Quintessence From The Decay of a Superheavy Dark Matter

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To Memory of
Changoul and her
Love

Abstract

We investigate the possibility of replacing the cosmological constant with gradual condensation of a scalar field produced during the decay of a superheavy dark matter. The advantage of this class of models to the ordinary quintessence is that the evolution of the dark energy and the dark energy are correlated and cosmological coincidence problem is solved. This model does not need a special form for the quintessence potential and even a simple $\phi^4$ theory or an axion like scalar is enough to explain the existence of the Dark Energy. We show that the model has an intrinsic feedback between energy density of the dark matter and the scalar field such that for a large volume of the parameter space the equation of state of the scalar field from very early in the history of the Universe is very close to a cosmological constant. Other aspects of this model are consistent with recent CMB and LSS observations.

1 Introduction

Quintessence models are alternatives to a Cosmological Constant i.e. a non-zero vacuum energy density. They are not however flawless. Even in models with tracking solutions the potential of the scalar field must somehow be fine-tuned to explain its smallness and its slow variation until today. In addition, many of them can not address the coincidence problem i.e. why the density of Dark Matter (DM) and Dark Energy (DE) evolve in such a way that they become comparable just after galaxy formation.

Recently a number of authors have proposed interaction between dark matter and quintessence field to explain the coincidence. L.P. Chimento et al. [1] based on an earlier work by L.P. Chimento et al. [2] and W. Zimdahl et al. [3] suggest an asymptotic scaling law between density of DE and DM. In their model due to a dissipative interaction between dark matter and quintessence scalar field $\phi_q$, $\rho_{dm}/\rho_q$ $\rightarrow$ cte, where $\rho_{dm}$ and $\rho_q$ are respectively DM and scalar field density. Assuming this “strong coincidence” [1], they find the class of potentials $V_q(\phi_q)$ such that the equation of state have a solution with scaling behavior. Then, using constraints from nucleosynthesis, they find that this category of models have $w_q \gtrsim -0.7$. This value is marginally compatible with WMAP data and far from publicly available SN-Ia data which prefers $w_q \sim -1$. In another version of the same model, W. Zimdahl et al. [3] consider a non-static scaling solution, $\rho_{dm}/\rho_q \propto (a_0/a)^{\eta}$. The model with $\eta = 1$ solves the coincidence paradigm but the standard $\Lambda$CDM fits the SN-Ia data better and their best fit has $w_q \sim -0.7$.

L. Amendola et al. [4] have extensively studied the interaction of quintessence field and dark matter in models with tracking solutions and $w_q > -1$. They show that these models are equivalent to a Brans-Dicke Lagrangian with power law potential and look like a “Fifth Force”. Modification of the CMB anisotropy spectra by such interactions is observable and put stringent constraints on their parameters.

D. Comelli et al. [5] study a model in which the effect of interaction between quintessence scalar and dark matter appears as time dependence of DM particles mass. This explains the extreme adjustment
of dark matter and dark energy densities during cosmological evolution. The coupling between two fields increases the parameter space for both and reduces by orders of magnitudes the amount of fine tuning. In this respect, as we will see below, their model is similar to what we propose in this work. However, there are a number of issues that these authors have not addressed. Cosmological observations put strict limits on the variation of fundamental parameters including the DM mass. In their model the largest amount of variation happens around and after matter domination epoch. The mass variation must leave an imprint on the CMB and large structure formation which was not observed.

In addition to the lack of explanation for coincidence in many quintessence models, it is difficult to find a scalar field with necessary characteristics in the frame of known particle physics models without some fine tuning of the potential [6]. In general, it is assumed that quintessence field is axion with high-order, thus non-renormalizable, interactions with the Standard Model particles (or its supersymmetric extension) which is highly suppressed at low energies. However, D. Chung et al. [7] show that any supergravity induced interaction between $\phi_q$ and other scalars with VEV of the order of Plank mass can increase the very tiny mass of the $\phi_q$ ($m_q \sim 10^{-33} eV$) expected in many models, unless a discrete global symmetry prevents their contribution to the mass.

In a very recent work, G.R. Farrar & P.J. Peebles [8] study models with a Yukawa interaction between DM and quintessence scalar field. Like D. Comelli et al. model, this interaction affects the mass of the dark matter particles. The general behavior of these models is close to ΛCDM with some differences which can distinguish them. One of the special cases with a 2-component CDM imitates the ΛCDM very closely. Many aspects of this model is similar to the model studied in the present work but without considering the source of the intimate relation between DM and DE in contrast (we believe) to the present work. Moreover, the necessity of having a very special self-interaction potential for the quintessence field is not removed.

What we propose here is a model for dark energy somehow different from previous quintessence models (A preliminary investigation of this model has been presented in [9]). We assume that DE is the result of the condensation of a scalar field produced during very slow decay of a massive particle. In most of quintessence models the scalar field is produced during inflation or reheating period in large amount such that to control its contribution to the total energy of the Universe, its potential must be a negative exponential (in most cases sum of two exponentials) or a negative power function [6]. We show that in the present model very small production rate of the scalar field replaces the fine tuning of the potential and practically any scalar field even without a self-interaction has a tracking solution for a large part of its parameter space.

The main motivation for this class of models is the possibility of a top-down solution [10] [11] [12] for the mystery of Ultra High Energy Cosmic Rays (UHECRs) [13] [14] [15]. If a very small part of the decay remnants which make the primaries of UHECRs is composed of a scalar field $\phi_q$, its condensation can have all the characteristics of a quintessence field. We show that in this model the most natural equation of state for the quintessence scalar is very close to a cosmological constant, at least until the age of the Universe is much smaller than the lifetime of the Superheavy Dark Matter (SDM, WIMPZILLA) which is the origin of the quintessence field.

Another motivation is the fact that a dark energy with $w_q \lesssim -1$ fits the SN-IIa data better than a cosmological constant [16] [17] [18]. Although the sensitivity of CMB data to the equation of state of the dark energy is much less than SNs, with 95% confidence WMAP data gives the range $-1 \pm 0.22$ for the $w_q$ [19] [20]. Estimation from galaxy clusters evolution is also in agreement with this range [21]. On the other hand, it has been demonstrated that the cosmological equation of state for a decaying dark matter in presence of a cosmological constant is similar to a quintessence with $w_q \lesssim -1$ [18]. Both observations therefore seem to encourage a top-down solution which explains simultaneously the dark energy and the UHECRs.

Like other models with interaction between DM and DE, the coincidence in this model is solved
without fine-tuning. Parameters can be changed by many orders of magnitude without destroying the
general behavior of the equation of state or the extreme relation between the energy density of dark
energy and the total energy density in the early Universe.

In Sec.2 we solve the evolution equations for dark matter and dark energy. For two asymptotic regime
we find analytical solutions for the evolution of $\phi_q$. In Sec.4 we present the results of numerical
solution of the evolution equations including the baryonic matter and we show that both approaches
lead essentially to the same conclusion. We study also the extent of the parameter space. The effect
of DM anisotropy on the energy density of the dark energy is studied in Sec.4. We show that the
perturbation of dark energy in this model is very small and very far from the resolution of present or
near future observations. The late time decoherence of the scalar field is discussed in Sec.2.1. We give
a qualitative estimation of the necessary conditions and leave a proper investigation of this issue as
well as the possible candidates for $\phi_q$ to future works.

2 Cosmological Evolution of a Decaying Dark Matter and a Quintessence
Field

Consider that at very early epoch in the history of the Universe, just after inflation, the cosmological
“soup” consists of 2 species: a superheavy dark matter (SDM) - $X$ particles - decoupled from the rest
of the “soup” since very early time and a second component which we don’t consider in detail. The
only constraint we need is that it must consist of light species including baryons, neutrinos, photons,
and light dark matter (by light we mean with respect to $X$). For simplicity we assume that $X$ is a
scalar field $\phi_x$. Considering $\phi_x$ to be a spinor or vector does not change the general conclusio ns of
this work. We also assume that $\phi_x$ is quasi-stable i.e. its lifetime is much longer than the present age
of the Universe. A very small part of its decay remnants is considered to be a scalar field $\phi_q$ with
negligibly weak interaction with other fields.

The effective Lagrangian can be written as:

$$\mathcal{L} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi_x \partial_\nu \phi_x + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi_q \partial_\nu \phi_q - V(\phi_x, \phi_q, J) \right] + \mathcal{L}_J \quad (1)$$

The field $J$ presents collectively other fields. The term $V(\phi_x, \phi_q, J)$ includes all interactions including
self-interaction potential for $\phi_x$ and $\phi_q$:

$$V(\phi_x, \phi_q, J) = V_q(\phi_q) + V_x(\phi_x) + g\phi_x^m \phi_q^n + W(\phi_x, \phi_q, J) \quad (2)$$

The term $g\phi_x^m \phi_q^n$ is important because it is responsible for annihilation of $X$ and back reaction of
quintessence field by reproducing them. $W(\phi_x, \phi_q, J)$ presents interactions which contribute to the
decay of $X$ to light fields and to $\phi_q$ (in addition to what is shown explicitly in (2)). The very long
lifetime of $X$ constrains this term and $g$. They must be strongly suppressed. For $n = 2$ and $m = 2$ the
g term contributes to the mass of $\phi_x$ and $\phi_q$. Because of the huge mass of $\phi_x$ (which must come from
another coupling) and its very small occupation number $\langle \phi_x^2 \rangle \sim 2\rho_x/m_x^2$, for sufficiently small $g$
the effect of this term on the mass of the SDM is very small. We discuss the rôle of this term in detail
later. If the interaction of other fields with $\phi_q$ is only through the exchange of $X$ (for instance due to
a conserved symmetry shared by both $X$ and $\phi_q$), the huge mass of $X$ suppresses the interaction and
therefore the modification of their mass. This solves the problem of “Fifth Force” in the dark [4] and
the SM sectors.

In a homogeneous universe the evolution equations for $\phi_q$ and $\phi_x$ are:

$$\ddot{\phi}_q + 3H\dot{\phi}_q + \frac{\partial V}{\partial \phi_q} = 0 \quad (3)$$
$$\ddot{\phi}_x + 3H\dot{\phi}_x + \frac{\partial V}{\partial \phi_x} = 0 \quad (4)$$
where dot means the comoving time derivative. In the rest of this work we treat $\phi_x$ and $J$ as classical particles and deal only with their density and equation of state. We assume that $X$ particles are non-relativistic (i.e. part of the CDM) with negligible self-interaction i.e.

$$V_x(\phi_x) = \frac{1}{2} m_x^2 \phi_x^2$$

Under this assumptions $\phi_x$ can be replaced by:

$$\phi_x \sim \left( \frac{2 \rho_x}{m_x^2} \right)^{\frac{1}{2}}$$

If $X$ is a spinor, the lowest order (Yukawa) interaction term in (1) is $g\phi_q \bar{\psi} \psi$. In the classical treatment of $X$:

$$\bar{\psi} \psi \sim \frac{\rho_x}{m_x}$$

The same argument about the negligible effect of the interaction on the mass of DM and SM particles is applied. For simplicity we consider only the scalar case.

For potential $V_q(\phi_q)$ we consider a simple $\phi^4$ model:

$$V_q(\phi_q) = \frac{1}{2} m_q^2 \phi_q^2 + \frac{\lambda}{4} \phi_q^4$$

Conservation of energy-momentum, Einstein and dynamic equations, give following system of equations for the fields:

$$\dot{\phi}_q[\ddot{\phi}_q + 3H \dot{\phi}_q + m_q^2 \phi_q + \lambda \phi_q^3] = -2g\phi_q \dot{\phi}_q \left( \frac{2 \rho_x}{m_x^2} \right) + \Gamma_q \rho_x$$

$$\rho_x + 3H \rho_x = - (\Gamma_q + \Gamma_J) \rho_x - \pi^4 g^2 \left( \frac{\rho_x^2}{m_x^3} - \frac{\rho_q^2}{m_q^3} \right)$$

$$\rho_J + 3H (\rho_J + P_J) = \Gamma_J \rho_x$$

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8 \pi G}{3} \left( \rho_x + \rho_J + \rho_q \right)$$

$$\rho_q = \frac{1}{2} m_q^2 \phi_q^2 + \frac{1}{2} m_q^2 \phi_q^2 + \frac{\lambda}{4} \phi_q^4$$

where (10) is the Boltzmann equation for $X$ particles. We calculate its right hand side in the appendix. $\rho_q'$ is the density of quintessence particles (not the classical field $\phi_q$) with an average energy larger than $m_x$ in the local inertial frame. Only interaction between these particles contribute to the reproduction of SDM. $\Gamma_q$ and $\Gamma_J$ are respectively the decay width of $X$ to $\phi_q$ and to other species. In (9) we have replaces $\phi_x$ with its classical approximation from (6). The effect of decay Lagrangian $W(\phi_x, \phi_q, J)$ appears as $(\Gamma_q + \Gamma_J) \rho_x$ which is the decay rate of $X$ particles (see equation (43) in the Appendix).

At very high temperatures when $\rho_x \gg \pi^4 g^2 m_x^3 \Gamma$, the annihilation and reproduction terms in (10) are dominant. $X$ particles however are non-relativistic up to temperatures close to their rest mass. Quintessence scalar particles at this time are relativistic and therefore their density falls faster than SDM density by a factor of $a(t)$. The probability of annihilation also decreases very rapidly. Consequently, from very early time only the decay term in (10) is important. The dominance of annihilation/reproduction can happen only if the production temperature of $X$ particles i.e. preheating/reheating temperature is very high. Such scenarios however can make dangerous amount of gravitinos [22]. For this reason, presumably the reheating temperature must be much smaller than $m_x$ and annihilation dominance never happens. This can not put the production of SDM in danger because it has been shown [23] that even with a very low reheating temperature they can be produced. It seems therefore reasonable to study the evolution of the fields only when the annihilation/reproduction is negligible. Another reason for this simplification is that we are interested in the decohered modes of
and therefore \( \phi \). When the self-annihilation of \( X \) particles is the dominant source of \( \phi \) most of particles are highly relativistic and their self interaction doesn’t have time to make long-wavelength modes. This claim needs however a detail investigation of the process of decoherence which we leave for another work.

The system of equations \( \text{(11)-(13)} \) is highly non-linear and an analytical solution can not be found easily. There are however two asymptotic regimes which permit an approximate analytical treatment. The first one happens at very early time just after the production of \( X \) (presumably after preheating \( \text{(11, 12)} \) and the decoherence of \( \phi \)'s long wavelength modes. In this epoch \( \phi \sim 0 \) and can be neglected. The other regime is when comoving time variation of \( \phi \) is very slow and one can neglect \( \dot{\phi} \). We show that the first regime leads to a saturation (tracking) solution where \( \phi \rightarrow \text{cte} \). It then can be treated as the initial condition for the second regime when \( \phi \) changes slowly.

The effect of last term in right hand side of \( \text{(10)} \) as we argued is negligible. The solution of \( \text{(10)} \) is then straightforward:

\[
\rho_x(t) = \rho_x(t_0)e^{-\Gamma(t-t_0)} \left( \frac{a(t_0)}{a(t)} \right)^3
\]  

(14)

where \( \Gamma \equiv \Gamma_q + \Gamma_J \) is the total decay width of \( X \). We consider \( t_0 \) to be the time after production and decoupling of \( X \). These two times can be very different, but with an extremely long lifetime for \( X \) and its weak interaction with other species, it is not important which one of them is selected as \( t_0 \).

After inserting the solution \( \text{(11)} \) and neglecting all the terms proportional to \( \phi \), equation \( \text{(9)} \) simplifies to:

\[
\dot{\phi} \frac{d}{dt}(a^3 \dot{\phi}) = \Gamma_q a^3(t_0)\rho_x(t_0)e^{-\Gamma(t-t_0)}
\]  

(15)

and can be solved:

\[
\frac{1}{2} \dot{\phi}^2(t) \equiv K_q(t) = \left( \frac{a(t_0)}{a(t)} \right)^6 \left[ K_q(t_0) + \Gamma_q \rho_x(t_0) \int_{t_0}^{t} dt \frac{a^3(t)}{a(t_0)}e^{-\Gamma(t-t_0)} \right]
\]  

(16)

For \( a \propto t^k \) the integral term in \( \text{(16)} \) decreases with time (i.e. \( \ddot{\phi} < 0 \)). This means that after a relatively short time \( \phi \) is saturated and its density does not change, in other words it behaves like a cosmological constant. The numerical simulation in the next section confirms this result. If \( \phi \) was a classical field the natural choice for the initial value of the kinetic energy \( K_q(t_0) \) was \( K_q(t_0) = 0 \) assuming a very rapid production of \( X \). However, in reality \( \phi \) is a quantum field and it gets time to decohere and to settle as a classical field. The initial value of \( K_q(t_0) \) can therefore be non-zero. Its exact value can only be determined by investigating the process of decoherence. In any case with the expansion of the Universe, its effect on \( \dot{\phi} \) decreases very rapidly because of \( a^{-6}(t) \) factor in \( \text{(16)} \).

Next we consider the regime where \( \phi \) changes very slowly and we can neglect \( \ddot{\phi} \) and higher orders of \( \dot{\phi} \). Equation \( \text{(15)} \) gets the following simplified form:

\[
\dot{\phi}(m_q^2 \phi_x + \lambda \phi_x^3) = -2g\dot{\phi}\phi_x \left( \frac{2\rho_x}{m_x^2} \right) + \Gamma_q \rho_x
\]  

(17)

We expect that self-interaction of \( \phi \) be much stronger than its coupling to \( X \). Neglecting the first term in the right hand side of \( \text{(17)} \), its \( \phi \)-dependent part can be integrated:

\[
\frac{d}{dt} \left( \frac{1}{2} m_q^2 \phi_x^2 + \frac{\lambda}{4} \phi_x^4 \right) = \frac{dV}{dt}(\phi_x) = \Gamma_q \rho_x
\]  

(18)

which then is easily solved:

\[
V_q(\phi_x) = V_q(\phi_x(t'_0)) + \Gamma_q \rho_x(t'_0) \int_{t'_0}^{t} dt \left( \frac{a(t'_0)}{a(t)} \right)^3 e^{-\Gamma(t-t'_0)}
\]  

(19)

Here \( V_q \) is the potential energy of \( \phi_q \). From \( \text{(18)} \) and \( \text{(19)} \) it is clear that the final value of the potential and therefore \( \phi \) energy density is driven by the decay term and not the self-interaction. Therefore the only vital condition for this model is the existence of a long life SDM and not the potential of \( \phi \).
In (19) the initial values \( t'_0 \) and \( \phi_q(t'_0) \) are different from equation (16). They correspond to the time and to the value of \( \phi_q \) in the first regime when it approaches to saturation. Similar to (16), the time dependence of \( \phi_q \) in (19) vanishes exponentially and the behavior of \( \phi_q \) approaches to a cosmological constant.

To estimate the asymptotic value of \( \phi_q \) we assume that \( a(t) \propto t^k \). Using (19) with the additional assumption that \( t_s - t'_0 \ll 1/\Gamma \), \( (t_s \) is the saturation time), we find:

\[
V(\phi_q) - V(\phi_q(t'_0)) \sim \frac{\Gamma q \rho_x(t'_0)}{(3k - 1)} \left( 1 - \left( \frac{t'_0}{t} \right)^{(3k-1)} \right).
\] (20)

If we define the saturation time as the time when \( V(\phi_q) - V(\phi_q(t'_0)) \) has 90% of its final value, for \( t_s \ll t_{eq} \) with \( t_{eq} \) the matter-radiation equilibrium time, \( k = 1/2 \) and:

\[
t_s \sim 100 t'_0
\] (21)

For \( t_s \gg t_{eq} \), \( k = 2/3 \) and:

\[
t_s \sim 10 t'_0
\] (22)

The interesting conclusion one can make from (20) is that the initial density of SDM, its production time, and its decay rate to \( \phi_q \) which are apparently independent quantities determine together the final value of the dark energy density. The long lifetime of SDM is expected to be due to a symmetry which is broken only by non-renormalizable high order weak coupling operators. They become important only at very large energy scales. These conditions are exactly what is needed to have a small dark energy density according to (20). In Sec. 3 we see that numerical calculation confirm these results.

We can also observe here the main difference between this model and other quintessence models. If \( \phi_q \) is produced during e.g. the decay of inflaton or from the decay of a short live particle in the early Universe, its final density should be much larger than observed dark energy unless either its production width was fine-tuned to unnaturally small values or its self-interaction was exponentially suppressed with some fine tuning of its rate.

### 2.1 Decoherence

Decoherence of scalar fields has been mainly studied in the context of phase transition \[24\] in a thermal system. Examples are phase transition in condensate matter \[21\] \[25\], and before, during and after inflation in the early Universe \[26\] \[27\]. In the latter case the aim is studying the inflation itself, production of defects and the reheating. Decoherence is the result of self-interaction as well as interaction between a field (regarded as order parameter after decoherence) and other fields in the environment. Long wavelength modes behave like a classical field i.e. don’t show “particle-like” behavior if quantum correlation between modes are negligible. More technically this happens when the density matrix for these modes is approximately diagonal. It has been shown \[26\] that interaction with higher modes is enough to decohere long wavelength modes (see Calzetta et al. \[27\] for a review). The classical order parameter corresponds to these modes after their decoherence. One can consider a cut-off in the mode space which separate the system (i.e. long wavelength modes) from environment (short wavelengths). The cutoff can be considered as an evolving scale which determines at each cosmological epoch the decoherent/coherent modes \[27\].

It has been shown \[25\] that the decoherence time in a thermal phase transition is shorter than the spinodal time i.e. the time after beginning of the phase transition when the scalar field or more precisely \( <\phi^2> \) settles at the minimum of the potential. The decoherence time in presence of external fields (with couplings of the same order as self-interaction) is

\[
t_d \sim \frac{1}{m}
\] (23)
By replacing Minkovski time with conformal time and considering a time dependent cut-off [25] one can show that modes with:

\[ \frac{k^2}{a^2} + m^2 \lesssim H^2 \]  

(24)
decohere and behave like a classical scalar field. The effect of coupling constant is logarithmic and less important.

If the SDM exists, it is produced during preheating [12] just after the end of the inflation presumably at \( T \sim 10^{14}eV - 10^{16}eV \) which correspond to:

\[ H \sim 10^{-6}eV - 10^{-4}eV \]  

(25)

From [24] this time range permits scalars with mass \( m \lesssim 10^{-6}eV \) to decohere. When the size of the Universe get larger, \( \phi_q \) stops decohering. This also helps having a very small dark energy density. If the preheating/reheating had happened when the Hubble constant was smaller, then \( m_q \) also must be smaller to have long wavelength modes which can decohere. We will see in the next section that in this case the main term in \( V(\phi_q) \) potential is the self-interaction. Moreover, \( \lambda \) can be larger which helps a faster decoherence of long wavelength modes.

The argument given here is evidently very qualitative and needs much deeper investigation. In the present work we take the possibility of decoherence as granted and study the evolution of \( \phi_q \) as a classical scalar field.

### 3 Numerical Solution

To have a better understanding of the behavior and the parameter space of this model, we have solved equations (9) to (13) numerically. We have also added the interaction between various species of the Standard Model particles to the simulation to be closer to real cosmological evolution and to obtain the equation of state of the remnants. This is specially important for constraining the lifetime of SDM [28]. Without considering the interaction between high energy remnants and the rest of the SM particles specially the CMB, the lifetime of SDM must be orders of magnitude larger than present age of the Universe.

Details of interaction simulation are discussed in [28] and we don’t repeat them here. The Boltzmann equation for SM species (equation (1) in [28]) replaces (11). Because of numerical limitations we switch on interactions only from \( z = 10^9 \) downward. For the same reason we had to begin the simulation of \( X \) decay from \( z \sim 10^{14} \) which is equivalent to a temperature of \( T = 10^{31}eV \). The expected reheating temperature is model dependent and varies from \( \sim 10^{22}eV \) to \( \sim 10^7eV \). For the time being no observational constraint on this large range is available. The change in the initial temperature however does not modify the results of the simulation significantly if \( f_q = \frac{\Gamma_q}{\Gamma} \) is rescaled inversely proportional to redshift and to the total decay width \( \Gamma \), and proportional to \( m_x \). In other words two models lead to very similar results for the quintessence field if:

\[ \frac{f_q}{f_q'} = \frac{z'\Gamma'm_x}{z\Gamma'm'_x} \]  

(26)

For the lifetime of \( X \) we use the results from [28] and [18] which show that a lifetime \( \tau = 5\tau_0 - 50\tau_0 \) (\( \tau_0 \) the present age of the Universe) can explain the observed flux of UHECRs as well as cosmic equation of state with \( w_q \lesssim -1 \). In the following we consider \( \tau = 5\tau_0 \). Our test shows that increasing \( \tau \) to 50\( \tau_0 \) does not significantly modifies the extent of the admissible parameter space or other main characteristics of the dark energy model. We consider only the models with \( m = 2, n = 2 \) and \( g = 10^{-15} \) in [2]. The results for \( 10^{-20} \leq g \leq 10^{-5} \) are roughly the same as what we present in this section and therefore they are not shown. The discussion in Sec[2] as well as Fig[2] show that the
Figure 1: Evolution of quintessence field (left), its derivative (center) and its total energy density (right) for $\Gamma_0 \equiv \Gamma_q/\Gamma = 10^{-16}$ (magenta) (see text for details), 5$\Gamma_0$ (cyan), 10$\Gamma_0$ (blue), 50$\Gamma_0$ (green), 100$\Gamma_0$ (red). Dash line is the observed value of the dark energy. $m_q = 10^{-6}\text{eV}$, $\lambda = 10^{-20}$.

contribution of the interaction with the SDM in the total energy density of $\phi_q$ is much smaller than other terms.

Fig.2 shows the evolution of $\phi_q$, its time derivative and its total energy density from the end of $X$ production to saturation redshift $z_s$. Here we have used as $z_s$ the redshift after which up to simulation precision the total energy density of $\phi_q$ does not change anymore. The result of the simulation is quite consistent with the approximate solutions discussed in Sec. 2. The final density energy of $\phi_q$ is practically proportional to $\Gamma_q/\Gamma$. The latter quantity encompasses 3 important parameters of the model: The fraction of energy of the remnants which changes to $\phi_q$, the fraction of energy in the long wavelength modes which can decohere and the coupling of these modes to the environment which contributes to $\phi_q$ yield and to the effective formation redshift of the classical quintessence field $\phi_q$. Therefore the effective volume of the parameter space presented by this simulation is much larger and the fine-tuning of parameters are much less than what is expected from just one parameter.

Figure 2: Evolution of the contribution to the total energy density of $\phi_q$ for $\Gamma_0 \equiv \Gamma_q/\Gamma = 10^{-16}$ and : Left, $m_q = 10^{-8}\text{eV}$ and $\lambda = 10^{-20}$; Center, $m_q = 10^{-6}\text{eV}$ and $\lambda = 10^{-20}$; Right, $m_q = 10^{-6}\text{eV}$ and $\lambda = 10^{-10}$. Curves are: mass (red), self-interaction (green), kinetic energy (cyan) and interaction with SDM (blue).

Fig.2 shows the evolution in the contribution of different terms of the Lagrangian (1) to the total energy of $\phi_q$. Very soon after beginning of production of quintessence field the potential takes over the kinetic energy and the latter begins to decrease. The relative contribution of each term and their time of dominance as this figure demonstrates, depends on the parameters specially $m_q$ and $\lambda$. Another conclusion from this plot is that changing these parameters by orders of magnitude does not change the general behavior of the model significantly and for a large part of the parameter space the final density of quintessence energy is close to the observed value. This can also be seen in Fig.3 and Fig.4 where the evolution of quintessence energy is shown for various combination of parameters.
Figure 3: Left: Evolution of total density with redshift for $\Gamma_0 \equiv \Gamma_q/\Gamma = 10^{-16}$ (magenta) (see text for details), $5\Gamma_0$ (cyan), $10\Gamma_0$ (blue), $50\Gamma_0$ (green), $100\Gamma_0$ (red). Dash line is the observed value of the dark energy. $m_q = 10^{-6}eV$, $\lambda = 10^{-20}$. Right: Relative density of dark energy and CDM as a function of $\Gamma_q/\Gamma$. The x-axis is normalized to $\Gamma_0 \equiv \Gamma_q/\Gamma = 10^{-16}$.

Figure 4: Quintessence energy density for: Left, $m_q = 10^{-3}eV$ (cyan), $m_q = 10^{-5}eV$ (magenta), $m_q = 10^{-6}eV$ (red) and $m_q = 10^{-8}eV$ (green), $\lambda = 10^{-20}$; Right, $\lambda = 10^{-10}$ (cyan), $\lambda = 10^{-15}$, $\lambda = 10^{-20}$ and $\lambda = 10^{-25}$ (green), $m_q = 10^{-6}eV$. The difference between quintessence density for the last 3 values of $\lambda$ is smaller than the resolution of the plot. Dash line is the observed energy density of the dark energy.

4 Perturbations

Large and medium scale observations show that the dark energy is quite smooth and uncorrelated from the clumpy dark matter [29]. If DE origin is the decay of the dark matter, the question arises whether it clumps around dark matter halos or has a large scale perturbation which is not observed in the present data. In this section we investigate the evolution of spatial perturbations in $\phi_q$ and show that they decrease with time. Another interest in doing such exercise is to investigate any imprint of the model on the power spectrum of matter and the CMB anisotropy.

We use the synchronous gauge metric:

$$ds^2 = dt^2 - a^2(t)(\delta_{ij} - h_{ij})dx^i dx^j$$

(27)

For small spatial fluctuations $\phi_q(x,t) = \bar{\phi}_q(t) + \delta \phi_q(x,t)$ where from now on barred quantities are the homogeneous component of the field depending only on $t$. We define the same decomposition for other fields.

We consider only scalar metric fluctuations $h \equiv \delta^{ij}h_{ij}$ and neglect vector and tensor components. The
Einstein equation gives following equation for the evolution of $h$:

$$\frac{1}{2} \dot{h} + \frac{\dot{a}}{a} h = 4\pi G(4\phi \delta \dot{\phi} - 2\delta V(\phi, \rho_x) + \delta \rho_x + \delta P_x + 3\delta P_J)$$  \hspace{1cm} (28)$$

where $\delta \rho_x$ is the fluctuation of $X$ particles density, $\delta P_J$ and $\delta P_J$ are respectively the collective density and pressure fluctuation of other fields. From the Lagrangian (1), the dynamic equation of $\phi_q$ is:

$$\partial_\mu (\sqrt{-g} g^{\mu \nu} \partial_\nu \phi_q) + \sqrt{-g} V'(\phi_q, \phi_x, J) = 0$$  \hspace{1cm} (29)$$

This equation and the energy momentum conservation determine the evolution of $\delta \phi_q(x, t)$:

$$\dot{\phi}_q \left[ \delta \ddot{\phi}_q + \partial_i \partial^i (\delta \phi_q) + V''(\bar{\phi}_q) \delta \phi_q + 2g \left( \frac{2\ddot{\rho}_x}{m^2} \right) \delta \phi_q + \frac{3\dot{a}}{a} \dot{\phi}_q \right] + \frac{2g \bar{\phi}_q \ddot{\rho}_x}{m^2} \left[ \delta \phi_q + \frac{\rho_q}{\bar{\rho}_q} \right] = \frac{\dot{h}}{a} \dot{\phi}_q - \frac{\dot{\phi}_q}{\rho_q} \bar{\rho}_q \dot{\phi}_q$$  \hspace{1cm} (30)$$

Like homogeneous case, we assume that SDM behaves like a pressure-less fluid:

$$T^{00}_x = \bar{\rho}_x + \delta \rho_x \quad T^{0i}_x = \bar{\rho}_x \delta u^i_x \quad T^{ij}_x = O(\delta^2) \approx 0$$  \hspace{1cm} (32)$$

where $\delta u^i_x$ is the velocity of SDM fluctuations with respect to homogeneous Hubble flow. Interaction terms are explicitly included in the energy-momentum conservation equation:

$$\partial_0 \left( \frac{\delta \rho_x}{\bar{\rho}_x} \right) + \partial_i (\delta u^i_x) - \frac{\dot{h}}{2} = -\pi^2 g^2 \left( \frac{3\delta \rho_x}{m^3} - \frac{2\dot{\rho}_q \delta \rho_x}{m^3 \bar{\rho}_x} - \frac{\rho_q^2 \delta \rho_x}{m^3 \bar{\rho}_x^2} \right)$$  \hspace{1cm} (33)$$

The effect of interactions in the right hand side of (33) is however very small, first because $X$ particles mass is very large and then because only high energy $\phi_q$ particles contribute to this term and their energy decreases with expansion of the Universe much faster than the SDM. The evolution of matter fluctuations is then practically the same as the standard $\Lambda$CDM case.

Using the conservation relation for other components of the energy-momentum in the limit when $\dot{\phi}_q \rightarrow 0$, we find the following relation between spatial fluctuation of $\delta \phi_q$ and $\delta u^i_x$:

$$-V'(\bar{\phi}_q, \bar{\rho}_x) \partial^i (\delta \phi_q) = \Gamma_q \bar{\rho}_x \delta u^i_x$$  \hspace{1cm} (34)$$

Equation (34) has a meaningful limit when $\dot{\phi}_q \rightarrow 0$ only if $\delta \dot{\phi}_q \rightarrow 0$. On the other hand, (34) shows that the divergence of quintessence field fluctuations $\partial^i \delta \phi_q$ follows the velocity dispersion of the dark matter with opposite direction. Their amplitude however is largely reduced due to the very small decay width $\Gamma_q$. In addition, with the expansion of the Universe, $V'(\bar{\phi}_q, \bar{\rho}_x)$ varies only very slightly - just the interaction between SDM and $\phi_q$ will change. In contrast, $\rho_x$ decreases by a factor of $a^{-3}(t)$ and even gradual increase of the dark matter clumping and therefore the velocity dispersion $\delta u^i_x$ [29] can not eliminate the effect of decreasing density. We conclude that the spatial variation of $\phi_q$ is very small from the beginning and is practically unobservable.

5 Closing Remarks

Since the original works on the production of superheavy particles after inflation [12], a number of investigations [28] have demonstrated that even with a reheating temperature as low as few MeV the production of superheavy particles is possible. We don’t discuss here the particle physics candidates for $\phi_q$, but for the sake of completeness we just mention that axion like particles are needed or at least can exist in large number of particle physics models (see [31] for some examples). The fact
that \( \phi_q \) does not need to have very special potential is one of the advantages of this model with respect to others and opens the way to a larger number of particle physics models as candidate for the quintessence field.

One of the arguments which is usually raised in the literature against a decaying dark matter is the observational constraints on the high energy gamma-ray and neutrino background. In \[28\] it has been shown that if \( m_x \gtrsim 10^{22} \text{eV} \) and its lifetime \( \tau \gtrsim 5\tau_0 \), and if simulations correctly take into account the energy dissipation of the high energy remnants, present observational limits are larger than expected flux from a decaying UHDM. Consequently the model is consistent with the available data.

The same fact is applied to the CMB and its anisotropy. The expected CMB distortion is of order \( 10^{-8} \), much smaller than sensitivity of present and near future measurements. As for the expected anisotropy in the arrival direction of UHECRs, the data is yet too scarce to give any conclusive answer. In \[30\] it has been shown that if the coupling constant is such that the high energy component of quintessence field is yet relativistic. As we have discussed in Sec.2, the production of this component from annihilation has been stopped very early in the history of the Universe and the contribution from decay of \( \phi_q \) is much smaller than the limits on the amount of Hot Dark Matter (as it has been shown in \[28\] for hot SM remnants). The small coupling of \( \phi_q \) with SM particles also suppresses the probability of its direct detection. However, the detection of an axion-like particle e.g. the QCD axion can be a positive sign for the possibility of existence of \( \phi_q \)-like particles in the Nature.

**Appendix**

Here we calculate the right hand side of the Boltzmann equation at lowest order of \( g \) coupling constant for annihilation and reproduction of \( X \) particles.

The Boltzmann equation for \( X \) particles is the following:

\[
p^{\mu} \partial_{\mu} f^{(X)}(x,p) - \Gamma_{\nu}^{\mu} p^{\nu} p^{\rho} \frac{\partial f^{(X)}}{\partial p^{\rho}} \equiv L[f] = -(A(x,p)+B(x,p))f^{(X)}(x,p)+C(x,p) \tag{35}
\]

\[
A(x,p) = \Gamma p^{\mu} u_\mu, \tag{36}
\]

\[
B(x,p) = \frac{1}{(2\pi)^3} \sum_i \int d\vec{p}_x f^{(i)}(x,p_x) A \sigma_{xi} \quad i = X, \phi_q \tag{37}
\]

\[
C(x,p) = \frac{1}{(2\pi)^3} \int d\vec{p}_q d\vec{p}_{\phi_q} f^{(q)}(x,p_q) f^{(\phi_q)}(x,p_{\phi_q}) \frac{A d\sigma_{\phi_q + \phi_q \rightarrow X + X}}{dp} \tag{38}
\]

\[
A = \sqrt{(p_1.p_2)^2 - m_1^2 m_2^2} \tag{39}
\]

The function \( f^{(X)}(x,p) \) is the distribution of \( X \) particles. The terms \( A, B \) and \( C \) are respectively the decay, the annihilation (self or in interaction with other species) and production rates. We assume that the interaction of \( X \) with other fields except \( \phi_q \) is negligible. According to Lagrangian \[1\] with \( n = 2 \) and \( m = 2 \) the lowest Feynman diagrams contributing to annihilation and production are:
The $S$ matrix for these diagrams is very simple:

$$ S = \frac{-i(2\pi)^4 g \delta(4)(\sum_i p_i)}{\prod_i 2 p_i^0} \quad (40) $$

and the differential cross-section:

$$ d\sigma = \frac{(2\pi)^{10} g^2 \delta(4)(\sum_i p_i)}{16 \sqrt{(p_1.p_2)^2 - m_1^2 m_2^2}} dp_3 dp_4 \quad d\vec{p}_i \equiv \frac{d^3 p_i}{(2\pi)^3 g_i p_i^0} \quad (41) $$

where $g_i$ is the number of internal degrees of freedom. Here we assume that $g_x = g_q = 1$. Using the relation:

$$ \left[ \int p^\mu p^{\mu_1} \ldots p^{\mu_n} f(x,p) dp \right]_{\mu} = \int p^{\mu_1} \ldots p^{\mu_n} L[f](x,p) dp $\quad (42) $$

and the definition of energy-momentum tensor $T^{\mu\nu}$ and number density of particles $n^\mu$, one obtains:

$$ T^{\mu\nu} = -\Gamma T^{\mu\nu} u_\nu - \pi^4 g^2 \left( n^\mu x \sum_i \int d\vec{p}_2 f^{(i)}(x,p_2) - \int d\vec{p}_1 d\vec{p}_2 p_1^\mu f^{(q)}(x,p_1) f^{(q)}(x,p_2) \theta(p_1^0 + p_2^0 - 2m_x) \right) \quad (43) $$

Both $f^{(X)}$ and $f^{(q)}$ have a large peak around the energies close to the mass of $X$. Therefore:

$$ \int d\vec{p}_i f^{(i)}(x,p) \approx \frac{n_i^{\nu} u_\nu}{m_i} \quad i = X, q \quad (44) $$

In the case of $\phi_q$ the density $n_q$ is only the density of particles with an average energy larger than $m_x$. Finally from (43) one can obtain the evolution equation of $\rho_x$ in a homogeneous cosmology i.e. equation [10] in Sec 2.

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