Gravity Cutoff in Theories with Large Discrete Symmetries

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Abstract

We set an upper bound on the gravitational cutoff in theories with exact quantum numbers of large $N$ periodicity, such as $\mathbb{Z}_N$ discrete symmetries. The bound stems from black hole physics. It is similar to the bound appearing in theories with $N$ particle species, though a priori, a large discrete symmetry does not imply a large number of species. Thus, there emerges a potentially wide class of new theories, that address the hierarchy problem by lowering the gravitational cutoff due to existence of large $\mathbb{Z}_{10^{32}}$-type symmetries.
1 Introduction

Black hole (BH) physics is a powerful tool for extracting non-perturbative information about microscopic structure of the theory. As examples of such use of BHs one may list the argument about violation of continuous global symmetries in gravitational theories [1], the bound on the entropy of bounded systems [2], and the constraints on possible violation of Lorentz invariance [3, 4]. Yet another example [5] is the restriction on the number \( N \) and mass \( M \) of particle species, which in the case of stable species and large \( N \) reads

\[
NM^2 \lesssim M_P^2 ,
\]

up to a factor that scales as \( \sim \ln N \). Here \( M_P \) is the Planck mass.

As also shown in [5], the same BH bound applies even to a single species of mass \( M \) that carries an exactly conserved quantum number (not associated with any long-range classical gauge force) of periodicity \( N \). An example of such a quantum number can be a discrete gauge symmetry \( Z_N \) [6,7], or a quantum hair under some massive integer spin field [8]. In what follows, we shall investigate this situation. Thus, unless otherwise stated, \( N \) will denote periodicity of \( Z_N \) (or of some other exact quantum number) and not the number of particle species and the bound (1) should be understood accordingly.

The above bound is applicable, in particular, to scalar fields and implies that masses of \( N \) scalar fields, or of a single scalar field charged under a \( Z_N \)-symmetry, are automatically limited by \( M_P/\sqrt{N} \). By naturalness arguments presence of light scalar fields suggests existence of some new stabilizing physics at that scale. For the case of many species this statement was made rigorous in [9] where it was shown that in a theory with large number of species the gravitational cutoff comes down to \( M_P/\sqrt{N} \). (This is consistent with earlier perturbative arguments [10, 11].) The purpose of this note is to prove a similar statement for the case of large discrete symmetry.

2 Black Hole Argument

Consider a (scalar) field \( \phi \) of mass \( M \) transforming under a discrete symmetry \( Z_N \),

\[
\phi \mapsto \phi e^{i 2\pi k/N} \quad k = 0, \ldots, N - 1.
\]

We assume that this \( Z_N \) symmetry is exact, i.e. it is not violated at any scale. A straightforward way to ensure this is to declare that \( Z_N \) is a gauge symmetry. However, for our reasoning it is unimportant what underlying physics guarantees exactness of \( Z_N \). We then make the two following assumptions:
$a)$ the particle $\phi$ has the largest charge to mass ratio among all the particles carrying $Z_N$ charge;  

$b)$ there are no BH remnants.

We are going to prove that there is a bound on the cutoff $\Lambda$ of the low energy theory,

$$\Lambda \leq \frac{M_P^2}{N M}.$$  \hspace{1cm} (3)

If $M \sim M_P/\sqrt{N}$ this bound coincides with the bound

$$\Lambda \leq \frac{M_P}{\sqrt{N}}$$ \hspace{1cm} (4)

implied by naturalness. However, in general, the bound (3) is weaker than (4). We show that the stronger bound (4) is obtained if one makes an additional assumption that the property of negative heat capacity of BHs persists in the high-energy theory.

Let us proceed to the proof of Eq. (3). One performs the same thought experiment [5] as in establishing the bound (1). Take a macroscopic (arbitrarily large) BH and throw a number $\sim N$ of $\phi$-particles into it. In this way we endow the BH with $Z_N$-charge of order $N$. Then one waits for the BH in question to evaporate. Since the $Z_N$-symmetry is exact at all scales and there are no remnants the BH eventually has to return the exact amount of the swallowed charge. Indeed, if the returned charge were not equal to the original one the BH would mediate a process that violates $Z_N$ explicitly, in contradiction with our assumption.

The crucial point is that as long as the BH is the usual Schwartzschild BH (SBH) it cannot give out any $Z_N$-charge. Indeed, radiation of SBH is thermal and contains as many $\phi$-particles as the antiparticles. Thus, to return back the $Z_N$-charge, the properties of BH must get modified when it reaches a certain size, $R_{BH} \sim \Lambda^{-1}$. This implies existence of a new physics at the scale $\Lambda$; in other words, $\Lambda$ is a cutoff of the low-energy theory.

Consider the BH that has just reached the cutoff scale. The mass of the BH at this moment must be sufficient to produce $\sim N$ of $\phi$-quanta,

$$M_{BH} \geq NM.$$ \hspace{1cm} (5)

On the other hand, the BH mass and size are still related at this moment by the standard Schwartzschild expression,

$$M_{BH} \sim R_{BH}M_P^2.$$ \hspace{1cm} (6)

Combining Eqs. (5), (6) one obtains the bound (3).

Notice that the above proof is UV-insensitive in the sense that it does not depend on the precise nature of BHs that are smaller than the cutoff scale $\Lambda^{-1}$. All we have used is the conservation of energy which is entirely a large-distance constraint.
The bound can be improved if we make an additional assumption that the property of negative heat capacity of BHs persists in the high-energy theory. More precisely, we assume that the BH, after it reaches the size $R_{BH} \sim \Lambda^{-1}$ corresponding to the Hawking temperature $T_H \sim \Lambda$, continues to radiate preferentially into modes with energies equal or higher than $\Lambda$. Then, Eq. (5) is replaced by

$$M_{BH} \geq N\Lambda.$$  \hspace{1cm} (7)

When combined with (6) it yields the stronger bound (4).

The above assumption about the BH spectrum appears to be natural. Its violation would imply very unusual properties of small BHs: they should be very cold and decay into quanta with inverse momenta greatly exceeding the size of the BH. Although we cannot exclude such a possibility, we conclude that under reasonable assumptions about the properties of small BHs the bound on the cutoff scale is (4).

It is worth comparing the argument presented in this section with the case of large number of species [9]. In the latter case the existence of the low cutoff can already be established in perturbation theory by considering the graviton propagator. The loop corrections to the propagator are amplified by the large number of species and the perturbative expansion goes out of control precisely at the scale $M_P/\sqrt{N}$ signaling that the cutoff is reduced to this value. The same conclusion can also be inferred directly from BH physics. The Hawking radiation of a BH of size $(M_P/\sqrt{N})^{-1}$ is drastically amplified as it can radiate $N$ species. As a consequence such BH would have a lifetime of order of its size and therefore it is not a classical object as in ordinary general relativity. On the other hand, in the case of a single field charged under a large discrete symmetry the perturbation theory does not show any sign of breaking down. Similarly, there is no indication of the radiation of a BH of the size $(M_P/\sqrt{N})^{-1}$ blowing up. Nevertheless as we have shown the consistency of the theory requires the presence of a low cutoff. The argument that enables to establish the existence of the cutoff is intrinsically non-perturbative and uses in an essential way the BH physics. This is reminiscent of the “gravity as the weakest force” conjecture [12]. It would be interesting to explore a possible connection between this conjecture and our work.

## 3 Explicit Examples

In this section we consider a few examples of theories with large discrete symmetries.

1) Consider a $U(1)$ gauge symmetry with two scalar fields, $\phi$ and $\chi$. The field $\phi$ has a unit charge $e$ while the charge of the field $\chi$ is $Ne$. Let the $\chi$-field develop a non-zero vacuum expectation value (VEV), thus breaking the $U(1)$ symmetry down to $Z_N$. The latter symmetry
acts on $\phi$ according to Eq. (2). The field $\phi$ is assumed to be lighter than the other fields, so it is the only degree of freedom at low energies.

It is important to notice that setting the ratio of charges of the fields $\chi$ and $\phi$ to a rational number (which we, for simplicity, took to be an integer) is not a fine-tuning. Rather, this is required by the bound (1). Indeed, if the ratio of charges were an irrational number the effective discrete symmetry would be $Z_\infty$, which is impossible.

In this theory the $Z_N$-charge inside a given volume of space can be monitored in the following way. Because of the non-trivial topological structure of the vacuum manifold (non-contractible loops) there are cosmic strings in this theory, around which the phase of the $\chi$ VEV winds by $2\pi$ multiple. These cosmic strings contain a unit flux of the gauge field. This allows to monitor the $Z_N$-charge of a system through the Aharonov-Bohm (AB) effect in the scattering of the cosmic strings from the system [6,7]. In particular, if the system collapses into a BH, the latter has a quantum $Z_N$ hair [6,7] that the AB effect can probe.

The proof of Sec. 2 implies that gravity in this model must be modified at distances $(M_P/\sqrt{N})^{-1}$. Indeed, from the proof it is clear that BHs with the size smaller than $(M_P/\sqrt{N})^{-1}$ have to acquire hair capable of producing $Z_N$-charge asymmetry in the BH evaporation. On the other hand, such hair are impossible in Einstein’s general relativity. At the classical level this follows from the no-hair theorems [1]. Quantum effects do not help either. Indeed, existence of quantum hair leads to polarization of vacuum around BHs with $Z_N$-charge [7]. This vacuum polarization is sensitive to the $Z_N$-charge of the BH and, a priori, can contribute to the asymmetry of the evaporation. However, at weak coupling, $Ne \ll 1$, the effect is exponentially suppressed [7] and is unable to produce the necessary asymmetry. Thus we conclude that the physics responsible for the cutoff at $M_P/\sqrt{N}$ must involve gravity in an essential way.

2) Existence of a large discrete symmetry may be accompanied by the presence of a large number of species in the theory. Then, the latter property, by itself, implies a low gravitational cutoff [9]. This point is illustrated by the following example.

Consider an $SU(2)$ gauge theory with two scalar fields, $\phi_j$ and $\chi_{j_1j_2...j_N}$, transforming as a fundamental and $N$-rank symmetric tensors respectively. Here $j = 1, 2$ and $j_k = 1, 2, \ k = 1...N$ are fundamental indices. We assume that the field $\chi$ acquires a VEV of only one component $\chi_{11...1}$. This VEV breaks the continuous $SU(2)$-symmetry down to a discrete $Z_N$ factor, under which $\phi_1 \mapsto \phi_1 e^{i\frac{2\pi}{N}}$ and $\phi_2 \mapsto \phi_2 e^{-i\frac{2\pi}{N}}$.

One may be tempted to apply our argument to show that the gravitational cutoff in this theory is low using the field $\phi_1$ (or $\phi_2$) in the proof. However, it would be incorrect: the proof of Sec. 2 is not directly applicable to this case. The reason is that the theory contains particles with arbitrarily large $Z_N$-charges and so the assumption a) of the proof is violated. The states with large $Z_N$-charges are the components of the field $\chi$. Indeed, a component $\chi_{j_1j_2...j_N}$ with $n$
indices equal to 1 and remaining \(N - n\) indices equal to 2 carry \(2n - N\) units of the \(Z_N\) charge. Correspondingly, a discrete charge of arbitrary \(2n - N < N\) number of the \(\phi_1\) fields can be recycled by a BH into a single \(\chi\) quantum. The corresponding gauge invariant operator has the form

\[
\bar{\phi}^{j_1} \cdots \bar{\phi}^{j_{2n}} \chi_{j_1 \cdots j_n a_{N-n} b_{N-n}} X_{j_{n+1} \cdots j_{2n} b_{N-n}} e^{a_1 b_1} \cdots e^{a_{N-n} b_{N-n}} .
\]  

(8)

However, the gravitational cutoff is still lowered down to \(M_P/\sqrt{N}\) in this model. This is due to the fact that the theory contains \(N\) species which are the \(N\) components of the symmetric tensor \(\chi\). Thus we again find in this example that large discrete symmetry implies cutoff \(M_P/\sqrt{N}\), though, in this case, indirectly: through a large number of species.

3) The previous example suggests that the condition a) of the proof can be replaced by a weaker one. Going through the proof it is straightforward to convince oneself that the following requirement for the structure of the theory is sufficient: it should be impossible to reproduce an arbitrary \(Z_N\)-charge with a small (much less than \(N\)) number of particles belonging to a small number of species. The following example proposed in [5] demonstrates that this condition cannot be weakened further.

Consider a theory with \(n\) scalar fields \(\Phi_k\), \(k = 1, \ldots, n\), with the following sequence of couplings,

\[
\Phi^3_1 + \Phi^*_1 \Phi^3_2 + \Phi^*_2 \Phi^3_3 + \cdots + \Phi^*_k \Phi^3_{k+1} + \cdots + \Phi^*_n \Phi^3_n + h.c. .
\]  

(9)

This theory is invariant under \(Z_N\) symmetry with \(N = 3^n\) and the transformation law,

\[
\Phi_k \mapsto \Phi_k e^{i(3^{n-k} 2\pi N)} .
\]  

(10)

In other words, the \(Z_N\)-charge of the field \(\Phi_k\) is \(3^{n-k}\).

Let \(\Phi_n\) be lighter than the rest of the fields. Then, integrating out the first \((n - 1)\) fields one obtains the effective \(Z_N\)-invariant theory for \(\Phi_n\). However, in this case one cannot use the BH argument to claim that the mass of \(\Phi_n\) is bounded by \(M_P/\sqrt{N}\), nor that there is a gravitational cutoff at \(M_P/\sqrt{N}\). Indeed, any \(Z_N\)-charge of order \(N\) thrown into the BH in the form of \(\Phi_n\) quanta can be radiated away in the form of just of order \(n \sim \ln N\) quanta of different \(\Phi_k\) fields at the late stage of BH evaporation. The arguments of [9] are also not applicable because the total number of species is only \(\sim \ln N\). The best one can do in these circumstances is to conclude that the cutoff of the theory does not exceed \(M_P/\sqrt{\ln N}\).

4) The bound [11] has been obtained without references to the explicit structure of the theory of quantum gravity. Hence, by consistency, it should be satisfied in the string theory. Here we propose a simple example which shows that this is indeed the case. Consider the setup where the \(Z_N\) group is generated by an isometry of compact space in string theory compactification. We take the string coupling to be of order one so that the 10-dimensional Planck mass is set by
the string scale $M_S$. Consider now a compactification on $T_6 \times M_4$, where $T_6$ is a 6-dimensional torus and $M_4$ is the 4-dimensional Minkowski space. The isometry group of this space is $U(1)^6$. We wish now to break one of the $U(1)$’s down to $Z_N$. Let the radius of the corresponding circle be $R$. We assume the radii of the other tori to be of order the string length. Then the relation between the 4-dimensional Planck mass and the string scale is

$$M_P^2 = M_S^2 (RM_S).$$

(11)

Let us break the $U(1)$ isometry on the $R$-circle down to $Z_N$ by creating $N$ fixed points around the circle. Alternatively this can be done by placing $N$ identical branes and requiring the exact symmetry under cyclic shifts. Since the distance between the fixed points or the branes is bounded by the string scale, the maximal number of them that can be fitted on the circle is $N \leq RM_S$. Recalling that $M_S$ is the cutoff of the low-energy effective theory one sees that (11) reproduces the bound (4).

4 Implication for the Hierarchy Problem

The results of this paper shed new light on the proposal [5] to solve the hierarchy problem by postulating a large discrete symmetry with $N \sim 10^{32}$. We find that in this case the gravitational cutoff of the theory is not far from the weak scale, thus the latter is automatically stabilized. A generic prediction of this solution to the hierarchy problem is appearance of strong gravitational physics not far from the weak scale. From the experimental point of view this physics is expected to manifest itself in softening of the scattering amplitudes at energies above the scale $M_P/\sqrt{N}$. We now briefly discuss some aspects of implementing the above idea in model building.

From the constructive point of view it is desirable to have an explicit mechanism ensuring the hierarchy between the Planck and the weak scales. In the Standard Model (SM) the Higgs boson cannot transform under any exact symmetry, so it is impossible to give the $Z_N$-charge to the Higgs itself to apply the bound (11) directly to its mass. Thus the idea is to ascribe the $Z_N$-charge to some other fields whose mass gets contributions from the Higgs VEV. The bound (11) on the mass of these particles then implies the bound on the Higgs VEV.

Let us stress that in pursuing this strategy one should be careful not to run into conflict with the low gravitational cutoff. To illustrate what we mean let us consider the following example. One can identify large $Z_N$ with the subgroups of the existing global symmetries of the SM that would appear exact in the absence of gravity. Ignoring gravity, the SM has two classically-exact continuous global symmetries that account for baryon and lepton number conservations. Thus one possibility is to declare that the $Z_N$ symmetry in question is a subgroup of some combination of baryon and lepton number symmetries. It is most straightforward to embed
$Z_N$ into the $B - L$ symmetry because the latter is automatically anomaly free. Then, from Eq. (1) the bound on $N$ is $M_P^2 / m_{\nu}^2 \gtrsim 10^{54}$ where $m_{\nu} \lesssim$ eV is the mass of the lightest neutrino. Postulating the $Z_N$ symmetry with $N \sim 10^{54}$ we would prevent the Higgs VEV to be larger than $10 - 100$ TeV since large Higgs VEV would make neutrino heavier than the BH upper bound for $N \sim 10^{54}$. However, according to the results of this paper, such a large $N$ would lower the gravitational cutoff below the weak scale, in contradiction with the observations. If we want the cutoff at an acceptable level, we have to choose $N \sim 10^{32}$. In this case the direct BH bound on the Higgs VEV is much higher than the bound on the cutoff.

The reader may be puzzled why one should worry about applying the BH bound (1) directly to the Higgs VEV, given the fact that in the above example with $N \sim 10^{32}$ the cutoff is at the needed level? Seemingly, the latter would suffice to solve the hierarchy problem. The point is that, in general, the cutoff controls the radiative stability of the weak scale but need not necessarily constrain its tree-level value. Correspondingly, if the tree-level value is large the physical scale will also be large even though the radiative corrections are small. Hence, the small cutoff does not necessarily guarantee the smallness of the physically observable weak scale, whereas the direct BH bound on the mass (1) does.

To make our reasoning more transparent it is useful to make a parallel with a much more familiar example of the low energy supersymmetry. The cutoff that controls the radiative corrections to the Higgs mass is the supersymmetry breaking scale in the observable sector, $m_{susy} \sim$ TeV. However, smallness of this cutoff cannot explain why there is no large tree-level contribution to the Higgs mass. The latter puzzle is the essence of the celebrated $\mu$-problem. Thus, in order to solve the hierarchy problem in supersymmetry, smallness of $m_{susy}$ is not enough. One needs an additional mechanism that would guarantee smallness of $\mu$. In our case the analog of $m_{susy}$ is the low gravity cutoff $M_P / \sqrt{N}$. However, the physical weak scale is restricted by the BH bound on the particle masses. Whenever we can directly apply this bound to the weak scale, the hierarchy problem is solved, with no need of any further assumptions about the tree-level masses versus cutoff.

An example when there is no large discrepancy between the cutoff and the direct BH bound on the Higgs VEV is obtained in the following way. The idea is to extend the SM by introducing new particles charged under $Z_N$ that get their masses from the Higgs VEV. These particles can be either scalars or fermions. In the latter case we can postulate the existence of a vector-like pair of left-handed lepton-like doublets $L, L'$ with opposite hypercharges and two pairs of right-handed singlets: “neutrinos” $\nu, \nu'$ and “electrons” $e, e'$. The couplings allowed by the gauge

\footnote{Note that in this construction neutrinos must be of the Dirac type since we assume an exact conservation of a subgroup of the $B - L$ symmetry.}
symmetry are similar to the ones of ordinary leptons\footnote{The bare mass terms can be forbidden by invariance under a parity transformation which changes the sign of the primed fields.}

\[ HL\nu + H^*\epsilon Le + H^*L'\nu' + H\epsilon L'e'. \]  \hfill (12)

The Yukawa couplings are assumed to be of order one. The only difference between these new vector-like fermions and the ordinary leptons is that they transform under an exact $Z_N$ symmetry with $N \sim 10^{32}$. This symmetry prevents mixings of the new fermions with other generations, and also implies the BH bound on their mass which translates into a bound on the Higgs VEV. In other words the BH bound on the particle masses guarantees the stable hierarchy between the Higgs VEV and the Planck mass.

An alternative, even simpler, possibility is to introduce a scalar $S$ transforming under $Z_N$-symmetry and having no charge under the SM gauge group. This scalar gets a mass from the Higgs VEV through the following coupling in the Lagrangian

\[ -\lambda H^*HS^*S - M^2S^*S, \]  \hfill (13)

where $\lambda \sim 1$ is the coupling constant and $M^2$ is the bare mass. Equation (11) yields a bound on the total mass of $S$,

\[ \lambda H^*H + M^2 \lesssim M_P^2/N, \]

which for $\lambda > 0, M^2 > 0$ translates into the bound on the Higgs VEV $\langle H \rangle \lesssim M_P/\sqrt{N}$. The latter is of the same order as the bound on the cutoff.

To avoid confusion let us stress that the solution of the hierarchy problem considered above, based on the existence of a discrete symmetry $Z_N$ with large $N$, is physically different from the explanation of the hierarchy considered in Refs. [9, 13] where the large number $N$ was the number of particle species. In particular, the arguments presented in [13] to show that the many-species scenario can simultaneously solve the strong CP problem, are not directly applicable to the case of large discrete symmetry. It would be interesting to understand whether an alternative argument exists which could explain the smallness of the strong CP parameter in the large $Z_N$ case as well.

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