Object Manipulation using Robotic Hands with Varying Degrees of Grasp Knowledge

Wenceslao Shaw-Cortez\textsuperscript{a,}\textsuperscript{*}, Denny Oetomo\textsuperscript{b}, Chris Manzie\textsuperscript{a}, Peter Choong\textsuperscript{b}

\textsuperscript{a}School of Electrical, Mechanical, and Infrastructure Engineering, The University of Melbourne, 3010, VIC, Australia
\textsuperscript{b}Department of Surgery, St. Vincent’s Hospital, 3065, VIC, Australia

Abstract

To date, there is limited rigorous analysis of object manipulation using robotic hands, where more focus has been placed on heuristic and experiment-based approaches. In this paper, we analyze the effects of grasp uncertainties based on realistic assumptions and propose a robust control framework for object manipulation. The framework considers a hand-object system subject to disturbances resulting from uncertainties in the object center of mass/inertia, hand kinematics, external wrenches, and contact locations. The proposed framework is then applied to practical object manipulation scenarios with different levels of uncertainty related to the sensors available to the robotic hand. These scenarios include when the hand-object system is known perfectly; when vision sensors are available; when tactile sensors are available; and when no vision/tactile sensors are available to the robotic hand (i.e. blind grasping). The analysis also addresses the internal force control in relation to the various practical cases. Simulation and experimental results validate the effectiveness of the proposed approach.

Keywords: Dexterous Manipulation, Robotic Grasping, Robust Control, Multi-fingered Hands

1. Introduction

Object manipulation via robotic hands is an ability that has been pursued for decades. One specific type of object manipulation is in-hand manipulation in which the object is translated and/or rotated within a grasp, and is only in contact with the fingertips. In-hand manipulation, as opposed to static grasping, requires more precise control of the robotic hand to apply the appropriate contact forces to move the object. In addition to moving the object, the robotic hand is responsible for ensuring the object stays within the grasp without slipping or losing contact with the fingertips. All of this must be accomplished despite the effects of rolling, inertial and Coriolis forces, and external disturbances that interplay the hand and object relationship.

In addition to the complexities inherent in object manipulation, this work is focused on a practical extension of in-hand manipulation, which deals with grasp uncertainties. When robots are deployed in the real world, it is unreasonable to assume intimate knowledge of every object that needs to be grasped, including its shape, inertia, center of mass, or possible wrench disturbances that can act on it. Thus the model of the object is at least partially unknown to the on-board controller. Similarly, modeling errors associated with the robotic hand affect how the hand can manipulate objects. Also, variables in the manipulation solution need to align with what can be measured or observed by the available sensors. For example, vision-based sensors cannot provide the object center of mass, but can track the object relative position and orientation.\textsuperscript{1} Tactile sensors can provide contact location.\textsuperscript{2} Also, the quality of the information provided by the available sensors will impact the ability of the robotic hand to perform manipulation. The objective of robust manipulation is then to perform in-hand manipulation in the presence of these uncertainties. One such application of robust manipulation is in the field of prosthetics.

State-of-the-art prosthetic hands are limited to grasping and making gestures, and lack the dexterity for fine manipulation skills. Thus a conventional prosthetic hand forces the amputee to perform gross arm motions for the same manipulation task that is generally performed using fine manipulation skills in able-bodied humans. These gross motions result in fatigue and poor performance,\textsuperscript{3} which reduces the quality of living for amputees. An amputee would benefit from dexterously capable prosthetic hands, however it is impossible to account for every object a person would like to grasp. Furthermore, external wrench disturbances other than gravity are present in everyday activities. These disturbances may result from interaction with the environment such as when using a key to unlock a door. It is impractical to assume every possible external disturbance can be anticipated a priori. Therefore a robust in-hand manipulation solution is required that can handle unknown disturbances as well as an uncertain model of the hand and object.

In the prosthetic setting, the human can be thought of as a high level planner to form grasps prior to manipulation. In a fully autonomous setup, the process of grasping and manipulation consists of the synthesis of the grasp followed by the control of the robotic hand to perform the manipulation task. Existing work has provided robust means of controlling end effector positions and performing grasp synthesis.\textsuperscript{4,5,6,7} In this work...
we focus on the analysis of in-hand manipulation control once a grasp has been formed. Initial work for in-hand manipulation proposed methodologies for modeling the hand-object system, as well as analyzing properties of the grasp [8,9,10,11]. However, due to the complexity in controlling robotic hands for in-hand manipulation, different types of solutions have emerged. Some existing solutions include grasp force optimization [12,13,14] and motion planning [15,16,17]. Those approaches typically assume properties such as object weight, contact friction, and center of mass are known a priori and that the hand-object system is quasi-static. Despite the novelty in those solutions, their inherent assumptions are overly constraining for the robust manipulation scenario presented here.

Furthermore, the quasi-static assumption used in much of the literature ignores the dynamics of the system, which is critical for manipulation tasks. That assumption simplifies the problem, but at the expense of not guaranteeing stability of the hand-object system. Stability in this sense refers to the hand manipulating the object to a desired set-point pose despite disturbances present in our scenario. Previous work on stabilizing controllers typically require a priori knowledge of the object, and/or assume the external disturbance is known or that none exist. In [9] a computed torque control law was presented for trajectory tracking, which assumed perfect knowledge of the entire hand-object system. In [18] the authors presented linearization-based controllers, which assumed the object center of mass and contact locations known. In [19], stability analysis related to internal force distribution is presented, but is limited to constant contact forces and neglects effects other than gravity. In [20] a PD control law with adaptive compensation was proposed to specifically deal with gravity, uncertain contact locations, and hand kinematics, but ignored general external wrenches acting on the hand-object. In [21] a passivity-based controller for two fingers that minimizes the contact angle was presented, but only applies in the plane, with no external disturbances. In [22] the authors presented a linearization-based controller formulated as a linear matrix inequality optimization problem that is specifically robust to contact uncertainties, but also ignored external disturbances on the hand-object system.

Other research considers the in-hand manipulation problem from a conservative perspective in which neither the object nor contact information is known, but also assume the external disturbance does not exist, or equivalently that it is known exactly to cancel it out. Work in this area is commonly referred to as “blind grasping” [23,24,25]. A passivity-based stabilizing controller was proposed for two fingers in the plane assuming that no external disturbance acts on the system. That passivity-based controller was then extended to specifically reject gravity disturbances [26], incorporate optimal contact angles without external disturbances [27], and include manipulation of arbitrary polyhedral objects in 3-D space also without external disturbances [28].

An additional difficulty associated with grasp uncertainty is the lack of object information to define a frame (herein referred to as the task frame) to define the manipulation task. When exact knowledge of the object is known, this task frame is simply the object frame attached to the object center of mass [9]. When the object center of mass is unknown, other methods are used to define the task frame with respect to the available sensors on the hand. For example, for vision-based manipulation, it is common to define the task frame as fixed to a point on the object that can be seen by the camera systems [29]. In blind grasping scenarios, the only available information is the hand configuration and so the task frame is defined based on the fingertip positions [30,31]. Those existing approaches are limited to specific grasp scenarios without analysis as to how the frame should be chosen in a general sense to guarantee stability. The related work summarized above focuses on sensor-specific task frames and requires access to a priori knowledge of object inertia and/or disturbances acting on the system. There has yet to be a framework that provides a solution for varying levels of uncertainty in the grasp scenario.

In previous work, we developed a manipulation controller that mitigates slip without knowledge of the friction coefficient, but did not provide any stability guarantees regarding grasp uncertainties [32]. A robust manipulation controller was then developed that handled unknown external disturbances, but was limited to grasping with tactile sensors and required the conservative assumption that the contact points do not roll [33]. In this paper we extend upon previous work to develop a robust control framework to perform in-hand manipulation with limited knowledge of the grasp scenario. The limited grasp knowledge manifests as disturbances arising from uncertainties in the hand-object model including object mass/inertia, hand kinematics, external wrenches, and contact locations all subject to the effects of rolling contacts. The proposed framework compensates for these uncertainties that have yet to be collectively addressed in the literature. The results presented here provide semi-global asymptotic/exponential stability of the system with appropriate tuning guidelines.

The layout of this paper is as follows. Section 2 outlines the hand-object and system dynamics with respect to the relevant assumptions, and the control objective. Section 3 presents the proposed control framework and subsequent stability analysis. The framework is general in that it is independent of the sensors available to the hand, and so Section 4 outlines the assumptions that must be satisfied such that the stability guarantees follow. Section 5 then applies the proposed framework to specific case studies when the hand-object system is known perfectly; vision-based sensing is available; tactile sensing is available; and when no vision/tactile information is available to the robotic hand (i.e. blind grasping). These case studies show how a designer should define the controller and task frame such that the stability guarantees of the proposed framework follow. Section 6 then applies the proposed framework in numerical simulations and in experiment to validate and analyze the proposed approach.

Notation

Throughout this paper, an indexed vector \( v_i \in \mathbb{R}^n \) has an associated concatenated vector \( v \in \mathbb{R}^{nk} \), where the index \( i \) is specifically used to index over the \( k \) contact points in the grasp. The notation \( v^c \) indicates that the vector \( v \) is written with respect to a frame \( \mathcal{E} \), and if there is no explicit frame defined, \( v \) is
written with respect to the inertial frame, $\mathcal{P}$. The operator $(\cdot)\times$ acting on a vector $v \in \mathbb{R}^3$ is defined by:

$$ (v)\times = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix} $$ (1)

where $v_j$ indicates the $j$th element of $v$. $SO(3)$ denotes the special orthogonal group of dimension 3. The centroid of the vector $v \in \mathbb{R}^3$ is defined by $\bar{v} = \frac{1}{2} \sum_{i=1}^{k} v_i$. The approximation of $v$ is denoted $\hat{v}$. The minimum and maximum eigenvalues of a positive-definite matrix, $B$, are respectively denoted by $\lambda_{\text{min}}(B)$, and $\lambda_{\text{max}}(B)$. The kernel or null-space of a matrix, $B$, is denoted by $\text{Ker}(B)$. The Moore-Penrose generalized inverse of $B$ is denoted $B^+$. The $n \times n$ identity matrix is denoted $I_{n \times n}$.

2. System Model

2.1. Hand-Object System

Consider a fully-actuated, multi-fingered hand grasping a rigid, convex object at $k$ contact points. Each finger consists of $n_i$ joints with smooth, convex fingertips of high stiffness. Let the joint configuration be described by the joint angles, $q_i \in \mathbb{R}^{n_i}$. The full hand configuration is defined by the joint angle vector, $q = (q_1, q_2, ..., q_k)^T \in \mathbb{R}^m$, where $m = \sum_{i=1}^{k} n_i$ is the total number of joints. Let the inertial frame, $\mathcal{P}$, be fixed on the palm of the hand, and a fingertip frame, $\mathcal{F}_i$, fixed at the point $p_i \in \mathbb{R}^3$. The contact frame, $\mathcal{C}_i$, is located at the contact point, $p_i \in \mathbb{R}^3$. A visual representation of the contact geometry for the $i$th finger is shown in Figure 1.

![Figure 1: A visual representation of the contact geometry for contact $i$.](image)

Let $v_c \in \mathbb{R}^3$ denote the instantaneous velocity of the contact point $p_c$. The joint velocities, $q_i$, and the contact point velocity, $v_c$, are related via $J_h(q_i, p_{ci}) \in \mathbb{R}^{3 \times n_i}$:

$$ v_c = J_h(q_i, p_{ci})q_i $$ (2)

$$ J_h(q_i, p_{ci}) = \begin{bmatrix} I_{3 \times 3} & -(p_{ci})\times \end{bmatrix} J_h(q_i) $$ (3)

where $p_{ci} \in \mathbb{R}^3$ is the vector from $\mathcal{F}_i$ to $\mathcal{C}_i$, and $J_h(q_i) \in \mathbb{R}^{6 \times n_i}$ is the manipulator Jacobian relating $q_i$ with the translational and rotational velocities about $p_i$ (see Figure 1). The hand Jacobian, $J_h := J_h(q_i, p_{ci}) \in \mathbb{R}^{3 \times m}$, is constructed by combining each $J_h(q_i, p_{ci})$ into a block diagonal matrix.

When tactile information is not available on the robotic hand, an approximate contact location must be used. In such cases, it is difficult to define a best approximation, but an intuitive one is a point fixed on the fingertip contact surface. The position vector from $\mathcal{F}_i$ to this fixed point is $p_{fi} \in \mathbb{R}^3$, and related to $p_i$ via the inertial position vector $p_i = p_{fi} + p_{fi}$.

Let $O$ be a reference frame fixed at the object center of mass $p_o \in \mathbb{R}^3$, and $R_{po} \in SO(3)$ is the rotation matrix, which maps from $O$ to $\mathcal{P}$. The angular velocity of the object frame with respect to $\mathcal{P}$ is $\omega_o \in \mathbb{R}^3$. The object pose is defined by $\mathbf{x}_o \in \mathbb{R}^6$, with $\mathbf{x}_o = (p_o, \omega_o)$. The position vector from the object center of mass to the respective contact point is $p_{oc} \in \mathbb{R}^3$ (see Figure 1).

Each fingertip exerts a contact force, $f_c \in \mathbb{R}^3$, on the object at the contact point, $p_i \in \mathbb{R}^3$. Let the matrix $G_i(p_{oc}) \in \mathbb{R}^{6 \times 3}$ be the map from the contact force, $f_c$, to the corresponding wrench acting on the object. The transpose, $G_i(p_{oc})^T$, maps the object motion to the velocity of the $i$th contact point. Using a point contact with friction model, $G_i(p_{oc})^T$ can be computed by:

$$ G_i^T(p_{oc}) = \begin{bmatrix} I_{3 \times 3} & -(p_{oc})\times \end{bmatrix} $$ (4)

The grasp map, $G := G(p_{oc}) \in \mathbb{R}^{6 \times 3}$, maps the contact force vector, $f_c$, to the net object wrench, and is defined by:

$$ G(p_{oc}) = [G_1, G_2, ..., G_k] $$ (5)

The hand and object kinematics are related by the grasp constraint (11):

$$ v_c = J_h q = G_i^T \mathbf{x}_o $$ (6)

The following assumptions are made for the hand and object:

**Assumption 1.** The given hand has $m = 3k$, and never reaches a singular configuration.

**Remark 1.** Assumption 1 ensures $J_h$ is square and invertible, which is a common assumption in related work [22, 28, 30]. This assumption is made in order to not distract from the main contribution of the paper and can be relaxed by considering internal motion of the dynamics [11] with appropriate null space controllers [34, 35].

**Assumption 2.** The given multi-fingered grasp has $k > 2$ contact points, which are non-collinear.

**Remark 2.** Assumption 2 ensures $G$ is full rank [29]. Note, this requires that a grasp is already formed for a manipulation task. The motivation behind such a requirement is to propose a low-level control framework as part of a hierarchical grasping architecture such as [6, 21].

**Assumption 3.** The fingertips can roll on the contact surface with the object, but do not slip or lose contact.

**Remark 3.** Assumption 3 ensures (6) is always satisfied, and can be enforced by appropriate slip prevention algorithms.

**Assumption 4.** The fingertip and object surfaces at the contact points are locally smooth.
Under Assumptions\textsuperscript{[1] and [3]} the hand-object dynamics can be derived as in \textsuperscript{[11]}:
\[
M_{ho} \dot{x}_o + C_{ho} \dot{x}_o = G J_h^T (u + \tau_o) + w_e
\]
with
\[
M_{ho} = M_o + G J_h^T M_h J_h^G + \frac{d}{dt} (G J_h^G),
\]
\[
C_{ho} = C_o + G J_h^T (C_h J_h^G + M_h \frac{d}{dt} (J_h^G)),
\]
where \(M_h := M_h(q) \in \mathbb{R}^{n \times n}\), \(M_o := M_o(x_o) \in \mathbb{R}^{6 \times 6}\) are respectively the hand and object inertia matrices, \(C_h := C_h(q, \dot{q}) \in \mathbb{R}^{n \times n}\), \(C_o := C_o(x_o, \dot{x}_o) \in \mathbb{R}^{6 \times 6}\) are the respective hand and object Coriolis matrices, \(\tau_o := \tau_o(t, q, \dot{q}) \in \mathbb{R}^n\) is the sum of all dissipative and non-dissipative disturbance torques acting on the hand, \(w_e := w_e(t) \in \mathbb{R}^6\) is an external wrench disturbing the object, and \(u \in \mathbb{R}^m\) is the joint torque control input for a fully actuated hand. We use \(M_{ho} \) := \(M_{ho}(q, x_o) \in \mathbb{R}^{6 \times 6}\) and \(C_{ho} := C_{ho}(q, x_o, \dot{q}, \dot{x}_o) \in \mathbb{R}^{6 \times 6}\) to denote the hand-object inertia and Coriolis matrices, respectively.

Remark 4. It is important to note that for rolling contacts, \(p_a\) is a function of the hand configuration, object configuration, and geometry of the fingertip surfaces. For smooth, convex surfaces there exists a smooth local bijection between the geometry of the fingertip/object surfaces and the hand-object configurations such that \(p_a, J_o, \) and \(G\) can be expressed as functions of the hand-object state, \((q, x_o)\). \textsuperscript{[11]}

Lemma 1. Under Assumptions\textsuperscript{[1] and [2]} \(M_{ho}\) is positive definite, uniformly bounded such that there exists constants \(m_{\text{min}}, m_{\text{max}} \in \mathbb{R}\) that satisfy:
\[
0 < m_{\text{min}} \leq \|M_{ho}^{-1}\| \leq m_{\text{max}}
\]  
(10)
where \(\| \cdot \|\) denotes a general matrix norm.

Proof. From the positive-definiteness of \(M_{ho}\), the following statement holds:
\[
0 < \lambda_{\text{min}}(M_{ho}^{-1}) \leq \|M_{ho}^{-1}\| \leq \lambda_{\text{max}}(M_{ho}^{-1})
\]  
(11)
It is a well known fact that \(M_o\) and \(M_h\) are uniformly bounded, positive definite matrices \textsuperscript{[66]}. Thus by Assumptions\textsuperscript{[1] and [2]} \(M_{ho}\) is uniformly bounded.

2.2. Task Frame and Control Formulation

Due to the limited knowledge of the grasp scenario, the object center of mass may be unknown or difficult to estimate, and may change during the execution of a task. For example, when a robotic hand is used to drink from a glass, the location of the center of mass is unknown to the hand, and will change as the water is emptied from the glass. Also, some situations require the robotic hand to manipulate the object about a point other than the object center of mass. For example, when turning a screwdriver, the hand is applying a torque about the handle, which does not necessarily coincide with the screwdriver center of mass. This motivates the use of a task frame with which to define a reference for object manipulation.

Let \(\mathcal{A}\) be the task frame located at the point \(p_a \in \mathbb{R}^3\) with respect to \(P\). Let \(R_{pa} \in SO(3)\) be the rotation matrix mapping from frame \(\mathcal{A}\) to \(P\). Let \(v_a \in \mathbb{R}^3\) denote the velocity of \(p_a\), and \(\omega_a \in \mathbb{R}^3\) denote the angular velocity of frame \(\mathcal{A}\) with respect to \(P\). The task frame state \(x \in \mathbb{R}^6\) is defined by the position \(p_a\) and orientation of the task frame. For practical considerations, a local parameterization of \(SO(3)\) is used to define a notion of orientation error by defining \(\gamma_a \in \mathbb{R}^3\), such that \(R_{pa} = R_{pa}(\gamma_a)\). The task state is thus \(x = (p_a, \gamma_a)\). To incorporate this local parameterization in the kinematics, let \(S(\gamma_a) \in \mathbb{R}^{3 \times 3}\) denote the one-to-one mapping defined by:
\[
\omega_a = S(\gamma_a) \gamma_a
\]  
(12)
The matrix \(S(\gamma_a)\) is absorbed into \(P := \text{diag}(I_{3 \times 3}, S(\gamma_a))\) such that:
\[
\begin{bmatrix}
\dot{p}_a \\
\dot{\omega}_a
\end{bmatrix} = P \dot{x}
\]  
(13)
It is inherently assumed that the orientation \(\gamma_a\) does not pass through a singular configuration.

The definition of the task frame, \(\mathcal{A}\), is dependent on the sensors available to the robotic hand. The frame \(\mathcal{A}\) must be accompanied by a Jacobian, \(J_o := J_o(x, q, x_o) \in \mathbb{R}^{6 \times 3k}\) that satisfies the following assumption:

Assumption 5. The Jacobian, \(J_o\) is full rank, twice continuously differentiable, and satisfies:
\[
J^o_o \dot{x} = v_c + \sigma_1
\]  
(14)
where \(\sigma_1 := \sigma_1(x, q, x_o, \dot{x}, \dot{q}, \dot{x}_o) \in \mathbb{R}^3\) is a twice continuously differentiable disturbance term that satisfies:
\[
(\dot{x}, \dot{x}) = 0 \implies (\sigma_1, \dot{\sigma}_1, \ddot{\sigma}_1) = 0
\]  
(15)

Remark 5. Assumption\textsuperscript{[5]} is a condition that must be satisfied by the choice of the task frame. Examples of how the task frame and \(J_o\) are chosen based on the grasp scenario are shown in Section\textsuperscript{[2]}. In the examples to follow, \(J_o\) is effectively an approximation of the grasp map with \(\sigma_1\) being the resulting velocity error, and thus the condition\textsuperscript{[15]} restricts how the approximation should be made. We emphasize that \(J_o\) need not be an approximation of \(G\), but here it is treated as such for the purpose of clarity.

The relation between \(\dot{x}\) and \(\dot{x}_o\) is defined via\textsuperscript{[6] and [14]}:
\[
\dot{x}_o = G^{-T} (J^T_o \dot{x} - \sigma_1)
\]  
(16)

The dynamics of the task state are derived by differentiating\textsuperscript{[16]}:
\[
\ddot{x}_o = \frac{d}{dt} [G^{-T} J^T_o] \dot{x} + G^{-T} J^T_o \ddot{x} - \frac{d}{dt} [G^{-T}] \sigma_1 - G^{-T} \dot{\sigma}_1
\]  
(17)
A similar relation is derived between \((x, \dot{x})\) and \((q, \dot{q})\) by substituting\textsuperscript{[6]} into\textsuperscript{[14]} and differentiating:
\[
\dot{q} = J^+_h (J^h_\dot{x} - \sigma_1)
\]  
(18)
The disturbance terms, \( \tilde{x} \), satisfy for a class of external disturbances. Herein we make the

\[
\dot{q} = \frac{d}{dt} \left[ J_h^{-1} J_a^T \right] \ddot{x} + J_h^{-1} J_a^T \dot{x} - \frac{d}{dt} [J_h^{-1}] \sigma_1 - J_h^{-1} \dot{\sigma}_1
\]  

(19)

Substitution of (16) and (17) into (9), and pre-multiplication by \( J_a^T G^T \) results in the following system dynamics:

\[
M_a \ddot{x} + C_a \dot{x} = J_a G^T T (u + \tau_r) + J_a G^T w_r
\]

\[
+ J_a G^T M_h \left( \frac{d}{dt} [G^T] \sigma_1 + G^T \dot{\sigma}_1 \right)
\]

(20)

with

\[
M_a = J_a G^T M_h \left( \frac{d}{dt} [G^T] J_a^T \right),
\]

(21)

\[
C_a = J_a G^T \left( M_h \frac{d}{dt} [G^T] J_a^T + C_h G^T J_a^T \right),
\]

(22)

where \( M_a := M_h(x, q, x_o), C_a := C_h(x, q, x_o, \dot{x}, \ddot{x}) \) are the task inertia and Coriolis matrices, respectively.

The following lemma shows that \( M_a \) is positive definite and bounded, which is a key property that is exploited in the proposed framework:

**Lemma 2.** Under Assumptions 1, 2, and 5, \( M_a \) is positive definite, and uniformly bounded.

**Proof.** \( J_a \) is full rank from Assumption 5 and so the proof of positive definiteness follows from linear algebra. For the uniformly bounded statement, the proof follows a similar approach to that of Lemma 1.

The proposed solution enhances existing work by compensating for a class of external disturbances. Herein we make the following assumption:

**Assumption 6.** The disturbance terms, \( \tau_r, w_r \), are continuously differentiable, and satisfy:

\[
\langle \dot{x}, \ddot{x} \rangle = 0 \implies \tau_r, w_r = 0
\]

(23)

**Remark 6.** Common disturbances that satisfy Assumption 6 include gravity acting on both the hand and object, and viscous friction acting on the joints in the form of \( -\beta \dot{q} \) for \( \beta \in \mathbb{R}_{>0} \).

For set-point object manipulation, the state \( x \) must reach a desired reference \( r \in \mathbb{R}^6 \), where \( r, \dot{r} \equiv 0 \). Let \( e = x - r \) define the error. The objective of the proposed control algorithm is to asymptotically reach \( \langle e, \dot{e} \rangle = 0 \) in the presence of uncertain disturbances. The control problem is defined as follows:

**Problem 1.** Given a hand-object system that satisfies Assumptions 2, 6 determine a control law that semi-globally satisfies:

\[
\lim_{t \to \infty} \langle e(t), \dot{e}(t) \rangle \to 0
\]

(24)

3. Proposed Framework for Robust Object Manipulation

The proposed control law for this framework is given by:

\[
u = J_h^T \left( \hat{G}^T (J_0 \hat{G}^T) \right)^{-1} u_m + u_f
\]

(25)

where \( J_h^T \) and \( \hat{G} \) are full rank approximations of \( J_0 \) and \( G \) respectively, and \( u_m \in \mathbb{R}^m \) is the PID-based manipulation controller:

\[
u_m = -K_p e - K_i \int e \, dt - K_d \dot{e}
\]

(26)

where \( K_p, K_i, K_d \in \mathbb{R}^{6 \times 6} \) are the respective proportional, integral, and derivative positive-definite gain matrices. The internal force control input \( u_f \in \mathbb{R}^3 \) is used to control the internal forces of the grasp.

**Remark 7.** The internal force controller is used in practice to ensure the fingertips always apply positive contact forces to prevent slip, and thus ensure Assumption 3 holds. The choice of an internal force controller is not unique. One possible heuristic solution for \( u_f \) is the internal force control law:

\[
u_f = k_f (p_c - p_o, \dot{p}_c - \dot{p}_o, ..., \dot{p}_c - \dot{p}_o)^T
\]

(27)

where \( k_f \in \mathbb{R}_{>0} \) is a scalar gain. Another systematic method to address the internal force control is via grasp force optimization [12, 13, 14]. It is well known that the generalized inverse from (25) can be re-written as a quadratic program:

\[
u = J_h^T u_{j_c},
\]

(28)

\[
u_{j_c} = \arg\min \nu^T v \quad \text{s.t.} \quad J_0 v = u_m
\]

(29)

In grasp force optimization, additional constraints (including friction cone constraints) are incorporated into (29) to ensure Assumption 3 is satisfied. The cost is usually used to minimize internal forces or actuator effort. The internal force controller is incorporated in the optimization to deal with these additional constraints and grasp redundancies. We refer to the existing literature to appropriately define the optimization, and in Section 4 we discuss how the grasp force optimization techniques can be incorporated into the framework.

3.1. Stability Analysis

The stability proof presented here is achieved by exploiting the structural similarities between the hand-object system (20) and that of a robotic manipulator, and thus extending the results from [38] to object manipulation. In order to apply those results several conditions must be met. First, the inertia matrix, \( M_h \), must be positive definite and bounded, which is guaranteed via Lemma 2. Second, the system (20) is not only a function of \( x, \dot{x} \), but also of \( q, x_o, \dot{q}, \dot{x}_o \). It is necessary to show an intimate relation between \( x, \dot{x} = 0 \implies (x_o, \dot{x}_o, \dot{q}, \ddot{q}) = 0 \). Finally, any disturbances that arise in the system must be shown to be constant when the task frame is static. To start, the following lemma provides a relation between the task dynamics and the hand-object dynamics:
Lemma 3. Consider the plant dynamics, \( \ddot{x} \), defined by (20), and hand-object dynamics, \( (\ddot{q}, \dot{x}_o) \), defined by (19). Under Assumptions 1-5 if \((\ddot{x}, \dot{x}_o) = 0\), then \((\ddot{q}, \dot{x}_o) = 0\).

Proof. The relation between \((\ddot{x}, \dot{x}_o)\) and \((\ddot{q}, \dot{x}_o)\) follows from (17) and (19). The relation between \((\ddot{x}, \ddot{x}_o)\) and \((\ddot{q}, \dot{q})\) follows from (18) and (16) under Assumptions 2, 3. Thus \((\ddot{x}, \dot{x}_o) = 0 \implies (\ddot{q}, \dot{x}_o) = 0\).

Remark 8. In practice, this assumption means that the errors resulting from the approximations of \(J_0\) and \(G\) resulting from grasp uncertainty must be constant at the origin.

Proof. The continuously differentiable property of \(\psi\) is ensured by Assumptions 5 and 6. For (33), we begin by differentiating \(\psi\) with respect to time:

\[
\dot{\psi} = -\frac{d}{dt}[C_a]\dot{e} - C_e\ddot{e} + \frac{d}{dt}[J_aG^1G^1] \dot{\tau}_e + J_aG^1w_e
+ J_aG^1M_{\dot{h}}(\frac{d}{dt}[G^1]r_1 + [G^1]r_1) + \sigma_2
\]

where

\[
\sigma_2 = D_1u_m + D_2u_f
\]

with \(D_1 := D_1(x, q, x_o) \in \mathbb{R}^{6 \times 6}\) and \(D_2 := D_2(x, q, x_o) \in \mathbb{R}^{6 \times 6}\) represent the residual matrices that arise from the approximations of \(J_0\) and \(G\) multiplying their respective true inverses, and are specifically defined in the case studies of Section 4. Thus \(\sigma_2\) is the resulting residual disturbance related to the imperfect approximations \(\tilde{J}_0\) and \(\tilde{G}\) in the control, \(u\), and must satisfy the following assumption:

Assumption 7. The disturbance term \(\sigma_2\) is continuously differentiable and satisfies:

\[
(\dot{e}, \ddot{e}) = 0 \implies \sigma_2 = 0
\]

Lemma 4. Under Assumptions 1-7, \(\psi\) defined by (34) is continuously differentiable and the following condition holds:

\[
(\dot{e}, \ddot{e}) = 0 \implies \psi = 0
\]

Proof. The continuously differentiable property of \(\psi\) is ensured by Assumptions 4, 5, and 6. For (33), we begin by differentiating \(\psi\) with respect to time:
Applying stronger conditions on the gain matrices can lead to the following result:

**Corollary 1.** Under Assumptions 3-7, the system (20) with the control law (25), (26) is semi-globally exponentially stabilizable.

**Proof.** With Lemmas 2 and 4 the proof for semi-global exponential stabilizability follows directly from Corollary 3 of [38] and Theorem 11.4 of [39].

**Remark 9.** Exponential stability provides additional robustness to the system with respect to small perturbations that can relax the constant disturbance condition from Assumption 6. Such perturbations in the grasping scenario may arise from further modeling errors associated with the point contact with friction model, rigid contact surfaces, and other general external disturbances that may act on the system.

### 3.2. Gain Tuning

Theorem 1 and Corollary 1 ensure the existence of PID gains to guarantee semi-global asymptotic and semi-global exponential stability, respectively, for object manipulation. In [38], a systematic tuning method was presented for PID control in which $K_p$, $K_i$, $K_d$ are parameterized by a single variable, $\varepsilon \in \mathbb{R}_{>0}$. That approach, which is adopted here, restricts the degrees of freedom in choosing the gains to facilitate the design of the controller without compromising stability. Let the $K_p$, $K_i$, $K_d$ gains be defined by:

\[
K_p = \hat{M}(K_1 + \frac{1}{\varepsilon}K_2) \quad (38a)
\]

\[
K_i = \frac{1}{\varepsilon} \hat{M}K_1 \quad (38b)
\]

\[
K_d = \hat{M}(K_2 + \frac{1}{\varepsilon}I_{6\times 6}) \quad (38c)
\]

where $\hat{M} \in \mathbb{R}^{6\times 6}$ is a positive definite matrix and $K_1, K_2 \in \mathbb{R}^{6\times 6}$ are positive definite gain matrices. The structure defined in (38) facilitates the choice of each gain parameter. The gains $K_1$ and $K_2$ relate to the behavior of a linear system, and can be chosen based on the desired closed loop time constant and damping coefficient [38]. The parameter $\hat{M}$ is a constant estimate of $M_a$ which must satisfy:

\[
\|U_{6\times 6} - M_a^{-1}\hat{M}\| < 1 \quad (39)
\]

**Remark 10.** Lemma 2 guarantees the existence of an $\hat{M}$ that satisfies (39). An acceptable choice is $\hat{M} = \frac{M_0}{\text{max} I_{6\times 6}}$ [38]. In this case only an upper bound on the norm of $\hat{M}$ is required, which is applicable in object manipulation cases where the object model is not well known.

Finally, the parameter $\varepsilon$ dictates the size of the region of attraction. Once $K_1, K_2$ are defined, $\varepsilon$ is solely responsible for the system’s transient response. Thus Theorem 1 and Corollary 1 can be re-stated under the restricted tuning guidelines as:

**Corollary 2.** Consider the control (25), (26), (38) applied to the plant (20) with Assumptions 1-7. For any $\Delta \in \mathbb{R}_{>0}$ and for all $\|e(t), \dot{e}(t)\| < \Delta$, there exists a $\varepsilon^{*} \in \mathbb{R}_{>0}$ such that for $\varepsilon \in (0, \varepsilon^{*})$, the origin is semi-globally asymptotically stable with respect to the origin, $(e, \dot{e}) = 0$. Furthermore, there exists a $\varepsilon^{**} \in \mathbb{R}_{>0}, \varepsilon^{**} \leq \varepsilon^{*}$ such that for $\varepsilon \in (0, \varepsilon^{**})$ the origin is semi-globally exponentially stable.

**Remark 11.** Stability with respect to the single parameter $\varepsilon$ allows a simple, systematic way to improve the robustness of the system. However, in practice signal noise will provide a lower bound, $\varepsilon_{min} \in \mathbb{R}_{>0}$, on $\varepsilon$. In the case that noise levels are sufficiently high, the set defined by $\varepsilon_{min} < \varepsilon < \varepsilon^{*}$ may be empty.

### 4. Case Studies

In this section, we show how the proposed framework is applied to various grasping scenarios encountered in the literature. Depending on the sensors available, the robotic hand has access to different types of information and thus has an incomplete knowledge of the grasp. Short of the ideal grasping scenario, this incomplete grasp knowledge not only manifests as uncertainties, but also restricts how the task frame $\mathcal{A}$ can be defined. Much of the related work assumes a task frame and its Jacobian are given [20], others do not provide stability analysis related to their choice of $\mathcal{A}$ [30], and other frame definitions are specific to certain grasp scenarios [28, 31].

Here we outline the steps for a designer to appropriately define the task frame, internal force controller, and ultimately the robust controller, $u$ such that the stability guarantees from Section 3 follow. The steps for defining an appropriate $u$ with respect to the available grasp knowledge are:

**Step 1:** Define the task frame, $\mathcal{A}$.

**Step 2:** Define the approximations $\hat{J}_h, \hat{G}$.

**Step 3:** Define conditions for $u_f$.

**Step 4:** Define $J_a$ and show that Assumption 8 is satisfied.

**Step 5:** Use the control framework (25) to define $u$.

**Step 6:** Show that Assumption 7 is satisfied.

In the following case studies, $J_a$ is chosen as $J_a = P^T \hat{G}$, where the definition of $\hat{G}$ depends on the available grasp information. The substitution of $J_a = P^T \hat{G}$ in (25) results in the following controller:

\[
u = \hat{J}_h^T ((P^T \hat{G}) u_m + u_f) \quad (40)
\]

The choice of $J_a = P^T \hat{G}$ allows for an intuitive interpretation of $J_a$ as an approximate grasp map. We note however that the definition of $J_a$ is not restricted to this choice.
4.1. Ideal Grasping

In the ideal grasping scenario, the robotic hand has measurements of the hand-object states \((q, x_0, p_{fc})\), contact locations \(p_{fc}\), and the hand kinematics are exactly known. It is important to note that this ideal scenario is used as the benchmark for the case studies presented in this paper, but does not assume knowledge of the object inertia or external disturbances.

**Step 1-3**

Due to the knowledge of the hand kinematics and \(q, x_0, p_{fc}\) (and thus \(J_0, G\)), **Step 1** and **Step 2** are straightforward in that \(\mathcal{A} = O, \dot{J}_h = J_h\) and \(\dot{G} = G\) for **Step 3**, the internal force control can be defined by the heuristic approach or the grasp force optimization approach mentioned in Remark 2 such that \(u_f\) remains in the null space of \(G\). This capability is beneficial in that \(u_f\) has no effect on the manipulation of the object and can be used to ensure the contact points do not slip. This is formally stated as:

**Assumption 8.** The internal force control satisfies:

\[
u_f \in \text{Ker}(G)\] (41)

**Step 4**

The choice of \(J_0\) along with the choice of the task frame allow for the following choice of \(J_0\) that satisfies Assumption 5.

**Lemma 5.** Let \(\mathcal{A}\) be defined by \(p_o = p_o\) and \(R_{po} = R_{po}\), and let \(J_0 = P^T G\). Under Assumptions 2, 4, Assumption 5 holds.

**Proof.** Refer to Appendix 7.1.

**Step 5**

For \(J_0 = P^T G\), the proposed control for the ideal grasping scenario is defined by (40) with \(\dot{J}_h = J_h, \dot{G} = G\).

**Step 6**

From Assumption 8, it is straightforward to show that \(\sigma_3 \equiv 0\) upon substitution of (40) into (20), and thus Assumption 7 is satisfied directly.

Thus from Lemma 5 the stability guarantees follow directly from Theorem 1 and Corollary 1, and are formally stated as follows:

**Corollary 3. (Ideal Grasping)** Let \(\mathcal{A}\) be defined by \(p_o = p_o\), and \(R_{po} = R_{po}\). Let \(\dot{G} = G\) and \(\dot{J}_h = J_h\). Suppose measurements of \(q, x_0, x, p_{fc}\) are available and Assumptions 2, 4, and 5 hold. The system (20), with control law (40), is semi-globally exponentially stabilizable.

4.2. Vision-based Grasping with Tactile Sensors

In many practical applications, the robotic hand is not privy to knowledge of the object center of mass, especially when the object to be grasped is arbitrary in nature (i.e. it is not from a set of predetermined objects). Many scenarios in the literature use vision-based approaches to provide object manipulation solutions [40, 29, 7]. When vision systems are available, the control system is able to track a fixed point on the object and determine the object orientation. Additionally, there exist tactile sensors that provide information of the grasp such as measurements of contact locations and contact forces [2]. In this paper we restrict our attention to tactile sensors that provide the contact position, \(p_{fc}\).

**Step 1**

The frame \(\mathcal{A}\) is defined with \(p_o\) being a point that is fixed with respect to the object frame tracked by the vision system, and without loss of generality, we let \(R_{po} = R_{po}\).

**Step 2**

Due to lack of knowledge of the object’s center of mass, an approximation of the grasp map is required to implement the proposed control strategy. This approximate grasp map is defined by substituting \(p_o\) with \(p_o\) in (42):

\[
\hat{G} = \begin{bmatrix} I_{3 \times 3} & \cdots & I_{3 \times 3} \\ (p_{c1} - p_o)\times & \cdots & (p_{c3} - p_o)\times \end{bmatrix}
\] (42)

In practice there always exists an error between the model and actual system. To account for this error we introduce an approximation of the hand Jacobian:

\[
\dot{J}_h(q, p_{fc}) = \begin{bmatrix} I_{3 \times 3} & \cdots & I_{3 \times 3} \\ -(p_{fc})\times & \cdots & -(p_{fc})\times \end{bmatrix} \dot{J}_h(q_i)
\] (43)

where \(\dot{J}_h\) refers to the spatial Jacobian resulting from approximations in the link lengths and joint positions.

**Step 3**

Due to the uncertainties in \(J_0\) that arise from this grasp scenario, a further realistic condition must be satisfied by the internal force control. Namely, the internal force control should be constant when the task frame is static. The reason for this is that when \(u_f\) is multiplied by \(\dot{J}_h \neq \dot{J}_h\), in (25), there will be a residual disturbance term that arises in \(\sigma_3\) that is multiplied by \(u_f\). Thus to ensure Assumption 7 is satisfied, \(u_f\) must be constant at the origin so that the derivative of the residual disturbance disappears. This condition on \(u_f\) is formally stated as:

**Assumption 9.** The internal force control satisfies:

\[(\dot{x}, \dot{\mathbf{x}}) = 0 \implies u_f = 0\] (44)

It is important to note that Assumption 9 is required due to the errors associated with the hand model uncertainties. This assumption can be removed if an accurate hand kinematic model is available such that \(\dot{J}_h = J_h\).

For vision-based grasping with tactile sensors, the heuristic internal force control (27) can be directly applied to satisfy Assumptions 8 and 9. However conventional grasp force optimization methods require \(G\) to explicitly enforce Assumption 8. Instead the same optimization can be performed by substituting \(\dot{G}\) with \(\dot{G}\) from (42) to enforce \(G u_f = 0\). This result is stated in the following lemma.

**Lemma 6.** For \(\hat{G}\) defined by (42), if \(u_f \in \text{Ker}(\hat{G})\), then \(u_f\) satisfies Assumption 8.

**Proof.** Refer to Appendix 7.4.
Step 4

The Jacobian \( J_h \) can be defined in a similar fashion as Section 4.1 such that Assumption 5 is satisfied, regardless of the unknown center of mass:

**Lemma 7.** Let \( \mathcal{A} \) be defined by \( R_{pa} = R_{po} \) with \( p_c \) fixed with respect to \( O \), and let \( J_a = P^T \dot{G} \), for \( \dot{G} \) defined by (42). Under Assumption 24 Assumption 5 holds.

**Proof.** Refer to Appendix 7.2

Step 5

By Lemma 7, the proposed control law is defined by (40), (42), (43), (26).

Step 6

Substitution of (40) into (20) results in (30) with \( \sigma_2 \) defined by:

\[
\sigma_2 = P^T (\dot{G} - G)(I_{3\times3} - G^T \dot{G})u_m
- (P^T \dot{G})^T G^T \dot{h}^T (h_f - \hat{h}_f)(P^T \dot{G})u_m + u_f
\]

(45)

The following lemma ensures that \( \sigma_2 \) satisfies Assumption 7:

**Lemma 8.** Under Assumptions 4, 6, 8, 9 and \( \sigma_2 \) defined by (45) satisfies Assumption 7.

**Proof.** Refer to Appendix 7.3

With Lemmas 7 and 8 the stability guarantees from Section 3 follow:

**Corollary 4.** (Vision-based grasping with tactile sensing) Let \( \mathcal{A} \) be fixed with respect to the object frame, \( O \), at an arbitrary position/orientation. Suppose measurements of \( q, x, x, p_f \) are available and Assumptions 4, 6, 8, 9 hold. The system (20) with control law (40), (42), (43), (26), is semi-globally exponentially stabilizable.

4.3 Vision-based Grasping with Limited/No Tactile Sensors

Section 4.2 requires that measurements of \( p_c \) are available to compute \( J_a, \dot{G} \) and \( u_f \). However the robotic hand may not have access to tactile measurements, or the available sensors may have significant measurement error. In order to deal with uncertainties or lack of knowledge of the contact locations, a further approximation of \( J_h \) and \( \dot{G} \) is required.

Let \( \hat{p}_{fc} \in \mathbb{R}^3 \) be an approximation of \( p_{fc} \) and let the approximate contact location of \( p_c \) be:

\[
\hat{p}_c = p_f(q) + \hat{p}_{fc}
\]

(46)

To ensure Assumptions 5 and 7 hold, the contact location approximation must not be changing when the task frame is static:

**Assumption 10.** The contact location approximation, \( \hat{p}_{fc} \), satisfies:

\[
(\dot{x}, \dot{x}) = 0 \implies \dot{\hat{p}}_{fc} = 0
\]

(47)

**Remark 12.** If no tactile information is available, \( \hat{p}_{fc} \) can be arbitrarily approximated by a fixed point on the fingertip (i.e. \( \hat{p}_{fc} \equiv p_{fc} \)) to satisfy Assumption 10

Step 1

Due to the availability of vision sensors, the same task frame defined in Section 4.2 applies here, such that \( p_c \) is fixed on the object and without loss of generality \( R_{pa} = R_{po} \).

Step 2

For the approximation of \( G \) with uncertain contact locations, (46) is used in place of \( p_c \) in (42):

\[
\dot{\bar{G}} = \begin{bmatrix} I_{3 \times 3}, & \ldots, & I_{3 \times 3} \end{bmatrix} (\hat{p}_{c1} - p_{a})\times \ldots (\hat{p}_{cn} - p_{a})\times
\]

(48)

A similar approximation is used to approximate \( J_h \) in which the points \( p_{fc} \) are substituted by \( \hat{p}_{fc} \) from (43):

\[
\dot{\bar{J}}_h(q, \hat{p}_{fc}) = \begin{bmatrix} I_{3 \times 3} & -(\hat{p}_{fc})\times \end{bmatrix} J_h(q)
\]

(49)

Step 3

The absence of reliable contact location measurements prevents an appropriate choice of internal force control directly from Remark 7. One solution for the internal force control when using the grasp force optimization approach is to substitute \( G \) with \( \dot{G} \) from (48) to implement the constraint \( \dot{G} u_f = 0 \). Another solution is to substitute \( p_c \) by \( \hat{p}_c \) in (27):

\[
u_f = k_f (\hat{p}_c - \hat{p}_{c1}, \ldots, \hat{p}_c - \hat{p}_{cn})^T
\]

(50)

Regardless of the approach, there is no guaranteed choice of \( u_f \) to satisfy Assumption 8. This manifests as the additional term related to \( u_f \) in \( \sigma_2 \). The consequence is that more integral action is required to compensate for this disturbance. Furthermore, the absence of any tactile information may result in slip that would violate Assumption 3.

Step 4

The following lemma provides a suitable choice for \( J_a \) that satisfies Assumption 5.

**Lemma 9.** Let \( \mathcal{A} \) be defined by \( R_{pa} = R_{po} \) with \( p_c \) fixed with respect to \( O \), and let \( J_a = P^T \dot{G} \), for \( \dot{G} \) defined by (42). Under Assumptions 4, 6, 8, 9 and Assumption 5 holds.

**Proof.** Refer to Appendix 7.3

Step 5

By Lemma 9 the proposed control law is defined by (40), (48), (49), (26).

Step 6

To show that Assumption 7 is satisfied, (40) is substituted into (20) resulting in (30) with \( \sigma_2 \) defined by:

\[
\sigma_2 = P^T (\dot{G} - G)(I_{3\times3} - G^T \dot{G})u_m + u_f
- (P^T \dot{G})^T G^T \dot{h}^T (h_f - \hat{h}_f)(P^T \dot{G})u_m + u_f
\]

(51)

The disturbance \( \sigma_2 \) is shown to satisfy Assumption 7.

**Lemma 10.** Under Assumptions 4, 6, 8, 9 and 10 \( \sigma_2 \) defined by (51) satisfies Assumption 7.
Corollary 5. (Vision-based grasping with limited/no tactile sensing) Let $\mathcal{A}$ be fixed with respect to the object frame, $O$, at an arbitrary position/orientation. Suppose measurements of $q, x, \dot{x}, p_c$ are available and Assumptions 1-4, 6, 9, 11 hold. The system (20) with control law (40), (48), (49), (26) is semi-globally exponentially stabilizable.

4.4. Tactile-based Blind Grasping

In many grasping cases, vision systems are impaired due to motion of the hand that occludes the object, or no such vision systems are available. In these cases, the robotic hand only has access to sensors integrated into the hand itself, which include tactile and joint angle sensors. This restriction complicates the design of a manipulation controller because a fixed reference point on the object is not available to define $\mathcal{A}$.

Step 1
Motivated by the approaches used in [30,31], the task frame is defined with respect to the joint angles and such that $x$ is a continuously differentiable function of the joint angles, $q$. The choice of $x(q)$ is restricted by the following assumption to ensure motion of the hand results in motion of the task frame:

Assumption 11. The function $x(q)$ is continuously differentiable, and $\frac{\partial x}{\partial q}$ is full rank.

Assumption 11 is a common implicit assumption in related work, and an acceptable choice of $x(q)$ is to define the frame with respect to the fingertip positions [30,31]:

$$p_s(q) = \frac{1}{k} \sum_{i=0}^{k} p_i(q)$$

$$R_{pa}(q) = \{p_x, p_y, p_z\}$$

where $p_s = p_x \times p_y, p_c = \frac{p_s - p_0}{||p_s - p_0||}, p_0 = \frac{(p_0 - p_s) \times (p_s - p_0)}{||p_s - p_0||}$.

A local parameterization of $R_{pa}$ is then used to define $\gamma_a$, one example of which is:

$$\gamma_a(q) = \begin{bmatrix} \arctan(-p_{z}/p_{y}) \\ \sqrt{1 - p_{z}^2} \end{bmatrix}$$

Note that (53) and (54) define $x$ as solely a function of $q$. Differentiation of (52) and (54) then provides a relation between $x$ and $(q,q)$, respectively. Thus the motivation behind this choice of frame is that only sensors required to determine the hand configuration are needed to define the task state, $x$. The consequence is that the task frame is no longer fixed to the object resulting in an additional disturbance in $\sigma_1$ (shown in the proof of Lemma 11).

Step 2
Due to the availability of tactile sensors, the approximations (42) and (43) for $\hat{G}$ and $\hat{J}$, respectively, from Section 4.2 are suitable for this grasp scenario.

Step 3
Again, the availability of tactile sensors allows for the same choice of internal force controllers as discussed in Section 4.2 such that Assumptions 3 and 9 hold.

Step 4
The choice of $J_a$ to satisfy Assumption 5 is defined in the following lemma:

Lemma 11. Let $p_a, R_{pa}$ be respectively defined by (52), (53) and let $J_a = P^T \hat{G}$ for $\hat{G}$ defined by (42). Under Assumptions 1-4, 6, 9, and 11, Assumption 5 holds.

Proof. Refer to Appendix 7.4

Step 5
The resulting control for this grasp scenario is (40), (42), (43), (26) for the task frame defined by (52), (53).

Step 6
Finally, to show that Assumption 7 is satisfied, $\sigma_2$ can be shown to have the same form as (45) after substitution of (40) in (20).

Lemma 12. Under Assumptions 1-4, 6, 9, and 2, $\sigma_2$ as defined by (45) satisfies Assumption 7.

Proof. Refer to proof of Lemma 8 (Appendix 7.3)

Corollary 6. (Tactile-based blind grasping) Let $\mathcal{A}$ be defined by (52), (53). Suppose measurements of $q, x, p_c$ are available, and Assumptions 1-4, 6, 9, and 11, Assumption 5 hold. The system (20) with control law (40), (42), (43), (26) is semi-globally exponentially stabilizable.

4.5. Blind Grasping

In this final scenario, the robotic hand has access to tactile sensors with measurement uncertainties or no tactile sensors at all. The controller follows directly from the previous grasp scenarios, where the choice of the task frame is identical to that used for blind grasping, and the choice of $G, J_h$ is the same as the vision with no tactile sensing scenario. Thus in this section, we show that the combination of these two approaches still satisfy Assumptions 5 and 7 so the stability guarantees follow.

Step 1
The frame $\mathcal{A}$ and states are defined by (52), (53), which is the same as from Section 4.4.

Step 2
The same approximations (48), (49) for $\hat{G}$ and $\hat{J}$, respectively, from Section 4.3 are applied here.

Step 3
Refer to Step 3 of Section 4.3.
Step 4
The following lemma provides an appropriate definition for $J_d$ under such limited grasp knowledge:

**Lemma 13.** Let $p_{m}, R_{m}$ be respectively defined by (52), (53) and let $J_d = \hat{P}^T \hat{G}$ with $\hat{G}$ defined by (48). Under Assumptions 7 14 6 9 and 11 Assumption 5 holds.

*Proof.* Refer to proof of Lemma 11 (Appendix 7.7).

Step 5
The resulting controller used for this scenario is (40), (48), (49), (26), with the task frame (52)-(54).

Step 6
The lack of reliable tactile sensors leads to the same discussion of the internal force control from Section 4.3 resulting in the requirement of Assumptions 5 and 10. The resulting disturbance $\sigma_2$ is thus (51). For completeness, we formally state the satisfaction of Assumption 5 in the following lemma:

**Lemma 14.** Under Assumptions 7 14 6 9 10 and 11 $\sigma_2$ as defined by (51) satisfies Assumption 5.

*Proof.* Refer to proof of Lemma 10 (Appendix 7.6).

**Corollary 7.** (Blind grasping) Let $A$ be defined by (52), (53). Suppose measurements of $q, x, \dot{p}_f$, are available, and Assumptions 7 14 6 9 10 11 hold. The system (20) with control law (40), (48), (49), (26) is semi-globally exponentially stabilizable

**Remark 13.** In related blind grasping research, the control solution causes an induced rolling disturbance, which requires additional control terms for compensation (28). In the proposed formulation presented here, the effect of no external information manifests as a disturbance which similarly a function of the proposed manipulation and internal force control terms as seen in (51). However the proposed control neatly compensates for these disturbances without requiring any additional control terms, and furthermore rejects unknown external disturbances which are not accounted for in the related blind grasping work (27) (28) (30) (37).

4.6. Existing Disturbance Compensators

In addition to knowledge of the hand-object models, most related work regarding disturbance rejection assume $\tau_c$, $\omega_c$ to be exactly known or an estimator is incorporated for compensation. These assumptions fit nicely into the framework here that deals with grasping scenarios with varying degrees of knowledge. In order to incorporate these existing compensators, we can define an exogenous input as:

$$ u_c = -\tau_c - \hat{J}_h^T \hat{G} \hat{\omega}_c $$

(55)

where $\hat{\tau}_c \in \mathbb{R}^m$ and $\hat{\omega}_c \in \mathbb{R}^6$ are the respective approximations of $\tau_c$ and $\omega_c$.

Under the condition that $(\dot{x}, \ddot{x}) = 0 \implies \dot{\tau}_c, \dot{\omega}_c = 0$, and based on the previous analysis, it is clear that the superposition of $u_c$ with $u$ satisfies the conditions of Theorem 1. As such, $u_c$ can be similarly incorporated into each of the practical implementations mentioned previously. The effect of $u_c$ is then to reduce the uncertainty that is otherwise compensated for by the integral action of (26).

5. Results

The proposed framework provides in-hand manipulation solutions for grasping scenarios in which external wrenches, object center of mass, and contact locations are unknown or uncertain. This section highlights the application of this robust framework to common grasp scenarios through numerical simulation and hardware implementation. Without loss of generality, the numerical simulation uses a 10x scaled model of the Allegro Hand (41) and demonstrates how the proposed control framework is applied to each scenario from Sections 4.1 to 4.5. Additionally, results from Section 3.2 are applied to the blind grasping scenario to demonstrate how the systematic gain tuning and semi-global properties presented improve the response of the system despite the robotic hand being deprived of any grasp information. Finally, the proposed control is implemented on the hardware-version of the Allegro Hand to demonstrate the proposed control in the blind grasping scenario.

5.1. Simulation

In the simulations, the Allegro Hand is grasping a rectangular prism as depicted in Figure 2. The initial grasp is purposefully offset from the object center of mass, where gravity acts, to accentuate effects of an unknown center of mass and unknown external disturbance. To define a valid reference for varying task frame definitions, $r$ is decomposed into $r = x(0) + \Delta r$, where $x(0)$ is the initial state defined by the initial task frame position and orientation, and $\Delta r \in \mathbb{R}^6$ is the desired reference change. The reference provided for the controller is $\Delta r = (0, -1, -2, 0, 0, 0)$, which relates to translating the object $-1$ m in the Y-direction, $-2$ m in the Z-direction, while maintaining the same X-position and initial orientation. The full simulation parameters are listed in Table 1. The parameters of the hand can be found at [http://www.simlab.co.kr/Allegro-Hand.htm](http://www.simlab.co.kr/Allegro-Hand.htm).

![Figure 2: Simulation setup.](image)

The five simulated grasping scenarios include “ideal grasping” where the object center of mass is known along with perfect tactile sensing (Section 4.1), “vision+tactile grasping” where
Table 1: Simulation Parameters

| Parameter                  | Value                                      |
|----------------------------|--------------------------------------------|
| Object dimensions          | 2.0 m × 0.34 m × 0.34 m                   |
| Object mass                | 1.0 kg                                     |
| Object moment of inertia   | diag([0.019, 0.343, 0.343])kgm²            |
| Initial $p_o$              | (−0.263, 0.041, 0.973)m                   |
| Initial $\bar{p}_t$        | (0.224, 0.107, 1.01)m                     |
| Initial $\gamma_a$         | (0, 0, 0)rad                               |
| $\tau_e$                  | $-1 \cdot I_{6 \times 6} \cdot \dot{q}$ Nm |
| $w_e$                      | (0, 0, -9.81, 0, 0, 0) N                 |
| $k_f$                      | 10                                         |
| $K_1$                      | $1.0 \cdot I_{6 \times 6}$               |
| $K_2$                      | $2.5 \cdot I_{6 \times 6}$               |
| $\bar{M}$                  | $0.02 \cdot I_{6 \times 6}$             |

Table 2: Grasp Scenarios and Control Laws

| Grasp Scenario             | $p_a$ | $R_{pa}$ | $\hat{G}$ | $\hat{J}_h$ | $u_f$ |
|----------------------------|-------|----------|-----------|------------|-------|
| Ideal                      | $p_a = p_o$ | $R_{pa} = R_{po}$ | $G$       | $J_h$      | $2.7$ |
| Vision+tactile             | fixed to $O$ | $R_{pa} = R_{po}$ | $4.2$ | $4.3$ | $4.4$ |
| Vision-only                | $52$ | $53$ | $48$ | $49$ | $50$ |
| Tactile-only               | $52$ | $53$ | $48$ | $49$ | $50$ |
| Blind Grasping             | $52$ | $53$ | $48$ | $49$ | $50$ |

The simulations were performed using Matlab’s ode45 integrator, with a simulation time of 15 seconds. The value of $k_f$ was empirically chosen such that the contact points do not lose contact with the object. The gains, $K_p, K_i, K_d$ were determined by (38), with $M, K_1, K_2$ defined as in [38] for $\varepsilon = 0.0005$, which was determined empirically. The resulting PID gains were $K_p = 100 \cdot I_{6 \times 6}, K_i = 40 \cdot I_{6 \times 6}, K_d = 40 \cdot I_{6 \times 6}$. The resulting error response of the five simulations are shown in Figures 3-7.

The responses in all five simulations show the respective proposed controller asymptotically driving the system error to zero, which implies manipulating the object to the desired reference, while simultaneously rejecting external disturbances. The effect of the limited knowledge of the grasping scenario is portrayed in the transient response of each simulation. The initial increase in error for all plots is a result of the unknown external disturbance acting on the system. During this transient period, the proposed control compensates for the uncertainties and ultimately drives the system error to zero, as shown in Figures 3-7.
mately drives the error to zero. As expected, the ideal grasping scenario shows the best performance in terms of least overshoot due to the fact that all properties of the grasp sans the external disturbances are known. In the remaining scenarios, $p_e$ is offset from $p_o$, and this error acts as a lever arm, magnifying the effect of $w_e$. This magnified disturbance is reflected in the larger overshoot in the orientation error seen in Figures 4-7. The transient behavior in Figures 4-7 are similar in terms of overshoot and settling time. This suggests that disturbances from contact contact location uncertainty, are small in magnitude compared to the external disturbances, $\tau_e, w_e$ acting on the hand-object. The simulations respectively verify the ability of the control laws from Section 4 to provide asymptotic stability to the origin for varying degrees of grasp knowledge.

For decreasing values of $\varepsilon$, Figure 10 shows improved performance with respect to smaller overshoot and settling times, and Figure 11 shows smaller contact position deviation resulting from the uncertain external disturbance. This trend is consistent with the notion from Corollary 2 where as $\varepsilon$ is decreased ($K_p, K_i, K_d$ increased), improved stability conditions are guaranteed in the form of asymptotic then exponential stability. For a value of $\varepsilon = 0.002$, the unknown disturbance moves the object sufficiently far such that the fingertips roll off of the object surface. These plots show the benefit of the semi-global guarantees of the proposed control. Namely, decreasing $\varepsilon$ (increasing $K_p, K_i, K_d$), improves the system performance. For object manipulation purposes this not only refers to system performance in manipulating the object, but also in reducing how much the contact points travel so that they remain on the fingertip surface.

We note that the Allegro Hand is in fact a redundant hand with four joints per finger. This redundancy does not satisfy Assumption 1, however the results show that the proposed framework does indeed provide stability in such a case. The effect of
viscous friction in \(\tau_r\) acts to dampen out any null space motion of the hand. This friction effect can be artificially included in the controller via the exogenous input \(u_e = -\beta \ddot{q}\) as a heuristic approach for redundant hands.

Limitations of the proposed control result from practical implementations. The semi-global guarantees are beneficial from the design perspective, but in practice the robotic hand will not be able to reject external disturbances of all magnitudes due to actuator constraints. The designer must choose appropriate hardware/gains for the desired task/actuator constraints.

5.2. Experimental Results

In the numerical simulations, the proposed framework was applied to common grasp scenarios with varying levels of grasp knowledge. Here, the proposed control framework was applied to the Allegro hand under the worst case scenario, that of blind grasping in which the hand was commanded to manipulate the two objects shown in Figure 12 using only joint angle information. The Allegro Hand setup includes a NI USB-8473s High-Speed CAN, which operates at a fixed sampling frequency of 333 Hz. The two objects are a 0.051 kg spherically-shaped thermos lid, and an 0.086 kg apple.

To perform the experiments, the objects were placed in the robotic hand grasp prior to implementing the control. As shown in Figure 12(a), the only contact between the object and hand occurs at the fingertips. The reference change of \(\Delta \mathbf{r}\) at the fingertips. The reference change of \(\Delta \mathbf{r}\) about the palm, while maintaining the initial position. The initial state \(x(0)\) was computed via (52), (53) for the initial hand configuration.

![Figure 12: Experimental setup.](image)

The controller used in the experiment consisted of the control proposed from Section 4.3 augmented with a gravity compensation component, which only compensates for the effect of gravity on the hand, not the object. The resulting controller is:

\[
\mathbf{u} = J_i^T ((P^T \dot{\mathbf{G}}) \mathbf{u}_m + \ddot{\mathbf{r}}) + \ddot{\mathbf{g}}(\mathbf{q})
\]

where \(\ddot{\mathbf{g}}(\mathbf{q}) \in \mathbb{R}^m\) is the approximate torque induced by gravity acting on the hand, and \(\dot{\mathbf{G}}, J_i, k_i\) are respectively defined by (48), (49), and (50). The task frame is defined by (52), (53) with the local parameterization (54) used to defined \(y_r\). Note that (56) is based in continuous time. For hardware considerations, the proposed controllers were implemented in discrete time by using Euler’s method for numerical approximation. It was assumed that the sampling time is small enough such that the time delay from the zero order hold is negligible on the stability of the system. We assumed quantization errors from the on-board sensors to be negligible.

The gain tuning method from Section 3.2 was not used due to the effects of noise found in the experimental setup. Instead, the following PID gains were determined empirically:

\[
\begin{align*}
K_p &= \text{diag}\{100, 100, 100, 0.5, 0.5, 0.5\} \\
K_i &= \text{diag}\{30, 30, 30, 0.4, 0.4, 0.4\} \\
K_d &= \text{diag}\{0.005, 0.005, 0.005, 0.05, 0.05, 0.05\}
\end{align*}
\]

In practice, robotic hands may be subject to static friction, which is a dead-zone disturbance that does not satisfy the smoothness property of Assumption 6. There exist many compensation techniques in the literature to handle static friction. A dither-based static friction compensation is used here in which a small sinusoidal dither signal to the control, which acts to vibrate the system. The static friction compensating controller is:

\[
\mathbf{u} = J_i^T ((P^T \dot{\mathbf{G}}) \mathbf{u}_m + \ddot{\mathbf{r}}) + \ddot{\mathbf{g}}(\mathbf{q}) + \mathbf{d}(t)
\]

where \(\mathbf{d}(t) \in \mathbb{R}^m\) is a vector comprised of individual dither signals \(d_j(t) \in \mathbb{R}\) for \(j = 1, \ldots, m\) defined by

\[
d_j(t) = a_j \sin(bt) + c_j,
\]

where \(a_j, b_j, c_j \in \mathbb{R}\) are dither signal parameters that were determined empirically. Furthermore, to avoid integrator windup from occurring at small error values, an error bound was incorporated to the integral and dither terms of (57). Thus the integration and dither were applied to the system for position errors \(\|e_1, e_2, e_3\|_{\infty} > 0.0005\) m, and orientation errors \(\|e_4, e_5, e_6\|_{\infty} > 0.005\) rad.

Figures 13 and 14 show the responses of the system for both objects using the static friction compensation controller (57). The plots show the error converging to the prescribed error bounds. The orientation error for both objects is fastest to converge than the position error, which are most likely a result of the heuristic tuning used. The benefits of the proposed framework are portrayed in the ability to stably grasp both objects of varying size and weight using only joint angle sensors. Note the settling time appears to be affected by the dither-based static friction compensation in combination with the tight error bounds. This suggests that dither-based static friction compensation may not be beneficial for high performance manipulation tasks. Future work will investigate how to deal with disturbances such as static friction, sampling time, and noise that arise from practical implementation.

![Figure 13: Response of (56) when manipulating apple.](image)
6. Conclusion

In this paper a robust control framework for in-hand manipulation was proposed to handle disturbances that manifest from unknown/uncertain hand-object model, external wrenches, and contact locations. The framework provides semi-global asymptotic and exponential stability about a set pose reference point. Case studies were presented to show how the framework provides control solutions to common grasping scenarios found in the literature in which the robotic hand is equipped with sensors that provide vision, tactile, or no external measurements. Simulation and experimental results demonstrate the efficacy of the proposed framework.

7. Appendix

7.1. Proof of Lemma 5

Proof. \( J_\omega \) is full rank and twice continuously differentiable from Assumptions 2 and 4. Under Assumption 3, (14) is shown by direct calculation:

\[
J_\omega x = G^T \begin{bmatrix} \dot{p}_a \\ \omega_o \end{bmatrix} = G^T \begin{bmatrix} p_o \\ \omega_o \end{bmatrix} = v_c,
\]

with \( \sigma_1 \equiv 0 \).

7.2. Proof of Lemma 7

Proof. \( J_\omega \) is full rank, twice continuously differentiable directly from Assumptions 2 and 4. Note that because the \( \mathcal{A} \) frame is fixed with respect to the object, \( \omega_o = \omega_o \). Under Assumption 3, (14) is shown by direct calculation:

\[
J_\omega x = G^T \begin{bmatrix} p_o \\ \omega_o \end{bmatrix} = p_o + \omega_o \times (p_c - p_o), \forall i
\]

\[
= p_o - \omega_o \times (p_o - p_c) + \omega_o \times (p_c - p_o), \forall i
\]

\[
= p_o -\omega_o \times p_o + \omega_o \times p_c, \forall i
\]

\[
= \dot{x}_{\omega}, \forall i
\]

with \( \sigma_1 \equiv 0 \).

7.3. Proof of Lemma 8

Proof. The disturbance term \( \sigma_2 \) defined by (45) can be rewritten as:

\[
\sigma_2 = D_1 u_m + D_2 u_f
\]

where \( D_1 := D_1(x, q, x_s) \in \mathbb{R}^{6 \times 6} \) and \( D_2(x, q, x_s) \in \mathbb{R}^{6 \times 6} \) are defined respectively from (45). Differentiation of (58) results in:

\[
\dot{\sigma}_2 = \frac{d}{dt} [D_1 u_m + D_1 (K_o \omega + K_e \omega + K_e \bar{e})]
\]

\[
+ \frac{d}{dt} [D_2 u_f + D_2 \bar{u}_f] \tag{59}
\]

In order for the proof from Theorem 1 to follow, \( \dot{\sigma}_2 \) must equal 0 at \( (e, \bar{e}) = 0 \) to satisfy Assumption 7. It is clear that the second term disappears at \( (e, \bar{e}) = 0 \). By the same argument in Lemma 4, the derivative terms \( \frac{d}{dt} [D_1 u_m] \) and \( \frac{d}{dt} [D_2 u_f] \) also disappear. The final term is zero by Assumption 9. Thus \( \sigma_2 \) satisfies Assumption 7.

7.4. Proof of Lemma 9

Proof. The grasp map, \( G \), can be re-written with respect to \( \hat{G} \) by \( \delta G = \hat{G} + \delta \hat{G} \) where \( \delta \hat{G} \) is:

\[
\delta G = \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}
\]

The term \( \delta G \) can now be simplified using \( u_f \in \text{Ker} (\hat{G}) \) as follows:

\[
Gu_f = \delta Gu_f
\]

\[
= \sum_{i=1}^{k} \begin{bmatrix} (p_o - p_c) \times & \cdots & (p_o - p_a) \times \end{bmatrix} u_f
\]

\[
= \begin{bmatrix} 0 \\ (p_o - p_a) \times \sum_{i=1}^{k} u_f \end{bmatrix}
\]

\[
= 0
\]

The final step, \( \sum_{i=1}^{k} u_f = 0 \), is true from \( \hat{G} u_f = 0 \).

7.5. Proof of Lemma 9

Proof. \( J_\omega \) is full rank, twice continuously differentiable directly from Assumptions 2 and 4. Note that because the \( \mathcal{A} \) frame is fixed with respect to the object, \( \omega_o = \omega_o \). Let \( p_{bc} = \hat{p}_c - p_c \). Under Assumption 3, (14) is shown by direct calculation:

\[
J_\omega x = G^T \begin{bmatrix} p_o \\ \omega_o \end{bmatrix} = p_o + \omega_o \times (\hat{p}_c - p_c), \forall i
\]

\[
= p_o - \omega_o \times (p_o - p_c) + \omega_o \times (p_c - p_o), \forall i
\]

\[
= p_o - \omega_o \times p_o + \omega_o \times p_c, \forall i
\]

\[
= \dot{x}_\omega, \forall i
\]

where \( \sigma_1 \equiv 0 \).
\[ \sigma_1 = \omega_0 \times p + 2 \omega_0 \times \dot{p} + \omega_0 \times \dot{p} \]

From \( \omega_0 = \omega_o \), and the fact that \( p_0 \) is fixed on the object such that \( \dot{p}_0 = p_0 \), it is clear that \( (\dot{x}, \dot{x}) = 0 \implies (\dot{x}_o, \dot{x}_o) = 0 \). Thus \( (\dot{x}, \dot{x}) = 0 \implies (\omega_0, \omega_0) = 0 \implies (\sigma_1, \sigma_1) = 0 \).

To show that \( \sigma_t = 0 \) when \( \dot{x}_o = (\dot{v}, \omega_0) = 0 \), this is shown by first differentiating (7):

\[ \frac{d}{dt} [M_{ho} \ddot{x}_o + M_{ho} \dddot{x}_o + J_h \dddot{x}_o] = \frac{d}{dt} [G J_h^{-1} (u + \tau_c) + G J_h^{-1} (\dot{u} + \tau_c) + \dot{w}_c] \]

With Assumption 6 and for a constant reference, it follows that at \( (\dot{x}, \dot{x}) = 0 \) reduces to:

\[ M_{ho} \dddot{x}_o = \frac{d}{dt} [G J_h^{-1} (u + \tau_c) + G J_h^{-1} u] \]  

From Assumption 9 and the relation (19), it follows that \( (\dot{x}, \dot{x}) = 0 \implies (\dot{x}_o, \dot{x}_o) = 0 \). Thus the term \( \frac{d}{dt} G J_h^{-1} [u + \tau_c] \) is zero when \( (\dot{x}, \dot{x}) = 0 \). By the same argument in Lemma 8 and with Assumption 7 it follows that for a constant reference, \( u = 0 \) when \( \dot{x}_o = 0 \). From Lemma 9 it is then clear that \( \dot{x}_o = 0 \) when \( (\dot{x}, \dot{x}) = 0 \).

Thus it follows that \( (\dot{x}, \dot{x}) = 0 \implies (\sigma_1, \sigma_1, \sigma_1) = 0 \).

7.6. Proof of Lemma 10

Proof. The disturbance term \( \sigma_2 \) from (51) can be re-written as:

\[ \sigma_2 = D_1 u_m + D_2 u_f \]

where \( D_1 := D_1 (x, q, x_o, \dot{p}_f) \in \mathbb{R}^{6 \times 6} \) and \( D_2 := D_2 (x, q, x_o, \dot{p}_f) \in \mathbb{R}^{6 \times 3} \) are defined respectively from (51). By the same argument in Lemma 9 and with Assumption 10 the derivative terms \( \frac{d}{dt} [D_1] \) and \( \frac{d}{dt} [D_2] \) disappear when \( (\dot{e}, \dot{e}, \dot{e}) = 0 \). The proof follows from that of Lemma 8.

7.7. Proof of Lemma 11

Proof. From Assumptions 2 and 4, \( J_a \) is full rank, twice continuously differentiable. Note that unlike the vision-grasping case, \( p_0 \) is no longer fixed on the object, but is defined relative to the hand configuration. Under Assumptions 1 and 3 \( \dot{x} \) can be written with respect to \( \dot{x}_o \) via (6):

\[ \dot{x} = \frac{\partial x}{\partial q} \dot{q} \]

\[ = \frac{\partial x}{\partial q} J_{h}^{-1} G^T \dot{x}_o \]

Thus by Assumption 11 there is a one-to-one relation between \( \dot{x} \) and \( \dot{x}_o \), such that \( \dot{x} = 0 \implies \dot{x}_o = 0 \). Furthermore, differentiation of (64) is:

\[ \dot{x} = \frac{d}{dt} \frac{\partial x}{\partial q} J_{h}^{-1} G^T \dot{x}_o + \frac{\partial x}{\partial q} J_{h}^{-1} G^T \dot{x}_o \]

It is clear then that \( (\dot{x}, \dot{x}) = 0 \implies (\dot{x}_o, \dot{x}_o) = 0 \). By Assumptions 6 and 9 the same argument in Lemma 9 applies such that \( (\dot{x}, \dot{x}) = 0 \implies (\dot{x}_o, \dot{x}_o, \dot{x}_o) = 0 \).

Now that a relation between \( (\dot{x}, \dot{x}) \) and \( (\dot{x}_o, \dot{x}_o, \dot{x}_o) \) has been established, \( \sigma_1 \) is determined from (65) and (15):

\[ \sigma_1 = (J_a^T \frac{\partial x}{\partial q} J_{h}^{-1} - I_{3 \times 3}) G^T \dot{x}_o \]

Let \( B(x, q, x_o) = (J_a^T \frac{\partial x}{\partial q} J_{h}^{-1} - I_{3 \times 3}) G^T \), such that \( \sigma_1 \) and its derivative can be written as:

\[ \sigma_1 = B \dot{x}_o \]

\[ \sigma_1 = \frac{d}{dt} [B \dot{x}_o + B \dot{x}_o] \]

From the relation \( (\dot{x}, \dot{x}) = 0 \implies (\dot{x}_o, \dot{x}_o, \dot{x}_o) = 0 \) it is clear that \( (\sigma_1, \sigma_1, \sigma_1) = 0 \).

Acknowledgement

This work is supported by the Valma Angliss Trust.

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