IMPORTANCE OF NEUTRINOLESS DOUBLE BETA DECAY

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Abstract

A natural explanation for the smallness of the neutrino mass requires them to be Majorana particles violating lepton number by two units. Since lepton number violation can have several interesting consequences in particle physics and cosmology, it is of utmost importance to find out if there is lepton number violation in nature and what is its magnitude. The neutrinoless double beta decay experiment can answer these questions: if there is lepton number violation and if neutrinos are Majorana particles. In addition, the magnitude of neutrinoless double beta decay will constrain any other lepton number violating processes. This lepton number violation may also be related to the matter-antimatter asymmetry of the universe, dark matter and cosmological constant.
1. Introduction

Unlike any other particles, the existence of neutrinos was postulated to explain energy-momentum conservation in the beta decay. For our understanding of the macroscopic world, the existence of neutrinos are not required. All the phenomena we see around us can be explained by the electromagnetic and the gravitational interactions. In addition the strong interaction is required to explain how the positively charged protons could stay together inside the nucleus. Only the beta decay involved the neutrinos and it interacts very weakly.

The neutrinos are highly puzzling and it took more than sixty years to find out that it has a very small mass. The atmospheric and the solar neutrinos, combined with the Laboratory experiments like KamLAND, have now established that the mass-squared difference between any two of the three neutrinos are non-vanishing:

\[
\Delta m_{\text{atm}}^2 = 2.1 \times 10^{-3} \text{eV}^2 \quad \text{with } \sin^2 2\theta_{\text{atm}} > 0.92
\]
\[
\Delta m_{\text{sol}}^2 = 7.9 \times 10^{-5} \text{eV}^2 \quad \text{with } \tan^2 \theta_{\text{sol}} 0.4 \pm 0.1 ,
\]

where \(\theta_{\text{atm}}\) is the mixing angle between \(\nu_\mu\) and \(\nu_\tau\) and \(\theta_{\text{sol}}\) is the mixing angle between \(\nu_e\) and one of the other two physical states, which is an admixture of the states \(\nu_\mu\) and \(\nu_\tau\). The absolute mass of the neutrinos have not yet been determined, although there is an upper bound on the sum over all neutrino masses from cosmology:

\[
\sum_{i=e,\mu,\tau} m_{\nu_i} \leq 0.69 \text{eV}.
\] (2)

The neutrinoless double beta decay also gives an upper bound on the absolute mass of the neutrinos, but this bound is not valid if the neutrinos are Dirac particles. We shall come back to this discussion later.

2. Dirac and Majorana Neutrinos

The smallness of the neutrino mass can be naturally explained in the standard model, if the neutrinos are Majorana particles. A Majorana particle has the property that it is its own antiparticle. The main difference between a Majorana particle and a Dirac particle lies in their mass terms:

- Majorana particle: \(M_{\text{maj}} \Psi \Psi\)
- Dirac particle: \(M_{\text{dir}} \bar{\Psi} \Psi\).

All charged fermions are Dirac particles, since the Majorana mass terms do not conserve any charge. Only the neutrinos can have either Dirac or Majorana masses. Since the neutrinos carry lepton number, lepton number will be violated if neutrinos are Majorana particles. In the standard model, lepton number is exactly conserved and we have not observed any lepton number violation in nature so far.
Let us now write the mass terms in chiral notation. We define the left-handed and right-handed particles as:

\[ \psi_L = \frac{1 - \gamma_5}{2} \psi \quad \text{and} \quad \psi_R = \frac{1 + \gamma_5}{2} \psi. \]

We now denote the parity transformation \( [(x, t) \leftrightarrow (-x, t)] \) by \( \mathcal{P} \), charge conjugation [particle \( \leftrightarrow \) antiparticle] by \( \mathcal{C} \) and time reversal \( [(x, t) \leftrightarrow (x, -t)] \) as \( \mathcal{T} \). The chiral fields transform under \( \mathcal{C}, \mathcal{P} \) and \( \mathcal{CP} \) as:

\[
\begin{align*}
\psi_L & \xrightarrow{\mathcal{P}} \psi_R & (\psi^c)_L & \xrightarrow{\mathcal{P}} (\psi^c)_R \\
\psi_L & \xrightarrow{\mathcal{C}} \psi^c_L & \psi_R & \xrightarrow{\mathcal{C}} \psi^c_R \\
\psi_L & \xrightarrow{\mathcal{CP}} \psi^c_R & \psi_R & \xrightarrow{\mathcal{CP}} \psi^c_L,
\end{align*}
\]

where the charge conjugation is defined as \( \psi^c = C\overline{\psi}^T = C\gamma_0\psi^* \), with \( C = -i\gamma_2\gamma_0 \) so that \( (\psi^c)_L = (\psi_R)^c = \frac{1}{2}(1 - \gamma_5)\psi^c \) and \( \psi^c_R = (\psi_L)^c = \frac{1}{2}(1 + \gamma_5)\psi^c \). The \( \mathcal{CP}T \) theorem ensures that the \( \mathcal{CP} \) conjugate states of any field must always be present. So, any theory can have the left-handed fields \( \psi_L \) and its \( \mathcal{CP} \) conjugate state \( \psi^c_R \). The mass term requires the field \( \psi_R \) and its \( \mathcal{CP} \) conjugate state \( \psi^c_L \).

Denoting a neutrino by \( \psi \), the most general mass term can be written as

\[
\mathcal{L}_{\text{mass}} = \frac{1}{2} m_L \overline{\psi}_L \psi_L - \frac{1}{2} m_R \overline{\psi}_R \psi_R - m_D \overline{\psi}_R \psi_L + h.c.
\]

\[
= \frac{1}{2} m_L \overline{\psi}_L C^{-1} \psi_L + \frac{1}{2} m_R \overline{\psi}_R C^{-1} \psi_R + m_D \overline{\psi}_R C^{-1} \psi^c_L + h.c.
\]

\[
= \frac{1}{2} \begin{pmatrix} \psi & \psi^c \end{pmatrix}_L C^{-1} \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \psi \\ \psi^c \end{pmatrix}_L = \frac{1}{2} \overline{\Psi}_L C^{-1} M \Psi_L, \quad (3)
\]

where \( \Psi_L^T = (\psi \quad \psi^c)^T \) and \( M \) is a \( 2 \times 2 \) symmetric mass matrix \( M = M^T \).

This general mass term contains most of the information required for an understanding of the Dirac and Majorana masses of the neutrinos. Any theory can have only the left-handed neutrinos and its \( \mathcal{CP} \) conjugate state, but no right-handed neutrinos. In this case the neutrino could be massless or can have a Majorana mass \( m_L \). When both the left-handed and right-handed neutrinos are present, several possibilities can emerge.

- the neutrinos are massless, so there are two Weyl spinors.
- \( m_L = m_R = 0 \), so that the left-handed and the right-handed neutrinos combine to form a Dirac neutrino.
- \( m_L = m_R \neq 0 \) and \( m_D = 0 \), so that there are two Majorana neutrinos and the physical states are \( \psi_L \) and \( \psi_R \) with masses \( m_L \) and \( m_R \).
- \( m_L \) or \( m_R \) or both are non-vanishing, and \( m_D \neq 0 \). In this case also it corresponds to two Majorana neutrinos and the physical states are admixtures of the states \( \psi_L \) and \( \psi_R \) with masses obtained by diagonalising the mass matrix.
The difference between a Majorana and a Dirac particle is that, for a Dirac particle the mass term takes a left-handed particle ($\psi_L$) to a right-handed particle ($\psi_R$), while for a Majorana particle the mass term takes a left-handed particle ($\psi_L$) to a right-handed antiparticle ($\psi^c_R$, which is a CP conjugate of a left-handed particle) or takes a right-handed particle ($\psi_R$) to a left-handed antiparticle ($\psi^c_L$). Another important difference between a Dirac and Majorana particles is the conservation of charges. If neutrinos are Majorana particles, then the mass term violates lepton number by two units.

A direct consequence of the lepton number violation is neutrinoless double beta decay. In some even-even nuclei ordinary beta decay is forbidden, although double beta decay (with and without two neutrinos) could still be allowed

\[
\begin{align*}
    n + n & \rightarrow p + p + e^- + e^- + \nu_e + \nu_e, \\
    n + n & \rightarrow p + p + e^- + e^-.
\end{align*}
\]

The $2\nu\beta\beta$ decay (equation 4) has been observed, in which the total kinetic energy of the two electrons is less than the total energy available, while for the neutrinoless double beta ($0\nu\beta\beta$) decay the total kinetic energy of the two electrons is same as the $Q$ value of the decay. This makes it possible to distinguish these two processes.

![Diagram of Majorana mass of the neutrinos allowing neutrinoless double beta decay.](image)

Figure 1. Majorana mass of the neutrinos allowing neutrinoless double beta decay.

When the neutrinos are Majorana particles there will be total lepton number $L$ violation, which will allow neutrinoless double beta decay through the diagram given in figure 1. Here the neutrinos are virtual particles in the intermediate state, so the neutrino masses and mixing enter into the neutrino propagator. The half-life of the neutrinoless double beta decay thus depends on the effective neutrino mass that enters in the amplitude. The Heidelberg-Moscow $0\nu\beta\beta$ decay experiment looked for the decay mode

$$^{76}Ge \rightarrow ^{76}Se + 2e^-$$

with their high resolution Ge detectors and given a strong bound on the effective mass of the neutrinos, $m_{ee} < 0.2$ eV. They also reported a few events for the $0\nu\beta\beta$ decay, which is yet to be confirmed.
3. Lepton Number Violation

In the standard model, there are three left-handed neutrinos $\nu_{iL}, i = e, \mu, \tau$ that transform under $SU(3)_c \times SU(2)_L \times U(1)_Y$ as $(1, 2, -1)$. Thus the Majorana mass term is not allowed. Since there are no right-handed neutrinos, the Dirac mass of the neutrinos are also not allowed. Thus neutrinos are massless in the standard model. However, a natural explanation for the observed tiny neutrino mass is possible in some extensions of the standard model. A general approach to understand this is to consider the most general dimension-5 effective lepton-number violating operator in the standard model that can contribute to the Majorana masses of the neutrinos \(^6\)

$$\mathcal{L}_{\text{Maj}} = \Lambda^{-1}(\nu \phi^0 - e \phi^+)^2.$$  \hspace{1cm} (6)

Here $\Lambda$ is some lepton-number violating heavy scale in the theory and $\phi$ is the Higgs doublet scalar. The electroweak symmetry breaking ($\langle \phi \rangle \neq 0$) then induce a Majorana mass for the neutrinos

$$\mathcal{L}_{\text{Maj}} = m_{\nu} \nu_{iL}^T C^{-1} \nu_{iL},$$  \hspace{1cm} (7)

where $m_{\nu} = v^2 / \Lambda$. A large lepton number violating scale $\Lambda$ can thus explain naturally why $m_{\nu}$ is much smaller than the charged fermion masses. This also suggests that a Majorana mass of the neutrinos is more natural than a Dirac mass.

The simplest extension of the standard model in which the effective operator \(^6\) may be realized requires either right-handed neutrinos or triplet Higgs scalars. In models with the right-handed neutrinos, one extends the standard model with three right-handed neutrinos $N_{iR}, i = 1, 2, 3$, which are singlets under the standard model. The mass terms for the neutrinos are now given by

$$\mathcal{L}_{\text{mass}} = m_D \nu_L N^c_L + M_R N^c_L N^c_L + H.c.$$  \hspace{1cm} (8)

Here the $3 \times 3$ mass matrix $m_D$ originates from the standard model Higgs vacuum expectation value, so it is of the order of charged lepton masses. But the Majorana mass of the right-handed neutrinos, which is the lepton number violating scale in the theory, could be very large: $M_R \sim 10^{10}$ GeV. Thus assuming $m_D \ll M$, the eigenvalues of this mass matrix then become,

$$m_1 = -\frac{m_D^2}{M_R} \quad \text{and} \quad m_2 = M_R.$$  \hspace{1cm} (9)

We then get a light neutrino with mass $m_1 \sim 0.1$ eV, which is mostly the left-handed neutrino with a small mixing $\tan \theta = \frac{2M}{m_D}$ with the right-handed neutrino. This is also known as the see-saw mechanism of neutrino masses \(^7\). This small neutrino mass will contribute to the neutrinoless double beta decay. Thus the neutrinoless double beta decay can, in principle, determine the absolute mass scale of the neutrinos.

We shall now consider another equivalent realization of the effective operator \(^6\), where the standard model is extended to include a triplet Higgs scalar $\xi$, which transforms under $SU(3)_c \times SU(2)_L \times U(1)_Y$ as $[1, 3, +1]$ \(^8\) \(^9\). Its couplings to the
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Neutrinos and the standard model Higgs doublet $\phi$ break lepton number,

$$\mathcal{L}_{Yuk} = f_{ij} \xi_i \xi_j + \mu \xi_i^\dagger \phi \phi.$$  \hspace{1cm} (10)

We consider \cite{9} the possibility $\mu \neq 0$ ($\mu = 0$ models \cite{8} are ruled out by LEP data). The neutral component of $\xi$ will acquire a induced vev during the electroweak symmetry breaking $u = -\mu v^2 / M^2$, where $M$ is the mass of the triplet Higgs $\xi$. The lepton number is broken explicitly at a very high scale $M \sim \mu$. So, there are no Goldstone bosons corresponding to the broken lepton number symmetry. The mass of the left-handed neutrinos are then given by

$$m_{\nu ij} = f_{ij} u = -f_{ij} \frac{\mu v^2}{M^2},$$  \hspace{1cm} (11)

which is of the order of $\sim$ eV. The neutrino mass matrix is now directly proportional to the Yukawa couplings $f_{ij}$. The absolute mass scale can be determined by the neutrinoless double beta decay.

The smallness of the neutrino mass can thus be naturally explained if neutrinos are Majorana particles and lepton number is violated by two units. The Majorana nature of the neutrinos can be confirmed by the neutrinoless double beta decay. In fact, any lepton number violating processes can contribute to the neutrinoless double beta decay. So, all lepton number violating processes are constrained by the neutrinoless double beta decay \cite{10}. In left-right symmetric models the right-handed charged gauge boson mass and the right-handed neutrino mass could be constrained by the present bound on the $0\nu\beta\beta$ decay. The inverse beta decay are also strongly constrained. The leptoquarks, diquarks and other exotic scalar bilinears that couples to two fermions of the standard model are also constrained by the $0\nu\beta\beta$ decay. In supersymmetric models all the R-parity violating and lepton number violating couplings are strongly constrained by the $0\nu\beta\beta$ decay. Even in R-parity conserving supersymmetric models, there could be lepton number violation originating from the soft terms, which are also constrained by the $0\nu\beta\beta$ decay. The compositeness scale for some models with composite particles are also constrained by the $0\nu\beta\beta$ decay. Some of these constraints and the consequence of $0\nu\beta\beta$ decay in colliders will be reviewed in another article by Prof. S.D. Rindani in this proceedings. We shall now proceed to discuss some cosmological consequences of the lepton number violation.

4. Matter-Antimatter Asymmetry

Our universe is composed mainly of matter and very little antimatter. This matter dominance requires an explanation, since a natural choice would be to start with a universe that is neutral with respect to any conserved charges like baryon or lepton numbers. At present the most popular explanation of this matter dominance in the universe originates from the lepton number violation that is required for the Majorana neutrino masses. This is known as leptogenesis. The present limit on the amount of lepton number violation coming from the neutrinoless double beta decay is just enough to explain this matter dominance and this predictability makes this scenario more appealing. To establish this connection between the neutrinoless double beta decay...
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decay and leptogenesis, we shall discuss leptogenesis in the see-saw model and the
triplet Higgs scalar model discussed in the previous section.

The generation of the baryon asymmetry of the universe requires three ingredients:

i) Baryon number ($B$) violation,

ii) $CP$ violation, and

iii) departure of the $B$-violating interactions from equilibrium. On the other hand if lepton number ($L$) is violated satisfying all the three conditions, then that will generate a lepton asymmetry of the universe. In the standard model, both $B$ and $L$ are global symmetries, but $(B + L)$ is broken by quantum effects arising from anomalous triangle loop diagrams. These anomaly induced $B + L$ violating processes are suppressed by the quantum tunnelling probability. But at finite temperature, during the period $10^2 < T < 10^{12}$ GeV, these interactions become strong in the presence of some static topological field configuration called the sphalerons [11]. As a result any existing $L$ asymmetry of the universe will get converted to the required baryon asymmetry of the universe, before the electroweak phase transition [12, 13].

In the see-saw mechanism of neutrino masses, the Majorana masses of the heavy right-handed neutrinos violate lepton number. $CP$ violation comes from the complex Yukawa couplings and interference of tree level decays with one-loop diagrams. These interactions can also satisfy the out-of-equilibrium decays condition. Thus the decays of the right-handed neutrinos into a lepton ($N_{Ri} \to \ell_j L + \phi$) and also an antilepton ($N_{Ri} \to \ell_j L^c + \phi$) can generate a lepton asymmetry of the universe, which then get converted to a baryon asymmetry of the universe in the presence of the sphalerons.

The $CP$ violation comes from an interference of the tree level decays of the right-handed neutrinos and the one loop diagrams:

![Figure 2](image-url)

Figure 2. One loop (a) vertex and (b) self energy diagrams, which interferes with the tree level right-handed neutrino decays to produce CP violation.

(i) vertex diagram [12, 14] of figure 2, which is similar to the $CP$ violation coming from the penguin diagram in $K$-decays.

(ii) self energy diagram [15] of figure 2, which is similar to the $CP$ violation in $K - \bar{K}$ oscillation, entering in the mass matrix of the heavy Majorana neutrinos.

The interference gives an asymmetry

\[
\delta = \frac{\Gamma(N \to \ell \phi^\dagger) - \Gamma(N \to \ell^c \phi)}{\Gamma(N \to \ell \phi^\dagger) + \Gamma(N \to \ell^c \phi)},
\]  

(12)
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which, when satisfies the out-of-equilibrium condition:

\[ \Gamma(N \rightarrow \ell\ell) < 1.7 \sqrt{g_*} \frac{T^2}{M_P} \quad \text{at } T = M_N, \tag{13} \]

can generate the required lepton asymmetry of the universe. Here the right-hand-side correspond to the expansion rate of the universe and \( M_P \) is the Planck scale. The lepton asymmetry thus generated is same as the \((B - L)\) asymmetry of the universe, since there is no primordial baryon asymmetry at this time. The sphaleron interactions now convert this \((B - L)\) asymmetry to a baryon asymmetry of the universe.

The amount of lepton asymmetry depends on the Yukawa couplings and the out-of-equilibrium condition also depends on the Yukawa couplings. Both these conditions can be satisfied for certain range of parameters, which implies a neutrino mass of \( m_\nu < 0.2 \text{ eV} \) \cite{16}. Although this limit is consistent with the upper bound on the neutrinoless double beta decay, the reported events for the neutrinoless double beta decay is not consistent with this limit \cite{5}. Thus determination of the neutrinoless double beta decay half-life will tell us if the simplest version of leptogenesis is possible.

\[ \xi^{++}_1 \rightarrow \ell^+\ell^+ \quad \text{at tree level and in one-loop order, whose interference gives } \mathcal{C}\mathcal{P} \text{ violation.} \]

The triplet Higgs mechanism of neutrino masses \cite{9} can also allow leptogenesis. The decays of the triplet Higgs scalars \( \xi_a, a = 1, 2 \), two scalars are required for \( \mathcal{C}\mathcal{P} \text{ violation) violate lepton number}

\[ \xi^{++}_a \rightarrow \begin{cases} l^+_i l^+_j \quad (L = -2) \\ \phi^+\phi^+ \quad (L = 0) \end{cases} \tag{14} \]

\( \mathcal{C}\mathcal{P} \text{ violation from the interference of the tree-level decays and the self energy diagrams of figure}\text{5} \text{. The rate of } \xi_b \rightarrow \xi_a \text{ no longer remains to be the same as } \xi^*_b \rightarrow \xi^*_a. \text{ Since by } CPT \text{ theorem } \xi^*_b \rightarrow \xi^*_a = \xi_a \rightarrow \xi_b, \text{ it means}

\[ \Gamma[\xi_a \rightarrow \xi_b] \neq \Gamma[\xi_b \rightarrow \xi_a]. \tag{15} \]

This is a different kind of CP violation compared to the CP violation in models with right-handed neutrinos. The lepton asymmetry is now given by,

\[ \delta = \frac{\Gamma(\xi \rightarrow \ell\ell) - \Gamma(\xi^\dagger \rightarrow l^c\ell^c)}{\Gamma(\xi \rightarrow \ell\ell) + \Gamma(\xi^\dagger \rightarrow l^c\ell^c)}. \tag{16} \]

The out-of-equilibrium condition is satisfied when the triplet Higgs scalars are very heavy. In this case the required amount of lepton asymmetry do not constrain the neutrino masses.
5. Dark Matter and Dark Energy

The total matter in the universe is same as the critical density and about 25% of the matter is dark matter and about 70% of matter is in the form of dark energy. Only about 5% matter is baryonic matter, of which only a fraction is visible. There are several dark matter candidates, including the lightest supersymmetric particle, which is stable and very weakly interacting. One class of dark matter candidate is related to the neutrino masses. If some discrete symmetry forbids the Yukawa coupling relating the left-handed and the right-handed neutrinos, there could be a second Higgs doublet scalar that does not acquire any vev or interact with the charged fermions and remain inert. The lightest of these inert particles (LIP) then could be a dark matter candidate [17].

The problem with dark energy is that the large symmetry breaking scales in particle physics would contribute orders of magnitude large dark energy, while observations indicate that the dark energy is comparable to the dark matter content of the universe. A natural solution is thus to consider a scenario in which the dark energy varies with time starting from a very high value in the early universe. In a popular model, the mass density of a scalar field, called the quintessence, gives the dark energy [18, 19]. The potential of the quintessence field ensures that the decrease of the dark energy is slower than the mass densities of matter and radiation, so that the nucleosynthesis predictions are not altered. Recently it has been pointed out that a varying neutrino mass scenario can account for the dark energy of the universe [20, 21]. The variation of the neutrino mass can originate from some scalar field, which could be a pseudo-Nambu-Goldstone boson in the neutrino sector [22].

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