Hadron correlators with improved fermions

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We investigate point-to-point correlation functions for various mesonic and baryonic channels using the $O(a)$-improved Wilson action due to Sheikholeslami and Wohlert. We consider propagators to both time slices 0 and 1. We find that discretisation effects are more pronounced than those reported with unimproved Wilson fermions, but that the same procedure for removing finite size effects is successful. Extrapolating to the chiral limit, we see the notable features predicted phenomenologically: the ratio of interacting to free correlators in the vector channel is roughly constant to about 1 fm, while in the pseudoscalar channel the ratio increases markedly due to the strong binding.

1. Introduction

Traditionally, much of lattice gauge theory has been concerned with calculations of on-shell quantities such as masses and matrix elements. However, it has been emphasised by Shuryak that there is a lot of interesting physics to be had by looking at off-shell quantities. In the reviewed cited, the fundamental objects of interest are space-like correlations between quantum fields in the vacuum:

$$K(r_1 - r_2) \equiv \langle 0 | T J(r_1) \bar{J}(r_2) | 0 \rangle,$$

where $J(r)$ are currents corresponding to some suitable quantum numbers: in meson channels, $J(r) = \bar{\psi}(r) \Gamma \psi(r)$, while in baryon channels, $J(r) = \epsilon_{ijk} \psi(r) \Gamma_1 \psi(r) \Gamma_2 \psi(r)$, for some appropriate Dirac structure $\Gamma$, $\Gamma_1$ and $\Gamma_2$.

These correlations decay very fast; at large volume the dominant behaviour is exponential in the lightest bound-state mass. It is therefore convenient to normalise them against the same correlations calculated for non-interacting quarks, in other words using a frozen gauge configuration. We therefore define

$$R(r) \equiv \frac{K_{\text{interacting}}(r)}{K_{\text{free}}(r)}.$$

The correlations defined in this fashion contain in principle a great deal of information about all that is happening in the chosen channel. In this way we can show up, for example, the striking differences between the vector and pseudoscalar which are far from obvious if one looks only at masses or decay constants, and hence shed more light on the mechanisms underlying confinement and chiral symmetry breaking.

A detailed analysis of point-to-point correlations has been carried out by Chu et al.; we refer the reader to that paper for more detailed definitions, formulae and methods. Much of our methodology is inherited from theirs. Here we describe calculations using Sheikholeslami-Wohlert $O(a)$-improved (S-W) fermions. In addition, our lattice is slightly finer than that of ref. though our lattice size is correspondingly smaller.

We shall now describe our data and its analysis and comparisons with phenomenological results, then proceed to conclusions.

2. Analysis

Our data sets are on $24^3 \cdot 48$ quenched lattices at inverse coupling $\beta = 6.2$. The quark propagators, inverted using the S-W action, are at three hopping parameters $\kappa = 0.14144, 0.14226$ and 0.14262. On this lattice, $\kappa_{\text{critical}} = 0.14314$. We have currently 27 configurations at all three $\kappa$; for consistency we restrict ourselves here to these 27. The S-W action adds a two-link part to the Wilson action and also requires a rotation
of the fermion fields. This is known to improve the perturbative scaling behaviour \([4]\); however, there is no particular reason to suppose the type of analysis we are performing will benefit from this.

We then sum over all rotationally equivalent points on all configurations (taking account of antisymmetry in our baryon correlations), using a bootstrap over all data to generate the errors. We take the ratio of these with free propagators generated using an adaption of the method of Carpenter and Baillie \([5]\). Initially we used only propagators from the origin to all sites at \(t = 0\), i.e. on the same time slice as the origin.

We use the method of ref. \([6]\) to improve finite size effects by removing images of the source at the origin. This works well.

However, we have no equivalent quantitative way of dealing with anisotropy. Examination of the data shows that there are strong effects up to approximately six lattice units; these appear to be significantly larger than those seen by Chu et al. with the Wilson action \([2]\). These authors point out that propagators to points nearest the body-diagonals of the lattice include more paths and should therefore be nearer the continuum values. In our case it appears that even these points are suffering from significant anisotropy for distances \(r < 6\).

To shed further light on this, we calculate correlators to points on the next time slice, \(t = 1\). We have verified directly that the effect of the differing lattice size in the time direction has negligible impact on the correlations. We find that these points lie consistently lower than the \(t = 0\) points, so much so that it is highly unlikely that the ‘more paths means lower anisotropy’ ansatz can be simply applied in our case. On the other hand, the interpretation ‘more paths means lower correlations’ does seem to hold: in figure 1 we show \(R(r)\) from \([1]\) with the image correction applied. The different symbols denote co-ordinates with different numbers of zeroes: i.e., \((x, y, z, t)\), \((x, y, z, 0)\), etc., with \(x, y, z\) and \(t\) non-zero. The expectation is that the ratio remains roughly constant for small \(r\) (note that this is for quarks around the strange mass, for which the behaviour of the anisotropy is cleanest), in other words that from \(r = 0\) to 8 the ratio should be \(\sim 1.5\). There is no easy way to make the data fit this. Presumably this is an effect primarily of the rotation of the fermion fields. Indeed, the major additional problem appears to be that our correlations plummet for some small values of \(r\); apart from that the anisotropy is similar to the Wilson case.

Matters are improved slightly by the choice of Chu et al. to restrict points to those making an angle \(\theta\) with the body diagonal of the spatial lattice with \(\cos(\theta) > 0.9\). In this case the scatter of points is consistent with the statistical scatter from about \(r \sim 6\), and our fits are performed from this distance on.

2.1. Chiral extrapolations

We follow the usual UKQCD convention in extrapolating our results to zero quark mass, rather than the alternative of extrapolating to the physical pion mass. Our chiral extrapolations are limited by the fact that currently we only have three quark masses. We have tried various forms of chiral extrapolation and have settled on a fit which linearly extrapolates \(\log(R(r))\).

2.2. Fits to phenomenological forms

A reasonable phenomenological approximation to the data should be found be assuming a smooth continuum background at large energy plus some
number of resonance poles. In practice one pole is entirely adequate and it is unlikely that our data could be persuaded to yield a convincing second pole. For the channels shown here this gives four parameters to fit: an overall normalisation, the position \((m)\) and height \((\lambda)\) of the delta function pole, and the threshold for the continuum. Because we cannot fit for \(r < 0.6\) the estimation of the continuum threshold is generally poor and the fits shown just use a reasonable fixed value. Details of the forms are as in ref. \(^2\). Errors are found by jackknife, completely re-analysing the data with each set missing in turn.

We show the extrapolated data with the \(\cos(\theta) > 0.9\) cut off and fits for the vector, nucleon and pseudoscalar in figure \(^3\). One fermi corresponds to about fourteen of our lattice spacings. We find a slightly low rho mass \((620 \pm 90\text{ MeV})\) and a high nucleon mass \((1170 \pm 70\text{ MeV})\); also the pseudoscalar is rather heavy \((300 \pm 20\text{ MeV})\), throwing some doubt on the validity of extrapolating to zero quark mass for this type of analysis.

3. Conclusions

We have examined point-to-point hadron correlation functions using an \(O(a)\) improved action. We find more severe finite lattice spacing effects than reported using Wilson fermions. Nonetheless, the physical features of the results are similar: asymptotic freedom in the vector channel persists to about \(1\) fm, but is violated quickly in the pseudoscalar channel.

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Figure 2. Extrapolated data and fits for the pseudoscalar, vector and nucleon. The vertical scale is arbitrary: anisotropy makes normalisation difficult.