Gluon Condensates, Chiral Symmetry Breaking

And Pion Wave-function

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We consider here chiral symmetry breaking in quantum chromodynamics arising from gluon condensates in vacuum. Through coherent states of gluons simulating a mean field type of approximation, we show that the off-shell gluon condensates of vacuum generate a mass-like contribution for the quarks, giving rise to chiral symmetry breaking. We next note that spontaneous breaking of global chiral symmetry links the four component quark field operator to the pion wave function. This in turn yields many hadronic properties in the light quark sector in agreement with experiments, leading to the conclusion that low energy hadron properties are primarily driven by the vacuum structure of quantum chromodynamics.
I. INTRODUCTION

It is now accepted that quantum chromodynamics (QCD) is the correct theory for strong interaction physics of quarks and of hadrons. However, this being a nonabelian gauge theory of strong interactions, at present no reliable method other than lattice gauge theory [1] is known for the solution of such problems [2]. Although this is a beautiful and powerful method, the solutions finally need better computer capabilities compared to what is available now. It is thus desirable to consider alternative dynamical schemes which do not involve a discretisation of space-time. One such scheme with a limited objective has been the QCD sum rules [3] through duality principle. It optimises the use of a nonperturbative vacuum structure with perturbative QCD calculations [3], and reproduces nice experimental results. However, in a purely nonperturbative situation, the method is expected not to be applicable.

We have proposed earlier an alternative scheme with a variational method which is nonperturbative [4,5], and uses off-shell gluons for the existence of a nontrivial vacuum structure [6]. Calculations are done in Coulomb gauge. The inputs consist of the QCD Lagrangian, and QCD vacuum contains gluon condensates arising through a variational principle [6]. With a similar technique we have also considered chiral symmetry breaking in Nambu Jona Lasinio model, as well as for some quark antiquark phenomenological potentials used for hadron spectroscopy [7,8]. In the present paper we consider the same as arising from gluon condensates along with some low energy hadronic phenomena.

The paper is organised as follows. In section 2 we recapitulate [6] the nonperturbative gluon condensates in Coulomb gauge giving rise to a nontrivial vacuum structure and define notations. We then conjecture that through minimal coupling the quarks have a nonzero mass-like contribution, and obtain an approximate value for the same. In section 3 we derive the pion state as a quark antiquark pair through a constructional use of Goldstone theorem. In section 4 we proceed to use this relationship to obtain some low energy hadronic properties in the light quark sector. In section 5 we discuss the results.

The method considered here is a non-perturbative one as we use only equal time quantum
algebra but is limited by the choice of ansatz functions in the calculations. The techniques have been applied earlier to solvable cases to examine its reliability as well as to physically more relevant ground state structures in high energy physics and nuclear physics. Starting from the QCD Lagrangian we examine here the more complex problem of chiral symmetry breaking and its relationship to low energy hadronic properties.

II. GLUON CONDENSATES AND CHIRAL SYMMETRY BREAKING

In the present section we shall recapitulate the nontrivial vacuum structure in quantum chromodynamics through Bogoliubov transformation as considered earlier to set the notations, and derive some results.

The Lagrangian density including the quark fields is given as

\[ \mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{int}}. \]  

(2.1)

Here,

\[ \mathcal{L}_{\text{gauge}} = -\frac{1}{2} G^{a\mu\nu} (\partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{abc} W^b_\mu W^c_\nu) + \frac{1}{4} G^{a\mu\nu} G^{a\mu\nu}. \]

(2.2)

where \( W^a_\mu \) are the SU(3) colour gauge fields. Also the Lagrangian densities for the quark fields and interactions with gluons are given as

\[ \mathcal{L}_{\text{matter}} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi, \]

(2.3)

and

\[ \mathcal{L}_{\text{int}} = g \bar{\psi} \gamma^\mu t_a W^a_\mu \psi. \]

(2.4)

In order to do the quantisation in Coulomb gauge, we write the electric field, \( G^{a0}_i \) in terms of the transverse and longitudinal parts as

\[ G^{a0}_i = T G^{a0}_i + \partial_i f^a, \]

(2.5)
where the form of $f^a$ is to be determined. In the Coulomb gauge the subsidiary condition and the equal time algebra for the gauge fields are given as (12)

$$\partial_i W^a_i = 0$$

(2.6)

and

$$\left[ W^a_i(\vec{x}, t), T^a_{0j}(\vec{y}, t) \right] = i\delta^{ab}(\delta_{ij} - \frac{\partial_i\partial_j}{\partial^2})\delta(\vec{x} - \vec{y}).$$

(2.7)

We take the field expansions for $W^a_i$ and $T^a_{0i}$ at time $t=0$ as (13)

$$W^a_i(\vec{x}) = (2\pi)^{-3/2} \int d\vec{k} \frac{1}{\sqrt{2\omega(\vec{k})}} (a^a_i(\vec{k}) + a^a_i(-\vec{k})^\dagger) \exp(i\vec{k} \cdot \vec{x})$$

(2.8a)

and

$$T^a_{0i}(\vec{x}) = (2\pi)^{-3/2} i \int d\vec{k} \sqrt{\frac{\omega(\vec{k})}{2}} (-a^a_i(\vec{k}) + a^a_i(-\vec{k})^\dagger) \exp(i\vec{k} \cdot \vec{x}).$$

(2.8b)

From equation (2.7) these give the commutation relations for $a^a_i$ and $a^{b\dagger}_j$ as

$$\left[a^a_i(\vec{k}), a^{b\dagger}_j(\vec{k}')\right] = \delta^{ab}\Delta_{ij}(\vec{k})\delta(\vec{k} - \vec{k}'),$$

(2.9)

where, $\omega(k)$ is arbitrary, and,

$$\Delta_{ij}(\vec{k}) = \delta_{ij} - \frac{k_ik_j}{k^2}.$$  

(2.10)

In Coulomb gauge, the expression for the Hamiltonian density, $\mathcal{T}^{00}$ from equation (2.1) is given as (14)

$$\mathcal{T}^{00} = \frac{1}{2} T^a_{0i} T^a_{0i} + \frac{1}{2} W^a_i(-\vec{\nabla}^2)W^a_i + g f^{abc}W^b_iW^c_j\partial_i W^a_j$$

$$+ \frac{g^2}{4} f^{abc} f^{aef}W^b_i W^c_j W^e_i W^f_j + \frac{1}{2}(\partial_i f^a)(\partial_i f^a)$$

$$+ \bar{\psi}(-i\gamma^i\partial_i - m)\psi - g\bar{\psi}\gamma^i t_a W^a_i \psi,$$

(2.11)

where $:\ :)$ denotes the normal ordering with respect to the perturbative vacuum, say $|\text{vac}\rangle$, defined through $a^a_i(\vec{k}) |\text{vac}\rangle = 0$. The term $\frac{1}{2}(\partial_i f^a)(\partial_i f^a)$ automatically includes
interactions for both time-like and longitudinal gluons, here through the auxiliary field description, and $T^{00}$ is calculated after the elimination of the same $O$.

In Ref [3] the possibility of a nonperturbative vacuum having a lower energy was discussed as from the above equation. Such a vacuum contained gluon condensates. We thus considered earlier a trial state, $|\text{vac}'\rangle$ with such condensates over the perturbative vacuum, $|\text{vac}\rangle$ given as [3,6]

$$|\text{vac}'\rangle = U|\text{vac}\rangle,$$  \hspace{1cm} (2.12)

where

$$U = \exp(B^\dagger - B),$$  \hspace{1cm} (2.13)

with

$$B^\dagger = \frac{1}{2} \int f(\vec{k}) a_i^a(\vec{k})^\dagger a_i^a(\vec{k})^\dagger d\vec{k}.$$  \hspace{1cm} (2.14)

$B^\dagger$ contains gluon pair creation operators with an ansatz function $f(\vec{k})$ to be determined later. The operators, say, $b^a_i(\vec{k})$, which annihilate $|\text{vac}'\rangle$ are given as

$$b^a_i(\vec{k}) = U a^a_i(\vec{k}) U^{-1}.$$  \hspace{1cm} (2.15)

We can explicitly evaluate from the equations above the operators $b^a_i(\vec{k})$ in terms of $a^a_i(\vec{k})$. In fact, we then obtain the Bogoliubov transformations [3]

$$\begin{pmatrix} b^a_i(\vec{k}) \\ b^a_i(-\vec{k})^\dagger \end{pmatrix} = \begin{pmatrix} \cosh f(\vec{k}) & -\sinh f(\vec{k}) \\ -\sinh f(\vec{k}) & \cosh f(\vec{k}) \end{pmatrix} \begin{pmatrix} a^a_i(\vec{k}) \\ a^a_i(-\vec{k})^\dagger \end{pmatrix},$$  \hspace{1cm} (2.16)

where we have assumed that the function $f(\vec{k})$ is even and real. Using equations (2.9) and (2.16), we obtain the same commutation relation for the operators $b^a_i$ and $b^b_j^\dagger$ given as

$$\left[ b^a_i(\vec{k}), b^b_j(\vec{k})^\dagger \right] = \delta^{ab} \Delta_{ij}(\vec{k}) \delta(\vec{k} - \vec{k}'),$$  \hspace{1cm} (2.17)

which merely reflects that the Bogoliubov transformation (2.16) is a canonical transformation. In order to consider the stability of $|\text{vac}\rangle$ with respect to the transformation of
equation (2.12), the expectation value of $T^{00}$ with respect to $|vac'\rangle$ was evaluated and was then minimised over $f(\vec{k})$.

It was noticed \[6\] that under certain conditions the perturbative vacuum becomes unstable, where it was also seen that the general solution of $f(\vec{k})$ through minimisation is impossible to obtain analytically. We had taken an ansatz function $f(\vec{k})$ of equation (2.14) for $|vac'\rangle$ description as given by \[6\]

$$\sinh(f(\kappa)) = A e^{-B\kappa^2/2}$$  \hspace{1cm} (2.18)

and then had determined the parameter $A$ through a minimisation of vacuum energy density. The dimensional parameter $B$ was determined through the evaluation of the SVZ parameter \[6\]. We may note from equations (2.16) and (2.18) that

$$<vac'|a^a_i(\vec{k})^\dagger a^a_i(\vec{k}')|vac'> = 16A^2\exp(-B\kappa^2)\delta(\vec{k} - \vec{k}') \hspace{1cm} (2.19)$$

so that the ansatz of equation (2.18) merely implies a Gaussian distribution for the perturbative gluons in $|vac'\rangle$. The minimisation was done for pure gluon fields in Ref. \[6\] with $\alpha_s = g^2/(4\pi)$ of the Lagrangian as the input coupling strength. For example, we then have for input values of $\alpha_s = 0.5, 0.6, 0.8, 1.0$, the identification for the output parameters as $A_{\text{min}} = .178, .306, .557, .658, B^\frac{1}{2} = .40, .48, .62, .69$ in fermis, and $m = 232, 272, 325, 345$ in MeV respectively, where $\omega(\kappa) = \sqrt{\kappa^2 + m^2}$. “$m$” here is a mass-like parameter for $\omega(\kappa)$ of equations (2.8) which was determined through a self-consistent calculation. It is not associated with energy momentum four vector of a “free” gluon, which anyhow does not exist. We may note that the results of Ref. \[6\] are very similar to some of the earlier results \[13\], and that the method of solution for the auxiliary equation avoided Gribov ambiguity \[13\].

Here we shall be using the final output, the stable vacuum $|vac'\rangle$ as the physical vacuum. We shall assume that the vacuum structure in QCD in presence of quarks is mainly driven by gluon condensates, and study some consequences of the same.

We now proceed to consider the quark fields in the presence of the “dressing” of vacuum
with off-mass shell gluon quanta as considered above. We note that here with minimal coupling

\[ \vec{k}^2 \to \vec{k}'^2 + < vac' | g^2 t^a t^b W^a_i W^b_i | vac' >. \]  

(2.20)

With the mean field approximation for the gluon fields as above, we then see that the mass of the quarks \( m_Q \) is now given as

\[ m_Q^2 \approx m^2 + g^2 t_a t_a f_{ii}(\vec{0}), \]  

(2.21)

where we have substituted

\[ < vac' | W^a_i(\vec{x}) W^b_j(\vec{y}) | vac' > = \frac{\delta^{ab}}{(2\pi)^3} \int d\vec{k} e^{i\vec{k} \cdot (\vec{x} - \vec{y})} \frac{F_+(\vec{k})}{\omega(k)} \Delta_{ij}(\vec{k}) \]  

\[ \equiv \delta^{ab} f_{ij}(\vec{x} - \vec{y}). \]  

(2.22)

In the above

\[ F_+(\vec{k}) = \left( \frac{\sinh 2f(k)}{2} + \sinh^2 f(k) \right). \]  

(2.23)

With \( t_a t_a = 4/3 \), we then obtain that

\[ m_Q^2 = m^2 + \frac{16\pi\alpha_s}{3} f_{ii}(\vec{0}) \]  

(2.24)

where summation over the index \( i \) is understood. We thus note that even when \( m = 0 \) there is a finite mass of the quark, which implies breaking of chiral symmetry. We then obtain through numerical evaluation of \( f_{ii}(\vec{0}) \) that \( m_Q^2 = .0358 \text{ GeV}^2 \) for \( \alpha_s = 0.5 \), so that we then have

\[ m_Q = 189\text{MeV}. \]  

(2.25)

We may however note that the above result is deceptive. It is incomplete since we have made the mean field approximation with minimal coupling to generate a mass-like contribution, and do not know how it operates regarding details of dynamics. We use this only to illustrate that gluon condensates break chiral symmetry. We shall include this effect in the expansion of the quark field operators.
We then write the quark fields as

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \int \left[ U(\vec{k})q_I(\vec{k}) + V(-\vec{k})\bar{q}_I(-\vec{k}) \right] e^{i\vec{k} \cdot \vec{x}} d\vec{k}, \quad (2.26)$$

where \[17,18\]

$$U(\vec{k}) = \begin{pmatrix} \cos\chi(\vec{k})/2 \\ \vec{\sigma} \cdot \vec{k} \sin\chi(\vec{k})/2 \end{pmatrix}, \quad (2.27a)$$

and

$$V(-\vec{k}) = \begin{pmatrix} -\vec{\sigma} \cdot \vec{k} \sin\chi(\vec{k})/2 \\ \cos\chi(\vec{k})/2 \end{pmatrix}. \quad (2.27b)$$

The above form is so taken that it satisfies the equal time algebra \[17,18\]. In the above, for free quark fields of mass \(m_Q\), we have \[17\]

$$\cos\chi(\vec{k})/2 = (p^0 + m_Q)^{1/2}, \quad \sin\chi(\vec{k})/2 = (p^0 - m_Q)^{1/2}, \quad (2.28)$$

with \(p^0 = (m_Q^2 + \vec{k}^2)^{1/2}\). We may further note that when chiral symmetry is unbroken \(\chi(\vec{k}) = \pi/2\). However, since the quarks are not free, and, chiral symmetry is broken, \(\chi(\vec{k})\) shall be different from the above.

We would like to associate the quark field operators as in equation (2.26) explicitly with the presence of quark condensates generating chiral symmetry breaking. Parallel to the earlier operators for gluon condensates, let us consider the quark antiquark pair creation operator, with \(h(\vec{k})\) as a trial function,

$$B^I_Q = \int h(\vec{k}) q_I(\vec{k})^\dagger \vec{\sigma} \cdot \vec{k} \bar{q}_I(-\vec{k}) d\vec{k}. \quad (2.29)$$

Let us now consider the unitary transformation

$$U_Q = e^{B^I_Q - B^I_Q}, \quad (2.30)$$

which will operate on chiral vacuum. Also, let us write the corresponding quark field operator of equation (2.26) in momentum space as

$$\tilde{\psi}_0(\vec{k}) = \begin{pmatrix} \cos\chi_0(\vec{k}) q_I(\vec{k}) - \vec{\sigma} \cdot \vec{k} \sin\chi_0(\vec{k}) \bar{q}_I(-\vec{k}) \\ \vec{\sigma} \cdot \vec{k} \sin\chi_0(\vec{k}) q_I(\vec{k}) + \cos\chi_0(\vec{k}) \bar{q}_I(-\vec{k}) \end{pmatrix}. \quad (2.31)$$
When chiral symmetry is there, $\chi_0 = \pi/2$. We may now note that the unitary transformation $U_Q$ is equivalent to the Bogoliubov transformation given as

$$U_Q^\dagger \begin{pmatrix} q_I(\vec{k}) \\ \tilde{q}_I(-\vec{k}) \end{pmatrix} U_Q = \begin{pmatrix} \cos(h(\vec{k})) & \vec{\sigma} \cdot \vec{k} \sin(h(\vec{k})) \\ -\vec{\sigma} \cdot \vec{k} \sin(h(\vec{k})) & \cos(h(\vec{k})) \end{pmatrix} \begin{pmatrix} q_I(\vec{k}) \\ \tilde{q}_I(-\vec{k}) \end{pmatrix}. \quad (2.32)$$

Now, with $\tilde{\psi}(\vec{k}) = U_Q^\dagger \tilde{\psi}_0(\vec{k}) U_Q$, we generate equation (2.26) when we identify that

$$\frac{\chi(\vec{k})}{2} = \frac{\chi_0}{2} - h(\vec{k}). \quad (2.33)$$

We thus note that the form of equation (2.26) with (2.27) corresponding to chiral symmetry breaking can be interpreted as a destabilisation of chiral symmetric vacuum through equation (2.30) when $\chi_0 = \pi/2$. *Hence taking the quark field operators with chiral symmetry breaking or taking a condensate over chiral vacuum become equivalent.* The quark field operators after chiral symmetry breaking get related to the vacuum structure.

We may also note that

$$< \bar{\psi}(\vec{x}) \psi(\vec{y}) > = < \text{vac} | U_Q^\dagger \tilde{\psi}_0(\vec{x}) \psi_0(\vec{y}) U_Q | \text{vac} > = -\frac{12}{(2\pi)^3} \int \cos(\chi(\vec{k})) e^{i\vec{k} \cdot (\vec{x} - \vec{y})} d\vec{k}, \quad (2.34)$$

where, the factor 12 arises from colour, flavour and spin degrees of freedom. We may substitute $\cos(\chi(\vec{k})) = \sin(2h(\vec{k}))$, describing the above function explicitly in terms of the quark antiquark correlations. We also have

$$< \bar{\psi}(\vec{x}) \psi(\vec{x}) > = -\mu^3 \quad (2.35)$$

with $\mu$ determined in terms of the condensate function or $\cos(\chi(\vec{k}))$. The condensate function $h(\vec{k})$ vanishes when $\cos(\chi(\vec{k})) = 0$, as should be the case for chiral vacuum.

The form of $\chi(\vec{k})$ will, generally speaking, depend on interactions $[7,18]$, and, can be obtained through an extremisation $[8,19]$. Also, the function $\chi(\vec{k})$ will get related to the pion wave function using Goldstone theorem as shown below. This shall include vacuum structure as a *post facto* information linked to phenomenology without an extremisation of energy density containing highly nonlinear expressions $[3]$. 

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III. CHIRAL SYMMETRY BREAKING AND PION WAVE FUNCTION

We shall here first consider only two quark flavours and, the usual three colours. When chiral symmetry remains good, we note that

\[ Q_5^i < vac > = 0 \]  \hspace{1cm} (3.1)

where \( Q_5^i \) is the chiral charge operator given as

\[ Q_5^i = \int \psi(\vec{x})^\dagger \tau^i \gamma^5 \psi(\vec{x}) d\vec{x}. \]  \hspace{1cm} (3.2)

In this case quarks are massless and we have

\[ \cos \frac{\chi(\vec{k})}{2} = \sin \frac{\chi(\vec{k})}{2} = \frac{1}{\sqrt{2}}. \]  \hspace{1cm} (3.3)

For symmetry broken case however

\[ Q_5^i < vac' > \neq 0. \]  \hspace{1cm} (3.4)

This will correspond to the pion state. To show this we first note that

\[ [Q_5^i, H] = 0. \]  \hspace{1cm} (3.5)

Clearly, for the symmetry breaking phase, \(< vac' >\) is an approximate eigenstate of the total Hamiltonian \( H \) with \( EV \) as the approximate eigenvalue, \( V \) being the total volume and \( E \) the energy density. With \( H_{eff} = H - EV \), we then obtain from equation (3.5) that

\[ H_{eff} Q_5^i | vac' > = 0 \]  \hspace{1cm} (3.6)

i.e. the state \( Q_5^i | vac' > \) with zero momentum has also zero energy, which corresponds to the massless pion. Explicitly using equations (2.26) and (2.27), and with \( q_I \) now as two component isospin doublet corresponding to \((u,d)\) quarks above, we then obtain the pion states of zero momentum as

\[ | \pi^i(\vec{0}) > = N_\pi \cdot \frac{1}{\sqrt{6}} \int q_I(\vec{k})^\dagger \tau^i q_I(\vec{k}) cos(\chi(\vec{k})) d\vec{k} | vac' >, \]  \hspace{1cm} (3.7)
where, $N_\pi$ is a normalisation constant. The wave function for pion $\tilde{u}_\pi(\vec{k})$ thus is given as proportional to $\cos\chi(\vec{k})$. The colour, isospin and spin indices of $q^\dagger$ and $\bar{q}$ for quarks and antiquarks have been suppressed. Further, with

$$<\pi^i(0)|\pi^j(p)> = \delta^{ij}\delta(p),$$  \hfill (3.8)

the normalisation constant $N_\pi$ is given through [17]

$$N_\pi^2 \int \cos^2(\chi(\vec{k}))d\vec{k} = 1.$$  \hfill (3.9)

In the notations of Ref. [17] thus the pion wave function $\tilde{u}_\pi(\vec{k})$ is given as

$$\tilde{u}_\pi(\vec{k}) = N_\pi \cos(\chi(\vec{k})).$$  \hfill (3.10)

Clearly the state as in equation (3.7) as the Goldstone mode will be accurate to the extent we determine the vacuum structure sufficiently accurately through variational or any other method so that $|\text{vac}'\rangle$ is an eigenstate of the total Hamiltonian. The above results yield pion wave function from the vacuum structure for any example of chiral symmetry breaking, and is a new feature of looking at phase transition through vacuum realignment [7].

We had earlier considered low energy hadronic properties [17] with an assumed form of interacting quark field operators. This is equivalent to a choice of $\cos(\chi(\vec{k}))$. In addition, we had taken Gaussian wave functions for baryons and mesons, and then discussed the phenomenology. We shall here examine the consistency of the present picture with low energy hadron phenomenology where pion wave function and quark field operators get related as in equations (2.27) and (3.10).

The notations of Ref. [17], which we shall use now, correspond to

$$f_q(\vec{k}) = \cos\frac{\chi(\vec{k})}{2}; \quad |\vec{k}|g_q(\vec{k}) = \sin\frac{\chi(\vec{k})}{2}.$$  \hfill (3.11)

We may note that as in equation (2.26) with (2.27), we are retaining the fully relativistic four-component quark field operator. The phenomenological assumption here will consist of
explicitly taking a specific expression for \( \cos \chi(\vec{k}) \). We note that \( \cos \chi(\vec{k}) \) as in equation (2.34) is the correlation function for quark pairs in vacuum. Hence, parallel to the choice of gluon correlation functions in equation (2.19) as a Gaussian, we shall take here the ansatz

\[
\cos \chi(\vec{k}) = \exp \left( -\frac{R^2}{2\vec{k}^2} \right). \tag{3.12}
\]

It also implies through equation (3.10) that the pion wave function is a Gaussian. From equation (3.9), here we have

\[
N_\pi = \frac{R^{3/2}}{\pi^{3/4}} = 0.424 \times R^{3/2}. \tag{3.13}
\]

From equations (2.34) and (2.35) we also obtain that

\[
\mu = \frac{(12)^{1/3}}{\sqrt{2\pi}} R^{-1}_\pi = 0.913 \times R^{-1}_\pi. \tag{3.14}
\]

We would now like to see what may be the nature of dispersion curves for quarks in vacuum. We note that for free quark fields \( \sin \chi(\vec{k}) = \frac{\kappa}{\epsilon(\vec{k})} \). We may use this to obtain that

\[
\epsilon(\vec{k}) = \frac{\kappa}{\sqrt{1 - \cos^2 \chi(\vec{k})}}, \tag{3.15}
\]

giving the dispersion curve for quarks in vacuum as a medium. Thus for equation (3.12),

\[
\epsilon(\vec{k}) = \frac{\kappa}{\sqrt{1 - e^{R^2 \vec{k}^2}}}. \tag{3.16a}
\]

Hence, for small \( \kappa^2 \),

\[
\epsilon(\vec{k}) = \frac{1}{R_\pi} + \frac{R_\pi}{4} \kappa^2. \tag{3.16b}
\]

We thus see that \( \epsilon(\vec{k}) \) does not have the form of a free particle. We note that for large \( \kappa \), \( \epsilon(\vec{k}) \approx \kappa \), which follows for any choice of \( \cos \chi(\vec{k}) \) from equation (3.9).

We shall need later expansion of \( \cos \chi(\vec{k} + \vec{p}) \) for small \( |\vec{p}| \) for charge radius and magnetic moment. For this purpose we use the notations

\[
\partial_i \cos \chi(\vec{k}) = -\vec{k}_i b_1(\vec{k}); \quad \partial_i \partial_j \cos \chi(\vec{k}) = -\delta_{ij} b_1(\vec{k}) + \vec{k}_i \vec{k}_j b_2(\vec{k}). \tag{3.17}
\]

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In the above, we have used rotational invariance. We also note that

$$\vec{R}^2 b_1(\vec{k})^2 = (\nabla \cos \chi(\vec{k}))^2,$$  \hspace{1cm} (3.18)$$

and, define $c_2(\vec{k})$ such that

$$b_1(\vec{k}) - \frac{1}{3} \vec{k}^2 b_2(\vec{k}) \equiv c_2(\vec{k}) = -\frac{1}{3} \nabla^2 \cos \chi(\vec{k}).$$  \hspace{1cm} (3.19)$$

The above equations shall be useful if we wish to change the ansatz function for $\cos \chi(\vec{k})$. Clearly, for equation (3.12), we have

$$b_1 = R^2 \cos \chi(\vec{k}); \quad b_2 = -R^4 \cos \chi(\vec{k}).$$  \hspace{1cm} (3.20)$$

**IV. LOW ENERGY HADRONIC PROPERTIES**

In the context of chiral symmetry breaking, the model of Ref. [17] now has got modified since the pion wave function and the quark field operators in the light quark sector get related. This in many ways enriches the earlier model [17], as it decreases the number of independent quantities. The previous model had also been extended to incoherent processes involving structure function as well as hadronisation [20,21]. It has also been applied to the problem of strong CP violation recently [22]. We here consider different low energy hadronic properties parallel to [17] using equations (3.11).

**A. Pion decay constant**

The pion decay constant $c_{\pi}$ as calculated earlier in terms of the wave function with relativistic correction to Van Royen-Weisskopf relation [23] is given by [17]

$$\left| (1 + 2g_q^2 \nabla^2) u_{\pi}(\vec{0}) \right| = \frac{c_{\pi}(m_{\pi})^{1/2}}{\sqrt{6}},$$  \hspace{1cm} (4.1)$$

where the value of the wave function is taken at the space origin, and, $g_q$ is a differentiation operator. In fact taking the Fourier transform the left hand side of equation (4.1) becomes
\[
\frac{1}{(2\pi)^{3/2}} \int (1 - 2g_q(\vec{k})^2 \vec{k}^2) u_\pi(\vec{k}) d\vec{k}.
\]

From equation (3.11) however \(1 - 2g_q(\vec{k})^2 \vec{k}^2 = \cos(\chi(\vec{k}))\). Hence from equation (3.9) the normalisation constant \(N_\pi\) is given through \([17]\), with \(c_\pi = 92\) MeV,

\[
\frac{1}{(2\pi)^{3/2}} \cdot \frac{1}{N_\pi} = \frac{c_\pi (m_\pi)^{1/2}}{\sqrt{6}} \approx 0.0140\text{GeV}^{3/2},
\]

so that in equation (3.10)

\[
N_\pi = 4.55 \text{ GeV}^{-3/2}.
\]

Hence, from equations (3.13) and (3.14) we may note that, for the choice of equation (3.12),

\[
R_\pi^2 = \pi \times N_\pi^{4/3} = 23.69 \text{ GeV}^{-2}; \quad \mu = 187 \text{ MeV}.
\]

We may note that in equation (3.16b) if we take \(m_Q = 1/R_\pi\), then \(m_Q\) for the above equation is around 205 MeV, which may be compared with the earlier value of 189 MeV from the vacuum structure in QCD due to gluon condensates as in equation (2.25).

Thus the known value of the pion decay constant puts a severe constraint both on the pion wave function as well as on the form of the quark field operators.

### B. Pion charge radius

We note that the pion form factor is given by \([17]\)

\[
<\pi^+(-\vec{p})|J^0(0)|\pi^+(\vec{p})> = \frac{1}{(2\pi)^3} \frac{m_\pi}{p^0} G_E^\pi(t)
\]

where through direct evaluation \([17]\) we obtain that

\[
G_E^\pi(t) = \int \bar{u}_\pi(\vec{k}^\prime_1)^\dagger u_\pi(\vec{k}_1) \left( f_{q_1}^{\prime}(\vec{k}^\prime_1)f_q(\vec{k}_1) + \vec{k}^\prime_1 \cdot \vec{k}_1 g_q(\vec{k}^\prime_1)g_{\bar{q}}(\vec{k}_1) \right) d\vec{k}_1.
\]

In the above \([17]\),

\[
t = -4\vec{p}^2; \quad \vec{k}^\prime_1 = \vec{k}_1 - \frac{m_\pi}{p^0} \vec{p}.
\]
We shall here retain only terms up to $|\vec{p}|^2$ and use

$$G_E(t) = 1 + \frac{R_{ch}^2}{6}t.$$  (4.8)

For the expansion in powers of $t$, let us substitute in equation (4.6)

$$\tilde{k}'_1 = \tilde{k} - \frac{1}{2}\vec{p}; \quad \tilde{k}_1 = \tilde{k} + \frac{1}{2}\vec{p}$$  (4.9)

corresponding to equation (4.7). Using equations (3.17) and (3.19), we get that

$$\cos\chi(\tilde{k}'_1)\cos\chi(\tilde{k}_1) = \cos^2\chi(\tilde{k}) - \frac{|\vec{p}|^2}{4}\left(\cos\chi(\tilde{k})c_2(\tilde{k}) + \frac{1}{3}\tilde{k}^2b_1(\tilde{k})^2\right).$$  (4.10)

and,

$$\cos\chi(\tilde{k}'_1) + \cos\chi(\tilde{k}_1) = 2\cos\chi(\tilde{k}) - \frac{|\vec{p}|^2}{4}c_2(\tilde{k}).$$  (4.11)

We then have in the lowest order in $|\vec{p}|$,

$$\tilde{u}_x(\tilde{k}'_1)^\dagger\tilde{u}_x(\tilde{k}_1) = N^2\left[\cos^2\chi(\tilde{k}) - \frac{|\vec{p}|^2}{4}\left(\cos\chi(\tilde{k})c_2(\tilde{k}) + \frac{1}{3}\tilde{k}^2b_1(\tilde{k})^2\right)\right].$$  (4.12)

Also, by equations (3.11) and (4.9),

$$f_Q(\tilde{k}'_1)f_Q(\tilde{k}_1) = \frac{1 + \cos\chi(\tilde{k})}{2} - \frac{|\vec{p}|^2}{16}\left(c_2(\tilde{k}) + \frac{\tilde{k}^2b_1(\tilde{k})^2}{3(1 + \cos\chi(\tilde{k}))}\right),$$  (4.13)

with $\kappa'_1 = |\tilde{k}'_1|$ and $\kappa_1 = |\tilde{k}_1|$,

$$\kappa'_1 g_Q(\tilde{k}'_1)\kappa_1 g_Q(\tilde{k}_1) = \frac{1 - \cos\chi(\tilde{k})}{2} - \frac{|\vec{p}|^2}{16}\left(-c_2(\tilde{k}) + \frac{\tilde{k}^2b_1(\tilde{k})^2}{3(1 - \cos\chi(\tilde{k}))}\right),$$  (4.14)

and,

$$\hat{k}'_1 \cdot \hat{k}_1 = 1 - \frac{|\vec{p}|^2}{6k^2}.\quad (4.15)$$

We then easily obtain from equations (4.6), (4.7) and (4.8) that

$$R_{ch}^2 = R_1^2 + R_2^2$$  (4.16)

where, using equations (4.12), (4.13), (4.14) and (4.15) and simplifying,
\[ R_1^2 = \frac{N_\pi^2}{4} \int (\nabla^2 \cos \chi(\vec{k}))^2 d\vec{k} \quad (4.17) \]

is the contribution coming from the wave function alone, and,

\[ R_2^2 = \frac{N_\pi^2}{16} \int \cos^2 \chi(\vec{k}) \left[ \frac{\vec{k}^2 b_1(\vec{k})^2}{1 - \cos^2 \chi(\vec{k})} + \frac{2(1 - \cos \chi(\vec{k}))}{\vec{k}^2} \right] d\vec{k} \quad (4.18) \]

is the balance of the contribution.

For the choice of equation (3.12) we then have

\[ R_{ch}^2 = \frac{3}{8} R_\pi^2 + I \times R_\pi^2, \quad (4.19) \]

where \( I = .0159 \) as numerically evaluated. This yields that \( R_{ch}^2 = 9.26 \text{ GeV}^{-2} \), or, \( R_{ch} = .605 \text{ fms} \), which may be compared with the experimental value of \( R_{ch} = 0.66 \text{ fms} \) or, \( R_{ch}^2 = 11.22 \text{ GeV}^{-2} \) [24].

### C. \(|g_A/g_V|\)

We next consider some properties associated with proton and neutron. Let us approximate the proton state with a harmonic oscillator wave function, so that we take [17]

\[ \tilde{u}_p(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \left( \frac{3 R_p^4}{\pi^2} \right)^{3/4} \exp \left( -\frac{R_p^2}{6} \sum_{i<j}(\vec{k}_i - \vec{k}_j)^2 \right). \quad (4.20) \]

With the relativistic corrections, we then obtain that [17]

\[ |g_A/g_V| = \frac{5}{3} \left( \frac{3 R_p^2}{2\pi} \right)^{3/2} \int \exp \left( -\frac{3 R_p^2}{2\vec{k}^2} \right) \left( \frac{1}{3} + \frac{2}{3} \cos \chi(\vec{k}) \right) d\vec{k}. \quad (4.21) \]

We have obtained here the above expression through a direct evaluation of

\[ < p_{1/2}(\bar{0}) | J_{5_t}^+ | n_{1/2}(\bar{0}) > \]

retaining the four component quark fields in the current through equation (2.27), and have substituted from equation (3.11) that \( f_q^2 - \frac{1}{3} g_q^2 \vec{k}^2 \equiv \frac{1}{3} (1 + 2\cos(\chi(\vec{k}))) \). Taking \( f_q = 1 \) (and hence \( g_q = 0 \)) or, \( \cos \chi(\vec{k}) = 1 \), will give the result for nonrelativistic quark model. Here the
full relativistic structure of the quark field operator is being retained. \(|g_A/g_V| = 1.257\) puts a constraint on the quark field operators and the approximate size of the nucleons as below.

In fact, we can use the above to obtain the value of \(R_p^2\) for the choice of equation (3.12). We note that we then have

\[
1.257 = \frac{5}{9} \left[ 1 + 2 \left( \frac{3a}{3a + 1} \right)^{3/2} \right], \tag{4.22}
\]

where we have taken \(R_p^2 = aR_\pi^2\). This yields that \(R_p^2 = 0.5707 \times R_\pi^2 = 13.52\) GeV\(^{-2}\).

**D. Proton charge radius**

In order to make another use of the size of the proton, let us calculate its charge radius with relativistic corrections. The charge radius is given through the same identification as in equation (4.8) with \[17\]

\[
< p_r (\vec{p}) | J^0 (0) | p_s (\vec{p}) > = \delta_{rs} \frac{1}{(2\pi)^3} \frac{m_p}{p^0} G_p^p (t). \tag{4.23}
\]

In the above, a direct evaluation yields that, up to order \(|\vec{p}|^2\),

\[
G_p^p (t) = \left( \frac{3R_p^2}{2\pi} \right)^{3/2} \int e^{\frac{3R_p^2}{4}(\vec{k}_1^2 + \vec{k}_1^{'2})} \times \left[ f_q (\vec{k}_1^{'}) f_q (\vec{k}_1) + \vec{k}_1^{'} \cdot \vec{k}_1 g_q (\vec{k}_1^{'}) g_q (\vec{k}_1) \right] d\vec{k}. \tag{4.24}
\]

Here, in contrast to equation (4.9), we have \[17\]

\[
\vec{k}_1^{'1} = \vec{k} - \frac{2}{3} \vec{p}; \quad \vec{k}_1 = \vec{k} + \frac{2}{3} \vec{p}. \tag{4.25}
\]

We then easily see as before that the proton charge radius is given as

\[
R_{ch}^2 = R_1^2 + R_2^2 \tag{4.26}
\]

where \(R_1^2 = R_p^2\) is the nonrelativistic contribution from the proton wave function as may be seen with a contribution parallel to that of equation (4.17), and,
\[
R_s^2 = \frac{1}{9} \left( \frac{3R_p^2}{2\pi} \right)^{3/2} \int \exp \left( -\frac{3R_p^2}{2} \bar{k}^2 \right) \frac{\bar{k}^2 b_1(\bar{k})^2}{1 - \cos^2 \chi(\bar{k})} \left[ \frac{2(1 - \cos \chi(\bar{k}))}{\bar{k}} \right] d\bar{k} \tag{4.27}
\]

is the “relativistic” contribution.

We have determined \( R_p^2 \) in the previous subsection. Substituting the same and using the form of \( \cos \chi(\bar{k}) \) as earlier, we obtain through a numerical evaluation that \( R_s^2 = 3.08 \text{ GeV}^{-2} \), so that we obtain \( R_{ch}^2 = 16.6 \text{ GeV}^{-2} \). This gives that

\[
R_{ch} = 0.81 \text{ fms}, \tag{4.28}
\]

which is similar to the experimental value of the same.

**E. Proton magnetic moment**

We identify the magnetic moment \( \mu_p \) of the proton through the equation \cite{17}

\[
\frac{i}{(2\pi)^3} [\vec{\sigma} \times (-2\vec{p})]_{rs} \mu_p = \langle p_r(-\vec{p}) | J^i(0) | p_s(\vec{p}) \rangle, \tag{4.29}
\]

where on the right hand side above all terms involving \( |\vec{p}|^2 \) will be neglected. Such an evaluation in fact through the notations of Ref. \cite{17} yields that

\[
i [\vec{\sigma} \times (-2\vec{p})]_{rs} \mu_p = \left( \frac{3R_p^2}{2\pi} \right)^{3/2} \int \exp \left( -\frac{3R_p^2}{2} \bar{k}^2 \right) d\bar{k} \bar{u} L^{-1}(p)(\bar{k}_1^i) \gamma^i u L(p)(\bar{k}). \tag{4.30}
\]

In the above, the momenta \( \bar{k}_1 \) and \( \bar{k}_1' \) are given by equatin (4.25) as for the charge radius, and, \( u L(p)(\bar{k}) = S(L(p))u(\bar{k}) \) includes spin rotations of the quarks \cite{17}. On carrying out the simplification of the above expression with a straightforward but complicated algebra, we obtain that

\[
\mu_p = \mu_{p1} + \mu_{p2}, \tag{4.31}
\]

where
\[ \mu_{p1} = \frac{e}{2m} \left( \frac{3R^2_p}{2\pi} \right)^{3/2} \int \exp \left( -\frac{3R^2_p \vec{k}^2}{2} \right) \left( \frac{1}{3} + \frac{2}{3} \cos \chi(\vec{k}) \right) \, d\vec{k} \] (4.32)

and,

\[ \mu_{p2} = \frac{e}{9} \left( \frac{3R^2_p}{2\pi} \right)^{3/2} \int \exp \left( -\frac{3R^2_p \vec{k}^2}{2} \right) \left( \frac{2 \sin \chi(\vec{k})}{\kappa} + \frac{\kappa b_1(\vec{k})}{\sin \chi(\vec{k})} \right) \, d\vec{k}. \] (4.33)

In the above we have used equation (4.13) and (4.14) as earlier. We may note that \( \mu_{p1} \) above came from Lorentz boosting and in the nonrelativistic limit equals one nuclear magneton. \( \mu_{p2} \) in the nonrelativistic limit equals (2/3) of quark magnetic moment as in the earlier calculation \[17\]. We may note that

\[ \mu_{p1} = \frac{e}{2m} \times \frac{3}{5} \frac{g_A}{g_V}. \] (4.34)

With the evaluation of the integral in equation (4.33) using equation (3.12), we obtain from equation (4.31) that

\[ \mu_p = \frac{e}{2m} \times 2.88. \] (4.35)

and, from symmetry or through direct evaluation \[17\],

\[ \mu_n = \frac{e}{2m} \times -1.92. \] (4.36)

These results are higher than expected, but still in reasonable agreement with experiments.

V. DISCUSSIONS

We should note that the present approach to chiral symmetry breaking through vacuum destabilisation \[7,8\] is different from the earlier attempts \[25\], which mainly use Schwinger Dyson equations to get the gap equation, and current algebra to obtain some additional results. We have more conclusions because of the explicit construction of vacuum state, here even relating the low energy properties of hadrons to a distribution function for quarks and antiquarks in vacuum!
Our results here are only linked to the vacuum structure. There will be additional dressing of the hadrons by gluons \[1,26\] which will affect the wave function of the hadrons. Chiral symmetry breaking also is only approximate, which will change the results for pion \[8\]. Also, old $SU(6)$ symmetry in spin and flavour space has been used for the construction of proton and neutron states, where we know that this symmetry is not sufficiently good. This shall have additional effects not included here \[17,20\].

Here $\cos\chi(\vec{k})$ through equations (2.33) and (2.34) is really governed by the vacuum structure of QCD for chiral symmetry breaking. The surprising feature is that this structure yields many hadronic properties in the light quark sector in agreement with experiments, leading to the conclusion that the low energy hadronic properties are primarily driven by the vacuum structure of quantum chromodynamics!

The results suggest that we should try to get $\cos\chi(\vec{k})$ through a variational procedure by minimising energy density parallel to \[6\]. Such a programme would relate $R_\pi$ to the vacuum structure of QCD where the value of $R_\pi$ will depend on $\alpha_s$ \[6\]. Implementation of the same will enable us to recognise the coupling constant $\alpha_s$ for which it agrees with present phenomenological value, which will be an additional conclusion from the present calculations.
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