Selection of turbulence models via multiscaling analysis of an axisymmetric pipe flow and heat transfer

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ABSTRACT

To fully evaluate a turbulent flow, Direct Numerical Simulation (DNS) is the most accurate method by far and requires considerable computational power and time; not optimum for industry standards. Developing an alternative model, providing results with reasonable accuracy would resolve this issue. Reynolds Averaged Navier Stokes (RANS) modeling has proven its worth in addressing this phenomenon. In this study, we investigated the RANS turbulence models from COMSOL for fully developed single-phase flow in a two-dimensional axisymmetric pipe domain with constant heating at the wall and periodic boundary conditions at the inlet and outlet. Heat transfer in the fluid module has been added to address the heat transfer phenomenon. We evaluated the computed results with existing DNS data to match the accuracy of the RANS models. RANS simulations are conducted for friction Reynolds number, i.e., $Re_t = 180, 314, \text{ and } 395$ with varying Prandtl numbers, i.e., $Pr = 0.71, 2, 5, \text{ and } 7$. Multiscaling analyses in the flow's inner, outer, and meso scaling regions are performed for fluid and heat transfer profiles, i.e., mean streamwise velocity, Reynolds shear stress, mean streamwise temperature, and turbulent heat flux, to compare with the DNS data. The investigation reports the scaling analysis's effectiveness and shows that RANS turbulence models can be used to describe such flow with reasonable accuracy.

Keywords: Turbulent flow models, DNS, RANS, COMSOL Multiphysics.
### Nomenclature

| Symbol | Description                                                                                   |
|--------|-----------------------------------------------------------------------------------------------|
| 𝐶_ƒ   | Skin Friction coefficient, \( \tau_w / (0.5 \rho U_b^2) \)                                  |
| 𝐶_𝑝   | Specific Heat Capacity                                                                         |
| \( h \) | Heat transfer coefficient, \( q_w / (T_w - T_b) \)                                          |
| 𝐾     | Turbulent Kinetic Energy                                                                       |
| \( \text{Nu} \) | Nusselt Number, \( h^*2R/k \)                                                              |
| \( \text{Pr} \) | Prandtl Number \( \left( \mu C_p / K \right) \)                                              |
| \( q_w \) | Uniform wall heat flux                                                                         |
| \( \mu \) | Dynamic viscosity                                                                             |
| \( \beta \) | Coefficient of volume expansion                                                               |
| \( T_b \) | Bulk temperature                                                                             |
| \( \nu \) | Kinematic viscosity                                                                           |
| \( \rho \) | Density                                                                                       |
| \( \tau \) | Non-dimensional time                                                                           |
| \( T \) | Wall Temperature                                                                              |
| \( \tau_w \) | Wall shear stress                                                                             |
| \( \Theta^+ \) | Non-dimensional Temperature \( (T_w - T) / T_\tau \)                                        |
| \( \theta \) | Non-dimensional temperature \( (T_w - T) / (T_w - T_b) \)                                       |
| \( u_\tau \) | Wall shear velocity, \( (\tau_w / \rho)^{0.5} \)                                             |
| \( v \) | Fluctuation                                                                                   |
| \( y \) | Wall normal distance from the wall                                                              |
| \( p^+ \) | Non-dimensional pressure term \( (P/\rho u_\tau^2) \)                                           |
| \( Re_D \) | Bulk Reynolds Number \( (uD/\nu) \)                                                            |
| \( Re_\tau \) | Frictional Reynolds Number \( (u_\tau R/\nu) \)                                            |

- \( T_\tau \) is the non-dimensional temperature, \( q_w / (\rho C_p U_b) \).
- \( y^+ = y u_\tau / \nu \), \( u_\tau^+ = u_\tau / \nu \), and \( \theta^+ = (T_w - T) / T_\tau \).
- \( \text{Average over the } x - \phi \text{ plane and time} \)
1. Introduction

Turbulence modeling is one of the most challenging problems in fluid mechanics. Numerical simulation of turbulent flow in a smooth pipe geometry is considered an ideal fluid mechanics problem due to its simple and benchmark geometry. However, it received less focus than other domains like channel flow due to the complexity of solving the Navier Stokes equation in a cylindrical coordinate system[1]. Turbulence modeling of heat transfer in smooth and wavy pipe flow has been conducted using direct numerical simulation (DNS) for different flow and thermal conditions[2][3]. DNS is the most accurate solver available till date due to its highly precise analysis with high fidelity solution and a massive range of information, including higher-order statistics[4]. Turbulent pipe flow in a heated medium has applications in the oil industry, drilling, offshore plants, and many other engineering applications[5]. Although DNS properly provides insight into the turbulent physics with higher-order accuracy, it requires considerable computing power, which is time-consuming and not industry standard [6]. The mean optimum properties of a fluid flow and heat transfer simulation sometimes serve the designer's purpose rather than finding the time dependent fluctuating behavior of the same obtained in a DNS simulation. For such cases, the average streamwise results are more important than the details to grasp the condition of the flow.

On the contrary, solving the flow using the averaged equation model, i.e., based on Reynolds Averaged Navier Stokes Equation (RANS), requires less computational power and reasonably accurate results to find the mean flow solution[7]. The generation of RANS models developed from separating the fluctuated and mean flow properties from the original equation and treating them differently. The fluctuated velocity components in the RANS models introduce the unknown term Reynolds stress in the momentum equation, which has been parameterized by different properties generating multiple RANS turbulence models. For the heat transfer application, model using RANS and energy equation, an accurate prediction of the flow field in the near-wall region by implementing wall function can be performed. Several research works have been done concerning RANS model assessment with various geometries and different applications[8–11]. It indicates an attractive focus on average equation models like RANS due to their equivalence in accuracy and computational efficiency. However, a detailed evaluation of RANS turbulence models for heat transfer in a turbulent pipe flow in different scaling regions still lacks investigation. Also, a limited comparative study has been done between the existing models like DNS and RANS.

As a prevalent canonical problem in fluid mechanics, turbulent pipe flow features one essential wall-bounded flow in nature and has been investigated in literature in the past and present. With the addition of the heat transport phenomenon, both Reynolds and Prandtl numbers effect becomes significant, and only a handful of investigations have been done considering both effect variations. In the heat transport phenomenon, the existence of self-similar mean properties of turbulent thermal boundary layer reveals the multi-scale analysis of turbulent flow and heat transfer. By proper scaling, all the mean profiles at different positions collapse onto a single curve, which helps to reveal the flow's insights. However, investigation shows that the variation of properties like normalized mean temperature profile changes rapidly with Prandtl number than Reynolds number [3], which demands a study of the combined effect of both the
parameters. Satake et al. [12], Piller [13], Redjeem-Saad et al. [14], Saha [15], Peng et al. [6] carried out DNS of turbulent heat transfer in pipe flows. Zheng et al. [16] performed a DNS simulation for Non-Newtonian flow using OpenFOAM. Investigation of heat transfer in a turbulent pipe flow using the RANS equation is scarce. Thakre and Joshi [17] and Dhotre and Joshi [18] investigated turbulent heat transfer in pipe flow by solving the RANS equation using the finite volume method. Saha et al. [19,20] studied the accuracy of different numerical methods and the importance of the governing parameters, which influence the resulting thermal statistics and must be considered while validating DNS data with others. Literature review suggests investigations are available showing the effect of Reynolds number [21–23] and Prandtl number [14,24,25] in pipe flow using DNS. However, to our best knowledge, the combined effect of Reynolds and Prandtl number has not been systematically investigated in the context of RANS turbulence models for a turbulent pipe flow. A detailed study would undoubtedly fill the knowledge gap in choosing the best RANS models for simulating a pipe geometry.

The traditional representation of mean turbulent properties employs either inner or outer normalization. However, the study fails to result in a unique profile. At the same time, the non-dimensional parameters get changed, and they are not successful in the vicinity of peak turbulent heat flux. To define the thermal and momentum statistics accurately, a different length scale model, i.e., mesoscale analysis, an intermediate-length scale analysis first proposed by Afzal [26,27], in the transition between inner and outer scaling regions, proved to be worthy.

Wei et al. [28,29] studied the four-layer structure in turbulent pipe and channel flow with momentum equation-based properties like Reynolds shear stress in the vicinity of the meso-scaling region. Previous literature concerning the scaling behaviors of mean momentum and energy equation in pipe flow using DNS, LES, and other methods are reported [30–35]. However, a more comprehensive comparative study of the scaling behaviors of mean momentum and energy equation in the vicinity of RANS turbulence modeling is still lacking. To the best of our knowledge, no study has been done investigating the RANS turbulence models presented in COMSOL CFD modules [36], validating with the higher-order DNS data for a smooth pipe flow, and this motivates the present study.

In the current work, we present the RANS numerical simulation of a pressure-driven, fully developed turbulent flow in an axisymmetric pipe under periodic boundary conditions, with uniform heat flux in the wall. Here we performed a comprehensive systematic multiscale analysis of the mean fluid and heat transfer properties of a turbulent flow in a heated pipe using four different RANS models, namely Shear Stress Transport (SST), Spalart-Allmaras, Length VElocity (L-VEL), and Algebraic yPlus model available in COMSOL [36]. Results are described in two sections.; in the first section, we compared the four RANS models with DNS data to predict the best models, which have been used for further validation with varying Karman numbers and Prandtl numbers. The multiscale analysis clarifies the scaling properties admitted by the mean momentum and energy equation. To gain a better understanding of the invariant profiles of the mean flow, temperature, Reynolds shear stress, and turbulent heat flux; the current investigation uses the properties of distinct boundary layers
and a generalized form of the intermediate length scaling to reveal the comparative difference between different the RANS turbulence models and DNS of turbulent pipe flow and heat transfer by following the framework of Saha [19].

2. Mathematical Modelling

2.1 Physical model

The simulation geometry is composed of a smooth circular pipe of the radius (D= 2R) and length L coordinated in the cylindrical coordinate system. As the flow and thermal characteristics in the direction normal to the wall are symmetric about the central axis of the pipe, a two-dimensional axisymmetric model is considered, as shown in Fig. 1 for the analysis. The length to radius ratio (L/R) is taken to be 4π, and the periodic boundary conditions (PBC) are applied at two longitudinal extremes for fully developed turbulent pipe flow and heat transfer. At the wall no slip boundary condition with constant heat flux $q_w$ (W/m²) have been applied.

![Figure 1 Schematic diagram of the two-dimensional axisymmetric pipe with boundary conditions.](image)

2.2 Governing equations

Reynolds Averaged Navier-Stokes (RANS) equation is presented here in the vicinity of conservation of mass and momentum[36]. For a compressible and Newtonian fluid, the RANS equations are as follows:

$$ \rho \nabla \cdot \mathbf{u} = 0 $$

$$ \rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot (- \mathbf{P} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)) + \mathbf{F} $$

As the flow becomes turbulent, the RANS equation form with the average and fluctuating part as follows:

$$ \rho \nabla \mathbf{U} = 0 $$
\[
\frac{\delta \mathbf{U}}{\delta t} + \rho \mathbf{U} \cdot \nabla \mathbf{U} + \nabla (\rho \mathbf{u}' \otimes \mathbf{u}') = -\nabla P + \nabla \left( \mu (\nabla \mathbf{U} + (\nabla \mathbf{U})^T) \right) + \mathbf{F}
\]

Here, \( \mathbf{U} \) is the average velocity field, \( \otimes \) is the outer vector product, and \( \mathbf{F} \) is the streamwise force term.

The mean thermal energy equations

\[
\rho C_p \mathbf{U} \nabla T = \nabla [K \nabla T]
\]

Below are the non-dimensional parameters used to define the non-dimensionalized RANS equation.

\[
\begin{align*}
x^+ &= \frac{x}{D}; \quad u^+ = \frac{u}{u_c}; \quad \theta^+ = \frac{T_w - T}{T_T}; \quad p^+ = \frac{P}{\rho u_c^2} \\
Re_T &= \frac{u_T R}{v}; \quad Re_D = \frac{u D}{v}; \quad Pr = \frac{\mu C_p}{K}
\end{align*}
\]

Here, frictional temperature, \( T_T = \frac{q_w}{\rho c_p u_c} \).

2.4 Boundary conditions

At the two longitudinal ends of the geometry, the periodic boundary condition is considered where the fluid leaves through one end and enters from the other end. This boundary condition enables consideration of a smaller model to simulate fully developed pipe flow. In the model domain, flow is considered both hydro and thermally fully developed, meaning zero velocity and temperature gradient in the axial direction. No slip flow boundary condition is adopted in the pipe wall along with the zero dimensionless temperature. As the hydrodynamic and thermal properties are symmetric about its central axis in a pipe flow, only the top half of the two-dimensional pipe is modeled by imposing axisymmetric boundary conditions around the central axis.

2.5 Multiscale Analysis

Scaling analysis is used to provide insights into different important flow features that cannot be obtained from other sources. The flowing methods have been used by Saha [37].

**Inner scaling:** For inner scaling, the mean velocity \( u^+ = \frac{u}{u_c} \), mean temperature \( \theta^+ = \frac{T_w - T}{T_T} \), Reynolds shear stress \( -\langle u'_x u'_r \rangle^+ \) and radial heat flux \( -\langle u'_r \theta' \rangle^+ \) are plotted against \( y^+ = \frac{r - r_T}{R} Re_T \).

**Outer scaling:** For inner scaling, the mean velocity \( u = \frac{u}{u_c} \), mean temperature \( \theta^+ = \hat{\theta}^+ \), Reynolds shear stress \( -\langle u'_x u'_r \rangle^+ \) and radial heat flux \( -\langle u'_r \theta' \rangle^+ \). Mean velocity is plotted against \( r/R \), whereas others are plotted against \( \eta \).

**Meso scaling:** To define streamline temperature and radial heat flux, the following expressions are applied

\[
\hat{\theta} = (\theta^+ - \hat{\theta}_m^+)
\]
\[ \tilde{\phi} = \frac{\tau_\phi - \tau_{\phi m}}{\sqrt{\delta^+ p_r}} \]

These are plotted against \( \tilde{y} = \frac{1}{\delta^+} (y^+ - y_{m}^+) \)

For streamwise velocity profile and Reynolds shear stress following expressions are applied

\[ \begin{align*}
\tilde{u} &= (\tilde{U}^+ - \tilde{U}_{m}^+) \\
\tilde{T} &= \sqrt{\delta^+} (T^+ - T_{m}^+) 
\end{align*} \]

These are plotted against \( \tilde{y} = \frac{1}{\delta^+} (y^+ - y_{m}^+) \)

3. Simulation procedure

3.1 Finite element method

The finite element method (FEM) is a numerical procedure used to find the solution to boundary value problems containing elliptic partial differential equations. This method is not as complicated as others, e.g. Finite Difference method (FDM), because of the structured discretization of the physical geometry, which allows solving the equations in each element throughout the geometry. This means that FEM is more flexible when compared with the application window with respect to geometry.

3.2 Mesh generation

Two-dimensional axisymmetric geometry is segmented into small elements to apply FEM. Quadrilateral meshing is used to discretize the pipe domain. As the radial distance from the wall increases, the distribution of meshing is changed from finer to coarse. Extremely fine meshing is taken at the near wall, and coarse meshing is taken in the symmetry region. This distribution of meshing is considered to capture the radically changing velocity and thermal field in the near wall region with higher accuracy. Meshing applied to the geometry is shown in figure 2.

Figure 2 Meshing for two-dimensional axisymmetric model geometry.

The meshing used in these simulations is composed of 100×40 element i.e., 100 elements in the axial direction and 40 in the radial direction. The same simulation was carried out using 150×40 and 100×80 element meshing but variation in Nusselt number and skin friction...
coefficient is observed to be insignificant. So, meshing with 100×40 elements is accepted and used for higher $Re_\tau$ and $Pr$ cases.

4. Results and Discussion

Four different RANS turbulence models, i.e., Shear Stress Transport (SST), Spalart-Allmaras, Length VElocity (L-VEL from hereon), and Algebraic yPlus model available in COMSOL[36], are employed to simulate the two-dimensional axisymmetric pipe at different turbulent Reynolds numbers, i.e., $Re_\tau = 180, 314, \text{ and } 395$, and Prandtl numbers, i.e., $Pr = 0.71, 1, 2, 5, \text{ and } 7$. At different length scaling, mean flow and thermal properties are calculated and compared with the DNS data obtained by Saha [33] to evaluate the performance of the RANS models.

The present study is divided into two sections. In the first step, we have carried out a model assessment of the RANS turbulent models by comparing the mean velocity and Reynolds shear stress profile at the inner, outer, and meso scaling region at the Reynolds number $Re_\tau = 395$. Mean temperature and Radial heat flux is plotted for the same scaling for the highest Prandtl number $Pr = 7$ at $Re_\tau = 180$. Then by validating these parameters with the available DNS data best RANS model has been chosen. The selected model has been verified further at three flow and thermal field scaling regions. For validation of turbulent fluid flow, mean velocity and Reynolds shear stress are plotted at the inner, outer, and meso scaling for different Reynolds numbers, $Re_\tau = 180, 314, \text{ and } 395$. For validation of thermal field, mean temperature and radial heat flux are plotted at different Prandtl numbers, $Pr = 0.71, 2, 5, \text{ and } 7$ at $Re_\tau = 180$.

4.1 Model assessment

4.1.1 Flow field assessment

Flow field parameters, mean velocity and Reynolds shear stress at the inner scaling region is shown in Figure 3. The friction velocity is used to normalize the mean velocity. Figure 3 represents the comparison of results obtained from the RANS simulation performed in the study with the DNS data of Saha at $Re_\tau = 395$. Optimum accuracy was achieved for RANS models with no slip conditions. Indeed, Spalart-Allmaras and SST turbulence models shows better agreement with DNS data at the inner scale of mean velocity with minimal deviation at the outer flow region. Algebraic yPlus and L-VEL model shows moderate deviation at the outer flow region. Analogous to the velocity field, Spalart-Allmaras and SST turbulence models present better agreement with the DNS data than Algebraic yPlus and L-VEL model for Reynolds shear stress. Among Spalart-Allmaras and SST turbulence models, the SST model is more accurately aligned with the DNS data overall.
Figure 3 Comparison of inner-scaled mean velocity (a) and Reynolds shear stress (b) profile for different turbulence models with DNS data at \( \text{Re}_\tau = 395 \).

Figure 4 represents the mean velocity and Reynolds shear stress at the outer scale normalized distance from the wall where centreline velocity have been used to normalize the mean velocity. Spalart-Allmaras and SST models appeared to be in good agreement with the DNS data for the velocity profile. SST models appeared to match well with the DNS data at the outer scaled Reynolds shear stress profile.

Figure 4 Comparison of outer-scaled mean velocity (a) and Reynolds shear stress (b) profile for different turbulence models with DNS data at \( \text{Re}_\tau = 395 \). Symbols are as in fig 3(a).

Figure 5 portrays the meso-scaling profile of mean velocity and Reynolds shear stress at \( \text{Re}_\tau = 395 \). The meso-scaled mean velocity profile is in good agreement with DNS data for both
Spalart-Allmaras and SST models. In contrast, the Algebraic yPlus and L-VEL models deviate away from the DNS data, and the SST model is in good agreement with the DNS data at the meso scaling region for the Reynolds shear stress profile.

Figure 5 Comparison of meso scaled mean velocity (a) and Reynolds shear stress (b) profile for different turbulence models with DNS data at $Re_τ = 395$. Symbols are as in fig 3(a).

4.1.2 Thermal field assessment

To investigate the mean temperature distribution and turbulent radial heat flux profile, high Prandtl number $Pr= 7$ at $Re_τ= 180$ is considered and plotted against the inner, outer, and intermediate meso scaling region, as illustrated in Figures 6, 7, and 8.

Mean temperature distribution for high Prandtl numbers against wall-normal distance is presented in both inner and outer scaling illustrated in Figures 6 and 7. The mean temperature is normalized by the friction temperature in inner scaling, while in outer scaling, it is normalized by the centreline temperature. For intermediate meso scaling we follow the proposed work of Saha [15]. The comparison of four turbulence models with DNS data reveals interesting points for selecting the appropriate RANS turbulence model to accurately predict the mean thermal field of pipe flow. It is clear from fig. 6 that the Spalart-Allmaras model provides more satisfactory results in inner scaling than the other turbulence models. For turbulent radial heat flux, the SST model matches quite well with DNS data [15]. But the deviation of mean temperature profile in the inner scaling becomes significant in the outer region for high $Pr$. 
Figure 6 Comparison of inner-scaled mean temperature (a) and turbulent radial heat flux (b) profile for different turbulence models with DNS data at Pr = 7 and Re$_t$ = 180. Symbols are as in fig 3(a).

The profile of mean temperature and turbulent heat flux is plotted against outer-normalized distance from the wall and is depicted in Fig. 7 for different turbulence models and DNS data. The outer-scaled mean temperature is in good agreement with DNS data for both SST and Spalart-Allmaras model. For turbulent radial heat flux Algebraic yPlus model provide the best validation with DNS than other models.

Figure 7 Comparison of outer-scaled mean temperature (a) and turbulent radial heat flux (b) profile for different turbulence models with DNS data at Pr = 7 and Re$_t$ = 180. Symbols are as in fig 3(a).
The profile of mean temperature and turbulent heat flux is plotted against intermediate meso-normalized distance from the wall and is depicted in Fig. 8 for different turbulence models and DNS data. The meso-scaled mean temperature is in good agreement with DNS data for both Spalart-Allmaras model as well as with SST model. For turbulent radial heat flux too Spalart-Allmaras and SST model provide best validation with DNS than other models.

Figure 8 Comparison of meso-scaled mean temperature (a) and turbulent radial heat flux (b) profile for different turbulence models with DNS data at Pr = 7 and Reτ = 180. Symbols are as in fig 3(a).

The result of the following assessment has been included in table 2.

Table 2: Assessment of RANS turbulent models with DNS.

| Governing Parameters | Turbulent Statistics | Inner scaling       | Outer scaling      | Meso scaling       |
|----------------------|----------------------|---------------------|--------------------|--------------------|
| Reτ = 395            | Mean velocity        | Spalart-Allmaras    | SST, Spalart-     | SST, Spalart-     |
|                      |                      |                     | Allmaras           | Allmaras           |
|                      | Reynolds shear       | SST                 | SST               | SST               |
|                      | stress               |                     |                   |                   |
| Reτ = 180, Pr = 7    | Mean temperature     | Spalart-Allmaras    | SST, Spalart-     | SST, Spalart-     |
|                      |                      |                     | Allmaras           | Allmaras           |
|                      | Radial heat flux     | SST                 | Algebraic yPlus   | SST               |
|                      |                      |                     |                   |                   |

From the above assessment, it can be shown that the Spalart-Allmaras and SST model offers satisfactory results with DNS. The following section uses the SST model to further validate the flow and thermal properties using the scaling analysis with a higher Reynolds and Prandtl number variation.
4.2 Further validation of various scaling properties

4.2.1 Properties of turbulent fluid flow

As fluid flow is independent of Prandtl number, for verification of turbulent fluid flow properties of SST model, simulation results are validated with the DNS data of Saha[15] for different frictional Reynolds number \( Re_\tau = 180, 314, \) and 395. Mean velocity and Reynolds shear stress for different Reynolds numbers against wall-normal distance are presented in inner scaling illustrated in fig. 9. The figure reveals that the SST model provides satisfactory results compared with the corresponding DNS data for all the cases. The figure indicates that the matching at the inner region is good, but as the distance increases from the wall and the Reynolds number, the deviation increases.

![Figure 9](image)

(a) Comparison of Inner-scaled mean velocity (a) and Reynolds shear stress (b) profile for SST turbulence models with DNS at \( Re_\tau = 180, 314, \) and 395. Here ○ represents \( Re_\tau = 180, \) ◊ for 314 and □ for 395. All solid of the corresponding symbol represents the RANS SST data.

Mean velocity and Reynolds shear stress for different Reynolds numbers mentioned above has been presented in the outer scaling as illustrated in fig. 10. The figure reveals that the SST model provides satisfactory results for all the cases for mean velocity and deviation in the Reynold shear stress as the frictional Reynolds number increases and flow becomes more turbulent. This can be attributed to the fluctuating component of the Reynolds shear stress that has not been taken into the RANS model.
Figure 10 Comparison of Outer-scaled mean velocity (a) and Reynolds shear stress (b) profile for SST turbulence models with DNS at $Re_\tau = 180$, 314 and 395. Here ○ represents $Re_\tau = 180$, ◊ for 314 and □ for 395. All solid of the corresponding symbol represents the RANS SST data.

Mean velocity and Reynolds shear stress (RSS) at the meso scaling region are illustrated in fig. 11. It can be seen that the mean velocity follows a similar trend turned into a single graph with accurate validation with the DNS data. A similar observation can be seen for RSS, where little deviation can be seen between the DNS and RANS data at the outer region of the scaling. The figure reveals that the SST model provides satisfactory results for all the cases.

Figure 11 Comparison of meso-scaled mean velocity (a) and Reynolds shear stress (b) profile for SST turbulence models with DNS at $Re_\tau = 180$, 314 and 395. Here ○ represents $Re_\tau = 180$, ◊ for 314 and □ for 395. All solid of the corresponding symbol represents the RANS SST data.
4.2.2 Properties of turbulent Heat Transfer

For verification of turbulent heat transfer properties of the SST model, simulation results are validated with DNS data from moderately low to high Prandtl number numbers, $Pr = 0.71, 2, 5, 7$ for $Re_\tau = 180$ and 395 for inner, outer, and meso scaling. For $Re_\tau = 180$, simulations have been conducted for $Pr = 0.71, 2, 5,$ and 7 have been considered for RANS simulation as presented in Table 3.

Table 3: Symbols represent the various scaling of SST turbulence model with DNS [15].

| Simulation Procedure | $Re_\tau$ | $Pr$ | Symbol |
|----------------------|-----------|------|--------|
| DNS (SEM) [15]       | 180       | 0.71 | ○      |
|                      |           | 2    | ◊      |
|                      |           | 5    | □      |
|                      |           | 7    | △      |
| RANS : SST (FEM)     | 180       | 0.71 | ●      |
|                      |           | 2    | ♦      |
|                      |           | 5    | ■      |
|                      |           | 7    | ▲      |

Mean temperature and turbulent radial heat flux for different Prandtl numbers are plotted against wall-normal distance presented in inner scaling illustrated in fig. 12. The figure reveals that the SST turbulent model provides promised results for most cases. Deviation from DNS data increases as the Reynolds and Prandtl number goes higher but matches well at the inner level. The difference can be attributed to the fluctuating component in the mean thermal properties that have not been modeled in the RANS simulation. But the RANS simulation results follow a similar close trend to the DNS results, which validates the purpose of this study to give a sense of the mean properties rather than the exact solution at the expense of time and computation power.

Figure 12 Comparison of inner-scaled mean temperature (a) and turbulent radial heat flux (b) profile for SST turbulence model at $Pr = 0.71, 2, 5, 7$ at $Re_\tau = 180$ and $Re_\tau = 395$, $Pr = 0.71, 1$. Symbols are same as in table 3.
Mean temperature and radial heat flux for varying Prandtl numbers in the outer scaling are illustrated in fig. 13. DNS and RANS results follow a similar trend and validate that for SST turbulent model can provide promised results for all the cases.

Figure 13 Comparison of outer-scaled mean temperature (a) and turbulent radial heat flux (b) profile for SST turbulence model at Pr = 0.71, 2, 5, and 7 at Reτ = 180. Symbols are same as in table 3.

Mean temperature and radial heat flux for varying Prandtl numbers against the meso scaling region are illustrated in fig 14. Both the exact solution (DNS) and average solution (RANS) have been agreed upon well at both inner and outer parts of the meso scaled region.

Figure 14 Comparison of meso-scaled mean temperature (a) and turbulent radial heat flux (b) profile for SST turbulence model at Pr = 0.71, 2, 5, and 7 at Reτ = 180. Symbols are same as in table 3.
5. Conclusion

This study evaluated the effectiveness of four different turbulent models from COMSOL to accurately predict the mean flow and thermal properties in a two-dimensional axisymmetric pipe flow with a constant heat source at the wall and periodic boundary conditions at the inlet and outlet. We compared the simulation result with the accurate DNS data of similar geometry and orientation. Results show that the mean flow and thermal properties obtained from RANS simulation in the current study agree well with the DNS data. As the Reynolds and Prandtl number increases, the different turbulent profile in the inner, outer, and meso scaling region seems to deviate more from the DNS. Further study needs to be done using higher Reynolds and Prandtl numbers for flow and thermal fields. This study shows that the Spalart-Allamaras and SST model agrees well with DNS data [15] for both flow and thermal field statistics. In addition to that, it shows the effectiveness of the scaling analysis framework followed in this work to accurately describe the fluid and heat transfer phenomenon in the inner, outer, and meso scaling region of a turbulent pipe flow.

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