Structure of spin-dependent scattering amplitude
and spin effects at small angles at RHIC energies

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Abstract

Spin-dependent pomeron effects are analyzed for elastic \( pp \) scattering and calculations for spin-dependent differential cross sections, analyzing power and double-spin correlation parameters are carried out for the energy range of the Relativistic Heavy Ion Collider (RHIC) at BNL. In this energy range, \( 50 \leq \sqrt{s} \leq 500 \) GeV, the structure of pomeron-proton coupling can be measured at RHIC with colliding polarized proton beams.
I. INTRODUCTION

The spin structure of the pomeron is still an unresolved question in the diffractive scattering of polarized particles. There have been many observations of spin effects at high energies and at fixed momentum transfers \[1,2\]; several attempts to extract the spin-flip amplitude from the experimental data show that the ratio of spin-flip to spin-nonflip amplitudes can be non-negligible and may be independent of energy \[3,4\]. In all of these cases, the pomeron exchange is expected to contribute the observed spin effects at some level. The large rapidity events that are observed at CERN \[5\] and DESY \[6\], and all diffractive and elastic high energy reactions are predominated by pomeron exchange, thus making the pomeron a popular field of study. Extensive polarized physics programs are proposed at HERA, RHIC and LHC (see e.g. \[7–9\]) in order to shed light on these and other spin effects in hadron reactions.

It is generally believed, based on calculations of the simplest QCD diagrams, that the spin effects diminish as inverse power of center-of-mass energy, and that the pomeron exchange does not lead to appreciable spin effects in the diffraction region at super-high energies. Complete calculations of the full set of helicity scattering amplitudes in diffraction region cannot be carried out presently since they require extensive treatment of confinement and contributions from many diagrams. Semi-phenomenological models, however, have been developed with parameters which are expected to be fixed with the aid of data from experiments \[10,11\].

Vacuum \(t\)-channel amplitude is usually associated with two-gluon exchange in QCD \[12\]. The properties of the spinless pomeron were analyzed on the basis of a QCD model, by taking into account the non-perturbative properties of the theory \[13,14\]. We refer to this as the standard-pomeron model in this paper.

Some models predict non-zero spin effects as \(s \to \infty\) and \(|t|/s \to 0\) limit. In these studies, the spin-flip amplitudes which lead to weakly altered spin effects with increasing energy are connected with the structure of hadrons and their interactions at large distances \[10,11\].
reference [10], the spin-dependence of the pomeron term is constructed to model rotation of matter inside the proton. This approach is based on Chou and Yang’s concept of hadronic current density [15]. The model developed in reference [11] considers the contribution of a sea quark-antiquark pair to hadron interactions at large distances.

This picture can be related with the spin effects determined by higher-order $\alpha_s$ contributions in the framework of PQCD. Really, it has been shown in the framework of QCD analysis at fixed momentum transfer that these corrections to the simple two-gluon exchange [16] may lead to the spin-flip amplitude growing as $s$ in the limit of $s \to \infty$. The similar effects can be determined by the quark loops contributions [17].

The high energy two-particle amplitude determined by pomeron exchange can be written in the form:

$$T(s, t) = i s P(s, t) V_{h_1 h_1}^\mu P \otimes V_{h_2 h_2}^{h_2 P}. \quad (1)$$

Here $P$ is a function caused by the pomeron, with a weak energy dependence $\sim (\ln s)^n$; and $V_{h h}^{P}$ are the pomeron-hadron vertices. The perturbative calculation of the pomeron coupling structure is rather difficult and the non-perturbative contributions are important for momentum transfers of a few $(\text{GeV}/c)^2$.

The calculation of Eq.(1) in the non-perturbative two-gluon exchange model [13] and in the BFKL model [18] shows that the pomeron couplings have the following simple form:

$$V_{h h}^{P} = \beta_{h h}^{P} \gamma^{\mu}. \quad (2)$$

In this case, the pomeron contribution leads to a weak energy dependence of the differential cross section with parallel and antiparallel spins, and their difference drops as inverse power of $s$, leading us to conclude that the spin effects are suppressed as a power of $s$.

This situation changes dramatically when large-distance loop contributions are considered which lead to a more complicated spin structure of the pomeron coupling. As mentioned above, these effects can be determined by the hadron wave function for the pomeron-hadron couplings, or by the gluon-loop corrections for the quark-pomeron coupling [19]. As a result,
spin asymmetries appear that have weak energy dependence as $s \to \infty$. Additional spin-flip contributions to the quark-pomeron vertex may also have their origins in instantons, e.g. \cite{20,21}.

In the framework of the perturbative QCD, the analyzing power of hadron-hadron scattering was shown to be of the order:

$$A_N \propto m \alpha_s / \sqrt{p_t^2}$$

where $m$ is about a hadron mass \cite{22}. Hence, one would expect a large analyzing power for moderate $p_t^2$ where the spin-flip amplitudes are expected to be important for the diffractive processes.

In this paper, we examine the spin-dependent contribution of pomeron to the differential cross sections with parallel and antiparallel spins, their possible magnitudes and energy dependence. We also estimate the possible experimental precisions for these observables in the RHIC energy domain.

**II. THE MODEL AMPLITUDES**

We use the standard helicity representation for the hadron-hadron scattering amplitudes:

$f_1 = <++|M|++>$ and $f_3 = <+-|M|+->$, helicity nonflip amplitudes; $f_2 = <++|M|-->$ and $f_4 = <+-|M|+->$, double-flip amplitudes; and $f_5 = <++|M|+->$, single-flip amplitude. We assume, as usual, that at high energies the double-flip amplitudes are small with respect to the spin-nonflip one, $f_2(s,t) \sim f_4(s,t) \ll f_1(s,t)$ and that spin-nonflip amplitudes are approximately equal, $f_+(s,t) = f_1(s,t) \sim f_3(s,t)$. Consequently, the observables are determined by two amplitudes: $f_+(s,t)$ and $f_-(s,t) = f_5(s,t)$. These customary assumptions are also made for the models developed in \cite{10,11}.

We use the below normalization for the differential cross section:

$$\sigma_0 = \frac{d\sigma}{dt} = \frac{4\pi}{s(s-4m^2)}(|f_+|^2 + 2|f_-|^2),$$
and the analyzing power and the double spin correlation parameters are:

\[ \sigma_0 A_N = \frac{-8\pi}{s(s-4m^2)} Im[f_+^* f_+], \]
\[ \sigma_0 A_{NN} = \frac{4\pi}{s(s-4m^2)} |f_-|^2. \]

The measured spin-dependent differential cross sections can be written in the form:

\[ N(↑↑)/\mathcal{L} = \sigma(↑↑) = (\sigma_0/4)[1 + A_N(P_1 + P_2) \cos \phi + A_{NN}P_1P_2 \cos^2 \phi], \] (3)
\[ N(↑↓)/\mathcal{L} = \sigma(↑↓) = (\sigma_0/4)[1 - A_N(P_1 + P_2) \cos \phi + A_{NN}P_1P_2 \cos^2 \phi], \]
\[ N(↑↓)/\mathcal{L} = \sigma(↑↓) = (\sigma_0/4)[1 + A_N(P_1 + P_2) \cos \phi - A_{NN}P_1P_2 \cos^2 \phi], \]
\[ N(↑↓)/\mathcal{L} = \sigma(↑↓) = (\sigma_0/4)[1 - A_N(P_1 + P_2) \cos \phi - A_{NN}P_1P_2 \cos^2 \phi], \]
\[ \sigma_0 = [N(↑↑) + N(↑↓) + N(↑↓) + N(↑↓)]/\mathcal{L}. \]

\( \mathcal{L} \) is the luminosity and \( \sigma_0 \) is the normalized differential cross section. \( P_1 \) and \( P_2 \) refer to the degree of beam polarizations for the first and second beams, respectively, and \( \phi \) is the azimuthal scattering angle. The arrows indicate the transverse spin orientations of the interacting protons.

If we adapt the following notation:

\[ \sigma(↑) = [N(↑↑) + N(↑↓)]/\mathcal{L}, \quad \sigma(↓) = [N(↑↓) + N(↑↓)]/\mathcal{L}, \]
\[ \sigma(↑↑) = [N(↑↑) + N(↑↓)]/\mathcal{L}, \quad \sigma(↑↓) = [N(↑↓) + N(↑↓)]/\mathcal{L}, \]

then, the analyzing power, \( A_N \), and the double-spin correlation parameter, \( A_{NN} \), can be extracted from the experimental measurements:

\[ A_N = \frac{\sigma(↑) - \sigma(↓)}{\sigma(↑) + \sigma(↓)} = \frac{\Delta \sigma^s}{\sigma_0} = \frac{-2Im(f_+^* f_+)}{|f_+|^2 + 2|f_-|^2}, \] (4)
\[ A_{NN} = \frac{\sigma(↑↑) - \sigma(↑↓)}{\sigma(↑↑) + \sigma(↑↓)} = \frac{\Delta \sigma^d}{\sigma_0} = \frac{2|f_-|^2}{|f_+|^2 + 2|f_-|^2}. \] (5)

Hereafter, \( \Delta \sigma^s \) and \( \Delta \sigma^d \) refer to the single- and double-spin cross section differences. We follow the model developed in [11] closely and extend it further for the calculation of spin-dependent differential cross sections. Pomeron-proton coupling \( V_{pp}^\mu \) is primarily connected with the proton structure at large distances and the pomeron-proton coupling looks like:
\[ V_{\mu PP}^\mu(p, t) = mp_\mu A(t) + \gamma_\mu B(t), \]  
\[ \text{(6)} \]

where \( m \) is the proton’s mass, \( p \) is the hadron momentum and \( t \) is the four-momentum-transfer square. \( \gamma_\mu B(t) \) is a standard pomeron coupling that determines the spin-nonflip amplitude. The term \( mp_\mu A(t) \) is due to meson-cloud effects. This coupling leads to spin-flip at the pomeron vertex which does not vanish in the \( s \rightarrow \infty \) limit. Using Eq.(6), we can calculate the spin-nonflip and spin-flip amplitudes from the pomeron-proton vertex:

\[ |f_+(s, t)| \propto s |B(t)|, \]
\[ |f_-(s, t)| \propto m \sqrt{|t|} s |A(t)|. \]  
\[ \text{(7)} \]

Hence, \( V_{\mu PP}^\mu \) determine \( A_N \) and \( A_{NN} \) parameters. Both of the above amplitudes have the same energy dependence and the ratio of spin-flip to spin-nonflip amplitudes in this picture gives:

\[ \frac{m |f_-(s, t)|}{\sqrt{|t|} |f_+(s, t)|} \simeq m^2 |A(t)| |B(t)| \simeq 0.05 \text{ to } 0.07 \text{ for } |t| \sim 0.5 \text{ (GeV/c)}^2 \]  
\[ \text{(8)} \]

which is consistent with other estimates \[3\].

The amplitudes \( A \) and \( B \) have a phase shift caused by the soft pomeron rescattering. As a result, the analyzing power determined by the pomeron exchange

\[ A_N \frac{d\sigma}{dt} = 2m \sqrt{|t|} Im(AB^*) \]  
\[ \text{(9)} \]

and appears to have a weak energy dependence.

The model also takes into account the contributions of the Regge terms to both \( f_+ \) and \( f_- \) amplitudes. So, the scattering amplitudes are

\[ f_\pm(s, t) = is[f_\pm^{as} + \frac{(c_1 - ic_2)}{\sqrt{s}}f_\pm^{reg} + \frac{c_3(1 + i)}{\sqrt{s}}f_-^{reg}] = is[f_\pm^p + f_\pm^r], \]  
\[ \text{(10)} \]

where \( f_\pm^{as}, f_\pm^{reg} \) are functions which weakly depend on energy and \( c_i \) are parameters. The asymptotic terms \( f_\pm^{as} \) and \( f_\pm^{reg} \) were calculated in the framework of the model \[11\]. The Born amplitudes in the form of Eq.(7) are modified by pomeron rescattering. The Regge
contributions, Eq. (10), were represented in the simplest exponential form. These, and some of the asymptotic function parameters, were obtained from the fit to the experimental data (for details see [11]). The model quantitatively describes all the known experimental data of the proton-proton and proton-antiproton scattering, from $\sqrt{s} = 10$ GeV up to $\sqrt{s} = 1.8$ TeV [11, 24]. We thus expect the predictions for differential cross sections at RHIC energies to be reliable. We neglect a possible odderon contribution in these calculations.

III. SPIN CORRELATION EFFECTS

The meson-cloud [11] and the rotating matter current [10] models quantitatively describe the experimental data on elastic $pp$ scattering at fixed momentum transfers and can predict physical observables (cross sections and asymmetries) at high energies. The predictions, however, differ in size and sign for asymmetries.

We calculate the spin-dependent cross sections using the above described amplitudes in Eq. (10), and we use the parameters of the RHIC beams [9] for the estimation of statistical errors. We assume at $\sqrt{s} \sim 200$ GeV, the luminosity is $L = 3 \times 10^{31}$ cm$^{-2}$ sec$^{-1}$ and the average beam polarization per beam is 70%. The typical geometrical acceptance of the detector is taken to be 20% and the running time is about a month. The momentum transfer binning, we take $\Delta t = 0.05$ (GeV/c)$^2$ at $|t| \leq 1.3$ (GeV/c)$^2$ and $\Delta t = 0.1$ (GeV/c)$^2$ at $|t| \geq 1.3$ (GeV/c)$^2$.

The energy dependence of the real-part of the nonflip amplitudes around diffraction minimum ($Im f_+ (s, t) \sim 0$), is model dependent and may lead to different polarization predictions in this momentum transfer range.

The local dispersion relations have been used in [11] to determine the real-part of $T(s, t)$. The model amplitude obeys the $s - u$ crossing symmetry which permits us to describe quantitatively all the specific effects in the elastic proton-proton and proton-antiproton diffraction scattering in the wide energy region ($9.8 \text{GeV} \leq \sqrt{s} \leq 1800 \text{GeV}$). For example, model describe the difference of the differential cross sections for proton-proton and
proton-antiproton scattering in domain of the diffraction minimum, the known polarization phenomena at $\sqrt{s} = 9.8 \text{ GeV}$ and $\sqrt{s} = 52.8 \text{ GeV}$ and the experimental data of proton-antiproton scattering at $\sqrt{s} = 540$ and 630 GeV. All these quantities are sensitive to the real part of the scattering amplitude. So, we can believe that $\text{Re}[T(s, t)]$ determined correctly and practically model-independent.

The differential cross section calculations are shown in Fig. 1 a,b. The estimated errors are statistical and are less than 1%. At $\sqrt{s} = 50 \text{ GeV}$, the model described here quantitatively reproduces the ISR data. The diffraction minimum defined by the zero of the imaginary part of the spin-nonflip amplitude is filled by the contributions of the real-part of spin-nonflip and spin-flip amplitudes. At $\sqrt{s} = 120 \text{ GeV}$, the dip becomes a shoulder and increases by an order of magnitude when the energy goes from $\sqrt{s} = 120 \text{ GeV}$ to 500 GeV. The model gives the same asymptotical predictions for the proton-proton and proton-antiproton differential cross sections. Hence the cross section prediction at $\sqrt{s} = 500 \text{ GeV}$ approximately corresponds to the $Spp\bar{S}$ data at $\sqrt{s} = 540 \text{ GeV}$.

Fig.2 a,b show calculations for $\Delta \sigma^d$ as determined in Eq.(5). It can be seen that the shape of $\Delta \sigma^d$ is similar to the spin-averaged differential cross sections. It has a sharp dip and the magnitude of $\Delta \sigma^d$ at the dip grows by an order of magnitude in the energy interval $120 \leq \sqrt{s} \leq 500 \text{ GeV}$. Although the dip positions for differential cross sections and that of $\Delta \sigma^d$ seem to coincide at $\sqrt{s} = 50 \text{ GeV}$ but at higher energies they move apart from each other since the dip of $\Delta \sigma^d$ moves more slowly. The position of the $\Delta \sigma^d$ minimum strongly depends on the model parameters. However, in the ranges of $|t| \simeq 0.7$ to $1.0 (\text{GeV}/c)^2$ and $|t| \geq 1.8 (\text{GeV}/c)^2$, the results weakly depend on these parameters. In the regions far from the dips, both cross sections change slowly, especially at $|t| \geq 2 (\text{GeV}/c)^2$.

The energy dependence of $f^+_{\text{pp}}(s, t)$ and $f^-_{\text{pp}}(s, t)$ amplitudes are determined in Eq.(7) for the $V_{\text{pp}} \pi^\rho$ vertex. Hence, we have the same energy dependence for $\Delta \sigma^d$ and $\sigma'(\uparrow \downarrow)$ for the spin-pomeron models, and the ratio of these quantities will be almost energy independent, except around the diffraction minimum.

The calculated ratio of the spin-parallel and antiparallel cross sections, $R(s, t) =$
shows only a logarithmic dependence on energy in its first maximum and is almost energy independent at $|t| = 0.7 \ (\text{GeV}/c)^2$, see (Fig. 3 a,b). It is clear from these results that the spin effects can be sufficiently large at all RHIC energies.

Relative errors in $R(s, t)$ are shown in Table I at two selected momentum transfers and at $t_m$ where the first maximum of $R(t)$ is observed. It is worthwhile to note that the errors with the growth of energy decrease due to increase in the differential cross sections. Also note that in all standard pomeron models this ratio is predicted to be

$$R(s, t) = \sigma^{(\uparrow \uparrow)} / \sigma^{(\uparrow \downarrow)} \rightarrow 1 \quad \text{as} \quad s \rightarrow \infty.$$  

However, the experiment data show a large deviation from unity $[1]$. It is also interesting that at low-energies ($p_L = 11.7$ and 18 GeV/c $[25,26]$, $R(s, t)$ does not change much with energy at $|t| \sim 2 \ (\text{GeV}/c)^2$ and is similar to our predictions of $R(s, t)$ at the first maximum.

Our calculation for $A_N$ is shown in Fig. 4 a,b. The magnitude and the energy dependence of this parameter depend on the energy behavior of the zeros of the imaginary-part of spin-flip amplitude and the real-part of spin-nonflip amplitude.

The maximum negative values of $A_N$ coincide closely with the diffraction minimum (see Figs. 1 and 4). We find that the contribution of the spin-flip to the differential cross sections is much less than the contribution of the spin-nonflip amplitude in the examined region of momentum transfers from these figures. $A_N$ is determined in the domain of the diffraction dip by the ratio

$$A_N \sim \frac{\text{Im} f_-}{\text{Re} f_+}.$$  

The size of the analyzing power changes from $-45\%$ to $-50\%$ at $\sqrt{s} = 50 \ \text{GeV}$ up to $-25\%$ at $\sqrt{s} = 500 \ \text{GeV}$. These numbers give the magnitude of the ratio Eq.(11) that does not strongly depend on the phase between the spin-flip and spin-nonflip amplitudes. This picture implies that the diffraction minimum is filled mostly by the real-part of the spin-nonflip amplitude and that the imaginary-part of the spin-flip amplitude increases in this domain as well.
We observe that the dips are different in speed of displacements with energy from Figs. 1 and 2. In Fig. 4, one sees that at larger momentum transfers, $|t| \sim 2$ to $3 \ (\text{GeV}/c)^2$, the analyzing power depends on energy very weakly.

The expected errors for the analyzing power are small in nearly all momentum transfer ranges examined. They are summarized in Table II.

As pointed out before, the model $[10]$ predicts a similar absolute value but opposite sign for $A_N$ near the diffraction minimum. The future $PP2PP$ experiment at RHIC $[9]$ should be able to provide data and help resolve these issues on the mechanism of spin-effect generation at the pomeron-proton vertex.

**IV. CONCLUSION**

In the framework of the standard-pomeron model, the spin-flip amplitude is defined only by the secondary Regge poles and the ratio

$$\frac{[\sigma(\uparrow\downarrow) - \sigma(\uparrow\downarrow)]}{\sigma(\uparrow\downarrow)} \propto \frac{1}{s}$$

that rapidly decreases with growing $s$ due to the standard energy dependence of the spin-flip amplitude. If we drop the asymptotic term in the spin-flip amplitude from Eq.$[11]$ and keep only the second Regge terms that fall as $1/\sqrt{s}$, we obtain the pre-asymptotic Regge contributions to the analyzing power and the difference of the polarized cross sections (the results are shown in Fig. 5 and 6).

The spin-flip amplitude of the second Regge contributions has a large relative phase compared to the spin-nonflip amplitude of the pomeron. Under this condition, too, the analyzing power can be large. As one observes from Fig. 5, the spin-flip amplitude defined by the Regge contributions can describe the experimental data at $\sqrt{s} = 23.4 \ \text{GeV}$ and give large effects at $\sqrt{s} = 50 \ \text{GeV}$. At higher energies, however, the effect quickly falls and becomes insignificant. Note that at $\sqrt{s} = 50 \ \text{GeV}$, $A_N$ can be positive and that its magnitude greatly depends on the size of the asymptotic term which gives a negative contribution to the analyzing power.
If the spin-flip amplitude is determined by the standard-pomeron model, its contribution will be clearly seen in experiments (see Figs 5, 6) if performed. In this case, the minimum in $\Delta \sigma^d$ is at small momentum transfers $|t| \leq 0.7 \ (\text{GeV}/c)^2$, and is quickly shifted with growing energy. In the ordinary dip region, there is a maximum in the difference of the cross sections which falls as the inverse power of energy. Moreover, at $|t| \geq 2 \ (\text{GeV}/c)^2$, in the spin-pomeron model, $\sigma_0$ and $\Delta \sigma^d$ do not appreciably change with energy at fixed $t$ (Figs. 1,2). The difference spin-dependent cross sections fall with energy as quickly as the increasing momentum transfers (Fig. 6).

The energy dependence of cross sections $\sigma^{(\uparrow\uparrow)}$ and $\sigma^{(\uparrow\downarrow)}$ can be studied experimentally at RHIC. Note that significant spin effects can have small relative errors at momentum transfers $|t| \sim 2$ to $3 (\text{GeV}/c)^2$ and direct information about the nature of the spin-flip effects in the pomeron-proton coupling can be obtained. The future $PP2PP$ experiment at RHIC should be able to measure the spin-dependent cross section with parallel $\sigma^{(\uparrow\uparrow)}$ and antiparallel $\sigma^{(\uparrow\downarrow)}$ beam polarizations in proton-proton scattering and the energy dependence of the spin-flip and spin-nonflip amplitudes can be studied in the energy range of RHIC, $50 \leq \sqrt{s} \leq 500 \ \text{GeV}$.

The spin-structure of the pomeron couplings are determined by the large-distance gluon-loop correction or by the effects of hadron wave function. Tests of the spin structure of QCD at large distances can be carried out in diffractive reactions in future polarized experiments at HERA, RHIC and LHC.

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TABLES

TABLE I. The expected relative errors for \( R(s,t) = \sigma(↑↑)/\sigma(↑↓) \) are tabulated below at \(|t_1| = 0.5 \text{ (GeV}/c)^2\), \(|t_2| = 2.5 \text{ (GeV}/c)^2\) and at \(t_m\), where \( R(s,t) \) is maximum.

| \(\sqrt{s} \) [GeV] | \(\delta R(t_1)\) | \(\delta R(t_m)\) | \(\delta R(t_2)\) |
|---------------------|-----------------|-----------------|-----------------|
| 50                  | 0.05\%          | 5.5\%           | 4.1\%           |
| 120                 | 0.07\%          | 2.2\%           | 4.1\%           |
| 250                 | 0.07\%          | 1.1\%           | 3.7\%           |
| 500                 | 0.09\%          | 0.6\%           | 3.6\%           |

TABLE II. The expected relative errors for \( A_N \) are calculated at \(|t_1| = 0.5 \text{ (GeV}/c)^2\), \(|t_2| = 2.5 \text{ (GeV}/c)^2\) and at \(t_m\), where \( A_N \) is maximum.

| \(\sqrt{s} \) [GeV] | \(\delta A_N(t_1)\) | \(\delta A_N(t_m)\) | \(\delta A_N(t_2)\) |
|---------------------|-------------------|-------------------|-------------------|
| 50                  | 3.1\%             | 3.7\%             | 16\%              |
| 120                 | 2.3\%             | 2.0\%             | 20\%              |
| 250                 | 3.5\%             | 1.4\%             | 18\%              |
| 500                 | 12.4\%            | 1.1\%             | 17\%              |
FIGURES

FIG. 1.  a) The calculated differential cross sections of the $pp$ elastic scattering at $s^{1/2} = 50$ GeV (solid curve) and at $s^{1/2} = 120$ GeV (dashed curve); b) $s^{1/2} = 250$ GeV (dotted curve) and at $s^{1/2} = 500$ GeV (dashed curve). The energy range between 50 GeV and 500 GeV correspond to the entire RHIC energy for polarized protons.

FIG. 2.  a, b) The calculated difference of the double spin differential cross sections at the RHIC energy range. The error bars indicate possible statistical errors for a realistic experiment at RHIC.

FIG. 3.  a, b) The calculated ratio of the cross sections with parallel and antiparallel spins at the RHIC energy range. The error bars indicate possible statistical errors for a realistic experiment at RHIC.

FIG. 4.  The calculated analyzing power of the $pp$ elastic cross sections for the weak energy dependence obtained in the model for the ratio $|F^+(s,0)|/|F^+(s,0)| \propto f(ln s)$ a) at $s^{1/2} = 50$ GeV (solid curve) and at $s^{1/2} = 120$ GeV (dashed curve); b) $s^{1/2} = 250$ GeV (dotted curve) and at $s^{1/2} = 500$ GeV (dashed curve). The error bars indicate expected statistical errors for a realistic experiment at RHIC.

FIG. 5.  The analyzing power for the $pp$ elastic cross sections for the rapid energy dependence of the ratio $|F^{+-}(s,0)|/|F^{++}(s,0)| \propto s^{-1/2}$ (the experimental data at $P_L = 300$ GeV/c are represented by open circles) ; the results of calculations are shown for $P_L = 300$ GeV (solid curve); at $s^{1/2} = 50$ GeV (long-dashed curve); at $s^{1/2} = 120$ GeV (short-dashed curve); and at $s^{1/2} = 500$ GeV (dotted curve).

FIG. 6.  The calculated difference of the double spin differential for the rapid energy dependence of the ratio $|F^{+-}(s,0)|/|F^{++}(s,0)| \propto s^{-1/2}$ The center-of-mass energy span corresponds to the RHIC energy and each curve shows the results at four different $cms$ energies as indicated on the figure.
Fig. 1 a

Fig. 1 b
Fig. 2 a

Fig. 2 b
Fig. 3 a

Fig. 3 b
Fig. 4 a

Fig. 4 b
Fig. 5

Fig. 6