Optimization of bearing capital of structural roof slab

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Abstract. The problem of exploring the optimum alternative of bearing capital of structural roof slab design solution is set and worked out in the article. As a method of searching for the optimum alternative, the parametric optimization method is used. Previously, a number of examples illustrate the application of this method. Then one variable parameter is chosen so that its changing provides a graph of objective function behavior, which in turn provides an optimal constructive solution according to the assumed optimality criterion. Constraints of the problem are strength and sustainability regulatory requirements and also a number of design parameters. Metal consumption was assumed as an optimality criterion. The work presents efficiency and relevance of optimization procedures while searching for design solutions at the early (conceptual) stage of design, which provide for an engineer the necessary information to choose reasonable design solutions based on options analysis.

1. Introduction
The problem of searching for the most efficient design solutions is traditionally solved at the stage of trial design [1,2,3].

In engineering tasks the problem of finding an optimal solution can be written as follows [4]:

\[ \text{It is necessary to minimize } f_i(x), \ x \in \mathbb{R}, \ (i = 1,2,3,\ldots,N) \] 
(1)

On condition, that

\[ h_j(x) = 0, \ (j = 1,2,3,\ldots,f) \] 
(2)

\[ g_k(x) \leq 0, \ (k = 1,2,3,\ldots,K) \] 
(3)

Where \( x = (x_1,x_2,\ldots,x_n) \) represent the vector of searching for the solutions which contains design variables.

The objective function \( f_i(x) \) includes design constraints as equalities \( h_j(x) \) and inequalities \( g_k(x) \), which are the elements of the design vector \( x \).

2. Problem statement
In the optimization a number of various search algorithms are used [5,6], initial design parameters are set randomly, the constraints are defined and the objective function is calculated. The project constraints shape the objective function. The set of design variables, which form the design vector, generates alternative design solutions. This initial part of the process is the same for all search algorithms. They can be further divided into two major groups - parametric optimization and structural optimization. Parametric optimization methods are related to the selection of design variables [7-11],
structural optimization methods change the structural topology according to special principles, improving the objective function. The second group includes evolutionary algorithms [12-14].

In presented study the optimality criterion is metal consumption and the objective function is as follows:

$$\min G = A \cdot l \cdot p$$  \hspace{1cm} (4)

where

- $A$ – Cross-Sectional area, cm$^2$
- $l$ – Element length, cm
- $p$ – Steel density, gr/cm$^3$

3. Methods and results of the study

In this work the parametric optimization method is applied.

![Figure 1. Suspension structure scheme. a - application, b – design scheme](image_url)

Example 1 presents a steel suspension structure (fig.1). The two suspensions of the same type and cross-section size are hingedly fixed in the point A and operated by the force P. While changing the angle $\alpha$ between the suspensions, the forces $T_1$ and $T_2$ between the suspensions are also changing. Therefore, according to the law: $A = \frac{T}{Ry Ye}$, the demanded cross-section of the elements is changing. The more angle $\alpha$ is, the less operating force is. However if we put the design constraints on the base of suspension B, the length of suspensions will decrease and the overall mass changing law will not be apparent. Finding the design solution corresponding with the minimal mass of the structure, i.e. the optimal solution, demands the solution of trial design problem based on parametric optimization method.

Let us assume the following variant parameters of the suspension structure (tab. 1).

| Table 1. |
|-----------------|-----------------|
| **Objective function** | **Metal consumption** |
| Constraints | 1. The steel class is C245 2. Hinge attachment 3. The number of suspensions is n=2 4. Base of suspension is B=3500 мм |
| Variable parameter | Angle $\alpha$ |
The force \( P \) assumed as 500 kN.

According to the set of rules 16.13330.2017, the design constraints for using the steel with \( R_{yn} \leq 44 \) kN/cm² are:

\[
\frac{P}{A_n R_{y} \gamma_c} \leq 1
\]  

(5)

The strength calculation of stretched elements, which can be operated even after metal reaches its yield strength, and strength calculation of stretched steel elements with normal resistance \( R_{yn} > 44 \) kN/cm² should be done by the formula (5) changing the \( R_{y} \) to the \( R_{u}/\gamma_u \)

The objective function will be

\[ G = n(A \cdot l \cdot \rho) \rightarrow \text{min} \]  

(6)

Where \( n=2 \) – the number of suspensions.

Increasing the number of suspensions decreases proportionally the size of the demanded cross-section. And, considering the apparency and the linearity of the dependence, the number of suspensions \( n \) is assumed as a nonvariable parameter (constant) and is not considered as a design variable.

The block-scheme of the optimization algorithm is presented at fig.2.

![Block-scheme of the objective function optimization process](image)

**Figure 2.** Block-scheme of the objective function optimization process

The calculation results are represented in the table 2 and graphically (fig.3).

| Angle,° | Stretching force, kN | Suspension length, cm | Demanded cross-sectional area, cm² | Suspension mass, kg. |
|---------|----------------------|-----------------------|------------------------------------|---------------------|
| 25      | 591,5                | 193,0                 | 24,6                               | 74,2                |
| 30      | 500,0                | 202,0                 | 20,8                               | 65,7                |
| 35      | 435,8                | 213,6                 | 18,2                               | 60,5                |
| 40      | 388,92               | 228,5                 | 16,2                               | 57,8                |
| 45      | 353,55               | 247,5                 | 14,7                               | 56,9                |
Analyzing the graph (fig. 3), it appears as following: for the scheme at fig. 1, the optimal angle between the suspensions, when the structure has its minimal mass, is 45°. While increasing or decreasing the size of angle $\alpha$, the mass of the structure is increasing, but, as the graph shows, changing the angle by the amount of $\pm 5$ degrees increases the mass only by 4%, what is inconsequential. In this example the cross-sectional area is assumed as a continuous function not considering the type of cross-section. When using a cross-section of a rope, strand, angle etc., the function graph takes in the discrete values of the cross-sectional areas and due to the gradation of the assortment.

The following example examines the construction of suspensions fixed at two points. (fig. 4).

![Figure 3. Graph of the objective function behavior](image)

![Figure 4. Calculation scheme](image)

### Table 3. Optimization parameters of the suspension construction

| Objective function | Metal consumption |
|--------------------|-------------------|
| Limitations        | 1. Steel category C245  
|                    | 2. Hinge fastening  
|                    | 3. Number of suspensions |
Load P is assumed to be 500 kN.

Using the formula (5) we select the required cross-sectional area. The results of the calculations performed are shown in Table 4 and Figure 5.

### Table 4. Determining the mass of the construction

| Angle $\alpha$, ° | Tensile force $T$, kN | Length, $L$, cm | Area, $cm^2$ | Mass 1 Cable, kg | Total Mass, kg |
|------------------|----------------------|-----------------|--------------|-----------------|---------------|
| 15               | 2.52                 | 353.02          | 0.105        | 28.9            | 57.8          |
| 25               | 2.56                 | 358.49          | 0.107        | 29.8            | 59.7          |
| 30               | 2.59                 | 362.34          | 0.108        | 30.5            | 61.0          |
| 35               | 2.62                 | 366.98          | 0.109        | 31.2            | 62.5          |
| 40               | 2.66                 | 372.45          | 0.111        | 32.2            | 64.4          |
| 45               | 2.71                 | 378.83          | 0.113        | 33.4            | 66.7          |
| 55               | 2.82                 | 394.57          | 0.117        | 36.2            | 72.3          |
| 60               | 2.89                 | 404.12          | 0.120        | 38.0            | 75.9          |
| 75               | 3.15                 | 441.12          | 0.131        | 45.2            | 90.3          |
| 80               | 3.26                 | 456.84          | 0.136        | 48.4            | 96.8          |
| 85               | 3.39                 | 474.66          | 0.141        | 52.3            | 104.6         |
| 90               | 3.54                 | 494.90          | 0.147        | 56.9            | 113.9         |

**Figure 5.** Graph of the changes in the objective function

By analyzing the presented graph in fig. 5, it can be noted that for the scheme shown in fig. 4, the minimum steel consumption matches the minimum angle $\alpha$ between the cables.

Example 3 shows the bar system, where elements are subjected to the impact of tensile and compressive forces (see Figure 6). An example of such a design case is the support structure of structural slabs of the covering, when the reaction of the support N compresses the rods 1 and 2 and
stretches the rod 3. When the angle $\alpha$ between the supports changes, the forces $T_1$ and $T_2$ change according to law (6), and the force $T_3$ changes according to law (5).

$$\frac{N}{\varphi A R_y c} \leq 1$$ (6)

Determining the optimal angle $\alpha$, which matches the minimum steel consumption for the given structure. Variable parameters and limitations for the given task are shown in Table 5.

Figure 6. a – practical application, b – calculation scheme

Table 5. Optimization parameters of the support structures

| Objective function limitations | Metal consumption |
|-------------------------------|-------------------|
| 1. Steel category C245       |                  |
| 2. Hinge fastening            |                  |
| 3. Number of suspensions n=2  |                  |
| 4. Suspension base H=3500mm   |                  |

| Variable parameter           | H – construction height |
|-------------------------------|-------------------------|

Load $N$ is assumed to be 300 kN.

The results of the calculations performed are shown in Table 6 and the graph (fig. 7).

Table 6. Determining the mass of the construction 3

| Height, H, cm | Angle $\alpha$, $^\circ$ | Compressive force $T_1, T_2$, kN | Area $A_1= A_2$, cm$^2$ | Length, $L_1= L_2$, cm | Tensile force $T_3$, kN | Area $A_3$, cm$^2$ | Length, B, cm | Total, Mass, kg |
|---------------|-------------------------|----------------------------------|-------------------------|-------------------------|-----------------------|-----------------|---------------|----------------|
| 1             | 126,89                  | 335,41                           | 15,53                   | 223,61                  | 300,0                 | 12,50           | 400           | 93,17          |
| 1.5           | 106,28                  | 250,00                           | 11,57                   | 250,00                  | 200,0                 | 8,33            | 400           | 45,4           |
| 2             | 90,02                   | 212,13                           | 9,82                    | 282,84                  | 150,0                 | 6,25            | 400           | 43,53          |
| 2.5           | 77,33                   | 192,09                           | 8,89                    | 320,16                  | 120,0                 | 5,00            | 400           | 44,57          |
| 3             | 67,39                   | 180,28                           | 8,35                    | 360,56                  | 100,0                 | 4,17            | 400           | 47,07          |
| 3.5           | 59,50                   | 172,76                           | 8,00                    | 403,11                  | 85,7                  | 3,57            | 400           | 50,41          |
By analyzing the graph (fig.7), it can be concluded that, for the scheme shown in Figure 6, the optimal angle between elements 1 and 2 at which the structure has a minimum mass, is in the area of 70°.

When we design a structural slab resting on a column with a capital, a question arises of what height the capital should have. Now, using the graph shown in Figure 7, we can easily select the appropriate value for the dimensions of the structure.

4. Conclusion
The problems of finding structural solutions for building structures with a minimum mass are still relevant [15,16].

Using the parametric optimization method makes it possible to search the optimal design solutions at the stage of variant design In the presented article, on a number of examples, the possibility and effectiveness of the application of this method is shown, which makes it possible to obtain the most effective constructive solution in terms of economic efficiency. Further development of this methodology is the automation of the search process [9,17], which will allow, with a small cost of resources and time, to analyze even more options for design solutions.

References
[1] Alpatov V.Yu., Belyakova A.A. Variant design of the dome cover of the temple in the village of Bereza // Collection: Traditions and innovations in construction and architecture. Construction: Samara, SGASU, 2016.S. 35-38.
[2] Kirsanov M.N. To the choice of the lattice of the girder // Structural mechanics of engineering structures and architectural engineering. 2017. No. 3. P. 23-27
[3] Akimov P.A., Sidorov V.N., Tusnin A.R. Peculiarities of design and construction of high-rise buildings and other unique architectural engineering in China. Moscow 2013.

[4] Xin-She Yang, Gebrail Bekdaş, Siğan Melih Nigdeli. Metaheuristics and Optimization in Civil Engineering // Modeling and Optimization in Science and Technologies // Springer Cham Heidelberg New York Dordrecht London, 2016. DOI 10.1007 / 978-3-319-26245-1.

[5] Vasilkin A.A. Integration of structural and parametric optimization tools at the stage of exploratory design of steel structures // Scientific journal of construction and architecture. 2018. No. 1 (49). S. 22-28.

[6] Volkov A.A., Vasilkin A.A. Optimal design of the steel structure by the sequence of partial optimization // Procedia Engineering. 2016. Vol. 153. P. 850-855.

[7] Vasilkin A.A. The use of computer-aided design systems to find design solutions for steel structures // System engineering in construction. Cyber-physical building systems [Electronic resource]: a collection of materials of the seminar held within the framework of the VI International scientific conference "Integration, partnership and innovation in building science and education" (Moscow, November 14-16, 2018). Moscow: Publishing house MISS - MGSU, 2018. -URL: http: //mgsu.ru/resources/izdatelskaya-deyatelnost/izdaniya/izdaniya-otkr-dostupa/. - Title with title. screen. S. 20-25

[8] Vostrov V.K. Vasilkin A.A. Optimization of tank wall chord heights. // Assembly and special works in construction. 2005. No. 11. S. 37-40.

[9] Vasilkin A.A., Shcherbina S.V. Construction of a computer-aided design system for the optimization of steel trusses // Vestnik MGSU. 2015. no. 2. S. 21-37.

[10] Ibragimov A.M., Vasilkin A.A. Automated solution of design tasks for the layout of the space-planning solution of warehouse facilities. Part 1. Statement of the assignment // Scientific Review. 2016. No. 13. S. 32-36.

[11] Ibragimov A.M., Vasilkin A.A., Shcherbina S.V. Automated solution of design tasks for the layout of the space-planning solution of warehouse facilities. Part 2. Application of information technology in the layout of logistics complexes // Scientific Review. 2016. No. 17. P. 35-40.

[12] Vasilkin A.A. To the integration of structural optimization tools into CAD // Promyshlennoe i grazhdanskoie stroitel'stvo. 2018. No. 9. P. 55-60.

[13] Kociecki, M., Adeli, H. Two-phase genetic algorithm for topology optimization of free-form steel space-frame roof structures with complex curvatures // Eng. Appl. Artif. Intell. 32, 218-227 (2014).

[14] Yusheng Lin, Zheng Sun, Alexandru Dadalau, Alexander Verl. Efficient. Combination of Topology and Parameter Optimization // Open Journal of Optimization. 2014, 3, 19-25.

[15] Khomyak Y.U., Naumenko I.E., Zheglova V.V., Popov V. Minimizing the mass of a flat bottom of cylindrical apparatus // Eastern-European Journal of Enterprise Technologies, 2018.2/1 (92). - S. 42-50.

[16] Tumenova. THEM. Parametric optimization of a trapezoidal wooden truss with ascending braces on metal toothed plates // Engineering Bulletin of the Don, 2017. No. 2 (45).

[17] Dmitrieva T.I., Le Chang Minh Dat. Algorithms for solving the problem of optimizing a spatial metal structure and their software implementation // Bulletin of ISTU. 2016, No. 4 (111). S. 75-82.