An integral sliding mode fault tolerant control for a class of non-linear Lipschitz systems

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Abstract
This paper proposes an active fault-tolerant control (FTC) strategy for a class of non-linear Lipschitz systems. The proposed FTC approach employs the integral sliding mode control (ISMC) technique due to its inherent capability of dealing with system uncertainties. First, under the nominal fault-free condition, the linear matrix inequality technique is introduced to design the primary controller for the non-linear Lipschitz system. To accommodate the actuator faults/failures, the ISMC law is combined with a control allocation scheme that distributes the control signals to the redundant actuators. A non-linear octorotor system is then used as a test bench to validate the tolerance performance of the proposed FTC strategy. In particular, the proposed FTC strategy is applied for the inner-loop control, while in the outer loop, a fractional-order control approach is used to achieve the precise longitude and latitude control. Finally, various simulations are performed to justify the effectiveness of the proposed control scheme.

1 | INTRODUCTION

As intelligent autonomy develops rapidly these years [1–3], a consequent crucial issue is how to guarantee the system safety. Under such a circumstance, the fault-tolerant control (FTC) has gained substantial attention among researchers. Broadly speaking, there are mainly two kinds of FTCs as discussed in the literature: active FTC and passive FTC. By comparison, the passive FTC provides tolerance against a limited number of faults because of the insufficiency of fault or failure information as required for the active FTC. To guarantee the system safety, numerous FTC approaches for linear systems have been proposed recently [4, 5]. However, most practical systems are characterised by non-linearity, requiring a high-performance FTC scheme to guarantee the system reliability. In the recent years, the robust stabilisation problem of a non-linear system is considered for example [6] suggested adaptive neural finite-time tracking control meant for uncertain feedback non-linear systems thru input saturation. In [7], the authors designed a dynamic output feedback controller such that, for all admissible uncertainties, the resulting fuzzy closed-loop system is stable. In [8], local asymptotic tracking performance for the uncertain switched non-linear systems is studied under a fuzzy logic framework. For FTC analysis, the most recent results are obtained in [9] in which adaptive neural FTC is strategy is proposed for a class of uncertain switched non-strict-feedback non-linear systems.

The FTC strategies have been extensively used especially for sensitive systems like passenger aircraft and unmanned aerial vehicles (UAVs). In particular, multiple effective FTC methods have been proposed for commercial airplanes among the aviation community [10–13]. The controller design is based on a linearised model obtained at a single operating point. Besides the detailed analysis and satisfactory results, the controller based on the linearised system has certain limitations in performance when subject to the broad operating range. Moreover, the FTC algorithms have also been proposed for the control of quadrotor UAVs. However, due to a lack of control redundancy, they cannot withstand the total failure of a single actuator. Recently, the mathematical model of the octorotor integrating eight rotors was modelled in [14] and the linear quadratic regulator (LQR) based state feedback control technique was used for its stable flight. By introducing an effective compensation approach for matched disturbances or model uncertainties, [15] proposed a non-linear sliding mode control (SMC) for the stabilisation of the octorotor system. However, the system performance was

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not discussed in the case of fault or failure. To deal with the fault/failure issue, [16–18] introduced control allocation (CA) mechanisms and designed other SMC schemes for the octorotor. In [16, 18], the linear parameter varying (LPV) model of the non-linear system is taken into account and a fixed state feedback controller is designed over the entire operating range. Moreover, with the help of a sliding mode observer (SMO), [19] developed a fault detection and isolation (FDI) scheme such that the actuator fault in the octorotor system can be identified and then a FTC strategy was developed by using a super-twisting algorithm. The proposed method works well but this work lacked in closed-loop stability analysis. Besides, the higher-order SMC and the SMC augmented by the neural network technique were also proposed for the octorotor system subject to the actuator fault in [20, 21]. No doubt higher-order SMC technique is effective against chattering reduction but the inclusion of higher derivative terms can make the system sensitive to noise and disturbance. In the authors’ previous work [22] an adaptive SMC is designed for an octorotor system that can also handle the actuator faults in the system. The adaptive strategy is integrated with SMC law provided robustness against the unknown faults. The existing scheme does not require the information of faults, therefore passive in nature. It can only cater limited class of faults and the complete failure cannot be encountered. Although the SMC methodology possesses a good fault-tolerant capability to some extent, it cannot suppress the fault or uncertainty effect during the reaching phase. To guarantee the robustness all the time, the integral SMC (ISMC) is available, which has been widely used in practice. See [12, 13, 23–25] for examples. Compared with the traditional SMC, the ISMC has three advantages. In the first place, the ISMC eliminates the reaching phase as in the SMC, facilitating the satisfaction of the physical control constraint. In the second place, the ISMC can withstand the matched uncertainty effect without amplifying the unmatched one in the sense of Euclidean norm. Last but not the least, with the ISMC, the matched uncertainty makes little difference to the system response. Due to these remarkable advantages, several applications of the ISMC scheme have already been studied, for example, for the photovoltaic pumping system [23], the aircraft dissimilar redundant actuator system [10, 11], the aircraft actuation system [12, 13], and even the missile system [26].

Due to increasing performance requirements, control approaches are studied with more attention on general non-linear systems. As a particular class of non-linear systems, the non-linear Lipschitz systems have been of considerable interest [27–31]. In practice, numerous physical systems are characterised by the global or local Lipschitz attribute. Motivated by this fact, this paper focuses on the development of an active FTC approach to deal with the actuator fault/failure issue of a class of non-linear Lipschitz systems. In particular, the developed active FTC strategy combines the ISMC with the CA scheme. The ISMC is available to the robustness improvement while the CA scheme is beneficial for enhancing tolerance capability against actuator faults/failures. By taking various combinations of faults/failures into account, the sufficient condition for the closed-loop system stability is determined in terms of the linear matrix inequality (LMI) technique and small gain theory. Next, the proposed control scheme is applied to an octorotor system to demonstrate its effectiveness. The whole octorotor system is first segregated into two subsystems. The first outer-looptranslational subsystem is controlled by a fractional order controller, while the other inner-loop rotational subsystem is controlled by the proposed active FTC scheme. Compared with the current works, the proposed control scheme has the following advantages:

- The proposed FTC strategy extends the combination of the ISMC with the CA scheme for LTI systems [10, 12] and LPV systems [13, 16, 18] to a class of non-linear Lipschitz systems. To guarantee the system stability in the case of simultaneous actuator faults/failures, a CA scheme is designed for the reasonable control allocation, while an ISMC strategy is designed to achieve the stable control with high robustness against uncertainty and disturbance.
- The control scheme reconfigures the ISMC based on the information of the actuator efficiency level such that the controller reconfiguration is not necessary at each fault step.
- The proposed control scheme is applied to the octorotor system. In particular, it effectively improves the fault tolerance capability and robustness performance against the rotor fault/failure and disturbance.

The rest of this paper is arranged as follows. Section 2 proposes the FTC strategy for non-linear Lipschitz systems, including the designs of the CA scheme and the ISMC law. Section 3 applies the proposed FTC strategy to a non-linear octorotor system. Section 4 performs simulations under nominal and different faulty conditions. Section 5 draws the conclusions and proposes future work.

2 FTC SCHEME FOR NON-LINEAR LIPSCHITZ SYSTEM

2.1 Nominal control

An active FTC scheme is put forward in this section to solve the actuator fault/failure problem of non-linear Lipschitz systems. In particular, the proposed scheme combines ISMC design and CA method, while the basic controller for the nominal system is determined by using the LMI approach. Intuitively, the architecture of the FTC scheme is provided in Figure 1. Consider the following non-linear Lipschitz system considering actuator faults/failures:

$$\begin{align*}
\dot{x}_p(t) &= A_p x_p(t) + B_p z_p(t) u_p(t) + D p x(t) + f(x_p, u_p, t) \\
\gamma_p(t) &= C_p x_p(t)
\end{align*}$$

where $x_p(t) \in \mathbb{R}^n$, $u_p(t) \in \mathbb{R}^m$, and $\xi_p(x_p, t) \in \mathbb{R}^q$ are the system state, input and disturbance, respectively, $A_p \in \mathbb{R}^{n \times n}$, $B_p \in \mathbb{R}^{n \times m}$, $C_p \in \mathbb{R}^{q \times n}$ and $D \in \mathbb{R}^{q \times q}$ are the system matrix, input distribution matrix, output distribution matrix and disturbance input matrix, $Z_p(t) \in \mathbb{R}^{n \times n}$ is a positive semi-definite matrix parameterising the efficiency of each single actuator, and
where \( \varphi_i \) \((i = 1 \ldots m)\) represents the uncertainty corresponding to the \(i\)-th actuator effectiveness.

On the other hand, to provide CA, the input distribution matrix \(B_p\) is partitioned into the following form

\[
B_p = \begin{bmatrix}
B_{p1} \\
B_{p2}
\end{bmatrix}
\]

where \(B_{p1} \in \mathbb{R}^{(r-l) \times \omega}\) and \(B_{p2} \in \mathbb{R}^{l \times \omega}\) with rank \(l < m\).

**Assumption 2.** The pair \((A_p, B_p)\) is controllable where \(B_p = B_pB_p^T\). It is also assumed that the pair \((A_p, B_pZ_p)\) is controllable even in the case of faults.

**Assumption 3.** The partitioning of matrix \(B_p\) is such that the channel associated with \(B_{p2}\) provides the dominant contribution in delivering the control effort compared to the channel associated with \(B_{p1}\).

\[
\nu_p(t) = B_pu_p(t)
\]

which can be interpreted as a virtual control effort created by the actuators. The idea of introducing the virtual control input is to stabilise the system with the minimum set of actuators. With the help of the CA strategy, the signals can be redistributed to all the available actuators. In this way, the engineer/designers have much flexibility to determine an optimal virtual control law for the pre-selected system according to the performance requirement. The same control law will be used in case of faults/failures. Once the partition of \(B_p\) has been attained, the states can be scaled such that \(B_{p1}B_{p1}^T = I_l\). It can be achieved without losing generality because of rank\((B_{p2}) = l\). The state re-scaling is done to facilitate the subsequent closed-loop system analysis. By substituting the virtual control input (8) into (1) and assuming that all the actuators are healthy \((i.e. Z_p(t) = I_m)\), a nominal system can be written as

\[
\dot{x}_p(t) = A_px_p(t) + B_p\nu_p(t) + f(x_p, u_p, t) + D\varphi_p(x_p, t)
\]

where \(B_p = \begin{bmatrix}
B_{p1} \\
B_{p2}
\end{bmatrix}\). Under Assumption 2, it is trivial that there exists a control law

\[
\nu_p(t) = Yx_p(t)
\]
that stabilises the closed-loop system

\[
\dot{x}_p(t) = (A_p + B_p\mathbf{Y})x_p(t) + f(x_p, u_p, t)
\]

where \(\mathbf{Y}\) is a feedback gain designed specifically for the primary actuators. We next focus on the determination of a proper feedback gain \(\mathbf{Y}\) such that the asymptotic stability of the nominal system (11) without the disturbance is guaranteed first. And subsequently, the non-linear ISMC strategy is introduced to counteract the disturbance effect.

Remark 2. The virtual control law associated with the nominal system (11) is a linear part of control law which will be later segregated into linear component and ISMC component law (see 52). The linear control law (10) is responsible for the stability of the system (11) without the disturbance. And subsequently the non-linear ISMC strategy is introduced to counteract the disturbance effect.

2.2 | Design of feedback gain \(\mathbf{Y}\)

In this section, an appropriate feedback gain \(\mathbf{Y}\) is chosen based on the LMI technique such that the nominal system (11) is asymptotically stabilised.

Under Assumption 1, it is trivial that there exists a unique equilibrium \(x_{p_0}\) such that

\[
A_p x_{p_0} + B_p v_{p_0} + f(x_{p_0}, u_{p_0}, t) = 0
\]

\[
u_{p_0}(t) = B_p^T v_{p_0}(t), \quad v_{p_0}(t) = \mathbf{Y} x_{p_0}(t)
\]

Define the error variable \(\epsilon_p = x_p - x_{p_0}\). By virtue of (11) and (12), its dynamics can be derived as

\[
\dot{\epsilon}_p(t) = (A_p + B_p\mathbf{Y})\epsilon_p(t) + \bar{f}
\]

where \(\bar{f} = f(x_p, u_p, t) - f(x_{p_0}, u_{p_0}, t)\). It follows from Assumption 1 that

\[
\|\bar{f}\| \leq \|f(x_p, u_p, t) - f(x_{p_0}, u_{p_0}, t)\| + \|f(x_{p_0}, u_{p_0}, t) - f(x_{p_0}, u_{p_0}, t)\|
\leq \mu_{\phi_p} \|x_p - x_{p_0}\| + \mu_{\phi_n} \|u_p - u_{p_0}\|
\leq \mu_{\phi_p} \|\epsilon_p\|
\]

where \(\mu_{\phi_p} = \mu_{\phi_n} + \|\mathbf{Y}\|\). Before presenting the main result, a useful result is introduced as follows.

Proposition 1. Consider the error system (13) with the stabilisable pair \((A_p, B_p)\). Suppose that Assumptions 1-3 hold. If the feedback gain \(\mathbf{Y}\) is designed such that the system \(A_w = A_p + B_p\mathbf{Y}\) is Hurwitz and there exists \(\varepsilon_2\) such that the algebraic Riccati equation

\[
A_w^T Q + Q A_w^T + \mu_{\phi_p}^2 \mu_{\phi_n}^2 Q + I + I \varepsilon_2 = 0
\]

has a positive definite solution \(Q > 0\), and the error \(e(t)\) converges to zero asymptotically.

Proof. It is trivial that \((A_w^T, B_w^T)\) is detectable. By taking the stable matrix \(A_w = A_p + B_p\mathbf{Y}\) into account, the condition (15) is equivalent to \(\min_{\omega \in \mathbb{R}^+} \sigma_{\min}(A_w^T - j\omega I) > \mu_{\phi_p}\). Assign a Lyapunov function candidate

\[
\mathcal{V}_1 = e_p^T Q^{-1} e_p
\]

Differentiating \(\mathcal{V}_1\) along the error system (13) yields

\[
\dot{\mathcal{V}}_1 = 2 e_p^T Q^{-1} e_p = 2 e_p^T Q^{-1} (A_w e_p + \bar{f})
\]

It then follows from (14) that

\[
2 e_p^T Q^{-1} \bar{f} \leq 2 \mu_{\phi_p} \|Q^{-1} e_p\| \|\bar{f}\|
\leq \mu_{\phi_p}^2 \|e_p\|^2 + \|\bar{f}\| \|Q^{-1} e_p\|
\]

By substitution, \(\dot{\mathcal{V}}_1\) satisfies

\[
\dot{\mathcal{V}}_1 \leq e_p^T Q^{-1} e_p + \mu_{\phi_p}^2 \|e_p\|^2 + \|\bar{f}\| \|Q^{-1} e_p\|
\leq e_p^T Q^{-1} (Q A_w^T + A_w Q + \mu_{\phi_n}^2 Q + I) Q^{-1} e_p
\]

In terms of (16), it further follows that

\[
\dot{\mathcal{V}}_1 \leq -\varepsilon_2 e_p^T Q^{-1} e_p \leq -\varepsilon_2 \|Q^{-1} e_p\| < 0
\]

This implies that \(\lim_{t \to \infty} e_p(t) = 0\), resulting in the asymptotic convergence of \(x_p(t)\) to the equilibrium point \(x_{p_0}\).

Remark 3. According to Theorem 2 in [36], Proposition 1 presents a sufficient condition for the asymptotic stability for a class of Lipschitz systems, which also determines conservative applicability of the control scheme to the unmodelled dynamics \(f\).

Note from the definition of \(\mu_{\phi_p}\) that, both sides of (15) involve the gain matrix \(\mathbf{Y}\). To introduce the convex synthesis method to attain \(\mathbf{Y}\), an upper bound on the gain matrix \(\mathbf{Y}\) is required. This can be followed by the following theorem.

Theorem 1. Consider the closed-loop error system (13) with the stabilisable pair \((A_p, B_p)\). Given \(\beta > 0\), if there exist a scalar \(\alpha_\sigma > 0\) and
matrices $Q, N$ so as for the following LMI:

$$\begin{bmatrix} A_p Q + QA_p - B_i N - N B_i + \alpha_s I & * \\ \mu_{\chi_p} + \beta_{\mu_{\chi_p}} Q & * \end{bmatrix} \leq 0$$

(22)

$$\begin{bmatrix} \beta_p^2 & * \\ N^T Q & * \end{bmatrix} \geq 0, \quad Q \geq I$$

(23)

the feedback gain $Y = -Q^{-1} N$ is sufficient for the asymptotic stability of the closed-loop error system (13).

**Proof.** In terms of (23), it follows that $\beta_p^2 I - N^T Q^{-1} N \succeq 0$, and further that

$$Y^T Y \leq \|Y\|^2 \leq \|\beta\|_2^2$$

(24)

which implies that $\|Y\| \leq \beta$. Since $\|Y\| \leq \beta$, then by considering

$$\min_{\omega \in \mathbb{R}} \sigma(A_{\omega} - j\omega I) > \mu_{\chi_p} + \beta_{\mu_{\chi_p}}$$

(25)

and $\|Y\| \leq \beta$, that yields $\mu_{\chi_p} + \beta_{\mu_{\chi_p}} \geq \mu_{\chi_p}$, the inequality (15) holds and then guarantees the asymptotic stability of the error system (13). Thus, for a given $\alpha_s > 0$, by following the similar analysis given in Proposition 1, there exists a positive definite matrix $Q (\alpha_s^{-1} Q > 0)$ such that

$$A_{\omega} Q + QA_{\omega}^T + \alpha_s^{-1} (\mu_{\chi_p} + \beta_{\mu_{\chi_p}}^2) Q + \alpha_s I < 0$$

(26)

It is notable that if the pair $\{Q, \alpha\}$ is the solution of the inequality (26), so is $\{\eta Q, \eta \alpha\}$ for any $\eta > 0$. As a consequence, we can assume $Q \geq I$ without loss of generality. It is trivial that the schur complement on (22) with $\alpha_s > 0$ is sufficient to guarantee (26). Now (22) and (26) yields the asymptotic convergence of closed-loop dynamics. Therefore, according to Proposition 1, the closed-loop error system (13) is asymptotically stable. \qed

### 2.3 Control allocation scheme

By following the gain specification in the former section, the nominal non-linear system (11) has been asymptotically stabilised by the nominal control law $y_p(t)$. In this section, we intend to reconfigure the virtual control law via a CA scheme. The actual control signal allocated to each actuator can be obtained from (8) as

$$u_p(t) = B_p^T(t) y_p(t)$$

(27)

where $B_p^T(t) \in \mathbb{R}^{m \times \ell}$ represents the weighted pseudo-inverse of the matrix $B_p$ in (8). In particular, $B_p^T(t)$ gives freedom in delivering the virtual control signals to the practical physical actuators. By assuming that $B_p^T(t) B_p = I$, $B_p^T(t)$ chosen as

$$B_p^T(t) = Z_p(t) B_p^T(t) Z_p(t) B_p^T(t)$$

(28)

such that $(B_p^T Z_p(t) B_p^T)^{-1} \neq 0$. Note that the CA scheme, which allocates the designed control signals to the practical set of actuators, is dependent on the information of actuator efficiency matrix $Z_p(t)$. By substituting (27) into (1), it follows that

$$\dot{x}_p(t) = A_p x_p(t) + B_p (I_{\omega} - \varphi(t)) B_p^T(t) y_p(t) + f(x_p, u_p, t) + D_{\xi_p}(x_p, t)$$

(29)

where

$$B_p^T(t) = Z_p^T(t) B_p^T(t) B_p^T(t) Z_p(t) B_p^T(t)$$

(30)

It is noticeable that $B_p^T(t)$ is the weighted pseudo-inverse of $B_p$ and satisfies $B_p^T B_p(t) = I$. Moreover, if there is no fault and the estimation is accurate (i.e. $\varphi = 0$ and $Z_p(t) = Z_p(t) = I_{\omega})$, then $B_p^T(t) = B_p^T(t)$. To analyze the closed-loop system stability, the upper bound of $B_p^T(t)$ is needed. According to [37], it is determined by

$$\|B_p^T(t)\| = \|Z_p^T(t) B_p^T(t) B_p^T(t)\| < \eta_p$$

(31)

where $\eta_p$ is a positive constant.

### 2.4 ISMC design

In the ISMC design, the sliding surface is given as

$$\Xi = [x_p \in \mathbb{R}^m : S(t) = 0]$$

(32)

where the continuous sliding mode function $S(t)$ is defined as

$$S(t) = G_p x_p(t) - G_p x_0(t) - G_p \int_0^t \left( (A_p + B_p) x(t) + f(x_p(t), u_p(t), t) \right) dt$$

(33)

with $G_p \in \mathbb{R}^{\ell \times m}$ being a design freedom matrix. The integral term therein is used to eliminate the reaching phase while maintaining the sliding motion through the entire response. Choose the design freedom matrix $G_p$ as

$$G_p = B_p^T(t) B_p^T(t)$$

(34)

such that

$$G_p B_p = B_p^T(t) B_p^T(t)$$

(35)

Next, by substituting (29) into the derivative of (33) yields

$$\dot{x}_p(t) = G_p B_p(t) y_p(t) - Y x_p(t) + G_p D_{\xi_p}(x_p, t)$$

(36)
By equating \( \dot{\xi}(t) = 0 \), the equivalent control can be designed as
\[
\nu_{eq}(t) = -(G_p \dot{B}_p(t))^{-1} (G_p D_p \xi_p(x_p, t) - Y x_p(t)) \tag{37}
\]
Subtracting and adding \( B_t Y x_p(t) \) in (37) and substituting it into (29) yields
\[
\dot{x}_p(t) = (A_p - B_t Y) x_p(t) + (B_t - \dot{B}_p(t))(G_p \dot{B}_p(t))^{-1} Y x_p(t) + f(x_p, u_p, t)
\]
\[
+ f(x_p, u_p, t)
\tag{38}
\]
The matrix \( \Delta(t) \) in (38) can amplify the effect of the unmatched term. However, with the choice of \( G_p \) in (34) and based on Lemma 2 in [38], it can be obtained that \( \|\Delta\| = \|I - B_p(B_t^p B_t)B_t\| = 1 \). Therefore \( \|\Delta(t)D_p \xi_p(x_p, t)\| \leq \|D_p \xi_p(x_p, t)\| \). However, it is necessary to prove the stability of the closed-loop system (38) in the presence of uncertainty and faults/failures.

**Assumption 4.** The disturbance term \( \xi_p(x_p, t) \) can be decomposed into \( \xi_p(x_p, t) = \phi(x_p, t) \phi(x_p, t) \), where the term \( \phi(x_p, t) \) is bounded, i.e., \( \|\phi(x_p, t)\| \leq \phi \).

**Remark 4.** This assumption 4 is frequently used in the existing work on FTC schemes such as see [33, 34] and it shows the linear combination of system states and disturbance function.

Using Assumption 4, the closed-loop system (38) can be rewritten as
\[
\dot{x}_p(t) = (A_p - B_t Y) x_p(t) + f(x_p, u_p, t) + B_t \Delta_{af} Y x_p(t) \tag{39}
\]
where \( B_t = [B D], \Delta_{af} = [\Psi(t) 0 \phi(x_p, t)], Y = [\dot{Y}], \)
\[
\bar{B} = [B_t]
\tag{40}
\]
and the time-varying uncertainty \( \Psi(t) \) is given by
\[
\Psi(t) = B_p B_p^T - B_p (1 - \phi(t)) B_p^* (B_p (I - \phi(t)) B_p^* (t))^{-1}
\tag{41}
\]
Note that the first two terms in (39) is the nominal part. As shown before, by choosing a proper gain \( \Gamma \), the nominal system can be asymptotically stabilised. Moreover, \( \Delta_{af} \) in the last term models the uncertainty arising from actuator faults/failures.

### 2.5 Stability analysis of closed-loop system

In the case with accurate estimation (i.e. \( \tilde{Z}_p(t) = Z_p(t) \)) and no fault (i.e. \( \Psi(t) = 0 \)), the closed-loop sliding motion in (38) becomes
\[
\dot{x}_p(t) = (A_p - B_t Y) x_p(t) + f(x_p, u_p, t) + D_p \Delta_p(t)
\tag{42}
\]
Note that a proper choice of the feedback gain \( Y \) is sufficient for the asymptotic stability of the system (42). However, subject to faults/failures, the right side of (39) is non-zero. Therefore, it is necessary to analyze the closed-loop stability in such a fault/failure case. For this purpose, the sliding motion is rewritten as
\[
\dot{x}_p(t) = (A_p - B_t Y) x_p(t) + f(x_p, u_p, t) + B_t \tilde{u}_p(t)
\tag{43}
\]
where \( \tilde{u}_p(t) = \Psi(x_p, t) \). According to [36], the \( H_\infty \) norm between \( \tilde{u}_p(t) \) and \( \tilde{u}_p(t) \) is defined as
\[
\eta_2 = \|\tilde{G}(\cdot)\|_\infty \tag{44}
\]
where
\[
\tilde{G}(\cdot) = Y (I - A_f)^{-1} B
\tag{45}
\]
and \( A_f = (A_p - B_t Y + \mu_u I) \) is Hurwirz with the well-specified feedback gain \( Y \).

**Assumption 5.** The augmented uncertainty \( \Delta_{af} \) satisfies the following inequality
\[
\|\Delta_{af}\| \leq \eta_1 \tag{46}
\]
**Remark 5.** Assumption 5 determines the bound on uncertain terms. This uncertainty is occurred in the closed-loop system due to faults, failures and fault estimation error.

**Proposition 2.** For any combination of faults/failures, the closed-loop system (43) is asymptotically stable if the following condition is satisfied:
\[
\eta_2 \eta_1 < 1 \tag{47}
\]
where \( \eta_1 \) is defined in (46) and \( \eta_2 \) is given in (44).

**Proof.** The closed-loop sliding motion in (43) interconnects the closed-loop stable term \( \tilde{G}(\cdot) \) and the uncertain term \( \Delta_{af} \) as shown in Figure 2. According to the small gain theorem [39], the asymptotic stability of (43) is guaranteed by
\[
\|\Delta_{af}\| \|\tilde{G}(\cdot)\|_\infty < 1 \tag{48}
\]
For this purpose, it is necessary to verify the boundedness of $\Delta_{sf}(t)$. It follows from (41) that

$$
\|\hat{\Psi}(t)\| \leq \|B_{p_1}B_{p_2}\| - \|B_{p_1}(1 - \varphi(t))B^+_{p_2}(t)\|
$$

(49)

Due to the facts that $\|B_{p_2}\| = 1$, $B_{p_2}B^+_{p_2}(t) = I$, (49) becomes

$$
\|\hat{\Psi}(t)\| \leq \|B_{p_1}\| + \|B_{p_1}(1 + \varphi_{max})B^+_{p_2}(t)\|
$$

(50)

This is well-defined by $\|B_{p_2}\varphi_{max}B^+_{p_2}(t)\| < \|\varphi_{max}\psi_o\| < 1$. Since $\psi_o > \|B_{p_2}\|$, and $\|B_{p_1}\| = \psi_3$, (50) further becomes

$$
\|\hat{\Psi}(t)\| \leq \frac{\psi_3(1 + \psi_o)}{1 - \varphi_{max}\psi_o}
$$

(51)

It thus follows from (49) and (51) that $\Delta_{sf}$ is bounded. Therefore, the condition claimed in (47) is sufficient to guarantee the asymptotic stability of the closed-loop system (43).

Remark 6. The closed-loop stability of the uncertain Lipschitz non-linear system (43) depends on the $H_{\infty}$ norm defined in (44) and the norm on the augmented uncertainty in (46). Note that the choice of $\mathbf{Y}$ has a significant impact in computing the norm $\psi_2$. On the other hand, it can be seen from (51) that the size of $\|B_{p_1}\|$ makes an effort in the norm of non-linearity in the small gain feedback loop. Moreover, it is also notable that the small $\|B_{p_1}\| = \psi_3$ is chosen, the less strengthen requirement on the norm magnitude between $\hat{y}_f(t)$ and $\tilde{y}_f(t)$ is needed. In addition, we can impose reasonable constraints on the state feedback gain such that the transient performance can be improved without violating the closed-loop stability. This can also ease the restriction on system matrix partitioning term $\|B_{p_1}\|$ which is complex to obtain in practice.

2.6 ISMC law

This section deals with the design of the ISMC law which ensures the motion on the sliding surface. The control law is separated into the linear part $v_{pl}(t)$ and the non-linear part $v_{pal}(t)$. The linear part accounts for the stabilisation of the nominal system while the non-linear part plays a role in dealing with the uncertainty arising from the fault/failure. In particular, the control law $v_p(t)$ is written as

$$
v_p(t) = v_{pl}(t) + v_{pal}(t)
$$

(52)

where the linear part is

$$
v_{pl}(t) = \mathbf{Y}_{x_p}(t)
$$

(53)

and the non-linear part is

$$
v_{pal}(t) = -\Lambda(t, x_p)\text{sign}(\mathbf{f}(t))
$$

(54)

with $\Lambda(t, x_p)$ being the modulation gain.

Proposition 3. Assume that proposition 2 is satisfied. If the modulation gain $\Lambda(t, x_p)$ satisfies

$$
\Lambda(t, x_p) \geq \frac{((1 + \varphi_{max})\psi_o - 1)\|v_{pl}(t)\| + \|G_p\varepsilon_p(x_p, t)\|}{(1 + \varphi_{max})\psi_o}
$$

(55)

where $k > 0$, the ISMC law (52) guarantees the reachability on the sliding surface (33).

Proof. By substituting the ISMC law (52) with $G_p = B_p^T$ into (36), it follows that

$$
\dot{\tilde{y}}(t) = (G_pB_p(t))(v_{pl}(t) + v_{pal}(t)) - v_{pl}(t) + G_p\varepsilon_p(x_p, t)
$$

(56)

Substituting $G_pB_p(t) = B_{p_2}(1 - \rho)B^+_{p_2}(t)$ into (56) and doing further calculation yield

$$
\dot{\tilde{y}}(t) = B_{p_2}(1 - \rho)B^+_{p_2}(t)(v_{pl}(t) + v_{pal}(t)) - v_{pl}(t) + G_p\varepsilon_p(x_p, t)
$$

(57)

Consider the following Lyapunov function candidate

$$
V(t) = \frac{1}{2}\text{ST}(t)S(t)
$$

(58)

Its derivative along (57) satisfies

$$
\dot{V}(t) = ST(t)(B_{p_2}(1 - \rho)B^+_{p_2}(t)(v_{pl}(t) + v_{pal}(t)) - v_{pl}(t)
$$

$$
+ G_p\varepsilon_p(x_p, t))
$$

$$
\leq \|S\|(1 - \rho_{max})\psi_o - 1\|v_{pl}(t)\|
$$

(59)

$$
- \Lambda(t, x_p)\|(1 - \rho_{max})\psi_o + G_p\varepsilon_p(x_p, t)\|
$$

By choosing the modulation gain given in (55), $\dot{V}$ becomes

$$
\dot{V}(t) \leq -k\|S(t)\| = -k\sqrt{\dot{V}(t)}
$$

(60)

Therefore, with the designed ISMC law (54), the reachability condition is satisfied, which is sufficient to guarantee that the sliding surface is reached [12].

Remark 7. In this paper, the modulation gain is chosen greater than the norm of disturbance and uncertainty arising from the fault/failure and fault estimation error. To avoid the large value of the modulation gain, it is necessary to have as much fault knowledge about the actuator effectiveness level as possible.
However, the larger gain may bring in chattering phenomenon which is not desirable in practical applications. Therefore, it is more appropriate to incorporate an adaptive strategy with non-linear ISMC law in order to keep the sliding gain within limit under nominal or faulty condition, which will be the part of our future work.

Remark 8. Finally, the control law \( u_p(t) \) is obtained by combining the CA component described in (27-28) and the virtual control law (52). The linear control law (53) and non-linear ISMC law (54) gives us the following control law:

\[
\dot{u}_p(t) = B_p^+ \left(-Y_p(t) - \Gamma(b, x_p)\text{sign}(S(t))\right) \tag{61}
\]

### 3 APPLICATION TO OCTOROTOR SYSTEM

#### 3.1 Non-linear model of octorotor system

To check the effectiveness of the proposed control scheme, a star-shaped octorotor is used as a test bench as shown in Figure 3. It is composed of eight rotors spaced equally at 45°. For the convenience of the subsequent control design, some reasonable assumptions [16] for establishing the octorotor model are first made as follows:

- The rolling moments and hub forces are negligible.
- The drag coefficients and thrust are constants.
- The octorotor is symmetric and so is the inertia matrix.

According to [15], the non-linear model of an octorotor system is described by

\[
\dot{p}_x = (\sin \theta \cos \phi \cos \psi + \sin \psi \sin \phi) \frac{1}{m} \tau_1 \tag{62}
\]

\[
\dot{p}_y = (\sin \theta \cos \phi \sin \psi - \cos \psi \sin \phi) \frac{1}{m} \tau_1 \tag{63}
\]

\[
\dot{p}_z = (\cos \phi \cos \theta) \frac{1}{m} \tau_1 - g \tag{64}
\]

\[
\dot{\phi} = qr \frac{I_{yy} - I_{xx}}{I_{xx}} - \frac{J_r}{I_{yy}} \phi \Omega + \frac{1}{I_{yy}} \tau_2 \tag{65}
\]

\[
\dot{\theta} = pr \left(\frac{I_{zz} - I_{xx}}{I_{yy}}\right) - \frac{J_r}{I_{yy}} \theta \Omega + \frac{1}{I_{yy}} \tau_3 \tag{66}
\]

\[
\dot{\psi} = pq \left(\frac{I_{xx} - I_{yy}}{I_{zz}}\right) + \frac{1}{I_{zz}} \tau_4 \tag{67}
\]

where the parameter definitions are presented in Table 1. Moreover, motivated by [40], the residual propeller speed of unbalanced rotors is expressed as

\[
\Omega = -\Omega_1 - \Omega_2 - \Omega_3 - \Omega_5 + \Omega_4 + \Omega_7 + \Omega_8 \tag{68}
\]

Equivalently, the non-linear model (62)-(67) can be rewritten in the following non-affine form:

\[
\dot{X} = f(X) + g(X)\tau \tag{69}
\]

where

\[
X = \begin{bmatrix} p_x \ p_y \ p_z \ \phi \ \theta \ \psi \ \dot{p}_x \ \dot{p}_y \ \dot{p}_z \ p \ q \ r \end{bmatrix}^T \tag{70}
\]

and

\[
\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} \text{Total thrust} \\ \text{Roll torque} \\ \text{Pitch torque} \\ \text{Yaw torque} \end{bmatrix} \tag{71}
\]

| Symbol | Parameters | Symbol | Parameters |
|--------|------------|--------|------------|
| \( p_x, p_y, p_z \) | Position components | \( \phi, \theta, \psi \) | Roll, pitch and yaw |
| \( \tau_1, \tau_2, \tau_3 \) | Yaw, pitch and roll rate | \( b, d \) | Thrust and drag factor |
| \( g \) | Gravitational constant | \( m \) | Mass |
| \( I_{xx}, I_{yy}, I_{zz} \) | Inertia's on \( x, y, z \) axes | \( I_0, I_r \) | Body and rotor inertia |
| \( L, l \) | Arm and moment arm | \( M \) | Vehicle mass |
| \( \tau \) | Thrust | \( \Omega \) | Body angular velocity |
In addition, the control input $\tau$ is generating by adjusting the motor speeds of rotors. In particular,

$$\tau = B_\Omega u$$  \hspace{1cm} (72)

where $u = [\Omega_1^2, \Omega_2^2, \ldots, \Omega_8^2]^T$ and

$$B_\Omega = \begin{bmatrix}
    b & b & b & b & b & b & b \\
    0 & 0 & -bl & -bl & 0 & 0 & bl & bl \\
    bl & bl & 0 & 0 & -bl & -bl & 0 & 0 \\
    -d & -d & d & d & -d & -d & d & d
\end{bmatrix}$$

with $P = l\cos(22.5^\circ)$. Herein, the parameters of the octorotor model defined in (62)-(67) are taken from [14].

### 3.2 Model decomposition

The proposed control strategy for the octorotor involves two control loops based upon the time-scale separation principle. The first one is the inner loop that belongs to the faster system dynamics associated with the altitude and rotation motion states, that is,

$$\dot{x}_\text{in} = [p_\zeta \phi \theta \psi p_\zeta p q r]^T$$  \hspace{1cm} (73)

The second one is the outer loop that contains longitude and latitude motion states, that is,

$$\dot{x}_\text{out} = [p_\zeta p_\psi p_\zeta p_\psi]^T$$  \hspace{1cm} (74)

It can be observed from the non-linear model (62)-(67) that the inner-loop system is over-actuated. Therefore, the proposed FTC strategy provides the tolerance in the event of actuator faults/failures. Moreover, a fractional-order controller is used for the outer-loop control such that the desired position tracking control can be achieved. For intuitive illustration, the control structure is depicted in Figure 4.

### 3.2.1 Fault estimation scheme

This subsection investigates a method to estimate the fault/failure information to complement our FTC law designed for inner loop control of non-linear octorotor system. Consider first the linearised model of octorotor system at hovering position i.e. $(\phi, \theta, \psi) \approx 0$ defined as

$$\dot{x}_p(t) = A_p x_p(t) + B_p (1 - K_p(t)) u_p(t)$$  \hspace{1cm} (75)

where $K_p(t) = \text{diag}(k_1(t), \ldots, k_m(t))$ and $k_i(t) = 1 - \gamma_i(t)$. The fault estimator is now designed to estimate the fault term defined as

$$\dot{x}_z(t) = A_p x_z(t) + B_p u_p(t) + B_p \hat{K}_p(t) u_p(t) - L_p \dot{x}_p(t) + \zeta(t)$$  \hspace{1cm} (76)

where $x_z(t)$ is the estimate of states, and $\zeta(t)$ is the discontinuous term defined as

$$\zeta(t) = -\rho_s \frac{\ddot{x}_p(t)}{\|\ddot{x}_p(t)\|} \quad \text{for} \quad \ddot{x}_p(t) \neq 0$$  \hspace{1cm} (77)

where $\rho_s$ is the sliding gain and $\ddot{x}_p(t)$ is the state error defined as

$$\ddot{x}_p(t) = x_p(t) - x_z(t)$$  \hspace{1cm} (78)

and the time derivative is obtained as

$$\dot{\ddot{x}}_p(t) = (A_p + L_p C_p) \ddot{x}_p(t) - B_p \hat{K}_p(t) u_p(t) + \ddot{\zeta}(t)$$  \hspace{1cm} (79)
where \( \tilde{K}_p(t) = K_p(t) - \dot{K}_p(t) \). At sliding mode, \( \tilde{x}_p(t) = 0 \) will be enforced by discontinuous injection term \( \varsigma(t) \). Then during sliding, \( \tilde{x}_p(t) = \tilde{x}_p(t) = 0 \), therefore the fault signal \( \tilde{K}_p(t)u_p(t) \) is reconstructed as [34]

\[
-\tilde{K}_p(t)u_p(t) \approx (B_p^T B_p)^{-1} B_p^T \varsigma(t)
\]

Then the scalar \( k_\varsigma(t) \) is obtained by introducing a small threshold \( \epsilon_p \) such that if a time \( t_{\epsilon_p}, \|u_p(t)\| < \epsilon_p \), then

\[
K_p(t_{\epsilon_p}) = \begin{cases} 
\frac{(B_p^T B_p)^{-1} B_p^T \varsigma(t)}{u_p(t)} & \|u_p(t)\| < \epsilon_p \\
\hat{k}_\varsigma(t_{\epsilon_p}) & \text{otherwise}
\end{cases}
\]

Proof.

\[
V_2(t) = \frac{1}{2} \tilde{x}_p^T(t) \tilde{x}_p(t)
\]

\[
\dot{V}_2(t) = \tilde{x}_p^T(t)(A_p + L_p C_p) \tilde{x}_p(t) - B_p \tilde{K}_p(t)u_p(t) + \varsigma(t)
\]

where choosing the adaptive law (80), the time derivative (83) is written as

\[
\dot{V}_2(t) = \tilde{x}_p^T(t)(A_p + L_p C_p) \tilde{x}_p(t)
\]

where an appropriate choice of \( L_p \) can make \( (A_p + L_p C_p) \) Hurwitz which yields \( \dot{V}_2(t) < 0 \)

3.3 Parameter Settings

The proposed FTC strategy designed in the former section is applied to the non-linear model of the octotor system. To achieve the stable inner-loop tracking, the non-linear model is transformed into the Lipschitz form (14) (assuming \( \hat{\phi}, \hat{\theta}, \hat{\psi} \approx (\phi, \theta, \psi) \) for small variation of attitude angles) defined by

\[
A_p = \begin{bmatrix} 0 & I_4 & 0 \\ I_4 & 0 & 0 \\ 0 & 0 & B_p \end{bmatrix}
\]

where \( B_p = \text{diag}(1/m, 1/I_{xx}, 1/I_{yy}, 1/I_{zz}) \) and the Lipschitz non-linearity is represented by

\[
f(x_{up}, t) = \begin{bmatrix} 0 \cdots 0 \\ -g + (\cos \phi \cos \theta - 1) \frac{1}{m} \\ q \frac{t_p-t_{ex}}{t_{ex}} \\ p \frac{t_p-t_{ex}}{t_{ex}} \\ q \frac{t_p-t_{ex}}{t_{ex}} \end{bmatrix}
\]

and the disturbance term \( D \) is defined as

\[
D = \begin{bmatrix} 0_{d \times 2} \\ d_{d \times 2} \end{bmatrix}
\]

with \( d = \begin{bmatrix} 0 & f/\omega \omega \omega & 0 \\ 0 & 0 & 0 \end{bmatrix}^T \) and \( \xi_p(t, x) = [\phi \Omega \phi \Omega]^T \). The FTC strategy designed in the previous section is applied to the affine non-linear model of the octotor system. It aims to guarantee an accurate tracking performance under the fault/failure condition of the octotor system.

The planned approach uses the efficiency of actuators and provides reconfigured control signals to the faulty system. The effectiveness level of each actuator is assumed to be available by the FDI scheme [12, 32]. Due to the partition of the input distribution matrix (A.1), it conceivably ensures that \( B_{\hat{p}}^T B_{\hat{p}}^T = I \) and \( \|B_{\hat{p}}\| = 1 \). With the choice of \( n = 2.22 \), the LMI given by [12] is solved by using CVX toolbox in MATLAB. The solved feed-back gain \( Y \) is given in Appendix A. By using the infinity norm in (44), \( \eta_2 \) is calculated as \( \eta_2 = 4.34 \). By using the numerical search, the maximum bound on the uncertainty in (41) is \( \eta_1 = 0.21 \). The maximum estimation error that the proposed arrangement is able to bear deprived of losing the stability is \( q_{max} = 15 \). This value of uncertainty satisfies the closed-loop stability condition (47) (i.e., \( 0.91 < 1 \)). Therefore, for any combination of faults/failures, the closed-loop system is asymptotically stable.

On the other hand, to achieve the longitude and latitude controls, the FOPID control strategy is designed in a feed-forward configuration in the outer loop by taking into account the state \( x_{out} \) defined in (74). The output of the FOPID controller is exploited as a command input for the desired roll and pitch angle, and the yaw command is fixed at zero. In this paper, the FOPID controller design is adopted from the previous work done by [41], and the Nelder Mead (NM) optimisation technique is used to calculate the controller parameters. The structure of FOPID controller is given as

\[
\begin{bmatrix} \begin{bmatrix} \kappa_{d} \kappa_{i} \kappa_{d} \mu \end{bmatrix} \begin{bmatrix} \kappa_{c} \kappa_{i} \kappa_{d} \mu \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 105.1 \ 0.1 \ 10 \ 0.52 \ 1 \end{bmatrix} \begin{bmatrix} 161.6 \ 0.01 \ 10 \ 0.48 \ 1 \end{bmatrix} \end{bmatrix}
\]

The outputs of FOPID controllers are then used to obtained the desired roll \( \phi_d \) and pitch command \( \theta_d \) to the inner loop control (i.e., \( \phi_d = \sin^{-1}(n_{\phi} \cos(\psi) - n_{\phi} \sin(\psi)) \) and \( \theta_d = \sin^{-1}(n_{\phi} \cos(\psi) + n_{\phi} \sin(\psi) / \cos(\phi_d)) \).
Remark 9. In order to avoid chattering effect, the sigmoid approximation of non-linear discontinuous function \( S(t) \) will be used in the simulations and is defined as

\[
\text{sign}(S(t)) = \frac{S(t)}{||S(t)||} + \delta_o
\]

where \( \delta_o \) is a small positive scalar.

FTC law for octorotor system is composed of the following steps:

- At first step, the non-linear states are partitioned into inner loop states (73) and outer loop states (74). The inner loop subsystem is an over-actuated system whereas the outer loop subsystem is un-actuated.
- The proposed FTC scheme is applied to the inner loop subsystem where the nominal control law is designed using the LMI approach proposed in (22-23) where the states information are available to the control law.
- The next part is to design ISMC law that satisfies the reachability condition defined in (54). The ISM switching function is designed to implement ISMC law which again depends on the system state. Therefore the state information is fed into the ISMC law block.
- The virtual control law (linear part plus ISMC part) is combined to the input of the control allocation (CA) scheme (27) which requires the fault/failure information from the fault estimation unit. The CA scheme provides the actual control signals to the system. The Figure 1 is labeled for more clarification.
- The information of the actuator effectiveness level is obtained from (81) and update online the fault/failure information to the CA unit.
- The outer loop control law is based on the fractional-order PID control technique given in (88) which provides \( x-y \) position tracking by generating necessary roll and pitch command to the inner loop subsystem. The controller parameters are tuned using the Nelder Mead optimisation technique.

4 | SIMULATION RESULTS

This section provides detailed simulation results subject to faults/failures on the non-linear octorotor system. For the detailed elaboration, two different scenarios are considered; one is a nominal condition while the other is the failure of four rotors. The performance is compared with the existing fixed CA scheme proposed in [16] and the ISMC scheme without the CA component.

4.1 | Fault-free case

In the fault-free mode, eight actuators are working to drive the movement of the octorotor. Two-step signals are chosen as desired reference signals for the controls of the longitude and latitude motions by the FOPID controller, while they are taken as the desired roll and pitch references. Figure 6 shows the accurate tracking results of the longitude and latitude motions, while Figure 5 shows the excellent tracking performance of altitude, roll, pitch and yaw motions. Moreover, Figures 7 and 8 exhibit the rotor speeds of the primary and secondary actuators in the fault-free case, respectively. It can be seen that both primary and secondary actuators are contributing towards desired angular
motion. In the case of a fixed CA scheme and ISMC schemes, the tracking performance is still achieved nominally. The plot of switching function in Figure 9 shows that sliding motion is maintained over the entire system response.

4.2 Failure case

This scenario aims to validate the proposed FTC strategy. In particular, the most challenging situation is considered, where the failure of primary actuators occurs at 20s. The CA unit makes use of the failure information and directs more control signals to the available healthy actuators. Figures 12 and 13 reveal the drop in rotor speed level to zero at 20s, which indicates the total failure. The plot of the actuator effectiveness level is provided in Figure 14 which shows that when the $k_f$ reaches 1, then the associated rotor is subjected to the failure. It can be observed from Figures 10 and 11 that there is no evident alteration in the tracking performance between the fault-free and faulty cases, which indicates that the proposed strategy succeeds in maintaining the tracking performance regardless of the complete failure to four primary rotors. In the case of a fixed CA scheme, the rotor distribution is not occurred however, the presence of sufficient redundancy provides a suitable tracking performance even in failure condition. The plot of ISMC design

**FIGURE 8** Rotor speeds of secondary actuators in fault-free case

**FIGURE 9** Switching function plot in fault free condition

**FIGURE 10** Altitude and attitude motions in failure case

**FIGURE 11** Longitude and latitude motions in failure case

**FIGURE 12** Rotor speeds of primary actuators in failure case

**FIGURE 13** Rotor speeds of secondary actuators in failure case
is not shown in these simulations because it is not capable to handle complete failure. By analysing the tolerant capability of ISMC design, it is capable to handle only one actuator failure due to its robustness property. The plot of the switching function is provided in Figure 15 which clear that sliding motion is maintained.

5 | CONCLUSION

This paper presents a new active FTC strategy for a class of non-linear Lipschitz systems. The proposed strategy adopts the ISMC idea that has the inherent ability to deal with the faults. The integration of the ISMC with the CA scheme covers a wide range of actuator faults/failures, especially for redundant actuators. The proposed control scheme is then implemented on the non-linear octotorotor model. The simulation results demonstrate that the presented FTC approach is capable of achieving the inner-loop control of the non-linear octotorotor system. The simulation results also validate the tolerance capability of the proposed control scheme even in the worst scenario where the complete failure occurs primary actuators and estimation error. In particular, the developed strategy for the octotorotor system can easily be extended to general non-linear Lipschitz systems with sufficient control redundancy. In future work, an adaptive strategy will be incorporated with nonlinear ISMC law. Moreover, a different CA scheme will be proposed that does not require state transformation. Finally the closed loop performance will then be analyzed in the presence of sensor faults and wind gust condition.

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APPENDIX A

\[
B_p = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.63 & 0.17 & 0.21 & 0.1765 & -0.63 & 0.17 & 0.21 & 0.1765 \\
-0.0087 & -0.60 & 0.25 & 0.27 & -0.0087 & -0.60 & 0.25 & 0.27 \\
-0.057 & -0.016 & -0.54 & 0.45 & -0.057 & -0.016 & -0.54 & 0.45 \\
-0.32 & -0.33 & -0.32 & -0.43 & -0.32 & -0.33 & -0.32 & -0.433 \\
0.43 & -0.64 & -0.79 & -1.5 & 2.83 & 0.0002 & -0.0002 & -0.0001 \\
-0.2625 & 0.70 & -0.53 & -0.75 & 0.0001 & 2.23 & 0 & -0.0001 \\
0.37 & 0.34 & 0.03 & 0.14 & 0 & 0 & 1.55 & 0 \\
-0.0013 & -0.04 & -0.53 & 0.39 & 0 & 0 & 0 & 1.1
\end{bmatrix}
\]

\[
Y = \begin{bmatrix}
-0.63 & 0.17 & 0.21 & 0.1765 & -0.63 & 0.17 & 0.21 & 0.1765 \\
-0.0087 & -0.60 & 0.25 & 0.27 & -0.0087 & -0.60 & 0.25 & 0.27 \\
-0.057 & -0.016 & -0.54 & 0.45 & -0.057 & -0.016 & -0.54 & 0.45 \\
-0.32 & -0.33 & -0.32 & -0.43 & -0.32 & -0.33 & -0.32 & -0.433 \\
0.43 & -0.64 & -0.79 & -1.5 & 2.83 & 0.0002 & -0.0002 & -0.0001 \\
-0.2625 & 0.70 & -0.53 & -0.75 & 0.0001 & 2.23 & 0 & -0.0001 \\
0.37 & 0.34 & 0.03 & 0.14 & 0 & 0 & 1.55 & 0 \\
-0.0013 & -0.04 & -0.53 & 0.39 & 0 & 0 & 0 & 1.1
\end{bmatrix}
\]