CPT transformation properties of the exact effective Hamiltonian for neutral kaon and similar complexes.

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Abstract

CPT–symmetry properties of the exact effective Hamiltonian \( H_\parallel \) governing the time evolution in the \( K_0, \bar{K}_0 \) mesons subspace implied by such properties of the total Hamiltonian \( H \) of the system under consideration are examined. We show that \( H_\parallel \) can commute with CPT– operator only if \( H \) does not commute with it. We also find that, in contradistinction to the standard result of the Lee–Oehme–Yang (LOY) theory, \( \text{Re} < K_0 | H_\parallel | K_0 > = \text{Re} < \bar{K}_0 | H_\parallel | \bar{K}_0 > \), i.e.,

\[
(< K_0 | H_\parallel | K_0 > - < \bar{K}_0 | H_\parallel | \bar{K}_0 >) = 0,
\]

only if the total system does not preserve CPT–symmetry. Using more accurate approximation than Weisskopf–Wigner approximation, an estimation of the difference \( (< K_0 | H_\parallel | K_0 > - < \bar{K}_0 | H_\parallel | \bar{K}_0 >) \) is found for CPT–invariant generalized Fridrichs–Lee model.


1 Introduction.

The problem of testing CPT–invariance experimentally has attracted the attention of physicists, practically since the discovery of antiparticles. CPT symmetry is a fundamental theorem of axiomatic quantum field theory which follows from locality, Lorentz invariance, and unitarity [1]. Many tests of CPT–invariance consist in searching for decay process of neutral kaons. All known CP– and hypothetically possible CPT– violation effects in neutral kaon complex are described by solving the Schrödinger–like evolution equation [2] — [8] (we use \( \hbar = \frac{\alpha}{c} = 1 \) units)

\[
i \frac{\partial}{\partial t} |\psi; t > \parallel = H_{\parallel} |\psi; t > \parallel \tag{1}
\]

for \( |\psi; t > \parallel \) belonging to the subspace \( \mathcal{H}_{\parallel} \subset \mathcal{H} \) (where \( \mathcal{H} \) is the state space of the physical system under investigation), e.g., spanned by orthonormal neutral kaons states \( |K_0 >, |\overline{K}_0 >, \) and so on, (then states corresponding with the decay products belong to \( \mathcal{H} \otimes \mathcal{H}_{\parallel} \overset{\text{def}}{=} \mathcal{H}_{\perp} \)), and nonhermitean effective Hamiltonian \( H_{\parallel} \) obtained usually by means of the so–called LOY approach (within the Weisskopf–Wigner approximation (WW)) [2] — [5]:

\[
H_{\parallel} \equiv M - \frac{i}{2} \Gamma, \tag{2}
\]

where

\[
M = M^+, \quad \Gamma = \Gamma^+, \tag{3}
\]

are \((2 \times 2)\) matrices.

Solutions of Eq. (1) can be written in matrix form and such a matrix defines the evolution operator (which is usually nonunitary) \( U_{\parallel}(t) \) acting in \( \mathcal{H}_{\parallel} \):

\[
|\psi; t > \parallel = U_{\parallel}(t) |\psi; t_0 > \parallel, \quad U_{\parallel}(t) |\psi > \parallel, \tag{4}
\]

where,

\[
|\psi > \parallel \equiv q_1 |1 > + q_2 |2 >, \tag{5}
\]

and \( |1 > \) stands for the vectors of the \( |K_0 >, |B_0 > \) type and \( |2 > \) denotes antiparticles of the particle “1”: \( |\overline{K}_0 >, |\overline{B}_0 >, < j |k > = \delta_{jk}, \ j, k = 1, 2. \)

Relations between matrix elements of \( H_{\parallel} \) implied by CPT–transformation properties of the Hamiltonian \( H \) of the total system, containing neutral kaon
complex as a subsystem, are crucial for designing CPT– invariance and CP–
violation tests and for proper interpretation of their results. (Main properties
of the effective Hamiltonian $H_\parallel$ and formulae appearing in the LOY approach
will be described in short in Section 2). The aim of this paper is to examine
the properties of the exact $H_\parallel$ generated by the CPT–symmetry of the total
system under consideration and independent of the approximation used. In
order to realize this purpose, the method described and applied to study
the properties of time evolution in neutral kaon system in $[4] – [11]$ an-
and, especially, in $[12]$, will be used. For the readers convenience this method will
be sketched briefly in Section 3.

2 Preliminaries.

2.1 Properties of eigenstates of $H_\parallel$.
The eigenstates of $H_\parallel$, $|l>$ and $|s>$, for the eigenvalues $\mu_l$ and $\mu_s$ respectively $[2] – [8], [9] – [11]

$$\mu_{l(s)} = h_0 - (+)h \equiv m_{l(s)} - \frac{i}{2} \gamma_{l(s)}; \quad (6)$$

where $m_{l(s)}, \gamma_{l(s)}$ are real, and

$$h_0 = \frac{1}{2}(h_{11} + h_{22}), \quad (7)$$

$$h = \sqrt{h_z^2 + h_{12}h_{21}}, \quad (8)$$

$$h_z = \frac{1}{2}(h_{11} - h_{22}), \quad (9)$$

$$h_{jk} = <j|H_\parallel|k>, \quad (j,k = 1,2), \quad (10)$$

correspond to the long (the vector $|l>$) and short (the vector $|s>$) living
superpositions of $K_0$ and $\bar{K}_0$.

The following identity taking place for $\mu_l$ and $\mu_s$ will be needed in next
Sections:

$$\mu_l + \mu_s = h_{11} + h_{22}, \quad (11)$$

$$\mu_s - \mu_l = 2h \overset{\text{def}}{=} \Delta \mu, \quad (12)$$

$$\mu_l \mu_s = h_{11}h_{22} - h_{12}h_{21}, \quad (13)$$

3
Using the eigenvectors

\[ |K_{1(2)}\rangle \equiv 2^{-1/2}(|1\rangle + (-)|2\rangle), \] (14)

of the CP–transformation for the eigenvalues \( \pm 1 \) (we define \( \mathcal{CP}|1\rangle = -|2\rangle, \mathcal{CP}|2\rangle = -|1\rangle \)), vectors \( |l\rangle \) and \( |s\rangle \) can be expressed as follows \[4, 5, 6\]

\[ |l(s)\rangle \equiv (1 + |\varepsilon_{l(s)}|^2)^{-1/2}[|K_{2(1)}\rangle + \varepsilon_{l(s)}|K_{1(2)}\rangle], \] (15)

where

\[ \varepsilon_l = \frac{h_{12} - h_{11} + \mu_l}{h_{12} + h_{11} - \mu_l} \equiv -\frac{h_{21} - h_{22} + \mu_l}{h_{21} + h_{22} - \mu_l}, \] (16)

\[ \varepsilon_s = \frac{h_{12} + h_{11} - \mu_s}{h_{12} - h_{11} + \mu_l} \equiv -\frac{h_{21} + h_{22} - \mu_s}{h_{21} - h_{22} + \mu_s}. \] (17)

This form of \( |l\rangle \) and \( |s\rangle \) is used in many papers when possible departures from CP– or CPT–symmetry in the system considered are discussed. The following parameters are used to describe the scale of CP– and possible CPT–violation effects \[5, 6\):

\[ \varepsilon \overset{\text{def}}{=} \frac{1}{2}(\varepsilon_s + \varepsilon_l), \] (18)

\[ \delta \overset{\text{def}}{=} \frac{1}{2}(\varepsilon_s - \varepsilon_l). \] (19)

According to the standard meaning, \( \varepsilon \) describes violations of CP–symmetry and \( \delta \) is considered as a CPT–violating parameter \[3, 4, 5\]. Such an interpretation of these parameters follows from properties of LOY theory of time evolution in the subspace of neutral kaons \[2\]. We have

\[ \varepsilon = \frac{h_{12} - h_{21}}{D}, \] (20)

\[ \delta = \frac{h_{11} - h_{22}}{D} \equiv \frac{2h_z}{D}, \] (21)

where

\[ D \overset{\text{def}}{=} h_{12} + h_{21} + \Delta \mu. \] (22)

Starting from Eqs. (11) — (13) and (16), (17) and using some known identities for \( \mu_l, \mu_s \) one can express matrix elements \( h_{jk} \) of \( H_{\parallel} \) in terms of the
physical parameters $\varepsilon_{l(s)}$ and $\mu_{l(s)}$:

\[
\begin{align*}
  h_{11} &= \frac{\mu_s + \mu_l}{2} + \frac{\mu_s - \mu_l}{2} \frac{\varepsilon_s - \varepsilon_l}{1 - \varepsilon_l \varepsilon_s}, \\
  h_{22} &= \frac{\mu_s + \mu_l}{2} - \frac{\mu_s - \mu_l}{2} \frac{\varepsilon_s - \varepsilon_l}{1 - \varepsilon_l \varepsilon_s}, \\
  h_{12} &= \frac{\mu_s - \mu_l}{2} \frac{1 + \varepsilon_l(1 + \varepsilon_s)}{1 - \varepsilon_l \varepsilon_s}, \\
  h_{21} &= \frac{\mu_s - \mu_l}{2} \frac{1 - \varepsilon_l(1 - \varepsilon_s)}{1 - \varepsilon_l \varepsilon_s}.
\end{align*}
\]

These relations lead to the following equations

\[
\begin{align*}
  h_{11} - h_{22} &= \Delta \mu \frac{\varepsilon_s - \varepsilon_l}{1 - \varepsilon_l \varepsilon_s}, \\
  h_{12} + h_{21} &= \Delta \mu \frac{1 + \varepsilon_l \varepsilon_s}{1 - \varepsilon_l \varepsilon_s}, \\
  h_{12} - h_{21} &= \Delta \mu \frac{\varepsilon_s + \varepsilon_l}{1 - \varepsilon_l \varepsilon_s}.
\end{align*}
\]

Note that relations (23) — (29) are valid for arbitrary values of $\varepsilon_{l(s)}$. From (27) one infers that if $\Delta \mu \neq 0$ then:

\[
  h_{11} = h_{22} \iff \varepsilon_l = \varepsilon_s.
\]

Relation (29) enables us to conclude that parameters $\varepsilon_l$ and $\varepsilon_s$ need not be small, in order $\varepsilon = 0$ (20). Indeed, the identity (24) implies that for $\Delta \mu \neq 0$

\[
  h_{12} = h_{21} \iff \varepsilon_l = -\varepsilon_s,
\]

for any values of $|\varepsilon_l|, |\varepsilon_s|$.

It is appropriate to emphasize at this point that all relations (23) — (31) do not depend on a special form of the effective Hamiltonian $H_\parallel$. They are induced by geometric relations between various base vectors in two-dimensional subspace $H_\parallel$. On the other hand, the interpretation of above relations depends on properties of the matrix elements $h_{jk}$ of the effective Hamiltonian $H_\parallel$, i.e., if for example $H_\parallel \neq H_{LOY}$, where $H_{LOY}$ is the LOY effective Hamiltonian, then the interpretation of $\varepsilon$ (18) and $\delta$ (19) etc., need not be the same for $H_\parallel$ and for $H_{LOY}$.
Experimentally measured values of parameters $\varepsilon_l, \varepsilon_s$ are very small for neutral kaons. Assuming

$$|\varepsilon_l| \ll 1, \quad |\varepsilon_s| \ll 1,$$

from (27) one finds:

$$h_{11} - h_{22} \simeq (\mu_s - \mu_l)(\varepsilon_s - \varepsilon_l),$$

and (28) implies

$$h_{12} + h_{21} \simeq \mu_s - \mu_l,$$

and (29) gives

$$h_{12} - h_{21} \simeq (\mu_s - \mu_l)(\varepsilon_s + \varepsilon_l).$$

Relation (34) means that in the considered case of small values of parameters $|\varepsilon_l|, |\varepsilon_s|$ (32), the quantity $D$ (22) appearing in formulae for $\delta$ and $\varepsilon$ approximately equals

$$D \simeq 2(\mu_s - \mu_l) \equiv 2\Delta \mu.$$ (36)

Keeping in mind that $h_{jk} = M_{jk} - \frac{i}{2} \Gamma_{jk}, M_{kj} = M^*_{jk}, \Gamma_{kj} = \Gamma^*_{jk}$ and then starting from Eqs. (33) — (35) and separating real and imaginary parts one can find some useful relations:

$$2\text{Re}(M_{12}) \simeq m_s - m_l,$$ (37)

$$2\text{Re}(\Gamma_{12}) \simeq \gamma_s - \gamma_l,$$ (38)

$$2\text{Im}(M_{12}) \simeq -(\gamma_s - \gamma_l)[\text{Im}\left(\frac{\varepsilon_s + \varepsilon_l}{2}\right) + \tan \phi_{SW} \text{Re}\left(\frac{\varepsilon_s + \varepsilon_l}{2}\right)],$$ (39)

$$\text{Im}(\Gamma_{12}) \simeq -(\gamma_s - \gamma_l)[\tan \phi_{SW} \text{Re}\left(\frac{\varepsilon_s + \varepsilon_l}{2}\right) - \text{Im}\left(\frac{\varepsilon_s + \varepsilon_l}{2}\right)],$$ (40)

etc., where

$$\tan \phi_{SW} \equiv \frac{2(m_l - m_s)}{\gamma_s - \gamma_l}. \quad \text{(41)}$$

and

$$\text{Re}(h_{11} - h_{22}) \equiv M_{11} - M_{22}$$

$$\simeq -(\gamma_s - \gamma_l)[\tan \phi_{SW} \text{Re}\left(\frac{\varepsilon_s - \varepsilon_l}{2}\right)$$

$$- \text{Im}\left(\frac{\varepsilon_s - \varepsilon_l}{2}\right)], \quad \text{(42)}$$
etc.. One should remember that relations (37) — (40) and (42), (43) are valid only if condition (32) holds. Completing the system of these last six relations one can rewrite Eq. (11) to obtain

\[ M_{11} + M_{22} = m_l + m_s, \]  
\[ \Gamma_{11} + \Gamma_{22} = \gamma_l + \gamma_s. \] (44) (45)

These last two Equations are exact independently of whether the condition (32) holds or not.

### 2.2 \( H_{LOY} \) and CPT–symmetry.

Now, let us consider briefly some properties of the LOY model. Let \( H \) be total (selfadjoint) Hamiltonian, acting in \( \mathcal{H} \) — then the total unitary evolution operator \( U(t) \) fulfills the Schrödinger equation

\[ i\frac{\partial}{\partial t} U(t)|\phi\rangle = HU(t)|\phi\rangle, \quad U(0) = I, \] (46)

where \( I \) is the unit operator in \( \mathcal{H} \), \( |\phi\rangle \equiv |\phi; t_0 = 0 > \in \mathcal{H} \) is the initial state of the system:

\[ |\phi\rangle \equiv |\psi > \] (47)

in our case \( |\phi; t > = U(t)|\phi\rangle \). Let \( P \) denote the projection operator onto the subspace \( \mathcal{H}_\parallel \):

\[ P\mathcal{H} = \mathcal{H}_\parallel, \quad P = P^2 = P^+, \] (48)

then the subspace of decay products \( \mathcal{H}_\perp \) equals

\[ \mathcal{H}_\perp = (I - P)\mathcal{H} \equiv Q\mathcal{H}, \quad Q \equiv I - P. \] (49)

For the case of neutral kaons or neutral \( B \)-mesons, etc., the projector \( P \) can be chosen as follows:

\[ P \equiv |1 > < 1| + |2 > < 2|. \] (50)
We assume that time independent basis vectors $|K_0>$ and $|\overline{K}_0>$ are defined analogously to corresponding vectors used in LOY theory of time evolution in neutral kaon complex \cite{2}: Vectors $|K_0>$ and $|\overline{K}_0>$ can be identified with eigenvectors of the so-called free Hamiltonian $H^{(0)} \equiv H_{\text{strong}} = H - H_W$, where $H_W$ denotes weak interaction which is responsible for transitions between eigenvectors of $H^{(0)}$, i.e., for the decay process.

In the LOY approach it is assumed that vectors $|1>$, $|2>$ considered above are eigenstates of $H^{(0)}$ for 2-fold degenerate eigenvalue $m_0$:

$$H^{(0)}|j> = m_0|j>, \quad j = 1, 2. \quad (51)$$

This means that

$$[P, H^{(0)}] = 0. \quad (52)$$

The condition guaranteeing the occurrence of transitions between subspaces $\mathcal{H}_\parallel$ and $\mathcal{H}_\perp$, i.e., a decay process of states in $\mathcal{H}_\parallel$, can be written as follows

$$[P, H_W] \neq 0. \quad (53)$$

Usually, in LOY and related approaches, it is assumed that

$$\Theta H^{(0)} \Theta^{-1} = H^{(0)\dagger} \equiv H^{(0)}, \quad (54)$$

where $\Theta$ is the antiunitary operator:

$$\Theta \overset{\text{def}}{=} \mathcal{CPT}. \quad (55)$$

Relation (54) is a particular form of the general transformation rule \cite{13,14,1,7}:

$$\Theta L \Theta^{-1} \overset{\text{def}}{=} L_{\mathcal{CPT}}, \quad (56)$$

where $L$ is an arbitrary linear operator. Basic properties of anti-linear and linear operators, their products and commutators are described, eg., in \cite{13,14,1,7}. Generally, defining the commutator of anti-linear operator $\Theta$ and linear operators $L, L_1, L_2$, appearing, e.g., in formulae discussed in Sec.4, we follow \cite{13,14}:

$$[\Theta, L] \equiv \Theta L - L\Theta, \quad (57)$$
where \( L \) can be replaced by product \( L = L_1L_2 \), etc.. For such defined commutators all basic commutation rules hold (see [13], Chap. XV, §3 — §5) including the following one:

\[
[\Theta, L_1L_2] = [\Theta, L_1]L_2 + L_1[\Theta, L_2].
\]

(58)

On the other hand, to minimalize risk of confusion, using definitions (57) and relations of type (58) one should not forget that properties of products of anti–linear and linear operators of types \( \Theta L_1L_2, L_1\Theta L_2, \) etc., and (58), differ from properties of the transformation rule (56) which means that in a general case \( \Theta L - L^+_{CPT}\Theta = 0 \) (but does not mean that \( [\Theta, L] = 0 \)), and which implies that

\[
\Theta L_1L_2\Theta^{-1} \equiv (\Theta L_2\Theta^{-1})(\Theta L_1\Theta^{-1}).
\]

The subspace of neutral kaons \( \mathcal{H}_\parallel \) is assumed to be invariant under \( \Theta \):

\[
\Theta P\Theta^{-1} = P^+ \equiv P.
\]

(59)

In the kaon rest frame, the time evolution is governed by the Schrödinger equation (46), where the initial state of the system has the form (47), (5). Within assumptions (51) — (53) the Weisskopf–Wigner approach lead s to the following formula for \( H_{LOY} \) (e.g., see [2, 3, 4, 6]):

\[
H_{LOY} = m_0P + PH_WP - \Sigma(m_0) \equiv PHP - \Sigma(m_0),
\]

(60)

\[
= M_{LOY} - \frac{i}{2}\Gamma_{LOY}
\]

(61)

where

\[
\Sigma(\epsilon) = PHQ\frac{1}{QHQ - \epsilon - i0QHP}.
\]

(62)

The matrix elements \( h_{jk}^{LOY} \) of \( H_{LOY} \) are

\[
h_{jk}^{LOY} = H_{jk} - \Sigma_{jk}(m_0), \quad (j, k = 1, 2),
\]

(63)

\[
= M^{LOY}_{jk} - \frac{i}{2}\Gamma_{jk}^{LOY}
\]

(64)

where, in this case,

\[
H_{jk} = <j|H|k> \equiv <j|(H^{(0)} + H_W)|k> \equiv m_0\delta_{jk} + <j|H_W|k>,
\]

(65)
and \( \Sigma_{jk}(\epsilon) = \langle j \mid \Sigma(\epsilon) \mid k \rangle \).

Now, if \( \Theta H_W \Theta^{-1} = H_W^+ \equiv H_W \), then using, e.g., the following phase convention [3] — [4]

\[
\Theta|1\rangle \overset{\text{def}}{=} -|2\rangle, \quad \Theta|2\rangle \overset{\text{def}}{=} -|1\rangle,
\]

(66)

and taking into account that \( \langle \psi|\varphi \rangle = \langle \Theta \varphi|\Theta \psi \rangle \), one easily finds from (60) – (65) that

\[
h_{11}^{\text{LOY}} - h_{22}^{\text{LOY}} = 0
\]

(67)

in the CPT–invariant system. This is the standard result of the LOY approach and this is the picture which one meets in the literature [2] — [8]. Property (67) leads to the conclusion that

\[
\delta \simeq \delta^{\text{LOY}} \equiv 0.
\]

(68)

3 Properties of the exact \( H_\parallel \)

Eq. (46) means that \( U(t) = \exp(-itH) \). Now, knowing \( U(t) \), the evolution operator \( U_\parallel(t) \) for \( H_\parallel \) can be expressed using the projector \( P \) as follows

\[
U_\parallel(t) \equiv PU(t)P.
\]

(69)

We have \( U_\parallel(0) = I_\parallel \equiv P \), where \( I_\parallel \) is the unit operator in \( H_\parallel \). This relation must be fulfilled by solutions \( U_\parallel(t) \) of Eq. (1) with any effective Hamiltonian, and therefore with the exact \( H_\parallel \) too. In [15] an observation has been made that for every effective Hamiltonian \( H_\parallel \) governing the time evolution in subspace \( H_\parallel \equiv PH \), which in general can depend on time \( t \) [3] — [12], [16] the following identity holds:

\[
H_\parallel \equiv H_\parallel(t) \equiv i\frac{\partial U_\parallel(t)}{\partial t}[U_\parallel(t)]^{-1}.
\]

(70)

We define the operator \( [U_\parallel(t)]^{-1} \) as follows

\[
[U_\parallel(t)]^{-1}U_\parallel(t) = U_\parallel(t)[U_\parallel(t)]^{-1} \equiv P.
\]

(71)

(One finds that \( PU(t)P = PU^+(t)P = [U_\parallel(t)]^{-1} \) if and only if \( [P,U(t)] = 0 \). From this definition one can infer that

\[
P[U_\parallel(t)]^{-1} \equiv [U_\parallel(t)]^{-1}P.
\]
This observation together with the property (69) means that the identity (70) can be replaced by the following one:

$$H_{\parallel} \equiv H_{\parallel}(t) \equiv i \frac{\partial U_{\parallel}(t)}{\partial t} [U_{\parallel}(t)]^{-1} P.$$  (73)

It can be easily verified that $H_{\parallel} \equiv H_{LOY}$, fulfils the identities (70) and (73).

A density matrix approach for a description of time evolution in $K_0, K_0^{\star}$ is sometimes used [17, 18]. One can show that the effective Hamiltonian appearing and used in such an approach fulfils the same identity (70).

Let us notice that definition (71) means that operators $[U_{\parallel}(t)]^{-1}$, as well as $U_{\parallel}(t)$ and $H_{\parallel}(t)$ act not only in $H_{\parallel}$ but also in $H$. It is obvious that all these operators and their products exist in the case of one dimensional subspace $H_{\parallel}$. In the case of two dimensional $H_{\parallel}$, in which we are interested, assuming that $H$ is spanned by a complete set of orthonormal vectors $\{|e_l>\}_{l \in \mathcal{Y}}$ (where the cardinality of the set $\mathcal{Y}$ of indexes $l \in \mathcal{Y}$ is equal to the dimension of $H$): $<e_l|e_m> = \delta_{mn}$, e.g., by eigenvectors for $H^{(0)}$ introduced above, and $|e_1> \equiv |1>$, $|e_2> \equiv |2>$ we have

$$U_{\parallel}(t) = \sum_{j,k=1}^{2} u_{jk} |j><k|,$$  (74)

$$[U_{\parallel}(t)]^{-1} = \sum_{j,k=1}^{2} \tilde{u}_{jk} |j><k|,$$  (75)

$$H_{\parallel}(t) = \sum_{j,k=1}^{2} h_{jk}(t) |j><k|,$$  (76)

$$O = \sum_{l,m \in \mathcal{Y}} O_{lm} |e_l><e_m|,$$

where, $O$ is an arbitrary operator acting in $H$,

$$u_{jk} = <j|U_{\parallel}(t)|k>,$$

$$\equiv <j|U(t)|k>, \quad (j,k = 1,2),$$

$$\tilde{u}_{jk} = <j|[U_{\parallel}(t)]^{-1}|k>, \quad (j,k = 1,2),$$

$$h_{jk}(t) = <j|H_{\parallel}(t)|k>, \quad (j,k = 1,2),$$

$$O_{lm} = <e_l|O|e_m>, \quad \text{for all } l,m \in \mathcal{Y}.$$
and

\[ \begin{align*}
d\tilde{u}_{jk} &= -u_{jk}, \quad (j \neq k; j,k=1,2), \\
d\tilde{u}_{11} &= u_{22}, \\
d\tilde{u}_{22} &= u_{11},
\end{align*} \]

This realization means that all operators: \( P, U_\parallel, U_\parallel^{-1}, H_\parallel, \mathcal{O}, \) etc., are isomorphic with \((N \times N)\) "block" matrices \( M \):

\[ M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \]

where \( M_{11} \) is \((N_1 \times N_1)\) submatrix, \( M_{12} \) represents \((N_1 \times N_2)\) submatrix, \( M_{22} \) is formed by \((N_2 \times N_2)\) submatrix, etc., and \( N_1 + N_2 = N, N_1 = \text{dim} H_\parallel, N = \text{dim} \mathcal{H} \) (in our case \( N_1 = 2, N = \infty \)). \( M_{11} \neq 0 \) and \( M_{12} = 0, M_{21} = 0 \) and \( M_{22} = 0 \) in the case of operators \( P, U_\parallel, U_\parallel^{-1} \) and \( H_\parallel \), but, e.g., for the evolution operator \( U(t) \) one finds \( M_{jk} \neq 0, \quad (j, k = 1, 2) \). So, products of type: \( H_\parallel U_\parallel, \mathcal{O} U_\parallel^{-1}, U_\parallel^{-1} \mathcal{O}, \) etc., exist not only in \( H_\parallel \) but also in \( \mathcal{H} \), and they are well defined in \( \mathcal{H} \).

Relations (71) and (75) define uniquely the operator \( U_\parallel^{-1}(t) \) in the considered case of neutral kaons. Namely, properties of determinants enable us to rewrite (79) as follows

\[ \begin{align*}
det U_\parallel(t) &= \det[D^{-1}U_\parallel(t)D] \\
&= \det U_\parallel^d(t) \equiv \zeta_-(t) \zeta_+(t),
\end{align*} \]

where \( D \) is the matrix diagonalizing \( U_\parallel(t) \). \( U_\parallel^d(t) \) denotes the diagonal form of \( U_\parallel(t) \) and \( \zeta_-(t), \zeta_+(t) \) are eigenvalues for the matrix \( U_\parallel(t) \). In [13] the eigenvalue problem for the operator of type (69) has been discussed and \( \zeta_-(t), \zeta_+(t) \) have been given as functions of the matrix elements \( u_{kl}(t) \) of the total evolution operator \( U(t) = \exp(-itH) \). These matrix elements possess the property: \( u_{kl}(t)_{0<t<\infty} \neq 0 \) in the basis of eigenvectors of \( H^{(0)} \). From formulae for \( \zeta_-(t), \zeta_+(t) \) and from the properties of \( u_{kl}(t) \) [19] it follows that \( \zeta_-(t)_{0\leq t<\infty} \neq 0 \) and \( \zeta_+(t)_{0\leq t<\infty} \neq 0 \), which appears quite natural and obvious for the neutral kaon complex. Therefore we can conclude that in our case

\[ \det U_\parallel(t) \equiv \zeta_-(t) \zeta_+(t) \neq 0, \]

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which means that the operator \( U^{-1}(t) \) exists and is defined uniquely in the subspace \( H \) considered by relations (71) and (75).

Generally the operator \( U^{-1}(t) \) defined by (71) exists and is unique, and formula (70) holds in subspaces \( H \) of \( \dim H \geq 2 \) when the effective evolution operator \( U(t) \) (69) for the subsystem considered has only non-zero eigenvalues. Theory presented and discussed below can be extended for all such systems.

Discussing properties of \( U^{-1}(t) \) one should remember that relations (71) and (75) define this operator uniquely only in the subspace \( H \). Its extension (75) to the whole state space \( H \) is not unique. The most general "inverse" operator (let us denote it by \( U^{-1}(t) \)) fulfilling relations (71) has the form

\[
U^{-1}(t) = U^{-1}(t) + QO(t)Q,
\]

where \( U^{-1}(t) \) is given by formula (75), and \( O(t) \) is an arbitrary operator. Inserting (83) instead of (75) into the identity (70) does not change anything: Properties of the operator \( H(t) \) (70) are the same independently of whether the operator (75) or (83) is used to calculate \( H(t) \). Therefore searching for transformation properties of the exact effective Hamiltonian for neutral kaon complex one need not worry about all possible extensions of \( U^{-1}(t) \) to the whole \( H \). In order to describe correctly properties of \( H(t) \) it is sufficient that \( U^{-1}(t) \) is uniquely defined in the subspace \( H \).

In the nontrivial case

\[
[P, H] \neq 0,
\]

from (73), using (46) and (69) we find

\[
H(t) \equiv PHPU(t)[U(t)]^{-1}P \equiv PHP + PHPQU(t)[U(t)]^{-1}P \overset{\text{def}}{=} PHP + V(t).
\]

(Assumption (84) means that transitions of states from \( H \) into \( H \) and from \( H \) into \( H \), i.e., decay and regeneration processes, are allowed). Thus [11, 13, 16]

\[
H(t) = PHP, \quad V(t) = 0, \quad V(t \to 0) \simeq -itPHQHP,
\]

so, in general \( H(t) \neq H(t \to 0) \) [16, 3, 10] and \( V(t \neq 0) \neq V(t \neq 0), \quad H(t \neq 0) \neq H(t \neq 0) \).
According to the ideas of the standard scattering theory, one can state that operator $H(\infty) \equiv H(\infty)$ describes the bounded or quasistationary states of the subsystem considered and in this sense it corresponds to $H_{LOY}$.

4 CPT transformations and the exact $H_{\|}$.

Now let us pass on to the investigation of CPT–transformation properties of $H_{\|}$. In this Section the following assumptions are used: The total Hamiltonian $H$ of the system considered is selfadjoint (which means that the total evolution operator $U(t)$ is the unitary operator and it solves the Schrödinger equation (46)). Orthonormal basis vectors $|1\rangle, |2\rangle$ are time independent and they are not eigenvectors for the $H$, i.e., that for the projector $P$ the condition (50) holds. (This assumption means that stationary states will not be considered). It is also assumed that vectors $|1\rangle, |2\rangle$ are related to each other through the transformation (66). Besides there is only one assumption for the anti–linear operator $\Theta$ (55) describing CPT–transformation in $\mathcal{H}$. We require CPT–invariance of $H_{\|}$. This means that for projector $P$ defining this subspace the relation (59) must hold. Due to the property $P = P^+$, Eq.(59) can be replaced by the following relation

$$[\Theta, P] = 0. \quad (89)$$

Using assumption (89) and the identity (85), after some algebra, one finds

$$[\Theta, H_{\|}(t)] = \mathcal{A}(t) + \mathcal{B}(t), \quad (90)$$

where:

$$\mathcal{A}(t) = P[\Theta, H]U(t)P(U(t))^{-1}, \quad (91)$$

$$\mathcal{B}(t) = \left\{ P[H - PHU(t)P(U(t))^{-1}]P(U(t))^{-1} \right\}[\Theta, U(t)]P(U(t))^{-1} \quad (92)$$

$$\equiv \left\{ P[H - H_{\|}(t)P] \right\}[\Theta, U(t)]P(U(t))^{-1} \quad (93)$$

$$\equiv \left\{ PHQ - V_{\|}(t)P \right\}[\Theta, U(t)]P(U(t))^{-1} \quad (94)$$

We observe that $\mathcal{A}(0) \equiv P[\Theta, H]P$ and $\mathcal{B}(0) \equiv 0$. From definitions and general properties of operators $\mathcal{C}, \mathcal{P}$ and $\mathcal{T}$ it is known that
\(TU(t \neq 0) = U_+^\pm (t \neq 0)T \neq U(t \neq 0)T\) (Wigner’s definition for \(T\) is used), and thereby \(\Theta U(t \neq 0) = U_{\text{CP}}^+(t \neq 0)\Theta\) \text{[13, 14, 21]} i.e. \([\Theta, U(t \neq 0)] \neq 0\). So, the component \(B(t)\) in (90) is nonzero for \(t \neq 0\) and it is obvious that there is a chance for \(\Theta\)–operator to commute with the effective Hamiltonian \(H_{\|}(t \neq 0)\) only if \([\Theta, H]\neq 0\). On the other hand, the property \([\Theta, H]\neq 0\) does not imply that \([\Theta, H_{\|}(0)] = 0\) or \([\Theta, H_{\|}(0)] \neq 0\). These two possibilities are admissible, but if \([\Theta, H] = 0\) then there is only one possibility: \([\Theta, H_{\|}(0)] = 0\) \text{[12]}. From (90) we find

\[
\Theta H_{\|}(t)\Theta^{-1} - H_{\|}(t) \equiv (A(t) + B(t))\Theta^{-1}.
\] (95)

Now, keeping in mind that \(|2\> \equiv |\overline{K}_0\>\) is the antiparticle for \(|1\> \equiv |K_0\>\) and that, by definition, the (anti–unitary) \(\Theta\)–operator transforms \(|1\> \in |2\> \equiv |\overline{K}_0\>\) according to formulae (66), and \(<\psi|\varphi> = <\Theta\varphi|\Theta\psi>\), we obtain from (95) (see Appendix A)

\[
h_{11}(t)^* - h_{22}(t) = <2|(A(t) + B(t))\Theta^{-1}|2>,
\] (96)

Adding expression (96) to its complex conjugate one gets \text{[20]}

\[
\text{Re} \left(h_{11}(t) - h_{22}(t)\right) = \text{Re} \ <2|(A(t) + B(t))\Theta^{-1}|2>. \tag{97}
\]

Note that if to replace the the requirement (84) for the projector \(P\) (54) by the following one:

\[
[P, H] = 0, \tag{98}
\]

i.e., if to consider only stationary states instead of unstable states, then one immediately obtains from (91) — (94):

\[
A(t) = P[\Theta, H]P, \tag{99}
\]

\[
B(t) = 0. \tag{100}
\]

5 The case of conserved CPT–symmetry.

Let us assume that condition (84) holds and

\[
[\Theta, H] = 0. \tag{101}
\]
For the stationary states (98), this assumption, relations (99), (100) and (97) yield
\[ \text{Re} (h_{11}(t) - h_{22}(t)) = 0. \]

Now let us consider the case of unstable states, i.e., states \(|1\rangle, |2\rangle\), which lead to such projection operator \(P\) (50) that condition (84) holds. If in this case (101) also holds then \(A(t) \equiv 0\) and thus \([\Theta, H_{\|}(0)] = 0\), which is in agreement with an earlier, similar result [12]. In this case we have \(\Theta U(t) = U^+(t)\Theta\), which gives \(\Theta U_{\|}(t) = U^+_\| (t)\Theta\), \(\Theta U^{-1}_\| (t) = (U^+_\| (t))^{-1}\Theta\), and
\[ [\Theta, U(t)] = -2i(\text{Im} \, U(t))\Theta \quad (102) \]

This relation leads to the following result in the case of the conserved CPT–symmetry
\[ B(t) = -2iP\left\{ H - H_{\|}(t) \, P \right\}(\text{Im} \, U(t))P(U^+_\| (t))^{-1}\Theta \quad (103) \]
\[ \equiv -2i\left\{ PHQ - V_{\|}(t) \, P \right\}(\text{Im} \, U(t))P(U^+_\| (t))^{-1}\Theta. \quad (104) \]

The simplest, nontrivial case is the case of \(\dim \mathcal{H} = 3\): here \(\Upsilon = \{1, 2, 3\}\) and (see (15))
\[ Q = |e_3\rangle\langle e_3|, \]
and all operators acting in \(\mathcal{H}\) can be realized as \(3 \times 3\) matrices. In this representation:
\[
P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]
\[
H = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{12} & H_{22} & H_{23} \\ H_{13} & H_{23}^* & H_{33} \end{pmatrix}, \quad V_{\|} = \begin{pmatrix} v_{11} & v_{12} & 0 \\ v_{21} & v_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix},
\]
(\(v_{jk} \overset{\text{def}}{=} \langle j|V_{\|}(t)|k\rangle; j,k = 1,2\)) and therefore for \(\{PHQ - V_{\|}(t)P\}\) in (104) we obtain
\[ PHQ - V_{\|}(t)P = \begin{pmatrix} -v_{11} & -v_{12} & H_{13} \\ -v_{21} & -v_{22} & H_{23} \\ 0 & 0 & 0 \end{pmatrix}. \quad (105) \]
Denoting \((\text{Im } U(t))P(U_{||}^+(t))^{-1} \overset{\text{def}}{=} Z(t)\) one finds that

\[
Z(t) = \begin{pmatrix}
z_{11} & z_{12} & 0 \\
z_{21} & z_{22} & 0 \\
z_{31} & z_{32} & 0
\end{pmatrix},
\]

(106)

where \(z_{lm} \overset{\text{def}}{=} <e_l|Z|e_m>\), \((l,m = 1,2,3)\). It is clear that \(Z(0) = 0\) and \(Z(t) \neq 0\) for \(t > 0\). From (105) and (106) it is seen that in the considered case of \(\text{dim } H = 3\), the operator \(B(t)\) (104) has the form

\[
B(t) = -2i\left(PHQ - V_{||}(t)P\right)Z(t)\Theta \equiv -2i\begin{pmatrix} b_{11} & b_{12} & 0 \\ b_{21} & b_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}\Theta,
\]

(107)

and that \(b_{jk}(0) = 0\) and \(b_{jk}(t) \neq 0\) for \(t > 0\) in case of conserved CPT–symmetry (101). Thus, in our case, \(B(0) \equiv 0\) and \(B(t) \neq 0\) for \(t > 0\). Similar conclusion holds in the case of \(\text{dim } H > 3\). Generally, in any case \(B(t > 0) \neq 0\).

Formulae (103), (104) and the example considered above allow us to conclude that \(<2|B(0)\Theta^{-1}|2> = 0\) and \(\text{Re}<2|B(t > 0)\Theta^{-1}|2> \neq 0\), if condition (107) holds. This means that in this case it must be \(\text{Re}(h_{11}(t)) \neq \text{Re}(h_{22}(t))\) for \(t > 0\). So, there is no possibility for \(\text{Re}(h_{11})\) to equal \(\text{Re}(h_{22})\) for \(t > 0\) in the considered case of \(P\) fulfilling the condition (84) (i.e., for unstable states) when CPT–symmetry is conserved: It must be \(\text{Re}(h_{11}) \neq \text{Re}(h_{22})\) and thus \(h_{11} \neq h_{22}\) in such a case.

6 Discussion.

Assuming that condition (84) for \(P\) holds one finds that the only possibility for \(\text{Re}(h_{11} - h_{22})\) to be equal zero appears if the nonzero contribution of \(B(t > 0)|2>\) into \(\text{Re}(h_{11}(t) - h_{22}(t))\) is compensated by a nonzero contribution of \(A(t)|2>\) — see (97). It can be observed that \(\text{Re}<2|B(t > 0)|2> \neq 0\) irrespectively of whether \(\Theta\) commutes with \(H\) or not, but \(A(t) \neq 0\) and \(<2|A(t)|2> \neq 0\) only appear if \([\Theta, H] \neq 0\).
So, all the above considerations lead to the following conclusions for the matrix elements $h_{jk}$ of the exact effective Hamiltonian $H_{||}$ governing the time evolution in neutral kaons subspace:

**Conclusions 1:**

1.a) If $\text{Re}(h_{11}(t>0)) = \text{Re}(h_{22}(t>0))$ then it follows that $[\Theta, H] \neq 0$,

1.b) If $[\Theta, H] = 0$ then it follows that $\text{Re}(h_{11}(t>0)) \neq \text{Re}(h_{22}(t>0))$.

1.c) If $\text{Re}(h_{11}(t>0)) \neq \text{Re}(h_{22}(t>0))$ then the cases $[\Theta, H] \neq 0$ or $[\Theta, H] = 0$ are both possible.

One should remember that above conclusions derived from relation (97) concern only the real parts of $h_{11}(t>0)$ and $h_{22}(t>0)$. Relations (95) — (97) give us no information about the imaginary parts of $h_{11}$ and $h_{22}$. One cannot infer from (97) that $[\Theta, H] = 0$ follows $\text{Im}(h_{11}) \neq \text{Im}(h_{22})$. The case when $[\Theta, H] = 0$ follows $\text{Re}(h_{11}(t>0)) \neq \text{Re}(h_{22}(t>0))$ and $\text{Im}(h_{11}) = \text{Im}(h_{22})$, is not in conflict with relations (95) — (97). The equality of $\text{Im}(h_{11})$ and $\text{Im}(h_{22})$ need not imply the equality of $\text{Re}(h_{11})$ and $\text{Re}(h_{22})$ and vice versa. This means that Bell–Steinberger relations [22] do not contradict relations (95) — (97) and Conclusions 1.a) — 1.c) following from them: Bell and Steinberger formulae lead to the equality of $\text{Im}(h_{11})$ and $\text{Im}(h_{22})$ in the case of conserved CPT–symmetry and do not concern the real parts of diagonal matrix elements of $H_{||}$ and do not give relations between them.

Real parts of diagonal matrix elements of the mass matrix $H_{||}$, $h_{11}$ and $h_{22}$, are considered in the literature as masses of unstable particles $|1>,|2>$ (eg., mesons $K_0$ and $2K_0$). Conclusion 1.b) means that masses of a decaying particle ”1” and its antiparticle ”2” should be different if CPT–symmetry is conserved in the system containing these unstable particles. In other words, in the exact theory unstable states $|1>,|2>$ appear to be nondegenerate in mass if CPT–symmetry holds in the total system considered. At the same time, relations (98) — (101) suggest that in the CPT–invariant system masses of a given particle and its antiparticle are equal (i.e., appear to be degenerate) only in the case of stationary (stable) states $|1>,|2>$. The case, when vectors $|1>,|2>$ describe pairs of particles $p, \bar{p}$, or $e^-, e^+$, can be considered as an example of such states. All these conclusions contradict the standard result of the LOY and related approaches.

Results of Sections 4 and 5 and Conclusions 1.a) — 1.c) are not in conflict
with such implications of the CPT–invariance as the equality of particle and antiparticle decay rates. It can be easily verified that assuming (101) one gets (see (69) and (89))

\[ | < 2 | U(t) | 2 > |^2 = | < 1 | U(t) | 1 > |^2. \]  

This last relation means that decay laws, and thus decay rates, of particle ”1” and its antiparticle ”2” are equal.

The consequences (67) and (68) of the LOY theory are in conflict with the results of Sec. 4 and 5, and Conclusions 1.a) – 1.c) obtained without approximations. From Conclusions 1.a) — 1.c) we infer that for experimentally measured parameter \( \delta \) (21) following Conclusions should be valid [20]:

**Conclusions 2:**

2.a) If \( \delta = 0 \) (or \( \varepsilon_l = \varepsilon_s \)) then it follows that \( [\Theta, H] \neq 0 \).

2.b) If \( [\Theta, H] = 0 \) then it follows that \( \delta \neq 0 \) (or \( \varepsilon_l \neq \varepsilon_s \)).

2.c) If \( \delta \neq 0 \) (or \( \varepsilon_l \neq \varepsilon_s \)) then the cases \( [\Theta, H] \neq 0 \) or \( [\Theta, H] = 0 \) are both possible.

Properties of the real systems described in Conclusions 1 and 2 are unobservable for the LOY approximation. In order to obtain at least an estimation for effects described in these Conclusions, matrix elements of \( H \) should be calculated much more exactly than it is possible within the LOY theory. A proposal of more exact approximation is given in [10, 11] (see Appendix B). All CP – and CPT – transformation properties of the effective Hamiltonian \( H \) calculated within this approximation are consistent with similar properties of the exact effective Hamiltonian. For instance, using the formalism mentioned (and, for readers convenience, briefly described in Appendix B), one can find \( (h_{11} - h_{22}) \) for generalized Fridrichs–Lee model [8]. Assuming CPT–invariance (i.e., (101)), it has been found in [11] (see formulae (152), (144) and (145) in [11]) that

\[ 2h_z = \lim_{t \to \infty} (h_{11}(t) - h_{22}(t)) \]  

\[ \equiv \frac{m_{21} \Gamma_{12} - m_{12} \Gamma_{21}}{|4m_{12}|} (F_0(m_0 - \mu - |m_{12}|) - (F_0(m_0 - \mu + |m_{12}|)), \]  

where, \( \Gamma_{12}, \Gamma_{21} \) can be identified with those appearing in the LOY theory (11), (14), \( m_0 \equiv H_{11} = H_{22} \) can be considered as kaon mass [8], \( m_{jk} \equiv \]
$H_{jk}$ ($j, k = 1, 2$), $\mu$ can be treated as the mass of the decay products of the neutral kaon \[8\], and

$$
F_0(m) = im^{-1/2}a_1(0),
a_1(0) \equiv (m_0 - \mu)^{1/2},
$$

which, in the case of conserved CPT–symmetry, lead to the following estimation for $|m_{12}| \ll (m_0 - \mu)$:

$$
h_{11} - h_{22} \simeq \frac{i m_{21} \Gamma_{12} - m_{12} \Gamma_{21}}{4(m_0 - \mu)}
= \frac{\text{Im}(m_{12} \Gamma_{21}) \Gamma_{12}}{2(m_0 - \mu)} \equiv 2h_z^{FL}
= \text{Re}(2h_z^{FL}).
$$

(110)

An equivalent form of this estimation is the following one:

$$
\text{Re}(2h_z^{FL}) = \frac{-\text{Re}(m_{12}) \text{Im}(\Gamma_{12}) + \text{Im}(m_{12}) \text{Re}(\Gamma_{12})}{2(m_0 - \mu)}.
$$

(111)

Real properties of neutral $K$–complex enable us to replace $\tan \phi_{SW}$ \[11\] in (40) by ”1” \[23, 24\]: $\tan \phi_{SW} \approx 1$, and to use assumption (32) in formula (40) for $\text{Im}(\Gamma_{12})$. Therefore keeping in mind relations (38), (40) one can conclude that the contribution $\text{Im}(\Gamma_{12})$ in the numerator of (111) is negligibly small in comparison with the contribution of $\text{Re}(\Gamma_{12})$ in the case of neutral $K$–mesons considered. Finally, using (38), the estimation (111) takes the following form:

$$
\text{Re}(2h_z^{FL}) = \frac{\text{Im}(m_{12}) \gamma_s - \gamma_l}{4(m_0 - \mu)},
$$

(112)

For neutral $K$–system, to evaluate $2h_z^{FL}$ one can take $\tau_s \simeq 0, 89 \times 10^{-10}$ sec \[23\]. Hence $\gamma_s = \frac{\hbar}{\tau_s} \simeq 7, 4 \times 10^{-12}$ MeV and (following \[8\] ) $(m_0 - \mu) = m_K - 2m_\pi \simeq 200$ MeV \[8\]. Thus $2h_z^{FL} \sim 0, 93 \times 10^{-14} \text{Im}(m_{12}) \equiv 0, 93 \times 10^{-14} \text{Im}(H_{12})$.

Searching for another properties of the Friedrichs–Lee model it has been also found in \[10\] that $h_{jk}(t) \simeq h_{jk}$ practically for $t \geq T_{as} \simeq \frac{10^2}{\pi(m_0 - \mu - |m_{12}|)}$ (see \[11\], formula (153)). Therefore, within assumptions used above we obtain $T_{as} \sim 10^{-22}$ sec.
On the other hand, relations (112) can be rewritten to obtain an estimation for the matrix element \( H_{12} \) of the CPT–invariant Hamiltonian \( H \):

\[
\text{Im}(m_{12}) \equiv \text{Im}(H_{12}) \simeq 4 \frac{\text{Re}(h_{11} - h_{22})}{\gamma_s - \gamma_l} (m_0 - \mu). \tag{113}
\]

Using relation (45) one can express \( \text{Re}(h_{11} - h_{22}) \) in terms of physical parameters and then inserting into the formula (113) experimentally obtained present data for these parameters one can obtain a numerical value of \( \text{Re}(h_{11} - h_{22}) \) [23, 24]:

\[
\frac{|\text{Re}(h_{11} - h_{22})|}{m_{K_0\text{average}}} \equiv \frac{|M_{11} - M_{22}|}{m_{K_0\text{average}}} \sim 9 \times 10^{-19}. \tag{114}
\]

This, together with the estimation \( (m_0 - \mu) \sim 200 \text{ MeV} \) used above, leads to the following relation

\[
|\text{Im}(H_{12})| \sim 72 \times 10^{-17} \frac{m_{K_0\text{average}}}{\gamma_s} [\text{MeV}] \simeq 9, 73 \times 10^{-5} m_{K_0\text{average}}. \tag{115}
\]

Thus the estimation for imaginary part of matrix element \( \text{Im}H_{12} \equiv \text{Im} < K_0|H|K_0 > \) of the CPT–invariant Hamiltonian \( H \) has been found. This simple estimation should be fulfilled by every CPT–invariant model of weak interactions, which is expected to give a correct explanation of properties of neutral K–mesons. Therefore relation (115) can be also considered a criterion for selfconsistency of model Hamiltonians \( H \) of interactions leading to a decay process of neutral kaons.

Note that considering in detail the generalized Fridrichs–Lee model one finds that \( \Gamma_{jk} = 0 \), \( (j,k = 1, 2) \) for \( m_0 < \mu \), i.e., for bound states [8, 10]. This observation and relations (110), (111) imply that for bound (stable) states \( \text{Re}h^{FL}_z = 0 \). So, if CPT–symetry is conserved in this model, then particle and antiparticle bound states remain to be also degenerate in mass beyond the LOY approximation, whereas unstable states (i.e., states for which \( m_0 > \mu \)) appear to be nondegenerate in mass in this model if CPT–symetry holds. These observations confirm our earlier conclusions implied by properties (108) — (100).

In a general case, in contradistinction to the property (67) obtained within the LOY theory, one finds for diagonal matrix elements of \( H_\parallel \) calculated within the approximation briefly sketched in Appendix B that in CPT–invariant system (see (B13), (B15))

\[
h^{\Theta}_{11} \neq h^{\Theta}_{22}, \tag{116}
\]
where $h^\Theta_{jk}$ denotes matrix elements of $H^\Theta_{\parallel}$ and $H^\Theta_{\parallel}$ is the operator $H_{\parallel}$ when the property (101) occurs. (The other approximation improving WW formulae for $h_{jk}$ and used in [3] lead to the same result). Similarly to the case of Fridrichs–Lee model, assuming that,

$$|H_{12}| \ll |H_{0}|,$$

(117)

where

$$H_{0} = \frac{1}{2}(H_{11} + H_{22}),$$

(118)

($H_{0} = H_{11} = H_{22}$ if (101) holds) we find that (see (B18), (B19))

$$2h^\Theta_{z} \equiv h^\Theta_{11} - h^\Theta_{22} \simeq H_{12} \frac{\partial \Sigma_{21}(x)}{\partial x} \bigg|_{x=H_{0}} - H_{21} \frac{\partial \Sigma_{12}(x)}{\partial x} \bigg|_{x=H_{0}} \neq 0,$$

(119)

($2h^\Theta_{z} = 0$ only if $[CP, H] = 0$). This means that if CPT–symmetry is conserved in the system considered and (117) holds then the parameter $\delta (21)$ takes the form

$$\delta^\Theta \equiv \frac{2h^\Theta_{z}}{D^\Theta} \neq 0,$$

(120)

where

$$D^\Theta \simeq D^{LOY} - (H_{12} + H_{21}a^{-1})(1 + a)\frac{\partial \Sigma_{0}(x)}{\partial x} \bigg|_{x=H_{0}},$$

and $D^{LOY} = h_{12}^{LOY} + h_{21}^{LOY} + \Delta \mu^{LOY}$, $\Delta \mu^{LOY} \equiv \mu^{LOY}_{s} - \mu^{LOY}_{l}$, $\mu_{s}^{LOY}$ and $\mu_{l}^{LOY}$ are eigenvalues of $H^{LOY}$ for eigenstates $|K_{s}>$ and $|K_{l}>$ respectively, $\Sigma_{0}(H_{0})$ is defined by formula (B14),

$$a \equiv \left( \frac{h_{21}^{LOY}}{h_{12}^{LOY}} \right)^{1/2}.$$

(121)

Confronting relations (67) with (97), or (68) with Conclusions 2, one should remember that, in fact, $H^{LOY}$ can be considered as the lowest, non-trivial order approximation in the perturbation $H_{W}$: All the terms to higher orders than $(H_{W})^{2}$ are neglected in $H^{LOY}$ [2] — [7]. It is obvious that CPT– and other transformation properties of such an approximate effective Hamiltonian and of the exact one need not be the same. Taking into account all the above, it seems that for the proper understanding of CPT–invariance tests
and CPT–invariance, or possible CPT–violation phenomena it is necessary to consider higher order contributions into $H_\parallel$ than those contained in $H_{LOY}$.

Effects described above are, probably, beyond today’s experiments accuracy, nevertheless nobody can exclude that accuracy of future experiments will be much higher and the result $\text{Re}(h_{11} - h_{22}) \neq 0$ will be obtained and then the question how to interpret such a result could arise. The LOY theory is unable to give a correct interpretation of such a hypothetical experimental result. For the correct interpretation of such a result, matrix elements of $H_\parallel$ should be calculated much more exactly than it is possible within the LOY approach. The result (67) of the LOY approximation is model independent whereas, in the more exact theory, the magnitude of $\text{Re} (h_{11} - h_{22})$ depends on the model of interactions considered. So a new possibility of the verification of models of weak interactions arises (see formulae (113) — (115)).

It also seems, that above results have some meaning when attempts to describe possible deviations from conventional quantum mechanics are made and when possible experimental tests of such a phenomenon and CPT–invariance in neutral kaons system are considered [17, 18]. In such a case a very important role is played by nonzero contributions to $(h_{11} - h_{22})$ [17, 18]: The correct description of these deviations and experiments mentioned is impossible without taking into account results of this and above Sections 2, 4 and 5.

In the light of the above discussion it is clear that it will be essential for the result of experimental tests of the CPT–invariance to be $|h_{11} - h_{22}| \ll |2h_z^{FL}|$ (110). In contradistinction to the standard, conventional interpretation [3] — [7], such results will prove that $[\Theta, H] \neq 0$ in neutral kaons, or other similar, systems. The same conclusion will follow from the result $|h_{11} - h_{22}| \gg |2h_z^{FL}|$. There is a chance for the tested system that $[\Theta, H] = 0$ only if the experiment shows that $(h_{11} - h_{22}) \sim 2h_z^{FL}$. Such an interpretation follows from the results of Sections 4 and 5, Conclusions 1, and from properties of generalized Fridrichs–Lee model [3, 10]. In the general case, all above conclusions are valid if one replaces $h_z^{FL}$ by $h_z^{\Theta}$ (119).

Analogous consequences will follow from the following results of experiments: Results $|\delta| \ll |\delta^{\Theta}|$ or $|\delta| \gg |\delta^{\Theta}|$ (120) will prove that CPT–symmetry is violated by interactions causing decays of $K_0, \bar{K}_0$ mesons or similar systems. Only the result $\delta \sim \delta^{\Theta} \neq 0$ can be considered as the confirmation of CPT–invariance of the tested system.

The problem is whether the experimenter will be able to perform their
experiments with the accuracy guaranteeing the proper answer to the question of whether $|h_{11} - h_{22}| \ll |2h^F_z| (|h_{11} - h_{22}| \ll |2h^\Theta|)$ and $|\delta| \ll |\delta^\Theta|$ or $|h_{11} - h_{22}| \gg |2h^F_z| (|h_{11} - h_{22}| \gg |2h^\Theta|)$ and $|\delta| \gg |\delta^\Theta|$.

From Conclusions 1 and 2 it follows that only the interpretation of results $\text{Re.}(h_{11} - h_{22}) = 0$ and $\delta = 0$ is uncontrovertible. Therefore only such results can be understood independently of the model.

The proper interpretation of the results $\text{Re}(h_{11} - h_{22}) \neq 0$ and $\delta \neq 0$ depends on the model calculations of the quantity $(h_{11}(t) - h_{22}(t))$ or, which is equivalent, on the calculated values of matrix elements of type $\langle 2|A(t)|1 \rangle$ and $\langle 2|B(t)|1 \rangle$. This can not be performed within the LOY approach and requires more exact approximations. It seems that the approximation described in [1], the one described in Appendix B and exploited in [9] — [11] may be a more effective tool for this purpose.

Above considerations suggest that tests consisting of a comparison of the equality of the decay laws of $K_0$ and $\bar{K}_0$ mesons, i.e. verifying the relation (108), seem to be the only completely model independent tests for verifying the CPT–invariance in such and similar systems.

Taking into account all the above, it seems that all theories describing the time evolution of the neutral kaons and similar systems by means of the effective Hamiltonian $H_\parallel$ governing their time evolution, in which the CPT–invariance of the total system leads to the property (67) for this $H_\parallel$, (such as LOY theory [2] — [6] based on the WW approximation), are unable to give the exact and correct description of all aspects of the effects connected with the violation or nonviolation of the CP– and especially CPT–symmetries. (It occurs probably because of the fact that such theories cannot exactly satisfy unitarity [25] and lead to inconsistencies of CPT–symmetry properties of the $H_\parallel$ and the total Hamiltonians $H$ [26]). Also, it seems that results of the experiments with neutral kaons, etc., designed and carried out on the basis of expectations of theories within WW approximation, such as tests of CPT invariance (at least results of those in which CPT–invariance or CPT–noninvariance of $H_\parallel$ generated by such invariance properties of $H$ were essential), should be revised using other methods than the WW approach.

**Appendix A.**
The aim of this Appendix is to calculate the commutator $[\Theta, H_\parallel(t)]$ discussed
in Sec. 2 and to study some of its applications. In order to calculate this commutator it is convenient to express $H(t)$ by means of the formula (85), and then to use assumption (89), the definition of $[U(t)]^{-1}$ (71), property (72) and the following one

$$ P[U(t)]^{-1} = [U(t)]^{-1} P \equiv P[U(t)]^{-1} P, \quad (A1) $$

which is the consequence of (71) and (72).

Let us consider a commutator $[\Theta, P[U(t)]^{-1}]$. It is only nontrivial relation, necessary for the calculation of $[\Theta, H(t)]$. Using the property $U(t) = PU(t) = U(t)P = PU(t)P$ and relations (71), (A1) we find (here the assumption (89) is crucial)

$$ [\Theta, P[U(t)]^{-1}] = \Theta P[U(t)]^{-1} - P[U(t)]^{-1} \Theta $$

$$ = \Theta P[U(t)]^{-1} - P[U(t)]^{-1} P \Theta $$

$$ = PU^{-1} (U(t) - \Theta U) U^{-1} $$

$$ = -PU^{-1} [\Theta, U] U^{-1} $$

$$ \equiv -PU^{-1} P[\Theta, U] PU^{-1}. \quad (A2) $$

Relation (88), properties (A1) and expression (83) lead to the following formulae

$$ [\Theta, H(t)] = [\Theta, PHUPU^{-1}] $$

$$ = [\Theta, PH] UPU^{-1} + PH[\Theta, UPU^{-1}] $$

$$ = P[\Theta, H] UPU^{-1} \quad \text{(A3)} $$

$$ + PH\{[\Theta, UP] U^{-1} + UP[\Theta, PU^{-1}]\}. $$

All steps in the above formulae and in formulae leading to (A2) have been performed without changing the order of operators appearing in products of type $\Theta H, \Theta U(t)$, etc.. By virtue of the assumption (89) only the order of operators $\Theta$ and $P$ in products $\Theta P$, etc., can be changed when it is necessary.

Now, defining

$$ A(t) \overset{\text{def}}{=} P[\Theta, H] UPU^{-1}, \quad (A4) $$

25
(which equals (91)) and taking into account (A2), one can obtain formula (90) from (A3)

\[
[\Theta, H_{\parallel}(t)] \equiv A(t) + PH[\Theta, U]PU_{\parallel}^{-1}
+ \ PHUP\{ -U_{\parallel}^{-1}P[\Theta, U]PU_{\parallel}^{-1}\}
= A(t) + B(t),
\]

where (see (92))

\[
B(t) = \left\{ PH - PHUPU_{\parallel}^{-1}P\right\}[\Theta, U]PU_{\parallel}^{-1},
\]
or (by means of (85))

\[
B(t) \equiv \left\{ PH - H_{\parallel}P\right\}[\Theta, U]PU_{\parallel}^{-1},
\]

(i.e., simply (93)), and due to the properties (86), (87)

\[
B(t) = \left\{ PHQ - V_{\parallel}P\right\}[\Theta, U]PU_{\parallel}^{-1},
\]

that is formula (94).

Let us consider now some details of the derivation of the relation (96). Taking into account properties of the anti–unitary operator \(\Theta\) and CPT–transformation properties of states \(|K_0>, |\overline{K}_0>\), etc., (see Sec. 2), without any assumptions for the commutator \([\Theta, H]\), one can transform the matrix element \(<2|\Theta H_{\parallel}(t)\Theta^{-1}|2>\) appearing in (96) as follows

\[
<2|\Theta H_{\parallel}(t)\Theta^{-1}|2> \equiv <K_0|\Theta H_{\parallel}(t)\Theta^{-1}|K_0>
\equiv <\Theta K_0, \Theta H_{\parallel}(t)\Theta^{-1}\Theta K_0>
= <\Theta^{-1}\Theta H_{\parallel}(t)\Theta^{-1}K_0, \Theta^{-1}\Theta K_0>
= <H_{\parallel}(t)K_0, K_0>
= <K_0, H_{\parallel}(t)K_0>^*
\equiv <1|H_{\parallel}(t)|1>^* \equiv h_{11}(t)^*.
\]

This last relation and the following consequence of (95)

\[
<2|\Theta H_{\parallel}(t)\Theta^{-1}|2> - <2|H_{\parallel}(t)|2> \equiv <2|(A(t) + B(t))\Theta^{-1}|2>,
\]

26
yield
\[ h_{11}(t)^* - h_{22}(t) = < 2| (A(t) + B(t)) \Theta^{-1} | 2 >, \]  
(122)
i.e., the formula (103).

**Appendix B.**
The approximate formulae for \( H_{\parallel}(t) \) have been derived in [10, 11] using the Krolikowski–Rzewuski equation for the projection of a state vector [16], which results from the Schrödinger equation (46) for the total system under consideration, and, in the case of initial conditions of the type (47), takes the following form

\[ (i \frac{\partial}{\partial t} - PHP) U_{\parallel}(t) = -i \int_0^\infty K(t - \tau) U_{\parallel}(\tau) d\tau, \]  
(B1)

where \( U_{\parallel}(0) = P \),

\[ K(t) = \Theta(t) PHQ \exp(-itQHQ)QHP, \]  
(B2)

and \( \Theta(t) = \{1 \text{ for } t \geq 0, \quad 0 \text{ for } t < 0 \} \).

Taking into account (87) one finds from (1), (4) and (B1)

\[ V_{\parallel}(t) U_{\parallel}(t) = -i K^* \left[ 1 + \sum_{n=1}^{\infty} (-i)^n L \ast \ldots \ast L \right] * U_{\parallel}(t), \]  
(B3)

(Here the asterix * denotes the convolution: \( f * g(t) = \int_0^\infty f(t - \tau) g(\tau) d\tau \).

Next, using this relation and a retarded Green’s operator \( G(t) \) for the equation (B1)

\[ G(t) = -i \Theta(t) \exp(-itPHP) P, \]  
(B4)

one obtains [10, 11]

\[ V_{\parallel}(t) U_{\parallel}(t) = -i K \ast \left[ 1 + \sum_{n=1}^{\infty} (-i)^n L \ast \ldots \ast L \right] * U_{\parallel}(t), \]  
(B5)

where \( L \) is convoluted \( n \) times, \( 1 \equiv 1(t) \equiv \delta(t), \)

\[ L(t) = G \ast K(t), \]  
(B6)
and
\[ U^{(0)}_\parallel = \exp(-itPHP)P \] (B7)
is a "free" solution of Eq. (B1). Of course, the series (B5) is convergent if \( \| L(t) \| < 1 \). If for every \( t \geq 0 \)
\[ \| L(t) \| \ll 1, \] (B8)
then, to the lowest order of \( L(t) \), one finds from (B5) [10, 11]
\[ V_\parallel(t) \approx V^{(1)}_\parallel(t) \overset{\text{def}}{=} -i \int_0^\infty K(t-\tau) \exp[i(t-\tau)PHP]d\tau. \] (B9)
In the case of (50) of the projector \( P \), this approximate formula for \( V_\parallel(t) \) leads to the following expressions for the matrix elements \( v_{jk}(t \rightarrow \infty) \overset{\text{def}}{=} v_{jk} \) of \( V_\parallel(t \rightarrow \infty) \overset{\text{def}}{=} V_\parallel \approx V^{(1)}_\parallel(\infty) \) [10, 11],
\[
v_{j1} = - \frac{1}{2} \left( 1 + \frac{H_z}{\kappa} \right) \Sigma j_1(H_0 + \kappa) - \frac{1}{2} \left( 1 - \frac{H_z}{\kappa} \right) \Sigma j_1(H_0 - \kappa)
- \frac{H_{21}}{2\kappa} \Sigma j_2(H_0 + \kappa) + \frac{H_{21}}{2\kappa} \Sigma j_2(H_0 - \kappa),
\]
\[
v_{j2} = - \frac{1}{2} \left( 1 - \frac{H_z}{\kappa} \right) \Sigma j_2(H_0 + \kappa) - \frac{1}{2} \left( 1 + \frac{H_z}{\kappa} \right) \Sigma j_2(H_0 - \kappa)
- \frac{H_{12}}{2\kappa} \Sigma j_1(H_0 + \kappa) + \frac{H_{12}}{2\kappa} \Sigma j_1(H_0 - \kappa),
\]
where \( j, k = 1, 2 \),
\[ H_z = \frac{1}{2} (H_{11} - H_{22}), \] (B11)
and \( H_0 \) is defined by [118],
\[ \kappa = (|H_{12}|^2 + H_z^2)^{1/2}. \] (B12)
Hence, by [87]
\[ h_{jk} = H_{jk} + v_{jk}. \] (B13)
These formulae for \( v_{jk} \) and thus for \( h_{jk} \) have been derived without assuming any symmetries of the type CP–, T–, or CPT–symmetry for the total
Hamiltonian $H$ of the system considered. It should also be emphasized that all components of the expressions (B10) have the same order with respect to $\Sigma(\varepsilon)$.

In the case of preserved CPT–symmetry (101), one finds $H_{11} = H_{22}$ which implies that $\kappa \equiv |H_{12}|$, $H_z \equiv 0$ and $H_0 \equiv H_{11} \equiv H_{22}$, and

$$\Sigma_{11}(\varepsilon = \varepsilon^*) \equiv \Sigma_{22}(\varepsilon = \varepsilon^*) \overset{\text{def}}{=} \Sigma_0(\varepsilon = \varepsilon^*). \quad (B14)$$

Therefore matrix elements $v_{jk}^{\Theta}$ of operator $V^{\Theta}_\| (V^{\Theta}_\| \text{denotes } V_\| \text{ when (101) occurs})$ take the following form

$$v_{j1}^{\Theta} = -\frac{1}{2}\left\{ \Sigma_{j1}(H_0 + |H_{12}|) + \Sigma_{j1}(H_0 - |H_{12}|) \right\} + \frac{H_{21}}{|H_{12}|}\Sigma_{j2}(H_0 + |H_{12}|) - \frac{H_{21}}{|H_{12}|}\Sigma_{j2}(H_0 - |H_{12}|) \right\}, \quad (B15)$$

$$v_{j2}^{\Theta} = -\frac{1}{2}\left\{ \Sigma_{j2}(H_0 + |H_{12}|) + \Sigma_{j2}(H_0 - |H_{12}|) \right\} + \frac{H_{12}}{|H_{12}|}\Sigma_{j1}(H_0 + |H_{12}|) - \frac{H_{12}}{|H_{12}|}\Sigma_{j1}(H_0 - |H_{12}|) \right\}, \quad (B16)$$

Assuming (117), we find

$$v_{j1}^{\Theta} \simeq -\Sigma_{j1}(H_0) - H_{21}\frac{\partial \Sigma_{j2}(x)}{\partial x} \bigg|_{x=H_0}, \quad (B16)$$

$$v_{j2}^{\Theta} \simeq -\Sigma_{j2}(H_0) - H_{12}\frac{\partial \Sigma_{j1}(x)}{\partial x} \bigg|_{x=H_0}, \quad (B17)$$

where $j = 1, 2$. One should stress that due to a presence of resonance terms, derivatives $\frac{\partial}{\partial x}\Sigma_{jk}(x)$ need not be small and neither need the products $H_{jk}\frac{\partial}{\partial x}\Sigma_{jk}(x)$ in (B16).

Finally, assuming that (117)) holds and using relations (B16), (B13) and the expression (B3), we obtain for the CPT–invariant system

$$h_{j1}^{\Theta} \simeq h_{j1}^{L\Theta} - H_{21}\frac{\partial \Sigma_{j2}(x)}{\partial x} \bigg|_{x=H_0}, \quad (B18)$$
\[ h_{j_2}^\Theta \simeq h_{j_2}^{\text{LOY}} - H_{12} \frac{\partial \Sigma_{j_1}(x)}{\partial x} \bigg|_{x=H_0}, \quad (B19) \]

where \( j = 1, 2 \). From these formulae we conclude that, e.g., the difference between diagonal matrix elements of \( H_j^\Theta \), which plays an important role in designing CPT– invariance tests for the neutral kaons system, equals (119).

Analogously, to the lowest order of \( |H_{12}| \), for eigenvalues \( \mu_l, \mu_s \) of \( H_\parallel \), we obtain [11]

\[
\mu_s^\Theta \simeq \mu_s^{\text{LOY}} - \frac{1}{2} \left[ H_{12} \left( \frac{\partial \Sigma_{21}(x)}{\partial x} \bigg|_{x=H_0} + a \frac{\partial \Sigma_0(x)}{\partial x} \bigg|_{x=H_0} \right) + H_{21} \left( \frac{\partial \Sigma_{12}(x)}{\partial x} \bigg|_{x=H_0} + a^{-1} \frac{\partial \Sigma_0(x)}{\partial x} \bigg|_{x=H_0} \right) \right],
\]

\[
(B20)
\]

\[
\mu_l^\Theta \simeq \mu_l^{\text{LOY}} - \frac{1}{2} \left[ H_{12} \left( \frac{\partial \Sigma_{21}(x)}{\partial x} \bigg|_{x=H_0} + a^{-1} \frac{\partial \Sigma_0(x)}{\partial x} \bigg|_{x=H_0} \right) + H_{21} \left( \frac{\partial \Sigma_{12}(x)}{\partial x} \bigg|_{x=H_0} - a \frac{\partial \Sigma_0(x)}{\partial x} \bigg|_{x=H_0} \right) \right],
\]

where \( a \) is defined by (121).

References
[1] W. Pauli, in: ”Niels Bohr and the Development of Physics”. ed. W. Pauli (Pergamon Press, London, 1955), pp. 30 — 51. G. Luders, Ann. Phys. (NY) 2 (1957) 1. R. Jost, Helv. Phys. Acta 30 (1957) 409. R.F. Streater and A. S. Wightman, ”CPT, Spin, Statistics and All That” (Benjamin, New York, 1964). N. N. Bogolubov, A. A. Logunov and I. T. Todorov, ”Introduction to Axiomatic Field Theory” (Benjamin, New York, 1975).

[2] T. D. Lee, R. Oehme and C. N. Yang, Phys. Rev., 106, (1957) 340.

[3] T. D. Lee and C. S. Wu, Annual Review of Nuclear Science, 16, (1966) 471. Ed.: M. K. Gaillard and M. Nikolic, Weak Interactions, (INPN et
de Physique des Particules, Paris, 1977); Chapt. 5, Appendix A. S. M. Bilenkij, Particles and nucleus, vol. 1. No 1 (Dubna 1970), p. 227 [in Russian]. P. K. Kabir, The CP-puzzle, Academic Press, New York 1968.

[4] L. A. Khalfin, The theory of $K_0, \overline{K}_0 (D_0, \overline{D}_0$ and $T_0, \overline{T}_0$ mesons beyond the Weisskopf-Wigner approximation and the CP–problem, preprint LOMI P–4–80, Leningrad, February 1980 [in Russian].

[5] J. W. Cronin, Rev. Mod. Phys. 53, (1981) 373. J. W. Cronin, Acta Phys. Polon., B15, (1984) 419. V. V. Barmin, et al., Nucl. Phys. B247, (1984) 293. L. Lavoura, Ann. Phys. (NY), 207, (1991) 428. C. Buchanan, et al., Phys. Rev. D45, (1992) 4088. C. O. Dib, and R. D. Peccei, Phys. Rev., D46, (1992) 2265. R. D. Peccei, CP and CPT Violation: Status and Prospects, Preprint UCLA/93/TEP/19, University of California, June 1993.

[6] E. D. Comins and P. H. Bucksbaum, Weak interactions of Leptons and Quarks, (Cambridge University Press, 1983). T. P. Cheng and L. F. Li, Gauge Theory of Elementary Particle Physics, (Clarendon, Oxford 1984).

[7] Yu. V. Novozhilov, Introduction to the Theory of Elementary Particles (Nauka, Moskow 1972), (in Russian). W. M. Gibson and B. R. Pollard, Symmetry Principles in Elementary Particle Physics, (Cambridge University Press, 1976).

[8] C. B. Chiu and E. C. G. Sudarshan, Phys. Rev. D 42 (1990) 3712; E. C. G. Sudarshan, C. B. Chiu and G. Bhamathi, Unstable Systems in Generalized Quantum Theory, Preprint DOE-40757-023 and CPP-93-23, University of Texas, October 1993.

[9] K. Urbanowski, Int. J. Mod. Phys. A 7, (1992) 6299. K. Urbanowski, Phys. Lett. A171, (1992) 151.

[10] K. Urbanowski, Int. J. Mod. Phys. A 8, (1993) 3721.

[11] K. Urbanowski, Int. J. Mod. Phys. A 10, (1995) 1151.

[12] K. Urbanowski, Phys. Lett. B 313, (1993) 374.

[13] A. Messiah, Quantum Mechanics, vol. 2, (Wiley, New York 1966).

[14] A. Bohm, Quantum Mechanics: Foundations and Applications, 2nd ed., (Springer, New York 1986).

[15] L. P. Horwitz and J. P. Marchand, Helv. Phys. Acta 42 (1969) 801.

[16] W. Krolikowski and J. Rzewuski, Bull. Acad. Polon. Sci. 4 (1956) 19. W. Krolikowski and J. Rzewuski, Nuovo. Cim. B 25 (1975) 739 and references therein. K. Urbanowski, Acta Phys. Polon. B 14 (1983) 485. K. Urbanowski, Phys. Rev. A 50, (1994) 2847.
[17] J. Ellis, J. S. Hagelin, D. V. Nanopoulos and M. Srednicki, Nucl. Phys. B241, (1984) 381. J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B 293, (1992) 142. J. Ellis, J. L. Lopez, N. E. Mavromatos and D. V. Nanopoulos, Phys. Rev. D 53, (1996) 3846.

[18] P. Huet and M. E. Peskin, Nucl. Phys. B 434, (1995) 3. P. Huet, Testing Violation of CPT and Quantum Mechanics in the $K_0 - \overline{K}_0$ system, Preprint: SLAC–Pub–6491, May 1994.

[19] K. Urbanowski, Int. J. Mod. Phys. A 7, (1992) 6299.

[20] K. Urbanowski, Is the new interpretation of some standard CPT–violation parameters necessary?, Preprint of the Pedagogical University, No WSP–IF 94–39, Zielona Gora, May 1994.

[21] E. P. Wigner, in: ”Group Theoretical Concepts and Methods in Elementary Particle Physics”, ed.: F. Goresy, (New York 1964).

[22] J. S. Bell and J. Steinberger, in: ”Oxford Int. Conf. on Elementary Particles 19/25 September 1965: Proceedings”, Eds. T. R. Walsh, A. E. Taylor, R. G. Moorhouse and B. Southworth, (Rutherford High Energy Lab., Chilton, Didicot 1966), pp. 195 — 222.

[23] Review of Particle Physics, Phys. Rev. D54, (1996), No 1, Part 1.

[24] L. Maiani, in ”The Second DaeΦne Physics Handbook”, vol. 1, Eds. L. Maiani, G. Pancheri and N. Paver, SIS — Pubblicazioni, INFN — LNF, Frascati, 1995; pp. 3 — 26.

[25] P. K. Kabir and A. Pilaftsis, Phys. Rev. A 53, (1996), 66.

[26] K. Urbanowski, Is the standard interpretation of CPT–violation parameter for neutral kaon complex well–founded?, Preprint of the Pedagogical University, No WSP–IF 97–39, Zielona Gora, January 1997; Int. J. Mod. Phys. A, in press.