Electric-Field Breakdown of Absolute Negative Conductivity and Supersonic Streams in Two-Dimensional Electron Systems with Zero Resistance/Conductance States

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We calculate the current-voltage characteristic of a two-dimensional electron system (2DES) subjected to a magnetic field and strong microwave radiation. The occurrence of the effect of the so-called zero-resistance (ZR) and zero-conductance (ZC) states has been linked to the effect of absolute negative conductivity (ANC) when the dissipative dc conductivity \( \sigma_d(E) \) and microwave photoconductivity \( \sigma_{ph}(E) \) is negative \( \sigma_{ph}(E) < 0 \). As speculated, the mechanism of ANC responsible for the occurrence of ZR and ZC states is associated with the photon-assisted scattering of electrons on impurities, although alternative mechanisms, particularly the photon-assisted interaction of electrons with acoustic phonons, can be essential as well. It has long been shown that the photon-assisted impurity scattering can result in a negative dissipative photoconductivity \( \sigma_{ph}(E) \) in the situations in the microwave frequency \( \Omega \) somewhat exceeds the cyclotron frequency \( \Omega_c = eH/mc \) or its harmonics. According to the calculations, the dissipative microwave photoconductivity can be negative at sufficiently strong irradiation at \( 0 < \Omega - \Omega_c \leq eEL/h, \Gamma/h \). Here \( e \) and \( m \) are the electron charge and effective mass, respectively, \( c \) is the velocity of light, \( E \) and \( H \) are the electric and magnetic fields, \( L = (\hbar/eH)^{1/2} \) is the quantum Larmor radius, \( \Gamma \) is the Landau level (LL) broadening, and \( h \) is the Planck constant.

It is crucial for the explanations of ZR and ZC states, invoking the concept of ANC, in a 2DES with both the Hall bar and Corbino configurations that there is an electric field \( E_0 \) at which \( \sigma(E_0) = 0 \). The modulus of the dissipative photoconductivity decreases with increasing electric field when the latter becomes sufficiently large, namely, when \( E > E_0 = \max\{h(\Omega - \Delta\Omega_c)/eL, \Gamma/eL\} \). Hence, one can expect that \( E_0 \) is determined by the resonance detuning and the LL broadening with \( E_0 > E_b \). The characteristic field \( E_b \) is rather large. Indeed, at the magnetic field \( H = 2 \) kG, assuming that \( |\Omega - \Omega_c|/\Omega_c = 0.25 \), we have \( E_b \approx 20 \) V/cm. The estimated values of \( E_b \) significantly exceed the average electric field \( \langle E \rangle \) in the 2DES observed experimentally, which is in the range \( \langle E \rangle \approx 3 \times (10^{-3} - 10^{-1}) \) V/cm depending on the sample geometry. Taking into account the instability of uniform electric-field distributions at \( E < E_b \), one can conclude (see, for example, Refs. [7, 8, 11]), that in the case \( \langle E \rangle \ll E_b \), nearly periodic domain structures are formed as shown in Fig. 1. In these domain structures, \( |J_y| = |\sigma_H d|\langle E_x \rangle| \ll |\sigma_H dE_0|, \quad J_x = 0, \quad \text{and} \quad E_y = 0 \) in the Hall bar configuration, and \( |V_y| = |d(\langle E_y \rangle)| \ll dE_0, \quad J_y = 0, \quad \text{and} \quad E_x = 0 \) in the Corbino samples. Here \( \sigma_H \) is the Hall conductivity and \( d \) is the 2DES size. It would appear reasonable that the shape of the domains

![FIG. 1: Schematic view of possible domain structures in 2DES corresponding to (a) ZR states in the Hall bar configuration and input current \( J_y \) is the input current and \( V_y = 0 \) is the measured voltage) and (b) ZC states in the Corbino geometry (here \( V_y \) is the applied voltage). Arrows show directions of the Hall current in different high-field domains.](image)

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The point is that at $E < E_c$, separation energy of the acoustic phonons involved equals the LL energy of the electron-phonon interaction to the dissipative dark current at low temperatures is exponentially small, the mechanism providing the lowest value of $E_0$ can substantially govern the domain structure and, therefore, the observable macroscopic characteristics.

The electric-field dependence of the net dissipative conductivity can be determined not only by $\sigma_{ph}(E)$ but by $\sigma_d(E)$ as well. As shown in the following, the dark component can be a steep function of the electric field resulting in the breakdown of ANC if $\sigma_{ph}(E) < 0$. The contribution of the electron-impurity interaction to $\sigma_d(E)$ results in a smooth electric-field dependence up to very strong electric fields $E > E_s = \hbar \Omega_c/eL$, can lead to a sharp overshoot of the dissipative current (conductivity) at $E \simeq E_s = \hbar \Omega_c/eL$ (see also Ref. [18]). However, the contribution of the electron scattering on acoustic phonons, being an important factor at high temperatures and weak electric fields $E < E_s$, the selection rules allow only the electron-phonon scattering events accompanied by the inter-LL electron transitions. Since the energy of the acoustic phonons involved equals the LL separation $\hbar \Omega_c$, and therefore is rather large, the number of such phonons at low temperatures is exponentially small. This results in relatively small contribution of the electron-phonon interaction to the dissipative dark current (dissipative conductivity) at $E < E_s$. In contrast, at $E_s \lesssim E \ll E_c$, the intra-LL scattering transitions can significantly contribute to the dissipative dark current. Such transitions are schematically shown in Fig. 2.

This range of the electric fields corresponds to the electron Hall drift velocity $v_H = eE/H \geq s$. Assuming $s = 3 \times 10^5$ cm/s and $H = 1 - 2$ kG, one can obtain $E_s \simeq 3 - 6$ V/cm. A peculiarity of the electron transport at $E \approx E_s$ was pointed out previously, particularly in connection with the breakdown of the quantum Hall effect [21, 22, 23, 24, 25, 26, 27] (see also Refs. therein). The dissipative dark current density $j_d(E)$ and, consequently, the dissipative dark conductivity $\sigma_d(E) = j_d/E$ of a 2DES in the case of the electron-phonon interactions can be calculated using the following formula:

$$j_d(E) = j_1 + j_2 = \sum_{N \neq N'} j_{N,N'} + \sum_N j_{N,N},$$

where

$$j_{N,N'} = \frac{e}{h} f_N (1 - f_{N'}) \times \int d^3 q \, q_y |V_q|^2 |Q_{N,N'}(L^2 q_{\perp}^2/2)|^2 \times \{N_q \delta[(N-N')\hbar \Omega_c + \hbar \omega_q + eEL^2 q_y]$$

$$+ (N_q + 1) \delta[(N-N')\hbar \Omega_c - \hbar \omega_q + eEL^2 q_y]\}.$$  

Here $f_N$ and $N_q$, are the electron and phonon distribution functions, $N = 0, 1, 2, ...$ is the LL index, $q = (q_x, q_y, q_z)$, $q_{\perp} = (q_x, q_y)$, and $\omega_q = sq$ are the phonon wave vector, its in-plane component, and the phonon frequency, respectively, $\delta(\omega)$ is the LL form-factor determined by $\Gamma$, $|V_q|^2 = |C|^2 \exp(-L^2 q_{\perp}^2/2)/q$ describes the electron interaction with piezoelectric phonons, where $l$ is the width of the electron localization in the $z$-direction perpendicular to the 2DES plane and $|C|^2$ is a constant. At a strong localization in the quantum well at the heterointerface, $l \ll L$. $|Q_{N,N'}(L^2 q_{\perp}^2/2)| = |P_{N'}^{N-N}(L^2 q_{\perp}^2/2)|^2 \exp(-L^2 q_{\perp}^2/2)$ is determined by the overlap of the electron wave functions, and $|P_{N'}^{N-N}(L^2 q_{\perp}^2/2)|^2$ is proportional to a Laguerre polynomial $L_{N'}^{N-N}(L^2 q_{\perp}^2/2)$.

In the range of moderate electric fields (much smaller than $E_c$), the contribution of the electron scattering on acoustic phonons accompanied by the inter-LL transitions can be presented as [12]

$$j_1 \simeq \sigma_1 E,$$

$$\sigma_1 = \mu_1 \left(\frac{\hbar s}{T L}ight) \left(\frac{s}{L \Omega_c}ight) \exp\left(-\frac{\hbar \Omega_c}{T}\right) \exp\left(-\frac{p^2 \Omega_c}{2s^2}\right),$$

where $\mu_1 \propto C^2 \sum_N f_N (1 - f_{N+1})/\sqrt{N}$ and $T$ is the temperature (in energy units). The quantity $j_1$ remains...
When filling factors corresponding to the experimental conditions contrast to Ref. [25], we will focus on the case of large $F$ with large $N$ with large energy reckoned from the lowest LL. In this case, the LL's scattering selection rule $\hbar \omega_\alpha = eEL^2 q_\parallel$ is not met at low electric fields. At $E \geq E_s$, considering Eqs. (1) and (2), we obtain

$$j_2 = \frac{e|C|^2}{2\mu_s} \sum_N f_N (1 - f_N) \int d\Phi \sin \Phi \Phi dq_\perp \frac{q^2}{\sqrt{q^2 - q^2_\perp}}$$

$$\times \exp(-l^2 q^2/2) \exp[(l^2 - L^2) q^2_\perp/2] L_N^0 (L^2 q^2_\perp/2)^2$$

$$\times [N_q \delta(q + F q_\perp \sin \Phi) + (N_q + 1) \delta(q - F q_\perp \sin \Phi)]. \quad (5)$$

Here $F = eEL^2/h\gamma = cE/sH$ and $\sin \Phi = q_\perp/q_N$. In contrast to Ref. [23], we will focus on the case of large filling factors corresponding to the experimental conditions $[1,2,3]$, i.e., the case $\zeta \gg \hbar \Omega_c$, where $\zeta$ is the Fermi energy reckoned from the lowest LL. In this case, the LL’s with large $N$ yield the main contribution (with $N = N_m$ and $N = N_m + 1$, where $\zeta/\hbar \Omega_c \leq N_m < \zeta/\hbar \Omega_c + 1$). When $N \gg 1$, $|L_N^0 (L^2 q^2_\perp/2)|^2 \simeq J_0^2 (2\sqrt{2N} L q_\perp) \simeq 2 \cos^2(\sqrt{2N} L q_\perp - \pi/4)/\sqrt{2N} L q_\perp$, where $J_0(z)$ is the Bessel function. Taking this into account, after integrating over $q_\perp$ Eq. (5) can be reduced to

$$j_2 = \frac{\mu_s h\gamma}{2eL^2} \int d\Phi \sin \Phi \Phi dq_\perp \frac{\exp(-l^2 (F^2 \sin^2 \Phi - 1) q^2_\perp/2)}{\sqrt{F^2 \sin^2 \Phi - 1}}$$

$$\times \exp(-L^2 q^2_\perp/2) \cos^2(\sqrt{2N} L q_\perp - \pi/4). \quad (6)$$
oscillations in Fig. 4 are akin to the Shubnikov-de Haas oscillations.

The relative height of the current overshoot is given by

$$\frac{j_2}{j_1} \bigg|_{E=E_s} \simeq \left( \frac{\mu_2}{\mu_1} \right) \left( \frac{T}{m s^2} \right) \exp \left( \frac{\hbar \Omega_c}{T} \right) \gg 1. \quad (9)$$

The ratio $\mu_2/\mu_1$ varies with the magnetic field because the LL filling factor depends on the LL positions with respect to the Fermi level. However, when $T < \hbar \Omega_c$, the product $\mu_2/\mu_1 \exp(\hbar \Omega_c/T) \gg 1$ at all values of $(\xi - N_0 h \Omega_c)$. The parameter $T/m s^2$ exceeds unity at $T > 50$ mK.

A sharp increase in the dissipative current at $E = E_s$ can also occur due to the electron interaction with interface acoustic phonons. In this case, owing to specific electron-phonon scattering selection rules [30], the dissipative current can reveal even sharper overshoot at $E = E_s$.

Because of a strong overshoot of the dissipative dark current at $E = E_s$, the net dissipative conductivity of a 2DES irradiated with microwaves can change its sign at $E_0 \simeq E_s$. The net current voltage characteristic of a 2DES under microwave irradiation with $\sigma_{ph} < 0$ (associated, for example, with the photon-assisted scattering on impurities) is shown schematically in the inset in Fig. 3. The latter characteristic corresponds to an abrupt change in the sign of the net dissipative conductivity, i.e., the breakdown of ANC, at $E_0 \simeq 3$ V/cm. A sharp transition from $\sigma(E) < 0$ at $E < E_0 \simeq E_s$ to $\sigma(E) > 0$ when $E$ exceeds $E_s$ can significantly influence the domain structures resulting in ZR and ZC states. The remarkable feature of these domain structures is that the regions, where $E_x > E_s$ in the Hall bar configuration or $E_0 > E_s$ in the Corbino samples, shown in Fig. 1 by the shaded “lanes” with arrows, are sources of the acoustic phonons generated by the electron streams with supersonic Hall drift velocities. The generation of acoustic phonons by these streams can lead to a pronounced deviation of the phonon system from equilibrium. This, in turn, can affect the values and sign of the dissipative conductivity [17].

Worth noting the feasibility of formation of stable ZR states at relatively large currents. These states can correspond to $\langle E_s \rangle \simeq \pm E_0$ when the electric-field distributions are nearly uniform. The states in question can be formed when $J_y \approx \pm J_0 = \pm \sigma_H dE_0$. Taking into account that $\sigma_{ph} = \epsilon \Sigma / H$, where $\Sigma$ is the electron sheet concentration, and assuming $E_0 = E_s$, one can obtain $J_0 \approx \epsilon s \Sigma$. The quantity $J_0$ is by two orders of magnitude larger than the currents in ZR states observed experimentally. The states with smooth electric-field distributions can arise in the Corbino samples at the applied voltage $V' \simeq \pm V_0$ with $V_0 = dE_0 \simeq \pm dE_s$. If $d = 0.25$ cm, at $H = 1$ kG one obtains $V_0 \approx 0.75$ V. Due to the absence of dissipation, the Joule heating can be disregarded despite relatively large values of the current (applied voltage).

In summary, we calculated the electric-field dependence of the dissipative dark current associated with the scattering of electrons on acoustic phonons in a 2DES subjected to a magnetic field. The obtained current-voltage characteristic exhibits a strong overshoot at a certain electric field. This field corresponds to the Hall drift velocity equal to the speed of sound. The overshoot of the dark component of the dissipative current can suppress the negative microwave photoconductivity (breakdown of ANC) in the regions with high electric field, where supersonic electron streams occur, and affect the formation of domain structures with the zeroth resistance/conductance.

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