Daisuke Inotani\textsuperscript{1} · Manfred Sigrist\textsuperscript{2} · Yoji Ohashi\textsuperscript{1}

Superfluid properties of one-component Fermi gas with an anisotropic \textit{p}-wave interaction

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Abstract We investigate superfluid properties and strong-coupling effects in a one-component Fermi gas with an anisotropic \textit{p}-wave interaction. Within the framework of the Gaussian fluctuation theory, we determine the superfluid transition temperature $T_c$, as well as the temperature $T_0$ at which the phase transition from the \textit{p}$_x$-wave pairing state to the \textit{p}$_x$ + \textit{i}p$_y$-wave state occurs below $T_c$. We also show that while the anisotropy of the \textit{p}-wave interaction enhances $T_c$ in the strong-coupling regime, it suppresses $T_0$.

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1 Introduction

Since the realization of the \textit{s}-wave superfluid state in $^{40}$K and $^6$Li Fermi gases, the possibility of \textit{p}-wave superfluid Fermi gas has attracted much attention both theoretically and experimentally\textsuperscript{1,2,3,4,5,6,7,8,9,10,11,12,13}. A tunable \textit{p}-wave pairing interaction associated with a \textit{p}-wave Feshbach resonance has been realized in $^{40}$K\textsuperscript{1,2} and $^6$Li\textsuperscript{3,4} Fermi gases. It has been also observed in a $^{40}$K Fermi gas that a magnetic dipole-dipole interaction lifts the degeneracy of the \textit{p}-wave Feshbach resonance, leading to different resonance magnetic fields between the \textit{p}$_x$-component and the other \textit{p}$_y$ and \textit{p}$_z$ components, under an external magnetic field applied in the \textit{x}-direction\textsuperscript{1,2}. This split naturally leads to the anisotropy of the three \textit{p}-wave interaction channels as $U_x \neq U_y = U_z$ (where $U_j$ is the interaction strength in the \textit{p}$_j$-channel). In this case, a phase transition from the \textit{p}$_x$-wave pairing state to the \textit{...
$p_x + ip_y$-wave one has been theoretically predicted. Since such a phase transition never occurs in the case of $s$-wave superfluid, the realization of the $p$-wave superfluid Fermi gas would be useful for the study of a phase transition between different pairing states, from the weak-coupling regime to the strong-coupling limit in a unified manner.

Pairing fluctuations are usually suppressed in the superfluid phase, because of the opening of single-particle excitation gap. However, in the present case, even in the $p_x$-wave superfluid phase below $T_c$, pairing fluctuations in the $p_x + ip_y$-channel would become strong near $T_0$, especially in the intermediate coupling regime. Thus, the $p$-wave superfluid Fermi gas is also an interesting system to study strong pairing fluctuations appearing in the superfluid phase.

In this paper, we investigate the phase transition between the $p_x$-wave state and $p_x + ip_y$-wave state in a superfluid Fermi gas with a $p$-wave pairing interaction. So far, this problem has been examined within the Ginzburg-Landau theory. In this paper, we employ a fully microscopic approach, including strong-coupling effects within the Gaussian fluctuation approximation. We determine the superfluid phase transition temperature $T_c$, as well as the transition temperature $T_0$ from the $p_x$-wave state to $p_x + ip_y$-wave state below $T_c$.

2 Gaussian fluctuation theory for $p$-wave superfluid Fermi gas

We consider a one-component Fermi gas with a $p$-wave pairing interaction, described by the Hamiltonian

$$H = \sum_p \xi p c_p^\dagger c_p - \frac{1}{2} \sum_{pp'q=x,y,z} p_i U_i p'_i c_p^\dagger - \frac{1}{2} c_{p+\frac{q}{2}}^\dagger c_{p+\frac{q}{2}} - \frac{1}{2} c_{p'-\frac{q}{2}}^\dagger c_{p'-\frac{q}{2}}. \quad (1)$$

Here, $c_p^\dagger$ is the creation operator of a Fermi atom with the kinetic energy $\xi_p = p^2/(2m) - \mu$, measured from the chemical potential $\mu$. $-p_i U_i p'_i$ ($i = x, y, z$) are the three components of an assumed $p$-wave pairing interaction. In this paper, we ignore detailed Feshbach mechanism, and simply treat $U_i$ as a tunable parameter. However, we include the anisotropy of the interaction by the dipole-dipole interaction. That is, assuming that an external magnetic field is applied in the $x$-direction, we set $U_x > U_y = U_z$.

The strength of the $p$-wave interaction is conveniently measured in terms of the scattering volume $v_i$ ($i = x, y, z$) and the effective range $k_0$, that are given by, respectively,

$$\frac{4\pi v_i}{m} = -\frac{U_i}{3 - U_i \sum_p \frac{p^2}{2\xi_p}}, \quad (2)$$

$$k_0 = -\frac{4\pi}{m^2} \sum_p \frac{p^2}{2\xi_p^2} = -\frac{4}{\pi} p_c, \quad (3)$$

where $p_c$ is a momentum cutoff. We also introduce the anisotropy parameter,

$$\delta v_p^{-1} = v_x^{-1} - v_y^{-1}.$$
Fluctuation correction $\Omega_{\text{Gauss}}$ to the thermodynamic potential $\Omega$ in the $p$-wave Gaussian fluctuation theory. The solid line and the dashed line describe the $2 \times 2$-matrix single-particle thermal Green’s function $G_0$ in the mean field theory, and the $p$-wave interaction $-p_i U_j p'_j$ ($i = x, y, z$), respectively. $\tau_{\pm}$ is given by $\tau_{\pm} = \tau_1 \pm i \tau_2$, where $\tau_j$ is the Pauli matrix.

We include pairing fluctuations in the $p$-wave Cooper channel within the Gaussian fluctuation theory. In this strong-coupling theory, the thermodynamic potential $\Omega$ consists of the mean field part $\Omega_{\text{MF}}$ and the fluctuation part $\Omega_{\text{Gauss}}$. $\Omega_{\text{MF}}$ is given by

$$\Omega_{\text{MF}} = \frac{1}{2} \sum_{i=x,y,z} d_i^\dagger U_i^{-1} d_i + \frac{1}{2} \sum_p (\xi_p - E_p) - \frac{1}{\beta} \sum_p \ln \left[ 1 + e^{-\beta E_p} \right].$$  (4)

Here, $d = (d_x, d_y, d_z)$ is the $p$-wave superfluid order parameter, and $E_p = \sqrt{\xi_p^2 + |d \cdot p|^2}$ describes Bogoliubov single-particle excitations. The fluctuation part, $\Omega_{\text{Gauss}}$, is diagrammatically given in Fig. 1. Summing up these diagrams, one has

$$\Omega_{\text{Gauss}} = \frac{1}{2\beta} \ln \det \left[ 1 + \hat{W}^{\alpha\beta}(q, i\nu_n) \right],$$  (5)

where $\hat{W}^{\alpha\beta}_{ij} = U_i \delta_{ij} \delta_{\alpha\beta}$ ($\alpha, \beta = 1, 2$ and $i, j = x, y, z$). $\hat{\pi}^{\alpha\beta}_{ij}$ is the correlation function, having the form,

$$\pi_{ij}^{11}(q, i\nu_n) = \frac{1}{\beta} \sum_p p_i p_j \text{Tr} \left[ \tau_- G_0 \left( p + \frac{q}{2}, i\omega_n \right) \tau_+ G_0 \left( p - \frac{q}{2}, i\omega_n - i\nu_n \right) \right],$$  (6)

$$\pi_{ij}^{12}(q, i\nu_n) = \frac{1}{\beta} \sum_p p_i p_j \text{Tr} \left[ \tau_- G_0 \left( p + \frac{q}{2}, i\omega_n \right) \tau_- G_0 \left( p - \frac{q}{2}, i\omega_n - i\nu_n \right) \right],$$  (7)

$$\pi_{ij}^{22}(q, i\nu_n) = \pi_{ij}^{11}(q, i\nu_n),$$  (8)

$$\pi_{ij}^{21}(q, i\nu_n) = \pi_{ij}^{12}(q, i\nu_n).$$  (9)

Here, $G_0(p, i\omega_n)$ is the $2 \times 2$-matrix single-particle thermal Green’s function in the mean field theory, given by

$$G_0(p, i\omega_n) = \frac{1}{i\omega_n - \xi_p \tau_3 + \text{Re}(d \cdot p) \tau_1 + \text{Im}(d \cdot p) \tau_2},$$  (10)

where $\tau_j$ ($j = 1, 2, 3$) are the Pauli matrices acting on the particle-hole space, and $\tau_{\pm} = \tau_1 \pm i \tau_2$. 

**Fig. 1** Fluctuation correction $\Omega_{\text{Gauss}}$ to the thermodynamic potential $\Omega$ in the $p$-wave Gaussian fluctuation theory. The solid line and the dashed line describe the $2 \times 2$-matrix single-particle thermal Green’s function $G_0$ in the mean field theory, and the $p$-wave interaction $-p_i U_j p'_j$ ($i = x, y, z$), respectively. $\tau_{\pm}$ is given by $\tau_{\pm} = \tau_1 \pm i \tau_2$, where $\tau_j$ is the Pauli matrix.
As usual, we determine the superfluid order parameter \( d \) by solving the gap equation

\[
d_i = \sum_p U_p d \cdot p \frac{\text{tanh} \beta E_p}{2},
\]

(11)

together with the equation for the number \( N \) of Fermi atoms,

\[
N = -\frac{\partial \Omega}{\partial \mu} = \frac{1}{2} \sum_p \left[ 1 - \frac{\xi_p}{E_p} \text{tanh} \frac{\beta E_p}{2} \right]
- \frac{1}{2\beta} \sum_{q,i} \text{Tr} \left[ (W^{-1} + \hat{\pi}(q,i)\nu_n)^{-1} \frac{\partial \hat{\pi}(q,i)\nu_n}{\partial \mu} \right],
\]

(12)

and determine \( d \) and the Fermi chemical potential \( \mu \) self-consistently.

Since we are taking \( U_x > U_y = U_z \), the superfluid phase transition first occurs in the \( p_x \)-wave Cooper channel. Thus, the equation for the superfluid phase transition temperature \( T_c \) is given by setting \( i = x \) and \( d \to 0 \) in Eq. (11), as

\[
1 = U_x \sum_p \frac{p_x^2}{2\xi_p} \text{tanh} \frac{\beta \xi_p}{2}.
\]

(13)

We solve this equation, together with the number equation (12) with \( q = 0 \), to determine \( T_c \).
Fig. 3 (Color online) Effects of anisotropy \((\delta v^{-1}_p - v^{-1}_x - v^{-1}_y)\) on the superfluid transition temperature \(T_c\) and the phase transition temperature \(T_0\) from the \(p_x\)-wave state to the \(p_x + ip_y\)-wave state. The solid triangle and circle, respectively, show the critical value of \((\delta v_p p_F^3)^{-1}\) where \(T_0\) vanishes with \((v_x p_F^3)^{-1} = 4.0, -4.0\), calculated within the mean field theory.

3 Superfluid phase transition and transition between \(p_x\)-wave and \(p_x + ip_y\)-wave states

Figure 2 shows \(T_c\) as a function of the interaction strength. In this figure, the increase of the inverse scattering volume \((v_x p_F^3)^{-1}\) corresponds to the increase of the interaction strength. Starting from the weak-coupling regime, \(T_c\) gradually increases with increasing the strength of the pairing interaction, and it approaches a constant value when \((v_x p_F^3)^{-1} > \sim 0\). Apart from the values of \(T_c\), the overall behavior of \(T_c\) is close to the \(s\)-wave case.

In the weak-coupling regime, Fig. 2 shows that the anisotropy of the pairing interaction (which is described by the anisotropy parameter \(\delta v^{-1}_p = v^{-1}_x - v^{-1}_y\)) is not crucial for \(T_c\). In this regard, we note that, since the \(T_c\) equation (13) does not explicitly involve \(U_y\) nor \(U_z\), they only affect \(T_c\) through the Fermi chemical potential \(\mu\) determined by the number equation (12). However, the magnitude of \(\mu\) is actually close to the Fermi energy in the weak-coupling regime because of weak pairing fluctuations. Thus, the superfluid phase transition in this regime is only dominated by \(U_x\) (or \(v_x\)), so that \(T_c\) is insensitive to \(\delta v^{-1}_p = v^{-1}_x - v^{-1}_y\).

The anisotropy of the \(p\)-wave pairing interaction gradually becomes important, as one goes away from the weak-coupling regime. To understand this, it is convenient to consider the strong coupling limit. In this extreme case, the system may be viewed as a Bose gas, consisting of three kinds of tightly bound molecules that are formed by \(U_x, U_y,\) and \(U_z\) pairing interactions. \(T_c\) is then dominated by the Bose-Einstein condensation of one of the three components having the largest number \(N_B\) of Bose molecules. While \(N_B = N/6\) in the isotropic case (where \(N\) is the number of the Fermi atoms), \(N_B\) approaches \(N/2\) with increasing the magnitude of \(U_x\) compared with the other two interactions. Since the BEC phase transition temper-
nature of an ideal Bose gas is proportional to $N_B^{2/3}$, $T_c$ increases with increasing the anisotropy parameter $\delta v^{-1} = v_x^{-1} - v_y^{-1}$.

Although the $p_x$-wave superfluid phase is realized near $T_c$, this pairing symmetry changes into the $p_x + ip_y$-wave at a certain temperature ($\equiv T_0$) below $T_c$, as shown in Fig. 2. While $T_c$ is larger for a larger value of the anisotropy parameter $\delta v^{-1}$, $T_0$ for $(\delta v p_B^3)^{-1} = 0.4$ is found to be lower than that for $(\delta v p_B^3)^{-1} = 0.1$.

To see this more clearly, we show the $(\delta v p_B^3)^{-1}$-dependence of $T_0$ in Fig. 3. When the $p$-wave interaction is very anisotropic ($U_x \gg U_y = U_z$), the $p_x$-wave pairing becomes more and more favorable, so that the $p_x + ip_y$-wave state is suppressed.

Although it is difficult to examine the region far below $T_c$ based on the present strong-coupling theory because of computational problems, we briefly note that a critical value of $\delta v^{-1}$ at which $T_0$ vanishes can be obtained within the mean field theory.

4 Summary

To summarize, we have investigated the superfluid properties of a one-component Fermi gas with an anisotropic $p$-wave interaction. Within the framework of the Gaussian fluctuation theory, we determined the superfluid transition temperature $T_c$, as well as the phase transition temperature $T_0$ from the $p_x$-wave pairing state to the $p_x + ip_y$-wave state. While the anisotropy of the $p$-wave pairing interaction ($U_x > U_y = U_z$) is not crucial for $T_c$ in the weak-coupling regime, we showed that this anisotropy enhances $T_c$ in the strong-coupling regime. We also showed that, in contrast to the case of $T_c$, the anisotropy of the pairing interaction suppresses $T_0$.

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