Multi-variable $\mathcal{H}_\infty$ Control Approach for Voltage Ancillary Service in Autonomous Microgrids: Modelling, Design, and Sensitivity Analysis

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This work was financially supported in part by Duy Tan University and in part by the French Ministry of Higher Education, Research, and Innovation.

ABSTRACT This paper proposes a multi-variable robust control scheme for voltage regulation in a diesel-photovoltaic-supercapacitor hybrid power generation system operating in stand-alone mode. First, we study the influence of system parameters on the dynamic behavior of open-loop system measured outputs by means of a stability analysis method based on Monte Carlo simulation. Next, by applying $\mathcal{H}_\infty$ control theory, an $\mathcal{H}_\infty$-based voltage controller is proposed to robustly force the voltage magnitude of a point of common coupling such as to satisfy design specifications. A cascaded two-level control architecture, where this controller acts as an upper control level and provides references to classical PI-based current tracking controllers placed on a lower level, is developed. A comprehensive methodology that casts the specific engineering demands of microgrid operation into an $\mathcal{H}_\infty$ control formalism is detailed. Effectiveness of the proposed voltage robust control strategy is validated via MATLAB®/Simulink® closed-loop time-domain simulations. Finally, we perform a sensitivity analysis of robust performance of the designed $\mathcal{H}_\infty$ controller in the presence of various load disturbances and model uncertainties through a series of closed-loop time-domain simulations carried out in MATLAB®/Simulink®.

INDEX TERMS Autonomous microgrids, voltage ancillary service, stability analysis, multi-variable $\mathcal{H}_\infty$ robust control, uncertainties, sensitivity analysis.

I. INTRODUCTION

A. MICROGRID CONTROL

MICROGRIDS (MGs) concept is gaining high momentum as a major, cost-effective solution to integrate distributed energy resources (DERs) into power networks [1]. Moreover, the distinctive autonomous operational capability of MGs has brought in higher reliability measures in supplying power demands when the utility grid is not available. The stability and control issues of autonomous MGs are however among the main challenges due to low inertia, uncertainties, and intermittent nature of DERs [2]. One critical control task in autonomous operation mode is the regulation of the network frequency and voltage magnitude. High-speed storage systems – e.g., lithium-ion batteries, flywheels or supercapacitors – have thus become necessary, leading to new grid configurations, for which more complex robust control structures are needed for dealing with multiple constraints such as unexpected disturbances and model uncertainties.

B. LITERATURE REVIEW

Topics around voltage control in autonomous – otherwise called islanded or stand-alone MGs – are nowadays extensively explored by employing advanced control techniques; among them, robust control design has a major role. In the work of Mohamed et al. [3], a variable-
structure voltage controller is integrated with a droop-based power-sharing controller to improve the transient and steady-state response of an MG against voltage disturbances and power angle swings. Based on master/slave control structure, an $\mathcal{H}_\infty$-mixed-sensitivity-based robust voltage controller for the master unit in a single-bus MG is presented in the work of Babazadeh et al. [4]. The proposed decentralized controller provides the good tracking of reference signals and robustly maintains load voltage magnitude despite parameter uncertainties. Similarly, using linear $\mathcal{H}_\infty$ mixed sensitivity control method, a new robust two-degree-of-freedom feedback-feedforward control scheme is proposed for the islanded operation of an MG comprising several DG units in the work of Babazadeh et al. [5] to robustly regulate load voltage in the presence of unmodeled dynamics, for example, linear, nonlinear, and highly unbalanced loads. Also, $\mathcal{H}_\infty$-mixed-sensitivity-loop-shaping-based robust control design for current sharing improvement in an islanded MG composed of parallel-connected inverter-interfaced DG units is presented in the work of Taher et al. [6]. The designed robust controller is capable of guaranteeing robust stability and performance under line parameter uncertainties and different loads. Hamzeh et al. propose in [7] an $\mathcal{H}_\infty$-based robust controller along with particle swarm optimization algorithm for the autonomous operation of an MG comprising electronically coupled DG units under unbalanced and nonlinear load conditions with unknown dynamics. Voltage control strategy based on feedforward compensation and internal model robust feedback control applied to the autonomous operation of a three/single-phase hybrid multi-MG is detailed in the work of Wang et al. [8]. The designed controller can get better voltage output characteristics and robustness in dealing with micro-source output power fluctuations, loads abrupt change, or nonlinear loads and unbalanced loads. In the above-mentioned methods, the plant transfer function is strictly proper and parametrically uncertain. However, no problem-dependent uncertainty modelling is included in the control design procedure.

Motivated by the aforementioned limitation, Kahrobaeian et al. introduce in [9] a robust system-oriented control approach for voltage performance improvement and suppression of typical interaction dynamics in DG converter-based MGs. The proposed control scheme uses a robust $\mathcal{H}_\infty$ voltage controller which is designed under an extended model comprising different converter-MG interactions imposed on the output voltage, e.g., MG impedance variation, local load interactions, uncertainties in AC-side filter parameters.Unlike conventional droop controllers, the proposed strategy generates a two-degree-of-freedom controller, leading to stable and smooth power sharing performance over a wide range of loading conditions. In the work of Davari et al. [10], a robust multi-objective control approach based on $D-K$ iteration $\mu$-synthesis – whose plant nominal model is unstructured uncertain – is applied to a voltage-source-converter-based DC-voltage power port in hybrid AC/DC multi-terminal MGs. The developed controller ensures tracking performance, robust disturbance rejection, and robust stability against operating point and parameter variations, as well as has a simple structure, low order, and fixed parameters, which makes it very attractive for industrial applications. Similarly, on the basis of $\mu$-synthesis and $\mathcal{H}_\infty$ loop shaping control through genetic algorithm and particle swarm optimization, the reactive power compensation and voltage stability of a PV-wind-diesel hybrid power generation system – where unstructured uncertainty modelling is adopted – can be found in the work of Mohanty et al. [11]. The proposed controller combines robustness and simplicity, and it is quite adaptable in its nature under different wind power input and load variations.

Unlike the augmented unstructured uncertainty modelling approach used with conventional control, structured uncertainty modelling is adopted to enable the realization of a less conservative robust controller. In the work of Karimi et al. [12], a linear time-invariant robust servomechanism voltage controller is proposed for a single DG MG whose plant nominal model is structurally uncertain. Despite the uncertainty of load parameters, the proposed controller guarantees robust stability and pre-specified performance criteria, e.g., fast transient response, negligible interaction, and zero steady-state error. Similarly, fundamental concepts of a central power-management system and a linear, time-invariant, multi-variable, robust, decentralized, servomechanism control strategy – whose plant nominal model is structurally uncertain – are detailed for a multiple-DER MG in the work of Etemadi et al., [13] and [14] respectively. Each control agent guarantees fast tracking, zero steady-state error, and robust performance despite the uncertainties of MG parameter, topology, and operating point. On the basis of $\mathcal{H}_\infty$ control method, a robust multi-objective centralized control strategy – whose plant nominal model is norm-bounded uncertain with an integral quadratic constant – is presented in the work of Hossain et al. [15] to optimize the active and reactive power sharing performance of a PV-wind MG in the presence of network changes, nonlinear uncertainties, and interaction dynamics. Also, decentralized robust control strategies subject to polytopic uncertainty are designed for plug-and-play voltage stabilization in islanded inverter-interfaced MGs and in islanded DC MGs in the work of Sadabadi et al., [16] and [17] respectively. The performance of the proposed control strategy is verified in terms of voltage tracking, MG topology change, plug-and-play capability features, and load changes. Li et al. propose in [18] a robust control scheme for an MG with a power-factor correction capacitor connected at the point of common coupling (PCC). An integrated control strategy composed of an outer $\mathcal{H}_\infty$ voltage controller, an inner filter inductor current feedback controller, and a virtual resistance compensator is presented. By properly selecting weighting functions with parameter uncertainty being transformed to multiplicative output structure, the
designed $\mathcal{H}_\infty$ controller can effectively produce desired performance and explicitly specify the degree of robustness against effective shunt filter capacitance variations. However, it might be sensitive to system resonance caused by the switching of power-factor correction capacitor. Similarly, Kahrobaian et al. present in [19] a direct single-loop $\mu$-synthesis voltage control scheme for the suppression of multiple resonances caused by power-factor correction capacitors and residential capacitive loads in DG MGs. Compared to conventional $\mathcal{H}_\infty$ multi-loop control, the proposed single-loop $\mu$-synthesis controller, with structured uncertainty modelling, reduced sensor requirement, and no additional passive or active damping mechanism, can ensure the robust stability and performance of the MG subject to parameter uncertainties and uncertain resonant peaks. However, the controller obtained with this method has high order, therefore it leads to harder design and practical implementation. Another interesting approach can be found in the work of Li et al. [20], where a novel cascaded-loop strategy for the control of a grid-forming inverter is designed. Even if a precise model for the inverter system is not required, the proposed method can tackle uncertainties and LC filter resonance without any passive or active damping techniques. It comprises a sliding-mode-control-based inner current loop and a mixed-$H_2/H_\infty$-control-based outer voltage loop where its uncertain matrix is norm bounded, which provides the benefits of constant switching frequency, low total harmonic distortion, robustness against parameters variations, and fast transient response. Moreover, the proposed control strategy ensures better transient and steady performance compared to conventional PI-based nested-loop control. Nevertheless, it may not be sufficiently robust against large system parameter variation and exogenous disturbances as well as well-performing due to the issue of conservatism of $H_2/H_\infty$ control.

C. PAPER CONTRIBUTION

Unlike the majority of the aforementioned power sharing and voltage control schemes, where MG interaction dynamics are completely ignored or assumed as lump-sum external disturbance, a design approach that is more robust to interaction dynamics can be synthesized when interaction dynamics are qualitatively considered in the design process by using the accurate modelling of overall MG dynamics. MG voltage comprehensive control system development using a complete diesel-PV-supercapacitor/converter dynamic model to effectively reject the wide band of interaction dynamics and uncertainties is conceptually designed in this work.

In addition, this study addresses the voltage stability and regulation issues of islanded MGs with high penetration of renewable energy by making use of energy storage devices. As a matter of fact, voltage magnitude deviation can be significantly reduced by using relatively small storage units, as long as saturation conditions are avoided, by dynamically coordinating storage with other generation sources [8]. Similarly, in the work of Kim et al. [21], a cooperative control strategy has been proposed for sharing the reactive power demand among generation sources and fast-acting energy storage systems (ESSs) during the islanded mode of MGs operation. The PCC voltage magnitude can be regulated at the rated value by an adequate reactive-power-balance matching process, provided that the available capacity of the ESSs does not exceed their limitations. In this paper, an $H_\infty$-based multi-variable robust controller for PCC voltage magnitude regulation that copes with load reactive power variations in a diesel-PV-supercapacitor hybrid power generation system operating in stand-alone mode is proposed. A two-level control scheme, where this controller acts as an upper control level and generates references to current controllers placed on a lower level, is detailed, leading to the low order $H_\infty$ controller and facilitating the practical implementation. This paper extends results presented in [22], [23], namely by the following points:

- The influence of system parameters on the dynamic behavior of open-loop system measured outputs is studied by means of a stability analysis method based on Monte Carlo simulation;
- Also, in this work, a systematic design of a multi-variable robust control structure for voltage regulation in stand-alone MGs— with a comprehensive methodology casting the specific engineering demands of MG operation into an $H_\infty$ control formalism— is developed;
- For the considered MG, the performance and robustness of the designed $H_\infty$ controller in the presence of various load disturbances and model uncertainties are analyzed through a series of MATLAB®/Simulink® time-domain simulations.

The remainder of this paper is structured as follows. Section II describes system configuration and design specifications. Mathematical modelling for $H_\infty$ control is provided in Section III. Section IV is dedicated to a stability analysis of open-loop system behavior. Section V is devoted to $H_\infty$ control design along with some closed-loop time-domain simulations performed in the rated operating point. A robust performance analysis of the synthesized $H_\infty$ controller, subject to the uncertainties in the steady-state value of the supercapacitor voltage $V_{sc}$ and the length of the transmission line $l$ connecting the diesel engine generator to the PCC, is investigated in Section VI. Some concluding remarks and perspectives are stated in Section VII.

II. SYSTEM CONFIGURATION AND DESIGN SPECIFICATIONS

A. MICROGRID STRUCTURE

This paper focuses on an islanded MG constituted by a diesel engine generator, a PV source, and a supercapacitor-based energy storage unit. The studied MG setup is illustrated in Fig. 1, where power sources are connected in parallel to a PCC and feed a common load. The diesel engine generator is connected to the PCC via a three-phase step-up transformer.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2021.3119375, IEEE Access
and a transmission line. The PV panel bank is connected to the PCC through a power-electronic conversion system and a three-phase step-up transformer allowing only unidirectional power flow; whereas the supercapacitor-based energy storage unit is connected to the PCC by means of a three-phase step-up transformer, which allows charging or discharging. A three-phase step-down transformer connects the aggregated load to the PCC.

### B. DESIGN SPECIFICATIONS

In stand-alone MGs with high penetration of PV energy sources, the PCC voltage is risky to be unstable or its magnitude variations can be considerable and unacceptable. Properly designed ESSs can then be employed to stabilize the MG and/or improve the PCC voltage transient response by absorbing or injecting instantaneous reactive power subsequent to disturbances. In the considered MG, a control strategy where the ESS and the diesel engine generator have to fulfill distinct roles in voltage control is proposed. Thus, the ESS must act in high frequency by guaranteeing faster recovery of the PCC line-to-line voltage magnitude $U_{\text{PCC}}$ consequent to a load or PV output reactive power variation, i.e., to improve its dynamic performances (e.g., overshoot, response time, steady-state error). On the basis of grid code requirements [24], a template for the PCC line-to-line voltage magnitude deviation in response to a load reactive power step disturbance is defined in Fig. 2. Another reasonable performance specifications on the DC-bus voltage $V_{\text{dc}}$ are given in Fig. 3.
III. MATHEMATICAL MODELLING FOR $H_{\infty}$ CONTROL

The schematic diagram of the MG under consideration for voltage stability analysis is depicted in Fig. 4 [22]. In this work, dynamical equations and equivalent averaged models of power-electronic converters are used to describe the behavior of the studied MG. By linearizing these equations for a given equilibrium state, a linear averaged model can be obtained [23]. It is often more convenient to work with per-unit models of power systems [25], [26]. In particular, when it comes about designing control structures, per-unit models of power systems are generally better conditioned, especially when it is about high-order, possibly multi-scale systems [25], [26]. Moreover, the use of a per-unit system can improve numerical stability of automatic computation methods. Therefore, in this work, the dynamic model of the studied MG is converted from real unit into per unit in order to serve the control design procedure, which is detailed in Section V.

For the considered ESS, the base values for AC-side quantities are first selected and serve to determine the DC-side base values. The linear averaged model of the considered MG using the state variables shown in Fig. 4 can be expressed by the following per-unitized set of equations [23]

\[
\frac{d\Delta V_{eq}}{dt} = \frac{\omega_b}{C_f} \Delta I_{eq} - \frac{\omega_b}{R_{f_p} C_f} \Delta V_{eq} - \frac{\omega_b}{C_f} \Delta I_{eq} - \omega_b \omega_{grid} \Delta V_{rd} - \omega_b V_{eq} \Delta \omega_{grid},
\]

\[
\frac{d\Delta V_{PCCd}}{dt} = \frac{\omega_b}{C_l} \Delta I_{td} + \frac{\omega_b}{R_{t_p} C_l} \Delta V_{PCCd} + \omega_b \omega_{grid} \Delta V_{PCCq} + \omega_b V_{PCCq} \Delta \omega_{grid},
\]

\[
\frac{d\Delta V_{PCCq}}{dt} = \frac{\omega_b}{C_l} \Delta I_{tg} + \frac{\omega_b}{R_{t_p} C_l} \Delta V_{PCCq} + \omega_b \omega_{grid} \Delta V_{PCCd} - \omega_b V_{PCCq} \Delta \omega_{grid},
\]

\[
\frac{d\Delta I_{td}}{dt} = \frac{\omega_b}{L_t} \Delta V_{rd} - \frac{\omega_b R_t}{L_t} \Delta I_{td} - \frac{\omega_b}{L_t} \Delta V_{PCCd} + \omega_b \omega_{grid} \Delta I_{tg} + \omega_b I_{tg} \Delta \omega_{grid},
\]

\[
\frac{d\Delta I_{tg}}{dt} = \frac{\omega_b}{L_t} \Delta V_{eq} - \frac{\omega_b R_t}{L_t} \Delta I_{tg} - \omega_b V_{PCCq} \Delta \omega_{grid},
\]

\[
\frac{d\Delta I_{gd}}{dt} = \frac{\omega_b}{L_g} \Delta V_{eq} - \frac{\omega_b R_g}{L_g} \Delta I_{gd},
\]

\[
\frac{d\Delta I_{gq}}{dt} = \frac{\omega_b}{L_g} \Delta V_{PCCq} - \omega_b \omega_{grid} \Delta I_{gq} + \omega_b V_{PCCq} \Delta \omega_{grid},
\]

\[
\frac{d\Delta I_{s}}{dt} = \frac{\omega_b}{R_{sc} C_s} \Delta V_{sc},
\]

\[
\frac{d\Delta V_{dc}}{dt} = \frac{\omega_b}{C_{dc}} \Delta I_{dc} + \frac{\omega_b}{R_{sc} C_{dc}} \Delta V_{dc} + \frac{\omega_b}{C_{dc}} (\Delta I_{rd} + \Delta I_{rd} + \Delta \beta_d)
\]

\[
+ \frac{\omega_b}{R_{dc} C_{dc}} \Delta V_{dc},
\]

\[
\frac{d\Delta V_{rd}}{dt} = \frac{\omega_b}{C_f} \Delta I_{rd} - \frac{\omega_b}{R_{f_p} C_f} \Delta V_{rd} - \frac{\omega_b}{C_f} \Delta I_{rd} + \omega_b \omega_{grid} \Delta V_{rq} + \omega_b V_{rq} \Delta \omega_{grid},
\]
respectively. Let us define here that $P$ and $Q$ are the direct and quadrature components of the load and pulsation. The external disturbances applied to the system are diesel engine generator output active and reactive power, the supercapacitor output current variation $\Delta I_q$, and the inverter output current variations in $d$- and $q$-axis, respectively, the supercapacitor output current variation $\Delta I_s$, and the inverter output current variations in $d$- and $q$-axis, respectively, $\omega_{grid}$ is the MG pulsation. The external disturbances applied to the system are the direct and quadrature components of the load and PV system output current variations, i.e., $\Delta I_{loadd} = -\Delta I_{PVd}$ and $\Delta I_{loaddq} = -\Delta I_{PQv}$ respectively. It should be noted that this averaged model takes into consideration only the primary voltage control of the diesel engine generator.

Active and reactive power of the sources and load can be computed by the following equations

$$P_s = V_{sc}I_s - R_{sc}I_s^2,$$  \hspace{1cm} (14)

$$P_t = V_{PCCd}I_{td} + V_{PCCq}I_{tq},$$  \hspace{1cm} (15)

$$Q_t = V_{PCCd}I_{td} - V_{PCCq}I_{tq},$$  \hspace{1cm} (16)

$$P_g = V_{PCCd}I_{gd} + V_{PCCq}I_{gq},$$  \hspace{1cm} (17)

$$Q_g = V_{PCCd}I_{gd} - V_{PCCq}I_{gq},$$  \hspace{1cm} (18)

$$P_{PV} = V_{PCCd}I_{PVd} + V_{PCCq}I_{PVq},$$  \hspace{1cm} (19)

$$Q_{PV} = V_{PCCd}I_{PVd} - V_{PCCq}I_{PVq},$$  \hspace{1cm} (20)

$$P_{load} = V_{PCCd}I_{loadd} + V_{PCCq}I_{loaddq},$$  \hspace{1cm} (21)

$$Q_{load} = V_{PCCd}I_{loaddq} - V_{PCCq}I_{loadd},$$  \hspace{1cm} (22)

where $P_s$ is storage device active power, $P_t$ and $Q_t$ are step-up transformer output active and reactive power respectively, $P_g$ and $Q_g$ are diesel engine generator output active and reactive power respectively supplied at the PCC, $P_{PV}$ and $Q_{PV}$ are PV output active and reactive power respectively, and $P_{load}$ and $Q_{load}$ are load active and reactive power respectively. Let us define here that

- For sources: $P > 0$ if they supply active power, $P < 0$ if they absorb active power; $Q > 0$ if they supply reactive power, $Q < 0$ if they absorb reactive power;

- For loads: $P > 0$ if they absorb active power, $P < 0$ if they supply active power; $Q > 0$ if they absorb reactive power, $Q < 0$ if they supply reactive power.

### Table 1. Steady-state Real-unit Values of the Linearized System.

| Variable     | Value $V_{sa}$ | Value $V_{dc}$ | Value $V_{rd}$ | Value $V_{rdq}$ | Value $V_{ts}$ |
|--------------|----------------|----------------|----------------|----------------|----------------|
| Value        | 585 V          | 1000 V         | 400.13 V       | 0 V            |                |
| Value        | 400.13 V       | -2.53 A        | -18.67 A       | 0 A            |                |
| Value        | $\alpha_e$     | $\beta_d$      | $\beta_q$      | $\omega_{grid}$ |
| Value        | 0.585          | 0.4            | -0.0027        | -1.48 kW       |
| Value        | $P_e$          | $Q_e$          |                |                |                |
| Value        | -7.47 kW       | 0 kVar         |                |                |                |

| Variable     | Value $V_{PCCd}$ | Value $V_{PCCq}$ | Value $I_{td}$ | Value $I_{tq}$ | Value $I_{gq}$ | Value $I_{gd}$ | Value $I_{PVd}$ | Value $I_{PVq}$ | Value $I_{loadd}$ | Value $I_{loaddq}$ | Value $f_{grid}$ | Value $\omega_{grid}$ |
|--------------|------------------|------------------|----------------|----------------|----------------|----------------|----------------|----------------|------------------|------------------|----------------|----------------|
| Value        | 20.86 kW         | 552.43 V         | 20.67 kW       | 20 kW          |                |                |                |                | 10.01 A          | -7.48 A           |                |                |
| Value        | 37.6 V           | 20 kW            | -0.87 A        | -0.38 A        |                |                |                |                | -52.54 A         | 7.48 A            |                |                |
| Value        | 59.08 A          | -44.89 A         | 17.49 kW       | 7.66 kVar      |                |                |                |                | 60.08 A          | 7.66 kVar         |                |                |
| Value        | 1049.5 kW        | 724.75 kVar      | 200 kW         | 150 kVar       |                |                |                |                | 1200 kW          | 900 kVar          |                |                |

For loads: $P > 0$ if they absorb active power, $P < 0$ if they supply active power; $Q > 0$ if they absorb reactive power, $Q < 0$ if they supply reactive power.

The equations which express power balance at the PCC are given by

$$P_t + \left( P_g - \frac{V_{PCC}^2}{R_p} \right) + P_{PV} - P_{load} = 0, \quad (23)$$

$$Q_t + \left( Q_g - \frac{V_{PCC}^2}{X_p} \right) + Q_{PV} - Q_{load} = 0. \quad (24)$$

Subscript $e$ indicates the steady-state equilibrium point.

Steady-state real-unit values are given in Table 1. Note that the $V_{sa}$ value chosen for $H_{\infty}$ control design corresponds here to the midrange between the minimum and maximum supercapacitor voltages. In this case, $V_{sa} = 585$ V, where 390 V ($SoC_{sc} = 25\%$) is taken as the minimum $V_{sc}$ and 780 V ($SoC_{sc} = 100\%$) as the maximum $V_{sc}$. The minimum value of the transmission line length $l$ is chosen for $H_{\infty}$ control design. In this case, $l = l_{min} = 1$ km, where 10 km has been chosen as the maximum value of $l$ in order to ensure the voltage drop on the transmission line at the high-voltage level of 20 kV being less than or equal to the admissible limit of 8% of the rated voltage.

### IV. OPEN-LOOP SYSTEM BEHAVIOR

Based on Monte Carlo simulation, a stability analysis is detailed in this section in order to emphasize the influence of system parameters on the dynamic behavior of open-loop system measured outputs.

With MG parameter values, the zeros of the open-loop sensitivity functions $\Delta V_{PCCd}/(\Delta I_{loadd} - \Delta I_{PVd})$ or $\Delta V_{PCCq}/(\Delta I_{loaddq} - \Delta I_{PVq})$ and $\Delta V_{PCCd}/(\Delta I_{loadd} - \Delta I_{PVd})$ or $\Delta V_{PCCq}/(\Delta I_{loaddq} - \Delta I_{PVq})$ of the linearized system (1)–(13) are given in Table 2. The transfer function $\Delta V_{PCCd}/(\Delta I_{loadd} - \Delta I_{PVd})$ or
TABLE 2. Zeros of the Open-loop Sensitivity Functions of the Linearized System.

| Sensitivity function | \( \Delta V_{PCCq} / (\Delta I_{loadq} - \Delta I_{PVq}) \) or \( \Delta V_{PCCd} / (\Delta I_{loadd} - \Delta I_{PVd}) \) |
|----------------------|-------------------------------------------------------------------------------------------------------------------|
| Zero                 | \[ z_1 = -306.3 \pm 1703.6j, z_2 = -329.9 \pm 12105j, z_5 = -265.8 \pm 9632.9j, z_7 = -19.2 \pm 316.4j, z_9 = -96.6 \pm 4.8 \pm 27.9j, \] |
| Zero                 | \[ z_{10} = -80.0 \pm 27.9j, z_{11} = -96.6 \pm 4.8 \pm 27.9j, \] |
The article discusses the open-loop sensitivity function and its impact on the load current variation. Specifically, it analyzes the conjugate zero pairs $z_{k,k+1}$ of the left-half-plane complex conjugate, which is approximately equal to zero, i.e., $\Delta I_{\text{load}} \approx 0$. This reduces the dynamic effects in the load current variation, especially for $V_{\text{PCC}}$ and $U_{\text{PCC}}$. The diagrams illustrate the sensitivity functions for different load variations, showing how the open-loop sensitivity impacts the system's response.

![Diagram](https://example.com/diagram.png)

**Figure 5.** Values $\gamma_{k,k+1} = |\text{Im}(z_{k,k+1})/|\text{Re}(z_{k,k+1})|$ of the left-half-plane complex conjugate zero pairs $z_{k,k+1}$ of the open-loop sensitivity function $\Delta V_{\text{PCCd}}/(\Delta I_{\text{loadd}} - \Delta I_{\text{PVd}})$ or $\Delta V_{\text{PCCq}}/(\Delta I_{\text{loadq}} - \Delta I_{\text{PVq}})$. This work is licensed under a Creative Commons Attribution 4.0 License. For more information, see https://creativecommons.org/licenses/by/4.0/
function $\Delta V_{PCC_d}/(\Delta I_{\text{load}q} - \Delta I_{PVq})$ compared to the contributions of those of the left-half-plane complex conjugate zero pairs of the other transfer functions. As can be observed from Fig. 6, the values $\gamma_{7,8}$ are the highest among these values $\gamma_{k,k+1}$, which means that $\gamma_{7,8}$ have the most influence on the dynamic effects displayed in $\Delta U_{PCC}$ in comparison to the influences of the other values $\gamma_{k,k+1}$. The minimum sample of $|\text{Im}(z_{7,8})/\text{Re}(z_{7,8})|$ is equal to 6.8 at the 623rd sampling time (i.e., $\gamma_{623} = 6.8$), which corresponds to the 623rd sample set of the input variables $X_{623}$, where their relative deviations from the rated values are given in Table 3. This desired parameter combination of the studied whole MG allows to minimize the oscillation amplitude in $\Delta U_{PCC}$.

V. $H_\infty$ CONTROL DESIGN

This section details a robust control design approach used to satisfy the dynamic specifications in Subsection II-B. Here the basic idea is to consider the current variations $\Delta I_s$, $\Delta I_{rd}$, and $\Delta I_{eq}$ in (11)–(13) as control inputs for the linearized
system (1)–(10), which should result from the disturbance rejection requirement $\Delta I_{\text{loadd}} - \Delta I_{\text{PVd}}$ and $\Delta I_{\text{loadq}} - \Delta I_{\text{PVq}}$. A hierarchical two-level control strategy, where the outer control loop deals with output regulation imposing low-frequency dynamics (e.g., $\Delta V_{dc}$, $\Delta V_{PCCd}$, $\Delta V_{PCCq}$) and the inner loop concerns current reference tracking of high-frequency dynamics (e.g., $\Delta I_s$, $\Delta I_{rd}$, $\Delta I_{rq}$), is adopted for voltage control. The validity and effectiveness of the proposed robust control strategy are demonstrated through a series of closed-loop time-domain simulations carried out in MATLAB®/Simulink®.

**A. GLOBAL CONTROL CONFIGURATION BLOCK DIAGRAM**

As can be seen in most energy management systems and according to control objectives, it is preferred – from the application point of view – to consider power sources acting as current sources [28]. Therefore, a cascaded two-level control structure – where an $\mathcal{H}_\infty$-based multi-variable controller acts as an upper control level and provides references to classical PI-based current tracking controllers.
placed on a low control level – is developed to ensure the previous dynamic specifications. The proposed global control structure is illustrated in Fig. 7 [23]. From the control viewpoint, note that \( V_{dc} \) appears as a gain in the low-level current control loops, such that \( V_{dc} \) being constant is an assumption making that classical PI controllers to be sufficient as this level (i.e., if \( V_{dc} \) is not kept within admissible limits, classical PI controllers would not be able to ensure stability and the required tracking performance).

**B. CURRENT CONTROL LEVEL**

The current variations \( \Delta I_s, \Delta I_{rd}, \text{ and } \Delta I_{rq} \) must be controlled and prevented from exceeding admissible limits. Current control loops have fast closed-loop dynamics compared to the \( H_\infty \) control loop. 

\( \Delta I_{rq} \) in (12) appears as a high-frequency perturbation; the same applies for \( \Delta I_{rd} \) in (13). Their reciprocal influence is significantly reduced by the \( d-q \) decoupling structure. \( \Delta V_{sc}, \Delta V_{dc}, \Delta V_{rd}, \text{ and } \Delta V_{rq} \) in (11), (12), and (13) are regarded as low-frequency perturbations that are rejected by the upper-level control. \( \omega_{grid} \) is considered a time-invariant parameter in voltage regulation, i.e., \( \omega_{grid} = \omega_{gridc} \) or \( \Delta \omega_{grid} = 0 \) (the MG frequency is supposed to be well-regulated at its nominal value of 50 Hz). Hence, first-order transfer functions relating the current variations with the duty ratio variations (inner plants) are computed straightforwardly from (11), (12), and (13) and PI controllers with prefilters are effectively used to ensure both the zero steady-state error and the desired closed-loop bandwidth. Current reference variations \( \Delta I_{s}^{ref}, \Delta I_{rd}^{ref}, \text{ and } \Delta I_{rq}^{ref} \) are generated by the outer \( H_\infty \) control loop.

**C. \( H_{\infty} \) CONTROL LEVEL**

1) Linear State-space Representation

The current variations \( \Delta I_s, \Delta I_{rd}, \text{ and } \Delta I_{rq} \) are controlled by the very-fast-dynamic control loops tracking the current reference variations generated by the \( H_{\infty} \) controller, therefore, the outer \( H_{\infty} \) plant “sees” \( \Delta I_s \equiv \Delta I_{s}^{ref}, \Delta I_{rd} \equiv \Delta I_{rd}^{ref}, \text{ and } \Delta I_{rq} \equiv \Delta I_{rq}^{ref} \). The supercapacitor voltage \( V_{sc} \) is considered a time-invariant parameter in the \( H_{\infty} \) controller synthesis, i.e., \( \Delta V_{sc} = 0 \) (its dynamic equation \( \Delta V_{sc} = f(\Delta V_{sc}, \Delta I_s) \) in (1) being neglected). The diesel engine generator output voltage is supposed to be well regulated at its setpoint value, i.e., \( \Delta V_{g} = 0 \) and \( \Delta V_{g} = 0 \). Thus, the state-space form of the linear system (2)–(10) can be written as follows (with the terms \( \omega_b \frac{C_{dc}}{I_{sd}} \Delta \omega_c, \omega_b \frac{C_{dc}}{I_{rd}} \Delta \beta_d, \text{ and } \omega_b \frac{C_{dc}}{I_{rd}} \Delta \beta_q \) in (2) being neglected in the outer \( H_{\infty} \) control loop design)

\[
\begin{align*}
\Delta \dot{x} &= A \Delta x + B_1 \Delta u + B_2 \Delta w \\
\Delta y &= C \Delta x + D_1 \Delta u + D_2 \Delta w,
\end{align*}
\]

where \( \Delta x = [\Delta V_{dc} \quad \Delta V_{rd} \quad \Delta V_{rq} \quad \Delta V_{PCCd} \quad \Delta V_{PCCq}]^T \) is the state vector, \( \Delta u = [\Delta I_{s}^{ref} \quad \Delta I_{rd}^{ref} \quad \Delta I_{rq}^{ref}]^T \) the control input vector, \( \Delta w = [\Delta I_{load} \quad \Delta I_{PCCd} \quad \Delta I_{load} \quad \Delta I_{PV}]^T \) the disturbance input vector, and \( \Delta y = [\Delta V_{dc} \quad \Delta V_{PCCd} \quad \Delta V_{PCCq}]^T \) the measured output vector. Matrices \( A, B_1, B_2, C, D_1, \text{ and } D_2 \) in (25) are given as follows.

2) Analysis of Open-loop Zeros

With MG parameter values, the zeros of the open-loop sensitivity functions \( \Delta V_{PCCd}/(\Delta I_{load} - \Delta I_{PV}) \) or \( \Delta V_{PCCq}/(\Delta I_{load} - \Delta I_{PV}) \) and \( \Delta V_{PCCd}/(\Delta I_{load} - \Delta I_{PV}) \) or \( \Delta V_{PCCq}/(\Delta I_{load} - \Delta I_{PV}) \) of the linear system (25) are given in Table 4.

**TABLE 4. Zeros of the Open-loop Transfer Functions of the Linearized System (25).**

| Sensitivity function | \( \Delta V_{PCCd}/(\Delta I_{load} - \Delta I_{PV}) \) or \( \Delta V_{PCCq}/(\Delta I_{load} - \Delta I_{PV}) \) |
|---------------------|----------------------------------------------------------------------------------|
| Zero                | \( z_1 = -309.2 \pm 16928j \) \( z_2 = 267.3 \pm 9067.6j \) \( z_3 = -99.4 \) |

| Sensitivity function | \( \Delta V_{PCCd}/(\Delta I_{load} - \Delta I_{PV}) \) or \( \Delta V_{PCCq}/(\Delta I_{load} - \Delta I_{PV}) \) |
|---------------------|----------------------------------------------------------------------------------|
| Zero                | \( z_1 = 13665 \) \( z_2 = 13529 \) \( z_3 = -3706.6 \pm 10856j \) \( z_5 = 3191.2 \pm 10859j \) |

| Transfer function   | \( \Delta V_{PCCd}/(\Delta I_{load} - \Delta I_{PV}) \) or \( \Delta V_{PCCq}/(\Delta I_{load} - \Delta I_{PV}) \) |
|---------------------|----------------------------------------------------------------------------------|
| Zero                | \( z_1 = -309.5 \pm 16906j \) \( z_3 = -267.8 \pm 9103.8j \) \( z_5 = -99.6 \) |

| Transfer function   | \( \Delta V_{PCCd}/(\Delta I_{load} - \Delta I_{PV}) \) or \( \Delta V_{PCCq}/(\Delta I_{load} - \Delta I_{PV}) \) |
|---------------------|----------------------------------------------------------------------------------|
| Zero                | \( z_1 = -297.4 \pm 13018j \) \( z_3 = -6969.9 \) \( z_4 = 6662.8 \) |

The zeros of the open-loop transfer functions \( \Delta V_{PCCd}/(\Delta I_{load} - \Delta I_{PV}) \) or \( \Delta V_{PCCq}/(\Delta I_{load} - \Delta I_{PV}) \) and \( \Delta V_{PCCd}/(\Delta I_{load} - \Delta I_{PV}) \) or \( \Delta V_{PCCq}/(\Delta I_{load} - \Delta I_{PV}) \) of the linear plant \( G(s) \) in (25) are also given in Table 4. The measured output variation \( \Delta V_{PCCd}/(\Delta I_{load} - \Delta I_{PV}) \) exhibits non-minimum phase behavior in response to \( \Delta I_{load} \) or \( \Delta I_{rd} \) respectively, as shown by the associated transfer function \( \Delta V_{PCCd}/(\Delta I_{load} - \Delta I_{PV}) \) or \( \Delta V_{PCCq}/(\Delta I_{load} - \Delta I_{PV}) \) respectively, having a right-half-plane zero \( z_4 \). This results in a non-minimum phase behavior in \( \Delta U_{PCC} \). Suppose that the system operates in a closed loop with an \( H_{\infty} \) controller. It will not be possible to compensate exactly this zero in closed loop, because the controller itself will be unstable. In this case, the control requires specific solutions that preserve control quality, meanwhile avoiding controller instabilities. An analysis of left-half-plane complex conjugate zero pairs similar to the one carried.
out in Section IV allows to conclude that there is another dynamic effect (e.g., oscillation) exhibited in $\Delta U_{PCC}$ in response to the control input variation $\Delta I_{rd}^{ref}$ or $\Delta I_{cq}^{ref}$. The design procedure of an $\mathcal{H}_\infty$ controller hereafter proposed will take into account influences of these dynamic effects.

We may sometimes use the term “transmission zeros” to distinguish them from the zeros of the elements of the transfer matrix $G(s)$. The “transmission zeros” of the linear plant $G(s)$ in (25) are equal to $z_{1,2} = -309.5 \pm 1723.4j$, $z_{3,4} = -309.5 \pm 16605j$, $z_{5,6} = -268 \pm 9403.7j$, $z_{7,8} = -268 \pm 8775.4j$, and $z_{9,10} = -99.1 \pm 314.2j$. Influence of these left-half-plane complex conjugate zero pairs, usually corresponding to “overshoots” (or oscillations) in the time response, can be compensated by control action.

3) Control Configuration in the $P - K$ Form
Multi-variable $\mathcal{H}_\infty$ control design for the linear system (25) is cast into the formalism in Fig. 8. A linear approach using the small-signal state-space model of the system is here adopted. In this case, focus is on rejecting disturbance due to the direct and quadrature component variations of the load and PV system output currents. Moreover, it is supposed that the measurement noise is relatively small. The tracking problem is not considered and therefore the $S/KS$ mixed-sensitivity optimization problem in the standard regulation form must typically be solved [27]. The inclusion of $KS$ as a mechanism for limiting the size and bandwidth of the $\mathcal{H}_\infty$ controller, and hence the control energy used, is also important. The key of this design method is the appropriate definition of weighting functions $W_{per f}(s)$ to guarantee the performance specifications and weighting functions $W_{uu}(s)$ to translate the constraints imposed to the control inputs.

$$
A = \begin{bmatrix}
-\frac{\omega_b}{R_{dc}C_{dc}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\omega_b}{R_{f_p}C_f} & \omega_b\omega_{grid,s} & 0 & 0 & -\frac{\omega_b}{C_f} & 0 & 0 & 0 \\
0 & 0 & -\frac{\omega_b}{R_{f_p}C_f} & 0 & 0 & \omega_b\omega_{grid,s} & 0 & 0 & -\omega_b \\
\omega_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \omega_b & \frac{L_t}{\omega_b} & 0 & 0 & -\frac{\omega_b}{L_t} & 0 & 0 & 0 \\
0 & 0 & \omega_b & \frac{L_t}{\omega_b} & 0 & 0 & -\frac{\omega_b}{L_t} & 0 & 0 \\
0 & 0 & 0 & \omega_b & \frac{L_g}{\omega_b} & 0 & 0 & -\frac{\omega_b}{L_g} & 0 \\
0 & 0 & 0 & 0 & 0 & \omega_b & \frac{L_g}{\omega_b} & 0 & -\omega_b \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_b \\
\end{bmatrix}
$$

$$
B_1 = \begin{bmatrix}
\frac{\omega_b}{C_{dc}} \alpha_{ce} & -\frac{\omega_b}{C_{dc}} \beta_{de} & -\frac{\omega_b}{C_{dc}} \beta_{ae} \\
0 & -\frac{\omega_b}{C_{dc}} \alpha_{de} & -\frac{\omega_b}{C_{dc}} \beta_{de} \\
0 & 0 & \frac{\omega_b}{C_f} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix},
B_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

$$
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
D_1 = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix},
D_2 = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}.
$$
are chosen as performance outputs. Their vector is noted $\Delta z$ in Fig. 8.

The $S/KS$ mixed-sensitivity optimization of the $\mathcal{H}_\infty$ control framework consists in finding a stabilizing controller which minimizes the norm $\left\| W_{\text{perf}S} W_u K \right\|_{\infty}$.

4) Weighting Functions Selection

Performance specifications on the direct and quadrature components of the PCC voltage variation, $\Delta V_{\text{PCC}d}$ and $\Delta V_{\text{PCC}q}$ respectively, in response to a load reactive power step disturbance, illustrated in Fig. 9 and Fig. 10 respectively, are deduced from the grid code requirements on the PCC line-to-line voltage magnitude variation $\Delta U_{\text{PCC}}$, where $U_{\text{PCC}} = \sqrt{V^2_{\text{PCC}d} + V^2_{\text{PCC}q}}$. It should be noted here that since $V_{\text{PCC}q} \ll V_{\text{PCC}d}$, meaning that $U_{\text{PCC}} \approx V_{\text{PCC}d}$ or $\Delta U_{\text{PCC}} \approx \Delta V_{\text{PCC}d}$, therefore the performance template for $\Delta V_{\text{PCC}d}$ is chosen to be quasi-identical to that for $\Delta U_{\text{PCC}}$.

The choice of linear time-invariant weighting functions is the key to deal with the performance requirements. The DC-bus voltage variation $\Delta V_{dc}$ and the direct and quadrature components of the PCC voltage variation, $\Delta V_{\text{PCC}d}$ and $\Delta V_{\text{PCC}q}$ respectively,
\( \Delta V_{PCCd} \) respectively, are bounded by first-order weighting functions \( W_{\text{perf}}(s) \) of the following form [27]

\[
\frac{1}{W_{\text{perf}}(s)} = \frac{s + \omega_b A_c}{s/M_s + \omega_b}.
\]  

The function \( 1/W_{\text{perf}}(s) \) can be representative of time-domain response specifications, where the high-frequency gain \( M_s \) has an impact on the system overshoot, whereas the desired response time is tuned by the cut-off frequency \( \omega_b \) and the steady-state error can be limited by appropriate choice of the low-frequency gain \( A_c \) [29]. In order to satisfy the tight control requirement at low frequencies in the presence of the right-half-plane zeros of the plant, the selection of the value \( \omega_b \) of the function \( 1/W_{\text{perf}}(s) \) is of great importance. Let \( G(s) \) be a multiple input multiple output (MIMO) plant with one right-half-plane zero at \( s = z \) and \( W_{\text{perf}}(s) \) be a scalar weight, then closed-loop stability is ensured only if \( \| W_{\text{perf}}(s) S(s) \|_{\infty} \geq \| W_{\text{perf}}(z) \|_{\infty} \) [27]. For the design problem, if the \( H_\infty \) controller meets the requirements, then \( \| W_{\text{perf}}(s) S(s) \|_{\infty} \leq 1 \). Therefore, with \( W_{\text{perf}}(s) \) being given in (26), a necessary condition for closed-loop stability is \( |W_{\text{perf}}(z)| \leq 1 \), which corresponds to [27]

\[
\frac{z/M_s + \omega_b}{z + \omega_b A_c} \leq 1.
\]  

Consider the case when \( z \) is real and positive, then (27) is equivalent to

\[
\omega_b \left(1 - A_c \right) \leq z \left(1 - \frac{1}{M_s}\right).
\]

The power active injection or absorption of the ESS is controlled via \( I_s \). The DC-bus voltage and the direct and quadrature components of the PCC voltage respectively are regulated via \( I_{rd} \) and \( I_{rq} \). Thus, the supercapacitor output current reference variation \( \Delta I_{rd}^{\text{ref}} \) and the direct and quadrature components of the inverter output current reference variation, \( \Delta I_{rq}^{\text{ref}} \) and \( \Delta I_{rq}^{\text{ref}} \) respectively, are bounded by the following first-order weighting functions [27]

\[
\frac{1}{W_u(s)} = \frac{A_u s + \omega_{bc}}{s + \omega_{bc} M_u}.
\]  

The parameters of the weighting functions \( W_{\text{perf}}(s) \) and \( W_u(s) \) are given in Table 5 and Table 6 respectively [23], where \( 1/T_{d_0} \) and \( 1/T_{d_{rdq}} \) are \( \Delta I_s \) and \( \Delta I_{rdq} \) inner imposed closed-loop bandwidths, respectively. Let us note here that the selection of the values \( \omega_{bc} \) and \( \omega_b \) of the functions \( 1/W_{\text{perf}2}(s) \) and \( 1/W_{\text{perf}3}(s) \) respectively satisfies the condition (28), which means that closed-loop stability is implicitly ensured in the presence of the right-half-plane zero \( z_4 \) of the open-loop transfer function \( \Delta V_{PCCd}/\Delta I_{rq}^{\text{ref}} \).

### Table 5. Weighting Function Parameters \( W_{\text{perf}}(s) \) [23].

| Parameter | \( \Delta V_{PCCd} \) | \( \Delta V_{PCCq} \) | \( \Delta V_{PCCq} \) |
|-----------|----------------|----------------|----------------|
| \( M_u \) | 0.25 \( \mu \text{A} \) | 0.85 \( \mu \text{A} \) | 0.85 \( \mu \text{A} \) |
| \( A_c \) | 0.05 \( \mu \text{A} \) | 0.05 \( \mu \text{A} \) | 0.05 \( \mu \text{A} \) |
| \( \omega_{bc} \) | 1/ \( 40T_{d_{rdq}} \) | 1/ \( 50T_{d_{rdq}} \) | 1/ \( 50T_{d_{rdq}} \) |

### Table 6. Weighting Function Parameters \( W_u(s) \) [23].

| Parameter | \( \Delta I_{rd}^{\text{ref}} \) | \( \Delta I_{rq}^{\text{ref}} \) | \( \Delta I_{rq}^{\text{ref}} \) |
|-----------|----------------|----------------|----------------|
| \( M_u \) | 0.05 \( \mu \text{A} \) | 0.05 \( \mu \text{A} \) | 0.05 \( \mu \text{A} \) |
| \( A_u \) | 0.01 \( \mu \text{A} \) | 0.01 \( \mu \text{A} \) | 0.02 \( \mu \text{A} \) |
| \( \omega_{bc} \) | 1/ \( 10T_{d_0} \) | 1/ \( 10T_{d_{rdq}} \) | 1/ \( 10T_{d_{rdq}} \) |
or \( \Delta V_{PCCq}/\Delta I_{rd} \) of the linear plant \( G(s) \) in (25).
In addition, the frequencies \( |z_{k,k+1}| \) of the left-half-plane complex conjugate zero pairs \( z_{k,k+1} \) of the transfer functions \( \Delta V_{PCCd}/\Delta I_{rd} \) or \( \Delta V_{PCCq}/\Delta I_{rd} \) and \( \Delta V_{PCCd}/\Delta I_{rq} \) or \( \Delta V_{PCCq}/\Delta I_{rq} \) are all higher than the frequency \( |z_4| \) of \( z_4 \), therefore the oscillation exhibited in \( \Delta U_{PCC} \) in response to the control input variation \( \Delta I_{rd} \) or \( \Delta I_{rq} \) are implicitly filtered.

5) \( \mathcal{H}_\infty \) Controller Synthesis

The robust control toolbox in the MATLAB\textsuperscript{®} software environment is used to design a full-order \( \mathcal{H}_\infty \) controller based on the system modelling and the selected weighting functions. The obtained result corresponds to the minimization of the norm

\[
\|W_{perf}S\|_{\mathcal{W}_q/\mathcal{K}S_{\mathcal{H}_\infty}} < \gamma. \tag{30}
\]

Design procedure may yield unstable controllers. Therefore, the value \( \gamma_{\text{min}} \) must be slightly increased in this case to relax the constraint on the control and lead to stable controllers. The linear matrix inequalities (LMI) algorithm under the MATLAB\textsuperscript{®} software environment yields a solution after five iterations for a multi-variable full-order \( \mathcal{H}_\infty \) controller which has 15 states with a conditioning value of \( \gamma = 160.87 \) and an \( \mathcal{H}_\infty \) norm = 40.71. This optimization quality allows the synthesized \( \mathcal{H}_\infty \) controller to guarantee stable and robust performance in case of a load step of 5\% of \( I_{loadq} \), (i.e., \(-2.25 \) A) in the system, which is demonstrated in the sequel.

For the considered system shown in Fig. 8, sensitivity functions \( S_{11}(s) \), \( S_{12}(s) \), \( S_{21}(s) \), \( S_{22}(s) \), \( S_{31}(s) \), \( S_{32}(s) \) and complementary sensitivity functions \( KS_{11}(s) \), \( KS_{12}(s) \), \( KS_{21}(s) \), \( KS_{22}(s) \), \( KS_{31}(s) \), \( KS_{32}(s) \) are defined as follows

\[
S_{11}(s) = \frac{\Delta V_{dc}}{\Delta I_{load} - \Delta I_{PVd}}, \tag{31}
\]

\[
S_{12}(s) = \frac{\Delta I_{loadq} - \Delta I_{PVq}}{\Delta V_{PCCd}}, \tag{32}
\]

\[
S_{21}(s) = \frac{\Delta I_{load} - \Delta I_{PVd}}{\Delta V_{PCCd}}, \tag{33}
\]

\[
S_{22}(s) = \frac{\Delta V_{PCCq}}{\Delta I_{loadq} - \Delta I_{PVq}}, \tag{34}
\]

\[
S_{31}(s) = \frac{\Delta I_{load} - \Delta I_{PVd}}{\Delta V_{PCCq}}, \tag{35}
\]

\[
S_{32}(s) = \frac{\Delta V_{PCCq}}{\Delta I_{loadq} - \Delta I_{PVq}}, \tag{36}
\]

\[
KS_{11}(s) = \frac{\Delta I_{load} - \Delta I_{PVd}}{\Delta I_{rd}}, \tag{37}
\]

\[
KS_{12}(s) = \frac{\Delta I_{loadq} - \Delta I_{PVq}}{\Delta I_{rd}}, \tag{38}
\]

\[
KS_{21}(s) = \frac{\Delta I_{load} - \Delta I_{PVd}}{\Delta I_{rq}}, \tag{39}
\]

\[
KS_{22}(s) = \frac{\Delta I_{loadq} - \Delta I_{PVq}}{\Delta I_{rq}}, \tag{40}
\]

D. MODEL VALIDATION AND EFFECTIVENESS OF THE PROPOSED ROBUST CONTROL STRATEGY

A series of MATLAB\textsuperscript{®}/Simulink\textsuperscript{®} closed-loop time-domain simulations using the topological model, as well as the nonlinear and linear averaged models are performed to show the effectiveness of the proposed control approach. The topological model refers here to the complete model of the ESS considering the ideal PWM switching functions of the chopper and of the three-phase inverter as control inputs. Without loss of generality, a load step change of \(+5\%\) of the quadrature component of the load rated current \( I_{loadq} \), (i.e., \(-2.25 \) A) at \( t = 1 \) s is applied as a disturbance, whereas the direct component of the load current variation is regarded
to be approximately equal to zero, i.e., $\Delta I_{\text{load,dd}} \approx 0$, which corresponds to a load step change of +5% of the load rated reactive power (i.e., +45 kVAR). The supercapacitor voltage $V_{sc}$, initially regarded as a time-invariant parameter in the $H_\infty$ controller synthesis, is considered now as a time-variant one, i.e., $\Delta V_{sc} \neq 0$ (its dynamic equation $\dot{V}_{sc} = f(V_{sc}, I_s)$ being taken into account in the simulation models).

Fig. 11, Fig. 12, Fig. 13, and Fig. 14 present a comparison of topological and averaged modelling for the time-domain responses of the PCC line-to-line voltage magnitude $U_{PCC}$, the direct and quadrature components of the PCC voltage $V_{PCCd}$ and $V_{PCCq}$, and the DC-bus voltage $V_{dc}$ respectively. The results show a good concordance with both of these models despite high-frequency oscillations seen in the topological model. Therefore, the averaged models themselves are validated when compared to the topological model results. It can be observed from Fig. 12 and Fig. 13 that the desired time-domain performances of $V_{PCCd}$ and $V_{PCCq}$ are successfully achieved in terms of overshoot, response time, and steady-state error with respect to the choice of the weighting functions $W_{perf_{d}}(s)$ and $W_{perf_{q}}(s)$ respectively. Fig. 11 shows that the dynamic performance specification imposed on $U_{PCC}$ in Fig. 2 is well respected. The system is also always stable. It should be noted here that, since focus is on primary voltage control only, the PCC voltage error cancellation cannot be obtained, which means that the PCC voltage cannot be brought back to its initial steady-state value after the disturbance. It can be seen from Fig. 14 that the desired time-domain performances of $V_{dc}$ corresponding to the tuning of the weighting function $W_{perf_{1}}(s)$ are satisfied. Hence, the synthesized $H_\infty$ controller guarantees the desired performance specifications.

Fig. 15, Fig. 16, and Fig. 17 compare the time-domain responses of the supercapacitor output current $I_s$ and the
direct and quadrature components of the inverter output current $I_{rd}$ and $I_{rq}$, respectively, obtained with both topological and averaged models. The results show a good agreement between these models, except for high-frequency oscillations observed in the topological model. Therefore, the averaged models precisely represent the dynamics of the ESS in that frequency bandwidth that is interesting for control purposes. Moreover, it can be seen that the admissible limit is guaranteed for these currents, which means that their imposed dynamic performance specifications are satisfied.

For the sake of simplicity and without loss of generality, the nonlinear averaged model will from now on be used for time-domain simulation.

The time-domain responses of the active power variations of the sources and load are illustrated in Fig. 18. It can be observed that, due to large variations in $U_{PCC}$, $V_{PCCd}$, and $V_{PCCq}$ within a 10-ms interval consequent to the disturbance, the active power of the diesel engine generator system output $P_g$ and the load $P_{load}$ fluctuate considerably, however they return nearly to their initial values after 10 ms of disturbance; whereas the output active power of the ESS step-up transformer $P_t$ and the PV system $P_{PV}$ present slight fluctuations around their initial values within this interval.

Fig. 19 shows the time-domain responses of the reactive power variations of the sources and load. In a similar way, one can see that the reactive power of the diesel engine generator system output $Q_g$ and the load $Q_{load}$ present dramatic fluctuations resulting from large variations in $U_{PCC}$, $V_{PCCd}$, and $V_{PCCq}$ within a 10-ms interval subsequent to the disturbance; whereas the output reactive power of the ESS step-up transformer $Q_t$ and the PV system $Q_{PV}$ fluctuate slightly around their initial values within this interval. Moreover, the ESS participation in primary voltage control has reduced the reactive power variation of the diesel generator around the steady-state point after the disturbance, as shown in Fig. 19.

VI. ROBUST PERFORMANCE ANALYSIS

This section details a robust performance analysis of the $H_\infty$ controller, which was designed in the previous section while the system parameters are fixed at their rated values. First, the uncertainties in the steady-state value of the supercapacitor state of charge $SoC_{scu}$ (or supercapacitor voltage $V_{scu}$) and the length of the transmission line $l$ connecting the diesel engine generator to the PCC are considered. Then, a sensitivity analysis is performed so as to determine the maximum variation ranges of the values $SoC_{scu}$ (or $V_{scu}$) and $l$ for which the imposed closed-loop performances are guaranteed. Next, a series of MATLAB®/Simulink® closed-loop time-domain simulations are presented to validate the controller robustness and performance in the presence of various load disturbances and the uncertainties in $SoC_{scu}$ and $l$. 

FIGURE 17. Time-domain response of the $q$-component of the inverter output current $I_{rq}$ under a small load step disturbance of $+5\%$ of the load rated reactive power ($+45\text{kVAr}$) with respect to the rated operating point.

FIGURE 18. Time-domain response of the active power variations of the sources and load under a small load step disturbance of $+5\%$ of the load rated active power ($+45\text{kVAr}$) with respect to the rated operating point.

FIGURE 19. Time-domain response of the reactive power variations of the sources and load under a small load step disturbance of $+5\%$ of the load rated reactive power ($+45\text{kVAr}$) with respect to the rated operating point.
A. PARAMETRIC UNCERTAINTIES

In the case of the studied hybrid system, uncertainties could be associated first with the parameters of the ESS but could also be representative of the variation in the steady-state value of the supercapacitor state of charge SoC_{sc} (or supercapacitor voltage V_{sc}). They could also represent changes in the parameters of the diesel engine generator system (e.g., inductor L_{tg} and resistor R_{tg} of the step-up transformer; inductor L_{t}, resistor R_{t}, capacitor C_{t}, and parallel resistor R_{tp} of the transmission line). This results in changes in the steady-state operating point of the system.

Let us note that only the uncertainties in SoC_{sc} (or V_{sc}) and L_{t}, R_{t}, C_{t}, and R_{tp} (i.e., due to the change in the length of the transmission line l) are taken into account in the below sensitivity analysis. In order to ensure reliable, efficient, and safe operation of the supercapacitor and prolong its lifespan, an admissible range of [25, 100] \% should practically be required for SoC_{sc}. The limited variation range, corroborated with the well-known equation for SoC estimation, points out that the V_{sc} value must be controlled between 390 V and 780 V. The midrange between these two values, i.e., V_{sc} = 585 V, is a suitable choice for designing the nominal H_{\infty} controller whose robustness is further assessed. Moreover, the minimum value of the transmission line length l = l_{min} = 1 km is chosen for H_{\infty} control design, where 10 km is the maximum value of l. This maximum value is limited at 10 km so as to ensure the voltage drop on the transmission line at the high-voltage level of 20 kV being less than or equal to the admissible limit of 8% of the rated voltage.

B. SENSITIVITY ANALYSIS

This subsection is devoted to the sensitivity analysis of robust performance of the synthesized H_{\infty} controller to answer the question whether the closed-loop system remains robust or not (from a performance point of view) to a given...
parametric uncertainty level in the steady-state value of the supercapacitor voltage $V_{\text{sc}}$ around its design value, 585 V, or to the variation in the transmission line length $l$ from 1 km to 10 km.

Without loss of generality, MATLAB®/Simulink® closed-loop time-domain simulations using the nonlinear averaged model are performed with a load step change of $+5\%$ of the load rated reactive current $I_{\text{load}dq}$, (i.e., $-2.25 \text{ A}$) at $t = 1 \text{ s}$, whereas the direct component of the load current variation is regarded to be approximately equal to zero, i.e., $\Delta I_{\text{load}d} \approx 0$, which corresponds to a load step change of $+5\%$ of the load rated reactive power (i.e., $+45 \text{ kVAR}$). The supercapacitor voltage $V_{\text{sc}}$ appears as a time-variant parameter (i.e., $\Delta V_{\text{sc}} \neq 0$) in the simulation model. The initial value of $SoC_{\text{sc}}$ is varied between $25\%$ and $100\%$ in simulation. The length of the transmission line $l$ is changed between 1 km and 10 km.

1) Uncertainty in the Supercapacitor State of Charge $SoC_{\text{sc}}$

Fig. 20, Fig. 21, Fig. 22, and Fig. 23 present the time-domain responses of the PCC line-to-line voltage magnitude $U_{\text{PCC}}$, the direct and quadrature components of the PCC voltage $V_{\text{PCC}d}$ and $V_{\text{PCC}q}$, and the DC-bus voltage $V_{\text{dc}}$, respectively, taking into account the uncertainty in $V_{\text{sc}}$ (or $SoC_{\text{sc}}$), where the transmission line length is fixed at $l = 1 \text{ km}$. Let us note here that the five curves in each above mentioned figure are practically superposed. As presented in the above mentioned figures, the imposed closed-loop overshoot performances of $U_{\text{PCC}}$, $V_{\text{PCC}d}$, $V_{\text{PCC}q}$, and $V_{\text{dc}}$, with respect to the choice of the weighting functions $W_{\text{per},f}(s)$, are preserved regardless of the initial value of $SoC_{\text{sc}}$. Hence, the synthesized $H_{\infty}$ controller is robust in performance to $SoC_{\text{sc}} \in [25, 100] \%$ (or $V_{\text{sc}} \in [390, 780] \text{ V}$).

The time-domain responses of the supercapacitor output current $I_s$ and the direct and quadrature components of the inverter output current $I_{rd}$ and $I_{rq}$ taking into account the uncertainty in $V_{\text{sc}}$ (or $SoC_{\text{sc}}$) are given in Fig. 24, Fig. 25, and Fig. 26, respectively. Similarly, as depicted in the aforementioned figures, the admissible limit is ensured for these currents with respect to the choice of the weighting functions $W_{\text{h}}(s)$, which means that their imposed dynamic performances are fulfilled.

2) Uncertainty in the Transmission Line Length $l$

The time-domain responses of the PCC line-to-line voltage magnitude $U_{\text{PCC}}$, the direct and quadrature components of the PCC voltage $V_{\text{PCC}d}$ and $V_{\text{PCC}q}$, and the DC-bus voltage $V_{\text{dc}}$, taking into account the uncertainty in $l$, where the steady-state value of the supercapacitor voltage is chosen at $V_{\text{sc}} = 585 \text{ V}$ (or supercapacitor state of charge $SoC_{\text{sc}} = 0.56$), are illustrated in Fig. 27, Fig. 28, Fig. 29, and Fig. 30, respectively. As observed from the previously mentioned figures, the imposed closed-loop overshoot performances...
of $U_{PCC}$, $V_{PCCd}$, $V_{PCCq}$, and $V_{dc}$, corresponding to the tuning of the weighting functions $W_{perf}(s)$, are maintained irrespective of the value of $l$. Therefore, the designed $\mathcal{H}_\infty$ controller is demonstrated to be robust in performance to $l \in [1, 10]$ km. Let us remark here that the longer the length of the transmission line $l$ is, the more damped the oscillations of $U_{PCC}$, $V_{PCCd}$, and $V_{PCCq}$ are, as shown in Fig. 27, Fig. 28, and Fig. 29, respectively. This can be explained due to the fact that if the length of the transmission line is increased, its resistance value is therefore higher, which induces a rise in the damping coefficient of the system.

Fig. 31, Fig. 32, and Fig. 33 show the time-domain responses of the supercapacitor output current $I_d$ and the direct and quadrature components of the inverter output current $I_{rd}$ and $I_{rq}$, respectively, taking into account the uncertainty in $l$. Similarly, as shown in the aforementioned figures, the admissible limit is guaranteed for these currents corresponding to the tuning of the weighting functions $W_u(s)$, which means that their imposed dynamic performances are satisfied.

The time-domain responses of the active power variations of the ESS step-up transformer output $\Delta P_I$ and the diesel engine generator system output $\Delta P_g$ are depicted in Fig. 34 and Fig. 35, respectively. It can be seen that, due to large oscillations in $U_{PCC}$, $V_{PCCd}$, and $V_{PCCq}$ within a 10-ms interval in response to the disturbance, as shown in Fig. 27, Fig. 28, and Fig. 29, respectively, there is a dramatic fluctuation in the active power of the diesel engine generator system output $P_g$, nevertheless it returns nearly to its initial value after 10 ms of disturbance; whereas the active power of the ESS step-up transformer output $P_I$ presents a slight fluctuation around its initial value within this interval.

Fig. 36 and Fig. 37 depict the time-domain responses of the reactive power variations of the ESS step-up transformer output $\Delta Q_I$ and the diesel engine generator system output $\Delta Q_g$. Let us remark here that the longer the length of the transmission line $l$ is, the more damped the oscillations of $Q_I$, $Q_{Ird}$, and $Q_{Irq}$ are, as shown in Fig. 36, Fig. 37, and Fig. 38, respectively.
ΔQ = (U_PCC^2 - U_PCC_0^2) C_{l \omega_{grid}}. In a similar way, one can see that the reactive power of the diesel engine generator system output Q_l presents a considerable fluctuation which is caused by large oscillations in \( U_{PCC}, V_{PCC_d}, \) and \( V_{PCC_q} \) within a 10-ms interval subsequent to the disturbance, as shown in Fig. 27, Fig. 28, and Fig. 29, respectively; whereas the reactive power of the ESS step-up transformer output Q_t fluctuates slightly around its initial value within this interval. In addition, the ESS participation in primary voltage control has decreased the reactive power variation of the diesel generator around the steady-state point after the disturbance, as seen in Fig. 36 and Fig. 37. It should be noted here that the longer the length of the transmission line l is, the higher the reactive power variation of the ESS step-up transformer output ΔQ_t is and the lower the reactive power variation of the diesel engine generator system output ΔQ_l = (U_PCC^2 - U_PCC_0^2) C_{l \omega_{grid}} is, around their initial values after the disturbance.

VII. CONCLUSION AND PERSPECTIVES

In this paper, we have presented voltage-control-oriented modelling for the diesel-PV-storage hybrid power generation system operating in stand-alone mode. In particular, topological, as well as nonlinear and linear averaged models of each subsystem have first been provided, then a voltage-control-oriented model of the whole system has been obtained by aggregating these sub-models. We have also studied the influence of system parameters on the dynamic behavior of open-loop system measured outputs by means of a stability analysis method based on Monte Carlo simulation. Next, a voltage robust control design approach based on a cascaded two-level control structure – where classical PI-based current tracking controllers are placed on the low control level and receive references from an \( H_{\infty} \)-control-based upper level – has been developed in order to satisfy the required dynamic specifications. Effectiveness of the proposed voltage robust control strategy has been validated via MATLAB®/Simulink® closed-loop time-domain simulations. Finally, we have performed a sensitivity analysis of robust performance of the synthesized \( H_{\infty} \) controller to a given parametric uncertainty level in the steady-state value of the supercapacitor state of charge ScSoC (or supercapacitor voltage \( V_{sc} \)), as well as to a given variation in the transmission line length \( l \) connecting the diesel engine generator to the PCC. Through a series of MATLAB®/Simulink® closed-loop time-domain simulations in the presence of various load disturbances, it has been pointed out that the synthesized \( H_{\infty} \) controller remains robust in performance to a large uncertainty in the initial value of ScSoC (in particular ScSoC ∈ [25, 100] %), as well as to a large uncertainty in \( l \) (in particular \( l \in [1, 10] \) km).

The main prospect of this paper is to design a coordinated robust control strategy of the ESS, the diesel engine generator, the PV system, and the demand-side management
for stabilizing the PCC voltage. Practical implementation of the proposed voltage robust control algorithms on a real-time test bench is also envisaged.

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