Quantum electrodynamics of a superconductor-insulator phase transition

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A chain of Josephson junctions implements one of the simplest many-body models undergoing a superconductor-insulator quantum phase transition between states with zero and infinite resistance \[1-5\]. This phenomenon is central to our understanding of interacting bosons and fermions in one dimension \[6\]. Apart from zero resistance, the superconducting state is always accompanied by a sound-like mode due to collective oscillations of the phase of the complex-valued order parameter \[7-10\]. Surprisingly little is known about the fate of this mode upon entering the insulating state, where the order parameter’s amplitude remains non-zero, but the phase ordering is “melted” by quantum fluctuations \[11\]. Here we report momentum-resolved radio-frequency spectroscopy of collective modes in nanofabricated chains of Al/AlOx/Al tunnel junctions. Our key finding is that the GHz-frequency modes survive far into the insulating regime, i.e. an insulator can superconduct AC currents. The insulating state manifests itself by a weak decoherence of collective modes with an unusual frequency dependence: longer wavelengths decohere faster, in fact suggesting the absence of DC transport. Owing to an unprecedentedly large kinetic inductance per unit length, the observed phase mode represents microwave photons with a remarkably low speed of light (below \(8 \times 10^5\) m/s) and high wave impedance (above 23 kΩ). The latter exceeds the transition value for the Bose glass insulator, expected in this system, by an order of magnitude \[12-14\], which challenges theory to revisit the finite-energy condensate dynamics near the transition. More generally, the high impedance translates into a fine structure constant exceeding a unity, opening access to previously impossible regimes of quantum electrodynamics \[15\].

Our devices consist of two closely spaced parallel chains of over 33,000 junctions fabricated on an insulating silicon chip (Fig. 1a,b). The chains are short-circuited at one end and connected to a dipole antenna at the other end for coupling external signals. The chip is suspended in the center of a metallic waveguide box with a single broadband microwave input/output port (Fig. 1c). This wireless interface minimizes parasitic ground capacitances seen by the junctions and allows collection of all the energy radiated off-chip. In the superconducting state, the device can be viewed as a telegraph transmission line \[16\] defined by the capacitance \(c\) between the chains and the Josephson inductance \(l\) per unit length (Fig. 1d). The capacitance adds inertia to the phase degree of freedom and its value is approximately given by the vacuum permittivity \(\epsilon_0\) adjusted by the dielectric constant of silicon. The inductance \(l\) plays the role of the inverse phase-rigidity of the condensate and it can significantly exceed the vacuum permeability \(\mu_0\). The “sound” waves associated with the collective phase oscillations across the junctions are equivalent to the transverse electromagnetic modes (one-dimensional photons) of the transmission line with a velocity \(v = 1/\sqrt{\epsilon c}\) and a wave impedance \(Z = \sqrt{1/c}\).

In close analogy with vacuum quantum electrodynamics, zero-point fluctuations of fields in our onedimensional system are controlled by the effective fine structure constant \(\alpha = Z/R_Q\), where \(R_Q = h/(2e)^2 \approx 6.5\) kΩ is the resistance quantum for Cooper pairs \[17\]. The superconducting state is favored for \(\alpha \ll 1\), when the line mimics the usual weak-coupling electrodynamics of the free space, for which \(\alpha = 1/137.0\). This is not a coincidence: at a given frequency the Josephson relation links the fluctuation of phases across the junctions with the fluctuation of electric field between the chains, which in turn defines the strength of light-matter coupling. The model of a single chain coupled to a ground plane (Fig. 1e) predicts a transition to the gapped Mott insulator phase at \(\alpha_{\text{Mott}} \approx 1/4\) \[2\]. The transition belongs to the celebrated Berezinski-Kosterlitz-Thouless (BKT) type and is driven by the proliferation of \(2\pi\)-slips (instantons) in the phases across the junctions \[13\]. A more realistic model should include an oxide capacitance across every junction and a random offset charge at every island. In this case the chain is modeled using a general framework of disordered Tomonaga-Luttinger liquids with the fine structure constant \(\alpha\) replacing the inverse Luttinger interaction parameter \[13\]. Here theory predicts a transition to a compressible Bose glass insulator state at a slightly elevated value of the fine structure constant \(\alpha_{\text{BG}} = 1/3\) (\(Z = 2.2\) kΩ) \[12\].

The goal of this work is to explore for the first time how the superconductor-insulator transition manifests itself
in the propagation of electromagnetic waves in progressively higher impedance transmission lines. The minimal circuit model for our double-chain line (Fig. 1f) includes the Josephson energy $E_J$, the charging energy $E_C = e^2/2C_J$ due to the oxide capacitance $C_J$, and the inter-chain charging energy $E_0 = e^2/2C_0$, where $C_0$ is the per junction capacitance between the two chains. Introducing the size $a$ of the junction, the propagation parameters are now defined as $l \times a = 2(\hbar/2e)^2/E_J$ and $c \times a = C_0$. The junction plasma frequency $\omega_p = \sqrt{8E_JE_C/\hbar}$ defines the ultra-violet cut-off in our system. With these parameters, the phase transition boundary formulated for single chains should be applicable to our devices [19].

The example results of momentum-resolved spectroscopy are shown in Fig. 2. The experiment is performed using a standard two-tone dispersive reflectometry, taking advantage of the weak Kerr non-linearity of a Josephson junction [20] (Methods). Data reveals an ordered set of discrete resonances which we associate with the standing wave modes of the transmission line (Fig. 2b). By indexing the individual resonances and plotting the frequency as a function of index $n = 1, 2, \ldots$, we obtain the dispersive relation $\omega_n(k_n)$, where $k_n$ is the wavenumber defined as $k_{n+1} - k_n = \pi/L$, and $L = 10$ mm is the length of the line (Fig. 2b). The dispersion is in excellent agreement with a simple two parameter expression $\omega(k) = v k/\sqrt{1 + (v k/\omega_p)^2}$, describing ultra-slow photons with a velocity $v = 1.88 \times 10^6$ m/s and a band edge at the plasma frequency $\omega_p/2\pi = 24.8$ GHz. The $n = 1$ mode is clearly visible at 40 MHz. This frequency is almost three orders of magnitude below the plasma edge and several times lower than the thermal frequency associated with the 10 mK temperature of our setup. In addition, the $n = 1$ mode frequency is half the mode spacing, which correctly reflects the additional $\pi$ phase-shift due to the short-circuit boundary condition. This observation confirms that wave propagation occurs along the entire length of the system and the spectrum is gapless. Fluctuations of the mode spacing as a function of mode index are found to be within $5 - 10\%$ up to a wavelength as low as $L/100$ (Fig. 2c), showing no signals of Anderson localization [10].

Combining the measured values of $\omega_p$, $v$, and the known dimensions of the chains, we reliably extract the wave impedance along with other chain parameters in multiple devices (Suppl. Mat.). For the device from Fig. 2, we get $Z = 11.7 \, k\Omega$. This is equivalent to a strikingly
large value of the fine structure constant $\alpha = 1.8$, exceeding the theoretical transition value by more than a factor of 5. Yet, this large value of $\alpha$ is fully consistent with the individual junction parameters expected from the fabrication process. Spectroscopy thus concludes that down to a frequency of $\omega_p/600$ the propagation of collective modes is completely unaffected by the insulating phase.

To reveal the insulating phase we explore the decoherence of the collective modes in devices with progressively higher wave impedance (Fig. 3). This requires an accurate measurement of the real and imaginary parts of the reflection coefficient at the single-photon excitations. For technical reasons, this was possible in our setup in approximately $4 - 12$ GHz band. The higher impedance was achieved by reducing the chain width while keeping all other dimensions the same (Fig. 1b). We define the mode quality factor $Q$ as the ratio of mode frequency to its linewidth after subtracting the small contribution due to the radiation of photons into the measurement port (Methods). For devices with $Z \lesssim 12$ k$\Omega$ the quality factor grows with reducing the normalized frequency $\omega/\omega_p$ independently of other device parameters (Fig. 3 - red and orange markers; see also Suppl. Mat.). Towards the lower end of the band, it reaches a relatively high value of $Q = 10^5$. The suppression of the $Q$-factor towards the plasma frequency is likely linked to the presence of a large (of order $10^4$) number of modes at $\omega \approx \omega_p$.

The key observation of our experiment is that this dependence of $Q$-factor on frequency undergoes a remarkable reversal in weaker junction chains (Fig. 3 - blue and violet markers). For some intermediate chain width (device “c”) the $Q$-factor becomes flat in frequency and it develops a clear tendency to drop towards lower frequencies in smaller width chains (device “d” and “e”). In other words, in sufficiently weak junction chains, the longer the wavelength of an excitation, the faster it decoheres. Such a behavior is highly unusual for materials-related loss, but it is consistent with the insulator phase: extrapolating to zero frequency, the observed decoherence would indeed inhibit DC transport.

For devices with $E_{J}/E_{C} < 7$ (devices “c” and “f”) the mode spacing fluctuations are dramatically enhanced along with device-to-device variations (Suppl. Mat.), making it difficult to accurately recover the chain parameters. Nevertheless, the $Q$-factor in such devices can still exceed $10^3$. Interestingly, reducing the wave impedance $Z$ by about 20% without modifying the junction dimensions – achieved by shrinking the spacing between the chains from 10 $\mu$m to 2 $\mu$m – showed no effect on the $Q$-factor in these devices. This observation suggests that the observed decoherence is much less sensitive to impedance $Z$ than to the $E_{J}/E_{C}$ ratio.

As a control experiment, we demonstrate the reversible transition between the “superconducting” and “insulating” frequency dependence of the $Q$-factor in a single device (Fig. 4). A fresh Josephson transmission line was fabricated with relatively small value of $E_{J}/E_{C}$ such that it still shows $Q$-factor growing towards the low frequency. The device is then aged for about 1000 hours at ambient conditions, which reduces $E_{J}$ by about 25%. This is confirmed by the mode spacing data clearly showing about a 10% reduction in both $v$ and $\omega_p$, without a dramatic enhancement of frequency disorder. This turned out to be enough to observe the reversed, “insulating”
frequency dependence of $Q$. In a final step, we annealed the device on a hot plate, also at ambient conditions, which recovered the fresh value of $E_J$ together with the original “superconducting” frequency dependence of $Q$. Note that at $\omega/2\pi \approx 5$ GHz the quality factor underwent a remarkable swing by an order of magnitude. We confirm that same aging test performed on devices with $E_J/E_C > 70$ had no effect on the decoherence.

The high sensitivity of the quality factor on both $E_J$ and $E_J/E_C$ signals that the observed decoherence is probably caused by the scattering of photons on the phase-slip fluctuations [18]. Phase-slips are indeed the key ingredient behind the quantum BKT transition [14]. The main result of our experiment is that, whether elastic or inelastic, the scattering of photons on phase-slips is frequency-dependent and can be dramatically suppressed at GHz-frequencies. Consequently, we observed low-loss wave propagation in transmission lines with a measured wave impedance over 23 k$\Omega$ ($\alpha > 3.5$), an order of magnitude above the theoretical transition value.

Can the lack of insulating behavior in our chains be merely a finite size effect despite the large number of junctions $N > 33,000$? A recent transport measurement on chains an order of magnitude shorter than ours reported strongly insulating behavior of resistance for $E_J/E_C < 16$ [21]. Another recent experiment, also on chains an order of magnitude shorter than ours, reported voltage gaps as large as 0.5 meV (equivalent to a frequency of 120 GHz) for $E_J/E_C = 5$ [14]. Furthermore, the scaling of breakdown voltages agreed with the Bose glass insulator predictions, i.e. $\alpha_c = 1/3$. In our experiment, the characteristic frequency $\omega_S$ associated with the phase-slip process can be estimated in the limit $\alpha \gg 1$. Mott insulator scenario assumes constructive interference of phase slips at every junction, such that $\omega_S/\omega_p \approx N \exp(-\sqrt{8E_J/E_C})$. Taking $E_J/E_C = 10$ (Fig. 1), we get $\omega_S/2\pi \approx 100$ GHz, which seems inconsistent with microwave propagation. In a Bose glass insulator, by contrast, phase-slips add with random phases, caused by offset charges, and hence $N \rightarrow \sqrt{N}$. This yields a more reasonable estimate $\omega_S/2\pi = 500$ MHz. However, even this frequency is two orders of magnitude larger than the measured linewidth of collective modes. We conclude that the non-zero frequency of the photon plays a key role in decoupling it from phase-slips. Although superconductivity of finite-wavelength excitations by an insulator does not contradict the renormalization group view of a phase transition [11], theory of the observed decoherence is missing.

Collective mode spectroscopy measurement presented here can be used to explore superconductor-insulator transition in a broad variety superconducting nanowires and thin films, where many fundamental questions remain unresolved from transport data [22–27]. The availability of a practically infinite one-dimensional media with the fine structure constant $\alpha > 1$ and a speed of light reduced by over two orders of magnitude is a unique resource for quantum technology. It can be used to explore extreme regimes of light-matter coupling using superconducting circuits [28–29], quantum dots [30], spin qubits [31], and possibly trapped Rydberg atoms or polar molecules [32]. An immediate application is the analog simulation of the strongly-correlated dynamics of quantum impurity models [33–35].

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METHODS

Here we briefly summarize the important technical details of our experiment. The measured devices and their extracted parameters are summarized in the table below. Supplementary materials are available upon request.

| Device | $\nu$, $10^6$ m/sec | $\omega_p/2\pi$, GHz | $\Lambda$, $\omega_c$, kOhm | $E_J/E_C$ |
|--------|-----------------|-------------------|-----------------|------------|
| a      | 2.76            | 27.0              | 38.4            | 7.4        | 440        |
| b      | 1.88            | 24.8              | 28.4            | 12.6       | 80         |
| c      | 1.12            | 20.8              | 20.2            | 19.0       | 15.8       |
| d      | 1.16            | 22.3              | 19.4            | 21.3       | 11.3       |
| d'     | 0.99            | 19.3              | 19.3            | 23.6       | 8.7        |
| d''    | 1.12            | 21.6              | 19.4            | 21.8       | 10.6       |
| e      | 0.98*           | 20.8*             | 17.7*           | 23*        | 7*         |
| f      | 0.83*           | 20.8*             | 15*             | 19*        | 7*         |
| g      | 22.6            | 26.7              | 4.7             | 0.7        | 712        |
| h      | 8.20            | 21.5              | 8.4             | 2.3        | 211        |
| i      | 2.11            | 20.9              | 37.9            | 7.0        | 484        |

Device fabrication

The chains were fabricated using the standard Dolan bridge technique involving a MMA/PMMA bi-layer resist patterned by electron beam lithography with subsequent double-angle deposition of aluminium with an intermediate oxidation step. The substrate is a high-resistivity silicon wafer. Due to the large number of junctions in the chain, patterning was done by stitching multiple fields of view with a size of 100 $\mu$m. The stitching error is invisible in device images. Curiously, it can be clearly seen in Fig. 2 as sharp periodic shifts in the mode spacing data. We use this information to confirm the conversion between the standing wave index and the wavelength.

Wireless RF-spectroscopy setup

The chip hosting the chains is mounted at the center of a copper waveguide (Fig. 1). In order to launch microwaves we have designed a coaxial-to-waveguide transition launcher with a good matching in the range 7–12 GHz. The antenna attached to the chain is smaller than the free space wavelength at these frequencies. Therefore, the combination of the chip antenna, the waveguide box, and the launcher can be viewed as a semi-transparent “mirror” with a frequency-dependent...
finess. Note that the two chains of the transmission line are spaced by only a few micrometers, whereas the distance between a chain and a wall of the copper box is at least 5 mm, comparable to the full length of the chain. Such a setup minimizes typical parasitic capacitances due to the measurement circuitry seen by the junctions.

**One-tone spectroscopy**

We used a Rohde & Schwarz ZNB network analyzer to measure the frequency-dependent reflection amplitude and phase in a single-port reflection experiment. To fit the data in the vicinity of each resonance, we use the commonly known expression:

\[
S_{11}(\omega) = \frac{2i(\omega - \omega_0)/\omega_0 - Q_\text{ext}^{-1} + Q_\text{int}^{-1}}{2i(\omega - \omega_0)/\omega_0 + Q_\text{ext}^{-1} + Q_\text{int}^{-1}},
\]

where \(\omega_0\) is the resonance frequency, \(Q_\text{int} \equiv Q\) is the internal quality factor, and \(Q_\text{ext}\) is the external quality factor, which in general is a complex number. It’s real part can be viewed as a measure of opacity of the mirror at the antenna end of the chain. We found that at frequencies below 7 GHz, the opacity grows upon reducing the frequency which is consistent with the propagation cut-off of our copper waveguide. The opacity also has a tendency to increase as frequency grows above 10 GHz which may be related to a partial Anderson localization of microwaves in the chain and their decoupling from the antenna end. As a result, one-tone spectroscopy becomes inefficient far outside the 7 – 12 GHz pass-band of our coaxial-to-waveguide launching system. For this reason, the frequency range in Fig. 3 is limited. Finally, we note that our reflection data fits exceptionally well to the above expression for \(S_{11}(\omega)\), as described in the Supplementary material, which allows very accurate extraction of both \(\omega_0\) and \(Q\).

**Two-tone spectroscopy**

We use a weak cross-Kerr interaction between the photons in different modes to perform broadband spectroscopy shown in Fig. 2. First, the readout mode is selected in the pass-band. Reflection amplitude and phase at a properly chosen frequency near the resonance are measured as a function of the frequency of the second tone, which is scanned to look for other modes. The cross-Kerr effect results in the shifting of the frequency of the readout mode due to the photon occupation of every other mode in the system. Since expressions for frequency shift per photon are readily calculable, we use this information to approximately calibrate the measurement power down to a single-photon level. In chains with \(E_j/E_C \approx 10\), the spectrum below a frequency of about 1 GHz becomes difficult to interpret. Nevertheless, there are resonances down to a frequency of about 100 MHz.

**Extracting Z from dispersion relation**

We tried two methods for extracting \(Z\). First is based on our knowledge of junction areas and the fact that the oxide capacitance has a rather device-independent value of 45 fF/µm². The second method is based on the known formulas for the capacitance of two infinitely long coplanar strips. Both methods yield consistent results within 20%. We used the more conservative result (smaller values of \(Z\) and higher values of \(E_j/E_C\)) in the main text of the manuscript.

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