Twist-3 Single-Spin Asymmetry for SIDIS and its Azimuthal Structure

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Abstract. We derive the complete twist-3 single-spin-dependent cross section for semi-inclusive DIS, \( ep \to e \pi X \), associated with the complete set of the twist-3 quark-gluon correlation functions in the transversely polarized nucleon, extending our previous study. The cross section consists of five independent structure functions with different azimuthal dependences, consistently with the transverse-momentum-dependent (TMD) factorization approach in the low \( q_T \) region. Correspondence with the inclusive DIS limit and comparison with the TMD approach are briefly discussed.

Keywords: Single spin asymmetry, Twist-3, Azimuthal asymmetry in semi-inclusive DIS

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In our recent paper [1], we have presented the twist-3 single-spin-dependent cross section for the large-\( q_T \) pion production in semi-inclusive DIS (SIDIS), \( ep \to e \pi X \), in the framework of the collinear factorization, in particular, the cross section associated with the twist-3 quark-gluon correlation functions. There we have also clarified the gauge-invariance and the factorization property of the corresponding twist-3 cross section. This report updates the result, supplying the calculation not presented in [1].

For the kinematic variables of SIDIS, \( e(\ell) + p(p, S_\perp) \to e(\ell') + \pi(P_h) + X \), we use \( S_{ep} = (\ell + p)^2 \), \( Q^2 = -q^2 = -((\ell - \ell')^2, x_{bj} = \frac{Q^2}{2p_q}, z_f = \frac{p_q}{p_{\pi}^h} \) and \( q_T = \sqrt{-q_{\mu}^T} \) with \( q_\mu^T = q_\mu - \left( \frac{p_{\pi}^h}{p_{\pi}^h} \right) p_\mu - \left( \frac{p_q}{p_{\pi}^h} \right) P_{\mu}^h \). We work in a frame where the momenta of the virtual photon and the initial nucleon are collinear, and use the lepton plane as a reference plane to define the azimuthal angles \( \phi_h \) and \( \phi_S \) of the hadron plane and the transverse spin vector \( S_\mu^\perp \), respectively. In this frame the magnitude of the transverse momentum of the final pion is given by \( P_{h\perp} = z_f q_T \). With this notation, one can present the azimuthal dependence of the twist-3 single-spin-dependent cross section in the following form [2]:

\[
\frac{d^5 \sigma^{tw3}}{[d\omega]} = \sin(\phi_h - \phi_S)(\Delta \sigma_1 + \Delta \sigma_2 \cos(\phi_h) + \Delta \sigma_3 \cos(2\phi_h))
+ \cos(\phi_h - \phi_S)(\Delta \sigma_4 \sin(\phi_h) + \Delta \sigma_5 \sin(2\phi_h)),
\]

(1)

with the differential element \([d\omega] \equiv dx_{bj}dQ^2dz_fdq_T^2d\phi_h \). In [1], we have presented the result only for \( \Delta \sigma_{1,2} \). In addition, some diagrams that produce the “soft-fermion-pole (SFP)” contribution to \( \Delta \sigma_2 \) were missing as pointed out in [3]. In this report we will present the main features of the full result, taking these points into account.

In the twist-3 mechanism for SSA, quark-gluon correlation functions in the transversely polarized nucleon play the central role: There exist two independent twist-3 distribution functions \( \{ G_F(x_1, x_2), \bar{G}_F(x_1, x_2) \} \) defined from the Fourier transform of the
correlation function \( \langle pS_\perp | \bar{\psi}(0)F^{\alpha+}(\xi n)\psi(\lambda n)|pS_\perp \rangle \) on the light cone \((n^2 = 0, F^{\alpha\beta}\) the gluon’s field strength), where \(x_{1,2}\) denote the momentum fractions associated with the quark fields \(\psi, \bar{\psi}\). \(G_F(x_1,x_2)\) is symmetric, while \(\tilde{G}_F(x_1,x_2)\) is anti-symmetric, under \(x_1 \leftrightarrow x_2\). By replacing the above nonlocal operator \(\bar{\psi}F^{\alpha+}\psi\) by its charge conjugate, one can define the twist-3 distribution functions for the "anti-quark" flavour \([2]\). Also, replacing \(F^{\alpha+}\) by the covariant derivative \(D^\alpha\), one can define other twist-3 distributions; the relation between \(\{G_F, \tilde{G}_F\}\) and those other twist-3 distributions was also clarified in \([4]\).

In the twist-3 mechanism, the single-spin-dependent cross section occurs as pole contributions from an internal propagator in the partonic hard part. For SIDIS, three kinds of poles contribute, which are classified as soft-gluon-pole (SGP), hard-pole (HP) and SFP. Below we discuss the characteristic features of each contribution from these poles.

1. SGP contribution

The SGP contribution occurs from diagrams shown in Figs. 8 and 10 of \([1]\). Those diagrams have a propagator pole at \(x_1 = x_2\), and its evaluation gives rise to SGP contribution with \(G_F(x,x)\) only, since \(\tilde{G}_F(x,x) = 0\) due to the symmetry property. One can directly calculate the contribution to \(\Delta \sigma_{4,5}\) from these diagrams. Alternatively, one can use the fact that the SGP cross section for SIDIS can be derived directly from the twist-2 unpolarized cross section formula as proved in \([5]\). This way, we reported the full SGP result in the erratum of \([5]\) as

\[
\frac{d^5 \sigma^{\text{tw3,SGP}}}{[d\omega]} = \frac{\alpha_s^2 \alpha_g}{8\pi^2 b_j^2 s_{ep} Q^2} \frac{\pi M_N}{C_F} \sum_q \mathcal{C}_q \sum_{j=q,g} \mathcal{C}_j \int \frac{dz}{z} \int \frac{dx}{x} D_j(z) \left[ \frac{q_T}{Q^2} \sin(\phi_h - \phi_S) \right.
\]

\[
\times \sum_{k=1}^4 \mathcal{A}_k \left\{ \frac{\hat{x}}{1-\hat{z}} \tilde{\sigma}_k^{ijg} \frac{dG_F^{ij}(x,x)}{dx} + \left( \frac{1}{\hat{z}} \right) \frac{\partial \tilde{\sigma}_k^{ijg}}{\partial q_T^2} - \frac{\hat{x}}{1-\hat{z}} \frac{\partial \tilde{\sigma}_k^{ijg}}{\partial \hat{x}} \right\} G_F^{ij}(x,x) \right]
\]

\[
- \frac{\cos(\phi_h - \phi_S)}{z q_T} \left( \frac{1}{2} \mathcal{A}_5 \tilde{\sigma}_3^{ijg} + \mathcal{A}_9 \tilde{\sigma}_4^{ijg} \right) G_F^{ij}(x,x) \delta \left( \frac{q_T^2}{Q^2} - \left( \frac{1}{\hat{x}} - 1 \right) \left( \frac{1}{\hat{z}} - 1 \right) \right) \]  

(2)

where the sum over \(q\) runs over all quark and anti-quark flavours and \(\tilde{\sigma}_k^{ijg}(k = 1, \cdots, 4)\) are the partonic hard cross section for the twist-2 unpolarized cross section as listed, for example, in \([1]\). \(\mathcal{C}_q = -1/(2N)\) and \(\mathcal{C}_g = N/2\) are the color factors, and \(\hat{x} = x_{bj}/x, \hat{z} = z_f/z\). The factors \(\mathcal{A}_k (k = 1, \cdots, 4, 8, 9)\) are defined as \(\mathcal{A}_1 = 1 + \cosh^2 \psi, \mathcal{A}_2 = -2, \mathcal{A}_3 = -\cos \phi_h \sinh 2 \psi, \mathcal{A}_4 = \cos 2 \phi_h \sinh^2 \psi, \mathcal{A}_8 = -\sin \phi_h \sinh 2 \psi, \mathcal{A}_9 = \sin 2 \phi_h \sinh^2 \psi\) with \(\cosh \psi = 2 x_{bj} s_{ep}/Q^2 - 1\). From \((2)\), one sees that the SGP contributions to \(\Delta \sigma_{4,5}\) are related to those for \(\Delta \sigma_2\) and \(\Delta \sigma_3\), respectively, although \(\Delta \sigma_{4,5}\) do not receive “derivative” contribution of the SGP function, \(dG_F^{ij}(x,x)/dx\), unlike the \(\Delta \sigma_{2,3}\) terms.

2. HP contribution

Diagrams for the HP contributions are shown in Fig. 2 of \([1]\), whose internal propagator has a pole at \(x_1 = x_{bj}\). Evaluating it, both \(G_F(x_{bj}, x)\) and \(\tilde{G}_F(x_{bj}, x)\) contribute. By the direct calculation of the \(\gamma_{8,9}\) components in the expansion of the hadronic tensor from these diagrams (see Eq.(50) of \([1]\)), one can obtain the HP contribution to \(\Delta \sigma_{4,5}\) in \([1]\). It turned out that both \(G_F\) and \(\tilde{G}_F\) contribution satisfy the relation,

\[
\Delta \sigma_4^{\text{HP}} = \Delta \sigma_2^{\text{HP}}, \quad \Delta \sigma_5^{\text{HP}} = \Delta \sigma_3^{\text{HP}}.
\]

(3)
3. SFP contribution

The SFP contributions, from the propagator pole at \( x_1 = 0 \), exist only in the anti-quark fragmentation and gluon fragmentation channels for the “quark distribution”, \( G_F(0,x) \) and \( \tilde{G}_F(0,x) \) with \( x > 0 \). The diagrams for those are shown in Fig. 1. In the quark fragmentation channel with the same \( G_F(0,x) \), \( \tilde{G}_F(0,x) \), the diagrams in Fig. 6 of [3] and those in Fig. 1 of [3] cancel with each other and thus there is no SFP contribution as was shown in [3]. (By reversing the arrows of the fermion lines in Fig. 1, one obtains the SFP those in Fig. 1 of [3] cancel with each other and thus there is no SFP contribution as was shown in [3].)

We do not know if this relation holds or not in the higher-order calculation.

In Fig. 1, the left and right diagrams for anti-quark and gluon fragmentation channels, respectively, are different only in the position of the final-state cut, and thus the partonic hard part for the SFP cross sections differ only in the overall sign for the two channels.

By collecting all the pole contributions associated with \( G_F \) and \( \tilde{G}_F \), one obtains the complete cross section formula in the form of (1) [2]. To make connection with the transverse-momentum-dependent (TMD) factorization approach, we recast (1) into the TMD formula contains more number of (TMD) distributions of nucleon, while the TMD formula contains more number of (TMD) distributions of nucleon.

It is instructive to consider the limit of inclusive DIS from our result. As is well known [7], SSA vanishes in the inclusive DIS. In our formula this limit corresponds to setting formally all the twist-2 fragmentation functions equal to one, and to integrating over \( \phi_h \), \( q_T \) and \( z_f \). Integration over \( \phi_h \) keeps only the \( F^{\sin(\phi_h)} \)-term in (4). Since the SFP partonic hard cross sections have opposite signs with the same magnitude between the anti-quark and gluon fragmentation channels, no SFP contribution survives. In addition, owing to the relation (3), there is no HP contribution to \( F^{\sin(\phi_h)} \) from the beginning. The remaining SGP contribution also proves to vanish after integration over \( q_T^2 \) [5]. This way our formula is consistent with the fact that there is no SSA in inclusive DIS.

Finally we mention briefly the connection and consistency of the present result with that of the TMD factorization approach in the intermediate region of \( q_T \), \( \Lambda_{QCD} \ll q_T \ll Q \). For \( F^{\sin(\phi_h)} \) in (4), it’s been shown that the two frameworks give identical result [2,3] as in Drell-Yan process [9]. To make connection with the TMD factorization approach, it is necessary to look at the \( q_T \to 0 \) limit of the other structures in (4). This limit was studied in [10], using the result presented in [1,4]. With our present update, we obtain the small-\( q_T \) behavior of the structure functions in (4) as

\[
\frac{d^2\sigma^{t=3}}{d\phi} = \sin(\phi_h - \phi_S) F^{\sin(\phi_h - \phi_S)} + \sin(2\phi_h - \phi_S) F^{\sin(2\phi_h - \phi_S)} + \sin(\phi_S) F^{\sin(\phi_S)}
\]

\[
\quad + \sin(3\phi_h - \phi_S) F^{\sin(3\phi_h - \phi_S)} + \sin(\phi_h + \phi_S) F^{\sin(\phi_h + \phi_S)},
\]

with the obvious relations

\[
F^{\sin(\phi_h - \phi_S)} = \Delta \sigma_1, \quad F^{\sin(2\phi_h - \phi_S)} = (\Delta \sigma_2 + \Delta \sigma_4)/2, \quad F^{\sin(\phi_S)} = \Delta \sigma_3 + \Delta \sigma_5)/2, \quad F^{\sin(\phi_h + \phi_S)} = (-\Delta \sigma_3 + \Delta \sigma_5)/2.
\]

The five types of azimuthal dependence appearing in (4) are the same as what was derived by the TMD factorization approach [6]. We note, however, that our formula which is valid at \( q_T \sim Q \gg \Lambda_{QCD} \) contains only two independent (twist-3) distributions of nucleon, while the TMD formula contains more number of (TMD) distributions of nucleon.

We do not know if this relation holds or not in the higher-order calculation.
\(F_{\sin}(2\phi_h - \phi_S) \sim 1/q_T^2, \quad F_{\sin}(\phi_S) \sim O(1), \quad F_{\sin}(3\phi_h - \phi_S) \sim 1/q_T, \quad F_{\sin}(\phi_h + \phi_S) \sim 1/q_T.\) Here the behavior of \(F_{\sin}(\phi_S)\) is consistent with the limit of inclusive DIS mentioned above. We also note that the SFP contributions are always more power-suppressed compared to the SGP and HP contributions in all structure functions. In [4], the contribution from the twist-3 fragmentation function of the pion, \(\hat{E}_F(z_1, z_2)\), was calculated, keeping only the derivative term of the SGP contribution. This contribution gives the small-\(q_T\) behavior as \(F_{\sin}(\phi_S)(\hat{E}_F) \sim 1/q_T^2, \quad F_{\sin}(\phi_h + \phi_S)(\hat{E}_F) \sim 1/q_T^3\) and \(F_{\sin}(\phi_h - \phi_S)(\hat{E}_F) \sim 1/q_T\), with no contribution to the other two azimuthal structures. The first relation does not contradict with the inclusive-DIS limit, since \(\hat{E}_F \rightarrow 0\) in this limit. The leading term with \(F_{\sin}(\phi_h + \phi_S)(\hat{E}_F) \sim 1/q_T^3\) may be connected to the Collins effect in SIDIS, for which one needs more study.

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