Interpreted History Of Neutrino Theory Of Light And Its Future

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De Broglie’s original idea that a photon is composed of a neutrino-antineutrino pair bound by some interaction was severely modified by Jordan. Although Jordan addressed an important problem (photon statistics) that de Broglie had not considered, his modifications may have been detrimental to the development of a composite photon theory. His obsession with obtaining Bose statistics for the composite photon made it easy for Pryce to prove his theory untenable. Pryce also indicated that forming transversely-polarized photons from neutrino-antineutrino pairs was impossible, but others have shown that this is not a problem. Following Pryce, Berezinskii has proven that any composite photon theory (using fermions) is impossible if one accepts five assumptions. Thus, any successful composite theory must show which of Berezinskii’s assumptions is not valid. A method of forming composite particles based on Fermi and Yang’s model will be discussed. Such composite particles have properties similar to conventional photons. However, unlike the situation with conventional photon theory, Lorentz invariance can be satisfied without the need for gauge invariance or introducing non-physical photons.

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I. INTRODUCTION

The theoretical development of a composite photon theory (based on de Broglie’s idea that the photon is composed of a neutrino-antineutrino pair bound by some interaction) has had a stormy history with many negative papers. While de Broglie did not address the problem of statistics for the composite photon, “Jordan considered the essential part of the problem was to construct Bose-Einstein amplitudes from Fermi-Dirac amplitudes.” Furthermore, “he suggested that it is not the interaction between neutrinos and antineutrinos that binds them together into photons, but rather the manner in which they interact with charged particles that leads to the simplified description of light in terms of photons.”

Jordan’s hypothesis traded one problem for another. Although he eliminated the need for theorizing an unknown interaction, his hypothesis that the neutrino and antineutrino are emitted in exactly the same direction seems rather artificial and was criticized by Fock. His strong desire to obtain exact Bose commutation relations for the composite photon led him to work with a scalar or longitudinally polarized photon. Indeed, starting with Jordan’s ansatz,

\[ A_\mu(k) = \int_0^1 f(\lambda)\bar{\psi}((\lambda - 1)k)\gamma_\mu\psi(\lambda k)d\lambda, \]

(1)
Barbour et al. [4] showed that the resulting photon would be longitudinally polarized instead of transversely polarized like real photons. If Jordan had worked with a transversely polarized photon, he could not have obtained Bose commutation relations.

More recently, researchers [5–8] have been satisfied that composite particles formed of fermion pairs (quasi-bosons) such as the deuteron and Cooper pairs are only approximate bosons as they do not quite satisfy Bose commutation relations.

Quasi-bosons obey the commutation relations of the form,

\[
\begin{align*}
\left[ Q(k), Q(l) \right] &= 0, \\
\left[ Q^\dagger(k), Q^\dagger(l) \right] &= 0, \\
\left[ Q(k), Q^\dagger(l) \right] &= \delta(k - l) - \Delta(k, l),
\end{align*}
\]

These are identical to Bose commutation relations except for the additional \(\Delta(k, l)\) term (see Ref. [7]), whose value is very small. Thus, it is easy to envisage that Eq. (2) is a more accurate form of the commutation relations for integral spin particles.

As presented in many quantum mechanics texts it may appear that Bose commutation relations follow from basic principles, but it is really from the classical canonical formalism. This is not a reliable procedure as evidenced by the fact that it gives the completely wrong result for spin 1/2 particles. Furthermore, in extending the classical canonical formalism for the photon, it is necessary to deviate from the canonical rules (see Ref. [1], p. 98).

If Jordan had taken the approach that the composite photon is only an approximate boson, Pryce [10] could never have proven that the theory was impossible. Jordan’s hypothesis that a single neutrino or antineutrino can simulate a photon, which was needed to obtain Bose statistics, is not in agreement with experiment. For otherwise, the interaction cross-section for neutrinos with matter would be orders of magnitude greater than the measured value.

From reading Pryce’s paper one might get the impression that creating transversely-polarized photons from neutrinos is a problem for the theory. The real problem (as Pryce does shows) is in obtaining both Bose statistics and transversely-polarized photons. It is interesting to note in retrospect that Pryce’s four postulates do not include the key assumption: the photon obeys Bose commutation relations, which is essential to the theorem. This is not too surprising since Jordan [4] and those working with him on the theory also took Bose statistics as an absolute. Berezinskii [11] in reaffirming Pryce’s theorem, does include Bose statistics as one of his five assumptions. He argues that certain Bose commutation relations are necessary for the photon to be truly neutral. However, Perkins [8] has shown that a neutral photon in the usual sense can be obtained without Bose commutation relations.

Pryce made the difficulties look much worse than they are by accepting Jordan’s solution to the statistics problem which sealed the impossibility of solving the polarization problem. Case [12] and Berezinskii [11] agree that constructing transversely polarized photons is not the problem. Kronig [13] and Perkins [14,8] have explicitly constructed transversely-polarized photons from neutrinos.

In a remarkable paper [13] Kronig showed a second quantization method for obtaining the photon field from components of the neutrino and antineutrino fields. The photons thus obtained have transverse polarization and obey Maxwell’s equations. (Pryce’s criticisms do
not apply to these calculations.) In order to obtain the usual commutation relations for the electromagnetic field, he introduced a relation between the neutrino spinors \( A \) and \( C \) which is not invariant under rotations of the coordinate system. Although Pryce emphasized this error in Kronig’s paper, the results that Kronig was trying to obtain follow directly from plane-wave spinors as shown by Perkins \[14\].

Although conventional photon theory has been very successful, it has some shortcomings. To satisfy Lorentz invariance, it is necessary to introduce non-physical polarization states. The difficulty in conventional theory of quantizing the electromagnetic field has been noted \[9\]. “It is ironic that of the fields we shall consider it is the most difficult to quantize.” To avoid the problems of non-physical photons, Veltman \[15\] quantized the field with a small photon mass. More serious are the divergent problems which require renormalization. A composite photon might help in solving some of these problems.

Two major problems facing the theory are: (1) Understanding the neutrino-antineutrino interaction that “binds” them into a photon, and (2) showing that the non-Bose photon (which results from the theory) is not in contradiction with any physical laws.

This latter point is the subject of a recent paper \[10\] in which it is shown that the Blackbody radiation spectrum for composite photons is so similar to Planck’s law that existing experiments could not have detected the difference. The commutation relations for the fields do not satisfy space-like commutativity, but this is true of many integral spin particles and just indicates that composite particles have a finite extent \[16\].

According to the Standard Model, the neutrino is described by a two-component theory. To form a photon which satisfies parity and charge conjugation, we need both sets of two-component neutrinos, right-handed and left-handed neutrinos. Two sets of neutrinos have been observed, one that couples with electrons and one that couples with muons. These can be our two sets of two-component neutrinos if the positive muon is identified as the particle and the negative muon as the antiparticle (see Sec. IV of Ref. \[14\]).

II. FORMING COMPOSITE PARTICLE WAVE FUNCTIONS

Fermi and Yang \[17\] have outlined a method of forming a composite pion from a nucleon-antinucleon pair. They imposed some special requirements on the interaction. The attraction should not be a new force field, for this would require the quanta of that new field to be elementary particles themselves. Thus, only forces of zero range are compatible with relativistic invariance and the simplest relativistically invariant interactions between two fields are usual five types \[17\].

We want to use these interactions for the same reasons that Fermi and Yang suggested them and in addition because we know from our previous work \[14,8\] that they result in transversely-polarized photons. Differing from Fermi and Yang, we will use two-component neutrinos instead of nucleons and work at the two-component level. We prefer to work at the two-component level as all terms are meaningful there, unlike the 4-component level where most of the terms are zero.

The Weyl equations for the two types of neutrinos are:

\[
\sigma \cdot \mathbf{p} \phi_{\nu 1}(x) = i \frac{\partial \phi_{\nu 1}(x)}{\partial t},
\]
\[ \sigma \cdot p \phi_{\nu 2}(x) = -i \frac{\partial \phi_{\nu 2}(x)}{\partial t}, \]  
\[ (3) \]

where the \( \sigma \)'s are the Pauli 2x2 spin matrices. These can be put into a symmetric (covariant) form (similar to the 4-component \( \gamma_{\mu} p_{\mu} \Psi(x) = 0 \)) by defining \( \sigma^1_\mu = (\sigma, iI) \) and \( \sigma^2_\mu = (\sigma, -iI) \) with \( I \) being the unit matrix. The equations in (3) become,

\[ \sigma^1_\mu p_\mu \phi_{\nu 1}(x) = 0, \]
\[ \sigma^2_\mu p_\mu \phi_{\nu 2}(x) = 0. \]
\[ (4) \]

The spinors obtained from plane-wave solutions for \( \nu_1 \) and \( \nu_2 \) respectively (positive energy) are:

\[ u(p) = \sqrt{p_4 + ip_3} \left( \begin{array}{c} 1 \\ \frac{p_1 + ip_2}{p_3 - ip_4} \end{array} \right), \]
\[ v(p) = \sqrt{p_4 + ip_3} \left( \begin{array}{c} -\frac{p_1 + ip_2}{p_3 - ip_4} \\ 1 \end{array} \right), \]
\[ (5) \]

where \( p_\mu \) is \((p, ip)\). They have been normalized in the sense that \( \phi^\dagger \phi = 1 \). Note that \( u(p) \) and \( v(p) \) depend only upon the direction of the momentum \( n = p / |p| \), and not its magnitude. For negative momentum \((-n)\), we obtain from (4) the relations,

\[ u(-p) = v(p), \]
\[ v(-p) = u(p). \]
\[ (6) \]

Operating on these solutions with the spin operator shows that \( u(p) \) corresponds to spin parallel to direction of propagation while \( v(p) \) is antiparallel. To obtain the antineutrino wave functions, we take the negative energy solutions and operate on them with CP (see p. 260 of Ref. [18]),

\[ \phi_{CP}(x, t) = C \phi^*(-x, t) = \mp i \sigma^2 \phi^*(-x, t). \]
\[ (7) \]

Surprising, this results in each antineutrino spinor being identical with its neutrino spinor!

The combined wave function for both set of neutrinos is:

\[ \Psi(x) = \frac{1}{\sqrt{V}} \sum_k \left\{ [a_1(k)u(k) + a_2(k)v(k)] e^{ikx} + [c_1^*(k)u(k) + c_2^*(k)v(k)] e^{-ikx} \right\}, \]
\[ (8) \]

where \( kx \) stands for \( k \cdot x + k_4 x_4 = k \cdot x - \omega_k t \). \( a_1 \) and \( c_1 \) are the fermion annihilation operators for \( \nu_1 \) and \( \bar{\nu}_1 \) respectively, while \( a_2 \) and \( c_2 \) are the annihilation operators for \( \nu_2 \) and \( \bar{\nu}_2 \).

We will assume that during an interaction (creation or annihilation) with other particles, the neutrino-antineutrino pair have antiparallel momenta. All possible combinations of such neutrino-antineutrino pairs are indicated in Fig. 1. We take the composite particle field \( G_{int}(R) \) to be the superposition of all the combinations in Fig. 1 with \( O_{int} \) representing the neutrino-antineutrino interaction. The composite-particle coordinate is \( R \) and its momentum and energy are \( P \) and \( \omega_P \). One particle has momentum \( P + k \) while the other has momentum \(-k\) with \( k \) parallel to \( P \).
\[ G_{\text{int}}(R) = \sum_{\mathbf{P}} \frac{1}{2\sqrt{V\omega_p}} \sum_{k} \left\{ F^\dagger(k, \mathbf{n}) \right\} \]

The individual terms of Eq. (9) correspond to the neutrino-antineutrino pairs in Fig. 1. Only the first of the eight terms is shown in (9) for brevity. If we tried to combine massive particles in this manner, it would not work because \( u(\mathbf{P} + \mathbf{k}) \neq u(\mathbf{k}) \) in that case.

\[ [c_1(k, -\mathbf{n})u^\dagger(-\mathbf{n})O\text{int}a^\dagger_1(P + k, \mathbf{n})u(\mathbf{n}) + \ldots]e^{iPR} \]

\[ + F(k, \mathbf{n})[a^\dagger_1(P + k, \mathbf{n})u^\dagger(\mathbf{n})O\text{int}c^\dagger_1(k, -\mathbf{n})u(-\mathbf{n}) + \ldots]e^{-iPR} \].

FIG. 1. All neutrino-antineutrino combinations with antiparallel momenta. The upper arrows indicate the momentum direction with the longer arrows corresponding to \( \mathbf{P} + \mathbf{k} \) and the shorter ones to \(-\mathbf{k}\). The lower, wider arrows indicate spin direction. Each neutrino-antineutrino combination corresponds to an annihilation term and a creation term of Eq. (9).

If we try to identify the field of Eq. (9) with the photon, there is clearly a problem as some terms correspond to longitudinal polarization (spin-0). It has been noted that one could consider each transverse polarization as an independent particle (see Ref. [15], p. 173, 180-2). This is because the polarization vectors are independent degrees of freedom and under a Lorentz transformation change into themselves. (For invariance under parity one needs both transverse polarizations.) Therefore, it seems logical to consider that \( G_{\text{int}}(R) \) is composed of two fields: the photon and another particle with longitudinal polarization.

Using (6), the terms corresponding to spin-1 (transverse polarization) in Eq. (10) can be
put in the form:

\[ A_\mu(R) = \sum_P \frac{1}{\sqrt{2V\omega_\mu}} \left\{ \left[ \gamma_R(P)\epsilon_\mu^1(n) + \gamma_L(P)\epsilon_\mu^2(n) \right] e^{iPR} + \left[ \gamma_R^\dagger(P)\epsilon_\mu^{1*}(n) + \gamma_L^\dagger(P)\epsilon_\mu^{2*}(n) \right] e^{-iPR} \right\}, \]

where

\[ \gamma_R(P) = \sum_k F^\dagger(k,n) \left[ c_1(k, -n)a_1(P + k, n) + c_2(P + k, n)a_2(k, -n) \right], \]

\[ \gamma_L(P) = \sum_k F^\dagger(k,n) \left[ c_2(k, -n)a_2(P + k, n) + c_1(P + k, n)a_1(k, -n) \right]. \]

These polarization vectors satisfy the normalization relation, \( \epsilon_\mu^1(n) \cdot \epsilon_\mu^{1*}(n) = 1 \). The four-vectors, \( \epsilon_\mu^1(n) \) and \( \epsilon_\mu^2(n) \) are orthogonal as \( \epsilon_\mu^1(n) \cdot \epsilon_\mu^{2*}(n) = 0 \). The Lorentz-invariant dot products of the momentum \( P_\mu = |P|(n_1, n_2, n_3, i) \) with the polarization vectors are,

\[ P_\mu \epsilon_\mu^1(n) = 0, \quad P_\mu \epsilon_\mu^2(n) = 0, \]

and in three dimensions,

\[ n \cdot \epsilon^1(n) = n \cdot \epsilon^2(n) = 0, \quad \epsilon^1(n) \times \epsilon^2(n) = -in, \]

\[ n \times \epsilon^1(n) = -i\epsilon^1(n), \quad n \times \epsilon^2(n) = i\epsilon^2(n). \]

If the momentum is along the third axis, the polarization vectors reduce to,

\[ \epsilon_\mu^1(n) = \frac{1}{\sqrt{2}}(1, i, 0, 0), \]

\[ \epsilon_\mu^2(n) = \frac{1}{\sqrt{2}}(1, -i, 0, 0), \]

which are the usual polarization vectors for right and left circular-polarized photons respectively.
III. THE FIELD OF THE COMPOSITE PHOTON

In this section, we will compare the similarities and differences between the composite photon field and the conventional theory.

A. Similarities to Conventional Theory

In comparing the similarities, it should be noted that $\epsilon^1_\mu(n)$ and $\epsilon^2_\mu(n)$ are the same as the usual right and left circular polarization vectors for $n$ along the third axis. However, an advantage of the composite theory is that Eq. (13) gives the polarization vectors for any $n$.

From $A_\mu(R)$, we can readily obtain the Hermitian fields $E$ and $H$. They follow directly from differentiating $A_\mu(R)$, and using (15). Similarly, by using (15), it can be shown that the resulting $E$ and $H$ satisfy Maxwell’s equations.

In computing the commutation relations for the photon fields in conventional theory (see Ref. [9], p. 71-2), one cannot follow the straight canonical path. Instead, it is necessary to modify the momentum space expansion [9] or sum over all three spatial dimensions subtracting the longitudinal component to obtain the transverse sum [19]. For if one does not limit the electromagnetic field to two transverse components, the result is not consistent with Maxwell’s equations [9]. Since the polarization vectors for the composite photon come from combinations of fermion spinors, the only choice is to sum over the polarizations using (13). The result is the same as in the (modified) conventional theory,

$$2 \sum_{j=1}^{2} \epsilon^j_r(k) \epsilon^j_s(k) = \delta_{rs} - \frac{k_r k_s}{k^2}, \quad (17)$$

with $r, s = 1,2,3$. This is the result that Kronig [13] was trying to obtain with his Eq. (17) and (19), which are not rotationally invariant.

B. Differences from Conventional Theory

Unlike conventional theory, $A_\mu(R)$ is uniquely determined by Eq. (10), derived from the composite theory. The natural choice for the Lagrangian is,

$$\mathcal{L} = -\frac{1}{4} \left( \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \right) \left( \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \right) - \frac{1}{2} \left( \frac{\partial A_\mu}{\partial x_\mu} \right)^2, \quad (18)$$

which is relativistically covariant and does not require the additional constraint of gauge invariance. The second term vanishes without imposing any condition, because $(\partial A_\mu/\partial x_\mu) = 0$ follows directly from (14). In fact, $\Phi_A = -i A_4 = 0$ and $\nabla \cdot A = 0$, which is like the radiation gauge. The problem in conventional theory causing one to introduce two non-physical polarization states in order to satisfy Lorentz invariance [15], never arises in the composite theory. The composite photon theory is Lorentz invariant without the need for gauge invariance.

One of the most important differences between the composite theory and conventional theory is that the composite photon creation and annihilation operators do not obey Bose
commutation relations. The composite photon is a quasi-boson and the expectation values of its commutation relations are:

\[
\begin{align*}
[\gamma_R(p')\gamma_R(p)] &= 0, \\
[\gamma_L(p')\gamma_L(p)] &= 0,
\end{align*}
\]

\[
\begin{align*}
[\gamma_R(p')\gamma_R^\dagger(p)] &= \delta(p' - p)(1 - \Delta_{12}(p,p)), \\
[\gamma_L(p')\gamma_L^\dagger(p)] &= \delta(p' - p)(1 - \Delta_{21}(p,p)),
\end{align*}
\]

\[
\begin{align*}
[\gamma_R(p'),\gamma_L(p)] &= 0, \\
[\gamma_R(p'),\gamma_L^\dagger(p)] &= 0,
\end{align*}
\]

(19)

where

\[
\Delta_{12}(p,p) = \sum_k |F(k,n)|^2 \left[ a_1^\dagger(p+k,n)a_1(p+k,n) + c_1^\dagger(k,-n)c_1(k,-n) \\
+ c_2^\dagger(p+k,n)c_2(p+k,n) + a_2^\dagger(k,-n)a_2(k,-n) \right].
\]

(20)

The deviation from Bose statistics is caused by \(\Delta_{12}(p,p)\) and \(\Delta_{21}(p,p)\), which are functions of the neutrino creation and annihilation operators.

We can define linear polarization photon operators by,

\[
\xi(p) = \frac{1}{\sqrt{2}} \left[ \gamma_L(p) + \gamma_R(p) \right],
\]

\[
\eta(p) = \frac{i}{\sqrt{2}} \left[ \gamma_L(p) - \gamma_R(p) \right].
\]

(21)

A particularly interesting commutation relation is,

\[
[\xi(p'),\eta^\dagger(p)] = \frac{i}{2} \delta(p' - p)[\Delta_{21}(p,p) - \Delta_{12}(p,p)],
\]

(22)

which follows from (19) and (21). This commutator is usually small as it is zero for states with equal numbers of right and left handed photons. However it cannot vanish for all states. For if it does, one can readily prove that a composite photon theory is impossible (Pryce’s theorem). The proof [10,11] is as follows. If the commutator of (22) gives zero when applied to any state vector, then all the coefficients of \(N_{a_1}(k) = a_1^\dagger(k)a_1(k), N_{c_1}(k) = c_1^\dagger(k)c_1(k)\), etc. must vanish separately. This means \(F(k,n) = 0\), and the composite photon does not exist. Thus, as Pryce and Berezinkii have proven, the composite photon cannot be a boson.

Because of these non-Bose commutation relations, \(E\) and \(H\) of the composite photon field do not satisfy space-like commutativity [14]. This means that we have a non-local theory, which might be an advantage (see Ref. [1], pp. 3-5) because “it is widely felt that the divergences are symptomatic of a chronic disorder in the small-distance behavior of the [conventional] theory.”

There appears to be an inconsistence in the Standard Model regarding the statistics of the pion and other meson. On the one hand they are considered to be exact bosons, while on the other hand they are considered to be composed of fermions. We would say that the mesons are quasi-bosons.
IV. CONCLUSIONS

The goals of this paper are to explain the historic reasons for the rejection of the neutrino theory of light and to help advance composite photon theories. Our achievement is very small in comparison to what is required for development of a satisfactory theory. The crucial problem of a neutrino-antineutrino interaction and its experiment ramifications has not been addressed. Nevertheless we can draw some conclusions from this work.

1. Combining neutrino fields following a method similar to that of Fermi and Yang [17], resulted in transversely-polarized particles which have properties very similar to conventional photons.

2. Pryce’s theorem (stating that it is impossible to form composite photons from neutrinos) contains an assumption which is unsupported and probably incorrect. Present experimental evidence cannot differentiate between a quasi-boson photon and a boson photon.

3. Unlike the situation with conventional photon theory, Lorentz invariance can be satisfied by the composite photon theory without the need for gauge invariance or introducing non-physical photons.

4. While in conventional theory the polarization vectors of integral spin particles are formed in a somewhat ad hoc manner, the polarization vectors in the composite theory are combinations of spinors resulting from the underlying fermion structure.

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