Interaction of Mesoscopic Magnetic Textures with Superconductors.

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We analyze magnetic textures in a thin magnetic film and screening currents induced by these textures in a superconducting film. The two films are supposed to be parallel and close each to other, but interact only via magnetic fields. We consider also vortices inside superconducting film and their interaction with magnetic textures. As examples of magnetic textures we consider magnetic dot and magnetic vortex. We derive the condition at which spontaneous formation of a vortex or a coupled vortex-magnetic defect is energetically favorable. It is proven that the normal component of magnetic field generated by a magnetic dot changes sign in the presence of the superconducting vortex.

74.60.Ge, 74.76.-w, 74.25.Ha, 74.25.Dw

The fabrication and experimental study \cite{1} of mesoscopic heterogeneous magnetic/superconducting systems together with recent theoretical predictions \cite{2,3} open a new class of physical effects. Earlier two of us (I.F.L. and V.L.P.) proposed to separate superconductivity and magnetism in space employing the modern technique of nanofabrication \cite{4}. The proximity effect which suppresses both order parameters can be easily avoided by growing insulator oxide layers between ferromagnetic (FM) and superconducting (SC) components. Inhomogeneous magnetization of the magnetic texture generates magnetic field penetrating into the superconductor. The magnetic field from the SC currents interacts with magnetic sub-system \cite{5}. We have proposed different realizations of mesoscopic magno-superconducting systems: arrays of magnetic dots on the top of a superconducting film \cite{6}, magnetic/superconducting bi-layers \cite{7}, magnetic nanorods embedded into a superconductor \cite{8}.

In the majority of proposed systems a magnetic texture interacts with the superconducting current. An inhomogeneous magnetization generates magnetic field outside the magnet. The magnetic field generates screening currents in superconductors which, in turn, change the magnetic field. The problem must be solved self-consistently \cite{9}. In this article we develop a general formalism for interacting inhomogeneous magnetization (texture) and superconductors in the London’s approximation. Employing this formalism, we find two elementary solutions for a circular magnetic dot on the top of a superconducting film, magnetized perpendicularly to it, and for a coupled magnetic and superconducting vortices. London’s approximation works satisfactory since the sizes of all structures in the problem exceed remarkably the coherence length $\xi$.

Recent theoretical works \cite{10,11} consider problems close to ours. The main difference between our work and the Ref. \cite{12} is that they do not incorporate spontaneous vortices in the ground state. In comparison to the Refs. \cite{12,13} our approach is more general and universal. A more detailed comparison with these references will be given later.

So far only arrays of sub-micron size magnetic dots covered by thin superconducting films have been prepared and studied experimentally \cite{14,15}. The effect of commensurability on the transport properties was reported. This effect is not specific for magnets interacting with superconductors and was first found many years ago by Martinoli and his group \cite{16}. Predicted effects specific for FM/SC systems, including topological instability of homogeneous state in bi-layers \cite{17}, were not yet observed experimentally.

General formalism The total energy of a stationary magnet-superconducting system reads:

$$E = \int \left[ \frac{B^2}{8\pi} + \frac{m n_s v_s^2}{2} - B \cdot M \right] dV$$

(1)

where $B$ is the magnetic induction, $M$ is the magnetization, $n_s$ is the density of superconducting carriers, $v_s$ is their velocity and $m$ is their effective mass. We assume that the superconducting density $n_s$ and the magnetization $M$ are separated in space. We assume also that the magnetic field $B$ and its vector-potential $A$ asymptotically turn to zero at infinity. Employing Maxwell equation $\nabla \times B = \frac{4\pi}{c} j$, and $B = \nabla \times A$, the magnetic field energy can be transformed as follows:

$$\int \frac{B^2}{8\pi} dV = \int \frac{j \cdot A}{2e} dV$$

(2)

The current $j$ can be represented as a sum: $j = j_s + j_m$ of the superconducting and magnetic currents, respectively:

$$j_s = \frac{n_s \hbar e}{m} (\nabla \varphi - \frac{2\pi}{\phi_0} A); \quad j_m = e \nabla \times M.$$

(3)
In the gauge invariant Eq. (3) $\phi$ is the phase of the SC carriers wave-function and $\phi_0$ is the (SC) flux quantum. Plugging Eq. (3) into Eq. (4) and Eq. (5), after minor transformations, we arrive at following equation for the total energy:

$$H = \int \left[ \frac{n_s h^2}{2m} (\nabla \phi)^2 - \frac{n_s h e}{2mc} \nabla \phi \cdot \mathbf{A} - \frac{B \cdot M}{2} \right] dV$$ (4)

The phase gradient $\nabla \phi$ can be included into $\mathbf{A}$ as a gauge transformation with the exception of vortex lines, where $\phi$ is singular. Eq. (4) allows to separate the energy of vortices from the energy of magnetization induced currents and fields and from the interaction energy.

**Two dimensional textures and vortices.** Below we apply our general formalism to the case of two parallel films, one FM, another SC, both very thin and very close to each other. Neglecting their thickness, we assume that both films are located at $z = 0$. The super-carrier density $n_s(r)$ can be represented as $n_s(r) = \delta(z)n_s^{(2)}(r)$ and the magnetization $M(R)$ can be represented as $M(R) = \delta(z)m(r)$, where $n_s^{(2)}(r)$ is the super-carrier density per unit area, $m(r)$ is the magnetization per unit area and $r$ is the two-dimensional radius-vector. In what follows $n_s^{(2)}$ is assumed to be a constant and the index (2) is omitted. The energy (4) can be rewritten for this special case:

$$H = \int \left[ \frac{h^2}{2m} (\nabla \phi)^2 - \frac{h e}{mc} \nabla \phi \cdot \mathbf{a} - \frac{\mathbf{b} \cdot \mathbf{m}}{2} \right] d^2r$$ (5)

where $\mathbf{a} = \mathbf{A}(r, z = 0)$ and $\mathbf{b} = \mathbf{B}(r, z = 0)$. The vector-potential satisfies Maxwell-London’s equation:

$$\nabla \times (\nabla \times \mathbf{A}) = -\frac{1}{\lambda} \mathbf{A} \delta(z) + \frac{4\pi h e}{mc} \nabla \phi \delta(z)$$ (6)

$$+ 4\pi \nabla \times (\mathbf{m} \delta(z))$$

where $\lambda = \lambda_L / d_S$, is the effective screening length for the SC film, $\lambda_L$ is the London penetration depth and $d_S$ is the SC film thickness.

According to our arguments, the term proportional to $\nabla \phi$ in Eq. (6) describes vortices. A plane vortex characterized by its vorticity $q$ and by position of its center on the plane $r_0$ contributes a singular term $q \frac{\hat{z} \times (r - r_0)}{|r - r_0|}$ to $\nabla \phi$ and generates a standard vortex vector-potential:

$$\mathbf{A}^v(r) = \frac{q \phi_0}{2\pi} \frac{\hat{z} \times (r - r_0)}{|r - r_0|} \int_0^\infty J_1(k|r - r_0|) e^{-k|z|} dk$$ (7)

Different vortices contribute independently into the vector-potential and magnetic field. Below we calculate the contribution of a magnetic texture to the vector potential $\mathbf{A}^m(R)$ in a special gauge $A_z = 0$. The magnetization vector field can be represented as follows:

$$\mathbf{m}(r) = m_z(r) \hat{z} + \mathbf{m}^\parallel(r) + \mathbf{m}^\perp(r)$$ (8)

where $\mathbf{m}^\parallel(r)$ is the gradient part of $\mathbf{m}(r)$ and $\mathbf{m}^\perp(r)$ can be represented as a vector product $\hat{z} \times \nabla f$ ($f$ is an arbitrary function of $r$). This representation is unique. The corresponding contributions to $\mathbf{A}^m$ are:

$$\mathbf{A}^m(R) = \mathbf{A}^\parallel(r) + \mathbf{A}^\perp(r)$$ (9)

where

$$\mathbf{A}^\parallel(r) = 2\pi \mathbf{m}^\perp(r) \times \hat{z}$$ (10)

$$A^\perp_j(R) = \int G_{j\beta}(R - r') m_l(r') d^2r'$$

where non-zero component $G_{j\beta}(R - r')$ are $(R = (r, z))$:

$$G_{\alpha z} = \epsilon_{\alpha \beta \gamma} x_\beta - x_\alpha \frac{2}{|r - r'|} \frac{\lambda^2}{F_1\left(\frac{|r - r'|}{\lambda}, \frac{z}{\lambda}\right)}; \quad \alpha = x, y$$ (11)

$$G_{xy} = -G_{yx} = \frac{2}{\lambda^2} F_0\left(\frac{|r - r'|}{\lambda}, \frac{z}{\lambda}\right)$$

Here:

$$F_1(\eta, \zeta) = \int_0^\infty J_1(\eta k) e^{-k|\zeta|} k^2 dk; \quad \eta = 0, 1$$ (12)

$\epsilon_{\alpha \beta \gamma}$ is the second rank antisymmetric tensor and $J_1(x)$ are the Bessel functions.

Equations (13-12) can be further simplified if the magnetization has only $z$ and $r$ components and both depend only on $r$. Then the vector-potential $\mathbf{A}^m_0(R)$ is directed along azimuthal lines and its only non-zero component reads:

$$A^m_0(R) = \sum_{\alpha = 0}^1 \int K_\alpha(R, r') m_\alpha(r') d^2r'$$ (13)

where $\alpha = 0$ stays for $z$, and $\alpha = 1$ stays for $r$.

$$K_\alpha(r, r') = \frac{4\pi}{\lambda^2} f_\alpha\left(\frac{r}{\lambda}, \frac{r'}{\lambda}, \frac{z}{\lambda}\right)$$ (14)

and

$$f_\alpha(\xi, \eta, \zeta) = \int_0^\infty J_1(\xi k) J_\alpha(\eta k) e^{-k|\zeta|} k^2 dk.$$ (15)

Santos et al. [3] have developed a formalism for calculation of magnetic fields and screening currents generated by magnetic textures in superconductors similar to ours. However, they did not consider singular current distributions, i.e. vortices. Bulaevsky et al. [8] considered a special system of prepared domains in a thick magnetic slab contacting with a semi-infinite superconductor and generating vortices in it. The spontaneous formation of domains and chains of vortices was discussed earlier [10][4] for a bi-layer composed from very thin films.

**Magnetic Dot.** For infinitely thin circular magnetic dot with magnetization $\mathbf{m} = m \hat{z}$ and radius $R$ the magnetic
field can be calculated using Eq. [3][14]. Its z-component has the form:

\[ B_z(r, z) = 4\pi\lambda mR \int_0^\infty J_0(kr)J_1(kR)e^{-k|z|} \frac{k^2dk}{1 + 2\lambda k} + \frac{q\phi_0}{2\pi} \int_0^\infty J_0(kr)ke^{-k|z|} \frac{dk}{1 + 2\lambda k} \]  

(16)

Numerical calculation based on Eq. [16] show that \( B_z \) on the film \((z = 0)\) changes sign at some \( r > R \) if \( q \) is positive (see Fig. 1), but it is negative everywhere at \( r > R \) if \( q = 0 \). The physical reason for this behavior is simple: the dot itself, without vortex currents, generates dipolar field which has the sign opposite to dipolar moment in the plane passing through the dipole and perpendicular to it. The SC current resists this tendency. At small distances from the center the screening is negligible and the field generated by dot dominates. At large distances \( r \gg \lambda \) the field generated by a vortex decays like \( 1/r^3 \), whereas the screened field of the dot decays as \( 1/r^4 \). The position of node \( r_0 \) can be easily determined if \( R \ll \lambda \): then \( r_0 = 2\pi\sqrt{mR^2/\lambda\Phi_0} \). Thus, the measurement of magnetic field near the film may serve as a diagnostic tool to detect SC vortex bound by the dot. The experimental measurements [13] demonstrated that \( B_z \) changes sign, i.e. the dot indeed generates the vortex.

In the presence of a vortex with charge \( q \), the energy of the system can be calculated using Eqs. [1][14]. The result is:

\[ E = q^2\epsilon_0\frac{\lambda}{\xi} - q\epsilon_m \frac{R}{2\lambda} - 2\pi m^2 R\ln|\frac{R}{a}| \]  

(17)

where \( \epsilon_m = m\Phi_0 \) is the characteristic SC/FM interaction energy. The vortex becomes energy favorable if \( \epsilon_mR/(2\lambda) > \epsilon_0 \) or \( mR > |\Phi_0/(8\pi^2)|\ln(\lambda/\xi) \). The ratio

\[ \delta = \frac{\epsilon_m}{\epsilon_0} = S\frac{2n_md_m}{n_sd_s} \]  

(18)

is the relative strength of the SC/FM coupling. In Eq. (17) \( n_m \) is the density of magnetic atoms, \( d_{m,s} \) are the thicknesses of magnetic and superconducting films, \( S \) is the value of an elementary spin in the magnet and \( g \) is the Lande factor. Far from the threshold the vorticity \( q \) generated by the magnetic dot is an integer closest to the value

\[ \tilde{q} = \frac{\delta R}{4\ln(\frac{\lambda}{\xi})} \]  

(19)

The logarithmic factor in Eq. (19) varies in the range 3-8. For typical values \( R = 10^{-4}\text{cm} \), \( d_m = 10^{-6}\text{cm} \), and \( n_m = 10^{22}\text{cm}^{-3} \), we find \( \tilde{q} > 1 \). Eq. (19) shows that, apart the logarithmic factor, the SV generation is controlled by properties of the magnetic film. The characteristic scale in superconducting film is \( \lambda \). For this reason, at \( R \approx \lambda \), the condition \( \delta \approx 1 \) gives an approximate criterion for the spontaneous vortex formation.

\[ A_x(R) = -4\pi m\lambda \int_0^\infty \frac{J_1(kr)e^{-k|z|}}{1 + 2\lambda k}dk \]  

(21)

![Fig. 1. Normal to the dot magnetization as a function of distance from the dot's center measured in the units of dots radius R. Top: without vortex, for \( \lambda = 20R \); Bottom: with vortex the case when \( \phi_0/(4\pi^2m) = 4 \) and \( \lambda = 20R \).](image-url)

Sasik and Hwa [5] mimicked the magnetic dots by magnetic dipoles. This approach does not reflect properly the real dot geometry. As it is seen from our results, the magnetic field and energy can not be expressed in terms of the total dot magnetic moment \((\pi R^2 m)\) only.

**Coupled Magnetic/Superconducting Vortices.** Earlier we predicted spontaneous formation of the pairs of bound superconducting and magnetic vortices in the case of bilayer consisting from superconducting and magnetic film with easy plane anisotropy [3]. A detailed analysis given below confirms this prediction, but reveals more restrictive criterion for this phenomenon. The exchange Hamiltonian of the magnetic layer reads:

\[ \mathcal{H}_M = \frac{J_M}{2} \int d^2r(\nabla n)^2 \]  

(20)

where \( n(r) \) is the unit 2D vector directed along the local magnetization. Further we assume that the vector \( n \) lays in plane (XY symmetry.) In a single magnetic vortex (MV) the local magnetization is directed along the radius: \( n(r) = r/r \). The energy (20) is invariant under a global rotation of all spins by the same angle. The MV energy grows logarithmically with the system size \( L \): \( E \approx \pi J_M \ln(L/\xi) \). We employ general formulæ, [4][17] to calculate the vector-potential, magnetic field and the total energy. The vector-potential induced by the vortex, including the effect of screening currents is:

\[ A_x = -4\pi m\lambda \int_0^\infty \frac{J_1(kr)e^{-k|z|}}{1 + 2\lambda k}dk \]  

(21)
Employing the standard vortex vector potential Eq. [6] with $R=0$ and eq. [9], we calculate the energy of the MV-SV pair with logarithmic accuracy:
\[
E = q^2 \epsilon_0 \ln\left(\frac{\lambda}{\xi}\right) - q \frac{\epsilon_{m}}{2} \ln\left(\frac{L}{\lambda}\right) + 8\pi^2 m^2 \lambda \ln\left(\frac{L}{\lambda}\right) \tag{22}
\]

The exchange energy of the magnetic vortex $E_{ex}$ is typically by 1-2 order of magnitude smaller than $\epsilon_0$ and can be neglected. The energy is minimal at $q$ being an integer part of the value $\tilde{q} = \frac{mF_0 \ln(\frac{L}{\lambda})}{2\epsilon_0 \ln(\frac{L}{\lambda})}$. The coupled pair of SC/FM vortices becomes energy favorable when minimum is realized at $q = 1$. It means that $\delta$ must overcome a threshold value $\delta_c = \frac{\ln(\lambda/\xi)}{\ln(L/L)}$. In the limit $L \rightarrow \infty$, due to the logarithmically divergent second term in Eq. 22, the magnetic/superconducting vortex pair is always energetically favorable. However, in practice the logarithmic factor never exceeds $5 - 7$. Therefore, the value $\delta$ exceeding 1 triggers generation of the MV/SV pairs. The value of $\delta$ can be enhanced by either reducing the supercarrier density $n_s$ or increasing the thickness of the magnetic film.

In a bi-layer of XY ferromagnet and SC films with Curie temperature $T_c$ larger than SC transition temperature, the ferromagnetic state will be observed in the temperature interval between $T_c$ and $T_s$. Below $T_s$ the MV/SV vortices proliferate. They destroy both, ferromagnetism and superconductivity. However, the reentrant phase transition at $T = T_V$ leads to restoration of both order parameters if $\delta(T = 0)$ is smaller than the logarithms ratio.

The vortices of the same sign repulse each other, whereas the antivortex attracts vortex. Together they form a dense system, most likewise a crystal at low temperature, with the lattice constant $d \sim \lambda$, since $\lambda$ is the only characteristic length in the system. This is a new phase, never observed in 2d systems before. It differs from a vortex crystal in the 3d superconductors in magnetic field. It is also very different from the Berezinskii-Kosterlitz-Thouless zero-field liquid vortex phases having no bound vortex-antivortex pairs.

In conclusion, we presented a general formalism to treat the interaction between magnetic textures and superconductors. As applications, we have shown that vortices in superconducting films can be generated by magnetic dots with normal to the film magnetization. Superconducting vortices also appear in bound pairs with magnetic vortices in a homogeneous, easy-plane magnetic film. We have found how the magnetic dot size and its material control the vortex appearance. Spontaneous generation of the MV/SV pairs in the bi-layer of the XY magnetic and SC films starts when characteristic FM-SC interaction energy $\epsilon_{m}$ exceeds the energy $\epsilon_0$ necessary for creation of a single SV. We have proposed experiments to detect the bound state of the magnetic dot and vortex and coupled pairs of magnetic and superconducting vortices.

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