On Inflation and Reheating Features in the Higgs-$R^2$ Model

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Abstract

We investigated inflation in the Higgs-$R^2$ model and assumed the trajectory to follow a single-field approximation called minimal two-field mode. We tried to constrain the Higgs’ non-minimal coupling $\xi$ using this approximation. During inflation, we investigated the effect of $\xi$ on the non-gaussianity. We found that $\xi$ could not provide the large non-gaussianity. This paper divided the preheating stage into quadratic and quartic regimes. The gauge bosons’ production is the most dominant during the quadratic regime. However, it could not drain the whole inflaton’s energy. Thus, we introduced a dark matter candidate with a large coupling that could drain the whole inflaton’s energy. We also found that if $\xi < 4.2$, the oscillation on the quadratic regime continued to the quartic regime. The reheating temperature obtained by this mode can be reached at $\sim 10^{10}$ GeV and strongly depends on the remaining inflaton’s energy density. On the contrary, if $\xi > 4.2$, the oscillation only happens at the quadratic regime. As we assumed the Pauli exclusion principle does exist during the quadratic regime, the instant decay of gauge bosons to fermions is restricted. Hence we expected the reheating temperature to be low for about $\sim 10^9$ GeV.

I. INTRODUCTION

Among many models of inflation, Starobinsky inflation [1] and Higgs inflation [2] are the most promising candidates for inflationary models. The recent Planck result [3] has proved the fitness of these models yet disfavoring other models, including chaotic inflation and power-law inflation. Starobinsky inflation has been through so many discussions and striking the problem by solely using gravity as the main cause of inflation. Although theoretically, this model is rather subtle [4], at many points, its inflationary prediction shows a good approximation with data [3]. The model is just introducing the action with the $R^2$ term, which is sometimes it can be referred to by the $R^2$ model and also the simplest class of $f(R)$ theory[1] (see [9] [12]). Higgs inflation [2], on another side, is quite economical in the model since there is no particle introduced beyond the standard model (SM). It introduces the non-minimal coupling $\xi$ between the Higgs field and the Ricci scalar. Despite its advantages, Higgs inflation has a defect on the unitarity problem [13] [15] due to the largeness of the

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1 See also Refs. [5] [7] for $f(R)$ model related to axion. One may also find the study about the deviation of the $R^2$ model on Ref. [8].
non-minimal coupling\(^2\) \(\sim 10^5\). In contrast, \(R^2\) inflation needs to be extended by another field to get the full preheating and reheating scenario [17]. Lastly, one can see Ref. [18] and [19] to distinguish both models and their equivalence, respectively.

There are several studies have been done to settle the problems of both Higgs and \(R^2\) and inflation, for example, to combine them in Higgs-\(R^2\) (or generally a Scalar-\(R^2\)) inflation (see, e.g., Refs. [16, 17, 20–33]). The addition of \(R^2\) operator may 'heal' the non-minimal coupling of Higgs inflation [16] also 'taming' the spiky behavior of oscillating inflaton field during preheating [27]. In the Higgs-\(R^2\) inflation, it is found that the cut-off scale is as large as the Planck scale [34]. One of the exciting facts, once Higgs and \(R^2\) terms are combined to become Higgs-\(R^2\) inflation, pure Higgs inflation can't be obtained while pure \(R^2\) inflation is possible [34, 35].

Higgs-\(R^2\) inflation should have characteristics similar to multi-field inflation, as it is expected to produce a larger and detectable non-gaussianity [20, 23] compared to single-field inflation [36, 37]. This distinction can lead to the observable result (for example, by Planck satellite [3]), which can bring us to whether the inflation consists of single-field or multi-field. However, it is still arguable if multi-field inflation may lead to large non-gaussianity comparable to a single-field case. For example, Ref. [23] shows the insignificant effect of the multi-field model to produce non-gaussianity by insisting on the slow-roll condition. A similar result is also discussed in Ref. [21], in which enhancement of non-gaussianity can only be possible at the late stage of inflation before the inflation ends when a slow-roll condition is violated. On another side, a different approach has been studied by Refs. [38, 39], in which non-gaussianity can be largely enhanced during preheating.

The inflationary stage of the Higgs-\(R^2\) model has been thoroughly discussed in Refs. [16, 20, 26, 40]. We expect this model will follow the trajectory of effective single-field inflation called *minimal two-field mode* [27]. As for effective single-field inflation, it will provide a small non-gaussianity. In this paper, we will constrain the non-minimal coupling \(\xi\) of the Higgs operator using the smallness of the non-gaussianity in the minimal two-field mode. We will see whether \(\xi\) is affecting the non-gaussianity.

In the preheating stage, the research on the Higgs-\(R^2\) model has been done by many papers (see, for example, Refs. [27, 30]). For comparison, in the single-field inflation with large non-minimal coupling, the preheating is so violent that it leads to the more effective

\(^2\) Even so, the large \(\xi\) is naturally problematic [16]
drain of the inflaton’s potential energy \[41\]. However, in the Higgs-\(R^2\) model, the preheating is much smoother, and the inflaton’s energy is drained less effectively \[41\]. Also, during preheating, in the Higgs-\(R^2\) model, a tachyonic preheating existed \[28, 30\]. This way, it is expected to drain the inflaton’s energy more quickly. However, in the minimal two-field mode, the last effect is suppressed. Thus, the inefficient inflaton’s energy drain is unavoidable.

Inflation with large non-minimal coupling\(^3\) during the beginning of the preheating stage, which corresponds to the inflaton’s field value in the Einstein frame \(\lesssim 1 \, M_p\), the potential will be approximated as quadratic form. Many papers regard this quadratic potential as the dominant stage where the inflaton’s energy is drained effectively. One can see \[42\] for the Higgs inflation\(^4\) or \[41\] for the general single-field case. Also, Refs. \[27, 28\] for the Higgs-\(R^2\) model. In another case, the quadratic regime can be neglected for small non-minimal coupling. Thus, the preheating stage can be regarded in the Jordan frame, leaving the inflaton’s oscillation represented in quartic potential (see, for example, \[45, 46\]). In both cases, either in quadratic or quartic regimes, the reheating temperature is expected to be produced by the perturbative decay of the daughter fields from the product of the inflaton’s oscillation. The energy level between the quadratic and quartic regimes strongly depends on the non-minimal coupling in the single-field case. We will see in this paper the value of the non-minimal coupling can affect the preheating stage and the reheating temperature. Please note, in this paper, we only consider \(\xi > 1\).

II. THE HIGGS-\(R^2\) INFLATION: FEATURES IN THE INFLATIONARY STAGE

In this part, we will discuss the features of Higgs-\(R^2\) inflation. As it belongs to multi-field inflation, we will discuss its features and adopt it into the Higgs-\(R^2\) model. For further treatment, the non-gaussianity of the Higgs-\(R^2\) model will be considered. Finally, we will try to constrain the non-minimal coupling \(\xi\) due to the smallness of non-gaussianity produced by the effective single-field theory.

\(^3\) We say ”large” to be in \(O(10^2)\) or more and ”small” to be in \(O(10)\) or less

\(^4\) See also Ref. \[43\] and Ref. \[44\] for preheating of Higgs inflation with lattice.
A. The multi-field inflation and slow-roll parameters

In this subsection, we will discuss the case of multi-field inflation and then simplify it into the Higgs-$R^2$ model later in the next subsection. We start by writing the action with $d$ number of scalar fields $\psi^a$ ($a = 1, 2, 3, \ldots d$) as

$$S = \int d^dx \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} \gamma_{ab} g^{\mu\nu} \partial_{\mu} \psi^a \partial_{\nu} \psi^b - U(\psi) \right],$$

(1)

where $R$ is the Ricci scalar with metric tensor $g_{\mu\nu}$ (and determinant $g$). We also have the field metric $\gamma_{ab}$, corresponding to the mixing kinetic terms. $U(\psi)$ is the potential with the function of $\psi^a$. We also use $M_p = 1/\sqrt{8\pi G}$ as the reduced Planck mass.

To obtain the equation of motions, we take the field to be only time-dependent as $\psi^a(x,t) = \psi_0^a(t)$, hence we can write three background equations:

$$H^2 = \frac{1}{3M_p^2} \left[ \frac{1}{2} \dot{\psi}_0^2 + U(\psi) \right],$$

$$\dot{H} = -\frac{\dot{\psi}_0^2}{2M_p^2},$$

$$D_t \dot{\psi}_0^a + 3H \dot{\psi}_0^a + U^a = 0.$$ (2)

We have defined the derivative potential $U_a = \partial U/\partial \psi^a$ and Hubble parameter $H = \dot{a}/a$, where $a(t)$ is the scale factor. The dot (e.g., in $\dot{\psi}_0$ and $\dot{a}$) corresponds to the derivative over physical time $t$. Here we also introduce the covariant derivatives $D\Phi = d\Phi + \Gamma^a_{bc} \Phi^b d\psi^c_0$ and $D_t = D/dt$. Please note, the Christoffel symbols we used are coming from the field metric $\gamma_{ab}$ as $\Gamma^a_{bc} = (1/2) \gamma^{ad} (\partial_b \gamma_{dc} + \partial_c \gamma_{bd} - \partial_d \gamma_{bc})$. For this paper, the dot always represents the derivative with respect to physical time.

Later, we will define the tangent direction $T^a$ and the normal direction $N^a$ to discuss the trajectory features. Both terms are defined as [47]

$$T^a \equiv \frac{\dot{\psi}_0^a}{\psi_0},$$

$$N^a \equiv \text{sign}(t) \left( \gamma_{bc} D_t T^b D_t T^c \right)^{-1/2} D_t T^a,$$ (3)

where $\text{sign}(t) = \pm 1$ responsible to adjust the orientation of $N^a$ with respect to $D_t T^a$. Please note, $T^a$ and $N^a$ obey the relation $N^a N_a = T^a T_a = 1$ and $N^a T_a = 0$. In addition, we can write $D_t T^a$ with the help of Eq. (2) and (3). Hence we obtain

$$D_t T^a = -\frac{\dot{\psi}_0^a}{\psi_0} T^a - \frac{1}{\psi_0} \left( 3H \dot{\psi}_0^a + U^a \right).$$ (4)
Projecting the last equation to $T^a$ and $N^a$ separately, we get the new equations:

$$\ddot{\psi}_0 + 3H\dot{\psi}_0 + U_\psi = 0$$  \hspace{1cm} (5)$$

and

$$D_t T^a = -\frac{U_N N^a}{\psi_0},$$  \hspace{1cm} (6)$$

where in Eq. (5) and (6) we defined $U_\psi = T^a U_a$ and $U_N = N^a U_a$. In addition, they obey the relation $U_a = U_\psi T^a + U_N N^a$.

With all requirements fulfilled, we can determine the slow-roll parameters. Straightforwardly, we can write the slow-roll parameters as

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{\dot{\psi}_0^2}{2M_p^2 H^2}, \quad \eta^a = -\frac{1}{H\dot{\psi}_0} D_t \dot{\psi}^a.$$  \hspace{1cm} (7)$$

Additionally, $\eta^a$ can be decomposed into $\eta^a = \eta^\| T^a + \eta^\perp N^a$, thus we obtain two other parameters

$$\eta^\| \equiv -\frac{\ddot{\psi}_0}{H\dot{\psi}_0},$$  \hspace{1cm} (8)$$

which can be regarded as the usual second slow-roll parameter and

$$\eta^\perp \equiv \frac{U_N}{H\dot{\psi}_0},$$  \hspace{1cm} (9)$$

as the turn parameter, which bends the trajectory of inflaton. We can also have the second turn parameter $\xi^\perp$ as

$$\xi^\perp \equiv -\frac{\eta^\perp}{H\eta^\perp}.$$  \hspace{1cm} (10)$$

The last parameter will be redundant for the rest of this paper.

Please note that $\epsilon$ and $\eta^\|$ tend to be small during inflation for slow-roll conditions, but $\eta^\perp$ does not have any constraint \cite{17}, it could be large. Thus large bending from the geodesic is also possible. The radius of curvature $\sigma$ can be defined by

$$\frac{1}{\sigma} = \left( \gamma_{ab} \frac{DT^a}{d\psi_0} \frac{DT^b}{d\psi_0} \right)^{1/2} = \frac{1}{\psi_0} \left( \gamma_{ab} \frac{DT^a}{dt} \frac{DT^b}{dt} \right)^{1/2} = H\eta^\perp,$$  \hspace{1cm} (11)$$

where the last line is provided by the help of Eq. (3), (6), and (9). Also, using Eq. (7), we can obtain

$$|\eta^\perp| = \sqrt{2\epsilon} \frac{M_p}{\sigma \psi_0}.$$  \hspace{1cm} (12)$$

We will later use the last result to calculate the non-gaussianity of multi-field inflation. Furthermore, even though $\eta^\perp$ is not constrained, it is suppressed by the smallness of $\sqrt{\epsilon}$. Hence, the largeness of $\eta^\perp$ strongly depends on $\sigma$ and $\dot{\psi}$.
B. A short preview of Higgs-\(R^2\) inflation

We start from the following action in the Jordan frame of Higgs-\(R^2\) inflation as

\[
S_J = \int d^4x \sqrt{-g_J} \left[ \frac{1}{2} M_p^2 \left( R_J + \frac{R_J}{6M_p^2} \right) + \frac{\xi h^2 R_J}{2} - \frac{g_J^{\mu\nu}}{2} \partial_\mu h \partial_\nu h - \frac{1}{4} \lambda h^4 \right],
\]

where \(g_J\) and \(R_J\) are respectively the determinant of the metric \(g_J^{\mu\nu}\) and Ricci scalar in the Jordan frame. Here we take \(h\) as the Higgs field in the unitary gauge, \(\xi\) as the non-minimal coupling between the Higgs field with gravity\(^5\), and \(\lambda\) as Higgs quartic coupling. In this paper, we used \(\lambda = 0.01\).

We can transform the action in Jordan frame \(S_J\) to Einstein frame \(S_E\) via Weyl transformation as

\[
g^{\mu\nu} = \Omega g_J^{\mu\nu},
\]

\[
\Omega \equiv 1 + \frac{R_J}{3M_p^2} + \frac{\xi h^2}{M_p^2} \equiv e^{\sqrt{\frac{\chi}{2M_p}}},
\]

which we defined the scalaron field \(\phi\) here [16, 29, 34]. Please note, \(M\) in Eq. (13) and (14) correspond to the effective mass of the scalaron during the small field value. After transformation, we should obtain the action in the Einstein frame \(S_E\) as

\[
S_E = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} e^{-\chi} g^{\mu\nu} \partial_\mu h \partial_\nu h - U(\phi, h) \right],
\]

with potential

\[
U(\phi, h) = \frac{1}{4} \lambda h^4 e^{-2\chi} + \frac{3}{4} M_p^2 M^2 \left[ 1 - \left( 1 + \frac{\xi h^2}{M_p^2} \right) e^{-\chi} \right]^2.
\]

Please note, action in Eq. (15) is similar with action (1) for the Higgs-\(R^2\) model with

\[
\gamma_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-\chi} \end{pmatrix}, \quad \psi^a = (\phi, h).
\]

In our paper, we assume in Higgs-\(R^2\) inflation, the trajectory of inflaton will follow a single-field approximation called \textit{minimal two-field mode}. In this mode, we use relation\(^5\)

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\(^5\) One should not be confused by \(\xi^\perp\) as second turn parameter and \(\xi\) as the non-minimal coupling.
\[ h^2 = \frac{\xi R_f}{\lambda} \] \[ [27, 34] \]. Inserting this result to Eq. (14), we obtain

\[ h^2 = \frac{e^\chi - 1}{\xi M_p^2 \left( \frac{\xi^2}{\lambda} + \frac{M_p^2}{3M^2} \right)} . \] \[ (18) \]

In addition, the potential represents the minimal two-field mode is \[ [34] \]

\[ U(\phi) = \frac{M_p^4}{4} \frac{1}{\xi^2 + \frac{M_p^2}{3M^2}} (1 - e^{-\chi})^2 . \] \[ (19) \]

Based on Eq. (19), we can conclude the potential for a minimal two-field mode dominantly following the scalaron’s trajectory with some deviation due to the shifting parameters. Using the last potential on the power scalar spectrum through constraint from the Cosmic Microwave Background (CMB) \[ [3] \], we obtain

\[ \frac{\xi^2}{\lambda} + \frac{M_p^2}{3M^2} = \frac{N_f^2}{72\pi^2A_s} \equiv \frac{1}{C} \approx 2.1 \times 10^9 , \] \[ (20) \]

where we pick \( N_f = 56 \) correspond to the number of e-folds, \( A_s = 2.1 \times 10^{-9} \) refers to the primordial spectrum perturbation on the pivot scale \( k_o = 0.05\text{Mpc}^{-1} \) \[ [3, 27] \]. Based on these values, the parameter \( \xi \) is bounded from \( \xi = 1 \) (for nearly pure \( R^2 \) inflation) to \( \xi \lesssim 4500 \) (for nearly-pure Higgs inflation). With this in mind, we can rewrite Eq. (18) to the simpler form:

\[ h^2 = C \frac{\xi M_p^2}{\lambda} (e^\chi - 1) . \] \[ (21) \]

Such that mind saves us by using \( M \). Thus, we can work only by terms \( \xi \) and \( \lambda \), which are the properties of Higgs inflation. It is also important to show that \( \xi^2/\lambda > M_p^2/3M^2 \) which corresponds to Higgs-like inflation, and \( \xi^2/\lambda < M_p^2/3M^2 \) which corresponds to \( R^2 \)-like inflation \[ [27, 34] \]. One can see relation (21) is model-independent: It can be used for both Higgs-like inflation and \( R^2 \)-like inflation as long as the minimal two-field mode is preserved.

One can also analytically solve Eq. (21) during inflation. We can assume \( e^\chi \gg 1 \), hence we obtain the relation

\[ h^2 \approx C \frac{\xi M_p^2}{\lambda} e^\chi . \] \[ (22) \]

During the end of inflation (when \( \epsilon = 1 \) and \( \phi \approx 1 M_p \)), the inflaton is started to oscillate. Additionally, the Higgs field has a value

\[ \tilde{h}_{end}^2 \approx \sqrt{\frac{2}{3}} C \frac{\xi M_p}{\lambda} \tilde{\phi}_{end} . \] \[ (23) \]
where the tilde corresponds to the amplitude of the corresponding fields. Lastly, the critical field value\(^6\) is obtained when

\[
\tilde{\phi}_{\text{crit}} \approx \tilde{h}_{\text{crit}} \simeq \sqrt{\frac{2}{3}} \frac{g \xi M_p}{\lambda}.
\]

(24)

The last two equations are only valid if we preserve the minimal two-field mode, which is described shortly in the preheating stage (see section \[\text{II} \] and \[\text{IV} \]). Considering the large \(\xi\) corresponding to Higgs–like inflation, the critical field value can be pushed higher. For example, if we consider the nearly-pure Higgs inflation \(\xi = 4500\), the corresponding critical field value of Higgs is \(1.75 \times 10^{-4} M_p\). However, if we take the small minimal coupling \(\xi = 10\), we got only \(3.88 \times 10^{-7} M_p\). This determination is really important in the late stage of the preheating regime. We will show later the non-minimal coupling could potentially affect the reheating temperature.

C. The turn parameter in Higgs-\(R^2\) inflation.

In this part, we will explicitly describe the features of Higgs-\(R^2\) using the features of general multi-field inflation depicted in \[\text{II A} \]. Straightforwardly, we can obtain the tangent and normal trajectory respectively as

\[
T^a = \frac{\dot{\psi}_0}{\psi_0} = \frac{1}{\sqrt{\dot{\phi}^2 + e^{-\chi} \dot{h}^2}} \left( \dot{\phi}, \dot{h} \right),
\]

(25)

\[
N^a = \text{sign}(t) \left( \gamma_{bc} \frac{DT^b}{dt} \frac{DT^c}{dt} \right)^{-1/2} \frac{DT^a}{dt} = \frac{e^{\chi/2}}{\sqrt{\dot{\phi}^2 + e^{-\chi} \dot{h}^2}} \left( -e^{-\chi} \dot{h}, \dot{\phi} \right),
\]

(26)

where partially can solve

\[
\left( \gamma_{ab} \frac{DT^a}{dt} \frac{DT^b}{dt} \right) = e^{\chi} \frac{\left( \frac{\partial U}{\partial \phi} \dot{\phi} - e^{-\chi} \frac{\partial U}{\partial h} \dot{h} \right)^2}{\left( \dot{\phi}^2 + e^{-\chi} \dot{h}^2 \right)^2}.
\]

(27)

These tools can approximate the turn parameter \(\eta^\perp\) from Eq. (11) and (27). Finally, we obtain

\[
e^{\chi/2} \frac{\left( \frac{\partial U}{\partial \phi} \dot{\phi} - e^{-\chi} \frac{\partial U}{\partial h} \dot{h} \right)}{\left( \dot{\phi}^2 + e^{-\chi} \dot{h}^2 \right)} = H \eta^\perp.
\]

(28)

\(^6\) The critical field value is a condition when \(\tilde{\phi}\) drops until \(\tilde{\phi} = \tilde{h}\), from which Jordan and Einstein’s frames coincide. When such conditions are happening, the scalaron vanishes, and the action is evaluated by using the Jordan frame action (see Eq. (13)).
Solving the last result may take much effort. However, even if $\partial U / \partial \phi \gg \partial U / \partial h$, $\partial U / \partial \phi$ is suppressed by $e^{-\chi}$ and $\dot{h}$ which makes it several order smaller than $\partial U / \partial h$. Hence, we can obtain the simplified result by using $e^{-\chi} 3H \dot{h} \approx \partial U / \partial h$ (see Eq. (35)) as

$$3e^{-\chi/2} \frac{\dot{h}}{\dot{\phi}^2} \approx \eta^\perp.$$  \hspace{1cm} (29)

If we used the ratio of $\dot{h}/\dot{\phi}$ in Eq. (22), we finally get

$$\eta^\perp \approx 3e^{-\chi/2} \frac{\dot{h}}{\dot{\phi}} = 3\sqrt{\frac{2\xi}{3\lambda}} C.$$  \hspace{1cm} (30)

Thus, we obtain the relation on the turn parameter $\eta^\perp$. We found that turn parameter $\eta^\perp$ strongly depends on $\xi / \lambda$ which are properties of Higgs inflation.

D. The non-gaussianity in the Higgs-$R^2$ inflation

In this section, we will use the formalism of non-gaussianity which is introduced by Ref. [20] (see also [49] and [50]) for analytical and Ref [51] for the numerical calculation. The parameter $f_{NL}$ provided by this formalism is

$$f_{NL} = f(\epsilon, \eta^\parallel, \eta^\perp, \xi^\perp, U(\phi, h), N_f).$$  \hspace{1cm} (31)

It is better to write Eq. (31) more explicitly. Before proceeding, we should make an approximation because this case is quite complicated in the original paper in Ref. [20]. In this approximation, $f_{NL}$ can be written by

$$f_{NL} = 2(\epsilon + 3\eta^\parallel - \xi^\perp / \eta^\perp) + \Psi N_f,$$  \hspace{1cm} (32)

where $\Psi \approx 4(\eta^\perp)^2$.

The small non-gaussianity $f_{NL} \ll 1$ may be acquired since it is a function of slow-roll parameters. Thus, from Eq. (32), the large non-gaussianity can be expected at the end of inflation when $\epsilon = 1$. In that case, the dominant term for non-gaussianity during inflation $f_{NL} \geq 1$ can be achieved if $4(\eta^\perp)^2 N_f \geq 1$. As for reminder $N_f$ is the number of e-fold, which means it could produce quite a large number. $\eta^\perp$ doesn’t need to be large, as long it is not too small ($\eta^\perp \sim 0.07$ would be sufficient for $N_f \sim 50$). Using these criteria, we can simplify Eq. (32) to be

$$f_{NL} \approx 4(\eta^\perp)^2 N_f = 4 \left(3\sqrt{\frac{2\xi}{3\lambda}} C\right)^2 N_f,$$  \hspace{1cm} (33)
where in the last result we have substituted $\eta_\perp$ from Eq. (30). If we put non-minimal coupling to be nearly-pure Higgs inflation which $\xi = \mathcal{O}(10^3)$ for $\lambda = 0.01$, the contribution on the non-gaussianity is still $\ll 1$. With this result, it is convenient to say that the non-minimal coupling can be regarded as a free parameter since we can take it to be almost any value. Thus, the spectrum of this model can be extended from nearly-pure $R^2$ inflation to nearly-pure Higgs inflation. However, in the next section, we will try to constrain the non-minimal coupling $\xi$ depending on its impact on preheating and reheating.

In addition, it is very interesting that in the minimal two-field mode of the Higgs-$R^2$ model, the parameters of the Higgs operator play the most important part. At this point, we can make the parameter of $R^2$ inflation which is $M$, stay hidden under constant $C$.

III. PREHEATING IN THE QUADRATIC REGIME

A. The self-production of inflaton field

During the end of inflation, when the field value is $\phi < 1 M_p$, the inflaton starts to oscillate. At this stage, the inflaton will decay non-perturbatively via parametric resonance. The equation of motions corresponding to the scalaron and Higgs boson can be depicted as

\[
\ddot{\phi} + 3H\dot{\phi} + \frac{1}{\sqrt{6}M_p} e^{-\chi}\dot{h}^2 + \frac{\partial U}{\partial \phi} = 0
\]

\[
\ddot{h} + 3H\dot{h} - \sqrt{2} \dot{\phi} \dot{h} M_p + e^\chi \frac{\partial U}{\partial h} = 0
\]

and an extra

\[
3M_p^2 H^2 = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} e^{-\chi} \dot{h}^2 + U(\phi, h).
\]

The numerical calculation corresponding to the last three equations can be seen in \cite{27, 28, 30}. However, we want to simplify the result by assuming the inflaton’s trajectory in a single-field approximation\footnote{Our chosen analytical single-field trajectory is invalid if it is nearly pure Higgs inflation. However, such a condition is discouraged by Ref. \cite{34}. Hence, our approximation is valid for the general case.}. With our chosen inflaton’s direction, we may lose the complexity of the scalaron and Higgs directions as depicted in Ref. \cite{27, 28, 30, 52} for example, the tachyonic behavior due to Higgs’ negative mass and the burst of particle productions due to Higgs direction when $\phi < 0$. We expect the preheating case to be calculated semi-analytically in...
our fine-tuning conditions. With the selected fine-tuning, the equation of scalaron can be depicted by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial U}{\partial \phi} = 0,$$  \hspace{1cm} (37)

the potential is written by the function of \(\phi\) (see Eq. (19)) only. Furthermore, we obtain

$$U(\phi) = \frac{3}{4} M_p^2 \tilde{M}^2 (1 - e^{-\chi})^2, \quad \tilde{M}^2 = \frac{M^2}{1 + \frac{3\xi^2 M^2}{\lambda M_p^2}} = \frac{M_p^2}{3} \mathcal{C},$$  \hspace{1cm} (38)

which is valid only for \(\phi > 0\). We will discuss \(\phi < 0\) later, but now we first focus on the \(\phi > 0\) condition. If we consider the minimal two-field mode, the mass \(M\) is constrained by CMB (see Eq. (20)) and also depends on whether it is Higgs-like inflation or \(R^2\)-like inflation. Usually, \(M\) is taken to be \(\mathcal{O}(10^{-5}) M_p\) \([27, 28]\), which corresponds to the mass of scalaron during the small field value in the pure \(R^2\) model. In a minimal two-field mode, one can check the relation between \(\xi\) and \(M\) in Fig. 1.

When \(\phi < 1 M_p\), the potential can be approximated by

$$U(\phi) \simeq \frac{1}{2} \tilde{M}^2 \phi^2 - \frac{1}{2} \sqrt{\frac{2}{3}} \frac{M^2}{M_p} \phi^3,$$  \hspace{1cm} (39)

also automatically giving

$$\frac{\partial U}{\partial \phi} \simeq \tilde{M}^2 \phi - \sqrt{\frac{3}{2}} \tilde{M} \phi^2.$$  \hspace{1cm} (40)

Here we named this regime to be quadratic regime, corresponding to the first term in Eq. (39) from which the potential is described dominantly by \(\phi^2\) term. Also, if we consider the inflaton’s trajectory is following (37), the analytical solution is

$$\phi(t) \simeq \tilde{\phi} e^{\frac{-3Ht}{2}} \cos \left( \sqrt{\frac{2}{3}} M_p \phi \right),$$  \hspace{1cm} (41)
with $\tilde{\phi}$ refer to the amplitude of inflaton during the beginning of the preheating stage. With $\tilde{M} \gg H$, we can claim that the inflaton’s oscillation is light-damped during this stage.

If we decompose $\phi$ in Heisenberg representation, namely

$$\phi(x, t) = \frac{1}{(2\pi)^{3/2}} \int d^3 k \left( \hat{a}_k \phi_k(t) e^{-ik \cdot x} + \hat{a}^*_k \phi^*_k(t) e^{ik \cdot x} \right),$$

we can write the equation of motion for inflaton self-production by

$$\ddot{\phi}_k + 3H \dot{\phi}_k + \left( \frac{k^2}{a^2} + \tilde{M}^2 - \sqrt{3 \tilde{M}^2} \frac{\phi}{M_p} \right) \phi_k = 0.$$

If we rescaled and redefined

$$\varphi_k \equiv a^{3/2} \phi_k, \quad \kappa^2 \phi \equiv \frac{k^2}{a^2} + \tilde{M}^2, \quad \phi \equiv \tilde{\phi} \sin(\tilde{M}t), \quad \tilde{M}t = 2z + \frac{\pi}{2},$$

we can obtain

$$\frac{d^2 \varphi_k}{dz^2} + (A_{\phi} - 2q_{\phi} \cos(2z)) \varphi_k = 0,$$

where

$$A_{\phi} \equiv \frac{4}{M^2} \left( \frac{k^2}{a^2} + \tilde{M}^2 \right), \quad \text{and} \quad q_{\phi} \equiv 2\sqrt{\frac{3}{2}} \frac{\tilde{\phi}}{M_p},$$

found to be the Mathieu equation. $A_{\phi} \gg q_{\phi}$ for the first oscillation after the end of inflation and $q_{\phi}$ is much lower just after that, resulting in the resonance getting narrower. The self-production of the scalaron field belongs to the narrow resonance. As usually happens, the particle fluctuation during the first-several oscillations is a rather broad-like resonance\(^8\) the resonance self-production of the inflaton field and its decay is neglected. With these results, we can assume that the decay product to other channels could be dominant compared with inflaton self-production. One can also realize that the single-field approximation causes $\partial U/\partial h$ negligible in Eq. (35), this is resulting in the oscillation being stagnant, which means there is almost no Higgs production due to this mode.

We can approximate the potential energy during the end of inflation. Taking $\epsilon = 1$ corresponds to the end of inflation. We obtain the potential energy to be

$$U(\phi_{\text{end}}) < 3.7 \times 10^{-11} M_p^4,$$

which is independent of $\xi$. This result is evaluated by using Eq. (38). Taking this potential as the starting value of the preheating stage is rather subtle. If the scalaron’s field value during

\(^8\) see ref. [53, 54]
the end of inflation is $\phi_{\text{end}} \sim 1M_p$ after the first crossing, the scalaron’s field value drops significantly. Numerically, one can see Fig. 2 the field value lost about 70% after the first crossing but smoother after that. Hence, we will say that there should be a transition phase during the end of inflation and the start of the preheating stage, which happens somewhere before the first crossing. For rough estimation, we can take the field value of $\phi_{\text{pre}} \sim 0.5M_p$ to be the starting point of the preheating stage. This approximation corresponds to the potential

$$U(\phi_{\text{pre}}) \sim 1 \times 10^{-11}M_p^4. \quad (48)$$

We expect the preheating process could drain most of this energy before the perturbative reheating happens.

**B. The gauge bosons and fermions production**

As discussed in the previous subsection, the scalaron, and Higgs self-production are suppressed. In this case, the production of other fields is expected to be large. Hence we straightforwardly write the $W$-boson equation of motion by assuming that gauge bosons are scalar fields [42]. We assume in this quadratic regime, both $W$ and $Z$ bosons are identical and have the same coupling $g_W$. Hence we can write the $W$-boson’s equation of motion as

$$\ddot{W}_k + 3H\dot{W}_k + \left(\frac{k^2}{a^2} + m^2_W(t)\right)W_k = 0, \quad (49)$$

where $m^2_W$ can be defined by (see Eq. (23))

$$m^2_W = \frac{g^2_W}{4} \sqrt{\frac{2}{3}} \frac{\xi M_p}{\lambda} \phi \sin(\tilde{M}t). \quad (50)$$
Please note that the induced mass of $m_W$ was much larger than $\tilde{M}$ at the beginning of the preheating era. Indeed, the allowed time ($\delta t$) during the zero crossing in which gauge bosons are produced can be written by

$$\delta t \lesssim \frac{4}{g_W^2} \sqrt{\frac{3}{2}} \frac{\lambda \tilde{M}}{C \xi M_\nu \phi} \sim 1 \times 10^{-7}/\phi,$$

which is obtained by the conditions $m_W < \tilde{M}$. The mass of gauge bosons depends strictly on the amplitude $\tilde{\phi}(t)$. With those in mind, we approximate $\sin(\tilde{M}t) \simeq \tilde{M}t$. In addition, we can redefine Eq. (49) to be

$$W_k \equiv a^{3/2} W_k, \quad \kappa_W^2 \equiv \frac{k_W^2}{K_W^2 a^2}, \quad \tau_W = K_W t, \quad K_W \equiv \left[ \frac{g^2}{4} \frac{\sqrt{2}}{3} \frac{C \xi M_\nu \phi}{\lambda} \right]^{1/3},$$

thus we obtain

$$\frac{d^2 W_k}{d \tau_W^2} + \left( \kappa_W^2 + \tau_W \right) W_k = 0,$$

which belongs to the Airy function. This way, we obtain the solutions

$$Ai(\tau_W) = \frac{1}{3} i \sqrt{\tau_W} \left[ J_{-1/3}(b) + J_{+1/3}(b) \right],$$

$$Bi(\tau_W) = i \sqrt{\frac{\tau_W}{3}} \left[ J_{-1/3}(b) - J_{+1/3}(b) \right],$$

with $b = \frac{2}{3} \tau_W^{3/2}$ and $J_{\pm}(\tau_W)$ is the Bessel function of the order $\pm 1/3$. Both functions will run from conformal time $\tau_W \text{end} = 0$, as we taking the time when inflation is ended by $t = 0$, into

$$\tau_W \text{crit} = K \text{crit} t \text{crit} = \left[ \frac{g^2}{4} \frac{\sqrt{2}}{3} \frac{C \xi M_\nu \phi \tilde{M} \tilde{\phi}}{\lambda} \right]^{1/3} t \text{crit} \simeq 4.2 \times 10^5 \xi^{-1/3}.$$

Taking this result into a plot, we can check it in Fig. 3. In the figure, we only take $\tau_W = 50$ since it is more convenient.

Also, we can predict in $\tau$ more than 20, the amplitude is only slightly getting smaller, and the oscillation is still not terminated in the quadratic regime.

We can approximate the largest $W$-boson production for the first crossing:

$$\delta \rho_W = \int_0^\infty \frac{dk_W^3}{(2\pi)^3} \sqrt{k_W^2/a^2 + m_W^2} e^{-\pi/(k_W^2/k^2)} \simeq m_W \frac{K^3_W}{8\pi^3}.$$

This way, we obtain the largest energy drain for the first crossing to be

$$\delta \rho_W \simeq 4.52 \times 10^{-20} \xi^{3/2} M_p^4.$$

9 Taking $\tau_W \sim 10^5$ may disrupt the figure of both $Ai$ and $Bi$. 

15
FIG. 3. Plot of $Ai(\tau_W)$ (red color) and $Bi(\tau_W)$ (blue color). The figure only up to $\tau_W = 50$ to make clear between red and blue plots. Instead, the process continues until $\sim 10^5$ oscillations (for $\xi \sim 10^2 - 10^3$), which means until the end of the quadratic regime. The amplitude is in order of $M_p$.

Here we assumed $k_W^2 \ll m_W^2$ in our calculation, and $m_W$ is evaluated along the half oscillation $\int_0^\pi \sin(\tilde{M} t) = 1$. Also we used $\tilde{\phi} = \tilde{\phi}_{pre} = 0.5 M_p$.

In this calculation, we can guarantee that the gauge bosons production is inefficient in the quadratic regime. It is important to note that the created gauge bosons are still oscillating, creating more daughter fields by secondary parametric resonance production. The universe will constantly be filled with heavy matters due to these multiple resonances. It is also noted that the Pauli exclusion principle suppresses the fermions production during this regime, similar to Ref. [42]. This way, we can focus on the gauge bosons. Also, in Ref. [42] for Higgs-driven inflation, the reheating temperature is obtained by the decay of the gauge bosons to fermions in this regime. However, in our paper, we still assumed the produced fermions are similarly suppressed. It means the instant reheating will be restricted in this model. In addition, the production of Higgs bosons due to the oscillating gauge bosons is suppressed by the $\lambda/g_W^2$ term. This means that the Higgs boson production can be neglected in the quadratic regime. It is also important to mention that tachyonic preheating is possible. For instance, see Ref. [28, 30]. However, since we assumed the inflaton’s trajectory is following the two-field mode as the effective single-field mode, the tachyonic mode is suppressed due to the suppression of the Higgs field ($\partial U^2/\partial h^2 \simeq 0$) in the quadratic regime.
The inflaton’s energy can be approximated to be drained by two gauge bosons production by multiplication of the series of amplitudes and the number of oscillations. The number of oscillations in the quadratic regime can be approximated by calculating the time started from the beginning of preheating stage \( t_{\text{end}} \) until critical time \( t_{\text{crit}} \) (which indicates the beginning of the upcoming quartic regime\(^{10}\)) as

\[
t_{\text{crit}} - t_{\text{end}} \simeq \frac{2}{M_C} \frac{\lambda}{\xi}.
\]

With the period of oscillation can be approximated by \( T = \frac{2\pi}{M} \), the number of oscillations during the quadratic regime is approximated to be

\[
n_{\text{osc}} = \frac{t_{\text{crit}} - t_{\text{end}}}{T} = \frac{1}{\pi C} \frac{\lambda}{\xi}. \tag{58}
\]

Thus, we can estimate the total energy drained by the gauge bosons by using the formula

\[
\rho_{\text{gauge}} \approx \text{number of gauge bosons} \times \text{number of crossing} \times \frac{1}{3} (\delta \rho_W \times n_{\text{osc}}). \tag{59}
\]

The \( (\delta \rho_W \times n_{\text{osc}}) \) should be proportionally resemble Fig. 3 for \( \tau = \infty \). Thus, it is where the factor 1/3 numerically comes from. Finally we get

\[
\rho_{\text{gauge}} \sim 6 \times 10^{-14} \sqrt{\xi} M_p^4. \tag{60}
\]

In Eq. (60), if the non-minimal coupling is in \( \mathcal{O}(10^4) \), they could drain the whole inflaton’s energy. However, for the maximum value of \( \xi \) in this model (\( \xi \sim 4500 \)), it still could not drain the whole inflaton’s energy. In this case, we proposed a new field with a larger coupling than gauge bosons coupled with Higgs. We will discuss this possibility in the next subsection.

**C. The wear off gravitational effect, end of quadratic regime, and dark matter**

The universe is filled by nothing but inflaton during inflation, leaving most of it nearly a vacuum. As we consider the second Friedman equation

\[
\ddot{a} = -\frac{1}{6M_p^2}(\rho_\phi + 3P_\phi)a, \tag{61}
\]

the pressure \( P_\phi = -\rho_\phi \) causes acceleration. This condition happens until the kinetic term of scalaron \( \propto \dot{\phi}^2 \) gets large and comparable with the inflaton’s potential. The cosmological

\(^{10}\) See next section about this definition
constant is only responsible for the expansion in the post-inflationary era, which we omitted in the last equation. At this point, the kinetic term of Higgs is still suppressed by $e^{-\chi}$ and tends to be neglected. Thus, the scalaron remains dominant at this point. This condition happens simultaneously with the creation of massive fields which constantly fill the universe, leaving the universe to enter the (heavy) matter-dominated era. At the end of the quadratic regime, when the field value of the scalaron enters the critical point, the gravitational effect is wearing-off. It is shown when $\phi_{\text{crit}} \approx h_{\text{crit}}$. The gravitational effect remains implicitly in the second term of the coupling $\lambda + \frac{3\xi M^2}{M_P}$ (see next subsection on Eq. (82)). However, this term tends to be negated by other terms. Thus, the Higgs quartic coupling will solely depend on $\lambda$.

In this subsection, we will introduce a Lagrangian of the dark matter (DM), which we assumed only coupled with Higgs and invariant under global $Z_2$ symmetry. The importance of this DM is due to the fact that both gauge bosons failed to drain the inflaton’s energy. The $S$ field is considered the DM candidate in this model. Here we have $S = (0 \ s)^T/\sqrt{2}$ and $\langle S \rangle = 0$. The additional potential from Eq. (13) with $S$-field can be written by

$$\frac{1}{4} \lambda_{hs} h^2 s^2.$$ (62)

During the quadratic regime, Eq. (62) will be transformed by Weyl transformation on the Einstein frame into

$$\frac{1}{2\sqrt{6}} \frac{\xi M_P}{\lambda} \lambda_{hs} \phi \ s^2.$$ (63)

It means that during the quadratic regime, DM production can be produced by the perturbative decay of the scalaron at the tree level, which also plays the role of the dominant decay of the scalaron. However, this effect should never be important during the preheating stage since the parametric resonance of DM is taking over. Before we proceed, we assume $\lambda_{hs}$ value is $\lambda < g^2 < \lambda_{hs} \approx 1$. This assumption is based on the fact that we expect the DM production to be greater than gauge bosons and fermions. With the large coupling of $\lambda_{hs}$, we expect both parametric resonance and perturbative decay of scalaron to DM to be dominant compared to the gauge bosons and fermions productions.

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11 Even without cancellation, $\frac{3\xi M^2}{M_P}$ still much smaller than $\lambda$.
12 Here we assumed the term $\frac{1}{2} m_2^2 s^2$ and $\frac{1}{4} \lambda_s s^4$ are much smaller during this time.
13 For simplicity, we assumed that the additional fields (the DM) would not affect the Higgs’ running coupling and ruined our setting of $\lambda = 0.01$. 

18
The decay rate of scalaron to two DMs can be obtained as

\[ \Gamma_{\phi \rightarrow ss} = \frac{1}{384\pi} C^2 \xi^2 M_p^2 \lambda^2 \lambda_{hs}^2. \]  

One can write the Boltzmann equation for DM density via

\[ \frac{d}{dt}(n_s a^3) = 2n_\phi(a)\Gamma_{\phi \rightarrow ss} a^3. \]  

As we continue, using our previous constraint, the decay should be much lower than the Hubble parameter \( \Gamma_{\phi \rightarrow ss} \ll H \). The DM density only fills a minor part of the universe during the early preheating stage but grows significantly due to parametric resonance. In addition, it is truly remarkable that during preheating, particle production by perturbative decay should be much smaller than the resonance production for successful preheating.

However, when the parametric resonance is extremely narrow, the resulting particle production due to parametric resonance is suppressed at the end of the quadratic regime. The perturbative effect, enhanced by the Bose-Einstein condensation (BEC), drains the inflaton’s energy. This enhanced perturbative effect, even though it is small, is constantly draining the inflaton field. This effect becomes important during the transition of the quadratic and quartic regimes\(^ {15} \). Nevertheless, we could not confirm this condition in the numerical study, but we assumed such a condition existed during the transition. The reason is that during the end of the quadratic regime, the oscillation becomes extremely narrow, and a mechanism should exist to drain the inflaton’s energy.

The decay rate enhanced by BEC can be written by \( ^{55} \)

\[ \Gamma_{\text{eff}} \simeq \Gamma_{\phi \rightarrow ss}(1 + 2n_s k). \]  

We assume the initial condition of occupation number of scalaron found to be much larger than DM \( n_k \). In addition, the equation of motion in momentum space of DM coupled with scalaron can be written by

\[ \ddot{s}_k + \left( \frac{k_s^2}{a^2} + \frac{1}{\sqrt{6}} C \frac{\xi M_p}{\lambda} \lambda_{hs} \phi \sin(\tilde{M}t) \right) s_k = 0. \]  

The width of \( k_s \), which correspond to the width band of the created particles, can be evaluated by assuming the energy of a single \( S \) equals \( \tilde{M}/2 \), and we obtain

\[ \left( \frac{\tilde{M}}{2} \right) = \frac{k_s^2}{a^2} + \frac{1}{\sqrt{6}} C \frac{\xi M_p}{\lambda} \lambda_{hs} \phi. \]  

\(^ {14} \) Let us call it that way, which is perturbative effect enhanced by BEC.

\(^ {15} \) This will be defined shortly in the next section.
Finally, $\Delta k_s$ can be evaluated as

$$\Delta k_s = \sqrt{\frac{2}{3}} \mathcal{C} \frac{\xi M_p}{\lambda M} \lambda_{hs} \tilde{\phi}, \quad (69)$$

where we neglected the expansion of the universe. We can calculate the occupation number $n^s_k$ as

$$n^s_k = \frac{n_s}{4\pi k^2_\Delta k_s/(2\pi)^3} = \sqrt{\frac{3}{8}} \frac{\tilde{\phi}}{\pi \mathcal{C} \xi M_p \lambda_{hs} n_\phi}, \quad (70)$$

where we have evaluated Eq. (70) by using $k^*_s = \tilde{M}/2$ and $n_\phi = \frac{1}{2} \tilde{M} \tilde{\phi}^2$. For further usage, it is important to write Eq. (67) into the redefined form as

$$S_k \equiv a^{3/2} s_k, \quad \kappa^2_s \equiv \frac{k^2}{K^2_s} a^2, \quad \tau_s = K_s t, \quad K_s \equiv \left[\frac{\lambda_{hs} \mathcal{C} \xi M_p \tilde{M}}{\sqrt{6} \lambda \tilde{\phi}}\right]^{1/3}, \quad (71)$$

thus we obtain

$$\frac{d^2 S_k}{d\tau^2_s} + \left(\kappa^2_s + \tau_s\right) S_k = 0, \quad (72)$$

which also belongs to the Airy function. The parametric resonance from this equation can enhance the BEC on this perturbative decay. Thus, the particle number density in the conformal mode of the DM is calculated via

$$\bar{n}_s = \int_{0}^{\infty} \frac{d^3 k_s}{(2\pi)^3} e^{-\pi \frac{k^2_s}{\kappa^2_s}} = \frac{1}{8\pi^3} K^{3/2}_s, \quad (73)$$

which correspond to physical number density $n_s = \frac{1}{8\pi^3} K^{3}_s$. We can substitute the last result to Eq. (70). After $n^s_k$ is obtained, neglecting the expansion of the universe, and assuming that $n^s_k \gg 1$, we can obtain the Boltzmann equation from Eq. (65) for the DM density as

$$n_s \propto \exp \left(\frac{1}{128\pi \sqrt{2}} \sqrt{C \xi} \frac{\lambda_{hs}}{\lambda} \tilde{\phi} \tau_t\right). \quad (74)$$

Given $\tilde{\phi} = \frac{M_p}{\sqrt{3 \pi M_t}}$, the total exponential growth of $n_s$ can be approximated as

$$n_s \propto \exp \left(\frac{1}{128\pi \sqrt{2}} \frac{\lambda_{hs}}{\lambda} \tilde{\phi} \tau_t\right). \quad (75)$$

Before we proceed, if the enhanced decay rate at the end of the quadratic regime is taken into account, which is $\Gamma_{eff}$ (see Eq. (66)), we can compare its result to the Hubble parameter $H$ during the same period. The end of preheating can be evaluated by using relation $\Gamma_{eff} \gtrsim H$. The Hubble parameter can be obtained by using the relation

$$H^2 = \frac{\rho_\phi}{3M^2_p}, \quad (76)$$
where $\rho_\phi$ is evaluated by the energy density during the near end of the quadratic regime, we show that for the preheating to be followed by the quartic regime, the non-minimal coupling should be constrained as

$$\xi \lesssim 4.2, \quad (77)$$

where we have calculated the BEC effect with $n_s$ in Eq. (70) from which is evaluated using the idea that DM density in the late quadratic regime grows as much as scalaron during the beginning of the quadratic regime, thus $n_s/n_\phi \sim 1$.

To calculate the DM particle production due to parametric resonance, we can use Eq. (72). The solution of this equation also belongs to the Airy function. Furthermore, the energy density for each crossing can be calculated similarly with Eq. (56) as

$$\delta \rho_s = \int_0^\infty \frac{dk_s^3}{(2\pi)^3} \sqrt{k_s^2/a^2 + m_s^2 e^{-\pi(k_s/\sqrt{a})^2}} \approx m_s K_s^3 8\pi^3, \quad (78)$$

where the induced mass $m_s$ is evaluated from Eq. (63). Finally, the total energy drain by DM can be calculated similarly to Eq. (60) by

$$\rho_s \sim 6 \times 10^{-13} \sqrt{\xi} M_p^4, \quad (79)$$

Interestingly, only if $\xi \gtrsim 277$ the whole inflaton’s energy could be drained. It means if we used smaller non-minimal coupling, the inflaton’s energy could not be drained by resonance alone. This way, the remaining inflaton’s energy will be drained by perturbative decay. In addition, it will affect the reheating temperature. We will discuss this matter further in the reheating section $V$.

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**FIG. 4.** The plot of $\xi$ with the growth of number density in the logarithmic scale.
The mass of DM can be constrained by using the DM abundance \( \Omega h^2 = 0.12 \). \[ \text{Straightforwardly, we obtain} \]

\[
\Omega h^2 = \frac{1.55 \times 10^{-10}}{\langle \sigma_{ss\to hh} \nu \rangle} \text{GeV}^{-2}.
\] (80)

With \( \langle \sigma_{ss\to hh} \nu \rangle \) corresponding to the annihilation of the DM to 2 Higgs bosons, we can constrain the DM mass to be \( m_s \sim 500 \text{ GeV} \). We expect the DM to be this value can be detected in the near future, such as in the Large Hadron Collider (LHC) or International Linear Collider (ILC).

**IV. PREHEATING IN THE QUARTIC REGIME**

When the inflaton’s field value is critical, the preheating in the quadratic regime is ended. In addition, the action in Jordan’s and Einstein’s frames are indistinguishable. During this time, Higgs plays the main role as the oscillating inflaton field. As the inflaton is mainly caused by the Higgs field, the equation of motion on the self-production Higgs boson can be approximated by

\[
\ddot{h} + 3H \dot{h} + \frac{\partial U}{\partial h} = 0. \tag{81}
\]

The potential in Eq. (81) can be approximated from Eq. (16) by

\[
U(h) \simeq \frac{1}{4} \left( \lambda + \frac{3\xi^2 M^2}{M_p^2} \right) h^4. \tag{82}
\]

Even for the large \( \xi \), the \( \frac{3\xi^2 M^2}{M_p^2} \) is so small compared by \( \lambda \). Thus, the Higgs coupling will solely depend on \( \lambda \). In addition, we named this regime to be *quartic regime* based on this potential. In addition, we can approximate the remaining energy of the inflaton to be

\[
V(h) \simeq \frac{1}{4}\lambda h_{\text{crit}}^4 \approx 5.7 \times 10^{-33} \xi^4 M_p^4,
\]

which was found to be \( \xi \) dependent.

In addition, we can approximate Eq. (81) using the Heisenberg representation similar to Eq. (42) and obtain

\[
\ddot{h}_k + 3H \dot{h}_k + \left( \frac{h_k^2}{a^2} + 3\lambda h^2 \right) h_k = 0. \tag{84}
\]

\[16\] The details of this calculation can be found in Ref. [57].
Defining
\[ \mathbf{h}_k = a h_k \quad \kappa_h^2 \equiv \frac{k_h^2}{\lambda h^2}, \quad a(\tau) = \frac{1}{2\sqrt{3} M_p} \frac{\tilde{h}}{\tau}, \quad \tau \equiv (6\lambda M_p^2/\pi)^{1/4} \sqrt{t}, \]
we can write Eq. (84) in the more straightforward as
\[ \mathbf{h}_k'' + \left( \kappa_h^2 + 3cn^2 \left( \tau, \frac{1}{\sqrt{2}} \right) \right) \mathbf{h}_k = 0, \quad (86) \]
where \( \tilde{h} \) corresponds to the amplitude of the Higgs field, and the prime corresponds to the derivative in respect of conformal time \( \tau \). Also, the solution of \( \mathbf{h}_k(\tau) = \mathbf{h}_k f(\tau) \) is obtained in the same way as Ref. [58]:
\[ f(\tau) = cn \left( \tau, \frac{1}{\sqrt{2}} \right), \quad (87) \]
which is an elliptic cosine function.

In the same way, we want to investigate the number of gauge bosons and fermions produced during this period. To get this, it is important to start with the equation of motion of the \( W \)-boson in a similar way to Eq. (49) in the Heisenberg picture as
\[ \dot{W}_k + 3H W_k + \left( \frac{k_W^2}{a^2} + m_W^2(t) \right) W_k = 0, \quad m_W^2 = \frac{g^2}{4} h^2 \quad (88) \]
The analytical calculation of \( W \)-boson will be different in this regime compared to the quadratic ones. By following the redefinition in Eq. (85) and \( W_k = a W_k \), we arrived at
\[ W_k'' + \left( \kappa_W^2 + \frac{g^2}{4\lambda} cn^2 \left( \tau, \frac{1}{\sqrt{2}} \right) \right) W_k = 0. \quad (89) \]
We can generalize Eq. (86) and (89) so it can be used for any species, which is
\[ \varphi_k'' + \left( \kappa_\varphi^2 + \Upsilon cn^2 \left( \tau, \frac{1}{\sqrt{2}} \right) \right) \varphi_k = 0, \quad (90) \]
where (for instance) \( \Upsilon = 3 \) correspond to the Higgs self-production, \( \Upsilon = \frac{g^2}{4\lambda} \) for gauge bosons, \( \Upsilon = \frac{\nu_\psi^2}{4\lambda} \) for fermions, and \( \Upsilon = \frac{\nu_s^2}{4\lambda} \) for DM.

In the numerical calculation on Eq. (90), we are varying \( \Upsilon \) based on the several standard models (SM) particles and assuming that \( \kappa^2 = 0 \). Mainly, we are testing \( \Upsilon \) with SM particles and the DM (S-field) candidate. Previously, we assumed both \( W \) and \( Z \) bosons are identical to simplify our previous result. However, in our numerical calculation in Fig. [5] on the upper-left side, we made a clear distinction between their couplings. We showed with only slight differences in couplings, both \( W \) and \( Z \) bosons’ production are extremely different. Another
FIG. 5. Plot of varied $\Upsilon$ which refer to different particle productions. The color difference refers to different particles. blue: $W$ boson, orange: $Z$ boson, red: Higgs boson, green: Top quarks, brown: Tauon, black: Bottom quarks, yellow: Charm quarks, and purple: the DM ($S$-field). The values on the right of every plot correspond to the values of $\Upsilon$.

peculiarity happens in $\Upsilon = 1 \sim 2$, particle production with those values is much larger than the $Z$ boson. But since we did not introduce a particle with such couplings, we will not discuss this case further. On the contrary, we also investigate the small coupling’s particle (small $\Upsilon$ indeed), which is not to be considered since their production can be neglected. Please refer to Fig. 5 on the upper-right side. We also investigated the couplings of heavy quarks (in bottom-left Fig. 5), which surprisingly resulted in their small growth of particle productions. Lastly, in Fig. 5 on the bottom-right, we compare the DM production with $Z$ boson production.

The variation of $\kappa_{\phi}$ could enhance or diminish particle production. The details on these conditions can be seen in Ref. [58], from which the instability chart is also presented. But, in this paper, we will not consider such behavior. The analytically preheating in the quartic regime could be ceased around $\sim 10^4$ oscillations. This lead to inefficient preheating. Based on the remained energy density compared with the quadratic regime, we assume the calculation of the reheating temperature would indirectly depend on the preheating process.
V. THE REHEATING TEMPERATURE

The inflation scenario would not be completed without the reheating temperature. In this part, we should look back on the preheating in the quadratic regime. Due to the Pauli exclusion principle, we assumed that instant reheating from the gauge bosons’ decay to lighter fermion particles is suppressed. Thus for $\xi < 4.2$, the produced $Z$-boson by inflaton in the quartic regime would be responsible for producing the necessary reheating temperature by their decay mode. We also noted that we used the $Z \rightarrow b\bar{b}$ as the dominant decay mode.

The conformal mass of the $Z$ boson and its number density, respectively can be written by

$$\tilde{m}_W = \sqrt{\frac{g_W^2}{4\lambda}} f_0 \sin(cf_0\tau) \quad \text{and} \quad \tilde{n}_W = \frac{1}{8\pi^3} \left( \frac{g_W^2}{4\lambda} \right)^{3/2},$$

where $c = \frac{2\pi}{\tau_0}$ and $\tau_0 = 7.416$ and we assumed there is no change in $f_0$. Again, we assumed the $W$ and $Z$ bosons couplings and masses are indistinguishable to simplify the calculation\(^\text{17}\). The energy density of produced $Z$ boson in the first half-crossing can be approximated as

$$\delta \tilde{\rho}_Z = \int_0^{\tau_0/2} d\tau \tilde{\Gamma}_{Z \rightarrow \bar{b}b} \tilde{m}_W \tilde{n}_W e^{\int_0^\tau \tilde{\Gamma}_{Z \rightarrow \bar{b}b} d\tau'},$$

where $\tilde{\Gamma}_{Z \rightarrow \bar{b}b}$ correspond to the decay of $Z$ boson in the conformal mode. The energy density transferred to the light particles is averaged and estimated as

$$\tilde{\rho}_Z = \frac{2\delta \tilde{\rho}_Z}{\tau_0} \tau \simeq 0.04\tau.$$  \hspace{1cm} (92)

If the inflaton’s oscillation energy density in conformal mode is $\tilde{\rho}_h = \frac{1}{4\lambda}$ and $\tilde{\rho}_Z$ are conserved, means $\tilde{\rho}_h = \tilde{\rho}_Z$, it can be translated into physical unit $\rho_h$ as

$$\rho_h = \frac{1}{4\lambda} \left( \frac{\sqrt{\lambda h_{\text{crit}}}}{a} \right)^4 = \frac{\pi^2 g^*}{30} T_R^4,$$  \hspace{1cm} (93)

where $g^* \sim 100$ corresponds to the SM particle’s degrees of freedom. Using Eq. (85), we can obtain the reheating temperature $T_R$ as

$$T_R(\xi<4.2) \sim 1 \times 10^{15}\text{GeV}.$$  \hspace{1cm} (94)

One should note, that the total energy density of the inflaton during $h_{\text{crit}}$ is equal to Eq. (83). However, the total energy for half-oscillation $\rho_Z$ is much higher than the remaining

\(^{17}\) That is why we still used $\tilde{m}_W$ and $\tilde{n}_W$ instead of $\tilde{m}_Z$ and $\tilde{n}_Z$. 

25
inflaton’s energy in Eq. (83). It means the inflaton’s energy could be drained only by a single zero-crossing of the inflaton’s field. Thus, the calculation of the reheating temperature of Eq. (95) must be re-evaluated by using the total remaining energy of Eq. (83). Finally, we obtain the upper bound of the temperature to be

\[ T_{R(\xi<4.2)} \simeq 1.17 \times 10^{10} \xi \text{ GeV}. \] (96)

\( \xi \) only ranges from 1 < \( \xi \) < 4.2, thus the temperature range is still in range of 10^{10} GeV. The calculation of the reheating temperature is rather different from Higgs inflation depicted in Ref. [42], from which the decay of gauge bosons happens instantly after they are produced in the quadratic regime. However, in our case, the decay is delayed quite later in the quartic regime. Consequently, the total energy drain by single crossing is bounded by the total inflaton’s energy at that time\[^{18}\].

In another case, for \( \xi \gtrsim 4.2 \), the inflaton oscillation is stopped before reaching the quartic regime. As the gauge bosons were second to DM when dominating the universe during this time, they decay into relativistic particles playing a role in the reheating temperature. This way, the reheating temperature can be approximated by

\[ T_{R(\xi>4.2)} \approx 0.55 \sqrt{M_p \Gamma_W}, \]

where \( \Gamma_W \) corresponds to the total gauge bosons decays. Thus, the reheating temperature in this model is

\[ T_{R(\xi>4.2)} \sim 1 \times 10^9 \text{ GeV}. \] (97)

The remaining scalaron decays perturbatively to heavy fields for 4.2 < \( \xi \) < 277. Finally, the remaining scalaron density will reach \( \phi_{\text{crit}} = h_{\text{crit}} \) resulting in the gravity ceasing, and the remaining inflaton’s energy only consists of Higgs. The remaining Higgs boson after the preheating is Higgs as the inflaton’s remnant and Higgs as the heavy particles produced during preheating. Both Higgs’ are small in energy density and could not compete with the gauge bosons. Automatically, the reheating temperature is obtained dominantly by the decay of the gauge bosons. This way, the reheating temperature for 4.2 < \( \xi \) < 277 will not be \( \xi \)-dependent. Conversely, for \( \xi > 277 \), the inflaton is fully converted to heavy particles. Thus, the reheating temperature will strongly depend on the gauge bosons’ decay and the decay of some minor particles independent of larger \( \xi \)\[^{19}\].

\[^{18}\] The different approach for reheating temperature in the same model as this paper is presented In Ref. [52] but it was purely evaluated in the quadratic regime.

\[^{19}\] In Ref. [59], the similar \( \xi \)-dependent on the preheating of Higgs inflation was studied. As in our case, Higgs plays the role of the second field, the result of \( \xi \)-dependent is different.
VI. CONCLUSION

We have investigated the Higgs-$R^2$ inflation, which corresponds to this model’s inflation and reheating features. For this paper, we used the effective single-field approximation called minimal two-field mode on the inflaton’s trajectory. With this, the inflaton will follow the direction of such a mode. We investigated the model and showed the turn parameter $\eta^\perp$ is strongly dependent on the Higgs parameters, mainly by the non-minimal coupling $\xi$. The turn parameter could potentially affect the largeness of the non-gaussianity $f_{NL}$. However, in our analytical approximation shown, even with the large $\xi$, this contribution failed to produce a large non-gaussianity. Of course, it is expected for the effective single-field mode. This way, the non-minimal coupling $\xi$ can be regarded as a free parameter since no definite constraint is applied. Thus, the non-minimal coupling can run from small $\xi$ corresponding to the nearly-pure $R^2$ model to large $\xi$ for nearly-pure Higgs inflation.

In the preheating stage, we divided it into two regimes: quadratic regime and quartic regime. Considering the quadratic regime, we found that the gauge bosons dominate particle production. But, they failed to drain the whole inflaton’s energy. Thus, we introduced the DM candidate with a large coupling. We expected this DM candidate could potentially drain the whole inflaton’s energy. During this quadratic regime, it strongly depends on $\xi$ to obtain the different conditions in the preheating stage. If the non-minimal coupling $\xi$ is large, which is $\gtrsim 277$, the resonance production of the inflaton could potentially drain the whole inflaton’s energy density. On the contrary, if the non-minimal coupling $\xi$ is less than 4.2, the resonance is continued to the quartic regime. For the value in between (4.2 < $\xi$ < 277), the inflaton energy is partly converted to heavy particles via resonance, from which the amount is $\xi$ dependent.

We already calculated the reheating temperature by the 3 different ranges of the non-minimal coupling for reheating features. Since we assumed the fermions production is suppressed during the quadratic regime, the resonance production and the perturbative decay of the fermions are strongly suppressed. It means the reheating temperature evaluated by the gauge bosons production to fermions is delayed until the end of the quadratic regime. The reheating temperature when $\xi > 277$ is evaluated by the decay of the gauge bosons produced by the resonance to the fermions at the end of the quadratic regime. The re-

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20 Since we take $\lambda$ as the fixed value in this paper
maining inflaton’s energy existed in $4.2 < \xi < 277$. However, due to the dominance of the
gauge bosons during the quadratic regime, the reheating temperature obtained is almost
similar to when $\xi > 277$. Finally, for $\xi > 4.2$, the reheating temperature is only as large
as $T_{R(\xi>4.2)} \sim 10^9$ GeV. Lastly, for $\xi < 4.2$, the reheating temperature is evaluated by the
decay of the $Z$ boson, produced by resonance in the quartic regime to the fermions. The
reheating temperatures obtained by the last mode are bounded by the remaining inflaton’s
energy during $h_{\text{crit}}$. It would produce a higher temperature as large as $\sim 10^{10}$ GeV.

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