Particle Capture in Porous Medium

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Abstract. Filtration problems in porous media are important for studying the movement of groundwater in porous formations and the spreading of liquid concrete injected into porous soil. Deep bed filtration of a monodisperse suspension in a homogeneous porous medium with two simultaneously acting particle capture mechanisms is considered. A mathematical model of suspension flow through porous medium with pore blocking by size-exclusion and arched bridging is developed. Exact solutions are obtained on the concentration front and at the porous medium inlet. For the linear filtration function, exact and asymptotic solutions are constructed.

1. Introduction

The problem of filtering fluids in porous formations is the important part of underground hydromechanics; it has extensive applications in hydraulic engineering and hydrogeology. Knowledge of the laws of filtration is necessary when choosing the location of construction objects, designing tunnels and underground structures. To strengthen loose soil and create a solid foundation, liquid grout is pumped into the soil under pressure. The grout is filtered in porous rock and, after solidification, strengthens the soil [1-3].

Solid sedimentary rocks containing interconnected voids of various sizes form a porous medium. The flow of the suspension - a fluid with suspended fine solid particles through a porous medium is accompanied by pore blocking and particle capture. The retained particles form a deposit. The deposition of retained particles during long-term filtration can significantly affect the permeability of the rock [4-6]. Depending on the type of suspension and porous medium, electrostatic and gravitational forces, diffusion, viscosity, etc. play a decisive role in particle capture [7-9]. If the particle sizes and pore diameter are about the same, then the main cause of particle retention becomes size-exclusion mechanism [10, 11]. Particles freely pass through large pores and are retained at the entrance of pores whose diameter is smaller than the particle size (figure 1). It is assumed that a particle stuck in the pore throat blocks the flow of fluid and cannot be knocked out of the pore by a fluid flow or another particle. The particles retained in a porous medium form a deposit.

At low fluid speed, electrostatic forces do not allow suspended particles to join into groups, and solid particles move through the porous medium or get stuck in it alone. With increasing fluid velocity, hydrodynamic forces begin to overcome electrostatic repulsion, with some particles merging into groups and moving together. Such groups can block the necks of pores whose diameter is larger than the size...
of individual particles [12]. Groups of particles get stuck at the entrance of the pores, forming arched bridges (figure 1).

Figure 1. Pore blocking by single particles and arched bridges.

Two particle capture mechanisms act independently, blocking particles in pores of various sizes. The arched bridge cannot linger at the entrance of a pore whose diameter is smaller than the size of a single particle, and individual particles freely pass through large pores that can be blocked by a group of particles.

The mathematical model of deep bed filtration includes the mass balance equation for suspended and retained particles, as well as the kinetic equations that determine the rate of deposit growth [13]. Models with a single size-exclusion capture mechanism have been studied in detail [14-16]. In [17, 18], several simultaneously acting mechanisms of particle capture with deposit growth proportional to the first degree of suspended particles concentration were considered. In contrast to standard models, we assume that the deposit in a porous medium is formed due to two particle capture mechanisms with different concentration functions. Size-exclusion – the capture of individual particles is described by a kinetic equation with the deposit growth rate proportional to the concentration of suspended particles. For the group particle capture, the deposit growth is proportional to the concentrations of all particles of the group. The simplest stable arch bridge consists of three particles (figure 1), in this case the deposit growth rate is proportional to the cube of the suspended particles concentration. The proposed model corresponds to the simultaneous formation of two types of deposit when filtering a monodisperse suspension in a porous medium with pores of various sizes.

The proportionality coefficient between deposit growth and particle concentration in the kinetic equation is called the filtration function [19]. In the case of low concentrations and limited filtration time, the filtration function can be considered constant. However, with a significant concentration of suspended particles, the character of filtration changes with time: the more deposit formed, the less small free pores remain, and the process of particle retention slows down. After long-term filtration in a porous medium, there are practically no vacant small pores for the retention of particles, the suspension flow moves through large pores, and the accumulation of deposit stops. To describe this phenomenon, a blocking filtration function is used - a positive decreasing function depending on the concentration of total deposit, which vanishes at a certain limiting value. A model with several particle capture mechanisms and small filtration functions was studied in [20].

In this paper, exact solutions on the concentration front and at the porous medium inlet are obtained. For the linear filtration functions, exact and asymptotic solutions are constructed.

2. Mathematical model
In the domain \( \Omega = \{(x,t) : 0 \leq x \leq 1, t \geq 0\} \) the suspended and retained particles concentrations \( C(x,t), S(x,t) \) satisfy the nonlinear system of equations

\[
\frac{\partial C}{\partial t} + \frac{\partial C}{\partial x} + \frac{\partial S}{\partial t} = 0 ;
\]
\[ \frac{\partial S_1}{\partial t} = \Lambda_1(S)C; \quad \frac{\partial S_2}{\partial t} = \Lambda_2(S) f(C). \]  

The unique solution of the system is determined by initial and boundary conditions

\[ S_1|_{t=0} = S_2|_{t=0} = 0; \quad C|_{t=0} = 0. \]  

\[ C|_{t=0} = 1. \]  

Here \( S_i \) is the concentration of retained particles blocking small pores with single particles; \( S_2 \) - concentration of retained particles blocking large pores with arched bridges; \( S = S_1 + S_2 \) - full deposit concentration; \( \Lambda_i(S), i = 1,2 \) - filtration functions; the concentration function \( f(C) \) determines the growth of arched bridges on the concentration of suspended particles.

Conditions (3) mean that at the initial moment the porous medium does not contain suspended and retained particles; according to (4), a suspension with a constant concentration of suspended particles is injected at the porous medium inlet. From the inconsistency of conditions (3), (4) at the origin, it follows that the solution of the problem (1)-(4) has a discontinuity. The line of discontinuity \( t = x \) coincides with the characteristic of the equation (1) - the concentration front of suspended and retained particles.

Problem (1)-(4) has a zero solution before the concentration front at \( 0 \leq t < x < 1 \); the solution is positive behind the front \( 0 \leq x \leq 1; t > x \).

In characteristic variables \( \tau = t - x, \) the system takes the form

\[ \frac{\partial C}{\partial x} + \frac{\partial S_1}{\partial \tau} = 0; \quad \frac{\partial S_2}{\partial \tau} = \Lambda_2(S) f(C). \]  

3. Exact and asymptotic solutions

3.1. Solution on the concentration front and at the inlet

On the concentration front \( \tau = 0 \), partial deposits \( S_1, S_2 \) and full deposit \( S = S_1 + S_2 \) are zero. Substitution of (6) into (5) yields

\[ \frac{\partial C}{\partial x} + \Lambda_i(0)C + \Lambda_2(0) f(C) = 0. \]  

The solution of equation (7) with condition (4) defines the concentration of suspended particles on the concentration front

\[ \int_{C(x,0)}^{\hat{C}} \frac{dC}{\Lambda_i(0)C + \Lambda_2(0) f(C)} = x. \]  

For constant filtration functions \( \Lambda_i(S) = \Lambda_i(0) = \lambda_i, i = 1,2 \), formula (8) determines the time-independent suspended particles concentration \( C(x, \tau) = C(x, 0) \) behind the concentration front in the domain \( \hat{\Omega} = \{0 \leq x \leq 1, \tau \geq 0\} \). 

Partial deposit concentrations are obtained from equations (6)

\[ S_1(x, \tau) = \lambda_i C \tau; \quad S_2(x, \tau) = \lambda_2 f(C) \tau. \]  

To obtain the equation for the total deposit concentration \( S(0, \tau) \) at the porous medium inlet \( x = 0 \), add up equations (6) and use condition (4)
\[ \frac{\partial S}{\partial \tau} = \Lambda_1(S) + \Lambda_2(S) f(I) . \]  

(10)

The solution of equation (10) with zero condition at \( \tau = 0 \)

\[ \int_0^{S(0, \tau)} \frac{dS}{\Lambda_1(S) + \Lambda_2(S) f(I)} = \tau . \]  

(11)

For known \( S(0, \tau) \) the partial deposit concentrations are determined by formulas

\[ S_1(0, \tau) = \int_0^{S(0, \tau)} \frac{\Lambda_1(S)dS}{\Lambda_1(S) + \Lambda_2(S) f(I)}; \quad S_2(0, \tau) = \int_0^{S(0, \tau)} \frac{\Lambda_2(S) f(I)dS}{\Lambda_1(S) + \Lambda_2(S) f(I)} . \]  

(12)

3.2 Exact solution for linear filtration function

Consider the system (5), (6) with a linear filtration function independent of the type of capture mechanism \( \Lambda_i(S) = \lambda - S; \quad i = 1, 2 \). For a minimum three-particle arched bridge with the concentration function \( f(C) = kC^3 \), the solution given by formula (8) can be written in explicit form

\[ C(x, 0) = \frac{1}{\sqrt{(1 + k)e^{2\lambda x} - k}} . \]  

(13)

Substitution of (6) into equation (5) and expression of the total deposit \( S \) gives

\[ S = \lambda + \frac{\partial C}{\partial x} \sqrt{(C + kC^3)} . \]  

(14)

Differentiate (14) and substitute into equation (5):

\[ \frac{\partial C}{\partial x} + \frac{\partial}{\partial \tau} \left( \frac{\partial C}{\partial x} \sqrt{(C + kC^3)} \right) = 0 . \]

Change the of differentiation order yields

\[ \frac{\partial C}{\partial x} + \frac{\partial}{\partial \tau} \left( \frac{\partial C}{\partial x} \sqrt{(C + kC^3)} \right) = 0 . \]  

(15)

Integrating over the variable \( x \) and using condition (4), we obtain the first order equation

\[ C + \frac{\partial C}{\partial \tau} \sqrt{(C + kC^3)} = 1 . \]  

(16)

Integrating equation (16) with the condition \( C|_{\tau=0} = C(x, 0) \) where the function \( C(x, 0) \) is determined by formula (13) gives the solution in implicit form

\[ F\left(C(x, \tau)\right) = F\left(C(x, 0)\right) + \tau , \]  

(17)

where

\[ F(C) = \ln C - \frac{1}{1 + k} \ln(1 - C) - \frac{k}{2(1 + k)} \ln(1 + kC^2) + \frac{\sqrt{k}}{1 + k} \arctg(\sqrt{k}C) . \]

The total deposit \( S(x, \tau) \) is determined by equation (14), and the partial deposit concentrations are given by the formulas
\[ S_l(x, \tau) = \int_0^\tau (\lambda - S(x, \vartheta)) C(x, \vartheta) d\vartheta; \quad S_s(x, \tau) = k \int_0^\tau (\lambda - S(x, \vartheta)) C^3(x, \vartheta) d\vartheta. \]  

(18)

For large \( \tau \), the solution tends to limit values \( \lim_{\tau \to \infty} C(x, \tau) = 1; \lim_{\tau \to \infty} S(x, \tau) = \lambda \).

3.3 Asymptotic solution

Asymptotic solution is constructed in powers of a small parameter \( k \). The solution (13) on the concentration front in asymptotic form

\[ C(x, 0) = e^{-\lambda x} + e^{-3\lambda x} - e^{-\lambda x} \frac{k}{2} + O(k^2). \]  

(19)

From (18) it follows that the principal asymptotic terms of the total deposit \( S \) and the partial deposit \( S_l \) coincide. The asymptotic solution has the form

\[ C = C_0(x, \tau) + O(k); \quad S = S_l = S_0(x, \tau) + O(k); \quad S_s = kS_l(x, \tau) + O(k^2). \]  

(20)

Substitution of expansions (19), (20) into (17), (18) gives the equations for the principal terms of the asymptotics

\[ \ln \left( \frac{C_0}{1-C_0} \right) + \ln \left( e^{\lambda x} - 1 \right) = \tau; \quad S_0 = \lambda + \frac{\partial C_0}{\partial x}; \quad S_l = \int_0^\tau (\lambda - S_0(x, \vartheta)) C_0^3(x, \vartheta) d\vartheta. \]  

(21)

From (21), the asymptotic expansions of the suspended and retained particles concentrations are obtained

\[ C(x, t) = e^\tau + O(k); \quad S = S_l = \lambda e^\tau - 1 + O(k); \]  

\[ S_s = \lambda k \left( \frac{e^{\tau} - 1}{G} + e^{3\lambda} (e^{\lambda x} - 1) \left( \frac{1}{G^2} - e^{-2\lambda x} + \frac{e^{-3\lambda x}}{3G^3} \right) \right) + O(k^2); \]  

where \( G = e^\tau + e^{3\lambda} - 1 \).

4. Numerical calculation

Numerical calculation of the asymptotics at the porous medium outlet \( x=1 \) is performed for \( \lambda = 1, k = 0.01 \).

Figure 2 shows a graph of the suspended particles concentration. At the initial time, the filter contains no particles and the concentration is zero. The concentration front moves in a porous medium at the velocity \( v = 1 \) and reaches the outlet at the moment \( t = 1 \). At this moment, the concentration of suspended particles changes abruptly, then increases and tends to the limit value \( C = 1 \). According to the graph, the limit value is practically reached at the moment \( t = 7 \). This means that all suspended particles entering the porous medium at \( t \geq 6 \) are not retained and do not form a deposit.
The main asymptotic term of the total retained particles concentration $S$ and the partial retained particles concentration $S_1$ are zero before the concentration front at $t < 1$, increases behind the front at $t > 1$, and tends to the limit value $S_{\text{max}} = 1$. Figure 3 a) shows that the deposit almost reaches the limit value at the filter output at $t = 7$. The asymptotics of the retained particles concentration $S_2$ formed by arched bridges (Figure 3 b)) increases as $t > 1$ and exponentially approaches its limiting value.

5. Results and discussions

In the proposed filtration model, the suspension flow in a porous medium with two particle capture mechanisms is considered. Single particles are blocked in small pores, the size of which does not allow the particles to move beyond the pore neck. The capture of a particle groups with the formation of arched bridges occurs at the entrance of pores, whose dimensions exceed the diameter of a single particle. Such pores freely pass individual suspended particles and are blocked by bridges of several particles. The arched bridge rests on the edges of the pore and overlaps its inlet. It blocks the flow of fluid through the pore, which provides the structure with additional stability: free suspended particles of the suspension do not press on the bridge. Two mechanisms of particle retention act independently, since the capture of single particles and particle groups occur in pores of different sizes.

The model considered assumes the constancy of the accessibility factor of pores and the fractional flow of particles of a porous medium. In the general case the accessibility factor and the fractional flow depend on the retained particles concentration, and equation (1) contains additional variable coefficients [21]. For a monodisperse suspension with size-exclusion, the exact and asymptotic solutions of the general filtration problem were obtained in [22, 23]. A similar problem for a model with two capture mechanisms requires a separate study [24].

The model hypothesis assumes that the particles stick in the pores irreversibly. However, at high flow rates and with a large salinity of the fluid, the particles come off from the pore throats increasing the concentration of suspended particles [25]. In order to take this effect into account, the additional term responsible for the particle release must be added to the right-hand side of the mass transfer equation.
6. Conclusions
The nonlinear mathematical model describes the motion of a monodisperse suspension in a porous medium, accompanied by the formation of a deposit caused by two different particle capture mechanisms: size-exclusion and arched bridging.

The solution has a discontinuity on the concentrations front of the suspended particles due to inconsistency of boundary and initial conditions. Before the front, the solution is zero, the solution behind the front is positive. On the concentration front and at the porous medium inlet, an exact analytical solution has been obtained. For constant filtration functions, the solution is constructed behind the concentration front.

In the case of a linear filtration function, an exact solution of the filtration problem is obtained. For a low concentration function responsible for pore blocking with arched three-particle bridges, an asymptotic solution is constructed. Numerical calculations show that the asymptotic solution quickly approaches the limiting values.

The exact and asymptotic solutions can be used in the analysis and processing of data from laboratory and field experiments [26, 27].

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