Modelling of Islamic Calendar System based on Moon Phase

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Abstract. The determination of the Islamic calendar system still relies on the Gregorian calendar, so until now, there is no Islamic calendar system prevailing in the international scale. By making function based on synodic period of the Moon, Wada Date (Julian Date version of the Islamic calendar) can be made, and in the end, the Islamic calendar which based purely on the position of the Moon can be reconstructed. The purpose of this modelling is to create an Islamic calendar that has a higher accuracy than the Islamic calendar that we use today. The method that we chose here is the study of literature with the resources that come from articles, books, and internet. As for the analysis of its own data, we chose the numerical analysis method to approach the results of synodic period of the Moon data. From this analysis, we obtained a fact that the graph of synodic period of the Moon turned out to resemble a specific function. Based on the graph from original data, we selected two functions to approach this graph, and we named it the Fourier function and the Moon Phase function. Furthermore, these two functions will be compared to determine which of these functions will be selected. The function that has been selected will be used to make Wada Date which will be used to search for the length of each month in the Islamic calendar. If we have got the length of each month, the last process is to map out the dates for the month to a matrix that would produce an Islamic calendar.

1. Introduction
The Islamic calendar is a lunar calendar, a calendar which is based on cycles of the phase of the Moon, consisting of 12 months in a year of 354 or 355 days. The twelve months in Islamic calendar are Muharram, Safar, Rabiul awal, Rabiul akhir, Jumadil awal, Jumadil akhir, Rajab, Sya’ban, Ramadhan, Syawal, Dzulkaidah, and Dzulhijjah.[6] Each month in the Islamic calendar begins at the time of the monthly "conjunction", called Ijtima’, when the Moon is located on a straight line between the Earth and the Sun. So, the length of each month has either 29 or 30 days, usually in no discernible order. Traditionally, the first day of each month is the day (beginning at sunset) of the first sighting of the hilal (crescent moon) shortly after sunset. If the hilal is not observed immediately after the 29th day of a month, then the day that begins at that sunset is the 30th.

On the other hand, the Gregorian calendar is a solar calendar whose dates indicate the position of Earth on its revolution around the Sun. The length of each calendar month in Gregorian calendar has always remain the same in each year, except in February which length can be 28 or 29 days depends on the year's position in the leap year cycle.[5]

In the world of astronomy, we also know the term of Julian Date (JD). Julian Date is the continuous count of days since the beginning of the Julian Period used primarily by astronomers. JD starts from the
number 0 on 1st of January 4713 BC. The number of JD is incremented by 1 unit per 1 day, thus continuously. Always keep in mind that the numbers of JD always starts at 12:00 pm. It is very important to understand Julian Date because one of its functions is to determine the time of the new moon.

Until today, the determination of the Islamic calendar system still relies on the Gregorian calendar, yet it is clear that both of these calendars have different reference base. Moreover, unlike the Gregorian calendar, there is no Islamic calendar system that applicable on international scale. This causes the difference in the Islamic calendar applied in various regions of the world. By utilizing the length of synodic period of the Moon data in a few years, we can look for a function to create Wada Date (Julian Date version of the Islamic calendar). The purpose is to reconstruct the Islamic calendar system which purely based on the position of the Moon. Furthermore, this Islamic calendar system is expected to replace the past Islamic calendar system that less accurate. It lead us to our goal, so we can determine any important dates in the Islamic calendar more effectively, such as determining the first of Syawal and the first of Ramadhan.

2. The construction of synodic period of the Moon function

The data that we used as reference to construct the function is the list of synodic period of the Moon data that began in September 1924 AD to July 2005 AD in the Gregorian calendar, which has the total amount of 1000 data. Here is the result of the plot of the data, with the $x$-axis as months in Islamic calendar and the $y$-axis as the length of each month.

![Figure 1. Synodic period of the Moon starting from September 1924 AD - July 2005 AD in Gregorian calendar.](image)

If we observed graph of synodic period of the Moon above, it appears that it resembles a function in the form of a Fourier series or the sum of sin function. So, we used the `cftool` menu in MATLAB to compare the results of curve fitting using a Fourier function and the sum of sin function. But before doing curve fitting using `cftool`, the midpoint of the original data is calculated in advance to be transformed to the point $y = 0$. The goal is to make the graphics becoming more similar with the sin function that has the midpoint in the $y = 0$.

From the results of curve fitting which approaching the Fourier function, we obtained a function with $R^2 = 0.8088$. The function is:
\[ f(x) = 29.52776 + \sum_{i=1}^{8} (a_i \cos 0.06439ix + b_i \sin 0.06439ix) \]

with the value of each coefficient are in Table 1.

**Table 1.** The value of the coefficients on the \( f(x) \) function.

| \( i \) | \( a_i \)      | \( b_i \)      |
|-------|---------------|---------------|
| 1     | 0.0002284     | -0.00007051   |
| 2     | 0.0002969     | 0.0001431     |
| 3     | 0.0001086     | 0.0001913     |
| 4     | 0.0001976     | -0.00003377   |
| 5     | 0.000672      | 0.0002167     |
| 6     | 0.0005739     | 0.001125      |
| 7     | -0.009149     | 0.002935      |
| 8     | 0.000672      | 0.0002167     |

On the other hand, from the results of curve fitting which approaching sum of sin function, we obtained a function with \( R^2 = 0.9917 \). The function is:

\[ g(x) = 29.551977 + \sum_{i=1}^{4} a_i \sin(b_i x + c_i) \]

with the value of each coefficient are in Table 2.

**Table 2.** The value of the coefficients on the \( g(x) \) function.

| \( i \) | \( a_i \) | \( b_i \) | \( c_i \) |
|-------|----------|----------|----------|
| 1     | 0.1819   | 0.4506   | -3.537   |
| 2     | 0.08662  | 0.508    | -0.8411  |
| 3     | 0.00001028 | 0.4511 | 2.109    |
| 4     | 0.02179  | 0.000301 | -1.9     |

Since the \( g(x) \) function has a greater value of \( R^2 \) than the \( f(x) \) function, then we selected the \( g(x) \) function as the Moon phase function that will be used later to make Wada Date (WD).

**3. Assumptions used in this modelling**

Before we create Wada Date, we should notice that the data that we used is the synodic period of the Moon in September 1924 AD or coincide with the month of Safar 1343 H in the Islamic calendar. Meanwhile, we want to make the starting point of Wada Date (WD 1) at the start of the Islamic calendar days in accordance with the history, which falls on the 9th of Dzulhijjah 10 H or coincide with the 6th of March 632 M in the Gregorian calendar.[1,6] Therefore, the point \( x \) in the \( g(x) \) function needs to be shifted back so that \( x = 1 \) is the month of Dzulhijjah in the year 10 H. Due to the difference between the month of Dzulhijjah 10 H with Safar 1343 H is 1332 years 2 months, then the point \( x \) need to be moved back 15986 units. Thus, the Moon phase function becomes:

\[ g(x) = 29.551977 + \sum_{i=1}^{4} a_i \sin(b_i(x - 15986) + c_i) \]

with the values of each coefficient as listed in Table 2.
Besides that, based on Danjon Limit, the hilal will be visible if the angular separation (center to center) between Sun and Moon is $7^\circ$. It means, according to Danjon, at least the hilal will be visible 14 hours after Ijtima'. Since the number of days in each month in the Islamic calendar depends on the sighting of the hilal shortly after sunset, then it is assumed that the Ijtima' event occurred on the 29th of each month in the Islamic calendar. If 14 hours after Ijtima' falls before 6:00 pm, then the next day is the 1st day of next month. But if not, then the next day will be the 30th day of that month.

4. The construction of Wada Date (Julian Date version of the Islamic calendar)

We have known that Wada Date will be made to have the same rules with the Julian Date that has been made in advance by previous mathematicians. So the construction of Wada Date can be done by utilizing the Julian Date through the logic as follows:

- WD (1) = 9th of Dzulhijjah 10 H = 6th of March 632 AD.
- To construct Wada Date, we utilized the formula of Julian Date that already existed to build the Julian Date of WD (1). With JDX is the Julian Date of WD (1).

The final goal of this modelling is to reconstruct the Islamic calendar to the more accurate version and depends just to the movement of the Moon only. So, to determine the number of days in each month, it is necessary to find the time difference between two ijtima' point that adjacent to each other. The way that we can use to find it is to convert each of the ijtima' point to the Wada Date that we have build first, so that we can calculate the time difference. The algorithm logic to convert each Ijtima' point to its Wada Date is as follows:

- We assumed that the first Ijtima' was set at 29th of Dzulhijjah 10 H.
- Based on the Moon phase function that has been selected, the function can be interpreted as follows: $g(x) = \text{the function of time difference between the } x^{\text{th}} \text{ Ijtima' with the } (x-1)^{\text{th}} \text{ Ijtima'}.$
- From $g(x)$ above, we searched its cumulative function, $G(x) = \sum_{i=1}^{x} g(i)$.
- Thus, we obtained Wada Date as follows: $WD'(x) = WD$ in $x^{\text{th}}$ Ijtima' = $-8.5 + G(x) + TD$.

The number 8,5 obtained from the initial assumption that WD (1) is 9th of Dzulhijjah 10 H at 12:00 pm. While TD is the time difference between time of the 0-Ijtima' point in the data with time of the 0-Ijtima' point in the function.

Next, we will look for a function to convert Wada Date to the Islamic calendar, so that when we input the $x^{\text{th}}$ Wada Date then we will know if it falls on which day, month, and year in the Islamic calendar. The algorithm logic of these functions are as follows:

- Input $x$ ($x^{\text{th}}$ Wada Date that will be converted to Islamic calendar).
- Find the biggest $n_1^{\text{th}}$ Ijtima' and smallest $n_2^{\text{th}}$ Ijtima' so that $WD'(n_1) < x \leq WD'(n_2)$.
- Find the date of $x$ in Islamic calendar by:
  $$z = \text{date in the Islamic calendar} = 29 - ([WD'(n_2)] - [x]).$$
- Find the month of $x$ in Islamic calendar by:
  $$l = \text{month in the Islamic calendar} = mod(n_2, 12) - 1.$$
- Find the year of $x$ in Islamic calendar by:
  $$h = \text{year in the Islamic calendar} = \left\lfloor \frac{n_2+11}{12} \right\rfloor + 10.$$

The last stage is to implement the algorithm logic to reconstruct the Islamic calendar as follows:

- Build matrix $A := \{'\text{Monday}' , '\text{Tuesday}' , .. , '\text{Sunday}'\}$.
- Build matrix $B := 6 \times 7$ sized matrix.
- Map each 29th in every month of the following ways:
\[ g = \text{floor}(\text{mod}(WD'(n_1) + 4), 7) \]

and then map: \( 29 \mapsto B(1, g) \).

- Find the length of each month by:
  \[ t = \lfloor WD'(n_2) \rfloor - \lfloor WD'(n_1) \rfloor. \]

- Map the 30th of the same month or the 1st of next month by:
  
  If \( t = 30 \)
  
  If \( g + 1 = 8 \) \( \Rightarrow B(3,1) = 30 \).
  Otherwise \( B(1, g + 1) = 30 \).

  If \( t = 1 \),
  
  If \( g + 1 = 8 \) \( \Rightarrow B(3,1) = 1 \).
  Otherwise \( B(1, g + 1) = 1 \).

- Fill matrix \( B \) after the number 29 or 30 with the numbers 1 to 29.

- Combining matrices \( A \) and \( B \).

5. The error generated in the Islamic calendar
As previously described above that the value of \( R^2 \) at the Moon phase function created is 0.9917. It means that there is still an error between the length of each month, or in other words the number of days, in the Islamic calendar created with the original length of each month in the Islamic calendar. However, an error on this monthly basis is not an error that directly visible on the Islamic calendar created. Errors that occur on each month in the Islamic calendar are the sum of all errors in previous months. For example, suppose if we want to know the result of error from the Moon phase function when Ramadhan 1400 H. Then the error on this month in the Islamic calendar is the result of the sum of all errors in each month before Ramadhan 1400 H.

After the program created in MATLAB to calculate the error that will be inflicted on a monthly basis, it is known that the function \( g(x) \) only give a maximum error of ±0.21 days, or approximately 5 hours per month. It is caused by the result of the plot of errors in each month that resembles the sin function which has a fixed intervals.

References
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