Computations of Scattering Lengths in \textit{nnpp} System within Cluster Reduction Method for Yakubovsky Equations\textsuperscript{*}

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1 INTRODUCTION

The elastic and rearrangement processes in the four nucleon system with two clusters in the initial and final states can be treated adequately in framework of Yakubovsky differential equations (YDE) approach. Unfortunately, a direct application of YDE to the scattering problem requires huge computer resources. It is why, new methods allowing a reduction of complexity of YDE are of interests from the point of view of practical calculations. In papers [1], the authors proposed a method which reduces the YDE to the equations for the functions describing the relative motions of clusters. This method of cluster reduction (CRM) was successfully applied to calculations of $n$–$^3$H scattering in [1]. It is worth to note that the exact four nucleon calculations of $n$–$^3$H scattering were performed on a personal computer what can characterize an efficiency of CRM.

In this report we give a sketch of CRM and our recent results of calculations of scattering lengths in the four nucleon system.

2 CRM FORMALISM

In this section we present a brief review of CRM for the low energy scattering problem in the system of four neutral particles. The more comprehensive description can

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be found in the papers [1]. The generalization on the case of charged particles is straightforward following the results of [2].

The starting point is YDE approach. In framework of YDE the four body wave function should be decomposed into components \( \Psi_{a_3 a_2} \) in one to one correspondence to the all chains of three cluster \((a_3)\) and two cluster \((a_2)\) partitions. The components \( \Psi_{a_3 a_2} \) obey the YDE [3]

\[
(H_0 + V_{a_3} - E) \Psi_{a_3 a_2} + V_{a_3} \sum_{(c_3 \neq a_3) \subset a_2} \Psi_{c_3 a_2} = -V_{a_3} \sum_{d_2 \neq a_2} \sum_{(d_3 \neq a_3) \subset a_2} \Psi_{d_3 d_2} \tag{1}
\]

For the two clusters collisions the YDE admit a further reduction. Let

\[
H_0 = T_{a_2} + T^{a_2}
\]

be the separation of kinetic energy operator in the intrinsic \( T_{a_2} \) with respect to the clusters of \( a_2 \) part and the kinetic energy \( T^{a_2} \) of the relative motion of \( a_2 \) clusters. The cluster reduction procedure consists in expanding of the components \( \Psi_{a_3 a_2} \) along the basis of the solutions of Faddeev equations (FE) for subsystems of the partition \( a_2 \)

\[
(T_{a_2} + V_{a_3}) \psi_{a_2, k}^{a_3} + V_{a_3} \sum_{(c_3 \neq a_3) \subset a_2} \psi_{c_2, k}^{a_3} = \varepsilon_{a_2}^{a_3} \psi_{a_2, k}^{a_3}.
\]

The actual expansion has the form

\[
\Psi_{a_3 a_2} = \sum_{k \geq 0} \psi_{a_2, k}^{a_3} (x_{a_2}) F_{a_2}^{k} (z_{a_2}) \tag{2}
\]

Here the unknown amplitudes \( F_{a_2}^{k} (z_{a_2}) \) depend only on the relative position vector \( z_{a_2} \) between the clusters of the partition \( a_2 \). The basis of the solutions of FE is complete but not the orthogonal one [4] due to not Hermitness of FE. The biorthogonal basis is formed by the solutions of conjugated FE

\[
(T_{a_2} + V_{a_3}) \phi_{a_2, k}^{a_3} + \sum_{(c_3 \neq a_3) \subset a_2} V_{c_3} \phi_{a_2, k}^{a_3} = \varepsilon_{a_2}^{a_3} \phi_{a_2, k}^{a_3}.
\]

Introducing expansion (2) into YDE (1) and projecting onto the elements of biorthogonal basis \( \{ \phi_{a_2, k}^{a_3} \} \) lead to the resulting reduced YDE (1) for \( F_{a_2}^{k} (z_{a_2}) \):

\[
(T^{a_2} - E + \varepsilon_{a_2}^{a_3}) F_{a_2}^{k} = - \sum_{a_3 \subset a_2} \langle \phi_{a_2, k}^{a_3} | V_{a_3} \sum_{d_2 \neq a_2} \sum_{(d_3 \neq a_3) \subset a_2} \sum_{l \geq 0} \psi_{d_2, l}^{d_3} F_{d_2}^{l} \rangle. \tag{3}
\]

Here the brackets \( \langle .| . \rangle \) mean the integration over \( x_{a_2} \). The asymptotic boundary conditions for \( F_{a_2}^{k} (z_{a_2}) \) have the following two body form as \( |z_{a_2}| \rightarrow \infty \):

\[
F_{a_2}^{k} (z_{a_2}) \sim \delta_{k0} \delta_{a_2 b_2} \exp i (p_{a_2}, z_{a_2}) + A_{a_2 b_2} \frac{\exp i \sqrt{E - \varepsilon_{a_2}^{a_3}} |z_{a_2}|}{|z_{a_2}|}, \tag{4}
\]

where it is implied that the subsystems of partitions \( a_2 \) have only one bound state with \( \varepsilon_{a_2}^{0} \) being the respective binding energy. The subscript \( b_2 \) corresponds to the initial state and \( p_{a_2} \) is the conjugated to \( z_{a_2} \) momentum.
3 RESULTS of CALCULATIONS of SCATTERING LENGTHS in FOUR NUCLEONS SYSTEM

In this section we present the results of calculations of channels scattering lengths in \( N - NNN \) system (without Coulomb interactions) for the states with the total isospin \( T = 0 \), and results for scattering lengths in \( d-d \), \( n-^3\text{He} \) and \( p-^3\text{H} \) systems. The MT I-III potential model was used to describe the \( N-N \) interaction.

In the first case the total spin \( S \) and isospin \( T \) are the integrals of motion, so that the scattering lengths \( A_{ST} \) can be introduced. The results of our calculations together with data of other authors are collected in the Table 1.

Table 1. Results of calculations of \( A_{ST} \) for \( S = 0, 1, T = 0 \) states.

| Refs. | \( A_{10} \) fm | \( A_{00} \) fm |
|-------|----------------|----------------|
| This work | 2.8            | 14.7           |
| [5]    | 3.013          | 12.317         |
| [6]    | 3.09           | 14.95          |
| [7]    | 2.9            | 8.1            |
| [8]    | 3.25           | 14.75          |

The modified version of equations [8] followed from equations [2] was used for computations of scattering lengths in the \( nnpp \) system taking into account the Coulomb interaction between protons. The \( d-d \) and \( n-^3\text{He} \), \( p-^3\text{H} \) scattering lengths corresponding to the values of total spin \( S = 0, 2 \) for \( d-d \) scattering and \( S = 0, 1 \) for \( n-^3\text{He} \) and \( p-^3\text{H} \) scattering are collected in Tables 2,3 and 4.
Table 2. $d-d$ scattering lengths for $S = 0$ and $S = 2$ states.

| Refs.     | $A_{d-d}(S = 0)$ fm | $A_{d-d}(S = 2)$ fm |
|-----------|---------------------|---------------------|
| This work | $10.2 - 0.2 \, i$   | $7.5$               |

Table 3. $p-^3H$ and $n-^3He$ scattering lengths for $S = 0$ state.

| Refs. | $A_{p-^3H}$ fm. | $A_{n-^3He}$ fm. |
|-------|-----------------|------------------|
| This work | -22.6           | $7.5 - 4.2 \, i$ |
| [9]    | -21             | $7.16 - 3.9 \, i$ |
| [7]    | 4.2             | $6.05 - 0.72 \, i$ |
| Exp. [10] | -               | $6.53 \, (\pm 0.32) - 4.445 \, (\pm 0.003) \, i$ |

Table 4. $p-^3H$ and $n-^3He$ scattering lengths for $S = 1$ state.

| Refs. | $A_{p-^3H}$ fm. | $A_{n-^3He}$ fm. |
|-------|-----------------|------------------|
| This work | 0.5             | $2.9 - 0.0 \, i$ |
| [4]    | -               | $4.25 + 0.005 \, i$ |

The modified equations (3) were used to calculate the $0^+$ resonance state of $^4He$. The results of calculations are presented in the Table 5.
Table 5. Results of calculations for the energy $E = E_r + i\Gamma_2$
of $0^+$ resonance state of $^4$He.

| Refs.       | $E_r$, MeV. | $\Gamma$, MeV. |
|-------------|-------------|-----------------|
| This work   | 0.15        | 0.3             |
| [9]         | 0.12        | 0.26            |
| Exp. [11]   | 0.25        | 0.3             |

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