

Multi-Objective Evolutionary Approach to Grey-Box Identification of Buck Converter

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Abstract—The present study proposes a simple grey-box identification approach to model a real DC-DC buck converter operating in continuous conduction mode. The problem associated with the information void in the observed dynamical data, which is often obtained over a relatively narrow input range, is alleviated by exploiting the known static behavior of buck converter as a priori knowledge. A simple method is developed based on the concept of term clusters to determine the static response of the candidate models. The error in the static behavior is then directly embedded into the multi-objective framework for structure selection. In essence, the proposed approach casts grey-box identification problem into a multi-objective framework to balance bias-variance dilemma of model building while explicitly integrating a priori knowledge into the structure selection process. The results of the investigation, considering the case of practical buck converter, demonstrate that it is possible to identify parsimonious models which can capture both the dynamic and static behavior of the system over a wide input range.

Index Terms—Buck converter, dc-dc power conversion, grey-box identification, nonlinear systems, NARX model.

I. INTRODUCTION

Modeling is the first step for control, condition-monitoring and fault diagnosis of power electronic converters. Over the past few years, this field has attracted significant research attention, which range from circuit topology and linear analysis based modeling approaches to data-driven modeling such as system identification and neural networks [1]. Among these, data-driven modeling is particularly well suited to handle inherent non-linearities of converters and can successfully account for uncertainties associated with stray parameter changes and aging effects. This study, therefore, follows a system identification based approach to model DC-DC buck converter operating in continuous conduction mode.

System identification deals with the development of mathematical descriptors of system dynamics from the observed dynamical data [2]–[4]. To this end, it is essential that the system under investigation is persistently excited over wide operating conditions so that the system dynamics are captured in the observed data, and subsequently encoded into the identified model. However, in practice, it is often difficult to drive the system over a wide input range. In such a scenario, the model identified using only observed dynamical data may not generalize well as the observed data contains information over a relatively small range of system dynamics. In this study, we consider a practical case study of modeling buck converter, which falls under this category. In particular, the buck converter considered here is excited over a relatively narrow input range. Our previous investigations [5], [6] on this case study show that while the models identified following black-box identification (i.e., using only dynamical data) can capture the converter dynamics, they cannot preserve the static non-linearity of the converter beyond local input range. In this study, we propose possible remedies based on the philosophy of grey-box identification to aid the identification process in such scenarios.

When the observed dynamical data contain only limited information about the system behavior, the identification process can be augmented by including additional auxiliary information about the system under investigation which could either be obtained by first principle or steady state data, e.g., static function, number and location of fixed points [4]–[12]. This auxiliary information, often referred to as a priori knowledge, can provide vital information about system behavior and can aid the identification process. Given that only finite data points are available for the identification, any a priori knowledge about the system under consideration is a welcome feature. The focus of this study is, therefore, the grey-box identification approach, which explicitly utilizes such a priori knowledge.

The major challenge of grey-box identification is to develop a suitable framework that can articulate and embed a priori system knowledge into the identification process. To this end, such a priori knowledge can be integrated into either of the following steps of the model building process: 1) Structure Selection and 2) Parameter Estimation. Given that most of the system representations such as Volterra and Nonlinear Auto-regressive with eXogenous inputs (NARX) are linear-in-parameter, the parameters of such models can be estimated following least-squares based algorithms. In contrast, structure selection, which involves the identification of significant terms/basis functions, is a much more complex issue, and it is one of the fundamental problems of system identification. It is easy to follow that, the ‘quality’ of the identified grey-box models can significantly be improved if a priori knowledge is directly integrated into the fundamental step of structure selection. However, to the best of our knowledge, this issue is yet to be explored in grey-box identification. This has been the main motivation for this study.

In most of the existing grey-box identification approaches, it is assumed that the structure of the system under consideration
is known, and \textit{a priori} knowledge is embedded into the parameter estimation. For instance, \textit{a priori} information about static gain and fixed point is utilized to constrain the estimated parameters in [6], [8]–[10]. In [11], the parameter estimation is formulated as a bi-objective problem to incorporate known steady-state behavior of the system. A detailed treatment of such grey-box identification approaches can be found in [4]. Further, a few notable exceptions to parameter estimation based approaches can be found in [5], [12]. In [5], the pool of viable system terms is restricted beforehand, based on a known static gain of the system. This approach, however, involves a trade-off in the dynamic prediction capabilities. In contrast, this study identifies globally valid models, these often involve a trade-off in the dynamic prediction capabilities. In [11], the parameter estimation was reported in [5], while these earlier approaches could identify globally valid models, these often involve a trade-off in the dynamic prediction capabilities. In contrast, this study proposes the use of \textit{a priori} knowledge at the fundamental level of structure selection, and it is essentially a further step in grey-box identification. This is convincingly demonstrated by a detailed comparative evaluation on the same case study.

The rest of the article is organized as follows: The experimental setup to gather identification data from the buck converter is described in Section II. The polynomial NARX model, term clusters and the structure selection problem are discussed briefly in Section III. The proposed multi-objective structure selection approach is discussed in detail in Section IV. The results are discussed at length in Section V followed by the conclusions in Section VI.

II. MODELLING OF DC-DC CONVERTER DYNAMICS

The objective of this study is to find a nonlinear model which successfully captures the dynamic behavior of the buck converter. The identification data for this purpose is gathered from the experimental setup described in Section II-A. Further, the static behavior of the buck converter is known. The use of this \textit{a priori} information and the modelling objectives are discussed in Section II-B.

A. Data Acquisition

In this study, a buck converter operating in the \textit{continuous} conduction mode is considered. For this purpose, the buck converter is implemented as shown in Fig. 1. The input voltage, ‘$V_d$’, is regulated at 24V throughout the experiment. The output voltage, ‘$V_o$’, is controlled by the Pulse Width

![Fig. 1. The buck converter considered in this study. The converter is driven by MOSFET IRF840. The PWM switching is controlled by LM3524 at 33kHz.](image-url)
Modulation (PWM) switching of the MOSFET (IRF840). In the PWM, a signal level dc voltage, ‘$V_{control}$’, is compared to a triangular waveform to adjust the duty ratio, $D = \frac{T_{ON}}{T}$, as per the prevailing requirements. This is accomplished by the PWM controller (LM3524, not shown here) at the rate of $\frac{1}{T} = 33kHz$ to ensure the operation in the continuous conduction mode, i.e., the current through the inductor ‘$L_1$’ (Fig. 1) is never zero.

The main objective of this study is to capture the nonlinear dynamics of the output voltage $V_o$, which is dependent on the duty ratio ‘$D$’ and the consequent energy exchange among $L_1$, $C_1$ and $R_1$ (see Fig. 1). For this purpose, a model is identified with the signal level PWM dc voltage, ‘$V_{control}$’, as the input (hereafter denoted by ‘$u$’) and the voltage $V_o$ as the output (hereafter denoted by ‘$y$’).

For identification, it is crucial to ensure that the converter is persistently excited so that the essential information about converter dynamics can be gathered. To this end, a Pseudo Random Binary Sequences (PRBS) signal is used as the input, $u$, which drives the converter in the range of $2.2V \leq u(k) \leq 2.5V$. The consequent changes in the output are captured by a digital oscilloscope at the sampling frequency of $1MHz$. The identification data (shown in Fig. 2) is obtained by decimating the input-output data by a factor of 12 to avoid the oversampling issues. Further details about the experimental setup and the data acquisition can be found in [5].

**B. Modelling Objectives**

The main objective of the identified model is to capture the dynamic behavior of the output voltage. Further, the steady-state relationship between the input and output converter voltages are usually known a priori. It is therefore essential to induce such static behavior in the identified models. For example, the steady state voltage relation for the buck converter considered in this study is given by,

$$\overline{y} = \frac{4V_d}{3} - \frac{V_d}{3}\pi$$

(1)

where, $\pi$ and $\overline{y}$ respectively denote the steady state values of the input and output. It is clear that in addition to a good prediction capability, the identified model must have a steady state relation of the form $\overline{y} = f(\overline{u})$ in order to mimic the static behavior of the buck converter given by (1). This a priori information is crucial to the identification process, as will be discussed in Section IV-A.

A black-box identification approach is not adequate to achieve the modeling objectives of this study because such an identification approach relies only on the information extracted from the dynamical dataset and the a priori information about static behavior is not incorporated. Given that the input drives the system over a relatively narrow range, i.e., $u(k) \in [2.2V, 2.5V]$, the static behavior of the back-box models is valid only in this local input range [5].

Hence, in this study, a grey-box identification approach is followed which integrates the a priori information about the static behavior of the buck converter (i.e., $\overline{y} = f(\overline{u})$) into the identification process to obtain globally valid models, which will be discussed in the following subsections.

III. PRELIMINARIES

The first step of the identification is to select system representation amongst many representations, e.g., Volterra, Wiener, Polynomial/Rational Nonlinear Auto-Regressive with eXogenous inputs (NARX), Neural Network and others. This study focuses on the polynomial NARX representation [3]. The rationale behind is two-fold:

- The concept of term-clusters was originally developed in the context of NARX models [15]. This concept forms the basis of the proposed grey-box identification approach, as will be discussed in detail in Section IV-A and IV-B
- The part of this study focuses on comparative evaluation with the earlier investigations in [5], [6], which were also focused on the polynomial NARX models.

In the following the polynomial NARX model is briefly discussed in Section IV-A. Further, the concept of term cluster is essential to derive the static models form the NARX representation, which is discussed briefly in Section IV-B.

**A. The Polynomial NARX Model**

The NARX model represents a non-linear system as a function of recursive lagged input and output terms as follows:

$$y(k) = F^n\{ y(k-1), \ldots, y(k-n_y), \ldots, u(k-1), \ldots, u(k-n_u) \} + e(k)$$

where $y(k)$ and $u(k)$ respectively represent the output and input at time intervals $k$, $n_y$ and $n_u$ are corresponding lags and $F^n\{ \}$ is some nonlinear function of degree $n_l$. The total number of possible terms or model size ($n$) of the NARX model is given by,

$$n = n_0 + \sum_{i=1}^{n_1} \frac{n_i-1(n_y+n_u+i-1)}{i}, \quad n_0 = 1$$

(2)
This model is essentially linear-in-parameters and can be expressed as:

$$y(k) = \theta_1 + \sum_{i=2}^{n} \theta_i x_i(k) + e(k)$$  \hspace{1cm} (3)

where, $x_i(k) = \prod_{j=1}^{p_y} y(k-n_{y_j}) \prod_{k=1}^{q_u} u(k-n_{u_k})$

$p_y, q_u \geq 0; \ 1 \leq p_y + q_u \leq n_i; \ 1 \leq n_{y_j} \leq n_y, 1 \leq n_{u_k} \leq n_u$

$n_i$ is the degree of polynomial expansion; $k = 1, 2, \ldots, N$ and ‘$N$’ denotes the total number of data points.

**B. Term Clusters**

The NARX model in (3) can be represented as summation of terms of with $m^{th}$ order nonlinearity (1 $\leq m \leq n_i$) as follows [15]:

$$y(k) = \sum_{m=0}^{n_2} \sum_{p=0}^{n_1} c_{p,m-p}(n_1, \ldots, n_m) \prod_{l=1}^{p} y(k-n_l) \prod_{i=p+1}^{m} u(k-n_i)$$ \hspace{1cm} (4)

where, $\sum_{n_1, n_m} = \sum_{n_1=1}^{n_y} \cdots \sum_{n_m=1}^{n_u}$ and the upper limit is respectively $n_y$ and $n_u$ for factors $y(k-n_l)$ and $u(k-n_i)$.

If the model is excited by a constant input and it is asymptotically stable, then the following holds in the steady state,

$$\bar{y} = y(k-1) = y(k-2) = \cdots = y(k-n_y)$$

$$\bar{u} = u(k-1) = u(k-2) = \cdots = u(k-n_u)$$

For such condition, (4) can further be simplified as follows:

$$\bar{y} = \sum_{m=0}^{n_2} \sum_{p=0}^{n_1} c_{p,m}(n_1, \ldots, n_m) \bar{y}^p \bar{u}^m$$ \hspace{1cm} (5)

**Definition 1. Cluster Coefficients** [15]: The constants $c_{p,m}(n_1, \ldots, n_m)$ in (5) are the coefficients of the term clusters $\Omega_{y^p u^m-p}$, which contain terms of the form $y^p(k-i)u^m(k-j)$ for $m+p < n_i$. Such coefficients are called cluster coefficients and are denoted by $\Sigma_{y^p u^m}$.

Following these definitions, the NARX model in the steady state is given by:

$$\bar{y} = \Sigma_0 + \Sigma_{y}\bar{y} + \Sigma_{u}\bar{u} + \sum_{m=1}^{n_2-1} \sum_{p=1}^{n_1} \Sigma_{y^p u^m}\bar{y}^p \bar{u}^m$$

$$+ \sum_{p=2}^{n_1} \Sigma_{y^p}\bar{y}^p + \sum_{m=2}^{n_2} \Sigma_{u^m}\bar{u}^m$$ \hspace{1cm} (6)

where term clusters and coefficients are defined as follows: constant terms in $\Sigma_0$; linear terms in $y$, $\Sigma_{y}\bar{y}$; linear terms in $u$, $\Sigma_{u}\bar{u}$; cross-terms in $\sum_{m=1}^{n_2-1} \sum_{p=1}^{n_1} \Sigma_{y^p u^m}\bar{y}^p \bar{u}^m$; non-linear terms in $y$, $\sum_{p=2}^{n_1} \Sigma_{y^p}\bar{y}^p$; non-linear terms in $u$, $\sum_{m=2}^{n_2} \Sigma_{u^m}\bar{u}^m$.

**C. The Structure Selection Problem**

The identification of a system includes the following two steps: 1) Determination of a significant/system terms 2) Estimation of corresponding coefficients. Due to convenient linear-in-parameter form of the NARX models, the parameters can be estimated relatively easily with least-square based approaches.

In contrast, detection of significant terms is a comparatively challenging task and it is often referred to as the structure selection problem. This problem has been extensively studied for continuous, discrete and time-varying systems both in time and frequency domain [3], [13], [14], [19], [20].

To understand the structure selection problem, consider the identification of a nonlinear system represented by polynomial NARX model. Given a large model set with $n$ number of terms, denoted as,

$$\mathcal{X}_{\text{model}} = \left[ \begin{array}{c} x_1 \ x_2 \ \ldots \ x_n \end{array} \right]$$ \hspace{1cm} (7)

where, $x_1, x_2, \ldots, x_n$ represent any possible linear or non-linear term of the NARX model. The goal of the structure selection is to determine the optimum subset of terms, $\mathcal{X}^* \subset \mathcal{X}_{\text{model}}$, by minimizing a suitable criterion function, ‘$\mathcal{J}^{\text{c}}$’.

It is worth noting that the model set $\mathcal{X}_{\text{model}}$ is essentially the union of all the possible term clusters [15], i.e.,

$$\mathcal{X}_{\text{model}} = \bigcup_{m=0..p=0..m} \Omega_{y^p u^m-p} = \left\{ \Omega_0 \cup \Omega_y \cup \Omega_u \cup \Omega_{yu} \cup \Omega_{yu} \cup \ldots \right\}$$ \hspace{1cm} (8)

where, $\Omega_0$ denotes the constant term.

**D. Pareto Dominance**

It is often difficult to identify the optimal solution for multi-criteria/objective problems due to the contradictory nature of search objectives. In practice, the unique optimal solution to such problem may not exist, in contrast, there exist multiple solutions which are non-dominated or Pareto Optimal, i.e., the solutions which are not necessarily optimum for each objective however better than the other solutions when all objectives are simultaneously considered.

To understand the concept of Pareto dominance, consider two structures $\mathcal{X}_1$ and $\mathcal{X}_2$ with the corresponding criteria/objectives, as follows:

$$\mathcal{J}(\mathcal{X}_1) = \{ J_1(\mathcal{X}_1), \ J_2(\mathcal{X}_1), \ \ldots \ J_{n_{\text{obj}}}(\mathcal{X}_1) \}$$

and, $$\mathcal{J}(\mathcal{X}_2) = \{ J_1(\mathcal{X}_2), \ J_2(\mathcal{X}_2), \ \ldots \ J_{n_{\text{obj}}}(\mathcal{X}_2) \}$$

where, ‘$n_{\text{obj}}$’ denotes the number of search objectives.

The Pareto dominance for these structures can be determined on the basis of the objective values as follows: $\mathcal{X}_1$ dominates $\mathcal{X}_2$,

$$\text{iff} \ \forall p \in \{1 \ldots n_{\text{obj}}\} : J_p(\mathcal{X}_1) \leq J_p(\mathcal{X}_2)$$

$$\land \ \exists p \in \{1 \ldots n_{\text{obj}}\} : J_p(\mathcal{X}_1) < J_p(\mathcal{X}_2)$$ \hspace{1cm} (10)

This is denoted by $\mathcal{X}_1 \prec \mathcal{X}_2$.

**IV. PROPOSED GREY-BOX IDENTIFICATION APPROACH**

The main objective of this study is to identify a model which can yield a better dynamic prediction as well as provide a
valid static behavior of a buck converter over a wide input range. It has been shown that a priori information about the static behavior of the buck converter can be integrated into the structure selection process, albeit with a trade-off in dynamic prediction capability [5], [6]. The proposed approach, therefore, casts the grey-box identification problem into a multi-objective framework to obtain a better overall trade-off over the desired objectives. In this approach, both dynamic prediction capability and static behavior are explicitly formulated as the search objectives and integrated into the multi-objective structure selection procedure.

In particular, this study takes a two-pronged approach to exploit the a priori information about the static behavior. First, the static behavior is used to determine the set of viable term-clusters. This step leads to a significant reduction in the search space by removing non-essential clusters as will be discussed in Section IV-A. Next, the static function of the model under consideration is determined and compared with the known static behavior. This quantification of the static behavior is the key feature of the proposed approach where this is explicitly included as one of the search objectives.

A. Prior Knowledge

Given that the static input-output relation of the buck converter is known, it can be used to identify the viable term clusters. To this end, the static behavior in (1) can be represented in a polynomial form as follows:

\[
\bar{y} = b_0 + b_1 \bar{u} + \sum_{m=2}^{n_l} \sum_{p=1}^{n_i-m} \bar{u}^m \Phi^{p-1}
\]

where, \( b_0 = \frac{4V_d}{3} \), \( b_1 = -\nu_d \frac{3}{3} \)

It is, thus, clear that to induce such a static behavior in the identified model, the corresponding static function should be a polynomial of input, \( u \). Further, the static function of the NARX model can be determined from (6) as follows:

\[
\bar{y} = \frac{\sum_{m=2}^{n_l} \sum_{p=1}^{n_i-m} \bar{u}^m \Phi^{p-1}}{1 - \sum_{m=1}^{n_l-1} \sum_{p=1}^{n_i-m} \bar{u}^m \Phi^{p-1}}
\]

(11)

It is easy to see that the following conditions should be satisfied in order to induce the static behavior similar to (11).

\[
\sum_{u^{m}} = 0, \quad m = 1, \ldots, n_l - 1, \quad \text{and} \quad p = 1, \ldots, n_l - m \]

\[
\sum_{u^{m}} = 0, \quad p = 2, \ldots, n_l \]

which yields,

\[
\bar{y} = \frac{\sum_{m=2}^{n_l} \sum_{p=1}^{n_i-m} \bar{u}^m \Phi^{p-1}}{1 - \sum_{m=1}^{n_l} \sum_{p=1}^{n_i-m} \bar{u}^m \Phi^{p-1}}
\]

(13)

This can further be simplified as,

\[
\bar{y} = a_0 + a_1 \bar{u} + a_2 \bar{u}^2 + \cdots + a_{n_l} \bar{u}^{n_l}
\]

where, \( a_0 = \frac{\sum_{i=1}^{n_i} \bar{u} \Phi^{i-1}}{1 - \sum_{m=1}^{n_l} \sum_{p=1}^{n_i-m} \bar{u}^m \Phi^{p-1}} \)

(14)

And the static relation is now in the desired polynomial form. Further, the required conditions for this simplification (12), can easily be satisfied by excluding the terms from the non-linear output cluster and the cross-term clusters from the pool of candidate terms, i.e.,

\[
X_{model} = X_{model} \setminus \Omega_{y^p}, \quad p = 2, \ldots, n_l
\]

(15)

where, \( m = 1, \ldots, n_l - 1; \quad \text{and} \quad p = 1, \ldots, n_l - m \)

It is worth emphasizing that although this reduction in candidate terms is crucial to induce the desired static behavior, it often involves a trade-off in the dynamic prediction capabilities [5]. Hence, although further reduction in pool of candidate term is possible, it is not desirable. This issue is discussed through an illustrative example in Section V-E.

B. Multi-objective Structure Selection

The structure selection is inherently multi-objective in nature as it involves the following two decisions: 1) How many terms are required to represent the system dynamics? and 2) Which are the significant terms among candidate terms? These two issues are crucial to effectively address the bias-variance dilemma. Hence, the structure selection can be approached as the multi-objective optimization problem. Further, the criterion function to evaluate a subset of candidate terms or structure can be formulated as follows:

\[
\arg \min \hat{J}(\mathcal{X}_i) = [J_1(\mathcal{X}_i), \quad J_2(\mathcal{X}_i)]
\]

where, \( J_1(\mathcal{X}_i) = \xi_i, \quad J_2(\mathcal{X}_i) = \mathcal{E}_i \)

\('\mathcal{X}_i'\) denotes the ith structure under consideration; \( '\xi_i' \) denotes the cardinality (number of terms) in \( \mathcal{X}_i \); and \( '\mathcal{E}_i' \) denotes the free-run prediction error obtained over the validation data and it is given by,

\[
\mathcal{E}_i = \frac{1}{N_v^s} \sum_{k=1}^{N_v} [y(k) - \hat{y}(k)]^2
\]

(16)

where, \( '\hat{y}' \) denotes the model predicted (free run or simulated) output obtained with \( \mathcal{X}_i \); and \( 'N_v' \) denotes the length of the validation data.

It is worth noting that since the criterion function in (16) directly incorporates the free-run prediction error \( (\mathcal{E}_i) \) and the structure cardinality \( (\xi_i) \), the search process to optimize \( \hat{J}(\cdot) \) is likely to yield parsimonious models with a better dynamic prediction capability. Similarly, if somehow the static behavior can be quantified and explicitly formulated as one of the search objectives then the search can be directed to identify the models with all the desired 'qualities', i.e., compact models with better dynamic prediction and globally valid static behavior. This has been the main motivation for the proposed approach.

Given that the static behavior of the buck converter is known (given by (1)) and the same for a candidate model can be determined using (15), the static behavior can easily be quantified for the search purposes as follows:

\[
\mathcal{E}_i = \sum_{k=1}^{N_v} [\bar{y}(\mathcal{X}_i) - \bar{y}(k)]^2
\]

(17)
Algorithm 1: Reproduction procedures

Input: Population/Archive of 'ps' parents, \( \beta_1, \beta_2, \ldots, \beta_{ps} \)
Output: Population of 'ps' offspring, \( \hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_{ps} \)

*/ Selection & Crossover
1 for \( i = 1 \) to \( \frac{ps}{2} \) do
   */ NSGA-II: Crowded Tournament Selection
   \{\beta_p, \beta_b\} = CTS (population, ranks, crowding distance)
   */ SPEA-II: Binary Tournament Selection
   \{\beta_p, \beta_b\} = BTS (archive, pseudo fitness)
   */ Parameterized Uniform Crossover
   \( \hat{\beta}_p \leftarrow \beta_p, \hat{\beta}_q \leftarrow \beta_q \)
   if \( p_c > rand \) then
      for \( j = 1 \) to \( n \) do
         if \( 0.5 > rand \) then
            \( \hat{\beta}_{p,j} \leftarrow \beta_{q,j}, \hat{\beta}_{q,j} \leftarrow \beta_{p,j} \)
         end
      end
   end
end

*/ Mutation
13 for \( i = 1 \) to \( ps \) do
14   for \( j = 1 \) to \( n \) do
15     if \( p_m > rand \) then
16       \( \hat{\beta}_{i,m} = 1 - \beta_{i,m} \)
17     end
18   end
19   Evaluate the fitness of \( \hat{\beta}_i \) as per Algorithm 2
20 end

'CTS' denotes the Crowded Tournament Selection, see [17].
'BTS' denotes the Binary Tournament Selection, see [18].

where, \( \mathcal{N}_s \) denotes the length of the static validation data; \( \bar{y}_{\text{buck}}, \bar{y} \) denotes the steady state output of buck converter which is given by (1); \( \bar{y} \) denotes the steady state output of the \( i \)th structure \( \mathcal{X}_i \).

It is worth noting that the static behavior can still be quantified even when the explicit input-output static relation similar to (1) is not available. For such a scenario, the required static data could be obtained experimentally, by the steady-state input-output measurements.

Since the static behavior of the candidate structure can be quantified using (18), it is now possible to integrate static behavior as one of the search objectives, as follows:

\[
\arg\min \bar{J}(\mathcal{X}_i) = \left[ J_1(\mathcal{X}_i), J_2(\mathcal{X}_i), J_3(\mathcal{X}_i) \right] \quad (19)
\]

where, \( J_1(\mathcal{X}_i) = \xi_i, J_2(\mathcal{X}_i) = \mathcal{E}_i, J_3(\mathcal{X}_i) = \bar{y}_i \).

Note that essentially this is a combinatorial optimization problem. An exhaustive search of all possible term subsets to solve (19) is often intractable even for a moderate number of NARX terms ‘\( n \)’, as it requires the examination of \( 2^n \) term subsets/structures. Hence, it is clear that an effective search strategy is crucial to optimize the multi-objective structure selection problem given by (19). This can be accomplished by any multi-objective evolutionary algorithm such as NSGA-II [17], SPEA-II [18], MOEA/D [21] and others. The comparative analysis of these algorithms on the structure selection problem in [14] indicates that dominance based MOEAs (e.g., NSGA-II and SPEA-II) often yields an improved Pareto front in comparison to decomposition based MOEAs such as MOEA/D. Hence, in this study, NSGA-II and SPEA-II are selected to solve the structure selection problem given in (19).

To address the structure selection problem with \( n \)-number of NARX terms, each parent in MOEA encodes a candidate structure in an \( n \)-dimensional binary vector as follows:

\[
\beta_i = [\beta_{i,1}, \beta_{i,2}, \ldots, \beta_{i,n}] \quad (20)
\]

where, \( \beta_{i,m} \in \{0, 1\}, \ m = 1, 2, \ldots, n \)

where, the \( i \)th parent, \( \beta_i \), encodes \( i \)th structure \( \mathcal{X}_i \). The \( m \)th term \( (x_m) \) from \( \mathcal{X}_{\text{model}} \) is included into the candidate structure \( \mathcal{X}_i \) provided the corresponding bit in the parent, ‘\( \beta_{i,m} \)’ is set to ‘1’.

Drawing on the recommendations in [14], the qualitative and quantitative control parameters of MOEAs are set as follows:

- **NSGA-II**: Population Size: 50; Selection: crowded tournament selection [17]; Recombination: uniform crossover; crossover rate \( (p_c) \): 0.9; Mutation: flip-bit mutation; mutation rate \( (p_m) \): 0.006.
- **SPEA-II**: Population Size: 50; Selection: binary tournament selection [18]; Recombination: uniform crossover; crossover rate \( (p_c) \): 0.7; Mutation: flip-bit mutation; mutation rate \( (p_m) \): 0.008.

The reproduction operators being used in this study are shown in Algorithm 1, where ‘\( \hat{\beta}_i \)’ and ‘\( \hat{\beta}_i \)’ respectively denote

Algorithm 2: Evaluation of Criterion Function, \( \bar{J}(\cdot) \)

Input: Search Agent, \( \hat{\beta}_i \)
Output: \( \bar{J}(\mathcal{X}_i) = \{ J_1(\mathcal{X}_i), J_2(\mathcal{X}_i), J_3(\mathcal{X}_i) \} \)

1 Set the \( i \)th structure to null vector, i.e., \( \mathcal{X}_i \leftarrow \emptyset \) and \( \xi_i \leftarrow 0 \)
2 for \( m = 1 \) to \( n \) do
   if \( \beta_{i,m} = 1 \) then
      \( \mathcal{X}_i \leftarrow \{ \mathcal{X}_i \cup x_m \} \) /* add the \( m \)th term
      \( \xi_i \leftarrow \xi_i + 1 \)
   end
end
3 Estimate Coefficients, ‘\( \Theta \)’, corresponding to the terms in \( \mathcal{X}_i \) using Least Squares based algorithm (see [3])
4 Determine the dynamic prediction error \( E_i \) using (17)
5 Determine the error in the static behavior \( \bar{y}_i \) as per (18)
6 \( J_1(\mathcal{X}_i) \leftarrow \xi_i, J_2(\mathcal{X}_i) \leftarrow E_i, J_3(\mathcal{X}_i) \leftarrow \bar{y}_i \)
the $i$th parent and the corresponding offspring. Each parent under consideration is evaluated following the steps outlined in Algorithm 2. Note that the other search components of NSGA-II and SPEA-II such as non-dominated sorting, crowding distance and pseudo fitness are omitted here for sake of brevity. Further implementation details about MOEAs can be found in [14], [17], [18].

The overall procedures involved in the proposed approach are outlined in Algorithm 3. Because of the stochastic nature of the algorithm, $R$ independent runs are carried out. Each run is set to terminate after 25,000 Function Evaluations (FEs). In each run, non-dominated structures and the corresponding criterion function are respectively accumulated in $\Gamma$ and $\Lambda$, as outlined in Line 8 Algorithm 3. At the end of these runs, the dominance of the accumulated structures in $\Gamma$ is again determined and the non-dominated structures and the corresponding criterion functions are stored respectively in $\Gamma^*$ and $\Lambda^*$. It is clear that the identified non-dominated structures in $\Gamma^*$ essentially represent a trade-off of varying degree over the search objectives, hence the a posteriori selection of a particular structure from this pool is primarily dependent on the choice of the Decision Maker (DM). These issues are discussed in detail in the following subsection.

C. Preference Articulation

The a posteriori selection from the identified non-dominated structures in $\Gamma^*$ is primarily dependent on the choice of the Decision Maker (DM). To this end, two possible a posteriori scenarios are considered in this study: 1) DM is unbiased, i.e., an equal preference is given to each design objective. 2) DM is biased towards a particular search objective. In the following, two a posteriori solution selection techniques are briefly discussed which can accommodate these two distinct scenarios.

1) Minimum Manhattan Distance: The Minimum Manhattan Distance (MMD) [22] approach for a posteriori decision making is appropriate when an equal priority is assigned to each objective, i.e., the DM is unbiased. In this approach, the identified non-dominated structures in $\Gamma^*$ are ranked as follows: First, a hypothetical ideal point ($\bar{J}^*$), which consists of the best value of each objective in $\Lambda^*$, is located in the objective space:

$$\bar{J}^* = \{ J_{1}^{\min}, J_{2}^{\min}, \ldots, J_{n_{obj}}^{\min} \} \quad (21)$$

where, $J_{p}^{\min} = \min J_{p}(\lambda^*_i)$, $\forall \lambda^*_i \in \Gamma^*$ and, $p = 1, \ldots, n_{obj}$. Subsequently, for each non-dominated structure $\lambda^*_i \in \Gamma^*$, the Manhattan distance, $D(\cdot)$, is evaluated with respect to $\bar{J}^*$, as outlined in Line 5 Algorithm 4. Note that the Manhattan distance $D(\cdot)$ is determined in the normalized objective space to avoid scaling issues. In the final step, the solutions are ranked in the ascending order of $D(\cdot)$. Based on this ranking, a few top structures can be selected for further analysis to account for uncertainties associated with the measurement of the dynamical and the static data. However, in this study, only the structure corresponding to the minimum Manhattan distance, $D(\cdot)$, is selected for sake of brevity.

2) Formulation of Priority Weights: If the DM is biased towards a particular search objective, it is essential to embed such a preference in the a posteriori selection. However, the human preferences are often abstract and partial [23], hence the first step is to encode such preferences in a quantitative metric. To this end, the DM’s preferences are encoded...
Algorithm 5: MTD approach to a posteriori selection

Input: Pareto set, \( \Gamma^{*} = \{ \mathcal{X}_1, \mathcal{X}_2, \ldots \} \)
   Pareto front, \( \mathcal{X} = \{ \mathcal{J}(\mathcal{X}_1), \mathcal{J}(\mathcal{X}_2), \ldots \} \)
Output: Selected Structure, \( \mathcal{X}^{*} \)
   \(/\) Preference formulation
1. Specify the objective rankings, \( O = [O_\xi \quad O_\varepsilon \quad O_\zeta] \)
2. Select the preference intensity, \( I \in [1, 9] \)
3. for \( i = 1 \) to \( n_{obj} \) do
   4. for \( j = 1 \) to \( n_{obj} \) do
      5. \( \delta_0 = \frac{O_\xi - O_\varepsilon}{n_{obj} - 1} \)
      6. \( \tau_{i,j} = I^{\delta_0} \) \(/\) preference relations
      7. \( w_i = \left( \prod_{j=1}^{n_{obj}} \tau_{i,j} \right)^{1/n_{obj}} \)
   8. \( \bar{w} = \left[ w_1 \quad w_2 \ldots \quad w_{n_{obj}} \right] \) \(/\) priority weights
   end
10. \(/\) Tournament function
   for \( i = 1 \) to \( |\Gamma^{*}| \) do
      11. for \( p = 1 \) to \( n_{obj} \) do
         12. \( t_{i,p} \leftarrow 0 \)
         13. for \( j = 1 \) to \( |\Gamma^{*}| \) do
            14. if \( J_p(\mathcal{X}_i) - J_p(\mathcal{X}_j) > 0 \) then
               15. \( t_{i,p} \leftarrow t_{i,p} + 1 \)
            end
         end
      16. \( T_p(\mathcal{X}_i, \Gamma^{*}) = \frac{t_{i,p}}{|\Gamma^{*}|} - 1 \)
   end
20. Determine global rank, \( \mathcal{R}(\mathcal{X}_i) = \left( \prod_{p=1}^{n_{obj}} T_p(\mathcal{X}_i, \Gamma^{*}) w_p \right)^{1/n_{obj}} \)
21. Select the structure with the maximum global rank \( \mathcal{R}(\cdot) \), i.e., \( \mathcal{X}^{*} = \{ \mathcal{X}_i | \mathcal{R}(\mathcal{X}_i) = \arg\max \mathcal{R}(\mathcal{X}_i), \forall \mathcal{X}_i \in \Gamma^{*} \} \)

Table I

| Objective Rankings | Weight Vector |
|-------------------|---------------|
| \( O = [O_\xi \quad O_\varepsilon \quad O_\zeta] \) | \( \bar{w} = [w_\xi \quad w_\varepsilon \quad w_\zeta] \) |
| \( O_1 = [3 \quad 1 \quad 2] \) | \( \bar{w}_1 = [0.1214 \quad 0.6071 \quad 0.2715] \) |
| \( O_2 = [1 \quad 3 \quad 2] \) | \( \bar{w}_2 = [0.6071 \quad 0.1214 \quad 0.2715] \) |
| \( O_3 = [1 \quad 2 \quad 3] \) | \( \bar{w}_3 = [0.6071 \quad 0.2715 \quad 0.1214] \) |

V. Results

The goal of this study is to develop a new approach to embed a priori system knowledge directly into the fundamental step of structure selection for grey-box identification problems. The efficacy of the proposed approach is demonstrated by considering a practical case study of buck converter modeling. The known static behavior of buck converter is treated as a priori knowledge. In the following, the results of this case study are discussed in detail. First, the search behavior of MOEAs is compared in Section V-A. The results of a posteriori preference articulation are discussed next in Section V-B. The steady-state behavior of the identified models is determined in Section V-C. Next, the results of a detailed comparative evaluation with the earlier investigation are provided in Section V-D. Finally, the role of non-linear input clusters is discussed in Section V-E.
The objective function values of the selected models are obtained for identification purposes from the experimental buck converter setup described in Section II-A. Following the cross-validation principle, 100 data points are used for the estimation of coefficients and the remaining data points form the validation data, i.e., \( N_v = 68 \). The candidate set of 286 NARX terms (i.e., \( n = 286 \)) is generated by the following specifications of the NARX model in (3): \( n_u, n_y, n_i \) = \([5, 5, 3]\). Further, as discussed in Section IV-A all the terms in nonlinear output and cross-term clusters are removed from the candidate terms, as outlined in Line 2, Algorithm 3.

Following the steps outlined in Algorithm 3, a set of non-dominated structures are identified over 100 independent runs of MOEAs. A total of 168 data-points are obtained for identification purposes from the experimental buck converter setup described in Section II-A. Following the cross-validation principle, 100 data points are used for the estimation of coefficients and the remaining data points form the validation data, i.e., \( N_v = 68 \). The candidate set of 286 NARX terms (i.e., \( n = 286 \)) is generated by the following specifications of the NARX model in (3): \( n_u, n_y, n_i \) = \([5, 5, 3]\). Further, as discussed in Section IV-A all the terms in nonlinear output and cross-term clusters are removed from the candidate terms, as outlined in Line 2, Algorithm 3.

A. Search Outcome

The overall procedure followed to identify non-dominated structures is outlined in Algorithm 3. A total of 168 data-points are obtained for identification purposes from the experimental buck converter setup described in Section II-A. Following the cross-validation principle, 100 data points are used for the estimation of coefficients and the remaining data points form the validation data, i.e., \( N_v = 68 \). The candidate set of 286 NARX terms (i.e., \( n = 286 \)) is generated by the following specifications of the NARX model in (3): \( n_u, n_y, n_i \) = \([5, 5, 3]\). Further, as discussed in Section IV-A all the terms in nonlinear output and cross-term clusters are removed from the candidate terms, as outlined in Line 2, Algorithm 3.

Following the steps outlined in Algorithm 3, a set of non-dominated structures are identified over 100 independent runs of MOEAs. A total of 117 and 24 non-dominated structures are identified respectively by NSGA-II and SPEA-II. For sake of simplicity, let the set of non-dominated structures identified by NSGA-II and SPEA-II be denoted by \( \Gamma_A \) and \( \Gamma_B \), respectively. Similarly, denote the corresponding set of objective function vectors by ‘\( \Lambda_A \)’ and ‘\( \Lambda_B \)’, i.e.,

\[
\Gamma_A = \{X_1, X_2, \ldots, X_{117}\}, \quad \Lambda_A = \{\vec{J}(X_1), \vec{J}(X_2), \ldots, \vec{J}(X_{117})\} \\
\Gamma_B = \{X_1, X_2, \ldots, X_{24}\}, \quad \Lambda_B = \{\vec{J}(X_1), \vec{J}(X_2), \ldots, \vec{J}(X_{24})\}
\]

The approximate Pareto fronts obtained by NSGA-II (\( \Lambda_A \)) and SPEA-II (\( \Lambda_B \)) are shown in Fig. 3a and 3b, respectively. From these results, it is obvious that NSGA-II could identify significantly higher number of non-dominated structures, i.e., \( |\Gamma_A| > |\Gamma_B| \). Further, the set coverage metric \([26]\) is considered to compare the quality of the identified structures. This metric is denoted here by ‘\( C(\cdot) \)’, and it is defined as:

\[
C(A, B) = \left[ \frac{\{X_{B,i} \in \Gamma_B | \exists X_{A,j} \in \Gamma_A : X_{A,j} \preceq X_{B,i}\}}{|\Gamma_B|} \right]
\]

The metric \( C(A, B) \) essentially determines the number of structures in \( \Gamma_B \) which are dominated by the structures in \( \Gamma_A \). For the approximate Pareto fronts shown in Fig. 3, these metrics are determined to be: \( C(A, B) = 0.6250 \) and \( C(B, A) = 0.0854 \). It is easy to follow that \( C(A, B) > C(B, A) \) which implies that the search performance of NSGA-II is better than that of SPEA-II, i.e., \( \Gamma_A \preceq \Gamma_B \). Hence, for the rest of this study we focus on the non-dominated structures identified by NSGA-II.

B. A posteriori Preference Articulation

Each non-dominated structure essentially represents a varying degree of compromise over search objectives, as seen in Fig. 3. Especially, the contradiction between the dynamic prediction error (\( \xi \)) and the static error (\( \zeta \)) is worth noting. It is clear that improvement in dynamic/static performance comes with a trade-off in the static/dynamic performance. This further highlights the need for a multi-objective approach.

For further analysis, 3 structures are selected from the identified non-dominated structures in \( \Gamma_A \), following the \textit{a posteriori} selection approaches discussed in Section IV-C. The selected structures and corresponding coefficients are given in the following models:

\[
M_1 : y(k) = 12.047 + 0.9268 y(k - 1) - 0.26037 y(k - 3) - 4.9214 u(k - 2) + 1.0603 u^2(k - 3) + 12.289 u^3(k - 1) + 12.777 u^2(k - 3)u(k - 1) - 19.02 u(k - 4)u(k - 3)u(k - 1) - 12.831 u(k - 3)u^2(k - 2) + 13.662 u(k - 4)u^2(k - 2) + 5.366 u(k - 4)u(k - 3)u(k - 2) - 6.1856 u^2(k - 5)u(k - 2) - 36.694 u(k - 5)u^2(k - 1) + 40.953 u^2(k - 5)u(k - 1) - 11.064 u^3(k - 5)
\]

\[
M_2 : y(k) = 21.366 + 0.76405 y(k - 2) - 0.38755 y(k - 4) - 7.7188 u(k - 2) - 4.086 u^2(k - 1) + 2.5905 u(k - 2)u(k - 1) + 2.2637 u(k - 5)u^2(k - 1) - 0.054585 u(k - 5)u(k - 4)u(k - 1) + 2.8763 u^2(k - 5) + 2.1183 u^3(k - 1)
\]

\[
M_3 : y(k) = 14.986 + 0.72049 y(k - 1) - 0.12131 y(k - 5) - 6.6797 u(k - 2) + 1.6136 u^2(k - 5) + 1.8557 u(k - 2)u^2(k - 1) - 1.2517 u(k - 5)u^2(k - 1) - 1.6357 u(k - 3)u(k - 2)u(k - 1) + 0.80815 u^2(k - 3)u(k - 2)
\]
C. Steady State Relation of The Identified Models

Given that in this study degree of nonlinearity ($n_l$) is fixed to 3, the static input-output relation given in (26) can further be simplified as follows:

$$y = a_0 + a_1 u + a_2 u^2 + a_3 u^3,$$

where,

$$a_0 = \frac{\Sigma_0}{1 - \Sigma_y}, a_1 = \frac{\Sigma_u}{1 - \Sigma_y}, a_2 = \frac{\Sigma_{u^2}}{1 - \Sigma_y}, a_3 = \frac{\Sigma_{u^3}}{1 - \Sigma_y}$$

This gives general form of steady state relation of the identified models. The coefficients of (26) are dependent both on terms and the corresponding coefficients of the identified models. For the selected models, $\mathcal{M}_1 - \mathcal{M}_3$, these coefficients are shown in Table IV.

| Model | Number of Terms ($\xi$) | Dynamic Error ($\xi$) | Static Error ($\xi$) | Remark                  |
|-------|-------------------------|-----------------------|----------------------|-------------------------|
| $\mathcal{M}_1$ | 15                      | 14.26                 | 1.56                 | MMD, $O_1 + MTD$        |
| $\mathcal{M}_2$ | 10                      | 19.73                 | 1.39                 | $O_2 + MTD$             |
| $\mathcal{M}_3$ | 9                       | 16.80                 | 2.39                 | $O_3 + MTD$             |

D. Comparative Evaluation

For the purpose of the comparative evaluation, the selected models (i.e., $\mathcal{M}_1$, $\mathcal{M}_2$ and $\mathcal{M}_3$) are compared with the models identified for the same experimental setup and the identification data of the buck converter by a different grey-box identification approaches: OFR [5] and OFR-EA [6]. The models identified in these earlier investigations are as follows:

$$y(k) = \theta_0 + \theta_1 y(k - 1) + \theta_2 y(k - 2) + \theta_3 u^2(k - 1) + \theta_4 y(k - 3) + \theta_5 u^2(k - 1) u(k - 3) + \theta_6 u^3(k - 3) + \theta_7 u(k - 1) u(k - 3) + \theta_8 u^2(k - 3) u(k - 1)$$

(27)

The corresponding coefficients ‘$\theta$’ are given in Table V.

First, the dynamic prediction capability of the models is compared by calculating the model-predicted output over the validation data, as shown in Fig. 4. It is clear that the models identified using the proposed approach could yield comparatively better prediction performance. The prediction error with the identified models lie in the range of [14% – 20%]. In comparison, OFR [5] and OFR-EA [6] could yield approximately 33% prediction error, clearly a higher trade-off is made in the dynamic performance with these approaches.

Next, the static behavior of the models is evaluated as shown in Fig. 5. It is worth noting that, while the static behavior is evaluated over the valid input range of $[V – 4V]$, the identification data has been generated over the relatively narrow input range of $2.2V \leq u(k) \leq 2.5V$. Therefore, evaluation of the models beyond this input range can be considered as the evaluation of global validity. As seen in Fig. 5b-5d, the identified models mimic the static behavior of the buck converter over almost the entire valid input range.

Further, the identified models yield the static error $\bar{E}$ in the range of $[1.39 – 2.39]$, which is better than/comparable to OFR/OFR-EA.

Further, the degree of compromise over the search objectives is clearly visible in the dynamic and static behavior of the identified models. For example, among the identified models, the prediction capability of $\mathcal{M}_2$ is comparatively poor with $\bar{E} = 19.7\%$, as seen in Fig. 4c. However, with this trade-off, $\mathcal{M}_2$ could perfectly mimic the static behavior of the buck converter over the entire input range, as seen in Fig. 5c.

Nevertheless, it is interesting to see that all the identified models ($\mathcal{M}_1 - \mathcal{M}_3$) yield practically acceptable dynamic and
E. Role of Non-linear Input Clusters: Some Comments

In this study, prior to the structure selection, the nonlinear output and cross-term clusters are removed from $X_{model}$, as discussed earlier in Section IV-A. Further, a closer inspection of the static input-output relations in (11) and (14) shows that only the following three term clusters are required to induce the ‘perfect’ static behavior of buck converter: constant terms ($\Omega_0$), linear input ($\Omega_u$) and linear output ($\Omega_y$). Thus, if the terms belonging to the non-linear-input clusters (i.e., $\Omega_u^p$, $p = 2, \ldots, n_l$) are also removed, then (14) simplifies to,

$$\bar{y} = a_0 + a_1 \bar{u}$$

(28)

where, $a_0 = \frac{\Sigma_0}{1 - \Sigma_y}$, $a_1 = \frac{\Sigma_u}{1 - \Sigma_y}$

It is clear that this simplified static relation is similar to the static behavior of the buck converter in (11). This could also be explained by the ‘straight-line’ nature of the static input-output relationship.

However, it is interesting to see that all the identified models, $M_1 - M_3$, contain the terms from the non-linear input clusters (of $\Omega_u^p$). This implies that while the $\Omega_u^p$ clusters are not required for the static behavior, they may be essential for the dynamic prediction.

To further investigate the role of $\Omega_u^p$ clusters, consider the identification of buck converter with the similar procedure, outlined in Algorithm 3 except with one key difference: In these experiments the non-linear input clusters are also removed, i.e., $X_{model} = \{\Omega_0 \cup \Omega_u \cup \Omega_y\}$. The model identified following this procedure is as follows:

$$M_4: y(k) = 30.392 + 0.061677 y(k-3) - 5.6359 u(k-2) - 1.8699 u(k-3) - 0.080413 u(k-4)$$

(29)

The validation results for $M_4$ are shown in Fig. 6. As expected, this model mimics the static behavior of buck converter very well, as seen in Fig. 6a. This improvement, however, comes with a significant trade-off in the dynamic prediction capabilities, as seen in Fig. 6b.
This implies that only the first, fourth and fifth terms from the set \( \mathcal{X}_{\text{model}} \) are included into the structure/term subset. Thus, the structure ‘\( \mathcal{X} \)’ encoded by the particle \( \beta_i \) is given by,

\[
\mathcal{X}_i = \begin{bmatrix} x_1 & x_4 & x_5 \end{bmatrix} = \begin{bmatrix} y(k - 1) & y(k - 2)u(k - 2) & u(k - 3)^3 \end{bmatrix}
\]

\section*{APPENDIX B}

\subsection*{ILLUSTRATIVE EXAMPLE: PRIORITY WEIGHTS}

Let the objective rankings and the preference intensity specified by the DM be given by: \( [O_\xi, O_\xi, O_\xi] = [3, 1, 2] \) and \( T = 5 \). The corresponding multiplicative preference relations can be determined as follows (see Line 4, Algorithm 5):

\[
\begin{bmatrix}
\tau_{\xi, \xi} & \tau_{\xi, \xi} & \tau_{\xi, \xi} \\
\tau_{\xi, \xi} & \tau_{\xi, \xi} & \tau_{\xi, \xi} \\
\tau_{\xi, \xi} & \tau_{\xi, \xi} & \tau_{\xi, \xi}
\end{bmatrix} = \begin{bmatrix}
1 & \frac{1}{5} & \frac{1}{\sqrt{5}} \\
5 & 1 & \sqrt{5} \\
\sqrt{5} & \frac{1}{\sqrt{5}} & 1
\end{bmatrix}
\]

Consequently, the preference weights are determined as follows (see Line 8, Algorithm 5):

\[
\begin{bmatrix}
w_\xi \\
w_\xi \\
w_\xi
\end{bmatrix} = \begin{bmatrix} 0.4472 \\ 2.2361 \\ 1 \end{bmatrix}
\]

which yields, \( \bar{w} = \left( \frac{w_\xi w_\xi w_\xi}{\sum w} \right)^T = \begin{bmatrix} 0.1214 \\ 0.6071 \\ 0.2715 \end{bmatrix} \)

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