Modelling of torso and dual arms for a humanoid robot with fixed base by using screw theory for dexterous applications

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Abstract. Most of the humanoid robots are facing problems due to non-dexterous arms while working in complex environments. Kinematic and dexterity analyses help to study and solve the manipulation difficulties of a robot. This paper presents the kinematic and dexterity analyses of torso and dual arms for a tree type upper body humanoid robot with fixed base using screw theory formulation. Orientation workspace and reachability analyses of the dual arms are done for determining the dexterity of the robot. In this paper, orientation workspace of the tree type upper body is obtained by converting the rotation matrix to Euler angle set representation. The upper body of the humanoid robot is designed with three Degrees of freedom (DoF) in the torso, two DoF in the neck, five DoF in each arm.

1. Introduction

There has been a rapid rise in the development of humanoid robots for ages because of their adaptableness and interaction to the human environment. The humanoid robots are mainly used for interaction with humans and other systems. These robots are used for training purposes, personal assistance, understanding the human body structure and behavior, health care field, entertainment sector, military purposes, space explorations, etc. Humanoid robots with dexterous arms are needed for completing complex tasks. Humanoid robots employed in real life working environments require motions similar to human hand to complete given tasks. Human hands are dexterous in nature. Hence manipulators of humanoid robots are also to be dexterous for imitating human like motion capabilities. Workspace and reachability analyses help to solve the dexterity problems in robotic arms.

Screw theory formulations are used in various applications for deriving the kinematic model. Muller [1] explained the advantages of using screw theory formulations for modelling tree type systems. Yi et al. [2] determined the kinematic model of a planar mobile robot using screw theory approach. The computational time for deriving the jacobian matrix is less compared to other methods. Man et al. [3] derived the kinematic model of a ten DoF humanoid robot arm using screw theory formulation. The kinematic modelling of a spatial humanoid robot with floating base using screw theory is depicted in [4]. Toscano et al. [5] presented a kinetostatic model for humanoid robots using screw theory approach. The kinetostatic model also determines the forces and moments at various joints of humanoid robot. The mathematical modelling of a rehabilitation robot using screw theory approach is carried out by Liao et al. [6]. The comparative study of kinematic modelling for a stanford manipulator using Denavit- Hartenberg method and screw theory approach is done by Rocha et al. [7]. The screw theory approach is proved to be more efficient for complex systems with higher DoF as compared to DH
method. A review of human like arm motion generation robots are depicted in [8]. Patel et al. [9] developed a manipulator to satisfy the performance requirements within given joint constraints and with least energy consumption. The optimum geometry of the orientation structure is given in [10]. The method for determining dexterous workspace of a manipulator is clearly discussed. Guenter et al. [11] introduced a humanoid robot which can exhibit human like torso motions. The proposed model offered enough flexibility for manipulation of objects. The main disadvantage is the slow movement of the joints due to increased number of DOF. Mizuuchi et al. [12] proposed the design and control of a tendon-driven flexible-spine humanoid named “Kenta”. The joints are controlled by using eight muscles tendons. However, the movements exhibited are not flexible compared to human spine. Gini et al. [13] suggested a control system strategy and mechanics for a humanoid upper body. The developed actuation system imitated the arrangement of the human muscle structure. Dynamic behavior of pneumatic actuators used in this work is not linear and subjected to hysteresis.

Kinematic modelling with workspace analysis plays an important role in determining the reachability and dexterity of the upper body robot by obtaining the link dimensions. The performance of the humanoid robots can be increased by analyzing the shape and volume of workspace. Umetani et al. [14] studied the performance of a two-dimensional space robot by analyzing the workspace and manipulability of different configurations of the robot. A method for obtaining the workspace of three degrees of freedom planar robotic manipulators using limiting curves is described in [15]. The singular behavior of a robotic manipulator by analyzing the workspace and jacobian matrix is explained in [16]. Most of the works related to humanoid robots are based only on position workspace. However, orientation workspace also plays a crucial role in evaluating the dexterity of the end effectors of the humanoid robots. The method for determining the orientation workspace of a 6 DoF parallel robot is depicted in [17].

Mathematical modelling of humanoid robot is a difficult task as far as the stability is concerned. Kinematic and dexterity analyses are also difficult due to the complexity of higher degree of freedom mechanisms. This paper focuses on kinematic, workspace and reachability of a tree type upper body humanoid robot with fixed base using screw theory formulation. Analysis of orientation workspace along with position workspace increases the accuracy of determination of dexterity and manipulability of the robotic system.

2. Kinematic analysis of humanoid upper body
The upperbody of humanoid robot with fixed base is considered as a tree type mechanism for kinematic analysis using screw theory approach. The design of the humanoid robot is done by considering the standard human body ratio [18]. The upper body of the humanoid robot consists of three DoF torso, two DoF neck and five DoF arms as shown in figure 1.

![Figure 1. Upper body humanoid robot](image-url)
\( J_i \) represents the joints and \( l_j \) represents the corresponding link dimensions and distances. The two arms of tree type upper body robot are designed with identical joints. The three joints in hip are assumed to be in same origin. The three shoulder joints are also assumed to be in same origin. The joint limits of various upper body joints are given in table 1.

| Joints | Joint limits (in degrees) |
|--------|---------------------------|
| 1      | -90 to +90                |
| 2      | -60 to +60                |
| 3      | -75 to +75                |
| 4      | -90 to +90                |
| 5      | -60 to +60                |
| 6 & 11 | -180 to +180              |
| 7 & 12 | -20 to +135               |
| 8 & 13 | -180 to +180              |
| 9 & 14 | -90 to +90                |
| 10 & 15| -180 to +180              |

The Screw theory formulation [19] is used for the kinematic modeling. One frame is assigned to the base frame and other frame to the end effector. The transformation matrix obtained by using screw theory approach with screw axis, \( e \) is given by equation (1)

\[
e^{[\epsilon]\theta} = \begin{bmatrix} e^{[\tilde{\epsilon}]\theta} \\ 0 \\ 1 \end{bmatrix} (\theta + (1 - \cos\theta)[\alpha] + (\theta - \sin\theta)[\alpha]^2v)
\]

Where \( \theta \) is the angular rotations of respective joints, \( \tilde{\alpha} \) is the skew symmetric matrix of angular velocity matrix, \( \alpha \) and \( v \) is the linear velocity. The Rodrigues formula for deriving rotation matrix is given in equation (2)

\[
R(\tilde{\alpha}, \theta) = e^{[\epsilon]\theta} = I + \sin\theta[\tilde{\alpha}] + (1 - \cos\theta)[\tilde{\alpha}]^2
\]

The humanoid robot upper body is considered as a tree type mechanism as shown in figure 1. The upper body is divided into three branches for kinematic analysis. The first branch consists of three hip joints and two neck joints as shown in figure 2. The position of end effector with respect to base frame is denoted as \( N_i \), where i denotes the number of corresponding branches.

![Figure 2. Screw axes of Hip joints and Neck joints](image-url)
The pose of head with respect to hip, \( S_1(\theta) \) is given in equation (3)

\[
S_1(\theta) = e^{[\epsilon_1]B_1}e^{[\epsilon_2]B_2}e^{[\epsilon_3]B_3}e^{[\epsilon_4]B_4}e^{[\epsilon_5]B_5} \cdot N_1
\]  

(3)

The second branch consists of five right arm joints as shown in figure 3. The pose of right hand with respect to hip, \( S_2(\theta) \) is given in equation (4)

\[
S_2(\theta) = e^{[\epsilon_6]B_6}e^{[\epsilon_7]B_7}e^{[\epsilon_8]B_8}e^{[\epsilon_9]B_9}e^{[\epsilon_{10}]B_{10}} \cdot N_2
\]  

(4)

Figure 3. Screw axes of Right arm joints

The third branch consists of five left arm joints as shown in figure 4. The pose of left hand with respect to hip, \( S_3(\theta) \) is given in equation (5)

\[
S_3(\theta) = e^{[\epsilon_{11}]B_{11}}e^{[\epsilon_{12}]B_{12}}e^{[\epsilon_{13}]B_{13}}e^{[\epsilon_{14}]B_{14}}e^{[\epsilon_{15}]B_{15}} \cdot N_3
\]  

(5)

Figure 4. Screw axes of Left arm joints
3. Orientation workspace

Most of the previous works related to workspace analyses are based on constant orientation workspace or position workspace. The orientation workspace is the subset of overall workspace of the system. Determination of orientation workspace is important for obtaining the complete description of mobility of the system. The overall workspace of an end effector is the combination of position and orientation workspaces. Evaluating position workspace without considering the orientation workspace reduces the accuracy of workspace analysis. There are different alternatives for representing the orientation workspace. In this paper, Euler angle sets [17] are used for representing the orientation workspace. Euler angle sets are plotted in a spherical coordinate system.

Let $\varphi, \theta, h$ represents the euler angle sets. $\varphi$ and $\theta$ denotes the azimuth and zenith angles whereas $h$ denotes the length of ray as shown in figure 5.

![Figure 5. Representation of Euler angles in spherical coordinates](image)

The equations for converting spherical coordinates to cartesian coordinates are given in equations (6) to (8)

$$X = h \cos(\varphi)\sin(\theta)$$  \hspace{1cm} (6)
$$Y = h \sin(\varphi)\sin(\theta)$$  \hspace{1cm} (7)
$$Z = h \cos(\theta)$$  \hspace{1cm} (8)

The orientation workspace for various configurations can be analysed for determining the dexterity and manipulability of the robotic configurations. The voids inside the orientation workspace indicate the probability of existence of singular surfaces inside the workspace envelopes. The orientation workspace is plotted by considering the arm joint limits given in table 1. The left hand is taken to the corresponding positions and the orientations by giving different joint limits to elbow joint as shown in figure 6. Figure 6 shows three configurations of left hand by changing elbow joint angles to $(-45,0,45)$ degrees.

![Figure 6. Different arm configurations (in mm)](image)
4. Singularity analysis
The jacobian matrix of end effector with relative position, \( P_i \) and orientation, \( \theta_i \) is given in equation (9)

\[ j = [ P_i \ \theta_i ]^T \quad (9) \]

The jacobian of each branch is derived separately for determining the singularity based on rank deficiency criteria of jacobian matrix. The jacobian matrix of first branch is given in equation (10)

\[ j_{\text{hip}} = \begin{bmatrix} P_1 \\ \theta_1 \\ P_2 \\ \theta_2 \\ P_3 \\ \theta_3 \end{bmatrix} \quad (10) \]

Similarly, the singular configurations of other joints of each branch in upper body section are determined based on jacobian matrix. The non-square jacobian matrices \( J \) are converted to square matrix by using the equation given in (11)

\[ J_{\text{final}} = \sqrt{\det(j \times j^T)} \quad (11) \]

5. Reachability of Arms
Reachability measurement helps to determine the most optimal configurations of a robotic system for a given task. The reachability measurement of arms play an important role in determining the dexterous workspace of the upper body humanoid robot. The reachable workspace is defined as the possible set of all points accessible by the robot end effector. For most of the given task, the reachable workspace is limited as compared with the dexterous workspace due to the presence of movement constraints. Reachability function, \( R \) [9] is used for determining the reachability of a given point \( p_i \) in a task space with joint variable \( q_i \) with n DoF is given in equation (12)

\[ R = \max_{k} \prod_{i=1}^{n} \min \left[ \frac{|q_{i,\text{max}} - q_i|}{0.5|q_{i,\text{max}} - q_{i,\text{min}}|} \right] \quad (12) \]

Where \( k \) denotes the number of inverse kinematic solutions, \( j_i \) represents joint variables. Reachability function varies from zero to unity for a given configuration. The maximum value for a reachability function is unity and minimum is zero. The reachability function attains unity only and only if the end effector reaches the given point inside the taskspace with all joint displacements within the midrange of joint angle limits. The reachability function values for different location of point \( p_i \) is given in table 2.

| Table 2. Reachability function |
|--------------------------------|
| Location of point \( p_i \) | Value of reachability function, \( R \) |
| Point \( p_i \) is located inside the workspace and at least one of the solutions is within the joint limit | [0 1] |
| Point \( p_i \) is located inside the workspace and is reachable with joint limits midrange | 1 |
| Point \( p_i \) is located inside the workspace and is reachable with at least one of the joint angles in extreme position | 0 |

6. Results and discussion
6.1. Determination of singular configurations of hip joints
The jacobian matrix of hip joints are given in equation (13)
Let $s_i = \sin \theta_i$ and $c_i = \cos \theta_i$

$$j_{\text{hip}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -s_1 & c_1c_2 \\ 0 & c_1 & s_1c_2 \\ 1 & 1 & -s_2 \end{bmatrix}$$  (13)

The singular configurations of hip joints are given in equations (14) to (15)

$$\begin{bmatrix} 0 & -s_1 & c_1c_2 \\ 0 & c_1 & s_1c_2 \\ 1 & 1 & -s_2 \end{bmatrix} = 0$$  (14)

$$\theta_1 = 0; \theta_2 = \pm \left( 2n + 1 \right) \pi/2$$  (15)

### 6.2. Determination of Reachability function

The reachability function for a particular point $P_i(x, y, z)$ inside the reachable workspace indicates the reachability of the point by using different configurations of the robotic system. The reachability function can also be used for energy optimization for a given task. The reachability function, $R_i$ for the three configurations of upper body movements for a point $P$ (200,150,225), which is inside the workspace is calculated and the maximum reachability value for $P_i$ by using equation (15) is obtained as 0.8567. The high value of reachability function for $P_i$ indicates the reachability of the point with most of the different configurations of the end effectors. The function value approaches to zero, if the given point is along the workspace boundary.

### 6.3. Orientation workspace analysis

In this paper, joint limits given in table 1 are used for analyzing the orientation workspace. Euler angles sets are derived from rotation matrix obtained by inputting the joint limits. Orientation workspace of the upper body tree type robot is analysed and is plotted as shown in figure 7. The dense area indicates the dexterous region where as the void regions of the workspace is less dexterous as compared to center region. The orientation workspace with respect to orientation matrix is plotted by converting it into Euler angle sets $(\varphi, \theta, \psi)$ in radians and the coordinates are converted to cartesian coordinates with respect to position coordinates.

![Figure 7. Orientation workspace](image-url)
7. Conclusion and future scope
Screw theory based kinematic modeling of upper body humanoid robot was carried out in this paper. Singularity analysis of the hip, reachability of arms and orientation workspace are also presented. The screw theory approach is suitable for higher DoF mechanism due to the absence of intermediate frame assignments. Workspace and singularity analysis help to determine the overall workspace and voids inside the workspace. Reachability functions along with workspace and singularity analysis are used to determine the dexterity and manipulability of the upper body humanoid robot. These analyses can be used for the synthesis of robot mechanism. Modeling can be helpful for the stable humanoid locomotion and energy optimized trajectory planning. Better control strategies can also be implemented in the simulation and experimental mode of operations using a better dynamic model.

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