A new method for assessing the structural safety of refractory ceramics based on a three-factor comprehensive criterion

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Abstract
Traditional structural safety criteria are difficult to apply in assessing the structural safety of refractory ceramics. Herein, a three-factor comprehensive criterion for structural safety assessment of high-temperature refractory ceramics is proposed based on the failure mechanism of refractory ceramics. For the critical temperature difference criterion, the temperature of refractory ceramics changes sharply (530 °C) after the boiler stop for 120 s. Correspondingly, the critical crack length is found to be 21 μm and the crack finally propagates to 10.87 cm (with a crack density of $N = 1 \text{ cm}^{-3}$) under thermal shock. For strength attenuation criterion, the cracks initiate when the compressive and tensile stress values caused by thermal shock exceed the inherent strength of refractory ceramics. Hence, the average stress of 20 MPa can be selected as the applied stress level, where the calculated life of refractory ceramics is comparable to the actual life. For the stress intensity factor criterion, the stress intensity factor ($K_I$) increases with the increase of crack depth ($a$) and crack length ($c$) during crack propagation, but it decreases when $a$ and $c$ increase to certain values. These results reveal that the three-factor comprehensive criterion is highly applicable in analyzing the failure of refractory ceramics, providing support for the maintenance decisions during practical applications.

1. Introduction
The supercharged boiler is the power plant of steam-powered ships, and has high requirements for the performance of refractory ceramics in the boiler [1, 2]. Refractory ceramics (e.g. aluminosilicate, silicon carbide and aluminosilicate-bonded silicon carbide) are widely used as refractory materials for supercharged boilers due to their excellent high-temperature structural properties [3–5]. For example, Reddy et al [5] prepared silicon nitride-bonded silicon carbide and aluminosilicate-bonded silicon carbide refractory ceramics by reaction bonding method. However, frequent thermal shocks in the boiler and strength attenuation caused by long-time operation compromise the performance of ceramic materials and lead to premature failure [4–8]. Therefore, the structural safety analysis and life prediction for refractory ceramics are the hotspots of current research. Effective structural safety analysis can avoid serious safety accidents and waste caused by the premature replacement of new components [9, 10].

A variety of methods have been proposed for judging the structural safety of high-temperature components. A typical example is an R5 regulation proposed by the UK in the nuclear industry in the 1960s, which requires the significance analysis of the creep and fatigue effects on installations to simplify the complexity of crack growth assessment under creep-fatigue interactions [11, 12]. In addition, creep-fatigue assessment (based mainly on elastic analysis methods) was also specified in Sub-volumes III-NH of Handbook of American Society of Mechanical Engineers (AMSE) [13]. However, the abovementioned regulations and methods are mainly applicable to metals, such as pressure pipes. There is still a lack of corresponding safety criteria and regulations for ceramic materials, especially high-temperature refractory ceramics.
Ceramic materials possess a complex microstructure and discrete physical properties \([14–16]\). Hence, the assessment of structural safety of ceramic materials is more difficult than metals. Paris \textit{et al}\[17\] have analyzed a large amount of experimental data, and found that the power exponent of loading amplitude could express the crack growth rate in brittle materials. Furthermore, they have proposed a Paris empirical formula, which is widely used in lifetime prediction models of different ceramics \([15, 18, 19]\). Bažant \textit{et al}\[14, 15\] have proposed a finite weakest-link model to calculate the strength and life distribution of ceramic materials under loading. The results revealed that the average fatigue life of brittle materials was closely related to the geometric structure. At the same time, the life of ceramic materials was also greatly dependent on the service environment. Wang \textit{et al}\[20\] have investigated the influence of temperature on thermal expansion and anisotropic residual stress of ceramic materials, and established a thermal shock resistance model of ultra-high temperature ceramics based on the comprehensive effect of temperature and microstructure. In addition, numerical methods, such as finite element analysis, are also used for life prediction \([9, 21]\). However, there is less research on the safety criteria for high-temperature refractory ceramics. Currently, the safety assessment of refractory ceramics has become a challenging task, which needs to be tackled on urgent basis.

As traditional structural safety criteria are difficult to be applied for refractory ceramics, the current work proposes a three-factor comprehensive criterion for structural safety assessment of high-temperature refractory ceramics based on the failure mechanism, and establishes a safety assessment model. This work lays the foundation for the technical risk assessment and maintenance decision of refractory ceramics, and provides technical support for the subsequent development, utilization, and maintenance of refractory ceramics for supercharged boilers.

2. Three-factor comprehensive criterion theory

The main factors that lead to the failure of refractory ceramics include transient thermal shocks caused by variable load, strength attenuation due to long-time operation, and initiation and propagation of cracks. In general, the safety criterion based on a single factor cannot meet the requirements of safety assessment of high-temperature refractory materials. Therefore, in view of three key factors of transient thermal shock, strength attenuation and crack propagation, this paper proposed a three-factor comprehensive criterion for the structural safety assessment of refractory ceramics. The relevant safety assessment methods and assessment process are illustrated in figure 1.

The above mentioned technical process can mainly be divided into the following six steps:

(1) Determine the operating conditions and working environment to set the applied loads during the safety assessment of refractory materials.

(2) Analyze the material performance and stress level to assess the safety of refractory materials.

(3) Judge the refractory materials by safety criterion based on the critical temperature difference. The safety of refractory materials will be affected by the transient thermal shock caused by start-stop and emergency conditions. If the material is damaged, it will be repaired or replaced, and the safety assessment process is continued if the material is safe.

(4) Judge the refractory materials by the safety criterion based on strength attenuation. With the extension of running time, the strength of refractory materials decreases. If the material is damaged, it will be repaired or replaced, and the safety assessment process is continued if the material is safe.

(5) Judge the refractory materials based on the stress intensity factor. The stress intensity factor is a physical quantity related to the intensity of elastic stress field at the crack tip, which depends on the crack size, geometric characteristics of the member, and load. If the material is damaged, it will be repaired or replaced, and the safety assessment process is continued if the material is safe.

(6) Combine the safety criteria based on critical temperature difference, strength attenuation and stress intensity factor to comprehensively assess the structural safety of refractory materials and determine whether the refractory material needs to be replaced or not, ensuring safe and reliable operation of refractory materials.
3. Failure criteria of refractory ceramics

3.1. Critical temperature difference criterion

Hasselman et al. [19] have established a unified theory of failure initiation and crack propagation based on thermal shocks of brittle ceramics. They took the elastic energy stored in the material when it was subjected to thermal shock as the driving force for crack propagation. When the total elastic energy was less than the fracture energy required for crack growth to the size of sample interface, the fracture caused by thermal stress would not be catastrophic. Hence, the required critical temperature difference when the crack is unstable can be given as:

\[
\Delta T_c = \left[\frac{\pi G (1 - 2\nu)^2}{2E_0 \alpha^2 (1 - \nu^2)}\right]^{0.5} \left[1 + \frac{16(1 - \nu^2)NL}{9(1 - 2\nu)}\right]^{-1.5}
\]

where \(W_t\) represents the total energy per unit volume, \(G\) refers to the surface fracture energy, \(\Delta T\) corresponds to temperature difference, \(L\) denotes crack length; \(\alpha\), \(E_0\) and \(\nu\) refer to thermal expansion coefficient, elastic modulus without cracks and Poisson’s ratio without cracks, respectively, \(\Delta T_c\) corresponds to the critical temperature difference caused by thermal shocks, and \(N\) denotes the crack density.

Assume the final crack length after crack propagation corresponding to the initial short crack is \(l_f\), hence, it satisfies the following relationship:

\[
3\frac{(\alpha \Delta T)^2 E_0}{2(1 - 2\nu)} \left[1 + \frac{16(1 - \nu^2)NL}{9(1 - 2\nu)}\right]^{-1.5} - \left[1 + \frac{16(1 - \nu^2)NLl^3}{9(1 - 2\nu)}\right]^{-1} = 0
\]

\[
= 2\pi NL (l_f^2 - l_0^2) - \frac{1}{2} \left(1 - \nu^2\right) Nl^3
give \]

\[
l_f = \sqrt{3(1 - 2\nu) / 8(1 - \nu^2) Nl^3}
\]

Considering that the length of initial short crack is much less than the final length after crack growth, the final crack length can be obtained by simplifying the aforementioned equation:

It can be found that the final propagation length of the initial short crack does not depend on any other material properties except for Poisson’s ratio, and is a function of crack density and initial crack length.

Hence, the variation curves of crack length and material strength of refractory ceramics with temperature difference can be obtained, as shown in figure 2 [19]. As shown in figure 2(a), when the temperature difference caused by thermal shocks is less than the critical temperature difference (\(\Delta T_c\)) corresponding to the short crack length, the crack length remains unchanged. When the temperature difference caused by thermal shocks reaches \(\Delta T_c\), the fracture begins and rapid increase of the crack length occurs instantly due to the dynamic behavior of

Figure 1. Flowchart of structural safety assessment of refractory ceramic materials.
crack growth. At the same time, the material strength drops rapidly at the critical temperature difference of $\Delta T_c$ (figure 2(b)). At this moment, the crack length is greater than the right critical crack length corresponding to the temperature difference, and the crack is in the subcritical state. If the critical temperature difference corresponding to the crack length at this moment is $\Delta T'_c$, $\Delta T' > \Delta T_c$. When the temperature difference is between $\Delta T'_c$ and $\Delta T_c$, the crack is always in the subcritical state, i.e., the crack does not grow. As shown in figures 2(a) and (b), the curves are flat lines between $\Delta T'_c$ and $\Delta T_c$. When the temperature difference caused by thermal shock is greater than $\Delta T'_c$, the crack continues to propagate. At this moment, quasi-static propagation of crack occurs, showing a smooth curve after $\Delta T'_c$ in figures 2(a) and (b).

The above results reveal that when the initial length of micro-crack reaches the critical crack length ($l_0$), corresponding to the temperature difference $\Delta T_c$ due to thermal shock, the crack length will dynamically and instantly increase to a large value and the strength will rapidly decrease, leading to the material failure. Therefore, the left critical crack length corresponding to $\Delta T$ can be used as the failure criterion of the material under the action of thermal shock with a constant temperature difference.

3.2. Strength attenuation criterion
During the failure process of high-temperature refractory materials, micro-crack propagation, grain boundary creep, and increased macro-effect from microscopic defects manifest the decrease of residual strength. Herein, the residual strength refers to the strength that remained after the initial strength attenuation of the material due to fatigue. The crack initiation process and crack propagation process are affected by several uncertainty factors, such as applied stress, temperature, boiler atmosphere and external environment. For example, dense oxide film is easy to form on the surface of refractory ceramics in oxidizing atmosphere, which can effectively prevent the infiltration of oxygen and slow down the oxidation inside the material. However, some ions (e.g. $F^-$, $Na^+$, $S^-$) generated by impurities in heavy oil fuel may aggravate oxidation of the refractory ceramic, resulting in the reduction of the strength and the generation of cracks. As a result, micro-analysis based on fracture mechanics may lead to the deviation of analysis results from reality. However, it is intuitive and appropriate to use the reduction of residual strength to characterize the fatigue damage and the whole fatigue process, regardless of whether there is slow crack propagation or not during the fatigue process of refractory materials. The following strength-stress attenuation formula can be established [22]:

$$\frac{d\sigma_r}{dt} = -A \cdot \sigma(t)^n \cdot t^{-m}$$

(4)

where $\sigma_r$ refers to the residual strength, $A$, $n$, and $m$ are all material constants ($A > 0$, $n \geq 1$, $0 \leq m \leq 1$), $n$ denotes the stress exponent, which characterizes the influence of stress on the attenuation of residual strength, and $m$ corresponds to the time exponent, which characterizes the variation of residual strength attenuation with fatigue time.

The initial conditions are:

$$t = t_0, \sigma_r = \sigma_0$$

(5)

where $\sigma_0$ represents the strength value of material under initial conditions, namely the initial fatigue strength.

The fracture conditions are:

$$t = T, \sigma_r = \sigma(T)$$

(6)
Hence, after the fatigue effect at $T$, the material fails when the residual strength ($\sigma_r$) decreases to the same value as the applied stress level $\sigma(T)$.

For the Paris formulas of crack propagation rate \[ (7) \]
\[
\frac{da}{dt} = AK^n, \quad K_0 = \sigma \sqrt{\pi a} f(a, W, \cdots)
\]

where $A$ and $n$ are material constants, $\sigma$ refers to the nominal stress, $a$ represents the crack size, and $f(a, W, \cdots)$ denotes the geometric correction factor that reflects the influence of component dimensions and crack size on the crack stress field.

According to the aforementioned Paris formulas, the fatigue equation under cyclic loading can be given as:

\[
\sigma^m = C
\]

During the fatigue process, the fatigue stress periodically varies with the fatigue time. Assuming that the maximum and minimum stresses are $\sigma_{\text{max}}$ and $\sigma_{\text{min}}$, respectively, the stress ratio can be defined as $R = \sigma_{\text{min}} / \sigma_{\text{max}}$. Assuming that the cyclic fatigue life is $T_c = NH$, where $H$ refers to the cyclic period and $N$ represents the cycle index. When cyclic fatigue fracture stress $\sigma_c = \sigma_{\text{max}}$ and $m = 0$, we can obtain equation (9) by substituting $\sigma_c = \sigma_{\text{max}}$ and $m = 0$ into equation (4) and integrating equation (4):

\[
T_c = \frac{(\sigma_0 - \sigma_{\text{max}})(n + 1)}{A\sigma_{\text{max}}^n (1 + R + R^2 + \cdots + R^n)}
\]

Hence, when the residual strength is less than the applied stress level, the material fails and corresponding life becomes $T_c$.

The safety criterion based on strength attenuation comprehensively considers the influence of stress, defects, and cracks on the strength of refractory materials, and effectively overcomes the uncertainty of fracture mechanics analysis during crack initiation, crack propagation and fracture failure.

3.3. Stress intensity factor criterion

In the case of metals, the crack propagation process can be divided into three stages, as shown in figure 3(a) \[23\]. Herein, the crack propagation rate caused by periodic alternating loads is expressed as $da/dN$. When the amplitude of stress intensity factor ($\Delta K$) of crack is lower than the threshold ($\Delta K_{th}$) of fatigue crack propagation, the fatigue crack propagation rate is close to zero. Then, it can be regarded that there is no crack propagation, which is denoted as Region I. With the increase of $\Delta K$, the propagation rate of fatigue crack increases slowly at an appropriately constant rate. In general, this stage is called the middle steady-state propagation zone (denoted as Region II), which is the focus of fatigue fracture research. When the amplitude of stress intensity factor ($\Delta K$) continues to increase and exceeds the fatigue fracture toughness ($K_{IC}$), the fatigue crack propagation rate increases rapidly until fracture failure occurs, and this stage is called the rapid propagation zone (Region III).

In recent years, it has been found that the fatigue crack propagation curve of ceramics is similar to that of metals. Figure 3(b) shows the variation of fatigue crack propagation rate of several ceramic materials with the amplitude of stress intensity factor ($\Delta K$). It can be seen that the edge of curves is almost vertical to the abscissa axis. Hence, $\Delta K_{th}$ and $K_{IC}$ can be regarded as the upper and lower boundaries of the middle steady-state propagation region, respectively. The values of $\Delta K_{th}$ and $K_{IC}$ are related to the intrinsic properties of the
material. According to the previous experimental research results, the values of $\Delta K_{th}$ and $K_{IC}$ of refractory ceramics are $2.2 \text{ MPa m}^{1/2}$ and $6.5 \text{ MPa m}^{1/2}$, respectively.

Cracks in components are usually classified into three types, including opening (type I) cracks, sliding (type II) cracks, and tearing (III) cracks. The opening cracks bear the normal stress perpendicular to the crack, which is the most common and easiest cracking mode to cause fracture failure in engineering applications. Therefore, type I cracks (opening cracks) are selected as the research object in this study. In 1957, Irwin et al. [24] have proposed the fatigue fracture criterion by the stress intensity factor, which can be given as:

$$K_1 = K_{IC}$$

(10)

where $K_1$ refers to the stress intensity factor, which is same as in equation (7), and $K_{IC}$ represents the fracture toughness of the material. Equation (10) shows that, when the stress intensity factor of cracks reaches the fracture toughness, the cracks propagate unstably. Equation (7) shows that the crack size renders an important influence on the stress intensity factor. Hence, the variation rules of the stress intensity factor can be deduced by studying the changes in crack size. Furthermore, the crack propagation characteristics of refractory materials can be explored by combining with the fatigue crack propagation rate.

4. Experimental

4.1. Experimental results based on the critical temperature difference criterion

According to the critical temperature difference formula (equation (1)), the steady-state crack curve of refractory ceramics under thermal shocks can be obtained, as shown in figure 4. The critical crack length, corresponding to the temperature difference caused by thermal shocks, can be used as the safety criterion for the failure of materials. The unstable region of the crack at the upper part of the curve is the crack propagation region, and the crack stable region is at the lower part of the curve. The three curves in figure 4 are the temperature difference versus crack length curves, corresponding to different initial crack densities under the condition of same physical properties of materials. It can be found that the initial crack density of the material does not influence critical temperature difference, corresponding to the short crack. However, initial crack density renders a great influence on critical temperature difference corresponding to the long crack. For long cracks, the critical temperature difference increases with the increase of crack density in the material.

The thermal shock caused by the start and stop of supercharged boilers promotes the initiation and propagation of cracks around the pores of high-temperature refractory ceramics, especially during the thermal shock stage of boiler stop when the surface of refractory ceramics is subjected to huge tensile stress. The experimental results reveal that the temperature of refractory ceramics sharply changes within 120 s after the boiler is stopped, and the temperature difference reaches $530 \degree C$. The left critical crack length corresponding to the constant temperature difference of $530 \degree C$ is $0.0021 \text{ cm (21 \mu m)}$. Figure 5 presents the surface microstructure of the unused high-temperature refractory ceramics (reaction bonded silicon nitride-bonded silicon carbide). It can be found that the size of pores (initial defects) is significantly smaller than the above calculated left critical crack length under the constant temperature difference. During practical applications of refractory materials,
with the periodic action of thermal shocks, the abovementioned short cracks grow slowly until their length reaches the left critical crack length and, then, rapidly propagates in the form of carrying kinetic energy under the action of thermal shocks at a constant temperature difference. According to equation (3), the calculated final crack propagation length is \( l_f = 10.87 \text{ cm} \) (assuming crack density \( (N) = 1 \text{ cm}^{-3} \)), which is a macroscopically visible length. In this case, the strength of refractory ceramics decreases rapidly, leading to the failure of materials. In conclusion, the left critical crack length \( l_c = 21 \mu \text{m} \), obtained from the above calculations, can be used as the criterion for thermal shock failure of refractory ceramics under a constant temperature difference.

4.2. Experimental results based on strength attenuation criterion

The stress variation rules of high-temperature refractory ceramics can be obtained by conducting statistics on the service time of supercharged boiler. It is found that the service time of high-temperature refractory ceramics in supercharged boiler obeys normal distribution. Figure 6 shows the relevant statistical analysis data. The average value is easily affected by extreme values, the data were sorted and ranked, and the data at the ends were removed according to a certain proportion. Thus, only the data in the middle positions were used to calculate the average value, which was the truncated average value. The 5% truncated average value was calculated and the result was 380.21, which is approximately equal to the overall average value of 379.22, indicating that the influence of extreme values on the average value is basically offset. Therefore, the service time of refractory ceramics obeys the following normal distribution :

![Figure 5. The surface microstructure of refractory ceramics.](image)

![Figure 6. Histogram of the service time of refractory ceramics.](image)
where the high-temperature bending strength of refractory ceramics is $\sigma_0 = 52$ MPa, the applied stress level is $\sigma(T) = 20$ MPa, and the experimental results of $A$ and $n$ are $4.48 \times 10^{-13}$ and 5.01, respectively. The substitution of above parameters into equation (12) yields $T_c = 2.17 \times 10^7$ s, and the corresponding cycle number is $N_c \approx 951$ (cyclic time $= 380$ min). Based on these results, the cyclic period of refractory ceramics is 951 times and the total life is 6027.7 h for the stress level of supercharged boiler, which is comparable to the actual life.

4.3. Experimental results based on the stress intensity factor criterion

There are many factors affecting the stress intensity factor of fatigue cracks, such as the value of applied load, length and depth of fatigue crack, and shape of a fatigue crack. Considering practical engineering applications, this study mainly investigated the influence of fatigue crack length and depth on the stress intensity factor. Herein, the fatigue crack is simplified to elliptical surface crack and the crack generated in high-temperature refractory ceramics is shown in figure 8(a). Based on the above results, it can be seen that the fatigue crack propagation mainly occurs during the cooling process. So, the analysis process should be carried out by successively analyzing temperature field change, stress change and stress intensity factor at the fatigue crack. Figure 8(b) shows the change in temperature of supercharged boiler after the boiler stopped.

The fatigue crack depth and length can both affect the stress intensity factor of a fatigue crack. First, the influence of crack depth $a$ on stress intensity factor $K_I$ of fatigue crack is analyzed when the crack length $c$ is constant. Figure 9 shows the variation of $K_I$ with crack depth $a$ when the crack length is 2 mm, 50 mm and 100 mm. It can be found that the stress intensity factor $K_I$ increased and gradually reached the fatigue crack propagation threshold with the increase of crack tip depth, entering the steady-state propagation zone. However, when the crack depth increases to a certain extent, the stress intensity factor decreases. With the decrease of temperature, the internal shrinkage of refractory ceramics induces pressure on the crack tip, resulting in a smaller stress intensity factor and even crack closure.

Second, the influence of crack length $c$ on the stress intensity factor is studied when the crack depth $a$ is constant. Figures 10(a) and (b) show the variation of $K_I$ with crack length when the crack depth $a$ is 2 mm and 4
The results show that the $K_I$ initially increased with the increase of $c$ value and gradually decreased when the crack length reached a certain value. Based on the stress field of refractory ceramics, it can be known that, when refractory ceramics shrink due to temperature decrease, the surface zone under tensile stress is mainly concentrated in the middle region and that the tensile stress is gradually transformed into compressive stress in the region near the boundary, resulting in the gradual decrease of $K_I$ and even crack closure with the increase of $c$ value.

In order to more intuitively display the influence of crack size on stress intensity factor, a three-dimensional surface diagram was generated according to the above calculation results, as shown in figure 11. It can be found that the crack depth (a) renders a more significant influence on stress intensity factor during crack propagation than the crack length (c). Based on the previous analysis, when crack initiation occurs, the crack propagation of refractory ceramics is relatively slow and the stress intensity factor does not reach the fatigue fracture toughness that can induce rapid crack propagation. However, in practical operation, the refractory ceramics may break and

![Figure 8](image1.png)

**Figure 8.** (a) Schematic diagram of cracks in refractory ceramics and (b) change in surface temperature after boiler stop.

![Figure 9](image2.png)

**Figure 9.** The variation of $K_I$ with crack depth: (a) $c = 2$ mm, (b) $c = 50$ mm, (c) $c = 100$ mm.
fall off even if there is no rapid crack propagation. This phenomenon can be explained as follow: In practical applications, the actual fatigue fracture toughness may be lower than the theoretical value. If the stress intensity factor reaches the critical safety value, rapid crack propagation occurs. Notably, this critical safety value needs to be obtained from a large amount of practical experience.

5. Conclusions

In summary, a three-factor comprehensive criterion to assess the structural safety of refractory ceramics has been proposed by considering three main factors that induced the failure of refractory ceramics, i.e., transient thermal shocks, strength attenuation and crack propagation. Finally, a safety assessment model is established by using the proposed theory as a failure criterion. The key results can be summarized as:

(1) Based on the critical temperature difference criterion, the temperature of refractory ceramics changes sharply (530 °C) within 120 s after the boiler stop. The corresponding left critical crack length is found to be 21 μm. With the action of thermal shocks, the initial short crack propagates and the final length \((l_f)\) of crack reaches 10.87 cm \((N = 1 \text{ cm}^{-3})\), leading to the strength degradation and even failure of refractory ceramics.

(2) For strength attenuation criterion, when the maximum compressive or tensile stress exceeds the intrinsic strength of the material, the cracks initiate in refractory ceramics. The average stress (20 MPa) can be selected as the applied stress level for life prediction. The calculation results show that the total life of refractory ceramics is 4465 h, which is comparable to the actual service life.
(3) The analysis results based on the stress intensity factor criterion reveal that the stress intensity factor \( (K_I) \) initially increases with the increase of crack depth \( (a) \) and crack length \( (c) \) during crack propagation, but starts to decrease when \( a \) or \( c \) increases to a certain value.

Overall, the three-factor comprehensive criterion is helpful in solving the problem of traditional structural safety criteria, which cannot be applied in refractory ceramics. Also, the proposed criterion provides the basis for the technical risk assessment of refractory ceramics, which has important engineering significance.

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Data availability statement

All data that support the findings of this article are included within the article (and any supplementary files).

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