Building Long-Term Portfolio Benchmarks for Pension Funds in Emerging Economies

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The movement from a defined benefit to a defined contribution pension system has important implications in the area of portfolio allocation. While the focus of defined benefit pension funds is essentially in the long term, some defined contribution funds might have incentives to invest with shorter-term horizons. The case of open pension funds, such as the ones in several countries in Latin America and Central and Eastern European countries, shows that competition on short-term returns may bring pension funds into suboptimal portfolio strategies. As policy makers become increasingly interested in finding long-term sources of financing for infrastructure and other long-term projects, it becomes essential to upgrade the regulatory framework of open pension funds. This paper contributes to the literature by proposing an investment regulatory framework based on strategic asset allocation that can be applicable to open pension funds. Based on the use of lifecycle investment strategies, the paper proposes the implementation of common portfolio benchmarks for pension funds. Three elements are emphasized for implementation of strategic asset allocation: (a) a well-defined pension objective, (b) sound governance of the portfolio benchmark, and (c) a methodology for developing the benchmark. The paper proposes the use of the approximation methodology as a starting point for designing portfolio benchmarks, and illustrates step-by-step how to build these long-term portfolios in a didactical way.
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I. Introduction

The purpose of this paper is to serve as a guide for policy makers who are interested in upgrading the investment regulatory framework in defined contribution (DC) open pension fund systems. The paper emphasizes the need for implementing an investment regulatory framework based on the concept of Strategic Asset Allocation (SAA). While the main tool for implementing SAA is through portfolio benchmarks, this document emphasizes the importance of having a clear pension objective, an adequate governance structure for the construction of the benchmark, and an adequate methodology for constructing and maintaining the benchmark.

The need to move to investment regulations based on SAA arises because market based regulations lead inevitably to inefficient portfolios. Rudolph and de la Torre (2016) argue that due to agency problems, the competitive pension system is unable to align the interests of the pension fund management companies (PFMCs) with the long-term objectives of the contributors to the pension fund system. They propose the need to use pension fund portfolio benchmarks as the most effective way of solving the agency problem. From a different perspective, Rudolph and others (2010) have also proposed focusing the investment regulatory framework of open pension funds on the provision of common portfolio benchmarks for the pension fund industry.

Section II starts by explaining the concept of strategic asset allocation and how lifecycle investment strategies are the optimal way of addressing long-term pension objectives. The asset portfolio of lifecycle strategies start from portfolios highly invested in equities and gradually move to more long-term fixed income assets as individuals get closer to retirement age. The section also explains that lifecycle strategies are not as simple as a combination of lifestyle pension funds and examines how far the regulatory framework in some countries is from having in place a lifecycle strategy. Finally, it proposes a methodology for managing pension risk in a competitive pension system that could be operationalized in the context of an open pension system.

Section III analyzes the three elements that need to be taken into consideration at the time of implementing a lifecycle strategy in an open pension system, guided through a portfolio benchmark: (a) the pension objective; (b) governance of the portfolio benchmark; and (c) the methodology for developing the benchmark. Section IV proposes the use of the approximation method as a starting point for building portfolio benchmarks, and illustrates step by step the method for building them. The last section concludes.

II. Strategic asset allocation and benchmark portfolios

Modern portfolio theory for long-term portfolios is based on the concept of Strategic Asset Allocation (SAA). Investors decide, ex-ante, on the future asset allocation, based on long-term risk

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2 See Campbell and Viceira (2002).
3 All codes for programing in Matlab are available upon request.
and asset return expectations. Such an allocation is called strategic as it is intended to be long-term oriented and should be only adjusted when some major events occur. This approach is in contrast with Tactical Asset Allocation (TAA) that permits a range of percentages in each asset class and allows the PFMC to take advantage of market conditions to adjust the holdings within the predefined percentages. According to Brinston, Hood and Beebower (1986), and Blake, Cairns and Dowd (2008), more than 90 percent of the returns of a long-term portfolio are explained by the SAA and only a small part by the TAA, including asset selection and market timing. These results suggest the need to focus the investment discussion of the pension funds on the SAA. To the extent that the investment regulatory framework of the pension fund system creates incentives for TAA or short-term optimization, open pension systems will be unable to align the incentives of the PFMCs with the long-term objectives of their contributors. Pension funds will not be able to offer adequate pensions compared to their optimal level.

The economic literature has documented the inefficiencies of the existing regulatory framework in most countries with mandatory funded schemes with open pension schemes, from various angles. The study of Castaneda and Rudolph (2010) analyzes the asset allocation resulting from interactions among investors in a competitive market in the presence of relative return guarantees, and concludes that the equilibrium is typically short-term oriented portfolios. Rudolph and de la Torre (2016) analyze the principal-agent, collective action, and collective cognition problems existing in the regulatory framework of open pension funds, and conclude that competition among open pension funds is insufficient to solve the agency problem, and consequently pension fund portfolios may converge to suboptimal strategies.

It is essential to include the main concepts of SAA in the investment regulatory framework of open pension funds. SAA can be incorporated into investment regulations through the use of benchmark portfolios that are built with the objective of optimizing the pensions of individuals at retirement age. As demonstrated by Campbell and Viceira (2002), lifecycle investment strategies are those that are the most efficient in optimizing the value of pension funds at retirement age. Stewart (2014) provides examples of countries in the implementation of strategic asset allocation.

In a simple framework, the concept underpinning lifecycle strategies is that individuals have two types of wealth: human capital and financial wealth. The framework assumes that individuals have preferences for maintaining a constant ratio between fixed income and equity in the total value of their wealth. For an individual who is expected to have a stable job and regular wage increases according to productivity, the return on the human wealth has a similar performance compared with a fixed income security. Since at the beginning of their working life individuals have little financial wealth and a large amount of human wealth, their financial wealth needs to be heavily invested in equities in order to achieve the desired ratio between fixed income to equity. As individuals get older, human wealth decreases and financial wealth increases, and consequently individuals need to reduce the equity share in the portfolio of their financial wealth. At retirement age, when human wealth is closer to zero, the financial portfolio should reflect the fixed income to equity ratio of the total wealth.4

4 See Viceira (2010) for more details on this subject.
The use of portfolio benchmarks in the investment regulatory framework of pension funds is the most effective way of introducing SAA into the asset portfolios of pension funds. In order to achieve their objective, the benchmark portfolios need to be common to the pension fund industry, and designed exogenously. Thus, the pension portfolios managed by individual PFMCs compete against the benchmark portfolio.

Since the benchmark portfolio is built with the objective of optimizing the pensions of individuals at retirement age, the portfolios managed by the pension funds are also aligned with the long-term objective of the contributors. This methodology has also the advantage of facilitating the performance measurement of pension funds and their managers.

The benchmark portfolio should be built from a core set of well-diversified financial market indices. These market indices should include equity, bonds, and eventually alternative assets. Based on a set of market indices, it is possible to build investment strategies of different asset classes that achieve a certain pension objective, for example, a replacement rate. In more sophisticated markets, the instruments can reflect the performance of standardized portfolios such as ETFs for these products. In the absence of these indices in the domestic market, the exposure to domestic portfolios would need to be minimized. To the extent that the country has the expectation of using the pension system to develop the domestic capital market, the pension funds, the emphasis of the domestic fixed income regulation should be on the duration of the fixed income debt. The government should make available bonds in the expected maturities, promoting participation of other parties, including local and international investors.

The use of benchmark portfolios helps to separate market risks from excess returns. All investment returns can be seen as the combination of the market return (Beta) and excess returns (Alpha). Thus, the volatility of the benchmark portfolio reflects market volatility; and volatility of the Alpha—with respect to Beta—reflects the value added by the PFMC.

**Comparing lifecycle and lifestyle approaches**

The design of lifecycle portfolios is more than a chain of lifestyle portfolios. The duration of the fixed income portfolio is one element that distinguishes the typical lifestyle funds from the lifecycle portfolios at each moment in time. While lifecycle funds typically require very long durations in their fixed income portfolios, lifestyle portfolios offered by pension funds are characterized by lower duration of the fixed income portfolio.

A simple analysis of the pension fund system in Chile reveals that: (a) the portfolios offered by the multifund scheme are not balanced funds, and (b) the chain of lifestyle funds creates portfolios that are suboptimal compared with lifecycle portfolios that are designed to optimize the pensions at

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5 Lifestyle funds, also called balanced funds, are funds that maintain their asset allocation constant independent from the cycle. For example, these funds maintain a fixed proportion of equity and fixed income along the life of the fund. Lifestyle funds have to rebalance their asset allocation constantly over time. Lifecycle pension funds, instead rebalance automatically toward a target asset mix, which varies over time. Thus, lifecycle funds move automatically from equity intensive portfolios to fixed income portfolios along the life of individuals.
retirement age. The absence of SAA in each of the funds in the multifund scheme, and the short-term orientation of the investment decisions resulting from market interactions are insufficient to bring the multifund scheme portfolios to their optimal level.

Chilean pension funds do not necessarily rebalance their portfolios periodically in order to maintain the level of risk constant. As shown in Figure 1, between 2007 and 2014 Chilean pension funds were gradually increasing their exposure to international assets. Some of the fluctuations are explained by the volatility of international equity prices, but there is a clear trend towards more international exposure. Since the risk of the portfolio is changing over time, these pension funds can hardly be considered balanced funds.

Figure 1. Chile: Investments of pension funds in international securities
(as a percentage of total assets)

In addition, the increase in international fixed income in the pension portfolios in the post-financial crisis period cannot be explained by SAA. While the increase in international exposure might be considered necessary in the presence of a pension fund system that has outgrown the size of the domestic capital market, the level of risk of the international portfolio has not been constant over time. As shown in Figure 2, the exposure to international fixed income, including predominantly high yield instruments, has increased from less than 1 percent to 15 percent in the period after the crisis, in the three funds with greater equity exposure (Funds A, B and C). Currently the weight of international fixed income in the portfolio is greater than that of domestic equity.
In addition, the relative proportion of equity to emerging markets and developed markets has been fluctuating over time. As shown in Figure 3, the exposure to emerging market equity increased from 20 to 56 percent from 2003 to 2007, but then declined to 31 percent in 2014. This opportunistic behavior suggests the lack of SAA in the international portfolios. In fact, investment decisions seem to be guided by strong tactical allocations to international securities, including investments in actively managed mutual funds.

Figure 3. Chile: International equity allocation of pension funds:
Developed versus emerging markets
(as a percentage of international equity)

The opportunistic asset allocation has not been effective in offering competitive market returns. Despite their relatively complicated investment strategy, the returns of the pension funds may not
have been as good as those driven by a set of rather simple portfolio benchmarks.\(^6\) A comparison of the performance of the international portfolio of the pension funds against simple portfolio benchmarks may help to explain this point.

The investment strategies for international equity investments followed by the Chilean pension funds between 2007 and 2014 have not been able to add any significant value to the alternative of investing in simple international equity indices. We benchmarked the portfolios of the pension funds against an equity index of a developed economy (S&P500) and an equity index of emerging markets (MSCI Emerging Markets). We tested the existence of *Alpha* (returns in excess of market returns) and, not surprisingly, we found that in most of the cases they were negative, as shown in Table 1. Using monthly data, we tested the *Alpha* against portfolios that maintain the weights constant for the whole period between developed markets and emerging markets equity indices (50/50; 60/40; and 70/30), and also against a portfolio that uses the same weight as the pension funds at each moment in time. With the exception for the case of 50/50 in portfolio A and another one where the indicator is zero, the rest of the *Alphas* were negative.\(^7\)

| Table 1. Chile: Alphas of pension fund investments in international equity (against S&P500 and MSCI Emerging Markets indices) |
|---|
| **Fund A** | **Fund B** | **Fund C** | **Fund D** | **Fund E** |
| 50% S&P500 / 50% MSCI EM | 0.4% | -0.6% | -1.8% | -3.7% | 2.8% |
| Actual Weights\(^1\) | S&P500 / MSCI EM | -0.6% | -1.7% | -3.0% | -5.3% | 1.8% |
| 60% S&P500 / 40% MSCI EM | 0.0% | -1.0% | -2.2% | -4.1% | 2.3% |
| 70% S&P 500 / 30% MSCI EM | -0.4% | -1.4% | -2.5% | -4.5% | 1.8% |

Source: The authors’ calculations based on information from the Superintendencia de Pensiones and Bloomberg.

The inefficiency of the international investment strategies followed by the pension funds can be explained in several ways. First, the strategies are unable to outperform simple well-diversified portfolio indices. Second, the pension funds have been relying on inefficient TAA in conducting their investment strategies, which has resulted in higher fees paid to asset managers. Third, PFMCs have managed the pension portfolios with a different level of risk compared with those of well-diversified portfolios.\(^8\)

These results confirm the importance of adopting SAA as a way of guiding the investments of pension funds. Adopting SAA implies establishing long-term portfolio benchmarks with a clear strategic vision about optimizing future pensions of individuals. The portfolio benchmark is expected to guide the relative allocation of the different asset classes along the lifecycle of individuals. While

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\(^6\) As of September 2014, international equity portfolio of the pension funds is channeled through investments in 96 mutual funds, 41 indices, 33 domestic funds invested abroad, two ADRs, one share of a foreign company and one foreign investment fund.

\(^7\) Fund E is excluded because international investments are insignificant.

\(^8\) The tracking error of the pension portfolios against these international equity indices tends to be high, which means that the risk level is significantly different from those of these well-diversified portfolios. For example, the equity exposure to Latin America is higher than the exposure in the MSCI Emerging Market Index, which is riskier from the perspective of the future pensions of individuals.
there is some room for limited active portfolio management, especially in less developed markets, it is essential to preserve the risk level of well-diversified portfolios.

Measuring performance in the presence of a long-term portfolio benchmark

The creation of a long-term portfolio benchmark allows a separation between market returns (Beta) and excess returns (Alpha). Consequently, a regulatory framework based on a long-term portfolio benchmark might be able to guide the investments of pension funds with the objective of optimizing the value of the pensions at retirement age. In other words, the use of long-term portfolio benchmarks may incentive pension fund managers of DC funds to act like defined benefit (DB) managers. While this concept allows a closer monitoring of the performance of PFMCs, it shifts most of the attention to the market risks, which are captured by the portfolio benchmark. The separation between market returns and excess returns allows the pension funds to compete on excess return, while aligned with the desired market risk.

Once the portfolio benchmark has been selected, it is expected that in an open pension system, the different pension funds may offer portfolios that are similar in risk to the portfolios included in the benchmark. Pension funds are allowed to take riskier positions than the benchmark portfolio, while within the tracking error.

The tracking error, which measures the standard deviation of the difference between returns of the portfolio and the returns of the benchmark, is a useful tool for measuring performance of pension funds against a benchmark. The application of the benchmark implies imposing an upper limit on the level of tracking error in each different asset class. It is expected that portfolio managers adjust their risk exposure when their tracking error levels are approaching the upper limit; these adjustments include rebalancing their portfolios and cutting bets. The tracking error limit can be different depending on the different asset classes.

An alternative tool that can be used is the information ratio, which measures not only the ability of the portfolio manager to generate excess returns relative to a benchmark, but also attempts to identify the consistency of these investments. The information ratio is measured as the ratio of portfolio returns above the returns of a benchmark to the volatility of those returns (measured as the tracking error).

The challenges of grouping individuals

It is important to highlight that, from the optimality perspective, each person would need his or her own benchmark. Everybody would need a personal lifecycle strategy, because there are multiple variables that are involved in the design of a portfolio. For example,

Different earnings profiles. The earnings profile is typically very different between blue-collar and white-collar workers.
Different earnings risks. While the wage growth of civil servants can be represented by a fixed income instrument, the wage growth of a hedge fund manager would resemble more an equity instrument.

Different life expectancy. People with high incomes tend to have higher life expectancy than people with low incomes. Women also have longer life expectancy than men.

Different family composition. The underlying portfolio of an annuity is different depending on the age profile of the dependents. For example, two males of the same age with one with spouse of the same age, and the other with a spouse 10 years younger, would require a different portfolio composition (in the presence of survivor pensions) if they aim to buy joint annuities to provide benefits for their spouse. The second one would require a longer duration of the fixed income portfolio at retirement age to reflect the fact that the annuity provider will consider the longer life expectancy of the spouse.

While gender is a variable known ex-ante, many of the other variables are not known until later in a person’s working life. It would be ideal to have data on the years of education of individuals, as it is an important variable explaining future income. While gender, education and other socioeconomic variables may help to explain the different income profiles of individuals, policy makers would need to understand the trade-off between simplicity and comparability with the number of benchmark portfolios that would need to be made available to individuals.

Grouping individuals would require strong assumptions and consequently the resulting pensions might not be optimal for each particular person, but overall the portfolio optimization would be better compared with the alternative of pension portfolios invested in short term assets.

As described in Rudolph and de la Torre (2016), it is also important to keep in mind that these portfolios should be considered as default options, and that individuals should be allowed to invest in different alternatives.

### III. Designing benchmark portfolio in open pension systems

Designing a benchmark portfolio scheme in an open pension system is not as simple as putting in place different asset allocations for the pension funds. It is essential to have an objective, a tool, and a framework that ensures that the benchmark is resilient to market fluctuations.

Three elements need to be taken into consideration at the time of designing a benchmark portfolio scheme in a defined contribution pension system. These elements, which will be used to guide the asset allocation of pension funds, are:

a. A clear objective for the pension fund system;

b. A governance structure for building the benchmark portfolio;

c. An appropriate methodology for building the benchmark portfolio.
3.1 Objectives

The discussion about the pension objective is important, to the extent that, in the absence of a clear objective, the accumulation of assets might be insufficient, or too risky, or with excessive annuitization risk.\(^9\)

There are basically three measures in defining the objectives of a defined contribution system. Each of them has different implications in terms of asset allocation of the pension funds:

- a. A pension wealth at retirement age;
- b. A lifetime annuity to be payable after retirement age; and
- c. A replacement rate that would be payable after retirement age.

From the perspective of asset management, the objective of accumulating a certain pension wealth requires targeting the duration of the fixed income portfolio with the retirement age of the individual. While there is room for equity investments along the lifecycle, the duration of the fixed income portfolio is converted into money market instruments as the age of the individual approaches retirement age. This is due to the fact that, at retirement age, the pension fund should not be subject to any significant interest rate risk.

Targeting pension wealth, or a certain amount of money at retirement age, is equivalent to designing a system that will pay lump sum benefits. While Rocha and others (2012) argue against the creation of pension systems with these features, many funded pension schemes currently operate in this manner. The main problem with lump sum benefits is that individuals may outlive their savings in retirement, and suffer drastic falls in consumption once they spend their retirement money and can no longer work to earn additional income.

Targeting a lifelong annuity at retirement is a better objective compared with a lump sum amount, in most cases. Lifetime annuities ensure that retirees will not outlive their savings, but they eliminate the possibility of leaving bequests.

From the perspective of asset management, targeting a fixed annuity at retirement age requires the asset manager to try to immunize the value of the annuity before retirement age. Thus, the duration of the fixed income portfolio should be much longer than in the case of lump sum amounts. In fact, at the time of retirement the duration of the fixed income portfolio should be similar to the duration of the underlying portfolio of the annuity provider. In addition, portfolio selection needs to take into consideration the price of the deferred annuity, which is typically associated with interest rate evolution.

\(^9\) Annuitization risk is the risk that at the time of ‘locking in’ a fixed annuity payment contract, the level of initial financial assets used to purchase the annuity at that particular time has suffered a decline or loss in value (due to market, credit or other risks) and thus the ensuing annuity payment will be lower than if the asset value had remained stable.
The concept of moving to portfolios at the point of reaching retirement age that are similar to the underlying portfolio also applies in the case of variable annuities, however there are no annuitization risks in this case, because the amount of the pension depends on the value of the market index that has been chosen.

Targeting a replacement rate involves greater complexity in a defined contribution scheme, as the portfolio of the pension fund should not only target the duration of the annuity, but also should take into consideration the wage evolution of contributors. This exercise requires investing in instruments that are correlated with the wage profile of the contributors. In the absence of such financial instruments, investment should be directed to instruments that can replicate wage evolution. The behavior of wages can be quite complex, especially in transition economies, where wages are guided by the convergence to Western European countries.

In some cases, tracking wages to achieve a replacement rate objective may imply directing a larger part of investments into equity, compared with cases where the pension objective is reaching a certain annuity at retirement age. As highlighted later in this paper, it is essential to have a good assessment of the income profile of the participants.

3.2 The governance aspects of the benchmark

Most countries have not set up long-term benchmark portfolios, based on the expectation that the market will bring portfolios into their long-term equilibrium. Creative solutions have been tried, including the use of relative performance measures, such as the Minimum Relative Return, or the use of the Value-at-Risk. As shown by Castaneda and Rudolph (2010) and Berstein, Tapia and Villanova (2013), these mechanisms are not efficient at optimizing long-term returns.

Some pension plans in countries, such as Lithuania and the United States (in relation to 401[k] plans), have avoided the decision to define common portfolio benchmarks, by letting each asset manager define its own benchmark. Unfortunately, this approach defeats the purpose of guiding uninformed contributors to make decisions about the alternatives being offered. While pension funds may claim that they are beating their benchmarks, individuals are unable to compare the portfolio benchmarks among asset managers.

In order to ensure fair competition among different pension funds, it is essential to measure the performance of pension funds against common benchmarks. Pension benchmarks could be built for different age groups, in five or ten year bands. For example, a pension fund for the generation retiring in 2030, another for 2035, and so on. The target date funds, as they are known in the United States, have provided funds for age groups in ten-year bands. More recently, some companies have started offering funds that reduce the band to five years.

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10 In many cases wages decrease in periods closer to retirement age.
11 The 401(k) plans offer some safe harbor rules that help to guide portfolios toward life cycle schemes, while it is still an imperfect substitute of the use of common benchmarks.
While benchmarking with explicit portfolios based on market indices has not been implemented in open pension systems, it is a common practice in the asset management industry. The asset management industry typically benchmarks the performance and bonuses of asset managers, against market indices. In the case of large defined contribution pension funds, such as Norwegian pension funds and NEST in the United Kingdom, pension portfolios are benchmarked against market indices defined exogenously.

As noted, regulation to date has not concerned itself with the establishment of benchmarks, based on its view of the power of the market. Investment regulations in many countries have implicit the naïve perception that the market is going to be able to design optimal portfolios for pension funds. The works of Lusardi and Mitchell (2013), Madrian (2013), on behavioral economics and lack of capacity of individuals to make informed decisions; Berstein and Cabrita (2007) on the inelasticity of the demand for pension fund management services; and Basak and Makarov (2009) and Castaneda and Rudolph (2010) on interactions among portfolio managers, have helped to clarify that market interactions provide insufficient incentives for bringing investment portfolios into allocations that optimize the future pensions of contributors.

As the performance of funded pension schemes is finally going to be measured in terms of their capacity to provide adequate pensions at retirement age, it is essential from a public policy perspective to ensure that investment strategies are aligned with the long-term objective of the pension system. The use of long-term portfolio benchmarks aligns with this objective.

The creation of a benchmark portfolio needs to be aligned with the responsibility of providing adequate pensions in the future. While the system will continue operating as defined contributions, the benchmark portfolio provides a clearer guidance of future pension expectations. If explicit benchmarks are not used, funded pension systems may end up offering unsatisfactory pensions in the future due to poor asset allocation. The most affected segment of the population will be those who are unable to make informed portfolio decisions, which is a large segment of the population.

The credibility and good reputation of the creators of the portfolio benchmark are essential factors to ensure sustainability of the benchmark portfolio. While designing the optimal benchmark portfolio is an important challenge, maintaining the strategic asset allocation through periods of market volatility will be even more challenging. In the absence of an adequate level of financial literacy in the population, the creators of the portfolio benchmarks will be perceived as accountable for the volatility of the portfolio of the pension funds. The creation of the benchmark portfolio should be accepted as the result of the informed demand of the country, and consequently the participation of the government and the supervisory agency should be limited.

The credibility of the benchmark must be sustained on a solid governance structure that combines technical expertise and social awareness. Designing the optimal pension portfolio will require a proper governance structure to safeguard the long-term investment decisions in periods of financial volatility. First of all, it is necessary to form a group of financial and quantitative analysts with

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12 From past experiences, there will be calls for shifts to fixed income instruments right after the equity prices have bottomed, consolidating the losses for the contributors.
experience in designing long-term portfolios. While this expertise is scarce, there is a growing body of knowledge being built in this area, and training can be acquired. In addition, it will be necessary to design a governance structure led by a committee, which in this paper is referred to as the Commission of Wise Persons (CWP). The role of the CWP is to validate the structure of the strategic asset allocation for the benchmark portfolio.

The suggested mechanics of the creation of the CWP is as follows:

a. The government or the supervisory authority puts in place a technical group specialized in modeling long-term portfolios (financial economists and quantitative analysts). Under a clear mandate of achieving a certain pension objective (i.e. replacement rate), this group will prepare different asset allocations that are consistent with the optimization of the pension objective at retirement age.

b. The Minister of Finance (or other competent authority) will call for the creation of the high profile CWP to derive the benchmark portfolio. The role of the CWP is to recommend to the authorities the benchmark portfolio that will be used to evaluate the performance of pension funds of individuals who have not made an explicit selection of pension portfolios at the time of joining the pension fund system (those choosing the default option).

c. The CWP will be constituted by a group of independent individuals who represent the views of the financially informed demand of the country. This point is essential for explaining the role of the Commission: the benchmark portfolio is intended to represent the demand for pension fund management services and not something that is imposed by the government. This is an important distinction in settling the potential conflicts of interests.

d. The CWP will consist of a group of between five and seven individuals who are recognized by the public as persons who understand how the financial market works and have been successful in their professional careers. This could include an ex-chairperson of the Central Bank, a former chairperson of a Commercial or Investment Bank, a recognized Professor of Economics or Finance at a local or international university, a recognized Investment Bank Strategist, etc. The participants in the CWP should not receive monetary compensation for their services, but their role should be perceived as a service to the country.

e. The CWP will work with the technical group in the preparation of the benchmark portfolios. The technical group should be prepared to present alternatives and answer all the questions that come from the CWP members. The CWP may request additional work by the technical group.

f. At the end of their study and deliberation period, the CWP should present to the Minister of Finance (or relevant authority) a report with detailed recommendations about the

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13 Individuals should be allowed to join the plans that track the benchmark portfolio at any time during their working lives.

14 Governments may face conflicts of interest at the time of setting portfolio benchmarks. For example, in the case of Colombia, the funded pension system used to have a synthetic portfolio for guiding the investments of the pension funds in fixed income instruments. Unfortunately, the objective of such a portfolio was to ensure proper financing of the government deficits rather than ensuring an optimal pension for individuals.

15 Remuneration to the members could be misinterpreted by the public.
portfolio benchmark. The recommendations of the CWP should be publicly disclosed, so individuals and other stakeholders (specialized press) may evaluate and comment on the recommended benchmark portfolios.

g. The Minister of Finance (or other relevant authority) may decide to accept, comment or reject the recommendations of the CWP. All the public interactions between the Minister and the CWP should be disclosed publicly. If the recommendations are accepted, three things have been achieved:

i. Contributors—through the mass media—have been informed about the process, and they have benefited from some financial education. This process of financial education may be very significant for the population, and the authorities should take advantage of this possibility.

ii. The CWP has helped the government to quarantine the benchmark portfolio against future political forces that may agitate for changes in asset allocation in periods of market volatility. The benchmark portfolio is not perceived as a decision of the authority, but a consequence of a well-considered process that involved representatives of the informed demand of the country.

iii. It is very likely that a large percentage of the contributors may decide to stay with the default option, which implies that ex-ante they have vicariously been rewarded with the best alternatives for optimizing the expected value of the pensions at retirement age.\(^{16}\)

h. If the Minister of Finance rejects the recommendations of the CWP and for example decides to adopt a different portfolio benchmark, the government will be accountable for the asset allocation, and will have to defend the proposed asset allocations in periods of market volatility.

The nomination of the members of the Commission as well as all the public events with the participation of the CWP and the Minister of Finance will be used as opportunities to educate contributors about all the investment options available, beyond the default portfolio.

The interactions between the government and the CWP should aim to clarify that individuals have the option of deciding the portfolios by themselves, but in the absence of an active decision by the participants, the default option will apply. Contributors should understand that they are allowed to invest in portfolios different from the benchmark portfolio, and consequently they are finally responsible for their investment decisions, if they exercise their right to choose. This concept would also help to alleviate tension on the benchmark portfolio in periods of financial volatility.

The CWP should revise the recommendations at least once every two years, in cases of severe market shocks, or in cases where market conditions change due to financial innovation.\(^{17}\) For example, after the financial crisis the securitization market suffered a serious shock that could have affected a benchmark portfolio with significant exposure to housing. In such a case, the CWP could have recommended a change in the relevant benchmark. Sometimes some market indices become obsolete.

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16 The experience from many countries, including Denmark, Colombia and Peru, suggests that more than 90 percent of the individuals will use the default option.

17 Each member of the Commission is expected to serve for a period of 4 years, and succession should be staggered.
and are replaced by new alternatives. In addition, revisions of the benchmark may also be affected by developments in financial innovation that could have helped to complete markets. For example, the development of ETFs and hedge fund markets has helped to internalize some risks that were not traditionally captured by market risk. This is the so-called “alternative Beta,” which are returns that in the past may have been considered Alpha.

Both the technical group and the CWP need to understand all the steps involved in the creation of these benchmarks and should be able to explain them. The following section goes step-by-step on the technical aspects for building a benchmark portfolio. In an initial stage, the use of prepackaged software for simulating the optimal portfolios may rest ownership to the process of fully understanding the portfolio decisions.

Since benchmark portfolios may be highly scrutinized by some stakeholders, including Congress or Parliament, academics and mass media, and, consequently, it is essential that the CWP and the technical group understand all the processes associated with building the portfolio benchmark. In this regard and in the absence of standard software for doing the analysis, it would be desirable that the technical group devise their own methodologies for the simulation scenarios, instead of relying on prepackaged software. Prepackaged software may be useful to validate the findings, but it should not be seen as the main vehicle for conducting the simulations. Relying on black box software could be an impediment for a complete understanding of the strengths and limitations of the exercise.

3.3 Technical aspects of the benchmark portfolio: The Approximation Methodology

The methodological aspects of the portfolio benchmarks go from (relatively) low levels to high levels of complexity. This paper presents the Approximation Methodology, a relatively simple methodology for calculating portfolio benchmarks for pension funds. This methodology goes step-by-step through building the portfolio benchmarks, and it could be used as a starting point of the technical group in charge of developing these portfolio benchmarks, and also as mechanism for testing other methodologies, especially those provided in prepackaged software.

Building of the benchmark in this simplified framework, using the Approximation Methodology, requires several steps:

a. Modeling the earnings profile of the participants;
b. Modeling the rules of movement of the different asset classes in the portfolio and estimating the main parameters of the model;
c. Defining investment strategies;
d. Estimating the expected pension (i.e. replacement rate), the probability distribution function (PDF), and the expected utility function of each of the investment strategies;
e. Estimating by approximation the optimal portfolio consistent with the pension objective.
First step. Modeling the earnings profile

In order to develop a lifecycle investment structure it is important to find a representative earnings profile. Finding the earnings profiles requires processing large data sets from contributors. Depending on data availability, the representative profile might be a combination of a dynamic sample with a cross section.

The empirical formulation for the income structure typically used in the literature:\(^\text{18}\)

\[
\ln(y_{i,t}) = \alpha + \beta X_{i,t} + \mu_i + \omega_{i,t} \\
\omega_{i,t} = \rho \omega_{i,t-1} + \varepsilon_{i,t}
\]

where \(\ln(y_{i,t})\) is the natural log of the wage of cohort \(i\) at time \(t\); \(X_{i,t}\) is a vector socioeconomic characteristics of the group, including gender, age and educational level; \(\mu_i\) is a random variable with mean zero and variance \(\sigma^2\); \(\omega_{i,t}\) is an error term that captures the unobserved characteristics of income and follows an AR(1) process with; and \(\varepsilon_{i,t}\) is a white noise. The variables included in \(X_{i,t}\) can be polynomial in age, to capture the life-cycle profile of income. In addition, time dummies can be included in order to capture common period effects.

Second Step. Modeling the rules of motion of the different asset classes

Each asset class has its own rule of motion. The financial literature has made important progress in this area. In this section, we develop the most common methodologies for different asset classes. Since pension funds in emerging economies need to have well diversified portfolios, it is essential for them to invest in international assets. The returns in local currencies depend not only on the returns on international assets, but also on the exchange rate volatility. Consequently, it is essential also to find the rules of movement of inflation and the exchange rate.

Inflation

In countries with inflation targeting, it is possible to model annual inflation \((\pi_t)\) according to an Ornstein–Uhlenbeck process. This model allows convergence to the target inflation \((\bar{\pi})\), such that \(\alpha_{\pi}\) is the speed of convergence to the target and \(\sigma_{\pi}\) is the volatility of inflation.

\[
\pi_t = \pi_{t-1} + \alpha_{\pi}(\bar{\pi} - \pi_{t-1}) + \sigma_{\pi}\varepsilon_{\pi}
\]

where \(\varepsilon_{\pi}\) is a random variable of mean 0 and variance 1.

The importance of modeling inflation comes from the fact that most of future estimates are defined in real terms.

Fixed income\(^\text{19}\)

Several popular models have been proposed to represent the rules of movement of interest rates. The Vasicek (1977) formulation is relatively simple and one of the most frequently used by practitioners,

\(^{18}\) See for example Viceira (2010), Blake, Wright and Zhang (2008), Berstein and Tokman (2005).

\(^{19}\) This section presents the methodology for modeling risk free assets, and the Annex presents the one for corporate bonds, which can be extended to other types on non-governmental fixed income instruments.
as it describes short-term interest rates \((r_t)\) following a mean reversion to the long term interest rate \((\bar{r})\), with a speed of reversion determined by the parameter \(\alpha_r\), and with a volatility of \(\sigma_r\):

\[
dr_t = r_{t-1} + \alpha_r(\bar{r} - r_{t-1}) + \sigma_r dW_t
\]

(4)

where \(\alpha\) and \(\sigma > 0\) and \(W_t\) is a Brownian motion under risk-neutral measures.

The structure of the zero curve is defined by the price of the zero-coupon bond \(B(t,T)\) and the time to maturity \(\delta\) (T-t):

\[
r_{t,T} = -\frac{\ln(B(t,T))}{\delta}
\]

(5)

where,

\[
B_{t,T} = e^{(-A_{t,T}T + D_{t,T})}
\]

\[
A_{t,T} = 1 - e^{(-\alpha_r(T-t))}
\]

\[
D_{t,T} = \left(\bar{r} - \frac{\sigma_r^2}{2\alpha_r}\right)(A_{t,T} - (T-t)) - \frac{\sigma_r^2 A_{t,T}^2}{4\alpha_r}
\]

The Vasicek model belongs to the family of so-called *equilibrium models*, since investors price bonds by responding to the known expectations of future interest rates. Using the process for short-term interest rates, it is possible to determine the yield on longer-term bonds. To determine the full term structure, it is possible to price bonds of any maturity based on the expected evolution of short-term rates. While *equilibrium models* have the advantage of having closed-form analytic solutions for bond prices and provide consistent statements about interest rates over time, they are are inefficient for pricing derivatives, even if the parameters of the model are chosen carefully. *Arbitrage free models* are most useful for pricing purposes, especially interest rate derivatives. Equilibrium models, which generate the term structure as an output, compare with *arbitrage free models*, which take the term structure as an input.

**Equity**

As is the case with interest rates, many studies have analyzed the behavior of equity returns. Shiller (2000) and Seigel (2002) analyze long-term patterns in stock returns and provide helpful analyses of long-term trends. For simulating equity prices, we assume that prices follow a Brownian Geometric Movement, which satisfies the condition:

\[
dS_{t,t} = \mu S_{t,t} dt + \sigma S_{t,t} dW_t
\]

(6)

where \(\mu\) is the expected return (the drift), \(\sigma\) the volatility of the returns, and \(dW_t\) is a Wiener process or simple random walk. For an initial condition \(S_0\), the stochastic differential equation that describes the price trajectory has the following analytical solution:
Using price series, it is possible to estimate the parameters for $\mu$ and $\sigma$ for the equity prices used in the construction of the portfolios.

**Real Exchange Rate**

The real exchange rates can be modeled using the stochastic process used by Gorvett (2012), where the change in real exchange rates is a function of the interest rate differential and an stochastic component dependent of the volatility $\sigma_{RER}$:

\[
RER_t = \left( \frac{1+r_{t-1,1}}{1+i_{t-1,1}} - 1 \right) + \sigma_{RER} \varepsilon_{RER} \tag{8}
\]

where $r$ and $i$ are the foreign and domestic interest rates respectively.

This model can be implemented taking the 12-month variation in the percent of the real exchange rate as a function of the real interest rate differential.

**Third Step. Defining investment strategies**

As a starting point, the modeler may propose various investment strategies. Viceira (2010) proposes some scenarios that could be used as a starting point with combinations of four or five asset classes: domestic fixed income, domestic equity, international fixed income, international equity, and alternative assets. The number of asset classes can be increased as necessary.

Each of the asset classes (equity, fixed income) is expected to have a price index that would serve as a reference for measuring performance. In the presence of an international market with multiple asset price indices, it is essential to set up criteria for their selection. We propose the following criteria:

a. Investable indices. It is essential to use indices that can be replicated and commercialized in the market, for example through ETFs, mutual funds or similar.

b. Dominant indices in the active management industry. It is desirable to use dominant indices in markers, especially in cases where active management is expected by the pension funds. For example, pension funds my consider taking more active positions in equity investments in emerging markets. For enhancing competition and promoting comparison, dominant indices may incentivize the search for Alpha.

c. Structures that promote proper diversification of the assets. For example, 10/40 structure of indices in emerging markets. 10/40 structures constrain the weight of any single group entity at 10 percent, and the sum of the weights of all group entities representing more than 5 percent weight at 40 percent.

d. As specific benchmark structure as possible. Using structures such as MSCI World are not very useful for tracking error purposes, and consequently, it is more efficient to decompose the World index into four or five categories that can be more specific for purposes of error tracking.

Based on these considerations, it is possible to select market indices for each of the asset classes. Typically, pension systems in emerging markets will be interested in the following asset classes:
a. Local fixed income;
b. Local equity;
c. International fixed income;
d. International equity;
   i. United States and Canada
   ii. Europe
   iii. Japan
   iv. Australia
   v. Emerging markets
e. Alternative Assets.

Each of these categories may involve some sub-indices, for example, high yield US or commodities, to track the risk of the portfolios better. See the Annex for further references.

**Fourth step. Estimating the expected pension**

Traditional models assume that individuals have a certain objective (in this case replacement rate), and with a power utility function, defined as:

\[
U(W) = \frac{W^{1-\gamma}}{1-\gamma}
\]  
\(9\)

where \(W\) is the replacement rate at retirement age, and \(\gamma\) the coefficient of risk aversion.

The utility function is useful to capture the preferences of individuals given their risk tolerance. Individuals with low risk tolerance might not be receptive to a very high replacement rates, when the replacement rates in the tail distribution are too low, compared with more conservative scenarios.

Other authors, including Blake, Wright, and Zhang (2008) use Epstein and Zin utility functions, which have the main advantage of separating the coefficient of relative risk aversion from the elasticity of intertemporal substitution.

The cost of an annuity of a single unit at retirement age in real terms is equal to:

\[
CNUT = \sum_{t=1}^{\infty} PV_t f_t
\]  
\(10\)

where

\[
PV_t = \frac{1}{(1 + r_{T,t})^t}
\]

and where \(r_{T,t}\) corresponds to the zero coupon rate at retirement age and for a period \(t\), and \(f_t\) corresponds to the conditional probability of being alive in \(t\) years ahead.
Based on the estimated parameters for the asset classes, the modeler runs 10,000 simulations for each of the investment strategies and computes the average expected pension (i.e. replacement rate, pension wealth, or annuity), the PDF, and the utility function.

**Fifth step. Approximating the optimal portfolio**

Comparing the expected pensions with the pension objectives of the pension system (provided exogenously), for example, 70 percent replacement rate.

Modifying the investment strategies in order to achieve the expected pension outcome. Based on the simulations from the previous step and parameters, the investment strategies are modified in order to achieve the expected replacement rate. For each portfolio, it is necessary to run 10,000 simulations and then to calculate the average (close to the expected outcome), the probability density function (PDF) at retirement age and the value of the utility function.

The utility function plays a very important role, as risk averse individuals might be comfortable with lower pension objectives (i.e. replacement rates) compared with individuals who seek to maximize their future pensions. While the expected value of the utility function plays an important role in the selection of the optimal expected strategy, other factors might also be relevant, as explained in the next section.

**IV. A concrete example of the Approximation Methodology**

Using data for Chile, we calibrate a benchmark model for the pension system, but the results may be applicable to other economies as well.

**First step: Calculating the earnings profile**

We calculate the earnings profiles of individuals based on equations (1) and (2), represented in Figure 4. In the absence of long dataset that that may capture the different stages of the economic cycle, we followed a strategy similar to the one presented by Huneecus and Repetto (2004), which builds a synthetic panel based on cohorts. In the period 1990-2009, we follow the income process of individuals with similar gender, cohort and level of education.

We estimate of the autocorrelation coefficient and the variances of the permanent and transitory shocks of the income process using a panel regression with fixed effects for a male with 16.4 years of education (OECD 2014), as shown in Table 2. Parameters $\sigma_u$ and $\sigma_e$ are higher than the ones estimated by Gomes, Kotlikoff and Viceira (2008) for the case of the United States, which are 10.95% and 13.89% respectively. The higher value of these parameters is expected in emerging economies, subject to more frequent shocks and a labor market and a relatively high proportion of the labor force moving in and out from the informal labor market.

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20 We estimate the synthetic panel based on the data from the Quarterly Survey for the Great Santiago, published by the University of Chile. We used the survey of the 2nd quarter between 1990 and 2009.
Table 2. Estimated Parameters of Earnings Profile

| Parameter | Value |
|-----------|-------|
| $\rho$    | 0.71  |
| $\sigma_u$ | 25.7% |
| $\sigma_e$ | 30.1% |

Source: Authors’ calculations

The estimates of the deterministic component of the income are based on a regression of the Natural Log of the income and its determinants: age, and is second and third order effects, years of formal education, and gender. In most countries, earnings profiles have an inverted U shape that tends to peak when individuals are in the mid-40s. Figure 4 presents the deterministic income profile of an individual with 16.4 years of formal education.

Figure 4. Chile: Deterministic Earnings Profile of a Representative Individual

(Chilean pesos)

Source: Authors’ calculations

The density of contributions is typically a major problem in emerging economies and explains the low replacement rates of some segments of the employees. It is not unusual to find average density of contributions in the range of 50 to 60 percent in countries like Chile and Colombia. However, low density of contributions is not only a consequence of periods of high unemployment rate, but also reflects the switch between employment and self-employment, gender and fertility issues, and educational level.

In order to reflect the relatively low density of contributions that is typically found in emerging economies, Berstein, Fuentes and Villanova (2013) propose a methodology for calculating the probability of contributing at each moment of time. The probability of contribution is estimated as a function of observed socioeconomic variables, including age, gender, year of birth, unemployment rate, education level, marital status, and number of children with ages below six years old. While the methodology is relatively simple, it requires a complete dataset that can track the work trajectory of individuals. For the purpose of this note, we took the results of Berstein Fuentes and Villanova (2013), presented in Figure 5.  

21 While these systems are compulsory for employees, they are voluntary for self-employed contributors.  
22 These figures might be representative of some Latin American countries with similar types of labor markets, but extrapolating these figures to other regions might be risky.
Based on the expected earnings profile and the probability of contribution of individuals, we proceed to calculate the expected contributions along the working life. Each simulation calculates the earnings profile multiplied by a set of \([0, 1]\) numbers that follow the distribution that reflects the probability of contribution described in Figure 5. In the aggregate, this procedure is equivalent to multiply the earnings profile by the probability of contribution at each moment in time.

Second step. Modeling the rules of movement of the different asset classes

Tables 3 and 4 describe the parameters used for the simulations and the matrix of correlations, respectively. This is one of the most critical parts of the exercise, as it involves setting up the main parameters of the model. Based on the models with the rules of motion of the asset classes, we proceed to populate the Baseline Parameter Values. The parameters of international securities (fixed income and equities) are typically well documented and differences among models tend to be lower than differences in local markets. However, the value of the equity premium is always a matter of debate. The equity premium used in these simulations reflects a mid-point in current estimates.

### Table 3. Baseline Parameter Values

|                      | Rate   | Volatility | Velocity of mean reversion |
|----------------------|--------|------------|---------------------------|
| Local fixed income (real) | 2.27%  | 3.52%     | 1.59%                     |
| International fixed income (real) | 1%     | 3.40%     | 1.08%                     |
| Equity local (risk premium) | 5.50%  | 25%       | ..                        |
| Equity international (risk premium) | 5%     | 16.80%    | ..                        |

Source: Authors’ elaboration

23 It is important to highlight that the quantitative easing carried by central banks in most industrialized countries has imposed important distortions in the value of the money, which makes it harder to set long-term prices for the future.
Each of the parameters deserves to be discussed. The discussion about parameters of domestic securities is more complicated, as they should reflect forward-looking expectations, rather than simple historical averages. A reality check is necessary to compare forward-looking parameters with the ones estimated based on historical information using the methodologies described in the previous section. Some emerging economies have short historical records and past returns typically reflect periods of high illiquidity. As local equity markets develop and foreign investors become interested in local equities, there is a process of return equalization with international markets. While returns tend to converge to international levels, volatility tends to be higher than volatility in international markets, simply because of fewer possibilities for broader diversification. It is not rare to use a local equity premium slightly above the international premium, but with higher volatility than international equity indices. The same discussion could be extended to the cases where instead of investment in local markets, the investments are broadened to emerging economies or sub-regions.

### Table 4. Matrix of Correlations

| Source: Authors’ calculations |
|--------------------------------|
| Equity Local | Equity Int'l | RER | Short term rate local (n indexed) | Short term rate Int'l (n indexed) | term (n Credit local) | term (n Credit Int'l) | Risk | Credit Risk Int'l |
|----------------|--------------|-----|----------------------------------|-------------------------------|---------------------|---------------------|------|------------------|
| Equity Local | 1.00 | 0.55 | -0.32 | 0.11 | -0.15 | -0.26 | -0.51 |
| Equity Int'l | 0.55 | 1.00 | -0.55 | 0.07 | -0.16 | -0.31 | -0.70 |
| RER | -0.32 | -0.55 | 1.00 | -0.07 | 0.32 | 0.30 | 0.48 |
| Short term rate local (n indexed) | 0.11 | 0.07 | -0.07 | 1.00 | 0.33 | 0.05 | 0.08 |
| Short term rate Int'l (n indexed) | -0.15 | -0.16 | 0.32 | 0.33 | 1.00 | 0.19 | 0.38 |
| Credit risk local | -0.26 | -0.31 | 0.30 | 0.05 | 0.19 | 1.00 | 0.43 |
| Credit Risk Int'l | -0.51 | -0.70 | 0.48 | 0.08 | 0.38 | 0.43 | 1.00 |

Third Step. Defining investment strategies

The starting point of the approximation method is to put in place a set of different plausible investment strategies. We took, as starting point, some of the investment strategies proposed by Viceira (2010) and we add our own.

Figure 6 provides a graphic representation of the investment strategies, which are also explained below:

- **Conservative.** The conservative portfolio is a lifestyle fund invested 75 percent in the risk free asset and 25 percent in international fixed income.
- **Balanced local:** The balanced local is also a lifestyle fund invested 2/3 risk free assets and 1/3 domestic equity portfolio.
- **Balanced International:** This lifestyle portfolio invests 44 percent in risk free assets, 14 percent in international fixed income, 10 percent in local equity and 32 percent in international securities along the lifecycle of individuals.
- **Stylized Chilean fund.** This is a lifecycle portfolio that assumes a gradual migration from one fund to another within a 5 year time period (1/5 each year).
  - i. up to 31 years of age: 17.5 percent risk free assets; 55 percent international equity; 20% domestic equity; 7.5 percent international fixed income
  - ii. from 31 to 50 years of age: 28 percent risk free assets; 35 percent international equity;25 percent domestic equity; 12 percent international fixed income;
  - iii. From 50 to 65 years of age: 56 percent risk free assets; 13 percent international equity; 7 percent domestic equity; 24 percent international fixed income
e. **Stylized Peruvian fund**: This investment strategy assumes a gradual migration from one fund to another within a 5 year time period (1/5 each year).
   i. up to 35 years of age: 20 percent risk free assets; 35 percent international equity; 45 percent domestic equity
   ii. from 36 to 50 years of age: 35 percent risk free assets; 30 percent international equity; 35 percent domestic equity;
   iii. From 50 to 65: 55 percent years of age: risk free assets; 15 percent international equity; 30 percent domestic equity.

f. **Stylized Estonian fund**: The stylized Estonian fund assumes a gradual migration from one fund to another within a 5 year time period (1/5 each year).
   i. up to 35 years of age: 25 percent risk free assets; 75 percent international equity;  
   ii. From 36 to 50 years of age: 50 percent risk free assets; 50 percent international equity;  
   iii. From 50 to 65: 75 percent years of age: risk free assets; 25 percent international equity;  

g. **Default Chile**. This investment strategy, which resembles the default strategy for the Chilean pension fund, assumes a gradual migration from one fund to another within a 5 year time period (1/5 each year).
   i. up to 35 years of age: 35 percent risk free assets; 45 percent international equity; 20 percent domestic equity  
   ii. from 36 to 50 years of age: 50 percent risk free assets; 30 percent international equity; 20 percent domestic equity;  
   iii. From 50 to 65: 75 percent years of age: risk free assets; 17 percent international equity; 8 percent domestic equity.

h. **Annuity Immunization**: This investment strategy is highly aggressive until the point at which the portfolio tries to immunize the value of an annuity. The portfolio remains constant at 90/10 international equity /risk free assets up to 55 years of age, and then moves linearly to 10/90 during the following ten years.

i. **Base Scenario**. The base scenario assumes a gradual migration from one fund to another within a 5 year time period (1/5 each year).
   i. Starting from 7 percent risk free assets; 55 percent domestic equity; 35 percent international equity; 3 percent international fixed income;  
   ii. up to 35 years of age: 17 percent risk free assets; 41 percent domestic equity; 35 percent international equity; 7 percent international fixed income;
   iii. from 36 to 55 years of age: 32 percent risk free assets; 27.5 percent domestic equity; 27.5 percent international equity; 13 percent international fixed income;  
   iv. From 55 to 65 years of age: 35 percent risk free assets; 25 percent domestic equity; 25 percent international equity; 15 percent international fixed income.

j. **Aggressive**. The aggressive strategy follows an allocation of 7 percent risk free assets up to age 40; 70 percent allocation to international equity; 20 percent to domestic equity; and 3 percent international fixed income. After age 40, the fund start gradually moving to allocations of 35 percent risk free assets, 39 percent international equity, 11 percent local equity, and 15 percent international fixed income at retirement age.
Figure 6. Investment strategies: asset allocation along the lifecycle
(Asset class as a percentage of total assets)
Fourth step. Estimating the expected pension

The idea is to have a broad overview of the expected pension (replacement rates in this case) that could be reached with each of the investment strategies, and then to fine-tune the strategies in order to achieve the desired replacement rates (as discussed in the next step).

We proceed to estimate the expected contributions of the individuals along their life cycles, which are calculated as the rate of contribution times the expected earnings at each moment of time. For the purposes of this exercise, we assume a fixed 10 percent of wages contribution rate along the working life of the individuals. We proceed to simulate the outcome of investing these contributions in the different investment strategies identified in step 3, following the rules of movement of each of the asset classes (Tables 2 and 3). For simplicity, we assume that all fees are charged on the income (there are no asset management fees).

We proceed to simulate the value of the expected pensions at retirement age and the PDFs (at percentiles 5, 50 and 95) 10,000 times for each of the investment strategies. Replacement rates are calculated as the expected annuity at retirement age divided by the deterministic salary at retirement age.

We proceed to estimate the annuity of the individual. For simplicity, the simulations only include males, but the exercise is similar for females. We assume that individuals buy joint life annuities at retirement age. For the purpose of this exercise, we assume that the individual starts working at age 25, retires at age 65, and is married with a woman two year younger. We assume that in case that the pensioner dies before his wife, the widow would receive 70 percent of the pension.

24 For modeling purposes, the introduction of asset management fees would imply a lower return on assets. Pension fund management companies in countries like Chile and Colombia impose fees only on income.
25 Joint life annuities are annuities designed for couples, which pay until the death of both spouses. A joint life annuity is based on the longevity estimates for both spouses and assumes a reduction in payments to the surviving spouse by a pre-determined amount.
26 This little detail sounds irrelevant, but it is extremely important at the time of calculating the expected longevity of the woman.
Given the characteristics of the individual, we proceed to calculate the necessary capital for purchasing an annuity. This factor is a function of the technical interest rate for purchasing an annuity and the life expectancy of the annuitant(s). For the case of Chile, this factor is approximately equal to 16.5. We calculate the life expectancy at retirement age, based on the information provided by the National Institute of Statistics of Chile (INE), as shown in Figure 7.

![Figure 7. Life expectancy at retirement age](image)

Source: Authors’ calculations

The annuity is calculated as the amount of wealth at retirement age divided by the necessary capital for purchasing a constant stream of payments, and the replacement rate as the amount of the annuity divided by the last wage.

As shown in Table 5, the Aggressive strategy dominates in probability the other investment alternatives. While it provides an average replacement rate similar to the Annuity Immunization strategy (81 percent of the last wage), it provides better outcomes in the presence of less favorable market scenarios. The fifth percentile of the distribution provides a replacement rate of 27 percent in the aggressive strategy, while the annuity immunization strategy provides only 23 percent.

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27 This estimate depends on the market conditions and life expectancy of the annuitants.
While the PDF provides a useful indicator for evaluating the preferences of the contributors, the utility function provides a more precise indicator of the individual preferences. We proceed to calculate the value of the utility function (equation 9) for each of the investment strategies for different values of the risk aversion parameter ($\gamma$). As shown in Table 5, the aggressive strategy provides the highest value of the expected utility for different values of the risk aversion parameter. It is interesting to compare, for example, utility achieved by the Annuity Immunization strategy and the Stylized Peruvian Fund strategy. While the first one provides higher expected utility for low levels of risk aversion, the Stylized Peruvian Fund provides higher expected utility for higher levels of risk aversion. This is basically a consequence of the higher volatility if the returns in the Annuity Immunization strategy.

The main problem with simply using the specified utility function is the fact that it is unable to capture potential tail risk in the probability distribution of the asset prices and consequently is unable to capture the annuitization risk. The recent financial crisis has once again been a reminder that the probability distribution of equity prices may approximate more a curve with “fat tails.” Despite showing higher utility at higher levels of risk aversion, the Stylized Peruvian fund strategy brings individuals to retirement age with an equity allocation of 45 percent of the assets under management, compared with 10 percent in the case of the annuity immunization strategy.

However, there are no easy solutions. While the annuity immunization strategy has the advantage of reducing the annuitization risk, it implies that individuals would have to live with high levels of volatility in the value of their portfolios during their working life. As shown in the third column of Table 6, the annuity immunization strategy has the highest standard deviation among the ten investment strategies analyzed.
It is also interesting to note the low level of utility that is achieved by investing in the conservative strategy throughout the different levels of risk aversion. Despite the low levels of volatility, it also implies low levels of asset accumulation at retirement age. As shown in Table 5 the difference between the conservative strategy and the rest of the investment strategies become even wider at higher levels of risk aversion.

Consequently, finding the optimal portfolio is not as simple as selecting the investment strategy that maximizes the expected utility of the individuals, as other factors such as annuitization risk and asset volatility in the accumulation phase play may play an important role in selecting the decisions for the optimal portfolio. While volatility is partially captured in the utility functions, the annuitization risk is not.

**Fifth step. Approximating the optimal portfolio**

Since individuals may have some other sources of retirement income, the objective of the individual may be to achieve a certain replacement rate, say 70 percent of the last wage. This objective is different than maximizing the replacement rate. By setting up the replacement rate, individuals are revealing their degree of risk aversion. For the purpose of this exercise, it is assumed that the objective is to achieve a replacement rate of 70 percent.

We made a few adjustments in the asset allocation of the investment strategies that provided replacement rates in the range of 70 percent: the aggressive fund, the annuity immunization fund, the Stylized Chile fund and the Stylized Peruvian fund. Figure 8 shows the original (A) and modified versions (B) of the different pension strategies.

### Table 6. Expected utility and risk aversion at retirement age

| Strategy                      | γ=1.5 | γ=2  | γ=2.5 | γ=3  | γ=4  | γ=4.5 |
|-------------------------------|------|-----|------|-----|-----|------|
| Balanced International        | -2.94| -2.23| -2.32| -2.81| -3.73| -5.31|
| Base Scenario                 | -2.62| -1.83| -1.81| -2.14| -2.84| -4.10|
| Balanced Local                | -3.12| -2.52| -2.83| -3.70| -5.35| -8.37|
| Stylized Chilean Fund         | -2.83| -2.09| -2.13| -2.55| -3.37| -4.78|
| Default Chile                 | -2.90| -2.18| -2.26| -2.72| -3.59| -5.07|
| Stylized Peruvian Fund        | -2.68| -1.90| -1.92| -2.30| -3.09| -4.55|
| Stylized Estonian Fund        | -2.95| -2.27| -2.44| -3.05| -4.24| -6.32|
| Annuity Immunization          | -2.62| -1.90| -1.99| -2.52| -3.62| -5.71|
| Conservative                  | -3.76| -3.56| -4.55| -6.59| -10.29| -16.90|
| Aggressive                    | -2.53| -1.73| -1.70| -1.99| -2.62| -3.76|

Source: Authors’ calculations
Figure 8. Comparison of investment strategies between original (A) and modified (B) strategies.
We modified the asset allocation of these investment scenarios in order to reach replacement rates of 70 percent. This modification was achieved by fine tuning the asset allocation of the different portfolios along the lifecycle.

As shown in Table 7, the PDFs of these five investment strategies are relatively similar, despite having different asset compositions along the lifecycle. With the exception of the annuity immunization scenario, the standard deviation is in the range of 40 percent, and the fifth percentile reports similar results as those reported in original scenarios (27 percent of the last wage).

Table 7. Replacement rates in modified investment strategies
(as a percentage of last wage)

|                      | Average | Std. Dev. | P5    | P50   | P95  |
|----------------------|---------|-----------|-------|-------|------|
| Base Scenario B      | 70%     | 39%       | 27%   | 61%   | 148% |
| Stylized Chile B     | 70%     | 42%       | 27%   | 59%   | 147% |
| Stylized Peruvian Fund B | 70%     | 40%       | 27%   | 61%   | 147% |
| Annuity Immunization B | 70%     | 50%       | 23%   | 57%   | 164% |
| Aggressive B         | 70%     | 40%       | 27%   | 60%   | 141% |

The expected utility of the different investment strategies shows that the aggressive fund dominates the other investment strategies for the different values of the risk aversion parameter (Table 8). It is interesting to note that, in the case of the aggressive portfolio, the expected utility of the individual decreases for most of the values of the risk aversion parameter, but not in all the cases. For higher levels of risk aversion, the expected utility of individuals in the modified aggressive fund increases, despite receiving a lower replacement rate. This finding reflects that more risk averse individuals are willing to sacrifice a considerable amount of their replacement rates (in this case 11 percent) simply to have lower volatility.

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28 For $\gamma = 4.5$ the expected utility in the original and modified aggressive fund are -5.77 and it is -5.75 respectively.
Table 8. Expected utility in the modified investment strategies

|                     | y=1.5 | y=2  | y=2.5 | y=3  | y=3.5 | y=4  | y=4.5 |
|---------------------|-------|------|-------|------|-------|------|-------|
| Base Scenario B     | -2.64 | -1.85| -1.83 | -2.15| -2.84 | -4.08| -6.27 |
| Stylized Chile B    | -2.65 | -1.87| -1.87 | -2.21| -2.92 | -4.19| -6.40 |
| Stylized Peruvian Fund B | -2.65 | -1.87| -1.88 | -2.24| -3.01 | -4.42| -6.99 |
| Annuity Immunization B | -2.75 | -2.05| -2.19 | -2.82| -4.08 | -6.45| -10.92|
| Aggressive B        | -2.63 | -1.84| -1.81 | -2.10| -2.73 | -3.84| -5.75 |

Source: Authors’ calculations

Despite the fact that the modified aggressive strategy shows higher levels of expected utility compared with the other strategies, the equity allocation at retirement age is in the range of 40 percent of the assets under management, which is relatively high if individuals are expecting to annuitize their pensions. A similar problem is found in the case of the modified base scenario and modified stylized Peruvian fund, the equity allocation of which at retirement age is in the range of 47 percent of the assets under management.

While the Annuity Immunization strategy solves the problem of immunization risk, its expected utility is much lower compared with other investment strategies, which is a consequence of the high volatility of the asset prices during the working life of individuals.

Taking these alternatives into consideration, the best alternative might be to use the Modified Stylized Chile fund, which reports slightly lower utility than the aggressive fund, but with lower annuitization risk. The total equity exposure at retirement age is one-third of the total assets of the pension fund.

V. Conclusions

This paper provides an overall justification for implementing SAA through the use of portfolio benchmarks, and goes step by step through the procedure for implementing the portfolio benchmark. The emphasis of the portfolio benchmarks is a consequence of the market failure in guiding the investment strategies of pension funds towards optimal levels. This paper emphasizes the need of having in place three elements at the time of designing a benchmark portfolio framework in a DC open pension system:

a. A clear objective for the pension fund system. The pension objective is essential to ensure that the investments are guided by the long term interests of the participants;

b. A governance structure for building the benchmark portfolio. The governance structure ensures that the investment strategy is resilient to market fluctuations;

c. An appropriate methodology for building the benchmark portfolio. The proper methodology ensures that the portfolio decisions are guided by SAA.

The use of portfolio benchmarks is consistent with healthy competition in open pension systems, where PFMCs would measure their returns against the benchmark. The performance of the PFMCs is consequently associated with their capacity to generate returns in excess of the market portfolio (portfolio benchmark).
This paper proposes the use of the approximation methodology to find the optimal portfolio. While the set of tools presented in this paper may guide the design of optimal portfolios, these tools have limitations and broader criteria need to be taken into consideration at the time of deciding on the benchmark portfolio.

Monitoring of the performance of pension funds could be done with the use of tracking error against the benchmark portfolio. The risk differences between the pension funds and portfolios can be represented in terms of a traffic light system. Different degrees of risk deviation may have associated a different light (green, yellow, or red). Thus, the supervisory agency may require PFMCs to cut their (tactical) bets on the pension funds depending on the light of each fund at each moment in time.
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Annex. Corporate Bonds

Pension funds typically include in their portfolios fixed income instruments, other than government securities. This annex provides an overview of the methodology for estimating the rules of movement of corporate bonds.

In a standardized model, credit risk is the main variable that differentiates corporate bonds from government securities, and the way of approaching the problem is through modeling the spread over the treasury, which reflects the default probability of this asset class compared to the sovereign. It is important to separate corporate bonds by broad categories of credit rating, as the behavior of spreads in “AAA” instruments is likely to be different than the ones in “BB.” Thus, the price of a corporate bond can be represented by:

\[ P_{t,T} = e^{(-A_{t,T} r_t + D_{t,T} s_t)} \]

where, \( A_{t,T} \) and \( D_{t,T} \) are the parameters of the Vasicek model for the risk free assets; \( r_t \) is the short-term rate; and \( s_t \) is the credit spread. Assuming that the spread is independent of the length of the period, it is possible to model it through a Cox–Ingersoll–Ross process. Following O’Donoghue et al. (2014):

\[ ds_t = \theta (\gamma - s_t) dt + \sigma_s \sqrt{s_t} dW_t \]

where, \( \theta \) is the velocity of mean reversion of the spread, \( \gamma \) is the long-term spread, and \( \sigma_s \) its volatility.

The following example models the spreads of the corporate bonds that are typically held by the Chilean pension funds (Riskamerica Index), and the spreads of high yield bonds in the United States (J.P. Morgan US High Yield Index), which are instruments that have been increasingly attractive for Latin American pensions in the recent years.

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29 The non-negativity is a desirable feature of this process, since under normal conditions investors are not willing to invest in a risky instrument at a lower rate than a risk free asset.
Figure A1. Evolution of Corporate Spreads, 2000-2014

![Graph showing evolution of corporate spreads, 2000-2014](image_url)

Source: Riskamerica Index and J.P. Morgan US High Yield Index

Table A1. Estimated parameters of corporate bonds

|                | Local Spread | International Spread |
|----------------|--------------|----------------------|
| $\theta$      | 0.5          | 0.4                  |
| $\gamma$      | 1.27%        | 8.58%                |
| $\sigma_g$    | 3.08%        | 6.37%                |

Source: Authors’ calculations