1. INTRODUCTION

Epistemic reasoning about action and change is a crucial requirement for systems that deal with incomplete knowledge in the presence of abnormalities, unobservable processes, human-computer interaction and other real-world considerations. A particular type of epistemic inference which is required for diagnosis tasks in an epistemic context is postiction \cite{Eppe2013}. Postdiction determines the condition of an action by observing its effect, and is therefore a fundamental requirement to determine the context under which actions occur or to diagnose abnormalities. As an example, consider a smart home with a robotic wheelchair that can navigate autonomously within the environment. If the smart home recognizes that the wheelchair does not arrive at its destination after executing a driving request, it can postdict that there must be an abnormality (such as a blocked corridor or a flat tire) that prevents the wheelchair from driving. The scenario becomes more complicated if a person is sitting on the wheelchair because here ramifications (i.e. indirect side-effects of actions) are involved: if the person is sitting on the wheelchair, it will move as the wheelchair moves.

In this work, we are interested in the backward direction of such epistemic ramification chains. A smart home assistance system should be able to postdict that the person is sitting on the wheelchair if the system observes that the person’s location changes. This inference is useful to trigger other directly linked inferences about the conditions of the sit-down action: if the person is not sitting on the wheelchair, this could mean that the patient is unconscious and help is required.

In addition to incomplete knowledge and ramifications, the scenario suggests to model knowledge (e.g. knowledge about the location of the wheelchair and the person) in a functional manner, which allows for simpler and more elaboration tolerant modeling of the reasoning domain, as compared to a boolean knowledge model.

In order to model and to solve such reasoning problems, we extend the \textit{h-approximation} (\textit{HPX}) theory \cite{Eppe2013} and present an extended theory called \textit{HPX}. The extended theory is provided in terms of an operational semantics and an implementation as Answer Set Programming (ASP) \cite{Lee2014}. It covers ramifications, postdiction and functional fluents in an elaboration tolerant manner. The theory is particularly useful in practice because it does not require an exponential number of state variables to model the knowledge-state of an agent. The projection problem is therefore tractable, as opposed to existing action theories that are based on a possible-worlds semantics (\textit{PWS}).
Another improvement of \( \mathcal{HPXC} \) compared to \( \mathcal{HPC} \) is that the new ASP implementation allows one to model domain-specific causal laws directly in terms of ASP. This was not possible in the previous formulation, where a set of translation rules was required to generate the domain-specific causal laws from a high-level input language.

2. PRELIMINARIES AND RELATED WORK

The field of *Reasoning about Action and Change* (RAC) emanated from the seminal work on the Situation Calculus by McCarthy (1963). Action theories developed in this research area usually employ discrete state transition semantics, where a state transition emerges from the occurrence of an event. In this work we are primarily interested in epistemic action theories, where an action does not only change the world state, but also the knowledge state of an agent.

2.1. Epistemic Action Theory

Most epistemic action theories are based on a possible-world semantics (PWS) of knowledge (Moore, 1985). Such theories support postdiction, but they require an exponential number of state variables to represent the epistemic state of an agent. This exponential blowup leads to a high computational complexity. For example, Baral et al. (2000) have shown that under certain conditions the planning problem for the PWS-based action language \( A_k \) is \( \Sigma_2^p \)-complete. To this end, Son and Baral (2001) provide approximations of epistemic action languages, and Baral et al. (2000) has shown that the 0-approximation, reduces the planning problem to NP-completeness, but it does not support postdiction. Functional fluents are also not supported in most epistemic action theories. A recent exception is Ma et al. (2013), providing a functional PWS-based epistemic extension to the Event Calculus, but without SCL.

2.2. Ramifications Static Causal Laws

Ramification has been thoroughly investigated in the field of reasoning about action and change (RAC) throughout the last decades, and static causal laws (SCLs) have been proposed as additional language elements to capture indirect side-effect of actions (McCain and Turner, 1995). The authors argue that a causal theory of ramifications should account for implications, but not necessarily for the contrapositive of implications. McCain and Turner (1995) gives the example of Fred the turkey, who can be made not to walk by making him dead, but making him walk does not make him alive. The authors formalize this thought by describing a result function that computes possible next states, thereby also considering inconsistent and disjunctive effects of actions. Thereupon, a least fixed point procedure is applied to capture the least possible change generated by SCL.

There are various non-epistemic action languages that support SCLs (Giunchiglia et al., 2004), but there exists only little work on SCLs within epistemic action theory. The action language \( A_i \) (Tu et al., 2007) is a successful exception that realizes efficient epistemic reasoning with SCLs. But since this approach is based on the 0-approximation of knowledge, it does not consider postdictive reasoning in an elaboration tolerant manner, and it does not account for functional fluents.

\(^2\)It is possible to “emulate” functional fluents with boolean fluents, but this approach is not elaboration tolerant.

\(^3\)Postdiction be realized in \( A_i \) with ad-hoc definitions of SCLs. This is often a convenient workaround, but in general not elaboration tolerant (see Eppe et al., 2013a for details).
2.3. The h-approximation

The h-approximation (Eppe et al., 2013b; Eppe, 2013) ($\mathcal{HP}\mathcal{X}$) is an epistemic action theory based on discrete state transitions. It does not use the possible-worlds model of knowledge, and therefore it does not require an exponential number of state variables. Instead of $\mathcal{PWS}$, it uses a simple three-valued knowledge model, i.e. something is either known to be true, known to be false or unknown. Complexity results in (Eppe, 2013) show that the temporal projection problem can be solved in polynomial time and the plan existence problem is solvable in $\text{NP}$, as opposed to $\Sigma^p_2$ for the $\mathcal{PWS}$-based action language $\mathcal{A}_k$.

This comes at the cost that $\mathcal{HP}\mathcal{X}$ is incomplete, i.e. not all knowledge generated with a $\mathcal{PWS}$-based approach is also generated with the $\mathcal{HP}\mathcal{X}$ theory.

The $\mathcal{HP}\mathcal{X}$ formalism has also been formulated in terms of ASP, and it has been successfully integrated in a robotic assistance system for a Smart Home (Eppe and Bhatt, 2013).

3. OPERATIONAL SEMANTICS OF THE EXTENDED $\mathcal{HP}\mathcal{X}_F$

The extended formalism involves functional fluents and SCLs. Functional fluents require a modification of knowledge histories and inference mechanisms, and SCLs are additional language elements that are compiled into conditional action effects.

3.1. Language Elements

A reasoning domain $\mathcal{D}$ is an 8-tuple $\langle \mathcal{FR}, \mathcal{VP}, \mathcal{EP}, \mathcal{SCL}, \mathcal{KP}, \mathcal{EXC}, G^{\text{strong}}, G^{\text{weak}} \rangle$ that consists of the following language elements:

- **Fluent range specifications.** $\mathcal{FR}$ is a set of tuples $\langle f, v \rangle$ to denote that $v$ is in the functional range of $f$.

- **Value propositions.** $\mathcal{VP}$ is a set of tuples $\langle f, v \rangle$, denoting that initially (at $t = 0$) fluent $f$ has the value $v$.

- **Effect propositions** represents conditional action effects. Formally, $\mathcal{EP}$ is a set of triples $\langle a, \{\langle f^c_1, v^c_1 \rangle, \ldots, \langle f^c_k, v^c_k \rangle \}, \langle f^e, v^e \rangle \rangle$, denoting that $a$ causes $f^e$ to have the value $v^e$ under the condition that fluents $f^c_1, \ldots, f^c_k$ have the values $v^c_1, \ldots, v^c_k$. For an effect proposition $ep$, we write $c(ep)$ to denote the set of condition fluent-value-pairs $\{\langle f^c_1, v^c_1 \rangle, \ldots, \langle f^c_k, v^c_k \rangle \}$ and $e(ep)$ to denote the effect fluent $\langle f^e, v^e \rangle$.

- **Static causal laws** are used to reason about indirect action effects. Formally, $\mathcal{SCL}$ is a set of tuples $\{\langle f^c_1, v^c_1 \rangle, \ldots, \langle f^c_k, v^c_k \rangle \}, \langle f^e, v^e \rangle \rangle$, denoting that $f^e$ is caused to have the value $v^e$ if fluents $f^c_1, \ldots, f^c_k$ are caused to have the values $v^c_1, \ldots, v^c_k$. For a SCL $scl$, we write $c(scl)$ to denote the set of condition fluent-value-pairs $\{\langle f^c_1, v^c_1 \rangle, \ldots, \langle f^c_k, v^c_k \rangle \}$ and $e(scl)$ to denote the effect fluent-value-pair $\langle f^e, v^e \rangle$.

- **Knowledge propositions** represent sensing actions. $\mathcal{KP}$ is a set of tuples $\langle a, f \rangle$, denoting that a sensing action $a$ will determine the value of $f$.
- **Executable conditions** denote what an agent must know to execute an action. $EXC$ is a set of tuples $\{(f_x, v_1^x), \ldots, (f_k^x, v_k^x)\}$, denoting that an agent must know that fluents $f_1^x, \ldots, f_k^x$ have the values $v_1^x, \ldots, v_k^x$ in order to execute $a$. We say that $a$ is executable in $h$ if all fluent-value pairs in the executability condition of $a$ are known to hold.

- **Goal propositions** ($G^{\text{strong}}, G^{\text{weak}}$) are sets of fluent-value pairs that denote strong and weak goals. Weak goals denote that a plan has to be found which possibly achieves the goal. That is, there must be at least one leaf state in the transition tree where the goal is achieved. A strong goal must be achieved in all leaf states, i.e. a plan must necessarily achieve a goal.

3.2. **Knowledge Histories with Functional Fluents**

The operational semantics of $\mathcal{HPC}_\mathcal{F}$ is based on so-called $h$-states (denoted as $h$) that represent historical knowledge about past and present. Formally, an h-state is a pair $\langle \alpha, \kappa \rangle$, where $\alpha$ is the action history and $\kappa$ is the knowledge history of an agent.

The operational semantics explicitly considers knowledge that a fluent does not have a certain value, even if the actual value is unknown. Functional knowledge histories $\kappa$ consist of triples $\langle f, v, t \rangle$ and $\langle f, \neg v, t \rangle$, which denote that it is known that a fluent $f$ has the value $v$ at a step $t$, respectively, that it is known that a fluent $f$ does not have the value $v$ at step $t$.

**Definition 1 (Functional h-states).** A functional h-state $h$ is a pair $\langle \alpha, \kappa \rangle$. An action history $\alpha$ is a set of pairs of actions and time steps, and a knowledge history $\kappa$ is a set of triples of fluents $f$, values $v$ or $\neg v$ and time steps $t$. A knowledge history $\kappa$ is valid if it holds that for all triples $\langle f, v, t \rangle \in \kappa$ (i) there exists no triple $\langle f, v', t \rangle \in \kappa$ with $v \neq v'$ and (ii) there exists no triple $\langle f, \neg v, t \rangle \in \kappa$ and (iii) $v$ is in the range of $f$, according to the fluent range specifications ($FR$) of a given domain $D$.

To simplify our model of concurrent conditional effects we also define the effect history $\epsilon$ of an h-state (see Definition 2). As notational convention we write $\alpha(h)$, $\kappa(h)$ and $\epsilon(h)$ to denote the action history, knowledge history and effect history of an h-state $h$. To simplify notation, we sometimes transfer sub- and superscripts from $h$ to $\epsilon$, $\kappa$ and $\alpha$ (if clear from the context). For instance we write $\epsilon_n$ to denote $\epsilon(h_n)$.

**Definition 2 (Effect history $\epsilon$).** Let $\alpha = \{\langle a_1, t_1 \rangle, \ldots, \langle a_n, t_n \rangle\}$ be an action history and let $EPA$ denote the set of effect proposition of an action $a$. Then the effect history $\epsilon(h)$ of the h-state $h$ is given by (7).

$$\epsilon(h) = \{\langle ep, t \rangle \mid \exists \langle a, t \rangle \in \alpha(h) : ep \in EPA\}$$

(1)

Effect histories are used to simplify our model of concurrent action execution and used in the inference mechanisms described in Section 3.4.

The behavior of an action is modeled via a transition function $\Psi$ that takes a set of actions $\mathcal{A}$ and an h-state $h$ as input and returns a set of h-states as output. The transition function involves eight inference mechanisms IM.1 – IM.8, namely forward inertia, backward inertia, causation, positive postdiction, negative postdiction, positive exclusion, negative exclusion and static causal consequence. The inference mechanisms implement certain epistemic effects (in particular postdiction) that emerge from a possible-worlds model of knowledge. The advantage of implementing these effects manually with the eight IM is
that they avoid the exponential blowup of the epistemic state space. Before presenting details of the IM in Section 3.4 we describe our approach to handle SCLs which allows us to apply the IM on SCLs as well.

3.3. Compiling Static Causal Laws to Effect Propositions

Opposed to the approach by McCain and Turner (1995), our theory does not consider disjuctive and inconsistent effects, i.e. the effects of actions are always deterministic (see Eppe, 2013). Therefore $\mathcal{HPX}$ always generates one single possible successor state per possible sensing result, so that we do not need to employ a fixed point approach as proposed in McCain and Turner (1995). Instead of using a corresponding closure function as defined in McCain and Turner (1995), we compile SCLs into effect propositions (EPs) to reason about indirect effects of actions. By compiling SCLs away, we do not need to implement additional inference mechanisms for SCLs, because the transition function is based on EPs and therefore already performs all necessary reasoning tasks.

The following recursive function generates additional effect propositions that are used in the inference mechanisms.

$$\text{genEP}(\mathcal{EP}) = \begin{cases} \mathcal{EP} & \text{if } \text{addEP}(\mathcal{EP}) = \emptyset \\ \text{genEP}(\mathcal{EP} \cup \text{addEP}(\mathcal{EP})) & \text{otherwise} \end{cases}$$

where

$$\text{addEP}(\mathcal{EP}) = \{ \langle a, \{f^{scl}_{1}, v^{scl}_{1}\}, \ldots, \{f^{scl}_{k}, v^{scl}_{k}\}, \{f^{ep}_{1}, v^{ep}_{1}\}, \ldots, \{f^{ep}_{l}, v^{ep}_{l}\} \rangle \mid \langle\{f^{trig}_{1}, v^{trig}_{1}\}, \ldots, \{f^{trig}_{k}, v^{trig}_{k}\}, \{f^{e}, v^{e}\}\rangle \in \mathcal{SCL} \land \langle f^{trig}, v^{trig} \rangle \subseteq \{\{f^{scl}_{1}, v^{scl}_{1}\}, \ldots, \{f^{scl}_{k}, v^{scl}_{k}\}\} \land \langle a, \{f^{ep}_{1}, v^{ep}_{1}\}, \ldots, \{f^{ep}_{l}, v^{ep}_{l}\} \rangle, \langle f^{trig}, v^{trig} \rangle \in \mathcal{EP} \}$$

Intuitively, $\text{addEP}$ looks for fluent-value pairs $\langle f^{trig}, v^{trig} \rangle$ that trigger the condition of a SCL to become true, such that the effect of the SCL becomes true as well. The recursive nature of $\text{genEP}$ is required to also cope for “chained” SCL triggering. For example, consider an EP $\langle a, \{\rangle, f^{trig}, v^{trig}\rangle$ and two SCL $\langle\{f^{trig}, v^{trig}\}, f^{scl1}, v^{scl1}\rangle$ and $\langle\{f^{scl1}, v^{scl1}\}, f^{scl2}, v^{scl2}\rangle$. The first call will produce an EP $\langle a, \{\rangle, f^{scl1}, v^{scl1}\rangle$, and the second call will produce another EP $\langle a, \{\rangle, f^{scl2}, v^{scl2}\rangle$.

It is also possible to trigger a SCL with two trigger conditions. For example, consider two EPs $\langle a, \{\rangle, f^{trig1}, v^{trig1}\rangle$ and $\langle a, \{\rangle, f^{trig2}, v^{trig2}\rangle$ and a SCL $\langle\{f^{trig1}, v^{trig1}\}, f^{scl1}, v^{scl1}\rangle$. It is clearly the case, that this should result in another EP $\langle a, \{\rangle, f^{scl1}, v^{scl1}\rangle$. The first call of $\text{addEP}$ will generate the EPs $\langle a, \{f^{trig1}, v^{trig1}\}, f^{scl1}, v^{scl1}\rangle$ and $\langle a, \{f^{trig2}, v^{trig2}\}, f^{scl1}, v^{scl1}\rangle$. In the second call, $\text{addEP}$ will generate the desired EP $\langle a, \{f^{scl1}, v^{scl1}\}$.  

3.4. State transitions and Inference Mechanisms with Functional Fluents

The transition function (4) adds a set of actions $A$ to the action history $\alpha$ and then evaluates the knowledge-level effects of these actions.
As an auxiliary notion we write $\text{now}(\mathfrak{h})$ to refer to the current step number.

$$\begin{align*}
\text{now}(\mathfrak{h}) = \begin{cases} 
0 & \text{if } \alpha(\mathfrak{h}) = \emptyset \\
t + 1 & \text{if } \exists \langle a, t \rangle \in \alpha(\mathfrak{h}) : \forall \langle a', t' \rangle \in \alpha(\mathfrak{h}) : t' \leq t
\end{cases}
\end{align*}$$

(3)

This allows us to define the transition function (4).

$$\Psi(A, \mathfrak{h}) = \bigcup_{k \in \text{sense}(A^e, \mathfrak{h})} \text{eval}((\alpha', \kappa(\mathfrak{h}) \cup k))$$

where

- $\alpha' = \alpha(\mathfrak{h}) \cup \{\langle a, t \rangle | a \in A^e \land t = \text{now}(\mathfrak{h})\}$
- $A^e$ is the subset of actions of $A$ which are executable in $\mathfrak{h}$

The transition function calls two other functions, $\text{sense}$ and $\text{eval}$.

- $\text{eval}$ (7) is a re-evaluation function that refines the knowledge-history of an h-state by determining the knowledge-level effects of non-sensing actions using the eight inference mechanisms IM.1 – IM.8 described in Sections 3.4.1–3.4.5.

- $\text{sense}$ adds sensing results to the knowledge history. It is formally defined as follows. Let $t^s = \text{now}(\mathfrak{h})$, let $\mathcal{FR}$ be the fluent range specification, and let $\mathcal{KP}$ be the knowledge propositions of a reasoning domain, then:

$$\text{sense}(A, \mathfrak{h}) = \bigcup_{\langle f, v, t^s \rangle \in \mathcal{FR}} \{\langle f, v, t^s \rangle | \exists a : \langle a, f \rangle \in \mathcal{KP} \land \langle f, \neg v, t^s \rangle \notin \kappa(\mathfrak{h})\}$$

(5)

Note that we restrict $\text{sense}$ (5) (and thereby the $\mathcal{HP}_X_\mathcal{F}$ theory) to the case where there is only one fluent to sense per state transition. Without this restriction, $\text{sense}$ would generate an exponential number of successor states and the tractability of $\mathcal{HP}_X_\mathcal{F}$ would be destroyed.

Intuitively, $\text{sense}$ describes that knowledge is added to the h-state if it is not known that the possible sensing results does not hold.

The re-evaluation function $\text{eval}$ (7) consists of eight inference mechanisms, namely forward inertia (10), backward inertia (11), causation (12), positive postdiction (13), negative postdiction (14), positive exclusion (15), negative exclusion (16) and static causal consequence (17) that constitute the re-evaluation process. To collectively apply the seven inference mechanisms in one function we define an $\text{evalOnce}$ function that successively applies each of the inference mechanisms.

$$\text{evalOnce}(\mathfrak{h}) = \text{scl(exneg}(expos(pdneg(pdpos(cause(back(fwd(\mathfrak{h}))))))))$$

(6)

A problem is that inference mechanism may trigger each other in any order, so it is often not sufficient to apply IM.1 – IM.8 only once. To this end, re-evaluation is defined recursively (7) until convergence is reached.

$$\text{eval}(\mathfrak{h}) = \begin{cases} 
\mathfrak{h} & \text{if } \text{evalOnce}(\mathfrak{h}) = \mathfrak{h} \\
\text{eval}(\text{evalOnce}(\mathfrak{h})) & \text{otherwise}
\end{cases}$$

(7)
To describe the laws of forward and backward knowledge propagation by inertia we first state when a fluent value pair \( \langle f, v \rangle \) is inertial wrt. an h-state \( h \) and a step \( t \).

\[
\text{inertial}(f, v, t, h) \iff \langle f, v \rangle \in \mathcal{F} \mathcal{R} \land \forall \langle e, t \rangle \in \epsilon(h) : \\
(\epsilon(ep) = \langle f, v' \rangle \land v \neq v') \Rightarrow \\
(\exists \langle f', v' \rangle \in \epsilon(ep) : \langle f', v', t \rangle \in \kappa(h))
\]

(8)

We also define inertia of a fluent and a negative value \(-v\).

\[
\text{inertial}(f, -v, t, h) \iff \langle f, v \rangle \in \mathcal{F} \mathcal{R} \land \forall \langle e, t \rangle \in \epsilon(h) : \\
(\epsilon(ep) = \langle f, v \rangle) \Rightarrow \\
(\exists \langle f', v' \rangle \in \epsilon(ep) : \langle f', -v', t \rangle \in \kappa(h))
\]

(9)

Forward inertia describes that a fluent \( f \) is known to have a value \( v \) at a step \( t \) if it is known that \( f \) has the value \( v \) already at step \( t - 1 \) and that \( \langle f, v \rangle \) is inertial at \( t - 1 \).

\[
fwd(h) = \langle \alpha(h), \kappa(h) \cup \text{add}^{\text{pos}}_{\text{fwd}}(h) \cup \text{add}^{\text{neg}}_{\text{fwd}}(h) \rangle
\]

where

\[
\text{add}^{\text{pos}}_{\text{fwd}}(h) = \{ \langle f, v, t \rangle | \langle f, v, t - 1 \rangle \in \kappa(h) \land \text{inertial}(f, v, t - 1, h) \land t \leq \text{now}(h) \} \tag{10}
\]

\[
\text{add}^{\text{neg}}_{\text{fwd}}(h) = \{ \langle f, -v, t \rangle | \langle f, -v, t - 1 \rangle \in \kappa(h) \land \text{inertial}(f, -v, t - 1, h) \land t \leq \text{now}(h) \}
\]

Backward inertia describes that a fluent \( f \) is known to have a value \( v \) at a step \( t \) if it is known that \( f \) has the value \( v \) at step \( t + 1 \) and that \( \langle f, v \rangle \) was not set at \( t \).

\[
\text{back}(h) = \langle \alpha(h), \kappa(h) \cup \text{add}^{\text{pos}}_{\text{back}}(h) \cup \text{add}^{\text{neg}}_{\text{back}}(h) \rangle
\]

where

\[
\text{add}^{\text{pos}}_{\text{back}}(h) = \{ \langle f, v, t \rangle | \langle f, v, t + 1 \rangle \in \kappa(h) \land \text{inertial}(f, -v, t, h) \land t \geq 0 \}
\]

(11)

\[
\text{add}^{\text{neg}}_{\text{back}}(h) = \{ \langle f, -v, t \rangle | \langle f, -v, t + 1 \rangle \in \kappa(h) \land \text{inertial}(f, v, t, h) \land t \geq 0 \}
\]

3.4.2. IM.3 – Causation

\[
\text{cause}(h) = \langle \alpha(h), \kappa(h) \cup \text{add}_{\text{cause}}(h) \rangle
\]

where

\[
\text{add}_{\text{cause}}(h) = \{ \langle f', t \rangle | \exists \langle e, t - 1 \rangle \in \epsilon(h) : \{ \langle l_1^f, t - 1 \rangle, \ldots, \langle l_n^f, t - 1 \rangle \} \subseteq \kappa(h) \}
\]

(12)
3.4.3. **IM.4, IM.5 – Positive and Negative Postdiction**

Positive postdiction is the inference that knowledge about the conditions of an effect proposition is gained if 
(i) the effect is known to hold after the action and 
(ii) known not to hold before the action and 
(iii) no other effect proposition could have triggered the effect.

\[
pd_{\text{pos}}(h) = \langle \alpha(h), \kappa(h) \cup add_{pd_{\text{pos}}}(h) \rangle
\]

where

\[
add_{pd_{\text{pos}}}(h) = \{ (f^e, v^e, t) \mid \exists ep, t \in e(h) : \\
(f^e, v^e) \in c(ep) \land (f^e, v^e, t + 1) \in \kappa(h) \land (f^e, \neg v^e, t) \in \kappa(h) \\
\land (\forall \langle ep', t \rangle \in e(h) : (ep' = ep \lor e(ep') \neq t')) \}
\]

(13)

Negative postdiction generates knowledge that the condition of an EP does not hold if the effect does not hold after the EP is applied. Formally, negative postdiction is defined with (14).

\[
pd_{\text{neg}}(h) = \langle \alpha(h), \kappa(h) \cup add_{pd_{\text{neg}}}(h) \rangle
\]

where

\[
add_{pd_{\text{neg}}}(h) = \{ (f^e, \neg v^e, t) \mid \exists ep, t \in e(h) : \\
f^e, v^e \in c(ep) \land (f^e, v^e, t + 1) \in \kappa(h) \\
\land (\forall \langle f^e, v^e \rangle \in c(ep) \setminus \langle f^e, v^e', t \rangle : (f^e, v^e, t) \in \kappa(h)) \}
\]

(14)

3.4.4. **IM.6, IM.7 – Positive and Negative Exclusion**

Positive exclusion determines that a pair \( \langle f, v \rangle \) holds if all values in the range of \( f \) except \( v \) are known not to hold.

\[
ex_{\text{pos}}(h) = \langle \alpha(h), \kappa(h) \cup add_{ex_{\text{pos}}}(h) \rangle
\]

where

\[
add_{ex_{\text{pos}}}(h) = \{ (f, v, t) \mid \forall \langle f, v \rangle \in FR : (v \neq v' \Rightarrow \langle f, \neg v', t \rangle \in \kappa(h)) \}
\]

(15)

Negative exclusion generates knowledge that a pair \( \langle f, v \rangle \) does not hold if it is known that \( \langle f, v' \rangle \) holds where \( v \neq v' \).

\[
ex_{\text{neg}}(h) = \langle \alpha(h), \kappa(h) \cup add_{ex_{\text{neg}}}(h) \rangle
\]

where

\[
add_{ex_{\text{neg}}}(h) = \{ (f, \neg v, t) \mid \langle f, v \rangle \in FR \land \exists \langle f, v' \rangle \in \kappa(h) : v' \neq v \}
\]

(16)

3.4.5. **IM.8 – Static Causal Consequences**

The described approach to compile SCLs into EPs is not sufficient to cover all aspects of SCLs. It may also be possible to produce knowledge via applying SCLs on initial knowledge or knowledge generated by sensing. To this end, we require another inference mechanism that generates indirect knowledge from
SCLs by considering immediate consequences. This is captured in (17).

\[ scl(h) = \langle \alpha(h), \kappa(h) \cup add_{scl}(h) \rangle \]

where

\[ add_{scl} = \{(f^e, v^e, t) | \exists \{(f^c_1, v^c_1), \ldots, (f^c_k, v^c_k)\}, (f^e, v^e) \in SCL : (f^c_1, v^c_1, t), \ldots, (f^c_k, v^c_k, t) \subseteq \kappa(h)\} \]

(17)

4. FORMALIZATION AS ANSWER SET PROGRAMMING

The formalization is based on a foundational theory \( \Gamma_{hpx} \), and a domain-specific theory \( \Gamma_{world} \), based on the language elements specified in Section 3.1.

- Domain-dependent theory (\( \Gamma_{world} \)): It consists of a set of rules \( \Gamma_{ini} \) representing initial knowledge; \( \Gamma_{act} \) representing actions; \( \Gamma_{scl-world} \) representing SCLs; and \( \Gamma_{goals} \) representing goals.

- Domain-independent theory (\( \Gamma_{hpx} \)): This consists of a set of rules to handle inertia (\( \Gamma_{in} \)); postdiction (\( \Gamma_{post} \)); SCLs (\( \Gamma_{scl-hpx} \)); sensing (\( \Gamma_{sen} \)); concurrency (\( \Gamma_{conc} \)), plan verification (\( \Gamma_{verify} \)) as well as plan-generation \& optimization (\( \Gamma_{plan} \)).

The resulting Logic Program for a reasoning domain \( D \) is given as:

\[
LP(D) = \\
\left[ \Gamma_{ini} \cup \Gamma_{act} \cup \Gamma_{scl-world} \cup \Gamma_{goals} \right] \cup \\
\left[ \Gamma_{in} \cup \Gamma_{post} \cup \Gamma_{scl-hpx} \cup \Gamma_{sen} \cup \Gamma_{conc} \cup \Gamma_{verify} \cup \Gamma_{plan} \right]
\]

4.1. \( \Gamma_{world} \) – Domain Specific Theory (W.1) – (W.7)

The domain specific theory \( \Gamma_{world} \) is a set of facts that correspond to the reasoning domain specification \( D \), i.e. the language elements described in Section 3.1.

Fluent Range Specification (\( FR \)).

For every pair \( (f, v) \in FR \), \( LP(D) \) contains the fact:

\[ poss\, Val(f, v). \]  

(W.1)

Value propositions (\( VP \)).

For every pair \( (f, v) \in VP \), \( LP(D) \) contains the fact:

\[ knows(f, v, 0, 0, 0). \]  

(W.2)
Effect propositions (EP).

For every triple \( \langle a, \{\langle f^c_1, v^c_1 \rangle, \ldots, \langle f^c_k, v^c_k \rangle \}, \{f^e, v^e\} \rangle \), \( \text{LP}(D) \) contains the facts:

\[
\begin{align*}
\text{act}(a). \\
\text{hasEP}(a, ep). \\
\text{hasEff}(ep, f^e, v^e). \\
\text{hasCond}(ep, f^c_1, v^c_1). \ldots \text{hasCond}(ep, f^c_k, v^c_k).
\end{align*}
\]

(W.3)

where \( ep \) is an arbitrary unique identifier for the particular EP.

Static causal laws (SCL).

For every tuple \( \langle \{\langle f^c_1, v^c_1 \rangle, \ldots, \langle f^c_k, v^c_k \rangle \}, \{f^e, v^e\} \rangle \in \text{SCL} \), \( \text{LP}(D) \) contains the facts:

\[
\begin{align*}
\text{sclHasEff}(scl, f^e, v^e). \\
\text{sclHasCond}(scl, f^c_1, v^c_1). \ldots \text{sclHasCond}(scl, f^c_k, v^c_k).
\end{align*}
\]

(W.4)

where \( scl \) is an arbitrary unique identifier for the particular SCL.

Knowledge propositions (KP).

For every tuple \( \langle a, f \rangle \in \text{KP} \), \( \text{LP}(D) \) contains the fact:

\[
\text{hasKP}(a, f).
\]

(W.5)

Executability conditions (EXC).

For every tuple \( \langle a, \{\langle f^x_1, v^x_1 \rangle, \ldots, \langle f^x_l, v^x_l \rangle \} \rangle \in \text{EXC} \), \( \text{LP}(D) \) contains the integrity constraints:

\[
\begin{align*}
\leftarrow \text{occ}(a, N, B), \not\text{knows}(f^x_1, v^x_1, N, N, B). \\
\vdots \\
\leftarrow \text{occ}(a, N, B), \not\text{knows}(f^x_l, v^x_l, N, N, B).
\end{align*}
\]

(W.6)

Goal propositions (\( G^{\text{strong}}, G^{\text{weak}} \))

For every tuple \( \langle l^{wg}, v^{wg} \rangle \in G^{\text{weak}} \), resp. \( \langle l^{sg}, v^{sg} \rangle \in G^{\text{strong}} \), \( \text{LP}(D) \) contains the facts:

\[
\begin{align*}
\text{wGoal}(f^{wg}, v^{wg}). \\
\text{sGoal}(f^{sg}, v^{sg}).
\end{align*}
\]

(W.7)

4.2. \( \Gamma_{hpx} – \text{Foundational Theory (F.0) – (F.8)} \)

The foundational domain-independent \( \mathcal{HPX} \)-theory is constituted by rules (F.0) – (F.8). It covers concurrency, the eight inference mechanisms, sensing, goals, plan-generation and plan optimization.
F.0  **Auxiliaries** ($\Gamma_{aux}$)

The following facts are auxiliary definitions to declare step numbers and branch labels.

\[ s(0..\text{maxS}) \quad \text{and} \quad br(0..\text{maxB}). \] (F.0)

where \text{maxS} and \text{maxB} are constants that denote the maximum number of steps and branches respectively.

F.1  **Concurrency** ($\Gamma_{conc}$)

Rules (F.1) handle concurrency.

\[
\text{apply}(EP, N, N, B) \leftarrow \text{hasEP}(A, EP), \text{occ}(A, N, B).
\] (F.1a)

\[
\quad \leftarrow \text{apply}(EP_1, T, N, B), \text{hasEff}(EP_1, F, V), \text{apply}(EP_2, T, N, B),
\] (F.1b)

\[
\quad \text{hasEff}(EP_2, F, V), EP_1 \neq EP_2, \text{possVal}(F, V).
\]

\[
\text{apply}(EP, T, N + 1, B) \leftarrow \text{apply}(EP, T, N, B), N < \text{maxS}.
\] (F.1c)

(F.1a) states that all EPs of an action are applied if the action occurs. (F.1b) is a restriction that forbids the concurrent application of similar EPs. Two effect propositions are similar if they have the same effect. This restriction is necessary for rules capturing positive postdiction (F.3d) and inertia (F.2).

F.2  **Inertia** ($\Gamma_{in}$)

Inertia is applied in both forward and backward direction. We model inertia for knowledge that a fluent-value pair holds and knowledge that a fluent-value pair does not hold with rules (F.2).

We first define a notion for knowing that a fluent-value pair $\langle f, t \rangle$ is not set, i.e. that $\langle f, \neg v \rangle$ is inertial at a step $t$. This is possible for two reasons; (i) if no effect proposition with the effect $\langle f, v \rangle$ is applied (F.2a), (F.2b), and (ii) if an effect proposition with the effect $\langle f, v \rangle$ is applied but it is known that a condition does not hold (F.2c). Note that the latter is only possible because of restriction (F.1b).

Having defined when $\langle f, v \rangle$ is not set, i.e. that $\langle f, \neg v \rangle$ is inertial, we can define that $\langle f, v \rangle$ is inertial by counting the number of possible values $v'$ of $f$ and assuring that for all possible values $v' \neq v$ the pair $\langle f, \neg v' \rangle$ is inertial (F.2d). Note that to this end we employ the auxiliary predicate \text{numPossVal}/2 to count the size of the range of a fluent (see (F.4b)).

\[
\text{kMaySet}(F, V, T, N, B) \leftarrow \text{apply}(EP, T, N, B), \text{hasEff}(EP, F, V).
\] (F.2a)

\[
\text{kInertial}(F, \neg V, T, N, B) \leftarrow \neg \text{kMaySet}(F, V, T, N, B), \text{uBr}(N, B), s(T), \text{possVal}(F, V)
\] (F.2b)

\[
\text{kInertial}(F, \neg V, T, N, B) \leftarrow \text{apply}(EP, T, N, B), \text{hasEff}(EP, F, V),
\] (F.2c)

\[
\text{hasCond}(EP, F', V_1), \text{knows}(F', V_1, T, N, B), V_1 \neq V_2, s(T).
\]

\[
\text{kInertial}(F, V, T, N, B) \leftarrow \text{N}_V := \{ \text{kInertial}(F, \neg V', T, N, B) : \text{possVal}(F, V') : V' \neq V \},
\] (F.2d)

\[
\text{uBr}(N, B), s(T), \text{numPossVal}(F, N_V + 1), \text{possVal}(F, V).
\]
Having defined inertia of fluent-value pairs, we can define forward and backward propagation of knowledge as follows.

\[
\text{knows}(F, V, T, N, B) \leftarrow \text{knows}(F, V, T - 1, N, B), \ k\text{Inertial}(F, V, T - 1, N, B), \quad (F.2e) \\
T \leq N, s(T), \text{possVal}(F, V).
\]

\[
\text{knows}(F, V, T, N, B) \leftarrow \text{knows}(F, V, T + 1, N, B), \ k\text{Inertial}(F, -V, T + 1, N, B), \\
T < N, \text{possVal}(F, V). 
\]

\[
\text{knowsNot}(F, V, T, N, B) \leftarrow \text{knowsNot}(F, V, T - 1, N, B), \ k\text{Inertial}(F, -V, T - 1, N, B), \\
T < N, \text{possVal}(F, V). \quad (F.2h)
\]

\[
\text{knowsNot}(F, V, T, N, B) \leftarrow \text{knowsNot}(F, V, T + 1, N, B), \ k\text{Inertial}(F, V, T, N, B), \\
T < N, \text{possVal}(F, V). \quad (F.2i)
\]

\[
\text{knowsNot}(F, V, T, N, B) \leftarrow \text{knowsNot}(F, V, T, N + 1, B), s(N). \quad (F.2j)
\]

\[
\text{knowsNot}(F, V, T, N, B) \leftarrow \text{knowsNot}(F, V, T, N - 1, B), s(N). \quad (F.2k)
\]

\( (F.2e) \) defines forward propagation of knowledge that a fluent \( f \) has value \( v \). \( (F.2f) \) defines backward propagation of knowledge that a fluent \( f \) has value \( v \). \( (F.2h) \) defines forward propagation of knowledge that a fluent \( f \) does not have value \( v \). \( (F.2i) \) defines backward propagation of knowledge that a fluent \( f \) does not have value \( v \). Rules \( (F.2j), (F.2k) \) capture forward propagation of knowledge itself. If an agent knows that fluent \( f \) has value \( v \) at a step \( t \) while being in state \( n - 1 \), then it will still have this knowledge at a step \( n \).

\( F.3 \) Causation and Postdiction (\( \Gamma_{\text{act}} \))

Causation and Postdiction are the primary knowledge-level effects of actions \( (F.3) \). For their implementation we first define two auxiliary predicates \( \text{numKnownCond}/5 \) \( (F.3a) \) and \( \text{hasNumCond}/2 \) \( (F.3b) \) to count the number of (known) conditions of EPs.

\[
\text{numKnownCond}(EP, C, T, N, B) \leftarrow C := \text{knows}(F, V, T, N, B) : \text{hasCond}(EP, F, V), \\
\quad uBr(N, B), \text{apply}(EP, T, N, B).
\]

\[
\text{hasNumCond}(EP, C) \leftarrow C := \{\text{hasCond}(EP, F, V)\}, \text{hasCond}(EP, \_). \quad (F.3b)
\]

Knowledge is produced by causation if all conditions of an EP are known to hold \( (F.3c) \). Positive postdiction generates knowledge that the conditions of an EP hold, if the effect of an EP was known not to hold before the EP is applied and if the effect is known to hold after the application of the EP \( (F.3d) \). Note that this implementation of positive postdiction is only valid under restriction \( (F.1b) \) that forbids the concurrent application of two EPs with the same effect.

Negative postdiction produces knowledge that a fluent \( f \) does not have a value \( v \) if the effect of an EP is known not to hold after the EP is applied \( (F.3e) \).
\( k_Cause(F, V, T + 1, N, B) \leftarrow apply(EP, T, N, B), \) numKnownCond(EP, C, T, N, B),
\( hasNumCond(EP, C), hasEff(EP, F, V), uBr(N, B), N > T. \)  
\( k_Cause(F, V, T, N, B) \leftarrow apply(EP, T, N, B), uBr(N, B), hasCond(EP, F, V), 
\( hasEff(EP, F', V'), knows(F', V', T + 1, N, B), 
\( knowsNot(F', V', T, N, B), not knowsNot(F, V, T, N, B), N > T. \) 
\( k_notNegPost(F, V, T, N, B) \leftarrow apply(EP, T, N, B), hasEff(EP, F', V'), 
\( knowsNot(F', V', T + 1, N, B), uBr(N, B), N > T, 
\( hasCond(EP, F, V), hasNumCond(EP, C + 1), 
\( numKnownCond(EP, C, T, N, B), not knows(F, V, T, N, B). \) 

Rules (F.3f), (F.3g), (F.3h) assign knowledge generated by causation and postdiction to the knows/5 resp. knowsNot/5 predicates.

\( knows(F, V, T, N, B) \leftarrow k_Cause(F, V, T, N, B). \)  
\( knows(F, V, T, N, B) \leftarrow k_PosPost(F, V, T, N, B). \)  
\( knowsNot(F, V, T, N, B) \leftarrow k_NotNegPost(F, V, T, N, B). \)  

\( \text{F.4} \quad \text{Knowledge by exclusion } (\Gamma_{exc}) \)

To define rules that generate knowledge by exclusion we first define two auxiliary rules to count the number values of a fluent that are not known (F.4a) and to count the total number of possible values of a fluent (F.4b).

\( numKNF(F, KN, T, N, B) \leftarrow KN := \{ \text{knowsNot} (F, V, T, N, B) : \text{possVal}(F, V), \) 
\( uBr(N, B), s(T), \text{possVal}(F, \_). \) 
\( numPossVal(F, NV) \leftarrow NV := \{ \text{possVal}(F, V) \}, \text{possVal}(F, \_). \)  

We are now ready to define the rules that generate knowledge by positive exclusion (F.4c) and negative exclusion (F.4d). Rules (F.4e), (F.4f) assign knowledge generate by exclusion to the knows/5 (resp. knowsNot/5) predicate.
\[
\text{kPosEx}(F, V, T, N, B) \leftarrow \text{numKNF}(F, KN, T, N, B), \text{numPossVal}(F, KN + 1), \text{not knowsNot}(F, V, T, N, B), \text{possVal}(F, V).
\]
\[
\text{kNotNegEx}(F, V, T, N, B) \leftarrow \text{knows}(F, V', T, N, B), V \neq V', \text{possVal}(F, V').
\]
\[
\text{knows}(F, V, T, N, B) \leftarrow \text{kPosEx}(F, V, T, N, B).
\]
\[
\text{knowsNot}(F, V, T, N, B) \leftarrow \text{kNotNegEx}(F, V, T, N, B).
\]

**F5 Sensing and branching (Γ_{sense})**

Sensing is modeled for contingent planning (e.g. Hoffmann and Brafman, 2005) purposes, i.e. when during plan generation a sensing action is considered, then all possible outcomes of the sensing action are accounted for in separate branches. Branches are generated whenever a sensing action occurs in the plan. Potential sensing outcomes are modeled via the \text{sRes/5} predicate, i.e. \text{misRes}(f, v, n, b, b') denotes that in node \(\langle n, b \rangle\) a sensing action occurs that assigns the value \(v\) to a fluent \(f\) in the child-branch \(b'\).

First, we state rules (F.5a) – (F.5c) to denote that branch 0 is valid in the initial step, and that if no sensing action occurs in a certain node of the transition tree, then the branch is marked as valid in the successor node without branching.

\[
\text{uBr}(0, 0).
\]
\[
\text{sNextBr}(N, B1) \leftarrow \text{sRes}(\_, N, B1, B2).
\]
\[
\text{uBr}(N, B) \leftarrow \text{uBr}(N - 1, B), \text{not sNextBr}(N - 1, B), s(N).
\]

Next, we generate sensing results. Rule (F.5d) generates the actual \text{sRes/5} predicates by assigning one branch to each value in the range of the sensed fluent which is not known not to be actual value of the fluent. However, we have to be careful not to assign potential sensing outcomes to child branches that are already used. This is realized with different integrity constraints.

(F.5e) states that only one sensing result can be assigned to one branch. (F.5f) assures that no used branch (except the current branch) is assigned. (F.5g) prohibits that if multiple sensing actions happen in different nodes, a free branch can be double assigned with different sensing outcomes. (F.5h) is an optional constraint that assures that there is a sensing result assigned to the original branch in any case. This reduces the number of different possible branch-assignments and therefore the search space.
\[1\{sRes(F, V, N, B_1, B_2) : br(B_2)\} \leftarrow \text{occ}(A, N, B_1), \text{hasKP}(A, F), s(N), \text{possVal}(F, V), \quad (F.5d)\]
\[
\neg \text{knowsNot}(F, V, N, N, B_1).
\]
\[\leftarrow 2\{sRes(F, \neg N, B_1, B_2)\}, br(B_1), br(B_2), s(N). \quad (F.5e)\]
\[\leftarrow sRes(F, V, N, B_1, B_2), uBr(N, B_2), B_1 \neq B_2. \quad (F.5f)\]
\[\leftarrow sRes(F, V, N, B_1^P, B_2^C), sRes(F', V', N, B_2^P, B_2^C), \quad (F.5g)\]
\[B_1^P \neq B_2^P.\]
\[\leftarrow \{sRes(F, \neg N, B, B)\} 0, \text{occ}(A, N, B), \text{hasKP}(A, F), \quad (F.5h)\]
\[s(N).\]

Having defined how sensing results are generated, we mark new branches as used (F.5i) and assign the sensing result to the knowledge (F.5j). Finally, we restrict that not more than one fluent can be sensed at a time (F.5k).

\[uBr(N, B_2) \leftarrow sRes(F, V, N - 1, B_1, B_2), s(N). \quad (F.5i)\]
\[\text{knows}(F, V, N - 1, N, B_2) \leftarrow sRes(F, V, N - 1, B_1, B_2), s(N). \quad (F.5j)\]
\[\leftarrow 2\{\text{occ}(A, N, B) : \text{hasKP}(A, \_), \}, br(B), s(N). \quad (F.5k)\]

When a new branch is generated, then the knowledge of the original branch has to be transferred to the new branch. Towards this we implement *inheritance* rules that assign knowledge (F.5l) as well as application of effect propositions (F.5m) from the original to the child branches.

\[\text{knows}(F, V, T, N, B_2) \leftarrow sRes(\_, \_, N - 1, B_1, B_2), \text{knows}(F, V, T, N - 1, B_1), N \geq T, s(N). \quad (F.5l)\]
\[\text{apply}(EP, T, N, B_2) \leftarrow sRes(\_, \_, N, B_1, B_2), \text{apply}(EP, T, N, B_1), N \geq T, s(N). \quad (F.5m)\]

**F.6 Static causal laws (Γ_{scl→hp})**

As discussed wrt. the operational semantics of \(\mathcal{HPX}_\tau\), we compile static causal laws into EPs. Towards this, we define rule (F.6a) that generates a new effect proposition for actions that have an effect proposition with an effect that is identical to the condition of a SCL, and hence can trigger the SCL to cause an indirect effect.

Rule (F.6b) assigns the effect to the new effect proposition. Rule (F.6c) adds the conditions from the original EP to the new EP, and adds the conditions from the SCL to the new EP.
\[ \text{hasEP}(A, (\text{EP}, \text{SCL})) \leftarrow \text{hasEP}(A, \text{EP}), \text{hasEff}(\text{EP}, F^{\text{trig}}, V^{\text{trig}}), \quad (\text{F.6a}) \]

\[ \text{sclHasCond}(\text{SCL}, F^{\text{trig}}, V^{\text{trig}}). \]

\[ \text{hasEff}((\text{EP}, \text{SCL}), F^c, V^c) \leftarrow \text{hasEP}(A, \text{EP}), \text{hasEff}(\text{EP}, F^{\text{trig}}, V^{\text{trig}}), \quad (\text{F.6b}) \]

\[ \text{sclHasCond}(\text{SCL}, F^{\text{trig}}, V^{\text{trig}}), \text{sclHasEff}(\text{SCL}, F^c, V^c). \]

\[ \text{hasCond}((\text{EP}, \text{SCL}), F^c, V^c) \leftarrow \text{hasEff}(\text{EP}, F^{\text{trig}}, V^{\text{trig}}), \text{sclHasCond}(\text{SCL}, F^c, V^c), \quad (\text{F.6c}) \]

\[ \text{sclHasCond}(\text{SCL}, F^{\text{trig}}, V^{\text{trig}}). \]

\[ \text{sclHasCond}(\text{SCL}, F^{\text{trig}}, V^{\text{trig}}), F^{\text{trig}} \neq F^c, V^{\text{trig}} \neq V^c. \]

Compiling SCLs to EPs causes knowledge to be produced as indirect action effects. This however does not account for knowledge that is indirectly produced by considering SCLs in combination with sensing outcomes or initial knowledge. This is captured by three more rules. (\text{F.6e}) and (\text{F.6f}) are auxiliary rules that count how many conditions of a SCL are known to hold, and how many conditions a SCL has in total. Finally, rule (\text{F.6g}) generates indirect knowledge that does not emerge via causation due to EPs.

\[ \text{sclNumKnownCond}(\text{SCL}, C, T, N, B) \leftarrow C := \{ \text{knows}(F, V, T, N, B) : \text{sclHasCond}(\text{SCL}, F, V) \}, \text{uBr}(N, B), s(T), \text{sclHasEff}(\text{SCL}, \ldots \ldots), T \leq N. \quad (\text{F.6e}) \]

\[ \text{sclNumCond}(\text{SCL}, C) \leftarrow C := \{ \text{sclHasCond}(\text{SCL}, F, V) \}, \text{sclHasEff}(\text{SCL}, \ldots \ldots). \quad (\text{F.6f}) \]

\[ \text{knows}(F, V, T, N, B) \leftarrow \text{sclHasEff}(\text{SCL}, F, V), \text{sclNumKnownCond}(\text{SCL}, C, T, N, B), \text{sclNumCond}(\text{SCL}, C). \quad (\text{F.6g}) \]

\section*{Plan verification ($\Gamma_{\text{verify}}$)}

The ASP formalization supports both weak and strong goals. For weak goals there must exist one leaf where all goal literals are achieved and for strong goals the goal literals must be achieved in all leaves. Weak or strong goals are declared with the \text{wGoal} and \text{sGoal} predicates and defined through declarations (\text{W.7}) in the domain-specific part of an \text{HPX} program. (\text{F.7a}) defines atoms \text{notWG}(n, b) which denote that a weak goal is not achieved at step \emph{n} in branch \emph{b}. An atom \text{allWGAchieved}(n) reflects whether all weak goals are achieved at a step \emph{n} (\text{F.7b}). If they are not achieved at step \text{maxS}, then a corresponding model is not stable (\text{F.7c}).

\[ \text{notWG}(N, B) \leftarrow \text{wGoal}(F, V), \text{uBr}(N, B), \text{not knows}(F, V, N, B), \text{possVal}(F, V). \quad (\text{F.7a}) \]

\[ \text{allWGAchieved}(N) \leftarrow \text{not notWG}(N, B), \text{uBr}(N, B). \quad (\text{F.7b}) \]

\[ \leftarrow \text{not allWGAchieved}(\text{maxS}). \quad (\text{F.7c}) \]

Similarly, \text{notSG}(n, b) denotes that a strong goal is not achieved at step \emph{n} in branch \emph{b} (\text{F.7d}). In contrast
to weak goals, strong goals must be achieved in all used branches at the final step \( \text{maxS} \) (F.7e).

\[
\text{notSG}(N, B) \leftarrow \text{sGoal}(F, V), uBr(N, B), \text{not knows}(F, V, N, N, B), \text{possVal}(F, V). \\
\text{notSG}(\text{maxS}, B) \leftarrow \text{notSG}(\text{maxS}, B), uBr(\text{maxS}, B).
\] (F.7d) (F.7e)

Information about nodes where goals are not yet achieved is also generated (F.7f), (F.7g). This is used in the plan generation part for pruning (F.8a)–(F.8b).

\[
\text{notGoal}(N, B) \leftarrow \text{notWG}(N, B). \\
\text{notGoal}(N, B) \leftarrow \text{notSG}(N, B).
\] (F.7f) (F.7g)

\section*{F.8 \hspace{1em} Plan generation (\( \Gamma_{\text{plan}} \))}

In the generation part of the Logic Program, (F.8a) and (F.8b) implement sequential and concurrent planning respectively: for concurrent planning the choice rule’s upper bound “1” is simply removed.

\[
1\{\text{occ}(A, N, B) : \text{act}(A)\} \leftarrow uBr(N, B), \text{notGoal}(N, B), N < \text{maxS}. \\
1\{\text{occ}(A, N, B) : \text{act}(A)\} \leftarrow uBr(N, B), \text{notWG}(N, B), N < \text{maxS}.
\] (F.8a) (F.8b)

\section*{5. COMPUTATIONAL PROPERTIES}

We are interested in the number of state variables that constitute the knowledge state of an agent, and the computational worst-time complexity for the temporal projection problem.

\textbf{Notation.} \( n_F \) is the number of fluents, \( n_V \) an upper limit for range size of fluents and \( n_S \) the max. number of steps.

\textbf{Theorem 1 (Number of state variables).} \( n_F \cdot n_V \cdot n_S \) is the maximal number of epistemic state variables per h-state \( h \), i.e. \(|\kappa(h)| \leq n_F \cdot n_V \cdot n_S\).

\textbf{Proof sketch:} Within the transition function, only the eight IM functions and \textit{sense}\textsuperscript{5} can generate triples \( \langle f, v, t \rangle \). Herein, \( t \) is limited to \( 0 \leq t \leq \text{now}(h) \), where \( \text{now}(h) \) equals the number of steps \( n_f \). Since a set can not have duplicate entries and the number of fluents and possible values is finite due to language element \( \mathcal{FR} \), the theorem holds.

\textbf{Theorem 2 (Complexity of temporal projection).} Let \( A \) be a set of actions and \( h \) a valid h-state. It holds for all \( h' \in \Psi(A, h) \) that determining whether \( \langle f, v, t \rangle \in h' \) is polynomial.

\footnote{In an actual implementation the LP may of course only contain one of these two choice rules, depending on which kind of planning is desired.}
Proof sketch: The transition function \( \text{eval} \) calls \( \text{sense} \) and \( \text{eval} \). \( \text{sense} \) iterates over \( \mathcal{FR}, \mathcal{KP} \) and \( \kappa(h) \), and since \( |\kappa(h)| \leq n_F \cdot n_V \cdot n_S \) (Theorem 1) we have that \( \text{sense} \) is polynomial.

\( \text{eval} \) converges with a linear number of applications of the inference mechanisms, because due to Theorem there exists only a linear number of elements in the knowledge history and because no element is ever removed from the knowledge history with any of the inference mechanisms (see also Lemma B.5 in (Eppe, 2013) for the case of boolean fluents). All inference mechanisms execute in polynomial time, because they only quantify over language elements (sets) from the domain specification and the knowledge history \( \kappa(h) \), which are all of linear size wrt. \( \mathcal{D} \) (see also Lemma B.4 in (Eppe, 2013)). Note that the number of additional EP generated by \( \text{scl2ep} \) is also polynomial wrt. the size of \( \mathcal{SCL} \) and \( \mathcal{EP} \).

Applying the transition function polynomially often (e.g. for determining the outcome of a conditional plan of polynomial size) does not change the complexity class.

6. EXAMPLE SCENARIO

We have integrated the \( \mathcal{HPX} \)-approach as an assistance planning system for the robotic wheelchair \textit{Rolland} within the smart home \textit{BAALL} (\textit{?}). A typical use case within this environment involves a person calling the wheelchair to bring him to a destination. For example, a (sub-) problem of getting on the wheelchair and driving from the bath to the corridor can be modeled within \( \mathcal{HPX}_F \) as follows:

\begin{align*}
\mathcal{R} &= \{\text{bath, kit}\}, \mathcal{B} = \{\text{true, false}\} \\
\mathcal{FR} &= \{\langle wcAt, r \rangle | r \in \mathcal{R}\} \cup \{\langle pAt, r \rangle | r \in \mathcal{R}\} \\
&\quad \cup \{\langle \text{sitting, b} \rangle | b \in \mathcal{B}\} \cup \{\langle ab\text{-sit, b} \rangle | b \in \mathcal{B}\} \\
\mathcal{KP} &= \{\langle pAt, \text{bath} \rangle, \langle wcAt, \text{bath} \rangle, \langle \text{sitting, false} \rangle, \} \\
\mathcal{EP} &= \{\langle \text{drv}(\text{bath, kit}), \{\langle wcAt(\text{bath})\rangle\}, \langle wcAt, \text{kit} \rangle\} \} \\
&\quad \cup \{\langle \text{sit}, \{\langle wcAt, \text{bath} \rangle, \langle pAt, \text{bath} \rangle, \langle ab\text{-sit, false} \rangle, \langle \text{sitting} \rangle \}\} \\
\mathcal{SCL} &= \{\{\langle wcAt, r \rangle, \langle \text{sitting}, \text{true} \rangle, \langle pAt, r \rangle\} | r \in \mathcal{R}\} \\
\mathcal{KP} &= \{\langle \text{senseLoc, pAt} \rangle\}
\end{align*}

(D.1) declares objects of the types room (\( \mathcal{R} \)), and boolean (\( \mathcal{B} \)). \( \mathcal{FR} \) specifies the fluents \( wcAt \mapsto \mathcal{R} \) and \( pAt \mapsto \mathcal{R} \) to denote that wheelchair and person are at a location, \( sitting : \mathcal{P} \mapsto \mathcal{B} \) to denote that a person can sit on a wheelchair and \( ab\text{-sit} \mapsto \mathcal{B} \) to denote that the sitting action may be abnormal. (D.3) specifies that initially the person and the wheelchair are in the bath and the person is not sitting on the wheelchair. The effect propositions describe the \textit{drive} (D.4) and the \textit{sit} (D.5) action, where the latter is only successful if there is no abnormality. The SCL (D.6) states that a person will move with the wheelchair when sitting on it. In this setting, we would like to infer that an abnormality occurred if we observe that a person is not in the corridor after he was supposed to sit down on it and driving the wheelchair from bathroom to corridor. The additional EP generated by \( \text{scl2ep}(\mathcal{EP}) \) is \( \langle \text{drv}, \{\langle \text{sitting, true} \rangle, \langle pAt, \text{bath} \rangle, \langle wcAt, \text{bath} \rangle\}, \langle pAt, corr \rangle \rangle \). This additional EP will make it possible to perform the desired inference through the \( p\\text{d}^{\text{neg}} \) function.

For the transition tree consider Example 1. The initial h-state \( h_0 \) corresponds to (D.3). Then the person executes the \textit{sit} action to sit down on the wheelchair. In the successor state \( h_1 \) it is unknown whether the person sits on the wheelchair because it is unknown whether there was an abnormality with the sitting. The next state \( h_2 \) results from driving to \textit{corr}. Here it is unknown where the person is located because it was unknown whether the person is sitting on the wheelchair. The next action is the sensing of the
Example 1. Knowledge gain through indirect postdiction

\[
\begin{align*}
\kappa_0 &= \{ \langle p_{At}, \text{bath}, 0 \rangle, \\
&\quad \langle w_{At}, \text{bath}, 0 \rangle, \\
&\quad \langle \text{sitting}, \text{false}, 0 \rangle \} \\
\alpha_0 &= \{ \} \\
\kappa_1 &= \{ \langle p_{At}, \text{bath}, 0 \rangle, \langle p_{At}, \text{bath}, 1 \rangle \\
&\quad \langle w_{At}, \text{bath}, 0 \rangle, \langle w_{At}, \text{bath}, 1 \rangle \\
&\quad \langle \text{sitting}, \text{false}, 0 \rangle \} \\
\alpha_1 &= \{ \langle \text{sit(bath)}, 0 \rangle \} \\
\psi(\{\text{sit}\}, h_0) \ni h_1 \\
\kappa_2 &= \{ \langle p_{At}, \text{bath}, 0 \rangle, \langle p_{At}, \text{bath}, 1 \rangle, \\
&\quad \langle w_{At}, \text{bath}, 0 \rangle, \langle w_{At}, \text{bath}, 1 \rangle, \langle w_{At}, \text{corr}, 2 \rangle \\
&\quad \langle \text{sitting}, \text{false}, 0 \rangle \} \\
\alpha_2 &= \{ \langle \text{sit(bath)}, 0 \rangle, \langle \text{drv(bath, kit)}, 1 \rangle \} \\
\psi(\{\text{senseLoc}\}, h_2) \ni h_3 \\
\kappa_3 &= \{ \ldots, \langle p_{At}, \text{bath}, 2 \rangle, \langle p_{At}, \text{bath}, 3 \rangle, \\
&\quad \ldots, \langle w_{At}, \text{corr}, 2 \rangle, \langle w_{At}, \text{corr}, 3 \rangle, \\
&\quad \ldots, \langle \text{sitting}, \text{false}, 2 \rangle, \langle \text{sitting}, \text{false}, 3 \rangle, \\
&\quad \ldots, \langle \text{ab\textunderscore sit, true}, 2 \rangle, \langle \text{ab\textunderscore sit, true}, 3 \rangle \} \\
\alpha_3 &= \{ \langle \text{sit(bath)}, 0 \rangle, \langle \text{drv(bath, kit)}, 1 \rangle, \langle \text{senseLoc}, 2 \rangle \}
\end{align*}
\]

person’s location. In the figure we consider only the case where the person is still located in the corridor. This triggers the indirect postdiction through (13) that the person is not sitting on the wheelchair, which in turn again is used to postdict that there must be an abnormality. Information about the abnormality can be used to call the care personnel.

7. CONCLUSION

We present an epistemic action theory $\mathcal{HPX}_F$ that accounts for epistemic ramification, postdiction, and functional fluents. We improve the original $\mathcal{HPX}$, in the sense that translation rules described earlier approaches (Eppe et al., 2013a) are not required anymore; the domain specification can now be given entirely in terms of ASP, and is therefore not restricted to a fixed input syntax anymore.

We have demonstrated the action planning and reasoning capabilities of our approach in the backdrop of a smart home scenario, but we would like to emphasize that many other application domains require postdictive reasoning in combination with ramifications and functional knowledge. Examples for such applications include narrative interpretation, continuity checking in plot writing for novels and movies, as well as forensics and criminal reasoning (e.g. (?)). These applications usually involve many unknown world properties, and epistemic action theories that are based on $\mathcal{PW}S$ will require a number of epistemic state variables that is exponential with the number of unknown world properties. $\mathcal{HPX}_F$ only requires a linear number of state variables to model the knowledge state of an agent, and is therefore more appropriate than $\mathcal{PW}S$-based approaches in many practical application domains.
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