THE NUCLEON ELECTRIC DIPOLE MOMENT IN LIGHT-FRONT QCD

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I present an exact relationship between the electric dipole moment and anomalous magnetic moment of the nucleon in the light-front formalism of QCD and consider its consequences.

1. Introduction

Interpreting the electric dipole moments of leptons and baryons as constraints on fundamental, CP-violating Lagrangian parameters of various extensions of the Standard Model (SM) gives key insight into TeV-scale physics. In this contribution I report on work in collaboration with Stan Brodsky and Dae Sung Hwang, in which we sharpen the connection between the computed neutron electric dipole moment and fundamental CP violation by comparing its hadronic matrix element to that of the anomalous magnetic moment. In the context of the SM, and, specifically, of the Cabbibo-Kobayashi-Maskawa (CKM) mechanism of CP violation, the assessed values of the neutron electric dipole moment (EDM) have been disparate, ranging from $d_{\text{CKM}} \simeq 10^{-32}\text{e-cm}$ arising from a $\pi-N$ loop calculation in a chiral Lagrangian treatment, to $d_{\text{KM}} \simeq 10^{-34}\text{e-cm}$ for the EDM of the $d$-quark itself, computed to three-loop precision in leading-logarithmic approximation. These EDMs are much too small to be experimentally observable, so that the marked disparity is actually of little consequence. However, if we restrict ourselves to effective CP-violating operators of dimension five or less, the method of QCD sum rules can be employed to compute the EDM of the neutron, yielding a value for $d^n$, induced by a QCD $\theta$-term, e.g., commensurate in size with that of the chiral estimate, with a surety of $\sim 50\%$. The evaluation of $d^n$ and $d^p$, and the errors therein, is also important to interpreting the $^2H$ EDM. Here we analyze...
the nucleon electric dipole moment in the light-front formalism of QCD \(^1\), to the end of realizing an independent test of the methods used to compute \(d^N\) — and of their assessed errors.

2. Electromagnetic Form Factors in Light-Front QCD

Our study of the electric dipole form factor \(F_3(q^2)\) in the light-front formalism of QCD complements earlier studies of the Dirac and Pauli form factors \(^10\). The Pauli and electric dipole form factors emerge from the spin-flip matrix elements of the electromagnetic current \(J^\mu(0)\):

\[
\langle P', -S_z | J^\mu(0) | P, S_z \rangle = \bar{U}(P', -\lambda) \left[ \frac{i}{2M} \sigma^{\mu\alpha} \right. 
\times \left( F_2(q^2) + iF_3(q^2) \gamma_5 \right) q_\alpha \left. \right] U(P, \lambda),
\]

where \(U(P, \lambda)\) is a Dirac spinor for a nucleon of momentum \(P\) and helicity \(\lambda\), with \(S_z = \lambda/2\). Recall that the anomalous magnetic moment \(\kappa\) and the electric dipole moment \(d\) are given by \(\kappa = (e/2M)[F_2(0)]\) and \(d = (e/M)[F_3(0)]\). We find a close connection between \(\kappa\) and \(d\) \(^1\), as long anticipated \(^11\). Working in the \(q^+ = 0\) frame, with \(q = (q^+, q^-, q^\perp) = (0, -q^2/P^+, q^\perp)\) and \(P = (P^+, P^-, P^\perp) = (P^+, M^2/P^+, 0^\perp)\), in the interaction picture for \(J^+(0)\), and in the assumed simple vacuum of the light-front formalism, we find, noting \(q^R/L \equiv q^1 \pm iq^2\),

\[
\frac{1}{2M} \left( \frac{F_2(q^2)}{-iF_3(q^2)} \right) = \sum_a \int [d\varepsilon][d^2k^\perp] \sum_j \varepsilon_j \frac{1}{2} \left[ -\frac{1}{q^2} \psi^\dagger_a(x_i, k^\perp_{jli}, \lambda_i) \right.
\times \psi^\dagger_a(x_i, k^\perp_{jli}, \lambda_i) \pm \frac{1}{q^2} \psi^\dagger_a(x_i, k^\perp_{jli}, \lambda_i) \psi^\dagger_a(x_i, k^\perp_{jli}, \lambda_i) \right],
\]

where \(k^\perp_{jli} = k^\perp_{li} + (1 - x_j)q^\perp\) for the struck constituent \(j\) and \(k^\perp_{li} = k^\perp_{li} - x_iq^\perp\) for each spectator \((i \neq j)\). The electric dipole form factor \(F_3(q^2)\) vanishes if the usual light-front wave functions are employed, so that we must learn how parity- and time-reversal-violating effects can be included in the light-cone framework.

3. Discrete Symmetries on the Light Front and a Relation for the Electric Dipole Moment

We construct parity \(\mathcal{P}_\perp\) and time-reversal \(\mathcal{T}_\perp\) in the light-front formalism \(^1\) by noting that these operations should act on the \(k^\perp\) of a free particle alone,
so that $|k_\perp|$, $k^+$, and $k^-$ remain unchanged. We choose $\mathcal{P}_\perp$ so that the components of a vector $d^\mu$ transform as $d^1 \rightarrow -d^1$, $d^2 \rightarrow d^2$, or $d^{R,L} \rightarrow -d^{L,R}$, and $d^\pm \rightarrow d^\pm$. With this $\mathcal{P}_\perp$ is an unitary operator, though it flips the spin as well. We find that $F_2(q^2)$ is even and $F_3(q^2)$ is odd under $\mathcal{P}_\perp$. The choice of $\mathcal{T}_\perp$ is predicated by that for $\mathcal{P}_\perp$: a momentum vector $q^\mu$ transforms as $q^{R,L} \rightarrow -q^{L,R}$ and $q^\pm \rightarrow q^\pm$ under $\mathcal{T}_\perp$, so that the position vector $x^\mu \equiv (x^+, x^-, x^L, x^R) \rightarrow (-x^+, -x^-, x^R, -x^L)$. With this we find that $\mathcal{T}_\perp$ is antiunitary, but it does not flip the spin. With the charge-conjugation operator $C$ defined in the usual way, we note that all scalar fermion bilinears are invariant under $\mathcal{CP}_\perp \mathcal{T}_\perp$ as needed. We find that Re($F_2$) and Im($F_3$) are even and Re($F_3$) and Im($F_2$) are odd under $\mathcal{T}_\perp$. Thus to realize a non-zero electric dipole form factor, we must include a $\mathcal{T}_\perp$- and $\mathcal{P}_\perp$-odd parameter $\beta_a$ in the light-front wave function $\psi_a^{\perp i}(x_i, k_{\perp i}, \lambda_i)$; namely, $\psi_a^{\perp i}(x_i, k_{\perp i}, \lambda_i) = \phi_a^{\perp i}(x_i, k_{\perp i}, \lambda_i) \exp(i\lambda\beta_a)$, where $\phi_a^{\perp i}(x_i, k_{\perp i}, \lambda_i)$ is both $\mathcal{P}_\perp$ and $\mathcal{T}_\perp$ invariant. We assume $\mathcal{CP}_\perp$ is broken at scales much larger than those of interest, so that $M_{CP}^2 \gg g^2$ and that any $g^2$-dependence in $\beta_a$ can be neglected. With this we find, for a Fock component $a$,

$$[F_3(q^2)]_a = (\tan \beta_a) [F_2(q^2)]_a \quad \text{and} \quad d_a = 2\kappa_a \beta_a \quad \text{as} \quad q^2 \rightarrow 0, \quad (3)$$

since $\beta_a$ is small. Thus the EDM and the anomalous magnetic moment of the nucleon should both be computed with a given method, to test for consistency. If the method employed is unable to confront the empirical anomalous magnetic moments successfully, it cannot be trusted to predict the electric dipole moments reliably. We note in the case of the QCD sum rule method, that the computed anomalous magnetic moments, as long known, are in good agreement with experiment, namely, $\kappa_{th}^n = -2$ and $\kappa_{th}^p = +2^{12}$.

### 4. Implications

We now consider some specific consequences of Eq. (3). In what follows we consider the EDM induced through a QCD $\bar{\theta}$-term only. In a quark–scalar-diquark, $g(qq)_0$, model of the nucleon a single $\mathcal{P}_\perp$ and $\mathcal{T}_\perp$-violating parameter $\beta^N$ suffices to characterize $d^N$. Since $\delta L_{CP}$ is isoscalar, $\beta^N = \beta^p$, and we can employ the empirical anomalous magnetic moments $\kappa^N = -1.91$ and $\kappa^p = 1.79$, in units of $\mu_N$, to estimate $(d^n + d^p)/(d^p - d^n) = (\kappa^n + \kappa^p)/(\kappa^p - \kappa^n) \approx -0.12/3.70 \approx -0.03$. The isoscalar electric dipole moment of the nucleon is extremely small. This is in accord with the chiral Lagrangian estimate, for which it is zero — the relevant diagrams are mediated by a $\pi - N$ loop, which is logarithmically enhanced as $M_\pi \rightarrow 0^8$. 

Our estimate can be compared to the QCD sum rule calculation, for which \( (d^n + d^p)/(d^p - d^n) \approx -0.3 \), which is much larger. We note that the QCD sum rule method also predicts a zero isoscalar magnetic moment \(^{12}\); the method is less successful in reproducing a quantity which suffers partial cancellation. The \( ^2H \) magnetic moment is determined not only by the sum of \( d^n + d^p \) but also by a CP-violating meson-exchange current — the former is estimated to be numerically larger \(^9\). The efficacy of a EDM measurement in a particular system in bounding \( \bar{\theta} \) is determined by the size of the coefficient multiplying \( \bar{\theta} \). The larger the coefficient, the better the bound on \( \bar{\theta} \), for a given experimental limit. Were \( d^n + d^p \) smaller, the bound on \( \bar{\theta} \) from a putative \( ^2H \) EDM measurement would weaken.

5. Summary

In summary, we have analyzed the electromagnetic form factors in the light-front formalism of QCD, extending the earlier Drell-Yan-West-Brodsky framework to the analysis of \( \mathcal{P}_\perp \) and \( \mathcal{T}_\perp \)-odd observables. We have used the light-front formalism to find a general equality between the anomalous magnetic and electric dipole moments. The relation holds for spin-1/2 systems, in general: it is not specific to the neutron and is independent of the mechanism of CP violation. An earlier study noting the importance of the simultaneous study of the muon’s electric dipole and anomalous magnetic moments is given in Ref. \(^{13}\). The relation we derive implies that both the EDM and anomalous magnetic moment of the spin-1/2 system of interest should be calculated in a given model, to test for consistency. Ultimately, this can lead to sharpened constraints on models containing non-CKM sources of CP violation.

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