Influence of solid boundary conditions on the evolution of free and wall-bounded turbulent flows

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Abstract. The effect of the boundary conditions on the organization of vortex clusters is analyzed in two separate cases: rough channel flow and grid-generated turbulence. The aim is to understand how far the fluid structures are affected by the presence of roughness and the geometry of the grid. The grid-turbulence cases show that the single- and multi-scale geometries generate a flow strongly dominated by the shear at the beginning. The shear is initially caused by the presence of the body and, for the multi-scale grids, subsequently by the large differences between the scales. Further downstream from the grid these shear-dominated structures break up and form more isotropic clusters, whose dimensions seem to depend little on the particular geometry of the grid. For fractal grids, clusters are formed right downstream of the grid, resulting in a flow with less inhomogeneities than for single- and multi-scale grids. Eight different rough surfaces have been analyzed. In the smooth channel, both attached and detached clusters have been found and, depending on the geometry, the roughness affects the attached structures. Roughnesses made of aligned obstacles with a large separation seem to reduce the number of these structures in the flow. When the roughness elements are closely packed, both for transverse and aligned obstacles, the attached clusters are not able to reach within the roughness elements, and they seem to be anchored to the plane of the crests.

1. Introduction

Many engineering applications require the manipulation of flow features like turbulence or mixing far from the solid boundaries where actuators are placed. Most of these cases fall into one of two categories: wall-bounded turbulent flows or turbulent wakes past an obstacle. The underlying question in both cases is the same: how far from the boundary condition (actuator on the wall or obstacle) are we able to modify a turbulent flow.

From the point of view of the turbulent wake past an obstacle, it is possible to draw an analogy between the boundary conditions and a Cauchy problem: the interaction of the object with the fluid particles sets their initial conditions, and they are then advected downstream, where they are free to evolve by themselves. One of the canonical cases in this category is grid turbulence, where a uniform flow is forced through a solid grid. In the region close to the grid the flow is highly anisotropic, being affected by the shape of the grid. Then, depending of the size of the vortices generated, an energy cascade process takes place: the larger eddies break-up, forming smaller scales. During this process the flow loses information about the initial condition.
and tends to isotropy. The resulting turbulent flow decays freely, in a similar way as in freely decaying turbulence simulations. In this case, the parameters of interest are the maximum level of isotropy and how quickly that condition is achieved in terms of distance to the grid. This problem was first studied by Taylor (1935) [8] and von Kármán (1937) [7]. Since then, several groups have analyzed the effect that the shape of the grid has on the rate of decay of the resulting turbulence: Lavoie et al. (2007) [9], Seoud and Vassilicos (2007) [10] and Ertunc et al. (2010) [11], among others. Specially interesting is the work by Seoud and Vassilicos (2007) [10], Hurst and Vassilicos (2007) [12] and Mazellier et al. (2010) [13]. Using multi-scale grids (grids with holes of different sizes) they were able to produce flows with unusual turbulence properties, which might lead to the design of new industrial mixers or combustion chambers.

From the point of view of the modification of a turbulent boundary layer using actuators at the wall, the problem is understanding the effect of the solid boundary conditions in wall-bounded turbulence. There has been extensive work on this problem, starting with the classical theory by Townsend (1976) [1]. Townsend postulated that, if \( Re \) is sufficiently high, the extent of the direct influence of the surface is limited to the near-wall region, while the rest of the flow only sees the slip velocity at the upper limit of the wall region. Most of the studies of turbulence over rough walls support this idea (see review by Jiménez, 2004 [2]): the difference between the mean velocity profiles of smooth- and rough-wall cases is the so-called roughness function, a roughly constant velocity difference. This velocity difference has been linked to the root-mean-squared vertical velocity at the plane of the crests, a very important quantity that plays a major role in transition to turbulence (Orlandi (2011) [3]) and allows one to write a simplified expression for the velocity profile in the logarithmic layer (Orlandi and Leonardi (2006) [4]). Another very important aspect of the influence of the boundary condition is the forces that the wall imposes on the flow, namely the drag. Leonardi et al. (2007) [5] linked the drag components (friction and pressure) in each case with the behaviour of the velocity structures in the overlying flow (d-type or k-type roughness). This can be exploited also to reduce drag, as in riblets. Indeed, García-Mayoral and Jiménez (2011) [6] explained the mechanisms behind drag reduction and its disruption with the appearance of spanwise rollers close to the wall (\( y^+ < 20 \)). It is then apparent that, in wall bounded turbulence, it is possible to modify the roughness function (which determines the mass flux) and the drag (which determines the pressure gradient) from the wall. However, the question of how far from the wall can we change turbulence remains open.

The macroscopic influence of the boundary conditions on the properties of the turbulent flows discussed above are due to the effects that both the roughness of the wall and the shape of the grids have on the velocity structures interacting with them. In this article, we study the statistical properties of these velocity structures, applying a coherent-structure tracking algorithm developed by del Álamo et al. (2006) [14] to our database of rough-walled channels and grid turbulence cases. This method allows to extract individual structures, and to acquire all the statistics necessary to understand the effect of the boundary conditions on the their formation and evolution, as well as to link the macroscopic effect of the boundary condition to these structures.

This clustering algorithm was originally applied by del Álamo et al. (2006) [14] to smooth-walled turbulent channels, and later by Flores et al. (2007) [15] to turbulent channels with disturbances at the boundary conditions at the walls (artificial roughness). To the authors knowledge, this is the first time that this algorithm is applied to turbulent channels with fully resolved roughness, and to grid turbulence cases.

This work is structured as follows: in section 2 we summarize the characteristics of the simulations and describe the numerical method applied. In section 3, the clustering technique is explained, and the results of the percolation analysis are presented. The joint probability density functions (p.d.f.s) of the invariant \( Q \) and of the clusters statistics are presented in section 4, along with three-dimensional representations of the structures. Finally, section 5 provides conclusions.
2. Numerical experiments

The non-dimensional incompressible Navier–Stokes equations, solved in this work, are

\[ \frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_j^2} + \Pi \delta_{1i}, \quad \frac{\partial u_i}{\partial x_i} = 0, \]

where \( \Pi \) is the pressure gradient required to maintain a constant mass flow rate in the channel flow (\( \Pi \) is zero in the grid flow), \( u_i \) is the component of the velocity vector in the \( i \) direction, \( p \) is the kinematic pressure and \( Re \) is the Reynolds number. This set of equations has been discretized in an orthogonal coordinate system, with a staggered second-order finite-difference method. The integration in time uses a third-order low-storage Runge–Kutta algorithm, coupled with a second-order Crank–Nicolson scheme. To correct the non-divergence-free field, the fractional step procedure is used. Further details are found in Orlandi (2000) [16] and are not repeated here.

The interaction between the flow and the solid is reproduced with the immersed-boundary technique, which assumes that the velocity is zero inside the body, and applies a correction to the viscous term at the first grid point near the solid surface. This method is described in detail in Orlandi and Leonardi (2006) [4].

For the channel, the periodicity assumed in the streamwise and spanwise directions implies uniform grids in \( x \) and \( z \). No-slip conditions at the walls and at the roughness requires a non-uniform grid in the wall-normal direction. In grid turbulence, the same resolution is used in the three directions. The solid grid is normal to the flow direction \( (x) \), and periodicity in \( y \) and \( z \) is imposed. Radiative conditions in \( x \) are applied at the outlet.

2.1. Simulation details for the grid-generated turbulence

In order to mimic wind tunnel experiments, the inlet flow is uniform with random fluctuations of small amplitude superimposed at \( x = 0 \). The large scales and hence the energy-containing eddies are generated by the solid grid, placed at \( x_G = 2 \). The shape of the grid dictates the velocity distribution at its exit, which has been checked to be independent of its location. The number of points and the dimensions of the domain are listed in table 1.

The solid grids considered are shown in figure 1 with the geometric parameters listed in table 1. For the regular grids \( G_{4-4-S} \) and \( G_{4-4-C} \), and the multi-scale grids \( G_{12-4} \) and \( G_{32-4} \), the mesh length of the grid is defined as \( M = 2\pi/N_a \), where \( N_a \) indicates the number of holes in a row. The effective mesh length for the fractal grid is \( M_{eff} = 4(L_2 L_3)/\sqrt{1-r/P} = 0.43 \), where \( r \) is the solidity of the grid defined below and \( P \) is the fractal perimeter length. Further details can be found in Hurst and Vassilicos (2007) [12]. Table 1 also shows the solidity of the grid \( r = V_{solid}/V_{tot} \), defined as the ratio between the solid and total volumes. All the grid turbulence simulations were performed at Reynolds number based on the domain length \( L_2 \), \( Re_{L_2} = U_1 L_2/\nu = 3000 \), where \( U_1 \) is the uniform velocity imposed in the region upstream of the grid.

Figure 3(a) shows the decay of the turbulent kinetic energy with the distance \( x \) from the grid. It is possible to adjust this decay with a power law \( \langle q \rangle = (X/ML - X_0)^{-m} \) [12], where the value of the decay exponent \( m \) depends on the geometry. These exponents and the values of the turbulent kinetic energy at the origin \( \langle q_0 \rangle \) are also listed in table 1. The different origin for \( G_{frac} \) is a consequence of the different solidity of the grid \( r \), smaller for the fractal grid. This was verified in preliminary simulations, where a uniform grid \( G_{4-4} \) with \( r = 0.257 \) yielded a value for \( \langle q_0 \rangle \) closer to that of \( G_{frac} \). Physically, this simply states that the effect of the grid on the flow (i.e., the transfer of kinetic energy from the mean flow to turbulence) is smaller as the solidity decreases. The figure shows also that different geometries have different slopes of \( \langle q_0 \rangle \) at the exit from the grid. In particular, for the single scale grids, \( d\langle q_0 \rangle/dx \) is positive, as
expected, because of the production term. For the multiscale and fractal grids \( d\langle q_0 \rangle/dx < 0 \) because the grid generates smaller scales closer to the grid that decay faster. Since the plot reports the average on a plane parallel to the grid, this effect is also taken into account in the decrease of total energy.

2.2. Simulation details for the rough channel flow

Regarding the channel simulations, \( h = 1 \) is the channel half-height, not including the roughness layer, and the dimensions of the computational domain are \( L_1 = 8 \) in the stream-wise direction and \( L_3 = 4 \) in the span-wise direction. The roughness is located only on the lower wall \((-1.2 < y < -1)\), while on the upper boundary the surface is smooth. All the simulations are performed at \( Re_{bulk} = 2800 \), corresponding to a smooth channel with \( Re_\tau \approx 180 \). The different values of \( Re_\tau \) on the channel walls, along with other parameters of interest, are given in table 2. The different values of \( Re_\tau \)|R at the bottom wall are due to the different turbulent activity. The flow on the upper wall is not influenced by the lower wall, resulting in more similar values for \( Re_\tau \)|S. At this Reynolds number, the roughness height is \( k = 0.2 \), and \( k^+ \) varies from 32.3 up to 60.1.

Several geometries have been reproduced (figure 2). The smooth channel (CHA) acts as both validation of the results and reference for the other cases. The rough walls include a three-dimensional geometry made of prisms (PRSH), transverse (SQWK1) or aligned square bars with two aspect ratios (SQBAR, SQBSM), and aligned triangular bars with three aspect ratios (TRBAR, TRBSM, TRBSSM).

An interesting aspect of these cases relates to the pressure gradient. As shown in the last column of table 2, the three-dimensional (3D) geometry causes a large drag increase, mainly due to the form drag. This term is also oscillatory, which is a sign of greater turbulent activity, and has been linked to strong vertical velocity events. The longitudinal square bars cause a moderate increase of the pressure gradient. One of the triangular-bars cases yields a drag reduction of
Table 2. Roughness parameters. Subscripts \( R \) and \( S \) are relative to the bottom rough wall and upper smooth wall respectively. \( \Pi \) is the pressure gradient, with the subscript \( i \) representing the geometry.

| Geometry | \( k^+ \) | \( Re_\tau|_R \) | \( Re_\tau|_S \) | \( 10^6 \frac{\Pi_{CHA}}{\Pi_{CHA}} \) |
|----------|-----------|----------------|----------------|------------------|
| CHA      | 0.0       | 176.1          | 176.1          | 0.0              |
| PRSH     | 60.1      | 295.9          | 189.5          | 85.4             |
| SQBAR    | 39.7      | 199.8          | 177.2          | 9.9              |
| SQBSM    | 41.1      | 204.4          | 178.1          | 13.7             |
| SQWK1    | 40.8      | 203.7          | 177.3          | 12.5             |
| TRBAR    | 37.6      | 190.2          | 174.3          | 0.1              |
| TRBSM    | 32.3      | 162.8          | 171.8          | 13.7             |
| TRBSSM   | 36.1      | 177.8          | 174.2          | 3.9              |

Figure 2. Roughness in the cases of a) PRSH, b) SQBAR, c) SQBSM, d) TRBAR, e) TRBSM, f) TRBSSM. From b) to f) only a portion of the domain is represented. The transverse square bars case (SQWK1) is the same as b), inverting the \( z \) and \( x \) axes.

nearly 14%, and the others are in line with the smooth channel case. In the transverse bars case, the form drag is low because \( w/k = 1 \) (\( w \) is the spacing between bars). Leonardi et al. [17] demonstrated that the form drag has a maximum for \( w/k = 7 \).

3. Clusters identification and classification

To identify and classify vortex clusters, the method described by del Álamo et al. (2006) [14] has been used. The same technique has been already adopted by Flores et al. (2006) [15] to investigate the vorticity organization in the outer layer of turbulent channels. This process is based on the vortex eduction method by Chong et al. (1990) [18]. According to those authors, a vortex core is a region where the velocity gradient tensor \( \nabla u \) is dominated by its rotational part. Expressed in terms of the second invariant of \( \nabla u \), this condition yields \( Q > 0 \). To identify regions in which \( \omega_i\omega_j/2 \) is significantly larger than \( s_{ij}s_{ij} \), a threshold is used: \( Q > Q^* \). This eduction method depends on the value of the threshold and, if the flow has an inhomogeneous direction, the dependence on the threshold becomes troublesome, complicating the comparison of data at different values of the coordinate. Nagaosa and Handler (2003) [19], applying the \( Q \) criterion to wall-turbulence, found that the p.d.f of the second invariant, normalized with its standard deviation, \( Q/(Q^2)^{1/2} \), is homogeneous everywhere except in the viscous sublayer.
Starting from these results, del Álamo et al. (2006) [14], using $D$, the discriminant of $\nabla u$, presented a procedure in which a point $x$ belongs to a vortex if $D(x) > \alpha \langle \dot{Q}^2(x) \rangle^{1/2}$, where $\alpha$ is the actual thresholding parameter and $\langle \dot{Q}^2(y) \rangle^{1/2}$ is the standard deviation of $D$. This procedure solves the problem of the inhomogeneity, and yields a vortex volume fraction which depends much less on the inhomogeneous direction than the one obtained using a uniform threshold.

In the present work, the clustering technique just described has been applied to grid generated turbulence and channel flows, where the inhomogeneous directions are respectively $x$ (grid normal) and $y$ (wall normal).

To identify the clusters, the second invariant of $\nabla u$ has been used. A grid point belongs to a cluster if $Q(x) > \alpha \langle \dot{Q}^2(\xi) \rangle^{1/2}$, where $\xi$ is the inhomogeneous direction. Averaging in the plane normal to the $\xi$ axis, it is possible to obtain $\langle \dot{Q}^2(\xi) \rangle^{1/2}$, shown in figure 3(b) for grid turbulence.

An appropriate value for the threshold $\alpha$ can be obtained through the percolation analysis (Moisy and Jiménez (2004) [20] and Stauffer (1985) [21]). Figures 4(a,c) represent the ratio between the volume of the largest cluster $V_{\text{max}}$ and the overall volume $V$ occupied by the clusters, as function of the thresholding parameter. When $\alpha > 1$, only a few small objects are identified. Decreasing $\alpha$, new clusters appear while other coalesce, and the variation of $V_{\text{max}}/V$ is the result of the trade-off between the two processes. For grid turbulence, if $\alpha \approx 0.85$, $V_{\text{max}}/V$ remains roughly constant while the number of clusters that can be visualized is almost at its maximum (figures 4b,d). For the channel, the plot of the volume ratio is slightly more dispersed, while the peak in the number of clusters always occurs at $\alpha \approx 0.75$. Therefore, this value has been used in the subsequent analysis.

4. Clusters statistics

With the chosen value for the threshold, the identification algorithm has been applied to the domain. For grid turbulence, the interest is to detect clusters located immediately after the grid, and for this reason the analysis was performed on a domain $2.25 < x < 18\pi$ which excludes the grid. In the rough channel, to investigate the interaction of the flow structures with the roughness elements, all the points, except those inside the solid body, are used for the clustering algorithm. In both cases, to compute the statistics, only clusters whose volume is larger than thirty grid points have been considered.
4.1 Grid turbulence

4.1.1 P.d.f.s of $Q$ The clustering technique described in section 3 uses the standard deviation $(Q^2)^{1/2}$ to normalize $Q$, so that the p.d.f. does not vary with the inhomogeneous direction. To verify this normalization for the grid turbulence cases, figure 5 shows the p.d.f.s of $Q$ for all the geometries. The colours indicate the percentage of the data contained in the corresponding iso-probability contour. The good similarity in $x$ confirms the suitability of this technique in the case of grid turbulence.

The figure also shows that the shape of the grid influences the p.d.f. of $Q$, which for the multi-scale ($G_{12-4}$ and $G_{32-4}$) and single-scale grids ($G_{4-4-S}$ and $G_{4-4-C}$) are skewed towards negative values. From the definition of $Q = \omega_i \omega_j/2 - s_{ij}s_{ij}$, this suggest that close to the grid the flow is dominated by the shear $s_{ij}s_{ij}$, consistent with the presence of a wake and shear layers downstream of the grid. In the case of single-scale grids, this shear is powered only by the wake of the grid, and its survival depends on the size of its holes. In the case of multi-scale grids, in addition to the small-scale shear there is a large-scale shear that is produced by the different velocities and length scales downstream of the two different meshes. This happens because these two flows have a similar behaviour to a jet interfacing with a different flow. This is the reason for the negative peak of $Q$ that appears only in the p.d.f.s of $G_{12-4}$ and $G_{32-4}$. Figure 3(c) shows the profiles of the shear $\langle \partial U/\partial y \rangle$ at different distances from the grid for $G_{32-4}$. Initially, two peaks are located at the two interfaces between the large- and small-scale flows, and these two homogeneous regions interact forming a mixing layer (Veeravalli and Warhaft (1989) [22]).

Note that, due to this shear-dominated region, few clusters (if any) will be identified close downstream of the grid. This is made explicit in figure 5 by the vertical black line, indicating the adopted threshold $Q/(Q^2)^{1/2} = 0.85$. The only exception is the fractal grid, whose p.d.f. is roughly symmetric, suggesting that clusters will be found right downstream of the grid. This can be observed in figure 6, where a visualization of $Q/(Q^2)^{1/2}$ is shown. While single-scale and
multi-scale grids only show vortices relatively far from the grid, the fractal grid shows evidence of vortices in close proximity to the grid.

4.1.2. Clusters distribution, orientation and dimensions  

The clustering algorithm was run for approximately forty fields (approximately one washout), to extract approximately $N_c \approx 10^4$ clusters with more than 30 data points. With this database, it was possible to obtain converged statistics of their geometry and distribution. The lengths of the box enclosing the cluster are $L_x$, $L_y$ and $L_z$. Figure 7 shows joint p.d.f.s of the streamwise length of the structures ($L_x$) as function of the coordinate of its centre ($x_c$), evaluated as the average between the points of the cluster that are closest to and farthest from the grid. Note that the black dashed line ($L_x = 2x_c$)

![Figure 6. 3D contour plot of $Q/(Q')^{1/2} = 2$, coloured by the distance from the grid for
a) $G_{4-4-S}$ ($\frac{u_{eff}}{u_L} < \frac{x}{x_M} < 17.2$), b) $G_{32-4}$ ($\frac{u_{eff}}{u_L} < \frac{x}{x_M} < 16.5$).]
limits the plot, because the streamwise length of any cluster cannot be longer than twice the position of its centre.

Figure 7 confirms that there are no clusters attached to the solid grid for the single- and multi-scale grids. This supports that the geometry of the grid dictates whether or not the flow is initially dominated by the shear. For $G_{\text{frac}}$, vortical structures are attached to the grid, while for single- and multi-scale grids the near region is shear-dominated and vortical structures (and clusters) are only formed downstream. It is interesting to note in figure 6 that the clusters are predominantly generated downstream of the larger holes, while the flow downstream of the finer mesh appears to be roughly irrotational.

A further interesting question is whether the size distribution of clusters is influenced by the geometry of the grid. Depending on the exponent $n$ in $\Delta V/\eta^3 = (L_i/\eta)^n$, it is possible to infer the shape of these structures. If $n \approx 1$ the other two dimensions do not increase proportionally, meaning that the structure is elongated only in one direction (i.e. it is a tube). For $n \approx 2$ or 3 the structures have a sheet-like or cubical shape respectively and, if the exponent $n$ is similar for all three $L_i$, there is no preferential direction of alignment. Figure 8 shows the joint p.d.f.s relative to the volume of the clusters in Kolmogorov units, as functions of one of the lengths of the clusters, suggesting that the clusters are relatively isotropic objects, with circumscribing boxes that are approximately cubical. This observation can be made for all grids, suggesting that the influence of the grid geometry is small in the relative dimensions of the clusters.

It should be noted that figure 8 shows all the clusters, irrespective of their position downstream of the grid. In order to analyse the size distribution as a function of the distance to the grid, figure 9 shows the joint p.d.f. of volume and length of the clusters for three different regions downstream of the grid. These regions are the different stages of the decay of the turbulent kinetic energy $\langle q \rangle$ shown in figure 3(a), and can be identified by the changes in the
Joint p.d.f.s in log-log scale of the volume of cluster scaled with the Kolmogorov scale $\Delta V/\eta^3$, as function of the length: 1) $L_x/\eta$; 2) $L_y/\eta$; 3) $L_z/\eta$. For a) $G_{4-4-S}$; b) $G_{fract}$ with $Q(x) > \alpha(Q^2(x))^{1/2}$.

Colours as in previous figures. The black dashed line indicates the power law $\Delta V/\eta^3 = (L_i/\eta)^{2.5}$.

decay exponent.

It is clear in the figure 9 that the shape of the grid has a strong impact on the development of the clusters downstream of the grid. Consistent with the discussion in the previous paragraphs, the fractal grid is the only geometry producing clusters attached to the grid. Interestingly, their aspect ratio changes little with the distance from the grid, showing clusters with the same dimensions in the first ($0 < x < 1$, figure 9e1) and second ($1 < x < 16.59$, figure 9e2) regions.

For the single-scale grids, there are basically no clusters in the first region, closer to the grid. Further downstream, the grid made of squares ($G_{4-4-S}$) shows only relatively large clusters, while the mesh made of circles ($G_{4-4-C}$) shows a broader range of scales. On the other hand, the multi-scale grids show a faster generation of clusters with a broader range of sizes.

Finally, it is interesting to note that both multiscale and fractal grids show a family of objects whose volume increase roughly like $\Delta V/\eta^3 = L_i/\eta$, suggesting tube-like clusters. These objects only appear in the most downstream regions, and represent a relatively small part of the population.

4.2. Geometry and distribution of the clusters for rough channel flows

Figure 10 shows the joint p.d.f. of the minimum and maximum wall distances of the points of the cluster. The diagonal black line indicates the limit within which it is possible to find a cluster, representing the simple condition $y_{max} > y_{min}$. The vertical black line corresponds to $y_{min} = -1$, i.e. the plane of the crests, which is also the position of the wall in the smooth case. The colours correspond to the logarithm of the p.d.f. (i.e. the number of clusters in each bin), and are on the same scale in all the images, while the black contour lines represent the points at constant probability that contain 30%, 50%, 70% and 95% of the data.

In the first image, relative to the smooth channel (CHA), two families of clusters appear: the wall-detached, lying on the $y_{max} = y_{min}$ line, and the wall-attached. Considering the bottom wall, this last family is made of clusters that start from the wall (small $y_{min}$) and can have any $y_{max}$. Among these the most interesting are those tall ones whose height is comparable to the
Figure 9. Joint p.d.f.s as in the previous figure as function of the length \( L_x/\eta \). The p.d.f.s have been evaluated within three ranges for \( x \): Single-scale: 1) \( 0 < x < 3 \), 2) \( 3 < x < 15 \), 3) \( 15 < x < 54.27 \); Multi-scale: 1) \( 0 < x < 5 \), 2) \( 5 < x < 30 \), 3) \( 30 < x < 54.27 \); Fractal: 1) \( 0 < x < 1 \), 2) \( 1 < x < 16.59 \).
channel half-height ($y_{\text{max}} > -0.7$). All the attached clusters, both tall and short, are located near the $y$ axis ($y_{\text{min}} \approx -1$): the smaller ones are in the lower-left corner and the tall ones are plotted above the former. For the top wall, this family of attached clusters is shown in the figures in the top-right corner ($y_{\text{max}} \approx 1$), and this group is not changed by the boundary condition on the lower wall: both the shape of the iso-probability lines and the number of clusters in this region are the same for all the geometries. The three-dimensional geometry (PRSH) shows a peak just above the plane of the crests. The greater turbulence intensity in this region causes a number of clusters so large that the colour in the plot is skewed towards the lower-left corner.

In the aligned-square-bars flows (SQBAR and SQBSM), the formation of clusters attached to the bottom wall is reduced with respect to the smooth channel. In fact, in the figures, the iso-probability lines assume a lower maximum value at $y_{\text{min}} \approx -1.2$ than in CHA. On the other hand, in these cases, the peak of the joint p.d.f. is shifted to $y_{\text{min}} \approx -1$, that is the plane of the crests. This means that the larger attached clusters are not able to reach into the grooves. In both cases, this decrease of clusters attached to the bottom wall occurs because, between the bars, the mean velocity is smaller. The figure relative to the transverse bars (SQWK1) shows a large number of clusters that are located between the bars ($y_{\text{min}} < -1$ and $y_{\text{max}} \approx -1$, in red), that are essentially isolated from the overlying flow. Besides these clusters, there are also attached clusters, even if they are lower in number when compared to the square-bars cases (30% and 50% lines).

Similar results are obtained for triangular bars. For the largest separation (TRBAR), the family of attached clusters seems also reduced with respect the smooth wall, and it is formed by structures with $y_{\text{min}} \approx -1.2$, attached to the groove between the bars. As the separation between bars decreases, the attached clusters move from the groove to the crests, and a family of clusters embedded in the grooves start to emerge. The triangular bars with an intermediate separation (TRBSM) show attached clusters both reaching into the groove and anchored in the crest of the triangles. For the triangular bars with the narrowest separation (TRBSSM), the attached clusters again seem as numerous as in the smooth case, but very few of them reach into the grooves.

The results found with the analysis of the joint p.d.f.s are qualitatively confirmed with the visualizations of $Q/(Q'^2)^{1/2}$ in figure 11. The threshold $\alpha = 0.75$ was chosen in the percolation analysis to maximize the number of structures, but using this value for the threshold would not
Figure 11. Three-dimensional contour visualizations of $Q\div (Q'_{2})^{1/2}$. Flow is left to right, and the wall-normal direction is vertical. The values for the isosurfaces are $Q\div (Q'_{2})^{1/2} = 2.5$ for a) CHA, b) PRSH, c) SQWK1 and d) TRBSSM.

have allowed to have a clear figure of the single structures. For this reason the value for the isosurface is $Q\div (Q'_{2})^{1/2} = 2.5$ for smooth (CHA), 3D (PRSH), transverse square bars (SQWK1) and triangular aligned (TRBSSM). In the smooth channel, the structures grow from the wall towards the centre of the channel, as expected. The structures are the same on both walls, and are very similar to those on the upper wall of the other cases. In the 3D geometry (PRSH), the clusters near the wall are much smaller with respect to CHA, showing that this region is characterized by a greater turbulence activity. The transverse bars (SQWK1) show structures similar to the smooth-wall case, that are generated on the plane of the crests. The image does not show the rollers inside the cavities because the value of the threshold is too large to allow their presence. The aligned-bars cases (only the triangular TRBSSM is reported) show that the clusters tend to reach the plane of the crests, but not the bottom wall.

The joint p.d.f.s of the height $y_c = (y_{max} - y_{min})/2$ versus the streamwise and spanwise lengths of the clusters, conditional on those that have $y_{min}^+ < 20$, are shown in figure 12 and figure 13. It is important to stress the different origin of the outer and wall units. The outer unit $y$ is traslated to have the origin on the bottom wall (located at $y = -1.2$), in order to use log-scale plots. The wall unit $y^+ = (y - y_w)Re_{x,R}$ has the origin on the plane of the crests to allow the comparison with the channel flow. In these plots, the black curve represents the iso-probability level that contains 98% of the data, and the straight lines represent the self-similar scaling found in [14] and [15]: $y_c = L_x/6 = L_z/3$.

In the smooth channel, the red region in figure 12 indicates that there are $x$-elongated structures and, while the spanwise length can grow up to the order of $\delta$, for most of the clusters

Figure 12. Joint p.d.f. of $(y_c, L_x)$, conditioned to $y_{min}^+ < 20$. The black curve contains 98% of the data, the straight line is $y_c = L_x/6$
In figure 14, the joint p.d.f. of the length and width of the clusters is given. From the two self-similarity scalings found in the previous images, it follows that $L_z = L_x/2$, and this is indeed what we find in these plots. In the smooth channel, the intense red regions below the black line demonstrate that a large number of clusters have length about one order of magnitude greater than their width. The dimensions are in the range characteristic of streaks. A different family is identifiable in the figure, consisting of very short and wide structures that stay very close to the wall (see figures 12 and 13). In the 3D geometry (PRSH), $L_x$ and $L_z$ are of the same order of magnitude, and the clusters tend to be shorter than in the smooth wall case (CHA). In the transverse bars (SQWK1) we can identify relatively short and wide clusters, and figures 12 and 13 show that they correspond to clusters basically embedded in the grooves, or lying right on the plane of the crest and connecting one groove with the next. For the case of aligned bars (TRBSM), we find very long and narrow clusters, again mostly embedded within the grooves.
The density of the attached clusters as a function of height (Figure 15) shows that the attached clusters are affected by the surface in the roughness sublayer, close to the roughness elements. However, the different cases collapse above $y_c^+ \approx 50$, suggesting that the attached clusters above the roughness sublayer are not influenced by the roughness elements, similar to the conclusions reported by Flores et al. (2006) [15] for channels with disturbances on the wall boundary conditions. The present result shows further evidence of the Townsend similarity hypothesis for turbulent flows with rough walls. The noise above $y_c \approx -0.2$ is due to the small number of attached clusters with heights of the order of the channel half-height.

5. Conclusions
The vortex clusters in rough-channel and grid-generated turbulence have been analyzed, defining them as regions of the flow satisfying a criterion based on the second invariant of the velocity gradient tensor. The inhomogeneity of the flows suggests to scale $Q$ with its standard deviation, and the same percolation analysis proposed by [14] has been used to get the appropriate value of the threshold. This procedure has been applied to roughly 30 flow fields per geometry of DNS of grid-generated turbulence, and over 50 flow fields of DNS of turbulent channels with rough walls, to extract over $10^7$ objects that have been used to produce p.d.f. of their sizes and spatial distribution.

The main conclusion of this study is that, for both flows, the effect of the boundary condition on the clusters seems to be limited to a region relatively close to the solid surfaces. For instance, in the grid turbulence case, the geometry of the grid is responsible for the flow right downstream of the grid to be dominated by shear (single- and multi-scale grids) or by vorticity (fractal grids). Of course, this affects the rate of decay of turbulent kinetic energy and the distance needed to develop an isotropic turbulent flow. But sufficiently far downstream of the grid, the distribution of clusters is very similar for all the grids considered here. These clusters seem to be isotropic, circumscribed by roughly cubical boxes. Their volume is roughly proportional to their lengths to the power of 2.5, suggesting relatively full objects.

The analysis of the multi-scale grids also shows that these grids produce a strong inhomogeneity between the regions with large and small holes. Relatively large clusters are observed downstream of the larger holes, while the clusters formed downstream of the smaller holes decay faster. This inhomogeneity in the flow results in a secondary source of shear, that is likely responsible for the higher turbulent kinetic energy levels of the multi-scale grids far downstream of the grid. On the other hand, the fractal grid produces a flow with little inhomogeneity that rapidly develops a wide range of clusters.

In terms of the rough channels, two families of clusters are found: wall-attached and wall-detached. The latter are little influenced by the details of the wall. The former have lengths and widths that are approximately proportional to their height, and they are somewhat affected by
the details of the wall. For relatively open roughness, some attached clusters are found embedded within the grooves. When the roughness is aligned with the flow, they tend to form relatively long objects. When the roughness is transverse to the flow, they tend to form rollers. For very tightly spaced roughness, the distribution of attached clusters resembles that over smooth walls, but they are based on the crest of the roughness elements, not able to reach into the grooves of the roughness.

The differences between the distributions of clusters found over different roughness considered here mitigate when the tallest objects are considered. Indeed, the p.d.f of the density of attached clusters with a given height collapses for all rough surfaces once the clusters are larger than the roughness sublayer, in agreement with previous investigations [15]. This suggests, in agreement with the results of grid turbulence, that the influence of the boundary condition into the turbulent structures of the flow is limited to a region close to the boundary.

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