We develop a partial charge-spin separation fermion-spin theory implemented the gauge invariant dressed holon and spinon. In this novel approach, the physical electron is decoupled as the gauge invariant dressed holon and spinon, with the dressed holon behaviors like a spinful fermion, and represents the charge degree of freedom together with the phase part of the spin degree of freedom, while the dressed spinon is a hard-core boson, and represents the amplitude part of the spin degree of freedom, then the electron single occupancy local constraint is satisfied. Within this approach, the charge transport and spin response of the underdoped cuprates is studied. It is shown that the charge transport is mainly governed by the scattering from the dressed holons due to the dressed spinon fluctuation, while the scattering from the dressed spinons due to the dressed holon fluctuation dominates the spin response.

Very soon after the discovery of the high-temperature superconductivity (HTSC) in doped cuprates, Anderson proposed a scenario of HTSC based on the charge-spin separation in the two-dimension (2D)\(^1\), where the internal degrees of freedom of the electron are decoupled as the charge and spin degrees of freedom, while the elementary excitations are not quasi-particles but collective modes for the charge and spin degrees of freedom, i.e., the holon and spinon, then these holon and spinon might be responsible for the experimentally observed nonconventional behavior of doped cuprates. Many unusual features of doped cuprates are extensively studied following this line within the 2D \(t-J\) type model.

The decoupling of the charge and spin degrees of freedom of electron is undoubtedly correct in the one-dimensional (1D) interacting electron systems\(^2\). In particular, the typical behavior of the non-Fermi-liquid, i.e., the absence of the quasi-particle propagation and charge-spin separation, has been demonstrated theoretically within the 1D \(t-J\) model\(^3\). Moreover, the holon and spinon as the real elementary excitations in 1D cuprates has been observed directly by the angle-resolved photoemission spectroscopy (ARPES) experiment\(^4\). However, the case in 2D is very complex since there are many competing degrees of freedom\(^5\). As a consequence, both experimental investigation and theoretical understanding are extremely difficult. Among the unusual features of doped cuprates, a hallmark is the charge transport\(^5,6\), where the conductivity shows a non-Drude behavior at low energies, and is carried by \(x\) holes, with \(x\) is the hole doping concentration, while the resistivity exhibits a linear temperature behavior over a wide range of temperatures. This is strong experimental evidence supporting the notion of the charge-spin separation, since not even conventional electron-electron scattering would show the striking linear rise of scattering rate above the Debye frequency, and if there is no the charge-spin separation, the phonons should affect these properties\(^7\). Furthermore, a compelling evidence for the charge-spin separation in doped cuprates has been found from the experimental test of the Wiedemann-Franz law, where a clear departure from the universal Wiedemann-Franz law for the typical Fermi-liquid has been demonstrated\(^8\). In this case, a formal theory implemented the gauge invariant holon and spinon, i.e., the issue of whether the holon and spinon are real, is centrally important, since we must squarely face if there is to be a meaningful discussion of theories based on the charge-spin separation\(^9\). In this paper, we propose a partial charge-spin separation fermion-spin theory, and show that if the local single occupancy constraint is treated properly, then the electron operator can be decoupled by introducing the dressed holon and spinon. These dressed holon and spinon are gauge invariant, i.e., they are real in 2D.

We begin with the \(t-t'\)-\(J\) model defined on a square lattice as,

\[
H = -t \sum_{\langle ij \sigma \tau \sigma \tau \rangle} C^\dagger_{i \sigma} C_{j \tau} + t' \sum_{\langle ij \sigma \tau \sigma \tau \rangle} C^\dagger_{i \sigma} C_{j \tau} + \mu \sum_{i \sigma} C^\dagger_{i \sigma} C_{i \sigma} + J \sum_{i \hat{\eta}} \mathbf{S}_i \cdot \mathbf{S}_{i + \hat{\eta}},
\]

supplemented by the single occupancy local constraint \(\sum_{\sigma} C^\dagger_{i \sigma} C_{i \sigma} \leq 1\), where \(\hat{\eta} = \pm \hat{x}, \pm \hat{y}, \hat{r} = \pm \hat{x} \pm \hat{y}\), \(C^\dagger_{i \sigma}\) (\(C_{i \sigma}\)) is the electron creation (annihilation) operator, \(\mathbf{S}_i = C^\dagger_{i \sigma} \hat{\sigma} C_{i \sigma}/2\) is spin operator with \(\hat{\sigma} = (\sigma_x, \sigma_y, \sigma_z)\) as Pauli matrices, and \(\mu\) is the chemical potential. It has been shown that the local constraint can be treated properly in analytical calculations within the fermion-spin theory\(^{10}\). In this approach\(^{10}\), the constrained electron operator is decoupled in the CP\(^1\) representation as \(C_{i \sigma} = h^\dagger_ia_{i \sigma}\), with the local constraint \(\sum_{\sigma} a^\dagger_{i \sigma} a_{i \sigma} = 1\), where the fermion operator \(h_i\) keeps track of the charge degree of freedom, while the boson operator \(a_{i \sigma}\) keeps track of the spin degree of freedom, then the
electron local constraint \( \sum_i C_{i\sigma}^\dagger C_{i\sigma} = 1 - h_i^1 h_i \leq 1 \) is satisfied, with \( n_h = h_i^1 h_i \) is the holon number at site \( i \), equal to 1 or 0. In this formalism, the charge and spin degrees of freedom of electron may be separated at the mean field (MF) level, where the elementary charge and spin excitations are called the holon and spinon, respectively. We call such holon and spinon as bare holon and spinon, respectively, since an extra \( U(1) \) gauge degree of freedom related with the local constraint \( \sum_{\sigma} a_{i\sigma}^\dagger a_{i\sigma} = 1 \) appears, i.e., the CP\(^1\) representation is invariant under a local \( U(1) \) gauge transformation \( h_i \rightarrow h_i e^{i\theta_i}, a_{i\sigma} \rightarrow a_{i\sigma} e^{i\theta_i} \), and then all physical quantities should be invariant with respect to this transformation. Thus both bare holon \( h_i \) and bare spinon \( a_{i\sigma} \) are not gauge invariant, and they are strongly coupled by this \( U(1) \) gauge field fluctuations. In other words, these bare holon and spinon are not real.

However, the CP\(^1\) boson \( a_{i\sigma} \) with the constraint can be mapped exactly onto the pseudospin representation defined with an additional phase factor. This is because that the empty and doubly occupied spin states have been ruled out due to the constraint \( a_{i\sigma}^\dagger a_{i\sigma} = 1 \), and only the spin-up and spin-down singly occupied spin states are allowed, thus the original four-dimensional representation space is reduced to a 2D space. Due to the symmetry of the spin-up and spin-down states, \( \langle \text{occupied}\rangle_\uparrow = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \) and \( \langle \text{empty}\rangle_\uparrow = \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \) are singly-occupied and empty spin-up, while \( \langle \text{occupied}\rangle_\downarrow = \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \) and \( \langle \text{empty}\rangle_\downarrow = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \) are singly-occupied and empty spin-down states, respectively. In this case, the constrained CP\(^1\) boson operators \( a_{i\sigma} \) can be represented in this reduced 2D space as,

\[
\begin{align*}
  a_\uparrow = e^{i\Phi_\uparrow} a \quad & \langle \text{occupied}\rangle = e^{i\Phi_\uparrow} \left( \begin{array}{c} 0 \\ 1 \end{array} \right) = e^{i\Phi_\uparrow} S^-, \\
  a_\downarrow = e^{i\Phi_\downarrow} a \quad & \langle \text{occupied}\rangle = e^{i\Phi_\downarrow} \left( \begin{array}{c} 0 \\ 1 \end{array} \right) = e^{i\Phi_\downarrow} S^+,
\end{align*}
\]

with \( S^- \) is the \( S^z \) lowering operator, while \( S^+ \) is the \( S^z \) raising operator, and then the local constraint \( \sum_{\sigma} a_{i\sigma}^\dagger a_{i\sigma} = S_+^i S^-_i + S_-^i S^+_i = 1 \) is exactly satisfied. Obviously, the bare spinon contains both phase and amplitude parts, and the phase part is described by the phase factor \( e^{i\Phi_\sigma} \), while the amplitude part is described by the spin operator \( S_i \). In this case, the electron CP\(^1\) decoupling form with the constraint can be expressed as \( C_\uparrow = h_i^1 e^{i\Phi_\uparrow} S^-_i \) and \( C_\downarrow = h_i^1 e^{i\Phi_\downarrow} S^+_i \), while the local \( U(1) \) gauge transformation is replaced as \( h_i \rightarrow h_i e^{i\theta_i}, \Phi_\sigma \rightarrow \Phi_\sigma + \theta_i \). Furthermore, the phase factor of the bare spinon \( e^{i\Phi_\sigma} \) can be incorporated into the bare holon, then we obtain a new fermion-spin transformation as,

\[
C_\uparrow = h_i^1 S^-_i, \quad C_\downarrow = h_i^1 S^+_i,
\]

with the spinful fermion operator \( h_{i\sigma} = e^{-i\Phi_\sigma} h_i \) describes the charge degree of freedom together with the phase part of the spin degree of freedom (dressed holon), while the spin operator \( S_i \) describes the amplitude part of the spin degree of freedom (dressed spinon). In this case, only the amplitude part of the spin degree of freedom is separated from the electron operator, and thus the partial charge-spin separation is implemented in the electron decoupling form (3). These dressed holon and spinon are invariant under the local \( U(1) \) gauge transformation, and therefore all physical quantities from the dressed holon or dressed spinon also are invariant with respect to this gauge transformation. In this sense, these dressed holon and spinon are real as the new elementary particle excitations in the low-dimensional solid. This gauge invariant dressed holon is a magnetic dressing.

In other words, the gauge invariant dressed holon carries some spinon messages, i.e., it shares some effects of the spinon configuration rearrangements due to the presence of the hole itself. We emphasize that the present dressed holon \( h_{i\sigma} \) is a spinless fermion \( h_i \) (bare holon) incorporated a spinon cloud (magnetic flux), although in common sense \( h_{i\sigma} \) is not an real spinful fermion, its behaviors like a spinful fermion. In correspondence with these special physical properties, we find that \( h_i^1 h_{i\sigma} = h_i^1 e^{i\Phi_\sigma} e^{-i\Phi_\sigma} h_i = h_i^1 h_i \), which guarantees that the electron on-site local constraint, \( \sum_{\sigma} C_{i\sigma}^\dagger C_{i\sigma} = S_+^i h_i^1 S^-_i + S_-^i h_i^1 S^+_i = 1 - h_i^1 h_i \leq 1 \), is always satisfied in analytical calculations. Moreover the double spinful fermion occupancy, \( h_{i\sigma}^1 h_{i\sigma}^1 = e^{i\Phi_\sigma} e^{i\Phi_\sigma} = 0 \), \( h_{i\sigma} h_{i\sigma} = e^{-i\Phi_\sigma} e^{-i\Phi_\sigma} = 0 \), are ruled out automatically. Since the spinless fermion \( h_i \) and spin operators \( S_+^i \) and \( S_-^i \) obey the anticommutation relation and Pauli spin algebra, respectively, then it is easy to show that the spinful fermion \( h_{i\sigma} \) also obey the same anticommutation relation as the spinless fermion \( h_i \). In this partial charge-spin separation fermion-spin representation, the \( t-t' \)-J model (1) can be rewritten as,

\[
H = -t \sum_{ij\sigma\bar{\sigma}} (h_{i\sigma} S_{i\sigma}^+ h_{j\bar{\sigma}} S_{j\bar{\sigma}}^+ + h_{i\sigma} S_{i\sigma}^- h_{j\bar{\sigma}} S_{j\bar{\sigma}}^+) + t' \sum_{i\sigma\bar{\sigma}} (h_{i\sigma} S_{i\sigma}^+ S_{i\bar{\sigma}}^+ + h_{i\sigma} S_{i\sigma}^- S_{i\bar{\sigma}}^-)
\]
\[ -\mu \sum_{i\sigma} \hat{h}_{i\sigma} h_{i\sigma} + \frac{1}{2} J \sum_{i<j,\sigma}(\hat{h}_{i\sigma} \hat{h}_{j\sigma}) \mathbf{S}_i \cdot \mathbf{S}_{i+j}(h_{i} h_{i+\sigma} h_{i+\sigma}^{-1} h_{i+\sigma}^{-1}). \]  

(4)

The spirit of the present partial charge-spin separation fermion-spin theory is very similar to those of the bosonization in 1D interacting electron system, where the electron operators are mapped onto the boson (electron density) representation, and then the recast Hamiltonian is exactly solvable.

Although the choice CP representation is convenient, so long as \( h_i^\dagger h_i = 1 \), \( \sum_\sigma C^\dagger_{i\sigma} C_{i\sigma} = 0 \), no matter what the values of \( S^\dagger_i S^-_i \) and \( S^-_i S^+_i \) are, therefore it means that a "spin" even to an empty site has been assigned. It has been shown that this defect can be cured by the projection operator \( P_i \), i.e., the constrained electron operator can be mapped exactly using the fermion-spin transformation defined with the additional projection operator \( P_i \) as \( C_{i\sigma} = P_i h^\dagger_i S^-_i P_i \) and \( C_{i\sigma} = P_i h^\dagger_i S^+_i P_i \). However, this projection operator is cumbersome to handle, and we will drop it in the actual calculations. It also has been shown that such treatment leads to errors of the order \( x \) in counting the number of spin states, which is negligible for small dopings. Moreover, the electron local constraint still is exactly obeyed even in the MF approximation (MFA), and therefore the essential physics of the gauge invariant dressed holon and spinon also are kept. This is because that the constrained electron operator \( C_{i\sigma} \) in the \( t-J \) type model also can be mapped onto the slave-fermion formalism as \( C_{i\sigma} = h_i^\dagger b_{i\sigma} \) with the local constraint \( h_i^\dagger h_i + \sum_\sigma b^\dagger_{i\sigma} b_{i\sigma} = 1 \). We can solve this constraint by rewriting the boson operators \( b_{i\sigma} \) in terms of the CP boson operators \( a_{i\sigma} \) as \( b_{i\sigma} = a_{i\sigma} \sqrt{1 - h_i^\dagger h_i} \) supplemented by the local constraint \( \sum_\sigma a^\dagger_{i\sigma} a_{i\sigma} = 1 \). As mentioned above, the CP boson operators \( a_{i\uparrow} \) and \( a_{i\downarrow} \) with the local constraint can be identified with the pseudospin lowering and raising operators, respectively, defined with an additional phase factor, therefore the projection operator is approximately related to the holon number operator by \( P_i \sim \sqrt{1 - h_i^\dagger h_i} = \sqrt{1 - h_i^\dagger h_i} \), and its main role is to remove the spurious spin when there is a holon at the site \( i \).

In the Fermi-liquid, the electron carried both charge and spin degrees of freedom moves almost freely since the weak electron-electron interaction in the system, where both amplitude and phase parts of the spin degree of freedom are delocalized. However, in the doped Mott insulator, our present study indicates that the electron operator can be decoupled as the dressed holon and spinon, with the dressed spinon represents the bare spinon’s amplitude part, and is localized, while the dressed holon represents the bare holon together with the bare spinon’s phase part, and then can move almost freely in the background of the dressed spinon fluctuation. The present decoupling scheme seems to have been confirmed in the doped 1D Mott insulator, where although the charge and spin degrees of freedom are decoupled, the charge and spin degrees of freedom are manifested themself by the excitations of charge-density wave and spin-density wave, respectively. These charge-density wave and spin-density wave are described in terms of the density-density correlation function and spin-spin correlation, respectively, and both density operator and spin operator are gauge invariant.

As an application of the present theory, we discuss the charge transport and spin response of the underdoped cuprates. The one-particle dressed holon and spinon two-time Green’s functions are defined as,

\[ g_\sigma(i-j, t-t') = -i\theta(t-t')\langle[h_\sigma(t), h^\dagger_\sigma(t')]\rangle = \langle[h_\sigma(t); h^\dagger_\sigma(t')]\rangle, \tag{5a} \]
\[ D(i-j, t-t') = -i\theta(t-t')\langle[S^+_i(t), S^-_i(t')]\rangle = \langle[S^+_i(t); S^-_i(t')]\rangle, \tag{5b} \]

respectively. Since the dressed spinon operators obey the Pauli algebra, then our goal is to evaluate the dressed holon and spinon Green’s functions directly for the fermion and spin operators in terms of equation of motion method. In the framework of equation of motion, the time-Fourier transform of the two-time Green’s function \( G(\omega) = \langle[A; A^\dagger]\rangle_\omega \) satisfies the equation \( \omega \langle[A; A^\dagger]\rangle_\omega = \langle[A, A^\dagger]\rangle + \langle[A, H]; A^\dagger\rangle_\omega \). If we define the orthogonal operator \( L \) as \( [A, H] = \zeta A - iL \), with \( \langle[L, A^\dagger]\rangle = 0 \), then the full Green’s function can be expressed as,

\[ G(\omega) = G^{(2)}(\omega) + \frac{1}{\zeta^2} G^{(0)}(\omega) \langle[L; L^\dagger]\rangle_\omega G^{(0)}(\omega). \tag{6} \]

with the MF Green’s function \( G^{(0)}(\omega) = \zeta/(\omega - \zeta) \), where \( \zeta = \langle[A, A^\dagger]\rangle \). It has been shown that if the self-energy \( \Sigma(\omega) \) is identified as the irreducible part of \( \langle[L; L^\dagger]\rangle_\omega \), then the full Green’s function (5) can be evaluated as,

\[ G(\omega) = \frac{\zeta}{\omega - \zeta - \Sigma(\omega)}. \tag{7} \]

with \( \Sigma(\omega) = \langle[L; L^\dagger]\rangle_{\omega}^2 / \zeta \). In the framework of the diagrammatic technique, \( \Sigma(\omega) \) corresponds to the contribution of irreducible diagrams.
It has been shown from the experiments that the antiferromagnetic (AF) long-range-order (AFLRO) in the undoped cuprates is destroyed by hole doping in the order \( \sim 0.24 \), therefore there is no AFLRO in the underdoped regime \( 0.025 \leq x < 0.15 \), i.e., \( S_{zz}^z = 0 \). In this case, a MF theory of the \( t-J \) model has been discussed within the Kondo-Yamaji decoupling scheme. Following their discussions, we can obtain the MF dressed holon and spinon Green’s functions in the present case as, \( g^{(0)}_\sigma(k, \omega) = 1/(\omega - \xi_k) \) and \( D^{(0)}(k, \omega) = B_k/(\omega^2 - \omega_k^2) \), respectively, where \( B_k = \lambda_1 [2 \chi(\xi_k - 1) + \chi(\xi_k - 1)] - \lambda_2 (2 \chi^2 \xi_k - \chi_k) \), \( \lambda_1 = 2 \omega J_{sf} \), \( \lambda_2 = 4 \phi \phi' \), \( \phi_1 = (h_{\sigma}^{i\sigma} h_{\sigma}^{\alpha\sigma}) \), \( \phi_2 = (h_{\sigma}^{i\sigma} h_{\sigma}^{\alpha\sigma}) \), \( \epsilon = 1 + 2 \phi_1/\epsilon_{max} \), \( \phi = (1-x)^2 \), \( x = (h_{\sigma}^{i\sigma} h_{\sigma}^{\alpha\sigma}) \), \( \gamma_k = (1/Z) \sum_{\sigma} \gamma^{i\sigma}_{k} \), \( \gamma_k = (1/Z) \sum_{\sigma} \gamma^{i\sigma}_{k} \), \( Z \) is the number of the nearest neighbor or second-nearest neighbor sites, the MF dressed holon spectrum, \( \omega_k = \lambda(\xi_k - \eta^T \chi_{k} \eta) - \mu_k \), and the MF dressed spinon spectrum, \( \omega_k = A_1(\xi_k - \eta^T \chi_{k} \eta) + A_2(\xi_k - \eta^T \chi_{k} \eta) + A_3(\xi_k - \eta^T \chi_{k} \eta) + A_4(\xi_k - \eta^T \chi_{k} \eta) \), \( A_1 = \alpha_1 \lambda(\xi_k - \chi_k) \), \( A_2 = \alpha_2 \lambda_2(\chi_k - \xi_k + \xi_k) \), \( A_3 = -\alpha_3 \lambda(\xi_k - \chi_k) \), \( A_4 = -\alpha_4 \lambda_2(\xi_k - \chi_k) \), \( \chi_k = (1/Z) \sum_{\sigma} \chi^{i\sigma}_{k} \), \( \chi_k = (1/Z) \sum_{\sigma} \chi^{i\sigma}_{k} \), and \( \chi_k = (1/Z) \sum_{\sigma} \chi^{i\sigma}_{k} \), respectively, where \( \chi_k = (S_{zz}^{i\sigma} S_{zz}^{i\sigma}) \), \( \chi_k = (S_{zz}^{i\sigma} S_{zz}^{i\sigma}) \), \( C_1 = (1/Z^2) \sum_{\sigma, \eta} (S_{zz}^{i\sigma} S_{zz}^{i\sigma}) \), \( C_2 = (1/Z^2) \sum_{\tau, \tau'} (S_{zz}^{i\tau} S_{zz}^{i\tau'}) \), and \( C_3 = (1/Z) \sum_{\sigma} (S_{zz}^{i\sigma} S_{zz}^{i\sigma}) \), and \( \chi_k = (1/Z) \sum_{\sigma} (S_{zz}^{i\sigma} S_{zz}^{i\sigma}) \), and \( \chi_k = (1/Z) \sum_{\sigma} (S_{zz}^{i\sigma} S_{zz}^{i\sigma}) \). In order not to violate the sum rule of the correlation function \( \langle S_{zz}^{i\sigma} S_{zz}^{i\sigma} \rangle = 1/2 \) in the case without AFLRO, the important decoupling parameter \( \alpha \) has been introduced in the MF self-consistent calculation, which can be regarded as the vertex correction.

With the help of Eq. (7), the full dressed holon and spinon Green’s functions are obtained as,

\[
g_{\sigma}(k, \omega) = \frac{1}{\omega - \xi_k - \Sigma^{(2)}_{\sigma}(k, \omega)}, \quad D(k, \omega) = \frac{B_k}{\omega^2 - \omega_k^2 - \Sigma^{(2)}_{\sigma}(k, \omega)},
\]

respectively, where the dressed holon self-energy from the dressed holon pair bubble \( \Sigma^{(2)}_{h}(k, \omega) = \langle \langle L_{h}^{(1)}(t); L_{h}^{(1)}(t') \rangle \rangle \rangle \) with the orthogonal operator \( L_{h}^{(1)} = -t \sum_{\sigma, \eta} h_{i, \eta}(S_{zz}^{i\sigma} S_{zz}^{i\sigma}) - \chi_1 + t' \sum_{\tau, \tau'} h_{i, \tau}(S_{zz}^{i\tau} S_{zz}^{i\tau'}) - \chi_2 \), and can be evaluated as,

\[
\Sigma^{(2)}_{h}(k, \omega) = \frac{1}{2} \left( \frac{Z}{N} \right)^2 \sum_{pp'} \gamma_{12}(k, p, p') B_{p'} B_{p+p'} \omega_{p'p} \
\times \left( \frac{F_{1}^{(h)}(k, p, p')}{\omega + \omega_{p'} - \omega_p - \xi_{p+k}} + \frac{F_{2}^{(h)}(k, p, p')}{\omega + \omega_{p'} - \omega_p - \xi_{p+k}} \right),
\]

where \( \gamma_{12}(k, p, p') = (t \gamma p_{p'+p'} - t' \gamma p_{p'-p'+p+p+k}^2) + (t \gamma p_{p-k} - t' \gamma p_{p'-k})^2 \), \( F_{1}^{(h)}(k, p, p') = n_F(\xi_{p+k})[n_B(\omega_{p'}) - n_B(\omega_{p'+p})] + n_B(\omega_{p'})[1 + n_B(\omega_{p'})] \), \( F_{2}^{(h)}(k, p, p') = n_F(\xi_{p+k})[n_B(\omega_{p'}) - n_B(\omega_{p'+p})] + n_B(\omega_{p'})[1 + n_B(\omega_{p'})] \), \( F_{3}^{(h)}(k, p, p') = n_F(\xi_{p+k})[n_B(\omega_{p'}) - n_B(\omega_{p'+p})] + n_B(\omega_{p'})[1 + n_B(\omega_{p'})] + n_B(\omega_{p'}) \), and \( n_F(\xi_{p}) \) and \( n_B(\omega_{p'}) \) are the boson and fermion distribution functions, respectively. The calculation of the dressed spinon self-energy is quite tedious, so our start point is the dressed spinon MF solution within the Kondo-Yamaji decoupling scheme. The full dressed spinon Green’s function satisfies the relation, \( \omega^2 D(k, \omega) = B_k + \langle \langle [S_{zz}^z(t), H(t)], [H(t)]; S_{zz}^z(t') \rangle \rangle \rangle \), where \( [S_{zz}^z, H], H_k = \omega_k^2 S_{zz}^z - i e_{\gamma}^{(0)} \). In the disordered spin liquid state without AFLRO, the dressed holon-spinon interaction should dominate the essential physics, so the orthogonal operator for the dressed spinon can be selected as,

\[
L_{i}^{(s)}_{s} = -(2e \chi^2 + \chi_1) \lambda_1 \frac{1}{Z} \sum_{\eta, \hat{a}} t_{\eta} (h_{i, \eta}^{\dagger} h_{i, \eta + \hat{a}} + h_{i, \eta + \hat{a}}^{\dagger} h_{i, \eta} + 2 \phi_\delta) S_{i, \nu}^{z} + [(2 \chi^2 + \chi_1) \lambda_1 - \chi_2 \lambda_2] \sum_{\eta} t_{\eta} (h_{i, \eta}^{\dagger} h_{i, \eta + \hat{a}} + h_{i, \eta + \hat{a}}^{\dagger} h_{i, \eta} + 2 \phi_\delta) S_{i, \nu}^{z},
\]

where \( \hat{a} = \hat{t}, \hat{\tau} \), with \( t_{\eta} = t \), \( \phi_\delta = \phi_1 \), and \( t_{\tau} = t' \), \( \phi_\tau = \phi_2 \). After a straightforward calculation, we obtain the dressed spinon self-energy \( \Sigma^{(2)}_{s}(k, \omega) = \langle \langle L_{i}^{(s)}(t); L_{j}^{(s)}(t') \rangle \rangle \rangle \), as,

\[
\Sigma^{(2)}_{s}(k, \omega) = B_k \left( \frac{Z}{N} \right)^2 \sum_{pp'} \gamma_{12}^{(2)}(k, p, p') B_{k+p} \omega_{k+p} \
\times \left( \frac{F_{1}^{(s)}(k, p, p')}{\omega + \omega_{p} - \xi_{p+k}} + \frac{F_{2}^{(s)}(k, p, p')}{\omega + \omega_{p} - \xi_{p+k}} \right),
\]
with $F_{\sigma}^{(s)}(k, p, p') = n_F(\xi_{k+p})[1 - n_F(\xi_{p})] - n_B(\omega_{k+p})[n_F(\xi_{p}) - n_F(\xi_{p+p'})]$, and $F_{\sigma}^{(s)}(k, p, p') = n_F(\xi_{p+p'})[1 - n_F(\xi_{p})] + [1 + n_B(\omega_{k+p})][n_F(\xi_{p}) - n_F(\xi_{p+p'})]$. Within the diagrammatic technique, this dressed spinon self-energy $\Sigma^{(2)}_s(k, \omega)$ corresponds to the contribution from the dressed holon pair bubble.

In the present partial charge-spin separation theoretical framework, the external electronic field can only be coupled to the gauge invariant dressed holons, but the strong correlation between dressed holons and spinons still is considered self-consistently through the dressed spinon's order parameters entering in the dressed holon's propagator. In this case, the resistivity can be obtained as $\rho = 1/\sigma_{dc}$, with $\sigma_{dc} = \lim_{\omega \to 0} \sigma(\omega)$, and the optical conductivity

$$\sigma(\omega) = \frac{Ze^2}{2N} \sum_{k\sigma} \gamma^2_s(k) \int_{-\infty}^{\infty} \frac{d\omega' d\omega}{2\pi} A^{(h)}_{\sigma}(k, \omega' + \omega) A^{(h)*}_{\sigma}(k, \omega') \frac{n_F(\omega + \omega') - n_F(\omega')}{\omega},$$

where $\gamma^2_s(k) = [\sin^2 k_x(\chi_1 t - 2\chi_3 t' \cos k_y)^2 + \sin^2 k_y(\chi_1 t - 2\chi_2 t' \cos k_x)^2]/4$ and the dressed holon spectral function $A^{(h)}_{\sigma}(k, \omega) = -2\text{Im} g_{\sigma}(k, \omega)$. The results of the resistivity as a function of temperature at $x = 0.03$ (solid line), $x = 0.04$ (dashed line), and $x = 0.05$ (dotted line), and $x = 0.06$ (dash-dotted line) for $t/J = 2.5$ and $t'/t = 0.15$ are plotted in Fig. 1 in comparison with the experimental data$^6$ taken from La$_{2-x}$Sr$_x$CuO$_4$ (inset). It is shown obviously that the resistivity is characterized by a crossover from the moderate temperature metallic-like behavior to low temperature insulating-like behavior in the heavily underdoped regime, and a temperature linear dependence with deviations at low temperatures in the moderate underdoped regime. But even in the heavily underdoped regime, the resistivity exhibits the metallic-like behavior over a wide range of temperatures, which are in agreement with the experiments$^6$.

FIG. 1. The electron resistivity as a function of temperature at $x = 0.03$ (solid line), $x = 0.04$ (dashed line), $x = 0.05$ (dotted line), and $x = 0.06$ (dash-dotted line) with $t/J = 2.5$ and $t'/t = 0.15$. Inset: the experimental result of La$_{2-x}$Sr$_x$CuO$_4$ taken from Ref$^6$.

The doped holes into the Mott insulator can be considered as a competition between the kinetic energy ($xt$) and magnetic energy ($J$). The magnetic energy $J$ favors the magnetic order for spins and results in "frustration" of the kinetic energy, while the kinetic energy $xt$ favors delocalization of holes and tends to destroy the magnetic order. In the present partial charge-spin separation fermion-spin theory, the scattering of dressed holons dominates the charge transport. The dressed holon scattering rate is obtained from the dressed holon self-energy by considering the dressed holon-spinon interaction, while this dressed holon self-energy from the dressed spinon pair bubble characterizes
the competition between the kinetic energy and magnetic energy, therefore the striking behavior in the resistivity is
intriguingly related with this competition. In the heavily underdoped regime, the dressed holon kinetic energy is much
smaller than the dressed spinon magnetic energy in lower temperatures, therefore the dressed holons are localized,
and scattering rate from the dressed holon self-energy is severely reduced, this leads to the insulating-like behavior
in the resistivity. With increasing temperatures, the dressed holon kinetic energy is increased, while the dressed
spinon magnetic energy is decreased. In the region where the dressed holon kinetic energy is larger than the dressed
spinon magnetic energy at moderate temperatures, the dressed holons move almost freely, and then the dressed holon
scattering would give rise to the metallic-like behavior in the resistivity.

Now we discuss the dynamical spin response. In the present partial charge-spin separation fermion-spin theory,
the spin fluctuation couples only to dressed spinons, but the effect of dressed holons on dressed spinons has been
considered through the dressed holon’s order parameters entering in the dressed spinon propagator. In this case, we
can obtain the dynamical spin structure factor

\[ S(k, \omega) = -2[1 + n_B(\omega)]\text{Im}D(k, \omega) \]

with \( \text{Im}\Sigma_s^{(2)}(k, \omega) \) and \( \text{Re}\Sigma_s^{(2)}(k, \omega) \) are corresponding imaginary part and real part of the dressed spinon self-energy function \( \Sigma_s^{(2)}(k, \omega) \).

We plot the dynamical spin structure factor spectrum \( S(k, \omega) \) in the \((k_x, k_y)\) plane at \( x = 0.06 \) with \( T = 0.05J \) and \( \omega = 0.05J \) for \( t/J = 2.5 \) and \( t'/t = 0.15 \) in Fig. 2. It is shown that with dopings, there is a commensurate-

\[ \delta(x) \]

incommensurate (IC) transition in the spin fluctuation geometry, where all IC peaks lie on a circle of radius of \( \delta \). Although some IC satellite diagonal peaks appear, the main weight of the IC peaks is in the parallel direction, and these parallel peaks are located at \( [(1 \pm \delta)/2, 1/2] \) and \( [1/2, (1 \pm \delta)/2] \) (hereafter we use the units of \([2\pi, 2\pi]\)). The present dynamical spin structure factor spectrum \( S(k, \omega) \) has been used to extract the doping dependence of the incommensurability \( \delta(x) \), which is defined as the deviation of the peak position from AF wave vector position, and the result is shown in Fig. 3 in comparison with the experimental result\(^{17}\) taken from \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) (inset). Our results show that \( \delta(x) \) increases progressively with the hole concentration at lower dopings, but saturates at higher dopings, which are qualitatively consistent with the experimental observations\(^{17}\).

Our results also show that the effect of dressed holons on the dressed spinon part is critical in determining the
characteristic feature of the IC magnetic correlation. This can be understood from the properties of the dressed spinon

excitation spectrum $E_k^2 = \omega_k^2 + \text{Re}\Sigma_s^{(2)}(k, E_k)$. During the calculation of the dynamical spin structure factor spectrum $S(k, \omega)$, we find when $W(k, \omega) = [\omega^2 - \omega_{k\delta}^2 - \text{Re}\Sigma_s^{(2)}(k\delta, \omega)]^2 \sim 0$ at the critical wave vectors $\pm k\delta$ in low energies, the IC peaks appear, then the weight of the IC peaks is dominated by the inverse of the imaginary part of the dressed spinon self-energy $1/\text{Im}\Sigma_s^{(2)}(k\delta, \omega)$. In this case, the positions of the IC peaks are determined by both functions $W(k, \omega)$ and $\text{Im}\Sigma_s^{(2)}(k, \omega)$, where the zero points of $W(k, \omega)$ (then the critical wave vectors $k\delta$) is doping dependence. Near the half-filling, the zero point of $W(k, \omega)$ locates at the AF wave vector $[1/2, 1/2]$, so the commensurate AF peak appears there. With doping, the holes disturb the AF background. As a result of self-consistent motion of dressed holons and spinons, the IC magnetic correlation is developed beyond certain critical doping. As seen from $S(k, \omega)$, the physics is dominated by the dressed spinon self-energy renormalization due to dressed holon pair bubble. In this sense, the mobile dressed holons are the key factor leading to the IC magnetic correlation, i.e., the mechanism of the IC type of structure in doped cuprates away from the half-filling is most likely related to the dressed holon motion. This is why the position of the IC peaks can be determined in the present study within the $t$-$t'$-$J$ model, while the dressed spinon energy dependence is ascribed purely to the self-energy effects which arise from the dressed holon-spinon interaction.

In summary, we have developed a partial charge-spin separation fermion-spin theory to study the physical property of doped cuprates. In this novel approach, the electron operator is decoupled as the dressed holon and spinon, with the dressed holon keeps track of the charge degree of freedom together with the phase part of the spin degree of freedom, while the dressed spinon keeps track of the amplitude part of the spin degree of freedom, then the electron single occupancy local constraint is satisfied even in MFA. The dressed holon is a magnetic dressing, and then its behaviors like a spinful fermion, while the dressed spinon is neither boson nor fermion, but a hard-core boson. Moreover, both dressed holon and spinon are gauge invariant, and in this sense, they are real as the new elementary particle excitations in the low-dimensional solid. Within the $t$-$t'$-$J$ model, we have studied the charge transport and spin response of the underdoped cuprates, and results are qualitatively consistent with the experimental observations. These results also are qualitatively consistent with the previous results based on the fermion-spin theory, where the phase factor $e^{i\Phi_{i\sigma}}$ described the phase part of the spin degree of freedom was not considered. Our results also show that the charge transport is mainly governed by the scattering from the dressed holons due to the dressed spinon fluctuation, while the scattering from the dressed spinons due to the dressed holon fluctuation dominates the spin response. In this case, the charge transport and spin response are almost independent, and the perturbations that interact primarily with charge do not much affect spin, therefore the notion of the partial charge-spin separation naturally accounts for the qualitative features of doped cuprates.
The present partial charge-spin separation fermion-spin theory also indicates that the kinetic energy (t) term in the t-J type model gives the dressed holon and spinon dynamics in the doped regime without AFLRO, while the magnetic energy (J) term is only to form an adequate dressed spinon configuration. Although the projection operator has been dropped in the actual calculations, leading to an overcounting the number of spin states, the calculated results show that it does not matter in the low-energy sector\(^7\), this is because that the effect of the kinetic energy on the spinon configuration has been considered self-consistently in the partial charge-spin separation fermion-spin theory.

Irrespective of the coupling mechanism responsible for HTSC in doped cuprates, the superconducting state is characterized by the electron Cooper pairs\(^9\). It has been shown\(^{20}\) from ARPES that in the real space the gap function and pairing force have a range of one lattice spacing. In this case, the superconducting order parameter for the electron Cooper pair can be expressed in the present theory as \(\Delta_\eta = \langle C_\uparrow C_\uparrow \rangle - \langle C_\downarrow C_\downarrow \rangle = h_\uparrow, h_\uparrow \eta S_\uparrow \eta S_\uparrow - h_\downarrow, h_\downarrow \eta S_\downarrow \eta S_\downarrow \). In the doped regime without AFLRO, the dressed spinons form the disordered liquid state. In this case, \(\langle S_\uparrow S_\downarrow \rangle = \langle S_\uparrow S_\downarrow \rangle \), and \(\Delta_\eta = \Delta_\eta (S_\uparrow S_\downarrow) \) with the dressed holon Cooper pair \(\Delta_\eta (S_\uparrow S_\downarrow) = h_\uparrow(h_\uparrow \eta - h_\downarrow, h_\downarrow \eta)\), which shows that the symmetry of the electron Cooper pair is essentially determined by the symmetry of the dressed holon Cooper pair\(^21\). It then is possible that the kinetic energy terms in the higher powers of the doping concentration \(x\) cause the superconductivity\(^7\). Since this form of the electron Cooper pair is common, then there is a coexistence of the electron Cooper pair and the AF short-range fluctuation. In other words, the AF short-range correlation may play an important role in the mechanism for HTSC\(^5,17\). However, in the doped regime with AFLRO, where \(\langle S_\uparrow S_\downarrow \rangle \neq \langle S_\uparrow S_\downarrow \rangle \), then obviously the magnetic correlation with AFLRO is not favorable for the superconductivity.

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