O'Sullivan, Cormac
Stirling's approximation and a hidden link between two of Ramanujan’s approximations. (English) Zbl 07680223
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Summary: A conjectured relation between Ramanujan’s asymptotic approximations to the exponential function and the exponential integral is established. The proof involves Stirling numbers, second-order Eulerian numbers, modifications of both of these, and Stirling’s approximation to the gamma function. Our work provides new information about the coefficients in Stirling’s approximation and their connection to Ramanujan’s approximation coefficients. A more analytic second proof of the main result is also included in an appendix.

MSC:
11Bxx Sequences and sets
05Axx Enumerative combinatorics
41Axx Approximations and expansions

Keywords:
asymptotic approximation; gamma function; exponential function; exponential integral

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