Radiation pressure measurement using a macroscopic oscillator in an ambient environment

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In contrast to current efforts to quantify the radiation pressure of light using nano-micromechanical resonators in cryogenic conditions, we proposed and experimentally demonstrated the radiation pressure measurement in ambient conditions by utilizing a macroscopic mechanical longitudinal oscillator with an effective mass of the order of 20 g. The light pressure on a mirror attached to the oscillator was recorded in a Michelson interferometer and results showed, within the experimental accuracy of 3.9%, a good agreement with the harmonic oscillator model without free parameters.

According to Newton’s second law, the force $F$ on an object is well-known to be equal to the rate of change of the momentum $p$ of the object as $F = dp/dt$. From this fundamental law, we can expect the largest conversion of the optical momentum to the mechanical momentum of the object as equal to the rate of change of the optical momentum to the mechanical momentum of a medium when light is fully reflected from a mirror. The magnitude of this force on an ideal 100% reflecting mirror in a vacuum is given by

$$F = \frac{2P}{c},$$

where $P$ is the optical power and $c$ is the speed of light. This radiation pressure of light was first theoretically described by Maxwell [1] in 1873, and then experimentally measured independently by Lebedev [2] and by Nichols and Hull [3] in 1901, but the accuracy of these early experiments was very limited.

Despite being a century-old discovery, the radiation pressure continues to be one of the key research interests in current optomechanics, such as in cooling of mechanical resonators [4–7], solar sail development [9], ultra-high laser power measurements [10–11], and nano-scale cantilevers’ spring constant calibration [12–13], to name a few. Recently, there also has been renewed interest in the centennial Abraham-Minkowski controversy on the light momentum in a dielectric medium [14–24].

The main trend in light pressure studies in recent years has been to miniaturize a mechanical oscillator to the nano-micro scale for a higher sensitivity to the radiation pressure [4–6, 7, 25]. However, optical forces in those nano-micromechanical systems have been directly accompanied by photothermal effects due to short thermal time constants of the miniaturized resonators [6, 7, 26–28], which has required further sophisticated techniques to discern them from the radiation pressure effects. Therefore, various optical, mechanical and thermal techniques have been developed to overcome the trade-off between the radiation pressure and the photothermal effects such as complex resonator designs consisting of highly reflective multilayer coatings deposited on the cantilever to further increase the reflectivity [7, 12], attachment of an additional mass to increase the thermal time constant of the cantilever [13], or other ways to separate the optical force from the photothermal effects [29–30].

In this work, we attempted a new direction opposite to the current trends by achieving a quantitative measurement of the radiation pressure of light in an ambient environment at room temperature by utilizing a macroscopic mechanical harmonic oscillator, which is orders of magnitude heavier than oscillators in previous reports [29–31]. The experimental setup is illustrated in Fig. 1. In contrast to conventional torsional oscillators used in most of the previous measurements, our oscillator is longitudinal with a mass hanging on a spring. Here we varied the mass and the damping constant to verify the accuracy of the harmonic oscillator model in the radiation pressure measurements. Note that our method can obviate the elaborated process to calibrate the spring constants [29–30], as the only additional measurement of the oscillator parameters is the direct determination of the oscillator masses using a digital scale.

The oscillator is driven optically by the reflection of the modulated laser beam at the wavelength of 975 nm at a highly reflective dielectric mirror, which is the oscillator mirror 1 in Fig. 1. The reflectivity of this mirror was larger than 99.9% and Eq. (1) can be used to quantify the optical force. The longitudinal displacement of the oscillator was detected by a Michelson interferometer using another laser at the wavelength of 632.8 nm. The interferometer beam was reflected from the oscillator mirror 2. The shifts of the interference fringes were recorded using a camera at a frame rate of 200 frames per second, from which the displacement of the oscillator was estimated for various incident light powers. It is noteworthy, in particular, that photothermal effects can be excluded since light is reflected from a highly reflective dielectric mirror on a macroscopic mechanical oscillator whose thermal time constant is much longer than the modulation time of the laser field. A more complete de-
Newton’s equation of motion for the mechanical oscillator with an effective mass \( m \) given by \([4]\)

\[
\frac{d^2 z}{dt^2} + 2\zeta \omega_0 \frac{dz}{dt} + \omega_0^2 z = \frac{F}{m},
\]

where \( \omega_0 \) is the undamped resonance frequency of the harmonic oscillator, \( \zeta \) is the damping coefficient, and \( F \) is the net external force. The damping coefficient is related to the Q factor as \( Q = 1/(2\zeta) \). When the mass of the vertically aligned spring is not negligible, the effective mass of the oscillator is given by \( m = m_0 + m_s/3 \), where \( m_0 \) is the rest mass of the oscillator and \( m_s \) is the rest mass of the spring \([33]\).

If the force is harmonically modulated with the angular frequency \( \omega \) as \( F = F_0 \cos^2(\frac{\omega}{2} t) = \frac{1}{2} F_0 [1 + \cos(\omega t)] \), where \( F_0 \) is the peak to peak amplitude of the force, then the steady-state solution of Eq. (2) is given as

\[
z(t) = z(\omega) \cos(\varphi + \omega t) + F_0/(2\omega_0^2) \cos(\omega_0 t),
\]

where \( \varphi = \arctan[2\omega_0 \zeta/(\omega^2 - \omega_0^2)] \in [-\pi, 0] \) and the displacement amplitude \( z(\omega) \) is given by

\[
z(\omega) = \frac{F_0/m}{2\sqrt{\omega_0^2 \zeta^2 + (\omega^2 - \omega_0^2)^2}}.
\]

The resonance frequency of the significantly under-damped oscillator with \( \zeta < 1/\sqrt{2} \) is \( \omega_1 = \omega_0 \sqrt{1 - 2\zeta^2} \) \([33]\). At this frequency, the displacement amplitude of the oscillator in Eq. (3) obtains its peak value, given by

\[
z_0 = F_0/(4m\omega_0^2\zeta\sqrt{1 - \zeta^2}).
\]

Thus, from the measured peak value of the displacement amplitude, we can obtain the optical force as

\[
F_0 = 4m\omega_0^2\zeta\sqrt{1 - \zeta^2} z_0.
\]

Here, for our macroscopic oscillator, the undamped angular frequency and the damping constant can be accurately determined based on the position and width of the mechanical resonance peak and the effective mass of the oscillator can be determined from the oscillator and spring masses measured with a digital scale. The effective mass of the lower damping oscillator without the damper fibers in Fig. 1 is \( m = (18.363 \pm 0.001) \) g, while the effective mass of the higher damping oscillator with the damper fibers is \( m = (19.007 \pm 0.001) \) g. Note that the difference in the oscillator masses is mainly produced in their fabrication and it is not due to the damper fibers whose total mass is less than 0.2 g. The damper fiber is commercially available optical fiber (Thorlabs, FG105LCA), which provides a very high mechanical stability against tensile stress.

Figure 2 presents the experimental results. In Fig. 2(a), the measured displacement amplitude of the lower damping oscillator is plotted as a function of the modulation frequency of the driving laser field with an example peak to peak power amplitude of \( P_0 = 0.975 \) W. Fig. 2(b) presents the same plot for the higher damping oscillator. Each graph that is marked with a solid line is
FIG. 2. The measured displacement amplitude of the mechanical oscillator is plotted (a) for the lower damping oscillator and (b) for the higher damping oscillator as a function of the modulation frequency of the driving laser with an example peak to peak power amplitude of \( P_0 = 0.975 \) W. Each graph that is marked with a solid line is the averaged frequency spectrum of a measurement for a single modulation frequency. The graphs peak at the modulation frequency and the peak points are marked with red dots. The modulation frequency is varied around the resonance frequency of the mechanical harmonic oscillator. The peak points of the graphs form together a curve that is the response function of the mechanical harmonic oscillator. The fitted harmonic oscillator response function is marked with the dashed line. In (c) and (d), the measured peak to peak radiation force amplitude is plotted for the two oscillators as a function of the peak to peak laser power amplitude. The least-squares regression lines are marked with the solid lines. The linear theoretical curve \( F_0 = 2P_0/c \) is presented by the dashed lines.

In Figs. 2(a) and 2(b), one can see that the fitted harmonic oscillator response function in Eq. (3) accurately describes the experimental results of both oscillators. It can be noted that, in the presence of photothermal effects, this response function would be modified from the ideal harmonic oscillator form as described, e.g., in Refs. [29, 30]. Thus, the ideal harmonic oscillator form of the frequency response function in Fig. 2(a) indicates that photothermal effects are negligible in our macroscopic setting as expected. In Figs. 2(a) and 2(b), one can also see that the mechanical resonance peak is observable in the noise spectrum that is seen below the fitted harmonic oscillator response function.

In the least-squares fitting of the harmonic oscillator response function in the experimental data of the lower damping oscillator in Fig. 2(a), the undamped frequency of the mechanical oscillator is found to be \( f_0 = (1.730943 \pm 0.000018) \) Hz and the damping constant is found to be \( \zeta = 0.000750 \pm 0.000012 \), which corresponds to the Q-factor of \( Q = 667 \pm 11 \). The errors indicate the 68.27% confidence intervals, which correspond to one standard deviation of normally distributed quantities. Using the experimental data of the higher damping oscillator in Fig. 2(b), we respectively obtain the undamped oscillator frequency of \( f_0 = (1.696275 \pm 0.000073) \) Hz and the damping constant of \( \zeta = 0.003006 \pm 0.000051 \), which corresponds to \( Q = 166.3 \pm 2.8 \).

Figure 2(c) shows the measured peak to peak radiation
force amplitude of the lower damping oscillator following from Eq. (4) as a function of the peak to peak laser power amplitude. The corresponding radiation force amplitude graph for the higher damping oscillator is presented in Fig. 2(d). The slope of the least-squares regression line is \( \frac{dF_0}{dP_0} = (6.66 \pm 0.26) \times 10^{-9} \text{ s/m} = (1.998 \pm 0.077)/c \) for the lower damping oscillator. The relative error is 4.3%, from which 1.6% comes from the determination of the damping constant in the fitting of Fig. 2(a) and 2.7% comes from the determination of the peak displacement amplitude. These values include uncertainties related to the laser power fluctuations around the expected value. For the higher damping oscillator, the regression line is \( \frac{dF_0}{dP_0} = (6.66 \pm 0.26) \times 10^{-9} \text{ s/m} = (1.998 \pm 0.077)/c \), where the relative error is 3.9%, from which 1.7% comes from the determination of the damping constant and 2.2% comes from the determination of the peak displacement amplitude. The slope of the corresponding universal theoretical line is \( 2/c = 6.67 \times 10^{-9} \text{ s/m} \). Thus, the experimental results of both the lower and higher damping oscillators agree with the theory within the experimental accuracy.

Previously, in Ref. [20], the slope in the optical force-power graph was measured to be \( \frac{dF_0}{dP_0} = 1.7 \times 10^{-9} \text{ s/m} = 0.50/c \), which is smaller than our result due to the notably lower reflectivity of the mirror. In many other previous works [8, 12, 13], one has typically assumed the optical force-power relation in Eq. (1) as known and concentrated on the determination of the spring constant of the nano-micro scale oscillator based on that. Regarding the determination of the absolute radiation force, in contrast to our work, the main experimental uncertainties in the previous works have originated from the determination of the magnitudes of the small optical power and the spring constant of the oscillator.

In conclusion, we have have demonstrated that the radiation pressure of light can be accurately measured in ambient environment by utilizing a macroscopic mechanical oscillator and detecting how the modulation of the optical signal can be tuned to drive the nanoscale motion of the oscillator. We have carried out measurements for two oscillators with different masses and damping constants, and shown that the correspondence between the theory and experiment is obtained within the relative experimental accuracy. The introduced setup can also be used for probing optical forces when the oscillator is driven through the optical fibers used as the damper fibers that are part of the setup in Fig. [1] but these investigations related to the Abraham-Minkowski controversy are left as a topic of further work. Our macroscopic oscillator setup and its larger-scale variations can also be used for measuring high laser powers through the determination of the radiation pressure.

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