The Hagedorn temperature in a decoupled sector of AdS/CFT

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Abstract

We match the Hagedorn/deconfinement temperature of planar $\mathcal{N} = 4$ super Yang-Mills (SYM) on $\mathbb{R} \times S^3$ to the Hagedorn temperature of string theory on AdS$_5 \times S^5$. The match is done in a near-critical region where both gauge theory and string theory are weakly coupled. On the gauge theory side we are taking a decoupling limit found in Ref. [1] in which the physics of planar $\mathcal{N} = 4$ SYM is given exactly by the ferromagnetic $XXX_{1/2}$ Heisenberg spin chain. We find moreover a general relation between the Hagedorn/deconfinement temperature and the thermodynamics of the Heisenberg spin chain. On the string theory side, we identify the dual limit which is taken of string theory on a maximally symmetric pp-wave background with a flat direction, obtained from a Penrose limit of AdS$_5 \times S^5$. We compute the Hagedorn temperature of the string theory and find agreement with the Hagedorn/deconfinement temperature computed on the gauge theory side. Finally, we discuss a modified decoupling limit in which planar $\mathcal{N} = 4$ SYM reduces to the $XXX_{1/2}$ Heisenberg spin chain with an external magnetic field.
1 Introduction

The most beautiful example of the relation between gauge theories and string theories is the
AdS/CFT correspondence which asserts an exact duality between $SU(N)$ $\mathcal{N} = 4$ super Yang-Mills (SYM) on $\mathbb{R} \times S^3$ and type IIB string theory on $AdS_5 \times S^5$ \cite{2, 3, 4}. The AdS/CFT correspondence is a strong/weak coupling duality. This is the power of the correspondence but it also makes it difficult to verify its validity. Many of the checks have involved computing physical quantities on the gauge theory side, such as the expectation value of Wilson loops \cite{5, 6} or the anomalous dimensions of gauge theory operators \cite{7}, and extrapolating the results to strong coupling in order to compare with string theory.

In this talk we instead check the validity of the AdS/CFT correspondence avoiding the
extrapolation of the results to strong coupling, following the papers \cite{1, 8, 9}. The strategy
we use is to compute the Hagedorn/deconfinement temperature for planar $\mathcal{N} = 4$ SYM on
$\mathbb{R} \times S^3$ at weak coupling $\lambda \ll 1$ in a certain near-critical region found in \cite{1} and to match this
to the Hagedorn temperature computed in weakly coupled string theory on $AdS_5 \times S^5$, in the
 corresponding dual near-critical region. This successful match mostly relies on the matching
of the low energy spectra of the gauge theory and the string theory in the near-critical region.

In \cite{10, 11, 12, 13} a relation is conjectured between the Hagedorn/deconfinement temper-
ature of planar $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$ and the Hagedorn temperature of string theory on
$AdS_5 \times S^5$. This is due to the discovery of a confinement/deconfinement phase transition in
planar $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$ at weak coupling $\lambda \ll 1 \cite{10}$. In particular, at high energies
the theory has a Hagedorn density of states, with the Hagedorn temperature being equal to
the deconfinement temperature \cite{11, 12, 13}.

However, the fact that we do not know how to quantize string theory on $AdS_5 \times S^5$
means that we cannot directly test this conjecture. One hope comes from considering certain
Penrose limits where the $AdS_5 \times S^5$ background becomes a maximally supersymmetric pp-
wave background \cite{7, 14} where type IIB string theory can be quantized. In this case in fact
the Hagedorn temperature has been computed \cite{15, 16}. However, in order to obtain the
correspondence with string theory, it is necessary to consider a strong coupling limit on the
gauge theory side so that most of the gauge theory operators decouple keeping only those
dual to the string states.

In this paper we take a different route by taking a decoupling limit corresponding to being
in a certain near-critical region. Using this we find a gauge-theory/pp-wave correspondence
appropriate for verifying the relation between the Hagedorn/deconfinement temperature of
planar $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$ and the Hagedorn temperature of string theory on $AdS_5 \times S^5$.

We expect more generally that our decoupling limits can be used to study the thermo-
dynamics of $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$ and its string theory dual also above the Hagedorn
temperature. It could in particular be interesting to study the connection to black holes in
$AdS_5 \times S^5$ \cite{17, 1}.

\footnote{See also \cite{13} for a related study of black holes with R-charged chemical potentials.}
2 Gauge theory side

The solution to find the appropriate gauge-theory/pp-wave correspondence comes from a recently found decoupling limit of thermal $SU(N)$ $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$ [1] which is given by

\[ T \rightarrow 0 \ , \ \Omega \rightarrow 1 \ , \ \lambda \rightarrow 0 \ , \ \tilde{T} \equiv \frac{T}{1 - \Omega} \text{ fixed} \ , \ \tilde{\lambda} \equiv \frac{\lambda}{1 - \Omega} \text{ fixed} \ , \ N \text{ fixed} \quad (2.1) \]

where $T$ is the temperature for $N = 4$ SYM, $\Omega$ is the chemical potential associated to the three R-charges $J_1, J_2, J_3$ for the $SU(4)$ R-symmetry and it is defined as $(\Omega_1, \Omega_2, \Omega_3) = (\Omega, \Omega, 0)$. $\lambda = g_{YM}^2 N/4\pi^2$ is the ’t Hooft coupling. In the limit (2.1) only the states in the $SU(2)$ sector survive, and $SU(N)$ $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$ reduces to a quantum mechanical theory with temperature $\tilde{T}$ and coupling $\tilde{\lambda}$. In fact, consider the thermal partition function of $SU(N)$ $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$ with non-zero chemical potentials

\[ Z(\beta, \Omega_i) = \text{Tr} \left( e^{-\beta D + \beta \sum_{i=1}^3 \Omega_i J_i} \right) \quad (2.2) \]

where $\beta = 1/T$ is the inverse temperature, $D$ is the dilatation operator and the trace is taken over all gauge invariant states, corresponding to all the multi-trace operators. It is convenient to combine the R-charges $J_1$ and $J_2$ into the following charges

\[ J \equiv J_1 + J_2, \quad S_z \equiv \frac{1}{2}(J_1 - J_2). \quad (2.3) \]

At weak coupling and in the decoupling limit (2.1) the partition function (2.2) reduces to

\[ Z(\tilde{\beta}) = \text{Tr}_H \left( e^{-\tilde{\beta} H} \right) \quad (2.4) \]

with $H$ being the Hamiltonian $H = D_0 + \tilde{\lambda} D_2$ and $\tilde{\beta} = 1/\tilde{T}$. We see that $SU(N)$ $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$ in the limit (2.1) reduces to a quantum mechanical theory with Hilbert space $\mathcal{H}$ given by the $SU(2)$ sector. $\tilde{T}$ and $\tilde{\lambda}$ can be regarded as the effective temperature and coupling of the theory.

Moreover, in the planar limit $N = \infty$, $\tilde{\lambda} D_2$ becomes the Hamiltonian for the ferromagnetic $XXX_{1/2}$ Heisenberg spin chain (without magnetic field) where [19]

\[ D_2 = \frac{1}{2} \sum_{i=1}^L \left( I_{i,i+1} - P_{i,i+1} \right) \quad (2.5) \]

for a chain of length $L$, where $I_{i,i+1}$ and $P_{i,i+1}$ are the identity operator and the permutation operator acting on letters at position $i$ and $i + 1$. We can therefore write the single-trace partition function as [1]

\[ Z_{ST}(\tilde{\beta}) = \sum_{L=1}^\infty e^{-\tilde{\beta}L} Z_L^{(XXX)}(\tilde{\beta}) \quad (2.6) \]

where

\[ Z_L^{(XXX)}(\tilde{\beta}) = \text{Tr}_L \left( e^{-\tilde{\beta} \tilde{\lambda} D_2} \right) \quad (2.7) \]
is the partition function for the ferromagnetic $XXX_{1/2}$ Heisenberg spin chain of length $L$. Note that $\text{Tr}_L$ here refers to the trace over single-trace operators with $J = L$ in the $SU(2)$ sector. The spin chain is required to be periodic and translationally invariant in accordance with the cyclic symmetry of single-trace operators. Using the standard relation between the single-trace and multi-trace partition functions, we get \[1\]

\[
\log Z(\tilde{\beta}) = \sum_{n=1}^{\infty} \sum_{L=1}^{\infty} \frac{1}{n} e^{-\tilde{\beta} n L} Z_{L}(XXX)(n/\tilde{\beta})
\] (2.8)

Therefore, the partition function of planar $SU(N) \ N = 4$ SYM on $\mathbb{R} \times S^3$ in the decoupling limit (2.1) is given exactly by (2.8) from the partition function $Z_{L}(XXX)(\tilde{\beta})$ of the ferromagnetic $XXX_{1/2}$ Heisenberg spin chain [1]. Using this interesting result we obtain a direct connection between the Hagedorn/deconfinement temperature for finite $\tilde{\lambda}$ and the thermodynamics of the Heisenberg spin chain expressed by the relation \[8\]

\[
\tilde{T}_H = \frac{1}{V(\tilde{\lambda}^{-1}\tilde{T}_H)}
\] (2.9)

where $t = \tilde{\lambda}^{-1}\tilde{T}_H$ is the temperature for the ferromagnetic Heisenberg chain with Hamiltonian $D_2$ and $-tV(t)$ is the thermodynamic limit of the free energy per site for the Heisenberg chain. The previous relation can be used to compute the Hagedorn temperature as a function of the coupling $\tilde{\lambda}$. In the large $\tilde{\lambda}$ limit \[2\] the Hagedorn temperature corresponds to the low temperature limit of the Heisenberg chain, and we obtain \[8\]

\[
\tilde{T}_H = (2\pi)^{\frac{1}{3}} \left[ \zeta \left( \frac{3}{2} \right) \right]^{-\frac{2}{3}} \tilde{\lambda}^{\frac{1}{3}}
\] (2.10)

where $\zeta(x)$ is the Riemann zeta function. Note that the low energy behavior of the Heisenberg chain, and thereby of the gauge theory, is tied to the large $\tilde{\lambda}$ limit. In this region the dominant states for the $D_2$ Hamiltonian are the low energy states of the Heisenberg spin chain. In fact, the low energy spectrum consisting of the chiral primary vacua with the magnon spectrum gives rise to the Hagedorn temperature (2.10). In the next section we will show that the same result (2.10) can be obtained by a direct string theory computation.

Before moving to the string theory side, we want to comment on a more general situation where we study thermal $SU(N) \ N = 4$ SYM on $\mathbb{R} \times S^3$ with chemical potentials for the R-charges for the $SU(4)$ R-symmetry taken to be $(\Omega_1, \Omega_2, \Omega_3) = (\Omega + h, \Omega - h, 0)$. We see that for $h = 0$ we have $\Omega_1 = \Omega_2 = \Omega$ as previously considered. In this new situation the decoupling limit is given by \[9\]

\[
\Omega \to 1, \quad \tilde{T} \equiv \frac{T}{1 - \Omega} \text{ fixed}, \quad \tilde{h} \equiv \frac{h}{1 - \Omega} \text{ fixed}, \quad \tilde{\lambda} \equiv \frac{\lambda}{1 - \Omega} \text{ fixed}, \quad N \text{ fixed}
\] (2.11)

The partition function can be written as

\[
Z(\tilde{\beta}, \tilde{h}) = \text{Tr}_H \left( e^{-\tilde{\beta} H} \right)
\] (2.12)

\[2\] The small $\tilde{\lambda}$ regime is related to the high temperature limit of the Heisenberg model and it is analyzed in \[8\].
where the decoupled Hamiltonian \( H = D_0 + \lambda D_z - 2\hbar S_z \) is the Hamiltonian for the ferromagnetic XXX_{1/2} Heisenberg model in the presence of an external magnetic field of magnitude \( \hbar \).

The trace is again restricted to the SU(2) sector. Our new decoupling limit (2.11) generalizes the limit (2.1) found in [1]. In fact it reduces to that for \( \hbar = 0 \). We can in principle compute the full partition function (2.12) for any value of \( \lambda \) and \( \hbar \). We thus have an extra parameter \( \hbar \) that can be regarded both as a magnetic field, and also as an effective chemical potential.

To compute the Hagedorn temperature we then use the relation (2.9) and we obtain [9]

\[
\tilde{T}_H = \frac{(2\pi)^{\frac{1}{2}} (1 - \hbar)^{\frac{3}{2}}}{\zeta(\frac{3}{2})^{\frac{3}{2}}} \lambda^\frac{3}{2} + \frac{4 (2\pi)^{\frac{3}{2}} \sqrt{\hbar (1 - \hbar)}^{\frac{3}{2}}}{3 \zeta(\frac{3}{2})^{\frac{3}{2}}} \lambda^\frac{3}{2} + O(\lambda^0). \tag{2.13}
\]

## 3 String theory side

Using the AdS/CFT correspondence, we find the following decoupling limit of string theory on AdS_5 \times S^5, dual to the limit (2.1),

\[
\epsilon \to 0 \ , \quad \tilde{H} \equiv \frac{E - J}{\epsilon} \quad \text{fixed} \ , \quad \tilde{T}_{\text{str}} \equiv \frac{T_{\text{str}}}{\sqrt{\epsilon}} \quad \text{fixed} \ , \quad \tilde{g}_s \equiv \frac{g_s}{\epsilon} \quad \text{fixed} \ , \quad J_i \quad \text{fixed} \tag{3.1}
\]

Here \( E \) is the energy of the strings, \( J_i, i = 1, 2, 3 \), are the angular momenta for the five-sphere, \( J = J_1 + J_2 \), \( g_s \) is the string coupling and \( T_{\text{str}} = R^2/(4\pi l_s^2) = \sqrt{\lambda}/2 \) is the string tension with \( R \) being the AdS radius and \( l_s \) the string length. \( \tilde{H} \) is the effective Hamiltonian for the strings in the decoupling limit. We see that both the string tension \( T_{\text{str}} \) and the string coupling \( g_s \) go to zero in this limit.

As mentioned in the Introduction, we should now take a Penrose limit of the AdS_5 \times S^5 background and then consider the string theory on the resulting pp-wave background. This gives the following pp-wave background with 32 supersymmetries

\[
\frac{d^{2s}}{\sqrt{\epsilon}} = -4dx^+dx^- - \mu^2 \sum_{I=3}^{8} x^I x^I (dx^+)^2 + \sum_{i=1}^{8} dx^i dx^i - 4\mu x^2 dx^1 dx^+ \tag{3.2}
\]

\[
F_{(5)} = 2\mu dx^+ (dx^1 dx^3 dx^4 + dx^5 dx^6 dx^7 dx^8) \tag{3.3}
\]

This background was first found in [20]. It is important to note that in the pp-wave background (3.2)-(3.3) the direction \( x^1 \) is an explicit isometry of the pp-wave [20][14], hence we call this background a pp-wave with a flat direction. The resulting spectrum and level matching condition are given by [3]

\[
\frac{l^2 p^+}{\sqrt{\epsilon}} H_{\text{lc}} = 2f N_0 + \sum_{n \neq 0} [(\omega_n + f) N_n + (\omega_n - f) M_n] + \sum_{n \in \mathbb{Z}} \sum_{I=3}^{8} \omega_n N^{(l)}_n
\]

\[
+ \sum_{n \in \mathbb{Z}} \left[ \sum_{b=1}^{4} \left( \omega_n - \frac{1}{2} f \right) F_n^{(b)} + \sum_{b=5}^{8} \left( \omega_n + \frac{1}{2} f \right) F_n^{(b)} \right] \tag{3.4}
\]

\(^3\text{The pp-wave background (3.2)-(3.3) is related to the maximally supersymmetric pp-wave background of [21][17] by a coordinate transformation [20][14]. Even so, as we shall see in the following, the physics of this pp-wave is rather different, which basically origins in the fact that the coordinate transformation between them depends on } x^+, \text{ i.e. it is time-dependent. See [14] for more comments on this.}\)
where we have defined \( f = \mu l_2 p^+ \) and \( \omega_n = \sqrt{n^2 + f^2} \). Here \( N_n^{(I)} \), \( I = 3, \ldots, 8 \) and \( n \in \mathbb{Z} \), are the number operators for bosonic excitations for the six directions \( x^3, \ldots, x^8 \), while \( N_n, n \in \mathbb{Z}, \) and \( M_n, n \neq 0 \), are the number operators for the two directions \( x^1 \) and \( x^2 \). \( F_n^{(b)}, b = 1, \ldots, 8 \) and \( n \in \mathbb{Z} \), are the number operators for the fermions. It is important to note that there is a vacuum for each value of the momentum along the flat direction, and that momentum is moreover dual to \( J_1 - J_2 \). This is exactly as on the gauge theory/spin chain side where and we have a vacuum for each value of the total spin measured by \( J_1 - J_2 \). Moreover we have a pp-wave spectrum for which all states with \( E = J, J = J_1 + J_2, \) correspond to the string vacua, again as in the gauge theory side.

By then taking the large \( \mu \) limit of the pp-wave

\[
\epsilon \to 0 \ , \ \mu \to \infty \ , \ \tilde{\mu} \equiv \mu \sqrt{\epsilon} \text{ fixed} \ , \ \tilde{H}_\text{lc} \equiv \frac{H_{\text{lc}}}{\epsilon} \text{ fixed} \ , \ \tilde{g}_n \equiv \frac{g_n}{\epsilon} \text{ fixed} \ , \ l_s, p^+ \text{ fixed} \quad (3.6)
\]

which is an implementation of the limit (3.1), we have that the resulting spectrum, expressed in terms of gauge theory quantities via the AdS/CFT correspondence, is given by

\[
\frac{1}{\tilde{\mu}} \tilde{H}_\text{lc} = \frac{2\pi^2 \tilde{\lambda}}{J^2} \sum_{n \neq 0} n^2 M_n \ , \ \sum_{n \neq 0} n M_n = 0 \quad (3.7)
\]

This precisely matches the gauge theory spectrum for large \( \tilde{\lambda} \) and \( J \) in the decoupling limit (2.1). Thus, we can match the spectrum of weakly coupled string theory with weakly coupled gauge theory in the corresponding decoupling limits.

It is not difficult to show that from the matching of the spectra it follows the matching of the Hagedorn temperatures [8] which also on the string side is given by equation (2.10).

We have thus shown that the Hagedorn temperature of type IIB string theory on \( \text{AdS}_5 \times S^5 \) in the decoupling limit (3.1) matches with the Hagedorn/deconfinement temperature (2.10).

The computation of the string theory partition function and Hagedorn temperature can also be done using the full spectrum (3.4). In this case we get

\[
\log Z(a, b, \mu) = \sum_{n=1}^{\infty} \frac{1}{n} \text{Tr} \left( e^{-anH_{\text{lc}, \epsilon} - bn p^+} \right) \quad (3.8)
\]

where the parameters \( a \) and \( b \) can be viewed as inverse temperature and chemical potential, respectively, for the pp-wave strings. For related computations of the string theory partition function and Hagedorn temperature in the presence of background fields that play the role of chemical potentials for the corresponding momenta see for example Ref.s [22, 15]. From eq.n (3.8) we get that the Hagedorn temperature is defined by the following equation

\[
bv \sqrt{a} = l_2^2 \frac{3}{2} \sqrt{2\pi \tilde{\mu}} \quad (3.9)
\]

In order to compare (3.9) with the gauge theory result (2.10) we have to express the parameters \( a \) and \( b \) in terms of the gauge theory quantities and take the limit (3.6). It is easy to see that we get again the result (2.10).
computed in weakly coupled $\mathcal{N} = 4$ SYM in the dual decoupling limit (2.1). This is done in the regime of large $\tilde{\lambda}$. On the string side we obtained the Hagedorn temperature by considering the large $\tilde{\lambda}$ and $J$ limit corresponding to strings on the pp-wave background (3.2)-(3.3) in the decoupling limit (3.6). The result means that in the sector of AdS/CFT defined by the decoupling limits we can indeed show that the Hagedorn temperature for type IIB string theory on the AdS$_5 \times S^5$ background is mapped to the Hagedorn/deconfinement temperature of weakly coupled planar $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$. Thus we have direct evidence that the confinement/deconfinement transition found in weakly coupled planar $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$ is linked to a Hagedorn transition of string theory on AdS$_5 \times S^5$, as conjectured in [10, 11, 12, 13].

A similar computation for the spectrum and Hagedorn temperature can be done for the situation dual to the decoupling limit (2.11). However there are interesting differences between the two cases $\tilde{h} = 0$ and $\tilde{h} \neq 0$. In the first case the vacuum of the spin chain has an $L + 1$ fold degeneracy since the states $\text{Tr}(\text{sym}(Z^{L-M}X^M))$ all have the same energy for $0 \leq M \leq L$. In the case when an external magnetic field is present this degeneracy is removed by the Zeeman term $\tilde{h}S_z$ and $\text{Tr}(Z^L)$ becomes the unique vacuum. An analogous difference is also present on the dual string theory side. It is possible to show that the Hagedorn temperature for the string theory dual to the gauge theory in the decoupling limit (2.11) is given by (2.13).

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