On the optimal choice of wavelet function for multiscale honed surface characterization

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Abstract. Multiscale surface topography characterization is mostly suited than standard approaches because it is more adapted to the multi-stage process generation. Wavelet transform represents a power tool to perform the multiscale decomposition of the surface topography in a wide range of wavelength. However, characterization results depend closely on the topography data acquisition instrument (resolution, height accuracy, sensitivity…) and also on the wavelet analysis method (discrete or continuous transform). In particular, the choice of wavelet function can have significant effect on the analysis results. In this paper, we present experimental work on a number of popular wavelets functions with the aim of finding wavelets that exhibit optimal description of honed surface features when continuous wavelet transform is used. We demonstrate that the regularity property of wavelet function has a significant influence on the characterization performances. This comparative study shows also that the Morlet wavelet is the more adapted wavelet basis function for multiscale characterization of honed surfaces using continuous wavelet transform.

1. Introduction
Faced with the development of several varieties of wavelet functions, several studies in various applications have tried to identify the effect of the wavelet function on its performances. Indeed, since the wavelet transform involves correlating the signal being analyzed and a prototype wavelet function, the choice of the wavelet function may influence the performance of wavelet decomposition. Actually, there is not yet a general answer for all wavelet application. In fact, for example, in textures classification applications, several comparative studies conclude that the choice of the mother wavelet function has little effect on the results [1, 2]. While, other studies show that the analysis of cardiac signals is optimal when using "Meyer" wavelet [3]. In data compression applications, the choice of wavelet is not very critical as long as it is reasonably smooth [4]. So, the choice of the "best" mother wavelet is obviously application dependent. This suggests that wavelet function must be adapted to the characteristics of relevant patterns that differ from one application to another. According to Wickerhauser [5, 6], two desirable properties for adapted wavelet bases are necessary. First, the spatial localization of each feature patterns at each scale particularly in high scale (low frequency).
The second property is the frequency localization to guaranty the separation of multiple features into different scales. In this paper, various wavelet functions within a surface topography characterization approach based on 1D Continuous Wavelet Transform (CWT) are tested on honed topographic profiles. The effect of the mother wavelet choice is at first pointed out. Then, the capacity of each wavelet to provide the best frequency-space localization is highlighted. Their sensitivity to the roughness amplitude is also investigated. Optimal wavelet function for honed surface characterization using continuous wavelets transform is then identified.

2. Continuous wavelets-based characterization approach

We give here a brief description of wavelet based characterization approach. This decomposition used 1D continuous wavelet transform. The continuous wavelet transform can be interpreted as a multi-channel filter system. The surface topography components pass through a filter bank which is a set of the contracting wavelets. The number of wavelets corresponds to the number of iterations.

One defines the 1D continuous wavelet transform of a topographic signal \( f(x) \) by [7, 8]:

\[
W_{b,a}(x) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(x) \psi(\frac{x-b}{a})dx
\]

Where “\(a\)” is the contraction coefficients according to the \(x\)-direction, \(b\) the translation coefficients according to the \(x\)-direction.

The result of the decomposition makes it possible to identify the component of the surface at each scale after a 1D inverse wavelet transformation. The superposition of all scales of the decomposition creates a plane in \((x, \text{scale})\) basis. The characterization methodology consists of quantifying the arithmetic mean values at each scale of decomposition. A spectrum of roughness absolute magnitude “Ma” is obtained.

3. Effect of the “Mother” wavelet function on honed surface characterization

To determine the optimal wavelet function for the multiscale characterization of honed surfaces, several wavelet basis functions are considered namely, Morlet, Meyer and Gaussian derivative at different order. Figure 1 shows of wavelets families’ functions.

![Figure 1. Examples of wavelets families’ functions expressed in the spatial domain (a) 8th derivative Gaussian (DOG8), (b) Morlet et (c) Meyer.](image)

Then, topographic profiles of surfaces after finish honing are considered. They are sampled in four thousand points with step scale of 4µm. Each signal is decomposed by CWT in thirty two scales. Figure 2 shows the Ma spectrum obtained by the characterization approach described above and using different wavelet functions. Although all the applied wavelet functions lead to a low reconstruction error, the multiscale Ma spectrum vary from a wavelet basis function to another. The choice of the wavelet "Mother" has then a major influence on the multiscale characterization of surface features.
4. Identification of the optimal wavelet function choice

To find the Mother wavelet that leads to a complete and precise description of the honed surface topography, we must determine the wavelets family allowing precise detection of all elementary patterns of honed surfaces. This simply boils down to analyzing the spatial and frequency localization of the topographical signal patterns. To qualify the detection and localization quality of a given wavelet function, we identify firstly the basic and repeatable pattern shape of a honed surface (figure 3-b). The identified characteristic pattern represents the indentation trace left by one abrasive grit on the surface of the cylinder. It is common to all abrasive finishing processes. This suggests that the results of the wavelet choice study can be generalized to other abrasive processes (belt finishing, polishing ...).

A signal \( t(x) \) is generated for testing the performance of the different wavelet families (Fig. 3-b). This signal is composed by two elementary patterns in two different positions and at two different scales and it is defined by the following expression:

\[
  t(x) = f((x - b_1)/a_1) + \sqrt{2} f((x - b_2)/a_2)
\]

(2)

Then, two local maxima would be detected on the space-scale diagram of the wavelet transform:

- The first in the position \( b_1 = 20 \) and the scale \( a_1 = 8 \);
- The second in the position \( b_2 = 40 \) and the scale \( a_2 = 4 \).
Besides, honed surfaces exhibit different levels of roughness at each step of honing process. To qualify the sensitivity to the detection of the honed surface patterns in the presence of plateaus micro-roughness, an additional random noise is added to the signal $I(x)$ (Figure 4-c).

**Figure 4.** (a) Elementary motif of honed surface $f(x)$  (b) $I(x)$ signal constituted by two motifs at two scales in two distinguishes positions.

4.1. Spatial and frequency localization performance

As it was already mentioned, by means of the wavelet choice we can access different time and frequency resolution in the time-frequency plane. This is demonstrated here for the topographic signal using various wavelet functions. Figure 5 shows the coefficients of the time-frequency representation of the wavelet transform of the signal $I(x)$ using different wavelet functions. It summarizes the space-frequency localization performance.

**Figure 5.** Comparison of the spatial and frequency localization performances of honed patterns using different wavelets functions: (a) Mexican Hat, (b) Meyer, (c) Morlet, (d) 1st derivative of Gaussian function, (e) 4th derivative of Gaussian function, (f) 8th derivative of Gaussian function.
By comparing these different wavelets coefficients planes, we can state that:

- The "Mexican Hat" and "Meyer" wavelet are not suited because they are not adapted to detect honed surfaces patterns (Fig. 5-a). For example, the pattern located in b2=40 should be detected at scale 4 does not appear in the wavelet coefficients spectrum.
- The "Morlet" wavelet and 8th order of Gaussian wavelet function present the more accurate robustness for honed surface patterns detection.

Furthermore, the "Gaussian" wavelets studied with three derivative orders (two, four, and eight). With higher filter order, the regularity of wavelets increases and in turn increases the compaction energy and the number of vanishing moments. Results shows that better frequency localization are obtained by applying more regular wavelet (figure 5). Thus, the order of a wavelet is important in achieving a representative characterization.

4.2. Sensitivity to the roughness amplitude

Figure 6 shows, by comparison of maxima amplitude of wavelet coefficients, that honed patterns detection using Morlet wavelet is less sensitive to the micro-roughness than by using the 8th Gaussian derivative wavelet function.

![Figure 6](image)

Figure 6. Comparison of the spatial and frequency localization performances of honed patterns using different wavelets functions: (a) Meyer, (b) 8th derivative of Gaussian function, (c) Morlet.

5. Conclusion

The presented work highlights the importance of the choice of the wavelet basis function in the characterization of honed surfaces. It demonstrates that the regularity is a fundamental property in the analysis of topographic profiles from honed surfaces. Hence, more regular (higher order) wavelet functions yield to precise detection of honed surface patterns. Based on this observation, Morlet wavelet appears as the most appropriate wavelet function for analyzing such signals.

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