GRAVITATIONAL COLLAPSE, BLACK HOLES AND NAKED SINGULARITIES

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This article gives an elementary review of gravitational collapse and the cosmic censorship hypothesis. Known models of collapse resulting in the formation of black holes and naked singularities are summarized. These models, when taken together, suggest that the censorship hypothesis may not hold in classical general relativity. The nature of the quantum processes that take place near a naked singularity, and their possible implication for observations, is briefly discussed.

1 Introduction

After a star has exhausted its nuclear fuel, it can no longer remain in equilibrium and must ultimately undergo gravitational collapse. The star will end as a white dwarf if the mass of the collapsing core is less than the famous Chandrashekhar limit of 1.4 solar masses. It will end as a neutron star if the core has a mass greater than the Chandrashekhar limit and less than about 3-5 times the mass of the sun. It is often believed that a core heavier than about 5 solar masses will end, not as a white dwarf or as a neutron star, but as a black hole. However, this belief that a black hole will necessarily form is not based on any firm theoretical evidence. An alternate possibility allowed by the theory is that a naked singularity can form, and the purpose of the present article is to review our current understanding of gravitational collapse and the formation of black holes and naked singularities.

A black hole has been appropriately described by Chandrashekhar as the most beautiful macroscopic object known to man. Only a few parameters suffice to describe the most general black hole solution, and these objects have remarkable thermodynamic properties. Further, excellent observational evidence for their existence has developed over the years. Thus, there can be no doubt about the reality of black holes, and the gravitational collapse of very many sufficiently massive stars must end in the formation of a black hole.

However, the following question is still very much open. If the collapsing core is heavy enough to not end as a neutron star, does this guarantee that a black hole will necessarily form? The answer to this question has to come from the general theory of relativity, and unfortunately this remains an unsolved problem.
What we do know from general relativity about gravitational collapse is broadly contained in the celebrated singularity theorems of Geroch, Hawking and Penrose. It has been shown that under fairly general conditions, a sufficiently massive collapsing object will undergo continual gravitational collapse, resulting in the formation of a gravitational singularity. The energy density of the collapsing matter, as well as the curvature of spacetime, are expected to diverge at this singularity.

Is such a singularity necessarily surrounded by an invisible region of spacetime, i.e. has a black hole formed? The singularity theorems do not imply so. The singularity may or may not be visible to a far away observer. If the singularity is invisible to a far away observer, we say the star has ended as a black hole. If it is visible, we say the star has ended as a naked singularity. We need to have a better understanding of general relativity in order to decide whether collapse always ends in a black hole or whether naked singularities can sometimes form.

Given this situation, Penrose was led to ask whether there might exist a cosmic censor who forbids the existence of naked singularities, ‘clothing each one of them with a horizon’? Later, this led to the cosmic censorship hypothesis, which in broad physical terms states that the generic singularities arising in the gravitational collapse of physically reasonable matter are not naked. Till today, this hypothesis remains unproven in general relativity, neither is it clear that the hypothesis holds true in the theory. What is of course true is that the hypothesis forms the working basis for all of black hole physics and astrophysics. If cosmic censorship were to not hold, then some of the very massive stars will end as black holes, while others could end as naked singularities. As we will argue in Section 3, these two kinds of objects have very different observational properties.

There are various very important reasons for investigating whether or not cosmic censorship holds in classical general relativity. As we have mentioned above, the hypothesis is vital for black hole astrophysics. Unfortunately this fact is rarely appreciated by the astrophysics community. The hypothesis is also necessary for the proof of the black hole area theorem. It is not clear what the status of this theorem will be if the hypothesis were to not hold. If naked singularities do occur in classical relativity, they represent a breakdown of predictability, because one could not predict the evolution of spacetime beyond a naked singularity. Such singularities would then provide pointers towards a modification of classical general relativity, so that a suitable form of predictability is restored in the modified theory. Further, naked singularities might be observable in nature, if they are allowed by general relativity. Undoubtedly then, it is important to find out if the censorship hypothesis is
valid.

We wish to make two further remarks. Firstly, while a great deal is known about the properties of stationary black holes, we know very little about the process of black hole formation. In fact we know as little about the formation of black holes as we do about the formation of naked singularities. Secondly, it has sometimes been remarked that a theory of quantum gravity is likely to get rid of the singularities of classical general relativity, irrespective of whether these singularities are naked or covered. Why then does it eventually matter whether or not cosmic censorship holds? The answer to this legitimate objection is the following. A quantum gravity theory is expected to smear out a classical singularity and replace it by a region of very high, albeit finite, curvature. If the classical singularity is hidden behind a horizon (i.e. is a black hole), this quantum smeared region remains invisible to an external observer. However, if the classical singularity is naked, the smeared region of very high curvature will be visible to far away observers, and the physical processes taking place near this smeared region will be significantly different from those taking place outside the horizon of ordinary astrophysical black holes. Hence, from such an experimental standpoint, quantum gravity has little bearing on the question of cosmic censorship. To put it differently, quantum gravity is not expected to restore the event horizon, if the horizon is absent in the classical theory.

Since a theorem proving or disproving the hypothesis has not been found, attention has shifted to studying model examples of gravitational collapse, to find out whether the collapse ends in a black hole or a naked singularity. While specialised examples such as have been studied are nowhere near a general proof, they are really all that we have to go by, as of now. However, there does seem to be an underlying pattern in the results that have been found in these examples, which gives some indication of the general picture. It is interesting that all models studied to date admit both black hole and naked singularity solutions, depending on the choice of initial data. In the next section, we give a summary of what has been learnt from these examples and what they probably tell us about cosmic censorship. In the third section we will address the question of whether naked singularities might occur in nature, and if so, what they would look like to an observer.

The reader is also invited to study a few other excellent reviews on the subject of cosmic censorship, which have appeared in recent years. Some of these reviews emphasize aspects other than those presented here, and some arrive at conclusions on cosmic censorship other than those given here. I would also like to draw attention to an earlier review of mine on this topic, which is somewhat more detailed, though a little dated. Also, for a detailed bibliography the reader is requested to look up this earlier review; the references
in the present article are not exhaustive, and largely confined to the more recent papers.

2 Theoretical Evidence for the Formation of Black Holes and Naked Singularities

We consider the gravitational collapse of physically reasonable classical matter, where by ‘physically reasonable’ is meant that the matter satisfies one or more of the energy conditions (weak energy condition, strong energy condition and the dominant energy condition). Also, most of the collapse studies that have been carried out so far deal with spherical collapse - even this simplest of systems is poorly understood, in so far as cosmic censorship is concerned. It is of course true that spherical collapse, if allowed to proceed to completion, results in a Schwarzschild black hole. However, a spherical collapsing system can also admit timelike or null singularities which can be naked. From the point of view of an observer falling with the star this can happen if a singularity forms inside the star (say at its center) before the boundary of the star enters its Schwarzschild radius. Such a singularity can be naked.

Since the exact solution of Einstein equations for spherical collapse with a general form of matter is not known, collapse of matter with various equations of state has been studied. In the following pages, we review some of these results.

2.1 Spherical Dust Collapse

Historically, the earliest model of gravitational collapse is due to Oppenheimer and Snyder. They showed that the collapse of a homogeneous dust sphere results in the formation of a black hole (by dust is meant an idealized perfect fluid for which the pressure is zero). It was thought that the formation of the black hole will not be affected even if the specialized assumptions of this model (homogeneity, sphericity, dust equation of state) are relaxed. However, we now know that this is not so.

When the assumption of homogeneity is relaxed, there is an exact solution of Einstein equations - the Datt-Tolman-Bondi solution, which describes collapse of a dust sphere with non-uniform initial density. Two kinds of singularities can result - shell crossing and shell focusing. While the former have a Newtonian analog, at least some of the latter appear to be of purely relativistic origin. It has been shown by various authors that the shell-focusing singularities can be of both the black hole and naked type, depending on the initial conditions.
As an illustration, we mention the following interesting case. Consider the collapse of a dust sphere, starting from rest, and having an initial density profile near the center given by

$$\rho(R) = \rho_0 + \rho_1 R + \frac{1}{2} \rho_2 R^2 + \frac{1}{6} \rho_3 R^3 + \ldots$$

It turns out that the singularity is naked if $\rho_1$ is less than zero, and also if $\rho_1$ is equal to zero and $\rho_2$ is less than zero. If both $\rho_1$ and $\rho_2$ are zero and $\rho_3$ is negative, then we define a dimensionless quantity $\xi = |\rho_3|/\rho_0^{5/2}$. The singularity is naked if $\xi \geq 25.48$ and covered if $\xi$ is less than this number. If $\rho_1$, $\rho_2$ and $\rho_3$ are all zero, the singularity is covered, the Oppenheimer-Snyder collapse being a special case of this.

It is known that the collapse of null dust (directed radiation), described by the Vaidya spacetime, also gives rise to both black hole and naked singularity solutions, depending on the rate of infall.

2.2 Spherical Collapse of Fluids with Pressure

Exact solutions of Einstein equations describing collapse of fluids are rare. Hence little is known about the end state of collapse in these systems. To some degree, numerical methods have been used to integrate Einstein equations and study light propagation. In comoving coordinates, the energy-momentum tensor is diagonal and its components are the energy density, the radial pressure and the tangential pressure. For a perfect fluid, the two pressures are identical.

A significant development was the work of Ori and Piran, who investigated the self-similar gravitational collapse of a perfect fluid with an equation of state $p = k\rho$. It is readily shown that the collapse leads to the formation of a curvature singularity. The assumption of self-similarity reduces Einstein equations to ordinary differential equations which are solved numerically, along with the equations for radial and non-radial null geodesics. It is then shown that for every value of $k$ (in the range investigated: $0 \leq k \leq 0.4$) there are solutions with a naked singularity, as well as black hole solutions.

An analytical treatment for this problem was developed by Joshi and Dwivedi. After deriving the Einstein equations for the collapsing self-similar perfect fluid they reduce the geodesic equation, in the neighborhood of the singularity, to an algebraic equation. The free parameters in this algebraic equation are in principle determined by the initial data. The singularity will be naked for those values of the parameters for which this equation admits positive real roots. Since this is an algebraic equation, it will necessarily have positive roots for some of the values of the parameters, and for the initial data corresponding to such values of the parameters the singularity is naked.
Lifshitz and Khalatnikov (and Podurets) worked out the form of the solution near the singularity for the equation of state of radiation. This work is a precursor to the Belinskii-Lifshitz-Khalatnikov (BLK) series solutions near singularities. Following the method of Podurets, we investigated the nature of the non-central shell-focusing singularity which can form during the collapse of a fluid. It is easily shown that such a singularity is covered so long as the radial pressure is positive. By considering the case of a perfect fluid, we showed that negative pressure allows for a naked singularity if the ratio of the pressure to the density is $\leq -1/3$; and the singularity is covered if this ratio exceeds $-1/3$. We note that the weak energy condition allows for pressure to be negative, although it is questionable whether negative pressures could develop during the final stages of realistic stellar collapse. The BLK series solutions offer a promising avenue for investigating cosmic censorship, which deserves to be pursued further.

On physical grounds, imperfect fluids are more realistic than perfect ones; very little is known about their collapse properties though. An interesting paper is the one by Szekeres and Iyer, who do not start by assuming an equation of state. Instead they assume the metric components to have a certain power-law form, and also assume that collapse of physically reasonable fluids can be described by such metrics. The singularities resulting in the evolution are analysed, with attention being concentrated on shell-focusing singularities at $r > 0$. They find that although naked singularities can occur, this requires that the radial or tangential pressure must either be negative or equal in magnitude to the density.

Another model which has recently attracted some attention is the collapse of a fluid having only tangential pressure. The analysis of the Einstein equations is considerably simpler than the case in which radial pressure is also present. Hence this is a useful system for studying the stability of dust naked singularities against the introduction of pressure. It has been found that while certain equations of state admit only black hole type singularities, other state equations admit naked singularities as well.

We conclude this brief discussion of fluids by commenting on the issue of whether fluids are a reasonable form of matter in so far as cosmic censorship studies are concerned. An objection sometimes raised against fluids is that they form singularities even in Minkowski spacetime, and hence the naked singularities that have been found do not have anything to do with general relativity. However, while some of the singularities, like the shell-crossings, and possibly the weak shell-focusings, have Minkowskian analogs, it is by no means clear that all the singularities (for instance the strong curvature shell-focusings) have counterparts in flat spacetime evolution. At the very least,
this has to be investigated further, so as to get a precise distinction of the singularities that have Minkowskian analogs, from those that are of purely relativistic origin.

It is however more useful to examine cosmic censorship keeping the astrophysical context in mind, and here we know that a fluid description of stellar matter is physically quite appropriate. Thus, if a naked singularity were to result in the collapse of a real star made of fluid matter, we would be compelled to seriously pursue its observational consequences.

2.3 Collapse of a Massless Scalar Field

In a series of papers, Christodoulou has pioneered analytical studies of the spherical collapse of a self-gravitating massless scalar field. He established the global existence and uniqueness of solutions for the collapsing field, and also gave sufficient conditions for the formation of a trapped surface. For a self-similar scalar collapse he showed that there are initial conditions which result in the formation of naked singularities.

Christodoulou was also interested in the question of the mass of the black hole which might form during the collapse of the scalar wave-packet. Given a one parameter family $S[p]$ of solutions labeled by the parameter $p$ which controls the strength of interaction, it was expected that as $p$ is varied, there would be solutions with $p \to p_{\text{weak}}$ in which the collapsing wave-packet disperses again, and solutions with $p \to p_{\text{strong}}$ which have black hole formation. For a given family there was expected to be a critical value $p = p_*$ for which the first black hole appears as $p$ varies from the weak to the strong range. Do the smallest mass black holes have finite or infinitesimal mass? This issue would be of interest for censorship, since an infinitesimal mass would mean one could probe arbitrarily close to the singularity.

This problem was studied by Choptuik numerically and some remarkable results were found. He confirmed that the family $S[p]$ has dispersive solutions as well as those forming black holes, and a transition takes place from one class to the other at a critical $p = p_*$. The smallest black holes have infinitesimal mass. Near the critical region, the mass $M_{bh}$ of the black hole scales as $M_{bh} \approx (p - p_*)^{\gamma}$, where $\gamma$ is a universal constant (i.e. same for all families) having a value of about 0.37. The near critical evolution can be described by a universal solution of the field equations which also has a periodicity property called echoing, or discrete self-similarity. That is, it remains unchanged under a rescaling $(r,t) \to (e^{-\Delta r}, e^{-\Delta t})$ of spacetime coordinates. $n$ is an integer, and $\Delta$ is about 3.4. Subsequently, these results have been confirmed by others.

At the critical solution, the mass of the forming black hole goes to zero,
as \( p \to p_* \) from the right. This critical solution is a naked singularity. However, since the naked singularity is realised for a specific solution in the one parameter family, it is a subset of measure zero. As regards cosmic censorship, the more significant feature are the black holes of arbitrarily small mass, and hence arbitrarily high curvature that is visible to a far away observer. It is more physical to think of censorship in terms of whether or not regions of unbounded high curvature are generically visible, and not just whether singularities are visible. Looked at in this way, scalar collapse provides a serious counterexample to censorship.

Similar critical behaviour has also been found in numerical studies of collapse with other forms of matter. Axisymmetric collapse of gravitational waves was shown to have a \( \gamma \) of about 0.36, and \( \Delta \simeq 0.6 \). For spherical collapse of radiation (perfect fluid with equation of state \( p = \rho/3 \)) the critical solution has continuous self-similarity, and \( \gamma \) of about 0.36. However it has become clear now that the critical exponent \( \gamma \) is not independent of the choice of matter. A study of collapse for a perfect fluid with an equation of state \( p = k \rho \) shows that \( \gamma \) depends on \( k \). For a given form of matter, there appears to be a unique \( \gamma \), but the value changes as the form of \( T_{ik} \) is changed.

An important issue regarding the solutions with \( p > p_* \) is the following. These solutions are identified as black holes because of the presence of an apparent horizon. However, current numerical studies do not probe the singularity itself, and one cannot for now rule out the possibility that the the solutions with \( p > p_* \) fall into two classes: (a) those which have a Cauchy horizon lying outside the apparent horizon, and hence are naked singularities, and (b) those which are black holes. There is actually some evidence for this in the work of Brady [16], and this aspect needs to be investigated further.

A much more detailed discussion of scalar collapse and critical phenomena can be found in other recent reviews [17].

2.4 Spherical Collapse with General Form of Matter

There is a certain degree of similarity in the collapse behaviour of dust, fluids and scalar fields - in all cases some of the initial data lead to black holes, while other data lead to naked singularities. This would suggest an underlying pattern which is probably characterized, not by the form of matter, but by some invariants of the gravitational field. Hence investigations of collapse which put no restriction on \( T_{ik} \) apart from an energy condition should prove useful.

An interesting attempt in this direction was made by Dwivedi and Joshi [18]. They assumed a general \( T_{ik} \) obeying the weak energy condition, and also that the collapsing matter forms a curvature singularity. As we noted earlier, in
the comoving coordinate system, matter is described by its energy density and the radial and tangential pressures. Along with these three functions, three functions describing the metric enter a set of five Einstein equations, which are coupled with an equation of state in order to close the system. The geodesic equation for radial null geodesics is written in the limit of approach to the singularity, and it is shown that the occurrence of a visible singularity is equivalent to the occurrence of a positive real root for the geodesic equation, suitably written. Since this equation depends on free initial data, it follows that for a subset of the initial data there will be positive real roots and the singularity will be visible.

2.5 Null Geodesic Expansion and Cosmic Censorship

A line of investigation which may prove useful for studying collapse of a general form of matter is to examine the evolution of the expansion $\theta$ of a congruence of outgoing null geodesics. Some preliminary work has recently been done. Consider first the case of the collapsing spherical dust cloud. If a point on the cloud ends up as a covered singularity, then $\theta$ at this point evolves to a negative value, as expected, starting from its initial positive value. A naked singularity forms precisely in those cases for which the initially positive $\theta$ continues to remain positive all the way until singularity formation. We have given an argument suggesting that this property of $\theta$ (i.e. its remaining positive throughout the evolution for some initial data) is stable against small changes in the equation of state.

Hence, if dust admits a naked singularity for some initial data, a naked singularity will form also in the collapse of a fluid for which the ratio of pressure to density is small but non-zero, provided one starts from the same initial data. It may be possible to generalise these results, by using the Raychaudhri equation to predict which initial conditions lead to a black hole, and which ones to naked singularities. This is at present under investigation.

2.6 Non-spherical Gravitational Collapse

Amongst the very few studies of non-spherical collapse that have been carried out so far is the numerical work of Shapiro and Teukolsky on oblate and prolate collisionless spheroids. Since there really have been no recent developments in this direction, I refer the reader to the discussion of their work in Section 3 of my earlier review, and to the review by Wald.

A recent work on non-spherical perturbations of spherical collapse deserves mention. Iguchi et al. have shown that the naked singularities arising in
spherical dust collapse are marginally stable against odd-parity non-spherical gravitational wave perturbations.

Unlike the spherical case, very little is known about gravitational collapse and cosmic censorship for non-spherical systems.

### 2.7 Properties of Naked Singularities

There are now sufficiently many known examples of naked singularities for one to enquire about properties of such singularities. It may be that there are well-defined laws of ‘naked singularity mechanics’, just as there are the laws of black hole mechanics (though there is no indication at the moment that such a thing is true). At present there is only some scattered knowledge about properties like curvature strength, stability of the Cauchy horizon and redshift.

Examples of both weak curvature and strong curvature naked singularities have been found. While spacetime cannot be extended through the latter kind of singularity, it may possibly be extendible through a weak singularity. For a discussion of extendibility see Clarke.

If the Cauchy horizon accompanying a naked singularity were to be unstable, that could be evidence in favour of cosmic censorship. However, examples of stable as well as unstable Cauchy horizons are known in classical collapse. (See Penrose for some more discussion on Cauchy horizon stability).

The redshift of the null rays emanating from a singularity can be shown to be infinite, in the known examples, assuming that the standard redshift definition can be used all the way up to a singularity. In this sense, naked singularities are as black as black holes themselves. However, this does not appear to be a good way to preserve censorship because ultimately quantum effects near the naked singularity must be taken into account, and these will serve to distinguish a black hole from a naked singularity.

It can also be shown in a straightforward way that any shell-focusing naked singularities that might form in spherical collapse are necessarily massless.

It can be said that if a naked singularity forms, its most significant property is that regions of extremely high curvature are exposed. This will have observable consequences which will be essentially unaffected by the other properties mentioned above. Hence these other properties can only have secondary importance.

### 2.8 Status of the Cosmic Censorship Hypothesis

Until we learn something definite about non-spherical collapse, it is not possible to conclude about the validity of the hypothesis. However I would like to suggest, on the basis of what is known, that the hypothesis is unlikely to be
true in classical general relativity. We also note that we are regarding a visible region of unbounded high curvature as a violation of censorship, even if this visible region does not contain an actual singularity.

The examples of naked singularities known in spherical collapse arise for various forms of matter. This includes dust, perfect fluids, imperfect fluids and scalar fields. There are also some general arguments suggesting the occurrence of naked singularities for any form of matter satisfying the weak energy condition (e.g. an existence proof, and the behaviour of null geodesic expansion). None of this, taken by itself, constitutes a proof. But, taken together, these arguments strongly suggest that visible regions of unbounded curvature arise generically in spherical gravitational collapse. Also, black holes arise generically in spherical collapse.

Now, we know from the singularity theorems that the occurrence of singularities in spherical collapse is stable against the introduction of non-spherical perturbations. In view of this, it is very hard to see why the naked singularities arising in spherical collapse should be unstable against non-spherical perturbations, whereas the black holes forming in spherical collapse should be stable against such perturbations.

It is only fair to say that different people have drawn widely different conclusions about cosmic censorship from the currently known examples. Since our viewpoint is that censorship possibly does not hold in classical relativity, we would like to ask next if naked singularities could actually occur in nature, and if they do, what would they look like.

3 Are there Naked Singularities in Nature?

3.1 Maybe No ...

Even if general relativity were to generically admit naked singularity solutions, it does not follow that these singularities actually occur in nature. It could be that stars simply do not possess the initial conditions necessary for formation of naked singularities.

Furthermore, there could actually be some principle, over and above general relativity, which forbids naked singularities. This would be in the same spirit in which the advanced wave solutions of electrodynamics are forbidden. We have recently pursued a line of thought wherein the second law of thermodynamics prohibits naked singularities.

Our idea can be deduced from Penrose’s work on the second law of thermodynamics. As explained by Penrose, a fundamental understanding of the second law can be had only if we understand why the initial entropy of the Universe is so low, compared to the maximal value it could have had. The
matter, including radiation, was itself in a high entropy state because of the thermal equilibrium that prevailed soon after the Big Bang. Hence there must be an entropy associated with the gravitational field and this gravitational entropy must have been initially very low, so that the net entropy (matter plus gravity) becomes extremely small.

Such a gravitational entropy will have to be defined from the Riemann curvature. The Ricci part of the curvature diverges at a Friedmann Big Bang singularity. Since we are interested in a low gravity entropy at the Big Bang, it is plausible that this entropy is related to the Weyl part of the curvature, which is zero at the Friedmann singularity. This has come to be known as the Weyl Curvature Hypothesis: in order to have an explanation of the second law, the Weyl curvature must be zero (or at least negligible compared to the Ricci curvature) at the initial cosmological singularity. It should be said though that a concrete mathematical relation between the Weyl curvature and gravitational entropy has not yet been found.

It is possible to regard a naked singularity forming in collapse as an ‘initial’ singularity, because geodesics terminate in the past at the singularity. Hence it is reasonable to require that only those naked singularities can occur which satisfy the Weyl hypothesis. That is, a suitable quantity constructed from the Weyl curvature must go to zero as the naked singularity is approached in the past along an outgoing geodesic. If a naked singularity solution occurs in general relativity but violates the Weyl hypothesis then its existence in nature is forbidden by the second law. Such a naked singularity is a singularity with very high initial gravitational entropy, contrary to what is expected for the second law to hold.

We tested the behaviour of the Weyl scalar in a few simple examples of spherical naked singularities and found it to diverge at the singularity, along outgoing geodesics. It diverges as fast as the Ricci part, and hence violates the Weyl hypothesis. Since strong inhomogeneity tends to favour a high Weyl, and since it also favours naked singularities, it is likely that this divergence behaviour is generic to naked singularities. Thus naked singularities may be anti-thermodynamic entities.

I do not know of any easy way out of this line of argument. The argument may fail only if it turns out that there is actually no connection between gravitational entropy and the Weyl curvature.

3.2 Maybe Yes ...

It would of course be a much more interesting state of affairs if no principle forbids naked singularities, and if they were to be found in nature. Hence we
would like to enquire what the observational signatures of naked singularities will be. It is nearly certain that naked singularities will not emit significantly through classical processes, because of the extremely large redshifts. However, a quantum treatment of the matter and of gravity will be unavoidable near the singularity, and these quantum effects will result in an observable emission, in spite of the large classical redshift. In the absence of a quantum theory of gravity, the best one can do is compute the quantum particle creation in the classical gravitational field which becomes very strong as the singularity is reached. This semiclassical treatment will in fact suffice until the final Planck epoch prior to the singularity formation.

Thus, in effect one is asking what is the analog of Hawking radiation in the case when a star collapses to a naked singularity. Answering this is not as direct as the Hawking radiation calculation for a black hole, because of the presence of a Cauchy horizon. Part of the future null infinity is exposed to the naked singularity and hence one cannot perform the usual expansion of matter field modes in the future. This prevents the usual Bogoliubov transformation and the standard particle creation calculation from being carried out. This is one of the reasons why not much work has been done on this important problem and computation is still in its infancy.

One way out is to compute the quantum expectation value of the stress energy tensor - this can be calculated locally, and on future null infinity, in the approach to the Cauchy horizon. The outgoing flux of radiation is a measure of the emission from the naked singularity. In four dimensions an exact calculation is not possible, but the outgoing flux in the geometric optics approximation has been calculated for dust shell-crossing singularities. In this case the flux does not diverge in the approach to singularity formation. We have recently used the method of Ford and Parker to compute the flux of a massless scalar field on the Cauchy horizon resulting from a shell-focusing dust naked singularity. This time the flux diverges, suggesting that the back-reaction will avoid formation of the naked singularity. In an interesting paper, Vaz and Witten have calculated the spectrum of this radiation, and shown it to be very different from the black-body spectrum of Hawking radiation.

In two dimensions, as a result of the conformal anomaly, the outgoing flux can be calculated exactly, without having to resort to the geometric optics approximation. This was earlier done for the null-dust (Vaidya) naked singularity and repeated by us for the dust (Tolman-Bondi) naked singularity. In both cases the flux diverges on the Cauchy horizon. Similar features have been found in studies of the quantum behaviour of naked singularities in some string-inspired gravity models. The back-reaction calculation in all the above models is extremely hard to perform, as can be expected. But it is plausible
that essentially the back-reaction will remove the classical naked singularity, without significantly affecting the flux emitted to infinity.

The observable signature of a naked singularity appears to be the burst of radiation emitted as the Cauchy horizon is approached, and the characteristic non-thermal spectrum which this radiation possesses. This is to be contrasted with the slow evaporation of a quantum black hole via black-body radiation. It would be important to generalise the above results to find the typical signatures of quantum naked singularities and to explore if there are any astrophysical objects whose properties resemble those of a naked singularity.

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QUESTIONS

C. Sivaram: The third law of thermodynamics might have something to do with, for instance, non-formation of a naked singularity from an extremal black hole. You need an infinite number of steps to reduce $M$, such that $M^2 < Q^2$ or $a^2$, i.e. to destroy the event horizon.

T. P. Singh: You maybe right, but perhaps we should be a little cautious and note that there is no formal proof yet of the third law of black hole mechanics within classical general relativity. It could turn out that for such a proof to hold, cosmic censorship may have to be assumed, as for the second law of black hole mechanics.

C. Sivaram: In a collapsing universe merger of black holes would lead to an enormous increase of entropy, so that there is complete time asymmetry with respect to the expanding phase. What would happen if naked singularities are formed?

T. P. Singh: If a naked singularity forms and the formation of the horizon is prevented, then the huge entropy increase associated with the horizon area will not take place. But it is not clear how much entropy is to be associated with the naked singularity itself.

N. D. Hari Dass: I do not understand your remark that Choptuik’s result that arbitrarily small black holes can form is not good for cosmic censorship, because howsoever small the black hole is big enough to cover the singularity.

T. P. Singh: I am avoiding making a physical distinction between a singularity and a region of unbounded high curvature. The physical processes that would result because of the formation of a naked singularity should be much the same as those resulting from the formation of a visible region of arbitrarily high curvature but not in itself containing a singularity. Hence I think one should regard cosmic censorship as the statement that visible regions of arbitrarily high curvature do not develop generically in collapse.

Tariq Shahbaz: How much energy would you expect to be released in the formation of a naked singularity?

T. P. Singh: At this stage, it is difficult to give a precise answer to this question, and the answer will also be model dependent. A few general remarks can be made. The energy release will come not only because of the naked singularity, but also because of the high curvature regions surrounding it. Let there be a visible region of mass $M$, which has curvature high enough so that the associated curvature length scale is comparable to the Compton wavelength of some typical elementary particles. Then one can expect almost the entire mass $M$ to be converted by pair creation into the energy released during collapse.
R. Misra: How long will a naked singularity last?

T. P. Singh: If we stay within the limits of classical general relativity, spacetime comes to an end at a naked singularity, and one cannot answer the above question. Also, from the point of view of a far away observer, a classical naked singularity will take forever to form, because of the infinite redshift. But quantum pair creation in the collapsing object will be important, and then the naked singularity can be expected to radiate itself away in a finite time (as seen by the distant observer) as a result of the pair creation. We are trying to get estimates of how much this time will be in some simple collapse models. Keeping in mind the divergence of the radiated flux on the Cauchy horizon, one thing appears very likely - naked singularities are events (like explosions) and not objects with astronomical lifetimes.

Chris Clarke: How can you make any statement about the Weyl Curvature Hypothesis for a singularity when viewed in the future, given that in that region one is no longer in the domain of dependence on the initial conditions?

T. P. Singh: We are assuming an analytic continuation.

Sukanta Bose: Is it fair to compare a local and “intensive” quantity, namely the Weyl curvature, with a global and extensive quantity such as gravitational entropy? Will not a given amount of matter, whether in a mixed or pure state, affect the Weyl curvature identically?

T. P. Singh: The connection between Weyl curvature and entropy, if there is one, should be such that a non-local quantity constructed out of the Weyl has properties of entropy. As regards the second question, if we are considering classical matter, then we know that clumping (increase in inhomogeneity) generally leads to increase in Weyl curvature.

J. Pasupathy: Are the results of P. S. Joshi and T. P. Singh on dust collapse analytical or numerical? $\rho_1 < 0$ seems physically reasonable. Would you then say that naked singularities are more likely than black holes?

T. P. Singh: The results are analytical. It is true that a naked singularity is more likely than a black hole in the example I gave in the talk (and mentioned here in the article). However, when the general initial data for spherical dust collapse is examined, both black holes and naked singularities result from a non-zero measure of the initial data set.