Investigation of Jump Resonance of a Horizontal Axis Washing Machine for Nonlinear Vibration using the Harmonic Balance Method

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Abstract. In this work, a drum-type horizontal axis washing machine with two springs and two dampers has been considered to study the jump resonance phenomenon. Here two springs and two dampers are used and effects of nonlinear spring stiffness on vibration level in both horizontal and vertical directions have been investigated. In this study, a mathematical model of a horizontal axis washing machine is formed. A separate analysis is carried out in both vertical and horizontal directions by considering the whole system as a two degree of freedom system. As vibration magnitude is large due to unbalance mass of wet clothes, hence nonlinear stiffness of spring is considered. The analytical solution for the mathematical model is found using the harmonic balance method. One term expansion is used in this method to study the response of the system. To study the vibration response of the system, one term expansion is used. From this study, it has been found that the spring with cubic nonlinear stiffness increases the amplitude of vibration. Also, with the increase in cubic non-linear stiffness, the system tends to have multiple equilibrium positions. These equilibrium positions may be stable or unstable. It has been detected that the vibration amplitude has three different values for a few excitation frequencies due to cubic stiffness. So, the response of the system suddenly increases and then decreases due to cubic non-linearity which indicates the jump resonance.

Keywords: Resonance, Stiffness, Vibration

1. Introduction

The recent advances in technology have continuously eased the life of a human being. We are surrounded by several such examples in our daily life. Among such, the notable example is the washing machine [1]. Washing machines are categorized as horizontal axis and vertical axis washing machines based upon the viewpoint of the rotational axis of the drum during washing and drying operation [2]. A washing machine generally consists of two springs, two dampers or three springs, three dampers attached to the drum. But in this study, two spring and two dampers are used to minimize the manufacturing cost and assess the vibration level due to unbalancing of wet clothes during washing and drying operation. Throughout the washing operation, damp clothes cling to the inner side of the drum and create an unbalanced mass. This unbalance mass makes the vibration level severe and hence owing to the large deflection of spring, non-linear cubic stiffness comes into the picture. Resonance is an unwanted phenomenon associated with vibration. A very large amplitude of vibration is observed during the resonance conditions. Due to unbalance mass of wet clothes, the washing machine sometimes shakes violently, which may cause injury to the person and a breakdown of its part may take place [3]. Beatriz et al. studied the outcome of the padding in estimating the fleeting movement of the tub [4]. They provide a D-optimal design approach to anticipate the movement of the tub during resonance and used it in the design phase of the washing machine to avoid tub collision. Vibration amplitude in a washing machine can be minimized by employing a hydraulic balancer. This hydraulic balancer contains saltwater and acts as a counter mass to unbalance mass[5]. Cristiano et al. used the
control technique for magnetorheological dampers for noise and tremor lessening in the washing machine [6]. They have proposed to swap the standard passive dampers with electronically controlled one and used 3-axis MEMS sensors instrumented to the washing machine body to study the noise and vibration level. Baris and Haluk came up with a method to regulate the oscillation to deal with the dynamic constancy of a horizontal axis washing machine. They have achieved it using feedback control of step motors. The oscillation data was received by the step motors attached to the friction dampers and accordingly expand or narrow the bracelets radius positioned on dampers in a washing machine. Friction damper non-linearity was studied by Kang et al. by using Bouc-wen model. They have assumed the hysteresis behaviour of the damper for the nonlinear model [8]. Yasuaki et al. have predicted the resonant frequency range in both horizontal and vertical directions for the outer drum of the washing machine. They have proposed the G fall balancer to reduce the unbalance mass due to wet clothes and reduces the vibration level by 70 % at resonance [9]. Pinar and Mutlu optimized the oscillation features of the washing machine throughout the spinning cycle using a genetic algorithm [10]. The suspension block of the washing machine was modelled dynamically in 2D to study the genetic algorithm for vibration characteristics optimizations. The input was given as the spin speed and translations were obtained as the output in the x-z plane. Mingyu et al. predicted the rebalancing method using a robot balancer system to reduce the unbalance mass for the front-loading machine. Here rotor balancer detects the unbalance mass and moves the zigzag layer in reverse direction[11]. Walking instability and its control during the spinning cycle was predicted by Evangelos et al. They have proposed a design-based and active method to reduce the vibration amplitude [12].

So far, negligible work has been done for investing jump resonance and instability of a horizontal washing machine considering cubic stiffness. So present work focuses on predicting the jump resonance frequency of a washing machine with two springs and two dampers using the harmonic balance method. Here the effect of cubic stiffness on vibration amplitude, jump resonance frequency and the possibility of multiple equilibrium states in both horizontal and vertical directions have been investigated.

2. Mathematical Prototype

The mathematical prototype of the horizontal axis washing machine is obtained considering the whole system as two degrees of freedom system and it is displayed in Figure 1. Here two spring and two viscous dampers are used. Large deflection of spring and dampers from equilibrium position is assumed and equation of motion in both horizontal and vertical direction is derived using D’ Alembert’s principle. The spring stiffness is assumed as cubic. The mathematical prototype of the washing machine with cubic stiffness is represented by equation (1) and (4). As equation (1) and (4) does not obey the superposition principle, hence these are non-homogenous, nonlinear ordinary differential equation. To get the analytical solution for equations (1) and (4), method of harmonic balance is used. Here the solution of the system is assumed in the form of Fourier series expansion.
Figure 1. Mathematical prototype of the washing machine with two springs and two dampers.

\[
(m_u + m)\ddot{x} + 2(c_e)\dot{x} + 2(k_e)x + k_3x^3 = m_u\omega^2 \sin \omega t
\]

(1)

\[
x = x_1 \cos (\omega t + \phi)
\]

(2)

\[
x_1^2 \left[ \frac{3k_1x_1^2}{4} + (k - m\omega^2)^2 \right]^{1/2} + \omega^2 c_2 = F^2
\]

(3)

Here equation (2) represents the assumed solution in horizontal direction using harmonic balance method. One term expansion is used to get frequency amplitude relation. Equation (3) represents the final solution after putting equation (2) in (1) and comparing the coefficient of 'ω' and 'ϕ' terms.

\[
(m_u + m)\ddot{y} + 2(c_e)\dot{y} + 2(k_e)y + k_3y^3 = m_u\omega^2 \cos \omega t
\]

(4)

\[
y = y_1 \cos (\omega t + \phi)
\]

(5)

\[
y_1^2 \left[ \frac{3k_1y_1^2}{4} + (k - m\omega^2)^2 \right]^{1/2} + \omega^2 c_2 = F^2
\]

(6)

Here equation (5) represents the assumed solution in the vertical direction using the harmonic balance method. One term expansion is used to get frequency amplitude relation. Equation (6) represents the final solution after putting equation (5) in (4) and comparing the coefficient of 'ω' and 'ϕ' terms.

Here \(m\) is mass of drum, \(k\) is linear spring stiffness, \(k_3\) is spring stiffness due to cubic nonlinearity, \(c\) is damping coefficient, \(\omega\) is the speed of rotation of the drum, \(e\) is the eccentricity of unbalanced mass, \(m_u\) is unbalanced mass due to wet clothes, \(\theta\) is the inclination angle of spring with vertical and \(\beta\) is the inclination angle of the damper with vertical. \((k_e)_x\) and \((k_e)_y\) are the effective stiffness in \(x\) and \(y\)-direction respectively. The values of \((k_e)_x\) and \((k_e)_y\) are \(k \sin^2 \theta\) and \(k \cos^2 \theta\) respectively. Similarly, \((c_e)_x\) and \((c_e)_y\) are the effective damping coefficient in \(x\) and \(y\)-direction respectively. The values of
\((c_e)_x\) and \((c_e)_y\) are \(c \sin^2 \beta\) and \(c \cos^2 \beta\) respectively. The value of \(F\) is \(m_u e \omega^2\). The value of linear spring constant \(k\) along the spring direction is taken as 1500 N/m.

| \(m_u\) (kg) | \(m\) (kg) | \(e\) (m) | \(\theta\) (degree) | \(\beta\) (degree) |
|------------|-----------|----------|-----------------|-----------------|
| 7          | 10        | 0.066    | 30              | 30              |

### 3. Results and Discussion

The governing nonlinear differential equation obtained from the mathematical model is solved analytically. To get the amplitude-frequency relation, the harmonic balance method is used. In the harmonic balance method, the solution is assumed in the form of the Fourier series. Here one term expansion in the Fourier series is used. From the frequency versus amplitude relation, it has been observed that for a small value of cubic spring stiffness, the system has only one equilibrium position. With the increase in cubic stiffness, multiple equilibrium positions exist at a particular frequency. These equilibrium positions may be stable or unstable. The amplitude of vibration also increases with cubic stiffness. The results are shown below in Figures 2 and 3 respectively for horizontal and vertical directions.

![Figure 2. Frequency versus amplitude relation showing the jump resonance for cubic stiffness in the horizontal direction.](image)

From the results shown in Figures 2 and 3, it has been observed that the jump resonance frequency value is more for vertical direction response. Also, vibration amplitude first increases with excitation...
frequency become maximum at the resonant frequency and then become constant for any value of cubic stiffness. The pattern is the same for both horizontal and vertical direction deflection.

![Graph showing frequency versus amplitude relation for cubic stiffness](image)

**Figure 3.** Frequency versus amplitude relation showing the jump resonance for cubic stiffness in the vertical direction.

4. **Conclusion**

In this study, the jump resonance of a horizontal axis washing machine with two springs and two dampers have been investigated. Cubic non-linear stiffness of both the spring is considered. The non-linear differential equation of the mathematical model is solved by the harmonic balance method to get frequency amplitude relation. Separate analysis is carried out in both vertical and horizontal directions by considering the whole system as a two degree of freedom system. From this study, it has been found that, by increasing the cubic stiffness, oscillation amplitude increases in both horizontal and vertical directions. Also, the system (washing machine) starts having multiple equilibrium positions with an increase in cubic stiffness at particular excitation frequency. These multiple equilibrium positions may be stable or unstable. As the number of solution changes with cubic stiffness, so the cubic stiffness act as a bifurcation parameter. This study also predicts the jump resonance frequency range in both horizontal and vertical directions. This study can be extended to design a new control technique to minimize the system vibration amplitude in this jump frequency range. Also, this amplitude can be used by piezoelectric energy harvesting MEMS sensor to generate power for the internet of things sensors of washing machine.

**Informed consent:** Informed consent was obtained from all individual participants included in the study.

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References

[1] Somheil T 1992 *New Ways of Washing* Appliance pp.74-7
[2] Waverly W 2021 3 Types of Washing Machines Retrieved from https://www.duerdensappliance.com/blog/3-types-of-washing-machines
[3] Homepage 2021 Why does washing machine sometimes shake violently Retrieved from https://youaskweanswer.net/why-does-washing-machine-sometimes-shake-violently
[4] Tabuenca B, Galé C, Lladó J, Albero C and Latre R 2020 Washing machine dynamic model to prevent tub collision during transient state *Sensors* **20** 1–17
[5] Bae S, Lee M, Kang J, Kang S and Yun R 2002 Dynamic analysis of an automatic washing machine with a hydraulic balancer *J. Sound Vib.* **257** 3–18.
[6] Spelta C, Previdi F, Savaresi S, Fraternelle G and Gaudiano N 2009 Control of magnetorheological dampers for vibration reduction in a washing machine *Mechatron.* **19** 410–21
[7] Yalcin B C and Erol H 2015 Semiactive vibration control for horizontal axis washing machine. *Shock Vib.* **1**-10
[8] Kang D, Jung S, Nho G, Ok J, Yoo W 2010 Application of Bouc-Wen model to frequency-dependent nonlinear hysteretic friction damper *J. Mech. Sci. Technol.* **24** 1311–17
[9] Sonoda Y, Yamamoto H and Yokoi Y 2003 Development of the vibration control system g-fall balancer for a drum type washer/dryer *Proc. IEEE/ASME Inter. Conf. on Advanced Mechatronics* **2** 1140–44
[10] Boyraz P and Gündüz M 2013 Dynamic modeling of a horizontal washing machine and optimization of vibration characteristics using Genetic Algorithms. *Mechatron.* **23** 581–93
[11] Jo M, Kim J and Choi J 2020 Rebalancing Method for a Front-loading Washing Machine Using a Robot Balancer System. *Int J Control Autom Syst.* **18** 1053–60
[12] Papadopoulos E and Papadimitriou I 2001 Modeling, design and control of a portable washing machine during the spinning cycle. *Proc. IEEE/ASME Inter. Conf. on Advanced Intelligent Mechatronics* **2** 899–04