F-theory inspired GUTs with extra charged matter

J. Pawełczyk

Institute of Theoretical Physics, University of Warsaw,
ul. Hoża 69, 00-681 Warsaw, Poland

Abstract

We consider GUT models inspired by recent local F-theory constructions. We show that after switching on vevs to scalars the extra matter becomes messengers. We discuss conditions on these vevs under which the models do not lead to unacceptable baryon/lepton number violating processes.
1 Introduction

GUT models rise hope for better unification for long time [1]. The basic arguments supporting the idea are twofold: all known matter is organized in SU(5) multiplets and coupling constants seem to unify at some scale. It appears that SUSY GUTs provide better coupling unification and shift the unification scale $\Lambda_{GUT}$ to values acceptable for the proton stability under heavy gauge bosons exchange [2, 3]. Besides these successes there still remains troublesome problems such as the origin of the doublet-triplet splitting i.e. phenomenon of absence of a full GUT representation for Higgses of MSSM and the suppression mechanism of the baryon/lepton (B/L) number violating processes [4]. Strong suppression of these processes is a one of the crucial test for the candidate GUT. For the elimination of dangerous dimension four operators it is enough to impose e.g. R-parity - an extra symmetry of unknown origin. Suppression of higher-dimensional operates requires extra structure e.g. more symmetries [5].

In models constructed recently within the realm of F-theory unification [6] all known particle physics (excluding gravity effects) come from a single $E_8$ F-theory singularity [7]. The models realize SU(5) GUTs with some extra global U(1)’s originating from $SU(5)_{\perp}/\Gamma$ where $\Gamma$ is so-called monodromy group. They include SUSY breaking sector and its mediation through gauge forces (GMSB), the doublet-triplet splitting is achieved in a novel way by introduction of background fluxes on matter curves of the compact CY space. For some $\Gamma$’s the R-parity is a subgroup of the global U(1)’s. Moreover the global symmetries forbid dimension five B/L breaking operators. In realistic models these symmetries must be spontaneously broken because messengers masses are provided by vevs of certain scalars. In consequence this may lead at low energies to generation of dangerous B/L effective operators. Besides most of the models of [7] contain an extra charged matter which role is unclear at first sight.

The purpose of this letter is to discuss all Dirac scenarios F-theory GUTs from [7]. We shall show that switching on vevs for charged scalars the extra charged matter can be interpreted as messengers and under certain conditions on these vevs B/L breaking operators generated at low energies are strongly suppressed. All model possesses a bunch of extra neutral scalars which safely can be assumed to be very massive and decouple.
2 The $\mathbb{Z}_3/S_3$ models

We start with short description of the model $\mathbb{Z}_3$ ($S_3$ model is just simple reduction of the latter). Details are in [7]. The matter content is summarized in the presented table. We must recall that F-theory case the effective Lagrangian contains all the invariant coupling including Yukawas and trilinear terms in Kahler potential (divided by the GUT scale denoted here by $\Lambda_{GUT}$).

|        | Minimal | Extra |
|--------|--------|-------|
|        | $10_M$, $Y_{10}$ | $\tilde{5}_M$, $\tilde{5}_H$, $\tilde{5}_H$, $Y_{10}^a$, $X$, $N$ |
| $U(1)_{PQ}$ | $+1$, $+1$, $-2$, $-2$, $+3$, $-4$, $-3$ | $0$, $+1$, $+3$, $+1$, $-1$ |
| $U(1)_X$ | $-1$, $+3$, $+2$, $-2$, $+1$, $0$, $-5$ | $+4$, $+3$, $-3$, $-5$, $+5$ |

It must be stressed that the chirality of the spectrum of the model has origin in non–trivial F-theory fluxes through 2d-cycles where the matter is localized. Manipulating fluxes results in different matter content. We shall use this freedom in the paper. Existence of such fluxes and cycles is a global issue which has not been resolved yet. Following [7] we shall assume that the appropriate global construction exists.

The model has two extra global U(1) symmetries which are in fact remnants of the "anomalous" gauge symmetries. They provide selection rules for possible GUT invariants. It is easy to see that R-parity is subgroup of above: $R = (-1)^q$, where $q = Q_{PQ}$ or $q = Q_X$. We shall slightly modify the matter content compared to the original paper in order to cure SU(5) anomaly. The simplest modification is just addition of one extra $Y_{10}^a$ (both fields will be denoted by $Y_{10}^a$, $a = 1, 2$).

The model contains standard matter $10_M, \tilde{5}_M$ as well as appropriate Higgses $5_H, \tilde{5}_H$. These couple to matter in the conventional way

$$W \supset 10_M^2 5_H, \ 10_M \tilde{5}_M \tilde{5}_H$$

(1)

Recall that color triplets of Higgs fields get mass thought appropriate hypercharge background flux. Their masses are assumed to be of the order $\Lambda_{GUT}$. This will be discussed later. Thus 5-dimensional representations of Higgses split into light doublets and heavy triplets. We shall use somehow hybrid notation $5_H = (5_H)_2 + 3_H$, $\tilde{5}_H = (\tilde{5}_H)_2 + 3_H$. Of course $(5_H)_2 = H_u$, $(\tilde{5}_H)_2 = H_d$ of MSSM.

There is a scalar $X$ which receives non–zero vev $X = \langle X \rangle + \theta^2 \langle F_X \rangle$ and breaks SUSY. For the discussion of the potential for scalars including $X$ see App[A] Trilinear couplings

$$W \supset f_d Y_{10}^a Y_{10} X, \ Y_{\tilde{5}}^a Y_5 X$$

(2)
through $\langle X \rangle$ provides masses for messengers $Y_{10}, Y_5, Y_\tilde{5}$ and one linear combination $f_a Y_{10}^a$ hereafter called $Y_{10}$. The Kahler potential term $X^\dagger 5_H \tilde{5}_H / \Lambda_{\text{GUT}}$ produces $\mu$-term $\mu = \langle F_X \rangle / \Lambda_{\text{GUT}}$.

What about the extra charged $10_{(1)}$? It appears that the model has the following coupling

$$W \supset g_a Y_{10}^a 10_{(1)} N$$

We decompose $g_a Y_{10}^a$ into the $Y_{10}$ and a new field $\overline{10}_{(1)}$ i.e. $g_a Y_{10}^a = (Y_{10}, \overline{10}_{(1)})$ thus provides mass of the order $\langle N \rangle$ to the pair $\{ \overline{10}_{(1)}, 10_{(1)} \}$ thus turning the fields into extra messengers plus some mixing of the order $N/D$ between $Y_{10}$ and $10_{(10)}$.

Thus it seems that turning on vev of scalars we just obtain standard GUT model with minimal messenger sector. One must be careful though. It is apparent that switching on vevs for $X$ and $N$ scalars breaks $R$-parity what in consequence may lead to baryon/lepton violating processes. It is clear that the smaller are these vevs the smaller amount of violation one could expect. On the other hand the vevs provide masses for messengers thus there are natural lower bound for their values. Because we are going to work with an effective action below the GUT scale $\Lambda_{\text{GUT}}$ we assume that all vevs are much smaller than this scale. Thus we introduce small parameters: $x = \langle X \rangle / \Lambda_{\text{GUT}}$, $n = \langle N \rangle / \Lambda_{\text{GUT}}$, $d = \langle D_{(1)} \rangle / \Lambda_{\text{GUT}}$ (or $d = \langle \overline{D}_{(1)} \rangle / \Lambda_{\text{GUT}}$ in the second version of the model).

The possible form of the potential for the scalars and its properties including minima and masses are discussed in App. A.

2.1 B/L violation

In the rest of the paper we shall discuss effects of switching on vevs of the charged scalars $X, N, D$ i.e. vevs of both bosonic components of the chiral superfields e.g. for $X : \langle X \rangle + \theta^2 \langle F_X \rangle$, etc. This will break both U(1) symmetries spontaneously thus also the $R$-parity. In consequence it may lead to dangerous processes violating lepton/baryon numbers. We are going to discuss these issues in the following section.

The primary result of non–trivial vevs of $X, N, D$ is mixing between fields of different $Q_{PQ}, Q_\chi$ charges. The mixing may directly lead to B/L violation. Let us write down all trilinear coupling between fields charges under GUT group:

$$10 \times 10 \times 5$$

\[ (10_M, Y_{10})^2 5_H, (10_M, Y_{10}) 10_{(1)} (5_M, Y_{\tilde{5}}), (10_M, Y_{10}) Y_{10}^a \tilde{5}_H, 10_{(1)} 10_{10}^\dagger 5_\tilde{5} \]

\[ (10_M, Y_{10})^2 5_H, (10_M, Y_{10}) 10_{(1)} (5_M, Y_{\tilde{5}}), (10_M, Y_{10}) Y_{10}^a \tilde{5}_H, 10_{(1)} 10_{10}^\dagger 5_\tilde{5} \]

\[ (10_M, Y_{10})^2 5_H, (10_M, Y_{10}) 10_{(1)} (5_M, Y_{\tilde{5}}), (10_M, Y_{10}) Y_{10}^a \tilde{5}_H, 10_{(1)} 10_{10}^\dagger 5_\tilde{5} \]

1In order not to proliferate coefficients we denote as $(A, B)$ any linear combination of the fields $A, B$ with coefficients of the order 1. Below we have suppressed obvious conjugate expressions.
10 × 5 × 5 singlets:

\[(10_M, Y_{10})(\bar{5}_M, Y_{\bar{5}})\bar{5}_H, (10_M, Y_{10})Y^\dagger_5\bar{5}_H, Y^\dagger_{\bar{10}}(\bar{5}_M, Y_{\bar{5}})\bar{5}_H, 10_{(1)}\bar{5}_H\bar{5}_H\]  

(5)

Scanning the above one sees that mixing of \(\bar{5}_H\) with \(\bar{5}_M\) and \(10_{(1)}\) with \(10_M\) would lead to B/L violation linear in vev of scalars through \(10_M\bar{5}_M\) vertex of the superpotential \(W\) or \(10^2_M\bar{5}_H\) vertex of Kahler potential the latter being suppressed by the scale \(\Lambda_{GUT}\).

Short inspection of the model with \(D_{(1)}\) reveals existence of the following term

\[W \supset \bar{5}_M5_H\]  

(6)

After Higgs triplets are decoupled (see the next paragraph) and \(X\), \(D_{(1)}\) receive vevs we obtain

\[W \supset (\mu\bar{5}_H + \langle D_{(1)}\rangle\bar{5}_M)_25_H\]  

(7)

with \(\mu = F_X/\Lambda_{GUT}\). This can be put into canonical form \(\mu'\bar{5}_H5_H\) \((\mu' = \mu^2 + \langle D_{(1)}\rangle^2)\) by a rotation: \(\bar{5}_H \rightarrow (\mu\bar{5}_H - \langle D_{(1)}\rangle\bar{5}_M)/\mu'\). In consequence \([1]\) produces lepton/baryon number violation vertex

\[y\frac{\langle D_{(1)}\rangle}{\mu'}10_M\bar{5}_M(\bar{5}_M)_2\]  

(8)

where we have restored the Yukawa coupling \(y\) and the subscript 2 means that we keep only the MSSM doublet piece. The r.h.s. of the above contains R-parity breaking operators \(\bar{E}LL, QLD\) (but not \(\bar{U}DD\) which couplings are sometimes named \(\lambda, \lambda'\) [15]. The current limits on \(\lambda's\) taken from [16] imply that acceptable values of \(\langle D_{(1)}\rangle/\mu\) are smaller than \(10^{-6}\). But the analysis of our potential for the scalars shows that generically \(\langle N \rangle \sim \langle D_{(1)}\rangle\). If we recall that \(\langle N \rangle\) sets the mass of messengers we immediately conclude that the model is in conflict with phenomenology.

The model can be easily cured assuming that the fluxes through matter curves are such that the spectrum contains \(\bar{D}_{(1)}\) with opposite U(1) charges \((Q_{PQ} = -1, Q_\chi = +5)\). If so then instead of (6) we have

\[K \supset \frac{1}{\Lambda_{GUT}}5_M\bar{5}_H\bar{D}_{(1)}^\dagger\]  

(9)

This produces mixing with Higgs proportional to F-term of the superfield \(\bar{D}_{(1)}\) (which we will denote by \(F_D\)). Hence \(\langle D_{(1)}\rangle\) is replaced by \(\mu_D \equiv F_D/\Lambda_{GUT}\) so it is enough that \(\mu_D \ll 10^{-6}\mu\) to be in accord with phenomenology. Recalling that \(\mu = F_X/\Lambda_{GUT}\) we obtain

\[F_D < 10^{-6}F_X\]  

(10)

\(2(\bar{5}_M)_2\) denotes doublet of SU(2) inside \(\bar{5}_M\) and similar for \(\bar{5}_H\).
what is reasonable requirement. The rotation of the Higgs due to (6) is
\[ (\bar{5}_H)_2 \rightarrow (\bar{5}_H)_2 - \frac{F_D}{F_X} (\bar{5}_M)_2 \] (11)
From now on we are going to focus on this version of the model (11).

At this point let us discuss at some length the influence of Higgs color triplets. Their mass term is
\[ M_3 \bar{3}_H^3 + M' \bar{3}_H^3 + \mu \bar{3}_H^3 + X \bar{3}_H^3 \] (12)
where tilde fields are appropriate KK modes of F-theory compactification. Due to their charge they may couple to \( X \) too. Of course the latter will obtain vev: \( X \rightarrow \langle X \rangle \). \( M \) and \( M' \) are masses of the order \( \Lambda_{\text{GUT}} \). Adding the mixing (9) and diagonalizing the mass term we get rotation
\[ \bar{3}_H \rightarrow \bar{3}_H - \frac{\mu_D X}{M^2} (\bar{5}_M)_3 \] (13)
and the effective B violating vertices
\[ - \frac{\mu_D X}{M^2} \langle U \bar{U} D, QL \rangle. \] (14)
With \( \langle X \rangle \sim 10^{-2} \Lambda_{\text{GUT}}, \mu_D \ll 10^{-6} \sim 10^{-20} \Lambda_{\text{GUT}}, M \sim \Lambda_{\text{GUT}} \) the suppression factor \( \ll 10^{-22} \) is in agreement with phenomenology (see also [4] Table 2.).

This ends discussion of dimension 4 B/L–violating vertices which may appear in the model discussed.

### 2.2 Higher–dimensional operators

Here we are going to look for possible higher dimension B/L breaking operators. There is one dangerous dimension 5 operator in superpotential invariant under SU(5): \( 10^3_M \bar{5}_M \) which includes two dangerous MSSM operators: \( QQQL, \bar{U} \bar{U} \bar{E} \bar{D} \). The operator is invariant under \( U(1)_{\chi} \) but not under \( U(1)_{PQ} \). The possible dimension 6 operators are numerous and they may correct the superpotential as well as the Kahler potential. The primary source of these operators are exchange of heavy states with appropriate group structure. The universal contribution comes form heavy GUT gauge fields and their KK modes - these where discusses in e.g. [18] and they contribute to the Kahler potential only. The exchange of heavy color Higgses is strongly suppressed: the reasoning goes in

---

3Below we use the standard notation for MSSM models where \( \bar{D} \) denotes chiral superfield containing the down quark. I hope the reader will not confuse it with the scalar \( \bar{D}_{(1)} \).
similar way as presented in the previous section (see also [18]). The Higgs doublets gives no effects.

The remaining possibility are diagrams with exchange of messengers. Here we shall show that dimension 5 operator is not produces and that the only nonvanishing contribution is a dimension 6 correction to the Kahler potential with a very small coefficient.

Hence one has to find out all operators of the form $MM'Y$ where $M$’s denote matter fields and $Y$ a messenger. These operators appear as a result of mixing discussed in the previous section. We shall be interested in operators arising from single redefinition because each redefinition is accompanied by small factor of the order $n \equiv \langle N \rangle / \Lambda_{GUT}$, $d \equiv \langle D(1) \rangle / \Lambda_{GUT}$ or $F_D/F_X$ (see above).

Let us discuss the remaining (besides (11)) mixings between fields. The couplings of interest are

$$ W \ni 5_H Y_5 \langle D(1) \rangle, \mu 5_H 5_H, \langle X \rangle Y_5 Y_5, \mu_D Y_5 Y_5 $$

$$ K \ni (k1) : \frac{N}{\Lambda_{GUT}} (5_M, Y_5)^5 \, \bar{5}_H, \, (k2) : \frac{D(1)}{\Lambda_{GUT}} (10_M, Y_{10})^+ 10_{(1)}. $$

where $\mu = F_X/\Lambda_{GUT}$, $\mu_D = F_D/\Lambda_{GUT}$. Redefining $5_H \to 5_H - \frac{N}{\Lambda_{GUT}} (5_M, Y_5)$ one can get rid of (k1) in the expense of $-|\kappa_1|^2$. The latter can be completely removed when $N \to \langle N \rangle$ (what is assumed hereafter) redefining kinetic terms for $5_M$, $Y_5$. This results in $5_M \to 5_M + |n|^2 Y_5$ and small rescaling of $Y_5$ both irrelevant for our analysis. Furthermore the rotation: $5_H \to 5_H + n 5_M$, $5_M \to 5_M - n 5_H$ adds up to: $5_H \to 5_H - n Y_5$, $5_M \to 5_M - n 5_H$. Next we diagonalize the mass terms (15) which lead to irrelevant mixing between Higgses and messengers. Hence we are going to ignore these contributions. Similarly treatment of (k2) gives

$$ 10_{(1)} \to 10_{(1)} - d Y_{10}, \, 10_M \to 10_M - d 10_{(1)} \quad (17) $$

Scrutinizing (11) one finds the following MM’Y couplings

$$ \kappa_1 10_M 10_{(1)} 5_M \, \bar{5}_M, \, \kappa_2 10_{(1)} Y_{10} \, (5_M)_{2} \quad (18) $$

where $\kappa_1 = 1/\Lambda_{GUT}$, $\kappa_2 = F_D/(F_X \Lambda_{GUT})$ while from (5) one obtains

$$ y^{(1)}_1 Y_{10} 5_M (5_M)_{2}, \, y^{(1)}_5 10_M 5_M Y_5 \quad (19) $$

$^4 Y_5 5_H D(1)$ has negligible effect.

$^5$ We ignored here subleading terms from Eqs. (11,13).
where \( y_{10}^{(1)} = F_D/F_X \), \( y_5^{(1)} = \max(F_D/F_X, n) \). Integrating over the messengers \( Y_{10} \) we obtain only a single dimension 6 operator. Suppressing the family indices it has the form

\[
\delta K \supset y_1 y_2 \left( \frac{(F_D/F_X)^2 M_{PL}}{\Lambda_{GUT}^2} \right) 10_M \bar{5}_M (\bar{5}_M)_2 + c.c.
\] 

(20)

where \( y_i \) are the \( i \)-th family Yukawas. The above contains such B/L breaking MSSM operators as \( Q^I \bar{D}^2 L \). The effective coupling constant is of the order \( y_1 y_2 10^{-8}/\Lambda_{GUT}^2 \).

Taking into account that the Yukawa couplings for the first family can be as small as \( y_1 \sim 10^{-5} \) we obtain enormous suppression.

### 3 \( \mathbb{Z}_2/\mathbb{Z}_2 \times \mathbb{Z}_2 \) model

Here we are going to discuss the \( \mathbb{Z}_2 \) model of [7]. The \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) model is a reduced version thereof thus our analysis will work also in this case. The minimal matter is the same as in \( \mathbb{Z}_3/S_3 \) model of the previous section and it will not be displayed here. The possible extra matter is presented in the table. We are going to shorten the discussion here to issues related to that extra matter.

We shall denote the minimal \( \bar{5}_H \) as \( \bar{5}_H^1 \) and \( \bar{5}_H^{(1)} \) as \( \bar{5}_H^2 \).

| Extra     | 10(1) | \( \bar{5}_H^2 \) | \( \bar{5}_H^{(1)} \) | 5(3) | D(2) | D(4) |
|------------|-------|-------------------|---------------------|------|------|------|
| \( U(1)_{PQ} \) | +4    | -2                | +5                  | +6   | +4   | -7   |
| \( U(1)_\chi \) | +4    | -2                | +3                  | -2   | 0    | -5   |

(21)

The apparent differences lie in the distribution of charges among the extra matter 10(1) and in the sector of 5’s.

First one must notice that there is no way one can give mass to \( \bar{5}_H^{(2)} \) without serious distraction done for the minimal sector. Thus we assume the field is absent from the spectrum. Similarly we remove cumbersome \( D_2(2) \) which could form a mass term with \( X \). We guess the extra pair of 5’s will become messengers. The relevant couplings producing mass term are

\[
W \supset 5(3) \bar{5}_H D_{(4)} + f a \bar{5}_H X
\]

(22)

With obvious definition of \( \alpha \) the messenger is \( Y_5^{(3)} \sim \cos \alpha (f a \bar{5}_H) + \sin \alpha \bar{5}_M \). The light \( \bar{5}_M \) ’s are \( \bar{5}_M^1 \sim \cos \alpha \bar{5}_M - \sin \alpha (f a \bar{5}_H) \) and \( \bar{5}_M^2 \sim \epsilon_{ab} f a \bar{5}_H^b \). We expect that \( \cos \alpha \approx 1 \) i.e. the matter \( \bar{5}_M \) field will not vary much during the process of redefinition. To define physical Higgs and matter \( \bar{5}_M \) ’s we need take into account the only trilinear coupling in the Kahler potential

\[
\bar{5}_H (\bar{5}_H^1, \bar{5}_H^2) X^\dagger \sim 5_H (-\sin \alpha \bar{5}_M^1, \bar{5}_M^2) X^\dagger
\]

(23)
This finally defines MSSM Higgs $\bar{5}_H^f = (\sin \alpha \bar{5}_M^f, \bar{5}_M^f) \approx (\sin \alpha \bar{5}_M^f, \bar{5}_M^f)$.

As in the previous section the mixing between fields may generate dangerous B/L-violating vertices. Below we estimate the coupling constant of the leading dimension 4 operator. The operator of interest originates from $(\bar{5}_H^1, \bar{5}_H^2)_{10}M\bar{5}_M^f$ producing: $\sin \alpha \bar{5}_M^f M\bar{5}_M^f$

what gives

$$\frac{\langle D_{(4)} \rangle}{\langle X \rangle} \ll 10^{-6}$$

We expect that the analysis of higher-dimensional operators will give negligible B/L-violating effects.

It is easy to see that the new $10_{(1)}$ is the messenger coupled to $Y_{a \ 10}$ and $D_{(4)}$ thus receiving mass when the scalar $D_{(4)}$ acquire a vev.

4 Conclusions

The discussion presented shows that the F-theory GUT models of \cite{7} seem to by phenomenologically viable after small (but sensible from the point of view of F-theory) modifications i.e. at low energies they give MSSM with some extra sterile scalars and broken SUSY. Apparent lack of R-parity spontaneously broken just below the GUT scale does not lead to dangerous B/L breaking processes under some conditions put on scalar vevs. Of course the simple analysis presented in this paper does not say anything about such important issue as FCNC, dark matter candidates, soft-SUSY breaking terms and more. This would require deeper studies which go beyond this letter.
Appendix: scalar potential

Here we shall discuss the potential for the scalars leading to SUSY breaking [8, 9, 14]. We focus on the $Z_3$ model as the discussion for the $Z_2$ model would be very similar. According to the results of Sec.2.1 we must work with the version of (1) with $\mathcal{D}(1)$ field. Also it is necessary that in a global setting there will be instantons generating Polonyi terms for all the scalars [12]. Gauge invariance forces the Polonyi terms to be accompanied by appropriate closed string modes (denoted hereafter by $t$) which we choose here to be twisted moduli [10]. One could consider untwisted moduli too but then in order to achieve viable vacua one needs to generate FI-term [17] as in [13]. We shall not work out this possibility because this section serves merely as the illustration of the SUSY breaking generation mechanism.

\[ W = W_0 + f_X e^{-\tau_{PQ}} X + f_N e^{-\frac{3}{4}\tau_{PQ}} N + f_D e^{-\frac{1}{4}\tau_{PQ} + \tau_{\mathcal{D}(1)}} \] (25)

The Kahler potential except the standard piece $K_0$ gets contribution from the trilinear coupling $X^+\mathcal{D}(1)N$ as well as corrections due to the exchange of the anomalous U(1) gauge bosons.

\[ K = K_0 + \frac{1}{\Lambda_{GUT}} (X\mathcal{D}(1)^+N^+ + X^+\mathcal{D}(1)N) \]
\[ - \frac{g^2}{4\Lambda_{GUT}^2} (|X|^2 + \frac{3}{4}|N|^2 + \frac{1}{4}|\mathcal{D}(1)|^2)^2 + (|N|^2 - |\mathcal{D}(1)|^2)^2 \] (26)

Finally there are D-terms

\[ D_{PQ} = |X|^2 + \frac{3}{4}|N|^2 + \frac{1}{4}|\mathcal{D}(1)|^2 + \lambda^2 (t_{PQ} + \bar{t}_{PQ}) \] (27)
\[ D_X = |N|^2 - |\mathcal{D}(1)|^2 + \lambda^2 (t_X + \bar{t}_X) \] (28)

where $\lambda$ is the mass scale characterizing the anomalous massive gauge bosons. We expect $\lambda$ to be close to $\Lambda_{GUT}$. With $l \equiv \Lambda_{GUT}/\lambda$ the extreme of the potential for the scalars are

\[ \langle X \rangle = \frac{2}{g^2 + l^2} w_0 \Lambda_{GUT}^2 \]
\[ \langle N \rangle = \alpha_1 \frac{f_D}{f_X} \Lambda_{GUT} + \alpha_2 \frac{f_N}{f_X} w_0 \Lambda_{GUT}^2 \] (29)
\[ \langle \mathcal{D}(1) \rangle = \alpha_3 \frac{f_N}{f_X} \Lambda_{GUT} + \alpha_4 \frac{f_D}{f_X} w_0 \Lambda_{GUT}^2 \]

where $w_0 = W_0/f_X$ and coefficients $\alpha_i$ are of the order one.\[^6\] Consistency of the calculations require that $\langle X \rangle, \langle N \rangle, \langle \mathcal{D}(1) \rangle \ll \Lambda_{GUT}$ thus we need $f_N, f_D \ll f_X$. Notice that

\[^6\] Explicitly

\[ \frac{8}{3g^2 + 6l^2 + 8} \frac{2}{g^2 + l^2} \frac{2}{g^2 + 2l^2 + 8} \frac{16}{3g^2 + 6l^2 + 8} \]

\[ \frac{72g^2 + 5g^4 + 96l^2 + 18g^2l^2 + 16l^4}{(g^2 + l^2)(g^2 + 2l^2 + 8)(3g^2 + 6l^2 + 8)} \]
generically vevs of $N$, $\langle D_{(1)} \rangle$ are related: none of them vanish without fine tuning. The phenomenological constraint (10) $F_D < 10^{-6} F_X$ implies $f_D < 10^{-6} f_X$ thus also $\langle N \rangle \ll \langle X \rangle $. The value of $f_N$ is unconstraint thus also $\langle D_{(1)} \rangle$. The contributions to gauginos masses are $\frac{g^2}{16\pi^2} f_X / \langle X \rangle$ and $\frac{g^2}{16\pi^2} f_N / \langle N \rangle$. Due to smallness of $f_N / f_X$ we can neglect it in (30) obtaining $f_N / \langle N \rangle \sim f_X / \Lambda_{GUT} \sim f_X / \langle X \rangle$ thus enhancing the GMSB mechanism. All scalars have similar masses for $f_X \gg f_N$

$$m^2_X = g^2 \frac{f_X^2}{\Lambda_{GUT}^2}, 
\quad m^2_N = (8 + 3g^2) \frac{f_X^2}{8\Lambda_{GUT}^2}, 
\quad m^2_D = (8 + g^2) \frac{f_X^2}{8\Lambda_{GUT}^2}$$

With $f_x \sim 10^{-18} M_{Pl}$, $g \sim 0.3$ and $\Lambda_{GUT} = 10^{-2} M_{Pl}$ one gets $m_{N,D} \sim 100$ GeV, $m_X \sim 30$ GeV.

$$\frac{8}{g^2 + 2l^2 + 8} \frac{2(88g^2 + 21g^4 + 96l^2 + 66g^2l^2 + 48l^4)}{(g^2 + l^2)(g^2 + 2l^2 + 8)(3g^2 + 6l^2 + 8)}$$
Acknowledgments

The author would like to acknowledge stimulating discussions with Emilian Dudas, Jonathan Heckman, Tomasz Jeliński, Pran Nath, Stefan Pokorski, Tomasz Taylor and Krzysztof Turzyński. This work was partially supported by the EC 6th Framework Programme MRTN-CT-2006-035863 , TOK Project MTKD-CT-2005-029466 and Polish Ministry of Science MNiSW grant under contract N N202 091839 (2010-2013).

References

[1] P. Langacker, “Grand Unified Theories And Proton Decay,” Phys. Rept. 72 (1981) 185.

[2] C. Kounnas, A. B. Lahanas, D. V. Nanopoulos and M. Quiros, Nucl. Phys. B 236 (1984) 438.

[3] D. V. Nanopoulos, Phys. Rept. 105 (1984) 71.

[4] P. Nath and P. Fileviez Perez, Phys. Rept. 441 (2007) 191 [arXiv:hep-ph/0601023].

[5] S. Forste, H. P. Nilles, S. Ramos-Sanchez and P. K. S. Vaudrevange, [arXiv:1007.3915 [hep-ph]].

[6] R. Donagi, M. Wijnholt, [arXiv:0802.2969 [hep-th]]; C. Beasley, J.J. Heckman, C. Vafa, JHEP 0901:058 (2009), [arXiv:0802.3391 [hep-th]];
   H. Hayashi et al., Nucl. Phys. B 806:224 (2009), [arXiv:0805.1057 [hep-th]];
   C. Beasley, JHEP 0901:059 (2009), [arXiv:0806.0102 [hep-th]];
   R. Donagi, M. Wijnholt, [arXiv:0808.2223 [hep-th]].

[7] J.J. Heckman, A. Tavanfar, C. Vafa, [arXiv:0906.0581 [hep-th]].

[8] M. Ibe and R. Kitano, “Sweet Spot Supersymmetry,” JHEP 0708 (2007) 016 [arXiv:0705.3686 [hep-ph]].

[9] Y. Nomura and M. Papucci, “A Simple and Realistic Model of Supersymmetry Breaking,” Phys. Lett. B 661 (2008) 145 [arXiv:0709.4060 [hep-ph]].

[10] T. Jeliński, Z. Lalak and J. Pawelczyk, Phys. Lett. B 689 (2010) 186 [arXiv:0912.3735 [hep-ph]].
[11] E. Dudas, Y. Mambrini, S. Pokorski, A. Romagnoni and M. Trapletti, “Gauge vs. Gravity mediation in models with anomalous U(1)'s,” JHEP 0903 (2009) 011 [arXiv:0809.5064 [hep-th]].

[12] E. Witten, “Non-Perturbative Superpotentials In String Theory,” Nucl. Phys. B 474 (1996) 343 [arXiv:hep-th/9604030];
O. Aharony, S. Kachru and E. Silverstein, “Simple Stringy Dynamical SUSY Breaking,” Phys. Rev. D 76 (2007) 126009 [arXiv:0708.0493 [hep-th]];
L. E. Ibanez and A. M. Uranga, “Instanton Induced Open String Superpotentials and Branes at JHEP 0802 (2008) 103 [arXiv:0711.1316 [hep-th]];
T. Jeliński, J. Pawełczyk, “Multi-Instanton Corrections to Superpotentials in Type II Compactifications”, [arXiv:0810.4369 [hep-th]], Int. Jour. Mod. Phys. 24: 4671-4684, 2009.

[13] J. J. Heckman and C. Vafa, “F-theory, GUTs, and the Weak Scale,” JHEP 0909 (2009) 079 [arXiv:0809.1098 [hep-th]];
J. Marsano, N. Saulina and S. Schäfer-Nameki, “Gauge Mediation in F-Theory GUT Models,” Phys. Rev. D 80 (2009) 046006 [arXiv:0808.1571 [hep-th]].

[14] R. Kitano, “Gravitational gauge mediation,” Phys. Lett. B 641 (2006) 203 [arXiv:hep-ph/0607090].

[15] H. K. Dreiner, [arXiv:hep-ph/9707435].

[16] H. K. Dreiner, M. Hanussek and S. Grab, [ arXiv:1005.3309 [hep-ph]].

[17] Z. Komargodski and N. Seiberg, “Comments on the Fayet-Iliopoulos Term in Field Theory and Supergravity,” JHEP 0906 (2009) 007 [arXiv:0904.1159 [hep-th]].

[18] J. J. Heckman, [arXiv:1001.0577 [hep-th]].