Vector singlet leptoquark model facing recent LHCb and BABAR measurements

Cristian H. García-Duque,1,2,* J. M. Cabarcas,3,† J. H. Muñoz,4,‡ Néstor Quintero,5,§ and Eduardo Rojas6,¶
1Programa de Física, Universidad del Quindío, Carrera 15 Calle 12 Norte, Código Postal 630004, Armenia, Colombia
2Doctorado en Ciencias, Universidad del Quindío, Carrera 15 Calle 12 Norte, Código Postal 630004, Armenia, Colombia
3Departamento de Física, Universidad del Tolima, Código Postal 73006299, Ibagué, Colombia
4Facultad de Ciencias Básicas, Universidad Santiago de Cali, Campus Pampalinda, Carrera 5 No. 62-00, Código Postal 76001, Santiago de Cali, Colombia
5Departamento de Física, Universidad de Nariño, A.A. 1175, San Juan de Pasto, Colombia

Very recently the LHCb experiment released the first measurement of the ratio $R(\Lambda_c) = \frac{\text{BR}(\Lambda_b \rightarrow \Lambda_c \tau\bar{\nu}_\tau)}{\text{BR}(\Lambda_b \rightarrow \Lambda_c \mu\bar{\nu}_\mu)}$. Moreover, the BABAR experiment reported a new result of the leptonic decay ratio of Upsilon meson $\Upsilon(3S)$, namely, $R_{\Upsilon(3S)} = \frac{\text{BR}(\Upsilon(3S) \rightarrow \tau^+\tau^-)}{\text{BR}(\Upsilon(3S) \rightarrow \mu^+\mu^-)}$. Both measurements are below their corresponding Standard Model predictions (deficit), deviating by $\sim 1.1\sigma$ and $\sim 1.8\sigma$, respectively. Motivated by these new data, in this work we study their impact on the phenomenology of the vector singlet leptoquark ($U_1$) model addressing the hints of lepton flavor universality violation in the semileptonic decays of $B$ mesons ($B$ meson anomalies), by carrying out a global fit analysis. In general, we found that a minimal version of the $U_1$ model with a mass of 1.8 TeV can successfully explain the $B$ meson anomalies, while being compatible with all other flavor observables and LHC bounds. Interestingly, our study shows that the new observables $R(\Lambda_c)$ and $R_{\Upsilon(3S)}$ generate strong tension, leading to non-trivial effects on the global fit. Future improvements at the LHCb and Belle II experiments would help to understand their complementary. Moreover, we also analyze the impact of the expected sensitivity on flavor observables at Belle II to provide a further test of the $U_1$ model. Finally, we study the minimal assumptions under which the $U_1$ model could, in addition, provide a combined explanation of the anomalous magnetic moment of the muon.

I. INTRODUCTION

The Standard Model (SM) of particle physics is until the present, the best-known theory for describing the dynamics of the fundamental constituents of the universe, excluding gravity. Despite its success, there are still open questions that are not answered by the SM, such as the number of families, neutrino masses, dark matter candidates, among others, which lead us to think that it corresponds to a low-energy effective theory of a more fundamental one. In the same route, in the SM the lepton flavor universality (LFU) states that in weak decays, there is no preference among the three lepton flavors, several experiments have looked for evidence of LFU-violation (LFUV) and thus for hints or signatures of new physics (NP). Particularly, during the last decade, there has been an accumulation of experimental results regarding $B$ meson transitions in tension with the SM predictions, namely, the charged and neutral $B$-anomalies associated with $b \rightarrow c\tau\bar{\nu}_\tau$ [1–15] and $b \rightarrow s\mu^+\mu^-$ [16–28] transitions, respectively. For a recent review, see Ref. [29]. Such anomalies offer excellent scenarios to test some NP models in order to explain simultaneously these tensions.

The most recent charged-current LFU test is the observable

$$R(\Lambda_c) = \frac{\text{BR}(\Lambda_b \rightarrow \Lambda_c \tau\bar{\nu}_\tau)}{\text{BR}(\Lambda_b \rightarrow \Lambda_c \mu\bar{\nu}_\mu)},$$

measured by the LHCb experiment [30],

$$R(\Lambda_c) = \begin{cases} 
\text{LHCb: } 0.242 \pm 0.026 \pm 0.040 \pm 0.059 \ [30], \\
\text{SM: } 0.324 \pm 0.004 \ [31],
\end{cases}$$

where the experimental uncertainties are statistical, systematic, and due to the external branching ratio measurement $\Lambda_b \rightarrow \Lambda_c \mu\bar{\nu}_\mu$ from LEP data, respectively [30]. This measurement is $\sim 1.1\sigma$ below its corresponding SM prediction.

* chgarca@uniqindio.edu.co; (Corresponding author)
† josecabaras@usantotomas.edu.co
‡ jhmunoz@ut.edu.co
§ nestor.quintero01@usc.edu.co
¶ eduro4000@gmail.com
TABLE I. Experimental status and SM predictions of the ratios $R_{\Upsilon(nS)}$ ($n = 1, 2, 3$).

| Ratio       | Exp. measurement       | SM prediction [37] |
|-------------|------------------------|--------------------|
| $R_{\Upsilon(1S)}$ | $1.005 \pm 0.013 \pm 0.022$ (BABAR [39]) | 0.9924            |
| $R_{\Upsilon(2S)}$ | $1.04 \pm 0.04 \pm 0.05$ (CLEO [40]) | 0.9940            |
| $R_{\Upsilon(3S)}$ | $1.05 \pm 0.08 \pm 0.05$ (CLEO [40]) | 0.9948            |
|              | $0.966 \pm 0.008 \pm 0.014$ (BABAR [41]) | 0.968 \pm 0.016 (Average [38]) |

This means that the $\Lambda_b \to \Lambda_c \ell \bar{\nu}_\ell$ process has a preference to decay to muon over tau lepton, $R(\Lambda_c)_{\text{LHCb}} < R(\Lambda_c)_{\text{SM}}$. Very recently, in Ref. [32] was pointed out that by normalizing the LHCb measurement of $\Lambda_b \to \Lambda_c \tau \bar{\nu}_\tau$ to the SM prediction for $\Lambda_b \to \Lambda_c \mu \bar{\nu}_\mu$ (rather than LEP measurement), it provides a more consistent comparison with the SM prediction for $R(\Lambda_c)$. From this study was obtained a value of $R(\Lambda_c) = 0.285 \pm 0.073$ [32] with a higher central value and in agreement with SM at the 0.53$\sigma$ level. Nevertheless, this $R(\Lambda_c)$ value also shows a suppression respect to the SM. Intriguingly, this behaviour of $R(\Lambda_c)$ is contrary to the other $b \to c \tau \bar{\nu}_\tau$ observables such as the well known $R(D^{(*)})$ anomalies,

$$R(D^{(*)}) = \frac{\text{BR}(B \to D^{(*)} \tau \bar{\nu}_\tau)}{\text{BR}(B \to D^{(*)} \ell \bar{\nu}_\ell)}, \quad (\ell = \mu, e),$$  \hfill (3)

which world averages values reported by the Heavy Flavor Averaging Group (HFLAV) [14, 15]

$$R(D) = \begin{cases} 
    \text{HFLAV}: 0.339 \pm 0.030, \\
    \text{SM}: 0.298 \pm 0.004,
\end{cases} \hfill (4)$$

$$R(D^*) = \begin{cases} 
    \text{HFLAV}: 0.295 \pm 0.014, \\
    \text{SM}: 0.254 \pm 0.005,
\end{cases} \hfill (5)$$

exhibit a combined discrepancy of $\sim 3.3\sigma$ above (excess) the SM [14, 15], $R(D^{(*)})_{\text{HFLAV}} > R(D^{(*)})_{\text{SM}}$. The same is true for the ratio $R(J/\psi) = \frac{\text{BR}(B_c \to J/\psi \tau \bar{\nu}_\tau)}{\text{BR}(B_c \to J/\psi \mu \bar{\nu}_\mu)}$ [12], the $\tau$ lepton polarization $P_\tau(D^*)$ [10, 11] and the longitudinal polarization of the $D^*$ meson $F_L(D^*)$ [13] related with the channel $B \to D^* \tau \bar{\nu}_\tau$, and the inclusive decay ratio $R(X_c) = \frac{\text{BR}(B \to X_c \tau \bar{\nu}_\tau)}{\text{BR}(B \to X_c \mu \bar{\nu}_\mu)}$ [33], which also show a tension above the SM predictions [33–36].

Furthermore, it has been shown that the NP left-handed vector operator that explains $b \to c \tau \bar{\nu}_\tau$ data, also generates effects on the neutral transition $b \bar{b} \to \tau^+ \tau^-$ [37, 38]. The leptonic decay ratio of Upsilon mesons $\Upsilon(nS)$ ($n = 1, 2, 3$) defined as

$$R_{\Upsilon(nS)} = \frac{\text{BR}(\Upsilon(nS) \to \tau^+ \tau^-)}{\text{BR}(\Upsilon(nS) \to \ell^+ \ell^-)}, \quad (\ell = \mu, e),$$  \hfill (6)

provides a very clean test of LFU [37]. In Table I we summarize the current experimental measurements reported by BABAR and CLEO [39–41], and the SM predictions (with an uncertainty typically of the order $\sim O(10^{-5})$) [37]. These measurements are in good agreement with the SM estimations, except for the recent BABAR measurement on $R_{\Upsilon(3S)}$ that shows a tension at the 1.8$\sigma$ level [41]. In addition, the $R_{\Upsilon(3S)}$ average also deviates at the 1.7$\sigma$ level with respect to the SM prediction [38], showing a deficit ($R_{\Upsilon(3S)}^{\text{SM}} < R_{\Upsilon(3S)}^{\text{SM}}$).

The interesting fact that the $b \to c \tau \bar{\nu}_\tau$ data reflect an excess with respect to the SM, except for the ratio $R(\Lambda_c)$ reported by LHCb [30], and that the ratio $R_{\Upsilon(3S)}$ also shows a deficit [41], raises the question: How does it impact a global phenomenological analysis, in any model beyond the SM used to explain simultaneously both the charged- and neutral-current $B$ meson anomalies? In this work, we study such an impact in one of the most promising NP models for addressing these flavor anomalies, the well-known singlet vector leptoquark $U_1 \equiv U_1 \sim (3, 1, 2/3)$ [29, 42–61], which is a $SU(3)_c$ triplet, $SU(2)_L$ singlet, and hypercharge 2/3. In the existing literature, two general approaches for the description of the $U_1$ model have been done. The first one starts from a phenomenological approach which introduces particular textures for the couplings of the leptoquark to the left-handed (LH) and right-handed (RH) SM fermions. However, in this procedure there is a limitation of the estimation of some one loop leptoquark contributions to low-energy processes (such as $\tau \to \mu \gamma$, $B_s - B_c$ mixing). This reason motivates the second approach which is based on the construction of a complete ultra-violet (UV) model to achieve the desire pattern of couplings, where...
the introduction of additional flavor symmetries is often required, new vector-like families are also needed, as well as extra scalar fields to achieve properly the symmetry breaking mechanism, e.g., Refs. [52–55, 58–61]. In this study, we will work under the phenomenological approach based on the minimal setup of couplings (flavor-dependent) between U and LH fermions of the SM with vanishing RH quark-lepton couplings, without specifying the underlying theory. This is the so-called minimal U1 model and we will closely follow the notation of Refs. [48, 49].1 To the best of our knowledge, in the recent analysis of the minimal U1 model presented in Ref. [49] was predicted an increasing of R(Λc) with respect to the SM, R(Λc)/R(Λc)SM ∈ [1.05, 1.25], which is clearly in contradiction with the LHCb measurement that exhibit a suppression, R(Λc)LHCb/R(Λc)SM ∈ [0.51, 0.98] [30].

The main goal of this work is to perform a global fit analysis of the parametric space of the minimal U1 model by considering the impact of the new measurements R(Λc) [30] and RΥ(3S) [41] to the b → cτντ and b → sμ+μ− (C9bsμμ = −C10bsμμ) data. We take into account LHC constraints to the model [49] and several low-energy processes that are induced at the tree-level such as, lepton flavor violating (LFV) decays (B± → K±μ±τ∓, Bs → μ±τ∓, τ → µντ, Υ(nS) → µ±τ∓) and rare B decays (B → Kτ+τ−, Bµ → τ+τ−). Furthermore, we also analyze the expected sensitivity on flavor observables at Belle II (for an integrated luminosity of 50 ab−1 [66]) to provide further test of the U1 model. At the end, we analyze the additional assumptions under which the minimal U1 model can in addition provide a combined explanation of the anomalous magnetic moment of the muon (g − 2)μ, without affecting the parametric space addressing the B meson anomalies. Some previous works have studied the possibility of a common solution of (g − 2)μ and the B meson anomalies [63–65], in which the U1 leptoquark couples to both LH and RH fermions. We will show that by allowing only one RH coupling different from zero, it is possible to get a combined explanation within the minimal U1 model.

We structured this work as follows: In section II we give a brief description of the U1 vector leptoquark model. In section III we present the various relevant processes to which the minimal U1 model contributes (b → cτντ, b → sμ+μ−, Υ decays, LFV and rare decays). We then carry out our phenomenological analysis of the allowed parametric space in section IV. In section V we extent economically the model (by including a single right-handed coupling) to adjust (g − 2)μ data. Our main conclusions are presented in section VI.

II. SINGLET VECTOR LEPTOQUARK MODEL: U1 ∼ (3, 1, 2/3)

The interaction of the SU(2)L singlet vector leptoquark U1 ∼ U1 ∼ (3, 1, 2/3) with the SM fermions can be written as [48, 49]

\[ \Delta \mathcal{L}_{U1} = (x^L_2 i \bar{Q}_L \gamma_\mu L_jL + x^R_1 \bar{d}_R \tau_\mu \ell_jR)U_{1\mu} \],

where the LH and RH quark-lepton flavor couplings xL and xR are (in general) complex 3 × 3 matrices, QL and L are the LH quark and lepton doublets defined as

\[ Q_{iL} = \begin{pmatrix} V_{ki} & \bar{u}_kL \\ d_kL \end{pmatrix} \], \quad \bar{L}_{jL} = \begin{pmatrix} \nu_jL \\ \ell_jL \end{pmatrix}, \]

respectively, with V denoting the Cabibbo-Kobayashi-Maskawa (CKM) matrix; and ℓR and dR are the RH charged leptons and down-type quarks singlets. We will consider a minimalistic flavor structure of the LH coupling matrix xL and assume

\[ x_L = \begin{pmatrix} 0 & 0 & 0 \\ 0 & x^L_{1\mu} & x^L_{2\tau} \\ 0 & x^L_{3\mu} & x^L_{3\tau} \end{pmatrix} \],

neglecting couplings to the first generation of quarks and leptons. While for the RH sector, we will assume vanishing couplings (xR = 0).2 This is the so-called minimal U1 model [48, 49]. In this work, we will take these flavor-dependent couplings (involving only second and third generations) to be real. Let us stress that for the phenomenological analysis of the minimal U1 model, we will assume that RH couplings are zero; however, by allowing only one RH coupling to

1 Very recently, it was shown in Ref. [62] that the minimal U1 model can be extended with a scalar dark matter (DM) candidate that couples to the U1 to explain different DM observables.

2 For analyses taking into account non-vanishing RH couplings, see, e.g. Refs. [52, 53].
bottom-quark and muon different from zero ($x_R^{b\mu} \neq 0$), it is possible to obtain an enhanced effect on the anomalous magnetic moment of the muon, as we will discuss in Sec. V.

After integrating out the Lagrangian $\Delta L_{U_1}$, the flavor structure given by Eq. (9) generates tree-level contributions to neutral-current $b \to s\mu^+\bar{\mu}^-$ and charged-current $b \to c\tau^-\bar{\nu}_\tau$ processes. Moreover, this $U_1$ model allows to induce other flavor observables, such as LFV decays ($B^+ \to K^+\mu^+\tau^+$, $B_s \to \mu^+\tau^+$, $\tau \to \mu\phi$, $T(nS) \to \mu^+\tau^+$), and rare $B$ decays ($B \to K\tau^+\tau^-$, $B_s \to \tau^+\tau^-$). In addition, this scenario gives rise to the neutral-current $b\bar{b} \to \tau^+\tau^-$ transition, thus generating effects on the leptonic decay ratio of Upsilon mesons $R_{\Upsilon(nS)}$, see Eq. (6). In most of the recent studies of the $U_1$ model [48, 49, 52, 53], the implications of these bottomonium observables are not usually taken into account. We will properly include them in our study.

### III. FLAVOR OBSERVABLES

In this section, we present the various relevant processes to which the minimal $U_1$ model contributes and summarize all the experimental constraints. It is well known that the singlet LQ $U_1$ does not generates tree-level contributions to the FCNC transition $b \to s\mu\bar{\nu}$ ($B \to K^{(*)}\nu\bar{\nu}$ processes) [45]. For this reason we will not include it in our analysis.

#### A. Charged-current $b \to c\tau^-\bar{\nu}_\tau$ processes

The effective Hamiltonian responsible for the charged-current $b \to c\tau\bar{\nu}_\tau$ transition is given by

$$
\mathcal{H}_{\text{eff}}(b \to c\tau\bar{\nu}_\tau) = \frac{4G_F}{\sqrt{2}} V_{cb} \left[ (1 + C^{b\tau\nu}_V)(\bar{c}\gamma^\mu P_L b)(\bar{\tau}\gamma^\mu P_L \nu_\tau) \right],
$$

where $V_{cb}$ denotes the Cabbibo-Kobayashi-Maskawa (CKM) matrix element, $G_F$ is the Fermi coupling constant, and $C^{b\tau\nu}_V$ is the Wilson coefficient (WC) in the $U_1$ LQ scenario read as [48, 49]

$$
C^{b\tau\nu}_V = \frac{\sqrt{2}}{4G_F V_{cb} M_{U_1}^2 (V_{x_L})^{\tau}(x_{L_b}^{b\mu})^*},
$$

$$
= \frac{\sqrt{2}}{4G_F M_{U_1}^2} \left[ |x_{L_b}^{b\mu}|^2 + \frac{V_{cs}}{V_{cb}} x_{L_b}^{s\tau}(x_{L_b}^{b\mu})^* \right],
$$

with $M_{U_1}$ the vector $U_1$ mass. The contribution of $U_1$ model leads to a re-scaling of all $b \to c\tau\bar{\nu}_\tau$ observables, namely

$$
R(H) = R(H)_{\text{SM}} \left[ 1 + C^{b\tau\nu}_V \right]^2, \quad \text{(with } H = D, D^*, J/\psi, \Lambda_c \text{)}
$$

$$
F_L(D^*) = F_L(D^*)_{\text{SM}} \left( \frac{R(D^*)}{R(D^*)_{\text{SM}}} \right)^{-1} \left[ 1 + C^{b\tau\nu}_V \right] \right]^2,
$$

$$
P_\tau(D^*) = P_\tau(D^*)_{\text{SM}} \left( \frac{R(D^*)}{R(D^*)_{\text{SM}}} \right)^{-1} \left[ 1 + C^{b\tau\nu}_V \right]^2,
$$

$$
R(X_c) = R(X_c)_{\text{SM}} \left[ 1 + 2.294 \text{ Re}(C^{b\tau\nu}_V) + 1.147 |C^{b\tau\nu}_V|^2 \right].
$$

$$
\text{BR}(B^- \to \tau^-\bar{\nu}_\tau) = \text{BR}(B^- \to \tau^-\bar{\nu}_\tau)_{\text{SM}} \left[ 1 + C^{b\tau\nu}_V \right]^2.
$$

The experimental measurements and SM predictions of $R(D)$, $R(D^*)$, and $R(\Lambda_c)$ are given by Eqs. (4), (5), and (2), respectively. For the other $b \to c\tau\bar{\nu}_\tau$ observables, we collect both the experimental and theoretical values in Table II. The tauonic channel $B^- \to \tau^-\bar{\nu}_\tau$ has not been measured yet, but indirect constraints on $\text{BR}(B^- \to \tau^-\bar{\nu}_\tau)$ have been imposed using the lifetime of $B_s$ ($< 30\%$) [67] and from LEP data at the Z peak ($< 10\%$) [68]. In further analysis we will use the bound of $10\%$ [68].

#### B. Neutral-current $b \to s\mu^+\bar{\mu}^-$ processes

The $U_1$ vector LQ contributes at the tree-level to $b \to s\mu^+\bar{\mu}^-$ transitions via the effective Hamiltonian [44, 45, 48, 49]

$$
\mathcal{H}_{\text{eff}}(b \to s\mu^+\bar{\mu}^-) = \frac{\alpha_{em} G_F}{\sqrt{2} \pi} V_{tb} V_{ts} \left[ C_9^{b\mu\mu}(sP_L\gamma_\beta b)(\bar{\mu}\gamma^\beta \mu) + C_9^{b\mu\mu}(sP_L\gamma_\beta b)(\bar{\mu}\gamma^\beta \gamma_5 \mu) \right],
$$

where $\alpha_{em}$ is the electromagnetic fine-structure constant.
where $\alpha_{\text{em}}$ is the fine-constant structure and the WCs read as

$$C^{bs\mu\tau}_9 = -C^{bs\mu\tau}_{10} = -\frac{\pi}{\sqrt{2}G_F \alpha_{\text{em}} V_{tb} V_{ts}^*} \frac{x_L^\mu(x_L^b)^*}{M_{U_1}^2}. \quad (19)$$

These operators provide an excellent description of the most current $b \rightarrow s\mu^+\mu^-$ data [49, 70–76]. We will adopt the results of the recent global analysis performed in Ref. [71]. According to the fit [71], the allowed $1\sigma$ solution to the WC is

$$C^{bs\mu\tau}_9 = -C^{bs\mu\tau}_{10} \in [-0.46, -0.32]. \quad (20)$$

Thus, one obtains

$$-\frac{|x_L^\mu(x_L^b)^*|}{M_{U_1}^2} \in [4.8, 6.9] \times 10^{-4} \text{ TeV}^{-2}. \quad (21)$$

C. Upsilon decay ratio $R_{\Upsilon(nS)}$ ($n = 1, 2, 3$)

The tree-level $U_1$ LQ effects on the leptonic decay ratio of Upsilon mesons $R_{\Upsilon(nS)}$ (Eq. (6)) can be written as [37]

$$R_{\Upsilon(nS)} = \frac{(1 - 4x^2_r)^{1/2}}{|A_V^{\text{SM}}|^2} \left[ |A^b_{\tau\tau}|^2 (1 + 2x^2_r) + |B^r_{\tau\tau}|^2 (1 - 4x^2_r) \right],$$

with $x_r = m_r/m_{\Upsilon(nS)}$, $|A_V^{\text{SM}}| = -4\pi\alpha_{\text{em}} Q_b$ ($Q_b = -1/3$), and

$$A^b_{\tau\tau} = -4\pi\alpha_{\text{em}} Q_b + \frac{m_{\Upsilon(nS)}^2}{4} \left(-\frac{|x_L^\tau|^2}{M_{U_1}^2}\right), \quad (23)$$

$$B^r_{\tau\tau} = -\frac{m_{\Upsilon(nS)}^2}{2} \left(-\frac{|x_L^r|^2}{M_{U_1}^2}\right). \quad (24)$$

D. LFV decays

This section is dedicated to the study of LFV decay channels of the $B$ meson, $\tau$ lepton and Upsilon mesons $\Upsilon(nS)$ $(n = 1, 2, 3)$, which occur at the tree-level due to the exchange of the $U_1$ vector LQ. The effective Hamiltonian for the LFV transitions $b \rightarrow s\mu^+\mu^-$, $\tau^- \rightarrow \mu^- s\bar{s}$, and $b \rightarrow \mu^+\tau^- s\bar{s}$ can be generically written as [45]

$$H^{\text{LFV}}_{\text{eff}} = \frac{\alpha_{\text{em}} G_F}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left[ C^{bs\mu\tau}_9 \langle \bar{q'} P_L \gamma_\beta q \rangle \langle \bar{\mu}\gamma^\beta \tau \rangle + C^{c\mu\tau}_{10} \langle \bar{q'} P_L \gamma_\beta q \rangle \langle \bar{\mu}\gamma^\beta \gamma_5 \tau \rangle \right] \left(q^{(i)} = b, s\right), \quad (25)$$

where the WCs are

$$C^{bs\mu\tau}_9 = -C^{bs\mu\tau}_{10} = -\frac{\pi}{\sqrt{2}G_F \alpha_{\text{em}} V_{tb} V_{ts}^*} \frac{x_L^\mu(x_L^b)^*}{M_{U_1}^2}, \quad (26)$$

$$C^{cs\mu\tau}_9 = -C^{cs\mu\tau}_{10} = -\frac{\pi}{\sqrt{2}G_F \alpha_{\text{em}} V_{tb} V_{ts}^*} \frac{x_L^\tau(x_L^c)^*}{M_{U_1}^2}, \quad (27)$$

$$C^{bb\mu\tau}_9 = -C^{bb\mu\tau}_{10} = -\frac{\pi}{\sqrt{2}G_F \alpha_{\text{em}} V_{tb} V_{ts}^*} \frac{x_L^\tau(x_L^b)^*}{M_{U_1}^2}. \quad (28)$$

TABLE II. Experimental measurements and SM predictions on other $b \rightarrow c\tau\bar{\nu}_\tau$ observables.

| Observable | Expt. measurement | SM prediction |
|------------|-------------------|---------------|
| $R(J/\psi)$ | $0.71 \pm 0.17 \pm 0.18$ [12] | $0.2582 \pm 0.0038$ [34] |
| $P_x(D^*)$ | $-0.38 \pm 0.51^{+0.21}_{-0.16}$ [10, 11] | $-0.497 \pm 0.013$ [35] |
| $F_L(D^*)$ | $0.60 \pm 0.08 \pm 0.035$ [13] | $0.46 \pm 0.04$ [36] |
| $R(X_c)$ | $0.223 \pm 0.030$ [33] | $0.216 \pm 0.003$ [33] |
respectively. This leads to the following processes $B^+ \to K^+ \mu^+ \tau^-$, $B_s \to \mu^+ \tau^-$, $\tau \to \mu \phi$, and $\Upsilon(nS) \to \mu^+ \tau^-$. In Table III we list the current experimental upper limit (UL) on the branching ratios of these LFV decays [77–79]. We also show for some of these processes the Belle II experiment expected sensitivity for an integrated luminosity of 50 ab$^{-1}$ [66].

1. $B^+ \to K^+ \mu^+ \tau^-$ and $B_s \to \mu^+ \tau^-$

The branching ratio of the LFV decay $B^+ \to K^+ \mu^+ \tau^-$ can be expressed as [44–46]

$$\text{BR}(B^+ \to K^+ \mu^+ \tau^-) = \left( a_K |c_9^{bs\mu\tau}|^2 + b_K |C_{10}^{bs\mu\tau}|^2 \right) \times 10^{-9},$$

with $a_K = 9.6 \pm 1.0$ and $b_K = 10.0 \pm 1.3$, which are numerical coefficients that have been calculated using the $B \to K$ transitions form factors obtained from lattice QCD [46]. The decay channel with final state $\mu^- \tau^+$ can be easily obtained by replacing $\mu \leftrightarrow \tau$. Let us notice that the LHCb limit on BR($B^+ \to K^+ \mu^- \tau^+$) [78] is comparable with the one quoted from PDG [77] (see Table III).

As for the LFV leptonic decay $B_s \to \mu^+ \tau^-$, the branching ratio is written as [46]

$$\text{BR}(B_s^0 \to \mu^+ \tau^-) = \frac{f_{B_s}^2 m_B m_s^2}{32 \pi^3} \alpha_\text{em}^2 G_F^2 |V_{tb} V_{ts}^*|^2 \left( 1 - \frac{m_s^2}{m_{B_s}^2} \right)^2 \frac{|C_9^{bs\mu\tau}|^2 + |C_{10}^{bs\mu\tau}|^2}{M_{U_1}^2},$$

where $f_{B_s} = (230.3 \pm 1.3)$ MeV is the $B_s$ decay constant [14]. The last expression was obtained by using the limit $m_\tau \gg m_\mu$.

2. $\tau \to \mu \phi$

For the LFV hadronic $\tau$ decay $\tau \to \mu \phi$ ($\tau \to \mu s\bar{s}$ transition), the branching ratio is computed as [44, 45]

$$\text{BR}(\tau^- \to \mu^- \phi) = \frac{f_\phi^2 m_\phi^3}{128 \pi^3 m_\tau^2} \left( 1 + 2 \frac{m_\phi^2}{m_\tau^2} \right) \left( 1 - \frac{m_\phi^2}{m_\tau^2} \right)^2 \frac{|x_L^b(x_L^{b\mu})^*|^2}{M_{U_1}^2},$$

where $m_\phi$ and $f_\phi = (238 \pm 3)$ MeV [45] are the $\phi$ meson mass and decay constant, respectively.

3. $\Upsilon(nS) \to \mu^+ \tau^-$

The LFV leptonic $\Upsilon(nS) \equiv \Upsilon(nS) \ (n = 1, 2, 3)$ decay is given by [44, 45]

$$\text{BR}(\Upsilon \to \mu^+ \tau^-) = \frac{f_{\Upsilon}^2 m_{\Upsilon}^3}{48 \pi^3 f_\pi^2} \left( 2 + \frac{m_{\Upsilon}^2}{m_\tau^2} \right) \left( 1 - \frac{m_{\Upsilon}^2}{m_\tau^2} \right)^2 \frac{|x_L^b(x_L^{b\mu})^*|^2}{M_{U_1}^2},$$

TABLE III. Experimental status and Belle II future sensitivity of different LFV processes and rare $B$ decays.

| Channel | Current UL (at 90% CL) | Belle II future sensitivity |
|---------|------------------------|-----------------------------|
| $B^+ \to K^+ \mu^+ \tau^-$ | $4.5 \times 10^{-5}$ (PDG [77]) | $3.3 \times 10^{-6}$ |
| $B^+ \to K^+ \mu^- \tau^+$ | $2.8 \times 10^{-5}$ (PDG [77]) | $3.3 \times 10^{-6}$ |
| $B_s \to \mu^+ \tau^-$ | $3.9 \times 10^{-5}$ (LHCb [78]) | $3.3 \times 10^{-5}$ (LHCb [79]) |
| $\tau \to \mu \phi$ | $8.4 \times 10^{-8}$ (PDG [77]) | $\sim 2.0 \times 10^{-9}$ |
| $\Upsilon(1S) \to \mu^+ \tau^-$ | $6 \times 10^{-6}$ (PDG [77]) | $\sim 10^{-7}$ [44] |
| $\Upsilon(2S) \to \mu^+ \tau^-$ | $3.3 \times 10^{-6}$ (PDG [77]) | $\sim 2 \times 10^{-8}$ (PDG [77]) |
| $\Upsilon(3S) \to \mu^+ \tau^-$ | $3.1 \times 10^{-6}$ (PDG [77]) | $\sim 10^{-7}$ (PDG [77]) |
| $B \to K \tau^+ \tau^-$ | $6.8 \times 10^{-3}$ (LHCb [80]) | $8.1 \times 10^{-4}$ (for 5 ab$^{-1}$) |
| $B_s \to \tau^+ \tau^-$ | $2.25 \times 10^{-3}$ (PDG [77]) | $2.0 \times 10^{-6}$ |
where $f_\Upsilon$ and $m_\Upsilon$ are the Upsilon decay constant and mass, respectively. The decay constant values can be extracted from the experimental branching ratio measurements of the processes $\Upsilon \to e^- e^+$. Using current data from the Particle Data Group (PDG) [77], one obtains $f_{\Upsilon(1S)} = (659\pm17)\,\text{MeV}$, $f_{\Upsilon(2S)} = (468\pm27)\,\text{MeV}$, and $f_{\Upsilon(3S)} = (405\pm26)\,\text{MeV}$. In our analysis we will only take into account the UL from $\Upsilon(3S)$ that leads to the strongest bound. Belle II would be able to improve $\Upsilon(3S) \to \mu^\pm \tau^\mp$ down to $\sim 10^{-7}$ [44].

E. $B \to K^{\pm}\tau^\mp$ and $B_s \to \tau^+\tau^-$ decay

We also take into account the rare $B$ decays, namely $B \to K^{\pm}\tau^\mp$ and $B_s \to \tau^+\tau^-$, which are induced via the $b \to s\tau^+\tau^-$ transition. These decay channels have not been observed so far and the present reported bounds [77, 80] are shown in Table III, as well as the planned Belle II sensitivity [66]. In the case of $B_s \to \tau^+\tau^-$, an additional projected sensitivity of $\sim 5 \times 10^{-4}$ is expected at LHCb with 50 fb$^{-1}$ [81]. The branching fraction of semileptonic decay $B \to K^{\pm}\tau^\mp$ can be expressed by the numerical formula [52]

$$
\text{BR}(B \to K^{\pm}\tau^\mp) \simeq 1.5 \times 10^{-7} + 1.4 \times 10^{-3} \left( \frac{1}{2\sqrt{2}G_F} \right) \frac{\text{Re}[x_L^s(x_b^b)\tau]}{m_U^2} + 3.5 \left( \frac{1}{2\sqrt{2}G_F} \right) \left( \frac{1}{2\sqrt{2}G_F} \right) \frac{x_L^s(x_b^b)\tau}{m_U^2}.
$$

For the leptonic process $B_s \to \tau^+\tau^-$, the SM branching ratio is modified as [52]

$$
\text{BR}(B_s^0 \to \tau^+\tau^-) = \text{BR}(B_s^0 \to \tau^+\tau^-)_{\text{SM}} \left| 1 + \frac{\pi}{\sqrt{2}G_F\alpha_{em}V_{tb}V_{ts}^*C_{10}} \frac{x_L^s(x_b^b)\tau}{m_U^2} \right|^2,
$$

where $\text{BR}(B_s^0 \to \tau^+\tau^-)_{\text{SM}} = (7.73 \pm 0.49) \times 10^{-7}$ [82] and $C_{10}^{\text{SM}} \simeq -4.3$.

IV. PHENOMENOLOGICAL ANALYSIS

In this section we perform a global phenomenological analysis on the parametric space of the $U_1$ vector model (discussed in Sec. II) addressing the $b \to s\mu^+\mu^-$ and $b \to c\tau\bar{\nu}_\tau$ anomalies. For this analysis we define the data set $All\ data$, which includes:

$$
All\ data \in \begin{cases} 
    b \to c\tau\bar{\nu}_\tau \text{ data: } (R(D), R(D^*)), (J/\psi, F_L(D^*)), P_\tau(D^*), \text{BR}(B_{c^-} \to \tau^-\bar{\nu}_\tau) < 10\%, R(X_c) \\
    b \to s\mu^+\mu^- \text{ data: } (C_9^{bs\mu\mu} = C_{10}^{bs\mu\mu} \text{ solution}) \\
    \text{LFV decays: } B^+ \to K^{*}\mu^\pm\tau^\mp, B_s \to \mu^\pm\tau^\mp, \tau \to \mu, \tau \to \phi, \Upsilon(nS) \to \mu^\pm\tau^\mp \\
    \text{rare \ B \ decays: } B \to K^{(*)}\tau^+\tau^-, B_s \to \tau^+\tau^-
\end{cases}
$$

All of the observables were previously discussed in Sec. III. Thus, we have a total of 15 observables for all data and four free-parameters ($x_L^s, x_L^b, x_b^s, x_b^b$) of the $U_1$ LQ model to be fitted; therefore, the degrees of freedom ($N_{\text{dof}}$) of the analysis is $N_{\text{dof}} = 11$. We will include in our analysis the first measurement by LHCb on the ratio $R(\Lambda_c)$ [30], as well as the $R(\Lambda_c)$ normalization issue discussed in Ref. [32] (referred to by us as $R(\Lambda_c)_{\text{Revisited}}$). We also take into account the leptonic decay ratio of bottomonium meson $R_T = R_T(nS)$ ($n = 1, 2, 3$), which includes the new BABAR measurement of $R_T(3S)$ [38]. The step by step inclusion of these new observables will increase $N_{\text{dof}}$ to 12 and 15, respectively, allowing us to explore their impact on the $U_1$ model global fit. It is worth noticing that the implications of $R_T$ are usually ignored in most of the recent (and previous) studies of the minimal $U_1$ model [49]. As we pointed out above, NP scenarios aiming to provide an explanation to the anomalous $b \to c\tau\bar{\nu}_\tau$ data, also induce effects in the neutral-current transition $b\bar{b} \to \tau^+\tau^-$ [37]. As concerns $R(\Lambda_c)$, our study is the first $U_1$ model analysis taking into account the recent LHCb result [30]. Finally, we use in our analysis the benchmark LQ mass $M_{U_1} = 1.8\,\text{TeV}$. This mass value corresponds to the lower limit obtained in Ref. [49] from an analysis of recent LHC data based on direct and indirect high-$p_T$ searches.

We construct the corresponding $\chi^2$ function and obtain its minimum ($\chi_{\text{min}}^2$). By considering the different data sets, in Table IV we show the best fit $1\sigma$ regions for the $U_1$ model parameters. To analyze the goodness of the fit, we also report the $\chi_{\text{min}}^2/N_{\text{dof}}$ and p-value of each scenario. As can be seen, all data has the largest p-value, thus providing the best fit. Once one incorporates $R(\Lambda_c)_{\text{LHCb}}$ (or $R(\Lambda_c)_{\text{Revisited}}$) into the fit, the p-value is reduced, without affecting (mildly) the $1\sigma$ regions. As expected, with the inclusion of $R(\Lambda_c)_{\text{Revisited}}$ we get a larger p-value than with $R(\Lambda_c)_{\text{LHCb}}$. Moreover, the addition of the bottomonium observables $R_T$ (mainly due to $R_T(3S)$) also impact the p-value reducing its value and leading to a different $1\sigma$ confidence values. For all of the data sets considered, we
TABLE IV. The 1σ fit results of $U_1$ LQ couplings, $\chi^2_{\text{min}}/N_{\text{dof}}$, and $p$--value for different data sets. In all the cases considered, we have used the benchmark mass value of $M_{U_1} = 1.8$ TeV.

| Data set | $x_L^{pl}$ | $x_L^{\tau}$ | $x_L^{x \tau}$ | $x_L^{\mu}$ | $\chi^2_{\text{min}}/N_{\text{dof}}$ | $p$--value (%) |
|----------|------------|-------------|----------------|-------------|----------------------------------|----------------|
| All data | $-0.20, 0.17$ | $0.08, 0.18$ | $0.12, 0.16$ | $1.23, 1.85$ | 5.78/11 | 88.7 |
| All data + $R(\Lambda_c)_\text{LHCb}$ | $-0.20, 0.17$ | $0.08, 0.18$ | $0.12, 0.16$ | $1.22, 1.86$ | 8.56/12 | 74.0 |
| All data + $R(\Lambda_c)_\text{Revisited}$ | $-0.20, 0.17$ | $0.08, 0.18$ | $0.12, 0.16$ | $1.22, 1.86$ | 7.11/12 | 85.0 |
| All data + $R(\Lambda_c)_\text{LHCb} + R_T$ | $-0.32, 0.27$ | $0.16, 0.32$ | $0.08, 0.10$ | $0.75, 1.23$ | 12.3/15 | 65.6 |
| All data + $R(\Lambda_c)_\text{Revisited} + R_T$ | $-0.32, 0.27$ | $0.17, 0.32$ | $0.08, 0.10$ | $0.76, 1.24$ | 10.9/15 | 76.2 |

FIG. 1. Pulls of the $b \rightarrow c\tau\bar{\nu}_\tau$ and $b\bar{b} \rightarrow \tau^+\tau^-$ observables with respect to SM (blue bar) and the best-fit point of the $U_1$ model for three different data sets (green, yellow, and red bars).

observed that fit’s values exhibit a hierarchical pattern preferring large values for the $x_L^{\tau} \sim O(1)$ coupling, while small ones for $x_L^{\mu} \approx x_L^{x \tau} \approx x_L^{\mu} \sim O(10^{-1})$. In summary, after the inclusion of $R(\Lambda_c)$ and $R_T(3S)$ into the analysis, the final result with the full data is that the minimal $U_1$ model provides a good fit, making it still a viable explanation of the $B$ meson anomalies. In addition, we have shown that the incorporation of these new observables generate strong tension, leading to non-trivial effects to the global fit. Future improvements on the measurements of $R(\Lambda_c)$ and $R_T(3S)$ at the LHCb and Belle II experiments would help to understand the complementarity of these observables.

To complement the previous discussion we now consider Fig. 1 where the blue bars represent the pulls of the $b \rightarrow c\tau\bar{\nu}_\tau$ and $b\bar{b} \rightarrow \tau^+\tau^-$ observables with respect to SM, i.e., pull$_i = (\mathcal{O}_i^\text{exp} - \mathcal{O}_i^{\text{th}})/\Delta\mathcal{O}_i$, where $\mathcal{O}_i^\text{exp}$ stands for experimental measurement, $\mathcal{O}_i^{\text{th}}$ its corresponding prediction by the SM (or the NP model we are considering) and $\Delta\mathcal{O}_i = (\sigma^2_{\text{exp}} + \sigma^2_{\text{th}})^{1/2}$ is the summation in quadrature of the experimental and theoretical uncertainties. The pull is positive when $\mathcal{O}_i^\text{exp} > \mathcal{O}_i^{\text{th}}$ (excess) and negative when $\mathcal{O}_i^\text{exp} < \mathcal{O}_i^{\text{th}}$ (deficit), such is the case of the observables $R(\Lambda_c)$ and $R_T(3S)$ (see Fig. 1). The green, yellow, and red bars correspond to the best-fit point of the $U_1$ model for three different data sets. It is important to note that with the $U_1$ model the observables $R(D)$, $R(D^*)$, $R(J/\psi)$ and $F_L(D^*)$ decrease the tension with the experiment, particularly, there is an excellent improvement in the prediction of $R(D_{s}^{(*)})$. The observables $P_T(D^*)$ and $R(X_c)$ increase the pull but it is less than one, so the model $U_1$ remains consistent with the experiment. While $R_T(1S)$ and $R_T(2S)$ remain (almost) unchanged. The only observables for which the pull are increased (therefore, inconsistent with the $U_1$ model) are $R(\Lambda_c)$ and $R_T(3S)$. As pointed out above, this result shows that these observables are important in the analysis of the charged-current $B$ anomalies, and should be taken into account in future analyses.

Taking into account the previous results of full data fit (all data + $R(\Lambda_c)$ + $R_T$, where $R(\Lambda_c)$ stands either for $R(\Lambda_c)_\text{LHCb}$ or $R(\Lambda_c)_\text{Revisited}$), in Fig. 2 we show the allowed region (gray color) at the 95% confidence level (CL) of the planes $(x_L^{pl}, x_L^{\tau})$ [left] and $(x_L^{pl}, x_L^{\mu})$ [right] of $U_1$ vector model for $M_{U_1} = 1.8$ TeV, respectively. In addition, for further discussion, we also include the impact on $U_1$ model from the Belle II envisaged improvements on different observables previously discussed in Sec. III. These include the Belle II sensitivities on the branching fraction of LFV decays ($B^+ \rightarrow K^+\mu^+\tau^-$, $B_s \rightarrow \mu^+\tau^-$, $Y(nS) \rightarrow \mu^+\tau^-$, $\tau \rightarrow \mu\phi$), rare $B$ decays ($B \rightarrow K\tau^+\tau^-$, $B_s \rightarrow \tau^+\tau^-$), and
Belle II prospects on $R(D^{(*)})$ [66] in which $R(D^{(*)})$ keep the central values of Belle combination averages [9] with the uncertainties improvements for an integrated luminosity of 50 ab$^{-1}$ [66]. The Belle II–50 ab$^{-1}$ projection is depicted in Fig. 2 by the red region. We obtain that the parametric space would be narrowed by Belle II–50 ab$^{-1}$ but still allowing small room for NP. Particularly, the ($x_L^{b_L^r}, x_L^s$) plane would be severely constrained to small values of $x_L^{b_L^r}$ coupling. This is a consequence of the expected improvements on $\tau \rightarrow \mu \phi$ and $B \rightarrow K \tau^+ \tau^-$. While the ($x_L^{b_L^r}, x_L^s$) plane would be mainly affected by the $\tau \rightarrow \mu \phi$ decay. Thus, the searches at Belle II of these LFV and rare decays will be a matter of importance on proving the $U_1$ vector LQ explanation to the $B$ meson anomalies, as previously suggested in Ref. [54]. We want to stress that our analysis strengthens and complements the recent phenomenological analysis of the minimal $U_1$ model presented in Ref. [49].

V. IMPLICATIONS ON $a_\mu = \frac{1}{2}(g - 2)_\mu$

Recently, a new measurement of the anomalous magnetic moment of the muon, $a_\mu = \frac{1}{2}(g - 2)_\mu$, has been obtained by the Muon $g - 2$ collaboration at Fermilab [83], in excellent agreement with the previous measurement performed at BNL E821 [84]. The combined experimental average is $a_\mu^{\text{Exp}} = (116592061 \pm 41) \times 10^{-11}$, corresponding to $4.2\sigma$ deviation from the SM contribution [83]

$$\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}. \quad (36)$$

The $U_1$ vector LQ can contribute at the one-loop level to $(g - 2)_\mu$ [63–65]. In the heavy limit $M_{U_1} \gg m_\mu$, the dominant one-loop contribution can be written as [63, 64]

$$\Delta a_\mu^{U_1} = \frac{N_c m_\mu^2}{8\pi^2 M_{U_1}^2} \left[ \left( |x_L^{b_L^u}|^2 + |x_R^{b_L^u}|^2 \right) \left( \frac{4}{3} Q_b - \frac{5}{3} Q_{U_1} \right) + 2 \text{Re}[x_L^{b_L^u}(x_R^{b_L^u})^*] \frac{2m_b}{m_\mu} (Q_b - Q_{U_1}) \right], \quad (37)$$

where $m_\mu$ and $m_b$ are masses of the muon and bottom quark, respectively; $N_c = 3$ is a color factor; $Q_{U_1} = +2/3$ and $Q_b = -1/3$ are the leptoquark and bottom quark electric charges, respectively. Let us notice, that in addition to the left-handed coupling $x_L^{b_L^u}$ contribution, it is also necessary to add the contribution from the right-handed coupling $x_R^{b_L^u}$ to explain $\Delta a_\mu$. Considering only the coupling $x_L^{b_L^u}$, the effect in $\Delta a_\mu$ is small ($\Delta a_\mu \sim 10^{-12}$). The presence of both LQ couplings gives rise to an enhancement due to mass ratio $m_b/m_\mu$.

In the following, we will explore the implications of our previous phenomenological analysis of the minimal $U_1$ model on $\Delta a_\mu$. In Fig. 3 we show the parameter space in the ($x_R^{b_L^u}, x_L^{b_L^u}$) plane, where the regions in gray and yellow are
associated with the 1σ allowed regions of $\Delta a_\mu$ and $x_{R}^{b\mu}$ from global fit of full data, respectively. Our analysis yields to the 1σ solution

$$x_{R}^{b\mu} \in [-2.25, -1.25].$$

(38)

Such a large values are still below the perturbative regime $|x_{R}^{b\mu}| \leq \sqrt{4\pi}$. Therefore, we conclude that the minimal $U_1$ model can be economically extended with (large) right-handed coupling $x_{R}^{b\mu}$ to simultaneously address the anomalies in $a_\mu$ and $b \rightarrow c\tau\bar{\nu}_\tau$ and $b \rightarrow s\mu^+\mu^-$ data.

VI. CONCLUDING REMARKS

To the light of the very recent LHCb measurement on the ratio $R(\Lambda_c) = \text{BR}(\Lambda_c \rightarrow \Lambda_c\tau\bar{\nu}_\tau)/\text{BR}(\Lambda_c \rightarrow \Lambda_c\mu\bar{\nu}_\mu)$ ($b \rightarrow c\tau\bar{\nu}_\tau$, underlying transition) and the new BABAR measurement of leptonic decay ratio of bottomonium meson $\Upsilon(3S)$, $R_{\Upsilon(3S)} = \text{BR}(\Upsilon(3S) \rightarrow \tau^+\tau^-)/\text{BR}(\Upsilon(3S) \rightarrow \mu^+\mu^-)$, we have studied the combined explanation of the semileptonic $B$ meson anomalies within the singlet vector LQ model (the so-called minimal $U_1$ model). For the $b \rightarrow c\tau\bar{\nu}_\tau$, data, we have included $R(D^{(*)})$, $R(J/\psi)$, $F_L(D^*)$, $P_T(D^*)$, $\text{BR}(B^- \rightarrow \tau^-\bar{\nu}_\tau) < 10\%$, and $R(X_c)$ observables. While for the $b \rightarrow s\mu^+\mu^-$ data, we used the $C_{bs\mu^+\mu^-} = -C_{bs\mu^+\mu^-}$ solution preferred by the global fit analyses. The minimal $U_1$ model is also constrained by a number of tree-level induced processes such as, LFV decays ($B^+ \rightarrow K^+\mu^+\tau^+$, $B_s \rightarrow \mu^+\tau^+$, $\tau \rightarrow \mu\phi$, $\Upsilon(nS) \rightarrow \mu^+\tau^+$), rare $B$ decays ($B \rightarrow K\tau^+\tau^-$, $B_s \rightarrow \tau^+\tau^-$), and bottomonium ratios $R_{\Upsilon(nS)}$; which we have properly taken into account. In addition, we have incorporated in our analysis the LHC constraints and the expected improvements on different flavor processes at Belle II for an integrated luminosity of 50 ab$^{-1}$.

Keeping in mind all these data, we carried out a global fit of the phenomenology (allowed parametric space) of the relevant flavor-dependent couplings between $U_1$ and left-handed SM fermions $(x_{L}^{s\mu}, x_{L}^{c\tau}, x_{L}^{b\mu}, x_{L}^{b\tau})$. For a benchmark mass value of $M_{U_1} = 1.8$ TeV, the main finding of our study is that the inclusion of the new observables $R(\Lambda_c)$ and $R_{\Upsilon(3S)}$ generates a non-trivial tension into the global fit, yielding to a worsening of the $p$-value (goodness of the fit). Nevertheless, our results showed that the minimal $U_1$ model is still one of the simplest combined explanation of the $B$ meson anomalies, providing a good fit of the current full data. On the other hand, regarding the Belle II perspectives, we have also found that the parametric space would be narrowed as a result of the expected improvements on $\tau \rightarrow \mu\phi$ and $B \rightarrow K\tau^+\tau^-$ decays. Our study thus confirms the potential of Belle II to provide a complementary test of the $U_1$ model.

Finally, we have shown that the (long-standing) current tension in the anomalous magnetic moment of the muon $a_\mu$ can be also addressed by economically extending the minimal $U_1$ model with the addition of the right-handed bottom-muon coupling $(x_{R}^{b\mu} \neq 0)$ with large values. As a consequence, the $B$ meson anomalies ($b \rightarrow c\tau\bar{\nu}_\tau$ and $b \rightarrow s\mu^+\mu^-$ data) and $a_\mu$ can be simultaneously explained within this singlet vector LQ model.
ACKNOWLEDGMENTS

The work of N. Quintero has been financially supported by Universidad Santiago de Cali. J. H. Muñoz is grateful to Vicerrectoría de Investigaciones de Universidad del Tolima for financial support of Project No. 290130517. E. Rojas acknowledges financial support from the “Vicerrectoría de Investigaciones e Interacción Social VIIS de la Universidad de Nariño,” Projects No. 1928 and No. 2172.

[1] J. P. Lees et al. [BaBar Collaboration], Evidence for an excess of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$ decays, Phys. Rev. Lett. 109, 101802 (2012) [arXiv:1205.5442 [hep-ex]].

[2] J. P. Lees et al. [BaBar Collaboration], Measurement of an excess of $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$ decays and implications for charged Higgs bosons, Phys. Rev. D 88, no. 7, 072012 (2013) [arXiv:1303.0571 [hep-ex]].

[3] M. Huschle et al. [Belle Collaboration], Measurement of the branching ratio of $B \to D^{(*)} \tau^- \bar{\nu}_\tau$ relative to $B \to D^{(*)} \ell^- \bar{\nu}_\ell$ decays with hadronic tagging at Belle, Phys. Rev. D 92, no. 7, 072014 (2015) [arXiv:1507.03233 [hep-ex]].

[4] Y. Sato et al. [Belle Collaboration], Phys. Rev. D 94, no. 7, 072007 (2016) [arXiv:1607.07923 [hep-ex]].

[5] S. Hirose [Belle Collaboration], $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$ and Related Tauonic Topics at Belle, arXiv:1705.05100 [hep-ex].

[6] R. Aaij et al. [LHCb Collaboration], Measurement of the ratio of branching fractions $B(B^0 \to D^{(*)} \tau^- \bar{\nu}_\tau)/B(B^0 \to D^{(*)} \mu^- \bar{\nu}_\mu)$, Phys. Rev. Lett. 115, no. 11, 111803 (2015) Erratum: [Phys. Rev. Lett. 115, no. 15, 159901 (2015)] [arXiv:1506.08614 [hep-ex]].

[7] R. Aaij et al. [LHCb Collaboration], Test of Lepton Flavor Universality by the measurement of the $B^0 \to D^{(*)} \tau^+ \nu_\tau$ branching fraction using three-prong decays, Phys. Rev. D 97, no. 7, 072013 (2018) [arXiv:1711.02505 [hep-ex]].

[8] R. Aaij et al. [LHCb Collaboration], Measurement of the ratio of branching fractions $B(B^0 \to D^{(*)} \tau^- \bar{\nu}_\tau$ and $B^0 \to D^{(*)} \mu^- \bar{\nu}_\mu$ branching fractions using three-prong $\tau$-lepton decays, Phys. Rev. Lett. 120, no. 17, 171802 (2018) [arXiv:1708.08856 [hep-ex]].

[9] G. Caria et al. [Belle Collaboration], Measurement of $R(D)$ and $R(D^*)$ with a Semileptonic Tagging Method, Phys. Rev. Lett. 124 (2020) no.16, 161803 [arXiv:1910.05864 [hep-ex]].

[10] S. Hirose et al. [Belle Collaboration], Measurement of the $\tau$ lepton polarization and $R(D^*)$ in the decay $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$ with one-prong hadronic $\tau$ decays at Belle, Phys. Rev. D 97, no. 1, 012004 (2018) [arXiv:1709.00129 [hep-ex]].

[11] S. Hirose et al. [Belle Collaboration], Measurement of the $\tau$ lepton polarization and $R(D^*)$ in the decay $\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau$, Phys. Rev. Lett. 118, no. 21, 211801 (2017) [arXiv:1612.00529 [hep-ex]].

[12] R. Aaij et al. [LHCb Collaboration], Measurement of the ratio of branching fractions $B(B^+ \to J/\psi \tau^+ \nu_\tau)/B(B^+ \to J/\psi \mu^+ \nu_\mu)$, Phys. Rev. Lett. 120, 121801 (2018) [arXiv:1711.05623 [hep-ex]].

[13] A. Abdesselam et al. [Belle Collaboration], Measurement of the $D^{*-}$ polarization in the decay $B^0 \to D^{*-} \tau^- \bar{\nu}_\tau$, arXiv:1903.03102 [hep-ex].

[14] Y. Ahmed et al. [HFLAV], Averages of $b$-hadron, $c$-hadron, and $\tau$-lepton properties as of 2021, [arXiv:2206.07501 [hep-ex]].

[15] For updated results see HFLAV average of $R(D^{(*)})$ for Spring 2021 in https://hflav-eos.web.cern.ch/hflav-eos/semi/spring21/html/RD0Star/RDRDs.html.

[16] R. Aaij et al. [LHCb], Test of lepton universality using $B^+ \to K^+ \ell^+ \ell^- \bar{\nu}_\ell$ decays, Phys. Rev. Lett. 113, 151601 (2014) [arXiv:1406.6482 [hep-ex]].

[17] R. Aaij et al. [LHCb], Test of lepton universality in beauty-quark decays, Nature Phys. 18, no.3, 277-282 (2022) [arXiv:2103.11769 [hep-ex]].

[18] R. Aaij et al. [LHCb], Tests of lepton universality using $B^0 \to K^0_S \ell^+ \ell^- \bar{\nu}_\ell$ and $B^+ \to K^{*+} \ell^+ \ell^- \bar{\nu}_\ell$ decays, Phys. Rev. Lett. 128, no.19, 191802 (2022) [arXiv:2110.09501 [hep-ex]].

[19] R. Aaij et al. [LHCb], Search for lepton-universality violation in $B^+ \to K^+ \ell^+ \ell^- \bar{\nu}_\ell$ decays, Phys. Rev. Lett. 122, no.19, 191801 (2019) [arXiv:1903.09252 [hep-ex]].

[20] S. Choudhury et al. [BELLE], Test of lepton flavor universality and search for lepton flavor violation in $B \to K \ell \ell$ decays, JHEP 03, 105 (2021) [arXiv:1908.01848 [hep-ex]].

[21] R. Aaij et al. [LHCb], Test of lepton universality with $B^0 \to K^{*0} \ell^+ \ell^- \bar{\nu}_\ell$ decays, JHEP 08, 055 (2017) [arXiv:1705.05802 [hep-ex]].

[22] A. Abdesselam et al. [LHCb], Test of Lepton-Flavor Universality in $B \to K^* \ell^+ \ell^- \bar{\nu}_\ell$ Decays at Belle, Phys. Rev. Lett. 126, no.16, 161801 (2021) [arXiv:1904.02440 [hep-ex]].

[23] R. Aaij et al. [LHCb], Measurement of Form-Factor-Independent Observables in the Decay $B^0 \to K^{*0} \mu^+ \mu^-$, Phys. Rev. Lett. 111, 191801 (2013) [arXiv:1308.1707 [hep-ex]].

[24] R. Aaij et al. [LHCb], Angular analysis of the $B^0 \to K^{*0} \mu^+ \mu^-$ decay using 3 fb$^{-1}$ of integrated luminosity, Phys. Rev. Lett. 102, 104 (2016) [arXiv:1512.04442 [hep-ex]].

[25] R. Aaij et al. [LHCb], Measurement of CP-Averaged Observables in the $B^0 \to K^{*0} \mu^+ \mu^-$ Decay, Phys. Rev. Lett. 125, no.1, 011802 (2020) [arXiv:2003.04831 [hep-ex]].

[26] R. Aaij et al. [LHCb], Differential branching fraction and angular analysis of the decay $B^0 \to \phi \mu^+ \mu^-$, JHEP 07, 084 (2013) [arXiv:1305.2168 [hep-ex]].

[27] R. Aaij et al. [LHCb], Angular analysis and differential branching fraction of the decay $B^0 \to \phi \mu^+ \mu^-$, JHEP 09, 179 (2015) [arXiv:1506.08777 [hep-ex]].
