Dark Matter, Modified Gravity and the Mass of the Neutrino.

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It has been suggested that Einstein’s theory of General Relativity can be modified to accommodate mismatches between the gravitational field and luminous matter on a wide range of scales. Covariant theories of modified gravity generically predict the existence of extra degrees of freedom which may be interpreted as dark matter. We study a subclass of these theories where the overall energy density in these extra degrees of freedom is subdominant relative to the baryon density and show that they favour the presence of massive neutrinos. For some specific cases (such as a flat Universe with a cosmological constant) one finds a conservative lower bound on the neutrinos mass of \( m_\nu > 0.31 \text{ eV} \).

I. INTRODUCTION

There is compelling evidence that the baryons in the Universe are unable to generate the gravitational potentials that we observe on a wide range of scales. A simple paradigm can be used to explain this mismatch between light and gravity: the Universe is filled with an appreciable amount of matter which is cold (i.e. has non-relativistic velocities today) and does not interact with light. It has been shown that Cold Dark Matter (CDM) can explain a host of observation, from dynamics of clusters to the formation of the cosmic web [1].

The CDM paradigm has been proposed within the context of Newtonian gravity and Einstein’s theory of General Relativity. It has been argued that these theories may not be valid on all scales. Indeed, proposals for modifying gravity have been shown to fit much of the currently available data [2]. A plethora of covariant theories have been studied in detail: TeVeS gravity, modified Einstein-Aether theories, conformal gravity, higher derivative actions, etc, have been advocated as possible rival theories to the CDM scenario [3, 4, 5, 6, 7, 8, 9]. There has been considerable effort in studying the cosmological consequences of these theories [5, 6, 7, 8]. Given the level of precision of current cosmological data, it is possible to find severe constraints on these alternative theories and compare their ability to describe nature with the CDM scenario.

There is an important, generic feature of covariant theories of modified gravity which is often overlooked: although they tamper with the gravitational sector of the equations of motion, they also inevitably lead to the introduction of extra degrees of freedom which may be interpreted as an exotic form of dark matter. Let us exemplify. Theories which modify the Einstein-Hilbert action by, for example, replacing the Ricci scalar, \( R \), by a function of different curvature invariants, \( f(R, R_{\alpha\beta}R^{\alpha\beta}, \ldots) \), introduce higher derivative terms, and hence new modes. These new modes will contribute to the overall energy density. This is patently obvious in the case of theories where \( f \) is simply a function of \( R \); such theories can be mapped onto normal Einstein gravity with an additional scalar field. This also true of conformal gravity, where the action is now constructed from the Weyl tensor. A field must be added to fix the scale of gravity and the resulting low energy equations are fourth order [14]. More modern attempts at constructing theories of modified gravity have the same characteristics in a much more explicit way. In TeVeS [5], a scalar field and a vector field is introduced which not only modify the gravitational field equations but also source the very same field through their stress energy tensor. In generalized Einstein-Aether theories, a time-like vector field is introduced [7].

Given what we have just said, there is an obvious question: aren’t these extra degrees of freedom simply a contrived form of dark matter? It is conceivable that the extra degrees of freedom in modified theories of gravity may play such a role. If so, dark matter has been introduced through the back door. It turns out that the role of extra degrees of freedom in theories of modified gravity is more complicated than one might expect. In Skordis et al. [10], it was shown that the extra degrees of freedom in TeVeS can make a negligible contribution to the background (or overall) energy density. Indeed, if TeVeS is to be consistent with Big Bang nucleosynthesis, the fractional energy density in these extra degrees of freedom, \( \Omega_X \), must be under a percent. Yet even though \( \Omega_X \ll 1 \), fluctuations in the extra fields could have a significant impact on the growth of structure. In particular, due to the modified nature of gravity, they could source the growth of gravitational potentials and sustain them through Silk damping at recombination. These results were corroborated in Dodelson and Liguori [15], where the fluctuations in the vector field were found to play an important role.

Hence some theories of modified gravity can fit current observations of large scale structure, either from galaxy surveys or the cosmic microwave background, even though \( \Omega_X \ll 1 \). We would like to point out that the latter property is not generic. In some incarnations \( \Omega_B \ll \Omega_X \ll 1 \) where \( \Omega_B \) is the fractional energy density in baryons. These theories end up being a hybrid of the two paradigms, modified gravity and dark matter, and in principle should be harder to distinguish from dark matter theories (although there are some suggestions of specific tests) [7,8].

In this paper we will try to expand on an important feature of TeVeS pointed out in Skordis et al.: if one assumes that the Universe is flat and the only form of non relativistic matter consists of 5% baryons (consistent with Big Bang Nucleosynthesis), the angular power spectrum of the Cosmic Microwave Background (CMB) will differ significantly from observations. The only way to resolve this discrepancy is to introduce some form of non-relativistic matter, and the only one allowed within the known menagerie of fundamental con-
stiuents of the Universe is a massive neutrino. To match observations of the CMB, neutrinos with a mass of approximately 2 eV are needed. This result is clearly a hint of a more general statement that may be made about theories of modified gravity in which the extra degrees of freedom play a subdominant role: if these theories are to agree with measurements of the CMB then they require the presence of massive neutrinos. We wish to see if this implies a lower bound on the mass of the neutrino.

II. AN APPROXIMATE THEORY AND COSMOLOGICAL OBSERVABLES

Let us consider a generic modified theory of gravity in the limit of homogeneity and isotropy. The physical metric (i.e. the metric which is minimally coupled to the matter fields) can be parametrized in terms of a scale factor, $a(t)$ which has a logarithmic derivative, $H = \frac{d\ln(a)}{dt}$. The energy density of the Universe can be split into the normal degrees of freedom, $\rho$ (such as baryons, photons, neutrinos and dark energy), and the extra degrees of freedom that arise from the modifications, $\rho_X$. The modified Friedman equations look somewhat like

$$ F(a, H)H^2 = \frac{8\pi G}{3}(\rho + \rho_X) \quad (1) $$

where $G$ is Newton’s constant and $FH^2$ is a function that arises from varying the action for a particular theory. In fact it is convenient to rewrite the equation in a more familiar form by defining an effective Newton’s constant $G_{\text{eff}} = G/F$. For the purpose of what follows we use a parametrization such that $G_{\text{eff}} \simeq G_{\text{eff}}(a)$; with a sufficiently flexible choice of parameters we can encompass cases where $G_{\text{eff}}$ depends on $a, H$, etc.

We consider a subclass of the theories, in which $\rho_X < \rho_B$, (where $\rho_B$ is the Baryon density). We consider a parametrization such that

$$ \rho_X \simeq f_B \rho_B + f_R \rho_R \quad (2) $$

where $\rho_R$ is the energy density in radiation. We have that $f_B < 1$ and the correct abundance of light elements requires that $f_R < 10^{-2}$. We also include in $\rho$, a component that behaves like dark energy, $\rho_{DE}$, with an equation of state $w < 0$. We find it convenient to parametrize the equation of state of the dark energy component as $w = w_0 + w_1 z/(1 + z)$. Modifications to the gravitational sector may lead to accelerated expansion at late times (such as those proposed in [4, 8, 12]), meaning that the dark energy could also arise from the extra fields in the modified gravity sector. Our dark energy term includes all of these possibilities.

With the evolution of the scale factor in hand, there are a few observables that we may now calculate. Let us start off with the position of the first peak of the angular power spectrum of the Cosmic Microwave Background (CMB). It is a direct measure of the angular diameter distance and hence of the expansion history of the Universe from recombination until today and is the centrepiece of the analysis of this paper. Schematically we have the following picture [16]. Before recombination (which occurred at time $t_*$), photons and baryons were tightly coupled and underwent acoustic oscillations. During tight coupling the photon density contrast in the conformal Newtonian gauge, obeys the differential equation

$$ \ddot{\delta}_\gamma + \frac{3\rho_\gamma}{\delta \rho_\gamma + 4\rho_\gamma a} \dot{\delta}_\gamma + k^2 c_s^2 \delta_\gamma = S[\Phi, \bar{\Phi}, \Psi] \quad (3) $$

where $c_s^2 = \frac{4\rho_\gamma}{3(3\rho_\gamma + 4\rho_\gamma)}$ is the sound speed, $S[\Phi, \bar{\Phi}, \Psi]$ is a source (a function of the gravitational potentials, $\Phi$ and $\Psi$) and derivatives are with conformal time $\tau$. In the WKB approximation [16] the two linearly independent solutions to the homogeneous part are $\cos kr_*\sin kr_*$ and $\sin kr_*\cos kr_*$, and depend on the sound horizon $r_*(\tau) = \int_0^\tau c_s\,d\tau$. The important thing is that these solutions are valid for any theory of gravity for which photons and baryons see the same physical metric. All modifications are to gravity or additional fields and implicitly alter the inhomogenous part through the source term $S$ which only depends on the potentials $\Phi$ and $\Psi$ of that same physical metric.

The physical scale, $d_*$ of the acoustic waves is set by the sound horizon at time $t_*$, i.e. $d_* \simeq \int_0^{t_*} c_s(t)\,dt$. After recombination, photons decoupled from the baryons and freestreamed towards us, travelling a distance given by $d_0 = c_0 \int_0^{t_*} c_s\,dt/a = \frac{c_0}{a_0} \int_0^{a_0} da/a(a^2H)$ where the subscript 0 labels today. The angular size on the sky of the sound horizon, $\theta_*$, is given by $\theta_* \simeq \frac{a_0 d_*}{a t_*}$. The sound horizon at last scattering leaves a very distinct signature on the angular power spectrum of the CMB: a series of peaks and troughs. The spectrum generated at recombination is related to the spectrum today via a projection through a spherical Bessel function $j_1(k(\tau - t_*))$, where $ad\tau = dt$, (an ultraspherical bessel function in the curved case). Once again this is independent of the theory of gravity. The position of the peaks and troughs in the angular power spectrum is primarily dependent on the cosmological shift parameter, $\mathcal{R}$ which is related to the angular diameter distance and is given by

$$ \mathcal{R} = \frac{1}{2} \sqrt{\frac{\Omega_M(a_0)}{\Omega_K(a_0)} \sin y} $$

$$ y = \sqrt{\Omega_K(a_0)\int_{a_*}^{a_0} \frac{da}{a^2(\sum_i \Omega_i(a))^{1/2}}} \quad (4) $$

where $\Omega_i(a)$ is the fractional energy density of component $i$ as a function of scale factor ($K$ corresponds to the curvature and $M$ to non relativistic matter) [17].

Very few assumptions have gone into this calculation: the Universe underwent recombination and the horizon structure is a result of the expansion rate of the Universe. Current measurements of the CMB have reached a level of precision such that it is practically impossible to deviate from this simple picture [19]. Attempts at changing these fundamental assumptions inevitably lead to radical departures from this simple picture and a gross mismatch to the data. So any theory of modified gravity must lead to the basic picture of the CMB that
we infer from the data. Hence we can use our Eq. 1 to work out $R$ for theories of modified gravity to find the fractional energy densities must be rescaled by the effective Newton’s constant, i.e. we must replace $\Omega_i$ by $(G_{\text{eff}}/G_0)|\Omega_i$ in Equation 4. Throughout this analysis we consider a conservative lower bound on the shift parameter: $1.63 < R < 1.76$ [18].

Another useful observable, as measured from the Hubble diagram of distant supernovae, is the luminosity distance, $d_L$. It is related to the angular diameter distance, $d_A$, described above, through $d_L = (1 + z)d_A$, where $1 + z = a_0/a$ defines the redshift at a given value of the scale factor. While the CMB gives us one measure of $d_A$ at $z \approx 1100$, the Hubble diagram of distant supernovae gives us a series of measurements of $d_L$ out to $z \approx 1.8$. Again, as above we can use our modified Friedman equations to calculate $d_L$, and compare to the data. Given that we do not have to use any information about perturbations about the background, we make even fewer assumptions. We use the group of supernovae termed the ‘gold’ set, from the HST/Goods programme [20], complemented by the recently discovered higher redshift supernovae, reported in [21].

Finally, we consider two more measurements. We take into account the constraints on $\Omega_B$ from the abundance of light elements. We use $\Omega_B = 0.022 \pm 0.002$, where the Hubble constant is defined to be $H_0 = 100h\text{km s}^{-1}\text{Mpc}^{-1}$. Lastly we consider current constraints from the Key Project of the Hubble Space Telescope (HST) on the expansion rate today. We use $H_0 = 72 \pm 8\text{km s}^{-1}\text{Mpc}^{-1}$.

### III. EXPLORING PARAMETER SPACE

We are interested in seeing if the presence of massive neutrinos is a generic feature of the class of models that we are considering. We assume three families of neutrinos with identical masses, and we take the mass of each family, $m_\nu$, as the free parameter. An obvious first case to study is a generalization of the TeVeS result from Skordis et al, i.e. a Euclidean Universe with a cosmological constant and a constant effective Newton’s constant. Indeed we find that the posterior for $m_\nu$ is positive, centered at $m_\nu \approx 0.84 \text{eV}$ and we can set a lower bound on the neutrino mass at the 95% confidence level (CL) of $m_\nu > 0.31 \text{eV}$, as shown in Table 1. The value of $m_\nu$ proposed in Skordis et al lies comfortably in that range $0.31 < m_\nu < 1.48 \text{eV}$.

Relaxing the assumption that the acceleration is driven by a cosmological constant (i.e. freeing up $w_0$ and $w_1$), leads to a lower bound of $m_\nu > 0.92 \text{eV}$, slightly stronger than in the previous case. The supernovae data strongly constrain the parameters describing the nature of dark energy in the range $0 \leq z \leq 1.8$, the era where its contribution is dominant, and favour an effective $w(z) < -1$. This means that the contribution of the dark energy component diminishes more rapidly than $\Lambda$ as a function of $z$. To compensate, the neutrinos are required to be relativistic at the surface of last scattering and hence considerably more massive, giving rise to the observed shift in the distribution to higher masses. The increased freedom gives a broader distribution. When the supernovae data set is removed, a larger contribution to the total energy from the dark energy component is allowed, weakening the lower bound on $m_\nu$, as shown in Table 1. However, for this simple class of theories, strong statements can now be made: there is a definite lower bound on the mass of the neutrino, as can be seen from Table 1.

Up until now, studies of theories of modified gravity have been undertaken in the context of Euclidean Universes. Relaxing the assumption of spatial flatness greatly broadens the posterior distribution of $m_\nu$, in particular, extending it to as much as $1.8 \text{eV}$ at the 95% CL. These models correspond to closed Universes where $\Omega_K < 0$. In addition the dark energy parameters are less strongly peaked (due to the degeneracy with curvature). Models in which the Universe is open are however favoured, leading to a weakened lower bound of $m_\nu > 0.07 \text{eV}$ but only at the 68% CL. In the absence of the supernovae data, $\Omega_K$ is weakly constrained according with the increased freedom. This leads to a generally broader $m_\nu$ distribution with a similar peak.

We have been exploring the effect of the extra degrees of freedom but we should expect modifications to the left hand side of equation 1. We have parameterized this in terms of $G_{\text{eff}}$ that depends on the scale factor. In principle, the time dependence of $G_{\text{eff}}$ can be more complex, depending on the normal matter fields as well as the extra degrees of freedom. Furthermore, for any given theory of modified gravity, the Bianchi identities as well as the various couplings between $G_{\text{eff}}$ and the remaining sector, impose specific constraints on its time evolution [22]. I.e. we do no have complete freedom to vary $G_{\text{eff}}$.

In what follows, we will be conservative and jettison any constraints that come from consistency but we will consider two types of relatively general behaviour which encompass what we have found in a wide range of models. One simple parametrization is

$$G_{\text{eff}} = G_0(1 + z)^n$$

For example for TeVeS one finds that, for a sufficiently small $n$ one can adequately mimic the behaviour of $G_{\text{eff}}$. Note that this parametrization does lead to a monotonically changing $G_{\text{eff}}$, all the way back to recombination and so must really only be considered an approximation- if not, it might lead to substantial changes to the peak structure in the CMB at re-
A variable $G_{eff}$ can have a substantial effect on allowed neutrino masses. In particular, in a Euclidean Universe with cosmological constant, it lowers the required mass contribution from neutrinos significantly to $m_\nu < 0.35$ eV at the 2σ level such that the massless case is no longer ruled out. This implies an anti-correlation between $n$ and $m_\nu$. Extending the model further to include dark energy again requires larger neutrino masses (lower bound of 0.22 eV at 95% CL). However in non-flat Universe case, the freedom in parameter space means that a wide range of masses are tolerable, including the massless scenario and $m_\nu = 1.32$ eV at the 2σ level. We note that allowing for the possibility of a time-dependent $G_{eff}(z)$ parameterized as above and admitting spatial curvature will have similar effects on the Hubble equation. The primary effect of $G_{eff}$ is to shift the distribution of $m_\nu$ to lower values, while $\Omega_K$ increases the range of neutrino masses that can be tolerated. This is explicitly illustrated in Figure 1. The plot (a) compares the 68% and 95% confidence intervals in $(m_\nu, \Omega_K)$ space when $G_{eff}$ is time-independent (dashed lines) and dynamical (solid lines) and shows the shift in the allowed regions to more negative values of $\Omega_K$. Figure 1(a) illustrates the impact of spatial curvature on constraints on $m_\nu$ in the presence of a time-dependent $G_{eff}$. The 1σ and 2σ regions are significantly reduced when curvature is admitted.

Another possible parameterization is if $G_{eff}$ switches between two values at some point in the past. For example, if $G_{eff}$ is approximately six times larger during the baryon dominated era than it is now, the background evolution will be essentially equivalent to that of dark matter dominated Universe at that time. To mimic this effect we consider

$$G_{eff} = G_0 \left( 1 + \frac{\alpha z}{1 + \gamma z} \right)$$

At low redshift, $G_{eff}$ starts at $G_0$ and increases linearly with $z$. At large redshift ($z > 1/\gamma$), $G(z)$ tends towards a constant $G_0 \alpha / \gamma$. We limit the change in $G(z)$ from $z = 0$ to recombination by imposing the condition that $\alpha / \gamma < 5$ such that it does not change by factor of more than 6. We find that with this parametrization, which is reminiscent of a number of different models, that the results are almost identical to that of the previous proposal for $G_{eff}$. Indeed, it is the very late time behaviour of $G_{eff}$ that plays a significant role in changing the observables and in that respects the two parametrizations are very similar.

### IV. DISCUSSION

It has been claimed that modified theories of gravity inevitably require the presence of massive neutrinos and that these may be sufficiently massive to be measurable with up and coming neutrino experiments such as KATRIN [23]. This claim has been triggered by two pieces of anecdotal evidence. Firstly that the simplest TeVeS model needs neutrinos to fit the angular power spectrum of the CMB as shown in [10]. And secondly, that attempts at reconciling observed and inferred

| Model                | $68\%$ CL   | $95\%$ CL   |
|----------------------|-------------|-------------|
| $\Lambda$CDM        | $m_\nu \leq 0.16$ | $m_\nu \leq 0.38$ |
| $w$CDM              | $0.32 \leq m_\nu \leq 0.97$ | $0.064 \leq m_\nu \leq 1.27$ |
| $w$CDM + $\Omega_K$ | $0.02 \leq m_\nu \leq 1.27$ | $m_\nu \leq 3.065$ |
| $\Lambda$CDM + $G(z)$ | $m_\nu \leq 0.14$ | $m_\nu \leq 0.35$ |
| $w$CDM + $G(z)$      | $0.015 \leq m_\nu \leq 0.53$ | $m_\nu \leq 0.92$ |
| $w$CDM + $\Omega_K + G(z)$ | $m_\nu \leq 1.38$ | $m_\nu \leq 2.79$ |
| $\Lambda$CDM + $G(\alpha, \gamma, z)$ | $m_\nu \leq 0.16$ | $m_\nu \leq 0.35$ |

TABLE II: Results for different cosmological models for a compilation data set where the supernovae data is excluded.

![Figure 1](image-url)
masses of clusters requires the presence of a massive neutrino halo [24, 25]. In this letter we have attempted to extend the remit of the first piece of evidence. We have found that, although for a restricted set of models, we can place a lower bound on the mass of the neutrino, for more general ranges of parameters, it is possible to satisfy the subset of cosmological constraints without having to invoke massive neutrinos. This is not to say that specific models with, for example, a variable effective Newton’s constant might not lead to a tight constraint on the neutrino mass. But it is clearly not possible to make a definitive statement on the mass of the neutrino for general theories of modified gravity. Theories must be studied case by case and we have shown how this can be done in an economical way. It may be possible to come up with constraints on the neutrino masses from a different set of observables, related to the second piece of evidence. For example, in the simplest picture of a cluster in these theories, neutrinos seem to be inevitable to be able to make up dynamical mass measurements and weak lensing observations. This simple picture is incomplete and much of the work that has been done on clusters in the context of modified gravity has opted to ignore the extra degrees of freedom [26]. They can play a significant role and, in the same way as for large scale observations, may substantially weaken cluster constraints on the neutrino mass. A more detailed analysis of these systems must be undertaken before definitive conclusions can be inferred.

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