A Novel Method for Implementing Harmonic Compensation with Multi-Inverters in a Micro-Grid

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Abstract. This paper presents a new multi-inverter-based compensation scheme for the harmonic and reactive influences in a micro-grid, which not only realizes the integration of renewable energy, but also combines the functions of improving power quality at the point of common coupling (PCC) simultaneously. The proposed method uses the remaining power energy in the inverter to implement local harmonic compensation, which achieves the flexible customization of power quality and economics. Specifically, an accurate and stable modulated hopping Discrete Fourier Transform (mHDT) algorithm with merits of computation of a sparse DFT result is applied to measure the harmonic current. In addition, a proportional resonant (PR) controller selected to track the command current due to its infinite gain for a specific point ensures there is no steady-state error (ess). Besides, delay compensation and discretization by pre-swapping bilinear transformation are performed to improve the stability of the current loop and enable the controller for digital implementation respectively. The simulation results based on Matlab/Simulink are given to verify the performance of the proposed method.

1. Introduction
Electrical Power has become the life-line of our civilization. However, the use of modern power electronic devices, such as switching power supplies and adjustable speed motor drives, have caused a serious power quality problem [1]. Among them, the potential harm of harmonics is quite serious, it will increase the cost of electricity production, transmission and usage, affect the service life of various components, generate electromagnetic interference, make the relay protection device think that the circuit is faulty and cut off the circuit, and reducing the accuracy of the measuring instrument [2]. In addition, harmonics will also generate resonance in the power grid, it could even cause spontaneous combustion of the line when the harmonic current is too much. Moreover, suitable equipment to implement the compensation is also needed. Therefore, it has been a major challenge to measure the harmonics accurately and compensate for it rapidly [3]. The methods to measure the harmonics are mainly based on the Discrete Fourier Transform (DFT) [4-6], instantaneous reactive power theory [7,8], Short Time Fourier Transform (STFT) and Wavelet Transforms[9-11].
At the beginning of the 21st century, the IEC electromagnetic compatibility standard requires the spectrum analysis only for the first 40 harmonics [12], so the calculation result of the FFT will be partially wasted. However, the Goertzel Algorithm [13] and the sliding DFT (SDFT) can be used.
These methods can compute sparse DFT results, such as only a single complex DFT spectral bin value for every N input time samples when the applications only cover a subset of all period points[14-16]. The most common method to improve the power quality in a micro-grid is passive governance, which uses additional power equipment (such as active power filter, unified power quality conditioner, dynamic voltage restorer) to compensate for harmonics and reactive power[17-20]. This paper presents a new method to implement the harmonic compensation with multi inverters in micro-grid. The method combining the advantages of the two ways above and employs mHDFT algorithm to calculate the content of the required harmonic level, which not only reduces the amount of calculation, but also overcome instabilities and accumulated errors. Then a PR controller with delay compensation and discretization is designed to track the calculated command current, which make sure the accuracy of current loop. Finally, the method uses the remaining energy of the grid-connected inverter in the micro-grid (multiple inverters can be used in parallel) to perform fast and local compensation of harmonics and active current.

Compare to the traditional method, the strategy of the paper have the advantages that use the energy power which could be wasted to implement the compensation of harmonic and active current and the algorithm of mHDFT reduce much more calculation but not sacrifice the accuracy compare to the traditional FFT algorithm. The method of the paper improves the power quality of the micro-grid system and better adapts to the goal of flexible customization of power quality. The numerical experiments and simulation results demonstrate the precision and feasibility of the strategy.

The micro-grid act as an autonomous system with self-control and self-energy management can realize sending, distributing, and self-sufficient operation with distributed generation. It can also be flexibly switched between the two modes of grid-connected operation and isolated operation. However, harmonic and reactive power current in distributed generation systems and micro-grids seriously affect the power quality of the point of common coupling (PCC), even bring adverse effects into the control of the grid-connected inverters. To govern the quality of power at the PCC, the most common method is to install an active or passive filter, but this requires additional equipment and increases the size and cost of the system. Thus, this paper puts forward a new idea based on multi-converters.

Figure 1. A campus Micro-grid with multi grid-tied inverters system

Figure 2 shows a campus micro-grid system diagram. The wind power generator and the photovoltaic array are connected as a distributed power source to the multi-grid inverter system, and the direct current (DC) is converted into alternating current (AC) by the inverter, and then connected to the large power grid and the load. It is not difficult to see that a part of the power will remain in the inverter when the inverter is working from the structure. According to the concept of this paper, the inverter also undertakes the function of improving the power quality of the micro-grid at the same time, that is, to use the remaining energy to compensate for harmonics and reactive current. At this time such a multi-inverter system is called a multi-function grid-tied inverter (MFGTI) by some scholars.
Figure 2 shows an MFGTI topology. The following equation can be obtained from this topology:

\[
\begin{align*}
    L \frac{d i_a}{dt} + R_L i_a &= u_a N - e_a \\
    L \frac{d i_b}{dt} + R_L i_b &= u_b N - e_b \\
    L \frac{d i_c}{dt} + R_L i_c &= u_c N - e_c
\end{align*}
\]  

(1)

Using the Clark transform and equation (1), the model of the controlled object in the \(\alpha\beta\) two-phase stationary coordinate system is obtained:

\[
\begin{bmatrix}
    u_{\alpha} \\
    u_{\beta}
\end{bmatrix} = 
\begin{bmatrix}
    R_L & 0 \\
    0 & R_L
\end{bmatrix} \begin{bmatrix}
    i_{\alpha} \\
    i_{\beta}
\end{bmatrix} + 
\begin{bmatrix}
    L \\
    0
\end{bmatrix} \frac{d}{dt} \begin{bmatrix}
    i_{\alpha} \\
    i_{\beta}
\end{bmatrix} + \begin{bmatrix}
    e_{\alpha} \\
    e_{\beta}
\end{bmatrix}
\]

(2)

It can be expressed in vector form as follows:

\[
u_{\alpha\beta} = R_L i_{\alpha\beta} + L \frac{d i_{\alpha\beta}}{dt} + e_{\alpha\beta}
\]

(3)

Assuming that the DC-side voltage remains constant for one switching period, and the converter modulating wave voltage exhibits zero-order retention characteristics in one switching cycle. Then a discrete model of a single-inductor grid-tied inverter in a stationary coordinate system can be obtained by combining the formula (3).

\[
G_p(s) = \frac{1}{sL + R_L} \Rightarrow G_p(z) = \frac{1}{z - e^{-R_L T_s / L}} \frac{1 - e^{-R_L T_s / L}}{z - e^{-R_L T_s / L}}
\]

(4)

where \(T_s\) is the sampling period.

By analyzing the micro-grid system and topology of a single-inductor grid-tied inverter, some features can be found:

1. There will be residual energy when the multi-inverter system in the micro grid integrates renewable energy.
2. The discrete model of the single-inductance grid-tied inverter can contribute to the design of the current loop.
3. The load environment in the micro grid is ideal, most of them are air conditioners, computers, lights, etc., without large industrial equipment, which makes the power quality problem not very serious.

2. The Proposed Method

2.1. Harmonic Component Measurement

The DFT is the standard algorithm used for spectrum analysis. It is usually implemented by means of the fast Fourier transform, which reduces the amount of mathematical operations required and enhances the algorithm efficiency. Paper [14] also performed the DFT in an efficient way by using the SDFT, a recursive algorithm that calculates a unique spectral component of the signal over a sliding time window of length M.

The spectrum value of the kth frequency is:

\[
X_n(k) = W_M^k [X_{n-1}(k) + d(n)]
\]

(5)

where \(d(n) = x(n) - x(n - M)\).
This is a single sliding DFT algorithm (the sliding step length L is 1), which adopts a sample-by-sample calculation. However, in some signal processing applications where real-time requirements are not strict, the single-step sliding DFT is computationally inefficient in such a low-time signal processing application.

Paper [16] promotes the sliding step size to any step length. In order to facilitate the derivation, consider the case of \( L = 2^a, a \geq 0 \). After L consecutive passes, the Hopping SDFT expression is:

\[
X_n(k) = W_M^{-lk} [X_{n-L}(k) + D_n^L(k)]
\]  

(6)

The hopping SDFT is a recursive algorithm and computationally efficient, but the approach suffers from accumulated errors and potential instabilities caused by the finite precision representation of the twiddle factor.

Paper [15] finds that it is easy to observe that if frequency index \( k = 0 \) when \( L = 1 \), then the twiddle factor \( W_M^{-lk} \) will be null.

The \( X(k) \) DFT bin can be shifted to the index \( k = 0 \) (zero Hz) by using the input signal \( x(n) \) to multiply a modulation sequence \( W_M^{-km} \) in the time domain. The DFT output at time index \( n \) is calculated from the DFT result at the time \( n - L \). Finally, the DFT result of the \( n \)-time frequency point \( k \) is obtained by phase correction.

Combining the advantages of papers [14-16], a high precision, permanently stable modulated HDFT algorithm is presented in paper [21]. The modulated HDFT after frequency domain shift process will be:

\[
X_n(0) = X_{n-L}(0) + W_M^{-km} G_n^L(k)
\]  

(7)

where \( G_n^L(k) \) is the Revised updating vector transform (rUVT),

\[
G_n^L(k) = \sum_{t=L}^{L+1} d(n - t) W_M^{-(t+L-1)L}
\]  

(8)

Therefore, the recursive method used in the mHDFT signal processing structure is shown in Figure 3.

**Figure 3.** mHDFT structure with recursive computation of modulating sequence

Figure 4 shows the single-frequency-point mHDFT structure for \( L = 4 \) and \( M = 16 \).

**Figure 4.** mHDFT structure for \( L = 4 \) and \( M = 16 \)

### 2.2. Controller design

#### 2.2.1 Controller choose

Figure 6 shows the whole system current loop structure, \( G_c(z) \) represents the current controller (such as PR or VPI controller), and \( G_{pl}(z) \) is the transfer function of the discrete domain of the controlled object.
Figure 5. Discrete domain current loop diagram

$G_{PL}(z)$ is achieved by analyzing the topology structure of the MFGTI in section 2, the next step is to choose the current controller and the parameter design.

In this paper, considering that the proportional integral (PI) controller has infinitely gain for $\omega = 0$, thus ensuring there is no steady-state error. Nevertheless, the PR controller has infinite gain for the $\omega = \omega^*$ point to make sure there is no steady-state error, which is suitable to track command current. So the paper chooses the PR controller. The expression of the PR controller in the stationary coordinate system is:

$$G_{PR}(s) = K_p + K_i \frac{s}{s^2 + \omega_e^2}$$

where $K_p$ is the proportional integral gain, $K_i$ is the resonant integral gain, $\omega_e$ is the resonant frequency.

2.2.2 The delay compensation of PR controller. Considering that the PR controller is superposed by Positive Sequence Synchronous Reference System PI (PS-SRF-PI) and Negative Sequence Synchronous Reference System PI (NS-SRF-PI). So the delay compensation principle of the PS-SRF-PI controller is extended to NS-SRF-PI. Thus, the block diagram of the delay compensation method of the PR controller is achieved. Where $T_d$ is time constant of the delay term.

Further, using the SRF frequency transfer principle, combined with Figure 8, the PR controller transfer function after the delay compensation can be obtained, where $\phi$ is the compensation angle, which is defined as the phase difference between the after and before delay compensation at the resonance frequency $\omega_e$.

$$G_{PR}(s) = 2k_p + 2k_i \frac{s \cdot \cos(\phi) - \omega_e \cdot \sin(\phi)}{s^2 + \omega_e^2}$$

For the delay compensation problem of the PR controller, paper [22] proposes that set delay compensation angle equal to the sum of the digital control one beat delay and the controlled object delay.

2.2.3 The discretization of PR controller. The analysis above is performed in a continuous domain, but the controller must be discretized in digital implementation. Paper [23] compared and analyzed the characteristics of eight discretization methods from the aspects of resonance frequency matching, resonance peak, system stability and delay compensation accuracy to research the discretization of the resonant controller. Finally, the paper uses a pre-swapping bilinear transformation to ensure the accuracy of the resonant frequency before and after the discretization.
\[
\text{So:} \quad R_{tp}^{1d}(z) = \frac{1}{2} \left(1 - z^{-2}\right) \cdot \cos(\varphi) \cdot \sin(\omega_c \cdot T_S) - (1 + 2z^{-1} + z^{-2}) \cdot \sin(\varphi) \cdot \left\{\sin\left(\frac{\omega_c \cdot T_S}{2}\right)\right\}^2
\]

The two formulas above can be combined to obtain the z-domain expression of the PR controller after the delay compensation.

\[
G_{PR}^d(z) = K_p + K_i \cdot R_{tp}^{1d}(z)
\]

As for the value of \(K_p\) and \(K_i\), a rule called critical damping method in paper [24] can achieve nearly the minimum setting time in combination with negligible overshoot for reference change:

\[
\begin{align*}
K_p &= k \cdot L \\
K_i &= k \cdot R_L
\end{align*}
\]

where \(L\) is the value of the inductance, \(R_L\) is the value of the resistance and the coefficient \(k\) is calculated by:

\[
k \approx 0.246f_s
\]

where \(f_s\) is the sample frequency. The PR controller based on the above design can better guarantee the tracking of the command current signal.

3. Simulation Results

In order to verify the correctness of the proposed algorithm on the MFGTI platform, this section is based on MATLAB/Simulink to build the corresponding grid-connected inverter model and the algorithm structure, to carry out experimental analysis and give experimental results to test the accuracy of proposed method in this paper. Figure 10 shows the calculation flow of the command current.

The reference current consists of two parts: the active reference current and the harmonic reference current. The harmonic reference current is calculated by the mHDFT algorithm proposed above, and the active reference current is calculated as follows:

Firstly collecting a voltage signal between the grid and the load.

\[
\begin{bmatrix}
i_{fa}^* \\
i_{fb}^* \\
i_{fc}^*
\end{bmatrix} = T_{2s-3s} \ast \begin{bmatrix}
\tilde{i}_a^* \\
\tilde{i}_b^* \\
\tilde{i}_c^*
\end{bmatrix}
\]

Thus, the command reference current is fully calculated:

\[
i^* = i^f + i^h
\]
harmonic distortion (THD) rate is 25.16%. The overall current distortion is more serious and the power quality is poor.

Figure 9 shows the grid-side current signal and its FFT analysis after the compensation of 5th harmonic. The 5th harmonic is almost zero after compensation, the current distortion and power quality have a significant improvement, and the THD is also reduced to 15.57%. However, this does not meet the requirement that THD must be less than 5%.

For further governance, only a few MFGTIs are needed to be connected in parallel to compensate for the harmonics with large disturbances. As show in Figure 10, it is easily found that the current distortion is improved a lot after the compensation of several levels harmonics, the amount of each harmonic is significantly reduced, and the total THD is reduced to 2.48%.

The compensation results of several levels harmonic and active current are shown in Figure 11. As the active injection, the fundamental component changed to 93.62 from 107.2, which cause the increase of total harmonic distortion, still below 5%.

4. Conclusion
A new strategy for harmonic measurement and compensation is presented in this paper. The method uses the remaining energy in grid-tied inverters to implement local harmonic compensation innovatively, with greatly reduced economic costs. The paper introduces the theory of recursive operation based on DFT to compute a single complex DFT spectral bin value for every M input time sample. Then use the DFT modulation property to exclude the accumulated errors and potential instabilities. Besides, this paper also extends the recursive interval from $L=1$ to any step length to accommodate applications that do not require results output sample-by-sample. Moreover, this paper
selects a PR controller according to the characteristics of a no steady-state error at the \( \omega = \omega^* \) point, then a delay compensation and discretization are performed on the controller by pre-swapping bilinear transformation to improve the stability of the current loop and enable the controller to digital implementation. Most of all, several grid-tied inverters in parallel are used to achieve the goal of fast and local compensation.

The system is verified in MATLAB/Simulink with frequency steps (common in grid-connected transitions of micro-grid) and harmonics (produced by nonlinear loads) and the simulation results have demonstrated that the proposed method are robust and accurate in the case of highly distorted environments.

5. References

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