Finite Element Modelling and Analysis of Conventional Pultrusion Processes

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Abstract. Pultrusion is one of many composite manufacturing techniques and one of the most efficient methods for producing fiber reinforced polymer composite parts with a constant cross-section. Numerical simulation is helpful for understanding the manufacturing process and developing scientific means for the pultrusion tooling design. Numerical technique based on the finite element method has been developed for the simulation of pultrusion processes. It uses the general purpose finite element software ANSYS Mechanical. It is shown that the developed technique predicts the temperature and cure profiles, which are in good agreement with those published in the open literature.

1. Introduction

Pultrusion is a continuous and efficient process producing composite profiles with a constant cross-section. During pultrusion, fibre reinforcements are saturated with resin in a resin tank and then continuously pulled through a heated die by a puller. Electrical heaters are used for heating of the die in the conventional pultrusion technology. Inside the die, the resin gradually cures and solidifies to form a composite part with the same cross-section profile as in the die. After exit from the die, the finished product is cut to the desired length by saw integrated into the pultrusion tool. Schematic of the process is given in figure 1. An effective set up of pultrusion process is not possible without a numerical analysis of the technological process. A lot of numerical simulations have been done in the last twenty years, most of them focus on the analysis of the heat transfer and cure [1-5], on the pressure on the inner walls of the die [6,7] and on the problems related to impregnation of reinforcing fibres [8].
2. Theory of the pultrusion process

The object of study in this investigation is part of the pultrusion tool – heated die and composite material in the die.

2.1. Energy equation for the die

Heat transfer in the non-moving die is governed by the following equation in the Cartesian coordinate system:

\[
\rho c_p \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) - \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) - q_h - q_b = 0
\]  

(1)

where \( T \) is the temperature in Kelvin, \( \rho \) and \( c_p \) are the density and specific heat of the die material, \( k_x, k_y, k_z \) – thermal conductivities in \( x, y, z \) directions, \( q_h \) – rate of heat generation by heaters, \( q_b \) – rate of energy exchange at the boundary.

2.2. Energy equation for the composite

Resin curing is exothermic chemical reaction that gives off heat energy. Therefore it is necessary to introduce the source term in the energy equation for the composite. In pultrusion, the fibres are saturated with resin before entering the die. Therefore it is reasonable to assume that the resin does not flow and is moving with the fibres as a solid composite material. The three-dimensional transient energy equation for the pultruded composite can be written as:

\[
\bar{\rho} \bar{c}_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial x} \left( \bar{k}_x \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial y} \left( \bar{k}_y \frac{\partial T}{\partial y} \right) - \frac{\partial}{\partial z} \left( \bar{k}_z \frac{\partial T}{\partial z} \right) - q = 0
\]  

(2)

where \( T \) is the temperature in Kelvin, \( u \) is the pull speed, \( \bar{\rho} \) and \( \bar{c}_p \) are the lumped density and specific heat of the composite material, respectively, \( \bar{k}_x, \bar{k}_y, \bar{k}_z \) are the lumped thermal conductivities in \( x, y, z \) directions, and \( q \) is the generative term related to internal heat generation due to the exothermic resin reaction.

The lumped thermal properties of the composite are evaluated by the following rules of mixture:

\[
\bar{\rho} = (1-V_r)\rho_f + V_r\rho_r
\]

(3)
where material properties with index $f$ are fiber properties, and with index $r$ – resin properties.

2.3. Species equation for the resin

The generative term of the composite energy equation related to the resin curing reaction is written as:

$$q = V_r \rho_r H_r R_r$$

(6)

where $V_r$ is the resin volume fraction, $\rho_r$ is the resin density, $H_r$ is the total heat of reaction and $R_r$ is the rate of resin reaction:

$$R_r = \left( \frac{\partial \alpha}{\partial t} + u \frac{\partial \alpha}{\partial x} \right)$$

(7)

Equation (7) is chemical species equation where $\alpha$ is the degree of cure which is defined as a ratio of the heat amount evolved during the reaction up to present time $t$, $H(t)$, to the total heat of reaction $H_r$:

$$\alpha = \frac{H(t)}{H_r}$$

(8)

2.4. Curing kinetic model of the resin

Traditionally the resin curing reaction is modelled using the Arrhenius relationship $K(T)$ multiplied by a simple mathematical function $f(\alpha)$ that depends on the resin properties and varies with the resin reaction model:

$$R_r(\alpha, T) = \frac{\partial \alpha}{\partial t} = \frac{1}{H_r} \frac{dH(t)}{dt} = K(T) \cdot f(\alpha)$$

(9)

The Arrhenius relationship is written as:

$$K(T) = K_0 \exp \left( \frac{-E}{RT} \right)$$

(10)

where $T$ is the temperature in Kelvin, $R$ is the universal gas constant, $K_0$ and $E$ are, respectively, the frequency factor and the activation energy of the resin.

As a simple mathematical function $f(\alpha)$, 1st-order, $n$-th order, Prout-Tompkins [9] are more frequently used for the formulation of the curing kinetic model:
\[ f(\alpha) = (1 - \alpha) - \text{first order kinetic model}; \]
\[ f(\alpha) = (1 - \alpha)^n - n \text{-th order kinetic model}; \]
\[ f(\alpha) = (1 - \alpha)^n (1 + K_2 \alpha) - n \text{-th order kinetic model with autocatalysis}; \]
\[ f(\alpha) = \alpha^m (1 - \alpha)^n - \text{Prout - Tompkins autocatalytic model}. \]  

(11)

Here \( K_2 \), \( n \), \( m \) are the kinetic parameters of the resin.

### 3. Numeric implementation

Numerical technique based on the finite element method has been developed for the simulation of the conventional pultrusion processes. It uses the general purpose finite element software ANSYS Mechanical.

#### 3.1. Numerical procedure based on ANSYS mechanical

The flowchart of the numerical procedure is shown in figure 2. At the beginning of the procedure then \( t = 0 \), \( \alpha \) is pre-defined as \( \alpha_{j=0}^i = \alpha_0 \), where \( \alpha_0 \) is equal to 0 or small value depending on the used resin kinetic model, and \( T_{j=0}^i = T_0 \), where \( T_0 \) is room temperature. Here \( j \) is the number of FE nodal point (figure 3) and \( i \) is the time step index.

The species equation for the resin (7) in numerical procedure is integrated explicitly as:

\[ \frac{\partial \alpha}{\partial t} = \left[ R - u \frac{\partial \alpha}{\partial x} \right]^{i-1} \]  

(12)

Time step is calculated as:

\[ \Delta t = \frac{1}{p} \cdot \frac{l}{u} \]  

(13)

where \( l \) is the length of the FE in pull direction, \( u \) is the pull speed, \( p \) is the number of sub steps chosen by the convergence condition depending on the mesh size and the material properties.

Within each time step, FE software ANSYS is used for the solution of the remaining transient heat conduction problem to obtain the temperature. After obtaining the temperatures, the degree of cure and the effect of the exothermic term are calculated at each nodal point using the nodal control volume (NCV) method realized in the user program. The NCVs are created within the zone defining composites by connecting the centroid of a finite element to the centre points of its surfaces. The boundary of each NCV contains only one FE node. NCV mesh is shown in figure 3. In the NCV the distribution of field variables (\( T \), \( \partial \alpha / \partial t \) and \( \alpha \)) is assumed constant and unchangeable over a time step \( \Delta t \).

The rate of the cure for the NCV at any time step \( i \) is calculated using (9):

\[ \frac{\partial \alpha_j^i}{\partial t} = \left[ \frac{\Delta \alpha_j^i}{\Delta t} \right] = K(T_j^i) f(\alpha_j^{i-1}) \]  

(14)

The degree of cure \( \alpha \) for the NCV is obtained continuously as:
\[ \alpha_j^* = \alpha_j^{*-1} + \left[ \frac{\Delta \alpha_j^*}{\Delta t} \right] \Delta t \] 

(15)

\textbf{Figure 2.} Flowchart of the numerical procedure.

\textbf{Figure 3.} FE/NCV method.
The exothermic effects of cure reaction are evaluated as the source term for a NCV using (6):

\[ q = V_i \rho_i H_i R_i = V_i \rho_i H_i \frac{\partial \alpha_i^j}{\partial t} = V_i \rho_i H_i \left[ \frac{\Delta \alpha_i^j}{\Delta t} \right] \]  

(16)

Movement of resin-saturated composite is simulated as a semi-steady process. The user program shifts the temperature and the curing fields in the composite to distance \( l \) in pull direction during creation of initial data for the next FE run.

Iterative procedure shown in figure 2 continues until the difference of temperature and curing fields obtained at the present and previous time steps is less than the set convergence norm.

4. Numerical example
The developed procedure was validated by the temperature and degree of cure fields described in the literature studying the pultrusion of cylindrical rod [1-3], flat plate [3, 4] and I-beam profiles [5]. As an example, the numerical results are given for the pultrusion process of an I-beam.

4.1. Pultrusion of I-beam
This example was chosen to demonstrate an application of the developed numerical procedure for the simulation of the pultrusion process with the temperature control. The temperature control is implemented by turning the heaters on and off.

The scheme of the pultrusion die is presented in figure 4. Dimensions of the pultruded composite profile and die, as well as the locations and dimensions of the heaters and cooling holes are presented in figures 5 - 7. The materials used for production of the I-beam are glass fibers with the fiber volume content of 45% and thermoset vinyl ester resin VE3. Their thermal properties are given in table 1. The rate of resin reaction is described using the Prout-Tompkins curing kinetics model (11) with the parameters presented in table 2.

The pull speed is 30 cm/min. Heating is provided by four electrical heat platens placed on the top and bottom surfaces with the heater’s power of 1600 W and two electrical strip heaters placed on both die’s sides with the heater’s power of 1500 W. The heat platens are insulated except the surfaces contacting with the die. The exterior surfaces of the die, except those under the heaters, and the I-beam are assumed to be exposed to the air and hence are simulated as convective boundaries with a convective heat transfer coefficient of 10 W/m²·K. The ambient room temperature is 20°C. The die is cooled at the distance of 100 mm from the die entrance by water with the temperature of 20°C to prevent excessive curing at the die entrance.

| Table 1. Thermal properties of materials. |
|-----------------------------------------|
| \( \rho \) (kg/m³) | \( c_p \) (J/kg·K) | \( k_z \) (W/m·K) | \( k_z = k_x \) (W/m·K) |
|----------------|----------------|----------------|----------------|
| Vinyl ester | 1100 | 1640 | 0.169 | 0.169 |
| Glass fibers | 2560 | 670 | 11.4 | 1.04 |
| Lumped properties \( (V_r = 0.55) \) | 1757 | 1004 | 0.4773 | 0.3748 |
| Steel (die) | 7860 | 486 | 51 | 51 |
| Aluminium (heat platens) | 2700 | 896 | 180 | 180 |
Control temperature of the pultrusion process is 123°C. Unfortunately, temperature control points and algorithms of heaters work are not clearly formulated in [5]. The following algorithm was chosen in the simulation: the first heat platen works permanently, the work of the second heat platen and strip heater is controlled – they are switched ON if the maximal temperature on line C within the boundaries of the die is less than the control temperature 123°C and switched OFF otherwise.

The finite element model for the simulation of the I-beam pultrusion was created in ANSYS Mechanical using 3-D thermal solid finite element Solid 70. The element has eight nodes with a single degree of freedom, temperature at each node and the orthotropic material properties. Taking into account the symmetry of the pultrusion tool, only one quarter of it is modelled. The insulation materials surrounding the heat platens are not included into the finite element model, but the surfaces of the platens, except those that contact with the die, are assumed as fully insulated. The strip heaters located on both sides of the die are simulated as surface powers directly applied on the surface of the die. The finite element model is shown in figure 8. An additional length of 285 mm of the I-beam is modelled at the die exit to extend the thermo-chemical analysis to the post-die region. The finite element mesh is regular in pull direction and has 87 elements on the beam. Total number of finite elements is 10660 (1740 for composite, 7480 for die and 1440 for the heaters). The time step defined by (13) is 3 s. During this time the composite travels in the pull direction to the distance equal to the dimension of one element (15 mm). It is necessary to note that iterations (sub-steps) are not performed in the finite element analysis. Initial conditions are applied at time $t=0$ when all nodal points of the composite have the resin temperature of 20°C and $10^{-10}$ degree of cure.

Algorithm of heaters work and changes of maximal temperatures on the control lines in time domain are presented in figure 9. It is seen that the pultrusion die has great thermal inertia, because of which maximal temperatures on the control lines vary considerably in the steady manufacturing process. The predicted temperatures and degree of cure distributions at the control lines in general agree with the published results (figures 10 – 13). The largest difference is observed in the temperature predictions on lines A (figure 10) and B (figure 11) located in the die and at the boundary of the die and composite respectively (figure 6). This difference could be explained by different location of temperature control points and algorithm of the heaters work used in [5] and current investigation. It is important to note that there is good agreement between the present and published results for the

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**Table 2. Kinetics parameters of the resin.**

| $H_r$ (J/kg) | $K_0$ (1/s) | $E$ (J/mol) | $n$ | $m$ | $\alpha_{max}$ |
|--------------|-------------|-------------|-----|-----|----------------|
| 398440       | 186958305   | 71688       | 1.2853 | 0.7147 | 0.97          |

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Figure 4. Scheme of the pultrusion die.
temperature (figure 12) and the degree of cure (figure 13) distributions in the profile (control line C in figure 6). Higher temperature at the active reaction point is obtained because of a slightly higher rate of the exothermic curing reaction that, in turn, could be explained by different time integration schemes applied in the finite element modelling and different finite element software used for the simulation.

Figure 5. Longitudinal section of the pultrusion die.

Figure 6. Transverse section of the pultrusion die A-A, B-B, C-C – control lines.

Figure 7. Cross-section of the pultruded profile.

Figure 8. FE model created in Ansys mechanical.
Figure 9. Work of the heaters and maximal temperatures on the control lines A, B and C (within the boundaries of the die).

Figure 10. Temperature distribution on line A.

Figure 11. Temperature distribution on line B.

Figure 12. Temperature distribution on line C.

Figure 13. Curing distribution on line C.
5. Conclusions
A numerical procedure based on the general purpose finite element package ANSYS Mechanical has been developed for the thermo-chemical simulation of conventional pultrusion processes. Good agreement between the present finite element results and published numerical-experimental results for both temperature and degree of cure shows that the developed technique can be used successfully for an accurate simulation of the pultrusion processes with different initial data, boundary conditions and new technological requirements.

6. References
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Acknowledgement
The authors gratefully acknowledge the support of European Commission, FRAMEWORK 7 program, contract no. NMP2-SL-2013-609149, project “Development of an Innovative Manufacturing Process for the In-Line Coating of Pultruded Composites (COALINE)”.