Assisted Tachyonic Inflation

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Abstract

The model of inflation with a single tachyon field generates larger anisotropy and has difficulties in describing the formation of the Universe \textsuperscript{1}. In this paper we consider a model with multi tachyon fields and study the assisted inflationary solution. Our results show that this model satisfies the observations.

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Recently Sen\cite{3} has constructed a classical time-dependent solution which describes
the decay process of an unstable D-brane in the open string theory. During the decay process
the tachyonic field on the brane rolls down toward the minimum of the potential. There
have been recently a lot of studies on various cosmological effects of the rolling tachyon \cite{3}. In general the unconventional form of the tachyonic action makes the cosmology with
tachyonic field differ from that with a normal scalar field. Regarding the role of tachyon as
a inflaton\cite{4,5}, recently Kofman and Linde \cite{1} pointed out the difficulties in generating an
acceptable metric perturbation. In this paper, we propose a model of inflation with multi-
tachyon, and study the dynamics of this type of assisted inflation. Our results show that
some of the difficulties with a single tachyonic field are overcome and our model satisfies
the observations.

We start with a brief review on the problems of the single tachyonic inflation model. The 4D effective field theory of the tachyon field coupled to Einstein gravity is given by

\[ S = \frac{1}{2\kappa^2} \int d^4 x \sqrt{-g} R + S_{brane}, \]  

where \( \kappa^2 = \frac{1}{M_p^2} \) is the 4D Planck mass, \( S_{brane} \) the 4D effective field theory action of the
tachyon field. To a non-BPS D3-brane action in supersymmetric theory (BSFT)\cite{6}, one has

\[ S_{brane} = \tau_3 \int d^4 x \left( \alpha' \ln 2 e^{-\frac{x^2}{4}} \partial_\mu T \partial^\mu T + e^{-\frac{x^2}{4}} \right), \]  

where the tension of the non-BPS brane is

\[ \tau_3 = \frac{\sqrt{2} M_s^4}{(2\pi)^3 g_s}, \]  

with \( g_s \) being the string coupling, and \( M_s = l_s^{-1} = \frac{1}{\sqrt{\alpha'}} \) the fundamental string mass and
length scales. The Planck mass in this theory is given by a dimensional reduction \cite{7}

\[ M_p^2 = \frac{v M_s^2}{g_s^2}. \]  

In Eq.4, \( v = (M_s r)^d \), \( r \) is the radius of the compactification, and \( d \) is the number of the
compactified dimensions.

Redefining the tachyon field \( T \rightarrow \sqrt{2 \ln 2} T \), the effective action of the tachyon field in
the Born-Infeld form can be written as \cite{2,3,5}

\[ S_{brane} = \int d^4 x \sqrt{-g} V(T) \sqrt{1 + \alpha' \partial_\mu T \partial^\mu T}, \]  

For a scalar field \( \phi \) with standard kinetic term, the authors of \cite{3} have given a constraint on the potential
\( U \) of the scalar field: \( 2H^2 < \kappa^2 U < 3H^2 \). With this potential satisfying the constraint, the cosmological
inflation can occur. It is interesting to see whether there is a similar constraint on the potential \( V \) of the
tachyon in the action \cite{3}. A similar calculation yields a constraint: \( 0 < \kappa^2 V < 3\sqrt{3} H^2 \).
which is a close expression incorporating all higher order of powers of $\partial_\mu T$. The potential $V(T)$ around the maximum is

$$V(T) \simeq \tau_3 e^{-\frac{T^2}{8\ln 2}},$$

(6)

which has a maximum at $T = 0$ and at large $T$ in terms of Ref. [4], the potential should be exponential

$$V(T) \simeq e^{-T}.$$  

(7)

The potential $V(T)$ in the Born-Infeld type of action can be regarded as a smooth function which interpolates between two asymptotic expressions given by (6) at maximum and by (7) at infinity.

Assuming that inflation occurs near the top of the tachyon potential, for a tachyonic field which is spatially homogeneous but time-dependent the slow-rolling conditions are given by [5]

$$\epsilon = \frac{M^2_p}{2\alpha' V(T)} \left(\frac{V'(T)}{V(T)}\right)^2 < 1;$$

(8)

$$\eta = \frac{M^2_p}{\alpha' V(T)} \frac{V''}{V(T)} < 1.$$  

(9)

In the slow-rolling approximation, the Hubble expansion rate can be expressed as

$$H^2 \simeq \frac{k^2}{3} V(T),$$

(10)

and the equation of motion for the rolling tachyon in an expanding universe is

$$3H\dot{T} + (\alpha' V(T))^{-1} V' \simeq 0.$$  

(11)

Substituting (3) and (10) into (13), we have

$$g_s > \frac{(2\pi)^3 v}{4\sqrt{2 \ln 2}} \sim 10^2 v.$$  

(12)

The amplitude of the gravitational waves produced during inflation is [10]

$$P_G \sim \frac{H}{M_p} \leq 3.6 \times 10^{-5}. $$

(13)

Substituting (4) and (10) into (13), we have

$$g_s^3 \leq 10^{-7} v^2.$$  

(14)

Combining (12) and (14), one can see that for the inflation driven by the rolling of the tachyon to happen, the following condition

$$v < 10^{-13}$$  

(15)
must be satisfied. Since the 4D effective theory is applicable only if $v \gg 1$ [7], Eq.(15) implies that the single tachyonic inflation can not result in a reasonable observed universe.

To solve this problem, we introduce more tachyon fields into the model and consider an assisted inflation scenario with multi-tachyonic fields. Specifically we consider a system with $n$ non-coincident but parallel non-BPS D3-branes [11]. In this system, there are two kinds of open strings. One of them starts from and ends on the same brane; The other starts from a given brane, then ends on a different brane. If the distance between two branes are much larger than the string scale, one can ignore the second kind of open strings, leaving a tachyon on the world volume for every brane. Thus, we have $n$ tachyons without interaction. And the action is simply the sum of $n$ single-tachyonic actions:

$$S_{brane} = \sum_{i=1}^{n} \int d^4x \sqrt{-g} V(T_i) \sqrt{1 + \alpha' \partial_{\mu} T_i \partial^{\mu} T_i}. \quad (16)$$

With the multi-tachyonic fields the slow-rolling conditions are found to be

$$\epsilon = \frac{M_p^2}{2 \alpha' V_{sum}(T)} \left( \frac{V'(T_i)}{V(T_i)} \right)^2 < 1; \quad (17)$$

$$\eta = \frac{M_p^2}{\alpha' V(T_i)} \frac{V''(T_i)}{V_{sum}(T)} < 1, \quad (18)$$

where $V_{sum}(T) = \sum_{i=1}^{n} V(T_i)$. The Friedman equation is now

$$H^2 \simeq \frac{\kappa^2}{3} V_{sum}(T). \quad (19)$$

And the equation of motion for one of the tachyons is

$$3H \dot{T}_i + (\alpha' V(T_i))^{-1} V'(T_i) \simeq 0. \quad (20)$$

Similar to the discussions on the constraints on the model parameters above, for the assisted inflation to be successful we have

$$ng_s^3 \leq 10^{-7} v^2; \quad ng_s > 10^2 v. \quad (21)$$

Thus we get,

$$v < 10^{-13} n^2. \quad (22)$$

One can see that when $n \geq 10^7$, the condition, $v \gg 1$, required by the applicability of the 4D effective field theory is satisfied.

In the following, we will calculate the amplitude of the density perturbation of our model. For potential [B] and the equations of motion of the tachyon fields [20], we notice that the equation of motion for each of the tachyon fields follow a simple relationship

$$\frac{d \ln T_i}{dt} \simeq \frac{d \ln T_1}{dt}. \quad (23)$$
This means that if the slow-rolling conditions are satisfied, all the tachyons would follow a similar trajectory with a unique late attractor, i.e. \( T_1 \sim T_2 \ldots \sim T_n \equiv T \). The calculation of the density perturbation responsible for the anisotropy of CMB depends crucially on this late-time attractor of the fields. In this case, the Friedman equation (19) can be rewritten as

\[
H^2 \simeq \frac{k^2}{3} nV(T),
\]

where \( V(T) \) at maximum is given by (6). The number of e-folds during inflation is

\[
N = \int H dt \simeq -\int_{T_{60}}^{T_{\text{end}}} \frac{H^2 V(T)}{V'(T)} dT,
\]

where \( T_{60} \) is the field value corresponding to \( N \simeq 60 \) as required when the COBE scale exits the Hubble radius, and \( T_{\text{end}} \) is the value of the tachyon field at which inflation ends. And \( T_{\text{end}} \) is determined by

\[
\eta \simeq \frac{10^2 v}{ng_s} e^{\frac{2}{\sqrt{10}n}} \simeq 1.
\]

Thus, we have

\[
T_{\text{end}}^2 \simeq 8 \ln 2 \ln \frac{ng_s}{10^2 v}.
\]

Following the definition of the amplitudes of the density perturbation in Refs. [10] and [12], we have

\[
\mathcal{P}_S \sim \frac{1}{\sqrt{\pi nV(T)2\pi T}} H^2,
\]

and

\[
\mathcal{P}_S \sim \frac{ng_s^2}{v^2 T_{60}}.
\]

Taking \( n \sim 10^{11}, v \sim 10 \) and \( g_s \sim 10^{-8} \), from (23) and (27), we obtain that \( 10T_{60} \sim T_{\text{end}} \sim O(1) \). Substituting the values of these parameters into (29), we have the amplitudes of the density perturbation, \( \mathcal{P}_S \sim 10^{-6} \). This is consistent with the observation of COBE. In this case, from (4) and (3), the string mass scale \( M_s \sim 10^{-8} M_p \) and the total tension \( n\tau_3 \sim 10^{-16} M_p^4 \).

In summary we have proposed a multi-tachyonic inflation model. Our results shown that with the help of a large number of tachyons the problems with a single tachyonic field to satisfy the slow-rolling condition and to generate the amplitude of the gravitational waves constrained by the CMB anisotropy can be solved. Furthermore the energy scale associated with the inflation in our model is also much smaller than the Planck scale. We should point out, however that our model does not solve all of the problems raised in [1]. In fact \( M_s \leq H \) required in the single as well as multi tachyonic inflation model implies that the size of the de-Sitter horizon will be smaller than the string length i.e. \( \frac{1}{H} \leq l_s \), which makes it invalid.
to describe the tachyon condensation by using an effective field theory \[1\]. In the general inflation models, the inflaton mass is of the same order as or less than \(H\). However in the tachyonic inflation model the inflaton has a mass around the string scale \(M_s = l_s^{-1}\). There might be some possible ways to overcome this difficulty. For instance, one may extend the model studied in this paper by including the effects of tachyons resulting from the open string stretching between different branes. There could exist the possibility in such kind of multi-brane system that the inflaton (tachyon) mass scale be fine-tuned so that it is in the range required and smaller than the string scale \(M_s\). To realize this speculation with a model may not be easy and goes beyond the scope of this paper. Finally, we would also like to mention here that the usual reheating mechanism is problematic in the tachyon inflation model, as pointed out by Kofman and Linde \[1\], because the tachyon does not oscillate during the decay of non-BPS branes. This issue has been discussed recently in \[13\].

This work implies that the inflation and cosmological applications of the multi-tachyon/multi-brane may have more fruitful phenomena, which is worth studying further.

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