Modelling of induction motor incorporating magnetic saturation

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Abstract. The paper studies the need to take into account the magnetic saturation of a squirrel-cage induction motor (IM). A mathematical description of an IM with linear and nonlinear magnetic characteristics is given. The simulation of motor’s direct start, as well as the field oriented control (FOC) system in Simulink Matlab environment is presented. The results of simulation are presented: time diagrams and mechanical characteristics of the motor. The conclusion is made whether it is necessary to take into account saturation in the study of transients in an asynchronous squirrel-cage motor.

1. Introduction
At the modern stage of the development of science and technology, the electric drive occupies a leading position among drive devices and ensures reliable and trouble-free operation of machines and mechanisms in almost all industrial sectors.

A squirrel-cage induction motor (IM) is the most common drive motor [1], which is due to its reliability, ease of operation, low cost and acceptable overall dimensions [2]. The main disadvantage of IM is the difficulty in controlling. The reason for this is the nonlinear and multiparametric nature of the asynchronous machine, as well as the inability to directly measure some parameters of the state of the machine.

A modern IM drive is built on the basis of power semiconductor power technology using microprocessor control [3]. The capabilities of such electric drive systems allow one to adjust the output coordinates (speed, moment) in a wide range with high accuracy and speed.

Modern frequency converters allow one to implement traditional and create new software algorithms, as well as synthesize control systems with a wide range of operational characteristics. However, even as part of a frequency-controlled electric drive, operating modes with maximum energy indicators are not always ensured.

Outstanding Russian and foreign scientists made a significant contribution to the creation and development of the theory of asynchronous electric drive systems - A. A. Gorev, G. N. Petrov, R. T. Shreiner, R. V. Filts, I. P. Kopylov, G. G. Sokolovsky, V. M. Terekhov, K. P. Kovach, I. Ya. Braslavsky, L. A. Bessonov, A. E. Kozyaruk, G. M. Asher, F. Blaschke, W. Floter, J. Holtz, W. Leonard, T. A. Lipo, D. W. Novotny and many others.

However, a number of tasks remain unresolved. One of these tasks is to increase the accuracy of the mathematical description of an IM. When developing control systems for frequency-controlled asynchronous electric drives, in most cases a mathematical description is used, where the magnetic characteristic of the machine is described by a linear dependence [4 - 6]. In real electric drives, this
characteristic is non-linear, which is due to the phenomena of saturation and hysteresis. This fact must be taken into account when operating asynchronous electric drives, since saturation of the magnetic circuits can cause a sharp deterioration in the properties of the control system [7].

There are two main methods for constructing control algorithms for an asynchronous electric drive, allowing to level out the influence of saturation. The first method is to restrict the rotor flux so that the machine operates in the linear zone of its magnetic characteristic (which corresponds to the traditional mathematical description). The disadvantage of this method is that it does not provide efficient operation of the machine, especially under high loads, since the stator current in this case is unnecessarily high [8]. The second method is to establish the value of the rotor flux in the area of its nominal value (which is usually located in the nonlinear part of the magnetic characteristic) [9]. Then the parameters of the generally accepted model of an asynchronous machine should correspond to the working point of the rotor flux. The efficiency of the machine will be maximum when the moment on its shaft is close to the nominal. However, during the operation of some mechanisms and machines (for example, mining machines, special-purpose vehicles), the load is subject to significant changes both up and down [10-11]. At load values below the rated value, excess energy will accumulate in the stator windings of the motor, reducing its efficiency.

Thus, to achieve high performance in the control of asynchronous electric drives, it is necessary to develop control systems that take into account the nonlinear nature of the magnetic system of the motor.

2. Modeling an IM with linear magnetic characteristic

2.1. Two-phase squirrel-cage IM model

When developing mathematical models of electric machines we use the concept of a generalized electric machine. For this, any multiphase electric machine with the number of windings \( n \) on the stator and \( m \) on the rotor is represented as an equivalent two-phase bipolar electric machine as shown in Figure 1. In this case, the description of the processes in the generalized electric machine is carried out under the following basic assumptions:

- the machine has a smooth air gap without grooves in the stator and rotor;
- the real non-linear characteristic of the magnetization of the machine is replaced by a linear one;
- magnetomotive forces of the windings are sinusoidal;
- the magnetic circuit is not saturated;
- there are no power losses in the magnetic circuit;
- parameters of the stator and rotor windings are concentrated.

![Figure 1. Scheme of a generalized electric machine.](image)

![Figure 2. Scheme of a two-phase model of a squirrel-cage IM.](image)
To switch to a two-phase structural model of a squirrel-cage IM, it is necessary to close the rotor windings of a generalized electric machine as shown in Figure 2. Thus, the projection of the rotor voltage on the axis will be zero.

The mathematical description of the energy conversion processes in the resulting model in the most compact form has the form [12]:

\[
\begin{align*}
U_s &= R_s I_s + \frac{d}{dt} \Psi_s + j\omega_c \Psi_s; \\
0 &= R_r I_r + \frac{d}{dt} \Psi_r + j(\omega_c - \omega) \Psi_r; \\
M &= \frac{3}{2} \cdot p \cdot J \cdot m \cdot (\Psi_s \times I_s); \\
\frac{d}{dt} \omega &= M - M_r,
\end{align*}
\]

where \( I_s, I_r, \Psi_s, \Psi_r, U_s \) – generalized spatial vectors of currents, flux linkages and voltages of stator and rotor windings; \( R_s, R_r \) – phase resistances of stator and rotor windings; \( \omega, \omega_c \) – angular speeds of the rotor and coordinate axes; \( M, M_r \) – the electromagnetic moment of motor and the moment of resistance on its shaft; \( J \) – total moment of inertia; \( p \) – number of pole pairs.

It should be noted once again that the dependence of flux linkage inside the stator on the magnetization current \( \Psi_s = f(I_s) \) in this model is linear (this assumption was made at the very beginning of Section 2.1). The nature of this relationship is shown in Figure 3.

![Figure 3. Real (a) and simplified (b) magnetization characteristic of the machine.](image)

2.2. Simulation of starting an IM without saturation

The objective of this stage of the study was to evaluate the starting properties of the engine during its simulation without taking into account the saturation of the magnetic circuit. To do this, it was necessary to take the timing diagrams at start-up and build the mechanical characteristics of the motor.

The mechanical characteristic of an electric motor is the dependence of the rotor speed on the torque on the shaft \( \omega = f(M) \). This characteristic for an IM is a nonlinear dependence (Figure 4) and allows a detailed analysis of the operation of an electric machine. For this, the concept of stiffness is introduced in the theory of electrical machines - the degree of change in speed with a change in moment. As can be seen from Figure 4, the stiffness of the IM remains constant only in the area of stable operation of the electric motor. The same area is also called a worker area.
Simulation of a squirrel-cage IM was carried out in a Simulink Matlab environment. The model of an IM is based on a system of equations (1). To build such a system, it is first necessary to calculate constant coefficients (motor parameters) before the variables in the equations. The parameters of the investigated electric motor are shown in Table 1.

Table 1. Parameters of the studied electric motor

| Motor parameter                        | Parameter value |
|----------------------------------------|-----------------|
| Rated power, kW                        | 147.1           |
| Rated speed, rpm                       | 1785            |
| Power factor                           | 0.98            |
| Rated voltage, V                       | 460             |
| Mains frequency, Hz                    | 60              |
| The number of pole pairs               | 2               |
| Moment of inertia, kg·m²               | 3.1             |
| Resistance of stator windings, Ω       | 0.015           |
| Inductance of stator windings, mH      | 0.3             |
| The resistance of the rotor, Ω         | 0.009           |
| Inductance of the rotor windings, mH   | 0.3             |
| Mutual inductance, mH                  | 10.46           |

As a result of direct start simulation of the motor with a light load of 50 N·m for a period of 10 s, time diagrams were obtained, shown in Figure 5. The speed, torque on the shaft, and stator phase A current were measured. These diagrams make it possible to evaluate the starting properties of the motor: time to reach the steady state 6.3 s, moment jump up to 900 N·m, current jump up to 2 kA.

Based on the data obtained, the mechanical characteristic of the studied electric motor was constructed, where the influence of saturation was not taken into account (Figure 6). On the graph obtained, it is possible to clearly distinguish characteristic points and areas of engine operation similar to Figure 4, taking into account the modeling error.

Figure 4. Mechanical characteristic of an IM:
point 1 - perfect idle speed; point 2 - nominal operating mode; point 3 - critical mode; point 4 - starting mode; section 1-3 - steady work; section 3-4 - unstable work.

Figure 5. Timing diagrams of speed, torque and current of the stator of the motor with direct start without taking into account saturation.
3. Modeling an IM with a nonlinear magnetic characteristic

3.1. Accounting for saturation in the equations of an IM
For a sufficiently accurate calculation of the characteristics of an IM in a wide range of operating modes, it is important to take into account saturation. The mathematical model of an IM, taking into account the saturation of the magnetic circuit, will be non-linear for both dynamic and steady state. With sufficient accuracy, we can assume that saturation manifests itself only in a change in inductance - a coefficient that relates flux linkage and magnetization current (Figure 3). Thus, it is necessary to introduce into the mathematical model a nonlinear dependence of flux linkage on the magnetization current.

When saturation is taken into account in equations (1), it is necessary to take into account the time variation of the inductance value $L_s = f(t) = \frac{W_1}{I_s}$. In the Simulink Matlab environment, this change is introduced by introducing into the motor model the nonlinear part of the dependence of the linear voltage on the magnetization current (Figure 7).

3.2. Simulation of IM starting with saturation
The objective of this stage of the study was to evaluate the starting properties of the motor during its simulation, taking into account the saturation of the magnetic circuit. For this, as in Section 2.2, time diagrams were taken at start-up and the mechanical characteristic of the motor was constructed. The parameters of the motor are given in Table 1.

As a result of simulating direct start of the motor without load for a period of 5 s, time diagrams
were obtained, shown in Figure 8. The speed, torque on the shaft and stator phase A current were measured. At first glance, the starting properties of the motor during simulation taking into account saturation are similar to the results obtained in section 2.2. However, it is worth noting that the time to reach the steady state increased by about 0.2-0.3 s, which in the relative ratio does not exceed 5%.

![Figure 8. Timing diagrams of the speed, torque and current of the motor stator during direct start, taking into account saturation.](image)

According to the data obtained, the mechanical characteristic of the motor was constructed during simulation taking into account saturation (Figure 9), which is also identical to that obtained previously.

4. Vector control of squirrel-cage IM

4.1. Field oriented control system
Of particular interest is the vector control system of an IM, which is widely used in the practice of an electric drive. Among vector control, the most widely used ones are field oriented control (FOC) and direct torque control (DTC). We consider the effect of magnetic saturation on control quality at FOC.

FOC system has a closed speed control loop [13]. Speed is measured directly or indirectly. The
output values of this speed control loop are the electromagnetic moment and the motor rotor flux. The torque and flow are controlled by adjusting the stator current in the coordinate axes $dq$, where the axis rotation speed is equal to the rotor speed ($\omega_c = \omega_r$). The projection of the stator current on these coordinate axes is then used to form the switching functions of the keys of the inverter of the frequency converter.

The main advantage of this control algorithm is its high dynamic characteristic, since the torque and rotor flux are independently regulated; however, this control method requires high computing power [14]. The block diagram of the FOC system is shown in Figure 10.

![Block diagram of FOC system](image)

**Figure 10.** Block diagram of FOC system.

### 4.2. Simulation results

For the simulation, the speed and load diagrams shown in Figure 11 were used. The parameters of the motor under study are shown in Table 1. As a result of the simulation, the timing diagrams shown in Figure 12 were obtained.

![Load and speed diagrams](image)

**Figure 11.** Load and speed diagrams of a model of a FOC system.

![Timing diagrams](image)

**Figure 12.** Timing diagrams of the FOC system.
Figure 12. The simulation results of a FOC system with and without saturation: a) speed; b) moment; c) stator phase A current.

As can be seen from the obtained graphs, the control quality does not depend on whether the saturation of the magnetic circuit is taken into account in the mathematical model of motor or not. The moment jump at start-up in the first case is 1775 N·m, in the second case - 1900 N·m, the difference in relative values is about 6%. The starting current in the first case is 1790 A, in the second case – 1590 A, the difference in relative values is about 13%.

5. Conclusion
According to the results obtained, we can conclude that the saturation of the magnetic circuit in the conditions of simulation weakly affects the quality of control. When modeling a FOC system for an IM, the quality of speed and torque control does not change when the magnetic characteristics of the motor change, or this change does not exceed 5%, which in most practical tasks is an acceptable result.

Separately, it is worth mentioning the starting properties: there is an increase in the starting current of an engine with a nonlinear magnetic characteristic. This can be explained by the fact that to obtain a certain value of flux linkage in the saturation zone, a larger magnetizing current is required than to obtain the same value of flux linkage in an unsaturated machine. This fact must be taken into account when designing electric drive systems, especially when developing motor protections. Failure to take this phenomenon into account can be the reason for arbitrary shutdown of the electric drive by protection devices.

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