Anthropic predictions: the case of the cosmological constant

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Abstract

Anthropic models can give testable predictions, which can be confirmed or falsified at a specified confidence level. This is illustrated using the successful prediction of the cosmological constant as an example. The history and the nature of the prediction are reviewed. Inclusion of other variable parameters and implications for particle physics are briefly discussed.

I. INTRODUCTION

The parameters we call constants of Nature may in fact be stochastic variables taking different values in different parts of the universe. The observed values of these parameters are then determined by chance and by anthropic selection. It has been argued, at least for some of the constants, that only a narrow range of their values is consistent with the existence of life [1–5].

These arguments have not been taken very seriously and have often been ridiculed as handwaving and unpredictive. For one thing, the anthropic worldview assumes some sort of a “multiverse” ensemble, consisting of multiple universes or distant regions of the same universe, with constants of Nature varying from one member of this ensemble to another. Quantitative results cannot be obtained without a theory of the multiverse. Another criticism is that the anthropic approach does not make testable predictions; thus it is not falsifiable, and therefore not scientific.

While both of these criticisms had some force a couple of decades ago, much progress has been made since then, and the situation is now completely different. The first criticism no longer applies, because we now do have a theory of the multiverse. It is the theory of inflation. A remarkable feature of inflation is that, generically, it never ends completely. The end of inflation is a stochastic process; it occurs at different times in different parts of the universe, and at any time there are regions which are still inflating [6,7]. If some “constants” of Nature are related to dynamical fields and are allowed to vary, they are necessarily randomized by quantum fluctuations during inflation and take different values in different parts of the universe. Thus, inflationary cosmology gives a specific realization of the multiverse ensemble, and makes it essentially inevitable. (For a review see, e.g., [8].)

In this paper I am going to address the second criticism, that anthropic arguments are unpredictive. I will try to dispel this notion and outline how anthropic models can be used to make quantitative predictions. These predictions are of a statistical nature, but they
still allow models to be confirmed or falsified at a specified confidence level. I will focus on
the case of the cosmological constant, whose nonzero value was predicted anthropically well
before it was observed. This case is of great interest in its own right and is well suited to
illustrate the issues associated with anthropic predictions.

II. ANTHROPIC BOUNDS VS. ANTHROPIC PREDICTIONS

For terminological clarity, it is important to distinguish between anthropic bounds and
anthropic predictions. Suppose there is some parameter $X$, which varies from one place
in the universe to another. Suppose further that the value of $X$ affects the chances for
intelligent observers to evolve, and that the evolution of observers is possible only if $X$ is
within some interval

$$X_{\text{min}} < X < X_{\text{max}}.$$

Clearly, values of $X$ outside the interval (1) are not going to be observed, because such
values are inconsistent with the existence of observers. This statement is often called “the
anthropic principle”.

Although anthropic bounds, like Eq. (1), can have considerable explanatory power, they
can hardly be regarded as predictions: they are guaranteed to be right. And the “anthropic
principle”, as stated above, hardly deserves to be called a principle: it is trivially true. This
is not to say, however, that anthropic arguments cannot yield testable predictions.

Suppose we want to test a theory according to which the parameter $X$ varies from one
part of the universe to another. Then, instead of looking for the extreme values $X_{\text{min}}$
and $X_{\text{max}}$ that make observers impossible, we can try to predict what values of $X$
will be measured by typical observers. In other words, we can make statistical predictions, assigning
probabilities $P(X)$ to different values of $X$. [$P(X)$ is the probability that an observer
randomly picked in the universe will measure a given value of $X$.] If any principle needs to
be invoked here, it is what I called “the principle of mediocrity” [9] – the assumption that
we are typical among the observers in the universe. Quantitatively, this can be expressed as
the expectation that we should find ourselves, say, within the 95% range of the distribution.
This can be regarded as a prediction at a 95% confidence level. If instead we measure a
value outside the expected range, this should be regarded as evidence against the theory.

III. THE COSMOLOGICAL CONSTANT PROBLEM

The cosmological constant is (up to a factor) the energy density of the vacuum, $\rho_v$.
Below, I do not distinguish between the two and use the terms “cosmological constant” and
“vacuum energy density” interchangeably. By Einstein’s mass-energy relation, the energy
density is simply related to the mass density, and I will often express $\rho_v$ in units of g/cm$^3$.

The gravitational properties of the vacuum are rather unusual: for positive $\rho_v$, its gravita-
tional force is repulsive. This can be traced to the fact that, according to Einstein’s General

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1I assume for simplicity that $X$ is variable only in space, but not in time.
Relativity, the force of gravity is determined not solely by the energy (mass) density $\rho$, but rather by the combination $(\rho + 3P)$, where $P$ is the pressure. In ordinary astrophysical objects, like stars or galaxies, pressure is much smaller than the energy density, $P \ll \rho$, and its contribution to gravity can be neglected. But in the case of vacuum, the pressure is equal and opposite to $\rho_v$:  

$$P_v = -\rho_v,$$  

(2)

so that $\rho_v + 3P_v = -2\rho_v$. Pressure not only contributes significantly to the gravitational force produced by the mass, it also changes its sign.

The cosmological constant was introduced by Einstein in his 1917 paper [10], where he applied the newly developed theory of General Relativity to the universe as a whole. Einstein believed that the universe was static, but to his dismay he found that the theory had no static cosmological solutions. He concluded that the theory had to be modified and introduced the cosmological term, which amounted to endowing the vacuum with a positive energy density. The magnitude of $\rho_v$ was chosen so that its repulsive gravity exactly balanced the attractive gravity of matter, resulting in a static world. More than a decade later, after Hubble’s discovery of the expansion of the universe, Einstein abandoned the cosmological constant, calling it the greatest blunder of his life. But once the Genie was out of the bottle, it was not so easy to put it back.

Even if we do not introduce the vacuum energy “by hand”, fluctuations of quantum fields, like the electromagnetic field, would still make this energy nonzero. Adding up the energies of quantum fluctuations with shorter and shorter wavelengths gives a formally infinite answer for $\rho_v$. The sum has to be cut off at the Planck length, $l_P \sim 10^{-33}$ cm, where quantum gravity effects become important and the usual concepts of space and time no longer apply. This gives a finite, but absurdly large value, $\rho_v \sim 10^{94}$ g/cm$^3$. A cosmological constant of this magnitude would cause the universe to expand with a stupendous acceleration. If indeed our vacuum has energy, it should be at least 120 orders of magnitude smaller in order to be consistent with observations. In supersymmetric theories, the contributions of different fields partially cancel, and the discrepancy can be reduced to 60 orders of magnitude. This discrepancy between the expected and observed values of $\rho_v$ is called the cosmological constant problem. It is one of the most intriguing mysteries that we are now facing in theoretical physics.

**IV. THE ANTHROPIC BOUND**

A natural resolution to the cosmological constant problem is obtained in models where $\rho_v$ is a random variable. The idea is to introduce a dynamical dark energy component $X$ whose energy density $\rho_X$ varies from place to place, due to stochastic processes that occurred in the early universe. A possible model for $\rho_X$ is a scalar field with a very flat potential [11,12], such that the field is driven to its minimum on an extremely long timescale, much longer

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$^2$Since the vacuum energy is proportional to the volume $V$ it occupies, $E = \rho_v V$, the pressure is $P_v = -dE/dV = -\rho_v$.  

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than the present age of the universe. Another possibility is a discrete set of vacuum states. Transitions between different states can then occur through nucleation and expansion of bubbles bounded by domain walls [13,14]. The effective cosmological constant is given by \( \rho_v = \rho_\Lambda + \rho_X \), where \( \rho_\Lambda \) is the constant vacuum energy density, which may be as large as \(+\) or \(-\)10^{94} \text{ g/cm}^3. The cosmological constant problem now takes a different form: the puzzle is why we happen to live in a region where \( \rho_\Lambda \) is nearly cancelled by \( \rho_X \).

The key observation, due to Weinberg [15] (see also [3,11,16]) is that the cosmological constant can have a dramatic effect on the formation of structure in the universe. The observed structures - stars, galaxies, and galaxy clusters - evolved from small initial inhomogeneities, which grew over eons of cosmic time by gravitationally attracting matter from surrounding regions. As the universe expands, matter is diluted, so its density goes down as

\[
\rho_M = (1 + z)^3 \rho_{M0},
\]

where \( \rho_{M0} \) is the present matter density and \( z \) is the redshift.\(^3\) At the same time, the density contrast \( \sigma \equiv \delta \rho/\rho \) between overdense and underdense regions keeps growing. Gravitationally bound objects form where \( \sigma \sim 1 \). The first stars form in relatively small matter clumps of mass \( \sim 10^6 M_\odot \). The clumps then merge into larger and larger objects, leading to the formation of giant galaxies like ours and of galaxy clusters.

How is this picture modified in the presence of a cosmological constant? At early times, when the density of matter is high, \( \rho_M \gg \rho_v \), the vacuum energy has very little effect on structure formation. But as the universe expands and the matter density decreases, the vacuum density \( \rho_v \) remains constant and eventually becomes greater than \( \rho_M \). At this point the character of cosmic expansion changes. Prior to vacuum domination, the expansion is slowed down by gravity, but afterwards it begins to accelerate, due to the repulsive gravity of the vacuum. Weinberg showed that the growth of density inhomogeneities effectively stops at that epoch. If no structures were formed at earlier times, then none will ever be formed.

It seems reasonable to assume that the existence of stars is a necessary prerequisite for the evolution of observers. We also need to require that the stars belong to sufficiently large bound objects - galaxies - so that their gravity is strong enough to retain the heavy elements dispersed in supernova explosions. These elements are necessary for the formation of planets and of observers. An anthropic bound on the vacuum energy can then be obtained by requiring that \( \rho_v \) does not dominate before the redshift \( z_{\text{max}} \) when the earliest galaxies are formed. With the aid of Eq.(3), this gives

\[
\rho_v \lesssim (1 + z_{\text{max}})^3 \rho_{M0}.
\]

The most distant galaxies observed at the time when Weinberg wrote his paper had redshifts \( z \sim 4.5 \). Assuming that \( z_{\text{max}} \sim 4.5 \), Eq.(4) yields the bound \( \rho_v \lesssim 170 \rho_{M0} \). A more careful analysis by Weinberg showed that in order to prevent structure formation, \( \rho_v \) needs to be 3 times greater than suggested by Eq.(4); hence, a more accurate bound is [15]

\[
\rho_v \lesssim 500 \rho_{M0}.
\]

\(^3\)The redshift \( z \) is defined so that \( (1 + z) \) is the expansion factor of the universe between a given epoch and the present (earlier times correspond to larger redshifts).
Of course, observation of galaxies at \( z \sim 4.5 \) means only that \( z_{\text{max}} \gtrsim 4.5 \), and Weinberg referred to (5) as “a lower bound on the anthropic upper bound on \( \rho_v \).” At present, galaxies are observed at considerably higher redshifts, up to \( z \sim 10 \). The corresponding bound on \( \rho_v \) would be

\[
\rho_v \lesssim 4000 \rho_{M0}. \tag{6}
\]

For negative values of \( \rho_v \), the vacuum gravity is attractive, and vacuum domination leads to a rapid recollapse of the universe. An anthropic lower bound on \( \rho_v \) can be obtained in this case by requiring that the universe does not recollapse before life had a chance to develop [3,17]. Assuming that the timescale for life evolution is comparable to the present cosmic time, one finds \( \rho_v \gtrsim -\rho_{M0} \).

The anthropic bounds are narrower, by many orders of magnitude, than the particle physics estimates for \( \rho_v \). Moreover, as Weinberg noted, there is a prediction implicit in these bounds. He wrote [18]: “... if it is the anthropic principle that accounts for the smallness of the cosmological constant, then we would expect a vacuum energy density \( \rho_v \sim (10 - 100)\rho_{M0} \), because there is no anthropic reason for it to be any smaller.”

One has to admit, however, that the anthropic bounds fall short of the observational bound, \( (\rho_v)_{\text{obs}} \lesssim 4\rho_{M0} \), by a few orders of magnitude. If all the values in the anthropically allowed range were equally probable, an additional fine-tuning by a factor of \( 100 - 1000 \) would still be needed.

\**V. ANTHROPIC PREDICTIONS**

The anthropic bound (4) specifies the value of \( \rho_v \) which makes galaxy formation barely possible. However, if \( \rho_v \) varies in space, then most of the galaxies will not be in regions characterized by these marginal values, but rather in regions where \( \rho_v \) dominates after a substantial fraction of matter had already clustered into galaxies.

To make this quantitative, we define the probability distribution \( \mathcal{P}(\rho_v)d\rho_v \) as being proportional to the number of observers in the universe who will measure \( \rho_v \) in the interval \( d\rho_v \). This distribution can be represented as a product [9]

\[
\mathcal{P}(\rho_v)d\rho_v = n_{\text{obs}}(\rho_v)\mathcal{P}_{\text{prior}}(\rho_v)d\rho_v. \tag{7}
\]

Here, \( \mathcal{P}_{\text{prior}}(\rho_v)d\rho_v \) is the prior distribution, which is proportional to the volume of those parts of the universe where \( \rho_v \) takes values in the interval \( d\rho_v \), and \( n_{\text{obs}}(\rho_v) \) is the number of observers that are going to evolve per unit volume. The distribution (7) gives the probability that a randomly selected observer is located in a region where the effective cosmological constant is in the interval \( d\rho_v \).

Of course, we have no idea how to calculate \( n_{\text{obs}} \), but what comes to the rescue is the fact that the value of \( \rho_v \) does not directly affect the physics and chemistry of life. As a rough

\[4\text{An important distinction between positive and negative values of } \rho_v \text{ is that for } \rho_v > 0, \text{ galaxies that formed prior to vacuum domination can survive indefinitely in the vacuum-dominated universe.}\]
approximation, we can then assume that \( n_{\text{obs}}(\rho_v) \) is simply proportional to the fraction of matter \( f \) clustered in giant galaxies like ours (with mass \( M \gtrsim M_G = 10^{12} M_\odot \)),

\[
n_{\text{obs}}(\rho_v) \propto f(M_G, \rho_v).
\] (8)

The idea is that there is a certain number of stars per unit mass in a galaxy and certain number of observers per star. The choice of the galactic mass \( M_G \) is an important issue; I will comment on it in next section.

The calculation of the prior distribution \( P_{\text{prior}}(\rho_v) \) requires a particle physics model which allows \( \rho_v \) to vary and a cosmological “multiverse” model that would generate an ensemble of sub-universes with different values of \( \rho_v \). An example of a suitable particle theory is the superstring theory, which appears to admit an incredibly large number of vacua (possibly as large as \( 10^{1000} \) [19–21]) characterized by different values of particle masses, couplings, and other parameters, including the cosmological constant. When this is combined with the cosmic inflation scenario, one finds that bubbles of different vacua copiously nucleate and expand during inflation, producing exponentially large regions with all possible values of \( \rho_v \). Given a particle physics model and a model of inflation, one can in principle calculate \( P_{\text{prior}}(\rho_v) \). Examples of calculation for specific models have been given in [12,22,23].\(^5\) Needless to say, the details of the fundamental theory and of the inflationary dynamics are too uncertain for a definitive calculation of \( P_{\text{prior}} \). We shall instead rely on the following general argument [27,28].

Suppose some parameter \( X \) varies in the range \( \Delta X \) and is characterized by a prior distribution \( P_{\text{prior}}(X) \). Suppose further that \( X \) affects the number of observers in such a way that this number is non-negligible only in a very narrow range \( \Delta X_{\text{obs}} \ll \Delta X \). Then one can expect that the function \( P_{\text{prior}}(X) \) with a large characteristic range of variation should be very nearly a constant in the tiny interval \( \Delta X_{\text{obs}} \). In the case of \( \rho_v \), the range \( \Delta \rho_v \) is set by the Planck scale or by the supersymmetry breaking scale, and we have \( (\Delta \rho_v)_{\text{obs}}/\Delta \rho_v \sim 10^{-60} - 10^{-120} \). Hence, we expect

\[
P_{\text{prior}}(\rho_v) \approx \text{const.}
\] (9)

I emphasize that the assumption here is that the value \( \rho_v = 0 \) is not in any way special, as far as the fundamental theory is concerned, and is, therefore, not a singular point of \( P_{\text{prior}}(\rho_v) \).

Combining Eqs.(7),(8),(9), we obtain

\[
P(\rho_v) \propto f(M_G, \rho_v).
\] (10)

In Ref. [9], where I first introduced the anthropic probability distributions of the form (7), I did not attempt a detailed calculation of the distribution for \( \rho_v \), resorting instead to a rough estimate. If we denote by \( z_G \) the redshift at the epoch of galaxy formation, then most of the galaxies should be in regions where the vacuum energy dominates at \( z_v \lesssim z_G \).

\(^5\)There are still some unresolved issues regarding the calculation of \( P_{\text{prior}} \) for models with a discrete spectrum of variable “constants”. For a discussion see [24–26].
Regions with $z_v \gg z_G$ will have very few galaxies, while regions with $z_v \ll z_G$ will be rare, simply because they correspond to a very narrow range of $\rho_v$ near zero. Hence, we expect a typical galaxy to be located in a region where

$$z_v \sim z_G.$$  

(11)

The expected value of $\rho_v$ is then

$$\rho_v \sim (1 + z_G)^3 \rho_{M0}.$$  

(12)

The choice of the galaxy formation epoch $z_G$ is related to the choice of the galactic mass $M_G$ in (8). I used $z_G \sim 1$, obtaining $\rho_v \sim 8\rho_{M0}$.

A similar approach was later developed by Efstathiou [29]. The main difference is that he calculated the fraction of clustered matter $f$ at the time corresponding to the observed value of the microwave background temperature, $T_0 = 2.73$ K, while my suggestion was to use the asymptotic value of $f$ at $t \to \infty$. The two approaches correspond to different choices of the reference class of observers among whom we expect to be typical. Efstathiou’s choice includes (roughly) only observers that have evolved until present, while my choice is to include all observers throughout the history of the universe. If we are truly typical, and live at the time when most observers live, the two methods should give similar results. Indeed, one finds that the probability distributions calculated by these methods are nearly identical [30].

VI. COMPARISON WITH OBSERVATIONS

Despite a number of observational hints that the cosmological constant might be nonzero (see, e.g., [31]), its discovery still came as a great surprise to most physicists and astronomers. Observations of distant supernovae by two independent groups in 1997-98 provided strong evidence that the expansion of the universe is accelerating [32]. The simplest interpretation of the data was in terms of a cosmological constant with $\rho_v \sim 2.3\rho_{M0}$. Further evidence came from the cosmic microwave background and galaxy clustering observations, and by now the case for the cosmological constant is very strong.

The discovery of the cosmological constant was particularly shocking to particle physicists who almost universally believed that it should be equal to zero. They assumed that something so small could only be zero and searched for a new symmetry principle or a dynamical adjustment mechanism that would force $\rho_v$ to vanish. The observed value of $\rho_v$ brought yet another puzzle. The matter density $\rho_M$ and the vacuum energy density $\rho_v$ scale very differently with the expansion of the universe. In the early universe the matter density dominates, while in the asymptotic future it becomes negligible. There is only one epoch in the history of the universe when $\rho_M \sim \rho_v$. It is difficult to understand why we happen to live in this very special epoch. This is the so-called cosmic coincidence problem.

6The original calculation by Efstathiou gave a different result, but that calculation contained an error, which was later pointed out by Weinberg [28].
FIG. 1. The logarithmic probability distribution $dP/d(\log \rho_v)$. The lightly and densely shaded areas are the regions excluded at 68% and 95% level, respectively. The uncertainty in the observed value $\rho_v^*$ is indicated by the vertical strip.

The coincidence is easily understood in the framework of the anthropic approach [33,34]. The galaxy formation epoch, $z_G \sim 1 - 3$, is close to the present cosmic time, and the anthropic model predicts that the vacuum domination should begin at $z \sim z_G$ [see Eq. (11)]. This explains the coincidence.

The probability distribution for $\rho_v$ based on Eq.(10) was extensively analyzed in [35]. The distribution depends on the amplitude of galactic-scale density perturbations, $\sigma$, which can be specified at some suitably selected epoch (e.g., the epoch of recombination). Until recently, significant uncertainties in this quantity complicated the comparison of anthropic predictions with the data [35,23]. These uncertainties appear now to have been mostly resolved [36]. In Fig. 1 we plot, following [37], the resulting probability distribution per logarithmic interval of $\rho_v$. Only positive values of $\rho_v$ are considered, so this can be regarded as a conditional distribution, given that $\rho_v > 0$. On the horizontal axis, $\rho_v$ is plotted in units of the observed vacuum energy density, $\rho_v^* = 7 \times 10^{-30} \text{ g/cm}^3$. The 68% and 95% ranges of the distribution are indicated by light and dark shading, respectively.

We note that the confidence level ranges in Fig. 1 are rather broad. This corresponds to a genuine large variance in the cosmic distribution of $\rho_v$. The median value of the distribution is about 20 times greater than the observed value. But still, the observed value $\rho_v^*$ falls well within the range of anthropic prediction at 95% confidence level.

At this point, I would like to comment on two important assumptions that went into the successful prediction of the observed value of $\rho_v$. First, we assumed a flat prior probability distribution (9). Analysis of specific models shows that this assumption is indeed valid in a wide class of models, but it is not as automatic as one might expect [12,38,22,39]. In particular, it is not clear that it is applicable to the superstring-inspired models of the type discussed in [19–21] (more on this in Section VIII).
Second, we used the value of $M_G = 10^{12} M_\odot$ for the galactic mass in (10). This amounts to assuming that most observers live in giant galaxies like our Milky Way. We know from observations that some galaxies existed already at $z = 10$, and the theory predicts that some dwarf galaxies and dense central parts of giant galaxies could form as early as $z = 20$. If observers were as likely to evolve in early galaxies as in late ones, the value of $\rho_v$ indicated by Eq.(12) would be far greater than observed. Clearly, the agreement is much better if we assume that the conditions for civilizations to emerge arise mainly in galaxies which form at lower redshifts, $z_G \sim 1$.

Following [39], I will now point to some directions along which the choice of $z_G \sim 1$ may be justified. As already mentioned, one problem with dwarf galaxies is that their mass may be too small to retain the heavy elements dispersed in supernova explosions. Numerical simulations suggest that the fraction of heavy elements retained is $\sim 30\%$ for a $10^9 M_\odot$ galaxy and is negligible for much smaller galaxies [40]. Hence, we have to require that the structure formation hierarchy evolves up to mass scales $\sim 10^9 M_\odot$ or higher prior to the vacuum energy domination. This gives the condition $z_G \lesssim 3$, but falls short of explaining $z_G \sim 1$.

Another point to note is that smaller galaxies, formed at earlier times, have a higher density of matter. This may increase the danger of nearby supernova explosions and the rate of near encounters with stars, large molecular clouds, or dark matter clumps. Gravitational perturbations of planetary systems in such encounters could send a rain of comets from the Oort-type clouds towards the inner planets, causing mass extinctions.

Our own Galaxy has definitely passed the test for the evolution of observers, and the principle of mediocrity suggests that most observers may live in galaxies of this type. Our Milky Way is a giant spiral galaxy. The dense central parts of such galaxies were formed at a high redshift $z \gtrsim 5$, but their discs were assembled at $z \lesssim 1$ [41]. Our Sun is located in the disc, and if this situation is typical, then the relevant epoch to use in Eq.(12) is the epoch $z_G \sim 1$ associated with the formation of discs of giant galaxies.

These remarks may or may not be on the right track, but if the observed value of $\rho_v$ is due to anthropic selection, then, for one reason or another, the evolution of intelligent life should require conditions which are found mainly in giant galaxies, which completed their formation at $z \sim 1$. This is a prediction of the anthropic approach. It will be subject to test when our understanding of galactic evolution and of the conditions necessary to sustain habitable planetary systems will reach an adequate level – hopefully in not so distant future.

VII. PREDICTIONS FOR THE EQUATION OF STATE

A generic prediction of anthropic models for the vacuum energy is that the vacuum equation of state (2) should hold with a very high accuracy [39]. In models of discrete vacua, this equation of state is guaranteed by the fact that in each vacuum the energy density is a constant and can only change by nucleation of bubbles. If $\rho_X$ is a scalar field potential, it must satisfy the slow-roll condition – that the field should change slowly on the time scale of the present age of the universe. The slow-roll condition is likely to be satisfied by excess, by many orders of magnitude. Although it is possible to adjust the potential so that it is only marginally satisfied, it is satisfied by a very wide margin in generic models. This implies the equation of state (2).
There is also a related prediction, which is not likely to be tested anytime soon. In anthropic models, $\rho_v$ can take both positive and negative values, so the observed positive dark energy will eventually start decreasing and will turn negative, and our part of the universe will recollapse to a big crunch. Since the evolution of $\rho_v$ is expected to be very slow on the present Hubble scale, we do not expect this to happen sooner than in a trillion years from now [39].

It should be noted that the situation may be different in more complicated models, involving more than one scalar field. It has been shown in [23] that the equation of state in such models may significantly deviate from (2), and the recollapse may occur on a timescale comparable to the lifetime of the Sun. Observational tests allowing to distinguish between the two types of models have been discussed in [42–44]. Recent observations yield [36] $P_v/\rho_v = -1 \pm 0.1$, consistent with the simplest models.

VIII. IMPLICATIONS FOR PARTICLE PHYSICS

Anthropic models for the cosmological constant have nontrivial implications for particle physics. Scalar field models require the existence of fields with extremely flat potentials. Models with a discrete set of vacua require that the spectrum of values of $\rho_v$ should be very dense, so that there are many such values in the small anthropically allowed range. This points to the existence of very small parameters that are absent in familiar particle physics models. Some ideas on how such small parameters could arise have been suggested in [12,38,45–48].

A different possibility, which has now attracted much attention, is inspired by superstring theory. This theory presumably has an enormous number of different vacua, scattered over a vast “string theory landscape”. The spectrum of $\rho_v$ (and of other particle physics constants) can then be very dense without any small parameters, due to the sheer number of vacua [19–21]. This picture, however, entails a potential problem. Vacua with close values of $\rho_v$ are not expected to be close to one another in the “landscape”, and there seems to be no reason to expect that they will be chosen with equal probability by the inflationary dynamics. Hence, we can no longer argue that the prior probability distribution is flat. In fact, since inflation is characterized by an exponential expansion of the universe, and the expansion rate is different in different parts of the landscape, the probabilities for well separated vacua are likely to differ by large exponential factors. If indeed the prior distribution is very different from flat, this may destroy the successful anthropic prediction for $\rho_v$. This issue requires further study, and I am sure we are going to hear more about it.

IX. INCLUDING OTHER VARIABLES

If the cosmological constant is variable, then it is natural to expect that some other “constants” could vary as well, and it has been argued that including other variables may drastically modify the anthropic prediction for $\rho_v$ [4,49,50]. The idea is that the adverse effect on the evolution of observers due to a change in one variable may be compensated by an appropriate change in another variable. As a result the peak of the distribution may drift into a totally different area of the parameter space. While this is a legitimate concern,
specific models with more than one variable that have been analyzed so far suggest that the anthropic prediction for \( \rho_v \) is rather robust.

Suppose, for example, that \( \rho_v \) and the primordial density contrast \( \sigma \) (specified at recombination) are both allowed to vary. Then we are interested in the joint distribution

\[
P(\rho_v, \sigma) d\rho_v d\sigma.
\]

Using the same assumptions as in Section V\(^7\) and introducing a new variable \( y = \rho_v / \sigma^3 \), one finds [39] that this distribution factorizes to the form\(^8\)

\[
\sigma^3 P_{\text{prior}}(\sigma) d\sigma \cdot f(y) dy,
\]

where, \( f(y) \) is the fraction of matter clustered in galaxies (which depends only on the combination \( \rho_v / \sigma^3 = y \)).

After integration over \( \sigma \), we obtain essentially the same distribution as before, but for a new variable \( y \). The prediction now is not for a particular value of \( \rho_v \), but for a relation between \( \rho_v \) and \( \sigma \). Comparison of the predicted and observed values of \( y \) is given by the same graph as in Fig. 1, with a suitable rescaling of the horizontal axis. As before, the 95% confidence level prediction is in agreement with the data.

Another example is a model where the neutrino masses are assumed to be anthropic variables. Neutrinos are elusive light particles, which interact very weakly and whose masses are not precisely known. The current astrophysical upper bound on the neutrino mass is \( m_\nu \lesssim 0.5 \) eV [36], and the lower bound from the neutrino oscillation data is \( m_\nu \gtrsim 0.05 \) eV [51]. (Here and below \( m_\nu \) denotes the sum of the three neutrino masses.) It has been suggested in [52] that small values of the neutrino masses may be due to anthropic selection. A small increase of \( m_\nu \) can have a large effect on galaxy formation. Neutrinos stream out of overdense regions, slowing the growth of density perturbations. The fraction of mass that neutrinos contribute to the total density of the universe is proportional to \( m_\nu \). Thus, perturbations will grow slower, and there will be fewer galaxies, in regions with larger values of \( m_\nu \). A calculation along the same lines as in Section V yields a prediction \( 0.07 \) eV < \( m_\nu < 5.7 \) eV at 95% confidence level.

In Ref. [37] this model was extended, allowing both \( m_\nu \) and \( \rho_v \) to be anthropic variables. The resulting probability distribution \( P(\rho_v, m_\nu) \) is concentrated in a localized region of the parameter space. Its peak is not far off from the peaks of the individual distributions for \( \rho_v \)

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\(^7\)The assumption that the number of observers is simply proportional to the fraction of matter clustered into galaxies may not give a good approximation in regions where \( \sigma \) is very large. In such regions, galaxies form early and are very dense, so chances for life to evolve may be reduced. A more accurate calculation should await better estimates for the density of habitable stellar systems.

\(^8\)Note that there is no reason to expect the prior distribution for \( \sigma \) to be flat. The amplitude of density perturbations is related to the dynamics of the inflaton field that drives inflation and is therefore strongly correlated with the amount of inflationary expansion. Hence, we expect \( P_{\text{prior}} \) to be a nontrivial function of \( \sigma \). In fact, it follows from (14) that \( P_{\text{prior}}(\sigma) \) should decay at least as fast as \( \sigma^{-3} \) in order for the distribution to be integrable [33].
and $m_\nu$. In fact, inclusion of $m_\nu$ somewhat improves the agreement of the prediction for $\rho_v$ with the data.

The parameters $\rho_v$, $\sigma$ and $m_\nu$ share the property that they do not directly affect life processes. Other parameters of this sort include the mass of dark matter particles and of baryons per photon. The effects of varying these parameters have been discussed in [4,49]. In particular, Aguirre [49] argued that values of the baryon to photon ratio much higher than the observed may be anthropically favored. What he showed, in fact, is that this proposition cannot at present be excluded. This is an interesting issue and certainly deserves further study. Extensions to parameters like the electron mass or charge, which do affect life processes, is on a much shakier ground. Until these processes are much better understood, one will have to resort to qualitative arguments, as in [1–3,5].

X. CONCLUDING REMARKS

The case of the cosmological constant demonstrates that anthropic models can be subjected to observational tests and can be confirmed or ruled out at a specified confidence level. It also illustrates the limitations and difficulties of anthropic predictions.

The situation we are accustomed to in physics is that the agreement between theory and observations steadily improves, as the theoretical calculations are refined and the accuracy of measurements increases. Not so in anthropic models. Here, predictions are in the form of probability distributions, having an intrinsic variance which cannot be further reduced.

However, there is an ample possibility for anthropic models to be falsified. This could have happened in the case of the cosmological constant if the observed value turned out to be much smaller than it actually is. And this may still happen in the future, with improved understanding of the prior and anthropic factors in the distribution (7). Also, there is always a possibility that a compelling non-anthropic explanation for the observed value of $\rho_v$ will be discovered. As of today, no such explanation has been found, and the anthropic model for $\rho_v$ can certainly be regarded a success. This may be the first evidence that we have for the existence of a vast multiverse beyond our horizon.

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