A binary with a $\delta$ Scuti star and an oscillating red giant: orbit and asteroseismology of KIC 9773821

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ABSTRACT

We study the $\delta$ Scuti – red giant binary KIC 9773821, the first double-pulsator binary of its kind. It was observed by Kepler during its four-year mission. Our aims are to ascertain whether the system is bound, rather than a chance alignment, and to identify the evolutionary state of the red giant via asteroseismology. An extension of these aims is to determine a dynamical mass and an age prior for a $\delta$ Sct star, which may permit mode identification via further asteroseismic modelling. We determine spectroscopic parameters and radial velocities (RVs) for the red giant component using HERMES@Mercator spectroscopy. Light arrival-time delays from the $\delta$ Sct pulsations are used with the red-giant RVs to determine that the system is bound and to infer its orbital parameters, including the binary mass ratio. We use asteroseismology to model the individual frequencies of the red giant to give a mass of $2.1 \pm 0.20 \, M_\odot$ and an age of $1.08 \pm 0.08 \, Gyr$. We find that it is a helium-burning secondary clump star, confirm that it follows the standard $\nu_{\text{max}}$ scaling relation, and confirm its observed period spacings match their theoretical counterparts in the modelling code mesa. Our results also constrain the mass and age of the $\delta$ Sct star. We leverage these constraints to construct $\delta$ Sct models in a reduced parameter space and identify four of its five pulsation modes.

Key words: stars: oscillations — stars: variables: $\delta$ Scuti — asteroseismology — stars: binaries: spectroscopic

1 INTRODUCTION

Double-lined spectroscopic binaries (SB2s) provide critical tests for stellar evolution (Pols et al. 1997; Pourbaix 2000; Claret 2007; de Mink et al. 2007; Lastennet & Valls-Gabaud 2002). Their value lies in the multitude of information that can be extracted from the composite spectra, such as the individual stellar effective temperatures, the system metallicity, and the radial velocities (RVs) whose variations encode the dynamical masses and mass-ratio of the stars (Vogel 1889; Pickering 1890; Stebbins 1911). These RVs can further be used to infer the binary orbital parameters, whose distribution functions feed directly into our understanding of binary star formation (Duchêne & Kraus 2013; Moe & Di Stefano 2017; Murphy et al. 2018; Shahaf & Mazeh 2019), such as the spatial scales at which fragmentation of protostellar disks outcompetes fragmentation of molecular cores as the dominant binary star formation mechanism (Tohline 2002; Bate 2009; Kratter 2011; Tobin et al. 2016).

In parallel with spectroscopy, asteroseismology has established itself as an indispensable tool for astrophysical investigation (Aerts et al. 2010). For example, catalogues of stellar properties that combine spectroscopy and asteroseismology for red giants (Pinsonneault et al. 2014, 2018) have greatly informed Galactic dynamics (e.g. Spitoni et al. 2019; Hasselquist et al. 2020; Hayden et al. 2020; Ratcliffe et al. 2020), thanks largely to the high age precision typically achievable (Chaplin & Miglio 2013; Lebreton et al. 2014; Hekker & Christensen-Dalsgaard 2017; Basu & Hekker 2020).

Masses and radii for red giants are often determined from so-
called asteroseismic scaling relations (Stello et al. 2008; Kallinger et al. 2010), using the observed frequency of maximum power, $v_{\mathrm{max}}$, and large frequency spacing, $\Delta v$. Since $\Delta v$ scales with the square-root of the mean density of the star (Ulrich 1986), the evolutionary stage of red giants – that is, the extent of their ascension up the red giant branch (RGB) – is readily discernible from $\Delta v$ alone. However, there is another phase of red giant evolution that occurs after the helium flash known as the red clump (RC), during which stars have a similar density and their evolutionary tracks overlap with the RGB. Except for metal-poor stars, the core-helium burning RC stars are observationally indistinguishable from the hydrogen-shell burning stars sit in the so-called secondary clump, whose period spacings are similar to RGB counterparts of slightly higher mass (e.g. Stello et al. 2010; Mosser et al. 2014; Bossini et al. 2017). This further challenges the identification of their evolutionary status, and poses some important questions, such as whether scaling relations apply and what causes the period spacing.

By comparison, asteroseismology of $\delta$ Scuti stars is less advanced, with a major barrier being mode identification (Breger 2000; Bowman & Kurtz 2018). Pairs of low-radial-order ($l \leq n$) radial modes can have period ratios that enable those modes to be identified (Petersen & Christensen-Dalsgaard 1996), but not every mode in $\delta$ Scuti stars is driven to an observable amplitude, so those modes might not be present. A recent breakthrough has been the discovery that some $\delta$ Scuti stars pulsate in regular patterns that form ridges in an échelle diagram (Bedding et al. 2020). This allows the modes to be identified somewhat analogously to solar-like oscillations, but is only possible for some stars close to the zero-age main-sequence, including on the pre-main-sequence (Murphy et al. 2021a). Binary systems can also be of assistance in analysing $\delta$ Scuti stars via the provision of dynamical masses, which may lead to more constrained models to allow the observed modes to be identified (e.g. Streamer et al. 2018; Stein dred et al. 2020).

In this paper we analyse KIC 9773821, whose light curve shows the pulsations of both a $\delta$ Scuti and a red giant star (Fig. 1 shows its amplitude spectrum). Such double-pulsator binaries are named PB2s in analogy to the SB2s of spectroscopy (Murphy et al. 2016). Throughout our investigations we adopt the convention that the $\delta$ Scuti star is component 1 and the RG is component 2 (e.g. in the subscripts for RV amplitudes, $K_1$ and $K_2$). This is because our investigation began with the Kepler light curve, whose variability is dominated by the $\delta$ Scuti star (Fig. 1), and because light travel time variations were discovered in the $\delta$ Scuti pulsations, providing the first evidence that the stars may be bound. However, the red giant is the more luminous, more evolved and more massive object.

The Kepler observations are described in Sec. 2, demonstrating the double-pulsator nature of the system. Our spectroscopic observations and analysis, including radial velocity extraction, are described in Sec. 3. In Sec. 4 we measure light travel time variations for the $\delta$ Scuti star, and combine them with the radial velocities to show that the stars have the same orbital period, and are thus a bound system. The orbital analysis of Sec. 4 guides deeper asteroseismology of the red giant (Sec. 5) to infer its evolutionary state, and the $\delta$ Scuti star (Sec. 6) to identify its modes. Our conclusions are given in Sec. 7 and our data are given in an appendix.

### 2 Kepler Observations

KIC 9773821 (magnitude $K_p = 9.86$) has 4 yr of Kepler observations in long-cadence mode (29.45-min sampling), for a total duration of $\Delta T = 1470.47$ d. The corresponding frequency resolution is $1/(\Delta T) = 0.00068$ d$^{-1}$ (0.0079 Hz). We used the light curve processed with the ‘msMAP’ pipeline (Stumpe et al. 2014), which we downloaded from KASOC. We converted the fluxes to magnitudes and subtracted the mean magnitude from each of the 18 Kepler ‘quarters’, including the commissioning quarter, Q0.

The Fourier transform of the Kepler light curve is shown in Fig. 1, which shows both $\delta$ Scuti and red-giant oscillations. Five $\delta$ Scuti peaks are labelled, with their extracted frequencies and amplitudes given in Table 1; we describe these in Sec. 2.1. The broad power excess near $8.6$ d$^{-1}$ (100 Hz) consists of red-giant (RG) oscillations and is analysed in Sec. 2.2. Other key features in the Fourier transform are a rising background at low frequency, which is a typical feature of red-giant power spectra that arises from granulation, and three low frequencies starting at $1.8$ d$^{-1}$ (21 $\mu$Hz) that are almost harmonics of each other (Sec. 2.3). Although only long-cadence observations are available, barycentric corrections to the time-stamps of the data result in a modulated sampling rate as the Kepler spacecraft orbits the Sun, allowing us to assert that none of the extracted peaks are Nyquist aliases (Murphy et al. 2013).

The oscillations of both stars are affected by the orbital motion, which causes periodic Doppler shifts of each star’s pulsations. The effect in Fourier space is the generation of a frequency multiplet (Shibahashi & Kurtz 2012) which can be used to extract orbital information if the oscillations are coherent (Shibahashi et al. 2015). However, orbital information cannot be extracted from the low-amplitude, stochastic red giant oscillations (Compton et al. 2016).

#### 2.1 The $\delta$ Scuti Pulsations

The $\delta$ Scuti pulsations appear to have some regularity, which led us to search for signs of harmonics or sum frequencies (e.g. Pápics 2012; Kurtz et al. 2015), but we found none. However, there are some interesting patterns among the five $\delta$ Scuti peaks. Firstly, $f_3$ lies almost exactly halfway between $f_1$ and $f_2$. Equidistance would correspond to a frequency of $13.892$ d$^{-1}$, whereas $f_3$ lies $0.03$ d$^{-1}$ away at $13.862$ d$^{-1}$, which is 44 times the frequency resolution and suggests this is not a combination frequency. Similarly, $f_2$ lies $0.054$ d$^{-1}$ away from being the combination $2f_2 - f_1 = 18.490$ d$^{-1}$. We conclude that these near-combinations are coincidences and we explore identities for these modes in our $\delta$ Scuti modelling (Sec. 6).

#### Table 1. Pulsation frequencies and amplitudes from the $\delta$ Scuti component.

| ID | Frequency d$^{-1}$ | Frequency $\mu$Hz | Amplitude $\mu$mag |
|----|-------------------|-------------------|-------------------|
| $f_1$ | 12.35999 | 143.0554 | 85 |
| $f_2$ | 15.42488 | 178.5287 | 28 |
| $f_3$ | 13.86231 | 160.4434 | 10 |
| $f_4$ | 16.70400 | 193.3333 | 5 |
| $f_5$ | 18.54437 | 214.6339 | 3 |

1 https://kasoc.phys.au.dk/

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2.2 The red-giant oscillations

The oscillations from the red giant are clearly visible in the power spectrum with very high signal-to-noise, making it straightforward to measure and identify dozens of modes. The left panel of Fig. 2 shows the power spectrum in échelle format, with a large frequency separation of $\Delta \nu = 8.05 \muHz$, calculated from the autocorrelation of the power spectrum. We can easily recognise the patterns of modes with degrees $\ell = 0$ (red circles), $\ell = 1$ (blue squares) and $\ell = 2$ (red triangles). The right panel shows the period échelle diagram (Bedding et al. 2011), and we see the characteristic pattern of $\ell = 1$ modes. By aligning the structure of these $\ell = 1$ modes vertically we determined an asymptotic period spacing $\Delta_1 = 194.0 \text{s}$. We used this pattern to guide the selection of the $\ell = 1$ modes.

The extracted frequencies, their uncertainties, and their and mode identifications are given in Table A1. We measured the frequencies after smoothing the power spectrum slightly in order to smooth modes broadened by the finite lifetimes. Some of the $\ell = 1$ peaks are split into multiplets by rotation. Our asteroseismic analysis (Sec. 5.3) is based on non-rotating models and, by measuring peaks in the smoothed spectrum, we measured the central ($n = 0$) component. Uncertainties on the measured frequencies are generally lower for stronger peaks and we were guided by the equations in Montgomery & O’Donoghue (1999) and Kjeldsen & Bedding (2012). In order to be conservative, we adopted an uncertainty of 0.03 $\muHz$ for most modes. In practice, the actual values of the uncertainties do not influence the results of the modelling and those given in Table A1 are intended to be approximate values.

We measured the frequency of maximum power, $\nu_{\text{max}}$, in a two-step process using the SYD pipeline (Huber et al. 2009). First, the background of a power-density spectrum was modelled by a sum of two power laws and a white-noise component, and then subtracted from the original power-density spectrum. This spectrum was subsequently smoothed using a Gaussian with a full width at half-maximum of $2(\Delta \nu)$ and $\nu_{\text{max}}$ was taken as the peak of the heavily smoothed spectrum. Its uncertainty was calculated by perturbing the original power-density spectrum 500 times with a $\chi^2$ distribution with two degrees of freedom and repeating the above procedure on each perturbed spectrum. The standard deviation of the resulting distribution was adopted as the formal uncertainty (Huber et al. 2011). In this way, we calculated $\nu_{\text{max}} = 102.06 \pm 0.96 \muHz$.

2.3 Low-frequency peaks

The low frequency peaks at 1.8292, 3.6578, and 5.4860 $\muHz$ (21.171, 42.336, and 63.495 $\muHz$; Fig. 1) are not exact harmonics of each other and are broader than the frequency resolution of the data. It is therefore unlikely that they result from ellipsoidal variability and reflection effects in a compact binary (Colman et al. 2017). Bowman (2017) discussed similar features in other stars, which he called ‘organ pipe stars’, and argued against a rotational origin unless significant latitudinal differential rotation is present in A-type stars. Since these low frequencies are also dissimilar to the spacing of the $\delta$ Sct p modes in KIC9773821, their origin remains unclear.

3 SPECTROSCOPY

We obtained high-resolution ($R \approx 85000$) spectroscopy of the system using the HERMES spectrograph (Raskin et al. 2011) attached to the 1.2-m Mercator telescope at the Roque de los Muchachos observatory on La Palma, Spain. In total, 17 spectra of KIC9773821 at a S/N $\sim 80$ were obtained from July 2012 to October 2019. We computed Least-Squares Deconvolution (LSD) profiles (Donati et al. 1997, as implemented by Tkachenko et al. 2013) from the normalised spectra to determine if the radial velocities of the individual components could be extracted. We found that the profiles of the individual components were blended at all orbital phases, which is unsurprising for a long-period binary ($P_{\text{orb}} = 482 \text{d}$; Sec. 4). In addition, the more-luminous slowly-rotating red giant dominates the LSD profiles, preventing us from extracting reliable RVs for the more rapidly-rotating $\delta$ Sct component (Fig. 3). RVs for the red giant were extracted to a precision of $\sim 0.15 \text{km s}^{-1}$ (Table A2).

We determined the atmospheric parameters by first separating the component spectra into their individual components using the spectral disentangling technique (Simon & Sturm 1994; Hadrava 1995). We used the Fourier domain-based disentangling code FDBinary (Ilić et al. 2004), which allowed us to simultaneously optimise
the orbital parameters of the system while obtaining the component spectra. Allowing both \( K_1 \) and \( K_2 \) to vary in the disentangling resulted in unphysical component spectra, and we therefore had to fix \( K_1 \) at the value 23.91 km s\(^{-1}\) determined from phase modulation (described in Sec. 4). After \( K_1 \) was fixed, we obtained realistic component spectra.

The atmospheric parameters were determined by fitting synthetic spectra to the 4800–5700 Å region\(^2\) using the Grid Search in Stellar Parameters (gspsr) software package (Tkachenko 2015). gspsr is able to generate and fit synthetic spectra to an observed spectrum using the SyntV radiative transfer code (Tsymbal 1996) combined with a grid of atmospheric models from the LL/models code (Shulyak et al. 2004). Using this method, we determined the atmospheric parameters \( T_{\text{eff}} \), \( \log g \), microturbulent velocity \( (v_{\text{mic}}) \), macroturbulent velocity \( (v_{\text{mac}}) \), projected rotational velocity \( (v \sin i) \), and the light contribution for each star, as well as the global metallicity \((\text{[M/H]}\)). We determined uncertainties from the distribution of \( \chi^2 \) values of the fit of each synthetic spectrum to the observed spectrum. Due to the degeneracy between \([M/H]\) and the light ratio, we would have to place constraints on at least one of the stellar luminosities \( L_{\text{RG}} \) or \( L_{\delta\text{Sct}} \) to obtain \([M/H]\) directly. Nonetheless, we note that the inferred metallicity of each component was equal for light ratios \( L_{\text{RG}}/L_{\delta\text{Sct}} = 3.2 \pm 0.3 \), and they rapidly diverge outside this range. To obtain individual luminosity constraints, the atmospheric parameters were determined iteratively with the asteroseismic analysis of the red giant (Sec. 5), starting with the best-fitting \( T_{\text{eff}} \) of 5080±130 K determined at the (initially fixed) solar metallicity. The \( \delta\text{Sct} \) parameters were determined thereafter, and include a 2% systematic uncertainty on the effective temperature (Casagrande et al. 2014; White et al. 2018).

4 ORBITAL ANALYSIS

We applied the phase modulation method (Murphy et al. 2014; Murphy & Shibahashi 2015) to measure light arrival-time delays for the \( \delta\text{Sct} \) pulsations. The method works by measuring the pulsation frequencies as precisely as possible using the entire 4-yr \textit{Kepler} data set, then subdividing the light curve and measuring the pulsation phases in each subdivision whilst keeping the frequencies fixed. Phase shifts are readily converted into time delays, which contain information on the star’s position in its orbit. When the \( \delta\text{Sct} \) star is on the far side of the orbit, the arrival of its light at Earth is delayed, whereas light arrives early (a negative time delay) when the star is at the near side of its orbit. Each pulsation frequency should respond in the same way, that is, should give the same time delay curve, if the modulation is due to binary motion rather than intrinsic phase modulation. The method has been applied to hundreds of \( \delta\text{Sct} \) pulsators, whose coherent pulsations appear to be good ‘clocks’ in this regard (Murphy et al. 2018).
Figure 3. One of the observed (black) LSD profiles of KIC9773283. The sharp-lined RG and the broad-lined δ Sct profile are indicated by the red and blue synthetic LSD profiles calculated from synthetic spectra generated using gsssp.

A free parameter in phase-modulation analyses is the subdivision size. Longer subdivisions result in more precise phases and thus smaller uncertainties per point but will undersample the orbital signal if the orbital period is short. Hey et al. (2020b) developed a forward-modelling method to fit orbits to the light curve directly, which offers great improvement for short-period binaries (P<100 d). For longer periods as in this case, the subdividing approach remains sufficiently accurate and we used it for this analysis. Following application of the subdividing approach, we confirmed our results with the maelstrom forward-modelling code (Hey et al. 2020a).

We extracted time delays for f1 and f2, which are the strongest of the five extracted peaks (Table 1). The next-strongest peak, f3, is not statistically significant in 10-d subdivisions of the light curve, and f4 and f5 are weaker still. However, f1 is significant in 20-d subdivisions, and shows time delays consistent with f1 and f2, albeit with large scatter. Fig. 4 shows the time delays with their weighted average (weighting by the inverse square of the pulsation phase uncertainties), indicating a projected light travel time variation of ±500 s, corresponding to a projected semi-major axis (a sin i) of approximately 1 au. Note that the projected separation of the two stars, a sin i, is larger than this [a sin i = (a1 + a2) sin i], and note also that time-delay uncertainties are overestimated because there is considerable variance in the data from the red-giant oscillations.

Radial velocities, RV(t), are the time derivative of the time delays, τ(t),

\[ RV(t) = \frac{d\tau(t)}{dt}. \tag{1} \]

and depend on the same orbital parameters. Murphy et al. (2016) verified the PM method by comparison with RVs, and those authors and others (Lampens et al. 2018; Derekas et al. 2019) have presented joint modelling of RVs and time delays, similar to the one we perform here for KIC 9773821. A key difference here is that the RV curve belongs to the red giant and so is in exact anti-phase with that calculated from the time delays of the δ Sct star (Fig. 5). This proves that the two stars orbit each other and are not a chance alignment.

We determined the orbital parameters using the Markov chain Monte Carlo method based on the Metropolis-Hastings algorithm, as described by Murphy et al. (2016). In each iteration of the Markov chain the two components, as described by their respective time-delay and RV curves, had the same orbital period and eccentricity, while their longitudes of periastron σ differed by precisely π radians. Their projected semi-major axes were sampled via independent proposals at each iteration in the Markov chain, and the inverse ratio of those axes gives the mass ratio q of the system, that is,

\[ q = \frac{M_2}{M_1} = \frac{a_1 \sin i}{a_2 \sin i}. \tag{2} \]

The total χ² was used to evaluate each iteration, and no additional weights were applied to the time-delay or RV data sets. The chain was run for 25 000 iterations and manually checked for convergence, after which the total χ² was 39.8 from 143 time delays and 17 RV data. All orbital parameters were determined as the medians of their marginalised posteriors, and the uncertainties were the 15.9 and 84.1 percentiles of those posteriors, together bracketing the central 68.2% per cent of the data. Randomly-drawn samples from the converged chain are shown in Fig. 5 and the orbital parameters are given in Table 2.

Since the mass ratio, the mass function, and one of the individual component masses are known precisely (the modelled red giant mass for the secondary lump scenario, discussed in Sec. 5), it is possible to calculate the orbital inclination. To do this, we performed a 100 000-iteration Monte Carlo simulation involving each of the above parameters to sample sin i according to

\[ f(m_1, m_2, \sin i) = \frac{M_m}{(m_1 + m_2)^2} \frac{m_1^3 \sin^3 i}{(m_1 + m_2)^2} \tag{3} \]

which can be rewritten as

\[ \sin^3 i = \frac{fM_m}{m_2^3} (1 + 1/q^2). \tag{4} \]

The resulting stellar inclination, i, was 81° ± 10°, meaning the system is observed nearly edge-on.

### Table 2. Orbital parameters for the KIC 9773821 system

| Parameter | Units | Values |
|-----------|-------|--------|
| P_0th     | d     | 481.93 ± 0.13 |
| e         |       | 0.241 ± 0.003 |
| σ         | rad   | 2.154 ± 0.014 |
| t_0       | d     | 2455 ± 273.9 |
| γ         | km s⁻¹ | 8.86 |
| f(m₁, m₂, sin i) | M_⊙ | 0.623 ± 0.061 |
| f(m₂, m₁, sin i) | M_⊙ | 0.324 ± 0.028 |
| a₁ sin i/c | s     | 513 ± 15 |
| a₂ sin i/c | s     | 412.1 ± 12.0 |
| a₁ sin i | au | 1.03 ± 0.03 |
| a₂ sin i | au | 0.8259 ± 0.002 |
| K₁       | km s⁻¹ | 23.91 ± 0.75 |
| K₂       | km s⁻¹ | 19.21 ± 0.05 |
| q = M₂/M₁ |       | 1.245 ± 0.038 |
| i       | deg | 81 ± 10 |

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5 ASTEROSEISMIC AND SPECTROSCOPIC ANALYSIS

5.1 Iterative fitting method

The spectroscopic and asteroseismic analyses are interconnected, because the system metallicity and the luminosity ratio of the stars produce a similar effect on the observed depths of the red giant’s spectral lines: both a higher metallicity and a higher red giant luminosity result in deeper spectral lines for the red giant. The luminosity ratio is determined from the asteroseismology, while the red giant’s oscillations are sensitive to the metallicity. We therefore used an iterative approach to arrive at the system parameters, which can be summarised as follows:

(i) Determine the red giant’s luminosity, age and mass (initially assuming solar metallicity).
(ii) Calculate the $\delta$ Sct mass using the orbital mass ratio.
(iii) Using evolutionary tracks, find the luminosity of the $\delta$ Sct star given that age, $\delta$ Sct mass, and system metallicity.
(iv) Calculate the light ratio and its uncertainty.
(v) Use the new light ratio to refine the spectroscopic metallicity.
(vi) Repeat from step (i) with the new metallicity.

For the first iteration of step (i), we used the asteroseismic scaling relations with $\Delta \nu = 8.05 \pm 0.04 \mu$Hz and $\nu_{\text{max}} = 102.06 \pm 0.96 \mu$Hz for the red-giant oscillations. Using figure 1 of Mosser et al. (2014), figure 4 of Stello et al. (2013) and figure 2 of Yang et al. (2012), we inferred that the red giant is probably a secondary clump star (core He burning) with a mass of $2.45 \pm 0.11 M_\odot$ and an age of 500–700 Myr. Then (ii), using the binary mass ratio $q = 1.245 \pm 0.038$ we inferred a $\delta$ Sct mass of $1.97 \pm 0.1 M_\odot$. At solar metallicity, this gave a luminosity range of 16.6 to 26.4 $L_\odot$ for the $\delta$ Sct component (iii), compared with $L_{\text{RG}} = 47 L_\odot$ from the measured $T_{\text{eff}}$ and calculated $R_{\text{RG}}$, for a light ratio of $L_{\text{RG}}/L_\delta = 2.3 \pm 0.5$ (iv).

Using this light ratio, the spectroscopic parameters for the red giant were revised slightly (v), with the metallicity changing by $-0.11$ dex on the first iteration, and by a total of +0.02 dex on subsequent iterations (vi), to become those given in Table 3 at a light ratio of $L_{\text{RG}}/L_\delta = 3.2$. This value is consistent with the spectroscopic light ratio that results in equal metallicities for the two
components. After the first iteration, the asteroseismic modelling used the oscillation frequencies directly rather than relying only on scaling relations, as we now describe.

5.2 Stellar model calculations

Asteroseismic modelling of the red giant was carried out with the stellar evolution code MESA (Modules for Experiments in Stellar Astrophysics, version 12115, Paxton et al. 2011, 2013, 2015, 2018) and the oscillation code GYRE (version 5.1, Townsend & Teitler 2013). We adopted the solar chemical mixture ([Z/X]⊙ = 0.0181) provided by Asplund et al. (2009). The initial chemical composition was calculated by:

\[
\log(Z_{\text{init}}/X_{\text{init}}) = \log(Z/X)_{\odot} + [\text{Fe/H}].
\]

We used the MESA p – T tables based on the 2005 update of the OPAL equation of state tables (Rogers & Nayfonov 2002) and we used OPAL opacities supplemented by the low-temperature opacities from Ferguson et al. (2005). The MESA ‘simple’ photosphere was used for the set of boundary conditions for modelling the atmosphere; alternative model atmosphere choices do not strongly affect the results for solar-like oscillators (Yıldız 2007; Joyce & Chaboyer 2018; Nsamba et al. 2018; Viani et al. 2018) or for δ Sct stars (Murphy et al. 2021b). The mixing-length theory of convection was implemented, where \(\alpha_{\text{MLT}} = f_{\text{MLT}}/H_p\) is the mixing-length parameter. The exponential scheme by Herwig (2000) was adopted for the convective core overshooting, where the diffusion coefficient in the overshoot region is given as

\[
D_{\text{ON}} = D_{\text{conv,0}} \exp \left( \frac{2(r - r_0)}{(f_0 + f_{\text{ov}})H_p} \right). \tag{6}
\]

Here, \(D_{\text{conv,0}}\) is the diffusion coefficient from the mixing-length theory at a user-defined location near the Schwarzschild boundary. The switch from convection to overshooting is set to occur at \(r_0\). To consider the step taken inside the convective region, \((f_0 + f_{\text{ov}})H_p\) is used. In MESA, \(f_{\text{ov}}\) is a free parameter and \(f_0\) equals 0.5\(f_{\text{ov}}\). We set the overshooting parameter \(f_{\text{ov}} = (0.13M - 0.098)/9.0\) and adopted a fixed \(f_{\text{ov}}\) at 0.018 for models above \(M = 2.0 M_\odot\) following the mass-overshooting relation found by Magic et al. (2010). We also applied the MESA predictive mixing scheme in our model for a smooth convective boundary. The mass-loss rate on the red-giant branch with Reimers prescription is set as \(\eta = 0.2\), which is constrained by old open clusters NGC 6791 and NGC 6819 (Miglio et al. 2012).

### Table 3. Spectroscopic parameters for the red giant and δ Sct components, after three iterations of asteroseismic inference to determine the luminosity ratio of the stars (see Sect. 5). Values are provided for \(L_{\text{RG}}/L_{\delta\text{Sct}} = 3.2\).

| Parameter | Units | Values |
|-----------|-------|--------|
| \(T_{\text{eff}}\) | K | 5124±194 |
| \(\log g\) | (cgs) | 2.65±0.39 |
| \(\nu_{\text{mic}}\) | km s\(^{-1}\) | 0.38±0.38 |
| \(\nu_{\text{mic}}\) | km s\(^{-1}\) | 6.1±1.2 |
| \(v\sin i\) | km s\(^{-1}\) | 2.5±1.5 |
| \([\text{Fe/H}]\) | | −0.09±0.16 |

### Table 4. Input ranges and grid steps of the model grid.

| Input Parameter | Range From | Increment |
|-----------------|------------|-----------|
| \(M/M_\odot\)  | 1.80 | 3.00 | 0.02 |
| \([\text{Fe/H}]\) | −0.4 | 0.4 | 0.1 |
| \(Y_{\text{init}}\) | 0.24 | 0.32 | 0.02 |
| \(\alpha_{\text{MLT}}\) | 1.7 | 2.3 | 0.2 |

The model computation ranged from 1.8 to 3.0\(M_\odot\). We computed each stellar evolutionary track from the Hayashi line to the point on the red-giant branch where \(\log g = 1.5\) dex. Each evolutionary track includes both H-burning (red-giant branch) and He-burning (red-clump) phases. The grid includes four independent model inputs: stellar mass \(M\), initial helium fraction \(Y_{\text{init}}\), initial metallicity \([\text{Fe/H}]\), and the mixing-length parameter \(\alpha_{\text{MLT}}\). Ranges and grid steps of the four model inputs are summarized in Table 4.

Modelling results in red clump stars can be sensitive to the prescription of convective overshooting (e.g. Bossini et al. 2017). Our prescription is that demonstrated in figure 1 of (Constantino et al. 2015), who compared model predictions for \(\Delta \Pi\) as a function of stellar mass with observations from Mosser et al. (2014). We review the choice of overshooting parameters at the end of this section.

5.3 Oscillation Models

Theoretical stellar oscillations for \(\ell = 0, 1,\) and 2 were calculated with GYRE (version 5.1; Townsend & Teitler 2013), by solving the adiabatic pulsation equations with the structural models generated by MESA. Our computation was carried out in three steps as described below to avoid excessive computing time.

1. Searching for stellar models within a 3\(\sigma\) cube constrained by the observed \(T_{\text{eff}}, \log g,\) and \([\text{Fe/H}]\), calculating a spectroscopic likelihood \(L_{\text{spec}} (L = e^{-\chi^2}, \text{where } \chi^2 \text{ is the reduced } \chi \text{ square}) \) for each model.

2. Computing radial (\(\ell = 0\)) mode frequencies for models whose \(L_{\text{spec}} \geq 0.001\), fitting those frequencies and calculating seismic likelihood for the radial modes \((L_{\text{seis,}\ell=0})\), and then deriving a total likelihood \(L' = L_{\text{spec}} \times L_{\text{seis,}\ell=0}\).

3. Computing \(\ell = 1\) and 2 mode frequencies for models whose \(L'/L_{\text{max}} \geq 0.001\), fitting all observed modes and calculating the seismic likelihood for all modes \((L_{\text{seis}})\), and lastly deriving a final likelihood with the method described by Li et al. (2020b) for each model to estimate stellar parameters.

For correcting the surface term of the red giant oscillations only, we used the two-term expression described by Ball & Gizon (2014).

The parameters of the top 10 models for the red giant are given in Table 5.
5.4 Modelling results

We fitted the spectroscopic observations and asteroseismic mode frequencies by using the likelihood functions and the fitting procedure introduced by Li et al. (2020b). This fitting method accounts for the systematic offset between observed and model frequencies (which is larger than the observed uncertainties) and also applies a weighting factor depending on the $\nu/\nu_{\text{max}}$ ratio for each peak, which together give more sensible probability distributions for stellar parameters. Specifically, we determined the model systematic uncertainty as the median offset between the observations and the best-fitting model in the penultimate iteration, which was 0.04 for $\ell = 0$ modes, 0.11 for $\ell = 1$, and 0.05 for $\ell = 2$. However, unlike in Li et al. (2020b), we did not interpolate the model frequencies in this work. In addition, no perturbative or other formulation was applied to the frequencies to account for rotation.

The modelling results show that the red giant could be an RGB star or a secondary red-clump star (Fig. 6). The latter is more likely because of its considerably longer duration, with tracks in the middle of the observational error box spending approximately 100 times longer in the secondary RC phase than the RGB phase. While the two phases can be distinguished in lower-mass red giants by the period spacing of their gravity modes, secondary RC stars have similar period spacings to RGB stars. We investigate modelling results for both possibilities.

We start by noting that our measured period spacing ($\Delta\Pi = 194.0\pm5$ s; Sec. 2.2) matches very well with the period spacings of our top ten MESA models ($\Delta\Pi = 195\pm2$ s; Table 5). This indicates that $\Delta\Pi$ values calculated in MESA using the integral $\Delta\Pi = \sqrt{2}\pi^2 (\int N/r \, dr)^{-1}$, where $N$ is the buoyancy frequency, do not have a significant systematic offset.

In the final iteration of modelling, the red giant masses were calculated to be $2.26^{+0.10}_{-0.12} M_\odot$ (RGB) and $2.10^{+0.20}_{-0.10} M_\odot$ (RC), with ages of $0.64^{+0.08}_{-0.08}$ and $1.08^{+0.06}_{-0.24}$ Gyr, respectively. The probability distributions of mass and age are shown in Fig. 7. The metallicity from modelling lies in the range $[\text{Fe/H}] = -0.1 \pm 0.2$.

Under the assumption that this component is a RC star, the binary mass ratio and the RC mass imply a $\delta$ Sct mass of $1.69^{+0.11}_{-0.10} M_\odot$. Using a grid of evolutionary tracks of $[\text{Fe/H}] = -0.1$ dex with mass-intervals of 0.05 $M_\odot$ (Murphy et al. 2019), we ran a Monte-Carlo simulation to determine the effective temperature and luminosity of the $\delta$ Sct star and to check the validity of our joint asteroseismic and spectroscopic analysis. In this process, we generated 10 000 random anti-correlated masses and ages from the above distributions, such that larger masses accompanied younger ages, and determined the closest $T_{\text{eff}}$ and luminosity from the evolutionary tracks at fixed metallicity. Using the median and standard deviation of the 10 000 samples, we found that the $\delta$ Sct star has $T_{\text{eff}} = 7820^{+450}_{-120}$ K, and $L = 14.8^{+4.3}_{-3.4} L_\odot$. For a red giant luminosity of $41 \pm 2 L_\odot$, this gives a luminosity ratio of $L_{\text{RGB}}/L_{\delta\text{Sct}} = 2.8^{+0.8}_{-0.6}$ which is consistent with the ratio assumed for the same iteration of spectroscopic analysis ($L_{\text{RGB}}/L_{\delta\text{Sct}} = 3.2$). In the final iteration, we found that the red-giant modelling was dominated by the oscillation frequencies: small changes in metallicity of $-0.02$ dex resulting from small changes in the luminosity ratio had little impact on the red giant mass, luminosity or age. The physical parameters of each star from this final iteration are given in Table 6.

We note that if the red giant were on the RGB instead of the secondary clump, then the $\delta$ Sct star would have a larger mass, at $M_{\delta\text{Sct}} = 1.81 \pm 0.14 M_\odot$. It would be considerably hotter, with $T_{\text{eff}} = 8700^{+270}_{-240}$ K, while the younger age ($0.64^{+0.08}_{-0.06}$ Gyr) would make it less evolved and thus only slightly more luminous ($L = 16.9^{+4.9}_{-4.7} L_\odot$). We note that this inferred temperature is considerably hotter than the spectroscopic constraint (7500 K), and places the
Here we investigate whether secondary clump stars follow the widely-used \( \nu_{\text{max}} \) scaling relation, \( \nu_{\text{max}} \propto \sqrt{M/R} \) (Brown et al. 1991; Kjeldsen & Bedding 1995), since \( \nu_{\text{max}} \) was not used in the model calculations. We can write this relation as

\[
\frac{\nu_{\text{max}}}{\nu_{\text{max},\odot}} \approx \frac{M}{M_\odot} \left( \frac{R}{R_\odot} \right)^{-2} \left( \frac{T_{\text{eff}}}{5777 \text{ K}} \right)^{-0.5},
\]

where we adopt \( \nu_{\text{max},\odot} = 3090 \pm 30 \mu Hz \) (Huber et al. 2011).

In addition to the 1% uncertainty in \( \nu_{\text{max},\odot} \), we also included a systematic uncertainty in the scaling relation of 1.1%, as recently suggested by Li et al. (2020a). This gives a \( \nu_{\text{max}} \) value for the best-fitting RC model in Table 5 of 99.5 ± 1.5 \mu Hz, which agrees at 1.7\( \sigma \) with the measured value of 102.06 ± 0.96 \mu Hz. This is a useful confirmation of the \( \nu_{\text{max}} \) scaling relation for a star in the secondary clump, which is a regime that has not previously been tested in this way.

### 5.6 Convective overshooting schemes

Here we discuss different convective overshooting schemes for the red giant phase, and justify our choice of ‘standard overshooting’ for modelling the red giant component of KIC 9773821. According to Constantino et al. (2015, 2016) and Bossini et al. (2017), asymptotic g-mode period spacings predicted by models with the ‘standard overshooting’ scheme (proposed by Herwig 2000) during the core helium burning (CHeB) lifetime are, on average, smaller than those found in Kepler CHeB stars and open clusters. To remedy this, a
‘maximal-overshoot’ scheme has been proposed (Constantino et al. 2015), which does not pretend to be realistic but produces the most massive convective core possible. This scheme extends the CHeB lifetime of models and hence increases the period spacing at the end of CHeB phase so that it matches the upper boundary of the observed ΔΠ distribution, which is dominated by low-mass stars. On the other hand, the ‘standard overshooting’ scheme is well calibrated with turn-off stars in open clusters and with main-sequence stars in eclipsing binary systems, and better fits more massive stars (Deheuvels 2020). However, the calibrations for main-sequence stars could be unsuitable because of the dramatic changes in stellar structure between the main-sequence turn-off and the onset of CHeB.

A relatively comprehensive way to construct evolutionary models for CHeB stars could be to start with the standard overshooting scheme to provide the best accuracy during the main-sequence phase, then switch to the maximal overshooting scheme at some point before the core-helium burning phase. The switch of overshooting scheme necessarily complicates the model computation because it introduces at least two additional free parameters: the optimal time to make the switch, and the ‘best’ scheme to switch to. Given that we find the RG component of KIC 9773821 to be at 1.66 M☉ and [Fe/H] = −0.2, which are consistent with the observed constraints at the 1σ level. The best-fitting model in this scenario has the following properties: 

\[ M = 1.62 M_\odot, \quad \tau = 1.10 \text{ Gyr}, \quad T_{\text{eff}} = 7581 \text{ K}, \quad \log g = 3.84, \quad L = 18.9 L_\odot, \quad \text{and} \quad [\text{Fe/H}] = -0.2. \]

In the middle panel of Fig. 8, we show a comparison of the oscillation frequencies of this model with the observations. The observed peaks are well fitted except for \( f_4 \) at 16.7 d\(^{-1}\). The mode identifications of these 10 models are consistent: \( f_1 \) (at 12.35 d\(^{-1}\)) is the first radial overtone (\( \ell = 0, n_p = 2 \)); \( f_2 \) (15.43 d\(^{-1}\)) is the second radial overtone (\( \ell = 0, n_p = 3 \)); \( f_3 \) (18.54 d\(^{-1}\)) is the third radial overtone (\( \ell = 0, n_p = 4 \)), and \( f_4 \) (13.86 d\(^{-1}\)) is a dipole mixed mode (\( \ell = 1, n_{pg} = 1 \)).

The fundamental radial mode (\( \ell = 0, n = 1 \)) is predicted to have a frequency of 9.734 d\(^{-1}\), which we do not observe. This falls in the region of the red giant oscillations and must have a very low amplitude if it is excited at all.

The model frequencies may have small offsets from a hypothetical rotating model because our models are non-rotating (see, e.g. Di Criscienzo et al. 2008), which may explain some of the small differences between the calculated and the observed frequencies.

### 6.1 The ‘δ Sct+RC’ Scenario

We first studied the ‘δ Sct+RC’ scenario, where the red giant is core-He burning. According to the constraints from the RC models (Sec. 5.4), we set hard limits for mass and age for the δ Sct component as 1.69 ± 0.11 M☉ and 1.08\(^{+0.26}_{-0.22}\) Gyr. All δ Sct models are shown in the top panel of Fig. 8 by blue shading, and all are located on the main sequence before the overall contraction phase (‘hook’). Details on the use of gyr to compute radial and dipole mode frequencies were given in Sec. 5.3. We fitted each peak to the nearest theoretical frequency of each model without any prior, allowing each peak to have either \( \ell = 0 \) or \( \ell = 1 \). We then used a maximum likelihood estimation method to find the best-fitting models by minimising the differences between the observed and modelled frequencies. All of the top 10 models have masses from 1.62 to 1.66 M☉ and [Fe/H] = −0.2, which are consistent with the observed constraints at the 1σ level. The best-fitting model in this scenario has the following properties: \( M = 1.62 M_\odot, \quad \tau = 1.10 \text{ Gyr}, \quad T_{\text{eff}} = 7581 \text{ K}, \quad \log g = 3.84, \quad L = 18.9 L_\odot, \quad \text{and} \quad [\text{Fe/H}] = -0.2. \)

In the middle panel of Fig. 8, we show a comparison of the oscillation frequencies of this model with the observations. The observed peaks are well fitted except for \( f_4 \) at 16.7 d\(^{-1}\). The mode identifications of these 10 models are consistent: \( f_1 \) (at 12.35 d\(^{-1}\)) is the first radial overtone (\( \ell = 0, n_p = 2 \)); \( f_2 \) (15.43 d\(^{-1}\)) is the second radial overtone (\( \ell = 0, n_p = 3 \)); \( f_3 \) (18.54 d\(^{-1}\)) is the third radial overtone (\( \ell = 0, n_p = 4 \)), and \( f_4 \) (13.86 d\(^{-1}\)) is a dipole mixed mode (\( \ell = 1, n_{pg} = 1 \)).

The fundamental radial mode (\( \ell = 0, n = 1 \)) is predicted to have a frequency of 9.734 d\(^{-1}\), which we do not observe. This falls in the region of the red giant oscillations and must have a very low amplitude if it is excited at all.

The model frequencies may have small offsets from a hypothetical rotating model because our models are non-rotating (see, e.g. Di Criscienzo et al. 2008), which may explain some of the small differences between the calculated and the observed frequencies.

### 6.2 The ‘δ Sct+RGB’ Scenario

We also studied the ‘δ Sct+RGB’ scenario with the same method. The parameter limits for mass and age were 1.81 ± 0.14, and 0.64\(^{+0.08}_{-0.06}\) Gyr. All examined δ Sct models are shown with red shading in the top panel of Fig. 8. Compared with the δ Sct+RC scenario, these models are at earlier evolutionary stages because the δ Sct+RGB system age is younger. The best-fitting model in this scenario has \( M = 1.92 M_\odot, \quad \tau = 0.67 \text{ Gyr}, \quad T_{\text{eff}} = 8622 \text{ K}, \quad \log g = 3.961, \quad L = 28.6 L_\odot, \quad \text{and} \quad [\text{Fe/H}] = 0.0, \) and is illustrated in the bottom panel of Fig. 8. Even in the best model there is not good agreement with observations, with only one mode being unambiguously matched to a reasonable tolerance (\( f_2 \) = 18.54 d\(^{-1}\) as the n = 3 radial mode). The \( \chi^2 \) of the best-fitting frequencies for \( f_1 \) to \( f_5 \) is three times higher in the δ Sct+RGB scenario than in the δ Sct+RC scenario. Combined with the time-frame and Teff arguments we made previously, we therefore conclude that the δ Sct+RGB scenario does not explain the observations well.

### 7 CONCLUSIONS

We have used spectroscopic radial velocities and pulsation timing to determine that the two pulsators visible in the Kepler light curve of KIC 9773821 are in fact a bound system consisting of a secondary clump star and a main-sequence δ Sct star – the first double-pulsator binary of its kind. We have determined the orbital parameters, which
include a period of 481.9 d, an eccentricity of 0.241, and a mass ratio $q = M_{\delta\text{Sct}}/M_{\text{RG}} = 0.80$.

An iterative procedure involving frequency modelling and spectroscopic atmospheric parameter determination has constrained the evolutionary properties of the system. We found the system to be slightly metal poor, with [Fe/H] $\sim -0.1$, and we concluded that the red giant is a secondary clump (core-He burning) star with a mass of $2.10^{\pm0.20} M_{\odot}$ and an age of $1.08^{\pm0.06} \text{ Gyr}$ ($1\sigma$ uncertainties). These constraints have also allowed four of the five modes of the $\delta$ Sct star to be identified as a mixed $\ell = 1$ mode and three radial modes. Thus the combination of two different types of pulsator in one system has facilitated deeper analyses than would normally be possible in isolation.

We give three arguments to suggest that the red giant is a red clump (RC) star, rather than a star ascending the red giant branch (RGB): (i) the RC timescale is two orders of magnitude longer, so this phase is more likely to be observed; (ii) the corresponding properties of the $\delta$ Sct star are more consistent with this; and (iii) pulsation models for the younger (RGB) scenario are a poor fit to the $\delta$ Sct oscillations, whereas the older (RC) scenario fits very well with only simple assumptions.

The orbital inclination was found to be edge on, and a projected rotational velocity $v \sin i = 46 \pm 7 \text{ km s}^{-1}$ for the $\delta$ Sct star was measured, but our pulsation models did not include rotation because the potential for spin-orbit misalignment precludes independent measurement of the equatorial rotation velocity. The observed frequencies are well described with radial modes nonetheless, and future exploration of rotating models of a range of inclinations remains possible.

Our precision modelling of a low-mass secondary clump star (i.e. having a mass not much greater than 1.8 $M_{\odot}$) has offered observational tests in a regime that has not been well studied previously. For instance, we have been able to confirm the $v_{\text{max}}$ scaling relation for a secondary clump star for the first time, and that period spacings calculated in $\text{msa}$ are accurate.

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8 **DATA AVAILABILITY**

The pulsation frequencies and mode IDs for the red giant are provided in Table A1. Radial velocity data are given in Table A2. Time delays for the $\delta$ Sct component are made available as supplementary online information.

**REFERENCES**

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**Figure 8.** Top: Theoretical models for the $\delta$ Sct star on the HR diagram. Dots are evolutionary tracks of [Fe/H] $\sim -0.1$ dex computed with $\text{msa}$. We use black and grey dots to distinguish models before and after the 'hook'. Red and blue stars represent top models for $\delta$ Sct+RGB and $\delta$ Sct+RC scenarios, respectively. The red and light blue shading indicate the region of all $\delta$ Sct models examined. The top ten $\delta$ Sct models in each case happen to lie before the 'hook'. Middle: The best $\delta$ Sct model for the $\delta$ Sct+RC scenario. Black dashed lines are the five $\delta$ Sct peaks listed in Table 1. Open circles and triangles represent theoretical frequencies for $\ell = 0$ and 1. Bottom: As the middle panel, but for the $\delta$ Sct+RGB scenario.
APPENDIX A: DATA TABLES

The pulsation frequencies and mode identifications for the red giant are given in Table A1. Radial velocities for the red giant component are given in Table A2. A sample of the $\delta$ Sct time delays (supplementary online information) is provided in Table A3.

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Table A2. Radial velocities for the red-giant component of the system.

| Time (BJD) | RV   | $\sigma$RV |
|------------|------|------------|
| d          | km s$^{-1}$ | km s$^{-1}$ |
| 2456122.702771 | 25.180 | 0.150 |
| 2456123.612354 | 25.125 | 0.155 |
| 2456173.369422 | 18.490 | 0.160 |
| 2456173.390837 | 18.450 | 0.160 |
| 2458392.378968 | 14.915 | 0.155 |
| 2458405.440501 | 16.650 | 0.160 |
| 2458425.324385 | 19.150 | 0.160 |
| 2458592.710566 | 15.840 | 0.150 |
| 2458601.707377 | 13.160 | 0.160 |
| 2458629.610681 | 2.500 | 0.150 |
| 2458653.547702 | $-6.400$ | 0.150 |
| 2458680.465544 | $-12.200$ | 0.160 |
| 2458688.562511 | $-12.755$ | 0.165 |
| 2458701.474174 | $-12.840$ | 0.160 |
| 2458710.457154 | $-12.330$ | 0.150 |
| 2458724.422518 | $-10.785$ | 0.155 |
| 2458753.533187 | $-5.995$ | 0.155 |

Table A3. Weighted-average light arrival-time delays (TDs) for the $\delta$ Sct component of the system. Only the first ten rows are shown. The full table is available online.

| Time (BJD) | TD   | $\sigma$TD |
|------------|------|------------|
| d          | s    | s          |
| 2454958.39198 | 350.7 | 142.6 |
| 2454969.50828 | 305.2 | 143.5 |
| 2454979.52111 | 444.2 | 140.7 |
| 2454989.53392 | 355.1 | 153.5 |
| 2454999.54669 | 434.7 | 168.4 |
| 2455009.53899 | 385.1 | 127.4 |
| 2455021.71768 | 282.8 | 140.4 |
| 2455031.73030 | 324.5 | 130.8 |
| 2455041.74286 | 204.2 | 127.2 |
| 2455051.61232 | 174.2 | 115.6 |