\( \varepsilon'/\varepsilon \)-2018: A Christmas Story

Andrzej J. Buras

TUM Institute for Advanced Study, Lichtenbergstr. 2a, D-85748 Garching, Germany
Physik Department, TU München, James-Franck-Straße, D-85748 Garching, Germany
E-mail: aburas@ph.tum.de

Abstract

I was supposed to review the status of \( \varepsilon'/\varepsilon \) both at the CKM Workshop in September in Heidelberg and recently at the Discrete 2018 Conference in Vienna. Unfortunately I had to cancel both talks for family reasons. My main goal in these talks was to congratulate NA48 and KTeV collaborations for the discovery of new sources of CP violation through their heroic efforts to measure the ratio \( \varepsilon'/\varepsilon \) in the 1980s and 1990s with final results presented roughly 16 years ago. As I will not attend any other conferences this year I will reach this goal in this writing. In this context I will give arguments, why I am convinced about the presence of new physics in \( \varepsilon'/\varepsilon \) on the basis of my work with Jean-Marc Gérard within the context of the Dual QCD (DQCD) approach and why RBC-UKQCD lattice QCD collaboration and in particular Chiral Perturbation Theory practitioners are still unable to reach this conclusion. I will demonstrate that even in the presence of pion loops, as large as advocated recently by Gisbert and Pich, the value of \( \varepsilon'/\varepsilon \) is significantly below the data, when the main non-factorizable QCD dynamics at long distance scales, represented in DQCD by the meson evolution, is taken into account. As appropriate for a Christmas story, I will prophesy the final value of \( \varepsilon'/\varepsilon \) within the SM, which should include in addition to the correct matching between long and short distance contributions, isospin breaking effects, NNLO QCD corrections to both QCD penguin and electroweak penguin contributions and final state interactions. Such final SM result will probably be known from lattice QCD only in the middle of the 2020s, but already in 2019 we should be able to see some signs of NP in the next result on \( \varepsilon'/\varepsilon \) from RBC-UKQCD. In this presentation I try to avoid, as much as possible, the overlap with my recent review of Dual QCD in [I].
1 Introduction

The ratio $\varepsilon'/\varepsilon$ measures the size of the direct CP violation in $K_L \to \pi\pi$ decays relative to the indirect one described by $\varepsilon_K$ and is very sensitive to new sources of CP violation. It has been measured already 16 years ago with the experimental world average from NA48 [2] and KTeV [3,4] collaborations given by

$$(\varepsilon'/\varepsilon)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}. \quad (1)$$

In the Standard Model (SM) $\varepsilon'$ is governed by a positive contribution from QCD penguins but receives also an important contribution from electroweak (EW) penguins that enter $\varepsilon'$ with the opposite sign. This is best seen in an analytic formula for $\varepsilon'/\varepsilon$ [5]

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = 10^{-4} \left[ \frac{\text{Im}\lambda_t}{1.4 \cdot 10^{-4}} \right] \left[ a (1 - \hat{\Omega}_{\text{eff}}) ( -4.1 + 24.7 B_6^{(1/2)} ) + 2.2 - 10.4 B_8^{(3/2)} \right], \quad (2)$$

where the contributions from the dominant QCD penguin ($Q_6$) and the dominant EW penguin ($Q_8$) are the ones proportional to the hadronic parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$, respectively. Next

$$\text{Im}\lambda_t = \text{Im}(V_{td}V_{ts}^*) = |V_{ub}||V_{cb}| \sin \gamma \quad (3)$$

and [5,8]

$$a = 1.017, \quad \hat{\Omega}_{\text{eff}} = (14.8 \pm 8.0) \times 10^{-2}. \quad (4)$$

$|V_{ub}|$ and $|V_{cb}|$ are the elements of the CKM matrix and $\gamma$ is the known angle in the unitarity triangle. The parameters $a$ and $\hat{\Omega}_{\text{eff}}$ represent isospin breaking corrections [6,8]. See latter papers and [5] for details. $\hat{\Omega}_{\text{eff}}$ differs from $\Omega_{\text{eff}}$ in [6,7] that includes EW penguin contributions to $\text{Im}A_0$. We find it more natural to calculate $\text{Im}A_0$ including both QCD and EW penguin contributions as this allows to keep track of new physics (NP) contributions.

Setting all parameters, but $B_6^{(1/2)}$ and $B_8^{(3/2)}$, at their central values we find

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = 10^{-4} \left[ -3.6 + 21.4 B_6^{(1/2)} + 2.2 - 10.4 B_8^{(3/2)} \right]. \quad (5)$$

The first term $(-3.6)$ comes from $(V - A) \times (V - A)$ QCD penguins and the third one $(1.2)$ from $(V - A) \times (V - A)$ EW penguins. These two contributions have been determined in [5] from the experimental values of $\text{Re}A_0$ and $\text{Re}A_2$ with $A_{0,2}$ being isospin amplitudes. In doing this one makes a plausible assumption that the real parts of these amplitudes and the related $\Delta I = 1/2$ rule can be properly described within the SM. The main uncertainty in $\varepsilon'/\varepsilon$ resides then in the contribution of $(V - A) \times (V + A)$ QCD penguin operator $Q_6$ and the one from $(V - A) \times (V + A)$ EW penguin operator $Q_8$ that enters $\varepsilon'/\varepsilon$ with a negative sign.

In the strict large $N$ limit, $N$ being the number of colours, one finds [9,11]

$$B_6^{(1/2)} = B_8^{(3/2)} = 1, \quad \text{(large N Limit)}. \quad (6)$$

Inserting these values into (5) we find

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = 8.6 \times 10^{-4}, \quad \text{(using (6))}, \quad (7)$$
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roughly by a factor of two below the central experimental value in (1). In fact this was a typical value obtained by the Rome [12] and Munich [13] groups in the 1990s, which used the values in (6). However, the authors in [14–20] using the ideas from Chiral Perturbation Theory (χPT) made a strong claim that final state interactions (FSI) enhance $B_{6}^{(1/2)}$ above unity and suppress $B_{8}^{(3/2)}$ below it so that the SM value for $\varepsilon'/\varepsilon$ according to them is fully consistent with experiment. Albeit only a very inaccurate value $(17 \pm 9) \times 10^{-4}$ [20] could be obtained. As there are some different views in the literature what is meant by FSI, let me call the effects pointed out in these papers simply pion loops. In fact this is the wording used in recent papers by some of these authors. More about it later.

Therefore, I suspect, that it was a great surprise, in particular for χPT experts, when in 2015 the RBC-UKQCD collaboration [21, 22] presented their first results for $K \to \pi\pi$ hadronic matrix elements. From their results one could extract values of $B_{6}^{(1/2)}$ and $B_{8}^{(3/2)}$ for $\mu = 1.53 \text{ GeV}$. They are [5]

$$B_{6}^{(1/2)} = 0.57 \pm 0.19, \quad B_{8}^{(3/2)} = 0.76 \pm 0.05, \quad \text{(RBC - UKQCD).} \quad (8)$$

Inserting the central values into our formula (5) we find

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = 1.9 \times 10^{-4}, \quad (9)$$

that is one order of magnitude below the experimental value. This is in fact the central value for $\varepsilon'/\varepsilon$ in [5] which including various uncertainties found

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = (1.9 \pm 4.5) \times 10^{-4}, \quad \text{(BGJJ).} \quad (10)$$

This result has been confirmed within the uncertainties in [23]

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = (0.96 \pm 4.96) \times 10^{-4}, \quad \text{(KNT).} \quad (11)$$

It should be stressed that the RBC-UKQCD lattice collaboration, calculating directly hadronic matrix elements of all operators, but not including isospin breaking (I.B.) effects, found [21, 22]

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = (1.38 \pm 6.90) \times 10^{-4}, \quad \text{(RBC - UKQCD).} \quad (12)$$

The larger error than in (10) and (11) is related to the fact that RBC-UKQCD collaboration did not use experimental data for $\text{Re} A_{0}$ and $\text{Re} A_{2}$ for the extraction of the $(V - A) \times (V - A)$ contributions. Otherwise the error in (12) would be smaller. The comparison of RBC-UKQCD value, that does not include I.B. effects, with the values in (10) and (11) would seemingly indicate that such effects are small. But such a conclusion would be wrong. I.B. effects definitely suppress significantly $\varepsilon'/\varepsilon$, as we have seen in (2), but the first negative term in this formula, that is dominated by the QCD penguin operator $Q_{4}$, is found by RBC-UKQCD to be significantly larger in magnitude than extracted from the experimental value of $\text{Re} A_{0}$. This difference suppresses $\varepsilon'/\varepsilon$ at a similar level as I.B. effects. I expect that future RBC-UKQCD calculations will result in the value of the first term in (2) that is close to the one used by us.
At a flavour workshop in Mainz in January 2016 two important \( \chi \)PT experts Gilberto Colangelo and Toni Pich expressed serious doubts about the RBC-UKQCD result in (12), because the \((\pi\pi)_I\) phase shift \( \delta_0 \approx (24 \pm 5)^\circ \) obtained by RBC-UKQCD disagreed with \( \delta_0 \approx 34^\circ \) obtained by combining dispersion theory with experimental input [24].

This criticism appeared in print one year ago [25] and in two recent conference proceedings [26, 27]. It is in line with the one expressed many years ago in [14–20], but one should realize that with \( \delta_0 \approx 24^\circ \) a big portion of FSI has been taken into account already in (12). From my point of you it is unlikely that increasing \( \delta_0 \) up to its dispersive value \( \delta_0 \approx 34^\circ \) will shift \( \varepsilon'/\varepsilon \) upwards by one order of magnitude. Recently a new result for \( \delta_0 \) has been presented by the RBC-UKQCD collaboration that with \( \delta_0 = (30.9 \pm 3.4)^\circ \) is within \( 1 \sigma \) from its dispersive value\(^1\). Whether this change has a crucial impact on \( \varepsilon'/\varepsilon \) as expected in [25–27] remains to be seen but should be known within the next six months.

2 The DQCD View

Motivated by the RBC-UKQCD results in (8), Jean-Marc Gérard and myself calculated already in July 2015 \( 1/N \) corrections to the large \( N \) limit in (6) [28]. These corrections, loop corrections in the meson theory with a \textit{physical cut-off} \( \Lambda \approx 0.7 \) GeV, are the leading non-factorizable corrections to hadronic matrix elements of \( Q_6 \) and \( Q_8 \). Two main results in this paper are:

- Realization that the large-\( N \) result is not valid at scales \( O(1 \) GeV\), as assumed in all papers before [28], but at much lower scales \( O(m_\pi^2) \). In order to find it out one has to calculate these loops. In the large \( N \) limit one cannot determine the scale in \( B_6^{(1/2)} \) and \( B_8^{(3/2)} \) and as for \( \mu \geq 1 \) GeV the \( \mu \) dependence of these parameters is weak [29], without knowing \( 1/N \) corrections it was useful to neglect this dependence.

- Calculation of \( B_6^{(1/2)} \) and \( B_8^{(3/2)} \) at scales \( O(1 \) GeV\) by performing the meson evolution from the low factorization scale \( O(m_\pi^2) \) to the physical cutoff \( \Lambda \) of DQCD with the result

\[
B_6^{(1/2)} \leq 0.54, \quad B_8^{(3/2)} = 0.80 \pm 0.10,
\]

in perfect agreement with the RBC-UKQCD result in [8]. Explicitly we found

\[
B_6^{(1/2)} = 1 - 0.66 \ln(1 + \frac{\Lambda^2}{m_6}), \quad B_8^{(3/2)} = 1 - 0.17 \ln(1 + \frac{\Lambda^2}{m_8}),
\]

with pseudoscalar mass scale parameters \( m_{6,8} \) bounded necessarily by the cut-off \( \Lambda \): \( \tilde{m}_{6,8} \leq \Lambda \). The upper bound for \( B_6^{(1/2)} \) is obtained by setting \( \Lambda = \tilde{m}_6 \).

As already mentioned, for scales above 1 GeV both parameters decrease very slowly. This is known from the 1993 analysis in [29] but as seen in Figs. 11 and 12 of that paper \( B_6^{(1/2)} \) decreases faster with increasing scale than \( B_8^{(3/2)} \) in accordance with the pattern at low scales found by Jean-Marc and myself. This can also be shown analytically [28].

\(^1\)See talks by Ch.Kelly and T. Wang at Lattice 2018.
Unfortunately, not knowing $1/N$ corrections to $B_6^{(1/2)}$ and $B_8^{(3/2)}$ in 1993, both parameters have been set at $\mu = m_c$ in [29] to unity, which is clearly wrong.

While it is possible that the bound on $B_6^{(1/2)}$ could be violated by $1/N^2$ corrections and other effects not taken by us into account, one should notice that with only pseudoscalars included in the loops, the cut-off $\Lambda$ has to be chosen below 1 GeV so that these omitted effects, even if they would increase $B_6^{(1/2)}$, could still be compensated by the running to higher scales that are explored by lattice QCD.

The second result is in my view very important for the following reason. There is no other lattice collaboration calculating $B_6^{(1/2)}$ and $B_8^{(3/2)}$ at present, so that in the lattice world the result of the RBC-UKQCD collaboration for $\varepsilon'/\varepsilon$ cannot be tested at present. As we will see in Section 4, $\chi$PT by itself has no means to verify or disprove the RBC-UKQCD results for $B_6^{(1/2)}$ and $B_8^{(3/2)}$. Therefore, it is really important that DQCD provides the insight in the values of these parameters, which clearly cannot be provided by a purely numerical method like lattice QCD. According to our analysis in [28], the main QCD dynamics behind the lattice values in (8) is the meson evolution, analogous to the well known quark-gluon evolution at short distance scales.

Before I write a few lines about FSI in DQCD, let me just mention that the importance of the meson evolution is also seen in the evaluation of BSM matrix elements relevant for $K^0 - \bar{K}^0$ mixing. Indeed the RBC-UKQCD collaboration working at $\mu = 3 \text{ GeV}$ finds for four BSM parameters [30,32]

$$B_2 = 0.488(7)(17), \quad B_3 = 0.743(14)(65),$$

(15)

and

$$B_4 = 0.920(12)(16), \quad B_5 = 0.707(8)(44),$$

(16)

with the first error being statistical and the second systematic. Similar results are obtained by ETM [33] and SWME [34] collaborations, although the values for $B_4$ and $B_5$ from the ETM collaboration are visibly below the ones from RBC-UKQCD given above: $B_4 = 0.78(4)(3)$ and $B_5 = 0.49(4)(1)$. Except for $B_4$ all values differ significantly from unity prohibiting the use of the vacuum insertion method.

In DQCD in the large $N$ limit, in which only factorizable contributions are present, one finds on the other hand [35]

$$B_2 = 1.20, \quad B_3 = 3.0, \quad B_4 = 1.0, \quad B_5 = 0.2 \quad (\text{large } N \text{ limit}).$$

(17)

These results differ drastically from the lattice results given above except for $B_4$. For instance $B_3$ is by a factor of four larger than the lattice result and $B_5$ by a factor of 3.5 smaller. But they apply to $\mu = \mathcal{O}(m_c)$ while the lattice results where obtained at $\mu = 3 \text{ GeV}$. Very importantly, as demonstrated in [35], the meson evolution to scales $\mathcal{O}(1 \text{ GeV})$ followed by the usual quark-gluon evolution, allows again to understand the pattern of lattice values in (15) and (16) and agrees with them within 5−20% depending on the considered parameter. Again the meson evolution and not the quark-gluon evolution is responsible for this insight. Moreover, this insight in the lattice results has been obtained basically without any free parameters, except for the cut-off $\Lambda$ which being physical is in any case approximately known and in the ballpark of 0.7 GeV, if only pseudoscalars are present in the loops as done in [35]. Moreover, our calculation has been done in the chiral
limit and we expect that going beyond this limit the agreement with RBC-UKQCD values would be better. I should stress that our goal was by no means to compete or even verify that RBC-UKQCD results are correct because this has been done already by two other collaborations. The main goal of our paper, unfortunately missed by all referees of our paper in JHEP and EPJC, was to gain the following important lesson from this analysis:

- Working in the strict large $N$ limit in non-leptonic Kaon decays misses a very important QCD dynamics represented by meson evolution so that it is mandatory to take it into account in any sensible phenomenology. As demonstrated in [35] this dynamics is hidden in lattice QCD results for $K^0 - \bar{K}^0$ mixing but as we will see in Section 4 it is missed in the $\chi$PT framework.

Now the controversial FSI are absent in the $K^0 - \bar{K}^0$ mixing but are certainly present in $K \to \pi\pi$ decays and as claimed by $\chi$PT researchers they are responsible for a significant enhancement of $\varepsilon'/\varepsilon$ over the RBC-UKQCD results as described above. Jean-Marc and myself looked at this issue in [36] and reached the following conclusions:

- In $\chi$PT, without additional dynamics, it is impossible to separate the current-current operator $Q_2 - Q_1$, responsible dominantly for the $\Delta I = 1/2$ rule, from the QCD penguin operator $Q_6$ that is irrelevant for this rule at $\mu = m_c$ and amounts to at most 15% correction at $\mu = 1$ GeV. Therefore claiming that pion dynamics, which could indeed provide an enhancement of $\text{Re}A_0$, enhances the matrix element of $Q_6$ at the same level as $\text{Re}A_0$ is questionable.

- Our claim is that the suppression of $\varepsilon'/\varepsilon$ by the meson evolution, being of $O(p^0/N)$ in the case of the $Q_6$ operator, must be more important than the enhancement of $\varepsilon'/\varepsilon$ by FSI that are $O(p^2/N)$.

Finally, let me react to the statements made in [25,26] that in all papers I was involved in, all absorptive parts have been set to zero and that the analysis in [5] leading to (10) was done in the context of DQCD. This is fake news. The analysis in [5] used entirely RBC-UKQCD results that include absorptive parts as mentioned above.

What has been obtained in DQCD is the upper bound [5,28]

$$\langle \varepsilon'/\varepsilon \rangle_{\text{SM}} \leq (6.0 \pm 2.4) \times 10^{-4}, \quad \text{(DQCD)}$$

that follows from $B_6^{1/2} \leq B_8^{3/2}$ with $B_8^{3/2}$ taken from RBC-UKQCD collaboration. It includes other uncertainties, but not the NNLO QCD corrections which provide a downward shift of $\varepsilon'/\varepsilon$ by 1 – 2 units as we will discuss now.

3 Few messages from short distance

All present analyses of $\varepsilon'/\varepsilon$ are based on the Wilson coefficients of QCD and EW penguin operators evaluated already 25 years ago at NLO in [29,37–41]. While at this level some removal of unphysical renormalization scheme and scale dependences present at LO takes place in the case of QCD penguins, as pointed out in [42], this is not the case for EW penguin contributions. In fact, as emphasized in [43], all the existing estimates of $\varepsilon'/\varepsilon$
at NLO suffer from short-distance renormalization scheme uncertainties in EW penguin contributions and also scale uncertainties in $m_t(\mu)$ that were practically removed in the NNLO matching at the electroweak scale already 20 years ago [42]. In the naive dimensional regularization (NDR) scheme, used in all recent analyses, these corrections enhance the electroweak penguin contribution by roughly 16%, thereby leading to a negative shift of $-1.3 \times 10^{-4}$ in $\varepsilon'/\varepsilon$, decreasing further its value, similar to isospin breaking effects. This could appear small in view of other uncertainties. However, on the one hand, potential scale and renormalization scheme uncertainties have been removed in this manner and on the other hand, one day such corrections could turn out to be relevant. Finally, the fact that this correction further decreases $\varepsilon'/\varepsilon$ within the SM gives another motivation for the search for NP responsible for it.

The world is now waiting for the final results of NNLO QCD corrections to QCD penguin contributions to $\varepsilon'/\varepsilon$. Preliminary results appeared already in [44–46] and looking at Fig. 1 in [46] we observe that the scale uncertainties in $\varepsilon'/\varepsilon$ have been practically eliminated and that these corrections slightly suppress $\varepsilon'/\varepsilon$ further. Yet, in order to be able to put this observation on solid grounds, we badly need final result from these impressive efforts.

We now turn to the highlights of this presentation.

4 The anatomy of the Gisbert-Pich plot

In a detailed review [25] and two conference proceedings [26,27] Gisbert and Pich presented their view on the present status of $\varepsilon'/\varepsilon$ within the SM. They work within the context of $\chi$PT which at its basis uses low energy symmetries of QCD. However, it should be realized that without any additional dynamical input from lattice QCD or large $N$ approach, like DQCD, hadronic matrix elements relevant for $\varepsilon'/\varepsilon$ cannot be calculated in $\chi$PT.

At this point it should be stressed that DQCD is not a constrained $\chi$PT, as stated by one referee of our papers, and anybody who thinks like this has missed completely the main ideas of DQCD [2]. The differences between DQCD and $\chi$PT have been spelled out in many of our papers and I will not repeat them here. Probably the best comparison is given in Section 3.3 of [49]. Let me just mention two points:

- Even if in both DQCD and $\chi$PT meson loops are involved, DQCD uses a physical cut-off in the calculation of these loops and thereby allows to restrict the loop momenta to the region in which the truncated meson theory is valid. In this manner one achieves a clear separation between long and short distance contributions which in turn allows through the meson evolution a proper matching of hadronic matrix elements with short distance dynamics. On the other hand $\chi$PT uses dimensional regularization which results in counter terms represented by low energy constants which can either be extracted from experiment or calculated by lattice QCD. As we will see soon, a proper matching with short distance physics is very difficult in this approach.

\[^2\]For a pedagogical introduction see [47,48] and a more recent reviews are [1,49].
While in DQCD the $Q_6$ operator can easily be separated from current-current operators, as discussed in [36], this is not possible in $\chi$PT unless some dynamics like QCD at large $N$ is used. See our discussion in Section 2.

Central for [25] and the subsequent two papers is the plot in Fig. 1, to be termed GP-plot in what follows. It shows the dependence of $\varepsilon'/\varepsilon$ on the low-energy constant $L_5$. It is obtained by finding the $\chi$PT realization of the $\Delta S = 1$ effective Lagrangian in the large $N$ limit which allows to find the coupling $g_8$ in this limit as given in (47) of [25]. It is a linear combination of contributions of current-current operators and the penguin operators $Q_4$ and $Q_6$. The one involving $Q_6$ is proportional to $L_5$ which is varied in Fig. 1. In addition the authors include pion loops representing FSI but the effect of meson evolution is absent in their calculation as the inclusion of it is beyond the $\chi$PT framework.

For our purposes it will be useful to find an analytic formula for the central dotted red line in Fig. 1. It reads

$$\frac{\varepsilon'}{\varepsilon}_{SM} = (19.2 L_5 - 7.8) \cdot 10^{-4}. \quad (19)$$

In the absence of the meson evolution, which has nothing to do with pion loops of these authors, one has $L_5 = 1.84 \cdot 10^{-3}$ which corresponds to the large $N$ limit of $B_6^{(1/2)}$ in [6], that is $B_6^{(1/2)} = 1$. This implies SM value of $\varepsilon'/\varepsilon$ as large as $27.5 \cdot 10^{-4}$, roughly by a factor of three larger than in (7). Indeed the impact of pion loops on $\varepsilon'/\varepsilon$, as calculated in [25], is very large. But, in spite of the fact that (47) in [25] is valid only in the large $N$ limit, eventually the authors decide to use the current FLAG compilation for $L_5$ which reads

$$L_5^{\text{Latt}} = (1.19 \pm 0.25) \cdot 10^{-3}, \quad (20)$$

and through (19) implies the $1\sigma$ range

$$10.2 \cdot 10^{-4} \leq \frac{\varepsilon'}{\varepsilon}_{SM} \leq 19.8 \cdot 10^{-4}. \quad (21)$$

As $L_5$ from FLAG surely includes some $1/N$ corrections, and (47) in [25] has been obtained in the large $N$ limit, it is not evident that such a procedure is really self-consistent. But let us not enter this issue in our Christmas story.

In any case, after the inclusion of other uncertainties, their final result for $(\varepsilon'/\varepsilon)_{SM}$ reads

$$(\varepsilon'/\varepsilon)_{SM} = (15 \pm 7) \cdot 10^{-4}. \quad (22)$$

This according to the authors of [25] is the present status of $\varepsilon'/\varepsilon$ within the SM. I certainly disagree with it. It can only be their view what $\varepsilon'/\varepsilon$ in the SM is. As we will see soon, one cannot even state that this is the view of all $\chi$PT experts. But let us first see what are the implications of the result in (22) taken at face value:

- There is no $\varepsilon'/\varepsilon$ anomaly.
- There is still a large room for NP contributions, as even within $1\sigma$, values as low as $8 \cdot 10^{-4}$ and as high as $22 \cdot 10^{-4}$ are allowed.
- Consequently this result is not motivating for NP searches. NP enhancing significantly $\varepsilon'/\varepsilon$ and also NP suppressing it even by a factor of two are both still allowed.
Fortunately, the authors are aware of this problematic and our statements are supported by a sentence of Gisbert and Pich in [25] which they repeated in [26]:

*Our dominant uncertainty reflects our ignorance about $1/N$-suppressed contributions that we have missed in the matching process.*

In my language this refers to the absence of meson evolution in their calculation which they want to replace somehow by using the FLAG value of $L_5$ that is rather uncertain.

Let me then perform the following exercise. I take at face value the GP calculation of pion loops that with the large $N$ value for $L_5$ resulted in a large value of $\epsilon'/\epsilon$. But as we have seen in Section [2] in the case of $K^0 - \bar{K}^0$ mixing our calculation of hadronic matrix elements in the large-$N$ limit totally misrepresented their values obtained by three lattice QCD groups. I definitely do not need the FLAG value for $L_5$ as DQCD is not ignorant about $1/N$ suppressed contributions that are included by means of the meson evolution in the matching process. I can therefore find the impact of the meson evolution on the plot in Fig. [1] without any help from lattice QCD. It is shown in Fig. [2]. Not much is left from the GP-plot as a large part of this plot is eliminated by the meson evolution. Few comments are in order.

- Our result $L_5 \leq 1.0 \cdot 10^{-3}$, that follows from the upper bound on $B_6^{(1/2)}$ in [13] and the large $N$ value $L_5 = 1.84 \cdot 10^{-3}$ corresponding to $B_6^{(1/2)} = 1$, is consistent with...
the FLAG value of $L_5$, but is significantly below FLAG’s central value.

- In view of my comments on the size of the impact of FSI on $\epsilon'/\epsilon$, as calculated presently in $\chi$PT, in Section 2 I expect that in reality the black region is shifted significantly to the left.

- But even with the result in Fig. 2 adding NNLO QCD corrections to EW-penguins, absent in the GP-plot, I end up with an upper bound

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\text{SM}} < 11 \cdot 10^{-4},$$

which in my view is conservative.

Thus even including GP’s pion loops, the meson evolution implies a value of $\left(\frac{\epsilon'}{\epsilon}\right)_{\text{SM}}$ significantly below the data. This eliminates NP scenarios which suppress $\epsilon'/\epsilon$. I leave it to the readers to decide whether this result can be considered as a hint for an $\epsilon'/\epsilon$ anomaly or not.

But now comes a surprise. As discovered by Jean-Marc, in a 2014 paper two important $\chi$PT experts, Bijnens and Ecker, making apparently a more sophisticated analysis of $L_5$ than done by FLAG, found the range $0.5 \leq L_5 \leq 1.0$ \cite{50}. This through (19) implies the range

$$1.8 \times 10^{-4} \leq \left(\frac{\epsilon'}{\epsilon}\right)_{\text{SM}} \leq 11.4 \times 10^{-4}, \quad \text{(BE)}$$

Figure 2: The impact of meson evolution on the Gisbert-Pich plot.
in perfect agreement with the bound in (23) and the very low value for $(\varepsilon'/\varepsilon)_{SM}$ predicted within DQCD. It is beyond my skills to judge whether the FLAG number or the number following from [50], is correct, but it appears that the uncertainty in the estimate of $(\varepsilon'/\varepsilon)_{SM}$ by $\chi$PT is even larger than shown to us by Gisbert and Pich.

5 Congratulations to NA48 and KTeV

I have reached my goal now. On the basis of my work with Jean-Marc Gérard and the findings above I am ready to sign the statement in Fig. 3.

However, similar to the case of $R_K$ and $R_{K^*}$ anomalies hinted by the LHCb data, which require the confirmation by Belle II, the $\varepsilon'/\varepsilon$-anomaly hinted by DQCD requires its confirmation by lattice QCD before it is firmly established. Moreover, while DQCD provides presently a rough range for NP contribution to $\varepsilon'/\varepsilon$

$$7 \cdot 10^{-4} \leq (\varepsilon'/\varepsilon)_{NP} \leq 15 \cdot 10^{-4},$$

one should hope that LQCD will provide in due time a much more narrow range that is crucial for NP searches.

I already discussed in detail why $\chi$PT is not in a position to congratulate NA48 and KTeV for this important discovery, so let me explain in a few lines why this is also the case for lattice QCD. In fact none of the members of the RBC-UKQCD collaboration, I talked
to, claims that their result implies NP physics and some are surprised that I am looking for it. Let me stress again, my motivations for looking for NP in $\varepsilon'/\varepsilon$ are definitely not based on the results from RBC-UKQCD, but on our results in DQCD. Yet, it is gratifying that RBC-UKQCD gives some support for them. Now come few lines about lattice QCD in general.

Lattice QCD is like an experiment, you perform very sophisticated numerical calculations based on first principles of QCD, which eventually will tell us one day with high precision what is the value of $\varepsilon'/\varepsilon$ in the SM. I admire the RBC-UKQCD collaboration for their efforts and also other collaborations, in particular those which calculate hadronic matrix elements and weak decay constants relevant for $B$ physics. They all should get required financial support to reach their goals. Yet the following should be realized.

The numerical values of various hadronic parameters lattice groups present to us come from a Blackbox without any insight why these values are what they are. For flavour phenomenology this insight is generally irrelevant. One can just use these numbers at face value hoping that they are right. Similar to experiments, one needs results from at least two lattice collaborations which agree with each other to be sure that they are correct. Moreover, as far as $B$-physics and charm physics is concerned, I do not know any method which would allow us to look inside the lattice Blackboxes. But in $K \rightarrow \pi\pi$ decays the situation is different. We have DQCD, which indeed allows us to see, at least roughly, what happens inside these Blackboxes. I just want to quote two examples:

- The parameter $\hat{B}_K$ entering the phenomenology of $\varepsilon_K$ is 0.75 in the large $N$ limit [9]. It takes at most one minute for beginners and few seconds for flavour experts to find this out. In DQCD, including $1/N$ corrections, it takes, say, a week to find $\hat{B}_K = 0.67 \pm 0.07$ [51], when only the octet of pseudoscalar mesons is included. It takes another few weeks to include lowest lying vector mesons which allows to increase the cut-off $\Lambda$ to 0.85 GeV and to improve the matching with short distances to find $\hat{B}_K = 0.73 \pm 0.02$ [49]. Several lattice groups calculated $\hat{B}_K$ over almost 20 years resulting in the latest FLAG value $\hat{B}_K = 0.763 \pm 0.010$. I have asked many lattice experts what is their explanation of $\hat{B}_K$ being so close to its value in the large $N$ limit. I did not get any answer. In DQCD the answer is simple. The loops with pseudoscalars suppress $\hat{B}_K$, while the ones with vector mesons enhance it. But vector meson contributions are mass suppressed relative to pseudoscalar ones, so that they loose in this competition resulting in $\hat{B}_K < 0.75$. This argument is consistent with a more detailed analysis of Jean-Marc [52]. Thus DQCD has still another prediction: FLAG’s central value will go below 0.75 one day.

- The case of BSM parameters $B_{2-5}$ discussed already in Section 2, in which DQCD explained the pattern of their values found by lattice QCD [35]. I challenge both lattice experts and $\chi$PT experts to explain this pattern.

Other insights provided by DQCD, like the one related to the $\Delta I = 1/2$ rule, are described in particular in [1,49]. Yet, one of the referees of our analysis in [35] stated that the insight into the lattice values $B_{2-5}$, that we provided in that paper, is of no interest to the flavour community. It could be. In the 21st century possibly most people are only interested in numbers coming out from Blackboxes. But one should realize that this insight, involving four numbers, gives a very strong support to the relevance of the
meson evolution, which represents the crucial QCD dynamics at scales below 1 GeV and thereby gives an additional support to the $\varepsilon'/\varepsilon$ anomaly.

In this context I would like to mention our 2018 calculation of all BSM hadronic matrix elements contributing to $\varepsilon'/\varepsilon$ [53]. Also in this case the results in the large $N$ limit totally misrepresent the values of hadronic matrix elements at scales $\mathcal{O}(1 \text{ GeV})$ that we calculated in [53] by performing the meson evolution. Similarly to the case of $B_K$, $B_6^{(1/2)}$, $B_8^{(3/2)}$ and $B_{2-5}$, the pattern of meson evolution agrees well with the quark-gluon evolution above 1 GeV. No lattice QCD results for these matrix elements are known and it will be interesting to see what lattice QCD will find for them one day.

All this shows that with the help of DQCD we can probe, in the case of $K \to \pi\pi$, the dominant QCD effects in the Blackboxes of our lattice colleagues. As we have seen $\chi$PT has no means to do it.

6 Summary

My Christmas story approaches the end. Let me make some final comments

- Meson evolution and separation of $Q_6$ from $Q_2 - Q_1$ is crucial for the correct estimate of $\varepsilon'/\varepsilon$ within the SM. It is an important advantage of DQCD and lattice QCD over $\chi$PT. Lattice QCD experts are not aware of the fact that they include meson evolution but we demonstrated this in several papers.

- Even with pion loops of Gisbert and Pich, the inclusion of the meson evolution and NNLO QCD corrections to EW penguins results in $(\varepsilon'/\varepsilon)^{\text{SM}}$ significantly below the data. This is an important message for model builders. NP has to provide an upward shift in $\varepsilon'/\varepsilon$ in order to reproduce data.

- The fate of $(\varepsilon'/\varepsilon)_{NP}$ depends now on the improved values for $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from lattice QCD, improved treatment of FSI and the inclusion of isospin breaking effects.

- The calculation of isospin breaking effects is presently led by $\chi$PT and in spite of my critical comments on some other aspects of this approach, I am sure that $\chi$PT will provide much more precise value than given in [4] and this will happen likely ahead of lattice QCD. In fact one of the first calculations of these corrections was done in [11] by Jean-Marc and myself with the result in the ballpark of 0.25. This is consistent with [4], although a bit larger. The improvements on $L_5$ could also help $\chi$PT in predicting $(\varepsilon'/\varepsilon)^{\text{SM}}$.

- Last but not least the final result on NNLO QCD corrections to QCD penguin contributions will definitely reduce perturbative uncertainties. Also subleading contributions to NNLO QCD effects in EW penguins are still missing.

Personally, I am convinced about the presence of new physics in $\varepsilon'/\varepsilon$ and prophesy that one day everybody will agree that [54]

$$(\varepsilon'/\varepsilon)^{\text{SM}} = (5 \pm 2) \cdot 10^{-4}, \quad (2026),$$

(26)
which is also consistent with (18). But this will take still several years. This number is
based simply on the fact that in the large $N$ limit we have first $8.6 \cdot 10^{-4}$. The suppression
of $\varepsilon'/\varepsilon$ through meson evolution will be partly compensated by FSI so that including
these effects we will see numbers in the ballpark of $(7 \pm 1) \cdot 10^{-4}$. Including NNLO QCD
corrections will result in (26). All this is very rough, but after spending over 30 years
with $\varepsilon'/\varepsilon$ these are my gut feelings what is its value within the SM. Most importantly
they are supported by DQCD.

With this conviction in mind I plan to continue to search for NP responsible for $\varepsilon'/\varepsilon$
anomaly as I did in the last 3.5 years. In this context let me just mention our additional
2018 papers that were not mentioned above [43,55,56], which are reviewed in [1,54,57].
In particular in [54], which I plan to update, there is a table with references to papers
that performed several analyses of $\varepsilon'/\varepsilon$ in the context of various extensions of the SM.

The next decade should be very exciting for flavour phenomenology, not only at Belle
II [58] and LHCb [59], but also generally at CERN where also ATLAS and CMS will
contribute in an important manner [59,60]. I also expect that the results on $K^+ \to \pi^+\nu\bar{\nu}$
from NA62 collaboration at CERN [61] and on $K_L \to \pi^0\nu\bar{\nu}$ from KOTO [62] at J-PARC,
when correlated with $\varepsilon'/\varepsilon$, will allow a deep insight into possible NP at short distances [63].
This insight will be enriched by $(g-2)_\mu$ experiments at Fermilab and J-PARC and in the
context of various experiments probing charged lepton flavour violation at CERN, PSI,
KEK and J-PARC. I also hope that my prophetic statements about $\varepsilon'/\varepsilon$, very appropriate
for a Christmas story, will be confirmed by several lattice QCD groups before my 80th
birthday in 2026.

The discovery of new sources of CP violation by NA48 and KTeV collaborations is
very important because we need them in order to explain our existence. I really have no
idea whether NP in $\varepsilon'/\varepsilon$ is responsible for our existence, not only because this is presently
beyond my skills but also because we did not yet identify what this NP is. It could also
happen that my strong believe in the presence of NP in $\varepsilon'/\varepsilon$ will be refuted by lattice
QCD one day. Yet, without the measurements of $\varepsilon'/\varepsilon$ by NA48 and KTeV collaborations,
all these discussions between RBC-UKQCD, $\chi$PT and DQCD experts would be much less
exciting and we should thank these two experimental groups for the result in (1). I wish
everybody, who still reads these lines, a Merry Christmas and a Happy 2019!

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