Spin magnetosonic shocks in quantum plasmas

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Abstract

The one-dimensional shock structures of magnetosonic waves (MSWs) propagating in a dissipative quantum plasma medium is studied. A quantum magnetohydrodynamic (QMHD) model is used to take into account the quantum force term due to Bohm potential and the pressure-like spin force term for electrons. The nonlinear evolution (Korteweg de-Vries-Burger ) equation, derived to describe the dynamics of small amplitude MSWs, where the dissipation is provided by the plasma resistivity, is solved numerically to obtain both oscillatory and monotonic shock structures. The shock strength decreases with increasing the effects of collective tunneling and increases with increasing the effects of spin alignment. The theoretical results could be of importance for astrophysical (e.g., magnetars) as well as for ultracold laboratory plasmas (e.g., Rydberg plasmas).

Keywords: Magnetosonic waves, Shock waves, Quantum plasmas, KdVB equation

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The subject of quantum plasmas have received a great attention in investigating various collective quantum effects in plasmas [see e.g. Refs. 1-14]. The collective motion of Fermi particles in a magnetic field thus gives rise a natural extension to the classical theory of magnetohydrodynamics (MHD) in terms of the well-known quantum magnetoplasmas, which have potential applications in astrophysical plasmas, such as pulsar magnetospheres, magnetars. Moreover, the motion of particles with spin properties become important in strong magnetic fields as a probe of quantum physical phenomena [15-17] in the laboratory plasmas. Many of these studies are motivated on a single particle properties. It is thus expected that the collective spin effects can influence the propagation characteristics of waves in a strongly magnetized quantum plasma [18-20]. Moreover, recent progress in producing Rydberg plasmas may give rise to an interesting experimental evidence for the dynamics of quantum plasmas. However, in such magnetized plasmas the thermal energy of the particles can be very small compared to the typical Zeeman energy of the particles. Recent investigations indicate that the spin properties of the electrons and positrons can lead to interesting collective effects in quantum magnetoplasmas [19]. More recently, it has been shown that the electron spin 1/2 effect significantly modifies the dynamics [21] and modulational instability domain [22] of magnetosonic solitary waves and the collective effects in strongly magnetized plasmas [23].

There has also been much interests in investigating structures and dynamics of shock waves in various quantum plasma media [24-27]. The dynamics of classical shocks is governed by a Korteweg de-Vries-Burger (KdVB) equation. A stationary solution of the latter can be represented as an oscillatory shock. However, when the dissipation overwhelms the dispersion and when the dissipative effect is in balance with the nonlinearity, we indeed have the possibility of monotonic shock waves. Unlike the classical fluids, quantum plasmas typically exhibit dispersion due to the collective tunneling associated with the Bohm potential instead of dissipation. For this reason, even a quantum shock propagating with constant velocity in a uniform medium does not exhibit a stationary structure. Transition from initial to compressed quantum media occurs in the form of a train of solitons propagating with different velocities and with different amplitudes.

In this letter, we derive a governing equation that describes the dynamics of magnetosonic waves (MSWs) in a quantum electron-ion plasma. The governing KdVB equation contains both dispersive term due to Bohm potential and the dissipative term due to plasma
resistivity (neglecting other effects viz., thermal conduction, viscosity etc.), and also the pressure-like spin quantum force. Still, when the normalized zeeman energy $\sim 1$ and the plasma resistivity is small, we can recover monotonic transition of the oscillatory shocks. The stationary shock solutions exist for the Mach number $\gtrsim 15$. The effects of collective tunneling and spin alignment influence the strength of the shocks.

The basic set of equations governing the dynamics of the magneto-sonic waves in a quantum plasma reads [21,22]

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v) = 0, \quad (1)$$

$$\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right) = -B \frac{\partial B}{\partial x} - c_s^2 \rho \frac{\partial}{\partial x} (\ln \rho) + \beta \rho \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{\rho}} \frac{\partial \sqrt{\rho}}{\partial x} \right) + \frac{\varepsilon}{v_B^2} \rho \frac{\partial}{\partial x} [\rho B \tanh(\varepsilon B)], \quad (2)$$

$$\frac{\partial B}{\partial t} + \frac{\partial}{\partial x} (Bv) - \gamma \frac{\partial^2 B}{\partial x^2} = 0, \quad (3)$$

where $\mathbf{B}$ is the magnetic field along the $z$-axis, i.e. $\mathbf{B} = B(x,t)\hat{z}$, normalized by its equilibrium value $B_0$; $\rho \equiv (m_e n_e + m_i n_i)$ is the total mass density normalized by its equilibrium value $\rho_0$; and $\mathbf{v} \equiv (m_e n_e \mathbf{v}_e + m_i n_i \mathbf{v}_i)/\rho = v(x,t)\hat{x}$ is the center of mass fluid velocity normalized by the Alfvén speed $C_A = \sqrt{B_0^2/\mu_0 \rho_0}$. The space and time variables are normalized by, respectively, $C_A/\omega_{ci}$ and the ion gyroperiod $\omega_{ci}^{-1} \equiv (eB_0/m_i)^{-1}$.

Here $n_e(n_i)$ is the electron (ion) number density, $m_e(m_i)$ is the electron (ion) mass, $\mathbf{v}_e(\mathbf{v}_i)$ is the electron (ion) fluid velocity and $e$ is the magnitude of the electron charge. Also, $\beta = 2c^2(m_e/m_i)\omega_{ci}^2 \lambda_C^2 / C_A^4$, where $\lambda_C = c/\omega_C = h/2m_e c$ is the Compton wavelength, $\omega_C$ is the Compton frequency, $c$ is the speed of light in vacuum, $h$ is the Planck’s constant divided by $2\pi$, $c_s = \sqrt{k_B(T_e + T_i)/m_i}$ is the sound speed, where $T_e(T_i)$ is the electron (ion) temperature, and $k_B$ is the Boltzmann constant. Moreover, $\gamma = \eta \omega_{ci} / \mu_0 C_A^2$, where $\eta$ is the resistivity, $v_B^2 = k_B T_e/m_i C_A^2 = (1/\varepsilon) \mu_B B_0/m_i C_A^2$ with $\mu_B = e h / 2 m_e$ is the Bohr magneton and $\varepsilon = \mu_B B_0/k_B T_e$ is the temperature normalized Zeeman energy. Note that in Eq.(2) [the second term on the right hand side] we have used the isothermal equation of state for electrons as $P_e = k_B n_e T_e$ for one-dimensional magneto-sonic wave propagation across $B_0$. One can also use the equation of state for electrons as $P_e = m_e V_F^2 n_e^3 / 3 n_0^2$ for one-dimensional propagation [3] or $P_e = m_e V_F^2 n_e^{5/3} / 5 n_0^{2/3}$ in three-dimension, assuming a local zero-temperature Fermi distribution [28]. The last two terms in the right-hand side of Eq.(3)
are due to the effects of collective tunneling and spin alignment, respectively, and in Eq.(3) we have neglected the inertial term.

In order to investigate the dynamics of MSWs, we employ the standard reductive perturbation technique (RPT) with the following stretching

$$\xi = \epsilon^{1/2}(x - v_0 t), \tau = \epsilon^{3/2} t,$$

where $\epsilon$ is a small expansion parameter and $v_0$ is the wave phase velocity normalized by $C_A$. The dynamical variables are expanded as

$$\rho = 1 + \epsilon \rho_1 + \epsilon^{3/2} \rho_2 + \epsilon^2 \rho_3 + \ldots,$$

$$v = \epsilon v_1 + \epsilon^{3/2} v_2 + \epsilon^2 v_3 + \ldots,$$

$$B = 1 + \epsilon B_1 + \epsilon^{3/2} B_2 + \epsilon^2 B_3 + \ldots$$

(5)

Now, substituting the expressions [Eqs.(5)] into the Eqs. (1)-(3) and collecting the terms in different powers of $\epsilon$ we obtain in the lowest order of $\epsilon$

$$\rho_1 = B_1, \quad v_1 = v_0(B_1 - 1),$$

(6)

together with the linear dispersion relation:

$$v_0 = \sqrt{1 + \epsilon^2 - \frac{\epsilon}{v_B^2} (2 \tanh \epsilon - \epsilon \sec h^2 \epsilon)}.$$  

(7)

From the next order of $\epsilon$, we obtain

$$\rho_2 = B_2 + \frac{\gamma}{v_0} \frac{\partial B_1}{\partial \xi}, \quad v_2 = v_0 B_2 + \frac{\gamma}{v_B} \frac{\partial B_1}{\partial \xi} - v_0$$

(8)

and

$$v_0^2 + v_0 v_2 = \left[ 1 - \frac{\epsilon}{v_B^2} (\tanh \epsilon + \epsilon \sec h^2 \epsilon) \right] B_2 + \left( \epsilon^2 - \frac{\epsilon}{v_B^2} \tanh \epsilon \right) \rho_2$$

(9)

Inserting Eq.(8) into the Eq.(9) we obtain

$$\gamma \left( v_0^2 - v_s^2 + \frac{\epsilon}{v_B^2} \tanh \epsilon \right) \frac{\partial B_1}{\partial \xi} = 0$$

(10)

Since the second factor in Eq.(10) is non-zero by means of Eq.(7) and also $\partial B_1/\partial \xi \neq 0$, $\gamma$ should be at least of the first order of $\epsilon$, so that $\gamma \partial B_1/\partial \xi$ becomes of the order of $\epsilon^2$, and it will be included in the equations for the order of $\epsilon^2$. Collecting the terms in powers of $\epsilon^2$
and eliminating the quantities $\rho_3, v_3$ [the coefficient of $B_3$ becomes zero by Eq.(7)] we obtain
with the help of Eq.(6) the required KdVB equation

$$\frac{\partial b}{\partial \tau} + P b \frac{\partial b}{\partial \xi} + Q \frac{\partial^2 b}{\partial \xi^2} + R \frac{\partial^2 b}{\partial \xi^3} = 0,$$

(11)

where $b \equiv B_1$ and the coefficients $P, Q$ and $R$ are given by

$$P = \frac{1}{2v_0} \left[ 3 - v_0^2 + 2c_s^2 - \frac{\varepsilon}{v_B^2} \left( 8 \tanh \varepsilon + 7 \sec \varepsilon \varepsilon - 2 \varepsilon^2 \tanh \varepsilon \sec \varepsilon \right) \right],$$

(12)

$$Q = -\frac{\beta}{4v_0}, \quad R = \frac{\gamma}{2v_0^2} \left( c_s^2 - M^2 - \frac{\varepsilon}{v_B^2} \tanh \varepsilon \right).$$

(13)

Note that the spin quantum effects are embedded in all of $P, Q$ and $R$, whereas the dispersion due to quantum diffraction and dissipation due to plasma resistivity are in $Q$ and $R$ respectively. We now numerically solve the Eq.(11) directly in order to obtain nonstationary shock solutions. In the numerical scheme the KdVB equation (11) is advanced in time with a standard fourth-order Runge-Kutta scheme with a time step of $10^{-4}$ s. The spatial derivatives are approximated with centered second-order difference approximations with a spatial grid spacing of 0.2 m. The profile of the oscillatory shock solution of Eq.(11) for the parameter values $B_0 = 0.14T, T_e = 0.09K, \varepsilon = 1.04, n_0 = 10^{30}m^{-3}, \lambda \equiv \eta/\mu_0 = 0.001$ is shown in Fig.1. The train of oscillations propagates together with the shock with the same velocity. As the role of spin force increases, the shock strength decreases and the oscillations ahead the shock becomes less in number, in which the first few oscillations are very close to the magnetosonic solitons. The oscillations decay quite slow as the role of quantum diffraction increases. The plot with $\lambda = 0.01$ shows the monotonic transition (Fig.2) from the oscillatory shocks shown in Fig.1. Increasing further the role of quantum effects, we can not observe the oscillatory shock transition from the Fig.2.

The stationary solution of Eq.(11) can also be obtained by transforming to the moving frame of reference $\zeta = \xi - V \tau = \sqrt{\varepsilon}(\omega_{ei}/C_A) [x - C_A(v_0 + \varepsilon V)t]$. The KdVB equation then reduces to the following system:

$$\frac{db}{d\zeta} = a, \quad \frac{da}{d\zeta} = -\frac{1}{Q} \left( Ra + \frac{P}{2} b^2 - V b + V - \frac{P}{2} \right)$$

(14)

The system of equations (14) has two singular points, namely $(1,0)$ and $(2V/P - 1,0)$ which are stable node or focus according as $R^2 + 4VQ \leq 0$ (since $Q < 0$). A stable focus
corresponds to an oscillatory shock (Fig.3) (dispersion dominant), while a stable node gives rise monotonic shocks (Fig.4) (dissipation dominant). The shock strength is given by

$$[\epsilon b]_{\text{max}} = \epsilon \left( \frac{2V}{P} - 1 \right) = \frac{2v_0}{P}(M - 1) - \epsilon,$$

where we have defined the shock Mach number $M$ as the ratio of the velocity $C_A(v_0 + \epsilon V)$ of the nonlinear magnetosonic wave to the linear wave velocity $C_A v_0$ by

$$M = 1 + \epsilon \frac{V}{v_0}.$$  

Numerical solutions of Eq.(14) are shown in Figs. 3 and 4 for the same parameter values as in Figs.1 and 2 respectively, but for $V = 20$. We find that for these sets of parameters shock solutions exist for $V \geq 15$. As the value of $V$ increases, the number of oscillations ahead of the shock increases with decreasing the shock amplitude near $\zeta = 0$. Also, as the value of the zeeman energy $\epsilon$ decreases, the shock strength increases and the oscillations decay quite slowly forming long wave train, while for large value of $\epsilon$, the oscillations decay quite fast. Numerical simulation also reveals that the shock strength decreases with increasing the particle number density and decreasing the electron temperature, while it increases with the strength of the ambient magnetic field.

To conclude, we have investigated the effects of quantum tunneling and spin alignment on the magnetosonic shock structures in a dissipative quantum plasma medium. The numerical solutions of the KdVB equation exhibit both stationary and nonstationary oscillatory/monotonic shock solutions in the quantum regime. Such significant modifications of the shock structures in our quantum plasma are completely a new feature relevant for astrophysical and ultracold laboratory plasmas.

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