An Energy-Saving Home Energy Management Supporting Selling Operation and User Comfort

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Abstract—In this study, we investigate the operation of a home energy management system with integrated renewable energy system (RES) and energy storage system (ESS) in a smart home. In its operation, there is a focus on selling operation and user comfort, which are analyzed in detail. A multi-objective mixed integer nonlinear programming model is proposed to optimize different and conflicting objectives. Through incorporation of these varied objectives, our system not only reduces energy cost but also maintains user comfort and peak-to-average ratio (PAR) for residents. The effect of selling price on user comfort and PAR is also considered. Moreover, by applying the weight method of multi-objective optimization, we have more flexibility in setting trade-offs between the values of these objectives. A formula for the lower bound of energy cost is developed. This formula helps residents or engineers quickly choose best parameters of RES and ESS for their homes during the decision-making process. Performance of our system is verified through a number of simulations under different scenarios using real data, and simulation results are compared in terms of energy cost per day, PAR, user’s convenience and waiting time to use appliances.

Index Terms—home energy management systems; selling operation; power trading; user comfort; lower bound

I. INTRODUCTION

IN THE FUTURE, the smart grid (SG) will not only be a main component of electricity delivery between suppliers, prosumers and consumers but will also play a key role in reducing energy consumption [1]. By using advanced metering infrastructure (AMI) of the SG, residents can utilize external information sent by the utilities to improve their energy usage. On the other side, the SG also helps the utilities distribute power in more effective way and reduce the system’s peak-to-average ratio (PAR), a main contributing factor to an electricity crisis [2]. With the amazing development of renewable energy and energy storage technologies, more and more houses are able to set up home energy management systems (HEMSs) with integrated renewable energy systems (RESs) and energy storage systems (ESSs) to reduce their energy cost, PAR, and dependency on the main grid in an efficient and reliable manner. In the SG, participating in the electricity market is another efficient way to decrease energy costs and maximally utilize renewable energy from residential zones. Hence, the main role of HEMS is not only to control and manage all electrical devices, but also to fully support selling operations by the residents, fulfilling their various requirements. Due to the tremendous effects of HEMS on energy consumption of houses, there are numerous studies on the different problems HEMS may encounter. A large number of optimization models and scheduling schemes have also been proposed. For example, in [3], Sereen Althaher et al. proposed an optimization-based automated demand response (ADR) to be implemented in HEMS. Their ADR aimed to reduce the consumer’s electricity bill below a certain level, whilst increasing their comfort. However, in their study, RES and ESS were not considered, and thus selling operations were not mentioned. In [4], a HEMS with integrated battery storage system and photo-voltaic (PV) system were presented to minimize energy cost and satisfy the thermal and device constraints. In their study, an example set of appliances for a house was given. In their HEMS, a load management algorithm is proposed to control battery and thermal appliances to adapt to operations of other appliances which were turned on or off by residents. In their work, selling was not mentioned. In [5], an energy management system with integrated RES and ESS for a group of homes was proposed to optimize energy cost and allow power trading. At each home, a set of appliances was given and a model was also built for minimizing energy cost and scheduling these appliances. In this paper, selling operations between the homes were supported using the Nash equilibrium from game theory. However, utilization of the main grid and user comfort were not considered for each home in their model. In [6], [7], and [8], the authors built an optimization model for HEMS with integrated RES and ESS to optimize energy cost and user comfort. Their model was developed for considering both energy saving, thermal comfort and the user’s convenience. In their work, a detailed schedule of appliances was given. Although selling activities were mentioned, their HEMS failed to give a detailed schedule for selling operations in each time slot. Moreover, the effect of selling price on system performance and user comfort was not considered.

In our previous work [9], we presented an HEMS with integrated RES and ESS to optimize energy cost and PAR. By using particle swarm optimization (PSO) algorithm, a detailed schedule of utilization of the main grid and selling operations were given in each time slot but user comfort was not considered. Instead of using a PSO algorithm, there are many other studies that use meta-heuristic algorithms for scheduling appliances such as genetic algorithm (GA), wind driven optimization (WDO), harmony search algorithm (HSA), and so on [10]. [11], [12], [13], [14], [15]. Although energy cost and various kinds of user comfort were considered and optimized in these studies, all of them failed to consider...
the selling operation in their optimization problems.

To the best of our knowledge, none of the previous studies considered an optimization model which takes selling operation, user comfort, and PAR into account. Therefore, in this paper which is an extension of our previous work [9], a mixed integer nonlinear programming (MINLP) model, including difference objectives: energy cost per day, user comfort, and PAR, is proposed. Two kinds of user comfort are considered in our model: the user’s convenience and waiting time to use appliances. Utilization of the main grid and selling operation in our model: the user’s convenience and waiting time to use appliances. The main contributions of our works can be summarized as follows:

- A multi-objective MINLP model that jointly optimizes different objectives is built. This model aims to achieve a balance between minimizing energy cost and preservation of user comfort and PAR. A detailed schedule for the operation of appliances, utilization of the main grid and selling operation in each time slot are given to achieve this optimal balance. The effect of selling price on system performance is considered. Specifically, simulation results show that a decrease of selling price impacts user comfort and PAR slightly.

- A formula for calculating the lower bound of energy cost is developed. Due to its simplicity, this formula can help residents or engineers quickly estimate the economic benefit achieved using a HEMS with integrated specific RES and ESS. Thus, residents or engineers can easily choose the parameters of the RES and ESS that are the best fit for their homes. The numerical results show that the minimum value of our energy cost approaches very close to this lower bound.

- We show the impacts of different weight coefficients, which control energy cost, user comfort, and PAR on system operation. The weight method for multi-objective optimization allows us more flexibility in setting trade-offs among energy cost, user comfort, and PAR.

The remainder of this paper is organized as follows. A brief description of the HEMS system is shown in Section II. A detailed problem formulation and optimization model are built in Section III. Section IV investigates the lower bound of energy cost. In Section V, the scenarios and simulation results are provided. Finally, Section VI gives conclusions and potential future works.

II. System Description

In this paper, a smart home with HEMS and a collection of shiftable and non-shiftable electrical appliances is studied. Generally, key components of a HEMS include AMI, a main controller (MC), ESS and RES. External information can be collected through an AMI. This includes pricing information, forecast temperature, and solar irradiance. The MC uses this useful information to control all electrical devices including the ESS and RES. The RES is used to decrease the dependency on the main grid and reduce energy cost. In this paper, we assume that a PV system is set up as the RES. To be able to store surplus RES energy and utilize electricity from the main grid at low price times, the ESS is needed for our system. Moreover, our HEMS supports residents in selling the electricity. Fig.1 shows all electricity flows in our smart house.

III. Problem Formulation

In this section, we build mathematical formulas for the RES, ESS, appliances and our objectives over the course of a day from 0 A.M. to 12 P.M. We divide a day into \( T = 24 \) time slots and the duration of each time slot is \( \Delta t = 1h \).

A. Renewable Energy Source

In this work, our HEMS is equipped with a PV system as its RES. According to [9], the output energy, \( E_{RES}(t) \), from a PV system in kWh in any time slot \( t (1 \leq t \leq T) \) can be measured as

\[
E_{RES}(t) = \frac{GHI(\tau)}{24} \cdot S \cdot \eta_{RES} \cdot \Delta t.
\]

where \( GHI \) is the global horizontal irradiation (kW/m\(^2\)) at the location of the solar panels, \( \tau \) is the real time in time slot \( t \), \( S \) is the total area (m\(^2\)) of solar panels and \( \eta_{RES} \) is the solar conversion efficiency of the PV system.

As shown in Fig.1, this energy would be used for home load and ESS charging. Thus, we have the following equation.

\[
E_{RES}(t) = E_{load}^{RES}(t) + E_{charge}^{RES}(t)
\]

where \( E_{load}^{RES}(t) \) is the energy quantity used for home load in time slot \( t \). \( E_{charge}^{RES}(t) \) is the energy quantity used to charge the ESS in time slot \( t \). In real life, because every ESS is only able to store a limited amount of energy over a certain time, if our RES generates more energy than the sum of the energy which home appliances need and the energy which is able to be stored in the ESS in a time slot, the remaining energy of RES will be wasted. Hence, (2) should be changed to

\[
E_{RES}(t) \geq E_{load}^{RES}(t) + E_{charge}^{RES}(t).
\]

B. Energy Storage System

As described in Fig.1, our ESS is charged by the RES energy and energy from the main grid and is discharged for home load use and selling. Hence, with \( \forall t, 1 \leq t \leq T \), we have the following formulas.

\[
E_{Discharge}^{ESS}(t) = E_{load}^{ESS}(t) + E_{selling}^{ESS}(t)
\]

\[
E_{Charge}^{ESS}(t) = E_{charge}^{RES}(t) + E_{MG}^{ESS}(t)
\]
\[ E_{\text{ESS}}^{\text{Level}}(t) = E_{\text{ESS}}^{\text{Level}}(t-1) + E_{\text{ESS}}^{\text{Charge}}(t) \cdot \eta_{\text{ESS}} - E_{\text{ESS}}^{\text{Discharge}}(t) / \eta_{\text{ESS}} \]  

where \( E_{\text{ESS}}^{\text{Discharge}}(t) \) refers to the energy quantity which is drawn from the ESS in a time slot \( t \). \( E_{\text{ESS}}^{\text{Charge}}(t) \) refers to the energy quantity stored in the ESS in a time slot \( t \). \( E_{\text{ESS}}^{\text{Level}}(t) \) is the energy quantity for home load in a time slot \( t \). \( E_{\text{ESS}}^{\text{Discharge}}(t) \) is the energy quantity sold to the outside in a time slot \( t \). \( E_{\text{ESS}}^{\text{Level}}(t) \) is the energy quantity stored in the ESS in a time slot \( t \). \( \eta_{\text{ESS}} \) is the ESS efficiency.

When using the ESS, we must satisfy the following constraints.

- The charge/discharge rate of the ESS cannot exceed the \( C_{\text{rate}}/D_{\text{rate}} \). This means that we are only able to put in or draw a certain energy quantity in a time slot \( t \) with duration \( \Delta t \).
- The energy level of the ESS must be between \( E_{\text{min}} \) and \( E_{\text{max}} \).
- We should avoid simultaneous charging and discharging of our ESS.

From the above, we have the following constraints.

\[ 0 \leq E_{\text{ESS}}^{\text{Discharge}}(t) \leq D_{\text{rate}} \cdot \Delta t \cdot (1 - \text{mode}_{\text{ESS}}(t)) \] 
\[ 0 \leq E_{\text{ESS}}^{\text{Charge}}(t) \leq C_{\text{rate}} \cdot \Delta t \cdot \text{mode}_{\text{ESS}}(t) \] 
\[ E_{\text{min}} \leq E_{\text{ESS}}^{\text{Level}}(t) \leq E_{\text{max}} \] 

where \( \text{mode}_{\text{ESS}}(t) \) is binary variable that shows the mode of the ESS in time slot \( t \) ("0" = discharging and "1" = charging).

Since we only consider our system over the course of a day (with no net accumulation being carried over to the next day), the energy level must be returned to the initial energy level by the end of the day. Thus, we have this constraint.

\[ E_{\text{ESS}}^{\text{Level}}(T) = E_{0} \] 

We assume that all energy to be sold comes from the ESS. If we want to sell energy generated from the RES, it should be stored in the ESS before sale. The parameters of our ESS used in this paper are shown in Table I.

| Parameters | Meaning |
|------------|---------|
| \( \eta_{\text{ESS}} \) | ESS efficiency |
| \( C_{\text{rate}}/D_{\text{rate}} \) | maximum charge/discharge rate of the ESS |
| \( E_{0} \) | initial energy level of the ESS |
| \( E_{\text{min}} \) | minimum energy level of the ESS |
| \( E_{\text{max}} \) | maximum energy level of the ESS |

C. Home Appliances

In our system, we suppose that there are two different sets of appliances: shiftable appliances \( M \) and non-shiftable appliances \( N \). The set of shiftable devices \( M = \{a_{1}, a_{2}, a_{3}, ..., a_{m}\} \) includes the devices which can operate during any time slot whereby we can move the operation time of these devices to low price slots to save cost. The set of non-shiftable devices \( N = \{b_{1}, b_{2}, b_{3}, ..., b_{n}\} \) includes the devices which have fixed operation time slots defined by users. None of the appliances can be interrupted during their operation.

The operation time of each non-shiftable appliance \( b_{i} \) is defined by binary parameter \( O_{b_{i}}(t) \) which shows the status of device \( b_{i} \) in time slot \( t \) ("1" = ON and "0" = OFF). These parameters have a fixed value and are defined by users. Assuming \( PR_{b_{i}} \) is the power rating of device \( b_{i} \) given by producers, the energy consumption of non-shiftable set \( N \) in a time slot \( t \) is calculated as

\[ E_{N}(t) = \sum_{i=1}^{n} (PR_{b_{i}} \times O_{b_{i}}(t) \times \Delta t) \] 

(11)

The operation time of each shiftable appliance \( a_{i} \) is defined by one parameter and two variables; parameter \( LoT_{a_{i}} \), is the length of operation time in a day, integer variable \( st_{a_{i}} \) refers to the time slot in which device \( a_{i} \) starts to run, and binary variable \( O_{a_{i}}(t) \) shows the status of device \( a_{i} \) in time slot \( t \) ("1" = ON and "0" = OFF). In a time slot \( t \), the energy consumption of shiftable set \( M \) is calculated as

\[ E_{M}(t) = \sum_{i=1}^{m} (PR_{a_{i}} \times O_{a_{i}}(t) \times \Delta t) \] 

(12)

where \( PR_{a_{i}} \) refers to the power rating of devices \( a_{i} \) given by producers. There are several constraints which these variables must follow. First, each shiftable device must finish its operation within that day.

\[ st_{a_{i}} \leq T - LoT_{a_{i}} + 1 \] 

(13)

Second, every shiftable device must not be interrupted during its operation time. This means that the binary variable \( O_{a_{i}}(t) \) must satisfy the following constraint.

\[ O_{a_{i}}(t) = \begin{cases} 1 & \text{if } st_{a_{i}} \leq t \leq st_{a_{i}} + LoT_{a_{i}} - 1 \\ 0 & \text{otherwise} \end{cases} \] 

(14)

Third, a specific shiftable device \( a_{i} \) should be started after the device \( a_{j} \)'s operation is completed (consecutive tasks). As an example, the clothes dryer should be started after the washing machine has already stopped. We have the following constraint.

\[ st_{a_{j}} + LoT_{a_{j}} + D_{j,i} \leq st_{a_{i}} \] 

(15)

where parameter \( D_{j,i} \) is the minimum delay between the time device \( a_{j} \) is stopped and the time device \( a_{i} \) is started and this parameter is determined by the residents.

In a time slot \( t \) (\( 1 \leq t \leq T \)), the energy consumption of all the appliances in the house, \( E_{\text{total}}(t) \), is the sum of the energy consumption of the shiftable set \( M \), \( E_{M}(t) \), and non-shiftable set \( N \), \( E_{N}(t) \). We have:

\[ E_{\text{total}}(t) = E_{N}(t) + E_{M}(t) \] 

(16)

To provide enough energy for home appliances in time slot \( t \), we use three different sources: energy from RES, ESS, and the main grid as shown in Fig. I. Hence, we have the following formula.

\[ E_{\text{total}}(t) = E_{\text{RES}}^{\text{load}}(t) + E_{\text{ESS}}^{\text{load}}(t) + E_{\text{MG}}^{\text{load}}(t) \] 

(17)
From (16), we have
\[ E_{MG}^{load}(t) + E_{RES}^{load}(t) + E_{ESS}^{load}(t) = E_N(t) + E_M(t). \] (18)

As \( E_{MG}^{load}(t) \geq 0 \) and we assume that the main grid always provides enough electricity for the requirements of our home load, we have the following constraint.
\[ 0 \leq E_{RES}^{load}(t) + E_{ESS}^{load}(t) \leq E_N(t) + E_M(t) \] (19)

D. Objective Functions

1) Objective 1: Minimizing energy cost: We assume that the energy from the RES and ESS is complimentary, and thus we only need to consider energy from the main grid. The load needed from the main grid, \( E_{LD}(t) \), in a given time slot \( t \) is calculated as
\[ E_{LD}(t) = E_{MG}^{load}(t) + E_{MG}^{charge}(t). \] (20)

From (18), we have
\[ E_{LD}(t) = E_N(t) + E_M(t) + E_{MG}^{charge}(t) - E_{RES}^{load}(t) - E_{ESS}^{load}(t). \] (21)

In addition, in the time slot \( t \), we sell an amount of energy, \( E_{ESS}^{selling}(t) \), to the outside. Hence, the energy cost in the time slot \( t \), \( EC(t) \), is
\[ EC(t) = E_{LD}(t) \times P_{MG}(t) - E_{ESS}^{selling}(t) \times P_{sell}(t). \] (22)

where \( P_{MG}(t) \) is the day-ahead price of the main grid in the time slot \( t \). This value is determined by the electrical provider and sent to users through the AMI. \( P_{sell}(t) \) is the price of selling energy in time slot \( t \). This value is decided by residents. From (21), we have
\[ EC(t) = \left( E_N(t) + E_M(t) + E_{MG}^{charge}(t) - E_{RES}^{load}(t) - E_{ESS}^{load}(t) \right) \times P_{MG}(t) - E_{ESS}^{selling}(t) \times P_{sell}(t). \] (23)

Since our objective is to minimize the total energy cost over a day, \( C_{day} \), the objective function is defined as
\[
\min(C_{day}) = \min \sum_{t=1}^{T} EC(t) = \min \sum_{t=1}^{T} \left( E_N(t) + E_M(t) + E_{MG}^{charge}(t) - E_{RES}^{load}(t) - E_{ESS}^{load}(t) \right) \times P_{MG}(t) - E_{ESS}^{selling}(t) \times P_{sell}(t).
\] (24)

where \( E_N(t) \) has a fixed value and \( E_M(t) \) is calculated by (12). Usually, the price of electricity from the main grid is higher than the selling price. We assume that \( P_{sell}(t) = \alpha \times P_{MG}(t) \) with \( 0 < \alpha \leq 1 \). Thus, the objective function of our system becomes
\[
\min \sum_{t=1}^{T} \left( E_N(t) + E_M(t) + E_{MG}^{charge}(t) - E_{RES}^{load}(t) - E_{ESS}^{load}(t) - \alpha \times E_{ESS}^{selling}(t) \right) \times P_{MG}(t).
\] (25)

2) Objective 2: Maximizing user’s convenience: To be able to measure the user’s convenience (UC) when a shiftable device is scheduled to run at a specific time, we introduce two kinds of time range which are set by residents for each shiftable device \( a_i \): Utilization range \( UTR_{a_i} = [u_{a_i}, u_{a_i}] \) is the time slots this device can be run; Best time range \( BTR_{a_i} = [b_{a_i}, b_{a_i}] \) is the time slots which are best suited for the operation of this device. If a shiftable device \( a_i \) is run in time slot \( t \), we define user’s convenience of \( a_i \), \( UC_{a_i}(t) \), as
\[
UC_{a_i}(t) = \begin{cases} 
0 & t \leq u_{a_i}, \\
\frac{t-u_{a_i}}{u_{a_i}-b_{a_i}} & u_{a_i} \leq t \leq b_{a_i}, \\
1 & b_{a_i} \leq t \leq u_{a_i}, \\
\frac{t-u_{a_i}}{u_{a_i}-b_{a_i}} & b_{a_i} \leq t \leq u_{a_i}.
\end{cases}
\] (26)

Fig. 2 depicts \( UC_{a_i}(t) \). Our objective function of a user’s convenience can be determined as
\[
\max(UC) = \max \sum_{i=1}^{m} \sum_{t=1}^{T} \left( pr_{a_i} \times UC_{a_i}(t) \times O_{a_i}(t) \right).
\] (27)

where \( pr_{a_i} \in [1, 2, 3] \) is the priority of device \( a_i \) that has a highest priority of 3 down to a lowest priority of 1. This parameter is defined by users.

3) Objective 3: Minimizing PAR: PAR, related to the operation of the main grid, is the ratio of the peak load demand and the average of total load demand over a day. In our system, our objective function of PAR can be defined as
\[
\min(PAR) = \min \left( \frac{\max(E_{LD}(t))}{\frac{1}{T} \sum_{t=1}^{T} E_{LD}(t)} \right).
\] (28)

where \( E_{LD}(t) \) is calculated by (21).

4) Objective 4: Minimizing waiting time: For our appliances, some devices \( (a_j, a_i) \) should satisfy the same consecutive constraint as in (15). However, it is inconvenient if we wait a long time to start device \( a_i \), even though device \( a_j \) has already stopped. In this paper, we propose an objective function to minimize this waiting time
\[
\min(WT) = \min \sum (s_{a_i} - (s_{a_j} + LoT_{a_j} + D_{ji})).
\] (29)
5) Optimization Model: To optimize our HEMS, all mentioned objectives must be considered. Hence, a multi-objective function is proposed as our system’s optimization model.

\[
\text{min} \{\text{MO Function} \} = \min \left( \frac{C_{\text{day}}}{UC - PAR - WT} \right)
\]

This function must be optimized with all previously mentioned constraints of the RES, ESS, and appliances.

IV. A LOWER BOUND FOR ENERGY COST

In this section, a lower bound for the energy cost \(C_{\text{day}}\) that is described in (25) is discovered. Clearly, as \(0 < \alpha \leq 1\), we have

\[
C_{\text{day}} \geq \sum_{t=1}^{T} \left( E_N(t) + E_M(t) + E_{\text{MG}}^{\text{charge}}(t) - E_{\text{MG}}^{\text{load}}(t) - E_{\text{ESS}}^{\text{charge}}(t) - E_{\text{ESS}}^{\text{load}}(t) \right) \times P_{\text{MG}}(t).
\]

Naming:

\[
E(t) = E_{\text{MG}}^{\text{charge}}(t) - E_{\text{RES}}^{\text{load}}(t) - E_{\text{ESS}}^{\text{load}}(t) - E_{\text{ESS}}^{\text{selling}}(t)
\]

We then have

\[
C_{\text{day}} \geq \sum_{t=1}^{T} \left( E_N(t) + E_M(t) + E(t) \right) \times P_{\text{MG}}(t).
\]

From (12), we have

\[
\sum_{t=1}^{T} \left( E_M(t) \times P_{\text{MG}}(t) \right) \geq \sum_{i=1}^{m} \sum_{t=1}^{L_{\text{LoT}}_i} \left( P_{\text{Ra}_i} \times P_{\text{MG}}(t) \times \Delta t \right)
\]

\[
= \sum_{i=1}^{m} \left( P_{\text{Ra}_i} \times \Delta t \times \sum_{t=1}^{L_{\text{LoT}}_i} P_{\text{MG}}(t) \right)
\]

\[
\geq \sum_{i=1}^{m} \left( P_{\text{Ra}_i} \times \Delta t \times M_{\text{MG}}^{a_i} \right).
\]

From (33), (34) and (37), a lower bound of the energy cost \(C_{\text{day}}\) can be determined as

\[
C_{\text{day}} \geq \sum_{t=1}^{T} \left( E(t) \times P_{\text{MG}}(t) \right) \geq \sum_{i=1}^{m} \left( P_{\text{Ra}_i} \times \Delta t \times M_{\text{MG}}^{a_i} \right) + C_{\text{min}}^{\text{MIP}} - \sum_{t=1}^{T} \left( E_{\text{RES}}(t) \times P_{\text{MG}}(t) \right).
\]

Combining (33), (34) and (37), a lower bound of the energy cost \(C_{\text{day}}\) can be determined as

\[
C_{\text{day}} \geq \sum_{t=1}^{T} \left( E(t) \times P_{\text{MG}}(t) \right) \geq \sum_{i=1}^{m} \left( P_{\text{Ra}_i} \times \Delta t \times M_{\text{MG}}^{a_i} \right) + C_{\text{min}}^{\text{MIP}} - \sum_{t=1}^{T} \left( E_{\text{RES}}(t) \times P_{\text{MG}}(t) \right).
\]

The equal sign \(=\) is only valid if the equal signs \(=\) of (34), (37) are valid and \(\alpha = 1\). It is worth noting that all elements of this lower bound are calculated quickly, thus this lower bound is easily determined.

V. SIMULATIONS AND DISCUSSIONS

In this section, a house with a set of household appliances, an RES and an ESS is simulated over the course of a day. The parameters for the household appliances are shown in Table II and Table III where the units of PR and LoT are kW and hour, respectively. There are 17 appliances that are divided into two categories: shiftable and non-shiftable. The shiftable appliances are devices whose operating times can be shifted to low price time slots whereas operating times of non-shiftable devices cannot be changed. None of the appliances can be interrupted during operation. We further define three consecutive constraints: the clothes dryer (CD) should be started as soon as possible after the washing machine (WM) is finished (\(D_{\text{WM,CD}} = 0\)), the hair dryer (HD) should be started as soon as possible after the electric shower (ES) is finished (\(D_{\text{ES,HD}} = 0\)), and the dish washer (DW) should be started 1 hour after the rice cooker (RC) is finished (\(D_{\text{RC,DW}} = 1\)).

The parameters for our ESS are shown in Table IV. For the RES in our house, a PV system is used for electricity.
TABLE III
DESCRIPTION OF NON-SHIFTABLE APPLIANCES.

| Appliances      | PR | LoT | Start Time |
|-----------------|----|-----|------------|
| Personal computers | 0.2 | 14  | 8:00       |
| Security cameras  | 0.1 | 24  | 0:00       |
| Refrigerator        | 0.9 | 21  | 2:00       |
| Television          | 0.2 | 6   | 16:00      |
| Lights               | 0.1 | 7   | 17:00      |

TABLE IV
THE INPUT PARAMETERS OF OUR ESS IN THE SIMULATION.

| ηESS | Chrate/Dhrate | Eλ0 | Eλmin | Eλmax |
|------|---------------|-----|-------|-------|
| 95%  | 1.0 kW        | 0.5 kWh | 0.5 kWh | 10 kWh |

Fig. 3. Hourly estimated RES.

generation. Our day-ahead price (DAP) signal is defined by the electricity provider and the solar irradiance is obtained from METEONORM 6.1 for the Islamabad region of Pakistan [11]. We assume that total area of solar panels is S = 1m² and the solar conversion efficiency ηRES = 0.95. Using (1), the hourly energy quantity generated by the PV system over the course of a day is shown in Fig. 3.

The performance of our HEMS is compared under three different scenarios: normal, economic, and smart. The normal scenario describes a situation in which no HEMS is set up. The RES and ESS are also not set up in this scenario. Therefore, there is no ability for utilizing the DAP information, RES, and ESS. Shiftable devices are not controlled according to different objectives and they are run upon the resident’s requests. The economic scenario describes a situation in which the HEMS including the RES and ESS is set up. However, shiftable devices are scheduled to minimize energy cost only without considering other objectives. On the other hand, in the smart scenario, our HEMS is fully utilized to optimize the multi-objective function, as shown in Fig. 3. Shiftable devices are scheduled not only to reduce energy cost, but also to optimize the outcomes of other objectives. On the other hand, in the smart scenario, our HEMS is fully utilized to optimize the multi-objective function, as shown in Fig. 3. Shiftable devices are scheduled not only to reduce energy cost, but also to optimize the outcomes of other objectives. It is worth noting that in all three scenarios, the consecutive constraints of shiftable devices are always satisfied. All of our simulations were run on an Intel(R) Core(TM) i7-8700 CPU @ 3.20GHz and 16GB RAM with Windows 10 pro (64-bit). The mathematical programming software AIMMS [17] with Cplex/Conopt/Outer-Approximation [18] solvers was used to solve our optimization problem. The computational time taken for the economic and smart scenarios was approximately 10 seconds and 23 seconds, respectively.

A. Performance of our HEMS in the three scenarios

Fig. 4 shows our system’s performance for each scenario when $P_{sell}(t) = P_{MG}(t) \forall t$ which means that $\alpha = 1$. As depicted in this figure, the smart scenario demonstrates excellent performance in comparison with the other scenarios by taking four objectives into account. The multi-objective function value (MO Function) for the smart scenario is only 8.3, a significant improvement of 45% and 54% over the normal and economic scenarios. In detail, in the economic scenario, to achieve the minimum of energy cost (339.22 cents), the user’s convenience, waiting time, and PAR were worst in comparison with the normal scenario. PAR is increased more than double from 2.2 to 5.1 and residents must wait 1 hour to start to use shiftable devices. Especially, the economic scenario fails to fulfill the satisfaction of residents in terms of the user’s convenience index $(UC/UC_{max} \times 100)$. The user’s convenience in the economic scenario is decreased dramatically from 100% to 51%. However, in the smart scenario, the user’s convenience, waiting time, and PAR are quite close to those in the normal scenario with a small increase in energy cost.

Moreover, our system still works well in cases where the selling price $P_{sell}(t)$ is decreased as shown in Fig. 5 and Fig. 6. As observed from the simulation results, both the energy cost and multi-objective function value have a tiny increase of 2.5% when $P_{sell}(t)$ is steadily reduced. However, our system still maintains the user’s convenience, waiting time, and PAR like in the normal scenario. In particular, in both cases, the PAR and waiting time are the same as in the normal scenarios, being 2.2 and 0, respectively. Only the user’s convenience is slightly decreased to 97%. These reductions of selling price have a tiny affect on two kinds of user comfort and PAR.
To gain better insights into the performance of the smart scenario, the optimal operation of the ESS, and shiftable appliances in the smart scenario with $\alpha = 0.9$ are shown in Fig. 7, and Table V. In Fig. 7, in the high price time slots from 7:00 to 10:00, energy from the ESS is discharged to provide the home load while the surplus energy is also discharged to sell as much as possible to make profits. This ESS energy came from the main grid in the low price time slots from 0:00 to 7:00. Likewise, the operation of the appliances was scheduled to avoid these high price time slots, as shown in Table V. Besides, the optimal schedule for the appliances was created by effectively taking the consecutive constraints and time preferences of the residents into account. As an example, the clothes dryer is run after the washing machine finishes. More specifically, the washing machine is started at 15:00 and the clothes dryer is started at 17:00.

### B. Lower bound of energy cost

With energy generated by the RES, as shown in Fig. 3 and the set of appliances as shown in Table II and Table III, the lower bound of the energy cost in our system was 338.92 (cents) by applying (38) while the energy cost of the economic scenario in our system with $\alpha = 1$ was 339.22 (cents), as shown in Fig. 4. To explain where the tiny difference comes from, we consider schedules of all appliances by our HEMS. In this economic scenario, the input energy $E_{\text{ESS}}^{\text{Charge}}(t)$, output energy $E_{\text{ESS}}^{\text{Discharge}}(t)$, and mode $E_{\text{ESS}}(t)$ of our ESS in each time slot are scheduled to achieve the minimum value $C_{\text{MIP}}^{\text{min}}$ of (37). However, the shiftable appliances cannot be scheduled to achieve the minimum value of (34) because consecutive constraints are considered in our system while these constraints are not included in (34). These differences make the energy cost of the economic scenario unequal to the lower bound calculated in (38). It is worth noting that in this economic scenario, the RES energy is always smaller than the energy demand of all appliances during every time slot and is fully utilized by our system (no loss).

Now, we increase the total area of solar panels $S = 1.5m^2$. This means that the energy generated by the RES is increased by 50% in every time slot, as shown in Fig. 8. In (38), most of the elements are not changed except $E_{\text{RES}}(t)$. Thus, the new lower bound of the energy cost calculated by (38) is 250.16 (cents). For this case, in the economic scenario, if our ESS and appliances continue following the old schedules, a lot of RES energy is lost at the high price time slots and the energy cost will be increased dramatically. The reason for this loss is that, in these time slots, the RES energy is a lot larger than energy demand for all appliances and the ESS cannot store the surplus RES energy because the ESS is in discharge mode. Hence, to minimize the energy cost, our system makes two main changes to reduce the loss of RES energy. First, our system tries to change mode of the ESS from discharge mode to charge mode in some high price time slots. Second, our system also moves some devices to run in some high price time slots. The main reason for these changes is to consume the RES energy in these high price time slots. However, not all RES energy is used. Our system decides to lose some RES energy from 8:00 to 11:00 as shown in Fig. 8. Due to the changes and RES loss mentioned above, we cannot achieve the minimum values of (34) and (37) and the new minimum energy cost of our system in this scenario is only 275.2 (cents).

There is always a gap between lower bound from (38) and the minimum value of energy cost of our system. If we continue increasing the RES energy by increasing total area of solar panels, this gap will continue to increase and thus the loss

| Appliances      | Start Time | Appliances      | Start Time |
|-----------------|------------|-----------------|------------|
| Toaster         | 6:00       | Washing Machine | 15:00      |
| Iron            | 5:00       | Clothes Dryer   | 17:00      |
| Vacuum Cleaner  | 11:00      | Rice Cooker     | 18:00      |
| Microwave       | 12:00      | Dish Washer     | 21:00      |
| Electric Kettle | 6:00       | Electric Shower | 20:00      |
| Air Conditioner | 13:00      | Hair Dryer      | 21:00      |
of the RES energy will also continue to increase. In real life, residents do not want to lose RES energy, they usually want to set up a PV system which generates an energy quantity equal to or a little larger than the energy demand from all appliances for every time slot. As a result, our lower bound is very close to the minimum energy cost of our system and is useful for these houses. It can help residents or engineers estimate quickly how much cost they can save if they want to set up a specific HEMS for a house.

### C. Weight method used in optimization model

In (30), the user’s convenience, waiting time, and PAR have the same weight coefficient, 1. In Fig. 4 in the smart scenario, the PAR of our system is 2.6, we want to reduce this PAR to smaller than the PAR in normal scenario (2.2). To do this job, the weight method of multi-objective optimization (MOO) is used and a new model of optimization is introduced as follows.

\[
\min (\text{MO Function}) = \min \left( \frac{C_{\text{day}}}{w_1.UC - w_2.PAR - w_3.WT} \right)
\]

where \(w_1, w_2, w_3 \in [0, 1]\) are the weight coefficients of user’s convenience, PAR, and waiting time, respectively. These parameters are set by residents and \(w_1 + w_2 + w_3 = 1\).

As depicted in Fig. 5, the PAR of our system decreases to 2 when its weight coefficient is set to 0.7, the other weight coefficients for user’s convenience and waiting time are set to 0.2 and 0.1, respectively. If we continue increasing the weight coefficient of PAR, the value of PAR will continue to decrease. However, there is then a trade-off between the energy cost and PAR. The energy cost of our system will be increased if PAR is decreased. The main reason for this trade-off is that, to decrease PAR, our HEMS must move operations of some appliances for every time slot. As a result, our lower bound for energy cost was discovered. There is always a gap between this lower bound and the minimum value of the energy cost of our system. However, this gap is very small if the energy quantity generated by the RES is equal to or a little larger than the energy demands of all the appliances in every time slot. Residents can use this lower bound to quickly calculate an estimate for the cost, and thus they can choose which parameters for the RES and ESS are suitable for their homes.

In the future, we are interested in testing and improving our optimization model with larger systems of a group of houses. Another direction for our research is to use machine learning algorithms to adapt to real-time changes of energy consumption in the smart home.

### VI. CONCLUSION

In this study, a new multi-objective MILNP-based HEMS was mathematically modeled and validated in three different scenarios: normal, economic, and smart. The obtained results demonstrated that under certain user and device constraints, our system can not only decrease energy cost of the household, but also fulfill the requirements of the residents satisfactorily. We also considered the effect of selling price in our HEMS. We showed that two kinds of user comfort and PAR were affected slightly when the selling price is decreased. By applying the weight method from MOO, the simulation results showed that our system had more flexibility in balancing energy cost, user’s convenience, waiting time, and PAR based on the requirements of residents. In this paper, a lower bound for energy cost was discovered. There is always a gap between this lower bound and the minimum value of the energy cost of our system. However, this gap is very small if the energy quantity generated by the RES is equal to or a little larger than the energy demands of all the appliances in every time slot. Residents can use this lower bound to quickly calculate an estimate for the cost, and thus they can choose which parameters for the RES and ESS are suitable for their homes.

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