ANALYSIS OF THE VECTOR AND AXIALVECTOR $QQ\bar{Q}Q$ TETRAQUARK STATES WITH QCD SUM RULES

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Abstract

In this article, we construct the axialvector-diquark-axialvector-antidiquark type currents to study both the vector and axialvector $QQ\bar{Q}Q$ tetraquark states with the QCD sum rules, and obtain the masses $M_{Y_{(ccQQ,1^+--)}} = 6.05 \pm 0.08$ GeV, $M_{Y_{(ccQQ,1^-+-)}} = 6.11 \pm 0.08$ GeV, $M_{Y_{(bbQQ,1^+--)}} = 18.84 \pm 0.09$ GeV, $M_{Y_{(bbQQ,1^-+-)}} = 18.89 \pm 0.09$ GeV. The vector tetraquark states lie 40 MeV above the corresponding centroids of the $0^{++}$, $1^{+-}$ and $2^{++}$ tetraquark states, which is a typical feature of the vector tetraquark states consist of four heavy quarks.

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1 Introduction

The exotic charmonium-like and bottomonium-like states, such as the $Z_c(3900)$, $Z_c(4025)$, $Z_c(4200)$, $Z(4430)$, $Z_b(10610)$, $Z_b(10650)$, are excellent candidates for the multiquark states. If they are really tetraquark states, their constituents are two heavy quarks and two light quarks. Up to now, no exotic tetraquark candidate composed of more than two heavy quarks has been reported. Theoretically, there have been several approaches to study the masses and widths of the exotic states, such as the non-relativistic potential models, the constituent quark model with color-magnetic interaction, the (moment) QCD sum rules, etc. Experimentally, the ATLAS, CMS and LHCb collaborations have measured the cross section for double charmonium production. Recently, the LHCb collaboration studied the $\Upsilon \mu^+\mu^-$ invariant-mass distribution for a possible exotic tetraquark state composed of two $b$ quarks and two $\bar{b}$ quarks based on a data sample of $pp$ collisions recorded with the LHCb detector at center-of-mass energies $\sqrt{s} = 7,8$ and $13$ TeV corresponding to an integrated luminosity of $6.3$ fb$^{-1}$, and observed no significant excess. The decays to the final states $\Upsilon \mu^+\mu^-$ can take place through $Y_b(0^{++}/2^{++}) \to \Upsilon \Upsilon/\bar{\Upsilon} \Upsilon \to \Upsilon \mu^+\mu^-$ or $Y_b(1^{--}) \to \Upsilon \mu^+\mu^-$. In Ref. [11], Esposito and Polosa argue that the partial width for the $Y_b(2^{++}) \to \Upsilon \mu^+\mu^-$ decay is too small to be currently observed at the LHC. However, if the barrier between the diquark and antidiquark is very narrow and the tetraquark width is sufficiently small, the detection of such a state is still possible.

In 2013, the BESIII collaboration studied the process $e^+e^- \to \pi^+\pi^- J/\psi$ at a center-of-mass energy of 4.26 GeV, and observed a structure $Z_c^+(3900)$ in the $\pi^+J/\psi$ mass spectrum. Recently, the BESIII collaboration determined the spin and parity of the $Z_c^+(3900)$ state to be $J^P = 1^+$ with a statistical significance larger than 7$\sigma$ over other quantum numbers. Analogously, there may exist a tetraquark state $Y_{c/\bar{b}}(1^{++})$ which decays to the $\eta_c J/\psi$ or $\eta_b \Upsilon$.

The diquarks $c^{ij}q_{k}^{T}CTq_{k}^{T}$ have five structures in Dirac spinor space, where the $i, j$ and $k$ are color indexes, $CT = C\gamma_5$, $C\gamma_\mu\gamma_5$, $C\gamma_\mu$ and $C\sigma_{\mu\nu}$ for the scalar, pseudoscalar, vector, axialvector and tensor diquarks, respectively. The stable diquark configurations are the scalar ($C\gamma_5$) and axialvector ($C\gamma_\mu$) diquark states from the QCD sum rules. The QCD sum rules have been extensively applied to study the tetraquark states and molecular states. In Ref. [25], we study the mass and width of the $Z_c^+(3900)$ with the $C\gamma_\mu\otimes\gamma_5 C - C\gamma_5\otimes\gamma_\mu C$ type current with the QCD sum rules in details, and reproduce the experimental data satisfactorily. In Ref. [26], we study both the vector and axialvector tetraquark states with the $C\gamma_\mu\otimes\gamma_\nu C - C\gamma_\nu\otimes\gamma_\mu C$ type currents, and

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reproduce the experimental values of the masses of the $Y(4660)$ and $Z_c(4020/4025)$ satisfactorily. The double-heavy diquark states $\varepsilon^{ijk} Q^T_3 C \gamma_5 Q_k$ cannot exist due to the Pauli principle. In previous work, we took the double-heavy diquark states $\varepsilon^{ijk} Q^T_3 C \gamma_5 Q_k$ as basic constituents to construct the scalar and tensor tetraquark states with the QCD sum rules [14]. Now we extend our previous work to study the vector and axialvector tetraquark states $QQ\bar{Q}\bar{Q}$ with the $C \gamma_5 \otimes \gamma_5 C - C \gamma_5 \otimes \gamma_5 C$ type currents, which are expected to couple potentially to the lowest tetraquark states, especially for the vector tetraquark states.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the vector and axialvector $QQ\bar{Q}\bar{Q}$ tetraquark states in section 2; in section 3, we present the numerical results and discussions; section 4 is reserved for our conclusion.

2 QCD sum rules for the vector and axialvector tetraquark states

In the following, we write down the two-point correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ in the QCD sum rules,

$$\Pi_{\mu\nu\alpha\beta}(p) = i \int d^4x e^{ipx} \langle 0 | T \left\{ J_{\mu\nu}(x) J^\dagger_{\alpha\beta}(0) \right\} | 0 \rangle,$$

(1)

where

$$J_{\mu\nu}(x) = \varepsilon^{ijk} \varepsilon^{imn} \left\{ Q^T_j(x) C \gamma_\mu Q_k(x) \bar{Q}_m(x) \gamma_\nu C \bar{Q}_n^T(x) - Q^T_j(x) C \gamma_\mu Q_k(x) \bar{Q}_m(x) \gamma_\nu C \bar{Q}_n^T(x) \right\},$$

(2)

where the $i, j, k, m, n$ are color indexes, the $C$ is the charge conjugation matrix. In Ref.[13], Chen et al construct the $C \gamma_\mu \gamma_5 \otimes C - C \gamma_\mu \gamma_5 C$ type currents with the color structure $6_c \otimes 6_c$ and the $C \sigma_{\mu\nu} \gamma_5 \otimes \gamma^\nu C - C \gamma^\nu \otimes \sigma_{\mu\nu} \gamma_5 C$ type currents with the color structure $3_c \otimes 3_c$ to interpolate the $1^{++}$ tetraquark states, and construct the $C \gamma_\mu \gamma_5 \otimes \gamma_5 C - C \gamma_5 \otimes \gamma_\mu \gamma_5 C$ type currents with the color structure $6_c \otimes 6_c$ and the $C \sigma_{\mu\nu} \otimes \gamma^\nu C - C \gamma^\nu \otimes \sigma_{\mu\nu} C$ type currents with the color structure $3_c \otimes 3_c$ to interpolate the $1^{--}$ tetraquark states. The $C, C \gamma_\mu, C \gamma_5, C \sigma_{\mu\nu}, C \sigma_{\mu\nu} \gamma_5$ type diquark states are not as stable as the $C \gamma_5, C \gamma_\mu$ type diquark states. Furthermore, the attractive interaction induced by one-gluon exchange favors formation of the diquark states in color antitriplet [27], we prefer the currents with the color structure $3_c \otimes 3_c$, but sometimes we have to choose the color structure $6_c \otimes 6_c$ to construct currents to satisfy the Fermi-Dirac statistics. In this article, we construct the tensor currents $C \gamma_\mu \otimes \gamma_\nu C - C \gamma_\nu \otimes \gamma_\mu C$ with the color structure $3_c \otimes 3_c$ to interpolate both the $1^{++}$ and $1^{--}$ tetraquark states, which differ from the ones in Ref.[13] completely. Moreover, in this article, we study the tetraquark states with the Borel QCD sum rules, while in Ref.[14], Chen et al study the tetraquark states with the moment QCD sum rules.

At the phenomenological side, we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_{\mu\nu}(x)$ into the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ to obtain the hadronic representation [28, 29]. After isolating the ground state contributions of the axialvector and vector tetraquark states, we get the following results,

$$\Pi_{\mu\nu\alpha\beta}(p) = \frac{\lambda^2_+}{M^2_+ (M^2_+ - p^2)} \left( p^2 g_{\mu\alpha} g_{\nu\beta} - p^2 g_{\mu\beta} g_{\nu\alpha} - g_{\mu\alpha} p_\nu p_\beta - g_{\mu\beta} p_\nu p_\alpha + g_{\nu\alpha} p_\mu p_\beta \right)$$

$$+ \frac{\lambda^2_-}{M^2_- (M^2_- - p^2)} \left( -g_{\mu\alpha} p_\nu p_\beta - g_{\nu\beta} p_\mu p_\alpha + g_{\mu\beta} p_\nu p_\alpha + g_{\nu\alpha} p_\mu p_\beta \right) + \cdots,$$

(3)

where the $Y^+$ and $Y^-$ denote the axialvector and vector tetraquark states respectively, the pole
residues $\lambda_{Y^\pm}$ are defined by
\[
\langle 0|J_{\mu^+}(0)|Y^+(p)\rangle = \frac{\lambda_{Y^+}}{M_{Y^+}} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{\alpha\beta} p^\nu,
\]
\[
\langle 0|J_{\mu^+}(0)|Y^-(p)\rangle = \frac{\lambda_{Y^-}}{M_{Y^-}} (\varepsilon_{\mu\nu} p^\nu - \varepsilon^\nu p_\mu),
\]
the $\varepsilon_\mu$ are the polarization vectors of the vector and axialvector tetraquark states. We can rewrite the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ into the following form according to Lorentz covariance,
\[
\Pi_{\mu\nu\alpha\beta}(p) = \Pi_{Y^+}(p^2) \left( p^2 g_{\mu\alpha} g_{\nu\beta} - p^2 g_{\rho\beta} g_{\sigma\alpha} - g_{\mu\alpha} p_\rho p_\beta - g_{\rho\beta} p_\mu p_\alpha + g_{\rho\beta} p_\rho p_\alpha + g_{\sigma\alpha} p_\mu p_\beta \right) + \Pi_{Y^-}(p^2) \left( -g_{\rho\beta} p_\mu p_\alpha + g_{\rho\beta} p_\rho p_\alpha + g_{\sigma\alpha} p_\mu p_\beta \right) .
\]
Now we project out the components $\Pi_{Y^+}(p^2)$ and $\Pi_{Y^-}(p^2)$ by introducing the operators $P_{Y^+}^{\mu\nu\alpha\beta}$ and $P_{Y^-}^{\mu\nu\alpha\beta}$,
\[
\tilde{\Pi}_{Y^+}(p^2) = p^2 \Pi_{Y^+}(p^2) = P_{Y^+}^{\mu\nu\alpha\beta} \Pi_{\mu\nu\alpha\beta}(p), \]
\[
\tilde{\Pi}_{Y^-}(p^2) = p^2 \Pi_{Y^-}(p^2) = P_{Y^-}^{\mu\nu\alpha\beta} \Pi_{\mu\nu\alpha\beta}(p),
\]
\[
P_{Y^+}^{\mu\nu\alpha\beta} = \frac{1}{6} \left( g^{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2} \right) \left( g^{\nu\beta} - \frac{p^\nu p^\beta}{p^2} \right),
\]
\[
P_{Y^-}^{\mu\nu\alpha\beta} = \frac{1}{6} \left( g^{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2} \right) \left( g^{\nu\beta} - \frac{p^\nu p^\beta}{p^2} \right) - \frac{1}{6} g^{\mu\alpha} g^{\nu\beta}.
\]
In the following, we briefly outline the operator product expansion for the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ in perturbative QCD. We contract the heavy quark fields with Wick theorem and obtain the results:
\[
\Pi_{\mu\nu\alpha\beta}(p) = 4i\varepsilon^{ijk} \varepsilon^{mn} \varepsilon_i^j \varepsilon^m^k \varepsilon^n^l \int d^4x e^{ipx} \left\{ \Tr \left[ \gamma_5 S^{kk'}(x) \gamma_\alpha \gamma_\beta \gamma_i \gamma_j \gamma^T(x) C \right] \Tr \left[ \gamma_5 S^{nn'}(x) \gamma_\alpha \gamma_\beta \gamma_i \gamma_j \gamma^T(x) C \right] \right\},
\]
where the $S_{ij}(x)$ is the full Q quark propagator,
\[
S_{ij}(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ikx} \left\{ \frac{\delta_{ij}}{k - m_Q} - \frac{g_s G_{\alpha\beta}^n i_{ij}}{4} \sigma^{\alpha\beta} (k + m_Q) + (k + m_Q) \sigma^{\alpha\beta} \right\},
\]
\[
\frac{g_s^2 C_{\alpha\beta}^n G_{\alpha\beta}^n}{12} \delta_{ij} m_Q \left( \frac{k^2 + m_Q^2}{(k^2 - m_Q^2)^2} + \cdots \right),
\]
and $t^n = \frac{\lambda^n}{2}$, the $\lambda^n$ is the Gell-Mann matrix \cite{29}. Then we compute the integrals both in the coordinate and momentum spaces to obtain the correlation functions $\Pi_{\mu\nu\alpha\beta}(p)$ therefore the QCD spectral densities through dispersion relation,
\[
\rho_A(s) = \frac{\text{Im} \tilde{\Pi}_{Y^+}(s)}{\pi},
\]
\[
\rho_V(s) = \frac{\text{Im} \tilde{\Pi}_{Y^-}(s)}{\pi},
\]
We take the quark-hadron duality below the continuum thresholds $s_0$ and perform Borel transform with respect to the variable $P^2 = -p^2$ to obtain the QCD sum rules:

$$\tilde{\Pi}_+(p^2) = P_{\gamma+}^{\nu\alpha\beta} \Pi_{\nu\alpha\beta}(p),$$

$$\tilde{\Pi}_-(p^2) = P_{\gamma-}^{\nu\alpha\beta} \Pi_{\nu\alpha\beta}(p).$$

(11)

$$\chi^2_Y \exp \left( \frac{-M_Y^2}{T^2} \right) = \int_{m_Q^2}^{s_0} ds \int_0^{r_{tj}} dz \int_{r_{tj}}^{r_{tf}} dt \int_{r_{ij}}^{r_{ij}} d\rho(s, z, t, r) \exp \left( -\frac{s}{T^2} \right),$$

(12)

$$\rho_A(s, z, t, r) = \frac{3m_Q^4}{16\pi^6} (s - m_Q^2)^2 + \frac{tzm_Q^2}{8\pi^6} (s - m_Q^2)^2 (4s - m_Q^2)
+ rzt(1 - r - t - z) \frac{s}{16\pi^6} (s - m_Q^2)^2 (7s - 4m_Q^2)
+ m_Q^2 \alpha_{GG} \pi \left\{ - \frac{m_Q^4}{12\pi^4} (s - m_Q^2)^2 - \frac{1 - r - t - z}{r^2} m_Q^2 \right\} [1 + s \delta (s - m_Q^2)]
- \frac{tz m_Q^2}{r^2} \left[ 1 + \frac{1}{12\pi^4} [4s + s^2 \delta (s - m_Q^2)] \right]
+ \frac{1}{r^2} (2s - m_Q^2)
+ \frac{m_Q^2}{4\pi^4} \left\{ - \frac{m_Q^4}{16\pi^4} (4s - 3m_Q^2)^2 - \frac{r(1 - r - t - z)}{16\pi^4} (s - m_Q^2)^2
- \frac{r(1 - r - t - z)}{4\pi^4} s (7s - 6m_Q^2) + \frac{m_Q^4}{r^2} \frac{1}{48\pi^4} + \frac{t m_Q^2}{r 24\pi^4} (2s - m_Q^2)
+ \frac{t(1 - r - t - z)}{32\pi^4} (s - m_Q^2)^2 + \frac{t(1 - r - t - z)}{48\pi^4} s (6s - 5m_Q^2) \right\},$$

(13)

$$\rho_V(s, z, t, r) = \frac{3m_Q^4}{16\pi^6} (s - m_Q^2)^2 - \frac{tzm_Q^2}{8\pi^6} (s - m_Q^2)^3
+ rzt(1 - r - t - z) \frac{s}{16\pi^6} (s - m_Q^2)^2 (7s - 4m_Q^2)
+ m_Q^2 \alpha_{GG} \pi \left\{ \frac{m_Q^4}{r^3 12\pi^4} \delta (s - m_Q^2) + \frac{1 - r - t - z}{r^2} m_Q^2 \right\}
+ \frac{t m_Q^2}{r^3 12\pi^4} - \frac{tz}{r^2 12\pi^4} \left[ 4s + s^2 \delta (s - m_Q^2) \right]
- \frac{1}{r^2} \left\{ \frac{m_Q^4}{12\pi^4} (s - m_Q^2) \right\}
+ \frac{m_Q^2}{4\pi^4} \left\{ \frac{m_Q^4}{16\pi^4} (5s - 3m_Q^2)^2 + \frac{r(1 - r - t - z)}{16\pi^4} (s - m_Q^2)^2
+ \frac{r(1 - r - t - z)}{4\pi^4} s (7s - 6m_Q^2) - \frac{m_Q^4}{r^2} \frac{1}{48\pi^4} - \frac{t}{r 24\pi^4} (s - m_Q^2)
- \frac{t(1 - r - t - z)}{32\pi^4} (s - m_Q^2)^2 - \frac{t(1 - r - t - z)}{48\pi^4} s (s - m_Q^2) \right\},$$

(14)
where

\[
\overline{m}^2_Q = \frac{m_Q^2}{r} + \frac{m_Q^2}{t} + \frac{m_Q^2}{z} + \frac{m_Q^2}{1 - r - t - z},
\]

\[
r_{f/i} = \frac{1}{2} \left\{ 1 - z - t \pm \sqrt{(1 - z - t)^2 - 4 \left( \frac{1 - z - t}{\hat{s} - \frac{1}{4}} \right)} \right\},
\]

\[
t_{f/i} = \frac{1}{2 \left( \hat{s} - \frac{1}{4} \right)} \left\{ (1 - z) \left( \hat{s} - \frac{1}{4} \right) - 3 \pm \sqrt{\left( (1 - z) \left( \hat{s} - \frac{1}{4} \right) - 3 \right)^2 - 4 \left( 1 - z \right) \left( \hat{s} - \frac{1}{4} \right)} \right\},
\]

\[
z_{f/i} = \frac{1}{2 \hat{s}} \left\{ \hat{s} - 8 \pm \sqrt{(\hat{s} - 8)^2 - 4 \hat{s}} \right\},
\]

and \( \hat{s} = \frac{s}{m_Q^2} \).

We derive Eq.(12) with respect to \( \tau = \frac{1}{r} \), then eliminate the pole residues \( \lambda \gamma \), and obtain the QCD sum rules for the masses of the vector and axialvector \( QQQQ \) tetraquark states,

\[
M_{f/i}^0 = -\frac{d}{dr} \int_{16m_Q^2}^{r_0} ds \int_{z_i}^{s} dz \int_{\tau_i}^{s} d\tau \int_{r_i}^{t} dr \rho(s, z, t, r) \exp(-\tau s) \]

\[
\int_{16m_Q^2}^{r_0} ds \int_{z_i}^{s} dz \int_{\tau_i}^{s} d\tau \int_{r_i}^{t} dr \rho(s, z, t, r) \exp(-\tau s).
\]

\[
(16)
\]

### 3 Numerical results and discussions

We take the gluon condensate to be the standard value \([28, 29, 30]\), and take the \( MS \) masses \( m_c(m_c) = (1.28 \pm 0.03) \text{ GeV} \) and \( m_b(m_b) = (4.18 \pm 0.03) \text{ GeV} \) from the Particle Data Group \([1]\). We take into account the energy-scale dependence of the \( MS \) masses from the renormalization group equation,

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{\Delta}{\mu}},
\]

\[
m_b(\mu) = m_b(m_b) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_b)} \right]^{\frac{\Delta}{\mu}},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0} + \frac{b_3^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^2 t^2} \right],
\]

(17)

where \( t = \log \frac{s_0^2}{\Lambda^2}, b_0 = \frac{33 - 2n_f}{12\pi}, b_1 = \frac{153 - 19n_f}{24\pi}, b_2 = \frac{2857 - 663n_f - 225n_f^2}{128\pi}, \Lambda = 210 \text{ MeV}, 292 \text{ MeV} \)

and \( 332 \text{ MeV} \) for the flavors \( n_f = 5, 4 \) and \( 3 \), respectively \([1]\).

In previous work, we studied the energy scale dependence of the predicated masses of the scalar and tensor \( QQQQ \) tetraquark states with the QCD sum rules in details, and observed that the optimal energy scales of the QCD spectral densities are \( \mu = 2.0 \text{ GeV} \) and \( 3.1 \text{ GeV} \) for the \( c\bar{c}c\bar{c} \) and \( b\bar{b}b\bar{b} \) tetraquark states, respectively \([13]\). In this article, we choose the same energy scales for the \( c\bar{c}c\bar{c} \) and \( b\bar{b}b\bar{b} \) tetraquark states, respectively.

In the QCD sum rules, we usually take the continuum threshold parameters as \( \sqrt{s_0} = M_{gr} + (0.4 \sim 0.6) \text{ GeV} \), for the conventional mesons, where \( gr \) denotes the ground states. Experimentally, the energy gaps \( M_{c'}/M_{f/0} = 589 \text{ MeV} \), \( M_{c'}/M_{b/0} = 656 \text{ MeV} \), \( M_{Y}/M_{T} = 563 \text{ MeV} \) and \( M_{Y'/T'} - M_{T} = 600 \text{ MeV} \) from the Particle Data Group \([1]\). The QCD sum rules support assigning the \( Z_c(3900) \) and \( Z(4430) \) to be the ground state and the first radial excited state of the axial-vector tetraquark states with \( J^{PC} = 1^{+^-} \), respectively, and assigning the \( X(3915) \) and \( X(4500) \) to be the ground state and the first radial excited state of the scalar \( c\bar{c}c\bar{c} \) tetraquark states with \( J^{PC} = 0^{++} \), respectively \([31, 32]\). The mass gaps are \( M_{Z_{(4430)}} - M_{Z_{(3900)}} = 576 \text{ MeV} \) and \( M_{X_{(4500)}} - M_{X_{(3915)}} = 588 \text{ MeV} \), which also satisfy the relation \( \sqrt{s_0} = M_{gr} + (0.4 \sim 0.6) \text{ GeV} \).
Table 1: The Borel parameters, continuum threshold parameters, energy scales, pole contributions, masses and pole residues of the $QQQQ$ tetraquark states.

|       | $T^2$(GeV$^2$) | $s_0$(GeV$^2$) | $\mu$(GeV) | pole | $M_Y$(GeV) | $\lambda_Y$(GeV$^2$) |
|-------|----------------|----------------|-------------|------|------------|----------------------|
| $cc\bar{c}(1^{+-})$ | 4.9 - 4.9 | 43 ± 1 | 2.0 | (46 - 61)% | 6.05 ± 0.08 | (2.97 ± 0.44) × 10$^{-1}$ |
| $cc\bar{c}(1^{--})$ | 4.2 - 4.6 | 44 ± 1 | 2.0 | (46 - 62)% | 6.11 ± 0.08 | (1.82 ± 0.33) × 10$^{-1}$ |
| $bb\bar{b}(1^{++})$ | 13.3 - 13.9 | 374 ± 3 | 3.1 | (48 - 60)% | 18.84 ± 0.09 | 5.45 ± 1.01 |
| $bb\bar{b}(1^{--})$ | 11.7 - 12.3 | 376 ± 3 | 3.1 | (47 - 60)% | 18.89 ± 0.09 | 1.64 ± 0.36 |

In this article, we take the relation $\sqrt{s_0} = M_{gr} + (0.4 \sim 0.6)$ GeV as a constraint, and search for the optimal continuum thresholds $s_0$.

We search for the optimal Borel parameters $T^2$ and continuum threshold parameters $s_0$ to satisfy the two criteria of the QCD sum rules: pole dominance at the phenomenological side and convergence of the operator product expansion at the QCD side. The resulting Borel parameters, continuum threshold parameters, energy scales, pole contributions are shown explicitly in Table 1. From the Table, we can see that the pole contributions are about $(45 - 60)\%$, the same as that for the scalar and tensor tetraquark states $14$, the pole dominance at the phenomenological side is well satisfied.

In the Borel windows, the dominant contributions come from the perturbative terms, the contributions of the gluon condensate are about $-15\%$, $-3\%$, $-8\%$ and $-2\%$ for the tetraquark states $cc\bar{c}(1^{--})$, $cc\bar{c}(1^{--})$, $bb\bar{b}(1^{--})$ and $bb\bar{b}(1^{--})$, respectively, the operator product expansion is well convergent. As the dominant contributions come from the perturbative terms, perturbative $O(\alpha_s)$ corrections amount to multiplying the perturbative terms by a factor $\kappa$, which can be absorbed into the pole residues and cannot impair the predicted masses remarkably. Now the two criteria of the QCD sum rules are all satisfied, we expect to make reasonable predictions.

We take into account all uncertainties of the input parameters, and obtain the values of the ground state masses and pole residues, which are also shown explicitly in Table 1. From Table 1, we can see that the constraint $\sqrt{s_0} = M_{gr} + (0.4 \sim 0.6)$ GeV is also satisfied. In Figs.1-2, we plot the masses and pole residues with variations of the Borel parameters at larger intervals than the Borel windows shown in Table 1. From Figs.1-2, we can see that the predicted masses and pole residues are rather stable with variations of the Borel parameters, the uncertainties originate from the Borel parameters in the Borel windows are very small, there appear Borel platforms.

In Fig.3, we plot the masses $M_Y$ with variations of the energy scales $\mu$ for the central values of other parameters in Table 1. From the figure, we can see that the masses $M_Y$ decreases monotonously and slowly with increase of the energy scales $\mu$. In this article, we choose the same energy scales as the corresponding ones for the $0^{++}$ and $2^{++}$ tetraquark states $QQQQ$, the uncertainties originate from the energy scales will not impair the predicative ability remarkably.

In Table 2, we present all the masses of the $0^{++}$, $1^{+-}$, $2^{++}$ and $1^{--}$ tetraquark states $QQQQ$ from the QCD sum rules in Ref.$14$ and this work. The vector tetraquark states lie 40 MeV above the corresponding centroids of the $0^{++}$, $1^{+-}$ and $2^{++}$ tetraquark states. Naively, we expect that an additional P-wave costs about 500 MeV, which is much larger than the energy gap 40 MeV. This maybe a typical feature of the vector tetraquark states consist of four heavy quarks.

The calculations based on the QCD sum rules indicate that the $C\gamma_{\mu} \otimes \gamma_{\nu} C - C\gamma_{\nu} \otimes \gamma_{\mu} C$ type vector tetraquark state $cq\bar{q}c$ has a mass 4.66 ± 0.09 GeV, the $C \otimes \gamma_{\nu} C$ type vector tetraquark state $csc\bar{c}$ has a mass 4.66 ± 0.09 GeV, which are all consistent with the $Y(4660/4630)$, the $C\gamma_{\nu} \otimes \gamma_{\mu} C$ type vector tetraquark state $cq\bar{q}c$ has a mass 4.34 ± 0.08 GeV, which is consistent with the $Y(4360/4320)$ $26$, $33$. The energy gap between the vector and axialvector tetraquark states is about or larger than 440 MeV, which is much larger than 40 MeV.

The values of the thresholds are $2M_{cc} = 5966.8$ MeV, $2M_{J/\psi} = 6193.8$ MeV, $M_{cc} + M_{J/\psi} = 6080.3$ MeV, $2M_{b\bar{b}} = 18798.0$ MeV, $2M_T = 18920.6$ MeV, $M_{b\bar{b}} + M_T = 18859.3$ MeV from the
|          | $M_Y$(GeV) | Centroids (GeV) |
|----------|------------|-----------------|
| $cccc(0^{++})$ | $5.99 \pm 0.08$ | $6.07 \pm 0.08$ |
| $cccc(1^{+-})$ | $6.05 \pm 0.08$ |               |
| $cccc(2^{++})$ | $6.09 \pm 0.08$ |               |
| $bbbb(0^{++})$ | $18.84 \pm 0.09$ | $18.85 \pm 0.09$ |
| $bbbb(1^{+-})$ | $18.84 \pm 0.09$ |               |
| $bbbb(2^{++})$ | $18.85 \pm 0.09$ |               |
| $cccc(1^{-+})$ | $6.11 \pm 0.08$ |               |
| $bbbb(1^{-+})$ | $18.89 \pm 0.09$ |               |

Table 2: The masses of the tetraquark states $QQ\bar{Q}\bar{Q}$ from the QCD sum rules.

Particle Data Group [1]. The decays

\[
Y(cccc, 1^{+-}) \rightarrow \eta_{c} J/\psi \rightarrow \mu^{+} \mu^{-} + \text{light hadrons},
\]
\[
Y(bbbb, 1^{+-}) \rightarrow \eta_{b} \Upsilon \rightarrow \mu^{+} \mu^{-} + \text{light hadrons},
\]
\[
Y(bbbb, 1^{-+}) \rightarrow \Upsilon \Upsilon \rightarrow \mu^{+} \mu^{-} \mu^{+} \mu^{-},
\] (18)

can take place with very small phase spaces. The decays

\[
X(cccc, 1^{-+}) \rightarrow J/\psi J/\psi^{*} \rightarrow \mu^{+} \mu^{-} \mu^{+} \mu^{-},
\] (19)

can take place through the virtual $J/\psi^{*}$. We can search for the $Y(cccc, 1^{+-}/1^{-+})$ and $Y(bbbb, 1^{+-}/1^{-+})$ in the mass spectrum of the $\mu^{+} \mu^{-} \mu^{+} \mu^{-}$ or $\mu^{+} \mu^{-} \mu^{+} \mu^{-}$ light hadrons in the future.

4 Conclusion

In this article, we construct the axialvector-diquark-axialvector-antidiquark type currents to study both the vector and axialvector $QQ\bar{Q}\bar{Q}$ tetraquark states with the QCD sum rules, and obtain the predictions $M_Y(cccc, 1^{+-}) = 6.05 \pm 0.08$ GeV, $M_Y(cccc, 1^{-+}) = 6.11 \pm 0.08$ GeV, $M_Y(bbbb, 1^{+-}) = 18.84 \pm 0.09$ GeV, $M_Y(bbbb, 1^{-+}) = 18.89 \pm 0.09$ GeV. The vector tetraquark states lie 40 MeV above the corresponding centroids of the $0^{++}$, $1^{+-}$ and $2^{++}$ tetraquark states, which is a typical feature of the vector tetraquark states consist of four heavy quarks. We can search for the $J^{PC} = 1^{+-}$ and $1^{-+} QQ\bar{Q}\bar{Q}$ tetraquark states in the mass spectrum of the $\mu^{+} \mu^{-}$ light hadrons and $\mu^{+} \mu^{-} \mu^{+} \mu^{-}$ respectively in the future.

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Figure 1: The masses of the tetraquark states with variations of the Borel parameters \( T^2 \), where the \( A \), \( B \), \( C \) and \( D \) denote the \( c\bar{c}c\bar{c}(1^{+-}) \), \( c\bar{c}c\bar{c}(1^{--}) \), \( b\bar{b}b\bar{b}(1^{+-}) \) and \( b\bar{b}b\bar{b}(1^{--}) \), respectively.
Figure 2: The pole residues of the tetraquark states with variations of the Borel parameters $T^2$, where the $A$, $B$, $C$ and $D$ denote the $cc\bar{c}\bar{c}(1^{+-})$, $cc\bar{c}\bar{c}(1^{--})$, $bb\bar{b}\bar{b}(1^{+-})$ and $bb\bar{b}\bar{b}(1^{--})$, respectively.

Figure 3: The masses of the tetraquark states with variations of the energy scales $\mu$, where the $A$, $B$, $C$ and $D$ denote the $cc\bar{c}\bar{c}(1^{+-})$, $cc\bar{c}\bar{c}(1^{--})$, $bb\bar{b}\bar{b}(1^{+-})$ and $bb\bar{b}\bar{b}(1^{--})$, respectively.
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