Quantum mechanics and EPR paradox
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Abstract

The orthodox quantum mechanics has been commonly regarded as being supported decisively by the polarization EPR experiments, in which Bell’s inequalities have been violated. The given conclusion has been based, however, on several mistakes that have not been yet commonly known and sufficiently analyzed. The whole problem will be newly discussed and a corresponding solution will be proposed.

1. Introduction

It is commonly believed that the Copenhagen interpretation of quantum mechanical model represents the only possibility of describing physical processes of microworld. The main support for such statement is seen in the results of experiments proposed in principle by Einstein, Podolsky and Rosen in 1935 [1]. The way to their interpretation in agreement with Copenhagen school has been, however, paved at least by three mistakes.

The first one coming from von Neumann [2] in 1932 has been step-by-step discovered already earlier (G. Hermann in 1935 [3], D. Bohm in 1952 [4], J. Bell in 1964 [5]). However, the other two mistakes have not been known and discussed to a sufficient extent until now. One of them relates to Bell’s inequalities as their derivation has been based on one assumption, the impact of which has not been generally recognized; the assumption being hardly acceptable for the common type of polarization EPR experiments. It has not been possible to derive these inequalities without such an assumption. The corresponding analysis of the problem may be found in Ref. [6]. As to the third mistake (see the book of Belifante [7]) it has had probably yet more important impact on the conviction of the most physicists. F. Belifante argued in 1973 that practically any hidden-variable theory was to provide significantly different predictions from those derived with the help of standard quantum-mechanical model, which is not true. It will be shown in the following that already a very simple hidden-variable theory gives practically the same predictions for a pair of polarizers (including EPR coincidence experiments) as the standard quantum mechanics.

Some important differences between hidden-variable interpretation and quantum mechanics should exist, of course, e.g., in the light transmission through three polarizers. Such experiments inspired by preliminary theoretical results were performed and published in 1993 and 1994 (see [8, 9]); they have shown that the standard quantum-mechanical theory of polarized light should be regarded in principle as falsified by these experimental results. A way of interpreting these experimental results on a new basis should be looked for.

The problem concerning the derivation of Bell’s inequalities will be summarized shortly in Sec. 2. The misleading argument of Belifante will be discussed in Sec. 3; a very simple
hidden-variable model of light transmission through a polarizer pair will be described. A generalized model taking into account a polarization shift (towards the polarizer axis) during the light passage through a polarizer will be then proposed in Sec. 4. The transmission characteristics (for individual polarizers) derived to be in agreement with the Malus law will be then used in predicting transmission characteristics for a triple of polarizers (Sec. 5); the results will be compared to quantum-mechanical predictions. The consequences following from the experimental data obtained with three polarizers will be discussed in Sec. 6.

2. Light transmission through a polarizer

The transmission of light through a polarizer has been studied since the beginning of the 19th century. It was found that the intensity of unpolarized light passing through two polarizers was decreasing according to Malus law, i.e. as

\[ m(\alpha) = (1 - \varepsilon) \cos^2 \alpha + \varepsilon \]

where \( \alpha \) was the angle between polarizer axes. It was assumed that \( \varepsilon = 0 \) at least for the so called ideal polarizers; in any case \( \varepsilon \ll 1 \).

Einstein argued in the thirtieths years that the quantum mechanics was not a complete theory and that some more detailed characteristics were necessary to be added to describe fully a microscopic object. However, physical community had not accepted his critical point of view. The main support for standard quantum mechanics was seen at that time undoubtedly in the "proof" of von Neumann that any "hidden" variables were excluded by the quantum-mechanical model. It was not taken into account, either, that already in 1935 Grete Herrmann [3] showed that the approach of von Neumann was practically a "circle proof". The argument of D. Bohm [4] that a hidden variable was contained already in Schrödinger equation was accepted seriously by a very small number of the then physicists. Only the approach of J. Bell [5] met with greater attention, especially since formulas were presented that seemed to enable bringing a decision between the two (orthodox and ensemble) interpretations of the quantum-mechanical model on experimental basis.

However, in a broad physical community there has not been any interest to change a generally accepted paradigm. Any greater doubts about the standard theory have not been evoked from the fact, either, that it was navigated from the very beginning by the mistake of von Neumann. The firm belief in the so called EPR paradoxes seems to live still in a great part of physical community. The main reason may be seen in that they have been supported seemingly by other two already mentioned arguments, both being false.

The first of these two arguments has related (as already mentioned) to Bell's inequalities that have been believed to hold for any hidden-variable alternative. However, their application to the current coincidence polarization experiments cannot be regarded as regular. In their derivation a seemingly self-evident assumption has been made use of. To derive these inequalities it has been necessary to interchange always the transmission probabilities for photons belonging to different photon pairs, which has been equivalent to assuming for transition probabilities of individual photons to be independent of the
impact point into the internal (plane grid) polarizer structure; or to regarding the corresponding measuring devices in principle as at least half-black boxes. Detailed analysis of the problem may be found in [3].

Bell’s inequalities cannot be derived without the given assumption. Consequently, the violation of Bell’s inequalities in the common EPR experiments does not provide any argument against a hidden-variable alternative, as they do not correspond to a fully consistent hidden-variable (realistic) description.

However, as already mentioned there has been another mistake more that has had probably a yet more important impact in influencing the attitude of physical community towards the belief in EPR paradoxes, which will be discussed in the next section.

3. Malus law and photon transmission through one polarizer

Belifante argued in his book [7] that the standard quantum mechanics and a hidden-variable theory should lead to quite different predictions as to current EPR experiments. However, such a statement has not been true, which will be now demonstrated. The angle dependence of light transmission through a polarizer pair in a hidden-variable theory may be expressed as

\[ p_2(\alpha) = \int_{-\pi/2}^{\pi/2} \bar{p}_1(\lambda, 0) \bar{p}_2(\lambda, \alpha) \, d\lambda \]  

where \( \alpha \) is again the angle between the axes of polarizers and \( \lambda \) is photon polarization; \( \bar{p}_j(\lambda, \alpha) \) being transmission probability of a photon characterized by \( \lambda \) polarization through a polarizer deviated by angle \( \alpha \) from the same zero direction. Belifante has chosen quite arbitrarily

\[ \bar{p}_j(\lambda, \alpha) = p_1(\lambda - \alpha) \sim \cos^2(\lambda - \alpha), \]

which has led to fundamental deviations of \( p_2(\alpha) \) from the Malus law (1).

However, the problem should have to be solved in opposite way. The question has been, which function \( p_1(\lambda) \) corresponds to the Malus law. The actual solution of the problem is represented by the full line in Fig. 1. The given curve may be described, e.g., by the formula

\[ p_1(\lambda - \alpha) = [1 - \phi(|\lambda - \alpha|)] \]

where

\[ \phi(\gamma) = [1 - \exp(-a\gamma^e)]/[1 + c \exp(-a\gamma^e)]; \quad a, e, c > 0. \]

The transmission probability represented by the full line in Fig. 1 is given by the following values of free parameters in Eq. (3):

\[ a = 1.74, \quad e = 3.78, \quad c = 200. \]

The light distribution around the axis outgoing from the first polarizer is characterized by \( d(\lambda) \); comp. similar function obtained under more general conditions and shown in Fig. 2.

The dependence \( p_2(\alpha) \) of light transmission through a polarizer pair on mutual angle values corresponds to the generalized Malus law for higher values of \( \alpha \) very well; see
As to the deviations at smaller angles there have not been suitable data for a detailed comparison; it would be very interesting to perform a thorough comparison of theoretical predictions in the whole angle range. In any way, we must conclude that there is not surely any significant difference in predictions for available EPR coincidence measurements; Belifante’s graph must be denoted as false.

A better agreement with Malus law may be obtained with the help of probability function \( p_1(\lambda) \) represented by a greater number of free parameters. However, even in such a case some greater deviations remain in the region of small angles, which might indicate that or the Malus law is not fully exact at the given angles or some mechanism exists that changes polarization of a photon passing through a polarizer. The latter possibility will be followed in Sec. 4.

4. Generalized transmission model

In Sec. 3 we have assumed that the spin or polarization of a photon does not change its direction in passing through a polarizer. However, some data seem to indicate that polarization may shrink to polarizer axis more than given by mere transmission probabilities. Let us consider now such a possibility.

We will assume that \( \lambda \)-distribution of photons outgoing from the first polarizer is not given by the function \( d(\lambda) = \frac{p_1(\lambda)}{\pi/2} \) (see Fig. 1), but that it is given by

\[
d(\lambda) = \int_{-\pi/2}^{\pi/2} p_1(\lambda')c(\lambda, \lambda')d\lambda' \quad (6)
\]

where

\[
\int_{-\pi/2}^{\pi/2} c(\lambda, \lambda')d\lambda = 1 ; \quad (7)
\]

i.e. that a photon having had original polarization \( \lambda' \) has gained polarization \( \lambda \) with the probability \( c(\lambda, \lambda') \geq 0 \). It holds then

\[
p_2(\alpha) = \int_{-\pi/2}^{\pi/2} d(\lambda) \ p_1(\lambda - \alpha) \ d\lambda \quad (8)
\]

We have chosen the following parameterization for probability function:

\[
c(\lambda, \lambda') = A_\sigma(\lambda')e^{-\sigma(\lambda - \lambda_e)^2} \quad (9)
\]

where \( A_\sigma(\lambda') \) is normalization coefficient (guaranteeing the validity of Eq. (7)) and

\[
\lambda_e = \lambda[1 + \epsilon(\eta - \lambda)] , \quad 0 < \lambda \leq \eta , \quad (10)
\]

\[
\lambda_e = \frac{\pi}{2} - (\frac{\pi}{2} - \lambda)[1 + \epsilon(\lambda - \eta)] , \quad \eta < \lambda < \pi/2 ; \quad (11)
\]

\( \sigma, \epsilon \) and \( \eta \) are free parameters; \( \sim \pi/4 < \eta < \pi/2 \). We have assumed that a greater part of \( \lambda \)-polarizations shrinks towards the polarizer axis while a rest keeps around a perpendicular direction.
The fit obtained under such conditions is shown in Fig. 2. The given results corresponds to the following values of free parameters

\[ \sigma = 40.5, \quad \epsilon = 0.40, \quad \eta = 1.38, \]
\[ a = 2.38, \quad b = 2.54, \quad c = 186.8. \]

It holds also for total light transmissions
\[ \frac{I_1}{I_0} = \int_{-\pi/2}^{\pi/2} d(\lambda) d\lambda = 0.496, \quad \frac{I_2}{I_0} = \int_{-\pi/2}^{\pi/2} d(\lambda) d\lambda = 0.482. \]

5. Light transmission through a triple of polarizers

Both the theories (quantum mechanics and hidden-variable theory) give practically the same predictions for light transmission through a pair of polarizers. However, one must expect that these predictions may significantly differ in other experiments, e.g., for the transmission of light through three polarizers. To analyze this case we will return to the simpler version introduced in Sec. 3 (without a polarization change during the passage) and try to derive corresponding characteristics of light transmitted through three polarizers. In such a case it is possible to write (in hidden-variable alternative)

\[ I(\alpha, \beta) = \int_{-\pi/2}^{\pi/2} \tilde{p}_1(\lambda) \tilde{p}_2(\lambda - \alpha) \tilde{p}_3(\lambda - \beta) d\lambda \]  

(12)

where \( \alpha \) and \( \beta \) are angle deviations of the second and third polarizers to the axis of the first polarizer. We will assume that the transmission probabilities \( \tilde{p}_i(\lambda) = p_i(\lambda) \) are characterized by parameters derived in Sec. 3. The polarization shrinkage during light passage through a polarizer will be neglected.

According to standard quantum mechanics holding for ideal polarizers (or to electromagnetic light theory) it should hold

\[ I(\alpha, \beta) = \cos^2 \alpha \cdot \cos^2(\alpha - \beta). \]  

(13)

The comparison of predictions by both the theories has been given in Fig. 3. The dependence of light intensity on \( \alpha \) (for \( \beta = 0 \)) indicates that measurable differences should exist surely around \( \alpha \simeq 50 - 75^o \). It means that a decision between these two theoretical alternatives might be given on experimental grounds. There is not any doubt that one should come to reliable conclusions concerning the validity of individual theoretical alternatives when the experimental measurement is performed with the polarizers exhibiting very small value of parameter \( \epsilon \) in Eq. (1), being near to the so called ideal polarizers.

6. Experimental data with three polarizers

The experimental data enabling to compare the results derived in the preceding section have been published already earlier (see Refs. [8, 9]). Some consequences of these results
have been mentioned in Ref. [10]. The experimental results do not seem, however, to correspond well to any of the given predictions given in Sec. 5. Anyway, they are in a strong disagreement with quantum-mechanical predictions, which should be a challenge of looking for a new theoretical explanation of polarization phenomena.

A full agreement has not been obtained with the results of Mueller calculus based on the old phenomenological theory proposed by Stokes, either, even if the data and predictions have exhibited some similar features (see Ref. [9]). A more detailed analysis of the given experimental data will be given elsewhere later.

7. Conclusion

One can conclude that some mistakes have influenced the way to the contemporary theory of microscopic physical world, which concerns also the description of polarization phenomena. Having removed these mistakes one is forced to look for a better description, especially, of data concerning experimental results with different numbers of polarizers; with the help of a suitable kind of hidden-variable models.

It has been shown in Sec. 3 that a good approximate description may be obtained already with a very simple model; it has been assumed that photon passing through a polarizer does not change its polarization (or direction of its spin). A much better agreement with generalized Malus law may be obtained if some shrinkage of polarization to polarizer axes occurs during the light passage through a polarizer. Anyway, hidden-variable alternative seems to open good possibilities of explaining all experimentally established polarization phenomena.

References

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Figure 1: Transmission probability through a polarizer pair leading to Malus law: $p_1(\lambda)$ - dashed line; $p_2(\lambda)$ - dotted line; $d(\lambda)$ - full line; Malus law $m(\lambda)$ - individual points.

Figure 2: Extended model of transmission probability through a polarizer pair leading to Malus law: $p_1(\lambda)$ - dashed line; $p_2(\lambda)$ - dotted line; $d(\lambda)$ - full line; Malus law $m(\lambda)$ - individual points.
Figure 3: Light transmission through three polarizers (dependence on $\alpha$ at $\beta = 0$); full line - hidden-variable alternative, dashed line - quantum mechanics.