Nonlocal conductivity in the vortex-liquid regime

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We investigate the nonlocal conductivity calculated from the time-dependent Ginzburg-Landau equation. When fluctuations of the vector potential are negligible, the high-temperature (Gaussian) and low-temperature (flux-flow) forms of the uniform conductivity within an $ab$ plane ($\sigma_{yy}$) are essentially identical. We find to what extent the nonlocal conductivity shares this feature. The results suggest that for pure samples in these regimes the length scales of the nonlocal resistivity, $\rho_{yy}(y-y',z-z')$, remain short-ranged in the $y$ and $z$ directions in contrast to the assumptions made by the hydrodynamic modelling of the multiterminal transport measurements. On the other hand, the resistivity is seen to have a long length scale in the $x$ direction $\rho_{yy}(x-x')$. The implications for the interpretation of recent experiments are discussed.

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The conductivity is nonlocal if the current at a site $r$ is determined not only by $E(r)$ the electric field at $r$ but also by the fields at other sites $E(r')$. It is always nonlocal on some microscopic scale, but the term "nonlocal" is generally reserved for cases in which the length scale involved is much longer than some underlying microscopic one. A nonlocal conductivity implies a nonlocal resistivity (though the length scales on which each varies may differ). Since the resistivity of type II superconductors will be influenced by the motion of flux lines, it seems reasonable to expect nonlocal effects. In recent experiments on heavily twinned samples of YBCO, Safar et al. have observed the onset and enhancement of nonlocal conductivity as the temperature is lowered.

With experiments on clean samples, samples with columnar defects and films underway, it would be useful to sort out the basic form of the conductivity. But the vortex-liquid state of a type II superconductor is a problem with many length scales and identifying which is/are relevant to a particular measurement is by no means trivial.

In the Ohmic (linear response) regime, nonlocality implies that the (dc) conductivity $\sigma_{\mu\nu}(r,r') \neq \sigma_{\mu\nu}(r-r')$ or in momentum space: $\sigma_{\mu\nu}(k) \neq \text{const}$. Huse and Majumdar explored a "hydrodynamic" approach in which the small-$k$ expansion of the conductivity is truncated as follows:

$$\sigma_{\mu\nu}(k) = \sigma_{\mu\nu}(0) + S_{\mu\nu,\alpha\beta} k_\alpha k_\beta. \quad (1)$$

In the example they work out in detail to explain features of the multiterminal transport measurements, they include only one $S$: $S_{yy,yy}$. It models the "tilt viscosity" — which measures the influence of pancake vortices moving in one $ab$ plane on those moving in another when they travel with different velocities. The hydrodynamic picture began by viewing vortices as the analogues of polymers in a melt, concepts like viscosities naturally followed. Thus this picture would seem most appropriate in the London limit where the core size and magnetic screening length $\lambda$ are shorter than the distance between vortices, i.e. where vortices are most polymer-like. It is unclear, however, what fraction, if any, of the vortex-liquid regime is described by such a hydrodynamic approach.

Stability requires $\sigma_{\mu\nu}(k)$ to be a positive definite matrix; for the hydrodynamic form of the conductivity, it implies for instance that $S_{yy,yy}$ and $S_{yy,yy}$ be positive. When these conditions are met, one finds that the resistivity decays with a length scale of order $(\sqrt{S/\sigma})$, which is in turn the length scale found in the electric-field and current distributions. Recent calculations starting from the time-dependent Ginzburg-Landau (TDGL) equation find that many of the $S$’s do not have the requisite sign — at least for clean samples at high temperatures. We study the nonlocal conductivity within the $ab$ plane $\sigma_{yy}(k)$ in the lowest Landau level (LLL) limit, which is valid not close to $H_{c2}(T)$ as is sometimes stated in the literature but at lower temperatures in the vortex-liquid regime. When fluctuations of the vector potential are negligible, the uniform conductivity $\sigma_{yy}(0)$ is known to interpolate smoothly between its high-temperature (Gaussian) form and its low-temperature (flux-flow) form. We extend these results to the nonlocal conductivity and argue that $S_{yy,yy}$ and $S_{yy,yy}$ do not change sign as the temperature is lowered, suggesting that there may be a substantial region where the hydrodynamic approach does not apply.

Our starting point is the TDGL equation for the superconducting order parameter $\Psi$ in a constant magnetic field along the $z$ axis in the Landau gauge:

$$\begin{align*}
\frac{1}{\Gamma} \frac{\partial}{\partial t} + \frac{\hbar^2}{2m_{ab}} \left( \frac{\partial^2}{\partial x^2} \right) + \left( \frac{\partial}{\partial y} - \frac{ie^*B_x}{\hbar} \right)^2 & \\
- \frac{\hbar^2}{2m_c} \frac{\partial^2}{\partial z^2} + a & = \Psi(r,t) + b|\Psi(r,t)|^2\Psi(r,t) = \eta(r,t),
\end{align*} \quad (2)$$

The hydrodynamic expansion of the conductivity is truncated in the London limit where the core size and magnetic screening length $\lambda$ are shorter than the distance between vortices, i.e. where vortices are most polymer-like. It is unclear, however, what fraction, if any, of the vortex-liquid regime is described by such a hydrodynamic approach.

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- \frac{\hbar^2}{2m_c} \frac{\partial^2}{\partial z^2} + a & = \Psi(r,t) + b|\Psi(r,t)|^2\Psi(r,t) = \eta(r,t),
\end{align*} \quad (2)$$
where the \( \langle \eta(r, t) \rangle = 0 \) and has \( \delta \)-function correlations
\[
\langle \eta^* (r, t) \eta (r', t') \rangle = \frac{2k_BT}{\hbar} \delta (r - r') \delta (t - t'). \tag{3}
\]

At high temperatures, the nonlinear term \((\propto b)\) in \(2\) can be ignored. The “states” of the linear operator collapse into Landau levels
\[
\phi_n (r) = e^{i k_y y + i k_z z} u_n (x/\ell - k_y \ell), \tag{4}
\]
where \(u_n (v)\) are the harmonic-oscillator eigenstates and \(\ell = (\hbar/e^* B)^{1/2}\) the magnetic length. The corresponding energies are
\[
E_n (k_z) = \hbar^2 k_z^2 /2m_c + \alpha + n\hbar \omega_0, \tag{5}
\]
where \(\omega_0 = e^* B/m_{ab}\) is the cyclotron frequency and \(\alpha = \alpha + \hbar \omega_0 /2\) the energy of the LLL.

Some length scales are already apparent; clearly \(\ell\) is an \(ab\)-plane length scale related to the distance between flux lines. Two energy scales, \(\alpha\) and \(\hbar \omega_0\), appear in \(E_n (k_z)\); from these one can construct two \(c\)-axis length scales: \(\xi_c = (\hbar^2 /2m_c \alpha)^{1/2}\) and \(\ell_c = (\hbar/m_c \omega_0)^{1/2}\). The former is the mean-field \(c\)-axis correlation length; the latter a \(c\)-axis analogue to the magnetic length. The lengths, \(\ell_c\) and \(\ell\), depend only on the magnetic field; while \(\xi_c\) grows as the temperature \((T)\) is lowered.

The Kubo formula can be used to determine the contribution of the superconducting fluctuations to the dc conductivity (the Aslamazov-Larkin term):
\[
\sigma_{\mu\nu}^{(s)} (k) = \frac{1}{2k_BT} \int d\tau \int d\tau' \int \frac{e^{i k \cdot r'}}{2 \pi} \langle J_\mu^{(s)} (r, t) J_\nu^{(s)} (r', t') \rangle_c. \tag{6}
\]
The superconducting currents are given by:
\[
J_\mu^{(s)} (r, t) = \frac{e^* h}{2m^*_\mu} \left[ \Psi^* \left( -i \frac{\partial}{\partial r_\mu} - e^* A_\mu \right) \Psi + \text{c.c.} \right]. \tag{7}
\]
Each involves a \(\Psi\), a \(\Psi^*\) and a (gauge-invariant) derivative, so when performing the average in \(6\) one “contracts” \(\Psi (r)\) with \(\Psi^* (r')\) and likewise \(\Psi^* (r)\) with \(\Psi (r')\) resulting in two correlators. In diagrammatic form the result is a loop, the first term in Fig. 1(a).

The Gaussian result for \(\sigma_{yy}^{(s)} (0)\) is
\[
\sigma_{yy}^{(s)} (0) = \frac{e^2 k_BT m_c^{1/2}}{2^{5/2}\pi h^2 T \alpha^{1/2}} \sum_{n=0}^{\infty} \frac{(-1)^n(n+1)}{(1 + n\omega_0 /2\alpha)^{1/2}}. \tag{8}
\]

We will not reproduce the full expression for the Gaussian \(\sigma_{\mu\nu}^{(s)} (k)\) here as they appear in Ref. [5]. Terms in the small-\(k\) expansion are similar to that above in Eq. 3.

When \(\alpha \ll \omega_0\), these sums are dominated by their first terms, yielding:
\[
\sigma_{yy}^{(s)} (k) = \frac{\pi |\psi_0|^2}{\Phi_0 \Gamma B} \left[ 1 + \frac{3h^2 \omega_0^2 k^2}{\alpha^2} \frac{\ell^2 k_y^2}{4} - \frac{\ell^2 k_y^2}{4} - \frac{\ell^2 k_z^2}{4} + O(k^4) \right]. \tag{9}
\]

where \(\Phi_0 = h/2e\) and \(\langle |\psi_0|^2 \rangle\), the equilibrium average of the fluctuations of the order parameter in the LLL, is
\[
\langle |\psi|^2 \rangle = \frac{k_B T m_c^{1/2}}{2^{5/2}\pi h^2 \alpha^{1/2}}. \tag{10}
\]

First note that this high \(T\) result for \(\sigma_{yy}^{(s)} (0)\) is very reminiscent of the flux-flow conductivity \(\sigma_{ff} = \eta /\Phi_0 B\) (where \(\eta\) is a viscosity coefficient) [3]. The flux-flow conductivity is derived from translating an unpinned Abrikosov flux-line lattice [4], and its validity depends on whether one can neglect thermal fluctuations in the vector potential. Its similarity to the expression for \(\sigma_{yy}^{(s)}\) in eq. (8) occurs since lowering \(T\) has minimal effect on \(\sigma_{yy}^{(s)} (0)\). This feature is specific to the \(ab\) conductivity; it is not shared by the \(c\)-axis conductivity \(\sigma_{zz}^{(s)} (0)\). It does extend, however, to the nonlocal conductivity \(\sigma_{yy}^{(s)} (k_x = 0, k_y, k_z)\), though it no longer holds if \(k_x \neq 0\). We exploit this latter point to argue that \(S_{yyyy}\) and \(S_{zzz}\) do not change sign as \(T\) is lowered and that the associated length scales remain short.

So what is this property? In the calculation of \(\sigma_{yy}^{(s)} (0)\) the momentum operators occurring in the definition of the currents \(\frac{\partial}{\partial \mu}\) act like creation (or annihilation) operators on the \(\Psi\)’s and \(\Psi^*\)’s. They raise (or lower) the “states” with the following important outcome: of the two correlators that result, one is in the \(n\)th Landau level the other is in the \((n + 1)\)th. In the \(\alpha \ll \omega_0\) limit, the contribution with one \(n = 0\) correlator and one \(n = 1\) correlator dominates. (See Fig. 1(a).)
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ment is dominated by poles associated with mass
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The full
channel. (This choice does not affect the outcome, just
the terms involving
is “down.” (c) Diagram with vector-potential fluctuations
by a static
in the Kubo formula (6) becomes
Next recall that the eigenfunction expansion of the cor-
relator \( C(\mathbf{r}, \mathbf{r}'; t') = \langle \Psi(\mathbf{r}, t + t') \Psi^*(\mathbf{r}', t) \rangle \) is
\[
C(\mathbf{r}, \mathbf{r}'; t') \propto \int_{p_x} \int_{p_y} \sum_{n} \phi_n(\mathbf{r}) \phi_n^*(\mathbf{r}') \frac{e^{-\Gamma E_n(p_z)} e^{-\Gamma E_n(p_z)|t'|}}{E_n(p_z)}, \tag{11}
\]
From here we see that in the LLL limit the time integral
in the Kubo formula (3) becomes
\[
\int dt' e^{-\Gamma[E_0(p)+E_1(p-k)]|t'|}, \tag{12}
\]
where \( p \) is the \( z \) component of momentum running
through the \( n = 0 \) channel and \( p - k \) that through the
\( n = 1 \) channel. (See Fig. 1(a).) Note that \( E_0 \) is much
smaller than \( E_1 \) and we can drop it from the integral (12).
This last step is equivalent to using the static (equilib-
rium) \( n = 0 \) correlator and yields the factor of \( \langle |\psi_0|^2 \rangle \) in (3).

The main point is that one and only one of the corre-
lators is in the \( n = 0 \) level and that it can be replaced
by a static \( n = 0 \) correlator. We now extend these argu-
ments to the nonlinear conductivity. Consider \( S_{yxyy} \)
which featured in the hydrodynamic modelling of the
multiterminal transport measurements. The only modifi-
cation in calculating \( S_{yxyy} \) is that it requires two ad-
ditional derivatives with respect to \( z \). As these derivatives
do not affect the Landau-level structure, the above argu-
ments carry through. In fact, we take further advantage
of the disparity in “masses” (\( \alpha < \omega_0 \)) by insisting that
the external momentum \( k_z \) be sent through the \( n = 1 \)
channel. (This choice does not affect the outcome, just
makes it more apparent.) The internal momentum inte-
dominated by poles associated with mass \( \alpha \), and
the terms involving \( k_z \) and \( \omega_0 \) are left essentially intact.
The full \( k_z \) dependence in the LLL limit is:
\[
\sigma_{yy}^{(s)}(k_z) = \frac{\pi \langle |\psi_0|^2 \rangle}{\Phi_0 \Gamma B} \left[ 1 + k_z^2 e_c^2/2 \right]^{-2}. \tag{13}
\]
The structure is simply that of \( [E_1(k_z)]^{-2} \) with \( \alpha \to 0 \).

More detailed calculations reveal that \( S_{yxyy} \) has a sim-
ilar property. This time the extra derivatives (with re-
spect to \( y \)) have some effect on the Landau-level struc-
ture: while one correlator is in the \( n = 0 \) level, the other
is in either the \( n = 1 \) or \( n = 2 \) level. The important
point is that there is still no contribution with two \( n = 0 \)
correlators. This last point applies to all terms in the \( k_y \)
expansion; the series resums to give:
\[
\sigma_{yy}^{(s)}(k_y) = \frac{\pi \langle |\psi_0|^2 \rangle}{\Phi_0 \Gamma B} \left[ 1 - e^{-k_y^2 e_c^2/2} \right]^{-2}. \tag{14}
\]
In fact, all the terms in the small-\( k \) expansion
\( \sigma_{yy}(k_y, k_z) \) (so long as \( k_z \) is 
zero) share this property, and the combined \( k_y \) and \( k_z \)
dependence is:
\[
\frac{\sigma_{yy}^{(s)}(k_y, k_z)}{\sigma_{yy}^{(s)}(0)} = e^{-k_y^2 e_c^2/2} \sum_{n=0}^{\infty} \frac{(n + 1)(k_z^2 e_c^2/2)^n}{n![(n + 1) + k_z^2 e_c^2/2]^n}. \tag{15}
\]
On the other hand, the extra derivatives in \( S_{yxyy} \) (with
respect to \( x \)) have a more crucial effect, producing a
nonzero contribution when both correlators are in the
\( n = 0 \) state. The outcome, as seen previously, is that
\( S_{yxyy} \) is positive (hydrodynamic) and the long length
scale \( \xi \) arises.

We want to go beyond the Gaussian result, i.e. con-
sider the effect of the nonlinear term in the TDGL
equation. One question is: To what extent is the simple loop
affected? In the region of the phase diagram dominated by
the LLL, the answer is “not much,” as already hinted
at by the similarity of the Gaussian and flux-flow results.
Another question is: Where is this region? The Gauss-
ian calculation suggests that the LLL approximation is
valid near the \( H_{c2}(T) \) line (\( \alpha = 0 \)), but renormal-
ization effects will change this! A very thorough discussion
of the region of validity of the LLL appears in Refs. [7].

Our first attack on the problem of including the non-
linear terms, i.e. renormalization effects, will be the
Hartree-Fock (HF) approximation. The theory remains
effectively Gaussian but \( \alpha \to \tilde{\alpha} = \alpha + 2\beta(\psi_0^2) \), giving the
following self-consistent equation for \( \tilde{\alpha} = \alpha + \hbar \omega_0/2 \):
\[
\tilde{\alpha} = \alpha + \frac{b}{2} \frac{k_y T}{T_c} \frac{m_e^{1/2}}{\ell^2 \hbar^{1/2} \alpha^{1/2}}. \tag{16}
\]
Now we ask not where \( \alpha \) is small but where \( \tilde{\alpha} \) is small to
establish where the \( \tilde{\alpha} \) approximation is valid. The
renormalized mass \( \tilde{\alpha} \) grows small as the bare mass
\( \alpha \) grows large and negative, at temperatures below
\( H_{c2}(T) \). Of course, renormalization effects beyond the
HF approximation ought to be considered, but concern-
ing the issue of where the \( \tilde{\alpha} \) approximation is valid,
they are only refinements; the essential features are cap-
tured by HF.
To go beyond the HF approximation to $\sigma^{(s)}_{yy}(k_x = 0, k_y, k_z)$, we expand perturbatively around it. Contributions to the $m^{th}$ order require $2m$ additional contractions (m correlators and m propagators). The pivotal issue is in what Landau levels do these new correlators and propagators lie. If $\tilde{\alpha} \ll \omega_0$, then any contribution in which they are in higher Landau levels would be “down” (have a significantly lower weight) compared to those in which they are all in the LLL. Any insertion that disrupts the $n \geq 1$ correlator (as in Fig. 1(b)) automatically results in an extra higher Landau-level contraction. The only way in which all new terms are in the LLL is if they only dress up the $n = 0$ correlator. This result is shown schematically in Fig. 1(a).

As a consequence, in the LLL approximation, $\sigma_{yy}(k_x = 0, k_y, k_z)$ still involves only two correlators: a fully renormalized $n = 0$ correlator $\tilde{C}^{(0)}(r, r', t')$ and an $n \geq 1$ correlator untouched beyond the HF approximation. In the LLL limit $\tilde{C}^{(0)}(r, r', t')$ has exactly the same dependence on $x, x', y$ and $y'$ as its “bare” version; thus, the $k_y$-dependence of $\sigma^{(s)}_{yy}$ is exactly the same as that in expression (13). Moreover, a repetition of the arguments following eq. (13) shows that the $n = 0$ correlator can still be considered static, that the external momentum integral is still dominated by poles associated with mass $\tilde{\alpha}$. So as before the terms involving the external momentum $k_z$ and mass $\omega_0$ remain essentially intact. The upshot is that the $k_z$-dependence of $\sigma^{(s)}_{yy}$ is exactly the same as that in expression (13) and in particular $S_{yyyy}$ and $S_{yyyy}$ remain negative. Only $\langle |\psi|^2 \rangle$ gets renormalized.

The perturbative arguments above (summarized in Fig. 1(a)) do not hold when vector-potential fluctuations are no longer negligible. Then we would include vertices involving a $\Psi$, a $\Psi^*$ and an $A_\mu$, where $A_\mu$ is the fluctuating part of the vector potential. These vertices lead to diagrams like that shown in Fig. 1(c), where the wavy line represents an $A \cdot A$ correlator. In the Landau gauge, vertices associated with $A_\mu$ come with a factor of $\omega_0$ which is large in the LLL limit. They also involve a (gauge-invariant) derivative and so the $\Psi$ and $\Psi^*$ at such vertices are in different Landau levels (see Fig. 1(c)). Diagrams with $A \cdot A$ correlators disrupting the $n = 1$ $\Psi$-$\Psi$ correlator (as in Fig. 1(c)) are not down compared to those with $A_\mu$ correlators dressing up the the $n = 0$ correlator. Hence the previous arguments no longer apply if vector-potential fluctuations are important.

The effects of disorder will be considered in a future publication. Briefly, point disorder enhances the conductivity in the $ab$ plane; however, the $c$-axis length scale appears to remain short, $\ell_c$. (In a nutshell, the external momentum $k_z$ can still be sent entirely through an $n = 1$ channel in the leading diagrams.) On the other hand, in a case with correlated disorder, such as columnar defects or twin planes, the $c$-axis length scale grows as $T$ is lowered, suggesting an increased likelihood for nonlocal effects.

In summary the central result is that in the LLL approximation, which holds over a considerable portion of the $H - T$ diagram for YBCO, the nonlocal conductivity $\sigma^{(s)}_{yy}(k_y, k_z)$ has the form given in Eq. (15) with special cases given by Eqs. (13) and (14) with only $\langle |\psi|^2 \rangle$ having any temperature dependence. In particular, the length scales multiplying $k_y^2$ and $k_z^2$ in these expressions are short and temperature-independent; in addition, the signs of $S_{yyyy}$ and $S_{yyyy}$ remain negative (nonhydrodynamic) throughout the regime. Hence one would expect in experiments on clean samples with geometries that probe chiefly $\sigma_{yy}$ to see essentially local behavior, as may be borne out by recent experiments.

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