Density perturbation of unparticle dark matter in the flat Universe

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Abstract. The unparticle has been suggested as a candidate of dark matter. We investigated the growth rate of the density perturbation for the unparticle dark matter in the flat Universe. First, we consider the model in which unparticle is the sole dark matter and find that the growth factor can be approximated well by \( f = (1 + 3\omega_u) \Omega_u \), where \( \omega_u \) is the equation of state of unparticle. Our results show that the presence of \( \omega_u \) modifies the behavior of the growth factor \( f \). For the second model where unparticle co-exists with cold dark matter, the growth factor has a new approximation \( f = (1 + 3\omega_u) \Omega_u + \alpha \Omega_{o} \) and \( \alpha \) is a function of \( \omega_u \). Thus the growth factor of unparticle is quite different from that of usual dark matter. These information can help us know more about unparticle and the early evolution of the Universe.

PACS. 95.36.+x Dark energy – 98.80.-k Cosmology

1 Introduction

The current observations confirm that our Universe is in a phase of accelerated expansion \( [1] \). This may indicate that our Universe contains dark energy (DE) which is an exotic energy component with negative pressure and constitutes about 72% of present total cosmic energy. One simple interpretation of DE which is consistent with current observations is the cosmological constant with equation of state (EOS) \( \omega = -1 \) \( [2] \). Another major component in the universe is dark matter (DM) which accounts for about 25% of total cosmic energy today. It is widely believed that DM plays an important role in explaining why galaxies hold together. In general, the DM in the universe is supposed to be nonbaryonic which does not interact with ordinary matter via electromagnetic forces. There are two prominent hypotheses on DM called hot DM and cold DM. The hot DM is consist of particles that travel with ultrarelativistic velocities. The best candidate for the hot DM \( [3, 4, 5, 6] \) is the neutrino which has very small mass and does not take part in the electromagnetic interaction and the strong interaction. The cold DM is consist of objects with sufficiently massive so that they move at sub-relativistic velocities (such as neutralino) \( [3, 4, 5, 6] \). However, the nature of DM is still unclear at present.

Recent investigations show that unparticle can be treated theoretically as a candidate of DM in the Universe because that it interacts weakly with standard model particles. The concept of unparticle is proposed by Georgi \( [7] \), which is based on the hypothesis that there could exist an exact scale invariant hidden sector resisted at a high energy scale (for a recent review of unparticles, see \( [8, 9] \)). In spite of the fundamental energy scale of such a sector is far beyond the reach of today’s or near future accelerators, it is possible that this new sector affects the low energy phenomenology. These effects is described as the so-called unparticle in the effective low energy field theory because that the behaviors of these new degrees of freedom are quite a different from those of the ordinary particles. For example, their scaling dimension does not have to be an integer or half an integer. This implies that the behavior of unparticle DM (UDM) is distinctly different from the usual DM. One of interesting feature of unparticle is that it has not a definite mass and instead has a continuous spectral density as a consequence of scale invariance \( [7] \)

\[
\rho(P^2) = A_{d_u} \theta(P^0) \theta(P^2)(P^2)^{d_u - 2},
\]

where \( P \) is the 4-momentum, \( A_{d_u} \) is the normalization factor and \( d_u \) is the scaling dimension. The theoretical bounds of the scaling dimension \( d_u \) are \( 1 \leq d_u \leq 2 \) (for boson unparticle) or \( 3/2 \leq d_u \leq 5/2 \) (for fermion unparticle) \( [9] \). The pressure and energy density of the thermal boson unparticle are given by \( [10] \)

\[
p_u = g_s T^4 \left( \frac{T}{\Lambda_u} \right)^{2(d_u - 1)} \frac{C(d_u)}{4\pi^2},
\]

\[
\rho_u = (2d_u + 1) g_s T^4 \left( \frac{T}{\Lambda_u} \right)^{2(d_u - 1)} \frac{C(d_u)}{4\pi^2},
\]

where \( C(d_u) = B(3/2, d_u) \Gamma(2d_u + 2)\zeta(2d_u + 2), \) while \( B, \Gamma, \zeta \) are the Beta, Gamma and Zeta functions, respectively.

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Thus, the EoS of boson unparticle reads \[ \omega_u = \frac{1}{2a_u + 1} \] (3)

For the fermion unparticle, we find the EoS has the same form as that of boson one. Obviously, the EoS of unparticle \( \omega_u \) is positive which is different from that of DE and usual DM. This means that the evolution of UDM would differ from both DE and usual DM. Recent investigations show that the unparticles play an important role in the early universe and black hole physics. The new collider signals for unparticle physics has been also considered in [10-14]. Recently, the growth factor of DM density perturbation has been attracted much attention because that it plays a prominent role in discriminating various DE models and modified gravity [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. Since the unparticle can be regarded as a new candidate of DM, it is natural to ask whether the density perturbation of UDM has some peculiar behaviors in the evolution of Universe. The main purpose of this paper is to study the density perturbation of UDM and to calculate its growth factor. Here we consider two models. The first model (UDM model) is described by that unparticle is the sole DM in the Universe and the other one by that unparticle co-exists with CDM (UCDM model). We find that in both models the growth factors present a new form differed from that of usual DM.

The paper is organized as follows: in the following section we give a short review and present the evolution equation of the UDM density perturbation. In Sec.III, we discuss the UDM model and the DE with constant \( \omega \), and then calculate the growth factor of UDM. In Sec.IV, we consider the model in which unparticle co-exists with CDM. Finally in the last section we include our conclusions.

2 Density perturbational equation of UDM

Let us now to shortly review how to obtain the second order differential equations describing the evolution of the linear perturbations of unparticle dark matter in a spatially flat FLRW Universe (see \[ 30, 37, 38 \]). Adopting to the synchronous gauge, the perturbed metric in comoving coordinates is

\[ ds^2 = -dt^2 + a(t)^2(\delta_{ij} + h_{ij})dx^i dx^j, \] (4)

where \( h_{ij} \) denotes the perturbation and can be decomposed into a trace part \( h = h^i_i \) and a traceless one. In the UDM-dominated era, the zeroth equations can be written as

\[ H^2 = \frac{8\pi G}{3}(\rho_u + \rho_x), \]

\[ \frac{\dot{H}}{H^2} = -\frac{3}{2}\left[1 + \omega_u \Omega_u + \omega(1 - \Omega_u)\right], \] (5)

where \( \Omega_u = \rho_u/\rho_c, \rho_x \) and \( \omega \) are the density and EoS of DE respectively. We consider only the case \( \omega \) is a constant and assume that there is no perturbation of DE density. The divergence of the unparticle dark matter velocity in its own rest frame is zero by definition and therefore in Fourier space we can obtain \[ 30, 37, 38 \]

\[ \delta^3 \dot{R} + 2H \delta^3 R = 0, \]

\[ \dot{\delta} + \frac{1 + \omega_u}{H} \left[4\pi G \rho_u \delta + \frac{1}{4} \delta^3 R \right] = 0, \] (6)

at large scales. Here \( \delta = \delta \rho_u/\rho_u \) and \( \delta^3 R \) is first order perturbation of the spatial curvature scalar. Differentiating the second equation in Eq. (6) with respect to time \( t \), we get \[ 30, 37, 38 \]

\[ \ddot{\delta} + \frac{3}{2}(1 + \omega_u)H \ddot{\delta} - 6\pi G (1 + \omega_u)^2 \rho_u \delta \\
- \frac{1}{8}(1 + \omega_u)(1 - 3\omega_u) \delta^3 R = 0. \] (7)

Combining Eq. (7) with Eq. (6) and eliminating \( \delta^3 R \), we find that the density perturbation equation for unparticle dark matter is \[ 30, 37, 38 \]

\[ \ddot{\delta} + 2H \dot{\delta} - 4\pi G (1 + \omega_u)(1 + 3\omega_u) \rho_u \delta = 0. \] (8)

Making use of Eq. (8) and defining \( f = d\ln \delta/d\ln a \), we can calculate the growth factor \( f \) of density perturbation of UDM. Here we must point out that although we only consider the density perturbation of UDM, Eq. (8) is valid generally for the arbitrary density perturbation in the Universe. For example, as the EOS \( \omega_u = 0 \), this equation can be reduced to the density perturbational equation of usual DM. Moreover, we also find Eq. (8) can yield the density perturbation equation in the radiation-dominated era when the EOS \( \omega_u = 1/3 \). Comparing with usual DM and radiation, we find from Eq. (8) that the special properties of UDM are that its EOS could be an arbitrary positive number. Since the third term in Eq. (8) depends on the EOS \( \omega_u \), it is expected that there exists some special properties in the growth factor of the density perturbation of UDM in the evolution of Universe.

3 The growth factor of unparticle Dark Matter for UDM model

We are now in position to study the growth factor of density perturbation of UDM. Let us assume that the Universe is filled only with the unparticle dark matter (UDM) and DE with the constant \( \omega \).

From the energy conservation equation

\[ \dot{\rho}_u = -3H(1 + \omega_u)\rho_u, \] (9)

we can obtain easily

\[ \Omega'_u = 3\Omega_u (\omega - \omega_u)(1 - \Omega_u), \] (10)

where the prime denote derivatives with respect to \( \ln a \). In term of the growth factor \( f \), the matter density perturbation Eq. (8) can rewritten as

\[ f' + f^2 + \left(\frac{\dot{H}}{H} + 2\right)f = \frac{3}{2}(1 + \omega_u)(1 + 3\omega_u)\Omega_u, \] (11)
where \( f' = df/d\ln a \). Substituting Eqs. (5) and (10) into Eq. (11), we find
\[
3 \Omega_u(\omega - \omega_u)(1 - \Omega_u) \frac{df}{d\Omega_u} + f^2 + \frac{1 - 3 \omega_u}{2 - 2 \omega_u} \Omega_u \\
- \frac{3}{2} \omega(1 - \Omega_u) f = \frac{3}{2} (1 + \omega_u)(1 + 3 \omega_u) \Omega_u.
\] (12)

Like Eq. (8), this equation is also true generally for the arbitrary density perturbation. For the \( \omega_u = 0 \), this equation can be reduced to that of the usual DM and its solution can be approximated generally as \( f = \Omega_u^\gamma \). The parameter \( \gamma \) is the so-called growth index which can help us to distinguish the different theories of DE and the modified gravity. For example, the value of growth index is \( \gamma = 6/11 \) for \( \Lambda \)CDM model and is \( \gamma = 11/16 \) for the Dvali-Gabadadze-Porrati (DGP) brane-world model. Thus if the value of \( \gamma \) can be determined by observations, one can discriminate these models. However, for the UDM, it is not difficult to find that \( f = \Omega_u^\gamma \) is not a solution of Eq. (8). It is not surprising because that there exists the term depended on \( \omega_u \) in the density perturbation equation. We assume that for UDM the growth factor has a form \( f = \beta \Omega_u^\gamma \), the equation (12) can be rewritten as
\[
3 \Omega_u(\omega - \omega_u)(1 - \Omega_u) \ln \Omega_u \frac{d\gamma}{d\Omega_u} + \frac{1 - 3 \omega_u}{2 - 2 \omega_u} \beta \Omega_u \\
3(\omega - \omega_u)(1 - \Omega_u)(\gamma - \frac{1}{2}) = \frac{3}{2\beta} (1 + \omega_u)(1 + 3 \omega_u) \Omega_u^{1-\gamma}.
\] (13)

At high redshifts, the \( 1 - \Omega_u \) is a small quantity. Expanding the Eq. (13), we obtain
\[
\beta = 1 + 3 \omega_u,
\]

\[\gamma = \frac{3 - 3 \omega + 6 \omega_u}{5 - 6 \omega + 15 \omega_u} + \frac{3(1 - \omega - 2 \omega_u)}{2(5 - 12 \omega + 21 \omega_u)} \times \frac{(2 - 3 \omega - 9 \omega_u)(1 - 2 \omega_u)}{(5 - 6 \omega + 15 \omega_u)^2} (1 - \Omega_u). \] (14)

Obviously, the growth factor \( f \) depends both on the EOSs of unparticle DM and DE. The dependence of the growth factor \( f \) on the the EOS \( \omega \) of DE can provides the direct method to identify the different modes of DE. For example, for the \( \Lambda \)UDM model (i.e., \( \omega = -1 \)), we find that \( \beta = 1 \), \( \gamma = 6/11 + 77/493 \Omega_u(1 - \Omega_u) \) for \( \omega_u = 1/10 \) and \( \beta = 2.5 \); \( \gamma = 11/16 + 27/819(1 - \Omega_u) \) for \( \omega_u = 1/3 \), respectively. Thus, for different unparticles, the values of \( \beta \) and the coefficients in \( \gamma \) are different. Moreover, we also find that for \( \Lambda \)UDM model, the coefficient of \( (1 - \Omega_u) \) is positive for \( \omega_u < 1/3 \) and is negative for \( \omega_u > 1/3 \). This may provide another way to distinguish the different unparticles. Similarly, if we fix the EOS of unparticle \( \omega_u = 1/3 \), it is easy to obtain that \( \beta = 1.6 \); \( \gamma = 15/29 + 45/6862(1 - \Omega_u) \) for \( \omega = -0.6 \) and \( \beta = 1.6 \); \( \gamma = 32/75 + 27/819(1 - \Omega_u) \) for \( \omega = -2.1 \). It means that for different DE, the values of \( \beta \) and the coefficients in \( \gamma \) are also different. Therefore, if \( \beta \) and \( \gamma \) can be determined by observations, we can find that which components are contained in our Universe.

When \( \omega_u = 0 \), we find from Eq. (14) that
\[
\beta = 1, \\
\gamma = \frac{3 - 3 \omega + 6 \omega_u}{5 - 6 \omega + 15 \omega_u} + \frac{3(1 - \omega - 2 \omega_u)}{2(5 - 12 \omega + 21 \omega_u)} \times \frac{(2 - 3 \omega - 9 \omega_u)(1 - 2 \omega_u)}{(5 - 6 \omega + 15 \omega_u)^2} (1 - \Omega_u). \] (15)

It is consistent with the result of the usual DM \[33,39\].

When \( \omega_u = 1/3 \), our results show that \( f = 2 \) at the high redshift which agrees with the growth factor in the era dominated by the radiation. If there is no DE, \( \Omega_u = 1 \), then Eq. (13) presents us that \( \gamma = (3 + 6 \omega_u)/(5 + 15 \omega_u) \), which means that for the unparticle DM the growth index \( \gamma \) is smaller that of the usual DM. In Fig. (1), we plotted the change of the growth factor \( f = \beta \Omega_u^\gamma \) with different \( \omega_u \) and \( \omega \). We find that \( f \) increases with the EOS of the unparticle but decreases with that of the DE. That is to say, the effects of EOS of DE differ entirely from those of the UDM themselves. Moreover, for the UDM the growth factor \( f \) > 1 at high redshifts in which \( \Omega_u \rightarrow 1 \). The mathematical reason is that for unparticle \( \omega_u > 0 \) and \( \beta = 1 + 3 \omega_u > 1 \). This means that the behaviors of the growth factor of unparticle is quite different from that of usual DM. It could help us to distinguish whether in the early evolution our Universe is dominated by unparticle or not. Moreover, from Fig. (1) we also find that at the low redshift the values of \( f \) of the unparticle is larger than of usual DM, which provided a possible way for us to discern whether there exists unparticle matter in the present Universe.

Let us now to solve Eq. (12) numerically and to see how well the approximation \( f = \beta \Omega_u^\gamma \) fits the growth factor. Firstly, we introduce the dimensionless matter density \( \Omega_u \) and its evolution with the redshift \( z \) can be described by
\[
\Omega_u = \frac{\Omega_{u0}}{\Omega_{u0} + (1 - \Omega_{u0})(1 + z)^{3(\omega - \omega_u)}}. \] (16)
Making use of the above equation, we can solve Eq.\(^{(12)}\) numerically to obtain the growth factor \(f\) for different values of \(\Omega_m\), \(\omega_u\), and \(\omega\). And then we can check the accuracy of \(f = \beta \Omega'_u\) by comparing the approximation with \(f\). In Fig.(2) we plotted the change of the relative error \(\beta \Omega'_u / f - 1\) with \(\omega_u\) for the fixed \(\omega = -1\) and \(\Omega_u = 0.27\) and find that the quantity \(\beta \Omega'_u\) approximates \(f\) so well that the error is in the range of \(10^{-2}\).

4 The growth factor of unparticle Dark Matter for UCDM model

In this section, we consider the case that the Universe is supposed to be filled with unparticle, the cold DM and DE and then study the growth factor of unparticle DM. From the Friedman equation, we have

\[
\frac{H^2}{H^2_0} = -\frac{3}{2}\left[1 + \omega_u \Omega_u + \omega (1 - \Omega_u - \Omega_m)\right], \tag{17}
\]

where \(\Omega_m = \rho_m / \rho\) is the ratio of the cold DM density in the total Universe. Since the density of the cold DM is changed with the time \(t\), we must take \(\Omega_m\) as a variable. According to the energy conservation equation, we find that \(\Omega_u\) and \(\Omega_m\) satisfies

\[
\Omega'_u = 3\Omega_u [(\omega - \omega_u)(1 - \Omega_u) - \omega \Omega_m], \tag{18}
\]

and

\[
\Omega'_m = 3\Omega_m [\omega (1 - \Omega_m) - (\omega - \omega_u) \Omega_u], \tag{19}
\]

respectively. Substituting Eqs.\(^{(18)}\), \(^{(18)}\) and \(^{(19)}\) into Eq.\(^{(11)}\), we find the density perturbation equation of unparticle becomes

\[
3\Omega_u [(\omega - \omega_u)(1 - \Omega_u) - \omega \Omega_m] \frac{df}{d\Omega_u} + 3\Omega_m [\omega (1 - \Omega_m)] \\
-(\omega - \omega_u) \Omega_u \frac{df}{d\Omega_m} + f^2 + \left[\frac{1}{2} - \frac{3}{2} \omega_u \Omega_u\right] \\
- \frac{3}{2} \omega (1 - \Omega_u - \Omega_m) f = \frac{3}{2} (1 + \omega_u)(1 + 3 \omega_u) \Omega_u. \tag{20}
\]

Similarly, this equation is also valid generally for the arbitrary density perturbation. As in \(^{(33)}\), we take the approximation \(f = \beta \Omega'_u + \alpha \Omega_m\). Inserting it into Eq.\(^{(20)}\) and expanding all quantities around \(\Omega_u = 1\) and \(\Omega_m = 0\) (Since in the unparticle-dominated era the density of cold DM is small), we finally obtain that

\[
\alpha = \frac{3 \omega (1 - 3 \omega_u)}{5(5 - 6 \omega + 15 \omega_u)}, \quad \beta = 1 + 3 \omega_u, \quad \gamma = \frac{3 - 3 \omega + 6 \omega_u}{5 - 6 \omega + 15 \omega_u}. \tag{21}
\]

Therefore, the approximation for the growth factor can be expressed as

\[
f = (1 + 3 \omega_u) \Omega'_u + \frac{3 \omega (1 - 3 \omega_u)}{5(5 - 6 \omega + 15 \omega_u)} \Omega_m, \tag{22}
\]

\[
\gamma = \frac{3 - 3 \omega + 6 \omega_u}{5 - 6 \omega + 15 \omega_u}.
\]

The approximation \(^{(21)}\) is consistent with the previous result \(^{(13)}\) when \(\Omega_m = 0\). Moreover, we find the effect of the the cold DM on the growth factor \(f\) is very apparent as the redshift is lower. It is also shown in Fig.(3) in which we plotted the change of the growth factor \(f\) with different \(\Omega_u\) and \(\Omega_m\) for fixed \(\omega = -1\) and \(\omega_u = 0.1\). From Eq.\(^{(22)}\), we find that for AUCDM model \(f = \frac{13}{11} \Omega'_u - \frac{21}{625} \Omega_m\)
for $\omega_u = \frac{1}{10}$ and $f = \frac{5}{2} \Omega_u^2 + \frac{3}{185} \Omega_m$ for $\omega_u = \frac{1}{2}$. It is quite a different from those in UDM models. Our results also provide a possible way to distinguish the UDM and UCDM models.

Similarly, in the UCDM model, the dimensionless density of the UDM and the cold DM are described by

$$\Omega_u = \frac{\Omega_{u0}(1 + z)^{3\omega_u}}{\Omega_{m0} + \Omega_{u0}(1 + z)^{3\omega_u} + (1 - \Omega_{u0} - \Omega_{m0})(1 + z)^\omega},$$

$$\Omega_m = \frac{\Omega_{m0}}{\Omega_{m0} + \Omega_{u0}(1 + z)^{3\omega_u} + (1 - \Omega_{u0} - \Omega_{m0})(1 + z)^\omega}.$$

Combining Eqs. (5), (11) and (23), we can obtain the numerical solution of the growth factor $f$ and then calculate the accuracy of the approximation $\Omega_u^2 + \alpha \Omega_m$. From Fig. (4), we see that the error is under 5%, which means that Eq. (22) describes well the evolution of the growth factor $f$ at the low redshift.

5 Summary

In this paper, we treat the unparticle as a kind of DM and study its density perturbation growth factor in the UDM and UCDM models in the flat Universe. We find that $f$ can be approximated well by $(1 + 3\omega_u)\Omega_u^2$ for UDM model and by $(1 + 3\omega_u)\Omega_u^2 + \alpha \Omega_m$ for UCDM model. The growth index $\gamma$ depends not only on the EOS of DE, but also on the unparticle. When the redshift $z$ tends to infinite, the growth factor approaches to $1 + 3\omega_u$ which is quite different from that of usual DM. That is to say, the presence of $\omega_u$ modifies the behavior of the growth factor. Moreover, we find at the low redshift the values of quantity $f$ of the unparticle is larger than of usual DM. These could open a window to detect whether there exists the unparticle in the present Universe. Furthermore, our result also show the growth factor increases with the EOS of unparticle but decreases with of DE, which can help us to distinguish the unparticle and DE in the Universe.

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