Electric and magnetic axion quark nuggets, their stability and their detection

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Abstract

The present work studies the axion quark nuggets introduced in [1] and exploited in the works [3]-[14], but taking into account the possibility that they become ferromagnetic. This issue was raised in [15], although ferromagnetism may also take place due some anomaly terms found in [16]-[18]. The purpose of the present letter however, is not to give evidence in favor or against these statements. Instead, it is focused in some direct consequences of this ferromagnetic behavior, if it exists. The first is that the magnetic field of these lumps induces an electric field on the nugget due to the axion wall, which may induce pair production due to the Schwinger effect. A critical value for such magnetic field at the surface of the nugget is obtained, and it is argued that the value of the magnetic field of [15] is at the verge of stability but otherwise pass the test. In addition, the interaction of such magnetic and electric nugget with the troposphere of the earth is also analyzed. It is suggested that the cross section with the troposphere is enhanced in comparison with a non magnetic nugget but still, it does not violate the dark matter collision bounds. Consequently, these nuggets may be detected by impacts on water or by holes in the mountain craters [19].

1. Introduction

The existence of lumps of quark matter has been postulated long ago in [20]-[21]. These lumps are different than nuclear matter, which is composed by a large number of protons and neutrons whose main interaction is due to nuclear forces. Quark matter instead is approximately a Fermi gas composed of $3N_B$ quarks constituting a color singlet baryon of baryon number $N_B$, and their interaction is much weak, mainly gravitational. Some of these states of quark matter are composed only by $u$ and $d$ quarks, and these are known as non strange quark matter. Other form is strange matter, which contains $s$ quarks as well as $u$ and $d$ quarks, in such a way that flavor equilibrium is established by weak interactions $d \rightarrow u + e + \bar{\nu}$, $s \rightarrow u + e + \bar{\nu}$ and $s + u \rightarrow d + u$. Strange matter also contain gluons, and a small component of electrons whose role is insure electric charge neutrality [21]. These strange lumps may be formed during an hypothetical phase transition of the early universe from a quark gluon plasma state to a hadron phase. There is debate about the nature of this transition, it can be a first order or second order one, a crossover or even an spinodal type of transition. But there are segregation scenarios which predicts these lumps, irrespective on the order of the phase transition. An example is the reference [22].

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Strange form of matter has received special attention, since it is conjectured to have lower energy per baryon number than ordinary nuclei, thus it may be a stable state \cite{23,24}. In other words, the energy per baryon is less than 930 MeV, at least at very low temperatures. A hint about the stability comes from the fact that a non strange quark lump posses a Fermi momentum $p_f \sim 300-350$ MeV, while the strange quark mass is around $m_s \sim 80-130$ MeV. As the Fermi momentum is very high in comparison, it may be energetically favored for some non strange quarks to become strange. This conversion may lower the energy and the Fermi momentum of the system. The estimations given in \cite{23,24} suggest that the inclusion of strange matter decrease the energy per baryon by 50-70 MeV. The range of baryon numbers allowed for these objects is $10^2 < N_B < 10^{57}$ \cite{25}. The upper bound is due to the fact that higher baryon numbers collapse into a black hole. The lower bound arises due to shell effects which raise the energy per baryon.

The mass density of a quark nugget is $\rho \sim 35 \cdot 10^{16}$ kg/cm$^3$, and its mass is approximately $6m = N_B$ GeV. The surface electron cloud for nuggets with $N_B < 10^{15}$ extend to a distance $r \sim 10^{-8}$ cm from the surface while for very high baryon numbers this distance can be around 400 fm. Such nuggets therefore possess a Coulomb barrier, with a value close to 10 MeV. Due to this barrier, only neutrons at low energies may be absorbed by the nugget. There is also a probability of emission of a neutron. By use of detailed balance arguments, it is found that primordial nuggets with $N_B < 10^{52}$ evaporate. However \cite{29} suggested that nuggets with $N_B > 10^{46}$ survive hadron emission, and lower charge nuggets also survive due to reabsorption of hadrons. Further stability issues were considered in \cite{23,24,35}, and it is believed that stable nuggets have a mass in the range $10^{-8}$ kg < $m_n$ < $10^{20}$ kg. Thus, very large quark nuggets may masses of planetary order.

The fact that the nuggets could be stable implies that iron is not the ground state of nuclear interactions. This is a delicate aspect when studying nucleosynthesis and was initially considered in \cite{36}. For instance, for an inflationary universe with $\Omega = 1$, the prediction of the density of the $^3$He and D elements is below the observed value. This problem may be solved if there is some unknown mechanism of photodisintegration of $^4$He into this elements. This requires high energy photons which may be produced by decay of massive neutrinos by non standard interactions. Such channels may be sourced by the presence of quark nuggets through a weak interaction of high order, as in this case the decay rate will be close to zero. Despite these observations, it was suggested in \cite{37} that quark nuggets with $N_B > 10^{16}$ are consistent with a universe with $\Omega = 1$ and with the predicted abundances of the light elements. This motivated a special interest for studying the cosmological consequences of these objects, in particular, in dark matter applications.

Several years ago it was suggested that quark nuggets may reach a highly magnetized final state \cite{15}. This observation is partially motivated from the Bloch works about the possible ferromagnetism of an electron gas \cite{38}. This hypothesis was supported in \cite{39}, who showed that for an electron gas in a fully polarized state the ferromagnetic state is stable. These claims were subsequently tested for a quark liquid in \cite{15} and some evidence supporting a ferromagnetic state was collected for quark nuggets. The presence of an intrinsic magnetic field is an attractive feature, as it may make the nugget
more stable if \( B < \epsilon_{10}^{16} T \). A problem is that such highly magnetized magnetic fields may violate the collision requirements for dark matter. However, it was shown in [19] that due to the fall of the field as \( R^{-3} \), the resulting cross section is acceptable for representing dark matter.

The present work is devoted to a specific type of quark nuggets introduced in [1], in which the quarks are trapped in the bulk of an axion domain wall. There are several theoretical reasons for which these models are interesting. First, the axion particle is an attractive candidate for solving the CP problem in QCD [41]. However, the formation of axion domain walls is problematic, since they generate an energy density which largely overcomes the critical density. Nevertheless, there are some axion models for which the axion domain structure does not possess this problem [41]. These domain walls induce CP violating quark anti-quark scattering. Due to the surface tension, these wall tend to contract and segregate baryons and anti-baryons. It is likely that the contraction of these objects stops when the surface tension compensate the pressure difference and the resulting bubble enters into the CS phase [42]-[43]. These nuggets may be formed without relying on the order of the phase transition from the quark gluon plasma to the confined phase.

The organization of this letter is as follows. In section 2.1 the modified Maxwell equations for magnetized quark nuggets in presence of an axion domain wall are considered. It is argued that neither the magnetic field found in [15] nor the axion wall profile are considerably deformed by the interaction between each other. The resulting electric field in the nugget is then estimated. In section 2.2 the pair creation by Schwinger effect in the wall is calculated by use of several methods, and it is argued that the magnetic field [15] is at the verge of stability, but it may pass the test. In section 2.3 the fate of a quark nugget impacting in the earth is analyzed and its possible detection is specified. Section 3 contains the discussion of the results.

2. Ferromagnetic axion nuggets

2.1 The equations describing the configuration

In the present section, a hypothetically magnetized quark nugget surrounded by an axion domain wall is considered [1]-[14]. The domain wall profile function is denoted \( a(r) \), and interpolates between two axion vacuums \( a_0 \) and \( a_0 + 2\pi f_a N \) with \( f_a \) the axion constant, whose standard values run between \( 10^9 \text{GeV} < f_a < 10^{12} \text{GeV} \). The value of \( N \) is usually not far from unity. The explicit form of the function \( a(r) \) is not of too relevant in the following discussion, the important point is that \( a(r) \) changes in a range between

\[
R \sim (0.3 - 0.5) \frac{N_B^{1.7}}{160 \text{MeV}} (10^{-3} - 10^{-6}),
\]

the last factors take into account the axion mass range \( 10^{-6} \text{eV} < m_a < 10^{-3} \text{eV} \). The baryon number range that the references [1]-[14] employ is \( 10^{23} < N_B < 10^{32} \). For the larger baryon number one has \( 10^{-6} \text{cm} < R < 10^{-3} \text{cm} \), while for the smallest value \( 10^{-9} \text{cm} < R < 10^{-6} \text{cm} \). The range of masses goes from \( 10^{-6} \text{kg} < m_n < 10^5 \text{kg} \), thus these objects can be quite heavy.
In presence of an axion field \( a(x,t) \), the Maxwell equations are modified as follows

\[
\nabla \cdot E = -g_{a\gamma\gamma} \nabla \cdot (aB) + 4\pi \rho_a, \tag{2.1}
\]

\[
\nabla \times E = -\partial_t B, \tag{2.2}
\]

\[
\nabla \cdot B = 0, \tag{2.3}
\]

\[
\nabla \times B = \partial_t E + g_{a\gamma\gamma} \nabla \times E - g_{a\gamma\gamma} B \partial_t a + 4\pi J, \tag{2.4}
\]

\[
\Box a + \frac{\partial U}{\partial a} = -g_{a\gamma\gamma} E \cdot B. \tag{2.5}
\]

The axion dependent terms in these equations arise due to the interaction \( \mathcal{L}_a = g_{a\gamma\gamma} aE \cdot B \) between the axion and the electromagnetic field. Here \( g_{a\gamma\gamma} = c\alpha/\pi f_a \) with \( \alpha = 1/137 \) the fine structure constant.

The constant \( c \) is model dependent, but its value is not far from the unity. It is seen from (2.1) that \( \rho_a = -g_{a\gamma\gamma} \nabla \cdot (aB) \) acts as a new type of charge for the electric field. Thus, when a magnetic field passes through an axion wall, an electric field is induced.

If the arguments of [15] about a possible ferromagnetic nature of these nuggets are taken into account, then there should be a magnetic field \( B \) due to bulk effects in the object. For standard nuggets, the reference [15] presents a magnetic field that scales as

\[
B = \frac{B_0 r_n^3}{r^4}. \tag{2.6}
\]

Strictly speaking, a nugget does not have a spherically symmetric radial magnetic field. In fact it may depend on the polar \( \psi \) angle in spherical coordinates. The value (2.6) represents a extrema value say, at certain north or south pole drawn on the nugget. This extreme numerical value at the surface of the nugget \( r_n \) is estimated as \( B_0 \approx 10^{12\pm1} \text{T} \) or \( B_0 \approx 10^{16\pm1} \text{Gauss} \) [15]. For ordinary quark nuggets, the work [10] suggests that such large magnetic field increase the stability of the configuration, if it does not reach a critical value of the order \( B \approx 10^{16} \text{T} \).

The statements given above should be taken with care in presence of the axion domain wall \( a(r) \) by two reasons. First, the magnetic field (2.6) is modified due to the equation (2.4). Furthermore, an electric field \( E \) is induced, as is seen from equation (2.1). It is explicitly given by

\[
E = -g_{a\gamma\gamma} aB. \tag{2.7}
\]

The presence of the electric field may generate electron positron pairs due to Schwinger effect. The stability of the configuration is then compromised if the resulting electric field (2.7) is very strong.

Clearly, the axion profile \( a(r) \) should be modified in presence of the magnetic field \( B \), which is also deviated from its original form (2.6). In fact, the equation (2.2) shows that \( \nabla \times E = 0 \), as the magnetic field \( B \) is static. In other words, the electric lines for the configuration are never closed. This may be non true if \( \nabla \times (aB) \neq 0 \), as seen from (2.1), unless the profile \( a \) and the magnetic field \( B(r) \) are adapted for this to happen. Thus, the task of solving the explicit profiles may be a complicated one. For this reason, it will be assumed that the axion domain wall function \( a(r) \) keep
its shape approximately and that the magnetic field $B(r)$ do not deviate considerably from (2.6). But before considering the decay of the induced electric field by Schwinger effect, it may be convenient to justify these approximations.

Consider first the approximation that the magnetic field does not deviate considerably from the functional dependence (2.6). The term $J_a = g_{a\gamma\gamma} \nabla a \times E$ is the axion induced current in (2.3). If one assumes that the field $a$ make a sudden change of $a \rightarrow a + 2\pi f_a$ at the nugget surface, then the axion surface magnetization is proportional to $M \sim 2\pi f_a g_{a\gamma\gamma} E \times n$ with $n$ the surface unit normal. The induced magnetic field at the surface is of the order $|B_a| \sim |M|$. But $|M| \sim g_{a\gamma\gamma} |E|$ and, by taking into account that $E = -g_{a\gamma\gamma} a B$, it follows that $|B_a| \sim |M| \sim f_a^2 g_{a\gamma\gamma}^2 |B|^2 \sim \alpha |B|^2$. This suggest that the magnetic field $B_a$ induced by the axion current is of smaller order than the magnetic field (2.6) itself.

Another heuristic argument for this statement about $B_a$ comes from the study of the magnetic field of an infinite wire with radius $r_w \sim m_a^{-1}$ near its border. This magnetic field is $B_w \sim \mu_0 J_w / r_c$ with $r_c$ the standard cylindrical radius. If the field value in the border of the wire is near $B = 10^{12\pm1}$ TeV, then its current should be $J_w \sim 10^{12\pm1}$ TeVm$_a$. On the other hand, the axion current is $J_a = g_{a\gamma\gamma} \nabla a \times E$, this follows from (2.3). If the domain wall $a$ is not considerably modified, it should have a size of length $L_a \sim m_a^{-1}$ and therefore, the current is of order $|J_a| \sim g_{a\gamma\gamma} \Delta a m_a E$. But the equation (2.4) shows that $E = -g_{a\gamma\gamma} a B$ and therefore

$$|J_a| \sim g_{a\gamma\gamma}^2 \Delta a^2 m_a B \sim \frac{1}{(137)^2} m_a B.$$  

Here the fact that $\Delta a \sim 2N f_a \pi$ has been taken into account, with $N$ an integer not far from unity. This current is fourth orders of magnitude larger than $J \sim 10^{11\pm1}$ TeVm$_a$, which suggest that the induced magnetic field due to the axion wall is of smaller order than the magnetic field (2.6) itself.

The estimations given in the last two paragraphs assume that the profile $a(r)$ is not strongly modified in presence of a magnetic field. To see that this may be the case, one should check that the term $E \cdot B$ is not considerably larger than $\partial_a U$ in (2.3), otherwise the axion domain wall will be considerably deformed by this new term. Since $\partial_a U \sim m_a^2 a$ and $g_{a\gamma\gamma} E \cdot B \sim g_{a\gamma\gamma}^2 a B^2$, with $B \sim 10^{11\pm1}$T, one has to check that

$$m_a^2 \sim \frac{m_a^2 f_a^2}{f_a^2} > g_{a\gamma\gamma}^2 10^{22\pm2} T^2 \sim \frac{1}{(137)^2 f_a^2 \pi^2} 10^{22\pm2} T^2.$$  

But $m_a^2 f_a^2 = 10^{-4}$ GeV$^4$ and $10^{22\pm2}$ T$^2 \sim 10^{-10\pm2}$ GeV$^4$, and thus the last inequality is true. Therefore, the assumption that the magnetic field $B$ do not deviate from its form (2.6), and that the electric field $E$ does not induce a considerable deformation of the domain wall is, at least, consistent. This is the approximation to be used below.

The field (2.6) is induced by a ferromagnetic behavior of a quark liquid at the bulk of the nugget. On the other hand, the axion wall can be a source of a magnetic field as well. This is understood from an analysis of anomaly terms for axions [16]-[18], in presence of a non zero chemical potential, which is the case for a nugget in the Color Superconducting phase [42]-[43]. There are two anomaly terms,
one is the standard one \( L_1 \sim a F_{\mu\nu} \tilde{F}^{\mu\nu} \), which induces the decay \( a \to \gamma + \gamma \) by the ABJ anomaly diagram. However, in presence of a non zero chemical potential \( \mu \) there is a further WZW type of anomaly term given by [16]-[18]

\[
L_2 = \frac{e^2 C}{4\pi} \mu B \cdot \nabla a.
\]

Here \( B \) is an external magnetic field, and \( C \) a model dependent constant, but not far from unity. From the fact that \( \nabla \cdot B = 0 \) it is seen that this term is a total derivative, thus it does not contribute to the equations of motion. However, the presence of this term is not trivial. To see this, consider a flat axion domain wall. Then this term induces an extra energy contribution proportional to \( \mu BA \) with \( A \) the area of the wall. This energy is produced by a magnetic moment by unit area located at the wall, given by

\[
M = \frac{e^2 C}{4\pi} \mu.
\]

For a spherical wall, the total magnetic moment may be zero, since the gradient contributions of \( a \) at opposite points are equal and of opposite sign. This implies that the leading contribution to the external magnetic field is the quadrupole one. However, for a simple estimation, one may remember that for a magnetized sphere at the south and north poles the magnetic field is \( |B| = \mu_0 |M| \), with \( M \) its uniform permanent magnetization. If this value is employed as a guide for a magnitude order, then it is mandatory to estimate the value of \( M \). By a naive application of formula (57) of [17] one finds that

\[
M \sim 10^{12} T \frac{\mu}{1.5 GeV} \frac{\Delta}{30 MeV}.
\]

Here \( \Delta \) is a gap characterizing the formation Cooper color pairs, by assuming that the evolution of the object is such that the CFL phase is achieved [12]-[13]. A typical value for the chemical potential is 1 GeV and \( \Delta \sim 50 \) MeV. This implies that the magnetic field these references predict may be of the order of \( B \sim 10^{12} T \), which is pretty close to the value of [15]. The physics describing these fields is however different.

In view of the present discussion, a generic magnetic field will be considered below without relying whether it is induced by the axion wall or by the quark liquid inside the nugget. The working assumption is then that an electric field \( E = -g_{a\gamma\gamma} a B \) is induced, and the task is to estimate its decay probability due to electron positron pair production.

### 2.2 Schwinger effect and estimation of the critical magnetic field of the configuration

In order to study the electron positron creation by the induced electric field \( E \), one may avoid the complication of the inhomogeneity of the electric field \( E \) and simply assume that \( E \sim g_{a\gamma\gamma} a_0 B_0 \) with \( B_0 \) the surface magnetic field of the bubble. This is the roughest possible approximation. In this situation, the vacuum persistence probability

\[
P_0 = |\langle 0_f |0_i \rangle|^2 = \exp(-\int_M dx^4 \omega_e).
\]
Here the rate of pair creation \( w \) by a constant electric field \( E \) is given by the well known Schwinger formula

\[
w_e = \frac{2e^2 E^2}{\pi^2} \sum_{n=1} \frac{1}{n^2} e^{-\frac{m^2}{eE}}.
\]  

(2.8)

This formula implies that the vacuum transition probability is given by

\[
P_0 = | \langle 0 | i \mid 0_f \rangle |^2 = \exp \left( - \frac{2VT\alpha E^2}{\pi^2} \sum_{n=1} \frac{1}{n^2} e^{-\frac{m^2}{eE}} \right).
\]

The approximate volume occupied by the field is roughly \( V_f \sim 4\pi R^3/3 \). The critical field is

\[
E_c \sim \frac{m^2}{e}.
\]  

(2.9)

If the critical field is reached, then evaporation in electron positron pairs may be relevant. This will be the case when

\[
E = \frac{4cB}{137} = \frac{m^2}{e}.
\]

This happens when the numerical value of the magnetic field at the surface is \( B \sim 10^{11} \text{ T} \).

The conclusion given above is avoiding some subtle point. From the relation \( E = -g_{\alpha\gamma\gamma}aB \) it follows that the magnetic field and the electric field of the bubble are collinear. In this situation, it is known that the magnetic field \( B \) suppresses pair creation for scalar fields if it is large enough, but enhance it for fermions \([58]-[59]\). Thus, the pair production corresponding to the value \( B \) found above may be even more significant. To see that this is not the case, recall that the rate of electron positron pair creation when the collinear magnetic field is turned on is given by \([58]\)

\[
w_m = \frac{2c^2EB}{\pi^2} \sum_{n=1} \frac{1}{n} e^{-\frac{m^2}{eE}} \coth \frac{\pi nB}{E}.
\]

From this formula it is seen that a large magnetic field increases \( w \). In the present case, the magnetic field is large, but it is related to the electric field by \( E = -g_{\alpha\gamma\gamma}aB \) with \( a \) taking values close to \( 2\pi \).

By taking into account that \( g_{\alpha\gamma\gamma} = c\alpha/\pi f_a \) it follows that \( E \sim \alpha B \), with \( \alpha \) the fine structure constant. The rate given above is then

\[
w_m = \frac{274c^2E^2}{\pi^2} \sum_{n=1} \frac{1}{n} e^{-\frac{m^2}{eE}} \coth \left( \frac{137\pi n}{E} \right).
\]

(2.10)

The leading term of (2.10) and (2.8) represent the pair production rate by unit time and unit volume. These are

\[
N_e = \frac{2e^2 E^2}{\pi^2} e^{-\frac{m^2}{eE}}, \quad N_m = \frac{274c^2E^2}{\pi^2} e^{-\frac{m^2}{eE}} \coth(137\pi).
\]  

(2.11)

Now, a rough approach for estimating the value of the field \( E \) for which pair creation is significant may be to evaluate the first rate (2.11) at the value \( E_c = m_e^2/e \) and find the electric field \( E \) such that the second rate has the same numerical value. The cotangent factor in (2.10) is close to the unity and can be neglected. Then the numerical relation defining \( E \) is thus given by

\[
137E^2 e^{-\frac{m_e}{E}} = E_c^2 e^{-\pi}.
\]
The last equation can be cast in the Lambert form
\[ \frac{E_c}{2E} e^{\frac{E_c}{2E}} = \frac{1}{2} \sqrt{137 e^{\pi}}. \]
From here, the solution is given in terms of the Lambert function \( W(x) \) as follows
\[ \frac{E_c}{2E} = W\left(\frac{1}{2} \sqrt{137 e^{\pi}}\right) \sim 2.44. \]
This means that \( E \sim E_c/4.5 \). Thus, the magnetic field does not change considerably the value of the electrical field for which pair creation is appreciable. For this reason, the presence of the magnetic field when studying pair creation can be neglected in this specific case, even though the nugget magnetization is quite large.

The estimation made above assumes that the electric field is uniform in the volume \( V_f \sim 4\pi R^3/3 \). But this estimation may be not accurate, as the field \( E \) is varying inside the wall. Thus, it is of interest to study the effect of the inhomogeneity of the field inside the wall. A possible approach is to assume that the radius of the bubble is large enough and to analyze what happens at the pole. There are several works that derive results when such inhomogeneities are present [44]-[57], and in the following these references will be followed closely. In order to study the role of the inhomogeneities, the electric can be approximated by a one dimensional one \( E = E(x) \) where the axis \( \hat{x} \) connects the pole to the center of the sphere. The field to be considered is of the Sauter form
\[ E = E_0 \text{sech}^2 \frac{x}{R}. \]
This choice is simply for convenience, as Schwinger pair creation is understood for this types of potential [44]-[57]. In addition, this field is localized in a region of width \( R \), and this imitates the field living on the axion wall bulk. Of course the functional form postulated above correspond to a flat situation and not to a spherical one. But for the sough estimation it may be enough. An important point is that, if the instanton method is to be used for estimating the pair production, the two conditions \( eER >> m_e \) and \( m_e R >> 1 \) should be fulfilled. The second is immediately satisfied since \( l_e \sim m_e^{-1} \sim 10^{-13} \text{cm} \), which is much smaller that the nugget radius estimated above. The first condition can be rewritten as \( eE >> 1/Rl_e \). But for electric fields near to the critical value one has \( eE \sim 1/l_e^2 \) and therefore \( eER \sim R/l_e^2 \sim m_e \), as \( l_e << R \). Thus, both conditions are satisfied for fields near or larger than the Schwinger critical field. In these terms, there exists a formula for the pair production per area [55], which applied to the present case gives
\[ N_f = \frac{2(eE_0)^3}{2\pi^2 m_a^3} \int \int_R \left( \frac{\cosh Zy - \cosh Zx}{\cosh Z - \cosh Zx} \right) (y^2 - x^2) dxdy, \quad Z = \frac{2\pi eE_0}{m_a^2}. \]
The region \( R \) of the integration is given by
\[ R = \{(x, y)| -1 \leq -y \leq x \leq y \leq 1, \quad \epsilon^2 \leq (1 - x^2)(1 - y^2)\}, \quad \epsilon = \frac{m_e m_a}{eE_0}. \]
In order to understand if the spatial inhomogeneity decreases pair production, assume that the electric field is such that \( \delta = eE_0/\pi m_e^2 >> 1 \). This is equivalent to say that the pair production becomes
significant when the value of $E_0$ is considerably larger the critical field (2.9). Then the last formula can be approximated by

$$N_f \sim \frac{(eE_0)^{5/2}}{6\pi^2 \gamma m}.$$ 

In these terms, one may calculate a value $E_0$ such that the last expression is equal to the value of $N_e$ in equation (2.11) evaluated at $E_c$. This calculation yields the following result

$$e^\pi \left( \frac{E_0}{E_c} \right)^2 \sqrt{\delta} = 1.$$ 

But this equation should be applied only for $\delta >> 1$, as follows from the assumption that $E_0 >> \pi E_c$. This condition contradicts the last equation. This implies that $\delta \sim 1$ and therefore the sought field can not be much larger than $E_c$. This reasoning suggest that the spatial inhomogeneity does not alter considerably mean value of pair production on the axion wall.

The result given are somehow similar to the one related to scalar pair production for Sauter fields [44]. In this reference, it is found that the quotient of the rate of pair creation for a Sauter field $w_s$ and for a uniform electric field $w_e$ is given by

$$\frac{w_s}{w_e} = (1 - \gamma^2)^{\frac{5}{2}} e^{-\frac{m^2}{\pi \gamma R}} \left[ \frac{2}{1 + (1-\gamma^2)^{1/2}} \right]^{-1}, \quad \gamma = \frac{m_e}{eER}.$$ 

This quotient shows that the rate is suppressed due to the inhomogeneity parameter $\gamma$, which tends to zero when $R \to \infty$. But the inhomogeneity parameter for the quark nugget with size $R$ considered here is much less than unity, and there is no significant numerical difference between $w_s$ and $w_e$. The results of the previous paragraph generalize partially this result to the case in which there is electron positron pair production instead of scalar fields.

The above discussion suggest that a magnetic field of $B \sim 10^{11}$T may induce pair production considerably. This value is close to the values [15], or the values we have estimated by use of the references [16]-[18]. This value may be even higher. The point is that these objects may have very large masses, and gravitational effects may affect the matter distribution and the value of the magnetic field at the border. In addition, there are several approximations that were employed in order to obtain this numerical value, for instance, that the original domain wall profile is not changed. A proper, more quantitative calculation may throw a modified value for the magnetic field inside and in the border of the object. Furthermore, the presence of the axion field may modified the ferromagnetic properties of the quark liquid inside the nugget and change the value of the magnetic field in [15] at the border of the object. In any case, we suggest there is not a real obstruction for such nuggets to exist as dark matter candidates, even in a magnetized state.

### 2.3 Imprints of a nugget passing through the earth

After arguing that magnetized axion nuggets may be present in the current universe, the next task is to discuss their possible detection. Consider a magnetized quark nugget impacting the troposphere.
It is hard to detect such nugget by underground detectors, since a magnetic field of \( B \sim 10^{11} \) T may increase considerably its stopping power. Thus, other types of experiment should be considered for detecting them. It should be recalled that for a non magnetized quark nugget \([60]\) it is usually assumed that its cross section \( \sigma_n \) is given by the cross sectional area of its core mass density. In this terms the following formula for the energy loss of a QN

\[
\frac{dE}{ds} = -\sigma_n \rho_n v^2, \quad \sigma_n = \pi \left( \frac{3m}{4\pi \rho_n} \right)^{\frac{2}{3}},
\]

is found. As the quark nugget density \( \rho_n \sim 10^{18} \text{kg/m}^3 \), the cross section \( \sigma_n \) is very small and the nuggets are hardly detectable. However, a magnetized and electrified quark nugget has an enhanced cross section. Therefore it is worthy to ask if these nuggets are detectable and if they fulfill the collision bounds for dark matter.

In order to clarify this point, consider one of such nuggets entering into the earth. As these nuggets generate an electric field, a first attempt in order to analyze their energy deposition is to consider the Bethe-Bloch formula for stopping power. There are some analogous formulas for magnetic objects such as monopoles, which in principle may be also applied to a magnetized nugget \([61]\). Instead, it may be convenient to generalize the results obtained in \([19]\) for the stopping power of a magnetic nuggets, since it takes into account that this object is surrounded by a plasma when it enters in the troposphere.

Consider first a nugget that is only magnetized. When it enters into the 20 km region of the troposphere, it finds a neutral medium. There is oxygen and nitrogen in this region and, for instance, the oxygen binding energy is around 8 KeV. The Zeeman term that arises from the interaction of these atoms and the nugget magnetic field is given by \( L_z \sim \mu_b B \) with \( \mu_b \sim 9.3 \times 10^{-25} \text{J T}^{-1} \) the Bohr magneton. It is seen by passing to natural units that \( L_z \sim 6.10^{-5} \text{eV T}^{-1} \). Then the Zeeman energy will have a numerical value close to the keV scale when \( B \sim 10^{7\pm1} \) T. By taking into account that \( B \sim r^{-3} \) in the model \([14]\) and that at the surface \( B \sim 10^{11\pm1} \) T, one may assume that inside a radius \( r \sim (10 - 10^2) r_n \) there is enough Zeeman energy for ionizing oxygen and hydrogen. Thus, in this region, the surrounding medium is approximately an ionized plasma. At first sight the electron and ions of the plasma tend to make round trajectories when entering into a region with a magnetic field \( B \), however the plasma pressure slows down this trajectory. But in addition there is an induced electric field due to the axion wall. The plasma may be considered a fairly good conductor, and it will try to expel somehow this electric field by forming a charged surface, a shield. This surface acts as a source of a new electric field, which modifies the axion profile and the magnetic field of the nugget. For these reasons, the exact problem of the dynamics of a nugget plasma system may be very complicated.

Despite the technical problem just mentioned, some estimations can be made for the nugget plasma dynamics. Consider first a nugget which is only magnetic hitting the troposphere at a non relativistic velocity \( v \). In the system in which the bubble is at rest there is an incoming plasma with a speed \( v \). The particles tend to make a round trajectory, of Larmor type, when approaching the region where the magnetic field is strong, the plasma pressure does not allows this to fully happen. Thus, there is a
surface formed in which the plasma pressure equals the magnetic pressure. The plasma ram pressure is $P_r \sim \rho v^2$ up to a model dependent constant which is not far from unity. The magnetic pressure is $P_m \sim B^2/\mu_0$. By taking into account the dependence $B = B_0 r_n^3/r^3$ is considered, then the condition $P_m = P_r$ shows that the plasma particles are allowed to approach the nugget up to a radius

$$r_n \sim \left( \frac{B_0^2 r_n^2}{\mu_0 \rho_p v^2} \right)^{\frac{1}{3}}.$$  (2.12)

The cross section then can be approximated in natural units by the area determined by this radius

$$Q_n \sim \pi r_n^2 \sim \left( \frac{B_0^2 r_n^2}{\rho_p v^2} \right)^{\frac{1}{3}}.$$  (2.13)

Assume now that an electric field is turned on. Then there are two electric field contributions, the nugget original field $E_n$ and the induced plasma one $E_p$. The plasma one is intended to screen the nugget electric field, something analogous to the Debye effect. It may be fairly reasonable to assume that both fields are of the same order. The electric nugget field is then

$$E_n^2 = g_{\alpha \gamma \gamma} a^2 B^2.$$

By taking into account that $g_{\alpha \gamma \gamma} = c\alpha/f_p \pi$ and that $a = f_\theta \theta$ with $\theta$ an angular variable, it follows that

$$E_n^2 = \frac{c^2 \theta^2 B^2}{\pi^2 (137)^2}.$$

The constant $c$ is not far from unity and the angular variable $\theta$ can take values between $0 \leq \theta < 2\pi N$ with $N$ an integer, it is seen that the electric pressure $P_e \sim \epsilon_0 E^2$ is less or of the same order of magnitude than the magnetic one for models with controlled values of $N$. Thus, it is likely that the region determined by (2.12) is not significantly deformed, but the charge distribution is modified in order to screen the nugget electric field. Therefore, it may be considered that the formula (2.13) applies even for this type of electrified and magnetized nuggets. We turn now to a discussion of the consequences of these results.

### 3. Discussion

In the present work a ferromagnetic axionic quark nugget was considered. It was shown that the internal magnetic field induces an electric field on the axionic wall which decays into electron positron pairs if the magnetic field is above the critical value $B_c = 10^{10}$ T, which close to the range given in [15]. The possible role of the inhomogeneities of the electric field was also taken into account, but it was shown that the deviation from the standard Schwinger formula is small and the critical value remains also unaltered. On the other hand, the energy deposition on the earth due to these objects was studied. This was investigated by generalizing the magnetopause model considered in [19] to the case in which both electric and magnetic fields are present. This model follows an analogy between the plasma surrounding the nugget and the solar wind interacting with the magnetic field of the earth.
In the present work, the effect of the electric field was included and it was shown, by use of the modified Maxwell equations in presence of an axion, that the electric field pressure on the plasma is less than the magnetic one. Consequently, the cross section for a nugget interacting with a plasma is not considerably modified. A direct application of the results given in [19] implies that these nuggets do not violate the collision bounds for dark matter.

The fact that (2.13) applies for these objects have several experimental implications [19]. Quark nuggets with axion domain wall may be detected by their brightness and their track speed. In the reference [60] the authors assumed that the plasma behind the nugget is in thermal equilibrium, that expands with the average molecular velocity and that radiates as a black body. The luminosity that this reference founds is based on the geometric cross section, and assumes that there is a single mass contribution to dark matter. However, for dark matter matter with a continuum of masses there should be a distribution function which is unfortunately unknown. The atmospheric observations of [62], the Pierre Auger Observatory [63] or ICE CUBE [64] could throw light about this distribution function. The fact that the cross section is enhanced implies that there may be additional lower mass contributions to dark matter compatible with observations.

Another possibility is that these objects may be detected by impact craters [65]. This craters should be compatible with energetic events without the presence of meteorites. The group of authors [19] is searching for such events in County Donegal in Ireland, and have analyzed several reported events in India or Nicaragua, but where unable to find unambiguous results. A further source for data analysis is related to impacts in water. When a quark nugget with these masses hit the water, it may produce a shock wave which then generates an acoustic pulse. This results in a pressure wave that can be monitored with three or more, time-synchronized sensors, which are able to determine the distance to the impact point by interpolation. Knowing the pressure at each sensor and the distances to the impact point, the energy per meter depth can be computed and used to estimate the mass of the quark nugget [19]. This source of date may be the promising one for detecting such objects.

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