We present a new technique for the calculation of helicity $\pm 1/2$ gravitino production during preheating. It is based on the equivalence between goldstinos and helicity $\pm 1/2$ gravitinos at high energies. The problem is thus reduced to the standard (Majorana) fermion production after inflation. Comparison with the results obtained in the unitary gauge is also presented.

1 Introduction

From supersymmetry it is possible to build models of inflation in a relatively natural way. Since scalar fields appear as superpartners of fermionic matter fields, there is no need to introduce them \textit{ad hoc} as in other models. In addition, the potential energy of those scalar fields typically contain flat directions which make them natural inflaton candidates. Moreover, due to the nonrenormalization theorems, those flat directions are not spoiled by radiative corrections.

However, when promoting supersymmetry to a local symmetry (supergravity), new problems may appear. First, it is known that supergravity corrections can modify the slow-roll parameter $\eta$, thus spoiling the inflationary period (at least in minimal sugra models). Another problem that supergravity creates is related to the superpartner of the graviton field, the gravitino.

Gravitinos are weakly interacting particles with very long lifetimes (for a typical mass around 1 TeV, gravitinos can live as long as $10^5$ s). This implies that their decay products could destroy the nuclei created in the nucleosynthesis period. This imposes very stringent constraints on the primordial abundance of gravitinos. Thus, for instance, for $m_{3/2} \simeq 1$TeV, the number density to entropy density ratio should satisfy $n/s \lesssim 10^{-14}$. Since gravitinos can be created after inflation due to particle collisions in the thermal bath generated in the reheating period, this constraint imposes the well-know bound on the maximum reheating temperature $T_R \lesssim 10^9$ GeV.

However, apart from particle collisions, gravitinos can be generated directly from the inflation oscillations in the preheating period. This production is much more efficient than the thermal
one and therefore much more dangerous for nucleosynthesis. Here we review some of our recent results on how to calculate gravitino production during preheating, by means of the high energy equivalence between goldstinos and helicity $\pm 1/2$ gravitinos. For further details and references we refer the reader to that work.

2 Supergravity Lagrangian and gravitino helicities

We will consider minimal supergravity coupled to a single chiral superfield which contains an scalar field $\phi$ (inflaton) and a Majorana spinor $\eta$ (inflatino, goldstino). We give only the form of the fermionic Lagrangian up to quadratic terms in the fields

$$g^{-1/2} L_F = -\frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma + \frac{i}{2} \bar{\eta} \bar{D} \eta + e^{G/2} \left( \frac{i}{2} \bar{\psi}_\mu \sigma_{\mu\nu} \psi_\nu + \frac{1}{2} \left( -G_{\phi\phi} + G_{\phi}^2 \right) \bar{\eta} \eta \right) + \sqrt{2} e^{G/2} \bar{G}_{\phi} \bar{\psi}_\mu \gamma^n \eta,$$

where the Kähler potential is given by $G(\Phi, \Phi^\dagger) = \Phi^\dagger \Phi + \log |W|^2$. Note that the last two terms contain mixing between gravitinos $\psi_\mu$ and goldstinos, and therefore their equations of motion are coupled. In addition, and in order to have a consistent model, we will impose that the inflaton potential energy vanishes at the minimum, so that the cosmological constant is zero. In addition, we will require the derivative of the Kähler potential $G_{\phi\phi}$ to be $\sqrt{3}$ at the minimum $\phi = \phi_0$, i.e. to be different from zero and therefore to have broken supersymmetry. The above Lagrangian together with the corresponding bosonic one display a gauge invariance associated to local supersymmetric transformations. As usual, when the gauged supersymmetry is spontaneously broken, the gravitino field acquires a mass $m_{3/2} = e^{G_0/2}$, by means of the super-Higgs mechanism. Hence, since the gravitino now is a massive spin $3/2$ particle, it also has states with $\pm 1/2$ helicity, in addition to the helicity $\pm 3/2$ states already present in the massless case.

As long as we are interested in the production of gravitinos during the preheating era at the end of inflation, we will consider the equations of motion, derived from the above Lagrangian, in a Friedmann-Robertson-Walker background and when the inflaton is a homogeneous field only depending on time. Then, it can be seen that the equations of motion for $\pm 3/2$-helicity gravitinos reduce to the Dirac-like equation

$$\left( i \bar{D} - e^{G/2} \right) \psi_{\mu}^{3/2} = 0$$

Thus, the production of helicity $\pm 3/2$ gravitinos can be treated as the production of Dirac fermions during preheating. However, the helicity $\pm 1/2$ equation is still coupled to the goldstino and very complicated. Two approaches can be followed in this case. In the first one, the gauge is fixed by imposing $\eta = 0$, so that the goldstino disappears from the Lagrangian, and we are left with the equations of motion of a pure spin $3/2$ spinor. This gauge is called unitary since the unphysical degrees of freedom are not explicit. The second approach, which is the one we are going to present here, is based on the so-called $R_\xi$ gauges, where it is possible to find a high energy relation between the helicity $\pm 1/2$ gravitinos and goldstinos. Thus we will be able to calculate the helicity $\pm 1/2$ gravitino production from the much simpler production of goldstinos.

3 Gauge fixing and the equivalence theorem

Let us consider the following gauge condition:

$$\gamma^\mu \psi_\mu - \frac{1}{\sqrt{2} \xi} e^{G/2} G_{\phi} \eta + \frac{i}{G_{\phi}} e^{-G/2} \gamma^\mu (\bar{\phi} \phi) \psi_\mu = 0.$$

(3)
where \( \xi \) is an arbitrary parameter. We will assume in the following that the space-time is asymptotically flat and that also asymptotically in \( t \to \pm \infty \) the inflaton field settles down at the minimum of the potential \( \phi \to \phi_0 \). With these conditions we see that in those asymptotic \( \text{in}, \text{out} \) regions, the above condition simplifies to

\[
\begin{align*}
a_{\text{in}, \text{out}}^{-1} \partial_\tau \psi_\mu &= \sqrt{\frac{3}{2}} \left( m_{3/2} / \xi \right) \eta, \\
\end{align*}
\]  

where \( a_{\text{in}, \text{out}} \) are the asymptotic values of the scale factor. If we use the equations of motion for gravitinos and goldstinos, this gauge condition can be rewritten as

\[
\partial_\mu \psi_\mu = \sqrt{\frac{3}{2}} \frac{m_{3/2}}{\xi} \eta, 
\]

where we have redefined \( m = a_{\text{in}, \text{out}} m_{3/2} \) and \( m_\pm = m (1 \pm \sqrt{1 - 3/(2\xi)}) \). Note that there are two different relations for goldstinos with masses \( m_- \) and \( m_+ \), but the important point is that the derivative of the gravitino field is proportional to the goldstino.

Let us then introduce the equivalence theorem. In the asymptotic regions that we mentioned before it is expected that the general solution of the equations of motion for gravitinos and goldstinos can be written as linear superpositions of plane waves. In fact, at high energies, since the effects of the difference in masses between gravitinos and goldstinos is negligible, the solutions are

\[
\psi_\mu^p(x) = \frac{1}{a^{3/2} \sqrt{2\omega}} e^{i p x} \tilde{\psi}_\mu^p(p) + \mathcal{O} \left( \frac{m}{\omega} \right), \quad \eta^p(x) = \frac{1}{a^{3/2} \sqrt{2\omega}} e^{i p x} \eta(p) + \mathcal{O} \left( \frac{m}{\omega} \right),
\]

where we assume \( p_\mu p^\mu = m^2 \) and \( \omega \gg m \). Again at high energies, the helicity \( \pm 1/2 \) projector for gravitinos is

\[
P_{\mu}^{\pm 1/2} = \sqrt{\frac{2}{3}} P_\pm \frac{p_\mu}{m} + \mathcal{O} \left( \frac{m}{\omega} \right),
\]

so that, in momentum space, the helicity \( \pm 1/2 \) component is given by

\[
\tilde{\psi}^{\pm 1/2}(p) = P_{\pm 1/2} \tilde{\psi}_\mu^p(p) = \sqrt{\frac{2}{3}} P_\pm \frac{p_\mu}{m} \tilde{\psi}_\mu^p(p) + \mathcal{O} \left( \frac{m}{\omega} \right).
\]

We see that up to a correction negligible at high energies, the helicity \( \pm 1/2 \) gravitino is proportional to \( \partial_\mu \psi_\mu \), but from (5) this in turn is proportional to the goldstino field. Therefore we can write

\[
\tilde{\psi}^{\pm 1/2}(p) = \sum_{\pm,-} \left[ -i \frac{1}{\xi} \left( 1 - \xi \frac{m_{\pm,\mp}}{m} \right) P_{\pm 1/2} + \mathcal{O} \left( \frac{m}{\omega} \right) \right] \tilde{\eta}^{\mp,-}(p).
\]

This equation still relates the \( \pm 1/2 \) helicity gravitino with the two different goldstino solutions, either with \( m_- \) or \( m_+ \). However, by choosing \( \xi = 3/2 \) we obtain \( m_- = m_+ \) and there is a unique high energy relation between \( \pm 1/2 \) helicity gravitino and goldstinos. Another, even simpler, possibility is to choose the Landau gauge \( \xi \to \infty \), where there is only one \( m_+ \) solution. In this gauge, we can then write the equivalence theorem as

\[
\tilde{\psi}^{\pm 1/2}(p) = \left[ 2i P_\pm + \mathcal{O} \left( \frac{m}{\omega} \right) \right] \eta(p).
\]
4 Particle production

The previous expression valid in the asymptotic regions is sufficient to relate the production of helicity $\pm 1/2$ gravitinos to the production of goldstinos. In fact, let us consider a pure positive(negative) frequency mode solution for goldstinos in the $in$ region

$$\eta^p_i(x) \rightarrow \frac{1}{a^{3/2}_{in}} \frac{e^{i\omega_{in}t-i\vec{p}\cdot \vec{x}}}{\sqrt{2\omega_{in}}} u(\vec{p}, l). \quad (11)$$

Because of the presence of the oscillating inflaton field and the space-time curvature, this solution will no longer behave as pure positive(negative) frequency mode in the $out$ region, but it will be a linear superposition of positive and negative frequency modes

$$\eta^p_i(x) \rightarrow \frac{1}{a^{3/2}_{out}} \frac{e^{i\omega_{out}t-i\vec{p}\cdot \vec{x}}}{\sqrt{2\omega_{out}}} \left( \alpha^G_{p,l} e^{i\omega_{out}t-i\vec{p}\cdot \vec{x}} u(\vec{p}, l) + \beta^G_{-p,l} e^{-i\omega_{out}t+i\vec{p}\cdot \vec{x}} u^G(-\vec{p}, l) \right), \quad (12)$$

where $\alpha^G_{p,l}$ and $\beta^G_{-p,l}$ are known as Bogolyubov coefficients. Since the Fourier modes of goldstinos are related to the Fourier modes of gravitinos in the asymptotic regions, using (11) we can find a relation between the particle numbers of helicity $\pm 1/2$ gravitinos and goldstinos:

$$N^L_{p,l} = \left[ 1 + \mathcal{O}\left( \frac{m_p}{\Lambda} \right) \right] |\beta^G_{p,l}|^2. \quad (13)$$

Thus in order to obtain the helicity $\pm 1/2$ gravitino production $N^L_{p,l}$, we only need to know the goldstino coefficients $\beta^G_{p,l}$. With that purpose we have to solve the equation of motion for the goldstinos. However in the Landau gauge the equation of motion of the goldstinos reduces again to a Dirac-like equation, in particular

$$i \not\!D \eta - e^{G/2} \left( G,_{\phi\phi} + G,_{\phi} \right) \eta = 0. \quad (14)$$

This is an additional reason to use the Equivalence Theorem in the Landau gauge. The problem of helicity $\pm 1/2$ production is thus reduced as in the helicity $\pm 3/2$ case to a Dirac fermions calculation. In order to check these results, we can see that in the limit $|\phi| \ll M_P$, the equation for the goldstinos in the Landau gauge (14) can be approximated by

$$i \not\!D \eta - (\partial_\phi \partial_\phi W) \eta = 0, \quad (15)$$

which is the equation obtained in the unitary gauge for the helicity $\pm 1/2$ gravitinos (in the global supersymmetric limit). Therefore, the number of goldstinos, calculated in the Landau gauge, is the same as that of helicity $\pm 1/2$ gravitinos, calculated in the unitary gauge.

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