A new look at scalar mesons

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Light scalar mesons are found to fit rather well a diquark-antidiquark description. The resulting nonet obeys mass formulae which respect, to a good extent, the OZI rule. OZI allowed strong decays are reasonably reproduced by a single amplitude describing the switch of a $q\bar{q}$ pair, which transforms the state into two colourless pseudoscalar mesons. Predicted heavy states with one or more quarks replaced by charm or beauty are briefly described; they should give rise to narrow states with exotic quantum numbers.

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The well identified scalar mesons $a(980) \ (I=1, \text{ formerly called } \delta)$ and $f(980) \ (I=0)$, have been frequently associated with $P$-wave, $q\bar{q}$ states. The main reason for this assignment is no doubt the fact that the other $P$-wave states, the axial and tensor nonets, are all well identified. However, the $q\bar{q}$ assignment has never really worked in the scalar case. For one, $f$ is clearly associated to strange more than to up and down quarks, contrary to what the $I=0$ state degenerate to the $I=1$ one should do in a well-behaved $q\bar{q}$ nonet. Alternative identifications have been proposed in the past, notably the $f$ as a bound $K\bar{K}$ molecule or as a $(q\bar{q})^2$ state. Motivated by the recent discussion of exotic baryons as penta-quarks and by the clear evidence by the KLOE Collaboration of a low mass $f^0$ resonance, $\sigma(450)$, we examine in this paper the possibility that the lowest lying scalar mesons are $S$-wave bound states of a diquark-antidiquark pair. Following ref. the diquark is taken to be in the fully antisymmetric combination of all quantum numbers, i.e. a colour anti-triplet, flavour anti-triplet, spin zero. The $(q\bar{q})^2$ states make a flavour $SU(3)$ nonet. We propose to put the $\sigma$ in the remaining $I=S=0$ state, and to assign to the $S=\pm 1$ states the $\kappa(800)$, a $K\pi\pi$ resonance seen in several experiments, most recently in the $K\pi\pi$ spectrum from $D$ decays by the E791 Collaboration at FermiLab. In addition to the quantum numbers, we consider the mass spectrum and the strong decays of the scalar mesons. A simple hypothesis on the way the $(q\bar{q})^2$ states may transform into a pair of pseudo-scalar mesons is found to give a rather good, one parameter description of the decays allowed by the Okubo-Zweig-Iizuka et al. rule. Addition of the remaining $SU(3)$ invariant couplings improves the description. In synthesis, we propose that scalar mesons below 1 GeV are diquark-antidiquark states. The $q\bar{q}$ $P$-wave scalar states, the partners of the tensor and axial nonet, will have to be found at higher masses. Some previous work in this direction can be found in the papers listed in. We close the paper with a brief discussion of four-quark mesons with hidden and open charm or beauty, which should be characterized by narrow widths and spectacular decay modes.

Quantum numbers and mass formulae. We denote by $|q_1q_2\rangle$ the fully antisymmetric state of the two quarks $q_1$ and $q_2$. The composition of a few members of the nonet is as follows:

$$a^0(I=1, J_3=0) = \frac{1}{\sqrt 2} (|su|\bar{s}\bar{u} - |sd|\bar{s}\bar{d})$$

$$f_0(I=0) = \frac{1}{\sqrt 2} (|su|\bar{s}\bar{u} + |sd|\bar{s}\bar{d})$$

$$\sigma_0(I=0) = |ud|\bar{u}\bar{d}$$

$$\kappa = |ud|\bar{s}\bar{d},$$

where:

$$|f\rangle = \cos \phi |f_0\rangle + \sin \phi |\sigma_0\rangle$$

$$|\sigma\rangle = -\sin \phi |f_0\rangle + \cos \phi |\sigma_0\rangle.$$
definite composition in strange quark pairs (exact OZI rule).

Assuming octet symmetry breaking, masses depend on four parameters, according to the tensor expression (we use squared masses):

\[ M^2 = \frac{1}{2} \left\{ \text{Tr}(S^2 m) + \sqrt{3} \text{c} \text{Tr}(S \lambda_8) \text{Tr} S + \frac{3}{2} d [\text{Tr}(S \lambda_0)]^2 \right\}. \]  

(3)

S is the nonet scalar meson matrix, which we define according to:

\[ S = \begin{pmatrix} f & \alpha^0 & \kappa^+ \\ \sqrt{2} & a^+ & \kappa^0 \\ -\sqrt{2} & a^- & \kappa^0 \end{pmatrix}, \]  

(4)

\[ m = \text{diag}(\alpha, \alpha, \beta), \]  

with \( \alpha, \beta, c \) and \( d \) unknown coefficients, \( \lambda_{0,8} \) are Gell-Mann’s matrices and numerical coefficients have been introduced for convenience.

In the limit \( c = d = 0 \), the mass formula admits the states given above as mass eigenstates. In the more general case, for the \( I \neq 0 \) states we find (here and in the following, we indicate mass-squared with the particle’s symbol):

\[ a = \alpha; \quad \kappa = \frac{\alpha + \beta}{2}. \]  

(5)

The mass-squared matrix of \( I = 0 \) states is:

\[ \mu^2 = \begin{pmatrix} \alpha + 2(c + d) & \frac{1}{\sqrt{2}} (-c + 2d) \\ \frac{1}{\sqrt{2}} (-c + 2d) & \beta - 2c + d \end{pmatrix}. \]  

(6)

We can eliminate \( c \) and \( d \) in favor of physical masses and \( f - \sigma \) mixing. We remain with one overall relation which fixes the \( f - \sigma \) mixing angle as function of the masses. Taking, for simplicity, \( f(980) \) degenerate with \( a(980) \) we find:

\[ \cos 2\phi + 2\sqrt{2} \sin 2\phi = 1 + 4 \frac{a + \sigma - 2\kappa}{a - \sigma}. \]  

(7)

For the masses we take the values reported in Table I. The \( \sigma \) mass is known with large errors. From the above equation we find:

\[ \tan 2\phi = -0.07, \quad \sigma = (570 \text{ MeV})^2 \]  

\[ \tan 2\phi = -0.19, \quad \sigma = (470 \text{ MeV})^2 \]  

\[ \tan 2\phi = -0.31, \quad \sigma = (370 \text{ MeV})^2. \]  

(8)

Since we are holding \( f \) degenerate with \( a \), the \( \sigma \) mass is pushed down as mixing becomes more negative. The \( \sigma \) mass-squared gets to zero for \( \tan 2\phi = -0.48 \), which gives the lowest bound to the mixing angle. Mixing is small because the OZI rule is respected in the physical mass spectrum. The spectrum is inverted with respect to \( \bar{q}q \) nonets: the isolated \( I = 0 \) state is the lightest one and strange particles come next. This is a most evident indication in favour of the four-quark nature of the scalar states \( \pi^0 \) and with the \( K\bar{K} \) molecule picture \( \Xi \). In the latter case, however, the analogy is only superficial. The meson states we are considering correspond to quite different configurations than a \( K - \bar{K} \) molecule. Indeed, they are completely orthogonal to them. The amplitude \( A \) describes the tunneling from the bound diquark pair configuration to the meson-meson pair, made by the unbound, final state particles. With the aid of Fig. 1, the amplitudes for different decays are easily computed. For instance, we have:

\[ [su]_{\lambda_3} [\bar{s}d]_{\lambda_3} \rightarrow (su)_{\lambda_1} (s\bar{u})_{\lambda_1} - (\bar{s}s)_{\lambda_1} (d\bar{u})_{\lambda_1} = \bar{K}^0 K^+ - \pi^+ \eta*. \]  

(9)
For convenience we introduce the combinations:
\[ \eta_q = \frac{\bar{u}_q u + \bar{d}_q d}{\sqrt{2}}, \quad \eta_s = s \gamma_5 s, \]
which can be expressed in terms of the physical \( \eta \) and \( \eta' \) fields and of the pseudoscalar meson mixing angle (we use mass-squared formulae and correspondingly \( \sin \phi_{PS} = 0.19 \)). After that, we find:
\[ \text{Ampl.}(a^+ \to \bar{K}^0 K^+) = A; \]
\[ \text{Ampl.}(a^+ \to \pi^+ \eta) = A \left( -\sqrt{\frac{2}{3}} \cos \phi_{PS} + \sqrt{\frac{1}{3}} \sin \phi_{PS} \right) \approx -0.69 A. \]

For the relevant decays, we give the result in the form of an effective Lagrangian and in terms of the unmixed fields \( f_c \) and \( \sigma_c \). In this form, the amplitude \( A \) has the dimension of a mass.
\[ \mathcal{L} = A \left[ f_c \left( -\frac{K K}{\sqrt{2}} + \eta_q \eta_s \right) - \sigma_c \left( \frac{\pi^+ \pi^-}{2} + \frac{\eta^2}{2} \right) \right. \\
+ a^0 \left( \frac{K \tau_3 K}{\sqrt{2}} - \pi^0 \eta_s \right) + \left( \frac{K^+ \pi^-}{\sqrt{2}} + \bar{K}^0 \pi^- \right) \kappa^+ \]
\[ + \ldots \right]. \]

Decay rates are expressed as:
\[ \Gamma(S \to i) = \frac{A^2 p}{8 \pi M^2_x} x_{s \to i}, \]
where \( p \) is the decay momentum, \( M \) the mass of the scalar meson and \( x_{s \to i} \) a factor which includes numerical coefficients in the individual amplitudes and isospin multiplicities. Without attempting a systematic fit, we take for \( A \) the value: \( A = 2.6 \text{ GeV} \) and give in Table II the corresponding calculated rates, compared to the available experimental information. For simplicity, statistical and systematic errors in the experimental values have been combined in quadrature. Some comments are in order.

1. We have taken from ref. \[ \text{12} \] the total width \( \Gamma_{\text{tot}}(a_0) = 72 \pm 16 \text{ MeV} \) and the \( \bar{K}K \) branching ratio \( B(a_0 \to \bar{K}K) = 0.17 \pm 0.03 \) thus obtaining:
\[ \Gamma(a_0 \to \eta \pi) = 60 \pm 13 \text{ MeV}, \]
\[ \Gamma(a_0 \to \bar{K}K) = 12 \pm 3 \text{ MeV}. \]

2. We compute the decay momentum with the central values of the parent mass, with the exception of the decay \( a \to \bar{K}K \), which is below threshold at the central mass value. In this case we have averaged the decay momentum over a Breit-Wigner, using the \( \Gamma_{\text{tot}}(a) \) given above, and find: \( \langle p(a \to \bar{K}K) \rangle \approx 84 \text{ MeV} \), which gives the value of the partial width reported in Table II.

3. In the case of \( f \to \bar{K}K \) or \( \pi \pi \), the authors of ref. \[ \text{12} \] define:
\[ \Gamma(S \to i) = g_i p(M) \]

| \( g \) | \( \sigma \) | \( \pi \pi \) | \( \bar{K}K \) |
|-----|-----|-----|-----|
| 345 MeV | 324 ± 50 MeV | - | - |
| 3.4 \( g_x < 0.02 \) | 3.4 \( g_x = 0.19 \pm 0.05 \) | 3.4 \( g_K = 0.28 \) | 3.4 \( g_K = 0.40 \pm 0.6 \) |
| 43 MeV | 60 ± 13 MeV | 23 MeV | 12 ± 3 MeV |
| \( \eta \pi \) | \( \bar{K} \pi \) |
| 138 MeV | 410 ± 100 MeV | - | - |

TABLE II: For a single parameter \( A = 2.6 \text{ GeV} \). For \( g_x \) we have reported the upper limit to the decay rate obtained from the \( f \to \pi \) mixing considered previously, see text.

and fit the data to a Breit-Wigner formula with mass-dependant width, thus giving directly the values of \( g_i \) that we report in the Table II.

It is interesting to see if the agreement can be improved by introducing other \( SU(3) \) allowed couplings. In the exact \( SU(3) \) limit there are four couplings, but one refers to a pure singlet-to-singlets amplitude, which is not relevant to the above decays. Restricting to the other three couplings, we write the effective Lagrangian according to:
\[ \mathcal{L} = (S_i^j c_{i,m}) e^{ikn} (a M_{l}^n M_{n}^m + b \delta_{l}^n (M_{m}^2)^n + c \delta_{l}^n (M_{n}^m) \text{Tr} M_n), \]

\[ M \] represents the nonet pseudoscalar matrix, analogous to \( S \), and we have made explicit the four quark nature of the scalar nonet. The first coupling corresponds to the switch amplitude, Fig. 1. The other two couplings correspond to amplitudes where one pair annihilates into a flavour singlet (gluons) that transforms into a \( q \bar{q} \) flavour singlet pair, violating the OZI rule. For \( a = A, b = c = 0 \) we reproduce the previous results. We obtain the effective Lagrangian:
\[ \mathcal{L} = f_0 \left[ b \sqrt{2} \frac{\pi \cdot \pi}{2} - (a - 3b) \frac{\bar{K}K}{\sqrt{2}} + \ldots \right] \]
\[ + \sigma_c \left[ -(a - 2b) \frac{\pi \cdot \pi}{2} + b \bar{K}K + \ldots \right] \]
\[ + a^0 \left[ (a - b) \frac{K \tau_3 K}{\sqrt{2}} - (a - c) \eta \pi^0 \right. \]
\[ - \sqrt{2}(b - c) \eta \pi^0 + \ldots \]
\[ + (a - b) \left( \frac{K^+ \pi^-}{\sqrt{2}} + \bar{K}^0 \pi^- \right) \kappa^+ + \ldots \]

The amplitude for \( a \to \pi \) receives a new contribution from \( c \) and is now independent from the others. The three OZI allowed amplitudes \( \sigma \to \pi \pi \), \( a_0/f_0 \to \bar{K}K \) are now predicted to be linearly spaced with \( b \). From the experimental values in Table II we find:
\[ |a - 2b| = 2.6 \text{ GeV} (= A) \text{ (from } \sigma \to \pi \pi), \]
\[ |a - 3b| = 3.1 \text{ GeV (from } f_0 \to \bar{K}K), \]
\[ |a - b| = 1.8 \text{ GeV (from } a_0 \to \bar{K}K), \]

which are indeed equally spaced with: \( b = -0.7 \text{ GeV} \). We find further: \( \Gamma(\kappa) = 66 \text{ MeV}; \quad g_\pi = 0.06 \). The \( c \)
(annihilation) coupling should be small: with \( c \) exactly zero we get \( \Gamma(a \to \eta \pi) = 30 \text{ MeV} \); cfr. Table II. A better agreement is found with data for the OZI allowed channels, except for the \( \kappa \) width, which is too low (but also known with large uncertainties). The large value of \( A \) seems indicative of a short distance effect, making perhaps more justifiable the use of flavour \( SU(3) \) symmetry. These results reinforce considerably the case of the scalar mesons as \((q\bar{q})^2(q\bar{q})^2\) states. A notable exception is the OZI forbidden decay \( a/f \rightarrow \eta \pi \), which turns out to be too small, even allowing for the full \( SU(3) \) couplings. It is quite possible that this decay proceeds via a different mechanism. One possibility we would like to suggest is:

\[
f \to K\bar{K} \to \pi \pi,
\]

with the first step via off-shell \( K\bar{K} \) states and the second by a strong, OZI allowed process. A calculation of this effect, with the second step mediated by \( K^* \) and \( \kappa \) exchange is under consideration. It is a calculation that closely resembles those performed in the \( K\bar{K} \) molecule picture.

**Open and hidden charm scalar mesons.** A firm prediction of the present scheme is the existence of analogous states where one or more quarks are replaced by charm or beauty. We consider the case of charm, extension to beauty is obvious. Open charm scalar mesons of the form \( S = [cq][\bar{q}\bar{s}] \), fall into characteristic \( 6 \oplus 3 \) multiplets of \( SU(3)_f \). The \( 3 \) has the same conserved quantum numbers of \( cq \) states, but the \( 6 \) contains exotic states which should be very conspicuous.

Open charm states are classified as follows.

\( S = 1: \)

\[ a_{c\bar{s}}(I = 1), f_{c\bar{s}}(I = 0) = [cq][\bar{q}\bar{s}]. \]

They form a degenerate triplet-singlet similar to the \( a/f \) complex, but with charges \( 0, +1, +2 \). OZI allowed decays are:

\[ a_{c\bar{s}} \to D_\pi, DK \quad (E_{thr} = 2103.6, \ 2367 \text{ MeV}), \]
\[ f_{c\bar{s}} \to D\eta, DK \quad (E_{thr} = 2515.9, \ 2367 \text{ MeV}). \]

\( S = 0: \)

\[ \delta_c(I = 1/2) = [cs][\bar{q}\bar{s}], \]
\[ S_c(I = 1/2) = [cq][\bar{u}\bar{d}]. \]

The two isodoublets are superpositions of \( 6 \) and \( 3 \) components with decays:

\[ \delta_c \to D\eta, Ds\bar{K} \quad (E_{thr} = 2416.6, \ 2466.3 \text{ MeV}), \]
\[ S_c \to D\pi \quad (E_{thr} = 2004.3 \text{ MeV}). \]

Hidden charm states of the form: \([cq][\bar{c}\bar{q}]\) fall into \( 8 \oplus 1 \) multiplets of \( SU(3) \) again producing very exotic states. We simply mention the degenerate isorightlet-isosinglet complex: \( a_{c\bar{c}}(I = 1), f_{c\bar{c}}(I = 0) \), equal to the \( a/f \) complex with \( s \) replaced by \( c \), with characteristic decays:

\[ a_{c\bar{c}} \to \eta_c\pi, \ D\bar{D} \quad (E_{thr} = 3114.7, \ 3738.6 \text{ MeV}), \]
\[ f_{c\bar{c}} \to \eta_c\eta, \ D\bar{D} \quad (E_{thr} = 3527, \ 3738.6 \text{ MeV}). \]

We expect quark pair annihilation to be suppressed by asymptotic freedom. Thus the decay rates into exclusive channels should be well described by the simple switch amplitude (Fig. 1). By scaling from Eq. (13) one finds widths \( \approx O(10) \text{ MeV} \).

Narrow states decaying into open or hidden charm states and a pseudoscalar meson are being discovered at PEPII and BELLE and in fixed target experiments (FNAL). Analysis of these states in term of four quark states has been done in some cases [13]. Further experimental search for exotic states is crucial.

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