$^3S_1-^3D_1$ coupled channel $\Lambda_cN$ interactions: chiral effective field theory versus lattice QCD

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Abstract

We study the lattice QCD $\Lambda_cN$ phase shifts for the $^3S_1-^3D_1$ coupled channel using both the leading order covariant chiral effective theory and the next-to-leading order non-relativistic chiral effective field theory (ChEFT). We show that although it is possible to describe simultaneously the $^3S_1$ and $^3D_1$ phase shifts and the inelasticity $\eta$, the fitted energy range is quite small, only up to $E_{c.m.}=5$ MeV. This raises concerns regarding the consistency between leading/next-to-leading order ChEFT and the lattice QCD simulations.

Keywords: $\Lambda_cN$ interaction, covariant ChEFT, lattice QCD

(Some figures may appear in colour only in the online journal)

1. Introduction

The $\Lambda_c$ baryon as the lightest charm baryon has attracted a lot of attention, which may exist in finite nuclei to form $\Lambda_c$ hypernuclei. The HAL QCD Collaboration performed the first lattice QCD simulations of the $\Lambda_cN$ and $\Sigma_cN$ interactions for unphysical light quark masses ($m_\pi=410$, 570, 700 MeV) [1], which provided vital information on the interaction between a nucleon and a charmed baryon $\Lambda_c$ or $\Sigma_c$. Employing these lattice QCD results, extrapolations to the physical point have been performed using either the non-relativistic chiral effective field theory (ChEFT) at next-to-leading order (NLO) [2] or the covariant ChEFT at leading order (LO) [3]. In the covariant ChEFT, Lorentz covariance is maintained by employing the covariant chiral Lagrangians, the full form of Dirac spinors, and the relativistic scattering equation (the Kadyshevsky equation). It has been shown that the covariant ChEFT approach can provide reasonable descriptions of octet baryon-octet baryon interactions already at LO, including all the systems from strangeness $S=0$ to $S=-4$, at least in the low energy region [4–14]. A recent study [11] showed that one could reproduce both the physical $^1S_0$ and $^3S_1-^3D_1$ and the lattice QCD nucleon–nucleon partial wave phase shifts fairly well. In particular, for the physical nucleon–nucleon phase shifts and lattice QCD data at $m_\pi=469$ MeV, if one only fits to the $^3S_1$ phase shifts, the $^3D_1$ phase shifts generally not reproduce the lattice QCD results as well as the $^3S_1$ phase shifts.

$^8$ The next-to-next-to-leading order relativistic chiral nucleon–nucleon interaction is shown to be able to describe the neutron-proton scattering phaseshifts up to $T_{\text{lab}}=200$ MeV as well as the next-to-next-to-next-to-leading order non-relativistic chiral nucleon–nucleon interactions [15].
phase shifts and inelasticity $\eta_1$ can be predicted and vice versa, as shown in [16]. It implies that indeed the correlations induced by the imposed constraint of covariance in the covariant chiral potentials is reasonable.

In our previous study of the $\Lambda N$ interaction in the covariant ChEFT [3], the low energy constants (LECs) were determined by fitting to the lattice QCD data from the HAL QCD Collaboration, where the $S$-wave phase shifts up to $E_{c.m.} = 50$ MeV for $m_\Lambda = 410$ and $570$ MeV were considered. The results showed that the covariant ChEFT can describe the lattice QCD data fairly well at low energies. In addition, the phase shifts of the $\Lambda N$ $^3D_1$ partial wave and the inelasticity $\eta_1$, as well as their physical counterparts were predicted.

In a recent study [17], it was shown that the predicted $^3D_1$ phase shifts by the NLO non-relativistic ChEFT are in agreement with the lattice QCD data of [18] at higher energies, but not those of [3]. A closer examination of the lattice QCD data revealed, however, that although at higher energies, the predictions of [3] do not agree with the lattice QCD data, but at low energies close to the threshold, they do agree, both for the $^3D_1$ phase shifts and the inelasticity, at least qualitatively. On the other hand, the predictions of the NLO non-relativistic ChEFT [2] do not agree with the lattice QCD data at low energies.

In this work, we revisit the fits to the lattice QCD data and the corresponding extrapolations to the physical point. We study in detail the differences between the non-relativistic ChEFT and covariant ChEFT in the description of the $\Lambda N$ $^3D_1$ phase shifts and inelasticity $\eta_1$, including the effects of baryon masses and SD coupling in the contact terms, and the retardation effects in the one meson exchange term. In addition, we study extrapolations to the physical point employing different fitting strategies to the lattice QCD data. These results are important to better understand the $\Lambda N$ interaction and might be helpful to guide future hypernuclei experiments.

The paper is organized as follows. In section 2, we briefly introduce the non-relativistic and the covariant chiral EFT. In section 3 we perform fits to the lattice QCD data of [18], focusing on the low energy region, where ChEFT is expected to work. We summarize in section 4.

2. Theoretical framework

In this section, we briefly introduce the non-relativistic ChEFT and covariant ChEFT for the $YN$ interactions, where $Y = \Lambda$, $\Sigma$, and highlight the differences relevant for the present study.

In the non-relativistic ChEFT, the next-to-leading order potentials consist of non-derivative four-baryon contact terms (CT) and one-meson-exchanges (OME). The CT potentials for the $^1S_0$ and $^3S_1$–$^3D_1$ partial waves are [19]

$$V_{CT}^{YN,1S_0} = \tilde{C}_{1S_0} + \tilde{D}_{1S_0} m^2 + (C_{1S_0} + D_{1S_0} m^2)(p^2 + p'^2),$$

$$V_{CT}^{YN,3S_1} = \tilde{C}_{3S_1} + \tilde{D}_{3S_1} m^2 + (C_{3S_1} + D_{3S_1} m^2)(p^2 + p'^2),$$

$$V_{CT,3D1}^{YN} = C_{3\ell} p^2,$$

where $p = |p|$ and $p' = |p'|$ are the initial and final center-of-mass (c.m.) momenta of the $YN$ system, respectively. $C_i, D_i, C_\ell, D_\ell(i = 1, 3, 1, 1)$ are LECs that need to be fixed by fitting to either experimental or lattice QCD data. The OME potential reads,

$$V_{OME}^{YN \to Y', N'} = - \frac{g_N^{Y'Y} g_N^{NN}}{4f^2} (\mathbf{r}_1 \cdot \mathbf{q}) (\mathbf{r}_2 \cdot \mathbf{q}) \mathbf{q}^2 + m^2 \times \mathcal{I}_{YN \to Y', N'},$$

where $\mathbf{q} = p' - p$ is the transferred momentum. The coupling constants $g_N^{Y'Y}$ and $g_N^{NN}$ and the isospin factor $\mathcal{I}$ can be found in, e.g. [2, 20]. The scattering amplitudes are then obtained by solving the coupled-channel Lippmann-Schwinger equation

$$T_{J',J}^{\nu\nu'}(p', p; \sqrt{s}) = V_{J',J}^{\nu\nu'}(p', p) + \sum_{\nu''} \int_0^\infty \frac{dp'' p''^2}{(2\pi)^3} V_{J',J}^{\nu''\nu'}(p', p'') \times \frac{2\mu_{\rho}'}{p_{\rho}' - p_{\rho}'' + i\eta} T_{J',J}^{\nu''\nu'}(p'', p; \sqrt{s}),$$

where the labels $\nu, \nu', \nu''$ denote the particle channels, $\rho, \rho', \rho''$ denote the partial waves, and $\mu_{\rho}$ is the pertinent reduced mass. The on-shell momentum in the intermediate state, $p_{\rho}$, is defined by $\sqrt{s} = \sqrt{M_{\rho}^2 + p_{\rho}^2} + \sqrt{M_{\rho}^2 + p_{\rho}''^2}$. The potentials are regularized with an exponential form factor

$$f_{\Lambda_F}(p, p') = \exp \left[ - \left( \frac{p^4}{\Lambda_F^4} - \left( \frac{p'^4}{\Lambda_F^4} \right) \right) \right],$$

where $\Lambda_F$ is the cutoff whose value is in the range of 500–600 MeV. The partial wave $S$ matrix is related to the on-shell $T$ matrix by

$$S_{\nu \nu'}^{\rho \rho'} = b_{\rho \rho'} \delta_{\nu \nu'} - 2ia T_{\rho \rho'}^{\nu \nu'}, \quad a = \frac{\sqrt{p_{\text{cm}}^2 p_{\text{cm}}'} \mu_{\nu \nu'}^2}{16\pi^2},$$

where $p_{\text{cm}}$ is the C.M. three-momentum of the $\Lambda N$ system. The phase space factor $a$ is determined by the elastic unitarity of the scattering equation. For single channels, the phase shifts $\delta$ can be obtained from the on-shell $S$ matrix

$$S = \exp(2i\delta).$$
Table 1. Seven fitting strategies studied in this work, where ✓ indicates that the SD coupling in the CT potential is turned on, while × denotes that the SD coupling is turned off in the covariant ChEFT approach.

| Strategy   | Lattice QCD data fitted | Approach | $M_L [\text{MeV}]$ | $M_H [\text{MeV}]$ | SD coupling |
|------------|-------------------------|----------|---------------------|-------------------|-------------|
| 1          | $^3S_1$                 | cov. ChEFT | $\approx$           | ✓                 | ✓           |
| 2          |                         |          | $\approx$           | ×                 | ×           |
| 3          | $E_{c.m.} \leq 30 \text{ MeV}$ |          | =                   | ✓                 | ✓           |
| 4          |                        |          | =                   | ×                 | ×           |
| 5          | $^3S_1$, $^3D_1$, $\eta_1$ | cov. ChEFT | $\approx$           | ✓                 | ✓           |
| 6          | $E_{c.m.} \leq 5 \text{ MeV}$ |          | =                   | ✓                 | ✓           |
| 7          |                         |          | non-rel. ChEFT      |                   |             |

In order to calculate the phase shifts in coupled channels ($J > 0$), we use the ‘Stapp’- or ‘bar’-phase shifts parametrisation [21] of the $S$ matrix, which can be written as

$$S = \begin{bmatrix} S_{--} & S_{+-} \\ S_{+-} & S_{++} \end{bmatrix} = \begin{bmatrix} \exp(i\delta_c) & 0 \\ 0 & \exp(i\delta_v) \end{bmatrix} \begin{bmatrix} \cos(2\epsilon) & i\sin(2\epsilon) \\ i\sin(2\epsilon) & \cos(2\epsilon) \end{bmatrix} \begin{bmatrix} \exp(i\delta_c) & 0 \\ 0 & \exp(i\delta_v) \end{bmatrix},$$

(7)

where the subscript ‘+’ is $J + 1$, ‘−’ is $J - 1$. The resulting phase shifts and mixing angles are

$$\tan(2\delta_c) = \frac{\Im(S_{+-} / \cos(2\epsilon))}{\Re(S_{+-} / \cos(2\epsilon))},$$

$$\tan(2\epsilon) = \frac{-\sin\delta_y}{\sqrt{S_{++} - S_{--}}}.$$  

(8)

For more details about the non-relativistic ChEFT, please refer to [2, 17, 19, 20, 22–31].

In the covariant ChEFT, as discussed in [3], the $^1S_0$ and $^3S_1$–$^3D_1$ CT potentials for the $Y_{1N}$ system read,

$$V_{\text{CT,3S1}}^{Y,N} = \frac{\xi_{1N}}{9\sqrt{2}} [(C_{150} - C'_{150}) R_p^{N} R_p^{N} + R_p^{F} R_p^{F}] + C_{3S1} (R_p^{N} R_p^{N} R_p^{F} R_p^{F} + 1),$$

$$V_{\text{CT,3S1}}^{Y,N} = \frac{1}{9} \xi_{1N} (2(C_{150} - C'_{150})$$

$$\times (R_p^{F} R_p^{N} - R_p^{N} R_p^{F})$$

$$+ C_{3S1} (-6R_p^{N} R_p^{N} + 9R_p^{F} R_p^{F})$$

$$+ 9R_p^{N} R_p^{F} + 6R_p^{N} R_p^{N} R_p^{F})$$

$$+ 9C_{3S1} (R_p^{F} R_p^{N} R_p^{F} R_p^{N} - 2 + 2R_p^{N} R_p^{N} R_p^{F} 91),$$

$$V_{\text{CT,3D1–3S1}}^{Y,N} = \frac{\xi_{1N}}{9\sqrt{2}} [(C_{150} - C'_{150})$$

$$[R_p^{N} (R_p^{F} + 3R_p^{N}) - R_p^{N} (3R_p^{N} + R_p^{F})]$$

$$+ C_{3S1} [9R_p^{N} (R_p^{F} + 4R_p^{N})$$

$$+ 3R_p^{N} R_p^{F} - 3R_p^{N} (3R_p^{N} + R_p^{F})]$$

$$+ 9C_{3S1} [R_p^{N} (4R_p^{N} R_p^{F} + 3 + R_p^{F})$$

$$- R_p^{N} (R_p^{F} + 3R_p^{N})].$$

It should be noted that there exist three differences between the covariant and the non-relativistic ChEFT potentials as presented above: (1) the covariant chiral potentials explicitly contain the baryon masses $M_L$ and $M_H$, where $M_L = (M_{\Lambda_c} + M_{\Lambda_c})/2$, whose values are the same as those given in Table 1 of [3]; (2) because of the fact $M_L = M_H$, the LECs from the $^1S_0$ partial wave also contribute to those of the $^3S_1$–$^3D_1$ partial waves; (3) the LECs responsible for the SD coupling are correlated with those of the $^1S_0$ and $^3S_1$ potentials. It should be noted that the contributions from the $\xi_{1S}$ intermediate state in the CT potentials were set zero in both the non-relativistic ChEFT [2] and covariant ChEFT [3], since the limited lattice QCD data could not fix these contributions.

The leading-order OME potential reads

$$V_{\text{OME}}^{Y,N \rightarrow Y,N} = i\xi_{A}^{YY'} b_{\text{NN}}^{NN} \tilde{u}_{N}(p')$$

$$\times \left( \frac{\gamma^\mu \gamma_5 q_\mu}{2f_{\pi}} \right) u_{N}(p) \frac{i}{\Delta E - q^2 - m^2 + i\epsilon}$$

$$\times \tilde{u}_{N}(-p')(\gamma^\mu \gamma_5 q_\mu)_{\mu}(-p') \times \Sigma_{N \rightarrow Y,N},$$

(10)

where $\Delta E = E_{p'} - E_p$ is the transferred kinetic energy, i.e.
the retardation effect, and we adopt the complete form of the Dirac spinor for the baryons involved

\[ u_0(p, s) = \left( \frac{1}{\mathbf{E}_p + M_0} \right) \chi_v. \]

The coupled-channel Kadyshevsky equation [32] is solved to obtain the scattering amplitudes

\[ T^{\sigma\nu}_{\rho\rho'}(p', p; \sqrt{s}) = V^{\sigma\nu}_{\rho\rho'}(p', p) + \sum_{\nu\nu', \sigma} \int_0^\infty \frac{dp'p'^2}{(2\pi)^3} \times \frac{M_{R_{\sigma\nu}} M_{R_{\nu'\sigma}} V^{\sigma\nu}_{\rho\rho'}(p', p'') T^{\sigma\nu}_{\rho'\rho''}(p'', p'; \sqrt{s})}{E_{1,\sigma} E_{2,\nu} (\sqrt{s} - E_{1,\sigma} - E_{2,\nu} + \imath\epsilon)}, \]

where \( \sqrt{s} \) is the total energy of the two-baryon system in the center-of-mass frame and \( E_{n,\sigma} = \sqrt{p'^2 + M_{R_{\sigma\nu}}^2}, (n = 1, 2). \) In the numerical study, the potentials are regularized with the same exponential form factor as that of equation (4). The relation between the phase shifts and T-matrix is the same as explained above except for the phase space factor \( a \), which appears in the Kadyshevsky equation as

\[ a = \frac{1}{\frac{8\pi^2}{\alpha_{\text{em}}} (E_{1,\sigma} + E_{2,\nu})(E_{1,\sigma} - E_{2,\nu})}. \]

More details about the covariant ChEFT approach can be found in [4–15].

3. Fitting procedure

In [33], the HAL QCD Collaboration presented the \( S_0 \) and \( S_1 \) phase shifts of the \( \Lambda N \) interaction obtained from lattice QCD simulations with \( m_\pi = 410, 570, \) and 700 MeV. In addition, the corresponding \( D_1 \) partial wave phase shifts and inelasticity \( \eta_1 \) can be found in the PhD thesis of Takaya Miyamoto [18]. The results show that the \( \Lambda N \) \( D_1 \) phase shifts for \( m_\pi = 410, 570, \) and 700 MeV are slightly repulsive for the center of mass energy no larger than 15, 30, and 40 MeV, respectively and become attractive as \( E_{\text{cm}} \) increases, and the inelasticity \( \eta_1 \) is close to unity in the whole energy region. Fittings to the \( S_1 \) partial waves of \( m_\pi = 410 \) and 570 MeV (with \( E_{\text{cm}} \leq 30 \) MeV), and extrapolations to the physical point were performed in both the non-relativistic ChEFT [2] and covariant ChEFT [3]. The predictions for the \( D_1 \) phase shifts and inelasticity \( \eta_1 \) turn out to be dramatically different. The \( D_1 \) interaction in the former approach is attractive, while that in the latter is repulsive. In addition, both approaches predict a \( SD \) coupling stronger than that shown by the lattice QCD data.

In this study, we first investigate where such differences in the predicted \( \Lambda N \) \( D_1 \) phase shifts between the two approaches originate. In particular, we focus on the masses of \( Y_c \) and \( N \) and the \( SD \) coupling in the CT potential. We note that there are no baryon mass terms in the CT potential of the non-relativistic ChEFT, while \( M_B \) (\( M_{N_S} \)) appears in the baryon spinors of the covariant ChEFT. As \( M_{N_S} \geq M_B \), we used the 'physical' masses for \( Y_c \) and \( N \) in our previous study, which has the consequence that \( C_{150} \) (\( C_{150}' \)) also contributes to the \( ^3S_1-^3D_1 \) partial waves [3]. In addition, the \( SD \) coupling in the covariant ChEFT is correlated to the \( ^3S_1 \) potential, while a free LEC appears in the non-relativistic ChEFT. These two differences lead to in total \( 2^2 = 4 \) combinations that will be examined. In addition to our previous study [3], we perform three more fits to the same lattice QCD data, and make a systematic comparison of the results, to better understand how the results depend on the baryon masses and \( SD \) coupling in the CT potential.

Moreover, since both approaches fail to precisely reproduce the lattice QCD \( ^3D_1 \) phase shifts of \( \Lambda N \) at low energies, we adopt a new fitting strategy where the phase shifts of \( \Lambda N \) \( ^3S_1, ^3D_1 \) partial waves and inelasticity \( \eta_1 \) with \( E_{\text{cm}} \leq 5 \) MeV are simultaneously fitted. The new strategy can provide a closer look at the two approaches in the descriptions of low energy lattice QCD data. Note that we only consider the lattice QCD data with \( m_\pi = 410 \) and 570 MeV in all the aforementioned fittings. Details of the fitting strategies in this work are shown in table 1.

4. Results and discussions

4.1. Origin of the difference in predicting the \( \Lambda N ^3D_1 \) phase shifts

The fitted results of strategies 1–4 as described in the previous section are summarized qualitatively in table 2 and quantitatively in figure 1. It is noted that the treatment of the potentials in strategy 1 is that adopted in [3] [19], and strategy 4 is approximately the same as that of the non-relativistic ChEFT. The following conclusions can be obtained from the table: first, the baryon masses affect the \( ^3D_1 \) phase shifts for the large pion mass (\( m_\pi = 570 \) MeV), where negative phase shifts are obtained in strategies 1, 2 and they become positive if \( M_Y \) is taken to be the same as \( M_N \) (strategies 3, 4). Second, only when \( M_Y = M_N \) and the \( SD \) coupling in the CT potential is turned off, the \( ^3D_1 \) interaction becomes attractive in the unphysical region (strategy 4). Third, the \( SD \) coupling in the covariant ChEFT reduces the attraction in the \( ^3S_1 \) partial wave in the physical region, compared with the non-relativistic case, as shown in strategies 1 and 3.

4.2. Simultaneous fits to the \( \Lambda N ^3S_1-^3D_1 \) partial waves

In this subsection, we simultaneously fit to the phase shifts of \( \Lambda N ^3S_1, ^3D_1 \) and inelasticity \( \eta_1 \) of the lattice QCD data for \( m_\pi = 410 \) and 570 MeV with a smaller energy range from threshold up to \( E_{\text{cm}} = 5 \) MeV in order to achieve a \( \chi^2/\text{d.o.f.} \approx 1 \). With this new strategy, we aim to check whether the covariant ChEFT approach or the non-relativistic ChEFT approach can precisely describe the lattice QCD data at low energies, where they are believed to work the best. In the covariant ChEFT, we only consider two strategies: either \( M_Y = M_N \) or \( M_Y = M_0 \). The \( SD \) coupling appears naturally in the CT potentials, therefore we did not manually turn it off.

The \( \chi^2 \) shown in table 2 is larger than that in [3] because of different fitting strategies. The \( \chi^2 \) in [3] is obtained by fitting to the \( ^3S_1 \) phase slats, while the \( \chi^2 \) in table 2 includes the \( ^3D_1 \) and mixing angle data as well.
Figure 1. \(\Lambda cN^3S_1, 3^D_1\) phase shifts and inelasticity \(\eta_1\) for different pion masses. The results are obtained by fitting to the lattice QCD \(\Lambda cN\) wave phase shifts for \(E_{\text{c.m.}} \leq 30\) MeV. The bands are generated from the variation of \(\Lambda F\) from 600 to 700 MeV. Different labels denote the \(\Lambda cN\) phase shifts of strategies 1–4: ‘w/’ is the abbreviation for ‘with’, and ‘w/o.’ is the abbreviation for ‘without’.

Table 2. Dependence of the \(\Lambda cN^3S_1\) and \(3^D_1\) phase shifts on the baryon masses and \(SD\) coupling for different pion masses (in units of MeV). The ‘+’ and ‘−’ indicate the sign of the \(\Lambda cN^3S_1\) and \(3^D_1\) phase shifts within the fitting region \(E_{\text{c.m.}} \leq 30\) MeV, where ‘+’ and ‘−’ denote attractive and repulsive potentials, respectively. The values of the \(\chi^2/\text{d.o.f.}\) (in units of \(10^{-2}\)) are obtained with \(\Lambda F = 600/700\) MeV.

| Strategy | \(M_\Lambda\) | \(M_N\) | \(SD\) coupling | \(m_\pi\) | \(\delta_{3S_1}\) | \(\delta_{3D_1}\) | \(\chi^2/\text{d.o.f.}\) |
|----------|----------------|----------------|----------------|--------|----------------|----------------|----------------|
| 1        | \(=\)           | \(\checkmark\) | \(\times\)     | 138    | +              | −              | 1.30/1.32      |
|          |                 |                |                | 410    | +              | −              | 25.9/30.9      |
|          |                 |                |                | 570    | +              | −              | 0.16/0.08      |
| 2        | \(=\)           | \(\checkmark\) | \(\times\)     | 138    | +              | −              | 1.30/1.08      |
|          |                 |                |                | 410    | +              | −              | 0.16/0.08      |
|          |                 |                |                | 570    | +              | −              | 7.31/11.0      |
| 3        | \(=\)           | \(\checkmark\) | \(\times\)     | 138    | +              | −              | 3.41/18.8      |
|          |                 |                |                | 410    | +              | +              | 7.31/11.0      |
|          |                 |                |                | 570    | +              | +              | 0.17/0.14      |
| 4        | \(=\)           | \(\checkmark\) | \(\times\)     | 138    | +              | +              | 2.41/2.07      |
|          |                 |                |                | 410    | +              | +              | 0.17/0.14      |
|          |                 |                |                | 570    | +              | +              | 0.17/0.14      |

Notes.

\(^{1}\) Indicate that the \(\Lambda cN^3S_1\) interaction for \(m_\pi = 138\) MeV is weakly attractive only at the very low energy region (about \(E_{\text{c.m.}} = 3\) MeV) and then becomes repulsive as the kinetic energy increases.

\(^{2}\) The small \(\chi^2/\text{d.o.f.}\) compared with those of table 3 imply that it is easy to reproduce the lattice QCD \(3^S_1\) phase shifts than the coupled channel results.
The non-relativistic ChEFT approach is also applied to perform the fits for comparison. The fitted results of strategies 5−7, as described in table 1, are qualitatively shown in table 3 and quantitatively shown in figures 2 and 4.

### 4.2.1. Covariant ChEFT

First, we study how the use of ‘physical’ baryon masses affects the description of the $\Lambda cN$ interactions in the covariant ChEFT. The relevant fitting details and the corresponding values of the $\chi^2$/d.o.f. are obtained with $\Lambda F = 600/700$ MeV in the covariant ChEFT and $\Lambda F = 500/600$ MeV in the non-relativistic ChEFT.
summarized in table 3. For strategy 5, with lattice QCD $M_{\Lambda N}$, $M_{\Lambda N}$ in the $\Lambda N$ CT potentials within the fitting region $E_{c.m.} \leq 5$ MeV, we presented the phase shifts of $\Lambda N^3S_1$ and $^3D_1$ partial waves and inelasticity in figure 2. One can see that the $\Lambda N^3S_1$ and $^3D_1$ phase shifts agree quantitatively with the lattice QCD data within uncertainties, and the asymptotic behaviors of inelasticity are in good agreement with the lattice QCD data. Comparing these results with those of strategy 6 where $M_{\Lambda N} = M_{\Lambda N}$, shown in figure 2, one can see that the $\Lambda N^3S_1$ and $^3D_1$ interactions are attractive, contrary to the repulsive potential obtained in strategy 5.

In both cases, the extrapolation of the relativistic $\Lambda N^3S_1$ and $^3D_1$ partial waves phase shifts and inelasticity to the physical point shows that the $\Lambda N$ interaction is attractive in the $^3S_1$ partial wave within the fitting region. Comparing the above results with strategy 1 (our previous study), where the $\Lambda N^3S_1$ potential is repulsive, we conclude that the extrapolated phase shifts of $\Lambda N^3S_1$ are not very stable.

To investigate whether the energy region fitted can affect the extrapolations, we also fitted the lattice QCD data up to $E_{c.m.} \leq 20$ MeV in the covariant ChEFT approach. The results are shown in figure 3 in comparison with the results obtained by fitting only up to $E_{c.m.} \leq 5$ MeV. The two fits are qualitatively consistent with each other. Only $\eta_1$ is closer to unity in the new fit. In addition, the extrapolated $\delta_3S_1$ and $\delta_3D_1$ show some visible differences. At $m_{\pi} = 138$ MeV, $\delta_3S_1$ becomes smaller, and $\eta_1$ becomes more dependent on the cutoff.

4.2.2. Non-relativistic ChEFT. Focusing on the lattice QCD data with $E_{c.m.} \leq 5$ MeV, we show in figure 4 (the blue bands) the $\Lambda N^3S_1$, $^3D_1$ partial wave phase shifts and inelasticity obtained from strategy 7 in the non-relativistic ChEFT. The corresponding $\chi^2$/d.o.f. are listed in table 3. Here, we find that the non-relativistic phase shifts of $\Lambda N^3S_1$ partial wave and inelasticity are in qualitative agreement with the lattice QCD data in the region fitted. On the other hand, the $\Lambda N^3D_1$ phase shifts turn out to be positive, while the lattice QCD data are negative, though quite small. This is very different from the covariant case as shown in figure 3, where the $^3D_1$ phase shifts are negative for the energy region studied. According to the previous experience in the $NN$ sector [11, 16], the two
EFTs should behave similarly in the low-energy regime, while the covariant EFT usually agrees better with the lattice QCD data than the non-relativistic EFT in the relatively high-energy regime. The present results are in conflict with such expectations to some extent. A better understanding can only be achieved once more precise lattice data with realistic uncertainties become available.

When we extrapolate the non-relativistic results to the physical point, we find that the $\Lambda cN$ interaction is repulsive in the $^3S_1$ partial wave within the fitting region. Comparing these results with those of the covariant ChEFT and the left panel in Fig.4 of [2], we again conclude that the extrapolated $\Lambda cN^3S_1$ interactions are not very stable, i.e. sensitive to the adopted fitting strategies.

The HB $\Lambda N^3S_1,^3D_1$ phase shifts and inelasticity with LECs obtained by fitting to the lattice QCD data up to $E_{c.m.} \leq 20$ MeV (Fit 2) are compared to those obtained by fitting only up to $E_{c.m.} \leq 5$ MeV (Fit 1) in figure 4. Compared to Fit 1, the descriptions of $\delta_{3D_1}$ remain almost unchanged in Fit 2, but the $^3S_1$ phase shifts are very different. In Fit 1, $\delta_{3S_1}$ increases with $E_{c.m.}$ for the case of $m_\pi = 570$ MeV, while in Fit 2, it increases with $E_{c.m.}$ for $E_{c.m.} \leq 5$ MeV and then decreases with $E_{c.m.}$, which are in better agreement with the lattice QCD simulations at least for the energy region shown in this figure. Moreover, the difference for the case of $m_\pi = 570$ MeV eventually contributes to the completely different prediction of the physical $\delta_{3S_1}$, where the phase shifts become positive in Fit 2. As for the case of $m_\pi = 410$ MeV, the two results show no qualitative difference. For the inelasticity, the results obtained in Fit 2 are closer to unity and larger than the lattice QCD simulations for the energy region fitted and become more independent on $E_{c.m.}$ as the pion mass increases.

5. Conclusion

The $\Lambda N^3S_1,^3D_1$ interactions were studied in leading order covariant ChEFT and next-to-leading order non-relativistic ChEFT. The low-energy constants were determined in two different strategies by fitting to the HAL QCD lattice data, i.e. (a) by only fitting to the $\Lambda N^3S_1$ partial wave lattice data, (b) by a combined fit to the phase shifts of $^3S_1, ^3D_1$ and inelasticity $\eta_1$. It was shown that for the first strategy, the predicted
\( \Lambda, N \) \( {^3D_1} \) phase shifts from the covariant ChEFT were consistent with the low-energy lattice QCD data by using lattice QCD \( M_f, M_N \) and retaining the \( SD \) coupling in the contact potentials, while for the second strategy, one obtained results similar to those of the first strategy in the covariant ChEFT. However, the non-relativistic ChEFT predicts an attractive \( \Lambda, N \) \( {^3D_1} \) interaction in both cases, which is inconsistent with the low-energy lattice QCD data. In addition, we found that the extrapolated \( \Lambda, N \) \( {^3S_1} \) phase shifts in the physical region were very sensitive to the fitting strategies and the theoretical approaches used. The covariant ChEFT predicts a repulsive/attractive \( \Lambda, N \) \( {^3S_1} \) interaction depending on the fitting strategy (a)/(b), while the non-relativistic ChEFT predicts the opposite, which also depends on the energy region fitted. These results indicate that more refined lattice QCD data are needed to reach a firm conclusion about the \( \Lambda, N \) \( {^3S_1} \rightarrow {^3D_1} \) interactions.

It is necessary to point out that there are ongoing discussions on the validity of the HAL QCD method [34–36]. In our present work, we have of course assumed that the method is valid and the \( {^3D_1} \) phase shifts and particularly the inelasticity are correctly extracted with the precision claimed in [18]. Hopefully, the present study can motivate a closer look at the \( \Lambda, N \) interaction in the \( {^3S_1} \rightarrow {^3D_1} \) coupled channel.

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