ON THE ORIGIN AND PHYSICS OF GAMMA FLARES IN CRAB NEBULA

GEORGE MACHABELI, ANDRIA ROGAVA, AND DAVID SHAPAKIDZE
Centre of Theoretical Astrophysics, Institute of Theoretical Physics, Iliia State University, Tbilisi, Georgia; andria.rogava@iliami.edu.ge

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ABSTRACT

We consider parametric generation of electrostatic waves in the magnetosphere of the pulsar PSR0531. The suggested mechanism allows us to convert the pulsar rotational energy into the energy of Langmuir waves. The maximum growth rate is achieved in the “superluminal” area, where the phase velocity of perturbations exceeds the speed of light. Therefore, electromagnetic waves do not damp on particles. Instead, they create plasmon condensate, which is carried out outside of the pulsar magnetosphere and reaches the Crab Nebula. It is shown that the transfer of the energy of the plasmon condensate from the light cylinder to the active region of the nebula happens practically without losses. Unlike the plasma of the magnetosphere, the one of the nebula contains ions, i.e., it may sustain modulation instability, that leads to the collapse of the Langmuir condensate. Langmuir wave collapse, in turn, leads to the acceleration of the distribution function particles. Furthermore, the processes that lead to self-trapping of the synchrotron radiation are discussed. The self-trapping results in the growth of the radiation intensity, which manifests itself observationally as a flare. The condition for the self-trapping onset is derived, showing that if the phenomenon takes place at 100 MeV, then it does not happen at lower (or higher) energies. This specific kind of higher-lower-energy cutoff could explain why when we observe the flare at 100 MeV that no enhanced emission is observed at lower/higher energies!

Key words: pulsars: general – pulsars: individual (PSR0531)

1. INTRODUCTION

The Crab Nebula is a source of almost steady high-energy emission. Observations made from orbital probes (Fermi, SWIFT, and RXTE) found evidence of its variability in the X-ray range. Recently, the Fermi and AGILE satellites detected brief and strong bursts of gamma radiation above 100 MeV, with its source located in the Crab Nebula. Currently, several (fewer than 10) gamma-ray flares have been reported. The brightest flares were detected by AGILE and Fermi-LAT on 2011 April 12 and 16 (Striani et al. 2011; Buehler et al. 2012). Further, the remarkably powerful bursts were also detected in 2013 by Mayer et al. (2013) and Aliu et al. (2014). In Büchner & Blandford (2014), an up-to-date review was given of the high-energy emission of the Crab Nebula and the pulsar PSR 0532.

The one and only source of radiation energy in the nebula is the rotational energy damping of the pulsar PSR 0532:

$$\frac{dE}{dt} = I \Omega \frac{d\Omega}{dt} = 5 \times 10^{38} \text{erg s}^{-1},$$

where $\Omega = 200 \text{s}^{-1}$ is the angular frequency of the pulsar and $I = 10^{45} \text{g cm}^2$ is its moment of inertia. This is more than enough to explain the value of the total luminosity of the nebula $dW/dt = 5 \times 10^{38} \text{erg s}^{-1}$. Evidently, the luminosity of the nebula is related to the interaction between accelerated electrons/positrons and could, in principle, be directly compared to the total power injected by the pulsar (Komissarov 2013). Note that the luminosity of the pulsar radiation amounts for only about 1% of the total radiation intensity. In order to generate radiation with energy up to 100 MeV, it is necessary to have particles with energy of the order of PeV’s ($10^{15} \text{eV}$). Tademaru (1973) and Cocke (1975) noticed that particles injected by the pulsar in the nebula, due to the radiation losses, cannot have energies exceeding the TeV range ($10^{12} \text{eV}$). The possible mechanism of the particle acceleration up to TeV energies and further production of 100 MeV photons was suggested in Clausen-Brown & Lyutikov (2012), where gamma bursts were explained in terms of the Doppler enhancement due to the variation of the averaged mini-jet orientations in space. The latter, in turn, is caused by the magnetic field line reconnection in the region occupied by magnetically dominated plasma (Blackman & Field 1994; Lyutikov & Uzdensky 2003). It was also shown (Kirk 2004) that in current layers, where $E > B$, while sufficiently fast particles move along the electric field lines, they are entrapped by the layer and are accelerated. In Cerutti et al. (2012a, 2012b), a region was considered where the electric field exceeds the normal component of the magnetic field $E > B_\perp$. It was shown that the mechanism of particle acceleration is similar to reconnection. For the large-scale processes, of the order of $10^{16} \text{cm}$, particles are accelerated up to PeV energies that correspond to Lorentz factors of the order of $3 \times 10^5$. An obvious test of the proposed mechanism could be a considerable change of the structure of the magnetic field. So far, such a noticeable change of this kind was not observed; in particular, Chandra failed to find such a variation in the X-ray range (Mayer et al. 2013; Weisskopf et al. 2013).

Let us consider the possibility of the energy transfer to the nebula by the kinetic energy of particles. The mechanism of particle appearance and their acceleration in the pulsar magnetosphere were outlined and specified in a number of papers (Kleipkov 1954; Deutsch 1955; Erber 1966; Goldreich & Julian 1969; Sturrock 1971; Tademaru 1973; Ruderman & Sutherland 1975; Michel 1982). According to this mechanism, the maximum energy density of the electron flux is equal to $mc^2 n_{\text{GJ}}$, where $n_{\text{GJ}}$ is the number density of particles extracted from the pulsar surface by the electric field, generated by the neutron star rotation (Goldreich & Julian 1969).
According to this paper, the number density of particles measures as $n_{\text{GL}} \approx 7 \cdot 10^{-2} B / P \text{cm}^{-3}$, where $B$ is the component of the magnetic field parallel to the axis of rotation and $P$ is the pulsar rotation period. The magnetic field falls before the light cylinder according to the dipolar law. Therefore, near the light cylinder, $n_{\text{GL}} \sim 10^9 \text{cm}^{-3}$. Sturrock (1971) is one who first considered the possibility of electron–positron pair creation at the surface of a neutron star through the cascade process that forms the plasma of the pulsar magnetosphere. Also, the Lorentz factor of primordial particles, $\gamma_p \sim 10^3$, was estimated. If we multiply this by the volume of the light cylinder and divide by the time interval, $R_{\text{LC}} / c$, necessary for the transfer of the energy, we find out that the power provided by the particle flux is equal to

$$mc^3 n_{\text{GL}} \gamma_p R_{\text{LC}}^2 \approx 10^{33} \text{erg s}^{-1}. \quad (2)$$

Obviously, it is not sufficient (the observed luminosity $\sim 10^{38} \text{erg s}^{-1}$). Therefore, it is necessary to have an additional mechanism of particle acceleration. This mechanism is proposed in Section 2. In Section 3, long-wavelength electrostatic waves are suggested and considered as candidates for the energy transport, transferring energy from the pulsar to its nebula practically without any losses. In Section 4, we consider the process of particle acceleration in the Crab nebula related to the collapse of Langmuir waves (Zakharov 1972). The Langmuir wave-collapse phenomenon was widely discussed in the plasma theory (e.g., Arcimowich & Sagdeev 1979), but it is relatively unknown to the astrophysical community. In order to explain observed 100 MeV gamma bursts in Section 5, we argue that their appearance is related to yet another plasma nonlinear process—self-trapping—which could explain unusual properties of these bursts. Finally, in Section 6, we summarize our results.

2. PARAMETRIC GENERATION OF LENGMUIR WAVES IN THE PULSAR MAGNETOSPHERE

Recently, it was shown (Machabeli et al. 2005; Mahajan et al. 2013) that relativistic centrifugal force can generate Langmuir waves. According to Machabeli & Rogava (1994), when a particle moves along a straight, rotating field line, the direction of centrifugal acceleration changes if the particle’s initial velocity $v_0 \leq c/\sqrt{2}$ and the particle decelerates. It was shown that this regime of the motion leads to the parametric growth of Langmuir oscillations.

Let us write down the equation of motion in an inertial frame of reference:

$$\frac{\partial p_i}{\partial t} + (v \cdot \nabla) p_i = -c^2 \gamma (1 - \Omega^2 r^2)^{1/2} \nabla (1 - \Omega^2 r^2)^{1/2} + q \left( E + \frac{1}{c} v \times B \right), \quad (3)$$

where $p$, $E$, $B$ are momentum, electric field, and magnetic field, respectively; $\Omega$ is the angular velocity of rotation; $r$ is the distance from the center of the pulsar to the particle; $i = e^x$, $e^y$ is the electron and/or positron. In the weak turbulence approximation, if we consider instantaneous values of $p$, $E$, $B$ as sums of their regular and perturbational components, $p = p_0 + p$, $E = E_0 + E'$, $B = B_0$, then we find that

$$\frac{\partial v_0}{\partial t} = \Omega^2 r \left( 1 - \frac{2v_0^2}{c^2 (1 - \Omega^2 r^2/c^2)} \right), \quad (4)$$

which yields the following simple, periodic solution for relativistic velocities:

$$v_0 = c \cdot \cos(\Omega t + \phi), \quad (5)$$

where $\phi$ is an initial phase and $c$ is the speed of light.

We can also write the continuity equation and the Poisson equation, taking into account that $n$ is the fluctuation of number density and $\nu_0$ is the equilibrium value of this quantity: $\nu_0 = p_0/m$, and $v = p/m$. This way, we will obtain the closed set of equations for electrons and positrons. Further, let us consider a model of two streams with different Lorenz factors. One of them we denote with the index “$p$” and another with the index “$b$.” These two streams are connected to each other by means of the common electric field $E'$. Taking this fact into account and applying Fourier transform, we reduce the system of equations for both streams to the following equation (Mahajan et al. 2013):

$$\frac{(\omega^2 - \omega_p^2)}{\gamma_p^3} N_p(\omega) = \frac{\omega_b^2}{\gamma_b} \sum_{l,s} l_l(a) I_l(a) \exp \left[ i (l \phi_p - s \phi_b) \right] \times \left[ (\omega - (s - l)\Omega)^2 - (\omega - s\Omega)^2 \right] \delta_{lp} \delta_{sb}, \quad (6)$$

where $N_p(t) = n_p \exp[i \cdot a \cdot \sin(\Omega t + \phi_p)]$; $a \equiv kc/\Omega$, while $l_l(a)$ are Bessel functions, $s = 1, 2, 3, \ldots$, and the sum is taken from $-\infty$ to $+\infty$. Taking the average of Equation (6) by phases, we see that on the right-hand side of the equation, the terms with $\phi_p = \phi_b$ give a nonzero contribution when the harmonics coincide with $l = s$. The equality of phases means that particles at the initial moment of time, $t = 0$, are situated at an equal distance from the center of pulsar. The ultimate dispersion relation has the form:

$$\left( \omega^2 - \frac{\omega_p^2}{\gamma_p^3} \right) = \frac{\omega_b^2}{\gamma_b} \sum_{l,s} l_l^2(a) \frac{\omega^2}{(\omega - s\Omega)^2}. \quad (7)$$

Taking $\omega = \omega_0 + \Delta$, we find that $\omega_0^2 = 2\omega_p^2\gamma_p^3$ and $\omega_b = s\Omega$. Relation (7) contains two resonances that have to be satisfied simultaneously. For $\Delta$, we obtain the cubic equation, with its solution containing an imaginary part:

$$\text{Im} \Delta = \pm \sqrt{\frac{3}{2} (2 \omega_0^2 \omega_p^2)} \frac{l_l^2(a)}{2^{3/2}}, \quad (8)$$

where $\omega_p^2 = 4\pi e^2 n_0/m$, the Bessel function index is $s = \omega_0/\Omega$, and $\text{Im} \Delta$ is the growth rate—the quantity that specifies the growth rate of electrostatic perturbation. This quantity depends on the quantitative value of the Bessel function. Its argument $a = kc/\Omega \gg 1$. For large values of the argument, the Bessel function has the maximum value when the argument and index are equal, $s = a$ (Abramowitz & Stegun 1964). Therefore, when $\omega_0 = kc$, only very high-order harmonics, with $s \gg 1$, participate in the parametric interaction of particles with waves. It is important to know what fraction of the Langmuir wave energy is dissipated in the pulsar magnetospheric plasma. An effective energy dissipation mechanism is Landau damping,
which takes place when \( v_{ph} < v \). For this reason, we must know at which wavenumbers \( k_c \) dispersion curve crosses the \( \omega = kc \) line.

Electrostatic waves are described by the following equation:

\[
1 - \sum \omega_p^2 \int \frac{f dp}{(\omega - kv)^2} \gamma^{-3} = 0. \tag{9}
\]

Here, the summation is made by the particle species—plasma electrons and positrons, \( \omega_p^2 = 8\pi e^2 n_p/m \), while \( f \) is the plasma distribution function normalized on unity. Equation (9) can easily be solved in two extreme cases: \( \omega \gg kv \) and \( \omega \approx kc \). The first case describes Langmuir waves, with phase velocities exceeding the speed of light and

\[
\omega^2 = \omega_p^2 \gamma^{-3} + 3k^2c^2. \tag{10}
\]

For the second case, in \( (\omega - kv)^2 \), we substitute \( \omega \approx kc \) and \( v = c(1 - \gamma^2)^{-1/2} \). If we expand the quadratic root by the small parameter \( 1/\gamma^2 \), we derive from Equation (10) the expression for the value of wavenumber vector \( k_c \), at which the curve crosses the \( \omega = kc \) line:

\[
\omega^2/c^2 \equiv k_c^2 = 2\gamma \omega_p^2/c^2 \tag{11}
\]

and the spectrum has the following form (Lominadze et al. 1979):

\[
\omega = kc \left[ 1 - \alpha (k - k_c)/k \right]. \tag{12}
\]

From condition (8), it is evident that \( \omega_0 < kc \); the \( \omega_0 = kc \) condition is fulfilled only when \( k < k_c \). Therefore, the growth rate is maximum in the region where the phase velocity \( \omega_0/k > \omega_0/k_c \approx c \). However, there are no particles with such velocities; hence, Langmuir waves can interact with the particles of electron–positron plasma only via nonlinear effects.

We have to note that while the parametric instability is developed, the energy of the pulsar rotation is directly pumped into the energy of Langmuir waves. From Equation (4), it is easy to estimate the potential power of the parametric instability:

\[
\frac{dW}{dt} \simeq \omega_p^2 \gamma_c^3 (m_p m_{\text{LC}})/(c^2 - \Omega r^2) \text{ erg s}^{-1}. \tag{13}
\]

While the longitudinal velocity, \( v_{\parallel} \), is decreased, the linear velocity of rotation, \( \Omega r \), is increased and, consequently, \( W_t \) may reach the value of \( 5 \cdot 10^{38} \text{ erg} \) at the light cylinder. Hence, we see that due to the parametric interaction, strongly relativistic particles manage to pump the rotational energy of the pulsar directly into the Langmuir waves.

A natural question arises as to what causes the upper limit to the growth of the luminosity with the value of \( 5 \cdot 10^{38} \text{ erg s}^{-1} \). In our opinion, the limit is related to the reconstruction of the magnetic field from the dipole to the monopole configuration (Michel 1982; Bogovalov 2001; Rogava et al. 2003; Osmanov et al. 2008, 2009).

We found out that the parametric generation of Langmuir waves for \( \omega/k > c \) happens with a higher growth rate than for \( \omega/k \leq c \). Therefore, Langmuir waves do not damp. On the contrary, the Langmuir oscillations become accumulated, and they transfer energy along the magnetic field lines up to the Crab Nebula. This process is considered in detail in the next section.

3. TRANSFER OF LANGMUIR OSCILLATIONS TO THE NEBULA

Let us make sure that the energy losses are negligible when the waves are propagating toward the nebula.

It was assumed that the most probable mechanism of longitudinal wave–particle interaction, in the weak turbulence approximation, is a wave scattering by plasma particles. This statement is true if the scattering process is due to the resonance of the type

\[
(\omega_k - \omega_k)/ (k - k_i) = v. \tag{14}
\]

Here, if we take the values of the frequencies from Equations (10) and (11), then it appears that the waves with a spectrum of \( \omega \approx kc \) are scattered onto the thermal particles of relativistic plasma causing the energy pumping into the long-wave band.

The plasma oscillations can be considered a gas of quantum quasi-particles—plasmons (analogue of photons)—and the concept of the number of plasmons is introduced. Scattering does not change the number of plasmons, which is a conserved quantity:

\[
N_k = |E_k|^2/\omega_k \simeq \text{const.} \tag{15}
\]

The energy is pumped to the long-wavelength region of \( k \rightarrow 0 \). From the conservation of \( |E_k|^2/\omega_k \), it follows that \( |E_k|^2/|E_k|^2 \approx \omega_{k=0}/\omega_k \approx \gamma^{-2} \). Thus, the wave energy losses are quite significant. This result has been obtained by Lominadze et al. (1979), based on the assumption that the waves are generated in the subluminal region, where \( v_{ph} < c \).

However, in our case, the main bulk of energy is generated in the region, where \( v_{ph} \gg c \). Therefore, the nonlinear scattering of Langmuir waves onto the plasma particles should be considered in the region, where \( v_{ph} \gg c \). Substituting the value of electrostatic wave frequency from Equation (10) in the resonant condition of nonlinear scattering (see Equation (14)), we obtain the following expression:

\[
v = \frac{\omega_p \gamma_p}{\omega_{k=0} \gamma_{k=0}} \left( \frac{1 + 3k^2c^2\gamma_3}{2\omega_p^2} - \frac{1 + 3k'^2c^2\gamma_3}{2\omega_p^2} \right) \frac{1}{k - k'}. \tag{16}
\]

Taking into account that \( k^2 - k'^2 = (k - k')(k + k') \) and assuming \( k \approx k' \), we derive the following estimation: \( v/c \approx 3c/v_{ph} \ll 1 \). The thermal particles of this sort in ultrarelativistic plasma practically do not exist, and therefore the scattering is not effective.

Since the density of the plasma particles is decreasing as the wave propagates toward the nebula, \( \omega_p^2 \) decreases as well. However, \( \omega_p^2 \) should be conserved. In order for this to happen in the non-relativistic case, it is necessary to raise the value of \( k' \). However, in the relativistic case, the frequency approximately equals \( \omega_{k=0}/\gamma_{k=0}^{1/2} \), when \( k \rightarrow 0 \). Therefore, it is sufficient to have:

\[
\left( \frac{m_{\text{LC}}}{n_{17}} \right)^{1/2} \left( \frac{\gamma_{\text{LC}}}{\gamma_p} \right)^{-3/2} \simeq 1, \tag{17}
\]

where \( n_{17} \approx 10^{13} \text{ cm}^{-3} \) is the bulk plasma density at the pulsar light cylinder; \( n_{17} \sim 10^3 \text{ cm}^{-3} \) denotes the plasma density within the fibers of the Crab Nebula (Manchotsner & Taylor 1977). The frequency does not change while Langmuir waves are propagating toward the nebula. Therefore, as it
follows from condition (17), the thermal particles of plasma have to be cooled down to Lorentz factors of the order of $\sim 1-10$. It means that due to the ineffective wave–particle interaction, the energy of the long waves ($k \to 0$) is transferred to the distance $R_{\gamma} \sim 10^{17}$ cm, almost without any considerable losses.

In the nebula, the spectrum of the Langmuir oscillations is also situated beyond the light cylinder. Here, the energy of electrostatic oscillations has to be converted into the kinetic energy of the particles. This issue is considered in the following section.

4. COLLAPSE OF LANGMUIR OSCILLATIONS

“Condensate” is an intense gas of long-wave plasmons that is unstable when the background is modulated by low-frequency waves. However, there are no such waves in the pulsar magnetosphere since the magnetosphere consists exclusively of electrons and positrons. Indeed, Sturrock (1971) has considered the model where the entire electrostatic field potential difference takes place at the surface of the pulsar and is directed along the open magnetic field lines. Depending on the polarity of the electrostatic field, either electrons or protons are extracted from the pulsar surface. The problem of extraction of positive ions from the surface of the star was considered in Ginzburg & Usov (1972). The authors show that the extraction of positive ions is complicated. The model by Ruderman & Sutherland (1975), as well as Sturrock’s model, is based on the injection of electrons. Then, the primary beam of electrons causes the cascade process of the electron–positron pair production. Thus, in the framework of these models, the pulsar magnetosphere should not contain ions.

In the Crab Nebula, unlike the magnetosphere, there are ions, and they can ensure the low-frequency modulation of the plasmon background, which, in turn, leads to modulation instability (Vedenov & Rudakov 1964). The development of this instability manifests itself in the formation of cavities—regions of localization of excessive wave energy and reduced plasma density. Due to this instability, the energy of Langmuir waves is localized in the cavern. Plasmons appear to be locked in the cavity. High-frequency pressure pushes particles out of the cavern. Consequently, the cavern deepens, and it absorbs more and more plasmons. The process of spatial attraction of plasmons is accelerated. It is accompanied by the collapse of the cavity.

This phenomenon cannot be considered in the framework of the weak turbulence approximation. The dynamics of cavity collapse was studied in Gallev et al. (1977). A numerical experiment illustrating the dynamics of the cavity collapse was presented in Degtjarev et al. (1976). It turned out that during the process, the wave vector, $k$, grows until the collapse phase velocity equalled the speed of particles $\omega/k = v$. Particles, which fall in the resonance with the wave, extract the wave energy. Due to Landau damping, the cavity collapse stops developing, and the distribution function of the particles acquires a long tail (Pelletier 1982).

The above-described collapse scenario takes place in the plasma without an external magnetic field. In the Crab Nebula, the plasma number density is $n_0 \sim 10^3$ cm$^{-3}$, and the magnetic field is $B_0 \sim 10^{-4}$ G, implying that $\omega_p > \omega_B \equiv cB_0/mc$. Thus, the influence of the magnetic field on the collapse of Langmuir waves can be neglected. In this case, the equation for the amplitude of Langmuir oscillations can be written in the following way:

$$\frac{\partial^2 E}{\partial t^2} - \frac{\omega_p^2}{\gamma_p^3} E - \frac{3n_e^2 \omega_p^2}{\gamma_p^3} \frac{\partial^2 E}{\partial x^2} = \frac{\omega_p^2}{\gamma_p^3} \left( n - \frac{n_0}{n_0} \right) E. \quad (18)$$

On the other hand, the number density perturbation, resulting from the plasmon pressure, obeys the equation:

$$\frac{\partial^2 n}{\partial t^2} - \left( T_e/M \right) \frac{\partial^2 n}{\partial x^2} = \pi M \frac{\partial^2}{\partial x^2} |E|^2, \quad (19)$$

where $M$ is the ion mass. The auto-model solution of the system has a “burst-like” appearance:

$$|E|^2 = \frac{E_0^2}{(n_0 - \tau)^2}. \quad (20)$$

where $\tau = n_0T_e(\omega_p/\gamma_p^3/2)\tau E_0^2/8\pi$.

Here, $|E_0|^2$ is the field energy density at the initial stage of the modulation instability and $\tau_0$ is the time of the active phase of the collapse (Zakharov 1972). In the course of the collapse, when waves are being damped, the electric field is decreasing with the increase of the particle kinetic energy.

5. SYNCHROTRON RADIATION OF THE NEBULA
AND 100 MeV BURSTS

As early as 1953, I. S. Shklovsky, in order to explain the continuous radio emission of the Crab Nebula, suggested the synchrotron mechanism (Shklovsky 1953). Soon afterward, observations in the visible spectrum range revealed a substantial level of linear polarization (Dombrovsky 1954; Vashakidze 1954; Oort & Warvalen 1956). The variation of the polarization degree in various regions evidently indicates that the synchrotron mechanism indeed has the leading role in the generation of nebula radiation.

In Gould (1965), Weekes et al. (1989), and de Jager et al. (1996), the mechanism of Compton-synchrotron formation of spectra was proposed. The intensity of synchrotron emission is strongly diminished above a few hundred MeV, and starting from frequencies of the order of 1 GeV, the radiation is determined by the inverse-Compton scattering. In this paper, we restrict ourselves to consider solely frequencies of the order of 100 MeV, which means we deal only with the synchrotron mechanism of the radiation. However, the choice of the particular mechanism, whether it is the inverse-Compton effect or the synchrotron mechanism, does not diminish the solidness of the problem of the pulsar rotational energy transfer to the nebula. Not rejecting the explanation of gamma bursts in terms of the magnetic field line reconnection, which was mentioned in the Introduction, we would like to consider a number of nonlinear phenomena related to the influence of the powerful radiation on the medium (Ashkaryan 1962a; Chiao et al. 1964; Klimontovich 1966; Akhmanov et al. 1967; Whitham 1974; Zeiger et al. 1974). As it turns out, bursts with 100 MeV energies are not accompanied by any tangible increase of radiation intensity in any other ranges of the Crab Nebula radiation (Zenitani & Hoshino 2001, 2007; Aliu et al. 2014; Bietenholz et al. 2015). Such behavior of the bursts makes us surmise that this phenomenon can be explained by the self-trapping effect. In this case, the mysterious behavior of the bursts could be explained in a quite natural way.

Self-trapping is a well-studied phenomenon in nonlinear optics. Its theory was developed in the 1960s, accompanying
the appearance of powerful light sources—lasers. As mentioned above, in the Crab Nebula, a modulation instability is developed that leads to the appearance of cavities, where the energy of long-wavelength electrostatic oscillations is stored. Cavities are eventually collapsed. At the beginning of the process, the collapse timescale is determined by the growth rate of the modulation instability. Further, while \(\tau \rightarrow \tau_0\), according to Equation (20), the collapse rate increases in a burst-like way; the active phase \(\tau \sim \tau_0\) of the collapse starts and Landau damping makes the absorption of the energy substantial with cavity particles from Langmuir waves. Particles get accelerated up to required limits and the cavity collapse stops. The size of cavity particles from Langmuir waves. Particles get accelerated up to Lorentz factors \(\gamma \sim 5 \cdot 10^9\). Then the frequency of synchrotron radiation is situated in the right range:

\[
\omega_s = (eB/mc)\gamma^2 \approx 1.8 \text{ keV}.
\]  

(22)

Plugging the values of the number density and the Lorentz factor into Equation (21), we obtain \(r_D \approx 5 \cdot 10^{14} \text{ cm}\). An average distance between plasma particles is \(r_p \approx n^{-1/3}a_0^{-1}\), hence we can estimate the maximum quantity of particles in the Debye volume as \(N_D \approx (r_D/r_p)^3 \sim 10^{46}\).

The particles from the tail of distribution function also contribute to the process of the cavity collapse. These particles are being accelerated up to Lorentz factors \(\gamma \sim 3 \cdot 10^9\). From Equation (22), we can estimate that the particles, with such Lorentz factors, are emitting photons with energies about 100 MeV due to the synchrotron radiation mechanism. Assuming that \(n_p\gamma_p = n_p\gamma_0\), we can estimate the number of particles, \(N_c \sim 10^{38}\). It is close to the value found in observations by Aliu et al. (2014). Multiplying the number of particles on the radiation intensity of a single particle,

\[
\left(\frac{d\varepsilon}{dt}\right) = \frac{2e^2{\omega_0^2}}{3c} \gamma^2,
\]

(23)

we obtain the intensity of the synchrotron radiation of the order of 100 MeV.

\[
\left(\frac{d\varepsilon}{dt}\right) \sim 10^{36} \text{ erg s}^{-1}.
\]  

(24)

The electromagnetic wavelength can not be less than \(r_D\) since in the Debye volume, electric field is screened out due to the grouping of charged particles. Therefore, the polarization of the medium takes place. For convenience, let us imagine the following model: the medium within the Debye radius consists of dipoles with the same spatial orientation. In this case, the polarization vector is written as follows:

\[
P = N (\text{er}).
\]  

(25)

In the course of the collapse, the transverse size of the cavity (in the \(X\) and \(Y\) directions) is decreasing. The acceleration of electrons happens in the same directions, which, in turn, generates synchrotron photons. So, if we assume that local magnetic field is directed along the \(Z\) axis and the \(X\) axis is directed toward an observer, then the synchrotron radiation is directed in the \(X\) and \(Y\) directions. The electric field of the radiation, \(E(t)\), shifts charged particles. The shifting causes the appearance of the elastic counterforce \(f(t) = -\eta P(t)\), where \(\eta\) is the elasticity coefficient. For not too small values of \(E(t)\), the elasticity force has a nonlinear form:

\[
f(t) = -\eta P(t) - \eta P(t)^3.
\]  

(26)

Taking into account the effect of the elasticity force (see Equation (26)) in the relativistic equation of motion, the value of the electron shifting is determined by the following equation:

\[
m\gamma^2 \frac{d^2r}{dt^2} - m1 \frac{d^2r}{dt^2} + \eta P + \eta P^3 = eE,
\]

(27)

where the second term on the left-hand side defines the dissipation-damping rate. Taking into account the definition of the polarization vector (see Equation (25)) and \(\eta = m\gamma^2\omega_0\), we obtain

\[
d^2P
\]

\[
\frac{dt^2} + \Gamma^2 \frac{dP}{dt} + (\omega_0^2/\gamma^2) P + (q/mc^2N^{2.5}) P^3
\]

\[
= e^2 NE/m\gamma^3.
\]  

(28)

Here, \(\omega_0\) is the frequency of Langmuir oscillations \(\omega_0 = \omega_L/\gamma^{3/2}\). The \(E\) field is large, but it is still much less than the internal field of the cavity. In particular, the power of the wave energy of the cavity stored in Langmuir waves is of the order of \(10^{38} \text{ erg s}^{-1}\), whereas the synchrotron radiation intensity is of the order of \(10^{36} \text{ erg s}^{-1}\). Consequently, the electric field of the wave is small compared to the internal field.

In such circumstances, the nonlinear term can be considered small and the equation can be solved by the method of consecutive approximations. Splitting \(P = P_L + P_{NL}\) with \(P_L \gg P_{NL}\) and neglecting the nonlinear term, we get

\[
d^2P
\]

\[
\frac{dt^2} + \Gamma^2 \frac{dP}{dt} + (\omega_0^2/\gamma^2) P = e^2 NE/m\gamma^3.
\]  

(29)

Taking \(E = A \cos(\omega t)\), we find the following solution:

\[
P(t) = e^2 NA \cos(\omega t + \Phi)/m\gamma^3 \omega^2 \left(\omega^2 - \omega_0^2\right)^2 + 4\gamma^2 \omega^2,
\]

(30)

where \(fr\Phi = \Gamma\omega/(\omega^2 - \omega_0^2)\).

Note that the Langmuir frequency is much smaller than the synchrotron radiation frequency, \(\omega_0 \ll \omega\). Therefore, the range of frequencies we consider is far from the resonance: \(|\omega_0^2 - \omega^2| \gg 4\gamma^2\). It means that the contribution related to the dissipation can be neglected. The polarization vector, \(P\), is proportional to the electric field via the polarizability of the medium, \(\mu\), in the following way: \(P = \mu E\). This expression can
also be written as
\[ P(t) = \mu(\omega)E(t). \] (31)

Equation (28), in the nonlinear approximation, is written in the following form:
\[ \frac{\partial P_{NL}}{\partial t} + \omega_0^2 P_{NL} = -g\mu^2(\omega)E^3(t)/m\gamma^3e^2N^2. \] (32)

Let us rewrite \( E^3(t \cos \omega t + \cos 3\omega t) \). This way, on the right-hand side of Equation (32), we have two terms describing the input of the first and the third harmonics. Consequently, we can write
\[ P(t) = \mu(\omega, A)E(t), \]
\[ \mu(\omega, A) = \mu(\omega)[1 + 3q\mu^2(\omega)A^2/4mN^2e^2\omega^2] \] (33)
and \( \mu(\omega) \) is determined from the solution of Equation (29):
\[ \mu(\omega) = e^2N/m\gamma^3\omega^2. \] (34)

Note that in the series expansion of \( \mu(\omega, A) \), we retain the first non-vanishing terms.

The dielectric permittivity of the medium is described by the tensor \( \varepsilon_{ij}(\omega, E) \). The connection between this tensor and \( \mu_{ij}(\omega, A) \) is determined by the following expression:
\[ \varepsilon_{ij}(\omega, E) = \delta_{ij} + 4\pi\mu_{ij}(\omega, A). \] (35)

The induction vector \( D = E + 4\pi P \) or \( D_i = \varepsilon_{ij}(\omega, E)E_j \). After this is written down, taking into account Equations (33) and (35), we write down Ampere’s law from the Maxwell equations:
\[ \nabla \times B = (1/c)[\varepsilon(\omega) + 3\pi q\mu^3(\omega)A^2/mN^2e^2\omega^2] \frac{\partial E}{\partial t}. \] (36)

From this analysis, it is clear that the influence of the nonlinear term is equivalent to the change of the dielectric permittivity index or the refraction index of the medium. When the electromagnetic wave is propagating in the medium, the refraction index, \( H = c/\sqrt{\varepsilon} \), is a function of the dispersion. Therefore, the dispersion of light depends on the refraction index. From the expression \( H^2 = \varepsilon \), we find that in the cavity, which undergoes the active phase, the refraction index becomes equal to \( H = H_L + H_{NL} \), where \( H_0^2 = \varepsilon(\omega) \), \( H_{NL} = H_2 A^2 \), and \( H_2 = 6\pi q\mu^3(\omega)/mN^2e^2\omega^2 \). (37)

Therefore, if \( H_2 > 0 \), the refraction index in the cavity, \( H = H_L + H_{NL} \), turns out to be larger than the refraction index beyond the cavity, which is equal to \( H = H_L \). In the whole volume of the cavity, separate rays are directed toward the observer. These rays are generated by particles with Lorentz factors \( \gamma \approx 3 \times 10^9 \) and directed toward the observer. That is why they are emitted within the narrow cone with an opening angle of \( 1/\gamma \). Due to the linear diffraction, they have to diverge-diffuse in the direction that is normal to the direction of propagation. Before leaving the cavity, they will be located within the cone with an opening angle of \( 2 \theta_D \), where \( \theta_D \approx \gamma/\sqrt{\gamma}H_0 \). However, when these rays leave the nonlinear medium of the cavity and enter the space with a refraction index, \( H_L \), they undergo nonlinear refraction. If a ray falls on the boundary between the nonlinear, optically thicker medium and the linear, optically thinner medium and the incident angle satisfies the condition \( \theta_D > \theta_0 \), then all diffracted rays will undergo complete internal reflection. We are interested in the condition \( \theta_L \approx \theta_D \), which leads to a situation where the rays are combined to form a parallel beam and the observer witnesses an increase in the radiation intensity.

An asymptotic sliding angle for the complete reflection from the cavity boundary is determined in the following way: \( \cos \theta_0 = H_L/(H_0 + H_2 A^2) \). For the small angle, \( \theta \), we find that
\[ \theta_0 \approx 2(H_2/H_0)A^2. \] (38)

The condition \( \theta_0 \approx \theta_D \) determines the length of the waves that are focused in a parallel beam:
\[ \lambda \approx 2A^2H_2r_D. \] (39)

Substituting Equation (33) into (37), we find that \( H_2 \approx 1/\omega^2 \). Therefore, if the self-trapping appears in a narrow frequency range of a certain wave band, then it is unlikely that it will appear in other wave bands. This circumstance could explain the strange behavior of bursts in the 100 MeV region of the spectrum, while at the same time, in other frequency ranges, the increase of the intensity is not observed.

6. CONCLUSION

Here, we briefly summarize the contents and main results of this paper.

1. The possibility of direct pumping of energy from the neutron star’s vast rotational kinetic energy storage, \( \sim 5 \times 10^{38} \text{erg s}^{-1} \), to proper electrostatic plasma (Langmuir) oscillations is demonstrated.

2. It is shown that the growth rate of the perturbations is maximum in the “superluminal” area, where the phase velocity of perturbations exceeds the speed of light. That is why in this region the condensate of plasmons is formed, which is transferred to the Crab Nebula.

3. It is demonstrated that the transfer of the energy of the plasmon condensate from the pulsar magnetosphere to the nebula over a huge distance, \( \sim 3 \times 10^{17} \text{cm} \), takes place practically without any tangible losses.

4. In the nebula, unlike the pulsar magnetosphere, apart from electrons and positrons, there are also protons. That is why a modulation instability is developed, which leads to the collapse of Langmuir waves. A cavity is formed, which collapses, and on the final stage of the collapse, particles attain very high Lorentz factors, resulting in the powerful synchrotron emission of the nebula. The collapse stops at the scale of a few Debye radii.

5. Theoretical estimation of the cavity (radiating region) and the number of emitting particles coincides with evaluations made on the basis of observational results in Aliu et al. (2014).

6. It is shown that in the course of the active phase of the collapse in the cavity due to the influence of nonlinear processes on the polarization properties of the medium, self-trapping of the synchrotron radiation (generated within the cavity) takes place.

7. It is shown that if the conditions for the appearance of self-trapping phenomenon are fulfilled for certain values of emitted wave frequencies, for other, both higher and lower values of the frequency, they are not satisfied.

8. In our paper, an abrupt increase of the radiation intensity is explained by self-trapping in the framework of nonlinear optics (Askaryan 1962b effect). Waves propagating in the non-parallel direction to the line of sight can be bent and directed to the observer due to self-trapping.
It leads to the required increase of the radiation intensity. At the same time, unlike other mechanisms, no additional energy sources are needed.

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