Learning by experimenting as a good way to effective and student friendly mathematics education – experiences from Young Explorer's Club

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Abstract. Students in many countries have problems with learning of mathematics. Many students do not like mathematics. Teachers and parents are mostly aware that the style of teaching of mathematics and “the atmosphere” around mathematics are the main reasons for this state. For the society it is clear that something must be changed. Maybe methods of teaching, maybe the whole system? Copernicus Science Centre in Warsaw 18 years ago proposed the idea of Young Explorer's Club (YEC) as the environment where students can discover knowledge by making experiments. Learning by experimenting, by own activity of learners makes the whole educational process more interesting and less frustrating for students. This also applies to mathematics education. Experiences from mathematical Young Explorer's Clubs established at the Centre for Creative Mathematics Learning at the Faculty of Mathematics University of Białystok as the possible way to more effective mathematics education will be presented in the paper. Experimenting and using functional strategy of teaching of mathematics is the basic form of activity in the Clubs.

1. Introduction
Students in many countries have problems with learning of mathematics. A lot of students do not like mathematics. It is necessary to answer the question: Why does mathematics education cause so much problems? Why students do not like to learn mathematics?

At the Faculty of Mathematics, University of Białystok, Poland, lecturers cooperate very strongly with teachers and students from schools of all types on all educational levels and during this cooperation some problems with mathematics education were identified. The main of them (from students' point of view) are the following:

- too abstract (for many students) character of mathematics;
- lack of convincing about the usefulness of mathematics;
- image of mathematics as a set of separate theorems without any connections between them;
- learning to the memory;
- superficial learning, lack of the ability of the concentration on the text;
• lack of the self-confidence and lack of belief in the possibility of successful learning mathematics;
• the style of the lessons where teachers play the leading role with spoon-feeding teaching methods; students' role is to memorize knowledge and solve exercises;
• dissonance between freedom of access to information in everyday life and the school environment where modern technologies are still much less frequented among teaching/learning resources.

Conclusion from this list of students' opinions? It is necessary to change the style of mathematics education.

2. How to change the style of mathematics education – a proposal
At the first edition of The Cognitive Adventures Conference [11] organized by Copernicus Science Centre in Warsaw in 2015 there was given good proposal for this. One of the speakers proposed to arrange education (not only mathematics education) according the following:

• inspiration versus information;
• exploration versus explanation;
• self-motivation versus obligation.

It is known very well that now in schools mostly information, explanation, obligation is given our students. What will happen if we give them inspiration (to discovering knowledge), good conditions for exploration (in order to discover knowledge) and self motivation (instead of fear of exam or bad mark)? What will happen if we propose our students diversity of experience instead of learning theorems and formulas in memory? What will happen if we propose our students “learning by doing”, learning by activities – different activities, both physical and mental?

“Teaching is really about inspiration, not information. Effective teaching focuses on why and how, not what. The goal should be to spark each student’s imagination, to find a hook in their heart and mind so that they feel a need to learn the material. The rest is easy, because the student then drives his or her learning. My role as a teacher is to ask provocative questions, and to help the students make a path toward the answers. If they are motivated to find the path, they will carve it themselves. If I have to pull out a mental machete to expose the path, then I haven’t done my job.” – Tina Seelig says in the interview “Teaching – it's about inspiration, not information” [7].

Is it possible at all?

3. Strategy of functional teaching of mathematics
Experiences show that it is possible, especially that we have strong theoretical, scientific bases for learning by doing. As the theoretical background of this kind of mathematics education the following theories can be taken: theory of constructivism; strategy of functional teaching of mathematics; problem solving method.

Many teachers and organizers of educational process forgot already about strategy of functional teaching of mathematics, created by Zofia Krygowska, Polish great specialist in didactics of mathematics, and strongly popularized by Helena Siwek, also specialist in mathematics education. It is a pity, because this strategy is a very good methodological background for recommended changes in mathematics education (and sometimes in other subjects of education too).

In one of her most important books „Outline of the didactics of mathematics” Zofia Krygowska gave some advices that can lead to raising students' creativity and independence in learning mathematics, for example: “Putting the student in conflict situations in which the adopted patterns of conduct fail and in which the student must transform (adapt) the old pattern or develop a new one. (…) Consistent learning of free use of known operations and getting the student accustomed to the fact that only specific action, not just passive contemplation and waiting for "inspiration" leads to the solution of the problem”. [5]
Strategy of functional teaching of mathematics recommends using three kinds of activities in the process of formulating mathematical concepts \[5\], \[8\]:

- concrete activities (on real objects) – enactive representation (action-based) in Bruner's Learning Theory in Education;
- imaginary activities (on pictures, patterns and other graphic images of ideas) – iconic representation (image-based) in Bruner's Learning Theory in Education;
- abstract activities (on mathematical symbols) – symbolic representation (language-based) in Bruner's Learning Theory in Education.

In such an approach to mathematics education, the student participates in the whole process of formulating a particular concept, so understands this concept and will remember it, because they will remember the process and reasoning that led them to constructing this concept.

And such an approach fits very well to the “observe, measure, compare, conclude” method: “observe, measure” – concrete activities; “compare” – very frequently activities on pictures, patterns and other graphic images of ideas; “conclude” – abstract activities.

Strategy of functional teaching of mathematics is strongly connected with problem-solving methods and realistic mathematics teaching. Concrete activities mean experimentation. We do experiments in order to solve some problems; that means that we use problem-solving method and organize investigative work for students. Doing experiments leads to discussions, arguing, and making conclusions. It means also using a research method – it can be useful in education of many school subjects.

Using problem-solving methods require very frequently correlations between school subjects; we solve not only mathematical problems, but problems from other fields of science that often need mathematics. It is a very good way to make our students convinced that mathematics is useful everywhere, also in making analysis of the results of experiments and concluding about trends.

Of course, using strategy of functional teaching of mathematics, implementing problem-solving methods, inspiring students to investigative work requires from teachers creating proper didactic situations that consist of properly selected didactic tools (research tools for students) and an atmosphere of scientific work, full of inspiring questions, discussions, arguing, concluding. As for research tools, we can use any proper tools, both manual and technological. Usually the main aims of using technology are to make simulations in research work and to make analysis of the results of experiments in order to conclude about trends. \[6\]

4. Mathematical experiment

Many teachers ask: what is “mathematical experiment”? We can imagine experiment in biology, chemistry, physics, but in mathematics? Is it possible to make experiments at the lesson of mathematics?

Yes, it is. Probably the first man-made experiments are due to the time of Galilei (for instance experiments with pendula). In mathematics, models and instruments became important in the 19th century. In the last years, some mathematical exhibitions (and not only mathematical ones) have been developed and science centers have been opened. Mathematics is typically not presented there in the traditional way using the mathematical language. Visitors find there exhibits, in which they may see or explore mathematics. Visitors are challenged to perform “mathematical experiments”. Also some mathematical experiments have been known for a long time, mostly under the name of “mathematical games”. \[1\]

All of us know some of these experiments related to mathematics, for example Pythagoras puzzle (in order to prove Pythagoras theorem), origami (in order to create beautiful models of solids) etc. But many people (especially mathematicians who deal with pure mathematics, but also teachers and parents) ask two fundamental questions:

1) Are these experiments at all?
2) Is this at all mathematics?

Albrecht Beutelspacher in \[1\] asks these questions and answers first question in the following way:
“One of the main features of mathematics is that the truth of an assertion is obtained by a proof, that is by purely logical arguments, and not, for instance, by experiments. This distinguishes mathematics from sciences such as physics or chemistry, where experiments are used to verify a theory or to falsify a wrong hypothesis. Also, mathematical experiments are not used to simply illustrate a definition or a theorem. The role of a mathematical experiment is quite different. Its basic property is to stimulate thinking. In science centers, experiments do not come second (after a theory), but experiments come first. They provide a strong impulse. Basically, a person working with a mathematical experiment is challenged by a mathematical problem. As in research, one has to get the right conception, the right idea of what’s going on. And sometimes, after a while of thinking, and sometimes with luck, one finds the solution. (…) To put it short, a mathematical experiment works “bottom-up”: starting from experience, leading to insight. It is an impulse. If the experiment is good, this impulse is so strong that it enables the visitor to ask the right questions, to get the right conceptions and, finally to get by an “Aha-moment” the right insight.”

As the answer the second question he writes: “Certainly, it does not look like mathematics, in particular not like school mathematics. In fact, in Mathematikum we explicitly stated at the beginning that we want to make a place that doesn’t look like school. Mathematical experiments do not show the mathematical language: no point is called “P”, no variable is called x, in fact, there are no formulas. Also, no definitions, no theorems, no proofs. On the other hand, an important part of mathematical activity is clearly present, namely problems. And, if visitors solve the problems, they activate mathematics-related competences, such as arguing, and communicating. (…) To sum up, working with mathematical experiments is a first step into mathematics.”

Let’s repeat our question asked at the beginning of the section: Is it possible to make experiments at the lesson of mathematics?

Strategy of functional teaching of mathematics recommends starting from experimenting by doing concrete activities. In teaching “school mathematics” all activities of students and teachers must lead to possessing mathematical knowledge and skills. In this situation (according to the theory of functional teaching) experiments (concrete activities) are arranged in order to solve some mathematical problem and acquiring some new knowledge. In this sense of the whole process experiment means the beginning of investigative work of student, the beginning of research process. The structure of the research process (observation – curiosity – problem formulation – assumption – hypothesis verification – application) fits very well to the strategy of functional teaching: curiosity leads to setting problem, then students set the hypothesis, then verify their hypothesis by making experiment on real objects, then formulate conclusions in the language of mathematics (construct new knowledge) with the help of iconic representations, symbols and abstract thinking.

5. The idea of Young Explorer’s Club

Teachers say that they have no time for such way of teaching, because of many pupils in the group, too less number of lessons per week, necessity of good preparing to exams… They often say: “How can I change the style of lesson if I have to prepare my students to exams, to explain them everything, to teach them how to solve exercises of types that occur in tests and have so little time for this?”

But it is necessary to change something if we want to make students not to be afraid of mathematics and (it would be so good!) to like mathematics.

In this situation extracurricular classes organized in the schools or in any other places can be some solution of the problem.

Copernicus Science Centre in Warsaw 18 years ago proposed the idea of Young Explorer’s Club (YEC) as the environment where students can discover knowledge by making experiments.

“Young Explorer Club means meetings, gripping classes and atmosphere! Children and young people together experiment under the watchful eye of Supervisors. They gain knowledge on their own. All over Poland and abroad there are several hundred clubs. The Copernicus Science Centre, the coordinator of the programme, enhances the development of the YEC with the support of the Strategic Partner, the Polish-American Freedom Foundation.” – we can read at the website of this program. [2]
What is important in the idea of Young Explorer Club?

- During meetings a personal commitment is most important.
- Members of the Clubs are seeking subjects interesting for them by themselves, instead of looking into the textbooks.
- Participants are looking for solutions of the problems by experimenting.
- Research method is a core method; it helps to develop a lot of competence and abilities in the same time, allows for crossing school borders between objects and shows that making mistakes is valuable and teaches to solve problems.
- It is possible to examine phenomena from different fields.

In Poland there are more than 700 Clubs, in Georgia – more than 100, now this idea starts in Ukraine and Armenia. Most of these Clubs are based on biology, chemistry and physics.

6. Mathematical Young Explorer's Clubs at the University of Białystok

In order to give some support for those children and young people who are interested in discovering mathematics by themselves in June 2019 we (lecturers from Faculty of Mathematics) established at the Centre for Creative Mathematics Learning (Faculty of Mathematics, University of Białystok) two Clubs: “We, trackers of mathematics” (for kids from primary schools) and “We, mathematicians” (for students from secondary schools). During meetings participants do manual activities in order to explore new for them topics or applications of mathematics.

For the first meeting kids came with parents, grandparents or teachers. Especially parents and grandparents were very happy that in Białystok will be the place where their children/grandchildren will be able “do something concrete in mathematics, not only solve exercises from textbook” – as some of them said.

Meetings of Clubs at the Faculty of Mathematics, University of Białystok, take place one time per month for each Club. Parents participate very often in the meetings of children from primary schools and solve the same problems that kids solve. And they have a fun from this and very often say: “I never learned mathematics in that way! I had new experience today and understood the topic!”

Books by Ian Stewart are very good source of topics for investigative work that leads to mathematical discoveries [9], [10].

There are two main principles of arranging workshops in the Clubs at the Centre of Creative Math Learning:

1) to inspire students to discover mathematics by experimenting with using real objects;
2) to help them in learning “school mathematics” – it means that all their activities in the Clubs are somehow connected to the topics from school mathematics and help them to understand these topics better.

There are two reasons for these principles:

1) to convince students, parents and teachers that mathematics can be learned by experimenting and thinking, not only by listening and solving exercises from textbooks;
2) to start the process of transfer methods of learning and teaching from YEC to the lessons of schools, because we want to help teachers and students in more effective and less frustrating mathematics teaching and learning.

Now Copernicus Science Centre pays big attention to the process of transfer methodology of experimenting from YEC to schools.

At the beginning our intention was to arrange workshops for students from secondary schools on more advanced topics, so first topic was the introduction to topology, but during the school year these students found out topics of workshops for kids from primary schools, found them very interesting and asked to work on the same topics, so some topics were proposed for participants of both groups. Also it happened that students asked to explain them topic from the lesson of the school (for example after first lesson in the school about trigonometric functions they were so interested in the topic that asked to tell them about practical applications of these functions and we devoted all meeting of the Club to this matter).
7. Selected topics of Clubs' meetings

Below the selected topics of workshops with methodical comments and illustrating pictures are presented. In two cases using Strategy of functional teaching of mathematics is described in more detailed way. The workshops for Club “We, mathematicians” were conducted by author of this paper, for Club “We, trackers of mathematics” by author and Dr. Justyna Makowska, lecturer in Faculty of Mathematics together. Presented materials are structured in chronological order.

7.1. Is the coffee mug always different from bagel? – introduction to topology.

The sketch of scenario for the workshop for YEC “We, mathematicians”, carried out on June 11, 2019 at the Centre for Creative Mathematics Learning at the University of Białystok. Students of secondary schools (technical schools) from Białystok participated in the classes.

Aim of the workshop:

Who has not heard the term "computer network topology"? We know from computer science lessons that it's just a way of connecting computer devices. What does the word "topology" mean? During the workshop, we will investigate whether diametrically opposed – it would seem – different subjects have something in common and in this way we will learn the basics of a very important branch of mathematics, i.e. topology. Next, we will consider if the topology is visible around us.

Materials: plasticine, drawing supplies (a sheet of paper and a pen will be enough).

Introduction (arousing cognitive curiosity): A topologist is jokingly described as a mathematician who cannot distinguish a coffee mug from a bagel. Why? – a short discussion on this topic, leading to interest in the topic rather than answering the question, because the very definition of the topologist was very surprising for the students.

Short introduction to the concept of topology: Topology is a branch of mathematics that studies properties that do not change even after objects have been radically deformed (geometric figures, solids, and objects with more dimensions). Such properties are called topological invariants, and deformation is understood as meaning any deformation (bending, stretching, twisting), but without tearing and sticking together various parts. The deformation process is easiest to imagine assuming that the object is made of rubber.

Problem 1: How can you find out if a bagel is the equivalent of a cup (topologically)? Brief brainstorming on research strategies and working with plasticine. This stage of work is illustrated by the photo below:

![Figure 1. Students try to transform mug into bagel with using plasticine.](image)
Discussion and conclusion: Yes, we can turn a plasticine cup into a bagel without tearing it apart and sticking the parts together! We know how to make topological transformations!

Problem 2: Geometric concepts such as length, area, parallelism, and perpendicularity are not topological properties. Why?
Discussion on this topic supported by experiences with plasticine. The answer to the question was not a problem for the students. They quickly discovered that, for example, when stretching an object, the original dimensions were not preserved.

Problem 3: The dimension is the topological property. Why?
And here problems appeared – mainly with understanding the concept of dimension, which was a surprise to me. We explained to ourselves, using sheets of paper, pens, a blackboard, chalk, drawing appropriate figures, that a point is a zero-dimensional figure (it has no dimensions, we do not measure it in any way), a straight line, a half-line, a line segment are one-dimensional figures (we measure one dimension of the segment, i.e. length, in the case of a straight line or a half-line this "length" is infinite), a rectangle is two-dimensional (we measure its length and width, i.e. it is characterized by two numbers), a cuboid is three-dimensional (we measure its length, width and height, i.e. it is characterized by three numbers). Of course, we also talked a bit about 2D and 3D. Once we explained the concept of dimension, the following problem arose: can plasticine still be a good research tool for us in this exercise, if a point has no size, a straight line has only one dimension, and we can only create three-dimensional material objects from plasticine – yes, even if we they tried very hard, we will not create a completely flat object without thickness. The conclusion from the discussion was as follows: here plasticine will not be useful to us. We have to use our imagination and think abstractly. Using just abstract thinking, students said that dimension is a topological property, because with the transformations allowed by the topology, we will not obtain an object with a given dimension of an object with a different dimension: the straight line will always remain one-dimensional, and the rectangle will not "grow" a cuboid.

Problem 4: Are the circle and the triangle topologically equivalent? And the circle and the triangle? Why or why not?
There was no trouble here anymore. Plasticine came in handy, the circle was quickly transformed into a triangle, both figures are two-dimensional, so they are equivalent in a topological sense. In the case of the circle and triangle, the answer was no. Perhaps the Readers will wonder why?
At the end of the class, we talked a bit about topology applications and it turned out that students were familiar with the term "topology" in relation to, for example, the GIS system, but they never had any associations with mathematics on this occasion.
Summary of the lesson (from the students' side): We were discovering strange mathematics today, but it was interesting!

Methodical note:
During the classes a clear application of the strategy of functional learning of mathematics appeared: there were concrete activities (on objects made of plasticine), imaginary activities (in drawings, when we explained the issues related to the concept of dimension) and abstract activities (performed only in our minds when explaining why dimension is a topological property and we did not need concrete activities because it was not possible to use plasticine). All three kinds of activities were necessary for the students to discover new knowledge and draw conclusions. Overall: It is strongly encouraged for teachers to use a functional strategy. Students of all ages like to do concrete activities. Expecting from them only abstract activities often causes a lack of understanding of the subject and builds a picture of mathematics as a field inaccessible to the average student.
7.2. How to make “perpetual calendar” from two cubes?
Workshop for kids from primary school, participants of the Club “We, trackers of mathematics”, September 30th, 2019.

In 1957, John Singleton patented a desktop calendar that was supposed to show each day of the month from the 1st to the 31st with two cubes, but allowed that patent to expire in 1965. There are six figures on each cube, one on each side. You can place cubes with any face facing forward and in any order. What should the numbers on each of the cubes be? [9]

From mathematical side of view the problem is connected to combinatorics.

![Children solving the perpetual calendar problem](image)

**Figure 2.** Children solve the problem of eternal calendar. The main problem is: what digits should be on the sides of cubes if we want to express number of each day from each month?

After the workshop one father said: “My son has at home such calendar, but he never wondered what the mathematical principle of its construction was. Now he experienced this on his own.”

7.3. Is it possible to transform Greek cross into a square?
Workshop for kids from primary school, participants of the Club “We, trackers of mathematics”, October 23rd, 2019.

![Greek cross](image)

**Figure 3.** Greek cross.

Problem: Is it possible to transform it into a square?

Problem solving objectives:
- deepening students' knowledge of geometry;
- training problem-solving skills;
- developing the ability to build a strategy, plan implementation, critical approach to the result of your actions;
- developing the ability to discuss, argue, justify one's opinion.
Concrete activities:
First, students try to make a square from the cut parts of the Greek cross:

![Concrete activities](image1)

**Figure 4.** Concrete activities at first.

Imaginary activities:
Students present their ideas on the blackboard and try to justify that the presented "puzzle" is indeed a square (the figure on the right side of the blackboard):

![Imaginary activities](image2)

**Figure 5.** Activities on the pictures and discussion about proving our thesis in next steps.

Abstract activities:
We are discussing the transformation that actually leads to the formation of a square (figure in the center of the board). We have to show that the "cut" fragments "fit" to the places where we want to place them, that is – mathematically – that the two triangles are congruent. And here the problem arises if we conduct classes in a group of different age groups. During the reported classes, students aged 7 to 13 were present, so not all of them knew the concept of congruent figures. This concept had to be introduced on the basis of everyday situations, close to the kids. And it really happened: the participants assimilated the concept of congruent figures (alternatively: identical) on the example of patterns of the part of the dress that the tailor sews, closely matching the template placed on the fabric. Determining the congruence properties of triangles was already simple (What does it mean that the triangles are identical? What does it mean for their sides and for their angles?) Based on the formulated congruence properties, we reasoned that the two triangles shown in the figure are identical.
Now it had to be justified that the "puzzle" was a square. We had to show that all sides of the resulting quadrilateral are equal and all angles are right. In the case of justifying the equality of the sides, the situation is simple: we use the congruence of triangles. With angles, the situation is more complicated. For children in younger age, intuition and measuring can be used, and for the older ones, operations on the measures of angles expressed generally, with the help of symbols.

Classes on the same topic were also conducted with secondary schools students, members of the Young Explorer's Club "We, mathematicians". Generating ideas for turning the Greek cross into a square did not go any better here than it did for primary school children, but justifying that we got a square was much smoother because the students knew the concept of congruence, the Pythagorean theorem, and were more skilled at angular measurements. Moreover: one of the students proposed to prove that both figures (the original Greek cross and the resulting square) have the same fields, which of course we did.

Reflection:
No student came up with the idea of how to turn a Greek cross into a square – despite being prompted at some point. The drawing in the center of the board was made by the teacher (AR) at the time when it was already known that there were no new ideas from the participants. Why was it so difficult for the participants to develop the concept of transformation? Probably because they are used to drawing geometrical figures in such a way that some sides are parallel to the edge of the page or board, so then "the drawing is decent". This is what they wanted to get the square on the table. Meanwhile, here we got a "slightly rotated" square, that is, it was necessary to break with the stereotype.

7.4. What common have knot tied in the ribbon or strip of paper and an apple?
At the next stage of workshop conducted in October 23rd 2019 we researched what is the result of tying the ribbon? And the paper ribbon? What does a cut apple have to do with a knot on the ribbon?

![Figure 6. Pupils discover pentagon in the knot made from strip of paper.](image-url)
Figure 7. Pupils discover pentagon in the apple. Mathematics in the apple? Yes!

Of course, for students from secondary schools it was necessary to prove that this knot on the strip of paper is a regular pentagon. For some kids from primary school it was first meeting with pentagon, so they counted sides, angles and vertices of the figure and created the name of it (by analogy with the triangle that has three sides, three angles and three vertices).

But for all participants it was really surprise that mathematics can be discovered on the knot made on the ribbon and the same figure is present in the nature.

7.5. Moving butterfly that is we are searching for the centre of gravity
On February 19th, 2020, another class of the “We, trackers of mathematics” Club took place. This time it concerned physics and mathematics and was led by the Dean of the Faculty of Mathematics, prof. Alina Dobrogowska. We used a sheet of paper, coins and a match to build a butterfly, which not only stayed on the fingertip or the blade of scissors, but also moved in this position without losing its balance. Of course, it was necessary to find the center of gravity of the object, and we learned to look for this center by first considering the centers of gravity of various geometric figures. By the way, we discovered several properties of the triangle that were not so obvious “at first glance”.

And the most beautiful thing was that the parents worked and discovered mathematics with the same enthusiasm as their children.

Figure 8. I did this! Butterfly flies on the tip of my finger!
Figure 9. Parents of members of the Club are working with full commitment.

This workshop showed the participants that there are correlations between mathematics and physics and that mathematical discoveries can be useful in physics. Also it is very important that even 7-years-old kids discovered center of gravity of triangle and other figures with using functional learning method, not from definition.

Figure 10. We are looking for the center of gravity of a triangle.
7.6. Clubs during pandemic
During pandemic and quarantine we couldn't have our normal meetings, so each month materials for work in families were sent to the members of our Clubs. Topics of these materials: Geodesic lines; We improve our mathematical intuition and the sense of observation; You can create art as Escher; You can discover properties of polyhedrons as Euler.

Some of these materials were published at the website of YEC as inspiration for all teachers and students from the whole Poland who are interested in the topic [4].

Online meetings of the Clubs were not arranged, because there were many problems in families with online school lessons, so we did not want to overload kids and their parents with additional online activities.

8. Research
We did not conduct any well prepared research on effectiveness of mathematics education in this style that we propose in Centre for Creative Learning of Mathematics – more precisely in our mathematical Young Explorer's Clubs. Our Clubs are relatively young, also pandemic broke our normal functioning, so we even had no time to prepare and conduct full research process. During these few months we observed great interest and enthusiasm of the members of our Clubs and their parents.

On the website of Copernicus Science Centre the report from big research carried out in 2015 “School of experience. Report of study on Young Explorer's Clubs” has been published. There were twelve research questions in this investigation related to different aspects of functioning of YEC: their members, leaders, parents, cooperation with schools, society etc. Among the conclusions we can read that “YEC is the place of meetings of children active and curious about world, interested in original educational methods, not necessarily pupils having the highest evaluations and standing out during normal lessons.” Also “Participants of the study expressed the belief that the participation in YEC can have a positive effect on results in the learning of science in the school and the self-confidence transferring pupils to the higher activity both during lessons, as well as in competitions of different kind.” [3]

We (lecturers from the Centre for Creative Mathematics Learning at the Faculty of Mathematics, University of Bialystok) plan to conduct research on effectiveness of proposed educational method, but at first students have to be allowed to come back to regular meetings in the schools and at the University. Before that we have time to prepare methodology of the research.
9. Conclusions
During several months of operation of our Clubs about 30 members participate in workshops in two groups. Interest of this kind of activity is big, even methodologists from methodology centers for teachers invited the author of the paper to the regional conferences for teachers that always take place before new school year to give some talks on this topic in the autumn 2020, so we expect more participants next school year (if pandemic will stop, of course).

Our experiences gathered at our mathematical Young Explorer's Clubs show that style of learning based on different activities (including manual activities) and investigative work help students to discover mathematics, to understand mathematics and to enjoy mathematics. They start to look at mathematics from quite new side: not as at the domain accessible only for selected people, but as at the domain that can be discovered and is present everywhere around us – even in apples and knots. Also parents are happy that their children found enjoyable way to mathematics and could feel themselves as discoverers.

Now great task before us: to convince the teachers that such methods can be used at the lessons in the schools and to inspire them to implement this style of teaching in their everyday work with students.

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