Warming of Monolithic Structures in Winter

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Abstract. The present work attempts to develop a mathematical model for calculating the heat transfer coefficient of the fence of monolithic structures erected in winter. The urgency and, at the same time, the practical significance of the research lies in the fact that to date no simple, effective tool has been developed to ensure the elimination of the unfavorable thermally stressed state of a structure’s concrete from maximum equalization of temperatures across its cross-section. The main problem for concrete is a high temperature which leads to a sharp decrease in the quality of erected structures due to developing cracks. This paper based on the well-known Newton’s law and its differential equation demonstrates the formula of concrete cooling and the analysis of its proportionality coefficient. Based on the literature analysis, it is established that the proportionality coefficient is determined by the thermophysical properties of concrete, the size and shape of the structure, and the intensity of its heat exchange with the surrounding medium. A limitation was used on the temperature gradient over the section of the monolithic structure to derive a formula for calculating the reduced heat transfer coefficient of a concrete fence. All mathematical calculations are given for cooling monolithic constructions in the form of plates. At the end of the work an example is given for the calculation of the required reduced heat transfer coefficient for the fence ensuring compliance with the permissible concrete temperature gradient.

1. Introduction
One of the main reasons for the deterioration of the quality of monolithic concrete in winter is its unfavorable thermo-stressed state during hardening [1,2]. Among the number of factors that cause such an unfavorable state, the most important is the uneven distribution of temperatures over a concrete section [3,4]. Due to significant temperature changes moisture flows appear. Moisture in the pores of the concrete creates overpressure within them, which adversely affects the emerging structure of the concrete [5].

Thus, in the process of designing the technological parameters of winter concreting, it is necessary to avoid significant temperature fluctuations during the entire period of concrete curing. One of the ways to solve this problem is to choose the right thermal characteristics of concrete structures (formwork and insulation).

It can be assumed that this choice will be affected by a number of parameters from the rate of cooling of the concrete (cooling time), to the location of temperature measurement points in the structure.

At present, there are several reasonable methods for calculating the time of concrete cooling, but with a high complexity of calculations, which makes it difficult to use them in construction practice.
Great popularity was gained by the method of calculating the cooling time proposed by B.G. Skramtayev [8], obtained from the heat balance equation. However, it yields results often underestimated by almost half [9]. The most interesting technique was proposed in by the works [9-13], which formed the basis of this paper.

2. Analytical solution of the problem

To estimate the cooling time of concrete, it is possible to use the well-known Newton's law: the rate of change in body temperature is proportional to the difference between the body temperature and the ambient temperature. Thus, the following differential equation can be proposed:

$$\frac{dt}{d\tau} = (t - t_{\text{air}}) \cdot k$$

Integrating this equation, the following is obtained:

$$\tau = \frac{1}{k} \ln C(k \cdot t - k \cdot t_{\text{air}})$$

Expanding the natural logarithm, the value of the constant $C$ will be defined using the following boundary condition: at the initial instant of time ($\tau = 0$) the temperature of the concrete is $t_{c,s}$. Then

$$C = \frac{1}{k \cdot (t_{c,s} - t_{\text{air}})}$$

Thus, equation (1) takes the form

$$\frac{t - t_{\text{air}}}{t_{c,s} - t_{\text{air}}} = e^{k\tau}$$

The value of the proportionality coefficient $k$ is determined using the second boundary condition: after the time period of $\tau = \tau_x$, the temperature of the concrete decreased to $t = t_x$. Then

$$\frac{t_x - t_{\text{air}}}{t_{c,s} - t_{\text{air}}} = e^{k\tau_x}$$

Hence

$$k = \frac{\ln \left( \frac{t_x - t_{\text{air}}}{t_{c,s} - t_{\text{air}}} \right)}{\tau_x}$$

Substituting the value of the proportionality factor in the formula (1), a formula was obtained for estimating concrete cooling time from temperature $t_{c,s}$ to temperature $t_{c,f}$ at the outside air temperature $t_{\text{air}}$:

$$\tau_{\text{cool}} = \frac{\tau_x}{\ln \left( \frac{t_x - t_{\text{air}}}{t_{c,s} - t_{\text{air}}} \right)} \cdot \ln \left( \frac{t_{c,f} - t_{\text{air}}}{t_{c,s} - t_{\text{air}}} \right) \text{ or } \tau_{\text{cool}} = \frac{1}{k} \cdot \ln \left( \frac{t_{c,f} - t_{\text{air}}}{t_{c,s} - t_{\text{air}}} \right) = -\frac{1}{k} \cdot \ln \left( \frac{t_{c,s} - t_{\text{air}}}{t_{c,f} - t_{\text{air}}} \right)$$

Analyzing the obtained formula (2), it can be seen that the proportionality coefficient $k$ determines the slope angle of the cooling curve in the axes $\tau$ ($t$) and, in its essence, determines the rate of cooling of the concrete.
In works [9,11], an analogous formula was used, but with experimental-analytic derivation of the proportionality coefficient. In these works, the parameter for cooling rate $m$ was used, and a coefficient of 1.33, which takes into account the duration of the irregular and regular cooling stages:

$$\tau_{cool} = \frac{1.33}{m} \cdot \ln \left( \frac{t_{v,a} - t_{air}}{t_{v,f} - t_{air}} \right)$$  \hspace{1cm} (3)$$

The rate of cooling $m$ is determined by the thermophysical properties of the concrete, the size and shape of the structure, and the intensity of its heat exchange with the surrounding environment, and does not depend on the capacity of the internal heat sources (concrete exothermy) or their location in the structure.

Comparing formulas (2) and (3), it is possible to conclude that

$$m = -1.33k$$

According to Kondratiev's theorem [14], the rate of cooling of the structure is expressed by the formula

$$m = \psi \frac{\alpha \cdot F}{C_v}$$ \hspace{1cm} (4)$$

where $\psi$ – coefficient of uneven distribution of temperature in the body of the structure; $\alpha$ – surface heat transfer coefficient; $F$ – area of the structure through which heat losses occur; $C_v$ – total heat capacity of concrete.

Here

$$C_v = C \cdot \gamma \cdot V$$

where $V$ – volume of concrete.

Thus, given that 1 W-hour = 3.6 kJ, and the module structure surface $M_s$ is equal to the ratio of the area of the cooled surfaces to the volume of the element, the formula (4) takes the form

$$m = \psi \frac{3.6 \cdot \alpha \cdot F}{C \cdot \gamma \cdot V} = \psi \frac{3.6 \cdot \alpha \cdot M_s}{C \cdot \gamma}$$ \hspace{1cm} (5)$$

Sources [15,16] give the following formula for the rate of cooling:

$$m = \frac{3.6 \cdot \alpha \cdot M_s}{C \cdot \gamma \cdot \left(1 + k_s \frac{\alpha}{M_s \cdot \lambda} \right)}$$ \hspace{1cm} (6)$$

Here $k_s$ is the shape coefficient (for plates $k_s = 1.14$) and $\lambda$ – the coefficient of thermal conductivity (for concrete $\lambda = 2.6$ W/m°C).

If formulas (5) and (6) are compared, it can be seen that

$$\psi = \frac{1}{1 + k_s \frac{\alpha}{M_s \cdot \lambda}}$$ \hspace{1cm} (7)$$

The coefficient of uneven distribution of temperatures in the body of the structure can be represented as the ratio of the average temperature of the structure ($t_{av,v}$) to the maximum temperature ($t_{max}$):
\[ \psi = \frac{t_{av,v}}{t_{\text{max}}} \]  

(8)

The \( \psi \) coefficient can vary from 0 to 1. If \( \psi = 1 \), the temperature field is even. In general,

\[ t_{av,v} = \frac{\int_0^R t(x)dx}{R}, \]  

(9)

where \( R \) represents the determining size of the structure (for plates: \( R = \frac{b}{2} \), where \( b \) is plate thickness). But since the temperature distribution over the cross section, and, consequently, the equation of the curve \( t(x) \), is not known in advance, for further calculations it is possible to use the following, often encountered restriction [5]: \( \text{grad}(t) \leq 0.1^\circ\text{C/cm} \). It should be noted that in the normative literature there is no clearly limited temperature gradient in concrete [17]. In this case, the temperature gradient is understood as the temperature difference at two neighboring points, referred to the distance between these two points:

\[ \text{grad}(t) = \frac{\Delta t}{c} \]

Then formula (9) can be represented thusly:

\[ t_{av,v} = t_{\text{max}} - \frac{\text{grad}(t) \cdot R}{2} \]

Thus, equating formulas (7) and (8), and also taking into account that for the plates

\[ M_s = \frac{2}{a} = \frac{2}{2R} = \frac{1}{R} \]

it will be obtained that

\[ \frac{t_{\text{max}} - \frac{\text{grad}(t)}{2 \cdot M_s}}{t_{\text{max}}} = \frac{1}{1 + k_s \frac{\alpha}{M_s \cdot \lambda}} \]

From the given formula it is possible to receive the resulted factor of a heat transfer concrete protecting fences (formwork and insulator) providing admissible difference of temperatures on a section of a plate structure:

\[ \alpha = \left( \frac{t_{\text{max}} - \frac{\text{grad}(t)}{2 \cdot M_s}}{k_s} \right) - 1 \cdot \frac{M_s \cdot \lambda}{k_s} \]

(10)

For example, let us determine the necessary heat transfer coefficient of concrete floor covering to ensure the maximum temperature gradient \( 0.1^\circ\text{C/cm} = 10^\circ\text{C/m} \). The thickness of floor is 0.2 m, the maximum temperature in concrete is assumed to be 40°C. Then, in accordance with (10),
\[
\alpha = \left( \frac{40}{40 - \frac{10}{2\cdot10}} \right)^{-1} \left( \frac{10\cdot2.6}{1.14} \right) = 0.289 \text{ W/m}^2\text{°C}
\]

Calculations according to formula (10) under different temperature conditions, temperature gradients, sections of structures have shown the adequacy of the proposed mathematical model.

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**References**

[1] Krasnovsky B M 1986 Dynamics of the thermally stressed state of structures at winter concreting *Concrete and reinforced concrete* vol 12 (Moscow) pp 18–20

[2] Melnik A A 2002 Full-scale studies of concrete hardening in the wall during chamber heating in winter *Construction and education* vol 5 (Ekaterinburg: USTU-UPI) pp 200–201

[3] Pikus G A and Mozgalyov K M 2014 Estimation of the necessary number of control temperature points during the maintenance of monolithic plate constructions in winter *Academic Bulletin UralNIiproekt RAASN* vol 3 (Yekaterinburg) pp 82–83

[4] Pikus G A and Mozgalyov K M 2015 Monitoring of parameters of concrete kept in winter conditions *Bulletin of South Ural State University. Series of Building and Architecture* vol 15 (Chelyabinsk: South Ural State University) 1 pp 6–9

[5] Krasnovsky B M 2007 *Engineering physical basis of winter concreting methods* (Moscow: GASIS) p 512

[6] Aleksandrovsky S V 1966 *Calculation of concrete and reinforced concrete structures for temperature and humidity effects (including creep)* (Moscow: Gosstroizdat) p 443

[7] Mironov S A, Ivanova O S and Deeve E K 1975 Predicting the modes of cooling and hardening of reinforced concrete structures, concreted in winter conditions by the thermos method *Industrial and civil construction* vol 9 (Moscow) pp 8–12

[8] Kopylov V D 2014 *Construction of monolithic concrete structures at negative ambient temperatures* (Moscow: ASV) p 184

[9] Valt A B 1978 Research of technological parameters for keeping structures that are concreted in winter with electrical heating of mixture (on the example of columnar foundations) *Thesis for the degree of Candidate of Technical Sciences* (Chelyabinsk: Chelyabinsk Polytechnic Institute) p 189

[10] Yunusov N V 1986 Calculation of the temperature mode of monolithic structures under the influence of negative temperatures *Effective technology of concrete work under environmental conditions* (Chelyabinsk: Chelyabinsk Polytechnic Institute) pp 125–129

[11] Golovnev S G 1999 *Winter concreting technology: optimization of parameters and choice of methods* (Chelyabinsk: South Ural State University) p 148

[12] Golovnev S G 1983 *Optimization of winter concreting methods* (Leningrad: Stroyizdat) p 235

[13] Golovnev S G, Kapranov V V, Yunusov N V and Valeev A Kh 1974 *Winter concreting in the Southern Urals* (Chelyabinsk: South Ural book publishing house) p 136

[14] Kondratiev G M 1954 *Regular thermal mode* (Moscow: Gostekhizdat) p 408

[15] R - NP SRO SSK - 02 - 2015 *Recommendations for the production of concrete works in winter* (Chelyabinsk: Union of construction companies of Ural and Siberia) p 85

[16] Krylov B A, Ambartsumian S A and Zvezdov A I 2005 *Guide to the heating of concrete in monolithic structures* (Moscow: RAASN, NIIZhB) p 275

[17] *SP 70.13330-2012 Carrying fence constructions* (Moscow: Gosstroy) p 203