A\textsuperscript{1}-CURVES ON AFFINE COMPLETE INTERSECTIONS

XI CHEN AND YI ZHU

Abstract. We generalize the results of Clemens, Ein, and Voisin regarding rational curves and zero cycles on generic projective complete intersections to the logarithmic setup.

1. Introduction

In this paper, we work with varieties over the complex numbers. First we introduce the notion of smooth complete intersection pairs.

Definition 1.1. Let $X$ be a complete intersection in $\mathbb{P}^n$ of type $(d_1, \cdots , d_c)$. Let $D \subset X$ be a hypersurface section of degree $k$. We call the pair $(X, D)$ a smooth complete intersection pair of type $(d_1, \cdots , d_c; k)$ if both $X$ and $D$ are smooth. We define the total degree $d$ of the pair $(X, D)$ by

$$d = d_1 + \cdots + d_c + k.$$ 

When $k = 0$, the boundary is empty and we simply denote $(X, D)$ by $X$.

The existence of rational curves, algebraic hyperbolicity and rational equivalence of zero cycles on generic complete intersection of general type has been studied by Clemens [Cle86], Ein [Ein88, Ein91], and Voisin [Voi94, Voi96, Voi98].

Theorem 1.2 (Clemens,Ein,Voisin). Let $X$ be a generic complete intersection in $\mathbb{P}^n$ of type $(d_1, \cdots , d_c)$.

1. If $d \geq 2n - c$, $X$ has no rational curves;
2. If $d \geq 2n - c + 1$, $X$ is algebraically hyperbolic;
3. If $d \geq 2n - c + 2$, no two points of $X$ are rationally equivalent.

The bounds above are not optimal. Voisin [Voi98] further improved the bound (1) to $d \geq 2n - 2$ in case of hypersurfaces, which is optimal because hypersurfaces of degree $\leq 2n - 3$ always admit lines.

In this paper, we generalize Theorem 1.2 to smooth complete intersection pairs, where we study $\mathbb{A}^1$-curves and $\mathbb{A}^1$-equivalence of zero cycles instead. See Theorems 1.3, 1.6 and Corollary 1.5 below. They specialize to Theorem 1.2 when the boundary is empty.

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1.1. $\mathbb{A}^1$-curves. An $\mathbb{A}^1$-curve is an algebraic map from $\mathbb{A}^1$ to the interior of a pair. When the boundary is empty, $\mathbb{A}^1$-curves are simply rational curves. We first study $\mathbb{A}^1$-curves on generic complete intersection pairs of general type.

**Theorem 1.3.** Let $(X, D)$ be a generic complete intersection pair in $\mathbb{P}^n$ of type $(d_1, \cdots, d_c; k)$. If $d \geq 2n - c$, the interior $X - D$ contains no $\mathbb{A}^1$-curves.

When the boundary is nonempty, the bound in Theorem 1.3 is optimal because a general such pair in $\mathbb{P}^n$ with $d \leq 2n - c - 1$ always admits an $\mathbb{A}^1$-line. Furthermore, we complete the last step studying $\mathbb{A}^1$-curves on complete intersection surface pairs in $\mathbb{P}^n$ of total degree $d$, summarized as the table below.

| dim $X = 2$ | $(X, D)$ | $\mathbb{A}^1$-curves |
|-------------|----------------|------------------------|
| $d \leq n$  | log Fano       | log rationally connected [CZ14] |
| $d = n + 1$ | log K3         | generically countable [Che99, LL12, CZ16] |
| $d \geq n + 2$ | of log general type | generically none (Thm. 1.3) |

1.2. Algebraic hyperbolicity.

**Theorem 1.4.** Let $(X, D)$ be a generic complete intersection pair in $\mathbb{P}^n$ of type $(d_1, \cdots, d_c; k)$. If $d \geq 2n - c - l + 1$, any closed subvariety $Y$ of $X - D$ of dimension $l$ has an effective log canonical bundle on its desingularisation; and if the equality is strict, $Y$ has a big log canonical bundle on its desingularisation.

Theorem 1.4 implies algebraic hyperbolicity of such pairs.

**Corollary 1.5.** Let $X$ be a generic complete intersection in $\mathbb{P}^n$ of type $(d_1, \cdots, d_c; k)$. If $d \geq 2n - c + 1$, the interior $X - D$ is algebraically hyperbolic.

For generic complete intersection pairs of type $(1; k)$, Theorems 1.3, 1.4 and Corollary 1.5 are proved by the first named author [Che04] and Pacienza-Rousseau [PR07].

1.3. $\mathbb{A}^1$-equivalence of zero cycles. For open varieties, the right substitution for Chow group of zero cycles is Suslin’s 0-th homology group $h_0(U)$, that is, the group of zero cycles modulo $\mathbb{A}^1$-equivalences. When the boundary is empty, it coincides with the Chow group of zero cycles. For surface pairs, the log version of Mumford’s theorem and Bloch’s conjecture was studied in [Zhu15, YZ15]. For generic complete intersection pairs, we have the following stronger version of Theorem 1.3.

**Theorem 1.6.** Let $(X, D)$ be a generic complete intersection pair in $\mathbb{P}^n$ of type $(d_1, \cdots, d_c; k)$. If $d \geq 2n - c + 2$, no two points of the interior $X - D$ are $\mathbb{A}^1$-equivalent.
2. Global positivity

In this section, we generalize Voisin’s global positivity result \[\text{[Voi96, Prop. 1.1]}\] for smooth complete intersection pairs. For the rest of the paper, we fix the following notations.

**Notation 2.1.** With the same notations as in Definition 1.1, let \(k := d_{c+1}\). Let \(S_{d_i} := H^0(\mathcal{O}_{\mathbb{P}^n}(d_i))\) for \(i = 1, \ldots, c + 1\). Let \(S\) be the product vector space \(\prod_{i=1}^{c+1} S_{d_i}\). We denote by \(S^o\) the open subset of \(S\) parametrizing smooth complete intersection pairs. Let \((X, D) \subset \mathbb{P}^n\times S^o\) be the universal family of smooth complete intersection pair. Let \(O_X(1)\) be the pullback line bundle \(pr_1^*(\mathcal{O}_{\mathbb{P}^n}(1))\). For any \(t \in S^o\), denote by \((X_t, D_t)\) the smooth complete intersection pair parametrized by \(t\). We assume that \(\dim X_t \geq 2\).

**Lemma 2.2.** For any smooth complete intersection pair \((X, D)\) with \(\dim X \geq 2\), we have
\[H^0(\Omega^i_X(\log D)) = 0\]
for \(0 < i < \dim X\).

**Proof.** The long exact sequence of the residue sequence gives
\[H^0(\Omega_X^i) \to H^0(\Omega_X^i(\log D)) \to H^0(\Omega_D^{i-1}) \to H^1(\Omega_X^{i+1})\].
The first term vanishes by the Lefschetz hyperplane theorem. If \(\dim D \geq 2\), the third term vanishes by the Lefschetz hyperplane theorem as well. If \(\dim D = 1\), the last map is the Gysin map which is injective. Therefore, \(H^0(\Omega_X^i(\log D)) = 0\). \hfill \square

**Lemma 2.3.** If \(d \geq n + 2\), then \(h^0(T_{X_t}^!(1)) = 0\) for every \(t \in S^o\).

**Proof.** By Serre duality and Lemma 2.2, we have
\[h^0(T_{X_t}^!(1)) = h^0(\Omega_{X_t}^{n-1}(\log D_t) \otimes (-K_{X_t} - D_t) \otimes O(1))\]
\[= h^0(\Omega_{X_t}^{n-1}(\log D_t) \otimes O(n + 2 - d))\]
\[\leq h^0(\Omega_{X_t}^{n-1}(\log D)) = 0. \hfill \square\]

**Proposition 2.4.** The log tangent bundle
\[T\mathcal{X}^!(1)|_{X_t}\]
is globally generated for every \(t \in S^o\) if \(h^0(T_{X_t}^!(1)) = 0\).

**Proof.** By \(\text{[CZ14, Lem. 4.1]}\), we have the short exact sequence
\[0 \to O_D \to T\mathcal{X}^!(1)|_D \to T_D \to 0.\]
The global generation of \(T\mathcal{X}^!(1)|_{X_t}\) implies the global generation of \(T\mathcal{D}(1)|_{D_t}\). In particular, Proposition 2.4 for the nonempty boundary case implies the empty boundary case. For the rest of the proof, we assume that the boundary is nonempty.
Since \((X, D)\) is a log smooth family over \(S^0\), we have
\[
0 \longrightarrow T_{X^t_1}(1) \longrightarrow T\mathcal{X}_1(1)|_{X_t} \longrightarrow S \otimes \mathcal{O}_{X_t}(1) \longrightarrow 0.
\]

By [CZ14, Lemma 2.1], the log tangent bundle \(T\mathcal{X}_1\) is determined by the short exact sequence:
\[
0 \longrightarrow T\mathcal{X}_1 \longrightarrow \mathcal{O}_{\mathcal{X}_1}(1)^{\oplus (n+1)} \longrightarrow \sum_{i=1}^{c+1} \mathcal{O}_{\mathcal{X}_1}(d_i) \longrightarrow 0,
\]
where \(\alpha\) is given by the multiplication of the Jacobian. The above two sequences lead to the commutative diagram:

\[
\begin{array}{ccccccccc}
0 & \longrightarrow & T_{X^t_1}(1) & \longrightarrow & T\mathcal{X}_1(1)|_{X_t} & \longrightarrow & S \otimes \mathcal{O}_{X_t}(1) & \longrightarrow & 0 \\
& & \downarrow{id} & & \downarrow{ev} & & \downarrow{\alpha} & & \\
0 & \longrightarrow & T_{X^t_1}(1) & \longrightarrow & \mathcal{O}_{X_t}(1)^{\oplus (n+1)} & \longrightarrow & \sum_{i=1}^{c+1} \mathcal{O}_{\mathcal{X}_1}(d_i)|_{X_t} & \longrightarrow & 0.
\end{array}
\]

Since \(h^0(T_{X^t_1}(1)) = 0\), we obtain the corresponding long exact sequences
\[
0 \longrightarrow H^0(T\mathcal{X}_1(1)|_{X_t}) \longrightarrow S \otimes S^1 \longrightarrow H^1(T_{X^t_1}(1)) \longrightarrow H^1(T\mathcal{X}_1(1)|_{X_t}) \longrightarrow 0.
\]

We have the following properties:
(1) \(H^0(T\mathcal{X}_1(1)|_{X_t}) = \ker(\mu)\);
(2) \(\ker(\beta) = \prod_{i=1}^{c+1} S^{d_i-1}/\text{Im}(\alpha)\);
(3) since \(ev\) is surjective, \(\text{Im}(\mu) = \ker(\beta)\). Thus we have the map
\[
\mu : S \otimes S^1 \rightarrow \ker(\beta).
\]

Now for any point \(x \in X_t\), tensoring all the terms in the diagrams as above with the ideal sheaf \(\mathcal{I}_x\), we have another commutative diagram:

\[
\begin{array}{ccccccccc}
H^0(T\mathcal{X}_1(1)|_{X_t} \otimes \mathcal{I}_x) & \longrightarrow & S \otimes S^1_x \mu_x & \longrightarrow & H^1(T_{X^t_1}(1) \otimes \mathcal{I}_x) \longrightarrow & H^1(T\mathcal{X}_1(1)|_{X_t} \otimes \mathcal{I}_x) & \longrightarrow & 0 \\
& & \downarrow{ev_x} & & \downarrow{\mu_x} & & \downarrow{id} & & \\
H^0(\mathcal{O}_{X_t}(2)^{\oplus (n+1)} \otimes \mathcal{I}_x) & \longrightarrow & \prod_{i=1}^{c+1} S^{d_i+1}_x & \longrightarrow & H^1(T_{X^t_1}(1) \otimes \mathcal{I}_x) \beta_x & \longrightarrow & H^1(\mathcal{O}_{X_t}(2)^{\oplus (n+1)} \otimes \mathcal{I}_x),
\end{array}
\]

where \(S^m_x = H^0(\mathcal{O}_{X_t}(m) \otimes \mathcal{I}_x)\). We have the following properties:
(1) \(H^0(T\mathcal{X}_1(1)|_{X_t} \otimes \mathcal{I}_x) = \ker(\mu_x)\);
(2) \(\ker(\beta_x) = \prod_{i=1}^{c+1} S^{d_i+1}_x/\text{Im}(\alpha_x)\);
(3) since \(ev_x\) is surjective, we have \(\text{Im}(\mu_x) = \ker(\beta_x)\). Thus we write
\[
\mu_x : S \otimes S^1_x \rightarrow \ker(\beta_x).
\]
Finally, consider the commutative diagram:

\[
\begin{array}{cccccc}
0 & \rightarrow & H^0(T_{X_t}(1)|_x) & \rightarrow & H^1(T_{X_t}(1) \otimes I_x) & \rightarrow & H^1(T_{X_t}(1)) & \rightarrow & 0 \\
\downarrow & & \downarrow \gamma & & \downarrow \beta & & \downarrow & & \\
H^0(O_{X_i}(2)^{\oplus(n+1)}) & \rightarrow & H^0(O_{X_i}(2)^{\oplus(n+1)}|_x) & \rightarrow & H^1(O_{X_i}(2)^{\oplus(n+1)} \otimes I_x) & \rightarrow & H^1(O_{X_i}(2)^{\oplus(n+1)}) & \rightarrow & 0.
\end{array}
\]

Since \(O_{X_t}(2)^{\oplus n+1}\) is globally generated, the map \(u\) is surjective. In particular, the composite map \(\gamma\) is the zero map. Hence we get

\[
\begin{array}{cccccc}
0 & \rightarrow & H^0(T_{X_t}(1)|_x) & \rightarrow & \ker(\beta_x) & \rightarrow & \ker(\beta) & \rightarrow & 0.
\end{array}
\]

It follows that

\[
\dim \text{Im}(\mu_x) - \dim \text{Im}(\mu) = \dim X_t.
\]

On the other hand, we have

\[
\dim \ker(\mu) - \dim \ker(\mu_x) = \dim S \otimes S^1 - \dim S \otimes S^1_x + \dim \text{Im}(\mu_x) - \dim \text{Im}(\mu) = \dim S + \dim X_t = \dim X.
\]

Thus

\[
h^0(T\mathcal{X}^\dagger(1)|_{X_t}) - h^0(T\mathcal{X}^\dagger(1)|_{X_t} \otimes I_x) = \dim X.
\]

In particular, \(T\mathcal{X}^\dagger(1)|_{X_t}\) is globally generated. \(\square\)

Now Proposition 2.4 implies

**Corollary 2.5.** For any \(l \geq 0\), the bundle \(\wedge^l T\mathcal{X}^\dagger \otimes O_{X_t}(l)\) is globally generated and the bundle \(\wedge^l T\mathcal{X}^\dagger \otimes O_{X_t}(l + 1)\) is very ample if \(d \geq n + 2\). \(\square\)

**Corollary 2.6.** If \(d \geq n + 2\), then \(\Omega^{\dim X - 1}|_{X_t}\) is globally generated when \(d \geq l + n + 1\) and is very ample when the inequality is strict.

**Proof.** By Serre duality, we have

\[
\wedge^l T\mathcal{X}^\dagger \otimes O_{X_t}(l) = \Omega^{\dim X - l - 1}_{\mathcal{X}^\dagger} \otimes K^{-1}_{\mathcal{X}^\dagger} \otimes O_{X_t}(l) = \Omega^{\dim X - l - 1}_{\mathcal{X}^\dagger} \otimes O_{X_t}(l + n + 1 - d).
\]

Now the assertions follow from Corollary 2.5. \(\square\)

### 3. Proof of Main Theorems

#### 3.1. Proof of Theorems 1.3, 1.4

**Proof of Theorem 1.3.** If not, there exists a family of \(\mathbb{A}^1\)-curves

\[
\begin{array}{ccc}
V \times \mathbb{A}^1 & \xrightarrow{f} & (\mathcal{X}, \mathcal{D}) \\
\downarrow & & \downarrow \\
V & \xrightarrow{j} & S^0,
\end{array}
\]

where \(j\) is an étale dominant morphism. By [Zhu15, Lem. 3.1], the morphism \(f\) extends to a morphism of log pairs

\[
f : (V' \times \mathbb{P}^1, V' \times \{\infty\}) \rightarrow (\mathcal{X}, \mathcal{D}),
\]

...
where $V'$ is a dense open subset of $V$. Here the dimension of $(V' \times \mathbb{P}^1, V' \times \{\infty\})$ is $\dim X - (n - c) + 1$. Corollary 2.6 with $l = n - c - 1$ implies that $\Omega_{X, l}^\dim X - l |_{X_t}$ is globally generated if
\[ d \geq n - c - 1 + n + 1 = 2n - c. \]
Pullback via $f$ gives a nontrivial section of the log canonical bundle of $(V' \times \mathbb{P}^1, V' \times \{\infty\})$, which is absurd because the pair is log uniruled. □

**Proof of Theorem 1.4.** The bound $d \geq 2n - c - l + 1$ implies that $\Omega_{\dim X - l}^X |_{X_t}$ is very ample. Now Theorem 1.4 follows from the same proof as in [PR07, Cor. 3]. □

### 3.2. $A^1$-equivalence of two points

To prove Theorem 1.6, we follow Voisin’s approach [Voi94]. We first introduce a Mumford type invariant $\delta Z$ (Definition 3.2).

**Notation 3.1.** Let
\[ \pi : (X, D) \to B \]
be a log smooth family of log pairs of relative dimension $n \geq 2$ with $\dim B = N$ and $D \neq \emptyset$. Assume that for every geometric fiber $X_b$, $H^0(\Omega_{X_b}^\dim X (\log D)) = 0$ for $0 < i < n$. In particular, $H^1(\mathcal{O}_{X_b}) = 0$. Assume that there are two distinct sections $p, q : B \to X - D$
and denote the relative zero cycle $Z = p(B) - q(B)$.

The cycle $Z$ defines an $(n, n)$-form on $X$:
\[ [Z] \in H^n(\Omega_X^n). \]
Note that such form is supported on an open neighborhood of $Z$, in particular, it is away from $D$. We may assume that
\[ [Z] \in H^n(\Omega_X^n (\log D) \otimes \mathcal{O}_X (-D)). \]
Let $[Z_0]$ be the image of $[Z]$ via the natural map
\[ H^n(\Omega_X^n (\log D) \otimes \mathcal{O}_X (-D)) \to H^0(B, R^n\pi_* (\Omega_X^n (\log D) \otimes \mathcal{O}_X (-D))). \]
Now by log smoothness, consider the short exact sequences
\[ 0 \to K \to \Omega_X^n (\log D) \to \Omega_{X|B}^n (\log D) \to 0. \]
After tensoring by $\mathcal{O}_X (-D)$, we get
\[ 0 \to K(-D) \to \Omega_X^n (\log D)(-D) \to \Omega_{X|B}^n (\log D)(-D) \to 0. \]
Applying $R^n\pi_*$, since $R^n\pi_* (\Omega_{X|B}^n (\log D)(-D))|_b$ is isomorphic to $H^1(\mathcal{O}_{X_b})$, which is zero, we have
\[ 0 \to R^n\pi_* K(-D) \to R^n\pi_* \Omega_X^n (\log D)(-D) \to R^n\pi_* \Omega_{X|B}^n (\log D)(-D). \]
Lemma 3.5. We have

\[ \delta Z := [Z_0] \in H^0(R^n\pi_*(K(-D))). \]

Proof. By Serre duality, we have

\[ R^n\pi_*(\Omega^n_X(\log D)(-D))_b \cong H^n(\Omega^n_X(\log D)(-D)|_{X_b}) \]
\[ \cong H^0((\Omega^n_X(\log D)(-D))^{\vee}|_{X_b} \otimes K_{X_b})^* \]
\[ \cong H^0((\Omega^n_X(\log D) \otimes K_X)|_{X_b} \otimes K_{X_b})^* \]
\[ \cong H^0(\Omega^n_X(\log D) \otimes \pi^*K_B^{-1}|_{X_b})^*, \]

and

\[ R^n\pi_*\Omega^n_{X|B}(\log D)(-D)|_b \cong H^n(\Omega^n_{X_b}(\log D)(-D)) \cong H^0(\mathcal{O}_{X_b}) \cong \mathbb{C}. \]

Hence we have

\[ R^n\pi_*K(-D)|_b \cong H^n(K(-D)|_{X_b}) \cong (H^0(\Omega^n_X(\log D) \otimes \pi^*K_B^{-1}|_{X_b})/H^0(\mathcal{O}_{X_b}))^* \]
and the class \( \delta Z|_b \) gives an element in

\[ (H^0(\Omega^n_X(\log D) \otimes \pi^*K_B^{-1}|_{X_b})/H^0(\mathcal{O}_{X_b}))^*. \]

Lemma 3.3. The invariant \( \delta Z \) is the pullback \( p^* - q^* \) given as below:

\[ H^0(\Omega^n_X(\log D) \otimes \pi^*K_B^{-1}|_{X_b}) \rightarrow H^0(\Omega^n_B(\log D) \otimes K_B^{-1}|_{X_b}) \cong \mathbb{C} \]

Proof. Since both sections \( p \) and \( q \) are away from the boundary, the proof directly follows from [Voi94, Prop 1.16]. \( \square \)

Next consider the short exact sequence

\[ 0 \rightarrow \pi^*\Omega^n_B \rightarrow K \rightarrow Q \rightarrow 0, \]

where \( Q \) admits a natural decreasing filtration whose graded pieces are \( \Omega^i_{X|B}(\log D) \otimes \pi^*\Omega^{n-i}_B \), for \( 0 < i < n \). We have

(3.1)

\[ R^{n-1}\pi_*Q \rightarrow R^n\pi_*\mathcal{O}_X(-D) \rightarrow R^n\pi_*(K(-D)) \rightarrow R^n\pi_*Q(-D). \]

Lemma 3.4. If for any \( 0 < i < n \), \( H^0(\Omega^n_{X_b}(\log D)) = 0 \), then

\[ R^n\pi_*Q(-D) = 0. \]

Proof. It suffices to verify for the graded pieces of \( Q \). For each \( 0 < i < n \), we have

\[ R^n\pi_*\Omega^i_{X|B}(\log D)(-D) \otimes \Omega^{n-i}_B|_b \cong H^n(\Omega^i_{X_b}(\log D)(-D)) \otimes \Omega^{n-i}_B|_b \]
\[ \cong H^0(\Omega^n_{X_b}(\log D)) \otimes \Omega^{n-i}_B|_b = 0. \]

\( \square \)

Lemma 3.4 and (3.1) imply

Lemma 3.5. \( \delta Z \in H^0(\Omega^n_B \otimes R^n\pi_*\mathcal{O}_X(-D)/\text{Im} \psi). \)

\( \square \)
Lemma 3.6. If $|Z|$ is $\mathbb{A}^1$-equivalent to 0, then $\delta Z = 0$ on some open subset of $B$.

Proof. Lemma 3.5 implies that locally, $\delta Z$ is an $(n,n)$-form decomposed as $\Omega_B^n \otimes R^n \pi_* \mathcal{O}_X(-D)$ and vertically compactly supported on the tubular neighborhood $U$ of $Z$. Thus $\delta Z$ is equivalent to the morphism

$$\delta Z : H^0(\pi_* \Omega^n_{X|B}(\log D)) \to H^0(\Omega^n_B)$$

$$\omega \mapsto \int \omega \wedge \delta Z$$

by taking the wedge product and integrating along the fiber of $\pi$.

For any $\omega \in H^0(\pi_* \Omega^n_{X|B}(\log D))$, we know that

$$\int_Z \omega|_Z \wedge \eta = \int_X \omega \wedge \delta Z \wedge \eta$$

for any compactly supported closed form $\eta$. By the relative version of [Zhu15, Theorem 3.3], $\omega|_Z = 0$. ([Zhu15 Theorem 3.3] is stated for surface pairs, but the proof works for any pairs.) Thus the integration above is zero. We conclude that the form $\omega \wedge \delta Z$ is exact. It is also vertically compactly supported on the tubular neighborhood $U$.

Now the Thom isomorphism tells us integration along the vertical fiber of an exact form gives us an zero element in $H^n(B)$. Therefore, $\delta Z(\omega) = 0$. □

Proof of Theorem 1.6. If the conclusion is not true, there exists a smooth variety $S'$ étale dominant over an open subset of $S$ such that

- the base change $(X, D) \times S S'$ admits two distinct sections $p$ and $q$ over $S'$;
- the relative zero cycle $Z = p(S') - q(S')$ is trivial under $\mathbb{A}^1$-equivalence.

By Lemma 3.6, $\delta Z$ vanishes on some open subset of $S'$.

On the other hand, by Corollary 2.6, $\Omega^{\dim X - \dim X_t}|_{X_t}$ is very ample if

$$d \geq n - c + n + 1 + 1 = 2n - c + 2.$$ 

This implies that $p^* - q^*$ is not zero over a general point. Thus by Lemma 3.3, $\delta Z$ is not zero. We have a contradiction. □

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(Chen) 632 Central Academic Building, University of Alberta, Edmonton, Alberta T6G 2G1, CANADA

E-mail address: xichen@math.ualberta.ca

(Zhu) Pure Mathematics, University of Waterloo, Waterloo, ON N2L3G1, Canada

E-mail address: yi.zhu@uwaterloo.ca