figure 3, we consider only the following four: $\Gamma = 10$ and $t_0,0 = 0.01$ s (see Fig. 3a, top); $\Gamma = 10$ and $t_0,0 = 0.1$ s (Fig. 3b, top); $\Gamma = 100$ and $t_0,0 = 0.1$ s (Fig. 3b, bottom); and $\Gamma = 100$ and $t_0,0 = 1$ s (Fig. 3c, bottom). For the values of the new parameter, we take $t_0,\max = t_0,0 + 0.01 R_c/c$, and $t_0,\max = t_0,0 + 100 R_c/c$, and $t_0,\max = t_0,0 + 100 R_c/c$. The corresponding light curves are displayed in Figures 4–7, which correspond to the four sets of $\Gamma$ and $t_0,0$, respectively.

The top panels of these figures show that when the power-law emission is as short as 0.01 times the typical timescale of the fireball radius (say, when $\Delta t_0,0 = 0.01 R_c/c$ and the Lorentz factor is not so large (say, not larger than 100), the light curves are similar to those arising from the $\delta$-function emission. This suggests that in the framework of the curvature effect, the light-curve characteristics of emission with timescales as short as 0.01 times $R_c/c$ and Lorentz factors not larger than 100 are hard to distinguish from that of a $\delta$-function. (This is in agreement with what is shown in Fig. 2.)
Fig. 3.—Light curves arising from intrinsic emission with a temporal profile that is a power-law function of time with decay time of infinity (see eq. [24]), plotted for (a) $t_0 = 0.01 \text{s}$, (b) $t_0 = 0.1 \text{s}$, and (c) $t_0 = 1 \text{s}$, where $h(t)$ is given by eq. (28). The top and bottom panels in (a)–(c) correspond to $\Gamma = 10$ and $\Gamma = 100$, respectively, and the left and right panels correspond to $R_c = 10^{13} \text{ cm}$ and $R_c = 10^{15} \text{ cm}$. The black lines indicate five different intrinsic temporal power-law indices: dot-dashed line, $\alpha_0 = 2$ ($\alpha_0 = 1 + \beta$); double-dot–dashed line, $\alpha_0 = 2.5$ ($\alpha_0 = 1.5 + \beta$); solid line, $\alpha_0 = 3$ ($\alpha_0 = 2 + \beta$); dashed line, $\alpha_0 = 4$ ($\alpha_0 = 3 + \beta$); dotted line, $\alpha_0 = 5$ ($\alpha_0 = 4 + \beta$). For the sake of comparison, the light curves from Fig. 1 are also plotted (gray lines).
Fig. 4.—Light curves arising from intrinsic emission with a temporal profile that is a power-law function of time and with limited duration, plotted for $\Gamma = 10$ and $t_{0,0} = 0.01 \text{ s}$. Here we consider two values of the fireball radius and three timescales for the duration of the power-law emission (see the key in each panel). The equations are the same as adopted in Fig. 3, except that we used eq. (30) to replace eq. (29). The line styles are the same as in Fig. 3.

Fig. 5.—Same as Fig. 4, but for $\Gamma = 10$ and $t_{0,0} = 0.1 \text{ s}$.
The bottom panels arise from longer duration power-law emission ($\Delta t_{0,0} = 100R_c/c$). Some new features appear. A remarkable one is a light curve with a power-law decay curve followed by a shallow phase and then a steeper power-law phase (type III). Connecting the two latter phases is a remarkable time break (see the upper three black lines in each of the bottom panels of Figs 4–7). This tends to happen when the intrinsic temporal power-law index is relatively small. Otherwise, this kind of curve disappears (e.g., the two lower black lines in each bottom panel).

When the duration of the power-law emission is not so large and not so small (say, $\Delta t_{0,0} = R_c/c$), other forms of light curves are observed (see the middle panels of Figs. 4–7). In this situation, when the Lorentz factor is large enough (say, $\Gamma = 100$), light curves of type III with shorter shallow phases appear (Figs. 6 and 7, middle). This is expectable, since in the framework of the curvature effect the profile of a light curve depends only on the ratio between the observational timescale and the corresponding fireball radius timescale $R_c/c$ (see Qin et al. 2004), and because of the effect of time contraction (note that the face-on part of the fireball surface moves toward us when the fireball expands), a certain observational timescale corresponds to a longer comoving timescale for a larger Lorentz factor.

4. EXAMPLE APPLICATION

In our analysis above, we considered only simple power-law emission, for which the power-law index $\beta$ is assumed to be constant. The model leads us to expect that the observed spectrum will be a constant power law with the same index and the rapid decay light curve will be the well-known $t^{-(2+\beta)}$ curve. This constrains our application, since the spectra of many X-ray afterglows of GRBs are found to vary with time and the corresponding light curves are found not to follow the $t^{-(2+\beta)}$ curve (see Zhang et al. 2007). Instead of a power law, many light curves are bent. To apply our model, one must find a burst with a constant spectral index and a light curve that follows (or approximately follows) the $t^{-(2+\beta)}$ curve in its X-ray afterglow.

After checking the data provided at the UNLV GRB Group’s Web site (up to 2008 March 25), we find that GRB 050219A might fit our simple model. The data show that the spectral index does not vary with time, and its mean is $\beta = 0.907 \pm 0.051$. In addition, the first decay curve of the burst follows a power law, with an index of approximately $2 + \beta$. This phase is followed by a shallow one that starts at about 500 s. Comparing this light curve with those presented in Figure 3, we guess that, if it is due to the curvature effect, the fireball radius must be larger than $R_c = 10^{15}$ cm; otherwise, the start time of the shallow phase would be too small to satisfy the data (see Figs. 3a–3c, left). If the radius is $R_c = 10^{15}$ cm, then the Lorentz factor must be larger than 10; otherwise, the start time of the shallow phase would be too large (see Figs. 3a–3c, top right). As illustrated in Figure 3, there are four factors that affect the start time of the shallow phase: the fireball radius $R_c$, the Lorentz factor $\Gamma$, the intrinsic temporal power-law emission index $\alpha_0$, and the start time of the intrinsic temporal power-law emission $t_{0,0}$.

Fig. 6.—Same as Fig. 4, but for $\Gamma = 100$ and $t_{0,0} = 0.1$ s.

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3 See also the page maintained by the GRB Group at the University of Nevada, Las Vegas, http://grb.physics.unlv.edu.
There are 75 data points in the Swift X-Ray Telescope (XRT) light curve of GRB 050219A available at the UNLV group’s site. For the example fitting, we ignore the three data points at the largest times, since the gap between them and the majority of the data set is too large and they are not covered by the constant spectral index domain (as a result, one cannot tell if the spectral index in the corresponding interval is still constant). With the remaining 72 data points, we need only apply the equations adopted in § 3.3.1 for the case of a power-law temporal profile of the intrinsic emission with a decay time of infinity. The equations adopted in producing Figure 3 are employed to fit the data set, where the term $I_p \nu^{-\beta}$, which dominates the magnitude of the theoretical curve, is to be determined by the fit.

Since both the fireball radius and the Lorentz factor are sensitive to the value of the start time of the shallow phase, we deal with them one by one. We first fix the Lorentz factor at $\Gamma = 100$ and allow $R_c$ and $R_c/v$ to vary, since not only should the start time of the shallow phase be met, but the power-law index of the shallow phase should also be accounted for. In addition, we take $t_{\phi,0} = 1$ s, since $t_{\phi,0}$ is less sensitive to the start time of the shallow phase (see Figs. 3a–3c, right). The best-fitting curve is shown in Figure 8. We find that the XRT data for GRB 050219A can be roughly accounted for by a power-law temporal emission from an expanding fireball surface. Note that the corresponding fit parameters are not important, since other possibilities exist (see below).

Next, we fix the fireball radius to $R_c = 10^{15}$ cm (in fact, we take $R_c/v = 1 \times 10^5$ s, which corresponds to $R_c \approx 10^{15}$ cm) and allow $\Gamma$ and $\alpha_0$ to vary. Also, we take $t_{\phi,0} = 1$ s. The best fit is displayed in Figure 9, which illustrates that the result of the fit with a fixed fireball radius is difficult to distinguish from that with a fixed Lorentz factor. Therefore, the resulting fit parameters are not important in this stage of investigation.

There is a third choice: one could fix $\alpha_0$ and allow $R_c$ and $\Gamma$ to vary. We guess that this might yield a similar result.

5. DISCUSSION AND CONCLUSIONS

We have investigated in this paper how intrinsic emission with a power-law spectrum $I_{\nu,0}(t_{\phi}, \nu_0) = I_0(t_{\phi})\nu_0^{-\beta}$ from an expanding fireball surface gives rise to an observed flux density when the full curvature effect is considered. We find that if the spectrum of the intrinsic radiation follows a power law within the energy range corresponding to the observed energy channel due to the Doppler shifting, the resulting spectrum will be a power law as well, and the index will be exactly the same as that in the intrinsic spectrum, regardless of the real form of the temporal profile of the intrinsic emission. Accompanying the power-law spectrum, of index $\beta$, is the power-law light curve with index $2 + \beta$ expected from the curvature effect, which was known previously (see Fenimore et al. 1996; Kumar & Panaiteescu 2000). Such a light curve could be observed if the intrinsic emission is extremely short or if the emission arises from exponential cooling.

In particular, we considered a power-law cooling emission in the comoving frame (for which the intrinsic temporal profile is a power law). We find that if the power-law decay time is infinite, thanks to the contribution of the power-law cooling in the comoving frame, the observed light curve as influenced by the full curvature effect contains two phases: one is a rapid decay in which the light curve closely
follows the well-known $t^{-(2+\beta)}$ curve, and the other is a shallow decay phase in which the light curve is obviously shallower than in the rapid phase. If the power-law decay time is instead limited, there are several kinds of light curve; one of these is remarkable in that it contains three power-law phases (see Figs. 4–7): the first is rapid, with an index equal to or larger than that of the $t^{-(2+\beta)}$ curve; the second is a shallow decay with index obviously smaller than that in the first phase; and the third is a rapid decay with index equal to or less than that of the first phase. It may be possible that some GRBs containing such features in their afterglow light curves are due to expanding fireballs or face-on, uniform jets (see, e.g., Qin et al. 2004) emitting with a power-law spectrum and a power-law cooling (either infinite or time-limited). In the view of a comoving observer, the dynamic process of the merger of shells would be somewhat similar to what occurs in external shocks (the main difference being that in the case of inner shocks, a comoving observer observes only a limited volume of the medium, for which the density would evolve with time because of the enhancement of the fireball’s surface). Based on this argument, we suspect that the intrinsic emission of some of those bursts possessing early X-ray afterglows with a rapid decay phase soon followed by a shallow decay phase and then a rapid decay phase might be somewhat similar to the well-known, standard forward-shock model (Sari et al. 1998; Granot et al. 1999), while for some bursts with a rapid phase followed by a shallow phase in their late X-ray afterglows the emission might be that of the standard forward-shock model, influenced by the curvature effect. Necessary conditions for perceiving this mechanism include (1) a spectral index that is constant during the period concerned and (2) a temporal index in the first phase that is equal to or larger than that of the $t^{-(2+\beta)}$ curve.

As a sample application, we employed the XRT data for GRB 050219A to perform a fit, since the spectral index $\beta$ of this burst does not vary with time and the first decay phase of its light curve follows a power law with index of approximately $2 + \beta$. The result shows that the XRT data of this burst can be roughly accounted for by power-law temporal emission from an expanding fireball surface. Since there exist various possibilities, the parameters obtained from the fit are not unique. To fix these parameters, we need other independent estimates. According to the analysis above, a reliable value of the fireball radius can be obtained if one observes a “cutoff” feature following the $t^{-(2+\beta)}$ curve in the case of a constant spectral index $\beta$. In any event, the start time of the shallow phase can place a limit on

![Fig. 8.—XRT light curve of GRB 050219A. Eqs. (24) and (28)–(29), which describe light curves arising from intrinsic emission with a temporal profile that is a power-law function of time with decay time of infinity, were employed to fit the data, where we take $\Gamma = 100$. The solid line is the best fit. The corresponding fit parameters are $R_c = 1.05 \times 10^{16}$ cm, $\alpha_0 = 2.05$, and $\nu \Gamma^{\alpha_0} = 9.56 \times 10^{-4}$ (see eq. [24]). The reduced $\chi^2$ of the fit is $\chi^2_{\text{red}} = 1.39$.

![Fig. 9.—Another fit to the XRT light curve of GRB 050219A, where we have taken $R_c/\nu = 3 \times 10^7$ s. The equations adopted for this fit are the same as used in Fig. 8. The black line represents the best fit, and the gray line is the best fit from Fig. 8. The corresponding fit parameters are $\Gamma = 25.1$, $\alpha_0 = 1.98$, and $\nu \Gamma^{\alpha_0} = 8.88 \times 10^{-4}$ (see eq. [24]). The reduced $\chi^2$ of the fit is $\chi^2_{\text{red}} = 1.48$.](image-url)
the fireball’s radius (see Figs. 3–7). For GRB 050219A, if the XRT data are indeed due to the curvature effect, the radius corresponding to this emission must be larger than $R_c = 10^{13}$ cm. We have checked that taking $R_c/v = 1.4 \times 10^4$ s (which corresponds to $R_c \approx 10^{14}$ cm), $\Gamma = 8$, and $\alpha_0 = 2.157$ can also roughly account for the data. As the Lorentz factor is so small in this situation, we conclude that if the X-ray afterglow of GRB 050219A does arise from the emission of an expanding fireball surface, the radius of the fireball cannot be much less than $R_c = 10^{14}$ cm, because otherwise $\Gamma$ would be too small to be regarded as representing relativistic motion.

Why does a shallow phase emerge as a consequence of the curvature effect? We guess that while the first phase is dominated by geometric effects and therefore obeys the $t^{-2+0}$ curve, in the shallow phase the intrinsic emission dominates the observed light curve. Note $\alpha_0 = 2$ curves in Figure 3 (dot-dashed lines)—the shallow phase of these lines is parallel to the time axis. As the radius grows linearly with time when a constant Lorentz factor is assumed (see Qin 2002), the emission from a fireball surface in a certain solid angle increases as the square of time (the area of the surface is proportional to $R^2$). This in turn makes the total emission $I_0 \propto t^{-2}$ from the surface become constant. When the intrinsic emission overcomes the geometric effect in the shallow phase, one cannot expect a $t^{-\alpha_0}$ light curve; instead, we expect the light curve to follow $t^{-(\alpha_0-2)}$ (see Fig. 3).

It is known that a $\delta$-function intensity approximates the process of an extremely short emission. This will occur when the corresponding fireball shells are very thin and the cooling time is relatively short compared with the curvature timescale (for the timescale of the curvature effect, see Kocevski et al. 2003; Qin & Lu 2005). Two light-curve characteristics are associated with quasi-$\delta$-function emission. The first is a strict power-law decay curve with index $2+\beta$. The second is the limited time range for this curve. If the cooling time is not so short but is an exponential, then these characteristics are also expected (see Fig. 2). Note that an exponential cooling does not extend the $t^{-2+0}$ curve to a much larger timescale if the cooling itself is not very strong (say, in the case of $R_c = 10^{15}$ cm, $\sigma_d < 100$ s; see Fig. 2). Thus, one can estimate the fireball radius in bursts that possess these characteristics (note that the timescale of the $t^{-2+0}$ curve is independent of the Lorentz factor; see eqs. [15] and [17]). For candidate bursts of this kind, we propose fitting the spectrum with $\nu^{-\beta}$ and the light curve with $[T_R - (t - T_0)]/[t - (t - T_0) - (2+0)]$, where both $T_R$ and $T_0$ are free parameters. When the fitting is good enough, we can say that the intrinsic fireball emission is likely very short or the cooling is exponential and that the corresponding fireball radius is $R_c \approx c T_R$. When the expansion of the fireball is relativistic, we get $R_c \approx c T_R$. Therefore, using this method one obtains at least an upper limit on the fireball radius as long as the intrinsic emission is extremely short, or the cooling is exponential, and the intrinsic spectrum is a power law.

In the case of intrinsic temporal power-law emission, when the temporal index is large enough ($\alpha_0 > 2+\beta$), there should be a “cutoff” curve located exactly at the same time position of the speedily falling off tail in the light curve of a $\delta$-function emission. This feature can be used to estimate the fireball radius as well. Several bursts presented by Zhang et al. (2007) seem to possess this “cutoff” feature: GRBs 050724, 060211A, 060218, 060427, 060614, 060729, and 060814. If the interpretation proposed here can be applied, their radii would range from $10^{13}$ to $10^{15}$ cm. There is at least one reason that prevents us from reaching such a conclusion. The spectra of these bursts happen to vary quite significantly within the light curves associated with this feature. This conflicts with what we have assumed in this paper (a constant intrinsic spectrum). We thus appeal for further investigation of this issue, taking into account variations in the intrinsic spectrum, which might tell us whether the “cutoff” remains and/or its properties are maintained.

As seen in the literature, many Swift bursts are found to possess a bent light curve instead of a strict power-law one in their early X-ray afterglows (see, e.g., Chincarini et al. 2005; Liang et al. 2006; Nousek et al. 2006; O’Brien et al. 2006). In our analysis above, we seldom obtained bent light curves. This must be due our model’s being too simple, in that we consider only emissions with constant spectra. When the intrinsic spectrum varies with time, one would expect bursts with both variable spectra and bent light curves (the well-known $t^{-2+0}$ curve is independent of the Lorentz factor; see eqs. [15] and [17]). For candidate bursts of this kind, we propose fitting the spectrum with $\nu^{-\beta}$ and the light curve with $[T_R - (t - T_0)]/[t - (t - T_0) - (2+0)]$, where both $T_R$ and $T_0$ are free parameters. When the fitting is good enough, we can say that the intrinsic fireball emission is likely very short or the cooling is exponential and that the corresponding fireball radius is $R_c \approx c T_R$. When the expansion of the fireball is relativistic, we get $R_c \approx c T_R$. Therefore, using this method one obtains at least an upper limit on the fireball radius as long as the intrinsic emission is extremely short, or the cooling is exponential, and the intrinsic spectrum is a power law.

Since the early X-ray afterglow spectra of some bursts evolve with time whereas for some others there is no significant temporal evolution (Zhang et al. 2007; Butler & Kocevski 2007b), we suspect that there may be two kinds of mechanism that account for X-ray afterglow emission. It seems likely that the observed variations in spectra are due to an intrinsic spectral evolution. Such intrinsic evolution would probably lead to deviations of the light curves studied above (in Figs. 3–7). We thus suspect that the bursts with no spectral evolution might have “normal” temporal profiles, while the others might exhibit somewhat “abnormal” profiles. This seems to be the case according to Figures 1–3 of Zhang et al. (2007) and Figures 7–8 of Butler & Kocevski (2007b).

Our simple model tends to account for the kind of bursts whose X-ray afterglow spectra do not evolve with time. However, for many bursts with roughly constant spectra and power-law light curves, the curves are too shallow to be accounted for by the $t^{-2+0}$ curve (see the UNLV group’s site). Our model seems to be too simple to account for the majority of the XRT light-curve data of Swift bursts. It is therefore necessary to explore more complicated cases. For example, a variable Lorentz factor (which is expected when the intrinsic emission is long enough) might play a role. Would this affect the slope of the decay curve? We look forward to seeing more investigations of this issue in the near future.

Before ending, we would like to point out that the quantity $t_{0,c}$ is the comoving time measured by a comoving observer when the fireball radius reaches $R_c$ (see Qin 2002). Note that $t_{0,c} > t_{0,c}$. Therefore, when assigning $t_{0,c} = 0$, $t_{0,0} = 1$ means that 1 s comoving time has passed since $R_c = R_c$. When one analyzes the emission associated with $R_c = 10^{15}$ cm, $t_{0,0} = 0$ refers only to the emission at $t_{0,c} = 0$, which is the comoving time when $R_c = R_c$. Although we employed quite small values of $t_{0,0}$ in the above analysis, these does not correspond to early emission when we adopt $R_c = 10^{15}$ cm or $R_c = 10^{13}$ cm. Therefore, our analysis of emission from fireballs with $R_c = 10^{15}$ cm does not put forward any constraint on the prompt emission. The conclusion that the characteristics of the prompt emission of bursts with shallow decay phases are similar to those without shallow decay phases, obtained recently by Liang et al. (2007), is not violated by our findings.
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