A window into $\delta$ Sct stellar interiors: Understanding the eclipsing binary system TT Hor

Margaret Streamer$^1$*, Michael J. Ireland$^1$, Simon J. Murphy$^{2,3}$ and Joao Bento$^1$

$^1$ Research School of Astronomy and Astrophysics, Australian National University, Canberra, ACT, 2611, Australia
$^2$ Sydney Institute for Astronomy (SIfA), School of Physics, The University of Sydney, NSW 2006, Australia
$^3$ Stellar Astrophysics Centre, Department of Physics and Astronomy, Aarhus University, DK-8000 Aarhus C, Denmark

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
The semi-detached eclipsing binary system, TT Hor, has a $\delta$ Sct primary component (accretor) accreting mass from the secondary star (donor). We fit an eclipsing binary model from V, B and I photometry combined with spectroscopy using PHOEBE. Radial velocity variations of the center of mass of TT Hor AB over 2 years suggest the presence of a wide companion, consistent with a Kozai-Lidov resonance formation process for TT Hor AB. Evolutionary models computed with MESA give the initial mass of the donor as $\approx 1.6 M_\odot$ and that of the accretor as $\approx 1.3 M_\odot$. The initial binary orbit has a similar initial separation to the currently observed separation of 11.4 $R_\odot$. Mass transfer commences at an age of 2.5 Gyr when the donor is a subgiant. We model the accretor as a tidally-locked, 2.2 $\pm$ 0.2 - $M_\odot$ $\delta$ Sct pulsator which has accreted $\approx 0.9 M_\odot$ of slightly He-enriched material (mean Delta Y $< 0.01$) from the donor over the last 90 Myr. The best fit from all measured parameters and evolutionary state is for a system metallicity of [M/H] is 0.15. A pulsation model of the primary gives a self-consistent set of modes. Our observed oscillation frequencies match to within 0.3% and the system parameters within uncertainties. However, we cannot claim that our identified modes are definitive, and suggest follow-up time-series spectroscopy at high resolution in order to verify our identified modes. With the higher SNR and continuous observations with TESS, more reliable mode identification due to frequency and amplitude changes during the eclipse is likely.

Key words: stars: variables: $\delta$ Scuti - stars: binaries: eclipsing: TT Hor - asteroseismology.

1 INTRODUCTION
A-stars span the transitional area of energy transport and can exhibit both a tiny convective core and a tiny convective envelope overlying a radiative envelope. They are often modelled, for example, by assuming homogeneous heavy element (CNO) abundances and neglecting rotation (Noels et al. 2004). However, many A-stars are complex and rotate rapidly ($>100$ km s$^{-1}$) (Royer et al. 2007) and the ongoing challenge is to incorporate these complexities into structural models and stellar evolution codes.

A-stars that exhibit $\delta$ Sct (pressure-mode) pulsations present an opportunity to delve into the internal structure of these stars and improve models of their structure and evolution. In recent years, asteroseismology has given exquisite information about the internal structure of our Sun and other solar-like oscillators (see Bedding 2014, for a perspective). The evolutionary stages of red giant stars have also been investigated successfully using asteroseismology (e.g., Kallinger et al. 2012 and a recent review by Hekker & Christensen-Dalsgaard 2017).

Conversely, accurate asteroseismology of $\delta$ Sct stars remains elusive because of the difficulty of mode identification from the complex oscillation spectra. Models and observational data for $\delta$ Sct stars do not always coincide. For example, 4 CVn has been studied extensively for over 40 years without reaching a satisfactory model (e.g., (Schmid et al. 2014; Breger et al. 2017)). Rotation, convection, metallicity and magnetism all affect the type of $\delta$ Sct pulsations observed. Of these, rotational splitting of the non-radial modes, made even worse in fast rotating stars, has a major impact on mode identification (Aerts et al. 2010). However, to successfully interpret the internal structure of the $\delta$ Sct star...
from observed pulsation frequencies, the effect of rotation must be described correctly in the first place.

The number of observable frequencies detected from space missions (Kepler and CoRoT) have increased multifold with several theoretical considerations relating stellar properties to asteroseismology now published. One outstanding example is the analysis by Yu et al. (2018) of 16094 oscillating red giants observed by Kepler. This catalogue presents asteroseismic parameters from which mass, radius and surface gravity are derived for red giants. However, far from being enlightening, the plethora of data for δ Sct stars has presented challenges to their theoretical interpretation. García Hernández et al. (2015), found a scaling relation between the large frequency separation, Δν, and the mean density of δ Sct stars which confirmed the theoretical prediction of Snář et al. (2014). García Hernández et al. (2017) have also calculated logg values by correlation with mean density, independent of the rotation rate. These latter studies on scaling relations are valuable for the interpretation of the asteroseismology of δ Sct stars, but for now, we must resort to eclipsing binary systems for the fundamental parameters of these stars.

Clearly, the more knowledge we have about the structure of a δ Sct star, the better predictions from asteroseismic models will be. Eclipsing binary systems are ideal to constrain stellar parameters as the absolute masses and radii of the components can be determined with confidence using a combination of photometry and spectroscopy. If one of the components is pulsating then the system is perfect for probing stellar evolution theory and the internal structure leading to pulsations. If tidal locking can be assumed, then the rotational period of the pulsator is also known. With a δ Sct star that has well-characterised fundamental properties from a binary system and identified pulsation frequencies, more reliable modelling of pulsation modes will be possible. From this basis, models for single isolated δ Sct stars can be validated.

δ Sct stars exhibit multiple periods from 0.5 to 8hrs with amplitudes from a few mmag to more than 300 mmag (Rodríguez et al. 2000). They typically pulsate in non-radial, p modes. Using data from Kepler observations, Grigahcène et al. (2010) were the first to identify numerous hybrid δ Sct/γ Dor Dor pulsators. Later, Balona et al. (2015) suggested that lower frequencies characteristic of γ Dor pulsations (non-radial, g modes with periods from one to 5d) are present in all δ Sct stars. However, though rare, pure δ Sct with no γ Dor pulsations have been found (Bowman & Kurtz 2018).

Liakos & Niarchos (2017) have catalogued 199 cases of eclipsing binary systems with at least one component being a δ Sct star. Surprisingly few of these systems have been characterised to any degree. Some notable exceptions are KIC 3858884, a system in a highly eccentric orbit with a period of 26 days (Maceroni et al. 2014) and KIC 9851944, with a short orbital period of 2.16 days (Guo et al. 2016). Both these binary systems have stars of similar masses (~1.8 \( M_\odot \)) which exhibit both g- and p-mode pulsations. Additionally, Murphy et al. (2018) have published a catalogue of 314 δ Sct pulsators in non-eclipsing binaries. This catalogue triples the number of known intermediate-mass binaries with full orbital parameters.

Sixty systems in the Liakos catalogue were reliably described as semi-detached, Algol-type systems in which mass transfer occurs. In these systems, the donor star is the more massive of the pair initially, evolves fastest and then transfers mass to its less massive partner. Typically, we observe the system when the donor has evolved to the red giant phase, and lost significant mass. The accretor has gained mass, is hotter, and of larger radius than initially.

In this paper, we consider TT Hor: a low-mass Algol binary which exhibits δ Sct pulsations. It was first reported as a variable star by Strohmeier (1967) but the existence of δ Sct pulsations was not identified until 2013 (Moriarty et al. 2013). The system offers the opportunity to probe the structure and evolution of an A-type star undergoing mass transfer.

We have fully characterised the system by combining time-series photometry with spectroscopic observations and we have analysed the pulsations. We compare the evolution of the system and that of a single star of similar mass to the pulsator and compute the expected pulsations in both scenarios. We are able to match the computed frequencies to the observed ones with remarkable success.

2 OBSERVATIONS AND DATA REDUCTION

2.1 Photometry

Time-series photometry was performed with a 350-mm Meade Schmidt-Cassegrain telescope situated in Murrum-bateman, NSW, Australia, IAU observatory code EO7. The telescope was equipped with either an SBIG ST8XE or SBIG STT1603ME CCD camera with Johnson B and V filters, and a Cousins I filter.

DIMENSION 4 (Thinking Man Software, 1992 – 2014, http://www.thinkman.com/dimension4/) was used to synchronize the computer’s clock to UTC. A fast cadence was used to ensure good coverage of the pulsations and for accurate determination of the eclipse times of minima. Typical exposure times were of 60 to 120 s with delays between each exposure (for image download) of about 30 s. TT Hor was observed during secondary eclipses and out of eclipse for as long as possible in any one night to maximise the time span available for Fourier analysis of the pulsations. Data were collected in 2013, 2014 and 2016 and are available on the AAVSO website (https://www.aavso.org/).

TT Hor (RA: 03 27 04.388, Dec: −45 52 56.58) is in a sparse field and scrutiny of a high resolution image from the Digitized Sky Survey catalogue (https://archive.stsci.edu/cgi-bin/dss_form/) and 2MASS (Skrutskie et al. 2006) showed no background contaminating stars to affect the photometry.

The data were reduced with aperture photometry using MAXIM DL. GSC 8059 0284 was used as the comparison star: V = 10.71 ± 0.059 mag, B−V = 0.54 mag. GSC 8059 0309 was used as a check star: V = 11.997, B−V = 0.68 mag. Magnitudes are taken from the AAVSO Photometric All-Sky Survey (APASS). The APASS \( \nu' \) magnitude data were converted to Cousins I filter magnitudes using the conversions from Munari (2012, Henden, private communication). The data were then transformed using standard stars from Landolt fields (Landolt 2007).

TT Hor was not included in Gaia data release 1 and so
there are no parallax measurements to determine accurate
distances and the absolute magnitude of the system. TT Hor
is outside the Galactic plane at a latitude of $-54^\circ$ where
interstellar reddening is minimal and no more than the quoted
errors for the comparison and standard stars. Therefore the
colour magnitudes obtained were not corrected for any in-
terstellar reddening.

Data were also collected in 2016 November 10–20, us-
ing the Los Cumbres Observatory (LCO) 1.0-m telescopes in
Australia, South Africa and Chile. These multi-site observa-
tions were requested for complete phase coverage of the orbit and the
pulsations to allow a precise Fourier analysis. However, only 18% of the data was useful due to bad weather and
scheduling restrictions. For the usable LCO images, groups
of four 5-s exposures obtained with the V filter were stacked
prior to analysis and otherwise reduced as described above.

2.2 Spectroscopy

Spectra were obtained using the ANU 2.3-m telescope at
Siding Spring Observatory and the Wide-field spectrograph
(WiFeS) (Dopita et al. 2007, 2010). Observations were taken
over 8 nights in 2014 mid-November, with additional data
obtained in 2016 mid-October. Observations covered most
phases of the orbital cycle. Spectra were obtained using the
RT560 beam splitter and the B7000 and R7000 gratings
for radial velocity (RV) determination. Spectra for spec-
tral classification were taken using the RT560/B3000 and
RT480/U7000 combinations. The spectra were reduced with
pVWyFiS, the data reduction pipeline specific for the WiFeS
spectrograph (Childress et al. 2014). Blocks of 4 consecu-
tive TT Hor spectra were taken and alternated with Ne-Ar
arc spectra and subsequently co-added in the pipeline. This
reduced the effect of pulsations on the RVs, by integrating
over more than half a pulsation period.

Spectra were dominated by the primary star at all or-
bital phases, with the main absorption features being the
Balmer series. The orbit of TT Hor is inclined to 75$^\circ$ (see
Sect. 3) thus it is not possible to isolate the spectral features of
the secondary component during the primary eclipse min-
imum. There was no evidence of Hor emission in these spec-
tra.

The spectral type was determined as A4IV by compar-
ison of the TT Hor spectrum with spectral standards from
Gray & Corbally (2009). The standards of luminosity class IV
gave a marginally better fit than luminosity class V. To
determine atmospheric parameters, Local Thermodynamic
Equilibrium (LTE) model atmospheres were taken from at-
las9 (Castelli & Kurucz, 2004) and synthetic spectra were
calculated from these using spectrum (Gray & Corbally,
1994). The blue data were continuum normalised and rec-
tified prior to comparison to the synthetic spectra. The best
fit to the data was with a synthetic spectrum with $T_{\text{eff}} = 8800 \pm 200 \, \text{K}$, $\log g = 3.9 \pm 0.3$, $[\text{M/H}] = 0$, microturbulence
$= 2 \, \text{km} \, \text{s}^{-1}$ and $v \sin i = 50 \, \text{km} \, \text{s}^{-1}$.

Given the dominance of the Balmer features, RVs were
extracted from the B7000 data rather than the R7000
data. We used a custom Python interpretation of the
two-dimensional correlation function, TDCOR (Zucker and
Mazeh, 1994) to extract the RVs of both components. The
code uses template spectra that match the spectra of the two
components of the binary. Templates were generated from
the synthetic spectra from atlas9 as described above and a
grid-based search was used to determine $\alpha$, the contrast
ratio of the two template spectra. Maximum correlation was
found using $\alpha = 0.1$, the parameters of $T_{\text{eff}}$ log $g$, $v \sin i = [8500–9000 \, \text{K}, 4.0, 50 \, \text{km} \, \text{s}^{-1}]$ for the primary star and [4750–
5000 \, \text{K}, 3.5, 50 \, \text{km} \, \text{s}^{-1}] for the secondary star, both with solar
metallicity.

The primary star parameters are consistent with the
spectral typing given above, with RV measurements for this
star being more reliable than for the secondary. Uncertain-
ties are lowest at quadrature phases (0.25 and 0.75), where
the primary and secondary stars are at maximum velocity
along our line-of-sight and spectral lines are Doppler
shifted the most. The maximum velocities set the limit on
the masses of the two stars. RV uncertainties at these phases
for the primary and secondary components are no more than
1 \, \text{km} \, \text{s}^{-1} and 4 \, \text{km} \, \text{s}^{-1}, respectively. Standards stars analysed
via TDCOR gave RV uncertainties of 2.5 \, \text{km} \, \text{s}^{-1}. We therefore
add these uncertainties in quadrature to give uncertain-
ties for the radial velocity semi-amplitudes of 2.7 \, \text{km} \, \text{s}^{-1} for
$K_1$ and 4.7 \, \text{km} \, \text{s}^{-1} for $K_2$. A table of RV measurements are
given in the Appendix (Table A1).

3 BINARY MODEL

Composite light curves for TT Hor in V, B and I from the
2014 to 2016 data are shown in Fig. 1. The system is brightest
in the I passband and dimmest in blue with $B-V \approx 0.26$ during uneclipsed phases. The light curves are typical of a
semi-detached binary system. The ingress and egress to and
from the primary eclipse are deeper compared to those of the
secondary eclipse, an effect which is most evident in the I
data. The secondary eclipse is also deeper in the I passband.
These effects are indicative of a red secondary star having
filled its Roche lobe.

The binary light curves in the three passbands and the
radial velocity data were used together to determine the sys-
tem model using PHOEBE 0.31A (Pria & Twitte 2005). PHOEBE
is a modelling package for eclipsing binary systems based
on the Wilson and Devinney code (Wilson & Devinney 1971). Our
standardised data in three passbands maximise the power of
the PHOEBE code to preserve the colour indices and this constrains
the model to readily differentiate the effective temperatures of both
components.

The final model of the system uses the radial velocities de-
termined from the parameters detailed in Sect. 2.2. In addition, RV
data gathered in 2016 November compared to those collected
in 2014 October showed a consistent system velocity difference
of 12.5 \, \text{km} \, \text{s}^{-1} (Table 1) which could not be attributed to RV changes
due to pulsations. The 2016 data were corrected for this amount
and combined with the 2014 data to give the final data set used
(Fig. 2).

The difference in RVs between 2014 and 2016 is postulated to
result from a third, low-mass object in a wide orbit about the bi-
nary system. This is consistent with the predictions of Tokovinin
et al. (2006) that 96% of binaries with orbital periods less than 3
days have tertiary companions. The overall evolution of the sys-
tem is affected by Kozai cycles with tidal friction as illustrated by
Eggleton & Kiseleva-Eggleton (2001) for the semi-detached bi-
nary $\beta$ Per, the prototype semi-detached Algol system. However,
at the time of our observations, the Kozai cycles are no longer
influencing the binary, as the perturbation caused by the distor-
tion of the Roche lobe-filling component is much larger than the
perturbation due to any third body. Of course, the third body

MNRAS 000, 1–12 (2017)
was used in this mode. The orbital period (P) of the light curves are indicative of a semi-detached system and the corrections was used throughout the modelling process. The shapes change in RVs with time.

is still having a gravitational effect on the binary as seen by the change in RVs with time.

The PHOEBE minimisation fitting method of differential corrections was used throughout the modelling process. The shapes of the light curves are indicative of a semi-detached system and PHOEBE was used in this mode. The orbital period (P) for the system was determined previously as 2.608204 d (Moriarty et al. 2013) and, with the additional 2014-2016 data, we refined this to $P = 2.608223 \pm 0.000004$ d. Using this $P$ and the RV data, we derived and then fixed the semi-major axis, the mass ratio of secondary to primary ($q = m_2/m_1$) and the system velocity. The maximum velocities at the critical quadrature phases give $q$. With uncertainties for the RVs conservatively set at no more than 5 km s$^{-1}$ (3% uncertainty), this gives an $\approx 10\%$ uncertainty in the final mass determinations.

Importantly, the model of RVs during primary eclipse shows no clear evidence of the Rossiter-McLaughlin (RM) effect (Rossiter 1924). During a primary eclipse, parts of the rotating surface of the primary star are hidden. This results in a redshift from the receding stellar surface and then a blueshift from the advancing surface compared to the orbital motion of the star. The spectral changes reveal the projected stellar rotation speed ($v \sin i$) and the angle between the stellar and orbital spins. For the primary star RVs, an upper limit of 10 km s$^{-1}$ for the RM effect is seen which is consistent with the orbital and spin axes being aligned.

The surface morphology of the stars depends on their Kopal surface potentials ($\Omega$) (Kopal 1955) which were calculated by PHOEBE from the radii and mass ratios, assuming a circular orbit and synchronous rotation. The potentials were fixed initially from the radii of the two stars as determined from the photometry. When the primary $\Omega$ was set as a free parameter to refine the final model, the resultant normalised value changed by only 0.002. The greater surface potential of the primary star compared to the secondary reflects the smaller radius of the primary star which is bound within a separate equipotential surface from the lobe-filling secondary.

The best fit model was searched for with the primary $T_{\text{eff}}$ set as 8800 K and the secondary $T_{\text{eff}}$, together with the inclination angle $i$, set as free parameters. Models were also computed with $T_{\text{eff}}$ set as 9000 K or 8600 K. This change only affected the temperature of the secondary star, and enabled the temperature uncertainty to be propagated. Limb darkening coefficients for each filter and temperature were determined using the linear cosine law and van Hamme (1993) tables and then fixed. For the primary star, the albedo and gravity darkening power-law coefficients for mean intensity are set to 1.0, as appropriate for radiative envelopes. This disregards the likely presence of a very thin convective envelope in the primary component. For the secondary star, they were set as free parameters and minimised at 0.529 $\pm$ 0.009 and 0.761 $\pm$ 0.009, respectively, indicating a convective envelope as expected for this cooler component.

The final PHOEBE model as fitted to the data for each filter band, together with the residuals, are shown in Fig. 1. The residual magnitudes for each filter band are between 0.2 and 0.3% with the B filter having the highest residuals. The geometry of the TT Hor system at different phases is shown in Fig. 3. The primary star has the smaller diameter with the secondary star swollen to fill its Roche lobe. The 75° inclination of the orbital axis is evident at the eclipse phases when both stars are visible. The models show no indication of eccentricity and is consistent with a circular orbit.

Pulsations are clearly evident throughout the orbital cycle, particularly in the B and V filters, and contribute to the residuals from the PHOEBE model. The pulsations and other residuals were subtracted from the V data model and the main frequencies of the pulsations were then determined using PERIOD04 (Lenz & Breger 2005). PERIOD04 is a discrete Fourier transform algorithm designed specifically for the statistical analysis of large time-series containing gaps. However, the gaps in our data are mostly too large and the data sets from each night were analysed separately (Section 4). Some data sets are also too short for Fourier analysis and were not included in the final pre-whitened orbital data.

The model for the frequency fit was then subtracted from
Table 1. The parameters of TT Hor as determined by PHOEBE from combined photometry and spectroscopy. Parameters that have purely statistical uncertainties from PHOEBE are marked with \(^\wedge\).

| Parameter                              | Primary (accretor) | Secondary (donor) | System        |
|----------------------------------------|--------------------|-------------------|---------------|
| Orbital period (days)                  | -                  | -                 | 2.608223 ± 0.000004 |
| Semi-major axis \(R_0\)                | -                  | -                 | 11.40 ± 0.07   |
| \(M_2/M_1\)                           | -                  | -                 | 0.323 ± 0.019  |
| System velocity (km s\(^{-1}\))       | -                  | -                 | 27.2 ± 2.5     |
| 2014 + 2016                            | -                  | -                 | 27.0 ± 2.5     |
| 2014                                   | -                  | -                 | 15.3 ± 2.6     |
| Semi-amplitudes, \(K\) (km s\(^{-1}\))| 52.2 ± 2.7         | 161.7 ± 4.7       |               |
| Inclination, (°)                       | -                  | -                 | 75.00 ± 0.01\(^\wedge\) |
| \(M\) \((M_\odot)\)                  | 2.22 ± 0.17        | 0.72 ± 0.06       |               |
| \(R\) \((R_\odot)\)                   | 2.05 ± 0.06        | 3.26 ± 0.09       |               |
| \(T_{\text{eff}}\)                    | 8800 ± 200         | 4627 ± 50         |               |
| \(\log g\)                            | 4.16 ± 0.05        | 3.27 ± 0.05       |               |
| Surface albedo                         | 1.0 (fixed)        | 0.761 ± 0.009\(^\wedge\) |       |
| Gravity darkening coefficient          | 1.0 (fixed)        | 0.529 ± 0.009\(^\wedge\) |       |
| Normalised surface potential, \(\Omega\) | 5.912 ± 0.002\(^\wedge\) | 2.495\(^\wedge\) |               |
| \(\log(L/L_\odot)\)                  | 1.35±0.05          | 0.64±0.03         |               |

\(^\wedge\)Computed from masses and radii based on the assumption of Roche lobe overflow.

The pulsating binary, TT Hor

The original data sets to produce a new phase-folded light curve without pulsations (Fig. 4).

The residuals for this prewhitened model (minus pulsations) are reduced significantly to less than 10 mmag for all phases except for parts of the primary eclipse. The higher residuals in the latter are explained by the difficulty of determining the exact times of eclipse minima. Only the primary star pulsates but because of the inclined orbital axis, the eclipses are partial and pulsations are visible at both eclipse minima.

The parameters for the final binary model are given in Table 1. The radius uncertainty is dominated by the velocity uncertainty, which is integrated to give the physical orbital scale. For parameters that have purely statistical uncertainties from PHOEBE, we have clearly marked them as such with \(^\wedge\). These uncertainties come directly from the PHOEBE model fit, which assumes uncorrelated observational errors. Given that our photometric uncertainties are correlated on scales at least equal to the residual pulsation frequencies at \(\approx40\) d\(^{-1}\) or 6 observations, these uncertainties are scaled by a factor of at least two.

The PHOEBE model confirms the semi-detached nature of TT Hor, consisting of a 2.22 ± 0.17-M\(_\odot\) primary (hereafter termed the accretor) and a 0.72 ± 0.06-M\(_\odot\) secondary (hereafter termed the donor). From the temperatures and luminosities, the donor is likely a H-shell burning, K-giant. From the luminosity of the accretor, we estimate the distance of TT Hor to be \(\approx800\) pc. This estimate uses the \(V\) magnitude (10.98) at secondary eclipse and a bolometric correction of zero for an A-type star (Flower 1996). Because of the inclination of the orbital axis, the K-giant’s contribution to the \(V\) magnitude is not negligible at this phase, so the distance is probably under-estimated. Also no correction was made for extinction which, though slight for our results, would work to over-estimate the distance. Recently released Gaia DR2 data give a distance for TT Hor of 786 ± 18 pc (Gaia Collaboration et al. 2018), thus our estimated distance is within Gaia DR2 uncertainties.

The donor has evolved to fill its Roche lobe (radius = 3.26\(R_\odot\)) and is transferring mass to the accretor. The model is consistent

Figure 3. PHOEBE representation of the shape models during the phase changes of TT Hor. From left to right, top to bottom: phase 0 (primary eclipse); phase 0.25; phase 0.5 (secondary eclipse); phase 0.75 (+0.25).

Figure 4. Magnitude of TT Hor (green dots) in V filter after pre-whitening compared to the PHOEBE model (black line). The residuals from the model are shown below.
4 PULSATION ANALYSIS

Pulsations seen in data taken with the B and V filters are of similar amplitudes but are barely visible in the I data. Our V data are the most abundant and were used for pulsation analysis. The best data sets are those taken on nights of excellent seeing and >4 h duration. The orbital phase changes for each night’s data need to be removed prior to pulsation analysis. Initially, an attempt was made to remove the relevant phase by subtraction of the phoebe model but matching the phase exactly to data sets other than those used in the phase folded light curves was difficult. The subtraction of the binary model left an obvious inflection point with some data above and the rest below the zero axis. Finally a low order polynomial was fitted to each ‘good’ data set with removal of any points at the beginning and end of each data set if the polynomial fit was inconsistent.

The best data sets from 2013 (10 nights, 64.5 h) were combined for Fourier analysis using PERIOD04, as were those from 2014 (5 nights, 35.5 h). Observations obtained with the LCO in 2016 were too intermittent and not in themselves of sufficient cadence for Fourier analysis. However, two data sets of 8 and 7 hr duration were obtained on 2016 November 12 and 17, respectively, from Murrumbateman using the 350-mm Meade Schmidt-Cassegrain telescope and SBIG STT 1603 camera using the V filter only. When combined with the LCO data (30 h) between these two dates, Fourier analysis was successful. The frequencies extracted for the three years are given in Table 2. The four frequencies have different amplitudes for different years as indicated by their order of extraction from the Fourier transform. Extracting more than four frequencies resulted in either an extraction of sidelobes of the dominant four, or frequencies extracted in the low frequency, systematic noise floor of the power spectrum.

A combined analysis of all three years is also given in Table 2. This latter Fourier analysis suffers from multiple aliases but gives more reliable amplitudes. However, the highest and second-highest peak amongst the 1 yr−1 aliases are not distinguishable at our signal-to-noise values. For this reason, we conservatively assign 1 yr−1 (or 0.003 d−1) as the uncertainty in the combined frequency analysis.

Our ground-based data do not allow the detailed pulsation analysis possible with Kepler and CoRoT data but as all four frequencies are consistently present over the three years of observations, they are considered authentic. Although not ideal, they are useful for a preliminary identification of the modes (see Sect. 5.2).

As TT Hor is inclined at 75° to our line of sight, the eclipses are partial and a spatial filtering effect is expected during primary eclipse (Moriarty et al. 2013). This effect can lead to an amplitude increase in some oscillation modes, and could lead to modes being observed that are not observable outside of eclipse. For the V data (of highest signal to noise), we attempted to search for such an effect. Only one reliable frequency could be found in common amongst our 7 primary eclipses, of 37.7 d−1, with a dispersion of 0.8 d−1, and an amplitude of 9.8 mmag with a dispersion of 2.8 mmag. Given that a primary eclipse only lasts approximately 11 pulsation cycles, the identity of the mode or combination of modes which produced this apparently higher amplitude is unclear.

5 MODELLING

5.1 Evolutionary models

An evolutionary model to match the phoebe binary model is essential to fully understand the TT Hor system. We used the MESA stellar evolutionary code, version r-8845 (Paxton et al. 2011, 2013, 2015) to generate evolutionary models from different initial conditions of mass and orbital period. A grid search was performed with the total initial mass for the binary system set at 2.931 M⊙ (total mass of the TT Hor system to four significant figures), the initial mass of the donor varied from 1.6 to 1.9 M⊙ and the initial orbital period varied from 2.6 to 2.9 days. A final model was subsequently found with the donor mass varied from 1.61 to 1.63 M⊙ and that of the accretor from 1.301 to 1.331 M⊙. For computation of the evolutionary models, an additional decimal place for the masses of the binary stars was used for better discrimination between these models for all parameters.

The default solar values for hydrogen (X), helium (Y) and the metal fraction (Z = 0.02) were used with convective-core overshoot in both the donor and accretor. As the initial masses of the donor and accretor for these final models were less than 2 M⊙, αov was set at 0.1 as determined from Claret & Torres (2017), for a 1 M⊙ star. The other core-over-overshoot parameters for MESA were set as α = 0.008 (a non-zero value for the edge of the convective zone in terms of scale height) and f = 0.01 (the pressure scale height of overshoot above the convective zone).

Mass transfer can be either conservative or non-conservative. With non-conservative mass transfer, mass is lost from the system and either an accretion disc or a ‘hot-spot’ associated with the accretor may drive the outflow of mass (Deschamps et al. 2015). Asymmetries in the light curves of TT Cam, RZ Cas (Rodríguez et al. 2004) and KIC 5621294 (Lee et al. 2015) have been attributed to the existence of hot-spots. Our phoebe model of the TT Hor system did not require any hot (or cool) spots to explain the light curves and no Hα emission was seen in the spectra. No other evidence suggested that mass transfer is anything but conservative. Indeed, low-mass binary systems (total mass ≤ 3 M⊙) have been successfully modelled by Kolb & Ritter (1990) with mass transfer occurring via a fully conservative (Rodríguez et al. 1990) mechanism (i.e. without systemic mass loss). In these systems the rate of mass transfer is in the order of 10−10 M⊙/yr. Given the semi-detached nature of the system and the masses of the component stars, mass transfer was assumed to be conservative and we modelled this with the “Kolb” binary control in MESA, following the Kolb and Ritter formalism.

The properties of the MESA models closest to the observed binary system are shown in Table 3. Using solar metallicity (Z = 0.02), initial donor mass = 1.61 M⊙; accretor mass = 1.321 M⊙; initial period = 2.77 d, we are able to construct a MESA model (Model A) that matches our phoebe model for the masses of both components. While all other parameters differed by about 1% in this mass, the temperature and radius of the accretor differed by +8% and -4%, respectively. These differences are greater than the uncertainties given by the phoebe model (+ 2% and 3%, respectively) and too great to be disregarded and suggested that the metallicity of the system might be greater than solar. Therefore, Z was varied from 0.02 to 0.036 to search for a MESA model that gives a lower temperature and higher radius for the accretor. The closest fitting model (Model B) was for Z = 0.033; initial donor and accretor mass as above; initial period = 3.05 d.

Because of the uncertainties in our RV measurements, there is a possibility that the masses of the two components of TT Hor are up to 10% smaller than calculated by phoebe modelling. Therefore an additional MESA model (Model C) with the same mass ratio (0.323) as the phoebe model but with the final mass for the accretor and donor reduced by 10% (to 2.016 and 0.65 M⊙, respectively) was also determined using solar metallicity. With the initial donor mass = 1.44 M⊙; accretor mass = 1.226 M⊙; ini-
The pulsating binary, TT Hor

5.2 Mode identification

From the short-duration, ground-based observations, we were only able to identify four reliable frequencies (refer to Table 2) for which we would like to identify the corresponding oscillation modes. Our PHOEBE model of the TT Hor binary system matches the MESA evolutionary Model B well (within 1.5% in all parameters). In addition, the binary model shows no indication of orbital eccentricity, and the RM effect is minimal and consistent with the orbital and spin axes being aligned. We also know the rotation period of the pulsator, so we considered a preliminary investigation of the pulsation properties a worthwhile exercise. When TESS data become available a more rigorous analysis and identification of the pulsation modes will be undertaken.

We searched for the observed frequencies in models computed by GYRE (Townsend & Teitler 2013), the asteroseismology extension of MESA. GYRE solves first-order equations describing perturbations to the boundary conditions at each interval of time using the Magnus Multiple Shooting numerical scheme. We used this code to calculate the simple linear adiabatic frequencies for the accretor and single star profiles from their MESA evolutionary models for Z = 0.033 (Model B). We set the rotation rate in the GYRE code using the orbital period of the binary at 2.788 x 10^5 rad s^{-1}. We assumed the accretor rotates uniformly, with no adjustment for its slightly oblate shape. We also computed the non-adiabatic and quasiadiabatic frequencies but the corrections are very small (≤ 0.01%) and we are unable to discriminate between the different perturbation effects using our current data. Additionally, GYRE neglects perturbations to the turbulent pressure, which is known to be an additional driving mechanism for pulsations in at least some δ Sct stars (Antoci et al. 2014; Xiong et al. 2016) This mechanism, if operative for the TT Hor accretor, would not be modelled by GYRE.

Frequency models for the accretor in Model B could be matched to the observed frequencies after applying a slight scaling factor of 1.027 (Table 4 and Fig. 8). The scaling factor applied to the frequencies is 2.7%, and is essentially a 5.4% density scaling which is within the uncertainties in radius and mass of the primary.

1 A description of the dependence of the binary separation on mass ratio and envelope properties can be found at https://joe-antognini.github.io/astronomy/mass-transfer-modes.
the accretor given by the PHOEBE binary model (Table 1). To find equivalent frequencies in the single star matching Model B, a scaling factor of 1.102 was applied. For such a substantially different model, the maximum difference to the accretor model frequency was only 0.09 d\(^{-1}\). Mode identification for Model C accretor and single star gave essentially the same modes after applying a scaling factor of 0.981 and are not presented here.

This non-standard procedure was used because of the complexity of fine-tuning the input for the MESA binary evolutionary models. Small changes in metallicity of tens of percent should change the radius through opacity changes, in turn changing the density and scaling the frequencies. To confirm that this approach is reasonable, and also to find a fully self-consistent model that could also be an alternative, we use this with TESS or additional LCO data. Such additional frequencies should enable more definitive mode identification and discriminate between adiabatic and non-adiabatic perturbation effects. These, in turn, may lead to a more accurate binary model as we have shown how small changes in metallicity and mass can affect mode identification.

Continuous, photometric time series over many days is expected to reveal many more frequencies and we hope to pursue this with TESS or additional LCO data. Such additional frequencies should enable more definitive mode identification and discriminate between adiabatic and non-adiabatic perturbation effects. These, in turn, may lead to a more accurate binary model as we have shown how small changes in metallicity and mass can affect mode identification.

As the rotational and orbital axes are aligned in the tidally locked system, the pulsation axis will also be aligned. The visibility of pulsations with different azimuthal orders (\(m\)) is highly dependent on the inclination of the star to our line of sight. Gizon & Solanki (2003) showed that at high inclination (80\(°\)), the rotationally split modes, \(l = 1\), \(m = \pm 1\) and \(l = 2, m = \pm 2\), are the most likely observed in slowly rotating stars with angular velocities between 1 and 10 solar units (\(\approx 2 \times 20\) km s\(^{-1}\)). A sufficiently high rotation rate is needed to resolve azimuthal modes split by rotation, a condition met by TT Hor which is rotating at 41 km s\(^{-1}\). However, not all modes are necessarily excited and therefore visible to the observer. Additionally, our data do not give reliable amplitudes for each frequency extracted via Fourier analysis, therefore substantiating our mode identification from their amplitudes is not plausible.

The best matched frequency, \(f_3\), for Model B is identified as an \(l = 2\), \(m = -2\) mode which has the second highest amplitude in our observations. The lowest amplitude frequency is \(f_4\), \(l = 1\), \(m = 1\). These preliminary mode identifications are in contrast to

| Parameters                   | PHOEBE Model | Model A | Model B | Model C | Model D | Model A | Model B | Model C | Model D |
|------------------------------|--------------|---------|---------|---------|---------|---------|---------|---------|---------|
| Initial mass (donor) (\(M_2\)) | -            | 1.61    | 1.61    | 1.44    | 1.54    |         |         |         |         |
| Initial mass (accretor) (\(M_0\)) | -            | 1.321   | 1.321   | 1.226   | 1.287   |         |         |         |         |
| Initial orbital \(P\) (days)  | -            | 2.84    | 3.05    | 2.95    | 3.00    |         |         |         |         |
| \(Z\) fraction               | -            | 0.02    | 0.033   | 0.02    | 0.0281  | 0.02    | 0.033   | 0.02    | 0.0281  |
| Final orbital \(P\) (days)   | 2.608223     | 2.60353 | 2.61282 | 2.60349 | 2.56812 | -       | -       | -       | -       |
| Semi-major axis (\(R_0\))    | 11.40 ± 0.07 | 11.388  | 11.422  | 11.040  | 11.166  | -       | -       | -       | -       |
| Age (Gyr)                    | -            | 2.03    | 2.58    | 2.96    | 2.72    | 0.50    | 0.30    | 0.38    | 0.29    |
| \(M_2/M_1\)                  | 0.323        | 0.323   | 0.323   | 0.323   | 0.265   | -       | -       | -       | -       |
| Primary (accretor) \(M (M_0)\) | 2.22 ± 0.17 | 2.217   | 2.216   | 2.018   | 2.153   | 2.216   | 2.216   | 2.016   | 2.153   |
| \(R (R_0)\)                  | 2.05 ± 0.06  | 1.968   | 2.076   | 1.951   | 2.014   | 1.869/2.435 | 2.176 | 1.862   | 2.014   |
| \(T_{\text{eff}}\) (K)       | 8800 ± 200   | 9571    | 8787    | 8807    | 8827    | 9585/8805 | 8797  | 8820    | 8881    |
| \(\log g\)                   | 4.16 ± 0.05  | 4.20    | 4.15    | 4.16    | 4.16    | 4.24/4.01 | 4.11  | 4.20    | 4.16    |
| \(\log (L/\ell_0)\)          | 1.35 ± 0.05  | 1.46    | 1.36    | 1.31    | 1.34    | 1.42/1.51 | 1.41  | 1.28    | 1.36    |
| Secondary (donor) \(M (M_0)\) | 0.72 ± 0.06  | 0.714   | 0.715   | 0.648   | 0.682   |         |         |         |         |
| \(R (R_0)\)                  | 3.26 ± 0.09  | 3.254   | 3.257   | 3.150   | 3.183   |         |         |         |         |
| \(T_{\text{eff}}\) (K)       | 4627 ± 50    | 4712    | 4571    | 4666    | 4593    |         |         |         |         |
| \(\log g\)                   | 3.27 ± 0.05  | 3.27    | 3.27    | 3.25    | 3.27    |         |         |         |         |
| \(\log (L/\ell_0)\)          | 0.64 ± 0.03  | 0.67    | 0.62    | 0.63    | 0.61    |         |         |         |         |

Table 3. Comparison of PHOEBE binary model for TT Hor and MESA evolutionary models with different initial conditions of mass, orbital period and Z. Evolutionary models for single stars of corresponding masses and Z to the accretor are also given. For single star, Model A, the parameters are given for this model which match both MESA Model A and the PHOEBE binary model.
The pulsating binary, \textit{TT} Hor

Figure 5. HR diagram showing the evolutionary tracks of \textsc{mesa} models for the accretor and donor with different initial conditions as given in Table 3. Tracks for single stars (SS) of the same final mass as the accretor for each model are also shown. The inverted triangles indicate the current evolutionary state of each star. Error bars are shown for the accretor and donor from the \textsc{phoebe} binary model. The black stars indicate start of rapid mass transfer. The open circles indicate the equivalent evolutionary conditions in the $T_{\text{eff}}/\log(L/L_\odot)$ parameter space for the single stars.

Table 4. Identification of the modes of the observed frequencies in the accretor by comparison to modes identified by \textsc{gyre} after applying a scaling factor of either 2.7\% (Model B) or no scaling factor (Model D).

| Observed Frequency ($\text{d}^{-1}$) | Observed Amplitude V (mmag) | Model B | Model D | Mode ID |
|-------------------------------------|-----------------------------|---------|---------|---------|
|                                    |                             | Accretor | Single star | Accretor | Single star | n = 5, l = 2, m = 2 |
| $f_1$                              | 39.907 ± 0.003              | 3.8 ± 0.3 | 39.596 | 39.779 | 39.760 | n = 5, l = 2, m = -2 |
| $f_2$                              | 38.075 ± 0.003              | 2.8 ± 0.5 | 38.058 | 38.282 | 38.262 | n = 5, l = 2, m = -2 |
| $f_3$                              | 34.310 ± 0.003              | 2.2 ± 0.6 | 34.302 | 34.467 | 34.450 | n = 4, l = 2, m = 2 |
| $f_4$                              | 42.301 ± 0.003              | 2.0 ± 0.1 | 42.207 | 34.183 | 34.362 | n = 5, l = 0 |

In a circular orbit, the tidal effect is at equilibrium and not expected to excite g modes at high harmonics which are often found in systems with highly eccentric orbits (the so-called heartbeat stars, e.g. Hambleton et al. 2017). Such tidal effects have also been found in non-heartbeat stars such as CoRoT 105906206 (da Silva et al. 2014) and KIC 9851944 (Guo et al. 2016), both of which have circularised orbits. Tidally excited oscillations are most visible in stars with shallow surface convective zones when resonance-locking increases their amplitudes (Fuller et al. 2017). Resonance-locking may occur in circularised but non-synchronised binaries. We found no orbital harmonics in the TT Hor data.

the findings of Gizon & Solanki (2003) indicating that the $l = 1$, $m = \pm 1$ should have a slightly higher amplitude than the $l = 2$, $m = \pm 2$ modes at an inclination of 80°. Corroboration of $f_4$ as an $l = 1$, $m = 1$ mode is not possible with our low pulsation amplitudes. The frequency of 37.7 ($\pm 0.8$) d$^{-1}$, identified during primary eclipse, showed an indication of an amplitude increase that could be associated with a spatial filtering effect. This frequency is closest to $f_2 = 38.071$ d$^{-1}$, which would be consistent with its identification as an $l = 2$, $m = -2$ mode. However, this effect needs to be confirmed.
6 DISCUSSION AND CONCLUSIONS

We have determined a model of the semi-detached, eclipsing binary TT Hor determined from our photometry and RV measurements. TT Hor consists of a 2.22 $M_\odot$ primary and a 0.72 $M_\odot$ secondary star and the high probability of a third body in distant orbit around the binary. We confirm that the primary star (accretor) of the binary is the $\delta$ Sct pulsator and is an A4IV star.

Both our evolutionary models B and D match the binary model within uncertainties. However, mode identification using Model B is only possible using a small density scaling factor, whereas for Model D, no scaling factor was needed to identify the same modes. Model D is therefore our preferred model for TT Hor, indicating that the accretor and donor have evolved from an initial mass of $\sim 1.29 M_\odot$ and $\sim 1.55 M_\odot$, respectively. The donor has lost 0.866 $M_\odot$ of slightly He-enriched material to the accreting, pulsating component which is evolving up the main sequence as it continues accreting mass from its companion. TT Hor fits within the definition of Mkrtichian et al. (2002, 2004) of an ‘oEA star’, indicating pulsating, mass-accreting, main-sequence stars of spectral type B-F in semi-detached Algol systems.

The evolution of the $\delta$ Sct pulsator in TT Hor is quite different from that of a single, isolated $\delta$ Sct star, as shown by our evolutionary models. Despite these differences, we are able to match the fundamental properties of the TT Hor accretor to a single $\delta$ Sct differing primarily only in age (0.30 and 2.58 Gyr, respectively). We are able to make a tentative identification of the pulsation modes of our observed frequencies using a MESA evolutionary model that matches our binary model. TT Hor is a tidally-locked binary system, enabling an accurate determination of the rotation rate of the pulsator which is then applied to the mode identification. Rotation rates cannot be measured to equivalent certainty in isolated stars and this is one of the major impediments in mode identification and modelling for $\delta$ Sct stars. By applying the same rotation rate to the single star model, we are also able to associate our observed frequencies to the same pulsation modes.

The temperature of TT Hor ($T_{\text{eff}} = 8800 \pm 200$ K) locates it at the extreme of the blue edge of the instability strip, $T_{\text{eff}} = 8500$ K. Here the HeII ionisation zone is close to the surface at $T_{\text{eff}} = 40,000$ K where the $\kappa$ mechanism is the driving mechanism and non-radial $p$ modes are excited. Radial orders of $n = 5$ or 6 are typically found (Xiong et al. 2016), as is the case in our analysis. We tentatively identify dipole and quadrupole non-radial modes, with the possibility of a radial mode.

Using Kepler data, near-uniform surface-to-core rotation rates have been found in hybrid $\delta$ Sct and $\gamma$ Dor pulsators during their main-sequence evolution (Kurtz et al. 2014; Saio et al. 2015; Murphy et al. 2016). The stars in these studies have slow rotation periods, 27–100 d making it easy to identify the rotational splitting of the modes. Once TESS data are available, we hope to identify $g$ modes which would indicate any rotational differences in the core of TT Hor pulsator. However, the pulsator is rotating moderately fast (41 km s$^{-1}$) and disentangling the frequencies in the oscillation spectrum to identify rotational splitting may not be possible.

Despite mass accretion and age differences between the TT Hor accretor and an isolated star, we have made a preliminary identification of the modes. While these identified modes are by no means definitive, they are surprisingly close to the observed frequencies. Hence we have illustrated our premise that pulsators in Algol systems are powerful tools to validate models for isolated $\delta$ Sct stars. We have data for other Algol-type systems which have different orbital and pulsation periods and by comparing these systems we anticipate shedding further light on these enigmatic pulsators.
APPENDIX A: RADIAL VELOCITY MEASUREMENTS

This paper has been typeset from a TEX/LATEX file prepared by the author.

### Table A1. Radial velocities as determined with todcor. RV1 and RV2 are radial velocities for the primary and secondary stars, respectively.

| Date (HJD) | Orbital phase | RV1  | RV2  |
|------------|---------------|------|------|
| 2456968.95762 | 0.10         | 5.8  | 145.6 |
| 2456969.00862 | 0.12         | -2.4 | 145.2 |
| 2456969.19061 | 0.19         | -26.5 | 169.4 |
| 2456969.21061 | 0.20         | -25.0 | 181.0 |
| 2456969.89460 | 0.46         | 13.1  | 63.3  |
| 2456969.90860 | 0.47         | 11.5  | 65.8  |
| 2456969.92860 | 0.47         | 14.3  | 68.9  |
| 2456969.94460 | 0.48         | 14.1  | 64.7  |
| 2456970.91559 | 0.85         | 72.3  | -106.4 |
| 2456970.94259 | 0.86         | 72.9  | -92.0  |
| 2456970.96959 | 0.87         | 70.3  | -79.3  |
| 2456970.98758 | 0.88         | 67.2  | -68.4  |
| 2456971.23258 | 0.98         | 48.4  | 15.1  |
| 2456971.89857 | 0.23         | -28.2 | 189.2  |
| 2456971.92357 | 0.24         | -29.7 | 185.9  |
| 2456971.94557 | 0.25         | -28.6 | 190.8  |
| 2456971.96057 | 0.25         | -28.3 | 196.4  |
| 2456971.99057 | 0.27         | -25.5 | 201.4  |
| 2456972.18056 | 0.34         | -18.9 | 172.3  |
| 2456972.21456 | 0.35         | -17.8 | 166.0  |
| 2456972.22756 | 0.36         | -14.8 | 166.5  |
| 2456972.23356 | 0.36         | -14.9 | 166.1  |
| 2456972.24656 | 0.36         | -14.7 | 167.3  |
| 2456975.98349 | 0.80         | 86.3  | -129.5 |
| 2456976.15848 | 0.87         | 77.2  | -75.8  |
| 2456976.97547 | 0.18         | -20.1 | 168.1  |
| 2456977.91745 | 0.54         | 40.0  | 19.9   |
| 2456978.93543 | 0.93         | 60.4  | -14.8  |
| 2456978.95542 | 0.94         | 56.9  | -7.2   |
| 2456978.97242 | 0.94         | 50.2  | -3.8   |
| 2456979.00542 | 0.96         | 48.5  | 9.7    |
| 2456979.03242 | 0.97         | 46.3  | 15.9   |
| 2456979.05742 | 0.98         | 43.2  | 19.9   |
| 2456979.09042 | 0.99         | 39.0  | 27.3   |
| 2456979.11642 | 1.00         | 28.6  | 31.6   |
| 2456979.13442 | 0.02         | 18.5  | 41.7   |
| 2456979.21142 | 0.03         | 8.9   | 53.9   |
| 2456979.25241 | 0.05         | 6.6   | 64.3   |
| 2457678.08807 | 0.98         | 27.5  | 6.1    |
| 2457679.05127 | 0.36         | -24.1 | 159.6  |
| 2457679.06517 | 0.36         | -28.6 | 148.4  |
| 2457679.08207 | 0.37         | -22.9 | 146.1  |
| 2457679.12297 | 0.38         | -19.4 | 132.5  |
| 2457679.13897 | 0.39         | -24.6 | 121.6  |
| 2457679.15488 | 0.40         | -18.8 | 106.7  |
| 2457679.17107 | 0.41         | -18.2 | 99.0   |
| 2457679.19037 | 0.41         | -15.3 | 89.6   |
| 2457679.20407 | 0.41         | -21.8 | 70.8   |
| 2457679.22197 | 0.42         | -15.6 | 89.9   |
| 2457679.25297 | 0.43         | -0.2  | 60.1   |
| 2457680.09407 | 0.76         | 70.7  | -154.1 |
| 2457680.10897 | 0.76         | 72.9  | -147.3 |
| 2457680.12727 | 0.77         | 67.8  | -128.9 |
| 2457680.14747 | 0.78         | 65.5  | -154.2 |
| 2457680.15587 | 0.78         | 66.7  | -140.9 |
| 2457680.18897 | 0.79         | 68.8  | -138.0 |
| 2457680.20407 | 0.80         | 66.5  | -140.6 |
| 2457680.22417 | 0.81         | 65.5  | -136.4 |
| 2457680.24567 | 0.81         | 68.2  | -126.8 |
| 2457680.25987 | 0.82         | 67.1  | -128.9 |