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Abstract. Based on the correspondence between the $N = 1$ superstring compactifications with fluxes and the $N = 4$ gauged supergravities, we study effective $N = 1$ four-dimensional supergravity potentials arising from fluxes and gaugino condensates in the framework of orbifold limits of (generalized) Calabi–Yau compactifications. We give examples in heterotic and type II orientifolds in which combined fluxes and condensates lead to vacua with small supersymmetry breaking scale. We clarify the respective roles of fluxes and condensates in supersymmetry breaking, and analyze the scaling properties of the gravitino mass.

1. Introduction

Gaugino condensation provides a way to induce nonperturbative supersymmetry breaking in $N = 1$ four-dimensional vacua. This phenomenon occurs in the infrared regime of strongly coupled gauge sectors \cite{1, 2} and affects the superpotential of the effective supergravity. These nonperturbative contributions coexist, in the effective superpotential, with the perturbative and nonperturbative moduli-dependent terms produced in heterotic \cite{2, 3}, type IIA \cite{4} and type IIB \cite{5} compactifications with fluxes.

The inclusion of the nonperturbative corrections in the flux-induced superpotential has been proposed by several authors \cite{6}. However, the conclusions have been either controversial or incomplete, mainly due to the pathological behaviour of the vacuum due \textit{e.g.} to a runaway behaviour of the moduli involved in the condensate or to a fine-tuning problem associated with the quantization of the flux coefficients. Destabilization of the no-scale structure with undesired transitions to anti-de Sitter vacua are also usual caveats.

In order to overcome the above difficulties, we must treat simultaneously the flux and the condensate contributions in a formalism which allows to capture unambiguously the corresponding effects in the superpotential. Fluxes are data given at the superstring level. Their translation into an $N = 1$ effective superpotential is ensured by the gauging procedure of some algebra allowed by the massless content of the theory \cite{7, 8}. Having a generic and unambiguous structure of the effective superpotential in the presence of fluxes and gaugino condensates, one can show that the usual pathologies of the vacuum are indeed avoided in heterotic, IIA and IIB $N = 1$ effective supergravities \cite{9}. This is possible provided one realizes that the issues of moduli...
stabilization, supersymmetry breaking, gaugino condensation and positivity of the potential, although related, must be treated separately. A straightforward corollary of this observation is that the nonperturbative contributions are not always the source for supersymmetry breaking, which affects the scaling properties of the gravitino mass.

The present contribution is a summary of the above results.

2. Fluxes, gaugings and the description of condensates

Even with broken supersymmetry, the underlying ten-dimensional theory encodes the constraints of \( N \geq 4 \) supersymmetry, which can then be used to derive information on the structure of the effective \( N = 1 \) supergravity. In \( N \geq 4 \) supergravities, the only available tool for generating a potential is to turn abelian gauge symmetries, naturally associated with vector fields, into non-abelian ones. This procedure of gauging introduces in the theory a gauge algebra \( G \) acting on the vector fields in the gravitational and/or vector supermultiplets [10]. The important fact is that from the point of view of the “daughter” \( N = 1 \) supergravity obtained after orbifold and/or orientifold projection, the gauging modifications only affect the superpotential \( W \), whereas the Kähler potential \( K \) and hence the kinetic terms remain the same as in the ungauged theory.

We will focus here on a \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold projection in heterotic (plus orientifold \( \Omega \) projection in type II), which leads to the following Kähler manifold:

\[
K = \left( \frac{SU(1,1)}{U(1)} \right)_S \times \prod_{A=1}^{3} \left( \frac{SO(2,2+n_A)}{SO(2) \times SO(2+n_A)} \right)_{T_A,U_A,Z_A^I}. \tag{1}
\]

Each string compactification is characterized by its own parameterization of the scalar manifold in terms of the seven complex scalars \( S, T_A, U_A \), \( A = 1, 2, 3 \) and the matter scalar fields \( Z_A^I \). In the case of heterotic, the index \( A \) labels the three complex planes defined by the \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) symmetry used for the orbifold projection. For type II compactifications, this holds up to field redefinitions, which mix all \( S, T_A \) and \( U_A \). The structure of the scalar manifold remains however unaltered.

This choice of parameterization singles out the appropriately redefined geometric moduli and the dilaton as \( \Re T_A, \Re U_A, \Re S \). Neglecting for simplicity the matter scalar fields \( Z_A^I \), the manifold of the geometric moduli is reduced to

\[
K = \left( \frac{SU(1,1)}{U(1)} \right)_S \times \prod_{A=1}^{3} \left( \frac{SU(1,1)}{U(1)} \right)_{T_A} \times \prod_{A=1}^{3} \left( \frac{SU(1,1)}{U(1)} \right)_{U_A}. \tag{2}
\]

The Kähler potential associated to these field takes for all cases the following form:

\[
K = -\log (S + \bar{S}) - \sum_{A=1}^{3} \log (T_A + \bar{T}_A) (U_A + \bar{U}_A). \tag{3}
\]

Due to the \( SU(1,1)^7 \)-structure of the manifold, the scalar potential considerably simplifies and takes the following suggestive form,

\[
e^{-K}V = \sum_i |W_i(Z_i + \bar{Z}_i)|^2 - 3|W|^2, \tag{4}
\]

where \( \{Z_i\} \equiv \{S, T_A, U_A\} \) and \( W_i = \partial_{Z_i}W \).

The structure of the superpotential is also well understood. The perturbative part of the latter is polynomial in the moduli with coefficients related to the fluxes of the underlying string theory. The simplicity of the approach based on gauging of the underlying \( N = 4 \) supersymmetry
algebra allows to study exhaustively various situations and establish a precise dictionary among monomial coefficients in the superpotential and fluxes (including also spin-connection geometric fluxes). The precise analysis can be found in [7] where we concentrated on orientifolds of type IIA strings which offered the broadest structure of allowed fluxes and had been explored to a lesser extent [4].

The non-perturbative effects originating from gaugino condensation provide modifications in the superpotential of the effective supergravity theory. These are of the form

\[ W_{\text{nonpert}} = \mu^3 \exp \left( -\frac{24\pi^2 Z}{b_0} \right), \tag{5} \]

where \( b_0 \) is a one-loop beta-function coefficient, \( \mu \) a scale at which the Wilson coupling \( g^2(\mu) \) is defined and \( Z \) a modulus such that \( \Re Z = g^{-2}(\mu) \). The expectation value of the nonperturbative superpotential defines the renormalization-group-invariant transmutation scale \( \Lambda \) of the confining gauge sector in which gauginos condense, \( \langle W_{\text{nonpert}} \rangle = \Lambda^3 \). The nature of the modulus \( Z \) depends on the underlying string (or M-) theory compactification: in the heterotic string, it is identified with the dilaton \( S \) field, whereas in type II orientifolds \( Z \) is the redefined \( S \) field in IIA theories [7] and a combination of \( T \) and \( S \) in IIB (or F-theory) compactifications [2, 7, 11, 12]. The resulting exponent is a number of order ten or more and \( n \)-instanton corrections \((n > 1)\) are exponentially suppressed.

Two remarks are in order here. First, many gaugino condensates could form and the nonperturbative superpotential could include several similar terms involving various moduli. Second, the scale \( \mu \) in (5) is in general a modulus-dependent quantity.

We will not expand any longer on the general structure of the nonperturbative contributions to the superpotential. A comprehensive analysis can be found in [13]. We will instead focus on a simple situation where the superpotential reads:

\[ W = a + w(S), \quad w(S) = \mu^3 e^{-S} \tag{6} \]

\( (S \) has been rescaled according to \( 24\pi^2 S/b_0 \to S \), which leaves the corresponding kinetic terms unchanged and multiplies the scalar potential by an overall factor). The scale \( \mu \) and the quantity \( a \) are in general moduli-dependent; \( a \) includes the perturbative contributions induced by fluxes\(^6\).

3. The fine-tuning and the runaway problems

Consider now the situation in heterotic where the geometrical fluxes are absent and thus the superpotential is \( T_A \)-independent. The scalar potential becomes:

\[ e^{-K} V = \sum_{\{Z_i\} \equiv \{S, U_A\}} |W - W_i(Z_i + \bar{Z}_i)|^2. \tag{7} \]

This exhibits a no-scale structure [14], with a semi-positive-definite potential and flat directions \( \{T_A\} \). The \( U_A \) moduli are generically fixed by their minimization conditions and \( a \) and \( \mu^3 \) in Eqs. (6) are effectively constant. The remaining minimization condition for the \( S \) field,

\[ a + (S + S + 1) w(S) = 0, \tag{8} \]

determines the value of \( S \). Supersymmetry is broken in the \( T_A \)-directions, in Minkowski space. Equation (8) shows that an exponentially small value of \( w(S) \) necessarily implies \( |a| \ll 1 \), a stringent fine-tuning condition. This is a severe problem in situations (such as the \( Z_3 \) orbifold

\(^6\) It does not depend on \( S \) in heterotic compactifications.
in heterotic) where \( a \) is directly given by constant perturbative fluxes, hence it must either vanish or be of order one. A vanishing \( a \) is also problematic because it leads to a \textit{runaway} potential, \( V \propto |\mu|^6 \exp-(S + \bar{S}) \). The attempts that have been proposed in the past for improving this situation include \textit{e.g.} multiple gauge group condensations (without fluxes). These do not help in removing the fine-tuning problem with a non-zero \( a \).

The fine-tuning and runaway problems are facets of the vacuum structure of the theory. As such, they can be understood \textit{only} from a comprehensive analysis of the combined perturbative and nonperturbative contributions, which equally contribute in the structure of the vacuum. In other words, it makes little sense to focus first on stabilizing the moduli from the flux superpotential, and add the condensate contributions in a second stage. Usually the first-stage flux superpotential turns out to be “too stable” to lead to relevant phenomenology once condensates are added.

One can illustrate the above in a type IIB model with superpotential

\[
W = A [1 + U_1 U_2 + U_2 U_3 + U_3 U_1 + \gamma S (U_1 + U_2 + U_3 + U_1 U_2 U_3)] + i B [U_1 + U_2 + U_3 + U_1 U_2 U_3 + \gamma S (1 + U_1 U_2 + U_2 U_3 + U_3 U_1)],
\]

(9)

Here \( A, B \) and \( \gamma A, \gamma B \) are respectively proportional to R-R and NS-NS flux numbers. The no-scale structure with semi-positive-definite scalar potential is due to the absence of any \( T_A \) dependence. The moduli \( (U_A, \gamma S) \) are fixed to unity and supersymmetry is broken in flat space with gravitino mass \( m_{3/2}^2 \propto (A^2 + B^2)/\prod_A (T_A + T_A) \). Nonperturbative contributions to the superpotential (9) may originate from \( D_3 \)- or \( D_7 \)-branes. In the latter case, these contributions are of the form \( \exp(-\alpha T) \) and their presence spoils \textit{arbitrarily} the no-scale structure and destabilizes the Minkowski vacuum: the moduli \( T \) gets stabilized but the potential becomes negative, \( V = -3m_{3/2}^2 \), as required by unbroken supersymmetry in anti-de Sitter space.

4. \textbf{Stationary points and Minkowski vacuum}

When supersymmetry breaks, the analysis of the non-positive scalar potential as a function of seven complex fields is difficult. It is somewhat simpler under the assumption of vanishing of the potential at the minimum. In a general supergravity theory with Kähler potential \( K = -\sum_j \ln(Z_j + \bar{Z}_j) \), supersymmetry is spontaneously broken if the equations

\[
F_j \equiv W - (Z_j + \bar{Z}_j) W_j = 0
\]

(10)
cannot be solved for all scalar fields \( Z_j \) (and with \( \Re Z_j > 0 \)). If supersymmetry breaks in Minkowski space, we have also

\[
\langle V \rangle = 0, \quad \langle W \rangle \neq 0.
\]

(11)

A stationary point of the scalar potential is a solution of the equation \( \partial_j V = 0 \), for each scalar field \( Z_j \). The explicit analysis of these equations, under the assumptions (11), can be worked out explicitly. The scalar fields split in two categories for which we use lower- \((a,b,\ldots)\) and upper-case \((A,B,\ldots)\) indices respectively: either \( \langle W_a \rangle = 0 \) and \( \langle F_A \rangle = \langle W \rangle \neq 0 \), or \( \langle F_A \rangle = 0 \). Supersymmetry breaking is controlled by the first category only. The Minkowski condition, \( \langle V \rangle = 0 \), implies then that this category contains precisely \textit{three} fields: the contribution of each of these fields to \( \langle V \rangle \) cancels one unit of the negative term \( -3\langle WW \rangle \). The seven minimization equations finally read:

\[
0 = \sum_{a=1}^3 W_{aj} \Re Z_a \quad \forall j \ (a \text{ or } A),
\]

(12)

(the summation is restricted over moduli which break supersymmetry i.e. with \( \langle W_a \rangle = 0 \)).
5. Supersymmetry breaking independent of the gaugino condensation

Nonperturbative phenomena do not necessarily break supersymmetry, independently of their effect on the stabilization of the moduli and on the positivity of the potential. The introduction of fluxes can indeed stabilize some of the moduli in flat space without inducing supersymmetry breaking. To be concrete, let us consider a superpotential with “supersymmetric mass terms” only:

\[ W_{\text{susy}} = A(U_1 - U_2)(T_1 - T_2) + B(U_1 + U_2 - 2U_3)(T_1 + T_2 - 2T_3) + (T_1 + T_2 - 2T_3)w(S), \]

with \( w(S) \) as given in Eq. (6). The perturbative part of this superpotential is created by geometrical fluxes, either in heterotic or in type IIA. It can be directly generated at the string level using freely acting orbifold constructions. It selects four (complex) directions in the seven-dimensional space of the moduli fields and minimizing the potential tends to cancel the fields in these four directions. Supersymmetry is not broken and cancellation of auxiliary fields (\( \langle F_A \rangle = 0 \)) fixes \( U_A = U \) and \( T_A = T \).

The condensate term \( (T_1 + T_2 - 2T_3)w(S) \) can be understood from the general form of the \( N = 4 \) superpotential (for details, see [9]). The presence of \( w(S) \) leaves \( U_1 = U_2 = U \) but we now have \( U_3 = U + w(S)/2B \). The above conclusions about supersymmetry remain however unchanged: supersymmetry is unbroken and the gravitino is massless in flat background.

The previous example can be modified by the addition of further flux terms which break supersymmetry. Consider for example, in the heterotic or type IIA,

\[ W_{\text{total}} = W_{\text{susy}} + W_{\text{break}} \]

with \( W_{\text{susy}} \) as in Eq. (13) and

\[ W_{\text{break}} = R(T_1U_1 + T_2U_2). \]

The term \( W_{\text{break}} \) breaks supersymmetry even in the absence of \( w(S) \). The scalar potential has a minimum with real \( T_A, U_A \) and \( T_A = T, U_1 = U_2 = U, U_3 = U + w(S)/2B \). The potential vanishes along the flat directions \( S, T \) and \( U \). The goldstino field is a combination of the fermionic partners of \( S, T_3 \) and \( U_3 \). There is an “effective” no-scale structure: since \( W_1 \) vanishes in these three directions, their corresponding contributions to the potential cancel the gravitational contribution \(-3m_{3/2}^2\). Thus supersymmetry is broken in flat space–time with

\[ m_{3/2}^2 = \frac{|R|^2}{32ST_3U_3} \]

and the presence of the nonperturbative term \( w(S) \) only acts as a small perturbation on supersymmetry breaking induced by the modulus-dependent contribution \( W_{\text{break}} \). It however explicitly appears in mass terms.

Many other examples can be worked out in the same line of thought in heterotic, type IIA and type IIB, where the role of the gaugino condensate is not crucial for the supersymmetry breaking. An explicit breaking term, generated by a specific combination of fluxes, has to be superimposed to the mass terms, in order for the supersymmetry to be broken, independently of the presence of the condensate. This situation is not generic, however, and examples exist where supersymmetry breaking is triggered by the gaugino condensates.

6. Gaugino-induced supersymmetry breaking

We will now analyze situations where the gaugino condensate breaks supersymmetry with \( m_{3/2} \) being related to the gaugino scale \( w(S) \). In these cases the form of the superpotential will be again

\[ W = W_{\text{susy}} + \mu^3(Z_i) e^{-S}, \]

with \( \mu \) being the gaugino mass parameter.
but \( \mu^3(Z_i) \) will no longer vanish at the minimum.

6.1. Type II

Consider the following type IIA superpotential

\[
W = (T_1 - T_2) (-U_1 + U_2 - T_3 + 2S) + (U_1 T_3 - L) w(S),
\]

which is generated by geometric and \( F_2 \) fluxes. It falls in the class mentioned in Sec. 4, where there is a partition between directions which break supersymmetry (here \( T_1, T_2 \) and \( U_3 \)) and directions which preserve supersymmetry (\( T_3, U_1, U_2 \) and \( S \)).

The requirement \( \langle W_{T_1} \rangle = \langle W_{T_2} \rangle = 0 \) ensures \( \langle V \rangle = 0 \) since \( W \) is independent of \( U_3 \), and the resulting supersymmetry-breaking condition reads

\[
-U_1 + U_2 - T_3 + 2S = 0.
\]

The vanishing of the \( F \)-auxiliary fields in the directions \( T_3, U_1, U_2 \) and \( S \) leads to the following equations:

\[
\xi \left( U_1 + U_2 - T_3 + 2S \right) - (U_1 T_3 + L) w(S) = 0,
\]

(19)

\[
\xi \left( -U_1 + U_2 - T_3 + 2S \right) + (U_1 T_3 - L) w(S) = 0,
\]

(20)

\[
\xi \left( -U_1 + U_2 + T_3 + 2S \right) - (U_1 T_3 + L) w(S) = 0,
\]

(21)

\[
\xi \left( -U_1 + U_2 - T_3 - 2S \right) + (U_1 T_3 - L) \left( 1 + S + \bar{S} \right) w(S) = 0,
\]

(22)

where we have introduced

\[
\xi \equiv T_1 - T_2.
\]

(23)

The minimization condition (12) reads here

\[
\Re \xi = 0.
\]

(24)

Equations (18)–(24) must be solved for \( \xi, T_3, U_1, U_2 \) and \( S \). Combining Eqs. (19) and (21), one concludes that \( T_3 = U_1 = t \). The requirement (24) can be fulfilled by adjusting appropriately the imaginary part of the \( S \) field: \( S = s - \frac{2}{\xi} (\pi - 6 \varphi_\mu) \) (we have introduced \( \mu = |\mu| \exp i \varphi_\mu \)). This implies through Eq. (18) that \( U_2 = u + i (\pi - 6 \varphi_\mu) \). The final equations for \( t, u \) and \( s \) are (18), a combination of (19) and (20) as well as a combination of (18), (19) and (22):

\[
u + 2(s - t) = 0,
\]

(25)

\[
t \left( t^2 - L \right) - u \left( t^2 + L \right) = 0,
\]

(26)

\[
t^6 + 2Lt^3 - 4Lt^2 - 3L^2 t - 4L^2 = 0.
\]

(27)

We will not reproduce the full analysis of these equations here, but instead state the results (details are available in [9]). Assuming the flux number \( L \) large, the leading and sub-leading behaviour for \( t, u \) and \( s \) is

\[
t = \sqrt{L} + 1 + \mathcal{O} \left( \frac{1}{\sqrt{L}} \right),
\]

(28)

\[
u = 1 + \mathcal{O} \left( \frac{1}{L} \right),
\]

(29)

\[
s = \sqrt{L} + \frac{1}{2} + \mathcal{O} \left( \frac{1}{\sqrt{L}} \right).
\]

(30)
Next one can compute
\[ \Im \xi \approx \sqrt{L} |\mu|^3 e^{-\sqrt{L}}, \tag{31} \]
which measures the supersymmetry breaking. The gravitino mass scales as
\[ e^{-K/2} m_{3/2} \approx 2i\sqrt{L} |\mu|^3 e^{-\sqrt{L}}. \tag{32} \]
The last two equations show that the gaugino condensate is entirely responsible for the breaking of supersymmetry. Notice also that as advertised previously, the fluxes generating the superpotential (17) are not fine-tuned, and solutions for the moduli exist generically.

6.2. Heterotic
Because of the absence of perturbative \( S \)-contributions in the heterotic superpotential, heterotic and type II are drastically different when the breaking of supersymmetry is induced by a gaugino condensate. Let us concentrate on a superpotential of the type
\[ W = \hat{A} U_1 + \hat{B} U_2 + \hat{C} U_3 + \hat{D} U_4, \tag{33} \]
where \( U_4 = U_1 U_1 U_3 \). This superpotential is odd in the \( U_i \)’s and captures most of the heterotic compactifications considered here, with a gaugino condensate. We have introduced the following functions of \( T_1, T_2 \) and \( S \):
\[ \hat{A} = \left[ \alpha + \alpha' w(S) \right] \xi + A w(S), \tag{34} \]
\[ \hat{B} = \left[ \beta + \beta' w(S) \right] \xi + B w(S), \tag{35} \]
\[ \hat{C} = \left[ \gamma + \gamma' w(S) \right] \xi + C w(S), \tag{36} \]
\[ \hat{D} = \left[ \delta + \delta' w(S) \right] \xi + D w(S), \tag{37} \]
where \( \xi = T_1 - T_2 \) as defined in (23) and \( w(S) \) in (6).

The minimization condition (12) reads \( \Re \xi = 0 \), as in the above type IIA example. We will therefore choose \( S = s - i(\pi - 6\phi_\mu)/2 \) and \( U_i = u_i \) real. Everything is consistent provided \( \alpha, \beta, \gamma, \delta \) and \( A, B, C, D \) are real and \( \alpha', \beta', \gamma', \delta' \) are imaginary.

The no-scale requirement \( \langle V \rangle = 0 \) is fulfilled provided \( \langle W_{T_1} \rangle = \langle W_{T_2} \rangle = 0 \) (\( W \) is independent of \( T_3 \)). The corresponding condition reads:
\[ (\alpha + \alpha' w) u_1 + (\beta + \beta' w) u_2 + (\gamma + \gamma' w) u_3 + (\delta + \delta' w) u_4 = 0. \tag{38} \]
The vanishing of the \( U_A \)-auxiliary fields leads to
\[ -\hat{A} u_1 + \hat{B} u_2 + \hat{C} u_3 - \hat{D} u_4 = 0, \tag{39} \]
\[ \hat{A} u_1 - \hat{B} u_2 + \hat{C} u_3 - \hat{D} u_4 = 0, \tag{40} \]
\[ \hat{A} u_1 + \hat{B} u_2 - \hat{C} u_3 - \hat{D} u_4 = 0 \tag{41} \]
and the equation for the \( S \)-auxiliary field (after some simplification involving Eqs. (38)–(41)) reads:
\[ \frac{2}{s} = -4 - \left( \frac{\alpha'}{A} + \frac{\beta'}{B} + \frac{\gamma'}{C} + \frac{\delta'}{D} \right) \xi w. \tag{42} \]
The equations at hand can be solved. We will exhibit a solution in the plane-symmetric situation, where
\[ \alpha = \beta = \gamma, \quad \alpha' = \beta' = \gamma', \quad A = B = C, \tag{43} \]
which imply that $\hat{A} = \hat{B} = \hat{C}$ and consequently 

$$u \equiv u_1 = u_2 = u_3 = \sqrt{\frac{A}{D}} \quad \text{and} \quad u_4 = u^3. \quad (44)$$

The final set of equations for $\xi$, $u$ and $s$ is therefore (38), (42) and (44). Eliminating $u$ from (38) (by using (44)) leads to

$$4\xi = -\frac{3D\alpha + A\delta + (3D\alpha' + A\delta')w}{(\alpha + \alpha'w)(\delta + \delta'w)} w. \quad (45)$$

The latter can be further used in Eq. (42) (together with (34)) to obtain the central equation for the determination of $s$:

$$\frac{2}{s} = -4 - \frac{(\alpha'\delta - \delta'\alpha)w}{(\alpha + \alpha'w)(\delta + \delta'w)} \frac{3D\alpha + A\delta + (3\alpha'D + A\delta')w}{D\alpha - A\delta + (D\alpha' - A\delta')w}. \quad (46)$$

For further simplification we specialize to

$$\alpha' = i\alpha, \quad \delta' = -i\delta. \quad (47)$$

Our aim is to show that Eq. (46) indeed admits physically acceptable solutions for $s$, provided that the fluxes $\alpha, \delta, A$ and $D$ are large, while their ratios (such as $D\alpha/A\delta$) are of order unity. Under these assumptions, we can perform an expansion in powers of $w$ for all quantities. We find the following dominant contributions (Eqs. (44), (45) and (46)):

$$u \approx \sqrt{-\frac{3\alpha}{\delta}}, \quad (48)$$

$$\xi \approx -\frac{D}{\delta} w, \quad (49)$$

$$\frac{A\delta - D\alpha}{D\alpha} \approx 2\frac{4s + 1}{2s + 1} w(s). \quad (50)$$

The latter equation is compatible with large values of $s$. In that regime it further simplifies:

$$s \approx \log \left( \frac{4|\mu|^3D\alpha}{D\alpha - A\delta} \right) - \frac{1}{4 \log \left( \frac{4|\mu|^3D\alpha}{D\alpha - A\delta} \right)}, \quad (51)$$

Using finally Eqs. (33), (34) and (44), in the special case captured by (47) and within the above approximations, the gravitino mass reads:

$$e^{-K/2}m_{3/2} \approx i4D \left( -\frac{3\alpha}{\delta} \right)^{3/2} \frac{s}{2s + 1} w^2, \quad (52)$$

with $s$ given in Eq. (51). The gravitino mass scales as $w^2$ instead of $w$ like in previous type II example. This is due to the absence of flux-induced $S$-term in the superpotential, which is a generic feature in heterotic compactifications.
7. Conclusions

The important outcome of the above analysis is that the pathological behaviour of the vacuum in the presence of fluxes with nonperturbative corrections is not a generic property of the $N=1$ effective supergravity, with or without spontaneously broken supersymmetry: the usual caveats quoted previously can be avoided with a suitable combination of fluxes and nonperturbative contributions. When appropriate, such a combination is a valuable tool for circumventing the runaway behaviour of moduli or the fine-tuning problem. Hence, understanding these mechanisms can shed light on the nature of the vacuum and the stabilization of moduli.

Our analysis makes also clear the importance of investigating the full superpotential, including both flux-induced perturbative and nonperturbative contributions. Although nonperturbative corrections do not necessarily trigger supersymmetry breaking, they can alter various terms and must therefore be taken into account at a very early stage: they can drastically change the picture of moduli stabilization drawn by fluxes only.

On the practical side, the situations that we have investigated fall in at least three classes, according to the scaling behaviour of the gravitino mass:

(i) Situations where the nonperturbative corrections do not trigger the supersymmetry breaking – they modify the mass terms though – in which the gravitino mass scales like:

$$m_{3/2} = c / \sqrt{V},$$

where $V = \exp K$ is the volume of the moduli space and $c$ is related to the flux numbers. In these cases, any mass hierarchy strongly relies on the volume $V$ of the moduli space. This behaviour can occur in all types of string compactifications, type IIA,B and heterotic.

(ii) Situations where the nonperturbative contributions to the superpotential induce the supersymmetry breaking, where the gravitino mass is controlled by the nonperturbative superpotential $w(S)$, like the type IIA example presented in Sec. 6,

$$m_{3/2} = c w(S) / \sqrt{V},$$

as commonly expected from nonperturbative supersymmetry breaking. In these cases, the mass hierarchy can be created irrespectively of the size of the volume of the moduli space.

(iii) The third scaling behaviour is more unusual but still generic in heterotic. It appears whenever the gaugino condensate is the only source of $S$-dependence. The gravitino mass scales now as

$$m_{3/2} = c w(S)^2 / \sqrt{V}.$$  \hspace{1cm} (54)

Hence, the supersymmetry breaking creates a mass hierarchy stronger than in the two previous cases. The heterotic realizations under consideration are actually quite generic despite the fact that the ratios between flux coefficients are required to be of order one, while the coefficients themselves are large. However, it is clear that heterotic models deserve a more systematic investigation, where gauge condensates of the type $\langle F \rangle$ are taken into account, together with the gaugino ones. This should shed light on the validity of various gaugino-condensate-induced supersymmetry breaking scenarios advertised in the literature but not captured by the analysis presented here.

It should finally be stressed that the analysis of the $N=1$, low-energy soft-supersymmetry-breaking terms strongly depends on the class of model under consideration since their pattern is mostly controlled by the radiative corrections induced by the supersymmetry-breaking sector.
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