A FERMION MASS MATRIX ANSATZ
FOR THE FLIPPED SU(5) MODEL

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ABSTRACT

A fermion mass matrix ansatz is proposed in the context of Grand Unified Supersymmetric Theories (GUTs). The fermion mass matrices are evolved down to the electroweak scale by solving the renormalization group equations for the gauge and Yukawa couplings. Eight inputs are introduced at the GUT scale to predict the 13 arbitrary parameters of the Standard model. The constraints imposed by the charged fermion data are used to make predictions in the neutrino sector. In particular, the neutrino mass matrix is worked out in the case of the flipped SU(5) model and it is found that the generalized see-saw mechanism which occurs naturally in this model can provide a solution to the solar neutrino puzzle and give a sufficiently large $\nu_\tau$ mass to contribute as a hot dark matter component as indicated by the recent COBE data.

1. Introduction

Although the Standard Model of Strong and Electroweak interactions explains all known experimental data, it is now widely accepted that it is unlikely to be the fundamental theory of nature.

In a fundamental theory the various experimentally measurable parameters should be calculable only from few inputs which could be specified in terms of the basic principles of the theory. For example, in String theories we expect that all masses and mixing angles can be determined only from one input, namely the String gauge coupling $g_{\text{String}}$ at the String Unification scale $M_{\text{String}} \sim O(g_{\text{String}} \times M_{\text{Pl}})$. However, there is a huge number of String derived models and it is rather impossible to find the unique model among them which might lead to the correct - experimentally tested - predictions at low energies. Although the previously mentioned arbitrariness puts a great obstacle in string model builders, which is not likely to be solved in the near future, we are by now convinced that there is no real rival to string theory. From the phenomenologist’s point of view, the alternative way is to attempt to describe the low energy theory, using the experience from string model building and all the possible information that one can extract out of it.

We know already, that there is a number of constraints which should be respected on our way from $M_{\text{String}}$ down to $m_W$. Here there are listed few of them:

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Recent calculations taking into account threshold corrections from String massive states[3, 4, 5, 6, 7], show that the String Unification scale is relatively high, and close to the Plank scale. On the contrary, renormalization group calculations have indicated that minimal supersymmetric Grand Unified Theories (SUSY-GUTs) are in agreement with the precision LEP data when the SUSY-GUT scale $M_G$ is taken close to $M_G \approx 10^{16} GeV$[2]. Thus, the minimal supersymmetric standard model (MSSM) cannot probably be derived directly at the string scale; the above discrepancy between the two scales, would rather suggest that MSSM should be obtained through the spontaneous breaking of some intermediate GUT-like gauge group which breaks at the scale $M_G$.

As long as string model building is based on $k = 1$ level of Kac-Moody algebras, it was soon realised that ordinary GUTs (like the SU(5) model), could not be suitable candidates for a string derived model. The reason is that ordinary SU(5) needs necessarily Higgs fields in the adjoint in order to break down to the MSSM, while such representations are not available in $k = 1$ constructions.

The flipped SU(5) model[8, 9, 10], was a first attempt to overcome this difficulty, and has been proposed as a candidate some time ago[9], in the context of the free fermionic formulation of the four-dimensional superstring. It was subsequently discovered[11], that the GUT symmetry $SU(4) \times O(4)[11]$, based on the Pati-Salam model[12], can also break down to the standard gauge group without using higgs fields in the adjoint, and therefore could also serve as a candidate Superstring model. We should mention that it is also possible to obtain directly the Standard model gauge group from the string[13], but the proton decay problem[14] as well as the high string unification mentioned above, seem to be naturally solved in the presence of an intermediate gauge group. Moreover, the Grand Unified models inspired from strings[9, 11] offer natural mechanisms of suppressing the neutrino masses[15, 16], explain the baryon asymmetry[17] of the universe etc.

In view of the above theoretical and phenomenological constraints, in my opinion, the GUT model building issue should be reconsidered and worked out systematically. In the present talk, I will propose a specific fermion mass matrix ansatz[18], which was inspired from phenomenological calculations on the flipped string SU(5). The analysis of the charged fermion mass spectrum which is given in section 2, has been done on completely general grounds, and can apply to any SUSY-GUT model. Furthermore, in view of the revived interest[19, 21, 20, 22] of the fermion mass problem in GUTs, this work contributes also to the search[23] for all possible mass matrices with the maximum number of zero entries at the Unification Scale which give the correct predictions at $m_W$. The discussion on neutrino mass matrix is given in section 3, and applies mainly to the particular string constructions[9, 11, 13].

2. Structure of Fermion Mass Matrices at $M_{GUT}$

The texture of the quark and lepton mass matrices has the following form
at the GUT scale

\[ M_U = \begin{pmatrix} 0 & 0 & x \\ 0 & y & z \\ x & z & 1 \end{pmatrix} \lambda_{top}(t_0) \frac{v}{\sqrt{2}} \sin \beta, \quad (1) \]

\[ M_D = \begin{pmatrix} 0 & a e^{i \phi} & 0 \\ a e^{-i \phi} & b & 0 \\ 0 & 0 & 1 \end{pmatrix} \lambda_b(t_0) \frac{v}{\sqrt{2}} \cos \beta, \quad (2) \]

\[ M_E = \begin{pmatrix} 0 & a e^{i \phi} & 0 \\ a e^{-i \phi} & -3b & 0 \\ 0 & 0 & 1 \end{pmatrix} \lambda_{t}(t_0) \frac{v}{\sqrt{2}} \cos \beta, \quad (3) \]

\[ M_{\nu_{Dirac}} = M_U \quad (4) \]

with \( \tan \beta \equiv \frac{\langle h \rangle}{\langle \bar{h} \rangle} \), \( \lambda_b(t_0) = \lambda_{t}(t_0) \), and \( v = 246 \text{GeV} \). We have taken the up-quark matrix to be symmetric, and the down quark matrix to be hermitian. The Dirac neutrino mass matrix, has been taken to be identical to the up-quark mass matrix at the GUT scale, since both arise from the same Yukawa term in these theories. Furthermore, we have chosen to relate the charged lepton mass matrix at the GUT scale with the down quark mass matrix. As a matter of fact, the \( m_D \) and \( m_E \) matrix elements are not necessarily related in the flipped model. However, our choice minimizes the number of arbitrary parameters and moreover, it leads to definite predictions in the neutrino sector.

Let us discuss first the charged fermion mass matrices. Our ansatz has a total of five zeros in the quark sector (the leptonic sector is directly related to them); two zeros for the up and three for the down quark mass matrix (zeros in symmetric entries are counted only once). These zeros reduce the number of arbitrary parameters at the GUT-scale to eight, namely \( x, y, z, a, b, \phi, \lambda_b(t_0), \) and \( \lambda_{top}(t_0) \). These non-zero entries should serve to determine the thirteen arbitrary parameters of the standard model, i.e., nine quark and lepton masses, three mixing angles and the phase of the Cabbibo-Kobayashi-Maskawa (CKM) matrix. Thus, as far as the charged fermion sector is concerned, we end up with five predictions (we will discuss the predictions in the neutrino sector in the next section.). We may reduce the number of arbitrary parameters by one, if we impose more structure in the up-quark mass matrix. We may for example relate the \((13), (23)\) and \((22)\) entries, as follows

\[ y = nz^2 \quad (5) \]
\[ x = (n-1)z^2 \quad (6) \]

where \( n \) can be a number in the range \( n \sim (3, 10) \). Although this structure is imposed by hand and not actually necessary, the results are more presentable and calculations more easy to handle.

In order to find the structure of the mass matrices at the low energy scale and calculate the mass eigenstates as well as the mixing matrices and compare them with the experimental data, we need to evolve them down to \( m_W \), using the renormalization group equations. The renormalization group equations for the Yukawa couplings at the one-loop level, read
In this case all other mass matrices are also rotated by the same similarity trans-
formations.

\[ 16\pi^2 \frac{d}{dt} \lambda_U = \left( I \cdot Tr[3\lambda_U \lambda_U^T] + 3\lambda_U \lambda_U^T + \lambda_D \lambda_D^T - I \cdot G_U \right) \lambda_U, \]  
\[ 16\pi^2 \frac{d}{dt} \lambda_N = \left( I \cdot Tr[\lambda_N \lambda_N^T] + \lambda_E \lambda_E^T - I \cdot G_N \right) \lambda_N, \]  
\[ 16\pi^2 \frac{d}{dt} \lambda_D = \left( I \cdot Tr[3\lambda_D \lambda_D^T + \lambda_E \lambda_E^T] + 3\lambda_D \lambda_D^T + \lambda_U \lambda_U^T - I \cdot G_D \right) \lambda_D, \]  
\[ 16\pi^2 \frac{d}{dt} \lambda_E = \left( I \cdot Tr[3\lambda_E \lambda_E^T + 3\lambda_D \lambda_D^T] + 3\lambda_E \lambda_E^T - I \cdot G_E \right) \lambda_E, \]  

where \( \lambda_\alpha, \alpha = U, N, D, E, \) represent the 3x3 Yukawa matrices which are defined in terms of the mass matrices given in Eq. (14), and \( I \) is the 3x3 identity matrix. We have neglected one-loop corrections proportional to \( \lambda_i^4 \), \( t \equiv \ln(\mu/\mu_0) \), \( \mu \) is the scale at which the couplings are to be determined and \( \mu_0 \) is the reference scale, in our case the GUT scale. The gauge contributions are given by

\[ G_\alpha = \sum_{i=1}^{3} c^i_\alpha g_i^2(t), \]  
\[ g_i^2(t) = \frac{g_i^2(t_0)}{1 - b_i \pi^2 g_i^2(t_0)(t-t_0)}. \]  

The \( g_i \) are the three gauge coupling constants of the Standard Model and \( b_i \) are the corresponding supersymmetric beta functions. The coefficients \( c^i_\alpha \) are given by

\[ \{c_U\}_{i=1,2,3} = \left\{ \frac{13}{15}, \frac{16}{3}, \frac{16}{3} \right\}, \quad \{c_D\}_{i=1,2,3} = \left\{ \frac{7}{15}, \frac{3}{3}, \frac{3}{3} \right\}, \]  
\[ \{c_E\}_{i=1,2,3} = \left\{ \frac{9}{5}, 3, 0 \right\}, \quad \{c_N\}_{i=1,2,3} = \left\{ \frac{3}{5}, 3, 0 \right\}. \]  

In what follows, we will assume that \( \mu_0 = M_G \approx 10^{16} \text{GeV} \) and

\[ a_i(t_0) = \frac{g_i^2(t_0)}{4\pi} \approx \frac{1}{25}. \]

In order to evolve the equations (Eqs. (7-10)) down to low energies, we also need to do some plausible approximations. First, we find it convenient to diagonalize the up quark mass matrix at the GUT scale and obtain the eigenstates

\[ m_1(M_G) \approx -n(n-1)\tan^2\theta \sin^2\theta, \]  
\[ m_2(M_G) \approx (n-1)\tan^2\theta, \]  
\[ m_3(M_G) \approx \frac{p}{\cos^2\theta}, \]  

with diagonalizing matrix

\[ K = \left( \begin{array}{ccc} \frac{1}{D_1} & -\frac{\sin\theta}{D_2} & \frac{(n-1)\sin^2\theta}{D_3} \\ \frac{\sin\theta}{D_2} & \frac{1}{D_3} & \frac{-\sin\theta}{D_2} \\ \frac{n\sin\theta}{D_2} & \frac{n\sin\theta}{D_3} & 1-n\sin^2\theta \end{array} \right) \]  

where \( p = \lambda_i(t_0) \frac{1}{\sqrt{2}} \sin\beta, D_1 = \sqrt{1+\sin^2\theta \cos^2\theta}, D_2 = \sqrt{1+\sin^2\theta}, \) and \( D_3 \approx \sqrt{1-(2n-1)\sin^2\theta}. \) In this case all other mass matrices are also rotated by the same similarity transformation, thus

\[ \lambda_\alpha \rightarrow \tilde{\lambda}_\alpha = K^\dagger \lambda_\alpha K, \alpha = D, E, N \]  

(17)
Now assuming that the $\lambda_t$ coupling is much bigger than all other fermion Yukawa couplings, we may ignore the contributions of the latter in the RHS of the RGEs in Eqs (7-10). In this case all differential equations reduce to a simple uncoupled form. Thus the top-Yukawa coupling differential equation, for example, may be cast in the form:

$$16\pi^2 \frac{d}{dt} \tilde{\lambda}_{\text{top}} = \tilde{\lambda}_{\text{top}}(6\tilde{\lambda}_{\text{top}}^2 - G_U(t))$$

(18)

with the solution \[24, 20\]

$$\tilde{\lambda}_{\text{top}} = \tilde{\lambda}_{\text{top}}(t_0) \xi^6 \gamma_U(t)$$

(19)

where

$$\gamma_U(t) = \prod_{j=1}^{3} \left(1 - \left(1 - \frac{b_j,0(t - t_0)}{2\pi}\right)^{c_j/2b_j}\right),$$

(20)

$$\xi = \left(1 - \frac{6}{8\pi^2} \int_{t_0}^{t} \frac{\gamma_U^2(t)}{\gamma_U(t)} dt \right)^{-1/12}.$$

(21)

Thus the GUT up quark mass eigenstates given in Eq (13) renormalized down to their mass scale, are given by

$$m_u \approx \gamma_U \xi^3 \eta_u n(n-1) p tan^2 \theta sin^2 \theta$$

$$m_c \approx \gamma_U \xi^3 \eta_c n(n-1) p tan^2 \theta$$

$$m_t \approx \gamma_U \xi^6 \frac{p}{cos^2 \theta}$$

(22)

where $\eta_u$, $n_c$ are taking into account the renormalization effects from the $m_t$-scale down to the mass of the corresponding quark. We may combine the equations in Eq (22) and give predictions for the top quark mass and the mixing angle $\theta$ in terms of the low energy up and charm quark masses. We obtain

$$m_t \approx \xi^3 \frac{n}{n-1} \frac{m_u^2 \eta_u}{m_u \eta_c}$$

(23)

$$sin \theta = \sqrt{\frac{m_u}{nm_c}}$$

(24)

In the basis where the up-quark mass matrix is diagonal, the renormalized down-quark and charged lepton mass matrices are given by

$$m_D^{\text{ren}} \approx \gamma_D I_\xi K^1 m_D K$$

(25)

$$m_E^{\text{ren}} \approx \gamma_E K^1 m_E K$$

(26)

where $I_\xi = \text{Diagonal}(1, 1, \xi)$, and $m_D, E$, are given in Eqs (2) and (3). In order to make definite predictions in the fermion mass sector, we choose the lepton masses as inputs and express the arbitrary parameters $a, b, \lambda_t (= \lambda_0)$ in $m_E$ in terms of $m_e, m_\mu$, and $m_\tau$. Substituting into $m_D$ and diagonalizing $m_D^{\text{ren}}$, we obtain \[18\]

$$m_d \approx 6.3 \times \left(\frac{\eta_d}{2}\right) \text{MeV}$$

(27)
\[
m_s \approx 153 \times \left(\frac{\eta_d}{2}\right) \text{MeV} \quad (28)
\]
\[
m_b \approx \eta_b \frac{\gamma_D}{\gamma_E} \xi m_r \quad (29)
\]
Since \(\eta_d \approx \eta_s \approx 2\), the predictions for the light quarks \(m_{d,s}\) are within the expected range \[25\]. Now, in order to make a prediction for the bottom quark, we need to know the value of \(\xi\), but the latter depends on the top quark coupling at the GUT scale as well as on the top-mass, through Eq.(21). We can use, however, Eq.(29), to predict the range of the top mass. Thus, using the available limits on the bottom mass, \(4.15 \leq m_b \leq 4.35 \text{GeV}\), and \(\eta_b \approx 1.4\), we can obtain the following range for \(m_{top}\)
\[
125 \text{GeV} \leq m_{top} \leq (165 - 170) \text{GeV} \quad (30)
\]
The CKM - matrix can be found by diagonalizing \(m_D^{ren}\) in Eq(25). For all the above range of the \(m_{top}\) mass, it is always possible to find a set of the input parameters, which give CKM-mixing angles within the experimentally acceptable ranges. Its analytic form is rather complicated. As an example, we give the CKM-entries for the specific case where, \(m_t = 135 \text{GeV}\), \(\tan \beta = 1.1\), and \(\phi = \frac{\pi}{4}\). In this case \(m_b\) is predicted to be \(4.33 \text{GeV}\), while we obtain
\[
|V_{CKM}|_{ij} = \begin{pmatrix}
0.9754 & 0.22 & 0.0032 \\
0.22 & 0.9750 & 0.038 \\
0.01 & 0.037 & 0.9993
\end{pmatrix} \quad (31)
\]

2. Neutrino Masses

There is a lot of experimental evidence today, that the neutrinos have non-zero masses. For example, recent data from solar neutrino experiments \[26\] show that the deficiency of solar neutrino flux, i.e. the discrepancy between theoretical estimates and the experiment, is naturally explained if the \(\nu_e\) neutrino oscillates to another species during its flight to the earth. Furthermore, the COBE measurement \[27\] of the large scale microwave background anisotropy, might be explained \[28\] if one assumes an admixture of cold (\(~ 75\%) plus hot (\(~ 25\%) dark-matter. It is hopefully expected that some neutrino (most likely \(\nu_\tau\)) may be the natural candidate of the hot dark matter component.

Here we would like to address the question of neutrino masses in GUT models which arise \[9, 11\] in the free fermionic construction of four dimensional strings. Taking into account renormalization effects, it was recently shown \[16, 18\] that the general see-saw mechanism which occurs naturally in the flipped model, turns out to be consistent with the recent solar neutrino data, while on the other hand suggests that CHOROUS and NOMAD experiments at CERN may have a good chance of observing \(\nu_\mu \leftrightarrow \nu_\tau\) oscillations \[†\]

The various tree-level superpotential mass terms which contribute to the neutrino mass matrix in the flipped \(SU(5)\) model are the following:
\[
\lambda_{ij} F^i \tilde{f}^j \tilde{h} + \lambda_{ij}^{\phi^e} F^i \tilde{H} \phi^j + \lambda_{ij}^{\phi^0} \phi^0 \phi^j \quad (32)
\]

\[†\] for neutrinos in conventonal GUTs see \[29\]
where in the above terms $F^i, \tilde{F}^j$ are the 10,5 matter SU(5) fields while $\bar{H}, \bar{h}, h$ are the $\bar{10},5,5$ Higgs representations and $\phi^i$ are neutral $SU(5) \times U(1)$ singlets. The Higgs field $\bar{H}$ gets a vacuum expectation value (v.e.v.) of the order of the SU(5) breaking scale ($\sim 10^{16}$GeV), $\bar{h}, h$ contain the standard higgs doublets while $\phi^i$ acquires a v.e.v., most preferably at the electroweak scale. The neutrino mass matrix may also receive significant contributions from other sources. Of crucial importance, are the non-renormalizable contributions which may give a direct $M_{\nu c c} = M^{rad}$ contribution which is absent in the tree-level potential. Then, the general $9 \times 9$ neutrino mass matrix in the basis $(\nu_i, \nu^c_i, \phi_i)$, may be written as follows:

$$m_\nu = \begin{pmatrix} 0 & m_U & 0 \\ m_U & M^{rad} & M_{\nu c, \phi} \\ 0 & M_{\nu c, \phi} & \mu_\phi \end{pmatrix}$$  \hspace{1cm} (33)$$

where it is understood that all entries in Eq.(33) represent $3 \times 3$ matrices. The above neutrino matrix is different from that of standard see-saw matrix-form, since now there are three neutral SU(5) × U(1) singlets involved, one for each family.

It is clear that the matrix (33) depends on a relatively large number of parameters and a reliable estimate of the light neutrino masses and the mixing angles is a rather complicated task. We are going to use however our knowledge of the rest of the fermion spectrum to reduce sufficiently the number of parameters involved. Firstly, due to the GUT relation $m_U(M_{GUT}) = m_{\nu D}(M_{GUT})$, we can deduce the form of $m_{\nu D}(M_{GUT})$, at the GUT scale in terms of the up-quark masses. The heavy majorana $3 \times 3$ matrix $M^{rad}$, depends on the kind of the specific generating mechanism. Here we take it to be proportional to the down quark-matrix at the GUT scale:

$$M^{rad} = \Lambda^{rad} m_D(M_{GUT})$$ \hspace{1cm} (34)$$

The $M_{\nu c, \phi}$ and $\mu_\phi$ $3 \times 3$ submatrices are also model dependent. In most of the string models however, there is only one entry at the trilinear superpotential in the matrix $M_{\nu c, \phi}$, which is of the order $M_{GUT}$. Other terms, if any, usually arise from high order non-renormalizable terms. We will assume in this work only the existence of the trilinear term, since higher order ones will be comparable to $M^{rad}$ and are not going to change our predictions. In particular we will take $M_{\nu c, \phi} \sim \text{Diagonal}(M, 0, 0)$, and $\mu_\phi \sim \text{Diagonal}(\mu, 0, 0)$, with $\mu << M \sim M_{GUT}$, thus we will treat Eq.(33) as a $7 \times 7$ matrix.

To obtain the neutrino spectrum and lepton mixing, we must introduce values for the two additional parameters $M, \Lambda^{rad}$ of the neutrino mass matrix in Eq.(33). We assume naturally $M = <\bar{H}> \approx 10^{16}$GeV. The neutrino mass eigenvalues can now be predicted in terms of the scale quantity $\Lambda^{rad}$. Thus they can be written as:

$$m_{\nu e} \approx 0, m_{\nu \mu} = \frac{\Lambda^\mu}{\Lambda^{rad}} \times 10^{-2}eV, m_{\nu \tau} = \frac{\Lambda^\tau}{\Lambda^{rad}} \times 10eV$$  \hspace{1cm} (35)$$

For $m_t \approx 130GeV$ we get $\Lambda^\mu \approx .80 \times 10^{12}$ and $\Lambda^\tau \approx 1.85 \times 10^{12}$. 


For the oscillation probabilities, we find \[18\]

\[ P(\nu_e \leftrightarrow \nu_\mu) \approx 3.1 \times 10^{-2} \sin^2(\pi \frac{L}{l_{12}}) \] (36)

\[ P(\nu_\tau \rightarrow \nu_\mu) \approx 4.0 \times 10^{-3} \sin^2(\pi \frac{L}{l_{13}}) \] (37)

\[ P(\nu_e \rightarrow \nu_\tau) \approx 4.0 \times 10^{-5} \sin^2(\pi \frac{L}{l_{13}}) \] (38)

where \( L \) is the source–detector distance and

\[ l_{ij} = \frac{4\pi E_\nu}{|m_i^2 - m_j^2|} \] (39)

We can determine the range of \( \Lambda^{rad} \), using the available solar neutrino data (see for example ref\[31\] for a systematic analysis of the allowed regions using all available data)

\[ 5.0 \times 10^{-3} \leq \sin^22\theta_{ij} \leq 1.6 \times 10^{-2} \] (40)

\[ 0.32 \times 10^{-5} (eV)^2 \leq \delta m^2_{ij} \leq 1.2 \times 10^{-5} (eV)^2 \] (41)

Our result in Eq(36) is a bit outside the above range but the mass constraint can be easily satisfied by choosing \( \Lambda^{rad} \) in the range

\[ 0.7 \times 10^{12} \leq \Lambda^{rad} \leq 7 \times 10^{12} \] (42)

Our neutrino masses can also easily be made to fall into the range of the Frejus atmospheric neutrinos

\[ 10^{-3} (eV)^2 \leq \delta m^2_{ij} \leq 10^{-2} (eV)^2 \] (43)

but our mixing is much too small. Our results are also consistent with the data on \( \nu_\mu \leftrightarrow \nu_\tau \) oscillations\[32\]

\[ \sin^22\theta_{\mu\tau} \leq 4. \times 10^{-3}, \delta m^2_{\nu_\mu,\nu_\tau} \geq 50 (eV)^2 \] (44)

Our results however cannot be made to fall on the \( \sin^22\theta \) vs \( \delta m^2 \) of the BNL \( \nu_\mu \leftrightarrow \nu_e \) oscillation results\[33\].

Moreover, it is always possible to obtain \( m_{\nu_\tau} \approx (\text{few } \sim 20)eV \), hence one can obtain simultaneously the cosmological HOT-dark matter component, in agreement with the interpretation of the COBE data. Indeed an upper limit on the \( \nu_\tau \) mass can be obtained from the formula

\[ 7.5 \times 10^{-2} \leq \Omega_{\nu}h^2 \leq 0.3 \] (45)
Translating this into a constraint on $m_{\nu_{\tau}}$, arising from the relation $m_{\nu_{\tau}} \approx \Omega_{\nu} h^2 91.5 eV$ where $h = 0.5 \sim 1.0$ is the Hubble parameter, one gets the range

$$6.8 \leq m_{\nu_{\tau}} \leq 27 eV$$

which can be easily achieved with the above range of $\Lambda^{rad}$.

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