We investigate the azimuthal asymmetry $v_2$ of produced pions in $pp$ and $pA$ collisions at both RHIC and SPS energies. In our approach, based on the pQCD parton model and the light-cone QCD-dipole formalism, the azimuthal asymmetry results from a correlation between the color-dipole orientation and the impact parameter of the collision. We introduce the color-dipole orientation within an improved Born approximation and the saturation model which satisfies available DIS data, showing that the azimuthal asymmetry of partons and pions is very sensitive to the choice of the model, and that it is reduced in the saturation model. We find that $v_2$ of quarks and gluons in parton-nucleus collisions have very different patterns. The azimuthal asymmetry of gluons in gluon-nucleus collisions can be negative at small transverse momentum, changes the sign and becomes positive at high transverse momentum. The azimuthal asymmetry of quarks in quark-nucleus collisions is positive at all values of transverse momentum. We find that the azimuthal anisotropy $v_2$ of produced pions in both $pp$ and $pA$ collisions is positive, albeit rather small.

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I. INTRODUCTION

One of the most important discoveries in ultrarelativistic nucleus-nucleus collisions at the Relativistic Heavy Ion Collider (RHIC) was the observation of a large elliptic flow $v_2$, which is as large as the one predicted by ideal-fluid models (which assume zero viscosity) $^2$ $^3$. Microscopically, the elliptic flow results from the interactions between the produced particles, and therefore carries information about the dense matter produced at RHIC.

Various theoretical calculations $^4$ support the notion that collective flow is perhaps generated early in the nucleus-nucleus collision, and is present at the partonic level, before partons coalesce or the hadronic fragmentation stage. On the other hand, it has been shown that two-body interactions between partons cannot by themselves generate sufficient flow to explain the observations, unless partonic cross sections are artificially enhanced by more than an order of magnitude over perturbative QCD predictions $^5$, (see, however, the recent development on this line in Ref. $^6$). This indicates that the quark-gluon matter created at RHIC is strongly interacting, unlike the type of weakly interacting quark-gluon plasma expected to occur at very high temperatures on the basis of asymptotic freedom. Ultrarelativistic heavy ion collisions probe QCD in different regimes at different stages of the collisions. To be able to understand the underlying dynamics of the collisions and in particular the observed elliptic flow, one has to learn how to disentangle between the contributions of different physical subprocesses and their contributions to the observed azimuthal asymmetry.

We have proposed recently a scenario which produces an azimuthal asymmetry coming from the initial stage of relativistic nuclear collisions $^2$ $^8$. In our approach, the azimuthal asymmetry is related to the sensitivity of parton multiple interactions to the steep variation of the nuclear density at the edge of the nuclei via the color dipole orientation. This effect is also present in elementary $pp$ reactions where the correlation between the color-dipole orientation and the impact parameter of the collision leads to an azimuthal asymmetry. We have recently computed the azimuthal asymmetry of the prompt photons in nuclear collisions coming from this mechanism $^8$.

In this paper we use this idea to calculate the azimuthal asymmetry of pions produced in $pp$ and $pA$ collisions. Measuring $v_2$ of the produced particles in $pp$ and $pA$ collisions is a challenge for experimentalists, partly due to the difficulties associated with the identification of the reaction plane. Nevertheless, measurements of the azimuthal correlations from $pp$, $dAu$ and $AuAu$ collisions at RHIC indicate that the azimuthal asymmetries in $pp$ and $pA$ collisions can be quite different from those in $AA$ collisions $^8$.

Here we systematically study the azimuthal asymmetry of quarks and radiated gluons in parton-nucleus collisions, and show how they contribute to the azimuthal asymmetry of produced hadrons in $pp$ and $pA$ collisions. The basic tools of our calculations include the pQCD parton model combined with the light-cone QCD-dipole formalism. Such a study helps us to understand how the azimuthal asymmetry of the produced particles evolves from elementary $pp$ collisions to cold nuclear matter in $pA$ collisions and then to a hot quark-gluon plasma in $AA$ collisions. The azimuthal asymmetry of the produced particles in peripheral $AA$ collisions or at high $p_T$ is expected to be similar to those in $pA$ and $pp$ reactions. Therefore, this study may be also relevant for the physics of $AA$ reactions and can be used as a baseline for jet-quenching models, since in $pp$ and $pA$ collisions no hot and dense medium is created.

This paper is organized as follows: in Sec. I and II we introduce the color-dipole orientation in an improved Born approximation $^{11}$ and within the saturation model of Golec-Biernat and Wiśniewski $^{12}$. In Sec. IV, we calculate the
azimuthal asymmetry of quarks in quark-nucleus collisions. In this section, the broadening of a projectile parton in \( pA \) collisions will be introduced. This is the key ingredient of the Cronin effect and hadron production in \( pA \) collisions. In Sec. VI, we study the azimuthal asymmetry of the radiated gluons in gluon-nucleon and gluon-nucleus collisions.

In Sec. VII, we discuss hadron production in the short- and long-coherence regimes. We introduce two very different schemes for hadron production in \( pp \) and \( pA \) collisions: (1) pQCD-improved factorization and (2) the light-cone QCD-dipole factorization scheme. The numerical results and discussions are given in Sec. VII. Some concluding remarks are given in Sec. VIII. In an Appendix, we give some details of the calculations carried out in Sec V.

II. COLOR DIPOLE ORIENTATION IN BORN APPROXIMATION

A colorless \( \bar{q}q \) dipole is able to interact only due to the difference between the impact parameters of the \( q \) and \( \bar{q} \) relative to the scattering center. Therefore, a \( \bar{q}q \) fluctuation cannot be produced if both \( q \) and \( \bar{q} \) have the same impact parameter \( \vec{b} \) from the target, even if their transverse separation \( r \) is not zero. In terms of the partial elastic amplitude \( f_{\bar{q}q}(\vec{b}, \vec{r}) \), it means that the vectors \( \vec{r} \) and \( \vec{b} \) are correlated. This can be seen in a simple example of a dipole interacting with a quark in Born approximation. The partial elastic amplitude up to a factor \( \mathcal{N} \) reads,

\[
\text{Im} f_{\bar{q}q}(\vec{b}, \vec{r}) = \frac{\mathcal{N}}{(2\pi)^2} \int \frac{d^2q \, d^2q'}{(q^2 + m_g^2)(q'^2 + m_g^2)} \left[ e^{i\vec{q}(\vec{b} + \vec{r}/2)} - e^{i\vec{q}(\vec{b} - \vec{r}/2)} \right] \left[ e^{i\vec{q}'(\vec{b} + \vec{r}/2)} - e^{i\vec{q}'(\vec{b} - \vec{r}/2)} \right],
\]

where \( K_0(x) \) is the modified Bessel function and we introduced an effective gluon mass \( m_g \) to take into account some nonperturbative effects. It is obvious from the above expression that the partial elastic dipole amplitude exposes a correlation between \( \vec{r} \) and \( \vec{b} \), and the amplitude vanishes when \( \vec{b} \cdot \vec{r} = 0 \).

The Born amplitude is unrealistic, since it leads to an energy independent dipole cross section \( \sigma_{\bar{q}q} \). Therefore, a \( \bar{q}q \) fluctuation cannot be produced if both \( q \) and \( \bar{q} \) have the same impact parameter \( \vec{b} \) from the target, even if their transverse separation \( r \) is not zero. In terms of the partial elastic amplitude \( f_{\bar{q}q}(\vec{b}, \vec{r}) \), it means that the vectors \( \vec{r} \) and \( \vec{b} \) are correlated. This can be seen in a simple example of a dipole interacting with a quark in Born approximation. The partial elastic amplitude up to a factor \( \mathcal{N} \) reads,

\[
\lim_{r \to 0} \sigma_{\bar{q}q}(r, x) = 2 \int d^2\vec{b} \text{Im} f_{\bar{q}q}(\vec{b}, \vec{r}) \approx \mathcal{N} 2\pi (0.62 - \ln(m_g r)) r^2,
\]

the unknown coefficient \( \mathcal{N} \) can be then obtained by comparing the above expression with the small \( r \) expansion of the Golec-Biernat and Wüsthoff (GBW) dipole cross section \[12\]. Notice that at small \( r \) there is no logarithmic term in the GBW dipole cross section, therefore we fix the mean value of \( \vec{r} \), which depends on the process under consideration.

Thus, we obtain

\[
\mathcal{N} = \frac{\sigma_0}{2\pi (0.62 - \ln(m_g \bar{r})) R_0^2(x)},
\]

where \( \sigma_0 = 23.03 \text{ mb} \), \( R_0(x) = 0.4 \text{ fm} \times (x/x_0)^{0.144} \) with \( x_0 = 3.04 \times 10^{-4} \). Notice that the amplitude given in Eq. [11] is for the \( \bar{q}q \) dipole colliding with a quark target, not with a nucleon. Although by fixing the coefficient \( \mathcal{N} \) with the GBW cross section some of the missing effects are incorporated, still the above approximation is rather crude. In the next section, we introduce the dipole orientation without using any approximation.

III. COLOR DIPOLE ORIENTATION IN THE SATURATION MODEL

Here, we introduce the color dipole orientation within the phenomenological saturation model of GBW, which includes contributions from higher order perturbative corrections as well as non-perturbative effects contained in DIS data. The dipole elastic amplitude \( f_{\bar{q}q}^N \) of a \( \bar{q}q \) dipole colliding with a proton at impact parameter \( \vec{b} \) is given by \[7\]

\[
\text{Im} f_{\bar{q}q}^N(\vec{b}, \vec{r}, x, \beta) = \frac{1}{12\pi} \int \frac{d^2q \, d^2q'}{q^2 q'^2} \alpha_s(x_s \bar{q}) \mathcal{F}(x, \vec{q}, \vec{q'}) e^{i\vec{b}(\vec{q} - \vec{q'})} \left( e^{-i\vec{q} \cdot \vec{r} \beta} - e^{i\vec{q} \cdot \vec{r}(1-\beta)} \right) \left( e^{i\vec{q}' \cdot \vec{r} \beta} - e^{-i\vec{q}' \cdot \vec{r}(1-\beta)} \right),
\]

where we defined \( \alpha_s = \sqrt{\alpha_s(q^2)x_s(q'^2)} \), and \( \mathcal{F}(x, \vec{q}, \vec{q'}) \) is the generalized unintegrated gluon density. The fractional light-cone momenta of the quark and antiquark are denoted by \( \beta \) and \( 1 - \beta \), respectively. In the 2g model, Eq. [1],
we assumed that that $q$ and $\bar{q}$ have equal longitudinal momenta, i.e. they are equally distant from the dipole center of gravity, which corresponds to the case with a parameter $\beta = 1/2$. The generalized unintegrated gluon density was proposed in Ref. [7] assuming that the transverse momentum distributions of the two gluons do not correlate, except for the Pomeron-proton vertex, which is function of the total momentum transfer, and it is given by

$$
\mathcal{F}(x, \bar{q}, q') = \frac{3\sigma_0}{16\pi^2\alpha_s} q^2 q'^2 R_0^2(x) \exp\left[-\frac{1}{8} R_0^2(x) (q^2 + q'^2) \right] \exp\left[-R_0^2(x)(\bar{q} - q')^2/2 \right], \tag{5}
$$

This generalized unintegrated gluon density is related to the diagonal one by $\mathcal{F}(x, \bar{q}, q') = \mathcal{F}(x, q)$. We assume here that the transverse momentum dependence of the dipole-proton elastic amplitude has a Gaussian form. Comparison with the saturated form [12] of the dipole-proton cross section $\sigma_{qq}(r, x)$,

$$
\sigma_{qq}(r, x) = 2 \int d^2\vec{b} \text{Im} f_{qq}^N(\vec{b}, \vec{r}, x, \beta),
$$

$$
= \frac{4\pi}{3} \int d^2q \frac{q^2}{q^4} (1 - e^{-i\vec{q}\cdot\vec{r}}) \alpha_s(q^2) \mathcal{F}(x, q), \tag{6}
$$

fixes the form of $\mathcal{F}(x, \bar{q}, q')$ and the values of the parameters, except for $R_N^2(x)$.

Knowing the generalized unintegrated gluon density, we can now perform the integration in Eq. (4) and obtain the partial elastic dipole-proton amplitude,

$$
\text{Im} f_{qq}^N(\vec{b}, \vec{r}, x, \beta) = \frac{\sigma_0}{8\pi B(x)} \left\{ \exp \left[ \frac{-\vec{b} + \vec{r}(1 - \beta)^2}{2B(x)} \right] + \exp \left[ \frac{-\vec{b} - \vec{r}\beta}{2B(x)} \right] - 2 \exp \left[ \frac{-\vec{r}^2}{R_0^2(x)} - \frac{\vec{b} + (1/2 - \beta)\vec{r}}{2B(x)} \right] \right\}, \tag{7}
$$

(GBW model)

with the notation $B(x) = R_N^2(x) + R_D^2(x)/8$.

To fix the function $R_N^2(x)$ we use another observable, the $t$-slope $B_{el}^{qq}(x, r)$ of the elastic dipole-proton cross section (at $t = 0$) in the limit of vanishingly small dipole, $r \to 0$. In this limit,

$$
B_{el}^{qq}(x) = \frac{1}{2} \left( \frac{b^2}{2} \right)^{\frac{1}{2}} \int d^2b d^2\bar{b} \text{Im} f_{qq}^N(\vec{b}, \vec{r}, x, \beta = 1/2) = B(x) + \frac{r^2}{8(1 - e^{-r^2/R_0^2(x)})}, \tag{8}
$$

$$
B_{el}^{qq}(x, r \to 0) = B(x) + \frac{1}{8} R_0^2(x). \tag{9}
$$

In this expression we fixed $\beta = 1/2$ for the sake of simplicity. Equation (9) can be compared with the slope of the cross section of elastic electroproduction of $\rho$-mesons measured at HERA at small $x$ and high $Q^2$. It was observed that at $Q^2 \gg 1$ GeV$^2$ the slope saturates at the value $B_{\gamma^*p \to \rho p}(x, Q^2 \gg 1$ GeV$^2) \approx 5$ GeV$^{-2}$ [10], which can be compared with our result Eq. (9) in the limit $r \to 0$, since at high $Q^2$ the effective size of the dipole is vanishingly small. Therefore, we have $R_N^2(x) = B_{\gamma^*p \to \rho p}(x, Q^2 \gg 1$ GeV$^2) - \frac{1}{4} R_0^2(x)$.

Notice that the expression Eq. (8) at $r \to 0$ exposes the property of color transparency [11]. $f_{qq}^N(\vec{b}, \vec{r}, x, \beta) \propto r^2$. It also goes beyond the usual assumption that the dipole cross section is independent of the light-cone momentum sharing $\beta$. Although, the partial amplitude Eq. (8) does depend on $\beta$, this dependence disappears after integration over impact parameter $\vec{b}$ as shown in Eq. (7). From Eq. (8), it is seen that when the transverse dipole size $r$ and the impact parameter $b$ become comparable in size then the orientation becomes important. For very small $r$ or $b$, the dipole orientation is not present. The partial dipole amplitude behaviour also changes with the parameter $\beta$ [13].

One should note that the GBW model is a simple parametrization which has some restrictions. In particular, the model exhibits no power-law tails in momentum space in contradiction with QCD. Besides, it does not match the QCD evolution (DGLAP) at large values of $Q^2$. Therefore, one should be cautious applying this model at very high transverse momenta accessible at the energies of LHC.

In the above we relied on the saturation GBW model, which depends on Bjorken $x$. However, for soft reactions the c.m. energy $s$, rather than Bjorken $x$, is the proper variable. Similar to the GBW model, the $s$-dependent dipole cross section with a saturated shape fitted to data on DIS at $Q^2$ not high, and to real photo-absorption and photoproduction of vector mesons, was introduced in Ref. [14],

$$
\sigma_{qq}(r, s) = \sigma_0(s) \left[ 1 - e^{-r^2/R_0^2(s)} \right], \tag{10}
$$
where \( R_0(s) = 0.88 \text{fm} (s_0/s)^{0.14} \) with \( s_0 = 1000 \text{GeV}^2 \). The normalization factor \( \sigma_0(s) \) is fixed by demanding that the pion-proton total cross section be reproduced, that is \( \int d^2r |\Psi_p(r)|^2 \sigma_{q\bar{q}}(r, s) = \sigma_{\text{tot}}^p(s) \), where the pion wave function squared integrated over longitudinal quark momenta has the form,
\[
|\Psi_p(\vec{r})|^2 = \frac{3}{8\pi} \exp \left(-\frac{3r^2}{8\langle r_{ch}^2 \rangle}\right),
\]
with a mean pion charge radius squared \( \langle r_{ch}^2 \rangle = 0.44 \text{fm}^2 \) \[12\]. In this way, the normalization factor \( \sigma_0(s) \) is determined,
\[
\sigma_0(s) = \sigma_{\text{tot}}^p(s) \left(1 + \frac{3 R_0^2(s)}{8 \langle r_{ch}^2 \rangle}\right).
\]

We employ the parametrization of the fit in Ref. \[16\] for the Pomeron part of the cross section \( \sigma_{\text{tot}}^p(s) = 23.6(s/s_0)^{0.08} \text{mb} \), where \( s_0 = 1000 \text{GeV}^2 \).

We assume that for soft processes one can switch from \( x \) to \( s \)-dependence, keeping the same functional form of the dipole amplitude Eq. \[8\] but adjusting the parameters \( R_0^2(s) \) and \( \sigma_0(s) \) to observables in soft reactions. The first condition is that the \( s \)-dependent dipole partial amplitude reproduces the \( s \)-dependent pion-proton cross section. Another condition is the reproduction of the slope at \( t = 0 \), \( B_{el}^N(s) = \frac{1}{3}(b^2) \). These conditions will be satisfied by the following replacements:
\[
\text{Im} f_{q\bar{q}}^N(\vec{b}, \vec{r}, x, \beta) \Rightarrow \text{Im} f_{q\bar{q}}^N(\vec{b}, \vec{r}, s, \beta),
\]
\[
R_0(x) \Rightarrow R_0(s) = 0.88 \text{fm} (s_0/s)^{0.14},
\]
\[
R_N^2(x) \Rightarrow R_N^2(s) = B_{el}^N(s) - \frac{1}{4} R_0^2(s) - \frac{3}{4} \langle r_{ch}^2 \rangle,
\]
\[
\sigma_0 \Rightarrow \sigma_0(s) = \sigma_{\text{tot}}^p(s) \left(1 + \frac{3 R_0^2(s)}{8 \langle r_{ch}^2 \rangle}\right),
\]
(\text{KST model})

where we use a Regge parametrization for the elastic slope, \( B_{el}^N(s) = B_0 + 2\alpha'_N \text{In}s(\mu^2) \), with \( B_0 = 6 \text{GeV}^{-2} \), \( \alpha'_N = 0.25 \text{GeV}^{-2} \), and \( \mu^2 = 1 \text{GeV}^2 \). In what follows, we call the \( s \)-dependent dipole amplitude KST model.

### IV. \( v_2 \) of Quarks

Multiple interactions of projectile quarks in the target may proceed coherently or incoherently. In the former case the multiple interaction amplitude is a convolution of single scattering amplitudes, and in the latter case one should convolute differential cross sections, rather than amplitudes. The condition of coherence is exactly the same as in classical optics, namely the maximal longitudinal distance between different scattering centers should not considerably exceed the so called coherence length,
\[
l_c = \frac{2E_q}{p_T},
\]
(14)

where \( E_q \) is the quark energy in the nuclear rest frame and \( p_T \) is the total transverse momentum of the quark accumulated from the multiple rescatterings. Notice that at mid rapidities we have \( E_q = p_T \sqrt{s}/m_N \). Thus, one can assume a coherent regime of multiple interactions only for not very large transverse momenta, which is restricted at mid rapidities by
\[
p_T \lesssim \frac{\sqrt{s}}{m_N R_A}.
\]
(15)

At the RHIC energy, \( \sqrt{s} = 200 \text{GeV} \), only quarks with up to several GeV transverse momentum can be produced coherently.

The transverse momentum distribution of a parton after propagation through a nucleus is given by the square of the amplitude of multiple interactions. The quark impact parameters in these two amplitudes, direct and conjugated, are different. As a result, one can express the \( p_T \)-distribution in terms of the eikonalized partial elastic \( q\bar{q} \) dipole amplitude \[17\],
\[
\frac{d\sigma^t(qA \rightarrow qX)}{d^2p_T d^2\vec{b}}(b, p_T, x) = \frac{1}{(2\pi)^2} \int d^2\vec{r}_1 d^2\vec{r}_2 e^{i\vec{p}_T \cdot (\vec{r}_1 - \vec{r}_2)} \Omega_{\text{in}}(\vec{r}_1, \vec{r}_2) (1 - \text{Im} f_{q\bar{q}}^A(b, (\vec{r}_1 - \vec{r}_2), \beta)),
\]
(16)
where $\Omega_{in}^q(\vec{r}_1, \vec{r}_2)$ is the density matrix which describes the impact parameter distribution of the quark in the incident hadron,

$$\Omega_{in}^q(\vec{r}_1, \vec{r}_2) = \frac{\langle k_T^2 \rangle}{\pi} e^{-\frac{1}{2} (r_1^2 + r_2^2)\langle k_T^2 \rangle},$$

(17)

where $\langle k_T^2 \rangle$ denotes the mean value of the parton primordial transverse momentum squared. The function $\text{Im} f_{qq}^A$ in the above denotes the partial amplitude of a $q\bar{q}$ dipole colliding with a nucleus at impact parameter $\vec{b}$ and can be written, in the eikonal form, in terms of the dipole elastic amplitude $f_{qq}^N$ of a $q\bar{q}$ dipole colliding with a proton at impact parameter $\vec{b}$,

$$\text{Im} f_{qq}^A(\vec{b}, \vec{r}, \beta) = 1 - \exp[-\int d^2 \vec{s} \, \text{Im} f_{qq}^N(\vec{s}, \vec{r}, \beta) T_A(\vec{b} + \vec{s})],$$

(18)

where $T_A(\vec{b}) = \int dz \rho_A(\vec{b}, z)$ is the nuclear thickness function and $\rho_A(\vec{b}, z)$ denotes the nuclear density at a impact parameter $\vec{b}$ and a longitudinal coordinate $z$. The fractional light cone momentum $x$ of the target gluons is implicit in the above expression, and the parameter $\beta$ in the $q\bar{q}$ dipole amplitude of the saturation model is taken to be 1/2.

The integral over $\vec{r}_1$ in Eq. (16) can be readily done. In order to compute the remaining integrals, we choose a coordinate in which the angle between the impact parameter $\vec{b}$ and the dipole transverse vector $\vec{r}$. The azimuthal light cone momentum $x$ of the target gluons is implicit in the above expression, and the parameter $\beta$ in the $q\bar{q}$ dipole amplitude of the saturation model is taken to be 1/2.

The result of the convolution of the nuclear thickness with the dipole amplitude will explicitly depend on the angle $\delta$ between the impact parameter $\vec{b}$ and the dipole transverse vector $\vec{r}$. The azimuthal asymmetry resulting from a single quark passing through the nucleus is computed as a second order Fourier coefficient in a Fourier expansion of the azimuthal dependence of a single-particle spectra Eq. (16) around the beam direction,

$$v_2^q(p_T, b) = \frac{\int_{-\pi}^\pi d\phi \cos(2\phi) \frac{\frac{d^3\sigma^{qA\to qX}}{d^2p_T d^2b}}{\frac{d^3\sigma^{qA\to qX}}{d^2p_T d^2b}}}{\int_{-\pi}^\pi d\phi \frac{\frac{d^3\sigma^{qA\to qX}}{d^2p_T d^2b}}{\frac{d^3\sigma^{qA\to qX}}{d^2p_T d^2b}}}.$$  

(21)

Substituting the expression in Eq. (19) into Eq. (21), the integral over $\phi$ can then be performed analytically by using the identity given in Eq. (A5), getting

$$v_2^q(p_T, b) = -\frac{\int_0^{2\pi} d\delta \int_0^\infty rdr J_2(p_T r) \cos(2\delta) e^{-\frac{(k_T^2)}{x}r^2-I(b,r,\delta)}}{\int_0^{2\pi} d\delta \int_0^\infty rdr J_0(p_T r) e^{-\frac{(k_T^2)}{x}r^2-I(b,r,\delta)}}.$$  

(22)

where $J_n(x)$ denotes the Bessel function. In the above expression the angle dependence $\phi$ between the impact parameter $\vec{b}$ and the transverse momentum of the projectile quark $\vec{p}_T$ disappeared and the azimuthal asymmetry is directly related to the dipole orientation with respect to impact parameter $\vec{b}$ through the angle $\delta$. If one neglects the angular dependence of dipole cross section, then $v_2^q$ becomes identically zero regardless of a given nuclear profile and of dipole amplitude parametrization.

V. $v_2$ OF GLUONS

One could calculate the cross section of high-$p_T$ gluon production in the same way as was done for quarks. Namely gluons can also experience multiple coherent interactions provided that the final $p_T$ is restricted by the condition Eq. (15). One can neglect interaction with the spectator partons, like we did it for quarks in the previous section,
if the final $p_T$ of the parton is much larger than its primordial transverse momentum. Otherwise, interaction with spectators is important since color screening is at work. Such an approximation is valid for valence quarks, which have a primordial momentum of the order of $\Lambda_{QCD}$, starting from $p_T$ of several hundred MeV.

Nevertheless, the dynamics of gluon radiation is more involved. There are many experimental evidences for a much higher primordial momentum of gluons compared to quarks. Therefore, one can neglect spectators only at $p_T$ of several GeV. In order to be able to work at smaller $p_T$ one should include interaction with spectators, i.e., instead of "elastic" gluon scattering, $GN \rightarrow GX$, we need to consider bremsstrahlung subprocesses, $GN \rightarrow 2GX$, or $qN \rightarrow gGX$. The Born approximation for this processes includes three graphs (interactions with the initial and two final partons). The cross section of this reaction can be also expressed in terms of the dipole approach, and can be eikonalized on a nuclear target provided that the coherence length is sufficiently long.

The condition of coherence is derived differently from the analysis that lead to Eq. (13). In this case $l_c$ is simply the inverse longitudinal momentum transfer,

$$l_c = \frac{2E}{M^2},$$

(23)

where $E$ is the initial parton energy, and $M$ is the invariant mass of the two final partons, which reads,

$$M^2 = \frac{p_T^2}{\alpha(1-\alpha)}.$$  

(24)

Here $p_T$ is the relative transverse momentum of the final partons, $\alpha$ is the fractional light-cone momentum of one of the final partons, and parton masses are neglected. Since gluon radiation is dominated by small values of $\alpha \ll 1$, Eqs. (23) and (24) lead to the same expression for the coherence length Eq. (14), where $E_q$ should be replaced by the energy of the parton detected in the final state.

In the long coherence length (LCL) regime, the transverse momentum spectra of gluon bremsstrahlung for a high energy gluon interacting with a nucleon $N$ (nucleus $A$) target including the nonperturbative interactions of the radiated gluon reads

$$\frac{d\sigma(GN(A) \rightarrow G_1G_2X)}{d^2p_T d^2b} = \frac{1}{2(2\pi)^2} \int d^2r_1d^2r_2 e^{i\vec{r}_1 \cdot \vec{r}_2} \Psi_{GG}^*(\vec{r}_1, \alpha)\Psi_{GG}(\vec{r}_2, \alpha) \times \text{Im} \left[ f^{N(A)}_{3G}(\vec{b}, \vec{r}_1, x) + f^{N(A)}_{3G}(\vec{b}, \vec{r}_2, x) - f^{N(A)}_{3G}(\vec{b}, (\vec{r}_1 - \vec{r}_2), x) \right],$$

(25)

where $\alpha = P_+(G_1)/P_+(G)$ denotes the light-cone momentum fractional of the radiated gluon. The partial amplitude $f^{N(A)}_{3G}$ can be given in terms of the $qq\bar{q}$ dipole amplitude,

$$\text{Im} f^{N(A)}_{3G}(\vec{b}, \vec{r}, x) = \frac{9}{8} (\text{Im} f^{N}_{qq}(\vec{b}, \vec{r}, x) + \text{Im} f^{N}_{q\bar{q}}(\vec{b}, \alpha\vec{r}, x) + \text{Im} f^{N}_{q\bar{q}}(\vec{b}, (1-\alpha)\vec{r}, x)),$$

(26)

where the factor $9/8$ is the ratio of Casimir factors. Here the vectors $\vec{r}$, $\alpha\vec{r}$ and $(1-\alpha)\vec{r}$ denote the two gluon transverse separations $\vec{r}(G_1) - \vec{r}(G_2)$, $\vec{r}(G) - \vec{r}(G_2)$ and $\vec{r}(G) - \vec{r}(G_1)$, respectively. Notice that the parameter $\beta$ that is present in the dipole saturation model of Eq. (15) corresponds here to the fraction of the total 3G momentum carried by the $2G = G - G_2$ system, and is related to the light-cone momentum fractional of the radiated gluon $\alpha$ by

$$\beta = 1 - \frac{\alpha}{2}.$$  

(27)

We still have to specify the light-cone distribution function ($\Psi_{GG}$) for $GG$ Fock component fluctuations of the incoming gluon, which includes nonperturbative interactions of these gluons. This is given by

$$\Psi_{GG}(\vec{r}, \alpha) = \frac{\sqrt{8\alpha_s}}{\pi r^2} \exp \left[ -\frac{r^2}{2r_0^2} \left( \alpha\langle \vec{e}_1 \cdot \vec{e}_2^* \rangle \vec{r} + (1-\alpha)\langle \vec{e}_2^* \cdot \vec{e}_1 \rangle \vec{r} - \alpha(1-\alpha)\langle \vec{e}_1 \cdot \vec{e}_2^* \rangle (\vec{e} \cdot \vec{r}) \right) \right],$$

(28)

where $r_0 = 0.3$ fm is the parameter characterizing the strength of the nonperturbative interaction, and which has been fitted to data on diffractive $pp$ scattering. In Eq. (25) the product of the wave functions is averaged over the initial gluon polarization, $\vec{e}$, and summed over the final ones, $\vec{e}_{1,2}$.

We consider here a case relevant for high energy gluon radiation with $\alpha \rightarrow 0$. The azimuthal asymmetry of gluons $\psi_2^G$ coming from a gluon-nucleon collision can be defined in a similar way to the $\psi_2^q$ given in Eq. (21), although replacing
the particle spectra with the one in Eq. (23). After some algebra one obtains,

$$v_2^{gN}(p_T,b) = \frac{\int_0^\infty \, d\phi \, \cos(2\phi) \, \delta(\phi) \, \frac{d\sigma}{d^2p_T \, db} \, \frac{d\sigma(N \rightarrow G_i G_j X)}{d^2p_T \, db}}{\int_0^\infty \, d\phi \, \delta(\phi) \, \frac{d\sigma(N \rightarrow G_i G_j X)}{d^2p_T \, db}},$$

$$f_0 \, dr \, \int f_0 \, d\delta \, \cos(2\delta) \, \text{Im} \, f_N^G(b,\tilde{r}) \left\{ \frac{2\pi}{p_T} \left( 1 - e^{-p_T^2 r_\delta^2/2} \right) \left( J_1(p_TR) - J_3(p_TR) \right) e^{\pi r_\delta^2} + J_2(p_TR)e^{\pi r_\delta^2} f(r,\delta) \right\},$$

$$f_0 \, dr \, \int f_0 \, d\delta \, \text{Im} \, f_N^G(\tilde{b},\tilde{r}) \left\{ \frac{4\pi}{p_T} \left( 1 - e^{-p_T^2 r_\delta^2/2} \right) J_1(p_TR)e^{\pi r_\delta^2} - J_0(p_TR)e^{\pi r_\delta^2} f(r,\delta) \right\},$$

where the function $f(r,\delta)$ is defined as

$$f(r,\delta) = \int_0^\infty \, d\Delta \, \int_{-\pi}^{+\pi} \, d\theta \, \frac{(\Delta^2 - r^2) \Delta r}{(\Delta^2 + r^2)^2 - 4(\Delta r \cos(\delta - \theta))^2} e^{-\frac{\Delta^2}{4r_\delta^2}}.$$

In the case of a nuclear target Eq. (25) still holds, but the dipole amplitude on a nucleon target $f_{3G}^N$ should be replaced with the one on a nuclear target $f_{3G}^N$. The partial elastic amplitude $f_{3G}^N$, for a colorless three-gluon system colliding with a nucleus can be written in terms of the partial amplitude $f_{3G}^N$ of a three-gluon system colliding with a proton at impact parameter $b$,

$$\text{Im} \, f_{3G}^A(\tilde{b},\tilde{r},x) = 2 \left\{ 1 - \exp\left[ - \int d^2 \tilde{s} \, \text{Im} \, f_{3G}^N(\tilde{s},\tilde{r},x) T_A(\tilde{b} + \tilde{s}) \right] \right\},$$

where the 3G amplitude $f_{3G}^N$ is related to the $q\bar{q}$ dipole amplitude via Eq. (26). In a very similar fashion, one can obtain the gluons $v_2$ in a gluon-nucleus collision (see Appendix A for a derivation),

$$v_2^{gA}(p_T,b) = \frac{\int_0^\infty \, d\phi \, \cos(2\phi) \, \delta(\phi) \, \frac{d\sigma}{d^2p_T \, db} \, \frac{d\sigma(2G = G_i G_j X)}{d^2p_T \, db}}{\int_0^\infty \, d\phi \, \delta(\phi) \, \frac{d\sigma(2G = G_i G_j X)}{d^2p_T \, db}},$$

$$\int_0^\infty \, dr \, \int f_0 \, d\delta \, \text{Im} \, f_N^A(\tilde{b},\tilde{r}) \left\{ \frac{4\pi}{p_T} \left( 1 - e^{-p_T^2 r_\delta^2/2} \right) J_1(p_TR)e^{\pi r_\delta^2} - J_0(p_TR)e^{\pi r_\delta^2} f(r,\delta) \right\},$$

where we defined

$$\Psi_N(p_T,r,b,\delta) = -\frac{(2\pi)^2}{p_T} \left( 1 - e^{-p_T^2 r_\delta^2/2} \right) \left( J_1(p_TR) - J_3(p_TR) \right) e^{\pi r_\delta^2} - I_G(b,r,\delta) - 2\pi J_2(p_TR)e^{\pi r_\delta^2} - I_G(b,r,\delta) f(r,\delta),$$

$$\Psi_D(p_T,r,b,\delta) = -\frac{(2\pi)^2}{p_T} \left( 1 - e^{-p_T^2 r_\delta^2/2} \right) J_1(p_TR)e^{\pi r_\delta^2} - I_G(b,r,\delta) + 2\pi J_0(p_TR)e^{\pi r_\delta^2} - I_G(b,r,\delta) f(r,\delta)$$

$$g(p_T) = \frac{(2\pi)^2}{p_T} \left( 1 - e^{-p_T^2 r_\delta^2/2} \right)^2,$$

with the notation,

$$I_G(b,r,\delta) = \frac{9}{4} \int d^2 \tilde{s} \, \text{Im} \, f_{3G}^N(\tilde{s},\tilde{r}) T_A(\tilde{b} + \tilde{s}).$$

The remaining integrals in Eqs. (29,30,32,33) can be performed only numerically. From Eqs. (29,32) one can observe that similar to Eq. (22), the angle dependence $\phi$ between the impact parameter $\tilde{b}$ and the transverse momentum of the projectile gluon $\tilde{p}_T$ was replaced by the angle $\delta$ between $\tilde{b}$ and dipole vector $\tilde{r}$. As a consequence the azimuthal asymmetry of the radiated gluon is directly related to the orientation of the color dipole.

**VI. PIONS AZIMUTHAL ASYMMETRY IN pp AND pA COLLISIONS**

The invariant cross section for hadron production in $pp$ collisions can be described, in the pQCD-improved parton model based on factorization [21,22,23], by the expression

$$\frac{d\sigma_{pp \rightarrow h + X}}{dy \, d^2p_T} = \sum_{ijkl} \int dx \, dx_j \, d^2k_T \, d^2k_{T'} \, f_{ij/p}(x_i, Q^2) \, G_{ij/k}(Q^2) \, f_{j/p}(x_j, Q^2) \, G_{j/k}(Q^2) \, K \, \frac{d\sigma}{dt}(ij \rightarrow kl) \, \frac{D_{h/k}(z_k, Q^2)}{\pi z_k},$$

(35)
where we sum over different species of participating partons, and \( f_{ij/p}(x_i, Q^2) \) and \( f_{j/p}(x_j, Q^2) \) are the parton distribution functions (PDF) of the colliding protons, which depend on the light-cone momentum fractions \( x_i, x_j \) and the hard scale \( Q \). The function \( D_{h/k}(z_k, Q^2) \) is the fragmentation function of parton \( k \) to the final hadron \( h \) with a momentum fraction \( z_k \). The cross section \( \frac{d\sigma}{dt}(ij \rightarrow kl) \) of the hard process, which is a function of Mandelstam variables, can be calculated perturbatively.

The higher order perturbative corrections are taken into account via a \( K \)-factor, and the primordial momentum distributions \( G_p(k_T, Q^2) \), which are assumed to have the form

\[
G_p(k_T, Q^2) = \frac{\exp(-k_T^2/(\Lambda_{QCD}^2))}{\pi(k_T^2)},
\]

with the mean values \( \langle k_T^2 \rangle \), are taken to be independent of \( Q^2 \) for the sake of simplicity.

### A. Short coherence length regime

An important QCD prediction is the \( p_T \) power dependence of jet production, confirmed by data. Due to this property the multiple interactions of a large \( p_T \) produced parton do not share equally the total transferred momentum \( p_T \) (like it would be if the \( p_T \) dependence of each collision were Gaussian). In fact, there is one collision with a large transverse momentum, close to \( p_T \), while the others are mainly soft interactions with small transferred momenta. Equation \((16)\) fully includes this dynamics, although it is really valid only for coherent multiple rescatterings, when multiple interaction amplitudes, rather than cross sections, are convoluted. What happens if one decreases the energy, or increases \( p_T \), and eventually gets into a regime of short coherence length (SCL) \((14)\)? According to the specific QCD dynamics described above, only the single high-\( p_T \) collision becomes incoherent, while the other multiple soft collisions remain coherent. Thus, we arrive at a three step picture: soft multiple coherent interactions slightly increasing the transverse momentum of the parton, followed by a hard incoherent parton-nucleon collision with high \( p_T \), and eventually multiple soft final state interactions of the produced parton leading to an additional broadening. A proper tool for calculations of soft multiple broadening is the dipole approach \((17)\), which allows to use the phenomenology of soft interactions. At the same time, for the hard parton-nucleon collision we rely on the factorization based parton model, since at large \( p_T \) Bjorken \( x \) of the target is too large for the dipole technique to be valid.

Thus, the inclusive cross section of \( pA \rightarrow hX \) can be written as

\[
\frac{d\sigma_{pA→hX}}{dy d^2p_T d^2\hat{b}} = T_A(b) \sum_{ijkl} \int dx_i dx_j d^2k_T d^2k_T f_{ij/p}(x_i, Q^2) \tilde{G}_p(k_T, Q^2) \tilde{f}_{j/N}(x_j, Q^2) \tilde{G}_p(k_T, Q^2) \\
\times K \frac{d\sigma}{dt}(ij \rightarrow kl) \frac{D_{h/k}(z_k, Q^2)}{\pi z_k},
\]

\[(37)\]

The parton distribution function of a bound nucleon, \( \tilde{f}_{j/N}(x_j, Q^2) \), is known to be modified by the nuclear environment, a phenomenon called EMC effect \((24)\). Notice that shadowing effects should not be considered, since we need the PDF of a single bound nucleon. Moreover, in the SCL regime the coherence length is too short for any shadowing effects to appear. At large \( x_j \) isotopic effect may be important for the target nucleon PDF, therefore we average over the nucleus,

\[
\tilde{f}_{j/N}(x, Q^2) = \frac{Z}{A} \tilde{f}_{j/p}(x, Q^2) + \left(1 - \frac{Z}{A}\right) \tilde{f}_{j/n}(x, Q^2),
\]

\[(38)\]

with atomic and charge numbers \( A \) and \( Z \) respectively.

Initial/final state broadening of the projectile/ejectile partons is effectively taken into account via a modification of the primordial transverse momentum distribution, \( G_p(k_T) \Rightarrow \tilde{G}_p(k_T) \), where

\[
\tilde{G}_p(k_T) = \frac{dN^{iA→hX}(b)}{d^2k_T}.
\]

\[(39)\]

The \( k_T \)-distribution \( dN^{iA→hX}(b)/d^2k_T \), normalized to unity, is calculated using Eq. \((16)\) and the KST parametrization of the dipole cross section (with a Casimir factor \( 9/4 \) for gluons). Notice that in the SCL regime under consideration the process of broadening is dominated by soft coherent multiple interaction which have no relation to the hard scale \( Q^2 \) imposed by the high-\( p_T \) process occurring incoherently. Therefore in this case the density matrix Eq. \((17)\) should have a rather small mean value of the primordial quark momentum \( \langle \hat{k}_T^2 \rangle \sim \Lambda_{QCD}^2 \). To simplify the calculations we assume that the initial and final partons are the same, so the total nuclear thickness \( T_A(b) \) contributes to broadening.
B. Midrapidities at high energies

At high energies and midrapidities the parton fractional momenta in the beam and target are small, \( x_1 \sim x_2 \sim 2p_T/\sqrt{s} \ll 1 \), so hadron production is dominated by fragmentation of radiated gluons, and we can rely on the results of Section V. The cross section of hadron production in \( pp \) collisions at impact parameter \( b \) is then given by a convolution of the distribution function of the projectile gluon inside the proton with the gluon radiation cross section coming from \( GN \) collisions and also with the fragmentation function,

\[
\frac{d\sigma_{pp-h+X}}{dyd^2p_Tdb} = \int dx_g f_{G/p}(x_g, Q^2) \frac{d\sigma(Gp \to G_1G_2X)}{d^2k_T^2 db} D_{h/G_2}(z, Q^2) .
\]

To simplify the calculations we assume here that the projectile gluon has the same impact parameter relative to the target as the beam proton. At midrapidities we have \( x_g = 2k_T/\sqrt{s} \) and we take \( Q^2 = k_T^2 \). The cross section of gluon radiation in the above expression can be obtained by the master Eq. (25). This cross section reproduces well the measured pion cross section in \( pp \) collisions [19, 20]. In the LCL regime, a high \( p_T \) parton propagating through the nucleus is freed by the multiple coherent interactions, and the Cronin effect may be conceived as color-filtering. In this regime the cross section of hadron production in \( pA \) collisions has the form,

\[
\frac{d\sigma_{pA-h+X}}{dyd^2p_Tdb} = \int dx_g f_{G/p}(x_g, Q^2) \frac{d\sigma(GA \to G_1G_2X)}{d^2k_T^2 db} D_{h/G_2}(z, Q^2) ,
\]

where the cross section of gluon radiation in \( GA \) collisions can be obtained from Eqs. (26,31). In Eqs. (10,41), we will use the same PDFs and the fragmentation functions which are employed in Eq. (34).

C. Azimuthal asymmetry of produced hadrons

Notice that the asymptotic expressions (40, 41), supplemented with the gluon radiation cross-section given in Eq. (25) at \( \alpha \ll 1 \), are only reliable at very long coherence lengths, which is certainly the case at LHC energies. At RHIC energies, for hadrons produced at midrapidities with moderate \( p_T \), we are in the transition region between the regimes of long and short coherence lengths. However, for peripheral collisions where \( v_2 \) is not zero, we are in the LCL regime.

The azimuthal asymmetry of hadrons produced in \( pp \) and \( pA \) collisions at impact parameter \( b \) is computed as a second order Fourier coefficients in a Fourier expansion of the azimuthal dependence of the transverse momentum spectra of the inclusive hadron production, Eqs. (37,40,41), around the beam direction,

\[
v_2(p_T, b) = \frac{\int_{-\pi}^{\pi} d\phi \cos(2\phi) \frac{d\sigma_{pp-h+X}}{dyd^2p_Tdb}}{\int_{-\pi}^{\pi} d\phi \frac{d\sigma_{pp-h+X}}{dyd^2p_Tdb}} .
\]

In the SCL regime, the main element of the hadron production Eq. (37), which leads to an azimuthal asymmetry, is the angular dependence of the broadened projectile partons via Eq. (19). At the same time, in the LCL regime, the angle dependence of the gluon radiation cross section Eq. (25) induces via Eqs. (40,41) an azimuthal asymmetry for the produced hadrons. In both cases, the introduction of the color-dipole orientation is the key ingredient.

VII. NUMERICAL RESULTS

First, we show the \( v_2 \) of quarks and gluons in parton-nucleus collisions introduced in sections IV and V. The only external input here is the nuclear profile. We take first the Woods-Saxon (WS) profile, with a nuclear radius \( R_A = 6.5 \text{ fm} \) and a surface thickness \( \xi = 0.54 \text{ fm} \). In our approach, the profile of nuclear density at the edge is a very important input, since the elliptic asymmetry stems from the rapid change of nuclear density at the edge. In order to show this more clearly, we also show the results for the hard sphere (HS) nuclear profile with a constant density distribution, \( \rho_A = \rho_0\Theta(R_A - r) \), with the same nuclear radius as was taken for the WS profile.

In Fig. 11, we show \( v_2 \) of quarks at various impact parameters, within the saturation model I, and at the RHIC energy. A smaller primordial transverse momentum (\( k_T^2 \)) leads to a bigger broadening and more multi-scatterings, consequently the azimuthal asymmetry will be also bigger.

The main source of azimuthal asymmetry in the amplitude (18,31) is the interplay between multiple rescatering and the shape of the physical system. The key function which describes the effect of multiple interactions is the eikonal
exponential and the color dipole amplitude. The information about the shape of the system is incorporated through a convolution of the impact parameter dependent partial elastic amplitude and the nuclear thickness function. The initial space-time asymmetry gets then translated into a momentum space anisotropy by the double Fourier transform in Eqs. (10, 25). For more central collisions, the correlation between nuclear profile and dipole orientation is minimal. In fact if the nuclear thickness was constant, then the convolution between the nuclear profile and the dipole orientation would become trivial and there would be no azimuthal asymmetry. Therefore, the main source of azimuthal anisotropy is not present for central collisions where the nuclear density has only small variation. This can be seen in Figs. (1,2), where a pronounced elliptic anisotropy is observed for collisions with impact parameters close to the nuclear radius $R_A$, where the nuclear profile undergoes rapid changes. In Fig. (2), for comparison, we show the $v_2$ coming from the WS and the HS nuclear profile, within the GBW model. The $v_2$ of quark with the HS nuclear profile can be twice bigger than the one with WS nuclear profile. This indicates that the nuclear density profile is an important input.

In Fig. (3), we show the $v_2$ of quarks in $pA$ collisions within the GBW model and the improved Born approximation, the 2g model. For the 2g model, we show $v_2$ for two different effective gluon masses $m_g = 140$ and 650 MeV. There are growing evidences suggesting that the effective gluons mass should be bigger than the confinement scale. For instance, a small gluon correlation radius 0.35 fm is a result of lattice calculations [22], and it is also predicted by the instanton model [26]. Experimental data suggests an enhanced intrinsic motion of gluons in light hadrons compared to the inverse hadronic radius [19]. And also the smallness of the triple-Pomeron coupling can be only explained by a enhanced gluon interactions due to nonperturbative effects [18].

It is obvious from Fig. (2) that the azimuthal asymmetry of quarks is very sensitive to the effective gluon mass and the higher order corrections. The azimuthal asymmetry within the saturation model which includes higher order radiation corrections is significantly smaller than the one for the 2g model. Therefore, the inclusions of the higher-order radiation corrections or taking a bigger effective gluon mass reduces the azimuthal asymmetry. A quark propagating through a nucleus interacts by gluon exchange with different nucleons located at different azimuthal angles relative to the quark trajectory, and their contributions to $v_2$ tends to cancel each other, reducing the azimuthal symmetry.

In Figs. (4,5), we show the azimuthal asymmetry of gluon radiation for gluon-nucleon and gluon-nucleus collisions, given in Eqs. (20,32) at various impact parameters. The azimuthal asymmetry of gluons in both $G\nu$ and $G\nu$ has a rather similar trend; it can be negative at small $p_T$ for peripheral collisions and becomes positive at higher $p_T$. Again, for more peripheral collisions where the color dipole orientation becomes more important, the azimuthal asymmetry is bigger. In Fig. (4), on the left panel, we also show the gluonic $v_2$ in $G\nu$ collisions at a fixed impact parameter $b = 7$ fm, within various color dipole models. Notice that for the case of gluon bremsstrahlung we have already included some nonperturbative effects of gluon interactions through the $GG$ distribution function Eq. (28), via the parameter $r_0 = 0.3$ fm, which simulates the strength of the non-perturbative gluon interactions contained in the diffractive $pp$ scattering data. Therefore, the azimuthal asymmetries of gluons $v_2^G$ within the saturation dipole model and the
FIG. 2: The azimuthal anisotropy of quarks coming from quark-nucleus scatterings, for both the Woods-Saxon (WS) and hard sphere (HS) profiles, within the GBW dipole model and the improved Born approximation model, at the RHIC energy $\sqrt{s} = 200$ GeV. For the 2g model, we show the $v_2^q$ for two different effective gluon masses $m_g = 140$ and 650 MeV.

FIG. 3: The azimuthal anisotropy of gluons from gluon-nucleon collisions, given by Eq. (29), at various impact parameters, within the GBW dipole model, and with the WS nuclear profile.

improved Born approximation with effective gluon mass $m_g = 650$ MeV are rather similar.

Next, we calculate the azimuthal asymmetry of produced pions in $pp$ and $pA$ collisions, within two different schemes, the standard pQCD factorization Eqs. (35,37) (short-coherence) and light-cone factorization Eqs. (40,41) (long-coherence).

We first concentrate on the short-coherence scheme. For the parton distribution $f_{i/p}(x, Q^2)$ used in our calculations, we employ the MRST (2006 NNLO) set [27]. For the fragmentation function $D(z, Q^2)$, we use the parametrization given by Kretzer [28], with NLO corrections. In our calculation we will take $\langle k_T^2 \rangle = 3$ GeV$^2$. This value corresponds to an average transverse momentum of the parton $k_T \approx 1.5$ GeV, which coincides with the analysis of $k_T$ for photon and pion production given in Refs. [29, 30]. Notice also that different value of $\langle k_T^2 \rangle$ has been used in different approaches [31]. For the scale $Q$ of the hard process in Eqs. (35,37), we choose $Q = p_T/(1.2z_c)$. We take the $K$-factor $K \approx 2$, which gives a good approximation of the NLO order contribution in the $p_T$ region of interest. With this setup, we are able to describe pion production data for $pp$ collisions from the SPS to RHIC energies within 35% discrepancy, see Fig. (5) right panel.
FIG. 4: Right: The azimuthal anisotropy of gluons from gluon-nucleus collisions, given in Eq. (32), at various impact parameters, within the GBW dipole model. Left: $v_2^g$ at a fixed impact parameter $b = 7$ fm, within various dipole models. We used for all curves the WS nuclear profile.

FIG. 5: Right: the cross section of $p + p \rightarrow \pi^0 + X$ at RHIC for $\sqrt{s} = 200$ GeV, and that of $p + p \rightarrow \pi^+ + X$ at the SPS fixed target experiment for $E_{lab} = 300, 400$ and 800 GeV. Data for RHIC and SPS are from Ref. [32] and Refs. [33, 34], respectively. Left: The azimuthal anisotropy of the produced pions $v_2^\pi$ in $pA$ collisions, in the short-coherence regime, at energy $E_{lab} = 400$ GeV, for the Woods-Saxon (WS) nuclear profile, and within the KST dipole model.

In this way, all free phenomenological parameters for hadron production in $pA$ collisions, Eq. (37), are already fixed in reactions different from $pA$ collisions. We recall that the azimuthal asymmetry originates here from the rapid change of the nuclear density, and therefore only the peripheral tail of the nuclear profile contributes to the azimuthal asymmetry. At the very periphery, the nuclear parton distribution is unchanged compared to a free nucleon. Nevertheless, we have numerically verified that $v_2^\pi$ changes less than 20% with various PDF parametrizations for impact parameters bigger than $b > 6$ fm. In Fig. (5) we show the azimuthal asymmetry of the produced pions in $pA$ collision at midrapidity, for the fixed target energy $E_{lab} = 400$ GeV obtained in the SCL scheme, Eq. (37).

In Fig. (6), right panel, we show the azimuthal asymmetry of produced pions in $pp$ collisions at the RHIC energy $\sqrt{s} = 200$ GeV. A pronounced positive azimuthal asymmetry is observed, at an impact parameter around half of the proton size $b \sim 0.5$ fm. The source of the azimuthal asymmetry of pions in $pp$ collisions is the angle dependence of the gluon bremsstrahlung cross section in Eqs. (40), via the color-dipole orientation. The azimuthal asymmetry of the
fragmented pions in \( pp \) collisions is sizable at a \( p_T \) where the color-dipole size becomes compatible with the impact parameter and consequently the color-dipole orientation becomes important, see Eq. (8).

At the LCL limit, pions are entirely produced by gluons. It may be then puzzling that the pion’s \( v_2 \) is positive while gluons have negative \( v_2 \) in the same range of transverse momentum. Nevertheless, one should notice that the gluons transverse momentum \( k_{gT} \) is related to the transverse momentum of the fragmented pions via \( k_{gT} = p_T/z \). Therefore, high \( k_{gT} \) gluons with positive \( v_2^g \) are responsible for the produced pions at moderate \( p_T \), see Fig. (3).

In Fig. (6) left panel, we show the \( v_2^\pi \) of the produced pions in \( pAu \) collisions, obtained in the LCL scheme. The azimuthal asymmetry \( v_2^\pi \) increases from zero at an impact parameter around nuclear radius to a maximum value at an impact parameter around \( b \sim 7 \text{ fm} \), and then decreases again for a more peripheral collision.

VIII. SUMMARY AND CONCLUSIONS

This paper is the first attempt to calculate the azimuthal asymmetry \( v_2 \) of produced hadrons in \( pp \) and \( pA \) collisions. We generalized the hadron production scheme within the pQCD parton model and the light-cone QCD-dipole formalism by the inclusion of the color-dipole orientation. This is the key ingredient which leads to the azimuthal asymmetry both in \( pp \) and \( pA \) collisions. We introduced the color-dipole orientation into the improved Born approximation and within the saturation model of Golec-Biernat and Wüsthoff, which satisfies available DIS data.

In the short-coherence regime, we used the improved pQCD parton model. The salient part of hadron production in \( pA \) collisions at the short-coherence regime is the broadening of the projectile partons going through nucleus. This broadening is calculated here in a free-parameter formalism which uses the color-dipole approach, where the angle dependence of the broadened projectile partons via the color-dipole orientation leads to the azimuthal asymmetry of the produced hadron. In the long-coherence regime, we used the light-cone dipole formalism in the rest frame of the target, which is valid at small \( x_2 \). The main source of the azimuthal asymmetry in the long-coherence regime originates from the angle dependence of the radiated gluons cross section. Our results show that the azimuthal asymmetry of pions in both \( pp \) and \( pA \) collisions is rather small. This indicates that the contribution of the initial state effects, which is present in cold nuclear matter in the observed azimuthal asymmetry \( v_2 \) of the produced hadrons in \( AA \) collisions at RHIC, is very small.

We have also systematically studied the azimuthal asymmetry of partons in parton-nucleus collisions. The azimuthal asymmetry of partons in partons-nucleus collisions is very sensitive to the effective gluon mass and the higher-order radiation corrections. We found that the azimuthal asymmetry of gluons \( v_2^g \) at small \( p_T \) can be negative in \( Gp \) and \( GA \) collisions, and at higher \( p_T \) changes sign and becomes positive. This is in contrast with the \( v_2^q \) of quarks in \( qA \) collisions, where it is positive.

The technique presented in this paper can be also used to study the azimuthal asymmetry in DIS and in the production of dileptons. We plan to report on some of these problems in the near future.
FIG. 7: The angles defined in Eqs. (A3)–(A11).

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APPENDIX A

In this section we illustrate a derivation of Eq. (32). At $\alpha = 0$, up to a constant normalization $N$, the averaged product of the light-cone wave function Eq. (28) reads,

$$\Psi^*_{GG}(\vec{r}_1, \alpha)\Psi_{GG}(\vec{r}_2, \alpha) = N \exp \left[ -\frac{r_1^2 + r_2^2}{2r_0^2} \frac{\vec{r}_1 \cdot \vec{r}_2}{r_1^2 r_2^2} \right]. \quad (A1)$$

The azimuthal asymmetry is defined as,

$$v_2^{\sigma}(p_T, b) = \frac{\int_{-\pi}^{\pi} \cos(2\phi) d\phi \frac{d\sigma(GA\rightarrow GGX)}{d^2p_T d^2b}}{\int_{-\pi}^{\pi} d\phi \frac{d\sigma(GA\rightarrow GGX)}{d^2p_T d^2b}} = \frac{I_{N1} + I_{N2}}{I_{D1} + I_{D2} + I_{D3}}, \quad (A2)$$

where the gluon radiation cross section is defined in Eq. (25). The Fourier-integral is inconvenient for numerical calculation, but we can perform some of integral analytically in order to make the numerical task manageable. Let’s define

$$\mathcal{I}_G(b, r, \delta) = \frac{9}{4} \int d^2 \vec{s} \text{Im} f_N^N(\vec{s}, \vec{r}) T_A(\vec{b} + \vec{s}), \quad (A3)$$

where the factor $\frac{9}{4}$ comes from the definition Eq. (26) at $\alpha = 0$, and $\delta$ denotes the angle between the impact parameter $\vec{b}$ and the dipole vector $\vec{r}$, see Fig. 7.

We first perform the calculation for the first term in the numerator and the denominator $I_{N1}, I_{D1}$ in Eq. (A2),

$$\left( \begin{array}{c} I_{N1} \\ I_{D1} \end{array} \right) = -2 \int d\phi d^2r_1 d^2r_2 \left( \cos(2\phi) \right) e^{ip\vec{r}(\vec{r}_1 - \vec{r}_2)} e^{-\frac{r_1^2 + r_2^2}{2r_0^2} \frac{\vec{r}_1 \cdot \vec{r}_2}{r_1^2 r_2^2}} e^{-i\mathcal{I}_G(b, r_1, \delta)}$$

$$= -2 \int d\phi dr_1 dr_2 d\delta d\theta_2 \left( \cos(2\phi) \right) \cos(\delta - \theta_2) e^{ip_1 r_1 \cos(\phi - \delta) - \frac{r_1^2}{2r_0^2} - i\mathcal{I}_G(b, r_1, \delta)} e^{-ip_2 r_2 \cos(\phi - \theta_2) - \frac{r_2^2}{2r_0^2}}. \quad (A4)$$

The angles in the second Eq. (A4) are defined in Fig. 7. We can immediately perform the integral over $\theta_2$ in the above equation by using the following identity

$$e^{iA\cos(\phi)} = \sum_{n=-\infty}^{+\infty} t^n J_n(A) e^{i n \phi}, \quad (A5)$$
where $J_n(x)$ denotes the Bessel function. We have

$$
K_1 = \int d\theta_2 \cos(\delta - \theta_2) e^{-i p_T r_2 \cos(\phi - \theta_2)} = \sum_{i=-\infty}^{+\infty} i^n J_n(-p_T r_2) e^{i n \phi} \int d\theta_2 e^{-i n \theta_2} \cos(\delta - \theta_2)
$$

$$
= -2\pi i J_1(p_T r_2) \cos(\phi - \delta).
$$

Having plugged the above equation into Eq. (A4), we can then perform the integral over $\phi$,

$$
K_2 = \int d\phi \cos(\phi - \delta) \left(\cos(2\phi)\right) e^{i p_T r_1 \cos(\phi - \delta)} = \sum_{i=-\infty}^{+\infty} i^n J_n(p_T r_1) e^{-i n \delta} \int d\phi e^{i n \phi} \cos(\phi - \delta) \left(\cos(2\phi)\right)
$$

$$
= \pi i \left((J_1(p_T r_1) - J_3(p_T r_1)) \cos(2\delta)\right) / 2J_1(p_T r_1).
$$

Now we can simplify Eq. (A4) by using $K_1$ and $K_2$ given in Eqs. (A6, A7),

$$
\begin{align*}
\begin{pmatrix}
I_{N1} \\
I_{D1}
\end{pmatrix} & = -4\pi^2 \int d\delta dr_1 \left((J_1(p_T r_1) - J_3(p_T r_1)) \cos(2\delta)\right) \frac{e^{-\frac{r_1^2}{2\delta^2}}}{2J_1(p_T r_1)} \int dr_2 J_1(p_T r_2) e^{-\frac{r_2^2}{\delta^2}} \\
& = -\frac{(2\pi)^2}{p_T} \left(1 - e^{-\frac{r_1^2}{\delta^2}/2}\right) \int d\delta dr_1 \left((J_1(p_T r_1) - J_3(p_T r_1)) \cos(2\delta)\right) \frac{e^{-\frac{r_1^2}{\delta^2}}}{2J_1(p_T r_1)} e^{-I_G(b, r_1, \delta)}.
\end{align*}
$$

The remaining integrals in the above expression can be only done numerically. Now we calculate the second term in the denominator and numerator of $v_x^2$ in Eq. (A2),

$$
\begin{align*}
\begin{pmatrix}
I_{N2} \\
I_{D2}
\end{pmatrix} & = \int d^2 r_1 d^2 r_2 \left(\cos(2\phi)\right) e^{i p_T (r_1 - r_2)} e^{-\frac{r_1^2 - r_2^2}{2\delta^2}} \frac{r_1 \cdot r_2}{r_1^2 r_2^2} \exp \left[-\frac{9}{4} \int d^2 \vec{s} \text{ Im} f_{qq}^N(\vec{s}, \vec{r}_1 - \vec{r}_2) T_A(\vec{b} + \vec{s})\right].
\end{align*}
$$

We first change the variables in the above integrals by:

$$
\vec{r}_1 = \frac{\vec{A} + \vec{r}}{2}, \quad \vec{r}_2 = \frac{\vec{A} - \vec{r}}{2}.
$$

Therefore, we obtain,

$$
\begin{align*}
\begin{pmatrix}
I_{N2} \\
I_{D2}
\end{pmatrix} & = \frac{1}{4} \int d\phi d^2 \vec{r} d^2 \vec{r} \left(\cos(2\phi)\right) e^{i p_T \vec{r} \cdot \vec{r}} e^{-\frac{\vec{r}^2 - \Delta^2}{4\delta^2}} e^{-I_G(b, r, \delta)} \frac{4(\Delta^2 - r^2)}{(\Delta^2 + r^2)^2 - 4(\Delta r)^2} \frac{(\Delta^2 - r^2) \Delta r}{(\Delta^2 + r^2)^2 - 4(\Delta r \cos(\delta - \theta_2))^2}.
\end{align*}
$$

The angles in the above equation are defined in Fig. 7. Notice that the factor $\frac{1}{\Delta^2}$ in the above expression is the Jacobi determinant. The integral over $\phi$ can be immediately performed using the identity Eq. (A5),

$$
K_3 = \int d\phi \left(\cos(2\phi)\right) e^{i p_T r \cos(\phi - \delta)} = 2\pi \left(-J_2(p_T r) \cos(2\delta)\right) / J_0(p_T r).
$$

$I_{N2}, I_{D2}$ defined in Eq. (A11) can be then written in the following form

$$
\begin{align*}
\begin{pmatrix}
I_{N2} \\
I_{D2}
\end{pmatrix} & = 2\pi \int dr d\delta \left(-J_2(p_T r) \cos(2\delta)\right) \frac{e^{-\frac{r^2}{\Delta^2}}}{e^{-\frac{r^2}{\delta^2}}} e^{-I_G(b, r, \delta)} f(r, \delta).
\end{align*}
$$

where the function $f(r, \delta)$ is defined,

$$
f(r, \delta) = \int d\Delta d\theta_2 \frac{(\Delta^2 - r^2) \Delta r}{(\Delta^2 + r^2)^2 - 4(\Delta r \cos(\delta - \theta_2))^2} \frac{-\Delta^2}{e^{\frac{\Delta^2}{4\delta^2}}}.\quad (A14)
$$
The remaining multi-integrals in the above equation can be only carried out numerically. The last term in the denominator of $v_2^2$ is independent of impact parameter $b$,

$$I_{D3} = \int d\phi d^2r_1 d^2r_2 e^{i\vec{p}_T \cdot (\vec{r}_1 - \vec{r}_2)} e^{\frac{-r_1^2 - r_2^2}{2r_1^2 r_2^2}},$$

$$= 2\pi \int dr_1 dr_2 d\theta_1 d\theta_2 \cos(\theta_1) \cos(\theta_2) e^{ip_{T1} \cos(\theta_1)} e^{ip_{T2} \cos(\theta_2)} e^{\frac{r_1^2}{2r_0} e^{-ip_{T2} \cos(\theta_2)} e^{\frac{r_2^2}{2r_0}}}. \quad (A15)$$

Using the following integral,

$$\int d\theta e^{ip_{T1} \cos(\theta_1)} \cos(\theta_1) = 2\pi i J_1(p_T). \quad (A16)$$

we obtain

$$I_{D3} = (2\pi)^3 \left( \int dr J_1(p_T) e^{\frac{r^2}{2r_0}} \right)^2 = \frac{(2\pi)^3}{p_T^2} \left( 1 - e^{-r_0^2/2} \right)^2. \quad (A17)$$

To put all equations together, we can write $v_2^2$ defined in Eq. (A2) as $v_2^2 = v_2^N / v_2^D$ in a factorized form,

$$v_2^D = \frac{(2\pi)^3}{p_T^2} \left( 1 - e^{-r_0^2/2} \right)^2 - 2\frac{(2\pi)^2}{p_T} \left( 1 - e^{-r_0^2/2} \right) \int dr J_1(p_T) e^{\frac{r^2}{2r_0}} f_0(b, r) + (2\pi) \int dr J_0(p_T) e^{\frac{r^2}{2r_0}} f_1(r, b),$$

$$v_2^N = -\frac{(2\pi)^2}{p_T} \left( 1 - e^{-r_0^2/2} \right) \int dr (J_1(p_T) - J_3(p_T)) e^{\frac{r^2}{2r_0}} f_2(b, r) - (2\pi) \int dr J_2(p_T) e^{\frac{r^2}{2r_0}} f_3(b, r), \quad (A18)$$

where function $f(r, \delta)$ is defined in Eq. (A14) and $f_{0-3}(b, r)$ in the above equation are defined as follows

$$f_0(b, r) = \int d\delta e^{-\mathcal{I}_0(b, r, \delta)},$$

$$f_1(b, r) = \int d\delta e^{-\mathcal{I}_0(b, r, \delta)} f(r, \delta),$$

$$f_2(b, r) = \int d\delta \cos(2\delta) e^{-\mathcal{I}_0(b, r, \delta)},$$

$$f_3(b, r) = \int d\delta \cos(2\delta) e^{-\mathcal{I}_0(b, r, \delta)} f(r, \delta). \quad (A19)$$

In a very similar fashion one can also obtain Eq. (20).

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