Weak decays of heavy hadron molecules involving the $f_0(980)$

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We study weak decays of the charm- and bottom-strange mesons $D_{s0}^*(2317)$, $D_{s1}(2460)$, $B_{s0}^*(5725)$ and $B_{s1}(5778)$ with $f_0(980)$ in the final state by assuming a hadronic molecule interpretation for their structures. Since in the proposed framework the initial and final states are occupied by hadronic molecules, the predictions for observables can provide a sensitive tool to further test the hadronic molecule structure in future experiments.

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I. INTRODUCTION

Over the last decades it became clear that the meson mass spectrum shows a much richer structure than one might expect from the conventional constituent quark model assigning mesons as $q\bar{q}$ states. For example, the structure of the light scalar mesons below 1 GeV such as the $f_0(980)$ have been in the focus. The strong and electromagnetic decay properties of the scalar $f_0$ have been intensely studied in various models ranging from quarkonium and hybrid structures to compact tetraquarks and hadronic molecules (for overview see e.g. Ref. [1]).

Newer experiments delivering data in the heavier mass region also attracted interest on mesons with open and hidden charm flavor configurations. Within this context one has to mention the $D_s^*(2317)$ which has the favored spin-parity assignments $J^P = 0^+$ and which was first observed by BABAR at SLAC [2]. Shortly afterwards the CLEO collaboration [3] published their data on the axial $D_{s1}(2460)$. Both resonances have been confirmed by Belle [4]. Up to now the structure issue of the $D_s^*(2317)$ and $D_{s1}(2460)$ remains an open question. Both mesons have therefore been discussed within various structure assumptions and theoretical frameworks [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57].

Since their masses are located slightly below the $DK$ and $D^*K$ thresholds, the $D_s^*(2317)$ and $D_{s1}(2460)$ are clear candidates for hadronic molecules with the configurations $D_s^*(2317) = DK$ and $D_{s1}(2460) = D^*K$. In addition, extending this interpretation to the bottom sector, the scalar and axial-vector mesons $B_s^*(5725)$ and $B_{s1}(5778)$ are treated as the equivalents to the charm-strange mesons $D_s^*(2317)$ and $D_{s1}(2460)$. The bottom-strange counterparts $B_s^*(5725)$ and $B_{s1}(5778)$ are consequently also described as bound states with $B_s^*(5725) = BK$ and $B_{s1}(5778) = D^*K$. The decay properties of these hadronic molecules were studied within the same effective Lagrangian approach [51, 52, 53, 54, 55, 56, 57]. Within this covariant model for hadronic bound states, the molecular structure is considered by the compositeness condition $Z = 0$ [51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65] which implies that the renormalization constant of the hadronic molecule field is set equal to zero. The composite object therefore exists exclusively as a bound state of its constituents. This condition also provides a method to fix the coupling between the hadronic molecule and its constituent mesons in a self-consistent way. Furthermore, our theoretical framework also features finite size effects of the meson molecules controlled by size parameters which are the only adaptive variables.

In the present paper the $f_0(980)$ properties are studied in weak hadronic decays of the scalar $D_s^*(2317)$ and its bottom-strange counterpart $B_s^*(5725)$ as well as in the weak non-leptonic decay processes of the axial-vector mesons $D_{s1}(2460)$ and $B_{s1}(5778)$. Since we deal with transition processes between hadronic molecules, the decay properties involve twice the effect of meson bound states: In the initial heavy meson system and in the final scalar $f_0$. For this reason the results might provide a sensitive observable to test the issue of hadronic molecule structure accessible in future experiments.

The paper is organized as follows. In the next section II we give a short introduction to the effective Lagrangian approach we use for the description of hadronic bound states. In section III we deal with the weak non-leptonic decays of the scalar mesons $D_s^*(2317)$ and $B_s^*(5725)$, where the meson molecule $f_0$ appears in the final state. The $D^*K\pi$ coupling $g_{\pi}$, which we need for the $D_{s1}^*(2460) \rightarrow f_0\pi^+$ transition, is derived in Sec. III from the $D_s \rightarrow \pi f_0$ decay. Thereby we also obtain the $D^* \rightarrow K\pi$ decay width as a byproduct of our analysis. In Sec. IV we finally compute the $f_0$-production in hadronic decays of the axial-vector mesons $D_{s1}(2460)$ and $B_{s1}(5778)$.

II. BASICS OF THE MODEL

An assortment of mesons with masses lying close to two-body thresholds are good candidates for mesonic bound states and have therefore been studied assuming a hadronic molecule structure. For instance, in Refs. [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43] we developed a field-theoretical approach to study the properties of hadronic molecules ($f_0(980), D_s^*(2317), D_{s1}(2460), B_s^*(5725), B_{s1}(5778)$ and $X(3872)$) as bound states of two mesons. Since above states are close to the
corresponding thresholds, we used the following dominant composite structures:

\[
    |f_0\rangle = \frac{1}{\sqrt{2}}(|K^+K^-\rangle + |K^0\bar{K}^0\rangle), \\
    |D^{*+}_{s0}\rangle = \frac{1}{\sqrt{2}}(|D^+K^0\rangle + |D^0K^+\rangle), \\
    |D^{*+}_{s1}\rangle = \frac{1}{\sqrt{2}}(|D^*K^0\rangle + |D^*K^+\rangle), \\
    |B^{*0}_{s0}\rangle = \frac{1}{\sqrt{2}}(|B^+K^-\rangle + |B^0\bar{K}^0\rangle), \\
    |B^{*0}_{s1}\rangle = \frac{1}{\sqrt{2}}(|B^*K^-\rangle + |B^*\bar{K}^0\rangle).
\]

The model for hadronic molecules \( H = f_0(980), D^{*0}(2317), D_{s1}(2460), B^{*0}_{s0}(5725) \) composed of two meson constituents \( M_1 \) and \( M_2 \) is thereby based on the nonlocal interaction Lagrangians

\[
    \mathcal{L}_{HM_1M_2} = g_H H(x) \int dy \Phi_H(y^2) M_1^T(x + w_{21}y) M_2(x - w_{12}y) + \text{H.c.},
\]

where \( M_1 \) and \( M_2 \) are the doublets of the meson fields:

\[
    K = \left( \begin{array}{c} K^+ \\ K^0 \end{array} \right), \quad D = \left( \begin{array}{c} D^0 \\ D^+ \end{array} \right), \quad D^*_\mu = \left( \begin{array}{c} D^{*0} \\ D^{*+} \end{array} \right)_\mu, \quad B = \left( \begin{array}{c} B^+ \\ B^0 \end{array} \right), \quad B^*_\mu = \left( \begin{array}{c} B^{*0} \\ B^{*+} \end{array} \right)_\mu
\]

and their antiparticles. The symbol \( T \) refers to the transpose of \( M_1 \). The kinematic variable \( w_{ij} = m_i/(m_i + m_j) \) where \( m_1 \) and \( m_2 \) are the masses of \( M_1 \) and \( M_2 \).

The finite size of the hadronic molecule is introduced through the correlation function \( \Phi_H(y^2) \) which describes the distribution of the constituent mesons. Its Fourier transform \( \widetilde{\Phi}_H(k^2_E) \) appears as the form factor in our calculations, where, in the present analysis, we have chosen a Gaussian form

\[
    \widetilde{\Phi}_H(k^2_E) = \exp(-k^2_E/\Lambda_H^2)
\]

in Euclidean momentum space. The size parameter \( \Lambda_H \) controls the spatial extension of the hadronic molecule and is varied between 1 - 2 GeV. The local case (LC), describing point-like interaction, is defined for \( \Lambda_H \rightarrow \infty \). (Note this limit can be applied to convergent matrix elements only). The size parameters \( \Lambda_H \) are the only adjustable parameters in our framework.

The coupling constants between the hadronic molecules and its building blocks, the constituent mesons, are fixed self-consistently by the compositeness condition \([31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43]\). The dynamics of the bound state is therefore related to its constituents by setting the field renormalization constant to zero. Because of this constraint, the coupling constants are no input parameters but are fixed within this theoretical framework.

The number of free variables is therefore reduced to the size parameters \( \Lambda_H \). For the generic hadronic molecule \( H = (M_1 M_2) \), the compositeness condition is given by the relation

\[
    Z_H = 1 - \Sigma_H(m^2_H) = 0,
\]

where \( \Sigma_H(m^2_H) = g_H^2 \Pi_H(m^2_H) \) is the derivative of the mass operator (see Fig.1) and \( m_H \) is the mass of hadronic molecule.

In the mesonic molecule picture all decays proceed via intermediate states which are the composite mesons of the hadronic bound state. We describe the dynamics of the intermediate states by free propagators given by the standard expressions

\[
    iS_M(x - y) = \langle 0 | T M(x) M^\dagger(y) | 0 \rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x - y)} S_M(k), \quad S_M(k) = \frac{1}{m^2_M - k^2 - i\epsilon}
\]

for pseudoscalar and scalar fields \( M \) and by

\[
    iS_{M^\mu}(x - y) = \langle 0 | T M^\mu(x) M^{\ast \mu \dagger}(y) | 0 \rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x - y)} S_{M^\mu}(k), \quad S_{M^\mu}(k) = -g^\mu_\nu + k^\mu k^\nu/m^2_M \frac{1}{m^2_M - k^2 - i\epsilon},
\]

in case of vector and axial-vector fields \( M^\ast \).
when we remove the cutoff $\Lambda$.

The coupling constants of the $B$ for the $D$ mesons have already been calculated in Refs. [31, 32, 33]:

$$\begin{align*}
g_{D^{*0}} &= 11.26 \text{ GeV} \quad (\Lambda_{D^{*0}} = 1 \text{ GeV}), \\
g_{D^{*1}} &= 11.62 \text{ GeV} \quad (\Lambda_{D^{*1}} = 1 \text{ GeV}),
\end{align*}$$

(9)

The results for the couplings of the $B_{s0}^{*}$ and $B_{s1}$ mesons to their constituents for different size parameters $\Lambda$ are [34]:

$$\begin{align*}
g_{B_{s0}^{*}} &= 27.17 \text{ GeV} \quad (\Lambda_{B_{s0}^{*}} = 1 \text{ GeV}), \\
g_{B_{s1}} &= 25.64 \text{ GeV} \quad (\Lambda_{B_{s1}} = 1 \text{ GeV}).
\end{align*}$$

(11)

Below we list our previous predictions for the couplings $g_H$ obtained for the respective molecular states. In particular, for the $fK\bar{K}$-coupling we obtained [30]:

$$g_{f_0} = 3.09 \text{ GeV} \quad (\Lambda_{f_0} = 1 \text{ GeV}), \quad g_{f_0} = 2.9 \text{ GeV} \quad (\text{LC}).$$

The coupling constants of the $D_{s0}^{*}$ and $D_{s1}$ mesons have already been calculated in Refs. [31, 32, 33]:

$$\begin{align*}
g_{D_{s0}^{*}} &= 9.9 \text{ GeV} \quad (\Lambda_{D_{s0}^{*}} = 2 \text{ GeV}), \\
g_{D_{s1}} &= 10.17 \text{ GeV} \quad (\Lambda_{D_{s1}} = 2 \text{ GeV}).
\end{align*}$$

(10)

The coupling constants of the $B_{s0}^{*}$ and $B_{s1}$ mesons to their constituents for different size parameters $\Lambda$ are [34]:

$$\begin{align*}
g_{B_{s0}^{*}} &= 23.21 \text{ GeV} \quad (\Lambda_{B_{s0}^{*}} = 2 \text{ GeV}), \\
g_{B_{s1}} &= 22.14 \text{ GeV} \quad (\Lambda_{B_{s1}} = 2 \text{ GeV}).
\end{align*}$$

(11)

One should stress that the coupling constants $g_{f_0}$, $g_{D_{s0}^{*}}$ and $g_{B_{s0}^{*}}$ of the scalar mesons $f_0$, $D_{s0}^{*}$, and $B_{s0}^{*}$ remain finite when we remove the cutoff $\Lambda_H \to \infty$. For the axial mesons $D_{s1}$ and $B_{s1}$ the couplings $g_{D_{s1}}$ and $g_{B_{s1}}$ are finite in the local limit when we neglect the longitudinal part $k^\mu k^\nu / m_H^2$ of the constituent vector meson propagator. In this case all the couplings are given analytically by

$$\frac{1}{g_H^2} = \frac{2}{(4\pi m_H)^2} \left[ \frac{m_2^2 - m_1^2}{m_1 m_2} \ln \frac{m_1}{m_2} - 1 + \frac{m_H^2(m_1^2 + m_2^2) - (m_1^2 - m_2^2)^2}{m_1^2 + m_2^2 - 2m_1 m_2 - 2m_1^2 m_2 - 2m_2^2 m_1} \right] \sum_\pm \frac{z_\pm}{\sqrt{\lambda}}$$

(12)

where $z_\pm = m_H^2 \pm (m_1^2 - m_2^2)$ and

$$\lambda = \lambda(m_1^2, m_2^2) = m_1^4 + m_2^4 - 2m_1^2 m_2^2 - 2m_1^2 m_2^2 - 2m_1^2 m_2^2$$

(13)

is the Källen function. When writing the mass $m_H$ of the hadronic molecule in the form $m_H = m_1 + m_2 - \epsilon$, where $\epsilon$ represents the binding energy, we can perform an expansion of $g_H$ in powers of $\epsilon$. The leading-order $O(\sqrt{\epsilon})$ result

$$\frac{g_H^2}{4\pi} = \frac{(m_1 + m_2)^{5/2}}{\sqrt{m_1 m_2}} \sqrt{8\epsilon}$$

(14)

in agreement with the one derived in Refs. [38, 42, 13, 45] based on a formalism which also used the compositeness condition $Z_H = 0$.

Numerical results for the coupling constants $g_H$

$$\begin{align*}
g_{f_0} &= 2.74 \text{ GeV}, \\
g_{D_{s0}^{*}} &= 8.27 \text{ GeV}, \\
g_{D_{s1}} &= 8.63 \text{ GeV}, \\
g_{B_{s0}^{*}} &= 19.63 \text{ GeV}, \\
g_{B_{s1}} &= 19.01 \text{ GeV}.
\end{align*}$$

(15)

compare well with the results obtained in the local case without the $\epsilon$ expansion and in the nonlocal case (see Eqs. [9] and [10]). Note that in the calculation of $g_{f_0}$ we use the averaged kaon mass $\bar{m}_K = (m_{K^\pm} + m_{K^0})/2$. 

FIG. 1: Mass operator of the hadronic molecule.
For consistency we also analyze the couplings $g_H$ and $\tilde{g}_H$ in the heavy quark limit (HQL), where the masses of the heavy mesons together with the heavy quark masses go to infinity. The scaling of the coupling constant $g_{D^*_{so}}$ in the HQL was already discussed in [32]. It was shown that $g_{D^*_{so}}$ both for the nonlocal and the local case, is proportional to the charm quark mass or the mass of the $D^*$ meson (see Eqs.(57) and (58) of Ref. [32]). This result is simply extended to the cases of the $B^*_{so}$ coupling and of the couplings of the axial states $D_{s1}$ and $B_{s1}$. In particular, for the nonlocal case the result for $g_H$ in the HQL is:

$$\frac{1}{g_{H}^2} = \frac{1}{(4\pi m_H)^2} \int_0^\infty \frac{d\alpha}{1 + \mu_K^2 \alpha} \Phi_H^2(\alpha),$$

where $\mu_K = m_K/\Lambda_H$. In the local case the HQL reads as:

$$\frac{1}{g_{H}^2} = \frac{1}{(4\pi m_H)^2} \ln \frac{m_H^2}{m_K^2}.$$  \hspace{1cm} (17)

Hence, the coupling of the heavy-light molecules to the constituents is proportional to the heavy quark mass (or the molecule mass $m_H = m_Q + O(1)$). Therefore, we deduce the following relations between the coupling constants $g_H$ in the HQL:

$$\frac{g_{D_{so}}}{g_{H}} = \frac{g_{D_{s1}}}{g_{H}} = \frac{g_{B_{so}}}{g_{H}} = \frac{g_{B_{s1}}}{g_{H}} \sim \frac{m_{B_{so}}}{m_{B_{s1}}} \sim \frac{m_{D_{s1}}}{m_{D_{so}}}. \hspace{1cm} (18)$$

This scaling behavior is also evident from Eq. (14), where the couplings $\tilde{g}_H$ behave in the HQL as:

$$\frac{\tilde{g}_H^2}{4\pi} = m_H^2 \sqrt{\frac{8\epsilon}{m_K}}.$$  \hspace{1cm} (19)

Keeping in mind that the binding energy $\epsilon$ is approximately the same for all four states ($D_{so}^*, B_{so}^*, D_{s1}, B_{s1}$), we deduce that in the HQL the relations (18) are also valid for the leading-order couplings $\tilde{g}_H$. Using the previous numerical values for the $g_H$ and $\tilde{g}_H$ couplings one can see that the HQL relations (18) are fulfilled with a good accuracy. It also explains the phenomenon that the bottom meson couplings are 2.2 - 2.8 times larger than the charm ones.

III. $D_{so}^*(2317)$ AND $B_{so}^*(5725)$ DECAYS

In this section we deal with the $f_0$-production properties in weak hadronic decays of the heavy scalar mesons $D_{so}^*(2317)$ and $B_{so}^*(5725)$. Here the final states of the $D_{so}^* \rightarrow f_0 X$ decay are occupied by the charged mesons $X = \pi^+, K^+, \rho^+$ and the scalar $f_0$. The decay pattern of the neutral $B_{so}^*0$ decay is richer and we deal with final $\pi^0$, $K^0$, $\rho^0$, $\omega$, $\eta$ and $\eta'$ mesons besides the $f_0$.

Since both heavy quark systems are assumed to be of molecular structure the decays proceed via intermediate kaons and $D$ or $B$ mesons as indicated in the diagrams of Figs. 2 and 3.

![Diagram](image)

FIG. 2: Diagrams contributing to the $D_{so}^* \rightarrow f_0 X$ decays with $X = \pi^+, K^+$ and $\rho^+$. 

The couplings of the hadronic molecules to the constituent mesons in the loop are fixed by the compositeness condition. The coupling constants between the intermediate $K$, $D$ and $B$ mesons and the final decay products $\pi, K, \rho, \omega, \eta$ and $\eta'$ are obtained from the $D$ and $B$ meson partial decay widths. The latter constants are given by following expressions, where we distinguish between final pseudoscalar ($P$) and vector mesons ($V$):

$$g_{H KP}^{c(n)} = \frac{16 \pi \Gamma(H \to K P)m_H^2}{\lambda^2(m_H^2, m_K^2, m_P^2)}, \quad (P = K, \pi, \eta, \eta', \ H = D, B),$$

$$g_{H KV}^{c(n)} = \frac{64 \pi \Gamma(H \to K V)m_H^2}{\lambda^2(m_H^2, m_K^2, m_V^2)}, \quad (V = \rho, \omega, \ H = D, B),$$

with the Källen function $\lambda(x, y, z)$ defined in Eq. (19). The superscript $c$ ($n$) denotes the decays of the charged (neutral) $D$ and $B$ mesons.

The couplings governing the $D_{s0}^* \to f_0 P$ and $B_{s0}^* \to f_0 P$ decays we calculate from

$$g_{D_{s0}^* f_0 P} = \frac{g_{D_{s0}^* f_0 P}}{(4\pi)^2} \left[ g_{H KP}^{c(n)} I(m_{D_{s0}^*}^2, m_{K^0}) + g_{H KP}^{n} I(m_{D_{s0}^*}^2, m_{K^+}) \right],$$

$$g_{B_{s0}^* f_0 P} = \frac{g_{B_{s0}^* f_0 P}}{(4\pi)^2} \left[ g_{H KP}^{c(n)} I(m_{B_{s0}^*}^2, m_{K^0}) + g_{H KP}^{n} I(m_{B_{s0}^*}^2, m_{K^+}) \right],$$

where $I(m_H^2, m_K^2)$ denotes the loop integral

$$I(m_H^2, m_K^2) = \int \frac{d^4k}{\pi^2} \bar{\Phi}_f(-k^2) \Phi_{H_{s0}^*} \left( - (k - \frac{p}{2} + \omega p_{H_{s0}^*})^2 \right) S_H(k - \frac{p}{2} + p_{H_{s0}^*}) S_K(k - \frac{p}{2}) S_K(k + \frac{p}{2}),$$

with $H_{s0}^* = B_{s0}^0, D_{s0}^{*+}$.

The decay widths are finally obtained from

$$\Gamma(H_{s0}^* \to f_0 P) = \frac{g_{H_{s0}^* f_0 P}^2}{16\pi m_{H_{s0}^*}^2} \lambda^2(m_{H_{s0}^*}, m_{f_0}, m_P).$$

For the decays with a final vector meson, $D_{s0}^*/B_{s0}^* \to f_0 V$, we proceed in analogy. For simplicity, we restrict in the following to the $D_{s0}^{*+} \to f_0 \rho^+$ decay since the corresponding expressions for the bottom $B_{s0}^*$ decays only differ in the masses and couplings, while the structure remains the same.

Again, the Feynman integral

$$I^\mu(m_{D_{s0}^*}^2, m_{K^0}^2) = \int \frac{d^4k}{\pi^2} \bar{\Phi}_f(-k^2) \Phi_{D_{s0}^*} \left( - (k - \frac{p}{2} + \omega p_{D_{s0}^*})^2 \right) (2k + p_{D_{s0}^*})^\mu$$

$$\times S_D(k - \frac{p}{2} + p_{D_{s0}^*}) S_K(k - \frac{p}{2}) S_K(k + \frac{p}{2})$$

defines the transition matrix element $\mathcal{M}^\mu$ which is given by

$$\mathcal{M}^\mu = \frac{g_{D_{s0}^* f_0 P}}{(4\pi)^2} \left[ g_{H KP}^{c(n)} I^\mu(m_{D_{s0}^*}^2, m_{K^0}^2) + g_{H KP}^{n} I^\mu(m_{D_{s0}^*}^2, m_{K^+}^2) \right]$$

$$= F_1(m_{D_{s0}^*}^2, m_{f_0}^2, m_{\rho}^2, p_f^\mu) + F_2(m_{D_{s0}^*}^2, m_{f_0}^2, m_{\rho}^2, p_\rho^\mu).$$
In the second line $\mathcal{M}^\mu$ is expressed in terms of the form factors $F_1$ and $F_2$ by writing the matrix element as a linear combination of the $f_0$ and $\rho$ meson momenta $p_f$ and $p_\rho$. We perform this decomposition since the form factor $F_1$ defines the coupling constant of the decay

$$F_1(m^2_{D_{s0}}, m^2_{f_0}, m^2_\rho) \equiv g_{D_{s0}f_0\rho}$$

and therefore characterizes the decay width with

$$\Gamma(D^{*+}_{s0} \to f_0\rho^+) = \frac{g_{D^{*+}_{s0}f_0\rho}^2}{64\pi m_{D^{*+}_{s0}}m_{\rho}} \lambda^2(m^2_{D^{*+}_{s0}}, m^2_{f_0}, m^2_\rho).$$

First we indicate the results for the coupling constants at the secondary interaction vertex as deduced from the decays $B^0/D^0 \to KX$ ($X = \pi, K, \eta', \eta, \omega, \rho$). In Table I we summarize the branching ratios (Br) as taken from and the resulting couplings $g_X^{(n)}$ (via Eqs. (20) and (21)) involving charged (c) and neutral (n) B and D mesons.

| Channel | $f_0^{(c,n)}$ | $g_X$ |
|---------|--------------|-------|
| $D^0 \to \pi^+ K^-$ | (3.89 ± 0.05) % | 2.88 · 10^{-6} GeV |
| $D^0 \to K^+ K^-$ | (3.93 ± 0.08) · 10^{-3} | 0.83 · 10^{-6} GeV |
| $D^0 \to \rho^+ K^-$ | (10.8 ± 0.7) % | 2.92 · 10^{-6} |
| $B^0 \to K^0\pi^0$ | (9.8 ± 0.6) · 10^{-6} | 3.36 · 10^{-8} GeV |
| $B^0 \to K^0\eta'$ | (6.5 ± 0.4) · 10^{-5} | 0.91 · 10^{-7} GeV |
| $B^0 \to K^0\eta$ | < 1.9 · 10^{-6} | < 0.15 · 10^{-7} GeV |
| $B^0 \to K^0\bar{K}^0$ | (9.6 ± 2.0) · 10^{-7} | 1.06 · 10^{-8} GeV |
| $B^0 \to K^0\omega$ | (5.0 ± 0.6) · 10^{-6} | 1.41 · 10^{-9} |
| $B^0 \to K^0\rho^0$ | (5.4 ± 0.9) · 10^{-6} | 0.14 · 10^{-8} |

In Tables II and III we summarize the results for the coupling constants and decay widths of the $D_{s0}^{*+}$ (2317) and $B_{s0}^{*0}$ (5725) decays. We also indicate the dependence of the results for different sets of size parameters $\Lambda_H$. Compared to the local case (LC) finite size effects induce a reduction of the $D_{s0}^{*+}$ decay widths by up to 50%. For the $B_{s0}^{*0}$ decays inclusion of finite size parameters leads to a reduction of the partial decay widths by up to a factor of 10.

For the $D_{s0}^{*+}$ decays we predict a decay pattern with

$$\Gamma(f_0\rho^+) > \Gamma(f_0\pi) > \Gamma(f_0K^+),$$

where the shortened notation $\Gamma(D_{s0}^{*+} \to H_1H_2) = \Gamma(H_1H_2)$. In the case of $B_{s0}^{*0}$ the weak decay mode $B_{s0}^{*0} \to f_0\eta'$ dominates the transitions with the decay hierarchy

$$\Gamma(f_0\eta') > \Gamma(f_0\pi) \approx \Gamma(f_0\rho) \approx \Gamma(f_0\omega) > \Gamma(f_0K) \approx \Gamma(f_0\eta).$$

| $\Lambda_H$ [GeV] | $g_{D_{s0}^{*+}f_0\pi}$ [GeV] | $g_{D_{s0}^{*+}f_0K}$ [GeV] | $g_{D_{s0}^{*+}f_0\rho}$ [GeV] |
|-----------------|-----------------|-----------------|-----------------|
| LC              | 1.83 · 10^{-6}  | 2.35 · 10^{-14} | 6.51 · 10^{-7}  |
| $\Lambda_H = 2, \Lambda_{f_0} = 1$ | 1.34 · 10^{-6}  | 1.26 · 10^{-14} | 4.86 · 10^{-7}  |
| $\Lambda_{D_{s0}^{*+}} = 1, \Lambda_{f_0} = 1$ | 1.28 · 10^{-6}  | 1.14 · 10^{-14} | 4.68 · 10^{-7}  |

For the $D_{s0}^{*+}$ decays we predict a decay pattern with $X = \pi^+, K^+, \rho^+$. The $\Lambda_H$ values indicate the dependence of the results for different sets of size parameters $\Lambda_H$. Compared to the local case (LC) finite size effects induce a reduction of the $D_{s0}^{*+}$ decay widths by up to 50%. For the $B_{s0}^{*0}$ decays inclusion of finite size parameters leads to a reduction of the partial decay widths by up to a factor of 10.
Now, the $D^+_s \to f_0 \pi^+$ decay

In this section we analyze the $D^+_s \to f_0 \pi^+$ decay in order to derive a value for the $D^+K\pi$ coupling constant $g_\pi$. This coupling is needed for the calculation of the $D_{s1} \to f_0 \pi$ decay width discussed in the next section. In this context we also obtain the decay width $\Gamma(D^+ \to K\pi)$ as an additional result. The $D_s$-decay is illustrated by the Feynman diagrams of Fig. 4, where the decay width is defined as

$$\Gamma(D_s^+ \to f_0 \pi^+) = \frac{g^2_{D_s f_0 \pi}}{16 \pi m^3_{D_s}} \lambda^2(m^2_{D_s}, m^2_{f_0}, m^2_{\pi}).$$

The decay coupling

$$g_{D_s f_0 \pi} = \frac{g_{D} g_{D_s} g_{\pi}}{(4\pi)^2} \left[ I(m^2_{D^+}, m^2_{K^0}) + I(m^2_{D_s^+}, m^2_{K^+}) \right]$$

can be computed from the loop integral $I(m^2_{D^+}, m^2_{K^0})$ given by

$$I(m^2_{D^+}, m^2_{K^0}) = \int \frac{d^4k}{\pi^2k^4} \Phi_f(-k^2)(p_{\pi} - k - \frac{p}{2}) \mu(k - \frac{p}{2} - p_{D_s}) S^q_{D^+}(k - \frac{p}{2} + p_{D_s}) S_K(k + \frac{p}{2}).$$

The coupling constant $g_{D_s}$ of the $D_s D^+K$ interaction vertex has been estimated in two different QCD sum rule approaches \cite{47,48}, where both results do not vary significantly from each other. Here we use the result of the QCD sum rule approach in \cite{47} with $g_{D_s} = 2.02$. By using the branching ratio $\text{Br}(D^+_s \to f_0 \pi^+) = (6.0 \pm 2.4) \cdot 10^{-3}$ \cite{44}, corresponding to $\Gamma(D^+_s \to f_0 \pi^+) = 7.9 \cdot 10^{-15}$ GeV, $g_\pi$ can be easily derived from (32) and (33):

$$g_\pi = 6.41 \cdot 10^{-5}.$$

Now, the $D^+ \to K\pi$ decay width is immediately given by

$$\Gamma(D^+ \to K\pi) = \frac{g^2_\pi}{48\pi m^3_{D^+}} \lambda^2(m^2_{D^+}, m^2_{K^0}, m^2_{\pi})$$

which leads to $\Gamma(D^+ \to K\pi) = 4.45 \cdot 10^{-11}$ GeV.
V. \( D_{s1}(2460) \) AND \( B_{s1}(5778) \) DECAYS

In this section we study the properties of the weak transitions between the axial vector hadronic molecules \( D_{s1}(2460) \) and \( B_{s1}(5778) \) and the scalar \( f_0(980) \). The determination of \( g_\pi \) in the last section enables us to compute the decay \( D_{s1}^+(2460) \rightarrow f_0^+ \pi^+ \) within the \( K \) \( D^* \) bound state framework. The Feynman diagrams which contribute to this decay are illustrated in Fig. 5. In the first step we define the matrix element of the \( D_{s1}^+ \rightarrow f_0^+ \pi^+ \) transition in terms of the form factors \( F_\pm \) and \( p_\pm = p_f \pm p_\pi \):

\[
\mathcal{M}^\mu = \frac{g_f g_{D_{s1}^+} g_\pi}{(4\pi)^2} \left( I^\mu(m_{D_{s1}}^2, m_K^2) + I^\mu(m_{D_{s1}}^2, m_K^2) \right)
= F_+(m_{D_{s1}}^2, m_{f_0}^2)p_f^\mu + F_-(m_{D_{s1}}^2, m_{f_0}^2)p_\pi^\mu,
\]

where \( p_f \) and \( p_\pi \) are the \( f_0 \) and \( \pi \) momenta, respectively.

The loop integral involving the constituent kaons and \( D^* \) meson is of the structure

\[
I^\mu(m_{D_{s1}}^2, m_K^2) = \int \frac{dk}{\pi^2} \tilde{\Phi}_{D^*}(-k^2) \Phi_{D_{s1}}(-(k-p_{D_{s1}})^2) \left( \frac{p_\pi - k - \frac{p}{2}}{2} \right)_{\nu} \times S_{D^*}^\mu(k - \frac{p}{2} + p_{D_{s1}})S_K(k - \frac{p}{2} + p_{D_{s1}})S_K(k + \frac{p}{2}).
\]

The form factor \( F_- \) defines the coupling \( g_{D_{s1}f_0} = F_-(m_{D_{s1}}, m_{f_0}, m_{f_0}) \) which characterizes the decay width given by the expression

\[
\Gamma(D_{s1}^+ \rightarrow f_0^+ \pi^+) = \frac{g_{D_{s1}f_0}^2}{48\pi m_{D_{s1}}^3} \chi^2(m_{D_{s1}}^2, m_{f_0}^2, m_{\pi^+}^2).
\]

We compute the decay width for \( D_{s1}^+ \rightarrow f_0^+ \pi^+ \) for the \( f_0 \) size parameter \( \Lambda_{f_0} = 1 \text{ GeV} \) while \( \Lambda_{D_{s1}} \) is varied between 1 GeV and 2 GeV.

The results for the \( D_{s1} \rightarrow f_0 \pi \) decay width obtained within our hadronic molecule approach range from

\[
\Gamma(D_{s1} \rightarrow f_0 \pi) = 2.85 \cdot 10^{-11} \text{ GeV}, \quad \text{where } g_{D_{s1}f_0} = 5.46 \cdot 10^{-5} \quad \text{at } \Lambda_{D_{s1}} = 1 \text{ GeV}
\]

(40)

to

\[
\Gamma(D_{s1} \rightarrow f_0 \pi) = 4.35 \cdot 10^{-11} \text{ GeV}, \quad \text{where } g_{D_{s1}f_0} = 6.74 \cdot 10^{-5} \quad \text{at } \Lambda_{D_{s1}} = 2 \text{ GeV}.
\]

(41)

By analogy, we can also study the \( B_{s1} \rightarrow f_0 X \) decay, where \( P \) represents a pseudoscalar final state. However, since no data are available to determine the \( B^* f_0 P \) coupling strength \( g_{B^*} \), we quote the width and corresponding decay coupling in dependence on \( g_{B^*} \). Varying \( \Lambda_{B_{s1}} \) from 1.0 GeV to 2 GeV the width lies between

\[
\Gamma(B_{s1} \rightarrow f_0 \pi) = 8.82 \cdot 10^{-6} g_{B^*}^2, \text{ GeV, where } g_{B_{s1}f_0} = 0.016 g_{B^*} \quad \text{at } \Lambda_{B_{s1}} = 1 \text{ GeV}
\]

(42)

and

\[
\Gamma(B_{s1} \rightarrow f_0 \pi) = 4.03 \cdot 10^{-5} g_{B^*}^2, \text{ GeV, where } g_{B_{s1}f_0} = 0.034 g_{B^*} \quad \text{at } \Lambda_{B_{s1}} = 2 \text{ GeV}.
\]

(43)
VI. SUMMARY

In the present paper we focused on weak hadronic production processes of the scalar $f_0(980)$. For this purpose we studied the weak non-leptonic decays of the heavy mesons $D_s^{*+}$, $D_{s1}^+$ as well as the $B_{s0}$ and $B_{s1}$ mesons assigned as the corresponding states in the bottom-strange sector.

The formalism presented provides a clear and straightforward method to study the issue of hadronic molecules. Since all coupling constants are either fixed self-consistently by the compositeness condition or are deduced from experimental data, the only adaptive variables are the size parameters of the meson molecules which allow for their extended structure. Finite size effects are studied by varying the size parameters within a physically reasonable region between 1 and 2 GeV. Additionally we also compare the results with finite size effects to the local case related to point-like interactions.

The molecular interpretation of both, the initial heavy mesons and the final decay product - the kaonic bound state $f_0$ - in the weak decays possibly offers a sensitive tool to study the structure issue. In particular for the $D_{s0}^*(2317) \rightarrow f_0 X$ transitions we give clear predictions for the decay pattern arising in the hadronic molecule picture, both for $D_{s0}^*$ and $f_0$. Similarly, the result for the process $D_{s1} \rightarrow f_0 \pi$ is a straightforward consequence of the molecular interpretation. In addition the $D^* \rightarrow f_0 \pi$ decay properties can also be used to get information on the $f_0$ substructure.

Presently no comparative calculations, as for example in the full or partial quark-antiquark interpretation of the $D_{s0}^* \rightarrow D_{s1}$ and $f_0$ mesons, exist. Hence, the real sensitivity of the results for the weak processes studied here on details of the meson structure remains to be seen. But judging from previous model calculations of for example the dominant observed decay modes of the $D_{s0}$ and $D_{s1}$ a strong dependence on the structure models can be expected. Therefore, upcoming experiments measuring the weak production processes involving the scalar meson $f_0(980)$ could lead to new insights into the meson spectrum and its structure issue.

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