We present moments and transition probabilities in the neighboring odd-mass nuclei of $^{208}\text{Pb}$ calculated fully self-consistently from the s.p. properties of $^{208}\text{Pb}$ with polarization corrections from its excitations, both given from previous Skyrme-Hartree-Fock and RPA calculations. The electric results agree nicely with the data with two very interesting exceptions. In the magnetic case we obtain similar results. We discuss also polarization contributions to the $l$-forbidden $M1$ transitions, which are, however, much too small compared to the data. With a modified external field operator which accounts effectively for mesonic and many-body effects the description of the data can be substantially improved.

I. INTRODUCTION

Nuclear shell structure is intimately related to nuclear single-particle (s.p.) properties as, e.g., s.p. energies with spin-orbit splitting thereof, s.p. multipole moments, or magnetic moments, see e.g. [1, 2]. A proper description of s.p. properties had been a crucial benchmark for the development of the empirical nuclear shell model which has become textbook standard since long, see e.g. [1–5]. Early development used the properties of odd systems next to doubly magic nuclei directly as s.p. signal. Soon it was realized that the one nucleon added to or removed from the doubly-magic mother nucleus acts back on the core. It responds by polarization which is determined by low-lying collective electric and magnetic resonances of the core nucleus. The effect of core polarization had been taken into account by augmenting the shell model with empirical nuclear response theory [6]. This then allowed reliable calculations of s.p. moments and, more demanding, transitions strengths between different s.p. configurations in odd nuclei [7], the latter being important, e.g., in astro-physical reaction chains.

The next stage in nuclear model development came up with self-consistent models using effective interactions, better described as nuclear density functional theory (DFT). Nearly simultaneously appeared relativistic
and non-relativistic DFT models \cite{10,12}. The advantage of self-consistent models is that they can be extrapolated farther away from the valley of stable nuclei than empirical models, even up to neutron stars \cite{13,14}. Originally oriented on global nuclear properties, as energies and radii, nuclear DFT soon has been developed further to access more refined observables, e.g., nuclear resonance excitations within a self-consistent Random-Phase-Approximation (RPA) \cite{15,16}. Odd nuclei are also naturally in reach of nuclear DFT, see e.g. \cite{17,24}, although complicated by the need of blocking and scanning a large amount of competing configurations \cite{25}. So far, odd nuclei had a minority application in the world of nuclear DFT and, to the best of our knowledge, the topic of transition strengths between the s.p. states in odd nuclei has been addressed practically only in the context of the non-self-consistent approach \cite{26,30} (see, however, the recent paper \cite{31}). It is the aim of this paper to study the description of s.p. moments and transition strengths for odd nuclei next to $^{208}$Pb for Skyrme functionals. To this end, we use the strategy already explored in empirical models, namely to describe the odd nucleon (or hole) in the mean-field of the $^{208}$Pb core and consider the self-consistent rearrangement of the mean field perturbatively through the RPA response of the core to the extra nucleon (or hole). This approach is legitimate in $^{208}$Pb where one nucleon out of 208 constitutes a small perturbation, indeed.

The paper is outlined as follows: In Section II we present the theoretical background which is based the many body Green functions \cite{6,7}, explain briefly the numerical realization, and check the reproduction of low-lying resonances by the chosen RPA scheme. In Section III we present our numerical results and draw various conclusions. Finally we summarize our calculations.

II. FORMAL FRAMEWORK

A. Transition operators

We consider moments and transition strengths for the s.p. states of odd-mass nuclei for electric and magnetic multipole operators $Q$. Basis of the description are the s.p. states $\alpha$ of the even-even core nucleus. Here and in the following we label these s.p. states briefly by numerical indices (1, 2, 3, ...) as synonym for $\alpha_1, \alpha_2, ...$ which stand for the set of the quantum numbers of some single-particle basis. An important aspect in this paper is that we consider polarization corrections to the measuring operator, thus dealing with an effective operator

$$\hat{Q}_{12} = Q_{12} + \Delta Q_{\text{pol},12}.$$  \hspace{1cm} (1)

The structure of the effective measuring operator is sketched in figure 1. The message of the diagrams becomes clear in the following discussion of the contributions.

The form of the bare operators $Q$ for the electric and magnetic moments and transitions is given in Refs. \cite{2,7}. The electric operator of the multipolarity $L$ reads

$$Q_{r,LM}^{(E)}(r) = e_r^{(L)}L Y_{LM}(\mathbf{n}),$$  \hspace{1cm} (2a)

where $\mathbf{n} = r/r$: the $e_r^{(L)}$ are the effective charges for the protons and neutrons which serve here to account for the center-of-mass correction (see \cite{32}) which reads for a nucleus with $Z$ protons and $A - Z$ neutrons

$$e_p^{(L)} = e[(A-1)^L + (-1)^L(Z-1)]/A^L,$$  \hspace{1cm} (2b)

$$e_n^{(L)} = eZ(-1/A)^L.$$  \hspace{1cm} (2c)

In principle, the effective charges should also incorporate many-body effects (Landau renormalization). However, in case of electric operators Ward identities \cite{7,33} allow...
to identify the renormalized electric operator with the bare operators done above.

The magnetic operator is defined as follows

\[
Q_{\tau,LM}^{(M)}(\mathbf{r}) = \mu_N \sqrt{L(2L+1)} \rho^{L-1} \times \sum_{L',\mu,\nu} \langle L', \mu, 1, \nu | L, M \rangle Y_{L',\mu}(\mathbf{n}) \times \left[ \delta_{L',L-1} \left( \tilde{\gamma}_{\tau} \hat{\sigma}_{1,\nu} + \frac{2c_{\tau,1}}{L+1} \hat{l}_{1,\nu} \right) + \delta_{L,1} \delta_{L',2} \frac{\kappa_\tau}{\sqrt{3}} r^2 \hat{\sigma}_{1,\nu} \right], \tag{3a}
\]

\[
\tilde{\gamma}_p(n) = (1 - \xi_s) \gamma_p(n) + \xi_s \gamma_n(p), \tag{3b}
\]

\[
\xi_{n,t} = \xi_t, \quad \xi_{p,t} = 1 - \xi_t \tag{3c}
\]

\[
\kappa_p = \frac{\xi_t}{100 \text{ fm}^2}, \quad \kappa_n = -\frac{\xi_t}{100 \text{ fm}^2}, \tag{3d}
\]

where \(\mu_N = e\hbar/2m_p c\) is the nuclear magneton, \(\hat{\sigma}_{1,\nu}\) and \(\hat{l}_{1,\nu}\) are the spin Pauli matrix and the single-particle operator of the orbital angular momentum in the tensor representation. The nucleons spin gyro-magnetic moments and renormalization parameters are

\[
\gamma_p = 2.793, \quad \gamma_n = -1.913, \quad \xi_s = 0.1, \quad \xi_t = 0 \tag{3e}
\]

In the case of the magnetic operator, no corresponding conservation laws exists. Therefore, in addition to magnetic properties of bare nucleons two effects have to be taken into account:

(I) Landau renormalization \[7, 33\] and

(II) virtual exchange of mesons \[34\].

The renormalization constants \(\xi_s\) and \(\xi_t\) simulate the effects of type (I). The parameters \(\kappa_\tau\) in the tensor contribution \([Y_2 \times \hat{\sigma}_1]_1\) simulate both type (I) and (II) together. This term becomes important in the case of the \(l\)-forbidden \(M1\) transitions. In Ref. \[2\] the explicit form of the effective operators is given. Here one realizes that they consists of a linear part which is simulated by the \(\xi\)-parameters and a part with complicated many particle many hole components which give rise to (small) vertex corrections part of which can be simulated by a term like \([Y_2 \times \hat{\sigma}_1]_1\). In the calculations we used two values: \(\xi_t = 0\) for the case where meson effects are ignored and \(\xi_t = 1.5\) which was fitted from the condition of describing the \(l\)-forbidden \(M1\) transitions in the neighboring odd-mass nuclei of \(^{208}\text{Pb}\).

The moments and the transition probabilities for the states of the odd-mass nuclei are determined by the reduced matrix elements \(\tilde{Q}^{LM}_{(12)}\) of the multipole effective operator \(\tilde{Q}^{LM}\) for which the local external-field operator defined in Eqs. \[2\]–\[3\] serves as the zero-order approximation. These reduced matrix elements are defined as

\[
\tilde{Q}^{LM}_{(12)} = (-1)^j_2 - m_2 \left( \begin{array}{ccc} j_1 & j_2 & L \\ m_1 & -m_2 & M \end{array} \right) \tilde{Q}^{LM}_{(11)}, \tag{4}
\]

where \(j\) and \(m\) is the single-particle total angular moments and its projection.

For the moment \(\mu^L_{(1)}\) of the multipolarity \(L\) in the state with the set of the quantum numbers \(1 = \{(1), m_1\}\) and the occupation number \(n_{(1)}\) we have

\[
\mu^L_{(1)} = q_{(1)} \sqrt{\frac{16\pi}{2L+1}} \left( \begin{array}{ccc} j_1 & j_1 & L \\ j_1 & -j_1 & 0 \end{array} \right) \tilde{Q}^{LM}_{(11)}, \tag{5}
\]

where \(q_{(1)} = 1 - 2n_{(1)}\) for the electric operators and \(q_{(1)} = 1/2\) for the magnetic operators. The reduced transition probability \(B\) is defined through the transition amplitude \(\tilde{Q}^{LM}_{(12)}\) as

\[
B(L; \{1\} \to \{2\}) = \frac{1}{2j_1 + 1} \left( \tilde{Q}^{LM}_{(12)} \right)^2. \tag{6}
\]

### B. RPA treatment of core polarization

To lowest order approximation, the moments of an s.p. state \(\alpha_1\) in the odd system are given by \(Q_{11}\) and transition amplitudes as \(Q_{12}\). An important correction comes from core polarization within the RPA, illustrated in the second term of figure \[1\]. The respective formalism was developed within the Green-function method and described in Ref. \[7\]. In this model which is used in our present calculations, the matrix elements of the local external-field operator \(Q\), Eqs. \[2\]–\[3\], are replaced by the matrix elements of the effective (or the renormalized) operator \(\tilde{Q}\) which are determined by the solutions of the RPA equations. The result can be represented in the form \[26\]

\[
\Delta Q_{\text{pol},12} = \sum_n \langle Z^n | n \rangle_{12} \frac{\text{sgn}(\omega_n)}{\varepsilon_1 - \varepsilon_2 - \omega_n} \langle Z^n | Q \rangle, \tag{7a}
\]

\[
\langle Z^n | Q \rangle = \sum_{12} Z^n_{12} Q_{12}, \tag{7b}
\]

\[
\langle V | Z^n \rangle_{12} = \sum_{34} \bar{V}_{12,34} Z^n_{34}. \tag{7c}
\]

The entries of the polarization term are all quantities defined in the even-even core: the \(\omega_n\) are the RPA excitation energies, the \(Z^n\) the corresponding transition amplitudes, and the \(\varepsilon_1\) the s.p. energies from the mean-field.
Hamiltonian $\hat{h}$. The RPA equation determining $\omega_n$ and $Z^n$ reads:

$$\sum_{34}^{n} \Omega_{12,34}^{\text{RPA}} Z^n_{34} = \omega_n Z^n_{12}. \quad (8a)$$

The transition amplitudes are normalized by the condition

$$\langle Z^n | M^{\text{RPA}} | Z^n \rangle = \text{sgn}(\omega_n), \quad (8b)$$

where

$$M_{12,34}^{\text{RPA}} = \delta_{13} \rho_{42} - \rho_{13} \delta_{42} \quad (8c)$$

is the metric matrix in the RPA and $\rho$ is the single-particle density matrix in the ground state.

Mean-field ground state and RPA are derived self-consistently from the same given energy density functional (EDF) $E[\rho]$. The RPA matrix $\Omega^{\text{RPA}}$ is defined by

$$\Omega_{12,34}^{\text{RPA}} = h_{13} \delta_{42} - \delta_{13} h_{42} + \sum_{56}^{56} M_{12,56}^{\text{RPA}} V_{56,34}, \quad (9a)$$

where the single-particle Hamiltonian $\hat{h}$ is given by the first functional derivative of $E[\rho]$ and the residual interaction $\hat{V}$ by the second derivative as

$$h_{12} = \frac{\delta E[\rho]}{\delta \rho_{21}}, \quad V_{12,34} = \frac{\delta^2 E[\rho]}{\delta \rho_{21} \delta \rho_{34}}. \quad (9b)$$

The details of the solution of the equations given in this section are the same as in the series of our previous papers, see, e.g., Refs. [35–38]. The s.p. basis was computed on a spherical coordinate-space grid with box radius of 18 fm. The s.p. basis was limited to a maximum value of s.p. energy as $\varepsilon_{p}^{\text{max}} = 100$ MeV.

C. Choice of Skyrme parametrizations and the details of calculations

At the side of the EDF, we use three different Skyrme parametrizations: SLy4 [39] as an EDF with low effective mass, SV-bas as fit to a large set of spherical nuclei and electrical giant resonances in $^{208}$Pb (i.e. proper core response) [40], and SV-bas$_{m}$ which takes care additionally to reproduce magnetic $M1$ response [41].

As we will see, core polarization, created by the virtual excitation of the eigenmodes in $^{208}$Pb, is crucial. Therefore it is important that the excitation spectrum of $^{208}$Pb is well reproduced which is actually the case for the Skyrme parametrizations we choose. The E3 and E5 transitions are of special interest as the (theoretical) transition energies in the odd-mass nuclei and the excitation energy of the E3 and E5 resonances in $^{208}$Pb can be very similar which may give rise to resonance effects. Therefore the single-particle spectrum of the $^{208}$Pb $\pm 1$ are here of importance. In order to demonstrate this effect we calculated those quantities in some cases also within the Landau Migdal (LM) approach where experimental sp energies were used and the force parameters adjust to reproduce quantitatively excitation energies and transition probabilities. In Table II the excitation energies and transition probabilities for the first collective states of four multipolarities are shown to give an impression. These states give large, often dominant, contributions to the polarization effects (which include, of course, all RPA states).

The details of the solution of the equations given in Section III B are the same as in the series of our previous papers, see, e.g., Refs. [35–38]. The s.p. basis was computed on a spherical coordinate-space grid with box radius of 18 fm. The s.p. basis was limited to a maximum value of s.p. energy as $\varepsilon_{p}^{\text{max}} = 100$ MeV.

III. RESULTS

A. The electric case

Here and in the following section, results for electric and magnetic moments and transition probabilities are presented. We show only theoretical results which can be compared with data. Before starting the tour, we emphasize that the three Skyrme EDF are taken as published, so to say “from the shelves”. No re-tuning of any parameter was done.

The results for the electric quadrupole moments are shown in Table III. The agreement is excellent for the both 9/2 states and still acceptable for the 11/2 state in $^{209}$Bi (last line). The agreement is not too surprising because multipole moments, similar as ground state deformations in even-even nuclei, are predominantly topological quantities which are predominantly determined by shell structure. It happens not only here but also in
two spin flip transitions in times larger produced by theory. There might still be a problem with the spin-flip and non-spin-flip transitions is correctly repro-
ment with the data and the qualitative difference between spin-flip transition 2 and non-spin-flip transition because of additional vector coupling B. The "spin-flip" transition 2 which deviates by a factor of two is the different parameter sets give similar results. The one case where very different values are quoted including lower values which would be much closer to our theoretical results. The cited number in Table III is a weighted average. But also the other two transitions which are experimentally known are not well reproduced.

In order to understand the origin of this good agreement and the small variation of different parameter sets we investigate composition of ground state quadrupole moments and transitions in more detail. Table IV shows the contributions (in percentage) of the s.p elements of the multipole operator $Q_{sp}$, of the lowest excited $L_1^\pi$ state, and of the giant resonance of given multipolarity. We present the results for the SV-bas parametrization only because the other parameterizations give similar results. For the quadrupole moment of $^{209}$Bi one notices that the external field operators and the polarization contributions are of the same magnitude. We also realize that in both cases the polarization is dominated by the lowest $2_1^+$ resonance and the GQR. Due to the energy denominator in Eq. (7a), the contribution from the low-
lying states is of order two times larger than those from the GQR. Similar relations are found for the quadrupole transitions (second block in Table IV). What changes with s.p. state or transition is the s.p. contribution $Q_{sp}$.

Now we look at the results for the $E3$ transition in Table III the $1i_{13/2} \rightarrow 1h_9/2$ in $^{209}$Bi and $1j_{15/2} \rightarrow 2g_{9/2}$ in $^{209}$Pb. There is an interesting phenomenon connected with these transitions: The excitation energy of the collective $3^-$ resonance in $^{208}$Pb is of the same order as the energy of the transitions. Therefore the energy denominator in Eq. (7a) and the $B(E3)$ value of the resonance plays an important role in the polarization contribution. This too large polarization leads to a significant discrepancy between theory and experiment. To...
TABLE III. Energies (in MeV) and $B(EL)$ values for the electric multipole transitions in the odd-mass nuclei of the lead region.

| Nucleus | Transition | Energies | $B(EL)$       |
|---------|------------|----------|---------------|
|         |            | SV-bas  | SLy4 | Experiment | SV-bas  | SLy4 | LM | Experiment |
|         |            | SV-bas$m$ |       |            |         |      |    |            |
| $^{207}$Tl | $2d_{3/2}$ → $3s_{1/2}$ | 0.774  | 0.652  | 0.753  | 0.351  | 163  | 161  | 164  | 196(51) | [17] |
| $^{207}$Pb | $2f_{5/2}$ → $3p_{1/2}$ | 0.950  | 1.023  | 0.971  | 0.570  | 77   | 75   | 85   | 70.9(2) | [17] |
| $^{207}$Pb | $3p_{3/2}$ → $3p_{1/2}$ | 0.884  | 0.798  | 1.103  | 0.898  | 82   | 82   | 89   | 60.5(25) | [17] |
| $^{209}$Pb | $4s_{1/2}$ → $3d_{5/2}$ | 0.566  | 0.500  | 0.638  | 0.465  | 155  | 155  | 100  | 157(6) | [12] |
| $^{209}$Pb | $3d_{5/2}$ → $2g_{9/2}$ | 2.235  | 2.160  | 2.519  | 1.567  | 245  | 253  | 236  | 184(52) | [12] |
| $^{209}$Bi | $2f_{7/2}$ → $1h_{9/2}$ | 1.117  | 0.786  | 0.921  | 0.896  | 12   | 12   | 13   | 26.1(16) | [12] |
| $^{209}$Bi | $3p_{3/2}$ → $2f_{5/2}$ | 3.042  | 3.126  | 3.362  | 2.223  | 957  | 1100 | 934  | 520(400) | [12] |
| $^{209}$Bi | $3d_{5/2}$ → $2g_{9/2}$ | 3.328  | 3.281  | 3.466  | 2.826  | 661  | 672  | 610  | 324(44) | [12] |

$E_2$ transitions [$B(E2)$ in units of $\text{e}^2\text{fm}^4$]

| $^{209}$Pb | $1j_{15/2}$ → $2g_{9/2}$ | 2.001  | 2.195  | 2.607  | 1.423  | 133  | 169  | 257  | 58   | 67(16) | [12] |
| $^{209}$Bi | $1i_{13/2}$ → $1h_{9/2}$ | 2.208  | 1.637  | 2.359  | 1.609  | 25.9 | 12.8 | 23.1 | 10.2 | 15.0(15) | [12] |

$E_3$ transitions [$B(E3)$ in units of $10^3\text{e}^2\text{fm}^6$]

| $^{207}$Tl | $1h_{11/2}$ → $2d_{3/2}$ | 0.357  | 0.780  | 0.144  | 0.997  | 0.94 | 0.94 | 1.17 | 1.11 | 1.82(18) | [17] |

$E_5$ transitions [$B(E5)$ in units of $10^7\text{e}^2\text{fm}^{10}$]

investigate the effect we calculate the same quantities alternatively within the empirical LM approach where, as mentioned before, experimental s.p. energies were used and the force parameter adjusted to reproduce quantitatively the excitation energies and $B(EL)$ values of the lowest $E_3$ and $E_5$ modes in $^{208}$Pb. For this reason, no resonance effect exists and the strong overshooting for the $B(E3)$ values disappears. Note also that the theoretical value of the $^{209}$Bi transition derived with the SV-bas$m$ parametrization agrees nicely with the data. For that parametrization exists no resonance between s.p. energies and $E(3^-)$.

Finally, we look at the $B(E5)$ value of $^{207}$Tl in Table III. Here we encounter the problem that all theoretical results including the Landau-Migdal approach are about 50% to small.

B. The magnetic case

In this subsection, we go for magnetic moments and transitions. We recall from Sections II A and II B that we employ in this case an effective operator which simulates the effect of the Landau renormalization as well as the contributions of the virtual meson exchange [34].

Table V shows the theoretical results for magnetic dipole moments in comparison with experimental data. The agreement is very good throughout. This happens, again, because also the magnetic moments are dominated.
TABLE IV. Contributions (in percent) to the electric quadrupole moments and to the transition amplitudes of the \( EL \) transitions from the external-field \( EL \) operator (\( Q \)) and of the two RPA states of \(^{208}\text{Pb} \) entering into the polarization term of Eq. \((7a)\). The electric \( L^2 \) RPA states with the maximum contributions are shown. Here the \( L_n^0 \) is the first electric state of the respective multipolarity. The \( L_n^0 \) is: (i) the \( 2^+ \) state from the region of the giant quadrupole resonance with \( E = 10.92 \text{ MeV} \) for the electric quadrupole moments and for the \( E2 \) transitions; (ii) the \( 3^- \) state from the region of the giant octupole resonance with \( E = 5.52 \text{ MeV} \) for the \( E3 \) transitions; and (iii) the \( 5^- \) state with \( E = 6.65 \text{ MeV} \) for the \( E5 \) transition. Calculations with the SV-bas parameter set.

| Nucleus | State/Transition | \( Q \) | \( L_n^0 \) | \( L_5^0 \) |
|---------|------------------|--------|--------|--------|
| \(^{209}\text{Pb} \) | \( 2g_{9/2} \) | 0.2 | 46.3 | 24.5 |
| \(^{209}\text{Bi} \) | \( 1h_{9/2} \) | 56.9 | 34.6 | 15.4 |
| \(^{209}\text{Bi} \) | \( 1i_{13/2} \) | 64.7 | 29.4 | 16.2 |

\( E2 \) transitions

| Nucleus | State/Transition | \( L_n^0 \) | \( L_5^0 \) |
|---------|------------------|--------|--------|
| \(^{207}\text{Pb} \) | \( 2d_{5/2} \rightarrow 3s_{1/2} \) | 59.8 | 33.7 | 15.2 |
| \(^{207}\text{Bi} \) | \( 2f_{5/2} \rightarrow 3p_{1/2} \) | 0.2 | 48.5 | 22.4 |
| \(^{207}\text{Bi} \) | \( 3p_{3/2} \rightarrow 3p_{1/2} \) | 0.2 | 45.3 | 24.7 |
| \(^{209}\text{Pb} \) | \( 4s_{1/2} \rightarrow 3d_{5/2} \) | 0.4 | 47.4 | 27.5 |
| \(^{209}\text{Pb} \) | \( 3d_{5/2} \rightarrow 2g_{9/2} \) | 0.2 | 52.2 | 25.2 |
| \(^{209}\text{Bi} \) | \( 2f_{7/2} \rightarrow 1h_{9/2} \) | 51.6 | 40.7 | 16.0 |
| \(^{209}\text{Bi} \) | \( 3p_{3/2} \rightarrow 2f_{7/2} \) | 42.5 | 44.5 | 16.1 |
| \(^{209}\text{Bi} \) | \( 2f_{5/2} \rightarrow 1h_{9/2} \) | 36.6 | 56.7 | 13.5 |

| Nucleus | State/Transition | \( L_n^0 \) | \( L_5^0 \) |
|---------|------------------|--------|--------|
| \(^{209}\text{Pb} \) | \( 1j_{15/2} \rightarrow 2g_{9/2} \) | 0.0 | 82.2 | 2.7 |
| \(^{209}\text{Bi} \) | \( 1i_{13/2} \rightarrow 1h_{9/2} \) | 12.1 | 80.9 | 2.9 |

\( E3 \) transitions

\( E5 \) transition

by “topological” shell effects.

Table \( \text{VI} \) shows results for magnetic transitions. We look first at dipole transitions. The upper part of the dipole block sows the allowed \( M1 \) transitions. Theory and experiment are in fair agreement for \(^{207}\text{Pb} \) and for all three parameter sets with variations within the three parametrizations between 10 % and 20%. As we found already for electrical transitions, agreement with data is poor for \(^{209}\text{Bi} \) which indicates, again, that present EDFs still have weak points concerning high-spin s.p. states. Below the \( l \)-allowed \( M1 \) transitions follow the \( l \)-forbidden \( M1 \) transitions. The external-field operator \( Q \), Eq. \((8a)\), gives no contribution to lowest order, i.e. if we neglect the \( \{ Y_2 \times \delta \}_1 \) term (\( \xi_l = 0 \)). In that case, only polarization effects yield finite contributions. However, this contribution is one order of magnitude too small, except for \(^{209}\text{Bi} \) where polarization amounts to nearly 25% of the measured \( B(M1) \). Here is the place where the corrections through the tensor term \( \sim \{ Y_2 \times \delta \}_1 \) in the magnetic operator \((9)\) may become qualitatively important. We activate the term by setting \( \xi_l = 1.5 \) which improves the agreement with data dramatically. This encouraging result calls for further analysis of the meson-exchange currents \((34)\) in the measurement of magnetic observables.

The dominant relative contributions to magnetic moments and transitions are collected in Table \( \text{VII} \). We show (in percent) the contribution from the external-field \( ML \) operator \( Q \) and the first and second \( L^2 \) magnetic RPA states in \(^{208}\text{Pb} \) calculated with the SV-bas parameter set. For \( L = 1 \) these two states are isoscalar \( 1^+_1 \) with the calculated \( E(1^+_1) = 5.66 \text{ MeV} \) and \( B(M1; 1^+_1) = 5.6 \mu_N^2 \) and the isovector \( 1^+_2 \) resonance with \( E(1^+_2) = 7.95 \text{ MeV} \) and \( B(M1; 1^+_2) = 17.8 \mu_N^2 \). The contribution of the external-field \( M1 \) operator dominates in all moments and \( l \)-allowed transitions. Totally different look the \( l \)-forbidden \( M1 \) transitions where in the case \( \xi_l = 0 \) the \( M1 \) operator, by definition, allows no transitions. In the case \( \xi_l = 1.5 \), the contribution of the first term \( Q \) in the right-hand side of Eq. \((11)\) dominates. As can be seen from Table \( \text{VI} \) the choice \( \xi_l = 1.5 \) provides on the whole the reasonable description of the \( l \)-forbidden \( M1 \) transitions, though for the transition \( 2f_{7/2} \rightarrow 1h_{9/2} \) in \(^{209}\text{Bi} \) one obtains the exceeding of the \( B(M1) \) in about 3 times. The contribution of the \( 1^+_1 \) state is larger than the \( 1^+_1 \) state as the \( B(M1; 1^+_2) \) value is larger than the \( B(M1; 1^+_1) \) by more than a factor of three.

Finally we look at the results for \( M2 \) and \( M4 \) transitions in the lower part of Table \( \text{VI} \). The agreement with the data is not as satisfying as for the allowed \( M1 \) transitions. One observers also larger differences between the
TABLE V. Energies of the states (in MeV) and their magnetic dipole moments (in units of $\mu_N$) of the odd-mass nuclei next to $^{208}\text{Pb}$.

| Nucleus | State | SV-bas | SV-bas$_{sm}$ | SLy4 | Experiment | $\xi_t$ | SV-bas | SV-bas$_{sm}$ | SLy4 | Experiment |
|---------|-------|--------|---------------|------|------------|--------|--------|---------------|------|------------|
| $^{207}\text{Tl}$ | $3s_{1/2}$ | 0.0 | 0.0 | 0.0 | 0.0 | 0 | 1.92 | 2.03 | 1.88 | 1.876(5) |
|         |       |       |               |      |            | 1.5 | 1.94 | 2.05 | 1.90 | 1.876(5) |
| $^{207}\text{Pb}$ | $3p_{1/2}$ | 0.0 | 0.0 | 0.0 | 0.0 | 0 | 0.42 | 0.44 | 0.44 | 0.59104(16) |
|         |       |       |               |      |            | 1.5 | 0.65 | 0.69 | 0.68 | 0.59104(16) |
|         | $2f_{5/2}$ | 0.950 | 1.023 | 0.971 | 0.570 | 0 | 0.81 | 0.86 | 0.86 | 0.80(3) |
|         |       |       |               |      |            | 1.5 | 0.99 | 1.06 | 1.05 | 0.80(3) |
|         | $3p_{3/2}$ | 0.883 | 0.798 | 1.103 | 0.898 | 0 | −1.11 | −1.20 | −1.18 | −1.09(11) |
|         |       |       |               |      |            | 1.5 | −1.19 | −1.28 | −1.26 | −1.09(11) |
|         | $1i_{13/2}$ | 1.584 | 1.226 | 1.496 | 1.633 | 0 | −0.96 | −1.00 | −1.04 | −1.00(3) |
|         |       |       |               |      |            | 1.5 | −1.10 | −1.16 | −1.19 | −1.00(3) |
| $^{209}\text{Pb}$ | $2g_{9/2}$ | 0.0 | 0.0 | 0.0 | 0.0 | 0 | −1.14 | −1.20 | −1.18 | −1.4735(16) |
|         |       |       |               |      |            | 1.5 | −1.28 | −1.36 | −1.35 | −1.4735(16) |
| $^{209}\text{Bi}$ | $1h_{9/2}$ | 0.0 | 0.0 | 0.0 | 0.0 | 0 | 3.42 | 3.35 | 3.42 | 4.1087(5) |
|         |       |       |               |      |            | 1.5 | 3.25 | 3.19 | 3.24 | 4.1087(5) |
|         | $2f_{7/2}$ | 1.117 | 0.785 | 0.921 | 0.896 | 0 | 4.93 | 5.04 | 4.89 | 4.41 |
|         |       |       |               |      |            | 1.5 | 5.04 | 5.16 | 5.01 | 4.41 |
|         | $1i_{13/2}$ | 2.208 | 1.637 | 2.359 | 1.609 | 0 | 7.92 | 7.96 | 7.78 | 8.07(19) |
|         |       |       |               |      |            | 1.5 | 8.07 | 8.13 | 7.94 | 8.07(19) |

 parameter sets, e.g. the result of the SV-bas set for $^{209}\text{Pb}$ agrees with the experimental value but is a factor three to large for $^{209}\text{Bi}$. It is worthwhile to counter check the contributions to the total $M2$ and $M4$ transition amplitudes in Table VII. This reveals that the external field operator for $M2$ transitions alone produce results which are one order of magnitude larger than the experiment value. The polarization contributions correct that overestimation toward the data. One should not wonder that the numbers do not add up to 100%. What is missing is the accumulated further reduction by all other magnetic modes in the spectrum which is in that case obviously a large fraction. In the case of the known $M4$ transition, the external field contribution alone is five times larger than the experimental value $B(M4)$. Core polarization helps a large way to reduce the theoretical $B(ML)$ values thus reducing the disagreement between the external field results and the data appreciably, though not yet completely.

IV. CONCLUSION

We present results of self-consistent calculations for electric and magnetic moments and transition probabi-
TABLE VI. Energies (in MeV) and $B(ML)$ values for the magnetic transitions in the odd-mass nuclei next to $^{208}$Pb.

| Nucleus | Transition | Energies | $B(ML)$ |
|---------|------------|----------|----------|
|         |            | SV-bas   | SV-bas$_m$ | SLy4 | Experiment | SV-bas | SV-bas$_m$ | SLy4 | Experiment |
|         | $l$-allowed M1 transitions [$B(M1)$ in units of $\mu_N^2$] |         |        |        |        |        |        |        |
| $^{207}$Pb | $3p_{3/2} \rightarrow 3p_{1/2}$ | 0.884 | 0.798 | 1.103 | 0.898 | 0 | 0.36 | 0.43 | 0.41 | 0.45(7) | 47 |
|          |           | 1.5 | 0.32 | 0.38 | 0.36 | 0.45(7) | 47 |
| $^{207}$Pb | $2f_{7/2} \rightarrow 2f_{5/2}$ | 2.417 | 2.186 | 2.981 | 1.770 | 0 | 0.40 | 0.47 | 0.46 | 0.49(16) | 47 |
|          |           | 1.5 | 0.36 | 0.42 | 0.41 | 0.49(16) | 47 |
| $^{209}$Bi | $2f_{5/2} \rightarrow 2f_{7/2}$ | 2.211 | 2.496 | 2.544 | 1.930 | 0 | 0.90 | 1.13 | 0.86 | 0.222(34) | 42 |
|          |           | 1.5 | 0.83 | 1.03 | 0.79 | 0.222(34) | 42 |
|         | $l$-forbidden M1 transitions [$B(M1)$ in units of $10^{-3}\mu_N^2$] |         |        |        |        |        |        |        |
| $^{207}$Tl | $2d_{3/2} \rightarrow 3s_{1/2}$ | 0.774 | 0.652 | 0.753 | 0.351 | 0 | 2.12 | 1.12 | 2.52 | 23(5) | 47 |
|          |           | 1.5 | 20.2 | 18.0 | 22.8 | 23(5) | 47 |
| $^{207}$Pb | $3p_{3/2} \rightarrow 2f_{5/2}$ | -0.067 | -0.225 | 0.132 | 0.328 | 0 | 1.99 | 1.36 | 1.87 | 50(9) | 47 |
|          |           | 1.5 | 38.2 | 42.9 | 44.1 | 50(9) | 47 |
| $^{209}$Pb | $1i_{11/2} \rightarrow 2g_{9/2}$ | 1.379 | 1.020 | 1.554 | 0.779 | 0 | 0.18 | 0.08 | 0.51 | 9.8(11) | 42 |
|          |           | 1.5 | 11.6 | 14.1 | 19.2 | 9.8(11) | 42 |
| $^{209}$Bi | $2f_{7/2} \rightarrow 1h_{9/2}$ | 1.117 | 0.786 | 0.921 | 0.896 | 0 | 1.04 | 0.46 | 1.12 | 4.6(9) | 42 |
|          |           | 1.5 | 14.6 | 13.3 | 18.0 | 4.6(9) | 42 |
|         | $M2$ transitions [$B(M2)$ in units of $\mu_N^2$ fm$^4$] |         |        |        |        |        |        |        |
| $^{209}$Pb | $1j_{15/2} \rightarrow 1i_{11/2}$ | 0.623 | 1.175 | 1.052 | 0.644 | 33.1 | 44.7 | 60.9 | 33(8) | 42 |
| $^{209}$Bi | $1i_{13/2} \rightarrow 1h_{9/2}$ | 2.208 | 1.637 | 2.359 | 1.609 | 110.1 | 140.2 | 64.2 | 34(5) | 42 |
|          | $M4$ transitions [$B(M4)$ in units of $10^5\mu_N^3$ fm$^6$] |         |        |        |        |        |        |        |
| $^{207}$Tl | $1h_{11/2} \rightarrow 2d_{3/2}$ | 0.357 | 0.780 | 0.144 | 0.997 | 4.91 | 5.67 | 4.31 | 2.39(23) | 47 |

a The initial state, $E_i = 2339.921$, is a mixture of $\nu(2f_{7/2})$ and $\nu(1i_{13/2}) \otimes 3^-$.
b The initial state is a mixture of $\pi(1i_{13/2})$ and $\pi(1h_{9/2}) \otimes 3^-$.c Weighted mean of 18.6(104) and 38(5) [48].

ties in the neighboring odd-mass nuclei of $^{208}$Pb. Starting point are Skyrme-Hartree-Fock calculations for the ground state of $^{208}$Pb and subsequent Skyrme-RPA to obtain the excitation spectrum of $^{208}$Pb. From that, we deduce the electric and magnetic s.p. matrix elements of the odd system from those of $^{208}$Pb together with a polarization correction to account for the change of the mean field by the odd nucleon (hole). For the calculations, we use a Skyrme energy functional with parameter sets which had been previously optimized for nuclear structure properties. We obtain at once theoretical results which are in fair agreement with the data for the moments and the electric as well as allowed magnetic transitions.

We also investigated the impact of the various contributions to the final results. In the electric case for
TABLE VII. Contributions (in percent) to the magnetic dipole moments and to the transition amplitudes of the $ML$ transitions from the external-field $ML$ operator ($Q$) and of the two RPA states of $^{208}$Pb entering into the polarization term of Eq. (7a). The magnetic $L^\pi$ RPA states with the maximum contributions are shown. Here the $L^\pi_0$ is: the $1^+_a$ state with $E=7.95$ MeV (representing the isovector $M1$ resonance) for the magnetic dipole moments and for the $l$-allowed $M1$ transitions; the $2^-$ state from the region of the giant $M2$ resonance with $E=8.56$ MeV for the $M2$ transitions; the $4^-$ state from the region of the giant $M4$ resonance with $E=7.97$ MeV for the $M4$ transition. The $L^\pi_b$ is: the $1^+_a$ state with $E=5.66$ MeV for the magnetic dipole moments and for the $l$-allowed $M1$ transitions; the $2^-$ state from the region of the giant $M2$ resonance with $E=9.39$ MeV for the $M2$ transitions; the $4^-$ state from the region of the giant $M4$ resonance with $E=5.19$ MeV for the $M4$ transition. Calculations with the SV-bas parameter set and $\xi_t=0$.

| Nucleus | State/Transition | $Q$ | $L^\pi_0$ | $L^\pi_b$ |
|---------|------------------|-----|-----------|-----------|
| $^{207}$Tl | $3s_{1/2}$ | 120.9 | -12.4 | -7.8 |
| $^{207}$Pb | $3p_{1/2}$ | 114.0 | -11.4 | -3.4 |
| | $2f_{5/2}$ | 127.8 | -26.1 | -1.4 |
| | $3p_{3/2}$ | 129.4 | -25.7 | -3.8 |
| | $1i_{13/2}$ | 150.1 | -34.0 | -15.7 |
| $^{209}$Pb | $2g_{9/2}$ | 126.9 | -25.7 | -0.6 |
| $^{209}$Bi | $1h_{9/2}$ | 87.9 | 9.0 | 3.0 |
| | $2f_{7/2}$ | 108.0 | -4.8 | -3.1 |
| | $1i_{13/2}$ | 105.1 | -4.3 | -0.7 |

$l$-allowed $M1$ transitions

| $^{207}$Pb | $3p_{3/2} \rightarrow 3p_{1/2}$ | 135.4 | -30.5 | -4.4 |
| | $2f_{7/2} \rightarrow 2f_{5/2}$ | 145.5 | -40.3 | -3.6 |
| $^{209}$Bi | $2f_{5/2} \rightarrow 2f_{7/2}$ | 141.2 | -23.0 | -17.0 |

$M2$ transitions

| $^{209}$Pb | $1j_{15/2} \rightarrow 1i_{11/2}$ | 223.4 | -27.9 | -8.4 |
| $^{209}$Bi | $1i_{13/2} \rightarrow 1h_{9/2}$ | 160.0 | -18.1 | -4.3 |

$M4$ transition

| $^{207}$Tl | $1h_{11/2} \rightarrow 2d_{3/2}$ | 147.4 | -7.1 | -4.5 |

TABLE VIII. The same as in Table VII but for the $l$-forbidden $M1$ transitions with two variants of the choice of the parameter $\xi_t$. The $1^+_a$ is the $1^+_a$ RPA state of $^{208}$Pb for all the transitions. The $1^+_b$ is the $1^+_b$ RPA state for all the transitions except for the transition $3p_{3/2} \rightarrow 2f_{5/2}$ in $^{207}$Pb. For the latter transition, $1^+_a$ is the high-energy $1^+$ RPA state with $E=17.71$ MeV in the case of $\xi_t=0$ and with $E=28.95$ MeV in the case of $\xi_t=1.5$.

| Nucleus | Transition | $\xi_t$ | $Q$ | $1^+_a$ | $1^+_b$ |
|---------|------------|----------|-----|--------|--------|
| $^{207}$Tl | $2d_{3/2} \rightarrow 3s_{1/2}$ | 1.5 | 99.6 | 16.8 | 12.3 |
| $^{207}$Pb | $3p_{3/2} \rightarrow 2f_{5/2}$ | 1.5 | 110.8 | 20.2 | -4.2 |
| $^{209}$Pb | $1i_{11/2} \rightarrow 2g_{9/2}$ | 1.5 | 123.2 | 16.6 | -5.5 |
| $^{209}$Bi | $2f_{7/2} \rightarrow 1h_{9/2}$ | 1.5 | 106.6 | 15.0 | 9.7 |

odd-neutron neighbors, the whole effect comes from the polarization as the external field gives zero contributions (except for the very small center of mass corrections). For odd-proton neighbors, the polarization contributes roughly 50% to moments and transitions. In all cases, the low lying $2^+$ state of $^{208}$Pb contributes most because of the energy denominator in the response function. In the magnetic case, one has to distinguish between the $M1$ properties and the higher ML transitions. For $M1$, the core polarization is much smaller than in the electric case as only the two spin orbit partners contribute (as isoscalar and isovector state, respectively). For the higher $L$ values the polarization effects are large as many components contribute. We obtain very similar results for three different parameter sets which is no surprise because all three sets produce similar excitations in $^{208}$Pb.

A particular case are $l$-forbidden $M1$ transitions. Polarization effects produce a finite contribution which, however, is an order of magnitude too small to reproduce the data. It is here where mesonic contributions and vertex corrections dominate. We have simulated them by the empirically tuned tensor term and find that it considerably improves the agreement with data in our self-
consistent context. This preliminary result points toward the next task, a fully microscopic description of mesonic effects and vertex correction in magnetic transitions.

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