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Computational process to study the wave propagation In a non-linear medium by quasi-linearization

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Abstract: Two objects having distinct velocities come into contact an impact can occur. The impact study i.e., in the displacement of the objects after the impact, the impact force is a function of time ‘t’ which is behaves similar to compression force. The impact tenure is very short so impulses must be generated subsequently high stresses are generated. In this work we are examined the wave propagation inside the object after collision and measured the object non-linear behavior in the one-dimensional case. Wave transmission is studied by means of material acoustic parameter value. The objective of this paper is to present a computational study of propagating pulsation and harmonic waves in nonlinear media using quasi-linearization and subsequently utilized the central difference scheme. This study gives focus on longitudinal, one-dimensional wave propagation. In the finite difference scheme Non-linear system is reduced to a linear system by quasi-linearization method. The computed results exhibit good agreement on par with the selected non-linear wave propagation.

1. Introduction

Two bodies have distinct velocities in the same direction come into contact, an impact occurs. In the impact analysis i.e., in the displacement of the bodies after the impact, impact force is a function of time ‘t’ is acting like a density force [2]. The objective of this work is to present a computational study of propagating pulsed and harmonic waves in a nonlinear media by using a Finite difference scheme. This study aims on longitudinal, one-dimensional wave propagation. In the finite difference scheme Non-linear model is reduced to a linear system by quasi-linearization method. The numerically computed results exhibit good quality agreement with the non-linear wave equation character.

1.1 FORMULATION OF THE PROBLEM

An object of length L1 contacts another object of length ‘L2’. Both the objects have the same material configuration with non-linearity. The first object has an initial velocity of V0, whereas the second one is at rest.

Here c (u) ≥ 0, R_N ≤ 0 & R_N c (u) =0

(1)
Here Reaction force ($R_N$), Normal gap $c(u)$ are always perpendicular to another. This kind of literature is available in the monuments [2], [5], [8] & [10].

Materials by means of plastic deformation, Materials with distributed gap, linear elastic Hooke’s law is usually not adequate to describe nonlinear, inelastic nature. Here we can study the class of materials whose nature can be described by the following stress-strain equation

$$\frac{\partial \sigma(\varepsilon, \varepsilon')}{\partial \varepsilon} = g(\varepsilon) - \alpha s(\varepsilon - \varepsilon') - \alpha s \left[ \int g'(\tau) - \frac{df(\tau)}{d\tau} \right] e^{\alpha s(\tau - \varepsilon)} d\tau$$

(2)

Where $\varepsilon_0$ is the initial strain $s(\varepsilon')$, $\alpha$ is a constant, and $f(\varepsilon)$ and $g(\varepsilon)$ are functions to be evaluated.

Now in a particular case of (2) namely with no initial stress and strain is considered as

$$\frac{\partial \sigma(\varepsilon, \varepsilon^1)}{\partial \varepsilon} = g(\varepsilon) + \alpha s(0) e^{-\alpha s} - \alpha s \left[ \int g'(\tau) - \frac{df(\tau)}{d\tau} \right] e^{\alpha s[\tau - \varepsilon]} d\tau$$

(2a)

Here we consider

$$\frac{1}{C^2} \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = \frac{1}{E} \frac{\partial \sigma}{\partial \varepsilon} - 1$$

(3)

Where $E$ is the elastic Young’s modulus and ‘c’ can be considered as the phase velocity. The nonlinear equation solved by applying quasi-linearization method [3]. While the process is initiated then iteration across the time–step is introduced to reduce the non-linear equation into linear.

For nonlinear materials type

$$\sigma = E \left( \varepsilon - \frac{1}{2} \gamma \varepsilon^2 \right)$$

(4)

Clearly, when $\gamma = 0$, the material is linear elastic. The parameter $\gamma$ indicates the amount of material nonlinearity. The parameter $\gamma$ defined here is identical to the acoustic nonlinear parameter. The acoustic nonlinear parameter comes in metals due to lattice non-harmonicity which is usually very small in comparison to the elastic deformation of the metals. So we can study wave propagation nature for various acceptable values of $\gamma$. Here we are selected in the acceptable region, i.e. the values $\gamma = 10000$, $\gamma = 5000$ and $\gamma = 2500$ respectively.

Apply (3) in (4)
\[ \frac{\partial^2 u}{\partial t^2} = c^2 \left( 1 - \gamma \frac{\partial u}{\partial x} \right) \frac{\partial^2 u}{\partial x^2} \]  

(5)

Equation (5) is the non-linear wave equation developed by Gol’dberg.

2. Computational solution of non-linear wave equation using Quasi-linearization

Consider the non-linear wave equation with a small rearrangement

\[ \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x^2} \]  

(6)

(6) is of the form

\[ f(u, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}) = 0 \]  

(7)

Apply the quasi-linearization method [3] on the governing equation (7) we have

\[ f(u_x, u_{xx}, u_{tt}) \bigg|_{n} + (u_x^{n+1} - u_x^n) \frac{\partial f}{\partial u_x} \bigg|_{n} + (u_{xx}^{n+1} - u_{xx}^n) \frac{\partial f}{\partial u_{xx}} \bigg|_{n} + (u_{tt}^{n+1}) \frac{\partial f}{\partial u_{tt}} \bigg|_{n} = 0 \]  

(8)

\[ (u_{tt} - u_{xx} + \gamma u_x u_{xx})_{(n)} + (u_{x}^{n+1} - u_x^n) \gamma u_{xx} \bigg|_{(n)} + (u_{xx}^{n+1} - u_{xx}^n) \gamma u_x \bigg|_{(n)} + (u_{tt}^{n+1} - u_{tt}^n) \gamma u_x \bigg|_{(n)} = 0 \]  

(9)

equation (9) after simplification can be expressed as

\[ u_{tt}^{(n+1)} + \gamma u_x(n) u_{xx}(n+1) + \gamma u_{xx}(n) u_x(n+1) - u_{xx}(n) - \gamma u_x(n) u_{xx}(n) = 0 \]  

(10)  

\( (n+1) \)th stage is now iterative so that we can get the solution profiles at \( n=0, 1, 2, 3, 4 \ldots \) so that

\[ u_{i,j+1} = u_{i,j} + 2(1-\beta) u_{i,j} + \beta (u_{i+1,j} + u_{i-1,j}) \]  

(11)

With the defined initial and boundary conditions in mesh point notation transformed to equations (12) to (14) respectively.

\[ \Rightarrow u_{i,1} = u_{i,0} + v_0 \]  

(12)
\[ \Rightarrow u_{i,1} = u_{i,0} \]  

Initial displacement \( u(x, 0) = 0.125 \sin x \)

\[ \Rightarrow u(ih,0) = 0.125 \sin(ih) \]  

Apply the quasi-linearization technique on (11) with (12) to (14) one can get the following results. Also the wave propagation is plotted with various time levels with initial velocity \( v_0 = 5 \text{ m/s} \). We are selected the exponentially decreasing harmonic input, \( \frac{\partial u}{\partial x} = 0.005 \exp(-t/5) \sin t \). In (11) the function values at the \( j^{th} \) and \( j-1^{th} \) time levels are required in order to evaluate at the \( (j+1)^{th} \) time level. Here select the mesh ratio \( \alpha = \frac{k^2}{h^2} \leq 1 \) for consistent, stable and convergent solution profiles. From equation (10) & (11) with the given initial and boundary conditions (12) to (14) the subsequent computational results can be obtained by using simple Newton-Raphson method. While evaluating the solution profiles at each mesh point applied the Thomas algorithm to solve the system of linear equations. The wave propagation patterns are also plotted with various time levels with selected initial velocity.

3. Computational results

CASE-1: at \( \gamma = 10000 \)

| X   | LEVEL-1 | LEVEL-2 | LEVEL-3 | LEVEL-4 | LEVEL-5 |
|-----|---------|---------|---------|---------|---------|
| 0   | 0       | 0       | 0       | 0       | 0       |
| 0.5 | 0.559928| -0.17976| -2.03978| -28.8021| -518.078|
| 1   | 0.5     | 1.119854| 2.814675| 27.12049| 524.336 |
| 8   | 0       | 0.999981| 3.514522| 41.43012| 854.3651|
| 8.5 | 0       | 0       | -0.75729| -19.6799| -494.568|
| 9   | 0       | 0       | 0       | 2.735351| 142.7683|
| 9.5 | 0       | 0.0027  | 0       | 0       | -14.93  |
| 14.5| 0       | 0       | 0       | 0       | 0       |
| 15  | -3      | -3      | -3      | -3      | -3      |

Table.1(a) Displacement values
Figure 1(a): Non-linear wave propagation

CASE-II at $y = 5000$

| X   | LEVEL-1 | LEVEL-2 | LEVEL-3 | LEVEL-4 | LEVEL-5 |
|-----|---------|---------|---------|---------|---------|
| 0   | 0       | 0       | 0       | 0       | 0       |
| 0.5 | 0.559828| 0.403166| 0.278242| -0.63727| -9.03406|
| 1   | 0.5     | 1.063496| 1.537378| 3.049896| 12.76054|
| 1.5 | 0.5     | 1       | 1.507661| 1.998268| 0.67732 |
| 2   | 0.5     | 1       | 1.5     | 1.993664| 2.468401|
| 2.5 | 0.5     | 1       | 1.5     | 2       | 2.510962|
| 9.5 | 0       | 0       | 0       | 0       | 0.091954|
| 14.5| 0       | 0       | 0       | 0       | 0       |
| 15  | -3      | -3      | -3      | -3      | -3      |

Table 1(b): Displacement values
CASE-III: \( \gamma = 2500 \)

| X   | LEVEL-1(u) | LEVEL-2  | LEVEL-3 | LEVEL-4 | LEVEL-5  |
|-----|------------|----------|---------|---------|----------|
| 0   | 0          | 0        | 0       | 0       | 0        |
| 0.5 | 0.559928   | 0.111702 | 0.585477| 0.685795| 0.688854 |
| 1   | 0.5        | 1.091675 | 0.545387| 0.651041| 0.877135 |
| 1.5 | 0.5        | 1        | 1.597348| 1.508572| 1.378224 |
| 8.5 | 0          | 0        | 0.812207| 0.747197| 0.633375 |
| 14.5| 0          | 0        | 0       | 0       | 0        |
| 15  | -3         | -3       | -3      | -3      | -3       |

Table.1(c): Displacement of waves numerical computation
4. Analysis of the Computational results & Conclusions

1. At the lower and the upper places of the object the collision might be random; It indicates the inelastic collision. In other words, the loss of kinetic energy may be constant and converted into equivalent sound and/or heat dissipated in to the surroundings.
2. At the center level the objects collision may be uniform; it indicates the elastic collision. In other words gain of kinetic energy is sustained in the collision.

An impact taking place the velocities of the two objects are varies according to the starting compression force existed at the impact point. An impact occurs a longitudinal sound wave is generated, it propagates in the region up to free end of the second object. When it went to the free end a reflection occurs. So the boundary condition at the free end is selected as negative but very small in magnitude.

The displacement in terms of length of the impact system with respective to time is showed in the Figure 1(a) -1(d)

5. Observations

1) At \( \gamma =10000 \) at the second level the displacement \( u(x,t) \) exhibits non-linearity at the middle of the position of the objects and at all other time levels, no non-linearity is observed.

2) At \( \gamma =5000 \) complete time levels, displacement sustains with respect to the origin except at time level 5. At end positions at time level 5 non-linearity is observed.

3) At \( \gamma =2500 \) all the time levels shows the displacement with disturbance at end positions (0-2 cm and 7-10 cm) and the middle position the displacement is found to constant and rises with respective to the previous time level-1.

4) At lower and higher Acoustic values non-linearity is not observed clearly but it gives the tendency. At middle \( \gamma \) value the non-linearity behavior is clearly observed at higher time level-5.

5) For all the acoustic parameter values displacement \( u \) is observed to be constant at 2 to 6 units distance with respective to time level.

REFERENCES

[1] Non linear wave propagation in disordered media, Angel sanchez, Departmento de fisica telrica, Facultad de ciencias, Fisicas, Universidad complutense, Spain., 1991.

[2] Achenbach. J.D., 1999. Wave Propagation in Elastic Solids. Elsevier Science publishers BV, Amsterdam. Berntsen,J.,Tiotta.J.N.,Tjotta. S., 1984 Nearfield of a large acoustic transducer, Part IV: Second harmonic and sum frequency.

[3] R. Bellman, R. Kalaba, Quasilinearization and Nonlinear Boundary Value Problems. Elseiver, New York (1965)

[4] Bland, D.R., 1969. Nonlinear Dynamic Elasticity. Blasdell Publishing Company, Waltham. Massachusetts.

[5] Debnath, L., 1997. Nonlinear Partial differential Equations for Scientists and Engineers. Birkhauser.
[6] Drum Heller, 1998. *Introduction to wave propagation in Nonlinear Fluids and Solids*. Cambridge University Press, Cambridge.

[7] Gruttmann F., Sauer R. and Wagner W. Theory and numeric’s of three-dimensional beams with elastoplastic material behavior. *International Journal for Numerical Methods in Engineering*, 48:1675-1702 (2000).

[8] *Non-Linear Wave Propagation With Applications to Physics and Magneto hydro dynamics* by A. Jeffrey and T Taniuti, Volume 9, 1st Edition.

[9] Reduction of nonlinear contrast agent scattering due to nonlinear wave propagation, R. Hansen; B. A. J. Angelsen; T. F. Johansen 2001 *IEEE Ultrasonics Symposium. Proceedings. An International Symposium (Cat. No.01CH37263)*, 2001.

[10] Laursen T.A. and Meng X. Two dimensional mortar contact methods for large deformation frictional sliding. *International Journal for Numerical Methods in Engineering*, 62: 1183-1225 (2005).

[11] Christoph prueel, Jin-Yeon kim, Jiamin qu and Laurence J. Jacobs, Evaluation of fatigue damage using non-linear guided waves *Smart Materials and Structures* IOP publications (2009).