Quantum field theory of interacting plasmon–photon system

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Abstract
In the framework of functional integral approach, quantum theory of interacting plasmon–photon system was constructed on the basis of general postulates (axioms) called also first principles of electrodynamics and quantum theory of many-body systems. Since plasmons are complex quasiparticles appearing as the resonances in plasma oscillations of the electron gas in solids, we start from the general expression of total action functional of interacting system consisting of electron gas and electromagnetic field. The collective oscillations of electron gas are characterized by a real scalar field \( \phi(x) \) called the collective oscillation field. In the harmonic approximation the collective oscillations behave like the small fluctuations around a background field \( \phi_0(x) \). The difference between \( \phi(x) \) and \( \phi_0(x) \) is called the fluctuation field \( \zeta(x) \). In the case of a homogeneous and isotropic electron gas the fluctuation field \( \zeta(x) \) is a linear functional of another real scalar field \( \sigma(x) \) satisfying the wave equation similar to the Klein–Gordon equation in relativistic quantum field theory. The quanta of corresponding Hermitian scalar field \( \sigma(x) \) are called plasmons. The real scalar field \( \sigma(x) \) is called plasmonic field. The total action functional of the interacting system of plasmonic and electromagnetic field was derived.

Keywords: plasmon, plasmonics, electron gas, functional integral, action functional
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1. Introduction

During the last two decades research on the interaction of plasmon with photon and other quasiparticles in matter has stimulated the rapid development of a new scientific area: plasmonics [1, 2]. However, the authors of both experimental and theoretical works on plasmonic processes and phenomena often accepted corresponding interaction Hamiltonians. In our recent article [3] we have proposed to derive the interaction Hamiltonians or action functional determining matrix elements of all plasmonic processes from basic postulates (axioms), also called first principles, of electrodynamics and quantum theory. For this purpose the effective mathematical tool is functional integral technique [4–6]. The ideas proposed in [3] will be implemented in the present work. The concrete expression of the action functional determining matrix elements of many plasmon–photon interaction processes will be exactly derived from three fundamental postulates of quantum theory: (i) assumption on the explicit form of total functional integral of interacting electron–photon system, (ii) extreme action principle, and (iii) canonical quantization procedure.

The original physical system is that of itinerant electrons in solid with the quantum mechanical single electron Hamiltonian

\[
H = \frac{-i}{\hbar} \frac{\partial}{\partial x} - i \frac{\partial}{\partial x}^2 + U(x),
\]

where \( U(x) \) is potential energy of electron in electrostatic field of ions in the crystal lattice and \( m \) is effective mass of electron. We use the unit system with \( \hbar = c = 1 \). The above-mentioned electron system will also be called the electron gas.
Since we study only physical processes and phenomena in which the electron spin plays no role, we can consider electron as a spinless fermion with anticommuting wave function $\psi(x, t)$. Its Hermitian conjugate is denoted $\bar{\psi}(x, t)$. They are often called dynamical Grassmann variables. Being the photon wave function in transverse gauge, the electromagnetic field is a commuting vector field $A(x, t)$ with vanishing divergence

$$\frac{\partial A(x, t)}{\partial x} = 0. \tag{2}$$

For shortening formulae we shall use the following brief notations:

$$x = (x, t) = (x, x_0),$$

$$\int dx = \int dx \int dt = \int dx \int dx_0,$$

$$\psi(x) = \psi(x, t) = \psi(x, x_0),$$

$$\bar{\psi}(x) = \bar{\psi}(x, t) = \bar{\psi}(x, x_0),$$

$$A(x) = A(x, t) = A(x, x_0),$$

the integrations being performed over the whole volume or interval of values of corresponding integration variables.

In section 2 the functional integral technique will be applied to the study of interacting electron–photon system. The expression of total functional integral of this system will be presented.

In section 3 a real scalar function $\varphi(x)$ playing the role of the order parameter of collective oscillations in electron gas will be introduced. It will be called the collective oscillation field. The expression of interacting system of collective oscillation field $\varphi(x)$ and electromagnetic field $A(x)$ will be established.

In the harmonic approximation with respect to scalar field $\varphi(x)$ this field can be split into two parts, the background static field $\varphi_0(x)$ and the fluctuation field $\zeta(x)$. In section 4 the total action functional of interacting system of two fields $\zeta(x)$ and $A(x)$ will be derived.

In section 5 it will be shown that in the harmonic approximation with respect to collective oscillation field $\varphi(x)$ and at vanishing absolute temperature $T=0$ K, the fluctuation field $\zeta(x)$ is expressed in terms of a real scalar field $\sigma(x)$ such that the quanta of the corresponding quantized scalar field $\sigma(x)$ are plasmons. From the expression of total effective action functional of the system of two fields $\zeta(x)$ and $A(x)$ it is straightforward to derive the action functional of interacting system of two fields $\sigma(x)$ and $A(x)$ in the form of a functional power series with respect to three fields: $\varphi_0(x)$, $\sigma(x)$ and $A(x)$.

The conclusion and discussions will be presented in section 6.

2. Functional integral of interacting electron–photon system

We start from following expression of the functional integral of the interacting electron–photon system

$$Z_{\text{tot}} = \int [D\psi][\bar{D}\bar{\psi}] \int [DA] \delta\left(\frac{\partial A}{\partial x}\right) \times \exp\left\{i\int_0^t \bar{\psi}(x, t) \bar{\psi}(x, t) \bar{\psi}(x, t) \bar{\psi}(x, t) \right\} \tag{3}$$

where $I_{\text{tot}}[\psi, \bar{\psi}, A]$ is the total action functional of this system. Due to the presence of $\delta$-function of $\partial A/\partial x$, condition (2) is always satisfied. $I_{\text{tot}}[\psi, \bar{\psi}, A]$ is composed of three parts

$$I_{\text{tot}}[\psi, \bar{\psi}, A] = I_0^e[A] + I'[\psi', \bar{\psi}], \tag{4}$$

where $I_0^e[A]$ is the action of free electromagnetic field, $I'[\psi', \bar{\psi}]$ is that of electron system in the presence of electron–electron Coulomb interaction and $I_{\text{int}}[\psi, \bar{\psi}, A]$ is the action functional of electron–photon interaction. It was known [7] that in the transverse gauge

$$I_0^e[A] = \frac{1}{2} \int dx \left[ \left( \frac{\partial A(x)}{\partial x_0} \right)^2 - \left( \frac{\partial}{\partial x} \wedge A(x) \right)^2 \right]. \tag{5}$$

$I'[\psi, \bar{\psi}]$ is composed of two parts

$$I'[\psi, \bar{\psi}] = I_0'[\psi, \bar{\psi}] + I_{\text{int}}[\psi, \bar{\psi}], \tag{6}$$

where $I_0'[\psi, \bar{\psi}]$ is the action functional of free electron system

$$I_0'[\psi, \bar{\psi}] = \int dx \bar{\psi}(x) \left[ i \frac{\partial}{\partial x_0} - H\left(-i \frac{\partial}{\partial x}, x\right) \right] \psi(x). \tag{7}$$

and $I_{\text{int}}[\psi, \bar{\psi}]$ is that of electron–electron Coulomb interaction

$$I_{\text{int}}[\psi, \bar{\psi}] = -\frac{1}{2} \int dx \int dy \bar{\psi}(x) \psi(y) e(x-y) \times \bar{\psi}(y) \psi(y), \tag{8}$$

$$u(x - y) = \delta(x_0 - y_0) u(x - y), \tag{9}$$

$$u(x - y) = \frac{e^2}{|x - y|}, \tag{10}$$

$-e$ being the electron charge. The action functional of electron–photon interaction has the following expression

$$I_{\text{int}}[\psi, \bar{\psi}; A] = -\int dx \bar{\psi}(x) H_{\text{int}} \left(-i \frac{\partial}{\partial x}, A(x)\right) \psi(x), \tag{11}$$

where $H_{\text{int}}\left(-i \frac{\partial}{\partial x}, A(x)\right)$ is the quantum mechanical Hamiltonian of electron–photon interaction

$$H_{\text{int}}\left(-i \frac{\partial}{\partial x}, A(x)\right) = H\left(-i \frac{\partial}{\partial x} + eA(x), x\right)$$

$$-H\left(-i \frac{\partial}{\partial x}, x\right), \tag{12}$$
It is easy to verify that
\[
I_{\text{int}}[\psi, \bar{\psi}; \mathbf{A}] = \frac{e^2}{2m} \int dx \sum_i \left[ \bar{\psi}(x) \left( \frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_i} \right) \psi(x) \right] 
\]
\[\times A_i(x) - \frac{e^2}{2m} \int dx \bar{\psi}(x) \psi(x) \mathbf{A}(x)^2.\quad (13)
\]

In formula (13) we used the following notation
\[
\bar{\psi}(x) \frac{\partial}{\partial x_i} \psi(x) = \frac{\partial}{\partial x_i} \bar{\psi}(x) \psi(x).\quad (14)
\]

The total functional integral of interacting electron–photon system has the expression
\[
Z_{\text{tot}} = \int [DA] \delta \left( \frac{\partial A}{\partial \mathbf{x}} \right) \int [D\varphi] [D\bar{\psi}] 
\times \exp \{ I_{\text{int}}^0[\mathbf{A}] \} \exp \{ I_{\text{int}}^0[\bar{\psi}, \psi] \}
\times \exp \{ I_{\text{int}}^\dagger[\bar{\psi}, \psi] \} \exp \{ I_{\text{int}}[\bar{\psi}, \psi; \mathbf{A}] \}.\quad (15)
\]

When both electron–electron Coulomb interaction as well as electron–photon interaction are neglected, \(Z_{\text{tot}}\) becomes the product
\[
Z_{\text{tot}} = Z_0^0 Z_0^e
\]
of the functional integral of free electromagnetic field
\[
Z_0^e = \int [DA] \delta \left( \frac{\partial A}{\partial \mathbf{x}} \right) \exp \{ I_{\text{int}}^0[\mathbf{A}] \} \quad (16)
\]
and that of free electron system
\[
Z_0^\dagger = \int [D\varphi] [D\bar{\psi}] \exp \{ I_{\text{int}}^\dagger[\bar{\psi}, \psi] \}.\quad (17)
\]

If only electron–photon interaction is neglected, \(Z_{\text{tot}}\) is the product
\[
Z_{\text{tot}} = Z_0^0 Z_0^e
\]
of the functional integral \(Z_0^0\) of free electromagnetic field and that of electron system with only electron–electron Coulomb interaction
\[
Z_0^e = \int [D\varphi] [D\bar{\psi}] \exp \{ I_{\text{int}}^\dagger[\bar{\psi}, \psi] \}.\quad (18)
\]

The system with functional integral (18) was studied in [4, 5].

3. Functional integral of interacting system of collective oscillation field and electromagnetic field

The Coulomb interaction action functional (8) is bilinear with respect to the electron density \(\psi(x)\bar{\psi}(x)\). In order to linearize this expression we apply celebrated Hubbard–Stratonovich transformation [8, 9], as was proposed in [4, 5]:
\[
\exp \left\{ -\frac{i}{2} \int \! dx \int \! dy \psi(x)\bar{\psi}(x)u(x - y)\varphi(y)\psi(y) \right\}
\]
\[
F^{(2,0)}[\varphi, \mathbf{A}] = \frac{(-i)^2}{2} \int dx \int dx' \int dy \int dy' \varphi(x') \times u(x' - x) \varphi(y') \varphi(y' - y) \times [n(x)n(y) - G(x - y)G(y - x)],
\]
\[
F^{(0,1)}[\varphi, \mathbf{A}] = 0,
\]
\[
F^{(0,2)}[\varphi, \mathbf{A}] = -i \frac{e^2}{2m} \int dx n(x) \mathbf{A}(x)^2,
\]
\[
F^{(1,1)}[\varphi, \mathbf{A}] = -i \frac{e}{2m} \int dx dx' \int dy \varphi(x') \times u(x' - x) \left[ G(y - x) \frac{\partial G(y - x)}{\partial y} \right. \\
\left. - G(y - x) \frac{\partial G(y - x)}{\partial y} - G(z - x) \frac{\partial G(z - x)}{\partial z} \right] \mathbf{A}(y),
\]
\[
F^{(0,2)}[\varphi, \mathbf{A}] = \frac{e^2}{2m} \int dx dx' \int dy \varphi(x') u(x' - x) \times \left[ G(y - x) \frac{\partial G(y - x)}{\partial y} \right. \\
\left. - G(y - x) \frac{\partial G(y - x)}{\partial y} - G(z - x) \frac{\partial G(z - x)}{\partial z} \right] \mathbf{A}(y)^2,
\]
\[
F^{(2,1)}[\varphi, \mathbf{A}] = \frac{e^2}{2m} \int dx dx' \int dy \int dz \int dz' \varphi(x') \times u(x' - x) \varphi(y') \varphi(y' - y) \times \left[ G(y - z) \frac{\partial G(z - y)}{\partial z} \right. \\
\left. - G(z - y) \frac{\partial G(z - y)}{\partial z} - G(z - x) \frac{\partial G(z - x)}{\partial z} \right] \mathbf{A}(z),
\]
\[
F^{(2,2)}[\varphi, \mathbf{A}] = \frac{e^2}{2m} \int dx dx' \int dy \int dy' \int dz \int dz' \varphi(x') \times u(x' - x) \varphi(y') \varphi(y' - y) \times \left[ G(y - z) \frac{\partial G(z - y)}{\partial z} \right. \\
\left. - G(z - y) \frac{\partial G(z - y)}{\partial z} - G(z - x) \frac{\partial G(z - x)}{\partial z} \right] \mathbf{A}(z)^2.
\]

and so on. Function \(G(x - y)\) in formulæ (26), (29)–(32) is the two-point Green function of electron in electron gas

\[
G(x - y) = \frac{1}{Z_0} \int [D\psi] \left[ D\bar{\psi} \right] \psi(x) \psi(y) \times \exp \left[ \mu \int_0^1 \int_0^1 \right],
\]
\[
\text{Using formulæ (23)–(32) we obtain the expression}
\]

\[
F[\varphi, \mathbf{A}] = \exp \left[ iW[\varphi, \mathbf{A}] \right],
\]

where \(W[\varphi, \mathbf{A}]\) is the series

\[
W[\varphi, \mathbf{A}] = \sum_{m,n=0}^{\infty} W^{(m,n)}[\varphi, \mathbf{A}]
\]

with the following terms

\[
W^{(0,0)}[\varphi, \mathbf{A}] = 0,
\]
\[
W^{(1,0)}[\varphi, \mathbf{A}] = -\frac{e^2}{2m} \int dx \int dx' \varphi(x') u(x' - x) n(x),
\]
\[
W^{(1,0)}[\varphi, \mathbf{A}] = \frac{e}{2m} \int dx dx' \int dy \int dy' \varphi(x') \times u(x' - x) \Pi(x - y) u(y - y') \varphi(y'),
\]

where

\[
\Pi(x - y) = -iG(x - y)G(y - x),
\]
\[
W^{(0,1)}[\varphi, \mathbf{A}] = 0,
\]
\[
W^{(0,2)}[\varphi, \mathbf{A}] = -\frac{e^2}{2m} \int dx n(x) \mathbf{A}(x)^2,
\]
\[
W^{(1,1)}[\varphi, \mathbf{A}] = \frac{e}{2m} \int dx dx' \int dy \varphi(x') \times u(x' - x) \Pi(x - y) \mathbf{A}(y)^2,
\]

\[
W^{(1,2)}[\varphi, \mathbf{A}] = \frac{e^2}{2m} \int dx dx' \int dy \int dy' \int dz \varphi(x') \times u(x' - x) \Pi(x - y) \mathbf{A}(y)^2,
\]
\[
W^{(2,1)}[\varphi, \mathbf{A}] = \frac{e^2}{2m} \int dx dx' \int dy \int dy' \int dz \int dz' \varphi(x') \times u(x' - x) \varphi(y') \varphi(y' - y) \times \mathbf{A}(x - z, y - z) \mathbf{A}(z),
\]

\[
W^{(2,2)}[\varphi, \mathbf{A}] = -\frac{e^2}{2m} \int dx dx' \int dy \int dy' \int dz \varphi(x') \times (x') u(x' - x) \varphi(y') u(y' - y) \times \mathbf{A}(x - z, y - z) \mathbf{A}(z)^2,
\]

where

\[
A(x - z, y - z) = \frac{1}{2} \left[ G(x - y) G(y - z) G(z - x) + G(x - y) G(x - z) G(y - y) \right],
\]

and so on.
In terms of functional $W[\varphi, A]$ the expression (22) of $Z_{\text{tot}}$ can be rewritten as follows

$$Z_{\text{tot}} = \frac{Z_{\text{tot}}}{Z_{\varphi}} \int [DA] \delta \left( \frac{\partial A}{\partial x} \right) \int [D\varphi] \exp \left[ i I_{\text{tot}}[\varphi, A] \right] \tag{49}$$

with

$$I_{\text{tot}}[\varphi, A] = I_0^A[\varphi] + \frac{1}{2} \int \int dy \varphi(x) u \times (x - y) \varphi(y) + W[\varphi, A]. \tag{50}$$

Functional $I_{\text{tot}}[\varphi, A]$ can be interpreted as total action functional of the system of two fields $\varphi(x)$ and $A(x)$. It is a functional power series of the form

$$I_{\text{tot}}[\varphi, A] = \sum_{m,n=0}^{\infty} I^{(m,n)}[\varphi, A], \tag{51}$$

where the term $I^{(m,n)}[\varphi, A]$ is a homogeneous functional of $m$th order with respect to the field $\varphi(x)$ and $n$th order with respect to the field $A(x)$. Note that

$$I^{(0,0)}[\varphi, A] = 0. \tag{52}$$

4. Effective action functional of interacting fluctuation field and electromagnetic field

Before the study of two interacting fields we review some related known results concerning scalar field $\varphi(x)$ without its interaction with electromagnetic field and limit to the second order (harmonic) approximation for the simplicity of subsequent applications. In [3, 4] it was shown that scalar field $\varphi(x)$ is split into two parts

$$\varphi(x) = \varphi_0(x) + \zeta(x), \tag{53}$$

where $\varphi_0(x)$ is the background field corresponding to the extreme value of action functional $I_0[\varphi]$, and $\zeta(x)$ is the field of small fluctuations around background field $\varphi_0(x)$. We call $\zeta(x)$ the fluctuation field.

The background field $\varphi_0(x)$ is determined by the following equation

$$\int dy \, K(x - y) \varphi_0(y) = \int dy \, u(x - y) u(y) = \int dy \, u(x - y) u(y) \tag{54}$$

with

$$K(x - y) = u(x - y) + \int dx' \int dy' u(x - x') \times \Pi(x' - y') u(y' - y), \tag{55}$$

where the physical meaning of functions $u(x - y), u(x - y), n(x)$ and the definition of function $\Pi(x - y)$ were presented in preceding sections. In terms of background field $\varphi_0(x)$ and fluctuation field $\zeta(x)$ the action functional $I_0[\varphi]$ of collective oscillation field $\varphi(x)$ in the harmonic approximation has the expression

$$I_0[\varphi_0 + \zeta] = I_0[\varphi_0] + I_{\text{eff}}[\zeta], \tag{56}$$

where

$$I_0[\varphi_0] = -\frac{1}{2} \int dx \int dy \, \varphi_0(x) K(x - y) \varphi_0(y) \tag{57}$$

and

$$I_{\text{eff}}[\zeta] = \frac{1}{2} \int dx \int dy \, \zeta(x) K(x - y) \zeta(y). \tag{58}$$

Functional (58) can be considered as the effective action functional of fluctuation field $\zeta(x)$. The dynamical equation for this field is

$$\int dy \, K(x - y) \zeta(y) = 0. \tag{59}$$

From equation (54) it follows that $\varphi_0(x)$ is a static (time-dependent) field

$$\varphi_0(x) = \varphi_0(x). \tag{60}$$

Now we establish the expression of total effective action functional $I_{\text{tot}}[\zeta, A]$ of the interacting system of fluctuation field $\zeta(x)$ and electromagnetic field $A(x)$ in the spatial domain in which there exists static background field $\varphi_0(x)$. Using formula (22) for functional integral of the system of two interacting field $\varphi(x), A(x)$ and expression (34) of functional $F[\varphi, A]$, we obtain

$$I_{\text{tot}}[\zeta, A] = I_0^A[\zeta] + W[\varphi_0 + \zeta, A] + \frac{1}{2} \int dx \int dy \, \varphi_0(x) + \zeta(x) + u(x - y) \int dy \, \varphi_0(y) + \zeta(y). \tag{61}$$

Functional (61) can be considered as a functional power series of three functions $\varphi_0(x), \zeta(x)$ and $A(x)$:

$$I_{\text{tot}}[\zeta, A] = \sum_{l,m,n=0}^{\infty} I^{(l,m,n)}[\varphi_0, \zeta, A]. \tag{62}$$

the term $I^{(l,m,n)}[\varphi_0, \zeta, A]$ is a homogeneous functional power of $l$th, $m$th and $n$th orders with respect to the fields $\varphi_0(x), \zeta(x)$ and $A(x)$ as three functional variables, respectively. In particular

$$I^{(0,0,0)}[\varphi_0, \zeta, A] = I^{(0,0,1)}[\varphi_0, \zeta, A] = 0. \tag{63}$$

Calculating the terms $I^{(l,m,n)}[\varphi_0, \zeta, A]$ in the harmonic approximation with respect to collective oscillation field $\varphi(x)$, we obtain the following result:

$$\sum_{l,m=0}^{\infty} I^{(l,m,0)}[\varphi_0, \zeta, A] = I_0[\varphi_0] + I_{\text{eff}}[\zeta]. \tag{64}$$

$$I^{(0,0,2)}[\varphi_0, \zeta, A] = I_0^A[\zeta] - \frac{e^2}{2m} \int dx \, n(x) A(x)^2, \tag{65}$$

$$I^{(1,0,1)}[\varphi_0, \zeta, A] = \frac{e}{2m} \int dx \int dx' \int dy \, \varphi_0(x') \times u(x' - x) \Pi(x - y) A(y), \tag{66}$$

$$I^{(2,0,0)}[\varphi_0, \zeta, A] = \frac{e^2}{2m} \int dx \int dx' \int dy \, \varphi_0(x') \times u(x' - x) \Pi(x - y) A(y)$$
In order to establish the expression of effective action functional of the interacting system of plasmonic and electromagnetic fields let us start from effective action functional $I_{\text{eff}}[\zeta, \mathbf{A}]$ derived in the preceding section and express $\zeta(x)$ in terms of the plasmonic field. As a simple example let us consider the case of a homogeneous and isotropic electron gas with a constant electron density $n(x)=n$ at vanishing absolute temperature $T=0 \text{K}$ and denote $p_F$ the electron momentum at Fermi surface

$$n = \frac{1}{(2\pi)^3} \frac{4\pi}{3} p_F^3.$$  

Denote $\tilde{K}(k) = \tilde{K}(k, \omega)$ and $\tilde{\zeta}(k) = \tilde{\zeta}(k, \omega)$ Fourier transforms of $K(x-y)$ and $\zeta(x)$:

$$\begin{aligned}
K(x-y) &= \frac{1}{(2\pi)^2} \int dk \ e^{ik(x-y)} \tilde{K}(k) \\
&= \frac{1}{(2\pi)^2} \int dk \ \int d\omega \ e^{i(kx-\omega y)} \\
&\times \tilde{K}(k, \omega),
\end{aligned}$$

$$\begin{aligned}
\zeta(x) &= \frac{1}{(2\pi)^3} \int dk \ e^{ikx} \tilde{\zeta}(k) \\
&= \frac{1}{(2\pi)^3} \int dk \ \int d\omega \ e^{i(kx-\omega y)} \tilde{\zeta}(k, \omega).
\end{aligned}$$

In [3–5] it was shown that in the first-order approximation with respect to the ratio $k^2/\omega_F^2$, where $\omega_F$ is the plasma frequency of electron gas

$$\omega_F^2 = \frac{4\pi^2 n}{m},$$

$\tilde{K}(k, \omega)$ has the following expression

$$\begin{aligned}
\tilde{K}(k, \omega) &= \frac{4\pi^2}{k^2 \omega^2} \left( \omega^2 - \omega_F^2 - \gamma^2 k^2 \right),
\end{aligned}$$

where

$$\gamma^2 = \frac{3}{5} \frac{p_F^2}{m^2}.$$ 

In terms of the Fourier transforms $\tilde{K}(k, \omega)$ and $\tilde{\zeta}(k, \omega)$ formula (58) becomes

$$I_{\text{eff}}[\zeta] = \frac{1}{(2\pi)^2} \int dk \ \int d\omega \ \frac{1}{2} \tilde{z}(k, \omega)^*$$

$$\times \tilde{K}(k, \omega) \tilde{\zeta}(k, \omega).$$

Setting

$$\tilde{\sigma}(k, \omega) = \frac{4\pi e^2}{\omega_F^2 k^2} \tilde{\zeta}(k, \omega),$$

introducing the real scalar field $\sigma(x) = \sigma(x, t)$ with Fourier
transform $\sigma(k, \omega)$ and denoting

$$I_{pl}[\sigma] = I_{\text{eff}}[\xi]$$

(84)

the effective action functional of this new real scalar field $\sigma(x)$, we obtain

$$I_{pl}[\sigma] = \frac{1}{(2\pi)^3} \int \frac{dk}{2\pi} \int d\omega \frac{1}{2} \hat{\sigma}(k, \omega)^* \times \left[ \alpha^2 - \alpha^2_{\text{pl}} - \gamma^2 k^2 \right] \tilde{\sigma}(k, \omega).$$

(85)

In the space–time coordinate representation formula (85) becomes

$$I_{pl}[\sigma] = \frac{1}{2} \int dt \int dx \left\{ \left[ \frac{\partial \sigma(x, t)}{\partial t} \right]^2 - \gamma^2 \left[ \nabla \sigma(x, t) \right]^2 - \alpha^2_{\text{pl}} \sigma(x, t)^2 \right\}.$$ (86)

It has the same form as that of action functional of Klein–Gordon real scalar field in relativistic field theory [10–13] except for the scaling factor $\gamma$ of spatial coordinate $x$. Canonical generalized momentum $x(\sigma, t)$ corresponding to canonical generalized coordinate $\sigma(x, t)$, by definition, is the function

$$\pi(x, t) = \frac{\partial I[\sigma]}{\partial \frac{\partial \sigma(x, t)}{\partial t}} = \frac{\partial \sigma(x, t)}{\partial t}.$$ (87)

Let us now perform the canonical quantization procedure. Then real scalar functions $\sigma(x, t)$ and $\pi(x, t)$ are replaced by Hermitian operators $\hat{\sigma}(x, t)$ and $\hat{\pi}(x, t)$, satisfying the following equal-time canonical quantization rules:

$$\left[ \hat{\sigma}(x, t), \hat{\pi}(y, t) \right] = \left[ \hat{x}(x, t), \hat{\pi}(y, t) \right] = 0,$$

$$\left[ \hat{\sigma}(x, t), \hat{\pi}(y, t) \right] = -i \delta(x - y).$$ (88)

In the analogy with relativistic Klein–Gordon Hermitian scalar quantum field we decompose the generalized coordinate operator $\hat{\sigma}(x, t)$ as follows:

$$\hat{\sigma}(x, t) = \frac{1}{(2\pi)^{3/2}} \int \frac{dk}{\sqrt{2\omega}} e^{i(kx - \omega t)} \hat{a}(k) + e^{-i(kx - \omega t)} \hat{a}^+(k),$$

(89)

where

$$\omega = \sqrt{\alpha^2_{\text{pl}} + \gamma^2 k^2}.$$ (90)

Then the generalized momentum operator $\pi(x, t)$ has the following decomposition

$$\hat{\pi}(x, t) = -\frac{i}{(2\pi)^{3/2}} \int \frac{dk}{\sqrt{2\omega}} \left\{ e^{i(kx - \omega t)} \hat{a}(k) - e^{-i(kx - \omega t)} \hat{a}^+(k) \right\}.$$ (91)

Inverting Fourier transformation formulae of $\sigma(x, 0)$ and $\pi(x, 0)$, we obtain

$$\hat{a}(k) = \frac{1}{(2\pi)^{3/2}} \int dx \ e^{-ikx}$$

$$\times \left\{ \sqrt{\frac{\alpha}{2}} \hat{\sigma}(x, 0) + \frac{i}{\sqrt{2\omega}} \hat{\pi}(x, 0) \right\},$$

$$\hat{a}^+(k) = \frac{1}{(2\pi)^{3/2}} \int dx \ e^{ikx}$$

$$\times \left\{ \sqrt{\frac{\alpha}{2}} \hat{\sigma}(x, 0) - \frac{i}{\sqrt{2\omega}} \hat{\pi}(x, 0) \right\}.$$ (92)

Using expressions (92) and commutation relations (88) we derive commutation relation between operators $\hat{a}(k)$ and $\hat{a}^+(k)$:

$$\left[ \hat{a}(k), \hat{a}(l) \right] = \left[ \hat{a}(k), \hat{a}^+(l) \right] = 0,$$

$$\left[ \hat{a}^+(k), \hat{a}^+(l) \right] = \delta(k - l).$$ (93)

Operators $\hat{a}(k)$ and $\hat{a}^+(k)$ are interpreted as the destruction and creation operators of the quanta of Hermitian scalar field. These quanta are called plasmons, and scalar Hermitian field $\hat{\sigma}(x, t)$ is called plasmonic quantum field.

Thus, the existence of plasmons, the quasiparticles of a special type being quanta of scalar Hermitian field $\hat{\sigma}(x, t)$ and playing the role of elementary excitations in electron gas, was rigorously proved on the basis of first-principles of electrodynamics and quantum theory.

From formula (83) there follows a linear integral relation between two scalar fields $\xi(x) = \hat{\xi}(x, t)$ and $\sigma(x) = \sigma(x, t)$:

$$\hat{\xi}(x) = \int dx' Q(x - x') \sigma(x'),$$

(94)

where

$$Q(x - x') = Q(x - x', t - t') = \frac{1}{(2\pi)^{3/2}} \int \frac{dk}{\sqrt{2\omega}}$$

$$\times \int d\omega \frac{\alpha^2 k^2}{4\pi e^2} e^{i(k(x - x') - \omega(t - t'))}.$$ (95)

After canonical quantization the classical corresponding quantum fields $\hat{\sigma}(x)$, $\hat{\xi}(x)$ and $\hat{A}(x)$,

$$\hat{\xi}(x) = \int dx' Q(x - x') \hat{\sigma}(x'),$$

(96)

functional (62) becomes the total action functional $J_{\text{tot}}[\hat{\sigma}, \hat{A}]$ of interacting system of two quantum fields $\hat{\sigma}(x)$ and $\hat{A}(x)$. It is represented as a series

$$J_{\text{tot}}[\hat{\sigma}, \hat{A}] = \sum_{l,m,n=0} a \ J^{l,m,n}[q_{0}, \hat{\sigma}, \hat{A}].$$ (97)

The term $J^{l,m,n}[q_{0}, \hat{\sigma}, \hat{A}]$ is a homogeneous functional power of $l$th, $m$th and $n$th orders with respect to functional.
variables $\varphi_0(x), \hat{\sigma}(x)$ and $\hat{A}(x)$. For example

$$J_{(1,0)}[\varphi_0, \hat{\sigma}, \hat{A}] = \frac{e}{2m} \int dx \int dx' \int dy \varphi_0(x') \times u(x' - x) \Pi(x - y) \hat{A}(y),$$

$$J_{(0,1)}[\varphi_0, \hat{\sigma}, \hat{A}] = \frac{e}{2m} \int dx \int dx' \int dy \int dy' \int dz \varphi_0(x') u(x' - x) q_0(y) u \times (y' - y) \hat{A}(x - y, y - z) \hat{A}(z),$$

$$J_{(1,1)}[\varphi_0, \hat{\sigma}, \hat{A}] = \frac{e}{2m} \int dx \int dx' \int dy \int dy' \int dy'' \varphi_0(x') u(x' - x) u(y - y') \times \hat{Q}(y' - y') \hat{\sigma}(y') \hat{A}(x - z, y - z) + \hat{A}(y - z, x - z) \hat{A}(z),$$

$$J_{(2,0,0)}[\varphi_0, \hat{\sigma}, \hat{A}] = \frac{e^2}{2m} \int dx \int dx' \int dy \int dy' \int dy'' \varphi_0(x') u(x' - x) \varphi_0(y') u \times (y' - y) \hat{A}(x - y, y - z) \hat{A}(z)^2,$$

$$J_{(2,0,2)}[\varphi_0, \hat{\sigma}, \hat{A}] = -\frac{e^2}{2m} \int dx \int dx' \int dy \int dy' \int dy'' \varphi_0(x') u(x' - x) \varphi_0(y') u \times \hat{Q}(y' - y') \hat{\sigma}(y') \hat{A}(x - z, y - z) + y' \hat{A}(y - z, x - z) \hat{A}(z)^2,$$

$$J_{(1,1,2)}[\varphi_0, \hat{\sigma}, \hat{A}] = -\frac{e^2}{2m} \int dx \int dx' \int dy \int dy' \int dy'' \varphi_0(x') u(x' - x) \varphi_0(y') u \times \hat{Q}(y' - y') \hat{\sigma}(y') \hat{A}(x - z, y - z) + \hat{A}(y - z, x - z) \hat{A}(z)^2,$$

$$J_{(0,2,2)}[\varphi_0, \hat{\sigma}, \hat{A}] = -\frac{e^2}{2m} \int dx \int dx' \int dy \int dy' \int dy'' \varphi_0(x') u(x' - x) \varphi_0(y') u \times \hat{Q}(y' - y') \hat{\sigma}(y') \hat{A}(x - z, y - z) + \hat{A}(y - z, x - z) \hat{A}(z)^2,$$

and so on.

6. Conclusion and discussions

In the framework of functional integral approach we have established the quantum theory of plasmon and plasmon–photon interaction on the basis of general postulates (axioms), called also first principles, of electrodynamics and quantum theory of many-body systems. Since the plasmons are complex quasiparticles appearing as resonances in plasma oscillations of electron gas, the starting point of our study is the total functional integral of interacting system consisting of electron gas and electromagnetic field. The electron–electron Coulomb interaction was taken into account.

The action functional of the above-mentioned interacting system contains a terms bilinear with respect to electron density $\bar{\psi}(x)\psi(x)$. By means of the Hubbard–Stratonovich transformation, this bilinear term was linearized and rewritten in a new form containing a real scalar field $\varphi(x)$ playing the role of the order parameter of collective oscillations of electron gas. The total action functional of the new interacting system consisting of collective oscillation field $\varphi(x)$ and electromagnetic field $\hat{A}(x)$ was derived.

Then it was demonstrated that in harmonic approximation the collective oscillation field $\varphi(x)$ behaves like small fluctuations around its background $\varphi_0(x)$,

$$\varphi_0(x) = \varphi_0(x) + \zeta(x).$$

The new real scalar field $\zeta(x)$ was called fluctuation field. Total action functional of the interacting system consisting of fluctuation field $\zeta(x)$ and electromagnetic field $\hat{A}(x)$ was established.

Subsequently we have demonstrated that in the case of homogeneous and isotropic electron gas there exists a real scalar field $\sigma(x)$ satisfying the wave equation similar to Klein–Gordon equation in relativistic quantum field theory such that fluctuation field $\zeta(x)$ is a linear functional of $\sigma(x)$. The quanta of the corresponding quantized Hermitian scalar field $\hat{\sigma}(x)$ are plasmons, and this scalar field was called quantum plasmonic field. By substituting the linear expression of quantized fluctuation field $\hat{\zeta}(x)$ in terms of quantum plasmonic field $\hat{\sigma}(x)$ into the expression of total action functional $J_{\text{tot}}[\hat{\varphi}, \hat{A}]$ of the interacting system of two fields $\hat{\varphi}(x)$ and $\hat{A}(x)$, we have obtained the expression of total action functional $J_{\text{tot}}[\hat{\sigma}, \hat{A}]$ of interacting system of plasmonic and electromagnetic fields. This total action functional determines matrix elements of plasmon–photon interaction processes.

The expression of total action functional $J_{\text{tot}}[\hat{\sigma}, \hat{A}]$ shows that plasmon–photon interaction processes are nonlocal. The physical origin of this nonlocality is the spatial extension of each plasmon around its center. Moreover, interaction processes involving plasmons are also not instantaneous: each matrix element contains plasmonic and other fields at different times.

Thus, we have demonstrated that interaction processes involving plasmons are neither local nor instantaneous. This remark suggests that the assumption on the local and instantaneous phenomenological interaction Hamiltonians of various plasmonic processes should be revised.

Finally, let us note that the electron–phonon plays a certain important role in plasmonic processes, but until now there was almost no theoretical research on related subjects. It
is worth studying the plasmon–phonon interaction processes, and functional integral technique would be again the effective tool for this purpose.

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