Analysis of influence factors of $P\Delta$ effect considering vertical ground motion

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Abstract. With the development of structure optimization, building materials and construction technology, the study of the $P\Delta$ effect is becoming more and more important in the pursuit of layers, flexibility and light quality. Based on the research status of $P\Delta$ effect at home and abroad, this paper selects the negative property fictitious member method to calculate the $P\Delta$ effect of the multi-layer frame structure under the combined effect of horizontal and vertical ground motion. This paper discusses the influence of the stiffness-to-weight ratio, structural damping, and the vertical acceleration response of the structure. The results show that the damping and structural acceleration responses have a significant effect on the structural $P\Delta$ effect. Under the premise of considering both, the stiffness-to-weight ratio of the structure, that is, the frequency of the structure, has a greater influence on the $P\Delta$ effect of the structure under earthquakes. With the increase of the stiffness-to-weight ratio, the effect of $P\Delta$ effect on structural response is increased and decreased.

1. Introduction
Second-order effect, as shown in Figure 1, can be divided into $P\delta$ effects and $P\Delta$ effects. Generally, the results of internal force and deformation obtained by the analysis of the second-order effect of the structure are greater than the results of the first-order analysis. The effect of the second-order effect can reach 25% to 30% in some cases[1]. Yi W J[2] et al. used OpenSees to study the second-order effects of the frame structure under near-fault ground motions and far-field ground motions. The results show that the second-order effect of near field pulse earthquake has the most significant impact on the seismic performance of the structure.

Figure 1. $P\delta$ and $P\Delta$ effect.

The $P\Delta$ effect is the main cause of the growth of the lateral deformation of the structure and the increase of the corresponding internal force in the structure, which is the main content of the second-order effect considered in the design, from the middle, low to high level to the super high rise...
building structure[3]. As the height of the building structure increases and the relative lateral stiffness decreases, the \( P-\Delta \) effect will increase [4]. This article focuses on the \( P-\Delta \) effect of the second-order effects.

At present, there are several methods to solve the \( P-\Delta \) effect. In addition to the iterative method [5], the \( P-\Delta \) effect is solved according to the actual situation. Other methods are equivalent transformations of the \( P-\Delta \) effect. From the perspective of the conversion object, they are mainly divided into two types. One is to deal with the internal force of the structure, for example, the additional bending moment of the vertical force is equivalent to the horizontal shear force, and the iterative calculation is carried out, or the first order bending is carried out, such as the amplification coefficient method[6] and the direct solution method[7]; the second is to modify the structural characteristics of the structure, such as the negative property fictitious member method[8], some scholars have called this method negative support rod method [9] (ie, the equivalent stiffness matrix method). The effect of the vertical force on the structure is equivalent to the stiffness added to the original horizontal stiffness matrix.

For the calculation of the \( P-\Delta \) effect, Wilson[1] proposed an iterative-free solution method that converts the additional bending moment generated by the vertical force on the horizontal displacement into an equivalent horizontal load, as shown in Figure 2, which is essentially negative. The negative property fictitious member method is used in this paper to analyze the \( P-\Delta \) effect of the structure.

![Figure 2](image)

(a) Additional moment and corresponding (b) Additional horizontal loads caused by layer weight.

2. The Influence of Structural Layer Stiffness-to-Weight Ratio and Vertical Acceleration Response on the Structure \( P-\Delta \) Effect

Studies have shown that the stiffness-to-weight ratio of structure has a great impact on the seismic response of the structure. In this paper, the north-south and vertical seismic records (abbreviated as Impvall (EL)) measured by the Imperial Valley earthquake in the United States in 1940 at the El centro station (abbreviated as Impvall (EL))and east-west and vertical seismic records measured at the Brawley Airport station in 1979(Abbreviated as Impvall (BRA)) are used to analyze the influence of the stiffness-to-weight ratio. Earthquake ground acceleration time history is shown as Figure 3 and Figure 4.

| Example | Layer weight \( m \) | Horizontal stiffness \( K_h \) | Axial stiffness \( K_v \) | Layer height \( h \) |
|---------|---------------------|-----------------|-------------------|-----------------|
| Example1 | \( 2.3226 \times 10^5 \text{kg} \) | \( 5.6449 \times 10^8 \text{N/m} \) | \( 2.4471 \times 10^8 \text{N/m} \) | 3.4m |
| Example2 | \( 2.3226 \times 10^4 \text{kg} \) | \( 5.6449 \times 10^7 \text{N/m} \) | \( 2.4471 \times 10^7 \text{N/m} \) | |
| Example3 | \( 2.3226 \times 10^5 \text{kg} \) | \( 5.6449 \times 10^7 \text{N/m} \) | \( 2.4471 \times 10^7 \text{N/m} \) | |

Table 1. Structural Characteristics of Examples 1, 2, and 3.

Figure 5 and Figure 6 respectively show the calculation results of the horizontal displacement of each layer under Impvall (EL) and Impvall (BRA) for example 1, 2 and 3. The structural characteristics of Examples 1, 2, and 3 are shown in Table 1. The layer stiffness-to-weight ratio for Examples 1 and 2 are the same, and the layer stiffness-to-weight ratio for Example 3 is one-tenth that of the former two. In the figures, \( u^1 \) refers to the first-order horizontal
displacement of the structure, $u^2$ refers to the second-order horizontal response under the influence of gravity and vertical ground motion, $u^g_2$ refers to the second-order horizontal response only due to gravity, and $u^v_2$ refers to the second-order horizontal response under the influence of vertical ground motion. From Figure 5 and Figure 6, it can be seen that when the layer stiffness-to-weight ratio of the structure is the same, the first-order reaction of the structure and the structural reaction under the influence of the $P$-$\Delta$ effect are basically the same, and after the change of the layer stiffness-to-weight ratio, the first-order reaction of the structure and the influence of the second-order reaction are all greatly affected.

![Figure 3](image3.png)  
**Figure 3.** (a) Horizontal and (b) Vertical time history of ground acceleration of the Impvall (EL) earthquake.

![Figure 4](image4.png)  
**Figure 4.** (a) Horizontal and (b) Vertical time history of ground acceleration of the Impvall (BRA) earthquake.

In addition, the expression of the vertical force is $F^v_i = G_i - m_i(\ddot{v}_g + \ddot{v}_r)$, where $\ddot{v}_g$ is the time history of vertical earthquake ground motion, and $\ddot{v}$ is the relative vertical acceleration response of the particle. In practical applications, sometimes the vertical acceleration is assumed to be equal to the vertical ground motion acceleration without considering the vertical dynamic magnification, i.e., $\ddot{v} = 0$ [10]. This article discusses here to study the effect of $\ddot{v}$ on the structural $P$-$\Delta$ effect. Figure 7(a) and Figure 7(b) are calculated results of case 3 under the effect of Impvall (EL) ground motion and Impvall (BRA) ground motion without considering the vertical dynamic magnification. From the comparison of Figure 5(c) with Figure 7(a), Figure 6(c) and Figure 7(b), it can be seen that under such conditions, the consideration of $\ddot{v}$ has a significant effect on $u^2_v$ (the second-order horizontal response only considering the influence of vertical ground motion). So in this paper, we consider the influence of relative vertical acceleration response on the calculation of $u^2_v$.

In summary, the layer stiffness-to-weight ratio has a greater influence on the first order reaction of structures and the structural response considering the effect of $P$-$\Delta$ effect.
3. Influence of Rayleigh Damping on P-Δ Effect

In order to study the effect of the P-Δ effect, an office building in Africa is chosen as a calculation model. The layout of the structure is shown in Figure 8. The structure is a three span and 17 storey
structure. In order to facilitate comparison, the height of the structure is taken as equal height, all of which are 3.4m. The concrete grade of the column is C40, and the concrete grade of beam and slab is C30. The plate is made of cast-in-place concrete plate with a thickness of 200mm. The floor is taken as 2kN/m². The self weight is 2500kg/m³. The cross-section of the column is 800mm × 800mm and the dimension of beam section is 300mm × 750mm, which is regarded as Example 4.

![Figure 8. Layout plan of structure.](image)

The influence of damping on the structure P-Δ effect is first discussed. On the basis of example 4, two analysis of Rayleigh damping and without considering Rayleigh damping are taken into account in this paper. The structure is entered into Impvall (BRA) and the results of the comparison of the α value are compared, as shown in Figure 9. According to the comparison of the curves in the graph, the Rayleigh damping has a great influence on the calculation result, which is due to the great change in the response spectrum of the ground motion after considering the damping, as shown in Figure 10. Figure 10 (a) is the response spectrum of the ground motion Impvall (BRA) without considering damping. From the diagram, it can be seen that the reaction of the structure increases and decreases when the structure cycle changes, which is in accordance with the change law of Figure 9 (a). Figure 10 (b) is the response spectrum of the ground motion Impvall (BRA) considering damping. Figure 9 (b) is the effect of the ground motion Impvall (BRA) on the α value when the damping is considered in the structure. The figure shows that the value of the α is greater than 1 when the stiffness ratio of the structural layer is greater than 397, which is basically consistent with the law of the seismic response spectrum given by Figure 10 (b). Therefore, the Rayleigh damping is considered in the study of the P-Δ effect later. At the same time, the analysis of ground motion response spectrum also explains why the effect of P-Δ effect on structural response has increased and decreased.
4. Conclusion

In this paper, the $P-\Delta$ effect of multi-layer structures is analyzed under the joint action of horizontal and vertical ground motions, and the effects of structural characteristics on the structure $P-\Delta$ effect are discussed. The main conclusions are as follows:

1. Structural damping has a great influence on the $P-\Delta$ effect of the structure. Considering damping, the response spectrum of the ground motion changes and the response of the structure is different.
2. The acceleration response of the structure under vertical earthquake has a significant effect on the second-order response of the structure.
3. The $P-\Delta$ effect on structural response can be increased and decreased in comparison with the structural first-order response, which depends mainly on the original frequency of the structure and the relative position of the structure frequency on the ground motion response spectrum after the structure $P-\Delta$ effect is taken into consideration.

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