Polarization and Dielectric Properties of BiFeO$_3$-BaTiO$_3$ Superlattice-Structured Ferroelectric Films

Yuji Noguchi $^{1,*}$ and Hiroki Matsuo $^{2,*}$

1. Division of Information and Energy, Faculty of Advanced Science and Technology, Kumamoto University, 2-39-1, Kurokami, Chuo-ku, Kumamoto 860-8555, Japan
2. International Research Organization for Advanced Science & Technology (IROAST), Kumamoto University, 2-39-1, Kurokami, Chuo-ku, Kumamoto 860-8555, Japan
* Correspondence: yuji19700126@gmail.com (Y.N.); matsuo_h@cs.kumamoto-u.ac.jp (H.M.)

Abstract: Superlattice-structured epitaxial thin films composed of Mn(5%)-doped BiFeO$_3$ and BaTiO$_3$ with a total thickness of 600 perovskite (ABO$_3$) unit cells were grown on single-crystal SrTiO$_3$ substrates by pulsed laser deposition, and their polarization and dielectric properties were investigated. When the layers of Mn-BiFeO$_3$ and BaTiO$_3$ have over 25 ABO$_3$ unit cells (N), the superlattice can be regarded as a simple series connection of their individual capacitors. The superlattices with an N of 5 or less behave as a unified ferroelectric, where the BaTiO$_3$ and Mn-BiFeO$_3$ layers are structurally and electronically coupled. Density functional theory calculations can explain the behavior of spontaneous polarization for the superlattices in this thin regime. We propose that a superlattice formation comprising two types of perovskite layers with different crystal symmetries opens a path to novel ferroelectrics that cannot be obtained in a solid solution system.

Keywords: perovskite; ferroelectric; polarization; dielectric; BiFeO$_3$; BaTiO$_3$; superlattice; film; epitaxial thin film

1. Introduction

Chemical tuning of the dielectric, ferroelectric, and piezoelectric properties of perovskite oxides (ABO$_3$) is traditionally based on the formation of solid solutions. Lead zirconate titanate, Pb(Zr, Ti)O$_3$, is representative, composed of ferroelectric PbTiO$_3$ in tetragonal symmetry, and antiferroelectric PbZrO$_3$ in rhombohedral symmetry [1,2]. In this system, the dielectric and piezoelectric properties are maximized near the composition-driven phase boundary [2], called the morphotropic phase boundary (MPB) [3], between the tetragonal and rhombohedral structures. The similar materials strategy has provided an extremely high piezoelectric response [4,5] in solid solutions such as Pb(Mg, Nb)O$_3$–PbTiO$_3$ and Pb(Zn, Nb)O$_3$–PbTiO$_3$, where an electric field (E) is considered to induce a rotation of spontaneous polarization ($P_s$) [6].

Recently, bismuth ferrite (BiFeO$_3$) [7,8] has attracted considerable attention because of its multiferroic nature [9,10], i.e., the simultaneous presence of ferroelectric $P_s$ and an incommensurate spin cycloid structure, even at room temperature. Bulk BiFeO$_3$ has a rhombohedral structure in space group R3c and possesses a large $P_s$ along the pseudo-cubic $[111]_r$ direction [11,12]. Moreover, BiFeO$_3$ exhibits an extremely high Curie temperature ($T_C$) of 830°C [11,12], which can provide piezoelectric devices operating at high temperatures. In analogy to Pb(Zr, Ti)O$_3$, considerable efforts have been made to investigate the solid solutions of rhombohedral BiFeO$_3$ and other perovskites in tetragonal symmetry. The BiFeO$_3$–BaTiO$_3$ system [13–17] has been widely studied mainly in ceramic form because the MPB is expected to appear between rhombohedral R3c and tetragonal P4mm. Detailed structural analysis reveals that an increase in the BaTiO$_3$ content causes a structural change from rhombohedral R3c to a pseudo-cubic structure [17], where the phase boundary is...
ambiguous at around 33% BaTiO$_3$ content. Moreover, it has been reported that BiFeO$_3$–BaTiO$_3$ solid solutions do not have a ferroelectric nature in the BaTiO$_3$ content range of 40–50% [18].

Another approach exploiting the interplay of two types of perovskite oxides is to build a superlattice by thin-film growth technology [19,20]. Epitaxially grown superlattices composed of BiFeO$_3$ and BaTiO$_3$ have been reported to show a high magnetoelectric coupling coefficient compared with pristine films or bulk ceramics of BiFeO$_3$, where the interface plays a crucial role [21,22]. At present, ferroelectric and dielectric properties of BiFeO$_3$–BaTiO$_3$ superlattices have been reported in a few reports [23–25], and thereby the fundamental questions remain unanswered concerning how the layers of BiFeO$_3$ and BaTiO$_3$ are structurally and ferroelectrically coupled, and how the coupling of the two layers is activated.

In this paper, we report the crystal structure, polarization, and dielectric properties of superlattice-structured epitaxial thin films composed of BaTiO$_3$ and BiFeO$_3$ on single-crystal SrTiO$_3$ substrates prepared by pulsed laser deposition (PLD) (Figure 1). Here, we adopted Mn(5%)-doped BiFeO$_3$ instead of BiFeO$_3$ to avoid a considerable influence of oxygen vacancies on the polarization and leakage current properties [23,26–28], because a trapping capability of oxygen vacancies by Mn$^{3+}$ at the Fe$^{3+}$ site, i.e., a strong attractive interaction between Mn$^{3+}$ and oxygen vacancy, inhibits the formation of an oxygen vacancy-rich layer at the interfaces. The total number of ABO$_3$ unit cells were fixed at 600, and that of the BaTiO$_3$ and Mn-BiFeO$_3$ layers (N) varied from 300 down to 1 (Figure 1b), while the average composition of the entire superlattices remained unchanged, i.e., 50% Mn-BiFeO$_3$–50% BaTiO$_3$. We found that the samples for an N greater than 25 can be regarded as a simple series connection of their individual capacitors, while those for an N of 5 or less behave as a unified ‘ferroelectric’, where the BaTiO$_3$ and Mn-BiFeO$_3$ layers are structurally and electronically coupled.

![Figure 1](image_url)

**Figure 1.** Schematics of the superlattice-structured thin film composed of layers of Mn(5%)-doped BiFeO$_3$ and BaTiO$_3$. The total thickness of the superlattice is fixed at 600 ABO$_3$ unit cells (a), and the number (N) of ABO$_3$ in the two layers varies from 300 to 1, see (b) for N = 3.

2. Materials and Methods

2.1. Experimental

Thin films of BaTiO$_3$ [29] and Mn(5%)-doped BiFeO$_3$ [28,30], a (Ba$_{0.2}$Sr$_{0.8}$)$_2$TiO$_3$ buffer layer [29], and (Ba$_{0.1}$Sr$_{0.9}$)$_2$RuO$_3$ electrodes [31] were fabricated on (100) SrTiO$_3$ single-crystal substrates (5 × 5 × 1 mm$^3$) by PLD (KrF excimer laser, $\lambda$ = 248 nm) using ceramic targets. The details of the deposition conditions are summarized in Supplementary Tables S1 and S2. Figure 1 displays the schematic of the superlattice composed of Mn-BiFeO$_3$ and BaTiO$_3$. The total number of ABO$_3$ unit cells was fixed at 600 (Figure 1a). The number
(N) of those in each layer forming the superlattice varied: \( N = 300, 50, 25, 10, 5, 3, \) and 1. Figure 1b depicts the structure of \( N = 3 \) as an example, where the superlattice is constructed by an alternate stacking of the thin layers of BaTiO\(_3\) and Mn-BiFeO\(_3\) with three ABO\(_3\) unit cells (\( N = 3 \)). For all the samples, the Mn-BiFeO\(_3\) layer was deposited on the bottom electrode because of its better in-plane lattice matching with it. As a result, the layer just beneath the top electrode was the BaTiO\(_3\) layer. During the deposition, the following condition was adopted: a substrate temperature \( T_{\text{sub}} \) of 640 °C, an oxygen pressure \( P(\text{O}_2) \) of 2.6 Pa, and a laser repetition rate of 1 Hz for BaTiO\(_3\) and 7 Hz for Mn-BiFeO\(_3\). The diameter of the top electrode was 0.1 mm. The polarization electric field (\( P-E \)) hysteresis properties were measured at 25 °C (3 kHz); the direction from the bottom to the top electrode was defined as positive for \( E \) and \( P \).

High-resolution X-ray diffraction (XRD) reciprocal space maps (RSMs) were observed by using a Cu-K\( \alpha_1 \) source. The data of the intensity profile \( I_i(q_x, q_z) \) of reflection \( i \) in the reciprocal space \((q_x, q_z)\) were used for the detailed analysis of the lattice parameters of the in-plane \((a)\) and the out-of-plane \((c)\) directions, where the parameters \( a \) and \( c \) denote those of the pseudo-cubic ABO\(_3\) unit cell. Throughout this paper, we adopted the pseudo-cubic notation unless otherwise stated.

### 2.2. DFT Calculations

Density functional theory (DFT) calculations were conducted using the generalized gradient approximation [32] with a plane wave basis set. We used the projector-augmented wave method [33] as implemented in the Vienna ab initio simulation package (VASP) [34]. We employed the Perdew–Burke–Ernzerhof gradient-corrected exchange correlation functional revised for solids (PBEsol) [35] and a plane wave cut-off energy of 520 eV. A gradient approximation [32] with a plane wave basis set. We used the projector-augmented

For building a superlattice cell, we took the following lattice constraint. Based on the experimental results of XRD for an \( N \) of 5 or less, the superlattice cell had a tetragonal structure with the lattice parameters of in-plane \( a_{\text{DFT}} \) and out-of-plane \( c_{\text{DFT}} \) in space group \( P4mm \); its \( a_{\text{DFT}} \) was fixed at the experimental \( a \) of 0.3985 nm, i.e., \( a_{\text{DFT}} = a \) (experiment). The parameter \( c_{\text{DFT}} \) is given by the following equation, \( c_{\text{DFT}} = Nc_{\text{BiFeO}_3} + Nc_{\text{BaTiO}_3} \), where \( c_{\text{BiFeO}_3} \) denotes the parameter \( c \) of the BiFeO\(_3\) unit cell, and \( c_{\text{BaTiO}_3} \) that of the BaTiO\(_3\) unit cell. The \( c_{\text{BiFeO}_3} \) and \( c_{\text{BaTiO}_3} \) were determined from the lattice volumes \((V)\) derived from the geometrical optimizations of the BaTiO\(_3\) cell \((5 \times 5 \times 5 \ k\)-point) and the BiFeO\(_3\) cell in \( P4mm \) symmetry. Considering the antiparallel spin configuration, we performed the optimization calculation of the BiFeO\(_3\) cell with \( 2c_{\text{BiFeO}_3} \) \((5 \times 5 \times 3 \ k\)-point) and regarded the half cell with \( c_{\text{BiFeO}_3} \) as the BiFeO\(_3\) unit cell. For imposing the antiparallel spin configuration for \( N = 1 \), the long lattice with \( 2c_{\text{super}} \) was taken as the superlattice cell, as depicted in Figure 2a. The structural optimizations were performed under a fixed \( a_{\text{DFT}} \) and \( c_{\text{DFT}} \) with \( 5 \times 5 \times 3 \ k\)-point mesh for all the supercells. From the structural parameters of the optimized cell, we obtained the atomic displacements \((\Delta z)\) from the corresponding positions in the hypothetical non-polar paraelectric lattice. We also calculated the Born effective charges \((Z^*)\) [38] in the superlattice cells by density-functional perturbation theory. We estimated \( P_s \), as expressed by the following equation:

\[
P_s = \sum_i m_i \Delta z_i Z_i^* / V, \tag{1}
\]
where \( m_i \) denotes the site multiplicity of the constituent atom \( i \), and \( \Delta z_i Z_i^* \) is its dipole moment. The summation in Equation (1) is taken over the superlattice cell with the cell volume \( V \).

**Figure 2.** Crystal structures of the superlattice cells with (a) \( N = 1 \), (b) \( N = 2 \), and (c) \( N = 4 \) obtained from the structural optimizations by DFT calculations, where BiFeO\(_3\) is employed instead of Mn-BiFeO\(_3\) for simplicity. The in-plane lattice parameter \( a_{\text{DFT}} \) of the superlattice cell was fixed at the experiment: \( a \) (experiment) = 0.3985 nm. The out-of-plane lattice parameter \( c_{\text{DFT}} \) of the superlattice cell was determined from the cell volume obtained by geometrical optimizations in our preceding calculations of BiFeO\(_3\) and BaTiO\(_3\) in tetragonal \( P4/mnm \) symmetry.

### 3. Results

#### 3.1. Crystal Structure

Supplementary Figure S1 shows the \( \theta-2\theta \) XRD patterns around the 002 reflection. In addition to the peaks of the SrTiO\(_3\) substrate at 46.5°, the (Ba\(_{0.7}\)Sr\(_{0.3}\))RuO\(_3\) electrodes at 46.4°, and the (Ba\(_{0.2}\)Sr\(_{0.3}\))TiO\(_3\) buffer at 44.7°, the sample with \( N = 300 \) exhibits peaks individual to the layers of BaTiO\(_3\) and Mn-BiFeO\(_3\) because their layers are sufficiently thick for providing their corresponding reflections. With decreasing \( N \), the integrated intensities of these peaks are weakened and eventually vanish for an \( N \) less than 5.

Supplementary Figure S2 shows the wide-area XRD-RSMs for \( N = 300 \) and 5. For \( N = 300 \) (Figure S2b), the apparent reflections of 3/2 3/2 1/2 and 1/2 1/2 3/2 of Mn-BiFeO\(_3\) in monoclinic symmetry appear, whereas those were not observed for an \( N \) of 5 (Figure S2d) or less. Figure 3 shows the integrated intensity of the 1/2 1/2 3/2 reflection as a function of \( N \). With decreasing \( N \), the intensity is weakened and then zero for \( N = 1-5 \). These results indicate that the monoclinic distortion, similar to the bulk (rhombohedral), is maintained in the Mn-BiFeO\(_3\) layer for the superlattice with \( N \geq 10 \), while that is lost with \( N \leq 5 \). The details of the structural analysis are described in Supplementary Note 2.

Figure 4 shows the high-resolution XRD-RSMs around the 103 reflections. For all the samples, the peak positions \((q_x, q_z)\) exhibit the following features: the (Ba\(_{0.7}\)Sr\(_{0.3}\))TiO\(_3\) buffer and the (Ba\(_{0.1}\)Sr\(_{0.9}\))RuO\(_3\) electrodes have an apparently small \( q_x \) compared with the SrTiO\(_3\) substrate, demonstrating that the parameter \( a \) of the (Ba\(_{0.7}\)Sr\(_{0.3}\))TiO\(_3\) buffer is sufficiently expanded to the bulk value, and also that the (Ba\(_{0.1}\)Sr\(_{0.9}\))RuO\(_3\) bottom electrode...
is coherently grown on the buffer. The detailed structural analysis for \(N = 300\) (Figure 4a along with the 113 reflection; see Supplementary Note 2) indicates that the Mn-BiFeO\(_3\) layer has a rhombohedral-like monoclinic \(M_A\) structure. The splitting into two peaks of the 103 reflection of the Mn-BiFeO\(_3\) layer stems from the ferroelastic domain variants. With further decreasing \(N\), the splitting of the Mn-BiFeO\(_3\) layer is smaller, and then the reflection can be regarded as a single peak for \(N = 25\) and 10. At the same time, the \(q_z\) of the Mn-BFO layer with \(N = 50, 25,\) and 10 becomes larger than that of \(N = 300\), suggesting a structural change from the \(M_A\) to monoclinic \(M_B\) phases owing to an in-plane tensile strain (see Supplementary Note 2). The experimental results, i.e., the single peak of the 103 reflection, the \(q_z\) shift, and the apparent 1/2 1/2 3/2 reflection (Figure 3), indicate that the Mn-BiFeO\(_3\) layer for \(N = 25\) and 10 has a pseudo-tetragonal structure, with a small monoclinic (\(M_B\)) distortion [39]. We note that for an \(N\) less than 5, the reflections from the Mn-BiFeO\(_3\) and BaTiO\(_3\) layers cannot be distinguished. These results enable us to consider that the superlattice has a unified tetragonal cell with a \(c/a\) of 1.01–1.02 as an average structure.

![Figure 3](image-url)

**Figure 3.** Integrated intensity of 1/2 1/2 3/2 reflection in the XRD-RSMs as a function of \(N\), where \(N\) denotes the number of ABO\(_3\) unit cells in the two layers of Mn-BiFeO\(_3\) and BaTiO\(_3\) comprising the superlattice. We confirmed that the integrated intensity of 1/2 1/2 3/2 reflection of the pristine Mn-BiFeO\(_3\) film (300 unit cell thickness) is almost the same as that for \(N = 300\).

### 3.2. Polarization and Dielectric Properties

Figure 5 shows the \(P-E\) loops (\(E/|[001]|\) at 3 kHz), and Figure 6a,b display the resultant remanent polarization (\(P_r\)) and the maximum polarization (\(P_{max}\)) at the highest positive \(E\) as a function of \(N\), respectively. It is interesting to note that the superlattice samples exhibit an apparent ferroelectric polarization with an apparent \(P_r\), which is completely different from the solid solutions in the same composition (50\% BaTiO\(_3\) content) featuring a non-ferroelectric nature [18]. The \(N = 300\) sample has a \(P_r\) of 22 \(\mu\)C cm\(^{-2}\). The \(P-E\) loop exhibits an imprint, i.e., a shift in the negative \(E\) direction. This behavior is assumed to stem from a flexoelectric effect [29,40,41], where a strain gradient in the out-of-plane direction in the ferroelectric layer stabilizes the upward polarization compared with the downward one. Compared with the buffered electrode with \(a = 0.3986\) nm, the BaTiO\(_3\) layer has the same \(a\), whereas the Mn-BiFeO\(_3\) layer possesses a slightly small \(a = 0.3965\) nm. This result indicates that a strain gradient driving the flexoelectric effect is present in the Mn-BiFeO\(_3\) layer adjacent to the boundary with the bottom electrode.
Figure 4. High-resolution XRD-RSMs around 103 reflection for (a) $N = 300$, (b) $N = 50$, (c) $N = 25$, (d) $N = 10$, (e) $N = 5$, (f) $N = 3$ and (g) $N = 1$, where the vertical axis is $q_z/\lbrack001\rbrack$ and the horizontal axis is $q_x/\lbrack100\rbrack$, where $N$ denotes the number of ABO$_3$ unit cells in the two layers of Mn-BiFeO$_3$ and BaTiO$_3$ comprising the superlattice. Here, [001] and [100] are the crystallographic directions of the (100) SrTiO$_3$ substrate.

From the data shown in Figure 6a–c, we think that the polarization and dielectric behavior can be divided into three regions: I. the simple series connection of the capacitors ($N \geq 25$, see Figure 7a), II. the transition region ($10 \leq N < 25$), and III. the unified ferroelectric regime ($N < 10$, see Figure 7b). In region I, with decreasing $N$, the hysteresis is slanted, and the resultant $P_r$ and $P_{\text{max}}$ are monotonically reduced (Figure 6a,b). We note that the relative dielectric permittivity ($\varepsilon_r$) remains constant at $\sim 120$. This constant $\varepsilon_r$ can be understood in terms of a simple series connection of the capacitors of the Mn-BiFeO$_3$ and the BaTiO$_3$ layers. Considering an $\varepsilon_r$ of 399 for the Mn-BiFeO$_3$ capacitor, and that of 93 for the BaTiO$_3$ one (those were measured individually for their respective capacitors), we obtain $\varepsilon_r \sim 150 \times \frac{\varepsilon_r(\text{Mn-BiFeO}_3) \cdot \varepsilon_r(\text{BaTiO}_3)}{\varepsilon_r(\text{Mn-BiFeO}_3) + \varepsilon_r(\text{BaTiO}_3)}$. This is qualitatively in good agreement with the experiment ($\varepsilon_r \sim 120$). In region III, with decreasing $N$, the $P_r$ is reduced, while the $\varepsilon_r$ is higher.
the number of ABO₃ unit cells in the two layers of Mn-BiFeO₃ and BaTiO₃ comprising the superlattice. The resultant maximum at the maximum electric field (E_{max}) of 3 kHz is applied along [001], where N denotes the number of ABO₃ unit cells in the two layers of Mn-BiFeO₃ and BaTiO₃ comprising the superlattice.

Figure 5. Polarization (P) electric field (E) hysteresis loops at 25 °C, for (a) N = 300, (b) N = 50, (c) N = 10, (d) N = 5, (e) N = 3 and (f) N = 1, where an E of 3 kHz is applied along [001], where N denotes the number of ABO₃ unit cells in the two layers of Mn-BiFeO₃ and BaTiO₃ comprising the superlattice.

Figure 6. (a) Remanent polarization (P_r), (b) relative dielectric permittivity (ε_r), (c) polarization maximum at the maximum E (P_{max}), and (d) spontaneous polarization (P_s) from DFT calculations of the unified unit cells in Figure 2.
BaTiO3 layers. Considering an εr of 399 for the Mn-BiFeO3 ... explained by the series connection of the capacitors of BaTiO3 and Mn-BiFeO3. It is reasonable to consider that the

Figure 7 shows the schematics of the superlattice structures along with the Ps component along the out-of-plane direction (Ps//[001]c). In region I (N ≥ 25), the presence of the 1/2 1/2 3/2 reflection from the Mn-BiFeO3 layer (Figure 3) and the polarization and dielectric properties (Figure 6) indicate that the superlattice can be regarded as the simple series connection of the capacitors of BaTiO3 and Mn-BiFeO3. In the BaTiO3 layer, the Ps vector is present along [001]c; our DFT calculations reveal that the Ps strength is 28.5 μC cm⁻², which is close to the bulk value [42]. In contrast, the Mn-BiFeO3 layer has a Ps nearly along [111]c, and the value is reported to be 90–100 μC cm⁻² [37]. As the polarization components along [001]c in these layers are markedly different, the interface effect plays an important role. It is assumed that the interface region of several to several tens of unit cells in width needs to accommodate the difference in the direction and strength of the Ps vector across it, as in ferroelastic domain walls [43–50]. As a result, a depolarization field (Edep) is built up in the interface region, where the Edep is present in a direction that prevents the change in the polarization component. Given that the Ps vectors are switched by an E application, the Ps is expected to be ~40 μC cm⁻². The Ps of 25 μC cm⁻² for N = 300 is smaller than this expected value, which is caused by a domain clamping by the Edep. In region I, the Ps is reduced when the N is smaller, which is because the volume fraction of the clamped domains is raised by a denser interface with the Edep.

In region III, the 1/2 1/2 3/2 reflection of the Mn-BiFeO3 layer is absent (Figure 3), and the polarization and dielectric properties (Figure 6) cannot be explained by the series connection of the capacitors of BaTiO3 and Mn-BiFeO3. It is reasonable to consider that the superlattice has a unified unit cell, where electronic orbitals of the BaTiO3 and the Mn-BiFeO3 layers are hybridized. In other words, these two layers are no longer distinguished, but the structural and electronic features are completely different from the solid solutions [18]. On the assumption that the superlattice has a unified unit cell (Figure 2), our DFT calculations show that the N = 1 cell has a Ps of 27.3 μC cm⁻², which is close to the

![Figure 7. Schematics of the crystal structures of the supercells with (a) N ≥ 25, and (b) N ≤ 5, along with the [001] component of P.s. In (a), the thickness of the layers of Mn-BiFeO3 and BaTiO3 is sufficiently thin, and thereby their polarization features are maintained inside them. In (b), the structural and electronic coupling of Mn-BiFeO3 and BaTiO3 is activated, and the two layers can no longer be distinguished, leading to a unified ferroelectric unit cell.](image-url)

4. Discussion

Figure 7 shows the schematics of the superlattice structures along with the Ps component along the out-of-plane direction (Ps//[001]c). In region I (N ≥ 25), the presence of the 1/2 1/2 3/2 reflection from the Mn-BiFeO3 layer (Figure 3) and the polarization and dielectric properties (Figure 6) indicate that the superlattice can be regarded as the simple series connection of the capacitors of BaTiO3 and Mn-BiFeO3. In the BaTiO3 layer, the Ps vector is present along [001]c; our DFT calculations reveal that the Ps strength is 28.5 μC cm⁻², which is close to the bulk value [42]. In contrast, the Mn-BiFeO3 layer has a Ps nearly along [111]c, and the value is reported to be 90–100 μC cm⁻² [37]. As the polarization components along [001]c in these layers are markedly different, the interface effect plays an important role. It is assumed that the interface region of several to several tens of unit cells in width needs to accommodate the difference in the direction and strength of the Ps vector across it, as in ferroelastic domain walls [43–50]. As a result, a depolarization field (Edep) is built up in the interface region, where the Edep is present in a direction that prevents the change in the polarization component. Given that the Ps vectors are switched by an E application, the Ps is expected to be ~40 μC cm⁻². The Ps of 25 μC cm⁻² for N = 300 is smaller than this expected value, which is caused by a domain clamping by the Edep. In region I, the Ps is reduced when the N is smaller, which is because the volume fraction of the clamped domains is raised by a denser interface with the Edep.

In region III, the 1/2 1/2 3/2 reflection of the Mn-BiFeO3 layer is absent (Figure 3), and the polarization and dielectric properties (Figure 6) cannot be explained by the series connection of the capacitors of BaTiO3 and Mn-BiFeO3. It is reasonable to consider that the superlattice has a unified unit cell, where electronic orbitals of the BaTiO3 and the Mn-BiFeO3 layers are hybridized. In other words, these two layers are no longer distinguished, but the structural and electronic features are completely different from the solid solutions [18]. On the assumption that the superlattice has a unified unit cell (Figure 2), our DFT calculations show that the N = 1 cell has a Ps of 27.3 μC cm⁻², which is close to the
experimental $P_t$ (21.6 $\mu$C cm$^{-2}$) of $N = 1$. Moreover, the enhancement in $P_t$ with increasing $N$ (Figure 6a) can be qualitatively explained by the theoretical calculations (Figure 6d): $P_s$ is 31.4 $\mu$C cm$^{-2}$ for the $N = 2$ cell, and 43.7 $\mu$C cm$^{-2}$ for the $N = 4$ cell.

Finally, we comment on an additional degree of freedom in superlattice design by adopting an unequal $N$ in the BaTiO$_3$ and the Mn-BiFeO$_3$ layers, where material properties can be tuned by different $N$(BaTiO$_3$) and $N$(Mn-BiFeO$_3$). For example, we can expect that $N$(BaTiO$_3$) < $N$(Mn-BiFeO$_3$) delivers an enhanced $P_t$ in a unified cell in the superlattice. Moreover, superlattice design based on different unit cell numbers is anticipated to provide a means to control the strain effect at will.

5. Conclusions

We investigated the crystal structure and dielectric and polarization properties of superlattice-structured epitaxial thin films composed of Mn(5%)-doped BiFeO$_3$ and BaTiO$_3$ with a total thickness of 600 perovskite (ABO$_3$) unit cells. The number of ABO$_3$ unit cell ($N$) in the layers of Mn-BiFeO$_3$ and BaTiO$_3$ varied from 300 down to 1. It was revealed that the superlattices for an $N$ greater than 25 can be regarded as a simple series connection of their individual capacitors. In the thin regime of an $N$ of five or less, the superlattice behaves as a unified ferroelectric, where the BaTiO$_3$ and Mn-BiFeO$_3$ layers are structurally and electronically coupled. With decreasing $N$ from five to one, the $cr$ is markedly enhanced, whereas the $P_t$ is reduced. DFT calculations show that the $P_s$ is suppressed with decreasing $N$, which is in good agreement with the experimental $P_t$. We conclude that superlattices formed by two types of perovskite layers with different crystal symmetries represent a path to novel ferroelectrics that cannot be obtained in a solid solution system.

Supplementary Materials: The following are available online at https://www.mdpi.com/article/10.3390/nano11071857/s1, Figure S1: 0-20 XRD patters around 002 reflection, Figure S2: Wide-area XRD-RSMs, Figure S3: Lattice parameters estimated from the peak positions of 103 reflection in the high-resolution XRD-RSMs as a function of $N$, Figure S4: (a) High-resolution XRD-RSM around 113 reflection for $N = 300$ where the vertical axis is $q_x$//[001] and the horizontal axis is $q_y$//[110]. Schematics of (b) ferroelastic domain structure, (c) the crystal structure of the monoclinic $M_A$ phase, and reciprocal lattice vectors of two domains comprising the ferroelastic domain structure projected onto (d) [100]; vs [001]; plane and (e) [110]; vs [001]; plane, Figure S5: Relationships between the rhombohedral-like monoclinic Mn-BiFeO$_3$ layer and the tetragonal BaTiO$_3$ layer, Table S1: Deposition conditions of substrate temperature ($T_{sub}$), oxygen partial pressure ($P_O$), laser repetition frequency, and laser fluence, Table S2: Number of laser shots to deposit one layers of Mn-BiFeO$_3$ and BaTiO$_3$ comprising the superlattice samples. Supplementary Note 1: Deposition conditions of PLD, Supplementary Note 2: Crystal structural analyses for the Mn (5%)-BiFeO$_3$ layer, Supplementary Note 3: Crystallographic relation between the monoclinic Mn-BiFeO$_3$ layer and the tetragonal BaTiO$_3$ layer, Supplementary Note 4: Superlattice structures and their ambiguity.

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