Dark Matter Bound States from Three-Body Recombination

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Abstract. The small-scale structure problems of the universe can be solved by self-interacting dark matter that becomes strongly interacting at low energies. A particularly predictive model is resonant short-range self-interactions, with a dark-matter mass of about 19 GeV and a large S-wave scattering length of about 17 fm. Such a model makes definite predictions for the few-body physics of weakly bound clusters of the dark-matter particles. We calculate the production of two-body bound clusters by three-body recombination in the early universe under the assumption that the dark matter particles are identical bosons, which is the most favorable case for forming larger clusters. The fraction of dark matter in the form of two-body bound clusters can increase by as much as 4 orders of magnitude when the dark-matter temperature falls below the binding energy, but its present value remains less than \(10^{-6}\).

Keywords: Dark matter, bound states, large scattering length, early universe.

1 Introduction

The model with collisionless cold dark matter and a cosmological constant provides an excellent description of the large-scale structure of the universe, but it has encountered problems at smaller scales associated with galaxies and clusters of galaxies. The problems involve the dark-matter distribution in the cores of galaxies and the properties of satellite galaxies. They can all be solved by self-interacting dark matter that is strongly interacting at low energies [1].

In Ref. [2], Kaplinghat, Tulin, and Yu deduced self-interaction reaction rates \(\langle v\sigma_{\text{elastic}} \rangle\) and mean velocities \(\langle v \rangle\) of dark matter particles for a number of dwarf galaxies, low-surface-brightness galaxies, and clusters of galaxies. Their results are shown in Fig. [1]. They fit their results with a simple self-interacting dark matter model with 3 parameters: the dark matter mass \(m_\chi\), a dark mediator
Fig. 1: Self-interaction reaction rate $\langle v \sigma_{\text{elastic}} \rangle$ for dark matter particles as a function of their mean velocity $\langle v \rangle$ (adapted from [3]). The data points from Ref. [2] are for dwarf galaxies (red), low-surface-brightness galaxies (blue), and galaxy clusters (green) [2]. The curves are the best fits to the model of Ref. [2] (dashed) and to Eq. (1) (solid). The diagonal lines are energy-independent cross sections.

mass $\mu$, and a Yukawa coupling $\alpha'$. Their best fit for the masses with fixed Yukawa coupling $\alpha' = 1/137$ was $m_\chi = 15$ GeV and $\mu = 17$ MeV [2].

In Ref. [3], we showed that the results in Fig. 1 can be fit equally well by a simpler self-interacting dark matter model with 2 parameters. The model has resonant short-range interactions with an S-wave resonance close to the scattering threshold [4]. The parameters are the dark matter mass $m_\chi$ and the scattering length $a$. This model has been applied previously to the direct detection of dark matter [5,6]. The self-interaction reaction rate as a function of the velocity $v$ is

$$v \sigma_{\text{elastic}}(v) = \frac{8\pi a^2 v}{1 + (am_\chi^2/2)^2 v^2}. \quad (1)$$

The best fit to the results in Fig. 1 is $m_\chi = 19$ GeV and $a = \pm 17$ fm [3].

The dark matter could all be in the form of individual dark matter particles $d$, but some (or all) of it could be bound into few-body clusters $d_N$, which we call dark nuclei. There are two basic formation mechanisms for larger dark nuclei. If there is a light mediator $\gamma_d$ for dark matter self-interactions, larger dark nuclei can be formed by radiative reactions: $d + d_{N-1} \rightarrow d_N + \gamma_d$. If there is no light mediator, as in our resonant short-range interaction model, larger dark nuclei must be formed instead by rearrangement reactions, such as 3-body recombination: $d + d + d_{N-1} \rightarrow d_N + d$. The formation of dark deuterons $d_2$ is a bottleneck for formation of larger dark nuclei $d_N$. 
2 Few-body Physics

The low-energy two-body physics of particles with resonant short-range interactions is very simple. It is completely determined by the large scattering length $a$. The cross section for the elastic scattering reaction $d + d \rightarrow d + d$ is given in Eq. (1). If $a$ is negative, there are no 2-body bound states. If $a$ is positive, there is a single 2-body bound state $d_2$ that we call the dark deuteron. Its binding energy is $E_2 = 1/m_\chi a^2$.

If the particles are identical bosons, the 3-body physics is much more intricate [3]. It is determined not only by the large scattering length $a$, but also by a 3-body parameter. There is a sequence of 3-body bound states called Efimov states. In the limit $a \rightarrow \pm \infty$, there are infinitely many Efimov states with an accumulation point at the 3-boson threshold and with the binding energy of each successive Efimov state smaller by a factor of $22^{7/2} = 515$. Three-boson reaction rates also have remarkable behavior. They depend log-periodically on a 3-body parameter $a_+$ with discrete scaling factor 22.7. If $a > 0$, a simple example is the rate for the 3-body recombination reaction $d + d + d \rightarrow d_2 + d + d$ at 0 collision energy:

$$R(E = 0) = \frac{399.8 \sin^2[s_0 \log(a/a_+)]}{1 - 0.00717 \sin^2[s_0 \log(a/a_+)]} a^4/m_\chi,$$

(2)

where $s_0 = 1.00624$. The 3-body recombination rate at nonzero collision energy $E$ has been calculated in Ref. [7] and in Ref. [8].

We consider a gas consisting of dark matter particles $d$ with number density $n_1$ and dark deuterons $d_2$ with number density $n_2$ in thermal equilibrium at temperature $T$. The rate of change in the number density of dark deuterons is

$$\frac{d}{dt} n_2 = + K_3(T) n_1^3 - K_2(T) n_1 n_2,$$

(3)

where $K_3(T)$ and $K_2(T)$ are the rate constants for 3-body recombination and for the dark deuteron breakup reaction $d + d_2 \rightarrow d + d + d$. These rate constants were calculated in Ref. [8]. The results for $K_3(T)$ are shown in Fig. 2.

3 Dark Matter in the Early Universe

We calculate the formation of dark deuterons in the Hubble expansion of the early universe, taking into account the 3-body recombination and dark deuteron breakup reactions. We calculate the number densities $n_1$ and $n_2$ as functions of the redshift $z$, which is a convenient time variable. The initial condition is that $n_2$ is negligible when dark matter decouples at a redshift of about $z_{dc} \approx m_\chi/20kT_{cmb}$, where $T_{cmb}$ is the present temperature of the cosmic microwave background [9]. For $m_\chi = 19$ GeV, this redshift is $z_{dc} \approx 10^{13}$. The dark-matter temperature as a function of $z$ is

$$T(z) = T_{cmb} \frac{(1 + z)^2}{1 + z_{dc}}.$$

(4)
The total number density of dark matter is determined by the present mass density $\rho_{\text{cdm}}$ of cold dark matter:

$$n_1(z) + 2n_2(z) = \frac{\rho_{\text{cdm}}}{m_\chi} (1 + z)^3. \quad (5)$$

It is convenient to express our results in terms of the dark deuteron mass fraction:

$$f_2(z) = \frac{2n_2(z)}{[n_1(z) + 2n_2(z)]}.$$

(6)

We assume the dark matter particles are identical bosons with mass $m_\chi = 19$ GeV, large scattering length $a = 17$ fm, and unknown 3-body parameter $a_+$. The dark deuteron mass fraction $f_2(z)$ is shown as a function of the redshift variable $z_{\text{dc}}/z$ in Fig. 3. The fraction increases by 3 or 4 orders of magnitude around the redshift $10^{-3}z_{\text{dc}}$ when $kT$ is equal to the binding energy $E_2 = 7$ keV of the dark deuteron. At smaller $1/z$, there is a plateau in $f_2$ at about $4 \times 10^{-11}$ from equilibrium between recombination and breakup. The final fraction $f_2(0)$ depends log-periodically on $a_+$, ranging from $5 \times 10^{-8}$ to $5 \times 10^{-7}$. It can be increased by ignoring the data from clusters of galaxies in Fig. 1 which means keeping $a = 17$ fm but allowing $m_\chi$ to decrease. The final fraction $f_2(0)$ can be increased to about $10^{-2}$ for $m_\chi = 0.4$ GeV. If $m_\chi$ is smaller, $f_2(0)$ is sensitive to the range of self-interactions.

Fig. 2: Rate coefficient $K_3(T)$ for three-body recombination as a function of the temperature $T$ (adapted from [3]). The upper band is the envelope of $K_3(T)$ for all possible values of the three-body parameter $a_+$. The dashed line is the extrapolation from the scaling behavior at high temperature. The lower band is the envelope of the $J = 0$ contribution to $K_3(T)$ for all possible values of $a_+$. The curves inside the lower band are for 8 values of $a_+$.
Fig. 3: Dark-deuteron mass fraction $f_2(z)$ as a function of $z_{dc}/z$ for $m_\chi = 19$ GeV and $a = 17$ fm (adapted from [3]). The band is the envelope of all possible values of $a_+$. The curves are for 8 values of $a_+$.

parameters $m_\chi \approx 19$ GeV and $a \approx \pm 17$ fm. Dark nuclei $d_N$ must be produced by rearrangement reactions, such as 3-body recombination: $d + d + d_{N-1} \rightarrow d_N + d$

The most favorable case for producing dark nuclei larger than the dark deuteron is for the dark matter particles to be identical bosons. We found that a significant fraction of dark deuterons cannot be formed in the early universe by 3-body recombination. Since the formation of the dark deuteron $d_2$ is a bottleneck for the formation of larger dark nuclei $d_N$, they cannot be formed either.

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