Irreducible Hamiltonian BRST analysis of Stueckelberg coupled $p$-form gauge theories

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Abstract

The irreducible Hamiltonian BRST symmetry for $p$-form gauge theories with Stueckelberg coupling is derived. The cornerstone of our approach is represented by the construction of an irreducible theory that is equivalent from the point of view of the BRST formalism with the original system. The equivalence makes permissible the substitution of the BRST quantization of the reducible model by that of the irreducible theory. Our procedure maintains the Lorentz covariance of the irreducible path integral.

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1 Introduction

It is well-known that the Hamiltonian BRST formalism [1]–[2] stands for one of the strongest and most popular quantization methods for gauge theories. In the irreducible context the ghosts can be interpreted like one-forms dual to the vector fields corresponding to the first-class constraints. This geometrical interpretation fails within the reducible framework due to the fact that the
vector fields are no longer independent, hence they cannot form a basis. The redundant behaviour generates the appearance of ghosts with ghost number greater than one, traditionally called ghosts for ghosts, of their canonical conjugated momenta, named antighosts, and, in the meantime, of a pyramidal non-minimal sector. The former objects, namely, the ghosts for ghosts, ensure a straightforward incorporation of the reducibility relations within the cohomology of the exterior derivative along the gauge orbits, while their antighosts are required in order to kill the higher resolution degree non trivial co-cycles from the homology of the Koszul-Tate differential.

A representative class of redundant systems is given by \( p \)-form gauge theories, that play an important role in string and superstring theory, supergravity and the gauge theory of gravity \([3]–[8]\), attracting much attention lately on behalf of some interesting aspects, like their characteristic cohomology \([9]\) or their applications to higher dimensional bosonisation \([10]\). The study of theoretical models with \( p \)-form gauge fields give an example of so-called ‘topological field theory’ and lead to the appearance of topological invariants, being thus in close relation to space-time topology, hence with lower dimensional quantum gravity \([3]\). In the meantime, antisymmetric tensor fields of various orders are included within the supergravity multiplets of many supergravity theories \([3]\), especially in 10 or 11 dimensions. It is known that the \( d = 11 \) supergravity is regarded as a sector of \( M \)-theory unification. Of the many special properties of \( d = 11 \) supergravity the most interesting is that it forbids a cosmological term extension in the context of lower-dimensional structures due precisely to the 4-form or ‘dual’ 7-form necessary to balance the degrees of freedom \([7]\). The construction of ‘dual’ Lagrangians involving \( p \)-forms is appears naturally in General Relativity and supergravity in order to render manifest the \( \text{SL}(2, \mathbb{R}) \) symmetry group of stationary solutions of Einstein’s vacuum equation, respectively to reveal some subtleties of ‘exact solutions’ for supergravity \([8]\). Interacting \( p \)-form gauge theories have been analyzed from the redundant Hamiltonian BRST point of view in \([11]\), where the ghost and auxiliary field structures required by the antifield BRST formalism are derived.

The purpose of this paper is to give a general irreducible approach to \( p \)-form gauge theories with Stueckelberg coupling in the Hamiltonian framework. This problem is solved mainly by replacing the original redundant Hamiltonian first-class system by an irreducible one, and by further quantizing the resulting theory in the Hamiltonian BRST context. The irreducible
first-class system is obtained by imposing that all the antighost number one co-cycles of the reducible Koszul-Tate differential should identically vanish under a suitable redefinition of the antighosts at antighost number one, and also by requesting that the number of physical degrees of freedom of the irreducible theory to be equal with that of the original reducible system. Initially, we analyze quadratic $p$-form gauge theories with Stueckelberg coupling [12], and then extend the results to the interacting case. The motivation for analyzing the Stueckelberg coupling and not the simpler case of free abelian $p$-forms is twofold. First, the Stueckelberg coupling is involved with the quantization of massive $p$-forms [12], and second, this class of models presents non-diagonal reducibility matrices as opposed to the free situation, which makes their irreducible treatment more interesting from the quantization point of view. Moreover, the free case can be obtained from the Stueckelberg coupling in the limit $M = 0$ (see [1]). The idea of converting a reducible Hamiltonian first-class theory into an irreducible one appears in [2] and [13], but it has not been either consistently developed or applied so far to the quantization of reducible first-class Hamiltonian systems.

Our paper is structured in four sections. Section 2 is dealing with the derivation of an irreducible Hamiltonian first-class system corresponding to the starting quadratic $p$-form gauge theory with Stueckelberg coupling on account of homological arguments, emphasizing that we can substitute the Hamiltonian BRST quantization of the original redundant model by that of the irreducible theory. In the end of this section we infer the path integral for the irreducible system in the context of the Hamiltonian BRST quantization. Section 3 investigates the extension of the analysis from Section 2 to the interacting case. In view of this, we propose a model of irreducible Hamiltonian theory associated to that in Section 2, and further determine its Lagrangian version, which relies on the original action and some Lorentz covariant irreducible gauge transformations. The interacting case is then addressed employing the above mentioned Lagrangian version as an appropriate starting point. Section 4 ends the paper with some conclusions.
2 Irreducible analysis of abelian $p$- and $(p - 1)$-forms with Stueckelberg coupling

In this section we derive the path integral of abelian $p$- and $(p - 1)$-forms with Stueckelberg coupling following an irreducible approach. Thus, we begin with the canonical analysis of the model, which is described by a quadratic Lagrangian action and displays a $(p - 1)$-stage reducible first-class constraint set. In subsection 2.2 we construct some irreducible first-class constraints associated with the original ones by means of homological arguments. More precisely, we require that all the non trivial antighost number one co-cycles of the reducible Koszul-Tate differential identically vanish under a suitable redefinition of the antighost number one antighosts while preserving the original number of physical degrees of freedom. The analysis is performed for $p = 2$ and $p = 3$, in order to emphasize various aspects of graduate complexity, which will be employed in order to generalize our results to arbitrary $p$. In this manner, we arrive at an irreducible set of first-class constraints, a corresponding first-class Hamiltonian and an irreducible Koszul-Tate differential associated with the starting reducible model. With these elements at hand, we construct in subsection 2.3 the irreducible BRST symmetry and show that it exists as it satisfies the general requirements of the homological perturbation theory. In the next subsection we elucidate the relationship between the reducible and irreducible BRST symmetries by proving that the physical observables deriving from the two contexts coincide, such that it is permissible to replace the Hamiltonian BRST quantization of the original reducible model by that of the resulting irreducible theory. In subsection 2.5 we apply the Hamiltonian BRST formalism to the irreducible model by using a proper non-minimal sector and a gauge-fixing fermion that finally lead to a manifestly covariant path integral.

2.1 Description of the model

Our starting point is the quadratic Lagrangian action

\[
S^L_0 \left[ A_{\mu_1...\mu_p}, H_{\mu_1...\mu_{p-1}} \right] = - \int d^d x \left( \frac{1}{2 \cdot (p + 1)!} F_{\mu_1...\mu_{p+1}} F^{\mu_1...\mu_{p+1}} + \frac{1}{2 \cdot p!} (MA_{\mu_1...\mu_p} - F_{\mu_1...\mu_p}) (MA^{\mu_1...\mu_p} - F^{\mu_1...\mu_p}) \right), \tag{1}
\]
where $F_{\mu_1 \ldots \mu_{p+1}}$ and $F_{\mu_1 \ldots \mu_p}$ represent the field strengths of $A_{\mu_1 \ldots \mu_p}$, respectively, $H_{\mu_1 \ldots \mu_{p-1}}$. Of course, it is understood that $d \geq p + 1$. Action (1) is invariant under the gauge transformations

$$\delta A^\mu_{\mu_1 \ldots \mu_p} = \partial^\mu [\epsilon^{\mu_1 \ldots \mu_p}],$$

$$\delta H^\mu_{\mu_1 \ldots \mu_p-1} = \partial^\mu [\epsilon^{\mu_1 \ldots \mu_p-1}] + M \epsilon^{\mu_1 \ldots \mu_p-1},$$

where $[\mu_1 \ldots \mu_k]$ signifies antisymmetry with respect to the indices between brackets.

Performing the canonical analysis of (1), one infers the first-class constraints

$$G_{i_1 \ldots i_{p-1}}^{(1)} \equiv \pi_{0i_1 \ldots i_{p-1}} \approx 0,$$

$$G_{i_1 \ldots i_{p-2}}^{(1)} \equiv \Pi_{0i_1 \ldots i_{p-2}} \approx 0,$$

$$G_{i_1 \ldots i_{p-1}}^{(2)} \equiv -p \partial^j \pi_{ii_1 \ldots i_{p-1}} + M \Pi_{ii_1 \ldots i_{p-1}} \approx 0,$$

$$G_{i_1 \ldots i_{p-2}}^{(2)} \equiv -(p - 1) \partial^j \Pi_{ii_1 \ldots i_{p-2}} \approx 0,$$

and the canonical Hamiltonian

$$\bar{H} = \int d^{d-1}x \left( -\frac{p!}{2} \pi_{ii_1 \ldots i_p} \pi^{ii_1 \ldots i_p} - \frac{(p - 1)!}{2} \Pi_{ii_1 \ldots i_{p-1}} \Pi^{ii_1 \ldots i_{p-1}} + A^{0i_1 \ldots i_{p-1}} G_{i_1 \ldots i_{p-1}}^{(2)} + \frac{1}{2} \cdot p! \left( MA_{i_1 \ldots i_p} - F_{i_1 \ldots i_p} \right) \left( MA^{ii_1 \ldots i_p} - F^{ii_1 \ldots i_p} \right) + \frac{1}{2} \cdot (p + 1)! F_{i_1 \ldots i_{p+1}} F^{ii_1 \ldots i_{p+1}} + H^{0i_1 \ldots i_{p-2}} G_{i_1 \ldots i_{p-2}}^{(2)} \right).$$

In (1–8) $\pi$ and $\Pi$ stand for the momenta of the corresponding $A$, respectively, $H$. Using the notations

$$G_{a_0}^{(2)} \equiv \left( G_{i_1 \ldots i_{p-1}}^{(2)}, G_{i_1 \ldots i_{p-2}}^{(2)} \right),$$

we find that the constraint functions (8) are $(p - 1)$-stage reducible

$$Z_{a_0}^{a_1} G_{a_0}^{(2)} = 0,$$

$$Z_{a_{k-2}}^{a_{k-1}} Z_{a_{k-1}}^{a_k} = 0, \ k = 2, \ldots, p - 1,$$

In (1–8) $\pi$ and $\Pi$ stand for the momenta of the corresponding $A$, respectively, $H$. Using the notations

$$G_{a_0}^{(2)} \equiv \left( G_{i_1 \ldots i_{p-1}}^{(2)}, G_{i_1 \ldots i_{p-2}}^{(2)} \right),$$

we find that the constraint functions (8) are $(p - 1)$-stage reducible

$$Z_{a_0}^{a_1} G_{a_0}^{(2)} = 0,$$

$$Z_{a_{k-2}}^{a_{k-1}} Z_{a_{k-1}}^{a_k} = 0, \ k = 2, \ldots, p - 1,$$
where the \( k \)th order reducibility functions are expressed by

\[
Z_{a_{k-1}}^{a_k} = \begin{pmatrix}
\frac{1}{(p-k-1)!} \partial^{i_1} \delta^{i_2}_{j_1} \cdots \delta^{i_{p-k}}_{j_{p-k-1}} & 0 \\
\frac{(-)^{k+1} M}{(p-k-1)!} \delta^{i_1}_{j_1} \cdots \delta^{i_{p-k-1}}_{j_{p-k-1}} & \frac{1}{(p-k-2)!} \partial^{i_1} \delta^{i_2}_{j_1} \cdots \delta^{i_{p-k-2}}_{j_{p-k-2}}
\end{pmatrix},
\]  

(12)

\( k = 1, \ldots, p - 1 \), and

\[
a_k = (j_1 \ldots j_{p-k-1}, j_1 \ldots j_{p-k-2}), \quad k = 0, \ldots, p - 1.
\]  

(13)

Throughout the paper we work with the conventions \( f^{i_1 \cdots i_m} = f \) if \( m = 0 \) and \( f^{i_1 \cdots i_m} = 0 \) if \( m < 0 \). This ends the canonical analysis of this model.

### 2.2 Irreducible constraints

The first step of our analysis consists in the derivation of an irreducible first-class theory associated with the original reducible one, which, in addition, preserves the initial number of physical degrees of freedom. In view of this we construct some irreducible first-class constraints starting with the reducible constraints (6–7). Clearly, the case \( p = 1 \) is irreducible and in consequence it will be not discussed in the sequel. For a deeper understanding of our procedure we initially expose the case \( p = 2 \), subsequently explore the situation \( p = 3 \), and finally generalize our results to an arbitrary \( p \).

#### 2.2.1 The case \( p = 2 \)

In this case the constraints (6–7) take the form

\[
G^{(2)}_i \equiv -2 \partial^j \pi_{ji} + M \Pi_i \approx 0, \quad G^{(2)} \equiv -\partial^i \Pi_i \approx 0,
\]  

(14)

and are first-stage reducible

\[
\partial^i G^{(2)}_i + MG^{(2)} = 0.
\]  

(15)

The reducible BRST symmetry

\[
s_R = \delta_R + \sigma_R + \cdots,
\]  

(16)

contains two basic differentials. The first one, \( \delta_R \), named the Koszul-Tate differential, realizes an homological resolution of smooth functions defined
on the constraint surface, while the second one, $\sigma_R$, represents a model of longitudinal derivative along the gauge orbits and accounts for the gauge invariances (generated by the first-class constraints). In order to realize a proper construction of $\delta_R$ we set the action of this operator on all phase-space variables to vanish and introduce some new generators, called antighosts and denoted by $P_{2i}$, $P_2$, and $\lambda$, accordingly to which we define

$$
\delta_R P_{2i} = -G_i^{(2)}, \quad \delta_R P_2 = -G^{(2)},
$$

(17)

$$
\delta_R \lambda = -\partial^i P_{2i} - MP_2.
$$

(18)

While $P_{2i}$ and $P_2$ are fermionic fields of antighost number one, the antighost $\lambda$ is bosonic and possesses antighost number two. The antighost $\lambda$ is required in order to make the co-cycle

$$
\mu = \partial^i P_{2i} + MP_2,
$$

(19)

$\delta_R$-exact, which will establish the acyclicity of the Koszul-Tate differential. Our main idea of passing to an irreducible treatment is to redefine the antighosts $P_{2i}$ and $P_2$ in such a way that the new co-cycle of the type (19) vanishes identically. If we implement this step, the new co-cycle at antighost number one will be trivially (identically vanishing) without introducing $\lambda$, hence the resulting theory will be indeed irreducible. The redefinition of the antighosts is performed through

$$
P_{2i} \rightarrow \tilde{P}_{2i} = D^i P_{2j} + \tilde{D}_i P_2, \quad P_2 \rightarrow \tilde{P}_2 = D^i P_{2j} + DP_2,
$$

(20)

where the quantities $D^i$, $\tilde{D}_i$, $D^j$ and $D$ are taken to satisfy the equations

$$
\partial^i D^j_{\ i} + D^j = 0, \quad \partial^i \tilde{D}_i + MD = 0,
$$

(21)

$$
D^i G^{(2)}_j + \tilde{D}_i G^{(2)} = G^{(2)}_i, \quad D^i G^{(2)}_j + DG^{(2)} = G^{(2)}.
$$

(22)

Taking into account (17), (20) and (22), after simple computation we find that

$$
\delta \tilde{P}_{2i} = -G_i^{(2)}, \quad \delta \tilde{P}_2 = -G^{(2)}.
$$

(23)

From (23) we obtain that the co-cycle of the type (19), $\tilde{\mu} = \partial^i \tilde{P}_{2i} + MP_2$, vanishes identically due to (21). We redenoted the Koszul-Tate differential
by $\delta$ in order to emphasize that it corresponds to an irreducible situation. In this way our scope, namely, to make $\tilde{\mu}$ vanish, can be attained if the system (21–22) is solvable. The solution to this system exists and is given by

$$D^j_i = \delta^j_i - \frac{\partial^i \partial_j}{\Delta + M^2}, \quad D^j = -M \frac{\partial^j}{\Delta + M^2},$$

$$\tilde{D}_i = -M \frac{\partial_i}{\Delta + M^2}, \quad D = 1 - \frac{M^2}{\Delta + M^2},$$

(24)

(25)

where $\Delta = \partial_k \partial^k$. Replacing (24–25) in (23) we arrive at

$$\delta P^2_i - \frac{\partial_i}{\Delta + M^2} \delta \left( \partial^j P^2_{2j} + MP_2 \right) = -G^{(2)}_i,$$

$$\delta P^2 - \frac{M}{\Delta + M^2} \delta \left( \partial^j P^2_{2j} + MP_2 \right) = -G^{(2)}.$$  

(26)

(27)

The last relations describe the action of $\delta$ corresponding to an irreducible model subject to some irreducible first-class constraints to be further determined. At this point we explore the requirement that the number of physical degrees of freedom should be preserved by passing to the irreducible theory. As the number of independent constraint functions (14) is equal to $(d - 1)$ and that of independent constraint functions implicitly involved with (26–27) is $d$, it results that we need an extra degree of freedom for the irreducible theory. We denote this supplementary degree of freedom by $(A, \pi)$, with $\pi$ the non-vanishing solution to the equation

$$\left( \Delta + M^2 \right) \pi = \delta \left( \partial^j P^2_{2j} + MP_2 \right).$$

(28)

Due to the invertibility of $(\Delta + M^2)$, the non-vanishing solution for $\pi$ enforces the irreducibility because the equation (28) possesses non-vanishing solutions if and only if $\delta \left( \partial^j P^2_{2j} + MP_2 \right) \neq 0$, hence if and only if (19) is not a co-cycle. Making use of (26–28) we get that

$$\delta P^2_i = -G^{(2)}_i + \partial_i \pi, \quad \delta P^2 = -G^{(2)} + M \pi.$$  

(29)

The above relations are nothing but the definitions of $\delta$ on the antighost number one antighosts corresponding to an irreducible theory subject to the irreducible first-class constraints

$$\gamma^{(2)}_i \equiv -2 \partial^j \pi_{ji} + M \Pi_i - \partial_i \pi \approx 0, \quad T^{(2)} \equiv -\partial^j \Pi_i - M \pi \approx 0.$$  

(30)
This solves the problem of constructing some irreducible first-class constraints deriving from (14) in the case \( p = 2 \).

It seems that our irreducible approach gives rise to some problems linked with locality. Indeed, \( \mathcal{P}_{2i} \) and \( P_2 \), explicitly written in (26–27), contain a non-local term. This is not a surprise as \( \mathcal{P}_{a_0} = (\mathcal{P}_{2i}, P_2) \) are nothing but the ‘transverse’ part of \( \mathcal{P}_{a_0} = (\mathcal{P}_{2i}, P_2) \) with respect to \( Z^{a_0} = \left( \frac{\partial^i}{M} \right) \), i.e., \( Z^{a_0} \mathcal{P}_{a_0} = 0 \), and it is known that the decomposition into transverse and longitudinal components generates non-locality. On the other hand, the solution of the equation (28) is generally non-local. However, the non-locality of the solution to (28) compensates in a certain sense the non-locality present in (26–27) such that the resulting irreducible constraints (30) (inferred via (29)) are local. Anticipating a bit, in the case of \( p \geq 3 \) the redefinition of the antighosts will consequently imply some ‘transverse’-type conditions with respect to the corresponding reducibility functions which lead to some non-local solutions. In order to compensate this non-locality and to further obtain some local irreducible constraints it will be also necessary to add some supplementary degrees of freedom that check some equations of the type (28). In general, the lack of locality in the BRST formalism can occur if the BRST charge or the gauge-fixing fermion are non-local. As it will be seen below, this does not happen in the context of our procedure (see (119) and (121)).

### 2.2.2 The case \( p = 3 \)

The guideline in this situation is the case \( p = 2 \). However, we will see that some new features arise. The starting reducible constraints have the form

\[
G_{ij}^{(2)} \equiv -3\partial^k \pi_{kij} + M\Pi_{ij} \approx 0, \quad G_i^{(2)} \equiv -2\partial^j \Pi_{ji} \approx 0.
\]

The definition of the reducible Koszul-Tate differential reads as

\[
\delta_R \mathcal{P}_{2ij} = -G_{ij}^{(2)}, \quad \delta_R P_{2i} = -G_i^{(2)},
\]

\[
\delta_R \lambda_i = -2\partial^j \mathcal{P}_{2ji} - MP_{2i}, \quad \delta_R \lambda = -\partial^i \mathcal{P}_{2i},
\]

\[
\delta_R \bar{\lambda} = -\partial^i \lambda_i + M\bar{\lambda},
\]

9
where $\mathcal{P}_{2ij}$ and $P_{2i}$ are fermionic of antighost number one, $\lambda_i$ and $\lambda$ are bosonic with antighost number two, and $\tilde{\lambda}$ is fermionic of antighost number three. The antighosts $\lambda_i$ and $\lambda$ must be introduced in order to enforce the $\delta_R$-exactness of the antighost number one co-cycles

$$\nu_i = 2\partial^j \mathcal{P}_{2ji} + MP_{2i}, \quad \nu = \partial^i P_{2i},$$

while the presence of $\tilde{\lambda}$ solves the exactness of the antighost number two co-cycle

$$\alpha = \partial^i \lambda_i - M\lambda.$$  

We apply the same idea like before, namely, we demand that $\nu_i$ and $\nu$ are no longer co-cycles in the irreducible context (described in terms of the irreducible Koszul-Tate operator $\delta$), so they must be no longer $\delta$-closed. This request can be satisfied if we add some new bosonic canonical pairs $(A^i, \pi_i)$, $(H, \Pi)$ and impose that the momenta $\pi_i$ and $\Pi$ are the non-vanishing solutions to the equations

$$\delta \left( 2\partial^j \mathcal{P}_{2ji} + MP_{2i} \right) = \left( \Delta + M^2 \right) \pi_i,$$

$$\delta \left( \partial^i P_{2i} \right) = \left( \Delta + M^2 \right) \Pi.$$  

From (37) we get that $\delta (M\partial^i P_{2i}) = (\Delta + M^2) \partial^i \pi_i$, which combined with (38) leads to

$$\partial^i \pi_i - M\Pi = 0,$$

on behalf of the invertibility of $\left( \Delta + M^2 \right)$. The last relation is a new constraint of the irreducible theory that ensures the preservation of the physical degrees of freedom with respect to the initial model. Indeed, the number of independent constraints (31) is $\left( d - 1 \right) \left( d - 2 \right) / 2$. By contrast, in the irreducible framework we will find precisely $\left( d - 1 \right) \left( d - 2 \right) / 2 + \left( d - 1 \right)$ irreducible constraints corresponding to (31) and a supplementary number of phase-space variables $(A^i, \pi_i)$, $(H, \Pi)$, which is equal to $2d$. Thus, in order to re-obtain the original number of degrees of freedom, it is necessary to add an extra constraint, which forms together with the others an irreducible first-class set. This constraint is nevertheless offered precisely by (39) and will be denoted by

$$\gamma^{(2)} \equiv -\partial^i \pi_i + M\Pi \approx 0.$$
We remark that from the entire set of first-class constraints (31) and (40), the latter is already irreducible, so its presence does not imply further co-cycles at antighost number one. The antighost corresponding to (40), $\tilde{P}_2$, is fermionic, has the antighost number equal to one, and must satisfy

$$\delta \tilde{P}_2 = -\gamma^{(2)}. \quad (41)$$

At this stage we redefine the antighost number one antighosts in order to make the co-cycles of the type (35) to vanish identically. The redefinition reads as

$$\tilde{P}_{2ij} \rightarrow \mathcal{P}_{2ij} = D_{ij}^{kl} \tilde{P}_{2kl} + \tilde{D}_{ij}^k P_{2k}, \quad (42)$$

$$\tilde{P}_{2i} \rightarrow P_{2i} = D_{i}^{kl} \mathcal{P}_{2kl} + D_{i}^k P_{2k}, \quad (43)$$

with $D_{ij}^{kl}$, $\tilde{D}_{ij}^k$, $D_{i}^{kl}$ and $D_{i}^k$ taken to fulfill

$$2\partial^i D_{ij}^{kl} + MD_{ij}^{kl} = 0, \quad 2\partial^i \tilde{D}_{ij}^k + MD_i^k = 0, \quad (44)$$

$$\partial^i D_i^l = 0, \quad \partial^i D_i^k = 0, \quad (45)$$

$$D_{ij}^{kl} G_{kl}^{(2)} + \tilde{D}_{ij}^k G_k^{(2)} = G_{ij}^{(2)}, \quad (46)$$

$$D_i^l G_{kl}^{(2)} + D_i^k G_k^{(2)} = G_i^{(2)}. \quad (47)$$

In consequence, the relations (32) become

$$\delta \tilde{P}_{2ij} = -G_{ij}^{(2)}, \quad \delta \tilde{P}_{2i} = -G_i^{(2)}, \quad (48)$$

which further lead to the co-cycles $\tilde{\nu}_i = 2\partial^i \tilde{P}_{2ji} + M \tilde{P}_{2i}$, $\tilde{\nu} = \partial^i \tilde{P}_{2i}$ that vanish identically due to (44–47). The solution of (44–47) takes the form

$$D_{ij}^{kl} = \frac{1}{2} \delta_i^k \delta_j^l - \frac{1}{2 (\Delta + M^2)} \delta_i^m \partial^k \delta_j^m \partial^l, \quad \tilde{D}_{ij}^k = -\frac{M}{2 (\Delta + M^2)} \delta_j^k \partial_i, \quad (49)$$

$$D_i^l = -\frac{M}{\Delta + M^2} \delta_i^l \partial^k, \quad D_i^k = \left(1 - \frac{M^2}{\Delta + M^2}\right) \delta_i^k - \frac{\partial_i \partial_i}{\Delta + M^2}. \quad (50)$$

Substituting (37 50) in (48) and taking into account (37 38), we finally deduce

$$\delta \mathcal{P}_{2ij} = -G_{ij}^{(2)} + \frac{1}{2} \partial_i \pi_j, \quad \delta P_{2i} = -G_i^{(2)} + M \pi_i + \partial_i \Pi. \quad (51)$$
The last formulas emphasize the irreducible first-class constraints

\[ \gamma^{(2)}_{ij} \equiv -3\partial^k \pi_{kij} + M\Pi_{ij} - \frac{1}{2} \partial_{[i} \pi_{j]} \approx 0, \quad (52) \]

\[ T^{(2)}_i \equiv -2\partial^j \Pi_{ji} - M\pi_i - \partial_i \Pi \approx 0, \quad (53) \]

which together with (51) form the searched for irreducible set associated with (31). The new feature arising in the \( p = 3 \) case is given by the appearance of the new constraint (51). Like in the case \( p = 2 \), the non-locality present in (37–38) compensates the non-locality produced by (49–50) in (48). We will see that in the more complex situations \( p \geq 4 \) we have to add more constraints instead of (51).

### 2.2.3 Generalization to arbitrary \( p \)

Acting along the line exposed above, we introduce the antisymmetric canonical pairs

\[ (A^{j_1 \ldots j_{p-2k-2}}, \pi^{j_1 \ldots j_{p-2k-2}}), (H^{j_1 \ldots j_{p-2k-3}}, \Pi^{j_1 \ldots j_{p-2k-3}}), \quad (54) \]

for \( k \geq 0 \) in order to prevent the appearance of any antighost number one co-cycle, and, by using some homological arguments similar with the previous ones, we construct the tower of irreducible first-class constraints associated with (31) under the form

\[ \gamma^{(2)}_{i_1 \ldots i_{p-2k-1}} \equiv -(p - 2k) \partial^i \pi_{i_1 \ldots i_{p-2k-1}} + M\Pi_{i_1 \ldots i_{p-2k-1}} - \frac{1}{p - 2k - 1} \partial_{[i_1} \pi_{i_2 \ldots i_{p-2k-1}]} \approx 0, \quad k = 0, \ldots, a, \quad (55) \]

\[ T^{(2)}_{i_1 \ldots i_{p-2k-2}} \equiv -(p - 2k - 1) \partial^i \Pi_{i_1 \ldots i_{p-2k-2}} - M\pi_{i_1 \ldots i_{p-2k-2}} - \frac{1}{p - 2k - 2} \partial_{[i_1} \Pi_{i_2 \ldots i_{p-2k-2}]} \approx 0, \quad k = 0, \ldots, c, \quad (56) \]

where we used the notations

\[ a = \begin{cases} \frac{p}{2} - 1, & \text{for } p \text{ even}, \\ \frac{p-1}{2}, & \text{for } p \text{ odd}, \end{cases} \quad c = \begin{cases} \frac{p}{2} - 1, & \text{for } p \text{ even}, \\ \frac{p-3}{2}, & \text{for } p \text{ odd}. \end{cases} \quad (57) \]
At this moment the constraints of the irreducible theory are expressed by (55) and (56).

Due to the fact that we intend to develop a covariant irreducible approach, it is necessary to further enlarge the phase-space by adding the antisymmetric canonical pairs

\[
(A^{0j_1\ldots j_{p-2k-1}}, \pi^{0j_1\ldots j_{p-2k-1}}), (H^{0j_1\ldots j_{p-2k-2}}, \Pi^{0j_1\ldots j_{p-2k-2}}),
\]

for \( k \geq 1 \), which we impose to be constrained by

\[
\pi^{0j_1\ldots j_{p-2k-1}} \approx 0, \; \Pi^{0j_1\ldots j_{p-2k-2}} \approx 0, \; k \geq 1.
\]

In this manner, the constraints of the irreducible theory are expressed by (55–56) and also by

\[
\gamma_{i_1\ldots i_{p-2k-1}}^{(1)} \equiv \pi_{i_1\ldots i_{p-2k-1}} \approx 0, \; i = 0, \ldots, a, \quad (60)
\]

\[
T_{i_1\ldots i_{p-2k-2}}^{(1)} \equiv \Pi_{i_1\ldots i_{p-2k-2}} \approx 0, \; i = 0, \ldots, c, \quad (61)
\]

which form a first-class, abelian and irreducible set. The first-class Hamiltonian with respect to the above first-class constraints will be taken of the form

\[
\bar{H} = \int d^{d-1}x \left( \frac{p!}{2} \pi^{0i_1\ldots i_p} \pi_{0i_1\ldots i_p} - \frac{(p-1)!}{2} \Pi_{i_1\ldots i_{p-1}} \Pi^{i_1\ldots i_{p-1}} + \sum_{k=0}^{a} A^{0i_1\ldots i_{p-2k-1}}_{i_1\ldots i_{p-2k-1}}^{(2)} + \frac{1}{2 \cdot p!} (MA_{i_1\ldots i_p} - F_{i_1\ldots i_p}) (MA^{i_1\ldots i_p} - F^{i_1\ldots i_p}) + \frac{1}{2 \cdot (p+1)!} F_{i_1\ldots i_{p+1}} F^{i_1\ldots i_{p+1}} + \sum_{k=0}^{c} H^{0i_1\ldots i_{p-2k-2}}_{i_1\ldots i_{p-2k-2}} T_{i_1\ldots i_{p-2k-2}}^{(2)} \right).
\]

At the level of the extended action of the initial model the gauge variations of the Lagrange multipliers for the reducible constraints contain some supplementary gauge parameters due to the reducibility, which ensure the covariance of the Lagrangian gauge transformations. On the contrary, in the framework of our irreducible treatment these additional gauge parameters are absent because of the irreducibility. This is why we cannot yet build a model of irreducible extended formalism that outputs some covariant Lagrangian gauge transformations. In order to restore the covariance, it is necessary to
add a number of supplementary pairs equal with double of the number of pairs (54)

\[
\left( B^{(1)}j_1...j_{p-2k-2}, \pi^{(1)}_{j_1...j_{p-2k-2}} \right), \left( V^{(1)}j_1...j_{p-2k-3}, \Pi^{(1)}_{j_1...j_{p-2k-3}} \right), \tag{63}
\]

\[
\left( B^{(2)}j_1...j_{p-2k-2}, \pi^{(2)}_{j_1...j_{p-2k-2}} \right), \left( V^{(2)}j_1...j_{p-2k-3}, \Pi^{(2)}_{j_1...j_{p-2k-3}} \right), \tag{64}
\]

with \( k \geq 0 \). In addition, we set the constraints

\[
\gamma^{(1)}_{i_1...i_{p-2k-2}} \equiv \pi^{(1)}_{i_1...i_{p-2k-2}} \approx 0, \ k = 0, \ldots, c, \tag{65}
\]

\[
T^{(1)}_{i_1...i_{p-2k-3}} \equiv \Pi^{(1)}_{i_1...i_{p-2k-3}} \approx 0, \ k = 0, \ldots, d, \tag{66}
\]

\[
\gamma^{(2)}_{i_1...i_{p-2k-2}} \equiv -\pi^{(2)}_{i_1...i_{p-2k-2}} \approx 0, \ k = 0, \ldots, c, \tag{67}
\]

\[
T^{(2)}_{i_1...i_{p-2k-3}} \equiv -\Pi^{(2)}_{i_1...i_{p-2k-3}} \approx 0, \ k = 0, \ldots, d, \tag{68}
\]

where

\[
d = \begin{cases} \frac{p}{2} - 2, & \text{for } p \text{ even}, \\ \frac{p-3}{2}, & \text{for } p \text{ odd}. \end{cases} \tag{69}
\]

It is well-known that one can always redefine the surface of first-class constraints up to a linear combination of constraints whose coefficients form an invertible matrix. In this respect, we remark that the canonical momenta \((\pi_{i_1...i_{p-2k-2}})_{k=0,...,c}\) and \((\Pi_{i_1...i_{p-2k-3}})_{k=0,...,d}\) can be expressed with the help of the constraints (63–66) under the form

\[
\pi_{i_1...i_{p-2k-2}} = -\frac{1}{M^2 + \Delta} \left( (p - 2k - 1) \partial^i \gamma^{(2)}_{i_1...i_{p-2k-2}} + MT^{(2)}_{i_1...i_{p-2k-2}} + \frac{1}{p - 2k - 2} \partial_{[i_1} \gamma^{(2)}_{i_2...i_{p-2k-2}]} \right), \tag{70}
\]

\[
\Pi_{i_1...i_{p-2k-3}} = -\frac{1}{M^2 + \Delta} \left( (p - 2k - 2) \partial^i T^{(2)}_{i_1...i_{p-2k-3}} - M\gamma^{(2)}_{i_1...i_{p-2k-3}} + \frac{1}{p - 2k - 3} \partial_{[i_1} T^{(2)}_{i_2...i_{p-2k-3}]} \right). \tag{71}
\]

Therefore, we can redefine the constraints (68) like

\[
\gamma^{(1)}_{i_1...i_{p-2k-2}} \equiv \pi^{(1)}_{i_1...i_{p-2k-2}} - \pi^{(1)}_{i_1...i_{p-2k-2}} \approx 0, \ k = 0, \ldots, c, \tag{72}
\]
\[ T_{i_1 \cdots i_{p-2k-3}}^{(1)} \equiv \Pi_{i_1 \cdots i_{p-2k-3}}^{(1)} - \Pi_{i_1 \cdots i_{p-2k-3}}^{(2)} \approx 0, \ k = 0, \ldots, d. \] (73)

It is clear that the constraints (55–56), (60–61), (67–68) and (72–73) are first-class and irreducible. The number of physical degrees of freedom of the last irreducible model coincides with that of the starting reducible theory. The first-class Hamiltonian corresponding to the theory possessing the above mentioned irreducible first-class constraints can be chosen of the type

\[ H' = \tilde{H} + \int d^{d-1}x \left( \sum_{k=0}^{c} A^{i_1 \cdots i_{p-2k-2}} H^{(2)}_{i_1 \cdots i_{p-2k-2}} + \sum_{k=0}^{d} H^{i_1 \cdots i_{p-2k-3}} \Pi_{i_1 \cdots i_{p-2k-3}}^{(2)} + \sum_{k=0}^{c} B^{(2)}_{i_1 \cdots i_{p-2k-2}} \left( (p-2k-1) \partial^2 \gamma^{(2)}_{i_1 \cdots i_{p-2k-2}} + MT_{i_1 \cdots i_{p-2k-2}}^{(2)} + \frac{1}{p-2k-2} \partial_{i_1} \gamma^{(2)}_{i_2 \cdots i_{p-2k-2}} \right) + \sum_{k=0}^{d} V^{(2)}_{i_1 \cdots i_{p-2k-3}} \left( (p-2k-2) \partial^2 T^{(2)}_{i_1 \cdots i_{p-2k-3}} - M \gamma^{(2)}_{i_1 \cdots i_{p-2k-3}} + \frac{1}{p-2k-3} \partial_{i_1} T^{(2)}_{i_2 \cdots i_{p-2k-3}} \right) \right) \equiv \int d^{d-1}x h'. \] (74)

In conclusion, starting from action (1) we derived an irreducible theory based on the first-class constraints (55–56), (60–61), (67–68), (72–73) and on the first-class Hamiltonian (74). We remark that the above first-class constraints and first-class Hamiltonian density are local functions.

### 2.3 Irreducible BRST symmetry

Here we point out the construction of the irreducible BRST symmetry for the irreducible theory built previously. The minimal antighost spectrum of the irreducible Koszul-Tate differential is organized as

\[ \left( P_{i_1 \cdots i_{p-2k-1}}, P_{i_1 \cdots i_{p-2k-1}} \right), \ k = 0, \ldots, a, \] (75)

(associated with (69), respectively, (63)),

\[ \left( P_{i_1 \cdots i_{p-2k-2}}, P_{i_1 \cdots i_{p-2k-2}} \right), \ k = 0, \ldots, c, \] (76)

(associated with (72), respectively, (67)),

\[ \left( P_{i_1 \cdots i_{p-2k-2}}, P_{i_1 \cdots i_{p-2k-2}} \right), \ k = 0, \ldots, c, \] (77)

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(corresponding to (61) and (56)), plus
\[ \left( P_{1i_1...i_{p-2k-3}}, P_{2i_1...i_{p-2k-3}} \right), \quad k = 0, \ldots, d, \quad (78) \]
(corresponding to (73) and (68)). All the previous fields are fermionic, with the \( P \)'s and \( P' \)'s of antighost number one. The usual definitions of \( \delta \) are given by
\[
\delta z^A = 0, \tag{79}
\]
\[
\delta P_{\Delta i_1...i_{p-2k-1}} = -\gamma^{(\Delta)}_{i_1...i_{p-2k-1}}, \quad \Delta = 1, 2, \quad k = 0, \ldots, a, \tag{80}
\]
\[
\delta P_{\Delta i_1...i_{p-2k-2}} = -\gamma^{(\Delta)}_{i_1...i_{p-2k-2}}, \quad \Delta = 1, 2, \quad k = 0, \ldots, c, \tag{81}
\]
\[
\delta P_{\Delta i_1...i_{p-2k-2}} = -T^{(\Delta)}_{i_1...i_{p-2k-2}}, \quad \Delta = 1, 2, \quad k = 0, \ldots, c, \tag{82}
\]
\[
\delta P_{\Delta i_1...i_{p-2k-3}} = -T^{(\Delta)}_{i_1...i_{p-2k-3}}, \quad \Delta = 1, 2, \quad k = 0, \ldots, d, \tag{83}
\]
where \( z^A \) generically denotes any original field/momentum or new variable in (54), (58) or (63–64). With the help of these definitions, \( \delta \) is found nilpotent and acyclic. The other differential involved with the BRST symmetry, namely, the longitudinal derivative along the gauge orbits, requires the minimal ghost spectrum
\[
\left( \eta_{i_1...i_{p-2k-1}}, \eta_{i_1...i_{p-2k-1}} \right), \quad k = 0, \ldots, a, \tag{84}
\]
(associated with (60), respectively, (55)),
\[
\left( \eta_{i_1...i_{p-2k-2}}, \eta_{i_1...i_{p-2k-2}} \right), \quad k = 0, \ldots, c, \tag{85}
\]
(associated with (72), respectively, (77)),
\[
\left( C_1^{i_1...i_{p-2k-2}}, C_2^{i_1...i_{p-2k-2}} \right), \quad k = 0, \ldots, c, \tag{86}
\]
(corresponding to (61) and (56)), and
\[
\left( C_1^{i_1...i_{p-2k-3}}, C_2^{i_1...i_{p-2k-3}} \right), \quad k = 0, \ldots, d, \tag{87}
\]
(corresponding to (73) and (68)). The above fields are fermionic and have the pure ghost number equal to one. The definitions of the longitudinal
derivative along the gauge orbits, $\sigma$, read as

$$
\sigma F = \sum_{\Delta=1}^{2} \left( \sum_{k=0}^{a} \left[ F, \gamma^{(\Delta)}_{i_{1} \cdots i_{p-2k-1}} \right] \eta^{i_{1} \cdots i_{p-2k-1}}_{\Delta} \right) + \sum_{k=0}^{c} \left[ F, \gamma^{(\Delta)}_{i_{1} \cdots i_{p-2k-2}} \right] \eta^{i_{1} \cdots i_{p-2k-2}}_{\Delta} + \\
\sum_{k=0}^{a} \left[ F, T^{(\Delta)}_{i_{1} \cdots i_{p-2k-2}} \right] C^{i_{1} \cdots i_{p-2k-2}}_{\Delta} + \sum_{k=0}^{d} \left[ F, T^{(\Delta)}_{i_{1} \cdots i_{p-2k-3}} \right] C^{i_{1} \cdots i_{p-2k-3}}_{\Delta},
$$

(88)

$$
\sigma G^{\Gamma} = 0,
$$

(89)

where $F$ is any function involving the original or newly added bosonic canonical pairs, and $G^{\Gamma}$ denote the minimal ghosts (84–87). The operator $\sigma$ is in this case strongly nilpotent. Extending $\sigma$ to the antighosts (75–78), generically denoted by $P^{\Gamma}$, through

$$
\sigma P^{\Gamma} = 0,
$$

(90)

and $\delta$ to the ghosts (84–87) by means of

$$
\delta G^{\Gamma} = 0,
$$

(91)

the homological perturbation theory [14] ensures that the irreducible BRST symmetry $s = \delta + \sigma$ exists and is nilpotent, $s^{2} = 0$. In conclusion, at this stage we constructed an irreducible BRST symmetry corresponding to the original reducible one. In the following we find its relationship with the standard reducible Hamiltonian BRST symmetry of the starting model.

### 2.4 Physical observables

Here, we establish the link between the reducible and irreducible BRST symmetries. In this light, we prove that the physical observables corresponding to the reducible, respectively, irreducible models coincide. We indicate below the line for $p$ even, the other situation being investigated in a similar way. Initially, we show that any observable associated with the irreducible theory is also an observable of the reducible system. Let $F$ be an observable of the irreducible model. Then, it fulfils the equations

$$
\left[ F, \gamma^{(1)}_{i_{1} \cdots i_{p-2k-1}} \right] \approx 0, \quad k = 0, \cdots, a,
$$

(92)

$$
\left[ F, T^{(1)}_{i_{1} \cdots i_{p-2k-2}} \right] \approx 0, \quad k = 0, \cdots, c
$$

(93)

$$
\left[ F, \gamma^{(2)}_{i_{1} \cdots i_{p-2k-1}} \right] \approx 0, \quad k = 0, \cdots, a,
$$

(94)
Equations (92) imply that

\[ F, T^{(2)}_{i_1 \ldots i_{p-2k-2}} \approx 0, \quad k = 0, \ldots, c, \]

(94)

\[ F, \gamma^{(1)}_{i_1 \ldots i_{p-2k-2}} \approx 0, \quad F, \gamma^{(2)}_{i_1 \ldots i_{p-2k-2}} \approx 0, \quad k = 0, \ldots, c, \]

(95)

\[ F, T^{(1)}_{i_1 \ldots i_{p-2k-3}} \approx 0, \quad F, T^{(2)}_{i_1 \ldots i_{p-2k-3}} \approx 0, \quad k = 0, \ldots, d. \]

(96)

Equations (92) imply that \( F \) does not depend (at least weakly) on any \( A^{0_1 \ldots 0_{p-2k-1}} \) or \( H^{0_1 \ldots 0_{p-2k-2}} \), while (95–96) ensure that \( F \) does not involve (at least weakly) any of the fields \( B^{(1)}_{1_1 \ldots 1_{p-2k-2}}, B^{(2)}_{1_1 \ldots 1_{p-2k-2}}, V^{(1)}_{1_1 \ldots 1_{p-2k-3}} \) or \( V^{(2)}_{1_1 \ldots 1_{p-2k-3}} \). Next, we explore the relations (93–94). We start from the last equations in (93–94) assuming that \( p \) is even

\[-2\partial_y^i [F (x), \pi_{ji} (y)] + M [F (x), \Pi_i (y)] - \partial_y^j [F (x), \pi (y)] \approx 0, \]

(97)

\[-\partial_y^j [F (x), \Pi_j (y)] - M [F (x), \pi (y)] \approx 0. \]

(98)

Applying \( \partial_y^i \) on (97), multiplying by \( M \) and adding the resulting equations, we arrive at \( \left( \partial_y^i \partial_y^i + M^2 \right) [F (x), \pi (y)] \approx 0 \), which yields

\[ [F (x), \pi (y)] \approx 0. \]

(99)

Replacing (99) back in (97, 98), these equations turn into

\[-2\partial_y^i [F (x), \pi_{ji} (y)] + M [F (x), \Pi_i (y)] \approx 0, \]

(100)

\[-\partial_y^j [F (x), \Pi_j (y)] \approx 0. \]

(101)

Taking into account the next equation from (94)

\[-3\partial_y^i [F (x), \Pi_{ij} (y)] - M [F (x), \pi_{ij} (y)] - \frac{1}{2} \left( \partial_y^i [F (x), \Pi_j (y)] - \partial_y^j [F (x), \Pi_i (y)] \right) \approx 0, \]

(102)

multiplied by \( \partial_y^i \) and employing (100, 101), we get \( \left( \partial_y^i \partial_y^i + M^2 \right) [F (x), \Pi_j (y)] \approx 0 \), hence

\[ [F (x), \Pi_j (y)] \approx 0. \]

(103)

Substituting the result (103) in (100) and (102), it follows

\[-2\partial_y^j [F (x), \pi_{ji} (y)] \approx 0, \]

(104)
Writing down the next equation from (93)

\[-3\partial_y^i [F(x), \Pi_{tij}(y)] - M [F(x), \pi_{ij}(y)] \approx 0. \tag{105}\]

on which we apply \(\partial_y^i\), and subsequently making use of (104–105), we finally deduce that

\[\left(\partial_y^i \partial_y^j + M^2\right) [F(x), \pi_{jk}(y)] \approx 0, \tag{107}\]

which then implies

\[\left[ F(x), \pi_{jk}(y) \right] \approx 0.\]

Inserting the previous result in (105–106) and going on with the procedure described above, we obtain

\[\left[ F(x), \pi_{i1\ldots ip-2k} \right] \approx 0, \quad k = 1, \ldots, b, \tag{108}\]

\[\left[ F(x), \Pi_{i1\ldots ip-2k-1} \right] \approx 0, \quad k = 1, \ldots, a, \tag{109}\]

which, replaced in the first equations from (93–94) (corresponding to \(k = 0\)), lead to

\[\left[ F, G^{(2)}_{i1\ldots ip-1} \right] \approx 0, \tag{110}\]

\[\left[ F, G^{(2)}_{i1\ldots ip-2} \right] \approx 0. \tag{111}\]

In (108) we employed the notation

\[b = \begin{cases} \frac{p}{2}, & \text{for } p \text{ even,} \\ \frac{p-1}{2}, & \text{for } p \text{ odd.} \end{cases} \tag{112}\]

The equations (108–109) show that \(F\) does not depend, at least weakly, on the fields \(A^{i1\ldots ip-2k}\) and \(H^{i1\ldots ip-2k-1}\), with \(k \geq 1\). As a consequence of our analysis, we managed to show that an observable \(F\) of the irreducible model does not depend (at least weakly) on the fields \(A^{0i1\ldots ip-2k-1}, H^{0i1\ldots ip-2k-2}\) (see (92) with \(k \geq 1\)), \((A^{i1\ldots ip-2k}, H^{i1\ldots ip-2k-1})_{k \geq 1}\) (see (103–104)), as well as on \((B^{(1)}j_1\ldots j_{p-2k-2}, B^{(2)}j_1\ldots j_{p-2k-2})_{k \geq 0}\), \((V^{(1)}j_1\ldots j_{p-2k-3}, V^{(2)}j_1\ldots j_{p-2k-3})_{k \geq 0}\) (see (95–96)), and, in addition, it satisfies the equations

\[\left[ F, \gamma^{(1)}_{i1\ldots ip-1} \right] \approx 0, \quad \left[ F, T^{(1)}_{i1\ldots ip-2} \right] \approx 0. \tag{113}\]
(see (110) with \( k = 0 \)) and (110–111), which are nothing but the equations verified by an observable of the redundant theory. Thus, we can conclude that any observable corresponding to the irreducible system stands for an observable of the original reducible model. The converse also holds, namely any observable of the redundant system remains so for the irreducible theory. This is because an observable \( \bar{F} \) of the original system checks the equations \( (110–111), (113) \) and does not depend on the newly introduced canonical pairs, such that \( (92-96) \) are automatically verified. In consequence, the two theories (reducible and irreducible) possess the same observables, such that the zeroth order cohomological groups of \( s_R \) and \( s_I \) coincide

\[
H^0 (s_R) = H^0 (s_I) .
\]

Thus, the irreducible and reducible theories are equivalent from the BRST formalism point of view, i.e., from the point of view of the basic equations underlying the BRST symmetry, \( s^2 = 0 \) and \( H^0 (s) = \{ \text{physical observables} \} \). This consideration yields the conclusion that we can replace the BRST quantization of the reducible model by that of the irreducible theory.

### 2.5 Irreducible path integral

Based on the last conclusion, we approach the Hamiltonian BRST quantization of the irreducible theory. The minimal antighost and ghost spectra are given in (75–78), respectively, (84–87). In addition, we further introduce the non-minimal sector

\[
\left( P^i_1...i_{p-2k-1} \bar{\eta}^{i_1...i_{p-2k-1}}, \ P^i_1...i_{p-2k-1} \right), \ k = 0, \ldots, a,
\]

\[
\left( P^i_1...i_{p-2k-1} b^{i_1...i_{p-2k-1}}, \ P^i_1...i_{p-2k-1} b^{i_1...i_{p-2k-1}} \right), \ k = 0, \ldots, a,
\]

\[
\left( P^i_1...i_{p-2k-2} \bar{C}^{i_1...i_{p-2k-2}}, \ P^i_1...i_{p-2k-2} \bar{C}^{i_1...i_{p-2k-2}} \right), \ k = 0, \ldots, c,
\]

\[
\left( P^i_1...i_{p-2k-2} \tilde{b}^{i_1...i_{p-2k-2}}, \ P^i_1...i_{p-2k-2} \tilde{b}^{i_1...i_{p-2k-2}} \right), \ k = 0, \ldots, c,
\]

The fields \( (116), (118) \) are all bosonic and possess ghost number zero, while \( (115), (117) \) are fermionic, the \( P \)'s having ghost number one, and the \( \bar{\eta} \)'s and \( \bar{C} \)'s displaying ghost number minus one. The ghost number is defined as the difference between the pure ghost number and the antighost number. The
non-minimal BRST charge, respectively, the BRST-invariant extension of $H'$ will consequently be expressed by

$$
\Omega = \int d^{d-1}x \left( \sum_{\Delta=1}^{2} \left( \sum_{k=1}^{p} \eta_{\Delta}^{i_{1} \ldots i_{p-k}} \gamma_{i_{1} \ldots i_{p-k}}^{(\Delta)} + \sum_{k=1}^{p-1} C_{\Delta}^{i_{1} \ldots i_{p-k-1}} T_{i_{1} \ldots i_{p-k-1}}^{(\Delta)} \right) + \sum_{k=0}^{a} \left( P_{\eta}^{i_{1} \ldots i_{p-2k-1}} b_{i_{1} \ldots i_{p-2k-1}} + P_{\eta}^{i_{1} \ldots i_{p-2k-1}} b_{1}^{1} \approx \sum_{k=0}^{c} \left( P_{C}^{i_{1} \ldots i_{p-2k-2}} b_{i_{1} \ldots i_{p-2k-2}} + P_{C}^{i_{1} \ldots i_{p-2k-2}} \tilde{b}_{i_{1} \ldots i_{p-2k-2}} \right) \right) \right),
$$

(119)

$$
H'_B = H' + \int d^{d-1}x \left( \sum_{k=0}^{c} \eta_{1}^{i_{1} \ldots i_{p-2k-2}} P_{2i_{1} \ldots i_{p-2k-2}} - \sum_{k=0}^{d} C_{1}^{i_{1} \ldots i_{p-2k-3}} P_{2i_{1} \ldots i_{p-2k-3}} + \frac{1}{p-1} \eta_{2}^{i_{1} \ldots i_{p-1}} \partial_{[i_{1}} P_{2i_{2} \ldots i_{p-1}]} + \sum_{k=1}^{a} \eta_{2}^{i_{1} \ldots i_{p-2k-1}} ((p-2k) \partial^{i} P_{2i_{1} \ldots i_{p-2k-1}} - M P_{2i_{1} \ldots i_{p-2k+1}} + \frac{1}{p-2} \partial_{[i_{1}} P_{2i_{2} \ldots i_{p-2k-2}]} + C_{1}^{i_{1} \ldots i_{p-2k-2}} \left( M P_{2i_{1} \ldots i_{p-2k-2}} + \frac{1}{p-2} \partial_{[i_{1}} P_{2i_{2} \ldots i_{p-2k-2}]} \right) + \sum_{k=1}^{c} C_{2}^{i_{1} \ldots i_{p-2k-2}} ((p-2k-1) \partial^{i} P_{2i_{1} \ldots i_{p-2k-2}} + M P_{2i_{1} \ldots i_{p-2k-2}} + \frac{1}{p-2} \partial_{[i_{1}} P_{2i_{2} \ldots i_{p-2k-2}]} ) - \sum_{k=0}^{c} \eta_{2}^{i_{1} \ldots i_{p-2k-2}} ((p-2k-1) \partial^{i} P_{2i_{1} \ldots i_{p-2k-2}} + M P_{2i_{1} \ldots i_{p-2k-2}} + \frac{1}{p-2k} \partial_{[i_{1}} P_{2i_{2} \ldots i_{p-2k-2}]} ) + \frac{1}{p-2k-2} \partial_{[i_{1}} P_{2i_{2} \ldots i_{p-2k-2}]} \right),
$$

(120)

We take the gauge-fixing fermion

$$
K = \int d^{d-1}x \left( P_{1i_{1} \ldots i_{p-1}} (\partial_{i} A_{i_{1} \ldots i_{p-1}} + M H_{i_{1} \ldots i_{p-1}} + \partial_{i} B^{(1) i_{1} \ldots i_{p-1}} ) + \right.
$$

21
\[\sum_{k=1}^{a} P_{1i_1 \ldots i_{p-2k-1}} \left( \partial_{i} B^{(1) i_{i_1} \ldots i_{p-2k-1}} + M V^{(1) i_{i_1} \ldots i_{p-2k-1}} + \partial_{i i_1} B^{(1) i_{2} \ldots i_{p-2k-1}} \right) +
\]
\[P_{1i_1 \ldots i_{p-2}} \left( \partial_{i} H^{i_{i_1} \ldots i_{p-2}} - MB^{(1) i_{i_1} \ldots i_{p-2}} + \partial_{i i_1} V^{(1) i_{2} \ldots i_{p-2}} \right) +
\]
\[\sum_{k=1}^{c} P_{1i_1 \ldots i_{p-2k-2}} \left( \partial_{i} V^{(1) i_{i_1} \ldots i_{p-2k-2}} - MB^{(1) i_{i_1} \ldots i_{p-2k-2}} + \partial_{i i_1} V^{(1) i_{2} \ldots i_{p-2k-2}} \right) +
\]
\[-(-)^{p+1} \sum_{k=0}^{c} P_{1i_1 \ldots i_{p-2k-2}} \left( \partial_{i} A^{i_{i_1} \ldots i_{p-2k-2}} - MA^{i_{i_1} \ldots i_{p-2k-3}} + \partial_{i i_1} H^{i_{i_2} \ldots i_{p-2k-3}} \right) +
\]
\[\sum_{k=0}^{a} D_{i}^{i_{i_1} \ldots i_{p-2k-1}} \left( P_{1i_1 \ldots i_{p-2k-1}} - \tilde{\eta}_{1i_1 \ldots i_{p-2k-1}} + \tilde{\eta}_{1i_1 \ldots i_{p-2k-1}} \right) +
\]
\[\sum_{k=0}^{a} b_{i}^{i_{i_1} \ldots i_{p-2k-1}} \left( \tilde{\eta}_{1i_1 \ldots i_{p-2k-1}} + \tilde{\eta}_{1i_1 \ldots i_{p-2k-1}} \right) +
\]
\[\sum_{k=0}^{c} b_{i}^{i_{i_1} \ldots i_{p-2k-2}} \left( C_{i_1 \ldots i_{p-2k-2}} + \tilde{C}_{i_1 \ldots i_{p-2k-2}} \right) +
\]
\[\sum_{k=0}^{c} b_{i}^{i_{i_1} \ldots i_{p-2k-2}} \left( P_{1i_1 \ldots i_{p-2k-2}} - \tilde{C}_{i_1 \ldots i_{p-2k-2}} + \tilde{C}_{i_1 \ldots i_{p-2k-2}} \right) \right), \quad (121)
\]

and obtain after some computation the path integral

\[Z_K = \int \mathcal{DA}^{\mu_1 \ldots \mu_p} \mathcal{D} H^{\mu_1 \ldots \mu_p} \left( \prod_{k=0}^{c} \mathcal{DB}^{(1) \mu_k \ldots \mu_{p-2k-2}} \right) \left( \prod_{k=0}^{d} \mathcal{DV}^{(1) \mu_k \ldots \mu_{p-2k-3}} \right) \times
\]
\[\left( \prod_{k=0}^{c} \mathcal{D} \tilde{\eta}_{1 \ldots \mu_{p-2k-2}} \mathcal{DC}^{\mu_1 \ldots \mu_{p-2k-2}} + \mathcal{DC}^{\mu_k \mu_{p-2k-3}} \right) \times
\]
\[\left( \prod_{k=0}^{a} \mathcal{DB}^{\mu_k \ldots \mu_{p-2k-1}} \mathcal{D} \eta_{1 \ldots \mu_{p-2k-1}} + \mathcal{D} \eta_{1 \ldots \mu_{p-2k-1}} \right) \exp i S_K, \quad (122)
\]

where

\[S_K = S_0^L + \int d^d x \left( - \sum_{k=0}^{a} \tilde{\eta}_{1 \ldots \mu_{p-2k-1}} \left( \square + M^2 \right) \eta_{1 \ldots \mu_{p-2k-1}} -
\]

\[22\]
\begin{align}
\sum_{k=0}^{c} C_{\mu_1, \ldots, \mu_{p-2k-2}} \left( \Box + M^2 \right) C^{\mu_1, \ldots, \mu_{p-2k-2}} + \\
b_{\mu_1, \ldots, \mu_{p-1}} \left( \partial_{\mu} A^{\mu_1, \ldots, \mu_{p-1}} + M H^{\mu_1, \ldots, \mu_{p-1}} + \partial^{\mu_1} B^{(1)} \mu_2, \ldots, \mu_{p-1} \right) + \\
\sum_{k=1}^{a} b_{\mu_1, \ldots, \mu_{p-2k-1}} \left( \partial_{\mu} B^{(1)} \mu_1, \ldots, \mu_{p-2k-1} + M V^{(1)} \mu_1, \ldots, \mu_{p-2k-1} + \\
\partial^{\mu_1} B^{(1)} \mu_2, \ldots, \mu_{p-2k-1} \right) + \tilde{b}_{\mu_1, \ldots, \mu_{p-2k-2}} \left( \partial_{\mu} H^{\mu_1, \ldots, \mu_{p-2}} + \\
MB^{(1)} \mu_1, \ldots, \mu_{p-2} + \partial^{\mu_1} V^{(1)} \mu_2, \ldots, \mu_{p-2} \right) + \\
\sum_{k=1}^{c} \tilde{b}_{\mu_1, \ldots, \mu_{p-2k-2}} \left( \partial_{\mu} V^{(1)} \mu_1, \ldots, \mu_{p-2k-2} - MB^{(1)} \mu_1, \ldots, \mu_{p-2k-2} + \\
\partial^{\mu_1} V^{(1)} \mu_2, \ldots, \mu_{p-2k-2} \right) \right)
\end{align}

(123)

and $S_0^L$ is given by \([\mathbb{I}]\). We remark that in deriving \((123)\) we realized the identifications

$$
B^{(1)} \mu_1, \ldots, \mu_{p-2k-2} \equiv \left( A^{01, \ldots, i_{p-2k-3}}, B^{(1)1, \ldots, i_{p-2k-2}} \right), \ k = 0, \ldots, c, 
$$

\(V^{(1)} \mu_1, \ldots, \mu_{p-2k-3} \equiv \left( H^{01, \ldots, i_{p-2k-4}}, V^{(1)1, \ldots, i_{p-2k-3}} \right), \ k = 0, \ldots, d, 

\(b_{\mu_1, \ldots, \mu_{p-2k-1}} \equiv \left( (p-2k-1) \pi_{1, \ldots, i_{p-2k-2}}, b_{1, \ldots, i_{p-2k-1}} \right), \ k = 0, \ldots, a, \)

\(\tilde{b}_{\mu_1, \ldots, \mu_{p-2k-2}} \equiv \left( (p-2k-2) \Pi_{1, \ldots, i_{p-2k-3}}, \tilde{b}_{1, \ldots, i_{p-2k-2}} \right), \ k = 0, \ldots, c, \)

\(\eta_{2}^{\mu_1, \ldots, \mu_{p-2k-1}} \equiv \left( i_{1, \ldots, i_{p-2k-2}}, \eta_{2}^{i, \ldots, i_{p-2k-1}} \right), \ k = 0, \ldots, a, \)

\(C_{2}^{\mu_1, \ldots, \mu_{p-2k-2}} \equiv \left( C_{2}^{i_1, \ldots, i_{p-2k-3}}, C_{2}^{i_1, \ldots, i_{p-2k-2}} \right), \ k = 0, \ldots, c, \)

\(\tilde{\eta}_{1, \ldots, \mu_{p-2k-1}} \equiv \left( - (p-2k-1) P_{1, \ldots, i_{p-2k-2}}, \tilde{\eta}_{1, \ldots, i_{p-2k-1}} \right), \ k = 0, \ldots, a, \)

\(\tilde{C}^{\mu_1, \ldots, \mu_{p-2k-2}} \equiv \left( - (p-2k-2) P_{1, \ldots, i_{p-2k-3}}, \tilde{C}_{1, \ldots, i_{p-2k-2}} \right), \ k = 0, \ldots, c. \)

In conclusion, we succeeded in deriving a gauge-fixed action with no residual gauge invariances without introducing the ghosts for ghosts, which moreover, is Lorentz covariant.

\[3\]
3 Interacting theories with Stueckelberg coupling

The treatment from Section 2 starts from the Lagrangian action (1), which is a quadratic action. In this section we study the possibility to perform an irreducible analysis in connection with interacting theories displaying Stueckelberg coupling. More precisely, we investigate what happens to our irreducible approach if we add to (1) some interaction terms which are invariant under the gauge transformations (2–3). An idea would be to realize the canonical analysis of the interacting theory and to further apply the irreducible treatment developed in the above. However, the interaction terms may contain higher order derivatives of the fields, such that the canonical analysis becomes intricate. In this context, it appears the question whether there exists a more direct irreducible method for interacting theories. As it will be seen in the sequel, the answer is affirmative. In this light, we prove that our irreducible Hamiltonian formalism induces an irreducible Lagrangian method that simplifies the approach to interacting theories. In view of this we investigate the gauge invariances of the Lagrangian action associated with the irreducible Hamiltonian formulation of abelian $p$- and $(p-1)$-forms with Stueckelberg coupling. From the point of view of the Hamiltonian BRST quantization, the split into primary and secondary constraints is not important. In this situation, what matters is the Hamiltonian gauge algebra. On the contrary, in order to obtain the gauge transformations of the Lagrangian action, it is necessary to distinguish between the primary and secondary first-class constraints. This is why we work with a model of Hamiltonian theory in the case of Stueckelberg coupling for which we assume that (60–61) and (72–73) are primary, while (55–56) and (67–68) are secondary constraints. The corresponding extended action takes the form

\[
S_{0}^{E} = \int d^{d}x \left( \sum_{k=0}^{b} \dot{A}^{\mu_{1}\cdots\mu_{p-2k}} \pi_{\mu_{1}\cdots\mu_{p-2k}} + \sum_{k=0}^{a} \dot{H}^{\mu_{1}\cdots\mu_{p-2k-1}} \Pi_{\mu_{1}\cdots\mu_{p-2k-1}} + \right.
\]

\[
\sum_{\Delta=1}^{2} \sum_{k=0}^{c} B^{(\Delta)}_{\mu_{1}\cdots\mu_{p-2k}} \pi^{(\Delta)}_{\mu_{1}\cdots\mu_{p-2k-2}} + \sum_{\Delta=1}^{2} \sum_{k=0}^{d} V^{(\Delta)}_{\mu_{1}\cdots\mu_{p-2k-2}} \Pi^{(\Delta)}_{\mu_{1}\cdots\mu_{p-2k-3}} - h - \right.
\]

\[
\sum_{\Delta=1}^{2} \sum_{k=0}^{a} U^{(\Delta)}_{\mu_{1}\cdots\mu_{p-2k-1}} \gamma^{(\Delta)}_{\mu_{1}\cdots\mu_{p-2k}} - \sum_{\Delta=1}^{2} \sum_{k=0}^{a} T^{(\Delta)}_{\mu_{1}\cdots\mu_{p-2k-2}} - \right.
\]

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\[ 
\sum_{\Delta=1}^{2} \sum_{k=0}^{c} u^{(\Delta)}_{1\ldots i_{p-2k-2}} \gamma^{(\Delta)}_{1\ldots i_{p-2k-2}} - \sum_{\Delta=1}^{2} \sum_{k=0}^{d} \bar{u}^{(\Delta)}_{1\ldots i_{p-2k-3}} f^{(\Delta)}_{1\ldots i_{p-2k-3}} \]  
(132)

where the \( u^{(\Delta)} \)'s and \( \bar{u}^{(\Delta)} \)'s denote the Lagrange multipliers of the corresponding constraints. From the extended action (132) we can obtain the so-called total action by setting all the multipliers of the type \( u^{(2)} \) and \( \bar{u}^{(2)} \) equal to zero. On the other hand, from the gauge transformations of (132) we can determine those of the total action by also taking the gauge variations of the \( u^{(2)} \)'s and \( \bar{u}^{(2)} \)'s equal to zero. Finally, from the gauge transformations of the total action we consequently arrive at the Lagrangian gauge invariances by eliminating the momenta and remaining multipliers on their equations of motion. In this way, starting from (132) we derive the corresponding Lagrangian gauge transformations

\[ 
\delta \epsilon A^{\mu_1\ldots \mu_p} = \partial [\epsilon^{\mu_1\ldots \mu_p}], 
\]  
(133)

\[ 
\delta \epsilon B^{(1)\mu_1\ldots \mu_p_{-2k-2}} = \partial [\epsilon^{\mu_1\ldots \mu_p_{-2k-2}}] + M \epsilon^{\mu_1\ldots \mu_p_{-2k-2}} + \partial \epsilon^{\mu_1\ldots \mu_p_{-2k-2}}, 
\]  
(134)

for \( k = 0, \ldots, c, \)

\[ 
\delta \epsilon H^{\mu_1\ldots \mu_p_{-1}} = \partial [\epsilon^{\mu_1\ldots \mu_p_{-1}}] + M \epsilon^{\mu_1\ldots \mu_p_{-1}}, 
\]  
(135)

\[ 
\delta \epsilon V^{(1)\mu_1\ldots \mu_p_{-2k-3}} = \partial [\epsilon^{\mu_1\ldots \mu_p_{-2k-3}}] + M \epsilon^{\mu_1\ldots \mu_p_{-2k-3}} + \partial \epsilon^{\mu_1\ldots \mu_p_{-2k-3}}, 
\]  
(136)

for \( k = 0, \ldots, d. \) The gauge parameters appearing in (133-136) are given by

\[ 
\epsilon^{\mu_1\ldots \mu_p_{-2k-1}} \equiv (\epsilon^{1\ldots i_{p-2k-2}}_{1\ldots i_{p-2k-1}}), \quad k = 0, \ldots, a, 
\]  
(137)

\[ 
\epsilon^{\mu_1\ldots \mu_p_{-2k-2}} \equiv (\epsilon^{1\ldots i_{p-2k-3}}_{1\ldots i_{p-2k-2}}), \quad k = 0, \ldots, c, 
\]  
(138)

with \( (\epsilon^{1\ldots i_{p-2k-2}}, \bar{\epsilon}^{1\ldots i_{p-2k-3}}) \) corresponding to the constraints (67), respectively, (68), and \( (\epsilon^{1\ldots i_{p-2k-1}}, \bar{\epsilon}^{1\ldots i_{p-2k-2}}) \) associated with (55), respectively, (56). The fields \( B^{(1)\mu_1\ldots \mu_p_{-2k-2}} \) and \( V^{(1)\mu_1\ldots \mu_p_{-2k-3}} \) are identified with

\[ 
B^{(1)\mu_1\ldots \mu_p_{-2k-2}} \equiv (A^{0i_1\ldots i_{p-2k-3}}, B^{(1)i_1\ldots i_{p-2k-2}}), \quad k = 0, \ldots, c, 
\]  
(139)

\[ 
V^{(1)\mu_1\ldots \mu_p_{-2k-3}} \equiv (H^{0i_1\ldots i_{p-2k-4}}, V^{(1)i_1\ldots i_{p-2k-3}}), \quad k = 0, \ldots, d. 
\]  
(140)

If we eliminate all the momenta, the Lagrange multipliers, all the fields carrying the superscript (2) and the fields \( (A^{1\ldots i_{p-2k-2}})_{k=0,\ldots,c}, (H^{1\ldots i_{p-2k-3}})_{k=0,\ldots,d} \)
on their equations of motion, we derive that the Lagrangian action resulting from (132) is identical with the original one, i.e.,

\[
S'_{L} \left[ A^{\mu_{1}...\mu_{p}}, H^{\mu_{1}...\mu_{p-1}}, B^{(1)\mu_{1}...\mu_{p-2k-2}}, V^{(1)\mu_{1}...\mu_{p-2k-3}} \right] = S_{L} \left[ A^{\mu_{1}...\mu_{p}}, H^{\mu_{1}...\mu_{p-1}} \right].
\] (141)

The theory based on action (141) and subject to the irreducible gauge transformations (133–136) represents the Lagrangian manifestation of the irreducible Hamiltonian model constructed in Section 2. The crucial feature of this irreducible Lagrangian theory is that it leads to (122–123) via the antifield BRST formalism by using an appropriate gauge-fixing fermion. The minimal solution to the master equation associated with the above irreducible Lagrangian theory is given by

\[
S = S_{L}^{0} \left[ A^{\mu_{1}...\mu_{p}}, H^{\mu_{1}...\mu_{p-1}} \right] + \int d^{d}x \left( A^{*}_{\mu_{1}...\mu_{p}} \partial^{[\mu_{1}} \eta^{\mu_{2}...\mu_{p}] + \right.
\]

\[
\sum_{k=0}^{c} B_{\mu_{1}...\mu_{p-2k-2}}^{(1)\ast} \left( \partial^{[\mu_{1}} \eta^{\mu_{2}...\mu_{p-2k-2}] - M C^{[\mu_{1}...\mu_{p-2k-2}]} + \partial_{\mu} \eta^{[\mu_{1}...\mu_{p-2k-2}] \right) +
\]

\[
H^{\ast}_{\mu_{1}...\mu_{p-1}} \left( \partial^{[\mu_{1}} C^{[\mu_{2}...\mu_{p-1}] + M T^{[\mu_{1}...\mu_{p-1}] \right) +
\]

\[
\sum_{k=0}^{d} V_{\mu_{1}...\mu_{p-2k-3}}^{(1)\ast} \left( \partial^{[\mu_{1}} C^{[\mu_{2}...\mu_{p-2k-3}] + M T^{[\mu_{1}...\mu_{p-2k-3} +
\]

\[
\partial_{\mu} C^{[\mu_{1}...\mu_{p-2k-3}] \right),
\] (142)

where \((\eta^{[\mu_{1}...\mu_{p-2k-1}]})_{k=0,...,c}\) and \((C^{[\mu_{1}...\mu_{p-2k-2}})_{k=0,...,c}\) represent the Lagrangian ghost number one ghosts, and the star variables stand for the antifields of the corresponding fields. Taking the non-minimal solution as

\[
S' = S - \int d^{d}x \left( \sum_{k=0}^{a} \tilde{\eta}^{[\mu_{1}...\mu_{p-2k-1}} \tilde{b}^{\mu_{1}...\mu_{p-2k-1} +
\]

\[
\sum_{k=0}^{c} \tilde{C}^{[\mu_{1}...\mu_{p-2k-2}} \tilde{b}^{\mu_{1}...\mu_{p-2k-2} \right),
\] (143)

and the gauge-fixing fermion of the form

\[
\psi = - \int d^{d}x \left( \sum_{k=0}^{a} \tilde{\eta}^{[\mu_{1}...\mu_{p-2k-1}} M^{[\mu_{1}...\mu_{p-2k-1} +
\]

\[
\sum_{k=0}^{c} \tilde{C}^{[\mu_{1}...\mu_{p-2k-2}} N^{[\mu_{1}...\mu_{p-2k-2} \right),
\] (144)
we get (122–123) modulo the identifications

\[ \eta^{\mu_1 \ldots \mu_{p-2k-1}} \equiv \eta_2^{\mu_1 \ldots \mu_{p-2k-1}}, \ C^{\mu_1 \ldots \mu_{p-2k-2}} \equiv C_2^{\mu_1 \ldots \mu_{p-2k-2}}. \quad (145) \]

In (144) the functions \( M \) and \( N \) read as

\[ M^{\mu_1 \ldots \mu_{p-1}} = \partial_\mu A^{\mu_1 \ldots \mu_{p-1}} + M H^{\mu_1 \ldots \mu_{p-1}} + \partial [\mu_1 B(1)_{\mu_2} \ldots \mu_{p-1}], \quad (146) \]

\[ M^{\mu_1 \ldots \mu_{p-2k-1}} = \partial_\mu B^{(1)\mu_1 \ldots \mu_{p-2k-1}} + MV^{(1)\mu_1 \ldots \mu_{p-2k-1}} + \partial [\mu_1 B(1)_{\mu_2} \ldots \mu_{p-2k-1}], \quad k \geq 1, \quad (147) \]

\[ N^{\mu_1 \ldots \mu_{p-2}} = \partial_\mu H^{\mu_1 \ldots \mu_{p-2}} - M B^{(1)\mu_1 \ldots \mu_{p-2}} + \partial [\mu_1 V(1)_{\mu_2} \ldots \mu_{p-2}], \quad (148) \]

\[ N^{\mu_1 \ldots \mu_{p-2k-2}} = \partial_\mu V^{(1)\mu_1 \ldots \mu_{p-2k-2}} + \partial [\mu_1 V(1)_{\mu_2} \ldots \mu_{p-2k-2}], \quad k \geq 1. \quad (149) \]

The fields \( \tilde{\eta}^{\mu_1 \ldots \mu_{p-2k-1}}, \tilde{C}^{\mu_1 \ldots \mu_{p-2k-2}}, \tilde{b}^{\mu_1 \ldots \mu_{p-2k-1}}, \tilde{c}^{\mu_1 \ldots \mu_{p-2k-2}} \) together with the attached antifields, which carry a star superscript, form the Lagrangian non-minimal sector. Hence, until now we proved that the irreducible Lagrangian version for the quadratic theory leads to the same path integral like the Hamiltonian one. Based on this result, we can simply solve the interacting case. If we add to action (1) any interaction terms that are gauge invariant under (2–3), the starting point of the irreducible approach is expressed by the interacting action, which is invariant under the gauge transformations (133–136). In this situation the non-minimal solution of the master equation can be obtained from (143) by adding the starting interaction terms. Using the same gauge-fixing fermion, namely, (144), we reach a gauge-fixed action that coincides with (123) apart from the starting Lagrangian action, which must include the gauge-invariant interaction pieces. This solves the interacting case discussed here.

### 4 Conclusion

To conclude with, in this paper we presented an irreducible BRST approach to interacting \( p \)-form gauge theories with Stueckelberg coupling. Our procedure includes two basic steps. The first one is relying on the irreducible
Hamiltonian analysis of the quadratic action describing Stueckelberg coupled $p$- and $(p-1)$-forms. The irreducible treatment is mainly based on some irreducible first-class constraints associated with the original reducible ones. The derivation of the irreducible constraint set is realized by requiring that all the antighost number one co-cycles of the Koszul-Tate differential identically vanish under an appropriate redefinition of the antighost number one antighosts, and, in the meantime, that the number of physical degrees of freedom should remain unchanged by passing to the irreducible context. The approach to interacting theories with Stueckelberg coupling is strongly related to the irreducible method developed for the quadratic action. Thus, beginning with the Hamiltonian gauge transformations generated by the irreducible first-class constraints we derive their Lagrangian version, which, in turn, is a good starting point for an irreducible BRST approach to interacting theories with Stueckelberg coupling.

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