Reasonable Application and Interpretation of Effective Stress Principle in Shale Reservoir Mining

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Abstract. This paper explores the effective stress characteristics of shale reservoir mining from the perspective of mechanics. The results show that the decrease of the pore fluid pressure and the increase of the effective stress acting on the active zone are not a simple linear relationship, but a nonlinear relationship that increases rapidly and then slows down. In the post-fracturing development process of shale reservoirs, the reservoirs in the early stage of production will produce uneven deformation and structural damage. The mechanical behavior of the reservoir is mainly characterized by plastic deformation. As mining progresses, reservoir failure and fracture areas are compacted or supported. The mechanical behavior of the reservoir is mainly characterized by elastic deformation. In the stress cloud diagram, the stress of the overlying strata presents a certain arched distribution. This structural effect makes the equivalent stress difference in the reformed area increase continuously, and the pressure bearing by the structure to the overlying strata will be greater. Therefore, the extent of the increase in the true effective stress borne by the transformation area will decrease.

1. Introduction
Shale reservoirs have typical ultra-low porosity and ultra-low permeability characteristics. Maximize the area of cracks in the reservoir by volumetric fracturing. Thereby greatly improving the effective permeability of the reservoir and increasing the production of the gas well [1]. However, when shale reservoirs are mined, as the pore pressure of the fluid decreases, the effective stress of the pay zone will increase, which will affect the reservoir permeability and porosity [2]. Regarding the effective stress characteristics of shale reservoir mining, domestic and foreign scholars have explored from the perspective of experiment and well testing. Kwon et al. [3] and Reyes et al. [4] tested the steady-state permeability of four shale cores in the Oklahoma area by changing the confining pressure. The experimental results show that the permeability is exponentially related to the effective stress. Luo et al. [5] combined the rock mineral analysis data and mercury intrusion test to study the microscopic pore structure of the reservoir. It is believed that the effective stress of the reservoir should be affected by the content of the framework particles, the pore throat radius and the clay mineral content. The stress-sensitive effects caused by the increase in effective stress should be considered in the development of reservoirs with ultra-low porosity and permeability. Zheng et al. [6] used the piecewise fitting and well testing analysis of production performance to study the variation of stress sensitive parameters. Studies have shown as production time and effective stress increase, all permeability, half-length of the crack and conductivity are reduced. Tian et al. [7] studied the
relationship between shale stress sensitivity and productivity. Studies have shown that the stress sensitivity curve of the shale fracture network is “L-shaped”. In the first stage, pseudoplastic deformation occurs mainly, and in the second stage, elastic compression deformation of the rock skeleton occurs mainly. The increase in effective stress during the middle and late stages of mining is relatively weak. Luo et al. [8] and Shen et al. [9] found that because of the low permeability and ultra-low permeability reservoir pore system consists of small pores, the capillary effect is stronger and the flow mechanism is complicated. Therefore, there is a strong starting pressure gradient, which has stronger stress sensitivity than the medium-high permeability reservoir. As for the effective stress itself, it is a bridge and bond to study the mechanical behavior of porous media using the theory of solid mechanics. Terzaghi proposed the first effective stress principle for loose soil media [10]. Its connotation is the general law between the measurable load effect of soil and the external load. When this principle is applied to geotechnical mechanics, scholars have questioned its applicability. Biot [11] introduced the effective stress coefficient α to modify Terzaghi’s effective stress principle. So far, Terzaghi’s effective stress principle has been widely used in Geomechanics. The understanding and definition of effective stress coefficient α is also gradually enriched, as shown in table 1.

Table 1. Effective stress coefficient α.

| Researcher                  | α                                      |
|-----------------------------|----------------------------------------|
| Terzaghi [10]               | 1                                      |
| Biot [11]                   | n (Void ratio)                         |
| Geertsma [12], Skempton [13], Zhang [14] | 1 – K/K₆ (K is the compression modulus of the rock matrix, Kₛ is the compression modulus of the solid particles) |
| Suklje [15], Lade and Boer [16] | 1 – (1 – n)K/Kₛ |

To a certain extent, the above studies reflect the changes in stress-sensitive strength caused by the increase of effective stress during shale reservoir mining, either locally or laterally. However, the study did not consider the reservoir rock mass background and has limitations. The change characteristics of the effective stress itself reflected by the decrease of reservoir pore pressure have not been correlated. Therefore, in order to further understand the stress-sensitive characteristics of shale reservoirs, it is necessary to study the effective stress characteristics of shale reservoir mining. Based on the knowledge of interface fracture mechanics and plate and shell theory, this paper explores the effective stress characteristics of shale reservoir mining from the perspective of mechanics and methods. The stress distribution and changes of the reconstructed area and surrounding areas during shale reservoir development were analyzed by modeling. The aim is to promote the understanding of shale reservoir development characteristics and promote efficient mining of shale gas.

2. Interface Fracture Mechanics Model

According to the equivalent percolation theory, the fracture network system after shale reservoir fracturing can be equivalent to a high permeability zone. The characteristics of fracture network system can be characterized by the quantity volume and permeability of high permeability zone [17].

Modeling at infinite stratigraphic scales. The reservoir is considered an elastic layer. Considering the longitudinal heterogeneity of shale reservoirs, it is simply assumed that the fracture network system traverses two different horizons. After the single-stage fracturing, the area of the fracture network is a high-permeability zone. The permeable zone may be equivalent to an interfacial crack zone that is subjected to fluid pore internal pressure and external contact pressure.

2.1. Mechanical Expression under External Contact Force [18]

The composite stress intensity factor K of an infinite plane containing a central crack of length L under a uniform stress field at infinity is given by equation (1).
\[ K = K_1 + iK_2 = (\sigma_{yy}^{\infty} + i\tau_{yy}^{\infty})(1 + 2i\varepsilon)(L)^{-i\varepsilon}\sqrt{\pi L/2} \]  

(1)

Here \( \varepsilon \) is the oscillation singularity index, which is calculated as shown in equation (2).

\[ \varepsilon = \frac{1}{2\pi} \ln[(k_1\mu_2 + \mu_1)/(k_2\mu_1 + \mu_2)] \]  

(2)

The displacement expression of the crack surface under the stress field is expressed by equation (3).

\[ (u_y + iu_x)_{\theta=\pi} - (u_y + iu_x)_{\theta=-\pi} = (c_1 + c_2)K r^{i\varepsilon}/[2\sqrt{2\pi}(1 + 2i\varepsilon)cosh(\pi\varepsilon)] \]  

(3)

Here, the shear modulus and Poisson's ratio of the upper and lower areas of the interface are \( \mu_1, \mu_2, v_1 \) and \( v_2 \), respectively. \( c_1 = (k_1 + 1)/\mu_1, c_2 = (k_2 + 1)/\mu_2 \). In the plane strain problem, \( k_1 = 3 - 4v_1, k_2 = 3 - 4v_2 \). In the plane stress problem, \( k_1 = (3 - v_1)/(1 + v_1), k_2 = (3 - v_2)/(1 + v_2) \).

2.2. Mechanical Expression under Fluid Internal Pressure

The composite stress intensity factor \( K' \) of an infinite plane containing a central crack of length \( L \) under a uniform compressive stress only on the crack surface is given by equation (4).

\[ K' = K_1 + iK_2 = T \sqrt{\pi L/2}(1 + 2i\varepsilon)(L)^{-i\varepsilon} \]  

(4)

Here \( T \) is the crack surface stress field under the force.

The displacement expression of the crack surface under the stress field is expressed by equation (5).

\[ (u_y + iu_x)_{\theta=\pi} - (u_y + iu_x)_{\theta=-\pi} = (c_1 + c_2)K' r^{i\varepsilon}/[2\sqrt{2\pi}(1 + 2i\varepsilon)cosh(\pi\varepsilon)] \]  

(5)

2.3. Mechanical Expression of Fluid Internal Pressure and External Contact Pressure

The superposition of uniform stress field at infinity and uniform compressive stress only on the crack surface is the expression of the fracture network high permeability zone which is equivalent to the central interface crack of infinite elastic layer after reservoir fracturing.

The total displacement of the crack faces obtained from the above formulas is as shown in equation (6).

\[ (u_y + iu_x)_{\theta=\pi} - (u_y + iu_x)_{\theta=-\pi} = (c_1 + c_2) (K + K') r^{i\varepsilon}/[2\sqrt{2\pi}(1 + 2i\varepsilon)cosh(\pi\varepsilon)] \]  

(6)

Equation (6) is organized as shown in equation (7).

\[ (u_y + iu_x)_{\theta=\pi} - (u_y + iu_x)_{\theta=-\pi} = \frac{c_1 + c_2}{2\sqrt{2\pi}cosh(\pi\varepsilon)}(\sigma_{yy}^{\infty} + i\tau_{yy}^{\infty} + T)sqrt{\pi L/2}(r/\ell)^{i\varepsilon} \]  

(7)

The stress field \( \sigma_{yy}^{\infty} + i\tau_{yy}^{\infty} + T \) can be expressed as \( \overline{\sigma_{yy}} + i\overline{\tau_{yy}} \), then the load phase angle \( \phi = tan^{-1}(\overline{\tau_{yy}}/\overline{\sigma_{yy}}) \). Assuming that the total displacement of the crack surface is zero, we solve the crack area scale relationship in the crack zone without deformation as in equation (8).

\[ \frac{c_1 + c_2}{2\sqrt{2\pi}cosh(\pi\varepsilon)}e^{i\phi}(r/\ell)^{i\varepsilon} = cos(\phi - \varepsilon ln((\ell/\ell)^{i\varepsilon})) = 0 \]  

(8)

Taking -\( \pi/2 < \phi < \pi/2 \), then equation (9) is obtained from equation (8).

\[ r/\ell = e^{-(\phi + \pi/2)/\varepsilon} \]  

(9)

For most elastic materials, \( 0 < \varepsilon < 0.175 \). Let \( \varepsilon = 0.175 \), when \( \phi = 0 \), the crack scale relationship is shown as \( r/\ell \approx 1.25 \times 10^{-4} \).
The mechanical parameters of shale reservoir are shown in table 2. Substituting the parameters of table 1 into equation (2) to obtain $\varepsilon = 0.101$. For $r/L = 0$, we get $r/L \approx 1/e^{157.08} \rightarrow 0$.

According to the maximum crack half-length and the minimum crack band width, the crack zone size relationship of the single-stage fracture area is shown as $r/L = 5/(2 \times 150) \approx 0.017 > 1/e^{157.08}$.

| Table 2. Mechanical parameters. |
|----------------------------------|
| Young’s modulus (MPa) | Poisson’s ratio | Shear modulus (MPa) |
| $E_1 = 2.265 \times 10^4$ MPa | $v_1 = 0.271$ | $\mu_1 = E_1/[2(1+v_1)] = 8.910 \times 10^3$ |
| $E_2 = 3.949 \times 10^4$ MPa | $v_2 = 0.185$ | $\mu_2 = E_2/[2(1+v_2)] = 1.666 \times 10^4$ |

That is, the results show that the crack length for the center crack which is subjected to uniform stress field at infinity without displacement deformation should be much longer than the crack height. By calculating the crack tip oscillation singularity index of the equivalent single-segment seam area, the actual seam size relationship under the uniform stress field is far from satisfying the theoretical fracture network scale relationship without displacement deformation. It shows that there is a considerable degree of strain effect in the actual fracture network region.

3. Modeling Based on Plate and Shell Theory

3.1. Transversely Isotropic Foundation Model

For sedimentary natural foundations such as rock reservoirs, there are large differences in the physical and chemical properties of rocks in the vertical direction. The horizontal difference in properties is relatively small. Therefore, a transversely isotropic foundation model is introduced to describe the state of the reservoir foundation [19].

In the Cartesian coordinate system, the basic expression of the axisymmetric space problem of the transversely isotropic elastomer is expressed by equations (10)-(12).

The equilibrium differential equations represented by the stress component are as shown in equation (10).

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = f_x, \quad \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \frac{\partial \tau_{xy}}{\partial x} = f_y, \quad \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = f_z$$

(10)

The geometric equation is as in equation (11).

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial u}{\partial y}, \quad \varepsilon_z = \frac{\partial u}{\partial z}, \quad \gamma_{yz} = \frac{\partial v}{\partial z}, \quad \gamma_{zx} = \frac{\partial w}{\partial z}, \quad \gamma_{xy} = \frac{\partial w}{\partial x}, \quad \gamma_{yx} = \frac{\partial v}{\partial x}$$

(11)

The physical equation is as in equation (12).

$$\begin{bmatrix}
\Delta \varepsilon_x \\
\Delta \varepsilon_y \\
\Delta \varepsilon_z \\
\Delta \gamma_{yz} \\
\Delta \gamma_{zx} \\
\Delta \gamma_{xy}
\end{bmatrix} = 
\begin{bmatrix}
1 & -\nu_{bh} & -\nu_{bh} & 0 & 0 & 0 \\
-\nu_{bh} & 1 & -\nu_{bh} & 0 & 0 & 0 \\
-\nu_{bh} & 1 & -\nu_{bh} & 0 & 0 & 0 \\
0 & 0 & 0 & 1/G_{bh} & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G_{bh} & 0 \\
0 & 0 & 0 & 0 & 0 & 2(1+\nu_{bh})/E_h
\end{bmatrix}
\begin{bmatrix}
\Delta \sigma_x \\
\Delta \sigma_y \\
\Delta \sigma_z \\
\Delta t_{yz} \\
\Delta t_{zx} \\
\Delta t_{xy}
\end{bmatrix}$$

(12)
Here $E_v$ is the elastic modulus perpendicular to the isotropic plane. $E_h$ is the elastic modulus parallel to the isotropic plane. $v_{vh}$ is the Poisson’s ratio of the horizontal strain caused by the application of the vertical strain. $v_{hh}$ is the Poisson’s ratio in an isotropic plane. $G_{vh}$ is the shear modulus perpendicular to the isotropic plane.

$\Delta \varepsilon_x$, $\Delta \varepsilon_y$, and $\Delta \varepsilon_z$ are strain increments. $\Delta \sigma_x$, $\Delta \sigma_y$, and $\Delta \sigma_z$ are positive stress increments. $\Delta \gamma_{yz}$, $\Delta \gamma_{zx}$, and $\Delta \gamma_{xy}$ are the shear strain increments. $\Delta \tau_{yz}$, $\Delta \tau_{zx}$, $\Delta \tau_{xy}$ are the shear stress increments.

3.2. Mechanical Model of Reservoir Reconstruction Area

The local interaction model of the reservoir after fracture is shown in figure 1.

![Figure 1. Local interaction model.](image)

A mechanical model is established based on the extended application of the Terzaghi effective stress principle [20] in reservoir mining (equation (13)).

$$\sigma = \sigma_e + \alpha \cdot p_p$$  \hspace{1cm} (13)

Here $\sigma$ is the overlying pressure, MPa. $\sigma_e$ is the effective stress, MPa. $\alpha$ is the effective stress coefficient, dimensionless. $p_p$ is the pore fluid pressure, MPa.

In figure 1, $\sigma = \sigma_e + \alpha \cdot p_p$, $\sigma' = \sigma_e' + \alpha \cdot p_p'$, and $\sigma = \sigma'$.

Take the single-stage fracturing area after shale reservoir fracturing as an example. The Reissner plate theory [21] was introduced, and the overlying strata acting in the transformation area was equivalent to the elastic plate supported by the uniform pressure and support, as shown in figure 2. The single-stage fracturing zone is an elliptical cylinder. The semi-major axis and semi-minor axis of the horizontal elliptical section are $a$ and $b$, respectively. In the mechanical model, the boundary of the same zone of the overlying strata corresponding to the single-stage fracturing zone is set as the clip side. It is assumed that the boundary of the overlying strata on a single-stage fracturing zone is approximated as elliptical. The equivalent thickness of the overlying strata is $h$, the average effective stress is $\bar{\sigma}_e$, the fluid pore pressure is $p_p$, and the contact reaction force with the fracturing zone is $p$.

The effective stress variation characteristics during the reservoir mining were simulated by solving the forces and deformations of the fracture network area and the corresponding overlying strata after single-stage fracturing.

![Figure 2. Equivalent mechanical model.](image)

The model system has the following assumptions:

1. It follows Reissner plate theory;
2. The reservoir is a transversely isotropic elastic half-space;
3. Displacement of the contact surface between the overlying strata and the fracturing zone remains consistent and continuous.
The boundary equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$, and the deflection is expressed as equation (14).

$$w = m \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right)^2$$ (14)

On the boundary, it satisfies the boundary condition $w = 0$, $\frac{\partial w}{\partial x} = 0$, $\frac{\partial w}{\partial y} = 0$.

The governing equations for elastic plates subjected to uniform pressure and supporting reaction force are as in equations (15-17) [22]:

$$D \nabla^4 w = q - \frac{h^2 (1-\mu)}{10} \nabla^2 q$$ (15)

$$D \left( \frac{\partial^2 \alpha}{\partial x^2} + \frac{1-\mu}{2} \frac{\partial^2 \alpha}{\partial y^2} + \frac{1+\mu}{2} \frac{\partial^2 \beta}{\partial x \partial y} \right) = \frac{5Gh}{6} \left( \alpha + \frac{\partial w}{\partial x} \right)$$ (16)

$$D \left( \frac{\partial^2 \beta}{\partial y^2} + \frac{1-\mu}{2} \frac{\partial^2 \beta}{\partial x^2} + \frac{1+\mu}{2} \frac{\partial^2 \alpha}{\partial x \partial y} \right) = \frac{5Gh}{6} \left( \alpha + \frac{\partial w}{\partial y} \right)$$ (17)

Here $w$ is the deflection of the plate; $q$ is the uniform pressure of the plate; $p$ is the supporting reaction force; $\alpha, \beta$ are the rotation angles of the normal of middle plane in the $x$ and $y$ directions, respectively; $D = EH^3/[12(1-\mu^2)]$ is the bending stiffness of the plate; $E$ is the elastic modulus; $G$ is the shear modulus; $\mu$ is the Poisson’s ratio; $h$ is the thickness of the plate.

According to the principle of effective stress, the overlying pressure $q$ is expressed as $q = \bar{\sigma}_e \uparrow + p_p$, and the supporting reaction force at the contact surface is shown in equation (18).

$$p = \sigma_e' + p_p' = \bar{\sigma}_e' + p_p'(\sigma_e' \neq \bar{\sigma}_e', p_p' \neq p_p')$$ (18)

Then the deflection expression to be solved is shown in equation (19).

$$w = \frac{(\sigma_e + p_p - \bar{\sigma}_e - p_p') \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right)}{BD (\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1)}$$ (19)

The solution satisfies the basic differential equations and boundary conditions and is correct.

The expressions of each stress component are shown in equation (20).

$$\sigma_x = \frac{12M_z}{\delta^3} z, \sigma_y = \frac{-12M_y}{\delta^3} z, \tau_{xy} = \tau_{yx} = \frac{-12M_{xy}}{\delta^3} z, \tau_{xz} = \frac{6F_{sx}}{\delta^3} \left( \frac{\delta^2}{4} - z^2 \right), \tau_{yz} = \frac{6F_{sy}}{\delta^3} \left( \frac{\delta^2}{4} - z^2 \right), \sigma_z = -2q \left( \frac{1}{2} - \frac{z}{\delta} \right) \left( 1 + \frac{z}{\delta} \right)$$ (20)

According to the same contact surface displacement between the overlying strata and the fracturing area, the coordination relationship between the contact surface deflections is established as $w = w'$.

Here $w$ is the deformation displacement of the overlying strata at the contact surface, and $w'$ is the deformation displacement of the fracturing zone at the contact surface.

The transversely isotropic foundation model is used to solve the deformation of the fracturing zone under the overburden pressure. The load and displacement of the model subjected to the circular uniformly distributed vertical load are shown in equations (21) and (22), respectively [23, 24].

The foundation load is expressed as in equation (21).

$$q(r) = \begin{cases} q & r \leq a \\ 0 & r > a \end{cases}$$ (21)

Here $a$ is the acting radius of the circular uniformly distributed vertical load.

The foundation displacement expression is expressed as equation (22).

$$w' = \frac{1}{d_{55} + d_{13}} \int_0^\infty \left[ -(d_{55}s_1^2 - d_{11})k_{11}e^{-s_1^2z^2} + (d_{55}s_2^2 - d_{11})k_{12}e^{-s_2^2z^2} \right] \frac{q_{aj}(x)}{x} f_0 \left( \frac{r}{a}x \right) dx$$ (22)
Here, $d_{11} = \lambda n(1 - n\mu_2^2)$, $d_{12} = \lambda n(\mu_1 - n\mu_2^2)$, $d_{13} = \lambda n\mu_2(1 + \mu_1)$, $d_{33} = \lambda(1 - \mu_1^2)$, $d_{55} = G_2$, $n = \frac{E_1}{E_2}$, $\lambda = \frac{E_2(1+\mu_1)}{(1-\mu_1)(1-2n\mu_2^2)}$, $K_{11} = \frac{K_3}{-K_3K_4+K_4}$, $K_{12} = \frac{K_3}{-K_2K_4+K_4}$, $K_1 = s_1(d_0 + e_0s_1^2)$, $K_2 = s_2(d_0 + e_0s_2^2)$, $K_3 = s_1(a_0 + c_0s_1^2)$, $K_4 = s_1(a_0 + c_0s_2^2)$, $a_0 = \frac{d_{11}d_{55}}{d_{13}+d_{55}}$, $c_0 = \frac{d_{13}(d_{13}+d_{55})-d_{11}d_{13}}{d_{13}+d_{55}}$, $e_0 = \frac{d_{33}d_{55}}{d_{13}+d_{55}}$, $s_1$ and $s_2$ are indicators of the anisotropy of the ground rock.

4. Model Solving and Analysis
Taking the reservoir single-stage fracturing model as an example, the effective stress characteristics during the reservoir mining were simulated and calculated. The model parameters are shown in Table 3.

| Material parameters for shale. |
|-------------------------------|
| $E_1$ (GPa) | $E_2$ (GPa) | $v_1$ | $v_2$ | $G_{12}$ (GPa) | $\rho$ (kg/m$^3$) |
| 44 | 32 | 0.2 | 0.25 | 8.35 | 2600 |

Take the half-width $b = 33$ m and half-length $a = 80$ m of the single-stage reformed area. According to the assumption of small deflection bending theory, the thickness $h \leq b/5 = 6.6$ m. The model's original stress field is initialized to a zero stress field.

The overall boundary of the model is a fixed displacement constraint. The intermediate transformation area is in substantial contact with the surrounding reservoirs. Grid densification is carried out in the contact area between the transformation area and the overlying strata.

Simulations were performed by increasing the overlying pressure to change the stress difference and increasing the bulk force to change the stress difference. Compare and analyze the changes of the true effective stress characteristics of the reservoir.

The loading mode of increasing the overburden pressure in the transformation zone is equivalent to changing the stress difference in the laboratory by increasing the confining pressure on the core with saturated fluid pressure. The simulation results of loading pressures of 1 MPa, 2 MPa, 3 MPa, and 4 MPa are shown in figures 3a, 4a, 5a and 6a, respectively. The loading mode of increasing the volume force in the transformation area is equivalent to an equivalent simulation of the increase of effective stress in the reservoir caused by the decrease in fluid pore pressure during reservoir production. The simulation results of the loading volume forces of 1 MPa, 2 MPa, 3 MPa, and 4 MPa are shown in figures 3b, 4b, 5b and 6b, respectively.

From the following stress cloud diagram, it can be seen that in addition to the stress concentration on the four corners of the boundary of the reformed area, as the pressure value increases, the stress in the transformation area caused by the two loading modes all increases. However, the reservoir area affected by the loading mode that increases the overburden pressure is mainly the transformation area and its lower rock area. The reservoir area affected by the loading mode that increases the bulk force is mainly the transformation area and its upper rock area. This shows that under the first loading mode, the force change is mainly transmitted to the lower rock layer through the transformation area. The stresses in the transformation area changed significantly with the increase of applied pressure. With the increase of applied pressure, compared with the change of stress in the transformation area under the first loading mode, the stress change in the transformation area under the second loading mode is weaker.

The change of equivalent stress in the transformation area during the increase of force under two loading modes was extracted to analyze the true effective stress characteristics of shale reservoir mining. As shown in figure 7, under the first loading mode, the true effective stress value of the reservoir increases linearly with the increase of the equivalent pressure difference. Under the second loading mode, the true effective stress value of the reservoir is a non-linear increase relationship that increases rapidly and then slows down as the equivalent stress difference increases. In the following, this feature is analyzed in combination with the extended application of Terzaghi’s effective stress principle in reservoir mining and the mechanical behavior of reservoir mining.
Figure 3. Stress contours: (a) The equivalent overburden pressure increases by 1 MPa; (b) The equivalent pore pressure decreases by 1 MPa.

Figure 4. Stress contours: (a) The equivalent overburden pressure increases by 2 MPa; (b) The equivalent pore pressure decreases by 2 MPa.

Figure 5. Stress contours: (a) The equivalent overburden pressure increases by 3 MPa; (b) The equivalent pore pressure decreases by 3 MPa.

Figure 6. Stress contours: (a) The equivalent overburden pressure increases by 4 MPa; (b) The equivalent pore pressure decreases by 4 MPa.
Figure 7. Effective stress curve.

The Terzaghi effective stress principle states that pore pressure transmitted through water and gas in the pores does not contribute to the strength and deformation of the rock mass. With the development of the reservoir, the pore fluid pressure $p_p'$ in the utilization area of the reservoir decreases, and the rock mass in the transformation area is compressed. The stress field at the interface between the transformation area and the overlying strata shows an increase in total support stress $\sigma'$. For overlying strata outside the reservoir utilization area, fluid pore pressure is constant. As a result, the effective stress $\sigma_e$ of the overlying strata must be increased due to the effect of the transformation area. The interaction of the stress transmission at the contact surface between the overlying strata and the reservoir utilization area is shown as a balance between the total compressive stress $\sigma'$ and the total support stress $\sigma$. Therefore, as the pore fluid pressure in the utilization area decreases, the effective stress $\sigma_e$ in the utilization area of the reservoir increases non-linearly with the change in compressive stress $\sigma'$ (equation (23)). This is consistent with the simulation results of the second loading mode.

$$\sigma_e' \uparrow + \alpha \cdot p_p' \downarrow = \sigma' \uparrow = \sigma \uparrow = \sigma_e \uparrow + \alpha \cdot p_p \tag{23}$$

The effective stress coefficient $\alpha$ given by the effective stress studies in previous experiments and calculations is a constant value related to the properties of the rock itself. In this case, the characteristics of the true effective stress of the reservoir are consistent with the simulation results of the first loading mode. However, the shale reservoir is buried deep, the rock mass is dense, and the characteristics of high temperature and high pressure are obvious. In the development process after reservoir fracturing, the decrease in pore fluid pressure can cause uneven deformation and destruction of the reservoir. As a result, anisotropy occurs in the value of the elastic modulus. In the early stage of production, the mechanical behavior of the reservoir is mainly plastic deformation. With the progress of mining, the damaged area of the reservoir is compacted or supported, and the mechanical behavior of the reservoir mainly manifests as elastic deformation. At the same time, throughout the development of the reservoir, whether it is local damage or deformation, the reservoir rock mass is always a structural whole. When the local force field or deformation increases, the overall structural effect of the reservoir rock mass will gradually become significant. It is shown in the stress cloud diagram that the overlying strata presents the pressure arch stress distribution effect. It also coincides with the related description of Wang et al. [25]. When the equivalent stress difference in the transformation area keeps increasing, this structural effect causes the structure to bear more pressure on the surrounding rock and the overlying strata, and the extent of the increase in the true effective stress borne by the transformation area will decrease. Therefore, in the process of reservoir development, the decrease of pore fluid pressure and the increase of effective stress are not a simple linear relationship. Instead, under the influence of elastoplastic mechanical behavior and structural mechanical effects of the reservoir rock mass, the two present a non-linear relationship of rapid increase first and then slower.
5. Conclusions
(1) An interface fracture mechanics model was established to determine that with the decrease of pore pressure, the increase of effective stress caused the reservoir to have a strong strain effect during the reservoir development. A transverse isotropic mechanical model of shale reservoir was established based on plate and shell theory. The mechanical strain model and the transverse isotropic foundation model established by the theory of medium and thick plates are used to solve the stress-strain effect of the transformation area during the reservoir exploitation.

(2) Based on the numerical simulation results, the effective stress principle and the mechanical behavior of the reservoir mining, the effective stress characteristics of the reservoir mining were comprehensively analyzed. Studies have shown that a decrease in the pore fluid pressure results in a significant increase in the effective stress acting on the utilization area. The change of pore fluid pressure and effective stress acting in the utilization area is not a simple linear relationship. It is a nonlinear relationship in which the effective stress increases rapidly and then slows down as the pore pressure decreases.

(3) Reservoir failure and deformation lead to anisotropy of the elastic modulus, and the effective stress coefficient $\alpha$ is not a constant value. At the same time, during the development of the reservoir, the overlying strata presents the pressure arch stress distribution effect, and the overall structure of the reservoir rock mass plays a significant role. When the equivalent stress difference in the transformation area keeps increasing, this structural effect causes the structure to bear more pressure on the surrounding rock and the overlying strata, and the extent of the increase in the true effective stress borne by the transformation area will decrease.

(4) Under the condition that shale reservoir fracturing is sufficient, the early stage of reservoir exploitation is greatly affected by plastic deformation. In order to avoid excessive damage to the reservoir and increase the degree of production, it is recommended that the production pressure should be reasonably controlled in the early stage of production.

Acknowledgments
This work is supported by the National Science and Technology Major Project of the Ministry of Science and Technology of China (Grant No. 2017ZX05037-001), the Demonstration Project of the National Science and Technology Major Project of the Ministry of Science and Technology of China (Grant No. 2016ZX05062-002-001).

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