Adiabatic quantum pumping and rectification effects in interacting quantum dots

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We derive a formula describing the adiabatically pumped charge through an interacting quantum dot within the scattering matrix and Green’s function approach. We show that when the tunneling rates between the leads and the dot are varied adiabatically in time, both in modulus and phase, the current induced in the dot consists of two terms, the pumping current and a rectification-like term. The last contribution arises from the time-derivative of the tunneling phase and can have even or odd parity with respect to the pumping phase ϕ. The rectification-like term is also discussed in relation to some recent experiments in quantum-dots.

I. INTRODUCTION

The idea of quantum pumping, i.e. of producing a dc current at zero bias voltage by time periodic modulation of two system parameters, dates back to the work of Thouless.⁹ If the parameters change slowly as compared to all internal time scales of the system, the pumping is adiabatic, and the average charge per period does not depend on the detailed time dependence of the parameters. Using the concept of emissivity proposed by Büttiker et al.,⁵ Brouwer⁴ related the charge pumped in a period to the derivatives of the instantaneous scattering matrix of the conductor with respect to the time-varying parameters. Since then, a general framework to compute the pumped charge through a conductor has been developed for noninteracting electrons. The interest in the pumping phenomenon has shifted then to the experimental investigations of confined nanostructures, as quantum dots, where the realization of the periodic time-dependent potential can be achieved by modulating gate voltages applied to the structure. In case of interacting electrons the computation of the pumped charge becomes rather involved and few works have addressed this issue for different systems and in specific regimes. As for the case of interacting quantum dots, the pumped charge in a period was calculated by Aono, by exploiting the zero-temperature mapping of the Kondo problem. A very general formalism was developed in Ref. [9] where an adiabatic expansion of the self-energy based on the average-time approximation was used to calculate the dot Green’s function while a linear response scheme was employed in Ref. [10]. More recently, another interesting study was performed aiming at generalizing Brouwer’s formula for interacting systems to include inelastic scattering events.

In this work we present a general expression for the adiabatic pumping current in the interacting quantum dot in terms of instantaneous properties of the system at equilibrium, generalizing the scattering approach for noninteracting particles and discuss the limit of its validity. To get a pumped current the two model parameters which are varied in time are the tunneling rates between the noninteracting leads and the quantum dot. In particular, we let them vary both in modulus and phase through the adiabatic and periodic modulation of two external parameters (e.g. gate voltages or magnetic fields) and show that a rectification-like term arises in the current due to the time-dependent tunneling phase.

The plan of the paper is the following. In Sec.II we introduce the model and relevant parameters. We develop the scattering matrix approach together with the Green’s function formalism to derive the formula of the pumped current through an interacting multilevel quantum dot in the adiabatic regime at very low-temperatures. In Sec.III we specialize on a single-level quantum dot and give the explicit expression of the pumped current. Conclusions are given in Sec.IV.

II. THE MODEL AND FORMALISM

We consider a multi-level quantum dot (QD) coupled to two noninteracting leads, with the external leads being in thermal equilibrium. The Hamiltonian of the system is given by:

\[ H = H_{\text{leads}} + H_{\text{dot}} + H_{\text{tun}}, \]

where

\[ H_{\text{leads}} = \sum_{k,\sigma,\beta} \epsilon_{\beta}(k) c_{k,\sigma,\beta}^\dagger c_{k,\sigma,\beta}, \]

with \( c_{k,\sigma,\beta}^\dagger \) the creation (annihilation) operator of an electron with spin \( \sigma = \uparrow, \downarrow \) in the lead \( \beta = L, R \) and dispersion \( \epsilon_{\beta}(k) \). The QD is described by the Hamiltonian

\[ H_{\text{dot}} = \sum_{j,\sigma} \epsilon_{j\sigma} n_{j\sigma} + U n_{\uparrow j} n_{\downarrow j}, \]

where \( n_{j\sigma} = d_{j\sigma}^\dagger d_{j\sigma} \) with \( d_{j\sigma}^\dagger d_{j\sigma} \) the creation (annihilation) operator of the electron with spin \( \sigma \) and \( \epsilon_{j\sigma} \) the dot \( j \)-th energy level. The on-site energy \( U \) describes the Coulomb interaction. The tunneling Hamiltonian is given by

\[ H_{\text{tun}} = \sum_{k,\sigma,\beta,j}[V_{k,\sigma,\beta,j}(t)c_{k,\sigma,\beta}^\dagger d_{j\sigma}^\dagger + H.c.], \]

with time-dependent tunnel matrix elements \( V_{k,\sigma,\beta,j}(t) \). For simplicity we assume that \( V_{k,\sigma,\beta,j} \) are spin independent, i.e.

\[ V_{k,\sigma,\beta,j} = V_{\beta,j} \]

and that both the modulus and the phase of \( V_{\beta,j}(t) \) vary in time with frequency \( \omega \), i.e. \( V_{\beta,j}(t) = |V_{\beta,j}(t)| \exp(i\Phi_{\beta,j}(t)) \). Their explicit time dependence is determined by two external parameters (e.g. two gate voltages applied at the barriers of the dot or a gate voltage and a magnetic field) which are varied...
adiabatically and periodically in time or by the presence of parasitic bias voltages. Two specific examples will be considered below. In particular, we will specialize on the case in which the tunneling phase can vary harmonically or linearly in time. The instantaneous strength of the coupling to the leads is instead characterized by the parameters $\Gamma_{nm}^\beta(t, t) = 2\pi \rho V_{\beta,m}(t)V_{\beta,n}(t)$, where $\rho$ is the density of states in the leads at the Fermi level. By varying in time $V_{\beta,j}(t)$ and $V_{\beta,j}(t)$ and keeping them out of phase, the charge $Q$ pumped in a period $T$ is related to the time dependent current $I^L(\tau)$ flowing through the left barrier, i.e. $Q = \int_0^T d\tau I^L(\tau)$.

While the exact formula for the current depends on time-dependent Green's function out of equilibrium, in the following we consider the adiabatic limit where the current depends only on the instantaneous equilibrium properties of the dot, i.e. on the retarded dot Green's function (GF). This situation is realized in the two following cases. First, let us consider that only the modulus of the tunneling matrix elements is varied in time, i.e. $V_{\beta,j}(t) = |V_{\beta,j}|e^{i\Phi_\beta(t)}$. Under the adiabatic condition, the tunneling rate varies slowly in time, and the quantum dot can be considered time in equilibrium with the external leads. The effect of quantum pumping is well described by an adiabatic expansion of the self-energy based on the average-time approximation as described in Ref. [9] and using the equilibrium relations to write the pumped current in terms of the retarded GF only. Let us consider now the situation in which the modulus of the tunneling terms is fixed while their phases are modulated in time, i.e. $V_{\beta,j}(t) = |V_{\beta,j}|e^{i\Phi_\beta(t)}$. This situation is equivalent (by a gauge transformation) to having a system biased with an ac external signal. In particular, the ac voltage applied to the leads is proportional to the time derivative of the tunneling phase $\partial_t \Phi_\beta(t)$ [12]. Since the tunneling terms are assumed to vary in time with frequency $\omega$, the ac signal forcing the quantum dot is proportional to the pumping frequency $\omega$ and thus can be considered as a small perturbation under the adiabatic condition. In particular, if one consider the case in which the tunneling phases vary linearly in time, $\Phi_\beta(t) = \pm V t/2$, this situation corresponds to an interacting quantum dot biased by a dc voltage $V$. Following the work by Meir and Wingreen [10], the current $I$ flowing through the interacting multilevel dot biased by a dc voltage $V$ can be written as:

$$I = \frac{e}{h} \int d\epsilon [f_L(\epsilon) - f_R(\epsilon)] Tr\{G^\alpha \Gamma^R G^\tau \Gamma^L R\}, \quad (2)$$

where $f_{L,R}$ are the Fermi functions, $\Gamma_{L,R}$ are the dot-leads coupling strengths and $R = \Sigma_0^{-1} \Sigma$ is the ratio between the fully interacting self-energy and the non-interacting one, responsible for the deviation from the Landauer-Büttiker formula (see Ref. [10], Eq. (10)). In the zero-temperature limit and for a weak bias, $\Sigma = \Sigma_0$ at the Fermi level and thus Eq. (2) can written as $I \propto VT r\{G^\alpha \Gamma^R G^\tau \Gamma^L \}$, which corresponds to the usual linear response form, even though the Green’s function are interacting ones. This argument is extensively discussed in Refs. [23] (see Eq.s (37) and (38)) and [10].

Thus in general we expect that, when both the modulus and the phase of the pumping parameters are varied in time, apart the usual dc pumping current a new term arises (that we call of rectification) which is proportional to the time derivative of the tunneling phase:

$$I_r = \frac{e}{h} Tr\{G^\alpha \Gamma^R G^\tau \Gamma^L \} \partial_t (U_L - U_R).$$

Since, as explained above, in adiabatic regime the current is determined by the instantaneous properties of the dot (retarded Green’s function) and since in the zero temperature limit the usual linear response formula can be adopted for the calculation of the current, the pumping and rectification currents through the interacting quantum dot can be calculated by the scattering matrix approach as well. In fact, as well known, the retarded Green’s function is related to scattering matrix by the Fisher-Lee relation. We thus employ the scattering matrix formalism developed in Ref. [11] where the charge current originated by an adiabatic pump is related to an expansion of the quantity $\{S(E, \tau)| f(E + i\partial_t/2) - f(E)\}S^\dagger(E, \tau)|_{\alpha\alpha}$ with respect to the time derivative operator $i\partial_t/2$ (here $S(E, \tau)$ is the scattering matrix and $f(E)$ is the Fermi function). The first order of this expansion reproduces the famous Brouwer’s formula. Let us only stress that the scattering matrix formalism is well defined, not only in the noninteracting case, but also for the interesting problem (e.g. see Ref. [12]). The expression of the pumped current $I^E(\epsilon)$ in terms of the time-dependent scattering matrix is [11]:

$$I^E(\epsilon) = \frac{q}{2\pi} Im \left\{ \sum_{\alpha} S^*_{\alpha\beta}(E, \tau) \partial_t S_{\beta\alpha}(E, \tau) \right\},$$

$$I^E(\tau) = \int \frac{dE}{2\pi} (-f') I^E(E, \tau), \quad (3)$$

where $f(E)$ is the Fermi function and $S(E, \tau)$ is the instantaneous $S$-matrix of the QD. It is given by the Wigner transform $S(E, \tau) = \int dt e^{iE t} S(\tau + t/2, \tau - t/2)$, where

$$S_{\alpha\beta}(t, t') = \delta_{\alpha\beta} \delta(t - t') - Tr\{M^{\alpha\beta}(t, t') G^\tau(t, t')\}. \quad (4)$$

Here $G^\tau$ is the full retarded QD Green’s function, $G^\tau_{nm}(t, t') = -i\theta(t - t')\langle\{d_n(t), d_\dagger_m(t')\}\rangle$ and $[M^{\alpha\beta}(t, t')]_{mn} = 2\pi i \rho V^*_{\alpha,m}(t)V_{\beta,n}(t')$. In the limit of the pumping frequency $\omega \ll \Gamma$, i.e. under the adiabatic condition, the scattering matrix $S(E, \tau)$ is expressed by the instantaneous Green’s function of the dot as [16].

$$S_{\alpha\beta}(E, \tau) = \delta_{\alpha\beta} - 2\pi i \rho \sum_{n,m} V^*_{\alpha,n}(\tau) G^\tau_{nm}(E, \tau) V_{\beta,n}(\tau). \quad (5)$$

When substituting (5) into (3) to compute the current we need the time-derivative of the QD Green’s function which satisfies the relation:

$$\partial_t G^\tau(E, \tau) = G^\tau(E, \tau) \Sigma^T(E, \tau) G^\tau(E, \tau), \quad (6)$$

where the dot symbol indicates a time-derivative and the matrix notation for the dot Green’s function has been
used. The final expression obtained for the $I^\beta(E,\tau)$ for a multi-level quantum dot is\cite{footnote2}:

$$I^\beta(E,\tau) = \frac{q}{2\pi} \left[ -i \text{Tr}\{\hat{I}^\beta\hat{G}^\dagger\} + \text{Im} \{i(\Phi_\beta - \Phi_\beta) \times \right. $$

$$\times \text{Tr}\{\Gamma^\beta \hat{G}^\dagger \Gamma^\beta \hat{G}^\dagger\} \left. + \Delta^\beta(E,\tau) \right]$$

where

$$\Delta^\beta(E,\tau) = \text{Im} \left\{ -i \text{Tr}\{\Gamma^\beta \hat{G}^\dagger \hat{G}^\dagger \} \right\}$$

$$+ \text{Im} \left\{ \text{Tr}\{\hat{I}^\beta \hat{G}^\dagger\} \text{Tr}\{\Gamma^\beta \hat{G}^\dagger\} \right\}$$

$$+ \sum_{s,m,n,p} \text{Im} \{\Gamma^\beta \hat{G}^a_{mn} \Gamma^\beta \hat{G}^a_{np} \times \}$$

$$\times \partial \tau \ln(|V_{\beta,s}||V_{\beta,p}|).$$

This expression has been obtained by considering explicitly the time dependence of the modulus and phase of the tunnel matrix elements, and consequently of the leads-dot coupling function $\Gamma(t)$. The symbol $\beta$ stands for the L,R lead in correspondence of $\beta=R,L$. The total dc current through the lead $\beta$ is given by:

$$I^\beta = \frac{\omega}{2\pi} \int dE \int_{-\infty}^{\infty} d\tau \text{I}^\beta(E,\tau)[-\partial_E f(E)].$$

The expression (7) represents our main result. It is valid for a multi-level QD and for any interaction strength in the zero temperature limit under the adiabatic condition. The first term in Eq. (7) represents the pumping current, while the second one, proportional to the time-derivative of the tunneling rate phase, is the effective rectification term we have discussed above. It can also be written as $G(\tau)V_{c,\ell}(\tau)$, where $G(\tau) \propto \text{Tr}\{\Gamma^\beta \hat{G}^\dagger \hat{G}^\dagger\}$ is the conductance of the structure, while $V_{c,\ell}(\tau) \propto (\Phi_\beta - \Phi_\beta\)$. The last term in (7) contains information on the time derivative of the retarded self-energy and is zero for a single-level quantum dot within the wide band limit.

III. TOTAL CURRENT FORMULA FOR A SINGLE LEVEL QD

Up to now we have developed a theory of the pumped current valid in the case of a multi-level QD. We now specialize Eq. (7) to the case of a single level QD. Eliminating the trace in (7) and considering the remaining quantities as c-numbers, the expression for the current simplifies to:

$$I^\beta(E,\tau) = -\frac{q}{2\pi} \left[ \text{Re}\{\hat{I}^\beta \hat{G}^\dagger\} + \Gamma^\beta |\hat{G}^\dagger|^2 \partial \tau \text{Re}\{\Sigma^\dagger\} + \right.$$

$$+ (\Phi_\beta - \Phi_\beta) \Gamma^\beta \Gamma^\beta |\hat{G}^\dagger|^2 \right].$$

When the time-derivative of the tunneling phase is neglected the above formula is equivalent to the self-energy adiabatic expansion\cite{footnote2}.

In the following we consider the case of a single level QD both in the strongly interacting and non-interacting case and describe the behavior of the charge $Q$ (in unit of the electron charge $q$) pumped per cycle in the zero-temperature limit. The Fermi energy $\mu$ is set to zero as reference energy level, while the static linewidth $\Gamma_0^{L/R}$ is assumed as energy unit (typical value for $\Gamma_0$ is $10\mu eV$).

In the noninteracting case, i.e. when the QD Green’s function becomes a scalar, the expression for the instantaneous pumping current is explicitly given by:

$$I^\beta(E,\tau) = \frac{q}{2\pi} \left[ -\frac{(E - \varepsilon_0) \Gamma^\beta}{(E - \varepsilon_0)^2 + (\Gamma/2)^2} \right.$$

$$+ \frac{\Gamma^\beta \Gamma^\beta (\Phi_\beta - \Phi_\beta)}{(E - \varepsilon_0)^2 + (\Gamma/2)^2}],$$

(11)

When $\Phi_\beta - \Phi_\beta = 0$, the charge pumped is zero when the level is resonant ($\varepsilon_0 = 0$).

In the case of a strongly interacting quantum dot, i.e. in the infinite-$U$ limit, we take the expression of the QD Green’s function as in Ref.\cite{13}. The current is:

$$I^\beta(E,\tau) = \frac{q}{2\pi} \left[ -\frac{(E - \varepsilon_0)(1 - n_\sigma) \Gamma^\beta}{(E - \varepsilon_0)^2 + (\Gamma/2)^2(1 - n_\sigma)^2} \right.$$

$$+ \frac{\Gamma^\beta \Gamma^\beta (\Phi_\beta - \Phi_\beta)(1 - n_\sigma)^2}{(E - \varepsilon_0)^2 + (\Gamma/2)^2(1 - n_\sigma)^2}],$$

(12)

where the occupation number $n_\sigma$ on the dot has to be determined self-consistently by the relation $n_\sigma = (2\pi i)^{-1} \int dE G^<_{\sigma \sigma}(E)$, where $G^<_{\sigma \sigma}$ is the QD lesser Green’s function.

In order to show the effects of the time-dependent tunneling phase we report below the numerical results of the charge $Q$ pumped per cycle. The pumping cycle is determined by the periodic time variation of the leads-dot coupling strength, where $\Gamma^a(t) = \Gamma_0^a + \Gamma_0^a \sin(\omega t + \varphi_\alpha)$, while for the tunneling phase two cases can be considered. Either it varies harmonically $\Phi_\alpha(t) = \Phi_\alpha(t) \sin(\omega t + \varphi_\alpha)$ with the same frequency of the two external gate voltages (this case is shown in Fig.\textbf{1}) or it varies linearly in time $\Phi_\alpha(t) = \Phi_\alpha t$, e.g. when a parasitic gate voltage is present (this case is shown in Fig.\textbf{2}). The quantity $\varphi_\alpha$ is the pumping phase that we take different from zero between L and R lead.

In Fig.\textbf{1} we plot $Q$ as a function of the energy level $\varepsilon_0$ by fixing the other parameters as: $\mu = 0, \varphi = \pi/2, \Gamma_0^L = 0.2, \Gamma_0^R = 0.4, \Gamma_0^R = \Gamma_0^R = 1, \Phi_\alpha = 0.3, \Phi_\alpha = 0.2$. In particular, in the upper panel, the charge induced by the pumping (triangle) and the charge due to the rectification term (box) is shown for a non-interacting dot ($U = 0$). The total charge (empty circle) is significantly modified by the presence of the rectification term which is a non-vanishing quantity at the Fermi energy ($\varepsilon_0 = 0$).

The lower panel in Fig.\textbf{1} shows the behavior of the rectification current as a function of $\varepsilon_0$ in the strongly interacting limit ($U \rightarrow \infty$) and by choosing the remaining
parameters as in the upper panel. While the general aspect of the total charge (empty circle) is only marginally modified by the strong correlations in the specified region of parameters, the rectified charge (box) shows a pronounced asymmetric behavior with respect to the level of the dot. In Fig.2 we focus on the case of time-linear

variation of the phase $\Phi_\alpha(t) = \Phi_\alpha^0 t$, and take the parameters as in Fig.1. In the upper panel, the pumped charge $Q$ is plotted as a function of the level $\varepsilon_0$ in the non-interacting case ($U = 0$). Contrary to the previous case, the rectification contribution (box) is dominant over the one induced by the pumping mechanism (triangle) and thus the total charge (empty circle) is only marginally affected by a resonant-like behavior. In the lower panel, the results for $U \to \infty$ are shown. Apart from a renormalization of the linewidth of the resonance induced by the factor $(1 - n_\beta)$ in the numerator of Eq. (12), a behavior similar to the one of the non-interacting system is found. A slave boson treatment with the inclusion of a renormalization of the dot energy level, could in principle modify this picture. Let us note that when the tunneling phase varies harmonically both the pumping current and the rectification current follow the same $\sin(\varphi)$ behavior w.r.t. the pumping phase $\varphi$. In Fig.3 we show the behavior of

the charge induced by the pumping term (upper panel) and by the rectification (lower panel) with respect to the pumping phase $\varphi$ and by fixing the dot level to $\varepsilon_0 = 0.3$ and the remaining parameters as done in Fig.2. The full line in both panels represents the result for the infinite-$U$ case. Let us note that while the pumping term follows the conventional $\sin(\varphi)$-behavior as in Brouwer theory, the rectification contribution takes the form $V_{eff}\{A + B\cos(\varphi)\}$, where the coefficients $A$ and $B$ for the $U = 0$ case are explicitly given by:

\[
A = \frac{\Gamma_0^0}{\xi + (\Gamma_0^0/2)^2}
\]

\[
B = \frac{\Gamma_0^0\Gamma_0^R[\Gamma_0^R - \Gamma_0^L] + 2\Gamma_0^R(4\xi - \Gamma_0^L) - 16\xi^2}{32\xi(\xi + (\Gamma_0^0/2)^2)^3},
\]

with $\xi = (\mu - \varepsilon_0)^2$. The different symmetry of the pump-

FIG. 1: Charge $Q$ pumped per pumping cycle as a function of the dot level $\varepsilon_0$. The pumping (triangle) and the rectification (box) contribution to the total charge (empty circle) are shown for the non interacting case ($U = 0$) in the upper panel and for strongly interacting dot ($U \to \infty$) in the lower panel. Both figures are computed for the following choice of parameters: $\mu = 0$, $\varphi = \pi/2$, $\Gamma_0^R = 0.2$, $\Gamma_0^L = 0.4$, $\Gamma_0^L = \Gamma_0^R = 1$, $\Phi_0^L = 0.3$, $\Phi_0^R = 0.2$. The pumping cycle is determined by: $\Gamma^\alpha(t) = \Gamma_0^\alpha + \Gamma_0^\alpha \sin(\omega t + \varphi_\alpha)$, $\Phi_\alpha(t) = \Phi_0^\alpha \sin(\omega t + \varphi_\alpha)$, with $\alpha = L$, $R$ and $\varphi_L = 0$, $\varphi_R = \varphi$.\n
FIG. 2: Charge $Q$ pumped per pumping cycle as a function of the dot level $\varepsilon_0$ in the non interacting case (upper panel) and strongly interacting case (lower panel). The pumping (triangle) and the rectification (box) contribution to the total charge (empty circle) are shown in the zero temperature-limit by setting the remaining parameters as follows: $\mu = 0$, $\varphi = \pi/2$, $\Gamma_0^R = 0.2$, $\Gamma_0^L = 0.4$, $\Gamma_0^L = \Gamma_0^R = 1$, $\Phi_0^L = 0.3$, $\Phi_0^R = 0.2$. Differently from Fig.1 the pumping cycle is determined by: $\Gamma^\alpha(t) = \Gamma_0^\alpha + \Gamma_0^\alpha \sin(\omega t + \varphi_\alpha)$, $\Phi_\alpha(t) = \Phi_0^\alpha t$, with $\alpha = L$, $R$ and $\varphi_L = 0$, $\varphi_R = \varphi$.\n
The pumping and rectification current has already been reported in experimental works in quantum dots \cite{Zhou99}, and some theoretical explanations have been proposed \cite{Entin-Wohlman02}. Furthermore, the analysis of the coefficient $B$ shows that the charge transferred by the rectification effect can be significantly increased by coupling the dot region to the leads in an asymmetric way ($\Gamma_0^L \neq \Gamma_0^R$), e.g. by using tunnel barriers with very different transparencies. The above results remain almost unchanged in the strongly interacting case ($U \to \infty$).

In the limiting case in which the phase difference between the tunneling barriers is kept zero, i.e. $\varphi_L = \varphi_R = 0$, the pumping term is exactly zero and the rectification contribution acts as a quantum ratchet \cite{Pothier92}.

IV. CONCLUSIONS

Within the Green’s function and scattering matrix approach we have analyzed the quantum pumping current through an interacting quantum dot when both the modulus and the phase of the model time-dependent parameters, in our case the leads-dot tunneling rate, is adiabatically varied. In this way it has been possible to derive an expression for the pumped current containing an effective rectification term due to the time-dependent phase. Such contribution can be written in a Landauer-Buttiker-like form, even though for the interacting system, when the zero temperature limit and the adiabatic conditions are met. The numerical analysis also show that when the tunneling phase varies linearly in time the rectification term is even with respect to the pumping phase $\varphi$, i.e. of the form $a + b \cos(\varphi)$, in contrast to the usual pumping term which is odd. The mentioned contribution could be related to the experimentally observed rectification effects in quantum dots \cite{Makhlin01}.

In particular, we have been considering an open system, but concerning closed systems (e.g. annular devices with a quantum dot), the tunneling phase contribution to the pumped charge could find a natural interpretation in a complex phase of geometric nature \cite{Levinson02}. This phase would be a Berry phase \cite{Thouless83}. Thus the detection of a rectification current in addition to the pumping one could be an indirect probe of a Berry phase.

The proposed analysis could be easily generalized up to the second order in the pumping frequency $\omega$ allowing to describe features involved in the moderate non-adiabatic limit.

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\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig3.png}
\caption{The behavior of the pumping current (upper panel) and of the rectification current (lower panel) with respect to $\varphi$ is reported for $\varepsilon_0 = 0.3$ and taking the remaining parameters as in Fig.\ref{fig2}. Notice that the pumping term is proportional to $\sin(\varphi)$, while the rectification one is proportional to $\cos(\varphi)$. Both in the upper and lower panel, the full line represents the non-interacting result, while the interacting case is indicated by full circles (•).}
\end{figure}

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In formula 16 the sum on the discretized dot levels has been explicitly written.

17 In deriving the expression for the instantaneous current we have been using the relation \( G' - G'' = -i G' \Gamma G' \) for the instantaneous Green’s function \( G'_{n,m}(E, \tau) \) of the QD, while we defined \( \Gamma_{nm} \tau) = 2\pi \rho V_{a,n}(\tau) V_{a,m}(\tau) \).

19 When an external d.c. or a.c. bias voltage is applied to the system, a gauge transformation can be performed to eliminate the bias and the tunneling term acquires a time-dependent phase proportional to the bias. In our case the time-dependent phase of the tunneling comes from the representation of a complex quantity.

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