Coexistence of Age and Throughput Optimizing Networks: Competition vs Cooperation

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Abstract—We investigate coexistence of an age optimizing network (AON) and a throughput optimizing network (TON) that access the same wireless spectrum band using a WiFi-like CSMA/CA based access. We consider two modes of long run coexistence: (a) networks compete with each other for spectrum access causing them to interfere and (b) networks cooperate and achieve non-interfering access.

To model competition, we define a non-cooperative stage game parameterized by average age of the AON at the beginning of the stage, derive its mixed strategy Nash equilibrium (MSNE), and analyze the evolution of age and throughput over an infinitely repeated game in which each network plays the MSNE in every stage. Cooperation has a coordination device use a coin toss during each stage to select the network that must access the medium. Networks use the grim trigger punishment strategy, reverting to playing the MSNE every stage forever, if the other disobeys the device. We determine if there exists a subgame perfect equilibrium, that is the networks obey the device forever as they find cooperation beneficial.

We find that networks choose to cooperate only when they have a small enough number of nodes. Else, they would rather disobey the device and compete.

I. INTRODUCTION

The ubiquity of Internet-of-Things (IoT) devices has led to the emergence of applications that require these devices to sense and communicate information (status updates) to a monitoring facility, or share with other devices, in a timely manner. These applications include real-time monitoring for disaster management, environmental monitoring and surveillance [1] references therein], which require timely delivery of updates to a ground station, and vehicular communications for safety, where each vehicle broadcasts its status (for example, position and velocity) so that they have timely information about each other to enable applications like collision avoidance [2].

Such networks often share the wireless spectrum with WiFi networks. For instance, the Federal Communications Commission (FCC) in the US opened up the 5.85 – 5.925 GHz band, previously reserved for vehicular communication, for use by high throughput WiFi (802.11 ac) devices, leading to the possibility of coexistence between WiFi and vehicular networks [3]. Similarly, IoT devices like Unmanned Aerial Vehicles (UAVs), equipped with 802.11 a/b/g/n technology, operate in the 2.4 and 5 GHz bands in use by WiFi networks.

While a network of IoT devices would like to optimize freshness of status updates, a WiFi network would like to provide high throughputs for its users. We quantify freshness using the metric of age of information [4] and refer to the former network as an age optimizing network (AON) and to the latter as a throughput optimizing network (TON).

In this work, we investigate coexistence of an AON and a TON from a MAC layer perspective when both networks use a WiFi-like CSMA/CA (Carrier Sense Multiple Access with Collision Avoidance) based wireless medium access to share a common spectrum resource. We analyze coexistence using a repeated game theoretic approach, where in each CSMA/CA slot is one stage of the game. We assume networks are selfish players and aim to optimize their utilities in the long run. While an AON wants to minimize the discounted sum average age of updates of its nodes (at a monitor), a TON wants to maximize the discounted sum average throughput.

We consider two modes of coexistence namely competition and cooperation. The former has nodes in the networks probabilistically interfere with those of the other as they access the shared medium. When cooperating, a coordination device schedules the networks to access the medium such that nodes belonging to different networks don’t interfere with each other. While one may view such cooperation as a coexistence scheme in its own right, more importantly it serves as a ploy that helps illuminate the surprising desirability of competition between an AON and a TON. This, as we will reveal in detail, is explained by the interplay of the networks as they attempt to optimize their distinct utilities.

When competing, the interaction between an AON and a TON in each CSMA/CA slot is modeled as a non-cooperative stage game. We define the stage game and derive it’s mixed strategy Nash equilibrium (MSNE). Our stage game is parameterized by the average age of the AON at the beginning of the stage. This captures the fact that age at the end of a stage is a function of the age at the beginning. Our analysis shows that the equilibrium strategy of the AON in each stage is a function of the average age seen at the beginning of the stage and the number of nodes in the networks, whereas, that of the TON is static and only a function of the number of nodes in its own network. We study the evolution of the equilibrium strategies over time, when players play the MSNE in each stage of the repeated game, and the resulting utilities of the networks.

We enable cooperation by introducing a coordination device, which uses a coin toss in every stage to recommend who between the AON and TON must access the medium during the slot. Also, a network punishes the other network in case the other does not adhere to the recommendation of the device. Specifically, it use a grim trigger that has both the
networks revert to using the MSNE every stage following the stage in which the other network disobeys the device. One would expect the threat of this punishment to have networks always obey the device if in fact they preferred cooperation to competition. We identify when networks prefer cooperation by checking if the strategy profile that results by obeying the device forms a subgame-perfect equilibrium (SPE) [5].

We apply our models for competition and cooperation to two cases of practical interest [9] (a) when networks use the basic access mechanism, and (b) when networks use the RTS/CTS based access mechanism. The former has collision slots (more than one node accesses the channel leading to all transmissions in error) that are at least as large as slots that see a successful (interference free) transmission by exactly one node. In the latter, collision slots are much smaller than a successful transmission slot. We show that for both the mechanisms the networks prefer cooperation when the number of nodes in them are small enough. However, as the number of nodes become larger, networks prefer competition.

Specifically, when networks use the basic access mechanism, TON finds competition more favorable than cooperation and when networks use the RTS/CTS based access mechanism, AON prefers cooperation over cooperation. When networks use the basic access mechanism, occasionally all AON nodes refrain from transmitting during a stage. If competing, such stages allow the TON interference free access to the medium. If cooperating, such stages are not available to the TON. Thus, competing improves TON’s payoff. In contrast, when using RTS/CTS, the AON nodes see benefit in accessing the medium aggressively. As a result, competition improves the AON payoff. However, the TON payoff reduces due to increased contention from nodes in the AON.

Next, in Section II we give an overview of related works. In Section III we describe the network model. This is followed by Section IV in which we discuss the formulation of the non-cooperative stage game, derive the mixed strategy Nash Equilibrium (MSNE) and analyze the repeated game with competition. We describe the proposed coexistence etiquette in detail in Section V. Computational analysis is carried out in Section VI where we describe the evaluation setup and also state our main results. We conclude in Section VII.

II. RELATED WORK

Recent works such as [3], [7]–[11] study the coexistence of DSRC based vehicular networks and WiFi. In [7] authors provide an overview of 5.9 GHz band sharing and discuss coexistence solutions that protect DSRC users. In [8] authors study the coexistence scenario of Wireless Access in Vehicular Environment (WAVE) systems with WiFi and propose a combined control of interference space and the receiver sensitivity on the WiFi devices to resolve the hidden terminal problem and bandwidth competition. In [9] authors evaluate the performance of two spectrum sharing mechanisms, i.e., detect & vacate and detect & mitigate, identify the delayed detection problem and propose addition of an extra idle period to the WiFi interference idle periods to improve the detection performance. In [3] and [10] authors study the impact of DSRC on WiFi and vice versa.

In these earlier works authors provide an in-depth study of the inherent differences between the two technologies, the coexistence challenges and propose solutions to better coexistence. However, the aforementioned works look at the coexistence of DSRC and WiFi as the coexistence of two CSMA/CA based networks, with different MAC parameters, where the packets of the DSRC network take precedence over that of the WiFi network. Also, authors in [3], [7]–[11] propose tweaking the MAC parameters of the WiFi network in order to protect the DSRC network. In contrast to [3], [7]–[11], we look at the coexistence problem as that of coexistence of networks which have equal access rights to the spectrum, use similar access mechanisms but have different objectives.

UAVs equipped with 802.11 a/b/g/n technology operate in the 2.4 and 5 GHz bands coexist with WiFi networks and are commonly used in real-time monitoring systems such as disaster management, environmental monitoring and surveillance [1], all of which rely on freshness of updates. Authors in [12] and [13] consider UAV applications with delay and achievable throughput as the performance metric. In contrast to [12] and [13], we employ the age of information metric, which adequately captures the freshness of updates critical to the aforementioned applications.

Works such as [14]–[16] investigate age of information in wireless networks. In [14], authors study optimal control of status updates from a source to a remote monitor via a network server to ensure data freshness. In [15], the author evaluates an updating policy between a source and a service facility using the status age timeliness metric and shows that a lazy updating policy is better than just in time updates as the latter wastes time in sending non-informative updates. In [15], authors analyze the age of information over multiaccess channels. In [4], authors investigate minimizing the age of status updates sent by vehicles over a carrier-sense multiple access (CSMA) network. As the main objective of the AON is the timely delivery of information, similar to [14]–[16], we also employ the metric of age to evaluate the performance of the AON.

Note that while throughput as the payoff function has been extensively studied from the game theoretic point of view (for example, see [17], [18]), age as a payoff function has not garnered much attention yet. In [19] and [20], authors study an adversarial setting where one player aims to maintain the freshness of information updates while the other player aims to prevent this. Also, in our preliminary work [21], we propose a game theoretic approach to study the coexistence of DSRC (Dedicated Short Range communication) aka vehicular communications and WiFi, where the DSRC network desires to minimize the time-average age of information and the WiFi network aims to maximize the average throughput. We studied the one-shot game and evaluated the Nash and Stackelberg equilibrium strategies. However, the model in [21] did not capture well the interaction of networks, evolution of their respective strategies and payoffs over time, which the repeated game model allows us to capture in this work. In this work, via the repeated game model we are able to shed better light on the AON-TON interaction and how their different utilities distinguish their coexistence from the coexistence of two utility maximizing CSMA/CA based networks.
Works such as [22]–[24] have employed repeated games in the context of coexistence. Since repeated games might foster cooperation, authors in [22] study a punishment-based repeated game to model cooperation between multiple networks in an unlicensed band and illustrate that under certain conditions selfish behavior incur negligible losses and whether the systems cooperate or not does not have much influence on the performance. Similar to [22], authors in [23] study a punishment-based repeated game to incorporate cooperation, however, they also propose mechanisms to ensure user honesty. Contrary to the above works, where coexisting networks have similar objectives and the equilibrium strategies are static, our work has different objectives and the equilibrium strategies, particularly that of the AON, as we show later, is dynamic and evolves over stages.

III. NETWORK MODEL

Our network consists of \( N_A \) age optimizing and \( N_T \) throughput optimizing nodes that contend for access to the shared wireless medium.

Both AON and TON nodes use a CSMA/CA based access mechanism in which nodes use contention windows (CW) and one or more backoff stages to gain access to the medium. We model this mechanism as in [6]. In this section, network represents a group of nodes that contend for the medium without reference to whether the nodes belong to the AON or the TON. We define events and probabilities that will appear in the context of both competitive and cooperative modes. We assume that all nodes can sense each other's packet transmissions. This allows modeling the CSMA/CA mechanism as a slotted multiaccess system. A slot may be an idle slot in which no node transmits a packet or it may be a slot that sees a successful transmission. This happens when exactly one node transmits. If more than one node transmits, none of the transmissions are successfully decoded and the slot sees a collision. Further, we assume that all nodes always have a packet to send. The modeling in [6] shows that the CSMA/CA settings of minimum contention window (\( CW_{min} \)), number of backoff stages and the number of nodes can be mapped to the probability with which a node attempts transmission in a slot. We use this probability to calculate the other probabilities of interest.

Let \( p_i \) be the probability of an idle slot, which is a slot in which no node transmits. Let \( p_S^{(i)} \) be the probability of a successful transmission by node \( i \) in a slot and let \( p_S \) be the probability of a successful transmission in a slot. We say that node \( i \) sees a busy slot if in the slot node \( i \) doesn’t transmit and exactly one other node transmits. Let \( p_S^{(-i)} \) be the probability that a busy slot is seen by node \( i \). Let \( p_C \) be the probability that a collision occurs in a slot.

Let \( \delta_I, \sigma_S \) and \( \sigma_C \) denote the lengths of an idle, successful, and collision slot, respectively. In this work, we study two scenarios of practical interest (a) when networks use the basic access mechanism and \( \sigma_S \leq \sigma_C \) and (b) when networks use the RTS/CTS based access mechanism and \( \sigma_S > \sigma_C \).

Next we define the throughput of a TON node and the age of an AON node, respectively, in terms of the above probabilities and slot lengths. We will detail the calculation of these probabilities for the competitive and the cooperative mode in Section V and Section VI respectively.

A. Throughput of a TON node over a slot

Let the rate of transmission be fixed to \( r \) bits/sec in any slot. Define the throughput \( \Gamma_i \) of a TON node \( i \in \{1,2,\ldots,N_T\} \) in a slot as the number of bits transmitted successfully in the slot. This is a random variable with probability mass function (PMF)

\[
P[\Gamma_i = \gamma] = \begin{cases} \bar{p}_S^{(i)} & \gamma = \sigma_S r, \\ 1 - \bar{p}_S^{(i)} & \gamma = 0, \\ 0 & \text{otherwise.} \end{cases}
\]

Using (1), we define the average throughput \( \bar{\Gamma}_i \) of node \( i \) as

\[
\bar{\Gamma}_i = \bar{p}_S^{(i)} \sigma_S r.
\]

The average throughput of the TON in a slot is

\[
\bar{\Gamma} = \frac{1}{N_T} \sum_{i=1}^{N_T} \bar{\Gamma}_i.
\]

We assume that the throughput in a slot is independent of that in the previous slots.

B. Age of an AON node over a slot

Let \( \Delta_i(t) \) be the status update age of an AON node \( i \in \{1,2,\ldots,N_A\} \) at other nodes in the AON at time \( t \). When the freshest update of node \( i \) at AON node \( j \in \{1,2,\ldots,N_A\} \) at time \( t \) is time-stamped \( u(t) \), the status update age, or simply the age, of node \( i \) at node \( j \) is defined as \( \Delta_i(t) = t - u(t) \). We assume that a status update packet that AON node \( i \) attempts to transmit in a slot contains an update that is fresh at the beginning of the slot. As a result, node \( i \)'s age at any other node \( j \) either resets to \( \sigma_S \) if a successful transmission occurs or increases by \( \sigma_I \), \( \sigma_C \) or \( \sigma_S \) at all other nodes in the AON, respectively, when an idle slot, collision slot or a busy slot occurs. Note that node \( i \)'s age at the end of a slot is determined by its age at the beginning of the slot and the type of the slot. Figure 1 shows an example sample path of the age \( \Delta_i(t) \) of an AON node \( i \). In what follows we will drop the explicit mention of time \( t \) and let \( \Delta_i \) be the age at the end and \( \Delta_i^\ominus \) be the age at the beginning of a given slot of node \( i \).

The age \( \Delta_i \), observed by node \( i \) at the end of a slot is thus a random variable with PMF conditioned on age at the beginning of a slot given by

\[
P[\Delta_i = \delta_i | \Delta_i^\ominus = \delta_i^\ominus] = \begin{cases} p_i & \delta_i = \delta_i^\ominus + \sigma_I, \\ p_C & \delta_i = \delta_i^\ominus + \sigma_C, \\ p_S^{(-i)} & \delta_i = \delta_i^\ominus + \sigma_S, \\ p_S^{(i)} & \delta_i = \sigma_S, \\ 0 & \text{otherwise.} \end{cases}
\]
Using (4), we define the conditional expected age
\[ \Delta_i \triangleq E[\Delta_i | \Delta_i = \delta_i] = (1 - p_S^{(i)}) \delta_i + (p_1 \sigma_1 + p_2 \sigma_S + p_C \sigma_C). \] (5)

The average age of status updates of nodes in an AON at the end of the slot, is
\[ \bar{\Delta} = \frac{1}{N_A} \sum_{i=1}^{N_A} \Delta_i. \] (6)

IV. THE REPEATED GAME

We define a repeated game to model the interaction between an AON and a TON. In every CSMA/CA slot, networks must compete for access with the goal of maximizing their expected payoff over an infinite horizon (a countably infinite number of slots). We capture the interaction in a slot as a non-cooperative stage game \( G_{NC} \), where NC stands for non-cooperation or competition and derive its mixed strategy Nash equilibrium (MSNE). The interaction over the infinite horizon is modeled as the stage game played repeatedly in every slot and is denoted by \( G_{NC}^\infty \). For each access mechanism, i.e. basic access and RTS/CTS, we study the effects of competition in \( G_{NC}^\infty \), when networks choose to play the MSNE in each stage. Next, we discuss these in detail.

A. Stage game

We define a parameterized strategic one-shot game [23] \( G_{NC} = (N, (S_k)_{k \in N}, (u_k)_{k \in N}, \Delta^-) \), where \( N \) is the set of players, \( S_k \) is the set of pure strategies of player \( k \), \( u_k \) is the payoff of player \( k \) and \( \Delta^- \) is the additional parameter input to the game \( G_{NC} \), which is the average age [6] of the AON seen at the beginning of the slot. We define the game \( G_{NC} \) in detail.

1. Players: The AON and the TON are the two player. We denote the former by A and the latter by T. Specifically, we have \( N = \{A, T\} \).

2. Strategy: Let \( T \) denote transmit and \( I \) denote idle. For AON comprising of \( N_A \) nodes, the set of pure strategies is \( S_A = S_1 \times S_2 \times \cdots \times S_{N_A} \), where \( S_i = \{T, I\} \) is the set of strategies for a certain AON node \( i \in \{1, 2, \ldots, N_A\} \). Similarly, for the TON comprising of \( N_T \) nodes, the set of pure strategies is \( S_T = S_1 \times S_2 \times \cdots \times S_{N_T} \).

We allow networks to play mixed strategies. In general, for the strategic game \( G_{NC} \), we can define \( \Phi_k \) as the set of all probability distributions over the set of strategies \( S_k \) of player \( k \), where \( k \in N \). A mixed strategy for player \( k \) is an element \( \phi_k \in \Phi_k \), such that \( \phi_k \) is a probability distribution over \( S_k \). For example, for AON with \( N_A = 2 \), the set of pure strategies for AON is \( S_A = S_1 \times S_2 = \{(T, T), (T, I), (I, T), (I, I)\} \) and the probability distribution over \( S_A \) is \( \phi_A \), such that \( \phi_A(s_A) \geq 0 \) for all \( s_A \in S_A \) and \( \sum_{s_A \in S_A} \phi_A(s_A) = 1 \).

In this work, we restrict ourselves to the space of probability distributions such that the mixed strategies of AON are a function of \( \tau_A \) and that of TON are a function of \( \tau_T \), where \( \tau_A \) and \( \tau_T \) are the probabilities with which nodes in AON and TON, respectively, attempt transmission in a slot. Specifically, we force all the nodes to choose the same probability to attempt transmission. As a result, the probability distribution for AON with \( N_A = 2 \), parameterized by \( \tau_A \), is \( \phi_A(\tau_A) = \{\tau_A^2, (1 - \tau_A)^2\} \), corresponding to the set of pure strategies \( S_A \) for the AON. Similarly, for TON with \( N_T = 2 \), the probability distribution parameterized by \( \tau_T \), is \( \phi_T(\tau_T) = \{\tau_T^2, (1 - \tau_T)^2\} \), corresponding to the set of pure strategies \( S_T \) for the TON.

3. Payoffs: We have \( N_T \) throughput optimizing nodes that attempt transmission with probability \( \tau_T \) and \( N_A \) age optimizing nodes that attempt transmission with probability \( \tau_A \). For the non-cooperative game \( G_{NC} \), let \( p_{i, NC} \) be the probability of an idle slot, where as stated earlier, \( \Phi_C \) stands for non-cooperation or competition. We have
\[ p_{i, NC} = (1 - \tau_A)^{N_A} (1 - \tau_T)^{N_T}. \] (7)

Let \( p_{S, NC} \) be the probability of a successful transmission in a slot, \( p_{S, NC}^{(i)} \) be the probability of a successful transmission by node \( i \), \( p_{S, NC}^{(-i)} \) be the probability of a busy slot. We have
\[ p_{S, NC} = N_T \tau_T (1 - \tau_T)^{N_T-1} (1 - \tau_A)^{N_A}, \] (8)
\[ p_{S, NC}^{(i)} = \left\{ \begin{array}{ll} \tau_A (1 - \tau_T) (1 - \tau_A)^{N_A-1} (1 - \tau_T)^{N_T}, & \forall i \in N_A, \\ \tau_T (1 - \tau_T) (1 - \tau_T)^{N_T-1} (1 - \tau_A)^{N_A}, & \forall i \in N_T. \end{array} \right. \] (9)
\[ p_{S, NC}^{(-i)} = \left\{ \begin{array}{ll} (N_A - 1) \tau_A (1 - \tau_T) (1 - \tau_A)^{N_A-1} (1 - \tau_T)^{N_T}, & \forall i \in N_A, \\ (N_T - 1) \tau_T (1 - \tau_T) (1 - \tau_T)^{N_T-1} (1 - \tau_A)^{N_A}, & \forall i \in N_T. \end{array} \right. \] (10)

The probabilities (7)-(10) can be substituted in (1)-(2) and (3)-(5), respectively, to calculate the average throughput (2) and average age (6). We use these to obtain the stage payoffs \( u_{T, NC} \) and \( u_{A, NC} \) of the TON and the AON. They
\[ u_{TNC}^* (\tau_A, \tau_T) = \tilde{\Gamma} (\tau_A, \tau_T), \quad u_{A,NC}^* (\tau_A, \tau_T) = -\tilde{\Delta} (\tau_A, \tau_T). \]

The networks would like to maximize their payoffs.

### B. Mixed Strategy Nash Equilibrium

Figure 2 shows the payoff matrix corresponding to the non-cooperative stage game \( G_{NC} \). Each network consists of a single node and can choose between pure strategies, transmit \( T \) and idle \( I \). As stated in [25], every finite non-cooperative game has a mixed strategy Nash equilibrium (MSNE). For the game \( G_{NC} \) defined in Section 4-A, a mixed-strategy profile \( \phi^*(\tau_A, \tau_T) = (\phi_A^*(\tau_A), \phi_T^*(\tau_T)) \) is a Nash equilibrium [25], if \( \phi_A^*(\tau_A) \) and \( \phi_T^*(\tau_T) \) are the best responses of player A and player T, respectively, to their respective opponents’ mixed strategy, i.e., \( \phi_T^*(\tau_A^T) \in \Phi_T \) and \( \phi_A^*(\tau_T^A) \in \Phi_A \), respectively. We have

\[
\begin{align*}
  u_{TNC}^* (\phi_A^*, \phi_T^*) & \geq u_{TNC}^* (\phi_A^*, \phi_T), \quad \forall \phi_T \in \Phi_T, \\
  u_{A,NC}^* (\phi_A^*, \phi_T^*) & \geq u_{A,NC}^* (\phi_A, \phi_T^*), \quad \forall \phi_A \in \Phi_A,
\end{align*}
\]

where \( \phi^*(\tau_A^*, \tau_T^*) \in \Phi = \prod_{k=1}^{N_A} \Phi_k \) is the profile of mixed strategy. Since the probability distributions \( \phi_A (\tau_A) \) and \( \phi_T (\tau_T) \) are parameterized by \( \tau_A \) and \( \tau_T \), respectively, we find \( \tau = [\tau_A, \tau_T] \), in Proposition 1 to compute the mixed strategy Nash equilibrium.

**Proposition 1.** The parameter \( \tau = [\tau_A^*, \tau_T^*] \) required to compute the mixed strategy Nash equilibrium \( \phi^*(\tau_A^*, \tau_T^*) = (\phi_A^*(\tau_A^*), \phi_T^*(\tau_T^*)) \) for the game \( G_{NC} \) is

\[
\tau_A^* = \begin{cases} 
1 & \text{if } \Delta^- > \Delta_a, \\
\frac{1 - \tau_T^* \Delta^- - N_A (\sigma_S - \sigma_T)}{N_A N_T \tau_T^* (\sigma_S - \sigma_C)} & \text{otherwise},
\end{cases}
\]

\[
\tau_T^* = \frac{1}{N_T}.
\]

where, \( \Delta_a = \max\{\Delta_{ah,0} + \sigma_S, \Delta_{ah,1}\} \), \( \Delta_{ah,0} = N_A (\sigma_S - \sigma_T) - N_A N_T \tau_T^* (\sigma_S - \sigma_C) \) and \( \Delta_{ah,1} = N_A (\sigma_S - \sigma_C) \).

**Proof:** The proof is given in Appendix 4.

As shown in (13a) and (13b), \( \tau_A^* \) in any slot is a function of average age observed at the beginning of the slot, i.e., \( \Delta^- \), the number of nodes in both the AON and the TON and \( \tau_T^* \), whereas, \( \tau_T^* \) is a function of nodes in the TON and is independent of the number of nodes in the AON. The threshold value \( \Delta_a \) can either take a value equal to \( \Delta_{ah,0} + \sigma_S \) or \( \Delta_{ah,1} \). For instance, when \( N_A = 1, N_T = 1 \), and \( \sigma_S \geq \sigma_C \), the threshold value \( \Delta_a \) is equal to \( \Delta_{ah,1} = (\sigma_S - \sigma_C) \), which is again less than the minimum value \( \sigma_S \) that \( \Delta^- \) can take, resulting in \( \tau_A^* = 1 \). In contrast to the prior scenarios where \( \sigma_S \geq \sigma_C \), when \( N_A = 1, N_T = 1 \), and \( \sigma_S < \sigma_C \), the threshold value \( \Delta_a \) is equal to \( \Delta_{ah,0} = \infty \) and since \( \Delta^- \) cannot exceed \( \infty \), \( \tau_T^* \) in this case is 0. Note that while the parameter \( \tau_A^* \) corresponding to the TON is equal to 1 for all selections of \( \sigma_C \), the AON chooses \( \tau_A^* = 1 \) when \( \sigma_S \geq \sigma_C \), and \( \tau_A^* = 0 \) when \( \sigma_S < \sigma_C \). This is because when \( \sigma_S > \sigma_C \) the increase in age due to a successful transmission by the TON, which has \( \tau_T^* = 1 \), is less than that due to a collision which would have happened if the AON chose \( \tau_A^* = 1 \). Corollary 1 gives the value of \( \sigma_C \) for which \( \Delta_{ah,0} + \sigma_S \) and \( \tau_T^* = 0 \) in the inequality \( \Delta_{ah,1} > \Delta_{ah,0} \), we get \( \sigma_C < \frac{\sigma_T + \sigma_S (N_T + \sigma_S - \sigma_T)}{1 + \sigma_T (N_T - 1)} \).

**Corollary 1.** In the game \( G_{NC} \), the parameter \( \tau_T^* \) required to compute the MSNE of the AON \( \phi_A^* \), lies in the interval \( [0, 1] \) if \( \sigma_C < \frac{\sigma_T + \sigma_S (N_T + \sigma_S - \sigma_T)}{1 + \sigma_T (N_T - 1)} \), otherwise, takes values in the interval \( [0, 1] \).

**Proof:** For \( \tau_A^* \in [0, 1] \), \( \Delta_a \) in (13a) should be equal to \( \Delta_{ah,1} \), i.e., \( \max\{\Delta_{ah,0}, \Delta_{ah,0}\} = \Delta_{ah,1} \), hence implying \( \Delta_{ah,1} > \Delta_{ah,0} \). On substituting the values of \( \Delta_{ah,0} \) and \( \Delta_{ah,1} \) from (13a) in the inequality \( \Delta_{ah,1} > \Delta_{ah,0} \), we get \( \sigma_C < \frac{\sigma_T + \sigma_S (N_T + \sigma_S - \sigma_T)}{1 + \sigma_T (N_T - 1)} \).

Therefore,

\[
\tau_T^* \in \begin{cases} 
(0, 1] & \text{if } \sigma_C < \frac{\sigma_T + \sigma_S (N_T + \sigma_S - \sigma_T)}{1 + \sigma_T (N_T - 1)}, \\
(0, 1] & \text{otherwise}.
\end{cases}
\]

A distinct feature of the stage game is the effect of self-contention and competition on the network utilities [2]. We define self-contention as the impact of nodes within one’s own network and competition as the impact of nodes in the other network, respectively, on the network utilities. Figure 3 shows the effect of self-contention and competition, when networks play their respective equilibrium strategies. We choose \( \Delta^- = \Delta_{ah,0} + \sigma_S \) as it gives \( \tau_T^* \in (0, 1] \) (see (13a)). As shown in Figure 3a and Figure 3c, while the access probability \( \tau_T^* \) for the TON is independent of the number of node in the AON, the payoff of the TON increases as the number of nodes in the AON increase. Intuitively, since increase in the number of AON nodes results in increase in competition, the payoff of the TON should decrease. However, the payoff of the TON

\[\text{1}\]

[2] We observed self-contention and competition in our earlier work [2] as well. However, the game in [2] was different from the one we study in this work.
Corollary 2. The parameter $\tau = [\tau^*_A, \tau^*_T]$ required to compute the mixed strategy Nash equilibrium $\phi^*(\tau^*_A, \tau^*_T) = (\phi^*_A(\tau^*_A), \phi^*_T(\tau^*_T))$, obtained by substituting $\sigma_S = \sigma_C$ in (13a) is

$$\begin{align*}
\tau^*_A &= \begin{cases}
\frac{N_A(\sigma_C - \sigma_S) + \Delta^-}{N_A(\sigma_C - \sigma_S + \Delta^-)} & \text{if } \Delta^- > N_A(\sigma_S - \sigma_I), \\
0 & \text{otherwise}.
\end{cases} \\
\tau^*_T &= \frac{1}{N_T}.
\end{align*}$$

As seen in (14a) and (14b), the access probabilities $\tau^*_A$ and $\tau^*_T$ required to compute the equilibrium strategies of the AON and the TON, respectively, when $\sigma_S = \sigma_C$ are independent of the number of nodes and access probability of the other network. As a result, the equilibrium strategy of each network is also its dominant strategy.

C. Repeated game

As the networks coexist over a long period of time, we consider an infinitely repeated game, defined as $G^{\infty}_{N_C}$, in which the one-shot game $G_{NC}$, where players play the MSNE (13), is played in every stage (slot) $n \in \{1, 2, \ldots\}$. We consider perfect monitoring [25], i.e., at the end of each stage, all players observe the action profile chosen by every other player. In addition to the action profiles, players also observe $\Delta_n^-$, i.e., the average age of the AON at the end of stage $(n - 1)$. The AON equilibrium strategy as shown in (13a) in any stage is a function of $\Delta_n^-$ and the dependence of the AON equilibrium strategy on it intertwines the utilities of the networks.

Figure 4 shows the payoffs (see (11)-(12)) and the access probabilities of the TON and the AON for the repeated game $G^{\infty}_{N_C}$ when (a) network use the RTS/CTS based access mechanism and $\sigma_S > \sigma_C$ (see Figures 4a-4c) and (b) when networks use the basic access mechanism and $\sigma_S \leq \sigma_C$ (see Figures 4d-4i). For each network, the access probability decreases with increase in the number of nodes in the AON.

For the AON, Figures 4a-4b, Figures 4e-4f and Figures 4g-4h illustrates the interlink between the access probability and age of the AON for different selections of $\sigma_C$. As shown in (13a) and (14a), $\tau^*_A$, is a function of age of observed in the beginning of a stage. The threshold values in (13a), i.e., $\Delta_{th,0}$ and $\Delta_{th,1}$, for $N_A = N_T = 5$, $\sigma_S = 1 + \beta$, $\sigma_C = 0.1\sigma_S$, $\sigma_I = \beta$ and $\Delta^- = 0.0812$ and 4.5450, respectively, resulting in $\Delta_{th} = \max(\Delta_{th,0}, \Delta_{th,1}) = 4.5450$. As a result, nodes in the AON access the medium with $\tau^*_A \in [0, 1)$ in any stage $n$ only if the average age in the $(n - 1)^{th}$ stage exceeds a threshold value, i.e., $\Delta_n^- > \Delta_{th}$, otherwise $\tau^*_A = 1$. For instance, in Figure 4a, $\tau^*_A = 1$ for $n \in [1, 36]$ since $\Delta_n^- < 4.5450$. However, for $n = 37$, $\tau^*_A = 0.9295$ as $\Delta_n^- > 4.6460$ exceeds the threshold value. Similarly, the threshold value in (14a), when $\sigma_S = \sigma_C$, is $N_A(\sigma_S - \sigma_I) = 5$. As a result, as shown in Figure 4c, nodes in the AON access the medium with $\tau^*_A \in [0, 1)$ in any stage $n$ only if the average age in the $(n - 1)^{th}$ stage exceeds the threshold value, i.e., $\Delta_n^- > 5$, otherwise $\tau^*_A = 0$.

Note that when $\sigma_S \leq \sigma_C$, nodes in the AON as shown in Figure 4d and Figure 4g occasionally refrain from transmission, i.e., choose $\tau^*_A = 0$ during a stage. In contrast, when $\sigma_S > \sigma_C$, nodes in the AON as shown in Figure 4a often access the medium aggressively, i.e., with $\tau^*_A = 1$ during a stage. Such a behavior of nodes in the AON is due to the presence of the TON. As the length of the collision slot decreases, the impact of collision on the age of the AON reduces. If nodes in the AON choose to refrain from transmission, the average age of the AON be a function of the events – successful transmission, collision or idle slot, happening in the TON, whereas if nodes in the AON choose to transmit aggressively, i.e., choose $\tau^*_A = 1$, the average age of the AON would only be
impact the collision slot. For instance, for a coexistence scenario with $N_A = N_T = 5$, $\sigma_S = 1 + \beta$, $\sigma_C = 0.1 \sigma_S$, $\sigma_1 = \beta$, $\beta = 0.01$ and $\Delta^- = \frac{5}{8}$, the average age in the stage computed using (10) is 1.4535, whereas, if $\tau^*_A = 1$, the average age is 1.1110. As a result, due to reduced impact of collision, nodes in the AON choose to contend with the TON aggressively for the medium and hence transmit with $\tau^*_A = 1$ during a stage.

Player $k$'s average discounted payoff for the game $G_{\text{NC}}$, where $k \in \mathcal{N}$ is

$$
U_{k,\text{NC}} = E_\phi \left\{ (1 - \alpha) \sum_{n=1}^{\infty} \alpha^{n-1} u_{k,\text{NC}}(\phi) \right\},
$$

where, the expectation is taken with respect to the strategy profile $\phi$, $u_{k,\text{NC}}(\phi)$ is player $k$'s payoff in stage $n$ and $0 < \alpha < 1$ is the discount factor. Note that a discount factor $\alpha$ closer to 1 means that the player values not only the stage payoff but also the payoff in the future, i.e., the player is far-sighted, whereas $\alpha$ closer to 0 means that the player is myopic and values only the current payoff. By substituting (12) and (11) in (15), we can obtain the average discounted payoffs $U_{A,\text{NC}}$ and $U_{T,\text{NC}}$ of the AON and the TON, respectively.

V. THE REPEATED GAME WITH COOPERATION

In general, selfish players choose the one-shot game MSNE (13). However, if they cooperate, they may achieve higher expected one-shot payoffs. For instance, consider the 2-player one-shot game shown in Figure 2. Figure 3 shows the convex hull of payoffs corresponding to it. The game has three pure strategy Nash Equilibria, i.e., $(T, T)$, $(T, I)$ and $(I, T)$ and a mixed strategy Nash Equilibrium, i.e., $\phi^*_A = \{1, 0\}$ and $\phi^*_T = \{1, 0\}$. Consider a coordination device which is identified with probability $P_R$ and whose recommendation is based on the flip of a fair coin such that $P_R = P[\text{Heads}]$ and $(1 - P_R) = P[\text{Tails}]$. This coordination device recommends the AON to play transmit $(T)$ if Heads $(\text{Heads})$ shows and play idle $(I)$ if Tails $(\text{Tails})$ shows, whereas, recommends the TON to do the vice versa, i.e., play idle $(I)$ if Heads $(\text{Heads})$ shows and transmit $(T)$ if Tails $(\text{Tails})$ show. If the players comply with the recommendation of the coordination device identified with $P_R = (1 - P_R) = 0.5$, the expected payoff of AON is $-0.91 \times 1.1 \times (1 - P_R) = -1.515$ and that of TON is $0 \times P_R + 0.91 \times (1 - P_R) = 0.505$, which is more than what the AON and the TON would get had they played the MSNE, i.e., $-2.02$ and 0, respectively.

Based on the above observations, we define a simple coexistence etiquette that enables cooperation between an AON and
a TON. Specifically, it employs a coordination device, which is a randomized signaling device $P_R$, based on the flip of a biased coin, that issues recommendations to the AON and the TON to access the spectrum in a non-overlapping manner, that is one at a time, thereby, eliminating the impact of competition and leaving the networks to deal with self-contention only. We consider

- The probability $P_R$ and $(1 - P_R)$, i.e., $P[\mathbb{H}]$ and $P[\mathbb{T}]$, respectively, are common knowledge to players.
- The recommendations issued to the players are
  - AON: play transmit ($\mathbb{T}$) if you are told that $\mathbb{H}$ has occurred and play idle ($\mathbb{I}$) otherwise.
  - TON: play transmit ($\mathbb{T}$) if you are told that $\mathbb{T}$ has occurred and play idle ($\mathbb{I}$) otherwise.
- The above recommendations are common knowledge to players.

While networks get an incentive to cooperate, they are punished if they are non-compliant of the recommendation of the coordination device. The proposed etiquette uses grim trigger strategies, which are the harshest punishment strategies in the class of Nash-reversion punishment strategies, to enforce cooperation. That is, if in any stage, one or both networks do not comply to the recommendation of the coordination device, the networks play their respective equilibrium strategies in each stage thereafter.

Further, we check if the cooperation strategy profile proposed under the coexistence etiquette is self-enforceable and if networks would comply to the recommendation of the coordination device under the threat of punishment. We confirm this by checking if the strategy profile proposed under the coexistence etiquette form a subgame-perfect equilibrium (SPE), i.e., we check that no player would want to deviate unilaterally at any stage of the game. We employ the one-shot deviation principle to do so.

A. Modeling cooperation between networks

We begin by modifying the network model defined in Section III to incorporate the recommendation of the coordination device $P_R$. As stated earlier, $P_R$ denotes the chance for nodes in the AON to transmit and $(1 - P_R)$ denotes the chance for nodes in the TON to transmit, while nodes in the competing network stays idle. Let $\hat{\tau}_A$ and $\hat{\tau}_T$ denote the optimal access probability for nodes in the AON and TON, respectively.

![Fig. 5: The convex hull of payoffs for the 2-player one-shot game (see Figure 2b).](image)

**Proposition 2.** The optimal strategy $\hat{\pi} = [\hat{\tau}_A, \hat{\tau}_T]$ of the one-shot game $G_C$, when networks cooperate is

$$\hat{\tau}_A = \begin{cases} \frac{\Delta^- - N_A(\sigma_S - \sigma_I)}{N_A(\Delta^- + (\sigma_I - \sigma_C)) - N_A(\sigma_S - \sigma_C)} & \Delta^- > \Delta_{th}, \\ 1 & \Delta^- < \Delta_{th} & \Delta_{th,0}, \\ 0 & \Delta^- < \Delta_{th} & \Delta_{th,1}. \end{cases}$$  

\(\hat{\tau}_T = \frac{1}{N_T},\)  

where, $\Delta_{th} = \max\{\Delta_{th,0}, \Delta_{th,1}\}$, $\Delta_{th,0} = N_A(\sigma_S - \sigma_I)$ and $\Delta_{th,1} = N_A(\sigma_S - \sigma_C)$.

**Proof:** The proof is given in Appendix \[R\].

Similar to (13a), the optimal strategy $\hat{\tau}_A$ in any slot is a function of average age observed at the beginning of the slot, i.e., $\bar{x}_-$. However, in contrast to (13a), in the cooperative game $G_C$, the optimal strategy $\hat{\tau}_A$ of the AON is a function of only the number of nodes in its own network, since the coordination device allows networks to access the medium one at a time. Similarly, the optimal strategy $\hat{\tau}_T$ of the TON is a function of the number of nodes in its own network and is independent of the number of nodes in the AON. The threshold value $\Delta_{th}$ can either take a value equal to $\Delta_{th,0}$ or $\Delta_{th,1}$. For instance, when $N_A = 1$, $N_T = 1$ and $\sigma_S = \sigma_C$, $\Delta_{th}$ takes a value equal to $\Delta_{th,0} = (\sigma_S - \sigma_I)$ and since $\Delta^-$ can only be equal to or greater than $\sigma_S$, which is larger than $(\sigma_S - \sigma_I)$, the AON chooses $\hat{\tau}_A = 1$.

Let $p_{l,c}$ be the probability of an idle slot

$$p_{l,c} = P_R(1 - \hat{\tau}_A)^{N_A} + (1 - P_R)(1 - \hat{\tau}_T)^{N_T}. \quad (17)$$

Similarly, let $p_{s,c}$ be the probability of a successful transmission in a slot, $p_{s,c}^{(i)}$ be the probability of a successful transmission by node $i$, and $p_{s,c}^{(i)}$ be the probability of a busy slot. We have

$$p_{s,c} = P_R N_A \tilde{\tau}_A(1 - \tilde{\tau}_A)^{(N_A - 1)} + (1 - P_R)N_T \tilde{\tau}_T(1 - \tilde{\tau}_T)^{(N_T - 1)}, \quad (18)$$

$$p_{s,c}^{(i)} = \begin{cases} P_R \tilde{\tau}_A(1 - \tilde{\tau}_A)^{(N_A - 1)}, & \forall i \in N_A, \\ (1 - P_R)\tilde{\tau}_T(1 - \tilde{\tau}_T)^{(N_T - 1)}, & \forall i \in N_T, \end{cases} \quad (19)$$

$$p_{s,c}^{(i)} = \begin{cases} (1 - P_R)N_T \tilde{\tau}_T(1 - \tilde{\tau}_T)^{(N_T - 1)} + P_R N_A - 1 \tilde{\tau}_A(1 - \tilde{\tau}_A)^{(N_A - 1)}, & \forall i \in N_A, \\ (1 - P_R)(N_T - 1)\tilde{\tau}_T(1 - \tilde{\tau}_T)^{(N_T - 1)} + P_R N_A \tilde{\tau}_A(1 - \tilde{\tau}_A)^{(N_A - 1)}, & \forall i \in N_T, \end{cases} \quad (20)$$

$$p_{c,c} = 1 - p_{s,c} - p_{l,c}. \quad (21)$$

By substituting (17), (21) in (3) and (6), we can obtain the stage utility of the TON and the AON, defined in (11) and (12), respectively, when networks cooperate. The resulting stage utility of the TON and the AON are

$$u_{T,c}(\hat{\tau}_A, \hat{\tau}_T) = \bar{u}(\hat{\tau}_A, \hat{\tau}_T), \quad (22)$$

$$u_{A,c}(\hat{\tau}_A, \hat{\tau}_T) = -\Delta(\hat{\tau}_A, \hat{\tau}_T). \quad (23)$$
Similar to the non-cooperative game $G_{NC}$, networks in the cooperative game $G_C$ would also like to maximize their payoffs.

B. Repeated game with cooperating networks

Contrary to the game defined in Section IV-C where networks play the MSNE in each stage, we define an infinitely repeated game $G_{\infty}^C$ where networks cooperate and comply to the recommendation of the coordination device $P_R$ in each stage. The course of action that the networks follow is as follows: Players in the beginning of each stage receive a recommendation $\mathbb{H}$ or $T$ from the coordination device $P_R$. In addition to the recommendation, players observe the action profile which comprises of player’s own action and the action of the other player in the previous stage. Consider the following grim trigger strategy profile defined inductively

$$
\alpha_n = \begin{cases} 
(\mathcal{I}, \mathcal{I}) & \text{if } a_{n-1} \in \{(\mathcal{I}, \mathcal{I}), (\mathcal{I}, \mathcal{T})\} \text{ and } R_n = \mathbb{H}, \\
(\mathcal{I}, \mathcal{T}) & \text{if } a_{n-1} \in \{(\mathcal{I}, \mathcal{I}), (\mathcal{I}, \mathcal{T})\} \text{ and } R_n = T, \\
(\phi_\mathcal{A}, \phi_\mathcal{T}) & \text{otherwise},
\end{cases}
$$

where, $a_{n-1}$ is the action profile observed by the networks in stage $(n-1)$ and $R_n$ corresponds to the signal from the coordination device $P_R$. As shown in (24), if $a_{n-1} \not\in \{(\mathcal{I}, \mathcal{I}), (\mathcal{I}, \mathcal{T})\}$ networks play $(\phi_\mathcal{A}, \phi_\mathcal{T})$.

In a nutshell, cooperation is sustained as long as the players comply to the recommendation of the coordination device $P_R$. However, if any player deviates, players revert to playing the MSNE and stick to it thereafter.

Figure 6 shows the payoffs (22)-(23) and the access probabilities of TON and AON for the repeated game $G_{\infty}^C$ when (a) network use the RTS/CTS based access mechanism and $\sigma_\mathcal{S} > \sigma_\mathcal{C}$ (Figures 6a-6c), and (b) when networks use the basic access mechanism and $\sigma_\mathcal{S} \leq \sigma_\mathcal{C}$ (Figures 6d-6f) corresponds to $\sigma_\mathcal{S} = \sigma_\mathcal{C}$ and Figures 6g-6i corresponds to $\sigma_\mathcal{S} < \sigma_\mathcal{C}$.

The results correspond to AON-TON coexistence with $N_A = N_T = 5$ and $P_R = 0.5$.

In contrast to the repeated game in Section IV-C where nodes in the AON choose to occasionally access the medium aggressively when $\sigma_\mathcal{S} > \sigma_\mathcal{C}$, in the repeated game $G_{\infty}^C$, nodes in the AON as shown in Figure 6a, Figure 6d and Figure 6g irrespective of the length of collision slot, never access the medium aggressively, i.e., do not choose $\tilde{\tau}_A = 1$, instead they occasionally refrain from transmission and choose $\tilde{\tau}_A = 0$ during a stage. This is due to the absence of contention from the TON when networks obey the recommendation of the coordination device. In the absence of contention from the TON, when nodes in the AON choose to refrain from transmission, the age of the AON only increases by the length of an idle slot. Since the benefits of idling surpasses that of contending aggressively, nodes in the AON occasionally choose to refrain from transmission irrespective of the length of collision slot.

C. Is cooperation self-enforceable?

We first validate our choice of the proposed coexistence etiquette by checking if in a stage game, networks find cooperation beneficial or not. In other words, we check if the stage utility of the players when they choose to cooperate is more than or at least as much as that when they choose to compete, i.e., $u_{T,C}(\tilde{\tau}_A, \tilde{\tau}_T) \geq u_{T,NC}(\tilde{\tau}_A, \tilde{\tau}_T)$ and $u_{A,C}(\tilde{\tau}_A, \tilde{\tau}_T) \geq u_{A,NC}(\tilde{\tau}_A, \tilde{\tau}_T)$. Using these inequalities, we determine the range of $P_R$, as shown in (25), over which networks prefer cooperation in the stage game.

We find that when $N_A = 1$ and $N_T = 1$, depending on the length of collision slot $\sigma_C$, the range of $P_R$ (25) varies. As discussed earlier in Section IV-C when networks compete and $\sigma_\mathcal{S} \geq \sigma_C$, $\tilde{\tau}_A = \tilde{\tau}_T = 1$, whereas, when $\sigma_\mathcal{S} < \sigma_C$, $\tilde{\tau}_A = 0$ and $\tilde{\tau}_T = 1$. In contrast, when $N_A = 1$ and $N_T = 1$, irrespective of the length of collision slot $\sigma_C$, when networks cooperate, $\tilde{\tau}_A = 1$ and $\tilde{\tau}_T = 1$. As a result, cooperation is beneficial for $P_R \in [0, 1]$ when $\sigma_\mathcal{S} \geq \sigma_C$, and only beneficial at $P_R = 0$ when $\sigma_\mathcal{S} < \sigma_C$. This is because when $\sigma_\mathcal{S} \geq \sigma_C$, while networks see a collision when they compete, they see a successful transmission if they choose to cooperate. In contrast, when $\sigma_\mathcal{S} < \sigma_C$, since the AON chooses not to access the medium when networks compete, the TON gets a contention free access to the medium and hence sees a successful transmission. As a result, the TON only cooperates if the AON doesn’t get a chance to access the medium under cooperation as well. Hence, when $\sigma_\mathcal{S} < \sigma_C$, cooperation is only beneficial if $P_R = 0$. Note that while the analysis for $N_A = 1$ and $N_T = 1$ as discussed above, is simple, (25) becomes intractable for $N_A > 1$ and $N_T > 1$. Therefore, we resort to computational analysis and show that as the number of nodes increases, when $\sigma_\mathcal{S} \leq \sigma_C$, cooperation is beneficial only at $P_R = 0$, whereas, when $\sigma_\mathcal{S} > \sigma_C$, it is beneficial only for higher values of $P_R$, i.e., for $P_R$ close to 1. We discuss this in detail in Section VI.

Next we check if the cooperation strategy profile defined in (24), proposed under the coexistence etiquette, is self-enforceable and if networks would comply to it under the threat of punishment. We confirm this by checking if the proposed path of play form a subgame-perfect equilibrium (5).

In other words, we check that no player would want to deviate unilaterally at any stage of the game.

We check for any deviations using the one-stage deviation principle defined as: Consider that in stage 1 the coordination device sees $\mathbb{H}$ and recommends the networks to play according to the path of play defined in (24). If players comply to the recommendation in stage 1 and thereafter, the discounted payoff is $U_{T,C} = (1 - \alpha) \sum_{n=1}^{\infty} \alpha^{n-1} u_{T,C}$ and $U_{A,C} = (1 - \alpha) \sum_{n=1}^{\infty} \alpha^{n-1} u_{A,C}$, respectively, where $u_{T,C}$ and $u_{A,C}$ are defined in (22) and (23). However, if any network deviates in stage 1, the grim trigger strategy comes into play, i.e., networks play the MSNE thereafter. Nevertheless, any selfish network would not deviate if the payoff earned from cooperation (C) surpasses that from non-cooperation (NC).

We check if cooperation can be self-enforced by considering the following four inequalities:

- **Case I:** $R_1 = \mathbb{H}$ and $a_1 = (\mathcal{I}, \mathcal{I})$: Consider in stage 1 the coordination device sees $\mathbb{H}$ and networks are recommended to follow the strategy $(\mathcal{I}, \mathcal{I})$. However, a player can choose to deviate. The right-hand side of the following two inequal-
Fig. 6: Illustration of per stage access probability $\hat{\tau}_k$ ($k \in N$) of the AON and the TON, age $\Delta(t_n)$ of the AON and throughput $\Gamma(t_n)$ of the TON wrt stage obtained from an independent run when $\sigma_C = 0.1\sigma_S$ (see Figures 6a, 6c), $\sigma_C = \sigma_S$ (see Figures 6d, 6f), and $\sigma_C = 2\sigma_S$ (see Figures 6g, 6h). The results correspond to $N_A = 5, N_T = 5, \sigma_S = 1 + \beta, \sigma_1 = \beta = 0.01$ and $P_R = 0.5$.

$$\Delta^- \tau_S^{(i)} = (\sigma_I - \sigma_C)(\tau_S^{(i)} - (1 - \hat{\tau}_T)^{N_T}) - (\sigma_S - \sigma_C)(\tau_S^{(i)} - N_T\tau_T(1 - \hat{\tau}_T)^{N_T - 1}) \leq P_R \leq 1 - (1 - \tau_A)^{N_A}$$

The non-cooperative discounted payoffs corresponding to the AON and the TON in (26) and (27) are $U_{A,N,C}^{(i)}(R_1 = H, a_1 = (I, \tau)) = (1 - \alpha)(\Delta + \sigma_1) + (1 - \alpha)\sum_{n=2}^{\infty} \alpha^{n-1} u_{A,N,C}$ and $U_{T,N,C}^{(i)}(R_1 = H, a_1 = (T, \tau)) = (1 - \alpha)\sum_{n=2}^{\infty} \alpha^{n-1} u_{T,N,C}$, respectively, where $u_{A,N,C}$ and $u_{T,N,C}$ are defined in (11) and (12).

- Case II: $R_1 = T$ and $a_1 = (I, \tau)$: We now consider the scenario when in stage 1 the coordination device sees $T$ and recommends the networks to play $(I, \tau)$. In this case, the right-hand side of the following two inequalities, (28) and (29), correspond to AON choosing to play transmit $(T)$ instead of idle $(I)$ and TON choosing to play idle $(I)$ instead of transmit $(T)$, respectively, and therefore triggering the grim strategy which results in the non-cooperative discounted payoff.

$$U_{A,N,C}(R_1 = T, a_1 = (I, \tau)) = U_{A,N,C}(R_1 = T, a_1 = (I, \tau))$$

$$U_{T,N,C}(R_1 = T, a_1 = (I, \tau)) = U_{T,N,C}(R_1 = T, a_1 = (I, \tau))$$

Similar to (26) and (27), the non-cooperative discounted payoffs corresponding to the AON and the TON in (28) and (29) are $U_{A,N,C}^{(i)}(R_1 = T, a_1 = (I, \tau)) = (1 - \alpha)(\Delta + \sigma_1) + (1 - \alpha)\sum_{n=2}^{\infty} \alpha^{n-1} u_{A,N,C}$ and $U_{T,N,C}^{(i)}(R_1 = H, a_1 = (T, \tau)) = (1 - \alpha)\sum_{n=2}^{\infty} \alpha^{n-1} u_{T,N,C}$, respectively.

We now formally specify the requirement for cooperation to be self-enforceable via threat of punishment.
Statement 1 (Cooperation is self-enforceable). There exists an \( \alpha < 1 \) such that for all \( \alpha > \pi \), there exists \( 0 < P_R < 1 \) with the property that the repeated game \( G_{\infty}^R \) has a subgame perfect equilibrium (SPE) supported with the coordination device \( P_R \) and strategies as specified in (27).

The parameters \((\pi, P_R)\), which makes Statement 1 true can be computed from the equilibrium incentive constraints (26)-(29). However, (26)-(29) are not theoretically tractable, therefore, we resort to computational analysis. Our analysis, as discussed in Section VII, shows that the existence of \((\pi, P_R)\) is dependent on the size of the AON and the TON and they exist for smaller networks only. In other words, there exist an SPE with coordination device \( P_R \) only when the networks have fewer nodes, whereas, as the size of networks increases, competition becomes more favorable than cooperation and the SPE with coordination device \( P_R \) ceases to exist, therefore, giving us Proposition 3 which states that

Proposition 3. Statement 1 is true for smaller networks, however, as the networks grow in size, Statement 1 becomes false.

VI. EVALUATION METHODOLOGY AND RESULTS

In this section, we first discuss the simulation setup and later the results. For what follows, we set \( \sigma_t = \beta, \ 0 < \beta < 1 \), and study two scenarios (a) when networks use RTS/CTS mechanism and \( \sigma_S > \sigma_C \) and (b) when networks use the basic access mechanism and \( \sigma_S \leq \sigma_C \). Specifically, we select (a) \( \sigma_S = (1 + \beta), \sigma_C = 0.1(1 + \beta) \), (b) \( \sigma_S = \sigma_C = (1 + \beta) \), and (c) \( \sigma_S = (1 + \beta), \sigma_C = 2(1 + \beta) \). In practice, the idle slot is much smaller than a collision or a successful transmission slot, that is, \( \beta \ll 1 \). We select \( \beta = 0.01 \) for the simulation results discussed ahead. We make different selections of \( N_A \) and \( N_T \) to illustrate the impact of self-contention and competition. Specifically, we simulate for \( N_A \in \{1, 2, 5, 10, 50\} \) and \( N_T \in \{1, 2, 5, 10, 50\} \). We consider \( \alpha \in [0.01, 0.99] \) and \( P_R \in [0.01, 0.99] \) for computing the discounted payoffs. Different selections of \( \alpha \) allow us to study the behavior of myopic and far-sighted players. We use Monte Carlo simulations to compute the average discounted payoff of the AON and the TON. We compute the average over 100,000 independent runs each comprising of 1000 stages. For each run, we fix the rate of transmission \( r \) for each node in the WiFi network to 1 bit/sec.

Our objective is to show via simulation if cooperation between networks is self-enforceable. We first study the impact of the length of collision slot \( \sigma_C \) on the average discounted payoff when (a) networks play the MSNE in each stage and compete for the medium, and (b) networks obey the recommendation of the coordination device \( P_R \) in each stage and hence cooperate. This allows us to understand how the choice of mechanism, i.e., RTS/CTS or basic access, impacts the behavior of networks under the competitive and cooperative mode for different selections of \( \sigma_C \). Specifically, we show that when networks compete, while nodes in the AON occasionally choose to refrain from transmitting during a stage under the basic access mechanism, hence, benefitting the TON, they choose to access the medium aggressively under the RTS/CTS mechanism improving their own payoff. Further, we discuss the scenarios when cooperation is beneficial in the stage game and self-enforceable in the repeated game. We show that in the stage game, when networks use the basic access mechanism, as the number of nodes in the networks increases, cooperation is beneficial only at \( P_R = 0 \), whereas, when networks use the RTS/CTS mechanism, it is beneficial only for higher values of \( P_R \), i.e., for \( P_R \) close to 1. Lastly, we illustrate the region of cooperation, i.e., the range of \( \alpha \) and \( P_R \) for which the inequalities (26)-(29) are satisfied and the repeated game has a SPE supported with the coordination device \( P_R \). We discuss how and why cooperation becomes unenforceable and the SPE ceases to exist in the repeated game as the size of networks increases.

Impact of \( \sigma_C \) on network payoffs in repeated game with competition: The length of collision slot \( \sigma_C \) impacts the payoff of the TON and the AON as shown in Figure 7. When networks use the basic access mechanism and \( \sigma_S \leq \sigma_C \), nodes in the AON occasionally refrain from transmitting during a stage. While such stages provide contention free access to the nodes in the TON and improves its payoff (see Figure 7a), the age of AON nodes increases, either by \( \sigma_S, \sigma_C \) or \( \sigma_t \), depending on whether nodes in the TON see a successful, collision or idle slot. However, as the length of the collision slot decreases, i.e., when networks use the RTS/CTS mechanism and \( \sigma_S > \sigma_C \), nodes in the AON choose to access the medium aggressively, therefore, improving its payoff (see Figure 7b) and eliminating the contention free slots available to the TON. As a result, payoff of the TON decreases with decrease in the length of the collision slot.

Figure 8 illustrates the empirical frequency of occurrence of \( \tau_A^* = 0, \tau_A^* = 1 \), successful transmission and collision for different sizes of AON and TON when networks choose to play the MSNE in each stage. The results correspond to different selections of \( \sigma_C \), i.e., \( \sigma_S = \sigma_C \) and \( \sigma_S > \sigma_C \). We skip the results for the case when \( \sigma_S < \sigma_C \), since the observations are similar to \( \sigma_S = \sigma_C \). From Figure 8 we observe the following: (a) empirical frequency of \( \tau_A^* = 0 \) and \( \tau_A^* = 1 \) increases as the size of networks increases, (b) empirical frequency of successful transmission decreases with increase in network size and decrease in \( \sigma_C \), and (c) empirical frequency of collision increases with increase in network size.

As shown in Figure 9a, the age threshold \( N_A(\sigma_S - \sigma_I) \) increases...
with increase in $N_A$. As a result, the frequency of occurrence of $\tau_A^* = 0$ increases with $N_A$. Similarly, as shown in (13a), the age threshold $N_A(\sigma_S - \sigma_C)$ also increases with $N_A$, therefore, increasing the empirical frequency of $\tau_A^* = 1$. We illustrate the increase in the frequency of occurrence of $\tau_A^* = 0$ and $\tau_A^* = 1$ with increasing number of AON nodes in Figure 8a and Figure 8c respectively. While the increase in the frequency of occurrence of $\tau_A^* = 0$ works in favour of a TON, the increase in the frequency of occurrence of $\tau_A^* = 1$ adversely impacts the TON as shown in Figure 8c and Figure 8f respectively. In contrast, the increase in the frequency of occurrence of $\tau_A^* = 0$ and $\tau_A^* = 1$ adversely impacts the AON as shown in Figure 8h and Figure 8i respectively. In addition, we also observe that the empirical frequency of successful transmission for the AON increases with increase in nodes in the TON and that for the TON increases with increase in nodes in the AON. Such a behavior can be credited to the self-contention effect, discussed in Section IV-B which forces a network to be conservative, therefore, benefitting the competing network.

**Impact of $\sigma_C$ on network payoffs in repeated game with cooperation:** Figure 9 shows the average discounted payoff of the TON and the AON when networks cooperate. As shown in Figure 9, while the impact of length of collision slot is insignificant on the payoff of the TON when networks obey the recommendation of the coordination device $P_R$, the payoff of the AON decreases with increase in the length of collision slot as $\alpha \to 1$ and $P_R \to 0$.

Figure 10 shows the gain of cooperation over competition for both the AON and TON when networks have 5 nodes each. As shown in Figure 10 when $\sigma_S > \sigma_C$, for all values of $\alpha$ and $P_R$, the payoff of the TON is higher when networks cooperate than when they compete. However, as $\sigma_C$ increases, cooperation is only beneficial for the TON when the network is far-sighted, i.e., $\alpha$ is close to 1 and it gets a higher chance to access the medium, i.e., $(1-P_R)$ is large. Similarly, for the AON, as $\sigma_C$ decreases, cooperation becomes favorable only when the network gets a higher chance to access the medium. Note that a positive value in Figure 10b and Figure 10d indicates that competition is more favorable, whereas a negative value indicates cooperation is better.

**Check if cooperation is beneficial in the stage game:** Table 1 shows the range of $P_R$, computed using (25), over which networks find cooperation beneficial in the stage game for different selections of $N_A$, $N_T$, $\Delta^-$ and $\sigma_C$. As shown in Table 1 when networks use the basic access mechanism ($\sigma_S = \sigma_C$), as the number of nodes in the networks increases, cooperation is beneficial only at $P_R = 0$, i.e., when the coordination device doesn’t give the AON chance to transmit, since in that case the TON gets the same utility as that if it chooses to compete. However, when networks use the RTS/CTS mechanism ($\sigma_S > \sigma_C$), networks cooperate when
the AON gets a higher chance to transmit, i.e., $P_R \to 1$. This is because under competition the AON accesses the medium with higher probability and would only cooperate if it gets a higher chance to access the medium during cooperation. We skip the results for the case when $\sigma_S < \sigma_C$, since the observations are similar to when $\sigma_S = \sigma_C$, i.e., cooperation is only beneficial when $P_R = 0$.

**Check if cooperation is self-enforceable in the repeated game:** Figure [11] and Figure [12] shows the range of $\alpha$ and $P_R$ for which cooperation can be self-enforced when networks use the basic access mechanism ($\sigma_S = \sigma_C$) and when they use RTS/CTS mechanism ($\sigma_S > \sigma_C$), respectively. The results correspond to different selections of $N_A$ and $N_T$ and region of cooperation is the range of $\alpha$ and $P_R$ for which the inequalities [25, 29] are satisfied and the game has a SPE supported with the coordination device $P_R$. We observe that for any selection of $\sigma_C$, the region of cooperation reduces as the size of networks increases despite the threat of punishment and the SPE ceases to exist when networks are large. However, networks behave differently under different access mechanism. We now discuss these in detail.

**Case I:** When networks use the basic access mechanism and $\sigma_S \leq \sigma_C$: As shown in Figure [11] cooperation is self-enforceable when networks use the basic access mechanism and have fewer nodes. However, as the number of nodes increases cooperation is no longer self-enforceable and the SPE supported with the coordination device $P_R$ ceases to exist. The range of $\alpha$ and $P_R$ corresponding to different selections of $N_A$ and $N_T$ and $\sigma_C = \sigma_S$ over which cooperation is self-enforceable is shown in Figure [11]. We skip the results for the case when $\sigma_S < \sigma_C$, since the observations are similar to when $\sigma_S = \sigma_C$, i.e., cooperation is not self-enforceable as the size of network increases.

The key to cooperation not being self-enforceable is the equilibrium strategy of the AON, which under the competition mode (see [14a] and Figure [4]), occasionally requires nodes in the AON to refrain from accessing the medium, i.e., choose $\tau_A^* = 0$, in a stage, therefore, allowing the TON competition free access to the medium in such stages. While the equilibrium strategy of the AON under the cooperative mode also refrains its nodes from transmitting in a stage, such stages merely lead to wastage of the medium since they cannot be accessed by the TON nodes. As a result, while the AON intends to cooperate as shown in Figure [11b] and [11e] and [11b] over a large range of $\alpha$ and $P_R$, the TON finds competition more favorable and cooperates only for larger values of $\alpha$ and $(1 - P_R)$ as shown in Figure [11f] and [11d]. Also, as the number of AON nodes increases, the frequency of $\tau_A^* = 0$ increases as shown in Figure [8a], therefore, increasing the benefits for TON under competition as shown in Figure [8c] hence, making cooperation unfavorable for TON. Consequently, the region of cooperation shrinks despite the threat of punishment with increasing number of nodes and the SPE ceases to exist as shown in Figure [11].

**Case II:** When networks use the RTS/CTS mechanism and $\sigma_S > \sigma_C$: Similar to the aforementioned scenario, where networks use the basic access mechanism, Figure [12] shows that while cooperation is self-enforceable when networks have fewer nodes, it is no longer beneficial for networks to cooperate as their size increases when networks use the RTS/CTS mechanism too. The range of $\alpha$ and $P_R$ corresponding to different selections of $N_A$ and $N_T$ over which cooperation is self-enforceable is shown in Figure [12].

The behavior, similar to the case when networks use the basic access mechanism, can again be credited to the equilibrium strategy of the AON which under the competitive mode forces its nodes to transmit aggressively, i.e., choose $\tau_A^* = 1$ in a stage, therefore, increasing competition for the TON in such stages. The empirical frequency of $\tau_A^* = 1$, as shown in Figure [8g] increases with increasing number of nodes in the AON, making competition worse for TON as networks grow in size (see Figure [8g]). In contrast, when networks cooperate, the TON at least gets slots with competition free access to the medium with probability $(1 - P_R)$, which allows it to attain a payoff higher than what it would achieve if networks were

---

**TABLE I:** Range of $P_R$, computed using [25], over which networks find cooperation beneficial in the stage game for different selections of $N_A$, $N_T$, $\Delta^-$, and $\sigma_C$. The results correspond to $\sigma_S = 1 + \beta$, $\sigma_1 = \beta$, $\sigma_C = \sigma_S, 0.1\sigma_S$ and $\beta = 0.01$.  

| $N_A$ | $N_T$ | $\Delta^-$ | $P_R$ | $\tau_A^*$ | $\tau_T^*$ | $\bar{\tau}_A$ | $\bar{\tau}_T$ |
|-------|-------|------------|-------|------------|------------|-------------|-------------|
| 2    | 2    | $\sigma_S$ | 0     | 0          | 0.50       | 0           | 0.50        |
| 10   | 10   | $\sigma_S$ | 0     | 0          | 0.10       | 0           | 0.10        |
| 50   | 50   | $\sigma_S$ | 0     | 0          | 0.02       | 0           | 0.02        |
| 2    | 100$\sigma_S$ [0.18, 0.69] | 0.445 | 0.50 | 0.445      | 0.50       |
| 10   | 100$\sigma_S$ 0.01 | 0.001 | 0.10  | 0.001      | 0.10       |
| 50   | 100$\sigma_S$ 0 | 0 | 0 | 0.02 | 0 | 0.02 |

---

**Fig. 10:** Gain of cooperation over competition for the TON and the AON for $N_T = N_A = 5$. The results correspond to $\sigma_S = 1 + \beta$, $\sigma_1 = \beta$, $\sigma_C = \{\sigma_S, 0.1\sigma_S\}$ and $\beta = 0.01$. 

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**Fig. 11:** Shows the range of $\sigma_C$ supported with the coordination device $P_R$ for different selections of $N_A$ and $N_T$.
correspondingly, the region of cooperation reduces as the size of networks grow.

Further, we employed the mechanism to study the effects of competition and cooperation on two cases of practical interests (a) when networks employ the basic access mechanism, and (b) when networks use the RTS/CTS based access mechanism. For each access mechanism, via computational analysis, we showed that while cooperation is self-enforceable between networks with fewer nodes, it is difficult to enforce as networks grow in size, even under the threat of grim trigger strategies. Consequently, the subgame perfect equilibrium (SPE) which can be supported by the coordination device also ceases to exist as networks grow in size. In short, competition is more favorable than cooperation.

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where, \( u_{A,NC} \) is the payoff of the AON defined as
\[
\begin{align*}
\tau_A, \mu \\
\tau_{A,NC} = & (1 - \tau_A)(1 - \tau_T)(1 - \tau_T)^{NT} \tilde{\Delta}^- \\
& + (1 - \tau_A)^{NA} (1 - \tau_T)^{NT} (\sigma_1 - \sigma_C) + \sigma_C \\
& + (N_A \tau_A (1 - \tau_A)(1 - \tau_T)^{NT} \\
& + N_T \tau_T (1 - \tau_T)(1 - \tau_A)^{NA})(\sigma_S - \sigma_C).
\end{align*}
\]

The Lagrangian of the optimization problem \((30)\) is
\[
\mathcal{L}(\tau_A, \mu) = u_{A,NC} - \mu_1 \tau_A + \mu_2 (\tau_A - 1).
\]
where \( \mu = [\mu_1, \mu_2]^T \) is the Karush-Kuhn-Tucker (KKT) multiplier vector. The first derivative of the objective function in \((30)\) is
\[
u_{A,NC} = -\tilde{\Delta}^-(1 - \tau_T)^{NT}((1 - \tau_A)(\sigma_A - \sigma_C)) - (\sigma_S - \sigma_C)(1 - \tau_T)^{NT} (N_A(1 - \tau_A)(\sigma_A - \sigma_C)) \\
- N_A(1 - \tau_A)(1 - \tau_T)(1 - \tau_T)^{NT} (N_A(1 - \tau_A)(\sigma_A - \sigma_C)).
\]

The KKT conditions can be written as
\[
u_{A,NC} = -\mu_1 + \mu_2 = 0, \quad (31a)
\]
\[
-\mu_1 \tau_A = 0, \quad (31b)
\]
\[
\mu_2 (\tau_A - 1) = 0, \quad (31c)
\]
\[
-\tau_A \leq 0, \quad (31d)
\]
\[
\tau_A - 1 \leq 0, \quad (31e)
\]
\[
\mu = [\mu_1, \mu_2]^T \geq 0. \quad (31f)
\]

We consider three cases. In case (i), we consider \( \mu_1 = \mu_2 = 0. \) From the stationarity condition \((31a)\), we get
\[
\tau_A = (1 - \tau_T) (\tilde{\Delta}^- - N_A(\sigma_S - \sigma_1) + N_A N_T \tau_T (\sigma_S - \sigma_C)) \\
\left( (1 - \tau_T) N_A (\tilde{\Delta}^- + (\sigma_1 - \sigma_C) - N_A (\sigma_S - \sigma_C)) \\
+ N_A N_T \tau_T (\sigma_S - \sigma_C) \right)
\]
\[
(32)
\]

In case (ii) we consider \( \mu_1 \geq 0, \mu_2 = 0. \) Again, using \((31a)\), we get \( \mu_1 = u_{A,NC} \). From \((31f)\), we have \( \mu_1 \geq 0, \) therefore, \( u_{A,NC} \geq 0. \) On solving this inequality on \( u_{A,NC} \), we get \( \tilde{\Delta}^- \leq \Delta_{th,0} \), where \( \Delta_{th,0} = N_A(\sigma_S - \sigma_1) - N_A N_T \tau_T (\sigma_S - \sigma_C) \).

Finally, in case (iii) we consider \( \mu_1 = 0, \mu_2 \geq 0. \) On solving \((31a)\), we get \( \tilde{\Delta}^- \leq \Delta_{th,1} \), where \( \Delta_{th,1} = N_A(\sigma_S - \sigma_C) \).

Therefore, the solution from the KKT condition is
\[
\tau_A = \begin{cases} 
(1 - \tau_T)(\tilde{\Delta}^- - N_A(\sigma_S - \sigma_1)) + N_A N_T \tau_T (\sigma_S - \sigma_C) \\
(1 - \tau_T) N_A (\tilde{\Delta}^- + (\sigma_1 - \sigma_C) - N_A (\sigma_S - \sigma_C)) \\
+ N_A N_T \tau_T (\sigma_S - \sigma_C) \\
\end{cases} \
\tilde{\Delta}^- > \Delta_{th,1}.
\]
\[
\tilde{\Delta}^- = \min \{\Delta_{th,0}, \Delta_{th,1}\}. \quad (33)
\]

where, \( \Delta_{th} = \max \{\Delta_{th,0}, \Delta_{th,1}\} \). Under the assumption that length of successful transmission is equal to the length of collision i.e. \( \sigma_S = \sigma_C \), \((33)\) reduces to
\[
\tau_A = \begin{cases} 
\frac{N_A(\sigma_S - \sigma_1) + \tilde{\Delta}^-}{N_A(\sigma_S - \sigma_1) + \Delta^-} \tilde{\Delta}^- > N_A(\sigma_S - \sigma_1), \\
0 \text{ otherwise .}
\end{cases}
\]

Similarly, we find \( \tau_I^* \) for the TON by solving the optimization problem.

**APPENDIX A**

**MIXED STRATEGY NASH EQUILIBRIUM (MSNE)**

We define \( \tau^* = [\tau_A^*, \tau_I^*] \) as the parameter required to compute the mixed strategy Nash equilibrium of the one-shot game. We begin by finding the \( \tau_A^* \) of the AON by solving the optimization problem

**OPT 1:** minimize \( u_{A,NC} \) subject to \( 0 \leq \tau_A \leq 1. \) (30)
tion problem

\[ \text{OPT II: minimize } -u_{T,NC}^{T} \]
\[ \text{subject to } 0 \leq \tau_{T} \leq 1. \tag{35} \]

where, \( u_{T,NC} \) is the payoff of the TON defined as
\[ u_{T,NC} = \tau_{T}(1 - \tau_{T})(N_{T} - 1)(1 - \tau_{A})^{N_{A}}. \]

The Lagrangian of the optimization problem (35) is
\[ \mathcal{L}(\tau_{T}, \mu) = -u_{T,NC} - \mu_{1}\tau_{T} + \mu_{2}(\tau_{T} - 1). \]
where \( \mu = [\mu_{1}, \mu_{2}]^{T} \) is the KKT multiplier vector. The first derivative of \( u_{T,NC} \) is
\[ u_{T,NC}' = (1 - \tau_{A})^{N_{A}}(1 - \tau_{T})(N_{T} - 1)\sigma_{S}. \]

The KKT conditions can be written as
\[ -u_{T,NC}' - \mu_{1} + \mu_{2} = 0, \tag{36a} \]
\[ -\mu_{1}\tau_{T} = 0, \tag{36b} \]
\[ \mu_{2}(\tau_{T} - 1) = 0, \tag{36c} \]
\[ -\tau_{T} \leq 0, \tag{36d} \]
\[ \tau_{T} - 1 \leq 0, \tag{36e} \]
\[ \mu = [\mu_{1}, \mu_{2}]^{T} \geq 0. \tag{36f} \]

We consider the case when \( \mu_{1} = \mu_{2} = 0 \). From (36a), we get \( u_{T,NC}' = 0 \). On solving the stationarity condition, we get \( \tau_{T}^{*} = 1/N_{T} \), which is also the solution of the KKT conditions.

**APPENDIX B**

**OPTIMAL STRATEGY UNDER COOPERATION**

We define \( \tilde{\tau} = [\tilde{\tau}_{A}, \tilde{\tau}_{T}] \) as the optimal strategy of the one-shot game when networks cooperate. We begin by finding the \( \tilde{\tau}_{A} \) of the AON by solving the optimization problem

\[ \text{OPT I: minimize } u_{A,c}^{A} \]
\[ \text{subject to } 0 \leq \tilde{\tau}_{A} \leq 1. \tag{37} \]

where, \( u_{A,c} \) is the payoff of AON defined as
\[ u_{A,c} = (1 - \tilde{P}_{R}\tau_{A}(1 - \tau_{A})(N_{A} - 1)\Delta^{+} + \sigma_{C} + (\tilde{P}_{R}(1 - \tau_{A})^{N_{A}} + (1 - \tilde{P}_{R})(1 - \tau_{T})^{N_{T}})(\sigma_{1} - \sigma_{C}) + (1 - \tilde{P}_{R})N_{A}\tau_{A}(1 - \tau_{A})(N_{A} - 1)\sigma_{S} - \sigma_{C}). \]

The Lagrangian of the optimization problem (37) is
\[ \mathcal{L}(\tau_{A}, \mu) = -u_{A,c} - \mu_{1}\tau_{A} + \mu_{2}(\tau_{A} - 1). \]
where \( \mu = [\mu_{1}, \mu_{2}]^{T} \) is the Karush-Kuhn-Tucker (KKT) multiplier vector. The first derivative of the objective function in (37) is
\[ u_{A,c}' = -\tilde{P}_{R}\Delta^{+}(1 - \tau_{A})^{N_{A} - 1} + (N_{A} - 1)\tau_{A}(1 - \tau_{A})(N_{A} - 2)\sigma_{S} - \sigma_{C}) + (\tilde{P}_{R}(1 - \tau_{A})^{N_{A}} - (N_{A} - 1)\tau_{A}(1 - \tau_{A})(N_{A} - 2)\sigma_{S} - \sigma_{C}) + (1 - \tilde{P}_{R})N_{A}\tau_{A}(1 - \tau_{A})(N_{A} - 1)\sigma_{S} - \sigma_{C}). \]

The KKT conditions can be written as
\[ u_{A,c}' - \mu_{1} + \mu_{2} = 0, \tag{38a} \]
\[ -\mu_{1}\tau_{A} = 0, \tag{38b} \]
\[ \mu_{2}(\tau_{A} - 1) = 0, \tag{38c} \]
\[ -\tau_{A} \leq 0, \tag{38d} \]
\[ \tau_{A} - 1 \leq 0, \tag{38e} \]
\[ \mu = [\mu_{1}, \mu_{2}]^{T} \geq 0. \tag{38f} \]

We consider three cases. In case (i), we consider \( \mu_{1} = \mu_{2} = 0 \). From the stationarity condition (38a), we get
\[ \tau_{A} = \frac{\Delta^{+} - N_{A}(\sigma_{S} - \sigma_{1})}{N_{A}(\Delta^{+} + (\sigma_{1} - \sigma_{C}) - N_{A}(\sigma_{S} - \sigma_{C}))}. \tag{39} \]

In case (ii) we consider \( \mu_{1} \geq 0, \mu_{2} = 0 \). Again, using (38a), we get \( \mu_{1} = u_{A,c}' \). From (38b), we have \( \mu_{1} \geq 0 \), therefore, \( u_{A,c}' \geq 0 \). On solving this inequality on \( u_{A,c}' \) we get, \( \Delta^{+} \leq \Delta_{n,0} \), where \( \Delta_{n,0} = N_{A}(\sigma_{S} - \sigma_{1}) \).

Finally, in case (iii) we consider \( \mu_{1} = 0, \mu_{2} \geq 0 \). On solving (38a), we get \( \Delta^{+} \leq \Delta_{n,1} \), where \( \Delta_{n,1} = N_{A}(\sigma_{S} - \sigma_{C}) \).

Therefore, the solution from the KKT condition is
\[ \tilde{\tau}_{A} = \begin{cases} \frac{\Delta^{+} - N_{A}(\sigma_{S} - \sigma_{1})}{N_{A}(\Delta^{+} + (\sigma_{1} - \sigma_{C}) - N_{A}(\sigma_{S} - \sigma_{C}))} & \text{if } \Delta^{+} > \Delta_{n,0}, \\ \Delta^{+} - \Delta_{n} & \text{if } \Delta_{n} = \Delta_{n,1}, \\ \Delta^{+} - \Delta_{n} & \text{if } \Delta_{n} = \Delta_{n,0}. \end{cases} \tag{40} \]

where, \( \Delta_{n} = \max\{\Delta_{n,0}, \Delta_{n,1}\} \). Under the assumption that length of successful transmission is equal to the length of collision i.e. \( \sigma_{S} = \sigma_{C} \), (40) reduces to
\[ \tilde{\tau}_{A} = \begin{cases} \frac{N_{A}(\sigma_{1} - \sigma_{S}) + \Delta^{+}}{N_{A}(\sigma_{1} - \sigma_{C} + \Delta^{+})} & \text{if } \Delta^{+} > N_{A}(\sigma_{S} - \sigma_{1}), \\ 0 & \text{otherwise}. \end{cases} \tag{41} \]

Similarly, we find \( \tilde{\tau}_{T} \) for the TON by solving the optimization problem

\[ \text{OPT II: minimize } -u_{T,c}^{T} \]
\[ \text{subject to } 0 \leq \tau_{T} \leq 1. \tag{42} \]

where, \( u_{T,c} \) is the payoff of TON defined as
\[ u_{T,c} = (1 - \tilde{P}_{R})\tau_{T}(1 - \tau_{T})(N_{T} - 1)\sigma_{S}. \]

The Lagrangian of the optimization problem (42) is
\[ \mathcal{L}(\tau_{T}, \mu) = -u_{T,c} - \mu_{1}\tau_{T} + \mu_{2}(\tau_{T} - 1). \]
where \( \mu = [\mu_{1}, \mu_{2}]^{T} \) is the KKT multiplier vector. The first derivative of \( u_{T,c} \) is
\[ u_{T,c}' = (1 - \tilde{P}_{R})\sigma_{S}(1 - \tau_{T})(N_{T} - 1)\sigma_{S} - (N_{T} - 1)\tau_{T}(1 - \tau_{T})(N_{T} - 2)] \]

- (37)}.
The KKT conditions can be written as

\[-u_{T,c}' - \mu_1 + \mu_2 = 0,\]  
\[-\mu_1 \tau_T = 0,\]  
\[\mu_2(\tau_T - 1) = 0,\]  
\[-\tau_T \leq 0,\]  
\[\tau_T - 1 \leq 0,\]  
\[\mu = [\mu_1, \mu_2]^T \geq 0.\]

We consider the case when \(\mu_1 = \mu_2 = 0\). From the (43a), we get \(u_{T,c}' = 0\). On solving the stationarity condition, we get \(\hat{\tau}_T = 1/N_T\), which is also the solution of the KKT conditions.