Simpson 1/3 rule based volume estimation of farm grown teak (Tectona grandis) in Cauvery delta zone

R Ravi Kumar, K Divya, M Vijayabhama, N Raja and S Selvanayaki

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Abstract

We concerned Tectona grandis (teak) growth and yield estimation on Cauvery delta zone. Scientifically sound growth models, based on advanced modelling techniques are often not available, although they are necessary for the successful management of teak stands in the country meanwhile long-term forest planning requires mathematical model. In this paper we present different yield models with respect to optimum rotations of 4 to 24 years of growth, among that Simpson 1/3 rule model is giving more accuracy of the volume than the existing other models.

Keywords: Simpson 1/3 rule model, volume estimation

1. Introduction

Teak (Tectona grandis, L.f) is one of the most important tropical timber species and is suitable for multiple end-use. It has excellent vegetation type and teak timber suitability is well documented and teak may grow more than 150 years under plantation conditions. In India, its natural zone of distribution is discontinuous and is mainly confined to the peninsular region south of 24 N latitude.

The most important teak forests in India are in Madhya Pradesh, Maharashtra, Karnataka, Tamil Nadu and Kerala. A high quality deciduous timber species are available in India, Burma and Indonesia. The important of teak plantations around the world can be appraised from the report “State of the World’s Forest Genetic Resources”. This report indicates tree species considered for conservation and management of forest genetic resources of teak and ranked more than 20 countries based on its economic value. In 2010 the global teak planted around 38 countries, conservatively 4.35 million ha, from which 80% grown in Asia.

In 2015 FAO (Food and Agriculture Organization of the United Nations) and ITTO (International Tropical Timber Organization) reported 6.89 million ha has planted teak as forest based. MAGAP (Ecuadorian Ministry of Agriculture, Livestock, Aquaculture and Fisheries) reported around of 50,000 ha teak planted, mainly located in Coastal region. This indicates information about pure teak plantation in Coastal region and economic price for ton, clear information should be actuated through MAGAP.

It is also predict economic value of teak and future demand of round wood export volumes. This information need through Mathematical model, to improve forestation and other management measures. In Tamil Nadu Thanjavur is most famous for teak. It is one of the important source of income for small scale farmers who plant the species on their farmers. The potential growth and merchantable timber size and economic benefits are undisputed. However, local knowledge on the growth and yield prediction of the species which will help in realising this potential is still lacking. Reliable of growth models are essential for effective long-term planning and decision making.

We present a biologically consistent for growth, based on state space approach, dominant height, volume estimation. In this way we can measure the yield of Cavuray delta region, such models could further silvicultural and economic studies, suitable ecological conditions to improve the wood production of living fences in the region.
2. Material and Methods

2.1. Study area

This research has been carried out Cauvery delta zones, the mean annual temperature is 29°C and total annual rainfall is 18000 to 2500mm. The samples are collected throughout tree growing areas in delta zones and their stand size, density to achieve a minimum of 30 trees per plot. The plots were re-measured annually during 2017-2020 and total of three annual measurement were available for each plot.

2.2. Tree volume estimation

In this model we assume that time is major roleto measure the behaviour of any stand teak evaluation. The dominant height (H), number of trees per hectare(N), base area (B) and site measure (Ω) are used in transition estimation function, which is predicting the growth of the tree. Afterwards, the main stem was cut in logs of 2m, thus measures are useful to find the commercial volume of trees.

The mortality transition function is directly proportional to the rate of change of N relative to increment of dominant height as well as current values of H, \( \log N = a_4 H^{3.5} N^{-0.5} \)

Differentiating the above equation, w. r. t. H

\[
\frac{1}{N} \frac{dN}{dH} = a_1 \left[ a_2 H^{a_2-1} N^{a_3} + H^{a_2} a_3 N^{a_3-1} \right] \frac{dN}{dH}
\]

\[
N^{-1} \frac{dN}{dH} - a_1 a_2 H^{a_2} N^{a_3-1} \frac{dN}{dH} = a_1 a_2 H^{a_2-1} N^{a_3}
\]

\[
N_{a_3} = a_1 a_2 H^{a_2} \log N_1 + a_1 a_2 (H^{a_2} - H^{a_2})
\]

\[
N_{a_2} = \left[ a_1 a_3 H^{a_3} \log N_1 + a_1 a_2 (H^{a_2} - H^{a_2}) \right]^{1/a_3}
\]

Here the value \( H_{a_2} \) indicates height at time \( t_{a_2}; H_{a_1} \) indicates height at time \( t_{a_1} \). If parameter restrictions were obtained by limiting values of self thinning slope base average square spacing is

\[
S = \frac{100}{\sqrt{N}}
\]

The rate of reduction for each site quality class has the following expression;

\[
\frac{dS}{dH} = \frac{\beta \gamma H^{\gamma-1}}{S^{\alpha-1}}
\]

\[
S^{-1} \log S = \beta' H^{\gamma'}
\]

\[
\left[ (y-1)S^{-1} \exp S + S^{-1} \right] dS = \beta' (y-1)H^{\gamma'} dH
\]

\[
\left[ (y-1)S^{-1} \exp S - (y-2)S^{-1} \exp S + (y-2)S^{-1} \right] dS = \beta' (y-1)(y-2)H^{\gamma'}
\]

\[
S^{-1} = \frac{1}{\exp(y-1)} \left[ (y-2)S^{-1} \exp S + (y-2)S^{-1} + \beta' (y-1)(y-2) \right] H^{\gamma'}
\]

\[
\frac{1}{S^{-2}} \log S = \beta' H^{\gamma'}
\]

The single entry equation is

\[
V = 1.66 \times 10^{-5} \times D^{3.1616} \times 0.9866^D
\]  

(1)

Where

\[
v = \text{tree volume}(m^3), \quad D = \text{dbh}(cm)
\]

and double entryequation is

\[
V = 1.66 \times 10^{-5} \times D^{1.2577} (H + [H - 1.3] \frac{d}{D})(H - [H - 1.3] \frac{d}{D})
\]  

(2)

Where

\[
v = \text{tree volume}(m^3) \text{to any upperstem diameter } d, \quad d_1 = \text{upperstem diameter}(cm), \quad H = \text{tree height}(m)
\]

As based the equation (1) and (2), Cauvery delta zone different age class data was used. It is showing as follows.

Fig 1: Cauvery delta zone different age class data with equation (1) and (2)

The first volume tables for teak in India were developed by Maitland in 1924 (Ram, 1942), further Ram (1942) was developed standard and commercial volume tables for the
“central provinces” in India. It presents an estimate of commercial bole volume and sapwood percentages in the commercial bole trees with respect of height and dbh classes. Singh (1981) developed volume table for stem include branch woods as follows

\[ V_1 = 0.19601K_1 + 0.25659D^2H - 0.10787 \]  

Where
- \( V_1 \) = total tree volume (standard timber) (m\(^3\))
- \( D \) = DBH (m)
- \( H \) = tree height (m)
- \( K_1 \) = 1 if the tree has

\[ V_2 = 0.20416K_1 + 0.25551D^2H + 0.10375 \]  

As based the equation (4), Cauvery delta zones different age class data was used. It is showing as follows.

The total volume calculated by the equations the following includes the total stem volume overbark to a top diameter of 20 cm as well as small wood volume, the volume of stem or branch wood longer than 1.5 m with a minimum top diameter of 5 cm.

As based the equation (6), Cauvery delta zone different age class data was used. It is showing as follows.

\[ \sqrt{V} = -0.1163 + 2.0813D \]  

\[ V = 0.16571 - 1.235D + 8.0855D^2 \]  

Where
- \( V \) = volume under bark (m\(^3\))
- \( D \) = mid point of diameter class (m)

The single entry equation is provided in Equation 7 and the double entry equation in Equation 8 as follows

\[ \ln(V) = -8.685 + 2.229 \ln(D) \]  

\[ \ln(V) = -9.733 + 2.055 \ln(D) + 0.773 \ln(H) \]  

Where
- \( V \) = total volume overbark (m\(^3\)),
- \( D \) = dbh (cm)
- \( H \) = tree height (m)

The double entry volume equation is preferred since the growth model allows for a choice of independent height and basal area growth functions. To determine the proportion of timber volume to total tree volume, the following two equations were developed:

\[ \frac{V_{tim}}{V} = -0.699 + 1.504(1 - \exp(-1.627(D^2H/10000))) \]

and

\[ \frac{V_{tim}}{V} = 0.949 - (2.557/D^2) \]

Where
- \( V_{tim}/V \) = ratio of timber volume to total volume

As based the equation (9), Cauvery delta zone different age class data was used. It is showing as follows.
A standard volume equation was developed and recommended for use in the whole country Nunifu and Murchison (1999); 

\[ V = -0.36 + 0.96D - 0.13D^2 + 0.05D^2H \]  

(10)

As based the equation (10), Cauvery delta zone different age class data was used. It is showing as follows.

Finally Caneleas and San Miguel (2004) developed growth and yield models for teak plantations and they said this following model is giving best result;

\[ V = -26.7721 + 0.02566D^2H \]  

(11)

where \( V \) = volume, \( D \) = diameter at breast height, \( H \) = total tree height (m)

As based the equation (11), Cauvery delta zone different age class data was used. It is showing as follows.

Further development made by Hurbers to measure the volume estimation through 

\[ v = \frac{\pi}{4} dm^2l \]  

(12)

Where 

\( dm \) is diameter at centre and \( l \) is tumbler total length.

As based the equation (12), Cauvery delta zone different age class data was used. It is showing as follows.

The Smalians suggested the following formula is better than Hurbers to measure the volume estimation;

\[ V = \frac{l}{2} \left[ \frac{\pi}{4} d_0^2 + \frac{\pi}{4} d_n^2 \right] \]  

(13)

Where 

\( d_0^2 \) is diameter at bottom and \( d_n^2 \) is tip diameter.
As based the equation (13), Cauvery delta zone different age class data was used. It is showing as follows.

\[ \text{volume} = \frac{1}{3} \left( r \log_2 + r \log_3 + 3(r \log_2 + r \log_3 + r \log_4 + \ldots + 2(r \log_3 + r \log_4 + \ldots) \right) \text{dbh} \]

Where \( h_1 \) is base stem height (m) and \( h_2 \) is stem height (m) that is greater than \( h_1 \). \( h_2 \) is diameter at DBH, \( r \log_2 \) is radius of log first, \( r \log_3 \) is radius of remaining logs. As based the equation (14), Cauvery delta zone different age class data was used. It is showing as follows.

![Fig 9: Cauvery delta zone different age class data with equation (13)](image)

A well structured Laser distance meter device is used to measure the height, diameter. In this study total tree volume is obtained by Simpson’s 1/3 rule base model, which is

\[ \text{volume} = \frac{1}{3} \left( r \log_2 + r \log_3 + 3(r \log_2 + r \log_3 + r \log_4 + \ldots + 2(r \log_3 + r \log_4 + \ldots) \right) \text{dbh} \]



![Fig 10: Cauvery delta zone different age class data with equation (14)](image)

As based on the above recommendation the models reported that average volume of teak in over all Tamil Nadu is given

| S. No | Model | Average volume of 4-5yrs | Average volume of 7-8yrs | Average volume of 12-13yrs | Average volume of 15-16yrs | Average volume of 18-19yrs | Average volume of 23-24yrs |
|-------|-------|--------------------------|--------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1.    | Equation(1) | -0.84338                | -0.843595                | -0.84387                    | 0.84                       | -0.84410                   | -0.84431                   |
| 2.    | Equation (4) | 0.04                     | 0.036093                 | 0.450103                    | 0.47                        | 0.42923                    | 0.480186                   |
| 3.    | Equation(6) | 0.01                     | 0.013483                 | 0.189468                    | 0.21                        | 0.24351                    | 0.245411                   |
| 4.    | Equation(8) | -0.09                    | -0.09235                 | -0.10                       | -0.10                       | -0.09992                   | -9.98849                   |
| 5.    | Equation (9) | -0.70                    | -0.69769                 | -0.69892                    | -1.61                       | -6.218                     | -6.2188                    |
| 6.    | Equation(10) | -1.61                    | -1.608                   | -1.608                      | -0.62                       | -6.218                     | -6.2188                    |
| 7.    | Humber’s formula | 0.13                     | 0.165098                 | 0.288204                    | 0.34                        | 0.47936                    | 0.536712                   |
| 8.    | Smallian’s formula | 0.15                     | 0.180781                 | 0.23961                     | 0.35                        | 0.45089                    | 0.580209                   |
| 9.    | Tip-diameter square | 0.13                     | 0.143783                 | 0.178614                    | 0.24                        | 0.227625                   | 0.31514                    |
| 10.   | Cylindrical | 0.068                    | 0.162                    | 0.213                       | 0.22                        | 0.31919                    | 0.440                      |
| 11.   | Simpson 1/3 rule | 0.06                     | 0.055813                 | 0.123377                    | 0.16                        | 0.26199                    | 0.37216                    |

4. Result and Discussion
Total volume of teak can be estimated based on dominant height and basal area. The different models were used for 3-4 years, 4-6 years, 6-8 years, 8-16 years, 16-18 years, 18-24 years of Cauvery delta zone. Among fourteen model Simpson’s 1/3 rule base model, tip diameter, cylindrical formula and equation (6) are coinciding almost nearly, Humber’s, Smalian’s are also coinciding nearly. The equation (1), equation (8) and equation (10) are giving negative values, those are given table 1. Therefore, Simpson’s 1/3 rule base is correlating to actual volume. Basal area per hectare was tested as a measure for stand density for the volume yield model, but was found to be statistically, significant and was eliminated from the model. Mean Annual Increment (MAI) was determined as the ratio of the volume yield model to age. The Current Annual Increment (CAI) model was obtained as a derivative of volume yield model with respect to age.

5. Conclusion
Cauvery delta zone with different ages of teak data was used in fourteen mathematical models. Among fourteen models, Simpson’s 1/3 rule base model is accurate than other models. Based on 15-16 yrs old tree data, the multiple linear regression formula and equation (6) are coinciding almost nearly.

\[ V = 0.223495 - 0.025470x_1 + 1.0346562x_2 \]

where \( x_1 = DBH \) and \( x_2 = \text{height} \) is more optimal to estimate volume, which is confirmed by \( R^2 (0.95) \). The increment of dominant height as well as current values of \( H \) is 15.65 m and DBH 0.183 m².

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