Mathematic Model and Error Analysis of Moving-Base Rotating Accelerometer Gravity Gradiometer

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Limited by manufacturing technology, in moving-base gravity gradiometry, accelerometer mounting errors and mismatch cause a rotating accelerometer gravity gradiometer (RAGG) to be susceptible to its own motion. In this study, we comprehensively consider accelerometer mounting errors, accelerometer linear scale factors imbalances, accelerometer second-order error coefficients, RAGG internal movement, and construct three RAGG models—namely, a numerical model, an 54 parameters analytical model, and a 25 parameters simplified analytical model. Moreover, by setting the gravitational gradient excitation of RAGG to zero, the RAGG motion error model is derived. The analytical model and motion error model are used to interpret the error propagation mechanism and develop RAGG technical solutions, such as motion error compensation, fault diagnosis, etc. The numerical model is used to facilitate the verification of the RAGG analytical model and RAGG technical solutions. We performed multifrequency gravitational gradient simulation experiment and dynamic sweep frequency experiments to verify the three RAGG models, and evaluate the dynamic noise floor of the analytical models. In dynamic experiments, standard deviation of RAGG vertical specific force grows from 10 to 40 mg (1 g = 9.8 m/s²); that of the lateral specific force grows from 5 to 20 mg; the noise density of the analytical model remains stable within 2.5 E/√Hz, and that of the simplified analytical model is in the range of 2.64–41.27 E/√Hz.

NOMENCLATURE
Coordinate frames

- \( a_{i}, \alpha_{i} \) (o-p-frame).
- \( o_{m}x_{m}y_{m}z_{m} \).
- \( o_{p}x_{p}y_{p}z_{p} \).
- \( o_{m1}x_{m1}y_{m1}z_{m1} \).
- \( x_{1}y_{1}z_{1} \).
- \( x_{y}y_{z}z_{x} \).
- \( o_{x}x_{i}y_{i}z_{i} \).

\( RAGG \) accelerometer mounting parameters

- \( \beta_{jx} \).
- \( \beta_{jy} \).
- \( \beta_{jz} \).
- \( \theta_{jx}, \theta_{jy}, \theta_{jz} \).
- \( R_{j} \).
- \( f_{jx}, f_{jy}, f_{jz} \).

Altitude angle of accelerometer \( A_{j} \), Initial phase angle of accelerometer \( A_{j} \), Attitude angles of \( A_{j} \) between the accelerometer measurement frame and the accelerometer nominal frame of the actual mounting position, Radial distance of accelerometer \( A_{j} \), Specific force in accelerometer nominal frame of actual mounting position of the accelerometer \( A_{j} \), in units of V/g.
Null bias of accelerometer \( A_j \), \( k_{j0} = K_{j1}K_{j0} \), individually in units of V and g.

Second-order error coefficients of accelerometer \( A_j \), in units of \( V/g^2 \).

Second-order error coefficients of accelerometer \( A_j \), in units of \( V/g^2 \).

Voltage output of the accelerometer \( A_j \), in units of V.

Angular velocity vector and angular acceleration of the m-frame with respect to the i-frame or the angular velocity vector and angular acceleration of the RAGG analytical model parameters.

\[ \omega_{im1}, \dot{\omega}_{im1} \]

Angular velocity vector and angular acceleration of the p-frame with respect to the inertial frame or the angular velocity vector and angular acceleration of the RAGG body with respect to the inertial frame.

Angular velocity vector and angular acceleration of the m-frame or the angular velocity vector and angular acceleration of the RAGG rotary stage plane driven by its motor rotation.

Angular velocity vector and angular acceleration of the m-frame or the angular velocity vector and angular acceleration of the RAGG rotary stage plane motion error model.

Gravitational acceleration of accelerometer \( A_j \).

Absolute acceleration of accelerometer \( A_j \) (acceleration with respect to the inertial frame).

Specific force of accelerometer \( A_j \).

Position vector from the origin of the m-frame to that of m1-frame; since the m-frame has the same origin with the m1-frame, \( r_{0oA_j} = 0 \).

Position vector from the RAGG rotary stage base to the center of the RAGG rotary stage plane; it is used to describe the motion of RAGG rotary stage plane (internal movement of RAGG).

Position vector from the center of the RAGG rotary stage plane to actual mounting position of the accelerometer \( A_j \).

Gravitational gradient tensor and components of gravitational gradient tensor.

Rotation speed of RAGG rotary stage plane driven by motor.

Specific force at the center of the RAGG rotary stage plane; \( f'_{cmn} = [a_x, a_y, a_z] \).

Sum of the four accelerometers of the terms denoted by subscript; for example, \( \sum k_{iR} = k_{11}R_1 + k_{21}R_2 + k_{31}R_3 + k_{41}R_4 \).

RAGG motion error model.

Imbalance terms denoted by subscript between two pairs accelerometers \( A_1, A_2 \) and \( A_3, A_4 \); for example, \( D_{1234} = k_{14}R_1 + k_{24}R_2 - k_{34}R_3 - k_{44}R_4 \).

Imbalance terms denoted by subscript between accelerometers \( A_1 \) and \( A_2 \); for example, \( D_{12} = k_{11}R_1 - k_{21}R_2 \).

Imbalance terms denoted by subscript between accelerometers \( A_3 \) and \( A_4 \); for example, \( D_{34} = k_{31}R_3 - k_{41}R_4 \).

Motion vector of RAGG 54 parameters motion error model.

Motion vector of RAGG 25 parameters motion error model.

Input vector of RAGG 25 parameters analytical model.

Input vector of RAGG 54 parameters analytical model.

Propagation parameters vector of RAGG 25 parameters analytical model.
1. INTRODUCTION

Airborne gravity gradiometry is an advanced technology for surveying a gravity field; it acquires gravity field information with high efficiency and high spatial resolution. Compared with gravity information, the gravity gradient tensor provides more information on the field source such as orientation, depth, and shape. The world’s first airborne gravity gradiometry was performed using the Falcon-AGG system in October 1999. Airborne gravity gradiometry has now been conducted for nearly 20 years, and the experience gained in airborne gravity gradiometry, and the analysis and interpretation of gravity gradient data have greatly promoted developments in geological science, resource exploration, high-precision navigation, and related fields [1]–[8]. The application value of gravity gradiometry has been recognized, and the associated technology and data interpretation have become of interest in scientific research, which has promoted development of gravity gradiometer. There are many different gravity gradiometers under development, for example, rotating accelerometer gravity gradiometers (RAGGs), superconducting gravity gradiometers, cold atomic interferometer gravity gradiometers, MEMS gravity gradiometers, and gravitec gravity gradiometers [9]–[14]. However, RAGGs are the only type successfully used in airborne surveys. Companies that operate commercial RAGG systems are—Lockheed martin corporation, Bell Geospace corporation, CGG corporation.

RAGG was developed by Ernest Metzger of Bell-Aerospace in the 1980s. The minimum configuration of an RAGG consists of two pairs of high-quality, low-noise, matched accelerometers, which are equi-spaced on a rotary stage with their sensitive axes tangential to the disk. The spin axis of the rotary stage is perpendicular to the plane of the disk, and passes through its center [15], [16]. The RAGG measures the gradients in the stage plane (RAGG input plane). The stage rotates at a constant speed, typically 0.25 Hz; this rotation results in the gravity gradient signal being modulated at 0.5 Hz, while the linear accelerations in the stage plane are modulated at 0.25 Hz. The sum of the diametrically opposed accelerometer reject linear accelerations in the stage plane, and the difference of the sum of the two pairs accelerometer cancel the angular acceleration about the spin axis and zero bias. Because of imperfection of RAGG, such as accelerometers input–output characteristics mismatch, sensitive axis misalignment, etc., RAGG is still sensitive to its motion.

The dynamic threshold is an important indicator of RAGG. It is defined as the worst dynamic working environment that maintains the RAGG measurement accuracy index; and it is determined by the motion sensitivity of the gravity gradiometer. For example, the RAGG with baseline distance 0.1 m, measurement accuracy 1 E, dynamic threshold 0.1 g, then its motion sensitivity should be better than $10^{-11}$. Without error compensation techniques, the motion sensitivity of $10^{-11}$ requires that the mismatch degree of the RAGG accelerometers should be better than $10^{-11}$ and the misalignment angles of RAGG accelerometer sensitive axis should be smaller than $10^{-11}$ rad. In the absence of error compensation technology, it is difficult to complete the development of an RAGG only relying on the current manufacturing technology. Motion error compensation is a common technical problem of all kinds of moving-base gravity gradiometers. To ensure the gravity gradient information to be used in desirable applications, the noise level of 7 E/$\sqrt{\text{Hz}}$ in the dynamic environment of survey flying is desirable. M. H. Dransfield reports the effect of turbulence on current gravity gradiometer: noise levels of an FTG GGI is about 13–23 E/$\sqrt{\text{Hz}}$ at 12–40 mg, and that of a FALCON GGI is about 3 ~ 4 E/$\sqrt{\text{Hz}}$ at 28–64 mg [17]. Currently, the turbulence threshold of airborne gravity gradiometry is about 70–100 mg.

The analytical model of the RAGG quantifies the mathematical relationship between the imperfection factors and the error propagation coefficients; it interprets the error propagation mechanism and is the theoretical basis for motion error compensation. The current RAGG error analysis is mainly single-factor perturbation analysis, which is used to analyze the accuracy requirements for RAGG manufacturing [18]; and there are no published literature that provides a complete RAGG analytical model or motion error model. In this article, we synthetically consider RAGG external movements, RAGG internal movements, accelerometer installation errors, accelerometer scale-factor imbalance, and accelerometer second-order error coefficients, and deduce three RAGG models: a numerical model, a analytical model, and a simplified analytical model. At the same time, we also derive the RAGG motion error model, which can be directly used for motion error compensation.

II. MODELS OF THE RAGG

A. RAGG Analytical Model

To facilitate understanding, we first briefly describe the input–output of RAGG, and briefly outline RAGG modeling process. Fig. 1(a) is the input and output model of the RAGG. The RAGG input includes gravitational gradient tensor, the RAGG internal movement, and external movement. The internal movement mainly include rotation and runout of the RAGG rotary stage plane; the external movement is the residual motion after motion isolation caused by moving base. Due to inaccurate manufacture of parts and assembly, the RAGG has many imperfection factors, such as accelerometer mounting position error, accelerometer input axis misalignment, accelerometer linear scale factors.
mismatch, accelerometer high order coefficients, circuit mismatch, frequency characteristic mismatch (flatness difference of amplitude spectrum and group delay difference). These imperfection factors determine the RAGG error propagation coefficients and RAGG scale factors, transferring the RAGG input to the output. The RAGG working environment (temperature, atmospheric pressure, etc.) acts on the RAGG unperfect factors, indirectly affecting the error propagations. Modeling the RAGG is to determine the relationship between its input and output, quantifying the relationship of error propagation coefficients, RAGG scale factors, the RAGG imperfection factors and RAGG working environment. Fig. 1(b) describes the modeling process of RAGG analytical model.

1) Accelerometer Installation Error and Input–Output Model: To simplify description of the RAGG accelerometer installation error and the RAGG internal movement (motion of the RAGG rotary stage), three frames are adopted: the RAGG rotary stage-fixed frame (p-frame), the RAGG measurement frame (m-frame), and the RAGG rotation frame (m1-frame). Fig. 2 shows the definition and the relative orientation of three frames. The origin of the RAGG measurement frame (m-frame) is at the center point of the rotary stage plane, and its x- and y-axes point, respectively, to the initial positions A1 and A3 of the accelerometer; its z-axis points to the actual rotation axis (instantaneous rotation axis).

The origin of the rotary stage-fixed frame (p-frame) is at the center point of the rotary stage base, in the initial state, the p-frame parallel to the RAGG measurement frame. The p-frame fixes to the RAGG rotary stage base, and only the RAGG external movements change its attitudes. Due to imperfect bearings, centroid deviation from axis, and other reasons, the RAGG rotary stage plane will runout and the rotation axis will deviate from its ideal rotation axis. Because of the linear movement and angular movement of RAGG rotary stage plane, the attitude of m-frame with respect to p-frame will keep changing. The RAGG rotation frame (m1-frame) has the same origin with the m-frame. In the initial state, the m1-frame coincides with the m-frame; it fixes to the rotary stage plane, and rotates with the rotary stage plane. The RAGG rotary stage plane is driven by motor rotation; using \( \Omega_1 \) represent the rotation speed of RAGG rotary stage motor, then the m1-frame results from the rotation of the m-frame about its z-axis by \( \Omega_1 t \).

The accelerometer mounting errors consist of mounting position errors and input-axis misalignments. We take the accelerometer A1 as an example for the mounting errors. In Fig. 3, A1, A2, A3, and A4 represent the nominal mounting positions, A1m represents the actual mounting position of the accelerometer A1 and the deviation from the nominal...
installation point. Another three reference coordinate systems are adopted: the accelerometer nominal frame of the nominal mounting position (xyz, axes marked in red), the accelerometer nominal frame of the actual mounting position (x1y1z1, axes marked in magenta), and the accelerometer measurement frame (a1a2a3, axes marked in orange). The accelerometer nominal frame of the nominal mounting position (xyz, axes marked in red) and the accelerometer nominal frame of the actual mounting position (x1y1z1, axes marked in magenta) are all the accelerometer nominal frame, these two reference coordinate systems are named after the location of the origin: the accelerometer nominal mounting position and the accelerometer actual mounting position.

The origin of the accelerometer nominal frame is located at the accelerometer mounting position; its x-axis is tangential to the rotary stage along the rotating direction, and its y-axis is from the stage center to the accelerometer position along the radial direction. Among the three frames, only the accelerometer measurement frame (a1a2a3) and the accelerometer nominal frame of the actual accelerometer mounting position (x1y1z1) are concentric frames.

The accelerometer mounting position error is the position difference between the actual mounting position and the nominal mounting position. Misalignment error is the orientation deviation between the accelerometer input axis (a1) and the tangential direction of the actual accelerometer mounting position (x1). We can use three parameters to determine the accelerometer mounting position with respect to the nominal mounting position: radial distance, initial phase angle, and altitude angle. The radial distance is the distance from the stage center (the origin of m-frame) to the accelerometer mounting position. We use the notation Rj to denote the radial distance of accelerometer Aj. The radial distance Rj of accelerometer Aj can also be expressed as R = R + dRj, where dRj is the radial distance error of accelerometer Aj, R is accelerometer nominal mounting distance. The nominal mounting positions of the four accelerometers are in the same plane, and we define this as the reference plane. The central angle from the accelerometer nominal mounting position to the projection of the actual accelerometer mounting position on the reference plane is defined as the initial phase angle. The notation βjz denotes the initial phase angle of accelerometer Aj. If the direction vector from the accelerometer nominal mounting position to the actual mounting position coincides with the rotating direction of the disk, then the initial phase angle βjz is positive; otherwise, the angle is negative. The angle between the radial distance line and the reference plane is defined as the altitude angle. The notation βjz represents the altitude angle of accelerometer Aj. If the z-coordinate of the actual accelerometer mounting position in the RAGG measurement frame is positive, then its corresponding altitude angle is positive; otherwise, the angle is negative.

The second type of mounting error is a misalignment error between the accelerometer measurement frame and the accelerometer nominal frame of the actual mounting position.

Three small angles θjx, θjy, and θjz, are used to describe it. The accelerometer measurement frame results from the rotation of the accelerometer nominal frame of the actual mounting position first about the x-axis by θjx, second about the y-axis by θjy, and then about the z-axis by θjz. Correspondingly, if we rotate the accelerometer measurement frame about its z-axis by −θjz and then about its y-axis by −θjy, then the input axis (a1) will coincide with the x-axis of the accelerometer nominal frame of the actual mounting position (tangential direction of the disk), so θjy and θjz are the misalignment error of accelerometer input axis. From the above, we can use the six parameters (Rj, βjx, βjy, βjz, θjx, and θjz) to determine the accelerometer mounting error of the RAGG.

The abbreviated version of the force rebalance accelerometer model equation is given [19]

\[ V_j/K_{j1} = f_{j1} + K_{j0} + K_{j2}f_{j2}^2 + K_{j5}f_{j0}^2 + K_{j7}f_{jp}^2 + K_{j4}f_{j1}f_{jp} + K_{j6}f_{j0}f_{j0} + K_{j8}f_{j0}f_{jp} + \cdots \]  \( (1) \)

where Vj is the electrical voltage output of accelerometer Aj; fj1, fj0, fjp are the applied specific forces in the directions of the input, output, and pendulous axes, respectively; \( K_{j1} \) is the linear scale factor (in units of V/g); \( K_{j0} \) is the null bias (in units of g); and \( K_{j2}, K_{j4}, K_{j5}, K_{j6}, K_{j7}, K_{j8} \) are the second-order error coefficients (in units of g/V^2). The RAGG with baseline distance 0.1 m, then disturbance acceleration of 10^-11 cause 1 E measurement error. Assume the RAGG dynamic threshold is 0.1 g, that is, the maximum input acceleration is 0.1 g. Without error compensation, the requirements of linear scale factors consistent degree should be smaller than \( 10^{-11}/(0.1g) = 10^{-10} \); the second-order error coefficients should be smaller than \( 10^{-11}g/(0.1g)^2 = 10^{-9}g/g^2 \); and the third-order error coefficients should be smaller than \( 10^{-11}g/(0.1g)^3 = 10^{-8}g/g^3 \). It is reported that with current manufacturing technology, the second-order error coefficients could achieve \( 10^{-6}g/g^2 \)[19]; usually the
higher order error coefficient is several orders of magnitude smaller than the second-order error coefficient; thus, in the RAGG modeling, the high-order error coefficient of the accelerometer is not considered.

The attitude angle (misalignment angle) between the accelerometer measurement frame and the accelerometer nominal frame the actual mounting position is three small angles: $\theta_{jx}$, $\theta_{jy}$, and $\theta_{jz}$. The transformation matrix from the accelerometer nominal frame of the actual mounting position to the accelerometer measurement frame is \[ C = \begin{bmatrix} 1 & \theta_{jz} & -\theta_{jy} \\ -\theta_{jz} & 1 & \theta_{jx} \\ \theta_{jy} & -\theta_{jx} & 1 \end{bmatrix}. \] (2)

Let $f_{jx}$, $f_{jy}$, $f_{jz}$ denote the coordinates of the specific force of accelerometer $A_j$ in the accelerometer nominal frame of the actual mounting position. So, we have

\[ f_{ji} = f_{jx} - f_{jx}\theta_{jy} + f_{jy}\theta_{jz} \]
\[ f_{j0} = f_{jy} + f_{jx}\theta_{jz} - f_{jz}\theta_{jx} \]
\[ f_{jp} = f_{jz} - f_{jy}\theta_{jx} + f_{jx}\theta_{jy}. \] (3)

Substituting (3) into (1), we get

\[ V_{j}/K_{ji} = f_{jx} + f_{jy}\theta_{jz}^* + f_{jz}\theta_{jy}^* + J_{j0} + K_{j1}f_{jx}^2 + K_{j2}f_{jy}^2 + K_{j3}f_{jx}f_{jy} + K_{j4}f_{jx}f_{jz} + K_{j5}f_{jy}f_{jx} + K_{j6}f_{jy}f_{jz} + K_{j7}f_{jx}f_{jz} + K_{j8}f_{jy}f_{jz} \]

where $\theta_{jx}^*$, $\theta_{jy}^*$, $K_{jx}$, $K_{jy}$, $K_{j6}$, $K_{j3}$, $K_{j4}$, $K_{j8}$ are given

\[ \theta_{jx}^* = (\theta_{jx} + \theta_{jx}\theta_{jy}) \]
\[ \theta_{jy}^* = \theta_{jy} - \theta_{jx}\theta_{jz} \]
\[ K_{j2} = K_{j2} + K_{j4}\theta_{jy} - K_{j6}\theta_{jz} + K_{j7}\theta_{jy}^2 - K_{j8}\theta_{jy}\theta_{jz} + K_{j5}\theta_{jz}^2 \]
\[ K_{j4} = (1 - \theta_{jy}^2 + \theta_{jx}\theta_{jy}\theta_{jz})K_{j4} + 2(\theta_{jx}\theta_{jy} - \theta_{jz})K_{j2} - 2(\theta_{jy}\theta_{jx} - \theta_{jz})K_{j3} + (\theta_{jx} + 2\theta_{jy}\theta_{jz})(\theta_{jx}\theta_{jy} - \theta_{jz})K_{j6} + 2(\theta_{jy} - \theta_{jz})K_{j7} + (\theta_{jx} - \theta_{jz})K_{j8} \]
\[ K_{j5} = K_{j5}(1 - 2\theta_{jx}\theta_{jy}\theta_{jz} + \theta_{jx}^2\theta_{jy}^2\theta_{jz}^2) - K_{j8}(\theta_{jx} - \theta_{jy}^2\theta_{jz}) \]
\[ K_{j6} = K_{j6} - K_{j4}(\theta_{jx}\theta_{jy} + \theta_{jx}\theta_{jz}) + K_{j2}(\theta_{jx}\theta_{jy}\theta_{jz}) + K_{j4}(\theta_{jx}\theta_{jy} + \theta_{jx}\theta_{jz}) \]
\[ K_{j7} = K_{j7}(1 - \theta_{jy}^2 + \theta_{jx}\theta_{jy}\theta_{jz})K_{j7} + 2(\theta_{jx}\theta_{jy} - \theta_{jz})K_{j2} + (\theta_{jx} + 2\theta_{jy}\theta_{jz})K_{j6} + 2(\theta_{jy} - \theta_{jz})K_{j7} + (\theta_{jx} - \theta_{jz})K_{j8} \]

Equation (4) reflects the relationship between the output of the RAGG accelerometer $A_j$ and the specific force in the accelerometer nominal frame of the actual mounting position. $\theta_{jx}^*$, $\theta_{jy}^*$ are scale factors of non-sensitive axis of accelerometer $A_j$ combined with misalignment angles. $K_{j2}$, $K_{j4}$, $K_{j5}$, $K_{j6}$, $K_{j7}$, $K_{j8}$ are equivalent second-order error coefficients combined with misalignment angles. To simplify the derivation of the RAGG model, (4) is written into the following form:

\[ V_j = k_{j1}f_{jx} + \theta_{jx}f_{jy} - \theta_{jy}f_{jz} + k_{j0} + k_{j3}f_{jx}^2 + k_{j5}f_{jy}^2 + k_{j4}f_{jx}f_{jy} + k_{j6}f_{jx}f_{jz} + k_{j7}f_{jy}f_{jx} + k_{j8}f_{jy}f_{jz} \]

where $k_{j1}$ is the linear scale factor (in units of V/g); $k_{j0}$ is the null bias (in units of V); and $k_{j2}$, $k_{j4}$, $k_{j6}$, $k_{j7}$, $k_{j8}$ are the second-order error coefficients (in units of V/g^2). Whether (2) is a simplified transformation matrix of small angle or not, (6) always has the same form; therefore, it does not affect the expression of RAGG analytical models.

2) Specific Force in the Accelerometer Nominal Frame of the Actual Mounting Position: The measurement of RAGG accelerometers is specific force, in other words, the difference between gravitational acceleration and absolute acceleration. Fig. 2 illustrates the position vector of the RAGG in the process of moving base gravity gradiometry. We choose the geocentric inertial coordinate system as the inertial frame, and denote the specific force measured by accelerometer $A_j$ as $f_j$

\[ f_j = a_{ji} - a_{sj} \] (8)

where $a_{ji}$ and $a_{sj}$ represent the absolute acceleration and gravitational acceleration of accelerometer $A_j$, respectively.

The absolute acceleration is the second derivative of the position vector of accelerometer $A_j$ with respect to the inertial frame

\[ a_{ji} = \frac{d^2s_A}{dt^2} \]
denotes the position vector from the origin of the inertial frame to accelerometer \(A_i\); \(r_{oA_i}\) denotes the position vector from the origin of the RAGG rotation frame to accelerometer \(A_j\); \(r_{oA_j}\) denotes the position vector from the origin of the inertial frame to the center of the RAGG rotary stage base; correspondingly, \(r_{oA_j}\) is the position vector from the origin of the p-frame to that of m-frame; \(r_{oA_{0A_j}}\) is the position vector from the origin of the m-frame to that of m1-frame. Since the m-frame, and the m1-frame are concentric frames, \(r_{oA_{0A_j}} = \overrightarrow{0}\). The second-order derivative of \(r_{oA_j}\) with respect to the i-frame is given by

\[
\frac{d^2r_{oA_j}}{dt^2} = \frac{d^2r_{oA_j}}{dt^2} + 2\omega_ip \times \frac{dr_{oA_j}}{dt} + \omega_ip \times (\omega_ip \times r_{oA_j}) + \dot{\omega}_ip \times r_{oA_j} \tag{10}
\]

where \(\omega_ip\) is the angular velocity of the RAGG rotary stage base with respect to the inertial frame, and it belongs to the angular movements of RAGG external movements; \(\frac{dr_{oA_j}}{dt}\), \(\frac{dr_{oA_j}}{dt}\), and \(\frac{dr_{oA_j}}{dt}\), respectively, are the runout acceleration and velocity of the RAGG rotary stage plane; and they belongs to the linear movements of RAGG internal movements. The last three terms of (10) are the cross-coupling terms of the external and internal motions. The second derivative of \(r_{oA_A_j}\) with respect to the i-frame is given by

\[
\frac{d^2r_{oA_A_j}}{dt^2} = \frac{d^2r_{oA_A_j}}{dt^2} + \omega_{im1} \times r_{oA_A_j} + \omega_{im1} \times (\omega_{im1} \times r_{oA_A_j}) \tag{11}
\]

where \(\omega_{im1}\) and \(\dot{\omega}_{im1}\) represent the angular velocity and angular acceleration of the RAGG accelerometers with respect to the inertial frame. The total angular movements of the RAGG accelerometers with respect to the inertial frame is the sum of RAGG external and internal angular movements.

Let \(\omega_{pm1}\), \(\omega_{pm}\) individually denote the angular speed and angular acceleration of the m-frame with respect to the p-frame; let \(\omega_{mm1}, \omega_{mm}\) individually denote the angular speed and angular acceleration of the m-frame with respect to the m-frame. The RAGG internal angular movements include three parts: nutation movement, precession movements and rotary stage plane rotation. The rotating RAGG rotary plane is similar to a mechanical gyroscope; some unperfect factors such as centroid deviation from axis, will cause precession and nutation; precession and nutation cause the angular movements of m-frame with respect to the p-frame \((\omega_{pm}, \dot{\omega}_{pm})\); correspondingly it causes the actual rotation axis deviating from the ideal rotation axis. The rotation of RAGG rotary stage plane is driven by the rotation of its motor; thus \(\omega_{mm1}\) equals the angular speed of rotary stage motor \((\omega_{mm1} = [0, 0, \Omega]^T)\); it is worth noting that \(\Omega\) is not constant, that is \(\dot{\Omega} \neq 0\). Based on the above analysis, we have

\[
\omega_{im1} = \omega_{ip} + \omega_{pm} + \omega_{mm1}
\]

\[
\dot{\omega}_{im1} = \dot{\omega}_{ip} + \dot{\omega}_{pm} + \dot{\omega}_{mm1} \tag{12}
\]

When mass is far enough away from the RAGG, the gravitational acceleration of accelerometer \(A_j\) is a linear approximation of the gravitational acceleration and gravitational gradient tensor at the center of the disk

\[
a_{g_j} \simeq a_{gm} + \Gamma \cdot r_{oA(A_j)} \tag{13}
\]

where \(a_{gm}\) and \(\Gamma\) individually represent the gravitational acceleration and the gravitational gradient tensor at the center of the RAGG rotary stage. Substituting (9), (11), and (13) into (8), we get

\[
f_j = f_{cmm} + \dot{\omega}_{im1} \times r_{oA(A_j)} + \omega_{im1} \times (\omega_{im1} \times r_{oA(A_j)} - \Gamma \cdot r_{oA(A_j)})
\]

\[
f_{cmm} = \frac{d^2r_{oA_j}}{dt^2} + a_{gm} = \frac{d^2r_{oA_j}}{dt^2} + 
\]

where \(f_{cmm}\) is the specific force at the center of the RAGG rotary stage plane, which consists of RAGG external linear movements \((\frac{d^2r_{oA_j}}{dt^2}_p \), RAGG internal linear movement, the cross-coupling terms of the external and internal movements, and gravitation acceleration. From (14), we can see that if the RAGG error propagation coefficients are known, applying high-precision accelerometer to monitor the specific force at the center of the RAGG rotary stage plane \((f_{cmm})\), and gyroscope to monitor angular velocity with respect to inertial frame \((\omega_{im1})\), then we can calculate and compensate the motion error of an RAGG. If the accuracy of RAGG motion error model is sufficient, the RAGG measurement accuracy is only determined by the inertial sensors that measure gravitational acceleration and RAGG motion status.

We can calculate the specific forces of accelerometer \(A_j\) in the directions of the \(x-, y-, \text{and } z\)-axes in the accelerometer nominal frame of the actual mounting position \(f_{jx}, f_{jy}, f_{jz}\)

\[
f_{jx} = f_j \cdot \tau_{jx}
\]

\[
f_{jy} = f_j \cdot \tau_{jy}
\]

\[
f_{jz} = f_j \cdot \tau_{jz} \tag{15}
\]

where \(\tau_{jx}, \tau_{jy}, \text{and } \tau_{jz}\) are unit vectors of the accelerometer nominal frame of the actual mounting position in the directions of the \(x-, y-, \text{and } z\)-axes. Writing the specific force, the angular velocity, and the angular acceleration of the RAGG with respect to the inertial frame in RAGG measurement frame in coordinate form gives

\[
f_{cmm} = [a_x, a_y, a_z]^T
\]

\[
\omega_{im1} = [\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z]^T \tag{16}
\]

\[
\dot{\omega}_{im1} = [\ddot{\omega}_x, \ddot{\omega}_y, \ddot{\omega}_z]^T.
\]

YU ET AL.: MATHEMATICAL MODEL AND ERROR ANALYSIS OF MOVING-BASE RAGG
Writing the gravitational gradient tensor at the center of the disk in coordinate form gives

\[
\Gamma = \begin{bmatrix}
\Gamma_{xx} & \Gamma_{xy} & \Gamma_{xz} \\
\Gamma_{yx} & \Gamma_{yy} & \Gamma_{yz} \\
\Gamma_{zx} & \Gamma_{zy} & \Gamma_{zz}
\end{bmatrix}.
\]  

(17)

Based on the configuration of the RAGG mentioned in Section II-A1, we can easily get the coordinates of the vectors \( r_{\alpha,i} \), \( \tau_{jx} \), and \( \tau_{jz} \). Substituting (14), (16), and (17) into (15), we can calculate the specific forces of accelerometers \( A_1 \sim A_4; a_{1x}, a_{1y}, a_{1z}, f_{2x}, f_{2y}, f_{2z}, f_{3x}, f_{3y}, f_{3z}, f_{4x}, f_{4y} \), and \( f_{4z} \). Substituting these specific forces into (6), we can calculate the output of the four accelerometers. Let \( V_1, V_2, V_3 \), and \( V_4 \), respectively, represent the output voltages of the four accelerometers; the output of the RAGG before demodulation is then given by

\[
G_{out} = V_1 + V_2 - V_3 - V_4.
\]  

(18)

To simplify the description, the notations \( T_1 \sim T_9 \) are adopted in RAGG analytical model

\[
\begin{align*}
T_1 &= 0.5(T_{xx} - T_{yy} + \omega_y^2 - \omega_x^2) \\
T_2 &= -T_{xx} + \omega_y \omega_x \\
T_3 &= (T_{yy} - \omega_x \omega_x + \omega_x) \\
T_4 &= 0.5(T_{xx} + T_{yy} + \omega_x^2 + 2\omega_y^2) \\
T_5 &= \omega_x \\
T_6 &= T_{zz} - \omega_x \omega_x - \omega_y \\
T_7 &= T_{zz} + \omega_x \omega_x + \omega_y \\
T_8 &= \omega_x^2 + \omega_y^2 + T_{zz}.
\end{align*}
\]  

(19)

It is worth noting that \( T_1 \sim T_9 \) consist of gravitational gradients \( (\Gamma_{xx}, \Gamma_{yy}, \text{etc.}) \), centrifugal gradients \( (\omega_x^2 - \omega_y^2, \omega_x \omega_y, \text{etc.}) \) and angular accelerations \( (\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z) \). As the magnitude of centrifugal gradients and angular accelerations are much larger than gravitational gradients, in the error analysis, we can treat \( T_1 \sim T_9 \) as angular motion error. Expanding (18) and collecting like terms, yields

\[
G_{out} = A_{4\Omega}(t) + A_{3\Omega}(t) + A_{2\Omega}(t) + A_{\Omega}(t) + A_{\text{com}}(t).
\]  

(20)

Equation (20) is RAGG analytical model; where \( A_{4\Omega}(t), A_{3\Omega}(t), A_{2\Omega}(t) \) and \( A_{\Omega}(t) \) are the signal components quadrature amplitude modulated by the carrier waves sine \( 4\Omega t \), cosine \( 4\Omega t \), sine \( 3\Omega t \), cosine \( 3\Omega t \), sine \( 2\Omega t \), cosine \( 2\Omega t \), sine \( \Omega t \), cosine \( \Omega t \), and \( A_{\text{com}}(t) \) is the signal component without modulation. \( A_{4\Omega}(t) \) is given

\[
A_{4\Omega}(t) = A_{4\Omega\text{mat}}(t) \mathbf{C}_{4\Omega\text{cor}}(t),
\]

where \( A_{4\Omega\text{mat}}(t) \) is \( 1 \times 2 \) input matrix, and its elements are \( A_{4\Omega\text{mat}}(1), A_{4\Omega\text{mat}}(2) \). \( \mathbf{C}_{4\Omega\text{cor}}(t) \) is \( 2 \times 1 \) coefficients matrix, and its elements are \( C_{4\Omega\text{cor}}(1), C_{4\Omega\text{cor}}(2) \). \( A_{3\Omega}(t) \) is given

\[
A_{3\Omega}(t) = A_{3\Omega\text{mat}}(t) \mathbf{C}_{3\Omega\text{cor}}(t)
\]

where \( A_{3\Omega\text{mat}}(t) \) is \( 1 \times 4 \) input matrix, and its elements are \( A_{3\Omega\text{mat}}(1), A_{3\Omega\text{mat}}(2), A_{3\Omega\text{mat}}(3), A_{3\Omega\text{mat}}(4) \). \( \mathbf{C}_{3\Omega\text{cor}}(t) \) is \( 4 \times 1 \) coefficients matrix, and its elements are \( C_{3\Omega\text{cor}}(1), C_{3\Omega\text{cor}}(2), C_{3\Omega\text{cor}}(3), C_{3\Omega\text{cor}}(4) \). \( A_{2\Omega}(t) \) is given

\[
A_{2\Omega}(t) = A_{2\Omega\text{mat}}(t) \mathbf{C}_{2\Omega\text{cor}}(t)
\]

where \( A_{2\Omega\text{mat}}(t) \) is \( 1 \times 4 \) input matrix, and its elements are \( A_{2\Omega\text{mat}}(1), A_{2\Omega\text{mat}}(2), \ldots, A_{2\Omega\text{mat}}(10) \). \( \mathbf{C}_{2\Omega\text{cor}}(t) \) is \( 4 \times 1 \) coefficients matrix, and its elements are \( C_{2\Omega\text{cor}}(1), C_{2\Omega\text{cor}}(2), \ldots, C_{2\Omega\text{cor}}(10) \). \( A_{\Omega}(t) \) is given

\[
A_{\Omega}(t) = A_{\Omega\text{mat}}(t) \mathbf{C}_{\Omega\text{cor}}(t)
\]

where \( A_{\Omega\text{mat}}(t) \) is \( 1 \times 10 \) input matrix, and its elements are \( A_{\Omega\text{mat}}(1), A_{\Omega\text{mat}}(2), \ldots, A_{\Omega\text{mat}}(10) \). \( \mathbf{C}_{\Omega\text{cor}}(t) \) is \( 10 \times 1 \) coefficients matrix, and its elements are \( C_{\Omega\text{cor}}(1), C_{\Omega\text{cor}}(2), \ldots, C_{\Omega\text{cor}}(10) \).
where $A_{\Omega mat}$ is a $1 \times 24$ input matrix, and its elements are $A_{\Omega mat}(1), A_{\Omega mat}(2), \ldots, A_{\Omega mat}(24)$. The elements of coefficients matrix $C_{concoe}$ are given

$$C_{\Omega mat}(1) = D_{1234}^{1234} = \begin{bmatrix} 1 & \phi & \theta & \psi \end{bmatrix}$$
$$C_{\Omega mat}(2) = D_{1234}^{1234} = -\phi$$
$$C_{\Omega mat}(3) = D_{1234}^{1234} = \theta$$
$$C_{\Omega mat}(4) = D_{1234}^{1234} = \psi$$
$$C_{\Omega mat}(5) = D_{1234}^{1234} = \beta_1$$
$$C_{\Omega mat}(6) = D_{1234}^{1234} = \beta_2$$
$$C_{\Omega mat}(7) = D_{1234}^{1234} = \beta_3$$
$$C_{\Omega mat}(8) = D_{1234}^{1234} = \beta_4$$
$$C_{\Omega mat}(9) = D_{1234}^{1234} = \gamma_1$$
$$C_{\Omega mat}(10) = D_{1234}^{1234} = \gamma_2$$
$$C_{\Omega mat}(11) = D_{1234}^{1234} = \gamma_3$$
$$C_{\Omega mat}(12) = D_{1234}^{1234} = \gamma_4$$
$$C_{\Omega mat}(13) = D_{1234}^{1234} = \delta_1$$
$$C_{\Omega mat}(14) = D_{1234}^{1234} = \delta_2$$
$$C_{\Omega mat}(15) = D_{1234}^{1234} = \delta_3$$
$$C_{\Omega mat}(16) = D_{1234}^{1234} = \delta_4$$
$$C_{\Omega mat}(17) = D_{1234}^{1234} = \epsilon_1$$
$$C_{\Omega mat}(18) = D_{1234}^{1234} = \epsilon_2$$
$$C_{\Omega mat}(19) = D_{1234}^{1234} = \epsilon_3$$
$$C_{\Omega mat}(20) = D_{1234}^{1234} = \epsilon_4$$
$$C_{\Omega mat}(21) = D_{1234}^{1234} = \xi_1$$
$$C_{\Omega mat}(22) = D_{1234}^{1234} = \xi_2$$
$$C_{\Omega mat}(23) = D_{1234}^{1234} = \xi_3$$
$$C_{\Omega mat}(24) = D_{1234}^{1234} = \xi_4$$

In order to facilitate RAGG error compensation and calibration, the RAGG analytical model $G_{mat}(t)$ is expressed as vector form

$$G_{mat}(t) = f(C_p, \Gamma, \phi_{conco}(t), \psi_{conco}(t), \alpha_{conco}(t))$$

$$A_{conco}(1) = (0.5T_T \alpha_x + 0.5T_T \alpha_y + T_y \alpha_z)$$
$$A_{conco}(2) = \alpha_x^2$$
$$A_{conco}(3) = (0.5T_T \alpha_x + 0.5T_T \alpha_y + T_y \alpha_z)$$
$$A_{conco}(4) = \alpha_x^2$$
$$A_{conco}(5) = \alpha_x^2$$
$$A_{conco}(6) = (0.5T_T^2 + 0.5T_T^2 + T_y^2)$$
$$A_{conco}(7) = (0.5T_T^2 + 0.5T_T^2 + T_y^2)$$
$$A_{conco}(8) = 0.5T_T^2 + 0.5T_T^2$$
$$A_{conco}(9) = T_y, A_{conco}(10) = (\omega_x^2 + \omega_y^2 + \omega_z^2)$$
$$A_{conco}(11) = -T_y, A_{conco}(12) = T_y$$
$$A_{conco}(13) = -T_y T_y, A_{conco}(14) = 1$$

where $A_{conco}$ is a $1 \times 14$ input matrix, and its elements are $A_{conco}(1), A_{conco}(2), \ldots, A_{conco}(14)$. The elements of $C_{conco}$ are given

$$C_{conco}(1) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(2) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(3) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(4) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(5) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(6) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(7) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(8) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(9) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(10) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(11) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(12) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(13) = D_{k_1 k_2}^{1234}$$
$$C_{conco}(14) = D_{k_1 k_2}^{1234}$$

where $C_p$ denotes analytical model coefficients vector; $A_{1 \times 54}$ is input vector; $C_{54 \times 1}$ is the analytical model coefficients vector.

In (21)–(25), the notation $D_{12}^{12}$ represents the imbalance terms denoted by subscript between accelerometers $A_1$ and $A_2$; the notation $D_{1234}^{1234}$ represents the imbalance terms denoted by subscript between accelerometers $A_3$ and $A_4$; the notation $D_{12}^{12}$ represents the imbalance terms denoted by subscript between two pairs accelerometers $A_1, A_2$ and $A_3, A_4$. For example $D_{k_1 k_2}^{1234}$ is the imbalance term of $k_1 R$ between accelerometers $A_1$ and $A_2$, that is $D_{k_1 k_2}^{1234} = k_1 r_1 - k_2 r_2$. $D_{k_1 k_2}^{1234}$ is the imbalance term between two pairs accelerometers that is $D_{k_1 k_2}^{1234} = k_1 r_1 + k_2 r_2 - k_3 r_3 - k_4 r_4$. Similarly, the notation $\sum k_1 k_2$ represents the sum of the four accelerometers of the terms denoted by subscript; for example, $\sum k_1 k_2$ is the sum of four accelerometers of $k_1 R$, that is, $\sum k_1 k_2 = k_1 r_1 + k_2 r_2 + k_3 r_3 + k_4 r_4$. In summary, the elements of coefficients vector are in the form of $D_{12}^{1234}$. 

YU ET AL.: MATHEMATICAL MODEL AND ERROR ANALYSIS OF MOVING-BASE RAGG 2149
where \( G_{\text{out}} \) is the ideal output of the RAGG before demodulation. Let \( k_{gg} \) denote the scale factors of the RAGG, thus, \( k_{gg} = \sum k_i R \). Since \( k_{gg} \) also is a error propagation coefficient of centrifugal gradient, the parameters of \( G_{\text{out}}, \sum_{s=4} \), \( D_{\text{subscript}}^{12}, D_{\text{subscript}}^{34}, \ldots \) are error propagation coefficients. Actually, in moving base gravity gradiometry, some error propagation coefficients have little effect on the sensitivity of RAGG and could be neglected. So, next we will further simplify the RAGG analytical model.

B. Simplified RAGG Analytical Model

Assuming that the RAGG sensitivity is 1 E and the nominal distance from the RAGG accelerometer to the center of the RAGG rotary stage is 0.1 m, the accelerometer mounting errors \( \beta_{1x}, \beta_{1z}, \theta_{1x}, \theta_{1z} \) are of the order of \( 10^{-4} \) rad, the accelerometer linear-scale-factor imbalance is of the order of \( 10^{-3} \), and the accelerometer second-order error coefficients are of the order of \( 10^{-6} \) g/g². Under these conditions, we simplified the RAGG analytical model by neglecting small error terms that have little effect on the RAGG sensitivity; the simplification of the RAGG analytical model is given in the appendix. The following is the simplified RAGG analytical model:

\[
G_{\text{out}} = A_{2\Omega_{\text{mat}}}^s(t) + A_{1\Omega_{\text{mat}}}^s(t) + A_{1\text{con}}^s(t) \tag{28}
\]

where \( G_{\text{out}}^s \) is the output of the simplified RAGG analytical model. \( A_{2\Omega_{\text{mat}}}^s(t) \) and \( A_{1\Omega_{\text{mat}}}^s(t) \) are the signal components quadrature amplitude modulated by the carrier waves sin2\( \Omega_t \), cos2\( \Omega_t \), sin\( \Omega_t \), and cos\( \Omega_t \); \( A_{1\text{con}}^s(t) \) is the signal components without modulation. \( A_{2\Omega_{\text{mat}}}^s(t) \) is given

\[
A_{2\Omega_{\text{mat}}}^s(t) = A_{2\Omega_{\text{mat}}^s(1)} C_{2\Omega_{\text{con}}^{s+1}}^s
\]

\[
A_{2\Omega_{\text{mat}}}^s(1) = T_1 \sin 2\Omega_t + T_2 \cos 2\Omega_t
\]

\[
A_{2\Omega_{\text{mat}}}^s(2) = T_2 \sin 2\Omega_t - T_1 \cos 2\Omega_t
\]

\[
A_{2\Omega_{\text{mat}}}^s(3) = a_x \sin 2\Omega_t + a_y \cos 2\Omega_t
\]

\[
A_{2\Omega_{\text{mat}}}^s(4) = -a_x \cos 2\Omega_t + a_y \sin 2\Omega_t
\]

\[
C_{2\Omega_{\text{con}}^{s+1}}^s = \sum_{k=1}^{s+1} k
\]

\[
C_{2\Omega_{\text{con}}^{s+1}}^s = \sum_{k=1}^{s+1} \frac{k}{k}
\]

where \( A_{2\Omega_{\text{mat}}}^s(1) \) is a \( 1 \times 4 \) matrix, and its elements are \( A_{2\Omega_{\text{mat}}}^s(1), A_{2\Omega_{\text{mat}}}^s(2), \ldots \)

\[
A_{1\Omega_{\text{mat}}}(1) = T_1 \sin \Omega_t - T_8 \cos \Omega_t
\]

\[
A_{1\Omega_{\text{mat}}}(2) = T_7 \cos \Omega_t + T_8 \sin \Omega_t
\]

The simplified RAGG analytical model \( G_{\text{out}}^s(t) \) can be expressed as vector form

\[
G_{\text{out}}(t) = f \left( C_p, \Gamma, f_{m_{\Omega_{\text{mat}}}}, \omega_{m_{\Omega_{\text{mat}}}}, \theta_{m_{\Omega_{\text{mat}}}} \right) = A_{1\times25}^s(t) C_{25\times1}^s
\]

\[
A_{1\times25}^s(t) = \left[ A_{2\Omega_{\text{mat}}}^s, A_{2\Omega_{\text{mat}}^s}, \ldots \right] C_{25\times1}^s = \left[ C_{2\Omega_{\text{con}}^{s+1}}^s, C_{2\Omega_{\text{con}}^{s+1}}^s, \ldots \right]^T \]

In the simplified analytical model, the error propagation coefficients in sin2\( \Omega_t \) and cos2\( \Omega_t \) are of the form \( \sum_{s=0}^{4} \).
C. RAGG Motion Error Model

In order to facilitate motion error compensation, we derive the RAGG motion error model by setting the gravitational gradient excitation of RAGG analytical model to zero. Correspondingly, the 54 parameters RAGG motion error model is given

\[ f_{\text{err}}(C, r_{\text{mm}}(t), a_{\text{im1}}(t), a_{\text{im2}}(t)) = m_{1 \times 54}(t)C_{54 \times 1}. \tag{33} \]

The elements of \( m_{1 \times 54}(t) \) are given

\[
m_1 = 0.5(\omega_xa_5^3 - \omega_ya_5^3)sin4\Omega t + (-0.125\omega_x^4 + 0.75\omega_x^2\omega_y^2 - 0.125\omega_y^4)cos4\Omega t
\]

\[
m_2 = (-0.125\omega_x^4 + 0.75\omega_x^2\omega_y^2 - 0.125\omega_y^4)sin4\Omega t + 0.5(\omega_xa_5^3 - \omega_ya_5^3)cos4\Omega t
\]

\[
m_3 = (0.25\omega_xa_5^3 - 0.25\omega_ya_5^3 - 0.75\omega_x\omega_ya_5^2
- 0.5\omega_xa_5^3 + 0.25\omega_x\omega_ya_5^2 + 0.75\omega_x^2\omega_ya_5^3
+ 0.75\omega_xa_5^3 + 0.25\omega_x\omega_ya_5^2)cos3\Omega t
\]

\[
m_4 = (0.5\omega_xa_5^3 + 0.25\omega_ya_5^3 - 0.75\omega_x\omega_ya_5^2
+ 0.25\omega_xa_5^3 + 0.25\omega_x\omega_ya_5^2 + 0.75\omega_x^2\omega_ya_5^3
+ 0.5\omega_xa_5^3 + 0.25\omega_x\omega_ya_5^2)cos3\Omega t
\]

\[
m_5 = (0.5a_5^2a_5^2 + a_xa_5^3a_5^2 - 0.5a_5a_5^2a_5^3)sin3\Omega t
+ (0.5a_5a_5^2 + a_xa_5^3 - 0.5a_5a_5^3)cos3\Omega t
\]

\[
m_6 = (0.5a_5a_5^2 + a_xa_5^3a_5^2 - 0.5a_5a_5^2a_5^3)sin3\Omega t
+ (0.5a_5a_5^2 + a_xa_5^3 - 0.5a_5a_5^3)cos3\Omega t
\]

\[
m_7 = 0.5(\omega_x^2 - \omega_y^2)sin2\Omega t + \omega_xa_5cos2\Omega t
\]

\[
m_8 = \omega_xa_5sin2\Omega t + 0.5(\omega_x^3 - \omega_y^3)cos2\Omega t
\]

\[
m_9 = a_xa_5sin2\Omega t + 0.5(a_5^3 - a_5^3)cos2\Omega t
\]

\[
m_{10} = 0.5(a_5^2 - a_5^2)sin2\Omega t + a_xa_5cos2\Omega t
\]

\[
m_{11} = (-\omega_xa_5^2 + \omega_ya_5^2)sin2\Omega t + 2\omega_xa_5\omega_ya_5cos2\Omega t
\]

\[
m_{12} = (-\omega_xa_5^2 + \omega_ya_5^2)sin2\Omega t + 2\omega_xa_5\omega_ya_5cos2\Omega t
+ (-0.5\omega_x^2 + \omega_ya_5^2 + 0.5\omega_y^2)cos2\Omega t
\]

\[
m_{13} = (0.25\omega_x^3 + 0.5\omega_x^2a_5^2 + \omega_xa_5^2
- 0.25\omega_y^3 - 0.5\omega_y^2a_5^2)sin2\Omega t
\]

\[
m_{14} = (\omega_xa_5^2 - \omega_ya_5^2 - 0.5a_5a_5^2\omega_x - 0.5a_5a_5^2\omega_y)sin2\Omega t
+ (0.5a_5a_5^2\omega_x^2 - a_xa_5^3\omega_x - 0.5a_5a_5^3\omega_x
- 0.5a_5a_5^3\omega_y)cos2\Omega t
\]

\[
m_{15} = (0.5a_5a_5^2 - 0.5a_5a_5^2 - 0.5a_5a_5^2
+ 0.5a_5a_5^2 + 0.5a_5a_5^2 + 0.5a_5a_5^2 + 0.5a_5a_5^2)
\]

\[
m_{16} = (0.5a_5a_5^2 - 0.5a_5a_5^2 - 0.5a_5a_5^2
+ 0.5a_5a_5^2 + 0.5a_5a_5^2 + 0.5a_5a_5^2 + 0.5a_5a_5^2)
\]

\[
m_{17} = (-\omega_xa_5 - \omega_ya_5)sin\Omega t + (\omega_xa_5 - \omega_ya_5)cos\Omega t
\]

\[
m_{18} = (\omega_x - \omega_ya_5)sin\Omega t + (-\omega_x - \omega_ya_5)cos\Omega t
\]

\[
m_{19} = a_xa_5sin\Omega t - a_xa_5cos\Omega t
\]

\[
m_{20} = a_xa_5sin\Omega t + a_xa_5cos\Omega t
\]

\[
m_{21} = a_xa_5sin\Omega t - a_xa_5cos\Omega t
\]

\[
m_{22} = -a_xa_5sin\Omega t + a_xa_5cos\Omega t
\]

\[
m_{23} = -2\omega_xsin\Omega t + 2\omega_xcos\Omega t
\]

\[
m_{24} = 2\omega_xsin\Omega t + 2\omega_xcos\Omega t
\]

\[
m_{25} = (\omega_x - \omega_ya_5)sin\Omega t + (\omega_xa_5 - \omega_ya_5)cos\Omega t
\]

\[
m_{26} = (\omega_x - \omega_ya_5)sin\Omega t + (\omega_xa_5 - \omega_ya_5)cos\Omega t
\]

\[
m_{27} = (0.25\omega_xa_5^3 - 0.25\omega_ya_5^3\omega_x + 0.5\omega_xa_5^2
- 0.25\omega_ya_5^3\omega_x + 0.25\omega_xa_5^3 - 0.25\omega_xa_5^3
+ 0.5\omega_xa_5^3 - 0.25\omega_xa_5^3)sin\Omega t
+ (0.25\omega_xa_5^3 - 0.25\omega_xa_5^3 - 0.5\omega_xa_5^3)
\]

\[
m_{28} = (0.5a_5^2a_5^2 + a_xa_5^3a_5^2 - 0.5a_5a_5^2a_5^3)
\]

\[
m_{29} = (0.5a_5^2a_5^2 + a_xa_5^3a_5^2 - 0.5a_5a_5^2a_5^3)
\]

\[
m_{30} = a_xa_5sin\Omega t - a_xa_5cos\Omega t
\]

\[
m_{31} = (0.5a_5a_5^2 + 0.5a_5a_5^2 + a_xa_5^2)sin\Omega t
+ (-a_xa_5^2 - 0.5a_5a_5^2 - 0.5a_5a_5^2)cos\Omega t
\]

\[
m_{32} = (0.5a_5a_5^2 + a_xa_5^3a_5^2 - 0.5a_5a_5^2)sin\Omega t
+ (0.5a_5a_5^2 - a_xa_5^3a_5^2 - 0.5a_5a_5^2)cos\Omega t
\]

\[
m_{33} = (-a_xa_5^2 - 0.5a_5a_5^2 - 0.5a_5a_5^2)sin\Omega t
+ (-a_xa_5^2 + 0.5a_5a_5^2 - 0.5a_5a_5^2)cos\Omega t
\]

\[
m_{34} = a_xa_5sin\Omega t + a_xa_5cos\Omega t
\]

\[
m_{35} = (2a_xa_5 - 2a_xa_5sin\Omega t
+ (2a_xa_5 - 2a_xa_5)cos\Omega t
\)
\[ m_{36} = (2a_x \omega_y \omega_z - 2a_z \omega_x \sin \Omega t + (-2a_x \omega_z - 2a_y \omega_x) \cos \Omega t, \]
\[ m_{37} = (\dot{\omega}_w \omega_z - \omega_x \omega_y \omega_z \sin \Omega t + (\dot{\omega}_w \omega_z + \omega_y \omega_x \omega_z) \cos \Omega t, \]
\[ m_{38} = (0.5a_x \omega_y \omega_z - 0.5a_z \omega_x \omega_z + 0.5a_y \omega_x \omega_z + a_x \omega_z^3 \]
\[ - 0.5a_x \omega_z^2 \omega_x - \omega_z \omega_x \sin \Omega t + (-0.5a_x \omega_z \omega^2 - 0.5a_z \omega^2 - 0.5a_y \omega^2 \omega_x - 0.5a \omega^2 \omega_z \omega_x \cos \Omega t, \]
\[ m_{39} = (-\dot{\omega}_w \omega_z - \omega_x \omega_y \omega_z \sin \Omega t + (\dot{\omega}_w \omega_z + \omega_y \omega_x \omega_z) \cos \Omega t, \]
\[ m_{40} = (0.5a_x \omega_y \omega_z - 0.5a_z \omega_x \omega_z + 0.5a_y \omega_x \omega_z + a_z \omega_x^3 \]
\[ + 0.5a_x \omega_x \omega_z - 0.5a_y \omega_x \omega_z + 0.5a \omega^2 \omega_x \omega_z \sin \Omega t \]
\[ + (0.5a^4 + 0.5a_y \omega^2 + 0.5a^2 \omega_x + 0.5a^2 \omega_z \omega_x \omega_z), \]
\[ m_{41} = 0.5a_x^2 + 0.5a_y^2, \]
\[ m_{42} = 0.5a_x \omega_x + 0.5a_y \omega_y - a_z \omega_z + 0.5a \omega_x \omega_z \]
\[ - 0.5a_x \omega_x \omega_z, \]
\[ m_{43} = 0.5a_x \omega^2 - 0.5a^2 \omega_x \omega_z + 0.5a_y \omega^2 - 0.5a_x \omega_x \omega_z \]
\[ + a_z \omega_x^2 + 0.5a_x \omega_x - 0.5a_x \omega_x, \]
\[ m_{44} = a_x^2 + a_z, \]
\[ m_{45} = a_z, \]
\[ m_{46} = 0.125a_x^4 + 0.25a_x^2 \omega_x^2 + 0.125a_x^4 + a_z^2, \]
\[ m_{47} = 0.375a_x^4 + 0.75a_x^2 \omega_x^2 + a_x^2 \omega_x^2 \]
\[ + 0.375a_x^2 + a_z^2 \omega_z^2 + a_x^4, \]
\[ m_{48} = 0.5a_x^2 \omega_z^2 - \omega_x \omega_x \omega_z + 0.5a_x^2 \omega_x^2 \]
\[ + a_x \omega_x \omega_z + 0.5a_x^2 + 0.5a_z^2, \]
\[ m_{49} = a_x^2 + a_z^2, \]
\[ m_{50} = -a_x^2 + 2a_x^2 \omega_x^2 - 2a_x^2 \omega_x^2 - a_y^2 - 2a_x^2 \omega_x^2 - a_z^2, \]
\[ m_{51} = -0.5a_x^2 - 0.5a_z^2 - a_z^2, \]
\[ m_{52} = a_z, \]
\[ m_{53} = -a_x a_z - 0.5a_z a_x - 0.5a_z a_x, m_{54} = 1 \] (34)

Correspondingly, the 25 parameters RAGG motion error model is given

\[ f_{merr}(C, \Omega_m^{m}(t), \omega_{im}^{m}(t), \dot{\omega}_{im}^{m}(t)) = P \times 25(t)C_{25 \times 1} \] (35)

The elements of \( P \times 25(t) \) are given

\[ p_1 = 0.5(a_x^2 - a_z^2) \sin 2\Omega t + a_z a_x \omega_x \cos 2\Omega t \]
\[ p_2 = a_x a_z \sin 2\Omega t + 0.5(a_z^2 - a_x^2) \cos 2\Omega t \]
\[ p_3 = a_x a_z \sin 2\Omega t + 0.5(a_z^2 - a_x^2) \cos 2\Omega t \]
\[ p_4 = 0.5(a_z^2 + a_x^2) \sin 2\Omega t + a_x a_z \cos 2\Omega t \]
\[ p_5 = (-\omega_x - a_x \omega_z - \omega_y \omega_x) \sin \Omega t \]
\[ p_6 = (\omega_x + a_x \omega_z) \sin \Omega t + (\omega_y + a_y \omega_z) \cos \Omega t \]
\[ p_7 = a_x \sin \Omega t - a_x \cos \Omega t \]
\[ p_8 = a_x \sin \Omega t + a_x \cos \Omega t \]
\[ p_9 = a_x a_z \sin \Omega t - a_x a_z \cos \Omega t \]
\[ p_{10} = a_x a_z \sin \Omega t + a_x a_z \cos \Omega t \]
\[ p_{11} = (\omega_x + a_x \omega_z) \sin \Omega t + (\omega_y + a_y \omega_z) \cos \Omega t \]
\[ p_{12} = (-\omega_x - a_x \omega_z - \omega_y \omega_x) \sin \Omega t \]
\[ p_{13} = a_x \omega_z \sin \Omega t - a_x \omega_z \cos \Omega t \]
\[ p_{14} = (0.5a_z^2 + 0.5a_x^2 + a_x \omega_z^2 - a_x \omega_z^2) \sin \Omega t \]
\[ p_{15} = (-0.5a_z^2 - 0.5a_x^2 - a_x \omega_z^2 + a_x \omega_z^2) \sin \Omega t \]
\[ p_{16} = a_z \omega_z \sin \Omega t - a_z \omega_z \cos \Omega t \]
\[ p_{17} = 0.5a_x^2 + 0.5a_z^2, p_{18} = -a_x \omega_z \]
\[ p_{19} = (0.5a_x a_z^2 + 0.5a_z a_x^2 + a_x \omega_z^2) \]
\[ p_{20} = a_x^2, p_{21} = a_z \]
\[ p_{22} = 0.25a_x^4 + 0.25a_x^2 + a_x^2 + 0.5a_x \omega^2 \]
\[ p_{23} = -0.5a_x^2 - 0.5a_z^2 - a_z^2 \]
\[ p_{24} = a_z, p_{25} = 1. \] (36)

The error propagation coefficients of motion error models consistent with that of RAGG analytical models. From the RAGG motion error models and the analytical models, we can apply the linear acceleration of a specific frequency to the RAGG, producing a detection signal for online continuous compensation of the error propagation coefficients or guiding the installation of RAGG accelerometers; by recording the RAGG motion status in gravity gradiometry and constructing a loss function, we can assume the RAGG error propagation coefficients and compensate the motion error off-line [21].

D. RAGG Numerical Model

We have established an RAGG numerical model [22]. In the numerical model, each accelerometer has six mounting error parameters: radial distance \( (R_j) \), initial phase angle \( (\beta_j) \), altitude angle \( (\beta_j) \), and misalignment error angles \( (\varphi_j, \varphi_j, \varphi_j) \). Among them, the radial distance \( (R_j) \), initial phase angle \( (\beta_j) \), and altitude angle \( (\beta_j) \) determine the mounting position of the accelerometer; the misalignment error angles \( (\varphi_j, \varphi_j, \varphi_j) \) determine the orientation deviation between the accelerometer measurement frame and the accelerometer nominal frame of the actual mounting position. Moreover, each accelerometer has eight other output model parameters: zero bias \( (k_j) \), linear scale factors \( (k_j) \), second-order error coefficients \( (k_j, k_j, k_j, k_j, k_j, k_j, k_j, k_j) \). In total, each accelerometer has 14 parameters.

We use a test mass to produce gravitational gradients to excite the RAGG. The specific force of the accelerometer \( A_j \) in the RAGG numerical model is given by

\[ f_j = f_{\text{cmm}} + \omega_{im} \times r_{A_j} + \omega_{im} \times (\omega_{im} \times r_{A_j}) \]
\[ - Gm A_j S / |A_j S|^3 \] (37)

where \( f_{\text{cmm}} \) is the specific force at the center of RAGG rotary stage plane; \( \omega_{im} \) is the angular acceleration of the
RAGG accelerometer with respect to the inertial frame; $\omega_{in}$ is the angular velocity of the RAGG accelerometer with respect to the inertial frame; $G$ is the gravitational constant; $r_{o_{in}A_j}$ is the position vector of accelerometer $A_j$ in the RAGG measurement; $m$ represents the weight of the test mass; and $A_jS$ is the position vector from accelerometer $A_j$ to the test mass. If the test mass is not a point mass, the gravitational acceleration that the RAGG accelerometers undergo produced by the test mass can be calculated using infinitesimal calculus. In addition, the test mass can be in motion with respect to the RAGG; in this case, $A_jS$ is time varying [23]. The specific forces of accelerometer $A_j$ in the accelerometer nominal frame of the actual mounting position ($f_{ix}$, $f_{iy}$, $f_{iz}$) can be calculated from

$$ f_{ix} = f_j \cdot \tau_{ix} $$
$$ f_{iy} = f_j \cdot \tau_{iy} $$
$$ f_{iz} = f_j \cdot \tau_{iz} $$

(38)

where $\tau_{ix}$, $\tau_{iy}$, and $\tau_{iz}$ are unit vectors of the accelerometer nominal frame of the actual mounting position in the directions of the $x$-, $y$-, and $z$-axes. The specific forces in the accelerometer measurement frame are (39) shown at the bottom of this page.

The accelerometer input and output model used in RAGG analytical derivation [see(6)] has the characteristics of all-pass and no delay, and it is difficult to describe the frequency characteristic mismatch of RAGG accelerometers (flatness difference of amplitude spectrum and group delay difference). So we improved the accelerometer model by introduce three transfer function; the accelerometer model used in RAGG numerical model is shown in Fig. 4. When $G_x(s) = G_y(s) = G_z(s) = 1$, the accelerometer model used in RAGG numerical model is consistent with that of the RAGG analytical model; so, in the RAGG model verification experiments, the above transfer functions are all set to 1. In Fig. 4, $f_{noise}$ is the accelerometer noise and it is simulated by a power spectral density model

$$ \Phi(f)_{\text{noise}} = a f^{-b} + \omega_T $$

(40)

where $a$ and $b$ represent the amplitude and low-frequency growth of the red noise, and $\omega_T$ denotes the amplitude of the white noise.

$$ \begin{bmatrix} f_{ix} \\ f_{iy} \\ f_{iz} \end{bmatrix} = \begin{bmatrix} \cos \phi_x \cos \phi_z \cos \phi_x \sin \phi_x \sin \phi_x \cos \phi_z - \cos \phi_z \sin \phi_x \cos \phi_x \sin \phi_x \sin \phi_z \sin \phi_x \\ -\sin \phi_x \cos \phi_y - \sin \phi_y \sin \phi_x \cos \phi_z \sin \phi_x \cos \phi_x \cos \phi_z \cos \phi_x \sin \phi_x \\ \sin \phi_y \cos \phi_x \sin \phi_x \cos \phi_x \cos \phi_x \sin \phi_x \cos \phi_x \cos \phi_x \sin \phi_x \cos \phi_x \end{bmatrix} \begin{bmatrix} f_j \end{bmatrix} $$

(39)

Fig. 5 is the flowchart in programming of the RAGG numerical model. First, the RAGG simulation parameters are set up, including test masses parameters, RAGG rotary stage parameters, accelerometer mounting parameters, accelerometer model parameters, RAGG motion parameters, etc. Then substituting the parameters into the formula (37)–(40) calculates the specific force in accelerometer measurement frame at time $t$. According to Fig. 4, calculating the output voltage of the RAGG accelerometer, the RAGG output before demodulation at time $t$ is calculated by: $G_{out}(t) = V_1(t) + V_2(t) - V_3(t) - V_4(t)$. The above process is repeated until time $t$ is equal to the simulation duration time. Finally, the RAGG output data is input to the quadrature amplitude modulation (QAM) demodulator to extract gravitational gradient. Let $\Gamma_{xx}$, $\Gamma_{xy}$, $\Gamma_{xz}$, $\Gamma_{yy}$, and $\Gamma_{yz}$ represent the five independent gravitational gradient elements at the origin of the RAGG measurement frame. When mass is far enough away from the RAGG, the gravitational acceleration measured by the RAGG accelerometers is a first-order approximation of the gravitational acceleration and gravitational gradient tensor at the center of the rotating disk; in this case, the inline channel measurement and the cross
In this section, a multifrequency gravitational gradient simulation experiment and dynamic sweep frequency experiments are designed to verify the correctness of the three RAGG models and to evaluate the performance of analytical models.

In multifrequency gravitational gradient simulation experiment, the three RAGG models simulate a measurement scene, in which a test mass rotates around a perfect RAGG for producing multifrequency gravitational gradient exciting the RAGG. In this case, the theoretical measurements of an RAGG are the center gravitational gradients; the outputs of the analytical models, that of the numerical model, and the center gravitational gradients can be consistent with each other, only when the analytical models and the numerical model do not have principle errors and calculation errors. Besides based on the properties of three RAGG models, the outputs of the analytical models are more closer to the center gravitational gradients than that of the numerical model. Compared the multifrequency gravitational gradient experimental results to the theoretical ones, we can preliminarily verify the correctness of the three RAGG models.

Multifrequency gravitational gradient simulation experiment only can verify the correctness of the three RAGG models when an RAGG does not have imperfect factors; furthermore dynamic sweep frequency experiment is designed to verify the correctness of the three RAGG models when an RAGG has imperfect factors. In the dynamic sweep frequency simulation experiment, the three RAGG models simulate a measurement scene, in which a test mass rotates around a perfect RAGG for producing multifrequency gravitational gradient excitations. Based on the angular velocity varying angular velocity producing multifrequency gravitational gradient tensor, the coupling error propagation coefficients concerning accelerometer second-order error coefficients and misalignment angles, it can’t fully simulate the frequency characteristic mismatch of RAGG accelerometers.

### Table I

| Model categories                  | Model characteristics                                                                                                                                 |
|-----------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| Numerical model                   | The imperfection factors considered in numerical model: accelerometer mounting position error, accelerometer input axis misalignment, circuit gain mismatch, accelerometer linear scale factor imbalance, accelerometer second-order error coefficients, high order gravitational gradient tensor, frequency characteristic mismatch, RAGG internal movement; it approaches the real RAGG. |
| Analytical model                  | Compared with the numerical model, it principally neglects high order gravitational gradient tensor, the coupling error propagation coefficients concerning accelerometer second-order error coefficients and misalignment angles; it can’t fully simulate the frequency characteristic mismatch of RAGG accelerometers. |
| Simplified analytical model       | Compared with the analytical model, it principally neglects high order gravitational gradient tensor, partial coupling terms concerning accelerometer second-order error coefficients, angular motions, and linear motions, partial coupling terms of accelerometer second-order error coefficients and angular motions; it also can’t fully simulate the frequency characteristic mismatch of RAGG accelerometers. |

In multifrequency gravitational gradient simulation experiment, a test mass rotates about the RAGG with time-varying angular velocity producing multifrequency gravitational gradient excitations. Based on the angular velocity of the test mass and its initial coordinate in the RAGG measurement frame, we can obtain the coordinates of the
test mass in the RAGG measurement frame at any time. We then calculate the gravitational gradient tensor at the origin of the RAGG measurement frame and then calculate the center gravitational gradients. The three RAGG models simulate a perfect RAGG with no accelerometer mounting errors, accelerometer scale-factor imbalances, or accelerometer second-order error coefficients, so we set the accelerometer mounting errors \((dR_j, \beta_{xj}, \beta_{zj}, \theta_{xj}, \theta_{yj}, \theta_{zj})\) and the accelerometer second-order error coefficients \((k_{jx}, k_{jz}, k_{x}, k_{y}, k_{z})\) to zero. The linear scale factor of the four accelerometers is \(k_j = 1000 \, \text{V/g}\), the nominal mounting radius \(R = 0.1 \, \text{m}\), and the rotation frequency of the RAGG disk is 0.25 \(\text{Hz}\). A point mass of 486 kg with an initial position in the RAGG measurement frame of \((0.8, 0.0)\), rotates about the RAGG with time-varying angular speed \(\omega(t) = 3600 + 560 \sin(0.0628t)^{0.5} / \text{h}\) producing multifrequency gravitational gradients.

Substituting the above experimental parameters into the analytical model (26) and the simplified analytical model (32) calculate the output of the analytical model and that of the simplified analytical model before demodulation. Inputting these experimental parameters into the numerical model, according to the program flow shown in Fig. 4, calculate the output of the numerical model before demodulation. Finally, the output of the analytical model, that of the simplified analytical model, that of the numerical model are demodulated by the same QAM demodulator.

Fig. 6(a) shows the voltage outputs of the four accelerometers in the RAGG numerical model excited by the rotating point mass. Fig. 6(b) shows the output voltage before demodulation of the numerical model, that of the analytical model, and that of the simplified analytical model; the output voltages before demodulation of the three RAGG models are consistent with each other. Fig. 6(c) and (d) shows the demodulated gravitational gradient comparison among the three RAGG models and the center gravitational gradients; from Fig. 6(c) and (d), we can see that the inline channel and the cross channel of the three RAGG models are consistent with those of the center gravitational gradients. Besides, we have calculated the differences between the center gravitational gradients and the outputs of the RAGG model. To simplify the description, the difference between outputs of the RAGG numerical model and the center gravitational gradients is called difference of the numerical model. This experimental phenomenons are consistent with the theoretical ones. It preliminarily verified the correctness of the RAGG models in case of RAGG without imperfect factors. These experimental phenomena are consistent with the theoretical ones. It preliminarily verified the correctness of the RAGG models in case of RAGG without imperfect factors.

### Table II

| Parameter name (unit) | Accelerometer name | Acc.1 | Acc.2 | Acc.3 | Acc.4 |
|-----------------------|--------------------|-------|-------|-------|-------|
| \(dR_j\) (\(\mu m\)) | 0 | 30 | 35 | 25 | 20 |
| \(\beta_{xj}\) (arcsec) | 16.6 | 13.5 | 6.2 | 21 | 20.7 |
| \(\beta_{zj}\) (arcsec) | 6.2 | 18.5 | 17.8 | 17.3 | 17.6 |
| \(\theta_{xj}\) (arcsec) | 14.1 | 0.7 | 5.2 | 13.7 | 17.3 |
| \(\theta_{yj}\) (arcsec) | 5 | 0 | 9 | 23 | 16.2 |
| \(k_{x}\) (\(\mu g/\text{g}^2\)) | 2.1 | 2.3 | 2.2 | 2.2 |
| \(k_{y}\) (\(\mu g/\text{g}^2\)) | 1000 | 1100 | 1150 | 1200 |
| \(k_{z}\) (\(\mu g/\text{g}^2\)) | 5.9 | 18.6 | 11.8 | 5.7 |
| \(k_{x}\) (\(\mu g/\text{g}^2\)) | 0.2 | 16.9 | 13.5 | 5.2 |
| \(k_{y}\) (\(\mu g/\text{g}^2\)) | 3.4 | 7.5 | 4.6 | 4.4 |
| \(k_{z}\) (\(\mu g/\text{g}^2\)) | 5.1 | 8.4 | 8.1 | 16.2 |
| \(k_{x}\) (\(\mu g/\text{g}^2\)) | 15.3 | 17.5 | 11.9 | 11 |
| \(k_{y}\) (\(\mu g/\text{g}^2\)) | 5.3 | 8.5 | 18.6 | 1.5 |
Fig. 6. Results of the multifrequency gravitational gradient simulation experiment. (a) Accelerometer voltage outputs in RAGG numerical model. (b) Output voltage before demodulation comparison among the numerical model, the analytical model, and the simplified analytical model. (c) Demodulated inline channel gravitational gradient comparison among the three RAGG models and the center gravitational gradients. (d) Demodulated cross channel gravitational gradient comparison among the three RAGG models and the center gravitational gradients. (e) Inline channel difference among the three RAGG models and the center gravitational gradients. (f) Cross channel difference among the three RAGG models and the center gravitational gradients.

TABLE III
Sweep Frequency Gaussian Random Linear Motion and Angular Motion Parameters in Sweep Frequency Experiments

| Motion type   | Physical quantity | Experiment 1        | Experiment 2        | Experiment 3        |
|---------------|-------------------|---------------------|---------------------|---------------------|
|               |                   | Mean | Standard deviation | Mean | Standard deviation | Mean | Standard deviation |
| Linear motion | \(a_x\)           | 0 mg | 5 mg               | 0 mg | 15 mg               | 30 mg | 20 mg               |
|               | \(a_y\)           | 0 mg | 5 mg               | 0 mg | 15 mg               | 30 mg | 20 mg               |
|               | \(a_z\)           | 1 g  | 10 mg              | 1 g  | 30 mg               | 1080 mg | 40 mg              |
| Angular motion| \(\omega_x\)      | 100 deg/h | 50 deg/h         | 200 deg/h | 100 deg/h         | 500 deg/h | 600 deg/h         |
|               | \(\omega_y\)      | 100 deg/h | 50 deg/h         | 200 deg/h | 100 deg/h         | 500 deg/h | 600 deg/h         |
|               | \(\omega_z\)      | 100 deg/h | 50 deg/h         | 200 deg/h | 100 deg/h         | 500 deg/h | 600 deg/h         |
and the standard deviation of angular speed components are 100 and 50 deg/h. The outputs of the analytical model and the simplified analytical model before demodulation are the combined signals of the four accelerometers. We treat the output before demodulation of the numerical model as that of the actual RAGG, and treat the differences between the analytical model and the numerical model as the model errors. Fig. 7 is the results of the first sweep frequency experiment. Fig. 7(a)–(c), respectively, shows the histogram of the lateral specific force, the vertical specific force and the vertical angular speed. Fig. 7(d) is the outputs before demodulation of the numerical model, the analytical model, and the simplified analytical model. Fig. 7(e) shows the model errors of the analytical model and the simplified analytical model after demodulation. The inline-channel and cross-channel errors standard deviation of the analytical model are 0.8029 E and 0.4097 E; the corresponding noise density of the analytical model are 2.539 E/√Hz and 1.2954 E/√Hz. The inline-channel and cross-channel errors standard deviation of the simplified analytical model are 1.6889 E and 0.8511 E, and the corresponding noise density of the analytical model are 5.3408 E/√Hz and 2.6916 E/√Hz. In the third sweep frequency experiment, the mean and the standard deviation of the vertical specific force ($a_z$) are, respectively, 1080 and 40 mg; that of the lateral specific force ($a_x$, $a_y$) are, respectively, 30 and 20 mg. The mean and the standard deviation of angular speed components are 500 deg/h and 600 deg/h. Fig. 8 is the results of the third sweep frequency experiment. The inline-channel and cross-channel errors...
Fig. 8. Results of the third sweep frequency experiment. (a) Histogram of lateral specific force. (b) Histogram of vertical specific force. (c) Histogram of vertical angular speed. (d) Output voltage before demodulation comparison among the three RAGG models. (e) The analytical model error and the simplified analytical model error after demodulation.

### TABLE IV

| RAGG model type   | Experiment 1 | Experiment 2 | Experiment 3 |
|-------------------|--------------|--------------|--------------|
|                   | Mean | SD   | NSD | Mean | SD   | NSD | Mean | SD   | NSD |
| analytical model  |      |      |     |      |      |     |      |      |     |
| inline channel    | 0.0069 | 0.7799 | 2.4663 | -0.0108 | 0.8029 | 2.539 | -0.0612 | 0.8035 | 2.5408 |
| cross channel     | 0.0093 | 0.3951 | 1.2496 | 0.0060 | 0.4097 | 1.2954 | 0.0112 | 0.4075 | 1.2887 |
| simplified model  |      |      |     |      |      |     |      |      |     |
| inline channel    | 0.0104 | 0.8357 | 2.6426 | -0.0466 | 1.6889 | 5.3408 | -1.8457 | 13.0498 | 41.2672 |
| cross channel     | 0.0129 | 0.4201 | 1.3285 | 0.0424 | 0.8511 | 2.6916 | 1.5661 | 6.4836 | 20.5029 |

1 SD is standard deviation; 2 NSD is noise spectrum power density. The unit of mean and SD is E, and that of NSD is E/√Hz.

The standard deviation of the analytical model are 0.8035 E and 0.4075 E, and the corresponding noise density of the analytical model are 2.5408 E/√Hz and 1.2887 E/√Hz. The inline-channel and cross-channel errors standard deviation of the simplified analytical model are 13.0498 E and 6.4836 E, and the corresponding noise density of the analytical model are 41.2672 E/√Hz and 20.5029 E/√Hz.

From the first sweep frequency experiment to the third sweep frequency experiment, although the linear motion and angular motion has greatly increased, in the Table IV, the standard deviation of the analytical model error have no significant changes and are about 0.8 E and 0.4 E; the corresponding noise density of the analytical model is about 2.5 E/√Hz. But the standard deviation of the simplified analytical model error increase as the linear motion and angular motion increase. This experiment result is consistent with the simplified analytical model properties; the simplified analytical model has neglected partial coupling terms concerning accelerometer second-order error coefficients, angular motions, and linear motions, partial coupling terms of accelerometer second-order error coefficients and angular motions; therefore, the greater the linear motion and angular motion, the greater the error of the simplified analytical model.

### IV. CONCLUSION

According to the measurement principle and configuration of RAGG, we considered almost all imperfection factors, such as accelerometer installation error, accelerometer scale factor imbalance, accelerometer second-order error coefficient, accelerometer frequency characteristic mismatch, RAGG internal movement and then developed a high-precision RAGG numerical model and RAGG analytical model. The analytical model directly gives the RAGG motion error propagation mechanism and is helpful for developing techniques such as on-line error compensation,
post error compensation, and fault diagnosis; the numerical model provides a tool for verifying different techniques in developing RAGG. In the third dynamic sweep frequency experiment, the RAGG vertical specific force is in the range of [960, 1200 mg], the noise density of the analytical model is 2.5 E/Hz. With using the error compensation techniques based on the analytical model, the turbulence threshold of survey flying is expected to be increased from the current 100 to 150 mg.

APPENDIX

SIMPLIFY THE RAGG ANALYTICAL MODEL

Assuming that the RAGG sensitivity is 1 E and the nominal distance from the RAGG accelerometer to the center of the RAGG rotary stage is 0.1 m, the accelerometer mounting errors $\beta_{x}, \beta_{y}, \beta_{z}, \vartheta_{x}$, and $\vartheta_{z}$ are of the order of $10^{-4}$ rad, the accelerometer linear-scale-factor imbalance is of the order of $10^{-4}$, and the accelerometer second-order error coefficients are of the order of $10^{-6} \text{g}/\text{g}^2$. Under these conditions, we will calculate the critical conditions of the RAGG motion parameters so that the error terms can be neglected. By comparing the critical conditions with those of the actual moving-base gravity gradiometry environment, we can determine whether the error terms should be ignored.

In the RAGG analytical model [see (21)–(25)], classified by error sources, all error terms can be divided into six categories: coupling error terms concerning second-order error coefficients and angular motion, coupling error terms concerning second-order error coefficients and linear motion, coupling error terms concerning linear scale-factor imbalance and linear motion, coupling error terms concerning second-order error coefficients, linear motion and angular motion, coupling error terms concerning misalignment angle, linear scale factors, and angular motion, coupling error terms concerning misalignment angles, linear scale factors, and linear motion. We simplify the RAGG analytical model by categories.

1) Coupling error terms concerning second-order error coefficients and angular motion. As mentioned in Section II-A1, the parameters $k_{jp}$ (j = 1, 2, 3, 4; p = 2, 4, 5, 6, 7, 8) in the accelerometer model represent the second-order error coefficients $k_{jp}$ of the accelerometer $A_{j}$. $T_{1} - T_{6}$ refer to the angular motion of an RAGG, so the basic coupling error terms concerning the second-order error coefficients and angular motion are of the form $k_{jp}T_{1}T_{6}R_{j}^{2}$ ($n_{1}, n_{2} = 1, 2, 3, 4, 5, 6, 7, 8, 9$; $p = 2, 4, 5, 6, 7, 8$). In the analytical model $G_{out}$, the terms $\sum_{k,p} T_{1}T_{6}R_{j}^{2}$ and $D_{k,p}^{1}T_{1}T_{6}$, consist of $k_{jp}T_{1}T_{6}R_{j}^{2}$ and belong to the coupling error terms concerning the second-order error coefficients and angular motion. The RAGG measurement error $M_{e1}$ due to $k_{jp}T_{1}T_{6}R_{j}^{2}$ can be expressed as

$$M_{e1} = k_{jp}T_{1}T_{6}R_{j}^{2}/k_{ggi}$$  \hspace{1cm} (41)

where $k_{ggi}$ is the RAGG scale factor. In the accelerometer model mentioned in Section II-A1, we have $k_{jp} = k_{j1}K_{jp}$, so we obtain

$$M_{e1} \approx 0.25K_{jp}T_{n_{1}}T_{n_{2}}R_{j}.$$  \hspace{1cm} (42)

$T_{n_{1}}$ and $T_{n_{2}}$ are of the same order, and we use $(T_{n_{1}}, T_{n_{2}})$ to denote the critical value of $T_{n_{1}}$ and $T_{n_{2}}$. To ensure a RAGG sensitivity of 1 E, it is reasonable to assume that the error contributed by $k_{jp}T_{n_{1}}T_{n_{2}}R_{j}^{2}$ is 0.1 E. Substituting $K_{jp} = 10^{-6} \text{g}/\text{g}^2$, $R_{j} = 0.1$ m, and $M_{e1} = 0.1$ E into (42), we calculate the critical condition for neglecting $k_{jp}T_{n_{1}}T_{n_{2}}R_{j}^{2}$

$$T_{n_{1}}T_{n_{2}} \leq 4 \times 10^{16}.$$  \hspace{1cm} (43)

Due to the angular speed of the RAGG rotary stage, $\omega_{t}$ and $\omega_{e}$ are much bigger than other angular motion components; correspondingly in $T_{1} \sim T_{6}$, $T_{5}$ and $T_{6}$ are more bigger. Assuming that the angular speed of the RAGG rotary stage plane is $\Omega = 1.57 \text{rad/s}$ (0.25Hz), and the angular acceleration is in the order of $\Omega = 10^{-3} \text{rad/s}^2$; based on the (19), $T_{5} = 2.54 \times 10^{8} \text{E}$, $T_{6} = 10^{8} \text{E}$. Equation (43) is equivalent to the following condition:

$$T_{1}T_{n_{1}} \leq T_{2}T_{5} \approx 0.5T_{5}(\omega_{t}^{2} - \omega_{e}^{2}) \leq 4 \times 10^{16}$$

$$T_{2}T_{n_{1}} \leq T_{3}T_{5} \approx T_{5}(\omega_{t}\omega_{e}) \leq 4 \times 10^{16}$$

$$T_{4}T_{n_{1}} \leq T_{5}T_{5} \approx T_{5}(\omega_{t}^{2} + \omega_{e}^{2}) \leq 4 \times 10^{16}$$

$$T_{2}T_{n_{1}} \leq T_{3}T_{5} \approx T_{5}(\omega_{t}\omega_{e} + \omega_{e}) \leq 4 \times 10^{16}$$

$$T_{4}T_{n_{1}} \leq T_{5}T_{5} \approx T_{5}(\omega_{t}\omega_{e} - \omega_{e}) \leq 4 \times 10^{16}$$

$$T_{2}T_{n_{1}} \leq T_{3}T_{5} \approx T_{5}(\omega_{t}\omega_{e} - \omega_{e}) \leq 4 \times 10^{16}.$$  \hspace{1cm} (44)

Assuming that the angular velocity and angular acceleration components are of the same order ($\omega_{t} = \omega_{t}$, $\omega_{e} = \omega_{e}$), and let $\omega_{critical}$ and $\omega_{critical}$ represent the critical angular velocity and the critical angular acceleration, respectively. Then by (44), we can calculate the critical value of the angular motion roughly as

$$\omega_{critical} = 0.0887 \text{rad/s} = 5.0842^\circ/s$$

$$\dot{\omega}_{critical} = 0.0079 \text{rad/s}^2 = 0.4511^\circ/s^2.$$  \hspace{1cm} (45)

Those harmonic components of the angular motion whose fundamental frequency equal the rotation rate of the rotary stage have considerable impact on the RAGG sensitivity. The angular acceleration is the derivative of the angular velocity. Therefore, we can denote the harmonic components of the angular motion as

$$\omega_{k}(t) = A_{k}\Omega \sin(k\Omega t)$$

$$\dot{\omega}_{k}(t) = k\Omega A_{k}\cos(k\Omega t)$$  \hspace{1cm} (46)

where $\omega_{k}(t)$ and $\dot{\omega}_{k}(t)$ are the $k$th-order harmonic components of angular velocity and angular acceleration, respectively, $\Omega$ is the angular frequency of the rotary stage, and $A_{k}\Omega$ is the magnitude of the $k$th-order harmonic component of angular velocity. Let $A_{ICritical}$ represent the critical magnitude of the $k$th-order harmonic component of angular velocity.
Obvious

$$A_{\text{critical}} \leq \min \{ \omega_{\text{critical}}, \omega_{\text{critical}}/k\Omega \}. \quad (47)$$

Substituting $k = 1$ and $\Omega = 1.57$ rad/s (the frequency of the rotary stage is 0.25 Hz) into (47), we calculate the critical magnitude of the fundamental frequency component as $A_{\text{critical}} = 0.005$ rad/s = 0.287°/s. Based on the above analysis, we list in Table V the conditions for neglecting $k_{jp}T_n T_{n2} R_j^2$. From the perspective of unit operation of physical quantity, the error terms concerning the second-order error coefficients and angular motion include $T_n T_{n2}$, and $T_{n1} T_{n2}$ will only be coupled to the accelerometer second-order error coefficients. In gravity gradiometry, the RAGG is mounted on a stabilized platform that is isolated from high-frequency vibrations by pneumatic mounting pads. It may be not difficult to meet the conditions listed in Table V, so in (21)–(25), any error terms containing $T_{n1} T_{n2}$ except $T_{n2}^2$ can be neglected.

2) Coupling error terms concerning second-order error coefficients and linear motion. We apply $k_{jp} a_{n1} a_{n2}$, $(n_1, n_2 = x, y, z; p = 2, 4, 5, 6, 7, 8)$ representing the basic coupling error terms concerning the second-order error coefficients and linear motion. In the analytical model $G_{out}$, the terms $D_{ij} a_{n1} a_{n2}$ and $\sum k_{jp} a_{n1} a_{n2}$ consist of $k_{jp} a_{n1} a_{n2}$ and belong to the coupling error terms concerning the second-order error coefficients and linear motion; correspondingly, the measurement errors contributed by $k_{jp} a_{n1} a_{n2}$ can be expressed as

$$M_{c2} = k_{jp} a_{n1} a_{n2} / g_{\text{ggi}}. \quad (48)$$

In the accelerometer, $k_{jp} = k_{j1} K_{jp}$, so (48) becomes

$$M_{c2} \approx 0.25 K_{jp} a_{n1} a_{n2} / R_j. \quad (49)$$

Let $a_{\text{critical}}$ denotes the critical accelerations of $k_{jp} a_{n1} a_{n2}$. Substituting $M_{c2} = 0.1$ E and $K_{jp} = 10^{-6}$ g/°g per to (49), we get $a_{\text{critical}} = 2 \times 10^{-3}$ g. In moving-base gradiometry, the linear acceleration $a_n$ is of the order of 0.1 g. Therefore, the coupling error terms concerning second-order error coefficients and linear motion cannot be neglected.

3) Coupling error terms concerning linear scale-factor imbalance and linear motion. We apply $k_{jp} a_{n1}$ ($n = x, y, z$) representing the basic coupling error terms concerning the linear-scale-factor imbalance and linear motion. In the analytical model $G_{out}$, the terms

$$\sum k_{ji} a_n \text{ and } D_{ji} a_{n1} a_{n2} \text{ consist of } dk_{ji} a_n \text{ and belong to the coupling error terms concerning the linear-scale-factor imbalance and linear motion. Correspondingly, the measurement errors contributed by } dk_{ji} a_n \text{ can be expressed as}$$

$$M_{c3} = dk_{ji} a_n / g_{\text{ggi}}. \quad (50)$$

The degree of linear-scale-factor imbalance before online adjustment is of the order of $10^{-4}$, i.e., $dk_{ji} / k_{ji} = 10^{-4}$, $g_{\text{ggi}}$ is the RAGG scale factor. So, (50) becomes

$$M_{c3} \approx 2.5 \times 10^{-5} a_n / R_j. \quad (51)$$

Let $a_{\text{critical}}$ denotes the critical accelerations of $dk_{ji} a_n$, respectively. Substituting $M_{c3} = 0.1$ E into (51), we get $a_{\text{critical}} = 4 \times 10^{-8}$ g. In moving-base gradiometry, the linear acceleration $a_n$ is of the order of 0.1 g. Therefore, the coupling error terms concerning the linear-scale-factor imbalance and linear motion cannot be neglected.

4) Coupling error terms concerning second-order error coefficients, linear motion, and angular motion. We denote the basic coupling error terms concerning second-order error coefficients, linear motion, and angular motion as $k_{jp} T_n R_j a_n$. In the RAGG analytical model, $D_{ji} T_n R_j a_n$ and $\sum k_{jp} T_n R_j a_n$ are the coupling error terms concerning the second-order error coefficients, linear motion, and angular motion. The measurement error contributed by $k_{jp} T_n R_j a_n$ is

$$M_{c4} = k_{jp} T_n R_j a_n / g_{\text{ggi}} \approx 0.25 K_{jp} T_n a_n. \quad (52)$$

In gravity gradiometry, the acceleration $a_n$ is of the order of 0.1 g. To ensure an RAGG sensitivity of 1 E and assuming $M_{c4} = 0.1$ E, $K_{jp} = 10^{-6}$ g/°g, and $a_n = 0.1$ g, based on (52), we obtain the critical value of $T_n$ for neglecting $k_{jp} T_n R_j a_n$, namely, $(T_{n, \text{critical}} = 4 \times 10^7$ E. Similar to the previous analysis, the critical angular velocity and angular accelerations are calculated and are listed in Table VI. The error terms concerning the second-order error coefficients, linear motion, and angular motion include $T_{n1} a_n$, and $T_{n2} a_n$ will only be coupled to the accelerometer second-order error coefficients. In moving-base gradiometry, it is relatively easy to satisfy the conditions listed in Table VI, so in (21)–(25), any term containing $T_{n1} a_n$, except $T_{n2} a_n$, and $T_{n1} a_n$ can be neglected.
5) Coupling error terms concerning mounting misalignment angle, linear scale factors, and angular motion. The mounting misalignment angles are \( \beta_{j1}, \beta_{j2}, \vartheta_{j1}, \) and \( \vartheta_{j2} \). The basic coupling error terms concerning the misalignment angle, linear scale factors, and angular motion are permutations of the misalignment angles \( (\beta_{j1}, \beta_{j2}, \vartheta_{j1}, \vartheta_{j2}) \), the linear scale factors \( (k_{j1}) \), and the angular motion \( (T_i \sim T_o) \). Because the magnitudes of the misalignment angles are of the order of \( 10^{-4} \), the more misalignment angles in an error term, the smaller its magnitude. Therefore, we analyze only those error terms with fewer than three misalignment angles. Coupling error terms with the same number of misalignment angles have the same order of magnitude. The basic coupling error terms with one misalignment angle are typically \( \beta_{jm}k_{j1}T_{pRj} \) and \( \vartheta_{jm}k_{j1}T_{pRj} \); the basic coupling error terms with two misalignment angles are typically \( \beta_{jm}\vartheta_{jm}k_{j1}T_{pRj} \) and \( \beta_{jm}\beta_{jm}k_{j1}T_{pRj} \); the basic coupling error term with three misalignment angles is typically \( \beta_{jm}\beta_{jm}\vartheta_{jm}k_{j1}T_{pRj} \). In the RAGG analytical model, \( D_{k_{j1}R_{p1}} \), \( \sum \beta_{jm}\vartheta_{jm}k_{j1}T_{pRj} \), \( \sum \beta_{jm}\beta_{jm}k_{j1}T_{pRj} \), \( D_{\beta_{jm}\beta_{jm}\vartheta_{jm}k_{j1}T_{pRj}} \), etc., are the error terms concerning the linear scale factors, misalignment angles, and angular motion. Here, we take \( \beta_{jm}k_{j1}T_{pRj} \), \( \beta_{jm}\vartheta_{jm}k_{j1}T_{pRj} \), and \( \beta_{jm}\beta_{jm}\vartheta_{jm}k_{j1}T_{pRj} \) as examples of analyzing the coupling error terms with one, two, and three misalignment angles, respectively. Let \( M_{51}, M_{52}, M_{53} \) represent the measurement errors contributed by the coupling error terms with one, two, and three misalignment angles, respectively

\[
M_{51} = \beta_{jm}k_{j1}T_{pRj}/k_{ggi} \\
M_{52} = \beta_{jm}\beta_{jm}k_{j1}T_{pRj}/k_{ggi} \\
M_{53} = \beta_{jm}\beta_{jm}\vartheta_{jm}k_{j1}T_{pRj}/k_{ggi}
\]  

where \( k_{ggi} \) is the RAGG scale factor. Substituting \( k_{ggi} = \sum_{j=1}^4 k_{j1}R_j \) into (53), we obtain

\[
M_{51} \approx 0.25\beta_{jm}T_p \\
M_{52} \approx 0.25\beta_{jm}\beta_{jm}T_p \\
M_{53} \approx 0.25\beta_{jm}\beta_{jm}\vartheta_{jm}T_p.
\]  

Let \( T_{1\text{critical}}, T_{2\text{critical}} \), and \( T_{3\text{critical}} \) represent the critical values for neglecting coupling error terms with one, two, and three misalignment angles, respectively. Similarly, assuming that \( M_{51} = M_{52} = M_{53} = 0.1 \) E and a misalignment angle of \( \beta_{jm} = \vartheta_{jm} = 10^{-4} \) rad, we calculate the critical values as \( T_{1\text{critical}} = 4 \times 10^3 \) E, \( T_{2\text{critical}} = 4 \times 10^7 \) E, and \( T_{3\text{critical}} = 4 \times 10^{11} \) E. Correspondingly, we have calculated the conditions for neglecting the coupling error terms and listed them in Table VII. Clearly, coupling error terms with more than one misalignment angle can be neglected.

6) Coupling error terms concerning linear scale factors, misalignment angles, and linear motion. The coupling error terms concerning linear scale factors, misalignment angles, and linear motion are permutations of the misalignment angles \( (\beta_{j1}, \beta_{j2}, \vartheta_{j1}, \vartheta_{j2}) \), the linear scale factors \( (k_{j1}) \), and the linear motion \( (\alpha_x, \alpha_y, \alpha_z) \). Similarly, we only analyze those coupling error terms that have fewer than three misalignment angles. Because coupling error terms that have the same number of misalignment angles also have the same magnitude, we take \( \beta_{jm}k_{j1}\alpha_z \), \( \beta_{jm}\beta_{jm}k_{j1}\alpha_z \), and \( \beta_{jm}\beta_{jm}\vartheta_{jm}k_{j1}\alpha_z \) as examples of analyzing coupling error terms with one, two, and three misalignment angles, respectively. Let \( M_{61}, M_{62}, \) and \( M_{63} \) represent the measurement errors contributed by error terms with one, two, and three misalignment angles, respectively

\[
M_{61} = \beta_{jm}k_{j1}\alpha_z/k_{ggi} \approx 0.25\beta_{jm}\alpha_z/R \\
M_{62} = \beta_{jm}\beta_{jm}k_{j1}\alpha_z/k_{ggi} \approx 0.25\beta_{jm}\beta_{jm}\alpha_z/R \\
M_{63} = \beta_{jm}\beta_{jm}\vartheta_{jm}k_{j1}\alpha_z/k_{ggi} \approx 0.25\beta_{jm}\beta_{jm}\vartheta_{jm}R.
\]  

Let \( a_{1\text{critical}}, a_{2\text{critical}}, \) and \( a_{3\text{critical}} \) represent the critical values for neglecting coupling error terms with one, two, and three misalignment angles, respectively. Similarly, assuming that \( M_{61} = M_{62} = M_{63} = 0.1 \) E, \( R = 0.1 \) m, and a misalignment angle of \( \beta_{jm} = \vartheta_{jm} = 10^{-4} \) rad, the critical values are calculated as \( a_{1\text{critical}} = 4 \times 10^{-3} \) g, \( a_{2\text{critical}} = 4 \times 10^{-4} \) g, and \( a_{3\text{critical}} = 4 \) g, respectively. Because in gravity gradiometry the RAGG acceleration is usually in the order of 0.1 g, we can neglect coupling error terms with more than two misalignment angles. Based on the previous analysis and neglecting the small coupling error terms, obtain the 25 parameters simplified RAGG analytical model.

**REFERENCES**

[1] L. Wu and J. Tian  
Automated gravity gradient tensor inversion for underwater object detection  
*J. Geophys. Eng.*, vol. 7, no. 4, pp. 410–416, 2010. [Online].  
Available: https://doi.org/10.1088/1742-2132/7/4/008

[2] T. C. Welker, M. Pachtler, and R. E. Huffman  
Gravity gradiometer integrated inertial navigation  
*Proc. Eur. Control Conf.*, 2013, pp. 846–851.

[3] D. Gao, B. Hu, L. Chang, F. Qin, and L. Xu  
An aided navigation method based on strapdown gravity gradiometer
Sensors, vol. 21, no. 3, 2021, Art. no. 829.

[4] X. Cheng, L. Xiong, and Y. Guo
Feasibility and error analysis of target detection based on gravity and gravity gradient
In Proc. 13th IEEE Conf. Ind. Electron. Appl., 2018, pp. 1900–1904.

[5] C. Cevallos
Automatic generation of 3D geophysical models using curvatures derived from airborne gravity gradient data
Geophysics, vol. 79, no. 5, pp. G49–G58, 2014.

[6] D. U. Carlos, M. A. Braga, H. F. Galbiatti, and W. R. Pereira
Airborne gravity gradiometry-data processing and interpretation
Revista Brasileira de Geofísica, vol. 31, no. 3, pp. 427–453, 2013.

[7] A. Araya et al.,
Gravity gradiometer implemented in AUV for detection of seafloor massive sulfides
In Proc. Oceans, Piscataway, NJ, USA, 2012, pp. 1–4, doi: 10.1109/OCEANS.2012.6405114.

[8] C. A. Affleck and A. Jircitano
Passive gravity gradiometer navigation system
In Proc. IEEE Symp. Position Location Navigation. Decade Excellence Navigation Sci., 1990, pp. 60–66.

[9] M. Talwani
Non linear inversion of gravity gradients and the GGI gradiometer
Open Geosci., vol. 3, no. 4, pp. 424–434, 2011. [Online]. Available: https://doi.org/10.2478/s13533-011-0041-3

[10] M. Moody
A superconducting gravity gradiometer for measurements from a moving vehicle
Rev. Sci. Instrum., vol. 82, no. 9, 2011, Art. no. 094501.

[11] J. Luo et al.,
An improved torque type gravity gradiometer with dynamic modulation
Acta Geodaetica et Geophysica, vol. 53, no. 2, pp. 171–187, 2018.

[12] H. Liu, W. Pike, and G. Dou
Design, fabrication and characterization of a micro-machined gravity gradiometer suspension
In Proc. Sensors, 2014, pp. 1611–1614.

[13] S.-G. Lee
A full-tensor superconducting gravity gradiometer system composed of levitation-type accelerometers
J. Korean Phys. Soc., vol. 75, no. 3, pp. 254–260, 2019.

[14] J. Flokstra, R. Cuperus, R. Wiegerrink, and M. van Essen
A MEMS-based gravity gradiometer for future planetary missions
Cryogenics, vol. 49, no. 11, pp. 665–668, 2009. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0011227508002154

[15] J. B. Lee
Falcon gravity gradiometer technology
Exploration Geophys., vol. 32, no. 3/4, pp. 247–250, 2001.

[16] H. Heard
The gravity gradiometer survey system (GGSS) : Eos, vol. 69, no. 8, pp. 105–117, 1988.

[17] M. H. Dransfield and A. N. Christensen
Performance of airborne gravity gradiometers
Leading Edge, vol. 32, no. 8, pp. 908–922, 2013.

[18] M. I. Evstifeev
Dynamics of onboard gravity gradiometers
Gyroscopy Navigation, vol. 11, no. 1, pp. 13–24, 2020. [Online]. Available: https://doi.org/10.1134/S207510872001006X

[19] E. Metzger
Recent gravity gradiometer developments
In Proc. Guid. Control Conf., 1977, pp. 306–315, Art. no. 77-1081.

[20] C. Jekeli
Inertial Navigation Systems with Geodetic Applications. Berlin, Germany: De Gruyter, 2012. [Online]. Available: https://doi.org/10.1515/9783110800234

[21] M. Yu, T. Cai, L. Tu, C. Hu, and L. Yu
Posterior compensation of moving-base rotating accelerometer gravity gradiometer
IEEE Trans. Instrum. Meas., vol. 70, pp. 1–10, Jul. 2021, doi: 10.1109/TIM.2020.3010194.

[22] M. Yu and T. Cai
Numerical Model of Moving Base Rotating Accelerometer Gravity Gradiometer. Berlin, Germany: Springer, 2020, pp. 1–5. [Online]. Available: https://doi.org/10.1007/978-3-030-20114

[23] M. Yu and T. Cai
Calibration of a rotating accelerometer gravity gradiometer using centrifugal gradients
Rev. Sci. Instrum., vol. 89, no. 5, 2018, Art. no. 54502.

[24] J. J. Catherines, J. S. Mixson, and H. F. Scholl
Vibrations measured in the passenger cabins of two jet transport aircraft
NASA Langley Research Center, Hampton, VA, USA, Tech. Rep. NASA-TN-D-7923, L-9531, 1975. [Online]. Available: https://trs.nasa.gov/search.jsp?R=19750020964

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