Abstract—Objective: This work investigates the hypothesis that focal seizures can be predicted using scalp electroencephalogram (EEG) data. Our first aim is to learn features that distinguish between the interictal and preictal regions. The second aim is to define a prediction horizon in which the prediction is as accurate and as early as possible, clearly two competing objectives. Methods: Convolutional filters on the wavelet transformation of the EEG signal are used to define and learn quantitative signatures for each period: interictal, preictal, and ictal. The optimal seizure prediction horizon is also learned from the data as opposed to making an a priori assumption. Results: Computational solutions to the optimization problem indicate a ten-minute seizure prediction horizon. This result is verified by measuring Kullback-Leibler divergence on the distributions of the automatically extracted features. Conclusion: The results on the EEG database of 204 recordings demonstrate that (i) the preictal phase transition occurs approximately ten minutes before seizure onset, and (ii) the prediction results on the test set are promising, with a sensitivity of 87.8% and a low false prediction rate of 0.142 FP/h. Our results significantly outperform a random predictor and other seizure prediction algorithms. Significance: We demonstrate that a robust set of features can be learned from scalp EEG that characterize the preictal state of focal seizures.

Index Terms—automatic feature extraction, convolutional neural networks, deep learning, focal seizures, preictal period, scalp EEG, seizure prediction

I. INTRODUCTION

Worldwide, there are approximately 65 million people with epilepsy, more than Parkinson’s disease, Alzheimer’s disease, and multiple sclerosis combined. Epileptic seizures are unpredictable, occurring often without warning. This contributes to the anxiety, morbidity, and mortality of the illness. The seizure prediction problem has, until recently, evaded success from computational approaches utilizing electroencephalogram (EEG) data. The difficulty of the problem arises from the lack of a general and specifiable definition of the phase transition between interictal and preictal periods of the EEG signal.

The shift from the hand-crafted design of features for machine learning systems to the merging of feature extraction with the learning process has proved successful on many interesting tasks, ranging from handwritten digit recognition to language translation. The idea of applying automatic feature extraction techniques to new data types other than images or natural language is a promising one. One example is the application of deep learning and automatic feature extraction to genomics data, yielding novel insights into patterns in DNA sequences [1].

In this work we apply automatic feature extraction techniques to predict seizures from scalp EEG, towards constructing a system to alert patients about oncoming seizures.

This paper is organized as follows. The rest of Section I covers the relevant prior work. Section II details the methods used in this study. Section III presents the discovered preictal phase transition and the results on the test set. Section IV compares this work with other seizure prediction methods. Finally, we discuss the results in Section V and conclude in Section VI.

A. Related Work

In the last two decades, research in the area of seizure prediction has matured as a result of the formalization of the problem and the availability of EEG data [2]. The underlying assumption of seizure prediction is that a difference exists in the brain waves between the interictal and preictal states. Many previous methods have failed to reliably predict seizures [3], [4]. However, the algorithms of multiple contestants in recent seizure prediction competitions on Kaggle [5] functioned at above random levels at accurately classifying interictal vs. preictal data, demonstrating the feasibility of seizure prediction. An implanted device [6] was the first study demonstrating prospective seizure prediction on long-term intracranial EEG collected in an ambulatory setting.

Work on the seizure prediction problem has been conducted on two sources of EEG data, intracranial EEG and scalp EEG. Due to the popularity of the open Freiburg EEG dataset [7] (now combined with EPILEPSIAE [8]), much of the early work on the seizure prediction problem utilized the available intracranial EEG data to develop algorithms for an implantable seizure prediction device. With the compilation of the EPILEPSIAE database, which contains a growing number of scalp EEGs, attention is being shifted towards the possibility of an external seizure prediction device.

The nature of data collected by intracranial EEG and scalp EEG differs greatly. Scalp EEG is readily available and is not invasive. However, it is more prone to artifacts introduced by shifting electrodes, muscle interference, and the effects of volume conduction. Intracranial EEG has a better signal-to-noise ratio than scalp EEG and can target specific areas of the brain directly. While most work has focused directly on human EEG, some studies have used canine intracranial EEG from...
Seizure prediction systems using intracranial or scalp EEG signals rely on moving window analysis on extracted features to generate predictions. One of the main challenges for accurate prediction is extracting and evaluating linear and nonlinear univariate and bivariate features from the signal. Seizure prediction methods have reported encouraging results using extracted linear features from the EEG signal. Autoregressive coefficients, spike rate, Hjorth parameters, spectral band power, and cross correlation are some linear features considered. The advent of the theory of dynamical systems introduced a number of nonlinear features, such as the dynamical similarity index, Lyapunov exponent, phase synchronization, attractor states, and combinations of non-linear features. Other extracted features include diffusion kernels and synchronization graphs.

Once a system extracts a set of features, two main approaches have been taken for prediction: statistical and algorithmic. Statistical methods, such as multivariate time series analysis techniques, attempt to estimate the usefulness of extracted features to the prediction problem retrospectively. Algorithmic methods use combinations of features or model parameters, such as parameters, and an explicit threshold to predict seizures prospectively. Support vector machines, random forests, autoencoders and other machine learning algorithms combined with feature extraction and selection methods have been applied to the seizure prediction problem. Formulating the problem as an instance of anomaly detection is also proposed, but due to the complexity of the underlying system, it is difficult to establish a robust baseline model that does not result in an overwhelming number of false positives, which is highly undesirable in this context. The closest work to this one applies convolutional neural networks to selected frequency bands on intracranial EEG. They report improved results using CNNs over support vector machines on extracted features, but do not use CNN to extract features directly from the EEG signal.

The length of the preictal period, the period before the seizure occurs in which it is possible to anticipate the seizure, has often been left as a design choice and has varied from hours to minutes. Some approaches to selecting the appropriate prediction horizon have been proposed, such as parameters, but suffer from being dependent on a specific group of features. Work by show that seizures captured by intracranial EEG recordings are patient-specific, but it is not clear if this affects the length of the preictal period or applies to seizures captured by scalp EEG data. Methods to statistically validate results based on the length of the preictal period have been developed and accepted, including time-series surrogates and comparison to an unspecific random predictor.

II. METHODS

A. Problem Definition

Underlying the search for the prediction horizon is the assumption that changes in the brain occur prior to seizure onset that make the seizure nearly inevitable. Despite this, seizure prediction based on EEG data has posed a challenge to the research community due to the absence of a clear and robust definition of the problem. The seizure prediction problem is defined as anticipating a seizure within some prediction horizon, or time window before seizure onset. Defining concretely the prediction horizon is difficult, since the optimal time window for prediction is not well understood. Our goal is to derive a justifiable patient-independent prediction horizon (preictal period) directly from the data while searching for early EEG predictors of the phase transition between the interictal and preictal periods. For the purpose of this study, we assume that a preictal phase exists for all focal seizures and that there is an inflection point between interictal and preictal states.

B. Datasets and Preprocessing

In this study, we trained our model and evaluated its performance on two independent datasets: (i) data collected from The Mount Sinai Epilepsy Center at the Mount Sinai Hospital (MSSM), and (ii) a subset of the public CHB-MIT EEG database (CHB-MIT). The datasets are composed of two types of recordings, interictal recordings without seizures and recordings with seizures. We use recordings with seizures to learn relevant features, determine the preictal period, and evaluate the sensitivity of the model. Interictal recordings are important to test the specificity of the model and estimate the false prediction rate. The system was trained and crossvalidated on 96 EEG recordings with seizures and 20 EEG recordings without seizures. Testing was done on 35 EEG recordings with seizures and 53 EEG recordings without seizures. In total, this study analyzed 131 recordings with seizures and 73 recordings without seizures. The recordings were collected using the standard 10-20 system for electrode placement with a bipolar montage. As a data preparation step, the EEG was filtered with a 128Hz low-pass filter and verified to have nonlinear structure using the BDS nonlinearity test with the appropriate ARIMA model.

1) MSSM EEG Dataset:

The MSSM dataset contained 86 scalp EEG recordings from 28 patients with epilepsy. The recordings were made from continuous EEG studies that utilized XLTEK equipment with 22 inputs and a sampling rate of 256Hz. The duration of the continuous EEG study varied from 2-8 days. All patient information was de-identified and the relevant EEG activity was converted into raw data using the European Data Share format.

The seizures were all focal with variable seizure onset zones, primarily temporal and frontal. All EEG data was selected, reviewed, and de-identified by two electroencephalographers. EEGs with electrode artifacts affecting more than one electrode were excluded. However, there was no attempt to remove sleep-wake transitions, eye blinks, movement, and chewing artifacts. Each subject was assigned a number from 1-28 without any identifying information. Sixty-one of the 86 recordings contained a seizure with onset times marked, and the remaining are interictal data only. The electroencephalographers were unable to distinguish any preictal signal on the
raw EEG in any of the recordings. The interictal recordings were typically 60 minutes in length. Prior to a seizure, 75 minutes of EEG data was included.

The types of seizures captured include subclinical as well as clinical focal seizures. No seizures with generalized onsets were used. Each patient in this study had more than one seizure during their recording.

2) CHB-MIT EEG Dataset:

The CHB-MIT database contains scalp EEG data collected from 22 patients with 9-42 recordings for each patient, originally collected for the seizure detection problem [38]. 136 of the recordings contain one or more labeled seizure and 509 of the recordings contain no seizure activity. The recordings were collected at a sampling rate of 256Hz. We selected a subset of recordings suitable for our study from this dataset for training and testing. Specifically, we selected 68 of the 129 seizure recordings which met the following two requirements: (a) contained only one seizure event and (b) contained at least 30 minutes of EEG data before the seizure event. We also randomly selected 50 recordings with no seizure activity from the CHB-MIT database to measure the specificity of the model.

3) From Raw Data to Wavelet Tensors:

The wavelet transform on an EEG signal transforms the signal from the time view to a combination of the time and frequency view, which is useful both in clinical and computational analysis [39]. Applying the continuous wavelet transform to each channel of the EEG yields a tensor of wavelet coefficients with three modes; time, scales, and channels as shown in Fig 1.

![Wavelet Tensor](image)

Fig. 1. EEG wavelet tensor. Wavelet tensor constructed by convolving each channel of the EEG signal with wavelet functions at different scales. There are 22 channels and 10 scales in each wavelet tensor.

We use the wavelet transform to provide a window into the frequency domain to aid in our analysis and to input to the convolutional neural network (CNN). This was motivated by previous success with the use of tensor decomposition and analysis on wavelet tensors for the seizure detection problem [40] and with a similar approach to genomic data [1]. Other benefits of using this transformed signal include revealing multiscale frequency information at each time point and isolating noise [41]. In theory, all of these benefits could be achieved by several convolutional layers trained on the raw EEG signal [42] but at the cost of needing more training data and training time. Training on the wavelet-transformed signal achieves a deeper network without the time and data penalty of training an additional set of convolutional layers. In addition, the training algorithm for the CNN is extended to include the scale parameters of the wavelet transform.

C. Methodology: Overview and Background

The first aim of our approach is to extract features from the EEG signal that can be used to distinguish between different functional states. We use CNN to extract features from the EEG signal and focus on explicitly differentiating between preictal, ictal, and interictal examples. Since the true preictal period length is unknown, the second aim is to estimate the length of the preictal period and the optimal prediction horizon. As there are many possible candidate preictal period lengths, we use cross-validation to choose the appropriate length.

1) Aim 1: Convolutional Neural Networks for Feature Extraction:

CNN have demonstrated considerable success due to their ability to model local dependencies in the input and reduce the number of trained parameters in a neural network through weight sharing. We leverage both of these properties on the wavelet-transformed EEG. We use convolutional filters to learn features that capture the short term temporal dependencies of the EEG, and at the same time look for relationships between close frequency bands. We will briefly describe some features of deep CNN that we take advantage of here, for a full discussion see [29].

In addition to learning filter maps, CNN also feature two other common components: max-pooling and dropout. The max-pooling method allows the network to learn features that are temporally (or spatially) invariant. The network can then identify patterns in the coefficients of the wavelet tensor without considering whether the pattern occurs in the beginning or end of the signal slice. Dropout [43] randomly sets the output of units in the network to zero during training, preventing those units from affecting the output or the gradient of the loss function for an update step. This serves as a regularization technique to improve generalization by ensuring the network does not overfit by depending on specific hidden units.

The CNN trained had six convolutional layers followed by two dense layers, as shown in Fig 2. The output layer consisted of three units with a soft-max activation representing a probability distribution over the three classes. We arrived at this architecture after experimenting with many different architectures, both shallower and wider. Deeper models proved difficult to train on the available hardware, while shallower models suffered from poor accuracy. The convolutional layers are stacked one after the other, decreasing the number of filters in each layer. Filter sizes were fixed at 3x3 and 2x2 as experiments with larger filters showed no improvement. Max-pooling layers were inserted after every other convolutional layer and dropout added after every layer. Instead of the typical sigmoid nonlinear activation, the rectified-linear unit is used for its non-saturating properties which inhibit the vanishing gradient problem [44].

2) Aim 2: Computing the Length of the Preictal Period:

The most crucial missing information in the seizure prediction problem is the actual length of the preictal period. Knowing the exact start and end points of the preictal period
Convolutional layers learn a set of filters (kernels) that are convolved with the output from the previous layer (feature map). Max-pooling layers downsample one or two dimensions of the feature map. For example, the third convolutional layer learns a set of 50 kernels of size 3x3 and is followed by a pooling layer along the time dimension. Fully connected layers have a connection to every element of the output from the previous layer. 

Fig. 2. CNN architecture

Convolutional layers learn a set of filters (kernels) that are convolved with the output from the previous layer (feature map). Max-pooling layers downsample one or two dimensions of the feature map. For example, the third convolutional layer learns a set of 50 kernels of size 3x3 and is followed by a pooling layer along the time dimension. Fully connected layers have a connection to every element of the output from the previous layer. The first fully connected layer has 250 units, each of these units has a weighted connection to the 5×5×16 elements in the previous feature map. Each unit in the output layer outputs a probability of belonging to one of the three classes. Generated with code from http://www.github.com/gwding/draw_convnet

during the training process would greatly improve supervised machine learning approaches.

Since supervised machine learning algorithms require training data in the form of labeled examples of the incident of interest, missing or incorrect labeling poses a crippling problem. This is well studied in the machine learning community [45]. In this context, the problem presents itself in two forms. (a) Assume the preictal period extends 8 minutes prior to seizure onset, but the labeler labels the period 15 minutes before seizure onset as belonging to the preictal class. Training a machine learning algorithm on such a dataset would result in very poor performance on the preictal class because the learner is given 7 minutes of signal that is interictal but labeled as preictal. This makes learning to differentiate between the interictal and preictal classes impossible. (b) If the opposite error is made and only 8 minutes are labeled as preictal but the preictal period extends 15 minutes before seizure onset the learner once again will fail. This is because the interictal class is now noisy with mislabeled examples from the preictal class. This is especially a problem in the context of automatic feature learning. Because the algorithm will find features that will perform well at the given task, in the presence of mislabeled data, the task the machine learns to do and the one we want it to do will not necessarily be the same.

The literature provides some estimates to the length of the preictal period, but the estimates are not shown to be general [33]. Our approach to deal with this problem is simple, we make the preictal length a parameter in the learning process and optimize over it using grid search. To further validate the automatic selection of the preictal length, we analyze the extracted features from different preictal lengths.

D. Methodology: A Prediction System

The input to the neural network is constructed by computing the wavelet transform on each recording to generate the tensors shown in Fig 1. Following [40], we use a set of didactic scales from 1 to 512 and the Mexican-hat mother wavelet. We introduce two parameters, epoch length and overlap percentage, and divide the tensor into overlapping windows of length $e$ seconds and overlapping by $o$ percent. Each of these windows becomes a separate training example to the neural network. We normalize by subtracting the mean and dividing by the standard deviation over each channel for each of the training examples.

In order to label the examples into the three classes; interictal, preictal, and ictal, we introduce an additional parameter $l$ representing the assumed preictal length. Using the provided seizure onset time, we label all windows that fall $l$ minutes before the seizure onset time as preictal and all windows after the seizure onset time as ictal (until the end of the recording). The remaining windows are considered interictal.

One of the common problems for machine learning algorithms is that they require balanced datasets in order to learn a non-trivial pattern from the data. The dataset is heavily imbalanced due to the ease of obtaining interictal data; the number of interictal examples outnumbers the other two classes by almost a factor of 8. Without balancing the classes, the classifier would achieve a low error rate by simply classifying all examples as interictal. This is a well-studied problem with many recommended techniques to balancing the dataset [46]. Examples of these approaches are oversampling the minority classes, undersampling the majority classes, or combinations of both to balance the dataset [47]. We adopt a simple undersampling scheme on the interictal class, randomly selecting a number of majority class examples such that the classes are balanced.

Before training the CNN on the labeled data, we split the data into a training and validation set using k-fold cross-validation. We set $k$ to 10 and split the data into 90% for training data, and 10% for validation data. We use the validation set for hyperparameter optimization and monitor the loss function on the validation set as the criteria for early stopping. Some of the important hyperparameters to optimize include; epoch length, overlap percentage, and preictal length. Early stopping is used to prevent overfitting the training dataset by halting training when loss on the validation set begins to increase.

Training the deep convolutional network was done using stochastic gradient descent with an adaptive learning rate [48]. Gradients were estimated using the Keras [49] wrapper for the Theano library [50]. It is important to note that the loss function for training was not based on seizure prediction performance (the network’s ability to detect the seizure before it occurred). Instead the network was trained using the categorical cross-entropy loss function over the three classes.

After training is complete, the network enters the inference stage where it generates a probability distribution over the three classes for a new input signal. We are interested in when the patient leaves the interictal state and use the output probabilities of the network to model this. Given $p_0$, $p_1$, and $p_2$ as the output probabilities for the interictal, preictal, and ictal classes respectively, where $p_0 + p_1 + p_2 = 1$, we compute the probability of an oncoming seizure as $p = p_1 + p_2 = 1 - p_0$.

Until this point, each input window has been treated independently. We introduce dependence between adjacent windows by allowing a previous prediction to influence the
current one and generate smooth outputs:
\[
s(0) = p(0)
\]
\[
s(t) = \alpha p(t) + (1 - \alpha)s(t - 1)
\]

Where \( \alpha \) is the exponential smoothing parameter. This effectively smooths the output over time and allows us to control the sensitivity of the predictor. The system declares a seizure imminent (within minutes) when the signal \( s \) crosses a threshold. We show results using an empirically determined threshold of 0.6 but evaluate the performance of the system using the area under the ROC curve (ROC-AUC).

The output of the last hidden layer of the trained network, the 100-unit fully connected layer of the network shown in Fig. 2, represents a compressed version of the input signal. We can observe the output of each unit in this layer for each epoch, yielding a \( T \times 100 \) matrix of extracted features. This is useful for analyzing the phase transition between the interictal and preictal state.

Hyperparameter optimization was done using a grid search on a range of values for each variable. The final set of hyperparameters was chosen as the set achieving the lowest average validation loss over the 10 fold cross-validation. The final hyperparameters after training are \( e = 1 \) second epoch length, \( \alpha = 0\% \) overlap between windows, \( \alpha = 0.7 \), and \( l = 10 \) minute preictal length. The preictal length optimized through cross-validation in the training phase is set as the prediction horizon for the system.

### III. Results

In addition to describing the performance of the system on the test datasets in this section, we verify that the features learned by the CNN capture the interictal to preictal phase transition using the Kullback-Leibler (KL) divergence.

#### A. Extracted Feature Analysis for Phase Transition

The features extracted by the network under different preictal lengths are analyzed to verify the preictal length value yielded by cross-validation. Specifically, we want to find the earliest time before seizure onset where a dramatic change in the distribution of extracted features occurs (i.e., a change point). After selecting a subset of the features and de-correlating them, two distributions are estimated in the extracted features; the distribution during interictal-only periods and the distribution around a time point \( t \). The interictal distribution is approximated by a multivariate Gaussian with mean \( \mu_0 \) and covariance \( \Sigma_0 \) calculated from the features of epochs in the interictal period. Similarly, we approximate the distribution of features at \( t \) by calculating the mean \( \mu_1 \) and covariance \( \Sigma_1 \) of the set of features from \( t - L \) to \( t \), where \( L \) is the number of samples. We then measure the divergence between the interictal distribution and the distribution at \( t \) using the following form of the KL divergence with \( k \) set as the number of features:

\[
D(t) = \frac{1}{2}(tr(\Sigma_1^{-1}\Sigma_0) + (\mu_1 - \mu_0)^T\Sigma_1^{-1}(\mu_1 - \mu_0) - k + \ln \frac{det\Sigma_1}{det\Sigma_0})
\]

Using this measure with \( \mu_0, \mu_1, \Sigma_0, \) and \( \Sigma_1 \) from above, the earliest time is found where the baseline and current distributions are significantly different; Fig 3 shows an example of this measure computed on a recording from the MSSM dataset for four preictal lengths. The divergence is small and constant during the first part of the recording but increases rapidly at approximately 9-10 minutes before the seizure onset. This means that there is a shift in the distribution of extracted features at that time. Furthermore, the shift occurs at the same location irrespective of the value of the preictal length parameter, indicating the features which are learned are capturing the transition in the underlying system. The same pattern also appears consistently in other recordings from the validation set. This affirms the prediction horizon determined using cross-validation and indicates the phase transition from the interictal state to the preictal state that is captured by these features occurs around 10 minutes before seizure onset.

![Fig. 3. Verifying the preictal period length KL-divergence between interictal feature distribution and feature distributions over a single recording for preictal lengths of 20, 15, 10, and 5 minutes. Notice the abrupt increase in divergence at around 9-10 mins before seizure onset (onset marked with red dashed line) irrespective of the preictal length parameter value. This pattern was observed across recordings indicating a approximate preictal length of 10 minutes.](image)

#### B. Test Set Results

The results on the MSSM and CHB-MIT test sets are summarized in Table 3 and Table 4 respectively. Table 3 lists the number of seizures and the total length of the recordings for each patient in the test set. Table 4 compares the specificity and sensitivity results for this work and three other seizure prediction methods. Sensitivity is defined as the percentage of seizures correctly predicted. Specificity is given by the false prediction rate (FPr) which is the number of false predictions divided by the total length of interictal-only periods. The prediction times of seizures from each patient tested aligned by the seizure onset time are shown in Fig 5. Fig 4 shows an example of the predictions generated by the system on a seizure recording using a fixed threshold \( (r = 0.6) \).

In addition to measuring seizure prediction performance, we trained an identical CNN on the raw data by feeding the raw EEG signal to the network (omitting the wavelet
transform step) to evaluate the advantages of training on wavelet-transformed data. Comparison of the two networks on the test set using Matthews correlation coefficient demonstrated that using wavelet-transformed EEG as input to the network resulted in improved performance across all test set recordings. One reason for this improvement is the filtering effect of the wavelet transform allows the CNN to learn features that involve activity across multiple frequency bands.

1) MSSM Test Set:

Three recordings in this test set are interictal-only from three different patients. The remaining 15 recordings contain seizures from 12 different patients. The average prediction time for the seizures was 8 minutes before seizure onset. One of the 15 seizures tested was not predicted by the system. The false prediction rate was .128/hr. Interestingly, false positives did not occur in interictal-only recordings but occurred in seizure recordings long before the 10 minute prediction horizon.

| Patient # | Type               | Pred. time (secs) | False pred. | ROC-AUC |
|-----------|--------------------|------------------|-------------|---------|
| 2         | interictal         | N/A              | 0           | N/A     |
| 4         | interictal         | 460              | 0           | 0.935   |
| 5         | right temporal     | 557              | 0           | 0.961   |
| 6         | interictal         | N/A              | 0           | N/A     |
| 11        | left temporal      | 586              | 0           | 0.941   |
|           | left frontotemporal| -35              | 0           | 0.448   |
| 18        | left temporal      | 234              | 0           | 0.795   |
| 19        | left temporal      | 515              | 0           | 0.955   |
| 22        | right frontotemporal| 512           | 0           | 0.934   |
| 24        | left temporal      | 569              | 1           | 0.949   |
| 25        | right temporal     | 532              | 0           | 0.945   |
| 26        | right temporal     | 536              | 0           | 0.937   |
| 27        | left temporal      | 520              | 1           | 0.903   |
| Avg.      |                    | 464.4            | 0.885       |

2) CHB-MIT Test Set:

The CHB-MIT test set consists of 18 seizure recordings and 50 interictal recordings, allowing us to evaluate the specificity and false prediction rate of the system accurately. The total of 68 recordings corresponds to approximately 70.5 hours of EEG, of which 50 hours are interictal only. Seizures were predicted on average 6 minutes before the seizure onset time with three of the 18 seizures not predicted. The false prediction rate was .147/hr. In this set of recordings, false predictions occurred within interictal only recordings.

C. Error Analysis

Analysis of the raw EEG signal revealed some causes of error, particularly false positives. This can be seen in the linear algebraic properties of the wavelet matrices obtained by slicing the wavelet tensor shown in Fig 1 along the channels mode. Each slice is a $T \times 10$ matrix of wavelet coefficients. In Fig 6, we show a scatterplot of the spectral gap, numerical rank, and condition number of these matrices. The spectral gap of a matrix can be calculated using the non-zero singular values $\sigma_1, \sigma_2, ..., \sigma_r$ of the matrix, with $r \leq 10$. Spectral gap is given by the ratio of the first and second singular values:

$$\frac{\sigma_2}{\sigma_1}$$

The numerical rank of the matrices is estimated by computing the squared ratio of the Frobenius norm and the spectral norm of the matrix $A$:

$$\frac{||A||_F^2}{||A||_2^2}$$
Fig. 5. **Prediction horizons for test patients** The prediction time of the system is shown for all test set recordings. Each row shows the prediction time the system achieved for all the recordings from a single patient. The vertical red line indicates the seizure onset time. Predictions colored black were reported after the seizure occurred. The standard deviation of prediction times across all the recordings is large ($\sigma = 98.7$). This indicates that while the preictal phase transition is captured by the system for most patients, a patient-specific system could decrease the spread of prediction times.

### TABLE III
Number of test seizures and recording lengths for each patient

| Patient | # of seizures | Length (mins) |
|---------|---------------|---------------|
| 6       | 1             | 85            |
| 4       | 1             | 85            |
| 11      | 2             | 170           |
| 5       | 1             | 85            |
| 14      | 1             | 85            |
| 18      | 3             | 255           |
| 19      | 1             | 85            |
| 22      | 1             | 85            |
| 24      | 1             | 85            |
| 25      | 1             | 85            |
| 26      | 1             | 85            |
| 27      | 1             | 85            |
| chb01   | 1             | 60            |
| chb02   | 1             | 60            |
| chb03   | 1             | 60            |
| chb04   | 1             | 60            |
| chb05   | 2             | 120           |
| chb07   | 1             | 60            |
| chb08   | 2             | 120           |
| chb10   | 1             | 60            |
| chb11   | 1             | 60            |
| chb13   | 1             | 60            |
| chb17a  | 2             | 120           |
| chb19   | 1             | 60            |
| chb21   | 1             | 60            |
| chb22   | 1             | 60            |
| chb24   | 1             | 60            |

The condition number of a matrix is given by the ratio of the largest singular value and the smallest singular value:

$$\frac{\sigma_1}{\sigma_r}$$

We observed that the false positives incurred on the MSSM test set are due to the poor conditioning of the matrices. This is indicated by the low spectral gap and high condition numbers for those recordings, shown as blue pentagons in Fig. 6. False positives from the CHBMIT dataset did not follow this pattern and are indistinguishable from recordings without error in this space. False negatives from both datasets are also scattered among recordings without error indicating the underlying causes are not captured by a linear analysis.

The four seizures not predicted by the system were also visually inspected. From the MSSM database, the single seizure not predicted was in a low amplitude EEG ($\leq 15$ uV) with some muscle artifact superimposed. From the CHB-MIT database, two of the seizures had substantial electrode and movement artifact, likely making prediction impossible. The third seizure not predicted had numerous sleep/wake transitions and frequent epileptiform bursts. Sleep/wake transitions and epileptiform bursts were also found in numerous EEGs in which the program was successful. Further data of failed studies may be helpful in refining the program.

D. **Sensitivity and Comparison to Unspecific Random Predictor**

The sensitivity of the system is shown in Table IV alongside the sensitivity range of an unspecific random predictor ($\sigma_{low} - \sigma_{up}$). The unspecific random predictor is used as a baseline to statistically validate the sensitivity reported by a model as significant. The sensitivity of the analytic random predictor is based on the performance of a predictor that uses a Poisson process to generate predictions. This is determined by a number of parameters; the seizure occurrence period (SOP - defined as the duration after a prediction in which a seizure must occur in order for the prediction to be correct), a fixed false prediction rate (FPR), and the number of independent features analyzed ($d$). The sensitivity range of the random predictor is given by:

$$\sigma_{low} = \frac{\lambda}{d}, \quad \sigma_{up} = \frac{\lambda}{d} + \sqrt{\frac{\lambda}{d^2}}$$

where $\lambda$ is the rate of seizures in the dataset.
prediction is given by (where $K$ is the number of analyzed seizures) [23]:

$$P = \text{SOP} \ast \text{FPr}$$

$$\sigma_{\text{low}} = \max_k \left( 1 - \left( \sum_{j<k} \binom{K}{j} P^j (1 - P)^{K-j} \right) \right)$$

$$\sigma_{\text{up}} = \max_k \left( 1 - \left( \sum_{j<k} \binom{K}{j} P^j (1 - P)^{K-j} \right)^d \right)$$

The ranges in the last column of Table IV were calculated using SOP = 10 minutes, FPr set to the observed false prediction rates in Table IV and a significance level $\alpha = 0.05$. The number of independent features $d$ is difficult to estimate and was set to the upper bound of $d = 100$ (corresponding to the number of units in the second to last layer of the CNN).

| Method       | Sensitivity | FPr (FP/h) | Random pred. $\sigma_{\text{low}}$ - $\sigma_{\text{high}}$ |
|--------------|-------------|------------|-----------------------------------------------------------|
| Kaggle1 [5]  | 72.7%       | 0.285      | 15.1% - 27.2%                                            |
| Kaggle2 [5]  | 75.8%       | 0.230      | 12.1% - 24.2%                                            |
| Cook et al. [6] | 66.7%     | 0.186      | 12.1% - 21.2%                                            |
| This work    | 87.8%       | 0.142      | 9.1% - 15.1%                                             |

### IV. Computational Comparison to Prior Work

Testing the learned features on an out-of-sample dataset achieves results with sensitivity and specificity superior to other published seizure prediction methods [14, 2]. We evaluated the performance of three other algorithms on our datasets (accessible online [http://www.dsrc.rpi.edu/page=databank](http://www.dsrc.rpi.edu/page=databank)). Two of the algorithms, Kaggle1 and Kaggle2 in Table IV are top performers from the Kaggle seizure prediction competition [5] ([https://www.kaggle.com/c/seizure-prediction](https://www.kaggle.com/c/seizure-prediction)). Participants were asked to develop algorithms that could distinguish between ictal and preictal EEG samples. The Kaggle1 algorithm uses a combination of time and frequency domain features, such as the spectral entropy of selected frequency bands, time correlation matrix, frequency correlation matrix, and higuchi fractal dimension, with logistic regression as a predictor. The Kaggle2 algorithm uses a similar approach but with fewer features and a support vector machine classifier. The third algorithm evaluated is the one proposed by Cook et al. [6], which calculates average energy, Teager-Kaiser energy, and line-length on an array of six filters over each channel. After a feature selection step, a hybrid decision-tree/k-nearest neighbor classifier is used for classification. Since we are not aware of any published codes for the algorithm, we implemented the algorithm in Python. As shown in Table IV each of the methods performed better than an unspecific random predictor. Our system achieves higher sensitivity (87.8%) than all of the other methods with a lower false prediction rate (0.142 FP/h).

### V. Discussion

Our results show that a preictal phase transition can be observed in scalp EEG data using features automatically extracted from the signal. The results also showed that typical EEG artifacts and changes, such as sleep-wake transitions, did not cause false positives. In addition, the phase transition is shown to occur approximately 10 minutes before seizure onset as reported in previous studies [20]. However, as shown in Fig 5 the spread of prediction time of the preictal transition is large ($\sigma = 98.7$). This spread is a drawback of training on data from many different patients, which was necessary because of the limited amount of patient specific data.

The datasets used in this study contain only scalp EEG recordings which, as mentioned previously, are easier to obtain but suffer drawbacks in signal quality. Applying our methodology to a dataset of intracranial EEG recordings could allow the system to learn features that detect the preictal phase transition further in advance and with less variability, similar to studies using intracranial EEG which report a prediction horizon of an hour or longer [14].

### VI. Conclusion

These results suggest real feasibility in creating a reliable seizure prediction system. The goal would be to create a wearable non-invasive EEG device which would alert patients, family members, and doctors to imminent seizures. This has the potential to enhance the safety of patients, decrease the rates of sudden unexpected death in epilepsy patients (SUDEP) and perhaps to allow some patients to take medications only when needed and not chronically.

To achieve a complete system, a number of hurdles still need to be overcome. The limited database of scalp EEG recordings needs to be extended in order to fully test the generalization ability of the system. In addition, a clear understanding of how the ictal state ends is required. This information will allow the predictor to reset itself after an ictal event. Creating an end to end deep neural network that directly optimizes over the prediction task is another step that we intend to pursue.

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