D-branes in the light-cone gauge and broken symmetries

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Boundary states for D-branes are constructed using the light cone gauge. The D-brane breaks half the spacetime supersymmetry giving rise to fermionic zero modes living on the brane. The nonlinear realization of the broken supersymmetry on the open string degrees of freedom is analysed and the influence of boundary terms coming from closed string vertex operators is discussed. (Contribution to the workshop "Gauge Theories, Applied Supersymmetry and Quantum Gravity", London Imperial College, July 1996)

1 Boundary states and broken symmetries

The introduction of boundaries breaks symmetries of conformal field theories because the boundary relates left-moving and right-moving degrees of freedom. The condition that the conformal invariance on the half plane $H = \{ z | \text{Im}(z) \leq 0 \}$ is not completely broken implies the continuity of the holomorphic and antiholomorphic parts of the stress energy tensor $\mathcal{T}(z) = \overline{\mathcal{T}}(\overline{z})$ for $\text{Im}(z) = 0$. More generally for a current $J, \overline{J}$ of conformal weight $(h, \bar{h})$ the condition that the boundary preserves the symmetry is given by the continuity $J(z) = \overline{J}(\overline{z})$ for $z = \overline{z}$. Mapping the half plane into the semi infinite cylinder via $z = \exp(\tau + i\sigma)$ we can translate this condition into a condition on a boundary state.

\begin{equation}
(J_n + (-1)^h \overline{J}_{-n}) | B \rangle = 0
\end{equation}

We can define two currents acting on the boundary, $J^\pm(\sigma) = J(z) \pm (-1)^h \overline{J}(\overline{z})$ where $J^-$ is the current broken by the boundary conditions. Conformal boundary conditions and boundary states are in one to one correspondence. In the case of toroidally compactified bosonic string theory with an enhanced gauge symmetry group $G \times G$ it was shown in $^1$ that the scalars $J^{-a}$ correspond to Goldstone bosons living in the coset $G \times G/G$ which is defined through the breaking of the $G \times G$ closed string symmetry to $G \times G/G$ by the boundary.

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D-branes have been identified with stringy solitons carrying RR-charges. A D-brane is a $p + 1$ dimensional hyperplane where open strings can end. The boundary conditions imposed on the string coordinates are $\partial_n X^\alpha = 0$ for $\alpha = 0, \cdots, p$ and $\partial_i X^i = 0$ for $i = p+1, \cdots, 9$. The boundary conditions for the world sheet fermions follows from (1) for the super stresstensor $T_F = \psi^\mu \partial x^\mu$. The space time supersymmetry of D-branes has been analysed using the R-NS formalism in.

2 Light cone supersymmetry

The light cone Green-Schwarz formulation of the superstring has manifest space time supersymmetry. We use a boundary state formulation introduced in where the string propagates on a cylinder and terminates at the proper time $\tau = 0$. The light cone gauge is imposed by $X^+ = x^+ + p^+ \tau$ and $X^+$ as well as $X^-$ satisfy Dirichlet boundary conditions. The coordinates transverse to the $\pm$ directions satisfy the boundary conditions

$$ (\partial X^I - M^I_{IJ} \partial X^J) |B\rangle = 0 $$

where $M^I_{IJ}$ is an element of $SO(8)$ and can be written as,

$$ M^I_{IJ} = \exp \{ \Omega_{KL} \Sigma^K_{IJ} \} $$

where $\Sigma^K_{IJ} = (\delta^K_i \delta^L_j - \delta^K_j \delta^L_i)$ are generators of $SO(8)$ transformations in the vector representation. In this talk we are concerned with D-branes without boundary condensates where the Neumann directions are $\alpha = I = 1, \cdots, p + 1$ and the Dirichlet directions are $i = I = p + 2, \cdots, 8$ $M^I_{IJ}$ can be written in block diagonal form,

$$ M^I_{IJ} = \begin{pmatrix} -I_{p+1} & 0 \\ 0 & I_{7-p} \end{pmatrix} $$

Each of the sixteen-component supercharges of the type II theories decompose in the light-cone gauge into two inequivalent $SO(8)$ spinors defined in terms of the world-sheet fields $X^I$ and $S^a$ ($a = 1, \cdots, 8$) by,

$$ Q^a = \frac{1}{\sqrt{2p^+}} \int_0^\pi d\sigma S^a(\sigma), \quad Q^{\dot{a}} = \frac{1}{\pi \sqrt{p^+}} \int_0^\pi d\sigma \gamma^I_{\dot{a}b} \partial X^I S^b(\sigma) $$

for the left-moving charges and similarly for the right-moving charges, $\dot{Q}^a$ and $\ddot{Q}^{\dot{a}}$ expressed in terms of the right-moving coordinates. We define the combinations of the supercharges

$$ Q^{\pm a} = (Q^a \pm iM_{ab} \dot{Q}^b), \quad Q^{\pm \dot{a}} = (Q^{\dot{a}} \pm iM_{\dot{a}b} \ddot{Q}^b) $$
The D-brane is a BPS-configuration which preserves half the spacetime supersymmetry which means that 16 of the 32 supercharges are annihilated by the boundary state.

\[ Q^{+a} | B \rangle = 0, \quad Q^{+\dot{a}} | B \rangle = 0 \]  

These two conditions and the consistency with the light cone \( N = 2 \) supersymmetry algebra are solved by \( SO(8) \) rotations acting on the spinors,

\[ M_{ab} = \exp \left\{ \frac{1}{2} \Omega_{IJ} \gamma^{IJ}_{ab} \right\}, \quad M_{\dot{a}\dot{b}} = \exp \left\{ \frac{1}{2} \Omega_{IJ} \gamma^{IJ}_{\dot{a}\dot{b}} \right\} \]

where \( \Omega_{IJ} \) is the same antisymmetric matrix (3) that defined the \( SO(8) \) rotation in the vector basis and \( \gamma^{IJ} = \frac{1}{2} (\gamma^I \gamma^J - \gamma^J \gamma^I) \). For the boundary condition given in (4) the solution is simply \( M_{ab} = \gamma^1 \ldots \gamma^{p+1} \).

The boundary state that solves (7) can be obtained

\[ |B\rangle = \exp \sum_{n>0} \left( \frac{1}{n} M_{IJ} \alpha^I_n \tilde{\alpha}^J_{-n} - i M_{ab} S^a_n \tilde{S}^b_{-n} \right) |B_0\rangle, \]

\[ = R(M) \exp \sum_{n>0} \left( \frac{1}{n} \alpha^I_n \tilde{\alpha}^J_{-n} - i S^a_n \tilde{S}^b_{-n} \right) |B_0\rangle \]

where the zero-mode factor is given by

\[ |B_0\rangle = C \left( M_{IJ} |I\rangle |J\rangle + i M_{ab} |\tilde{a}\rangle |\tilde{b}\rangle \right) \]

The normalisation factor \( C \) is determined by comparing the open string one loop partition function with the closed string vacuum to vacuum transition calculated using the boundary state (9). The fields of IIB supergravity expressed as a light-cone superfield by introducing Grassmann coordinates \( \theta^a = S^0_a - i \bar{S}^0_a \) and \( p^+ \partial / \partial \theta^a = S^0_a + i \bar{S}^0_a \) which satisfy \( \{ \partial / \partial \theta^a, \theta^b \} = \delta^{ab} \). To incorporate the D-brane boundary conditions modified Grassman coordinates are introduced by

\[ \hat{\theta} = \frac{1}{2} (1 + M)_{ab} \theta^b + \frac{p^+}{2} (1 - M)_{ab} \frac{\partial}{\partial \theta^b} \]

and its conjugate

\[ \frac{\partial}{\partial \hat{\theta}^a} = \frac{1}{2p^+} (1 - M)_{ab} \theta^b + \frac{1}{2} (1 + M)_{ab} \frac{\partial}{\partial \theta^b}. \]

The conditions (10) imply \( \partial / \partial \hat{\theta}^a |B_0\rangle = 0 \) which means that the D-brane can be interpreted as a bottom part of a IIB multiplet. Note that the higher terms in the multiplet are obtained by multiplying with up to eight \( \hat{\theta} \) which corresponds to the emission of a zero momentum fermions. The higher terms in the multiplet are important when one considers scattering of D-branes with helicity flip which constitutes a change in the D-brane state during the scattering process.
3 Nonlinear realization of supersymmetry

The massless bosonic and fermionic open-string vertex operator are given by

\[ V_B(\zeta, k) = (\zeta^I B^I - \zeta^-) e^{ikX}, \quad V_F(u, k) = (u^a F^a + u^\dot{a} F^{\dot{a}}) e^{ikX}, \]  

where

\[ B^I = \partial X^I - \frac{1}{2} S^a(z) \gamma^I_a J^b(z) k^j, \quad F^a = S^a(z) \]  

\[ F^{\dot{a}} = \gamma^{I\dot{a}} S^a(z) \partial X^I + \frac{1}{6} :\gamma^{I\dot{a}} S^a(z) S^b(z) \gamma^{IJ} S^c(z) : k^j \]  

The 16 components of the supersymmetry act on the vertex operators in the following way,

\[ \delta_\eta V_B = [\eta^a Q^a, V_B(\zeta)] = V_F(\tilde{u}) \]  

\[ \delta_\eta V_F = [\eta^a Q^a, V_F(u)] = V_B(\tilde{\zeta}) \]  

\[ \delta_v V_B = [e^\dot{a} Q^{\dot{a}}, V_B(\zeta)] = V_F(\tilde{u}) + \epsilon^\dot{a} \partial_2 W_B^\dot{a}(\zeta, k, z) \]  

\[ \delta_v V_F = [e^\dot{a} Q^{\dot{a}}, V_F(u)] = V_B(\tilde{\zeta}) + \epsilon^\dot{a} \partial_2 W_F^\dot{a}(u, k, z) \]  

Supersymmetry transformations on the \( SO(8) \) components of the massless open-string fields take the form,

\[ \tilde{\zeta}^I = \eta^a \gamma^{I\dot{a}} u^{\dot{a}} \sim 0, \quad \tilde{\zeta} = \sqrt{\frac{1}{2}} e^{\dot{a}\gamma^{I\dot{a}}} u^a + \frac{\sqrt{2}}{k+} \epsilon^\dot{a} u^\dot{a} k^I = k^I \epsilon^\dot{a} \gamma^{IJ} \tilde{\zeta}^J \]  

\[ \tilde{v}^\dot{a} = \eta^a \gamma^{IJ} \zeta^I \tilde{v} = \frac{1}{k+} \sqrt{\frac{1}{2}} (\epsilon^\dot{a} \gamma^{IJ} k^I \tilde{\zeta}^J + \epsilon^\dot{a} \gamma^{IJ} k^I) \]  

The total derivative terms in (16) are given by

\[ W_B^\dot{a} = \sqrt{2} \gamma^{I\dot{a}} S^a e^{ikX} \]  

\[ W_F^\dot{a} = \sqrt{2} \epsilon^\dot{a} e^{ikX} + \frac{\sqrt{2}}{8} (\gamma^{IJ} u)^\dot{a} S^b \gamma^{IJ} S^c e^{ikX}. \]  

The amplitude with \( n \) bosonic open-string ground states has the form,

\[ A_n(\Psi \mid \zeta_1, \ldots, \zeta_n) = \int d\sigma_1 \ldots d\sigma_n \langle \Psi \mid V_B(\zeta_1, k_1, z_1) \cdots V_B(\zeta_n, k_n, z_n) \mid B \rangle. \]  

A supersymmetry transformation of this amplitude is obtained by substituting a transformed wavefunction \( \tilde{\zeta}_1 \) or \( \tilde{\zeta}_1 \) into \( A_n \). Using (16) the transformed vertex
operator can be written as a commutator of a supersymmetry generator and a fermionic vertex operator. Thus, for the linearly realized components of the conserved supersymmetry the vertex,

\[ V_B(\tilde{\zeta}) = \eta^a_+ Q^+ a V_F(u) - V_F(u) \eta^a_+ Q^+ a, \]  

(20)
can be inserted into (19) and the \( Q^+ \) in the first term acts to the left on the closed string state \( \Psi \) giving a transformed state, \( \delta \eta_+ \Psi \). The \( Q^+ \) in the second term is moved to the right and gives transformed open string vertex operators until it hits the boundary where it is annihilated by the boundary state. Thus the conserved supersymmetry relates S-matrix elements with \( n + 1 \) bosonic states (including the one (bosonic) closed-string end-state) to elements with \( n - 1 \) bosonic and 2 fermionic states,

\[ A_n(\Psi | \zeta_1, \zeta_2, \cdots, \zeta_n) = A_n(\delta \eta_+ \Psi | u_1, \zeta_2, \cdots, \zeta_n) + A_n(\Psi | u_1, \tilde{u}_2, \zeta_3 \cdots, \zeta_n) + \cdots + A_n(\Psi | u_1, \zeta_2, \cdots, \zeta_{n-1}, \tilde{u}_n). \]

This corresponds to the linearly realized supersymmetry which is not broken by the boundary state. A similar analysis applies to the non-linearly realized conserved supercharge, \( Q^{+ \dot{a}} \) with \( \tilde{\zeta} \) and \( \tilde{u} \) replaced by \( \tilde{\zeta} \) and \( \tilde{u} \).

The supercharge \( Q^{- \dot{a}} \) is not annihilated by the boundary so that similar manipulations for these supercharges leave a residual term. This is proportional to \( \eta^{\dot{a}} Q^{- a} | B \rangle \), which has the form of the fermion emission vertex (13) acting on the boundary, in which the supersymmetry parameter \( \eta^{\dot{a}} \) is the wave function. Therefore, the amplitude with \( n + 1 \) bosonic states is related by the \( Q^- \) supersymmetry to a sum of terms with \( n - 1 \) bosons and two fermions, together with an extra term which has an extra zero-momentum fermion insertion – it has a total of \( n \) bosons and two fermions. This term can also be interpreted as a shift in the fermionic collective coordinate of the boundary \( \delta \theta^{\dot{a}} = \eta^{- \dot{a}} \).

\[ A_n(\Psi | \tilde{\zeta}_1, \zeta_2, \cdots, \zeta_n) = A_n(\delta \eta^{-} \Psi | u_1, \zeta_2, \cdots, \zeta_n) + A_n(\Psi | u_1, \tilde{u}_2, \zeta_3 \cdots, \zeta_n) + \cdots + A_n(\Psi | u_1, \zeta_2, \cdots, \zeta_{n-1}, \tilde{u}_n) + A_{n+1}(\Psi | u_1, \zeta_2, \cdots, \zeta_n, \eta^{-}) \]  

(21)

This is the S-matrix statement of the nonlinear realization of the spontaneously broken \( Q^- \) supersymmetry. The corresponding analysis with the non-linearly realized supercharge \( Q^{+ \dot{a}} \) leads to the same relationship between amplitudes but with \( \tilde{\zeta} \) and \( \tilde{u} \) replaced with \( \tilde{\zeta} \) and \( \tilde{u} \). Higher-order terms give rise to S-matrix elements with arbitrary numbers of soft fermions.
4 Closed string symmetries and collective coordinates

The vertex operator for massless NS-NS tensors is given by a product of two open string vertices (13). Consider gauge transformations on the graviton

\[ \delta \zeta_{IJ} = k_I \Lambda_J \]

under which the corresponding vertex operator becomes a total derivative on the world sheet.

\[ \delta \zeta_{IJ} \int d^2 z V_I(z) \bar{V}_J(\bar{z}) | B \rangle = \Lambda_I (\delta^{IJ} + M^{IJ}) \oint d\sigma \partial X^J | B \rangle \]

The boundary term induces a shift of the position of the D-brane in the transverse directions

\[ \delta Y^I = u^a \gamma^I a \]

when one considers the limit \( k^I \to 0 \).

The undotted components of the two gravitini also produce boundary terms upon gauge transformation

\[ \delta \zeta_{Ia} \int d^2 z V_I(z) \bar{V}_{\bar{a}}(\bar{z}) = u_a \oint d\sigma S^a | B \rangle \]

The gauge transformation of the combination \( \zeta_{I a} + i M_{ab} \hat{\zeta}_{I b} \) induces a shift of the fermionic collective coordinate \( \delta \hat{\theta} = u^a \) given by the insertion of a zero momentum fermionic vertex operator at the boundary. The other gravitino given by \( \zeta_{I a} - i M_{ab} \hat{\zeta}_{I b} \) does not give rise to a boundary term and correspond to the unbroken supersymmetry. Similarly the combination \( \zeta_{I a} + i M_{ab} \hat{\zeta}_{I b} \) produces a boundary term proportional to \( Q^a \).

Applying the broken supersymmetry to the boundary states corresponds to inserting fermionic zero modes on the D-brane. The bosonic zero modes correspond to the broken translational and internal symmetries on the brane and should be generated by acting with the unbroken supersymmetry on the fermionic zero mode.

\[ u_1 Q^a + u_2 Q^b | B \rangle = u_1^a \gamma^I a u_2^b \oint (\partial X^I + M^{IJ} \bar{\partial} X^J) | B \rangle \]

The unbroken supersymmetry acting on a fermionic zero mode creates a vertex which is proportional to the momentum of the boundary state in the Dirichlet directions since \( p^I | B(y) \rangle = i\partial/\partial y^I | B(y) \rangle \) this term creates a shift in the transverse position of the D-brane \( \delta Y^I = u_1 \gamma^I u_2 \). In the Neumann directions the operator is proportional to the winding number of the boundary and it does not contribute for flat D-branes.
5 Contact terms

The fact that total derivative terms appear in the representation of SUSY on the massless vertex operators becomes important when one considers the supersymmetry in the presence of closed string vertex operators. For simplicity consider the D-instanton and the action of the unbroken supersymmetry on the NS-NS antisymmetric tensor.

\[
(Q^a + i\bar{Q}^a)\xi_{IJ} \int d^2z V^I(z)\bar{V}^J(\bar{z}) \mid B \rangle = \xi_{IJ} \int d\sigma (W^I \dot{a} V^J + iV^I \bar{W}^J \bar{a}) \mid B \rangle
\]

Inserting (14) and (18) into (24) then gives

\[
\xi_{IJ} \oint d\sigma \left( \gamma^I_{aa} S^a \partial_z X^J + i\gamma^J_{aa} \bar{S}^a \partial_{\bar{z}} X^I \right)
\]

\[
- \frac{1}{2} \xi_{IJ} \int d\sigma \left( \gamma^I_{aa} S^a k^N \bar{S}^b \gamma^J_{bc} \bar{S}^c - i\kappa^N S^b \gamma^J_{bc} S^c \gamma^I_{aa} \bar{S}^a \right)
\]

The boundary conditions can be used to express the right-moving fields in terms of the left-moving ones. It is then easy to see that the second term in (26) vanishes for an antisymmetric \( \xi_{IJ} \), but the first term is nonzero and corresponds to a violation of space time supersymmetry. This violation can be cancelled by adding a boundary term of the form

\[
\xi_{IJ} \oint d\sigma S^a(\sigma) \gamma^I_{ab} S^b(\sigma) \mid B \rangle = \xi_{IJ} R^I_{0} \mid B \rangle
\]

Note that the boundary term is a helicity operator which rotates the leftmoving fermionic modes \( S_a \) leaving the bosonic modes and the right-moving fermions untouched. The fact that the boundary condition on the bosonic coordinates \( X \) are not changed means that the modified boundary state represents still a D-instanton, but it couples to different closed string fields. The supersymmetry variation of this boundary term gives

\[
(Q^a + i\bar{Q}^a)\xi_{IJ} R^I_{0} \mid B \rangle = \xi_{IJ} \gamma^I_{ab} Q^a \mid B \rangle
\]

\[
(Q^a + i\bar{Q}^a)\xi_{IJ} R^I_{0} \mid B \rangle = \xi_{IJ} \gamma^I_{ab} Q^b + \xi_{IJ} \int d\sigma \partial_z X^I \gamma^J_{ab} S^a \mid B \rangle
\]

The presence of the boundary term hence induces a modification of the conservation of the supersymmetry charge by the modified boundary state and cancel the boundary term coming from the nonlinearly realized supersymmetry. For the massless part of the boundary state we have

\[
\xi_{IJ} R^{IJ} \mid (N) \rangle \mid \bar{N} \rangle + i \mid \dot{a} \rangle \mid \bar{a} \rangle = \xi^{IJ} \mid I \rangle \mid \bar{J} \rangle + i\xi^{IJ} \gamma^I_{ab} \mid \dot{a} \rangle \mid \bar{a} \rangle
\]
i.e. the boundary state couples to a specific combination of the NS-NS and R-R antisymmetric tensor states. It is also interesting to note that the bispinor states do not give a contact term. The supersymmetry transformation on a bispinor state is given by

\[ \epsilon^\dagger (Q^a + i\check{Q}^b)u^a\check{u}^b \int d^2z V^a(z)\check{V}^b(\bar{z}) | B \rangle = \int d\sigma \epsilon^\dagger (v^\dagger \check{v}^b S^b - i\check{v}^a u^b S^b)e^{iky} | B \rangle \]

(30)

We can define the new susyparameter \( \epsilon'^a = e^{\dagger} \gamma^I a \gamma^I a \). Using the boundary conditions (30) can be written in the following form

\[ \epsilon'^a \int (u^a \check{u}^b - \check{u}^a u^b)S^a | B \rangle = \epsilon'^a \xi^{IJ} \gamma^I a \gamma^I b Q^b | B \rangle \]

(31)

Where the antisymmetric part of the bispinor is written in terms of the corresponding AST wave function using \( u_{[a} u_{b]} = \xi^{IJ} \gamma^I a \gamma^I b \)

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