Frictionless UV-finite Instantons in Curved Spacetime

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ABSTRACT

We identify a new class of UV-complete instanton solutions that describe the false vacuum decay of a real scalar field in a particular curved spacetime background. To this end, we consider a simple scalar theory with a Coleman potential and calculate the Euclidean action $S_E$ by assuming an O(4)-symmetric curved spacetime. The function $a(r)$ dictating the geometry of spacetime may consistently be chosen to be a constant, thereby eliminating the drag forces from the equations of motion and ensuring that the gravitational backgrounds of both the false vacuum and bounce solutions are identical. By employing standard WKB and Gelfand-Yaglom methods, we compute the corresponding prefactor due to quantum fluctuations around this frictionless bounce solution which becomes UV finite after renormalization. The possible consequences of such frictionless UV-finite instantons are discussed.

KEYWORDS: Quantum field theory, Instantons, Vacuum decay

I. INTRODUCTION

In Quantum Field Theory, the mechanism for which phenomena of vacuum decay occur is described by the emergence of instantons. In $D$ dimensions, these are O($D$)-symmetric classical solutions to the equation of motion that possess a finite action. They describe the quantum mechanical process of the tunnelling of a scalar field through a barrier that separates two minima of the potential. Conservation laws favour the tunnelling from the false vacuum to the true vacuum of the theory. Vacuum decay plays an important role in our understanding of the Universe, as such decays contribute to cosmological phase transitions governing the evolution of the Universe [1–15]. Moreover, considerations on the stability of the vacuum provide bounds on the numerical values of the physical constants in particle physics models, including the well-established Standard Model [16–20].

Coleman was first to formulate a successful field theoretic approach to the problem of vacuum decay in flat spacetime [1], by introducing the so-called “thin wall” solution corresponding to a potential defined such that the energy difference between the two minima is small compared to the height of the barrier. Together with Callan [2], he then extended the previous results by computing the quantum corrections corresponding to fluctuations around
the bounce and false vacuum solutions. Three years later, Coleman and De Luccia developed
the mathematical framework for evaluating such decays in curved spacetime [4].

In a previous work [21], we have studied the effects of Goldstone modes on the stability
of the vacuum. We concluded that energy conservation prevents the existence of non-zero
Goldstone modes in flat spacetime. However, Goldstone bosons do play a role in wormhole-
induced vacuum decays in a curved spacetime background, in which case the local ground
state was found to be extremely short-lived. Furthermore, there is an asymmetry in tun-
nelling rates depending on the relative field values between false and true vacua. These
findings have motivated us to compute quantum corrections to such decays in gravitational
backgrounds.

The tunnelling rate for a false vacuum decay is given by [1–12]
\[
\Gamma = A e^{-B},
\]
(I.1)
where \(A\) is a prefactor determined by corrections originating from quantum fluctuations
around the bounce solution, and 
\(B = S_E(\phi_c) - S_E(\phi_{fv})\), where \(\phi_c\) and \(\phi_{fv}\) are the bounce
(classical) and the false vacuum solutions, respectively. A general \(O(4)\) symmetric Euclidean
metric, in spherical coordinates, is given by [4, 20]
\[
ds^2 = dr^2 + a^2(r) d\Omega_3^2.
\]
(I.2)

In the above, \(a(r)\) is a function that depends on 
\(r \equiv \sqrt{\tau^2 + x^2 + y^2 + z^2}\), with \(\tau \equiv -it\), and
describes the geometry of Euclidean spacetime. Several papers have evaluated the prefactor
\(A\) in the flat case limit where the gravitational constant \(\kappa \to 0\) and \(a(r) = r\), as there is
no back-reaction to the metric and the profile of \(a(r)\) is identical in the false vacuum and
classical solutions [26–29].

In this paper we consider a new class of solutions, where the profile of \(a(r)\) is a constant
for all \(r\). The profound consequence of such an extraordinary but very simple choice for the
scaling factor \(a\) will be the elimination of the drag forces from the equations of motion. The
resulting dynamics is analogous to a particle being subject to a conservative potential where
the Hamiltonian \(T_{rr}\) is a constant of motion. Like in the flat case mentioned above, this last
property ensures that there is no back-reaction to the metric and that the profiles of \(a(r)\)
in the false vacuum solution and in the classical solution are the same, namely the same
constant. This particular feature of such solutions eliminates many of the mathematical
complications that arise while calculating the fluctuation determinant.

The layout of this paper will be as follows. In Section [I], we go through the theoretical
and mathematical foundations concerning the problem. First, we compute the quantum
fluctuations and the resultant corrections around the bounce solution. Then, we use the
Gelfand-Yaglom method [30] to compute the emerging fluctuation determinants, where we
go through processes of regularization and renormalisation. In Section [III] we examine the
Coleman potential and derive analytical formulations and approximations. In the same sec-
tion, we then find numerical solutions and discuss the tabulated results. Finally, Section [IV]
summarises our conclusions and discusses the possible consequences that such frictionless
instantons might have.
II. THEORETICAL BACKGROUND

The Lagrangian of a real scalar field $\phi$ minimally coupled to gravity, with a local metric signature $(+,+,+,+)$, is given by

$$L = \sqrt{g} \left( g^{\mu \nu} \frac{1}{2} \partial_\mu \phi \partial_\nu \phi + V(\phi) - \frac{R}{2\kappa} \right). \quad (\text{II.1})$$

The first variation of the action is given by

$$\delta S = \int d^4 x \left[ \frac{\partial L}{\partial g^{\mu \nu}} \delta g^{\mu \nu} + \frac{\partial L}{\partial \phi} \delta \phi + \frac{\partial L}{\partial \partial_\mu \phi} \delta \partial_\mu \phi \right] \quad (\text{II.2})$$

$$= \int d^4 x \left[ \frac{\sqrt{g}}{2\kappa} \left( -R_{\mu \nu} + \frac{1}{2} g_{\mu \nu} R - \kappa T_{\mu \nu} \right) \delta g^{\mu \nu} + \left(-\partial_\mu (\sqrt{g} g^{\mu \nu} \partial_\nu \phi) + \sqrt{g} \frac{\partial V}{\partial \phi} \right) \delta \phi \right],$$

where $T_{\mu \nu} = -\partial_\mu \phi \partial_\nu \phi + g_{\mu \nu} \left[ \frac{1}{2} g^{\sigma \lambda} (\partial_\sigma \phi \partial_\lambda \phi) + V \right] [4, 20]$ is the energy-momentum tensor. To calculate the corresponding quantum corrections to the classical solutions (denoted as $g_c$ and $\phi_c$), we must compute the second variation of the action. Nevertheless, finding UV-finite solutions for dynamical gravitational backgrounds proved to be mathematically difficult, possibly due to the non-renormalizability of quantum gravity.

To avoid the aforementioned difficulty, we explore in this paper the possibility of a static gravitational field with no backreaction. This amounts to considering $\delta g^{\mu \nu} = 0$ and $g_c^{\mu \nu} = g_{\text{fv}}$. With this assumption in mind, the second variation of the action gives

$$\delta^2 S|_{g_c,\phi_c} = \int d^4 x \left[ \frac{\partial^2 L}{\partial \phi \partial \phi} \delta \phi \delta \phi - \partial_\nu \left( \frac{\partial^2 L}{\partial \partial_\nu \phi \partial \partial_\mu \phi} \delta \partial_\mu \phi \right) \right]_{g_c,\phi_c} \quad (\text{II.3})$$

$$= \int d^4 x \left[ \delta \phi \left( g^{\frac{\partial^2 V}{\partial \phi^2}} - \sqrt{g} \partial \Box \right) \delta \phi \right]_{g_c,\phi_c},$$

where $\Box \equiv g^{-1/2} \partial_\mu (\sqrt{g} g^{\mu \nu} \partial_\nu)$ is the d’Alembertian operator. We may now write [2]

$$\phi(x) = \phi_c(x) + \delta \phi(x) \quad (\text{II.4})$$

where $\delta \phi(x) \equiv \sum_{n,l,m} c_n^{l,m} \phi_n^{l,m}(x)$ is a sum of O(4)-symmetric orthonormal functions which vanish at the boundaries. The functions $\phi_n^{l,m}(x)$ may be conveniently rewritten as

$$\phi_n^{l,m}(r,\theta_1,\theta_2,\theta_3) = R_n(r) Y_n^{l,m}(\theta_1,\theta_2,\theta_3). \quad (\text{II.5})$$

In the above, $\phi_n^{l,m}(x)$ is expressed as a product of a radial function $R_n$ and a 4D spherical harmonics $Y_n^{l,m}$, with the quantum numbers taking integer values in the intervals: $0 \leq l \leq n$ and $-l \leq m \leq l$. In particular, we require that $\phi_n^{l,m}(x)$ be orthonormal eigenfunctions of the second variation of the action, viz.

$$\mathcal{O}_c \phi_n^{l,m} \equiv \left[ g^{\frac{\partial^2 V}{\partial \phi^2}} - \sqrt{g} \Box \right]_{\phi_c,g_c} \phi_n^{l,m} = \lambda_n \phi_n^{l,m}, \quad (\text{II.6})$$
by obeying the following properties \[22\]:

\[
\int d^4x \, \phi_n^{lm}(x) \phi_{n'}^{l'm'}(x) = \delta_{nn'} \delta_{ll'} \delta_{mm'}, \quad \lim_{r \to \infty} \phi_n^{lm}(r, \theta_1, \theta_2, \theta_3) = 0 .
\] (II.7)

With the help of these properties, the second variation of the action evaluates to

\[
\delta^2 S|_{g_c, \phi_c} = \int d^4x \sum_{n,n',l,l',m,m'} c_n^{lm} \phi_n^{lm} \lambda_n c_{n'}^{l'm'} \phi_{n'}^{l'm'} = \sum_{n,l,m} \lambda_n (c_n^{lm})^2 .
\] (II.8)

On the other hand, the path integral measure is now given by effectively replacing \(D\phi\) with \(dc_n/\sqrt{2\pi}\). We note that \(\text{det} M = \prod_n \lambda_n\), where \(M\) is a matrix and \(\lambda_n\) are its eigenvalues. Upon accounting for the degeneracy factor \((n+1)^2\) for a given eigenvalue \(\lambda_n\), it is then straightforward to perform the Gaussian integrals that occur in the transition:

\[
\langle \phi_{fv}, r = \infty | \phi_{fv}, r = 0 \rangle = e^{-SE} \prod_n \sqrt{\lambda_n^{-1}} .
\] (II.9)

However, these eigenvalues include zero and negative modes which need special mathematical treatment, as opposed to the trivial Gaussian treatment of the positive modes. In Subsection A we first extract the zero modes from the fluctuation determinant. Then, in Subsection B we evaluate the contributions from the negative modes. Subsequently, we study in Subsection C the Gelfand-Yaglom method \[30\] which will be employed to compute the fluctuation determinant pertinent to a new class of instantons. Finally, we discuss the process of regularisation and renormalisation in Subsection D.

### A. Zero Modes

The eigenfunctions corresponding to zero eigenvalues are four, given by the set: \(\{\phi_n^{lm}\} = \{\phi_0^{00}, \phi_1^{1,-1}, \phi_1^{10}, \phi_1^{11}\}\). These four eigenfunctions may be expressed in terms of \(\nabla_\mu \phi_c\), where the index \(\mu = 1, 2, 3, 4\) runs over the four spacetime Euclidean coordinates that also label individually the four zero modes. In fact, the latter can be easily verified as follows:

\[
\sqrt{g} \left(\frac{\partial^2 V}{\partial \phi_c^2}\right) \nabla_\mu \phi_c - \sqrt{g} \nabla_\mu \phi_c = \sqrt{g} \frac{\partial}{\partial \phi_c} \left(\frac{\partial V}{\partial \phi_c}\right) \nabla_\mu \phi_c - \nabla_\mu \left(\partial_\alpha (\sqrt{g} g^{\alpha\beta} \partial^\beta \phi_c)\right)
\]
\[
= \nabla_\mu \left(\sqrt{g} \frac{\partial V}{\partial \phi_c} - \partial_\alpha (\sqrt{g} g^{\alpha\beta} \partial^\beta \phi_c)\right) = 0 ,
\] (II.10)

where we have used the identities: \([\nabla_\mu, \Box] = 0\) and \(\nabla_\mu g = 0\).

In order to determine the normalisation constants \(C_{(\mu)}\) associated to the four zero modes \(\nabla_\mu \phi_c = \partial_\mu \phi_c\), we first notice that because of the spherical symmetry \(O(4)\) of the bounce, one has

\[
\int d^4x \sqrt{g} \partial_\mu \phi \partial_\nu \phi = \frac{1}{4} \delta_{\mu\nu} \int d^4x \sqrt{g} \partial_\sigma \phi \partial_\sigma \phi .
\] (II.11)
By virtue of the scaling properties of the action under \( x^\mu \rightarrow \alpha x^\mu \) \cite{23, 24}, one gets
\[
0 = \left. \frac{dS_E}{d\alpha} \right|_{\alpha=1} = -2 \int d^4x \frac{1}{2} \sqrt{g} \partial_\mu \phi_c \partial_\mu \phi_c - 4 \int d^4x \sqrt{g} V + 2 \int d^4x \sqrt{g} \frac{R}{2\kappa} . \tag{II.12}
\]
The latter implies
\[
\frac{R}{2\kappa} = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + 2V(\phi) . \tag{II.13}
\]
With the aid of this last relation, we may derive
\[
B = S_E(\phi_c) - S_E(\phi_{fv}) = \frac{1}{4} \int d^4x \sqrt{g} \partial_\mu \phi_c \partial_\mu \phi_c . \tag{II.14}
\]
Consequently, requiring that
\[
1 = C_\mu^2 \int d^4x \sqrt{g} (\partial_\mu \phi_c)^2 = C_\mu^2 B , \tag{II.15}
\]
we easily arrive at \( C_\mu = 1/\sqrt{B} \). In light of this last result, the would-be Gaussian integral over the four path-integral variables \( \{c_1^{lm}\} \), which we collectively denote as \( c_1 \), may be performed as follows:
\[
\int dc_1 = \int \frac{\sqrt{B}}{\partial_\mu \phi} d\phi = \int_{-X^\mu}^{X^\mu} \sqrt{B} dx^\mu = 2X^\mu \sqrt{B} . \tag{II.16}
\]
Since we have four spatial translations, we have four factors of \( C_\mu \) contributing to the tunnelling rate. Therefore, the product contains a factor: \( 2^4 X_1 X^2 X^3 X^4 = VT \), which emanates from the four zero modes.

**B. Negative Modes**

The survival probability for the ground state to remain in the false vacuum \(|\phi_{fv}\rangle\) may be determined as
\[
\lim_{t \rightarrow \infty} \langle \phi_{fv}, t/2|e^{-iHt}|\phi_{fv}, -t/2\rangle|^2 = \lim_{t \rightarrow \infty} e^{2\text{Im}E_0 t} \approx \lim_{t \rightarrow \infty} 1 + 2 \text{Im}E_0 t , \tag{II.17}
\]
where \( E_0 \) is the ground state energy of the false vacuum. Hence, the probability of its decay to the true vacuum \(|\phi_{tv}\rangle\) will be the unitary complement of (II.17), i.e.
\[
\lim_{t \rightarrow \infty} \Gamma t = \lim_{t \rightarrow \infty} \langle \phi_{tv}, t/2|e^{-iHt}|\phi_{tv}, -t/2\rangle|^2 = \lim_{t \rightarrow \infty} \left( -2 \text{Im}E_0 t \right) . \tag{II.18}
\]
Note that by inserting a complete set of energy eigenstates, \(|E_i\rangle\), we can find the relation between the ground state of the false vacuum and the partition function as \cite{25}
\[
\lim_{\tau \rightarrow \infty} Z(\tau) \equiv \lim_{\tau \rightarrow \infty} \int \mathcal{D}\phi \exp \left( -\int_{-\tau/2}^{\tau/2} d\tau' L_E \right) = \lim_{\tau \rightarrow \infty} \langle \phi_{fv}, \tau_f = \tau/2|e^{-H\tau}|\phi_{fv}, \tau_i = -\tau/2\rangle
= \sum_i \lim_{\tau \rightarrow \infty} \langle \phi_{fv}, \tau_f = \tau/2|e^{-H\tau}|E_i\rangle \langle E_i|\phi_{fv}, \tau_i = -\tau/2\rangle = \lim_{\tau \rightarrow \infty} e^{-E_0\tau} \tag{II.19}
\]
As a consequence, we obtain the useful relation
\[
\lim_{\tau \to \infty} \Gamma_{\tau} = \lim_{\tau \to \infty} \left( -2 \text{Im} E_0 \tau \right) = \lim_{\tau \to \infty} 2 \text{Im} \left( \ln Z(\tau) \right). \tag{II.20}
\]

On the other hand, we have
\[
\lim_{\tau \to \infty} \text{Im} \left( \ln Z(\tau) \right) = \lim_{\tau \to \infty} \text{Im} \left( \ln |Z(\tau)| + i \text{arg} Z(\tau) \right) = \lim_{\tau \to \infty} \frac{\text{Im} Z(\tau)}{\text{Re} Z(\tau)}, \tag{II.21}
\]
which is valid in the limit where the real part is much larger than the imaginary part of \( Z \). In the large \( \tau \) limit, the real part of \( Z \) is determined by the constant path \( \phi = \phi_{fv} \) and its fluctuations. In this limit, we therefore have
\[
\text{Re} Z = \det \left[ \sqrt{g} \frac{\partial^2 V}{\partial \phi^2} - \sqrt{g} \Box \right]_{\phi_{fv}, g_{fv}}. \tag{II.22}
\]

To find the imaginary part of the partition function \( Z(\tau) \) in the same large \( \tau \) limit, we should first find the contribution from the negative modes, \( \phi_0 \), associated with the negative eigenvalue \( \lambda_0 \). To this end, let us consider the integral
\[
\int_{-\infty}^{\infty} \frac{dc_0}{\sqrt{2\pi}} e^{-\frac{1}{2} \lambda_0 c_0^2}. \tag{II.23}
\]

Using analytical continuation techniques, we rotate the contour such that \( \lambda_0 c_0^2 > 0 \) which in turn gives rise to the imaginary part,
\[
\text{Im} \left( \int_{-\infty}^{\infty} \frac{dc_0}{\sqrt{2\pi}} e^{-\frac{1}{2} \lambda_0 c_0^2} \right) = \frac{1}{2} \sqrt{\frac{1}{|\lambda_0|}}. \tag{II.24}
\]

Hence, the imaginary part of \( Z \) is given by
\[
\text{Im} Z = e^{-S_E} \frac{S_E^2}{4\pi^2} \left| \det' \left[ \sqrt{g} \frac{\partial^2 V}{\partial \phi^2} - \sqrt{g} \Box \right]_{\phi_{c}, g_{c}} \right|^{-1/2} V T, \tag{II.25}
\]
where the prime denotes the removal of the zero eigenvalues. Note that the use of the absolute value is required in (II.25) to correctly account for the negative mode.

We are now in a position to write down the full expression for the decay probability per unit volume and per unit time as
\[
\Gamma / V = \frac{B^2}{4\pi^2} e^{-B - S_{\text{CT}}} \left| \det' \left[ \sqrt{g} \frac{\partial^2 V}{\partial \phi^2} - \sqrt{g} \Box \right]_{g_{c}, \phi_{c}} \right|^{-1/2} \equiv \frac{B^2}{4\pi^2} e^{-B - S_{\text{CT}}} D^{-1/2}. \tag{II.26}
\]

Note that \( S_{\text{CT}} \) is the action that contains the counter-terms (CTs) of renormalisation. With the help of the latter, we can explicitly write down the form of the prefactor \( A \) that occurs in (I.1) as
\[
A = \frac{B^2}{4\pi^2} e^{-S_{\text{CT}}} D^{-1/2}. \tag{II.27}
\]
C. Gelfand-Yaglom Method

In curved spacetime, the relevant operator resulting from the second variation of the action is given by

$$\mathcal{O} = -a^3 \frac{d^2}{dr^2} - 3a'a^2 \frac{d}{dr} - a \nabla_S + a^3 \frac{\partial^2 V}{\partial \phi^2}. \quad (I1.28)$$

This operator acting on $\phi_{lm}^{jn}$ gives

$$\left(-a^3 \frac{d^2}{dr^2} - 3a'a^2 \frac{d}{dr} + n(n + 2)a + a^3 \frac{\partial^2 V}{\partial \phi^2}\right) R_n = \lambda_n R_n. \quad (I1.29)$$

The spectral zeta function corresponding to an operator like $\mathcal{O}$ is given by \[26–34\]

$$\zeta_\mathcal{O}(s) = \sum_n \lambda_n^{-s}, \quad (I1.30)$$

where $\lambda_n$ is the $n$th non-degenerate eigenvalue of $\mathcal{O}$ with eigenfunctions $R_n$. The boundary conditions are given by $R_n(\infty) = 0$ and $R_n'(\infty) = 1$. We can write the spectral zeta function as

$$\zeta_\mathcal{O}(s) = \frac{1}{2\pi i} \oint d\lambda \lambda^{-s} \frac{d}{d\lambda} \ln R_\lambda(\infty). \quad (I1.31)$$

Since the poles of $\ln R_\lambda(\infty)$ are located at the eigenvalues $\lambda_n$, and since these are non-degenerate, these poles are simple. For $s > -1$, the difference of two zeta functions corresponding to two given operators $\mathcal{O}_1, \mathcal{O}_2$ is given by

$$\zeta_{\mathcal{O}_1}(s) - \zeta_{\mathcal{O}_2}(s) = \left(\frac{e^{i\pi s}}{2\pi i} - \frac{e^{-i\pi s}}{2\pi i}\right) \int_0^\infty d\lambda \lambda^{-s} \frac{d}{d\lambda} \ln \frac{R_{1,\lambda}(\infty)}{R_{2,\lambda}(\infty)}. \quad (I1.32)$$

Differentiating with respect to $s$ and setting $s = 0$ gives

$$\frac{d}{ds} \left(\zeta_{\mathcal{O}_1}(s) - \zeta_{\mathcal{O}_2}(s)\right)\bigg|_{s=0} = -\ln \frac{R_{1,\lambda=0}(\infty)}{R_{2,\lambda=0}(\infty)}. \quad (I1.33)$$

Given that

$$\frac{d}{ds} \zeta_\mathcal{O}(s)\bigg|_{s=0} = -\ln \prod_n \lambda_n = -\ln \det \mathcal{O}, \quad (I1.34)$$

we arrive at the expression

$$\frac{\det(\mathcal{O}_1)}{\det(\mathcal{O}_2)} = \frac{R_{1,\lambda=0}(\infty)}{R_{2,\lambda=0}(\infty)}. \quad (I1.35)$$

Note that $R_n$ is $(n + 1)^2$ degenerate in 4D spacetime. Here, we are interested in the positive eigenvalue contributions ($n \geq 2$). Taking these considerations into account, we compute \[26–29, 31, 34\]

$$\Sigma \equiv \ln \mathcal{D} = \ln \prod_{n=2}^{\infty} \left[\frac{\det(\mathcal{O}_{c,n})}{\det(\mathcal{O}_{IV,n})}\right]^{(n+1)^2} = \sum_{n=2}^{\infty} (n + 1)^2 \ln \frac{R_{1,\lambda=0}(\infty)}{R_{2,\lambda=0}(\infty)} \equiv \Sigma_1(N) + \Sigma_2(N). \quad (I1.36)$$

where $\Sigma_1(N) = \sum_{n=N}^{\infty} (n + 1)^2 \ln \frac{R_{1,\lambda=0}(\infty)}{R_{2,\lambda=0}(\infty)}$ and $\Sigma_2(N) = \sum_{n=N+1}^{\infty} (n + 1)^2 \ln \frac{R_{1,\lambda=0}(\infty)}{R_{2,\lambda=0}(\infty)}$. 


D. Regularisation and Renormalisation

In this paper, we consider a static gravitational background with no back-reaction, i.e.
\[
\partial g^{\mu\nu}/\partial r = 0, \quad a(r) = a_{fv}.
\] (II.37)

Furthermore, we ignore quantum gravity effects on renormalisation of the $\phi^4$ theory. It is useful to define the quantities
\[
U(r) \equiv \frac{\partial^2 V}{\partial \phi^2}(\phi_c(r)) - M,
\] (II.38)
\[
M \equiv \frac{\partial^2 V}{\partial \phi^2}(\phi_{fv}),
\] (II.39)
such that $U(\infty) = 0$. Now, we define the operators $O_{c,n}$ and $O_{fv,n}$ as
\[
O_{c,n} \equiv -\frac{d^2}{dr^2} + \frac{n(n+2)}{a_{fv}^2} + U(r) + M,
\] (II.40)
\[
O_{fv,n} \equiv -\frac{d^2}{dr^2} + \frac{n(n+2)}{a_{fv}^2} + M.
\] (II.41)

Note that these operators are derived from (II.28) after dividing by factors of the constant function $a(r) = a_{fv}$. The quantity of interest $\Sigma$ is an infinite sum which needs to be regularised. So, we introduce a mass regulator $\mu$ and write
\[
\ln \left[ \frac{\det(O_{c,n})}{\det(O_{fv,n})} \right]_{\text{reg}} = -(n+1)^2 \int_0^\infty \frac{ds}{s} (\mu^2 s)^\epsilon \text{Tr} \left[ e^{-sO_{c,n}} - e^{-sO_{fv,n}} \right].
\] (II.42)

For large $n$, say $N$, we may define the parameter
\[
\delta_N^{-2} = N(N+2).
\] (II.43)

Then, by employing standard WKB techniques, we find
\[
\ln \frac{R_{c,\lambda=0}(\infty)}{R_{fv,\lambda=0}(\infty)} = \delta_N \frac{1}{2} \int_0^\infty dr \ a_{fv} U - \delta_N^3 \frac{1}{8} \int_0^\infty \left[ \frac{d}{dr} a_{fv}^3 \right] U(U + 2M) + O(\delta_N^5).
\] (II.44)

We also make use of the following identities:
\[
\sum_{n=2}^{n=N} (n+1) = \frac{1}{2} (N+1)(N+2) - 3, \quad \sum_{n=2}^{n=N} \frac{1}{n+1} \simeq \ln N + \gamma_E - \frac{3}{2},
\] (II.45)

where $\gamma_E$ is the Euler-Mascheroni constant. Combining these results, alongside dimensional regularisation techniques, we obtain the expression
\[
\Sigma_2(N) \simeq -\frac{(N+1)(N+2)}{4} \int_0^\infty \frac{d}{dr} a_{fv} U + \frac{\ln N}{8} \int dr \ a_{fv}^3 U(U + 2M)
- \frac{1}{16} \int dr \ a_{fv}^3 \left( \frac{1}{\epsilon} + 2 - \gamma_E + \ln \frac{r^2}{4} \right) U(U + 2M).
\] (II.46)
FIG. 1: The figure displays a Coleman potential determined by the parameters $\lambda = 0.1, \epsilon = 0.5 \text{ GeV}^4, V_0 = 10 \text{ GeV}^4$ and $\phi_{fv} = 5 \text{ GeV}$. The blue line shows the trajectory of the bounce solution with the blue circle indicating the tunnelling point $\phi_0$. The value of $\epsilon$ is chosen to highlight the difference between the two minima in the plot, for the remainder of this paper we will choose a smaller value given by $\epsilon = 0.001 \text{ GeV}^4$.

We note that this result coincides with the one quoted in the literature [28], once $a_{fv}$ is replaced with $r$.

The divergences corresponding to the terms proportional to $N^2, N$, and $\ln N$ that occur in $\Sigma_2(N)$ will be cancelled against the divergence in the $N$-finite sum $\Sigma_1$. The latter will be computed numerically, as $N \to \infty$. The divergence in the last line of (II.46) will be cancelled identically by the CTs that arise from the renormalisation of the $\phi^4$ theory, as we highlight in the next section.

III. COLEMAN POTENTIAL

To showcase our new class of UV-finite instanton solutions, we consider a theory with a real scalar field. This simple scalar theory is described by a Coleman potential given by [1]

$$V(\phi) = \frac{\lambda}{8} (\phi^2 - \phi_{fv}^2)^2 - \frac{\epsilon}{2\phi_{fv}} (\phi - \phi_{fv}) + V_0 ,$$

(III.1)

with $\lambda, \epsilon > 0$. The term associated with the constant $\epsilon$ represents a constant external source. As can easily be inferred from Figure [I] the potential, $V(\phi)$, exhibits two minima, namely the true vacuum $\phi_{tv}$ and the false vacuum $\phi_{fv}$. The difference in potential energy between the two minima is small compared to the height of the barrier between them. From [II.13],
we obtain
\[ a'^2 = 1 + \frac{\kappa}{3} a^2 \left( \frac{1}{2} \phi'^2 - V(\phi) \right). \quad (\text{III.2}) \]

Adopting the usual boundary conditions \( \phi'_c(0) = 0 \) and \( a'_c(0) = 0 \), combined with (III.2), we obtain the expression
\[ a^2_{fv} = \left( \frac{\kappa}{3} V(\phi_{fv}) \right)^{-1}. \quad (\text{III.3}) \]

The exotic geometry described by the function \( a(r) \) held constant at \( a_{fv} \), i.e. \( a_e(r) = a_{fv} \) for all \( r \), seems to represent the geometry of a surface of a wormhole in 4D Euclidean spacetime. Similar exotic geometries have been studied in the literature [35–37]. It is important to remark here that the classical solution \( \phi_c(r) \) to the equations of motion is described by the motion of a particle subject to a conservative potential, i.e.
\[ \phi''_c - \frac{\partial V}{\partial \phi} = 0, \quad (\text{III.4}) \]
as the would-be drag forces vanish when \( a'(r) = 0 \). The boundary conditions of \( \phi_c(r) \) are given by \( \phi'_c(0) = 0, \ \phi_c(\infty) = \phi_{fv} \). The profile of the bounce solution is shown in Fig. 2 where the particle starts sliding near the true vacuum \( \phi_{tv} \) and asymptotes towards the false vacuum \( \phi_{fv} \). We can easily deduce that \( \phi_c(r = 0) \) satisfies \( V(\phi_c(r = 0)) = V(\phi_{fv}) \). In this way, the expression for the quantity \( B \) takes on the equivalent form,
\[ B = S_E(\phi_c) - S_E(\phi_{fv}) = -2\pi^2 a^3_{fv} \int_0^\infty dr \left( V(\phi_c(r)) - V(\phi_{fv}) \right). \quad (\text{III.5}) \]

Let us now move on to compute the prefactor \( A \). To this end, we use the Gelfand-Yaglom method in order to evaluate the determinant,
\[ \det \left( \frac{\partial^2 L}{\partial \phi \partial \phi} \right) = \det \left( -\sqrt{g} \Box - \frac{3}{2} \sqrt{g} \lambda \phi^2_{fv} - \frac{1}{2} \sqrt{g} \lambda \phi^2 \right). \quad (\text{III.6}) \]

To do so, we solve
\[ \left( -\frac{d^2}{dr^2} + \frac{n(n+2)}{a^2_{fv}} + M \right) R_{n,fv} = 0, \quad (\text{III.7}) \]
\[ \left( -\frac{d^2}{dr^2} + \frac{n(n+2)}{a^2_{fv}} + U(r) + M \right) R_n = 0, \quad (\text{III.8}) \]
where \( M = \lambda \phi^2_{fv} \), and \( U(r) \) is given by
\[ U(r) = \frac{3}{2} \lambda \phi^2_c(r) - \frac{1}{2} \lambda \phi^2_{fv} - M. \quad (\text{III.9}) \]

By regularity, the boundary conditions are now given by \( R(0) = 0 \) and \( R'(0) = 1 \). The new set of boundary conditions is a consequence of the Gelfand-Yaglom treatment of the
FIG. 2: The bounce solution $\phi(r)$ for a particle undergoing a Coleman potential parametrised $\lambda = 0.1, \epsilon = 0.001 \text{ GeV}^4, V_0 = 10 \text{ GeV}^4$ and $\phi_{fv} = 5 \text{ GeV}$.

problem as we set all eigenvalues to zero. The first of these differential equations can be solved analytically and the solution is given by

$$R_{n,fv}(r) = \frac{\sinh (k_n r)}{k_n}, \quad (\text{III.10})$$

where

$$k_n = \sqrt{\frac{n(n+2)}{a_{fv}^2} + M}. \quad (\text{III.11})$$

Evidently, the obtained solution is exponentially growing with $r$. But this does now cause a problem, since we are only interested in the ratio: $R_n(\infty)/R_{n,fv}(\infty)$. Thus, we define the quantity

$$T_n(r) \equiv \frac{R_n(r)}{R_{n,fv}(r)}, \quad (\text{III.12})$$

which enables one to recast the two differential equations, (III.7) and (III.8), into the single equation,

$$-T''_n - 2k_n \coth (k_n r) T'_n + U(r) T_n = 0. \quad (\text{III.13})$$

Again, by virtue of regularity, we have the boundary conditions $T(0) = 1, T'(0) = 0$. At the one-loop level, the $\phi^4$ theory under study may be renormalised by means of the following
two CTs:

\[ \delta \lambda = \frac{9 \lambda}{32 \pi^2} \left( \frac{1}{\epsilon} - \gamma_E \right), \quad (\text{III.14}) \]

\[ \delta (\lambda \phi^2) = \frac{3 \lambda^2 \phi^2}{32 \pi^2} \left( \frac{1}{\epsilon} - \gamma_E \right). \quad (\text{III.15}) \]

These CTs give rise to the following action \cite{26, 38}:

\[ S_{\text{CT}} = -2 \pi^2 \int_0^\infty dr a^3_{\text{iv}} \left[ \frac{\delta \lambda}{8} (\phi^2 - \phi^2_{\text{iv}})^2 - \frac{\lambda \delta \phi^2_{\text{iv}}}{4} (\phi^2 - \phi^2_{\text{iv}}) \right] \]

\[ = -\frac{1}{16} \int_0^\infty dr a^3_{\text{iv}} \left( \frac{1}{\epsilon} - \gamma_E \right) U(U + 2M). \quad (\text{III.16}) \]

Adopting the \( \overline{\text{MS}} \) scheme and combining with the regularised expression of the zeta function given in (II.36), we may renormalise \( \Sigma \) to become a UV-finite expression as

\[ \Sigma_{\text{fin}} \equiv S_{\text{CT}} + \Sigma = \lim_{N \to \infty} \Sigma_1(N) - \frac{(N+1)(N+2)}{4} \int_0^\infty dr a^3_{\text{iv}} U + \frac{\ln N}{8} \int_0^\infty dr a^3_{\text{iv}} U(U + 2M) \]

\[ - \frac{1}{16} \int_0^\infty dr a^3_{\text{iv}} \left( 2 + \ln \frac{r^2}{4} \right) U(U + 2M). \quad (\text{III.17}) \]

Figure 3 highlights how the asymptotic behavior of \( T_n(r) \) depends on \( n \). For lower values of \( n \), the ratio is very small, approaching zero. However, for very large values of \( n \), the ratio asymptotes 1. Figure 4 shows the logarithm of the asymptotes of these ratios, \( \ln T_n(\infty) \),
FIG. 4: The figure displays the behaviour of $\ln(T_n(r))$ as $r \to \infty$, it asymptotes to 0 as the value of $n$ grows indicating convergence of the results.

$$B = \frac{\pi^2 \phi_{fv}^{12}}{6\epsilon^3 \lambda^4}.$$  \hspace{1cm} \text{(III.18)}$

For the parameters chosen in our analysis, $\lambda = 0.1, \epsilon = 0.001$ GeV$^4$, and $\phi_{fv} = 5$ GeV, we get the value $B = 4.016 \times 10^{21}$. 

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$V_0$ [GeV$^4$] & $a_{fv}$ [GeV$^{-1}$] & $B$ & $\Sigma_{\text{fin}}$ \\
\hline
10 & $1.33 \times 10^{18}$ & $-6.173 \times 10^{60}$ & $-6.035 \times 10^{57}$ \\
10$^2$ & $4.21 \times 10^{17}$ & $-1.952 \times 10^{59}$ & $-1.211 \times 10^{55}$ \\
10$^5$ & $1.33 \times 10^{16}$ & $-6.173 \times 10^{54}$ & $-3.906 \times 10^{50}$ \\
10$^{10}$ & $4.21 \times 10^{13}$ & $-1.952 \times 10^{47}$ & $-1.101 \times 10^{43}$ \\
\hline
\end{tabular}
\caption{Numerical estimates of the Euclidean action $B$ and the corresponding corrections $\Sigma$ for different input values of $V_0, a_{fv}$, for a Coleman potential fixed by the values $\lambda = 0.1, \epsilon = 0.001$ GeV$^4$ and $\phi_{fv} = 5$ GeV.}
\end{table}

for a range of values for $n$. As expected, we find that the logarithm asymptotes zero for sufficiently large values of $n$. In Table I we give a few representative results that show how the Euclidean Action $B$ scales with $a_{fv}$, which in turn scales with the value of $V_0$ as $V_0 \propto a_{fv}^{-2} \propto B^{-6}$. In all numerical evaluations, we notice that the corrections $\Sigma$ fall short by 3 to 4 orders of magnitude in comparison with the Euclidean action $B$.

The theory predicts a very short lived vacuum compared to the values we get from calculating the Euclidean action in flat spacetime. To illustrate this, we quote the expression from the literature:

\begin{align*}
B &= \frac{\pi^2 \phi_{fv}^{12}}{6\epsilon^3 \lambda^4}.
\end{align*}  \hspace{1cm} \text{(III.18)}$

For the parameters chosen in our analysis, $\lambda = 0.1, \epsilon = 0.001$ GeV$^4$, and $\phi_{fv} = 5$ GeV, we get the value $B = 4.016 \times 10^{21}$. 

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For the parameters chosen in our analysis, $\lambda = 0.1, \epsilon = 0.001$ GeV$^4$, and $\phi_{fv} = 5$ GeV, we get the value $B = 4.016 \times 10^{21}$. 

\begin{align*}
B &= \frac{\pi^2 \phi_{fv}^{12}}{6\epsilon^3 \lambda^4}.
\end{align*}  \hspace{1cm} \text{(III.18)}$
The situation does not change noticeably, when a curved spacetime described by a dynamical gravitational background \((g'(r) \neq 0)\) is considered. In this case, the expression for the quantity \(B\) is given by

\[ B = \frac{\pi^2 \phi_{12}^{12}}{6\epsilon^3 \lambda^4 [1 - (\phi_{12}^3/2\epsilon \sqrt{\Lambda})^2]^2}, \]

where \(\Lambda = (\kappa \epsilon/3)^{-1/2}\). This expression gives roughly the same value as the flat case since gravitational effects are negligible for the range of values chosen for the parameters.

The large gap between the predicted values for the quantity \(B\) governing the lifetime of the vacuum between the ordinary flat spacetime decay given in (III.18) and the new class of solutions given in Table I is due to the elimination of drag forces from the equations of motion by setting \(a'(r) = 0\). The latter ensures that there are no positive contributions to the Euclidean action since the potential of the bounce solution stays positive, i.e. \(V(\phi_c(r)) > 0\), for all \(r\). This is contrary to the bounce solution in the flat-metric case where the instanton quasi-particle spends large amounts of “time” \(r\) near the the true vacuum giving a large positive contribution to \(S_E\).

\[ \text{IV. DISCUSSION} \]

We have found a new class of instanton solutions describing the decay of the vacuum in a gravitational background. These solutions are characterised by a constant value for the function \(a(r)\) which describes the geometry of spacetime. This exotic geometry seems to resemble the surface of a wormhole with a fixed size \(a_{ev}\) in 4D Euclidean spacetime.

Choosing such a metric configuration ensures that there is no backreaction to the metric, resulting in an identical metric for both the false vacuum and the bounce solution. This helps to avoid many of the mathematical complications that arise from having two different metric configurations for the two solutions, leading to relatively simple expressions for the quantum corrections to the Euclidean action and to instanton solutions that are UV complete. To the best of our knowledge, this is the first possible UV-finite computation of the fluctuation determinant \(A\) given in (II.27) on a curved, albeit exotic, gravitational background.

The results predict an extremely short lived vacuum when compared to the longer lived ground state that occurs in the standard flat spacetime geometry. This is due to the elimination of drag forces from the equations of motion by choosing a constant value for \(a(r)\), which ensures that the potential of the bounce solution is always positive, that is \(V(\phi_c(r)) > 0\), for all \(r\). This means that there are no positive contributions to the Euclidean action \(S_E\).

As exhibited in Table I, the quantum corrections \(\Sigma_{\text{fin}}\) are found to reduce the tree-level quantity \(B\), given in (II.14), by 3 to 4 orders of magnitude. However, these quantum-corrected predictions do not alter the generic feature of this new frictionless instanton solution which describes an extremely short-lived false vacuum.

The mechanism by which the vacuum decay occurs is through the nucleation of ever-growing bubbles, where the interior of these bubbles resides in the true vacuum. Such explosive decays may manifest themselves in the form of gravitational waves which might be detectable using...
laser interferometers that are planned to be launched into space in the early 2030s under the LISA project [39].

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