Fano resonances in nanoscale structures

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Modern nanotechnology allows to scale down various important devices (sensors, chips, fibres, etc), and, thus, opens up new horizons for their applications. The efficiency of most of them is based on fundamental physical phenomena, such as transport of wave excitations and resonances. Short propagation distances make phase coherent processes of waves important. Often the scattering of waves involves propagation along different paths, and, as a consequence, results in interference phenomena, where constructive interference corresponds to resonant enhancement and destructive interference to resonant suppression of the transmission. Recently, a variety of experimental and theoretical work has revealed such patterns in different physical settings. The purpose of this Review is to relate resonant scattering to Fano resonances, known from atomic physics. One of the main features of the Fano resonance is its asymmetric line profile. The asymmetry originates from a close coexistence of resonant transmission and resonant reflection, and can be reduced to the interaction of a discrete (localized) state with a continuum of propagation modes. We will introduce the basic concepts of Fano resonances, explain their geometrical and/or dynamical origin, and review theoretical and experimental studies for light propagation in photonic devices, charge transport through quantum dots, plasmon scattering in Josephson junction networks, and matter wave scattering in ultracold atom systems, among others.

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I. HISTORICAL REMARKS

One of the important diagnostic tools in physics is scattering of radiation (waves) by matter. It allows to investigate properties of matter and to control the radiation. For example, Rydberg spectral lines (1888) of the hydrogen atom allowed Niels Bohr to deduce his model of an atom (1913), which layed the basis of quantum mechanics. Later, Beutler (1935) observed that some of the Rydberg spectral atomic lines exhibit sharp asymmetric profiles in the absorption. It was Ugo Fano (1935) who suggested the first theoretical explanation of this effect and suggested a formula (also known as the Beutler-Fano formula) which predicts the shape of spectral lines based on a superposition principle from quantum mechanics. The complexity of the physical phenomena was encapsulated in a few key parameters, which made this formula a workhorse in many fields of physics, including nuclear, atomic, molecular, and condensed matter physics. According to Fano: "the Beutler spectra showed unusual intensity profiles which struck me as reflecting interference between alternative mechanisms of excitation" (Fano, 1977). The interpretation provided by Fano...
of these "strange looking shapes" of spectral absorption lines is based on the interaction of a discrete excited state of an atom with a continuum sharing the same energy level, which results in interference phenomena. The first paper with the derivation of the line-shape formula (Fano, 1935), was published in 1935, when Ugo Fano was a young postdoctoral fellow in the group of Enrico Fermi. Fano has acknowledged the influence of his teacher on the derivation of this key result. The second much more elaborated paper (Fano, 1961) became one of the most important publications in the physics of the XX century, rated between the first three most relevant works published in The Physical Review (Redner, 2004), with over 5300 citations by now (October 2008). "The paper appears to owe its success to accidental circumstances, such as the timing of its publication and some successful features of its formulation. The timing coincided with a rapid expansion of atomic and condensed matter spectroscopy, both optical and collisional. The formulation drew attention to the generality of the ingredients of the phenomena under consideration. In fact, however, the paper was a rehash of work done 25 years earlier ..." (Fano, 1977).

In his pioneering papers, Ugo Fano introduced an important new ingredient of matter-radiation interaction in atomic physics, making him a key player in XX century physics. This was also acknowledged by the Fermi Award in 1995 for "his seemingly formal use of fundamental theory" leading to "the underpinning of a vast variety of practical results which developed naturally from this understanding".

Remarkably, the first observation of the asymmetric line-shapes can be traced back to the discovery made by Wood in 1902, namely, the presence of unexpected narrow bright and dark bands in the spectrum of an optical reflection grating illuminated by a slowly varying light source (Wood, 1902). Wood was astounded to see that under special illumination conditions the grating efficiency in a given order dropped from maximum to minimum illumination, within a wavelength range not greater than the distance between the sodium lines. These rapid variations of intensities of the various diffracted spectral orders in certain narrow frequency bands were termed anomalies, since the effects could not be explained by the conventional grating theory (Wood, 1935). The first theoretical treatment of these anomalies is due to Lord Rayleigh (1907). His "dynamical theory of the grating" was based on an expansion of the scattered electromagnetic field in terms of outgoing waves only. This theory correctly predicted the wavelength (Rayleigh wavelengths) at which anomalies occurred. However, one of the limitations of Rayleigh's approach was that it yields a singularity at the Rayleigh wavelength, and, therefore, does not give the shape of the bands associated with the anomaly. Fano tried to overcome this difficulty in a series of papers (Fano, 1936, 1937, 1938, 1941) by assuming a grating consisting of lossy dielectric material, and suggesting that anomalies could be associated with the excitation of a surface wave along the grating. The resonant excitation of leaky surface waves near the grating, which occurs when a suitable phase matching between the incident plane wave and the guided wave is satisfied, leads to a strong enhancement of the field near the grating surface (de Abajo, 2007; Hessel and Oliner, 1965; Sarrazin et al., 2003). As it was pointed out by Sarrazin et al. (2003), the observed asymmetric profiles can be fitted by the Fano formula with very good accuracy. Thus, the interaction of excited leaky modes with an incoming radiation leads to similar interference phenomena as in absorption by Rydberg atoms, where a leaky mode can be associated with a discrete state, and the incoming radiation with a continuum. These examples reveal the universality of Fano's approach in describing the origin of asymmetric line-shapes in terms of interference phenomena, regardless of the nature of the constituting waves, as well as in predicting both the position and the width of the resonance.

Similar asymmetric profiles were observed in various other systems and settings. But sometimes it is not obvious to determine the origin of the interference. In the present survey paper, we provide a very general explanation of appearance of the Fano resonances in various physical systems based on a simple model, which sheds light on the origin of the interference phenomena, which is well along the lines of Steven Weinberg: "our job in physics is to see things simply, to understand many complicated phenomena in a unified way, in terms of a few simple principles." (1979 Nobel Prize Lecture).
II. THE FANO RESONANCE

A. Two oscillators with a driving force

Usually, a resonance is thought to be an enhancement of the response of a system to an external excitation at a particular frequency. It is referred to as the resonant frequency, or natural frequency of the system. One of the simplest examples is a harmonic oscillator with periodic forcing. When the frequency of the driving force is close to the eigenfrequency of the oscillator, the amplitude of the latter is growing towards its maximal value. Often many physical systems may also exhibit the opposite phenomenon, when their response is suppressed if some resonance condition is met (which lead even to the term antiresonance). This can be illustrated by using two weakly coupled underdamped harmonic oscillators, where one of them is driven by a periodic force [see Fig. 2(a)]. In such a system, in general, there are two resonances located close to eigenfrequencies $\omega_1$ and $\omega_2$ of the oscillators \cite{Joe2006}. One of the resonances of the forced oscillator demonstrates the standard enhancement of the amplitude near its eigenfrequency $\omega_1$, while the other resonance exhibits an unusual sharp suppression of the amplitude near the eigenfrequency of the second oscillator $\omega_2$ [see Fig. 2(b,c)]. The first resonance is characterized by a symmetric profile, described by Lorentzian function, and known as a Breit-Wigner resonance \cite{BreitWigner1936}. The second resonance is characterized by an asymmetric profile.
The case of zero asymmetry parameter is the possibility of destructive interference, leading to asymmetric line shapes. Fano used a perturbation approach to explain the appearance of asymmetric resonances. He considered a so-called prediagonalised state by putting the coupling between a discrete bound state, which is degenerate in energy with a continuum of states, to zero. Such a prediagonalized state may or may not have a clear physical analogy, but serves in any case as a convenient mathematical construction, which allows to solve the problem. As a result Fano obtained the formula for the shape of the resonance profile (Fano 1935, 1961) of a scattering cross-section

$$
\sigma = \frac{(\epsilon + q)^2}{\epsilon^2 + 1} \quad (1)
$$

using a phenomenological shape parameter $q$ and a reduced energy $\epsilon$ defined by $2(E - E_F)/\Gamma$. $E_F$ is a resonant energy, and $\Gamma$ is the width of the auto-ionized state. Formula (1) suggests that there are exactly one maximum and one minimum in the Fano profile

$$
\sigma_{\text{min}} = 0, \quad \text{at } \epsilon = -q
$$

$$
\sigma_{\text{max}} = 1 + q^2, \quad \text{at } \epsilon = 1/q. \quad (2)
$$

In his original paper Fano (1961) has introduced the asymmetry parameter $q$ as a ratio of transition probabilities to the mixed state and to the continuum. In the limit $|q| \to \infty$ the transition to the continuum is very weak, and the lineshape is entirely determined by the transition through the discrete state only with the standard Lorentzian profile of a Breit-Wigner resonance. When the asymmetry parameter $q$ is order of unity both the continuum and discrete transition are of the same strength resulting is the asymmetric profile (1), with the maximum value at $E_{\text{max}} = E_F + \Gamma/(2q)$ and minimum value at $E_{\text{min}} = E_F - \Gamma q/2$. The case of zero asymmetry parameter $q = 0$ is very unique to the Fano resonance and describes a symmetrical dip, sometimes called an anti-resonance (see Fig. 5). The main feature of the Fano resonance is the possibility of destructive interference, leading to asymmetric line shapes (Bandopadhyay et al., 2004).
1992: Margulis and Pyataev, 2004: Marinho et al.
2001: Mehlor, 1998: Meijerink and Blasse, 1990: Nockel and Stone, 1994: Nussenzweig, 2001: Mehlhorn, 1998: Meijerink and Blasse, 1989: Oliveira and Wilkins, 1985: Patthey et al., 1990: Pichl et al., 2000: Ramaker and Schrader, 1974: Roney, 1994a,b, 1995: S. J. Xu and Zheng, 2006: Sanchez and Martin, 1994: Siegner et al., 1995: Simonian et al., 1973: Smith et al., 1992: Sturm et al., 1987: Taylor and Johnson, 1993: Ueda, 1987: Waligorski et al., 1997: Wickenhauser et al., 2005: Winstead and Langhoff, 1991: Yaefet, 1981: thus, revealing the underlying mechanism of the observed resonances in terms of quantum-mechanical interaction between discrete and continuous states. In nuclear and atomic physics, interferences are often originating from the interaction of open (continuum) and closed (discrete levels) channels (Feshbach, 1958, 1962). Bhatia and Temkin (1984) unified the approaches of Fano and Feshbach with ab initio calculations and derived a rigorous expression of the asymmetry parameter q (Bhatia and Temkin, 1984).

There are limitations to the applicability of the Fano formula (Connerade, 1998). First, it can be applied to describe single, isolated resonances. The appearance of more than two propagation paths will change the profiles. Second, the width of the discrete level should be narrow enough compared to other resonant structures in the scattering profile.

In general, the Coulomb interaction between an outgoing electron e^- and a charged ion core A^+ during auto-ionization leads to a renormalization of the energy levels of the many-electron system. Such a renormalization is known as the quantum defect of Rydberg series. To precisely describe the positions and width of the resonances, a multichannel quantum defect theory was developed by Seaton (1960) and Fano (1970), which provides a rigorous description of the process. It allows to derive all asymptotic quantities such as phase shifts or amplitudes of the auto-ionized levels. Eq. (1) was derived by Fano by neglecting effects due to long-range Coulomb interaction. Still it provides a physical insight into the auto-ionization process in terms of quantum-mechanical interference of discrete and continuum states.

At the resonance the phase of the scattering wave changes sharply by \( \pi \). Thus, the interaction of scattering waves will result in constructive and destructive interference phenomena located very close to each other, corresponding to a maximum \( E_{\text{max}} \) and a minimum \( E_{\text{min}} \) of the transmission (absorption), respectively. The width of the resonance is proportional to the distance between them \( \Gamma \sim |E_{\text{max}} - E_{\text{min}}| \). In principle, they may be located very close to each other \( E_{\text{max}} \approx E_{\text{min}} \) resulting in a very narrow resonance \( \Gamma \approx 0 \). Corresponding to a very long-lived quasi-bound state \( \text{Stillinger and Herrick, 1975} \), using artificial one-dimensional potentials one can even achieve \( \Gamma = 0 \) \( \text{von Neumann and Wigner, 1929} \), as a proof of concept. By applying Feshbach’s theory of resonances to two overlapping Fano resonances, Friedrich and Wintgen (1985a,b) demonstrated that the interference of several auto-ionizing levels of a Rydberg atom may lead to the formation of bound states in the continuum with anomalously narrow resonances.

C. Light and structured matter

Experiments on the absorption cross-section of a single quantum dot, which is often considered as an artificial atom, have revealed that the asymmetry parameter \( q \) can be continuously tuned with the power of the laser (Kroner et al., 2008). In this system, the transition rate to the discrete level saturates at high power, while the rate of the continuum transition does not (Zhang et al., 2006). Eventually, the initially weak continuum transition rate will match the saturated transition rate to the discrete level with increasing laser power. As a result, a symmetric Lorentzian profile at low power will transform to an asymmetric Fano profile at sufficiently large power (see Fig. 6).

In biased semiconductor superlattices the Fano coupling parameter \( \Gamma \) between the discrete state and the continuum can be continuously tuned by varying the applied electric field (Holfeld et al., 1998). The external bias gives rise to Wannier-Stark states, which interact with excitons, and result in asymmetric absorption spectra of Wannier-Stark transitions (Hino and Toshima, 2005, S. J. Xu and Zheng, 2006). The external bias determines the energy spacing of a Wannier-Stark subband, and, thus, controls the effective coupling between the discrete states and the continua. It allows to study the dephasing dynamics of the Fano resonance.

In general, the asymmetry parameter \( q \) is not restricted to be only real. In systems with broken time reversal
symmetry transition amplitudes to the discrete level and to the continuum may become complex, and so does the asymmetry parameter. The Fano resonance in such systems can be studied by analyzing the dynamical response. In particular, Misochko et al. (2003) found that the time-dependent reflection of light from a bismuth single crystal after the excitation by an ultrashort laser pulse exhibits Fano asymmetric profiles in the Fourier transform of a time-periodic signal. They demonstrated that the asymmetric parameter varies periodically with the time delay between pump and probe pulses. The breaking of time reversal symmetry is indicated by the change of the sign of the asymmetry parameter.

Asymmetric lineshapes were also observed in Raman spectra of heavily doped semiconductors (Bechstedt and Peuker, 1975; Bell et al., 1973; Cerdeira et al., 1973a; Chandrasekhar et al., 1978; Hopfield et al., 1967; Magidson and Beserman, 2002) and high-Tc superconductors (Friedl et al., 1990; Limonov et al., 1998, 2000; Misochko et al., 2000). Although, almost any asymmetric profile of these spectra can be fitted by the Fano formula [Aleshkin et al., 2007; Belitsky et al., 1997; Cardona, 1983; Cardona et al., 1974; Cerdeira et al., 1973b; Hase et al., 2006b; Jin and Xu, 2007; Jin et al., 2001; Lee et al., 2006; Menéndez and Cardona, 1987], a suitable theory for a quantitative description of these cases is still lacking. The general qualitative understanding is that the absorbed photon can initiate two kinds of processes. The first one is the inter- or intra-band electronic transition from the ground state to the continuum. The second process is the transition to an intermediate state followed by a one-phonon Raman emission and electron transition to either the initial ground state or to the excited donor state. Thus, the interference of two processes may in principle result in the Fano resonance.

D. Atoms and atoms

When two atoms collide with each other a quasi-bound state can be formed, which is characterized by a complex energy \( E = E_F + i\Gamma \). In scattering theory this quasi-bound state is called a resonance since it possesses a finite life-time \( \Gamma \). The quasi-bound state is formed due to the excitation and sharing of electrons, and can interpreted as an interaction between discrete and continuous states [see Fig. 3(b)]. In a similar manner, the observed asymmetric resonances in pre-dissociation (Bandrauk and Laplante, 1976; Cotting et al., 1994; Lebech et al., 2006; Lewis et al., 2001; Palffy et al., 2007) (or fragmentation) of molecules were explained by Rice (1953) in terms of auto-ionization. The concept was introduced by Feshbach (1958) in the context of reactions forming a compound nucleus. A Feshbach resonance in a two-particle collision appears whenever a bound state in a closed channel is coupled resonantly with a scattering continuum of an open channel (Bloch et al., 2008). The scattered particles are temporarily captured in the quasibound state, and the associated long time delay gives rise to a Breit-Wigner-type resonance in the scattering cross section (see Fig. 7).

A series of recent studies was devoted to the explicit calculation of scattering states for one-dimensional chains with two interacting bosons or fermions (Grupp et al., 2007; Nygaard et al., 2008a; Valiente and Petrov, 2009). These systems allow for two-particle continuum states, but also for bound states of two particles. Tuning the Bloch wave number, the bound state dissolves with the two-particle continuum. However, its trace inside the continuum remains, leading to a \( \pi \) phase shift of the scattering phase, and to corresponding Fano or Feshbach resonances in the scattering length. Notably in these problems a clear notion of resonant transport is absent, since there is no difference between a probe beam and a target due to indistinguishability of the two particles.

Efimov predicted that a three-body quantum system can support weakly bound states (trimer) under conditions when none of the three constituting pairs are bound (Efimov, 1971). Efimov trimer states appear in the limit where the two-body interaction is too weak to support a two-body bound state (dimer). Such trimer states should exist regardless of the nature of the two-body interaction, and, thus, are generic in few-body systems. Recently, the first experimental obser-
vation of Efimov states has been reported in ultracold cesium trimers [Kraemer et al. 2006], by measuring the three-body recombination process Cs+Cs+Cs→Cs2+Cs. The fingerprint of Efimov trimers in this system appears as a resonant enhancement and suppression of three-body collisions as a function of the two-atom interaction strength (Esry and Greene, 2000; Kraemer et al. 2006), with typical asymmetric profiles. Mazumdar et al. (2006) explained this asymmetric response in terms of a Fano resonance, suggesting that the asymmetry can be used as a diagnostic tool for the Efimov effect.

III. MODELING: COMPLEX GEOMETRIES

One possibility to model a Fano resonance is to choose the geometry of a given system in such a way that (at least) two scattering paths are available. In this Section we will consider the basic geometries which will do the job, and discuss several extensions.

A. Fano-Anderson model

One of the simplest models which describes the physics and the main features of the Fano resonance is the Fano-Anderson model (Mahan 1993) which mimics the energy level structures [see Fig. 3(a)] of the model proposed by Fano (1961). In a simplified version [Miroshnichenko et al., 2005a] it can be described by the following Hamiltonian

\[ H = C \sum_n (\phi_n \phi_n^* + c.c.) + E_F |\psi|^2 + V_F (\psi^* \phi_0 + c.c.) , \]

where the asterisk denotes complex conjugation. This model describes the interaction of two subsystems. One is a linear discrete chain with the complex field amplitude \( \phi_n \) at site \( n \) and nearest-neighbor coupling with strength \( C \). This system supports propagation of plane waves with dispersion \( \omega_k = 2C \cos k \). The second subsystem consists of a single Fano state \( \psi \) with the energy \( E_F \). The interaction between these two subsystems is given by the coupling coefficient \( V_F \) between the state \( \psi \) and one site of the discrete chain \( \phi_0 \). A propagating wave may directly pass through the chain, or instead visit the Fano state, return back and continue with propagation. These two paths are the ingredients of the Fano resonance.

The lattice Hamiltonian (3) generates the following differential equations:

\[ i \dot{\phi}_n = C(\phi_{n-1} + \phi_{n+1}) + V_F \psi \delta_{n0} , \]

\[ i \dot{\psi} = E_F \psi + V_F \phi_0 . \]

With the ansatz

\[ \phi_n(\tau) = A_n e^{-i\omega \tau} , \quad \psi(\tau) = B e^{-i\omega \tau} , \]

we obtain a set of algebraic equations for the amplitudes:

\[ \omega A_n = C(A_{n-1} + A_{n+1}) + V_F B \delta_{n0} , \]

\[ \omega B = E_F B + V_F A_0 . \]

For a scattering problem, the system (6) should be solved for frequencies chosen from the propagation band \( \omega = \omega_k \) with the following boundary conditions

\[ A_n = \begin{cases} I e^{i kn} + r e^{-i kn}, & n < 0, \\ \tau e^{i kn}, & n > 0, \end{cases} \]

where \( I, r, \) and \( \tau \) have the meaning of the incoming, reflected and transmitted wave amplitudes, respectively. From (6) it follows

\[ B = \frac{V_F A_0}{\omega_k - E_F} , \]

and finally

\[ \omega_k A_n = C(A_{n-1} + A_{n+1}) + \frac{V_F^2}{\omega_k - E_F} A_0 \delta_{n0} . \]

The main resulting action of the Fano state is that the strength of the effective scattering potential \( V_F^2 / (\omega_k - E_F) \) resonantly depends on the frequency of the incoming wave \( \omega_k \). If \( E_F \) lies inside the propagation band of the linear chain \( |E_F| < 2C \), the scattering potential will become infinitely large for \( \omega_k = E_F \), completely blocking propagation. Therefore meeting the resonance condition leads to a resonant suppression of the transmission, which is the main feature of the Fano resonance.

The transmission coefficient \( T = \frac{|\tau|}{|I|^2} \) can be computed by using the transfer matrix approach (Tong et al., 1999), and expressed in the following
form (Miroshnichenko et al., 2005b)

\[ T = \frac{\alpha_k^2}{\alpha_k^2 + 1}, \]  

(10)

where

\[ \alpha_k = c_k(E_F - \omega_k)/V_F^2, \quad c_k = 2C \sin k. \]  

(11)

Transmission vanishes at \( \omega_k = E_F \). The expression of the transmission coefficient (10) corresponds to the Fano formula (1) with \( q = 0 \), where \( \alpha_k \) corresponds to the dimensionless energy, and \( E_F \) is the resonant frequency. The Fano state is an additional degree of freedom which allows waves propagating in the chain to interfere with those propagating through the discrete state.

The width of the resonance is defined as

\[ \Gamma = \frac{V_F^2}{C \sin k_F}, \]  

(12)

where \( k_F \) is the wavenumber at the resonance, \( E_F = \omega_k \). The width of the resonance is proportional to the square of the coupling strength \( V_F^2 \).

The Fano-Anderson model (3) is perhaps the simplest one-dimensional model, which shows up with a Fano resonance. Since its asymmetry parameter \( q = 0 \), the location of the maximum in the Fano profile is tuned to infinity. The essence of the Fano resonance - destructive interference - is therefore not encapsulated in an asymmetric scattering profile with both a maximum and a minimum. It is the minimum which is generated by interference along several propagation paths. Due to its analytical simplicity the model may serve as a guideline for the analysis of more complicated physical models. There are many variations of this model (Burioni et al., 2005, 2006; Chakrabarti, 2006; Miroshnichenko and Kivshar, 2005a) studied recently.

B. Tuning the asymmetry parameter

The Fano-Anderson model (3) describes the resonant suppression of the transmission with a symmetric lineshape (\( q = 0 \)), emphasizing the main property of the Fano resonance which is destructive interference (resonant reflection). It can be easily extended in order to obtain a nonzero asymmetry parameter \( q \) with asymmetric lineshapes, such that both resonant suppression and resonant enhancement of the transmission will be located close to each other. Introducing a defect \( E_L \delta_L \delta_n L \) in the main array (3) [see Fig. 9(a)], both paths for scattering waves will yield phase shifts. As a result, both constructive and destructive interference phenomena may coexist, generating asymmetric transmission profiles [see Fig. 9(b)]. As observed in Fig. 9(b) the sign of the asymmetry parameter \( q \) alternates with the distance between the side-coupled defect and the defect in the main array (which is known as \( q \)-reversal (Kim and Yoshihara, 1993))

\[ \text{sign}(\omega_{T_{\text{max}}} - \omega_{T_{\text{min}}}) = (-1)^L. \]

(13)
D. Nonlinear Fano resonance

The Fano state amplitude becomes largest

$$|B_{\text{max}}|^2 = 4V_F^2|I|^2/I^2,$$

exactly at the resonant value of the wave number $k_F$, and it diverges (and is therefore much larger than the amplitudes in the chain which are bounded by $I$) in the limit of small coupling strength $V_F$.

Whatever the physical origin of the waves whose scattering is studied, large amplitudes call for corrections - either many-body interactions in a quantum setting, or nonlinear response corrections in a classical setting. Notably these corrections apply in first order only for the Fano state. Taking the classical setting, nonlinear Fano resonances [Miroshnichenko et al. (2005b)] were studied by introducing nonlinear corrections to the evolution equation for the Fano state only

$$\omega B = E_F B + \lambda |B|^2 B + V_F A_0.$$

The nonlinear transmission coefficient can be expressed in the following form [Miroshnichenko et al. (2005b)]

$$T = \frac{x^2}{x^2 + 1},$$

where $x = -\cot \delta(k)$ is a function of the scattering phase $\delta(k)$, and satisfies the cubic equation

$$(x^2 + 1)(x - \alpha k) - \gamma_k = 0,$$

with the parameter $\gamma_k = \lambda |k|^3 |I|^2/V_F^4$. The nonlinear Fano resonance condition corresponds to $x = 0$ in Eq.(17), which needs the condition $\gamma_k = -\alpha_k$ to be satisfied [see Fig.8(c)]. The transmission coefficient depends not only on the frequency of the incoming wave $\omega_k$, but on its intensity $|I|^2$ as well. The presence of nonlinearity leads to a renormalization of the self-energy of the Fano state, and consequently to an intensity-dependent shift of the resonance. Miroshnichenko et al. (2005b) have shown that the nonlinear Fano resonance exists for any value of the input intensity $|I|^2$ [see Fig.8(c)]. Therefore, nonlinearity allows to tune the location of the Fano resonance by changing the intensity of the input waves. In general, there exist up to three solutions of the cubic Eq.(17), which will result in bistable transmission [see Fig.8(d)].

E. Resonant reflection of pulses and solitons

So far we discussed the scattering of monochromatic plane waves. Consider a pulse instead which is launched towards the scattering region. The more narrow the pulse is in real space, the broader is its spectral decomposition in Fourier (plane wave) space $k$, which is characterized by the maximum frequency $\omega_m$ and the spectral width $\Delta \omega$. Each pulse component in Fourier space $k$ will scatter as discussed above. The spectral width $\Delta \omega$ has to be compared with the width of a Fano resonance $\Gamma$. If $\Delta \omega \ll \Gamma$, tuning $\omega_m$ into resonance with a Fano resonance will lead to a practically complete reflection of the pulse. If on the contrary $\Delta \omega \gg \Gamma$, only a narrow part of the spectral

FIG. 10 Resonant reflection of a soliton in topological networks (a) The reflection coefficient versus wavenumber $k$ for two Cayley trees of length $M = 5$ (line) and $M = 6$ (line) attached to the discrete array. Empty circles and stars correspond to direct numerical simulations of the soliton propagation. (b) Example of the soliton reflection by a Fano-like defect. From Burioni et al. (2005).
component of the pulse will be reflected, while the rest will be transmitted with a spectral hole 'burned' into it.

If nonlinearities are added into the propagation channel, they lead to an interaction between the various plane waves constituting the pulse and may ultimately yield nondispersing solitons. Their scattering by Fano defects was studied as well (Burioni et al., 2003, 2006; Miroshnichenko et al., 2003; Wulf and Skalozub, 2005). There are two characteristic time scales important for the scattering of solitons. One of them is the time the soliton resides in the vicinity of the defect $\tau_{rs}$, which is inversely proportional to its spectral width $\Delta \omega$ and the soliton velocity $v$. The second one is set by the nonlinearity. It is the time scale on which the plane wave which constitute the soliton interact with each other $\tau_{int}$ (Miroshnichenko et al., 2003). For fast propagating solitons the residence time is much smaller than the interaction time $\tau_{rs} \ll \tau_{int}$. Then, during the scattering process the soliton can be considered as a set of noninteracting plane waves, and the results of the above pulse scattering apply (Burioni et al., 2003, 2006; Miroshnichenko et al., 2003) [see Fig. 10(b)]. In the opposite case, when the residence time is much larger than the interaction time $\tau_{rs} \gg \tau_{int}$, the nonlinearity-induced mode-mode interaction becomes crucial during the scattering process. In general a nonlinear interaction between many degrees of freedom (modes or plane waves) will lead to chaotic dynamics, and consequently to a dephasing of individual plane waves. Therefore phase coherence will not be maintained during the scattering, and interference effects will vanish. Therefore the Fano resonance should quickly deteriorate as the soliton parameters are tuned into the region of validity of the second case. This was numerically confirmed by Miroshnichenko et al. (2003).

F. Quadratic nonlinearities

Consider the wave scattering in an array of channel waveguides with quadratic nonlinearity generated by periodic poling of several waveguides (Miroshnichenko et al., 2005a). When the matching conditions are satisfied, the fundamental-frequency (FF) mode with frequency $\omega$ can parametrically generate a second-harmonic (SH) wave with the frequency $2\omega$ [see Fig. 11(a)], such that a structure with several poled waveguides may behave as a nonlinear defect with spatially confined quadratic nonlinearity (Iwanow et al., 2004). The waveguide array can be described by a discrete model of weakly coupled linear waveguides with several waveguides having a quadratic nonlinear response (Iwanow et al., 2004; Miroshnichenko et al., 2005a), which is very similar to the Fano-Anderson model (4). The fundamental mode in this case can be considered as a continuum of propagating states, while the generated second harmonic can be either extended or effectively localized depending on the phase matching condition (Miroshnichenko et al., 2005a). In the latter case the excited second harmonic will act as a discrete state in the continuum, leading to the appearance of a Fano resonance in the transmission [see Fig. 11(b)]. Results of the direct numerical simulations of the Gaussian beam scattering are in a good agreement with the plane wave analysis [see Fig. 11(b)]. Figures 11(c,d) show the evolution of the fundamental and second harmonic of the Gaussian beam scattering at resonance. A part of the fundamental harmonic of the Gaussian beam is resonantly reflected by a single nonlinear defect showing the resonant reflection part of the beam at the fundamental frequency (c), and resonant excitation of the second harmonic (d). Adapted from Miroshnichenko et al. (2005a).

FIG. 11 (Color online) Light scattering in an array of channel waveguides with quadratic nonlinearity. (a) Schematic view of a one-dimensional array of channel waveguides with nonlinear defects, created by periodic poling. Arrows indicate the scattering process. (b) Comparison of the transmission coefficients of plane waves (solid line) and a Gaussian beam (crosses). Bottom: Example of the Gaussian beam scattering by a single nonlinear defect showing the resonant reflection part of the beam at the fundamental frequency (c), and resonant excitation of the second harmonic (d). Adapted from Miroshnichenko et al. (2005a).

IV. MODELING: COMPLEX DYNAMICS

Several propagation pathes and interference phenomena can be generated not only by imprinting complex geometries, but also by using complex dynamics. Nonlinear wave excitations, e.g. discrete solitons, when scat-
tering small amplitude waves, generate several propagation pathes purely dynamically. The reason is that the scattering potentials are time-dependent (in fact usually time-periodic). The amplitude and the temporal period can be tuned by controlling the characteristics of the nonlinear excitations (Emmanouilidou and Reichl, 2002; Li and Reichl, 1999; Martinez and Reichl, 2001). Total resonant reflection was also observed (Babev and Lake, 1992). This is because the time-periodic scattering potential generates several harmonics. In general, these harmonics will correspond to open and closed propagation channels, respectively. The presence of such dynamically generated channels is equivalent to a local increase of the spatial dimensionality, discussed in the previous Section. In other words, each new channel generates an alternative pathway for the scattering wave to propagate. The spectrum of excitations in each additional closed channel may contain discrete (localized) states, which happen to resonate with the continuum of the original open channel. As a result, Fano resonances can be expected, where the Fano state is the discrete state from a dynamically generated closed channel.

A. Scattering by discrete breathers

Discrete breathers (DBs) are known as time-periodic and spatially localized solutions of nonlinear wave equations on lattices (Aubry, 1997; Flach and Gorbach, 2008; Flach and Wilks, 1998; MacKay and Aubry, 1994). They originate from a constructive interplay between nonlinearity and discreteness. DBs exist independent of the lattice dimension, and are not relying on integrability properties. In return, these excitations can not freely move through lattices. Therefore, they act as scattering centers for small amplitude plane waves. Tuning the amplitude of the DB excitation, one tunes its temporal period, and all other characteristics of the resulting time-periodic scattering potential. DBs were detected and studied experimentally in interacting Josephson junction networks (Binder et al., 2000; Trías et al., 2000), coupled nonlinear optical waveguides (Eisenberg et al., 1998), lattice vibrations in crystals (Swanson et al., 1999), anti-ferromagnetic structures (Schwarz et al., 1999), micro-mechanical cantilever arrays (Sato et al., 2003), Bose-Einstein condensates loaded on optical lattices (Eiermann et al., 2004), and many others (Flach and Gorbach, 2008).

Resonant scattering of plane waves by DBs was studied and showed Fano resonances with zero transmission \( T = 0 \) (Flach et al., 2003a, 2003b; Kim and Kim, 1999, 2000; Lee and Kim, 2000b; Miroshnichenko et al., 2005c). Below we will demonstrate the concept using a particular example of wave scattering by DBs in the discrete nonlinear Schrödinger model (DNLS) (Flach et al., 2003a).

The equations of motion for the DNLS are given by

\[
i\hat{\Psi}_n = C(\hat{\Psi}_{n+1} + \hat{\Psi}_{n-1}) + |\hat{\Psi}_n|^2\hat{\Psi}_n , \tag{18}
\]

where \( n \) is an integer labeling the lattice sites, \( \hat{\Psi}_n \) is a complex scalar variable and \( C \) describes the nearest neighbor interaction (hopping) on the lattice. The last term in (18) is a cubic nonlinearity. For small amplitude waves \( \hat{\Psi}_n(t) = e^{i(\omega_n t - k n)} \) the dispersion relation

\[
\omega_k = -2C \cos k \tag{19}
\]

follows from Eq. (18).

The DNLS model supports DB solutions with a single harmonic

\[
\hat{\Psi}_n(t) = \hat{A}_n e^{-\delta_n t}, \quad \hat{A}_{|n|} \to \infty \to 0 , \tag{20}
\]

where the time-independent amplitude \( \hat{A}_n \) can be taken real valued, and the breather frequency \( \Omega_b \neq \omega_k \) is some function of the maximum amplitude \( \hat{A}_0 \). The spatial localization is given by an exponential law \( \hat{A}_n \sim e^{-\lambda |n|} \) where \( \cos \lambda = |\Omega_b|/2C \). Thus the DB can be approximated by a single-site excitation if \( |\Omega_b| \gg C \). In this case the relation between the single-site amplitude \( \hat{A}_0 \) and \( \Omega_b \) becomes \( \Omega_b = \hat{A}_0^2 \). In the following, the DB amplitudes for \( n \neq 0 \) will be neglected, i.e. \( \hat{A}_{n \neq 0} \approx 0 \), since \( \hat{A}_{\pm 1} \approx (C/\Omega_b)\hat{A}_0 \ll \hat{A}_0 \).

Let us perturb the breather solution with small fluctuations \( \phi_n(t) \)

\[
\hat{\Psi}_n(t) = \hat{\Psi}_n(t) + \phi_n(t) \tag{21}
\]

and substitute this ansatz into (18). Linearization in the small fluctuating perturbation leads to the following set of equations:

\[
i\phi_n = C(\phi_{n+1} + \phi_{n-1}) + \Omega_b\delta_{n,0}(2\phi_0 + e^{-2i\Omega_b t}\phi_0) \tag{22}
\]

with \( \delta_{n,m} \) being the Kronecker symbol. The DB generates a scattering potential that consists of two parts: a static (dc) one, which depends on the breather intensity only \( \sim \Omega_b = \hat{A}_0^2 \), and a dynamical (ac) one, which depends periodically on time \( \sim \Omega_b e^{-2i\Omega_b t} \). With the two channel ansatz

\[
\phi_n(t) = X_n e^{i\omega t} + Y_n e^{-i(2\Omega_b + \omega)t} \tag{23}
\]

Eq. (22) is reduced to a set of algebraic equations for the complex channel amplitudes \( X_n \) and \( Y_n \)

\[
-\omega X_n = C(X_{n+1} + X_{n-1}) + \Omega_b\delta_{n,0}(2X_0 + Y_0) , \tag{24}
\]

\[
(2\Omega_b + \omega)Y_n = C(Y_{n+1} + Y_{n-1}) + \Omega_b\delta_{n,0}(2Y_0 + X_0) . \tag{25}
\]

For propagating the frequency \( \omega \) should be chosen from the propagation band \( \omega_k \). As a result, the channel \( X_n \) supports extended waves, while for \( Y_n \) channel does not, since the frequency \( -(2\Omega_b + \omega) \) is outside the propagation band \( \omega_k \) (Flach et al., 2003a) [see Fig. 12(a)]. Therefore the scattering takes place with an open channel \( X_n \) which interacts with a closed channel \( Y_n \).

Let us consider the more general set of equations

\[
-\omega_k X_n = C(X_{n+1} + X_{n-1}) - \delta_{n,0}(V_x X_0 + V_n Y_0) , \tag{26}
\]

\[
(\Omega + \omega_k)Y_n = C(Y_{n+1} + Y_{n-1}) - \delta_{n,0}(V_y Y_0 + V_n X_0) . \tag{27}
\]
The transmission coefficient for the general case is given by Eq. (30) and vanishes, when the condition \( F \) from Eq. (30) it follows that the transmission coefficient is satisfied, which is equivalent to requesting the resonance condition

\[
\omega_k = \omega_L^{(y)} .
\]  

The conclusion is, that total reflection takes place when a local mode, originating from the closed \( Y \)-channel, resonates with the plane wave spectrum \( \omega_k \) of the open \( X \)-channel. The resonance condition is not renormalized by the actual value of \( V_a \). The existence of local modes which originate from the \( X \)-channel for nonzero \( V_x \) and possibly resonate with the closed \( Y \)-channel is evidently not of any relevance. The resonant total reflection is a Fano resonance, as it is unambiguously related to a local state resonating and interacting with a continuum of extended states. The fact that the resonance is independent of \( V_a \) is due to the local coupling between the Fano state (originating from the \( Y \)-channel) and the open channel, and originates from the approximative DB solution in the limit \( |\Omega| \gg C \). Corrections to the DB solution will increase the range of coupling between the Fano state and the continuum, and correspondingly lead to a renormalization of the resonance location \( \Omega_{\mathrm{FB, RN}} \). Therefore we conclude, that the resonance location is not significantly renormalized, if the wavelength of the propagating wave is large compared to the extension of the space region where the coupling between a Fano state and a continuum occurs \( \Omega_{\mathrm{FB, RN}} \).

If the closed channel is reduced to the localized discrete Fano state \( Y \) only, the equations for the amplitudes take the form

\[
-\omega X_n = C(X_{n-1} + X_{n+1}) + V_a Y \delta_{n0},
\]

\[
-\omega Y = E_F Y - V_a X_0 .
\]  

The different signs in front of the coupling between the chain and the Fano state are due to the fact, that time-periodic scattering potentials correspond to eigenvalue problems with a symplectic propagator. At variance, complex geometries \( \Omega \) do not leave the grounds of unitary propagators. Remarkably, these differences in the symmetries of the underlying dynamical processes do not alter the final result of destructive interference and Fano resonances.

The above analysis leads to a recipe of finding the position of resonances. One first calculates the localized states of closed channels decoupled from the open one \( \Omega_{\mathrm{FB, RN}} \). Switching on the coupling again, Fano resonances will take place exactly at the eigenfrequencies of the localized states for weak coupling. For stronger coupling the positions of the resonances will renormalize. In general, there is an infinite number of harmonics of the DB, which generate an infinite number of closed channels \( \Omega_{\mathrm{FB, RN}} \). The approach described above is rather generic and can be applied to the scattering through many types of oscillating barriers, self-induced (like DBs) or parametrically driven (by external forces) \( \Omega \). All of them produce similar scattering potentials with an open and a number of closed channels for small amplitude scattering waves.

### B. Light scattering by optical solitons

The above concept of scattering by solitary excitations was applied to predict resonant light scattering by optical solitons in a slab waveguide with an inhomogeneous refractive index core \( \Omega_{\mathrm{FB, RN}} \). The soliton is generated in a nonlinear planar waveguide by a laser beam injected into the slab along the \( z \)-direction \( \Omega_{\mathrm{FB, RN}} \). The soliton beam is confined in the \( y \)-direction by total internal reflection. The localization in the \( x \)-direction is achieved by a balance between linear diffraction and an instantaneous Kerr-type nonlinearity.
The analogy with the discussed above scattering problem by time-periodic potentials comes from the possibility to interpret the spatial propagation along the z-axis, while the probe beam propagates in the xz-plane at some angle to the soliton; by time-periodic potentials comes from the possibility to interpret the spatial propagation along the z-axis, while the probe beam propagates in the xz-plane at some angle to the soliton; (b) top view of the scattering process; (c) transmission coefficient vs kx for plane waves under oblique incidence. There is total suppression of the transmission near kx ≈ 0.181; (d) Fourier spectrum of the incident (dashed line) and transmitted (solid line) beams. The suppression of the resonant frequency [see plot (c)] in the spectrum is observed. Adapted from Flach et al. (2005).

Another theoretical prediction concerns the plasmon scattering by DBs in Josephson junction ladders (JJLs). JJLs are formed by an array of small Josephson junctions that are arranged along the spars and rungs of a ladder [see Fig. 14(a)]. Each junction consists of two small weakly coupled superconducting islands. The dynamical state of a junction is described by the phase difference \( \phi(t) \) (Josephson phase) of the superconducting order parameters of two neighbouring islands. When the difference does not vary in time \( \phi(t) = \text{const} \), the junction is traversed by a superconducting current only, with zero voltage drop. Otherwise, the junction is traversed in addition by a resistive current component with a nonzero voltage drop \( V \propto \phi(t) \). It was observed experimentally that JJLs support dynamic localized states (DBs) [Binder et al. 2000, Trias et al. 2000]. A discrete breather is characterized by a few junctions being in the resistive state \( \langle \hat{\phi} \rangle \neq 0 \) while the others reside in the superconducting state \( \langle \hat{\phi} \rangle = 0 \). The frequency of a DB is proportional to the average voltage drop across the resistive junctions \( \Omega_b \propto \langle \hat{\phi} \rangle \). Miroshnichenko et al. (2005d) have recently proposed an experimental setup to measure Fano resonances in that transmission line. Small amplitude waves are generated in a JJL with open ends by applying locally a time-periodic current \( \gamma_1(t) = \gamma_{ac} \cos(\omega t) \). The local current acts as a local parametric drive. It excites edge junctions at a frequency \( \omega \). This tail extends into the ladder. To monitor the linear wave propagation in the system, the time-averaged oscillation power \( P_{ac,R} = \langle \hat{\phi}_n^2 \rangle \) is measured. The transmission coefficient can be obtained by relating the oscillation power at the right boundary with and without an excited DB in the system

\[
T = \frac{P_{ac,R}(\text{with DB})}{P_{ac,R}(\text{without DB})}. \tag{34}
\]
The analysis in Ref. [Miroshnichenko et al. 2005c] reveals that they correspond to Fano resonances, which originate from localized states of closed channels of the time-periodic scattering potential which is generated by the DB.

D. Matter wave scattering in Bose-Einstein condensates

Over the last couple of years, it has been shown that optical lattices, generated by counter-propagating laser beams and providing a periodic potential modulation for the atoms, introduce many interesting and potentially useful effects by modifying single atom properties and enhancing correlations between atoms (Morsch and Oberthaler, 2006). Using about 1000 $^{87}\text{Rb}$ atoms in a quasi one-dimensional optical lattice, Eiermann et al. (2004) obtained a spatially localized Bose-Einstein condensate (BEC) which is an experimental manifestation of a gap soliton, or a discrete breather. The solitary state exists due to the atom-atom interaction, which can be tuned in various ways experimentally. Vicencio et al. (2007) considered a BEC on a lattice, where interactions between atoms are present only in a very localized region (see Fig. 15). Such a situation could be realized experimentally by combining optical lattices with atom-chip technology (Hänzel et al. 2001; Ott et al. 2001) or in optical micro-lens arrays (Dumke et al. 2002). The system is described by the discrete nonlinear Schrödinger (DNLS) equation, a classical variant of the Bose-Hubbard model appropriate for a BEC in a periodic potential in the tight binding limit (Morsch and Oberthaler, 2006). With interactions being present only on site number $n_c$, it follows

$$i \frac{d \Psi_n}{dt} = - (\Psi_{n+1} + \Psi_{n-1}) - \gamma |\Psi_{n_c}|^2 \Psi_n, \delta_{n,n_c},$$  \hspace{1cm} (35)

where $\Psi_n$ is the complex amplitude of the condensate field at site $n$ and $-\gamma = U/J$ is the interaction strength on site $n_c$, where $J$ is the tunneling energy between the lattice cites and $U$ is on-site interaction energy per atom.

Equations (35) support a localized state $\Psi_n(t) = bx^{m-n_c} \exp(-iE_b t)$, where $x = -\frac{1}{2}(E_b + g)$ with $g = \gamma b^2$, $b$ is the condensate amplitude and $E_b = -(4 + g^2)^{1/2}$ is the chemical potential.

The scattering of propagating atomic matter waves with the energy $E_k = -2 \cos k$ by this localized BEC were calculated analytically within the framework of the Bogolyubov-DeGenne equations (Vicencio et al. 2007). The transmission $T(k)$ is shown in Fig. 16 for three values of $g$ (solid curves). As $g$ increases, the width and the position of the resonance increase. Furthermore, the more localized the BEC becomes, the stronger it reflects the atom beam off resonance. By tuning the nonlinear parameter $g$, we can thus choose the amount of the beam which passes through the BEC. Off resonance (for larger values of $k$), we can select the percentage of the incoming beam that is transmitted for a defined quasi-momentum. Therefore, the actual setup can be used as a 100% blockade or as a selective filter.

The analytical results have been confirmed by numerical simulation of Eq. (35) using wave packets for $g = 0.36$ (line and boxes), $g = 0.6$ (line and diamonds), and $g = 0.9$ (line and triangles). From Vicencio et al. (2007).

V. LIGHT PROPAGATION IN PHOTONIC DEVICES

Optical microcavity structures are of great interest for device applications, and many of these structures
We consider a photonic crystal created by a periodic square lattice of infinite cylindrical rods parallel to the $z$ axis. We neglect the material dispersion and assume the dielectric constant $\epsilon(\vec{r})$ to be periodic in two transverse directions, $\vec{r} = (x, y)$. The evolution of the $E$-polarized electric field propagating in the $(x, y)$ plane is governed by the scalar wave equation

$$\nabla^2 E_z(\vec{r}, \tau) - \frac{1}{c^2} \partial^2_{\tau} [\epsilon(\vec{r}) E_z(\vec{r}, \tau)] = 0,$$  \hspace{1cm} (36)$$

where $\nabla^2 = \partial^2_x + \partial^2_y$. We assume that the light field propagating in such structures can be separated into fast and slow components, $E_z(\vec{r}, \tau) = e^{-i\omega \tau} E(\vec{r}, \tau | \omega)$, where $E(\vec{r}, \tau | \omega)$ is a slowly varying envelope of the electric field, i.e., $\partial^2_{\tau} E(\vec{r}, \tau | \omega) \ll \omega \partial_{\tau} E(\vec{r}, \tau | \omega)$. This allows to simplify Eq. (36) to the following form

$$\left[ \nabla^2 + \epsilon(\vec{r}) \left( \frac{\omega}{c} \right)^2 \right] E(\vec{r}, \tau | \omega) \simeq -2i\epsilon(\vec{r}) \frac{\omega}{c^2} \frac{\partial E(\vec{r}, \tau | \omega)}{\partial \tau} ,$$ \hspace{1cm} (37)$$

Both the straight waveguide and the side-coupled cavity are created by introducing defect rods into a perfect two-dimensional periodic structure. Therefore, the dielectric constant can be represented as a sum of two components, describing the periodic and defect structures $\epsilon(\vec{r}) = \epsilon_{pc} + \delta \epsilon$. We employ the Green's function of the two-dimensional periodic structure without defects, and rewrite Eq. (37) in the integral form

$$E(x, \tau | \omega) = \int d^2 y G(x, y | \omega) \hat{L} E(y, \tau, \omega) ,$$ \hspace{1cm} (38)$$

where we introduce the linear operator

$$\hat{L} = \left( \frac{\omega}{c} \right)^2 \delta \epsilon(\vec{r}) + 2i\epsilon(\vec{r}) \frac{\omega}{c^2} \frac{\partial}{\partial \tau} ,$$ \hspace{1cm} (39)$$

and consider the time evolution of the slowly varying envelope as a perturbation to the steady state.

The defect rods introduced into the periodic structure can formally be described as follows:

$$\delta \epsilon(\vec{r}) = \sum_{n,m} \left[ \delta \epsilon^{(0)}_{m,n} + \chi^{(3)} |E(x, \tau | \omega)|^2 \right] \theta(x - x_{n,m}) ,$$ \hspace{1cm} (40)$$

where we use the $\theta$-function to describe the position of a defect rod at site $n, m$, with $\theta(x) = 1$ for $x$ inside the defect rods, and $\theta(x) = 0$ otherwise. $\delta \epsilon^{(0)}_{m,n}$ is the variation of the dielectric constant of the defect rod $(m, n)$. Importantly, this approach allows us to incorporate a nonlinear response in a straightforward manner, which is assumed to be of the Kerr type being described by the term $\chi^{(3)} |E|^2$.

Substituting Eq. (40) into the integral equation (38) and assuming that the electric field does not change inside the dielectric rods, we can evaluate the integral at the right hand side of Eq. (38) and derive a set of discrete nonlinear equations.

![FIG. 17 Schematic setup for (a) a waveguide directly coupled to a cavity and (b) a waveguide side-coupled to a cavity.](image-url)
\[ i\sigma \frac{\partial}{\partial \tau} E_{n,m} - E_{n,m} + \sum_{k,l} J_{n-k,m-l}(\omega)(\delta\epsilon_{k,l}^{(0)} + \chi^{(3)}|E_{k,l}|^2) E_{k,l} = 0, \]

(41)

for the amplitudes of the electric field \( E_{n,m}(\tau|\omega) = E(x_{n,m},\tau|\omega) \) calculated at the defect rods. The parameters \( \sigma \) and \( J_{k,l}(\omega) \) are determined by using the corresponding integrals of the Green’s function, where the whole information about the photonic crystal dispersion is now hidden in their specific frequency dependencies, which can be found in Refs. (Mingaleev and Kivshar, 2001; Mingaleev et al., 2000). In this way, the Green’s function needs to be calculated only once for a given photonic structure, e.g. by employing the approach outlined in Ref. (Ward and Pendry, 1998), and then it can be used to study any photonic circuit in that structure.

For the simple system when the photonic crystal has a waveguide side coupled to a single defect see Fig. 18(a), the problem describes a discrete system studied earlier [see Fig. 18(b)], and the transmission shows a Fano resonance [see Fig. 18(c)], analyzed in details in Refs. (Mingaleev et al., 2006; Miroshnichenko et al., 2005).

In a general case, the effective interaction between defect rods is of long-range nature (Mingaleev and Kivshar, 2002a; Mingaleev et al., 2006). However, the coupling strength decays exponentially with the distance and, as a result, for coupled-resonators optical waveguides the specific discrete arrays with nearest-neighbor interactions (at \( L = 1 \)) give already an excellent agreement with direct FDTD simulations (Mingaleev and Kivshar, 2002).

B. Defects in the waveguide

The two basic geometries shown in Figs. 17(a,b) can be further improved by placing partially reflecting elements into the waveguides (Fan, 2002; Khelif et al., 2003). These elements allow creating sharp and asymmetric response line shapes. In such systems, the transmission coefficient can vary from 0% to 100% in a frequency range narrower than the full width of the resonance itself.

To illustrate the effect of defects, Fan (Fan, 2002) simulated the response of the structure shown in Fig. 19(a) using a FDTD scheme with perfectly matched layer boundary conditions. A pulse is excited by a monopole source at one end of the waveguide. The transmission coefficient is then calculated by Fourier transforming the amplitude of the fields at the other end, and is shown as a solid line in Fig. 19(b). In comparison, the transmission spectra for the same structure, but without the two small cylinders in the waveguide, is shown by a dashed line.

Importantly, no detailed tuning of either the resonant
frequency or the coupling between the cavity and the waveguide is required to achieve asymmetric line shapes. Also, since the reflectivity of the partially reflecting elements need not to be large, the underlying physics here differs from typical coupled-cavity systems, and resembles Fano resonances involving interference between a continuum and a discrete level.

C. Sharp bends

One of the most fascinating properties of photonic crystals is their ability to guide electromagnetic waves in narrow waveguides created by a sequence of line defects, including light propagation through extremely sharp waveguide bends with nearly perfect power transmission [Lin et al., 1998; Mekis et al., 1996]. It is believed that the low-loss transmission through sharp waveguide bends in photonic crystals is one of the most promising approaches to combine several devices inside a compact nanoscale optical chip.

Interestingly, the transmission through sharp bends in photonic crystal waveguides can be reduced to a simple model with Fano resonances, where the waveguide bend hosts a specific localized defect. Miroshnichenko and Kivshar (2005b) derived effective discrete equations for two types of the waveguide bends in two-dimensional photonic crystals and obtained exact analytical solutions for the resonant transmission and reflection.

D. Add-drop filters

Fano resonances can be employed for a variety of photonic devices based on resonant tunneling. In particular, if two waveguides interact through a coupling element which supports a localized mode, a channel add-drop filter can be realized via the resonant tunneling between the waveguides [Fan et al., 1998; 1999; Soljačić et al., 2003]. The schematic diagram of a generic coupled system of this kind is shown in Fig. 20(a). At Fano resonance, the propagating state excites the resonant modes, which in turn decay into both waveguides. The transmitted signal in the first waveguide is made up of the directly propagating signal and the signal which originates from the second path which visits the coupling region. In order to achieve complete transfer from one waveguide to the other one, these two signal components must interfere destructively. The reflected amplitude, on the other hand, originates entirely from the second path into the coupling region. Hence, at least two states in the coupling region are needed to achieve also destructive interference of backscattered waves in the first waveguide. With these conditions satisfied, one may resonantly transfer the excitation from the first into the second waveguide.

This concept was developed by Fan et al. (1998) for the propagation of electromagnetic waves in a two-dimensional photonic crystal. To realize this concept, they used two photonic crystal waveguides and two coupled single-mode high-Q cavities, as shown in Figure 20(b). The photonic crystal is made of a square lattice of high-index dielectric rods, and the waveguides are formed by removing two rows of dielectric rods. The cavities are introduced between the waveguides by reducing the radius of two rods. The resonant states have different symmetry. An accidental degeneracy, caused by an exact cancellation between the two coupling mechanisms, is enforced by reducing the dielectric constant of four specific rods in the photonic crystal. The cancellation could equally have been accomplished by reducing the size of the rods instead of their dielectric constant.

Figure 20(b) shows the field pattern at resonance. The quality factor is larger than $10^3$. The backward transferred signal is almost completely absent over the entire frequency range.

This type of four-port photonic crystal systems can be employed for optical bistability, being particularly suitable for integration with other active devices on a chip (Soljačić et al., 2003). A similar concept can be employed for the realization of all-optical switching action in a nonlinear photonic crystal cross-waveguide geometry with instantaneous Kerr nonlinearity. There the transmission of a signal can be reversibly switched on and off by a control input (Yanik et al., 2003).

E. All-optical switching and bistability

A powerful principle that could be explored to implement all-optical transistors, switches, and logical gates is based on the concept of optical bistability. The use of photonic crystals enables the system to be of a size of the order of the wavelength of light, consume only a few milliwatts of power, and have a recovery and response time smaller than 1 ps. Several theoretical and experimental studies explored nonlinear Fano resonances for designing optimal bistable switching in nonlinear photonic crystals (Cowan and Young, 2003).
One of the great advantages in using nonlinear photonic-crystal cavities is the enhancement of nonlinear optical processes, including nonlinear photonic-crystal cavities. Such an enhancement can be very efficient in the regime of the slow-light propagation, that was demonstrated experimentally with the smallest achieved group velocity \( c/1000 \) (Gersen et al., 2003; Jacobsen et al., 2005; Notomi et al., 2001; Vlasov et al., 2002). Because of this success, the interest in slow-light applications based on photonic-crystal waveguides is rapidly growing, and posing problems of a design of different types of functional optical devices which would efficiently operate in the slow-light regime.

Recently, Mingaleev et al. (2007) have studied the resonant transmission of light through a photonic-crystal waveguide coupled to a nonlinear cavity, and demonstrated how to modify the structure geometry for achieving bistability and all-optical switching at ultra-low powers in the slow-light regime. This can be achieved by placing a side-coupled cavity between two defects of a photonic-crystal waveguide assuming that all the defect modes and the cavity mode have the same symmetry. In this structure the quality factor grows inversely proportional to the group velocity of light at the resonant frequency and, accordingly, the power threshold required for all-optical switching vanishes as a square of the group velocity (see Fig. 22).

The numerically obtained dependence \( Q(v_{gr}) \sim 1/v_{gr} \) is shown in Fig. 22(a), and it is in an excellent agreement with the theoretical predictions. Since the bistability threshold power of the incoming light in waveguide-cavity structures scales as \( P_{th} \sim 1/Q^2 \) (Mingaleev et al., 2006), one observes a rapid diminishing of \( P_{th} \sim v_{gr}^2 \) when the resonance frequency approaches the band edge, as shown in numerical calculations summarized in Figs. 22(b,c).

By now, several experimental observations of optical bistability enhanced through Fano interferences have been reported (Weidner et al., 2007; Yang et al., 2007). In particular, Yang et al. (2007) employed a high-\( Q \) cavity mode \( (Q = 30000) \) in a silicon photonic crystal and demonstrated Fano resonance based bistable states and switching with thresholds of 185\( \mu \)W and 4.5 fJ internally stored cavity energy that might be useful for scalable optical buffering and logic.

It is important to note, that the nonlinear Fano resonance shows dynamical instabilities with plane wave excitations (Miroshnichenko et al., 2009). Near the resonance the intensity of the scattered wave starts to grow in time, leading to modulational instability, while far from resonance it converges to a steady-state solution (see Fig. 23). However, as it was demonstrated by Miroshnichenko et al. (2009) this instability can be suppressed for temporal Gaussian pulses excitations, providing with an effective method of recovering the bistable transmission.

![FIG. 21](image1.png)

FIG. 21 (Color online) Electric field distributions in a photonic crystal for (a) high and (b) low transmission states. Red and blue colors represent large positive or negative electric fields, respectively. The same color scale is used for both panels. The black circles indicate the positions of the dielectric rods. From Yanik et al. (2003a).

![FIG. 22](image2.png)

FIG. 22 (Color online) Ultra-low all-optical switching in the slow-light regime. (a) Quality factor \( Q \) vs. group velocity \( v_{gr} \) at resonance for the waveguide-cavity structure. (b) Nonlinear bistable transmission at the frequencies with 80% of linear light transmission vs. the incoming light power for different values of the rod radius; (c) Switch-off bistability threshold vs. the group velocity at resonance. From Mingaleev et al. (2007).
Reflection coefficients which can be presented in the form characterize the light transmission by the transmission and (Boyd and Gauthier, 2006).

Towards the development of integrated all-optical chips providing an efficiently tunable transparency on an optical

resonators (Tomita et al., 2006), in a cavity with at least two resonant modes (Franzon and Hendrickson, 2006), and in integrated photonic chips with two microring resonators (Naweed et al., 2005), although the early work (Opatrný and Welsch, 2001) suggested already an idea of macroscopic double-resonator optical system exhibiting the EIT-like effect. Recently, the CRIT effect has been observed experimentally in the system of two interacting microresonators (glass spheres of about 400 μm in diameter) with whispering-gallery modes (Naweed et al., 2005), in a cavity with at least two resonant modes (Franzon and Hendrickson, 2006), and in integrated photonic chips with two microring resonators (Tomita et al., 2009; Xu et al., 2006). Providing an efficiently tunable transparency on an optical chip, such CRIT devices are considered as a crucial step towards the development of integrated all-optical chips (Boyd and Gauthier, 2006).

To explain the origin of CRIT resonances, we characterize the light transmission by the transmission and reflection coefficients which can be presented in the form

\[ T(\omega) = \frac{\sigma^2(\omega)}{\sigma^2(\omega) + 1}, \quad R(\omega) = \frac{1}{\sigma^2(\omega) + 1}, \quad (42) \]

where the detuning function \( \sigma(\omega) \) may have quite different type of frequency dependence for different types of waveguide-cavity structures. Zero transmission (total reflection) corresponds to the condition \( \sigma(\omega) = 0 \), while perfect transmission (zero reflection) corresponds to the condition \( \sigma(\omega) = \pm \infty \).

\[ \sigma(\omega) \approx \frac{\omega_\alpha - \omega}{\Gamma(\omega - \omega)}, \quad (43) \]

where \( \omega_\alpha \) is the eigenfrequency of the localized cavity mode of an isolated cavity \( \alpha \). The spectral width \( \gamma_\alpha \) of the resonance is determined by the overlap integral between the cavity mode and the guided mode at the resonant frequency.

To find \( \sigma(\omega) \) for the two-cavity structure, one can apply a variety of methods but the simplest approach is based on the transfer-matrix technique (Fan, 2002). When two cavities are separated by the distance \( d = 2\pi m/k(\omega_i) \), where \( k(\omega) \) is the waveguide’s dispersion relation, \( m \) is any integer number, and the frequency \( \omega_i \) is defined below, and there is no direct coupling between the cavities, we obtain

\[ \sigma(\omega) \approx \frac{(\omega_\alpha - \omega)(\omega_\beta - \omega)}{\Gamma(\omega - \omega)}, \quad (44) \]

with the total resonance width \( \Gamma = \gamma_\alpha + \gamma_\beta \) and the frequency of perfect transmission \( \omega_\alpha = (\gamma_\alpha \omega_\beta + \gamma_\beta \omega_\alpha)(\gamma_\alpha + \gamma_\beta)^{-1} \), lying in between the two cavity frequencies, \( \omega_\alpha \) and \( \omega_\beta \), of zero transmission.

In the case when the cavities \( \alpha \) and \( \beta \) are identical, we obtain a single-cavity resonance and the only effect of using two cavities is the doubling of the spectral width, \( \Gamma = 2\gamma_\alpha \), of the resonant reflection line, as it is illustrated in Fig. 25(a). However, introducing even the smallest difference between two cavities leads to the opening of an extremely narrow resonant transmission line on the background of this broader reflection line, as it is illustrated in Fig. 25(c). Indeed, for slightly different cavities we may rewrite Eq. (44) in the vicinity of the resonant transmission frequency, \( \omega_i = \omega_\alpha + \delta \omega/2 \), as \( \sigma(\omega) \approx \Gamma_i/(\omega - \omega_i) \), with the line width \( \Gamma_i = \delta \omega^2/8\gamma_\alpha \), which can easily be controlled by tuning the frequency difference \( \delta \omega \). The quality factor of this transmission line, \( Q_i = \omega_i/2\Gamma_i \approx 4\gamma_\alpha\omega_\alpha/\delta \omega^2 \), grows indefinitely when \( \delta \omega \) vanishes. As mentioned above, this effect is the all-optical analogue of the electromagnetically-induced...
transparency and is now often referred to as the effect of coupled-resonator-induced transparency (Smith et al., 2004).

In contrast, the inter-coupling between two cavities, as shown in Fig. 25(b) manifests itself as a qualitatively new effect of coupled-resonator-induced reflection (CRIR): for small detuning $\delta \omega = \omega_\beta - \omega_\alpha$, one of the resonant reflection frequencies shifts very close to the perfect transmission frequency, $\omega_1$, producing a narrow resonant reflection line, as is illustrated in Fig. 25(d). The frequency of this line is always close to the frequency $\omega_\beta$ of the cavity mode, while its spectral width is determined by the frequency difference $\delta \omega$, growing indefinitely as $\delta \omega$ vanishes (Landroba, Y. Mario and Chin, 2006; Mingaleev et al., 2008).

It should be emphasized that despite such a qualitative difference in their spectral manifestations, both CRIT and CRIR effects have the same physical origin which can be attributed to the Fano-Feshbach resonances (Feshbach, 1958; 1962; Mies, 1968) which are known to originate from the interaction of two or more resonances (e.g., two Fano resonances) in the overlapping regime, where the spectral widths of resonances are comparable to or larger than the frequency separation between them. In a general situation it leads to a drastic deformation of the transmission spectrum and the formation of additional resonances with sharp peaks. The Fano-Feshbach resonances are associated with a collective response of multiple interacting resonant degrees of freedom, and they have numerous evidences in quantum mechanical systems (Magunov et al., 2003; Raoult and Mies, 2004).

Finally, we discuss the interaction between two Fano resonances (Hino, 2001; Miroshnichenko, 2009) which can be employed to stop and store light coherently, with an all-optical adiabatic and reversible pulse bandwidth compression process (Yanik and Fan, 2004; Yanik et al., 2004). Such a process overcomes the fundamental bandwidth delay constraint in optics and can generate arbitrarily small group velocities for any light pulse with a given bandwidth, without any coherent or resonant light-matter interaction. The mechanism can be realized in a system consisting of a waveguide side coupled to tunable resonators, which generates a photonic band structure that represents a classical EIT analogue (Maes et al., 2005; Yanik et al., 2004).

G. Guided resonances in photonic crystal slabs

Scattering of light by photonic crystal slabs leads to another class of Fano resonances associated with the presence of guided resonances in periodic structures. A photonic crystal slab consists of a two-dimensional periodic index contrast introduced into a high-index guiding layer Fig. 26(a). Such modulated structures support in-plane guided modes that are completely confined by the slab without any coupling to external radiations (Magnusson and Wang, 1992). In addition to in-plane waveguiding, the slabs can also interact with external radiations in a complex and interesting way (Fan and Joannopoulos, 2002; Fan et al., 2003; Koshino, 2003). Of particular importance is the presence of guided resonances in the structures. Guided resonances can provide an efficient way to channel light from within the slab to the external environment. In addition, guided resonances can significantly affect the transmission and reflection of external incident light, resulting in complex resonant line shapes which can be linked to Fano resonances.

Fan and Joannopoulos (2002) calculated the transmission and reflection coefficients at various $k$ points for the structure shown in Fig. 26(a). The calculated spectra for s-polarized incident waves are shown in Figs. 25(b,c). The spectra consist of sharp resonant features superimposed upon a smoothly varying background. The background resembles Fabry-Perot oscillations when light interacts with a uniform dielectric slab. To clearly see this, the background is fitted to the spectra of a uniform slab, which are shown as dashed lines in Figs. 26(b,c). The uniform slab has the same thickness as the photonic crystal. Resonances can be described by employing the Fano-type formulas, with the effective dielectric constant as the only fitting parameter. The fitting agrees very well with the numerical simulations (see also Koshino, 2003).

By introducing a nonlinear layer into the slab with a periodic lateral structure, we can generate a bistable transmission for significant intensity ranges due to Fano resonances, and achieve a strong frequency-dependent transparency variation related to the transfer via guided
modes. A self-consistent simulation tool which allows for the computation of multivalued transmission has been developed by Lousse and Vigneron (2004). It explained the peculiar shape of the hysteresis loops associated with nonlinear Fano resonances.

Complex resonant line shapes due to Fano resonances were observed experimentally in several settings (Chen et al., 2009; Grillet et al., 2006; Harbers et al., 2007; Qiang et al., 2008; Yang et al., 2008). In particular, Grillet et al. (2006) observed Fano resonances in the optical transmission spectrum of a chalcogenide glass photonic crystal membrane and demonstrated, for the first time, the suppression of optical transmission by over 40 dB, the strongest reported so far, and a remarkable result for a dielectric structure with a thickness of only 330 nm. These results will allow further progress towards the engineering of very sharp resonances and, combined with the large intrinsic nonlinearity of the chalcogenide glasses, should allow for the observation of optical bistability in a photonic-crystal mirror.

Recently, it was experimentally demonstrated that the shape of the Fano resonance in the light scattering by a high-Q planar photonic crystal nanocavity can be controlled by varying the waste of the Gaussian beam (Galli et al., 2009). For a tightly focused beam with a spot diameter $d_1 \approx 2 \mu m$ a strong asymmetric Fano resonance was observed with the asymmetry parameter $q_1 = -0.348$ [see Fig. 27(a)]. On the other hand, for a slightly defocused Gaussian beam with the spot diameter $d_2 \approx 10 \mu m$ a symmetric Fano resonance was observed with $q_2 = -0.016$ [see Fig. 27(b)]. In this geometry the light reflected from the nanocavity mimics the scattering through a discrete level, while the light reflected from the photonic crystal pattern, can be considered as the scattering to the continuum. The interference of these two reflected components leads to the Fano resonance. The variation of the Fano profile with the increase of the excitation area can be understood as an enhancement of the scattering to the continuum, leading to the decrease of the asymmetry parameter $q$. Indeed, the variation of the asymmetry parameter $q_1/q_2 \sim 22$ is proportional to the variation of the excitation areas $(d_2/d_1)^2 \sim 25$. Thus, by changing the excitation conditions it is possible to tune the Fano resonance in the scattering by photonic crystal nanocavity.

H. Light scattering by spherical nanoparticles

Light scattering by an obstacle is one of the fundamental problems of electrodynamics, see, e.g., monographs Bohren and Huffman, 1998; Born and Wolf, 1999; van der Hulst, 1981. It was first described by Lord Rayleigh and is characterized by a sharp increase
in scattering intensity with increasing the light frequency \( \frac{\omega}{\omega_p} \). It is used to explain why we can enjoy the blue sky during day time (the intensely scattered blue component of the sunlight), and scarlet sunrises and sunsets at dawn and dusk (the weakly scattered red component). Lord Rayleigh’s studies were generalized by Gustav Mie who obtained the complete analytical solution of Maxwell’s equations for the scattering of electromagnetic radiation by a spherical particle valid for any ratio of diameter to the wavelength (Mie, 1908).

A common assumption is that the general Mie solution transforms into that of Rayleigh when particles are small. However, recent studies of resonant scattering by small particles with weak dissipation rates (Basheov et al., 2005; Tribelsky and Luk’yanchuk, 2006) have revealed new and unexpected features, namely giant optical resonances with an inverse hierarchy (the quadrupole resonance is much stronger than the dipole one, etc.), a complicated near-field structure with vortices, unusual frequency and size dependencies, which allow to name such a scattering anomalous. Tribelsky et al. (2008) revealed that the physical picture of this anomalous scattering is analogous to the physics of Fano resonances. This analogy sheds new light to the phenomenon. It allows to employ powerful methods developed in the theory of the Fano resonances (such as, e.g., the Feshbach-Fano partitioning theory) to describe the resonant light scattering. It also easily explains certain features of the anomalous scattering and related problems, namely sharp changes in the scattering diagrams upon small changes in \( \omega \) (see Fig. 28). Tribelsky et al. (2008) analytically obtained an asymmetric profile of the resonance lines by analyzing the exact Mie solution of the light scattering problem by a spherical nanoparticle (Miroshnichenko et al., 2008).

Figure 28 demonstrates light scattering by a potassium colloidal nanoparticle immersed in a KCl crystal, calculated with a realistic dependence \( \epsilon(\omega) \) and fitting actual experimental data (Luk’yanchuk et al., 2008; Tribelsky et al., 2008). A slight variation of the incident light frequency in the vicinity of the quadrupole resonance drastically changes the scattering pattern (see Fig. 28), resulting in asymmetric Fano-like profiles for intensities of the forward and backward scattered light. In this case, excited localized plasmons (polaritons) are equivalent to the discrete levels in Fano’s approach, while the radiative decay of these excitations is similar to the tunneling to the continuum. In general, it may lead to a significant suppression of the scattering along any given direction. Note, that in accordance with the theoretical expression obtained from the Mie formula, the points of destructive interference for the forward and backward scattering lie on different sides of the corresponding resonant peaks.

I. Plasmonic nanocavities and tunable Fano resonance

Recent progress in the fabrication and visualization of nano-sized structures gave rise to a novel and rapidly emerging field of nanoplasmonics. The optical properties of metals are governed by coherent oscillations of conduction-band electrons, known as plasmons (Bohm and Pines, 1951). The interaction between light and metallic nanoparticles is mostly dominated by charge-density oscillations on the closed surfaces of the particles, called localized surface plasmon resonances (LSPs). The studies of LSPs in noble-metal nanoparticles, such as gold and silver, extended applications from various surface-enhanced spectroscopies (Moskovits, 1985) to novel nanometer optical devices and waveguides (Barnes et al., 2003; Ozbay, 2006). One of the most important properties of LSPs is the possibility of strong spatial localization of the electron oscillations, combined with their high frequencies varying from UV to IR ranges. LSPs have the ability to strongly scatter, absorb, and squeeze light on nanometer scales, producing huge enhancement of electromagnetic field amplitudes. Such unique properties of nanomate-
materials are essential for the development of novel material functions with potential technological and medical applications with specific optical, magnetic, and reactivity properties.

Plasmonic nanostructures can be considered as a physical realization of coupled oscillator systems at the nanoscale. The energies and linewidths of the LSPs depend mostly on the nanoparticle geometries, such as size and shape. Thus, the spectral tunability of LSPs has been widely investigated. As it was suggested by Hao et al. (2008), promising geometries for fine tuning are rings and disks. In such structures the dipole-like resonance can be tuned into the near-infrared region by changing the width of the metallic ring, for example. One of the important issues of nanoplasmonics is the effect of symmetry breaking, which allows to excite higher-order multipolar modes leading to larger electromagnetic field enhancements. The symmetry breaking can be easily achieved in metallic ring/disk cavity structures by displacing the disk with respect to the center of the ring. The plasmon resonances of a ring/disk cavity system can be understood in terms of the interaction or hybridization of the single ring and disk cavity plasmons. This hybridization leads to a low energy symmetric plasmon and high energy anti-symmetric plasmon (Hao et al. 2007). The latter one is superradiant, i.e. it strongly radiates because disk and ring dipolar plasmons are aligned and oscillate in phase. The low energy symmetric plasmon is subradiant because of opposite alignment of dipolar moments. It turns out that in a symmetry-broken structure, the quadrupole ring resonance couples to the superradiant high energy anti-symmetric disk-ring dipole mode (Hao et al. 2008). The direct coupling interferes with the dispersive coupling between the quadrupolar ring mode and the superradiant mode, resulting in a Fano resonance in the extinction spectrum (see Fig. 29). By varying the incident angle, the shape of the Fano resonance can be altered from asymmetric to a symmetric one.

Other examples of nanoplasmonic structures supporting the asymmetric Fano resonance are metallic nanoshells near a metallic film (Le et al. 2007), and heterogeneous dimers composed of gold and silver nanoparticles (Bachelier et al. 2008). Both structures show up with a highly tunable plasmonic Fano resonance, accompanied by large local electric field enhancement. Thus, the strong response of LSP resonances may be effectively used for biological and medical sensing applications.

A novel type of nonlinear Fano resonance has been found in hybrid molecules composed of semiconductor and metal nanoparticles (Zhang et al. 2006). The latter ones support surface plasmons with a continuous spectrum, while the former ones support discrete interband excitations. Plasmons and excitons become strongly coupled via Förster energy transfer. At high light intensities, the absorption spectrum demonstrates a sharp asymmetric profile, which originates from the coherent interparticle Coulomb interaction, and can be understood in terms of a nonlinear Fano resonance.

J. Extraordinary transmission of light through metallic gratings

The scattering by metallic gratings was the subject of extensive research for over a century. One of the important early achievements of the optics of metallic gratings was the discovery and understanding of Wood’s anomalities (Rayleigh, 1907; Wood, 1902, 1935). One type of anomaly is due to the excitation of surface plasmon-polaritons propagating on the metallic surface. Another one is the diffractive anomaly, when a diffracted order becomes tangential to the plane of the grating. It is characterized by a rapid variation of the diffracted order intensity, corresponding to the onset or disappearance of a particular spectral order (Wood, 1935). This resonant behaviour of the Wood’s anomaly can be understood in terms of the coupling of the incoming waves with the surface-bound states of periodic arrays (Fano, 1936, 1937, 1938, 1941; Hessel and Oliner, 1965). Thus, by considering a surface-bound state as a discrete level and scattered waves as a continuum, Wood’s anomaly can be interpreted as a Fano resonance (Billaudeau et al. 2009).

It was demonstrated that for a periodic thin-film metallic grating, formed from a two-dimensional array of holes, the transmitted fraction of the incident light can exceed the open fraction of the array for certain wavelengths (Ebbesen et al. 1998; Ghaemi et al. 1998). The enhancement in the transmitted zero-order beam is reported to be several orders of magnitude larger than that from pure metallic slab without holes. This phenomenon has been called extraordinary transmission through periodic arrays of subwavelengths holes in metallic films.

The common understanding of the extraordinary transmission is due to a resonant excitation...
of surface plasmon-polaritons by incoming radiation (Ghaemi et al., 1998; van der Molen et al., 2005). In addition to the resonant enhancement of the transmission the resonant suppression was observed as well. It was demonstrated that these transmission minima correspond exactly to loci of Wood’s anomaly (see Fig. 30) (Ghaemi et al., 1998). According to experimental observations, each extraordinary transmission is accompanied by resonant suppression of transmission resulting in asymmetric lineshapes, which can be perfectly fitted by the Fano formula (de Abajo, 2007). Moreover, it was theoretically demonstrated by Spevak et al. (2009) that periodically modulated ultrathin metal films may exhibit resonant suppression of the transmittance, emphasizing the Wood’s anomaly effect. Thus, the extraordinary resonant scattering of light by modulated metal film can be described in terms of the Fano resonance, revealing the interference nature of the phenomenon.

Kobyakov et al. (2009) suggested to use active layers to simultaneously enhance both transmittance and reflectance at the resonance in subwavelength periodic planar bimetallic grating by exciting gain-assisted surface plasmons.

K. Resonant four-wave mixing induced autoionization

Four-wave mixing involves the interaction of three laser beams to produce a nonlinear polarization via the cubic electric susceptibility $\chi^{(3)}$. The induced polarization acts as the source of a fourth coherent light beam, detected as the signal. Four-wave mixing can be considered as the formation and scattering from laser-induced gratings.

The grating is formed by two laser beams, called grating beams, with wavevector $k_g$. The third probe beam with the wavevector $k_p$ is then scattered off the laser-induced grating and produces the fourth scattered beam, which is detected as the four-wave mixing signal. Due to energy conservation, the frequency of the signal beam must be equal to the frequency of the probe beam $\omega_s = \omega_p$. Momentum conservation results in a phase-matching condition for the signal wavevector

$$|k_s| = |k_{g1} - k_{g2} + k_p| = \frac{\omega_p}{c},$$

and the Bragg-scattering angular condition

$$\frac{\omega_p}{\omega_g} = \sin\left(\frac{\theta_g}{2}\right) \sin\left(\frac{\theta_{p-s}}{2}\right),$$

where $\theta_g$ is the angle between two grating beams, and $\theta_{p-s}$ is the angle between the probe and signal beams (see Fig. 31).

In general, the four-wave mixing process can take place in any material. When the frequency of the incident laser beams matches the transition resonances of the medium, a drastic enhancement of the signal intensity can be observed. Such processes are called resonant four-wave mixings (RFWMs), and they are used as spectroscopic and diagnostic tools for probing stable and transient molecular species. Armstrong and Wynne (1974) studied experimentally four-wave mixing involving an autoionizing resonance in alkali-metal atomic vapor. In their experiment a two-photon transition between two bound states of the metal was excited, followed by a single-photon absorption to the autoionizing level. The detected signal demonstrated a characteristic asymmetric response. Using the Fano formalism, the authors derived an expression for the line-shape and fitted it with the Fano formula (Armstrong and Wynne, 1974), which allows to obtain the width and asymmetry parameter for the autoionizing states (Agarwal and Lakshmi, 1983; Alber and Zoller, 1983; Armstrong and Beers, 1975; Crance and Armstrong, 1982a,b; Haan and Agarwal, 1987; Meier et al., 1995). Thus, this form of RFWM...
can be considered as one of the techniques to study autoionizing levels.

A double resonance version of RFWM is called two-color RFWM (TC-RFWM) and takes place, when two optical fields have frequencies in resonance with two different transitions. It yields a variety of excitation schemes, which are very useful for high-resolution spectroscopy. In Figure 32 possible TC-RFWM excitation schemes are shown, where the grating beams are in resonance with the lower transition and the probe is tuned to the upper transition [see Fig. 32(a)], and vice-versa [see Fig. 32(b)]. Because of the presence of autoionizing states in overall the FWM process, in both cases TC-RFWM exhibits asymmetric profiles, which can be approximated by the Fano formula [see Fig. 32(c)]. Unlike the Fano profile, the TC-RFWM spectral lines have no exact zeroes. This can be explained using the dephasing during nonlinear parametric conversions, which is a key difference to the usual Fano resonance case. Nevertheless, TC-RFWM provides with an efficient way to coherently control the signal lineshape (McCormack et al., 1998).

VI. CHARGE TRANSPORT THROUGH QUANTUM DOTS

In recent decades charge transport through quantum dots (QD) has been extensively studied both theoretically and experimentally (Altshuler et al., 1991; Hanson et al., 2007; Kastner, 1992; Koch and Lübbing, 1992; Reimann and Manninen, 2002). One of the reasons of that interest is the further miniaturization of electronic device components. A comprehensive picture of a big variety of underlying physical phenomena has emerged (see e.g. Aleiner et al. (2002); Alhassid (2000) and references therein). The finite size of the dot is responsible for a dense but discrete set of single particle levels. Confinement of electrons in small quantum dots leads to the necessity of taking into account their Coulomb repulsion. As a result, at temperatures below the charging energy the Coulomb blockade emerges (Aleiner et al., 2002; Alhassid, 2000). At even lower temperatures the phase coherence of the excitations in the quantum dot is preserved during the scattering, and additional interference phenomena appear, depending on the coupling strength to the leads. In view of the enormous literature available, we will briefly introduce the main physics, and focus on results which are directly related to the finding of destructive interferences and Fano resonances.
A. From a single electron transistor to quantum interference

A quantum dot is a small confinement region for electrons (typically almost two-dimensional) with leads coupled to it. The manufacturing of a huge variety of geometries is easily possible. In the simplest case two leads are used, and a voltage \( V \) is applied, resulting in a current of electrons which enter the dot through one lead, and eventually exit into the second lead. Various gate voltages can be additionally applied, e.g. \( V_g \) which controls the energy of the electrons in the dot relative to the leads, and others which control the strength of the coupling between the leads and the dot. Here we will consider only situations where the applied voltage \( V \) between the two leads is small so that the energy \( eV \) is smaller than all other relevant energy scales. This is also called the equilibrium case, at variance to the non-equilibrium case which is also frequently studied.

Let us consider a closed dot with linear size \( L \), when the leads are decoupled. If one neglects the contribution from Coulomb interaction, the spectrum of many body states in a quantum dot can be obtained from the solution of the single particle problem. The single particle level spacing \( \Delta_{sp} = \pi \hbar^2/m^* L^2 \) (Alhassid, 2000). The effective mass of an electron in GaAs is rather low: \( m^* = 0.067 m_e \) (Alhassid, 2000). For \( L = 100 \text{nm} \) one obtains \( \Delta_{sp} \approx 2 K \), while for \( L = 500 \text{nm} \) the spacing is reduced to \( \Delta_{sp} \approx 90 mK \). Adding one electron to the closed dot therefore leads to an energy increase of the order of \( \Delta_{sp} \). Now take Coulomb interaction into account. If the number of electrons in the dot is \( N \), then the charging energy of adding one additional electron is \( E_c \sim N e^2/L \). Therefore, for large values of \( N \), and not too small values of \( L \), \( E_c \gg \Delta_{sp} \). Note that typical dot sizes are of the order \( 100 \text{nm} - 1 \mu \text{m} \). \( N \) can strongly vary, with values \( N \sim 10^2 - 10^3 \). Characteristic values of the charging energy are in the range \( E_c \sim 100 - 400 K \) (12 - 50meV). Therefore, for all practical purposes, \( E_c \gg \Delta_{sp} \).

The number of electrons in a quantum dot is defined by minimizing the energy of the dot with respect to \( N \). This energy is given by (Alhassid, 2000)

\[
E(N) = -NeV_g + N^2 e^2/2C , \tag{47}
\]

where \( C \) is the total capacitance between the dot and its surroundings. Apart from special values of the gate voltage, there will be a given electron number \( N \) with smallest energy, and changing the number of electrons will cost an amount about one charging energy \( E_c \). For particular values of the gate voltage \( V_g(n) \) however degeneracies between \( E(N) \) and \( E(N+1) \) appear.

Consider an experimental geometry shown in Fig.33. If the coupling to the leads is weak enough and the temperature \( kT < E_c \) the Coulomb blockade regime sets in. As long as \( V_g \neq V_g(n) \) the charging energy prevents lead electrons from entering the dot, and the conductance \( G \) is practically zero. However, when \( V_g = V_g(n) \), the degeneracy between \( N \) and \( N + 1 \) electron states on the dot sets in. Therefore, electrons can pass through the dot one by one, and the conductance takes the universal value \( G = 2e^2/h \) (here the factor 2 accounts for spin degeneracy). Note that the Coulomb interaction is treated in a mean-field type way, therefore no phase coherence of dot electrons is required.

Lowering the temperature further, the phase coherence of the dot electrons becomes essential [see e.g. Aikawa et al., 2004; Li et al., 2000]. Note that the typical electron mean free path can be of the order of \( 10 \mu \text{m} \), one-two orders of magnitude larger than the dot size. It may also be possible to reduce decoherence effects within some suitable range by increasing the coupling of the dot to the leads, which may lead to a shorter residence time of electrons inside the dot and therefore to less scattering. With the option of having several channels which electrons use to pass through the dot, phase coherence

FIG. 34 Conductance versus gate voltage. Comparison of conductance measurements in the (a) Fano regime, (b) intermediate regime, and (c) Coulomb blockade regime. From (c) to (a) the lead-dot coupling increases. Fits to the Fano formula (11) are shown for the center and right resonances in (a). The respective asymmetry parameters are \( q = 0.03 \) and \( q = -0.99 \). From Gores et al. (2000).
will lead to interference effects, and therefore to possible Fano resonances.

If a magnetic field is added, orbital and spin effects have to be considered as well. The Zeeman energy $E_Z = g \mu_B H$ sets another temperature scale. Depending on the Lande factor $g$, which can vary strongly from sample to sample, the corresponding Zeeman energy $E_Z$ is of the order of $100-200 mK$ for $B = 1T$. Allowing the electrons to traverse the dot along different paths, an Aharonov-Bohm phase shift $\phi$ occurs due to a nonzero magnetic flux penetrating the area $S$ enclosed by them $[\text{Altshuler et al. 1980}]: \phi = \frac{\pi}{2} B S$. With $S = L^2$ we find for $L = 100 nm$ that $\phi/2\pi = 0.38 B/T$, and for $L = 1 \mu m$ that $\phi/2\pi = 38 B/T$.

Therefore, for $L = 100 nm$ and $B = 1 T$ it follows $E_Z \ll \Delta_{sp}$. Then at low temperatures $kT < E_Z$ of the order of $T \sim 50-100 mK$ and at a magnetic field $B \sim 1T$ the Coulomb blockaded dot has a well defined spin: either $|S_z| = 1/2$, or $S_z = 0$. Changing the gate voltage and reaching the next degeneracy $E(N) = E(N + 1)$, an electron with a well-defined spin is allowed to enter the dot - either spin up or spin down. The allowed spin value alternates as one further tunes the gate voltage to the next degeneracy. If the phase coherence of electrons is preserved during the scattering, one may again expect interference phenomena - but this time, depending on the chosen value of $V_g$, only electrons with spin up (respectively spin down) will interfere along different channels. Increasing the coupling to the leads may cause spin-selective destructive interference for a given spin species, while the other spin species is freely passing through. The orbital effect of the magnetic field leads to an additional phase shift of the order of $0.8\pi$, independent of applied gate voltages.

For $L = 1 \mu m$ the single particle spacing $\Delta_{sp} \approx 20 mK$. Therefore at $B = 1 T$ it follows $E_Z \gg \Delta_{sp}$. Then at temperatures $T \sim 50-100 mK$ the Coulomb blockaded dot is magnetized, but electrons which enter the dot can have any spin, preventing spin-selective destructive interference. The orbital effect of the magnetic field is huge with a $2\pi$ phase shift every $25 mT$ upon changing the magnetic field.

Before proceeding, let us briefly mention related studies of the Kondo effect in transport through quantum dots. In the Coulomb blockade, the number of electrons on the dot is well defined, and either even or odd. Assuming a ground state only, the total electronic spin is either $1/2$ (odd number of electrons) or zero (even number). In the absence of a magnetic field and for odd numbers of electrons, the whole dot could be viewed as some magnetic impurity with spin $1/2$, which scatters conduction electrons passing from one lead to another. That calls for an analogy with the well known Kondo effect which is observed in the low temperature properties of the conductivity of electrons in metals with magnetic impurities $[\text{Hewson, 1993}]$. The resistivity in metals usually drops with lowering the temperature, since the number of phonons, which are responsible for electron scattering due to electron-phonon interaction, decreases. At around $30 K$ a minimum in the resistivity appears for some metals, and subsequently the resistivity increases again with further lowering the temperature. This increase is due to scattering of electrons by magnetic impurities, and originates from an exchange interaction of the conductance electron spin with the spin of the magnetic impurity. The exchange interaction sets an energy and temperature scale (the Kondo temperature $T_K$), which is typically of the order of $T_K \sim 100 mK - 1 K$, similar to the Zeeman energy of an electronic spin $1/2$ in a magnetic field of $1$ Tesla. For temperatures $T < T_K$ the impurity spin is screened by a cloud of renormalized conduction electrons. The Kondo temperature depends sensitively on the coupling strength (hybridization) $\Gamma$ between the conduction electrons and the magnetic impurities. For weak coupling $T_K$ is exponentially small in $-1/\Gamma$. This analogy stirred ideas to observe the Kondo effect in the conductance of electrons through quantum dots. For that low temperatures have to be used, and the coupling of the leads to the dot has to be increased (in order to increase $T_K$). An enormous amount of theoretical studies was performed $[\text{Aleiner et al. 2002}]$. Experimental results showed a deviation from the Coulomb blockade regime for strong lead-dot coupling (see below). The relation to theoretical models based on Kondo mechanisms is still debated $[\text{Aleiner et al. 2002, Ji et al. 2000}]$. 

![FIG. 35 Conductance versus gate voltage. (a) Temperature dependence of the conductance for two Fano resonances. (b) Conductance as a function of the gate voltage for various magnetic fields applied perpendicular to the two-dimensional electron gas. Adapted from Göres et al. (2000).](Image 354x498 to 525x740)
B. From Coulomb blockade to Fano resonances

A number of experimental studies report on the observation of Coulomb blockade in various quantum dot realizations on the basis of AlGaAs heterostructures \cite{1998Goldhaber-Gordon, 1998etCronenwett, 2001etCronenwett, 1998etGores, 1998etKobayashi, 2002etSchmid}. The charging energies are in the range \( E_c \approx 100 - 300 \text{K} \). Temperatures were as low as 30\( mK \), applied magnetic fields up to 1T, and higher. Therefore, the Zeeman energy \( E_Z \) is 2-3 orders of magnitude lower than the charging energy \( E_c \). The Coulomb blockade regime is usually observed in the case of weak coupling between the leads and the dot. In Fig. 34 the results of \cite{2000etGores} are shown, which correspond to the setup in Fig. 33. For weak lead-dot coupling (c) the Coulomb blockade regime is nicely observed (temperatures are around 100\( mK \), and the drain-source voltage \( V_D \approx 5 \mu V \ll V_g \)). With increasing coupling the sharp peak structure is smeared out (b), which has been discussed in relation to the Kondo effect. Even further increasing the coupling, Fano resonances are observed in the strong coupling case (a). A fitting yields asymmetry parameters \( q = -0.03 \) and \( q = -0.99 \) for the center and right resonances, respectively. Note that also the peaks in (c) separating Coulomb blockades with different numbers of electrons on the dot, are clearly asymmetric. The same authors studied the temperature and weak magnetic field dependence of the Fano profiles in the strong coupling regime for even larger absolute values of the gate voltage, shown in Fig. 35.

The fitting of the resonances in Fig. 35(a) yields an almost linear decrease of the linewidth \( \Gamma \) with temperature, reaching values of 2\( meV \) at 100\( mK \). The depth of the Fano resonance increases with decreasing temperature, making the Fano resonance sharper and deeper at low temperatures. The Fano resonances show very strong dependence on the value of a weak applied magnetic field Fig. 35(b). Note that the largest applied fields are at 50\( mT \), which corresponds to a Zeeman energy of the order of 10\( mK \) or less.

The origin of the observed Fano resonances is interferences of electrons along several channels (paths) traversing the quantum dot. When the lead-dot coupling is weak, the background conductance is very small [see Fig. 35(c)]. Still an asymmetric line shape is observed. The Fano resonance (dip) may either be hard to be detected with that background, or simply be absent, since essentially only one path is active. Another possibility is that the antiresonance is extremely narrow (weak coupling to a dot state). Since the Fano resonances are well observed at large lead-dot coupling, phase coherence of electrons passing through the dot is therefore established, and is further increased with lowering the temperature.

The dramatic change of the resonance shape at weak magnetic fields is attributed to a suppression of the coupling into the dot states \cite{2000etGores}. That leads to an enhancement of the asymmetry parameter \( q \), and respectively to a shifting of the Fano resonance (dip) out of the window of available gate voltages. The alternative explanation of losing phase coherence of traversing electrons does not account for the extremely low-field scale at which the change occurs \cite{2000etGores}. In a similar way one can exclude orbital Aharonov-Bohm effects, since the expected phase shifts are of the order of \( \phi \lesssim 0.12 \).

C. From Fano to Aharonov-Bohm interferometers

In the above described experiments, the quantum dot design allowed essentially only to control the lead-dot coupling. To further advance in the tunability of Fano resonances with quantum dots, interferometer devices have been manufactured. In addition to a small quantum dot, which can be traversed by electrons, a second region (second dot, or additional channel, or additional arm) is coupled in a controlled way. Therefore, the coupling to a second channel can be tuned systematically. Of course there may be already several channels involved in the traversing of electrons through the primary dot.

Impressive results have been obtained by \cite{2004etJohnson} in designing a tunable Fano interferometer which consists of a quantum dot and an additional tunnel-coupled channel (see Fig. 36). A sequence of several Fano resonances was observed, and well fitted with the Fano formula \cite{10}. Moreover, \cite{2004etJohnson} performed careful fittings of various resonance shapes as shown in Fig. 37. In panel (a) another set of resonances is observed. Upon variation of the gate voltage the asymmetry of the resonance shape clearly changes, as also seen in panel (e). In addition, also the line width \( \Gamma \) is changing (panel (d)). In another gate voltage window [panel (f)] these changes are even more
drastic. Indeed, the fit yields a change of the sign of $q$ with $V_g$ (panel (g)). Note, that according to \[1\] at $q = 0$ a symmetric resonant reflection, with no resonant transmission, is predicted. Indeed, around the value $V_g \approx -1900mV$ the conductance in panel (f) shows practically a dip only.

Yet another twist was taken by \[Kobayashi et al (2002)\] with a qualitatively similar geometry but an additional magnetic field penetrating the interferometer area and turning it into an Aharonov-Bohm (AB) device (see Fig.38). The current through the quantum dot and the additional arm (channel) can be controlled independently. Magnetic fields were around 1 Tesla. With the arm switched off, a series of Coulomb blockade peaks is observed (see Fig.39).

When making the arm transmittable, clear interference effects are observed through asymmetric Fano line shapes (see Fig.39). In that system, the discrete level and the continuum are spatially separated, allowing to control Fano interference via the magnetic field piercing the ring as shown in Fig.10. The line shape changes periodically with the AB period $\sim 3.8mT$, which agrees with the expected value using the ring dimension \[Kobayashi et al (2002)\]. As magnetic field $B$ is swept, an asymmetric line shape with negative $q$ continuously changes to a symmetric one and then to an asymmetric one with positive $q$. \[Kobayashi et al (2002)\] argue, that due to the breaking of time reversal symmetry in the presence of a magnetic field, the matrix elements defining $q$ are not real as usually assumed, but complex, therefore leading to complex $q$ values. This confirms theoretical investigations for the noninteracting single particle AB interferometer case \[Aharony et al, 2002, 2003; Entin-Wohlman et al, 2002a,b; Sasada and Hatano, 2005\].

D. Correlations

An enormous bulk of theoretical literature on various facets of the conductance properties of quantum dots is available. We will discuss some of these results below. Let us remind the reader about some characteristic scales. The Coulomb energy (charging energy) of quantum dots is of the order of $50meV$ ($380K$). The Kondo temperature in a typical metal with magnetic impurities is of the order of $10\mu eV$ ($100mK$), comparable to the Zeeman energy of a spin 1/2 electron in a magnetic field of around 1 Tesla. Therefore, when operating at temperatures of the order of the Zeeman energy, the charge on a typical quantum dot is extremely well fixed by the number of electrons. The next question is whether a con-
ductance electron, when penetrating the quantum dot, is able to efficiently interact with an excess spin $1/2$ particle for odd electron numbers, or whether it will usually follow a path which avoids strong exchange interaction. These, partly open, issues make it sometimes hard to judge the relevance of many interesting theories.

The simplest model, which keeps the effect of Coulomb interactions and correlations, uses exactly one level from the quantum dot, adds links to leads (left and right), and takes Coulomb interaction of spin up and spin down electrons into account - but only on the dot (see Fig. 41A). The resulting Hamiltonian has the following form:

$$ H_s = H_D + H_W, \quad H_D = \epsilon_d \sum_{\sigma} n_{\sigma} + U n_{\uparrow} n_{\downarrow}, \quad (48) $$

$$ H_W = \sum_{k \sigma r} \epsilon_{k \sigma r} c_{k \sigma r}^\dagger c_{k \sigma r} + (V_c c_{k \sigma r}^\dagger d_{\sigma} + H.c.). \quad (49) $$

Here $n_{\sigma} = d_{\sigma}^\dagger d_{\sigma}$ measures the number of electrons on the quantum dot level, which interact with each other with strength $U$. The left and right leads are denoted by $r = L(R)$. The level energy $\epsilon_d$ is measured from the Fermi energy of the leads. The lead states are chosen in the momentum representation. All fermionic creation and annihilation operators $c, c^\dagger, d, d^\dagger$ obey the standard anticommutation relations.

E. Interference

There are many ways to incorporate interference and multiple paths, in order to reach Fano resonances. One of the simplest ones is a T-shaped scheme, which is a small change of the above model by side-coupling the quantum dot to the quantum wire (leads) (see Fig. 41B):

$$ H_T = -t \sum_{n,\sigma} (c_{n,\sigma}^\dagger c_{n+1,\sigma} + c_{n+1,\sigma}^\dagger c_{n,\sigma}) + \sum_{\sigma} \epsilon_d n_{\sigma} + \sum_{\sigma} (V d_{\sigma}^\dagger c_{0,\sigma} + V^* c_{0,\sigma}^\dagger d_{\sigma}) + Un_{\uparrow} n_{\downarrow}. \quad (50) $$

The lead states are chosen in the coordinate representation. Interference is possible because electrons can directly pass from the left to the right, but can also visit the side dot and exit again. These two paths are enough for destructive interference.

Another possibility is to extend the serial dot scheme (48)-(49) by adding a direct path (arm) for electrons to transit from the left to the right leads (Hofstetter et al., 2001):

$$ H_{AB} = H_s + H_a, \quad H_a = \sum_{k \sigma r} We^{i \phi} c_{k \sigma r}^\dagger c_{k \sigma r} + H.c. \quad (51) $$

The phase $\phi$ models a magnetic flux which is encompassed by the loop of the direct path and the path via the quantum dot.

The Hamiltonians (48)-(49),(50) belong to the class of Anderson Hamiltonians (Anderson, 1961). Thus the thermodynamic properties of both models are similar, e.g. the average number of (spin up and spin down) electrons on the dot $\langle n_{\sigma} \rangle$. However, the transport properties depend crucially on the chosen geometry (Luo et al., 2004). Note that changing the dot level $\epsilon_d$ is qualitatively similar to varying the gate voltage of a quantum dot. The dot level is capable of accepting at most one spin up and one spin down electron.

Wiegmann and Tsvelick (1983) obtained analytical results for $\langle n_{\sigma} \rangle$ assuming a linearized spectrum of lead electrons, which is not a crucial constraint, as long as the lead electron bands are partially filled (ideally half-filling), and as long as the temperature is much smaller than the distance from the Fermi energy to the band edge. In addition, there exist various numerical methods to compute $\langle n_{\sigma} \rangle$ approximately.

With standard scattering matrix approaches, as well as using the Friedel sum rule (Hewson, 1993; Langreth, 1966), the conductance of the serial dot scheme (48)-(49) at zero temperature can be expressed in the following way (Glazman and Raikh, 1988; Ng and Lee, 1988):

$$ g_\sigma = \left( \frac{2V_L V_R}{V_L^2 + V_R^2} \right)^2 \sin^2 \pi \langle n_{\sigma} \rangle. \quad (52) $$

Hofstetter et al. (2001) studied Fano resonances in transport through the the AB interferometer model (51).
at zero temperature. The schematic view of the AB interferometer is similar to Fig. 38(a). For zero AB phase \( \phi = 0 \), and the direct path being switched off \( W = 0 \), there are three states of a Coulomb blockade to be expected upon variation of the gate voltage \( \epsilon_d \): the dot contains either zero, one, or two electrons, with sharp transitions between them. We remind again, that the empty dot is almost not conducting (Coulomb energy too large), and the dot filled with two electrons as well (Pauli principle). When there is one electron on the dot, a second can enter while the first leaves. Despite of applying a magnetic field, model \( 51 \) is invariant under spin reversal (because the bare dot levels are not Zeeman splitted). This may be not easy to be achieved in an experiment. Therefore, when there is one electron on the dot, it can have either spin up or spin down, and on average \( \langle n_\sigma \rangle = 1/2 \) in that case. For \( \epsilon_d > 0 \) (Fermi energy is placed at zero) the dot is empty, and the conductance is zero. When \( -U < \epsilon_d < 0 \), one electron can enter the dot, but not two. Then additional electrons can tunnel through, giving maximal conductance. Finally, for \( \epsilon_d < -U \), two electrons occupy the dot, and the conductance is zero again. This broad region of almost perfect conductance is due to spin exchange processes on the quantum dot level, and can be therefore related to the discussed above Kondo effect. Indeed, in Fig. 42 this is observed for \( T_b = 0 \) with \( T_b = 4x/(1+x)^2 \) being the background transmission probability, where \( x = \pi^2 W^2 N_L N_R \), and \( N_{L,R} \) is the density of states in the left (right) lead. With increasing \( T_b \) the curves change dramatically. Most importantly, a Fano resonance is appearing in the studied energy window, qualitatively similar to experimental observations (Kobayashi et al. 2002). For the considered model the resonance location is shifting towards \( -U/2 \), and its width tends to \( -U \), as \( T_b \) further increases Fig. 42. A variation of the AB phase \( \phi \) in some intermediate \( T_b \) regime yields the possibility to change the sign of the asymmetry parameter \( q \).

F. Spin filters

When a magnetic field is applied to the AB interferometer setup in Fig. 38(a), it is reasonable to consider also its action on the quantum dot region itself, which leads to a Zeeman splitting of the dot level. This is incorporated in the side dot model \( 50 \) with specifying

\[
\epsilon_{d, \uparrow} = \epsilon_d + \Delta/2 , \quad \epsilon_{d, \downarrow} = \epsilon_d - \Delta/2 ,
\]

where \( \Delta \) is the Zeeman energy up to which the single particle level is splitted for spin down and spin up electrons. It is easy to incorporate the AB phase shift as well, we will discuss it below.

For \( U = 0 \) \( 51 \) is reduced to the Fano-Anderson model \( 9 \), and the transmission is computed within the one-particle picture for an electron moving at the Fermi energy \( \epsilon_F \):

\[
- \epsilon_F \phi_i = t(\phi_{n-1} + \phi_{n+1}) + V^* \varphi \delta_{n0} ,
\]

\[
- \epsilon_F \varphi = - \epsilon_{d,\sigma} \varphi + V \phi_0 ,
\]

where \( \phi_n \) refers to the amplitude of a single particle at site \( n \) in the conducting channel and \( \varphi \) is the amplitude at the side dot. With the help of the Friedel sum rule (Hewson 1993, Langreth 1966) one arrives at (Torio et al. 2004)

\[
g_\sigma = \cos^2 \pi \langle n_\sigma \rangle .
\]

This relation has a geometric origin and actually holds for arbitrary \( U \) (at zero temperatures). For a nonzero magnetic field \( \Delta \gg \Gamma \) the two Fano resonances for spin up and spin down electrons are energetically separated. Therefore, the current through the channel is completely polarized at \( \epsilon_F = \epsilon_{d, \uparrow} \) and \( \epsilon_F = \epsilon_{d, \downarrow} \). The AB phase can be easily included into the model \( 50 \) similar to \( 51 \). Remarkably it will not change the position of the resonances (cf. also \( 50 \)), since the position of the Fano resonance is entirely determined by the matching condition between the dot level(s) and the Fermi energy.

The obtained spin filter will operate at temperatures \( kT \ll \Delta \). For a field of a few Tesla that implies temperatures less than \( 100 \) \( \mu K \). While that is possible in principle, two more problems appear. First, to control such a spin filter, one would have to control the gate voltage on the scale of \( \mu eV \) (because the spin polarized Fano resonances are separated in the gate voltage by the same amount of the Zeeman energy). Second, as discussed above, Coulomb interactions have to be taken into account.

For nonzero \( U \) and \( \Delta \), the results for the mean number of particles on the dot, and for the spin-resolved conductance, have been obtained by (Torio et al. 2004), and are shown in Fig. 43. The main outcome is, that the presence of a strong Coulomb interaction is shifting the two Fano resonances for spin up and spin down electrons further apart. Therefore, the current through the channel is completely polarized at \( \epsilon_F = \epsilon_{d, \uparrow} + U \) and \( \epsilon_F = \epsilon_{d, \downarrow} \).
For $U \gg \Delta$ the distance between the two spin polarized Fano resonances is of the order of the charging energy (and not the Zeeman energy). At the same time, the Kondo regime is completely suppressed. For $\epsilon_F < \epsilon_{d,\downarrow}$, the dot level is empty, and electrons pass directly from the left to the right lead (background transmission). For $\epsilon_F = \epsilon_{d,\downarrow}$ the dot is opening for spin down electrons. A Fano resonance appears, and its width is determined solely by $\Gamma = 2|V|^2/|v_F|$ where $v_F = d\epsilon/dq|\epsilon_F|$ is the Fermi velocity. For $\epsilon_{d,\downarrow} < \epsilon_F < \epsilon_{d,\downarrow} + U$ the dot level is filled with one spin down electron, and does not contribute to the conductance, leading to direct transmission from left to right leads. For $\epsilon_F = \epsilon_{d,\downarrow} + U$ the dot is opening for spin up electrons. A Fano resonance appears, with the same width as for the previous case. Finally, for $\epsilon_F > \epsilon_{d,\downarrow} + U$, the dot is filled with two electrons and does not contribute to the conductance, leading to direct transmission from left to right leads.

For typical quantum dots with $L \approx 100nm$ and $B \approx 1T$, the spin filter effect is expected to be active for temperatures below $100mK$, with a distance between the spin polarized Fano resonances of the order of $20 - 50meV$. To observe it, one needs to monitor experimentally the spin-resolved flow of electrons with a spatial resolution less than the dot dimension.

**G. Perspectives**

Gurvitz and Levinson (1993) obtained resonant reflection and transmission within a generalized description of a conducting channel (with several transverse modes) with a single impurity.

Extensions of the theoretical models in order to include many dot levels were performed by Stefanski et al. (2004) for very large ($0.1eV$) charging energies. Two dots with rather small charging energies ($1meV$) were discussed by Stefanski (2003). A series of authors considered the limit $U \rightarrow \infty$ (Balke and Stefanski, 2001; Kang et al., 2001; Kang and Shin, 2000). It remains to be clarified, whether such models can be used to discuss temperature effects on transport properties through quantum dots.

Lee and Brulei (2006) extended the spin filter model by including spin-orbit interactions and extending the side dot into a side ring with many levels. Estimates of Kondo temperatures, and general temperature effects, have been discussed by Aligia and Salgueiro (2004). Lobos and Aiglicia (2008) included Rashba spin-orbit coupling into the consideration of AB interferometers (see also Chi et al. (2007); Gong et al. (2008); Sanchez and Serra (2006); Serra and Sánchez (2007)). Spin inversion devices in a quasi-two-dimensional semiconductor wave guide under sectionally constant magnetic fields and spin-orbit interactions were discussed by Cardoso and Perevra (2008).

Experimental progress was reported by Neel et al. (2007) through contacting the tip of a low-temperature scanning tunneling microscope with individual cobalt atoms adsorbed on Cu(100), where Fano resonances have been observed.

Single-molecule devices attracted attention recently. There one sandwiches various molecules between gold electrodes and studies their conductance properties. Impressive Fano resonances (with the background transmission dropping by several orders of magnitude) were reported recently by Finch et al. (2009). The additional influence of Andreev reflection at low temperatures, when the metallic contacts turn superconducting, was studied by Kormányos et al. (2009).

Since Fano resonances rely on phase coherence of electrons traversing the structure along different paths, several authors investigated the influence of phonons on decoherence in quantum dots (Pastawski et al., 2002; Torres et al., 2006). Clerk et al. (2001) studied the possibility to extract phase decoherence properties from measurements on the $q$-factor of the Fano resonance.

During the last decades, carbon nanotubes have been studied extensively because of their unconventional properties (Saito et al., 1998). For applications to nanoscale electronic devices, researchers have fabricated various forms of carbon nanotubes to engineer their physical properties, including new morphologies such as X- and T-shaped junctions (Terrones et al., 2000). These developments offer interesting opportunities to study phase coherent transport in novel geometries. Carbon nanotubes are excellent objects for observing phase coherence phenomena and Fano effects, and there are many theoretical studies and experimental signatures of the Fano effect in different types of carbon nanotubes (Babic and Schonenberger, 2004).
VII. CONCLUSIONS

This Review offers a bird’s-eye view on the Fano resonances in various physical systems. All examples presented here share the same basic feature – coexistence of resonant and nonresonant paths for scattering wave to propagate. It results in constructive and destructive interference phenomena and asymmetric line shapes, first quantitatively described by Ugo Fano. It turns out to be a very common situation in any complex system describing wave propagation, either on a classical footing, or on a quantum mechanical one. This makes the Fano resonance a very generic phenomenon. The characteristic fingerprints of the Fano resonance are usually assumed to be related to an asymmetric profile of a cross-section or transmission as a function of some relevant control parameters. A detailed study of the problem shows, that symmetric profiles are allowed as well, and therefore a Fano resonance is indicating its presence whenever a resonant suppression of forward scattering (transmission) is observed. It is intimately related to the presence of a quasi-bound state resonantly interacting with a continuum of scattering states. The pinning down of such a bound state may or may not be an obvious undertaking, depending on the given physical setting. In particular, such quasi-bound states can be generated by geometrical means, and in more complicated settings by many body interactions. We focussed here on the study of Fano resonances in light propagation through artificial nanoscale optical devices, and in charge transport through quantum dots. Several other potential applications were discussed as well, touching such areas as superconductivity, Bose-Einstein condensates in optical lattices, among others.

Despite being interference in nature the Fano resonance we should pointed out here that it is quite different to other interference phenomena, such as, for instance, double slit experiment or weak localization in disordered media (Gantmakher, 2003). The latter two share the common feature of interference between two open channels (or broad continuums) represented by similar diffraction pattern of the slits in the first case, or identical length of the two counter-propagating paths along a loop in the second. The phase of a scattering wave varies relatively slowly along a continuum. Therefore, for nearly identical continua the phase accumulation during propagation along two paths will be practically the same. The constructive/destructive interference takes place when the sum of these two phases become equal to zero or \(\pi\), and, in general, are very well separated from each other. In the case of a Fano resonance the situation is quite different. Along the discrete level path the phase undergoes sharp variation (in comparison with the continuum) with a consequent change of its sign. It results in a very strong asymmetric profile where constructive and destructive interferences are located very close to each other. Several detailed examples considered in this Review demonstrate that systems which support Fano resonance can be mapped onto the Fano-Anderson model. This model is very simple and provides with a core understanding of the phenomenon. It can be considered as a guideline for explanation of the Fano resonance in a particular system.

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