Precision study of positronium and precision tests of the bound state QED

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Abstract

Despite its very short lifetime positronium provides us with a number of accurate tests of the bound state QED. In this note a brief overview of QED theory and precision experiments on the spectrum and annihilation decay of the positronium atom is presented. Special attention is paid to the accuracy of theoretical predictions.

1 Introduction

Positronium is a very specific two-body atomic system, which in some respects is similar to hydrogen, but some its features are quite different from those of the hydrogen atom. In particular, positronium is an unstable atom and the lifetime of its ground state is about $10^{-7}$ or $10^{-10}$ sec depending on its spin. Because of that any experiments with positronium are much more complicated than those with hydrogen and considerably less precise. In this brief overview of positronium studies we will try to demonstrate that despite the existing experimental problems positronium is worth studying.

The theory which describes simple atomic systems is bound state Quantum Electrodynamics (QED) and the questions we are trying to answer in this paper are

- Why do we have to study and test bound state QED?
- What are essential problems of present-day bound state QED?
- What is specific with positronium tests of QED?

Bound state QED is Quantum Electrodynamics for the bound states and a problem of the bound states is a complicated one even in the case of classical physics. The bound state QED theory has been mainly developed to describe two-body and three-body atoms. Even in the case of free mass-shell particles, QED as a theory of interactions between leptons (electrons and muons) and photons is indeed incomplete. It faces a lack of pure QED description of the nucleon structure, hadronic vacuum polarization and other hadronic effects. Such effects are unavoidable while calculating the spectrum of hydrogen,
muonium and most of other simple atoms [1]. For some applications the weak interaction is also involved into calculations, but in such a case it can often be done \textit{ab initio}, while strong interaction effects require model-dependent evaluations, experimental data and phenomenological approaches.

QED cannot predict a number to be compared with some experimental value, producing instead some expressions which need appropriate values of fundamental constants. These constants as well as a number of auxiliary nuclear parameters should be determined experimentally. Measurements of the constants and parameters are apparently a problem beyond QED. For most of the so called tests of bound state QED the accuracy is usually limited by factors related to non-QED phenomena and the actual goal of the study is rather to determine the constants and nuclear properties.

A progress of theory is required if one intends to separate nuclear and QED effects in order to determine the nuclear parameters (like e.g. the proton charge radius). It is also needed to improve the accuracy of the fundamental constants obtained this way (like the Rydberg constant, the fine structure constant $\alpha$ etc.).

Presently QED calculations have some essential problems to be solved. Those are related to the bound-state effects of atomic states. Theory involves a number of small parameters such as the fine structure constant $\alpha$, the Coulomb strength $Z\alpha$, the electron-to-nucleus mass ratio $m/M$ etc. Because of that no calculation can be exact and some effective expansion over small parameters is involved. Higher order corrections become more and more complicated and at any time there are some which cannot be calculated because of their complicated analytic structure, huge amount of diagrams, numerous lengthy computations etc. That faces the basic problem of real QED, \textit{how to estimate the corrections we cannot calculate}? The problem exists for both QED of a free particle and the bound state QED. However, in the case of the bound state problem the expansion over $Z\alpha$ and $m/M$ is not analytic and involves large logarithmic factors and big numerical coefficients. The front line of the today calculations is the study of three kinds of corrections:

- the higher-order two-loop corrections of order $\alpha^2(Z\alpha)^6m$ crucial for theory of the hydrogen Lamb shift;
- radiative-recoil corrections of order $\alpha(Z\alpha)^6m^3/M^2$ which are critical contributions to the muonium hyperfine structure (HFS);
- pure recoil corrections of order $(Z\alpha)^7m^3/M^2$ which are to be calculated to improve the accuracy of the muonium HFS.

Since for most of simple atoms any precision test of the bound state of QED involves numerous effects beyond QED, it should be of great interest to develop some test which will be related to a pure QED quantity. Positronium is one of very few atoms which offer such an opportunity. Few features of positronium make it a very useful system to test the bound state QED. Like muonium, it is a pure leptonic atom with no nuclear structure. However, in contrast to muonium, the electron-to-nucleus mass ratio is unity and that allows to study accurately higher order recoil effects performing relatively low accuracy experiments. E.g. a study of the recoil contributions to the positronium HFS in order $\alpha(Z\alpha)^6m^3/M^2$ and $(Z\alpha)^7m^3/M^2$ with the same precision as for muonium ($m/M \sim 1/200$) requires a fractional accuracy two orders of magnitudes below that for
muonium. The relatively low accuracy offers a possibility to perform all calculations and measurements at the level above 1 ppm and to separate QED problems from a problem of the determination of the fine structure constant. The latter is related to the level of accuracy significantly below 1 ppm. Absence of any serious problems with the accurate determination of fundamental constants is a significant advantage of positronium studies.

One further advantage is a great variety of quantities which can in principle be investigated with high accuracy. A number of spectral values can be measured (1S − 2S interval, fine structure at n = 2 and 1S HFS and some others) and they are not sensitive to any new physics beyond the Standard Model. That is a direct result of the lightness of the nuclear mass (positronium mass) which does not allow any high momentum in any virtual effects. However, physics beyond the Standard model can be studied within non-spectral experiments with positronium, particularly in the case of exotic decay modes. A possibility to clearly separate quantities insensitive and possibly sensitive to the new physics is another advantage of the positronium studies.

2 Present status of precision physics of positronium

About ten years ago the experimental accuracy was essentially better than the theoretical one for most of the quantities under study. The last decade delivered a great theoretical improvement. We have collected most of important theoretical references in Table 1 and will briefly describe the recent progress in this section. In the case of the positronium spectrum the critical contributions to the 1S HFS and to the 1S − 2S interval are of the order α^6m and their calculation was completed some years ago. Some results were obtained numerically and later improved by analytic calculations (see e.g. Refs. [2, 10]). However, minor shifts (even equal to few standard deviations of the numerical integration) did not affect the final results because considerably higher uncertainty arose because of another effect related to uncalculated higher-order terms. It was an exception related to the so-called pure recoil spin-dependent α^6m contribution, calculated by several authors with contradicting results. The later analytic calculations [10] confirmed the numerical result from Ref. [11].

| Value            | Ref. to α^6m or α^2Γ₀ | Ref. to α^7m ln^2 α | Ref. to α^7m ln αΓ₀ |
|------------------|------------------------|---------------------|---------------------|
| 1S − 2S fine structure | 2                      | 3                   | unknown             |
| 1S HFS           | 4                      | 5                   | 6                   |
| Γ(p−Ps)          | 7                      | 8                   | 8                   |
| Γ(o−Ps)          | 9                      | 8                   | 8                   |

Table 1: References to recent progress in positronium theory. The contributions to the spectrum are classified by the electron mass, while those to the decay are presented in units of the leading contribution Γ₀.

The uncertainty of any theoretical calculation is determined by a possible value of
unknown higher-order corrections which are expected to have large coefficients. There is a number of corrections enhanced by a big double logarithmic factor \( \ln^2 \alpha \simeq 24 \) \cite{5} and the higher-order terms should be studied to better understand the accuracy of theory. It is also necessary to investigate terms beyond the leading logarithms. The higher-order terms have been known only in part.

In the case of decay theoretical problems are related to corrections of the relative order \( \alpha^2 \) which were calculated only recently. The calculation of the \( \alpha^3 \) contributions is now in progress and they are known in the logarithmic approximation.

| Quantity | Prediction |
|----------|------------|
| \( \Delta \nu(1S - 2S) \) | 1233 607 222.2(6) MHz |
| \( \Delta \nu_{HFS}(1S) \) | 203 391.7(5) MHz |
| \( \Delta \nu(2^3S_1 - 2^1P_0) \) | 18 498.25(9) MHz |
| \( \Delta \nu(2^3S_1 - 2^3P_1) \) | 13 012.41(9) MHz |
| \( \Delta \nu(2^3S_1 - 2^3P_2) \) | 8 625.70(9) MHz |
| \( \Delta \nu(2^3S_1 - 2^1P_1) \) | 11 185.37(9) MHz |
| \( \Gamma(p - Ps) \) | 7 989.32(2) \( \mu s^{-1} \) |
| \( \Gamma(o - Ps) \) | 7 040 07(2) \( \mu s^{-1} \) |

Table 2: Theoretical predictions for positronium.

We collected all theoretical predictions in Table 2. The results were published and presented in different compilations. What we would like to underline here is our estimation of uncertainty. For most of the quantities not only the leading logarithmic corrections (e.g. in the case of spectrum that is \( \alpha^7 m \ln^2 \alpha \)) are known, but also the next-to-leading term (\( \alpha^7 m \ln^2 \alpha \)). However, that cannot reduce the uncertainty because the leading term originates from a single source and its magnitude is characteristic of the correction, while the next-to-leading term used is a result of cancelation between different contributions and can be sometimes small. But that smallness is misleading and the constant following the single logarithm is not small. Our estimation of the uncertainty is based on a value of the double logarithmic term in any case (see Ref. \cite{12} for more detail).

3 Summary of positronium study

Studies of the spectrum and decay rates of positronium provide us with a number of the strong tests of bound state QED, some of which are among the most accurate. Some theoretical predictions from Table 2 can be compared with accurate experimental data, a review of which can be found in Refs. \cite{13, 14}. The most accurately measured spectroscopic data are related to the \( 1S - 2S \) interval (see Fig. 3) and to the ground state HFS (see Fig. 1). There are some minor discrepancies between experimental and theoretical data.

The current experimental situation with the orthopositronium decay (see Fig. 3) is not acceptable at all. The main problem is inconsistency of various experiments. Note that we included a new result from Tokyo \cite{15} and corrected a gas value from Ann Arbor according to the preliminary analysis in Ref. \cite{13}. In contrast, theory and experiment are in a fair agreement for the parapositronium decay (see Fig. 4). References to experimental
results can be found in Refs. [13, 14]. These two papers also review experiments on the fine structure in positronium performed at $2S_1 - 2P$ intervals which were less accurate than experiments at $1S$ HFS and $1s - 2S$ intervals.

4 What we can learn from positronium?

To understand the role of positronium tests of the bound state QED, let us have a look at crucial contributions which can be studied in different tests. They are collected in Table 3.

One sees that a spectroscopic study with positronium allows a few high-precision tests for higher-order recoil effects needed for other atoms. Positronium theory is a good training field for a number of other atomic systems. First of all, let us note that the positronium is in some sense an only ‘true’ two-body atom among all hydrogen-like QED systems. In the case of hydrogen, muonium and others most of the calculations are performed for an electron bound by an external Coulomb field, and only for a few corrections the two-body

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**Figure 1:** $1S$ HFS in positronium

**Figure 2:** $1S - 2S$ interval in positronium

**Figure 3:** Decay of orthopositronium
effects are significant. In contrast, for positronium, the two-body phenomena are essential from the very beginning of any calculation. Due to that positronium studies have been important for understanding higher-order QED effects in helium, a system which can mainly be treated as a system of two electrons in an external field.

| Value                      | Order          |
|----------------------------|----------------|
| hydrogen (gross structure) | $\alpha^8 m$  |
| hydrogen (fine structure)  | $\alpha^8 m$  |
| hydrogen (Lamb shift)      | $\alpha^8 m$  |
| He$^+$ (Lamb shift)        | $\alpha^8 m$  |
| nitrogen (fine structure)  | $\alpha^8 m$  |
| $^3$He$^+$ HFS             | $\alpha^8 m^2/M$, $\alpha^7 m^3/M^2$ |
| muonium HFS                | $\alpha^8 m^2/M$, $\alpha^7 m^3/M^2$ |
| positronium HFS            | $\alpha^7 m$  |
| positronium (gross structure) | $\alpha^7 m$  |
| positronium (fine structure)| $\alpha^7 m$  |
| parapositronium (decay rate)| $\alpha^7 m$  |
| orthopositronium (decay rate)| $\alpha^8 m$  |
| parapositronium (4$\gamma$ branching) | $\alpha^8 m$  |
| orthopositronium (5$\gamma$ branching) | $\alpha^8 m$  |

Table 3: Crucial QED contributions for most important tests of the bound state QED. Tests related to positronium are exact in $m/M$ since $m/M = 1$.

Development of the bound state QED is also fruitful for a better understanding of hadronic systems, like deuteron (proton-neutron system) and mesons (quark-antiquark systems). In both cases a consideration of the $m/M$ value close to unity is of particular interest.

Successful development of theory during the last decade has made it more accurate than the experiment and we hope that some progress from the experimental side will come. That is in particular related to the fine structure at $n = 2$. 
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