Naturally small Dirac neutrino masses in supergravity

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We show that Dirac neutrino masses of the right size can arise from the Kähler potential of supergravity. They are proportional to the supersymmetry and the electroweak breaking scales. We find that they have the experimentally observed value provided that the ultraviolet cut-off of the Minimal Supersymmetric Standard Model (MSSM) is between the Grand Unification (GUT) scale and the heterotic string scale. If lepton number is not conserved, then relatively suppressed Majorana masses can also be present, resulting in pseudo-Dirac neutrino masses.

I. INTRODUCTION

We have recently learned a great deal about mixing in the neutrino sector\textsuperscript{1}. However we have thus far learned relatively little about why the neutrino masses are so small, or their relation to the other much higher scales in particle physics. The presence of such vastly different mass scales remains a great puzzle. The current favoured explanation for small neutrino masses is the “see-saw” mechanism\textsuperscript{2}. In this picture a large Majorana mass for the right handed neutrino suppresses the mass of the light states, and the active neutrinos we observe today are therefore almost pure Majorana. However, in this framework there is no room for Dirac or pseudo-Dirac neutrinos, and so it is worth examining alternative ways to generate neutrino masses.

There is one other instance in supersymmetry where it was possible drastically to suppress a mass scale, the solution of the \( \mu \)- problem by Giudice and Masiero\textsuperscript{3}. The \( \mu H_u H_d \) term in the superpotential is a mass term for the Higgs fields required for electroweak symmetry breaking. The parameter \( \mu \), which has dimensions of mass, therefore has to be of the order of 1 TeV. But in global supersymmetry it is apparently independent of the supersymmetry breaking terms which also have to be of order 1 TeV, appearing as it does in the superpotential. At first glance there is no connection between supersymmetry breaking and the parameter \( \mu \). However the problem is resolved if the \( H_u H_d \) interaction appears in the Kähler potential of supergravity rather than the superpotential. Then an effective \( \mu \)-term is generated only upon supersymmetry breaking and is of the order of the gravitino mass \( m_{3/2} \sim 1 \text{ TeV} \). The crucial ingredient of this solution to the \( \mu \) problem is the absence of this term in the superpotential of unbroken supergravity, and its subsequent generation through an analogous coupling in the Kähler potential, once supersymmetry is broken.

Could such a Kähler suppression be responsible for the smallness of neutrino masses as well? The numbers certainly suggest that it could be as has been occasionally noted in the literature in the context of global supersymmetry or globally supersymmetric approximations to supergravity\textsuperscript{4,5,6,7,8}. Consider for example a contribution to the Kähler potential of the form

\[
K \supset \frac{L H_u \bar{N}}{M} + \frac{L H_d \bar{N}}{M} + \text{H.c.}
\]

where \( \bar{N} \) is the right handed neutrino and \( M \) is the scale at which higher dimensional operators first make their appearance in the Kähler potential. For the sake of argument assume that \( M = M_P = (8 \pi G_N)^{-1/2} = 2.44 \times 10^{18} \text{ GeV} \). One would expect the effective neutrino mass to be suppressed by a factor \( m_{3/2}/M \) which (taking \( \langle H_u \rangle = m_{\text{top}} \)) gives a neutrino mass \( m_\nu \sim 10^{-4} \text{ eV} \). This is rather small but intriguingly quite close to the measured\textsuperscript{21} value of (0.04 – 0.05) eV (within 1\( \sigma \)). Even more intriguingly, the measured value corresponds to taking \( M = 5 \times 10^{15} \text{ GeV} \), just below the GUT scale. We think that this coincidence deserves more careful inspection in the context of full supergravity\textsuperscript{2}.

The above operators are expected to be generated in various ways (perhaps from some kind of GUT theory or by the underlying string theory) and so \( M \) does not have to be close to \( M_P \). Because of this the scale \( M \) (when it was not set by some model building assumption or other) has always been treated as a moveable parameter. In this letter we take a more phenomenological approach. If the operators above are indeed responsible for the neutrino masses, what does the scale \( M \) of new physics have to be? Exploiting supergravity as a possible breakdown scenario of supersymmetry, we find that within this scenario, the scale \( M \) may differ by two orders of magnitude from the naive expectation above. Indeed, for gravitino masses of 100 GeV < \( m_{3/2} < 10 \text{ TeV} \), the correct mass automatically arises from the general couplings of supergravity if the scale \( M \) is in the range

\[
M = (4 \times 10^{16} - 5 \times 10^{17}) \text{ GeV}.
\]

This range, remarkably, is between the GUT scale and the heterotic string scale of old. At tree level the relation for the latter is \( M_s = 9_{\text{GUT}} M_P \approx 10^{18} \text{ GeV} \) if \( \alpha_{\text{GUT}} = 1/24 \). Including threshold effects in the \( \overline{\text{MS}} \) scheme gives\textsuperscript{10} \( M_s = 3.8 \times 10^{17} \text{ GeV} \). To find this result, we need to consider the contributions to fermion masses in full supergravity.

II. FERMION MASSES IN SUPERGRAVITY

Consider a set of chiral superfields \( \{ S_i, y_a \} \). The fields \( S_i \) are those fields of the hidden sector that are responsible for the spontaneous breaking of supergravity. They are assumed to be singlets of the gauge group, and we make no other assumptions about them or their superpotential, apart from that they eventually acquire a v.e.v. of order \( S_i \approx M \). It is convenient to set \( S_i = M \sigma_i \). The superfields \( y_a \) are those of the observable sector, namely \( y_a = \{ Q, U, D, L, E, N, H_u, H_d \} \). The most general superpotential, \( W \), and Kähler potential, \( K \), read \([11]\)

\[
W(\sigma, y) = W^{(h)}(\sigma) + W^{(o)}(\sigma, y),
\]

\[
K(\sigma, \sigma^*, y, y^\dagger) = K^{(h)}(\sigma, \sigma^*) + K^{(o)}(\sigma, \sigma^*, y, y^\dagger)
\]

where the superscript \((h)\) and \((o)\) denote hidden or observable superpotential and Kähler potentials, respectively.

\[
m_{\alpha\beta} = \frac{1}{2} \left\{ \frac{\partial^2 W^{(o)}}{\partial y^\alpha \partial y^\beta} - g^{\gamma\delta} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^\gamma \partial y^\beta} \frac{\partial W^{(o)}}{\partial y^\gamma} - \frac{1}{M^2} \left[ g^{ij\ast} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^i \partial y^{j\ast}} \frac{\partial W^{(h)}}{\partial \sigma^i} \right] \right. 
\]

\[
- \frac{1}{M} \left[ g^{\gamma\delta} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^\gamma \partial y^{\delta\ast}} \frac{\partial W^{(o)}}{\partial y^\delta} + g^{\delta\ast\gamma} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^\delta \partial y^{\gamma\ast}} \frac{\partial W^{(h)}}{\partial \sigma^\gamma} \right] \right.
\]

\[
- \frac{m_{3/2}}{2} g^{\gamma\delta} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^\gamma \partial y^{\delta\ast}} \frac{\partial K^{(o)}}{\partial y^\gamma} - \frac{\partial^2 K^{(o)}}{\partial y^\alpha \partial y^\gamma} \frac{\partial K^{(o)}}{\partial y^\gamma} + \frac{1}{M^2} \left[ g^{ij\ast} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^i \partial y^{j\ast}} \frac{\partial K^{(h)}}{\partial \sigma^i} \right] 
\]

\[
+ \frac{1}{M} \left[ g^{\gamma\delta} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^\gamma \partial y^{\delta\ast}} \frac{\partial K^{(o)}}{\partial y^\delta} + g^{\delta\ast\gamma} \frac{\partial^3 K^{(o)}}{\partial y^\alpha \partial y^\delta \partial y^{\gamma\ast}} \frac{\partial K^{(h)}}{\partial \sigma^\gamma} \right] \right\}. 
\]

In the above \( m_{3/2} \) is the gravitino mass given by

\[
m_{3/2} = \frac{\langle W^{(h)} \rangle}{M_P^2} \exp(K^{(h)}/2M_P^2),
\]

and we have taken the flat limit, \( M_P \to \infty \) and \( m_{3/2} \to \text{const.} \) We should remark here that we have made no other approximations in deriving Eq. (4). Contributions to the visible fermion masses in Eq. (4) arise from both the hidden and the observable sectors. We have divided the contributions to the fermion masses into two classes:

i) terms which are not proportional to the gravitino mass and survive in the global supersymmetry limit \( m_{3/2} \to 0, m_{3/2}M_P \to \text{const.} \) (the first two lines of Eq. (4)). Of these terms the first can be recognized as the standard term present in global supersymmetry. The second term arises purely from the observable sector. It was used by the authors of Ref. [6] in order to induce Majorana neutrino masses from dimension six Kähler operators. In our scenario, the third term in the first line of Eq. (4) is precisely the term that produces the dominant contribution to the neutrino masses. Note that this term has not previously been considered in the context of neutrino masses, and can significantly change any estimates that one might make within the framework of Supergravity. It vanishes in the limit of exact local supersymmetry transformations as it should. Terms in the second line of Eq. (4) can only be non-zero if the v.e.v. of the Kähler metric mixes fields from the visible sector with fields from the hidden sector. We shall not consider this possibility here.

ii) terms that are proportional to the gravitino mass (the last two lines of Eq. (4)) and exist only in the framework of supergravity. They depend only on the structure of the Kähler potential. Of these terms the second gives rise to a relatively suppressed Dirac neutrino mass and was used (in a different context) in Ref. [8]. Actually it is obvious that all terms in the third line of Eq. (4) can contribute to Dirac neutrino masses. The terms in the
fourth line of Eq. (4) require mixed hidden and observable sector kinetic terms and as with the terms in the second line of Eq. (4) we assume they are absent. They are only relevant when \( K^{(0)} \) and/or \( W^{(0)} \) contain a tadpole gauge singlet.

III. (PSEUDO-) DIRAC NEUTRINO MASSES

An obvious starting point for a theory of small Dirac neutrino masses is to prevent them from appearing directly in the superpotential. A natural solution to the \( \mu \)-problem would require in addition the non-existence of the operator \( H_u H_d \) in the superpotential. This can naturally be done with a discrete \( R \)-symmetry or perhaps some other symmetry. As a working example, let us consider an \( R \)-symmetry with \( R \)-characters for the matter superfields given by Table 1. To these we have added a right handed gauge singlet superfield \( N \) with \( R \)-character \( R(N) = n \). The symmetry has to be chosen so that the operators \( L H_u \bar{N} + H_d N \) are forbidden in the superpotential but are present in the Kähler potential. In addition we will for definiteness suppose that the singlet \( S \) has a non-zero \( R \) character as well, so that its appearance in the superpotential will be limited as we’ll see shortly. (Zero \( R \)-character for this singlet is also possible but necessitates other hidden sector fields.) The visible superpotential has \( R(W) = 2 \). The Kähler potential is \( R(K) = 0 \). We shall choose \( n = -1 \). For the moment we shall also assume lepton number conservation. The allowed terms are then

\[
W^{(o)}(\sigma, y) \supset Y_E L H_d \bar{E} + Y_D Q H_d \bar{D} + Y_U Q H_u \bar{U} + W^\sigma, \\
K^{(o)}(\sigma, \sigma^*, y, y^*) \supset c_1(\sigma, \sigma^*) H_u H_d + \frac{c_2(\sigma, \sigma^*)}{M} L H_u \bar{N} + \frac{c_3(\sigma, \sigma^*)}{M} H_d \bar{N} + H.c, 
\]

where \( M \) is our ultraviolet cutoff and \( W^\sigma \) is the \( \sigma \) dependent part of the superpotential which will be responsible for supersymmetry breaking (to be discussed later). As an example if \( R(s) = 2 \) then this could be a Polonyi-like term \( \beta S \) where \( \beta \) is some constant. The \( c(\sigma, \sigma^*) \) coefficients are the result of all perturbative and non-perturbative contributions to the Kähler potential so we do not need to insist that \( \sigma < 1 \) although this is where we need to be to have perturbative control. We may quite reasonably assume these coefficients and their derivatives to be of order one. Of course \( W^{(o)} \) and \( K^{(o)} \) contain other non-renormalizable terms, irrelevant to neutrino masses, of order \( 1/M \) and higher.

We can now use the master formula of Eq. (4) together with Eq. (7) to obtain the relevant terms for the Dirac neutrino masses. Consider for simplicity one singlet, \( \sigma \), and one generation of neutrinos with \( \chi^\alpha = \bar{\nu}_R \chi^\beta = \nu_L \);

\[
m_\nu^D = v \left( \frac{m_{3/2}}{M} \right) \sin \beta \left[ c_2(\sigma, \sigma^*) - c_1(\sigma, \sigma^*)c_3(\sigma, \sigma^*) \right] - v \left( \frac{F_S}{M^2} \right) \sin \beta \left[ \partial_\sigma^* c_2(\sigma, \sigma^*) + \cot \beta \partial_\sigma^* c_3(\sigma, \sigma^*) \right],
\]

where \( \partial_\sigma = \partial/\partial \sigma \) and

\[
F_S = \partial_S W^{(h)} + m_{3/2} \partial_S K^{(h)}. 
\]

Notice that in Eq. (8) there exists a source for Dirac neutrino masses which survives even in the global supersymmetric limit. In this limit, only the term proportional to \( \partial_S W^{(h)} \) remains and for \( m_\nu^D = (0.04 - 0.05) \text{ eV} \) we find

\[
\frac{F_S}{M^2} \simeq \frac{m_\nu^D}{v \sin \beta (1 + \cot \beta)} = (1.6 - 2.8) \times 10^{-13},
\]

where we used \( v = 174.1 \text{ GeV} \) and \( \tan \beta = 1 - 60 \) and have assumed that the v.e.v. of all the \( c \)'s and their

| \( Q \) | \( \bar{U} \) | \( \bar{D} \) | \( L \) | \( \bar{E} \) | \( \bar{N} \) | \( H_u \) | \( H_d \) |
|------|------|------|------|------|------|------|------|
| \( 2 - d - h \) | \( d + 2h \) | \( d - h - 2n \) | \( 2 - 2h + n \) | \( n - h \) | \( h \) |
derivatives are unity (there is the possibility of cancellation). This is a rather model independent result. In local supersymmetry for example, vanishing of the vacuum energy implies that $F_S = \sqrt{3} M_p m_{3/2}$, and varying 100 GeV $< m_{3/2} < 10$ TeV we obtain

$$4 \times 10^{16} \text{ GeV} < M < 5 \times 10^{17} \text{ GeV}. \quad (11)$$

The terms in the second line of Eq. (8) are enhanced by a factor $M_p/M$ relative to the terms in the first line, and thus are the dominant ones for any scenario. This is important for example in no-scale models where the gravitino can be quite light. In addition, one should note that the new non-holomorphic term proportional to $\partial_\sigma c_3$ dominates in Eq. (8), if $c_1$ and $c_2$ take on small values.

We should remark that the soft breaking masses of other particles such as squarks are proportional to $\tilde{m}_{2}$ if there are terms desirable, which requires some degree of tuning. For example it could be forbidden by hidden sector symmetries. This is model dependent and we will not present a detailed discussion of this as we would like to preserve our phenomenological approach.

We emphasize that the constraint of Eq. (10) survives in the global supersymmetric limit. This is an important condition for incorporating Dirac neutrinos in models with low scale supersymmetry breaking as for example in the case of gauge mediated supersymmetry breaking.

The values obtained for $M$ in Eq. (11), naturally lie between $M_{GUT}$ and the heterotic string scale $M_s$ for a very wide range of parameters, the result anticipated in the introduction. Note that the neutrino mass in Eq. (8) varies as the square of $M$ so that the value is rather accurately determined. This is the central point of this paper, that in supergravity small neutrino masses of the experimentally observed size can arise, and that neutrinos are predominantly Dirac fermions. In contrast, the operation of a see-saw mechanism demands the introduction of extra scale(s) in order to obtain the correct order of magnitude. Of course, the input in our case was an $R$-symmetry which forbade direct neutrino masses. However this requirement is more general than the neutrino mass problem at hand, since it is also necessary to resolve the $\mu$-problem.

If we relax the assumption of lepton number conservation then the Dirac neutrinos obtained from the Kähler potential can be “polluted” by the presence of active Majorana neutrino masses derived from extra non-renormalizable terms in addition to those in Eqs. (8-11):

$$W^{(o)}(\sigma, y) \supset \frac{g_4(\sigma)}{M} (L\tilde{H}_u)(L\tilde{H}_u), \quad (12)$$

$$K^{(o)}(\sigma, \sigma^*, y, y^*) \supset \frac{c_4(\sigma, \sigma^*)}{M^3} W^{(h)} N^2 + \text{H.c.} \quad (13)$$

Assume that only the first term is present. (The second term is quite high order to get a zero $R$-charge, so one could argue that such terms become suppressed.) Then from the first term in Eq. (12) with $\chi^a = \nu^*_L, \chi^b = \nu_L$ we obtain

$$m_{\nu}^D = g_4(\sigma) \frac{v^2}{M} \sin^2 \beta. \quad (14)$$

For the range of $M$ above we obtain $m_{\nu}^D = (3 \times 10^{-5} - 7 \times 10^{-4})$ eV. In summary, Dirac and Majorana neutrino masses (we consider one generation of neutrinos) are combined in the basis $(\chi^a = \nu_L + \nu^*_R, \omega = \nu_R + \nu^*_R)$

$$\langle \chi, \omega \rangle \left( \begin{array}{c} m_{\nu}^D \\ m_{\nu}^D \end{array} \right) \left( \begin{array}{c} \chi \\ \omega \end{array} \right), \quad (15)$$

with eigenvalues close to $m_{\nu}^D$. The small mass splitting between the two physical eigenstates is

$$\delta m^2 \simeq 2 m_{\nu}^D m_{\nu}^L = (3 \times 10^{-6} - 5 \times 10^{-5}) \text{ eV}^2, \quad (16)$$

and the mixing angle $\tan 2\theta = \frac{2 m_{\nu}^D}{m_{\nu}^L}$ very close to maximal, $\sin 2\theta = 1$. Thus, neutrinos are pseudo-Dirac [15, 16]; the Dirac neutrino splits into a pair of two maximally mixed Majorana neutrinos with almost equal masses. Furthermore, the effective mass for the neutrinoless double beta decay is given by [17],

$$\langle m_{\text{eff}} \rangle = \frac{1}{2} \sum_j U_{e j}^2 \delta m^2_{2m_j}, \quad (17)$$

where $U$ is the neutrino mixing matrix determined by the solar and atmospheric neutrino oscillations. Using the numbers quoted above, we find that Eq. (17) gives $\langle m_{\text{eff}} \rangle = (10^{-5} - 3 \times 10^{-4})$ eV. One cannot detect neutrinoless double $\beta$ decay of such small magnitudes, and these contributions are therefore unobservable for the foreseeable future. One may instead have to resort to astrophysical techniques to distinguish pseudo-Dirac from Dirac neutrinos [17]. Furthermore, if the $N^2$ operator of Eq. (13) is present and of equal size, the situation becomes highly involved, with the three generations of neutrinos having a general $6 \times 6$ mass matrix.

IV. QUESTIONS AND CONCLUSIONS

There are a number of questions that arise. The most pressing concerns the source of the non-renormalizable terms in the Kähler potential of Eq. (7). The analysis
presented here leads us to suspect that the required operators may appear simply as effective operators in heterotic string theories in much the same way as the \( \mu \) term does. The scale \( M \) may also appear radiatively in the Kähler potential, along the lines discussed in [18] or explicitly by construction in a GUT model. One aspect of this picture that we find appealing is that, in contrast with the see-saw picture, the connection with string or GUT scale physics is rather immediate. The neutrino masses and mixings are not filtered through unknown Majorana terms but carry direct information about the structure of the Kähler metric. This fact certainly offers new opportunities for neutrino model building.

In this paper we have been arguing that the scale of neutrino masses may quite easily be associated with the scale of supersymmetry breaking and hence the Weak/Planck scale hierarchy, in the very same way that the \( \mu \)-term can. Although this is a general observation, we are obliged to present a simple model of supersymmetry breaking where \( F_\mu \) is generated with the correct size with the charges we have been using. Consider for example an \( R \)-charge for the singlet \( R(S) = 1 \). In this case the supersymmetry breaking part of the potential can take the form

\[
W^\sigma = \beta S^2
\]

where \( \beta \) is a dimensionful coupling of order \( M_W \). The fact that this represents a fine-tuning is of course the usual tuning problem associated with supersymmetry breaking. \( S \) needs to get a v.e.v and in order for this to happen we may further suppose that the \( R \)-symmetry we are using is gauged and anomalous. Such models were considered in ref. [19], and it is known that such a symmetry must be broken at scales \( M \lesssim M_P \), and that there are no effects from gauging the \( R \)-symmetry remaining at low energies. Because of the \( R \)-charge of \( S \) it is now perfectly natural for \( S \) to get a v.e.v of order \( M \) from the Fayet-Iliopoulos \( D \)-term of the \( R \)-symmetry, especially as it has no other \( D \) terms to force it to zero v.e.v. This then gives

\[
(W^\sigma) \sim \beta M^2; \quad F_S = \beta M^2 \frac{K_S}{M_P^2} + W_S \sim \beta M.
\]

The value of \( F_S \) may now be tuned to \( \sqrt{3}M_W M_P \) get zero cosmological constant as usual. But the point is of course that we now have to make no additional tuning to get the Dirac neutrino masses of the right order and this is the main finding of the paper.

Another important question is how to account for non-trivial (maximal) neutrino mixing matrix. The answer to this question may be linked to the fact that the Kähler potential parameters are not protected by the non-renormalization theorem, and vertex corrections may induce large flavour mixing through Renormalization Group running.

To summarize, we have shown that minimal supergravity naturally allows Dirac masses without the ad-hoc addition of any new mass scales. If there is lepton number conservation, then the MSSM naturally contains pure Dirac neutrino masses that are comparable to the atmospheric neutrino mass. The only other remnant would be a slowly decaying right-handed s-neutrino with mass \( \sim 1 \) TeV. We have throughout been focussing on the atmospheric neutrino mass, but the remaining masses and mixings could be generated by the Yukawa couplings in the Kähler potential of Eq. (1) in much the same way as the quark masses and mixings. If lepton number is violated, then we have seen that it is possible to get either pseudo-Dirac neutrinos or a general \( 6 \times 6 \) Majorana mass matrix structure with naturally small elements. Finally, we should remark that baryogenesis can be accommodated via leptogenesis with Dirac neutrinos [20].

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[21] We will throughout be assuming that the measured mass-squared differences are indicative of the actual masses. We focus on the atmospheric neutrino mass.
[22] Here $\chi_\alpha$ are the fermion superpartners of the scalar fields $y_\alpha$ in the observable sector.
[23] If lepton number is violated, the $R$-character of the superfield $N$ classifies the additional neutrino mass operators and, although we do not present them here, models with other less phenomenologically appealing choices are possible.