Transition radiation from graphene

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Abstract. Transition radiation from graphene monolayer is studied theoretically. The cases of normal, oblique and grazing incidence are considered in ultrarelativistic case. The spectral-angular radiation distribution is studied in detail, both for the forward and for backward radiation. The radiation along the monolayer is predicted to exist. The radiation at grazing incidence is demonstrated to be considerably more intensive than at normal and moderately oblique incidence.

Introduction

Discovering of graphene gave an impulse to wide research of this material. The experimental studies [1, 2] demonstrate that graphene conductivity is higher than conductivity of the other two-dimensional structures. On the other hand, graphene is a monolayer in its natural form, whereas usually transition radiation is considered for rather thick targets. This difference can lead to the changing the angular radiation distribution comparatively with standard TR models. Besides, having the TR from graphene studied, one can investigate more complicated structures made of graphene. So, from theoretical point of view graphene can be of interest as an object of exploration. We consider the case when conductivity of the graphene monolayer is set a priori [3], which allows us to make numerical calculations of TR characteristics from graphene.

1. Field of TR from monolayer

It is necessary to find TR characteristics from infinite graphene monolayer, which is placed in vacuum \((z = 0)\) (figure 1). The source of TR are polarization currents, which are produced by the field of a charged particle \(e\) moving uniformly through the monolayer.

![Figure 1. Principle scheme.](image-url)
Maxwell equations for Fourier images can be written in form:
\[
\begin{align*}
\text{rot} \mathbf{H}(\mathbf{r}, \omega) &= \frac{4\pi}{c} j(\mathbf{r}, \omega) - \frac{i\omega}{c} \mathbf{E}(\mathbf{r}, \omega), \\
\text{rot} \mathbf{E}(\mathbf{r}, \omega) &= \frac{i\omega}{c} \mathbf{H}(\mathbf{r}, \omega).
\end{align*}
\] (1)

From (1) one can get expression for magnetic field of our system:
\[
(\Delta + k^2) \mathbf{H}(\mathbf{r}, \omega) = -\frac{4\pi}{c} \text{rot} j(\mathbf{r}, \omega),
\] (2)

where \( k^2 = \omega^2 / c^2 \). Solution of this equation in the wave zone \(|\mathbf{r}| \gg |\mathbf{r}'|, \lambda\) can be written as:
\[
\mathbf{H}(\mathbf{r}, \omega) = \frac{1}{cr} \exp(ikr) \int d^3r' \left( k \times j(r', \omega) \right) \exp(-ikr').
\] (3)

where \( k = n_k = \frac{r}{r} k \) – a wave vector of the radiation field.

Spectral and angular radiation distribution is given by the expression [4]:
\[
\frac{d^2W(n, \omega)}{d\Omega d\omega} = \frac{1}{c} \int d^3r \exp(-ikr)(k \times j(r, \omega))^2.
\] (4)

where \( j(k, \omega) \) is the polarization currents density. To describe the polarization currents density we will use an expression:
\[
j(r, \omega) = \sigma(\omega) \delta(z) E_o(r, \omega),
\] (5)

where \( \sigma(\omega) \) – is the graphene monolayer conductivity, \( E_o(r, \omega) \) – the own moving charge field. The form of the graphene conductivity can be found, for example, in [3].

The radiation distribution from the graphene monolayer can be obtained from (5) and (4):
\[
\frac{d^2W(n, \omega)}{d\Omega d\omega} = \frac{(2\pi)^4}{c} \left| \sigma(\omega) \right|^2 \left| k \times E_o(k_z, z = 0, \omega) \right|^2 \]
(6)

The Fourier image of the own charge field of the in (6) can be found as:
\[
E_o(k_z, z = 0, \omega) = -\frac{ie}{2\pi^2} A \left(\omega v - c^2 Q \right),
\]
\[
Q = k_z + ae_z,
\]
\[
A = \left[ c^2 \left( k_z^2 + a^2 \right) \left( \frac{\omega}{c} \right)^2 \right]^{-1},
\]
\[
a = (\omega - v_z k_z) v_z^{-1}.
\] (7)

Therefore one can get a common expression for the distribution of the TR from graphene monolayer over the angles and frequencies:
\[
\frac{d^2 W(n, \omega)}{d \Omega d \omega} = \frac{(2e)^2}{c} |\sigma(\omega)|^2 \left( \frac{A}{\nu} \right)^2 |\mathbf{k} \times (\omega \mathbf{v} - c^2 \mathbf{Q})|^2. \tag{8}
\]

### 2. Normal incidence case

First we consider normal incidence of charged particle on the monolayer. In this case velocity vector has only one component:

\[\mathbf{v} = \{v_x, v_y, v_z\} = \{0, 0, v\} \tag{9}\]

The radiation distribution takes the form:

\[
\frac{d^2 W(n, \omega)}{d \Omega d \omega} = \frac{(2e)^2}{c} |\sigma(\omega)|^2 A^2 k_z^2 \left( \frac{\nu}{v} \left( \frac{\omega}{v} + k_z \right) \right)^2, \tag{10}\]

where we use the definitions from (7).

In spherical coordinates (see figure 1) equation (10) gets the form:

\[
\frac{d^2 W(n, \omega)}{d \Omega d \omega} = \frac{(2e)^2}{c} |\sigma(\omega)|^2 A^2 \left( \frac{\nu}{v} \right)^2 \left( \beta^{-1} \left( \beta^{-1} + \cos \theta \right) \right)^2 \sin^2 \theta, \tag{11}\]

where

\[A = \left[ \nu \left( \sin^2 \theta + \beta^{-2} \right) - 1 \right]^{-1}. \tag{12}\]

For the ultrarelativistic case one can easily get:

\[
\frac{d^2 W(n, \omega)}{d \Omega d \omega} = \frac{(2e)^2}{c} |\sigma(\omega)|^2 \frac{\sin^2 \theta \cos^2 \theta}{\left( \sin^2 \theta + \gamma^{-2} \right)^2}. \tag{13}\]

In optical frequency range the graphene conductivity takes the simple form [3]:

\[\sigma(\omega) = \frac{e^2}{4\hbar}. \tag{14}\]

and the radiation distribution expression is:

\[
\frac{d^2 W(n, \omega)}{d \Omega d \omega} = \frac{e^2}{c} \frac{1}{\hbar \nu} \left( \frac{e^2}{\hbar c} \right)^2 \frac{\sin^2 \theta \cos^2 \theta}{\left( \sin^2 \theta + \gamma^{-2} \right)^2}. \tag{15}\]

where \(\frac{e^2}{\hbar c} = \frac{1}{137}\) is the fine structure constant.

The radiation distribution as a function of the angle \(\theta\) (figure 1) is demonstrated by figure 2.
It is worth noting that (15) resembles the one from the work [5], where TR from the monolayer was also investigated. The cosine in (15) is not usual for TR from half-infinite space; however, the same cosine was obtained in [4] in a different way, by means of microscopic theory. This cosine defines a shape of TR along the monolayer. In spite of weakness of this radiation, it might be of use for diagnostic purposes – of course, only if it is possible to register this radiation experimentally.

The other thing that differs (15) from the standard TR expressions [6] is a square of fine structure constant. The presence of such a small parameter is expectable, it defines the distinctions between the radiation from monolayer and from macroscopically thick targets.

As one can see, in the normal incidence case the spectral-angular density of radiation does not depend on the azimuth angle $\phi$. The peaks are equally high and distribution is symmetrical. It is notable also, that radiation is concentrated within the two narrow ranges near $\theta = 0$ and $\theta = \pi$. These ranges characterize “backward” and “forward” radiation emitted in the directions that can be obtained according to expression [6]:

$$\Delta = \pi/2 \pm \alpha,$$  \hfill (16)

which in case of normal incidence $\alpha = \pi/2$ are equal to $\pi$ and $0$.

The radiation along the monolayer, i.e. at $\theta = \pi/2$ and near $\theta = \pi/2$, is extremely weak and can be negligible in comparison with the radiation maxima.

The dependence of radiation maxima from Lorentz factor is usual: Lorentz factor increasing leads to radiation increasing and to narrowing of radiation distribution peaks: the higher Lorentz factor, the narrower ranges for distribution peaks.

3. Oblique incidence case
Let us consider an oblique incidence in a relativistic case. In this case velocity vector has two components:

$$v = \begin{cases} v_x, v_y, v_z \end{cases} = \begin{cases} 0, v \cos \alpha, -v \sin \alpha \end{cases}.$$  \hfill (17)

The distribution of TR over the angles and frequencies can be written in form:
\[
\frac{d^2 W(n, \omega)}{d\Omega d\omega} = \frac{e^2}{c} 4 |\sigma(\omega)|^2 \left( \omega^2 A \right)^2 \times \left[ n_x^2 \left( \sin \alpha - \frac{1}{\sin \alpha} \left( 1 - n_y \cos \alpha \right) + n_z \right) \right]^2 + \cos^2 \alpha + \\
+ n_y^2 \left( \sin \alpha - \frac{1}{\sin \alpha} \left( 1 - n_y \cos \alpha \right) + n_z \right) + \left( n_x/n_y \right) \cos^2 \alpha \right],
\]

where

\[ A = \left[ \omega^2 \left( \sin^2 \theta + \frac{1-\sin^2 \alpha}{\sin^2 \alpha} + \gamma^{-2} \right) - n_y \frac{\text{ctg} \alpha}{\sin \alpha} \left( 2 - n_y \cos \alpha \right) \right]^{-1}. \]

In the case \( \alpha = \pi/2 \) (18) transforms into (13). The distribution shape over the angles is demonstrated in figure 3.

\[ d^2 W(\theta, \phi, \delta, \varphi) \]

\[ \alpha = \pi/4 \]

\[ \gamma = 20 \]

\[ \delta = 0.07; \]

\( \sigma(\omega) \) is taken from (14).

As well as in the normal incidence case it is seen that the radiation is concentrated in two narrow peaks, the angular directions of which are described by (16). These peaks characterize “forward” and “backward” radiation. Asymmetry of the shape reflects asymmetry of initial problem (geometry) in the oblique incidence case. The radiation peaks are equally high as in for normal incidence, but in the oblique case peaks are higher than in the normal incidence case.

Radiation directed along the monolayer near \( \theta = \pi/2 \) is not always extremely low and negligible in comparison with distribution maxima as opposed to normal incidence case. For example, at \( \alpha = \pi/4 \) one can neglect the radiation along the monolayer (\( \varphi = 1.14 \)), but for \( \alpha = \pi/5 \) the radiation along the monolayer becomes comparable with the radiation emitted in forward and backward direction. The Lorentz factor does not play important role for the shape of the radiation distribution.

It is also interesting to notice, that if the incidence angle is small enough (\( \alpha = \pi/10 \)), the radiation concentrates near \( \theta = \pi/2 \), that means that radiation is also directed along the monolayer.
4. Grazing incidence case

Let us transform angular part of (18) for the case when the incidence angle $\alpha$ is extremely small, but not equal to zero. The radiation distribution for this case is shown in figures 4. One can see that for incidence angle $\alpha = \pi/100$, the distribution asymmetry can be seen still (figure 4 (a)), and radiation along (in) the monolayer is considerable ($\theta = \pi/2$). Figures 4 (b) and 4 (c) demonstrate that the asymmetry dies away when the incidence angle is much less than the previous quantity. Also, we can see that in grazing incidence case considerable part of the radiation is directed along the monolayer.

![Figure 4](image)

**Figure 4.** TR distribution in case of grazing incidence ($\varphi = \pi/2$); $\sigma(\omega)$ is taken from (14).

Also in grazing case Lorentz factor influences on the distribution shape more intensively. Figures 5 demonstrate that Lorentz factor increasing leads to increasing of the radiation along the monolayer, and the maximum of TR is even higher than in oblique incidence case. The distribution of TR over the angles and frequencies at $\alpha << 1$ can be written as

$$\frac{d^2 W(n, \omega)}{d\Omega d\omega} = \frac{e^2}{c} \left( \frac{e^2}{2hc} \right)^2 \frac{1}{\left[ \left( \sin^2 \theta + \gamma^{-2} \right) \sin^2 \alpha + 1 - \sin^2 \alpha \right]} \times $$

$$\times \left\{ \sin^2 \theta \left( \sin^3 \alpha - \sin^2 \alpha \left( \frac{1}{\sin \alpha} \left( 1 - n_y + n_z \right) \right) \right)^2 + \right.$$  

$$ + \left( n_x^2 + n_z^2 \right) \sin^4 \alpha + 2 \sin^4 \alpha \left( \sin \alpha - \left( \frac{1}{\sin \alpha} \left( 1 - n_y + n_z \right) \right) n_y n_z \right) \right\}.  \quad \text{(20)}$$
6. Conclusion
TR characteristics from graphene monolayer in ultrarelativistic case is considered. The cases of normal and oblique incidence, including the grazing one, are studied.

In normal incidence it is shown that:
1) the spectral-angular radiation distribution is independent of the azimuth angle; 
2) highness of both peaks are equal, and the radiation distribution has clear-cut symmetry; 
3) radiation is concentrated in two narrow ranges: near $\theta = 0$ and $\theta = \pi$. These ranges characterize “backward” and “forward” radiation; 
4) with the Lorentz factor increasing the radiation maximum increases.

In oblique incidence it is shown:
1). radiation distribution over the angles is asymmetrical; 
2). radiation directed along the monolayer near $\theta = \pi/2$ depends on the incidence angle and does not depend on the Lorentz factor; 
3). if the incidence angle is small enough ($\alpha = \pi/10$), radiation is concentrated near $\theta = \pi/2$, i.e. directed along the monolayer.

In grazing incidence case the variety of possible ways of radiation propagation is demonstrated. It is also shown, that the Lorentz factor influences on the distribution radiation shape more intensively than for both previous incidence cases, and more complicated behavior for radiation propagation along the monolayer takes place. The radiation at grazing incidence proves to be more intensive than at moderately oblique and normal incidence.

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