Spin asymmetries in jet-hyperon production at LHC

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We consider polarized Λ hyperon production in proton-proton scattering, \( p p \to (\Lambda^\uparrow\text{jet})\) jet \(X\), in the kinematical region of the LHC experiments, in particular the ALICE experiment. We present a new Λ polarization observable that arises from the Sivers effect in the fragmentation process. It can be large even at midrapidity and therefore, is of interest for high energy hadron collider experiments. Apart from its potential to shed light on the mechanisms behind the phenomenon of Λ polarization arising in unpolarized hadronic collisions, the proposed observable in principle also allows to test the possible color flow dependence of single spin asymmetries and the (non)universality of transverse momentum dependent fragmentation functions.

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I. INTRODUCTION

Since the observation of large transverse polarization of produced Λ hyperons in the inclusive reactions \(pp \to \Lambda^\uparrow X\) [1] and \(pBe \to \Lambda^\uparrow X\) [2] in the middle of the 1970’s, there have been many experimental and theoretical investigations aimed at understanding this striking polarization phenomenon [3, 4]. The polarization measurements of Λ hyperons produced in these inclusive reactions have been performed in fixed target experiments, and the data showed that the Λ polarizations are large only for large \(x_F\). This poses a problem if one wants to investigate this observable further using high energy colliders such as RHIC, Tevatron or LHC, where the capabilities to measure Λ polarization are restricted to the midrapidity region, where \(x_F\) is very small. However, in this paper we point out a way to by-pass this problem by making a less inclusive measurement: to select two-jet events and to measure the jet momenta \(K_j\) and \(K_j'\) in addition to the momentum \(K_\Lambda\) and polarization \(S_\Lambda\) of the Λ that is part of either of the two jets. An asymmetry proportional to \(\epsilon_{\mu\nu\alpha\beta}K_\mu^j K_\nu^{j'} K_\alpha^\Lambda S_\beta^\Lambda\) can then arise, which is neither power suppressed, nor needs to be zero at vanishing \(x_F\) of the Λ. The observable is proportional to \((K_j \times K_\Lambda) \cdot S_\Lambda\) in the center of mass frame of the two jets. Within a factorized description of the process under consideration, such an asymmetry can arise from spin and transverse momentum dependent fragmentation functions, which in turn can arise from two types of interactions. We will point out how the measurement of the suggested asymmetry in principle can allow for a differentiation between these two types of mechanisms. In this way also hadron collider experiments can contribute to our understanding of the underlying mechanisms that lead to Λ polarization, in addition to experiments with electron-positron and lepton-hadron collisions.

It is well-known that transverse momentum dependent distribution and fragmentation functions, nowadays commonly referred to as TMDs, can have a nontrivial spin dependence and that the so-called “T-odd” TMDs can lead to single spin asymmetries [3, 4, 5, 6]. They are also often referred to as “naively T-odd”, because the appearance of these functions does not imply a violation of time-reversal invariance. The Sivers distribution function \(f_1^{T_\perp}\), schematically depicted in Fig. 1 is the oldest example of such functions. It describes the difference between the momentum distri-
The Sivers distribution function was put forward \cite{5, 7} as a possible explanation for the large single spin asymmetries observed in $p^\uparrow p \to \pi X$ experiments \cite{9}. Furthermore, it generates single spin asymmetries in semi-inclusive DIS \cite{8, 10}, which have also been measured to be nonzero \cite{11}, and it results e.g. in asymmetric di-jet correlations in $p^\uparrow p \to$ jet jet $X$ \cite{12, 13}, which however are not yet visible in the data analyzed \cite{14}.

The fragmentation analogue of the Sivers distribution function is called $D_{1T}^\perp$ \cite{15}. It describes the distribution of transversely polarized spin-$1/2$ hadrons, such as $\Lambda$'s, inside the jet of a fragmenting unpolarized quark, cf. Fig. 2. For this reason it has been referred to as “polarizing fragmentation function” in Ref. \cite{16}. It is an odd function of the transverse momentum of the observed hadron w.r.t. the quark direction, or equivalently, the jet direction. Despite the similarity between the definitions of $D_{1T}^\perp$ and $f_{1T}^\perp$, there are some important differences. Nonvanishing $T$-odd distribution functions require soft gluonic interactions between the target remnants and the active partons \cite{10}. These interactions can be resummed into Wilson lines (gauge links), ensuring the gauge invariance of the operator definitions of the distribution functions \cite{17, 18, 19, 20}. On the other hand, there are two mechanisms to generate $T$-odd effects in the fragmentation process: through final-state interactions within the jet, e.g. between the observed outgoing hadron and the rest of the jet \cite{6}, or through soft gluonic interactions between the jet and the hard scattering part (sometimes also referred to as final-state interactions), see Figs 3a and 3b respectively. As for the distribution functions, the latter interactions give rise to Wilson lines. Both effects can be expressed in terms of a TMD fragmentation function. The $\Lambda$ polarization observable that is the subject of this paper in principle allows for a differentiation between these two effects. There is considerable interest in this issue, as it could shed light on the color flow dependence of single spin asymmetries and the (non)universality of transverse momentum dependent fragmentation functions.

In recent years it has become apparent that $T$-odd TMDs enter in different ways in different processes, depending on the color flow in the partonic subprocesses\footnote{Although the possibility of such effects for $T$-even TMDs are not excluded, they will not be considered here.}. The sign relation between the Sivers function appearing in semi-inclusive DIS and in Drell-Yan scattering was the first example of such process dependence \cite{17, 21}. More complicated
relations were discussed soon afterwards for hadron production processes in hadronic collisions \cite{22, 23, 24, 25}. Process dependence may seem at odds with factorization \cite{26}, but since the color flow dependence can be explicitly taken into account through the determination of the Wilson line structure, the process dependence is explicitly calculable and hence, predictive power may be retained albeit in a less straightforward manner. For the specific example of the single spin asymmetry in $p^+_p \rightarrow \text{jet jet}$ the effects of color flow dependence have been taken into account \cite{27}, leading to a suppression w.r.t. the standard parton model expectation. For fragmentation functions the situation is complicated further by the fact that only one of the two mechanisms leading to nonzero T-odd fragmentation functions depends on the color flow structure \cite{20}. However, a claim based on a model calculation \cite{28} and on more general arguments \cite{29} has been put forward that for T-odd fragmentation functions no color flow dependence arises at all. This makes an experimental test of this dependence, or of its absence, all the more interesting.

We propose to measure it in the process $pp \rightarrow (\Lambda^\uparrow \text{jet})$ jet $X$, where the brackets indicate that the $\Lambda$ is part of one of the two observed jets which are almost back-to-back in the plane perpendicular to the beam axis and its polarization is measured. This measurement can be done for instance at LHC and RHIC, although –as we will point out– the situation at LHC is more straightforward thanks to the dominance of gluon-gluon scattering in the region of interest.

The $\Lambda$ polarization observable we put forward here is similar to the Sivers effect observable in $p^+_p \rightarrow \text{jet jet} X$ \cite{12, 13}, but the $\Lambda$ polarization observable does not depend on a non-collinearity of jets in the transverse plane. In other words, even if one were to assume the initial partons to be collinear to the parent hadrons, in which case the jets are exactly back-to-back in the plane perpendicular to the beam axis, an asymmetry can still arise. Therefore, it is expected that the $\Lambda$ polarization observable is less sensitive to effects that would smear out the partonic $2 \rightarrow 2$ kinematics. This prompts us to ignore contributions that depend on nontrivial spin and transverse momentum dependent effects in the initial states (which would be proportional to $h_1^T H_1$ in the notation of Ref. \cite{8}).

The observable turns out to be very sensitive to the unpolarized $\Lambda$ fragmentation functions. The relative importance of contributing partonic subprocesses differs strongly depending on whether one uses for instance the fragmentation functions by De Florian et al. \cite{30} (FSV) or by Albino et al. \cite{31} (AKK). Therefore, even if no polarization effect is seen, the unpolarized data obtained by the proposed measurement can in any case lead to a considerable improvement in the determination of the unpolarized $\Lambda$ fragmentation functions.

We will explain why it is necessary to measure both jet moments and also how it differs from the process $pp \rightarrow \Lambda^\uparrow X$. We will start with a discussion of that process. Anselmino et al. \cite{16} have attempted to describe the latter in terms of $D_{1T}^\uparrow$, but although a satisfactory description of the $\Lambda$ polarization at larger values of the transverse momentum of the $\Lambda$ could be obtained, considerable doubts about the applicability of the assumed factorization have arisen afterwards. A similar problem is unlikely to arise for the $\Lambda$ polarization study in the process $pp \rightarrow (\Lambda^\uparrow \text{jet})$ jet $X$ at LHC and RHIC.

II. $\Lambda$ POLARIZATION IN $pp \rightarrow \Lambda^\uparrow X$

The $\Lambda$‘s measured in the process $pp \rightarrow \Lambda^\uparrow X$ \cite{1} by the fixed target experiments have transverse momenta $p_T$ w.r.t. the beam direction ranging up to 4 GeV/$c$. The $\Lambda$ polarization transverse to the production plane first grows linearly with $p_T$, but around $p_T \approx 1$ GeV/$c$ it levels off and remains approximately constant. For large $p_T$ one expects the polarization to fall off as $1/p_T$, based on theoretical (collinear expansion) arguments \cite{32}. The high energy collider experiments would be ideally suited to demonstrate this power-law fall-off behavior, if it were not for the small $x_F$ values probed in those experiments.

The linearly rising behavior at very low $p_T$ is well-described by recombination models employing spin-orbit couplings \cite{4}. However, the plateau region for $p_T \sim 1$–4 GeV/$c$ is less straightforward to incorporate in such soft physics models. Anselmino et al. \cite{16} assumed that the $p_T$ in this region may be considered a hard scale which then would allow for factorization of the cross section expression. However, since the scale is not too large either, instead of restricting to collinear factorization, transverse momentum dependence at the parton level was included. In this way the $\Lambda$ polarization could be described in terms of the fragmentation function $D_{1T}^\uparrow$. Although a satisfactory description of the higher-$p_T$ data was obtained in this way, a preliminary study of the unpolarized cross section within the same factorized approach resulted in considerable disagreement with the data, as mentioned in Ref. \cite{33}. Apparently, at large $x_F$ a $p_T$ value of 3 or 4 GeV/$c$ is not yet large enough to allow for a factorized description in the case of $\Lambda$ production, as opposed to for instance pion production. Inclusion of intrinsic transverse momentum in the initial state parton distributions may lead to less discrepancy with the data, as studied for pion production \cite{34}, but remains to be done for $\Lambda$ production. Despite the doubts about whether $D_{1T}^\uparrow$ can be extracted from the existing fixed-target $\Lambda$ polarization data, the collider experiments at RHIC and LHC are in a kinematic region that should allow for a factorized description of the cross section and thus can be considered safe. As mentioned, they do have the drawback that $\Lambda$ measurements are generally restricted to small values of $x_F$, where the $\Lambda$ polarization in $pp \rightarrow \Lambda^\uparrow X$ will be close to zero. Therefore, we
next discuss a different process, where this drawback does not appear.

The main difference between the traditional process $pp \to \Lambda^1 X$ and the one considered in this paper, $pp \to (\Lambda^1\text{jet}) X$ comes from the way $D_{T}^{1/2}$ enters in the asymmetry expressions. In the latter process it enters in an unintegrated way, i.e. the asymmetry is proportional to $D_{T}^{1/2}(z,k_{T}^2)$, where $z$ and $k_{T}$ are observed. In the traditional process on the other hand, there is an integration over $k_{T}$, which implies that one essentially probes the first derivative of the partonic cross section (in case of collinear initial partons), which results in the aforementioned $1/p_{T}$ dependence. In $pp \to (\Lambda^1\text{jet}) X$ this $1/p_{T}$ factor does not arise.

III. A POLARIZATION IN $pp \to (\Lambda^1\text{jet}) X$

We consider the process $p(P_1)+p(P_2)\to (\Lambda^1(K_\Lambda)\text{jet}(K_j)) +\text{jet}(K_{j'})+X$. As indicated by the brackets, the $\Lambda$ is part of the jet with observed momentum $K_j$, where the two jets are almost back-to-back in the plane perpendicular to the beam axis. The projections of the outgoing momenta in this plane will be denoted by \( x \) of the jet with observed momentum $K_j$ and the Boer-Mulders effect and the Sivers fragmentation effect. In the Boer-Mulders mechanism \cite{8} the single spin scales in the process (such as $\sqrt{s}$). In principle there are several effects that contribute to the spin dependence of the cross section in Eq. (2), viz. the Boer-Mulders effect and the Sivers fragmentation effect. In the Boer-Mulders mechanism \cite{8} the single spin...
asymmetry arises as a consequence of the transverse momentum and transverse spin of a quark inside an unpolarized initial hadron. Hence, it relies on a $k_T$-effect that will have to translate into an asymmetry in the final state. Through the hard partonic scattering there can be $k_T$-smearing, leading to a dilution of the asymmetry in the final state. We will, therefore, assume that the Boer-Mulders effect is less important than the Sivers fragmentation effect. The latter is a $k_T$-effect that is directly in the final state, such that there is no dilution from $k_T$-smearing in the partonic scattering. In addition, one may expect that the chiral-odd contributions ($\propto h_\perp H_1$) are generally smaller than the chiral-even ones ($\propto f_1 D_{1T}^\perp$), similar to what was found in Ref. [24] for the process $p^1 p \to \gamma j X$ using positivity bounds. Based on these considerations, we will simply neglect the intrinsic transverse momenta $p_{1T}$ of the incoming particles, such that only the Sivers fragmentation mechanism contributes.

As mentioned in the introduction, this effect is described by the TMD fragmentation function $D_{1T}^\perp(z,K_\perp^2)$, which enters the expression in Eq. (2) through the parameterization of the fragmentation correlator [15, 30, 37] (suppressing Wilson lines and with the convention $\delta^{123} = +1$)

$$\Delta(z,k_T;K,A,S) = \sum_X \frac{1}{z} \int \frac{d(\xi)K_A}{(2\pi)^3} e^{-i k \cdot \xi} \langle 0 | ;\Lambda_X(\xi) \Lambda_X(0) | 0 \rangle_{\xi,n=0} \tag{4a}$$

$$= \left( D_1(z,K_\perp^2) - D_{1T}^\perp(z,K_\perp^2) \frac{\epsilon^{\mu\nu\rho\sigma}K_{A\mu}b_{A\nu}S_{A\rho\sigma}}{M_A (K_\perp-n)} \right) K_A + \text{other functions} \tag{4b}$$

$$= \left( D_1(z,K_\perp^2) + D_{1T}^\perp(z,K_\perp^2) \frac{\langle k \times K_A \rangle \cdot S_A}{z M_A} \right) K_A + \text{other functions} , \tag{4c}$$

in the case of quark fragmentation. Here $k$ denotes the quark momentum, which equals the jet momentum $K_j$, and $n$ is the Sudakov vector discussed before. The last line (4c) holds in any reference frame where $n$ and $\hat{k}$ point in opposite directions. Sometimes the notation $\Delta_N D_{1T}/a = |K_A T| D_{1T}^\perp / M_A$ is used to indicate the Sivers fragmentation function [10].

As mentioned in the introduction, there is a complication specifically associated with TMD fragmentation functions. The Sivers fragmentation function of Eqs. (4b) and (4c) is actually a combination of the two functions in Fig. 3, arising from two different types of interactions. In the hadronic cross section the function $D_{1T}^\perp(z,K_\perp^2)$ generated by the final-state interactions will appear convoluted with the usual partonic scattering cross sections. However, due to the dependence of the soft gluon interactions on the partonic subprocesses (in particular on its color flow structure), the TMD fragmentation function $D_{1T}^\perp(z,K_\perp^2)$ generated by this effect will appear convoluted with modified partonic scattering cross sections. The contribution of the Sivers fragmentation mechanism to the hadronic cross section will thus have the form

$$\frac{d\tilde{\sigma}_{ab-\text{cd}}}{dt} D_{1T}^\perp(z,K_\perp^2) + \frac{d\tilde{\sigma}_{ab-\text{cd}}}{dt} D_{1T}^\perp(z,K_\perp^2) \tag{5}$$

where the $d\tilde{\sigma}_{ab-\text{cd}}$ are the usual partonic scattering cross sections and the $d\tilde{\sigma}_{ab-\text{cd}}$ are the modified partonic scattering cross sections (referred to as gluonic pole cross sections) calculated in Refs. [12, 24]. Therefore, the Sivers fragmentation functions in Eqs. (4b) and (4c) should be read as $D_{1T}^\perp \to D_{1T}^\perp + c_\perp G D_{1T}^\perp$, where the $c_\perp$ are color fractions that depend on the particular term in the partonic squared amplitude $H^*H$ which is given by the cut Feynman diagram $D$. These factors are perturbatively calculable and, once absorbed in the partonic scattering cross sections, lead to the gluonic pole cross sections. The gluonic pole cross sections, then, are gauge invariant weighted sums of cut Feynman diagrams $D$ with the color fractions $c_\perp$ as weight factors.

After neglecting intrinsic transverse momentum effects for the incoming particles the Sivers fragmentation contri-

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3 For T-odd gluon correlators in principle two distinct gluonic pole cross sections $d\hat{\theta}(f)$ and $d\hat{\theta}(f)$ appear. However, the latter cross section vanishes for the gluon-gluon scattering contribution [24], the only contribution considered in Sec. [IV]. Therefore, the function $D_{1T}^\perp$ in this paper refers to $D_{1T}^\perp(f)$.  

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bution to the cross section Eq. \((2)\) in the jet-jet c.o.m.-frame becomes

\[
\frac{E_\perp E_j E_j'}{d^3 K_A d^3 K_j d^3 K_j'} \frac{d\sigma_{pp\to(\Lambda')X}}{d^3 K_A d^3 K_j d^3 K_j'} = \frac{1}{\pi} \delta^2(K_{j\perp} + K_{j'\perp}) \sum_{a,\cdots,d} x_1 f_1^a(x_1) x_2 f_1^b(x_2) \left\{ \frac{d\hat{\sigma}_{ab\rightarrow cd}}{dt} z D_1^c(z,(K_A - z K_J)^2) + \frac{S_A(\hat{K}_j \times K_A)}{z M_A} \left[ \frac{d\hat{\sigma}_{ab\rightarrow cd}}{dt} z D_1^c(z,(K_A - z K_J)^2) + \frac{d\hat{\sigma}_{ab\rightarrow |c|d}}{dt} \tilde{D}_1^c(z,(K_A - z K_J)^2) \right] \right\}, \tag{6}
\]

where the momentum fractions \(x_1\) and \(x_2\) have been fixed by the first two delta functions on the r.h.s. of Eq. \((5)\).

The partonic scattering cross sections \(d\hat{\sigma}_{ab\rightarrow cd}\) and the gluonic pole cross sections \(d\hat{\sigma}_{ab\rightarrow |c|d}\) are functions of the Mandelstam variables \(s = (p_1 + p_2)^2\), \(t = (k - p_1)^2\) and \(u = (k' - p_1)^2\) or, alternatively, of \(\hat{s}\) and the variable \(y\) defined through

\[
y = \frac{\hat{t}}{\hat{s}} = \frac{1}{e^{\eta_t - \eta_s} + 1}, \quad \text{and} \quad -\frac{\hat{u}}{\hat{s}} = 1 - y. \tag{7}
\]

Note that the expression of \(y\) in terms of \(\eta_t\) and \(\eta_s\) in Eq. \((7)\) holds in the center of mass frame of the initial hadrons. Since we are ignoring masses w.r.t. \(|K_{j\perp}|\), pseudorapidity and rapidity are the same and since differences of rapidities are invariant under boosts, Eq. \((7)\) also holds in the jet-jet c.o.m.-frame. It should be emphasized that Eq. \((6)\) does not hold in the c.o.m.-frame of the incoming hadrons.

Eq. \((6)\) forms the central result of this paper. Any nonzero measurement of the \(S_A(\hat{K}_j \times K_A)\) asymmetry implies a nonzero Sivers effect in the fragmentation process and will thus be very interesting. It will imply nonzero \(\tilde{D}_1^c\) and/or \(\tilde{D}_1^c\). Due to the different partonic cross sections appearing in front of these two different Sivers fragmenta-

\[\text{tion functions, measuring the } y \text{ dependence of the asymmetry in principle offers the possibility to separate the two contributions, and to learn about their relative importance. This we will explore further in the next section.}\]

If one were to study instead the process \(p(P_1) + p(P_2) \to (\Lambda'(K_A)\text{jet}(K_j)) + X\) without constructing \(K_j\), then one does not have the full partonic kinematic information and one will be probing the combination

\[
D_1^c(z,K^2_{j,l,T}) + \frac{d\hat{\sigma}_{ab\rightarrow |c|d}}{dt} \tilde{D}_1^c(z,K^2_{j,l,T}) \tag{8}
\]

averaged over \(y\). In addition, referring to a correlation \(S_A(\hat{K}_j \times K_A)\) in the jet-jet c.o.m.-frame, involving three-vectors (corresponding to the choice \(n \propto K_{j,l}\)) would not make sense. One could for instance choose \(n \propto P_1\) and consider this correlation in terms of three-vectors in any frame in which \(\hat{P}_1\) and \(\hat{K}_j\) point in opposite directions. In the remainder of this article we will focus on the case where \(K_j\) is measured and present results in the jet-jet c.o.m.-frame.

### IV. PHENOMENOLOGY

One can decompose the hadronic cross section in a spin-independent and a spin-dependent part:

\[
\frac{d\sigma}{d^3 K_A d^3 K_j d^3 K_j'} = \frac{d\sigma_U}{d^3 K_A d^3 K_j d^3 K_j'} + \frac{S_A(\hat{K}_j \times K_A)}{z M_A} \frac{d\sigma_T}{d^3 K_A d^3 K_j d^3 K_j'}. \tag{9}
\]

Comparing to Eq. \((4)\) it follows that within our approximations the ratio of the spin-dependent and spin-independent cross sections is given by

\[
\frac{d\sigma_T}{d\sigma_U} = \frac{\sum_{a,\cdots,d} x_1 f_1^a(x_1) x_2 f_1^b(x_2) \left\{ \frac{d\hat{\sigma}_{ab\rightarrow cd}}{dt} D_1^c(z,(K_A - z K_J)^2) + \frac{d\hat{\sigma}_{ab\rightarrow |c|d}}{dt} \tilde{D}_1^c(z,(K_A - z K_J)^2) \right\}}{\sum_{a,\cdots,d} x_1 f_1^a(x_1) x_2 f_1^b(x_2) d\hat{\sigma}_{ab\rightarrow cd} D_1^c(z,(K_A - z K_J)^2)} . \tag{10}
\]

This ratio appears in the single spin asymmetry

\[
\text{SSA} = \frac{d\sigma(\pm S_A)}{d\sigma(\pm S_A) + d\sigma(-S_A)} = \frac{S_A(\hat{K}_j \times K_A)}{z M_A} \frac{d\sigma_T}{d\sigma_U}. \tag{11}
\]
The relevant partonic hard functions for our study are [24]

\[
\frac{d\hat{\sigma}_{gg-gg}}{dt} = \frac{\alpha_s^2 \pi^2 N^2}{\frac{2 N^2 - 1}{2} s^2} \left( \frac{y^2 + (1-y)^2}{y^2 + (1-y)^2} \right), \quad \frac{d\hat{\sigma}_{gg-|g|g}}{dt} = \frac{\alpha_s^2 \pi^2 N^2}{\frac{2 N^2 - 1}{2} s^2} \left( \frac{y^2 + (1-y)^2}{y^2 + (1-y)^2} \right), \quad \frac{d\hat{\sigma}_{gg-|g|g}}{dt} = \frac{\alpha_s^2 \pi^2 N^2}{\frac{2 N^2 - 1}{2} s^2} \left( \frac{y^2 + (1-y)^2}{y^2 + (1-y)^2} \right).
\]

(12a)

All cross sections are given for massless partons. The cross sections with the initial or final state particles interchanged are obtained from these by a $t \leftrightarrow \bar{t}$ substitution. We also define the ratios (displayed in Fig. 4b)

\[
a_g(y) = \frac{d\hat{\sigma}_{gg-|g|g}}{d\hat{\sigma}_{gg-gg}} = \frac{s^2 y^2 + (1-y)^2}{s^2 y^2 + (1-y)^2} = \frac{1 + y^2 + (1-y)^2}{1 + y^2 + (1-y)^2}, \quad a_q(y) = \frac{d\hat{\sigma}_{gg-|g|g}}{d\hat{\sigma}_{gg-gg}} = \frac{N^2+1}{N^2-1} \frac{s^2}{s^2 y^2 + (1-y)^2}, \quad b(y) = \frac{d\hat{\sigma}_{gg-gg}}{d\hat{\sigma}_{gg-gg}} = \frac{N^2-1}{N^2-1} \frac{y^2 + (1-y)^2}{y^2 + (1-y)^2}.
\]

(15a)

In Fig. 5 we show the expressions $x_1 f_1^q(x_1) x_2 f_1^q(x_2) d\hat{\sigma}_{ab-cd}$ appearing in the denominator of Eq. (10), integrated over the pseudorapidity ranges $-\eta \leq \eta \leq +1$, normalized to the corresponding quantity for gluon-gluon scattering (Fig. 5a) and quark-gluon scattering (Fig. 5b). The pseudorapidity integration range is inspired by the $\eta$ coverage of the ALICE detector at LHC, which has the particle identification capabilities that allow to measure $\Lambda$'s with transverse momenta $p_T$ of several GeV/c. We have taken $\sqrt{s} = 14$ TeV as is relevant to the LHC situation and for the unpolarized distribution functions we have used the CTEQ5L parametrizations evaluated at the scale 10 GeV. The figures do not change significantly with the scale in this region (see Figure 5). The first and second terms in the equation are the gluon and quark contributions appearing in $\frac{d\hat{\sigma}_{ab-cd}}{d\hat{\sigma}_{ab-cd}}$, respectively. The third term is the total contribution to the $\hat{\sigma}_{ab-cd}$ cross section appearing in this particular kinematic region. The quark-gluon ($gg \rightarrow gg$) scattering contribution is on the order of 10% of the $(gg \rightarrow gg)$ contribution at perpendicular scales $x_1$ typical for LHC, if we consider jets with transverse momenta between 30 and 100 GeV/c. Similarly, the other partonic contributions are of order 10% or less of the $gg \rightarrow gg$ contribution (Fig. 5b).

Whether or not the unpolarized $\Lambda$ production cross section $d\hat{\sigma}_U$ can also be approximated by the gluon-gluon scattering contribution, depends heavily on the $\Lambda$ fragmentation functions employed. Several such functions have been fitted to experimental data [31, 32, 33, 34], unfortunately with vastly different results for the ratios $D_1^q / D_1^g$ for the various quark and antiquark flavors. If one restricts to the subprocesses $gg \rightarrow gg$ and $gg \rightarrow gg$, then the unpolarized cross section $d\hat{\sigma}_U$ contains

\[
d\hat{\sigma}_U \sim x_1 f_1^q(x_1) x_2 f_1^q(x_2) d\hat{\sigma}_{gg-gg} D_1^q(z) \left\{ 1 + \varepsilon(\eta_1, \eta_2) \right\},
\]

with

\[
\varepsilon = \sum_q x_1 f_1^q(x_1) x_2 f_1^q(x_2) \left[ d\hat{\sigma}_{gg-gg} D_1^q(z) + d\hat{\sigma}_{gg-|g|g} D_1^q(z) \right] x_1 f_1^q(x_1) x_2 f_1^q(x_2) d\hat{\sigma}_{gg-|g|g} D_1^q(z)
\]

\[
+ \sum_q x_1 f_1^q(x_1) x_2 f_1^q(x_2) \left[ d\hat{\sigma}_{gg-gg} D_1^q(z) + d\hat{\sigma}_{gg-|g|g} D_1^q(z) \right] x_1 f_1^q(x_1) x_2 f_1^q(x_2) d\hat{\sigma}_{gg-|g|g} D_1^q(z)
\]

\[
= b(y) \sum_q f_1^q(x_1) D_1^q(z) + b(1-y) \sum_q f_1^q(x_1) D_1^q(z) + b(y) \sum_q f_1^q(x_2) D_1^q(z) + b(1-y) \sum_q f_1^q(x_2) D_1^q(z).
\]

(15a)
FIG. 5: (a) The partonic cross sections multiplied by the unpolarized collinear distribution functions, integrated over $-1 \leq \eta_{j/j'} \leq 1$, normalized to the corresponding quantity for gluon-gluon scattering (numerator and denominator integrated separately). Solid line: $qg \rightarrow qg/gg \rightarrow gg$, dashed line: $qq' \rightarrow qg/gg \rightarrow gg$. For the unpolarized distribution functions we have used the CTEQ5L distribution functions evaluated at 10 GeV. For comparison $qq' \rightarrow qg/gg \rightarrow gg$ at 20 GeV was also included (dotted line). We have approximated $x_{j} \approx x_{j'} \equiv x_{\perp}$. (b) As in (a), but now with $qq' \rightarrow qq/qg \rightarrow qg$ (solid line), $gg \rightarrow q\bar{q}/qg \rightarrow qg$ (dashed line) and $qq \rightarrow qq/qg \rightarrow qg$ (dash-dotted line). Over the entire kinematical region considered here all other partonic channels are dominated by those shown in the figure. For comparison $qq' \rightarrow qg/gg \rightarrow gg$ at 20 GeV was also included (dotted line).

FIG. 6: Expression in Eq. (15a) integrated over $-1 \leq \eta_{j/j'} \leq 1$ (numerator and denominator integrated separately) with (a) AKK and (b) FSV Λ-fragmentation functions, for $z = 0.1$ (solid line), $z = 0.3$ (dashed line), $z = 0.5$ (dash-dotted line) and $z = 0.7$ (dotted line). Distribution and fragmentation functions are evaluated at 10 GeV.

The function $\varepsilon$ is quite sensitive to the choice of unpolarized fragmentation functions, as demonstrated in Fig. 6, which shows a comparison using the functions by De Florian et al. [30] (FSV) and by Albino et al. [31] (AKK), for a factorization scale of 10 GeV.

For the AKK set the assumption of gluon-gluon scattering dominance may be good to 30%, but for the FSV set one can reach this conclusion only for small $z$ values. At ALICE Λ’s with transverse momentum of several GeV are expected to be measured in sufficiently large quantities. A typical $z$ value could be 0.1 for a 5 GeV Λ within a 50 GeV jet. Therefore, we expect the midrapidity data to be restricted to small $z$ values at LHC and the assumption of gluon-gluon scattering dominance to be valid despite the large variation in Λ fragmentation function sets.

A future investigation of Λ production at LHC or RHIC should, therefore, first focus on describing the unpolarized cross section correctly, especially regarding the $y$-dependence, before the $y$-dependence of the polarization effect is studied. Therefore, even if no polarization effect is seen, the unpolarized data obtained by the proposed measurement can in any case lead to a considerable improvement in the determination of the unpolarized Λ fragmentation functions.

To illustrate the main idea regarding the $y$ dependence of the asymmetry, we will simply assume that the unpolarized cross section is known sufficiently well and that gluon-gluon scattering dominates, and consider two distinct scenarios: 1) $D^{g\perp}_{1T} \neq 0$; 2) $D^{g\perp}_{1T} = 0$ or more generally, $D^{g\perp}_{1T} \ll D^{g\perp}_{1T}$. 


In scenario 1 ($D_{1T}^{y \perp} \neq 0$) one finds from Eq. (10):

$$\frac{d\sigma}{d\sigma_U} = \frac{x_1 f_1^o(x_1) x_2 f_1^b(x_2) d\sigma_{gq-gg} D_{1T}^{y \perp}(z,K_Z^2) + x_1 f_1^o(x_1) x_2 f_1^q(x_2) d\sigma_{gq-\eta g} D_{1T}^{y \perp}(z,K_Z^2)}{x_1 f_1^o(x_1) x_2 f_1^b(x_2) d\sigma_{gq-gg} D_1^y(z,K_Z^2) + a_g(y) D_{1T}^{y \perp}(z,K_Z^2)}.$$  

(16)

If there would be no Wilson line or color flow dependence in the fragmentation process in the way as has been claimed in Refs. 28, 29, $D_{1T}^{y \perp}$ would arise with the usual partonic cross section. In that case there is no need to consider two separate functions to begin with and one should not find any $y$ dependence at all. Observing the $y$ dependence $a_g(y)$ (which is rather weak unfortunately) would be a signal for color flow dependence of the $T$-odd effects in the fragmentation process.

In scenario 2 ($D_{1T}^{y \perp} \ll D_{1T}^{y \perp}$) one finds on the other hand:

$$\frac{d\sigma}{d\sigma_U} \approx \sum_q \left( \frac{x_1 f_1^q(x_1) x_2 f_1^b(x_2) d\sigma_{gq-gg} D_{1T}^{y \perp}(z,K_Z^2) + x_1 f_1^q(x_1) x_2 f_1^q(x_2) d\sigma_{gq-\eta g} D_{1T}^{y \perp}(z,K_Z^2)}{x_1 f_1^q(x_1) x_2 f_1^b(x_2) d\sigma_{gq-gg} D_1^q(z,K_Z^2)} \right)$$

$$+ \sum_q \left( \frac{x_1 f_1^q(x_1) x_2 f_1^q(x_2) d\sigma_{gq-\eta g} D_{1T}^{y \perp}(z,K_Z^2) + x_1 f_1^q(x_1) x_2 f_1^q(x_2) d\sigma_{gq-\eta g} D_{1T}^{y \perp}(z,K_Z^2)}{x_1 f_1^q(x_1) x_2 f_1^q(x_2) d\sigma_{gq-gg} D_1^q(z,K_Z^2)} \right)$$

$$= b(y) \frac{\sum_q f_1^q(x_1) D_{1T}^{y \perp}(z,K_Z^2)}{f_1^q(x_1) D_1^q(z,K_Z^2)} + b(y) a_q(y) \frac{\sum_q f_1^q(x_1) \tilde{D}_{1T}^{y \perp}(z,K_Z^2)}{f_1^q(x_1) D_1^q(z,K_Z^2)}$$

$$+ b(1-y) \frac{\sum_q f_1^q(x_2) D_{1T}^{y \perp}(z,K_Z^2)}{f_1^q(x_2) D_1^q(z,K_Z^2)} + b(1-y) a_q(1-y) \frac{\sum_q f_1^q(x_2) \tilde{D}_{1T}^{y \perp}(z,K_Z^2)}{f_1^q(x_2) D_1^q(z,K_Z^2)}.$$  

(17)

This expression has a rather involved $y$ dependence. It is probably too difficult to disentangle the separate contributions. Therefore, we suggest a further step. Since the momentum fractions $x_1$ and $x_2$ are fixed by the jet momenta, one can select those events where they are equal, which corresponds to $\eta = \eta_j$. In that case the expression simplifies

$$\frac{d\sigma}{d\sigma_U} = \sum_q f_1^q(x_1) \left\{ \left[ b(y) + b(1-y) \right] D_{1T}^{y \perp}(z,K_Z^2) + \left[ b(y) a_q(y) + b(1-y) a_q(1-y) \right] \tilde{D}_{1T}^{y \perp}(z,K_Z^2) \right\}.$$  

(18)

where now $y = (e^{2\eta_j}+1)^{-1}$ and $x_1 = x_2 = x_\perp/2\sqrt{y(1-y)}$. The two different combinations of $y$-dependent terms are depicted in Fig. 4(b) as a function of $\eta_j$. One can see that one term in Eq. (18) varies more strongly with $\eta_j$ than the other term. This may possibly allow a discrimination of the two effects.

V. DISCUSSION AND CONCLUSIONS

In this paper we have proposed a measurement of Sivers polarization in the process $pp \to (\Lambda^1\text{jet})$ jet $X$, where the $\Lambda$ is part of one of the two observed jets which are almost back-to-back in the plane perpendicular to the beam axis. Unlike the traditional measurement in the process $pp \to \Lambda^1X$, this new observable need not vanish at midrapidity. This makes it of interest for high energy collider experiments, such as RHIC, Tevatron and LHC, as they typically can detect $\Lambda$’s at midrapidity. We have studied the asymmetry for the ALICE experiment specifically, due to the fact that ALICE has good PID capabilities that allow detection of $\Lambda$’s of momenta of several GeV/c. The experimental situation at LHC furthermore leads to the dominance of gluon-gluon and gluon-quark scattering, allowing for a simplification of the expressions of good approximation.

Observation of a nonzero $S_{\Lambda}(\vec{K}_j \times \vec{K}_\Lambda)$ asymmetry in the jet-jet c.o.m. frame indicates a nonzero Sivers effect in the fragmentation process. The $y$ dependence of this observable in principle could allow for a study of the relative importance of the two types of interactions that could exhibit such a Sivers effect. It allows for a study of possible color-flow dependence of the asymmetry and of the (non)universality of the Sivers fragmentation functions, issues currently of much interest and that cannot be addressed in $pp \to \Lambda^1X$. But even if no asymmetry is observed, the unpolarized data that is obtained from the polarization measurement would help to constrain the unpolarized $\Lambda$ fragmentation functions, which are currently not well-determined. High energy collider experiments such as to be performed at LHC would be very helpful in this respect, as the question of factorization of the unpolarized cross section would not pose a problem.
The study of color flow dependence we suggest here can be done without an actual extraction of $D_{1T}$, namely by just studying the dependence of the observable on particular kinematic variables. This should prove useful even if a trustworthy extraction of the Sivers fragmentation functions themselves turns out to be too difficult.

Currently no knowledge on the magnitude of the various Sivers fragmentation functions is available, therefore, no predictions can be given for the actual size of the asymmetry. But the $y$ dependence of the various contributing terms could be derived. They were discussed for two different scenarios: one in which the gluon Sivers fragmentation function is important to include and one in which it is irrelevant compared to the quark function.

The proposed measurement allows for a determination of the $z$ and $k_T^2$ dependence of the Sivers fragmentation functions. For completeness, we mention that there are other processes from which this information can be extracted. This can be done in electron-positron annihilation experiments via the process $e^+e^- \rightarrow (\Lambda')\text{jet}X$ in a straightforward manner [4]. It allows to probe the combination $D_{1T} + \tilde{D}_{1T}$. It can also be done via neutral or charged current semi-inclusive DIS: $\ell p \rightarrow \ell' (\Lambda')\text{jet}X$, where the combination $D_{1T} - \tilde{D}_{1T}$ is probed. The comparison to the functions from $pp \rightarrow (\Lambda')\text{jet}X$ would allow for another test of the (non)universality of the Sivers fragmentation functions.

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