Re-examining sin2β and ∆m_d from evolution of B_d^0 mesons with decoherence

Ashutosh Kumar Alok, Subhashish Banerjee, and S. Uma Sankar

1 Indian Institute of Technology Jodhpur, Jodhpur 342011, India
2 Indian Institute of Technology Bombay, Mumbai - 400076, India

(Dated: April 14, 2015)

In the time evolution of neutral meson systems, a perfect quantum coherence is usually assumed. The important quantities of the B_d^0 system, such as sin2β and ∆m_d, are determined under this assumption. However, the meson system interacts with its environment. This interaction can lead to decoherence in the entangled mesons even before they decay. In our formalism this decoherence is modelled by a single parameter λ. It is desirable to re-examine the procedures of determination of sin2β and ∆m_d in meson systems with decoherence. We find that the present values of these two quantities are modulated by λ. Re-analysis of B_d^0 data from B-factories and LHCb can lead to a clear determination of λ, sin2β and ∆m_d.

Introduction. — In neutral meson systems, quantum coherence plays a crucial role in the determination of many observables. However, any real system interacts with its environment and this interaction can lead to a loss of quantum coherence. The environmental effects may arise at fundamental level, such as the fluctuations in a quantum gravity space-time background [1,2]. They may also arise due to the detector environment itself. Irrespective of the origin of the environment, its effect on the neutral meson systems can be taken into account by using the ideas of open quantum systems [3,4]. This formalism enables the inclusion of effects such as decoherence and dissipation in a systematic manner [5].

The time evolution of neutral mesons, which are coherently produced in meson factories, are used to measure a number of parameters of the standard model of particle physics and also to search for physics beyond the standard model. With the inclusion of the decoherence effects, the measured values of some of these parameters can change. In this letter, we study the effect of decoherence on the important observables in the B_d^0 meson system, such as the CP violating parameter sin2β and the B_d^0 – B_d^0̄ mixing parameter ∆m_d. We also suggest methods which will enable clean determination of such observables.

The evolution of the B_d^0 system is built up from first principles. The effect of the environment forces the evolution to be a semi-group rather than a unitary one [6–8]. We use the density matrix formalism to represent the time evolution of the B_d^0 system. This ensures the complete positivity of the state of the system and hence its physical validity. In this formalism, the decoherence is modelled by a single parameter λ. By construction, the density matrices are trace preserving.

The work presented here, we hope, would lead to the inclusion of the effects of decoherence in the analysis of data from the B_d^0 systems. It may be worthwhile to reanalyze the data from the B factories and LHCb to verify if a singature of decoherence is already inherent in it. Given the wealth of data expected from the KEK Super B factory, it is conceivable that a signal for the decoherence may well be found. Thus a detailed study of B_d^0 observables can lead to tests of physics at scales much higher than those typical of flavour physics.

We first study the parameter sin2β, whose measurement is the first signal for CP violation outside the neutral kaon system. The precision measurement of its value is the corner stone in establishing the CKM mechanism for CP violation. With the inclusion of the decoherence effects, it turns out that the experimentally measured CP asymmetry depends both on the decoherence parameter λ and the angle β of the unitarity triangle. Next we study ∆m_d, which denotes the mixing in the B_d^0 system and is an important input in extracting sin2β from the measured time dependent CP asymmetry. We find that the measured value of ∆m_d is also affected by the decoherence effects. Finally, we suggest a method of analysis by which the three quantities, (a) λ, (b) ∆m_d and (c) sin2β can all be measured.

Determination of sin2β. — In the following, we develop the formalism which applicable to B_d^0 as well as B_s^0 mesons. We are interested in the decays of B^0 and B_s^0 mesons as well as B^0 ↔ B̄^0 oscillations. To describe the time evolution of all these transitions, we need a basis of three states: |B^0⟩, |B^0⟩̄ and |0⟩, where |0⟩ represents a state with no B meson and is required for describing the decays. In this basis, we can define ρ_{B^0|B^0}(0), the initial density matrix for the state which starts out as B^0(B^0). The time evolution of these matrices is governed by the Kraus operators K_i(t) as ρ(t) = ∑_i K_i(t) ρ(0) K_i(t)^†. The Kraus operators are constructed taking into account the decoherence in the system which occurs due to the evolution under the influence of the environment [10,11]. The time dependent density matrices are

\[
\rho_{B_d^0}(t) = \begin{pmatrix} a_{ch} + e^{-\lambda t} a_c & -a_{sh} - ie^{-\lambda t} a_s & 0 \\ -a_{sh} + ie^{-\lambda t} a_s & a_{ch} - e^{-\lambda t} a_c & 0 \\ 0 & 0 & 2(e^{\lambda t} - a_c) \end{pmatrix} \]

\[
\rho_{B_d^0}(t) = \begin{pmatrix} a_{ch} - e^{-\lambda t} a_c & -a_{sh} + ie^{-\lambda t} a_s & 0 \\ -a_{sh} - ie^{-\lambda t} a_s & a_{ch} + e^{-\lambda t} a_c & 0 \\ 0 & 0 & 2(e^{\lambda t} - a_c) \end{pmatrix} \]
for $B^0$ and $\bar{B}^0$ respectively. In the above equation, $a_{\lambda h} = \cosh \left( \frac{\lambda t}{2} \right)$, $\alpha_s = \sinh \left( \frac{\lambda t}{2} \right)$, $a_c = \cos \left( \Delta m t \right)$, $a_s = \sin \left( \Delta m t \right)$, $\Gamma = (\Gamma_L + \Gamma_H)/2$, $\Delta \Gamma = \Gamma_L - \Gamma_H$, where $\Gamma_L$ and $\Gamma_H$ are the respective decay widths of the decay eigenstates $B_d^0$ and $\bar{B}_d^0$. Also $\lambda$ is the decoherence parameter, representing interaction between one-particle system and its environment. Here we assume no CP violation in mixing which is a valid approximation in $B$ systems.

We define the decay amplitudes $A_f \equiv A(B^0 \to f)$ and $A_{\bar{f}} \equiv A(\bar{B}^0 \to f)$. The hermitian operator describing the decays of the $B^0$ and $\bar{B}^0$ mesons into $f$ is

$$O_f = \begin{pmatrix}
|A_f|^2 & A_f^\ast \bar{A}_f & 0 \\
A_f \bar{A}_f & |\bar{A}_f|^2 & 0 \\
0 & 0 & 0
\end{pmatrix}.$$  \hspace{1cm} (2)

The probability, $P_f(t/(B^0/\bar{B}^0); t)$, of an initial $B^0/\bar{B}^0$ decaying into the state $f$ at time $t$ is given by $\text{Tr} \left[ O_f P_f(t) \right]$. Let us now consider $B_d^0 \to J/\psi K_S$ decay. One can define a CP violating observable

$$A_{J/\psi K_S}(t) = \frac{P_{J/\psi K_S}(B_d^0; t) - P_{J/\psi K_S}(\bar{B}_d^0; t)}{P_{J/\psi K_S}(B_d^0; t) + P_{J/\psi K_S}(\bar{B}_d^0; t)}.$$  \hspace{1cm} (3)

Calculating the probabilities using Eqs. (1) and (2) we get

$$\frac{A_{J/\psi K_S}(t)}{e^{-\lambda t}} = \frac{(|\lambda_f|^2 - 1) \cos \left( \Delta m_d t \right) + 2 \text{Im}(\lambda_f) \sin \left( \Delta m_d t \right)}{(1 + |\lambda_f|^2) \cos \left( \frac{\Delta \Gamma_d}{2} \right) - 2 \text{Re}(\lambda_f) \sin \left( \frac{\Delta \Gamma_d}{2} \right)}.$$  \hspace{1cm} (4)

where $\lambda_f = A(B_d^0 \to J/\psi K_S)/A(B_d^0 \to J/\psi K_S)$. For $B_d^0$ meson system, $\Delta \Gamma_d \approx 0$. With this approximation, the above expression simplifies to

$$\frac{A_{J/\psi K_S}(t)}{e^{-\lambda t} \cos \left( \Delta m_d t \right)} = \frac{1 - |\lambda_f|^2}{(1 + |\lambda_f|^2)} + \frac{2 \text{Im}(\lambda_f) \tan \left( \Delta m_d t \right)}{(1 + |\lambda_f|^2)}.$$  \hspace{1cm} (5)

Putting $\lambda = 0$ in the above equation, we get the usual expression for CP asymmetry in the interference of mixing and decay. Thus the presence of decoherence modifies the expression for CP asymmetry in the interference of mixing and decay.

For $B_d^0 \to J/\psi K_S$, $\text{Im}(\lambda_f) = \sin 2\beta$. Assuming $|\lambda_f| = 1$, i.e., no direct CP asymmetry, we get

$$A_{J/\psi K_S}(t) = \sin 2\beta \ e^{-\lambda t} \sin \left( \Delta m_d t \right).$$  \hspace{1cm} (6)

Therefore we see that the coefficient of $\sin \left( \Delta m_d t \right)$ in the CP asymmetry is $\sin 2\beta e^{-\lambda t}$ and not $\sin 2\beta$! The measurement of $\sin 2\beta$ is masked by the presence of decoherence. Thus in order to have a clean determination of $\sin 2\beta$, an understanding of $\lambda$ is imperative.

In [2, 14], it was shown that there can be some discrepancy between the experimental value and the standard model (SM) theoretical prediction of $\sin 2\beta$. This discrepancy may hint towards possible physics beyond the SM. However, if $\lambda$ is not small enough to be neglected, this discrepancy can also be attributed to the ignorance of decoherence in the analysis.

**Determination of $\Delta m_d$**—It is obvious that in order to determine $\sin 2\beta$, we need to know $\Delta m_d$ and $\lambda$. If $\Delta m_d$ is measured using observables which are independent of $\lambda$, then we only need to determine $\lambda$ for the clean extraction of $\sin 2\beta$. If the determination of $\Delta m_d$ is also masked by the presence of decoherence then we need to have a clean determination of $\Delta m_d$. The present world average of $\Delta m_d$ quoted in PDG is $(0.510 \pm 0.003) \text{ s}^{-1}$ which is an average of measurements of $\Delta m_d$ from OPAL [12], ALEPH [15], DELPHI [17], L3 [18], CDF [19], BaBar [20], Belle [21], D0 [22] and LHCb [23] experiments. There are several ways in which $\Delta m_d$ can be determined experimentally. LHCb, CDF and D0 experiments determine $\Delta m_d$ by measuring rates that a state that is pure $B_d^0$ at time $t = 0$, decays as either $B_d^0$ or $\bar{B}_d^0$ as function of proper decay time. In the presence of decoherence, the survival (oscillation) probability of initial $B_d^0$ meson to decay as $B_d^0(\bar{B}_d^0)$ at a proper decay time $t$ is given by

$$P_{\pm}(t, \lambda) = \frac{e^{-\lambda t}}{2} \left( 1 \pm e^{-\lambda t} \cos \left( \Delta m_d t \right) \right),$$  \hspace{1cm} (7)

where we have neglected the width difference $\Delta \Gamma_d$. The positive sign applies when the $B_d^0$ meson decays with the same flavor as its production and the negative sign when the particle decays with opposite flavor to its production. We see that the survival (oscillation) probability of $B_d^0$ is $\lambda$ dependent! The time dependent mixing asymmetry, used to determine $\Delta m_d$, is then given by

$$A_{\text{mix}}(t, \lambda) = P_{\pm}(t, \lambda) - P_{\pm}(t, \lambda) = e^{-\lambda t} \cos \left( \Delta m_d t \right).$$  \hspace{1cm} (8)

Thus we see that the otherwise pure cosine dependence of mixing asymmetry is modulated by $e^{-\lambda t}$. Belle and BaBar experiments determine $\Delta m_d$ by measuring time dependent probability $P_{\pm}(t)$ of observing unoscillated $B_d^0/\bar{B}_d^0$ events and $P_{\pm}(t)$ of observing oscillated $B_d^0(\bar{B}_d^0)/B_d^0(\bar{B}_d^0)$ events for two neutral $B_d$ mesons produced in an entangled state in the decay of the $Y(4S)$ resonance. The expressions for $P_{\pm}(t)$, in the presence of decoherence, are the same as those given in Eq. (7), except that the proper time $t$ is replaced by the proper decay-time difference $\Delta t$ between the decays of the two neutral $B_d$ mesons. Therefore, we see that the determination of $\Delta m_d$ at LHCb, CDF, D0, Belle and BaBar experiments is also masked by the presence of $\lambda$. The true value of $\Delta m_d$ can be determined by a two parameter $(\Delta m_d, \lambda)$ fit to the time dependent mixing asymmetry $A_{\text{mix}}(t, \lambda)$ defined in Eq. (8). This in turn will enable a determination of true value of $\sin 2\beta$ using Eq. (6).

**Determination of $\Delta m_d$ in the LEP experiments** is mainly based on time independent measurement, i.e.,
from the ratio of the total same-sign to opposite-sign semileptonic rates \( (R) \) or the total \( B_d^0 - B_d^0 \) mixing probability \( (\chi) \). We shall now see that these observables are also \( \lambda \) dependent. Therefore all the methods used to determine \( \Delta m_d \) depend upon \( \lambda \).

Correlated \( B_d^0 \) meson semi-leptonic decays. — The entangled \( B_d^0 - B_d^0 \) mesons, produced in the decay of the \( T(4S) \) resonance, can both decay semi-leptonically. The effects of decoherence on the resulting dilepton signal was studied in \cite{21}. Here we calculate these effects using the formalism described in the previous section. The entangled \( B_d^0 - B_d^0 \) state can be written as

\[
|\psi(0)\rangle = \frac{1}{\sqrt{2}} \left( |B_d^0 B_d^0\rangle - |\bar{B}_d^0 B_d^0\rangle \right) .
\]

The time evolution of the above state is described by the following density matrix:

\[
\rho(t_{1}, t_{2}) = \frac{1}{2} \left( \rho_{1}(t_{1}) \otimes \rho_{2}(t_{2}) + \rho_{2}(t_{1}) \otimes \rho_{1}(t_{2}) \right)
\]

where \( \rho_{1}(t) = \rho_{B_d^0}(t) \), \( \rho_{2}(t) = \rho_{\bar{B}_d^0}(t) \) which are given in Eq. \ref{1}, while \( \rho_{3/4}(t) = \sum_{i} K_i \rho_{3/4}(0) K_i^\dagger \), where \( \rho_{3/4}(0) = |B_0^0(B_0^0)\rangle \langle B_0^0(B_0^0)| \) and are given by

\[
\rho_{3}(t) = \frac{1}{2e^{-\Gamma t}} \begin{pmatrix}
-a_{sh} - i e^{-\lambda t} a_{sh} & a_{ch} + e^{-\lambda t} a_{ch} & 0 \\
-a_{ch} - e^{-\lambda t} a_{ch} & -a_{sh} + i e^{-\lambda t} a_{sh} & 0 \\
0 & 0 & 2a_{sh}
\end{pmatrix},
\]

\[
\rho_{4}(t) = \frac{1}{2e^{-\Gamma t}} \begin{pmatrix}
-a_{sh} + i e^{-\lambda t} a_{sh} & a_{ch} - e^{-\lambda t} a_{ch} & 0 \\
-a_{ch} + e^{-\lambda t} a_{ch} & -a_{sh} - i e^{-\lambda t} a_{sh} & 0 \\
0 & 0 & 2a_{sh}
\end{pmatrix}
\]

Here the parameters are as in Eq. \ref{1}. The double decay rate, \( G(f, t_{1}; g, t_{2}) \), that the left-moving meson decays at proper time \( t_{1} \) into a final state \( f \), while the right-moving meson decays at proper time \( t_{2} \) into the final state \( g \), is then given by \( \text{Tr} \left[ (O_{f} \otimes O_{g}) \rho(t_{1}, t_{2}) \right] \). From this a very useful quantity called the single time distribution, \( \Gamma(f; g; t) \), can be defined as \( \Gamma(f; g; t) = \int_{0}^{\infty} d\tau G(f; \tau + t; g; \tau) \), where \( t_{1} - t_{2} \) is taken to be positive.

We now consider the decays of \( B_d^0 \) mesons into semileptonic states \( h l \nu \), where \( h \) stands for any allowed charged hadronic state. Under the assumption of no CP violation in mixing, CPT conservation and no violation of \( \Delta B = \Delta Q \) rule, the amplitudes for \( B_d^0 / B_d^0 \) into \( h l^{-}\nu \) can be written as

\[
A \left( B_d^0 \rightarrow h l^{-}\nu \right) = M_{h} , \quad A \left( B_d^0 \rightarrow h l^{+}\nu \right) = 0 ,
\]

whereas the amplitudes for \( B_d^0 / B_d^0 \) into \( h l^{+}\bar{\nu} \) are

\[
A \left( B_d^0 \rightarrow h l^{+}\bar{\nu} \right) = 0 , \quad A \left( \bar{B}_d^0 \rightarrow h l^{+}\bar{\nu} \right) = M_{h}^\ast .
\]

There are two important observables which can be affected by interaction with the environment. One is the ratio of the total same-sign to opposite-sign semileptonic rates

\[
R = \frac{\Gamma(h^{+}, h^{+}) + \Gamma(h^{-}, h^{-})}{\Gamma(h^{+}, h^{-}) + \Gamma(h^{-}, h^{+})} ,
\]

and the other is the total \( B_d^0 - B_d^0 \) mixing probability

\[
\chi = \frac{\Gamma(h^{+}, h^{+}) + \Gamma(h^{-}, h^{-})}{\Gamma(h^{+}, h^{-}) + \Gamma(h^{-}, h^{+})} .
\]
functions of the decoherence parameter $\lambda$. Hence it is imperative to measure $\lambda$ for a clean determination of these quantities. We suggest a re-analysis of the data on the above asymmetries for an accurate measurement of all the three quantities $\lambda$, $\sin 2\beta$ and $\Delta m_d$. The present analysis can easily be extended to the $B^0_d$ system as well.

Acknowledgments. — We thank K. Mazumdar for helpful discussions on several parts of this analysis. The work of AKA and SB is supported by CSIR, Government of India, grant no: 03(1255)/12/EMR-II.

\[\text{[1]}\] S. W. Hawking, Commun. Math. Phys. 87 (1982) 395.

[2] J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B 293 (1992) 142 [hep-ph/9207268].

[3] U. Weiss, Quantum Dissipative Systems, Third Edition (World Scientific 2008).

[4] H.-P. Breuer and F. Petruccione, The Theory of Open Quantum Systems (Oxford University Press 2002).

[5] S. Banerjee and R. Ghosh, Phys. Rev. E 67, 056120 (2003).

[6] S. Banerjee, A. K. Alok and R. Mackenzie, arXiv:1409.1034 [hep-ph].

[7] G. Lindblad, Commun. Math. Phys. 48, 119 (1976).

[8] V. Gorini, A. Kossakowski and E. C. G. Sudarshan, J. Math. Phys. 17, 821 (1976).

[9] R. Alicki and K. Lendi, Quantum Dynamical Semigroups and Applications (Lect. Notes Phys. 717 (Springer, Berlin Heidelberg 2007)).

[10] A. K. Alok et al., Work in progress.

[11] E. C. G. Sudarshan, P. M. Mathews and J. Rau, Phys. Rev. 121, 920 (1961); K. Kraus, States, Effects and Operations: Fundamental Notions of Quantum Theory (Springer Verlag 1983).

[12] E. Lunghi and A. Soni, Phys. Lett. B 666 (2008) 162 [arXiv:0803.4340 [hep-ph]].

[13] E. Lunghi and A. Soni, Phys. Rev. Lett. 104 (2010) 251802 [arXiv:0912.0002 [hep-ph]].

[14] E. Lunghi and A. Soni, Phys. Lett. B 697 (2011) 323 [arXiv:1010.6069 [hep-ph]].

[15] G. Abbiendi et al. [OPAL Collaboration], Phys. Lett. B 493, 266 (2000) [hep-ex/0010013].

[16] D. Buskulic et al. [ALEPH Collaboration], Z. Phys. C 75, 397 (1997).

[17] J. Abdallah et al. [DELPHI Collaboration], Eur. Phys. J. C 28, 155 (2003) [hep-ex/0303032].

[18] M. Acciarri et al. [L3 Collaboration], Eur. Phys. J. C 5, 195 (1998).

[19] F. Abe et al. [CDF Collaboration], Phys. Rev. D 60, 072003 (1999) [hep-ex/9903011].

[20] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 73, 012004 (2006) [hep-ex/0507054].

[21] K. Abe et al. [BELLE Collaboration], Phys. Rev. D 71, 072003 (2005) [Erratum-ibid. D 71, 079903 (2005)] [hep-ex/0408113].

[22] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. D 74, 112002 (2006) [hep-ex/0609034].

[23] R. Aaij et al. [LHCb Collaboration], Phys. Lett. B 709, 177 (2012) [arXiv:1112.4311 [hep-ex]]; R. Aaij et al. [LHCb Collaboration], Eur. Phys. J. C 73, no. 12, 2655 (2013) [arXiv:1308.1302 [hep-ex]]; R. Aaij et al. [LHCb Collaboration], Phys. Lett. B 719, 318 (2013) [arXiv:1210.6750 [hep-ex]].

[24] R. A. Bertlmann and W. Grimus, Phys. Rev. D 64, 056004 (2001) [hep-ph/0101160].

[25] H. Albrecht et al. [ARGUS Collaboration], Phys. Lett. B 324, 249 (1994); J. E. Bartelt et al. [CLEO Collaboration], Phys. Rev. Lett. 71, 1680 (1993).