Flavour Physics and CP Violation

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Abstract

After listing basic properties of the Standard Model (SM) that play the crucial role in the field of flavour and CP violation, we discuss the following topics: 1) CKM matrix and the unitarity triangle. 2) Theoretical framework in a non-technical manner, classifying various extensions of the SM. 3) Particle-Antiparticle mixing and various types of CP violation. 4) Standard analysis of the unitarity triangle. 5) Strategies for the determination of the angles $\alpha$, $\beta$, and $\gamma$ in non-leptonic $B$ decays. 6) The rare decays $K^+ \to \pi^+$ and $K_L \to \pi^0$. 7) Models with minimal flavour violation (MFV). 8) Models with new complex phases, addressing in particular possible signals of new physics in the $B \to \tau \nu$ data and their implications for rare $K$ and $B$ decays. A personal shopping list for the rest of this decade closes these lectures.

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Abstract
After listing basic properties of the Standard Model (SM) that play the crucial role in the field of flavour and CP violation, we discuss the following topics: 1) CKM matrix and the unitarity triangle. 2) Theoretical framework in a non-technical manner, classifying various extensions of the SM. 3) Particle-Antiparticle mixing and various types of CP violation. 4) Standard analysis of the unitarity triangle. 5) Strategies for the determination of the angles and in non-leptonic B decays. 6) The rare decays K+ ! and K_L ! . 7) Models with minimal flavour violation (MFV). 8) Models with new complex phases, addressing in particular possible signals of new physics in the B ! K data and their implications for rare K and B decays. A personal shopping list for the rest of this decade closes these lectures.

1 Introduction
Flavour physics and CP violation in K and B meson decays are among the central topics in particle physics. In particular particle–antiparticle mixing and CP violation in K ! decays have been of fundamental importance for the construction and testing of the Standard Model (SM). They have also proven often to be undefeatable challenges for suggested extensions of this model.

In this context a very important role is played by the Glashow-Iliopoulos-Maiani (GIM) mechanism [1] for the suppression of flavour changing neutral current (FCNC) processes that in turn proceed first at the one–loop level and are consequently sensitive to the short distance structure of the SM and its possible extensions. In particular from the calculation of the K_L ! K_S mass difference, Gaillard and Lee [2] were able to estimate the value of the charm quark mass before charm discovery. On the other hand the first measurement of the size of the B_d ! B_d mixing [3] gave the first indication of a large top quark mass.

The pattern of flavour and CP violations, both in the charged current and FCNC processes, is governed by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [4,5] that parametrizes the weak charged current interactions of quarks. In particular this matrix contains the single CP-violating phase that within the SM and its simplest extensions is supposed to describe all CP-violating processes.

One of the important questions still to be answered, is whether the CKM matrix is capable to describe with its four parameters all weak decays that include in addition to tree level decays mediated by W–bosons, a vast number of FCNC processes in which the so-called penguin and box diagrams play the central role. This sector of the SM has not yet been sufficiently tested and one should be prepared for surprises, in spite of the fact that the last three years of experimental and theoretical investigations indicate that the CKM matrix is likely to be the dominant source of flavour and CP violation. The present and future studies of CP violation in B decays, of theoretically clean rare decays K+ ! and K_L ! , and of a number of rare decays such as K_L ! 0+1+ , B ! X_sud , B ! X_sud+1+ , B ! X_sud and B_sud ! should give decisive answers already in this decade. These studies will
be complemented by the investigations of flavour violation in the lepton sector, its possible relation to
flavour violation in the quark sector, studies of $D^0$-$\bar{D}^0$ mixing and of the electric dipole moments.

It should be emphasized that all these efforts are very challenging because the relevant rare and CP-
violating decays have small branching ratios and consequently are very difficult to measure. Moreover, as
mesons are bound states of quarks and antiquarks, the determination of the CKM parameters requires in
many cases a quantitative control over QCD effects at long distances where the existing non-perturbative
methods are not yet satisfactory.

In spite of these difficulties, we strongly believe that the picture of flavour and CP violation in the
quark sector will be much clearer at the end of this decade and certainly in ten years from now with the
studies of these phenomena in the leptonic sector probably requiring more time. This belief is based on
an impressive progress in the experimental measurements in this field and on a similar progress made by
theorists in perturbative and to a lesser extend non-perturbative QCD calculations. The development of
various strategies for the determination of the CKM parameters, that are essentially free from hadronic
uncertainties, is also an important ingredient in this progress. The last account of these joined efforts by
experimentalists and theorists appeared in \[6\] and certainly other accounts of this type will follow in the
coming years.

These lecture notes provide a rather non-technical up to date description of flavour and CP vo-
lution in the SM and in its simplest extensions. In particular we will discuss the decays that are best
suited for the determination of the CKM matrix. There is unavoidably an overlap with our Les Houches
\[7\], Lake Louise \[8\], Erice \[9\], Zacatecas \[10\], Schladming \[11\] and Zakopane \[12\] lectures and with the
reviews \[13\], \[14\] and \[15\]. On the other hand new developments until the appearance of this article have
been taken into account, as far as the space allowed for it, and all numerical results have been updated.
In particular with respect to the Schladming lectures we present an extended discussion of $B \to \tau^+$,
$B \to K$ and $K \to \pi^+$ decays. Moreover, the discussion of the physics beyond the SM is significantly
extended.

The material of these lectures is organized as follows. In Section 2 we recall those facts about the
SM that are fundamental for the topics discussed here. In Section 3 we describe the CKM matrix and
the Unitarity Triangle. Section 4 summarizes briefly the general aspects of the theoretical framework
for weak decays. Here it will turn out to be useful to make a classification of various extensions of the
SM. In Section 5 the particle–antiparticle mixing and various types of CP violation are presented. In
Section 6 we describe the so–called standard analysis of the unitarity triangle, present the shape of the
2004 unitarity triangle and comment briefly on the ratio $\omega_{23}$.

In Section 7 a number of strategies for the determination of the angles $\beta$, $\gamma$ and in non-leptonic
$B$ decays are discussed in some detail. In Section 8 the rare decays $K^+ \to \pi^+$ and $K_L \to \pi^0$ are
reviewed with particular emphasis put on their important virtues with respect to a clean determination of
the parameters of the CKM matrix. In Section 9 we discuss briefly the models with minimal flavour
violation (MFV). Section 10 discusses new developments in $B \to \tau^+$ and $B \to K$ decays, with the
latter possibly indicating some new physics not only beyond the SM but more generally beyond the MFV
framework. This new physics implies spectacular effects in rare $K$ and $B$ decays. A shopping list closes
our lectures.

We hope that these lecture notes will be helpful in following the new developments in this exciting
field. In this respect the books \[16\] \[17\] \[18\] \[19\], the working group reports \[6\] \[20\] \[21\] \[22\] \[23\] and the
reviews \[24\] \[25\] \[26\] \[27\] are also strongly recommended.
2 Basic Facts about the Standard Model

In the first part of these lectures we will dominantly work in the context of the SM with three generations of quarks and leptons and the interactions described by the gauge group \( SU(3)_C \times SU(2)_L \times U(1)_Y \) spontaneously broken to \( SU(3)_C \times U(1)_{\text{EM}} \). There are many textbooks on the dynamics of the SM. At this school excellent lectures have been given by Toni Pich [28].

Here we will only collect those ingredients of the SM which are fundamental for the subject of these lectures.

The strong interactions are mediated by eight gluons \( G_a \), the electroweak interactions by \( W \) and \( Z^0 \).

Concerning Electroweak Interactions, the left-handed leptons and quarks are put into \( SU(2)_L \) doublets:

\[
\begin{align*}
\ell^L & \quad \ell^L & \quad \ell^L \\
u^L & \quad c^L & \quad t^L \\
d^L & \quad s^L & \quad b^L
\end{align*}
\]

with the corresponding right-handed fields transforming as singlets under \( SU(2)_L \). The primes in (2.2) indicate that the down quark fields \((d^0; s^0; b^0)\) placed in these doublets are weak (flavour) eigenstates. They differ from the mass eigenstates \((d; s; b)\). Without loss of generality one can set \( u = u^0; c = c^0; t = t^0 \). Similarly one can set the weak and the mass eigenstates for \((\ell^L; \ell^L; \ell^L)\) to be equal to each other. Then the flavour eigenstates \((\ell^L; \ell^L; \ell^L)\) must differ from the corresponding mass eigenstates \((1; 2; 2)\) because of the observed neutrino oscillations. The latter topic is beyond the scope of these lectures.

The charged current processes mediated by \( W \) are flavour violating with the strength of violation given by the gauge coupling \( g_2 \) and effectively at low energies by the Fermi constant (see Section 4)

\[
\frac{G_F}{2} = \frac{g_2^2}{8M_W^2}
\]  

and a unitary \( 3 \times 3 \) CKM matrix.

The CKM matrix [4,5] connects the weak eigenstates \((d^0; s^0; b^0)\) and the corresponding mass eigenstates \(d; s; b\) through

\[
0 \quad 1 \quad 0 \quad V_{ud} \quad V_{us} \quad V_{ub} \quad 1 \quad 0 \quad 1 \quad 0 \quad 1
\]

\[
\begin{bmatrix}
B & d^0 & c^0 & \bar{d} & \bar{s} & \bar{b} & V_{cK} & \bar{V}_{cK} & s & \bar{s} & \bar{c} & \bar{C}
\end{bmatrix}
\]

\[
\begin{bmatrix}
d & d & d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

In the leptonic sector the analogous mixing matrix is the MNS matrix [29], but due to the possibility of neutrinos being Majorana particles, two additional complex phases could be present.

The unitarity of the CKM matrix assures the absence of flavour changing neutral current transitions at the tree level. This means that the elementary vertices involving neutral gauge bosons \((G_a, Z^0, \gamma)\) and the neutral Higgs are flavour conserving. This property is known under the name of GIM mechanism [1].

The fact that the \( V_{ij} \)'s can a priori be complex numbers allows CP violation in the SM [5].

Feynman rules for charged current and neutral current vertices are given in Fig. [31] where \( T^a \) are colour matrices and

\[
v_\ell = T_3^f \quad 2Q_\ell \sin^2 \theta_W \quad a_\ell = T_3^f.
\]
Here $Q_f$ and $T_3^f$ denote the charge and the third component of the weak isospin of the left-handed fermion $f_L$, respectively. These electroweak charges are collected in Table 1. It should be noted that the photonic and gluonic vertices are vectorlike (V), the $W$ vertices are purely $V$ and $A$ and the $Z^0$ vertices involve both $V$ and $V + A$ structures.

An important property of the strong interactions described by Quantum Chromodynamics (QCD) is the asymptotic freedom [30]. This property implies that at short distance scales $O(1 \text{ GeV})$ the strong interaction effects in weak decays can be evaluated by means of perturbative methods with the expansion parameter $s_{\frac{M_Z}{2}} = 0.01187 \pm 0.0020 [32, 33]$. The value of $s(\mu)$ for $\mu = M_Z$ can be calculated by means of

$$s(\mu) = \frac{s(M_Z)}{v(\mu)} = 1 - \frac{1}{4} s(M_Z) \frac{\ln(v(\mu))}{v(\mu)}; \tag{2.6}$$

where

$$v(\mu) = 1 + \frac{s(M_Z)}{2} \ln \frac{M_Z}{\mu}; \tag{2.7}$$

Here

$$0 = 11 \frac{2}{3} \varepsilon; \quad 1 = 102 \frac{38}{3} \varepsilon \tag{2.8}$$

with $\varepsilon$ denoting the number of flavours.

At long distances, corresponding to $O(1 \text{ GeV})$, $s(\mu)$ becomes large and QCD effects in weak decays relevant to these scales can only be evaluated by means of non-perturbative methods. As we will see in the course of these lectures, this is the main difficulty in the description of weak decays of mesons. We will address this problem in Section 4.

3 CKM Matrix and the Unitarity Triangle

3.1 Preliminaries

Many parametrizations of the CKM matrix have been proposed in the literature. The classification of different parametrizations can be found in [34]. While the so called standard parametrization [35]

$$\hat{V}_{\text{CKM}} = \begin{pmatrix} 0 & c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{i \phi} \\ c_{12} s_{13} & c_{23} & -s_{23} s_{13} e^{i \phi} & c_{13} \\ s_{12} c_{23} & s_{23} c_{13} & s_{23} s_{13} e^{i \phi} & c_{13} \\ s_{12} s_{23} & c_{23} s_{13} e^{i \phi} & -s_{23} c_{13} & c_{13} \end{pmatrix}, \tag{3.1}$$

with $c_{ij} = \cos i j$ and $s_{ij} = \sin i j$ ($i, j = 1, 2, 3$) and the complex phase necessary for CP violation, should be recommended [12] for any numerical analysis, a generalization of the Wolfenstein parametrization [36], as presented in [37], is more suitable for these lectures. On the one hand it is more transparent than the standard parametrization and on the other hand it satisfies the unitarity of the CKM matrix to higher accuracy than the original parametrization in [36]. Let us then discuss it in some details.
3.2 Generalized Wolfenstein Parametrization

In order to find this parametrization we make the following change of variables in the standard parametrization (3.1) \[37, 38\]

\[
s_{12} = \; \; s_{23} = A^{2}; \; \; s_{13}e^{i} = A^{3}(\% \; \; i); \quad (3.2)
\]

where

\[
; \; \; A; \; \; %; \quad (3.3)
\]

are the Wolfenstein parameters with \(0 \leq 2\) being an expansion parameter. We find then

\[
V_{ud} = 1 \; \frac{1}{2} \; 2 \; \frac{1}{8}; \; \; V_{cs} = 1 \; \frac{1}{2} \; 2 \; \frac{1}{8} \; (1 + 4A^{2}); \quad (3.4)
\]

\[
V_{tb} = 1 \; \frac{1}{2} \; A^{2} \; 4; \; \; V_{cd} = \; + \; \frac{1}{2} \; A^{2} \; 5[1 \; 2(\% \; \; i)]; \quad (3.5)
\]

\[
V_{us} = \; + \; O(7); \; \; V_{ub} = A^{3}(\% \; \; i); \; \; V_{tb} = A^{2} + O(8); \quad (3.6)
\]

\[
V_{ts} = A^{2} + \frac{1}{2} \; A \; 4[1 \; 2(\% \; \; i)]; \; \; V_{td} = A^{3}(\% \; \; i); \quad (3.7)
\]

where terms \(O(6)\) and higher order terms have been neglected. A non-vanishing \(\%\) is responsible for CP violation in the SM. It plays the role of \(\%\) in the standard parametrization. Finally, the barred variables in (3.7) are given by \[37\]

\[
\% = \% (1 \; \frac{2}{2}); \quad = \; (1 \; \frac{2}{2}); \quad (3.8)
\]
The advantage of this generalization of the Wolfenstein parametrization over other generalizations found in the literature is the absence of relevant corrections to \( V_{us} \), \( V_{cb} \) and \( V_{ub} \) and an elegant change in \( V_{td} \) which allows a simple generalization of the so-called unitarity triangle beyond LO. For these reasons this generalization of the Wolfenstein parametrization has been adopted by most authors in the literature.

Finally let us collect useful approximate analytic expressions for \( i = V_{td}V_{is} \) with \( i = c \tau \tau \):

\[
\begin{align*}
\text{Im } t &= \text{Im } c = A^2 \sin \theta \; ; \\
\text{Re } c &= (1 - 2\sin^2 \theta) \; ; \\
\text{Re } t &= (1 + 2\sin^2 \theta)A^2 \sin \theta \; ; \\
\end{align*}
\]

(3.9)

(3.10)

(3.11)

Expressions (3.9) and (3.10) represent to an accuracy of 0.2% the exact formulae obtained using (3.1). The expression (3.11) deviates by at most 0.5% from the exact formula in the full range of parameters considered. After inserting the expressions (3.9)–(3.11) in the exact formulae for quantities of interest, a further expansion in \( \theta \) should not be made.

3.3 Unitarity Triangle

The unitarity of the CKM-matrix implies various relations between its elements. In particular, we have

\[
V_{ud}V_{ub} + V_{cd}V_{cb} + V_{td}V_{tb} = 0: 
\]

(3.12)

Phenomenologically this relation is very interesting as it involves simultaneously the elements \( V_{ub} \), \( V_{cb} \) and \( V_{td} \) which are under extensive discussion at present.

The relation (3.12) can be represented as a “unitarity” triangle in the complex \( (\Re; \Im) \) plane. The invariance of (3.12) under any phase-transformations implies that the corresponding triangle is rotated in the \( (\Re; \Im) \) plane under such transformations. Since the angles and the sides (given by the moduli of the elements of the mixing matrix) in this triangle remain unchanged, they are phase convention independent and are physical observables. Consequently they can be measured directly in suitable experiments. One can construct additional five unitarity triangles corresponding to other orthogonality relations, like the one in (3.12). They are discussed in [39]. Some of them should be useful when LHC-B experiment will provide data. The areas of all unitarity triangles are equal and related to the measure of CP violation \( J_{CP} \):

\[
|J_{CP}| = 2A \; ; 
\]

(3.13)

where \( A \) denotes the area of the unitarity triangle.

The construction of the unitarity triangle proceeds as follows:

We note first that

\[
V_{cd}V_{cb} = A^3 + O(\theta^7): 
\]

(3.14)

Thus to an excellent accuracy \( V_{cd}V_{cb} \) is real with \( V_{cd}V_{cb} = A^3 \).

Keeping \( O(\theta^5) \) corrections and rescaling all terms in (3.12) by \( A^3 \) we find

\[
\begin{align*}
\frac{1}{A^3}V_{ud}V_{ub} &= \% + i \; ; \\
\frac{1}{A^3}V_{td}V_{tb} &= 1 \quad (\% + i) \\
\end{align*}
\]

(3.15)

with \( \% \) and \( i \) defined in (3.8).
Thus we can represent $A=(\bar{\rho},\bar{\eta})$ as the unitarity triangle in the complex $(\% ; \%)$ plane as shown in Fig. 2.

Let us collect useful formulae related to this triangle:

- We can express the angles $\alpha, \beta, \gamma$ in terms of $(\% ; \%)$. In particular:
  
  $$\sin(2\alpha) = \frac{2(1 - \%}{(1 + \%)^2};$$  

  The lengths $CA$ and $BA$ are given respectively by

  $$R_b = \frac{\mathcal{V}_{td}V_{ub}}{\mathcal{V}_{cd}V_{cb}} = \frac{q}{\%^2 + 2} = (\frac{1}{2})^1 \frac{1}{V_{ub}};$$

  $$R_t = \frac{\mathcal{V}_{td}V_{td}}{\mathcal{V}_{cd}V_{cb}} = \frac{q}{\%^2 + 2} = \frac{1}{V_{td}};$$

  The angles and $\gamma$ of the unitarity triangle are related directly to the complex phases of the CKM-elements $V_{td}$ and $V_{ub}$, respectively, through

  $$V_{td} = \mathcal{V}_{td} e^{\frac{i}{\%}}; \quad V_{ub} = \mathcal{V}_{ub} e^{\frac{i}{\%}};$$

  The unitarity relation (3.12) can be rewritten as

  $$R_b e^{\frac{i}{\%}} + R_t e^{\frac{i}{\%}} = 1;$$

  The angle $\gamma$ can be obtained through the relation

  $$\alpha + \beta + \gamma = 180;$$

  Formula (3.20) shows transparently that the knowledge of $(R_t; \%)$ allows to determine $(R_b; \%)$ through

  $$R_b = \frac{q}{1 + R_t^2} \frac{2R_t \cos}{2R_b \cos}; \quad \cot = \frac{1}{R_b \sin};$$

  Similarly, $(R_t; \%)$ can be expressed through $(R_b; \%)$:

  $$R_t = \frac{q}{1 + R_b^2} \frac{2R_b \cos}{2R_t \cos}; \quad \cot = \frac{1}{R_b \sin};$$
These relations are remarkable. They imply that the knowledge of the coupling $V_{td}$ between $t$ and $d$ quarks allows to deduce the strength of the corresponding coupling $V_{ub}$ between $u$ and $b$ quarks and vice versa.

The triangle depicted in Fig. 2, $y_{us,j}$ and $y_{cb,j}$ give the full description of the CKM matrix. Looking at the expressions for $R_b$ and $R_t$, we observe that within the SM the measurements of four CP conserving decays sensitive to $y_{us,j}$, $y_{ub,j}$, $y_{cb,j}$ and $y_{td,j}$ can tell us whether CP violation ($\theta \neq 0$ or $\theta = 0$) is predicted in the SM. This fact is often used to determine the angles of the unitarity triangle without the study of CP-violating quantities.

### 3.4 A Different Parametrization

Parallel to the use of the Wolfenstein parameters it is sometimes useful to express the CKM elements $V_{td}$ and $V_{ts}$ as follows \[41\]

$$V_{td} = A R_t e^{i \phi}; \quad V_{ts} = \frac{q}{1 + 2 (2\% - 1)} 0.985; \quad (3.24)$$

with $\tan s = t^2$. The smallness of $s$ follows from the CKM phase conventions and the unitarity of the CKM matrix. Consequently it is valid beyond the SM if three generation unitarity is assumed.

We have then

$$V_{ts} V_{td} = \Re \, y_{cb,j} R_t e^{i \phi} \quad \text{with} \quad \Re = \frac{V_{ts}}{V_{cb}} = q \left( \begin{array}{cc} 1 + 2 (2\% - 1) & 0.985 \end{array} \right); \quad (3.25)$$

where in order to avoid high powers of $s$ we expressed the parameter $A$ through $y_{cb,j}$ Consequently

$$\Im \, t = \Re \, y_{cb,j} R_t \sin (\phi); \quad \Re \, t = \Re \, y_{cb,j} R_t \cos (\phi) \quad (3.26)$$

with $e^\phi = s$.

### 3.5 Leading Strategies for ($%; \theta$)

Next, we have the following useful relations, that correspond to the best strategies for the determination of ($%; \theta$) considered in \[42\]:

**(R $t$; $\theta$) Strategy:**

$$\% = 1 \quad R_t \cos \theta = R_t \sin \theta \quad (3.27)$$

with $R_t$ determined through (6.7) below and through the CP asymmetry $A_{CP}^{m \ell \nu}$ ($K_S$) as discussed in Section 6. In this strategy, $R_b$ and $R_t$ are given by (3.22).

**(R $b$; $\theta$) Strategy:**

$$\% = R_b \cos \theta = R_b \sin \theta \quad (3.28)$$

with $R_b$ (see Fig. 2), determined through clean strategies in tree dominated $B$-decays \[20\] \[21\] \[22\] \[23\]. In this strategy, $R_t$ and $R_b$ are given by (3.23).

**(R $t$; $\theta$) Strategy:**

Formulae in (3.27) and

$$R_t = \frac{\sin \theta \sin (\phi + \theta)}{\sin \phi} \quad (3.29)$$
with and determined through $A_{CP}^{m s, l s} (K_S)$ and clean strategies for as in (3.28). In this strategy, the length $R_b$ and $Y_{ub}$ can be determined through

$$R_b = \frac{\sin \theta_{12}}{\sin \theta_{13}}; \quad \frac{V_{ub}}{V_{cb}} = \frac{1 - 2\sin^2 \theta_{23}}{2}; \quad R_b,$$

(3.30)

( ; ) Strategy:

$$\% = \tan \phi$$

(3.31)

with determined for instance through $B \rightarrow K_L ! 0$) as discussed in Section 8 and as in the two strategies above.

As demonstrated in [42], the $(R; ; )$ strategy will be very useful as soon as the $B_s^0$ mixing mass difference $M_{s}$ has been measured. However, the remaining three strategies turn out to be more efficient in determining $(%; )$. The strategies $(; )$ and $(; )$ are theoretically cleanest as and can be measured precisely in two body $B$ decays one day and can be extracted from $B \rightarrow K_L ! 0$) subject only to uncertainty in $Y_{cb}$. Combining these two strategies offers a precise determination of the CKM matrix including $Y_{cb}$ and $Y_{ub}$. On the other hand, these two strategies are subject to uncertainties coming from new physics that can enter through $Y_{ub}$ and $Y_{cb}$. The angle , the phase of $V_{ub}$, can be determined in principle without these uncertainties.

The strategy $(R_v; )$, on the other hand, while subject to hadronic uncertainties in the determination of $R_v$, is not polluted by new physics contributions as, in addition to , also $R_b$ can be determined from tree level decays. This strategy results in the so-called reference unitarity triangle as proposed and discussed in [44]. We will return to some of these strategies in the course of our lectures.

3.6 The Special Role of $Y_{us}$, $Y_{ub}$ and $Y_{cb}$

What do we know about the CKM matrix and the unitarity triangle on the basis of tree level decays? Here the semi-leptonic $K$ and $B$ decays play the decisive role. The present situation can be summarized by [6, 15]

$$Y_{us} = 0.2240 \pm 0.0036; \quad Y_{ub} = (0.15 \pm 0.08) = 1^\circ; \quad (3.32)$$

$$\frac{Y_{ub}}{Y_{cb}} = 0.092 \pm 0.012; \quad V_{ub} = (0.81 \pm 0.46) = 1^\circ; \quad (3.33)$$

implying

$$A = 0.83 \pm 0.02; \quad R_b = 0.40 \pm 0.06; \quad (3.34)$$

There is an impressive work done by theorists and experimentalists hidden behind these numbers. We refer to [6] for details. See also [32] and the recent improved determinations of $V_{us}$ [43].

The information given above tells us only that the apex $A$ of the unitarity triangle lies in the band shown in Fig. 3. While this information appears at first sight to be rather limited, it is very important for the following reason. As $Y_{us}, Y_{ub}, Y_{cb}$ and consequently $R_b$ are determined here from tree level decays, their values given above are to an excellent accuracy independent of any new physics contributions. They are universal fundamental constants valid in any extension of the SM. Therefore their precise determinations are of utmost importance. In order to answer the question where the apex $A$ lies on the “unitarity clock” in Fig. 3 we have to look at other decays. Most promising in this respect are the so-called “loop induced” decays and transitions and CP-violating $B$ decays. These decays are
sensitive to the angles and as well as to the length $R_t$ and measuring only one of these three quantities allows to find the unitarity triangle provided the universal $R_b$ is known.

Of course any pair among $(R_t, R_b, \gamma)$ is sufficient to construct the UT without any knowledge of $R_b$. Yet the special role of $R_b$ among these variables lies in its universality, whereas the other three variables are generally sensitive functions of possible new physics contributions. This means that assuming three generation unitarity of the CKM matrix and that the SM is a part of a grander theory, the apex of the unitarity triangle has to be eventually placed on the unitarity clock with the radius $R_b$ obtained from tree level decays. That is even if using SM expressions for loop induced processes, $(\% ; \%)$ would be found outside the unitarity clock, the corresponding expressions of the grander theory must include appropriate new contributions so that the apex of the unitarity triangle is shifted back to the band in Fig. 3. In the case of CP asymmetries this could be achieved by realizing that the measured angles, and are not the true angles of the unitarity triangle but sums of the true angles and new complex phases present in extensions of the SM. The better $R_b$ is known, the thinner the band in Fig. 3 will be, selecting in this manner efficiently the correct theory. On the other hand as the branching ratios for rare and CP-violating decays depend sensitively on the parameter $A$, the precise knowledge of $V_{cb}$ is also very important.

4 Theoretical Framework

4.1 General View

The basic starting point for any serious phenomenology of weak decays of hadrons is the effective weak Hamiltonian which has the following generic structure

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i V^\dagger_{CKM} C_i(Q_i).$$

(4.1)

Here $G_F$ is the Fermi constant and $Q_i$ are the relevant local operators which govern the decays in question. The Cabibbo-Kobayashi-Maskawa factors $V^\dagger_{CKM}$ and the Wilson Coefficients $C_i$ describe the strength with which a given operator enters the Hamiltonian. We will soon give a more intuitive names to $C_i$ and $Q_i$.

In the simplest case of the $\pi^-$ decay, $H_{\text{eff}}$ takes the familiar form

$$H_{\text{eff}}^{\pi^-} = \frac{G_F}{\sqrt{2}} \cos \theta_{13} (1 \ s) d \ e (1 \ s) e;$$

(4.2)

where $V_{ud}$ has been expressed in terms of the Cabibbo angle. In this particular case the Wilson Coefficient is equal to unity and the local operator, the object between the square brackets, is given by a product
of two $V_A$ currents. This local operator is represented by the diagram (b) in Fig. 4. Equation (4.2)

![Diagram](https://via.placeholder.com/150)

represents the Fermi theory for $\beta$-decays as formulated by Sudarshan and Marshak [48] and Feynman and Gell-Mann [49] almost fifty years ago, except that in (4.2) the quark language has been used and following Cabibbo a small departure of $V_{ud}$ from unity has been incorporated. In this context the basic formula (4.1) can be regarded as a generalization of the Fermi Theory to include all known quarks and leptons as well as their strong and electroweak interactions as summarized by the SM. It should be stressed that the formulation of weak decays in terms of effective Hamiltonians is very suitable for the inclusion of new physics effects. We will discuss this issue later on.

Now, I am aware of the fact that the formal operator language used here is hated by experimentalists and frequently disliked by more phenomenologically minded theorists. Consequently the literature on weak decays, in particular on $B$-meson decays, is governed by Feynman diagram drawings with $W^\pm$, $Z^0$- and top quark exchanges, rather than by the operators in (4.1). In the case of the $\beta$-decay we have the diagram (a) in Fig. 4. Yet such Feynman diagrams with full $W^\pm$-propagators, $Z^0$-propagators and top-quark propagators really represent the situation at very short distance scales $\mathcal{O}(M_W, M_Z, m_t)$, whereas the true picture of a decaying hadron with masses $\mathcal{O}(m_b, m_c, m_K)$ is more properly described by effective point-like vertices which are represented by the local operators $Q_i$. The Wilson coefficients $C_i$ can then be regarded as coupling constants associated with these effective vertices.

Thus $H_{eff}$ in (4.1) is simply a series of effective vertices multiplied by effective coupling constants $C_i$. This series is known under the name of the operator product expansion (OPE) [46, 47, 50]. Due to the interplay of electroweak and strong interactions the structure of the local operators (vertices) is much richer than in the case of the $\beta$-decay. They can be classified with respect to the Dirac structure, colour structure and the type of quarks and leptons relevant for a given decay. Of particular interest are the operators involving quarks only. They govern the non-leptonic decays.

As an example we give a list of operators that play the role in weak $B$ decays. Typical diagrams in the full theory from which these operators originate are shown in Fig. 5. The cross in Fig. 5d indicates that magnetic penguins originate from the mass-term on the external line in the usual QCD or QED penguin diagrams. The six classes of operators are given as follows (and are colour indices):

**Current–Current (Fig. 5a):**

\begin{align}
Q_1 &= (\bar{c}_b \gamma_V)_V \quad (\bar{s}_c \gamma_V)_V \\
Q_2 &= (\bar{c}_b)_V \quad (\bar{s}_c)_V
\end{align}

(4.3)
and contributions from other heavy particles such as \( \bar{\nu}_e \)-decays the typical choice is

to use the physics contributions from scales higher than

dimensions and the heavy top quark
coupling the neutral Higgs may also contribute. Consequently

can be found by evaluating so-called

\begin{equation}
Q_3 = (s b)_{V} A \quad (q q)_{V} A \quad Q_4 = (s b)_{V} A \quad (q q)_{V} A
\end{equation}

\begin{equation}
Q_5 = (s b)_{V} A \quad (q q)_{V + A} \quad Q_6 = (s b)_{V} A \quad (q q)_{V + A}
\end{equation}

Electroweak Penguins (Fig. 5c):

\begin{equation}
Q_7 = \frac{3}{2} (s b)_{V} A \quad X \quad e_{q} (q q)_{V + A} \quad Q_8 = \frac{3}{2} (s b)_{V} A \quad X \quad e_{q} (q q)_{V + A}
\end{equation}

\begin{equation}
Q_9 = \frac{3}{2} (s b)_{V} A \quad X \quad e_{q} (q q)_{V + A} \quad Q_{10} = \frac{3}{2} (s b)_{V} A \quad X \quad e_{q} (q q)_{V + A}
\end{equation}

Magnetic Penguins (Fig. 5d):

\begin{equation}
Q_7 = \frac{e}{8} \left( \frac{1}{2} \right)_{B} F \quad Q_{8G} = \frac{g}{8} \left( \frac{1}{2} \right)_{B} S \quad (1 + \tau) T^a \quad B \quad G^a
\end{equation}

\( S = 2 \) and \( B = 2 \) Operators (Fig. 5e):

\begin{equation}
Q (S = 2) = (s d)_{V} A \quad (s d)_{V} A \quad Q (B = 2) = (b d)_{V} A \quad (b d)_{V} A
\end{equation}

Semi–Leptonic Operators (Fig. 5f):

\begin{equation}
Q_{9V} = (s b)_{V} A \quad (s b)_{V} A \quad Q_{10A} = (s b)_{A} A \quad (s b)_{A} A
\end{equation}

\begin{equation}
Q = (s b)_{V} A \quad (s b)_{V} A \quad Q = (s b)_{V} A \quad (s b)_{V} A
\end{equation}

Now what about the couplings \( C_{1}( ) \) and the scale \( \lambda v \)? The important point is that \( C_{1}( ) \) summarize the physics contributions from scales higher than \( \lambda v \) and due to asymptotic freedom of QCD they can be calculated in perturbation theory as long as \( \lambda v \) is not too small. \( C_{1} \) include the top quark contributions and contributions from other heavy particles such as \( W, Z \)-bosons, charged Higgs particles, supersymmetric particles in the supersymmetric extensions of the SM, Kaluza–Klein modes in models with extra dimensions and the heavy top quark \( T \) in the Little Higgs models. At higher orders in the electroweak coupling the neutral Higgs may also contribute. Consequently \( C_{1}( ) \) depend generally on \( m_t \) and also on the masses of new particles if extensions of the Standard Model are considered. This dependence can be found by evaluating so-called box and penguin diagrams with full \( W \)-, \( Z \)-, top- and new particles exchanges (see Fig. 5) and properly including short distance QCD effects. The latter govern the dependence of the couplings \( C_{1}( ) \).

The value of \( \lambda v \) can be chosen arbitrarily. It serves to separate the physics contributions to a given decay amplitude into short-distance contributions at scales higher than \( \lambda v \) and long-distance contributions corresponding to scales lower than \( \lambda v \). It is customary to choose \( \lambda v \) to be of the order of the mass of the decaying hadron. This is \( \mathcal{O}(m_b) \) and \( \mathcal{O}(m_c) \) for \( B \)-decays and \( D \)-decays respectively. In the case of \( K \)-decays the typical choice is \( \mathcal{O}(1 \quad 2 \quad GeV) \) instead of \( \mathcal{O}(m_K) \), which is much too low for any perturbative calculation of the couplings \( C_{1} \).

Now due to the fact that \( M_W \), \( m_t \), large logarithms \( \ln M_W \) = compensate in the evaluation of \( C_{1}( ) \) the smallness of the QCD coupling constant \( \alpha_s \) and terms \( n_s (\ln M_W = )^n \)
The scale factors reside in the matrix elements \( C_i \) of the \( C_K M \) factors and the long-distance contributions contained in \( h Q_1(\cdot) \). As indicated in (4.12) these matrix elements depend similarly to \( C_1(\cdot) \) on \( h Q_1(\cdot) \). They summarize the physics contributions to the amplitude \( A(\cdot \cdot \cdot \cdot \cdot \cdot \cdot) \) from scales lower than \( \cdot \).

We realize now the essential virtue of OPE: it allows to separate the problem of calculating the amplitude \( A(\cdot \cdot \cdot \cdot \cdot \cdot \cdot) \) into two distinct parts: the short distance (perturbative) calculation of the couplings \( C_i \) and the long-distance (generally non-perturbative) calculation of the matrix elements \( h Q_1(\cdot) \). The scale \( \cdot \), as advertised above, separates then the physics contributions into short distance contributions contained in \( C_i \) and the long distance contributions contained in \( h Q_1(\cdot) \). By evolving this scale from \( \cdot \cdot \cdot \cdot \cdot \cdot \cdot \) down to lower values one simply transforms the physics contributions at scales higher than \( \cdot \cdot \cdot \) from the hadronic matrix elements into \( C_i \). Since no information is lost this way the full amplitude cannot depend on \( \cdot \). Therefore the dependence of the couplings \( C_i(\cdot) \) has to cancel the dependence of \( h Q_1(\cdot) \). In other words it is a matter of choice what exactly belongs to \( C_i(\cdot) \) and what to \( h Q_1(\cdot) \). This cancellation of dependence involves generally several terms in the expansion in (4.12).

Clearly, in order to calculate the amplitude \( A(\cdot \cdot \cdot \cdot \cdot \cdot \cdot) \), the matrix elements \( h Q_1(\cdot) \) have to be evaluated. Since they involve long distance contributions one is forced in this case to use non-perturbative methods such as lattice calculations, the 1-N expansion (\( N \) is the number of colours), QCD sum rules, hadronic sum rules, chiral perturbation theory and so on. In the case of certain \( B \)-meson decays, the Heavy Quark Effective Theory (HQET) and recent approaches to non-leptonic decays like QCDF [51], PQCD [52] and SCET [53] also turn out to be useful tools. Needless to say, all these non-perturbative methods have some limitations. Consequently the dominant theoretical uncertainties in the decay amplitudes reside in the matrix elements \( h Q_1(\cdot) \).

The fact that in most cases the matrix elements \( h Q_1(\cdot) \) cannot be reliably calculated at present is very unfortunate. One of the main goals of the experimental studies of weak decays is the determination of the CKM factors \( V_{CKM}^i \) and the search for the physics beyond the SM. Without a reliable estimate of \( h Q_1(\cdot) \) this goal cannot be achieved unless these matrix elements can be determined experimentally or removed from the final measurable quantities by taking the ratios or suitable combinations of amplitudes or branching ratios. However, this can be achieved only in a handful of decays and generally one has to face directly the calculation of \( h Q_1(\cdot) \).

Now in the case of semi-leptonic decays, in which there is at most one hadron in the final state,
the chiral perturbation theory in the case of $K$ -decays and HQET in the case of $B$ -decays have already provided useful estimates of the relevant matrix elements. This way it was possible to achieve satisfactory determinations of the CKM elements $V_{us}$ and $V_{cb}$ in $K \rightarrow e$ and $B \rightarrow D e$ respectively. We will also see that some rare decays like $K \rightarrow \mu$ and $B \rightarrow \tau$ can be calculated very reliably.

The case of non-leptonic decays in which the final state consists exclusively of hadrons is a completely different story. Here even the matrix elements entering the simplest decays, the two-body decays like $K \rightarrow \mu$ or $B \rightarrow$ cannot be calculated in QCD satisfactorily at present. For this reason approximative schemes for these decays like QCDF, PQCD and SCET can be found in the literature. In the case of $B$ decays they use the fact that the mass $m_b$ is much larger than the typical hadronic scale. We will also see in later sections that purely phenomenological approaches supplemented by isospin symmetry, the approximate SU(3) flavour symmetry and various plausible dynamical assumptions can provide useful results.

Returning to the Wilson coefficients $C_i$ it should be stressed that similar to the effective coupling constants they do not depend only on the scale but also on the renormalization scheme used: this time on the scheme for the renormalization of local operators. That the local operators undergo renormalization is not surprising. After all they represent effective vertices and as the usual vertices in a field theory they have to be renormalized when quantum corrections like QCD or QED corrections are taken into account. As a consequence of this, the hadronic matrix elements $h_Q$ are renormalization scheme dependent and this scheme dependence must be cancelled by the one of $C_i$ so that the physical amplitudes are renormalization scheme independent. Again, as in the case of the $\mu$ -dependence, the cancellation of the renormalization scheme dependence involves generally several terms in the expansion (4.12).

Now the and the renormalization scheme dependences of the couplings $C_i$ can be evaluated efficiently in the renormalization group improved perturbation theory. Unfortunately the incorporation of these dependences in the non-perturbative evaluation of the matrix elements remains as an important challenge for non-perturbative methods but during the last years some progress has been done also here.

So far I have discussed only exclusive decays. It turns out that in the case of inclusive decays of heavy mesons, like $B$ -mesons, things turn out to be easier. In an inclusive decay one sums over all (or over a special class) of accessible final states and eventually one can show that the resulting branching ratio can be calculated in the expansion in inverse powers of $m_b$ with the leading term described by the spectator model in which the $B$ -meson decay is modelled by the decay of the $b$-quark:

$$ Br(B \rightarrow X) = Br(b \rightarrow q) + O \left( \frac{1}{m_b^2} \right) :$$

This formula is known under the name of the Heavy Quark Expansion (HQE). Since the leading term in this expansion represents the decay of the quark, it can be calculated in perturbation theory or more correctly in the renormalization group improved perturbation theory. It should be realized that also here the basic starting point is the effective Hamiltonian and that the knowledge of the couplings $C_i$ is essential for the evaluation of the leading term in (4.13). But there is an important difference relative to the exclusive case: the matrix elements of the operators $Q_i$ can be "effectively" evaluated in perturbation theory. This means, in particular, that their and renormalization scheme dependences can be evaluated and the cancellation of these dependences by those present in $C_i$ can be investigated.

Clearly in order to complete the evaluation of $Br(B \rightarrow X)$ also the remaining terms in (4.13) have to be considered. These terms are of a non-perturbative origin, but fortunately they are suppressed.
Indeed, the box diagrams have the Dirac structure $Y$ often by two powers of $m_\ell$. They have been studied by several authors in the literature with the result that they affect various branching ratios by less than 10% and often by only a few percent. Consequently the inclusive decays give generally more precise theoretical predictions at present than the exclusive decays. On the other hand their measurements are harder. There are of course some important theoretical issues related to the validity of HQE in \ref{eq:HQE1} which appear in the literature under the name of quark-hadron duality but I will not discuss them here.

We have learned now that the matrix elements of $Q_\lambda$ are easier to handle in inclusive decays than in the exclusive ones. On the other hand the evaluation of the couplings $C_\lambda(\ell)$ is equally difficult in both cases although as stated above it can be done in a perturbative framework. Still in order to achieve sufficient precision for the theoretical predictions it is desirable to have accurate values of these couplings. Indeed it has been realized at the end of the 1980’s that the leading term (LO) in the renormalization group improved perturbation theory, in which the terms $\frac{n}{\Lambda} (\ln M_W)^n$ are summed, is generally insufficient and the inclusion of next-to-leading corrections (NLO) which correspond to summing the terms $\frac{n}{\Lambda} (\ln M_W)^n \frac{1}{2}$ is necessary. In particular, unphysical left-over dependences in the decay amplitudes and branching ratios resulting from the truncation of the perturbative series are considerably reduced by including NLO corrections. These corrections are known by now for the most important and interesting decays. Reviews can be found in \cite{13,7,9,27}.

### 4.2 Penguin Box Expansion

Formula \ref{eq:Formula} can be cast into a more intuitive master formula for weak decay amplitudes in the SM \cite{57}

\[ A(\Delta \text{ decay}) = X_{\ell} \cdot B_{\Delta \text{CD}} V^{\Delta}_{\text{SM}} \cdot F_{\lambda}(\kappa_{\ell}); \quad (4.14) \]

where $\kappa_{\ell} = m_\ell^2 = \frac{2}{M_W}$ and we suppressed $G_F$. Here non-perturbative parameters $B_{\Delta \text{CD}}$ represent the matrix elements of local operators present in the SM. For instance in the case of $K^0 - K^0$ mixing, the matrix element of the operator $s (\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}) d = (\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}) d$ is represented by the parameter $B_{\Delta \text{CD}}$. There are other non-perturbative parameters in the SM that represent matrix elements of operators $Q_\lambda$ with different colour and Dirac structures. The objects $X_{\Delta \text{CD}}$ are the QCD factors resulting from RG-analysis of the corresponding operators and $F_{\lambda}(\kappa_{\ell})$ stand for the so-called Inami-Lim functions \cite{58} that result from the calculations of various box and penguin diagrams in the SM shown in Fig. 5. They depend on the top-quark mass. Analogous diagrams are present in the extensions of the SM.

In order to find the functions $F_{\lambda}(\kappa_{\ell})$, one first looks at various functions resulting from penguin diagrams: $C$ ($Z^0$ penguin), $D$ (penguin), $E$ (gluon penguin), $D^0$ (magnetic penguin) and $E^0$ (chromomagnetic penguin). Subsequently box diagrams have to be considered. Here we have the box function $S (F = 2 \text{ transitions})$, as well as $F = 1$ box functions $B$ and $B^\prime$ relevant for decays with and in the final state, respectively.

While the $F = 2$ box function $S$ and the penguin functions $E, D^0, E^0$ are gauge independent, this is not the case for $C, D$ and the $F = 1$ box diagram functions $B$ and $B^\prime$. In the phenomenological applications it is more convenient to work with gauge independent functions \cite{57}

\[ X(\kappa_{\ell}) = C(\kappa_{\ell}) + B(\kappa_{\ell}); \quad Y(\kappa_{\ell}) = C(\kappa_{\ell}) + B(\kappa_{\ell}); \quad Z(\kappa_{\ell}) = C(\kappa_{\ell}) + \frac{1}{4} D(\kappa_{\ell}); \quad (4.15) \]

Indeed, the box diagrams have the Dirac structure $(\bar{\nu} A) (\bar{\nu} A)$, the $Z$ penguin diagram has the $(\bar{\nu} A) (\bar{\nu} A)$ and $(\bar{\nu} A) (\bar{\nu} A)$ components and the penguin is pure $(\bar{\nu} A) (\bar{\nu} A)$. The $X$ and $Y$ correspond then to linear combinations of the $(\bar{\nu} A) (\bar{\nu} A)$ component of the $Z$ penguin

\[ V(\kappa_{\ell}) = C(\kappa_{\ell}) + \frac{1}{2} B(\kappa_{\ell}); \quad (4.16) \]
diagram and box diagrams with final quarks and leptons having weak isospin $T_3 = 1/2$ and $T_3 = -1/2$, respectively. $Z$ corresponds to the linear combination of the $(V \ A \ V)$ component of the $Z^0$ penguin diagram and the $W$ penguin.

Then the set of seven gauge independent functions which govern the FCNC processes in the SM models is given by

$$S(\kappa_t), X(\kappa_t), Y(\kappa_t), Z(\kappa_t), E(\kappa_t), D^0(\kappa_t), E^0(\kappa_t).$$

In the SM we have to a very good approximation:

$$S_0(\kappa_t) = 2.46 \frac{m_t}{170 \text{ GeV}}^{1.28};$$

$$X_0(\kappa_t) = 1.57 \frac{m_t}{170 \text{ GeV}}^{1.25}; Y_0(\kappa_t) = 1.02 \frac{m_t}{170 \text{ GeV}}^{1.56};$$

$$Z_0(\kappa_t) = 0.71 \frac{m_t}{170 \text{ GeV}}^{1.26}; E_0(\kappa_t) = 0.26 \frac{m_t}{170 \text{ GeV}}^{1.22};$$

$$D^0_0(\kappa_t) = 0.38 \frac{m_t}{170 \text{ GeV}}^{0.50}; E^0_0(\kappa_t) = 0.19 \frac{m_t}{170 \text{ GeV}}^{0.38}.$$ 

The subscript “0” indicates that these functions do not include QCD corrections to the relevant penguin and box diagrams. Exact expressions for all functions can be found in [7].

Generally, several master functions contribute to a given decay, although decays exist which depend only on a single function. This will be discussed in the context of models with minimal flavour violation below.
4.3 Master Formula Beyond the SM

Formula (4.14) can be generalized to a master formula for weak decay amplitudes that goes beyond the SM [59]. It reads

$$A(\text{Decay}) = \sum_i B_i^{\text{QCD}} V_i^{\text{CKM}} F_i^{\text{SM}} + F_i^{\text{New}} + \sum_k B_k^{\text{New}} \{ k^{\text{QCD}} N^{\text{New}} C_k^{\text{New}} \};$$

(4.21)

where the first terms represent the SM contributions with $F_i^{\text{SM}} = F_i^{\text{SM}}(x_t)$ given explicitly in (4.17)–(4.20).

New physics can contribute to our master formula in two ways. It can modify the importance of a given operator, present already in the SM, through the new short distance functions $F_i^{\text{New}}$ that depend on the new parameters in the extensions of the SM like the masses of charginos, squarks, charged Higgs particles and $\tan \beta = v_2/v_1$ in the MSSM. These new particles enter the new box and penguin diagrams. In more complicated extensions of the SM new operators (Dirac structures) that are either absent or very strongly suppressed in the SM, can become important. Their contributions are described by the second sum in (4.21) with $B_k^{\text{New}} \{ k^{\text{QCD}} N^{\text{New}} C_k^{\text{New}} \}$ being analogs of the corresponding objects in the first sum of the master formula. The $V_k^{\text{New}}$ show explicitly that the second sum describes generally new sources of flavour and CP violation beyond the CKM matrix. This sum may, however, also include contributions governed by the CKM matrix that are strongly suppressed in the SM but become important in some extensions of the SM. In this case $V_k^{\text{New}} = V_k^{\text{CKM}}$. Clearly the new functions $F_i^{\text{New}}$ and $G_k^{\text{New}}$ as well as the factors $V_k^{\text{New}}$ may depend on new CP violating phases complicating considerably phenomenological analyses.

4.4 Classification of New Physics

Classification of new physics (NP) contributions can be done in various ways. Having at hand the formula (4.21) let us classify these contributions from the point of view of the operator structure of the effective weak Hamiltonian, the complex phases present in the Wilson coefficients of the relevant operators and the distinction whether the flavour changing transitions are governed by the CKM matrix or by new sources of flavour violation [60]. For the first four classes below we assume that there are only three generations of quarks and leptons. The last class allows for more generations.

Class A

This is the simplest class to which also the SM belongs. In this class there are no new complex phases and flavour changing transitions are governed by the CKM matrix. Moreover, the only relevant operators are those that are relevant in the SM. Consequently NP enters only through the Wilson coefficients of the SM operators that can receive new contributions through diagrams involving new internal particles.

The models with these properties will be called Minimal Flavour Violation (MFV) models, as defined in [61]. Other definitions can be found in [62, 63]. In this case our master formula simplifies to

$$A(\text{Decay}) = \sum_i B_i^{\text{QCD}} V_i^{\text{CKM}} F_1(v); \quad F_1 = F_1^{\text{SM}} + F_1^{\text{New}} \text{(real)};$$

(4.22)

where $F_1(v)$ are the master functions of MFV models [12]

$$S(v); X(v); Y(v); Z(v); E(v); D^0(v); E^0(v)$$

(4.23)

with $v$ denoting collectively the parameters of a given MFV model. A very detailed account of the MFV can be found in [12]. In Section [9] some of its main features will be recalled. Examples of models in this
class are the Two Higgs Doublet Model II and the MSSM without new sources of flavour violation and for \( \tan \beta \) not too large. Also models with one extra universal dimension and the simplest little Higgs models are of MFV type.

We have the following correspondence between the most interesting FCNC processes and the master functions in the MFV models [12, 64]:

\[
\begin{align*}
K^0 & \rightarrow K^0 \text{-mixing (} \kappa_k \text{)} & S (\nu) \\
B_{d_{ps}}^0 & \rightarrow B_{d_{ps}}^0 \text{-mixing (} M_{s_{ps}} \text{)} & S (\nu) \\
K^+ & \rightarrow B^+ X_{d_{ps}} & X (\nu) \\
K_L^+ & \rightarrow B_{d_{ps}}^0 \ell \nu & Y (\nu) \\
K_L^0 & \rightarrow 0e^+ e^- & Y (\nu), Z (\nu), E (\nu) \\
B^0 & \rightarrow X_s & X (\nu), Y (\nu), Z (\nu), E (\nu) \\
B^0 & \rightarrow X_s \gamma \mu \nu & E^0 (\nu) \\
B^0 & \rightarrow X \mu l^+ l^- & Y (\nu), Z (\nu), E (\nu), D^0 (\nu), E^0 (\nu)
\end{align*}
\]

This table means that the observables like branching ratios, mass differences \( M_{d_{ps}} \) in \( B_{d_{ps}}^0 \rightarrow B_{d_{ps}}^0 \) mixing and the CP violation parameters \( \kappa_k \) and \( \kappa^0 \), all can be to a very good approximation expressed in terms of the corresponding master functions and the relevant CKM factors. The remaining entries in the relevant formulae for these observables are low energy parameters such as the parameters \( B_i \) that can be calculated within the SM and the QCD factors \( \Gamma_{QCD} \) describing the renormalization group evolution of operators for scales \( M_W \). These factors being universal can be calculated, similarly to \( B_i \), in the SM. The remaining, model specific QCD corrections can be absorbed in the functions \( F_i \). Further simplifications are discussed in [12].

The formulae for the processes listed above in the SM, given in terms of the master functions and CKM factors can be calculated within the SM and the QCD factors \( \Gamma_{QCD} \) being real. Typical examples of new Dirac structures are the operators \((V A)\) and \((V + A)\), \((S P)\) and \((S P)\), contributing to \( B_{d_{ps}}^0 \rightarrow B_{d_{ps}}^0 \) mixings that become relevant in the MSSM with a large \( \tan \beta \). A subset of relevant references can be found in [84, 85, 86, 87, 88, 89].

### Class B

This class of models differs from class A through the contributions of new operators not present in the SM. It is assumed, however, that no new complex phases beyond the CKM phase are present. We have then

\[
A (\text{Decay}) = X \left( B_{1} \Gamma_{QCD} V_{CKM}^{1} F_{SM}^{1} + F_{\text{New}}^{1} \right) + X \left( B_{k}^{\text{New}} \left( \Gamma_{QCD}^{k} F_{\text{New}}^{k} V_{CKM}^{k} G_{\text{New}}^{k} \right) \right)
\]

(4.24)

with all the functions \( F_{SM}^{1}, F_{\text{New}}^{1} \) and \( G_{\text{New}}^{k} \) being real. Typical examples of new Dirac structures are the operators \((V A)\) and \((V + A)\). A subset of relevant references can be found in [84, 85, 86, 87, 88, 89].

### Class C


This class of models differs from class A through the presence of new complex phases in the Wilson coefficients of the usual SM operators. Contributions of new operators can be, however, neglected. An example is the MSSM with a small $\tan\beta$ and with non-diagonal elements in the squark mass matrices. This class can be summarized by

$$A \ (\text{decay}) = \prod_i B \Gamma_{\text{QD}} V_{\text{CKM}} F_i(v); \quad F_i(v) \ (\text{complex})$$

A simple example of this class will be discussed in the context of $B \to K$ decays in Section 10.

**Class D**

Here we group models with new complex phases, new operators and new flavour changing contributions which are not governed by the CKM matrix. As now the amplitudes are given by the most general expression (4.21), the phenomenology in this class of models is more involved than in the classes A–C [90, 91]. Examples of models in class D are multi-Higgs models with complex phases in the Higgs sector, general SUSY models, models with spontaneous CP violation and left-right symmetric models.

**Class E**

Here we group models in which the unitarity of the three generation CKM matrix does not hold. Examples are four generation models and models with tree level FCNC transitions. If this type of physics is present, the unitarity triangle does not close.

We observe that the structure of weak decays beyond the SM could be very rich. However, until today, there are no fully convincing signs in the existing data for any contributions beyond the SM. Exceptions could be some hints for new physics seen in the $B \to K$ data with possible spectacular implications for rare $K$ and $B$ decays. They will be discussed in Section 10. Also the decays $B \to K, s$ and $B \to K^*$ give some signs of new physics but we should wait until the data improve.

### 5 Particle-Antiparticle Mixing and Various Types of CP Violation

#### 5.1 Preliminaries

Let us next discuss the formalism of particle–antiparticle mixing and CP violation. Much more elaborate discussion can be found in two books [17, 18]. We will concentrate here on $K^0 \bar{K}^0$ mixing, $B^0_{d,s} \to B^0_{d,s}$ mixings and CP violation in $K$ -meson and $B$ -meson decays. Due to GIM mechanism [11] the phenomena discussed in this section appear first at the one–loop level and as such they are sensitive measures of the top quark couplings $V_{ti} (i = d,s,t)$ and in particular of the phase $\delta$. They allow then to construct the unitarity triangle as explicitly demonstrated in Section 6.

![Box diagrams contributing to $K^0 \bar{K}^0$ mixing in the SM.](image)
5.2 Express Review of $K^0$ $K^0$ Mixing

$K^0 = (scl)$ and $K^0 = (scl)$ are flavour eigenstates which in the SM may mix via weak interactions through the box diagrams in Fig. [6](#). We will choose the phase conventions so that

$$\text{CP} K^0_i = K^0_i; \quad \text{CP} K^0_i = K^0_i;$$

(5.1)

In the absence of mixing the time evolution of $K^0(t)i$ is given by

$$K^0(t)i = K^0(0)i \exp(-iHt); \quad H = M + \frac{i}{2};$$

(5.2)

where $M$ is the mass and the width of $K^0$. A similar formula exists for $K^0$.

On the other hand, in the presence of flavour mixing the time evolution of the $K^0$ $K^0$ system is described by

$$\frac{d}{dt} K^0(t)i = \hat{H}(t); \quad \hat{H} = M_{ii}^\dagger - \frac{i}{2} = M_{11}^\dagger + \frac{i}{2} M_{12}^\dagger + \frac{i}{2} M_{22}^\dagger;$$

(5.3)

where $\hat{H}$ and $M$ being hermitian matrices having positive (real) eigenvalues in analogy with $M$ and $M$. $M_{1j}$ and $M_{ij}$ are the transition matrix elements from virtual and physical intermediate states respectively. Using

$$M_{11} = M_{22}; \quad M_{21} = M_{12}; \quad \text{(hermiticity)}$$

$$M_{11} = M_{21}; \quad M_{12} = M_{22}; \quad \text{(CPT)}$$

(5.5)

(5.6)

we have

$$\hat{H} = M_{11}^\dagger M_{12}^\dagger - M_{12} M_{11}^\dagger;$$

(5.7)

Diagonalizing (5.3) we find:

**Eigenstates:**

$$K_{L,S} = \frac{1}{\sqrt{2}} (1 + "iK^0 (1 - "iK^0;$$

(5.8)

where $"$ is a small complex parameter given by

$$\frac{1}{1 + "} = \frac{\sqrt{ \left| M_{12}^\dagger M_{12} \right| ^2 + \left| M_{22}^\dagger M_{22} \right| ^2 + \frac{1}{2} \frac{1}{2} M_{12}^\dagger M_{12} + \frac{1}{2} \frac{1}{2} M_{22}^\dagger M_{22}^- \exp(i\theta);$$

(5.9)

with $M_{12}$ and $M_{22}$ given below.

**Eigenvalues:**

$$M_{L,S} = M \text{ Re}Q; \quad L,S = 2\text{ Im}Q$$

(5.10)

where

$$Q = \left| M_{12} \right|^2 \frac{1}{2} \frac{1}{2} M_{12} \frac{1}{2} \frac{1}{2} M_{22} \frac{1}{2} \frac{1}{2} M_{22}^-;$$

(5.11)

Consequently we have

$$M = M_L \quad M_S = 2\text{ Re}Q; \quad L,S = 4\text{ Im}Q;$$

(5.12)
It should be noted that the mass eigenstates \( K_S \) and \( K_L \) differ from the CP eigenstates

\[
K_1 = \frac{1}{2} (K^0 + K^0) \quad \text{and} \quad \text{CP} K_1 i = K_1 i \quad (5.13)
\]

\[
K_2 = \frac{1}{2} (K^0 + K^0) \quad \text{and} \quad \text{CP} K_2 i = K_2 i \quad (5.14)
\]

by a small admixture of the other CP eigenstate:

\[
K_S = \frac{K_1 + K_2}{1 + \gamma} \quad \text{and} \quad K_L = \frac{K_2 + K_1}{1 + \gamma} \quad (5.15)
\]

Since \( \gamma \) is \( \mathcal{O}(10^{-3}) \), one has to a very good approximation:

\[
M_{K} = 2 \text{Re} M_{12} \quad \text{and} \quad \rho = 2 \text{Re} \rho_{12} \quad (5.16)
\]

where we have introduced the subscript \( K \) to stress that these formulae apply only to the \( K^0 \) \( K^0 \) system.

The \( K_L \) \( K_S \) mass difference is experimentally measured to be \( 32 \)

\[
M_K = M(K_L) \quad M(K_S) = (3 \times 90 \quad 0 \times 06) \quad 1 \text{GeV} : \quad (5.17)
\]

In the SM roughly 80% of the measured \( M_K \) is described by the real parts of the box diagrams with charm quark and top quark exchanges, whereby the contribution of the charm exchanges is by far dominant. The remaining 20% of the measured \( M_K \) is attributed to long distance contributions which are difficult to estimate \( 92 \). Further information with the relevant references can be found in \( 93 \). The situation with \( K \) is rather different. It is fully dominated by long distance effects. Experimentally one has \( M_K \).

Generally to observe CP violation one needs an interference between various amplitudes that carry complex phases. As these phases are obviously convention dependent, the CP-violating effects depend only on the differences of these phases. In particular the parameter \( \gamma \) depends on the phase convention chosen for \( K^0 \) and \( K^0 \). Therefore it may not be taken as a physical measure of CP violation. On the other hand \( \text{Re} \gamma \) and \( \rho \), defined in \( 5.9 \), are independent of phase conventions. In fact the departure of \( \rho \) from 1 measures CP violation in the \( K^0 \) \( K^0 \) mixing:

\[
\rho = 1 + \frac{2 j_{12}}{4 M_{12} j_{12} + j_{12} j_{12}} \text{Im} \frac{M_{12}}{12} 1 \text{ Im} \frac{M_{12}}{12} : \quad (5.18)
\]

This type of CP violation can be best isolated in semi-leptonic decays of the \( K_L \) meson. The non-vanishing asymmetry \( a_{\text{SL}} (K_L) \):

\[
\frac{\langle K_L \mid e^+ e \rangle}{\langle K_L \mid e^+ e \rangle} + \frac{\langle K_L \mid e^+ e \rangle}{\langle K_L \mid e^+ e \rangle} = \text{Im} \frac{M_{12}}{12} K = 2 \text{Re} \gamma \quad (5.19)
\]

signals this type of CP violation. Note that \( a_{\text{SL}} (K_L) \) is determined purely by the quantities related to \( K^0 \) \( K^0 \) mixing. Specifically, it measures the difference between the phases of \( 12 \) and \( M_{12} \).

That a non-vanishing \( a_{\text{SL}} (K_L) \) is indeed a signal of CP violation can also be understood in the following manner. \( K_L \), that should be a CP eigenstate \( K_2 \) in the case of CP conservation, decays into CP conjugate final states with different rates. As \( \text{Re} \gamma > 0 \), \( K_L \) prefers slightly to decay into \( e^+ e \) than \( e^+ e \). This would not be possible in a CP-conserving world.
5.3 The First Look at $^{*}$ and $^{0}$

Since two pion final states, $^{+}$ and $^{0}$, are CP even while the three pion final state $^{0}$ is CP odd, $K_S$ and $K_L$ decay to $^{2}$ and $^{3}$, respectively via the following CP-conserving decay modes:

$$K_L ! 3^{0} \ (via \ K_2); \quad K_S ! 2^{0} \ (via \ K_1);$$

(5.20)

Moreover, $K_L ! ^{+} 0$ is also CP conserving provided the orbital angular momentum of $^{+}$ is even. This difference between $K_L$ and $K_S$ decays is responsible for the large disparity in their lifetimes. A factor of 579. However, $K_L$ and $K_S$ are not CP eigenstates and may decay with small branching fractions as follows:

$$K_L ! 2^{0} \ (via \ K_1); \quad K_S ! 3^{0} \ (via \ K_2);$$

(5.21)

This violation of CP is called \textit{indirect} as it proceeds not via explicit breaking of the CP symmetry in the decay itself but via the admixture of the CP state with opposite CP parity to the dominant one. The measure for this indirect CP violation is defined as (I=isospin)

$$\frac{A(K_L ! ^{+} 0)}{A(K_S ! ^{0})};$$

(5.22)

Note that the decay $K_S ! ^{+} 0$ is CP violating (conserving) if the orbital angular momentum of $^{+}$ is even (odd).

Following the derivation in [94] one finds

$$n = n^* + i = \frac{\exp(i \cdot 4)}{2 \sqrt{M_K}} (\Im M_{12} + 2 \Re M_{12}) = \frac{\Im A_0}{\Re A_0};$$

(5.23)

The phase convention dependence of $n$ cancels the one of $n^*$ so that $n$ is free from this dependence. The isospin amplitude $A_0$ is defined below.

The important point in the definition (5.22) is that only the transition to $(I=0)_{I=0}$ enters. The transition to $(I=2)_{I=0}$ is absent. This allows to remove a certain type of CP violation that originates in decays only. Yet as $n \neq n^*$ and only $\Re n = \Re n^*$, it is clear that $n$ includes a type of CP violation represented by $\Im n^*$ which is absent in the semileptonic asymmetry (5.19). We will identify this type of CP violation in Section 5.7 where a more systematic classification of different types of CP violation will be given.

While \textit{indirect} CP violation reflects the fact that the mass eigenstates are not CP eigenstates, so-called \textit{direct} CP violation is realized via a direct transition of a CP odd to a CP even state: $K_2 ! ^{+} 0$. A measure of such a direct CP violation in $K_L ! ^{+} 0$ is characterized by a complex parameter $n^0$ defined as

$$n^0 = \frac{1}{2} \frac{A_{20L}}{A_{00S}} \frac{A_{20S}}{A_{00L}} A_{00L}^* A_{00S}^*$$

(5.24)

where $A_{I,L} = A(K_L ! (I)_{I})$ and $A_{I,S} = A(K_S ! (I)_{I})$.

This time the transitions to $(I=0)_{I=0}$ and $(I=2)_{I=2}$ are included which allows to study CP violation in the decay itself. We will discuss this issue in general terms in Section 5.7. It is useful to cast (5.24) into

$$n^0 = \frac{1}{2} \Im \frac{A_2}{A_0} \exp(i \cdot \omega); \quad \omega = \frac{\Im}{2} + \frac{\Re}{2} 0;$$

(5.25)

where the isospin amplitudes $A_I$ in $K ! ^{+} 0$ decays are introduced through

$$A(K ! ^{+} 0) = \frac{r}{2} A_2 e^{i \omega};$$

(5.26)
Here the subscript \( I = 0; 2 \) denotes states with isospin 0; 2 equivalent to \( I = 1=2 \) and \( I = 3=2 \) transitions, respectively, and \( \omega_2 \) are the corresponding strong phases. The weak CKM phases are contained in \( A_0 \) and \( A_2 \). The isospin amplitudes \( A_I \) are complex quantities which depend on phase conventions. On the other hand, \( \omega_0 \) measures the difference between the phases of \( A_2 \) and \( A_0 \) and is a physical quantity. The strong phases \( \omega_2 \) can be extracted from scattering. Then \( \omega_0 = 4 \). See [9] for more details.

Experimentally " and \( \omega_0 \) can be found by measuring the ratios

\[
00 = \frac{A (K_L ! 0 0)}{A (K_S ! 0 0)} = \frac{A (K_L ! + +)}{A (K_S ! + +)} ;
\]

(5.29)

Indeed, assuming " and \( \omega_0 \) to be small numbers one finds

\[
00 = " \frac{2 \omega_0}{1 + ! = 2} ; + = " + \frac{\omega_0}{1 + ! = 2} ;
\]

(5.30)

where ! = \( Re A_2 = Re A_0 = 0 \pm 45 \). In the absence of direct CP violation \( 00 = \). The ratio \( \omega_0 \) can then be measured through

\[
Re (m_0 \omega_0) = \frac{1}{6 (1 + ! = 2)} 1 + \frac{00}{2} ;
\]

(5.31)

### 5.4 Basic Formula for "

With all this information at hand one can derive a formula for " which can be efficiently used in phenomenological applications. As this derivation has been presented in detail in [9], we will be very brief here.

Calculating the box diagrams of Fig. 5 and including leading and next-to-leading QCD corrections one finds

\[
M_{12} = \frac{G_F^2}{12} \frac{p_F^2}{2} B_K m_K M \frac{h}{w} c_{1} S_0 (x_c) + \frac{t}{2} S_0 (x_t) + 2 \frac{c}{t} \frac{3}{3} S_0 (x_c; x_t) ;
\]

(5.32)

where \( F_K = 160 \text{ MeV} \) is the K-meson decay constant and \( m_K \) the K-meson mass. Next, the renormalization group invariant parameter \( B_K \) is defined by [71, 96]

\[
B_K = B_K ( ) \frac{h^{(3)}}{s^{(3)}} ( ) \frac{1}{2} \frac{9}{4} J_3 ;
\]

(5.33)

\[
(h^{(3)}) ( ) = \frac{8}{3} B_K ( ) F_K^2 m_K^2 ;
\]

(5.34)

where \( s^{(3)} \) is the strong coupling constant in an effective three flavour theory and \( J_3 = 1.895 \) in the NDR scheme [71]. The CKM factors are given by \( i = \frac{V_{18}}{V_{18}} \) and the functions \( S_0 \) by \( (x_4 = m_4^2 M_w^2) \)

\[
S_0 (x_c) = 2 \sqrt{6} \frac{m_c}{170 \text{ GeV}} ; \quad S_0 (x_t) = x_t ;
\]

(5.35)

\[
S_0 (x_c; x_t) = x_c \frac{3 x_c}{4 (1 - x_c)} - \frac{3 x_t}{4 (1 - x_c)^2} ;
\]

(5.36)
Short-distance NLO QCD effects are described through the correction factors \(1, 2, 3\) \(93, 97, 98\):

\[
1 = (1.32 \ 0.32) \ \frac{1.30 \text{ GeV}}{m_c \langle m_c \rangle} ; \quad 2 = 0.57 \ 0.01 ; \quad 3 = 0.47 \ 0.05 ; \quad (5.37)
\]

To proceed further we neglect the last term in \(5.23\) as it constitutes at most a 5% correction to "\(\)". A recent discussion can be found in \(99\). This is justified in view of other uncertainties, in particular those connected with \(\mathcal{B}\). Inserting \(5.32\) into \(5.23\) we find

\[
\" = C^* \hat{B}_K \ \text{Im} \ \text{Fr} \ e^{c_i \left[ \begin{array}{cc} 1S_0 (\xi_c) & 3S_0 (\xi_c; \xi_t) \end{array} \right]} \ \text{Re} \ e^{t_2 (\xi_c) g e^{-i \theta}} ; \quad (5.38)
\]

where the numerical constant \(C^*\) is given by

\[
C^* = \frac{G_F^2 \frac{\mu}{\pi} \frac{\mu}{\pi}}{2 \ 2 \ 2} M_K^2 = 3.837 \ \text{f0} ; \quad (5.39)
\]

Comparing \(5.38\) with the experimental value for "\(\)\(32\)

\[
\"_{\text{exp}} = (2.280 \ 0.013) \ 1^0 \ \text{exp} i \ \theta ; \quad \theta = \frac{\pi}{4} ; \quad (5.40)
\]

one obtains a constraint on the unitarity triangle in Fig. 2. See Section 6.

### 5.5 Express Review of \(B^0_{d\bar{s}} - B^0_{d\bar{s}}\) Mixing

The flavour eigenstates in this case are

\[
B^0_d = (\phi d) ; \quad B^0_s = (\phi s) ; \quad B^0_s = (\phi s) ; \quad B^0_s = (\phi s) ; \quad (5.41)
\]

They mix via the box diagrams in Fig. 6 with \(s\) replaced by \(b\) in the case of \(B^0_{d\bar{s}} - B^0_{d\bar{s}}\) mixing. In the case of \(B^0_{d\bar{s}} - B^0_{d\bar{s}}\) mixing also \(d\) has to be replaced by \(s\).

Dropping the subscripts \(d; s\) for a moment, it is customary to denote the mass eigenstates by

\[
B_H = pB^0 + qB^0 ; \quad B_L = pB^0 - qB^0 ; \quad (5.42)
\]

\[
p = \frac{1 \ + \ \"}{2 (1 + \ \"^2) ^2} ; \quad q = \frac{1 \ - \ \"}{2 (1 + \ \"^2) ^2} ; \quad (5.43)
\]

with \(\"\) corresponding to " in the \(K^0 - K^0\) system. Here "H" and "L" denote Heavy and Light respectively. As in the \(B^0 - B^0\) system one has \(M\), it is more suitable to distinguish the mass eigenstates by their masses than by the corresponding life-times.

The strength of the \(B^0_{d\bar{s}} - B^0_{d\bar{s}}\) mixings is described by the mass differences

\[
M_{d\bar{s}} = M_H - M_L - (5.44)
\]

In contrast to \(M_K\), in this case the long distance contributions are estimated to be very small and \(M_{d\bar{s}}\) is very well approximated by the relevant box diagrams. Moreover, due to \(m_{u,c} \approx m_t\) only the top sector is relevant.

\(M_{d\bar{s}}\) can be expressed in terms of the off-diagonal element in the neutral \(B\) -meson mass matrix by using the formulae developed previously for the \(K\) -meson system. One finds \(96\)

\[
M_q = 2 M_{12} ; \quad q = 2 \text{Re} \left( \frac{M_{12}}{M_{12}^2} \right) \quad M_q i \quad q = d; s ; \quad (5.45)
\]
These formulae differ from (5.16) because in the B-system $\pi_\text{B}$ cannot be neglected and $M_{12}$. We also have

$$\frac{q}{p} = \frac{M_{12}}{M_{12}} \left( \frac{\hat{d}_{12}}{\hat{d}_{12}} \right) = 2M_{12} \frac{1}{M_{12}} \left( \frac{i_{12}}{i_{12}} \right) = M_{12} \left( \frac{1}{M_{12}} \right) \frac{1}{M_{12}} \left( \frac{1}{M_{12}} \right) = \frac{1}{2} \Im \frac{M_{12}}{M_{12}}$$

(5.46)

where higher order terms in the small quantity $M_{12}$ have been neglected.

As $\Im (12 M_{12}) < O(10^{-3})$,

The semileptonic asymmetry $\bar{B}(\pi_B)$ discussed a few pages below is even smaller than $a_{SL}(K_L)$. Typically $O(10^{-4})$. These are bad news.

The ratio $q/p$ is a pure phase to an excellent approximation. These are very good news as we will see below.

Calculating the relevant box diagrams we find

$$\Phi_{12}^q = \frac{G_F^2}{12} F_{B_q}^2 \beta_{B_q} m_{B_q} M_{12}^2 (V_{tq} V_{tb})^2 S_0 (x_t) \beta_B$$

(5.47)

where $F_{B_q}$ is the $B_q$-meson decay constant, $\beta_{B_q}$ renormalization group invariant parameters defined in analogy to (5.33) and (5.34) and $\beta_B$ stands for short distance QCD corrections [71, 100].

Consequently,

$$\Phi_{12}^d / (V_{td} V_{tb})^2 ; \Phi_{12}^s / (V_{ts} V_{tb})^2 :$$

(5.49)

Now, from Section 3 we know that

$$V_{td} = \beta_{td}^1 ; V_{ts} = \beta_{ts}^1 :$$

(5.50)

with $s = O(10^{-2})$. Consequently to an excellent approximation

$$\frac{q}{p} = e^{i2 d_{ps} M} ; d = ; s = s :$$

(5.51)

with $d_{ps} M$ given entirely by the weak phases in the CKM matrix.

### 5.6 Basic Formulae for $M_{d_{ps}}$

From (5.45) and (5.47) we have with $V_{tb} = 1$

$$M_q = \frac{G_F^2}{6} \beta_{B_q} \beta_{B_q} F_{B_q}^2 m_{B_q} M_{12}^2 (V_{tq} V_{tb})^2 S_0 (x_t) \beta_B$$

(5.52)

Using (5.35) and (5.52) we obtain two useful formulae

$$M_d = 0.55 \pm 0.01 :$$

(5.53)

and

$$M_s = 172 \pm 0.040 :$$

(5.54)
5.7 Classification of CP Violation

5.7.1 Preliminaries

We have mentioned in Section 4 that due to the presence of hadronic matrix elements, various decay amplitudes contain large theoretical uncertainties. It is of interest to investigate which measurements of CP-violating effects do not suffer from hadronic uncertainties. To this end it is useful to make a classification of CP-violating effects that is more transparent than the division into the indirect and direct CP violation considered so far. A nice detailed presentation has been given by Nir [25].

Generally complex phases may enter particle–antiparticle mixing and the decay process itself. It is then natural to consider three types of CP violation:

- CP Violation in Mixing
- CP Violation in Decay
- CP Violation in the Interference of Mixing and Decay

As the phases in mixing and decay are convention dependent, the CP-violating effects depend only on the differences of these phases. This is clearly seen in the classification given below.

5.7.2 CP Violation in Mixing

This type of CP violation can be best isolated in semi-leptonic decays of neutral $B$ and $K$ mesons. We have discussed the asymmetry $a_{\text{SL}}(K_L)$ before. In the case of $B$ decays the non-vanishing asymmetry $a_{\text{SL}}(B)$ (we suppress the indices $(d; s)$),

\[
\frac{\langle B^0 (t) \mid \bar{l}^+ | X \rangle}{\langle B^0 (t) \mid l^+ | X \rangle} + \frac{\langle B^0 (t) \mid l^- | X \rangle}{\langle B^0 (t) \mid \bar{l}^- | X \rangle} = \frac{1}{1 + \hat{f}_{\text{weak}} \hat{f}} = \mu_{M_{12}} \frac{12}{M_{12}} \tag{5.55}
\]

signals this type of CP violation. Here $B^0(0) = B^0$, $B^0(0) = B^0$. For $t \neq 0$ the formulae analogous to (5.44) should be used. Note that the final states in (5.55) contain “wrong charge” leptons and can only be reached in the presence of $B^0 \to B^0$ mixing. That is one studies effectively the difference between the rates for $B^0 \to B^0 \to l^+ | X$ and $B^0 \to B^0 \to l^- | X$. As the phases in the transitions $B^0 \to B^0$ and $B^0 \to B^0$ differ from each other, a non-vanishing CP asymmetry follows. Specifically $a_{\text{SL}}(B)$ measures the difference between the phases of $M_{12}$ and $M_{12}$.

As $M_{12}$ and in particular $M_{12}$ suffer from large hadronic uncertainties, no precise extraction of CP-violating phases from this type of CP violation can be expected. Moreover as $\hat{f}_{\text{weak}}$ is almost a pure phase, see (5.46) and (5.51), the asymmetry is very small and very difficult to measure.

5.7.3 CP Violation in Decay

This type of CP violation is best isolated in charged $B$ and charged $K$ decays as mixing effects do not enter here. However, it can also be measured in neutral $B$ and $K$ decays. The relevant asymmetry is given by

\[
A_{CP}^{d} = \frac{\langle B^+ \mid l \rangle}{\langle B^+ \mid \bar{l} \rangle} = \frac{1}{1 + \hat{f}_{\text{weak}} \hat{f}} = \mu_{M_{12}} \frac{12}{M_{12}} \tag{5.56}
\]

where

\[
A_{\ell} = \frac{\langle B^+ \mid l \rangle}{\langle B^+ \mid \bar{l} \rangle} \quad A_{\bar{\ell}} = \frac{\langle B^+ \mid \bar{l} \rangle}{\langle B^+ \mid l \rangle}
\]

For this asymmetry to be non-zero one needs at least two different contributions with different weak ($\hat{f}$) and strong ($\hat{f}$) phases. These could be for instance two tree diagrams, two penguin diagrams or one tree
and one penguin. Indeed writing the decay amplitude $A_{\varepsilon}$ and its CP conjugate $A_{\varepsilon}^{\dagger}$ as

$$A_{\varepsilon} = X A_{1} e^{i(\lambda_1 \pm)}; \quad A_{\varepsilon}^{\dagger} = X A_{1} e^{i(\lambda_1 \mp)};$$  \hspace{1cm} (5.58)

with $A_{1}$ being real, one finds

$$A_{CP}^{\text{dir}}(B \to f) = \frac{2A_{1}A_{2} \sin \left( \frac{1}{2} \right) \sin \left( \frac{1}{2} \right)}{A_{2}^{*} + 2A_{1}A_{2} \cos \left( \frac{1}{2} \right) \cos \left( \frac{1}{2} \right)};$$  \hspace{1cm} (5.59)

The sign of the strong phases $\lambda$ is the same for $A_{\varepsilon}$ and $A_{\varepsilon}^{\dagger}$ because CP is conserved by strong interactions. The weak phases have opposite signs.

The presence of hadronic uncertainties in $A_{1}$ and of strong phases $\lambda$ complicates the extraction of the phases $\lambda$ from data. An example of this type of CP violation in $K$ decays is $\pi^{0}$. We will demonstrate this below.

5.7.4 CP Violation in the Interference of Mixing and Decay

This type of CP violation is only possible in neutral $B$ and $K$ decays. We will use $B$ decays for illustration suppressing the subscripts $d$ and $s$. Moreover, we set $= 0$. Formulae with $= 0$ can be found in [14, 24].

Most interesting are the decays into final states which are CP-eigenstates. Then a time dependent asymmetry defined by

$$A_{CP}(t;f) = \frac{B^{0}(t) \rightarrow f}{B^{0}(t) \rightarrow \bar{f}} + \frac{B^{0}(t) \rightarrow \bar{f}}{B^{0}(t) \rightarrow f}$$  \hspace{1cm} (5.60)

is given by

$$A_{CP}(t;f) = A_{CP}^{\text{dir}}(f) \cos (M t) + A_{CP}^{\text{mix}}(f) \sin (M t)$$  \hspace{1cm} (5.61)

where we have separated the decay CP-violating contributions ($A_{CP}^{\text{dir}}(f)$) from those describing CP violation in the interference of mixing and decay ($A_{CP}^{\text{mix}}(f)$). The latter type of CP violation is usually called the mixing-induced CP violation. One has

$$A_{CP}^{\text{dir}}(f) = \frac{1}{1 + j_{\varepsilon}^{2}} C_{\varepsilon}; \quad A_{CP}^{\text{mix}}(f) = \frac{2 \Im(A_{\varepsilon} f)}{1 + j_{\varepsilon}^{2}} S_{\varepsilon}$$  \hspace{1cm} (5.62)

where $C_{\varepsilon}$ and $S_{\varepsilon}$ are popular notations found in the literature. Unfortunately, the signs in the literature differ from paper to paper and some papers interchange $B^{0}$ and $\bar{B}^{0}$ with respect to the one used by us in the definition of the asymmetry in (5.60). Therefore in Table 2 provided by Stefan Recksiegel, we give the relations between various definitions.

| These Lectures | $A_{CP}^{\text{dir}}(\varepsilon)$ | $A_{CP}^{\text{mix}}(\varepsilon)$ |
|----------------|---------------------------------|---------------------------------|
| BaBar Book     | $C_{\varepsilon}$               | $S_{\varepsilon}$               |
| BaBar          | $C_{\varepsilon}$               | $S_{\varepsilon}$               |
| Belle          | $A_{\varepsilon}$               | $S_{\varepsilon}$               |

Table 2: Comparison between various definitions present in the literature.

The quantity $\varepsilon$ containing all the information needed to evaluate the asymmetries (5.62) is given by

$$\varepsilon = \frac{q_{A}(B^{0} \rightarrow f)}{p_{A}(B^{0} \rightarrow f)} = \exp(i2\pi \frac{A(B^{0} \rightarrow f)}{A(B^{0} \rightarrow f)})$$  \hspace{1cm} (5.63)
dealing with the interference of mixing and decay. As \( \mathcal{B}^{0} \rightarrow \mathcal{B}^{0} \) mixing, \( A(\mathcal{B}^{0} \rightarrow \mathcal{B}^{0} f) \) and \( A(\mathcal{B}^{0} ! \rightarrow \mathcal{B}^{0} f) \) are decay amplitudes. The time dependence of \( A_{CP}(t;f) \) allows to extract \( A_{CP}^{dir} \) and \( A_{CP}^{mix} \) as coefficients of \( \cos(M t) \) and \( \sin(M t) \), respectively.

Generally several decay mechanisms with different weak and strong phases contribute to \( A(\mathcal{B}^{0} ! \rightarrow \mathcal{B}^{0} f) \). These are tree diagram (current-current) contributions, QCD penguin contributions and electroweak penguin contributions. If they contribute with similar strength to a given decay amplitude the resulting CP asymmetries suffer from hadronic uncertainties related to matrix elements of the relevant operators \( Q_{L} \). The situation is then analogous to the class just discussed. Indeed

\[
\frac{A(\mathcal{B}^{0} ! \rightarrow \mathcal{B}^{0} f)}{A(\mathcal{B}^{0} ! \rightarrow \mathcal{B}^{0} f)} = \epsilon \frac{A_{T} e^{i(\tau+\varphi)} + A_{P} e^{i(\tau-\varphi)}}{A_{T} e^{i(\tau+\varphi)} + A_{P} e^{i(\tau-\varphi)}}
\]

(5.64)

with \( \epsilon = 1 \) being the CP-parity of the final state, depends on the strong phases \( \tau, \varphi \) and the hadronic matrix elements present in \( A_{T, P} \). Thus the measurement of the asymmetry does not allow a clean determination of the weak phases \( \tau, \varphi \). The minus sign in (5.64) follows from our CP phase convention \( CP(\mathcal{B}^{0} ! = \mathcal{B}^{0} f) \), that has also been used in writing the phase factor in (5.63). Only is phase convention independent. See Section 8.4.1 of [14] for details.

An interesting case arises when a single mechanism dominates the decay amplitude or the contributing mechanisms have the same weak phases. Then the hadronic matrix elements and strong phases drop out and

\[
\frac{A(\mathcal{B}^{0} ! \rightarrow \mathcal{B}^{0} f)}{A(\mathcal{B}^{0} ! \rightarrow \mathcal{B}^{0} f)} = \epsilon e^{i\omega_{D}}
\]

(5.65)

is a pure phase with \( D \) being the weak phase in \( A(\mathcal{B}^{0} ! \rightarrow \mathcal{B}^{0} f) \). Consequently

\[
\epsilon = \epsilon \exp(i\omega_{M}) \exp(i\omega_{D}); \quad j_{f}f_{f} = 1;
\]

(5.66)

In this particular case \( A_{CP}^{dir}(f) = C_{f} \) vanishes and the CP asymmetry is given entirely in terms of the weak phases \( M \) and \( D \):

\[
A_{CP}(t;f) = A_{CP}^{mix}(f) \sin(M t); \quad A_{CP}^{mix}(f) = \Im \epsilon = \epsilon \sin(2D - 2M) = S_{f};
\]

(5.67)

Thus the corresponding measurement of weak phases is free from hadronic uncertainties. A well known example is the decay \( B_{d} ! \rightarrow K_{S} \). Here \( M = 0 \) and \( D = 0 \). As in this case \( f = 1 \), we find

\[
A_{CP}(t;f) = \sin(2t) \sin(M t); \quad S_{f} = \sin(2t)
\]

(5.68)

which allows a very clean measurement of the angle \( \epsilon \) in the unitarity triangle. We will discuss other examples in Section 7.

We observe that the asymmetry \( A_{CP}^{mix}(f) \) measures directly the difference between the phases of the \( \mathcal{B}^{0} \rightarrow \mathcal{B}^{0}\)-mixing \( \varphi_{M} \) and of the decay amplitude \( \varphi_{D} \). This tells us immediately that we are dealing with the interference of mixing and decay. As \( M \) and \( D \) are phase convention dependent quantities, only their difference is physical, it is impossible to state on the basis of a single asymmetry whether CP violation takes place in the decay or in the mixing. To this end at least two asymmetries for \( \mathcal{B}^{0} \rightarrow \mathcal{B}^{0} \) decays to different final states \( f_{1}, f_{2} \) have to be measured. As \( M \) does not depend on the final state, \( \Im \epsilon_{1} ; \Im \epsilon_{2} \) is a signal of CP violation in the decay.

We will see in Section 7 that the ideal situation presented above does not always take place and two or more different mechanisms with different weak and strong phases contribute to the CP asymmetry. One finds then

\[
A_{CP}(t;f) = C_{f} \cos(M t) \quad S_{f} \sin(M t);
\]

(5.69)
\[ C_f = 2r \sin \left( \begin{array}{cc} 1 & 2 \end{array} \right) \sin \left( \begin{array}{cc} 1 & 2 \end{array} \right); \]  
\[ S_f = f \left[ \sin 2 \left( \begin{array}{cc} 1 & M \end{array} \right) + 2r \cos 2 \left( \begin{array}{cc} 1 & M \end{array} \right) \sin \left( \begin{array}{cc} 1 & 2 \end{array} \right) \cos \left( \begin{array}{cc} 1 & 2 \end{array} \right) \right] \]  
where \( r = A_2 = A_1 \) has been assumed and \( 1 \) and \( 4 \) are weak and strong phases, respectively. For \( r = 0 \) the previous formulae are obtained.

In the case of \( K \) decays, this type of CP violation can be cleanly measured in the rare decay \( K_L \to \pi^0 \). Here the difference between the weak phase in the \( K^0 \to K^0 \) mixing and in the decay \( s \to d \) matters. We will discuss this decay in Section 8.

We can now compare the two classifications of different types of CP violation. CP violation in mixing is a manifestation of indirect CP violation. CP violation in decay is a manifestation of direct CP violation. The third type contains elements of both the indirect and direct CP violation.

It is clear from this discussion that only in the case of the third type of CP violation there are possibilities to measure directly weak phases without hadronic uncertainties and moreover without invoking sophisticated methods. This takes place provided a single mechanism (diagram) is responsible for the decay or the contributing decay mechanisms have the same weak phases. However, we will see in Section 7 that there are other strategies, involving also decays to CP non-eigenstates, that provide clean measurements of the weak phases.

### 5.7.5 Another Look at \( * \) and \( *^0 \)

Let us finally investigate what type of CP violation is represented by \( * \) and \( *^0 \). Here instead of different mechanisms it is sufficient to talk about different isospin amplitudes.

In the case of \( * \), CP violation in decay is not possible as only the isospin amplitude \( A_0 \) is involved. See (5.22). We also know that only \( \text{Re} * = \text{Re} *^0 \) is related to CP violation in mixing. Consequently:

- \( \text{Re} * \) represents CP violation in mixing,
- \( \text{Im} * \) represents CP violation in the interference of mixing and decay.

In order to analyze the case of \( *^0 \) we use the formula (5.25) to find

\[
\text{Re} *^0 = \frac{1}{\sqrt{2}} \frac{A_2}{A_0} \sin \left( \begin{array}{cc} 2 & 0 \end{array} \right) \sin \left( \begin{array}{cc} 2 & 0 \end{array} \right); \]  
\[
\text{Im} *^0 = \frac{1}{\sqrt{2}} \frac{A_2}{A_0} \sin \left( \begin{array}{cc} 2 & 0 \end{array} \right) \cos \left( \begin{array}{cc} 2 & 0 \end{array} \right); \]  

Consequently:

- \( \text{Re} *^0 \) represents CP violation in decay as it is only non zero provided simultaneously \( 2 \neq 0 \) and \( 2 \neq 0 \).
- \( \text{Im} *^0 \) exists even for \( 2 = 0 \) but as it requires \( 2 \neq 0 \) it represents CP violation in decay as well.

Experimentally \( 2 \neq 0 \). Within the SM, \( 2 \) and \( 0 \) are connected with electroweak penguins and QCD penguins, respectively. We will briefly discuss the ratio \( \frac{\text{Im} *^0}{\text{Re} *^0} \) at the end of Section 6.

### 6 Standard Analysis of the Unitarity Triangle (UT)

#### 6.1 General Procedure

After this general discussion of basic concepts let us concentrate on the standard analysis of the Unitarity Triangle (see Fig. 2) within the SM. A very detailed description of this analysis with the participation of the leading experimentalists and theorists in this field can be found in [6].
Setting $\mathcal{V}_{us} = 0.224$, the analysis proceeds in the following five steps:

**Step 1:**
From the $b \to c$ transition in inclusive and exclusive leading $B$-meson decays one finds $\mathcal{V}_{cb}$ and consequently the scale of the UT:

$$\mathcal{V}_{cb} = \mathcal{V}_{cb} = 3A \quad \text{(6.1)}$$

**Step 2:**
From the $b \to u$ transition in inclusive and exclusive $B$ meson decays one finds $\mathcal{V}_{ub} = \mathcal{V}_{cb}$ and consequently using (6.1) the side $C = R_b$ of the UT:

$$\frac{\mathcal{V}_{ub}}{\mathcal{V}_{cb}} = \frac{q}{\alpha^2 + 2} = 4.35 \quad \text{(6.2)}$$

**Step 3:**
From the experimental value of $\delta_K$ in (5.40) and the formula (5.38) rewritten in terms of Wolfenstein parameters one derives the constraint on $(\lambda')$ [101] and summarizing the contributions of box diagrams with two charm quark exchanges and the mixed charm-top exchanges.

As seen in Fig. 7, equation (6.3) specifies a hyperbola in the $(\lambda'; \lambda')$ plane. The position of the hyperbola depends on $m_t$, $\mathcal{V}_{cb} = A^2$ and $\delta_K^0$. With decreasing $m_t$, $\mathcal{V}_{cb}$ and $\delta_K^0$ it moves away from the origin of the $(\lambda'; \lambda')$ plane.

**Fig. 7: Schematic determination of the Unitarity Triangle.**

**Step 4:**
From the measured $M_d$ and the formula \(15.53\), the side $AB = R_t$ of the UT can be determined:

\[
R_t = \frac{\frac{1}{2} \frac{\mathcal{V}_{ud}^j}{\mathcal{V}_{cb}^j} = 0.85}{\mathcal{V}_{ud}^j \mathcal{V}_{cb}^j} = 0.041 \frac{7 \mathcal{M} eV}{10} \; \frac{M_d}{15 \mathcal{M} eV}; \tag{6.5}
\]

\[
\mathcal{V}_{ud}^j = 78 \times 10^{-4} \frac{230 \mathcal{M} eV}{\mathcal{B}_{B_d} \mathcal{F}_{B_d}} \; \frac{167 \mathcal{G} eV}{m_t(m_t)}; \tag{6.6}
\]

with all symbols defined in the previous Section. $m_t(m_t) = (168 \pm 4) \; \text{GeV}$. Note that $R_t$ suffers from the additional uncertainty in $\mathcal{V}_{cb}^j$ which is absent in the determination of $\mathcal{V}_{ud}^j$ this way. The constraint in the $(\% \; \pm)$ plane coming from this step is illustrated in Fig. 7.

**Step 5:**

The measurement of $M_s$ together with $M_d$ allows to determine $R_t$ in a different manner:

\[
R_t = 0.90 \frac{\frac{18 \mathcal{M} eV}{124}}{\frac{M_d}{0.50 \mathcal{M} eV}}; \tag{6.7}
\]

One should note that $m_t$ and $\mathcal{V}_{cb}^j$ dependences have been eliminated this way and that $M_s$ should in principle contain much smaller theoretical uncertainties than the hadronic matrix elements in $M_d$ and $M_s$ separately.

The main uncertainties in these steps originate in the theoretical uncertainties in $\mathcal{B}_K$ and $\mathcal{B}_{B_d}$ and to a lesser extent in $\mathcal{B}_s$:

\[
\mathcal{B}_K = 0.96 \; 0.15; \quad \mathcal{B}_{B_d} = (235^{+33}_{-41}) \; \mathcal{M} eV; \quad = 1.24 \; 0.08; \tag{6.8}
\]

It should also be emphasized here that the most recent approach is to use $\mathcal{B}_{B_s}$ instead of $\mathcal{B}_{B_d}$ as it is subject to smaller theoretical uncertainties in lattice calculations that the latter quantity. The most recent value used by the UTfit collaboration $10.24$ is $\mathcal{B}_{B_d} = 276 \; 38 \mathcal{M} eV$ with being unchanged. This results in a slightly lower value for $\mathcal{B}_{B_d}$ that given above but well within the quoted errors.

Also the uncertainties due to $\mathcal{V}_{ub}^j = \mathcal{V}_{cb}^j$ in step 2 are substantial. The QCD sum rules results for the parameters in question are similar and can be found in $6$. Finally $6$

\[
M_d = (0.503 \; 0.006) \; \mathcal{M} \mathcal{eV}; \quad M_s > 14 \mathcal{M} \mathcal{eV} \text{ at } 95\% \; \text{C.L.}; \tag{6.9}
\]

### 6.2 The Angle from $B_d$! $K_S$

One of the highlights of the last two years were the considerably improved measurements of $\sin(2 \theta)$ by means of the time-dependent CP asymmetry

\[
A_{CP} (K_S; t) = A_{CP}^{\text{ext}} (K_S) \sin (M_d t) = \sin(2 \; \sin (M_d t); \tag{6.10}
\]

The BaBar $10.3$ and Belle $10.4$ collaborations find

\[
\langle \sin (2 \; \theta) \rangle_{K_S} = 0.741 \; 0.067 (\text{stat}) \; 0.033 (\text{syst}) \; \text{(BaBar)}
\]

\[
0.719 \; 0.074 (\text{stat}) \; 0.035 (\text{syst}) \; \text{(Belle)}.
\]

Combining these results with earlier measurements by CDF $(0.79^{+0.41}_{-0.34})$, ALEPH $(0.74^{+0.82}_{-1.24})$ and OPAL gives the grand average

\[
\langle \sin (2 \; \theta) \rangle_{K_S} = 0.726 \; 0.037; \tag{6.11}
\]
This is a milestone in the field of CP violation and in the tests of the SM as we will see in a moment. Not only violation of this symmetry has been confidently established in the $B$ system, but also its size has been measured very accurately. Moreover in contrast to the five constraints listed above, the determination of the angle in this manner is theoretically very clean.

6.3 The Sign of $M_d$ and $\sin 2$

The result in (6.11) leads to a two fold ambiguity in the value of

$$C_{\text{KM}} = 23 \pm 1 \pm 6$$

with the second possibility inconsistent with the SM expectations as discussed below. Measuring $\cos 2\theta$ will tell us which of these two solutions is chosen by nature. The first direct experimental result of BaBar [105] for $\cos 2\theta$ and other analyses [106, 110, 107] disfavour in fact the second solution in (6.12).

In extracting the value given in (6.12) it is usually assumed that the mass difference $M_d = M_1 - M_2 > 0$ with $M_1$ and $M_2$ denoting the masses of the neutral $B$ meson eigenstates. See (5.44) and (5.45). As the sign of $M_d$ is not known experimentally by itself, it is instructive to see whether the sign of $M_d$ matters at all here. If it mattered one would conclude that with $M_d < 0$, the BaBar and Belle data imply $\sin 2\theta = 0.726 \pm 0.037$. This would mean that instead of only two solutions in (6.12) two additional solutions for the angle exist. These findings, if correct, would weaken significantly the present believe that the BaBar and Belle data combined with the standard analysis of the unitarity triangle imply that the CKM matrix is very likely the dominant source of CP violation observed in low energy experiments.

Fortunately, it can be straightforwardly shown that the sign of $M_d$ is irrelevant for the determination of $\sin 2\theta$ [108] and the only relevant quantity for this determination is the weak phase of the mixing amplitude $M_{12}$.

Let us recall this derivation here. Using the expressions of the previous Section but not assuming $M_d$ to be positive we have

$$M_d = M_1 - M_2 = 2 \text{Re} \left( \frac{q}{p} M_{12} \right) = 2\bar{M}_{12} \hat{j}$$

(6.13)

$$B_1 = pB^0 + qB^0; \quad B_2 = pB^0 - q\Phi^0;$$

(6.14)

where $B_1$ and $B_2$ denote the mass eigenstates and

$$\frac{q}{p} = \frac{M_{12}}{M_{12}} = \frac{M_{12}}{M_{12}^2} = 2 \frac{M_{12}}{M_d}$$

(6.15)

with corresponding to (6.13). (6.15) is the generalization of (5.46) to include both signs except that we have used the fact that the width difference and $M_{12}$ can be neglected. Consequently,

$$k_p = \frac{qA(\bar{p}B^0 + f)}{pA(\bar{p}B^0 + f)} = \frac{q}{p} = 2 \frac{M_{12}}{M_d}$$

(6.16)

where we have used (5.65) with $k_p = 1$ and $p = 0$.

Inserting (6.16) into (5.67) we find

$$A_{\text{CP}} (K_S; t) = 2 \text{Im} \frac{M_{12}}{M_d} \sin (M_{12} t);$$

(6.17)

32
This formula demonstrates explicitly that the sign of $M_d$ is irrelevant and only the phase of $M_{12}$ matters. Therefore let us have a look at $M_{12}$ including already some new physics effects. Assuming that $M_{12}$ is governed by the usual $(V_A V_A) (V_A V_A)$ operator, we have quite generally (compare with (6.47))

$$M_{12} = \frac{G_F^2}{12} F_B m_B m_{\bar{B}} M_W^2 (V_{td} V_{tb})^2 S_0 (x_t) Q_{CD} \text{r} e^{i \phi};$$

(6.18)

with all symbols defined in Section 5. The last factor in (6.18) describes possible new physics contributions to the Wilson coefficient of the $(V_A V_A) (V_A V_A)$ operator that have been discussed at various occasions in the literature [109, 110, 111, 112, 113, 15]. Without loss of generality we take $r > 0$. $d$ is a new weak phase.

Using

$$V_{tb} = 1; \quad V_{td} = \sqrt{3};$$

and inserting (6.18) into (6.17) we find

$$A_{CP} (K_S; t) = \text{sign}(B_B) \sin 2(\phi + d) \text{sign}(M_B) \sin(M_B t);$$

(6.20)

This formula generalizes and summarizes various discussions of $A_{CP} (K_S; t)$ in the SM and its simplest extensions that appeared in the literature. In particular in [114] the relevance of the sign of $B_B$ has been discussed. In these extensions only the usual $(V_A V_A) (V_A V_A)$ operator is present and as new physics has no impact on its matrix element between $B^0$ and $\bar{B}^0$ states, $B_B > 0$ [6]. With $d = 0$ formula (6.20) reduces to the usual formula used by BaBar and Belle, except that sign $(M_d)$ in front of $\sin(M_d t)$ demonstrates that the sign of $M_d$ is immaterial.

With $d = 90$ one recovers a particular minimal flavour violation scenario of [115] in which the sign of $S_0 (x_t)$ is reversed. In this case indeed the BaBar and Belle measurement implies $\sin 2 = 0.726 \pm 0.037$, but this has nothing to do with the sign of $M_d$.

### 6.4 Unitarity Triangle 2004

We are now in the position to combine all these constraints in order to construct the unitarity triangle and determine various quantities of interest. In this context the important issue is the error analysis of these formulae, in particular the treatment of theoretical uncertainties. In the literature the most popular are the Bayesian approach [116] and the frequentist approach [117]. For the PDG analysis see [32]. A critical comparison of these and other methods can be found in [6]. I can recommend this reading. Most recent analyses of the UT by the bayesians (“UTfitters”) and frequentists (“CKMfitters”) can be found in [102] and [107], respectively.

In Fig. 8 we show the result of an analysis in collaboration with Felix Schwab and Selma Uhlig [15]. The allowed region for $(\% ; r)$ is the area inside the ellipse. We observe that the region $\% < 0$ is disfavoured by the lower bound on $M_s$. It is clear from this figure that the measurement of $M_s$ giving $R_t$ through (6.7) will have a large impact on the plot in Fig. 8. Most importantly there is an excellent agreement between the direct measurement in (6.11) and the standard analysis of the UT within the SM. This gives a strong indication that the CKM matrix is very likely the dominant source of CP violation in flavour violating decays. In order to be sure whether this is indeed the case other theoretically clean quantities have to be measured. In particular the angle $\theta$. We will discuss other processes that are useful for the determination of the UT in the next two sections.

Finally, the ranges for various quantities that result from this analysis are given in the SM column of table 3. The UUT column will be discussed in Section 9.
The ratio $\frac{\bar{\eta}}{\rho}$ that parametrizes the size of direct CP violation with respect to the indirect CP violation in $K_L \to \pi^+\pi^-$ decays has been the subject of very intensive experimental and theoretical studies in the last three decades. After tremendous efforts, on the experimental side the world average based on the results from NA48 [118] and KTeV [119], and previous results from NA31 [120] and E731 [121], reads

$$\frac{\bar{\eta}}{\rho} = (16^{+6}_{-1}) \times 10^{-4} \quad (2003)$$

On the other hand, the theoretical estimates of this ratio are subject to very large hadronic uncertainties. While several analyzes of recent years within the Standard Model (SM) find results that are compatible with $\frac{\bar{\eta}}{\rho}$ [6.21, 122, 67, 123, 124, 125, 126, 127, 128], it is fair to say that the chapter on the theoretical calculations of $\frac{\bar{\eta}}{\rho}$ is far from being closed. A full historical account of the theoretical efforts before 1998 can for example be found in [7, 129]. Most recent reviews can be found in [130, 131].
7 The Angles, and from B Decays

7.1 Preliminaries

CP violation in B decays is certainly one of the most important targets of B-factories and of dedicated B-experiments at hadron facilities. It is well known that CP-violating effects are expected to occur in a large number of channels at a level attainable experimentally and in fact as we have seen above and we will see below, clear signals of CP violation in B decays have already been observed. Moreover there exist channels which offer the determination of CKM phases essentially without any hadronic uncertainties.

The results on $\sin 2\beta$ from BaBar and Belle are very encouraging. These results should be further improved over the coming years through new measurements of $A_{CP}(t; K_S)$ by both collaborations and by CDF and D0 at Fermilab. Moreover measurements of CP asymmetries in other B decays and the measurements of the angles $\gamma$ and $\beta$ by means of various strategies using two-body B decays should contribute substantially to our understanding of CP violation and will test the KM picture of CP violation. In fact some interesting results are already available. They will be discussed later on.

The various types of CP violation have been already classified in Section 5.7. It turned out that CP violation in the interference of mixing and decay, in a B meson decay into a CP eigenstate, is very suitable for a theoretically clean determination of the angles of the unitarity triangle provided a single CKM phase governs the decay. However as we will see below several useful strategies for the determination of the angles $\gamma$ and $\beta$ have been developed that are effective also in the presence of competing CKM phases and when the final state is not a CP eigenstate. The notes below should only be considered as an introduction to this rich field. For more details the references in Section 1 should be contacted.

7.2 Classification of Elementary Processes

Non-leptonic B decays are caused by elementary decays of b quarks that are represented by tree and penguin diagrams in Fig. 9. Generally we have

$$b \rightarrow q_1 q_2 d(s); \quad b \rightarrow q q d(s) \quad (7.1)$$

for tree and penguin diagrams, respectively.

There are twelve basic transitions that can be divided into three classes:

Class I: both tree and penguin diagrams contribute. Here $q_1 = q_2 = q = u;c$ and consequently the basic transitions are

$$b \rightarrow \psi s; \quad b \rightarrow \omega d; \quad b \rightarrow uus; \quad b \rightarrow uud \quad (7.2)$$
Class II: only tree diagrams contribute. Here \( q_1 \neq q_2 \) and
\[
\begin{align*}
 b &! c u_s; \quad b &! c u_d; \quad b &! u c s; \quad b &! u c d; \quad (7.3)
\end{align*}
\]

Class III: only penguin diagrams contribute. Here \( q = d; s \) and
\[
\begin{align*}
 b &! s s s; \quad b &! s s d; \quad b &! d d s; \quad b &! d d d; \quad (7.4)
\end{align*}
\]

Now in presenting various decays below, we did not show the corresponding diagrams on purpose. After all these are lectures and the exercise for the students is to draw these diagrams by embedding the elementary diagrams of Fig. 9 into a given \( B \) meson decay. In case of difficulties the student should look at [22, 24] where these diagrams can be found.

7.3 Neutral B Decays into CP eigenstates

7.3.1 \( B_0^d \to K_S \) and

The amplitude for this decay can be written as follows
\[
A(\mathcal{B}_0^d \to K_S) = V_{cs}V_{cb}(A_T + P_c) + V_{us}V_{ub}P_u + V_{ts}V_{tb}P_t
\]
where \( A_T \) denotes tree diagram contributions and \( P_i \) with \( i = u; c; t \) stand for penguin diagram contributions with internal \( u, c \) and \( t \) quarks. Now
\[
V_{cs}V_{db} = A^2; \quad V_{us}V_{ub} = A^4 R_\theta e^{i \phi}\; ; \quad V_{ts}V_{tb} = V_{us}V_{ub}(P_u + P_t) \quad (7.5)
\]
with the last relation following from the unitarity of the CKM matrix. Thus
\[
A(\mathcal{B}_0^d \to K_S) = V_{cs}V_{db}(A_T + P_c P_t) + V_{us}V_{ub}(P_u + P_t) \quad (7.6)
\]

We next note that
\[
\frac{V_{us}V_{ub}}{V_{cs}V_{db}} 0 < 2; \quad \frac{P_u}{A_T + P_c P_t} < 1 \quad (7.8)
\]
where the last inequality is very plausible as the Wilson coefficients of the current–current operators responsible for \( A_T \) are much larger than the ones of the penguin operators [7, 13]. Consequently this decay is dominated by a single CKM factor and as discussed in Section 5.7, a clean determination of the relevant CKM phase is possible. Indeed in this decay \( D = 0 \) and \( M = 0 \). Using (5.67) we find once more (\( K_S = 1 \))
\[
A_{CP}^m(K_S) = K_S \sin(2D, 2M) = \sin 2; \quad (7.9)
\]
\[
C_{K_S} = 0; \quad S_{K_S} = \sin 2 \quad (7.10)
\]
that is confirmed by experiment as discussed in the previous Section.

7.3.2 \( B_s^0 \to K_S \) and \( s \)

This decay differs from the previous one by the spectator quark, with \( d ! s \) so that the formulae above remain unchanged except that now \( M = s \). A complication arises as the final state is a mixture of \( CP = + \) and \( CP = - \) states. This issue can be resolved experimentally [22]. Choosing \( = 1 \) we then find
\[
A_{CP}^m(\ ) = \sin(2D, 2M) = 2^2 = 2^2 = 0.03; \quad C = 0 \quad (7.11)
\]
Thus this asymmetry measures the phase of \( V_{ts} \) that is predicted to be very small from the unitarity of the CKM matrix alone. Because of this there is a lot of room for new physics contributions to \( A_{CP}^m(\ ) \).
7.3.3 $B_0^d \to K_S$ and $B_0^d \to K_L$

This decay proceeds entirely through penguin diagrams and consequently should be much more sensitive to new physics contributions than the decay $B_d \to K_S$. Assuming $= (ss)$, the decay amplitude is given by (7.7) with $A_T$ removed:

$$A(B_d \to K_S) = V_{cs} V_{cb} (P_c P_t) + V_{us} V_{ub} (P_u P_t)$$

With

$$\frac{V_{us} V_{ub}}{V_{cs} V_{cb}} = 0 \pm 2; \quad \frac{P_u}{P_c} = 0.1$$

also in this decay a single CKM phase dominates and as $d$ and $M$ are the same as in $B_d \to K_S$ we find

$$C_{K_S} = 0; \quad S_{K_S} = S_{K_S} = \sin 2 :$$

The equality of these two asymmetries need not be perfect as the meson is not entirely a $ss$ state and the approximation of neglecting the second amplitude in (7.12) could be only true within a few percent. However, a detailed analysis shows [132] that these two asymmetries should be very close to each other within the SM: $\mathcal{F}_{K_S} S_{\pi K_S} J = 0.04$. Any strong violation of this bound would be a signal for new physics.

In view of this prediction, the first results on this asymmetry from BaBar [133] and Belle [134] were truly exciting:

$$\begin{align*}
(\sin 2 )_{K_S} &= 0.1 \pm 0.1 \text{ (stat)} \pm 0.09 \text{ (syst)} \quad \text{(BaBar)} \\
&= 0.73 \pm 0.04 \text{ (stat)} \pm 0.18 \text{ (syst)} \quad \text{(Belle)},
\end{align*}$$

implying

$$S_{K_S} = 0.29 \pm 0.01; \quad C_{K_S} = 0.56 \pm 0.03;$$

$$\mathcal{F}_{K_S} S_{\pi K_S} j = 1.12 \pm 0.1$$

and the violation of the bound $\mathcal{F}_{K_S} S_{\pi K_S} j = 0.04$ by 2.7. These results invited a number of theorists to speculate what kind of new physics could be responsible for this difference. Some references are given in [135]. Enhanced QCD penguins, enhanced $Z^0$ penguins, rather involved supersymmetric scenarios have been suggested as possible origins of the departure from the SM prediction.

Unfortunately the new data presented at the 2004 summer conferences by both collaborations look much closer to the SM predictions

$$\begin{align*}
(\sin 2 )_{K_S} &= 0.60 \pm 0.25 \text{ (stat)} \pm 0.06 \text{ (syst)} \quad \text{(BaBar)} \\
&= 0.06 \pm 0.03 \text{ (stat)} \pm 0.09 \text{ (syst)} \quad \text{(Belle)},
\end{align*}$$

implying

$$S_{K_S} = 0.34 \pm 0.20; \quad C_{K_S} = 0.04 \pm 0.07;$$

In particular the BaBar result is in full agreement with the SM. Still some room for new physics contributions is left and it will be interesting to follow the development in the values of $S_{K_S}$ and $C_{K_S}$ and similar values in other channels such as $B \to K_S$. Some recent discussions can be found in [136].
7.3.4 $B^0_d ! \to +$ and $B^0_d ! \to$ and
This decay receives the contributions from both tree and penguin diagrams. The amplitude can be written as follows
\[
A(B^0_d ! \to +) = V_{ud}V_{ub}(A_T + P_u) + V_{cd}V_{cb}P_c + V_{td}V_{tb}P_t \quad (7.18)
\]
where
\[
V_{cd}V_{cb}A^3; V_{ud}V_{ub}A^3R_{b}; V_{td}V_{tb}=V_{ud}V_{ub}V_{cd}V_{cb} \quad (7.19)
\]
Consequently
\[
A(B^0_d ! \to +) = V_{ud}V_{ub}(A_T + P_u + P_c) + V_{cd}V_{cb}(P_c + P_t) \quad (7.20)
\]
We next note that
\[
\frac{V_{cd}V_{cb}}{V_{ud}V_{ub}} = \frac{1}{R_b} = 2S; \quad \frac{P_c}{A_T + P_u + P_c} = \frac{P_t}{P_T} \quad (7.21)
\]
Now the dominance of a single CKM amplitude in contrast to the cases considered until now is very uncertain and takes only place provided $P_T$. Let us assume first that this is indeed the case. Then this decay is dominated by a single CKM factor and a cleandetermination of the relevant CKM phase is possible. Indeed in this decay $D = \frac{A_T}{A_T + P_u + P_c}$ and $M = \frac{P_{c}}{A_T + P_u + P_c}$. Using (5.67) we find then ($= 1$)
\[
A_{CP}^{m\bar{m}}(+) = \sin(2D \frac{A_T}{A_T + P_u + P_c} 2M) = \sin 2(+) = \sin 2 \quad (7.22)
\]
and
\[
C = 0; \quad S = \sin 2 \quad (7.23)
\]
This should be compared with the 2004 results from BaBar [237] and Belle [238]:
\[
\begin{align*}
C & = 0.09 \pm 0.15 \text{(stat)} \pm 0.04 \text{(syst)} \quad \text{(BaBar)} \\
S & = 1.00 \pm 0.21 \text{(stat)} \pm 0.07 \text{(syst)} \quad \text{(Belle)}
\end{align*}
\]
giving the averages
\[
C = 0.37 \pm 0.11; \quad S = 0.61 \pm 0.14 \quad (7.24)
\]
The results from BaBar are consistent with earlier expectations that the direct CP violation is very small. After all from the UT fit is in the ballpark of 90°. On the other hand Belle results indicate a non-zero asymmetry and a sizable contribution of the penguin diagrams invalidating our assumption $P_T$. While the results from BaBar and Belle are not fully compatible with each other, the average in (7.24) did not change by much over the last two years.

The “QCD penguin pollution” discussed above has to be taken care of in order to extract $\sin 2\theta$. The well known strategy to deal with this “penguin problem” is the isospin analysis of Gronau and London [137]. It requires however the detailed measurements of all $B^0 ! \to$ modes. For this reason several other strategies for extraction of $\sin 2\theta$ have been proposed. They are reviewed in [14, 20, 21, 22, 24]. Some recent analyses of $B^0 ! \to$ data can be found in [138, 107]. Others will be discussed in Section 10.

While it is not clear that a precise value of $\sin 2\theta$ will follow in a foreseeable future from this enterprise, one should stress [139, 140, 141, 42] that only a moderately precise measurement of $\sin 2\theta$ can be as useful for the UT as a precise measurement of the angle $\theta$. 
From my point of view a more promising approach is to use the full $B^0$ system in conjunction with the known value of in order to determine the angle and subsequently $(\% ; \% )$. Some analyses of this type have already been presented in $[6; 142; 111]$, but recently in view of the new $B^0$ data they have been generalized and improved. One of such analyses done in collaboration with Robert Fleischer, Stefan Recksiegel and Felix Schwab will be presented in Section 10.

7.4 Decays to CP Non-Eigenstates

7.4.1 Preliminaries

The strategies discussed below have the following general properties:

$B^0_d (B^0_s)$ and their antiparticles $B^0_d (B^0_s)$ can decay to the same final state,

Only tree diagrams contribute to the decay amplitudes,

A full time dependent analysis of the four processes is required:

$$B^0_{d(s)}(t) \to f; \quad B^0_{d(s)}(t) \to f; \quad B^0_{d(s)}(t) \to f; \quad B^0_{d(s)}(t) \to f; \quad \text{(7.25)}$$

The latter analysis allows to measure

$$\varphi = \exp \left( i \frac{2}{M} \right) \frac{A (B^0 \to f)}{A (B^0 \to f)}; \quad \varphi = \exp \left( i \frac{2}{M} \right) \frac{A (B^0 \to f)}{A (B^0 \to f)}; \quad \text{(7.26)}$$

It turns out then that

$$\varphi \varphi = F \left( \varphi \right) \quad \text{(7.27)}$$

without any hadronic uncertainties, so that determining $M$ from other decays as discussed above, allows the determination of $\varphi$. Let us show this and find an explicit expression for $F$.

7.4.2 $B^0_d \to D^{*} \quad , \quad B^0_d \to D^{*} \quad and$

With $\varphi = D^{*}$ the four decay amplitudes are given by

$$A (B^0 \to D^{*}) = M_{\varphi}A \frac{4}{R_{\varphi}}e^{i\varphi}; \quad A (B^0 \to D^{*}) = M_{\varphi}A \frac{4}{R_{\varphi}}e^{i\varphi}; \quad \text{(7.28)}$$

$$A (B^0 \to D^{*}) = M_{\varphi}A \frac{4}{R_{\varphi}}e^{i\varphi}; \quad A (B^0 \to D^{*}) = M_{\varphi}A \frac{4}{R_{\varphi}}e^{i\varphi}; \quad \text{(7.29)}$$

where we have factored out the CKM parameters, $A$ is a Wolfenstein parameter and $M$ stand for the rest of the amplitudes that generally are subject to large hadronic uncertainties. The important point is that each of these transitions receives the contribution from a single phase so that

$$\varphi \varphi = e^{i\frac{2}{M} \varphi}; \quad \varphi \varphi = e^{i\frac{2}{M} \varphi}; \quad \text{(7.30)}$$

Now, as CP is conserved in QCD we simply have

$$M_{\varphi} = M_{\varphi}; \quad M_{\varphi} = M_{\varphi}; \quad \text{(7.31)}$$

and consequently $[143]$

$$\varphi \varphi = e^{i\frac{2}{M} \varphi}; \quad \varphi \varphi = e^{i\frac{2}{M} \varphi}; \quad \text{(7.32)}$$

as promised. The phase is already known with high precision and consequently can be determined.

Unfortunately as seen in (7.28) and (7.29), the relevant interferences are $O \left( \frac{2}{M} \right)$ and the execution of this strategy is a very difficult experimental task. See [144] for an interesting discussion.
7.4.3 $B_s^0$ ! $D_s K$ , $B_s^0$ ! $D_s K$ and

Replacing the $d$-quark by the $s$-quark in the strategy just discussed allows to enhance the interference between various contributions. With $f = D_s^0 K$ equations (7.28) and (7.29) are replaced by

$$A(\bar{B}_s^0 ! D_s^+ K^-) = M_f A^3 e^{i \epsilon_3}$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (7.33)

$$A(\bar{B}_s^0 ! D_s^- K^+) = M_f A^3 e^{i \epsilon_1}$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (7.34)

Proceeding as in the previous strategy one finds [145]

$$\epsilon_f^{(a)} = e^{i \epsilon_2 (2 \epsilon^{(s)})}$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (7.35)

with $\epsilon_f^{(a)}$ and $\epsilon_f^{(a)}$ being the analogs of $\epsilon_f^{(a)}$ and $\epsilon_f^{(a)}$, respectively. Now, all interfering amplitudes are of a similar size. With $s$ extracted one day from the asymmetry in $B_s^0 (\bar{B}_s^0)$!, the angle $\epsilon$ can be determined.

7.4.4 $B$ ! $D^0 K$ , $B$ ! $D^0 K$ and

By replacing the spectator $s$-quark in the last strategy through a $u$-quark one arrives at decays of $B$ that can be used to extract $d$. Also this strategy is unaffected by penguin contributions. Moreover, as particle-antiparticle mixing is absent here, $s$ can be measured directly without any need for phases in the mixing. Both these features make it plausible that this strategy, not involving to first approximation any loop diagrams, is particularly suited for the determination of $\epsilon$ without any new physics pollution.

By considering six decay rates $B$ ! $D^0 K$ , $B$ ! $D^0 K$ and $B$ ! $D^0 K$ and $B$ ! $D^0 K$ where $D^0$ = $D^0 + D^0$ is a CP eigenstate, and noting that

$$A(\bar{B}^+ ! D^0 K^+) = A(\bar{B} ! D^0 K)$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (7.36)

$$A(\bar{B}^+ ! D^0 K^+) = A(\bar{B} ! D^0 K e^{2i \epsilon_1})$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (7.37)

the well known triangle construction due to Gronau and Wyler [146] allows to determine $\epsilon$. However, the method is not without problems. The detection of $D^0_{CP}$, that is necessary for this determination because $K^+ D^0$, $K^+ D^0$, is experimentally challenging. Moreover, the small branching ratios of the colour supressed channels in (7.37) and the absence of this suppression in the two remaining channels in (7.36) imply a rather squashed triangle thereby making the extraction of $\epsilon$ very difficult. Variants of this method that could be more promising are discussed in [147, 148].

7.4.5 Other Clean Strategies for $s$ and

The three strategies discussed above can be generalized to other decays. In particular [147, 149]

$$2s^+$$ and can be measured in

$$B_d^0 ! K_s D^0; K_s D^0; B_d^0 ! D^0 D^0; 0 D^0$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (7.38)

and the corresponding CP conjugated channels,

$$2s^+$$ and can be measured in

$$B_s^0 ! D^0; D^0; B_s^0 ! K_s D^0; K_s D^0$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} (7.39)

and the corresponding CP conjugated channels,
can be measured by generalizing the Gronau–Wyler construction to \( B \rightarrow D^0 \pi^0 \) and to \( B_c \) decays \[150]:
\[
B_c \rightarrow D^0 s \pi^0 ; D^0 s \pi^0 ; B_c \rightarrow D^0 D_s ; D^0 D_s ;
\]
(7.40)
In this context I can recommend the papers by Fleischer \[149\] that while discussing these decays go far beyond the methods presented here. It appears that the methods discussed in this subsection may give useful results at later stages of CP-B investigations, in particular at LHC-B.

7.5 U–Spin Strategies

7.5.1 Preliminaries

Useful strategies for using the U-spin symmetry have been proposed by Robert Fleischer in \[151, 152\]. The first strategy involves the decays \( B^0 \rightarrow K^0 \pi^0 \) and \( B^0 \rightarrow K^0 \), \( B^0 \rightarrow \pi^0 K^0 \). The second strategy involves \( B_s \rightarrow K^0 \pi^0 \) and \( B_d \rightarrow K^0 \pi^0 \). They are unaffected by FSI and are only limited by U-spin breaking effects. They are promising for Run II at FNAL and in particular for LHC-B.

A method of determining \( \Delta \), using \( B^+ \rightarrow K^0 \pi^0 \) and the U-spin related processes \( B^0 \rightarrow K^0 \pi^0 \) and \( B^0 \rightarrow K^0 \pi^0 \), was presented in \[153\]. A general discussion of U-spin symmetry in charmless \( B \) decays and more references to this topic can be found in \[24, 154\]. I will only briefly discuss the method in \[152\].

7.5.2 \( B^0_d \rightarrow K^0 \pi^0 \), \( B^0_s \rightarrow K^0 \pi^0 \) and \( (\,;\, ) \)

Replacing in \( B^0_d \rightarrow K^0 \pi^0 \) the \( c \) quark by an \( s \) quark we obtain the decay \( B^0_s \rightarrow K^0 \pi^0 \). The amplitude can be then written in analogy to (7.20) as follows
\[
A(B^0_s \rightarrow K^0 \pi^0 ) = V_{us} V_{ub}^* \left( P_{u} A_T + P_{c} P_{t} \right) + V_{cs} V_{cb}^* \left( P_{c} P_{t} \right)
\]
(7.41)
This formula differs from (7.20) only by \( d \) and the \( c \) quark by an \( s \) quark for both decays and extracting \( s \) from \( B^0_s \rightarrow K^0 \pi^0 \), we can determine four unknowns: \( d \), \( S_f \), \( C_f \) and \( \phi \) subject mainly to U-spin breaking corrections. An analysis using these ideas can be found in \[111\].
7.6 Constraints for \( B \to K \)

The four modes \( B \to K \), \( B \to K \), \( B \to K \), \( B \to K \) have been observed by the CLEO, BaBar and Belle collaborations and allow us already now to obtain some direct information on this. This information will improve when the errors on branching ratios and the CP asymmetries decrease. The progress on the accuracy of these measurements is slow but steady and they begin to give, in addition to , an interesting insight into the flavour and QCD dynamics. In particular, one of the highlights of 2004 was the observation of the direct CP violation in \( B_d \to K \) decays [155, 156]. Other experimental results for \( B \to K \) decays will be discussed in Section 10.

There has been a large theoretical activity in this field during the last seven years. The main issues here are the final state interactions (FSI), SU(3) symmetry breaking effects and the importance of electroweak penguin contributions. Several interesting ideas have been put forward to extract the angle in spite of large hadronic uncertainties in \( B \to K \) decays [157, 158, 159, 160, 161, 162].

Three strategies for bounding and determining have been proposed. The “mixed” strategy [157] uses \( B_d \to K \) and \( B \to K \). The “charged” strategy [162] involves \( B \to K \) and the “neutral” strategy [160] the modes \( B_d \to K \), \( B \to K \). General parametrizations for the study of the FSI, SU(3) symmetry breaking effects and of the electroweak penguin contributions in these channels have been presented in [159, 160, 161]. Moreover, general parametrizations by means of Wick contractions [163, 164] have been proposed. They can be used for all two-body B-decays. These parametrizations should turn out to be useful when the data improve. Finally, recently a graphical approach using SCET ideas has been proposed in [165].

Parallel to these efforts an important progress has been made by developing approaches for the calculation of the hadronic matrix elements of local operators in QCD beyond the standard factorization method. These are in particular the QCD factorization approach [51], the perturbative QCD approach [52] and the soft-collinear effective theory [53]. Moreover new methods to calculate exclusive hadronic matrix elements from QCD light-cone sum rules have been developed in [166, 167]. While, in my opinion, an important progress in evaluating non-leptonic amplitudes has been made in these papers, the usefulness of this recent progress at the quantitative level has still to be demonstrated when the data improve. In fact as discussed in Section 10 the most recent data indicate that the present theoretical frameworks have real problems in certain channels.

As demonstrated in a number of papers [157, 159, 160, 161, 162] in the past, these strategies imply within the SM interesting bounds on that do not necessarily agree with the values extracted from the UT analysis of Section 6. In particular already in 2000 combining the neutral and charged strategies [168] we have found that the 2000 data on \( B \to K \) favoured in the second quadrant, which was in conflict with the standard analysis of the unitarity triangle that implied \( \cos \theta \approx 0 \). Other arguments for \( \cos \theta < 0 \) using \( PP, PP, PV \) and \( VV \) decays were given also in [169] [6] [170].

On the other hand it has been emphasized by various authors that in view of sizable theoretical uncertainties in the analyses of \( B \to K \) and of still significant experimental errors in the corresponding branching ratios it is not yet clear whether the discrepancy in question is serious. For instance [171] sizable contributions of the so-called charming penguins [172, 173] to the \( B \to K \) amplitudes could in principle shift extracted from these decays below 90 but at present these contributions cannot be calculated reliably. A similar role could be played by annihilation contributions [52] and large non-factorizable SU(3) breaking effects [168].

However, a much more attractive solution to all these problems appears to me the proposal made by Robert Fleischer and myself already in 2000 that the puzzling features of the \( B \to K \) data indicate
new physics contributions in the electroweak penguin sector \[168\]. In order to address this issue in the presence of improved data, we have recently developed a strategy in collaboration with Stefan Recksiegel and Felix Schwab that allows to analyze not only the \(B^+ \rightarrow K^0_s\) system but also the \(B^+ \rightarrow K_s\) system and subsequently study possible implications of the modified electroweak penguin sectors on rare \(K\) and \(B\) decays. We will discuss this strategy in Section \[10\]. In order to be prepared for this discussion we turn now to the analysis of \(K^+ \rightarrow \pi^+\) and \(K_L \rightarrow \pi^0\) decays.

8 \(K^+ \rightarrow \pi^+\) and \(K_L \rightarrow \pi^0\)

8.1 Preliminaries

The rare decays \(K^+ \rightarrow \pi^+\) and \(K_L \rightarrow \pi^0\) are very promising probes of flavour physics within the SM and possible extensions, since they are governed by short distance interactions. They proceed through \(Z^0\)-penguin and box diagrams in Fig. 10. As the required hadronic matrix elements can be extracted from the leading semileptonic decays and other long distance contributions turn out to be small \[173, 175, 176, 177, 178, 179, 180, 181\], the relevant branching ratios can be computed to an exceptionally high degree of precision \[182, 74, 75\]. The main theoretical uncertainty in the CP conserving decay \(K^+ \rightarrow \pi^+\) originates in the value of \(c\) in \(m_c\). It has been reduced through NLO corrections down to \(8\%\) \[182, 74\] at the level of the branching ratio. The dominantly CP-violating decay \(K_L \rightarrow \pi^0\) is even cleaner as only the internal top contributions matter. The theoretical error for \(B \pi (K_L \rightarrow \pi^0)\) amounts to \(2\%\) and is safely negligible.

![Fig. 10: Penguin and box diagrams contributing to \(K^+ \rightarrow \pi^+\) and \(K_L \rightarrow \pi^0\).](image)

On the experimental side the AGS E787 collaboration at Brookhaven was the first to observe the decay \(K^+ \rightarrow \pi^+\) \[184\]. The resulting branching ratio based on two events and published in 2002 was...
In 2004 a new $K^+ \! \! \! +$ experiment, AGS E949 \[186\], released its first results that are based on the 2002 running. One additional event has been observed. Including the result of AGS E787 the present branching ratio reads

\[
Br(K^+ \! \! \! +) = (15.7^{+1.5}_{-1.2}) \times 10^3 \quad (2002): \tag{8.1}
\]

It is not clear, at present, how this result will be improved in the coming years but AGS E949 should be able to collect in total 10 SM events. One should also hope that the efforts at Fermilab around the CKM experiment \[187\], the corresponding efforts at CERN around the NA48 collaboration \[188\] and at J-PARC in Japan \[189\] will provide additional 50-100 events in the next five years.

The situation is different for $K_L \! \! \! 0$. While the present upper bound on its branching ratio from KTeV \[190\],

\[
Br(K_L \! \! \! 0) < 5 \times 10^{-9}; \tag{8.3}
\]

is about four orders of magnitude above the SM expectation, the prospects for an accurate measurement of $K_L \! \! \! 0$ appear almost better than for $K^+ \! \! \! +$ from the present perspective.

Indeed, a $K_L \! \! \! 0$ experiment at KEK, E391a \[191\], which just started taking data, should in its first stage improve the bound in \(8.3\) by three orders of magnitude. While this is insufficient to reach the SM level, a few events could be observed if $Br(K_L \! \! \! 0)$ turned out to be by one order of magnitude larger due to new physics contributions.

Next, a very interesting experiment at Brookhaven, KOPIO \[192\], should in due time provide 40-60 events of $K_L \! \! \! 0$ at the SM level. Finally, the second stage of the E391 experiment could, using the high intensity 50 GeV/c proton beam from J-PARC \[189\], provide more than 100 SM events of $K_L \! \! \! 0$, which would be truly fantastic! Perspectives of a search for $K_L \! \! \! 0$ at a -factory have been discussed in \[193\]. Further reviews on experimental prospects for $K \! \! \!$ can be found in \[192, 194\].

In view of these prospects a new very detailed review of $K \! \! \!$ decays in collaboration with Felix Schwab and Selma Uhlig has been presented in \[15\]. Essentially everything that is known about these decays on the theoretical side can be found there. Therefore, we will only summarize the main virtues of $K \! \! \!$ decays here. Other reviews can be found in \[195\]. For a very recent summary of the exceptional virtues of $K_L \! \! \! 0$ in probing new physics see \[196\].

### 8.2 Branching Ratios

The basic formulae for the branching ratios are given as follows \[74\]

\[
Br(K^+ \! \! \! +) = \left(\frac{\text{Im}(\tau) \times (\kappa_L)}{5 \tau \times (\kappa_L)}\right) + \frac{\text{Re}(\tau) \times (\kappa_L)}{5 \tau \times (\kappa_L)} \quad (\kappa_L) \tag{8.4}
\]

\[
Br(K_L \! \! \! 0) = \left(\frac{\text{Im}(\tau) \times (\kappa_L)}{5 \tau \times (\kappa_L)}\right) \quad (\kappa_L) \tag{8.5}
\]

Here $\kappa_L = \frac{m^2}{2M^2_W}$, $\tau = V_{ts}V_{td}$ and

\[
+ = (4 \pm 4 \quad 0.06) \quad 1 \bar{0}^3; \quad L = (2.12 \quad 0.03) \quad 10^0 \tag{8.6}
\]
include isospin breaking corrections in relating $K^0 \to \pi^0$ and $K_L \to \pi^0$, respectively [197]. Due to the update of input parameters made in [15], the numbers in (8.6) differ from the ones in the original papers [140, 74].

Next

$$X (m_c) = 1.53 \pm 0.04$$

represents the internal top contribution and $P_c (X)$ results from the internal charm contribution [182]. A recent analysis of the uncertainties in $P_c (X)$ resulted in [15]

$$P_c (X) = 0.389 \pm 0.033_{m_c} \pm 0.045_{c} \pm 0.010_s = 0.39 \pm 0.07;$$

(8.8)

where the errors correspond to $m_c (m_c)$, $c$ and $s (M_Z)$, respectively. The latter parameters have been varied as follows

$$1.25_G eV \; m_c (m_c) \; 1.35_G eV; \; 1.0_G eV \; c \; 3.0_G eV;$$

$$0 \pm 16 \; s (M_Z) \; 0 \pm 20;$$

(8.9)

The result in (8.8) does not include the recently calculated [180, 181] contributions of dimension-eight four fermion operators generated at the charm scale and genuine long distance contributions which can be described within the framework of chiral perturbation theory. Including these contributions one finds

$$P_c (X) = 0.43 \pm 0.07;$$

(8.11)

We anticipate that all remaining long distance uncertainties, that are well below the error in (8.11), are already included in the error quoted above.

We observe that the error in $P_c (X)$ is dominated by the left-over scale uncertainty ($m_c$), implying that a calculation of $P_c (X)$ at the NNLO level is certainly desirable. The uncertainty due to $m_c$ is smaller but still significant. On the other hand, the uncertainty due to $s$ is small.

We expect that a NNLO calculation, that is now in progress [198], will reduce the error in $P_c (X)$ due to $m_c$ by a factor of 2-3 and the reduction of the error in $s (M_Z)$ to $0.001$ will decrease the corresponding error to $0.005$, making it negligible. Concerning the error due to $m_c (m_c)$, we have to a good approximation [15]

$$P_c (X) = \frac{0.57}{G eV} (m_c (m_c));$$

(8.12)

This discussion shows that after a NNLO analysis has been performed, the main uncertainty in $P_c (X)$ will be due to $m_c$. From the present perspective, unless some important advances in the determination of $m_c (m_c)$ will be made, it will be very difficult to decrease the error on $P_c (X)$ below $0.03$, although $0.02$ cannot be fully excluded.

Imposing all existing constraints on the CKM matrix one finds using (8.8) [15]

$$B r (K^+ \to \pi^0) = (7.77 \pm 0.52_c \pm 0.51) \quad 1^{\pm} = (7.8 \pm 1.2) \quad 1^{\pm};$$

(8.13)

$$B r (K_L \to \pi^0) = (3.0 \pm 0.5) \quad 1^{\pm}$$

(8.14)

Similar results are found in [195, 199].

On the other hand, using the most recent result on $P_c (X)$ in (8.11) one finds

$$B r (K^+ \to \pi^0) = (8.3 \pm 1.2) \quad 1^{\pm};$$

(8.15)

45
without any change in (8.14).

The central value of \( \mathcal{B}(K^+ \to +) \) in (8.14) is below the central experimental value in (8.2), but within theoretical, parametric and experimental uncertainties, the SM result is fully consistent with the data. We also observe that the error in \( \mathbb{P}_c(K) \) constitutes still a significant portion of the full error.

The present upper bound on \( \mathcal{B}(K_L \to 0) \) from the KTeV experiment at Fermilab [190] and given in (8.3) is about four orders of magnitude above the SM expectation (8.14). Moreover this bound is substantially weaker than the model independent bound [200] from isospin symmetry:

\[
\mathcal{B}(K_L \to 0) < 4 \times \quad (8.16)
\]

which through (8.2) gives

\[
\mathcal{B}(K_L \to 0) < 1 \times 10^{-6} \quad (90\% \ C.L.) \quad (8.17)
\]

![Unitarity triangle from K !](image)

**8.3 Unitarity Triangle, \( \sin^2 \) and from \( K ! \)**

The measurement of \( \mathcal{B}(K^+ \to +) \) and \( \mathcal{B}(K_L \to 0) \) can determine the unitarity triangle completely, (see Fig. 11 [139] [140]. The explicit formulae can be found in [37] [9] [139] [140] [15]. Most interesting in this context are very clean determinations of \( \sin^2 \) and \( \text{Im} \) that are free not only from hadronic uncertainties but also parametric uncertainties like \( \mathcal{V}_{cb} \) and \( m_c \). The determination of \( \mathcal{V}_{td} \) is also theoretically clean but its precision depends on the accuracy with which \( \mathcal{V}_{cb} \) and \( m_c \) are known. Also the scale uncertainties in \( \mathcal{V}_{td} \) amount to 4-5% at the NLO [140] [15]. They should be significantly reduced through a calculation of NNLO corrections [198] to the charm contribution that is in progress and should be available in 2005.

As emphasized in [15] an interesting determination of the angle \( \alpha \) can also be made by means of \( K! \). Assuming that the branching ratios will be known to within 10% we expect the following accuracy [15]

\[
(\sin^2 \alpha) = 0.05; \quad (\text{Im} \ \alpha) = 5\%; \quad (\mathcal{V}_{td}) = 7\%; \quad (\gamma) = 11: \quad (8.18)
\]

With the measurements of the branching ratios at the 5% level these estimates change to

\[
(\sin^2 \alpha) = 0.03; \quad (\text{Im} \ \alpha) = 3\%; \quad (\mathcal{V}_{td}) = 4\%; \quad (\gamma) = 6: \quad (8.19)
\]
Further details can be found in [15].

Clearly the UT resulted from $K^+$ decays could significantly differ from the one obtained in Fig. 8 by means of the standard UT analysis. This we show in Fig. 12 where each crossing point between the horizontal lines from $K_L^0$ and the circles from $K^+\!\!\!^+\!\!\!^+$ represents possible apex of the UT. The UT shown in this figure is the one of Fig. 8.

![Fig. 12: The UT from $K^0$ and $K^+\!\!\!^+\!\!\!^+$][15]

### 8.4 Golden Relations

The comparison of these results with the corresponding determinations in $B$ decays will offer a very good test of flavour dynamics and CP violation in the SM and a powerful tool to probe the physics beyond it. To this end a number of theoretically clean relations will play an important role. We list them here for completeness.

In [74] an upper bound on $\mathcal{B}(K^+\!\!\!^+\!\!\!^+)\!\!\!^+$ has been derived within the SM. This bound depends only on $\mathcal{Y}_{cb}^2 X$, and $M_{ud} = M_s$. With the precise value for the angle $\psi_{K_S}$ now available this bound can be turned into a useful formula for $\mathcal{B}(K^+\!\!\!^+\!\!\!^+)\!\!\!^+$ [112] that expresses this branching ratio in terms of theoretically clean observables. In the SM and any MFV model this formula reads:

$$\mathcal{B}(K^+\!\!\!^+) = \frac{\mathcal{Y}_{cb}^2 X^2}{4} \frac{1}{\mathcal{R}_t \sin^2 + 1} R_t \cos + \frac{4 \mathcal{P}_c \mathcal{Y}_{cb} \mathcal{X}}{\mathcal{Y}_{cb}^2 X} ; \tag{8.20}$$

where

$$+ = \frac{+}{g} = (7 \pm 6 \quad 0 \pm 9) \quad 1^0 ; \quad = \frac{1}{2} \quad \# \quad \frac{1}{2} ; \tag{8.21}$$

It can be considered as the fundamental formula for a correlation between $\mathcal{B}(K^+\!\!\!^+\!\!\!^+)$ and any observable used to determine $R_t$. This formula is theoretically very clean with the uncertainties residing only in $\mathcal{Y}_{cb}$ and $\mathcal{P}_c \mathcal{X}$. However, when one relates $R_t$ to some observable new uncertainties could enter. In [74] and [112] it has been proposed to express $R_t$ through $M_{ud} = M_s$ by means of (6.7). This implies an additional uncertainty due to the value of $\psi_{K_S}$.

In [15] it has been pointed out that if the strategy ( ; ) of Section 3 is used to determine $R_t$ by means of (3.29), the resulting formula that relates $\mathcal{B}(K^+\!\!\!^+\!\!\!^+)$ and $R_t$ is even cleaner than the
one that relates $\mathcal{B}r(K^+!^+)\), and $M_{d}=M_{s}$. We have then \[15\]

$$\begin{align*}
\mathcal{B}r(K^+!^+) & = \frac{4\mathcal{F}_dX}{\mathcal{Y}_{dd}^{-2}X^2}T_1^2 + \frac{1}{T_2 + \frac{4\mathcal{F}_dX}{\mathcal{Y}_{dd}}};
\end{align*}$$

(8.22)

where

$$\begin{align*}
T_1 & = \frac{\sin \sin (\theta)}{\sin (\theta)}; \quad T_2 = \frac{\cos \sin (\theta)}{\sin (\theta)};
\end{align*}$$

(8.23)

Next, the branching ratio for $K_L^0$ allows to determine

$$\begin{align*}
\mathcal{B}r(K_L^0) & = 0.351 \frac{3.34^{10}_L}{1.53^{10}_L \times \langle \alpha_t \rangle} \frac{0.0415^2}{3 \times 10^1};
\end{align*}$$

(8.24)

where

$$L = \frac{\mathcal{L}}{8} = (3.34, 0.05) \quad 10^1.$$  \hspace{1cm} (8.25)

The determination of in this manner requires the knowledge of $\mathcal{Y}_{dd}$ and $m_t$. With the improved determination of these two parameters a useful determination of should be possible.

On the other hand, the uncertainty due to $\mathcal{Y}_{dd}$ is not present in the determination of $\mathcal{M}_t$ as \[140\]:

$$\mathcal{M}_t = 1.39 \quad 10^{0.224}\left(3.34^{10}_L \times \langle \alpha_t \rangle \frac{0.0415^2}{3 \times 10^1}\right);$$

(8.26)

This formula offers the cleanest method to measure $\mathcal{M}_t$ in the SM and all MFV models in which the function $X$ takes generally different values than $X(\alpha_t)$. This determination is even better than the one with the help of the CP asymmetries in $B$ decays that require the knowledge of $\mathcal{Y}_{dd}$ to determine $\mathcal{M}_t$. Measuring $\mathcal{B}r(K_L^0)$ with 10% accuracy allows to determine $\mathcal{M}_t$ with an error of 5% \[13, 140\].

Next, in the spirit of the analysis in \[43\] we can use the clean CP asymmetries in $B$ decays and determine through the $(\omega, \omega)$ strategy. Using (8.27) and (8.29) in (8.24) we obtain a new “golden relation” \[15\]

$$\frac{\sin \sin (\theta)}{\sin (\theta)} = 0.351 \frac{3.34^{10}_L}{1.53^{10}_L \times \langle \alpha_t \rangle} \frac{0.0415^2}{3 \times 10^1};$$

(8.27)

This relation between , and $\mathcal{B}r(K_L^0)$, is very clean and offers an excellent test of the SM and of its extensions. Similarly to the “golden relation” in (8.29) it connects the observables in $B$ decays with those in $K$ decays and has other important virtues that are discussed in \[15\].

Finally, defining reduced branching ratios

$$B_1 = \frac{\mathcal{B}r(K^+!^+)}{\mathcal{L}}; \quad B_2 = \frac{\mathcal{B}r(K_L^0)}{\mathcal{L}};$$

(8.28)

one has \[39\]

$$\sin^2 = \frac{2r_3}{1 + r_3^2}; \quad r_3 = \mathcal{P}_{\mathcal{L}} \left(\frac{B_1}{B_2}\right) = \cot^2;$$

(8.29)

Thus, within the approximation of \[37\], is independent of $V_{dd}$ (or $A$) and $m_t$ and as shown in \[15\] these dependences are fully negligible.

It should be stressed that $\sin^2$ determined this way depends only on two measurable branching ratios and on the parameter $\mathcal{P}_{\mathcal{L}}(\mathcal{X})$ which is completely calculable in perturbation theory as discussed
in the previous section. Consequently this determination is free from any hadronic uncertainties and its accuracy can be estimated with a high degree of confidence. The calculation of NNLO QCD corrections to $P_c$ in [198] will certainly improve the accuracy of the determination of $\sin 2\theta$ from the $K^+$ complex.

8.5 Concluding Remarks

As the theorists were able to calculate the branching ratios for these decays rather precisely, the future of this field is in the hands of experimentalists and depends on the financial support that is badly needed.

9 Minimal Flavour Violation Models

9.1 Preliminaries

As discussed in Section 4 these are the simplest extensions of the SM. A detailed review of these models can be found in [12]. Here I would like to list five interesting properties of these models that are independent of particular parameters present in these models. Other relations can be found in [201, 12]. Here I would like first to list five interesting properties of these models that are independent of particular parameters present in these models. Other relations can be found in [201, 12].

There exists a universal unitarity triangle (UUT) [61] common to all these models and the SM that can be constructed by using measurable quantities that depend on the CKM parameters but are not polluted by the new parameters present in the extensions of the SM. The UUT can be constructed, for instance, by using measurable quantities that depend on the CKM parameters but are not polluted by the new parameters present in these models [12]. The relevant formulae can be found in Section 6 and in [61, 115], where also other quantities suitable for the determination of the UUT are discussed.

The golden relation [139, 115]:

$$\sin 2\theta_K = \sin 2\theta$$  \hspace{1cm} (9.1)

For given $\sin 2\theta$ and $B \tau (K^0, K^0, K^0)$ only two values of $B \tau (K_L, K_L, K_L)$ are possible in the full class of MFV models, independently of any new parameters present in these models [115]. These two values correspond to two signs of the function $X(v)$. Consequently, measuring $B \tau (K_L, K_L, K_L)$ will either select one of these two possible values or rule out all MFV models. A very recent analysis shows that the case of $X(v) < 0$ is very unlikely [202], living basically only one value for $B \tau (K_L, K_L, K_L)$ once $B \tau (K^+!K^+, K^+!K^+, K^+!K^+)$ has been precisely measured.

There exists a correlation between $B \tau (B_\text{dbs}!, B_\text{dbs}!)$, and $M_{d_{bs}}$ [203]:

$$\frac{B \tau (B_\text{dbs}!, B_\text{dbs}!)}{M_{d_{bs}}} = \frac{\beta_{d_{bs}}}{\beta_{d_{bs}}} \frac{M_{d}}{M_{s}}$$  \hspace{1cm} (9.2)

where $(B_q)$ are $B$-meson life-times and $\beta_q$ non-perturbative parameters in $M_q$ with $\beta_d = 1.34$, $0.12$ and $\beta_s = 1.34$, $0.12$ obtained in lattice simulations [6]. This correlation is practically free of theoretical uncertainties as $\beta_s = \beta_d = 1$ up to small SU (3) breaking corrections.

Similar correlations between $B \tau (B_\text{dbs}!, B_\text{dbs}!)$ and $M_{d_{bs}}$ respectively, allow rather precise predictions for $B \tau (B_\text{dbs}!, B_\text{dbs}!)$ within the MFV models once $M_{d_{bs}}$ are known [203]. Indeed one finds (q = s, d):

$$\frac{B \tau (B_q!, M_q)}{F(v)} = 4 \lambda^0 \frac{B_q}{F(v)}; \hspace{1cm} F(v) = \frac{Y^2(v)}{S(v)}$$  \hspace{1cm} (9.3)

In the SM, $F_\text{SM} = 0.40$. 

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9.2 Universal Unitarity Triangle

The presently available quantities that do not depend on the new physics parameters within the MFV models and therefore can be used to determine the UUT are $R_t$ from $M_d = M_s$ by means of (6.7), $R_b$ from $y_{ub} = y_{cb}$ by means of (6.17) and $\sin 2\beta$ extracted from the CP asymmetry in $B^0_d \to K_S$. Using only these three quantities, we show in the UUT column of table 3 the results for various quantities of interest related to this UUT [15]. A similar analysis has been done in [42, 62, 102]. In particular [102] finds

$$\begin{align*}
\sin 2\beta & = 0.353 \pm 0.028; \\
\sin 2\beta & = 0.191 \pm 0.046
\end{align*}$$

(9.4)

in a good agreement with the results of table 3.

It should be stressed that any MFV model that is inconsistent with the UUT column in table 3 is ruled out. We observe that there is little room for MFV models that in their predictions for UT differ significantly from the SM. It is also clear that to distinguish the SM from the MFV models on the basis of the analysis of the UT of Section 6, will require a considerable reduction of theoretical uncertainties.

9.3 Models with Universal Extra Dimensions

In view of the difficulty in distinguishing various MFV models on the basis of the standard analysis of UT from each other, it is essential to study other FCNC processes as rare $B$ and $K$ decays and radiative $B$ decays like $B \to X_s$ and $B \to X_s^+$ + . In the case of MSSM at low $\tan \beta$ such analyses can be found in [67, 204]. In 2003 a very extensive analysis of all relevant FCNC processes in a SM with one universal extra dimension [205] has been presented in [65, 66]. In this model all standard model fields can propagate in the fifth dimension and the FCNC processes are affected by the exchange of the Kaluza-Klein particles in loop diagrams. The most interesting results of [65, 66] are the enhancements of $\text{Br}(B \to X_s^+)$ and $\text{Br}(B \to X_s^+)$, strong suppressions of $\text{Br}(B \to X_s^-)$ (see also [206]) and $\text{Br}(B \to X_s^+)$, and a significant downward shift of the zero $\rho_0$ in the forward-backward asymmetry in $\text{Br}(B \to X_s^+)$. As pointed out in [66], this downward shift of $\rho_0$ is correlated with the suppression of $\text{Br}(B \to X_s^-)$. This is seen in Fig. 13. This property is characteristic for all MFV models and is verified in a MSSM with MFV even after the inclusion of NNLO QCD corrections [80].

![Fig. 13: Correlation between $\text{Br}(B \to X_s^-)$ and $\rho_0$ [66]. The dots are the results in the ACD model (see below) with the compactification scale 200; 250; 300; 350; 400; 600 and 1000 GeV and the star denotes the SM value.](image-url)
9.4 FCNC Processes in the Littlest Higgs Model

The little Higgs models [207]-[211] offer an attractive and a rather simple solution to the gauge hierarchy problem. In these models the electroweak Higgs boson is regarded as a pseudo-goldstone boson of a certain global symmetry that is broken spontaneously at a scale \( 4 \ell \sim 10 \text{ TeV} \), much higher than the vacuum expectation value \( v \) of the standard Higgs doublet. The Higgs field remains then light, being protected by the approximate global symmetry from acquiring quadratically divergent contributions to its mass at the one-loop level. On the diagramatic level the new heavy particles present in these models allow to cancel, analogously to supersymmetric particles, the quadratic divergencies in question. Reviews of the little Higgs models can be found in [212].

One of the simplest models of this type is the “Littlest Higgs” model [210] (LH) in which, in addition to the Standard Model (SM) particles, new charged heavy vector bosons \( (W_H) \), a neutral heavy vector boson \( (Z_H) \), a heavy photon \( (A_H) \), a heavy top quark \( (T) \) and a triplet of heavy Higgs scalars \( (++, +, 0) \) are present. The details of this model have been worked out in [213] and the constraints from various processes, in particular from electroweak precision observables and direct new particles searches, have been extensively discussed in [213]-[219]. It has been found that except for the heavy photon \( A_H \), that could still be as “light” as 500 GeV, the masses of the remaining particles are constrained to be significantly larger than 1 TeV.

The new particles can also contribute to FCNC processes. In [81], in collaboration with Anton Poschenrieder and Selma Uhlig, the \( K^0 \rightarrow K^0, B_{d,s}^0 \rightarrow B_{d,s}^0 \) mixing mass differences \( M_{K^0}, M_{B_{d,s}^0} \) and the CP-violating parameter \( \epsilon_K \) have been calculated in the LH model. We have found that even for \( f=v \) as low as 5, the enhancement of \( M_{ud} \) for the \( t \rightarrow T \) mixing parameter \( x_L \) amounts to at most 20%. Similar comments apply to \( M_{s} \) and \( \epsilon_K \). The correction to \( M_K \) is negligible. Our results have recently been confirmed in [220]. Only for \( x_L \) \( 0.05 \) significantly larger effects could be found [81][82]. Significant effects could be present in \( D^0 \rightarrow D^0 \) mixing [221], where in contrast to processes involving external down quarks, FCNC transitions are already present at the tree level.

Concerning FCNC decay processes, the corrections to \( B^0 \rightarrow X_s^0 \) have been found to be small [222], while the analysis of \( K^0 \rightarrow X_s^0 \) in [223] resulted in a large enhancement of the relevant branching ratio.

In [83] we have extended our study of FCNC processes in the LH model to the rare decays \( K^+ \rightarrow +, K_L^0 \rightarrow 0, B_{s,d}^{+} \rightarrow +, B^0 \rightarrow X_{s,d}^{0} \). Preliminary results appeared in [82]. Our results differ from those of [223] in that we find only very small corrections to \( K^+ \) and \( K_L^0 \). On the other hand a significant enhancement of \( B^0 \rightarrow (B^0 \rightarrow X_s^0) \) has been identified.

The details of this analysis will appear in [83]. Although no dramatic modifications of the SM expectations for FCNC decays have been found, the analysis is interesting from the technical point of view and can be considered as a symphony of new particle contributions to rare decays. Indeed, this analysis involving many diagrams, in particular the \( Z^0, Z_H \) and \( A_H \) penguins with the heavy \( T, W_H \), and the ordinary quarks exchanges has a certain beauty in view of only three new parameters involved.

9.5 Upper Bounds on rare K and B Decays from MFV

Very recently a detailed analysis of several branching ratios for rare \( K \) and \( B \) decays in MFV models has been performed in [202]. Using the presently available information on the UUT, summarized in [9.4], and from the measurements of \( B^0 \rightarrow (B^0 \rightarrow X_s^0) \), \( B^0 \rightarrow (B^0 \rightarrow X_s^0) \) and \( B^0 \rightarrow (K^+ \rightarrow +, K_L^0 \rightarrow 0 \), the upper bounds on various branching ratios within the MFV scenario have been found. They are collected in
Table 4: Upper bounds for rare decays in MFV models at 95% probability, the corresponding values in the SM (using inputs from the UUT analysis) and the available experimental information. See [202] for details.

| Branching Ratios | MFV (95%) | SM (68%) | SM (95%) | exp |
|------------------|-----------|----------|----------|-----|
| Br(K⁺ ! s⁺) | 10⁻¹ | < 11.9 | 8.3 | 1.2 | [6.1;10.9] | (14.7 ± 3.3) | 186 |
| Br(K L ! 0 ) | 10⁻¹ | < 4.5 | 3.0 | 0.6 | [2.0;3.4] | < 5.9 | 10 | 190 |
| Br(K L ! 0 ) | 10⁻² | < 1.3 | 0.8 | 0.1 | [0.6;1.5] | - |
| Br(B ! X s ) | 10⁻² | < 5.1 | 3.6 | 0.2 | [2.5;4.0] | < 6.4 | 262 |
| Br(B ! X d ) | 10⁻² | < 2.1 | 1.5 | 0.1 | [1.0;2.0] | - |
| Br(B d ! 0 ) | 10⁻² | < 7.4 | 3.7 | 1.0 | [1.9;5.0] | < 2.7 | 10 | 264 |
| Br(B d ! 0 ) | 10⁻³ | < 2.2 | 1.0 | 0.1 | [0.8;1.2] | < 1.5 | 10 | 264 |

Table 4 together with the results within the SM. Moreover with 95% probability one finds

\[
X (\nu)_{m ax} = 1.95; \quad Y (\nu)_{m ax} = 1.43; \quad Z (\nu)_{m ax} = 1.46; \quad (9.5)
\]

to be compared with \( X = 1.54, Y = 0.99 \) and \( Z = 0.69 \) in the SM. Finally, anticipating that the leading role in constraining this kind of physics will eventually be taken over by \( K⁺ ! s⁺, K_L ! 0 \) and \( B_{sd} ! + + \), that are dominated by the function \( C (\nu) \), reference [202] provides plots for several branching ratios as functions of \( C (\nu) \).

The main message from from [202] is the following one:

The existing constraints coming from \( K⁺ ! s⁺, B ! X s \) and \( B ! X s^⁺ l \) do not allow within the MFV scenario of [61] for substantial departures of the branching ratios for all rare \( K \) and \( B \) decays from the SM estimates. This is evident from Table 4.

This could be at first sight a rather pessimistic message. On the other hand it implies that finding practically any branching ratio enhanced by more than a factor of two with respect to the SM will automatically signal either the presence of new CP-violating phases or new operators, strongly suppressed within the SM, at work. In particular, recalling that in most extensions of the SM the decays \( K⁺ ! s⁺, K_L ! 0 \) are governed by the single \( (V A)(V A) \) operator, the violation of the upper bounds on at least one of the \( K⁺ ! s⁺ \) branching ratios, will either signal the presence of new complex weak phases at work or new contributions that violate the correlations between the \( B \) decays and \( K \) decays present in models with MFV.

9.6 Final Comments on MFV

Assuming that the MFV scenario will survive future tests, the next step will be to identify the correct model in this class. Clearly, direct searches at high energy colliders can rule out or identify specific extensions of the SM. But also FCNC processes can play an important role in this context, provided the theoretical and experimental uncertainties in some of them will be sufficiently decreased. In this case, by studying simultaneously several branching ratios it should be in principle possible to select the correct MFV models by just identifying the pattern of enhancements and suppressions relative to the SM that is specific to a given model. If this pattern is independent of the values of the parameters defining the model, no detailed quantitative analysis of the enhancements and suppressions is required in order to rule it out. As an example the distinction between the MSSM with MFV and the models with one universal extra dimension should be straightforward:

In the MSSM with MFV the branching ratios for \( K⁺ ! s⁺, K_L ! 0 \) and \( B ! X_d \) and \( B_d ! + + \) are generally suppressed relative to the SM expectations, while those governed by
Finally, if MFV will be confirmed, and some new particles will be observed, the rare processes discussed in this work will constitute a most powerful tool to probe the spectrum of the NP model, which might not be entirely accessible via direct studies at the LHC.

10 New Weak Phases

10.1 Preliminaries

We will now consider the class C of Section 5.7. In this class of models the dominant operators are as in the MFV models (class A) but the master functions become now complex quantities. If the new weak phases are large, the deviations from the SM can be spectacular as we will see below.

Let us consider first $F = 2$ transitions. In the MFV scenario discussed in the previous section, the NP effects enter universally in $K^0$, $B^0_d$, $B^0_s$ and $B^0_s$ mixings through the single real function $S(v)$, implying strong correlations between new physics effects in the $F = 2$ observables of $K$ and $B$ decays. When new complex weak phases are present, the situation could be more involved with $S(v)$ replaced by

$$S_K(v) = \mathcal{F}_K(v) \mathcal{V}^i_v, \quad S_d(v) = \mathcal{F}_d(v) \mathcal{V}^i_d, \quad S_s(v) = \mathcal{F}_s(v) \mathcal{V}^i_s; \quad (10.1)$$

for $K^0$, $B^0_d$, $B^0_s$ and $B^0_s$ mixing, respectively. If these three functions are different from each other, some universal properties found in the MFV models A are lost. In addition the mixing induced CP asymmetries in $B$ decays will not measure the angles of the UT but only sums of these angles and of $\theta$. An example is given in (6.20). Yet, within each class of $K$, $B_d$ and $B_s$ decays, the NP effects of this sort will be universal. Scenarios of this type have been considered for instance in 109, 110, 111, 112, 113, 115.

New weak phases could enter also decay amplitudes. As now these effects enter in principle differently in each decay, the situation can be very involved with many free parameters, no universal effects and little predictive power.

Here I will only discuss one scenario, discussed first in 224–228 and recently in the context of a simultaneous analysis of prominent non–leptonic $B$ decays like $B^0 \rightarrow X_s$, $B^+ \rightarrow X_s$, $K^0 \rightarrow X_s$ and $B^+ \rightarrow X_{sd}$ and equally prominent rare decays like $K^+ \rightarrow 0e^+e^−$, $B_{sd}^0 \rightarrow X_{sd}$ and $n^\text{Isr}$ in 411, 229. It is the scenario of enhanced $Z^0$ penguins with a large complex weak phase in which the only modification with respect to the MFV models is the replacement in the $Z^0$ penguin function $C(v)$ of $\mathcal{F}(v) \mathcal{V}^i_c$ that makes the master functions $X(v)$, $Y(v)$ and $Z(v)$ of Section 4 to be complex quantities:

$$X(v) = \mathcal{F}(v) \mathcal{V}^i_c X; \quad Y(v) = \mathcal{F}(v) \mathcal{V}^i_c Y; \quad Z(v) = \mathcal{F}(v) \mathcal{V}^i_c Z; \quad (10.2)$$

The magnitudes and phases of these three functions are correlated with each other as they depend only on $\mathcal{F}(v) \mathcal{V}^i_c$ and other smaller contributions that can be set to their SM values. This new analysis has been motivated by an interesting experimental situation in $B^0 \rightarrow X_s$ and $B^+ \rightarrow K$ decays that we will summarize below.
10.2 Basic Strategy

The present studies of non-leptonic two-body $B$ decays and of rare $K$ and $B$ decays are very important as they will teach us both about the non-perturbative aspects of QCD and about the perturbative electroweak physics at very short distances. For the analysis of these modes, it is essential to have a strategy available that could clearly distinguish between non-perturbative QCD effects and short-distance electroweak effects. A strategy that in the case of deviations from the SM expectations would allow us transparently to identify a possible necessity for modifications in our understanding of hadronic effects and for a change of the SM picture of electroweak flavour-changing interactions at short-distance scales.

In [41, 229], we have developed a strategy that allows us to address these questions in a systematic manner. It encompasses non-leptonic $B$ and $K$ decays and rare $K$ and $B$ decays but has been at present used primarily for the analysis of $B \to \pi^0 K^0$ systems and rare $K$ and $B$ decays. In what follows I will summarize the basic ingredients of our strategy and list the most important results. The basic concepts can be found in [229, 41] and an update has been presented in [230]. The discussion presented below is entirely based on these papers and borrows a lot from the talk in [231]. In order to illustrate our strategy in explicit terms, we shall consider a simple extension of the SM in which new physics (NP) enters dominantly through enhanced $Z^0$ penguins involving a CP-violating weak phase, as advertised above. As we will see below, this choice is dictated by the pattern of the data on the $B \to \pi^0 K^0$ observables and the great predictivity of this scenario. Our strategy consists of three interrelated steps, and has the following logical structure:

Step 1:
Since $B \to \pi^0 K^0$ decays and the usual analysis of the unitarity triangle (UT) are only insignificantly affected by electroweak (EW) penguins, the $B \to \pi^0 K^0$ system can be described as in the SM. Employing the $SU(2)$ isospin flavour symmetry of strong interactions and the information on $\theta_{12}$ from the UT fits, we may extract the hadronic parameters of the $B \to \pi^0 K^0$ system, and find large non-factorizable contributions, which are in particular reflected by large CP-conserving strong phases. Having these parameters at hand, we may then also predict the direct and mixing-induced CP asymmetries of the $B_d \to \pi^0 K^0$ channel. See Table 7. A future measurement of one of these observables allows a determination of $\theta_{12}$.

Step 2:
If we use the $SU(3)$ flavour symmetry and plausible dynamical assumptions, we can determine the hadronic $B \to K$ parameters from the hadronic parameters of the $B \to \pi^0 K^0$ system found in Step 1. Subsequently, this allows us to calculate the $B \to K$ observables in the SM. Interestingly, we find agreement with the pattern of the $B$-factory data for those observables where EW penguins play only a minor rôle. On the other hand, the observables receiving significant EW penguin contributions do not agree with the experimental picture, thereby suggesting NP in the EW penguin sector. Indeed, a detailed analysis shows [41, 229, 230] that we may describe all the currently available data through moderately enhanced EW penguins with a large CP-violating NP phase around $90^\circ$. A future test of this scenario will be provided by the CP-violating $B_d \to \pi^0 K^0_S$ observables, which we may predict. See Table 7. Moreover, we may obtain valuable insights into $SU(3)$-breaking effects, which support our working assumptions, and may also determine the UT angle $\theta_{13}$, in remarkable agreement with the UT fit of Section 6.

Step 3:
In turn, the modified EW penguins with their large CP-violating NP phase have important implications
for rare $K$ and $B$ decays. Interestingly, several predictions differ significantly from the SM expectations and should easily be identified once the data improve. Similarly, we may explore specific NP patterns in other non-leptonic $B$ decays such as $B_d \to K_S$.

A chart of the three steps in question is given in Fig. 14. Before going into the details it is important to emphasize that our strategy is valid both in the SM and all SM extensions in which NP enters predominantly through the EW penguin sector. This means that even if the presently observed deviations from the SM in the $B \to K$ sector would diminish with improved data, our strategy would still be useful in correlating the phenomena in $B \to \pi K$, $B \to K$ and rare $K$ and $B$ decays within the SM. If, on the other hand, the observed deviations from the SM in $B \to \pi K$ decays would not be attributed to the modification in hadron dynamics but to NP contributions, our approach should be properly generalized.

### 10.3 $B \to \pi K$ decays

The central quantities for our analysis of the $B \to \pi K$ decays are the ratios

\[
R_+ = \frac{2 \left( \frac{\text{Br}(B_d \to \pi^+ K^-)}{\text{Br}(B_d \to \pi^+ K^-)} + \frac{\text{Br}(B_d \to \pi^0 K^-)}{\text{Br}(B_d \to \pi^0 K^-)} \right)}{1 + \frac{\text{Br}(B_d \to \pi^0 K^-)}{\text{Br}(B_d \to \pi^0 K^-)}}
\]

(10.3)

\[
R_{00} = \frac{2 \left( \frac{\text{Br}(B_d \to 0^+ K^-)}{\text{Br}(B_d \to 0^+ K^-)} + \frac{\text{Br}(B_d \to 0^0 K^-)}{\text{Br}(B_d \to 0^0 K^-)} \right)}{1 + \frac{\text{Br}(B_d \to 0^0 K^-)}{\text{Br}(B_d \to 0^0 K^-)}}
\]

(10.4)
of the CP-averaged branching ratios, and the CP-violating observables provided by the time-dependent rate asymmetry

\[
\frac{\text{Br} \left( B^+ \to \mu^+ \nu \right)}{\text{Br} \left( B^0 \to \mu^+ \nu \right)} + \frac{\text{Br} \left( B^0 \to \mu^+ \nu \right)}{\text{Br} \left( B^+ \to \mu^+ \nu \right)} = A_{CP}^{\mu} \left( B^+ \to \mu^+ \nu \right) \sin(M_{B^0} t) + A_{CP}^{\mu} \left( B^0 \to \mu^+ \nu \right) \sin(M_{B^0} t); \tag{10.5}
\]

The current status of the $B^+$ data together with the relevant references can be found in Table 5. The so-called “$B^+$ puzzle” is reflected in a surprisingly large value of $B^+ \text{Br} \left( B^0 \to \mu^+ \nu \right)$ and a somewhat small value of $B^+ \text{Br} \left( B^0 \to \mu^+ \nu \right)$, which results in large values of both $R_{00}$ and $R_{+}$. For instance, the central values calculated within QCD factorization (QCDF) give $R_{+} = 1.24$ and $R_{00} = 0.97$, although in the scenario “S4” of values 2.0 and 0.2, respectively, can be obtained. As pointed out in [229], these data indicate important non-factorizable contributions rather than NP effects, and can be perfectly accommodated in the SM. The same applies to the NP scenario considered in [229] [41], in which the EW penguin contributions to $B^+$ are marginal.

In order to address this issue in explicit terms, we use the isospin symmetry to find

\[
P = 2A \left( B^+ \to \mu^+ \nu \right) \left( 1 + 0 \right), \quad T + C^\prime = [T + C]; \tag{10.6}
\]

\[
A \left( B^0 \to \mu^+ \nu \right) = T + P \tag{10.7}
\]

\[
P = 2A \left( B^0 \to \mu^+ \nu \right) \left( 1 + 0 \right), \quad C^\prime = [C + \left] E \right]; \tag{10.8}
\]

The individual amplitudes of (10.6)–(10.8) can be expressed as

\[
P = 3A \left( P_\tau \ P_c \right) \left( P_\tau \ P_c \right) \tag{10.9}
\]

\[
T = 3A \left( P_\tau \ P_\mu \ E \right) \tag{10.10}
\]

\[
C^\prime = 3A \left( P_\tau \ P_\mu \ E \right) \tag{10.11}
\]

where $A$ and $R_b$ have been defined in Section 3. The $P_q$ describe the strong amplitudes of QCDF penguins with internal $q$-quark exchanges ($q \ 2 \ f t ; C ; u g$), including annihilation and exchange penguins, while $T$ and $C^\prime$ are the strong amplitudes of colour-allowed and colour-suppressed tree-diagram-like topologies, respectively, and $E$ denotes the strong amplitude of an exchange topology. The amplitudes $T$ and $C^\prime$ differ from

\[
T = 3A \left( P_\tau \ P_\mu \right) T; \quad C^\prime = 3A \left( P_\tau \ P_\mu \right) C \tag{10.12}
\]
through the $\mathcal{P}_{ui} - \mathcal{E}$ pieces, which may play an important rôles in the GIM penguins with internal up-quark exchanges, whereas their "charming penguin" counterparts enter in $\mathcal{P}$ through $\mathcal{P}_{ci}$, as can be seen in (10.9) [172, 173, 240, 241].

In order to characterize the dynamics of the $B^+$ system, we introduce four hadronic parameters $c_d$, $x$ and $\theta$ through

$$
d e^i = \frac{P}{T} e^{i(\theta - r)}; \quad x e^i = \frac{C}{T} e^{i(\phi - r)};
$$

(10.13)

with $\theta$ being strong phases. Using this parametrization, we have

$$
R_+ = F_1 (d; \; \chi; \; 2); \quad R_{00} = F_2 (d; \; \chi; \; 2);
$$

(10.14)

$$
A_{CP}^{dK} (B_d ! +) = G_1 (d; \; 2); \quad A_{CP}^{mK} (B_d ! +) = G_2 (d; \; 2);
$$

(10.15)

with explicit expressions for $F_1$, $F_2$, $G_1$ and $G_2$ given in [41]. Taking then as the input

$$
(65 \quad 7); \quad d = 2 = 46 \mp \frac{32}{32};
$$

(10.16)

and the data for $R_+$, $R_{00}$, $A_{CP}^{dK}$ and $A_{CP}^{mK}$ of Table 5, we obtain a set of four equations with the four unknowns $d$, $\chi$ and $\theta$. Interestingly, as demonstrated in [41, 230], a unique solution for these parameters can be found

$$
d = 0 \pm 0.26; \quad \chi = 140 \pm 14; \quad \chi = 1 \pm 0.18; \quad \chi = 59 \pm 19;
$$

(10.17)

The large values of the strong phases and $\chi$ and the large values of $d$ and $\chi$ signal departures from the picture of QCDF. Going back to (10.10) and (10.11) we observe that these effects can be attributed to the enhancement of the $\mathcal{P}_{ui} - \mathcal{E}$ terms that in turn suppresses $\mathcal{T}$ and enhances $\mathcal{C}$. In this manner, $\mathcal{B}(B_d ! +)$ and $\mathcal{B}(B_d ! 0)$ are suppressed and enhanced, respectively.

With the hadronic parameters at hand, we can predict the direct and mixing-induced CP asymmetries of the $B_d ! 0$ channel. They are given in Table 7. As seen there these predictions, while still subject to large uncertainties, have been confirmed by the most recent data. On the other hand, as illustrated in [41], a future precise measurement of $A_{CP}^{dK} (B_d ! 0)$ or $A_{CP}^{mK} (B_d ! 0)$ allows a theoretically clean determination of $\chi$.

The large non-factorizable effects found in [229] have been discussed at length in [41, 230], and have been confirmed in [241, 242, 243, 244, 245]. For the following discussion, the most important outcome of this analysis is the values of the hadronic parameters $d$, $\chi$ and $\theta$. These quantities allow us – with the help of the SU(3) flavour symmetry – to determine the corresponding hadronic parameters of the $B ! K$ system.

### 10.4 $B ! K$ decays

The key observables for our discussion are the following ratios:

$$
\begin{align*}
R_{B^+} & = \frac{\mathcal{B}(B_d ! K^+) + \mathcal{B}(B_d ! + K)}{\mathcal{B}(B_d ! + K^0) + \mathcal{B}(B_d ! K^0)} \quad \# \quad B_d^+ \quad (10.18) \\
R_{B^0} & = \frac{2 \mathcal{B}(B_d ! 0K^+) + \mathcal{B}(B_d ! ^0K)}{\mathcal{B}(B_d ! + K^0) + \mathcal{B}(B_d ! K^0)} \quad \# \quad B_d^0 \quad (10.19) \\
R_{K^+} & = \frac{1 + \mathcal{B}(B_d ! K^+) + \mathcal{B}(B_d ! + K)}{2 \mathcal{B}(B_d ! 0K^0) + \mathcal{B}(B_d ! 0K^0)} \quad ; \quad (10.20)
\end{align*}
$$
Table 6: The current status of the CP-averaged $B \rightarrow K$ branching ratios, with averages taken from [232]. We also give the values of the ratios $R$, $R_c$ and $R_n$, introduced in [10.18], [10.19] and [10.20], where $R$ refers again to $B = B_d = 0.016 \pm 0.017$. In the last column we also give the values of $R_i$ at the time of the analyses in [230] [31].

where the current status of the relevant branching ratios and the corresponding values of the $R_i$ is summarized in Table 6. The so-called “$B \rightarrow K$ puzzle”, which was already pointed out in [168], is reflected in the small value of $R_n$ that is significantly lower than $R_c$. We will return to this below.

In order to analyze this issue, we neglect for simplicity the colour-suppressed EW penguins and use the $SU(3)$ flavour symmetry to find:

\[
A (B_d^0 \rightarrow K^+) = P^h_0 \Re e^i \quad (10.21)
\]

\[
A (B_d^0 \rightarrow K^0) = P^h_0 \quad (10.22)
\]

\[
A (B^+ \rightarrow B^0 K^+) = P^h_0 1 e^i \quad (10.23)
\]

\[
A (B_d^0 \rightarrow K^0) = P^h_0 1 + n e^i + n e^i \quad (10.24)
\]

Here, $P^h_0$ is a QCD penguin amplitude that does not concern us as it cancels in the ratios $R_i$ and in the CP asymmetries considered. The parameters $r_c$ and $r_n$ are of hadronic origin. If they were considered as completely free, the predictive power of $(10.21)$–$(10.24)$ would be rather low. Fortunately, using the $SU(3)$ flavour symmetry, they can be related to the parameters $d$, $x$ and $s$ in $(10.17)$. Explicit expressions for these relations can be found in [41]. In this manner, the values of $r_c$ and $r_n$ can be found. The specific numerical values for these parameters are not of particular interest here and can be found in [230]. It suffices to say that they also signal large non-factorizable hadronic effects.

The most important recent experimental result concerning the $B \rightarrow K$ system is the observation of direct CP violation in $B_d \rightarrow K$ decays [155] [156]. This phenomenon is described by the following rate asymmetry:

\[
A_{CP}^{dir} (B_d \rightarrow K) = \frac{B (B_d^0 \rightarrow K^+)}{B (B_d^0 \rightarrow K^0)} = +0.113_{-0.019}^{+0.021}
\]

where the numerical value is the average compiled in [232]. Using the values of $r_c$ and $r_n$ as determined above and $(10.21)$, we obtain $A_{CP}^{dir} (B_d \rightarrow K) = +0.127^{+0.021}_{-0.026}$, which is in nice accordance with the experimental result. Following the lines of [248] [249], we may determine the angle of the UT by complementing the CP-violating $B_d \rightarrow K$ observables with either the ratio of the CP-averaged branching ratios $B (B_d \rightarrow K)$ and $B (B_d \rightarrow K)$ or the direct CP-asymmetry $A_{CP}^{dir} (B_d \rightarrow K)$. These avenues, where the latter is theoretically more favourable, yield the following results:

\[
\delta = 63.3^{+7.7}_{-15.8} \quad ; \quad \delta_{CP}^{dir} = 66.6^{+11.0}_{-11.8} \quad : (10.26)
\]
which are nicely consistent with each other. Moreover, these ranges are in remarkable accordance with the results of Section 6. A similar extraction of can be found in [252].

Apart from $A_{CP}^{\text{LF}}(B_d \rightarrow K)$, two observables are left that are only marginally affected by EW penguins: the ratio $R$ introduced in (10.18) and the direct CP asymmetry of $B ightarrow K$ modes. These observables may be affected by another hadronic parameter $\epsilon^{\pm} c$, which is expected to play a minor rôle and was neglected in (10.22) and (10.23). In this approximation, the direct $B ightarrow K$ CP asymmetry vanishes — in accordance with the experimental value of $+0.020 \pm 0.034$ — and the new experimental value of $R = 0.92 \pm 0.06$, which is on the lower side, can be converted into $\left( 64^{+4}_{-6} \right)$ with the help of the bound derived in [157]. On the other hand, if we use the values of $\epsilon$ and $c$ as determined above, we obtain

$$R = 0.943^{+0.028}_{-0.021},$$

which is sizeably larger than the experimental value. The nice agreement of the data with our prediction of $A_{CP}^{\text{LF}}(B_d \rightarrow K)$, which is independent of $\epsilon^c$, suggests that this parameter is actually the origin of the deviation of $R$. In fact, as discussed in detail in [230], the emerging signal for $B \rightarrow K \bar{K}$ decays, which provide direct access to $\epsilon^c$, shows that our value of $R$ in (10.27) is shifted towards the experimental value through this parameter, thereby essentially resolving this small discrepancy. A nice related discussion can be found in [250] [251].

It is important to emphasize that we could accommodate all the $B \rightarrow K$ data so far nicely in the SM. Moreover, as discussed in detail in [41] [230], there are also a couple of internal consistency checks of our working assumptions, which work very well within the current uncertainties.

Let us now turn our attention to (10.23) and (10.24). The only variables in these formulae that we did not discuss so far are the parameters $\eta$ and $\epsilon$ that parametrize the EW penguin sector. The fact that EW penguins cannot be neglected here is related to the simple fact that a $K$ meson can be emitted directly in these colour-allowed EW penguin topologies, while the corresponding emission with the help of QCD penguins is colour-suppressed. In the SM, the parameters $\eta$ and $\epsilon$ can be determined with the help of the $SU(3)$ flavour symmetry of strong interactions [162], yielding

$$\eta = 0 \pm 9 \quad \text{and} \quad \epsilon = 0 : \quad (10.28)$$

In this manner, predictions for $R_{C}$ and $R_{n}$ can be made [229] [41], which read as follows [230]:

$$R_{C} = 1.14 \pm 0.05 ; \quad R_{n} = 1.14^{+0.04}_{-0.05} : \quad (10.29)$$

Comparing with the experimental results in Table 6, we observe that there is only a marginal discrepancy in the case of $R_{C}$, whereas the value of $R_{n}$ in (10.29) is definitely too large. The "B -> K" puzzle is seen here in explicit terms.

The disagreement of the SM with the data can be resolved in the scenario of enhanced EW penguins carrying a non-vanishing phase $\eta$. Indeed, using the measured values of $R_{n}$ and $R_{C}$, we find [230]:

$$\eta = 1.88^{+0.04}_{-0.02} \quad \text{and} \quad \epsilon = 137^{+137}_{-192} : \quad (10.30)$$

We observe that — while $\eta$ is consistent with the SM estimate within the errors — the large phase $\epsilon$ is a spectacular signal of possible NP contributions. It is useful to consider the $R_{n} - R_{C}$ plane, as we have done in Fig. 15. There we show contours corresponding to different values of $\eta$, which is independent of $\epsilon$ as determined above, and SM ranges.

We close this section with a list of predictions for the CP asymmetries in the $B \rightarrow K$ systems, which are summarized in Table 7.
Our Prediction

Fig. 15: The situation in the $R_c$–$R_e$ plane. We show contours for values of $q_1 = 0.69$, $q_2 = 1.22$ and $q_3 = 1.75$, with $2 \, \sigma$ intervals. The experimental ranges for $R_c$ and $R_e$ and those predicted in the SM are indicated in grey, the dashed lines serve as a reminder of the corresponding ranges in [41].

| Quantity | Our Prediction | Experiment |
|----------|----------------|------------|
| $A_{CP}^{\text{dir}} (B_d \to 0^0)$ | $0.28 \pm 0.037$ | $0.28 \pm 0.039$ |
| $A_{CP}^{\text{mix}} (B_d \to 0^0)$ | $0.63 \pm 0.043$ | $0.48 \pm 0.048$ |
| $A_{CP}^{\text{dir}} (B \to K)$ | $0.127 \pm 0.002$ | $0.113 \pm 0.019$ |
| $A_{CP}^{\text{mix}} (B \to K)$ | $0.10 \pm 0.005$ | $0.04 \pm 0.04$ |
| $A_{CP}^{\text{dir}} (B \to K_S)$ | $0.01 \pm 0.015$ | $0.09 \pm 0.14$ |
| $A_{CP}^{\text{mix}} (B \to K_S)$ | $0.08 \pm 0.04$ | $0.02 \pm 0.02$ |

Table 7: Compilation of our predictions for the CP-violating $B \to K$ asymmetries.

10.5 Implications for Rare $K$ and $B$ Decays

The rates for rare $K$ and $B$ decays are sensitive functions of the EW penguin contributions. We have discussed it in detail in Section 8 in the context of $K \to \pi^0 \pi^0$ decays. In a simple scenario in which NP enters the EW penguin sector predominantly through $Z^0$ penguins and the local operators in the effective Hamiltonians for rare decays are the same as in the SM, there is a strict relation [254, 229, 41] between the EW penguin effects in the $B \to K$ system and the corresponding effects in rare $K$ and $B$ decays. The $Z^0$-penguin function $C(x_e)$ of the SM generalizes to $C(v)$ with

$$C(v) = \frac{\mathcal{F}(v) \mathcal{F}^{\dagger}}{\mathcal{F}(v) \mathcal{F}^{\dagger}} = 2 \sqrt{35} q e^{i \phi} \bar{u} \ell = 0.82; \quad q = q \frac{V_{ub}^{\text{m}}}{V_{cb}} = 0.086 .$$

In turn, the functions $X, Y, Z$ that govern rare decays in the scenarios in question become explicit functions of $q$ and $\phi$.

$$X = \frac{\mathcal{K}^{\dagger} \mathcal{K}}{\mathcal{F}(v) \mathcal{F}^{\dagger}} = 2 \sqrt{35} q e^{i \phi} \bar{u} \ell = 0.82; \quad q = q \frac{V_{ub}^{\text{m}}}{V_{cb}} = 0.086 .$$

See [253] for a discussion of the $B \to K$ system in a slightly different scenario involving an additional $Z^0$ boson.
\[
Y = \chi \, \frac{1}{2} e^{i \frac{\pi}{4}} = 2.35 q e^{i 0.64}; \\
Z = \tilde{\chi} \, \frac{1}{2} e^{i \frac{3}{4} \pi} = 2.35 q e^{i 0.94};
\]

(10.33) (10.34)

If the phase \( \phi \) was zero (the case considered in [254]), the functions \( X, Y, Z \) would remain to be real quantities as in the SM and the MFV model but the enhancement of \( q \) would imply enhancements of \( X, Y, Z \) as well. As in the scenario considered \( X, Y, Z \) are not independent of one another, it is sufficient to constrain one of them from rare decays in order to see whether the enhancement of \( q \) is consistent with the existing data on rare decays. It turns out that the data on inclusive \( B \to X_s l^+ \) decays [255, 256] are presently most powerful to constrain \( X, Y, Z \), but as demonstrated very recently in [202] their impact is considerably increased when combined with data on \( B \to X_s K^+ l^+ \). One finds then in particular \( X_{\text{max}} = 1.95 \) and \( Y_{\text{max}} = 1.43 \) to be compared with \( 1.54 \) and \( 0.99 \) in the SM. Correspondingly as seen in Table 4, the enhancements of rare branching ratios within the MFV scenario of [61] by more than a factor of two over the SM expectations are no longer possible.

The situation changes drastically if \( \phi \) is required to be non-zero, in particular when its value is in the ball park of \( 0.90 \) as found above. In this case, \( X, Y \) and \( Z \) become complex quantities, as seen in (10.32)–(10.34), with the phases \( \phi \) in the ball park of \( 0.22 \) [41]:

\[
x = (86, 12); \quad y = (100, 12); \quad z = (108, 12);
\]

(10.35)

Actually, the data for the \( B \to X_s K \) decays used in our first analysis [41] were such that \( q = 1.75^{+1.27}_{-0.99} \) and \( q = (85.1^{+11.1}_{-14}) \) were required, implying \( \sqrt{2} \, j \), \( \sqrt{2} \, j \), \( \sqrt{2} \, j \), \( \sqrt{2} \, j \), \( 4_{12}^{13} \), barely consistent with the data. Choosing \( \sqrt{2} \, j = 2.2 \) as high as possible but still consistent with the data on \( B \to X_s l^+ l^- \) at the time of the analysis in [41], we found

\[
q = 0.92^{+0.69}_{-0.69}; \quad \phi = (85.1^{+11.1}_{-14})
\]

(10.36)

which has been already taken into account in obtaining the values in (10.35). This in turn made us expect that the experimental values \( R_C = 1.17 \pm 0.12 \) and \( R_R = 0.76 \pm 0.10 \) known at the time of the analysis in [41] could change (see Fig. 15) once the data improve. Indeed, our expectation [41],

\[
R_C = 1.70^{+0.12}_{-0.08} \quad R_R = 0.82^{+0.12}_{-0.12};
\]

(10.37)

has been confirmed by the most recent results in Table 6 making the overall description of the \( B \to X_s K \) and \( B \to X_s l^+ l^- \) rare decays within our approach significantly better with respect to our previous analysis.

The very recent analysis of MFV in [202], indicating allowed values for \( Y \) below 2.2, will have some indirect impact also on our analysis that goes beyond MFV. However, a preliminary analysis indicates that the impact on our \( B \to X_s K \) results is rather insignificant. In particular the phase \( \phi \) remains very large. We will return elsewhere to these modifications once the data on \( B \) and \( B \to X_s K \) improve.

There is a characteristic pattern of modifications of branching ratios relative to the case of \( \phi = 0 \) and \( \phi = 0 \):

The formulae for the branching ratios proportional to \( \sqrt{2} \, j \) like \( B \to X_s \), and \( \sqrt{2} \, j \), like \( B \to X_s \), remain unchanged relative to the case \( \phi = 0 \), except that the correlation between \( \sqrt{2} \, j \) and \( \sqrt{2} \, j \) for \( \phi \neq 0 \) differs from the one in the MFV models.

In CP-conserving transitions in which in addition to top-like contributions also charm contribution plays some rôle, the \( constructive \) interference between top and charm contributions in the SM becomes \( destructive \) or very small if the new phases \( \phi \) are large, thereby compensating for the
enhancements of $X$, $Y$, and $Z$. In particular, $B \tau (K^+ \! \to \! e+\nu\bar{\nu})$ turns out to be rather close to the SM estimates, and the short-distance part of $B \tau (K_L^+ \! \to \! e+\nu\bar{\nu})$ is even smaller than in the SM.

Not surprisingly, the most spectacular impact of large phases $\phi$ is seen in CP-violating quantities sensitive to EW penguins.

In particular, one finds \[ \frac{B \tau (K_L^+ \! \to \! 0)}{B \tau (K_L^+ \! \to \! 0)} = \frac{X}{X_{SM}} \frac{\sin (\phi)}{\sin (\phi)} \] with the two factors on the right-hand side in the ball park of 2 and 5, respectively. Consequently, $B \tau (K_L^+ \! \to \! 0)$ can be enhanced over the SM prediction even by an order of magnitude and is expected to be roughly by a factor of 4 larger than $B \tau (K^+ \! \to \! e+\nu\bar{\nu})$. We would like to emphasize that this pattern is only moderately affected by the results in [202] as the maximal value of $\phi$ used in [41] is only slightly higher than the upper bound found in [202].

In the SM and most MFV models the pattern is totally different with $B \tau (K_L^+ \! \to \! 0)$ smaller than $B \tau (K^+ \! \to \! e+\nu\bar{\nu})$ by a factor of 2–3 [15, 139, 140, 115, 15, 115]. On the other hand a recent analysis shows that a pattern of $B \tau (K_L^+ \! \to \! 0)$ expected in our NP scenario can be obtained in a general MSSM [258]. In Fig. 16 we show the ratio of $K_L^+ \! \to \! 0$ and $K^+ \! \to \! e+\nu\bar{\nu}$ branching ratios as a function of $\beta_X$ for different values of $\phi$. It is clear from this plot that accurate measurements of both branching ratios will give a precise value of the new phase $\phi$ without essentially any theoretical uncertainties.

The result in a general MSSM is shown in Fig. 17 [258]. The points in this figure for which both branching ratios are large and $B \tau (K_L^+ \! \to \! 0)$ and $B \tau (K^+ \! \to \! e+\nu\bar{\nu})$ correspond to large complex phases in the non-diagonal terms $12_{LL}$, $13_{ULR}$, $23_{ULR}$ in the squark mass matrix.

![Fig. 16: The ratio of $K_L^+ \! \to \! 0$ and $K^+ \! \to \! e+\nu\bar{\nu}$ branching ratios as a function of $\beta_X$ for different values of $\phi$.](image)

We note that $B \tau (K_L^+ \! \to \! 0)$ is predicted to be rather close to its model-independent upper bound [200] in [16]. Moreover, another spectacular implication of these findings is a strong violation...
of the relation [139][140]
\[
(\sin 2\theta) = (\sin 2\theta)_{\text{SM}};
\]
which is valid in the SM and any model with MFV. Indeed, we find [229][41]
\[
(\sin 2\theta) = \sin 2\theta (x) = (0.07 \pm 0.23);\quad (10.40)
\]
in striking disagreement with \((\sin 2\theta)_{\text{SM}} = 0.0725 \pm 0.037\). Fig [18] shows what happens when \(x\) is varied.

Even if eventually the departures from the SM and MFV pictures could turn out to be smaller than estimated here, finding \(B \to K_L \gamma\) to be larger than \(B \to K^+ \gamma\) would be a clear signal of new complex phases at work. For a more general discussion of the \(K^0\) decays beyond the SM, see [15].

Similarly, as seen in Table 8, interesting enhancements are found in \(K_L \to 0\gamma\) [41][265][181] and the forward–backward CP asymmetry in \(B \to X_s\eta\) as discussed in [41]. The impact of the very recent analysis in [202] is to moderately suppress the enhancements seen in this table. The exception is \(B \to (\bar{B} \to s)\) for which values higher than \(1\) are not possible any longer also in our scenario. Further implications for rare decays in our scenario can be found in [266].

| Decay | SM prediction | Our scenario | Exp. bound (90% C.L.) |
|-------|---------------|--------------|----------------------|
| \(K^+ \to \pi^0\) | \((7.8 \pm 1.2)\) \(10^3\) | \((7.5 \pm 2.1)\) \(10^3\) | \((14.7^{+13.4}_{-9.9})\) \(10^3\) |
| \(K_L \to 0\gamma\) | \((3.9 \pm 0.6)\) \(10^3\) | \((3.6 \pm 1.0)\) \(10^3\) | < \(5.9 \cdot 10^3\) |
| \(K_L \to \eta^0 e^+ e^-\) | \((3.7^{+1.2}_{-0.9})\) \(10^3\) | \((9.0 \pm 1.6)\) \(10^3\) | < \(2.8 \cdot 10^3\) |
| \(B \to X_s\eta\) | \((1.5 \pm 0.3)\) \(10^3\) | \((4.3 \pm 0.7)\) \(10^3\) | < \(3.8 \cdot 10^3\) |
| \(B \to \eta\) | \((3.5 \pm 0.5)\) \(10^0\) | \(7 \cdot 10^3\) | < \(6.4 \cdot 10^3\) |
| \(B \to X_s\) | \((3.42 \pm 0.53)\) \(10^3\) | 17 \(10^3\) | < \(5.0 \cdot 10^3\) |

Table 8: Predictions for various rare decays in the scenario considered compared with the SM expectations and experimental bounds. For a theoretical update on \(K_L \to 0\gamma\) and a discussion of \(K_L \to 0\gamma\), see [265].

As emphasized above, the new data on \(B \to K^0\) improved the overall description of \(B \to K^0\) rare decays within our approach. However, the most interesting question is whether the large negative values of \(\tan \beta\) and \(\lambda_1\) will be reinforced by the future more accurate data. This would be a very spectacular signal of NP!

We have also explored the implications for the decay \(B_d \to K_S\) [41]. Large hadronic uncertainties preclude a precise prediction, but assuming that the sign of the cosine of a strong phase agrees with factorization, we find that \((\sin 2\theta)_{K_S} > (\sin 2\theta)_{K_S}\). This pattern contradicted the first data on \((\sin 2\theta)_{K_S}\) in [7.15] but the most recent data in [7.17] are fully consistent with our expectations. A future confirmation of \((\sin 2\theta)_{K_S} > (\sin 2\theta)_{K_S}\) could be another signal of enhanced CP-violating \(Z^0\) penguins with a large weak complex phase at work. A very recent analysis of the \(B \to K^0\) decays and the correlation with \(B \to K^0\) in supersymmetric theories can be found in [267].

10.6 Outlook

We have presented a strategy for analyzing \(B \to K^0\), \(B \to K^0\) decays and rare \(K\) and \(B\) decays. Within a simple NP scenario of enhanced CP-violating EW penguins considered here, the NP contributions enter significantly only in certain \(B \to K^0\) decays and rare \(K\) and \(B\) decays, while the \(B \to K^0\) system
Fig. 17: Distributions of $X$, $\text{Br}(K^L \rightarrow \pi^0 \nu\bar{\nu})$ and $\text{Br}(K^+ \rightarrow \pi^+ \nu\bar{\nu})$ for $\tan\beta = 2$ in the 63-parameter scan [55].
Fig. 18: $B \to K^+ \pi^-$ as a function of $B \to K_L^+ 0$ for various values of $\chi = \chi_{11}$. The dotted horizontal lines indicate the lower part of the experimental range and the grey area the SM prediction. We also show the bound of $[200]$. 

is practically unaffected by these contributions and can be described within the SM. The confrontation of this strategy with the most recent data on $B^+ \to d \phi K_S$ and $B^+ \to d \phi K^-$ modes from the BaBar and Belle collaborations is very encouraging. In particular, our earlier predictions for the direct CP asymmetries of $B_d^+ 0 0$ and $B_d^+ K$ have been confirmed within the theoretical and experimental uncertainties, and the shift in the experimental values of $R_c$ and $R_n$ took place as expected.

It will be exciting to follow the experimental progress on $B^+ K^-$ and $B^+ K$ decays and the corresponding efforts in rare decays. In particular, new messages from BaBar and Belle that the present central values of $R_c$ and $R_n$ have been confirmed at a high confidence level, a slight increase of $R$ and $\alpha$, and a message from KEK $[189]$ in the next two years that the decay $K_L^+ 0$ has been observed would give a strong support to the NP scenario considered here.

11 Shopping List

Flavour physics and CP violation have been with us for almost 50 years but only in the present decade we can expect to be able to test the CKM picture of flavour and CP violation at a satisfactory level. There is of course hope that at a certain level of precision some deviations from the SM expectations will be observed, signalling new physics contributions of the MFV type or even beyond it.

Here is my shopping list for the rest of the decade, that I present here not necessarily in a chronological order.

There has been an impressive progress on the determination of $\mathcal{Y}_{ub,j}$ and $\mathcal{Y}_{ub,j}$ in the last years but both determinations, in particular the one of $\mathcal{Y}_{ub,j}$ will hopefully be improved.

The angle $\beta$ in the UT is already known with a high precision. Yet, it is important to measure it more accurately as the corresponding corner of the UT is placed far from its apex, where the main action in the $\beta$; $\gamma$ takes place. Even more importantly, one should find out whether the solution for $\beta$ chosen by the SM is chosen by nature and whether new phases in the $B_d^0 B_d^0$ mixing could pollute this measurement.

An important milestone in the physics of flavour violation will be undoubtedly the measurement of $M_A$ that in combination with $M_d$ will offer a rather clean measurement of $\mathcal{Y}_{td,j}$ and indirectly of the angle $\alpha$ in the UT.
Another important message will be the measurement of the angle $\theta_{13}$ by means of the strategies discussed in Section 7. At Tevatron the $U$-spin strategies seem to be promising. Also the $B^+ - K^+$ systems offer interesting results. But of course these extractions will be surpassed one day by very clean measurements of this angle in $B^+ - D K^+$ decays.

A precise determination of $\theta_{13}$ without hadronic and new physics uncertainties that is available by means of the tree level $B^+ - D K^+$ decays, will be an important milestone in the tests of the KM picture of CP violation. Combined with hopefully precise value of $\Re V_{ub} = V_{ub}^{\text{true}}$ it will allow the construction of the reference UT (RUT) [44]. This triangle together with $\Re V_{us}$ and $\Re V_{cd,j}$ will give us the true CKM matrix without essentially any new physics pollution. First steps in this direction have been made in [102].

These results will be confronted one day with the accurate measurements of $B^r K^0$ and $B^r (K_L^0 \to 0^-)$. To this end a number of golden relations will allow us to test the SM and MFV scenarios. We have listed them in Section 8. It should be emphasized that although theoretically very clean, the decays $K^+$, in contrast to tree level decays relevant for the RUT, are sensitive to new physics contributions. Consequently the determination of the UT by means of these decays is subject to new physics uncertainties. But precisely the comparison of the UT from $K^+$ with the RUT will teach us about new physics in a clean environment.

Of course the simultaneous comparison of various determinations of the UT with the RUT will allow us not only to discover possible new physics contributions but also to identify the type of this new physics.

An important issue is a better clarification of the measurement of $(\sin^2 \theta_{13})_{K_S}$. The confirmation of the significant departure of $(\sin^2 \theta_{13})_{K_S}$ from the already accurate value of $(\sin^2 \theta_{13})_{K_S}$, would be a clear signal of new physics that cannot be accommodated within classes A and B. Also in a particular scenario of class C, discussed in the previous Section, one finds $(\sin^2 \theta_{13})_{K_S} > (\sin^2 \theta_{13})_{K_S}$.

Also very important are the measurements of $B^r (B_{d,s} \to K_{d,s}^0)$ by factors as high as 100 in some versions of supersymmetric models, are the largest enhancements in the field of $K^+$ and $B^+$ decays, that are still consistent with all available data. A recent review can be found in [89]. The correlation of the measurement of $B^r (B_{d,s} \to K_{d,s}^0)$ with the one of $M_{s,t}$ may teach us something about the nature of new physics contributions. Finding $M_{s,t}$ below $(M_{s,t})_{\text{SM}}$ and $B^r (B_{d,s} \to K_{d,s}^0)$ well above the SM expectations would be a nice confirmation of a SUSY scenario with a large $\tan \beta$ that has been considered in [86].

The improved measurements of several $B^+$ and $B^+ - K^+$ observables are very important in order to see whether the theoretical approaches like QCDF [51], PQCD [52] and SCET [53] in addition to their nice theoretical structures are also phenomenologically useful. On the other hand, independently of the outcome of these measurements, the pure phenomenological strategy [41, 229] presented in Section 10 will be useful in correlating the experimental results for $B^+$ and $B^+ - K^+$ with those for rare $K^+$ and $B^+$ decays, $B_{d,s} \to K_{d,s}$ and $M_{d,s}^*$.

Assuming that the future more accurate data on $B^+$ and $B^+ - K^+$ will not modify significantly the presently observed pattern in these decays, the scenario of enhanced $Z^0$ penguins with a new large complex weak phase will remain to be an attractive possibility. While the enhancement of $B^r (K_L^0 \to 0^-)$ by one order of magnitude would be very welcome to our experimental colleagues and $(\sin^2 \theta_{13}) < 0$ would be a very spectacular signal of NP, even more moderate departures of this sort from the SM and the MFV expectations could be easily identified in the very clean $K^+$ decays as clear signals of NP. This is seen in Figs. 16 and 18.
The improved measurements of $\mathcal{B}(B \to \phi X_s\ell\bar{\nu}_\ell)$ and $\mathcal{B} (K^+ \to \phi X_s\ell\bar{\nu}_\ell)$ in the coming years will efficiently bound the possible enhancements of $Z^0$ penguins, at least within the scenarios A–C discussed here. A very recent analysis in [202], that we briefly summarized in Section 9, demonstrates very clearly that the existing constraints on $Z^0$-penguins within the MFV scenario are already rather strong.

Also very important is an improved measurement of $\mathcal{B}(B \to \phi X_s\ell\bar{\nu}_\ell)$ as well as the removal of its sensitivity to $c$ in $m_c$ through a NNLO calculation. This would increase the precision on the MFV correlation between $\mathcal{B}(B \to \phi X_s\ell\bar{\nu}_\ell)$ and the zero $\delta_0$ in the forward–backward asymmetry $A_{FB}(\phi)$ in $B \to \phi X_s\ell\bar{\nu}_\ell$ [60]. A 20% suppression of $\mathcal{B}(B \to \phi X_s\ell\bar{\nu}_\ell)$ with respect to the SM accompanied by a downward shift of $\delta_0$ would be an interesting confirmation of the correlation in question and consistent with the effects of universal extra dimensions with a low compactification scale of order few hundred GeV. On the other hand finding no zero in $A_{FB}(\phi)$ would likely point towards flavour violation beyond the MFV.

Finally, improved bounds and/or measurements of processes not existing or very strongly suppressed in the SM, like various electric dipole moments and FCNC transitions in the charm sector will be very important in the search for new physics. The same applies to $\mathcal{B}(B \to X_{cs}e\bar{\nu}_e)$ and generally lepton flavour violation.

We could continue this list for ever, in particular in view of the expected progress at Belle and BaBar, charm physics at Cornell, experimental program at LHCb and the planned rare $K$ physics experiments. But the upper bound on the maximal number of pages for these lectures has been saturated which is a clear signal that I should conclude here. The conclusion is not unexpected: in this decade, it will be very exciting to follow the development in this field and to monitor the values of various observables provided by our experimental colleagues by using the strategies presented here and other strategies found in the rich literature.

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