Algorithm for obtaining palindromes of the binary number system

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Abstract. The aim of this paper is to describe a new method for searching all binary palindromes that have a given number of the binary digits. The main result is presented in the form of an algorithm. The obtained algorithm is based on the use of the regularity revealed in the course of the study, based on the application of basic arithmetic operations with palindromes that have a various number of the binary digits. The paper also provides examples of using this algorithm to obtain binary palindromes with different number of the binary digits. The algorithm formulated here can be applied to analyse computer data written in the form of a binary code.

1. Introduction
The mathematical term "palindrome" was formulated in the 17th century (from Greek πάλιν "back, again" and δρόμοι "movement") [1]. In number theory, a palindrome is understood as a natural number, the digits of which are arranged symmetrically, in a mirror image. Examples of the palindromes are integer numbers 121, 10101, 143341, etc. To obtain palindromes, there is an algorithm [2], which allows one to obtain a palindrome from any number, as suggested by the authors of [3]. The mathematician Charles Trigg proved the opposite: he found numbers from which palindromes are not obtained in the first hundred iterations. One of these numbers is the number 196. Also, this algorithm does not allow obtaining palindromes from numbers in the binary number system. [3]. In particular, the binary number 10110 will never make a palindrome. The study of numerical palindromes can be applied in palindromic matrices considered in publications [4], [5], [6]. The numbers of palindromes of the binary number system for even and odd digits were determined in the article [7]. The article [8] considers obtaining expressions for calculating the probability of random events consisting in the formation of binary palindromes on a finite equally probable combination of zeros and ones.

The aim of the work is to find a new method that allows you to directly obtain the palindromes of the binary number system, and not select them from the totality of palindromes of other number systems, in particular, the decimal one. The resulting method is based on the application of a regularity linking palindromes of different categories with each other. Before describing the algorithm, let's consider this pattern in more detail.

2. Mutual dependence of palindromes of the binary number system that have a different number of digits among themselves
For convenience, we will write the palindromes of the binary number system in figure 1. The row number of this table corresponds to the ordinal number of the i-th position of the palindromes that make
it up. It should be noted that palindromes are indicated for the ordinal numbers of the category \( n = 1, 2 \ldots 7 \), since this is sufficient for understanding the regularity described below.

![Figure 1. Table of binary palindromes.](image)

In figure 2, each element is obtained by subtracting each palindrome from the following.

![Figure 2. Table of differences of adjacent palindromes.](image)

It should be noted that in each row of figure 2 the numbers are arranged symmetrically. For example, the last line consists of numbers 1000, 9100, 1000; from the central number 88910; after - the same numbers that stand before the central number - 1000, 9100, 1000. It is worth noting that each line consists of the modified previous ones: the central number with increasing line number \( n \) (starting from the fifth) is the sum of two numbers: the central number the previous line \( n-1 \) and; all numbers up to the central number in the row make up the string \( n-2 \) multiplied by 10. Thus, using the above-described pattern and knowing the first four rows of figure 2, you can get the rest of the rows (the differences of neighboring palindromes for digits above 4), and therefore, and the palindromes themselves. It is this pattern that underlies the algorithm for obtaining palindromes.

3. **Algorithm for obtaining palindromes of the binary number system**

Problem statement: To formulate a new algorithm that allows finding all binary palindromes of an arbitrary given finite bit \( n \).

Algorithm (rule for searching palindromes):

- Multiply by 10 each element of row number \( n-2 \) of figure 2.
- 2. Calculate the number \( z = \begin{cases} 910, & \text{if } n = 5 \\ \sum_{k=3}^{n-3} 8 \cdot 10^{n-k} + 910, & \text{if } n > 5 \end{cases} \).
- Write a row (from left to right) of palindromes, consisting of: the string obtained in step 1; the number obtained in paragraph 2; and the row obtained from the string of palindromes in clause 1, in reverse order.
- To the first palindrome of this category \( n=(10n-1+1) \) add each element of the line item 3 in turn.
• Write down all received palindromes of category \( n \).

Remark 1: The presented algorithm assumes the presence of a table of differences of adjacent palindromes. In the absence of this table, before applying the algorithm, it is required to determine the necessary elements of the table of differences of adjacent palindromes.

Remark 2: If the condition \( n - 2 > 4 \) is fulfilled, it is required to repeat steps 1-3 for palindromes of the \( m = n - 2 \) category.

4. Numerical simulation

Here are some examples of applying the resulting rule to find palindromes of different orders.

Example 1. It is required to get the palindromes of the binary numeral system of the 8th bit (\( n = 8 \)).

Solution: Since \( n - 2 > 4 \), perform steps 1-3 for \( m = n - 2 = 6 \).

1. Let's multiply each element of the row of the difference of adjacent palindromes of 4 digits (figure 2) by 10: 1100.
2. Applying the formula of item 2 of the algorithm, we obtain: \( z = 8 \times 103 + 910 = 8910 \).
3. Let's write the line: 1100 8910 1100
   Received the line of the difference of adjacent palindromes for the 6th digit.

Now, when there is a row of the difference of adjacent palindromes for the 6th bit, we apply the algorithm for \( n = 8 \).

1. Let's multiply each element of the row of the difference of adjacent palindromes of the 6th grade by 10: 11000 89000 11000.
2. Applying the formula of item 2 of the algorithm, we obtain: \( z = 8 \times 105 + 8 \times 104 + 8 \times 103 + 910 = 888910 \).
3. Let's write down the line of the difference of adjacent palindromes for the 8th position: 11000 89000 88910 11000 89100 11000

4. To the first palindrome, one by one, add each element of line item 3:
   - 10000001 + 1100 = 10011001
   - 10011001 + 89100 = 10100101
   - 10100101 + 11000 = 10111101
   - 10111101 + 88910 = 11000011
   - 11000011 + 11000 = 11011011
   - 11011011 + 89100 = 11100111
   - 11100111 + 11000 = 11111111

As a result, all 8 palindromes of the 8th order were obtained.

The answer: 10000001, 10011001, 10100101, 10111101, 11000011, 11011011, 11100111, 11111111.

Example 2. It is required to get the palindromes of the binary numeral system of the 9th bit (\( n = 9 \)).

Solution: Since \( n - 2 > 4 \), then we carry out items 1-3 for \( m = n - 2 = 7 \). In this case \( m - 2 > 4 \), therefore, it is more rational to consistently perform items 1-item 3 for \( n = 5, 7 \) and 9.

1. Let's multiply each element of the line of the difference of adjacent palindromes of 3 digits by 10: 100.
2. Applying the formula of item 2 of the algorithm, we obtain: \( z = 910 \).
3. Let's write the line: 100 910 100
   Received a line of the difference of adjacent palindromes for the 5th digit.

Now let's apply the algorithm for \( n = 7 \).

1. Let's multiply each element of the row of the difference of adjacent palindromes of the 5th grade by 10: 1000 9100 1000.
2. Applying the formula of item 2 of the algorithm, we obtain: \( z = 8 \times 104 + 8 \times 103 + 910 = 88910 \).
3. Let's write the line: 1000 9100 1000 88910 1000 9100 1000
   Received a line of the difference of adjacent palindromes for the 7th bit.

Now let's apply the algorithm for \( n = 9 \).
1. Multiply each element of the row of the difference of adjacent palindromes of the 7th grade by 10:
   10000 91000 10000 889100 10000 91000 10000
2. Applying the formula of claim 2 of the algorithm, we obtain: 
   \[ z = 8 \times 106 + 8 \times 105 + 8 \times 104 + 8 \times 103 + 910 = 888910 \]
3. Let's write the line: 
   10000 91000 10000 889100 10000 91000 10000 8888910 10000 91000 10000
4. To the first palindrome, add each element of the line item 4 in turn:
   10000 91000 10000 889100 10000 91000 10000 8888910 10000 91000 10000
   889100 10000 91000 10000
   91000 10000
   10000
   91000
As a result, we got all 16 palindromes of the 9th order.
   The answer: 100000001, 100010001, 100101001, 100111001, 101000101, 101010101, 101101101, 101111101, 110000011, 110010011, 110101011, 110111011, 111000111, 111010111, 111101111, 111111111.

Example 3. It is required to get the palindromes of the binary numeral system of the 5th bit (n = 5).
Solution:
1. Let's multiply each element of the line of the difference of adjacent palindromes of 3 digits by 10:
   100.
2. Applying the formula of item 2 of the algorithm, we obtain: 
   \[ z = 910 \]
3. Let's write the line: 100 910 100
   Received the line of the difference of adjacent palindromes for the 6th digit.
4. To the first palindrome, add each element of line item 3 in turn:
   10001 + 100 = 10101
   10101 + 910 = 11011
   11011 + 100 = 11111
As a result, all 4 palindromes of the 5th order were obtained.
   The answer: 10001, 10101, 11011, 11111.

5. Conclusion
In the process of studying palindromes of the binary number system, a method was developed for finding all palindromes with a given number of digits. This method is an algorithm based on a certain pattern of differences between adjacent palindromes of the same order. When calculating palindromes according to the specified algorithm, the authors failed to find not one bit of binary palindromes, in which the presented algorithm does not allow finding all palindromes with a given number of digits. The obtained algorithm is supposed to be used in the analysis of computer data in the form of a binary code.

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