Research Article

Hexagon-Based Generalized Voronoi Diagrams Generation for Path Planning of Intelligent Agents

Fen Tang, Xiong You, Xin Zhang, and Kunwei Li

College of Geospatial Information, Information Engineering University, Henan, Zhengzhou 450052, China

Correspondence should be addressed to Xiong You; youarexiong@163.com

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Grid-based Generalized Voronoi Diagrams (GVDs) are widely used to represent the surrounding environment of intelligent agents in the fields of robotics, computer games, and military simulations, which improve the efficiency of path planning of intelligent agents. Current studies mainly focus on square-grid-based GVD construction approaches, and little attention has been paid to constructing GVDs from hexagonal grids. In this paper, an algorithm named hexagon-based crystal growth (HCG) is presented to extract GVDs from hexagonal grids. In addition, two thinning patterns for obtaining one-cell-wide GVDs from rough hexagon-based GVDs are proposed. On the basis of the principles of a leading square-grid-based algorithm named Brushfire, a hexagon-based Brushfire algorithm is realized. A comparison of the HCG and the hexagon-based Brushfire algorithm shows that HCG is much more efficient. Further, the usefulness of hexagon-based GVDs for the path planning tasks of intelligent agents is demonstrated using several representative simulation experiments.

1. Introduction

Spatial representation is considered to be a fundamental subject in the fields of robotics, computer games, and military simulation. In applications in these fields, one or more intelligent agents frequently implement spatial reasoning tasks such as route planning, self-localisation, and collision avoidance. Generally speaking, the representation method of the underlying working space significantly influences the efficiency of spatial reasoning algorithms of the intelligent agents. Therefore, a sparse and well-structured spatial representation is needed for these agents.

There are many ways to represent the spatial environment, such as regular and irregular grids [1], waypoint graphs [2], probabilistic roadmaps [3], and GVDs [4]. GVDs can represent the sparse skeleton of the entire spatial environment, which reduces the search space and improves the efficiency of path planning algorithms. In addition, GVDs provide the maximum clearance in regions with obstacle so that robots can avoid them. Because of these advantages, research on efficiently constructing the GVDs has recently drawn significant attention.

Triangles, squares, and hexagons are the only regular polygons that can be used to tessellate a continuous two-dimensional (2D) environment [5]. Among them, square grids are the most commonly used, and several GVDs construction algorithms based on them have been proposed, e.g., the Brushfire algorithm [6] and its improved versions (the dynamic Brushfire algorithm [7] and the method developed by Lau et al. [8]). However, the Brushfire algorithm provides no efficient mechanism for updating partial areas where local changes occur; it abandons the existing GVDs and builds a new one from scratch, making it unacceptable in a dynamic environment. To solve this problem, Kalra et al. proposed the dynamic Brushfire algorithm [7], and Lau et al. developed a novel method (referred to as the BL algorithm in this paper) [8]. Both algorithms can efficiently update GVDs when the underlying environment changes. An algorithm named dynamic topology detector (DTD) was proposed by Qin et al. [9]. In addition to generating GVDs, it extracts the connectivity among the edges and vertices of the GVDs and provides an efficient repair mechanism to dealing with local changes.

As one of the three 2D regular grids, hexagonal grids have been well studied and widely used in many fields. For
example, the hexagonal grid has long been known to be superior to the more traditional rectangular grid system in many aspects in image processing and machine vision fields [10]. In [11], a hexagonal image processing framework was proposed, advantages and disadvantages of which were also explained. In [12], a simple formula was derived for the distance between two points on a hexagonal grid, in terms of coordinates with respect to a pair of oblique axes. In [13], the binary tomography reconstruction problem of images on the hexagonal grid was studied and a (near) optimal solution was found. In addition to the applications in image processing, hexagonal grids have important applications in other fields. For example, Quijano and Garrido [14] simulated robot exploration algorithms based on hexagonal grids. They proved that these algorithms outperform those on quadrangular grids for both single and multiagent problems.

Chrpa and Komenda [15] proposed a method to smooth the trajectory of a helicopter based on hexagonal grids and extended their method to multiagent pathfinding [16]. A number of computer games, e.g., Wartile and World of Warships, use hexagonal grids to represent the game world. In military simulations, the Joint Theater Level Simulation (JTLS) system of the United States also uses hexagonal grids to represent the terrain of the battlefield. Hexagonal grids have many advantages over square grids. First, when dividing the same area, hexagonal grids provide a higher spatial resolution than square grids [17]. Second, each cell in a hexagonal grid has six neighbours whose cell centroids are at the same distance [18], as seen in Figure 1. Third, hexagonal grids suffer less from orientation bias and sampling bias from edge effects since the distances to the centroids of the six adjacent cells are the same [19].

The square-grid-based GVDs generated from square-grid-based map can help to improve the efficiency of path planning tasks of intelligent agents. However, methods that employ this GVDs construction technique are not used in robotics, computer games, and military simulations, where the environments are represented with hexagonal grids. To date, there are no related studies that have focused on hexagon-based GVD generation for the path planning of intelligent agents. Pathfinding tasks on hexagonal grids are quantitatively implemented by intelligent agents in these fields, with the Morris [20], A∗, and iterative deepening A∗ (IDA∗) [21] algorithms being the most commonly used. However, their time and space complexities dramatically increase when the search area becomes larger or the number of hexagonal grids increases. In order to improve the efficiency of path planning tasks in a hexagon-based environment, the ability to construct hexagon-based GVDs is crucial for an intelligent agent that moves in a large area.

The main focus of this paper is to construct GVDs from a preexisting hexagon-based representation of an environment. An algorithm named hexagon-based crystal growth (HCG) is presented, which extracts GVDs from a hexagonal grid map. Further, two thinning patterns are proposed to obtain one-cell-wide GVDs from rough hexagon-based GVDs. Then, HCG is compared to the hexagon-based Brushfire algorithm. The usefulness of hexagon-based GVDs for the path planning tasks of agents is also demonstrated with some simulation scenarios.

The outline of this paper is as follows: First, existing GVD construction algorithms are briefly reviewed. Among the leading algorithms for extracting GVDs from square grid maps, the Brushfire algorithm is the most fundamental. Owing to geometric differences between square and hexagonal grids, the Brushfire algorithm cannot be directly applied to hexagon-based GVD construction and needs to be modified to accommodate hexagonal grids. Next, the process of constructing a hexagonal grid occupation map is detailed. Then, the data structure employed by the presented algorithms is presented. After that, the hexagon-based Brushfire algorithm, HCG, and two thinning patterns are presented, all of which are illustrated through pseudocode. Next, HCG is compared with the hexagon-based Brushfire algorithm, and the usefulness of hexagon-based GVD metrics for path planning tasks is tested. Finally, the conclusions are presented.

2. Related Work

Voronoi diagrams, named after Georgy Voronoi, have been used to address different problems in various fields, including anthropology, archaeology, astronomy, biology, cartography, chemistry, computational geometry, geography, robotics, and planning [22]. To address the complexity of real-world problems, a number of advanced Voronoi diagrams have been developed, e.g., weighted Voronoi diagrams [23], city Voronoi diagrams [24], and GVDs [4]. In the field of robotics, GVDs have been extensively used to plan a path that stays as far away from obstacles as possible. As a reduced search space, it can help to reduce computation time significantly.

Existing algorithms for building 2D GVDs can be roughly divided into two types according to the type of input data: vector data and raster data, which are called vector- and raster-based algorithms, respectively [25]. GVDs built by vector-based algorithms are accurately and sparsely represented as a set of parametric lines or curves, which separate different sites in space [26, 27]. There are also local update mechanisms for various local changes, e.g., moving sites [28] and inserted or deleted sites [29]. Despite these advantages, vector-based algorithms are not suitable for robots whose working spaces are represented as grid maps.

Raster-based algorithms are very practical for grid maps, and they have received a significant amount of attention. As discussed in the previous section, most existing raster-based algorithms, e.g., the Brushfire, dynamic Brushfire, BL, and DTD algorithms, are based on square-grid maps. Although the performance and application scenarios of these algorithms are different, similarities in their algorithmic principles exist. They all need to generate three metric matrices (dist_s, obst_s, and voro_s) to represent a GVD. The matrix dist_s stores the discrete or actual Euclidean distance between an arbitrary entry (denoted by s) and the site cell from which s
The matrix \( \text{obst} \), registers the site identifier and the coordinates of the exact site cell to which \( s \) is currently the closest. Finally, the matrix \( \text{voro} \), is a Boolean matrix that indicates whether \( s \) is a GVD cell [9]. Figure 2 shows the three matrices. Some other researchers have improved some of these algorithms. For example, Scherer et al. propagated the actual Euclidean distance instead of the grid steps used by the dynamic Brushfire algorithm from the exact source cell to greatly reduce the relative error [30]. By using the “thinning patterns” proposed in [31], Lau et al. provided additional thinning steps to obtain one-cell-wide edges, which makes the resulting GVD edges sparser.

3. Construction of a Hexagonal-Grid Occupation Map

Hexagonal grids are used to discretise the maximum active space of intelligent agents. As illustrated in Figure 3, the maximum manoeuvring range of an agent is a rectangular area, the length and width of which are denoted by \( L \) and \( W \), respectively. The length of the edge of each hexagon is denoted by \( r \), and the distance from the centre of a hexagon to its edge is denoted by \( h \).

The hexagonal grid is indexed as “ColID–RowID,” where “ColID” and “RowID” are the indices of the column and row in the grid, respectively. Given the maximum ColID \( \text{col} \) and maximum RowID \( \text{row} \), \( r \) can be calculated by

\[
    r = \min \left( \frac{2 \times L}{3 \times \text{col} + 1} \frac{W}{\sqrt{3} \times \text{row}} \right). 
\]

For the cells in the even columns, the coordinates of the centre point of each hexagonal grid can be calculated by

\[
    x = \left\lfloor \text{int} \left( \frac{\text{ColID}}{2} \right) \times 3 + 1 \right\rfloor \times r, 
\]

\[
    y = (2 \times \text{RowID} + 1) \times h. 
\]

For the cells in the odd columns, the coordinates of the centre point of each hexagonal grid can be calculated by
\[
\begin{align*}
    x &= \left\lfloor \left( \frac{\text{ColID}}{2} \right) \times 3 + 2 \right\rfloor \times r + \frac{r}{2} \\
    y &= (2 \times \text{RowID} + 1) \times h
\end{align*}
\] (3)

As illustrated in Figure 3, the grey and white hexagons represent obstacles and free cells, respectively. In this paper, we use a raster-based occupation map, which represents the working space of intelligent agents, as the data source for constructing a hexagonal map. For intelligent agents, the black part in the occupation map is impassable, and the white part is passable. The main processes for hexagonal map construction are the generation of the hexagonal grid that overlies the occupation map and the calculation of the area of an obstacle that falls into each hexagon in the hexagonal grid. If the ratio of the obstacle area to the area of the hexagon is larger than a preset threshold, then the hexagon is set as an obstacle that is impassable. In contrast, if the ratio is smaller than the preset threshold, the hexagon is set as a free hexagon that is passable. In this paper, the threshold is set as 50%.

ArcGIS 10.6 (a geographic information system) is used to convert the raster-based occupation map into a hexagonal map owing to its powerful spatial analysis capabilities. Figure 4 shows the whole construction process. First, by using the ArcGIS tool “Raster to Polygon,” the raster occupation map is converted into a vector map. Then, according to the range of the generated vector map, the ArcGIS 10.6 tool “Generate Tessellation” is used to generate a hexagonal grid, which is also in a vector format. Using the generated vector map and the hexagonal grid as input data, the ArcGIS 10.6 tool “Intersection” is employed to retrieve the intersections of the obstacles and the hexagonal grid. At the same time, the area attribute of each intersection can be obtained. All intersections whose areas are larger than 30% of the area of a single hexagon are selected by the ArcGIS 10.6 tool “Select by Area” and are exported as a new polygon feature layer, which is further converted into a point feature using the ArcGIS 10.6 tool “Feature to Point.” Using the point feature layer and hexagonal grid as input data, the ArcGIS 10.6 tool “Spatial Joining” is employed to obtain the hexagonal grid occupation map, whose “Joint_Count” attribute indicates whether a hexagon is impassable (if the value of “Joint_Count” in the attribute table is 0) or passable (if the value of “Joint_Count” in the attribute table is 1).

4. Employed Data Structure

A data structure is employed to represent the components of GVDs, as shown in Figure 5. Hash tables are used to store the components that are extracted during the construction of GVDs to ensure efficient retrieval. Each component instance inserted into a table is assigned with a unique identifier. For example, vertices are identified by their coordinates in the hexagonal grid map, and edges are identified by the ID pairs that indicate the two sites it divides. The semantics of the related data objects and their attributes that will be quoted by the two algorithms are listed in Table 1.
Figure 4: Process of constructing a hexagonal grid occupation map using ArcGIS 10.6.

Figure 5: Class diagram for the data structure employed by DTD.
5. Algorithms

In this section, on the basis of the principles and steps of the Brushfire algorithm based on a regular quadrilateral grid, this algorithm is adaptively adjusted to achieve GVD generation on a regular hexagonal grid, and a hexagon-based Brushfire algorithm is realized. After that, the HCG algorithm is proposed to generate GVDs on a hexagonal grid. A comparison between these two algorithms is also made to determine which one is more efficient.

5.1. Hexagon-Based Brushfire Algorithm. Figure 6(a) shows a flowchart of the main steps of the hexagon-based Brushfire algorithm. In Step 1, the \( \text{dist} \) matrix, where each entry stores the integral distance to its nearest site cell, is built. In Step 2, the \( \text{obst} \) matrix, where each entry stores the site identifier and the exact coordinates of its nearest site cell, is built. In Step 3, the \( \text{Edge} \) matrix, where each entry shows whether the site cell belongs to the GVDs (registered as 1) or not (registered as 0), is built. The width of the generated initial rough GVD edges may be 1, 2, or more cells. In Step 4, by thinning the rough result obtained in Step 3, the one-cell-wide GVD edges are generated. Figure 6(b) shows the transitions of the GVDs in order from Step 1 to Step 4.

In the hexagon-based Brushfire algorithm in Algorithm 1, \( \text{bfQueueList} \) is initialised as a list of cells that are adjacent to all of the sites in the working space and is sorted in ascending order according to the distance from the cell to its nearest obstacle. That is, a higher rank is given to a cell closer to the obstacle. If \( \text{bfQueueList} \) is not empty, the first cell \( s \) (lines 1-2) in it is removed. \( \text{Dist} \) is the distance from \( s \) to its nearest obstacle. If \( \text{dist} \) is not 0 (which means that \( s \) is not a site cell), then the six cells adjacent to \( s \) are added to the list \( \text{adjCellList} \) (which stores the neighbouring cells of the cells in \( \text{bfQueueList} \)) and are sorted in ascending order of distance from each cell to its nearest obstacle (lines 3–5). On this basis, the neighbouring cell \( n \) with the smallest distance to its nearest obstacle can be taken from \( \text{adjCellList} \); then, the attributes of \( s \), including the distance to the nearest obstacle \( \text{dist} \), the nearest obstacle \( \text{obst} \), and the parent cell \( \text{parent} \), can be updated according to the attributes of cell \( n \) (lines 6–9). For each cell in \( \text{adjCellList} \), if it is not an obstacle cell, if its distance to the nearest obstacle is the initial value \( \text{MAXDIS} \), and if \( a \) is not in the list \( \text{bfQueueList} \), then the nearest obstacle is \( \text{Obst} \) and will be added to the list \( \text{bfQueueList} \) (lines 10–15). After the above process, the lists \( \text{dist} \) and \( \text{obst} \) can be obtained.

The function \( \text{markBrushfireRoughGVDEdges()} \) (line 16) is used to determine whether a cell is a GVD edge cell on the basis of the site identifier of each cell. The attribute values of \( \text{bEdgeCell} \) for all cells are initialised as false, which means each cell in the working space is not a GVD edge cell at first. For each cell \( s \) in \( \text{allCellList} \), the six cells adjacent to \( s \) are removed. For each adjacent cell \( n \) of \( s \), if the site identifiers to which \( s \) and \( n \) belong are different and cell \( s \) is not a site cell, then the attribute of \( \text{bEdgeCell} \) is set to true, which means that \( s \) is a GVD edge cell (lines 17–21). Then, cell \( s \) is added to the boundary cell list. After that, it is necessary to determine the GVD edge that \( s \) belongs to and add it to the list of cells that make up the GVD edge. However, if the GVD edge to which \( s \)
Figure 6: (a) Flowchart of the process of building a hexagon-based GVD. (b) Transitions of the hexagon-based GVDs from Step 1 to Step 4.

5.2. Crystal-Growth Algorithm on Hexagonal Grid. The function GenerateCrystalGrowthRoughGVDEdges() in Algorithm 2 is used to generate the initial GVD edges, the width of which may be 1, 2, or more cells. UnCrystalCellList is a list of all cells except the site cells in the working space, which remain unhandled. BoundCellList is a list of adjacent cells of all site cells. For list sbcl in boundCellList, each cell s is removed. For each adjacent cell n of s (lines 1–4), if n is not a site cell and is not occupied, the attribute values typen, obstn, and distn of n are reset (lines 5–9). Then, cell n is added to the temporary surrounding cell list tempSBC and is removed from boundCellList (lines 10–11). However, if n is not a site cell but is already occupied and if the site identifier s is different from that of n, then the type of n is set as EDGE (lines 12–14). After the above process, the list tempSBC is added to the new surrounding cell list newSBCL, and newSBCL is used to replace boundCellList (lines 15–17). At this point, a round of growth is finished, and the growth process will not stop until all of the cells in the working space are handled. The function markCrystalGrowthRoughGVDEdges() determines whether a cell is a GVD edge cell on the basis of the site identifier of the cells. The list allCellList is a list of all of the cells after the previous stage (lines 1–16). First, all of the cells in allCellList are traversed to check whether the type of cell s is EDGE. For all EDGE cells, their corresponding edge e is queried. If edge e does not exist, a new edge e is created, and one of the site identifiers of edge e is set equal to obstn. Then, edge e is added to the edge list (lines 18–24). After that, EDGE cell s is added to edge e. The function insertQueue(roughQueue, s, dist) inserts s into roughQueue with priority dist, (lines 25–26). To refine the initial boundary in the future, cell s is added to the priority list roughQueue, which is sorted in ascending order by the distance from s to the nearest site.

5.3. Thinning the Rough Edges. Two thinning patterns (shown in Figure 7) are proposed and are employed by the function pruningEdgeCell() in Algorithm 3 to obtain one-cell-wide hexagon-based GVD edges. The input for thinning is roughQueue, which involves all hexagonal edge cells created by GenerateBrushfireRoughGVDEdges() or GenerateCrystalGrowthRoughGVDEdges(). All cells in
GenerateRoughBrushfire GVDEdges()
(1) while bfQueueList ≠ Ø do
(2) \( s \leftarrow \text{pop(bfQueueList)} \)
(3) if \( \text{dist} \neq 0 \) then
(4) adjCellList ← Adj6(s)
(5) sort(adjCellList)
(6) \( n \leftarrow \text{pop(adjCellList)} \)
(7) \( \text{dist}_n = \text{dist}_s + 1 \)
(8) obst_n = obst_s
(9) parent_n = n
(10) for all \( a \in \text{adjCellList} \) do
(11) if type_a ≠ OBST then
(12) if \( \text{dist}_a = \text{MAXDIS} \) then
(13) if \( a \neq \text{bfQueueList} \) then
(14) obst_a = obst_s
(15) insert(bfQueueList, a)
(16) markBrushfire RoughGVDEdges()
markBrushfire RoughGVDEdges()
(17) for all \( s \in \text{allCellList} \) do
(18) bEdgeCell ← false
(19) for all \( n \in \text{Adj6(s)} \) do
(20) if obst_n \neq obl_n ∧ type_n ≠ OBST then
(21) bEdgeCell ← true
(22) if \( bEdgeCell \) then
(23) \( e \leftarrow \text{findEdge(s)} \)
(24) if \( e = \text{Ø} \) then
(25) \( e \leftarrow \text{Edge()} \)
(26) obst_e = obst_s
(27) insertEdge(e)
(28) insertCell(e, s)
(29) insertQueue(roughQueue, s, dist_s)
(30) pruningEdgeCell()

Algorithm 1: Pseudocode for the hexagon-based Brushfire algorithm.

roughQueue are processed in two phases. First, for each cell \( s \) in the priority queue roughQueue, if \( s \) matches the two thinning patterns in Figure 7, then \( s \) is retained and the next one in roughQueue is processed (lines 1–3). Second, if \( s \) does not satisfy the two thinning patterns in Figure 7, it is removed from the edge list to which \( s \) belongs. After all of the cells in the roughQueue are processed, the one-cell-wide GVDs are obtained (lines 4–6). Following the entire description of the above thinning process, the details of the two thinning patterns will be introduced.

The function thinningPatternOne(s) in Algorithm 3 is used to realise thinning pattern one. For each edge cell \( s \) in the roughQueue, if cell \( n \), one of the six cells adjacent to \( s \), is an edge cell, and the two cells \( a \) and \( b \), which are both adjacent to \( n \) and \( s \), are both unoccupied, then \( s \) will be retained in roughQueue (lines 7–11). This means that \( s \) belongs to the final one-cell-wide GVD edge. The function thinningPatternTwo(s) is used to realise thinning pattern two. For each edge cell \( s \) in the roughQueue, if cell \( n \), one of six cells adjacent to \( s \), is a not edge cell, and the two cells \( a \) and \( b \), which are both adjacent to \( n \) and \( s \), are both occupied (one of them is an edge cell, and the other is an edge cell or a site cell), then \( s \) will be retained in roughQueue (lines 12–17). This means that \( s \) belongs to the final one-cell-wide GVD edge.

6. Experiments and Analysis

In this section, statistical methods are employed to compare the hexagon-based Brushfire and HCG algorithms for some simulated scenarios. The usefulness of the hexagon-based GVDs for high-level path planning tasks is also demonstrated.

6.1. Comparison to the Hexagon-Based Brushfire Algorithm

We compared the HCG and hexagon-based Brushfire algorithms for seven scenarios, as shown in Figure 8. All scenarios are located in a static environment with a fixed size but with seven different hexagonal grid resolutions, which are 72 × 100, 108 × 150, 144 × 200, 180 × 250, 216 × 300, 252 × 350, and 288 × 400. In each scenario, all sites are predefined and fixed, and two algorithms were executed 10 times for each scenario. All tests were carried out with Python implementations of the algorithms running on an Intel Xeon processor.

Comparisons of the performance of the two algorithms are presented in Table 2 and Figure 9 for the computation time. From these tables and figures, it is concluded that (1) the GVD construction time of both the hexagon-based Brushfire and HCG algorithms increases in proportion to the hexagonal grid resolution for a fixed-size environment.
**Algorithm 2: Pseudocode for the hexagon-based crystal-growth algorithm.**

```plaintext
GenerateCrystalGrowthRoughGVDEdges()
(1) while unCrystalCellList ≠ Ø do
(2) for all sbcl ∈ boundCellList do
(3) for all s ∈ sbcl.cSiteCells do
(4) for n ∈ Adj(s) do
(5) if typeₙ ≠ OBST then
(6) if typeₙ == EMPTY then
(7) typeₙ ← OCCUPY
(8) obstₙ ← obstₙ
(9) distₙ ← distₙ + 1
(10) insert(tempSBC. cSiteCells, n)
(11) unCrystalCellList.pop(n)
(12) else if typeₙ == OCCUPY then
(13) if obstₙ ≠ obst then
(14) typeₙ ← EDGE
(15) insert(newSBCL, tempSBC)
(16) boundCellList ← newSBCL
(17) markCrystalRoughEdge()
markCrystalGrowthRoughGVDEdges()
(18) for all s ∈ allCellList do
(19) if typeₙ == EDGE then
(20) e ← findEdge(s)
(21) if e ≠ Ø then
(22) e ← Edge()
(23) obst_e ← obst_
(24) insertEdge(e)
(25) insertCell(e, s)
(26) insertQueue(roughQueue, s, dist_
(27) pruningEdgeCell()
```

**Figure 7:** Patterns used by edge thinning, (a) pattern one; (b) pattern two.
and (2) the HCG algorithm is much more efficient and requires less time than the hexagon-based Brushfire algorithm for each scenario with the same grid resolution.

6.2. Application Test for Path Planning. In order to demonstrate the usefulness of our algorithm for path planning tasks, seven mobile robots operating in a fixed-size map with

| Algorithm                | Resolution                  |
|--------------------------|-----------------------------|
|                          | 72 x 100                    | 108 x 150                    | 144 x 200                    | 180 x 250                    | 216 x 300                    | 252 x 350                    | 288 x 400                    |
| Hexagon-based Brushfire  | 7.5838                      | 22.2965                      | 51.6161                      | 111.6368                     | 182.0032                     | 294.6933                     | 448.1999                     |
| Hexagon-based crystal growth | 1.4945                      | 3.5701                       | 7.3051                       | 14.7915                      | 23.5950                      | 37.4815                      | 57.4377                      |

Figure 8: Original raster map and seven hexagonal grid maps with different resolutions. (a) First line, left to right: the original raster map and grid maps with resolutions of 72 x 100, 108 x 150, and 144 x 200. (b) Second line, left to right: grid maps with resolutions of 180 x 250, 216 x 300, 252 x 350, and 288 x 400.
seven different resolutions (shown in Figure 10) were simulated. The starting points of the seven robots were at the same absolute coordinates at the bottom left of the map. For each search task, each robot was given a unique destination point with the same absolute coordinates at the top right of the map. However, despite the same absolute coordinates, the hexagonal grid coordinates of the seven starting points were different owing to the different resolutions. The grid coordinates of the starting and destination cells of the seven agents are listed in Table 3.
The search space adopted by these agents was (1) the entire grid map and (2) the hexagon-based GVD metrics generated by HCG. The \( A^* \) algorithm was employed by the agents to search routes. The simulation results are listed in Table 4. We can see that the agents that adopted the \( A^* \) algorithm to search the entire hexagonal grid map have a higher computation time and more cell visits than those that adopted the \( A^* \) algorithm to search the hexagon-based GVD metrics. The entire hexagonal grid map provides no further information about the maximum clearance to sites, making the resulting paths (in yellow in Figure 10) contain several cells near the sites, which will lead to collisions when the physical size of the agent exceeds the limited clearance. The agents that adopted the \( A^* \) algorithm to search the hexagon-based GVD metrics only explore the hexagon-based GVD edge cells, significantly reducing the computation time. Each of the resulting paths (in yellow in Figure 10) consists of (1) an initial route from the starting cell to the nearest hexagon-based GVD cell, (2) a set of connecting hexagon-based GVD edges ensuring the reachability of the hexagon-based GVD departure cell, which is nearest the destination cell, and (3) a final route from the hexagon-based departure cell to the destination cell. The path planning results of the seven simulation scenarios suggest that the ratio of the computation time for searching the whole map with the \( A^* \) algorithm to that searching the GVD metrics increases as the hexagonal grid resolution increases. The result is also obtained for the ratio of the number of cells visits for searching the whole map with the \( A^* \) algorithm to that searching the GVD metrics. This means that \( A^* \) searching of the GVDs becomes more efficient than searching the entire hexagonal grid map as the hexagonal grid resolution increases.

### 7. Conclusions

In this paper, an algorithm named HCG was proposed to construct GVDs from hexagonal grid maps. Several simulation experiments were conducted to compare the HCG algorithm with the hexagon-based Brushfire algorithms (a leading grid-based GVD construction algorithm), and the results suggest that, in a hexagonal grid map with the same range and resolution, the HCG algorithm is much more efficient, requiring less time and fewer cell visits to construct hexagon-based GVDs. Moreover, two thinning patterns for obtaining one-cell-wide GVDs from rough hexagonal GVDs were proposed and were applied to both the hexagon-based Brushfire and HCG algorithms. The usefulness of the hexagonal GVD metrics in path planning was further illustrated using several representative simulation scenarios, and we found that it can significantly improve the efficiency of path planning of intelligent agents.

The proposed HCG algorithm could be applicable to the path planning of intelligent agents in fields of robotics, computer games, and military simulations, where high computing performance needs to be guaranteed. In the future, a dynamic HCG algorithm will be further explored to efficiently construct GVDs from hexagonal grid maps in which local changes may occur.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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