Atmospheric Neutrinos as a Probe of CPT Violation

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Abstract

We show that atmospheric neutrinos can provide a sensitive and robust probe of CPT violation (CPTV). We perform realistic event-rate calculations and study the variations of the ratio of total muon to antimuon survival rates with $L/E$ and $L$ ($L \equiv$ baseline length, $E \equiv$ neutrino energy) in a detector capable of identifying the muon charge. We demonstrate that measurements of these ratios when coupled with the significant $L$ and $E$ range which characterizes the atmospheric neutrino spectrum provides a method of both detecting the presence of such violations and putting bounds on them which compare very favourably with those possible from a future neutrino factory.

PACS numbers: 11.30.Er.,11.30.Cp.,14.60.Pq,13.15.+g

1 Introduction

The CPT theorem is a cornerstone of quantum field theory in general and particle physics in particular. It rests on principles whose generality and scope makes them pillars of modern physics, like Lorentz invariance, the spin-statistics theorem, and the local and hermitian nature of the Lagrangian [1]. Tests of CPT invariance thus assume importance not only because of the almost sacrosanct nature of these principles, but because any violation of CPT would signal radical new physics and force a re-thinking of foundational aspects of field theory and particle physics [2, 3]. In particular, Greenberg [2] has shown that CPT violation necessarily implies violation of Lorentz invariance.

For over three decades, particle physics has focussed its efforts on testing the predictions of the Standard Model (SM) and seeking the next physics frontier beyond it. One of the conclusions to emerge from this multi-pronged effort over the past decade is that the neutrino sector, both via

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theory and via experiment, provides us with an almost unmatched window to physics beyond the SM [4].

The role of neutrinos as probes of Lorentz and CPT invariance was discussed in a general framework by Colladay and Kostelecky [5] and by Coleman and Glashow [6]. Recent papers [7] have studied possible mechanisms beyond the SM which could lead to CPTV in the neutrino sector. CPTV, in the form of different masses for neutrinos and anti-neutrinos has been invoked to explain all the neutrino anomalies simultaneously, including the LSND result [8]. Bounds possible on such violations from reactor and solar experiments were discussed in [9] and the Dirac and Majorana nature of neutrinos in their presence was studied in [10].

As first proposed in [11] and also discussed in, for instance, [12], significant bounds on CPTV parameters can be set in neutrino factory experiments due to their expected high luminosities and low backgrounds [13]. However, with all their advantages, neutrino factories are a tool which may become available to us only about fifteen or twenty years from now.

In contrast to this, detectors capable of accurately detecting the charge, direction and energy of a muon employ well understood and familiar technology. For instance, large mass magnetized iron calorimeter neutrino detectors were considered in [14] to study atmospheric neutrino interactions in great detail. At least one such detector is being currently actively planned to begin data-taking five years from now [15]. We show that such detectors, or variants thereof, can, in conjunction with the by now well understood atmospheric neutrinos, form an ideal tool to detect CPTV in the neutrino sector. We focus on the survival probabilities for $\nu_\mu$ and $\bar{\nu}_\mu$. A difference in these quantities is a signal for CPTV. By calculating the ratio of their event-rates, we show that comprehensive tests of CPTV are possible in the atmospheric neutrino sector, with sensitivities which compare very favourably with those projected for neutrino factory experiments.

2 CPT Violation in $\nu$ Interactions

We consider the effective C and CPT-odd interaction terms $\bar{\nu}_\mu^\alpha b_{\alpha\beta}^\mu \gamma_\mu \nu_\beta^\beta$, where $\alpha$ and $\beta$ are flavour indices [11]. In presence of this CPTV term, the neutrino energy acquires an additional term which comes from the matrix $b_{\alpha\beta}^\beta$. For anti-neutrinos, this term has the opposite sign. The energy eigenvalues of neutrinos (in ultra-relativistic limit) are obtained by diagonalizing the Hermitian matrix given by

$$A = \frac{m^2}{2p} + b,$$

where $m^2 \equiv m m^\dagger$ is the Hermitian mass squared matrix and we have dropped the superscript 0 from $b^0$.

We assume equal masses for neutrinos and anti-neutrinos. For simplicity we have assumed that the two mixing angles that diagonalize the matrices $m^2$ and $b$ are equal (i.e. $\theta_m = \theta_b = \theta$). In addition, the additional phase that arises due to the two different unitary matrices needed to diagonalize the $\delta m^2$ and $\delta b$ matrices is set to zero.

\footnote{Only one of the two phases can be absorbed by a redefinition of neutrino states.}
For atmospheric neutrinos, it is at times (but not always) a good approximation to consider two flavours only, depending on the parameters which one is studying. We adopt this in our calculations. This is tantamount to assuming that $\sin^2 2\theta_{13}$ is small (below the CHOOZ [16] bound) and so is $\delta m^2_{21}$ (compared to $\delta m^2_{32}$) and thus matter and related three-flavour effects can be safely neglected. As shown in [17] matter effects show up in atmospheric neutrinos for $\sin^2 2\theta_{13} \sim 0.1$ and baselines above 7000 km. The expression for survival probability for the case of CPTV 2-flavour oscillations then becomes

$$P_{\alpha\alpha}(L) = 1 - \sin^2 2\theta \sin^2 \left( \left( \frac{\delta m^2}{4E} + \frac{\delta b}{2} \right) L \right)$$

where $\delta m^2$ and $\delta b$ are the differences between the eigenvalues of the matrices $m^2$ and $b$, respectively and $\alpha$ corresponds to $\mu$ or $\tau$ flavours. Note that $\delta b$ has units of energy (GeV). For $\bar{\nu}$, the sign of $\delta b$ is reversed. The difference between $P_{\alpha\alpha}$ and $P_{\bar{\alpha}\bar{\alpha}}$ is given by,

$$\Delta P_{\alpha\alpha}^{\text{CPT}} = -\sin^2 2\theta \sin \left( \frac{\delta m^2 L}{2E} \right) \sin(\delta b L)$$

An important consequence of the modified dispersion relation in presence of CPTV is that the characteristic $L/E$ behaviour of neutrino oscillations is lost. Hence depending on which term is larger for a given set of parameters and the energy, the mixing angle and oscillation length can vary dramatically with $E$. Thus precision oscillation measurements can set unprecedented bounds on such effects. Also, in order to see any observable effect of CPTV, one must have both CPT-even and CPT-odd terms to be non-zero.

### 3 Calculations

In order to quantitatively demonstrate the feasibility of using atmospheric neutrinos as a source of detecting and putting bounds on CPT violation, we focus on a typical detector which can detect muon energy and direction and also identify its charge. The simplest choice of a suitable prototype is an iron calorimeter, which employs well-understood technology. Such a detector was proposed for Gran Sasso (MONOLITH) [14] and is also currently being planned for a location in India (INO) [15], with initial data-taking by 2007. It is contemplated as both a detector for atmospheric neutrinos and as a future end detector for a neutrino factory beam.

The atmospheric neutrino physics program previously studied in the literature in the context of a Magnetized Iron Tracking Calorimeter includes attempting to obtain conclusive proof that neutrinos oscillate by observation of a $L/E$ dip in the up-down ratio of atmospheric neutrino induced muons, and a more accurate pinning down of oscillation parameters. However, its usefulness as a detector for CPTV parameters using atmospheric neutrinos has not been studied earlier.

Our prototype is a 50 kT Iron detector, with detection and charge discrimination capability for muons, provided by a B field of about 1.2 Tesla. We have assumed a (modest) 50% efficiency of the detector for muon detection and a muon energy resolution of within 5%. We have factored in a resolution in $L/E$ of 50% at Full Width Half Maximum, and incorporated the requisite smearing.
in our event-rate calculations. The resolution function is best parameterised by an exponential
damping term given by, \( R(\delta m^2, L/E) = \exp(-0.25 \delta m^2 L/E) \) [14].

In the calculations presented here, we have assumed that the atmospheric neutrino prob-
lem is resolved by \( \nu_\mu \rightarrow \nu_\tau \) oscillations. Specifically, we use the following input parameters:
\( \delta m^2_{32} = 0.002 \text{ eV}^2 \), \( \sin^2 2\theta_{23} = 1 \), which are consistent with best fit values determined by the most
recent analyses of atmospheric data combined with CHOOZ bounds [18]. In addition we have
used the Bartol atmospheric flux [19] and set a muon detection threshold of 1 GeV. For neutrino
ergies below 1.8 GeV the quasi-elastic \( \nu \)-nucleon crosssection has been used, while above this
energy we have put in the DIS value of the crosssection. The number of muon events have been
calculated using

\[
N = N_n \times M_d \int \sigma^{CC}_{\nu_\mu-N} \frac{dN_\nu}{dE_\nu} dE_\nu
\]

where \( N_n = 6.023 \times 10^{32} \) is the number of (isoscalar) nucleons in 1kT of target material and \( M_d \) is the detector mass. Our results are obtained from a simple parton level monte-carlo event
generator. We have used CTEQ4LQ [20] parametrisations for the parton distribution functions
to estimate the DIS crosssection.

Finally, we comment on the exposure time necessary to see a dependable signal. Since the
number of \( \bar{\nu} \) atmospheric events will be significantly smaller than the number of \( \nu \) events, reducing
the statistical error in the ratio will require an exposure time that enables observations of a
sufficient number of \( \bar{\nu} \) events. Our calculations indicate that an exposure of 400 kT-yr would be
sufficient for statistically significant signals to emerge.

4 Results and Discussions

Figure 1 shows the variation of the ratio of total (up + down) muon survival events to those of
anti-muons, plotted vs \( L \) for various values of \( \delta b \). The solid line in each of the plots is the (CPT
conserving) \( \delta b = 0 \) case, shown for comparison. The overall shape and position of this (solid)
curve is representative of the ratio of the two crosssections (\( \nu \) vs \( \bar{\nu} \)) at the relatively low (few
GeV) energies which dominate the event-rates. The small wiggles and variations are a result of
the various energies and lengths involved and angular differences in fluxes which characterize the
overall atmospheric neutrino spectrum.

From Equation 3, we see that the CPTV difference in probabilities will become zero whenever
\( \delta b L = n\pi \) \((n=\text{integer})\), resulting in a node \((\text{i.e. an intersection with the} \ \delta b = 0 \ \text{curve})\). The
positions and the number of nodes for the various curves nicely correspond to these expected
“zeros” of CPTV and also provide a way of distinguishing between them. Clearly, parameter
values of the order of \( \delta b = 3 \times 10^{-22} \) GeV should be nicely discernible in these observations. We
note that here the effects of the CPTV parameters are maximal at baseline length \( L \approx 1000 \text{ km} \),
thus neglecting matter effects is justified even if \( \theta_{13} \) is close to the CHOOZ upper bound.
Figure 1: The ratio of total muon to anti-muon events plotted against $\log_{10}(L)$ for different values of $\delta b$ (in GeV). The oscillation parameters used in all the plots are: $\delta m^2 = 2 \times 10^{-3} \text{ eV}^2$ and $\sin^2 2\theta_{13} = 1$.

In Figure 2, we plot the same ratio of event-rates vs $L/E$. The nodal position is now dictated by the term $\sin(\delta m^2 L / 2E)$, resulting in a common node for the various $\delta b$ values at $\delta m^2 L / 2E = n\pi$. The plots also show a significant dip near $L/E \approx 310 \text{ km/GeV}$. This is explained by the fact that $\delta m^2 L / 4E = \pi/4$ for this value. In Equation 2, the sine function has its maximum slope at this value of its argument, and hence the survival probabilities for $\nu$ and $\bar{\nu}$ differ maximally here due to the sign difference of the $\delta b$ terms, providing highest sensitivity to the presence of CPTV parameters. We note that in the vicinity of the dip the antineutrino event-rate increases and the neutrino rate decreases, which consequently tends to reduce the statistical error in the ratio, aiding detection. This set of curves provides heightened sensitivity to the presence of CPTV, without the same discriminating sensitivity (between various $\delta b$ values) of the plots in Figure 1. For instance, CPTV induced by parameter values as low as $\delta b = 3 \times 10^{-23}$ can be detected. For a lower value of $\delta b$, say $10^{-23}$, the curve tends to creep back closer to the $\delta b = 0$ solid line. Very recently, the authors of ref. [21] considered the bounds on various types of new physics coming from Super-K and K2K data. They obtain $\delta b \leq 5 \times 10^{-23} \text{ GeV}$. 
Figure 2: The ratio of total muon to anti-muon events plotted against $\log_{10}(L/E)$ for different values of $\delta b$ (in GeV).

**Conclusions**: Atmospheric neutrinos in a detector capable of measuring muon energy and direction and identifying its charge can allow us to set significant bounds on all types of CPTV in the neutrino sector. These bounds compare very favourably with those possible from future neutrino factories [11]. Specifically, the charge discrimination capability of such a detector when coupled with the significant $L$ and $E$ ranges which characterize the atmospheric neutrino spectrum provides a potent and sensitive probe of such violations. By calculating the ratios of muon and anti-muon events and studying their variation with $L$ and $L/E$ we have shown that the presence of CPTV can be detected provided $\delta b > 3 \times 10^{-23}$ GeV. For somewhat higher values of $\delta b$, it is also possible to obtain a measure of their magnitudes by studying their minima and zeros as discussed in the text.

**Acknowledgments**: PM would like to acknowledge Council for Scientific and Industrial Research, India for partial financial support and thank HRI and INO for hospitality.
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