DARK ENERGY MODELS TOWARD OBSERVATIONAL TESTS AND DATA

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Abstract

A huge amount of good quality data converges towards the picture of a spatially flat universe undergoing the today observed phase of accelerated expansion. This new observational trend is commonly addressed as Precision Cosmology. Despite of the excellent surveys, the nature of dark energy, dominating the matter-energy content of the universe, is still unknown and a lot of different scenarios are viable candidates to explain cosmic acceleration. Methods to test these cosmological models are based on distance measurements and lookback time toward astronomical objects used as standard candles. The related degeneracy problem is the signal that more data at low ($z \sim 0 \div 1$), medium ($1 < z < 10$) and high ($10 < z < 1000$) redshift are needed to definitively select realistic models.

1 Introduction

The increasing bulk of data that have been accumulated in the last few years have paved the way to the emergence of a new standard cosmological model usually referred to as the concordance model. The Hubble diagram of Type Ia Supernovae (hereafter SNeIa) has been the first evidence that the universe is undergoing a phase of accelerated expansion. On the other
hand, balloon born experiments determined the location of the first and second peak in the anisotropy spectrum of cosmic microwave background radiation (CMBR) strongly pointing out that the geometry of the universe is spatially flat. If combined with constraints coming from galaxy clusters on the matter density parameter $\Omega_M$, these data indicate that the universe is dominated by a non-clustered fluid with negative pressure, generically dubbed dark energy, which is able to drive the accelerated expansion. This picture has been further strengthened by more precise measurements of the CMBR spectrum, due to the WMAP experiment [1], and by the extension of the SNeIa Hubble diagram to redshifts higher than 1.

After these observational evidences, an overwhelming flood of papers, presenting a great variety of models trying to explain this phenomenon, has appeared; in any case, the simplest explanation is claiming for the well known cosmological constant $\Lambda$. Although the best fit to most of the available data [1], the $\Lambda$CDM model failed in explaining why the inferred value of $\Lambda$ is so tiny (120 orders of magnitude lower) compared to the typical vacuum energy values predicted by particle physics and why its energy density is today comparable to the matter density (the so called coincidence problem). As a tentative solution, many authors have replaced the cosmological constant with a scalar field rolling down its potential and giving rise to the class of models now referred to as quintessence. Even if successful in fitting the data, the quintessence approach to dark energy is still plagued by the coincidence problem since the dark energy and matter densities evolve differently and reach comparable values for a very limited portion of the universe evolution, coinciding at present era. In this case, the coincidence problem is replaced with a fine-tuning problem. Moreover, it is not clear where this scalar field originates from, thus leaving a great uncertainty on the choice of the scalar field potential.

The subtle and elusive nature of dark energy has led many authors to look for completely different scenarios able to give a quintessential behavior without the need of exotic components. To this aim, it is worth stressing that the acceleration of the universe only claims for a negative pressure dominant component, but does not tell anything about the nature and the number of cosmic fluids filling the universe. This consideration suggests that it could be possible to explain the accelerated expansion by introducing a single cosmic fluid with an equation of state causing it to act like dark matter at high densities and dark energy at low densities. An attractive feature of these models, usually referred to as Unified Dark Energy (UDE) or Unified Dark Matter (UDM) models, is that such an approach naturally solves, at least
phenomenologically, the coincidence problem. Some interesting examples are the generalized Chaplygin gas and the tachyon fields. A different class of UDE models has been proposed [3] where a single fluid is considered whose energy density scales with the redshift in such a way that the radiation dominated era, the matter dominated era and the accelerating phase can be naturally achieved. It is worth noting that such class of models are extremely versatile since they can be interpreted both in the framework of UDE models and as a two-fluid scenario with dark matter and scalar field dark energy. The main ingredient of the approach is that a generalized equation of state can be always obtained and observational data can be fitted.

Actually, there is still a different way to face the problem of cosmic acceleration. It is possible that the observed acceleration is not the manifestation of another ingredient in the cosmic pie, but rather the first signal of a breakdown of our understanding of the laws of gravitation. From this point of view, it is thus tempting to modify the Friedmann equations to see whether it is possible to fit the astrophysical data with models comprising only the standard matter. An interesting example of this kind is the DGP gravity [4].

Moving in this framework, it is possible to find alternative schemes where a quintessential behavior is obtained by taking into account effective models coming from fundamental physics giving rise to generalized or higher order gravity actions (see for example [5]). SNeIa data could also be efficiently fitted including higher-order curvature invariants in the gravity Lagrangian.

It is worth noticing that these alternative schemes provide naturally a cosmological component with negative pressure whose origin is related to the geometry of the universe thus overcoming the problems linked to the physical significance of the scalar field.

It is evident, from this short overview, the high number of cosmological models which are viable candidates to explain the observed accelerated expansion. This abundance of models is from one hand the signal of the fact that we have a limited number of cosmological tests to discriminate among rival theories, and from the other hand that a urgent degeneracy problem has to be faced. To this aim, it is useful to remember that both the SNeIa Hubble diagram and the angular size-redshift relation of compact radio sources are distance based methods to probe cosmological models so then systematic errors and biases could be iterated. From this point of view, it is interesting to look for tests based on time-dependent observables. For example, one can take into account the lookback time to distant objects since this quantity can discriminate among different cosmological models.
The lookback time is observationally estimated as the difference between the present day age of the universe and the age of a given object at redshift $z$. Such an estimate is possible if the object is a galaxy observed in more than one photometric band since its color is determined by its age as a consequence of stellar evolution. It is thus possible to get an estimate of the galaxy age by measuring its magnitude in different bands and then using stellar evolutionary codes to choose the model that reproduces the observed colors at best. A similar approach was pursued by Lima & Alcaniz [6] who used the age (rather than the lookback time) of old high redshift galaxies to constrain the dark energy equation of state. It is worth noting, however, that the estimate of the age of a single galaxy may be affected by systematic errors which are difficult to control. Actually, this problem can be overcome by considering a sample of galaxies belonging to the same cluster. In this way, by averaging the estimates of all galaxies, one obtains an estimate of the cluster age and reduces the systematic errors. Such a method was first proposed in [7] and then used in [8] to test a class of models where a scalar field is coupled with the matter term giving rise to a particular quintessence model.

In this report, I shortly discuss the dark energy ”paradigm” and some methods to constrain it toward observational data. Far from being exhaustive and complete, my aim is to point out the degeneracy problem and the fact that we need further and self-consistent observational surveys at all redshifts to remove it.

2 The dark energy ”paradigm”

Many rival theories have been advocated to fit the observed accelerated expansion and to solve the puzzle of the dark energy. As a simple classification scheme, we may divide the different cosmological models in three wide classes. According to the models of the first class, the dark energy is a new ingredient of the cosmic Hubble flow, the simplest case being the ΛCDM scenario and its quintessential generalization which we will refer to as QCDM models.

This is in sharp contrast with the assumption of UDE models (the second class) where there is a single fluid described by an equation of state comprehensive of all regimes of cosmic evolution [3] which I will consider here referring to it as the parametric density models or generalized Equation of State $EoS$ models.
Finally, according to the third class models, accelerated expansion could be the first evidence of a breakdown of the Einstein General Relativity (and thus of the Friedmann equations) which has to be considered as a particular case of a more general theory of gravity. As an example of this kind of models, one can consider the $f(R)$-gravity [5].

Far from being exhaustive, considering these three classes of models allows to explore very different scenarios proposed to explain the observed cosmic acceleration. However, the "paradigm" is the $\Lambda$CDM model.

Cosmological constant $\Lambda$ has become a textbook candidate to drive the accelerated expansion of the spatially flat universe. Despite its conceptual problems, the $\Lambda$CDM model turns out to be the best fit to a combined analysis of completely different astrophysical data ranging from SNeIa to CMBR anisotropy spectrum and galaxy clustering [1, 9].

As a simple generalization, one may consider the QCDM scenario in which the barotropic factor $w \equiv p/\rho$ takes at a certain epoch a negative value with $w = -1$ corresponding to the standard cosmological constant. Testing whether such a barotropic factor deviate or not from $-1$ is one of the main issue of modern observational cosmology. How such a negative pressure fluid drives the cosmic acceleration may be easily understood by looking at the Friedmann equations:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_M + \rho_Q), \quad (1)$$

$$2\frac{\ddot{a}}{a} + H^2 = -8\pi G \rho_Q = -8\pi G w \rho_Q,$$  \quad (2)

where the dot denotes differentiation with respect to cosmic time $t$, $H$ is the Hubble parameter and the universe is assumed spatially flat as suggested by the position of the first peak in the CMBR anisotropy spectrum.

From the continuity equation, $\dot{\rho} + 3H(\rho + p) = 0$, we get for the $i$-th fluid with $p_i = w_i \rho_i$:

$$\Omega_i = \Omega_{i,0} a^{-3(1+w_i)} = \Omega_{i,0} (1 + z)^{3(1+w_i)},$$  \quad (3)

where $z \equiv 1/a - 1$ is the redshift, $\Omega_i = \rho_i/\rho_{crit}$ is the density parameter for the $i$-th fluid in terms of the critical density $\rho_{crit} = 3H_0^2/8\pi G$ and, hereafter, I label all the quantities evaluated today with a subscript 0. Inserting this result into Eq.(1), one gets:

$$H(z) = H_0 \sqrt{\Omega_{M,0}(1 + z)^3 + \Omega_{Q,0}(1 + z)^{3(1+w)}}.$$  \quad (4)
Using Eqs. (1), (2) and the definition of the deceleration parameter $q \equiv -\dot{a}/a^2$, one finds:

$$q_0 = \frac{1}{2} + \frac{3}{2}w(1 - \Omega_{M,0}) \ .$$

(5)

The SNeIa Hubble diagram, the large scale galaxy clustering and the CMBR anisotropy spectrum can all be fitted by the $\Lambda$CDM model with $\left(\Omega_{M,0}, \Omega_Q\right) \simeq (0.3, 0.7)$ thus giving $q_0 \simeq -0.55$, i.e. the universe turns out to be in an accelerated expansion phase. The simplicity of the model and its capability of fitting the most of the data are the reasons why the $\Lambda$CDM scenario is the leading candidate to explain the dark energy cosmology. Nonetheless, its generalization, QCDM models, i.e. mechanisms allowing the evolution of $\Lambda$ from the past are invoked to remove the $\Lambda$-problem and the coincidence problem.

3 Methods to constrain models

Let us discuss now how cosmological models can be constrained using suitable distance and/or time indicators. As a general remark, solutions coming from cosmological models have to be matched with observations by using the redshift $z$ as the natural time variable for the Hubble parameter, i.e.

$$H(z) = -\frac{\dot{z}}{z + 1} \ .$$

(6)

Interesting ranges for $z$ are: $100 < z < 1000$ for early universe (CMBR data), $10 < z < 100$ (LSS), $0 < z < 10$ (SNeIa, radio-galaxies, GRBs, etc.). The method consists in building up a reasonable patchwork of data coming from different epochs and then matching them with the same cosmological solution ranging, in principle, from inflation to present accelerated era.

In order to constrain the parameters characterizing the cosmological solution, a reasonable approach is to maximize the following likelihood function:

$$L \propto \exp \left[ -\frac{\chi^2(p)}{2} \right]$$

(7)

where $p$ are the parameters assigning the cosmological solution. The $\chi^2$ merit function can be defined as:
\[
\chi^2(p) = \sum_{i=1}^{N} \left[ \frac{y_i^{th}(z_i, p) - y_i^{obs}}{\sigma_i} \right]^2 + \left[ \frac{R(p) - 1.716}{0.062} \right]^2 + \left[ \frac{A(p) - 0.469}{0.017} \right]^2.
\]

Terms entering Eq.(8) can be characterized as follows. For example, the dimensionless coordinate distances \( y \) to objects at redshifts \( z \) are considered in the first term. They are defined as:

\[
y(z) = \int_0^z \frac{dz'}{E(z')}
\]

where \( E(z) = H(z)/H_0 \) is the normalized Hubble parameter. This is the main quantity which allows to compare the theoretical results with data. The function \( y \) is related to the luminosity distance \( D_L = (1 + z) r(z) \). A sample of data on \( y(z) \) for the 157 SNeIa is discussed in the Riess et al. [2] Gold dataset and 20 radio-galaxies are in [11]. These authors fit with good accuracy the linear Hubble law at low redshift \((z < 0.1)\) obtaining the Hubble dimensionless parameter \( h = 0.664 \pm 0.008 \). Such a number can be consistently taken into account at low redshift. This value is in agreement with \( H_0 = 72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1} \) given by the HST Key project [10] based on the local distance ladder and estimates coming from time delays in multiply imaged quasars and Sunyaev-Zel’dovich effect in X-ray emitting clusters [13].

The second term in Eq.(8) allows to extend the \( z \)-range to probe \( y(z) \) up to the last scattering surface \((z \geq 1000)\). The \textit{shift parameter} [14, 15] \( R = \sqrt{\Omega_M y(z_{ls})} \) can be determined from the CMBR anisotropy spectrum, where \( z_{ls} \) is the redshift of the last scattering surface which can be approximated as \( z_{ls} = 1048 \left( 1 + 0.00124 \omega_b^{-0.738} \right) \left( 1 + g_1 \omega_g^{0.5} \right) \) with \( \omega_i = \Omega_i h^2 \) (with \( i = b, M \) for baryons and total matter respectively) and \( (g_1, g_2) \) given in [16]. The parameter \( \omega_b \) is constrained by the baryogenesis calculations contrasted to the observed abundances of primordial elements. Using this method, the value \( \omega_b = 0.0214 \pm 0.0020 \) is found [17]. In any case, it is worth noting that the exact value of \( z_{ls} \) has a negligible impact on the results and setting \( z_{ls} = 1100 \) does not change constraints and priors on the other parameters of the given model. The third term in the function \( \chi^2 \) takes into account the \textit{acoustic peak} of the large scale correlation function at 100 \( h^{-1} \text{ Mpc} \) separation, detected by using 46748 luminous red galaxies (LRG) selected from the SDSS Main Sample [18]. The quantity
\[ A = \frac{\sqrt{\Omega_M}}{z_{LRG}} \left[ \frac{z_{LRG}}{E(z_{LRG})} y^2(z_{LRG}) \right]^{1/3} \] (10)

is related to the position of acoustic peak where \( z_{LRG} = 0.35 \) is the effective redshift of the above sample. The parameter \( A \) depends on the dimensionless coordinate distance (and thus on the integrated expansion rate), on \( \Omega_M \) and \( E(z) \). This dependence removes some of the degeneracies intrinsic in distance fitting methods. Due to this reason, it is particularly interesting to include \( A \) as a further constraint on the model parameters using its measured value \( A = 0.469 \pm 0.017 \) [18]. Note that, although similar to the usual \( \chi^2 \) introduced in statistics, the reduced \( \chi^2 \) (i.e., the ratio between the \( \chi^2 \) and the number of degrees of freedom) is not forced to be 1 for the best fit model because of the presence of the priors on \( R \) and \( A \) and since the uncertainties \( \sigma_i \) are not Gaussian distributed, but take care of both statistical errors and systematic uncertainties. With the definition (7) of the likelihood function, the best fit model parameters are those that maximize \( L(p) \).

Using the method sketched above, the several classes of models can be constrained and selected by observations. However, most of the tests recently used to constrain cosmological parameters (such as the SNeIa Hubble diagram and the angular size-redshift) are essentially distance-based methods. The proposal of Dalal et al. [7] to use the lookback time to high redshift objects is thus particularly interesting since it relies on a completely different observable. The lookback time is defined as the difference between the present day age of the universe and its age at redshift \( z \) and may be computed as:

\[ t_L(z, p) = t_H \int_0^z \frac{dz'}{(1 + z') E(z', p)} \] (11)

where \( t_H = 1/H_0 = 9.78h^{-1} \) Gyr is the Hubble time (with \( h \) the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\)), and, as above, \( E(z, p) = H(z)/H_0 \) is the dimensionless Hubble parameter and \( \{p\} \) the set of parameters characterizing the cosmological model. It is worth noting that, by definition, the lookback time is not sensible to the present day age of the universe \( t_0 \) so that it is (at least in principle) possible that a model fits well the data on the lookback time, but nonetheless it predicts a completely wrong value for \( t_0 \). This latter parameter can be evaluated from Eq.(11) by simply changing the upper integration limit from \( z \) to infinity. This shows that it is indeed a different quantity since it depends on the full evolution of the universe and
not only on how the universe evolves from the redshift \( z \) to now. That is why this quantity can be explicitly introduced as a further constraint. As an example, let us discuss how to use the lookback time and the age of the universe to test a given cosmological model. To this end, let us consider an object \( i \) at redshift \( z \) and denote by \( t_i(z) \) its age defined as the difference between the age of the universe when the object was born, i.e. at the formation redshift \( z_F \), and the one at \( z \). It is:

\[
t_i(z) = \int_z^{z_F} \frac{dz'}{(1 + z')E(z', p)} = t_L(z_F) - t_L(z). \tag{12}
\]

where I have used the definition (11) of the lookback time. Suppose now we have \( N \) objects and we have been able to estimate the age \( t_i \) of the object at redshift \( z_i \) for \( i = 1, 2, \ldots, N \). Using the previous relation, we can estimate the lookback time \( t^\text{obs}_L(z_i) \) as:

\[
t^\text{obs}_L(z_i) = t^\text{obs}_0 - t_i(z) - df, \tag{13}
\]

where \( t^\text{obs}_0 \) is the today estimated age of the universe and a delay factor can be defined as \( df = t^\text{obs}_0 - t_L(z_F) \). The delay factor is introduced to take into account our ignorance of the formation redshift \( z_F \) of the object. Actually, what can be measured is the age \( t_i(z) \) of the object at redshift \( z \). To estimate \( z_F \), one should use Eq.(12) assuming a background cosmological model. Since our aim is to determine what is the background cosmological model, it is clear that we cannot infer \( z_F \) from the measured age so that this quantity is completely undetermined. It is worth stressing that, in principle, \( df \) should be different for each object in the sample unless there is a theoretical reason to assume the same redshift at the formation of all the objects. If this is indeed the case, then it is computationally convenient to consider \( df \) rather than \( z_F \) as the unknown parameter to be determined from the data. Again a likelihood function can be defined as:

\[
\mathcal{L}_lt(p, df) \propto \exp \left(-\chi^2_{lt}(p, df)/2\right) \tag{14}
\]

with:

\[
\chi^2_{lt} = \frac{1}{N - N_p + 1} \left\{ \frac{\left[ t^\text{theor}(p) - t^\text{obs}_0 \right]^2}{\sigma^2_{t^\text{obs}_0}} + \sum_{i=1}^{N} \frac{\left[ t^\text{theor}(z_i, p) - t^\text{obs}_L(z_i) \right]^2}{\sigma^2_i + \sigma^2_t} \right\} \tag{15}
\]
where \( N_p \) is the number of parameters of the model, \( \sigma_t \) is the uncertainty on \( t_0^{\text{obs}} \), \( \sigma_i \) the one on \( t_L^{\text{obs}}(z_i) \) and the superscript \( \text{theor} \) denotes the predicted values of a given quantity. Note that the delay factor enters the definition of \( \chi^2_H \) since it determines \( t_L^{\text{obs}}(z_i) \) from \( t_i(z) \) in virtue of Eq.(13), but the theoretical lookback time does not depend on \( df \). In principle, such a method should work efficiently to discriminate among the various dark energy models. Actually, this is not exactly the case due to the paucity of the available data which leads to large uncertainties on the estimated parameters. In order to partially alleviate this problem, it is convenient to add further constraints on the models by using Gaussian priors\(^1\) on the Hubble constant, i.e. redefining the likelihood function as:

\[
\mathcal{L}(\mathbf{p}) \propto \mathcal{L}_H(\mathbf{p}) \exp \left[ -\frac{1}{2} \left( \frac{h - h^{\text{obs}}}{\sigma_h} \right)^2 \right] \propto \exp \left[ -\chi^2(\mathbf{p}) / 2 \right] \quad (16)
\]

where we have absorbed \( df \) in the set of parameters \( \mathbf{p} \) and have defined:

\[
\chi^2 = \chi_H^2 + \left( \frac{h - h^{\text{obs}}}{\sigma_h} \right)^2 \quad (17)
\]

with \( h^{\text{obs}} \) the estimated value of \( h \) and \( \sigma_h \) its uncertainty. The HST Key project results [10] can be used setting \((h, \sigma_h) = (0.72, 0.08)\). Note that this estimate is independent of the cosmological model since it has been obtained from local distance ladder methods. The best fit model parameters \( \mathbf{p} \) may be obtained by maximizing \( \mathcal{L}(\mathbf{p}) \) which is equivalent to minimize the \( \chi^2 \) defined in Eq.(17). It is worth stressing that such a function should not be considered as a statistical \( \chi^2 \) in the sense that it is not forced to be of order 1 for the best fit model to consider a fit as successful. Actually, such an interpretation is not possible since the errors on the measured quantities (both \( t_i \) and \( t_0 \)) are not Gaussian distributed and, moreover, there are uncontrolled systematic uncertainties that may also dominate the error budget. Nonetheless, a qualitative comparison among different models may be obtained by comparing the values of this pseudo \( \chi^2 \) even if this should not be considered as a definitive evidence against a given model. Having more than one parameter, one obtains the best fit value of each single parameter \( p_i \) as

\(^{1}\) The need for priors to reduce the parameter uncertainties is often advocated for cosmological tests. For instance, in [6] a strong prior on \( \Omega_M \) is introduced to constrain the dark energy equation of state. It is likely, that extending the dataset to higher redshifts and reducing the uncertainties on the age estimate will allow to avoid resorting to priors.
the value which maximizes the marginalized likelihood for that parameter defined as:

\[ L_{p_i} \propto \int dp_1 \ldots \int dp_{i-1} \int dp_{i+1} \ldots \int dp_n L(p). \] (18)

After having normalized the marginalized likelihood to 1 at maximum, one computes the 1σ and 2σ confidence limits on that parameter by solving \( L_{p_i} = \exp(-0.5) \) and \( L_{p_i} = \exp(-2) \) respectively. In summary, taking into account the above procedures for distance and time measurements, one can reasonably constrain a given cosmological model. In any case, the main and obvious issue is to have at disposal sufficient and good quality data sets.

4 Conclusions

The impressive amount of data indicating a spatially flat universe in accelerated expansion has posed the problem of dark energy and stimulated the search for cosmological models able to explain such unexpected behavior. Several theories have been proposed to solve the puzzle of the nature of dark energy ranging from a rolling scalar field to a unified picture where a single exotic fluid accounts for the whole dark sector (dark matter and dark energy). Moreover, modifications of the gravity action has also been advocated. Although deeply different in their underlying physics, all these scenarios share the common feature of well reproducing the available astrophysical data giving rise to a degeneracy problem. It is worth stressing, however, that the most widely used cosmological tests (in particular the SNeIa Hubble diagram and the angular size-redshift relation) are essentially based on distance measurements to high redshift objects and are thus affected by similar systematic errors. It is hence particularly interesting to look for methods which are related to the estimates of different quantities. Being affected by other kinds of observational problems, such methods could be considered as cross checks for the results obtained by the usual tests and they should represent complementary probes for cosmological models. Among these alternative methods, the lookback time is related to a different astrophysics than the distance based methods and it is thus free from any problem connected with the evolution of standard candles (such as the SNeIa absolute magnitude and the intrinsic linear size of radio sources [19]).

In any case, from one hand we need some experimentum crucis capable of removing the degeneracy for a reasonably large redshift range and, from the theoretical viewpoint, we need a physically reliable cosmological model,
emerging from some fundamental theory, without the conceptual shortcomings of ΛCDM.

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