Constraining the Konoplya-Rezzolla-Zhidenko deformation parameters I: limits from supermassive black hole X-ray data

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Abstract

X-ray reflection spectroscopy is a powerful technique for probing the nature of gravity around black holes in the so-called strong-field regime. One popular approach is to look at theory-agnostic deviations away from the Kerr solution, which is the only astrophysically relevant black hole solution within classical general relativity, in order to verify whether astrophysical black holes are described by the Kerr metric. We have recently extended our X-ray reflection spectroscopy framework to a class of very general axisymmetric non-Kerr black holes proposed by Konoplya, Rezzolla & Zhidenko (Phys. Rev. D93, 064015, 2016). Here, we analyze XMM-Newton and NuSTAR observations of the supermassive black hole in the Seyfert 1 galaxy MCG–06–30–15 with six different deviation parameters of this extended model. We recover the Kerr solution in all cases, but some deformation parameters are poorly constrained. We discuss the implications of this verification and future possibilities.

I. INTRODUCTION

One of the most important predictions of Einstein’s theory of gravity, also known as general relativity (GR), is the existence of black holes (BHs). While originally thought of as mathematical idealizations, BHs are now expected to be present in myriad numbers throughout the Universe. With developments in technology, the ability to detect astrophysical systems has progressed remarkably over the last decade. Although there existed a quite general and powerful framework to study the behavior of gravity in the weak-field regime from long ago, these new developments have enabled novel and much more precise (than before) probes of the behavior of strong-field gravity. BHs provide the best environments to perform such probes. One way to see this is to use the potential-curvature plot [1–4], where we can classify astrophysical systems according to their characteristic curvature scale and characteristic potential scale. Following Refs. [3, 4], we define the characteristic curvature $R = M/L^3$ and the characteristic potential $\phi = M/L$, where $M$ is the characteristic mass scale and $L$ the characteristic length scale of the astrophysical system under consideration. Fig. 1 shows a range of astrophysical systems, which have been used to test GR, on such a plot. Among all the systems in the right half of the plot, corresponding to the strong-field regime, a majority have one or more BHs in the system.

As a consequence of the “no-hair” theorems (see, for instance, [5] and references therein for their assumptions), four-dimensional GR predicts that isolated BHs in our Universe are described by only two parameters, which are refereed to as its mass and spin angular momentum, and defined by the Kerr solution.1 This is known as the Kerr hypothesis. The hypothesis is expected to hold even for BHs surrounded by accretion disks, since the gravitational effects of the disk are normally negligible compared to those of the BH [8]. This gives rise to an interesting possibility for testing GR in the strong-field regime with BHs – consider a metric which parametrically deviates away from Kerr, i.e., the deviation, or deformation, away from Kerr is controlled with a set of (possibly infinite) parameters. This new metric may or may not be the solution of a known theory of gravity (but see Ref. [9] for an interesting approach to mapping parametrically deformed metrics to some scalar-tensor theory of gravity). By analyzing astrophysical data against this new metric, one can try to constrain the deviation parameters and perform verification tests of GR in this theory-agnostic approach [10].

There are several techniques in vogue today which probe BHs and their environments. Theory-agnostic tests of gravity have been performed with gravitational waves (GWs) [4, 11, 12], X-ray spectroscopy [13–16], BH imaging [17–19], and infrared observations of the Galactic Center [20]. Our focus in this work is on X-ray reflection spectroscopy (XRS). XRS is based on the idea of extracting information about the BH from relativistic reflection spectrum of accretion disks. The technique is well established for measuring the spin of Kerr BHs, and has recently been extended to perform both theory-agnostic and theory-specific tests of GR. In the presence of accurate models and high-quality data, XRS can be a very powerful technique for constraining deviations from the

1A third parameter, the electric charge, though allowed within GR, is expected to be negligible in macroscopic astrophysical BHs [6].
2The inverse is not true, i.e., the existence of BHs which satisfy the Kerr solution does not automatically validate GR, since there are theories that differ from GR but whose BH solutions coincide with those of GR [7].
Kerr metric. One of the most attractive aspects of XRS that sets it apart from other techniques, is its applicability in both stellar-mass and supermassive BHs. This means that the whole of the right half of the potential-curvature phase space shown in Fig. 1 is accessible to XRS-based tests. This is highlighted by marking one representative low-mass X-ray binary (GX 339–4 [14]), a typical AGN (MCG–06–30–15 [21]) and a heavy AGN (Fairall 9 [22]). GWs from ground-based detectors, on the other hand, only cover the upper-right quadrant of this phase space, and BH imaging techniques only the lower-right quadrant.

One of the most popular theory-agnostic metrics in the market today is the metric proposed in Ref. [23] by Kono-plya, Rezzolla & Zhidenko (KRZ metric hereafter). The KRZ metric is a stationary axisymmetric metric written in Boyer-Lindquist-like coordinates. Notably, it does not always possess a Killing tensor and, as such, the equations of motion are not always separable. This makes it a better choice for verification tests of GR than those metrics that always have a Carter constant, since it captures a larger variety of deviations from Kerr. In addition, the metric deformation functions are expressed in terms of continued-fraction expansions, which has superior convergence compared to the more common $M/r$-based power series expansion. This feature provides significant advantage when dealing with rapidly rotating BHs where the characteristic length scales (the innermost stable circular orbit, the photon orbit, etc.) are $\sim M$ and higher-orders terms in the $M/r$ expansion become non-negligible. In a recent work, we implemented this metric in the XRS framework and put constraints on possible deviations from GR in terms of six distinct deviation parameters of the KRZ metric [24].

![FIG. 1. A potential-curvature plot showing several astrophysical systems which have been used to test GR. Sources analyzed with X-rays have been marked in red. A GW event appears as a dynamic system in this plot, and is denoted with a line instead of a point. See Tab I for details on the characteristic mass and length scales of the systems shown, and the text for discussion.](image)

| System and main reference | $M \ [M_\odot]$ | $L$ |
|---------------------------|-----------------|-----|
| Cassini [25]              | 1               | $1.1 \times 10^6$ km |
| Mercury’s perihelion [26] | 1               | $5.8 \times 10^7$ km |
| Binary pulsar (Shapiro) [3] | 1.34       | $1.04 \times 10^4$ km |
| PSR J0030+0451 [27, 28]   | 1.44           | 13 km |
| GX 339–4 [14]             | 10             | $R_{\text{ISCO}}$ |
| GW150914 [29]             | 65.3           | $385 - 1300$ km |
| MCG–06–30–15 [21]         | $2.8 \times 10^6$ | $R_{\text{ISCO}}$ |
| Fairall 9 [22]            | $2.55 \times 10^8$ | $R_{\text{ISCO}}$ |
| M87 [17]                  | $6.5 \times 10^9$ | $R_{\text{ph}}$ |

TABLE I. The characteristic mass and length scales of astrophysical systems shown in Fig. 1. For BHs analyzed with X-rays, the characteristic length scale is taken to be $R_{\text{ISCO}}$, i.e. the radius of the innermost stable circular orbit (ISCO), while those analyzed with imaging have their characteristic length scale at $R_{\text{ph}}$, i.e., the photon orbit. The binary pulsar data point is related to the Shapiro delay at the impact parameter [9].

In the present work, we analyze the X-ray spectra of the Seyfert 1 galaxy MCG–06–30–15 as observed simultaneously by XMM-Newton and NuSTAR telescopes in 2013. We use the reflection model relxill.nk, a public model developed by us, to model the reflection component and constrain parameters of the KRZ metric. Our aim is to verify the Kerr hypothesis, namely, to verify whether, and how well, we can constrain the deviations to the Kerr solution using the KRZ metric. The presence of a very prominent and broad iron line in the spectrum and the unprecedented high quality of simultaneous XMM-Newton and NuSTAR observations make the 2013 data of MCG–06–30–15 particularly suitable for our test.

This article is organized as follows. Sec. II gives a review of the reflection model and the KRZ metric and its deformation parameters. Sec. III presents the source properties and the details of the observation. Details of data analysis and results are given in Sec. IV, and the results are discussed in Sec. V. Through the article, we use geometrized units, namely $c = G = 1$, and use the metric signature $(- +++)$. Additionally, since XRS is independent of the mass of the BH and its distance from Earth, we set the BH mass $M = 1$.

II. THE RELXILL.NK MODEL

relxill.nk is an extension of relxill, the standard X-ray reflection model for Kerr BHs [30, 31], to metrics beyond the Kerr solution [32–34]. relxill itself combines the radiative transfer code xillver that balances the microphysics inside the accretion disk in a rigorous way and provides a local spectrum [35] and the relativistic blurring code relconv that evolves the local spectrum.
along null geodesics, on a Kerr background, to calculate the spectrum as seen by a distant observer \cite{36, 37}. \texttt{relxill\_nk} modifies \texttt{relconv} to evolve the local spectrum on non-Kerr backgrounds.

\texttt{Thermal component}

\texttt{Power law component}

\texttt{Reflected component}

\begin{equation}
F_o(\nu_o) = \int I_o(\nu_o, X, Y) d\Omega,
\tag{1}
\end{equation}

where $I_o$ is the intensity received by the observer and depends on the photon frequency at the observer, $X$ and $Y$ are Cartesian coordinates on the plane of the observer, and $d\Omega$ is the integration element on this plane. Since the intensity is known at the point of emission (given by Planck’s law in the case of the thermal component and by \texttt{xillver}, for instance, in the case of the reflection component), we relate $I_o$ to the intensity at emission $I_e$ with Liouville’s theorem as follows

\begin{equation}
I_o(\nu_o) = g^3 I_e(\nu_e)
\tag{2}
\end{equation}

where $g = \nu_o/\nu_e$ is the redshift the photons experience on their way from emission to observation.

At this stage, calculation of flux involves raytracing photons every time the flux has to be calculated. This can be extremely time-consuming, especially for non-Kerr metric backgrounds where any simplification of the geodesic evolution equations may not be possible, and cumbersome for data analysis. The \texttt{relxill} and \texttt{relxill\_nk} suites of models use a transfer function which acts as an integration kernel and considerably speeds up computation of the flux. It is defined as \cite{40}

\begin{equation}
f(g^*, r_e, \iota) = \frac{1}{\pi r_e^2} g^\ast \sqrt{1 - g^\ast} \left| \frac{\partial (X,Y)}{\partial (g^*, r_e)} \right|, \tag{3}
\end{equation}

where $r_e$ is the radial coordinate on the disk, $\iota$ is the inclination of the observer relative to the BH spin axis, $g^∗$ is the normalized redshift factor, defined as

\begin{equation}
g^* = \frac{g - g_{\text{min}}}{g_{\text{max}} - g_{\text{min}}}, \tag{4}
\end{equation}

where $g_{\text{min}}$ and $g_{\text{max}}$ are, respectively, the minimum and maximum redshift at a constant $r_e$ and $\iota$, and $\frac{\partial (X,Y)}{\partial (g^*, r_e)}$ is the Jacobian relating quantities at the observer and the disk.

The metric we use here to test the Kerr hypothesis is given in Boyer-Lindquist-like coordinates $(t, r, \theta, \phi)$ as \cite{23, 24, 41}

\begin{equation}
ds^2 = - \frac{N^2 - W^2 \sin^2 \theta}{K^2} dt^2 - 2W r \sin^2 \theta dt d\phi \\
+ K^2 r^2 \sin^2 \theta d\phi^2 + \frac{\Sigma B^2}{N^2} dr^2 + \Sigma r^2 d\theta^2, \tag{5}
\end{equation}

where the metric functions are defined as
\[ N^2 = \left(1 - \frac{r_0}{r}\right) \left(1 - \frac{\epsilon a r_0}{r} + \left(k_{00} - \epsilon_0\right) \frac{r_0^2}{r^2} + \frac{\delta r_0^3}{\alpha^2}\right) + \left(\frac{a_20 r_0^3}{r^4} + \frac{a_21 r_0^4}{r^3} + \frac{k_{21} r_0^3}{r^2 \left(1 + \frac{k_{22}(1 - \frac{\epsilon}{a})}{1 + k_{22}(1 - \frac{a}{M})}\right)}\right) \cos^2 \theta, \]

\[ K^2 = 1 + \frac{a_* W}{r} + \frac{1}{\Sigma} \left(\frac{k_{00} r_0^2}{r^2} + \left(\frac{k_{20} r_0^2}{r^2} + \frac{k_{21} r_0^3}{r^3 \left(1 + \frac{k_{22}(1 - \frac{\epsilon}{a})}{1 + k_{22}(1 - \frac{a}{M})}\right)}\right) \cos^2 \theta\right), \]

\[ W = 1 \left(\frac{a_0 W}{r^2} + \frac{\delta r_0^3}{\alpha^2} \cos \theta\right), \quad B = 1 + \frac{\delta r_0^2}{r^2} + \frac{\delta r_0^2}{r^2} \cos \theta, \quad \Sigma = 1 + \frac{a_*^2}{r^2} \cos^2 \theta. \]

Here \( a_* = J/M^2 \) is the dimensionless BH spin, and

\[ r_0 = 1 + \sqrt{1 - a_*^2}, \quad a_{21} = -\frac{a_*^4}{r_0^2} + \delta_6, \quad k_{21} = \frac{a_*^4}{r_0^2} - 2a_*^2 - \delta_6. \]

The metric contains six parameters, denoted by \( \{\delta_i\} \) \( i = 1, 2, \ldots, 6 \), quantifying deviations away from the Kerr solution. The remaining parameters are defined such that Eq. 5 reduces to the Kerr metric when all \( \{\delta_i\} \) are identically zero. Their exact expressions can be found in Ref. [24]. (Note that the expressions given in Ref. [23] do not reduce to the Kerr metric, and the correct expressions are given in Ref. [24].) Therein are also given bounds on \( \{\delta_i\} \) that are required to ensure regularity of the spacetime outside the horizon (e.g., a negative definite metric determinant, a positive definite \( g_{\phi\phi} \), and a nonzero \( N^2 \)). In particular,

\[ \delta_1 > \frac{4r_0 - 3r_0^2 - a_*^2}{r_0^2}, \quad \delta_2, \delta_3 \left\{ \begin{array}{ll}
\geq -\frac{4}{a_*^2}(1 - \sqrt{1 - a_*^2}) & \text{if } a_* > 0 \\
< \frac{4}{a_*^2}(1 - \sqrt{1 - a_*^2}) & \text{if } a_* < 0, \\
\delta_4, \delta_5 > -1.
\end{array} \right. \]

The bounds on \( \delta_6 \) turn out to be stronger than what is reported in Ref. [24]. While the new bound cannot be expressed analytically, it is easily evaluated numerically. The following analysis takes this new bound into account and restricts the parameter exploration to only the allowed region.

We note that \( \delta_1 \) is associated to a deformation of the metric coefficient \( g_{tt}, \delta_2 \) and \( \delta_3 \) to deformations related to the BH rotation, \( \delta_4 \) and \( \delta_5 \) to deformations of \( g_{rr} \), and \( \delta_6 \) alters the shape of the BH event horizon. Since the structure of an infinitesimally thin disk (in particular, ISCO radius and orbital velocity of the gas) are determined by \( g_{tt}, g_{t\phi}, \) and \( g_{\phi\phi} \), only \( \delta_1, \delta_2, \delta_3, \) and \( \delta_6 \) can modify the motion of the gas in the disk. However, \( \delta_1 \) and \( \delta_2 \) have a large impact on the disk, while the effect of \( \delta_3 \) and \( \delta_6 \) is quite weak. \( \delta_1 \) and \( \delta_2 \) do not have any effect on the disk and only change the motion of the X-ray photons from the emission point in the disk to the detection point far from the source. The impact of the deformation parameters \( \{\delta_i\} \) on the reflection spectrum of a disk was shown in Ref. [24].

The \texttt{relxill.nk} model has two parameters that control the non-Kerr nature of the BH. One parameter is used to decide the type of deviation (e.g. the \( i \) in \( \delta_i \)), and the other decides the size of the deviation. Since \texttt{relxill.nk} allows for one type of deviation at a time, the analysis is performed for each \( \delta_i \) separately. In the following sections, we will use the model to analyze some X-ray data.

### III. SOURCE AND OBSERVATION OVERVIEW

MCG–06–30–15 is a bright Active Galactic Nucleus (AGN) in which a broad iron line was clearly detected by ASCA for the first time [42]. The iron Kα line was extended to lower energies which indicates its origin in the innermost regions of the BH [43, 44]. MCG–06–30–15 has been observed by many X-ray missions like BeppoSAX [45], RXTE [46, 47], XMM-Newton [48–52], Suzaku [53, 54], and NuSTAR [55]. The observation of MCG–06–30–15 by NuSTAR, along with the simultaneous XMM-Newton observation, displays a prominent Compton hump around 20-30 keV and the iron Kα line peaked at 6-7-keV. The presence of these features make this source suitable for testing general relativity using X-ray reflection spectroscopy. Ref. [21] analyzed the same dataset for testing the Kerr hypothesis using the Johannsen metric [56] as the background metric. The spectrum of this source at lower energies is very complex due to absorption by warm ionized winds [57]. High resolution Chandra and XMM-Newton studies confirmed the presence of absorbers around the source [58–62]. Besides these complexities, the source is also found to be extremely variable [21].

#### A. Observations and Data Reduction

XMM-Newton [63] with its EPIC CCD detectors Pn [64] and MOS1/2 [65] observed MCG–06–30–15 for three consecutive revolutions (obs. ID 0693781301 and 0693781401) starting 2013 January 29 for about 315 ks. The Pn raw data for these revolutions are downloaded from the HEASARC website and is processed into cleaned event files using Science Analysis Software (SAS) v16.0.0. MOS data is not included in this
analysis because it is severely affected by pileup. TABTIGEN is used to generate good time intervals (GTIs). A source region of radius 40 arcsec is taken around the center of the source. A background region of 50 arcsec is taken as far as possible from the source to avoid any contamination from source photons. The corresponding ancillary and response files are generated using the SAS routines ARFGEN and RMFGEN, respectively. Finally, the source spectra is rebinned such that it oversamples the instrumental resolution by a factor of 3 and has a minimum of 50 counts per bin.

NuSTAR [66] with its two detectors FPMA and FPMB observed this source simultaneously with XMM-Newton for about 360 ks (obs. ID 60001047002, 60001047003, and 60001047005). The raw data from both detectors are processed into cleaned event files using the NUPIPELINE routine of the NuSTAR data analysis software (NuSTARDAS), which is distributed as part of the high energy analysis software (HEASOFT). We use the latest Calibration files from the Calibration database (CALDB) v20180312. A source region of 70 arcsec is extracted from the cleaned event files around the center of the source. A background region of radius 100 arcsec is taken on the same detector and as far as possible from the source. Source spectra, background spectra, and response files are generated using the NUPRODUCTS routine. The source spectra is rebinned to 70 counts per bin to improve the signal-to-noise ratio and to apply the \( \chi^2 \) statistic.

Due to the extreme variability of the source using strictly simultaneous flux resolved data [21] is necessary. We combined the GTIs from both XMM-Newton and NuSTAR cleaned event files using the tool MGTIME. The data from EPIC-Pn, FPMA, and FPMB are divided into four flux states. These flux states are divided such that the counts in each state for each instrument is similar.

IV. SPECTRAL ANALYSIS

For further work, we used the X-ray spectral analysis package XSPEC v12.11.1 [67], WILMS abundance [68], and VERN cross-section [69] distributed as part of HEASOFT v6.28.

For each of the Pn, FPMA, and FPMB instruments, we have four spectra corresponding to four flux states (low, medium, high, very high). So, there are twelve spectra in total which are fitted simultaneously. For each flux state, the cross-calibration constant for XMM-Newton is frozen to 1 leaving the cross-calibration constant for FPMA (\( C_{FPMA} \)) and FPMB (\( C_{FPMB} \)) free to vary. Throughout our analysis, the values of \( C_{FPMA} \) and \( C_{FPMB} \) are within 5% of each other which is in agreement with the standard calibration of instruments. For NuSTAR, we used the energy range of 3.0-80.0 keV where the quality of the data is considered to be suitable for spectral studies. For XMM-Newton, data in the 0.5-10.0 keV energy range is used. Due to poor data quality below 0.5 keV and background domination above 10 keV, these energy ranges are excluded during analysis. The energy range 1.5-2.5 of Pn data is not used because of the calibration issues discussed in [55] and [21].

To display the features present in the observation, we fit the data with the absorbed power-law. Fig. 3 shows the ratio of the lowest flux state data to the model \( tbabs \times cutoffpl +relxill\_n\_k\) to the absorbed power-law to fit the reflection component. To address the residuals at lower energies, we add two warm absorbers and one dusty neutral absorber. Narrow line emissions around 7 keV are also present and are modeled with a distant reflector that is non-relativistic in nature. A narrow emission line and absorption line can also be seen after adding these components. In XSPEC, the model describing the source is written as :

\[
tbabs \times dustyabs \times warmabs_1 \times warmabs_2 \times (cutoffpl + relxill\_n\_k \times xillver\_n\_k + zgau\_n\_k)
\]

\( tbabs \) accounts for galactic absorption along the line of sight of the observer and has column density (\( N_H \)) as its only free parameter [68]. We freeze its value to \( 3.9 \times 10^{20} \text{cm}^{-3} \) obtained by other independent measurements [75]. \( dustyabs \) accounts for the neutral dust absorber and has iron density (log \( N_{Fe} \)) as a free parameter. This multiplicative table has been made especially for this source using the high-resolution Chandra data. Please see [59] for more details about absorption by dust in MCG–06–30–15. \( warmabs_1 \) and \( warmabs_2 \) describes the two warm absorbers modeled with the multiplicative table constructed using xstar. Each warm absorber is modeled as an ionized zone characterized by column density (\( N_H \)) and ionization parameter (log \( \xi \)). \( cutoffpl \) corresponds to the power-law continuum with the photon index \( \Gamma \), cut-off energy of the continuum (\( E_{cut} \)), and the normalization as free parameters. \( relxill\_n\_k \) describes the reflection coming from the inner regions of the accretion disk where the relativistic effects are significant [32, 33]. In this work, we used \( relxill\_n\_k \) using the KRZ metric as the background metric. \( xillver \) describes the reflection from the region far away from the source where the relativistic effects are negligible [35]. \( zgau\_n\_k \) models the red-shifted Gaussian line. Here, one of the Gaussians represents the emission line at 0.81 keV which is believed to be oxygen line emission due to relativistic outflow [76]. The other Gaussian corresponds to the absorption line at 1.24 keV which is most likely the blue-shifted oxygen absorption.

Column density, the only parameter in \( tbabs \) model, is kept frozen and constant for all flux states. The col-
FIG. 3. Data to model ratio for the absorbed power-law in the low flux state. Magenta, blue, and green curves correspond to Pn, FPMA and FPMB data, respectively.

FIG. 4. The best-fit model (upper quadrant) and data to best-fit model ratio (lower quadrant) for different flux states. In the upper quadrant, the black, magenta, red and green curves correspond to total theoretical model, cutoffpl, relxill_nk and xillver respectively. In the lower quadrant, magenta, blue and green crosses represent Pn, FPMA and FPMB data respectively.

The reflection component varies over small timescales when calculated near the BH due to relativistic effects. The relativistic reflection model relxill_nk assumes the emissivity profile in the form of a broken power-law modeled with three parameters: inner emissivity \( q_{in} \), outer emissivity \( q_{out} \), and break radius \( R_{br} \). This is the standard description for a corona of unknown geometry. These three parameters vary among the four flux states because different reflected flux is likely to be the result of different emissivity profiles. Spin, inclination, and iron abundance are tied among the different states as these parameters are not expected to vary over such

umn density and ionization parameter of warmabs_1 and warmabs_2 model are free to vary among flux states as the warm absorbers are expected to vary over small timescales. The iron density of dustyabs is free to vary but tied among the four flux states. The power-law emission, represented by cutoffpl, also varies among flux states because coronal emission is expected to vary in order to produce the different flux states.

The reflection component varies over small timescales when calculated near the BH due to relativistic effects.
The ionization parameter \( \xi \) is in units erg cm s\(^{-1}\). The reported uncertainties correspond to the 90\% confidence level for one relevant parameter. * indicates that the parameter is frozen. The data sets 1, 2, 3, and 4 correspond, respectively, to the low, medium, high, and very-high flux states. See the text for more details.

small timescales. The ionization parameter varies among the flux states as it is the property associated with flux. Deformation parameter is a property of spacetime and is not expected to change over the flux variations. So, it is linked among the flux states. The reflection fraction is frozen to −1 in order to return only the reflected component as the power-law emission is modeled with cutoffpl.

\( \Gamma \) and \( E_{\text{cut}} \) of xillver are tied to the coronal emission of the corresponding flux state. As we are considering only the reflected component far away from BH, we freeze the reflection fraction to −1. \( \log \xi \) is frozen to 0 as it is assumed that there will be no ionization far away from BH. The iron abundance is assumed to be solar. The emission line at 0.81 keV and absorption line at 1.24 keV are modeled with zgauss.

Fig. 4 shows the best-fit model (upper quadrant) and data to best-fit model ratio (lower quadrant) for all four flux states (low, medium, high, very high) for deformation parameter \( \delta_1 \). We do not show the corresponding ratio plots for other deformation parameters as they are very similar to Fig. 4. The best fit parameters values obtained for the best fit model are given in tables II to IV for all six deformation parameters. Fig. 5 shows the confidence contours in the spin and deformation parameter plane for all six cases. The red, green, and blue curves show 68\%, 90\%, and 99\% confidence, respectively. The black horizontal line corresponds to the Kerr solution.

| Model | \( \delta_1 \) | \( \delta_2 \) |
|---|---|---|
| xillver | 0.039 | 0.039 |
| cutoffpl | 1.740\( ^{+0.007}_{-0.022} \) | 1.741\( ^{+0.019}_{-0.029} \) |
| dustyabs | log (\( N_{\text{Fe}}/10^{21} \text{ cm}^{-2} \)) | 3.22 \( ^{+0.06}_{-0.06} \) | 3.24 \( ^{+0.06}_{-0.06} \) |
| warmabs \( \gamma \) | 1.84\( ^{+0.04}_{-0.02} \) | 1.954\( ^{+0.008}_{-0.004} \) | 1.973\( ^{+0.005}_{-0.004} \) | 2.016\( ^{+0.004}_{-0.004} \) | 2.029\( ^{+0.005}_{-0.005} \) | 2.016\( ^{+0.004}_{-0.004} \) | 2.029\( ^{+0.004}_{-0.004} \) |
| warmabs \( \nu \) | 0.64\( ^{+0.02}_{-0.02} \) | 1.92\( ^{+0.02}_{-0.04} \) | 3.1 \( ^{+0.5}_{-0.6} \) | 3.2 \( ^{+0.5}_{-0.6} \) | 2.48 \( ^{+0.5}_{-0.6} \) | 2.48 \( ^{+0.5}_{-0.6} \) |
| dustabs | log (\( N_{\text{Fe}}/10^{22} \text{ cm}^{-2} \)) | 0.039 | 0.039 |
| warmabs | log (\( N_{\text{Fe}}/10^{22} \text{ cm}^{-2} \)) | 0.039 | 0.039 |

**TABLE II.** Summary of the best-fit values for the models with \( \delta_1 \) and \( \delta_2 \). The ionization parameter \( \xi \) is in units erg cm s\(^{-1}\). The reported uncertainties correspond to the 90\% confidence level for one relevant parameter. * indicates that the parameter is frozen. The data sets 1, 2, 3, and 4 correspond, respectively, to the low, medium, high, and very-high flux states. See the text for more details.
As shown in Fig. 5, the results are consistent with a Kerr BH solution. While MCG–06–30–15 has been used to verify the Kerr solution before, this work is significant for a few reasons. This is the first time that a test of the Kerr hypothesis using MCG–06–30–15 has been performed in the context of the KRZ metric. The KRZ metric is quite generic and has fewer symmetries than the Kerr solution. It is, thus, capable of capturing a larger variety of potential violations of the Kerr solution and of GR. This also makes it a better proxy for the BHs of some of the most popular modified theories of gravity, which do not possess all the symmetries of the Kerr solution. A verification of the Kerr solution in this context is, therefore, an important step forward towards testing modified theories of gravity. Considering the properties of MCG–06–30–15 and the excellent quality of the 2013 data of XMM-Newton and NuSTAR, the analysis reported in this paper is presumably the best we can do today for testing the Kerr hypothesis with supermassive BHs using XRS. The dataset we studied here is quite complete, requiring as many as three absorption components and split across four different flux states. That the Kerr solution is recovered, in most cases at the 1-σ confidence level itself, is remarkable and adds to the robustness of the result.

From Fig. 5, we see that the deformation parameters \( \delta_3 \), \( \delta_4 \), and \( \delta_6 \) are poorly constrained: eventually, their constraints are set by the boundaries of the regular spacetime region rather than by our fits. In order to figure out whether better data than those available can constrain these parameters or whether the reflection spectrum is not very sensitive to these deformations from the Kerr metric, we simulated a 300 ks simultaneous observation of MCG–06–30–15 with the X-IFU instrument on board of...
### Table IV

| Model          | $\delta_5$ | $\delta_6$ | $\delta_7$ | $\delta_8$ |
|----------------|------------|------------|------------|------------|
| $N_H/10^{22} \text{ cm}^{-2}$ | 0.039*     | 0.039*     | 0.039*     | 0.039*     |
| waribs         | 0.46$^{+0.19}_{-0.10}$ | 1.18$^{+0.04}_{-0.05}$ | 1.01$^{+0.04}_{-0.06}$ | 0.74$^{+0.10}_{-0.08}$ |
| log $\xi_2$   | 1.85$^{+0.13}_{-0.03}$ | 1.954$^{+0.016}_{-0.027}$ | 1.919$^{+0.021}_{-0.033}$ | 1.83$^{+0.03}_{-0.03}$ |
| log $\xi_2$   | 0.66$^{+0.18}_{-0.04}$ | 0.02$^{+0.02}_{-0.02}$ | 0.52$^{+0.19}_{-0.15}$ | 0.25$^{+0.06}_{-0.05}$ |
| log $\xi_2$   | 1.91$^{+0.03}_{-0.08}$ | 3.1$^{+0.8}_{-0.8}$ | 3.23$^{+0.8}_{-0.8}$ | 2.48$^{+0.03}_{-0.13}$ |
| $N_{\text{ps}}/10^{21} \text{ cm}^{-2}$ | 17.40$^{+0.03}_{-0.03}$ | 17.40$^{+0.029}_{-0.015}$ |
| cutoffpl       | 1.954$^{+0.012}_{-0.013}$ | 1.971$^{+0.018}_{-0.011}$ | 2.01$^{+0.012}_{-0.011}$ | 2.026$^{+0.011}_{-0.011}$ |
| $E_{\text{cut}}$ [keV] | 200$^{+33}_{-25}$ | 155$^{+40}_{-8}$ | 164$^{+41}_{-9}$ | 280$^{+47}_{-77}$ |
| norm (10$^3$)  | 8.39$^{+0.15}_{-0.16}$ | 12.12$^{+0.80}_{-0.20}$ | 15.2$^{+0.6}_{0.6}$ | 20.9$^{+0.18}_{-0.6}$ |
| relxill        | 2.5$^{+0.05}_{-0.03}$ | 2.8$^{+0.08}_{-0.02}$ | 3.3$^{+0.05}_{-0.11}$ | 3.3$^{+0.05}_{-0.11}$ |
| $a_*$          | 31.4$^{+1.5}_{-1.6}$ | 0.96$^{+0.014}_{-0.014}$ | 0.8$^{+0.02}_{-0.02}$ | 0.8$^{+0.02}_{-0.02}$ |
| $\delta$      | 0.007749$^*$       | 0.007749$^*$       | 0.007749$^*$       | 0.007749$^*$       |
| $A_{\text{Fe}}$ | 0.049$^{+0.006}_{-0.006}$ | 0.063$^{+0.001}_{-0.001}$ | 0.104$^{+0.009}_{-0.009}$ | 0.131$^{+0.002}_{-0.002}$ |
| norm (10$^{-3}$) | 0$^*$     | 0$^*$      | 0$^*$      | 0$^*$      |
| z              | 0.057$^{+0.007}_{-0.007}$ | 0.058$^{+0.008}_{-0.007}$ |
| $E_{\text{line}}$ [keV] | 0.8130$^{+0.0015}_{-0.0009}$ | 0.814$^{+0.001}_{-0.0009}$ |
| $\chi^2$/dof   | 3027.76/2685 = 1.12766 | 3027.81/2685 = 1.12767 |

*We note that the fit for $\delta_5$ (top-left panel of Fig. 6) does not seem to recover the Kerr solution well even if the input model assumes the Kerr metric ($\delta_3 = 0$). We investigated the reason and it seems related to the combination of the complicated absorption model of the source and the response of the instruments. Repeating the simulation without absorbers, we find the situation in the top-right panel of Fig. 6, which is the result that we would expect from a simulation.*
FIG. 5. Constraints on the BH spin and on the deformation parameters. The red, green, and blue curves correspond, respectively, to the 68%, 90%, and 99% confidence level contours for two relevant parameters. The black horizontal line at $\delta_i = 0$ corresponds to the Kerr solution. The gray region is not included in our analysis because the spacetime is not regular there, see Eq. (9).

We note that the errors reported in tables II to IV and Fig. 5 are only the statistical errors. Systematic errors, in particular those related to the theoretical model, are not included [38]. However, most modeling uncertainties are quite under control and are expected to be subdominant for the quality of the data available today, where the statistical error is the main source of uncertainty. Our model assumes that the disk is infinitesimally thin, with the inner edge at the ISCO, and that there is no emission of radiation inside the ISCO\(^4\). The impact of the thickness of the disk was studied in Ref. [81] for this dataset, with

\(^4\)We note that there are attempts to construct more sophisticated models, where the accretion disk is obtained from GRMHD simulations and the corona and the illumination of the disk are calculated self-consistently; see, e.g., Refs. [79, 80]. However, these models are not yet suitable to analyze data and can only simulate some spectra.
FIG. 6. Constraints on the BH spin and on the deformation parameters $\delta_3$, $\delta_4$, and $\delta_6$ from our simulations of a simultaneous 300 ks observation with X-IFU/Athena and LAD/eXTP. The red, green, and blue curves correspond, respectively, to the 68%, 90%, and 99% confidence level contours for two relevant parameters. The black horizontal line corresponds to the Kerr solution. The gray region is not included in our analysis because the spacetime is not regular there, see Eq. (9). In the case of $\delta_3$, we show the results of the simulation of the full model (top-left panel) and of the simulation without absorbers (top-right panel); see the text for more details.

the conclusion that the infinitesimally thin disk approximation does not produce any significant bias in the estimate of the properties of the source. The material in the plunging region is expected to be fully ionized and therefore its reflection spectrum has no features: neglecting the radiation from the plunging region in the analysis of MCG–06–30–15 should not affect our measurements [82]. The ionization parameter is constant over the whole disk in our analysis, while it would be natural to expect a non-vanishing ionization gradient. However, even the assumption of a constant ionization parameter should not affect our capability of constraining deformations from the Kerr solution [83]. As of now, the impact of the returning radiation (the radiation emitted by the disk and returning to the disk because of the strong light bending near the black hole) is likely the least understood source of uncertainty, since there are only partial studies in the literature; see [84] and reference therein.

XRS-based tests of GR in general, and the KRZ metric-based exploration in particular, are in early stages of development, with a lot of scope for the future. With the technique in general, significant progress is possible with MHD simulations of the BH neighborhood in non-GR backgrounds [85, 86], implementation of numerically evaluated BH solutions from modified theories of gravity, etc. With the KRZ metric in particular, it is possible to explore things like chaos [87, 88] as well as some BHs from modified theories of gravity that can be mapped to the KRZ metric [89].

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