The effects of Majorana phases in three-generation neutrinos

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Abstract

Neutrino-oscillation solutions for the atmospheric neutrino anomaly and the solar neutrino deficit can determine the texture of the neutrino mass matrix according to three types of neutrino mass hierarchies as Type A: $m_1 \ll m_2 \ll m_3$, Type B: $m_1 \sim m_2 \gg m_3$, and Type C: $m_1 \sim m_2 \sim m_3$, where $m_i$ is the $i$-th generation neutrino absolute mass. The relative sign assignments of neutrino masses in each type of mass hierarchies play the crucial roles for the stability against quantum corrections. Actually, two physical Majorana phases in the lepton flavor mixing matrix connect among the relative sign assignments of neutrino masses. Therefore, in this paper we analyze the stability of mixing angles against quantum corrections according to three types of neutrino mass hierarchies (Type A, B, C) and two Majorana phases. Two phases play the crucial roles for the stability of the mixing angles against the quantum corrections.

PACS:12.15.Ff, 14.60.Pq, 23.40.Bw.
1 Introduction

Recent neutrino oscillation experiments suggest the strong evidences of tiny neutrino masses and lepton flavor mixings[1, 2, 3, 4]. Studies of the lepton flavor mixing matrix, which is so-called Maki-Nakagawa-Sakata (MNS) matrix[5], will give us important cues of the physics beyond the standard model. One of the most important studies is the analysis of the quantum correction on the MNS matrix[6]–[13].

In order to explain both the solar and the atmospheric neutrino problems, two mass squared differences are needed, which implies

\[ \Delta m^2_{\text{solar}} \equiv |m^2_2 - m^2_1|, \quad \text{and} \quad \Delta m^2_{\text{ATM}} \equiv |m^2_3 - m^2_2|, \]  

(1)

where \( m_i \) is the \( i \)-th (\( i = 1 \sim 3 \)) generation neutrino mass (\( m_i \geq 0 \)). \( \Delta m^2_{\text{solar}} \) and \( \Delta m^2_{\text{ATM}} \) stand for the mass-squared differences of the solar neutrino [1] and the atmospheric neutrino solutions[2, 3], respectively. Then there are the following three possible types of neutrino mass hierarchies [14]:

Type A : \( m_1 \ll m_2 \ll m_3 \),  
Type B : \( m_1 \sim m_2 \gg m_3 \),  
Type C : \( m_1 \sim m_2 \sim m_3 \),

(2)

where \( m_i \) is the \( i \)-th generation neutrino absolute mass. In Ref. [11], it has been studied whether the lepton-flavor mixing angles are stable or not against quantum corrections for all three types of mass hierarchies with all considerable relative sign assignments, which are shown below, in the minimal supersymmetric standard model (MSSM) with an effective dimension-five operator which gives the Majorana masses of neutrinos.

1. Type A:

\[ \text{case(a1)} : \quad m^a_{\nu} = \text{diag.}(0, m_2, m_3), \]  
\[ \text{case(a2)} : \quad m^a_{\nu} = \text{diag.}(0, -m_2, m_3). \]  

(3)

\( m_1 = 0, \quad m_2 = \sqrt{\Delta m^2_{\text{solar}}}, \quad m_3 = \sqrt{\Delta m^2_{\text{solar}} + \Delta m^2_{\text{ATM}}} \)

2. Type B:

\[ \text{case(b1)} : \quad m^b_{\nu} = \text{diag.}(m_1, m_2, 0), \]  
\[ \text{case(b2)} : \quad m^b_{\nu} = \text{diag.}(m_1, -m_2, 0). \]  

(5)

\( m_1 = \sqrt{\Delta m^2_{\text{ATM}}}, \quad m_2 = \sqrt{\Delta m^2_{\text{solar}} + \Delta m^2_{\text{ATM}}}, \quad m_3 = 0 \)
3. Type C:

\[ \begin{align*}
\text{case (c1): } m^c_{\nu} &= \text{diag.}(-m_1, m_2, m_3), \\
\text{case (c2): } m^c_{\nu} &= \text{diag.}(m_1, -m_2, m_3), \\
\text{case (c3): } m^c_{\nu} &= \text{diag.}(-m_1, -m_2, m_3), \\
\text{case (c4): } m^c_{\nu} &= \text{diag.}(m_1, m_2, m_3).
\end{align*} \]

\[ m_1 = m_0, \quad m_2 = \sqrt{m_0^2 + \Delta m_{\text{solar}}^2}, \quad m_3 = \sqrt{m_0^2 + \Delta m_{\text{solar}}^2 + \Delta m_{\text{ATM}}^2}. \]

In Ref. [11], it has been found that the above relative sign assignments of neutrino masses in each type play the crucial roles for the stability of the mixing angles against quantum corrections. Actually, two physical Majorana phases in the lepton flavor mixing matrix connect among the above relative sign assignments of neutrino masses. Therefore, in this paper we analyze the stability of mixing angles against quantum corrections according to three types of neutrino mass hierarchies (Type A, B, C) and two Majorana phases. Two phases play the crucial roles for the stability of the mixing angles against the quantum corrections. In Refs. [12, 13], it has been already analyzed that the effect of a Majorana phase plays an important role for the stability against the quantum corrections in the two-generation neutrinos.

2 Quantum corrections to neutrino mass matrix

In the MSSM with the effective dimension-five operator which gives Majorana masses of neutrinos, the superpotential of the lepton-Higgs interactions is given by

\[ W = y_{ij}^e (H_d L_i) E_j - \frac{1}{2} \kappa_{ij} (H_u L_i) (H_u L_j). \]

Here the indices \( i, j \) (= 1 \sim 3) stand for the generation number. \( L_i \) and \( E_i \) are chiral superfields of \( i \)-th generation lepton doublet and right-handed charged-lepton, respectively. \( H_u \) (\( H_d \)) is the Higgs doublet which gives Dirac masses to the up- (down-) type fermions. The neutrino mass matrix of the three generations, \( \kappa \) is diagonalized as

\[ U^T \kappa U = D_\kappa, \]

where \( D_\kappa \) is given by

\[ D_\kappa = \begin{pmatrix}
  m_1 & 0 & 0 \\
  0 & m_2 & 0 \\
  0 & 0 & m_3
\end{pmatrix}, \]
with $m_i \geq 0$. The unitary matrix $U$ is defined as

$$U = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
e^{i\phi_1} & 0 & 0 \\
0 & e^{i\phi_2} & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

(14)

where $\phi_{1,2}$ denote the physical Majorana phases of the lepton sector. In the diagonal base of charged lepton masses, $U$ is just the MNS matrix. We can easily show that one Majorana phase connects between cases of (a1) and (a2), (b1) and (b2), and two Majorana phases connect among cases of (c1)∼(c4). Thus, the stabilities of mixing angles against quantum corrections are completely determined by three types of neutrino mass hierarchies (Type A, B, C) and two Majorana phases $\phi_{1,2}$ in stead of the classifications of Eqs. (3)∼(10).

We will analyze whether the lepton flavor mixing angles are changed or not by the quantum corrections by fitting the low energy data. We determine the MNS matrix at $m_Z$ scale as

$$U = \begin{pmatrix}
cos \theta_{12} & \sin \theta_{12} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}
\begin{pmatrix}
e^{i\phi_1} & 0 & 0 \\
0 & e^{i\phi_2} & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

(15)

where we input $\sin \theta_{23} = 1/\sqrt{2}$ and $\sin \theta_{13} = 0$ which values are suitable for the atmospheric neutrino experiments [2, 3] and for the CHOOZ experiment [4], respectively. The mixing angle $\theta_{12}$ depends on the solar neutrino solutions of the large angle MSW solution (MSW-L), the small angle MSW solution (MSW-S) and vacuum oscillation solution (VO), which are given by

$$\sin \theta_{12} = \begin{cases}
0.0042 & (\theta = 0.0042) \quad (\text{MSW-S}), \\
\frac{1}{\sqrt{2}} & (\theta = \frac{\pi}{4}) \quad (\text{MSW-L}), \\
\frac{1}{\sqrt{2}} & (\theta = \frac{\pi}{4}) \quad (\text{VO}).
\end{cases}
$$

(16)

We also use the following values of mass-squared differences in the numerical analyses.

$$\Delta m^2_{\text{SOLAR}} \simeq \begin{cases}
0.8 \times 10^{-5} \text{ eV}^2 & (\text{MSW-S}), \\
1.8 \times 10^{-5} \text{ eV}^2 & (\text{MSW-L}), \\
0.85 \times 10^{-10} \text{ eV}^2 & (\text{VO}),
\end{cases}
$$

(17)

$$\Delta m^2_{\text{ATM}} \simeq 3.7 \times 10^{-3} \text{ eV}^2.
$$

(18)
The quantum corrections change the neutrino mass matrix, and it is given by\cite{9,10}

\[
\hat{\kappa}(m_R) = \frac{\hat{\kappa}(m_R)_{33}}{\kappa(m_Z)_{33}} \begin{pmatrix}
1 - \epsilon & 0 & 0 \\
0 & 1 - \epsilon & 0 \\
0 & 0 & 1
\end{pmatrix} \kappa(m_Z) \begin{pmatrix}
1 - \epsilon & 0 & 0 \\
0 & 1 - \epsilon & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad (19)
\]

at the high energy scale $m_R$, where $\epsilon$ can be estimated as

\[
\epsilon \simeq 1 - \exp\left(-\frac{1}{16\pi^2} \int_{\ln(m_Z)}^{\ln(m_R)} y^2_{\tau} dt\right),
\]
\[
\simeq \frac{1}{8\pi^2} \frac{m_{\tau}^2}{v^2} \left(1 + \tan^2 \beta\right) \ln\left(\frac{m_R}{m_Z}\right). \quad (20)
\]

where $y_{\tau}$ is the Yukawa coupling of $\tau$, $v^2 \equiv v_u^2 + v_d^2$ and $\tan \beta \equiv v_u / v_d$ ($v_u$ and $v_d$ are vacuum expectation values of Higgs bosons, $H_u$ and $H_d$, respectively). We neglect the Yukawa couplings of $e$ and $\mu$ in Eqs.\cite{19} and \cite{20}, since those contributions to the renormalization group equations are negligibly small comparing to that of $\tau$.\cite{11}. The magnitude of $\epsilon$ can be determined by the value of $\tan \beta$ and the scale of $m_R$. The unitary matrix $\hat{U}$ which diagonalizes $\hat{\kappa}$ shows us whether the lepton flavor mixing angles are stable against quantum corrections or not.

3 Type A ($m_1 \ll m_2 \ll m_3$)

In both (a1) and (a2) cases, all mixing angles are stable against quantum corrections in each sign assignment\cite{11}. This is understood from the analogy of two-generation analysis, which shows the mixing angle of $2 \times 2$ mass matrix is not changed drastically by the quantum corrections when there is the large mass hierarchy between two neutrinos\cite{11}. This situation is not changed when we consider the Majorana phase contribution as shown in two-generation neutrinos\cite{12}. Cases (a1) and (a2) are connected with each other by the Majorana phase of $\phi_2$. Where $\phi_1$ is rotated out, since $m_1 = 0$. The case of $\phi_2 = 0$ corresponds to (a1), while the case of $\phi_2 = \pi/2$ corresponds to (a2). Since Type A has the large mass hierarchies, all mixing angles are supposed to be stable against quantum corrections independently of the value of the Majorana phase $\phi_2$. This is really confirmed by numerical analyses as shown in Table\cite{1}, where we use $m_R = 10^{13}$ GeV and $\tan \beta = 60$.

\footnote{Hereafter, we denote the mixing angles and the other physical parameters at the $m_R$ scale are written with $\hat{\ }$ mark.}
Table 1: Stabilities of the mixing angles with the Type A mass hierarchy according to the change of $\phi_2$ from 0 to $\pi$ in the case of $m_R = 10^{13}$ GeV and $\tan \beta = 60$.

4 Type B ($m_1 \sim m_2 \gg m_3$)

In Type B mass hierarchy, all mixing angles except for $\sin \theta_{12}$ of (b2) are stable against quantum corrections [11]. The analogy of two-generation neutrinos analysis shows that mixing angles of $\sin \theta_{13}$ and $\sin \theta_{23}$ are stable against quantum corrections, since there are large mass hierarchies between the first and the third generations, and between the second and third generations. This is the same situation as that of Type A. This situation is not changed by including the Majorana phase contributions of $\phi_{1,2}$ as shown in Table 2, which shows the results of the numerical analyses in the case of $m_R = 10^{13}$ GeV and $\tan \beta = 60$.

On the other hand, the mixing angle of $\theta_{12}$ can receive significant quantum corrections dependently on the relative sign assignment of $m_2$ as shown in Ref [11]. The mixing angle of $\sin \theta_{12}$ of (b1) receives the quantum correction while that of (b2) does not. Now we understand that two cases of (b1) and (b2) are connected with each other by the phase of $\phi \equiv \phi_1 - \phi_2$, which is the only physical phase, since $m_3 = 0$. The case of $\phi = 0$ corresponds to (b1), while the case of $\phi = \pi/2$ corresponds to (b2). The phase $\phi$ is the parameter which determines whether the mixing angle $\theta_{12}$ is stable against quantum corrections or not.

Now let us show the analytic estimations for the stabilities of the mixing angles in Type B mass hierarchy. The neutrino mass matrix of Type B which is diagonalized is given by

$$D^{(B)} = m_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 + \xi_b & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

(21)
where

\[ \xi_b \equiv \frac{m_2 - m_1}{m_1} \simeq \frac{1}{2} \frac{\Delta m_{\text{solar}}^2}{\Delta m_{\text{ATM}}^2}. \]  

(22)

We can determine the mass matrix of \( \kappa^{(B)} \) by using Eqns. (12) and (15). Then Eq. (19) gives the mass matrix of \( \hat{\kappa}^{(B)} \) at the high energy scale \( m_R \).

The MNS matrix \( \hat{U}^{(B)} \) which diagonalizes \( \hat{\kappa}^{(B)} \) is given by

\[
\hat{U}^{(B)} = \begin{pmatrix}
1 & 0 & 0 \\
0 & \left(1 - \epsilon \right)/\sqrt{1 + \left(1 - \epsilon \right)^2} & 1/\sqrt{1 + \left(1 - \epsilon \right)^2} \\
0 & -1/\sqrt{1 + \left(1 - \epsilon \right)^2} & \left(1 - \epsilon \right)/\sqrt{1 + \left(1 - \epsilon \right)^2}
\end{pmatrix}
\times \begin{pmatrix}
\cos \hat{\theta}_{12} & \sin \hat{\theta}_{12} & 0 \\
-\sin \hat{\theta}_{12} & \cos \hat{\theta}_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\times \begin{pmatrix}
e^{i \hat{\phi}_1} & 0 & 0 \\
0 & e^{i \hat{\phi}_2} & 0 \\
0 & 0 & 1
\end{pmatrix},
\]  

(23)

which means that the mixing angle between the first and the third generations, which is zero, is unchanged by quantum corrections. The mixing angle of \( \hat{\theta}_{23} \) is given by

\[ \sin^2 2\hat{\theta}_{23} = \left( \frac{2\left(1 - \epsilon \right)}{1 + \left(1 - \epsilon \right)^2} \right)^2, \]

(24)

which indicates that the large mixing between the second and the third generations is stable with respect to quantum corrections. By using Eq. (24), we can estimate that \( \sin^2 2\hat{\theta}_{23} \simeq 0.99 \) in the case of \( m_R = 10^{13} \) GeV and \( \tan \beta = 60 \), which is consistent with the numerical analysis in Table 2. Therefore the mixing between the first and the third generations and the mixing between the second and the third generations are stable with respect to quantum corrections as shown in Table 2.

How about the mixing between the first and the second generations?

For the MSW-L and the VO solutions, where \( \sin \theta_{12} = \cos \theta_{12} = 1/\sqrt{2} \) at \( m_Z \) scale, the mixing angle of \( \tan \hat{\theta}_{12} \) is given by

\[ \tan 2\hat{\theta}_{12} \simeq \left(1 - \epsilon \right) \sqrt{1 - \epsilon} \frac{\sqrt{4\epsilon^2 + \epsilon^2 \sin^2 2\phi}}{\epsilon (1 + \cos 2\phi)}, \]

(25)

where we use the approximation which neglects the higher order corrections of \( \epsilon^2 \), \( \epsilon \xi_b \), and \( \xi_b^2 \). When \( \phi = \pi/2 \), the mixing angle \( \hat{\theta}_{12} \) becomes

\[ \tan 2\hat{\theta}_{12} = \infty, \]

(26)
Figure 1: Majorana phase dependence of $\sin^2 2\hat{\theta}_{12}$ for the MSW-L and the VO solutions in Type B mass hierarchy in the case of $m_R = 10^{13}$ GeV and $\tan \beta = 60$.

which means the maximal mixing is stable against quantum corrections. On the other hand, when $\phi = 0$,

$$\tan 2\hat{\theta}_{12} \simeq \frac{\xi_b}{\epsilon}, \quad (27)$$

which shows that the mixing angle of $\hat{\theta}_{12}$ strongly depends on the magnitude of $\epsilon$. The large mixing is spoiled when $\xi_b \leq \epsilon$, which corresponds the region of $\tan \beta \geq 10$ for the MSW-L solution, and any value of $\tan \beta$ for the VO solution when we take $m_R = 10^{13}$ GeV. In Fig.1, we show the change of $\sin^2 2\hat{\theta}_{12}$ due to the continuous change of Majorana phase $\phi$ in the case of $\tan \beta = 60$ and $m_R = 10^{13}$ GeV. As the Majorana phase $\phi$ changes from 0 to $\pi/2$, the value of $\sin^2 2\hat{\theta}_{12}$ changes from 0 to 1. The large deviation from 1 of $\sin^2 2\hat{\theta}_{12}$ means that the mixing angle $\theta_{12}$ is unstable with respect to the quantum corrections. Figure 1 shows that the mixing angle $\theta_{12}$ changes from being unstable to being stable as the change $\phi$ from 0 to $\pi$. The lines of the MSW-L and the VO solutions are almost overlapping in Fig.1, since the discrepancy of $\xi_b$‘s for the two solutions is negligible compared with the quantum correction, $\epsilon = 0.1$, when $\tan \beta = 60$ and $m_R = 10^{13}$ GeV.

As for the MSW-S solution, $\phi = 0$ induces

$$\tan 2\hat{\theta}_{12} \simeq \tan 2\theta_{12} \left(1 + \frac{1}{\cos 2\theta_{12} \xi_b} \epsilon \right)^{-1}, \quad (28)$$

while $\phi = \pi/2$ induces

$$\tan 2\hat{\theta}_{12} \simeq \tan 2\theta_{12} . \quad (29)$$

Equations (28) and (29) show that the mixing angle of $\theta_{12}$ is not changed in the region of $\tan \beta \leq 10$ when $\phi = 0$, while it is not changed independently of $\tan \beta$ when $\phi = \pi/2$. 
Above conclusions are the same as those of Ref. [11]. In Fig. 2, we show the value of \( \sin^2 2\theta_{12} \) at \( m_R = 10^{13} \) GeV scale in the case of \( \tan \beta = 60 \) according to the continuous change of \( \phi \) from 0 to \( \pi \). Figure 2 shows that the mixing angle \( \theta_{12} \) changes from being unstable to being stable corresponding to the change of \( \phi \) from 0 to \( \pi \).

5 Type C \((m_1 \sim m_2 \sim m_3)\)

In Type C mass hierarchy, it has been shown in Ref. [11] that the MNS matrix approaches the definite unitary matrix according to the relative sign assignments of the neutrino mass eigenvalues, as the effects of quantum corrections become large enough to neglect the mass-squared differences of neutrinos. Independent parameters of the MNS matrix at the \( m_R \) scale approach the following fixed values in the large limit of quantum corrections:

**case (c1):** \( \text{diag}.(-m_1, m_2, m_3) \)

\[
U_{e2} = \frac{\sin \theta_{12}}{\sqrt{1 + \cos^2 \theta_{12}}} , \quad U_{e3} = -\frac{1}{2} \frac{\sin 2\theta_{12}}{\sqrt{1 + \cos^2 \theta_{12}}} , \quad U_{\mu 3} = \frac{1}{\sqrt{2}} \frac{\sin^2 \theta_{12}}{\sqrt{1 + \cos^2 \theta_{12}}} . \tag{30}
\]

**case (c2):** \( \text{diag}.(m_1, -m_2, m_3) \)

\[
U_{e2} = \sin \theta_{12} , \quad U_{e3} = \frac{1}{2} \frac{\sin 2\theta_{12}}{\sqrt{1 + \sin^2 \theta_{12}}} , \quad U_{\mu 3} = \frac{1}{\sqrt{2}} \frac{\cos^2 \theta_{12}}{\sqrt{1 + \sin^2 \theta_{12}}} . \tag{31}
\]
case (c3): \( \text{diag}(-m_1, -m_2, m_3) \)

\[
U_{e2} = 0, \quad U_{e3} = 0, \quad U_{\mu3} = \frac{1}{\sqrt{2}}.
\] (32)

case (c4): \( \text{diag}(m_1, m_2, m_3) \)

\[
U_{e2} = 0, \quad U_{e3} = 0, \quad U_{\mu3} = 0.
\] (33)

We can easily obtain the values of the mixing angles by using relations of [15],

\[
\sin^2 2\theta_{12} = 4 \frac{U_{e2}^2}{1 - |U_{e3}|^2} \left( 1 - \frac{U_{e2}^2}{1 - |U_{e3}|^2} \right),
\] (34)

\[
\sin^2 2\theta_{13} = 4 |U_{e3}|^2 \left( 1 - |U_{e3}|^2 \right),
\] (35)

\[
\sin^2 2\theta_{23} = 4 \frac{U_{\mu3}^2}{1 - |U_{e3}|^2} \left( 1 - \frac{U_{\mu3}^2}{1 - |U_{e3}|^2} \right).
\] (36)

As shown above, the cases of (c1)\textendash(c4) are connected by Majorana phases of \( \phi_1 \) and \( \phi_2 \).

Figure 3 shows that the values of mixing angles at the high energy scale \( m_R = 10^{13} \) GeV for the MSW-L and the VO solutions according to continuous changes of Majorana phases \( \phi_1 \) and \( \phi_2 \) in the case of \( \tan \beta = 60 \). Under the conditions that the effects of quantum corrections are large enough to neglect the mass-squared differences of neutrinos, the results of the MSW-L solution are the same as those of the VO solution [11]. Table 3 shows the fixed values of the mixing angles for the MSW-L and the VO solutions in the large limit of quantum corrections which are obtained from Eqs.(30) \textendash (33) by using Eqs.(34) \textendash (36). The deviations from the values at \( m_Z \) scale, \( \sin^2 2\theta_{12} = 1, \sin^2 2\theta_{13} = 0 \)

\[
\begin{array}{|c|c|c|c|}
\hline
\text{ } & (c1) & (c2) & (c3) & (c4) \\
\hline
\sin^2 2\theta_{12} & 0.96 & 0.96 & 0.0 & 0.0 \\
\sin^2 2\theta_{13} & 0.56 & 0.56 & 0.0 & 0.0 \\
\sin^2 2\theta_{23} & 0.36 & 0.36 & 1.0 & 0.0 \\
\hline
\end{array}
\]

Table 3: The fixed values of the mixing angles for the MSW-L and the VO solutions in the large limit of quantum corrections given by Eqs.(30) \textendash (36).

and \( \sin^2 2\theta_{23} = 1 \), indicate that mixing angles receive significant quantum corrections. For \( \sin^2 2\theta_{12} \), Table 3 shows that the cases of (c1) and (c2) conserve the maximal mixing, while the cases of (c3) and (c4) do not, in the large limit of quantum corrections. From Eq.(15), we can show that the change of \( \phi_1 \) form 0 to \( \pi/2 \) with the relation \( |\phi_2 - \phi_1| = 0 \)
Figure 3: The contour plots of (a): $\sin^2 2\hat{\theta}_{12}$, (b): $\sin^2 2\hat{\theta}_{13}$ and (c): $\sin^2 2\hat{\theta}_{23}$, at $m_R = 10^{13}$ GeV in the case of the MSW-L and the VO solutions with $\tan \beta = 60$.

$(|\phi_2 - \phi_1| = \pi/2)$ corresponds to the change of (c4) to (c3) ((c2) to (c1)). Figure 3(a) shows that the unstable region of $\sin^2 2\hat{\theta}_{12} < 0.1$ exists around the line of $|\phi_2 - \phi_1| = 0$, and the stable region of $\sin^2 2\hat{\theta}_{12} \sim 1.0$ exists around the line of $|\phi_2 - \phi_1| = \pi/2$. Since the cases of (c1) and (c2) have masses with opposite signs between the first and second generations, the mixing angle is stable from the analogy of two-generation neutrinos. Therefore the maximal mixing between the first and second generations is conserved in the continuous region preserving the relation of $|\phi_2 - \phi_1| = \pi/2$. As for stability of $\sin^2 2\hat{\theta}_{13}$, Table 3 shows that the cases of (c3) and (c4) conserve the zero mixing, while the cases of (c1) and (c2) do not. Figure 3(b) shows that the stable region exists around the line of $|\phi_2 - \phi_1| = 0$ which connects (c3) and (c4), and the unstable region exists around the line of $|\phi_2 - \phi_1| = \pi/2$ which connects (c1) and (c2). For the stability of $\sin^2 2\hat{\theta}_{23}$, Table 3 shows that the case of (c3) only conserves the maximal mixing, and the case of (c4) induces the zero mixing. Both
cases of (c1) and (c2) induce $\sin^2 2\theta_{23} \sim 0.36$. These situations are connected continuously by two Majorana phases $\phi_1$ and $\phi_2$ as shown Fig.3(c).

Figures 4 shows the values of mixing angles at high energy scale $m_R = 10^{13}$ GeV for the continuous change of the Majorana phases for the MSW-S solution in the case of $\tan \beta = 60$. Table 4 shows the fixed values of the mixing angles for the MSW-S solution in the large limit of quantum corrections, which are obtained from Eqns. (31) $\sim$ (33) by using Eqns. (34) $\sim$ (36). The deviations from the values at $m_Z$ scale, $\sin^2 2\theta_{12} = 7.1 \times 10^{-5}$, $\sin^2 2\theta_{13} = 0$ and $\sin^2 2\theta_{23} = 1$, indicate that the mixing angles receive significant quantum corrections. For $\sin^2 2\theta_{12}$, Table 4 shows that all the cases of (c1) $\sim$ (c4) make it zero in the large limit of quantum corrections. Figure 4(a) shows that the unstable region of $\sin^2 2\theta_{12} > 0.2$ exists around the points of $(\phi_1, \phi_2) \simeq (\pi/2, \pi/30)$ and $(\phi_1, \phi_2) \simeq (\pi/2, 29\pi/30)$. For the stability of $\sin^2 2\theta_{13}$, Table 4 shows that all the cases of (c1) $\sim$ (c4) conserve the zero
mixing. Figure 4(b) shows that \( \sin^2 2\hat{\theta}_{13} \) is stable with respect to the quantum corrections for any values of two Majorana phases. For the stability of \( \sin^2 2\hat{\theta}_{23} \) Table 4 shows that the cases of (c2) and (c3) conserve the maximal mixing, while the cases of (c1) and (c4) do not, in the large limit of quantum corrections. Figure 4(c) shows that the stable region exists around lines of \( \phi_2 = \pi/2 \) which connect (c2) and (c3) by changing \( \phi_1 \) from 0 to \( \pi/2 \), and the unstable region exists around the lines \( \phi_2 = 0 \) and \( \phi_2 = \pi \) which connect (c1) and (c4) by changing \( \phi_1 \) from 0 to \( \pi/2 \).

6 Summary

Neutrino-oscillation solutions for the atmospheric neutrino anomaly and the solar neutrino deficit can determine the texture of the neutrino mass matrix according to three types of neutrino mass hierarchies \([14] \) as Type A: \( m_1 \ll m_2 \ll m_3 \), Type B: \( m_1 \sim m_2 \gg m_3 \), and Type C: \( m_1 \sim m_2 \sim m_3 \). We found that the relative sign assignments of neutrino masses in each type of mass hierarchies play the crucial roles for the stability against quantum corrections. Actually, two physical Majorana phases in the lepton flavor mixing matrix connect among the relative sign assignments of neutrino masses. Therefore, in this paper we analyze the stability of mixing angles against quantum corrections according to three types of neutrino mass hierarchies (Type A, B, C) and two Majorana phases. Two phases play the crucial roles for the stability of the mixing angles against the quantum corrections. The results in Ref.[11], where the stabilities of the mixing angles in (a1) and (a2), (b1) and (b2), (c1) \( \sim \) (c4) with respect to quantum corrections are argued, are reproduced by taking the definite values of two Majorana phases.

Acknowledgment

The work of NO is supported by the JSPS Research Fellowship for Young Scientists, No.2996.

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