Extracting Dynamical Green’s Function of Ultracold Quantum Gases via Electromagnetically Induced Transparency

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(Dated: March 13, 2014)

The essential quantum many-body physics of an ultracold quantum gas relies on the single-particle Green’s functions. We demonstrate that it can be extracted by the spectrum of electromagnetically induced transparency (EIT). The single-particle Green’s function depends on the convolution of EIT Green’s functions. We demonstrate that it can be extracted by the spectrum of electromagnetically induced transparency (EIT). The single-particle Green’s function depends on the convolution of EIT Green’s functions. We demonstrate that it can be extracted by the spectrum of electromagnetically induced transparency (EIT).

The ultracold quantum gases have been the test bed to investigate the many-body physics \([1]\). Thanks to the versatile advances in the experiments, the interaction strength \([2]\), the optical lattice depth, and the dimensionality can be manipulated to study strongly correlated quantum gases. To name a few, the typical atomic systems involve Luttinger liquid of one-dimensional (1D) Bose gases \([3–8]\), a three-dimensional (3D) Bose-Mott insulator \([9, 10]\), and the superfluid state of two-component Fermi gases \([11]\). Conventional experimental measurements of the atomic many-body systems include the time-of-flight experiment, the noise correlations measurement \([12]\), the Bragg scattering spectroscopy \([13]\), and the in-situ imaging \([14, 15]\). Recently, we investigate the spectrum of electromagnetically induced transparency (EIT) \([16, 17]\) in the strongly correlated atomic systems \([18]\) as an alternative and non-destructive method \([18, 19]\) to probe the quantum many-body physics.

The EIT spectrum is shown to be solely determined by the single-particle Green’s function of the ground-state atoms. Non-trivial many-body effect for the spectrum is predicted \([18, 20]\) when the atoms are virtually coupled to the low-lying Rydberg states \([21, 22]\). The single-particle Green’s function is crucial for many-body systems, from which the observable of any single-particle operator, the ground state energy, and the excitation spectrum can be extracted \([11]\). Similar to the well-known angle-resolved photoemission spectroscopy (ARPES) \([23]\), where the single-particle Green’s function is probed to investigate the electronic structure of the surface in the solids, the EIT spectroscopy accesses the information of it as well. What differs is the single-particle Green’s function in ARPES is directly measured whereas the EIT spectrum involves an integral of the Green’s function and the functional laser parameters \([18]\).

In this Rapid Communication, we demonstrate that the dynamical single-particle Green’s function of the many-body system can be determined by EIT spectroscopy. We propose an experimental setup to realize the reconstruction of the single-particle Green’s function, and also discuss the effects of finite temperature and finite size. Our results suggest a universal determination on the essential single-particle Green’s function of the ultracold quantum gases in parallel to the ARPES in the solids.

We consider the conventional EIT setup (Λ type scheme) as shown in Fig. 1. The probe (\(\Omega^1\)) and control (\(\Omega^2\)) fields couple the ground state (|g\rangle) to the other hyperfine ground state (|s\rangle) and an excited state (|e\rangle). The

\[
\begin{align*}
\Delta_2^* \\
\Delta_1 \\
\Omega^2 \\
\Omega^1 \\
|g\rangle \\
|s\rangle \\
\text{B.S.} \\
\end{align*}
\]

\(\Gamma\)

\(\text{Spatiotemporal interferometry.}\)

\[
G^\text{c}(0,0;x,y,t)
\]

\(\Omega^1(x,y,t)\)

\(\Omega^2(r,t)\)

FIG. 1: (Color online) Schematic setup to extract single-particle Green’s function \([G^\text{c}(0,0;x,y,t)]\) via an EIT experiment with a spatial-temporal probe pulse. The experiment is conducted with a probe pulse in the ultracold quantum gas with an atomic Λ-type configuration: The counter-propagating control (\(\Omega^2\)) and the probe (\(\Omega^1(x,y,t)\)) fields in \(z\) direction couple two hyperfine ground states (|g\rangle and |s\rangle) with the excited state (|e\rangle) (detunings are \(\Delta_2^*\) and \(\Delta_1\), respectively). \(\Gamma\) is the spontaneous decay rate of |e\rangle. B.S. represents the beam splitter. Open circles denote the fiber coupling that guide the reference and transmitted pulses into the spatial-temporal interferometry. From the interferometry box, \(G^\text{c}(0,0;x,y,t)\) is extracted from the complete characterization of the output probe pulse \(\Omega^1(x,y,t)\).
detunings are $\Delta_1$ and $\Delta_2$ for the probe and control fields respectively. They are defined as the differences from the laser central ($\omega_1$) to the atomic transition frequencies. In the linear response of the probe field, the electric susceptibility is derived as $\chi(q, \omega) = \delta \langle \hat{P}(q, \omega) \rangle / \Omega_1(q, \omega)$ where $\Omega_1(q, \omega)$ is the Fourier transform of slow-varying $\Omega_1(r, t)$, and $\delta \langle \hat{P}(q, \omega) \rangle$ is the polarization operator calculated by the perturbation of a weak probe field [11].

In the momentum-frequency space, the electric susceptibility is obtained as ($\hbar = 1$) [18]

$$\chi(q, \omega) = - \frac{d_0}{V} \sum_k \int_{-\infty}^{\infty} d\tilde{\omega} i \tilde{G}^<(k, \tilde{\omega}) M(k + q, \omega - \tilde{\omega}),$$

(1)

where $V$ is the quantization volume, $d_0$ is the dipole moment of the probe field transition, and $\tilde{G}^<(k, \omega)$ is the Fourier transform of the Green’s function $G^<(0, 0; r, t)$ at zero temperature [11]. The functional $M(k + q, \omega - \tilde{\omega})$ that includes the laser parameters is

$$M(k + q, \omega - \tilde{\omega}) = \frac{\cos^2 \phi_{k+q}}{\omega - \omega - \epsilon_-(k + q)} + \frac{\sin^2 \phi_{k+q}}{\omega - \omega - \epsilon_+(k + q)},$$

(2)

where the mixing angle is

$$\cos \phi_k = \sqrt{\frac{\epsilon_+(k) - \epsilon_0_{k+k_z}}{\epsilon_-(k) - \epsilon_0_{k+k_z}}}.$$

(3)

The eigenenergies from the unperturbed Hamiltonian in the EIT setup ($\Omega_1 = 0$) is

$$\epsilon_{\pm}(k) = -\Delta_1 + \frac{\tilde{\Delta}_2 + \epsilon_0_{k+k_z}}{2} \pm \sqrt{\left(\frac{\tilde{\Delta}_2 + \epsilon_0_{k+k_z}}{2}\right)^2 + 4 \Omega_1^2},$$

(4)

The recoil-energy shifted detuning is $\tilde{\Delta}_2 \equiv \Delta_2 + \epsilon_0_{k+k_z}$, the kinetic energy is $\epsilon_0_{k+k_z} \equiv k^2/(2m) - \mu$, and the recoil momentum is $k_r \equiv k_1 - k_2$. The chemical potential is $\mu$, and $k_{1,2}$ are the central momenta of the probe and control fields respectively. A phenomenological spontaneous decay rate ($\Gamma$) of the excited state can be added by replacing $\epsilon_0_{k+k_z}$, with $\epsilon_0_{k+k_z} - i\Gamma$. Note that we assume negligible dephasing rate between the two hyperfine ground states.

The above Eq. (1) provides the crucial recipe of the many-body effects on the EIT spectrum for generally any strongly correlated ultracold quantum gases [18]. The nontrivial power-law dependence of the EIT spectrum near the resonance is shown for a Luttinger liquid, and the significant frequency shift and asymmetric absorption spectrum can be identified for a Bose-Mott insulator phase [18]. This non-destructive EIT measurement is proposed to detect the Fermi paring in a Bardeen-Cooper-Schrieffer (BCS) superfluid state of two-component Fermi gases [19]. The gap energy can be also measured from the transparency position [18]. As long as the single-particle Green’s function is known, the EIT spectrum can be derived. On the other hand, the EIT spectroscopy can be an efficient method to extract the single-particle Green’s function.

The functional $M(k + q, \omega - \tilde{\omega})$ incorporates all the information of the laser parameters in the EIT setup, which is well defined. We find that Eq. (1) has a simple form of the convolution between the single-particle Green’s function and $M$ in momentum-frequency space. Since the dispersion of the probe field has $\omega = q_{c} \tilde{\omega}$ in the propagating direction, the momentum (position) of $q_{c} (z)$ is not able to be resolved independently from frequency (time) in the EIT spectroscopy. We let $q_{c} = 0$ in Eq. (1) which is valid if the spread of frequency $\omega \ll \omega_1$ so $\chi(q_{c} = |\omega|/c) \approx \chi(q_{c} = 0)$. We then derive the single-particle Green’s function as

$$iG^<(0, 0; -\bar{r}, t) = -\frac{F^{-1} [\chi(q_{c}, \omega)] (\bar{r}, t)}{2\pi d_0 M(\bar{r}, t)},$$

(5)

where $M(\bar{r}, t)$ is the inverse Fourier transform ($F^{-1}$) of $M(k, \omega)$ with $\bar{r} \equiv (x, y)$ and $q_{c} \equiv (q_{x}, q_{y})$. The above result indicates that the dynamical single-particle Green’s function is fully determined by the dynamical EIT spectrum $\chi(q_{c}, \omega)$. It can be reconstructed by the inverse Fourier transform of the spectrum divided by the known functional $M(\bar{r}, t)$. Although the propagating direction in the Green’s function can not be extracted, the transverse space and real-time information in $G^<(0, 0; -\bar{r}, t)$ is sufficient to provide the features that characterize the many-body systems.

In Fig. 1 we demonstrate an experimental setup for EIT spectroscopy. We use a spatio-temporal pulse to probe the ultracold quantum gas. The probe pulse goes through a 50:50 beam splitter (B.S.) to interact with the atoms, and the reflected one acts as a reference pulse. The transmitted probe pulse is then guided by the fiber to interfere with the reference one. We denote the process as the interferometry box that characterizes the spatial and temporal information of the transmitted pulse. This technique of measuring the spatial and temporal electric field is not new for the ultrafast optical society. The single trace of interferometry is done through the input fiber at the specific position ($x, y, z$), the positions of the fiber is scanned ($x, y$), and the wavelengths of the control field ($\tilde{\omega}$) with $\mu \equiv (q_{x}, q_{y})$. The above result indicates that the dynamical single-particle Green’s function is fully determined by the dynamical EIT spectrum $\chi(q_{c}, \omega)$. It can be reconstructed by the inverse Fourier transform of the spectrum divided by the known functional $M(\bar{r}, t)$. Although the propagating direction in the Green’s function can not be extracted, the transverse space and real-time information in $G^<(0, 0; -\bar{r}, t)$ is sufficient to provide the features that characterize the many-body systems.
purpose of extracting the dynamical Green’s function in the strongly correlated quantum gases, the temporal range of the probe pulse requires less than the natural lifetime of the probe field transition. We choose the low-laying Rydberg transition in EIT measurements to have significant many-body effects from the quantum gases [18, 20]. The temporal range is then required to be the inverse of the EIT spectrum spread \( \sim 1/(4\Omega_{2}) \). It is in the order of 1/\( \Gamma \) where \( \Gamma \) is several kHz for rubidium atoms with a low-lying Rydberg transition \( |n'_{3}/2 \rangle \) \( (n' \approx 20) \) [21]. The \( \mu \) pulse is sufficient to probe the entire dynamical EIT spectrum in frequency space. The range of the probe pulse in transverse momentum space can be estimated by the inverse of the transverse distribution of atom clouds.

In practice, the finite temperature effect should be taken into account in the ultracold quantum gases. We follow the formalism of linear response theory at finite temperature [11], and find that the EIT spectrum depends on the single-particle temperature Green’s function,

\[
i G^<(r, t) = \text{Tr}\{\hat{\rho}_G \hat{\psi}(r, t) \hat{\psi}(0)\},
\]

where \( \hat{\rho}_G \equiv \exp(-\beta \hat{K})/\text{Tr}(\exp(-\beta \hat{K})) \) is the density operator for the grand canonical ensemble, and \( \hat{\psi}(r, t) \) is the ground state operator for the Hamiltonian \( \hat{K} = \hat{H} - \mu \hat{N} \). \( \hat{H} \) is the unperturbed Hamiltonian for the EIT setup without the probe light, and \( \beta = 1/(k_B T) \) where \( k_B \) is the Boltzmann constant, and \( T \) is the temperature of the gas. The electric susceptibility of EIT at finite temperature is then similar to Eq. 1 with the replacement of the single-particle Green’s function by a finite temperature one in Eq. 4.

To derive the single-particle temperature Green’s function, we may utilize the spectral function \( A(k, \omega) \) [27] that

\[
i G^<(k, \omega) = \pi f_{F/B}(\omega) A(k, \omega),
\]

\[
A(k, \omega) = -2\text{Im}[G_{\text{ret}}(k, \omega)],
\]

where \( f_{F/B}(\omega) \) is the Fermi or Boson number distributions at finite temperature \( T \), and \( G_{\text{ret}} \) is the retarded Green’s function.

As an example we consider the Mott-Insulator (MI) of strongly interacting bosons in a 3D optical lattice. In a single band Hubbard model (HM) [9], the ground state field operator is \( \hat{\psi}_g(r, t) = \sum_{\mathbf{R}} \hat{g}_\mathbf{R}(t) \hat{w}_\mathbf{R}(r) \) where \( \hat{w}_\mathbf{R}(r) \) is the Wannier function, and \( \hat{g}_\mathbf{R}(t) \) is the field operator at site \( \mathbf{R} \). When deep inside the MI state, we use the three-state model [28, 29] to obtain the Green’s function at finite temperature in the limit of large on-site interaction

\[
U,
\]

\[
i G^<(k, \omega) = \sum_{\mathbf{R}} |\tilde{w}_\mathbf{R}(k)|^2 e^{-i\omega t/2}\theta(t) \left[ f_B \left( U \over 2 \right) (n_0 + 1) + \left( 1 + f_B \left( U \over 2 \right) \right) n_0 \right],
\]

where for convenience we calculate the real time single-particle temperature Green’s function in momentum space. \( \theta(t) \) is a step function, \( n_0 \) is the integer filling fraction, and \( \tilde{w}_\mathbf{R}(k) \) is the Fourier transform of \( w_\mathbf{R}(r) \). Note that the particle and hole excitation energies \( \epsilon_{p/h}(k) \approx U/2 \) when deep inside the Mott state.

The first and second parts in the bracket of Eq. 8 represent the contributions from the particle and hole excitations respectively. At zero temperature, only the hole excitation remains as the quantum many-body effect on the EIT spectrum [18]. The Green’s function of MI has the distribution of Wannier function in momentum space, which implies the range of spatial coherence. In Fig. 2 we demonstrate the real part of the Green’s function for a MI at \( k = 0 \). The real-time dynamics of Green’s function indicates a modulation of the period which is the inverse of the on-site interaction energy. In this way we can determine the gap order parameter of the MI phase as the alternative approach to lattice potential gradient [10] or modulation spectroscopy [7]. The finite temperature effect on the MI is the fluctuation of the total number of particles, so \( i G^<(k, t) \) is not much different from \( i G^<(k, t) \) in Fig. 2 in the deep Mott regime. The spatial coherence in the MI phase reflects on the modulation amplitude in Fig. 2 where the amplitude of 1/2 indicates a full-width half-maximum of the Wannier function if it is approximated by a Gaussian profile.
In Fig. 3 we show the real-time single-particle Green’s function of LL. For a weakly interacting quantum gas ($\kappa = 10$), the relatively flat correlation function indicates the long-range order in the superfluid. The strongly interacting quantum gas ($\kappa = 0.6, 1$) on the other hand shows a noninteger power-law decay in the Green’s function. This quasi-long-range order originates from the collective excitations in a LL model with a linear dispersion $\omega(\mathbf{q}) = |\mathbf{q}| v$ in the low-temperature and long-wavelength limit. In our proposed experiment, the EIT spectroscopy provides a nondestructive way to extract the LL parameter $\kappa$ which is essential to characterize the 1D bosonic quantum gas.

Our proposed EIT spectroscopy may be also applied to the repulsive 1D two-component fermions. It can provide a genuine observation of the two-fold linear excitations in the spin and the charge sectors from the extraction of the single-particle Green’s function II similar to our case of a bosonic LL. For attractive 1D spin-$1/2$ fermions, the spin gap order parameter is also observable in the EIT spectroscopy in analogy to the BCS superfluid of two-component Fermi gases 18, 19. Therefore, the EIT spectroscopy can be applied to investigate on a plethora of low-dimensional quantum many-body systems with arbitrary interactions.

In summary, we show that the single-particle Green’s function can be extracted by an efficient and nondestructive EIT spectroscopy. We propose to utilize an electric field interferometry to characterize the spatial and temporal profiles of the probe fields. From the information of the transmitted probe pulse or equivalently the EIT dynamical response, we may deduce the essential single-particle Green’s function of the ultracold quantum gases. In the examples we take 3D Mott insulator and 1D LL compatible experiments. In our proposed experiment, the EIT spectroscopy may be also applied to the repulsive 1D two-component fermions. It can provide a genuine observation of the two-fold linear excitations in the spin and the charge sectors from the extraction of the single-particle Green’s function II similar to our case of a bosonic LL. For attractive 1D spin-$1/2$ fermions, the spin gap order parameter is also observable in the EIT spectroscopy in analogy to the BCS superfluid of two-component Fermi gases 18, 19. Therefore, the EIT spectroscopy can be applied to investigate on a plethora of low-dimensional quantum many-body systems with arbitrary interactions.

For a finite size system, there would be transverse and longitudinal confinements. For the transverse direction, the finite width ($\sigma$) of the density distribution will result in a finite transverse momentum distribution ($1/\sigma$). However, it will not affect Eq. 11 if the applied probe field is along the longitudinal direction.

The longitudinal density distribution will indicate a spatial dependence for the electric susceptibility in Eq. 11. If we average out the spatial dependence, we expect the qualitatively same EIT spectrum as in a uniform gas, except for an extra inhomogeneous broadening in the spectrum due to the momentum/density distribution 18. We note that in the light propagation dynamics, the edge effect appears in the EIT setup in a Bose-Einstein condensate (BEC) 31. The spin wave inside the BEC will occupy near the cloud edge with a low density, which affects the fidelity of the storage process of the probe pulse 31. As in our case for the EIT spectroscopy, though the finite size effect influences the overall efficiency, it does not significantly modify the structure of the dynamical response of the transmitted probe field.

For the last example, we demonstrate the single-particle Green’s function of the Luttinger liquid (LL) which is a universal one-dimensional (1D) effective model 3, 8. The single-particle Green’s function can be exactly calculated by the Bosonization method 3, 32, which is

$$iG_{\mu \nu}^{\rm LL}(x,t) = \frac{a n^{1/(2\kappa)}}{|x^2 + (1 + ivt)^2|^{1/(4\kappa)}}.$$  

The Luttinger parameter is $\kappa$, $v$ is the phonon velocity, $a^{-1}$ is the system-dependent momentum cutoff, and $n$ is the atomic density. $1 < \kappa < \infty$ is for a short-range repulsive interaction, while $\kappa$ can be smaller than one if the interaction is long ranged.

In Fig. 3 we show the real-time single-particle Green’s function of LL. For a weakly interacting quantum gas ($\kappa = 10$), the relatively flat correlation function indicates the long-range order in the superfluid. The strongly interacting quantum gas ($\kappa = 0.6, 1$) on the other hand shows a noninteger power-law decay in the Green’s function. This quasi-long-range order originates from the collective excitations in a LL model with a linear dispersion $\omega(\mathbf{q}) = |\mathbf{q}| v$ in the low-temperature and long-wavelength limit. In our proposed experiment, the EIT spectroscopy provides a nondestructive way to extract the LL parameter $\kappa$ which is essential to characterize the 1D bosonic quantum gas.

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