EARLY COSMIC FORMATION OF MASSIVE BLACK HOLES

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submitted to
Astrophysical Journal Letters
ABSTRACT

The evolution of nonlinear density fluctuations around the Jeans mass shortly after cosmological recombination is analyzed using a 3D hydrodynamics/dark–matter code. The Cosmic Background Radiation (CBR) exerts Compton friction on free electrons due to peculiar velocities. The dynamics therefore depends strongly on the gas ionization history. Under a variety of ionization conditions and in systems with or without non-baryonic components, the baryons lose angular momentum efficiently and collapse to form a compact optically–thick object which would probably quickly evolve into a massive black hole. Attention is concentrated on elucidating some of the novel physical effects in early cosmological collapses, but ways in which more realistic calculations might be made and in which the scenario could be incorporated into a more complete cosmogonic model are discussed.

1. INTRODUCTION

Although it has long been known (Dicke and Peebles 1968) that the Jeans mass in the cosmic plasma just after recombination is

$$M_J = 1.27 \times 10^6 M_\odot \Omega_B \Omega_0^{-1/2} h^{-1}$$

in baryons (where $\Omega_B$ and $\Omega_0$ are respectively the baryonic and the total density parameters at present and $h$ is the present Hubble constant in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$), almost all recent discussions of cosmic structure formation concentrate on much larger mass scales collapsing at much later times. The basic justification has been that CBR limits on recombination era density fluctuations on mass scales of

$$\gtrsim 6 \times 10^{12} M_\odot \Omega_0^{-2} h^{-1},$$

combined with popular theoretical fluctuation power spectra, extrapolate to quite small amplitudes at the Jeans mass scale, thus implying that a long period of linear growth must ensue before anything of interest occurs. However, while not unreasonable, these considerations are by no means compelling. In fact, there are no significant direct limits on the fluctuation amplitudes on small mass scales at recombination. Moreover, theoretical knowledge of the power spectrum of initial density fluctuations is tentative and speculative at best, and non-linear perturbations on the Jeans mass scale at recombination are at least a logical possibility. In fact, such an amplitude on small mass scales is compatible with cosmological models involving Primordial Isocurvature Baryon (PIB) perturbations (Peebles 1987) or topological defects (e.g. Albrecht & Stebbins 1992).

More positively, there are several semi–empirical indications that very early structure formation on small mass scales might have actually occurred. The existence of apparently high mass ($\gtrsim 10^8 M_\odot$) black holes needed to power observed quasars at redshifts up to at least 5 (Turner 1990) could be more easily understood if some population of far smaller mass black
holes formed early enough to grow to the required size by accretion. The increasingly stringent limits on small angular scale fluctuations in the CBR could be more easily accommodated in many structure formation scenarios if the cosmic plasma were re-ionized at redshifts 30–100, thus hiding the primordial signal (Doroshkevich, Zel’dovich, & Novikov 1967; Peebles 1987; Gouda & Sugiyama 1992). The fact that the IGM is observed to be ionized out to redshifts of at least 5 (Schneider, Schmidt, & Gunn 1991) adds plausibility to the idea. Aside from the possibility of re-ionization by decaying particles (Fukugita & Kawasaki 1992, and references therein), these scenarios require substantial early structure formation to provide the energy needed to ionize the IGM via stars or black hole accretion. Another intriguing model, recently suggested by Gnedin and Ostriker (1992), would allow the dynamically detected dark matter in the universe to be baryonic given a just post-recombination population of black holes producing a gamma-ray radiation field that would modify the results of Big Bang light element nucleosynthesis. Also, black holes could be a possible resolution for the composition of the dark halos around galaxies and the heating sources of stellar disks (Lacy and Ostriker 1985). Furthermore, the existence of substantial heavy element abundances in intergalactic clouds at redshifts above 3 (Steidel and Sargent 1988), in intracluster gas far from the galaxies at low redshift (Hatsukade 1989), and in Pop II stars (da Costa 1991; Carney 1992; Suntzeff 1992) also points to an early, pre-galactic era of structure formation (Matsuda, Sato, & Takeda 1969; Truran & Cameron 1971; Yoneyama 1972; Peebles 1974; Silk 1977, 1983; Fall 1979; Carr, Bond, & Arnet 1984). Finally, even if *typical* fluctuation amplitudes on the Jeans scale were as small as usually imagined at recombination, there would still be a small amount of mass in rare, many $\sigma$ peaks; these unusual structures might be particularly important or interesting precisely because they could form and influence their surroundings long before most of the action took place.

For all of these reasons, and also simply because it has been insufficiently explored to date, we here report a preliminary numerical investigation of structure formation on Jeans mass scales immediately following the recombination epoch. Loeb (1993) recently pointed out that structure formation at redshifts above $\sim 160$ will be strongly influenced by radiative drag effects on the baryonic components of any fluctuations if the gas becomes ionized or contains dust. In particular, his analytic treatment of the evolution of very idealized perturbations showed that the Compton drag could suppress the centrifical barriers produced by the mutual tidal spin ups of neighboring objects expected during gravitational structure formation (Hoyle 1949; Peebles 1969; Ryden 1988; Heavens & Peacock 1988). This would make the formation
of very compact systems, and in particular black holes, much more likely (Fowler 1966; Fricke 1973). In this Letter, we report a preliminary numerical study of more realistic, though still quite idealized, early structure formation using a newly developed code (Umemura et al. 1993) devised specifically for this purpose. At this stage our goal is not to present a well developed theory, but rather to make an initial survey of some of the major physical processes and of how they affect the resulting structures. We shall concentrate particularly on the possibility of efficiently forming very compact objects, which might be or rapidly evolve into black holes.

§2 of this Letter describes the calculations we have performed, the physical processes included, the models considered, and the numerical techniques employed. The results are presented in §3 and conclusions and discussion are given in §4.

2. FORMULATION OF THE PROBLEM

The Compton friction force exerted by the Cosmic Background Radiation (CBR) at a redshift $z$ is proportional to the electron peculiar velocity by the factor, $\alpha(z) = \alpha_0 (1+z)^4$, 
\[ \alpha_0 = \frac{4 \sigma_T \varepsilon_{\gamma 0} \chi_e}{3 \mu m_p c} \]  
where $\sigma_T$ is the Thomson cross section, $\varepsilon_{\gamma 0}$ is the present energy density of the CBR, $\chi_e$ is the ionization fraction, and $\mu$ is the molecular weight of the gas. The corresponding equation of motion for the plasma is,
\[ \frac{dv}{dt} = -\nabla \Phi - \frac{1}{\rho} \nabla p + F \times r - \alpha (v - Hr), \]  
where $v$, $\rho$ and $p$ are the gas velocity, mass density, and pressure, $\Phi$ is the net gravitational potential (including the dark matter), $H(t)$ is the Hubble parameter, and $F \times r$ is an external tidal force. The dark matter particles follow, $d v / d t = -\nabla \Phi + F \times r$. Finally, the thermal history of the gas is described by the energy equation,
\[ \frac{\rho}{\gamma - 1} \frac{d}{dt} \left( \frac{P}{\rho} \right) - \frac{P}{\rho} \frac{d \rho}{dt} = -\Lambda, \]  
where $\gamma = 5/3$ is the adiabatic exponent, and $\Lambda$ is the cooling function combining Compton, Bremsstrahlung, recombination, and line cooling for a primordial abundance with hydrogen and helium (Umemura 1993 et al.).

For simplicity, we consider a quasi–spherical top–hat density perturbation that acquires angular momentum in the $z$–direction from an external tidal field, $|F| = 0.28 (t/t_i)^{2/3} GM_T/R_i^3$,
where $M_T = 2 \times 10^7 M_\odot$ is the total mass of the perturbation and $R_i = 28.4 \text{pc}$ is its initial radius. The amplitude of the external torque is fixed so that it increases the spin parameter of the dark matter up to a value of $\lambda \equiv \left| J_T \right| E_T^{-1/2} G^{-1} M_T^{-5/2} = 0.05$ at maximum expansion, where $E_T$ and $J_T$ are the total energy and angular momentum (Peebles 1971, Barnes & Efthathiou 1986, Gunn 1987, Zurek, Quinn, & Salmon 1988), and it rises in time according to linear theory (Hoyle 1949; Peebles 1969; Ryden 1988). The initial amplitude of the perturbation shortly after the cosmological recombination ($z = 10^3$) is set to $\delta \rho / \rho = 2$ for both the baryons and the dark matter on the corresponding mass scale.

The effects of the Compton coupling to the CBR depend strongly on the ionization history of the gas involved in the perturbation. We parametrize this dependence by assuming partial ionization of $\chi_{HII} = 4 \times 10^{-4}$, $\chi_{HeII} = 5 \times 10^{-5}$, and $\chi_{HeIII} = 10^{-12}$ before a redshift $z_{ion}$ and complete ionization afterwards. Different examples are tabulated in Table 1. Models 2, 4, & 5 with $z_{ion}$ later than the collapsing redshift, $z_c = 463$, can be associated with internal sources of UV radiation like massive stars; model 3 with $z_{ion} = 1000$ assumes an external source of ionizing radiation. In all cases the Hubble constant is $H_0 = 50 \text{km s}^{-1} \text{Mpc}^{-1}$.

During the evolution of the gas cloud we calculate its radial optical depth around the center of mass (averaged over angles), $\tau \equiv \int \langle n_e \rangle \sigma_T dr$, where $n_e \equiv \rho \chi_e / m_p$ is the electron density. If $\tau$ exceeds unity in a sphere of radius $r$ the Compton coupling of the gas enclosed by this sphere to the CBR is turned–off (in reality, the CBR affects only the outer layer of one optical depth in thickness). The non–sphericity of the gas distribution is averaged out because of numerical difficulties in applying the above algorithm in 3D. If the gas cloud becomes thick with a small amount of rotation it will eventually cool and form a quasi–spherical supermassive star which is supported by radiation pressure and eventually collapses to a black hole after $\sim 10^4 \text{yr} \times (M / 10^5 M_\odot)^{-1}$ (Bisnovatyi–Kogan, Zeldovich & Novikov 1967, Shapiro & Teukolsky 1983). The radius of the star is related to its central temperature $T_c$ and mass $M_s$ by the relation (Wagoner 1969), $R_s = 0.06 \text{pc} \times (M_s / 10^5 M_\odot)^{1/2} (T_c / 10^4 \text{K})^{-1}$; the star becomes gravitationally unstable to a free–fall collapse into a black hole when $R_s = 0.5 \text{A.U.} \times (M_s / 10^5 M_\odot)^{3/2}$ (Shapiro & Teukolsky 1983). On the other hand if rotation is dominant, the cloud would cool and form a low–entropy rotationally–supported configuration (a supermassive disk) that could slowly shrink to form a black hole at its center due to angular momentum transport by viscous effects (Wagoner 1969; Loeb & Rasio 1993).

The nonlinear dynamics of the gas is followed by a Smooth Particle Hydrodynamics (SPH) code that was optimized for the present calculation (Umemura et al. 1993). The SPH method
is combined with an $N$-body scheme in which the gravitational interactions are calculated by direct summations using the special purpose processor GRAPE-1A (Fukushige et al. 1992). The combined code was tested extensively (Umemura et al. 1993); in particular, simulations with several thousand particles conserve the total energy and momentum to an accuracy of $0.2 - 0.4\%$ and $2 - 5 \times 10^{-5}$, respectively. In this letter we use 5000 particles that are randomly distributed inside a sphere of radius $R_i$ at the initial redshift, $z = 10^3$. In the dark–matter cosmologies (Models 1-4) only half of the particles are gaseous, so that the mass of a gaseous particle is $400M_\odot$ and that of a dark–matter particle is $7600M_\odot$. Also, the spatial resolution is limited in terms of the size of smoothed particles. The typical scale-height of smoothed particles is about 0.08 pc in dense regions.

3. RESULTS

The top–hat perturbation prescribed in §2 would have reached maximum expansion at $z = 752$ and collapsed by $z = 473$ if it were purely spherical. The actual collapse is delayed to $z_c = 463$ due to the induced angular momentum. After the collapse, the dark matter component undergoes violent relaxation to form a core–halo configuration with a core radius of 1.5 pc. Without Compton drag, the baryonic component dissipates its energy and collapses to a disk which is stabilized by the extended dark–matter halo. (This mechanism is similar to that reported by Katz and Gunn (1991)). The disk is characterized by a solid–body rotation law inside 1.8pc with a rotation period of $4.7 \times 10^4$ yr (the age of the universe equals $1.3 \times 10^6$ yr at $z_c$), and by a flat rotation curve for $1.8 < r < 2.7$pc with a rotation velocity of 235 km s$^{-1}$.

In analogy with galactic systems, it is plausible that the collapse to a stable disk will trigger star formation activity. In Models 2 and 4 the gas is assumed to be entirely ionized after the disk forms (see Loeb (1993) for the energetic requirements). For a fully–ionized gas the Compton drag timescale is shorter than the cosmological expansion time by $0.1 \times ([1 + z]/400)^{-5/2}\Omega_0^{1/2}$. Therefore, the disk loses angular momentum very effectively. Figure 1 presents the temporal evolution of the total angular momentum for Model 2. The angular momentum of the dark matter increases until turn around ($z = 752$) due to the external tidal torque. As the cloud collapses, angular momentum is exchanged between dark–matter and baryonic clumps. Although both components show rapid variations, the sum of their angular momenta declines smoothly due to the action of the CBR on the free electrons. Most of the angular momentum is extracted from the baryons. Since the rotation period is shorter than the Compton drag timescale, the gaseous disk shrinks adiabatically. Figure 2 compares the
disk without (Model 1) and with (Model 2) the CBR drag. Between $z = 395$ and 342 the ionized disk shrinks by about a factor of 2 in radius while completing $\sim 8$ rotations. Figure 3 shows the radial distributions of the mass, the centrifugal barrier radius, the density, and the specific angular momentum of the baryons. The centrifugal barrier radius, $j_B^2/GM(r)$, is normalized by the smallest radius of a stable supermassive star with a mass of $10^5 M_\odot$. The gaseous disk in Model 2 shrinks with a nearly solid body rotation due to the potential of the dark matter core, namely

$$\frac{v_\phi^2}{r} = \frac{4\pi}{3} \rho_c r,$$

where $\rho_c \approx \text{const}$ is the core density. Therefore,

$$\frac{\dot{r}}{r} = \frac{\dot{v}_\phi}{v_\phi} = -\alpha(t),$$

This relation holds as long as the disk is optically–thin and supported by a uniform central density. With a constant ionization fraction, equations (4) and (5) yield (Loeb 1993),

$$r_2/r_1 = \exp\left[\frac{2}{5} H_0^{-1} \alpha_0 \left\{(1 + z_2)^{5/2} - (1 + z_1)^{5/2}\right\}\right].$$

For example, $z_1 = 400$ and $z_2 = 342$ provide $r_2/r_1 = 0.2$ for a fully ionized plasma. The predicted decrease in radius is larger by a factor of $\sim 2.5$ than found in Figures 1 and 2b. The actual change is smaller because 35% of the baryonic mass becomes optically–thick by $z = 342$ and the baryons share a fraction of their angular momentum loss with the dark matter. The specific angular momentum of the baryons follows,

$$\frac{j}{j} = \frac{\dot{r}}{r} + \frac{\dot{v}_\phi}{v_\phi} = 2 \frac{\dot{r}}{r}. $$

For purely optically–thin baryons $\dot{j}$ would have been reduced to 5% of its original value in the above redshift interval but the actual reduction is $\sim 2.5^2 \times 5\% = 30\%$ as indicated by Figure 1. The top panel of Figure 3 indicates by arrows the radii at which the optical depth from the center equals unity. Table 1 gives the mass of the central dense core in the gas distribution $M_c$ that is optically thick. This core can potentially lose its angular momentum by shedding mass along the equatorial plane and can eventually collapse to form a massive black hole (Bisnovatyi–Kogan, Zel’dovich & Novikov 1967, Loeb & Rasio 1993). If the ionization precedes the collapse, the dynamics of the the gas cloud is qualitatively different. Model 3 assumes that the ionization takes place at $z = 10^3$. Initially the baryons are dragged with the CBR; although their overdensity does not grow, they lose angular momentum effectively. The
dark matter collapses as before to form a core halo configuration. Later on, the gas near the center falls into the dark matter potential well (Figure 2c). This effect results from the fact that the amount of mass enclosed within the inner baryonic shells is increased appreciably due to the collapse of the dark matter (Loeb 1993). Finally, a compact quasi-spherical object forms as shown in Fig. 3. The inner optically-thick region of the system is likely to form a supermassive star with a mass of $2.7 \times 10^5 M_\odot$ which is 27% of the total baryonic mass. This star could accrete additional mass while cooling at the Eddington luminosity and collapsing to form a black hole on a relatively short timescale (Bisnovatyi-Kogan, Zel’dovich & Novikov 1967, Loeb & Rasio 1993). The mass of the optically-thick core is plotted as a function of redshift in Figure 4.

When the ionization occurs shortly after the collapse (Model 4) the disk is subject to a self-gravitational instability due to its high density. Consequently, it breaks into two parts with masses $\sim 1.5 \times 10^5 M_\odot$. The resulting dense objects are optically-thick, so that their orbital angular momentum cannot be dissipated effectively by Compton drag. This configuration is likely to evolve into a binary system of supermassive stars. Dynamical friction and gravitational radiation could later reduce the orbital angular momentum of the binary.

Finally, we simulated the evolution of a pure baryonic perturbation (Model 5). Since $\Omega(z) = (1 + [\Omega_0^{-1} - 1]/(1 + z))^{-1}$ and standard nucleosynthesis provides $\Omega_{B_0} \approx 0.05 h_{50}^{-2}$ (Walker et al. 1991) we use $\Omega_{B} \approx 1$ at the relevant redshift interval. The ionization is assumed to occur shortly after the collapse. Due to the high central density, the computational timestep is $\sim 10^2$ yr and the dynamical evolution was followed only down to $z = 459$. Even at this relatively early time, a dense compact object is found near the center with $\tau > 1$ and a mass of $1.6 \times 10^7 M_\odot$ which is 80% of the total mass. In fact, as shown in Fig. 3, this model produces a more compact and rapidly spinning central object than any other model. Of course, there is no increase in the system’s total angular momentum (compared to Model 1); rather the collapse just brings higher specific angular momentum material into the inner regions. The equilibrium rotationally supported disk requires higher angular momentum at a given radius to support it than in Model 1 because all of the mass is in collapsing baryons rather than largely in a more diffuse dark matter halo. Also, the Compton friction is suppressed somewhat by the large optical depths implied by all baryonic mass. Most of the Model 5 system is optically shielded immediately at the epoch of collapse and ionization.

4. CONCLUSIONS AND DISCUSSION

The primary conclusion of this investigation is that highly compact baryonic objects, plau-
sible progenitors of massive black holes, can form efficiently shortly after the cosmic recombination epoch in the collapse of density fluctuations having approximately the Jeans mass if the gas component is re-ionized. This conclusion holds in a reasonably wide range of scenarios corresponding to various ionization histories and to models with and without dominant non-baryonic dark matter components. Compton friction of the re-ionized gas against the CBR is a key physical process which can effectively remove angular momentum, thus facilitating the formation of more compact objects. The specialized 3D hydrodynamic and N-body numerical code (Umemura et al. 1993) therefore allows us to confirm the main results and implications of 1D analytic studies of more idealized situations (Loeb 1993).

The simulations have also revealed the rich and complex dynamical behavior which can result from the addition of Compton drag forces to gravitational collapses. In cases in which the gravitationally dominant dark matter component does not feel the drag but the physically crucial baryonic component does, the interplay between the two can be particularly subtle. An example is the way in which the rotating dark matter component can act as a reservoir of angular momentum for the baryons which are constantly losing their spin to the photons (see Figure 1).

The preliminary nature of this first numerical investigation must be emphasized. The simulations could be made much more realistic in many ways; less symmetric initial conditions could be used, more realistic ionization and tidal torque histories could be imposed, more natural and chaotic "shapes" (i.e., not "top hats") could be used, interaction with material outside the original perturbation (i.e., infall) might be considered, star formation resulting in gas depletion and ionization could be modeled, and so forth. Rather than attempting a maximally realistic calculation, we have first tried to concentrate on elucidating the primary physical effects and their consequences.

Furthermore, for the present we have not attempted to place the study of these early collapsing structures in the context of any more general cosmogonic theory. It is nevertheless clear that they are of potential interest in a number of such contexts. For example, structure formation theories such as PIB (Peebles 1987) or those which invoke topological defects will produce substantial nonlinearities on small mass scales (with possibly non–Gaussian probabilities) at the recombination epoch. More generally, if the initial power spectrum of Gaussian density fluctuations is a power law of index \( n > -1 \) at the recombination epoch, then collapses such as those studied here will be important. Even in more conventional cosmogonic scenarios in which most material does not find itself in a collapsing structure until much later epochs...
(when the drag is no longer important), a few very rare such objects can be expected to form and may be of special interest precisely because they are so unusual.

In addition to explaining the origin of the fluctuations, a more complete theory will have to give some account of the subsequent evolution of the compact baryonic objects (supermassive stars or disks) which result from their collapse. It is expected that evolution of a convective supermassive star (Wagoner 1969) will lead to mass shedding and loss of angular momentum (Loeb & Rasio 1993) through marginally stable configurations and cooling will lead to a supermassive black hole. The radiative energy released in this process and by subsequent accretion onto the black holes may have profound cosmological consequences including erasure of CBR anisotropies (Doroshkevich, Zel’dovich, & Novikov 1967; Peebles 1987; Gouda & Sugiyama 1992) and possibly even modification of the light element abundances (Gnedin and Ostriker 1992). The black holes may grow through accretion to become the progenitors of high redshift quasars (Turner 1991). Star formation associated with the predicted cold, high density gas disks could produce further ionization and significant heavy elements at high redshift. This could not only help explain various observations (metals in the oldest known stars and the high redshift IGM, for example) but also might give rise to dust at high redshift. Such dust would further increase the coupling to the CBR and could extend the dynamical importance of the radiative drag to redshifts as low as 60 (Loeb 1993).

In a sense then, our most general conclusion is that the evolution of the universe between redshifts of roughly 1000 to 100 need not have been simple, quiet and uninteresting merely because it is (currently) so difficult to observe and thus given little attention in most models.

ACKNOWLEDGEMENTS

We are grateful to J. E. Gunn, J. P. Ostriker, and D. Weinberg for helpful discussions. One of the authors (MU) appreciates a great deal of kind hospitality at Princeton University Observatory. This work was supported in part by a JSPS/NSF US–Japan Cooperative Research Grant INT-9116745, the Grants-in-Aid of the Ministry of Education, Science, and Culture No. 02740131 and 03740133 (MU), a W. M. Keck Foundation Fellowship (AL), and NASA grants NAGW-2448 and NAGW-2173 (ELT).
| Model | $\Omega_D$ | $\Omega_B$ | $z_{ion}$ | $z_f$  | $M_c$         |
|-------|------------|------------|-----------|--------|---------------|
| 1.     | 0.95       | 0.05       | 0         | 395    | 0             |
| 2.     | 0.95       | 0.05       | 400       | 342    | $3.5 \times 10^5 M_\odot$ |
| 3.     | 0.95       | 0.05       | 1000      | 188    | $2.7 \times 10^5 M_\odot$ |
| 4.     | 0.95       | 0.05       | 463       | 340    | $1.3 \times 10^6 M_\odot$ |
| 5.     | 0          | 1.0        | 463       | 459    | $1.6 \times 10^7 M_\odot$ |
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FIGURE CAPTIONS

FIG. 1 — Angular momenta versus redshift for Model 2. The upper panel shows the total (dotted) and the dark–matter (solid) angular momenta after the re–ionization redshift $z_{ion} = 400$. The lower panel shows the angular momentum of the baryonic component.

FIG. 2 — The spatial distributions of the dark–matter and the baryons at the final stages of the calculation, namely: a) $z = 395$ for Model 1, b) $z = 342$ for Model 2, and c) $z = 188$ for Model 3. The box size is 10 pc.

FIG. 3 — Final radial distribution (around the center of mass) of the gas properties. Top panel shows the baryonic mass within a radius $r$. The mass unit is $10^5 M_\odot$ for Models 1, 2, and 3, and $2 \times 10^6 M_\odot$ for Model 5. The short arrows in this panel show the radius at which the optical depth equals unity. The second panel shows the ratio of the centrifugal barrier radius to the radius of a supermassive star of the mass enclosed within that radius at the onset of gravitational instability $R_s \equiv 0.5 A.U. \times (M_s/10^5 M_\odot)^{3/2}$ (Shapiro & Teukolsky 1983). The third panel shows the mass density, and the bottom panel presents the specific angular momentum of the baryons in the $z$–direction. The unit of specific angular momentum is $pc^2 yr^{-1}$. The various lines refer to the final time of Models 1 (solid), 2 (dot-dashed), 3 (dotted), and 5 (dashed).

FIG. 4 — Mass of the optically–thick core as a function of redshift for Model 3.