Electroproduction of the $\Delta(1232)$ resonance at high momentum transfer.

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(Received November 28, 2021)

We studied the electroproduction of the $\Delta(1232)$ resonance via the reaction $p(e,e'p)p^0$ at four-momentum transfers $Q^2 = 2.8$ and $4.0$ GeV$^2$. This is the highest $Q^2$ for which exclusive resonance electroproduction has ever been observed. Decay angular distributions for $\Delta \rightarrow p\pi^0$ were measured over a wide range of barycentric energies covering the resonance. The $N - \Delta$ transition form factor $G_{SM}^3$ and ratios of resonant multipoles $E_{1+}/M_{1+}$ and $S_{1+}/M_{1+}$ were extracted from the decay angular distributions. These ratios remain small, indicating that perturbative QCD is not applicable for this reaction at these momentum transfers.

PACS numbers: 13.60.Le, 13.60.Rj, 14.20.Gk, 13.40.Gp.

An important concern in hadron physics is the determination of the appropriate degrees of freedom to describe exclusive reactions in the experimentally accessible range of momentum transfers. It is widely agreed that perturbative QCD (pQCD) should apply at sufficiently high momentum transfer; however, there is no general agreement about how high the momentum transfer must be. Many exclusive reactions exhibit scaling behavior which has been interpreted by some authors as the onset of pQCD. Others have argued pQCD would apply only at much higher $Q^2$, and that the observed scaling can be explained by contributions of the soft Feynman mechanism.

The evolution from soft non-perturbative physics towards hard pQCD can be studied using the excitation of the $\Delta(1232)$ resonance. In addition to the $Q^2$ dependence of the transition form factor, $G_M^3$, pQCD makes definite predictions about the relative contributions of the magnetic dipole, $M_{1+}$, electric quadrupole, $E_{1+}$, and Coulomb quadrupole, $S_{1+}$, multipole amplitudes. At low $Q^2$, according to the quark model the $N - \Delta$ transition is due primarily to a single quark spin flip so that $M_{1+}$ would dominate, and the contributions of $E_{1+}$ and $S_{1+}$ would be very small. Near $Q^2 = 0$, recent experiments confirm this prediction: $R_{EM} \equiv E_{1+}/M_{1+} \sim -0.03$ and $R_{SM} \equiv S_{1+}/M_{1+} \sim -0.11$, where $E_{1+}$ and $M_{1+}$ are evaluated at the resonance position. At high $Q^2$, according to valence pQCD, only helicity-conserving amplitudes should contribute, leading to the predictions $R_{EM} = +1$ and $R_{SM} \rightarrow \text{constant}$. An evaluation of earlier data from DESY at $Q^2 = 3.2$ GeV$^2$ suggests that $R_{EM}$ is small, but with large errors.

To address these issues, we measured the differential cross section of the neutral pion decay channel in electroproduction of the $\Delta(1232)$ at $Q^2 = 2.8$ and $4.0$ GeV$^2$. This experiment was performed with the 100% duty factor electron beam in Hall C at the Thomas Jefferson National Accelerator Facility. The availability of a multigi electron beam having high duty factor, high luminosity and excellent beam energy resolution allowed us for the first time to measure this exclusive reaction with high statistical precision in this range of $Q^2$.

Electron beams having energies 3.2 and 4.0 GeV, with currents 100 and 80 $\mu$A at $Q^2 = 2.8$ and $4.0$ GeV$^2$, respectively, were incident on a liquid hydrogen target of thickness 4 cm. The beam current was measured to 1% accuracy by two resonant cavities and a parametric current transformer.

At these $Q^2$ the protons emerge in a narrow cone around the momentum transfer vector $\vec{q}$. Thus, a significant fraction of the full c.m. solid angle was obtained in the decay of the $\Delta \rightarrow p\pi^0$. Scattered electrons were detected in the Short Orbit Spectrometer (SOS) and protons were detected in the High Momentum Spectrometer (HMS). The SOS central momentum and angle were fixed throughout the data taking at each $Q^2$ point. The HMS central momentum and angle were varied to cover the proton’s full momentum range and most of its decay.
angle cone. There were a total of 45 kinematic settings at $Q^2 = 2.8$ GeV$^2$ and 28 settings at 4.0 GeV$^2$. The momentum and angular acceptances of adjacent settings were overlapped for redundancy.

Electrons were identified utilizing a gas threshold Čerenkov counter and a lead glass shower counter. Protons were identified from the coincidence time between electron and hadron arms, and by single arm time-offlight.

The experiment is described in detail in refs. [7,8]. The experiment is described in detail in refs. [7,8]. The experiment is described in detail in refs. [7,8].

A brief outline of the analysis follows. For the $p\pi^0$ system, we calculated the invariant mass distribution $m_{\pi^0}$, and the out-of-plane angle $\phi^cm_{\pi^0}$. Most of the background from the elastic radiative tail was removed by accepting only those events with $|\phi^cm_{\pi^0}| > 8^\circ$ and which passed the missing mass $\pi^0$ cut. The remaining contribution ($\sim 5\%$) was modeled and subtracted by a Monte Carlo simulation. The accidental background ($\sim 1\%$) and the background associated with events rescattered on the spectrometer collimator, vacuum chamber, and magnet ($\sim 5\%$) were obtained from experimental data which were outside the timing and track reconstruction cuts, and subtracted on a bin-by-bin basis. The data were corrected also for tracking inefficiency ($\sim 9\%$), trigger efficiency ($< 1\%$), dead time ($\sim 2.5\%$), nuclear absorption ($\sim 3.5\%$), local heating effects on target density ($\sim 4\%$), and target wall contributions ($\sim 1.5\%$). The acceptance and radiative corrections were obtained from a Monte Carlo simulation of the entire experiment. The product of the target thickness and beam current was monitored throughout the experiment by the electron single arm rate. The ratio of the number of SOS counts to the beam charge was stable to 1% from run to run. To check the absolute normalization, elastic scattering measurements were performed. The result is consistent with world data [8] to within 2%.

Assuming the one-photon-exchange approximation, the differential cross section of single pion electroproduction is related to the center-of-mass differential cross section for pion production by virtual photons, $d\sigma/dQ^2$, as follows:

$$\frac{d\sigma}{dW dQ^2 d\Omega_{\pi^0}} = \Gamma_\nu \frac{d\sigma}{d\Omega_{\pi^0}},$$

where $\Gamma_\nu$ is the virtual photon flux factor. The center-of-mass angles, $\theta^cm_{\pi^0}$ and $\phi^cm_{\pi^0}$, were reconstructed from the detected proton laboratory angles. Because at these $Q^2$ the protons emerge in a narrow cone in the laboratory, accurate measurements of the proton and electron momenta and angles are necessary to produce good angular resolution in the c.m. The resolutions obtained were typically $\sigma_W = 15$ MeV, $\sigma_{Q^2} = 0.006$ GeV$^2$, $\sigma_{\cos\theta^cm} = 0.03$ and $\sigma_{\phi^cm} = 3^\circ$.

The events with invariant mass $1.1 < W < 1.4$ GeV were binned with $\Delta W = 30$ MeV, $\Delta \cos\theta^cm = 0.2$, and $\Delta \phi^cm = 30^\circ$. The $Q^2$ bin size ($\approx 0.5$ GeV$^2$), was determined by the apparatus acceptance. Measurements of $d\sigma/d\Omega_{\pi^0}$ were obtained, with the Hand convention for $\Gamma_\nu$, at 751 intervals of $\cos\theta^cm_{\pi^0}$, $\phi^cm_{\pi^0}$, and $W$ at $Q^2 = 2.8$ and 867 intervals at $Q^2 = 4.0$ GeV$^2$. Examples of the angular distribution are shown in Fig. 2. Errors shown in the figure are statistical only; they do not include an estimated systematic uncertainty in the overall normalization of 5%.

We extracted information about the contributing multipoles by two methods. One method consisted of making model-independent empirical multipoles fits to the angular distributions independently at each $W$, assuming $M_{1+}$ dominance at the resonance pole, and only S and P wave contributions, as, for example in [10]. The quality of the fits is indicated in Fig. 2. Over the entire dataset $\chi^2 \approx 1.36$ per degree of freedom at $Q^2 = 2.8$ GeV$^2$ and 1.21 per degree of freedom at $Q^2 = 4.0$ GeV$^2$. The resulting extracted amplitudes confirm the $M_{1+}$ dominance at the resonance. In particular at $W = 1.235$ GeV, at $Q^2 = 2.8$ GeV$^2$, $R_{EM} = -0.01 \pm 0.01$ and $R_{SM} = -0.06 \pm 0.01$, and at $Q^2 = 4.0$ GeV$^2$, $R_{EM} = -0.02 \pm 0.01$ and $R_{SM} = -0.11 \pm 0.01$. This result shows clearly that at the resonance position $R_{EM}$ is very small and $R_{SM}$ is moderately small and negative.

A more sophisticated extraction of the amplitudes was performed using an effective Lagrangian (EL) theoretical basis [11]. In this approach, the s- and u-channel contributions of the nucleon and $\Delta$ are taken into account, along with the $t-$channel vector meson contributions. The full $\gamma N\Delta$ and $\pi N\Delta$ vertices are retained, and the characteristic “off-shell” parameters for the $\Delta$ and the electromagnetic gauge couplings are fitted to the data at each $Q^2$ [13], maintaining unitarity. Similar fits to existing data at lower $Q^2$ have been performed by other authors with results close to what we give here [13]. The EL extracted values of $G_M^3/3G_D$, $R_{EM}$, and
for the present data are given in Table 1, where

\[ G_D \equiv \frac{1}{(1 + Q^2/0.71)^2}. \]

They are in good agreement with the empirical fit. The resonance contributions to the amplitudes were also obtained within the framework of the model. We note that at the resonance position the resonance amplitudes account for nearly all of \( G_M^*, R_{EM}, \) and \( R_{SM}, \) as indicated in Table 1. These results for the resonance are shown in Fig. 3. The first errors shown are statistical. The second are systematic errors estimated from variations in the empirical fits of experimental quantities resulting from our estimates of systematic errors in the cross sections.

\[ \frac{d\sigma}{d\Omega_{\pi}} = \frac{G_M^*}{3G_D} \]

\[ \frac{d\sigma}{d\Omega_{\pi}} = \frac{R_{EM}}{3G_D} \]

\[ \frac{d\sigma}{d\Omega_{\pi}} = \frac{R_{SM}}{3G_D} \]

Table I. \( G_M^*, R_{EM}, \) and \( R_{SM} \) extracted from the present data by means of the effective Lagrangian fits discussed in the text.

| \( Q^2 \) | \( G_M^*/3G_D \) | \( R_{EM} \) | \( R_{SM} \) |
|---------|----------------|-----------|-----------|
| 2.8     | 0.70 ± 0.02 ± 0.02 | -0.023 ± 0.012 ± 0.005 | -0.114 ± 0.013 ± 0.01 |
| 4.0     | 0.59 ± 0.02 ± 0.02 | -0.035 ± 0.012 ± 0.005 | -0.150 ± 0.013 ± 0.01 |

| Resonance Only |
|----------------|
| 2.8 0.70 ± 0.02 ± 0.02 | -0.20 ± 0.012 ± 0.005 | -0.112 ± 0.013 ± 0.01 |
| 4.0 0.59 ± 0.02 ± 0.02 | -0.031 ± 0.012 ± 0.005 | -0.148 ± 0.013 ± 0.01 |

From Fig. 3a, it is seen that the \( N - \Delta \) transition form factor \( G_M^* \) is decreasing with \( Q^2 \) faster than the dipole
form, which characterizes the elastic nucleon form factor and those for the strongly excited resonances in the second and third resonance region. This result corroborates earlier fits to the peaks above the non-resonant continuum in available single arm inclusive electron scattering data. This \(Q^2\) dependence of \(G_M^\pi\) is in disagreement with pQCD constituent scaling predictions. The \(N-\Delta\) transition was previously studied theoretically in a pQCD framework, which resulted in an anomalously small helicity conserving amplitude compared to that for elastic and other resonance amplitudes. Thus they conclude that the pQCD description for the \(\Delta(1232)\) might become important only at a higher \(Q^2\) than for the elastic or other resonance transitions.

The dot-dash curve in Figure 3a is the result of a relativistic quark model calculation, and clearly does not describe the data. The solid curve, which more accurately describes the \(Q^2\) dependence of \(G_M^\pi\) is due to a relativistic quark model calculation, as in, but with a quark form factor added. The dashed curve is the result of a QCD sum-rule calculation, which utilizes duality between quark and hadron spectral densities to parameterize the dominant non-perturbative transition matrix elements.

Regarding the ratios \(R_{EM}\) and \(R_{SM}\), both are small, as are the quark model predictions, even taking into account calculational uncertainties. Beyond that neither quark model calculation describes all the data well. On the other hand, the pQCD prediction \((R_{EM} = +1)\) can be ruled out unambiguously.

Recently there has been an attempt to describe the small value of \(R_{EM}\) in the few GeV\(^2\) range of \(Q^2\) by interpolating between the pQCD calculated asymptotic amplitudes, and the measured photoproduction \((Q^2 = 0)\) amplitude. For the helicity non-conserving amplitude they used a dipole form with a faster falloff than the helicity conserving amplitude. The results for \(R_{EM}\) remain relatively small at moderately large momentum transfer, but are still in significant disagreement with the data.

To summarize, we studied exclusive \(\pi^0\) electroproduction in the region of the \(\Delta(1232)\) resonance at \(Q^2 = 2.8\) and 4.0 GeV\(^2\). We find that the ratio \(R_{EM}\) remains small, the ratio \(R_{SM}\) is small but nonzero, and the \(N-\Delta\) transition form factor \(G_M^\pi\) is decreasing with \(Q^2\) faster than the dipole form factor, as suggested by previous analyses of inclusive data.

These results indicate that the hadron helicity is not conserved in this reaction and that pQCD is not applicable for this reaction at these momentum transfers; on the other hand, it is not clear how high in \(Q^2\) the use of constituent quark models is appropriate. There has been recent progress in treating exclusive reactions in terms of off-forward parton distributions. These are generalizations of deep inelastic scattering parton distribution functions, and include exclusive reactions and form factors in the \(Q^2\) region where soft processes dominate. The application of these results to the present reaction, and to the higher \(Q^2\) extensions planned for Jefferson Lab are anticipated.

The authors acknowledge the support of the staff of the Accelerator Division of Thomas Jefferson National Accelerator Facility. This work was supported in part by the U.S. Department of Energy, the National Science Foundation, and the Korean Science and Engineering Foundation (KOSEF). We would like to thank S. Capstick, C. E. Carlson, B. Keister, T.-S. H. Lee, A. V. Radyushkin, and S. Simula for communicating results of their calculations, as well as for many fruitful discussions.

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[1] S. J. Brodsky and G. P. Lepage, Phys. Rev. D, 23, 1152 (1981); C. E. Carlson and J. L. Poor, Phys. Rev. D, 38, 2758 (1988); C. E. Carlson and N. C. Mukhopadhyay, Phys. Rev. D, R41, 2434 (1990); Phys. Rev. Lett. 74, 1288 (1995).
[2] N. Isgur and C. H. Llewellyn-Smith, Phys. Rev. Lett. 52, 1080 (1984); A. V. Radyushkin, Nucl. Phys. A527, 153 (1991).
[3] C. Becchi and G. Morpurgo, Phys. Lett. 17, 352 (1969).
[4] R. Beck et al., Phys. Rev. Lett 78, 606 (1997); G. Blanpied et al., Phys. Rev. Lett. 79, 4337 (1997); R. M. Davidson et al. submitted to Phys. Rev. Lett.
[5] F. Kallelicher et al., Z. Phys. A359, 201 (1997).
[6] R. Haidan, PhD Thesis, DESY report F21-79/03 (1979).
[7] V. F. Frolov, PhD thesis, Rensselaer Polytechnic Institute (1998), unpublished.
[8] C. S. Armstrong, PhD thesis, College of William & Mary (1998), unpublished; C. S. Armstrong et al., to be published.
[9] P. E. Bosted, Phys. Rev. C 51, 409 (1996).
[10] J. C. Alder et al., Nucl. Phys. B46, 415 (1972).
[11] R. M. Davidson, N. C. Mukhopadhyay and R. Wittman, Phys. Rev. Lett 56, 804 (1986); Phys. Rev. D, 43, 71 (1991).
[12] R. M. Davidson and N. C. Mukhopadhyay, Phys. Lett. B353, 131 (1995).
[13] T. Sato and T.-S. H. Lee, Phys. Rev. C, 54, 2660 (1996).
[14] P. Stoler, Physics Reports 226, 103 (1993); Phys. Rev. D, 44, 73 (1991).
[15] C. Keppel, Workshop on CEBAF at Higher Energies, CEBAF pp. 237 (1994).
[16] C. E. Carlson, Phys. Rev. D, 34, 2704 (1986).
[17] S. Capstick and B. D. Keister, Phys. Rev. D, 51, 3598 (1995), and private communication.
[18] F. Cardarelli, E. Pace, G. Salme, S. Simula, Phys. Lett. B371, 7 (1996).
[19] V. M. Belyaev and A. V. Radyushkin, Phys. Rev. D, 53, 6509 (1996).
[20] C. E. Carlson and N. C. Mukhopadhyay, hep-ph 9804356, to be published.
[21] R. M. Davidson, private communication, to be published.
[22] V. Burkert and L. Elouadrhiri, Phys. Rev. Lett. 75, 3614 (1995); R. M. Davidson and N. C. Mukhopadhyay, private communication.
[23] A. Radyushkin, Phys. Lett B, 385, 333 (1996). X. Ji, Phys. Rev. Letters, 78, 610 (1997), and references therein.