Casimir–Polder (CP) forces between dipolar bodies and macroscopic objects arise from zero-point fluctuations of the electromagnetic field $\mathbf{E}$ and $\mathbf{B}$. These forces, along with more general Casimir forces, have been experimentally measured in systems including planar substrates, gratings, Rydberg atoms, molecules, and Bose–Einstein condensates [1–3,12–14]. Owing to their strong and complicated dependence on geometry, prior works have sought means of modifying the magnitude and sign of these forces (beyond the typical attractive and monotonically decaying power laws [1–3,5,15]) via nanostructuring. In particular, outside of systems satisfying the Dzyaloshinskii–Lifshitz–Pitaevskii permittivity criterion for repulsion [16–20] (requiring an intervening medium, e.g., fluids), repulsive Casimir and CP forces have been predicted for bodies separated in vacuum mainly for anisotropic dipoles at small separations [21, 22], planar magnetic media [20,23,25], metallic rectangular gratings [26], metallic or dielectric plates with circular holes [21,27,28], and other metallic surfaces with sharp edges [29, 30]. Likewise, beyond the general no-go theorem for repulsion in mirror-symmetric systems in vacuum [31] or recent generalizations of Earnshaw’s theorem setting constraints on stable equilibria [32], quantitative limits on attractive or repulsive Casimir forces have mainly been restricted to uniform planar dielectric and magnetic media [33,34]. Understanding bounds on CP forces is crucial for designing surfaces to trap, adsorb, or suspend atoms, molecules, and quantum emitters [1–3,14].

In this paper, we present upper and lower bounds on the Casimir–Polder force between an anisotropic dipolar body and a macroscopic body separated by vacuum via algebraic properties of Maxwell’s equations. These bounds require only a coarse characterization of the system—the material composition of the macroscopic object, the polarizability of the dipole, and any convenient partition between the two objects—to encompass all structuring possibilities. We find that the attractive Casimir–Polder force between a polarizable dipole and a uniform planar semi-infinite bulk medium always comes within 10% of the lower bound, implying that nanostructuring is of limited use for increasing attraction. In contrast, the possibility of repulsion is observed even for isotropic dipoles, and is routinely found to be several orders of magnitude larger than any known design, including recently predicted geometries involving conductors with sharp edges. Our results have ramifications for the design of surfaces to trap, suspend, or adsorb ultracold gases.

Fundamental limits to attractive and repulsive Casimir–Polder forces

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We derive upper and lower bounds on the Casimir–Polder force between an anisotropic dipolar body and a macroscopic body separated by vacuum via algebraic properties of Maxwell’s equations. These bounds require only a coarse characterization of the system—the material composition of the macroscopic object, the polarizability of the dipole, and any convenient partition between the two objects—to encompass all structuring possibilities. We find that the attractive Casimir–Polder force between a polarizable dipole and a uniform planar semi-infinite bulk medium always comes within 10% of the lower bound, implying that nanostructuring is of limited use for increasing attraction. In contrast, the possibility of repulsion is observed even for isotropic dipoles, and is routinely found to be several orders of magnitude larger than any known design, including recently predicted geometries involving conductors with sharp edges. Our results have ramifications for the design of surfaces to trap, suspend, or adsorb ultracold gases.

As notation, a vector field $\mathbf{v}(\mathbf{x})$ will be denoted as $\langle \mathbf{v} \rangle$. At $\omega = i \xi$, all relevant polarization and field quantities can be defined to be real-valued in position space without loss of generality, so we define the unconjugated inner product $\langle \mathbf{u}, \mathbf{v} \rangle = \int d^3x \mathbf{u}(\mathbf{x}) \cdot \mathbf{v}(\mathbf{x})$. An operator $\mathcal{A}(\mathbf{x}, \mathbf{x}')$ will be denoted as $\mathcal{A}$, with $\int d^3x' \mathcal{A}(\mathbf{x}, \mathbf{x}') \cdot \mathbf{v}(\mathbf{x'})$ denoted as $\mathcal{A} \langle \mathbf{v} \rangle$. Unless stated otherwise, all quantities are taken to implicitly depend on $i \xi$.

Main result.—Consider the CP force on a dipole of susceptibility $\mathcal{V}_{\text{dipole}} = \sum_\beta \alpha_\beta \langle \mathbf{u}^{(\beta)}(\mathbf{x}) \rangle \langle \mathbf{u}^{(\beta)}(\mathbf{R}) \rangle$, where the polarizabilities $\alpha_\beta(i \xi)$ are positive and the basis functions $\mathbf{u}^{(\beta)}(\mathbf{x}) = n_\beta \delta^3(\mathbf{x} - \mathbf{R})$ are given in terms of the dipole location $\mathbf{R}$ and the principal axes $n_\beta$. The upper and lower bounds (repulsion and attraction) on the CP force along Cartesian axis $k$, respectively $F_k^+$ and $F_k^−$, are shown to depend on $\mathcal{V}_{\text{dipole}}$, the macroscopic susceptibility $\chi(i \xi)$ (assumed to be homogeneous, local, and isotropic), and the choice of domain enclosing the macroscopic body, all of which are completely general, independent of any particular material dispersion model or body shapes. As argued in the following derivation, the lower bounds can never increase, and the upper bounds never decrease, when the chosen domain is enlarged, so that as a whole the bounds are domain monotonic. Based on this fact,
onto a domain which contains the support of body, which has the same support as We define larizable dipolar body of parallel (perpendicular) polarizability $\alpha_{\parallel}$ ($\alpha_{\perp}$) above any nanostructured medium of susceptibility $\chi$ within a given domain.

the bounds on the CP force can be written as

$$F_k^\pm = \frac{\hbar}{2\pi} \int_0^\infty \sum_\beta \alpha_\beta \left[ \left\langle \frac{\partial \mathbf{u}^{(\beta)}}{\partial R_k}, G^{\text{sca}} \mathbf{u}^{(\beta)} \right\rangle \pm \left\langle \mathbf{u}^{(\beta)}, G^{\text{sca}} \mathbf{u}^{(\beta)} \frac{\partial}{\partial R_k} \mathbf{u}^{(\beta)} \right\rangle^{1/2} \right] \, d\xi,$$  \hspace{1cm} (1)

where again all quantities are evaluated at $\omega = i\xi$. Here, $G^{\text{sca}}$ is the scattering Green’s function of the equivalent object created by filling the domain in question uniformly with a material of susceptibility $\chi(i\xi)$, and $\mathbf{u}$ the scattering Green’s function of any object possibly contained within the domain. (Crucially, (1) is not the expression of the CP force for any particular geometry.) These bounds need not have a definite sign either: for combinations of dipole polarizability, position and domain choice where the upper bound is positive and the lower bound is negative, there are potentially structures producing either attractive or repulsive CP forces.

Technical Derivation.—The CP force $F_k = \frac{\partial}{\partial R_k} \int (\mathbf{E}(t,x) \cdot \mathbf{P}(t,x)) \, d^3x$ by a macroscopic body of susceptibility $\mathbf{V}$ on a dipole of susceptibility $\mathbf{V}_{\text{dipole}}$ as above can be derived briefly as follows, where $\langle ... \rangle$ refers to a thermodynamic average. We take the Fourier transform to real $\omega$ and solve the integral form of Maxwell’s equations $\mathbf{E} = \mathbf{E}^{(0)} + G^{\text{vac}} \mathbf{P}$ simultaneously with $\mathbf{P} = \mathbf{P}^{(0)} + (\mathbf{V} + \mathbf{V}_{\text{dipole}}) \mathbf{E}$ for $\mathbf{E}$ and $\mathbf{P}$ for real $\omega$, where $G^{\text{vac}}$ solves $(\mathbf{\nabla} \times (\mathbf{\nabla} \times \omega/c)^2) G^{\text{vac}} = -(\omega/c)^2$. We define $\mathbf{T} = (\mathbf{I} - \mathbf{V} G^{\text{vac}})^{-1} \mathbf{V}$ for the macroscopic body, which has the same support as $\mathbf{V}$ and satisfies $\mathbf{T} = T(V^{-1} - G^{\text{vac}}) T$, where $G^{\text{vac}}$ is implicitly projected onto a domain which contains the support of $\mathbf{V}$. Finally, we use the zero-temperature fluctuation–dissipation relations

$$\langle \mathbf{E}^{(0)}(\omega) | \mathbf{E}^{(0)}(\omega') \rangle = \hbar \text{Im}(G^{\text{vac}}(\omega)) \times 2\pi(\omega - \omega')$$

and

$$\langle \mathbf{P}^{(0)}(\omega) | \mathbf{P}^{(0)}(\omega') \rangle = \hbar \text{Im}(\mathbf{V}(\omega) + \mathbf{V}_{\text{dipole}}(\omega)) \times 2\pi(\omega - \omega'),$$

perform a Wick rotation to $\omega = i\xi$ as the integrand is analytic in the upper-half plane of $\omega$, and then expand to lowest order in scattering between the dipole and macroscopic body to yield

$$F_k = \frac{\hbar}{2\pi} \int_0^\infty \sum_\beta \alpha_\beta \left[ \left\langle \frac{\partial \mathbf{u}^{(\beta)}}{\partial R_k}, G^{\text{vac}} T G^{\text{vac}} \mathbf{u}^{(\beta)} \right\rangle \right] \, d\xi \hspace{1cm} (2)$$

as the CP force. Henceforth, we assume that $\mathbf{V}$ represents a scalar (homogeneous, local, isotropic) susceptibility $\chi$. Our goal then is to find bounds such that $F_k \in [F_k^-, F_k^+]$. We note that at $\omega = i\xi$, $\mathbf{V}$, $G^{\text{vac}}$, and $\mathbf{T}$ in general are real-symmetric operators in position space, with $\mathbf{V}$ and $\mathbf{T}$ being positive-definite while $G^{\text{vac}}$ is negative-definite (and this applies to its diagonal projected blocks too).

We first consider the problem of optimizing $\frac{\partial}{\partial R_k} \langle \mathbf{E}^{\text{inc}} | \mathbf{TE}^{\text{inc}} \rangle = 2 \langle \frac{\partial \mathbf{E}^{\text{inc}}}{\partial R_k} | \mathbf{TE}^{\text{inc}} \rangle$ for an arbitrary incident field $\mathbf{E}^{\text{inc}}$. In particular, we define the action of $\mathbf{T}$ to be a new vector $| \mathbf{P} \rangle = T | \mathbf{E}^{\text{inc}} \rangle$, and optimize the quantity $2 \langle \frac{\partial \mathbf{E}^{\text{inc}}}{\partial R_k} | \mathbf{P} \rangle$ with respect to $| \mathbf{P} \rangle$, assuming that the response $| \mathbf{P} \rangle$ can be chosen arbitrarily given its support. However, we also take care to impose the equality constraint $\mathbf{T} = T(V^{-1} - G^{\text{vac}}) T$ to ensure physical consistency: evaluating this with respect to $| \mathbf{E}^{\text{inc}} \rangle$ gives $\langle \mathbf{E}^{\text{inc}} | \mathbf{P} \rangle = \chi^{-1} (\mathbf{P} | \mathbf{P} ) - (\mathbf{P} | G^{\text{vac}} | \mathbf{P} )$, and this quantity is positive as $\mathbf{T}$ is positive-definite. For convenience, we define the eigenvalue decomposition of the projection of $G^{\text{vac}}$ into the given domain as $G^{\text{vac}} = -\sum_{\mu} \rho_{\mu} | N(\mu) \rangle \langle N(\mu) |$, where $\rho_{\mu} > 0$ and $\langle N(\mu) | N(\nu) \rangle = \delta_{\mu\nu}$, and define the basis expansions $v_{\mu} = \langle N(\mu) | \mathbf{E}^{\text{inc}} \rangle = \langle \mathbf{E}^{\text{inc}} | N(\mu) \rangle$ and $t_{\mu} = \langle N(\mu) | \mathbf{P} \rangle = (\mathbf{P} | N(\mu) \rangle$. As the domain choice is independent of $\mathbf{R}$, then $2 \langle \frac{\partial \mathbf{E}^{\text{inc}}}{\partial R_k} | \mathbf{P} \rangle = 2 \sum_{\mu} \frac{\partial v_{\mu}}{\partial R_k} t_{\mu}$. This leads to the constrained optimization of the objective

$$L = \sum_{\mu} \left[ 2 \frac{\partial v_{\mu}}{\partial R_k} t_{\mu} - \lambda (t_{\mu} v_{\mu} - (\chi^{-1} + \rho_{\mu}) t_{\mu}^2) \right] \hspace{1cm} (3)$$

where $\lambda$ is a Lagrange multiplier. As we have chosen the domain into which we project $G^{\text{vac}}$ to contain the support of $| \mathbf{P} \rangle$—encoded in the expansion coefficients $\langle \mu |$—enlarging the domain into which we project $G^{\text{vac}}$ cannot affect the equality constraint. Similarly, the magnitude of the objective cannot decrease with increasing domain because $| \mathbf{P} \rangle$ can access the smaller domain. If no better performance is possible, $| \mathbf{P} \rangle$ can always be taken to be the previous solution. Thus, our bound is domain monotonic, and so any domain with projection operator $I_1$ that fully encloses all possible object designs of interest can be used to generate bounds.

Carrying out the optimization yields the equations $2 \frac{\partial v_{\mu}}{\partial R_k} - \lambda (v_{\mu} - 2(\chi^{-1} + \rho_{\mu}) t_{\mu}) = 0$ and $\sum_{\mu} (t_{\mu} v_{\mu} - (\chi^{-1} + \rho_{\mu}) t_{\mu}^2) = 0$. The first equation gives $t_{\mu} = \frac{1}{\chi^{-1} + \rho_{\mu}} (v_{\mu}^2 - \lambda \frac{\partial v_{\mu}}{\partial R_k})$, and
plugging this into the second equation gives
\[ \lambda \in \pm 2 \sqrt{\frac{\langle \partial E^{\text{inc}} / \partial R_k, (\chi^{-1} 1_4 - G^{\text{vac}})^{-1} \partial E^{\text{inc}} \rangle}{\langle E^{\text{inc}}, (\chi^{-1} 1_4 - G^{\text{vac}})^{-1} E^{\text{inc}} \rangle}}. \]

The constrained objective has \( \delta^2 L_{\mu\nu} = 2\lambda (\chi^{-1} + \rho_\mu) \delta_{\mu\nu} \), so
\[ L^\pm = \left( \frac{\partial E^{\text{inc}}}{\partial R_k}, (\chi^{-1} 1_4 - G^{\text{vac}})^{-1} E^{\text{inc}} \right) \pm \sqrt{\left( \frac{\partial E^{\text{inc}}}{\partial R_k}, (\chi^{-1} 1_4 - G^{\text{vac}})^{-1} \partial E^{\text{inc}} \right) \langle E^{\text{inc}}, (\chi^{-1} 1_4 - G^{\text{vac}})^{-1} E^{\text{inc}} \rangle}. \] (4)

For our problem of interest we set \( E^{\text{inc}} = G^{\text{vac}} [1(\beta)] \) and identify \( G^{\text{sc}} = G^{\text{vac}} (\chi^{-1} 1_4 - G^{\text{vac}})^{-1} G^{\text{vac}} \) as the scattering Green’s function of the equivalent object formed by filling the entire domain of interest with the susceptibility \( \chi \). As each \( \alpha_\beta (\xi) > 0 \), the net upper bound cannot rise above the upper bound applied to each channel \( \beta \), just as the net lower bound cannot fall below the per-channel lower bound. This argument also applies to integration over \( \xi \) and so (1) follows.

Discussion.—Domain monotonicity allows us to choose the largest domain enclosing any desired design. As gratings, plates with apertures, wedges, and knife-edge geometries have all been studied in the context of CP repulsion, and since experiments typically consider extended nanostructured media, we take the domain to be a planar semi-infinite half-space; this choice ensures the existence of a separating plane between the dipole and the macroscopic object, in contrast to interleaved geometries allowing effective repulsion through lateral forces. For this choice, \( G^{\text{sc}} \) of the equivalent object admits semi-analytical expressions. Additionally, the CP force and its bounds are linear functionals of the polarizabilities \( \alpha_\beta (\xi) \), and become simple linear functions if the polarizabilities are assumed to be dispersionless. Thus, for simplicity, we consistently choose the principal axes to align with the Cartesian axes, and consider \( \alpha_\perp (0) = \alpha_\parallel (0) = \alpha_\parallel \) and \( \alpha_\perp (0) = \alpha_\perp \). The dipole location is taken to be \( \mathbf{R} = d \mathbf{e}_z \), where \( d \) is the minimum separation of the dipole from the design domain, and the force direction of interest to lie along \( \mathbf{e}_z \). This choice of polarizabilities allows us to decompose the force into parallel and perpendicular components, \( F_\parallel = g^\perp_\perp \alpha_\perp / \alpha_\parallel \) for appropriate functions \( g^\perp_\perp \) and \( g^\perp_\parallel \) which are linearly proportional to \( \alpha_\parallel \), so \( F_\parallel / \alpha_\parallel \) is an affine function of the polarizability ratio (similar to an aspect ratio) \( \alpha_\perp / \alpha_\parallel \). These assumptions make evaluation and analysis of the CP force bounds particularly convenient.

We begin by considering a macroscopic body of dispersionless susceptibility \( \chi (\xi) = \chi_0 \). This leads to the simple result that the bounds \( F_\parallel^\pm \) for this domain, as well as the CP force for a nondispersive dipole above a planar semi-infinite bulk of susceptibility \( \chi_0 \), both scale as \( d^{-5} \). Therefore, we need only consider the dependence of these bounds on \( \chi_0 \) as well as the polarizability ratio \( \alpha_\perp / \alpha_\parallel \). Figure 2 shows the bounds, as well as the actual attractive CP force above a planar semi-infinite bulk of susceptibility \( \chi_0 \), as a function of \( \alpha_\perp / \alpha_\parallel \) for multiple \( \chi_0 \) (a), and as a function of \( \chi_0 \) for the isotropic case \( \alpha_\perp = \alpha_\parallel \) (b). As expected, for any nonzero \( \chi_0 \) and \( \alpha_\parallel \), the bounds and planar force (normalized by the dependence on \( d \) and \( \alpha_\parallel \)) attain a nonzero value for \( \alpha_\perp = 0 \).
and increase linearly with \( \alpha_\perp/\alpha_\parallel \); moreover, the bounds increase monotonically with \( \chi_0 \), saturating at finite values in the perfect electrically conducting (PEC) limit \( \chi_0 \to \infty \). Stun-
ningly, the actual force is consistently within 10% of the lower bound for all \( \chi_0 \) and \( \alpha_\perp/\alpha_\parallel \), indicating that nanostructuring can only weakly enhance attractive CP forces in extended geometries. In (1), the first term in the summand is half of the actual force above a planar semi-infinite bulk, so the second term is crucial to making the bounds valid and tight for this domain choice. Conversely, at every \( \chi_0 \) and \( \alpha_\perp/\alpha_\parallel \), the upper bound is positive, suggesting that at any \( d \) and for any polarizability ratio and \( \chi_0 \), there are in fact potential macroscopically geometries that can meaningfully repel dipoles. The tightness of the lower bounds indicates that these limits capture essential physics. Hence, it is fairly plausible that tailored macroscopic geometries approaching the upper bound do exist. It is also worth mentioning that in the few geometries where repulsion is predicted for strongly anisotropic dipoles, it is prohibited for isotropic dipoles, but our upper bounds do not rule out the existence of other repulsive geometries even for isotropic dipoles. Finally, we point out that the magnitude of the upper bounds are consistently more than an order of magnitude smaller than the magnitude of the lower bounds.

Next, we relax the assumption that \( \chi(i\xi) \) is nondispersive, and consider the particular case of a gold medium, for which \( \chi(i\xi) = \chi_{\text{p}}^2/(\xi^2 + \gamma \xi) \) for \( \chi_{\text{p}} = 1.37 \times 10^{16} \text{ rad/s} \) and \( \gamma = 5.32 \times 10^{13} \text{ rad/s} \). For simplicity, we continue to neglect dispersion in \( \alpha_\parallel \) and \( \alpha_\perp \), so the linear scaling of the bounds with \( \alpha_\parallel \) and affine linear scaling with \( \alpha_\perp/\alpha_\parallel \) are preserved. The introduction of dispersion means that the bounds no longer scale uniformly as \( d^{-5} \): as seen in Fig. 5, the bounds transition from the nonretarded scaling of \( d^{-4} \) toward \( d^{-5} \) as the separation increases. The linear increase in the bounds with \( \alpha_\perp/\alpha_\parallel \) is also clear. More importantly, while more than an order of magnitude smaller than the lower bounds, for a dispersive metal like gold the possibility of repulsion is still not ruled out as the upper bounds remain positive for all \( d \) and \( \alpha_\perp/\alpha_\parallel \). For attraction, the actual forces produce from the planar geometry are again within 10% of the correspond-

**Figure 3. Distance dependence of bounds on Casimir–Polder forces for gold nanostructures.** Repulsive and attractive bounds on CP forces as in Fig. 2 but with the macroscopic susceptibility \( \chi \) corresponding to that of gold and \( \alpha_\perp/\alpha_\parallel \) increasing logarithmically from \( 10^{-2} \) to \( 10^2 \) (lighter to darker shades). Also shown is the CP force on a gold needle above a gold plate with a circular aperture from Ref. [27] (dark blue star), corresponding to a static anisotropic polarizability ratio \( \alpha_\perp/\alpha_\parallel \approx 51.1 \) at \( d = 200 \text{ nm} \); bounds for \( \alpha_\perp/\alpha_\parallel = 50 \) are marked in lines with circles.

In summary, we have derived bounds for the CP force on a general anisotropic dipolar body by a macroscopic body of susceptibility \( \chi \) enclosed within a prescribed domain, and have evaluated these bounds specifically for a planar semi-

infinite half-space domain. The lower bounds are nearly achieved by the typical geometry of a dipole above a uniform planar body, whereas existing predictions of repulsive CP forces in geometries involving conductors with sharp edges fall nearly two orders of magnitude below the limits on repulsion. We expect that similar to other nanophotonic phe-

nomena like local density of states modifications and radiative heat transfer, optimal structures for attraction or repul-

sion found through brute-force techniques such as inverse de-

sign [40–42] will look very different from the high-symmetry geometries proposed thus far, and that the tightness of the lower bounds for known structures suggests that appropriately designed structures may indeed approach the upper bounds and yield measurable repulsive CP forces even for relatively isotropic dipoles like Rydberg atoms, in contrast to existing designs [1][2][21][27][28]. Additionally, we point out that the Casimir energy between a dipolar particle and a macroscopic object in vacuum is always negative and goes to zero at asymptotically large separations, precluding a macroscopic geometry that repels a dipole for every location (as the force must be attractive sufficiently far away); this suggests that the upper bounds could be further tightened. Though our results focused on the force normal to the plane separating the dipolar and extended bodies, [1] can be employed to bound lateral...
forces, the subject of much recent interest \[13-16\], as well as forces involving compact objects. Finally, we point out that our bounds can be easily generalized to finite temperature equilibrium CP forces by replacing the frequency integration with a Matsubara summation \[2,3\].

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