Moving-Horizon Predictive Input Design for Closed-Loop Identification

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Abstract: This paper presents a new approach to input design for closed-loop identification. The idea is to maximize the trace of the Fisher information matrix associated with the plant model, while enforcing explicit constraints on both inputs and outputs. The result is the richest possible excitation signal for which the operation of a running closed-loop system remains within acceptable bounds. The function to be maximized is a convex quadratic. A Moving Horizon Predictive (MHP) framework is used to solve the input design problem at each sample time. The method can be combined with a fixed model variable regressor technique to estimate time delays. The suggested technique is implemented and used to identify machine-directional processes in an industrial paper machine.

Keywords: Closed-Loop Identification, Input Design, Optimization Problems, Parameter Estimation, Recursive Estimation

1. INTRODUCTION

The accuracy of an estimated model depends on the richness of the excitation signals used to probe the system. The identifiability condition (Soderstrom et al. (1976)) quantifies the degree of excitation required. Identification in industry often proceeds by adding a PRBS1 to the plant input. Of course, any perturbation of the process inputs also affects the outputs, and hence the quality of the output products. Thus, identification experiments are not free; the need to extract sufficient information about the process at an acceptable cost has driven many researchers to study optimal input design for closed loop identification. For instance, Wahlberg et al. (2010) calculate a minimum power input sequence to obtain an estimated model which satisfies the control performance. See also, Pronzato (2000), Forssell and Ljung (2000), Belforte and Gay (2002), Jauberthie et al. (2006), Rojas et al. (2007), Franceschini and Macchietto (2008), Pronzato (2008).

Typically, one tries to find an input with minimum variance that optimizes some function of the covariance matrix of estimated parameters and guarantees the identifiability of a related system. Jansson and Hjalmarsson (2004a) and Jansson and Hjalmarsson (2004b) propose a general framework to deal with such problems. Jansson and Hjalmarsson (2005) also suggest a method to solve the input design problem by parameterization of the covariance matrix. Gopaluni et al. (2011) use a particle filtering approach to solve the input design problem for nonlinear stochastic dynamic systems. Valenzuela et al. (2013) use results from graph theory to decrease the computational difficulties in optimal input design. Bombois et al. (2005) specify a condition for costless identification which is not applicable in most practical cases. Yan et al. (2009) show how a process with an MPC controller can be sufficiently excited by changing the controller parameters instead of modifying the input signal directly.

Most of the above mentioned input design methods rely on knowledge of true systems. In addition, they are very complex and complicated to use in practice. Patwardhan and Gopaluni (2014) suggest a completely different technique. They show that for a process controlled by an MPC controller, the MPC objective function is equal to the trace of the Fisher information matrix, assuming the process is compatible with an ARIX model. Therefore, if we switch the MPC optimization problem from minimizing the objective function to maximizing the objective function, the process will be sufficiently exciting for identification purposes. Due to the fact that we keep all MPC constraints, the process inputs and outputs never diverge. However, the main characteristic of this technique that may concern users is that the controller does not work in a normal operating mode. Technically, in this situation, the process does not operate under closed-loop control and some concerns about the stability and other control performance criteria show up.

The method proposed in this paper needs only an initial approximation for the process model and an estimate of the maximum perturbation from the true model to ensure robustness. This method is implemented in a moving horizon framework. At each sample time, by estimating the process model recursively,
using a direct method, the model used in the input design algorithm gets updated. Over time, it converges to the true model. The approach is compatible with output constraints along with amplitude and frequency constraints on the inputs. Thus, it complies with requirements on safety and financial concerns of industrial practice.

In the rest of this paper, we show that the trace of the Fisher information matrix is a convex quadratic function of the optimization parameters. For such functions, constrained minimization is a well-understood problem, for which efficient computational algorithms exist. Here, however, the goal is to maximize the trace of the information matrix subject to given constraints. This is considerably more challenging, especially when the vector of parameters to be chosen is high-dimensional. Special characteristics of typical control systems make it possible to suggest a practical method that decreases the dimension of the problem by splitting it into a sequence of lower-dimensional problems. The suggested technique is practical, easy to implement, and applicable to closed-loop systems built around any type of controller.

2. PARAMETER ESTIMATION

We will consider a linear MIMO system in discrete time, described by the following ARX model:

\[ A(q, \theta)y_t = B(q, \theta)u_t + e_t, \]  

(1)

Here \( y_t \) is a (column) vector of \( n \) output components, \( u_t \) is a vector of \( m \) inputs, \( e_t \) is an \( n \)-component vector of white noise with invertible covariance matrix \( \Sigma_e \). The system matrices \( A(q, \theta) \) and \( B(q, \theta) \) have dimensions \( n \times n \) and \( n \times m \), and depend on the delay operator \( q^{-1} \) as follows:

\[ A(q, \theta) = I - A_1 q^{-1} - \cdots - A_n q^{-n_A} \]

(2)

\[ B(q, \theta) = B_1 q^{-d} + \cdots + B_n q^{-d-n_B}. \]

(3)

The positive integers \( n_A \) and \( n_B \) define the orders of the polynomial entries in matrices \( A \) and \( B \), and \( d \geq 1 \) is the minimum time delay in matrix \( B \). The coefficient matrices in (2)–(3) constitute the parameters of the system model, and can be gathered into the block matrix \( \theta \) of shape \( n \times (n_A + n_B) \) defined as

\[ \theta = [A_1 \ldots A_n B_1 \ldots B_n]. \]

(4)

With these definitions, the output vector at time \( t \) satisfies

\[ y_t = \theta \psi_t, \]

(5)

where \( \psi_t \) is the \( (n_A + n_B) \times 1 \) vector defined by

\[ \psi_t = \left[ y_{t-1} \ldots y_{t-n_A} u_{t-d} \ldots u_{t-d-n_B} \right]. \]

(6)

It is assumed that the true system has the form described above for some (unknown) value \( \theta = \theta_0 \). The one-step ahead predictor for the model (1) is

\[ \hat{y}_t = (I - A(q, \theta)) y_t + B(q, \theta) u_t \]

(7)

and the vector of prediction errors is

\[ e(t, \theta) = y_t - \hat{y}_t. \]

(8)

The parameters can be estimated by minimizing the mean squared prediction error Ljung (1999). Given \( N \) data values, this produces the estimate

\[ \hat{\theta}_N = \arg \min_{\theta} \frac{1}{N} \sum_{i=1}^{N} [e(t, \theta) e(t, \theta)]^\top. \]

(9)

Well-known methods Ljung (1999) lead to the explicit formula

\[ \hat{\theta}_N = Y \Psi^\top (\Psi \Psi^\top)^{-1} \]

(10)

where

\[ Y = \begin{bmatrix} y_1 & \cdots & y_{N-1} \end{bmatrix}, \]

(11)

\[ \Psi = \begin{bmatrix} \psi_1 & \cdots & \psi_{N-1} \end{bmatrix}. \]

(12)

The above process can be implemented recursively (recursive least squares) to update the parameter matrix \( \hat{\theta}_N \) at each sample time. The accuracy of the estimate depends on the covariance of the estimated parameters. According to the mean square consistency theorem (Lai et al. (1985)), if the process is stable and the error \( e_t \) is normally distributed with mean 0 and \( \text{cov}(e_t) = \sigma^2 I \), then the least square estimate is mean square consistent if

\[ \lim_{N \to \infty} \text{var} (\text{cov}(\hat{\theta}_N)^{-1}) = 0, \]

(13)

where \( \text{cov}(\theta) = (\Psi \Psi^\top)^{-1} \). Accordingly, smaller values for \( \text{cov}(\theta) \) correspond to better estimates. Meanwhile, equations (6), (12) show that the covariance of the estimated parameters depends on the inputs. Thus, the design of the input signal has a direct effect on the quality of the resulting estimate.

3. INPUT DESIGN

As noted in (13), the quality of an estimate depends on the covariance of the estimated parameters. The covariance, in turn, depends on the input signal—recall (6), (12), (13). Thus, input design strategies for identification purposes are often formulated to minimize some scalar associated with the size of the covariance matrix. For instance, one can minimize the trace, the largest eigenvalue, or the determinant of the covariance matrix. In the large literature on optimal input design, a typical goal is to find the minimum-power input sequence that guarantees process identifiability (Soderstrom et al. (1976)). This is a challenging and complex problem.

In this work, we do not minimize the trace of the covariance matrix. Instead, we maximize the trace of its inverse—i.e., the trace of the Fisher information matrix

\[ F = \text{cov}(\hat{\theta}_N)^{-1} = \Psi \Psi^\top. \]

(14)

Clearly trace(\( F \)) is an unbounded function of the inputs, so suitable constraints must be introduced to produce a well-posed maximization problem.

In detail, \( R_d = \text{trace}(F) \) is given by

\[ R_d = \sum_{q=1}^{n_A} \sum_{j=1}^{n_B} \left[ \text{cov}(y_t | q) \text{cov}(u_t) + \text{cov}(u_t | q) \text{cov}(y_t) \right] = \sum_{q=1}^{n_A} \sum_{j=1}^{n_B} \sum_{i=1}^{n_B} \sum_{j=1}^{n_B} \left[ \text{cov}(y_t | q) \text{cov}(u_t) \right]. \]

(15)

where \( y_t \) and \( u_t \) denote the \( q \)th output and input in output and input vectors, respectively. We are assuming that the true process model has the form described in section 2. It follows that the process outputs for times \( t - N \) through \( t - 1 \) satisfy

\[ Y_t = X U_{t-d}. \]

(16)

where \( Y_t = [y_{t-N}, \ldots, y_{t-1} | q), U_{t-d} = [u_{t-d}, \ldots, u_{t-d-n_B}], \) and \( X \) is an \( Nn \times Nm \) matrix whose elements are generated by the true system parameters. Using (16) and assuming the input before time \( t - d - N \) is zero, the \( R_d = \text{trace}(F) \) can be written as follows:

\[ R_d = \sum_{i=1}^{n_B} \sum_{i=1}^{n_B} X_{i,j} U_{i,j} \]

(17)

Here \( X_i \) and \( U_i \) are block matrices of shapes \( Nn \times Nm \) and \( Nm \times Nmm \) matrices, respectively, defined by
\[
\chi_i = \begin{bmatrix}
O_{m \times Nm} & I_{(N-i) \times m} \times \chi
\end{bmatrix},
\]
\[
t_i = \begin{bmatrix}
O_{(i-1) \times Nm} & I_{(N-i+1) \times (i-1)m} \times \chi
\end{bmatrix},
\]
where \( I \) and \( O \) are identity and zero matrices, respectively. Defining the symmetric, positive-semidefinite matrix
\[
G = \sum_{i=1}^{n_A} \chi_i^T \chi_i + \sum_{i=1}^{n_B} t_i^T t_i
\]
leads to a succinct presentation of (17):
\[
R_u = U_{i-d}^T G U_{i-d}.
\]
This is a convex quadratic function of the vector variable \( U_{i-d} \).

Since \( R_u \) is a convex quadratic function, any critical point will be a global minimizer. However, we seek a maximizer. Existence of a solution requires the set of competing inputs to be closed and bounded. Thus the mathematical formulation corresponds to the need to explicitly recognize the limitations on permissible inputs for the process under study. In closed-loop identification, the designed input signal will be added to the control signal. To keep output variations small and to control the bandwidth of the excitation can be adjusted by filtering. As shown in Fig. 1, the controller’s output \( u_i^* \) is augmented by \( \tilde{u}_i^* \), where
\[
\tilde{u}_i^* = H(q^{-1}) u_i^*
\]
for some filter \( H(q^{-1}) \) chosen to satisfy the frequency constraints. By analogy with (16), the filtered input can be written as
\[
\tilde{U}_i = \vartheta U_i,
\]
where \( U_i = [u_i^* \ldots u_{i-N+1}^*] \) and \( \vartheta \) is an \( Nm \times Nm \) matrix which is built based on \( H(q^{-1}) \). Accordingly, the matrix \( G \) first defined in (20), is modified as follows,
\[
G = \sum_{i=1}^{n_A} \chi_i^T \chi_i + \sum_{i=1}^{n_B} \vartheta_i^T \vartheta_i,
\]
where
\[
\vartheta_i = \begin{bmatrix}
O_{(i-1) \times Nm} & I_{(N-i+1) \times (i-1)m} \times \vartheta
\end{bmatrix}.
\]
Further, the matrices \( \chi_i \) in (26) have the same block structure as in (18), but now the matrix \( \vartheta \) used to build them must be made from the best current estimate of the system parameters (the true parameter values are unknown).

In summary, the input design problem is the following non-standard quadratic optimization problem with convex constraints:
\[
\begin{align*}
\text{maximize} & \quad R_u \\
\text{subject to} & \quad y_L \leq y_t \leq y_H, \\
& \quad u_L \leq u_t \leq u_H.
\end{align*}
\]
A maximizer in this problem provides an input sequence that gives maximum information about the process subject to the given input/output constraints. This information can be used to estimate the best possible model of the process under these conditions.

Fig. 1 shows the closed-loop system during the modelling experiment. In this block diagram, \( P(q^{-1}) \) and \( C(q^{-1}) \) denote the process and the controller, respectively. Once the raw signal \( u_i^* \) is designed, it is filtered and added to the control signal \( u_t^* \). Their sum becomes the process input. Subsequently, by measuring the input and output signals, the process can be identified using the direct method recursively. When the estimated parameters appear to settle on values we can use, we can stop exciting the process and use the estimated model for other purposes.

### 3.1 Moving Horizon Framework

The length of the excitation signal we design for identification purposes can be called a prediction horizon. In the suggested input design technique, we build the Fisher information matrix by predicting the future outputs. The output prediction over the prediction horizon is carried out based on the initial approximation of the process model. Due to the existence of output disturbances and the differences between the true model and the model used to formulate the input design, when the system is excited with the designed signal, there is no guarantee that the outputs do not go beyond constraints.

To remedy the problem, we can implement the algorithm in moving horizon fashion. Using this framework, instead of solving the input design problem once, it can be solved at each sample time. Then the first element of the designed input sequence is sent through the process to provide the excitation and the remaining terms are discarded. In the next sample time, the process model is updated using the recursive estimation technique and the updated model is used in the input design problem to generate a new excitation signal. Fig. 2 depicts the moving horizon approach to input design over time. Fig. 3 illustrates the structure of the closed-loop system during the closed-loop identification experiment while the moving horizon framework is used.

### 4. ROBUSTNESS ISSUES

An identification experiment is typically initiated when the current model of the process fails to accurately describe the process’s behaviour. Since this questionable current model is a key element of the suggested input design technique, one might be concerned that the output constraint may not be met when the designed signal is applied to the true process at the beginning of the experiment. To deal with this issue, the output constraints in (22) must be changed as follows. Let \( \theta_{\text{base}} \) denote the matrix of parameters in the current model, on which the excitation signal is based. Then choose a matrix \( \Delta \) of...
Fig. 2. Illustration of moving horizon approach in input design compatible dimension that specifies a maximum variation from the nominal parameters to the true ones, so that
\[
\theta = \theta_{\text{nom}} + \Delta
\]  
(29)
\Delta can be estimated based on some close loop performance measures. Accordingly, the output constraint in (22) is replaced as follows,
\[
y_L \leq (\theta_{\text{nom}} + \Delta) y_{t} \leq y_H,
\]  
(30)
and the output constraint in problem (28) becomes
\[
y_L - \Delta y_{t} \leq y_{t} \leq y_H - \Delta y_{t}.
\]  
(31)

5. TIME DELAY ESTIMATION

The time delay plays an important role in the input design formulation discussed in the previous sections, where it was assumed to be known. Thus, it is important to have an acceptable estimate of the time delay before starting to design an excitation signal. A number of methods (Elmaggar (1990), Ferretti et al. (1991)) have been suggested for online estimation of the time delay \(d\) in (1)-(3). Elmaggar (1990) proposes Fixed Model Variable Regressor Estimation (FMVRE) for this purpose in industrial processes. Lynch and Dumont (1996) demonstrate the effectiveness of FMVRE for finding the time delay in the performance monitoring of industrial processes. On the strength of this proven success, ease of implementation, rapid convergence, and independence of the estimation approaches for other model parameters, we will use FMVRE to estimate the time delay in (1).

When the delay \(d\) is known only to lie in an interval \([d_{\min}, d_{\max}]\), an estimate \(\hat{d}\) can be obtained by solving the following minimization problem:
\[
\hat{d} = \arg \min_{d'} \mathbb{E} [e(t, d')],
\]  
(32)
Here \(e(t, d')\) is the prediction error from (8). Elmaggar (1990) shows that for an auxiliary model of first order and a process with a positive gain, the minimization in (32) is equivalent to the maximization of
\[
E_1(d') = r_{xy}(d') - r_{xy}(d' - 1)
= \mathbb{E} [(y_{t} - y_{t-1})' u_{t-d'}],
\]  
(33)
where \(r_{xy}(d')\) is the cross-correlation between input and output signals at lag \(d'\) (if the process gain is negative, the function to be minimized is \(-E_1\)). To solve this estimation problem recursively, introduce the time-varying quantity
\[
E_1(t, d') = \lambda E_1(t-1, d') + [(y_{t} - y_{t-1})' u_{t-d'}],
\]
where \(\lambda\) is a tuning parameter which affects the rate of convergence. Then, instead of the one-shot maximization of \(E_1\) in (33), generate a sequence of delay estimates as follows:
\[
\hat{d}(t) = \arg \max_{d'} \{E_1(t, d')\} \quad d' \in [d_{\min}, d_{\max}]\).  
(34)
Elmaggar (1990) shows that to obtain a good estimate of the time delay, the input must be sufficiently rich.

As discussed above, one of the advantages of the FMVRE method for estimating time delay is the rapid convergence of the algorithm. Therefore, we can design an excitation signal based on the current model of a process. Using the designed signal, we can estimate the time delay of the process. Based on the updated time delay, the excitation signal can be redesigned and used to estimate the other parameters of the process.

6. SIMULATION RESULTS

In this section, we use the technique described above to identify machine directional processes in a paper machine operating in closed-loop. The system has two inputs and two outputs, namely,
\[
u^{(1)} - \text{stock flow}, \quad y^{(1)} - \text{dry weight},
\]
\[
u^{(2)} - \text{dryer pressure}, \quad y^{(2)} - \text{size press moisture}.
\]
As in many processes, each input/output relation can be captured by a first order model with dead time. We postulate
\[
\begin{pmatrix}
y^{(1)}_t \\
y^{(2)}_t
\end{pmatrix}
= \begin{pmatrix}
\frac{b_1 q^{-d_1}}{1-a_1 q^{-1}} & 0 \\
\frac{b_{21} q^{-d_1}}{1-a_1 q^{-1}} & \frac{b_{22} q^{-d_2}}{1-a_2 q^{-1}}
\end{pmatrix}
\begin{pmatrix}
u^{(1)}_t \\
u^{(2)}_t
\end{pmatrix},
\]  
(35)
The above model for paper machines is obtained from Yousefi et al. (2014). (35) can be rearranged to produce the deterministic elements of the standard ARX model (1). The constraints on the designed excitation signals and the output signals are
\[
\begin{pmatrix}
-12 \\
-10
\end{pmatrix} \leq y_t \leq \begin{pmatrix}
12 \\
10
\end{pmatrix}
\]  
(36)
\[
\begin{pmatrix}
-10 \\
-10
\end{pmatrix} \leq u_t \leq \begin{pmatrix}
10 \\
10
\end{pmatrix}
\]  
(37)
\[
f_b \leq 0.159 \text{ rad/s}
\]  
(38)

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where $f_b$ denotes the bandwidth of the designed signal. Now MHP is used to generate the excitation signal at each sample time and the designed signal is added to the control signal to provide the excitation needed for identification. Fig. 5 illustrates a sequence of the calculated excitation signal over time.

Fig. 4 shows the outputs of the process under the closed-loop condition. It can be seen that the outputs sometimes cross the upper and lower limits. This happens because we add the excitation signal to the control signal, and the feedback controller acts to reject disturbances. Thus, these two signals together cause an extra perturbation in the outputs. Indeed, we are trying to keep the outputs within the tight limits. Therefore, if we tighten the output constraints, we will make sure that the output values will stay inside the specified bounds.

Once the optimal excitation signals are determined and added to the process inputs, we start using the measured inputs and outputs of the process to estimate the model parameters recursively. See Fig. 1. In the first step, we estimate the time delay from each input to each output using FMVRE. Fig. 6 shows the results. As discussed in section 5, the estimated values converge to the true values very quickly. Since the estimated time delays are the same as the given model, there is no need to update the time delay in the optimization problem equations.

Given the estimated time delays, we estimate the parameters of the model using recursive least squares. Fig. 7 shows how the estimated parameters converge to the true values.

As mentioned earlier, the suggested approach to input design is independent of the type of controller used in the closed-loop system. In addition, since the signal is added to the control signal, it does not inhibit the controller from rejecting disturbances appearing in the outputs. Fig. 8 shows that if a big step disturbance happens during the modeling experiment, although it causes the output to cross the upper limits, the controller brings it back within the desired bounds. In this figure, a step disturbance of size 10 occurs at sample time 18.

7. CONCLUSION AND FUTURE WORK

In this paper a Moving Horizon Predictive technique was proposed for generating sequences of input perturbations for the purpose of closed-loop process identification. The method maximizes the trace of the Fisher information matrix, a quadratic function of the system inputs, subject to constraints on process inputs and outputs over the horizon of the planned experiment. It can be implemented in parallel with any type of controller, and allows for adjustment of the constraints to ensure robustness to the unavoidable mismatch between the existing process model and the true plant characteristics.

Finding a method to solve the constrained quadratic problem defined in the proposed input design technique can be suggested as a future work. This is a high dimensional and non-convex problem to solve. Indeed, to generate $N$ input samples for a process whose input vector $u$ has dimension $m$, the number of scalar choice variables is $Nm$. So it is a very difficult problem to solve for quadratic solvers that can handle non-
Fig. 8. Dry weight signal perturbed with a step disturbance convex function, e.g., \textit{fmincon} function in MATLAB, and there is no guarantee to get a global optimum solution. The function \( R_u \) is convex, so its maximum over any compact convex domain will be achieved at an extreme point. The constraints in (28) determine a polyhedron in the space of possible inputs and the extreme points of this set are the vertices of the polyhedron. It is easy to construct examples in which a given vertex provides a local maximum but not a global one and hence situations in which standard optimization solvers fail to identify a true maximizer. The number of vertices in (28) grows exponentially with the dimension of the problem, so any strategy based on simply visiting each one is out of the question. Therefore, it helps a lot if one can suggest a solver which reduces the computational burden of solving the high-dimensional non-convex optimization problem and guarantees the global optimum answer.

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