Heat to Electricity Conversion by a Graphene Stripe with Heavy Chiral Fermions

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(Dated: August 22, 2012)

A conversion of thermal energy into electricity is considered in the electrically polarized graphene stripes with zigzag edges where the heavy chiral fermion (HCF) states are formed. The stripes are characterized by a high electric conductance \( G_e \) and a significant Seebeck coefficient \( S \). The electric current in the stripes is induced due to a non-equilibrium thermal injection of \textit{“hot”} electrons. This thermoelectric generation process might be utilized for building of thermoelectric generators with an exceptionally high figure of merit \( Z \delta T \sim 100 \gg 1 \) and with an appreciable electric power densities \( \sim 1 \text{ MW/cm}^2 \).

PACS numbers: 84.60.Rb, 73.40.Gk, 73.63.Kv, 44.20.+b

The heat energy can be immediately converted into electricity by thermoelectric generators (TEG) [1–4]. However, available solid state TEGs typically have a low figure of merit \( Q \). The key element of the suggested thermoelectric generator (TEG) is the graphene stripe polarized by the transverse electric field \( E \) (see Fig. 1). The graphene TEG exploits the gate voltage-controlled heavy chiral fermion (HCF) states formed inside \( \mathcal{G} \) and it also benefits from the electric/heat current filtering. The design strategy is illustrated by an expression for the figure of merit \( Z \delta T = G_e S^2 \delta T / \Lambda \) for a symmetric TEG. [1, 2] Here \( \mathcal{S} \) is the Seebeck coefficient, \( G_e \) the electric conductance, \( \Lambda = \Lambda_e + \Lambda_{ph} \) the thermal conductance due to the electrons (e) and phonons (ph). In typical TEG devices, [3–6] \( \Lambda_{ph} / \Lambda_e \sim 10^3 – 10^4 \), providing that \( \Lambda \simeq \Lambda_{ph} \gg \Lambda_e \). Since normally \( \Lambda \gg \mathcal{G}\mathcal{S}^2 \delta T^2 \), one gets \( Z \delta T \leq 1 \). In the TEG reported here, instead of regular electrons and holes, we exploit “heavy” HCF particles to boost the electric conductance \( G_e \propto \sqrt{m^*/m_e} \) and Seebeck coefficient \( S \) (see Supplement). In contrast to a pristine graphene, the HCF mass \( m^* \) in \( \mathcal{G}\)-TEG exceeds the free electron mass \( m_e \) by a huge factor \( m^*/m_e \sim 10^2 – 10^6 \gg 1 \). We also implement special multilayered “hot” metallic electrodes with an aim of decimating \( \Lambda_{ph} \). It allows reducing of the phonon part \( \Lambda_{ph} \) the heat energy flow by factor \( \sim 10^{-2} – 10^{-3} \). The combined approach yields \( Z \delta T \geq 1 \), or even \( Z \delta T \gg 1 \), which manifests a significant improvement over existing TEG devices.

The whole TEG circuit represents a 2D array formed by three interdigitated combs \( \mathcal{G}, \mathcal{G}_A, \) and \( \mathcal{G}_B \) as shown in Fig. 1d. The comb \( \mathcal{G} \) consists of parallel graphene stripes alternating with metallic split-gate stripes \( \mathcal{G}_{A(B)} \) which form the other two combs. The \( \mathcal{G}_A \) and \( \mathcal{G}_B \) combs serve as split-gates to creating of the transverse electric field \( E \neq 0 \) across \( \mathcal{G} \). Additionally, there are three other metallic combs, \( \mathcal{C}_e \) and \( \mathcal{C}_h \), which are oriented in perpendicular to the former \( \mathcal{G} \) and \( \mathcal{G}_{A(B)} \) combs. The two combs, \( \mathcal{C}_e \) and \( \mathcal{C}_h \), act as heat sinks, while the H comb represents a heat source. Besides, there are two more back gate combs, \( \mathcal{G}_L \) and \( \mathcal{G}_R \), which are placed underneath of the dielectric substrate and which are also oriented in-perpendicular to the \( \mathcal{G}_{A(B)} \) and \( \mathcal{G} \) combs.

Let us consider an elementary cell of the TEG device sketched in Fig. 1a (we call it \( \mathcal{G}\)-TEG) and its diagrams shown in Fig. 1b. The \( \mathcal{G}\)-TEG cell consists of two field effect transistors, [3–12] left (\( \mathcal{FET}_L \)) and right (\( \mathcal{FET}_R \)), both being fabricated on the same section of the graphene stripe \( \mathcal{G} \). The two transistors, \( \mathcal{FET}_L \) and \( \mathcal{FET}_R \), are connected electrically in a sequence, but thermally in parallel (see Fig. 1e). [4] The edges of each \( \mathcal{G} \)-stripe have an atomic zigzag shape. [7] The opposite edges of the same stripe are terminated by carbon atoms belonging to two distinct sublattices \( A \) and \( B \). [10] The split-gate voltage \( V_{SG} = V_{GA} - V_{GB} \) polarizes the \( \mathcal{G} \)-stripe in the transverse \( y \)-direction. The left (\( \mathcal{G}_L \)) and right (\( \mathcal{G}_R \)) back-gate electrodes are controlling the concentration and type of the charge carriers inside the left (\( x < -L_0 / 2 \)) and right (\( x > L_0 / 2 \)) \( \mathcal{G}\)-TEG sections, which are forming \( \mathcal{FET}_L \) and \( \mathcal{FET}_R \), correspondingly. The local gate voltage \( V_G \) induces an \( x \)-dependent change \( U(x) \) of the electrochemical potential along the \( \mathcal{G} \)-stripe as shown at the bottom of Fig. 1b.

The sharp distinctive HCF levels at energies \( E_p = \pm \Delta \) result from reflections at the atomic zigzag edges [7] of the graphene stripes \( \mathcal{G} \). The \( \mathcal{G} \)-stripes are polarized in the transverse \( \hat{y} \)-direction by a finite electric field \( E \neq 0 \) which controls the value \( \Delta \). The origin of the HCF resonances is illustrated in terms of a mere mean field approach [4, 22]. The method [4, 22] describes the electronic excitation spectrum \( E_p \) of the graphene stripe in terms of the Dirac equation for bipartite sublattices. One adds a symmetry breaking term \( \propto \Delta \) into the Hamiltonian

\[
\mathcal{H} = -i \hbar v \left( (\hat{\sigma}_x \otimes \hat{1}) \partial_x + (\hat{\sigma}_y \otimes \hat{\tau}_z) \partial_y \right) + U(x) (\hat{1} \otimes \hat{1}) + \Delta (\hat{\sigma}_z \otimes \hat{\tau}_z)
\]  

(1)
where \( v = 8.1 \times 10^5 \text{m/s} \approx c/300 \) is the massless fermion speed, \( \hat{\sigma}_z \) and \( \hat{\tau}_k \) are the Pauli matrices, \( \otimes \) is the Kronecker product, \( \{i,k\} = 1 \ldots 3 \), and \( \Delta = \epsilon |E|/W = \epsilon V_{SG}/2 = \epsilon (V_{GA} - V_{GB})/2 \), \( V_{SG} \) is the split gate voltage, \( W \) is the stripe width. The pseudospin polarization \( \Psi \) means that the electric charge is depleted in one zigzag edge while is accumulated in the other. Then an electric dipole is formed as soon as \( V_{SG} = 2\Delta/e \neq 0 \). At \( V_{SG} = 0 \) there are two flat bands giving rise to a large density of states at the Fermi energy and being associated to zigzag edge states. When \( V_{SG} \neq 0 \), the resulting pseudo-spin polarization driven by the \( \Delta \) term in Eq. (1) yields an excitation spectrum \( \varepsilon_{\pm} = \pm v \sqrt{k^2 + q^2 + (\Delta/e)^2} \pm U \) characterized by the energy gap \( 2\Delta \). The electron wavefunction \( \Psi \) satisfies the boundary conditions \( \Psi_A(0) = \Psi_{A'}(0) = 0 \) at \( y = 0 \) and \( \Psi_B(W) = \Psi_{B'}(0) = 0 \) at \( y = W \) for zigzag atomic edges. It gives the transversal quantization condition (QC) in the form \( k = q_p/\tan(q_p W) \) where \( q_p = \pm \sqrt{(E_p \mp U)^2/e^2 - k^2 - (\Delta/e)^2} \). The QC actually determines the localized energy levels when is being solved in respect to the excitation energy \( E_p \). Thus, for a finite \( \Delta \neq 0 \), the graphene zigzag stripes are band insulators with the pseudospin polarization. The steady state energy spectrum of the \( G \)-stripe features two flat bottom bands \( E_p = \pm \Delta \) which represent the highest occupied and lowest un-occupied bands. The flat bands correspond to a very high effective electron mass \( m^* = (10^{-2} - 10^{-6}) m_e >> m_e \).

The sign of \( U(x) \) determines whether the HCF charge carriers inside of each FET \( \text{L,R} \) are electrons or holes. In Figs. 1b, the \( V_G(x) \) profile is step-wise, so \( V_G(x) = -U_0/\alpha_G \) for \( x < -L_0/2 \) (inside FET \( \text{L} \)) and \( V_G(x) = U_0/\alpha_G \) for \( x > L_0/2 \) (inside FET \( \text{R} \)). Here \( U_0 \) is the shift of the electron electrochemical potential and \( \alpha_G \) is the back gate efficiency. The \( G \)-stripe RR section located immediately under the central hot H-electrode \( ( -L_0/2 < x < L_0/2) \) remains neutral, since \( V_G(x) = 0 \). This part of the \( G \)-stripe serves as an electron-hole recombination region (RR).
FIG. 2: Color online. (a) The electron energy subbands in the graphene stripe $G$ with zigzag edges. The transverse split gate voltage $V_{SG}$ induces an energy gap $2\Delta = eV_{SG} = e(V_{GA} - V_{GB})$ which splits the narrow zero-energy level. (b) Density of electron states $N_{e}$ for three inelastic scattering rates $\gamma_{1,2,3} = 0.01, 0.001$ and 0.0001 in units of $\Delta$. The two sharp peaks constitute the Heavy Chiral Fermion (HCF) excitations arising at energies $\pm \Delta$. The corresponding peak height $N_{e = \pm \Delta}$ depends on $\gamma$. It exceeds the Van Hove singularity (VHS) peaks $N_{VHS}$ by the big factors $N_{e = \pm \Delta}/N_{VHS} = \sqrt{m^*/m_e} = 10, 100$ and 1000. (c) The phonon transmission coefficient $\zeta$ for multilayered metallic electrode H (shown in Fig. 1c) versus the number of layers $N$. (d) The transmission coefficient $T_{RR/G}(\varepsilon)$ between the RR and uncovered $G$ sections for different electron subbands.

In the TEG shown in Fig. 1d, the heat is supplied via the "hot" H-comb, while it is drained by the "cold" C$_{e}$ and C$_{h}$ sink combs. The phonon fraction of the heat flow is filtered out by the multilayered H electrode (see Supplement). Since the electron temperature $T_H$ in H is much higher than the electron temperature in the $G$-stripe, $T_G$, one achieves the non-equilibrium thermal injection $Q_{eh}$ of the "hot" electrons and holes from H into RR (see Supplement for details). Although the contact Seebeck coefficient $S_{H/RR}$ and contact electric conductance $G_{H/RR}$ through the H/RR interface vanish (i.e., $S_{H/RR} = 0$ and $G_{H/RR} = 0$, as shown in Supplement), the contact thermal conductance $\Lambda_{H/RR}$ remains essentially finite. After being thermally injected from H into RR, the electrons and holes are quickly (during time $\sim 10^{-13}$ s) converted into the HCF excitations. In the RR region, the non-equilibrium HCF electrons...
and holes populate the levels $\pm \Delta$ inversely: the upper $+\Delta$ level is populated by excessive HCF electrons while the lower $-\Delta$ level by the excessive HCF holes. The quasiparticle distribution function $n^R_E$ is obtained as a solution of the quantum kinetic equation \cite{22} (see Supplement).

In the configuration shown in Fig. 1, the holes ballistically propagate from RR toward $C_h$ while the electrons in the left $G$-section proceed from RR toward $C_e$. Thus, the latter stage implies a chiral transmission of the excessive non-equilibrium HCF electrons from the upper $+\Delta$ level localized in RR into the upper $-\Delta - U_0$ level located in the uncovered $G$-section adjacent to $C_e$ (see the diagram at the top of Fig. 1b) where $U_0 \leq \gamma$, $\gamma$ is the localized level width. Simultaneously, the excessive holes are transmitted from the lower $-\Delta$ level in RR into the lower $-\Delta + U_0$ level localized near $C_h$. In this way, the full thermal flow $Q_e^{th}$ from H to RR is eventually split between the $C_h$ and $C_e$ sections of the graphene stripe $G$. In the uncovered $G$-section, broadening $\gamma$ of the HCF level $E_0$ originates from coupling of the HCF states to the phonons. It yields $\gamma = \gamma_{ph} = h/\tau_{ph}$ (typically $\tau_{ph} \approx 10^{-12}$ s at $T = 10$ K). In the RR section, there is an additional coupling $\gamma$ to the electron states in H which gives $\gamma = \gamma_{ph} + \gamma_H$.

Both the electrons and holes are supplied into the electrically neutral RR by the thermal injection from H. Then the electric current $J$ in FET$_R$ actually emerges due to the HCF holes ballistically propagating from the RR region toward the $C_h$ electrode. Simultaneously, the electric current $J$ in FET$_L$ also consists of HCF electrons ballistically propagating in the opposite direction from RR toward $C_e$. In this way, both the HCF electrons and holes transfer the heat from the hot (H) to the cold ($C_{e,h}$) electrodes. Then the combined thermal flow $Q^{th}$ of electrons and holes generates an electric current accompanied by a finite voltage drop $V_{SD} = \sum_n V_{SD}^n$. All the $G$-stripes are connected in parallel, thus the net electric current through the whole TEG is $I = \sum_k I_k$ where $I_k$ is the current through the $k$-th $G$-stripe. The voltage drop on the two FET$_{L,R}$ is $V_{SD}^2 = 2S_G \delta T_n$. Because the heat flow is transferred by heavy charged HCF particles ($m^*/m_e \gg 1$) it ensures a considerable value of Seebeck coefficient $S_E$ (see Supplement). It in turn yields a high thermoelectric voltage $V_{SD}^2 = 2S_G \delta T_n$ generated by the heat flow when a finite temperature difference $\delta T_n$ is maintained between the H and $C_{e,h}$ electrodes of the $n$-th $G$-TEG. Then the $G$-TEG generates the electric power $Q_{G}^{thelian} - TEG = \sum_{k,n} I_k V_{SD}^n$ which is considerably higher than in TEG devices where conventional electrons and holes are engaged.

Another increase of $Z\delta T$ is accomplished when the phonon part of the heat conductance $\Lambda_{ph}$ is strongly reduced. A visible decimation of $\Lambda_{ph}$ by a few orders of magnitude is achieved, e.g., if the hot H-electrodes are metallic multilayers fabricated as sketched in Fig. 1c. The multilayered structure shown in Fig. 1c serves for decimating the phonon transport through the TEG by the factor $\zeta$ which we plot in Fig. 2c versus the number of layers $N$ (see Supplement). Because the multilayered metal contacts H and $C_{e,h}$ are reducing the phonon component of the heat flow, the figure of merit $Z\delta T$ can additionally be increased by orders of magnitude.

One computes the net electric power generated by the whole TEG with equivalent $G$-stripes and equivalent $G$-TEGs as $Q_{elec} = N_G \times IV_{SD} = N_G \times G_e(2S_G \delta T)^2$ where $N_G$ is the total number of $G$-TEG elements in the TEG array and we have omitted index $n$. The corresponding thermal power is $Q_{heat} = N_G \times \delta T$. Both the quantities, $Q_{electr}$ and $Q_{heat}$, are expressed in terms of Seebeck coefficient $S$, temperature differences $\delta T$, electric $G_e$, and thermal $\Lambda$ conductivities of the individual $G$-TEG sections. The TEG conversion efficiency is then estimated as $(Z\delta T)_{TEG} = Q_{electr}/Q_{heat} = 4N_G S_G^2 G_e \delta T/A$. The electron part, $\Lambda_e$, of the thermal conductance $\Lambda$ is obtained as $1/\Lambda_e = 2/\Lambda_e' + 1/\Lambda_G$ and is typically $10^4 - 10^5$ times smaller than the phonon part, which is $1/\Lambda_{ph} = 2/\Lambda_e' + 1/\Lambda_G' + 1/\Lambda_{SiO_2}$. Here $\Lambda$'s indices e, ph, c, G, and SiO$_2$ relate it to the electrons, phonons, contacts, $G$-stripe, and dielectric SiO$_2$ substrate respectively. There are only two conducting HCF channels per each $G$-stripe. For one conducting channel in a regular graphene stripe by width $W = 10$ nm, one gets Seebeck coefficient at most $S \sim 10^{-3}$ V/K. For a graphene stripe with zigzag edges by the same width $W \simeq 10$ nm and the split gate voltage $V_{SG} = 0.1$ V, where the HCF resonances are invented, one improves it to $S \sim 10^{-3} - 10^{-1}$ V/K (see Supplement). In combination with the multilayered “hot” electrode it yields as much as $Z\delta T \sim 10^2$.

The electric power generated by a single $G$-TEG is evaluated as $Q_e^{th} \simeq \kappa_{HCF} G_q V_{SG}^2 \simeq 1$ mW, where $\kappa_{HCF} = \sqrt{m^*/m_e} (\delta T_{H}/RR) / (\delta T_{RR}/C)$, $G_q = 2e^2/h = 7.75 \times 10^{-5}$ $\Omega^{-1}$, and we have used $m^*/m_e = 10^2$ (which corresponds to $\gamma = 1$ meV), $\delta T_{H}/RR = 300$ K, $\delta T_{RR}/C = 30$ K, $V_{SG} = 0.1$ V. Let us assume that the length of a single $G$-TEG element is $L_G \simeq 2.5$ $\mu$m, the $G$-stripe width is $W_G = 10$ nm, and the period in the y-direction is $W_p = 40$ nm. Then the TEG device by size $1$ cm$\times$1 cm contains $N_G = 10^8$ $G$-TEG elements which might generate the electric power about 1 MW. One can see that the $G$-TEG efficiency largely depends on the maximum value of $V_{SG}$ and on reducing of $\Lambda_{ph}$. For practical realizations, instead of d.c. gate voltages, one can operate the TEG at industrial a.c. frequency $f = 60$ or 50 Hz. Then required $V_G$ and $V_{SG}$ might be generated immediately during the heat to electricity conversion.

The obtained result suggests that the $G$-TEG devices which exploit filtering of the electric/heat currents and of the generated output electric power by 2-3 orders of magnitude as compared to presently known devices.
I wish to thank V. Chandrasekhar for fruitful discussions. This work had been supported by the AFOSR grant FA9550-11-1-0311.

SUPPLEMENTARY INFORMATION

Blocking the phonon flow by multilayered electrodes.

The goal here is to reducing of the phonon transport considerably while stimulating the electron transport at the same time. We design the filter pads to separate the G-section both from the external electrodes and from the substrate thermally, but not electrically. We consider two different types of the heat/electric current valve pads. One design involves metallic multi-layers Pb/Al with the layer thickness ∼ 10 nm. Another method is to depositing of pads made of SrHfO$_3$ and/or SrRuO$_3$. The layered materials have an appreciable electric conductance while their thermal conductance along the c-axis is remarkably low. [23, 24] Planting of the H/RR pad between the metallic electrodes and graphene stripe would reduce the effective Λ$^{ph}$ significantly, because the phonons which provide large heat conductance Λ$^{ph}$ between the hot and cold electrodes are eliminated from the thermoelectric path. Then the net heat conductance which involves the path HOT ⇒ RR ⇒ G ⇒ C.e,h ⇒ COLD will be considerably diminished. Thus placing of the H/RR pad with a sufficient number of nanolayers allows to decimating of the phonon part Λ$^{ph}$ the whole thermal conductance Λ. Optimal G-TEG geometry is also determined by the electric and thermal transfer lengths which are estimated [6, 11] correspondingly as $L_{el}$ ≃ 10 − 100 nm and $L_{th}$ ≃ 100 − 250 nm.

Molecular dynamics simulations.

The phonon part of the thermal transport through the TEG had been examined as follows. We describe the non-equilibrium thermal transport through the G-stripe in presence of multiple scattering on lattice defects, boundaries, and electrons. A finite temperature difference $\delta T$ between the opposite ends of each G-stripe induces the thermal flow given as a sum of contributions of the individual phononic subbands. The phonon density of states $F_\beta(\omega)$ related to the phonon subband $\beta$ is mismatched in adjacent layers of the H electrode sketched in Fig. 1c. Inside the G-stripe, the phonon distribution function $N(\omega)$ is non-equilibrium which means that $N(\omega)$ deviates from the Bose-Einstein distribution in the hot (H) and cold (C) ends. For a "clean" graphene stripe, the phonon mean free path exceeds the G-stripe length $L$. Therefore the non-equilibrium effect does not influence the final results. The equilibrium phonon distribution at the G-stripe ends is established due to a free phonon diffusion into the bulk of attached metallic contacts and dielectric substrate. The thermal conductance $\Lambda^{th}_G$ of the G-stripe had been computed by using the phonon density of states $F_\beta(\omega)$ preliminary obtained for each phonon subband $\beta$.

The thermolectric characteristics are found by solving the Dirac equation for chiral fermions in graphene (see above). The analytical model is verified by numeric calculations based on the density functional theory. [7] The electron and phonon excitation spectra are obtained considering influence of the inelastic electron-phonon and elastic electron-impurity scatterings. They are taken into account along with processes of the electron tunneling through the interface barriers. The electron-impurity and electron-phonon scatterings are included within the Keldysh-Green function technique [25] which allows deriving of the quantum kinetic equations (see below for details).

Non-equilibrium thermal injection.

The heat energy flow $Q^{th}$ from the "hot" electrode H with temperature $T_H$ into a much colder heat sinks $C_{e(h)}$ proceeds along the G-stripe with temperature $T_G$ ($T_H >> T_G$). The process happens in two stages (i) and (ii) as illustrated by the energy diagrams at the top of Fig. 1b. In the diagram we show the distribution function of "hot" excessive quasiparticles $\delta n_{e,h}$ in H where $\delta n_{e,h} = [n(\epsilon/T_H) - n(\epsilon/T_G)]\theta(\epsilon - \epsilon_F)$ corresponds to the thermally excited excessive electrons while $\delta n_h = -[n(\epsilon/T_H) - n(\epsilon/T_G)]\theta(\epsilon_F - \epsilon)$ describes the excessive hole thermal excitations [here $n(x) = 1/(e^x + 1)$]. (i) The thermal injection H⇒RR: The temperature difference $\delta T = T_H - T_G$ initializes the non-equilibrium "thermal injection" of "hot" electrons and holes from H into the two levels ±$\Delta$ localized inside the recombination region RR. As a result, the upper +$\Delta$ level becomes inversely populated by the non-equilibrium electrons while the lower level −$\Delta$ is populated with non-equilibrium holes. The process is described by the quantum
kinetic equation [25]
\[
\frac{\partial n_\varepsilon}{\partial t} = \mathcal{L}_{H/RR} \{\varepsilon, n_\varepsilon\} + \mathcal{L}_{ep} \{\varepsilon, n_\varepsilon\} + \mathcal{L}_{eh} \{\varepsilon, n_\varepsilon\} + \mathcal{L}_{RR/C_{e,h}} \{\varepsilon, n_\varepsilon\}.
\]  
(2)

In the quasistationary case, one sets \(\partial n_\varepsilon/\partial t \equiv 0\) which gives
\[
\Gamma_{H/RR}(n_\varepsilon^H - n_\varepsilon^{RR}) + \Gamma_{RR/C_{e,h}}(n_\varepsilon^{RR} - n_\varepsilon^{h}) - \frac{n_\varepsilon^{RR} - n_\varepsilon^{F}}{\tau_{eh}} - \frac{n_\varepsilon^{RR} - n_\varepsilon^{F}}{\tau_{ep}} = 0,
\]  
(3)

where we have used \(\mathcal{L}_{H/RR} = \Gamma_{H/RR}(n_\varepsilon^H - n_\varepsilon^{RR})\) for the non-equilibrium thermal injection, \(\mathcal{L}_{ep} = (n_\varepsilon^{RR} - n_\varepsilon^{F})/\tau_{ep}\) is the electron-phonon collision term, \(\mathcal{L}_{eh} = (n_\varepsilon^{RR} - n_\varepsilon^{F})/\tau_{eh}\) describes the electron-hole recombination, [14] and \(\mathcal{L}_{RR/C_{e,h}} = \Gamma_{RR/C_{e,h}}(n_\varepsilon^{RR} - n_\varepsilon^{h})\) accounts for the HCF electron/hole escapes from RR into directions of \(C_{e,h}\).

The corresponding electron distribution functions are approximated as \(n_\varepsilon^{h} = 1/(\varepsilon^H/\hbar + 1)\) in the H electrode and \(n_\varepsilon^{c,h} = 1/(\varepsilon^{c,h}/\hbar + 1)\) in the \(C_{e,h}\) shoulders where \(\hbar \equiv \sqrt{\varepsilon/G/\alpha}\) is the back gate-induced shift of the electron electrochemical potential, \(\alpha\) is the back gate efficiency, \(n_\varepsilon^{F}\) is the Fermi function. Besides, \(T_{H(c)}\) are the effective electron temperatures in the H and \(C_{e,h}\)-adjacent regions. The non-equilibrium distribution function \(n_\varepsilon^{RR}\) in the RR region then is
\[
n_\varepsilon^{RR} = \frac{\Gamma_{RR/C_{e,h}} n_\varepsilon^{h} - \Gamma_{H/RR} n_\varepsilon^{H} - \Gamma_{RR} n_\varepsilon^{F}}{\Gamma_{RR/C_{e,h}} - \Gamma_{H/RR} - \Gamma_{RR}}
\]  
(4)

where the electron energy broadening is \(\varepsilon_\tau = \hbar/\tau_{eh} + \hbar/\tau_{ep}\). Typically one gets [11] \(\Gamma_{H/RR} = 5\) meV for a rough \(\mathcal{G}/\text{Pd-interface while } \Gamma_{H/RR} = 60\) meV for a smooth \(\mathcal{G}/\text{Pd Interface, } \Gamma_{RR/C_{e,h}} = 100\) meV, \(\Gamma_{RR} = 1.5\) meV. The temperatures are taken as \(T_H = 630\) K, \(T_{RR} = 330\) K, and \(T_G = 300\) K, the level position \(\Delta = 50\) meV, which corresponds to the split gate voltage \(V_{SG} = 0.1\) V. The above formulas allows computing the electric of the thermal and currents. Both types of the currents are inhomogeneous in vicinity of the H and \(C_{e,h}\) electrodes on the corresponding spatial lengths \(L_{m} \sim 10 - 100\) nm and \(L_{eh} \sim 100 - 250\) nm. [6,11] The "hot" conventional electrons coming from H in RR are converted inside RR into the "heavy" HCF excitations during the short time [11] \(\tau_{\varepsilon} \approx \hbar/\Gamma\) (in Ref. [11] \(\tau_{\varepsilon} \approx 10^{-13}\) s, due to the energy broadening because the tunneling coupling between H and \(\mathcal{G}\). One finds \(\Gamma = \hbar v_H T_{H/RR}/(4d_\hbar) \approx 5\) meV where \(T_{H/RR}\) is the H/RR interface transparency, \(\hbar v_H\) is the Fermi velocity in H.

(ii) During the next chiral transmission process, RR \(\rightarrow \mathcal{G}\), which occurs on the longer timescale \(\tau_{eh} \leq L/v \approx 10^{-12}\) s, most of the HCF electrons and holes inside RR are captured by the adjacent FET_L/R. Simultaneously, minor fractions of HCF electrons and holes annihilate with each other [14] during the time \(\tau_{eh} \geq 10^{-12}\) s [typically \(\tau_{eh} \sim (3 - 7) \cdot \tau_{eh}\)]. It sets a requirement to the spatial dimension of RR as \(L_0 = v_{\varepsilon} \tau_{eh} \leq 1\) \(\mu\)m. Since the thermal injection is essentially a non-equilibrium process, the transforming of electron states between the stages (i) and (ii) is coherent.

Knowing \(n_\varepsilon^{RR}\), one computes the electric conductance, \(G_{\varepsilon}\), Seebeck coefficient, \(S\), and the electronic part of the heat conductance, \(\Lambda_{\varepsilon}\). The electric contact conductance \(G_{H/RR}\) across the H/RR interface vanishes \((G_{H/RR} \equiv 0)\) because the electron part of the electric current is compensated by the hole part. The H/RR contact Seebeck coefficient also vanishes since it is \(S_{H/RR} = -(1/\varepsilon T)(L_{H/RR}(1)/L_{H/RR}(0)) = V_{H/RR}/\delta T_{H/RR} \equiv 0\) where \(V_{H/RR}\) is the bias voltage and \(\delta T_{H/RR}\) is the temperature difference across the H/RR interface. Here we have introduced auxiliary functions \(L_{e}^{(a)}(\varepsilon) = (2e^2/\hbar) \int d\varepsilon \cdot (\varepsilon - \mu)^a M_\varepsilon(\varepsilon) \mathcal{T}_\varepsilon(\varepsilon) \mathcal{Y}_{H/RR}(\varepsilon)\) where \(\mathcal{Y}_{H/RR}(\varepsilon) = -\partial [n_\varepsilon^H - n_\varepsilon^{RR}] / \partial \varepsilon\) is the driving factor, \(a = 1 \ldots 3\), \(M_\varepsilon(E) = N_G(E)(\hbar v/L)\) is the number of modes, \(N_G(E)\) is the electron density of states shown in Fig. 2b, \(L\) is the \(G\)-stripe length, \(\mathcal{T}_\varepsilon(\varepsilon)\) is the contact transparency. Then \(L_{e,H/RR}^{(1)} \equiv 0\), because the driving factor \(\mathcal{Y}_{H/RR}(\varepsilon)\) vanishes as it is an even function of \(\varepsilon\). Unlike to \(G_{H/RR}\) and \(S_{H/RR}\), which vanish at the H/RR contact, the thermal contact conductance \(\Lambda_{H/RR} = (\Gamma_{H/RR}^{(2)} - \mathcal{L}_{H/RR}^{(1)})(1/\hbar v_L)\) remains essentially finite.

Using the model form \(M_\varepsilon(\varepsilon) = \sqrt{m_e^*/m_e} \delta(\varepsilon - \Delta)\) one finds \(\Lambda_{H/RR} \approx \mathcal{F}_{H/RR} \sqrt{m_e^*/m_e} (2\Delta^2/\hbar^2) \delta T_{H/RR}\) where \(\delta T_{H/RR}\) is an effective electron temperature difference across the H/RR-interface, and \(\mathcal{F}_{H/RR} = (\partial [n_\varepsilon^H - n_\varepsilon^{RR}] / \partial \varepsilon)_{\varepsilon=\Delta} \approx 0.1\).

Along the \(G\)-stripe, the TEG parameters \(S, G_{\varepsilon}, \Lambda_{\varepsilon}\) are determined purely by the electron and hole transport. The underlying physical mechanism is the chiral transmission of the HCF electrons and holes from the neutral RR section to the voltage p- and n-doped \(G\)-sections. In the same approximation, one evaluates the electric conductance of FET_L/R along the \(G\)-stripe between the RR and \(C_{e,h}\) as
\[
G_{RR/C} = \frac{L_{RR/C}^{(0)}}{\Gamma_{RR/C}} = \frac{2e^2}{h} \int d\varepsilon M(\varepsilon) \mathcal{T}_{RR/C_{e,h}}(\varepsilon) \mathcal{Y}_{RR/C}(\varepsilon)
\]  
(5)
\[
= \frac{2e^2}{h} \sqrt{m_e^*/m_e} \mathcal{F}_{RR/C_{e,h}}(\varepsilon) \mathcal{Y}_{RR/C}(\varepsilon)|_{\varepsilon=\Delta}
\]
where $\mathbb{T}_{RR/C}(\varepsilon)$ is shown in Fig. 3d. Analogously, one finds Seebeck coefficient along the $G$-stripe $S_{RR/C} \simeq (\Delta/\varepsilon)/\delta T_{RR/C} = V_{SG}/\delta T_{RR/C}$ and $\Lambda_{RR/C} = L^{(2)}_{RR/C}/e^{2}T - TS_{G}^{2} \varepsilon \simeq 2\sqrt{m^{*}/m_{e}} \left(\Delta^{2}/\delta T_{RR/C}\right)/\hbar - 2\sqrt{m^{*}/m_{e}} \left(\Delta^{2}/\delta T_{RR/C}\right)/\hbar \rightarrow 0$. The last result indicates that Seebeck coefficient $S_{RR/C}$ could be huge while the electron/hole part of the thermal conductance $\Lambda_{RR/C}$ along the stripe is typically low. The phonon part of the heat energy flow is $Q_{pp}^{ph} = \Lambda_{ph} \delta T_{RR/C} = N_{ph} \delta T_{RR/C}$ where $\delta T_{RR/C}$ also depends on the temperature and the stripe geometry.

The electron/hole heat energy flow is

$$Q_{ph} = \delta T^{2} \cdot S_{e(h)}^{2} G_{e(h)} = \delta T^{2} \cdot S_{RR/C}^{2} G_{RR/C} \simeq 2e^{2}/h V_{SG}^{2} \left(\delta T_{H/RR}/\delta T_{RR/C}\right)^{2} \frac{e^{2}}{h} V_{SG}^{2} \cdot \kappa_{HCF}$$

where we have used $V_{SG} = 2\Delta/\varepsilon$, $S_{RR/C} = V_{SG}/\delta T_{RR/C}$, $G_{RR/C} = \sqrt{m^{*}/m_{e}}(2e^{2}/h)$, and we have defined the factor $\kappa_{HCF}$. Because $\kappa_{HCF}$ can be big, one might achieve huge values of $Q_{ph}$. Typically $\Lambda_{ph}^{H/RR} \ll \Lambda_{ph}^{RR/C}$, therefore the $G$-TEG net ”phonon” heat conductance $[6]$ is $\Lambda_{ph} \simeq \Lambda_{ph}^{H/RR}$, which can be comparable to the contact electron heat conductance $\Lambda_{H/RR} = 2\Delta^{2}\sqrt{m^{*}/m_{e}} F_{H/RR}/(h \cdot \delta T_{H/RR})$. It means that only the contact electron/hole and phonon heat conductances actually contribute into $\Lambda$. Summarizing the above estimates one arrives at $Z\delta T >> 1$.

\textbf{Transparency of the H/RR interface}

is directly related to the thermal injection efficiency. The interface barriers which contribute into $\Lambda_{H/RR}$ originate from the difference of the workfunctions in the metallic H electrode and the graphene $G$-stripe right beneath it [11,15]. The heat-conducting $C_{h}$, $H$, and $C_{e}$ electrodes are deposited at the top of the $G$-stripe, as schematically shown in Fig. 1. Another factor is the change in number of the conducting channels when electrons and holes tunnel from the 3D metallic H electrode into the 2D graphene $G$-stripe. [21] Conversion of the regular electrons and holes into the HCF excitations also contributes to $\Lambda_{H/RR}$. Thus, for the $G$-TEG, $\Lambda_{H/RR}$ depends on the 3D/2D conversion efficiency $\eta$ and on the spatial distribution of charge carriers near the H/RR interface. The contact thermal conductance problem and its solution are illustrated in Fig. 3.

In Fig. 3a we plot the transmission probability $T(\varepsilon)$ as a function of the electron energy $\varepsilon$ for the conventional electrons penetrating a non-chiral potential barrier (curve 1), and the quantum well (curve 2). Curve 3 shows $T(\varepsilon)$ for the non-chiral heavy fermions transmitting via a potential well. One can see that $T(\varepsilon)$ is strongly suppressed in the latter case. For such a reason, the contact conductance for conventional ”heavy“ fermions is low. Quite a different behavior $T(\varepsilon)$ takes place if instead of the conventional ”heavy“ electrons there are the ”heavy“ chiral (HCF) particles as is evident from Fig. 3b. In Fig. 3b we compare $T(\varepsilon)$ for the conventional chiral fermions penetrating the chiral barrier (curves 1 and 3) with the same characteristics for HCF particles (curves 2 and 4). One can see that $T(\varepsilon)$ is fairly good for both types of the chiral particles if the incidence angle is small, i.e., $\varphi = \pi/8$ (curves 1 and 2). For bigger incidence angles, i.e., $\varphi = 3\pi/8$ (curves 3 and 4), for the HCF particles $T(\varepsilon)$ becomes suppressed (curve 4). The electron thermal conductance $\Lambda_{e}$ of $G$-TEG is determined by $2\Lambda_{H/RR}$. The dominant contribution into $\Lambda_{H/RR}$ comes at the low angles $\varphi$, therefore using of the HCF particles helps to maintaining of $\Lambda_{e}$ at a decent level.

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