Bilinear R-parity Violation and $\tau^{\mp}\tilde{\kappa}^{\pm}$ mixing production in $e^+e^-$ colliders

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Abstract

A R-parity breaking SUSY model characterized by an effective bilinear violation with only $\tau$-lepton number breaking in the superpotential is outlined. The CP-odd Higgs boson masses and those of charged Higgs bosons are discussed. In the model, several interesting mass mixings else such as the mixing between $\tau$ lepton and charginos etc in the model are discussed too. Being one of example, we have computed the mixing production $e^+e^- \rightarrow \tau^{\mp}\tilde{\kappa}_i^{\pm}(i = 1, 2)$ in $e^+e^-$ colliders, where $\tau^{\mp}, \tilde{\kappa}_i^{\pm}(i = 1, 2)$ denote the physical $\tau$ lepton and charginos.

12.60.Jv, 13.10.+q, 14.80.Ly
I. INTRODUCTION

The minimal supersymmetric extension of the Standard Model (MSSM) [1] is believed one of the most attractive candidates beyond the Standard Model (SM) now. In the usual MSSM an additional quantum number, the so-called R parity of a particle: \( R = (-1)^{2S+3B+L} \) [2], is assumed to be conserved, where, besides the spin quantum number \( S \), \( L \) is the lepton number and \( B \) is the baryon number. In such a case with R-parity conserved, all supersymmetric particles must be pair-produced, while the lightest of super-partners must be stable. Whether or not with a conserved R-parity, the supersymmetric realization is an open dynamical question, sensitive to physics at a more fundamental scale [3]. Whereas if relaxing the R-parity conservation and the relaxing will not conflict with all the observations such as the proton decay and the other rare decays for quarks, leptons and weak bosons etc., we may have new insight to see the long standing problems of particle physics, such as the neutrinos masses problem etc and can make the supersymmetric realization to occur at a comparative lower energy scale. Remarkably, for instance the neutrino can acquire the tree level supersymmetric masses via the mixing with the neutralinos at the weak scale in the R-parity violation framework [4–8]. This mechanism does not involve in the physics at the large energy scale \( M_{int} \sim O(10^{12} \text{GeV}) \). It is, in contrast to the see-saw mechanism, relate the neutrino mass to the weak-scale physics that is more accessible for experimental observations [9].

The R-parity can be broken explicitly [10] or spontaneously [11], that depends on the superpotential and the soft SUSY breaking pattern precisely of the model. The first option allows one to establish very general phenomenological consequences of R-parity violation while the second one, R-parity is kept at the Lagrangian level as a fundamental symmetry, but it is broken by the vacuum i.e. the ground state of the world. For the second, there are quite a lot of possible virtues being added, such as a possibility of having a dynamical origin for the breaking of R-parity through radiative corrections if SUSY has been broken already, that is very similar to certain models for the electroweak symmetry breaking [12] etc.
In this paper we focus on the truncated version of such a model, namely in which the violation of R-parity is effectively introduced by a bilinear superpotential term $\epsilon^I \varepsilon_{ij} \hat{L}_i \hat{H}_j^2$ with proper soft SUSY breaking pattern that the R-parity violation is not only originated to the precise R-parity breaking term in the superpotential but also to the vacuum. Here $\hat{L}_I (I = 1, 2, 3)$ denote three generations of the leptonic SU(2) supersymmetric fields, thus the term also breaks the leptonic numbers. Whereas we will assume that only one generation of the lepton number, i.e. the $\tau$ lepton number, is broken for simplicity. To deduct free parameters in the model so ‘artificially’ by the assumption here is because we may argue and believe the third generation is special based on the fact that the third generation is very heavy, especially, the top-quark mass so heavy $m_t \simeq 175 GeV$ close to the electroweak broken scale already. In addition, we think the general feature of the model in phenomenology can still be kept, even the leptonic numbers of the other two generations are broken occasionally in the same way. In this effective truncated model, the all superfield contents are exactly the same as those of the MSSM but the R-parity violation is broken and realized by the bilinear R-parity violation in the superpotential. Generally the superpotential and the relevant soft breaking terms of the model may also lead to two scenarios: the vacuum expectation values(VEVs) of the sneutrino field i). being zero; ii). being non-zero. In the paper we would like to explore the more complex scenario with non-zero VEVs for the sneutrino field of the third generation. In the sneutrino, as results, mixings of lepton-gaugino-Higgsino and slepton-Higgs etc, and a number of interesting phenomena are issued [13,14]. If the R-parity violation is originated from the vacuum only without the breaking in the Lagrangian, then certain continual quantum number such as lepton number or else must be associated to be broken, so there will be certain physical Goldstone particle occurring and a lot of phenomenological difficulties cannot be avoided hence in the paper we will not discuss the case. Indeed one will see that in the present sneutrino, there is no physical Goldstone boson associating the breaking of R-parity. Here in the paper, taking an interesting example, we will consider a consequences of the bilinear slepton-Higgs R-parity violation terms on the Higgs masses and the mixed production $e^+ e^- \rightarrow \tau^\pm \kappa_1^\pm$ which is forbidden in the MSSM,
and $\tilde{\kappa}_1^\pm$ is to denote the lightest charginos.

The paper is organized as follows. Basic ingredients of the R-parity violation MSSM with the explicit R-parity violation are briefly described in Section II. In Section III, we will discuss the masses of CP-odd Higgs and charged Higgs sectors etc. The required massless Goldstone boson ‘eaten’ by the electroweak gauge fields in unitary gauge is obtained naturally, and the gauge-fixing terms in 't Hooft-Feynman gauge are derived. Furthermore, we take into account the effect of $e^+e^- \rightarrow \tau^+\tilde{\kappa}_1^\pm$ in $e^+e^-$ colliders. In Section V, we will present the numerical results calculated under certain assumptions and discussions. In addition, we close our discussions with short comments on certain implications of the model for the other experiments.

II. MINIMAL SUSY MODEL WITH BILINEAR R-PARITY VIOLATION

The supersymmetric Lagrangian is specified by the superpotential $W$ that is given by \[8, 15:\]

$$W = \mu \varepsilon_{ij} \hat{H}_i^1 \hat{H}_j^2 + \varepsilon_{ij} l_{ij} \hat{H}_i^1 \hat{L}_j^I \hat{R}_j^I + \varepsilon_{ij} d_{ij} \hat{H}_i^1 \hat{Q}_j^I \hat{D}_j^J + \varepsilon_{ij} u_{ij} \hat{H}_i^2 \hat{L}_j^I \hat{U}_j^I + \epsilon' \varepsilon_{ij} \hat{H}_i^2$$

where $I, J = 1, 2, 3$ are generation indices, $i, j = 1, 2$ are SU(2) indices, and $\varepsilon$ is a completely antisymmetric $2 \times 2$ matrix, with $\varepsilon_{12} = 1$. The capital letters covered by a symbol "hat" denote superfields: $\hat{Q}_I^I, \hat{L}_I^I, \hat{H}_i^1$, and $\hat{H}_i^2$ being the SU(2) doublets with hyper-charges $1/3, -1, -1, 1$ respectively; $\hat{U}, \hat{D},$ and $\hat{R}$ being SU(2) singlets with hyper-charges $-4/3, 2/3$, and 2 respectively. The couplings $u_{ij}, d_{ij},$ and $l_{ij}$ are $3 \times 3$ Yukawa matrices, and $\mu, \epsilon''$ are parameters with units of mass. The first four terms in the superpotential are those as the MSSM, and the last one is the R-parity violating term.

As MSSM, general and possible soft SUSY-breaking terms to break SUSY need to be introduced:

$$\mathcal{L}_{\text{soft}} = -m^2_{H_i} H_i^{1*} H_i^1 - m^2_{H_2} H_2^{2*} H_i^2 - m^2_{L_i} \tilde{L}_i^{*} \tilde{L}_i$$
\[-m_{R_i}^2 \tilde{R}_i^{I*} \tilde{R}_i^I - m_{Q_i}^2 \tilde{Q}_i^{I*} \tilde{Q}_i^I - m_{D_i}^2 \tilde{D}_i^{I*} \tilde{D}_i^I \]
\[-m_{U_i}^2 \tilde{U}_i^{I*} \tilde{U}_i^I + (m_1 \lambda_B \lambda_B + m_2 \lambda_A \lambda_A + m_3 \lambda_G^2 \lambda_G + h.c.) + (B \mu \varepsilon_{ij} H_1^I H_2^I + B_1 \epsilon^{II} \varepsilon_{ij} H_1^j \tilde{L}_j^I) \]
\[+ \varepsilon_{ij} \lambda_1 \mu H_1^I \tilde{L}_j^I \tilde{R}_i^I + \varepsilon_{ij} \lambda_2 \mu H_1^j \tilde{Q}_i^I \tilde{D}_j^I \]
\[+ \varepsilon_{ij} \lambda_3 \mu H_1^j \tilde{Q}_i^I \tilde{U}_j^I + h.c. \]
\] (2)

where $m_{H_1}^2, m_{H_2}^2, m_{L_I}^2, m_{R_I}^2, m_{Q_I}^2, m_{D_I}^2,$ and $m_{U_I}^2$ are the parameters with units of mass squared while $m_1, m_2, m_3$ denote the masses of the $SU(3) \times SU(2) \times U(1)$ gauginos $\lambda_G, \lambda_A,$ and $\lambda_B,$ B and $B_1$ are free parameters with units of mass.

In order to eliminate unnecessary degrees of freedom, we assume that the soft-breaking parameters and $\mu, \epsilon^{II} (I = 1, 2, 3)$ are real and perform an operation that is the same as in the standard model by the redefinition of the fields $[14]$:

\[\hat{Q}_i^I \to V_{Q_i}^{IJ} \hat{Q}_i^J,\]
\[\hat{U}_i^I \to V_{U_i}^{IJ} \hat{U}_j^J,\]
\[\hat{D}_i^I \to V_{D_i}^{IJ} \hat{D}_j^J,\]
\[\hat{L}_i^I \to V_{L_i}^{IJ} \hat{L}_j^J,\]
\[\hat{R}_i^I \to V_{R_i}^{IJ} \hat{R}_j^J\]
\] (3)

One can diagonalize the matrices $l_{IJ}, u_{IJ},$ and $d_{IJ},$ the superpotential has the form:

\[W = \mu \varepsilon_{ij} \hat{H}_i^1 \hat{H}_j^2 + l_{IJ} \hat{H}_i^1 \tilde{L}_j^I \hat{R}_i^I - u_I (\hat{H}_i^2 C^{IJ} \hat{Q}_2^I) \]
\[\hat{Q}_2^I \) \hat{L}_i^I - d_{IJ} (\hat{H}_i^1 \hat{Q}_2^I - \hat{H}_2^1 C^{IJ} \hat{Q}_1^I) \hat{D}_j^I \]
\[+ \epsilon^{IJ} \varepsilon_{ij} \hat{H}_i^2 \hat{L}_j^I\]
\] (4)

and the Kobayashi-Maskawa matrix $C$ and $\epsilon^I$ have the definition as:

\[C = V_{Q_2}^I V_{Q_1},\]
\[\epsilon^I = \epsilon^{IJ} V_{L_1}^{IJ}\]
\] (5)

and correspondingly the soft SUSY breaking sector has the form:
\[ \mathcal{L}_{\text{soft}} = -m_H^2 H_1^+ H_1^+ - m_H^2 H_2^+ H_2^+ - m_{H_1}^2 \tilde{L}_1^\dagger \tilde{L}_1^\dagger - m_{H_2}^2 \tilde{R}_1^\dagger \tilde{R}_1^\dagger \\
+ m_{Q_1}^2 Q_1^\dagger Q_1^\dagger - m_{D_1}^2 \tilde{D}_1^\dagger \tilde{D}_1^\dagger - m_{U_1}^2 \tilde{U}_1^\dagger \tilde{U}_1^\dagger + (m_A \lambda_A + m_3 \lambda_G + \text{h.c.}) + \{ B \mu \varepsilon_{ij} H_1^+ H_1^+ + B_1 \varepsilon_{ij} H_1^+ \tilde{L}_j \}
+ \varepsilon_{ij} \mu H_1^+ \tilde{L}_j^\dagger + d_{ij} \mu (-H_1^+ \tilde{Q}_j^\dagger + C^{iK} H_2^+ \tilde{Q}_1^\dagger) \tilde{D}_j^\dagger \\
+ u_{ij} \mu (-C^{iK} H_1^+ \tilde{Q}_2^\dagger + H_2^+ \tilde{Q}_1^\dagger) \tilde{U}_j^\dagger + \text{h.c.}\} \tag{6} \]

As pointed out at the above, from now on we take \( \epsilon_1 = \epsilon_2 = 0 \) always. In this way, only \( \tau \)-lepton number is violated. The electroweak symmetry may be broken spontaneously in a general way that the two Higgs doublets \( H_1, H_2 \), and the \( \tau \)-sneutrino as well acquire vacuum expectation values (VEVs):

\[
H^1 = \begin{pmatrix}
\frac{1}{\sqrt{2}}(\chi_1^0 + v_1 + i\varphi_1^0)

H_2^1
\end{pmatrix}
\tag{7}
\]

\[
H^2 = \begin{pmatrix}
H_1^2
\frac{1}{\sqrt{2}}(\chi_2^0 + v_2 + i\varphi_2^0)
\end{pmatrix}
\tag{8}
\]

\[
\tilde{L}_3 = \begin{pmatrix}
\frac{1}{\sqrt{2}}(\chi_3^0 + v_3 + i\varphi_3^0)

\tilde{\tau}^-
\end{pmatrix}
\tag{9}
\]

It is easy to recognize the fact that the gauge bosons \( W \) and \( Z \) acquire masses given by \( m_W^2 = \frac{1}{4}g^2 v^2 \) and \( m_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2 \), where \( v^2 = v_1^2 + v_2^2 + v_3^2 \) and \( g, g' \) are coupling constants of SU(2) and U(1), if one writes the rest sectors for the model relating to the gauge fields. Let us introduce the following notation in spherical coordinates \[3\]:

\[
v_1 = v \sin \theta_v \cos \beta \\
v_2 = v \sin \theta_v \sin \beta \\
v_3 = v \cos \theta_v
\tag{10}\]

which preserves the MSSM definition \( \tan \beta = \frac{v_2}{v_1} \). If furthermore the angle \( \theta_v \) equals to \( \frac{\pi}{2} \), this sector will change back to the MSSM limit exactly. Note that in the literature
many authors choose a special direction $\theta_\nu = \frac{\pi}{2}$ by field redefinitions \[21\], whereas we are considering the model with leptonic number either conserved or violated in the soft SUSY breaking sector and only a bilinear R-parity breaking term in superpotential thus here we leave the angle $\theta_\nu$ as a parameter to be determined phenomenologically.

The full scalar potential may be written as:

$$V_{\text{tree}} = \sum_i \left| \frac{\partial W}{\partial A_i} \right|^2 + V_D + V_{\text{soft}}$$

$$= V_F + V_D + V_{\text{soft}}. \quad (11)$$

where $A_i$ denotes any one of the scalar fields in the theory, $V_D$ are the usual D-terms, $V_{\text{soft}}$ are the SUSY soft breaking terms given in Eq. (6). Here, we do not consider the radiative corrections to the scalar potential at all.

The scalar term potential contains linear terms:

$$V_{\text{linear}} = t_1^0 \chi_1^0 + t_2^0 \chi_2^0 + t_3^0 \chi_3^0 \quad (12)$$

where

$$t_1^0 = \frac{1}{4}(g^2 + g'^2)v_1(v_1^2 - v_2^2 + v_3^2) + \frac{1}{2} |\mu|^2 v_1 + \frac{1}{2} m_{H_1}^2 v_1 + \frac{1}{2} B_1 v_1 + \frac{1}{2} B_1 \epsilon_3 v_3 + \frac{1}{2} \epsilon_3^2 v_2$$

$$t_2^0 = -\frac{1}{4}(g^2 + g'^2)v_2(v_1^2 - v_2^2 + v_3^2) + \frac{1}{2} |\mu|^2 v_2 + \frac{1}{2} B_1 v_1 + \frac{1}{2} m_{H_2}^2 v_2 - \frac{1}{2} B_1 \epsilon_3 v_3 + \frac{1}{2} \epsilon_3^2 v_2$$

$$t_3^0 = \frac{1}{4}(g^2 + g'^2)v_3(v_1^2 - v_2^2 + v_3^2) + \frac{1}{2} m_{L_3}^2 v_3 + \frac{1}{2} \epsilon_3^2 v_3 + \frac{1}{2} \epsilon_3^2 v_1 - \frac{1}{2} B_1 \epsilon_3 v_2. \quad (13)$$

These $t_i^0, i = 1, 2, 3$ are the tree level tadpoles, and the VEVs of the neutral scalar fields satisfy the condition $t_i^0 = 0, i = 1, 2, 3$, we can obtain:

$$m_{H_1}^2 = -(|\mu|^2 + \epsilon_3^2 \frac{v_3}{v_1} + B_1 \frac{v_3}{v_1} + \frac{1}{2}(g^2 + g'^2)(v_1^2 - v_2^2 + v_3^2))$$

$$m_{H_2}^2 = -(|\mu|^2 + \epsilon_3^2 - B_1 \epsilon_3 \frac{v_3}{v_2} + B_1 \frac{v_1}{v_2} - \frac{1}{2}(g^2 + g'^2)(v_1^2 - v_2^2 + v_3^2))$$

$$m_{L_3}^2 = -(\frac{1}{2}(g^2 + g'^2)(v_1^2 - v_2^2 + v_3^2) + \epsilon_3^2 + \frac{\epsilon_3 \mu v_1}{v_3} - B_1 \epsilon_3 \frac{v_2}{v_3}). \quad (14)$$

An impact of the R-parity violation on the low energy phenomenology is twofold. Firstly, it leads the lepton number violation(LNV). Secondly, the bilinear R-parity violation term in
the superpotential and that in soft breaking terms generate the non-zero vacuum expectation value for the sneutrino fields \( \langle \tilde{\nu}_i \rangle \neq 0 \). As the consequences, not only the neutrino-neutralino and electron-chargino mixing, but also various scalar mixings, such as those of the charged Higgs sector and the stau sector etc are caused. In the discussions below, we always take the ‘new’ parameters (those besides MSSM) at the weak interaction scale and impose on the restriction \( m_{\nu_{\tau}} \leq 24\text{MeV} \).

III. SOME PHENOMENOLOGY OF THE BRPV MODEL

A. CP-odd neutral scalars and Charged Higgs-stau mixing

The neutral scalar sector of the \( \epsilon^- \) model differs from that of the R-parity conserved MSSM: the Higgs bosons mix with the tau sneutrino. The CP-even sector is a mixture of the real part of the \( H_1^1, H_2^2 \), and \( \tilde{L}_3^3 \), the mass matrix is given in Ref \[3\]. Similarly, the CP-odd sector is a mixture of the imaginary part of the \( H_1^1, H_2^2 \), and \( \tilde{L}_3^3 \), after the mixing there must be a linear combination corresponding to the unphysical and massless Goldstone boson that is requested for electroweak breaking.

Let us see the fact precisely. In the original basis, where \( \Phi_{\text{odd}} = (\varphi_1^0, \varphi_2^0, \varphi_3^0) \), the scalar potential contains the following mass term: linear combination being the unphysical Goldstone boson. In the original basis, where \( \Phi_{\text{odd}} = (\varphi_1^0, \varphi_2^0, \varphi_3^0) \), the scalar potential contains the following mass term:

\[
\mathcal{L}_{\text{odd}}^m = -\Phi_{\text{odd}}^\dagger M_{\text{CP-odd}}^2 \Phi_{\text{odd}} \tag{15}
\]

where the \( 3 \times 3 \) mass mixing matrix can be like this:

\[
M_{\text{CP-odd}}^2 = \begin{pmatrix}
    r_{11} & -B \mu & \epsilon_3 \mu \\
    -B \mu & r_{22} & B_1 \epsilon_3 \\
    \epsilon_3 \mu & B_1 \epsilon_3 & r_{33}
\end{pmatrix} \tag{16}
\]

with
Using Eq. (14), we can rewrite the matrix as below:

\[
\mathcal{M}_{CP-odd}^2 = \begin{pmatrix}
-B\mu \frac{v_2}{v_1} - \epsilon_3 \mu \frac{v_3}{v_1} & -B\mu & \epsilon_3 \mu \\
-B\mu & B_1 \epsilon_3 \frac{v_3}{v_2} - B\mu \frac{v_3}{v_2} & B_1 \epsilon_3 \\
\epsilon_3 \mu & B_1 \epsilon_3 & B_1 \epsilon_3 \frac{v_3}{v_1} - \epsilon_3 \mu \frac{v_1}{v_3}
\end{pmatrix}
\]

The above matrix has an eigenstate:

\[
G^0 = \sum_{i=1}^{3} Z_{odd}^i \varphi_i^0 \\
= \frac{1}{v_2} (v_1 \varphi_1^0 - v_2 \varphi_2^0 + \varphi_3^0) \\
= \sin \theta_v \cos \beta \varphi_1^0 - \sin \theta_v \sin \beta \varphi_2^0 + \cos \theta_v \varphi_3^0.
\]

which is corresponding to the massless Goldstone boson which will disappear if the unitary gauge is taken. The other two mass-eigenstates can be written as:

\[
A_i^0 (i = 1, 2) = \sum_{j=1}^{3} Z_{odd}^{i,j} \varphi_j^0
\]

where \(Z_{odd}^{i,j} (i, j = 1, 2, 3)\) is the transformation matrix that rotates from the original basis into the mass-eigenstates. As we expected, all the \(A_i^0 (i = 1, 2)\) acquire masses.

In the model the complex scalar \(H_2^1^*\), \(H_1^2\) mix with the left and right \(\tau\)-slepton. In the original basis, where \(\Phi_c = (H_2^1^*, H_1^2, \tilde{\tau}_L^*, \tilde{\tau}_R)\), the scalar potential contains the following masses term:

\[
\mathcal{L}_m^C = -\Phi_c^i \mathcal{M}_c^2 \Phi_c
\]

where the 4×4 mass matrix of the charged scalar sector can be divided into three components for the model \([13, 17]\)
\[\mathcal{M}_c^2 = \begin{pmatrix} M_{H_1}^2 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & M_\tau^2 \end{pmatrix} + \begin{pmatrix} 0 & M_\epsilon^2 \\ M_\epsilon^2 & 0 \end{pmatrix} \]

where \(M_{H_1}^2\) is a 2 × 2 charged Higgs masses matrix for MSSM and \(M_\tau^2\) is the 2 × 2 stau masses matrix. The \(M_\epsilon^2\) component does not present in MSSM, which cases a mixing of the charged \(H_2^1\), \(H_1^2\) and the \(\tau\)-slepton sector. The matrix \(\mathcal{M}_c^2\) can be obtained in the model (here the matrix is too big to be written precisely so we write it by each element individually):

\[
\mathcal{M}_{c,1,1} = g^2 v_1^2 - \frac{g^2 - g'^2}{2} (v_1^2 - v_2^2 + v_3^2) + |\mu|^2 + \frac{1}{2} v_3^2 l_{(I=3)}^2 + m_{H_1}^2,
\]

\[
= g^2 (v_1^2 - v_3^2) + \frac{1}{2} v_3^2 l_{(I=3)}^2 - \epsilon_3 \mu \frac{v_3}{v_1} - B_\mu \frac{v_3}{v_1},
\]

\[
\mathcal{M}_{c,1,2} = g^2 v_1 v_2 - B \mu,
\]

\[
\mathcal{M}_{c,1,3} = g^2 v_1 v_3 + \epsilon_3 \mu - \frac{1}{2} l_{(I=3)} v_1 v_3,
\]

\[
\mathcal{M}_{c,1,4} = \epsilon_3 l_{(I=3)} \frac{v_2}{\sqrt{2}} - l_{s(I=3)} \frac{\mu v_3}{\sqrt{2}},
\]

\[
\mathcal{M}_{c,2,2} = g^2 v_2^2 + \frac{1}{2} (g^2 + g'^2) (v_1^2 - v_2^2 + v_3^2) + |\mu|^2 + m_{H_2}^2,
\]

\[
= g^2 (v_1^2 + v_3^2) + B_1 \epsilon_3 \frac{v_3}{v_2} - B_\mu \frac{v_3}{v_2},
\]

\[
\mathcal{M}_{c,2,3} = g^2 v_2 v_3 + B_1 \epsilon_3,
\]

\[
\mathcal{M}_{c,2,4} = \frac{l_{(I=3)}}{\sqrt{2}} \mu v_3 + \frac{l_{(I=3)}}{\sqrt{2}} \epsilon_3 v_1,
\]

\[
\mathcal{M}_{c,3,3} = g^2 v_3^2 + \frac{1}{2} (g^2 + g'^2) (v_1^2 - v_2^2 + v_3^2) + \epsilon_3 l_{(I=3)} \frac{v_3}{2} + m_{L_3}^2,
\]

\[
= g^2 (v_2^2 - v_1^2) - \epsilon_3 \frac{\mu v_1}{v_3} + B_1 \epsilon_3 \frac{v_3}{v_3} + \frac{l_{(I=3)}^2}{2} v_1^2,
\]

\[
\mathcal{M}_{c,3,4} = \frac{1}{\sqrt{2}} l_{(I=3)} \mu v_2 + \frac{1}{\sqrt{2}} l_{s(I=3)} \mu v_1,
\]

\[
\mathcal{M}_{c,4,2} = -g'^2 (v_1^2 - v_2^2 + v_3^2) + \frac{1}{2} l_{(I=3)}^2 (v_1^2 + v_3^2) + m_{R_3}^2,
\]

\[
\mathcal{M}_{c,2,1} = \mathcal{M}_{c,1,2},
\]

\[
\mathcal{M}_{c,3,1} = \mathcal{M}_{c,1,3},
\]

\[
\mathcal{M}_{c,4,1} = \mathcal{M}_{c,1,4},
\]

\[
\mathcal{M}_{c,2,3} = \mathcal{M}_{c,3,2},
\]

\[
\mathcal{M}_{c,1,2} = \mathcal{M}_{c,2,1},
\]

\[
\mathcal{M}_{c,2,4} = \mathcal{M}_{c,4,2},
\]
$$\mathcal{M}_{c_{1,3}}^2 = \mathcal{M}_{c_{3,4}}^2$$  \hfill (21)

where the Eq. [14] is used sometimes.

This matrix has an eigenstate:

$$G^+ = \sum_{i=1}^{4} Z_{n,i}^c \phi_{i}^c$$
$$= \frac{1}{\nu} (\nu_1 H_2^{1*} - \nu_2 H_1^2 + \nu_3 \tilde{\tau}_L^*)$$
$$= \sin \theta \cos \beta H_2^{1*} - \sin \theta \sin \beta H_1^2 + \cos \theta \tilde{\tau}_L^*$$  \hfill (22)

with zero eigenvalue, and being the massless charged Goldstone boson it will be absorbed by W bosons and disappear in the physical (unitary) gauge. The other three eigenstates

$$H^+, \tilde{\tau}_1, \tilde{\tau}_2$$

can be expressed as:

$$H^+ = \sum_{i=1}^{4} Z_{2,i}^c \phi_{i}^c$$
$$\tilde{\tau}_1 = \sum_{i=1}^{4} Z_{3,i}^c \phi_{i}^c$$
$$\tilde{\tau}_2 = \sum_{i=1}^{4} Z_{4,i}^c \phi_{i}^c.$$  \hfill (23)

If a process is calculated only up to the tree approximation in a spontaneously broken gauge theory, the most convenient choice is to take the unitary gauge in which the unphysical Goldstone bosons will be absent in the Lagrangian and Feynman rules. Whereas when calculating higher order corrections, it is convenient to choose a renormalizable gauge, commonly the so-called 't Hooft-Feynman gauge is favored [3], in which the Goldstone fields appear explicitly. For our later calculations, the appropriate choice for gauge fixing:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi} (\partial^\mu A_\mu^0 + \xi m_z \cos \theta_W G^0)^2 - \frac{1}{2\xi} (\partial^\mu B_\mu - \xi m_z \sin \theta_W G^0)^2 - \frac{1}{2\xi} (\partial^\mu A_\mu^1 + \frac{i}{\sqrt{2}} \xi m_W (G^+ - G^-))^2 - \frac{1}{2\xi} (\partial^\mu A_\mu^2 - \frac{1}{\sqrt{2}} \xi m_W (G^+ + G^-))^2$$
$$= \{ -\frac{1}{2\xi} (\partial^\mu Z_\mu)^2 - \frac{1}{2\xi} (\partial^\mu F_\mu)^2 - \frac{1}{4\xi} (\partial^\mu W^+)(\partial^\mu W^-) \} - \{ m_z G^0 \partial^\mu Z_\mu + i m_W (G^+ \partial^\mu W^- - G^- \partial^\mu W^+) \} -
\[
\left\{ \frac{1}{2} \xi m_Z^2 (G^0)^2 - \xi m_W^2 G^+ G^- \right\}
\]  
\hspace{1cm} (24)

is taken. Here

\[ \cos^2 \theta_W = \frac{m_W^2}{m_Z^2} \]

and with \( G^0, G^\pm \) are defined as above. The first part of the above expression is identical to the usual gauge-fixed terms; the second part cancels the off-diagonal vertices for Higgs-boson-gauge-boson remaining in the Lagrangian after symmetry breaking; and the third part ‘gives’ masses to the Goldstone bosons in the gauge.

**B. The Mixed Production \( e^+ e^- \rightarrow \tilde{\kappa}_1^\pm \tau^\pm \) in the \( e^+ e^- \) Colliders**

Similarly to the Higgs bosons, charginos mix with the \( \tau \) lepton and form a set of the charged fermions \( \tau^-, \tilde{\kappa}_1^-, \tilde{\kappa}_2^- \) \[13,19\]. In the original basis where \( \psi^T = (-i\lambda^+, \bar{H}_1^2, \tau_R^+) \) and \( \psi^{-T} = (-i\lambda^-, \bar{H}_1^2, \tau_L^-) \), the charged fermion mass terms in the Lagrangian are:

\[
\mathcal{L}_m = -\psi^{-T} \mathcal{M}_f \psi^+ \hspace{1cm} (25)
\]

with the mass matrix given by \[13,19\]:

\[
\mathcal{M}_f = \begin{pmatrix}
2m_2 & \frac{e\nu_2}{\sqrt{2} S_W} & 0 \\
\frac{e\nu_1}{\sqrt{2} S_W} & \mu & \frac{l_{(l=3)\nu_3}}{\sqrt{2}} \\
\frac{e\nu_3}{\sqrt{2} S_W} & \frac{l_{(l=3)\nu_3}}{\sqrt{2}} & \epsilon_3
\end{pmatrix}
\]  
\hspace{1cm} (26)

where \( S_W = \sin \theta_W \) and \( \lambda^\pm = \frac{\lambda_1^\pm + \lambda_3^\pm}{\sqrt{2}} \). Thus two mixing matrices \( Z^+ \) and \( Z^- \) appear, and they are defined by the condition that the product \( (Z^+)^T \mathcal{M}_f Z^- \) should be a diagonal matrix:

\[
(Z^+)^T \mathcal{M}_f Z^- = \begin{pmatrix}
m_\tau & 0 & 0 \\
0 & m_{\tilde{\kappa}_1^-} & 0 \\
0 & 0 & m_{\tilde{\kappa}_2^-}
\end{pmatrix}
\]  
\hspace{1cm} (27)
The unitary matrices $Z^+$ and $Z^-$ are not uniquely specified if changing their relative phases and the order of the eigenvalues. It is possible to choose $m_\tau, m_{\tilde{\kappa}_i}$ positive and to have the order $m_{\tilde{\kappa}_2} \geq m_{\tilde{\kappa}_1} \geq m_\tau$, and we do so only for fixing the irrelevant freedoms. Due to the mixing between $\tau$ and charginos, it is possible to occur the mixed production $e^+e^- \rightarrow \tilde{\kappa}_1^\pm \tau^\mp$ which is forbidden in MSSM. In the present model, the Feynman diagrams that contribute to the lowest-order amplitude are given in Fig. 1, and the contribution is mainly from the s-channel with the exchange of Z-gauge bosons and t-channel with the exchange of sneutrinos, whereas the contribution due to exchange of the other scalars is small, except crossing their corresponding resonances respectively.

**IV. NUMERICAL RESULTS AND DISCUSSIONS**

Throughout the paper, we consider all the independent parameters to take the values at the weak interaction scale. As known, there are many parameters in the model needed to be fixed still, so for simplicity but not losing the key features, in the numerical evaluation we assume there are some constraints among the parameters defined in Eqs. (4, 5) as follow (all are at the weak interaction scale):

\[
\begin{align*}
\frac{l_{(I=3)}}{l_{s(I=3)}} &= \frac{d_{(I=3)}}{d_{s(I=3)}} = \frac{u_{(I=3)}}{u_{s(I=3)}}, \\
B &= -B_1 = \frac{l_{s(I=3)}}{l_{(I=3)}} \mu - 1, \\
m^2_{H^1} = m^2_{H^2} = m^2_{L^3} = m^2_{R^3} = m^2_{Q^3} = m^2_{U^3} = m^2_{D^3}, \\
m_3 = m_2 = m_1 = M_\frac{1}{2}
\end{align*}
\]

With these constraints and Eq. (14) together, only four free parameters in this model are left. Hence we may choose $\tan \beta, \nu_3, \epsilon_3$ and $M_\frac{1}{2}$ to be the four.

As for the other parameters that used in the numerical evaluation, we take $\alpha \equiv \frac{e^2}{4\pi} = \frac{1}{128}$, $m_e = 0.51 MeV$, $m_\tau = 1.78 GeV$, $M_Z = 91.19 GeV$, $M_W = 80.23 GeV$.

In order to find out the allowed region in the parameter space, one has to take all the experimental constraints into account. First, we would like to note that $\epsilon_3, M_\frac{1}{2}$, and $\nu_3$
are the three free parameters which enter into the chargino and neutralino mass matrices. Then a very strong restriction on the parameters comes from the fact that the $\tau$ mass has been measured very precisely $^{20}$, therefore, for any combination of the $\epsilon_3$, $M_2$, and $\nu_3$, the lowest eigenvalue of Eq. (26) should agree with $m_\tau$. Also, $\nu_\tau$ has a laboratory upper bound on its mass $m_{\nu_\tau} \leq 24 MeV$. These two restrictions, together with the positive-definite condition for the Higgs mass matrices, the restrict the allowed parameter space very seriously. Furthermore, since we are interested in relatively light charginos, thus in the numerical calculation we take $M_2 \sim 300 GeV$ so small.

Fig. 2, Fig. 3 show mass squared of the lightest CP-odd Higgs and mass squared of the lightest charged Higgs varied with the parameter $\epsilon_3$. With the assumption Eq. (28), the $\epsilon_3$ must be less than zero. The main point to note is that $M_{H^+}^2$ can be lower than the expected value in MSSM due to the fact that in the model the sneutrino acquires nonzero VEVs so a negative contribution from the R-parity violating stau-Higgs mixing results in. It in fact is controlled by the parameter $\epsilon_3$ and $\nu_3$. From Fig. 3, one may see that the charged Higgs mass may turn to small when $\epsilon_3$ approaches to a certain value, varying with the other parameters taken. It is also because we have made the assumption the Eq. (28). From the scalar potential Eq. (11), we have:

$$V_{tree} = m_{L_3}^2 \tilde{L}_1^3 \tilde{L}_3^3 + \epsilon_3 \tilde{L}_1^3 \tilde{L}_3^3 + (B_1 \epsilon_3 H_2^2 \tilde{L}_1^3 + h.c.) - (\mu \epsilon_3 H_1^1 \tilde{L}_1^3 + h.c.) +$$

$$\frac{g_1^2 + g_2^2}{8} \{(\tilde{L}_1^3 \tilde{L}_3^3)^2 + 2 \tilde{L}_1^3 \tilde{L}_3^3 H_1^1 H_1^1 - 2 \tilde{L}_1^3 \tilde{L}_3^3 H_2^2 H_2^2 \} + \cdots$$  (29)

If the $\tau$-sneutrino has a non-zero vacuum expectation value, $\epsilon_3^2 + m_{L_3}^2$ must be negative. Because we interest the case that $\epsilon_3$ parameter is real, so $m_{L_3}^2$ is a negative number. Under conditions Eq. (28), $m_{R_3}^2$ is a negative number. Furthermore, from the Eq. (14) and the relations in Eq. (28):

$$B = -B_1 = \frac{l_{(I=3)} \mu}{l_{(I=3)}} - 1$$

and

$$m_{H^1}^2 = m_{H^2}^2 = m_{L_3}^2$$
the mass matrix of charged Higgs depends on the $\epsilon_3$ in a very complicated manner. The two reasons make the mass matrix of charged Higgs is not positive-definite when $\epsilon_3$ approaches to the value when $\tan \beta$, $\nu_3$ and $M_2$ are given. We can understand the Fig. 2 in a similar way. Fig. 4 shows the mass of the lightest chargino varied with $\epsilon_3$, the minimum of $m_{\tilde{\kappa}_1}$ is about 100GeV. If we don’t consider the constraint Eq. (28), the value of $\epsilon_3$ can be larger than zero, this case has been discussed by Ref [13] and Ref [17].

Finally, let us discuss the mixing production $\tilde{\kappa}_1^\pm \tau^\mp$ as the typical consequences of the bilinear R-violating terms. In Fig. 5, we plot the $\sigma_{e^{-}e^{+}\rightarrow \tilde{\kappa}_1^\pm \tau^\mp}$ against $\epsilon_3$ (in GeV), $\sigma_{e^{-}e^{+}\rightarrow \tilde{\kappa}_1^\pm \tau^\mp} \rightarrow 0$ when $|\epsilon_3| \rightarrow 0$. The $\sigma_{e^{-}e^{+}\rightarrow \tilde{\kappa}_1^\pm \tau^\mp}$ varied with $\nu_3$ is plotted in Fig. 6. From Fig. 5 and Fig. 6, we find that the $\sigma_{e^{-}e^{+}\rightarrow \tilde{\kappa}_1^\pm \tau^\mp}$ depend on the parameters $\tan \beta$ and $\nu_3$ strongly. If we release Eq.(29), the case is very involved.

In summary, it is shown that the Bilinear R-parity Violation Model is one of the simplest extension of MSSM, in which the R-parity violation is introduced by two folds: a violation term in the Lagrangian and the VEVs of the sneutrino. In the model there are two massless Goldstone $G^0, G^\pm$, requested to be ‘eaten’ by week bosons as the manner in SM and MSSM in the unitary physical gauge. As a quite large value of $\epsilon$ and $\nu_3$ is allowed in the model, so we are quite sure that with the parameters one can find certain differences of the model from MSSM in phenomenology at tree and/or one-loop level. Being a consequence of the bilinear R-violation term, the $e^{-}e^{+} \rightarrow \tilde{\kappa}_1^\pm \tau^\mp$ can occur, and the cross section is typical in order $10^{-4}pb$ when $|\epsilon_3|$ is so large as $|\epsilon_3| \sim 100GeV$.

**Acknowledgement** This work was supported in part by the National Natural Science Foundation of China and the Grant No. LWLZ-1298 of the Chinese Academy of Sciences.
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FIG. 1. The Feynman-diagrams for mixing production $e^- e^+ \rightarrow \tilde{\kappa}_1^+ \tau^-$.
FIG. 2. the $M_{A_1}^2$ vary with $\epsilon_3$.

(a) dot-line: $\tan \beta = 1.5, M_{1/2} = 300 GeV, \epsilon_3 = 10 GeV$

(b) dash-line: $\tan \beta = 4.5, M_{1/2} = 300 GeV, \epsilon_3 = 10 GeV$

(c) dot-dash-line: $\tan \beta = 12.5, M_{1/2} = 300 GeV, \epsilon_3 = 10 GeV$

(a) dot-line: $\epsilon_3 = 5 GeV, M_{1/2} = 300 GeV, \tan \beta = 4.5$

(b) dash-line: $\epsilon_3 = 20 GeV, M_{1/2} = 300 GeV, \tan \beta = 4.5$

FIG. 2. the $M_{A_1}^2$ vary with $\epsilon_3$. 
FIG. 3. the $M_{H^+}^2$ vary with $\epsilon_3$.
FIG. 4. the $\kappa_1$ varied with $\epsilon_3$

(a) dot-line: $\tan \beta = 1.5, M_1 = 300 GeV, \nu_3 = 10 GeV$

(b) dash-line: $\tan \beta = 4.5, M_1 = 300 GeV, \nu_3 = 10 GeV$

(c) dot-dash-line: $\tan \beta = 12.5, M_1 = 300 GeV, \nu_3 = 10 GeV$

(a) dot-line: $\nu_3 = 5 GeV, M_1 = 300 GeV, \tan \beta = 4.5$

(b) dash-line: $\nu_3 = 20 GeV, M_1 = 300 GeV, \tan \beta = 4.5$

FIG. 4. the $M_{\kappa_1}$ varied with $\epsilon_3$
FIG. 5. The lowest-order cross sections of mixing production $\tau^\pm \tilde{\kappa}_1^\pm$ against $\varepsilon_3$ with

(a) dot-line: $\tan \beta = 4.5, M_{\tilde{\tau}} = 300 GeV, v_3 = 5 GeV, \sqrt{S} = 1000 GeV$;

(b) dash-line: $\tan \beta = 12.5, M_{\tilde{\tau}} = 300 GeV, v_3 = 5 GeV, \sqrt{S} = 1000 GeV$;

(c) dot-dash-line: $\tan \beta = 40.5, M_{\tilde{\tau}} = 300 GeV, v_3 = 5 GeV, \sqrt{S} = 1000 GeV$. 

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FIG. 6. The lowest-order cross sections of mixing production $\tau^\pm \tilde{\kappa}_1^\pm$ against $\nu_3$ with

(a) dot-line: $\epsilon_3 = -100 GeV, M_{1/2} = 300 GeV, \tan \beta = 4.5, \sqrt{S} = 1000 GeV$;

(b) dash-line: $\epsilon_3 = -100 GeV, M_{1/2} = 300 GeV, \tan \beta = 12.5, \sqrt{S} = 1000 GeV$;

(c) dash-dot-line: $\epsilon_3 = -100 GeV, M_{1/2} = 300 GeV, \tan \beta = 40.5, \sqrt{S} = 1000 GeV$. 