Simulation study for cross-sectional absorption distribution in turbid medium using spatially resolved backscattered light with lateral scanning and one-dimensional solution of nonlinear inverse problem

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For a practical technique of cross-sectional imaging of animal bodies, we developed a new method using spatially resolved backscattered light. This method is based on the solution of the one-dimensional nonlinear inverse problem, and on the lateral scan of the source–detector pair along the object surface. Using this method, unknown variables in inverse problems can be reduced more greatly than when using conventional methods. A stable solution for the inverse problem becomes possible. The possibility of using the proposed method was assessed using simulation analysis. The results verified that cross-sectional imaging from several to 10 millimeter depths is possible for animal tissue. This analysis clarified the specific spatial resolution and accuracy in the estimated absorption coefficient. Distortionless imaging was confirmed. Results suggest that the proposed method represents new options as a stable and practical method for biological cross-sectional imaging.

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1. Introduction

Near-infrared (NIR) light with the wavelength of 700–1200 nm can penetrate deeper into animal tissue than visible light because of the low absorption of water and hemoglobin. Optical techniques with NIR light are useful for various physiological measurements and imaging for biomedical applications. Using NIR light with different wavelengths, we can perform a quantitative analysis of biological components using the spectroscopy principle. Nevertheless, the light is scattered strongly in tissues. Obtaining absorption information selectively and quantitatively is not easy. Many studies and efforts have been undertaken to resolve these and related difficulties.¹–¹¹ Because of the importance of biomedical imaging, studies using NIR imaging have been undertaken actively.¹²–¹⁴ One method is diffuse optical tomography (DOT),¹⁵ which realizes cross-sectional imaging of the breast and the neonatal head.¹⁶,¹⁷

For DOT, light is illuminated from many directions around the animal body. A cross-sectional image is reconstructed using light received at multiple points around the body, including transmitted light.¹⁷ However, in a human body, parts through which the transmitted light is available are very scarce. Therefore, its application is limited. To address these shortcomings, imaging with backscattered light applicable to thicker parts has been proposed. Typical examples include optical coherence tomography (OCT)¹⁸ and photoacoustic tomography (PAT).¹⁹ In general, OCT can yield images to 1–2 mm depth from the surface with a high spatial resolution of micrometer order. By contrast, PAT can produce images for several millimeters of depth from surfaces with a resolution of 10–100 μm order. However, for some applications such as the evaluation of inflamed areas under the skin surface, or skin cancer penetration depth, more macroscopic tomography is necessary. If one were able to obtain cross-sectional images of light absorption to 1–2 cm under the skin with millimeter spatial resolution, then one could expect different merits from OCT and PAT. Several methods have been reported for similar macroscopic optical cross-sectional imaging.²⁰–²⁵

Earlier, we proposed a method to obtain a one-dimensional (1D) distribution of absorption coefficient in the depth direction of animal tissue using a single source–detector (S–D) pair for spatially resolved measurement of backscattered light.²⁶–²⁸ This method is based on the CW measurement of reflectance or backscattered light intensity. It makes the device simple, and measurement highly stable and repeatable. However, the application of 1D absorption distribution is limited, because it assumes the optical uniformity in the estimated layers in animal tissue. Therefore, in this paper, we propose a new method to extend the earlier method to estimate the two-dimensional (2D) distribution of the absorption coefficient. In this method, we measure the spatially resolved backscattered light by scanning the S–D pair laterally along the object surface. In the absorption estimation, we divide the object into many regions and the absorption coefficient of each region is estimated by resolving the inverse problem. To estimate many absorption coefficients stably, a new algorithm is also proposed in this work. The algorithm greatly reduces the number of unknown variables that must be estimated. Moreover, the estimation accuracy has been improved more than by extension to two dimensions by S–D scanning. The feasibility of the developed technique was examined through simulation. Its limits and characteristics have been analyzed.

2. Principle of the proposed method

2.1. Estimation of 1D absorption distribution

Figure 1 presents the principle of 1D absorption coefficient distribution estimation by spatially resolved measurement of backscattered light. The object of the turbid medium is divided into virtual layers. The attenuation of light intensity through n layers is expressed as a function of reflectance as

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The method for determining the absorption coefficient distribution is based on scattering measurements. Both the absorption coefficient and the scattering coefficient of the biological tissues depend on the light wavelength. However, the optical path length distribution does depend not on the absorption coefficient: only on the scattering coefficient. 

Therefore, when obtaining the absorption coefficient distribution, we should obtain only the wavelength dependence of the optical path length distribution.

Equation (1) shows that we can obtain the depth distribution of the absorption coefficient ($\mu_a$) from the reflectance or the spatially resolved backscattered light intensity using the parameters, the number of photons and the optical path length distribution. The scattering coefficient ($\mu_s$) and $\mu_a$ vary with the light wavelength. In the Monte Carlo simulation we use, the parameters $N_0$, $N(\rho)$ and $l_i(\rho)$ are dependent on $\mu_s$ and independent on $\mu_a$. Therefore, we use the known $\mu_s$ of a specific wavelength of the light to obtain the parameters, and the estimated $\mu_a$ is the value of the specific wavelength.

### 2.2. Expansion to 2D imaging

When a light beam is illuminated on the turbid medium surface, the light propagates in the turbid medium. Some part of the light comes out from the upper surface as backscattered light. The spatial distribution of light intensity on the surface or the reflectance $R(\rho)$ is measured as a function of S–D distance $\rho$. In the three-dimensional orthogonal coordinate system, the depth direction is designated by $z$.

Using Eq. (1), we can estimate the absorption coefficient ($\mu_a$) distribution along the $z$ direction. If one divides the $x$–$z$ plane into rectangular segments as shown in Fig. 2 and makes Eq. (1) accommodate two dimensions, then the $\mu_a$ of each pixel region in the $x$–$z$ plane can be estimated simultaneously, in principle. If we accommodate Eq. (1) for three dimensions, then the $\mu_a$ in each voxel region can be estimated simultaneously. Most earlier reports are based on this methodology. However, by this policy, the number of unknowns $\mu_a$ increases rapidly with the number of segmentations. Then the solutions of the inverse problem become unstable. A unique solution becomes difficult to obtain.
Therefore, we decided not to extend Eq. (1) to two dimensions, but to use lateral scanning of the S–D pair for 2D imaging through a turbid medium. If S–D aligned in the y direction is scanned in the x direction, then cross-sectional imaging in the x–z plane becomes possible. To verify this possibility for the present study, we attempted cross-sectional imaging in the x–z plane using the S–D pair in the y direction and scanning it in the x direction.

As depicted in Fig. 2, the turbid medium under consideration is divided into regions in x and z directions. The numbers of divided regions in the x and z directions are \( n_x \) and \( n_z \), respectively. In addition, \( \mu_{a\text{j}x} \) stands for the absorption coefficient of the region, which is the \( j \)th in the x direction and the \( i \)th in the z direction. When the S–D pair is oriented in y-direction on the center of the \( j \)th region in the x direction and the source is at position \( y = 0 \), the average path length of the photons propagated in the region just below the S–D pair reaches a maximum value \( I_{jx}(y) \) and becomes smaller with the distance from \( j \)th region. Considering that the changes in \( \mu_a \) along this light path are small, Eq. (1) is approximated as Eq. (2). Consequently, plural regions in the x direction can be regarded as a single region. If one estimates the absorption distribution in the z direction using Eq. (2) and scans the S–D pair in the x direction, then 2D imaging with a greatly reduced number of unknowns becomes possible.

\[
\sum_{j=1}^{n_z} \left[ \mu_{a\text{ij}x} \sum_{i=1}^{n_x} I_{jx}(y) \right] = \ln \left[ \frac{1}{N_0R_{jx}(y)} \sum_{i=1}^{N(y)} \exp \left( \sum_{j=1}^{n_z} \left[ \mu_{a\text{ij}x} \sum_{i=1}^{n_x} \delta_{jx}(y) \right] \right) \right].
\]  

(2)

Next, we devised a technique to obtain the \( \mu_a \) distribution more stably and accurately than before by reducing the number of unknowns in solving the inverse problem. The proposed method is based on a solution of the simultaneous equation, Eq. (1), to obtain as many \( \mu_a \)’s as the number of layers. As presented in Fig. 1, the measured reflectance has more information on shallower layers than deep layers. This results in more accurate \( \mu_a \) estimation in shallower layers. Therefore, in the proposed technique, we first estimate the \( \mu_a \) at a shallow layer. Then, we successively estimate the deeper layer using the estimated \( \mu_a \) values of the shallower layers as known variables. The object is divided into a few layers consisting of a shallow layer and deep layers. We combine many deep layers into a single deep layer with the averaged \( \mu_a \). Particularly, we combine from \( p_x \)th to \( n_x \)th layers into a single \( p_x \)'th layer and approximate the left-hand-side of Eq. (2) into Eq. (3), as

\[
\sum_{j=1}^{n_z} \left[ \mu_{a\text{ij}x} \sum_{i=1}^{n_x} I_{jx}(y) \right] \approx \ln \left[ \frac{1}{N_0R_{jx}(y)} \sum_{i=1}^{N(y)} \exp \left( \sum_{j=1}^{n_z} \left[ \mu_{a\text{ij}x} \sum_{i=1}^{n_x} \delta_{jx}(y) \right] \right) \right].
\]  

(3)

where \( 1 < p_x < n_x, 1 < p'_x < n_x, \) and \( n_x, n_z, p_x \) are all integers. In actual sequential estimation, the unknown variables are determined one-by-one sequentially. Therefore, one can reduce the unknown variables in Eq. (3) to as few as two to resolve them stably. Using the approximation of Eq. (3), we can redefine Eq. (2) as shown below.

\[
\sum_{j=1}^{n_z} \left[ \mu_{a\text{ij}x} \sum_{i=1}^{n_x} I_{jx}(y) \right] = \ln \left[ \frac{1}{N_0R_{jx}(y)} \sum_{i=1}^{N(y)} \exp \left( \sum_{j=1}^{n_z} \left[ \mu_{a\text{ij}x} \sum_{i=1}^{n_x} \delta_{jx}(y) \right] \right) \right].
\]  

(4)

It is noteworthy that the number of unknown variables should be reduced to three layers in this paper. They are the layer to determine, the buffer layer, and the remaining layer. The purpose of the buffer layer is to reduce the error caused by the influence of the combined layer.

Figure 3 presents this process. First, the deeper layers are combined into a single layer to make a few layers. Then, on the region \( jx \), the \( \mu_a \)'s for all layers are obtained using Eq. (4). Then, we pick up the \( \mu_a \) of the first layer and determine it as the absorption coefficient of the first layer \( \mu_{a1jx} \). We repeat the same layout arrangement with the remaining layers to make them a few layers. With these new layers, we solve simultaneous equation Eq. (4) and obtain \( \mu_a \)'s for all layers. The \( \mu_a \) of the top unknown layer becomes the absorption coefficient of the second layer \( \mu_{a2jx} \). This sequence is repeated until the unknown of the final layer \( \mu_{a3jx} \) is found. Although we must rearrange the simultaneous equation at each repetition, we can estimate the \( \mu_a \) distribution in the depth direction with a small number of unknowns. In this way, more accurate \( \mu_a \) can be obtained for large numbers of layers more stably than solving the simultaneous equation with numerous unknowns all at once. Particularly, we replace \( p_x \) in Eq. (4) by \( p'_x + m_x \) for the case in which the \( \mu_a \) are known from the top to \( m_x \)th layer. Therefore, in this case, Eq. (4) can be transformed to Eq. (5). The absorption terms for 1st to

![Fig. 3. Estimation process of absorption coefficients by variable layer reduction. \( \mu_{a1jx} \) to \( \mu_{a4jx} \) are obtained by solving simultaneous equations for three unknowns respectively. \( \mu_{a5jx} \) and \( \mu_{a6jx} \) are obtained by solving simultaneous equations for two unknowns.](image)
where $m_r^{th}$ layers on the left side of Eq. (4) move to the right side in Eq. (5) as the known terms.

\[
\sum_{j=m+1}^{p} \mu_{ij,j} \sum_{l=1}^{n_l} I_{ij,l}(y) = \ln \left( \frac{1}{N_0 R_{ij}(y)} \right) \sum_{l=1}^{n} \exp \left( - \sum_{j=m+1}^{p} \mu_{ij,j} \sum_{l=1}^{n_l} \delta_{ij,l}(y) \right) - \sum_{l=1}^{n_l} \sum_{j=1}^{m_r} \mu_{ij,j} \tilde{I}_{ij,l}(y) 
\]

Therefore, to estimate the robust $\mu_i$ on the right side is the determined 2D distribution of absorption coefficient in the $x$-$z$ plane. It is obtained by solving the S–D pair in the $x$ direction from $j_x=1$ to $n_x$. In the repeated computation of the proposed method, the primary task is to solve simultaneous equation Eq. (5) or to calculate an inverse matrix. Measurement noise strongly affects the simple inverse matrix calculation. Therefore, to estimate the robust $\mu_i$ for each layer, we used Eq. (6), which was transformed from Eq. (5) into an integration type as

\[
\begin{pmatrix}
\mu_{1j,j} \\
\mu_{2j,j} \\
\vdots \\
\mu_{mj,j}
\end{pmatrix} = A^{-1}B,
\]

where $A$ and $B$ are following matrices,

\[
A = \begin{pmatrix}
\int_{r_1}^{r_2} \left[ \sum_{l=1}^{n_l} I_{ij,l}(y) \right] dy \\
\int_{r_1}^{r_2} \left[ \sum_{l=1}^{n_l} I_{ij,l}(y) \right] dy \\
\vdots \\
\int_{r_1}^{r_2} \left[ \sum_{l=1}^{n_l} I_{ij,l}(y) \right] dy \\
\int_{r_1}^{r_2} \left[ \sum_{l=1}^{n_l} I_{ij,l}(y) \right] dy \\
\end{pmatrix}
\]

\[
B = \begin{pmatrix}
\int_{r_1}^{r_2} \left[ \sum_{l=1}^{n_l} \frac{1}{N_0 R_{ij}(y)} \sum_{l=1}^{N_l} \exp \left( - \sum_{j=m+1}^{p} \mu_{ij,j} \sum_{l=1}^{n_l} \delta_{ij,l}(y) \right) \right] dy \\
\int_{r_1}^{r_2} \left[ \sum_{l=1}^{n_l} \frac{1}{N_0 R_{ij}(y)} \sum_{l=1}^{N_l} \exp \left( - \sum_{j=m+1}^{p} \mu_{ij,j} \sum_{l=1}^{n_l} \delta_{ij,l}(y) \right) \right] dy \\
\vdots \\
\int_{r_1}^{r_2} \left[ \sum_{l=1}^{n_l} \frac{1}{N_0 R_{ij}(y)} \sum_{l=1}^{N_l} \exp \left( - \sum_{j=m+1}^{p} \mu_{ij,j} \sum_{l=1}^{n_l} \delta_{ij,l}(y) \right) \right] dy \\
\int_{r_1}^{r_2} \left[ \sum_{l=1}^{n_l} \frac{1}{N_0 R_{ij}(y)} \sum_{l=1}^{N_l} \exp \left( - \sum_{j=m+1}^{p} \mu_{ij,j} \sum_{l=1}^{n_l} \delta_{ij,l}(y) \right) \right] dy \\
\end{pmatrix}
\]

$r_1$ and $r_2$ respectively represent the starting and ending points of the integration. They are determined by practical considerations as described in chapter 3.

2.3. Cross-sectional imaging procedures

Figure 4 presents estimation procedures in the form of a flowchart. Using the proposed technique, we divide the object turbid medium in many layers in the $x$-$z$ plane. Then we obtained the values of $N(y), I_{ij,l}(y), \delta_{ij,l}(y)$ through Monte Carlo simulation. The structure of many layers was arranged by combining the deeper layers into a single layer. The corresponding $I_{ij,l}(y), \delta_{ij,l}(y)$ are calculated as the sum of the values of the respective layer components. We have a three-layer model if we combine the layers deeper than the third layer into a single layer. Then, with the S–D pair on the region $j_x$, we measured the reflectance along the $y$-direction, or the spatial distribution of backscattered light $R_{ij}(y)$. With this reflectance, we can obtain the unknowns $\mu_{1j,j}, \mu_{2j,j}$, and $\mu_{Nj,j}$ on the region $j_x$ by solving simultaneous equations Eq. (4) or its integral for three unknowns. This nonlinear inverse problem is solvable by iterative operation. Moreover, one can ascertain the absorption coefficient of the top layer on the region $j_x$, $\mu_{1j,j}$. We repeat this process different S–D positions by shifting the S–D pair in the $x$-direction: we measure the reflectances $R_{ij}(y)$ at $y$-direction at the positions $j_x=1$ to $n_x$, and determine the distribution of the first layer absorption coefficient $\mu_{1j,j}$.

Then we rearrange the layer structure as depicted in Fig. 3. In the next step, the second layer becomes a new first unknown layer. The deeper layers are combined into new second and third unknown layers. The new parameters $N(y), I_{ij,l}(y), \delta_{ij,l}(y)$ are calculated easily from the values for each layer obtained in the Monte Carlo simulation. This requires the time-consuming Monte Carlo simulation only once.
initially to obtain these parameters for each layer. Using these parameters, we can obtain the unknown $\mu_{a2j}$, $\mu_{a3j}$, and $\mu_{a4j}$ for this new layer arrangement by solving simultaneous equations Eq. (5) or Eq. (6) for three unknowns. From those solutions, we ascertain the absorption coefficient of the second layer $\mu_{a2j}$. In the same manner as that presented above, we repeat this process for different S–D positions in the x-direction and determine the distribution of the second layer absorption coefficient $\mu_{a2j}$.

By repeating this process successively for deeper layers, we can obtain the absorption distribution in the x and z-directions to have the cross-sectional image in the x–z plane. This technique presents the following benefits. Compared with other methods [30–32] in which the $\mu_a$ of many pixels or voxels are obtained simultaneously in solving the inverse problem, a stable unique solution can be obtained because the unknowns are far fewer. The computational load is markedly lessened because calculation of the forward model is not included in an iteration loop. A result that is close to an exact solution can be obtained because the higher-order nonlinear terms for the unknown $\mu_a$ are handled in Eqs. (5) and (6). In contrast, we must obtain the average photon propagation path lengths in each layer in simulation or measurement before solving Eqs. (5) or (6) for $\mu_a$. They are obtainable through simulation.

In the validity test of the proposed technique in simulation, the values of the reflectance $R_{jx}(y)$ are required. For this study, they were obtained in separate Monte Carlo simulations for photon propagation.

3. Fundamental analysis of photon propagation

When we illuminate light on the turbid medium surface, it propagates in the medium and returns, partially, to the surface by backscattering. Reflectance $R_{jx}(y)$ represents the spatial distribution of the backscattered light intensity as a function of S–D distance $y$. Therefore, for cross-sectional imaging, it is important to know how the detected light propagates in the medium. For example, the extent of $y$ can be judged for the measurement of $R_{jx}(y)$. Here in simulation, we analyzed the photon propagation in a turbid medium in conditions of human body tissues. Figure 5 presents the numerical phantom model used in the simulation. The object was a rectangular parallelepiped with $400 \times 400 \times 200 \text{mm}^3$. The x and z directions were divided respectively into 29 and 5 sections to make $1 \times 2 \text{ mm}^2$ pixels. The area outside of the divided area was lumped until the end of the phantom model. The reduced scattering coefficient of the whole phantom was $\mu_s' = 1.0 \text{ mm}^{-1}$, which is a typical value of the general body tissue in NIR wavelengths. The anisotropy parameter was $g = 0$. Forward scattering is generally dominant in animal tissue, or $g$ is close to 1. However, because of the large scattering coefficient, the photon mean free path is smaller than 1 mm. In our analysis, the size of each region is larger.
than $1 \times 2$ mm$^2$. Therefore, we used the values $\mu_s' = 1.0$ mm$^{-1}$ and $g = 0$ commonly used in NIR wavelengths assuming isotropic scattering in the analysis region. The Monte Carlo method was used to simulate photon propagation. We injected $2 \times 10^9$ photons at a point on the top horizontal plane and obtained the intensity of backscattered photons caught by the detector on the horizontal surface plane. Each detector’s diameter was 0.25 mm. The center of each detector was aligned in the $y$ direction from $y = 2.125$ mm to $y = 31.875$ mm with a 0.25 mm interval.

In the simulation, we attempted to clarify the propagation region of photons received by the detector, and to optimize the range of spatial integration in Eq. (6). Figure 6 portrays the spatial distribution of the average path lengths in the $x$–$z$ plane for different $S$–$D$ distances. The segmentation size in the $z$ direction here was made as small as 1 mm to present the distribution in detail. On the top plane ($z = 0$), the light detectors with different $S$–$D$ distance $y$ were aligned in the $y$ direction along the line ($x = 0, z = 0$). The average path lengths of the photons received by the detector at a specific $y$ were calculated through Monte Carlo simulation.29) Here, each photon is considered to have the same energy in the calculation. The result is shown in terms of brightness for each $1 \times 1$ mm$^2$ pixel in the $x$–$z$ plane. The average path lengths in each pixel correspond to $l_{ij}(\rho)$ in Eq. (5). Figure 6 shows that the photons received by the detector with small $S$–$D$ distance $y$ propagated mainly in surface layers.
Furthermore, it shows how much the photon paths extend to a deeper layer with the distance \( y \). One can understand how deep the detected photons can reach before arriving at the surface, and how much information they can carry from the depths. Through these analyses, we obtained the following facts. In the reflectance measurement presented in Fig. 1, the ratio of photons that pass the deep area increases with S–D distance \( y \). Therefore, we inferred that different suitable S–D distances exist for different depths. To find the suitable distance, we obtained the depths of the pixel in which the average path length is maximum. Figure 7 presents the result. From this result, we can ascertain the appropriate integration range between starting point \( r_1 \) and ending point \( r_2 \) for Eq. (6). For example, to estimate 4 mm depth, the range should be set to include \( y = 15 \) mm. For 8 mm depth, the range should include \( y = 27 \) mm as a rule of thumb. Based on this result, we ascertained the integration range as 2.125–3.625 mm, 2.125–10.625 mm, 2.125–17.875 mm, 2.125–25.125 mm, and 2.125–31.875 mm, respectively, for the first, second, third, fourth, and over fifth layers.

These values depend on the scattering coefficient of the scattering medium. Therefore, the scattering coefficient at the used wavelength has to be known beforehand. It is available from the literature or from an independent experiment. In addition, the wavelength dependence of the scattering coefficient is known much smaller than that of the absorption coefficient in the NIR wavelengths. It should be noted if the interval range is too narrow, i.e., if the measurement points are not enough, then the convergent calculation is likely to diverge. In such a case, it is possible to calculate it again by extending the range to some extent.

4. Cross-sectional imaging by layer estimation

4.1. Imaging characteristics in depth-direction
To examine the possibility of cross-sectional imaging using the proposed technique, we obtained the reflectance in Monte Carlo simulation and solved the inverse problem to obtain a cross-sectional image from the reflectance. The phantom
model is the same as that presented in Fig. 5 with one or two light-absorbing inclusions. The inclusion width in the x direction was set to 1 mm, 2 mm, and 3 mm. The \( \mu_a \) values for the inclusions were either 0.025 mm\(^{-1} \) or 0.050 mm\(^{-1} \), whereas that of the surrounding medium was 0.01 mm\(^{-1} \), respectively assuming typical absorption coefficients of the fiber cystic, carcinoma, and fatty normal tissues.\(^{33-38} \) Other characteristics of the phantom model were the same as those presented in Chap. 3.

To check the imaging characteristics in the depth direction, we analyzed the cross-sectional images of a single absorber at different depths. Figure 8 shows examples of images for 2 mm width and a 0.025 mm\(^{-1} \) absorption coefficient of an absorber. The depth dependence of the correlation coefficient for each combination of width and absorption coefficient of an absorber is shown in Fig. 9. The correlation coefficient \( r_{IG} \) between the reconstructed image \( \mu_{aj, i}^G \) and the given image \( \mu_{aj, i} \) is given as,

\[
r_{IG} = \frac{\sum_j \sum_i (\mu_{aj, i}^G - \bar{\mu}_a^G)(\mu_{aj, i} - \bar{\mu}_a)}{\sqrt{\left(\sum_j \sum_i (\mu_{aj, i} - \bar{\mu}_a^G)^2\right)\left(\sum_j \sum_i (\mu_{aj, i} - \bar{\mu}_a)^2\right)}}.
\]

where \( \bar{\mu}_a^G \) and \( \bar{\mu}_a \) respectively stand for the mean value of the absorption coefficients over all the elements of the reconstructed image \( \mu_{aj, i}^G \), and that of the given image \( \mu_{aj, i} \).

By setting the criterion for the detectability to \( r_{IG} > 0.5 \), we can estimate the detectable depth for the absorber. Figure 9 shows that we can detect the absorber as deep as 4, 8, 10 mm respectively for 1, 2, 3 mm absorber widths. These are the cases when the absorption coefficients for the absorber and the surrounding medium are respectively 0.025 and 0.01 mm\(^{-1} \). When this contrast increases as 0.05 and 0.01 mm\(^{-1} \), the detectable depth becomes larger. As the depth of the absorber increases, image artifacts around the absorber increase, resulting in a lower correlation coefficient. Particularly, the effect is noticeable in the area which is a shadow of the absorber viewed from the surface. This is apparently unavoidable property for cross-sectional imaging by the reflection measurement from a single direction such as shadowing phenomena in ultrasonic echo imaging. However, the proposed technique is based on scattered light propagation instead of wave reflection in ultrasonic echo imaging. Because the photons propagated behind the absorber are also included in the measured reflectance, measurement of the shadow region is not impossible. These results suggest that cross-sectional imaging is possible up to 10 mm in conditions associated with general body tissues.

4.2. Basic characteristics of cross-sectional images

To clarify the possibilities and limitations of the proposed technique, we analyzed the basic characteristics of the estimated cross-sectional images. First, the spatial resolution of estimated images was evaluated in the following manner. The 2D point spread function (PSF) was obtained in deconvolution of the estimated image with the given 2D distribution of \( \mu_a \). Actually, PSF was determined with the 2D deconvolution operation of the reconstructed absorption coefficient distribution by the given absorption coefficient distribution using the Richardson–Lucy algorithm.\(^{39} \) We evaluated the full width at half maximum of the PSF in the depth direction as the spatial resolution of the Rayleigh criterion. Figure 10 presents the result. Although the spatial resolution becomes poorer with depth, one can expect spatial resolution of less than 1.5 mm even for 8–10 mm depth.

Next, we analyzed the sensitivity of the proposed method to a change in \( \mu_a \) of the absorber. Figure 11 presents examples of the estimated images. It shows that we can visualize the absorption change in the absorber as a change in the contrast in the estimated image. Figure 12 portrays the examples of the estimated images. It shows that we can visualize the absorption change in the absorber as a change in the contrast in the estimated image. Figure 12 shows that the contrast decreases with the absorber depth. The decrease rate is greater in the case of a smaller absorber. That is, the detectable absorption coefficient depends on the depth and the size of the absorber. Considering the consistency with the detectable depth presented in Sect. 4.1 with Fig. 9, we set the detectable limit of the absorption contrast to 0.05. Then, the detectable absorption coefficient was estimated as, less than 0.015 mm\(^{-1} \) up to 4 mm depth, less than 0.015 mm\(^{-1} \) up to 8 mm depth, and less than 0.020 mm\(^{-1} \) up to 10 mm depth, respectively for 1, 2, 3 mm width absorbers.

The absolute value of the estimated absorption coefficient was not always correct, or smaller than the given value. It seems that some kind of calibration is required to obtain an accurate absorption coefficient value.

4.3. Imaging characteristics for plural absorbers

The analysis introduced up to this point has dealt with the case in which a single absorber exists in a turbid medium. However, in NIR imaging of abdominal parts of animal bodies, for example, kidneys of both sides act as absorbers. In such a case with a plural number of absorbers, mutual...
image-interaction of absorbers might occur in an image estimated with measurement from one direction, such as a proposed technique. Therefore, we analyzed the images of the case in which there were two absorbers in a turbid medium. Figures 13 and 14 respectively present examples of cross-sectional images, when two absorbers exist in horizontal and vertical directions. In the deep part, horizontally close absorber images are overlapped because of the poor resolution. This horizontal-overlap effect on the estimated absorption coefficient was further analyzed. The values of the absorption coefficient at the absorber positions were compared with those obtained in the case of a single absorber. The arithmetic mean of the absorption coefficient at the two absorber positions was calculated, and the ratio to the absorption coefficient estimated in the single absorber case was analyzed. Figure 15 shows the results of the analysis. The horizontal spacing between the two identical absorbers was varied from 1 to 3 times as large as the absorber width. The results show the tendency of larger value estimation than the single absorber case. This tendency was enhanced with closer and deeper absorbers. Considering the detectable depth obtained in Sect. 4.1, these results suggest the overlap effect of two absorbers is within 10% of the absorption coefficient.

Figure 14 shows that the imaging behind the shallow absorber is possible even with the shadowing effect of the shallow absorber because this technique is based on the
propagation of scattered light and not on the reflected wave. The scattered light component can exist behind a shallow absorber, even with low intensity, and the absorber is detectable. This vertical effect on the estimated absorption coefficient value was further analyzed. The absorption coefficients of the absorbers at the first layer and the deeper layer were 0.025 mm$^{-1}$ and 0.050 mm$^{-1}$ respectively or vice versa in the medium with 0.01 mm$^{-1}$. The widths of all the absorbers were set to 2 mm as shown in Fig. 14. The absorption coefficient at the deeper absorber position was estimated with and without the absorber at the first layer. The ratio of estimated absorption coefficients in these two cases was obtained. Figure 16 presents the results of the analysis.

Fig. 13. Cross-sectional imaging of horizontal double absorbers for resolution test: true images (left) and estimated images (right). Absorbers of $\mu_a = 0.025$ mm$^{-1}$ in medium of $\mu_a = 0.01$ mm$^{-1}$. Spacing between two absorbers is 2 mm.

Fig. 14. Cross-sectional imaging of vertical double absorbers for shadowing test: true images (left) and estimated images (right). Absorbers of $\mu_a = 0.025$ mm$^{-1}$ in medium of $\mu_a = 0.01$ mm$^{-1}$. Spacings between two absorbers are (a) 2 mm and (b) 4 mm.
The estimated absorption coefficient became larger owing to the absorber located above. The value became 42% larger in the worst case in the analysis, or in the estimation of the absorber of $\mu_a = 0.025 \text{ mm}^{-1}$ at the position just below the absorber of $\mu_a = 0.05 \text{ mm}^{-1}$. However, this effect diminished rapidly with the distance from the above absorber. If the absorption coefficient of the above absorber was as low as $0.025 \text{ mm}^{-1}$, the effect was as small as around 20 to a few percent. These results suggest the following. The absorption coefficient estimation is influenced by the existence of another absorber located in a shallower position than the absorber of interest. However, we can still detect the existence of the deeper absorber, and the influence is limited in close proximity to the shallower absorber. It should be noted that no completely invisible region exists behind an absorber in this method.

Finally, to examine the effects of the asymmetrical absorber position in both vertical and horizontal directions, we attempted imaging with absorbers shifted vertically and horizontally. Figure 17 presents the result. The correlation coefficients between the 2-dimensional given and estimated absorption distributions were analyzed. The absorption coefficients of the absorber and the medium were respectively 0.025 mm$^{-1}$ and 0.01 mm$^{-1}$. Figures 17(a) and 17(b) respectively show the results when one absorber is fixed and the other one is shifted vertically and horizontally. The size of the absorber was 2 $\times$ 2 mm$^2$ and the horizontal or vertical distance between the two absorber centers was set to 4 mm. The same analysis was made exchanging the shifting absorber from the left to the right. There was no difference found in the correlation coefficients in this left to right change. These results suggest that no directionality exists in the proposed method and that distortionless cross-sectional imaging is possible.

Through the analysis described in Sects. 4.1–4.3, the solution never diverged, at least in the cases investigated in the present study. The feasibility of macroscopic 2D cross-sectional imaging with significant stability was confirmed. Although the technique using backscattered light is applicable to a thick body, and although it is very useful for practical biomedical applications, deterioration of spatial resolution and of estimation accuracy is unavoidable. To resolve these difficulties, we can formulate these deteriorations beforehand in experiments or simulations. We can also...
calibrate the estimated results. For example, we can obtain deterioration in spatial resolution in terms of the PSF for each depth beforehand. Then by deconvolution with the corresponding PSF, the image deterioration in deep parts of the turbid medium can be improved. By the same principle, we can improve the errors in the estimated value of absorption coefficient in deep parts using calibration factors obtained beforehand.

5. Conclusions

To visualize the internal structure of an animal body with safe NIR light, we devised a method to reconstruct the cross-sectional image through practical spatially resolved measurements of backscattered light. This method requires no large or complicated instruments such as those for temporary resolved measurement and frequency modulated measurement. It can be realized with simple and compact CW optics and the lateral scan of an LED and low-speed photodiodes. Because it measures backscattered light, it is applicable to thick body parts through which transmitted light cannot be obtained. Using the proposed technique, we can estimate the absorption coefficients of a few virtual layers instead of dividing the object turbid medium into numerous voxels. Therefore, many difficulties associated with the solution of inverse problems with an enormous number of unknowns can be resolved. In solving the inverse problem using this technique, the linearization error is suppressed effectively by the nonlinear solution using converging iteration. In this paper, the proposed method was applied to 2D cross-sectional imaging. Light source and detectors (S–D) aligned in the y direction was scanned in the x direction, then cross-sectional imaging in the x–z plane became possible. Cross-sectional imaging in the y–z plane becomes possible if S–D aligned in the x direction is scanned in the y direction. This imaging is based on the lateral scanning of 1D distribution estimation. Therefore, its extension to 3D imaging is straightforward. For example, using both modes of scanning and image processing, one can visualize the absorption distribution in a turbid medium in 3D.

To test the feasibility of the proposed method, we analyzed the results of estimation in the simulation incorporated with the Monte Carlo simulation of photon propagation. As a result, the feasibility of cross-sectional imaging using the proposed method was confirmed. From analyses of estimated images, the following facts were clarified. The shadowing effect is not severe, in principle. The spatial resolution and estimation error in the absolute value of $\mu_a$ deteriorates with depth in a turbid medium. However, they are consistent and are apparently correctable, respectively, by suitable deconvolution and calibration. Furthermore, results clarified that we can visualize the change in $\mu_a$ of the absorber as the contrast of the estimated image, and clarified that no field-interference effect exists between the plural number of absorber images and no distortion in a total image. These analyses verified that practical cross-sectional imaging is feasible when using the proposed method. Investigation of the applicability of this method for experiments with real animal bodies remains a task for future research.

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