Partially Composite Dark Matter

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Abstract

In a class of theories where the Higgs field emerges as a pseudo Nambu-Goldstone boson, it is often assumed that interactions to generate the top Yukawa coupling provide the Higgs potential as well. Such a scenario generically requires a little cancellation in the leading contribution to the Higgs potential, and the electroweak scale is generated by the balance between the leading and the subleading contributions. We, instead, consider the possibility that the contribution from the dark matter particle balances against that from the top quark. The thermal relic of the new particle explains the abundance of dark matter in a consistent region of the parameter space, and the direct detection is found to be promising.
1 Introduction

The discovery of the Higgs boson \cite{1,2} completes the list of the particles in the Standard Model, but, nevertheless, the origin of the Higgs field and its potential remains as mysteries. The Higgs boson mass, \( m_h = 126 \text{ GeV} \) \cite{3,4}, which is smaller than its vacuum expectation value (VEV), \( v = 246 \text{ GeV} \), is far from the naive expectation from the analogy of QCD, \( m_h \sim 4\pi v \), which naturally leads us to consider the possibility that the Higgs field as a pseudo Nambu-Goldstone boson \cite{5}.

The hypothesis of the partially composite fermions \cite{6} provides a consistent picture for the scenario of the light Higgs boson as a pseudo Nambu-Goldstone boson. The dynamically broken global symmetry to ensure the massless NG boson is explicitly but weakly broken by couplings between the fermions in the Standard Model and operators in the dynamical sector in addition to the gauge interactions in the Standard Model. The coupling induces mixings between the elementary fermions and composite hadronic states in the dynamical sector, and the Yukawa interactions as well as the Higgs potential are generated through the mixing. The top quark should give the most important contribution to the Higgs potential since its Yukawa coupling is the largest. It has been studied that various types of models can indeed reproduce the 126 GeV Higgs boson while explaining the top quark and the \( W \) boson masses. (For a recent review, see Ref. \cite{7}.)

The TeV scale new physics motivated by the origin of the Higgs boson also provides a natural explanation of dark matter of the Universe as thermal relic of a stable TeV or weak scale particle. If there is such a particle in the scenario of the pseudo Nambu-Goldstone Higgs, one should also consider the contribution to the Higgs potential from the dark matter particle. Interestingly, such a contribution is somewhat needed to generate a realistic potential. The constraints from the electroweak precision measurements prefer to have a little hierarchy between the scale of dynamical symmetry breaking, \( f \), and the electroweak VEV, \( v \), so that the Standard Model is realized as a good low energy effective theory. If a single source of the potential is dominated such as from the top quark, the naive expectation of the Higgs VEV is zero or of the order of \( \pi f \) due to the periodicity of the Higgs potential. Both are clearly not acceptable. A little hierarchy can be accommodated by assuming a cancellation in the leading contribution so that the subleading one becomes important. The presence of the dark matter particle provides another possibility. The Higgs potential is destabilized at the origin by the contribution from the top quark, and stabilized at a small value \( v \) by that from the dark matter particle. Dark matter candidates in the models of dynamical electroweak
symmetry breaking have been considered in the literatures; e.g., “technibaryon” \cite{8,9} and “topological dark matter” \cite{10,11}. Also, a Majorana fermion in the strong dynamics as the dark matter particle has been discussed in Ref. \cite{13}.

In this paper, we study the contributions to the Higgs potential from the weakly interacting massive particle (WIMP) dark matter. As a concrete example, we consider the $SO(5)/SO(4)$ model for the Nambu-Goldstone Higgs field \cite{14}, and introduce a gauge singlet Majorana fermion as the dark matter particle which couples to the strong sector in a way that $SO(5)$ symmetry is explicitly broken. The dark matter generates the Higgs potential of the $\sin^2 h/f$ type at the leading order of the coupling. This contribution can balance against the $\cos h/f$ type potential generated from the top quark. We find that in the parameter region where the correct size of the Higgs potential is generated, the dark matter abundance is explained simultaneously through the induced coupling between dark matter and the Higgs field. The predicted spin-independent cross sections for the direct detection experiments are found to be consistent with the current experimental bounds, but are large enough to be covered by the future experiments.

This paper is organized as follows. In the next section, we review the minimal composite Higgs model \cite{14} which we use for the basis of our study. In Section 3, we calculate the dark-matter contribution to the Higgs potential, and discuss the consistent parameter region in Section 4. The abundance of dark matter and the possibility of the direct detection are studied in Section 5. Section 6 is devoted to summary and discussion.

2 The $SO(5)/SO(4)$ model

We consider the composite Higgs model associated with the $SO(5) \to SO(4)$ symmetry breaking \cite{14}. The unbroken global symmetry $SO(4)$ together with $U(1)_{B-L}$ global symmetry contains $SU(2)_L \times U(1)_Y$ gauge group as a subgroup. The unbroken $SO(4)$ symmetry ensures the custodial symmetry in the strong sector, and thus there is no severe constraints from the $T$-parameter.

The Nambu-Goldstone field, $\pi(x)$, is introduced as

$$\xi(x) = e^{i\pi^a(x)X^a},$$

where $X^a$, $a = 1, \cdots, 4$, are generators of $SO(5)/SO(4)$ in the vector representation, 5, of

\*See also Ref. \cite{12}
$SO(5)$ \cite{15}. The $\xi$ field transforms under $SO(5)$ symmetry as
\[
\xi \rightarrow \hat{g} \xi \hat{h}^{-1}(\pi, \hat{g}),
\] (2)
where $\hat{g} \in SO(5)$ and $\hat{h} \in SO(4)$. We take the basis where unbroken $SO(4)$ generators are embedded as
\[
S^a = \left( \begin{array}{cc} T^a & 0 \\ 0 & 0 \end{array} \right), \quad a = 1, \cdots, 6.
\] (3)
Therefore the group element $h$ takes the form of
\[
\hat{h} = \left( \begin{array}{cc} * & 0 \\ 0 & 1 \end{array} \right).
\] (4)

The Higgs field $\Sigma(x)$ is defined as
\[
\Sigma(x) = \xi(x) \left( \begin{array}{cccc} 0 \\ 0 \\ 0 \\ 1 \end{array} \right) = \frac{\sin(h/f)}{h} \left( \begin{array}{c} h_1 \\ h_2 \\ h_3 \\ h_4 \end{array} \right).
\] (5)
This field transforms homogeneously as $\Sigma \rightarrow \hat{g} \Sigma$ and the upper four components have the quantum numbers of the Higgs field in the Standard Model, and $h^2 = h_1^2 + h_2^2 + h_3^2 + h_4^2$. The electroweak symmetry breaking is described as $\langle h \rangle = \langle h_3 \rangle \neq 0$, where $f \sin(h/f) = v = 246$ GeV.

In the original minimal composite Higgs model where the top and bottom quarks couple to the operators in the spinorial representation, 4, of $SO(5)$, the Higgs potential with the following form is generated
\[
V(h) \simeq \alpha_t \frac{h}{f} \cos \frac{h}{f} - \beta_t \sin^2 \frac{h}{f},
\] (6)
where we ignore the small contributions from the SM gauge interactions. The first and the second terms are the leading and sub-leading contributions in terms of the expansion with the coupling constants of the interaction terms between the top quark and the dynamical sector. These couplings break the $SO(5)$ symmetry explicitly since the top and bottom quarks do not fill the complete multiplet of $SO(5)$. The Higgs potential is generated though the explicit breaking.

By denoting $\lambda_q$ and $\lambda_u$, respectively, as the dimensionless couplings of $q = (t, b)$ and $t^c$ to the dynamical sector, the naive estimates of $\alpha_t$ and $\beta_t$ are
\[
\alpha_t = \frac{c_q \lambda_q^2 + c_u \lambda_u^2}{(4\pi)^2} N_c m_t f_t, \quad \beta_t = \frac{c_q \lambda_q^2 \lambda_u^2}{(4\pi)^2} N_c f_t^4,
\] (7)
where \( c_q, c_u \) and \( c_\beta \) are unknown \( O(1) \) parameters, and \( f_\nu \) and \( m_\nu \) are coupling and masses of the lowest resonance to which the operator couples. The top Yukawa coupling is written as

\[
y_t = \frac{c_t \lambda_q \lambda_u f_\nu^2}{m_\nu f},
\]

with an \( O(1) \) parameter, \( c_t \).

The coefficients \( \alpha_t \) and \( \beta_t \) are \( O(\lambda^2_{q,u}) \) and \( O(\lambda^4_{q,u}) \). On the other hand, from the minimization of the potential, we find

\[
v = 246 \text{ GeV} = \sqrt{1 - \frac{\alpha_t^2}{4 \beta_t^2}} \times f.
\]

We need \( \alpha_t < 2 \beta_t \) for the vacuum to be stable, which either means that the perturbative expansions in terms of \( \lambda \)'s are violated or there is some accidental cancellation in \( \alpha_t \). Phenomenologically, one needs \( v/f \lesssim 0.25 \) to satisfy experimental constraints, especially from the \( Zb\bar{b} \) coupling \([16]\), which means \( \alpha_t \simeq 2 \beta_t \).

In the following we consider the possibility that \( \alpha_t \gg \beta_t \) as expected from the perturbative expansion, but a large \( \sin^2 \left( \frac{h}{f} \right) \) term necessary for the electroweak symmetry breaking is supplied by the contribution from the dark matter particle.

The models in which SM fermions couple to operators in the fundamental, 5, or the antisymmetric, 10 representation have also been proposed \([17, 18]\) and reported that such models relax the constraints from the \( Zb\bar{b} \) coupling, \( v/f < 0.3 - 0.4 \). For other possibilities see, e.g., Ref. \([19]\). In this paper, we consider the original model in which top and bottom quarks couple to the operators in the spinorial representation, 4, since that is the simplest option to incorporate the dark matter particle. By embedding the Standard Model singlet fermion in the 5 representation as we explain later, one can generate the potential which can balance against the contributions from the top quark. See also Refs. \([14, 19, 20, 21, 22, 23, 24, 25]\) for other possible ways for natural electroweak symmetry breaking.

### 3 Higgs potential from Dark Matter

We introduce the dark matter field, \( \psi_S \), which is a Majorana fermion and singlet under the SM gauge group. We assume that the dark matter field couples to the dynamical sector as

\[
\mathcal{L} \ni -\frac{m}{2} \bar{\psi}_S \psi_S + \lambda \bar{\psi}_S \mathcal{O}_5 + i \lambda' \bar{\psi}_S \gamma_5 \mathcal{O}_5,
\]
where $O_5$ is a Majorana fermionic operator in the dynamical sector, and is a component of $SO(5)$ vector representation,

\[
O = \begin{pmatrix}
O_1 \\
O_2 \\
O_3 \\
O_4 \\
O_5
\end{pmatrix}.
\]  

The real valued couplings $\lambda$ and $\lambda'$ break the $SO(5)$ symmetry explicitly. As we will see later, the interaction between the dark matter and the dynamical sector gives a mass to the dark matter which we assume to be the dominant contribution. In that case, one can ignore the mass term $m$ in Eq. (10), which in turn makes it possible to eliminate the $\lambda'$ term by a field redefinition of $\psi_S$.

The 2-point function of $\psi_S$ is written as,

\[
\langle \psi_S(x) \bar{\psi}_S(0) \rangle = -\int \frac{d^4k}{i(2\pi)^4} \frac{e^{-ikx}}{k^2 + \lambda^2 \Pi_{55}(k)},
\]  

where

\[
\Pi_{ij}(q) = i \int d^4x \langle O_i(x) \bar{O}_j(0) \rangle e^{iqx} = \Pi_4(q)(\delta_{ij} - \Sigma_i \Sigma_j) + \Pi_1(q)\Sigma_i \Sigma_j.
\]  

In the last expression of Eq. (13), we decompose $\Pi$’s in terms of the unbroken $SO(4)$ symmetry. The field $\Sigma$ is treated as an external field. The $\Pi_4$ and $\Pi_1$ functions can be expressed in terms of the spectral functions such as

\[
\Pi_4(q) = -\int_0^\infty ds \frac{\tilde{\rho}_4(s) + \tilde{\rho}_4(s) + i\gamma_5\tilde{\rho}_{4,5}(s)}{q^2 - s + i\epsilon} + \cdots,
\]  

\[
\Pi_1(q) = -\int_0^\infty ds \frac{\tilde{\rho}_1(s) + \tilde{\rho}_1(s) + i\gamma_5\tilde{\rho}_{1,5}(s)}{q^2 - s + i\epsilon} + \cdots,
\]

where the ellipsis are regular functions of $q^2$, representing the contact term.

In the case where there is an effective description in terms of weakly coupled composite states, such as in a large $N$ theory, spectral functions are approximated as collections of hadron poles:

\[
\rho_4(s) = \sum_i f_{4,i}^2 \delta(s - |m_{4,i}|^2), \quad \rho_1(s) = \sum_i f_{1,i}^2 \delta(s - |m_{1,i}|^2),
\]  

\[
\Pi(q) = -C \Pi^T(\gamma^5-q)C, \quad \Pi^1(q) = \gamma^0 \Pi(q) \gamma^0,
\]  

which forbid the terms proportional to $\gamma_5\gamma_5$. 

† The two point functions of Majorana operators satisfy

$\Pi(q) = -C \Pi^T(\gamma^5-q)C, \quad \Pi^1(q) = \gamma^0 \Pi(q) \gamma^0$,
\[ \tilde{\rho}_4(s) = \sum_i f_{i,4}^2 \text{Re}[m_{4,i}]\delta(s - |m_{4,i}|^2), \quad \tilde{\rho}_1(s) = \sum_i f_{i,1}^2 \text{Re}[m_{1,i}]\delta(s - |m_{1,i}|^2), \quad \tag{17} \]

\[ \tilde{\rho}_{4,5}(s) = \sum_i f_{i,4}^2 \text{Im}[m_{4,i}]\delta(s - |m_{4,i}|^2), \quad \tilde{\rho}_{1,5}(s) = \sum_i f_{i,1}^2 \text{Im}[m_{1,i}]\delta(s - |m_{1,i}|^2). \quad \tag{18} \]

We assume that $SO(5)$ symmetry is broken by a VEV of some composite operator, $X$, with the mass dimension $d$. Then it contributes to $\Pi_4(q) - \Pi_1(q)$ as $\propto \langle X^\dagger X \rangle / q^{2d-1}$ for a large $q$. This condition gives the Weinberg sum rules for the spectral functions:

\[ \int_0^\infty ds (\rho_4(s) - \rho_1(s)) = 0, \quad (d > 1), \quad \tag{19} \]
\[ \int_0^\infty ds (\tilde{\rho}_4(s) - \tilde{\rho}_1(s)) = 0, \quad \int_0^\infty ds (\tilde{\rho}_{4,5}(s) - \tilde{\rho}_{1,5}(s)) = 0, \quad (d > 3/2), \quad \tag{20} \]
\[ \int_0^\infty ds \cdot s (\rho_4(s) - \rho_1(s)) = 0, \quad (d > 2), \quad \tag{21} \]
\[ \int_0^\infty ds \cdot s (\tilde{\rho}_4(s) - \tilde{\rho}_1(s)) = 0, \quad \int_0^\infty ds \cdot s (\tilde{\rho}_{4,5}(s) - \tilde{\rho}_{1,5}(s)) = 0, \quad (d > 5/2). \quad \tag{22} \]

For example, if $X$ is a fermion pair in an asymptotically free theory, $d = 3$, and the above six sum rules apply. One can also obtain a relation that the contact terms in Eqs. (14) and (15) are common for $\Pi_4(q)$ and $\Pi_1(q)$.

The Higgs potential can be calculated by using the Coleman-Weinberg formula:

\[ V(h) = -\frac{1}{2} \int \frac{d^4k}{i(2\pi)^4} \text{Tr} \log \left[ \frac{k + \lambda^2 \Pi_{55}(k) + i\epsilon}{k + i\epsilon} \right] \]
\[ = \text{const.} - \frac{1}{2} \int \frac{d^4k}{i(2\pi)^4} \text{Tr} \left[ \frac{-\lambda^2}{k + i\epsilon} (\Pi_4(k) - \Pi_1(k)) \Sigma_5 \Sigma_5 \right] + O(\lambda^4) \]
\[ \equiv \text{const.} - \beta \sin^2 \frac{h}{f} + O(\lambda^4), \quad \tag{23} \]

where

\[ \beta = -\frac{1}{2} \cdot \lambda^2 \int_0^\infty ds \int \frac{d^4k}{i(2\pi)^4} \frac{4k^2(\rho_4(s) - \rho_1(s))}{(k^2 + s + i\epsilon)(k^2 - s + i\epsilon)}. \quad \tag{24} \]

The Weinberg sum rules make the momentum integral converge. The piece which is non-vanishing under the Weinberg sum rules (19) and (21) is

\[ \beta = \frac{1}{2} \cdot \frac{4\lambda^2}{(4\pi)^2} \int_0^\infty ds \cdot s (\rho_4(s) - \rho_1(s)) \log \frac{s}{s_0}. \quad \tag{25} \]

where $s_0$ is an arbitrary number. The $s_0$ independence is ensured by Eq. (21).
When we set \( s_0 \) as the mass squared of the lowest resonance to couple the operator \( \mathcal{O}_i \), \( m_{\mathcal{O}} \),

\[
\beta = \frac{1}{2} \frac{4\lambda^2}{(4\pi)^2} \int_{m_{\mathcal{O}}^2}^{\infty} ds \cdot s(\rho_4(s) - \rho_1(s)) \log \frac{s}{m_{\mathcal{O}}^2} \\
= -\frac{1}{2} \frac{4\lambda^2}{(4\pi)^2} \int_{m_{\mathcal{O}}^2}^{\infty} ds \cdot \Delta(s) \left( 1 + \log \frac{s}{m_{\mathcal{O}}^2} \right),
\]

(26)

where

\[
\Delta(s) = \int_{m_{\mathcal{O}}^2}^{s} ds' (\rho_4(s') - \rho_1(s')).
\]

(27)

The function \( \Delta(s) \) goes to zero as \( s \to \infty \) because of Eq. (19). Therefore, the integration in Eq. (26) should be dominated by the lower resonances. Therefore, we expect

\[
\beta = \frac{1}{2} \frac{4\lambda^2}{(4\pi)^2} c_{\beta} m_{\mathcal{O}}^2 f_{\mathcal{O}}^2,
\]

(28)

where \( c_{\beta} \) is an \( O(1) \) coefficient, and \( f_{\mathcal{O}} \) is the coupling of the lowest resonance which couples to the operator \( \mathcal{O}_i \). The overall sign depends on that of \( \rho_4(s) - \rho_1(s) \) near \( s \sim m_{\mathcal{O}}^2 \). We assume \( \beta > 0 \) which is necessary for the vacuum to be stable.

### 4 Electroweak symmetry breaking

Adding the contribution in Eq. (23) to Eq. (6), the total Higgs potential is obtained as

\[
V(h) = \alpha_t \cos \frac{h}{f} - (\beta + \beta_t) \sin^2 \frac{h}{f}.
\]

(29)

The minimization of the potential gives the electroweak VEV and the Higgs mass as follows:

\[
v = 246 \text{ GeV} = \sqrt{1 - \frac{\alpha_t^2}{4(\beta + \beta_t)^2}} \times f \equiv \epsilon f,
\]

(30)

\[
m_h^2 = (126 \text{ GeV})^2 = \frac{2(\beta + \beta_t)\epsilon^2}{f^2}.
\]

(31)

Therefore, in the case where the dark matter contribution exists, the stable minimum can be found for \( \alpha_t \gg \beta_t \). A small \( \epsilon \) can be obtained when \( \beta \sim \alpha_t \gg \beta_t \). When \( \beta_t \) is negligible, from Eq. (28), we find

\[
m_h^2 = c_{\beta} \cdot \epsilon^2 \cdot \frac{4\lambda^2}{(4\pi)^2} m_{\mathcal{O}}^2 \left( \frac{f_{\mathcal{O}}}{f} \right)^2.
\]

(32)
From this, we obtain the mass of the first resonance to be
\[ m_O = 4.9 \text{ TeV} \cdot c_{\beta}^{-1/2} \left( \frac{\lambda_{fO}}{1 \text{ TeV}} \right)^{-1} \left( \frac{\epsilon}{0.2} \right)^{-2}. \] (33)

On the other hand, from Eq. (30), \( \alpha_t \) is required to satisfy:
\[ \alpha_t = 2(\beta + \beta_t)\sqrt{1 - \epsilon^2} \simeq 2 \beta = \frac{m_h^2 v^2}{\epsilon^4}. \] (34)

The mass of the lowest top partner resonance is, therefore, given by
\[ m_{t'} = 2.4 \text{ TeV} \left( \frac{c_t \cdot 2 \lambda_q \lambda_u}{c_q \lambda_q^2 + c_u \lambda_u^2} \right)^{1/3} \left( \frac{\epsilon}{0.2} \right)^{-1} \leq 2.4 \text{ TeV} \left( \frac{c_t}{\sqrt{c_q c_u}} \right)^{1/3} \left( \frac{\epsilon}{0.2} \right)^{-1}. \] (35)

Here, we have used \( m_h = 126 \text{ GeV} \) and \( m_t = 173 \text{ GeV} \). The correct top quark mass requires
\[ \frac{\lambda_q \lambda_u f_{\psi}^2}{m_{t'}^2} = 0.5 \cdot c_t^{-1} \left( \frac{\epsilon}{0.2} \right)^{-1} \left( \frac{m_{t'}}{2.4 \text{ TeV}} \right)^{-1}. \] (36)

The assumption that perturbative expansions by \( \lambda, \lambda_q \) and \( \lambda_u \) make sense requires
\[ \frac{\lambda^2 f_{\psi}^2}{m_{t'}^2} < 1, \quad \frac{\lambda_{q,u} f_{\psi}^2}{m_{t'}^2} < 1. \] (37)

Compared with Eq. (36), we need somewhat large \( c_t \) and/or \( m_{t'} \) for reliable perturbative estimates while explaining the top quark mass. We also expect that \( m_O \sim m_{t'} \) since they are both hadrons in the same dynamics. Putting altogether, we find \( \lambda f_{\psi} \sim 1 - 2 \text{ TeV} \) and \( m_O \sim m_{t'} \sim 2 - 4 \text{ TeV} \) provides the successful electroweak symmetry breaking within the perturbative regime. We will see below that the correct abundance of dark matter is obtained in the same parameter region.

### 5 Dark matter abundance and prospects for direct detection

By integrating out the dynamical sector, the mass and coupling of the dark matter particle are generated such as
\[ L_{\text{eff}} = -\frac{m_{DM}}{2} \bar{\psi}_S \psi_S - i \frac{m_{DM,5}}{2} \bar{\psi}_S \gamma_5 \psi_S \]
\[ + \frac{\kappa}{2} \bar{\psi}_S \gamma_5 \psi_S \sin^2 \frac{h}{f} + i \frac{\kappa_5}{2} \bar{\psi}_S \gamma_5 \psi_S \sin^2 \frac{h}{f}. \] (38)

At the leading order in \( \lambda \) and \( \epsilon \), they are given by
\[ m_{DM} = -\lambda^2 \int_0^{\infty} ds \frac{\tilde{p}_{1}(s)}{s}, \quad m_{DM,5} = -\lambda^2 \int_0^{\infty} ds \frac{\tilde{p}_{1,5}(s)}{s}, \] (39)
\[ \kappa = \lambda^2 \int_0^\infty ds \frac{\tilde{\rho}_4(s) - \tilde{\rho}_1(s)}{s}, \quad \kappa_5 = \lambda^2 \int_0^\infty ds \frac{\tilde{\rho}_{4,5}(s) - \tilde{\rho}_{1,5}(s)}{s}. \]  

(40)

The couplings to the gauge bosons are suppressed by \( \epsilon^2 \). One can eliminate the \( m_{\DM,5} \) term by the redefinition of \( \psi_S \). In that basis, \( m_{\DM}, \kappa \) and \( \kappa_5 \) are shifted to

\[ m_{\DM} \to \sqrt{m_{\DM}^2 + m_{\DM,5}^2}, \quad \kappa \to \kappa \cos \tilde{\theta} + \kappa_5 \sin \tilde{\theta}, \quad \kappa_5 \to \kappa \sin \tilde{\theta} + \kappa_5 \cos \tilde{\theta}, \]  

(41)

where \( \tan \tilde{\theta} = m_{\DM}/m_{\DM,5} \). In general, \( \kappa_5 \) cannot be eliminated simultaneously. Since we expect from the dimensional analysis that the dark matter mass and couplings are \( O(\lambda^2 f_O^2/\MO) \), we take the parameters in Eq. (38) as

\[ m_{\DM} = c_{\DM} \frac{\lambda^2 f_O^2}{\MO}, \quad m_{\DM,5} = 0, \quad \kappa = c_\kappa \frac{\lambda^2 f_O^2}{\MO}, \quad \kappa_5 = c_{\kappa_5} \frac{\lambda^2 f_O^2}{\MO}, \]  

(42)

with \( O(1) \) parameters, \( c_{\DM}, c_\kappa \) and \( c_{\kappa_5} \).

### 5.1 Relic abundance

Since we expect \( \kappa_5 \sim \kappa \), the annihilation via the \( \kappa_5 \) coupling mainly contributes to determine the relic density of dark matter because it is an \( s \)-wave process whereas the one with \( \kappa \) is \( p \)-wave. The annihilation cross section is given by

\[ \langle \sigma_{\text{ann}} , v \rangle \simeq 4s \left( \frac{\kappa_5}{f^2} \right)^2 \frac{v^2}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2} \frac{\Gamma_h|_{m_h = \sqrt{s}}}{\sqrt{s}} \]

\[ + \frac{1}{8\pi} \left( \frac{\kappa_5}{f^2} \right)^2 \left( 1 + \frac{3m_h^2}{s - m_h^2} \right)^2 \left( 1 - \frac{4m_h^2}{s} \right)^{1/2}, \]  

(43)

for \( m_{\DM} > m_h \), and \( s \) and \( \Gamma_h \) are the center of mass energy (\( \sim 2m_{\DM} \)) and the total decay width of the Higgs boson, respectively. In the limit of heavy dark matter, \( m_{\DM} \gg m_h \), it simplifies to

\[ \langle \sigma_{\text{ann}} , v \rangle \simeq \frac{1}{2\pi} \left( \frac{\kappa_5}{f^2} \right)^2. \]  

(44)

The dependence on the \( m_{\DM} \) disappears.

Requiring \( \Omega_{\DM} h^2 = 0.12 \) \([30]\), we find

\[ \kappa_5 = 160 \text{ GeV} \left( \frac{\epsilon}{0.2} \right)^{-2}, \]  

(45)

‡ Phenomenology of dark matter candidates which have a similar effective interactions has been studied in Refs. [26, 27, 28, 29].
Figure 1: Consistent parameter regions are shown for $\epsilon = 0.2$. The bands are drawn by taking $O(1)$ parameters to range $1/3 < c_X < 3$. which means

$$\lambda f_O = 900 \text{ GeV} \cdot c_{\kappa_5}^{-1/2} \left( \frac{m_O}{5 \text{ TeV}} \right)^{1/2} \left( \frac{\epsilon}{0.2} \right)^{-1} ,$$

from Eq. (42). Barring $O(1)$ uncertainties in various estimates, the value is comfortably consistent with the requirements from the Higgs and top quark masses. We summarize the consistent parameter regions in Fig. 1 where we allow $O(1)$ parameters to range $1/3 < c < 3$. The requirements from the top quark mass, the Higgs boson mass, the dark matter abundance all agree in the region where the perturbative expansion is reliable.

### 5.2 Direct detection cross section

From Eqs. (42) and (45), the dark matter mass is obtained as

$$m_{\text{DM}} = 160 \text{ GeV} \left( \frac{c_{\text{DM}}}{c_{\kappa_5}} \right) \left( \frac{\epsilon}{0.2} \right)^{-2} .$$

The on-going direct detection experiments have good sensitivity for such a weak scale dark matter.

In contrast to the case of the annihilation process, the $\kappa$ coupling, rather than $\kappa_5$, mainly contributes to the scattering processes for the direct detection since the $\kappa_5$ coupling only
contributes to the spin-dependent part in the non-relativistic limit. The spin independent cross section per nucleon is given by

$$\sigma_{SI} = \frac{4}{\pi} \left( \frac{m_N m_{DM}}{m_N + m_{DM}} \right)^2 \frac{[Z f_p + (A - Z) f_n]^2}{A^2},$$

where $m_N$ ($N = p, n$) is the nucleon mass and $A$ and $Z$ are the mass and atomic number of the target nucleus, respectively. The factor $f_N^q$ are matrix elements, $f_N^q = \langle m_q/m_N \rangle \langle N | \bar{q} q | N \rangle$. Assuming $f_p^u = f_p^d$ and taking the following values, $f_p^u = 0.021$, $f_p^d = 0.029$ and $f_p^s = 0.009$ [31, 32], which provide us with conservative estimates, the cross section is given by

$$\sigma_{SI} \simeq 1.2 \times 10^{-45} \text{cm}^2 \left( \frac{c_6}{c_{\kappa_5}} \right)^2,$$

where we use the information of the annihilation cross section in Eq. (45).

Fig. 2 shows the spin independent cross section of the dark matter scattering on proton. In the shaded region, the thermal dark matter abundance is consistent with the current observation and, in this figure, we assume $1/3 < c_\kappa/c_{\kappa_5} < 3$. The solid line and dashed

\footnote{If we take other values in the literatures, e.g., in Refs. [33, 34], the cross section changes by $O(10\%)$.}
line show the upper bound on the cross section from the LUX experiment and the expected upper bound from future LUX 300-day run, respectively \cite{35,36}. The dotted line denotes the expected upper bound from the future XENON 1T experiment \cite{37}. As one can see, a large parameter region can be covered by direct detection experiments in near future.

Here, we comment on other possible signatures of the model. The compositeness of the Higgs boson affects the coupling of the Higgs boson. In particular, the coupling to electroweak gauge bosons can be measured with a good accuracy. It has been studied that the sensitivity can reach to $\epsilon \sim 0.1$ (0.01) at the LHC (ILC) \cite{7}. In the $m_{\text{DM}} < m_h/2$ region, the invisible branching ratio of the Higgs boson decay is also expected. The current searches at the LHC \cite{40,41,42,43} put an upper bound around 20\% \cite{44} from the global fit assuming that the coupling constants are not modified from the Standard Model ones.

Multi-TeV spin-1/2 resonances are expected in the top quark and dark matter sector as discussed above. The top partner with the same charge of top quark may be accessible at future collider experiments ($<3.2$ TeV by LHC 33 TeV with 3 ab$^{-1}$ \cite{45,46}).

6 Summary

Dark matter of the Universe and the Higgs boson are two mysterious items in particle physics, and probably hints for deeper understanding of particle physics are hidden there. Indeed, the size of the interaction required to explain the abundance of dark matter by the thermal relic is of the order of the weak interaction, that is characterized by the Higgs VEV. This may be telling us that the nature of dark matter and that of the Higgs boson are tightly related.

We consider the possibility that dark matter is the one which is responsible for creating the potential of the Higgs field. We see that in the minimal composite Higgs model, the balance between the potentials made by the top quark and the dark matter can trigger the successful electroweak symmetry breaking while explaining the abundance of the dark matter.

We have studied the effective theory where the Higgs field is described as the pseudo Nambu-Goldstone boson. In the language of the effective theory, the Higgs field is already introduced as effective degrees of freedom, and we assume that the dark matter particle couples to it through some interaction term to break the global symmetry. In a full dynamical description, however, the picture may be more dramatic; the Higgs field may actually be the condensation of dark matter. For example, it has been studied recently that the scenario

\footnote{The constraints from indirect detections of dark matter turn out to be not quite strong \cite{38}.}
\footnote{For details, see also Ref. \cite{39}.}
of the top quark condensation can have a picture of the pseudo Nambu-Goldstone Higgs boson\cite{47,48}. It is promising that the realization of the dark-matter condensation as the Higgs field is also possible.

In the parameter region where the Higgs boson mass and the abundance of the dark matter is explained, the spin-independent cross section for the direct detection experiments turns out to be quite large, just below the experimental constraints. If there is no significant fine-tuning, we expect to see the detection quite soon.

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