Deep Sea Bubble Stream Characterization
Using Wide-Baseline Stereo Photogrammetry

Mengkun She, Yifan Song, Tim Weiß, Jens Greinert, Kevin Köser
GEOMAR Helmholtz Centre for Ocean Research Kiel, Wischhofstr. 1-3, 24148 Kiel

Abstract

Reliable quantification of natural and anthropogenic gas release (e.g. CO$_2$, methane) from the seafloor into the ocean, and ultimately, the atmosphere, is a challenging task. While ship-based echo sounders allow detection of free gas in the water even from a larger distance, exact quantification requires parameters such as rise speed and bubble size distribution not obtainable by such sensors. Optical methods are complementary in the sense that they can provide high temporal and spatial resolution of single bubbles or bubble streams from close distance. In this contribution we introduce a complete instrument and evaluation method for optical bubble stream characterization. The dedicated instrument employs a high-speed deep sea stereo camera system that can record terabytes of bubble imagery when deployed at a seep site for later automated analysis. Bubble characteristics can be obtained for short sequences of few minutes, then relocating the instrument to other locations, or in autonomous mode of intervals up to several days, in order to capture variations due to current and pressure changes and across tidal cycles. Beside reporting the steps to make bubble characterization robust and autonomous, we carefully evaluate the reachable accuracy and propose a novel calibration procedure that, due to the lack of point correspondences, uses only the silhouettes of bubbles. The system has been operated successfully in up to 1000m water depth in the Pacific Ocean to assess methane fluxes. Besides sample results we also report failure cases and lessons learnt during development.

Keywords: gas bubbles, bubble quantification, underwater vision, free gas
1. Introduction

Greenhouse gases such as methane or CO$_2$ play a key role in climate change. At the ocean floor they can escape from natural reservoirs\textsuperscript{1}, or from leaky or abandoned drill or carbon storage sites, or participate in or result from chemical reactions, geological processes or microbial metabolism. Gas release, as well as formation and destruction of hydrates, can also influence the mechanical stability of seafloor areas, e.g. at continental slopes, and can have important impact on the local habitats. For all these reasons, exact quantification and monitoring of gas release from the ocean floor, as well as understanding the controlling conditions, are important research questions.

When released continuously from the seabed into the ocean as bubbles, methane and CO$_2$ form bubble streams that can rise towards the atmosphere\textsuperscript{2} (see Fig. 2). Key quantification techniques for such free gas include physical collection of the gas with funnels, acoustical and optical methods, all of which
Figure 2: Methane escaping from the seafloor in the North Sea and forming bubble streams (left: horizontally looking ROV camera. right: multibeam echo sounder at a location with two bubble streams). The visual sensor can be positioned on top of one of these streams to capture the stream parameters such as rise speed and size distribution. Such streams can be detected also in ship-based echo sounders\cite{3} as can be seen in the right image. Jointly, optical and acoustical observations can be used for large scale quantification. TODO: better picture???

have advantages and drawbacks. Already a short-term, small-scale measuring of gas emissions with a collector is a challenging and costly task as it usually requires remotely operated vehicles (ROVs) to perform the mission and the volume measured is an integrative measure that does not provide information on individual bubbles. Long-term automated monitoring of gas seeps over a large area using such a technique is unpractical. Since acoustical-based approaches can detect free gas underwater from a large distance, they have become the most efficient tools to find gas flux in a lake or a particular area of the ocean\cite{4,5,6,7} and to map gas activity over large areas\cite{3}. However, exactly quantifying the gas flux requires prior knowledge of some essential parameters such as the bubble size distributions and rising speeds\cite{8,9}, which cannot be observed from the acoustic data itself. On top, these approaches require careful calibration, are sensitive to wavelength/size variations and noise and have limited spatial resolution\cite{10,3}. Quite complementary, rising bubble streams can be photographed by high speed cameras and characteristic parameters can be obtained using photogrammetric techniques, which facilitates the acoustic inversions. In addition, visual information provides a better understanding of the bubble characteristics and behaviors, such as deformation and motion patterns
of the bubbles\cite{10,11}.

Early works have focused on quantifying bubbly flow in a laboratory setup\cite{12, 13, 14, 15, 10}, where imaging and instrumentation conditions are much better controlled than in the ocean. Fewer works exist that actually design and deploy in-situ imaging systems for the ocean, including telecentric lenses for very small bubbles \cite{16}, a monocular camera \cite{11} and a small baseline stereo system \cite{17}. All approaches require a lot of manual interaction to obtain bubble information, and their focus was not on robust, automatic techniques that can work on gigabytes of image data. Consequently, extracting the bubble information from the video sequences accurately and robustly remains a challenging task and a complete and robust pipeline is required to automatically analyze many thousands of images that are obtained under uncontrollable, and often sub-optimal, conditions.

In this paper, we build on our preliminary lab studies for bubble measurements\cite{10} and propose the following contributions: We (i) present a deep-sea wide baseline bubble measurement system, including a robust and complete pipeline that is able to automatically characterize bubble streams from long-term video sequences. We (ii) propose a new silhouette-based calibration approach that can adjust the calibration without point correspondences, i.e. only from bubble observations. We show that the same technique can be used for accurate bubble ellipsoid estimation. We (iii) carefully evaluate the system performance using ground truth measurements and show results and challenges on real data acquired from deep sea missions up to 1000\(m\) depth.

2. Related Work

3D Measurement of Bubbles. Quantifying multi-phase flow parameters has been an interesting topic in many natural and industrial applications, and Particle Image Velocimetry (PIV) is a common technique to tackle such problems\cite{19}. The gas/liquid flow is the basic scenario of two-phase flow, where the bubbles can be identified in the image sequences and the bubble features can be
extracted using 3D image processing techniques. Therefore in some laboratory-based works, special setups were built to photograph bubbly flow in a tube with high-speed cameras. These approaches generally consist of three major steps which include bubble identification, bubble 3D size measurement and bubble tracking over time. Zelenka et al. and Fu et al. [14, 19] have focused on the bubble outline extraction, but they both use a single camera to estimate the 3D volume of the bubble. Bian et al. [12, 13] assume the bubble is composed by two semi-ellipsoids and they have proposed an approach to extract the characteristic parameters of a single rising bubble from a pair of stereo images. The stereo camera system can significantly improve the 3D bubble size estimation but also poses other challenges, for instance, finding the correspondence of the same bubble in the stereo image pair. However, bubbles are not easily distinguishable, and there is no rich texture information, consequently, traditional feature-based (e.g. SIFT [20]) or pixel-wise (e.g. SGM [21]) matching approaches can hardly be applied in this scenario. Therefore, lab approaches can often only reconstruct a single rising bubble. To save the second camera, and to avoid synchronization while sacrificing image area, Xue et al. [22, 15] have constructed a camera-mirror system, where the mirrors are used to generate a reflection of the bubble stream as seen from a different perspective. They have also proposed an equal-height heuristic to disambiguate multiple match candidates across the stereo image pair, making strong assumptions on bubble position. In a laboratory environment, many complex setups can be built to better reconstruct the shape of the bubble. For instance, Fu et al. [23] have developed a space carving algorithm to reconstruct the free form surface of a single large rising bubble using multiple cameras. A similar system is developed by Masuk et al. [24] but with cameras and mirrors to create 4 virtual views for the space carving algorithm. These systems and algorithms are very attractive, but it can be difficult to transfer them into in-situ deep sea bubble stream characterization systems that can be used reliably in the ocean.
Optical Bubble Measurement Systems in the Ocean. Leifer et al. [16] have presented a deep sea optical bubble meter to analyze sizes and motions of bubbles with a single camera, and a manual workflow to extract the bubble volume according to the ellipsoid projection assumption is introduced. Using a telecentric lens avoids scale ambiguity, but restricts the observation space to an extremely small volume. Later, Sahling et al. [25] develop an optical device and apply it to measure the gas discharge of a bubble stream using a perspective lens from a camera mounted on an ROV. Thomanek et al. [11] improved the system in terms of hardware design and proposed a more complete image processing workflow to extract the bubble features. These works assume the distances of the bubbles to the camera to be constant, hence, the pixel-to-object scale can be calibrated using a ruler or reference. However, the bubbles in the ocean often rise in a zigzag way or the entire stream can bend with currents, consequently, the object distances can vary significantly. Also, large bubbles are often not spherical, the extent of the bubble in the camera's viewing direction is not observed then. Therefore, monocular systems can suffer from relatively large uncertainty when measuring the size and shape of a bubble. To address these issues Wang et al. [17] propose a short-baseline stereo camera system to obtain more 3D information, and later successfully deploy the system in an expedition [26]. Jordt et al. [10] geometrically analyze the uncertainty of the triangulation estimation in a short-baseline stereo setup and proposed a novel 90° wide-baseline stereo camera system based bubble measurement setup and prove feasibility by lab experiments. This configuration can observe both the frontal and the side view of the bubble to remedy shape and distance uncertainty. They also discuss things like image pre-processing, camera calibration considering flat port refraction [27, 28], bubble stereo matching and an ellipsoid fitting approach. While a complete workflow under ideal lab conditions is presented, and a prototype metal frame with GoPro cameras has been set up, the system was not evaluated in the ocean, where many challenges remain. Later, She et al. [29] analyse replacing the flat port camera housings with dome ports, and discuss centering and calibration of the dome port cameras [30, 31]. In this
contribution we build on top of this work and describe a practical deep ocean bubble measurement system, that, once positioned approximately on a bubble seep spot, captures bubbles using stereo photogrammetry. The system is robust against inevitable nuisances such as dirt, disturbed illumination from upstirred sediment, offsets of the bubble stream, wobbly bubble ascent or temporal shift of the stream position. We provide automated methods for synchronization and robust background removal, as well as a completely new approach to instrument self-calibration without point correspondences: we use only the outlines of bubbles for wide-baseline recalibration from in-situ data. While previous systems have been qualitatively validated, we show a reconstruction accuracy of better than 1% when estimating the diameters of glass spheres sinking through the box that have a diameter of millimeters to centimeters.
3. System Design

3.1. Hardware Overview

Before diving into the gas flow quantification approach, we first summarize the deep sea in-situ bubble stream characterization instrument. The instrument follows the wide-baseline setting proposed in [10] and is a box shaped stereo recording device that can be deployed by a robot arm of an ROV as shown in Fig. 3b or can also be lowered from a surface vessel and then be positioned by divers. It contains a vertical corridor (64cm² cross section) in the box center which allows bubble streams rising through. Two deep sea titanium housings with dome ports are mounted at two adjacent sides of the box. Inside each of the housings, there is a high-speed machine vision camera recording images of 1024×800 pixels resolution and a field of view of around 60°. The frame rate of the camera can be set to 80Hz−100Hz (larger if smaller image areas are used), thus, each bubble can be photographed about 40 times since the bubbles can rise with a speed of 25−35cm/s [32]. Since both gas and water are largely transparent, the cameras can only observe refraction and reflection effects at the bubble surface, and previous studies[10] have shown that background illumination (also known as bright-field illumination in microscopy) is advantageous for photographing bubbles.

Therefore, two panels of green LEDs (550nm) are mounted at the camera-opposite faces of the box behind acrylic diffusor plates. They provide backlight illumination such that the outline of the bubble produces a dark rim in the image. Since each bubble is observed from 90° different perspectives, no photometric properties such as color or texture can be used for matching, and the outline in one camera only provides a weak hint about the size of the outline in the other camera and no explicit point correspondences can be obtained. A sample stereo image pair is shown in Fig. 4 where the left image and the right image are concatenated horizontally.
Figure 4: Sample stereo image pair captured by the instrument. The left image and the right image are concatenated horizontally (If not mentioned particularly, in the later figures, the left image and the right image are shown concatenated).

3.1.1. System Weight, Size, ROV-switch, Pressure-proof

The system size is 81cm (height, without funnel: 62cm) × 43cm × 43cm (see Fig. 5). The weight is about 60kg in air and 24kg in water. The box was designed to work at up to 1000m depth, but could be updated easily for use in up to 6000m, for which the main components, like the camera housings, are already built. The overall power consumption during 80Hz recording is 70W on average (100W peak during flash). It can be powered remotely or by battery, where the batteries last for 5h-10h of permanent flashing, depending on water temperature. Recording time can be extended as explained in the next paragraph.

3.1.2. Computers and Synchronization

The box consists of two pressure housings that each contain an intel NUC computer and a Basler Ace acA1300-200um machine vision camera behind a dome port. The 8mm C-Mount lenses (AZURE) have been carefully centered in the domes [30, 31]. We crop the images to 1024×800 pixels to avoid frame-drops when writing to the 1TB SSDs inside each housing at high frame rates. The cameras are triggered by a micro-controller that starts exposure time and
flashes on the rising edge of the trigger signal and stops exposure on the falling edge. Exposure time is set to 1ms to avoid motion blur. Every 5000 images, exposure and flash time is set to 10 microseconds only, which produces a black image in both cameras. These ”black flash” images are exploited to synchronize the two streams. Before operation, the computer clocks are synchronized via network time protocol up to 1 second and we write the time stamp of the incoming image into the filename and save the raw image as pgm. At 80Hz the overall system currently produces a data rate of 1 gigabit per second or 0.45 terabyte per hour. The images can be downloaded after the mission via gigabit ethernet which takes approximately the same time as taken for recording.

The box is equipped with a long-term-mode that modifies the microcontroller’s trigger signal. Rather than permanent mode, the microcontroller can create intervals such as the first five minutes of each hour. Outside these intervals LEDs are only flashed once every few seconds as a standby visualization. This reduces the storage needs by an order of magnitude and allows to operate
Figure 6: Left: Vertical section of the box showing the entrance funnel at the bottom leading to a 80mm wide square-shaped rise corridor. Right: Camera inside dome port looking through a mirror into the box. Also the background illumination for the other camera (not displayed here) using LEDs behind an acrylic plate is displayed.

the box for more than one day.

3.1.3. Cameras and Mirrors

As can be seen in Fig. 6, the rise corridor into which the bubbles are directed at the bottom is 8cm wide. The effective resolution is 5.7 pixel per millimeter in the center of the observation corridor. Since the camera housing is elongated, they are mounted vertically to the frame to save space, consequently, the cameras are looking upwards through two mirrors at roughly 45° angle to create two virtual horizontal views.

3.1.4. Adjustment and Calibration

Each dome port camera is centered according to the method of [30] and it has been verified that refraction effects are insignificant in the observation corridor [31] and the perspective camera model can be used. After dome centering, we mount the stereo camera rigidly and perform stereo calibration. Due
to the bright field setting, stereo calibration is performed underwater using a transparent calibration target within the bubble corridor. As shown in Fig. 7, both cameras observe the same target multiple times, and the camera intrinsics $K_1, K_2$ are calibrated using the traditional approach [33]. Note that the remaining tiny decenterings of the dome ports can be safely absorbed by distortion parameters as we are observing objects at a relatively fixed distance. Then, let the left camera be the origin of the world coordinate system as $T_1 = [I_{3 \times 3} \mid 0]$.

1Classically, the calibration matrix $K$ represents the perspective camera intrinsics such as focal lengths and principal points. For ease of notation we use the symbol $K$ to represent all intrinsics, including lens distortion parameters.

Figure 7: Stereo calibration of the instrument. Top: A calibration target is photographed by both two cameras multiple times underwater. Hence, the relative position and orientation as well as the camera intrinsics can be obtained. Bottom: sample calibration photos.
The right camera has the pose of $\mathbf{T}_2 = [R \mid t]$. We can determine the pose of the right camera and do final refinement on both camera intrinsics by projecting the 3D target points onto the stereo images and minimizing the residuals between the projections and the identified corresponding points, as common for target-based calibration:

$$E = \sum_{i}^{n} \sum_{j}^{m} (\|\pi(\mathbf{X}_i, \mathbf{T}_1) - \mathbf{x}_{i,j}^1\|^2 + \|\pi(\mathbf{X}_i, \mathbf{T}_2) - \mathbf{x}_{i,j}^2\|^2) \quad (1)$$

Here, $\mathbf{x}_{i,j}^1$ and $\mathbf{x}_{i,j}^2$ indicate the $i^{th}$ point on the target photographed by the $j^{th}$ image, and the superscript 1 and 2 indicate the left camera and the right camera. $\pi()$ represents the perspective projection function.

### 3.2. Bubble Stream Characterization Method

During in-situ operation, image data is recorded by the instrument. Processing is done afterwards using different modules written in C++ and CUDA, all of which can also run inside a docker-container on a remote server with GPU capabilities. In this case access works through a Jupyter Notebook interface. Automated, and efficient, processing is important, since we record more than half a million images per hour. After processing, the software is able to generate a report on the important bubble stream characteristics such as overall volume of gas released, bubble size distribution and rise velocity. In this section, we will give details on the approach for bubble stream characterization, and Fig. 8 illustrates an overview of the working pipeline.

#### 3.2.1. Temporal Synchronization of Stereo Data

The two cameras are operated by independent computers and record time-stamped data. The time-stamps of the computers agree up to one, or for long-term deployments at most a few, seconds. Since the goal is to record with 80Hz-100Hz, we would need a clock agreement of less than 5ms in order to unambiguously associate matching frames in the recorded sequences. Therefore, every 5000 images, the micro-controller will generate a very short flash time (10 microsecond rather than 1000 microseconds), which leads to a black image.
Those black images are now used as identifiers for synchronization of the two photo sequences. To achieve that, the software first iterates through both photo sequences and extracts the timestamp of each image and also detects the black images. This way the damage of potential frame drops in one camera is limited and the moment it happened can be detected easily. Next, the average time offset between the two computers can be calculated by the timestamps of the aligned black images. Afterwards, for each image in one of the sequences, we compute the expected timestamp when the corresponding image is captured, and then search the corresponding image in the other sequence by finding the minimum time difference. Finally, two photo sequences are aligned and a sequence of stereo image pairs is created. Particularly, when looking for black
images, we sample the main-diagonal and counter-diagonal pixels of the image and check their intensities. If all pixel intensities are smaller than 8 (empirical threshold), the image is considered a synchronization frame.

3.2.2. Background Learning and Removal

The original images usually contain complex background structures, for instance, sediments stuck on the dome ports, bubbles trapped and stuck somewhere and markers on the frame of the instrument, which makes bubble detection more complicated. But the background information can be learned and therefore removed if the background objects stay static in a certain time interval. Assuming that we have only sparse bubble observations, at each pixel position in the image we will see the background in the majority of the images. Consequently, a robust temporal median filter algorithm can be leveraged to compute a background image from a series of bubbles images (see Fig. 9). Since the background can change over time, we apply the Median background estimation using a sliding-window approach. We then subtract the ’learned’ background image from each raw bubble image to obtain only the moving objects, i.e. the bubbles. In this step the images can also be radially undistorted, in case radial distortion is present. These steps can be implemented efficiently in CUDA using a streaming architecture.

3.2.3. Bubble Detection

Bubble detection aims at finding positions and 2D shapes of the bubbles in an image, more specifically, the bubble contours. Due to the back illumination, the bubbles appear as dark rims with a brighter area inside the contour in the foreground images. The image processing technique used here is similar to [10][11], first, the Canny edge detector is run to find those pixels and group them into edges. Then, the convex hull is determined for each bubble and an ellipse is fitted to the convex hull afterwards. The workflow of bubble detection is shown in Fig. [10]. Since there are always single noisy pixels or dirt particles inside the water, a threshold for minimum sizes of bubbles has to be used. Since
Figure 9: Principal of the temporal median filtering for background learning. If the background structures are stable in a certain time interval, they can be learned and removed by estimating the background image. For each pixel in the background image, the intensity value is determined by taking the median value along the temporal time series.

A 1mm-diameter spherical bubble corresponds to a circle of about 31 pixels circumference, the default setting is to reject contours with less than 30 pixels length as moving particles, in order to reduce mis-detection (see Fig. 10e). This
can be adapted of course, e.g. when the goal is to measure very tiny bubbles in very clear water.

3.2.4. Epipolar Geometry and Stereo Matching

To estimate the volumes of the bubbles, it is required to find the corresponding bubble outlines across the stereo image pairs prior. Traditional feature-based [20] or pixel-wise [21] matching approaches find correspondences by computing similarity of local image patches. Since bubbles lack texture or other specific appearance information, only geometric constraints can be used for matching, such as epipolar geometry. As is shown in Fig. [11] a bubble is observed by two cameras, and the pose of the second camera with respect to the first camera is related by a rotation and translation $R, t$, which is obtained from stereo calibration. The second bubble’s projection is therefore constrained by epipolar geometry. The matching we use is the same as described in [10], searching for candidates in a corridor around the epipolar line, and then solving the matching problem of all bubbles in an image pair at once using a bipartite graph.
Figure 11: Epipolar geometry constraint for stereo bubble matching. The epipolar lines $l_1$ and $l_2$ are the projections of light rays seeing by the pixel $B_2$ and $B_1$. $C_1$ and $C_2$ are the camera centers. The baseline between two camera centers and the light rays of a corresponding bubble span an epipolar plane. The epipolar constraint can reduce the searching space of the corresponding bubble.

After finding the bubble correspondences in the stereo image pair, we can estimate the bubble shape.

### 3.2.5. Ellipsoid Initialization

We model all bubbles in 3D as ellipsoidal, and initialize a 3D ellipsoid from the 2D ellipse parameters in the two images, before we optimize the 3D ellipsoid position and axes to fit both projections.

As can be seen from Fig. 12 the center of the ellipsoid is triangulated from the projection center of the ellipses, therefore, the distances of the bubble center to the cameras are also computed as $d_1$ and $d_2$. Then, we specify a set of feature points $A_1, A_2, B_1, B_2, C_1, C_2$ as the end point of the ellipsoid axes, among which $A_1, A_2$ are back-projected from $a_1, a_2$ in the left image with a distance of $d_1$. Similarly, $B_1, B_2$ are back-projected from $b_1, b_2$ in the right image with a distance of $d_2$. The vector $\overrightarrow{A_1A_2}$ and $\overrightarrow{B_1B_2}$ form a plane, and the third vector $\overrightarrow{C_1C_2}$ is constructed as the cross product of the other two vectors,
such that $\vec{C_1C_2}$ is orthogonal to the plane. Since $\vec{A_1A_2}$ and $\vec{B_1B_2}$ are not orthogonal, we setup a $3 \times 3$ matrix $R$ with three columns being these vectors, and map to a rotation matrix where the three columns are orthonormal. Then, we obtain the axes of the ellipsoid. Finally, we recompute the lengths of the axes by mapping the pre-computed feature points to the axis vectors, and the volume of the bubble can be obtained as:

$$V = \frac{4}{3} \pi abc$$

where $a, b, c$ are the lengths of the ellipsoid axes.

Of course, the triangulated ellipsoid is an initial solution which can be refined further using bundle adjustment without point correspondences, which will be described in the next subsection.

3.2.6. Bubble Adjustment without Point Correspondences

We first represent the ellipsoid surface as a quadric, which is a $4 \times 4$ symmetric matrix $Q$ in the 3D projective space $\mathbb{P}^3$. The quadric can be initialized with
the triangulated ellipsoid parameters as following:

\[ Q = H^{-T} Q_u H^{-1} \]  \hspace{1cm} (3)

where \( Q_u = \text{diag}\{1, 1, 1, -1\} \) denotes the unit sphere. \( H \) is the point transformation matrix which is composed by the orientation, translation, and the lengths of the ellipsoid axes:

\[ H = \begin{bmatrix} D_e & t_e \\ 0^T & 1 \end{bmatrix} \]  \hspace{1cm} (4)

where \( t_e \) is the translation of the ellipsoid center, and \( D_e \) is:

\[ D_e = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \cdot R_e \]  \hspace{1cm} (5)

with \( R_e \) being the orientation of the ellipsoid. Since we have calibrated the stereo camera system, we have obtained the camera intrinsics, as well as the camera poses, which are \( K_1, K_2, T_1, T_2 \). Therefore, we can construct the projection matrices of the stereo camera as \( P_1 = K_1 T_1 \) and \( P_2 = K_2 T_2 \). Now, given the projection matrix \( P \) of a camera, the quadric can be projected onto the image as:

\[ C^* = P Q^* P^T \]  \hspace{1cm} (6)

where \( C^*, Q^* \) is the dual conic of \( C \), and dual quadric of \( Q \) respectively, and they can be obtained through:

\[ Q^* = Q^{-1} \hspace{1cm} C^* = C^{-1} \]  \hspace{1cm} (7)

Therefore, we can project the triangulated ellipsoid to the left image and the right image and obtain their 2D conics \( C_1 \) and \( C_2 \). To optimize the quadric, we minimize the difference between the projected conics and the detected conics. Since we have detected the bubble and fit an ellipse around the contour in Sect. 3.2.3, we sample points on the ellipse uniformly and define the cost function as the Mahalanobis distance of the sampled points \( x \) (lens distortion was removed.
in the background removal step) to the conic:

$$x^T C x$$

Therefore, the optimal quadric $Q$ can be solved via minimizing the following:

$$E = \sum_i \|x_{i,1}^T C_1 x_{i,1}\|^2 + \|x_{i,2}^T C_2 x_{i,2}\|^2$$

where $x_{i,1}$ and $x_{i,2}$ are the sampled points on the detected ellipse in the left image and the right image. After optimizing the quadric, we retrieve the ellipsoid parameters from the quadric representation.

### 3.2.7. Bubble Tracking

![Diagram of bubble tracking](image)

Figure 13: The principal of the data association step in the bubble tracking. The blue ellipses denote the bubbles in the current frame, the brown ellipses represent the identified bubbles in the next frame. The IoU is computed between the predicted and detected bounding box.

To measure the rise speed of each bubble, and also to avoid multiple countings, the bubbles need to be tracked over the image sequence. We utilize the Tracking-by-Detection [35] framework to address this issue where the interesting target is identified in each frame and associated with its previous trajectory. Therefore, the tracking problem is essentially converted to a matching problem which is similar to Sect. 3.2.4. The bubbles always rise upwards with a
small oscillation in the sideward direction, which can be used as a constraint to reduce the search space in the data association step. In this contribution, we employ the SORT (Simple Online and Realtime Tracking) tracker [36]. The bubble position in the next frame is first predicted by a Kalman Filter that holds each bubble’s parameters, then the IoU (Intersection of Union) is utilized as the weight for constructing the bipartite graph.

To assign the newly identified bubbles to their previous trajectories, we treat the bubbles in the current frame as the source group and the new bubbles in the next frame as the target group, and construct a bipartite matching graph like in Sect. 3.2.4. To impose the upwards and the sideward motion constraints, we discard the edges where the new bubbles are below the old bubbles, and also discard the edges where the sideward motion exceeds a threshold. Then, for each bubble in the current frame, its expected position and bounding box are predicted by the Kalman Filter, and we compute the intersection area of union (IoU) between the predicted bounding box and the new bubble, and use it as the weight of the edge in the graph. As illustrated in Fig. 13, the IoU expresses the similarity between the predicted bubble and the detected bubble. Finally, the best matches from the next frame to the current frame can be found via the Hungarian algorithm. For those bubbles which have no assignment to the previous trajectories, new trajectories are initialized.

3.2.8. Counting at Reference Surface

To avoid multiple counting of the bubbles, and to treat fast and slow rising bubbles in a consistent manner, we only count a valid bubble and calculate its characteristics when its trajectory passes a virtual horizontal plane, the counting reference surface. This surface is defined by selecting a certain image row in the first camera. Consequently, we disregard bubble trajectories that start above the counting reference surface or bubbles that dissolve before reaching the reference surface.
4. Evaluation

4.1. Recalibration by Bubble Adjustment

Figure 14: Top: Before recalibration, the projections of the square photogrammetric markers are off from their true positions. Bottom: After recalibration, the projections match the actual image.

One lesson learned from the actual deep sea experiment is that the relative orientation and position of the stereo camera system could vary due to significant environmental changes. Under extremely high water pressure and very low temperature, the metal frames and the plastic panels where the cameras and the mirrors are mounted can behave different from the water pool in room temperature, where we perform the stereo camera calibration step. Therefore the results are not directly applicable to the deep sea environment, which can be proved by comparing the stereo images acquired in the water pool and the in-situ data. As shown in Fig. 14 top, the black square photogrammetric markers are attached rigidly to the acrylic panels and they are both visible by the stereo cameras. Since we know the ground truth positions of the markers, we can project the outer corners of the markers onto the images using the stereo
calibration results obtained in Sect. 3.1.4 and it is clear that the projections are off from the in-situ images (see the red arrows). Similarly, the stereo epipolar matching results have shown that the epipolar lines of the bubble contour masses are tangent to the corresponding bubble outlines (see Fig. 15a). Consequently, a recalibration is necessary, however, performing in-situ calibration is infeasible. Therefore, we propose a self-calibration approach to refine the relative orientation and translation of the stereo cameras utilizing directly the
The self-calibration is essentially an extension to Sect. 3.2.6 where we use bundle adjustment with ellipse constraints to optimize the quadric representation of the bubble. Now, leaving the cost function untouched, we add multiple bubbles (quadrics) into the equation system (see below) to jointly optimize all quadrics and the relative orientation and position of the stereo camera.

$$E = \sum_k \sum_i \| x_{i,1}^T C_{1i} x_{i,1} \|^2 + \| x_{i,2}^T C_{2i} x_{i,2} \|^2$$  \hspace{1cm} (10)

now, $C_{1i}^k$ and $C_{2i}^k$ are the 2D conics projected from $k^{th}$ bubble (quadric) in the left and right image. By accumulating multiple bubbles from multiple frames, we can re-calibrate the stereo camera system and resolve the environmental change issue (note that we don’t need to track those bubbles in this scenario).

As can be seen from Fig. 14 bottom and Fig. 15b that the projections of the photogrammetric markers are in the correct positions, and the epipolar lines of the bubble contour masses now intersect with their corresponding bubble outlines.

4.2. Dark Frames

We have verified that the dark frame detection works robustly and reliably. Using a frame rate of 80Hz we do not observe frame drops, and we can see in the image counters that a dark frame appears every 5000 images. At 100Hz frame drops appear rarely. In Fig. 16 we exemplify the dark frame detection and the timing during a short experiment of taking about 40000 images in both cameras at 80Hz. Before the experiment, we deliberately did not synchronize the clocks of the two computers and they differ about 2.7 seconds. We subtract both the frame counters and the timestamps recorded independently by both computers when a black frame is detected. It can be seen that the counter difference is constant, whereas the time is almost constant, but drifting slightly. For this short experiment that takes 437.5 seconds, we observe a clock drift between the two computers by 1.3ms, which means 0.27 seconds per day. Since the frame counter offset stays stable at the detected dark frames we can still associate
Figure 16: Frame offset measured by black frames (top / blue curve) and clock difference of the computers in seconds (bottom / red curve). We observe that the black frames are detected reliably and no frames are lost. However, the recorded time stamp difference increases slightly due to clock drift.

the correct images and they were triggered at the same time. However, one of the computer’s clocks runs 0.0005% faster than the trigger signal, the other one 0.0002%. For computing the rise speeds and flow rates, this consistency is by far good enough.

4.3. Background Removal

Fig. 17 shows two sample results of the background removal from two different data sets. As can be seen that the temporal median filtering algorithm works perfectly when the water body is clean and clear (see Fig. 17a), and it is also evidently that the algorithm is robust enough to cope with images like in Fig. 17b where sediments stuck on the camera housings and block part of the cameras’ view.

4.4. Known Reference Objects

To evaluate the accuracy of the proposed ellipsoid triangulation technique and later bundle adjustment with ellipse constraints, we first conduct an experi-
Figure 17: Two sample results of the background removal. In each sub-figure, the top row shows the original stereo images and the bottom row shows the resulting foreground images.
Table 1: Evaluation results of the glass marbles.

| Marble | Proposed Method [mm] | Vernier Caliper [mm] | Err. [%] |
|--------|----------------------|----------------------|----------|
| R17    | 34.33 ± 0.09         | 34.72 ± 0.10         | 1.12     |
| R13    | 25.00 ± 0.20         | 25.10 ± 0.01         | 0.39     |
| R8     | 16.01 ± 0.16         | 15.91 ± 0.25         | 0.63     |
| R6     | 12.68 ± 0.16         | 12.41 ± 0.13         | 2.2      |

Figure 18: Evaluation on the known reference objects (4 glass marbles with different radiiusses).

The evaluation using the instrument to photograph glass marbles of approximate radiiusses 6mm, 8mm, 13mm and 17mm, falling through the corridor of the instrument in water (see Fig. 18). The diameter of each marble were measured by a vernier caliper. Since the marbles are also not perfectly spherical, to obtain an accurate reference, we measure each glass marble 10 times while rotating it and use the average diameter as the reference. Next, we drop each glass marble through the corridor of the instrument 10 times to obtain over 100 samples. Then, we perform bubble detection, epipolar geometry matching, ellipsoid triangulation and bundle adjustment on images where we can see and identify the glass marbles, and the evaluation results of the equivalent diameter are shown in Tab. 1.

It is clearly shown that the estimation accuracy of a known reference object is in the range of 1 − 2% in the equivalent diameter under ideal conditions. Some sample intermediate results can be found in Fig. 19, where the epipolar

\[ \text{The equivalent diameter of a non-spherical particle is equal to a diameter of a spherical particle that exhibits identical properties to that of the investigated non-spherical particle.} \]
Frame 19: Sample intermediate results of the known reference objects evaluation (The stereo images are concatenated into one image). From top to bottom: selected one sample for each glass marble. From left to right: stereo epipolar geometry matching and the re-projected ellipsoid. In the left part of each sub-figure, the glass marbles are identified and marked as red outlines and a blue bounding box; The lilac lines connect the bubble correspondences and the yellow lines are the epipolar lines of the contour mass in the other image (Note that the contour masses of the corresponding bubbles are not corresponding points). In the right part of each sub-figure, the final reconstructed 3D ellipsoid is projected onto the image, and its $X$, $Y$, $Z$-axis are shown in red, green, blue lines respectively.

d (a) Glass marble R17 (radius $\approx 17$mm)  
(b) Glass marble R13 (radius $\approx 13$mm)  
(c) Glass marble R8 (radius $\approx 8$mm)  
(d) Glass marble R6 (radius $\approx 6$mm)

geometry matching results are shown in the left part of the sub-figures, while the re-projected ellipsoids are shown in the right. Here, to re-project an ellipsoid,
we first project its center point onto the image, and project the 6 end points of the ellipsoid axes and then connect them.

4.5. Controlled Experiments

Next, to demonstrate the effectiveness of the complete workflow, we first conduct a controlled experiment. We set up the instrument in a water pool with an air-bubble generator attached underneath the instrument. The generator is able to produce air bubbles at different flow rates with different sizes to control the density of the bubble stream. A cylinder is used to collect the released gas bubbles on top of the instrument and to measure the overall volume. The instrument records photos of the rising bubbles as if it was in the ocean, and the image sequences are analyzed by the bubble stream characterization approach which is proposed in Sect. 3.2. Then, we compare the estimated overall volume with the cylinder measurement. To evaluate the accuracy and robustness of the bubble stream characterization approach, we start with low bubble density and gradually increase the gas flow of the bubble generator. An overview of the sequences can be seen in Fig. 20 and the experiment results are shown in Tab. 2.

As can be seen from Tab. 2, when there is only one single bubble plume, the overall volume estimation is accurate as it shows a relative volume error of 4.2% (sequence 1) and 4.0% (sequence 2). Since the volume increases with the third power of the radius, this can be interpreted as an equivalent radius error
of slightly more than 1%, which is close to the glass marble experiment results and also shows that the ellipsoid model assumption works well in this bubble size range. The sample intermediate results are shown in Fig. 21a and it can be seen that the ellipsoid re-projections well fit the identified bounding boxes of the bubbles. But we can see that the accuracy decreases with increasing flow rate due to bubble contours overlapping in the image, especially in sequence 4 where the bubble stream density is too high. One explanation is that the overlapping contours in the image are merged into a bigger contour which encloses the bubble cluster such that the ellipsoid size is overestimated. In addition, it also introduces ambiguity both in stereo epipolar matching and bubble tracking (non-equal number of bubbles identified, see Fig. 21b), which also adds error to the final volume estimation. Nevertheless, for a moderate amount of overlapping bubbles in the image (like in sequence 3), we still obtain a result of an overall volume error of 10% (or, equivalent radius error of around 3%). Note that also the physical collection of the gas comes with some uncertainty. In particular for the more turbulent high flow rate sequences 3 and 4 a few bubbles already stick to the cylinder entrance and do not get collected, and also the cylinder measurement has some uncertainty (surface tension, different water pressure leading to different gas expansions). We therefore consider an overall volume

Figure 20: An overview of the sequences for the controlled experiments.
Figure 21: Sample results of stereo epipolar geometry matching and ellipsoid re-projection from the controlled experiments.
Table 3: Sample results of bubble stream characterization on data from the Pacific Ocean.

| date, time    | count | volume [ml] | flow rate [ml/s] | diameter [mm] | velocity [cm/s] |
|--------------|-------|-------------|------------------|---------------|----------------|
| 06/19, 16:50 | 370   | 40.33       | 0.646            | 5.71 ± 0.51   | 26.21          |
| 06/19, 17:05 | 349   | 36.78       | 0.591            | 5.70 ± 0.50   | 26.33          |
| 06/20, 09:09 | 453   | 37.31       | 0.591            | 5.20 ± 0.62   | 28.22          |
| 06/20, 09:24 | 462   | 33.93       | 0.565            | 5.00 ± 0.60   | 28.65          |
| 06/20, 19:11 | 544   | 41.97       | 0.699            | 5.03 ± 0.56   | 27.79          |
| 06/20, 19:26 | 520   | 41.13       | 0.685            | 5.08 ± 0.54   | 27.86          |

4.6. Data analysis from the Pacific Ocean

The instrument has been deployed in the Pacific Ocean during Falkor Cruise 'Observing Seafloor Methane Seeps at the Edge of Hydrate Stability' (FK190612), jointly with several other instruments to analyze bubbles and hydrate. A detailed analysis of the observations of the scientific cruise is out of the scope of this paper and will be discussed in [37]. Instead, we only exemplify some results here. The instrument was deployed at the seep sites by an ROV (also see Fig. 1). We selected 6 sequences from a few interesting time points and report the evaluation results in Tab. 3.

Bubble stream characterization

Figure 22: Bubble stream characterization results for sequence 06/19, 16:50.
Figure 23: Sample results of stereo epipolar geometry matching and ellipsoid re-projection for sequence 06/19, 16:50.
particularly, we show the bubble stream characterization results and some sample intermediate results for the first sequence in Fig. 22 and Fig. 23. It is clear that the algorithm is able to find correct bubble correspondences, and the re-projected ellipsoids are well located inside the bounding boxes of the identified bubbles. Since it is very difficult to obtain a ground truth reference for the estimated bubble streams, we also positioned a scale-bar board in-situ right next to the bubble streams as a second examination, although we understand that this is only a weak validation, it cannot provide an accurate reference. Still, by calculating roughly the bubble radiusses and the rising velocity, we have verified that our estimation is in a reasonable range.

Figure 24: A scale-bar board was used to roughly examine the estimated bubble sizes and rising velocity. Picture taken by ROV SuBastian, Schmidt Ocean Institute.

5. Discussion

both the real data as well as the ground truth experiments indicate that the overall approach works well in the deep sea and the measurement process is robust even in presence of dirt and other nuisances. The achieved accuracy under good conditions is much better than can be expected from monocular
methods. However, there are some limitations and failure cases in practical applications:

**Both Cameras Must See the Bubbles**

For stereo evaluation, both cameras have to see every bubble. In case the view of a single camera is blocked by an animal, or the water is too turbid, the stereo matching procedure fails (see Fig. 25a). Also, in case the box is not deployed in an upright orientation, e.g. at a slope or in case it sinks into the sediment, bubbles will not rise exactly through the corridor and might get out of sight for one of the cameras, as shown in Fig. 25b. In this case single view approximations (shape and position assumptions) have to be used. In future versions we will use a wider image, and not crop it strictly to the 8cm corridor that works well only in lab conditions. This should increase robustness further.

**Density of Bubble Stream**

The entire system is designed for a stream of bubbles where bubbles rise one by one. In case multiple or many bubbles are seen at the same height, they tend to occlude each other and it is difficult to disambiguate the bubbles. In this case small bubbles tend to be overlooked, which might bias the bubble size distribution, whereas the overall flow will probably only be slightly affected.

The opposite problem occurs in case the water contains many small dirt particles (e.g. upstirred sediment). From a geometrical point of view they are difficult to distinguish from small bubbles, so a minimum bubble size threshold was used. This is a general specificity versus sensitivity problem. Potentially, a solution could by to learn motion patterns and appearances, but this might not generalize to previously unseen ocean locations.

**Technical Limitations**

A current limitation of the instrument is that it is still using too much power in standby-mode since only the flashes are disabled during standby, but the computers are still running. This limits maximum runtime. Similarly, recording full images rather than saving only the bubbles quickly fills the storage space. We
Figure 25: Failure cases. (a) Left, a sea urchin is sitting on the dome port. Right, a large piece of sediment is blocking the camera’s view. (b) The bubble stream does not rise upwards, due to the instrument standing on a slope or is sunken into the sediment. When the bubbles get out of sight for the left camera stereo quantification does not work any more, and one has to use single view approximations.
have tried an experimental real-time encoding using live background subtraction, but processing 0.8 gigapixel per second reliably is at the performance limit of the current hardware and capturing data at sea is so expensive that we decided not to risk deleting the raw data.

6. Conclusion

We presented and discussed a robust photogrammetric bubble stream characterization system for deep ocean deployment. The overall system has been deployed in up to 1000m water depth and was used to quantify methane fluxes offshore Oregon. In a test tank we have verified the accuracy of the bubble radius estimation to be correct up to very few percent using hand-measured glass marbles and an air bubble test with the gas additionally collected by a funnel. Besides the robust steps we have also presented a new calibration procedure that does not rely on point correspondences, but works with silhouettes in a wide baseline setting.

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