Continental drift of the rotation pole: observation and theory

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Abstract. The Earth rotation pole moves with respect to the crust. Along with a nearly seasonal wobble of about 0.2'' and other sub-seasonal oscillations of smaller amplitude, it drifts at longer periods. In light of astro-geodetic observations going back to 1890, this drift is composed of a linear time varying term and of a multi-decadal wobble. Linear trend is interpreted as the visco-elastic response of the Earth to the last deglaciation, however tectonic processes with mantle downwelling or upwelling could also account for a part of it. In this respect, it provides constrains on law mantle viscosity. According to last studies, decadal change of the mean pole path mostly reflects climatic changes through variation in the continental ice coverage.

1. Introduction
Owing to the flattening of the Earth of and its diurnal rotation, the rotation pole should freely wobble with respect to the Earth at a period of 304 days. This prediction was made by Euler in the middle of eighteen century by widening Newtonian mechanics to an extended rigid body. The motion of the rotation pole was searched for as latitude variations. Indeed, if rotation axis moves with respect to continents, the astronomical colatitude, namely the angle between the rotation axis and the local vertical, should change accordingly. This phenomena was finally discovered at the end of the nineteen century, but not with the expected period of 303 days. American astronomer Chandler put forward a cycle of about 0.3'' or 300 milli-arc-seconds (mas) in 430 days, accompanied by an annual oscillation of smaller amplitude. The alternative destructive and constructive interference between these two modes produces an amplitude modulation of 6.4 years, appearing in time series of the pole coordinates $x_p$ and $y_p$ displayed in Figure 1. Pole coordinates $x_p$ and $y_p$ are the cosine director of the rotation axis with respect to the axes $Gx$ and $ Gy$ of the terrestrial frame $Gxyz$ respectively. The terrestrial frame is a crust tied Cartesian system, which the center is the center of mass of the Earth, and in which the crust presents only deformations. The $Gz$ axis defines the geographic axis, $Gx$ and $Gy$ axes define the geographic equator with $Gx$ lying in the prime meridian (the Greenwich meridian), and $Gy$ at 90° East. At this time, astrometric pointing had a precision of 0.03'. According to the contemporaneous interpretation, worked out by Newcomb (1992) and other prominent scientists of this epoch like Zhukovsky1, the Chandler wobble is the free wobble, which the period is lengthened by the non-rigid feed-back of the Earth to the centrifugal variation generated by the wobble itself. Especially it reflects the quasi-elasticity of the mantle and the hydro-static deformation of the ocean at the level of 1 cm. In reason of the interest raised by this phenomenon, its implication for Earth

1During conference, Pr. VyacheslavIvashkin let us aware of the Zhukovsky contribution [6] and offered us a copy of it in Keldish Institute
rheology, and possibly internal structure, observations of the astronomical latitude started to be coordinated in 1900 through a set of 5 observatories, all located at about the same latitude (39° 8′ N). The International Latitude Service (ILS) was born. Meanwhile the rotation pole lost its status of fixed point with respect to the continents, and to which the latitudes were reckoned. In this respect astronomers and geodesists defined the geographic pole as the mean pole of the epoch 1900-1906, free of its wobble. In practice, the mean pole was obtained by estimating the mean latitudes of ILS stations over a time span of 5-6 years. Yet, knowledge of this epoch was restricted to annual and 430 day wobbles, that had been related to simultaneous mass redistribution, like the one taking place in the oceans and the atmosphere at seasonal scale. But the polar drift was still unknown.

2. Discovery of rotation pole drift

The drift of the mean pole was discovered in 1922 by American astronomer Walter D. Lambert (1879-1968) [2]. It was confirmed in the 1960’s by Markowitz, E. Fedorov and Proverbio, who had longer time series at hand. So, the rotation axis slowly drifts with respect to the crust at an angular rate of 4 mas/year (equivalently the rotation pole moves by about 12 cm/year) towards Greenland. Actually, over some decades this drift is not uniform: the rate varies between 3 and 4 mas/year and longitude lies between 70° and 80° West (see Figure 2). Before the advent of space geodetic technique in the 1960’s, this variability cannot be attributed to physical factors only, but it partly stems from the reduction of optical astrometric observation. Indeed, the latitude variation are relying on local verticals, which the direction can vary in the terrestrial frame because of miscellaneous mass transports within the Earth or at its surface in an unknown way. So, latitude variations embody the uncertainty on the direction of vertical, and this uncertainty propagate into the motion of the rotation axis in the terrestrial frame.

Until the 1960’s the precision of optical determination did not increase much (30 mas). The outbreak of space geodetic techniques was accompanied by a drop of the EOP uncertainty by a factor 10 over the period 1965-1985. From 1965 the Doppler satellite tracking permitted to divide the uncertainty by a factor 2, and yielded the pole coordinates with a precision of about 50 cm (15 mas). However this technique was surpassed from 1975 by Satellite Laser Ranging (SLR), and from 1985 the Very Long Baseline radio Interferometry (VLBI) on extragalactic radio sources gave the pole coordinates with an uncertainty of about 2 mas. Since the middle of the 1980’s, the optical astrometry is almost no longer used. As latitude determination fade in polar motion observations, ILS was replaced by International Polar Motion Service (IPMS) in 1967. In 1987 IPMS and Bureau International de l’Heure (BIH) were merged into the "International Earth rotation and Reference System Service" (IERS). From 1993 the Global Navigation Satellite Systems had become gradually the best technique for determining the pole coordinates, which the present accuracy is about 0.05 mas.

As the external torque that the Moon and the Sun exert on the equatorial bulge do not explain anyhow the polar drift, its cause lies in mass redistributions. A better understanding of the problem requires the lineaments of the polar motion theory, starting from a rigid Earth.

3. Dynamical Euler Equation for a rigid Earth

As Euler did, let us consider a rigid Earth and study the rotation of the principal inertia axes frame $Gxyz$, in which the inertia matrix reads

$$
\bar{I} = \begin{bmatrix}
A & 0 & 0 \\
0 & B & 0 \\
0 & 0 & C
\end{bmatrix},
$$

(1)
Figure 1. Time series of the pole coordinates $x = x_p$ and $y = y_p$ from 1890 to nowadays. Source: http://iers.obspm.fr/eop-pc

Figure 2. Polar motion over 2015-2017 superimposed to the filtered polar motion from 1900 (cut-off period of 10 years).

where $C$ is the largest moment of inertia and $B = A$ (biaxial or axi-symmetric Earth). The relative difference between axial and equatorial moment of inertia define the dynamical ellipticity $e = (C - A) A^{-1} = 1/304.5$.

In this geocentric frame, rotating with respect to the celestial system with the instantaneous rotation vector $\overset{\rightharpoonup}{\omega}$, the angular momentum balance reads
\[
\frac{dH}{dt} + \omega \wedge H = L,
\]

where \(H, \omega, L\) are column matrices of the components of the corresponding vectors in \(Gxyz\). As \(H = I \omega\), we obtain

\[
\begin{align*}
A\dot{\omega}_x - (C - A)\omega_y \omega_z &= L_x \\
A\dot{\omega}_y + (C - A)\omega_x \omega_z &= L_y \\
C \omega_z &= L_z
\end{align*}
\]

The free solution reads

\[
\begin{align*}
\omega_z &= \text{Constant} = \Omega \\
\omega_x + i\omega_y &= \omega_0 e^{i\sigma_\varepsilon t}
\end{align*}
\]

where \(\Omega = 2\pi \text{ rad/sidereal day}\) is the constant diurnal rotation, and \(\sigma_\varepsilon = \epsilon \Omega \sim \Omega/304.5\) is the so-called Euler angular frequency. The rotation axis is described by the complex equatorial coordinate \(\omega_x + i\omega_y\). It presents a free wobble at the period of 304.5 sidereal day, or 304 mean solar days. Whereas this simple derivation can account for the existence of a free polar motion, it does not account for the period of the main cycle in 430 days. A better theory has to include the non-rigid Earth response to polar motion, as show now.

4. Linear theory of the rotation of a non rigid Earth

Let us recall the lineaments of the polar motion theory for a real Earth, as exposèd in [3]. As we do not know the terrestrial mass distribution perfectly, we do not have access to the exact orientation of the main principal axes frame with respect to the crust. Moreover this mass distributions is not constant. In practice the terrestrial frame \(Gxyz\) is attached to the ground and is dissociated from the main principal axes frame. We only know that in \(Gxyz\), the inertia moment matrix almost is almost diagonal, reading

\[
\bar{I}(t) = \begin{bmatrix}
A + c_{11} & c_{12} & c_{13} \\
c_{21} & A + c_{22} & c_{23} \\
c_{31} & c_{32} & C + c_{33}
\end{bmatrix},
\]

where the \(c_{ij}\) are variable quantities smaller than \(10^{-7}A\) at time scale smaller than 100 years. The winds in the atmosphere, current in oceans create a relative angular momentum that will be noted \(\bar{h}\), and the total angular momentum of the Earth is expressed by \(\bar{H} = \bar{I}\omega + \bar{h}\). The instantaneous rotation vector \(\omega\) is now written

\[
\omega = \Omega(m_1, m_2, 1 + m_3),
\]

where it is easy to show that \(m_1 = x_p\) and \(m_2 = y_p\). At sub-secular scale, observations show that \(m_1, m_2 \leq 10^{-6}\) and \(m_3 \leq 10^{-8}\). Accounting for non-rigid effect, the basic equation (2) becomes the Liouville equation

\[
\frac{d(\bar{I}(t)\omega + \bar{h}(t))}{dt} + \omega \wedge (\bar{I}(t)\omega + \bar{h}(t)) = L.
\]
\[ m + \frac{i}{\sigma_e} \dot{m} = \chi - \frac{i}{\Omega} \dot{\chi} + i \frac{L}{\Omega^2 (C - A)} \]  

(8)

where

\[ m = m_1 + i m_2 \]

\[ \chi = \frac{c_{13} + ic_{23}}{C - A} = \frac{h_1 + ih_2}{L} \]

\[ C - A = h_1 + ih_2 \]

(9)

Now, we have to notice that the complex off-diagonal moment of inertia \( c = c_{12} + ic_{23} \) in RHS of (8) will depend on polar motion \( m \) if the Earth is non rigid. Indeed, at a given location, the distance to the axis of rotation varies, centrifugal forces change consequently and the Earth undergoes a deformation called pole tide. This deformation is commonly modelled as hydrostatic for the ocean and quasi-elastic for the solid part of the Earth. In turn, the pole tide causes a change \( c_{rot} \) of the inertia moment \( c \), which can be shown to be linear with respect to the pole displacement \( m \):

\[ \frac{c_{rot}}{C - A} = \frac{k}{k_s} m, \]

(10)

where \( k \approx 0.35 \) and \( k_s = 0.94 \). After reporting this contribution in the LHS of (8), we obtain

\[ m + \frac{i}{\sigma_e} \dot{m} = \frac{k_s}{k_s - k} \chi - \frac{i}{\Omega} \dot{\chi}, \]

(11)

where the equatorial external torque \( L \) has been cast aside for the considered problem of a mass transport in the mechanical system, and

\[ \sigma_c = \left( 1 - \frac{k}{k_s} \right) \sigma_e, \]

(12)

corresponding to a period of about 480 days for the free wobble. By considering moreover that the liquid core does not present any pole tide, one gets the observed period of 430 days. That is the essence of the polar motion theory for time scale smaller than 100 years.

According to our contemporaneous understanding, at sub-secular time scale polar motion results from transports in the surface layer of the Earth: atmosphere, ocean, in-land water and continental ice (see Figure 4). Models based upon the assimilation of observations like wind, ocean current, temperature,...allow to construct 3D carts of the state of these layers at regularly sampled times. Then integration yields induced moment of inertia \( c_l \) (the index \( l \) means load) or relative angular momentum \( h = h_1 + ih_2 \) for each of these layers with a time resolution generally longer than 3 hours. Such time series then provide a good account for sub-decadal polar motion (period smaller than 10 year) adopting the framework of a quasi-elastic Earth resulting in Equation (11) for polar motion. For a low frequency mass load \( (\sigma \leq \Omega) \), (11) becomes

\[ m = \frac{k_s}{k_s - k} \frac{c_l}{C - A} \approx 1.6 \frac{c_l}{C - A}. \]

(13)

In \( c_l \) we include the (elastic) deformation of the solid Earth under the loading itself.
At decadal time scale, the quasi-elastic behaviour of the Earth is still valid. But the causes of polar motion are partly elucidated. Change in the land water distribution or ice coverage seem to be the major contributor. For instance, it accounts for the deflection of the rotation pole since 2000 towards Europe shown in Figure 2 [1].

Around 30 years appears a clockwise oscillation with an amplitude of 10 mas, called Markowitz wobble, and captured by the wavelet transform of Figure 3. Some study speculate on its causes. It could originate in electromagnetic coupling between the iron-nickel core and the thin conducting layer located at the bottom of the mantle.

![Figure 3. Wavelet transform on the 2D polar motion in the band 10 year - 40 years. Amplitude is expressed in mas.](image)

![Figure 4. Synoptic of geophysical processes causing polar motion from 12 h to one million years](image)

5. **Visco-elastic Earth and secular drift**

The trend observed in mass redistributions, either located in the atmosphere or in the hydrological layer, are too small to induce the observed secular drift. The most plausible explanation is an enormous moment of inertia change, almost reaching the Earth triaxiality \((A - B \sim 10^{-5}A)\), that occurred long time in the past, but which the effect continues because of the visco-elastic response of the Earth. The longest is the period of the exciting process, the largest will be delayed the deformation response. The relation (10) becomes frequency dependent:

\[
c_{\text{rot}}(\sigma) = \frac{k(\sigma)}{k_c} (C - A)m(\sigma) .
\]  

(14)
In time domain this is equivalent to a convolution product. Rheological models yield expression having the form [4]

\[ k(t) = k_0 \delta(t) + \sum_{j=1}^{j+1} H(t)k_j e^{-s_j t}, \]  

(15)

where \( H(t) \) is the Heaviside function. The elastic/hydrostatic response is accounted by \( k_0 \delta(t) \approx 0.035 \delta(t) \) and the viscosity is embodied in the terms \( k_j e^{-s_j t} \). In frequency domain we have

\[ k(\sigma) = k_0 - \sum_{j=1}^{j+1} \frac{k_j}{s_j + i\sigma}, \]  

(16)

In the quasi-elastic approximation, the viscosity term \( \sum \frac{k_j}{s_j + i\sigma} \) is taken as a constant complex number, independent from frequency.

As the variations of all pertaining quantities are still small in regard to the total Earth inertia moment or to the Earth angular velocity, linearity of Liouville holds. At this time scale excitation is limited to off-diagonal inertia change \( c \) in (ref{EL1}). As the pole tide effect becomes a convolution product in time domain, it is easier to solve (ref{EL1}) in Laplace or Fourier domain, and go back to time domain. It can be shown that the solution takes the form

\[ m(t) = H(t)(\rho_0 + \sum_{j=1}^{j+1} \rho_j e^{-\sigma_j t}) \frac{c_i(t)}{C - A}. \]  

(17)

with \( \rho_0 \approx 1 \). Considering the extension of ice coverage during the last glaciation and its thickness (up to 1000 m), models gives for \( c_i \) a maximum of \( 10^{32} \) kg m². For a unique mode, occurring for a mono-layered Earth for instance, we have

\[ k(t) = k_0 \delta(t) + H(t)k_1 e^{-s_1 t}, \]  

(18)

and we obtain the polar motion

\[ m(t) \approx H(t) \sum_{j=1}^{j+1} \rho_j e^{-\sigma_j t} \frac{c_i(t)}{C - A}. \]  

(19)

To the largest mode of rheological Model A in \cite{wu_peltier_1984}, namely \( s_1 \approx 1.92 \) ky\(^{-1}\) and \( k_1 \approx 0.94 \) ky\(^{-1}\) corresponds the solution \( \sigma_1 \approx 0.46, \rho_1 \approx 2.12, \sigma_2 \approx 1.46 - i5.176 \) \( 10^3 \), \( \rho_2 \approx -2.12 - 17.558 \) \( 10^3 \), with all these coefficients expressed in ky\(^{-1}\).

Here the mode of complex frequency \( \sigma_2 \) models the forced Chandler wobble, and the contribution of the visco-elasticity to the secular polar motion is given by the mode of frequency \( \sigma_1 \), namely

\[ m^{ve}(t) = H(t)\rho_1 e^{-s_1 t} \frac{c_i(t)}{C - A} = \rho_1 \int_0^t e^{-\sigma_1 u} \frac{c_i(t - u)}{C - A} du. \]  

(20)

At epoch \( t \), this is the weighted sum of moment of inertia at all prior instants back to the first instant \( t_0 = 0 \) of the mass redistribution, which the weights decrease exponentially with offset from the present epoch. Now we see the striking contrast with a rigid or elasto-fluid Earth, where the secular pole \( m(t) \) is always proportional to \( c_i(t) \).
If we grossly model the evolution of $c_l$ as a saw-like time function lasting 100 kyr, reaching its maximum 90 kyr years after the glaciation has started, and melting in 10 kyr (see top of Figure 5), then numerical evaluation of (17) gives the polar motion from $-100$ kyr (starting of the glaciation) to $+100$ kyr, considering both a mono-layer Earth (having an homogeneous rheology), and a refined multi-layer model. The corresponding evolution of $m_1$ and $m_2$ pole coordinates are represented at the top of Figure 5. Here $t = 0$ is for the end of the melting. Just below we show the velocity of the polar drift, indicating its amplitude in degree/Myr and its longitude. This velocity is zoomed in the bottom plots from year 0 to 20 kyr. If we assume that our epoch is $+6$ kyr after the end of the ice recession, we obtain a polar drift of about 3 mas/year towards $80^\circ$ W, comparable with the contemporaneous order of magnitude. So, in this view, pole drift is the footprint of a mass load, that has totally disappeared, but which the rotational effect is still visible.

6. Qualitative description

As shown by the upper plots of Figure 5, for a visco-elastic Earth, polar drift accelerates with time, while the one estimated for an elastic Earth according to (13) increases uniformly with time as $c(t)/(C - A)$. How can we interpret this simply in light of the basic equation (8)? Let us consider successive equally spaced instant, $t_0 = 0, t_1, t_2, t_n ...$. During each time step $\tau = [t_i, t_{i+1}]$, off-diagonal moment of inertia of the load increases by $\Delta c$. To illustrate our demonstration, let us take a positive increase of $\Delta c = \Delta c_{23} = - \int yz dm$ in the plane and symbolized by a trapezoidal form in Figure 6. For the sake of simplicity we assume that at time $t_0$ the inertia pole, the geographic pole (corresponding to the $z$ axis of the terrestrial frame) and the Earth rotation pole coincide at the point $R_0$. At time $t_1 = \tau$, the Earth axis moves to the position $R_1^T$ by $\Delta m = \Delta c_1/(C - A)$ for a rigid Earth. In
the case of a solid elastic Earth, partially covered by hydrostatic oceans, the pole tide is instantaneous. Its account - as well as a possible elastic deformation in the case of a surface loading occurring at time scales larger than 10 years - yields Euler-Liouville equation (13), and does not change this expression much. The shift is amplified, and rotation pole is now in the position \( R^e_\alpha \). For the next interval \( [t_1, t_2] \) the same increment happens for \( c \), the pole shift is doubled for a rigid or an elastic/fluid Earth. The new pole position are respectively \( R^e_2 \) and \( R^e_3 \).

For a visco-elastic rheology, the first step \([0, t_1]\) did not present a full adjustment of the equatorial bulge around the new direction of the Earth axis. This delayed pole tide begins during the period \([t_1, t_2]\), and it causes a supplementary pole tide with the same sign than the \( c(t) \). Again, this reinforces the purely elastic or hydrostatic effect, and at time \( t_2 \) rotation pole lies in the more shifted position and \( R^e_2 \). Iterating this argument at instant \( t_3 \), we see that \( R^e_2 \) integrates the delayed pole tide effect during period \([t_1, t_2]\) and \([t_0, t_1]\) in a less measure, so that at time \( t_n \), \( R^e_n \) results not only from the rigid polar shift and elastic pole tide but also from the delayed pole tide over the time intervals \([t_0, t_1], [t_1, t_2], \ldots, [t_{n-2}, t_{n-1}]\). So the displacement of \( R^n \) is an accelerated function of \( t \), unlike \( R^e \) or \( R^v \).

Let us consider the simplistic rheological model (18). According to (10) the visco-elastic rotational effect on inertia moment reads then

\[
c_r^{(ve)}(t) = \frac{H(t)k_1 e^{-st}t}{k_s} (C - A)* m(t) = \frac{k_1}{C - A} \int_0^t e^{-s_i u}m(t - u)du.
\]

Approximating the exponential time decrease \( e^{-s_i u} \) by a step wise function \( H(t - u) \) dropping from 1 to zero after relaxation time \( \tau = 2\pi/s_1 < 3 \) kyears, we have

\[
c_r^{(ve)}(t) = \frac{k_1}{C - A} \int_0^t m(t - u)du \approx \frac{k_1}{C - A} \tau m(t - \tau).
\]

Here \( \tau \) is small with respect to the duration of the glaciation, which is 30 times longer. Put another way, the visco-elastic pole tide in the interval \( \tau = [t_{l-1}, t_l] \) is a constant fraction of the total polar shift \( m(t_{l-1}) \) at time \( t_{l-1} \): \( \Delta c_r^{(ve)}(t_l) = \alpha m(t_{l-1}) \), with \( 0 < \alpha \leq 1 \). So, at instant \( t_2 \) the total displacement of the visco-elastic pole \( R^v \) is \( R_0 R^v_2 = (2 + \alpha) \Delta m \), whereas \( R^e \) has only moved by \( 2\Delta m \). At time \( t_3 \), the rotation pole \( R^v_3 \) has moved by

\[
R_0 R^v_3 = R_0 R^v_2 + \Delta m + \alpha R_0 R^v_2 = (2 + \alpha) \Delta m + \Delta m + 2\alpha \Delta m = (3 + 3\alpha) \Delta m.
\]

Iterating this argument at equally spaced instants, we find that the full displacement is \( (n + \alpha(n - 1)n/2) \Delta m \) at instant \( t_n \), exceeding the rigid or the elastic one by \((n - 1)n/2) \alpha \Delta m \). In this expression \( n \) and \( t \) are proportional, so that the departure for the elastic/fluid pole \( R^f \) is quadratic function of time, while the moment of inertia \( c_l \) linearly increases with time. This qualitative approach well accounts for the parabolic acceleration of the pole shift observed in Figure 5 for the glaciation phase.

7. Summary and discussion

The low frequency polar motion, embodying periods above 10 years, reflects miscellaneous geophysical processes, and is a valuable source of information for the Earth rheology. Multi-annual oscillations are associated with changes in land water storage and continental ice coverage. The 25-30 wobble is not explained yet, whereas some study suggest the interaction between the fluid core and the mantle. Finally, the secular pole drift results from the visco-elastic response of the Earth to the last glaciation cycle. Notwithstanding, the value of the secular polar drift depends on the viscosity of the
lower mantle, which the value is poorly known (between $10^{21}$ and $10^{22}$ Pas s). In turn the reconstructed pole drift can vary by an order of magnitude. Meanwhile, we provided a qualitative description of the visco-elastic mechanism of the secular pole drift. In parallel to the elastic/fluid pole tide linearly amplifies the polar displacement of a rigid Earth, the visco-elasticy let appear a delayed tide reflecting the pole displacements at all former instants. In the case of a mass load increase or decrease in a given geographic area (like an ice cap), these visco-elastic pole tide effects are added and tend to strongly accentuate the pole shift.

Over million year, the effect of tectonic mass redistribution takes precedence over the effect of glaciations. Moreover at this time scale, the polar displacement is blurred by the continent sliding over the mantle, which is grossly reconstructed from geological observations, especially assuming that some volcano ranges (like the one of Hawaii islands) reflect the trace on the Earth crust of magmatic plumes originating from the mantle, the hot spots. Yet, it is not excluded that the continents present of global rotation with respect to the mantle up to a few mas/year, then partially or totally invalidating the role of glaciation in the observed drift. For instance, if the continents globally rotate towards $180^\circ - 80^\circ = 100^\circ$ East, then the rotation axis, fixed with respect to the mantle will shift in the observed direction with respect to the crust.

![Figure 6](image)

**Figure 6.** Schematic representation of the polar shift mechanism when off-diagonal moment of inertia $c_{23}$ linearly increases with time. At time $t_1 = t_0 + \tau$ positive inertia moment change $c_{23}$ (in the plane $yz$) is added, pictured by a trapezoidal form. A time $t_2 = t_1 + \tau$ this moment inertia variation is multiplied by 2, and so one. The subsequent displacement of the rotation pole $R$ are represented (magnified) for three Earth models: $R^e$ for a rigid Earth, $R^f$ for a elastic/fluid Earth and $R^{ve}$ for a visco-elastic Earth.

**References**

[1] Adhikari S and Ivins E R 2016 Climate-driven polar motion: 2003-2015, *Science Advances* Vol. 2, no. 4, e1501693, DOI: 10.1126/sciadv.1501693

[2] Lambert W D 1922 The interpretation of apparent changes in mean latitude, *Astronomical Journal* vol. 34 iss. 804 pp 103-110
[3] Munk W H and Mac Donald G 1960 *The rotation of the Earth* (Cambridge University Press)

[4] Sabadini R, Vermeersen B and Cambiotti G 2016 *Global Dynamics of the Earth: Applications of Viscoelastic Relaxation Theory to Solid-Earth and Planetary Geophysics*, Springer, pp 335.

[5] Wu P and Peltier W R 1984 Pleistocene deglaciation and the Earth's rotation: a new analysis *Geophys. J.R. astr. Soc.* 76 pp 753-791

[6] Zhukovsky (1904), *Geometricheskaya interpretsiya teorii dvizheniya polyusov vrashcheniya zemli po ee poverxnosti*, Collected paper of Zukovsky, Vol. 1, edited by A.P. Kolesnikov, Editorial Officie for Aviation Litterature (1937).

**Acknowledgments**

We would like to acknowledge the financial support of our journey in Moscow by the High School of Economy. We also express our gratitude to Bauman Institute for its invitation to give this talk, and Guy Berthault for encouraging us to undertake the study of rotation pole tilt in view of its coupling with marine recessions and transgressions.