Output Coupling For an Atom Laser by State Change

G.M. Mox and C.M. Savage

Department of Physics and Theoretical Physics, The Australian National University, Australian Capital Territory 0200, Australia.
(March 31, 2022)

We calculate the spectrum of a beam of atoms output from a single mode atomic cavity. The output coupling uses an internal state change to an untrapped state. We present an analytical solution for the output energy spectrum from a broadband coupler of this type. An example of such an output coupler, which we discuss in detail uses a Raman transition to produce a non-trapped state.

As a result of recent experiments in which a Bose Einstein Condensate (BEC) has been produced in the lab there has been considerable interest in coupling the atoms in a BEC out of a trap. This could produce a continuous, coherent, directional beam of atoms - an atom laser beam. While initial experiments have succeeded in coupling atoms out of a BEC by changing the internal state of the atoms to a non-trapped state, there is still much to be understood about the output beam. In this paper we present an analytical solution for the output energy spectrum of atoms in a single trapped mode coupled to free space by a change of internal state. Our analysis is based on the atom field input-output theory presented by Hope. We discuss the dependence of the spectrum on output coupling strength, and relate these findings to the MIT atom laser experiment.

In a BEC a large number of bosonic atoms are cooled into a single energy eigenstate of a trap. This is an important step towards producing a monoenergetic beam of atoms. Nevertheless we still have the problem of how to coherently couple the atoms out of such a trap in a way that preserves their monoenergetic nature.

There are many ways in which atoms can be coupled out of a trap. The simplest method is to turn off the trap. The result of rapidly turning off the trap is to reproduce the BEC wavefunction in free space. In particular, the wavefunction momentum width is conserved. As a result, the atoms have the corresponding range of energies in free space and the monoenergetic nature of the original BEC is lost. Fortunately, energy conserving output coupling is possible. One example is quantum mechanical tunneling of atoms through the trap walls. This is the atomic analogue to the use of partially transparent mirrors on an optical laser. Such a process has been considered in a model of an atom laser proposed by Wiseman. It would be difficult in practice, however, to use tunneling to produce sufficient fluxes of atoms due to the exponential dependence of the tunneling rate on the trap potential barrier.

Another approach to the output coupling problem would be to change the internal state of the trapped atoms to an untrapped state. Experimentally such a method has been used by implementing radio-frequency pulses to induce spin flips on trapped atoms in a BEC. Furthermore the use of Raman transitions as a method of output coupling has been suggested. Raman transitions have a number of advantages. A Raman transition can have an extremely narrow linewidth so that lasers can be tuned so as to only couple atoms from a particular trap mode, due to energy conservation. Moreover when Raman beams are oriented so that they are counter propagating, they provide a momentum kick of size $2\hbar k$. This could be used to provide directionality to the atomic output beam if atoms were supported against gravity, for instance in a hollow optical fiber.

We model here an output coupler based on change of state, focusing initially on the specific case of a Raman output coupler which uses two lasers tuned to a two-photon resonance to couple atoms between an initial atomic state, and a final atomic state. There is a third, excited, atomic state which mediates the Raman transition. We assume that each of the lasers is far detuned from single photon resonance. In this far detuned limit we can adiabatically eliminate the third state to produce an effective two level Hamiltonian. In this Hamiltonian we ignore the energies of higher atomic modes of the trap. Initially these other modes are empty as we assume all the atoms are condensed in the ground mode. Ignoring these higher energy modes for later time is valid for very narrow linewidth Raman lasers which are only on resonance with the ground trap mode. This ensures that higher modes do not become populated by atoms in the output state transferring back into the initial state at later times. In addition population of other modes is suppressed by Bose enhancement of transitions into the ground mode. We also ignore the effects of atom-atom interactions. The resulting effective Hamiltonian is then of the form

$$H_{\text{eff}} = H_{\text{sys}} + H_{\text{ext}} + H_{\text{int}},$$

$$H_{\text{sys}} = \hbar \tilde{\omega}_0 a^\dagger a,$$

$$H_{\text{ext}} = \int dk \hbar \tilde{\omega}_k b_k^\dagger b_k,$$

$$H_{\text{int}} = -i\hbar \int dk (\kappa(k, t) b_k a^\dagger - \kappa^*(k, t) b_k^\dagger a),$$

with
\[ \tilde{\omega}_0 = \omega_1 + \omega_2 - \frac{\Omega_1^2}{\Delta_1}, \tag{5} \]
\[ \tilde{\omega}_k = \omega_2 + \frac{\hbar k^2}{2m} - \frac{\Omega_2^2}{\Delta_2}, \tag{6} \]
\[ \kappa(k, t) = \Gamma^+(\tilde{\omega}_0 - \omega_L, k - k_L - k_{2L}), \tag{7} \]
\[ \Gamma^+ = \frac{\Omega_1 \Omega_2}{\Delta_1}. \tag{8} \]

Here, the single trap mode is described by the creation operator, \( a^\dagger \) and is coupled by the Raman lasers to a continuous spectrum of external modes described by creation operators, \( b^\dagger_k \). \( \hbar \omega_1 \) (\( \hbar \omega_2 \)) is the energy of the trap (output) atomic state. \( \hbar \omega_0 \) is the ground state trap energy, \( m \) is the mass of the trapped atoms. \( \hbar k_L \) and \( \hbar k_{2L} \) are the momenta of the two lasers inducing the Raman transition, with frequencies \( \omega_{1L} \) and \( \omega_{2L} \) respectively. Thus \( \hbar (k_{1L} + k_{2L}) \) is the total momentum kick received by atoms making the Raman transition. \( \Omega_1 \) (\( \Omega_2 \)) is the Rabi frequency of the transition between the trapped (output) state and the excited state which mediates the Raman transition. \( \Delta_1 \) and \( \Delta_2 \) are the detunings of the two Raman lasers from the excited state. We have assumed these are large in adiabatically eliminating the upper level. If the lasers are tuned close to the two-photon resonance, \( \Delta_1 \approx \Delta_2 \), \( \psi(k) \) is the momentum space wavefunction of the ground state of the trap. \( \Gamma \) is a coupling strength, given here in terms of the Rabi frequencies and single photon detuning.

The form of the Hamiltonian, Eqs. (3 - 6), is valid in the more general case of an arbitrary output coupling through state change involving a single mode system coupled to a continuous spectrum of external modes. In this more general case \( \hbar \tilde{\omega}_0 \) gives the energy of the trapped atoms, and \( \hbar \tilde{\omega}_k \) gives the energy of the free atoms. The coupling strength is more generally defined through \( \kappa(k, t) = \Gamma^+ \kappa^*(k, t) \) where \( \kappa^*(k, t) \) describes only the shape of the coupling and is normalised to unity. The form of \( \kappa(k, t) \) for a general interaction describing a change of state is \( \kappa(k, t) = \psi^*(k - k_0) \) \( \Gamma^+ \). Here, \( \psi(k) \) is the ground state momentum space wavefunction of the single mode system and \( k_0 \) describes a possible fixed momentum kick applied to the atoms in the state change process. In the following we discuss the Raman coupling case, given by Eqs. (3 - 6) for definiteness. The results, however, are valid for a general output coupler in the regime where the coupling strength, \( \Gamma \), and the energies \( \hbar \tilde{\omega}_0 \) and \( \hbar \tilde{\omega}_k \) are suitably defined.

We are interested in the output energy spectrum, \( \langle b^\dagger_k b_k \rangle \) which is the mean population density of the continuum of free space momentum eigenstate modes, labelled by the momentum \( \hbar k \). We obtain this by solving the Heisenberg equations of motion for the operators, \( b_k(t) \). In general, such a solution is difficult to obtain, however recently Hope [14] has presented a solution in terms of inverse Laplace transforms. Using these solutions, the output spectrum, in the case where initially the external modes are empty is given by

\[ \langle b^\dagger_k(t) b_k(t) \rangle = |\kappa(k, t)|^2 \langle a^\dagger(0) a(0) \rangle |M_k(t)|^2, \tag{9} \]

where

\[ M_k(t) = \mathcal{L}^{-1} \left\{ \frac{1}{(s + \mathcal{L}(f')(s))} \frac{1}{(s + i\delta_k)} \right\} (t), \tag{10} \]

\[ f'(t) = \int dk |\kappa(k, t)|^2 e^{-i\delta_k t}, \tag{11} \]

\[ \delta_k = \tilde{\omega}_k - \tilde{\omega}_0 - \omega_{1L} + \omega_{2L} = \frac{\hbar k^2}{2m} - \omega_0. \tag{12} \]

The final equality holds for the case when the lasers are tuned to the two photon resonance in free space, which we assume here. \( \mathcal{L} \) and \( \mathcal{L}^{-1} \) are the Laplace transform and inverse Laplace transform respectively.

We present an analytic solution for the spectrum in the limit of broadband coupling. For simplicity, we consider the case where the total momentum kick from the Raman lasers is very small. That is we assume \( k_{1L} \approx -k_{2L} \). This is analogous to the MIT output coupling experiments in which the atoms receive a negligible momentum kick in changing state [12,13]. We also assume that the coupling function \( \kappa(k, t) \) is broad. The shape of \( \kappa(k, t) \) is given by the ground state momentum wavefunction of the trap, \( \psi(k) \). We consider here a harmonic trap, with a gaussian ground state of standard deviation \( \sigma_k \) in wavenumber space. We can calculate an exact value for \( \mathcal{L}(f')(s) \) from the definition given in Eq. (11), however we must simplify \( \mathcal{L}(f')(s) \) in order to evaluate Eq. (10). In the regime where \( \text{Im}(s) << h\sigma_k^2/m \) we can approximate \( \mathcal{L}(f')(s) \) by

\[ \mathcal{L}(f')(s) \approx \frac{\Gamma e^{-\sqrt{s - i\omega_0}}}{\sqrt{s - i\omega_0}}, \tag{13} \]

\[ c = -i \left( \frac{m\pi}{\hbar \sigma_k^2} \right)^{1/2}. \tag{14} \]

Using this approximation to calculate \( M_k(t) \) is equivalent to discarding high (\( > h\sigma_k^2/m \)) frequency information in the Laplace transform space. As we increase the width of our coupling in momentum space, given by \( \sigma_k \), our solution for \( M_k(t) \) becomes valid for increasingly high frequencies. For an infinitely broad coupling our expression becomes exact, and is equivalent to the form of the general broadband coupling discussed by Hope [14]. Using the above expression for \( \mathcal{L}(f')(s) \) we find the inverse Laplace transform, \( M_k(t) \) to be

\[ M_k(t) = -e^{i\omega_k t} \frac{i\sqrt{\Gamma} c}{(\omega_k \Delta^2_k - \Gamma^2 c^2)^{1/2}} \frac{1}{\beta^2} \]
\[ + e^{-i\Delta_k t} \frac{i\omega_k \Delta_k}{\omega_k \Delta^2_k - \Gamma^2 c^2} \]
\[ + e^{-i\Delta_k t} \frac{1}{2} \sqrt{\frac{\pi}{t \omega_k \Delta^2_k - \Gamma^2 c^2}} L_{\gamma^2/2}(i\omega_k t) \]
\[ + \frac{\alpha^2 \exp \left[ i(\alpha^2 + i\omega_k) t \right]}{(\beta - \alpha)(\gamma - \alpha)(\alpha^2 + i\omega_k)(1 + \text{Erf}(\alpha\sqrt{t}))} \]
\[ + \frac{\alpha^2 \exp \left[ i(\alpha^2 + i\omega_k) t \right]}{(\beta - \alpha)(\gamma - \alpha)(\alpha^2 + i\omega_k)(1 + \text{Erf}(\alpha\sqrt{t}))} \]
where we have defined $\omega_k = \hbar k^2/(2m)$ and $\Delta_k = \omega_k - \omega_0$. The function $L^\gamma_m(x)$ is a Laguerre polynomial, Erf is the error function and $\alpha, \beta$ and $\gamma$ are the roots of the equation $s^3 + i\omega_0 s + \Gamma c/v = 0$.

Fig. 1 shows the behaviour of $|M_k(t)|^2$ as a function of $\omega_k$ and time after we turn on the output coupling interaction. Initially $|M_k(t)|^2$ is small, and for short enough times, arbitrarily broad in $k$-space. Initially $|M_k(t)|^2$ agrees with the perturbative solutions presented by Hope above. For longer times, we can see that the spectrum reaches a stable shape. For very large values of the coupling strength, the long time limit becomes very broad in $k$-space. As a result, the shape of the output spectrum, as given by Eq. (3), simply reflects the momentum distribution of the cavity wave-function, $\psi(k)$. As a result, there is no narrowing of linewidth in momentum space. The recent MIT experiments [12,13] are an example of an output coupling with an extremely large coupling strength. In these experiments a short, $5\mu$s RF pulse was used to couple atoms out of a BEC, making a pulsed atom laser.

We consider here a continuous coupler, turned on at time $t = 0$, and examine the resulting long time spectrum in the external modes described by $b_k^\dagger$. We observe in Fig. 2 that for longer times $|M_k(t)|^2$ narrows into a sinc function centered about the trap ground state frequency, $\omega_0$. Eventually $|M_k(t)|^2$ reaches a stationary state with a lorentzian like profile as shown in Fig. 3. This long time behaviour is given by

$$\lim_{t \to \infty} M_k(t) = \frac{i\sqrt{\omega_k}e^{-\Delta_k t}}{\sqrt{\omega_k \Delta_k - \Gamma c}} + \frac{2\gamma^2 e^{(i\omega_0 + \gamma^2)t}}{(\alpha - \gamma)(\beta - \gamma)(\gamma^2 + i\omega_k)}$$

where $\gamma$ is the particular solution to the cubic discussed above, given by the expression

$$\gamma = e^{\frac{\xi}{2}} \left( \frac{2\xi}{\xi^2} - \frac{\xi^2}{32} \right),$$

$$\xi = -27\Gamma c + (27\Gamma c)^2 + 108\omega_0^3)^{\frac{3}{2}}.$$

The long time expression for $M_k(t)$, Eq. (18) contains two terms. The first of these terms dominates in the case of small $\Gamma$, while the second dominates for very large $\Gamma$. As a result, the long time spectrum has two distinct behaviours depending on the strength of the coupling. We consider the case of slow coupling (small $\Gamma$) initially. In this case, the long time expression for $M_k(t)$ is dominated by the first term in Eq. (18) above, and the resulting long time spectrum is given by

$$\langle b_k^\dagger b_k \rangle = \Gamma |\psi(k)|^2 \frac{1}{(\Delta_k^2 + |\Gamma c|^2/\omega_k)}.$$

A plot of the long time spectrum, Eq. (18) as a function of $\omega_k$ is presented in Fig. 2 for various coupling strengths. Fig. 2 shows that for increasing coupling strength the linewidth of the long time spectrum increases. The values for $\Gamma$ chosen correspond approximately to values of Raman laser Rabi frequencies, $\Omega_1 \approx 2\pi \times 50$ kHz and $\Omega_2 \approx 2\pi \times 1.6$ MHz and detuning, $\Delta_1 = \approx 2\pi \times 2.5$ GHz similar to values presented in [13]. However, much smaller or larger coupling strengths can be achieved by suitably adjusting the intensities of the lasers and their detunings.

For each of the graphs shown in Fig. 2, the lorentzian like spectrum is centred about $\omega_0$, the ground state frequency of the single mode trap, with the width of the spectrum dependent on the strength of the coupling as mentioned above. In all cases, however, the linewidth is much less than that which would be obtained if the trap was rapidly turned off, that is $\sigma_{\omega_k} \approx 10^3 s^{-1}$. We see from Eq. (18) that the distribution isn’t exactly lorentzian due to the presence of $\omega_k$ in the second part of the denominator. However for large $\omega_0$ the spectrum is well approximated by a lorentzian distribution of width $|\Gamma c|/\sqrt{\omega_0}$.

We have already noted that for large coupling rates, the width of the long time limit of $|M_k|^2$, and hence of the long time spectrum is increased. When $\Gamma$ is very large, $|\Gamma c|/\sqrt{\omega_0} >> \sigma_{\omega_k}$, the width of $M_k(t)$ becomes large compared with $\kappa(k,t)$ and the spectrum becomes dominated by the cavity momentum spread $\psi(k)$. As a result, for sufficiently fast coupling (large $\Gamma$) the output spectrum changes significantly from the lorentzian shape considered above, and instead reflects the momentum spread of the cavity. This is shown in Fig. 3. For very large $\Gamma$ the spectrum is centred about zero, and falls away exponentially in $\omega_k$ space, as required for a gaussian distribution in momentum space given by $\psi(k)$.

We have shown that the long time spectrum from an output coupler based on state change depends on the strength of the output coupling. For very strong coupling, the output spectrum is given by the cavity spectrum, and is very broad in momentum space. The spectrum is then centered about the zero of momentum when there is no net momentum kick from the lasers. As the strength of the coupling is reduced, however, the long time linewidth is correspondingly reduced. For small coupling strengths the final linewidth is effectively lorentzian, centred about the energy of the cavity with a linewidth proportional to the coupling strength $\Gamma$.

The authors would like to thank Joseph Hope for much advice and many thoughtful discussions.
* Email address: Glenn.Moy@anu.edu.au

[1] M.H. Anderson et al., Science 269, 198 (1995).
[2] C.C. Bradley et al., Phys. Rev. Lett. 75, 1687 (1995).
[3] K.B. Davis et al., Phys. Rev. Lett. 75, 3969 (1995).
[4] M.O. Mewes et al., Phys. Rev. Lett. 77, 416 (1996).
[5] M. Holland et al., Phys. Rev. A 54, R1757 (1996).
[6] H.M. Wiseman and M.J. Collett, Physics Lett. A 202, 246 (1995).
[7] H.M. Wiseman et al., Quantum Semiclass. Opt. 8, 737 (1996).
[8] A.M. Guzman et al., Phys. Rev. A 53, 977 (1996).
[9] R.J.C. Spreeuw et al., Europhysics Letters 32, 469 (1995).
[10] M. Olshanii et al., Proc. of the 12th Int. Conference on Laser Spectroscopy, edited by M. Inguscio, M. Allegrini and A. Sasso (1995).
[11] G.M. Moy et al., Phys. Rev. A 55, May (1997).
[12] M.-O. Mewes et al., Phys. Rev. Lett. 78, 582 (1997).
[13] M.R. Andrews et al., Science 275, 637 (1997).
[14] J.J. Hope, Phys. Rev. A 55, April (1997).
[15] S. Marksteiner et al., Phys. Rev. A 50, 2680 (1994).
[16] H. Ito et al., Optics Comm. 115, 57 (1995).
[17] M. Renn et al., Phys. Rev. A 53, R648 (1996).
[18] M. -O. Mewes et al., Phys. Rev. Lett. 77, 416 (1996).

FIG. 1. Plot of $|M_k(t)|^2$ as a function of $\omega_k$ and time for $t = 0s$ to $t = 5s$, and $\omega_k$ ranging from $762s^{-1}$ to $783s^{-1}$ about the single mode trap frequency, $\omega_0 \approx 772s^{-1}$. $\Gamma = 1.8 \times 10^3 s^{-2}$.

FIG. 2. Plot of the long time behaviour of $\langle b_k^\dagger b_k \rangle$ as a function of $\omega_k$ for various coupling strengths, $\Gamma = 10^4 s^{-2}$ (dotted line), $\Gamma = 3 \times 10^4 s^{-2}$ (solid line) and $\Gamma = 5 \times 10^4 s^{-2}$ (dashed line).

FIG. 3. Plot of the steady state behaviour of $\langle b_k^\dagger b_k \rangle$ as a function of $\omega_k$ for the large coupling limit ($\Gamma \approx 10^{13} s^{-2}$).
\[
\langle b_k^\dagger b_k \rangle \quad (\text{m}^{-1})
\]

\[
\omega_k \quad (\text{s}^{-1})
\]

\[
(\times 10^{-9})
\]