Cosmological Constraint from the 2dF QSO Spatial Power Spectrum

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in original form 2002 January 15

ABSTRACT

In this paper we obtain constraints on the cosmological density parameters, $\Omega_m$ and $\Omega_b$, by comparing the preliminary measurement of the QSO power spectrum from the two degree field QSO redshift survey with results from an analytic technique of power spectrum estimation, described in this paper. We demonstrate the validity of the analytic approach by comparing the results with the power spectrum of an N-body simulation. We find a better fit to the shape of the QSO power spectrum for low density models with $\Omega_m = 0.1 - 0.4$. We show that a finite baryon fraction $\Omega_b/\Omega_m = 0.2$, consistent with observations of the CMB anisotropies and nucleosynthesis, fits the observational result of the 2QZ survey better, though the constraint is not very tight. By using the Fisher matrix technique, we investigate just how a survey would be required before a significant constraint on the density parameters can be made. We demonstrate that the constraint could be significantly improved if the survey was four times larger.

Key words: cosmology – large-scale structures of Universe: quasars.
1 INTRODUCTION

Since the pioneering work of Osmer in the 1980’s (Osmer 1981), QSOs have been used to probe the high redshift Universe. It was soon shown that QSOs were clustered (Shaver 1984; Shanks et al. 1987; Iovino & Shaver 1988) but results were limited due to the relatively small QSO samples available, see Croom & Shanks (1996) and references therein for a summary of early QSO clustering results. The two degree Field QSO Redshift Survey (2QZ) will dramatically improve things as it will contain at least a factor of 25 more QSOs than than previous QSO surveys. The 2QZ group has recently reported preliminary clustering results based on an initial sample of 10,000 QSOs (Croom et al., 2001, Hoyle et al. 2002; 2001, Outram et al. 2001).

Observational results and theoretical predictions are most directly compared through the use of numerical simulations. Indeed, the 2QZ group has utilized one of the Hubble Volume simulations run by the Virgo consortium (Frenk et al. 2000) to interpret their results. It is a huge N-body simulation of horizon box size, containing 1 billion mass particles output along a light-cone. However, numerical simulations of this size are slow and expensive to run. A simple, semi-analytic formula that reproduces numerical results but which doesn’t require intensive computation would be useful as it allows a wide range of parameter space to be explored in a short time frame. Fortunately, several examples appear in the literature (e.g. Yamamoto et al. 1999; Suto et al. 2000; Hamana et al. 2001b; and references therein).

In this paper, we describe such an analytic approach. We demonstrate the validity of this technique by comparing the semi-analytic formula with results from N-body simulations and mock 2QZ samples. As a demonstration of the usefulness of this approach, we use the analytic formula to place constraints on the cosmological density parameters by comparing the theoretical predictions to the 2QZ power spectrum of Hoyle et al. (2002). By using the Fisher matrix approach we also make predictions for the size of the survey that would be required before the density parameters can be tightly constrained. (see e.g., Tegmark 1997, Tegmark et al. 1998)

This paper is organized as follows: In section 2, we briefly summarize a theoretical formula for the power spectrum that incorporates the light-cone effect, the linear redshift-space distortions and the geometric distortion. In section 3, we compare the analytical power spectrum formula with power spectra results from the N-body simulation and mock QSO samples. In section 4 we place constraints on the density parameters, $\Omega_m$ and $\Omega_b$, by fitting
the analytic formula to the QSO power spectrum measured from the 10k 2QZ sample. In section 5 we make predictions for how well the density parameters may be constrained with yet larger surveys and Section 6 is devoted to summary and conclusions. Throughout this paper we use the unit in which the light velocity \( c \) equals 1.

## 2 ANALYTIC FORMULAS

In this section we briefly review an analytic formula for the spatial power spectrum of cosmological objects. The analytic form of the power spectrum has to include light-cone effects, redshift-space distortions and geometric distortions. We follow, for example, Suto et al. (2000) and define

\[
P^{LC}(k) = \frac{\int dz W(z) P^a_0(k, z)}{\int dz W(z)},
\]

where

\[
W(z) = \left(\frac{dN}{dz}\right)^2 \left(\frac{1}{S^2}\right) \left(\frac{dz}{ds}\right),
\]

\(dN/dz\) is the number density of objects per unit redshift per unit solid angle, \(s = s(z)\) is the distance-redshift relation defined by equation (3), and \(P^a_0(k, z)\) is the local power spectrum defined on a constant time hypersurface at redshift \(z\). Thus the light-cone effect is taken into account by averaging the local power spectrum \(P^a_0(k, z)\) over the redshift range. The distance \(s = s(z)\) is the comoving distance which, in a specially flat universe, is given by

\[
s(z) = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + 1 - \Omega_m}},
\]

where \(H_0 = 100h\) km/s/Mpc, the Hubble parameter, and \(\Omega_m\) is the total matter density parameter. In the present paper we fix \(\Omega_m = 0.3\) and \(h = 0.7\) in equation (3), consistent with the values used in the analysis by Hoyle et al. (2002). For the local power spectrum, \(P^a_0(k, z)\), we model

\[
P^a_0(k, z) = \frac{1}{c_{||}(z)c_{\perp}(z)^2} \times \int_0^1 d\mu P_{QSO}\left(q_{||} \to \frac{k\mu}{c_{||}}, q_{\perp} \to \frac{k\sqrt{1 - \mu^2}}{c_{\perp}}, z\right),
\]

where \(P_{QSO}(q_{||}, q_{\perp}, z)\) is the QSO power spectrum, \(q_{||}\) and \(q_{\perp}\) are the wave number components parallel and perpendicular to the line-of-sight direction in the real space, and we define

\[
c_{||}(z) = \frac{dr(z)}{ds(z)}
\]
where $r(z)$ is the comoving distance of the universe, found using equation (3) but with varying $\Omega_m$. In the present paper, we only consider the spatially flat cosmological model with a cosmological constant. Because an analysis of the observational data is performed by fixing the cosmological model, i.e., by equation (3), we need to take the geometric distortion effect into account when comparing the observational result with models using different cosmological parameters. The geometric distortion effect is included by scaling the wave number in equation (4) by the factors $c_\parallel(z)$ and $c_\perp(z)$.

We model the QSO power spectrum of the distribution with the linear distortion (Kaiser 1987) by

$$P_{QSO}(q_\parallel, q_\perp, z) = \left(1 + \frac{f(z)}{b(z)} \frac{q^2_\parallel}{q^2} \right) b(z)^2 P_{\text{mass}}(q, z),$$

where $q^2 = q^2_\parallel + q^2_\perp$, $b(z)$ is a scale independent bias factor, $P_{\text{mass}}(q, z)$ is the CDM mass power spectrum, and we defined $f(z) = d \ln D(z)/d \ln a(z)$ with the linear growth rate $D(z)$ and the scale factor $a(z)$. For a phenomenological correction for the nonlinear velocity field, a damping function might be multiplied on the right hand side of equation (7). This is a very small correction on the scales we consider in this work so we neglect it here. (see e.g., Mo, Jing & Börner 1997; Magira, Jing & Suto 2000)

The two-point correlation function, corresponding to the spectrum (1), can be written

$$\xi^{LC}(R) = \frac{1}{2\pi^2} \int_0^\infty dk k^2 j_0(kR) P^{LC}(k).$$

Formula (8) is obtained under the short distance approximation, $R(= 2\pi/k) \ll L$, where $L$ is the size of a survey area. Hence the validity of the use is limited to small length scales. The finite size effect means we cannot obtain a robust estimation of the correlation function or the power spectrum on length scales larger than the size of a survey. For the 2QZ survey, the limit is $L \sim 10^3 h^{-1}\text{Mpc}$. For the correlation function (8), it was reported that the analytic expression is in good agreement with results of the N-body numerical simulation (Hamana et al. 2001a; 2001b), though the comparison might be limited to a finite range of length scales.

We only consider the linear power spectrum as nonlinear effects are negligible on the length scales on which the power spectrum from QSOs can be measured. Hence we adopt

$$P_{\text{mass}}(q, z) = A q^n T(q, \Omega_m, \Omega_b, h)^2 D(z)^2,$$
where $A$ is a normalization constant, $T$ is the transfer function, and $n$ is the index of the initial spectrum, for which we assume $n = 1$ in the present paper. We adopt the fitting formula of the cold dark matter transfer function by Eisenstein & Hu (1998) which is robust even when the baryon fraction is large.

3 COMPARISON WITH N-BODY NUMERICAL SIMULATION

In this section we investigate how well the analytic power spectrum reproduces a power spectrum measured from an N-body numerical simulation and a power spectrum measured from mock 2QZ catalogues.

The simulation is one of the Hubble Volume (HV) simulations. It has a $\Lambda$CDM cosmology with $\Omega_m = 0.3$, $\Omega_b = 0.04$ and $\Omega_{\Lambda} = 0.7$, $h = 0.7$ and normalized to $\sigma_8 = 0.9$ at present day and with a shape parameter of $\Gamma = 0.17$. The simulation has a light-cone output such that the evolution of the dark matter is fully accounted for. It extends to redshift $\approx 4$, and covers an area of $15 \times 75 \text{deg}^2$, which we split into three $5 \times 75 \text{deg}^2$ strips to match the geometry of the 2QZ strips.

For the first comparison, we simply take the dark matter points in real space, over the range $0.3 \leq z \leq 2.5$. We apply the selection function of the 2QZ survey shown, for example, in Hoyle et al. (2002), to the points and then measure the power spectrum. We do this in each of the three $5 \times 75 \text{deg}^2$ strips and average the result together. The analytic power spectrum assumes the same $dN/dz$ distribution, that the points are in real space and assumes the same cosmology. Figure 1 shows a comparison of these two power spectra. The solid and dotted lines, respectively, show the results of the analytic prescription and the numerical simulation. There is good agreement (within $\sim 10\%$) on scales $0.01 h \text{Mpc}^{-1} \lesssim k \lesssim 0.1 h \text{Mpc}^{-1}$.

As a second comparison, we compare the analytic power spectrum to the power spectrum of mock QSO catalogues drawn from the HV. The mock catalogues are constructed as follows: First, the selection function of the 2QZ is applied to the dark matter particles in redshift-space over the redshift range $0.3 \leq z \leq 2.5$. Then the dark matter particles are biased such that the clustering approximately matches that of the 2QZ. It is assumed that the QSO clustering amplitude remains constant with redshift, consistent with the results of Croom et al. (2001), when the mock catalogues are constructed. The points are then sparse sampled to match the number density of the 2QZ. Again the power spectrum is measure from each
strip and the results are averaged together. Full details on the construction of the mock
catalogues are given in Hoyle (2000).

For this comparison, the amplitude of the analytic power spectrum has to be determined.
To do this, we minimizing $\chi^2$ defined by equation (10).

$$
\chi^2 = \sum_i^N \frac{(P^{LC}(k_i) - P^{sim}(k_i))^2}{\Delta P(k_i)^2}
$$

(10)

where $P^{sim}(k_i)$ is the value of the power spectrum from the simulation at $k_i$, $\Delta P(k_i)^2$ is
the variance of errors in Figure 2, and $N$ is the number of the data points. In the analytic
modeling of the clustering bias, we assumed the form

$$
b(z) = \frac{b_0}{[D(z)]^p}
$$

(11)

where $b_0$ and $p$ are the parameters. We determine $b_0$ to minimize $\chi^2$ for each pair of $\Omega_m$ and
$\Omega_b$. Croom et al (2001) report that the QSO clustering amplitude does not show significant
time-evolution, therefore we set $p = 1$. However, this choice does not alter our conclusions.
For example, when we set $p = 2$ the values of $\chi^2$ change by only a few percent. This confirms
the result that the power spectrum is only sensitive to a mean amplitude of the bias and
is insensitive to the speed of the redshift evolution (Yamamoto & Nishioka 2001). Figure 2
shows that a more realistic numerical result can still be fit by the analytic approach even
though the bias is calculated in a different way.

Figure 3 shows the $\chi^2$ contours found when different values of the cosmological density
parameters $\Omega_m - \Omega_b/\Omega_m$ are used in the analytic formula. Each panel corresponds to
the constraint using (a) the 17 points ($N = 17$), (b) the 16 points, (c) the 15 points and (d) the
14 points, respectively, of the data from left to right in Figure 2. The cross point in each
panel is the target parameter of the simulation. In the case where we including data points
of small length scales, the minimum of $\chi^2$ is slightly inconsistent with the target parameter.
The disagreement on these small scales $k \simeq 0.2$ can not be explained by including the
nonlinear corrections of the density perturbations and the finger of God effect. It might be
inferred that long mean separations between QSOs cause this feature. In general, however,
agreement of the numerical simulation and the analytic approach is acceptable over the
range of scales $0.01h\text{Mpc}^{-1} \lesssim k \lesssim 0.1h\text{Mpc}^{-1}$. 
4 COMPARISON WITH AN OBSERVATIONAL RESULT

As an application, we compare the power spectrum measured from the 10k 2QZ dataset to the analytic formula as a way to constrain cosmological parameters. We simply define $\chi^2$ in the similar way as equation (10) but replace $P_{\text{sim}}(k_i)$ with $P_{\text{obs}}(k_i)$, i.e., the observational data, for which we use the result of Hoyle et al. (2002). Figure 4 shows the observational data points from Hoyle et al. (2002) compared to the best fit analytic power spectrum. Figure 5 displays the contours of $\chi^2$ for various cosmological models on the $\Omega_m - \Omega_b/\Omega_m$ plane. Each panel shows the contour using the data of (a) the 17 points, (b) the 16 points, (c) the 15 points and (d) the 14 points, respectively, from left to right in Figure 4, where the wave numbers of the observational data used in Figure 5 are same as those in Figure 3. From Figure 5 it is clear that the QSO power spectrum favors a low density universe with $\Omega_m = 0.1 - 0.4$ rather than the standard CDM model with $\Omega_m = 1$. The minimum of the $\chi^2$ is located at $\Omega_m \approx 0.2$ and $\Omega_b/\Omega_m \approx 0.2$. The dashed curve shows $\Omega_b = 0.04$, inferred from results of the comic microwave background (CMB) anisotropies and the big bang nucleosynthesis (BBN) (see e.g., Turner 2001). Our result $\Omega_m \approx 0.2$ is consistent with the previous result of the 2QZ power spectrum, which reports the best fit value $\Omega_m h = 0.1$ (Hoyle et al. 2002, cf. Outram et al. 2001), but, is smaller than $\Omega_m \approx 0.3$, which is obtained from the large scale structures of galaxies (e.g., Peacock et al. 2001). It is unclear whether the discrepancy has any physical meaning or not, because the $1-\sigma$ contour is broad. An interesting fact is that the QSO power spectrum is better explained with the finite baryonic component, though the peak of $\chi^2$ is broad and thus the constraint is not that tight (cf. Miller et al. 2001, Peacock et al. 2001).

5 TIGHTNESS OF THE CONSTRAINT

It is useful to investigate how much larger the 2QZ Survey would have to be before a tight constraint on the density parameters can be obtained. For this purpose, we adopt the Fisher matrix approach. The Fisher matrix analysis provides a technique by which statistical errors on parameters from a given data set can be estimated. In general, the Fisher matrix is defined by

$$F_{ij} = \left\langle -\frac{\partial^2 \ln L}{\partial \theta_i \partial \theta_j} \right\rangle,$$

(12)
where \( L \) is the probability distribution function of a data set, given model parameters. Here, we follow the work of Tegmark (1997). Using the Fisher matrix approach, Tegmark investigated the accuracy with which cosmological parameters can be measured from galaxy surveys. Following Tegmark (1997), we can write the Fisher matrix element,

\[
F_{ij} = \frac{\kappa}{4\pi^2} \int_{k_{\text{min}}}^{k_{\text{max}}} dkk^2 \frac{\partial P_{\text{LC}}(k)}{\partial \theta_i} \frac{\partial P_{\text{LC}}(k)}{\partial \theta_j},
\]

where we define

\[
\kappa = \Delta \Omega \int dz W(z),
\]

where \( \Delta \Omega \) is the solid angle of the survey area and \( \theta_i \) denotes the theoretical parameters. In deriving equation (13), we use the fact that \( \bar{n}P_{\text{QSO}}(k) \lesssim O(0.1) \) where \( \bar{n} \) is the comoving mean number density of the QSOs. Here we set \( k_{\text{max}} = 0.2h\text{Mpc}^{-1} \) and \( k_{\text{min}} = 0.01h\text{Mpc}^{-1} \).

By using the Bayes theorem, the probability distribution in the parameter space can be written \( P(\theta_i) \propto \exp[-\sum_{ij}(\theta_i - \theta_i^r)F_{ij}(\theta_j - \theta_j^r)/2] \), where \( \theta_i^r \) is the target model parameters. Here, the Fisher matrix is the 2 \( \times \) 2 matrix, \( \Omega_m \) and \( \Omega_b \) matrix. We show how accurately these two parameters can be determined for the final 2QZ sample and for a sample that is four times larger in Figure 6. Curves in each panel indicate the 68\%, 95\%, and 99.7\% confidence region on the \( \Omega_m - \Omega_b/\Omega_m \) plane. Both panels assume the same \( dN/dz \) as the 10k 2QZ catalogue but the left panel assumes \( \Delta \Omega = 750 \text{ deg}^2 \) and the right panel assumes \( \Delta \Omega = 3000 \text{ deg}^2 \). This figure shows an example of the tightness of the constraint possible from the QSO spatial power spectrum alone. Note that the constraint will be tighter if we combine the results with other constraints, such as those from the Alcock-Paczynski test (Alcock & Paczynski 1979; Outram et al. 2001; Hoyle et al. 2002).

\section{Summary and Conclusions}

In this paper we have shown that an analytic approach can accurately reproduce the power spectrum of an N-body simulation and mock QSO catalogues over the range of length scales \( k \lesssim 0.1h^{-1}\text{Mpc} \). By performing a simple \( \chi^2 \) test we have also shown that the observational QSO power spectrum is consistent with a simply biased mass power spectrum based on the popular CDM cosmology with a cosmological constant. We find that the analytic formula better matches the 2QZ power spectrum if the baryon fraction is around 20\% of the mass fraction. The constraint is not very tight because our analysis is based on just the initial sample of 10,000 QSOs from the 2QZ survey. Accumulation of a number of samples will
improve the statistical errors. Tighter constraints will be possible if we compare the results with other cosmological constraints such as those obtained from the A-test.

Acknowledgments: This work was supported by fellowships for Japan Scholar and Researcher abroad from Ministry of Education, Science and Culture of Japan and the Deutscher Akademischer Austauschdienst (DAAD). The author thanks Prof. S. D. M. White, Dr. H. J. Mo, and the people at Max-Planck-Institute for Astrophysics (MPA) for their hospitality and useful discussions and comments. He is grateful to the referee for useful comments on the earlier manuscript, which helped improve it. He also thanks Y. Kojima for encouragement. The author is extremely grateful to Fiona Hoyle for many useful discussions and communications, and for providing me the results of the numerical simulation and the power spectrum data used in the present paper. This work couldn’t be done without the collaborations with her. He acknowledge the efforts of everyone involved in the 2QZ project for producing a unique and valuable dataset.
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Figure 1. Comparison of dark matter power spectra in real space with the 2QZ selection function applied. The dotted curve shows the result from the numerical simulation and the solid curve shows the result from the analytic approach. The dashed curves are the power spectra $P^a(k, z = 0)$ (the upper curve) and $P^a(k, z = 2.5)$ (the lower curve).
Figure 2. Comparison of the power spectra of biased particles in redshift space with the 2QZ selection function applied. The points with error bars show the mock catalogue power spectrum from Hoyle et al. (2002) and the solid curve shows the result using the analytic approach.
Figure 3. The contours of $\chi^2$ show the level to which the power spectrum of the numerical simulation and the analytic formula agree. The number of points used in calculating $\chi^2$ is shown in each panel. These are the $N$ points from the left shown in figure 2. The cross point is the target parameter of the simulation.
Figure 4. The observational data, from Hoyle et al. (2002), which we used in constraining the cosmological parameters. The solid curve shows the analytic formula using the best fit parameters $\Omega_m = 0.2$ and $\Omega_b = 0.04$. 
Figure 5. Contours of $\chi^2$ found by comparing the observed QSO power spectrum with the analytic formula. The format of this figure is the same as Figure 3. The number of data points, $N$, and the wave numbers of the observed power spectrum, $k_i$, are the same as those in Figure 3. The dashed curve shows $\Omega_b = 0.04$, which is consistent with results from the cosmic microwave background anisotropies and big bang nucleosynthesis. (see e.g., Turner 2001).
Figure 6. Confidence regions from data sets of QSO samples. The curves show the 68%, 95% and 99.7% confidence regions. The left panel assumes a solid angle of $\Delta \Omega = 750 \, \text{deg}^2$ and the right panel assumes a solid angle of $\Delta \Omega = 3000 \, \text{deg}^2$ for the survey region. In both cases, we assume the $dN/dz$ of the 2QZ survey, and we set $k_{\text{max}} = 0.2 h \text{Mpc}^{-1}$ and $k_{\text{min}} = 0.01 h \text{Mpc}^{-1}$. The target parameters are $\Omega_m = 0.2$ and $\Omega_b = 0.04$. 