A POSSIBLE INTRINSIC FLUENCE-DURATION POWER-LAW RELATION IN GAMMA-RAY BURSTS

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ABSTRACT

We argue that the distributions of both the intrinsic fluence and the intrinsic duration of the $\gamma$-ray emission in gamma-ray bursts from the BATSE sample are well represented by log-normal distributions, in which the intrinsic dispersion is much larger than the cosmological time dilatation and redshift effects. We perform separate bivariate log-normal distribution fits to the BATSE short and long burst samples. The bivariate log-normal behavior results in an ellipsoidal distribution, whose major axis determines an overall statistical relation between the fluence and the duration. We show that this fit provides evidence for a power-law dependence between the fluence and the duration, with a statistically significant different index for the long and short groups. We discuss possible biases which might affect this result, and argue that the effect is probably real. This may provide a potentially useful constraint for models of long and short bursts.

Subject headings: gamma-rays: bursts – methods: statistical – methods: data analysis

1. Introduction

The simplest grouping of gamma-ray bursts (GRBs), which is still lacking a clear physical interpretation, is given by their well-known bimodal duration distribution. This divides bursts
into long \((T > 2 \text{ s})\) and short \((T \leq 2 \text{ s})\) duration groups \cite{kouveliotou1993}, defined through some specific duration definition such as \(T_{90}, T_{50}\) or similar. The bursts measured with the BATSE instrument on the Compton Gamma-Ray Observatory are usually characterized by 9 observational quantities, i.e. 2 durations, 4 fluences and 3 peak fluxes \cite{meegan1996, paciesas1999, meegan2000a}. In a previous paper \cite{bagoly1998}, we used the principal components analysis (PCA) technique to show that these 9 quantities can be reduced to only two significant independent variables, or principal components (PCs). These PCs can be interpreted as principal vectors, which are made up of some subset of the original observational quantities. The most important PC is made up essentially by the durations and the fluences, while the second, weaker PC is largely made up of the peak fluxes. This simple observational fact, that the dominant principal component consists mainly of the durations and the fluences, may be of consequence for the physical modeling of the burst mechanism.

In this paper we investigate in greater depth the nature of this principal component decomposition, and in particular, we analyze quantitatively the relationship between the fluences and durations implied by the first PC. In our previous PCA treatment of the BATSE Catalog \cite{paciesas1999}, we used logarithmic variables, since these are useful for dealing with the wide dynamic ranges involved. Since the logarithms of the durations and the fluences can be explained by only one quantity (the first PC), one might suspect the existence of only one physical variable responsible for both of these observed quantities. The PCA assumes a linear relationship between the observed quantities and the PC variables. The fact that the logarithmic durations and fluences can be adequately described by only one PC implies a proportionality between them and, consequently, a power law relation between the observed durations and fluences.

We analyze the distribution of the observed fluences and durations of the long and the short bursts, and we present arguments indicating that the intrinsic durations and fluences are well represented by log-normal distributions. The implied bivariate log-normal distribution represents an ellipsoid in these two variables, whose major axis inclinations are statistically different for the long and the short bursts. An analysis of the possible biases and complications is made, leading to the conclusion that the relationship between the durations and fluences appears to be intrinsic, and may thus be related to the physical properties of the sources themselves. We calculate the exponent in the power-laws for the two types of bursts, and find that for the short bursts the energy fluence is roughly proportional to the intrinsic duration, while for the long ones the fluences are roughly proportional to the square of intrinsic durations. The possible implications for GRB models are briefly discussed.

2. Analysis of the Duration Distribution

Our GRB sample is selected from the current BATSE Gamma-Ray Burst Catalog according to two criteria, namely, that they have both measured \(T_{90}\) durations and fluences (for the definition of these quantities see \cite{meegan2000a}, henceforth referred to as the Catalog). The Catalog
in its final version lists 2041 bursts for which a value of $T_{90}$ is given. The fluences are given in four different energy channels, $F_1$, $F_2$, $F_3$, $F_4$, whose energy bands correspond to $[25, 50]$ keV, $[50, 100]$ keV, $[100, 300]$ keV and $>300$ keV. The “total” fluence is defined as $F_{\text{tot}} = F_1 + F_2 + F_3 + F_4$, and we restrict our sample to include only those GRBs which have $F_i > 0$ values in at least the channels $F_1, F_2, F_3$. Concerning the fourth channel, whose energy band is $>300$ keV, if we had required $F_4 > 0$ as well this would have reduced the number of eligible GRBs by $\approx 20\%$. Hence, we decided to accept also the bursts with $F_4 = 0$, rather than deleting them from the sample. (With this choice we also keep in the sample the no-high-energy (NHE) subgroup defined by Pendleton et al. 1997.) Our choice of $F \equiv F_{\text{tot}}$, instead of some other quantity as the main variable, is motivated by two arguments. First, as discussed in Bagoly et al. 1998, $F_{\text{tot}}$ is the main constituent of one of the two PCs which represent the data embodied in the BATSE catalog, and hence it can be considered as a primary quantity, rather than some other combination or subset of its constituents. Second, Petrosian and collaborators in a series of articles (Efron & Petrosian 1992, Petrosian & Lee 1996, Lee & Petrosian 1996, Lee & Petrosian 1997) have also argued for the use of the fluence as the primary quantity instead of, e.g., the peak flux. Using therefore these two cuts, we are left with $N = 1929$ GRBs, all of which have defined $T_{90}$ and $F_{\text{tot}}$, as well as peak fluxes $P_{256}$. This is the sample that we study in this paper.

The distribution of the logarithm of the observed $T_{90}$ displays two prominent peaks\footnote{There is also evidence for the existence of a third subgroup as part of the long duration group, see Horváth 1998, Mukherjee et al. 1998, Hakkila et al. 2000a, Hakkila et al. 2000c which shows a distinct sky angular distribution (Mészáros, 2000a,2000b). We do not deal with this third group here.}, which is interpreted as reflecting the existence of two groups of GRBs (Kouveliotou et al. 1993). This bimodal distribution can be well fitted by means of two Gaussian distributions (e.g., Horváth 1998), indicating that both the long and the short bursts are individually well fitted by pure Gaussian distributions in the logarithmic durations.

The fact that the distribution of the BATSE $T_{90}$ quantities within a group is log-normal is of interest, since we can show that this property may be extended to the intrinsic durations as well. Let us denote the observed duration of a GRB with $T_{90}$ (which may be subject to cosmological time dilatation), and denote with $t_{90}$ the duration which would be measured by a comoving observer, i.e. the intrinsic duration. One has then

$$T_{90} = t_{90} f(z)$$

where $z$ is the redshift, and $f(z)$ measures the time dilatation. For the concrete form of $f(z)$ one can take $f(z) = (1 + z)^k$, where $k = 1$ or $k = 0.6$, depending on whether energy stretching is included or not (see Fenimore & Bloom 1995 and Mészáros & Mészáros 1996). If energy stretching is included, for different photon frequencies $\nu$ the $t_{90}$ depends on these frequencies as

$$t_{90}(\nu) = t_{90}(\nu_0)(\nu/\nu_0)^{-0.4} \propto \nu^{-0.4},$$

where $\nu_0$ is an arbitrary frequency in the measured range (i.e. for higher frequencies the intrinsic duration is shorter). The observed duration at $\nu$ is
simply \((1 + z)\) times the intrinsic duration at \(\nu \times (1 + z)\). Thus, \(T_{90}(\nu) = t_{90}(\nu(1 + z))(1 + z)\) = \(t_{90}(\nu_0)(\nu(1 + z)/\nu_0)^{-0.4}(1 + z) = t_{90}(\nu)(1 + z)^{0.6}\). Hence, when stretching is included, \(f(z) = (1 + z)^{0.6}\) is used.

Taking the logarithms of both sides of equation (1) one obtains the logarithmic duration as a sum of two independent stochastic variables. According to a theorem of Cramér 1937 (see also Rényi 1962), if a variable \(\zeta\) which has a Gaussian distribution is given by the sum of two independent variables, e.g. \(\zeta = \xi + \eta\), then both \(\xi\) and \(\eta\) have Gaussian distributions. Therefore, the Gaussian distribution of \(\log T_{90}\) (confirmed for the long and short groups separately, Horváth 1998) implies that the same type of distribution exists for the variables \(\log t_{90}\) and \(\log f(z)\).

However, unless the space-time geometry has a very particular structure, the distribution of \(\log f(z)\) cannot be Gaussian. This means that the Gaussian nature of the distribution of \(\log T_{90}\) must be dominated by the distribution of \(\log t_{90}\), and therefore the latter must then necessarily have a Gaussian distribution. This holds for both duration groups separately. This also implies that the cosmological time dilatation should not affect significantly the observed distribution of \(T_{90}\), which therefore is not expected to differ statistically from that of \(t_{90}\). (We note that several other authors, e.g. Wijers & Paczynski 1994, Norris et al. 1994, Norris et al. 1995, have already suggested that the distribution of \(T_{90}\) reflects predominantly the distribution of \(t_{90}\).)

One can check the above statement quantitatively by calculating the standard deviation of \(f(z)\), using the available observed redshifts of GRB optical afterglows. The number of the latter is, however, relatively modest, and so far they have been obtained only for long bursts. There are currently upwards of 18 GRBs with known redshifts (Norris & Marani 2000, Bloom et al. 2001, Bloom et al. 2001b). The calculated standard deviation is \(\sigma_{\log f(z)} = 0.14\), assuming \(\log f(z) = \log(1 + z)\). Comparing the variance \(\sigma_{\log f(z)}^2\) with that of the group of long burst durations (see Table 1, which gives \(\sigma_{\log T_{90}} = 0.37\)), one infers that the standard deviation of \(\log f(z)\), or \(\log(1 + z)\), can explain only about \((0.14/0.37)^2 \approx 14\%\) of the total variance of the logarithmic durations. (If \(f(z) = (1 + z)^{0.6}\), then the standard deviation of \(\log f(z)\) can only explain an even smaller amount than 14\%, because \(\sigma_{\log f(z)} = 0.6 \times 0.14\).) This comparison gives support to the conclusion obtained by applying Cramér’s theorem to the long duration group.

### 3. Distribution of the Energy Fluences

The observed total fluence \(F_{\text{tot}}\) can be expressed as

\[
F_{\text{tot}} = \frac{(1 + z)E_{\text{tot}}}{4\pi d_l^2(z)} = c(z)E_{\text{tot}}.
\]  

Here \(E_{\text{tot}}\) is the total emitted energy of the GRB at the source in ergs, the total fluence has dimension of erg/cm\(^2\), and \(d_l(z)\) is the luminosity distance corresponding to \(z\) for which analytical expressions exist in any given Friedmann model (e.g. Weinberg 1972, Peebles 1993). (We note that the considerations in this paper are valid for any Friedmann model. Note also that the
usual relation between the luminosity and flux is given by a similar equation without the extra 
\((1 + z)\) term in the numerator. Here this extra term is needed because both the left-hand-side is 
integrated over observer-frame time and the right-hand-side is integrated over time at the source).

Assuming as the null hypothesis that the log \(F_{\text{tot}}\) of the short bursts has a Gaussian 
distribution, for the sample of 447 bursts with \(T_{90} < 2\) s, a \(\chi^2\) test with 26 degrees of freedom 
gives an excellent fit with \(\chi^2 = 20.17\). Accepting the hypothesis of a Gaussian distribution within 
this group, one can apply again Cramer’s theorem similarly to what was done for the lognormal of durations. This leads to the conclusion that either both the distribution of log \(c(z)\) and the 
distribution of log \(E_{\text{tot}}\) are Gaussian, or else the variance of one of these quantities is negligible 
compared to the other, which then must be mainly responsible for the Gaussian behavior.

The above argument, however, should be taken with some caution. As shown in Bagoly et al. 
1998, the stochastic variable corresponding to the duration is independent from that of the peak 
flux. This means that a fixed level of detection, given by the peak fluxes, does not have significant 
influence on the shape of the detected distribution of the durations (e.g. Efron & Petrosian 
1992, Wijers & Paczyński 1994, Norris et al. 1994, Norris et al. 1995, Petrosian & Lee 1996, 
Lee & Petrosian 1996, Lee & Petrosian 1997). In the case of the fluences, however, a detection 
threshold in the peak fluxes induces a bias on the true distribution, since they are stochastically 
not independent. Therefore the log-normal distribution recognized in the data does not necessarily 
implies the same behavior for the true distribution of fluences occurring at the detector. A further 
complication arises from the differences of the spectral distribution among GRBs. A discussion of 
these problems can be found in a series of papers published by Petrosian and collaborators (Efron 
& Petrosian 1992, Petrosian & Lee 1996, Lee & Petrosian 1996, Lee & Petrosian 1997, Lloyd 
& Petrosian 1999). Despite these difficulties, there are substantial reasons to argue that the observed 
distribution of fluences is dominated by the intrinsic distribution. This assumption can be tested 
by comparing the variance of the fluences with those obtained for \(c(z)\) considering the GRBS that 
have measured \(z\). We will return to this problem in more detail in §5 dealing with the effect of 
possible observational biases.

A Gaussian behavior of log \(c(z)\) can almost certainly be excluded. One can do this on the 
basis of the current observed distribution of redshifts (e.g. Bloom et al. 2001, Bloom et al. 2001b), 
or on the basis of fits of the number vs. peak flux distributions (e.g. Fenimore & Bloom 1995, 
Ulmer & Wijers 1995, Horváth et al. 1996, Reichart & Mészáros 1997). In such fits, using a 
number density \(n(z) \propto (1 + z)^D\) with \(D \simeq (3 - 5)\), one finds no evidence for the stopping of this 
increase with increasing \(z\) (up to \(z \simeq (5 - 20)\)). Hence, it would be contrived to deduce from this 
result that the distribution of log \(c(z)\) is normal. In order to do this, one would need several ad hoc 
assumptions. First, the increasing of number density would need to stop around some unknown 
high \(z\). This was studied, e.g. by Mészáros & Mészáros 1995, Horváth et al. 1996, 
Mészáros & Mészáros 1996, and no such effect was found. Second, even if this were the case, above this \(z\) 
the decrease of \(n(z)\) should mimic the behavior of a log-normal distribution for \(c(z)\), without any 
obvious justification. Third, below this \(z\) one must again have a log-normal behavior for \(c(z)\), in
contradiction with the various number vs. peak flux fits. Fourth, this behavior should occur for any subclass separately. Hence, the assumption of log-normal distribution of \( c(z) \) appears highly improbable, and this holds for both groups of GRBs.

Thus, for the short bursts the variance of log \( c(z) \) must be negligible compared to the variance of \( \log E_{tot} \). The latter possibility means that the observed distribution of the logarithmic fluences is essentially intrinsic, and therefore \( \log E_{tot} \) should have a Gaussian distribution for the group of short bursts.

In the case of the long bursts, a fit to a Gaussian distribution of logarithmic fluences does not give a significance level which is as convincing as for the short duration group. For the 1482 GRBs with \( T_{90} > 2 \) s a \( \chi^2 \) test on \( \log F_{tot} \) with 22 degrees of freedom gives a fit with \( \chi^2 = 35.12 \). Therefore, in this case the \( \chi^2 \) test gives only a low probability of 3.5% for a Gaussian distribution. This circumstance prevent us from applying Cramér’s theorem directly in the same way as we did with the short duration group. Calculating the variance of log \( c(z) \) for the GRBs with known redshifts [Bloom et al. 2001b] one obtains \( \sigma_{\log c(z)} = 0.43 \). From Table 1 of this article it follows that \( \sigma_{\log E_{tot}} = 0.66 \). Hence, the variance of \( c(z) \) gives roughly a \((0.43/0.66)^2 \simeq 43\%\) contribution to the entire variance, which is also a larger value than in the case of the durations. We return to this question in §4.

4. Fitting the Logarithmic Fluences and Durations by the Superposition of two Bivariate Distributions

We assume here as a working hypothesis that the distributions of the variables \( T_{90} \) and \( F_{tot} \), for both the short and long groups, can be approximated by log-normals. In this case, it is possible to fit simultaneously the values of \( \log F_{tot} \) and \( \log T_{90} \) by a single two-dimensional (bivariate) normal distribution. This distribution has five parameters (two means, two dispersions, and the correlation coefficient). Its standard form is

\[
 f(x, y) \, dx \, dy = \frac{N}{2\pi \sigma_x \sigma_y \sqrt{1 - r^2}} \times \\
 \exp \left[ -\frac{1}{2(1 - r^2)} \left( \frac{(x - a_x)^2}{\sigma_x^2} + \frac{(y - a_y)^2}{\sigma_y^2} - \frac{2r(x - a_x)(y - a_y)}{\sigma_x \sigma_y} \right) \right] \, dx \, dy ,
\]

where \( x = \log T_{90}, \ y = \log F_{tot} \) \( a_x, a_y \) are the means, \( \sigma_x, \sigma_y \) are the dispersions, and \( r \) is the correlation coefficient [Trumpler & Weaver 1953: Chapt. 1.25]. An equivalent set of parameters consists of taking the same two means with two other dispersions \( \sigma_x^2, \sigma_y^2 \), and (instead of the correlation coefficient) the angle \( \alpha \) between the axis \( \log T_{90} \) and the semi-major axis of the “dispersion ellipse”. (In the case of bivariate normal distributions, the constant probability curves define ellipses with well-defined axis directions). In this case \( \alpha \) and the correlation coefficient are
related unambiguously through the analytical formula

$$\tan 2\alpha = \frac{2r\sigma_x \sigma_y}{\sigma_y^2 - \sigma_x^2}$$

(4)

and the relations among the variances are also given by analytical formulae (Trumpler & Weaver 1953, Chapt. 1.26). If the data are well fitted by this bivariate normal distribution, then the distributions of each of the variables by themselves must also be univariate normal distributions (the marginal distributions are also normal).

A crucial point in this analysis is that, when the $r$-correlation coefficient differs from zero, then the semi-major axis of the dispersion ellipse represents a linear relationship between log $T_{90}$ and log $F_{tot}$, with a slope of $m = \tan \alpha$. This linear relationship between the logarithmic variables implies a power law relation of form $F_{tot} = (T_{90})^m$ between the fluence and the duration, where $m$ may be different for the two groups. As we have shown in §2, a similar relation will exist between $t_{90}$ and $E_{tot}$.

Fitting the data with the superposition of two bivariate log-normal distributions can be done by a standard search for 11 parameters with $N = 1929$ measured points (cf. Press et al. 1992, Chapt. 15). (Both log-normal distributions have five parameters; the eleventh parameter defines the weight of the first log-normal distribution.) We will use tan $\alpha$ as the fifth parameter for both partial distributions ("terms"). Figure 1. shows the values of $x = \log T_{90}$ and $y = \log F_{tot}$ for the $N = 1929$ GRBs. Each GRB defines a point in the $x, y$ plane with coordinates $x_i, y_i$ ($i = 1, 2, ..., N$). The fitted function $f_2(x, y, a_{xk}, a_{yk}, \sigma_{xk}, \sigma_{yk}, \alpha_k, W) \ (k = 1, 2)$ is a sum of two normal distributions as given in Eq.(3). The normalization constant of the first [second] term is $NW \ [N(1 - W)]$, where $W$ is the weight ($0 \leq W \leq 1$). For the first (second) term the parameters are $a_{x1}, a_{y1}, \sigma_{x1}, \sigma_{y1}, \alpha_1$ ($a_{x2}, a_{y2}, \sigma_{x2}, \sigma_{y2}, \alpha_2$).

We obtain the best fit to the 11 parameters through a maximum likelihood (ML) estimation (e.g., Kendall & Stuart 1976, Vol.2., pp.57-58). We search for the maximum of the formula

$$L_2 = \sum_{i=1}^{N} \ln f_2(x_i, y_i, a_{xk}, a_{yk}, \sigma_{xk}, \sigma_{yk}, \alpha_k, W)$$

(5)

using a simplex numerical procedure (Press et al. 1992, Chapt. 10.4); the index “2” in $L_2$ shows that we have a sum of two log-normal distributions of type given by Eq.(3). The results of this fit are shown in Table 1. We discuss this fit in the next §5, together with a discussion of instrumental biases.

5. Bias Effects and Fit Results

Several papers discuss the biases in the BATSE values of $F_{tot}$ and $T_{90}$ (cf. Efron & Petrosian 1992, Lamb et al. 1993, Lee & Petrosian 1996, Petrosian & Lee 1996, Lee & Petrosian 1997).
Paciesas et al. 1999, Hakkila et al. 2000b, Meegan et al. 2000b). The effect of these biases is non-negligible, and they may in principle have an impact on the correlations between fluence and duration. In other words, it could be that the correlations between the measured fluences and measured durations do not necessarily reflect (due to several instrumental effects) the actual correlations between the real fluences and durations (i.e. between the ideal data which would be obtained by ideal bias-free instruments). In this Section we discuss several tests which indicate that these biases do not significantly influence the final results presented.

Roughly speaking, there are two kinds of instrumental biases. First, some of the faint GRB below the detection threshold may not be detected, and these missing GRBs - if they were detected - could change (at least in principle) the statistical relations between the durations and fluences. Second, even for GRB which are detected, the measured \( F_{\text{tot}} \) and \( T_{90} \) do not reproduce the real values of these quantities, due to the different background noise effects and other complications mainly in the values of \( F_{\text{tot}} \) (for a more detailed survey of biases see the review of Meegan et al. 2000b).

Both of these types of biases are particularly important for the fainter GRBs. To evaluate the impact of these effects, without going into instrumentation details, we will perform two different sets of test calculations. First, we will truncate the whole sample of GRBs with respect to the peak flux, and we will restrict ourselves to the brighter ones. For a sufficiently high truncation limit, this truncated sample should be free from the effects of the first type of bias above. Second, we will modify the measured values of \( F_{\text{tot}} \) and \( T_{90} \) in order to approximate them by real bias-free values. Then we repeat the calculations of the previous Section for these test samples.

To restrict ourselves to the brighter GRBs we do two truncations. First, we take only GRBs with \( P_{256} > 0.65 \) photon/(cm\(^2\)s) (N = 1571); second, we take only GRBs with \( P_{256} > 1.26 \) photon/(cm\(^2\)s) (N = 994). The first choice is motivated by the analysis of Pendleton et al. 1997, and this should already cancel the impact of biases of first type (Stern et al. 1999, Pendleton et al. 1997). The second choice is an ad-hoc one, and is motivated by two opposite requirements. Clearly, to avoid the impact of biases of second type, the truncation should be done on the highest possible peak flux value. On the other hand, the number of remaining bright GRBs should not be small compared with the whole sample. A sample with \( P_{256} > 1.26 \) photon/(cm\(^2\)s) (N = 994) appears to be a reasonable choice from these opposite points of view: for these high peak fluxes the biases in the values of \( F_{\text{tot}} \) and \( T_{90} \) should be largely negligible.

The results after the first truncation are collected in Table 2 and can be seen in Figure 2. We see that all the values are practically identical with the values of Table 1. To complete the ML estimation, we need to calculate also the uncertainties in the obtained best fit parameters. For our present purposes, it is enough to obtain these uncertainties for \( \alpha_1 \) and \( \alpha_2 \), respectively. For this we use the fact that

\[
2(L_{2,\text{max}} - L_2) \simeq \chi^2_1, \tag{6}
\]

where \( L_{2,\text{max}} \) is the likelihood for the best parameters, and \( L_2 \) is the likelihood for the true
values of the parameters, cf. Kendall & Stuart 1976. The $\chi^2_{11}$ is the value of $\chi^2$ function for 11 degrees of freedom (with the degrees of freedom given by the number of parameters). Taking the values of $\chi^2_{11}$ corresponding to 1σ probability ($\simeq 68\%$) one obtains a 10 dimensional hypersurface in the 11 dimensional parameter-space around the point defined by the best fit parameters. This hypersurface can be obtained through computer simulations by changing the values of the parameters around the best fit values and for any new set calculating the value of the likelihood. Then, using Eq.(6), one can estimate the 1σ uncertainty in the parameters. This procedure leads to the values $0.87 \leq \tan \alpha_1 \leq 1.21$ and $2.02 \leq \tan \alpha_2 \leq 2.52$, respectively. In other words, there is a 68% probability that the tangents of angles are in these intervals.

The random probability of obtaining identical angles for both groups may be calculated as follows. We do again an ML estimation, similarly to the procedure used to derive Table 2, except for one difference. That is, we consider only 10 independent parameters, because we require that the two angles be identical. Doing this, we obtain that the $L_{2,max}$ value is smaller by 4.46 than the value obtained for 11 parameters. (The concrete values of the 10 parameters are unimportant here.) Then the one degree of freedom $\chi^2 = 8.92$, corresponding to the difference between the 11 and 10 parameter ML estimation (see Eq.(6)) defines a 0.3% probability. This is the random probability that the two angles are identical (Kendall & Stuart 1976). The two angles are therefore definitely different, with a high level of significance.

Thus, for the two groups the dependence of the total fluence on the measured duration is

$$F_{\text{tot}} \propto \begin{cases} (T_{90})^{1} & ; \text{(short bursts)}; \\ (T_{90})^{2.3} & ; \text{(long bursts)} \end{cases},$$

and the exponents of the two groups are different at a high level of confidence, with $\lesssim 0.3\%$ probability of random occurrence.

The results obtained with the second truncation are collected in Table 3. We see that all values are again similar to the values of Table 1 and Table 2. The difference in the averages of the durations and fluences understandable, because we take brighter GRBs. However the changes in them are small, and our conclusions therefore remain the same.

To further verify this result, we also perform a second type of test. We modify the values of the measured $F_{\text{tot}}$ and $T_{90}$ in order to approximate them by their real bias-free values. We consider a very simplified model of GRB pulse shapes, namely, we assume that the measured time behavior of a GRB may be described by a triangle. If a GRB pulse arises at a time instant $t_1$, after this time its flux increases linearly, and reaches its peak flux (denoted by $P$) at time $t_2$. Then it again decreases linearly, and the flux becomes zero at a time instant $t_3$. Then the real measured duration is $t = t_3 - t_1$, and the real fluence is $F = t \times P/2$. Assume now that there is a background noise, causing a flux $P_o$. Then, clearly, the GRB will be detected only when the flux $P$ is larger than $P_o$. This means that the measured duration will be $t' = (1 - P_o/P)t$. Also, the measured fluence will be lower, because no flux is seen during the time when the flux is smaller than $P_o$. The value of the measured fluence will be given in this case by $F' = (1 - (P_o/P)^2)F$. Taking this into account,
we will do the modifications $F_{\text{tot};\text{mod}} = F_{\text{tot}}/(1 - (P_o/P_{256})^2)$ and $T_{90;\text{mod}} = T_{90}/(1 - P_o/P_{256})$, respectively. This means that the modified values (which are expected to be closer to the real ones) are larger than the measured ones. There are of course possible objections that can be raised against such a procedure. One might argue that these modifications are ad-hoc for several reasons, e.g., that $t'$ and $t$ in the previous consideration are not identical to $T_{90}$ and $T_{90;\text{mod}}$; or similarly, that $F$ and $F'$ are not identical to $F_{\text{tot}}$ and $T_{\text{tot};\text{mod}}$; also that $P$ cannot be substituted automatically by $P_{256}$; that the concrete value of $P_o$ is subject to change; that the "triangle" approximation is arbitrary; etc. Nevertheless, keeping all these caveats in mind, it is still useful to see what would be the change, if the calculation of the previous Section is repeated for these modified fluences and durations.

The results of this second test calculation are collected in Table 4. The concrete value of $P_o = 0.3$ photon/(cm$^2$s) is based on the results of Stern et al. 1999 and Pendleton et al. 1997, which suggest that the background noise is $\simeq (0.2 - 0.3)$ photon/(cm$^2$s). In order to avoid the problem with GRBs which have $P_{256} < P_o$, here we do not use the whole sample of $N = 1929$, but a sample with $P_{256} > 0.65$ photon/(cm$^2$s) ($N = 1571$). This truncation does not lead to any essential changes, as was already seen earlier in this Section. The values of Table 4 are again very similar to those of Table 2. Omitting the calculation of the uncertainties in the best parameters, we calculate only the probability of having identical angles; this probability is 0.4% (the value of $\chi^2$ drops by 4.22). This tends to support, again, the earlier results obtained with the larger sample.

Therefore, from these two different tests we are led to conclude that the instrumental biases do not change the basic results. The discussion in this Section gives support to the interpretation that the correlation between the logarithms of the durations and the logarithms of the fluences are real, that they are different for the long and the short bursts, and that these conclusions remain valid even after taking into account instrumental biases.

6. Are the Correlations Actually Intrinsic?

In the previous Section we presented arguments showing that the different power law relations between $F_{\text{tot}}$ and $T_{90}$, expressed through Eq.(7), are real, and are not substantially affected by instrumental bias effects. In §3 it was shown that these same power-law relations hold between the intrinsic $t_{90}$ and $E_{\text{tot}}$ values. Since, however, for the long bursts the validity of a log-normal representation of the fluences is not so obvious, we return here for the sake of completeness to this question.

In general, the PCA analysis shows that the logarithm of any measured quantity can be represented by a linear combination of PCs. In the PCA analysis of Bagoly et al. 1998 the whole BATSE sample of GRBs (lumping together the long and short groups) it was shown that there are two important PCs, which may be identified - to a high accuracy - with the log $T_{90}$ and log $P_{256}$.
variables. This means that also the logarithm of fluence may be written as

$$\log F_{\text{tot}} = a_1 \log T_{90} + a_2 \log P_{256} + e,$$  \hspace{1cm} (8)

where $a_1$ and $a_2$ are some constants (defining the importance of the PC) and $e$ is some noise term (see Bagoly et al. 1998).

As shown in §2, the distribution of the measured $T_{90}$ is well be described by the superposition of two log-normal distributions. It is also shown in §2 that the intrinsic durations $t_{90}$ for the two groups (short and long) separately are distributed log-normally. Hence, if it were the case that $a_2$ were negligibly small with respect to $a_1$, it would be possible to conclude (i.e. without any further separate investigation of $\log F_{\text{tot}}$ itself) that $\log F_{\text{tot}}$ is given by the superposition of two log-normal distributions. However, the smallness of $a_2$ is not fulfilled generally. Therefore, an additional study of $F_{\text{tot}}$ is needed, and this is done in §3. It is shown there that, because $c(z)$ in §2 is very unlikely to obey a log-normal distribution (which holds for both groups separately), from the log-normal behavior of $\log F_{\text{tot}}$ it follows that $E_{\text{tot}}$ must also have log-normal distribution. More precisely, the log-normal distribution of $E_{\text{tot}}$ is well justified for the short group, but not so well for the long group. Therefore, a further analysis, based on the PCA method, may help to clarify the situation.

As mentioned, the coefficient $a_2$ in equation (8) is not always negligible with respect to $a_1$. Nonetheless, a simple trick can be used which gets one around this obstacle. If we take a narrow enough interval of $\log P_{256}$ so that in it it is approximately valid to take $P_{256} = \text{const}$, then $a_2 \log P_{256}$ is also constant there, and will not play a role in the form of the distribution of $\log F_{\text{tot}}$. Keeping in mind this possibility, we again do truncations in $P_{256}$ (as in the previous Section), but here we restrict $P_{256}$ from both sides (i.e. there is also an upper limit). In addition, we need to take as narrow an interval of $\log P_{256}$ as possible; of course, the number of GRBs remaining cannot be too small (we will require at least $N > 500$), otherwise a statistical study is not possible. Because PCA studies were done for the whole sample, we also do a fitting for the whole sample, similarly to what was done in previous Sections.

Results of the truncation with $0.65 < P_{256} < 1.26$ (where $P_{256}$ has units of photons/(cm$^2$ s)) is given in Table 5. We see that the results are again similar to the whole sample presented in Table 1; of course, some concrete values may differ from that of Table 1. due to the choice of sample. For our purpose here it is the most essential conclusion that the fitting with the superposition of two two-dimensional log-normal distributions may again be done well. This means that the distribution in $\log F_{\text{tot}}$ is therefore also a sum of two log-normal distributions.

The results using a truncation with $1.26 < P_{256} < 3.98$ is given in Table 6. We again see that the results are similar to those of the whole sample given in Table 1. For our purposes here, it is again essential that the data is well fitted with a superposition of two two-dimensional log-normal distributions; the distribution in $\log F_{\text{tot}}$ is therefore also a sum of two log-normal distributions.

Having thus concluded that the distribution of $F_{\text{tot}}$ is a sum of two log-normal distributions, since $c(z)$ is very unlikely to be log-normally distributed (for both groups separately), $E_{\text{tot}}$ should
also be distributed log-normally for both subclasses separately.

It is appropriate to discuss, in addition, how these results relate to those showing a cosmological time dilatation and redshift signature. If it were the case that the correlation were dominantly caused by a factor depending on $z$, then the GRBs with lower fluence would have systematically longer durations. Norris et al. 1994, Norris et al. 1995, Lee & Petrosian 1997 find an inverse trend between the peak flux $P_{256}$ and the values of $T_{50}$ of long bursts $T_{50} > 1.5s$, and note that the dispersion is substantially larger than the inferred cosmological dilatation effect. Our results are not in conflict with that of these authors, although our purpose is different from theirs. In the present work we have used a more extended burst sample than these authors. Our methodology also differs from that of Norris et al. in at least three respects: we used fluences instead of peak fluxes, since we are interested in total energies; we used $T_{90}$ instead of $T_{50}$, since that is more useful for separating the short and long bursts; and we did not normalize the noise to the same level in all the bursts, nor did we perform selection cuts designed to find a cosmological dilatation. As mentioned, in the distribution of the $T_{90}$ we find that the dispersion is dominantly intrinsic, which agrees with Norris et al. 1994, Norris et al. 1995, and we find that the positive fluence-duration correlation is statistically stronger in the short bursts, for which both Norris et al. 1994, Norris et al. 1995 and ourselves find that the dispersion would mask a cosmological dilatation signal. In the Appendix, as a check, we compare the behavior of the $T_{50}$ and the $T_{90}$ durations and their dispersions as a function of the peak flux. As seen from Table 7 of the Appendix, for the long bursts the $T_{50}$ vs. $P_{256}$ does show (even with our cuts, which are not optimized for that purpose) a slight trend in the sense of cosmological time dilatation, which is substantially weaker than the dispersion. A similar, but much fainter trend may be also be seen in the $T_{90}$ vs. $P_{256}$, which is even more strongly dominated by the dispersion. For the short bursts (Table 8 of the Appendix), not surprisingly, this weak trend cannot be recognized. This is in qualitative agreement with the results of Norris et al. 1994, Norris et al. 1995. We may also ask whether the correlations based on the $T_{90}$ measurements that we discuss here would differ from those derived using $T_{50}$. An inspection of Tables 7 and 8 of the Appendix indicates that the character of the dispersions in both variables are essentially the same. Therefore the correlations and the angles (power law indices) would be the same, regardless of whether $T_{90}$ or $T_{50}$ is used.

7. Discussion

We have presented evidence indicating that there is a power-law relationship between the logarithmic fluences and the $T_{90}$ durations of the GRBs in the current BATSE Catalog, based on a maximum likelihood estimation of the parameters of the bivariate distribution of these measured quantities. As shown in the Appendix, the dispersions of the $T_{90}$ do not differ significantly from those of the $T_{50}$ distributions, and therefore the same correlations and the same power-law relations would be expected if one used the $T_{50}$ instead of the $T_{90}$. We have also evaluated the possible impact of instrumental biases, with the results that the conclusions do not change.
significantly when these effects are taken into account.

An intriguing corollary of these results is that the exponents in the power-law dependence between fluence and duration differ significantly for the two groups of short ($T_{90} < 2$ s) and long ($T_{90} > 2$ s) bursts. As shown in §5, this also means that the same power law relations hold between the total energy emitted ($E_{\text{tot}}$) and the intrinsic durations ($t_{90}$) of the two groups. The intrinsic nature of this relation is also confirmed by further calculations based on a principal component analysis.

While an understanding of such power-law relations in terms of physical models of GRB would require more elaborate considerations, without going into details it may be noted here that the results are compatible with a simple interpretation where the short bursts involve a wind outflow leading to internal shocks responsible for the gamma-rays (Rees & Mészáros 1994, Piran 1999), in which the luminosity is approximately constant over the duration $t$ of the outflow, so that both the total energy $E_{\text{tot}}$ and the fluence $F_{\text{tot}}$ are $\propto t$. If an external shock were involved, e.g. Mészáros & Rees 1993, Piran 1999, for a sufficiently short intrinsic duration (impulsive approximation) there would be a simple relationship between the observed duration and the total energy, $t \propto E_{\text{tot}}^{1/3}$, resulting from the self-similar behavior of the explosion and the time delay of the pulse arrival from over the width of the blast wave from across the light cone. This relationship is steeper than the one we deduced for long bursts. However, the observed $F_{\text{tot}} \propto t^{2.3}$ behavior could possibly be the result of having the observed gamma-rays due to some combination of internal and external shocks.

In summary, we have presented quantitative arguments in support of two new results, namely that there is a power law relation between the fluence and duration of GRBs which appears to be physical, and that this relation is significantly different for the two groups of short and long bursts. For the short ones, the total energy released is proportional to the duration of the gamma-ray emission, while for the long ones it is proportional roughly to the square of the duration. This may indicate that two different types of central engines are at work, or perhaps two different types of progenitor systems are involved. It is often argued that those bursts for which X-ray, optical and radio afterglows have been found, all of which belong to the long-duration group, may be due to the collapse of a massive stellar progenitor (e.g. Paczyński 1998, Fryer et al. 1999). The short bursts, none of which have as of March 2001 yielded afterglows, may be hypothetically associated with neutron star mergers (e.g. Fryer et al. 1999) or perhaps other systems. While the nature of the progenitors remains so far indeterminate, our results provide new evidence suggesting an intrinsic difference between the long and short bursts, which probably reflects a difference in the physical character of the energy release process. This result is completely model-independent, and if confirmed, it would provide a potentially useful constraint on the types of models used to describe the two groups of bursts.

We are indebted to Dr. Gábor Tusnády (Rényi Institute for Mathematics), Dr. Chryssa Kouveliotou and Dr. Michael Gibbs (NASA MSFC) and the referee for useful discussions and
critique. Research supported in part through OTKA grants T024027 (L.G.B.), F029461 (I.H.) and T034549, NASA NAG5-2857, Guggenheim Foundation and Sackler Foundation (P.M.), GA ČR grant 202/98/0522 and Domus Hungarica Scientiarum et Artium grant (A.M.).

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**Table 1:** The best fit parameters of the sum of two bivariate log-normal distributions for $x = \log T_{90}$ and $y = \log F_{tot}$ for the whole sample with $N = 1929$, giving a value of likelihood $\log L_2 = 10664.82$ (for more details see the text).

|     | $a_{x1}$  | $a_{x2}$  | $a_{y1}$  | $a_{y2}$  | $\sigma'_{x1}$ | $\sigma'_{x2}$ | $\sigma'_{y1}$ | $\sigma'_{y2}$ | $\tan \alpha_1$ | $\tan \alpha_2$ | $W$   |
|-----|-----------|-----------|-----------|-----------|----------------|----------------|----------------|----------------|-----------------|----------------|------|
|     | -0.08     | 1.54      | -6.22     | -5.29     | 0.73          | 0.67           | 0.46           | 0.37           | 0.91            | 2.29           | 0.32 |
Table 2: The best fit parameters of the sum of two bivariate log-normal distributions for \( x = \log T_{90} \) and \( y = \log F_{\text{tot}} \) for the sample with \( P_{256} > 0.65 \text{ photon}/(\text{cm}^2\text{s}) \) \((N = 1571)\), giving a value of likelihood \( \log L_2 = 8388.33 \) (for more details see the text).

| \( a_{x1} \) | \( a_{x2} \) | \( a_{y1} \) | \( a_{y2} \) | \( \sigma_{x1} \) | \( \sigma_{x2} \) | \( \sigma_{y1} \) | \( \sigma_{y2} \) | \( \tan \alpha_1 \) | \( \tan \alpha_2 \) | \( W \) |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| -0.12 | 1.55 | -6.18 | -5.16 | 0.66 | 0.65 | 0.45 | 0.38 | 1.05 | 2.30 | 0.32 |

Table 3: The best fit parameters of the sum of two bivariate log-normal distributions for \( x = \log T_{90} \) and \( y = \log F_{\text{tot}} \) for the sample with \( P_{256} > 1.26 \text{ photon}/(\text{cm}^2\text{s}) \) \((N = 994)\), giving a value of likelihood \( \log L_2 = 4906.82 \) (for more details see the text).

| \( a_{x1} \) | \( a_{x2} \) | \( a_{y1} \) | \( a_{y2} \) | \( \sigma_{x1} \) | \( \sigma_{x2} \) | \( \sigma_{y1} \) | \( \sigma_{y2} \) | \( \tan \alpha_1 \) | \( \tan \alpha_2 \) | \( W \) |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| -0.15 | 1.52 | -5.99 | -4.96 | 0.55 | 0.65 | 0.43 | 0.39 | 1.06 | 1.92 | 0.30 |

Table 4: The best fit parameters of the sum of two bivariate log-normal distributions for the modified \( x = \log T_{90} \) and \( y = \log F_{\text{tot}} \) for the sample with \( P_{256} > 0.65 \text{ photon}/(\text{cm}^2\text{s}) \) \((N = 1571)\), giving a value of likelihood \( \log L_2 = 83512.70 \) (for more details see the text).

| \( a_{x1} \) | \( a_{x2} \) | \( a_{y1} \) | \( a_{y2} \) | \( \sigma_{x1} \) | \( \sigma_{x2} \) | \( \sigma_{y1} \) | \( \sigma_{y2} \) | \( \tan \alpha_1 \) | \( \tan \alpha_2 \) | \( W \) |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| -0.01 | 1.66 | -6.15 | -5.13 | 0.65 | 0.62 | 0.47 | 0.41 | 0.94 | 2.60 | 0.33 |
Table 5: The best fit parameters of the sum of two bivariate log-normal distributions for $x = \log T_{90}$ and $y = \log F_{tot}$ for the sample with $1.26 > P_{256} > 0.65$ photon/$(cm^2 s)$ ($N = 577$), giving a value of likelihood $\log L_2 = 2765.64$ (for more details see the text).

|       | $a_{x1}$ | $a_{x2}$ | $a_{y1}$ | $a_{y2}$ | $\sigma_{x1}$ | $\sigma_{x2}$ | $\sigma_{y1}$ | $\sigma_{y2}$ | $\tan \alpha_1$ | $\tan \alpha_2$ | $W$ |
|-------|----------|----------|----------|----------|----------------|----------------|----------------|----------------|------------------|------------------|-----|
|       | 0.19     | 1.62     | -6.44    | -5.44    | 0.91          | 0.45           | 0.36           | 0.22           | 0.40             | 1.04             | 0.43|

Table 6: The best fit parameters of the sum of two bivariate log-normal distributions for $x = \log T_{90}$ and $y = \log F_{tot}$ for the sample with $3.98 > P_{256} > 1.26$ photon/$(cm^2 s)$ ($N = 667$), giving a value of likelihood $\log L_2 = 3204.19.64$ (for more details see the text).

|       | $a_{x1}$ | $a_{x2}$ | $a_{y1}$ | $a_{y2}$ | $\sigma_{x1}$ | $\sigma_{x2}$ | $\sigma_{y1}$ | $\sigma_{y2}$ | $\tan \alpha_1$ | $\tan \alpha_2$ | $W$ |
|-------|----------|----------|----------|----------|----------------|----------------|----------------|----------------|------------------|------------------|-----|
|       | -0.17    | 1.54     | -6.15    | -6.15    | 0.52           | 0.36           | 0.36           | 0.52           | 0.73             | 1.06             | 0.33|
Fig. 1.— The best fit of two bivariate log-normal distributions for the whole BATSE sample (1929 GRBs). The ellipses give the 1σ and 2σ probabilities (for more details see the text).
Fig. 2.— The best fit of two bivariate log-normal distributions for the 1571 GRBs which fulfil the $P_{256} > 0.65$ photon/(cm$^2$s) criterion. The ellipses give the 1σ and 2σ probabilities (for more details see the text).
8. APPENDIX: Comparison of $T_{90}$ and $T_{50}$ Statistical Properties

In order to check whether there is some influence of the time dilatation on the distribution of $T_{90}$ or $T_{50}$ we compare here the basic properties of these two quantities in our sample for the long and the short bursts, separately. We grouped the data, using the 256 ms peak flux values, into 0.2 bins in $P_{256}$, and summarized in Tables 7 and 8 the mean values and the corresponding standard deviations of the logarithmic durations of GRBs in each bin. We stress that this does not include any equalization of the noise level in the various $p$ bins, and is not intended as a test of the time dilatation hypothesis, but rather as a test of whether dilatation, would have any effect on our results.

Inspecting the durations of long ($T_{90} > 2s$) GRBs summarized in Table 7 one sees that, except from the brightest and faintest bins, there is no significant difference in log $T_{90}$. The decrease of the duration in the faintest bin is probably due to the biasing of the determination, namely, the fainter parts of the bursts cannot be discriminated against the background and the duration obtained is systematically shorter. There is a remarkable homogeneity and no trend in the standard deviations of the log $T_{90}$.

In the case of the long burst $T_{50}$ durations, this quantity shows an increasing trend towards bursts of fainter peak flux. The shortening in the faintest bin is probably also due to selection effects. Similarly to the log $T_{90}$ values the same homogeneity can be observed in the standard deviations also in case of log $T_{50}$. The standard deviations are almost the same in both log $T_{90}$ and log $T_{50}$.

One can test whether, within our analysis methodology and with our sample, there is a significant difference among the binned $T_{90}$ values, and whether the slight trend in the $T_{50}$ significantly differs from zero. To evaluate the significance of these data we performed a one way analysis of variance with the ANOVA program from a standard SPSS package. The ANOVA compares the variances within sub-samples of the data (in our case within bins), with the variances between the sub-samples (bins).

In the case of log $T_{90}$ the probability that the difference is accidental is 66%. In the case of the $T_{50}$ durations the same quantities (variances within and between bins) gives a probability of 98.5% for being a real difference between bins, or a probability of 1.5% that there is no difference between the bins. This figure gives some significance for the reality of a trend in the data; however, this value of 0.2 explains less than 1/6 of the variance of $T_{50}$ within one bin. We may conclude that even in this case the variance is mainly intrinsic.

Inspecting the same data in the case of the short duration bursts (Table 8) we come to a similar conclusion, i.e. there is no sign of trends in the durations of the different bins. Dropping the two faintest bins, which are definitely affected by biases, and dropping the poorly populated brightest bins, we arrive by the analysis of variances with ANOVA to probabilities of 53 % and 92.1 % for the difference being purely accidental between bins in $T_{90}$ and $T_{50}$, respectively.
Table 7: GRBs of long duration ($T_{90} > 2s$)

| log $P_{256}$ | log $T_{90}$ | log $T_{50}$ | $\sigma_{\log T_{90}}$ | $\sigma_{\log T_{50}}$ | No. of GRB |
|--------------|-------------|--------------|--------------------------|--------------------------|------------|
| -.50         | 1.24        | .85          | .48                      | .47                      | 49         |
| -.30         | 1.42        | 1.00         | .47                      | .50                      | 230        |
| -.10         | 1.48        | 1.08         | .49                      | .53                      | 309        |
| .10          | 1.46        | 1.02         | .51                      | .57                      | 272        |
| .30          | 1.51        | 1.01         | .52                      | .61                      | 194        |
| .50          | 1.43        | .94          | .51                      | .59                      | 161        |
| .70          | 1.45        | .96          | .48                      | .56                      | 104        |
| .90          | 1.42        | .83          | .54                      | .62                      | 56         |
| 1.10         | 1.41        | .83          | .50                      | .49                      | 44         |
| 1.30         | 1.44        | .88          | .50                      | .53                      | 34         |
| >1.40        | 1.21        | .68          | .41                      | .50                      | 29         |

Table 8: GRBs of short duration ($T_{90} < 2s$)

| log $P_{256}$ | log $T_{90}$ | log $T_{50}$ | $\sigma_{\log T_{90}}$ | $\sigma_{\log T_{50}}$ | No. of GRB |
|--------------|-------------|--------------|--------------------------|--------------------------|------------|
| -.50         | -.57        | -.87         | .55                      | .60                      | 7          |
| -.30         | -.65        | -1.01        | .53                      | .57                      | 43         |
| -.10         | -.40        | -.77         | .49                      | .51                      | 103        |
| .10          | -.35        | -.74         | .35                      | .32                      | 105        |
| .30          | -.33        | -.75         | .39                      | .41                      | 75         |
| .50          | -.27        | -.69         | .35                      | .36                      | 54         |
| .70          | -.29        | -.72         | .36                      | .34                      | 25         |
| .90          | -.35        | -.76         | .39                      | .36                      | 22         |
| 1.10         | -.18        | -.72         | .44                      | .39                      | 7          |
| 1.30         | -.74        | -1.21        | .31                      | .43                      | 5          |
| >1.40        | -.72        | -.90         | .00                      | .00                      | 1          |