Quantifying the nucleon’s pion cloud with transverse charge densities

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The transverse densities in a fast-moving nucleon offer a model-independent framework for analyzing the spatial structure of the pion cloud and its role in current matrix elements. We calculate the chiral large-distance component of the charge density using a dispersion representation of the form factor and discuss its partonic interpretation. The non-chiral core is dominant up to surprisingly large distances $\sim 2\,\text{fm}$. The chiral component can be probed in precision low-$Q^2$ elastic $eN$ scattering or in peripheral deep-inelastic processes which resolve its quark/gluon content.

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The large-distance behavior of strong interactions is governed by the spontaneous breaking of chiral symmetry in QCD, through which the pion appears as an almost massless Goldstone boson, coupling weakly to hadronic matter. The resulting effective dynamics explains numerous observations in low-energy $\pi\pi$ and $\pi N$ scattering, the $NN$ interaction at large distances, as well as weak and electromagnetic processes. From the perspective of nucleon structure, these are often summarized in an intuitive spatial picture of the nucleon as consisting of a non-chiral “core” and a “pion cloud” of size $1/M_\pi$. Despite its widespread appeal and textbook-level status, this spatial picture has proved surprisingly difficult to quantify. Effective field theory (chiral perturbation theory, or ChPT) provides a systematic method to calculate dynamical effects at the scale $1/M_\pi$ but does not resolve the structure of the core, which is encoded in local counter terms [1]. Chiral soliton models of the nucleon provide a spatial picture of its structure, but are restricted to the large-$N_c$ limit of QCD (semiclassical approximation) and subject to model assumptions about short-distance dynamics [2].

The lack of an unambiguous spatial representation of the pion cloud is felt most acutely in elastic $eN$ scattering, which measures the charge and magnetization form factor of the nucleon. What is needed is a model-independent and fully quantitative formulation of the spatial structure of the nucleon’s chiral component appropriate for the analysis of such measurements.

A new approach to this problem is possible with the recently proposed concept of transverse densities [3]. Defined as 2-dimensional Fourier transforms of the elastic form factors, they describe the distribution of charge and magnetization in the plane transverse to the direction of motion of a fast nucleon. In contrast to the traditional representation of form factors through 3-dimensional spatial densities in the Breit frame (zero energy transfer) [4, 5], the transverse densities provide an unambiguous spatial interpretation also for systems in which the motion of the constituents is essentially relativistic. They are closely related to the parton picture of hadron structure in high-energy processes and correspond to a reduction of the generalized parton distributions (or GPDs) describing the distribution of quarks/antiquarks with respect to longitudinal momentum and transverse position [3]. In this way they establish an interesting connection between low-energy elastic $eN$ scattering and deep-inelastic processes sensitive to the transverse size of the nucleon, such as exclusive and diffractive processes in high-energy $eN$ and $NN$ scattering [2], and enable comprehensive studies of the nucleon’s spatial structure with several independent observables.

In this Letter we analyze the spatial structure of the nucleon’s pion cloud and its role in elastic $eN$ scattering using the framework of transverse charge densities. We calculate the chiral component of the charge density in a $t$-channel representation of the form factor, which relates the large-distance behavior to the singularities in the timelike region. It is shown that this formulation is equivalent to the partonic picture in the $s$-channel, where the large-distance behavior is governed by $\pi N$ and $\pi \Delta$ configurations in the nucleon’s light-cone wave function. We find that the non-chiral core of the charge density is numerically dominant up to surprisingly large distances $\sim 2\,\text{fm}$, and discuss the prospects for probing the chiral component in precision low-$Q^2$ elastic scattering. A detailed account will be given in a forthcoming article.

The transverse charge density is defined as the 2-dimensional Fourier transform of the Dirac form factor of the vector current ($b \equiv |b|$) [3]

$$
\rho(b) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i(\Delta_\perp \cdot b)} F_1(t = -\Delta_\perp^2). \quad (1)
$$

In a frame where the nucleon is moving fast and the momentum transfer $\Delta_\perp$ is in the transverse direction, $b$ may be interpreted as the transverse position at which the current measures the charge density. However, since the form factor is Lorentz-invariant, the quantity defined by Eq. (1) may be evaluated in more general ways. In fact, the form factor $F_1(t)$ is an analytic function of the invariant momentum transfer $t$, with singularities (branch cuts, poles) in the timelike region $t > 0$. By performing the angular integral in Eq. (1), and deforming the integration contour in the radial variable $\Delta_\perp \equiv |\Delta_\perp|$ to
the charge density at large distances behaves as

$$\rho(b) = \int_0^\infty dt \frac{\text{Im}F_1(t+i0)}{2\pi} K_0(\sqrt{tb}).$$

(2)

It is particularly useful for discussing the asymptotic behavior of the charge distribution at large \( b \). For a branch cut singularity at \( t \sim \kappa^2 \) with \( \text{Im}F(t+i0) \propto (t-\kappa^2)^\nu \) the charge density at large distances behaves as \( \rho(b) \propto e^{-\nu b}/(\kappa b)^{\nu+3/2} \), with a coefficient which is easily calculable from Eq. (2). The exponent is determined by the position of the singularity only, while the pre-exponential factor depends on the power–like behavior of the imaginary part near threshold.

The leading singularity of the form factor in the timelike region is the two–pion cut at \( t > \kappa^2 \). For a branch cut singularity at \( t \sim \kappa^2 \) with\( \text{Im}F(t+i0) \propto (t-\kappa^2)^\nu \) the charge density at large distances behaves as \( \rho(b) \propto e^{-\nu b}/(\kappa b)^{\nu+3/2} \), with a coefficient which is easily calculable from Eq. (2). The exponent is determined by the position of the singularity only, while the pre-exponential factor depends on the power–like behavior of the imaginary part near threshold.

The leading singularity of the form factor in the timelike region is the two–pion cut at \( t > 4M_N^2 \), which is of isovector nature. Because it lies in the unphysical region its strength can only be calculated theoretically. In order to reliably describe the charge density at \( b \leq 1/M_x \) we need an approximation to the imaginary part which is generally accurate in the region \( t-4M_N^2 \sim M_x^2 \) and has the correct threshold behavior in the limit \( t \to 4M_N^2 \). It is provided by the amplitudes of Fig. 1, where the \( \pi \) and \( N \) are pointlike and the couplings are those of the leading–order relativistic chiral Lagrangian of Ref. [8]. The loop integrals are evaluated without expanding in the particle masses; the resulting expression smoothly interpolates between the region \( t-4M_N^2 \sim M_x^2 \), where it contains the leading term in heavy–baryon ChPT, and the near–threshold region where the heavy–baryon expansion does not converge; see Ref. [8] for a detailed discussion. Up to negligible orders of \( t/M_N^2 \), the result can be stated as (see also Ref. [8])

$$\text{Im}F_1(t+i0) \propto \left( \frac{2}{2\pi F_\pi^2} \right)^2 M_N \sqrt{t} (x-\arctan x) \quad (3)$$

$$+ \frac{(1-g_A^2)(t-4M_N^2)^{3/2}}{6(4\pi F_\pi^2)^2 \sqrt{t}} \quad (4)$$

where \( g_A = 1.26 \) is the nucleon isovector axial coupling, \( F_\pi = 93 \text{ MeV} \) the pion decay constant, and \( x \equiv M_N \sqrt{t-4M_N^2}/(t-2M_N^2) \). The first term, Eq. (3), arises from the diagram of Fig. 1a and describes the contribution of physical \( \pi N \) intermediate state in the \( s \)–channel; the second term, Eq. (4), comes from the contact interaction of Fig. 1b and the part of diagram Fig. 1a in which the nucleon pole is canceled. The transverse charge density resulting from Eqs. (3) and (4) is shown in Fig. 2. One sees that the \( b \)–dependence is far from a simple exponential decay in the region shown here, indicating strong variations in the pre-exponential factor.

The leading term in the pre-exponential factor dominates only in the region \( b \gg M_N^2/M_\pi^2 \), corresponding to extremely large distances of the order \( \sim 10^2 \) fm; already for \( b \sim M_N^2/M_\pi^2 \) the terms in the pre-exponential factor need to be summed up. The unusually slow convergence can be traced to an anomalous unphysical threshold close to \( t = 4M_N^2 \) in Eq. (3); cf. also the discussion in Ref. [8].

Alternatively, one may expand Eqs. (3) and (4) in powers of \( M_x/M_N \) in the region \( t \sim M_x^2 \), not necessarily close to threshold, corresponding to the leading term in heavy–baryon ChPT [8]. This leads to an approximation for \( \rho^{\pi-n}(b) \) valid in the region \( b \sim 1/M_x \), which, however, reproduces the finite–mass result only within a factor of \( \sim 2 \) in the region shown in Fig. 2. Together, these observations affirm the rationale of our interpolating approximation in numerical studies of the chiral component.

It is worth noting that in our dispersive approach the values of \( t \) in Eq. (2) are automatically restricted to...
\( \sqrt{t} \sim 1/b \), with exponential suppression of large \( t \). No external cutoff is needed. For values \( b \sim 1/M_\pi \) the imaginary part is only sampled in a region where finite–size effects are negligible and it can safely be computed in the point particle approximation to chiral dynamics, cf. the numerical studies in Ref. \[10\]. The transverse distance \( b \) thus acts as an external parameter justifying the chiral expansion, similar to angular momentum in the partial–wave expansion of low–energy \( \pi N \) scattering.

Excitation of \( \Delta \) resonances is known to play an important role in the two–pion cut; in particular, it ensures the proper scaling of the nucleon’s isovector vector charge in the large–\( N_c \) limit of QCD where \( N \) and \( \Delta \) become degenerate \[11\]. Using an empirical \( \pi N \Delta \) coupling as in Ref. \[12\], it is straightforward to include the \( \Delta \) in the process of Fig. 1a. We obtain an expression similar to Eqs. (3) and (4), which has opposite sign and can be shown to cancel the \( N \) diagram in the large–\( N_c \) limit where \( M_{\pi N} \sim N_c, M_{\Delta} - M_{NN} \sim N_c^{-1}, t \sim M_\pi^2 \sim N_0^0 \), and the couplings are simply related \[12\]. The transverse charge density from the \( \Delta \) is shown in Fig. 2. One sees that at \( b \leq 1 \) fm it is comparable to that from intermediate \( N \) states, resulting in significant cancellations, but that at large \( b \) the \( N \) becomes numerically dominant.

To assess the numerical relevance of the chiral component at large \( b \) we need to compare it to the bulk of the charge density unrelated to chiral dynamics. In the representation Eq. (2), the latter is generated by the higher–mass singularities of the form factor at \( t > 0 \) and can be calculated in a systematic fashion. The leading one is the \( \rho \) meson, which produces an asymptotic charge density

\[
p_{\rho}^{<}(b) \sim M_\rho^2 e^{-M_\rho b} / \sqrt{8 \pi M_\rho b},
\]

where \( M_\rho = 770 \) MeV and the coupling has been chosen to ensure charge conservation (we neglect the finite width). Higher–mass states \( \rho' \) etc. observed in the photon spectral function, which build up the 1/4\( b^4 \) behavior of the form factor at large \( |t| \), are expected to have a negligible effect at the distance of interest here, as is indeed confirmed by numerical studies. Chiral and non–chiral components of the charge density are compared in Fig. 2.

One sees that the non-chiral component is dominant up to distances \( b \sim 2 \) fm, the reason being the large coupling of the \( \rho \) compared to two pions. This result runs counter to naive expectations which place the region of the nucleon’s “pion cloud” at distances \( > 1 \) fm.

To what extent could present or future nucleon form factor measurements in the spacelike region \( t < 0 \) probe the chiral component in the transverse charge density? To answer this question, it is instructive to consider the \( b^2 \)–moments of the charge density, which are proportional to derivatives of the isovector form factor at \( t = 0 \):

\[
\langle b^{2n} \rangle = \int d^2 b b^{2n} p_{\rho}^{<}(b) \quad (n = 1, 2)
\]

\[
\langle b^2 \rangle = 4F_1'(0), \quad \langle b^4 \rangle = 32F_2'(0).
\]

The moments of the chiral component can easily be computed by integrating Eq. (2) over \( b \). Restricting the integral to values \( b > b_0 = \sqrt{\langle b^2 \rangle}_{\rho\text{-exch}} = 2/M_\pi \), we find \( \langle b^2 \rangle_{\text{chir}} = 0.08 \) fm\(^2\) (including both intermediate \( N \) and \( \Delta \), which amounts to only 15% of the experimental value \( \langle b^2 \rangle_{\text{exp}} = 0.57 \) fm\(^2\). This value is consistent with the uncorrelated two–pion contribution estimated in the dispersion analysis of Ref. \[3\]. Detecting the chiral component through an effect on the charge radius thus seems difficult. As an aside, we note that the expression for \( \langle b^2 \rangle \) obtained from Eqs. (2,3) formally reproduces the well–known divergence of the “3–dimensional” isovector charge radius in the chiral limit \( M_\pi \to 0 \) \[11, 13\],

\[
\langle r^2 \rangle = \frac{3}{2} \langle b^2 \rangle \sim -\frac{(1 + g_\rho^2)}{(4\pi F_\rho)} \ln M_\pi^2,
\]

however, this is of little consequence for the numerical results at the physical pion mass. A more promising chiral observable is \( \langle b^4 \rangle \), which receives most of its contributions from distances \( > 1 \) fm. We find \( \langle b^4 \rangle_{\text{chir}} = 3.4 \) fm\(^4\) \( \approx \langle b^2 \rangle_{\text{exp}} \) (including both \( N \) and \( \Delta \)), a chiral contribution comparable to the “natural” non–chiral value estimated from the charge radius. One should thus be able to see the chiral component in the behavior of the second derivative of the form factor at \( t = 0 \). The present form factor data at finite \( t \) consistently extrapolate to \( F_1(0) = 1 \) with the slope (charge radius) measured in atomic physics experiments \[14\], indicating that the second derivative could be extracted without tension given sufficiently precise data.

The chiral large–distance component of the transverse charge density can also be discussed in the infinite–momentum frame, where it relates to the traditional concept of the “pion cloud” in the nucleon’s partonic structure \[15\]; see Ref. \[10\] for a detailed discussion. Rewriting the invariant integrals for the form factor in terms of partonic variables, we find that the term Eq. (3) (coming from Fig. 1a), and the corresponding one with the intermediate \( \Delta \), can equivalently be expressed as

\[
p_{\pi N}^{<}(y, b) = \int_0^1 dy \left[ \frac{3}{8} f_{\pi N}(y, b) - \frac{3}{8} f_{\pi \Delta}(y, b) \right],
\]

where \( f_{\pi N}(y, b) (B = N, \Delta) \) are the distribution of pions in the fast–moving nucleon as a function of their longitudinal momentum fraction \( y \) and transverse position \( b \). In this representation the chiral charge density arises as the cumulative effect of pions in the nucleon’s light–cone wave function at transverse distances \( b \sim 1/M_\pi \) — a very intuitive picture, see Fig. 3. The contact term in the form factor, Eq. (4) (coming from Figs. 1a and b), when expressed in partonic variables, corresponds to a delta function–type contribution \( f_{\pi B}(y, b)_{\text{contact}} \sim \delta(y) \rho(b)_{\text{contact}} \); such terms indeed arise from the formal operator definition of the isovector pion momentum distribution in the nucleon. At first sight,
this appears to contradict the partonic interpretation of the transverse charge density in QCD, as only part of its chiral component seems to correspond to constituents carrying a finite fraction of the nucleon’s momentum. The paradox is resolved when one realizes that the “true” pion density in the nucleon at small \( y \) really involves high–mass intermediate states beyond the \( N \) and \( \Delta \). In a complete theory, the sum over such states would produce a smooth \( y \)–distribution of pions, whose integral then gives the chiral charge density at large \( b \). In the present calculation based on point particles and the chiral Lagrangian, the effect of these high–mass states is summed up by contact terms. In this sense, they represent legitimate contributions to the partonic charge density which are simply not “resolved” at the present level of approximations. (An explicit resummation of higher–mass multipion states was performed for a pion target within the leading logarithmic approximation of ChPT \[10\].) Most of the nucleon’s charge at large \( b \) resides in the \( \pi N \) component of the wave function, in which the contact term Eq. (4) contributes \( \lesssim 10\% \) at \( b > 1 \text{ fm} \) (in the \( \pi \Delta \) component it contributes about half). Thus, even at the present level of approximation, most of the charge density at large \( b \) can be directly interpreted in the partonic picture.

More generally, our approach suggests an interesting connection between low–energy elastic \( eN \) scattering and the physics of peripheral high–momentum transfer processes in \( eN, \gamma N \) and hadron–N scattering, which can resolve the partonic content of the pion cloud at a given momentum fraction, \( x \). Examples are exclusive meson/photon production \( eN \rightarrow e'N + M(M = \rho, \phi, J/\psi, \gamma) \) at \( Q^2 \gg 1 \text{ GeV}^2 \) and \( |t| \ll 1 \text{ GeV}^2 \), or corresponding processes in which the production happens on a pion at distances \( b \sim 1/\langle M \rangle \), which is knocked out and observed in the final state \[12\]. Such processes can measure the isoscalar quark and gluon density in the pion cloud, in which \( N \) and \( \Delta \) states contribute with the same sign and produce a sizable chiral component at \( x < \langle M \rangle /M \pi \) \[10, 12\].

In sum, the concept of transverse charge densities provides a rigorous framework for analyzing the spatial structure of the nucleon’s pion cloud and its contribution to current matrix elements. Our results quantify the impact of chiral dynamics on the analysis of empirical charge densities \[8, 17\]. The coordinate–space approach developed here can be extended to many other observables, such as magnetic form factors and the nucleon’s orbital angular momentum content, gravitational form factors and the matter density \[18\], and \( N \rightarrow \Delta \) transition form factors \[19\].

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