Kink dynamics in a system of two coupled scalar fields in two space-time dimensions

A. Alonso

A brief history of two-component kinks

A system of two coupled scalar fields in two space-time dimensions

Kink variety in a system of two coupled scalar fields in two space-time dimensions

Kink dynamics in a system of two coupled scalar fields in two space-time dimensions

Compilation Album of The Kinks

Alberto Alonso-Izquierdo

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6th International Workshop on New Challenges in Quantum Mechanics: Integrability and Supersymmetry
A BRIEF HISTORY OF TWO-COMPONENT KINKS

where the concept kink is defined and allows us to introduce
A BRIEF HISTORY OF TWO-COMPONENT KINKS

1. A BRIEF HISTORY OF TWO-COMPONENT KINKS

   where the concept kink is defined and allows us to introduce

2. A SYSTEM OF TWO COUPLED SCALAR FIELDS IN TWO SPACE-TIME DIMENSIONS
A BRIEF HISTORY OF TWO-COMPONENT KINKS

1. A BRIEF HISTORY OF TWO-COMPONENT KINKS
where the concept kink is defined and allows us to introduce

2. A SYSTEM OF TWO COUPLED SCALAR FIELDS IN TWO SPACE-TIME DIMENSIONS
where we describe

3. STATIC KINK VARIETY
which comprises

3.1 Vacuum solutions
3.2 Static kink solutions
A brief history of two-component kinks

A system of two coupled scalar fields in two space-time dimensions

Kink dynamics in a system of two coupled scalar fields in two space-time dimensions

A brief history of two-component kinks

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A system of two coupled scalar fields in two space-time dimensions

Kink variety in a system of two coupled scalar fields in two space-time dimensions

Kink dynamics in a system of two coupled scalar fields in two space-time dimensions
**Definition.** A 1-component Kink is a non-singular solution of the nonlinear Klein-Gordon equation

\[
\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = -\frac{\partial U}{\partial \phi}
\]

which can be interpreted as an extended particle.

- The previous equations are the Euler-Lagrange equations of a (1+1) relativistic scalar field theory model with action functional

\[
S = \int d^2 x \left\{ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - U(\phi) \right\}
\]

- The total energy is defined as

\[
E = \int dx \left\{ \frac{1}{2} \frac{\partial \phi}{\partial t} \frac{\partial \phi}{\partial t} + \frac{1}{2} \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} + U(\phi) \right\}
\]

The energy density is localized around one point.

- The configuration space \( C = \{ \phi (x, t) \in \mathbb{R} : E[\phi (x, t)] < +\infty \} \)

- Examples:
  - \( \phi^4 \)-Model: \( \phi_{tt} - \phi_{xx} = 2\phi(1 - \phi^2) \)
  - sine-Gordon Model: \( \phi_{tt} - \phi_{xx} = -\sin \phi \)
A BRIEF HISTORY OF TWO-COMPONENT KINKS

1 Structural phase transitions in ferroelectric and ferromagnetic materials.
   A.H. Eschenfelder, Magnetic Bubble Technology, (1981) Berlin, Springer-Verlag.
   F. Jona and G. Shirane, Ferroelectric Crystals, (1993) New York, Dover
   E.K. Salje, Phase Transitions in Ferroelastic and Co-Elastic Crystals, Cambridge, UK, Cambridge University Press.
   B.A. Strukov and A. Levanyuk, Ferroelectric Phenomena in Crystals, Berlin, Springer-Verlag.

2 Topological excitations in quasi-one-dimensional systems like biological macromolecules and hydrogen bonded chains, or polymers, etc.
   J.M. Harris, Poly(ethylene glycol) chemistry: Biotechnical and Biomedical Applications, (1992) New York, Plenum.
   A. S. Davydov, Solitons in molecular systems, (1985) Dordrech, D. Reidel.

3 Electric charge fractionization phenomena in polyacetilen
   Jackiw R and Schrieffer R 1981 Nucl. Phys. B 190 253

4 Nonlinear excitations in Bose-Einstein condensates
   J. Belmonte-Beitia, V. M. Perez-Garcia, V. Vekslerchik, and V. V. Konotop, Physical Review Letters, 100, (2008) 16, 164102.
   A. T. Avelar, D. Bazeia, and W. B. Cardoso, Phys. Rev. E79, 2 (2009) 025602R.

5 Hydrodynamics and solitary waves
   Scott A, Chu F and McLaughlin D 1973 Proc. IEEE 61 1443

6 Cosmology: Study of the Early Universe
   A. Vilenkin and E.P.S. Shellard, Cosmic strings and other topological defects, Cambridge University Press, 1994
A brief history of two-component kinks

Definition. A $N$-component Kink is a non-singular solution of the nonlinear Klein-Gordon equations

$$\frac{\partial^2 \phi^a}{\partial t^2} - \frac{\partial^2 \phi^a}{\partial x^2} = - \frac{\partial U}{\partial \phi^a} \quad a = 1, \ldots, N$$

which can be interpreted as an extended particle.

- The previous equations are the Euler-Lagrange equations of a (1+1) relativistic scalar field theory model with action functional

$$S = \int d^2 x \left\{ \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - U(\phi_1, \ldots, \phi_N) \right\}$$

- The total energy is defined as

$$E = \int dx \left\{ \frac{1}{2} \frac{\partial \phi^a}{\partial t} \frac{\partial \phi^a}{\partial t} + \frac{1}{2} \frac{\partial \phi^a}{\partial x} \frac{\partial \phi^a}{\partial x} + U(\phi_1, \ldots, \phi_N) \right\}$$

The energy density is localized around one point.

- The configuration space $C = \{ \phi^a(x, t) \in \mathbb{R} : E[\phi^a(x, t)] < +\infty \}$

- Examples:

???
A BRIEF HISTORY OF TWO-COMPONENT KINKS

Two coupled scalar field theory system:

\[
\begin{align*}
\phi_{tt} - \phi_{xx} &= -U_\phi(\phi, \psi) \\
\psi_{tt} - \psi_{xx} &= -U_\psi(\phi, \psi)
\end{align*}
\]

Rajaraman: Solitons and Instantons

Let us move on to the next level of complexity, i.e. static solutions to systems of coupled scalar fields in two space-time dimensions. This already brings us to the stage where no general methods are available for obtaining all localised static solutions, given the field equations. However, some solutions, but by no means all, can be obtained for a class of such Lagrangians using a little trial and error (Rajaraman 1979).
A brief history of two-component kinks

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Strategy: Choose a non-negative potential term \( U(\phi, \psi) \), which gives rise to a completely integrable dynamical system in a \((\phi, \psi)\)-space.
A brief history of two-component kinks

Two coupled scalar field theory system:

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\phi_{tt} - \phi_{xx} = -U_\phi(\phi, \psi) \\
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UIC MathPhys-CYL (section Usal)

Strategy: Choose a non-negative potential term \( U(\phi, \psi) \), which gives rise to

A HAMILTON-JACOBI SEPARABLE DYNAMICAL SYSTEM

in a \((\phi, \psi)\)-space.
A BRIEF HISTORY OF TWO-COMPONENT KINKS

References:

1. A. Alonso-Izquierdo, M.A. González León and J. Mateos Guilarte, Kink manifolds in (1+1)-dimensional scalar field theory, Journal of Physics A 31 (1998), 209-229.

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4. A. Alonso Izquierdo, M.A. González León and J. Mateos Guilarte, Stability of Kink Defects in a Deformed $O(3)$ Linear Sigma Model, Nonlinearity 15 (2002), 1097-1125.

5. A. Alonso Izquierdo, M.A. González León y M. de la Torre Mayado, Adiabatic motion of two-component BPS kinks, Physical Review D 66 (2002) 105022, 1-9.

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A SYSTEM OF TWO COUPLED SCALAR FIELDS

- **Action functional:**
  \[ S = \int d^2x \left\{ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \partial_\mu \psi \partial^\mu \psi - U(\phi, \psi) \right\} \]

- **Potential term:**
  \[ U(\phi, \psi) = (4\phi^2 + \psi^2 - 1)^2 + 4\phi^2\psi^2 \]

- **Klein-Gordon equations:**
  \[ \frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = - \frac{\partial U}{\partial \phi} = -16\phi \left[ 4\phi^2 + \frac{3}{2} \psi^2 - 1 \right] \]
  \[ \frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial x^2} = - \frac{\partial U}{\partial \psi} = -4\psi \left[ 6\phi^2 + \psi^2 - 1 \right] \]

- **System invariants:**
  1. **Total Energy:**
     \[ E[\phi, \psi] = \int dx \left[ \frac{1}{2} \left( \frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial \psi}{\partial t} \right)^2 + \frac{1}{2} \left( \frac{\partial \psi}{\partial x} \right)^2 + U(\phi, \psi) \right] \]
  2. **Total Momentum:**
     \[ P[\phi, \psi] = \int dx \left[ \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial t} + \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial t} \right] \]
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VACUUM SOLUTIONS

Solutions of the model:

\[ \phi_{tt} - \phi_{xx} = -16\phi\left[4\phi^2 + \frac{3}{2}\psi^2 - 1\right] , \quad \psi_{tt} - \psi_{xx} = -4\psi\left[6\phi^2 + \psi^2 - 1\right] \]

in the configuration space: \( C = \{ \phi(x, t) \in \mathbb{R} : E[\phi(x, t)] < +\infty \} \)
Vacuum solutions

Solutions of the model:

\[ 0 = -16\phi [4\phi^2 + \frac{3}{2}\psi^2 - 1] , \quad 0 = -4\psi [6\phi^2 + \psi^2 - 1] \]

in the configuration space: \( C = \{ \phi(x, t) \in \mathbb{R} : E[\phi(x, t)] < +\infty \} \)
Vacuum solutions

Solutions (static and homogeneous) of the model:

\[ 0 = -16\phi [4\phi^2 + \frac{3}{2}\psi^2 - 1], \quad 0 = -4\psi [6\phi^2 + \psi^2 - 1] \]

in the configuration space: \( C = \{ \phi(x, t) \in \mathbb{R} : E[\phi(x, t)] < +\infty \} \).

• Zero Energy Solutions:

\[ \mathcal{M} = \{ (\phi_0, \psi_0) \in \mathbb{R}^2 : U(\phi_0, \psi_0) = 0 \} = \{ A_1 = \left( \frac{1}{2}, 0 \right), A_2 = \left( -\frac{1}{2}, 0 \right), B_1 = (1, 0), B_2 = (-1, 0) \} \]
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Vacuum solutions

Kink statics:
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\[ \phi_{tt} - \phi_{xx} = -16\phi\left[4\phi^2 + \frac{3}{2}\psi^2 - 1\right] , \quad \psi_{tt} - \psi_{xx} = -4\psi\left[6\phi^2 + \psi^2 - 1\right] \]

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**Static Kinks**

Solutions of the model:

$$0 - \phi_{xx} = -16\phi[4\phi^2 + \frac{3}{2}\psi^2 - 1], \quad 0 - \psi_{xx} = -4\psi[6\phi^2 + \psi^2 - 1]$$

in the configuration space: $$C = \{\phi(x, t) \in \mathbb{R} : E[\phi(x, t)] < +\infty\}$$
Static kinks of the model:

\[ 0 - \phi_{xx} = -16\phi \left[ 4\phi^2 + \frac{3}{2}\psi^2 - 1 \right] , \quad 0 - \psi_{xx} = -4\psi \left[ 6\phi^2 + \psi^2 - 1 \right] \]

in the configuration space: \( C = \{ \phi(x, t) \in \mathbb{R} : E[\phi(x, t)] < +\infty \} \).

**Newton equations of an analogue mechanical system**
Static kinks of the model:

\[ 0 - \phi_{xx} = -16\phi[4\phi^2 + \frac{3}{2}\psi^2 - 1] \quad , \quad 0 - \psi_{xx} = -4\psi[6\phi^2 + \psi^2 - 1] \]

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**Newton equations of an analogue mechanical system**

\[ -U(\phi, \psi) = -(4\phi^2 + \psi^2 - 1)^2 - 4\phi^2\psi^2 \]

**Completely integrable mechanical system**
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**Static kinks**

Static kinks of the model:

\[ 0 - \phi_{xx} = -16\phi [4\phi^2 + \frac{3}{2}\psi^2 - 1] \quad , \quad 0 - \psi_{xx} = -4\psi [6\phi^2 + \psi^2 - 1] \]

in the configuration space: \( C = \{ \phi(x, t) \in \mathbb{R} : E[\phi(x, t)] < +\infty \} \).

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The Newton equations of an analogue mechanical system:

\[-U(\phi, \psi) = -(4\phi^2 + \psi^2 - 1)^2 - 4\phi^2\psi^2\]

---

The completely integrable mechanical system:

Hamilton-Jacobi separable system

Parabolic coordinates

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Static kinks with one energy density lump
Static kinks with two energy density lumps
Static kinks with four energy density lumps

Notation: \( \bar{x} = x - x_0 \)
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Kink dynamics in a system of two coupled scalar fields in two space-time dimensions

\[ K_{\text{static}}^{(q_1, q_2, \lambda)}(\bar{x}) = \left( \frac{q_1}{4} [\lambda + \tanh(\sqrt{2} \bar{x})], -\lambda q_2 \sqrt{\frac{1}{2} [1 - \lambda \tanh[\sqrt{2} \bar{x}]]} \right), \quad q_i, \lambda = \pm 1 \]
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**Static single kinks**

\[ K_{\text{static}}^{(q_1, q_2, \lambda)}(\bar{x}) = \left( \frac{q_1}{4} \left[ \lambda + \tanh(\sqrt{2} \bar{x}) \right], -\lambda q_2 \sqrt{\frac{1}{2} \left[ 1 - \lambda \tanh(\sqrt{2} \bar{x}) \right]} \right), \quad q_i, \lambda = \pm 1 \]
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A system of two coupled scalar fields in two space-time dimensions

Kink variety in a system of two coupled scalar fields in two space-time dimensions

Kink dynamics in a system of two coupled scalar fields in two space-time dimensions

**Static single kinks**

\[ K^{(q_1, q_2, \lambda)}_{\text{static}}(\bar{x}) = \left( \frac{q_1}{4} [\lambda + \tanh(\sqrt{2} \bar{x})], -\lambda q_2 \sqrt{\frac{1}{2} [1 - \lambda \tanh(\sqrt{2} \bar{x})]} \right), \quad q_i, \lambda = \pm 1 \]
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**Static single kinks**

\[ K_{\text{static}}^{(q_1, q_2, \lambda)}(x) = \left( \frac{q_1}{4} \left[ \lambda + \tanh(\sqrt{2}x) \right], -\lambda q_2 \sqrt{\frac{1}{2} \left[ 1 - \lambda \tanh(\sqrt{2}x) \right]} \right), \quad q_i, \lambda = \pm 1 \]
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\[ K_{\text{static}}^{(q_1, q_2, \lambda)}(\vec{x}) = \left( \frac{q_1}{4} [\lambda + \tanh(\sqrt{2} \vec{x})], -\lambda q_2 \sqrt{\frac{1}{2} \left[ 1 - \lambda \tanh(\sqrt{2} \vec{x}) \right] } \right), \quad q_i, \lambda = \pm 1 \]
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**STATIC SINGLE KINKS**

\[
K_{\text{static}}^{(q_1, q_2, \lambda)}(x) = \left( \frac{q_1}{4} \left[ \lambda + \tanh(\sqrt{2}x) \right], -\lambda q_2 \sqrt{\frac{1}{2} \left[ 1 - \lambda \tanh[\sqrt{2}x] \right]} \right), \quad q_i, \lambda = \pm 1
\]
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### Static Single Kinks

\[
K^{(q_1, q_2, \lambda)}_{\text{static}}(x) = \left( \frac{q_1}{4} \left[ \lambda + \tanh(\sqrt{2}x) \right], -\lambda q_2 \sqrt{\frac{1}{2} \left[ 1 - \lambda \tanh[\sqrt{2}x] \right]} \right), \quad q_i, \lambda = \pm 1
\]
**FUNDAMENTAL PARTICLES**

**LIST OF FUNDAMENTAL EXTENDED PARTICLES**

| Particles | First topological charge | Second topological charge |
|-----------|--------------------------|---------------------------|
| $K_{\text{static}}^{-1,1,-1}$ | -1 | 1 |
| $K_{\text{static}}^{1,1,-1}$ | 1 | 1 |
| $K_{\text{static}}^{1,-1,-1}$ | 1 | -1 |
| $K_{\text{static}}^{-1,-1,-1}$ | -1 | -1 |

| Antiparticles | First topological charge | Second topological charge |
|---------------|--------------------------|---------------------------|
| $K_{\text{static}}^{1,-1,1}$ | 1 | -1 |
| $K_{\text{static}}^{-1,-1,1}$ | -1 | -1 |
| $K_{\text{static}}^{-1,1,1}$ | -1 | 1 |
| $K_{\text{static}}^{1,1,1}$ | 1 | 1 |
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\[ \bar{K}_{\text{static}}^{(q_1,0)}(\bar{x}, b) = \left( \frac{q_1}{4} \frac{\sinh(2\sqrt{2} \bar{x})}{\cosh(2\sqrt{2} \bar{x}) + b^2}, \frac{b}{[b^2 + \cosh(2\sqrt{2} \bar{x})]^{\frac{1}{2}}} \right), \quad q_1 = \pm 2, \ b \in \mathbb{R} \]
STATIC TWO-LUMP COMPOSITE KINKS

\[ \overline{K}^{(q_1,0)}_{\text{static}}(x,b) = \left( \frac{q_1}{4} \frac{\sinh(2\sqrt{2}x)}{\cosh(2\sqrt{2}x) + b^2}, \frac{b}{[b^2 + \cosh(2\sqrt{2}x)]^{\frac{1}{2}}} \right), \quad q_1 = \pm 2, \ b \in \mathbb{R} \]
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**STATIC TWO-LUMP COMPOSITE KINKS**

\[
K_{\text{static}}^{(q_1,0)}(\bar{x}, b) = \left( \frac{q_1}{4} \frac{\sinh(2\sqrt{2}\bar{x})}{\cosh(2\sqrt{2}\bar{x}) + b^2}, \frac{b}{[b^2 + \cosh(2\sqrt{2}\bar{x})]^\frac{1}{2}} \right), \quad q_1 = \pm 2, \ b \in \mathbb{R}
\]
Static two-lump composite kinks

\[ \overline{K}_{\text{static}}^{(q_1,0)}(\vec{x}, b) = \left( \frac{q_1}{4} \frac{\sinh(2\sqrt{2} \vec{x})}{\cosh(2\sqrt{2} \vec{x}) + b^2}, \frac{b}{[b^2 + \cosh(2\sqrt{2} \vec{x})]^{\frac{1}{2}}} \right), \quad q_1 = \pm 2, \quad b \in \mathbb{R} \]
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**STATIC TWO-LUMP COMPOSITE KINKS**

\[
\overline{K}^{(q_1,0)}_{\text{static}}(\bar{x}, b) = \left( \frac{q_1}{4} \frac{\sinh(2\sqrt{2} \bar{x})}{\cosh(2\sqrt{2} \bar{x}) + b^2}, \frac{b}{[b^2 + \cosh(2\sqrt{2} \bar{x})]^\frac{1}{2}} \right), \quad q_1 = \pm 2, \ b \in \mathbb{R}
\]
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\[ \overline{K}_{\text{static}}^{(0,q_2)}(\bar{x}, c) = \left( \frac{\sinh 2\sqrt{c} \sinh 2\sqrt{2\bar{x}}}{\cosh^2 2\sqrt{2\bar{x}} + 2 \cosh 2\sqrt{2c} \cosh 2\sqrt{2\bar{x}} + 1}, \begin{array}{c} q_2 \frac{\sinh 2\sqrt{2\bar{x}}}{[\cosh^2 2\sqrt{2\bar{x}} + 2 \cosh 2\sqrt{2c} \cosh 2\sqrt{2\bar{x}} + 1]^{\frac{1}{2}}} \\ q_2 = \pm 2, \ c \in \mathbb{R} \end{array} \right) \]
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**Static Four-Lump Composite Kinks**

\[
\mathcal{K}_{\text{static}}^{(0,q_2)}(\bar{x}, c) = \left( \frac{\sinh 2\sqrt{2}c \sinh 2\sqrt{2\bar{x}}}{\cosh^2 2\sqrt{2\bar{x}} + 2 \cosh 2\sqrt{2}c \cosh 2\sqrt{2\bar{x}} + 1}, q_2 \left( \frac{\sinh 2\sqrt{2\bar{x}}}{\cosh^2 2\sqrt{2\bar{x}} + 2 \cosh 2\sqrt{2}c \cosh 2\sqrt{2\bar{x}} + 1} \right)^{\frac{1}{2}} \right)
\]

\( q_2 = \pm 2, \quad c \in \mathbb{R} \)
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**STATIC FOUR-LUMP COMPOSITE KINKS**

\[
\overline{K}^{(0, q_2)}_{\text{static}} (\vec{x}, c) = \left( \frac{\sinh 2\sqrt{2}c \cdot \sinh 2\sqrt{2}\vec{x}}{\cosh^2 2\sqrt{2}\vec{x} + 2 \cosh 2\sqrt{2}c \cosh 2\sqrt{2}\vec{x} + 1}, \frac{q_2}{2} \frac{\sinh 2\sqrt{2}\vec{x}}{[\cosh^2 2\sqrt{2}\vec{x} + 2 \cosh 2\sqrt{2}c \cosh 2\sqrt{2}\vec{x} + 1]^{\frac{1}{2}}} \right)
\]

\[q_2 = \pm 2, \ c \in \mathbb{R}\]
**STATIC FOUR-LUMP COMPOSITE KINKS**

\[
\overline{K}_{\text{static}}^{(0,q_2)}(\bar{x}, c) = \left( \frac{\sinh 2\sqrt{2}c \sinh 2\sqrt{2\bar{x}}}{\cosh^2 2\sqrt{2\bar{x}} + 2 \cosh 2\sqrt{2\bar{x}} \cosh 2\sqrt{2\bar{x}} + 1}, \frac{q_2}{2} \frac{\sinh 2\sqrt{2\bar{x}}}{[\cosh^2 2\sqrt{2\bar{x}} + 2 \cosh 2\sqrt{2\bar{x}} \cosh 2\sqrt{2\bar{x}} + 1]^{\frac{1}{2}}} \right),
\]

\[q_2 = \pm 2, \quad c \in \mathbb{R}\]
**Static Four-Lump Composite Kinks**

\[
K_{\text{static}}^{(0,q_2)}(\bar{x}, c) = \left( \frac{\sinh 2\sqrt{2}c \sinh 2\sqrt{2}\bar{x}}{\cosh^2 2\sqrt{2}\bar{x} + 2 \cosh 2\sqrt{2}c \cosh 2\sqrt{2}\bar{x} + 1}, \frac{q_2}{2} \right) \frac{\sinh 2\sqrt{2}\bar{x}}{[\cosh^2 2\sqrt{2}\bar{x} + 2 \cosh 2\sqrt{2}c \cosh 2\sqrt{2}\bar{x} + 1]^{1/2}}
\]

\[q_2 = \pm 2, \quad c \in \mathbb{R}\]
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**STATIC FOUR-LUMP COMPOSITE KINKS**

\[
\overline{K}_{\text{static}}^{(0,q_2)}(\bar{x}, c) = \left( \frac{\sinh 2\sqrt{2}c \sinh 2\sqrt{2}\bar{x}}{\cosh^2 2\sqrt{2}\bar{x} + 2 \cosh 2\sqrt{2}c \cosh 2\sqrt{2}\bar{x} + 1}, \frac{q_2}{2} \frac{\sinh 2\sqrt{2}\bar{x}}{[\cosh^2 2\sqrt{2}\bar{x} + 2 \cosh 2\sqrt{2}c \cosh 2\sqrt{2}\bar{x} + 1]^\frac{1}{2}} \right)
\]

\[q_2 = \pm 2, \; c \in \mathbb{R}\]
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Kink dynamics:

[Image of a camera]
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Kink dynamics

Solutions of the model:

\[
\phi_{tt} - \phi_{xx} = -16\phi[4\phi^2 + \frac{3}{2}\psi^2 - 1], \quad \psi_{tt} - \psi_{xx} = -4\psi[6\phi^2 + \psi^2 - 1]
\]

in the configuration space: \( C = \{ \phi(x, t) \in \mathbb{R} : E[\phi(x, t)] < +\infty \} \)
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**Kink dynamics**

Solutions of the model:

\[ \phi_{tt} - \phi_{xx} = -16\phi[4\phi^2 + \frac{3}{2} \psi^2 - 1] \quad \psi_{tt} - \psi_{xx} = -4\psi[6\phi^2 + \psi^2 - 1] \]

in the configuration space: \[ C = \{ \phi(x, t) \in \mathbb{R} : E[\phi(x, t)] < +\infty \} \]

Numerical method: \[ \phi_j^n = \phi(x_m + j\delta, n\tau) \quad \text{and} \quad \psi_j^n = \psi(x_m + j\delta, n\tau), \quad j = 0, \ldots, J \]

Energy conservative Second order finite difference scheme

\[ \frac{\phi_{j+1}^n - 2\phi_j^n + \phi_{j-1}^n}{\tau^2} - \frac{\phi_{j+1}^{n+1} - 2\phi_j^{n+1} + \phi_{j-1}^{n+1}}{\tau^2} + \frac{U[\phi_{j+1}^{n+1}, \psi_j^n] - U[\phi_{j+1}^n, \psi_j^n]}{\phi_{j+1}^{n+1} - \phi_j^n} = 0 \]

\[ \frac{\psi_{j+1}^n - 2\psi_j^n + \psi_{j-1}^n}{\tau^2} - \frac{\psi_{j+1}^{n+1} - 2\psi_j^{n+1} + \psi_{j-1}^{n+1}}{\tau^2} + \frac{U[\phi_j^n, \psi_{j+1}^{n+1}] - U[\phi_j^n, \psi_{j-1}^{n+1}]}{\psi_{j+1}^{n+1} - \psi_j^{n+1}} = 0 \]

Mur absorbing contour conditions

\[ \phi_0^{n+1} - \phi_1^n - \frac{n_c - 1}{n_c + 1} (\phi_1^{n+1} - \phi_0^n) = 0 \]

\[ \phi_J^{n+1} - \phi_{J-1}^n - \frac{n_c - 1}{n_c + 1} (\phi_{J-1}^{n+1} - \phi_J^n) = 0 \]

\[ \psi_0^{n+1} - \psi_1^n - \frac{n_c - 1}{n_c + 1} (\psi_1^{n+1} - \psi_0^n) = 0 \]

\[ \psi_J^{n+1} - \psi_{J-1}^n - \frac{n_c - 1}{n_c + 1} (\psi_{J-1}^{n+1} - \psi_J^n) = 0 \]
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# Fundamental Two-Particle Scattering

## List of Fundamental Two-Particle Scattering

| Two-lump scattering processes | Arrangement                  |
|------------------------------|------------------------------|
| **Type**                     | **kink-antikink**            | **antikink-kink**            |
| different type               | KINK-ANTIKINK OF THE DIFFERENT TYPE | ANTIKINK-KINK OF THE DIFFERENT TYPE |
| same type                    | KINK-ANTIKINK OF THE SAME TYPE | ANTIKINK-KINK OF THE SAME TYPE |
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**Kink-antikink (of different type) scattering**

\[ K^{(q_1, q_2, -1)}(v_0) \cup K^{(q_1, -q_2, 1)}(-v_0) \rightarrow \]
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**Kink-antikink (of different type) scattering**

\[ K^{(q_1, q_2, -1)}(v_0) \cup K^{(q_1, -q_2, 1)}(-v_0) \rightarrow \]
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**Kink-antikink (of different type) scattering**

\[ K^{(q_1, q_2, -1)}(v_0) \cup K^{(q_1, -q_2, 1)}(-v_0) \rightarrow \]
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Kink-antikink (of different type) scattering

\[ K^{(q_1, q_2, -1)}(v_0) \cup K^{(q_1, -q_2, 1)}(-v_0) \rightarrow \]
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**Kink-antikink (of different type) scattering**

\[ K^{(q_1, q_2, -1)}(v_0) \cup K^{(q_1, -q_2, 1)}(-v) \rightarrow \]
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**Kink-antikink (of different type) scattering**

\[ K^{q_1, q_2, -1} (v_0) \cup K^{q_1, -q_2, 1} (-v_0) \rightarrow K^{q_1, -q_2, -1} (-v_0) \cup K^{q_1, q_2, 1} (v_0) \]
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\[ K^{(q_1, q_2, 1)}(v_0) \cup K^{(-q_1, q_2, -1)}(-v_0) \rightarrow \]

**ANTIPLICIT-KINK (OF DIFFERENT TYPE) SCATTERING**
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**ANTI-KINK-KINK (OF DIFFERENT TYPE) SCATTERING**

$$K^{(q_1, q_2, 1)}(v_0) \cup K^{(-q_1, q_2, 1)}(-v_0) \rightarrow K^{(q_1, q_2, 1)}(-v_0) \cup K^{(-q_1, q_2, 1)}(v_0)$$
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Kink-antikink (of different type) scattering

\[ K^{(q_1, q_2, -1)}(v_0) \cup K^{(q_1, -q_2, 1)}(-v_0) \rightarrow \]
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**Kink-antikink (of same type) scattering**

\[ K^{(q_1,q_2,-1)}(v_0) \cup K^{(-q_1,-q_2,1)}(-v_0) \rightarrow \]
KINK-ANTI KINK (OF SAME TYPE) SCATTERING

\[ K^{(q_1, q_2, -1)}(v_0) \cup K^{(-q_1, -q_2, 1)}(-v_0) \rightarrow K^{(q_1, q_2, -1)}(-v_0) \cup K^{(-q_1, -q_2, 1)}(v_0) + \nu \]
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**ANTI KINK-KINK (OF SAME TYPE) SCATTERING**

\[ K^{(q_1, q_2, 1)}(v_0) \cup K^{(-q_1, -q_2, -1)}(-v_0) \rightarrow \]
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ANTIKINK-KINK (OF SAME TYPE) SCATTERING

\[ K^{(q_1, q_2, 1)}(v_0) \cup K^{(-q_1, -q_2, -1)}(-v_0) \rightarrow \]
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**Antikink-kink (of same type) scattering**

\[ K^{(q_1, q_2, 1)}(v_0) \cup K^{(-q_1, -q_2, -1)}(-v_0) \rightarrow K^{(q_1, q_2, 1)} \cup K^{(-q_1, -q_2, -1)} \]
**Conclusions: Kink Dynamics**

1. There exist four basic extended particles together with its corresponding antiparticles described by kinks and antikinks in this model.

2. There exist attractive forces between kinks and antikinks of the same type, which can form bound states. The kink-antikink interaction is stronger at short distances than the antikink-kink force although this last one has a longer range. The kink-antikink pair formation involves radiation emission.

3. The repulsive forces manage the antikink-kink interaction when the involved lumps are of different type.

4. The kink-antikink (of different type) interaction is almost absent although an exchange of the second topological charge is induced.

Thank your for your attention!
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**THREE LUMP SCATTERING**

\[ K(q_1, q_2, -1) \cdot K(q_1, -q_2, 1) \cdot K(-q_1, -q_2, -1) \text{ scattering} \]

\[ K(q_1, q_2, -1) \cdot K(q_1, -q_2, 1) \cdot K(-q_1, -q_2, -1) \text{ scattering} \]

\[ K(q_1, q_2, -1) \cdot K(-q_1, -q_2, 1) \cdot K(q_1, -q_2, -1) \text{ scattering} \]

\[ K(q_1, q_2, -1) \cdot K(-q_1, -q_2, 1) \cdot K(q_1, q_2, -1) \text{ scattering} \]