Super-Yang-Mills Theory in SIM(1) Superspace

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Abstract

In this paper, we will analyse three dimensional supersymmetric Yang-Mills theory coupled to matter fields in $SIM(1)$ superspace formalism. The original theory which is invariant under the full Lorentz group has $\mathcal{N} = 1$ supersymmetry. However, when we break the Lorentz symmetry down to $SIM(1)$ group, the $SIM(1)$ superspace will break half the supersymmetry of the original theory. Thus, the resultant theory in $SIM(1)$ superspace will have $\mathcal{N} = 1/2$ supersymmetry. This is the first time that $\mathcal{N} = 1$ supersymmetry will be broken down to $\mathcal{N} = 1/2$ supersymmetry, for a three dimensional theory, on a manifold without a boundary. This is because it is not possible to use non-anticommutativity to break $\mathcal{N} = 1$ supersymmetry down to $\mathcal{N} = 1/2$ supersymmetry in three dimensions.

1 Introduction

Lorentz symmetry is one of the most important symmetries in nature. However, there are strong theoretical indications it might only be an effective symmetry and it might break at Planck scale. These theoretical indications come from various approaches to quantum gravity. For example, in string theory the unstable perturbative string vacuum is expected to break Lorentz symmetry \[^{[1]}-^{[2]}\]. This happens as in this case certain tensors acquire non-zero vacuum expectation values, which in turn induce a preferential direction in spacetime. In fact, as string theory is related to noncommutativity and non-commutativity is expected to break Lorentz symmetry, it is not a surprise that unstable perturbative string vacuum can break Lorentz symmetry \[^{[3]}-^{[4]}\]. In most approaches to quantum gravity the Lorentz symmetry is expected to break at Planck scale \[^{[5]}\]. One way to observe that is to note that the gravity is not renormalizable. It can only be made renormalizable by adding higher terms to original action \[^{[6]}\], which in turn break the unitarity of the theory \[^{[7]}\]. The unitarity can be preserved by taking a different Lifshitz scaling for space and time, thus, adding higher order spatial derivatives to the theory without adding any term containing higher order temporal derivative. This theory is called Hořava-Lifshitz gravity, and it obviously breaks Lorentz symmetry \[^{[8]}-^{[9]}\]. In fact,
even in loop quantum gravity, Lorentz symmetry is expected to break at Planck scale \[10\]-\[11\]. There have been attempts to study a model where a system is only invariant under subgroups of the Lorentz group, such that this subgroup still preserves enough symmetry for the constancy of the velocity of light. This theory is called very special relativity (VSR) \[12\]. In this theory the whole Lorentz group is recovered if CP symmetry is also postulated as a symmetry of the system. Two subgroups of Lorentz group called the $SIM(2)$ and $HOM(2)$ have been studied in this regard. The advantage of using these subgroups is that the dispersion relations, time delay and all classical tests of special relativity are valid for these subgroups.

The VSR can also be realized as the part of the Poincare symmetry preserved on a noncommutative Moyal plane with light-like noncommutativity \[13\]. In fact, the three subgroups relevant to the VSR can also be realized in the noncommutative spacetime setting. Quantum field theory with abelian gauge symmetry have been studied in spacetime with the symmetry group corresponding to VSR \[13\]. This work has been recently generalized to include non-abelian gauge theories \[15\]. Four dimensional supersymmetric theories have been analysed in $SIM(2)$ \[16\]. In fact, a superspace construction \[17\] and supergraph rules \[18\] for such theories have also been developed. This $SIM(2)$ superspace formalism has been used for analysing gauge theories \[19\].

It may be noted that if the Lorentz invariant is broken down to invariance under $SIM(2)$ group, the resultant $SIM(2)$ superspace breaks half the supersymmetry of the original theory. Thus, if we modify a four dimensional Lorentz invariant theory with $\mathcal{N} = 1$ supersymmetry, to $SIM(2)$ superspace, the resultant theory has $\mathcal{N} = 1/2$ supersymmetry. The terminology $\mathcal{N} = 1/2$ supersymmetry is borrowed from non-anticommutative deformation of a theory in four dimensions. This is because in four dimensions, it is also possible to break half the supersymmetry of a theory by deforming the theory to a non-anticommutative superspace \[20\]-\[25\]. So, if non-anticommutativity is imposed on a four dimensional theory with $\mathcal{N} = 1$ supersymmetry, the resultant theory is called a theory with $\mathcal{N} = 1/2$ supersymmetry, as it preserves only half the supersymmetry of the original theory. The breaking of the Lorentz group down to $SIM(2)$ group also breaks half the supersymmetry of the original Lorentz invariant theory. So, the amount of supersymmetric broken by breaking the Lorentz symmetry of a theory to $SIM(2)$ superspace is the same as the amount of supersymmetric broken by deforming it by imposing non-anticommutativity. Thus, if the Lorentz symmetry of four dimensional theory with $\mathcal{N} = 1$ supersymmetry is broken down to $SIM(2)$ group, the resultant theory will also be called called a theory with $\mathcal{N} = 1/2$ supersymmetry.

It is not possible to break the supersymmetry of a three dimensional theory from $\mathcal{N} = 1$ supersymmetry to $\mathcal{N} = 1/2$ supersymmetry by deforming it to a non-anticommutative superspace. This is because there are not enough anticommutative degrees of freedom to perform such a deformation. Any non-anticommutative deformation of a three dimensional supersymmetric theory with $\mathcal{N} = 1$ supersymmetry will break all the supersymmetry of the theory. It is possible to break the supersymmetry of a three dimensional theory with $\mathcal{N} = 2$ supersymmetry down to $\mathcal{N} = 1$ supersymmetry by imposing non-anticommutativity \[26\]. However, a three dimensional theory with $\mathcal{N} = 1/2$ supersym-
metry can be constructed on a manifold with a boundary [27]-[30]. This is because the boundary effects break half the supersymmetry of the original theory. So, if a theory has $\mathcal{N} = 1$ supersymmetry in absence of a boundary, the same theory will only have $\mathcal{N} = 1/2$ supersymmetry in presence of a boundary. Furthermore, in presence of a boundary, we can also use projections to construct a theory with $\mathcal{N} = (1, 1)$ supersymmetry. As both the boundary effects and non-anticommutativity breaks half the supersymmetry, it is possible to use a different projection to impose non-anticommutativity from the projection used to preserve half the supersymmetry on the boundary. So, for a three dimensional theory with $\mathcal{N} = (1, 1)$ supersymmetry, it is also possible break the supersymmetry down to $\mathcal{N} = (1/2, 0)$ supersymmetry by combining non-anticommutativity with boundary effects [31]. However, the advantage of using $\text{SIM}(1)$ superspace is that, we will be able construct a three dimensional theory with $\mathcal{N} = 1/2$ supersymmetry by modifying a theory with $\mathcal{N} = 1$ supersymmetry on a manifold without a boundary. Thus, it is the first time a three dimensional theory with $\mathcal{N} = 1/2$ supersymmetry will be constructed from a theory $\mathcal{N} = 1$ supersymmetry on a manifold without a boundary.

2 Subgroup of the Lorentz group preserving light-like direction

A Very Special Relativity [12] works with space-time symmetry reduced to a subgroup of the Lorentz group. In four dimensions the largest such subgroup is $\text{SIM}(2)$ a group that preserves fixed light-like vector up to its rescaling. It is possible to consider subgroups of the Lorentz group determined by such condition also in dimensions other than four. We will examine this possibility in this section.

An infinitesimal transformation of a vector $x$ under the group $\text{SO}(D-1, 1)$ is given as $\delta x^a = \omega^{ab} x^b$, where $\omega^{ab}$ are infinitesimal parameters chosen such that the size of the vector is not changed. This means that $0 = \delta(x^2) = x^a(\omega^c_{\quad ab} \eta_{bc} + \omega^c_{\quad cb} \eta_{ac}) x^b$, because this must hold for any vector, the expression inside brackets must vanish. If we use the metric $\eta_{ab}$ for rising and lowering of indices we get the condition that $\omega$ must be antisymmetric $\omega^{ab} + \omega^{ba} = 0$.

In addition to invariance of size of vectors we impose the condition that some null vector $n$ is preserved up to rescaling. This can be written as

$$\delta n^a = \omega^{ab} n_b = -2A n^a, \quad A \in \mathbb{R}. \quad (1)$$

It is convenient to work with light-cone coordinates $x^\pm = \frac{1}{\sqrt{2}} (x^0 \pm x^1)$, where the indices take values in the set $+, -, 2, \ldots, D-1$ and metric is

$$\eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (2)$$

where $1_{D-2}$ denotes $(D-2) \times (D-2)$ unit matrix. We choose the null-vector such that it has only one nonzero coordinate $n^+ = -n_- = \frac{1}{\sqrt{2}}$, and the remaining coordinates
 vanish $n^- = -n_+ = 0$, $n^a = n_a = 0$ for $a = 2, \ldots, D - 1$. The condition Eq. (1) then leads to

$$\omega^+b n_b = A n^+ \quad \Rightarrow \quad \omega^+ = 2A,$$
$$\omega^-b n_b = A n^- \quad \Rightarrow \quad 0 = 0,$$
$$\omega^{ab} n_b = A n^a \quad \Rightarrow \quad \omega^a = 0, \quad \text{for } a = 2, \ldots, D - 1. \quad (3)$$

The third condition is the only one that restricts the infinitesimal parameters, it sets $D - 2$ of them to zero. Thus the dimension of the resulting group is \(\frac{D(D - 1)}{2} - (D - 2)\).

In the case of $D = 3$ the dimension of the group is 2 and the matrix $\omega$ has the form

$$\omega^{ab} = \begin{pmatrix}
0 & \omega^+ & \omega^{+2} \\
\omega^- & 0 & \omega^{-2} \\
\omega^{2+} & \omega^{2-} & 0
\end{pmatrix} = \begin{pmatrix}
0 & 2A & -\sqrt{2}B \\
-2A & 0 & 0 \\
\sqrt{2}B & 0 & 0
\end{pmatrix} \quad (4)$$

where $A, B \in \mathbb{R}$. The exponentiation of the infinitesimal transformation gives the transformation

$$x^+ \to e^{-2A} x^+ - \sqrt{2} e^{-A} B x^2 + B^2 x^-, \quad x^- \to e^{2A} x^-, \quad x^2 \to x^2 - \sqrt{2} e^{A} B x^- \quad (5)$$

Another way how to arrive to this group is to represent vectors by two-dimensional symmetric matrices

$$x = \begin{pmatrix}
x^0 + x^1 \\
x^2 \\
x^0 - x^1
\end{pmatrix} = \begin{pmatrix}
\sqrt{2} x^+ \\
x^2 \\
\sqrt{2} x^-
\end{pmatrix}, \quad (6)$$

the size of the vector is $x^2 = -\det x$ and the size-preserving transformations are given as

$$x' = g x g^T, \quad g \in SL(2, \mathbb{R}). \quad (7)$$

Any null-vector can be written as $n^{\alpha\beta} = \xi^\alpha \xi^\beta$ where the commuting spinor $\xi$ is determined uniquely up to a sign. A convenient choice of a null vector is to choose $\xi^+ = 1$, $\xi^- = 0$. The condition that this null-vector is preserved up to a rescaling can be now written as

$$g \xi = \pm e^{-A} \xi, \quad A \in \mathbb{R}. \quad (8)$$

Only matrices from $SL(2, \mathbb{R})$ that satisfy this criteria are

$$g = \pm \begin{pmatrix}
e^{-A} & -B \\
0 & e^A
\end{pmatrix}, \quad (9)$$

the meaning of $A$ and $B$ is the same as in Eq. (5).

The difference between $D = 3$ and $D = 4$ is that in the case of $D = 4$ we worked with complex matrices from $SL(2, \mathbb{C})$, while in the case of $D = 3$ we have real matrices from $SL(2, \mathbb{R})$. In the four dimensional case we called this group $SIM(2)$ a group of similarity transformations in two dimensions (consisting of rotation, scaling and shift).
In our $D = 3$ case we can identify this group with a group of orientation preserving similarity transformations in one dimension (consisting of scaling and shift). In order to show that we identify a point in one-dimensional space determined by coordinate $z$ with a point in projective space $\mathbb{RP}^1$ represented by $(z \ 1)$. An action of the group given by left multiplication by $g$ then gives

$$
\left( \begin{array}{c} z \\ 1 \end{array} \right) \to \ g \left( \begin{array}{c} z \\ 1 \end{array} \right) = \pm \left( \begin{array}{c} e^{-A}z - B \\ e^A \end{array} \right) \sim \left( \begin{array}{c} e^{-2A}z - e^{-A}B \\ 1 \end{array} \right) = \left( \begin{array}{c} z' \\ 1 \end{array} \right).
$$

(10)

The change $z \to z'$ indeed describes the orientation preserving similarity transformation.

3 Three dimensional Supersymmetry

In this section, we will study three dimensional superspace. In the Lorentz invariant theory the supersymmetry will be generated by $Q_\alpha = \partial_\alpha - (\gamma^a \theta)_\alpha \partial_a$. Now as was shown in Eq. (9), the $SIM(1)$-transformation of spinors is given as

$$
\psi' = e^{-A} \psi - e^{-A}B \left( \begin{array}{c} \psi \\ 1 \end{array} \right) \leftrightarrow \psi' = e^A \left( \begin{array}{c} 0 \\ \psi \end{array} \right) = e^A \left( \begin{array}{c} 0 \\ \psi \end{array} \right)
$$

with $A, B \in \mathbb{R}$. Spinors that satisfy the condition

$$
\eta \psi = 0 \quad \Rightarrow \quad \psi = \left( \begin{array}{c} 0 \\ \psi \end{array} \right)
$$

consist a space that is invariant under $SIM(1)$ transformations. Let us denote the space of all spinors as $S$, and the invariant space that we have just described as $S_{\text{invariant}}$. We also define a space $S_{\text{quotient}} = S/S_{\text{invariant}}$. A convenient description of this space is provided by choosing a representative

$$
\psi = \left( \begin{array}{c} \psi_+ \\ 0 \end{array} \right).
$$

(13)

in each equivalence class. Both spaces $S_{\text{invariant}}$ and $S_{\text{quotient}}$ carry a representation of the $SIM(1)$-group, they transform as

$$
\left( \begin{array}{c} 0 \\ \psi_- \end{array} \right) = e^{-A} \left( \begin{array}{c} 0 \\ \psi_- \end{array} \right), \quad \left( \begin{array}{c} \psi_+ \\ 0 \end{array} \right) = e^A \left( \begin{array}{c} \psi_+ \\ 0 \end{array} \right).
$$

(14)

Thus we have two distinct one-dimensional representations.

The reduction of supersymmetry will be done following the same steps as in the four dimensional case [16], [17]. We can summarize the necessary steps as:

- The space-time symmetry will be reduced to $SIM(1)$-subgroup.
- The supersymmetry transformations will be reduced to those that correspond to symmetry generator $\epsilon Q$ with anticommuting parameter $\epsilon$ satisfying the condition $\gamma \epsilon = 0$. There will remain only one supersymmetry generator proportional to $\gamma Q$. It will transforms under $SIM(1)$ in the same way as spinors from $S_{\text{quotient}}$.

- Only the anticommuting $\theta$ coordinates that satisfy $\gamma \theta = 0$ are kept in the superspace. There will be only one anticommuting coordinate. It will transform under $SIM(1)$ in the same way as spinors from $S_{\text{invariant}}$.

- The covariant derivatives are reduced in a similar way as supersymmetry generators. There are two things we have to take care of. Firstly, we only keep covariant spinor derivatives that are proportional to $\gamma D$. Secondly, because the resulting superspace is reduced we have to make a projection that removes anticommuting coordinates that are no longer part of it (ie. projection that sets $\gamma \theta = 0$). There will be one spinor covariant derivative that will transform under $SIM(1)$ in the same way as spinors from $S_{\text{quotient}}$.

The above steps can be easily done if we introduce another null-vector $\tilde{n}$ that satisfies the relation $n \cdot \tilde{n} = 1$. This allows us to define projectors that split any spinor into two parts

$$
\psi = \frac{1}{2} \gamma \psi + \frac{1}{2} \gamma \tilde{n} \psi \quad \Leftrightarrow \quad \psi_\alpha = -\tilde{n}_{\alpha\beta} n^\beta \gamma \psi_\gamma - n_{\alpha\beta} \tilde{n}^\beta \gamma \psi_\gamma.
$$

With the choice of $n$ and $\tilde{n}$ in which only the components $n^{++} = i$, $\tilde{n}^{--} = -i$ are nonzero we get

$$
\frac{1}{2} \gamma \psi = \begin{pmatrix} \psi_+ \\ 0 \end{pmatrix}, \quad \frac{1}{2} \gamma \tilde{n} \psi = \begin{pmatrix} 0 \\ \psi_- \end{pmatrix}.
$$

The supersymmetry generator $S$, the anticommuting superspace coordinate $\zeta$ and spinor derivative $d$ are defined as

$$
S = \frac{1}{2} \gamma \gamma Q, \quad \zeta = \frac{1}{2} \gamma \tilde{n} \theta, \quad d = \frac{1}{2} \gamma \gamma D |_{\gamma \theta = 0}.
$$

Each of them have only one nonzero component

$$
S_+ = \partial_+ + i \zeta_- \partial_+ \quad \zeta_- = \theta_-, \quad d_+ = \partial_+ - i \zeta_- \partial_+ \quad (18)
$$

and they satisfy

$$
\{S_+, S_+\} = 2 \partial_+, \quad \{S_+, d_+\} = 0, \quad \{d_+, d_+\} = -2 \partial_+, \quad \partial_+ \zeta_- = -i. \quad (19)
$$

It may be noted that this modification breaks half the supersymmetry of the original theory. Thus, as our original theory had $N = 1$ supersymmetry, the resultant theory after this modification only has $N = 1/2$ supersymmetry. Unlike the four dimensional case [20]-[25], we cannot break the supersymmetry of a three dimensional theory with $N = 1$ supersymmetry to $N = 1/2$ supersymmetry by using non-anticommutativity.
This is because in four dimensions there are enough degrees of freedom to partially break the $N = 1$ supersymmetry. For a four dimensional theory with $N = 1$ supersymmetry, there are four independent anticommutating coordinates. So, if non-anticommutativity is imposed between two of them, still the supersymmetry corresponding to the other two is preserved. However, for three dimensional theory with $N = 1$ supersymmetry there are only two independent anticommutating coordinates, and so, any non-anticommutativity will break all the supersymmetry of such a three dimensional theory. Hence, a three dimensional theory with $N = 1/2$ supersymmetry can be obtained by breaking the Lorentz symmetry down to $SIM(1)$ group.

4 Superfield decomposition

In this section we are going to establish correspondence between superfields that appear in $SO(2,1)$ superspace and superfields that we use in $SIM(1)$ superspace. There are two things we have to resolve in order to establish this correspondence. Firstly, the $SO(2,1)$ superspace is bigger than $SIM(1)$ superspace. This means that if we write a $SO(2,1)$ theory in $SIM(1)$ superspace then to each $SO(2,1)$ superfield there will correspond multiple $SIM(1)$ superfields, otherwise we lose some degrees of freedom. In fact, we will observe that for each $SO(2,1)$ superfield there are two $SIM(1)$ superfields. Secondly, if the $SO(2,1)$ superfield carries some space-time indices then we have to handle them specially, otherwise we will get $SIM(1)$ superfields that transform in a very complicated way under $SIM(1)$ group.

Let us start with a scalar $SO(2,1)$ superfield $\Phi$. The projections

$$\phi = \Phi|_{\theta_+ = 0}, \quad \tilde{\phi}_- = (D_- \Phi)|_{\theta_+ = 0},$$

contain all information carried by $\Phi$. This is most easily seen from the fact that the superfield $\Phi$ could be written as

$$\Phi = \phi - i\theta_+ \left( \tilde{\phi}_- + i\theta_- \partial_+ \phi \right).$$

The $SIM(1)$ rotations change these superfields as (prime denotes transformed quantities)

$$\phi'(x', \theta') = \phi(x, \theta), \quad \tilde{\phi}_-'(x', \theta') = e^{-A} \tilde{\phi}_-(x, \theta) + B \phi(x, \theta).$$

The superfield $\phi$ transforms nicely but the superfield $\tilde{\phi}_-$ transforms into combination of both $\phi$ and $\tilde{\phi}_-$ which makes it unsuitable for description of $SIM(1)$ theories because it makes $SIM(1)$ invariance non-trivial. This behavior originates in the fact that in order to define it we need the projectors Eq. (17). The definition of these projectors requires the null-vector $\tilde{n}$ that introduces another preferred direction (apart from the direction of $n$) which further breaks $SIM(1)$ symmetry.

However, we can change the projection $\tilde{\phi}_-$ in such a way that it has better transformation properties with respect to $SIM(1)$ group. We introduce an operator

$$\hat{q} = \frac{i\partial}{2n \cdot D},$$

7
which has only one nonzero component

\[ \hat{q}_- = D_- - \frac{\partial_{++}}{\partial_{++}} D_+. \]  

(24)

The improved SIM(1) superfield is defined as

\[ \hat{\phi}_- = (\hat{q}_- \Phi)|_{\theta_+ = 0}. \]  

(25)

The new SIM(1) transformation rule

\[ \hat{\phi}'_- (x', \theta') = e^{-A} \hat{\phi}_-(x, \theta), \]  

(26)

does not suffer from mixing with the other superfield \( \phi \). In the case of gauge theory we are going to replace the derivatives in Eq. (23) with covariant ones. This makes things more complicated because the noncommutativity of covariant derivatives makes the definition of this operator ordering dependent. In this text we will use the following variant \(^3\)

\[ q = \eta \sqrt{\frac{1}{2n}} \nabla, \]  

(27)

which leads to the definition of the superfield

\[ \phi_- = (q_- \Phi)|_{\theta_+ = 0} = \left( \left( \nabla_- - \nabla_+ \frac{\nabla^+}{\sqrt{\nabla_{++}}} \right) \Phi \right)|_{\theta_+ = 0}. \]  

(28)

The SIM(1) transformation properties are the same as in the case of \( \hat{\phi}_- \).

If the SIM(1) superfield carries space-time indices we arrive at basically the same problem as in the case of \( q_- \). We will illustrate this problem on the superfield \( W_\alpha \).\(^3\)

The projections

\[ w_+ = W_+|_{\theta_+ = 0}, \quad \tilde{w}_- = W_-|_{\theta_+ = 0}, \]  

(29)

transform under the action of SIM(1) group as

\[ w'_+(x', \theta') = e^A w_+(x, \theta), \quad \tilde{w}'_- (x', \theta') = e^{-A} \tilde{w}_-(x, \theta) + Bw_+(x, \theta). \]  

(30)

the projection \( \tilde{w}_- \) has the same ugly transformation rule as we had for \( \hat{\phi}_- \). The transformation properties can be improved by the same trick that we used above. We introduce

\(^1\) In 3 + 1 dimensions \(^1\) the ordering ambiguity is resolved if we want the \( \phi_- \) projection of a covariantly chiral superfield \( \nabla_\alpha \Phi = 0 \) satisfy the SIM(2) chiral-covariant condition \( \nabla_+ \phi_- = 0 \). This forces us to choose the ordering

\[ \phi_- = \left( \left( \nabla_- - \nabla_+ \frac{1}{\sqrt{\nabla_{++}}} \right) \Phi \right)|_{\theta_+ = 0}. \]  

\(^2\) We should also consider \( \hat{q}_- \) projections, but in the case of field strengths in gauge theory we do not need them.
an operator \( \Delta \)

\[
\Delta = i \left( \frac{1}{2n \cdot \nabla} \frac{\nabla}{\nabla \cdot \nabla} \right).
\]  

(31)

the only nonzero components of \( \Delta_{\alpha\beta} \) are

\[
\Delta_{--} = 1, \quad \Delta_{-+} = 1, \quad \Delta_{-+} = \frac{1}{2} \nabla_{++} - \frac{1}{2} \nabla_{++} \nabla_{+-}.
\]  

(32)

We define

\[
w_{-} = i(\Delta_{-} \alpha W_{\alpha})|_{\theta_{+}=0} = (W_{-} - \Delta_{--} W_{+})|_{\theta_{+}=0}.
\]  

(33)

If there are more space-time indices we have to repeat this procedure for each index, in particular we will need to do this for the superfield \( F_{\alpha\beta} \). We define

\[
f_{++} = F_{++}|_{\theta_{+}=0},
\]

\[
f_{+-} = i(\Delta_{-} \alpha F_{+\alpha})|_{\theta_{+}=0} = (F_{+-} - \Delta_{--} F_{++})|_{\theta_{+}=0},
\]

\[
f_{-} = -(\Delta_{-} \alpha \Delta_{-} \beta F_{\alpha\beta})|_{\theta_{+}=0} = (F_{-} - 2\Delta_{--} F_{++} + \Delta_{--} \Delta_{--} F_{++})|_{\theta_{+}=0},
\]  

(34)

The superfields that we obtain in this way have very simple transformation properties under action of \( SIM(1) \). For a general superfield \( \psi_{+++---} \) we can schematically write this rule as

\[
\psi'_{+++---}(x', \theta') = e^{A(\# \text{ of } +\text{ indices minus } \# \text{ of } -\text{ indices})} \psi_{+++---}(x, \theta).
\]  

(35)

5 Gauge theory with \( \mathcal{N} = 1/2 \) supersymmetry

In this section, we will use \( SIM(1) \) superspace to study super-Yang-Mills theory coupled to matter field. As the reduction of the \( \mathcal{N} = 1 \) superspace to \( SIM(1) \) superspace breaks the supersymmetry from \( \mathcal{N} = 1 \) supersymmetry to \( \mathcal{N} = 1/2 \) supersymmetry, the Yang-Mills theory coupled to matter fields will have \( \mathcal{N} = 1/2 \) supersymmetry. It may be noted that if we do not break the Lorentz symmetry but use the \( SIM(1) \) formalism for analysing the super-Yang-Mills theory coupled to matter field, we can recover the full \( \mathcal{N} = 1 \) supersymmetry. In this case only \( \mathcal{N} = 1/2 \) supersymmetry will be manifested in the superspace formalism. The other half of the symmetry can be considered as an accidental symmetry that would disappear once we use this formalism in its intended role – to study effects that break the Lorentz symmetry but preserve \( SIM(1) \) symmetry.

We will consider the action for matter fields as follows

\[
S_{m} = \frac{1}{2} \int d^{3}x \nabla^{2} \left[ (\nabla^{\alpha} \Phi^{\dagger}) (\nabla_{\alpha} \Phi) \right],
\]  

(36)

The ordering in this definition was chosen in this way because it results in a very simple rule for integration by parts:

\[
\int d^{3}x \nabla_{+} ((\Delta_{\alpha\beta} f) g) = \int d^{3}x \nabla_{+} (f (\Delta_{\alpha\beta} g)) + \text{surface terms},
\]

where \( f \) and \( g \) are arbitrary superfunctions.
and the action for the gauge superfield as

$$S_g = \text{tr} \int d^3x \nabla^2 [W^2].$$  \hspace{1cm} (37)

When the space-time covariant derivatives appear in the \(SIM(1)\) action, we have to understand them as projections \(\nabla_{\alpha\beta}\big|_{\theta_\pm=0}\) of \(SO(2,1)\) derivatives. The matter field action in the \(SIM(1)\) superspace formalism is

$$S_m = \frac{1}{2} \int d^3x \nabla_+ \left[ (\nabla_+ \phi_{-}^\dagger) \phi_{-} + \phi_{-}^\dagger (\nabla_+ \phi_{-}) - 2i \phi_{-}^\dagger \left( w_+ \frac{\nabla_+}{\nabla_+} \phi \right) - 2i \left( w_+ \frac{\nabla_+}{\nabla_+} \phi^\dagger \right) \phi_{-} ight]$$

$$+ \left( \square_{\text{cov}} \phi \right) \left( \frac{\nabla_+}{\nabla_+} \phi \right) + \left( \nabla_+ \phi^\dagger \right) \left( \square_{\text{cov}} \phi \right)$$

$$- i \left( \frac{\nabla_+}{\nabla_+} \phi^\dagger \right) \left( \nabla_+ w_- \phi \right) + \frac{1}{2} \left( \nabla_+ \phi^\dagger \right) \left[ f_{++}, \frac{1}{\nabla_+} w_+ \left( \nabla_+ \phi \right) \right]$$

$$+ \text{surface terms}, \hspace{1cm} (38)$$

and we can say that it is explicitly \(SIM(1)\) invariant. In fact each term that appears in the action is separately \(SIM(1)\) invariant. The covariant d’Alambertian operator is defined as

$$\square_{\text{cov}} = -\frac{1}{2} \nabla_\alpha \nabla_\alpha.$$ 

Now we can write the action for gauge sector of the theory as

$$S_g = \text{tr} \int d^3x \nabla_+ \left[ -\tilde{f}_{++} \tilde{w}_- + w_+ \tilde{f}_{--} \right] + \text{surface terms}. \hspace{1cm} (39)$$

This form, that uses projectors Eq. (29), does not show manifest \(SIM(1)\) invariance. The \(SIM(1)\) transformations change, according to Eq. (30), this action to

$$S'_g = S_g + (e^A B) \text{tr} \int d^3x \nabla_+ \left( w_+ \tilde{f}_{++} - \tilde{f}_{++} \tilde{w}_- \right)$$

$$+ (e^{2A} B^2) \text{tr} \int d^3x \nabla_+ \left( w_+ f_{++} - f_{++} w_+ \right)$$

$$= S_g + (e^A B) \text{tr} \int d^3x \nabla_+ \left( -\nabla_+ (w_+ \tilde{w}_-) \right) = S_g + \text{surface terms}, \hspace{1cm} (40)$$

where we used \(\nabla_+ w_+ = f_{++}\) and \(\nabla_+ \tilde{w}_- = \tilde{f}_{++}\). We see that the \(SIM(1)\) invariance is not obvious at first glance. This is a reason why it is better to write the action in terms of \(SIM(1)\) superfields that have simple transformation properties

$$S_g = \text{tr} \int d^3x \nabla_+ \left[ -f_{--} w_- + w_+ f_{--} - \frac{i}{2} \left( w_+, \frac{1}{\nabla_+} w_+ \right) w_- - \frac{i}{2} \left( \frac{1}{\nabla_+} \left( \nabla_+ w_+ \right) \right) \right] + \text{surface terms}. \hspace{1cm} (41)$$
Each term in this action is separately SIM(1) invariant. The verification of the SIM(1) invariance is easy because the superfields $w_+, w_-, f_{++}, f_{+-}, f_{--}$ and derivatives $\nabla_+, \nabla_{++}$ transform under SIM(1) according to the rule Eq. (33). Thus the expressions are invariant if they contain the same number of lower plus indices as there are lower minus indices. The only exception to this rule are the operators $\triangle_{-\alpha}$ that appear in the last term. This term is equal to

\[-\frac{i}{2} (\triangle_{-} w_+) \left\{ w_+, \frac{1}{\nabla_{++}} w_+ \right\} + \frac{i}{2} w_+ \left\{ w_+, \frac{1}{\nabla_{++}} (\triangle_{-} w_+) \right\} \tag{42}\]

where we used that $\triangle_{+-} = 1$. Using the transformation rule $\triangle_{--} \rightarrow e^{-2A} \triangle_{--} + e^{-A} B \triangle_{--}$ we find that SIM(1) transformations change this term as

$$
\delta \left( \text{tr} \int d^3 x \nabla_+ \left[ -\frac{i}{2} (\triangle_{-} w_+) \left\{ w_+, \frac{1}{\nabla_{++}} w_+ \right\} + \frac{i}{2} w_+ \left\{ w_+, \frac{1}{\nabla_{++}} (\triangle_{-} w_+) \right\} \right] \right) = e^{A} B \text{tr} \int d^3 x \nabla_+ \left[ -\frac{i}{2} w_+ \left\{ w_+, \frac{1}{\nabla_{++}} w_+ \right\} + \frac{i}{2} w_+ \left\{ w_+, \frac{1}{\nabla_{++}} w_+ \right\} \right] = 0, \tag{43}\]

so this term is also SIM(1) invariant. Thus, we have been able to write the action of super-Yang-Mills theory coupled to matter fields in SIM(1) superspace.

So far we have worked with a particular choice of the vector $n$, but we could also write the results in a form that shows explicit dependence on this vector. Thus, the action for the matter sector can be written as

$$
S_m = \frac{1}{2} \int d^3 x \nabla^\alpha \left[ - (\nabla^\beta q_{\beta} \Phi^i) (q_\alpha \Phi) - (q_\alpha \Phi^i) (\nabla^\beta q_\beta \Phi) \\
+ 2 (q_\alpha \Phi^i) \left( W^\beta \frac{n_\beta}{\sqrt{2 n \cdot \nabla}} \nabla_\gamma \Phi \right) + 2 \left( W^\beta \frac{n_\beta}{\sqrt{2 n \cdot \nabla}} \nabla_\gamma \Phi^i \right) (q_\alpha \Phi) \\
+ (\Box_{\text{cov}} \Phi^i) \left( \frac{n_\alpha}{\sqrt{2 n \cdot \nabla}} \nabla_\beta \Phi \right) + \left( \frac{n_\alpha}{\sqrt{2 n \cdot \nabla}} \nabla_\beta \Phi^i \right) (\Box_{\text{cov}} \Phi) \\
- i \left( \frac{n_\alpha}{\sqrt{2 n \cdot \nabla}} \nabla_\beta \Phi^i \right) \left( \left( \frac{1}{\sqrt{2 n \cdot \nabla}} \right) \triangle_{-} \left( \frac{1}{\sqrt{2 n \cdot \nabla}} \nabla_\gamma \Phi \right) \right) \\
+ \frac{1}{2} \left( \frac{n_\alpha}{\sqrt{2 n \cdot \nabla}} \nabla_\beta \Phi^i \right) \left[ \left( n^\delta f_{\gamma \delta}, \frac{1}{\sqrt{2 n \cdot \nabla}} \right) W_\sigma \left( \frac{n_\epsilon}{\sqrt{2 n \cdot \nabla}} \nabla_\epsilon \Phi \right) \right] + \text{surface terms}, \tag{44}\]

and the action for the gauge sector can be written as

$$
S_g = \text{tr} \int d^3 x \nabla^\alpha \left[ (\triangle^\delta F_{\gamma \delta}) (\triangle_\alpha \beta W_\delta) + (\triangle_\alpha \gamma \triangle_\beta \delta F_{\gamma \delta}) W^\beta \\
- \frac{i}{2} \left\{ W_\gamma, \frac{n_\gamma}{\sqrt{2 n \cdot \nabla}} \right\} (\triangle_\alpha \beta W_\delta) - \frac{i}{2} \left\{ \frac{n_\gamma}{\sqrt{2 n \cdot \nabla}}, W_\gamma \right\} (\triangle_\alpha \beta W_\delta) \\
- \frac{1}{2} \left( \triangle_\gamma \left( \frac{n_\gamma}{\sqrt{2 n \cdot \nabla}} \right) W_\gamma \right) \left\{ W_\sigma, \frac{n_\sigma}{\sqrt{2 n \cdot \nabla}} (\triangle_\delta \beta W_\delta) \right\} \right] + \text{surface terms}. \tag{45}\]

The fact that we could write the action in this form proves that the supersymmetry is broken only due to presence of the preferred light-like direction determined by $n$. 

11
6 Conclusion

In this paper, we analysed three dimensional super-Yang-Mills theory in $SIM(1)$ superspace. The original Lorentz invariant theory had $\mathcal{N} = 1$ supersymmetry. However, when the Lorentz symmetry was broken down to $SIM(1)$ group, the resultant theory preserved only half the supersymmetry of the original theory. As the original theory had $\mathcal{N} = 1$, so, the theory in $SIM(1)$ superspace has $\mathcal{N} = 1/2$ supersymmetry. This was the first time that $\mathcal{N} = 1$ supersymmetry was broken down to $\mathcal{N} = 1/2$ supersymmetry in three dimensions, on a manifold without a boundary. This is because for a manifold without a boundary, the other way to obtain a theory with $\mathcal{N} = 1/2$ supersymmetry is by imposing non-anticommutativity. However, in three dimensions there are not enough superspace degrees to allow this partial breaking of supersymmetry. So, any non-anticommutative deformation of a three dimensional theory with $\mathcal{N} = 1$ supersymmetry, will break all the supersymmetry of the resultant theory. It would be interesting to analyse a theory on a manifold with boundaries with $SIM(1)$ superspace. It is known that the presence of a boundary also breaks half the supersymmetry of a theory \cite{30,31}. It is possible that both the boundary effects and the modification of the superspace to $SIM(1)$ superspace will break the same supercharges and hence will preserve half the supersymmetry of the original theory. It is also possible that a similar effect can be generated by studying non-anticommutativity in $SIM(1)$ superspace.

Three dimensional superspace is important as it has been used for studying three dimensional superconformal field theories. Three dimensional superconformal field theory with $\mathcal{N} = 8$ supersymmetry is thought to describe the low energy action for multiple M2-branes. This is because apart from a constant closed 7-form on $S^7$, $AdS_4 \times S_7 \sim SO(2,3) \times SO(1,2)/SO(8) \times SO(7) \subset OSp(8|4)/SO(1,3) \times SO(7)$, and so, $OSp(8|4)$ symmetry of the eleven dimensional supergravity on $AdS_4 \times S_7$ gets realized as $\mathcal{N} = 8$ supersymmetry of its dual superconformal field theory. There are further constraints on this superconformal field theory which are satisfied by a theory called the BLG theory \cite{32,33,34,35,36}. However, the gauge symmetry of the BLG theory is generated by a Lie 3-algebra, and it only describes two M2-branes. It is possible to generalize the BLG theory to a theory describing any number of M2-branes and this theory is called the ABJM theory \cite{35,36,37,38,39,40,41,42,43,44}. Even though the ABJM theory has only $\mathcal{N} = 6$ supersymmetry, it is expected that its supersymmetry might get enhanced to full $\mathcal{N} = 8$ supersymmetry \cite{39,40,41,42,43,44}. It is also possible to use a Mukhi-Papageorgakis novel higgs mechanism to obtain a theory of multiple D2-branes from a theory of multiple M2-branes \cite{43,44,45,46}. The gauge sector for the low energy action of multiple D2-branes is described by a super-Yang-Mills theory. As it is known that certain unstable string theory vacuum states break Lorentz symmetry \cite{1,2}, it will be interesting to analyse the action of multiple D2-branes in $SIM(1)$ superspace. It would also be interesting to analyse the theory of multiple M2-branes and the Mukhi-Papageorgakis novel higgs mechanism in $SIM(1)$ superspace.
Appendix

A spinor $\theta_\alpha$ is real (Majorana), spinor metric is antisymmetric and imaginary, the rules for raising and lowering of spinor indices are

$$\theta^\alpha = \theta_\beta C^{\beta\alpha}, \quad \theta_\alpha = \theta^\beta C_{\beta\alpha}. \quad (46)$$

Gamma matrices are real (Majorana)

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab}, \quad (\gamma^a)^* = \gamma^a, \quad (47)$$

with space-time metric $\eta$ having signature $-1, +1, +1$. The notation with spinor indices is related to the notation with matrix multiplication by identifying

$$\theta \sim \theta_\alpha, \quad \bar{\theta} = \theta^\dagger i\gamma^0 = \theta^T C \sim \theta^\alpha, \quad C \sim C^{\alpha\beta}, \quad C^{-1} \sim C_{\alpha\beta}, \quad \gamma^a \sim (\gamma^a)_\alpha^\beta. \quad (48)$$

We also define

$$\theta^2 = \frac{1}{2} \bar{\theta} \theta = \frac{1}{2} \theta^\alpha \theta_\alpha, \quad \bar{\gamma}^a = v_a \gamma^a, \quad \gamma^a = (\gamma^a)_\alpha^\beta C_{\alpha\beta}, \quad \gamma^a_\alpha = -(\gamma^a)_\alpha^\beta C^{\alpha\beta}. \quad (49)$$

There is a lot of useful relations

$$(\theta^\alpha)_\alpha^\gamma = \theta^\gamma, \quad (\theta^\alpha)^* = -\theta^\alpha, \quad C_{\alpha\beta} = -C^*_{\alpha\beta}, \quad \delta_{\alpha\beta} = -\delta^*_{\alpha\beta}, \quad \gamma^a_{\alpha\beta} = \gamma^a_{\beta\alpha} = -(\gamma^a)_{\alpha\beta}^*, \quad \gamma^a_{\alpha\beta} = \gamma_{\alpha\beta}^a = -(\gamma_{\alpha\beta}^a)^*, \quad \gamma^a_{\alpha\beta} \gamma^b_{\alpha\delta} = -2\delta^a_{\beta\delta}, \quad \gamma^a_{\alpha\beta} \gamma^b_{\gamma\delta} = -\delta^a_{\alpha\beta} \delta^b_{\gamma\delta}. \quad (50)$$

The explicit form of spinor metric and gamma matrices can be, for example, chosen as

$$C_{\alpha\beta} = \sigma_2 = C_{\alpha\beta}, \quad (\gamma^a)_\alpha^\beta = (i\sigma_2, \sigma_1, -\sigma_3), \quad \gamma^a_{\alpha\beta} = \gamma_{\alpha\beta}^a = (i1, i\sigma_3, i\sigma_1). \quad (51)$$

The correspondence between spinor and vector indices for coordinates, derivatives and other vectors (represented by $n$)

$$x^{\alpha\beta} = \frac{1}{2} \gamma_{\alpha\beta} x^a, \quad \partial_{\alpha\beta} = \gamma_{\alpha\beta} \partial_a, \quad n^{\alpha\beta} = \frac{1}{\sqrt{2}} \gamma_{\alpha\beta} n^a \quad (52)$$

or if we need the inverse relations

$$x^a = -\gamma^a_{\alpha\beta} x^{\alpha\beta}, \quad \partial_a = \frac{1}{2} \gamma^a_{\alpha\beta} \partial_{\alpha\beta}, \quad n^a = \frac{1}{\sqrt{2}} \gamma_{\alpha\beta} n^{\alpha\beta}. \quad (53)$$

With these rules we have

$$\partial_{\alpha\beta} x^{\gamma\delta} = -\frac{1}{2} \delta^\gamma_{\alpha} \delta^\delta_{\beta}, \quad \partial^{\alpha\gamma} \partial_{\beta\gamma} = -\delta^\delta_{\beta} \square. \quad (54)$$
Supersymmetry generator and super-derivative for a three-dimensional theory with $N = 1$ supersymmetry can be written as

\[ Q_\alpha = \partial_\alpha - (\gamma^a \theta)_a \partial_a = \partial_\alpha + \gamma^a_{\alpha \beta} \theta^\beta \partial_a, \quad D_\alpha = \partial_\alpha + (\gamma^a \theta)_a \partial_a = \partial_\alpha - \gamma^a_{\alpha \beta} \theta^\beta \partial_a, \] \hspace{1cm} (55)

such that they satisfy the relations

\[
\{Q_\alpha, Q_\beta\} = 2\gamma^a_{\alpha \beta} \partial_a, \quad \{Q_\alpha, D_\beta\} = 0, \quad \{D_\alpha, D_\beta\} = -2\gamma^a_{\alpha \beta} \partial_a, \]

\[
D_\alpha \theta^\beta = \delta^\beta_\alpha, \quad D^2 \theta^2 = -1. \] \hspace{1cm} (56)

Now we can write the gauge covariant derivatives as

\[ \nabla_\alpha = D_\alpha - i\Gamma_\alpha, \quad \nabla_{\alpha \beta} = \partial_{\alpha \beta} - i\Gamma_{\alpha \beta}, \] \hspace{1cm} (57)

such that there (anti)commutators are given by

\[
\{\nabla_\alpha, \nabla_\beta\} = -2\nabla_{\alpha \beta}, \quad [\nabla_\alpha, \nabla_\gamma] = C_{\alpha \beta W_\gamma}, \quad [\nabla_{\alpha \beta}, \nabla_{\gamma \delta}] = -\frac{1}{2} C_{\alpha \gamma F_{\beta \delta}} - \frac{1}{2} C_{\alpha \delta F_{\beta \gamma}} - \frac{1}{2} C_{\beta \delta F_{\alpha \gamma}} - \frac{1}{2} C_{\beta \gamma F_{\alpha \delta}}, \] \hspace{1cm} (58)

where

\[ \Gamma_{\alpha \beta} = -\frac{1}{2} \left( D_{(\alpha} \Gamma_{\beta)} - i \{\Gamma_\alpha, \Gamma_\beta\} \right), \]

\[ W_\alpha = -\frac{i}{2} D_\beta D_\alpha \Gamma_\beta - \frac{1}{2} [\Gamma_\alpha, D_\beta \Gamma_\alpha] + \frac{i}{6} \{\Gamma_\beta, \{\Gamma_\alpha, \Gamma_\beta\}\}, \quad \nabla^\alpha W_\alpha = 0, \]

\[ F_{\alpha \beta} = \frac{1}{2} \nabla_{(\alpha} W_{\beta)}. \] \hspace{1cm} (59)

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