Quantum-state transfer between atom and cavity field in Jaynes-Cummings model

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We present a scheme for transferring quantum state between atom and cavity field in Jaynes-Cummings model. It is based on the fact that the atom in a cavity can induce the generation of modified coherent states, which can be shown to be macroscopically distinguishable. The application on two-cavity system provides an alternative scheme for preparation of non-local superpositions of quasi-classical light states. Numerical simulation shows that the proposed schemes are efficient.

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I. INTRODUCTION

Coherent transfer between an arbitrary state of a qubit and a superposition of two quasi-classical coherent states is of fundamental conceptual interest in many fields of physics [1–4]. Coherent states provide a close connection between classical and quantum mechanics, which has been introduced in a physical context, first as quasi-classical states in quantum mechanics, then as the backbone of quantum optics [5]. The non-classical nature of such states appears since two coherent states correspond to two different values of a macroscopic variable, such as the quasi-probability distribution in phase space. Recently, in a new branch of quantum computing, two phase-opposite coherent states are exploited to be the macroscopic qubits [6–9]. Much attention has been paid on obtaining such superposed coherent states [10–13].

In this paper, we propose a type of modified coherent state based on the canonical coherent state, which also demonstrate the non-classical nature. We show that an arbitrary atom state can be mapped onto the field as an atom-cavity Schrödinger cat state. This make it possible to realize entangled pairs of macroscopic objects, nonlocal Schrödinger cat state.

This paper is organized as follows. In Section II we present a modified coherent state which is shown to be equivalent to a canonical coherent state. In Section III, we investigate the JC model and propose the effective Hamiltonian for a type of states. In Section IV, we study the dynamics of the system for the initial field state being a coherent state. In Section V, as an application we investigate the entanglement transfer between atoms and fields in the two-cavity system. Finally, we give a summary in Section VI.

II. MODIFIED COHERENT STATE

We start with a canonical coherent state

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \quad (1)$$

which is the eigen state of the boson annihilation operator $a$, and the Fock state is defined as $|n\rangle = (a^\dagger)^n / \sqrt{n!} |\text{Vac}\rangle$ with $|\text{Vac}\rangle$ being the vacuum state of $a$. It is a Gaussian wavepacket in the coordinate representations $x = 1/\sqrt{2} (a^\dagger + a)$, whose center is shifted $\sqrt{2} \text{Re}(\alpha)$ from the origin. The amplitude $|\alpha|$ characterizes the distance between $|\alpha\rangle$ and $|-\alpha\rangle$ in phase space. Then the states $|\alpha\rangle$ and $|-\alpha\rangle$ are sufficiently distinguishable for large $|\alpha|$, and present many advantages compared with discrete variable qubit states $|0\rangle$ and $|1\rangle$.

Now we consider a type of modified coherent state (MCS)

$$|\alpha, g\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} e^{-ig(n)} |n\rangle, \quad (2)$$

where $g(n)$ is an arbitrary real function. Here the term modified just indicates the difference from the canonical coherent state $|\alpha\rangle$, which is the simplest case of the MCS with $g(n) = 0$, i.e., $|\alpha\rangle = |\alpha, 0\rangle$. It is worth noting that the MCS $|\alpha, g\rangle$ is equivalent to the state $|\alpha, 0\rangle$ when associated with a Hamiltonian in the form of $\hat{h} = h(a^\dagger a)$, where $h$ is an arbitrary function.

Based on the original boson operator $a$, let us now define a class of boson operator

$$\hat{b} = e^{i[g(a^\dagger a) - g(a^\dagger a + 1)]} a, \quad (3)$$

with an arbitrary real function $g$ as long as $g(0) = 0$. It turns out that $\hat{b}$ satisfies the commutation relation

$$[\hat{b}, \hat{b}^\dagger] = 1. \quad (4)$$

Then the Fock state associated with the annihilation operator $\hat{b}$ can be written as

$$|n, g\rangle = \left( \frac{\hat{b}^\dagger}{\sqrt{n!}} \right)^n |\text{Vac}\rangle, \quad (5)$$
based on that the modified coherent state $|\alpha, g\rangle$ has the standard form
\[ |\alpha, g\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} |n, g\rangle. \tag{6} \]

We see that the MCSs $|\alpha, g\rangle$ and $|\alpha, 0\rangle$ are connected by the transformation in Eq. (5). Interestingly, the Hamiltonian $h$ is invariant under the same transformation, i.e., $h(a^\dagger a) = h(b^\dagger b)$. In this sense, coherent states $|\alpha, 0\rangle$ and $|\alpha, g\rangle$ are absolutely equivalent for a system described by $h$. Hereafter we will not distinguish between the standard and the modified coherent states.

In this paper, we focus on a simple case with
\[ g(n) = \gamma \sqrt{n}, \tag{7} \]
which generates the coherent state
\[ |\alpha, \gamma\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} e^{-i\gamma \sqrt{n}} |n\rangle. \tag{8} \]

According to the above analysis, coherent states $|\pm\alpha, 0\rangle$ are sufficiently distinguishable for large $|\alpha|$. In this paper, we are interested in the pair of states $|\alpha, \pm\gamma\rangle$. We will show that the atom-field coupling can induce the generation of state $|\alpha, \pm\gamma\rangle$ from $|\alpha, 0\rangle$ by natural time evolution and two states $|\alpha, \pm\gamma\rangle$ are macroscopically distinguishable as that of $|\pm\alpha, 0\rangle$. To this end, we compute the Wigner quasiprobability distribution $W_{\alpha, \gamma}(x, p)$ in phase space, where $x = (a + a^\dagger) / \sqrt{2}$, $p = (a - a^\dagger) / (i\sqrt{2})$ are the quadrature operators of the cavity field.

By taking $\alpha = 7, \gamma \in [-4\alpha \pi, 4\alpha \pi]$, we plot the probability distribution
\[ P(p, \gamma) = |\psi_{\gamma}(p)|^2 \]
\[ = e^{-|\alpha|^2} \sum_n \frac{\alpha^n}{\sqrt{n!}} e^{-i\gamma \sqrt{n}} |\phi_n(p)|^2, \tag{9} \]
where $\phi_n(p)$ is the eigenfunction of the harmonic oscillator system in $p$ space.

We plot the probability distribution $W_{\alpha, \gamma}(x, p)$ for $\alpha = \pm 7, \gamma = 0; \alpha = 7, \gamma = \pm \alpha \pi$. Fig. 1 shows that $|\alpha, \pm\gamma\rangle$ are also sufficiently distinguishable when $\gamma$ is around the values of $\alpha \pi (2l + 1)$, with $l = 0, \pm 1, \pm 2, \ldots$. It is not true for the cases of $|l| \gg 1$, which is beyond our concern because they cannot be precisely prepared as $|l|$ increases in our scheme. It indicates that the state $c_1 |\alpha, \gamma\rangle + c_2 |\alpha, -\gamma\rangle$ can be considered as a Schrödinger cat state. Moreover, modified coherent state $|\alpha, \gamma\rangle$ is physically relevant. It can be prepared from the canonical coherent state $|\alpha, 0\rangle$ by natural time evolution in a system with $\sqrt{n}$ spectrum. In the following section, we will demonstrate that the atom-field interaction can induce effective nonlinear spectrum of the photon. The cat state can be prepared by mapping the qubit state onto the field.

### III. EFFECTIVE SEPARATION OF ATOM AND FIELD

Consider a single-cavity JC model
\[ H = \lambda (\sigma_+ a + \sigma_- a^\dagger) + \frac{1}{2} \omega_0 \sigma_z + \omega a^\dagger a, \tag{10} \]
\[ \sigma_+ = (\sigma_-)^\dagger = |e\rangle \langle g|, \sigma_z = |e\rangle \langle e| - |g\rangle \langle g|, \tag{11} \]
where $\omega$ is photon frequency, $|g\rangle$ and $|e\rangle$ denote the ground and excited states of atom with transition frequency $\omega_0$, and $\lambda$ is the atom-field coupling constant. Under the resonance condition $\omega = \omega_0$, it can be reduced to a simple form
\[ H = \lambda (\sigma_+ a + \sigma_- a^\dagger). \tag{12} \]
We notice that the Jaynes-Cummings model has been realized in the laboratory in several well-known ways [14–16].
We note that the excitation number
\[ N = \frac{1}{2} \sigma_z + a^\dagger a + \frac{1}{2}, \]
is a conservative quantity, i.e., \([N, H] = 0\). So the Hamiltonian can be diagonalized in each \(2 \times 2\) invariant subspace. We start our analysis from Hamiltonian (12), which can be rewritten in the form
\[ H = H_1 + H_2, \]
(14)

\[ H_1 = \lambda \left( e^{i\phi} |e\rangle \langle g| \sum_n \sqrt{n+1} |n\rangle \langle n| - e^{-i\phi} |g\rangle \langle e| \sum_n \sqrt{n} |n\rangle \langle n| \right), \]
(15)

\[ H_2 = \lambda \sum_n \sqrt{n+1} \left( e^{i\phi} |e\rangle \langle g| n + |g\rangle \langle e| n + 1\right) \left( e^{-i\phi} |n+1\rangle - |n\rangle \right), \]
(16)

by taking \(n + 1) = e^{i\phi} \langle n + e^{-i\phi} (n + 1) - \langle n|\), \(\langle n| = e^{-i\phi} |n+1\rangle - e^{-i\phi} |n+1\rangle + \langle n|\), where \(\phi\) is an arbitrary real number.

For a separated state
\[ |\phi(0)\rangle = (c_1 |g\rangle + c_2 |e\rangle) \sum_n f_n |n\rangle, \]
(17)

where \(|c_1|^2 + |c_2|^2 = 1\), and \(\sum_n |f_n|^2 = 1\), one can reduce the Hamiltonian to the simple form under certain conditions. We note that if
\[ f_n \approx e^{i\phi} f_{n-1}, \]
(18)

\[ |f_n|^2 \ll 1, \]
(19)

we have
\[ |H_2 |\phi(0)\rangle| \ll |H_1 |\phi(0)\rangle|, \]
(20)

i.e., the dynamics of state \(|\phi(0)\rangle\) is governed by the Hamiltonian \(H_1\) approximately. Now we focus on the Hamiltonian \(H_1\). In addition to the condition in Eq. (18), when we consider the field state satisfying
\[ n \gg \Delta n \gg 1, \]
(21)

where \(n\) denotes the average photon number,
\[ \bar{n} = \sum_{n_1}^{n_2} n \left| f_n \right|^2, \]
(22)

\[ \Delta n = n_2 - n_1, \]
(23)

we can have
\[ H_1 = H_+ P_+ + H_- P_. \]
(24)

Here the projection operators \(P_\pm\) for atom state are
\[ P_\pm = \frac{1}{2} \left( |g\rangle \pm e^{i\phi} |e\rangle \right) \left( |g\rangle \pm e^{-i\phi} \langle e| \right), \]
(25)

which satisfy
\[ P_+ + P_- = 1, \]
(26)

\[ P_+ P_- = 0. \]
(27)

In the derivation, we have used the approximation
\[ (n + 1)^{1/2} \approx n^{1/2} + \frac{1}{2 n^{1/2}}, \]
(28)

under the condition in Eq. (21). The sub-Hamiltonians \(H_\pm\) are in the form
\[ H_+ = -H_- = \frac{\lambda}{2 n^{1/2}} |e\rangle \langle e| + \lambda \sum_n \sqrt{n} |n\rangle \langle n|. \]
(29)

Before further discussion of the implication of the obtained result, two distinguishing features need to be mentioned. First, the spectra of the photons in \(H_\pm\) are \(\pm \lambda \sqrt{n}\), which are related to the preparation of states \(|\alpha, \pm \gamma\rangle\) from \(|\alpha, 0\rangle\) by natural time evolution, as mentioned above. This is crucial for the scheme to write the qubit state in the field. Second, we note that \(H_+\) and \(H_-\) have opposite sign, which leads to the time evolution of photons in two reverse directions respectively. This is crucial for the scheme to read the state from the field.

IV. QUANTUM STATE TRANSFER BETWEEN ATOM AND FIELD

A state in the form of Eq. (17) can be rewritten as
\[ |\phi(0)\rangle = \frac{1}{\sqrt{2}} \left[A \left(|g\rangle + e^{i\phi} |e\rangle\right) + B \left(|g\rangle - e^{i\phi} |e\rangle\right)\right] \times \sum_n |f_n| e^{i n \phi} |n\rangle, \]
(30)

where \(A = \frac{1}{\sqrt{2}} (c_1 + e^{-i\phi} c_2)\), and \(B = \frac{1}{\sqrt{2}} (c_1 - e^{-i\phi} c_2)\). Then we have
\[ |\phi(t)\rangle = \frac{1}{\sqrt{2}} \left[ A \left(|g\rangle + e^{-i\phi} |e\rangle\right) \right. \sum_n |f_n| e^{-i \sqrt{\lambda/2} t} e^{i n \phi} |n\rangle \]
\[ + \frac{1}{\sqrt{2}} B \left(|g\rangle - e^{i\phi} |e\rangle\right) \sum_n |f_n| e^{i \sqrt{\lambda/2} t} e^{i n \phi} |n\rangle. \]
(31)

It is the superposition state of two independent evolution processes in which there are no interactions between atom
Cavity Atom \[ \tau \]

\[
\text{where the evolution alone this path, as expected in a general scheme,}
\]

hence, the stored information can be employed to transfer or store the quantum information to the field. However, the stored information can never go back to the atom as expected. This procedure explains the coupling constant \( \lambda(t) \) is explained in the text.

Cavity \[ \alpha, \alpha \pi \]
Cavity \[ \alpha, -\alpha \pi \]

As one can see in the formula above, an arbitrary atom and field. At the instants, \( t_l = (2l + 1) \tau, l = 0, 1, 2, ... \), \( \tau = \pi \sqrt{n}/\lambda \), we have

\[
|\phi(t_l)\rangle = \frac{|g \rangle - (-1)^l ie^{i\varphi} |e\rangle}{\sqrt{2}} (A |\Phi_+^{l}\rangle + B |\Phi_-^{l}\rangle), \tag{32}
\]

where

\[
|\Phi_\pm^l\rangle = \sum_n |f_n| e^{\mp i\sqrt{n} \lambda t} e^{in\varphi} |n\rangle. \tag{33}
\]

As one can see in the formula above, an arbitrary atom state can retrieve a pure state at the instants \( t_l \). Remarkably, the initial atomic state \((A, B)\) is mapped on the field state if \( |\Phi_+^l\rangle \) and \( |\Phi_-^l\rangle \) are orthogonal, while the atom is always in the state \( |g \rangle - ie^{i\varphi} |e\rangle / \sqrt{2} \). This is termed as “attractor” in [17]. Then the initial state \((A, B)\) can never go back to the atom as expected. This procedure can be employed to transfer or store the quantum information to the field. However, the stored information can not be read out from the field via the further time evolution alone this path, as expected in a general scheme, the initial state is revival periodically.

Considering the initial field state as a coherent state \( |\alpha,0\rangle \), with \( f_n = e^{-|\alpha|^2/2} \alpha^n / \sqrt{n!} \), \( \varphi = 0 \), we will have

\[
|\phi(\tau)\rangle = \frac{1}{\sqrt{2}} \left(|g\rangle - i |e\rangle\right) (A |\alpha, \alpha\pi\rangle + B |\alpha, -\alpha\pi\rangle). \tag{34}
\]

Then the quantum information in the atom is encoded into the field. We would like to point that, the initial state of the atom cannot be revival again as expected, not like the swap gate.

Another interesting feature in such dynamic process is that two effective Hamiltonians for atoms differ only in an opposite sign, i.e., \( H_+ = -H_- \). It shows that two atom states \( (|g\rangle + ie^{i\varphi} |e\rangle) / \sqrt{2} \) and \( (|g\rangle - ie^{i\varphi} |e\rangle) / \sqrt{2} \) evolve in the same way but in opposite time directions. It is crucial for the scheme of reading out the information from the field.

A trivial way to retrieve the initial atomic state is to switch the sign of interaction strength \( \lambda \) to realize the reversed time evolution. Alternative operation can also implement the same task: flipping the states \( (|g\rangle - ie^{i\varphi} |e\rangle) / \sqrt{2} \rightarrow (|g\rangle + ie^{i\varphi} |e\rangle) / \sqrt{2} \) by the external pulse. (rotation operator \( e^{i\pi/4}(\sigma_z + 1) = -\sigma_z \)) Then taking the state

\[
|\rho(\tau)\rangle = \frac{1}{\sqrt{2}} (|g\rangle + i |e\rangle) (A |\alpha, \alpha\pi\rangle + B |\alpha, -\alpha\pi\rangle), \tag{35}
\]

as the initial state, we have

\[
|\rho(\tau)\rangle = -\sigma_z |\rho(0)\rangle. \tag{36}
\]

Although this conclusion is achieved in the framework of approximation, we would like to point that such a time-reversal process is exact. The underlying mechanism is similar to the phenomenon of spin echo. We note that the Hamiltonian [12] obeys the anti-commutation relation

\[
\{\sigma_z, H\} = 0. \tag{37}
\]

Then for an arbitrary initial state \( \psi(0) \) and an arbitrary time interval \( t \), we have

\[
\psi(t) = (-\sigma_z) e^{-iHt} (-\sigma_z) e^{-iHt} \psi(0) = e^{iHt} e^{-iHt} \psi(0) = \psi(0). \tag{38}
\]

It represents a process similar to the spin echo, refocusing the information spreading to the field. The atom retrieves its initial state under the operation of spin flip at half time. We note that the relation in Eq. \( 37 \) requires the resonance condition in an atom-field system. Then resonance is crucial for the reversal process. This feature can be applied to the atom-field system without rotating-wave-approximation.

We would like to point that the separation of atom and field is approximate. In fact, the atom and field always interact with each other. The effective separation is the result of interference. We note that the approximation condition require that the phase between \( |n\rangle \) and \( |n + 1\rangle \) is arbitrary but identical, i.e., \( \varphi \) is \( n \)-independent. However, the evolved state will acquire an extra phase being proportional to \( \sqrt{n} t \) rather than \( nt \). Then, as time increases, the deviation of the evolved wave function from the approximation condition get large. The effective Hamiltonian is available in a short time.

In order to quantitatively evaluate the extent of approximation of the effective Hamiltonian and demonstrate the write and read scheme, the numerical method
is employed to simulate the dynamic processes of quantum state transfer.

For the writing process, we take the initial state as

$$|\phi(0)\rangle = \frac{1}{\sqrt{2}} (|g\rangle + |e\rangle) |\alpha, 0\rangle. \quad (39)$$

The fidelity of the state transfer is

$$F_W(t) = |\langle \phi(0) | e^{iHt} | \phi_W \rangle|, \quad (40)$$

where $|\phi_W\rangle = \frac{1}{\sqrt{2}} (|g\rangle - i |e\rangle) |\alpha, \alpha\pi\rangle$ is the target state. Similarly, the fidelity for the reading process is defined as

$$F_R(t) = |\langle \psi(0) | e^{iHt} | \psi_R \rangle|, \quad (41)$$

where the initial state $|\psi(0)\rangle = -\sigma_z |\phi_W\rangle$ and the target state is

$$|\psi_R\rangle = -\sigma_z |\phi(0)\rangle = \frac{1}{\sqrt{2}} (|g\rangle - |e\rangle) |\alpha, 0\rangle. \quad (42)$$

Straightforward derivation shows that $F_W(t) = F_R(t) \equiv F(t)$. In Fig. 3, $F(t)$ is plotted for the cases of $\alpha = 3, 5, 7, \text{ and } 10$, which shows that the fidelity increases with $\alpha$ and the QST approaches to perfect when the average photon number is more than two dozen.

V. ENTANGLEMENT TRANSFER

A straightforward application of the above analysis is generation of a non-local Schrödinger cat state, which is a fundamental resource in fault-tolerant quantum computing and quantum communication. In this section, we study the dynamics of entanglement transfer in a system composed of two initially entangled atoms, each located in one of two non-interacting cavities. A schematic illustration of the system is given in Fig. 2. The Hamiltonian of the set-up is

$$H_{AB} = \lambda (a^\dagger \sigma_A^+ + b^\dagger \sigma_B^+ + \text{H.c.}), \quad (43)$$

where $\sigma_A^+$ ($\sigma_B^+$) and $a^\dagger$ ($b^\dagger$) are the corresponding operators of the atom and field in the cavity $A$ ($B$), respectively, and $\lambda$ is the coupling constant between the atoms and their cavities. Here we only consider the case of resonance.

The separation of systems $A$ and $B$ makes the dynamics of whole system easier to be understood. It is predictable that the entanglement of two atoms can be perfectly transferred to the fields $a^\dagger$ and $b^\dagger$, which leads to generate a non-local Schrödinger cat state in large $\alpha$ limit.

For finite $\alpha$, to demonstrate the process of entanglement transfer, we compute the purity $p_{ab}$ of the field states in two cavities, as well as that $p_a$ (or $p_b$) for a single cavity. The former is the measure of the entanglement between the atoms and the fields, while the later is that between fields in two cavities. Here the purities for an arbitrary state $|\phi\rangle$ are defined as

$$p_{ab} = \text{Tr} (\rho_{ab})^2, \quad p_a = \text{Tr} (\rho_a)^2, \quad (44)$$

with

$$\rho_{ab} = \text{Tr}_{AB} (|\phi\rangle \langle \phi|), \quad \rho_a = \text{Tr}_b (\rho_{ab}), \quad (45)$$

where $\text{Tr}_{AB} (\cdot)$ and $\text{Tr}_b (\cdot)$ denote the operation of tracing out all atomic states and field $b$ states, respectively.

Now we examine the efficiency of the entanglement transfer from the atoms to the field for finite $\alpha$. The initial state is

$$|\phi(0)\rangle = \frac{|e\rangle_A |g\rangle_B - |g\rangle_A |e\rangle_B |\alpha, 0\rangle_a |\alpha, 0\rangle_b}{\sqrt{2}}, \quad (46)$$

which denotes a maximally entangled $AB$ state, but an unentangled $ab$ state. According to our analysis above
Furthermore, fields are unentangled. Additionally, fields are separable from the state of atoms. Purity is maximal if each cavity is regarded as a qubit. Quantities (in unit of $\alpha = 0.75$) and parameter $\lambda$ indicate the maximal entanglement of the state evolving from the initial state given by Eq. (46) as a function of time $t/\tau$.

We can see that the atoms $AB$ and $ab$ are maximally entangled if each cavity is regarded as a qubit. Quantities $p_{ab}(t)$ and $p_a(t)$ approach to 1 and 0.5, respectively, at time $t/\tau$ as the coupling strength $\lambda$ increases. This indicates that the entanglement can be perfectly transferred from atoms to fields. It also provides an alternative scheme for preparation of non-local superpositions of quasi-classical light states.

Before ending this paper, we want to stress that there is an important feature of the scenario. Actually, the dynamic processes in above schemes are invariant if the coupling strength $\lambda$ is time dependent. All the derivation above is still true if we replace $\lambda \Delta t$ by $\int_0^{\Delta t} \lambda(t) \, dt$. One can take the interaction in the form $\lambda(t) = \lambda_0 \exp(-\alpha^2 t^2)$ which can be used to turn an initial coherent field into a Schrödinger cat state. It ensures the switching control feasible for schemes in experiment. In addition, the switching processes of $\lambda$ in two cavities cannot be simultaneous. This allows the generation of entanglement between a cavity and a distant atom.

VI. SUMMARY

In summary, we have presented a scheme for state transfer between atom and cavity field in Jaynes-Cummings model. It is shown that the nonlinearity arising from the atom-field coupling can induce the generation of modified coherent states, which can be shown to be macroscopically distinguishable as standard coherent states. We have shown that an arbitrary atom state can be mapped onto the field as Schrödinger cat state via natural time evolution. This result can be extended to non-interacting multi-cavity system. Analytical and numerical calculations have demonstrated that the dynamic process on two-cavity system can provide an alternative scheme for preparation of non-local superpositions of quasi-classical light states.

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