What if a specific neutrinoless double beta decay is absent

Takehiko Asaka\textsuperscript{1}, Hiroyuki Ishida\textsuperscript{2}, and Kazuki Tanaka\textsuperscript{3}

\textsuperscript{1}Department of Physics, Niigata University, Niigata 950-2181, Japan
\textsuperscript{2}KEK Theory Center, IPNS, Tsukuba, Ibaraki 305-0801, Japan
\textsuperscript{3}Graduate School of Science and Technology, Niigata University, Niigata 950-2181, Japan

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Abstract

We consider the seesaw model with two right-handed neutrinos $N_1$ and $N_2$ which masses are hierarchical, and investigate their contribution to the neutrinoless double beta ($0\nu\beta\beta$) decay. Although the lepton number is broken by the Majorana masses of right-handed neutrinos, such decay processes can be absent in some cases. We present a possibility where the lighter $N_1$ gives a destructive contribution to that of active neutrinos by choosing the specific mixing elements of $N_1$, while $N_2$ is sufficiently heavy not to contribute to the $0\nu\beta\beta$ decay. In this case the mixing elements of $N_1$ in the charged current interaction are determined by its mass and the Majorana phase of active neutrinos. We then study the impacts of such a possibility on the direct search for $N_1$. In addition, we discuss the consequence of the case when the $0\nu\beta\beta$ decay in one specific nucleus is absent.

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1 Introduction

The seesaw mechanism \cite{1-7} by right-handed neutrinos with Majorana masses is very attractive because it can provide a natural explanation of the tiny neutrino masses which have been confirmed by various oscillation experiments. One of the most important consequences is the Majorana nature of active neutrinos as well as the heavier states which we call as heavy neutral leptons (HNLs), and then the lepton number is violated by two units. In the Standard Model (SM), although the lepton and baryon numbers are accidental symmetries of the Lagrangian, both are violated by the non-perturbative quantum effect by the anomaly \cite{8,9}. However, its breaking effect is highly suppressed at zero temperature and is essentially negligible in the experimental processes. Thus, the experimental tests for the lepton number conservation are crucially important to verify the seesaw mechanism.

The well-known example of such tests is the search for the neutrinoless double beta (0νββ) decay: 
\((Z,A) \rightarrow (Z+2,A) + 2e^-\) which violates the lepton number by two units (see, for example, reviews \cite{10-13}). The current limit of the 0νββ decay half-life is \(\tau_{1/2} > 1.07 \times 10^{26}\) yr by the KamLAND-Zen with \(^{136}\text{Xe}\) \cite{36}. The decay is mediated by massive active neutrinos if they are Majorana particles, and their contribution is parameterized by the effective neutrino mass \(m_{\text{eff}}\). The above half-life limit is translated into the upper bound on the effective mass as \(61 - 165\) meV \cite{36}. Note that the bound on the effective mass receives the uncertainty in the nuclear matrix element of the decay process.

In the seesaw mechanism HNLs may participate the 0νββ decay which is quantified as an additional part to \(m_{\text{eff}}\). When the masses and mixing elements of HNLs are sufficiently light and large, their contribution to \(m_{\text{eff}}\) can be comparable to that of active neutrinos, and the cancellation in \(m_{\text{eff}}\) between these contributions happens in some cases. Notice that the cancellation in \(m_{\text{eff}}\) between three active neutrino is possible if the masses of active neutrinos are in the normal hierarchy and the lightest active neutrino has a specific mass. To achieve this possibility, HNLs must be completely decoupled from the decay. In this paper we discuss the seesaw mechanism with two right-handed neutrinos in which the lightest active neutrino becomes massless. In this case such a cancellation does not occur and then we shall disregard the possibility.

One simple possibility is that all the HNLs participating in the seesaw mechanism are lighter than about 0.1 GeV scale (which is the typical momentum scale in the 0νββ decay). In this case \(m_{\text{eff}} = 0\) is ensured by the intrinsic property in the seesaw mechanism \cite{37}. If this is the case, the processes of the 0νββ is absent although the lepton number is violated and active neutrinos induce the sizable contribution to \(m_{\text{eff}}\).

In a recent article \cite{38} we have pointed out another possibility. It is shown that, in the minimal choice of the seesaw mechanism with two right-handed neutrinos, \(m_{\text{eff}} = 0\) is realized when the heavier HNL decouples from the 0νββ decay but the lighter one being lighter than the typical momentum scale of the decay gives a destructive contribution. The purpose of this paper is to extend the discussion to more
general cases, especially to the case in which the lighter one is heavy as $\mathcal{O}(1-10)$ GeV scale. It will be shown that the cancellation in $m_{\text{eff}}$ is possible for such a heavy mass region and the required mixing elements are relatively large so that such a HNL is a good target for the future search experiments.

The rest of this paper is organized as follow. In Sec. 2 we explain the model in the present analysis. In Sec. 3 we describe the contributions to the effective neutrino mass in the $0\nu\beta\beta$ decay from active neutrinos as well as HNLs, and then show how the cancellation in the effective neutrino mass is realized by the lighter HNL. In addition, we suggest a possibility that even if the $0\nu\beta\beta$ decay is not observed at an experiment using a specific element, other experiments which use different elements can observe the decay due to an enhancement originated in the difference of the nuclear matrix elements. It is then discussed in Sec. 4 that the implication of such a cancellation to the direct search of HNLs. Finally, Sec. 5 is devoted to discussions and conclusions. We add Appendices A and B to present the physical region of the model parameters and the predicted upper and lower bounds of mixing elements in each flavor for the HNL $N_1$.

## 2 Seesaw model with two right-handed neutrinos

In this paper we consider the minimal seesaw scenario where the SM extended by two right-handed neutrinos $\nu_{RI}$ ($I = 1, 2$). The number of right-handed neutrinos must be larger than or equal to two in order to explain the observed mass squared differences in neutrino oscillations. The model is described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i \bar{\nu}_{RI} \gamma^\mu \partial_\mu \nu_{RI} - \left( F_{aI} \ell_a \Phi \nu_{RI} + \frac{M_I}{2} v_{RI}^c \nu_{RI} + h.c. \right), \quad (1)$$

Here $\mathcal{L}_{\text{SM}}$ is the SM Lagrangian. $\Phi$ and $\ell_a$ ($a = e, \mu, \tau$) are the Higgs and lepton doublets of the weak SU(2). Neutrino Yukawa coupling constants and Majorana masses of right-handed neutrinos are denoted by $F_{aI}$ and $M_I$, respectively. Here and hereafter, we work in the basis where the mass matrices of charged leptons and right-handed neutrinos are diagonal.

When the neutrino masses of the Dirac type $[M_D]_{aI} = F_{aI} \langle \Phi \rangle$ is much smaller than the Majorana masses $M_I$, i.e., $|[M_D]_{aI}| \ll M_I$, the seesaw mechanism is realized, and the mass matrix of active neutrinos $\nu_i$ ($i = 1, 2, 3$) is given by

$$[M_\nu]_{\alpha\beta} = -\frac{[M_D]_{aI} [M_D^T]_{aI}}{M_I}, \quad (2)$$

which is diagonalized by the neutrino mixing matrix $U$ called as the PMNS matrix $[39]$. The remaining mass eigenstates are HNLs denoted by $N_I$ which almost correspond to right-handed neutrino states. The seesaw mechanism tells that the mass of $N_I$ is given by $M_I$.

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#2 Throughout this paper, we neglect higher order corrections in the expansion by $[M_D]_{aI} / M_I$ to both active neutrino and HNL masses.
part in the weak gauge interactions through the mixing effect as
\[ \nu_{La} = U_{al } \nu_i + \Theta_{al } N_i^c, \]  
(3)

where the mixing elements of \( N_i \) is given by
\[ \Theta_{\alpha I} = \begin{bmatrix} M_\alpha^D \end{bmatrix}_{\alpha I} = \begin{bmatrix} M_\alpha^D - 1 \end{bmatrix}_{\alpha I}. \]

We apply the parametrization of the couplings by Casas and Ibarra \[41, 42\] as
\[ F = \frac{i}{\langle \Phi \rangle} U D_{\nu}^{1/2} \Omega D_N^{1/2}, \]  
(4)

where \( D_{\nu} = \text{diag}(m_1, m_2, m_3) \). In the considering case with two right-handed neutrinos the lightest active neutrino becomes massless and then the possible mass orderings are \( m_3 > m_2 > m_1 = 0 \) for the normal hierarchy (NH) case and \( m_2 > m_1 > m_3 = 0 \) for the inverted hierarchy (IH) case, respectively. The mixing matrix of active neutrinos is expressed as
\[ U = \begin{pmatrix} c_{12} c_{13} & c_{13} e^{-i\delta} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{23} c_{12} s_{13} e^{i\delta} & c_{23} c_{12} - s_{23} s_{12} s_{13} e^{i\delta} & -s_{23} c_{12} - c_{23} s_{12} s_{13} e^{i\delta} \\ s_{23} s_{12} - c_{23} c_{12} s_{13} e^{i\delta} & -s_{23} c_{12} - c_{23} s_{12} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix} \times \text{diag}(1, e^{i\eta}, 1), \]  
(5)

with \( s_{ij} = \sin \theta_{ij} \) and \( c_{ij} = \cos \theta_{ij} \). \( \delta \) and \( \eta \) are the Dirac and Majorana phases, respectively. \( D_N = \text{diag}(M_1, M_2) \) is the mass matrix of HNLs. The \( 3 \times 2 \) matrix \( \Omega \) can be taken in the form
\[ \Omega = \begin{pmatrix} 0 & 0 \\ \cos \omega & -\sin \omega \\ \xi \sin \omega & \xi \cos \omega \end{pmatrix} \quad \text{for the NH case}, \]
\[ \begin{pmatrix} \cos \omega & -\sin \omega \\ \xi \sin \omega & \xi \cos \omega \\ 0 & 0 \end{pmatrix} \quad \text{for the IH case}, \]  
(6)

where \( \xi = \pm 1 \) is sign parameter and \( \omega \) is a complex parameter
\[ \omega = \omega_r + i \omega_i, \]  
(7)

with \( \omega_r (\omega_i) \) as the real (imaginary) part of \( \omega \). In Eq. (4) the masses and mixing angles of active neutrinos, which are relevant for the neutrino oscillations, are automatically satisfied reproducing the experimental results being independent of the choice of \( D_N \) and \( \Omega \). Further, it is convenient to introduce
\[ X_\omega = \exp(\omega_i), \]  
(8)

because it represents the overall strength of the Yukawa coupling matrix (see, for example, the discussion in Ref. [43]). In practice the Yukawa coupling constants can be enhanced by taking large values of \( |\omega_i| \), as \( F \propto X_\omega \) or \( X_{\omega}^{-1} \) for \( X_\omega > 1 \) or \( \ll 1 \). As a result, the enhancement of the Yukawa coupling constants is reflected to the enhancement of the mixing elements \( \Theta_{\alpha I} \).

In the present analysis we adopt the convention where the ranges of the parameters are \( \omega_r \in [-\frac{\pi}{2}, \frac{\pi}{2}] \) and \( X_\omega \in [0, +\infty) \) with fixing \( \xi = +1 \) (see the discussion in Appendix A). Note that the Majorana phase takes a value in \( \eta \in [0, \pi] \).

\[ ^{#3}\text{Technically speaking, } X_\omega \text{ (or } \omega_i) \text{ can take the value in the whole range } [0, +\infty], \text{ however, the region is restricted not to exceed the limits of the mixing elements from the direct search experiments and the perturbative limit of the neutrino Yukawa coupling constants in practice.} \]
3 Neutrinoless double beta decay

Now let us discuss the $0\nu\beta\beta$ decay in the model under consideration. The half-life of the decay is parameterized as

$$\tau_{1/2}^{-1} = G |\mathcal{M}| m_{\text{eff}}^2,$$

(9)

where $G$ is the phase-space factor, $\mathcal{M}$ is the nuclear matrix element of active neutrinos and $m_{\text{eff}}$ is the effective neutrino mass in the $0\nu\beta\beta$ decay. Since all Majorana neutrinos, i.e., not only active neutrinos but also HNLs, contribute to the decay, the effective mass is given by

$$m_{\text{eff}} = m_{\text{eff}}^v + m_{\text{eff}}^{N_1} + m_{\text{eff}}^{N_2}.$$

(10)

Note that we consider $m_{\text{eff}}$ as a complex number while the observed value has to be its absolute value. The contribution from active neutrinos is

$$m_{\text{eff}}^v = \sum_i U_{ei}^2 m_i.$$

(11)

By taking the central values of the mass squared differences, the mixing angles and the Dirac phase in the PMNS matrix given in Ref. [44] and by varying the Majorana phase $\eta$ we find that $|m_{\text{eff}}^v| = 1.45–3.68$ meV and 18.6–48.4 meV for the NH and IH cases, respectively. On the other hand, the effective mass from each HNL is

$$m_{\text{eff}}^{N_i} = \Theta_{ei}^2 M_i f_\beta(M_i),$$

(12)

which can be written explicitly as

$$m_{\text{eff}}^{N_1} = \Theta_{e1}^2 M_1 f_\beta(M_1) = \begin{cases} -\left(U_{e2} m_2^{1/2} \cos \omega + U_{e3} m_3^{1/2} \sin \omega\right)^2 f_\beta(M_1) & \text{for the NH case}, \\ -\left(U_{e1} m_1^{1/2} \cos \omega + U_{e2} m_2^{1/2} \sin \omega\right)^2 f_\beta(M_1) & \text{for the IH case} \end{cases},$$

(13)

$$m_{\text{eff}}^{N_2} = \Theta_{e2}^2 M_2 f_\beta(M_2) = \begin{cases} -\left(U_{e2} m_2^{1/2} \sin \omega - U_{e3} m_3^{1/2} \cos \omega\right)^2 f_\beta(M_2) & \text{for the NH case}, \\ -\left(U_{e1} m_1^{1/2} \sin \omega - U_{e2} m_2^{1/2} \cos \omega\right)^2 f_\beta(M_2) & \text{for the IH case} \end{cases},$$

(14)

where $f_\beta(M_i)$ represents the suppression in the nuclear matrix element by the propagator of $N_i$. Here we follow the results in Refs. [45][46] and take the form

$$f_\beta(M_i) = \frac{\Lambda^2_\beta}{\Lambda^2_\beta + M_i^2},$$

(15)

where $\Lambda_\beta$ is the typical scale of the Fermi momentum and we take $\Lambda_\beta = 200$ MeV as a reference value throughout this analysis. We will discuss the impact of the change of the Fermi momentum later.
It is known that \( m_{\text{eff}} = 0 \) is possible even if \( m_{\nu_{\text{eff}}}^\nu \neq 0 \) due to the presence of HNLs participating the seesaw mechanism. Namely, when both \( N_1 \) and \( N_2 \) are sufficiently lighter than \( \Lambda_\beta \), \( f_\beta(M_{1,2}) = 1 \) and then the seesaw mechanism guarantees the following equality \cite{37}:

\[
m_{\text{eff}} = \sum_i U_{ei}^2 m_i + \sum_I \Theta_{eI}^2 M_I = 0. \quad (16)
\]

Thus, even though the lepton number is violated by the Majorana masses of right-handed neutrinos, the \( 0\nu\beta\beta \) decay processes are absent being independent of how large \( \Lambda_\beta \) is.

We have pointed out in Ref. \cite{38} another possibility in the case when the masses of \( N_1 \) and \( N_2 \) are hierarchical. It is assumed that the mass and mixing elements of \( N_2 \) are sufficiently heavy and small so that \( m_{\nu_{\text{eff}}}^{N_2} \) can be neglected. In that analysis we have also assumed that \( M_1 \ll \Lambda_\beta \) which gives \( f_\beta(M_1) = 1 \) approximately. The effective mass in this case is abridged as

\[
m_{\text{eff}} = \begin{cases} 
(U_{e2} m_2^{1/2} \sin \omega - U_{e3} m_3^{1/2} \cos \omega)^2 & \text{for the NH case} \\
(U_{e1} m_1^{1/2} \sin \omega - U_{e2} m_2^{1/2} \cos \omega)^2 & \text{for the IH case}
\end{cases} . \quad (17)
\]

It is found that \( m_{\text{eff}} \) vanishes if the complex parameter \( \omega \) satisfies

\[
\tan \omega = A = \begin{cases} 
U_{e3} m_3^{1/2} & \text{for the NH case} \\
U_{e2} m_2^{1/2} & \text{for the NH case} \\
U_{e1} m_1^{1/2} & \text{for the IH case}
\end{cases} . \quad (18)
\]

As described in Ref. \cite{38}, the mixing elements of \( N_1, \Theta_{e1} \), are determined from the mass \( M_1 \) and the Majorana phase \( \eta \) of active neutrinos in this situation and the flavor structure among them is different in each mass hierarchy of active neutrinos. Then, the relative sizes of \( \Theta_{e1} \), if they will be measured in future experiments, give the important information of \( \eta \) which is not basically determined by neutrino oscillation experiments as well as the mass hierarchy of active neutrinos.

In this paper we would like to extend the above discussions to more general cases. We assume again \( f_\beta(M_2) = 0 \)\footnote{Our results do not change much as long as the contribution from \( N_2 \) is sufficiently suppressed.}, but consider the case \( f_\beta(M_1) \neq 1 \) which is valid for heavier \( N_1 \) with \( M_1 \gg \Lambda_\beta \). Hereafter we present the analytic results in the NH case of active neutrino masses and the extension to the IH case is straightforward. In this case, the effective mass can be written as

\[
m_{\text{eff}} = (U_{e2} m_2^{1/2} \sin \omega - U_{e3} m_3^{1/2} \cos \omega)^2 + (U_{e2} m_2^{1/2} \cos \omega + U_{e3} m_3^{1/2} \sin \omega)^2 \times \delta_f^2 , \quad (19)
\]

where we have introduced \( \delta_f (1 > \delta_f > 0) \) as

\[
f_\beta(M_1) = 1 - \delta_f^2 . \quad (20)
\]
Figure 1: The required value of $\omega_r$ for the vanishing effective neutrino mass in the NH (red-solid line) or IH (blue-dashed line) case.

It is then found that the effective mass vanishes if

$$\tan \omega = \frac{A \pm i\delta_f}{1 \pm i\delta_f A} \equiv \tan \omega_\pm.$$  \hspace{1cm} (21)

Note that this condition gives Eq. (18) for $\delta_f \to 0$, as it should be. This result itself can be applied to both mass hierarchies by taking proper $A$ defined in Eq. (18).

Interestingly, we find that the required value for the real part $\omega_r$ is independent of $\delta_f$ (i.e., the mass of $N_1$):

$$\tan 2\omega_r = \frac{2\text{Re}A}{1 - |A|^2}. \hspace{1cm} (22)$$

This relation holds for both signs in Eq. (21). Since it can be applied to the case $\delta_f = 0$ (i.e., $f_\beta(M_1) = 1$), the result concerning on $\omega_r$ in Ref. [38] is substantiated even in this case. The required value of $\omega_r$ is determined by the Majorana phase, which is shown in Fig. 1. On the other hand, the required value of $\omega_i$ is represented by using $X_\omega$ as

$$X_\omega^2 = \frac{1 \pm \delta_f}{1 \mp \delta_f} \sqrt{\frac{1 + |A|^2 + 2\text{Im}A}{1 + |A|^2 - 2\text{Im}A}},$$

where the upper/lower sign corresponds to that in Eq. (21).

When $\delta_f \to 0$ (i.e., $f_\beta(M_1) \to 1$), it becomes independent of $M_1$ and is determined by the Majorana phase and turns out to be the results in Ref. [38]. This point is shown in Fig. 2. It is seen that the required value $X_\omega$ is of the order of the unity and it takes the maximal value at $\eta = 0.41\pi$ or $\pi/2$ in the NH or IH case, respectively. This can simply be read from the dependence on the CP phases $\delta$ and $\eta$ in each mass hierarchy. We find a simple relationship between $\omega$ and the CP phases in the matrix $U$ under the cancellation condition

$$\frac{\sinh 2\omega_i}{\sin 2\omega_r} = \frac{1}{2} (X_\omega^2 - X_{\omega}^{-2}) \frac{\text{Im}A}{\text{Re}A} = \left\{ \begin{array}{cl} -\tan(\delta + \eta) & \text{for the NH case} \\ \tan \eta & \text{for the IH case} \end{array} \right.. \hspace{1cm} (24)$$
When $\delta_f \neq 0$, there are two possibilities for $X_\omega$ depending on the sign in Eq. (23). The result with $\eta = 0.3\pi$ is shown in Fig. 3. It is seen that, when $M_1$ becomes smaller than $\Lambda_\beta$, $X_\omega$ approaches to the value shown in Fig. 2. On the other hand, as $M_1 \gg \Lambda_\beta$, the required value of $X_\omega$ is proportional to $M_1$ or $M_1^{-1}$ for the upper or lower sign in Eq. (23). This behavior can be understood from the fact that $\delta_f$ approaches to zero as $M_1$ goes smaller enough than $\Lambda_\beta$ and monotonically depends on $M_1$ as it gets larger. This is one of the important results in the present analysis, especially when we discuss the impacts on the direct search of $N_1$ in the next section. This is because $X_\omega$ (or $X_{\omega^{-1}}$) sets the overall scale of the mixing elements of HNLs. See, for example, the discussion in Ref. [43].

As similar equation to Eq. (24), we can also get the simple relationship even when $\delta_f \neq 0$ as

$$\frac{\sinh 2\omega_l}{\sin 2\omega_r} = \frac{1}{2} \left( X_\omega^2 - X_{\omega^{-2}}^2 \right) \frac{1}{\sin 2\omega_r} = \frac{(1 + \delta_f^2) \text{Im} A \pm \delta_f (1 + |A|^2)}{(1 - \delta_f^2) \text{Re} A},$$

which can be applied to both NH and IH cases depending on the choice of $A$. By using this expression,
Figure 4: The range of the effective mass $|m_{\text{eff}}|$ (red-region) when the $0\nu\beta\beta$ decay of a nucleus with $\Lambda_\beta = \Lambda_\beta^{\text{cr}} = 200$ MeV is absent. The range of the effective mass from active neutrinos $|m_{\nu_{\text{eff}}}|$ (gray-region) is also shown. Here we take $M_1 = 1$ GeV and $M_2 = 200$ GeV.

we can get

$$X^2_\omega = \zeta + (\zeta^2 + 1)^{1/2}, \quad (26)$$

where $\zeta$ is defined as

$$\zeta = \frac{(1 + \delta_f^2) \text{Im} A \pm \delta_f (1 + |A|^2)}{(1 - \delta_f^2) \sqrt{(1 - |A|^2)^2 + 4 \text{Re} A^2}}, \quad (27)$$

which is consistent with Eq. (23).

Before closing this section, we should mention the form of the function $f_\beta$, the choice of the scale $\Lambda_\beta$ and its impacts on the observations. These quantities must be evaluated by the detailed calculation of the nuclear matrix element for a given $0\nu\beta\beta$ decay nucleus. Especially, the value of $\Lambda_\beta$ varies depending on the nucleus of each experiments and receives an uncertainty of the nuclear physics (see, for example, Refs. [45, 46]).

As for the case when $M_1 \ll \Lambda_\beta$, the cancellation (or a partial cancellation) occurs universally since $f_\beta(M_1) \approx 1$ for all the $0\nu\beta\beta$ nuclei. In this case, thus, if the decay is absent for a specific nucleus, the same thing happens for other nuclei. This is the main observation in Ref. [38].

On the other hand, when $M_1 \gtrsim \Lambda_\beta$, the situation is more involved. Let consider the case when the cancellation in $m_{\text{eff}}$ occurs as described above for one nucleus with $\Lambda_\beta = \Lambda_\beta^{\text{cr}}$ which we have take to be 200 MeV as a reference value in our main discussions. Then, the rates of the $0\nu\beta\beta$ decays for other nuclei can be suppressed or even enhanced depending on the value of $\Lambda_\beta$ for the nuclei.

\footnote{This point has been discussed in earlier papers [47, 48] in the context of one light and one heavy Majorana neutrinos. Here we consider the case which is consistent with the oscillation experiments in the minimal seesaw mechanism.}
by denoting \( f^{\text{cr}}(M_1) = f_\beta(M_1) |_{\Lambda_\beta = \Lambda_\beta^{\text{cr}}} \), the cancellation condition gives us that

\[
M_1 \Theta_{e1}^2 = - \frac{m^\nu_{\text{eff}}}{f^{\text{cr}}_\beta(M_1)},
\]

(28)

By inserting this expression into Eq. (10) with dropping \( N_2 \) contribution, we get

\[
m^\nu_{\text{eff}} = m^\nu_{\text{eff}} \left( 1 - \frac{f_\beta(M_1)}{f^{\text{cr}}_\beta(M_1)} \right),
\]

(29)

where \( f_\beta(M_1) \) can be different from the critical value depending on the elements. Namely, if \( f_\beta(M_1) \) becomes bigger than \( f^{\text{cr}}_\beta \), which corresponds to \( \Lambda_\beta \gtrsim \sqrt{2} \Lambda_\beta^{\text{cr}} \), the predicted effective mass can be greater than \( |m^\nu_{\text{eff}}| \) at another experiment. As shown in Fig. 4, the predicted effective mass can overcome \( |m^\nu_{\text{eff}}| \) when \( \Lambda_\beta \gtrsim 290 \text{ MeV} \) in the both mass hierarchy.

4 Impacts on search for heavy neutrino

Next, we turn to discuss the consequences of the no 0\(\nu\beta\beta \) decay due to the destructive contribution of \( N_1 \). Since the complex parameter \( \omega \) is fixed as shown in Eq. (21), the mixing elements of \( N_1, \Theta_{a1} \), are predicted by its mass and the Majorana phase when we use the central values of mixing angles, mass squared differences and the Dirac phase from the oscillation experiments [44].

Especially, among all flavor mixing elements, the electron-type mixing element is simply given by

\[
|\Theta_{e1}|^2 = \frac{|m^\nu_{\text{eff}}|}{M_1 f_\beta(M_1)},
\]

(30)

which is a direct consequence of \( m^\nu_{\text{eff}} = 0 \). Note that \( |\Theta_{e1}|^2 \) is independent of the choice of \( \omega = \omega_+ \) or \( \omega_- \) and is uniquely determined solely by the mass \( M_1 \) as well as active neutrino parameters in \( m^\nu_{\text{eff}} \). On the other hand, the elements with \( \omega = \omega_+ \) are larger than those with \( \omega = \omega_- \) for \( |\Theta_{a1}|^2 \) and \( |\Theta_{r1}|^2 \) in the wide region of \( \eta \) as shown in Fig. 5 where we fix \( M_1 = 1 \text{ GeV} \). This feature is almost independent of the choice of \( M_1 \). Depending on the choice of \( \omega \), the muon- and tau-type elements are very different, namely whether making a peak or a bump, which is helpful to identify the value of \( X_\omega (i.e., \) the imaginary part of \( \omega \)).

Further, the relative sizes of the mixing elements are very sensitive to the Majorana phase \( \eta \). In Fig. 5 we show the relative size of each mixing elements in terms of the Majorana phase. In the NH case, the electron-type mixing element is always smallest among all flavors, in the IH case, on the other hand, there is some possibilities where the electron-type element can be the largest when \( \omega = \omega_+ \) and further it is always largest when \( \omega = \omega_- \). In addition to that, the order between muon- and tau-type elements depends on \( \eta \). When \( \omega = \omega_+ \) in the NH case, the electron element becomes far below than others. When \( \omega = \omega_- \) in the NH case, the magnitude of all flavor mixing elements gets closer but the flavor structure is different from those of \( \omega_+ \) in wide region of \( \eta \). On the other hand, in the IH case,
\[ \omega_\alpha = \omega + |\Theta_{\alpha 1}|^2 \frac{\eta}{\pi} \]

\[ \omega_\alpha = \omega - |\Theta_{\alpha 1}|^2 \frac{\eta}{\pi} \]

Here we take the $N_1$ mass $M_1 = 1$ GeV.

\( \omega = \omega_+ \) gives degenerated magnitude for all flavor mixing elements and relatively stronger than any other cases. When \( \omega = \omega_- \) in the IH case, other than the electron mixing element become far below than the electron mixing elements by taking specific $\eta$. Therefore, muon- and tau-type mixing elements have feebly chance to be detected.

In Fig. 5, we show the range of the mixing element $|\Theta_{\alpha 1}|^2$ by varying the Majorana phase from $\eta = 0$ to $\pi$ in terms of $M_1$. The dependence on $M_1$ drastically changes at around $M_1 = \Lambda_\beta$ correlating with Eq. (15). Namely, since $M_1$ gets exceed $\Lambda_\beta$, $f_\beta$ works as a suppression factor, the mixing element $|\Theta_{\alpha 1}|^2$ has to become larger (by enlarging $X_\omega$ or $X_\omega^{-1}$) to realize the cancellation of the effective mass. This feature is advantageous for the direct search experiments. In Fig. 5 we also show the current bounds from various searches [49–51] and also the sensitivities by the future experiments [52–55]. It is seen that a wide range of $|\Theta_{\alpha 1}|^2$ can be probed by the future experiments, especially for the IH case. On the other hand, the results of other elements, $|\Theta_{\mu 1}|^2$ and $|\Theta_{\tau 1}|^2$, are shown in Appendix B.
The region of the mixing element $|\Theta_{e1}|^2$ for the vanishing effective neutrino mass (between two red lines) in the NH (left panel) or IH (right panel) case. Here we vary the Majorana phase $\eta = 0$ to $\pi$. The shaded regions are excluded by the direct searches for HNL. The dotted lines shows the sensitivities on $|\Theta_{e1}|^2$ by future search experiments.

Furthermore, the sum of the $N_i$ mixing elements is given by

$$|\Theta_1|^2 = \sum_{\alpha} |\Theta_{\alpha 1}|^2 = \left\{ \begin{array}{ll}
\frac{1}{M_1} \left[ \frac{m_3 + m_2}{4} (X_\omega^2 + X_\omega^*) - \frac{m_3 - m_2}{2} \cos(2\omega_r) \right] & \text{for the NH case} \\
\frac{1}{M_1} \left[ \frac{m_2 + m_1}{4} (X_\omega^2 + X_\omega^*) - \frac{m_2 - m_1}{2} \cos(2\omega_r) \right] & \text{for the IH case}
\end{array} \right..$$

(31)

Since the cancellation in $m_{\text{eff}}$ requires the specific values of $\omega_r$ and $X_\omega$ as shown in Eqs. (22) and (26), we obtain

$$|\Theta_1|^2 = \left\{ \begin{array}{ll}
\frac{1}{M_1} \left[ \frac{m_3 + m_2}{2} (\zeta^2 + 1)^{1/2} \pm \frac{m_3 - m_2}{2} \frac{1 - |A|^2}{\sqrt{(1 - |A|^2)^2 + 4ReA^2}} \right] & \text{for the NH case} \\
\frac{1}{M_1} \left[ \frac{m_2 + m_1}{2} (\zeta^2 + 1)^{1/2} \pm \frac{m_2 - m_1}{2} \frac{1 - |A|^2}{\sqrt{(1 - |A|^2)^2 + 4ReA^2}} \right] & \text{for the IH case}
\end{array} \right..$$

(32)

We show in Fig. 7 the maximal and minimum values of $|\Theta_1|^2$ by varying the value of $\eta$ as a free parameter. Note here that $|\Theta_1|^2$ in the considering case is bounded from below [56] by considering $X_\omega = 1$ and $\omega_r = 0$ as

$$|\Theta_1|^2 \geq \frac{m_*}{M_1},$$

(33)

where $m_* = m_2$ or $m_1$ for the NH or IH case, respectively. This bound is also shown in Fig. 7 as the black line. It is thus found that $|\Theta_1|^2$ becomes proportional to $M_1$ for $M_1 \gtrsim \Lambda_\beta$, and hence a wide region of our possibility can be tested by future experiments together with the null observation of the $0\nu\beta\beta$ decay.

Finally, we mention the properties of heavier HNL $N_2$. We have assumed so far that its mass is much heavier than $\Lambda_\beta$ so that $N_2$ decouples from the $0\nu\beta\beta$ decay process. On the other hand, since $X_\omega \gg 1$
Figure 7: The maximal and minimal values of the mixing elements $|\Theta_1|^2$ for the vanishing effective neutrino mass by varying the Majorana phase. The left or right panel is for the NH or IH case. The red-solid or blue-dashed lines are for the case with $\omega = \omega_\pm$ or $\omega_-$ in Eq. (21).

or $X_\omega^{-1} \gg 1$ as $M_1$ gets heavier, the Yukawa coupling constants of $N_2$ become rather large and exceed the perturbative values when the mass of $N_2$ becomes large. See, for example, Ref. [56]. Since all other parameters than $M_2$ are already fixed by the conditions related to $N_1$ or observables of the neutrino oscillation experiments, there is no way to suppress the Yukawa coupling constants of $N_2$.

$$\sum_\alpha |F_{\alpha 2}|^2 = \begin{cases} 
\frac{M_2}{\langle \Phi \rangle^2} \left[ \frac{m_3 + m_2}{4} (X_\omega^2 + X_\omega^{-2}) + \frac{m_3 - m_2}{2} \cos(2\omega_r) \right] & \text{for the NH case} \\
\frac{M_2}{\langle \Phi \rangle^2} \left[ \frac{m_2 + m_1}{4} (X_\omega^2 + X_\omega^{-2}) + \frac{m_2 - m_1}{2} \cos(2\omega_r) \right] & \text{for the IH case} 
\end{cases}$$

(34)

Note again that the cancellation in $m_{\text{eff}}$ fixes the values of $X_\omega$ and $\omega_r$. Then, by requiring $\sum_\alpha |F_{\alpha 2}|^2 < 4\pi$ the upper bound on $M_2$ is obtained which is shown in Fig. 8. It is seen that the bound becomes stringent for $M_1 \gg \Lambda_\beta$ and $M_2 < \mathcal{O}(10^{10} - 10^{11})$ GeV when $M_1 = 10^2$ GeV simply because it makes $X_\omega$ to be the largest in the considered mass range of $M_1$.

5 Discussions and conclusions

In this paper we have examined the neutrinoless double beta decay in the seesaw mechanism with two right-handed neutrinos. The Majorana nature of active neutrinos and heavy neutral leptons breaks the lepton number of the theory, which may lead to the neutrinoless double beta decay. In this framework, the cancellation among active neutrino contributions cannot occur since the lightest active neutrino is massless. heavy neutral leptons can give a significant contribution in addition to those from active neutrinos, and the effective neutrino mass can vanish in some cases. One possibility is that all the heavy neutral leptons are lighter than $\Lambda_\beta$. It has been shown that the lighter heavy neutral lepton $N_1$ can obliterate the neutrinoless double beta decay even if active neutrinos do contribute it. In such a case the required mixing elements becomes proportional to the mass for $M_1 \gtrsim 0.1$ GeV. Then, the future
Figure 8: The upper bound on the mass of heavier HNL $N_2$ from the perturbative limit $\sum_\alpha |F_{\alpha 2}|^2 < 4\pi$ for the NH (left) or IH (right) case. The red solid and blue dashed lines are the bounds for $\omega = \omega_+$ and $\omega_-$, respectively. Here we take $\eta = 0.3\pi$.

experiments like DUNE, SHiP, and FCC-ee can test the present scenario even if the neutrinoless double beta decay processes are not observed. If $N_1$ becomes light enough, the suggested values of the lifetime are so long that $N_1$ decays after the onset of the big-bang nucleosynthesis (BBN) and would destroy the success of the BBN and/or conflict with the observational data of the cosmic microwave background radiation. One possibility to avoid this difficulty is the dilution of the $N_1$ abundance by the late time entropy production. Such an additional production may be realized by the decay of the heavier heavy neutral lepton $N_2$\textsuperscript{[57]}. Such a bound, however, becomes safe enough when $M_1 \gtrsim 1$ GeV. Namely, the suppression in the neutrinoless double beta decay rate can be easily achieved in wide region of $M_1$ without conflicting any constraints at the moment.

It has also been shown that even if the neutrinoless double beta decay at an experiment, namely $N_1$ contribution conceals the decay in a specific element, there is still a possibility to detect the decay at other experiments which use other nuclei. If the neutrinoless double beta decay have been observed at an experiment while another one cannot do, it may mean the missing decay signal is caused by the heavy neutral lepton contribution.

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A Physical region of the model parameters

In this Appendix we explain the physical region of the model parameters, especially, parameters for the right-handed neutrinos based on the discussion in Ref. [58]. First of all, we observe that the Lagrangian is invariant under $\omega \rightarrow \omega + \pi$, $N_1 \rightarrow -N_1$ and $N_2 \rightarrow -N_2$. Thus, the two ranges of $\text{Re}\omega$, $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and $[\frac{\pi}{2}, \frac{3\pi}{2}]$, are physically equivalent and it is enough to consider the range $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Second, the Lagrangian is also invariant under $\omega \rightarrow \frac{\pi}{2} - \omega$, $\xi \rightarrow -\xi$, $\nu_{R1} \rightarrow -\nu_{R2}$ and $\nu_{R2} \rightarrow -\nu_{R1}$ (i.e., the change of the mass ordering $M_2 > M_1 \rightarrow M_1 < M_2$). It is therefore found that we can take $M_2 > M_1$ without loss of generality. Third, the change of Majorana phase $\eta \rightarrow \eta + \pi$ can be compensated by $\xi \rightarrow -\xi$, and then the physical range of $\eta$ is taken to be $[0, \pi]$ in general. Finally, a set of the transformations $\omega \rightarrow -\omega$, $\xi \rightarrow -\xi$ and $\nu_{R2} \rightarrow -\nu_{R2}$ is another symmetry of the Lagrangian, which restricts the physical range of the parameter space.

We find that there are 16 independent choices for the physical regions. Here we show the three useful choices:

1. Fix $\xi = +1$ (or $\xi = -1$), consider the both ranges of $\text{Re}\omega [-\frac{\pi}{2}, 0] \text{ and } [0, \frac{\pi}{2}]$, and consider the whole range of $\text{Im}\omega [-\infty, \infty]$.

2. Consider both signs of $\xi$, consider the half range of $\text{Re}\omega [0, \frac{\pi}{2}]$ (or $[-\frac{\pi}{2}, 0]$), and the whole range of $\text{Im}\omega [-\infty, \infty]$.

3. Consider both signs of $\xi$, consider the both ranges of $\text{Re}\omega [0, \frac{\pi}{2}] \text{ and } [-\frac{\pi}{2}, 0]$, and the half range of $\text{Im}\omega [0, \infty]$ (or $[-\infty, 0]$).

In the analysis [43] the option (3) has been selected. These choices of the physical region of parameters $\xi$, $\omega$ and $\eta$ are summarized in Tab. 1. In the present analysis we choose the option (1) by fixing the sign parameter $\xi = +1$.

|   | $\xi$ | $\omega_r$ | $\omega_i$ | $X_{\omega} = \exp(\omega_i)$ | $\eta$ |
|---|------|----------|----------|-----------------|------|
| 1 | +1   | $[-\frac{\pi}{2}, \frac{\pi}{2}]$ | $[-\infty, +\infty]$ | $[0, +\infty]$ | $[0, \pi]$ |
| 2 | ±1   | $[0, \frac{\pi}{2}]$ | $[-\infty, +\infty]$ | $[0, +\infty]$ | $[0, \pi]$ |
| 3 | ±1   | $[-\frac{\pi}{2}, \frac{\pi}{2}]$ | $[0, +\infty]$ | $[1, +\infty]$ | $[0, \pi]$ |

Table 1: Three options of the physical region of parameters $\xi$, $\text{Re}\omega$, $\text{Im}\omega$, $X_{\omega}$ and $\eta$.

B Upper and lower bounds of mixing elements in each flavor

As shown in the main text, the cancellation condition is directly tied to the value of the electron-type mixing element, whereas mixing elements of other flavor can also be determined since all the free parameters such as $\omega$ can be fixed. In this Appendix, we show the predicted values of mixing elements for
all flavor components in Figs. 9 and 10 for the NH and IH case, respectively, together with the regions where have already been excluded by previous experiments and the future experiments can search.

Figure 9: Upper and lower bounds on the mixing elements $|\theta_{e1}|^2$ (left, red solid lines), $|\theta_{\mu1}|^2$ (middle, blue dashed lines) and $|\theta_{\tau1}|^2$ (right, green dot-dashed lines) for vanishing $m_{\text{eff}}$ in the NH case. Here take $\omega = \omega_+$ (upper panel) and $\omega_-$ (lower panel).
Figure 10: Upper and lower bounds on the mixing elements $|\Theta_{e1}|^2$ (left, red solid lines), $|\Theta_{\mu 1}|^2$ (middle, blue dashed lines) and $|\Theta_{\tau 1}|^2$ (right, green dot-dashed lines) for vanishing $m_{\text{eff}}$ in the IH case. Here take $\omega = \omega_+$ (upper panel) and $\omega_-$ (lower panel).
References

[1] P. Minkowski, Phys. Lett. B 67, 421 (1977).

[2] T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Ibaraki 305- 0801 Japan, 1979) p. 95.

[3] T. Yanagida, Prog. Theor. Phys. 64, 1103 (1980).

[4] P. Ramond, [arXiv:hep-ph/9809459 [hep-ph]].

[5] M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by P. van Niewwenhuizen and D. Freedman (North Holland, Amsterdam, 1979) [arXiv:1306.4669 [hep-th]].

[6] S. L. Glashow, NATO Sci. Ser. B 61 (1980), 687 doi:10.1007/978-1-4684-7197-7_15

[7] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.

[8] G. ’t Hooft, Phys. Rev. Lett. 37 (1976), 8-11.

[9] G. ’t Hooft, Phys. Rev. D 14 (1976), 3432-3450 doi:10.1103/PhysRevD.14.3432

[10] M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Suppl. 83 (1985), 1 doi:10.1143/PTPS.83.1

[11] H. Päs and W. Rodejohann, New J. Phys. 17 (2015) no.11, 115010 doi:10.1088/1367-2630/17/11/115010 [arXiv:1507.00170 [hep-ph]].

[12] S. Dell’Oro, S. Marcocci, M. Viel and F. Vissani, Adv. High Energy Phys. 2016 (2016), 2162659 doi:10.1155/2016/2162659 [arXiv:1601.07512 [hep-ph]].

[13] M. J. Dolinski, A. W. Poon and W. Rodejohann, Ann. Rev. Nucl. Part. Sci. 69 (2019), 219-251 doi:10.1146/annurev-nucl-101918-023407 [arXiv:1902.04097 [nucl-ex]].

[14] T. G. Rizzo, Phys. Lett. B 116 (1982), 23-28 doi:10.1016/0370-2693(82)90027-2

[15] D. London, G. Belanger and J. N. Ng, Phys. Lett. B 188 (1987), 155-158 doi:10.1016/0370-2693(87)90723-4.

[16] D. A. Dicus, D. D. Karatas and P. Roy, Phys. Rev. D 44 (1991), 2033-2037 doi:10.1103/PhysRevD.44.2033.

[17] G. Belanger, F. Boudjema, D. London and H. Nadeau, Phys. Rev. D 53 (1996), 6292-6301 doi:10.1103/PhysRevD.53.6292 [arXiv:hep-ph/9508317 [hep-ph]].

[18] J. Gluza and M. Zralek, Phys. Rev. D 52 (1995), 6238-6248 doi:10.1103/PhysRevD.52.6238 [arXiv:hep-ph/9502284 [hep-ph]].
[19] J. Gluza and M. Zralek, Phys. Lett. B 362 (1995), 148-154 doi:10.1016/0370-2693(95)01158-M [arXiv:hep-ph/9507269 [hep-ph]].

[20] J. Gluza and M. Zralek, Phys. Lett. B 372 (1996), 259-264 doi:10.1016/0370-2693(96)00074-3 [arXiv:hep-ph/9510407 [hep-ph]].

[21] C. Greub and P. Minkowski, eConf C960625 (1996), NEW149 doi:10.1142/S0217751X98001153 [arXiv:hep-ph/9612340 [hep-ph]].

[22] W. Rodejohann, Phys. Rev. D 81 (2010), 114001 doi:10.1103/PhysRevD.81.114001 [arXiv:1005.2854 [hep-ph]].

[23] S. Banerjee, P. S. B. Dev, A. Ibarra, T. Mandal and M. Mitra, Phys. Rev. D 92 (2015), 075002 doi:10.1103/PhysRevD.92.075002 [arXiv:1503.05491 [hep-ph]].

[24] T. Asaka and T. Tsuyuki, Phys. Rev. D 92 (2015) no.9, 094012 doi:10.1103/PhysRevD.92.094012 [arXiv:1508.04937 [hep-ph]].

[25] K. Wang, T. Xu and L. Zhang, Phys. Rev. D 95 (2017) no.7, 075021 doi:10.1103/PhysRevD.95.075021 [arXiv:1610.02618 [hep-ph]].

[26] A. Ilakovac, B. A. Kniehl and A. Pilaftsis, Phys. Rev. D 52 (1995), 3993-4005 doi:10.1103/PhysRevD.52.3993 [arXiv:hep-ph/9503456 [hep-ph]].

[27] A. Ilakovac and A. Pilaftsis, Nucl. Phys. B 437 (1995), 491 doi:10.1016/0550-3213(94)00567-X [arXiv:hep-ph/9403398 [hep-ph]].

[28] A. Ilakovac, Phys. Rev. D 54 (1996), 5653-5673 doi:10.1103/PhysRevD.54.5653 [arXiv:hep-ph/9608218 [hep-ph]].

[29] V. Gribanov, S. Kovalenko and I. Schmidt, Nucl. Phys. B 607 (2001), 355-368 doi:10.1016/S0550-3213(01)00169-9 [arXiv:hep-ph/0102155 [hep-ph]].

[30] A. Atre, V. Barger and T. Han, Phys. Rev. D 71 (2005), 113014 doi:10.1103/PhysRevD.71.113014 [arXiv:hep-ph/0502163 [hep-ph]].

[31] J. N. Ng and A. N. Kamal, Phys. Rev. D 18 (1978), 3412 doi:10.1103/PhysRevD.18.3412.

[32] J. Abad, J. G. Esteve and A. F. Pacheco, Phys. Rev. D 30 (1984), 1488 doi:10.1103/PhysRevD.30.1488.

[33] C. Dib, V. Gribanov, S. Kovalenko and I. Schmidt, Phys. Lett. B 493 (2000), 82-87 doi:10.1016/S0370-2693(00)01134-5 [arXiv:hep-ph/0006277 [hep-ph]].

[34] A. Ali, A. V. Borisov and N. B. Zamorin, Eur. Phys. J. C 21 (2001), 123-132 doi:10.1007/s100520100702 [arXiv:hep-ph/0104123 [hep-ph]].
[35] T. Asaka and H. Ishida, Phys. Lett. B 763 (2016), 393-396 doi:10.1016/j.physletb.2016.10.070 [arXiv:1609.06113 [hep-ph]].

[36] A. Gando et al. [KamLAND-Zen], Phys. Rev. Lett. 117 (2016) no.8, 082503 doi:10.1103/PhysRevLett.117.082503 [arXiv:1605.02889 [hep-ex]].

[37] M. Blennow, E. Fernandez-Martinez, J. Lopez-Pavon and J. Menendez, JHEP 07 (2010), 096 doi:10.1007/JHEP07(2010)096 [arXiv:1005.3240 [hep-ph]].

[38] T. Asaka, H. Ishida and K. Tanaka, [arXiv:2012.12564 [hep-ph]].

[39] B. Pontecorvo, Sov. Phys. JETP 7 (1958), 172-173.

[40] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. 28 (1962) 870.

[41] J. A. Casas and A. Ibarra, Nucl. Phys. B 618 (2001) 171 [arXiv:hep-ph/0103065].

[42] A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, JHEP 0609 (2006) 010 [arXiv:hep-ph/0605281].

[43] T. Asaka, S. Eijima and H. Ishida, JHEP 1104 (2011) 011 doi:10.1007/JHEP04(2011)011 [arXiv:1101.1382 [hep-ph]].

[44] I. Esteban, M. Gonzalez-Garcia, A. Hernandez-Cabezudo, M. Maltoni and T. Schwetz, JHEP 01 (2019), 106 doi:10.1007/JHEP01(2019)106 [arXiv:1811.05487 [hep-ph]], “NuFiT 5.0: Three-neutrino fit based on data available in July 2020,” www.nu-fit.org.

[45] A. Faessler, M. Gonzalez, S. Kovalenko and F. Simkovic, Phys. Rev. D 90 (2014) no.9, 096010 doi:10.1103/PhysRevD.90.096010 [arXiv:1408.6077 [hep-ph]].

[46] J. Barea, J. Kotila and F. Iachello, Phys. Rev. D 92 (2015), 093001 doi:10.1103/PhysRevD.92.093001 [arXiv:1509.01925 [hep-ph]].

[47] A. Halprin, S. T. Petcov and S. P. Rosen, Phys. Lett. B 125 (1983), 335-338 doi:10.1016/0370-2693(83)91296-0

[48] C. N. Leung and S. T. Petcov, Phys. Lett. B 145 (1984), 416-420 doi:10.1016/0370-2693(84)90071-6

[49] M. Aoki et al. [PIENU], Phys. Rev. D 84 (2011), 052002 doi:10.1103/PhysRevD.84.052002 [arXiv:1106.4055 [hep-ex]].

[50] A. Aguilar-Arevalo et al. [PIENU], Phys. Rev. D 97 (2018) no.7, 072012 doi:10.1103/PhysRevD.97.072012 [arXiv:1712.03275 [hep-ex]].

[51] E. Cortina Gil et al. [NA62], Phys. Lett. B 807 (2020), 135599 doi:10.1016/j.physletb.2020.135599 [arXiv:2005.09575 [hep-ex]].
[52] A. Blondel et al. [FCC-ee study Team], Nucl. Part. Phys. Proc. 273-275 (2016), 1883-1890 doi:10.1016/j.nuclphysbps.2015.09.304 [arXiv:1411.5230 [hep-ex]].

[53] C. Ahdida et al. [SHiP], JHEP 04 (2019), 077 doi:10.1007/JHEP04(2019)077 [arXiv:1811.00930 [hep-ph]].

[54] I. Krasnov, Phys. Rev. D 100 (2019) no.7, 075023 doi:10.1103/PhysRevD.100.075023 [arXiv:1902.06099 [hep-ph]].

[55] C. Alpigiani et al. [MATHUSLA], [arXiv:2009.01693 [physics.ins-det]].

[56] T. Asaka and T. Tsuyuki, Phys. Lett. B 753 (2016), 147-149 doi:10.1016/j.physletb.2015.12.013 [arXiv:1509.02678 [hep-ph]].

[57] T. Asaka, M. Shaposhnikov and A. Kusenko, Phys. Lett. B 638 (2006), 401-406 doi:10.1016/j.physletb.2006.05.067 [arXiv:hep-ph/0602150 [hep-ph]].

[58] A. de Gouvea, A. Friedland and H. Murayama, Phys. Lett. B 490 (2000) 125 [hep-ph/0002064].