Higgs alignment of visible and dark gauge groups

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Abstract. We discuss a dark family of lepton-like particles with their own “private” gauge
bosons $X_\mu$ and $C_\mu$ under a local $SU'(2) \times U'(1)$ symmetry. The product of dark and visible
gauge groups $SU'(2) \times U'(1) \times SU_w(2) \times U_Y(1)$ is broken dynamically to the diagonal (vector-like)
subgroup $SU(2) \times U(1)$ through the coupling of two fields $M_i$ to the Higgs field and the dark
lepton-like particles. This defines a new Higgs portal, where the “dark leptons” can contribute
to the dark matter and interact with Standard Model matter through Higgs exchange.

1. Introduction
The identification of the dark matter which dominates the large scale structure in the universe
remains an open problem. Higgs exchange had been proposed as an option for non-gravitational
interactions between dark matter and ordinary matter, and Higgs portal models with couplings
to the scalar product $H^0 + H$ of the Higgs doublet $H = (H^+, H^0)$ have been discussed extensively
for bosonic and fermionic dark matter, see e.g. [1, 2, 3, 4, 5]. In the present paper we suggest
studies of another kind of Higgs portal which does not involve the scalar product $H^0 + H$ of the
Higgs doublet, but may contribute to the masses of dark fermions, although the Higgs field and
the dark fermions transform under a priori separate $SU(2)$ transformations at high energies.

It is an intriguing question whether $SU(2)$ breaking by the Standard Model (SM) Higgs
boson could also generate the dark matter masses without violating the Standard Model gauge
symmetries, and yet be safe from constraints arising through couplings to the electroweak gauge
bosons. We propose a mechanism to achieve this. The key idea is to have an a priori separate
$SU'(2) \times U'(1)$ gauge symmetry in the dark sector with its own gauge bosons $X_\mu$ and $C_\mu$.
Dynamical breaking of the dark symmetry, e.g. through scalar fields $M_i \equiv \{M_i, ab\}$, which are
charged both under the dark and visible gauge groups, then induces standard chiral Yukawa
couplings of dark left-handed $SU'(2)$ lepton-like doublets and right-handed $SU'(2)$ singlets with
the Standard Model Higgs doublet,

$$L_{H-DM} = \frac{\sqrt{2}}{v_h} \left( m_2 \overline{\Psi}_L \cdot H \cdot \psi_{2,R} + m_1 \overline{\Psi}_L \cdot \xi \cdot H^* \cdot \psi_{1,R} \right) + \text{h.c.} \quad \quad (1)$$

These Yukawa couplings in turn break the gauge symmetry of the coupled SM+dark matter
model to $SU_c(3) \times SU_w(2) \times U_Y(1)$, because the Higgs couplings align the local $SU(2) \times U(1)$
gauge transformations in the visible and dark sectors.
Breaking of dark gauge symmetries occurs in particular if the Higgs boson couples both to the visible and dark electroweak type gauge bosons,

\[ D_\mu H(x) = \partial_\mu H(x) - ig_\mu W_\mu(x) \cdot \frac{\sigma}{2} H(x) - ig_2 X_\mu(x) \cdot \frac{\sigma}{2} H(x) - i \frac{g_Y}{2} B_\mu(x) H(x) \]

Combined with Eq. (1), this defines a class of renormalizable Higgs portal models for dark matter with SU\(_w(2) \times U_Y(1) \times SU'\(_2(2) \times U'\(_1(1)\) gauge symmetry and spontaneously broken SU\(_2(2)\) factors. This model would not be ruled out (yet) through bounds on the invisible Higgs decay width if all massive particles in the dark sector are heavier than \(m_b/2\), and if all dark particles with masses below \(m_b/2\) have small masses, \(m \ll m_b/2\). However, in the present investigation we will focus on the case that the Higgs boson does not directly couple to the SU\(_2(2)\) gauge bosons, but is only charged under the Standard Model gauge group,

\[ D_\mu H(x) = \partial_\mu H(x) - ig_\mu W_\mu(x) \cdot \frac{\sigma}{2} H(x) - i \frac{g_Y}{2} B_\mu(x) H(x). \]

2. A dynamical mechanism for alignment of visible and dark gauge symmetries

A dynamical mechanism to generate the coupling (1) with a Higgs field which is not charged under the dark gauge groups, cf. (3), can be constructed with scalar fields \(M_a \equiv \{M_{a,b}\}\) which are charged both under the dark gauge group and the Standard Model gauge group. The first index \(a\) refers to the fundamental representation of SU\(_2(2)\) from the left, while the second index \(b\) refers to SU\(_w(2)\) acting through the adjoint SU\(_w(2)\)-matrices from the right. The U\(_1\) charges are given in terms of the dark fermion charges by \(Y_{i}^{(M)} = Y_{i}^{L} - Y_{i}^{R}\), and the U\(_Y(1)\) charges are \(Y_{i}^{(M)} = -Y_{i} = -1\). The covariant derivatives of the \(M_{i}\)-fields are therefore

\[ D_\mu M_{a} = \partial_\mu M_{a} - ig_2 X_\mu \cdot \frac{\sigma}{2} M_{a} - i \frac{g_Y}{2} (Y_{L} - Y_{R}^{'} \cdot C_\mu M_{a}) + igW_{\mu} \cdot \frac{\sigma}{2} M_{a} + i \frac{g_Y}{2} B_\mu M_{a}. \]

The following coupling term between the visible and the dark sector is then invariant under the full gauge group SU\(_2(2)\) × U\(_Y(1)\) × SU\(_w(2)\) × U\(_Y(1)\),

\[ \mathcal{L}_{MH-DM} = -\frac{\sqrt{\gamma}}{v_h} \left( \nabla_L \cdot M_{a} \cdot H \cdot \psi_{2,R} + \nabla_L \cdot M_{1} \cdot \xi \cdot H^{*} \cdot \psi_{1,R} \right) + h.c. \]

This yields the new Higgs portal coupling (1) in the low energy sector through the potential

\[ V(M_{1},M_{2}) = \frac{1}{4} \sum_{i=1}^{2} \lambda_i \left[ \text{Tr} \left( M_{i} \cdot M_{i}^{*} \right) - 2 \text{Det} M_{i} \right]^2. \]

A 2 \times 2 matrix \(M_{i}\) satisfies the ground state condition \(\text{Tr} \left( M_{i} \cdot M_{i}^{*} \right) = 2 \text{Det} M_{i}\) if and only if the matrix is proportional to the unit matrix, \(M_{i} = M_{i,1}\), up to an additional possible unitary factor \(\frac{\sqrt{\gamma}}{v_h} \). The parameters \(m_i\) can be chosen to satisfy \(m_i \geq 0\). These results can easily be proved using the polar decomposition \(M_{i} = H_{i} \cdot V_{i}\) of the matrix \(M_{i}\) into a positive semidefinite hermitian factor \(H_{i}\) and a unitary factor \(V_{i}\).
3. Dark matter masses from the Standard Model Higgs boson

A priori the dark gauge group \( SU'(2) \times U'(1) \) acting on the fields \( \Psi_L \) and \( \psi_{i,R} \) and the electroweak gauge group are different symmetries, with \( SU'(2) \times U'(1) \) acting only in the dark sector while the electroweak symmetry only acts in the visible sector. However, the Yukawa couplings (1) of the Higgs doublet align the transformations in both symmetry groups at low energies, thus breaking the direct product of symmetry groups to its diagonal component,

\[
SU'(2) \times U'(1) \times SU_w(2) \times U_Y(1) \rightarrow SU'(2) \times U'(1) = SU_w(2) \times U_Y(1).
\]

The corresponding dark hypercharge \((U'(1))\) assignments have to satisfy

\[
Y_L' - Y_{1,R}' = -Y_h = -1, \quad Y_L' - Y_{2,R}' = Y_h = 1,
\]

where \( Y_h = 1 \) is the weak hypercharge of the Higgs doublet.

The gauge symmetry \( SU_w(2) \times U_Y(1) \) is therefore implemented for low energy in the visible and dark sectors through the \( SU(2) \) transformations \( U(x) = \exp[i\varphi(x)\cdot \sigma/2] \) and the \( U(1) \) transformations \( \exp[i\alpha(x)\kappa/[2], \exp[i\alpha(x)\kappa]/2] \). The action of the SM fields is as usual,

\[
H'(x) = \exp[i\alpha(x)/2]U(x) \cdot H(x), \quad \left( \begin{array}{c} \nu_L'(x) \\ e_L'(x) \end{array} \right) = \exp[-i\alpha(x)/2]U(x) \cdot \left( \begin{array}{c} \nu_L(x) \\ e_L(x) \end{array} \right),
\]

\[
\nu_R'(x) = \nu_R(x), \quad e_R'(x) = \exp[-i\alpha(x)]e_R(x), \ldots
\]

\[
W'_\mu(x) \cdot \sigma = U(x) \cdot (W_\mu(x) \cdot \sigma) \cdot U^+(x) + \frac{2i}{g_w} U(x) \cdot \partial_\mu U^+(x), \quad B'_\mu(x) = B_\mu(x) + \frac{1}{g_Y} \partial_\mu \alpha(x),
\]

and the corresponding action in the dark sector is

\[
\Psi_L'(x) = \exp[iY_L'\alpha/2]U(x) \cdot \left( \begin{array}{c} \psi_{1,L}(x) \\ \psi_{2,L}(x) \end{array} \right),
\]

\[
\psi_{1,R}'(x) = \exp[i(Y_L' + 1)\alpha/2]\psi_{1,R}(x), \quad \psi_{2,R}'(x) = \exp[i(Y_L' - 1)\alpha/2]\psi_{2,R}(x),
\]

\[
X'_\mu(x) \cdot \sigma = U(x) \cdot (X_\mu(x) \cdot \sigma) \cdot U^+(x) + \frac{2i}{q_2} U(x) \cdot \partial_\mu U^+(x),
\]

\[
C'_\mu(x) = C_\mu(x) + \frac{1}{q_1} \partial_\mu \alpha(x).
\]

Here \( q_2 \) and \( q_1 \) are the gauge couplings of the dark \( SU'(2) \times U'(1) \) symmetry group, and the covariant derivatives on the dark matter fields are

\[
D_\mu \Psi_L(x) = \partial_\mu \Psi_L(x) - iq_2 X_\mu(x) \cdot \sigma/2 \Psi_L(x) - iq_1/2 Y_L' C_\mu(x) \Psi_L(x),
\]

\[
D_\mu \psi_{i,R}(x) = \left( \partial_\mu - iq_1/2 Y_{i,R}' C_\mu(x) \right) \psi_{i,R}(x), \quad Y_{i,R}' = Y'_R - (-)^i.
\]
4. Dark atoms

The same technique which is used in QED in Coulomb gauge [6] also applies here to identify any attractive particle-particle combinations in the dark sector. In the non-relativistic limit, the equations (with \( \psi_i^+ \psi_i \equiv \psi_i^{+L} \psi_i^{+L} + \psi_i^{+R} \psi_i^{+R} \))

\[ \partial_\mu C^{\mu 0} = -q_C = -\frac{q_1}{2} \left[ Y_L^2 \left( \psi_1^+ \psi_1 + \psi_2^+ \psi_2 \right) + \psi_{1,R}^+ \psi_{1,R} - \psi_{2,R}^+ \psi_{2,R} \right], \]

\[ D_\mu X^{\mu 0} = -\theta_a = -\frac{q_2}{2} \frac{\Psi^+_L}{\Psi_L} \cdot \sigma_a \cdot \Psi_L, \]

yield the dark sector Coulomb operator \( H_{AC} = \int d^3x \int d^3x' \mathcal{O}_A^2(x, x')/8\pi|x - x'| \) with the charged density-density correlation operator

\[ \mathcal{O}_A^2(x, x') = \mathcal{O}_C^2(x, x') + \sum_{a=1}^{3} \mathcal{O}_a^2(x, x') \]

\[ = \frac{q_2}{4} \left[ \psi_{1,L}^+(x) \psi_{1,L}(x') \psi_{1,L}(x') \psi_{1,L}(x) + \psi_{2,L}^+(x) \psi_{2,L}(x') \psi_{2,L}(x') \psi_{2,L}(x) \right. \]

\[ - 2 \psi_{1,L}^+(x) \psi_{2,L}(x') \psi_{1,L}(x') \psi_{2,L}(x) + 4 \psi_{1,L}^+(x) \psi_{1,L}(x') \psi_{1,L}(x') \psi_{2,L}(x) \]

\[ + \frac{q_2}{4} Y_L^2 \left[ \psi_{1,L}^+(x) \psi_{1,L}(x') \psi_{1,L}(x') \psi_{1,L}(x) \right. \]

\[ + 2 \psi_{1,L}^+(x) \psi_{2,L}(x') \psi_{2,L}(x') \psi_{2,L}(x) \]

\[ + \frac{q_2}{4} \left[ (Y_L^2 + 1)^2 \psi_{1,R}^+(x) \psi_{1,R}(x') \psi_{1,R}(x') \psi_{1,R}(x) \right. \]

\[ + (Y_L^2 - 1)^2 \psi_{2,R}^+(x) \psi_{2,R}(x') \psi_{2,R}(x') \psi_{2,R}(x) \]

\[ + 2(Y_L^2 - 1) \psi_{1,R}^+(x) \psi_{2,R}(x') \psi_{2,R}(x') \psi_{1,R}(x) \]

\[ + \frac{q_2}{2} Y_L Y_L' + 1 \left[ \psi_{1,L}^+(x) \psi_{1,R}(x') \psi_{1,R}(x') \psi_{1,L}(x) \right. \]

\[ + \psi_{2,L}^+(x) \psi_{1,R}(x') \psi_{1,R}(x') \psi_{2,L}(x) \]

\[ + \psi_{1,L}^+(x) \psi_{1,R}(x') \psi_{2,R}(x') \psi_{1,L}(x) \]

\[ + \frac{q_2}{2} Y_L Y_L' - 1 \left[ \psi_{1,L}^+(x) \psi_{1,R}(x') \psi_{2,R}(x') \psi_{1,L}(x) \right. \]

\[ + \psi_{2,L}^+(x) \psi_{2,R}(x') \psi_{2,R}(x') \psi_{2,L}(x) \]. \hspace{1cm} (12)

The resulting Coulomb term for the dark two-lepton states \( \psi_{1,R}^+(x) \psi_{2,R}(x') |0\rangle \),

\[ H_{1R,2R} = \int d^3x \int d^3x' \frac{q_2(Y_L'^2 - 1)}{16\pi|x - x'|} \psi_{1,R}^+(x) \psi_{2,R}(x') \psi_{2,R}(x') \psi_{1,R}(x) \]

\[ = 1 + \delta_W m_v^2 \sqrt{\frac{m_v^2}{\Phi_0} \sqrt{s - 4m_v^2} \sqrt{s - 4m_v^2} (s - 2m_v^2)^2 + 8m_v^4}} \]

\[ \sigma_{\psi \bar{\psi} \to VV}(s) = \frac{\delta_W}{64\pi} \frac{m_v^2}{\Phi_0} \sqrt{\frac{m_v^2}{\Phi_0} \sqrt{s - 4m_v^2} \sqrt{s - 4m_v^2} (s - 2m_v^2)^2 + 8m_v^4}}, \hspace{1cm} (13) \]
1. $\langle \sigma v \rangle_{\psi \bar{\psi} \rightarrow f \bar{f}}(s) = \frac{N_c}{16\pi v_h^2 s} \frac{(s - 4m_f^2)^{3/2}}{(s - m_f^2)^2 + m_f^2 \Gamma_f^2} \left( \frac{s}{s - 4m_i^2} \right)^{1/2}. \quad (15)$

2. $\langle \sigma v \rangle_{\psi \bar{\psi} \rightarrow hh}(s) = \frac{9m_i^4 m_h^4}{64\pi v_h^2 s} \sqrt{\frac{s - 4m_h^2}{s - 4m_i^2}} \sqrt{\frac{s - 4m_i^2}{s - 4m_h^2}} \left( \frac{s}{s - m_h^2} \right)^{3/2} + m_h^2 \frac{\Gamma_h^2}{v_h^2}. \quad (16)$

The total annihilation cross section increases with mass $m_i$ for masses above 80 GeV, and therefore the heavier dark lepton species will determine both the mass $M$ of the dark atoms and the freeze out temperature. We will assume $m_i \lesssim 0.01 m_2$ and therefore $M \approx m_2$.

The requirement of thermal freeze out then determines $M \approx 96$ GeV, see Fig. 1, where the thermally averaged annihilation cross section for a particle with mass $m_2 \approx M$ is compared to the required value from thermal dark matter creation. This dark matter model is even more predictive than the standard Higgs portal dark matter models because the coupling to the Higgs field is already determined in terms of the mass, $g = m_2/v_h \approx M/v_h$. The requirement of thermal dark matter creation therefore does not yield a parametrization $g(M)$ of the Higgs portal coupling as a function of the dark matter mass, but determines $M$.

5. Conclusions

Alignment of gauge symmetries in the visible and dark sectors through the new Higgs portal is an interesting new tool for dark matter model building. It can arise as a consequence of dynamical symmetry breaking in gauge theories, and it opens a door to fermionic Higgs portal models without the need of higher mass-dimension effective vertices. The construction presented here opens the Higgs portal into much more complicated and rich dark sectors, even with the possibility of $CP$ and time-reversal reciprocity between the visible and dark sectors.

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