Quasar X-ray and UV flux, baryon acoustic oscillation, and Hubble parameter measurement constraints on cosmological model parameters

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ABSTRACT
We use the Risaliti & Lusso (2015) compilation of 808 X-ray and UV flux measurements of quasars (QSOs) in the redshift range $0.061 \leq z \leq 6.28$, alone and in conjunction with baryon acoustic oscillation (BAO) and Hubble parameter $[H(z)]$ measurements, to constrain cosmological parameters in six cosmological models. The QSO data constraints are significantly weaker than, but consistent with, those from the $H(z) + $BAO data. A joint analysis of the QSO + $H(z) + $BAO data is consistent with the current standard model, spatially-flat $\Lambda$CDM, but mildly favors closed spatial hypersurfaces and dynamical dark energy.

Key words: (cosmology:) cosmological parameters – (cosmology:) observations – (cosmology:) dark energy

1 INTRODUCTION
Type Ia supernova (SNIa) apparent magnitude measurements provided the first convincing evidence for accelerated cosmological expansion (see Scolnic et al. (2018) for a recent discussion). Supporting evidence soon came from other cosmological probes, the most significant being cosmic microwave background (CMB) anisotropy data (Plank Collaboration 2018), baryon acoustic oscillation (BAO) distance measurements (Alam et al. 2017), and Hubble parameter $[H(z)]$ observations (Moresco et al. 2016; Farooq et al. 2017). If general relativity is an accurate model of gravitation, hypothetical dark energy is responsible for the observed acceleration of the cosmological expansion. There are many different dark energy models. In this paper we consider three of them and also consider flat and non-flat spatial hypersurfaces in each case, for a total of six cosmological models.

The simplest observationally-consistent dark energy model is the flat $\Lambda$CDM model, the current standard model (Peebles 1984). In this model the accelerated expansion is powered by the spatially homogenous cosmological constant ($\Lambda$) energy density which is constant in time. This model is consistent with most observations when about 70% of the current cosmological energy budget is contributed by dark energy, with about 25% coming from the cold dark matter (CDM), and the remaining 5% due to baryons. The standard model assumes flat spatial hypersurfaces. Current observations allow a little spatial curvature,1 so we can generalise the standard model to the non-flat $\Lambda$CDM model which allows for non-zero spatial curvature energy density.

While the $\Lambda$CDM model is consistent with many observations, its assumption of a time-independent and spatially-homogeneous dark energy density is difficult to theoretically motivate. Also, observations do not require that the dark energy density be time independent, and models in which the dark energy density decreases with time have been studied. Here we consider two dynamical dark energy models, the XCDM parametrization in which an $X$-fluid is the dynamical dark energy and the $\phi$CDM model in which a scalar field $\phi$ is the dynamical dark energy. We also study spatially flat and non-flat versions of both the XCDM parametrization and the $\phi$CDM model.

The main goal of our paper is to use the Risaliti & Lusso (2015) quasar (QSO) X-ray and UV flux measurements to constrain cosmological parameters. Risaliti & Lusso (2015) for discussion of observational constraints on spatial curvature, see Farooq et al. (2015), Chen et al. (2016), Yu & Wang (2016), Wei & Wu (2017), Rana et al. (2017), Ooba et al. (2018a,b,c), DES Collaboration (2018a), Witzemann et al. (2018), Yu et al. (2018), Park & Ratra (2018a,b,c,d, 2019), Mitra et al. (2018), Penton et al. (2018), Xu et al. (2019), Zheng et al. (2019), Ruan et al. (2019), Giambó et al. (2019), Giambó et al. (2019), Coley (2019), Eingorn et al. (2019), Jesus et al. (2019), Handley et al. (2019), and references therein.

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consider cosmological parameter constraints in the non-flat \( \Lambda \)CDM model; here we also consider cosmological parameter constraints in five other cosmological models. In addition, we examine the effect of different Hubble constant priors on the cosmological parameter constraints. By studying constraints in a number of models, we are able to draw somewhat model-independent conclusions about the QSO data constraints. We find that the Risaliti & Lusso (2015) QSO data by themselves do not provide very restrictive constraints on cosmological parameters. However, the QSO constraints are largely consistent with those that follow from the \( H(z) \) + BAO data, and when jointly analyzed the QSO data slightly tighten and shift the \( H(z) \) + BAO data constraints.

The QSO + \( H(z) \) + BAO data are consistent with the standard flat \( \Lambda \)CDM cosmological model although they mildly favor closed spatial hypersurfaces over flat ones and dynamical dark energy over a cosmological constant.

While current QSO data by themselves do not provide restrictive cosmological parameter constraints, the new Risaliti & Lusso (2018) compilation of 1598 QSO measurements will provide tighter constraints, that should be improved upon by near-future QSO data. Currently, CMB anisotropy, BAO, SNIa, and \( H(z) \) data provide the most restrictive constraints on cosmological parameters. To test consistency, and to help tighten cosmological parameter constraints, it is essential that additional cosmological probes, such as the QSO data studied here, be developed.

This paper is organized as follows. In Sec. 2 we describe the models that we use. In Sec. 3 we discuss the data that we use to constrain cosmological parameters in these models. In Sec. 4 we describe the methodology adopted for these analyses. In Sec. 5 we present our results and conclude in Sec. 6.

2 MODELS

In this paper we constrain cosmological parameters of the spatially-flat and non-flat versions of three different dark energy cosmological models, for a total of six cosmological models. For the dark energy we consider a cosmological constant \( \Lambda \) in the \( \Lambda \) cold dark matter (\( \Lambda \)CDM) model, as well as a decreasing dark energy density modeled as an \( X \)-fluid in the XCDM parametrization, or as a scalar field \( \phi \) in the \( \phi \)CDM model.

In the \( \Lambda \)CDM model the Hubble parameter, as a function of redshift \( z \), is

\[
H(z) = H_0 \sqrt{\Omega_{m0}(1 + z)^3 + \Omega_{k0}(1 + z)^2 + \Omega_\Lambda},
\]

and

\[
\Omega_{m0} + \Omega_{k0} + \Omega_\Lambda = 1.
\]

Here \( H_0 \) is the Hubble constant, \( \Omega_{m0} \) and \( \Omega_{k0} \) are the current values of the non-relativistic matter and the spatial curvature energy density parameters and \( \Omega_\Lambda \) is the dark energy density parameter. For the spatially-flat \( \Lambda \)CDM model the free parameters are chosen to be \( \Omega_{m0} \) and \( H_0 \). For the spatially non-flat \( \Lambda \)CDM model the free parameters are chosen to be \( \Omega_{m0}, \Omega_\Lambda, \) and \( H_0 \).

In the XCDM parametrization, the dark energy density is dynamical and decreases with time. The equation of state for the dark energy fluid is \( P_X = \omega_X \rho_X \). Here \( P_X \) is the pressure of the \( X \)-fluid, \( \rho_X \) is the energy density of that fluid, and \( \omega_X \) is the equation of state parameter whose value is negative (\( \omega_X < -1/3 \)). In this model the Hubble parameter is

\[
H(z) = H_0 \sqrt{\Omega_{m0}(1 + z)^3 + \Omega_{k0}(1 + z)^2 + \Omega_\Lambda(1 + z)^{3(1 + \omega_X)}},
\]

and

\[
\Omega_{m0} + \Omega_{k0} + \Omega_{\Lambda0} = 1.
\]

Here \( \Omega_{\Lambda0} \) is the current value of the \( X \)-fluid energy density parameter. For the spatially-flat case the free parameters are \( \Omega_{m0}, \omega_X, \) and \( H_0 \). For the non-flat case the free parameters are \( \Omega_{m0}, \Omega_{k0}, \omega_X, \) and \( H_0 \). In the \( \omega_X = -1 \) limit the XCDM model is the \( \Lambda \)CDM model.

In the \( \phi \)CDM model dark energy is modeled as a scalar field \( \phi \) with potential energy density \( V(\phi) \) (Peebles & Ratra 1988; Ratra & Peebles 1988; Pavlov et al. 2013). In this model the dark energy density is dynamical and decreases with time. A widely used \( V(\phi) \) of the inverse power law form,

\[
V(\phi) = \frac{1}{2} \kappa m_p^2 \phi^{-\alpha},
\]

where \( m_p \) is the Planck mass, \( \alpha \) is a positive parameter, and

\[
\kappa = \frac{8}{3} \left( \frac{\alpha + 4}{\alpha + 2} \right)^{2} \left[ \frac{3}{2} \alpha (\alpha + 2) \right]^{1/2}.
\]

In this model the equations of motion are

\[
\ddot{\phi} + \dot{\phi} \frac{3a}{\dot{a}} - \frac{1}{2} \kappa m_p^2 \phi^{-\alpha-1} = 0,
\]

and

\[
\left( \frac{a}{\dot{a}} \right)^2 = \frac{8\pi G}{3} \left( \rho_m + \rho_\phi \right) - \frac{k}{a^2},
\]

where \( a \) is the scale factor, overdots denote derivatives with respect to time, \( k \) is positive, zero, and negative for closed, flat, and open spatial hypersurfaces, \( \rho_m \) is the non-relativistic matter density, and the scalar field energy density is

\[
\rho_\phi = \frac{m_p^2}{32\pi} \left[ \dot{\phi}^2 + \kappa m_p^2 \phi^{-\alpha} \right].
\]

So, the Hubble parameter in this model is

\[
H(z) = H_0 \sqrt{\Omega_{m0}(1 + z)^3 + \Omega_{k0}(1 + z)^2 + \Omega_\phi(z, \alpha)},
\]

where the scalar field energy density parameter

\[
\Omega_\phi(z, \alpha) = \frac{8\pi G \rho_\phi}{3H_0^2}
\]

Discussion of constraints on the \( \phi \)CDM model may be traced through Chen & Ratra (2004), Samushia et al. (2007), Yashar et al. (2009), Samushia & Ratra (2010), Samushia et al. (2010), Chen & Ratra (2011b), Campanelli et al. (2012), Farooq & Ratra (2013), Farooq et al. (2013), Avsajanishvili et al. (2015), Sōla et al. (2017), Sōla Peracaula et al. (2018, 2019), Zhai et al. (2017), Sangwan et al. (2018), Singh et al. (2019), Mitra (2019), and references therein.
where $G$ is the gravitational constant, and $\Omega_m(z, \alpha)$ has to be numerically computed. For the spatially non-flat $\phi$CDM model the free parameters are $\Omega_m0, \Omega_k0, \alpha,$ and $H_0$. For the spatially-flat $\phi$CDM model the free parameters are $\Omega_m0, \alpha,$ and $H_0$. In the limit $\alpha \to 0$, the $\phi$CDM model reduces to the $\Lambda$CDM model.

### 3 DATA

We use three different data sets to constrain cosmological parameters. The main purpose of our paper is to use the 808 QSO X-ray and UV luminosities of quasars. This relation is $\log L_X = \beta + \gamma \log L_{UV}$, extending over a redshift range of $0.061 \leq z \leq 6.28$, to determine cosmological parameter constraints, and to compare these QSO cosmological parameter constraints to those determined from more widely used BAO distance measurements and $H(z)$ observations. The BAO and $H(z)$ data we use are listed in Tables 1 and 2 of Ryan et al. (2018) and consist of 11 BAO measurements over the redshift range $0.106 \leq z \leq 2.36$ and 31 $H(z)$ measurements over the redshift range $0.07 \leq z \leq 1.965$.

### 4 METHOD

As described in Risaliti & Lusso (2015), the method of analysis depends on the non-linear relation between the X-ray and UV luminosities of quasars. This relation is

$$\log(L_X) = \beta + \gamma \log(L_{UV}),$$

(12)

where $\log = \log_{10}$ and $L_X$ and $L_{UV}$ are the QSO X-ray and UV luminosities, $\beta$ and $\gamma$ are free parameters to be determined by using the data.

Expressing the luminosity in terms of the flux, we obtain

$$\log(F_X) = \beta + (\gamma - 1) \log(4\pi) + \gamma \log(F_{UV}) + 2(\gamma - 1) \log(D_L).$$

(13)

where $F_X$ and $F_{UV}$ are the X-ray and UV fluxes respectively. Here $D_L$ is the luminosity distance, which is a function of redshift and cosmological parameters, which will allow us to constrain the cosmological model parameters. The luminosity distance $D_L(z, p)$ is given by

$$\frac{H_0 \sqrt{\Omega_k0 D_L(z, p)}}{1 + z} = \sinh[g(z)]$$

$$\begin{cases} 
\text{if } \Omega_k0 > 0, \\
\text{if } \Omega_k0 = 0, \\
\sin[g(z)] 
\end{cases}$$

(14)

where $p$ is the set of cosmological model parameters, $g(z) = H_0 \sqrt{\Omega_k0} \int_z^\infty \frac{dz'}{H(z')}$

(15)

and $H(z)$, which is a function of cosmological model parameters, is given in Sec. 2 for the six cosmological models we study in this paper.

We determine the best-fit values and uncertainty of the parameters for a given model by maximizing the likelihood function. For QSO data, we have the observed X-ray flux and we can predict the X-ray flux at given redshift as a function of cosmological parameters by using eqs. (13) and (14). So, the likelihood function (LF) for QSO data is

$$\ln(\text{LF}) = -\frac{1}{2} \sum_{i=1}^{808} \left[ \log(F^\text{obs}_{X,i}) - \log(F^\text{th}_{X,i}) \right]^2 + \ln(2\pi \sigma_i^2),$$

(16)

where $\ln = \log_{e}$ and $\sigma_i^2 = \sigma^2 + \delta^2$, where $\sigma_i$ and $\delta$ are the measurement error on $F^\text{th}_{X,i}$ and the global intrinsic dispersion respectively. We treat $\delta$ as a free parameter to be determined by the data, along with the other two free parameters, $\beta$ and $\gamma$, which characterise the $L_X - L_{UV}$ relation in eq. (12). In eq. (16) $F^\text{th}_{X,i}$ is the corresponding model prediction defined through eq. (13), and is a function of $F^\text{th}_{UV}$ and $D_L(z, p)$.

This method of analysis follows Ryan et al. (2018). The likelihood function for the uncorrelated BAO and $H(z)$ data is

$$\ln(\text{LF}) = -\frac{1}{2} \sum_{i=1}^{N} \left[ A_{\text{obs}}(z_i) - A_{\text{th}}(z_i, p) \right]^2,$$

(17)

where $A_{\text{obs}}(z_i)$ and $\sigma_i$ are the measured quantity and error at redshift $z_i$ and $A_{\text{th}}(z_i, p)$ is the corresponding model-predicted value. The measurements in the first six lines of Table 1 of Ryan et al. (2018) are correlated and the likelihood function for those data points is

$$\ln(\text{LF}) = -\frac{1}{2} \sum_{i=1}^{N} \left[ A_{\text{obs}}(z_i) - A_{\text{th}}(z_i, p) \right]^2 C^{-1}[A_{\text{obs}}(z_i) - A_{\text{th}}(z_i, p)],$$

(18)

where $C^{-1}$ is the inverse of the covariance matrix $C$ (Ryan et al. 2019).

$$\begin{bmatrix}
624.707 & 23.729 & 325.332 & 8.34963 & 157.386 & 3.57778 \\
23.729 & 5.60873 & 11.6429 & 2.33996 & 6.39263 & 0.968056 \\
325.332 & 11.6429 & 905.777 & 29.3392 & 515.271 & 14.1013 \\
8.34963 & 2.33996 & 29.3392 & 5.42327 & 16.1422 & 2.85334 \\
157.386 & 6.39263 & 515.271 & 16.1422 & 1375.12 & 40.4327 \\
3.57778 & 0.968056 & 14.1013 & 2.85334 & 40.4327 & 6.25936
\end{bmatrix}$$

(19)

For all parameters except for $H_0$, we assume top hat priors, non-zero over $0 \leq \Omega_m0 \leq 1, 0 \leq \Omega_k \leq 1.3, -0.7 \leq k \leq 0.7, -5 \leq \omega_X \leq 5, 0 \leq a \leq 1.2, -20 \leq \ln \delta \leq 10, 0 \leq \beta \leq 11,$ and $-2 \leq \gamma \leq 2$. Here $k = -\Omega_k0 a^2$ where $a_0$ is the current value of the scale factor. For the Hubble constant we use two different Gaussian priors, $H_0 = 68 \pm 2.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ corresponding to the results of a median statistics analysis of a large compilation of $H_0$ measurements (Chen & Ratra 2011a), and $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from a recent local expansion rate measurement (Riess et al. 2016).
The likelihood analysis is performed using the Markov chain Monte Carlo (MCMC) method as implemented in the emcee package (Foreman-Mackey et al. 2013) in Python 3.7. By using the maximum likelihood value \( L_{\text{max}} \) we compute the minimum \( \chi^2_{\text{min}} \) value \(-2\ln(L_{\text{max}})\). In addition to \( \chi^2_{\text{min}} \) we also use the Akaike Information Criterion

\[
AIC = \chi^2_{\text{min}} + 2d
\]

and the Bayes Information Criterion

\[
BIC = \chi^2_{\text{min}} + d \ln N
\]

(Ryan et al. 2018), where \( d \) is the number of free parameters, and \( N \) is the number of data points. The \( AIC \) and \( BIC \) penalize models with a larger number of free parameters.

5 RESULTS

5.1 \( H(z) + \) BAO constraints

Results for the \( H(z) + \) BAO data set are listed in Tables 1–3. The un marginalized best-fit parameters are in Tables 1 and 2. The marginalized one-dimensional best-fit parameter values with 1σ error bars are given in Table 3. These results are consistent with those of Ryan et al. (2018). The slight differences between the two sets of results are the consequence of the different analysis techniques used. Our computations are done using the MCMC method while Ryan et al. (2018) used a grid-based \( \chi^2 \) technique.

The one-dimensional likelihoods and two-dimensional 1σ, 2σ, and 3σ confidence contours for all parameters determined by using \( H(z) + \) BAO data are shown in red in Figs. 1–12. Plots for the \( \phi \)CDM model differ slightly from the corresponding plots of Ryan et al. (2018) because of the difference we discussed above and because we have used a smaller range of \( a \) than what Ryan et al. (2018) used. The \( H(z) + \) BAO data reduced \( \chi^2 \) values are \( \sim 0.6-0.8 \), smaller than unity largely because of the \( H(z) \) data.

5.2 QSO constraints

Use of the QSO data to constrain cosmological parameters is based on the assumed validity of the \( L_X - L_{\text{UV}} \) relation in eq. (12). This assumption is tested by Risaliti & Lusso (2015). By fitting this relation in cosmological models we have found, in agreement with Risaliti & Lusso (2015), that the slope \( \gamma = 0.60 \pm 0.02 \), the intercept \( \beta \) is between 8 and 9, and the global intrinsic dispersion \( \delta = 0.32 \pm 0.08 \). The global intrinsic dispersion is large and so cosmological parameter determination done using these data is not as precise as that done by using, for example, the SNIa data. But the main advantage of using the quasar sample is that it covers a very large redshift range and eventually with more and better quality data it should result in tight constraints.

The QSO data determined cosmological parameter results are given in Tables 1, 2, and 4. The un marginalized best-fit parameters are given in the Tables 1 and 2 for the \( H_0 = 68 \pm 2.8 \text{ km s}^{-1}\text{Mpc}^{-1} \) and 73.24 \pm 1.74 \text{ km s}^{-1}\text{Mpc}^{-1} \) priors respectively. The one-dimensional likelihoods and the two-dimensional confidence contours are shown in grey in the left panels of Figs. 1–12. The cosmological parameter constraints are insensitive to the \( H_0 \) prior used. Risaliti & Lusso (2015) have determined cosmological parameters for the non-flat \( \Lambda \)CDM model. Our QSO data constraints in the \( \Omega_m - \Omega_m \) sub-panels of our Figs. 3 and 4 agree well with the corresponding constraints in Fig. 6 of Risaliti & Lusso (2015). For the non-flat \( \Lambda \)CDM model we find \( \Omega_m = 0.24_{-0.10}^{+0.16} \) and \( \Omega_\Lambda = 0.93_{-0.39}^{+0.18} \) also in good agreement with the corresponding Risaliti & Lusso (2015) values of \( \Omega_m = 0.22_{-0.08}^{+0.10} \) and \( \Omega_\Lambda = 0.92_{-0.30}^{+0.18} \).

The cosmological parameters obtained by using these QSO data have relatively high uncertainty for all models but they are mostly consistent with the results obtained by using the BAO + \( H(z) \) data set, which are shown in red in Figs. 1–12. The QSO data reduced \( \chi^2 \) values are also small \( \sim 0.6-0.7 \); it is of interest to understand why this is so. The QSO data \( \chi^2_{\text{min}} \) values do not change significantly from model to model.

5.3 QSO + \( H(z) + \) BAO constraints

From Figs. 1–12 we see that the constraints from \( H(z) + \) BAO data and those from QSO data alone are largely mutually consistent, except for the \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1}\text{Mpc}^{-1} \) prior case in the flat \( \Lambda \)CDM and flat \( \phi \)CDM models, see the bottom left sub-panels in the left panels of Figs. 2 and 10. The \( H(z) + \) BAO data constraint cosmological parameters quite tightly while the QSO data result in very loose constraints on these parameters. Although the QSO data alone are not able to provide restrictive constraints, they can help tighten constraints when used in combination with \( H(z) + \) BAO data.

Given that the QSO and the \( H(z) + \) BAO constraints are mostly consistent, it is reasonable to determine joint QSO + \( H(z) + \) BAO constraints. These results are given in Tables 1, 2, and 5. The QSO + \( H(z) + \) BAO one-dimensional likelihoods and two-dimensional confidence contours for all the cosmological parameters are shown in blue in the right panels of Figs. 1–12. These figures also show the \( H(z) + \) BAO data constraint contours in red. Adding the QSO data to the \( H(z) + \) BAO data and deriving joint constraints on cosmological parameters, results in bigger effects for the case of the \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1}\text{Mpc}^{-1} \) prior (Figs. 2, 4, 6, 8, 10, & 12), and in the flat \( \phi \)CDM model for both priors (Figs. 9 & 10).

Using the combined data set we see, from Table 5, the non-relativistic matter density parameter is measured to lie in the range \( \Omega_m = 0.30_{-0.01}^{+0.01} \) to \( 0.31_{-0.01}^{+0.01} \) (\( \Omega_m = 0.30_{-0.01}^{+0.01} \) to \( 0.32_{-0.01}^{+0.02} \) for flat (non-flat) models and the \( H_0 = 68 \pm 2.8 \text{ km s}^{-1}\text{Mpc}^{-1} \) prior and to lie in the range \( \Omega_m = 0.29_{-0.01}^{+0.01} \) to \( 0.32_{-0.01}^{+0.02} \) (flat (non-flat) models and the \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1}\text{Mpc}^{-1} \) prior). In some cases these results differ slightly from the \( H(z) + \) BAO data results of Table 3. These results are consistent with those derived using other data.

The Hubble constant is found to lie in the range \( H_0 = 67.06_{-1.15}^{+1.15} \) to \( 68.11_{-0.91}^{+0.91} \) (\( H_0 = 67.93_{-1.53}^{+1.53} \) to \( 68.84_{-1.48}^{+1.48} \) km s\(^{-1}\)Mpc\(^{-1}\) for flat (non-flat) models and the \( H_0 = 68 \pm 2.8 \text{ km s}^{-1}\text{Mpc}^{-1} \) prior and to lie in the range \( H_0 = 70.07_{-0.84}^{+0.83} \) to \( 72.35 \pm 1.13 \) (\( H_0 = 71.95_{-1.07}^{+1.08} \) to \( 72.31_{-1.08}^{+1.09} \) km s\(^{-1}\)Mpc\(^{-1}\) for flat (non-flat) models and the \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1}\text{Mpc}^{-1} \) prior. As expected, for the \( H_0 = 73.24 \pm 1.74 \text{ km s}^{-1}\text{Mpc}^{-1} \) prior case, the measured value of \( H_0 \) is pulled lower than the
Table 1. Unmarginalized best-fit parameters of all models for the $H_0 = 68 \pm 2.8$ km s$^{-1}$ Mpc$^{-1}$ prior.

| Model          | Data set | $\Omega_{m0}$ | $\Omega_A$ | $\Omega_{X0}$ | $\omega_X$ | $\alpha$ | $H_0$ | $\delta$ | $\beta$ | $\gamma$ | $\chi_{min}^2$ | AIC | BIC |
|----------------|----------|---------------|------------|---------------|------------|----------|-------|----------|---------|----------|----------------|-----|-----|
| Flat ΛCDM      | $H(z) +$ BAO | 0.30 | 0.70 | - | - | - | 68.14 | - | - | - | 25.65 | 29.65 | 33.13 |
|                | QSO      | 0.20 | 0.80 | - | - | - | 68.00 | 0.32 | 8.29 | 0.59 | 468.94 | 478.94 | 502.41 |
| Non-flat ΛCDM  | $H(z) +$ BAO | 0.30 | 0.70 | - | - | - | 68.10 | 0.32 | 8.54 | 0.58 | 499.08 | 509.08 | 532.81 |
| Flat ΦCDM      | $H(z) +$ BAO | 0.31 | 0.69 | - | -0.98 | - | 67.69 | - | - | - | 25.58 | 31.58 | 36.78 |
| Non-flat ΦCDM  | $H(z) +$ BAO | 0.33 | - | -0.25 | -0.73 | - | 65.78 | - | - | - | 23.52 | 31.52 | 38.47 |
| Flat XCDM      | $H(z) +$ BAO | 0.31 | 0.69 | - | -0.98 | - | 67.69 | - | - | - | 25.58 | 31.58 | 36.78 |
| Non-flat XCDM  | $H(z) +$ BAO | 0.33 | - | -0.25 | -0.73 | - | 65.78 | - | - | - | 23.52 | 31.52 | 38.47 |
| Flat $\phi$CDM | $H(z) +$ BAO | 0.33 | - | -0.25 | -0.67 | - | 68.00 | 0.32 | 8.65 | 0.58 | 463.10 | 477.10 | 509.96 |
| Non-flat $\phi$CDM | $H(z) +$ BAO | 0.33 | - | -0.25 | -0.78 | - | 67.91 | 0.32 | 8.77 | 0.57 | 493.33 | 507.33 | 540.55 |

$\Omega$ km s$^{-1}$Mpc$^{-1}$.

Table 2. Unmarginalized best-fit parameters of all models for the $H_0 = 73.24 \pm 1.74$ km s$^{-1}$ Mpc$^{-1}$ prior.

| Model          | Data set | $\Omega_{m0}$ | $\Omega_A$ | $\Omega_{X0}$ | $\omega_X$ | $\alpha$ | $H_0$ | $\delta$ | $\beta$ | $\gamma$ | $\chi_{min}^2$ | AIC | BIC |
|----------------|----------|---------------|------------|---------------|------------|----------|-------|----------|---------|----------|----------------|-----|-----|
| Flat ΛCDM      | $H(z) +$ BAO | 0.31 | 0.69 | - | - | - | 69.55 | - | - | - | 30.89 | 34.89 | 38.37 |
|                | QSO      | 0.20 | 0.80 | - | - | - | 73.24 | 0.32 | 8.21 | 0.59 | 468.94 | 478.94 | 502.41 |
| Non-flat ΛCDM  | $H(z) +$ BAO | 0.31 | 0.78 | -0.09 | - | - | 71.68 | - | - | - | 26.79 | 32.79 | 38.37 |
| Flat ΦCDM      | $H(z) +$ BAO | 0.31 | 0.69 | - | - | - | 70.35 | 0.32 | 8.54 | 0.58 | 506.97 | 516.08 | 540.69 |
| Non-flat ΦCDM  | $H(z) +$ BAO | 0.33 | 0.79 | -0.10 | - | - | 72.34 | 0.32 | 8.63 | 0.58 | 499.26 | 511.26 | 543.73 |
| Flat XCDM      | $H(z) +$ BAO | 0.30 | 0.70 | -1.13 | - | - | 71.55 | - | - | - | 27.95 | 33.95 | 39.16 |
| Non-flat XCDM  | $H(z) +$ BAO | 0.30 | 0.70 | -1.13 | - | - | 71.55 | - | - | - | 27.95 | 33.95 | 39.16 |
| Flat $\phi$CDM | $H(z) +$ BAO | 0.30 | 0.70 | -1.13 | - | - | 71.55 | - | - | - | 27.95 | 33.95 | 39.16 |
| Non-flat $\phi$CDM | $H(z) +$ BAO | 0.33 | 0.69 | - | -0.99 | - | 67.91 | 0.32 | 8.77 | 0.57 | 493.33 | 507.33 | 540.55 |

$\Omega$ km s$^{-1}$Mpc$^{-1}$.

Prior value because the $H(z)$ and BAO data favor a lower $H_0$.

For the non-flat ΛCDM model the curvature energy density parameter is measured to be $\Omega_k = -0.01^{+0.06}_{-0.07}$ and $-0.10 \pm 0.05$ for the $H_0 = 68 \pm 2.8$ km s$^{-1}$Mpc$^{-1}$ and $73.24 \pm 1.74$ km s$^{-1}$Mpc$^{-1}$ priors respectively. The curvature energy density parameter is found to be $\Omega_k = -0.16^{+0.14}_{-0.17}$ and $-0.15 \pm 0.09$ for the non-flat XCDM and non-flat $\phi$CDM models for the $H_0 = 68 \pm 2.8$ km s$^{-1}$Mpc$^{-1}$ prior and $\Omega_k = -0.12^{+0.12}_{-0.14}$ and $-0.21^{+0.09}_{-0.11}$ for the non-flat ΛCDM and non-flat $\phi$CDM models for the $H_0 = 73.24 \pm 1.74$ km s$^{-1}$Mpc$^{-1}$ prior. It is interesting that in all cases closed spatial hypersurfaces are favored, albeit just barely in the non-flat ΛCDM model with the $H_0 = 68 \pm 2.8$ km s$^{-1}$Mpc$^{-1}$ prior, and only at 1σ for most other cases, but at more than 2σ for the non-flat $\phi$CDM model and the $H_0 = 73.24 \pm 1.74$ km s$^{-1}$Mpc$^{-1}$ prior. This preference for closed spatial hypersurfaces is largely driven by the $H(z) +$ BAO data (Park &
Table 3. Marginalized one-dimensional best-fit parameters with 1σ confidence intervals for all models using BAO and $H(z)$ data.

| $H_0^{a}$ prior | Model | $\Omega_{m0}$ | $\Omega_{\Lambda}$ | $\Omega_{k0}$ | $\omega_X$ | $\alpha$ | $H_0^{a}$ |
|-----------------|-------|----------------|-------------------|--------------|-----------|-------|-----------|
| Flat ΛCDM       | 0.28±0.17 | 0.70±0.01 | -0.01±0.00 | -0.32±0.01 | 0.53±0.02 | 0.59±0.02 |
| Non-flat ΛCDM   | 0.28±0.12 | 0.93±0.18 | -0.17±0.34 | -0.32±0.07 | 0.54±0.43 | 0.45±0.02 |
| Flat XCDM       | 0.20±0.13 | 0.11±0.66 | -0.21±0.11 | -0.32±0.09 | 0.55±0.43 | 0.45±0.02 |
| Non-flat XCDM   | 0.22±0.11 | 0.32±0.02 | -0.23±0.11 | -0.32±0.09 | 0.55±0.43 | 0.45±0.02 |

$^a$ km s$^{-1}$Mpc$^{-1}$.

Table 4. Marginalized one-dimensional best-fit parameters with 1σ confidence intervals for all models using QSO data.

| $H_0^{a}$ prior | Model | $\Omega_{m0}$ | $\Omega_{\Lambda}$ | $\Omega_{k0}$ | $\omega_X$ | $\alpha$ | $H_0^{a}$ |
|-----------------|-------|----------------|-------------------|--------------|-----------|-------|-----------|
| Flat ΛCDM       | 0.26±0.17 | 0.70±0.01 | -0.01±0.00 | -0.32±0.01 | 0.53±0.02 | 0.59±0.02 |
| Non-flat ΛCDM   | 0.28±0.12 | 0.93±0.18 | -0.17±0.34 | -0.32±0.07 | 0.54±0.43 | 0.45±0.02 |
| Flat XCDM       | 0.20±0.13 | 0.11±0.66 | -0.21±0.11 | -0.32±0.09 | 0.55±0.43 | 0.45±0.02 |
| Non-flat XCDM   | 0.22±0.11 | 0.32±0.02 | -0.23±0.11 | -0.32±0.09 | 0.55±0.43 | 0.45±0.02 |

$^a$ km s$^{-1}$Mpc$^{-1}$.

Table 5. Marginalized one-dimensional best-fit parameters with 1σ confidence intervals for all models using QSO+$H(z)$+BAO data.

| $H_0^{a}$ prior | Model | $\Omega_{m0}$ | $\Omega_{\Lambda}$ | $\Omega_{k0}$ | $\omega_X$ | $\alpha$ | $H_0^{a}$ |
|-----------------|-------|----------------|-------------------|--------------|-----------|-------|-----------|
| Flat ΛCDM       | 0.30±0.01 | 0.70±0.01 | -0.01±0.00 | -0.32±0.01 | 0.53±0.02 | 0.59±0.02 |
| Non-flat ΛCDM   | 0.31±0.01 | 0.71±0.06 | -0.01±0.06 | -0.32±0.09 | 0.55±0.43 | 0.45±0.02 |
| Flat XCDM       | 0.33±0.02 | 0.16±0.11 | -0.17±0.11 | -0.32±0.09 | 0.55±0.43 | 0.45±0.02 |
| Non-flat XCDM   | 0.34±0.03 | 0.17±0.09 | -0.32±0.09 | -0.32±0.09 | 0.55±0.43 | 0.45±0.02 |

$^a$ km s$^{-1}$Mpc$^{-1}$.

Ratra 2018d; Ryan et al. 2019). Mildly closed spatial hypersurfaces are also consistent with CMB anisotropy measurements (Ooba et al. 2018a,b,c; Park & Ratra 2018a,b,c, 2019).

The cosmological constant density parameter for the flat (non-flat) ΛCDM model is determined to be $\Omega_{\Lambda} = 0.70 \pm 0.01(0.71^{+0.06}_{-0.05})$ and $0.68 \pm 0.01 (0.79^{+0.04}_{-0.04})$ for the $H_0 = 68 \pm 2.8$ km s$^{-1}$Mpc$^{-1}$ and 73.24 ± 1.74 km s$^{-1}$Mpc$^{-1}$ priors respectively.
closer to those of a time-independent $\Lambda$ when we jointly analyze the QSO data with the $H(z) + \text{BAO}$ data, compared to the corresponding QSO data alone case. From the analyses of the QSO + $H(z) + \text{BAO}$ data the equation of state parameter for the flat (non-flat) XCDM parametrization is determined to be $\omega_X = -0.99 \pm 0.07 \text{ (} -0.81 \pm 0.16 \text{)}$ and $-1.17 \pm 0.07 \text{ (} -0.96 \pm 0.15 \text{)}$ for the $H_0 = 68 \pm 2.8 \text{ km s}^{-1} \text{Mpc}^{-1}$ and $73.24 \pm 1.74 \text{ km s}^{-1} \text{Mpc}^{-1}$ priors respectively. The parameter $\sigma$ in the flat (non-flat) $\delta$CDM model is determined to be $\sigma = 0.16^{+0.18}_{-0.11} \text{ (} 0.68^{+0.35}_{-0.46} \text{)}$ and $0.04^{+0.08}_{-0.03} \text{ (} 0.51^{+0.43}_{-0.35} \text{)}$ for the $H_0 = 68 \pm 2.8 \text{ km s}^{-1} \text{Mpc}^{-1}$ and $73.24 \pm 1.74 \text{ km s}^{-1} \text{Mpc}^{-1}$ priors respectively. Of these 8 cases, 6 favor dark energy dynamics over a time-independent cosmological constant energy density; for the $68 \pm 2.8 \text{ km s}^{-1} \text{Mpc}^{-1}$ prior dynamical dark energy is favored at 1.5$\sigma$ to 1.7$\sigma$ in the flat $\delta$CDM and non-flat XCDM and $\delta$CDM models, while for the $73.24\pm 1.74 \text{ km s}^{-1} \text{Mpc}^{-1}$ prior, dark energy dynamics is favored at 1.3$\sigma$ to 2.4$\sigma$ in the flat and non-flat $\delta$CDM models and the flat XCDM cases. Other data also favor mild dark energy dynamics (Ooba et al. 2018d; Park & Ratra 2018b,c).

6 CONCLUSION

We have used the Risaliti & Lusso (2015) compilation of 808 X-ray and UV QSO flux measurements to constrain cosmological parameters in six cosmological models. These QSO data constraints are much less restrictive than, but mostly consistent with those obtained from the joint analyses of 31 Hubble parameter and 11 BAO distance measurements.

We find that joint analyses of the QSO and $H(z) + \text{BAO}$ data tightens (and in some models, alters) constraints on cosmological parameters derived using just the $H(z) + \text{BAO}$ data. In general, the tightening effect is more significant in models with a larger number of free parameters. The joint QSO + $H(z) + \text{BAO}$ data constraints are consistent with the current standard flat $\Lambda$CDM model, although they weakly favor closed over flat spatial hypersurfaces and dynamical dark energy over a cosmological constant.

While cosmological parameter constraints from the QSO data we have used here are not that restrictive, the new Risaliti & Lusso (2018) QSO data compilation (of 1598 measurements over the redshift range $0.04 \leq z \leq 5.1$) will result in more restrictive cosmological parameter constraints that near-future QSO data should improve upon.

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This paper has been typeset from a \TeX/L\LaTeX file prepared by the author.
Figure 1. Flat ΛCDM model constraints from QSO (grey), $H(z) + BAO$ (red), and QSO + $H(z) + BAO$ (blue) data. Left panel shows 1, 2, and 3σ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_m$ and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 68 \pm 2.8$ km s$^{-1}$Mpc$^{-1}$ prior.

Figure 2. Flat ΛCDM model constraints from QSO (grey), $H(z) + BAO$ (red), and QSO + $H(z) + BAO$ (blue) data. Left panel shows 1, 2, and 3σ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_m$ and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 73.24 \pm 1.74$ km s$^{-1}$Mpc$^{-1}$ prior.
Figure 3. Non-flat ΛCDM model constraints from QSO (grey), $H(z)$ + BAO (red), and QSO + $H(z)$ + BAO (blue) data. Left panel shows 1, 2, and 3σ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for cosmological parameters $\Omega_m$, $\Omega$, and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 68 \pm 2.8$ km s$^{-1}$Mpc$^{-1}$ prior. The black dotted straight lines corresponds to the flat ΛCDM model.

Figure 4. Non-flat ΛCDM model constraints from QSO (grey), $H(z)$ + BAO (red), and QSO + $H(z)$ + BAO (blue) data. Left panel shows 1, 2, and 3σ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_m$, $\Omega$, and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 73.24 \pm 1.74$ km s$^{-1}$Mpc$^{-1}$ prior. The black dotted straight lines corresponds to the flat ΛCDM model.
Figure 5. Flat XCDM model constraints from QSO (grey), $H(z)$ + BAO (red), and QSO + $H(z)$ + BAO (blue) data. Left panel shows 1, 2, and 3σ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_m$, $\omega_X$, and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 68 \pm 2.8$ km s$^{-1}$Mpc$^{-1}$ prior. The green dotted straight lines represent $\omega_X = -1$.

Figure 6. Flat XCDM model constraints from QSO (grey), $H(z)$ + BAO (red), and QSO + $H(z)$ + BAO (blue) data. Left panel shows 1, 2, and 3σ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_m$, $\omega_X$, and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 73.24 \pm 1.74$ km s$^{-1}$Mpc$^{-1}$ prior. The green dotted straight lines represent $\omega_X = -1$. 

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Figure 7. Non-flat XCDM model constraints from QSO (grey), $H(z)$ + BAO (red), and QSO + $H(z)$ + BAO (blue) data. Left panel shows 1, 2, and 3σ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_m0$, $\Omega_k0$, $\omega_X$, and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 68 \pm 2.8$ km s$^{-1}$Mpc$^{-1}$ prior. The black dashed straight lines and the green dotted straight lines are $\Omega_k0 = 0$ and $\omega_X = -1$ lines.

Figure 8. Non-flat XCDM model constraints from QSO (grey), $H(z)$ + BAO (red), and QSO + $H(z)$ + BAO (blue) data. Left panel shows 1, 2, and 3σ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_m0$, $\Omega_k0$, $\omega_X$, and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 73.24 \pm 1.74$ km s$^{-1}$Mpc$^{-1}$ prior. The black dashed straight lines and the green dotted straight lines are $\Omega_k0 = 0$ and $\omega_X = -1$ lines.
Figure 9. Flat $\phi$CDM model constraints from QSO (grey), $H(z) + \text{BAO}$ (red), and QSO + $H(z) + \text{BAO}$ (blue) data. Left panel shows 1, 2, and 3$\sigma$ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_m$, $\alpha$, and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 68 \pm 2.8$ km s$^{-1}$Mpc$^{-1}$ prior.

Figure 10. Flat $\phi$CDM model constraints from QSO (grey), $H(z) + \text{BAO}$ (red), and QSO + $H(z) + \text{BAO}$ (blue) data. Left panel shows 1, 2, and 3$\sigma$ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_m$, $\alpha$, and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 73.24 \pm 1.74$ km s$^{-1}$Mpc$^{-1}$ prior.
Figure 11. Non-flat $\phi$CDM model constraints from QSO (grey), $H(z) +$ BAO (red), and QSO + $H(z) +$ BAO (blue) data. Left panel shows 1, 2, and 3$\sigma$ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_{m0}$, $\Omega_{k0}$, $\alpha$, and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 68 \pm 2.8$ km s$^{-1}$Mpc$^{-1}$ prior. The black dashed straight lines are $\Omega_{k0} = 0$ lines.

Figure 12. Non-Flat $\phi$CDM model constraints from QSO (grey), $H(z) +$ BAO (red), and QSO + $H(z) +$ BAO (blue) data. Left panel shows 1, 2, and 3$\sigma$ confidence contours and one-dimensional likelihoods for all free parameters. Right panel shows magnified plots for only cosmological parameters $\Omega_{m0}$, $\Omega_{k0}$, $\alpha$, and $H_0$, without the QSO-only constraints. These plots are for the $H_0 = 73.24 \pm 1.74$ km s$^{-1}$Mpc$^{-1}$ prior. The black dashed straight lines are $\Omega_{k0} = 0$ lines.