SOME RESULTS ON ALMOST L-WEAKLY AND ALMOST M-WEAKLY COMPACT OPERATORS

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Abstract. In this paper, we present some necessary and sufficient conditions for semi-compact operators being almost L-weakly compact (resp. almost M-weakly compact) and the converse. Mainly, we prove that if X is a nonzero Banach space, then every semi-compact operator \( T : X \to E \) is almost L-weakly compact if and only if the norm of \( E \) is order continuous. And every positive semi-compact operator \( T : E \to F \) is almost M-weakly compact if and only if the norm of \( E' \) is order continuous. Moreover, we investigate the relationships between almost L-weakly compact operators and Dunford-Pettis (resp. almost Dunford-Pettis) operators.

1. INTRODUCTION

Throughout this paper, \( X \) and \( Y \) will denote real Banach spaces, \( E \) and \( F \) will denote real Banach lattices. \( B_X \) (resp. \( B_E \)) is the closed unit of Banach space \( X \) (resp. Banach lattice \( E \)) and \( \text{Sol}(A) \) denotes the solid hull of a subset \( A \) of a Banach lattice.

Recall that a continuous operator \( T : X \to E \) from a Banach space to a Banach lattice is said \textit{semi-compact} if and only if for each \( \varepsilon > 0 \) there exists some \( u \in E_+ \) such that \( T(B_X) \subset [-u, u] + \varepsilon B_E \). In recent years, K. Bouras et al. [4] introduced two classes of operators of almost L-weakly and almost M-weakly compact. Recall that an operator \( T \) from a Banach space \( X \) into a Banach lattice \( F \) is called \textit{almost L-weakly compact} if \( T \) carries relatively weakly compact subsets of \( X \) onto L-weakly compact subsets of \( F \). An operator \( T \) from a Banach lattice \( E \) into a Banach space \( Y \) is called \textit{almost M-weakly compact} if for every disjoint sequence \( (x_n) \) in \( B_E \) and every weakly convergent sequence \( (f_n) \) of \( Y' \), we have \( f_n(T(x_n)) \to 0 \).

They proved in [4] that an operator \( T \) from a Banach space \( X \) into a Banach lattice \( F \) is almost L-weakly compact if and only if \( f_n(T(x_n)) \to 0 \).
0 for every weakly convergent sequence \((x_n)\) of \(X\) and every disjoint sequence \((f_n)\) of \(B_{F'}\) ([4, Theorem 2.2]). After that, A. Elbour et al. [6] gave a useful characterization of almost L-weakly compact operator. An operator \(T\) from a Banach space \(X\) into a Banach lattice \(F\) is almost L-weakly compact if and only if \(T(X) \subseteq F^a\) and \(f_n(T(x_n)) \to 0\) for every weakly null sequence \((x_n)\) of \(X\) and every disjoint sequence \((f_n)\) of \(B_{F'}\) ([6, Proposition 1]).

Recall that a norm \(\|\cdot\|\) of a Banach lattice \(E\) is order continuous if for each net \((x_\alpha)\) in \(E\) with \(x_\alpha \downarrow 0\), one has \(\|x_\alpha\| \downarrow 0\). It is easy to see that if \(E\) has an order continuous norm, \(E = E^a\). A Banach lattice is said to have weakly sequentially continuous lattice operations whenever \(x_n \stackrel{w}{\to} 0\) implies \(|x_n| \stackrel{w}{\to} 0\). Every AM-space has this property. A Banach space is said to have the Schur property whenever every weakly null sequence is norm null, i.e., whenever \(x_n \stackrel{w}{\to} 0\) implies \(\|x_n\| \to 0\). A Banach space is said to have the positive Schur property whenever every disjoint weakly null sequence is norm null. In [4], it was proved that the identity operator \(Id_E\) is almost L-weakly compact if and only if \(E\) has the positive Schur property ([4, Proposition 2.2]). And the identity operator \(Id_E\) is almost M-weakly compact if and only if \(E'\) has the positive Schur property ([4, Corollary 2.1]).

Following from these conclusions, it is easy to see that there exist operators which are semi-compact but not almost L-weakly compact or almost M-weakly compact. And there also exist operators which are almost L-weakly compact (resp. almost M-weakly compact) but not semi-compact.

In this paper, we establish some necessary and sufficient conditions for semi-compact operator being almost L-weakly compact (resp. almost M-weakly compact) and the converse. More precisely, we prove that every semi-compact operator \(T\) from a nonzero Banach space \(X\) to a Banach lattice \(E\) is almost L-weakly compact if and only if the norm of \(E\) is order continuous (Theorem 2.1). And every positive semi-compact operator from a Banach lattice \(E\) into a nonzero Banach lattice \(F\) is almost M-weakly compact if and only if the norm of \(E'\) is order continuous (Theorem 2.2). We also investigate the conditions under which each almost L-weakly compact operator is semi-compact (Theorems 2.3, 2.5). Moreover, we show each positive almost Dunford-Pettis operator \(T : E \to F\) is almost L-weakly compact if and only if \(F\) has an order continuous norm (Proposition 2.8).

All operators in this paper are assumed to be continuous. We refer to [1,7] for all unexplained terminology and standard facts on vector
2. MAIN RESULTS

There exist operators which are semi-compact but not almost L-weakly compact. For example, the identity operator $Id_c : c \to c$ is semi-compact since $c$ is an AM-space with unit. But it is not almost L-weakly compact since $c$ doesn’t have the positive Schur property.

The following Theorem gives a necessary and sufficient condition under which every semi-compact operator is almost L-weakly compact.

**Theorem 2.1.** Let $X$ be a nonzero Banach space and $E$ be a Banach lattice. Then the following statements are equivalent:

1. Every semi-compact operator $T : X \to E$ is almost L-weakly compact;
2. The norm of $E$ is order continuous.

**Proof.** (2) $\Rightarrow$ (1) If the norm of $E$ is order continuous, then by Corollary 3.6.14 of [7], semi-compact operator $T : X \to E$ is L-weakly compact. It is obvious that every L-weakly compact operator is almost L-weakly compact. Hence $T$ is almost L-weakly compact.

(1) $\Rightarrow$ (2) Assume by way of contradiction that the norm of $E$ is not order continuous, we need to construct an operator which is semi-compact but not almost L-weakly compact.

Since the norm of $E$ is not order continuous, by Theorem 4.14 of [1], there exists a vector $y \in E_+$ and a disjoint sequence $(y_n) \subset [-y, y]$ such that $\|y_n\| \nrightarrow 0$. On the other hand, as $X$ is nonzero, we may fix $u \in X$ and pick a $\phi \in X'$ such that $\phi(u) = \|u\| = 1$ holds.

Now, we consider operator $T : X \to E$ defined by

$$T(x) = \phi(x) \cdot y$$

for each $x \in X$. Obviously, $T$ is semi-compact as it is compact (its rank is one). But it is not an almost L-weakly compact operator. If not, as the singleton $\{u\}$ is a weakly compact subset of $X$, and $T(u) = \phi(u) \cdot y = y$, the singleton $\{y\}$ is an L-weakly compact subset of $E$. Since disjoint sequence $(y_n) \subset sol(\{y\})$, we have $\|y_n\| \to 0$, which is a contradiction. $\square$

There exist operators which are semi-compact but not almost M-weakly compact. For example, the operator $T : \ell_1 \to \ell_\infty$ defined by

$$T(\lambda_n) = \left( \sum_{n=1}^{\infty} \lambda_n \right) \cdot e$$
for each \((\lambda_n) \in \ell_1\), where \(e = (1, 1, \ldots)\) is the constant sequence with value 1 [1, p. 322]. Obviously, \(T\) is semi-compact as it is compact (its rank is one). But based on the argument in [6, p. 3], we know that \(T\) is not an almost M-weakly compact operator.

The following Theorem gives a necessary and sufficient condition under which every semi-compact operator is almost M-weakly compact.

**Theorem 2.2.** Let \(E\) and \(F\) be two nonzero Banach lattices. Then the following statements are equivalent:

1. Every positive semi-compact operator \(T : E \to F\) is almost M-weakly compact;
2. The norm of \(E'\) is order continuous.

**Proof.** (2) \(\Rightarrow\) (1) Since positive operator \(T : E \to F\) is semi-compact, following from Corollary 3.3 of [2], \(T' : F' \to E'\) is an almost Dunford-Pettis operator. As the norm of \(E'\) is order continuous, by Proposition 2.4 of [4], \(T'\) is an almost L-weakly compact operator. Following from Theorem 2.5(1) of [4], \(T\) is almost M-weakly compact.

(1) \(\Rightarrow\) (2) Assume by way of contradiction that the norm of \(E'\) is not order continuous, we need to construct a positive operator which is semi-compact but not almost M-weakly compact.

Since the norm of \(E'\) is not order continuous, by Theorem 4.14 of [1], there exists a vector \(\phi \in E'_+\) and a disjoint sequence \((\phi_n) \subset [-\phi, \phi]\) such that \(\|\phi_n\| \to 0\). On the other hand, as \(F\) is nonzero, we may fix \(y \in F_+\) and pick a vector \(g \in (F')_+\) such that \(g(y) = \|y\| = 1\) holds.

Now, we consider operator \(T : E \to F\) defined by

\[T(x) = \phi(x) \cdot y,\]

for each \(x \in E\). Obviously, \(T\) is positive and semi-compact operator as it is compact (its rank is one). But it is not an almost M-weakly compact operator. In fact, by Theorem 2.5(1) of [4], we only need to show that its adjoint \(T' : F' \to E'\) defined by

\[T'(f) = f(y) \cdot \phi\]

for any \(f \in F'\) is almost L-weakly compact. If not, as the singleton \(\{g\}\) is a weakly compact subset of \(X'\), and \(T'(g) = g(y) \cdot \phi = \phi\), the singleton \(\{\phi\}\) is an L-weakly compact subset of \(E'\). Since disjoint sequence \((\phi_n) \subset sol(\{\phi\})\), we have \(\|\phi_n\| \to 0\), which is a contradiction.

There also exist operators which are almost L-weakly compact but not semi-compact. For instance, the identity operator \(Id_{\ell_1} : \ell_1 \to \ell_1\) is almost L-weakly compact since \(\ell_1\) has the positive Schur property. But
it is not semi-compact. If not, as \( \ell_1 \) is discrete with order continuous norm, \( Id_{\ell_1} \) is compact, which is impossible.

Next, denote \( T : E \to F \) as a continuous operator, we investigate the conditions under which each almost L-weakly compact operator \( T \) is semi-compact.

Based on Theorem 4 of [6], we know that if \( E' \) has an order continuous norm, then each positive almost L-weakly compact operator \( T \) is M-weakly compact, hence semi-compact. Now, we claim that if \( E \) is reflexive then each almost L-weakly compact operator \( T \) is semi-compact. In fact, if \( E \) is reflexive, then \( B_E \) is a relatively weakly compact subset of \( E \). As \( T \) is almost L-weakly compact, \( T(B_E) \) is an L-weakly compact subset of \( F \). By Proposition 3.6.2 of [7], for every \( \varepsilon > 0 \), there exists a vector \( u \in F^+ \subset F^+ \) such that \( \phi(u_n) > \varepsilon \) for all \( n \).

The following Theorem gives the conditions under which each positive almost L-weakly compact operator \( T \) from \( E \) to \( E \) is semi-compact.

**Theorem 2.3.** Let \( E \) be a Banach lattice with an order continuous norm. Then the following assertions are equivalent:

1. Each positive almost L-weakly compact operator \( T \) from \( E \) to \( E \) is semi-compact.
2. The norm of \( E' \) is order continuous.

**Proof.** (2) \( \Rightarrow \) (1) Follows from Theorem 4 of [6].

(1) \( \Rightarrow \) (2) Assume by way of contradiction that the norm of \( E' \) is not order continuous. To finish the proof, we need to construct a positive almost L-weakly compact operator \( T : E \to E \) which is not semi-compact.

Since the norm of \( E' \) is not order continuous, it follows from Theorem 116.1 of [8] that there exists a norm bounded disjoint sequence \( (u_n) \) of positive elements in \( E \) which does not weakly converge to zero. Without loss of generality, we may assume that \( \|u_n\| \leq 1 \) for any \( n \). And there exist \( \varepsilon > 0 \) and \( 0 \leq \phi \in E' \) such that \( \phi(u_n) > \varepsilon \) for all \( n \).

Then by Theorem 116.3 of [8], we know that the components \( \phi_n \) of \( \phi \) in the carriers \( C_{u_n} \) form an order bounded disjoint sequence in \( (E')_+ \) such that

\[
\phi_n(u_n) = \phi(u_n) \quad \text{for all } n \quad \text{and} \quad \phi_n(u_m) = 0 \quad \text{if} \quad n \neq m. \tag{*}
\]

Define the positive operator \( S_1 : E \to \ell_1 \) as follows:

\[
S_1(x) = \left( \frac{\phi_n(x)}{\phi(u_n)} \right)_{n=1}^\infty
\]

for all \( x \in E \). Since
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\[ \sum_{n=1}^{\infty} \frac{\phi_n(x)}{\phi(u_n)} \leq \frac{1}{\varepsilon} \sum_{n=1}^{\infty} \phi_n(|x|) \leq \frac{1}{\varepsilon} \phi(|x|) \]

holds for all \( x \in E \), the operator \( S_1 \) is well defined and it is also easy to see that \( S_1 \) is a positive operator.

Now define the operator \( S_2 : \ell_1 \to E \) as follows:

\[ S_2(\lambda_n) = \sum_{n=1}^{\infty} \lambda_n u_n \]

for all \( (\lambda_n) \in \ell_1 \). As \( \sum_{n=1}^{\infty} \|\lambda_n u_n\| \leq \sum_{n=1}^{\infty} |\lambda_n| < \infty \), \( S_2 \) is well defined and is positive.

Next, we consider the composed operator \( T = S_2 \circ S_1 : E \to \ell_1 \to E \) defined by

\[ T(x) = \sum_{n=1}^{\infty} \frac{\phi_n(x)}{\phi(u_n)} u_n \]

for all \( x \in E \). Now we claim \( T \) is an almost L-weakly compact operator. Since \( E \) has an order continuous norm, \( E = E^a \). It suffices to show that \( T \) satisfies the condition (b) of Proposition 1 of [6]. Let \( x_n \overset{w}{\to} 0 \) in \( E \) and \( (f_n) \) be a disjoint sequence in \( B_{E'} \). It is obvious that \( S_1(x_n) \) is a weakly null sequence in \( \ell_1 \). As \( \ell_1 \) has the Schur property, \( \|S_1 x_n\| \to 0 \). Hence \( \|T(x_n)\| = \|S_2(S_1(x_n))\| \to 0 \). Now following from the inequality

\[ |f_n(T(x_n))| \leq \|f_n\| \|T(x_n)\| \leq \|Tx_n\|, \]

we obtain that \( T \) is an almost L-weakly compact operator.

But \( T \) is not a semi-compact operator. In fact, note that \( \|u_n\| \leq 1 \) and \( T(u_n) = u_n \) for all \( n \) following from (\( \ast \)). So, if \( T \) is semi-compact, then \( T(B_E) \) is almost order bounded in \( E \). Hence, \( (u_n) \subset T(B_E) \) is also almost order bounded. So, \( u_n \to 0 \) in \( \sigma(E,E') \), which is a contradiction. \( \square \)

To investigate the necessary and sufficient conditions under which each almost L-weakly compact operator \( T : E \to F \) is semi-compact, we first give the following useful Lemma.

**Lemma 2.4.** Let \( E \) be a Banach lattice with an order continuous norm. If \( (u_n) \) is a norm bounded disjoint sequence of \( E \) such that the set \( \{u_n\} \) is almost order bounded in \( E \), then \( (u_n) \) converges to zero in norm.

**Proof.** Since \( A := \{u_n : n \in \mathcal{N}\} \) is almost order bounded, there exists some \( u \in E_+ \) such that

\[ \|(|u_n| - u)^+\| \leq \varepsilon \text{ for all } n. \]
On the other hand, since \((|u_n| \land u)\) is an order bounded disjoint sequence in \(E\) and the norm of \(E\) is order continuous, following from Theorem 4.14 of [1], \((|u_n| \land u)\) converges to zero in norm. Hence, there exists some \(n_0\) such that
\[
\| |u_n| \land u \| \leq \varepsilon \quad \text{for all} \quad n \geq n_0.
\]
Now, following from the equality \(|u_n| = (|u_n| - u)^+ + |u_n| \land u\), we obtain that
\[
\|u_n\| \leq \|(|u_n| - u)^+\| + \| |u_n| \land u \| \leq 2\varepsilon
\]
holds for all \(n \geq n_0\). So, \(u_n \to 0\) in norm. \(\Box\)

**Theorem 2.5.** Let \(E\) and \(F\) be two Banach lattices such that the norm of \(F\) is order continuous. Then the following assertions are equivalent:

1. Each positive almost L-weakly compact operator \(T : E \to F\) is semi-compact;

2. One of the following conditions is valid:
   a. The norm of \(E'\) is order continuous;
   b. \(E\) or \(F\) is finite dimensional.

**Proof.** (1) \(\Rightarrow\) (2) Assume \(E\) and \(F\) are both infinite dimensional. We have to show that the norm of \(E'\) is order continuous. If not, to finish the proof, we need to construct a positive almost L-weakly compact operator \(T : E \to F\) which is not semi-compact.

Since the norm of \(E'\) is not order continuous, similarly with the proof of Theorem 2.3, we define the positive operator \(S_1 : E \to \ell_1\) as follows:
\[
S_1(x) = \left( \frac{\phi_n(x)}{\phi(u_n)} \right)_{n=1}^{\infty}
\]
for all \(x \in E\). And the operator \(S_1\) is well defined.

On the other hand, since \(F\) is infinite dimensional, by Lemma 2.3 of [3], there exists a disjoint sequence \((y_n) \subset (B_F)_+\) such that \(\|y_n\| = 1\).

Now define the operator \(S_3 : \ell_1 \to F\) as follows:
\[
S_3(\lambda_n) = \sum_{n=1}^{\infty} \lambda_n y_n
\]
for all \((\lambda_n) \in \ell_1\). As \(\sum_{n=1}^{\infty} \|\lambda_n y_n\| \leq \sum_{n=1}^{\infty} |\lambda_n| < \infty\), \(S_3\) is well defined and is positive.

Next, we consider the composed operator \(T = S_3 \circ S_1 : E \to \ell_1 \to F\) defined by
\[
T(x) = \sum_{n=1}^{\infty} \frac{\phi_n(x)}{\phi(u_n)} y_n
\]
for all \( x \in E \). Now we claim \( T \) is an almost \( L \)-weakly compact operator. Since \( F \) has an order continuous norm, \( F = F^\alpha \). It suffices to show that \( T \) satisfies the condition (b) of Proposition 1 of [6]. Let \( x_n \underset{w}{\to} 0 \) in \( E \) and \( (f_n) \) be a disjoint sequence in \( B_{F'} \). It is obvious that \( S_1(x_n) \) is a weakly null sequence in \( \ell_1 \). As \( \ell_1 \) has the Schur property, \( \| S_1 x_n \| \to 0 \). Hence \( \| T(x_n) \| = \| S_3(S_1(x_n)) \| \to 0 \). Now following from the inequality

\[
| f_n(T(x_n)) | \leq \| f_n \| \| T(x_n) \| \leq \| T x_n \|,
\]

we obtain that \( T \) is an almost \( L \)-weakly compact operator.

But \( T \) is not a semi-compact operator. In fact, note that \( \| u_n \| \leq 1 \) and \( T(u_n) = y_n \) for all \( n \) following from \((*)\). So, if \( T \) is semi-compact, then \( T(B_E) \) is almost order bounded in \( F \). Hence, \( (y_n) \subseteq T(B_E) \) is almost order bounded. By Lemma 2.4, \( y_n \to 0 \) in norm, which is a contradiction.

(2a) \( \Rightarrow \) (1) Follows from Theorem 4 of [6].

(2b) \( \Rightarrow \) (1) Let \( T : E \to F \) be a positive operator. If \( E \) is finite dimensional, \( T \) is \( M \)-weakly compact. Also, if \( F \) is finite dimensional, \( T \) is \( L \)-weakly compact. Hence, \( T \) is semi-compact. \( \Box \)

There exist operators which are almost \( M \)-weakly compact but not semi-compact. For instance, the identity operator \( Id_{c_0} : c_0 \to c_0 \) is almost \( M \)-weakly compact since \((c_0)' = \ell_1 \) has the Positive Schur property. But it is not semi-compact. If not, as \( c_0 \) is discrete with order continuous norm, \( Id_{c_0} \) is compact, which is impossible. Following by Corollary 5 of [6], we have the following assertion.

**Theorem 2.6.** Let \( E \) and \( F \) be two Banach lattices. And let \( T : E \to F \) be an order bounded almost \( M \)-weakly compact operator. If \( F'' \) has order continuous norm, then \( T \) is semi-compact.

By Theorem 1 of [6], we know that every Dunford-Pettis operator \( T : X \to E \) is almost \( L \)-weakly compact if and only if \( E \) has an order continuous norm. Next, we investigate the conditions under which each almost \( L \)-weakly compact operator is Dunford-Pettis.

There exist operators which are almost \( L \)-weakly compact but not Dunford-Pettis. For example, the identity operator \( Id_{L_1[0,1]} : L_1[0,1] \to L_1[0,1] \) is almost \( L \)-weakly compact as \( L_1[0,1] \) has the positive Schur property. But it is not Dunford-Pettis as \( L_1[0,1] \) does not have the Schur property. However, we have the following result.

**Proposition 2.7.** Let \( X \) be a Banach space and \( E \) be a Banach lattice. If \( E \) has weakly sequentially continuous lattice operations, then each almost \( L \)-weakly compact operator \( T : X \to E \) is Dunford-Pettis.
Proof. Let \((x_n)\) be a weakly null sequence, we have to show that \(T(x_n) \to 0\) in norm. Based on Corollary 2.6 of [5], it suffices to show \(|T(x_n)| \to 0\) in \(E\) and \(f_n(T(x_n)) \to 0\) for each positive disjoint sequence \((f_n)\) in \(B_{E'}\).

As \(x_n \to 0\) in \(X\) and \(T\) is an almost L-weakly compact operator, for each positive disjoint sequence \((f_n)\) in \(B_{E'}\), \(f_n(T(x_n)) \to 0\). Hence, we get that \(T\) is Dunford-Pettis. \(\square\)

At last, we give a conclusion about the relationships between almost L-weakly compact operators and almost Dunford-Pettis operators. K. Bouras et al. show that if \(F\) has an order continuous norm, each positive almost Dunford-Pettis operator \(T : E \to F\) is almost L-weakly compact ([1, Proposition 2.4]). We show that the condition of “\(F\) has an order continuous norm” is also necessary.

**Proposition 2.8.** Let \(E\) and \(F\) be two nonzero Banach lattices. Then the following statements are equivalent:

1. Each positive almost Dunford-Pettis operator \(T : E \to F\) is almost L-weakly compact.
2. The norm of \(F\) is order continuous.

**Proof.** (2) \(\Rightarrow\) (1) Follows from Proposition 2.4 of [4].

(1) \(\Rightarrow\) (2) Assume by way of contradiction that the norm of \(F\) is not order continuous. we need to construct a positive operator which is almost Dunford-Pettis but not almost L-weakly compact.

Similarly with the proof of Theorem 2.1. Since the norm of \(F\) is not order continuous, by Theorem 4.14 of [1], there exists a vector \(y \in F_+\) and a disjoint sequence \((y_n) \subset [-y, y]\) such that \(||y_n|| \to 0\). On the other hand, as \(E\) is nonzero, we may fix \(u \in E_+\) and pick a \(\phi \in (E')_+\) such that \(\phi(u) = ||u|| = 1\) holds.

Now, we consider operator \(T : E \to F\) defined by

\[ T(x) = \phi(x) \cdot y \]

for each \(x \in E\). Obviously, \(T\) is a positive operator and is compact (its rank is one). Hence, it is almost Dunford-Pettis. But it is not an almost L-weakly compact operator. If not, as the singleton \(\{u\}\) is a weakly compact subset of \(E\), and \(T(u) = \phi(u) \cdot y = y\), the singleton \(\{y\}\) is an L-weakly compact subset of \(F\). Since disjoint sequence \((y_n) \subset \text{sol}(\{y\})\), we have \(||y_n|| \to 0\), which is a contradiction. \(\square\)
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