This review outlines the observations that now provide an overwhelming scientific case that the center of our Milky Way Galaxy harbors a supermassive black hole. Observations at infrared wavelength trace stars that orbit about a common focal position and require a central mass ($M$) of $4 \times 10^6 M_\odot$ within a radius of 100 AU. Orbital speeds have been observed to exceed 5,000 km s$^{-1}$. At the focal position there is an extremely compact radio source (Sgr A*), whose apparent size is near the Schwarzschild radius ($2GM/c^2$). This radio source is motionless at the $\sim 1$ km s$^{-1}$ level at the dynamical center of the Galaxy. The mass density required by these observations is now approaching the ultimate limit of a supermassive black hole within the last stable orbit for matter near the event horizon.

Keywords: supermassive black hole; Sgr A*; Galactic center.

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1. Introduction

In the late 18th century, the English naturalist John Michell and the French mathematician Pierre Simon Laplace considered what would happen if a huge mass were placed in an incredibly small volume. They conjectured that gravitational forces might not allow anything, even light, to escape. Two centuries later, Albert Einstein’s theory of General Relativity provided the theoretical foundation for such conjectures, and in the 1960s John Archibald Wheeler introduced the term “black hole” to describe the effects of mass at such an extreme density.

The concept of a black hole formed by the explosive collapse of a dying star is astounding. The possibility that matter from millions and even billions of stars can condense into a single supermassive black hole (SMBH) is even more fantastic. Yet we are now confident that supermassive black holes exist and in fact are commonplace, occupying the centers of many, if not all, of the $\sim 10^{11}$ galaxies in the Universe. Indeed, SMBHs may hold more than 0.01% of the baryonic mass of the Universe.

2. Evidence for Supermassive Black Holes

Early evidence for SMBHs closely paralleled the development of radio astronomy. Very strong sources of radio waves were discovered in the early years of radio astronomy. Accurate positions of these sources revealed that they were often centered on distant galaxies. In the 1950s, radio interferometers revealed a totally unexpected picture of these “radio galaxies.” The radio waves did not come from the galaxy itself, but from two giant “lobes” symmetrically placed about, but well separated from, the parent galaxy (see Fig. 1). These lobes can be among the largest structures in the Universe, hundreds of times the size of the parent galaxy.

How are immense radio lobes energized? Their symmetrical placement about galaxies indicated a galactic link. In the 1960s, sensitive radio interferometers confirmed the previously circumstantial case by detecting faint trails (called “jets”) of radio emission from the lobes back toward the parent galaxy. Radio jets often lead back to a point-like source at the precise center of the galaxy. These point-like sources were found to be variable on timescales less than one year, implying sizes less than a light-year. Martin Rees, in a remarkably prescient paper, showed that the source energetics and variability required synchrotron emitting plasma that has been accelerated to bulk relativistic speeds. One predicted consequence was that such sources would display apparent motions on the sky that were faster than the speed of light (so called super-luminal motion discussed below).

In the late 1960s, Very Long Baseline Interferometry (VLBI) extended baselines to the size of the Earth, achieving resolution ($\approx \lambda/D$, where $\lambda$ is the observing wavelength and $D$ is the maximum separation of interferometer elements) better than 0.001 arcseconds. This was achieved by replacing cables between antennas of the interferometer with tape recorders synchronized with atomic oscillators. Radio images made from VLBI observations revealed that the sources at the centers of
radio galaxies are incredibly small, even smaller than the distance between the Sun and the nearest star.\(^5\)

Simple calculations of the minimum energy needed to power giant radio lobes require the total conversion of up to \(10^7\) stars into energy.\(^6\) Since nuclear reactions convert less than 1% of mass to energy, trying to explain a radio galaxy with nuclear power would require channeling more than \(10^9\) stars through a region smaller than the distance between the Sun and the nearest star. Because of these requirements, astronomers began considering a more efficient energy source: a supermassive black hole.

The small measured sizes of radio sources at the centers of galaxies were consistent with those inferred from the rapid variability. Additionally, VLBI data taken months to years apart indicated that “blobs” of synchrotron emitting plasma appeared to move across the sky at speeds exceeding that of light.\(^7\) This phenomenon, called super-luminal motion, has now been well documented in numerous sources.\(^9\) Super-luminal motion is simply explained as an “optical illusion” that occurs when light emitting plasma moves toward us, nearly along our line of sight, at speeds near, but below, that of light. This phenomenon finds a natural explanation in the acceleration of material from regions very close to the event horizon of a black hole.\(^10\)

In 1990s, images made with the Hubble Space Telescope\(^11\) showed that ma-
Material located within 18 parsecs of the center of the galaxy M 87 was moving with speeds of about 750 km s\(^{-1}\). Assuming the motions are from material in orbit about a large mass concentration, the Virial theorem implies a central mass of \(\approx 2.4 \times 10^9\ M_\odot\) (1 solar mass or \(M_\odot = 2 \times 10^{30}\ kg\)) and an average mass density of \(10^5\ M_\odot\ pc^{-3}\). While this density is very high, it is not above stellar densities in dense clusters and does not require a supermassive black hole.

In the 1980s, maser emission was discovered toward the centers of nearby galaxies\(^{13}\). This emission comes from clouds of gas containing trace amounts of water molecules whose level populations for the \(6_{16} - 5_{23}\) transition at 22 GHz become inverted. In the mid-1990s, water masers in the galaxy NGC 4258 were discovered to have internal motions exceeding 1000 km s\(^{-1}\)\(^{13}\). Subsequent VLBI
images of the masers showed that they came from a rotating disk of material \( \text{[15][16][17]} \). The variation of velocity \( (v) \) with radius \( (r) \) followed \( v \propto 1/\sqrt{r} \) to better than 1% accuracy, indicating gravitational orbits about a compact central mass. The rotation speed was about 1000 km s\(^{-1}\) at a radius of about 0.13 parsecs, requiring a central gravitational mass of \( 4 \times 10^7 \) M\(_\odot\). The corresponding enclosed mass density is \( 4 \times 10^9 \) M\(_\odot\) pc\(^{-3}\). Were one to place \( 4 \times 10^7 \) stars inside a radius of 0.13 parsecs, the system would be dynamically unstable with less massive stars being expelled (“evaporated”) and more massive stars sinking to the center, colliding and possibly forming a black hole. The timescale for the cluster to evaporate would be fairly short, \( \sim 10^9 \) years, making it unlikely that a cluster of stars could provide the central gravitational mass \( \text{[18]} \). However, the densities were not extreme enough to conclusively rule out clusters of some types of objects or more exotic speculations involving dense condensations of elementary particles.

In 1994, the ASCA satellite’s x-ray telescope recorded a spectrum of the nucleus of the galaxy MCG-6-30-15, showing a broad line at 6.4 keV from iron K\( \alpha \) emission \( \text{[19]} \). As shown in Fig. 3, the full width at zero intensity was \( \sim 10^5 \) km s\(^{-1}\), or about 30% of the speed of light! The line is asymmetric with most of the flux appearing redshifted with respect to the rest motion of the galaxy. The simplest interpretation of this line is that it comes from fluorescence of iron atoms in an accretion disk irradiated by a halo of hot gas and all within \( \sim 10R_{\text{Sch}} \) of a massive black hole, where \( R_{\text{Sch}} = 2GM/c^2 \) is the Schwarzschild radius. Thus, by the mid-1990s extremely strong, but perhaps not overwhelming, evidence for supermassive black holes existed.

3. Our Galactic Center

Surveys of the sky at radio wavelengths in the 1950s revealed a strong radio source in the constellation Sagittarius toward the center of the Milky Way. This radio source was named Sagittarius A (Sgr A), where the letter “A” denoted the strongest source in the constellation. Early radio telescopes did not have the angular resolution to resolve this source, and its nature remained a mystery for some time. With the advent of radio interferometry, Sgr A was revealed to contain multiple components \( \text{[20]} \). Fig. 4 shows recent images of this complicated region made the Very Large Array radio interferometer. The left-hand image reveals at least two strong and extended sources: Sgr A-East, a non-thermal (synchrotron emitting) supernova remnant and Sgr A-West, a “spiral-shaped” source of thermal bremsstrahlung emission from ionized gas.

3.1. The Discovery of Sgr A*

In 1974, a very compact radio source, smaller than 1 arcsecond in diameter, was discovered toward Sagittarius A and later given the name Sagittarius A* (Sgr A*) \( \text{[21]} \). Early VLBI observations established that Sgr A* was extremely compact – less than
Fig. 3. Fluorescence Fe Kα line toward the galaxy MCG-6-30-15 from Tanaka et al. (1995). The line is extremely broad (∼10^5 km s^{-1}), asymmetric, and mostly redshifted from the rest energy of about 6.4 keV. This line probably arises from material in an accretion disk inside of 10R_Sch of a SMBH where Doppler and gravitational redshifts are large. (Reprinted by permission of Macmillan Publishers Ltd: Nature, 375, 1995, 659)

Interestingly, even before the discovery of a compact radio source at the center of the Milky Way, it had been speculated that the center of our Galaxy contained a supermassive black hole. This speculation was based primarily on two arguments: 1) the presumed similarity between the nuclei of radio (and other active) galaxies and that of our Galactic center and 2) the possibility of explaining most of the luminosity arising from the Galactic center as owing ultimately to accretion of material onto a black hole. While argument 1) has been borne out, argument 2) has not, since most of the luminosity of the Galactic center can be traced to stellar origins. Unlike the centers of active galaxies believed powered by supermassive black
Fig. 4. Radio wavelength negative images of the center of the Milky Way with East toward the left and North toward the top. Left Panel: The multi-component source Sgr A. The diffuse elliptical shell-like source Sgr A-East, a supernova remnant from an exploded star, fills most of the left side of this panel, which measures ≈ 7 parsecs across. The bright spiral-shaped emission toward the right-center of the panel is called Sgr A-West and comes from plasma spiraling inward to the center. Right Panel: Expanded view of the central 0.25 parsecs, showing the central region of the Sgr A-West spiral. Also seen at the center of the panel is a point-like source called Sgr A*, a candidate for a supermassive black hole. (Images courtesy of J.-H. Zhao.)

holes, Sgr A* is extremely under-luminous$^{24}$. 

### 3.2. The Growing Case for a Dark Mass at the Galactic Center

In the late 1970s, infrared observations of gas motions provided the first clues for a very large mass concentration at the center of the Milky Way. Analysis of the center velocities of fine-structure lines of singly-ionized neon atoms (at a wavelength of 12.8 µm) across Sgr A-West showed differences of ±260 km s$^{-1}$. Assuming that the gas clouds were in circular orbits indicated “a central point-like mass of several ×10^6 M⊙ in addition to several ×10^6 M⊙ of stars within 1 parsec of the center”$^{25}$. This proved to be a remarkably accurate inference. While these observations provided the first solid observational evidence for a supermassive “object” at the center of the Milky Way, other possibilities were recognized. Firstly, while a point-mass component fit the data best, a distributed mass of ~10^7 stars could not be ruled out. Secondly, gas is susceptible to non-gravitational forces, and the assumption of gravitationally bound orbits was questioned. Finally, the “point-mass” component needed only to be smaller than ≈ 0.2 parsecs, and as discussed later in §5, this does not require a SMBH.

While gas is susceptible to non-gravitational forces, stars are not, and soon measurements of radial velocities (Doppler shifts) of stars confirmed the presence of several million solar masses located within ~ 0.1 parsecs of the center of the
Milky Way. There is indeed a dense cluster of stars at the center of the Milky Way, which cannot be seen at optical wavelengths because visible light is totally absorbed by dust between the Galactic center and the Sun. However, these stars can be seen at infrared wavelengths, where \( \sim 10\% \) of the 2 \( \mu \)m wavelength light is received. Using novel techniques that allow diffraction-limited imaging in the infrared, groups led by Reinhard Genzel in Germany and Andrea Ghez in the USA have been measuring positions of these stars for more than a decade. These results showed that stars projected very close to the position of Sgr A* were moving very rapidly across the sky.

Fig. 5 plots the stellar velocity dispersions available in 1998 and the inferred enclosed mass as a function of projected distance from the position of Sgr A*. Measurements of stellar radial velocity sample projected distances from 4 to 0.1 parsecs and proper motions (motions on the plane of the sky) sample from 0.3 to 0.01 parsecs. Between projected radii of 0.2 and 0.01 parsecs, the velocity dispersion, \( \sigma \), increases as \( \sigma \propto 1/\sqrt{r} \). The enclosed mass estimated for virialized material is indeed nearly constant between these radii, greatly strengthening the case for a large “point mass.” The implied mass density, while very high (comparable to that inferred from the water masers in the galaxy NGC 4258 discussed in §2), still could not rule some alternatives to a SMBH (see §5).

![Figure 5](image_url)
4. Recent Overwhelming Evidence that Sgr A* is a SMBH

Evidence for the existence of SMBHs, especially for Sgr A*, has been steadily growing over the years, but recently observational constraints have become so strong that there can be almost no doubt that Sgr A* is a supermassive black hole.

4.1. Stars Orbiting an Immense Unseen Mass Concentration

![False color infrared image taken with by the European Southern Observatory's Very Large Telescope of the central few parsecs of the Milky Way. Superposed with a 100-times finer scale is the orbital track of one star named S2. The orbital period of S2 is 15.8 years, and recently a complete and closed elliptical orbit has been observed. The orbit requires an unseen mass of $\approx 4 \times 10^6 M_\odot$ at the focal position, indicated by the arrow. The focal position is coincident with the position of the compact radio source Sgr A* as discussed in $\S 4.2$. (Image courtesy R. Genzel.)](image)

Fig. 6. False color infrared image taken with by the European Southern Observatory’s Very Large Telescope of the central few parsecs of the Milky Way. Superposed with a 100-times finer scale is the orbital track of one star named S2. The orbital period of S2 is 15.8 years, and recently a complete and closed elliptical orbit has been observed. The orbit requires an unseen mass of $\approx 4 \times 10^6 M_\odot$ at the focal position, indicated by the arrow. The focal position is coincident with the position of the compact radio source Sgr A* as discussed in $\S 4.2$. (Image courtesy R. Genzel.)

Continued monitoring of the positions of stars, with increasing positional accuracy, led to clear detections of acceleration (i.e., curving motions on the sky). Importantly, the directions of the acceleration vectors “pointed” to a common central gravitational source very close to the position of Sgr A*. Recently these observations culminated in the discovery that stars are executing elliptical paths (orbits) on the sky. One star named S2 (a.k.a. S0-2) and has now been observed over one complete 15.8-year elliptical orbit (see Fig. 6). All stellar orbits are well fit by a single enclosed mass and focal position (see Fig. 7). Two stars have
been observed to approach within 100 AU\textsuperscript{1} of the focal position, moving at nearly $10^4$ km s$^{-1}$. The orbital solutions leave no doubt that the stars are responding to an unseen compact mass of $\approx 4 \times 10^6 M_{\odot}$.

We note that the central mass estimated from stellar velocity dispersions ($\approx 2.5 \times 10^6 M_{\odot}$) has been lower than the value obtained from fitting Keplerian orbits ($\approx 4 \times 10^6 M_{\odot}$). Transforming one-dimensional velocity dispersions to enclosed masses can be accomplished with a standard Virial analysis or using alternative statistical mass estimators: e.g. Bahcall-Tremaine.\textsuperscript{31} These methods are usually based on an assumption of isotropic motions. While this is well justified for old stars orbiting the Galactic center, it does not work well for young stars that still “remember” the orbital plane of the gas cloud from which they recently formed. At least one such collection of young stars has been discovered.\textsuperscript{34,42} An alternative statistical mass estimator, the Leonard–Merritt approach, does not depend on isotropic distributions of stellar velocities. However, as for all projected mass estimators, it assumes measurements over all radii. For the central stellar cluster, which is distributed as a power-law over the range of observed radii, normalization problems arise. Empirical corrections to the Leonard–Merritt estimator, for the limited range of projected orbital radii of stars that have been measured, increase the central mass estimate and resolve the discrepancy with orbital fitting.\textsuperscript{34}

The possibility of a combination of a point mass and an extended ($>100$ AU) distribution of mass has been considered.\textsuperscript{46,47,48} Based on the small deviations from an elliptical orbit for star S2 allowed by measurement uncertainty, any extended component within 0.01 parsecs of the center must be $<10\%$ of the point mass. Limits on deviations of orbits from those responding to a pure point-mass will undoubtedly improve rapidly as the star S2 proceeds on a second cycle and other stellar orbits are better traced. Because the orbital paths are almost perfect ellipses, most of the unseen mass must be contained within a radius of about 100 AU (0.0005 pc). This implies a mass density of $>8 \times 10^{15} M_{\odot}$ pc$^{-3}$, which is so great that one can rule out the exotic speculation that a “ball” of Fermions with rest energies of $\sim 15$ keV might provide the extreme central mass for Sgr A* and other, more massive, galactic nuclei (see §5)\textsuperscript{.33}

4.2. The Unseen Mass is Centered on Sgr A*

The infrared results just described are beautifully complemented by observations at radio wavelengths. It is crucial to know the position of Sgr A* on infrared images. However, while Sgr A* is a strong radio source, it is extremely dim at infrared wavelengths. In general, the infrared coordinate system is not known to better than about 0.1 arcseconds relative to the International Celestial Reference Frame determined by VLBI observations at radio wavelengths. Unfortunately, there are

\textsuperscript{1} Astronomical Unit (AU), the mean distance from the Earth to the Sun, equals $1.5 \times 10^{11}$ m or $5 \times 10^{-6}$ parsecs.
Fig. 7. Stars within the 0.02 parsecs of the Galactic center orbiting an unseen mass. Yearly positions of seven stars are indicated with filled colored circles. Both curved paths and accelerations (note the non-uniform spacings between yearly points) are evident. Partial and complete elliptical orbital fits for these stars are indicated with lines. All orbital fits require the same central mass of \( \approx 4 \times 10^6 \, M_{\odot} \) and a common focus at the center of the image, the position of the radio source Sgr A*. (Image courtesy A. Ghez.)

How can one transfer Sgr A*'s radio position to infrared images with better than the 0.01 arcseconds accuracy needed to clearly identify candidates? The key is to find sources visible at both radio and infrared wavelengths. This has been accomplished with red giant stars that are bright in the infrared and have molecular maser emission at radio wavelengths from their circumstellar material. Fig. 8 shows an infrared image of the central ±20 arcseconds of the Galaxy and the positions and measured motions of SiO maser emission at radio wavelengths for nine stars.
This novel combination of infrared and radio observations has allowed the position of the compact radio source at the center of the Galaxy, Sgr A*, to be transferred accurately (±0.01 arcseconds or ±80 AU) to infrared images \[49, 50, 51\], where Sgr A* is extremely dim. The position of Sgr A*, determined in this manner, is circled at the center Fig. [8]. Fig. [9] shows one infrared frame covering the inner-most ±1 arcsecond (±0.04 parsecs) of the Galactic center. Many stars, including some that are known to orbit Sgr A*, are visible. However, at the position of Sgr A* no steady emission was detected, although on this frame the diffraction-limited image of star...
S3 overlaps slightly with Sgr A*. Recently, using improved infrared observing techniques, some weak varying emission has been seen toward the position of Sgr A*. (see §6). However, the infrared emission of Sgr A* is much weaker than individual stars!

The radio position of Sgr A* transfered to infrared images matches the focal position determined from the stellar orbital fits to within the 0.01 arcseconds measurement accuracy. Could Sgr A* be projected toward the Galactic center, but in reality not be located there? The fact that Sgr A*, a nearly unique source in the Milky Way, lies in projection within 0.01 arcseconds (ie, within a solid angle of $\sim 10^{-14}$ steradians) of the gravitational focus of the stellar orbits makes it extraordinarily unlikely that it is simply a projection effect. In addition, Sgr A*'s apparent

Fig. 9. An infrared frame from July 1995 covering the inner ±1 arcsecond from Reid et al. (2003). Three of the stars that orbit the Galactic center, S1, S2, and S3, are labeled, as is the position of Sgr A*. The image of star S3 overlaps slightly with Sgr A*, whose emission is much weaker than a single star. (Reproduced by permission of AAS.)
motion with respect to distant quasars is consistent with it being at the dynamical center of the Milky Way (see §4.4), ensuring that it is indeed at that location.

4.3. **Sgr A*'s Emission is Extremely Compact**

The cm-wavelength emission from Sgr A* is strongly affected by scattering from interstellar electrons, which increases its apparent size. At mm-wavelengths the scattering is reduced, and the true size of the source can be measured. Many groups have analyzed VLBI data for Sgr A* and find that the intrinsic size is less than 1 AU at mm-wavelengths. However, the absence of strong refractive scintillations for Sgr A* provides a lower limit of $\sim 0.1$ AU for the intrinsic size of the cm-wave emission. Thus, the intrinsic radius of the radio emission from Sgr A* near 1 cm wavelength is between 1 and 6 Schwarzschild radii for a $4 \times 10^6 M_\odot$ black hole.

4.4. **Sgr A* is Motionless at Dynamical Center of Milky Way**

How much of the unseen mass in the central 100 AU region can be tied directly to Sgr A*? Were Sgr A* just a stellar mass object in the central stellar cluster, it would be moving at $\sim 10^4$ km s$^{-1}$ in the strong gravitational potential as stars are observed to do. Only if Sgr A* is extremely massive could it move slowly. Early measurements of Sgr A*'s motion revealed it to be moving $< 20$ km s$^{-1}$. Fig. 10 shows the latest data on the apparent motion of Sgr A* on the sky relative to a background quasar. This includes the data published by Reid & Brunthaler in 2004 and new data taken in 2007 that confirms the published results. The apparent movement is as expected for an object that is stationary at the dynamical center of the Milky Way and viewed from the Sun-Earth system, which orbits the Galaxy with a period of $\approx 210$ My at a radius of about 8 kpc and a speed of 240 km s$^{-1}$. It is amazing that VLBA can detect this motion in only a few weeks time.

The Sun’s orbit is almost entirely in the plane of the Galaxy, but the orbital speed in the plane is not easily measured from within the Galaxy and is uncertain by $\approx 20$ km s$^{-1}$. However, the small component of the Sun’s motion perpendicular to the plane of the Galaxy is very accurately known ($7.16 \pm 0.38$ km s$^{-1}$) from observations of $10^4$ stars in the solar neighborhood by the astrometric satellite HIPPARCOS. Thus, the contribution of the Sun’s motion perpendicular to the plane of the Galaxy can be removed from the apparent motion of Sgr A* with very high accuracy. When this is done, Sgr A*'s intrinsic motion perpendicular to the plane of the Galaxy is $-0.4 \pm 0.9$ km s$^{-1}$. The extremely small intrinsic motion (essentially an upper limit) for Sgr A* is close to the expected motion for a SMBH in the presence of its dense cluster of surrounding stars.

The discovery that Sgr A* is nearly stationary at the Galactic center requires that Sgr A* must contain a significant fraction of the unseen mass indicated by the orbiting stars. The calculation of the expected motion of a massive object within
Fig. 10. The apparent motion on the sky of the compact radio source Sgr A* relative to a distant quasar (J1745−283). The dashed line is the variance-weighted best-fit motion of $0.006379 \pm 0.000024$ arcseconds per year for the data published by Reid & Brunthaler through 2004. Recent data from 2007 shown here confirms the published result. All of the apparent motion of Sgr A* can be accounted for by the $\approx 210$ My period orbit of the Sun about the Galactic center. The solid line gives the orientation of the Galactic plane, and the difference in orientation of the two lines is caused by the 7.16 km s$^{-1}$ motion of the Sun perpendicular to the Galactic plane. The residual, intrinsic motion of Sgr A* perpendicular to the Galactic plane is extremely small: $-0.4 \pm 0.9$ km s$^{-1}$. A SMBH perturbed by stars orbiting within its gravitation sphere of influence is expected to move $\approx 0.2$ km s$^{-1}$ in each coordinate. Only a supermassive object could be motionless in the presence of the $4 \times 10^6$ M$_\odot$ known from infrared stellar orbital fits to occupy this region.
its bound stellar cluster (which allows the limit on the motion of Sgr A* to give a minimum mass) can be cast as a gravitational Brownian-motion problem \(^{62,63,64}\). Such a system comes into “thermal” equilibrium, resulting in equipartition of kinetic energy among its constituents. For the Galactic center case in particular, both an analytical result, with only a few simplifications, and a numerical result for fully realistic cases allow estimation of the minimum expected motion of a SMBH surrounded by a dense stellar cluster \(^{60}\). For a \(4 \times 10^6 \, M_\odot\) black hole, one expects \(\approx 0.2 \, \text{km s}^{-1}\) for each component of its motion. This is a very conservative estimate as it does not include possible clustering of the perturbing stars nor any contribution from a cluster of dark stellar remnants, which are expected to have accumulated in this region \(^{65}\). Including either of these effects would increase the expected motion of the black hole, bringing it very close to the measured limit.

Both analytical and numerical estimates of the mass of the motionless radio source Sgr A* yield \(> 4 \times 10^5 \, M_\odot\). Assuming this mass is contained within the observed source size (see \(^{11,12}\)), the mass density is a staggering \(> 7 \times 10^{21} \, M_\odot \, \text{pc}^{-3}\). This density is within about three orders of magnitude of the ultimate limit of \(4 \times 10^6 \, M_\odot\) within its Schwarzschild radius, \(R_{\text{Sch}}\). For the simplest case of a non-rotating black hole, stable gravitational orbits exist only outside of \(3R_{\text{Sch}}\). Any material inside \(3R_{\text{Sch}}\) cannot orbit and falls rapidly into the hole. Thus, the effective volume of such a black hole is \(3^3\)-times that defined by the Schwarzschild radius, resulting in a corresponding decrease in the “ultimate” mass density. In this case, the measured density is within only two orders of magnitude of the black hole limit. At this density the evidence is overwhelming that Sgr A* is a supermassive black hole (see \(^{37,38,39,40}\)).

| Object                  | Mass (\(M_\odot\)) | Radius (AU) | \(M/R\) (kg/m) | Density (\(M_\odot\) pc\(^{-3}\)) | Reference |
|-------------------------|---------------------|-------------|----------------|----------------------------------|-----------|
| Globular Cluster        | \(1 \times 10^6\)   | \(2 \times 10^5\) | \(7 \times 10^{19}\) | \(2 \times 10^5\)                | \([11,12]\) |
| M 87                    | \(2 \times 10^9\)   | \(4 \times 10^6\) | \(7 \times 10^{21}\) | \(1 \times 10^5\)                | \([15]\)   |
| NGC 4258                | \(4 \times 10^7\)   | \(2 \times 10^4\) | \(3 \times 10^{22}\) | \(1 \times 10^{10}\)             | \([37,38,39,40]\) |
| Sgr A* (orbits)         | \(4 \times 10^6\)   | <100        | \(> 5 \times 10^{24}\) | \(> 8 \times 10^{15}\)           | \([37,38,39,40]\) |
| Sgr A* (motion)         | >\(4 \times 10^5\)  | <0.5        | >\(1 \times 10^{25}\) | >\(7 \times 10^{21}\)            | \([60]\)   |
| SMBH (3\(R_{\text{Sch}}\)) | \(4 \times 10^6\) | 0.24       | 2 \times 10^{26} | 7 \times 10^{23}               |           |

For comparison, Table 1 lists observed densities, in standard astronomical units, for dense stellar clusters (i.e., globular clusters), some SMBH candidates, and a Schwarzschild SMBH within its inner-most stable orbit. Also listed are the mass-to-radius (\(M/R\)) ratios, in standard physical units. For a black hole, \(M/3R_{\text{Sch}} = c^2/6G\).

The above density calculation assumes that the size of the emitting region is equal to or greater than the size of the mass in Sgr A*. This is true for almost all astrophysical sources. Notable exceptions for radio sources are solar flares and
pulsars. However, these sources 1) are sporadic (either flares or pulses), 2) are characterized by highly-polarized gyrosynchroton emission and 3) have spectra with flux density falling very rapidly with increasing observing frequency. Sgr A* shares none of these characteristics; its radio emission is generally slowly varying, almost unpolarized, and has a rising spectrum.

5. Excluding Alternatives to a SMBH

Are there alternatives to a SMBH that are consistent with the extraordinarily high mass density of Sgr A*? The most obvious possibility is a cluster of dark stars. As a point of reference, globular clusters are spherical collections of upwards of $10^6$ stars within a radius of $\sim 1$ parsec. These are long-lived systems, some nearly the age of the Universe, and their stellar densities are truly astounding – imagine placing $\sim 10^6$ stars between the Sun and the nearest star! However, while such a density is extremely high, it is nearly a factor of $10^{18}$ times less than for a SMBH of Sgr A*'s mass. (Also, normal, luminous stars are easily ruled out by the dearth of infrared emission from the position of Sgr A*.)

Dense clusters of stars undergo significant gravitational interactions, including core-collapse, collisions and evaporation of stars. Both the evaporation or collisional timescales can provide approximate upper limits for the lifetimes of such systems. If the cluster members have typical masses of $\sim 1$ M$_\odot$, then the evaporation timescale for a cluster that satisfies the Galactic center stellar orbital data would be $< 10^6$ years. The existence of a cluster with such a short lifetime is extremely implausible.

Evaporation timescales are approximately inversely proportional to the typical mass of a cluster member. Thus, by postulating very low member masses, the evaporation timescale can be made arbitrarily long. In order to have an evaporation timescale be a reasonable fraction of the age of the Galaxy ($\approx 13 \times 10^9$ years), member masses would need to be $< 0.001$ M$_\odot$. Thus, one would need $\sim 10^{10}$ planets like Jupiter to make a long-lived $4 \times 10^6$ M$_\odot$ cluster. This would seem impossible to arrange.

The argument for excluding dense stellar clusters based on their rapid evaporation timescales is based on the implicit assumption of an isolated system. Perhaps a quasi-steady state condition could occur for which the cluster is fed stars from the outside at a rate comparable to the evaporation rate. A $4 \times 10^6$ M$_\odot$ cluster that evaporates in $10^6$ years would require the addition of only a few stars per year to offset evaporation losses. This possibility seems unlikely but has yet to be critically analyzed.

Are there other possibilities for extreme concentrations of matter at the Galactic center? Hypothetical concentrations of exotic dark particles have been considered as alternatives to supermassive black holes in the centers of galaxies. For example, a “ball” of heavy Fermions supported by degeneracy pressure has been proposed. One attractive aspect of this alternative to supermassive black holes was
that Fermions of rest energy $\sim 15$ keV could naturally explain the range of masses ($\sim 3 \times 10^6$ to $\sim 3 \times 10^9 M_\odot$) observed at the centers of galaxies. However, a Fermion ball cannot achieve extraordinarily high central densities and its gravitational potential is softer than that of a black hole and flattens at the center. In 2002, Munyaneza & Viollier showed that the then existing stellar motion data could only be fit by a restricted range of parameter space for a supermassive black hole, but could be easily fit by Fermion ball models. Subsequent observations of stellar orbits showed that the supermassive black hole predictions were indeed met, and that the stars moved with speeds greater than allowed by their Fermion ball model.

We note that the density limit employed in this section was the one obtained only from the orbits of stars. The additional constraints from the radio observations provide nearly a factor of $10^6$ more stringent limit on density, making all of the above arguments vastly stronger. Since the density limits are now within only about two orders of magnitude of that of a SMBH (see Table 1), any alternatives to a SMBH must allow similarly high densities. At such densities it is difficult to avoid gravitational collapse and proposed alternatives, such as “boson stars’ and “magnetospheric eternally collapsing objects,” or others, possibly involving “new physics,” would be even more fantastic than a “mundane” SMBH.

Finally, there is now strong evidence for the existence of an event horizon in both stellar-mass black holes and for SMBHs. Many galactic nuclei, in which high mass-densities indicative of SMBHs are observed, are under-luminous and best explained with radiatively inefficient accretion flows in which energy can vanish through the event horizon. Without an event horizon, energy liberated by the accretion process cannot vanish. Indeed, Sgr A* is such an under-luminous object, and evidence from the dearth of infrared emission and an extremely small intrinsic size strongly point to an event horizon.

6. Other Evidence for a SMBH

While quiescent emission from Sgr A* has been difficult to detect outside of cm to sub-millimeter wavelengths, it does produce detectable “flares” of short duration at radio, infrared, X-ray wavelengths and x-ray energies. This flaring is thought to be associated with material occasionally spiraling inward, converting a significant fraction of its total energy into heat, and radiating profusely near the inner-most stable orbit around a black hole. Of particular interest is the hint of a quasi-periodic flaring at infrared wavelengths, which could arise during the final few orbits of material falling into a black hole. Since, the radius of the inner-most stable orbit depends on black hole spin and whether the material is orbiting in a prograde or retrograde sense, this may provide an observational approach to measure black hole spin.

In 1988, J. G. Hills considered the fate of tightly bound binary stars that encounter a supermassive black hole at the Galactic center. If the encounter is close enough, he predicted that one of the stars could become bound to the black hole,
while the other could be ejected at high speeds of up to 4000 km s$^{-1}$. Detailed calculations of ejection rates have now been made. Recently, such hyper-velocity stars have been discovered leaving the Milky Way, confirming Hills’ prediction. While this may be a strong confirmation of the existence of a supermassive black hole in the Galactic center, it is probable that large numbers of stellar-mass black holes have migrated to the inner 0.1 parsec of the center and hyper-velocity stars can also be ejected from interactions with these lower mass black holes.

7. Broader Implications of SMBHs at Centers of Galaxies

The highly complementary discoveries at infrared wavelengths (of stars orbiting an unseen massive object) and at radio wavelengths (that the stellar orbital focus coincides with Sgr A*, that the size of Sgr A* is comparable to $R_{\text{Sch}}$, and that Sgr A* is essentially motionless at the dynamical center of the Galaxy) establish with near certainty that the center of the Milky Way is anchored by a supermassive black hole. Of course the Milky Way cannot be unique in the Universe in having a supermassive black hole at its center, and, indeed, the Hubble Space Telescope finds evidence for a SMBH at the center of all galaxies nearby enough so that the telescope can resolve the gravitational sphere of influence of the SMBH. While the centers of most, if not all, galaxies contain a SMBH, this is not to say that all galaxies are identical. The mass and spin of the black hole and the density of stars and gas within its gravitational sphere of influence lead to a rich variety of phenomena, which are collectively called active galactic nuclei. Sgr A*, being the nearest SMBH, serves as the archetypal source for understanding other galaxies.

Astronomers now believe that when supermassive black holes accrete matter at a high rate they become extremely luminous and can outshine their entire host galaxy of $\sim 10^{11}$ stars (ie, the quasar phenomenon). Quasars have been found at high redshifts, indicating they existed as early as 1 billion years after the Big Bang, when the Universe was less than 8% of its current age. While it is not yet understood how supermassive black holes form so quickly, they now appear to be an integral part of the generation of structure in the early Universe.

Recent observations show a strong correlation between the masses of supermassive black holes and the masses and motions of stellar bulges which protrude from disk galaxies. These observations and large computational simulations of how galaxies form indicate that supermassive black holes may shape the evolution of galaxies. Thus, supermassive black holes are not only fascinating objects from the perspective of fundamental physics, they also are exceptionally important from an astrophysical perspective for determining the nature of galaxies.

8. Future Observations

We know that galaxies interact gravitationally and sometimes collide and merge. The merger of two galaxies, each containing a SMBH, would be followed by orbital
decay of the two nuclei, owing first to dynamical friction and ultimately to gravitational radiation (when a tight black hole binary is reached). The final merging of two SMBHs produces very strong gravitational waves, which might be detected with the future space-based gravity-wave detector LISA\textsuperscript{101}.

Returning to the nearest SMBH, Sgr A*, an obvious goal for astronomers is to image the region within the inner-most stable orbit with resolution of $\sim R_{\text{Sch}}$. As this review was being assembled, interferometer fringes were reported toward Sgr A* at a wavelength of 1.3 mm and a fringe spacing of 0.00005 arcseconds\textsuperscript{77}. This demonstrates that a significant fraction of the mm-wave emission of Sgr A* comes from a region with a radius $< 0.2$ AU, which is $< 3R_{\text{Sch}}$. When “snapshot” images can be made at slightly shorter wavelengths (eg, 0.8 mm), they should reveal the “shadow” caused by extreme bending of space around a black hole\textsuperscript{102} and might show highly dynamic activity associated with the final in-spiral of plasma toward the event horizon\textsuperscript{103}.

Astrometric observations of the region within $\sim 10R_{\text{Sch}}$ can also be made with infrared interferometry. Plans for such an interferometer experiment, called GRAVITY\textsuperscript{104}, are well underway. Since the radius, and hence period, of the inner-most stable orbit depends on the black hole spin, measuring the in-spiraling material motion can yield determinations of spin.

Stars orbiting with short periods ($\sim 1$ yr) in highly elliptical orbits would approach very close to Sgr A* at pericenter and should display general relativistic precession of their orbital planes. Diffraction-limited imaging of the Galactic center in the infrared with future extremely large (> 30 m diameter) telescopes should be able to make such observations\textsuperscript{105} and also allow a test of the “no hair” theorem for a black hole\textsuperscript{110}.

9. Summary

The major observational results that provide overwhelming evidence that Sgr A* is a SMBH are as follows:

- Stars near Sgr A* move on elliptical orbits with a common focal position.
- The required central mass is $4 \times 10^6 M_\odot$ within a radius of 100 AU.
- The position of Sgr A* agrees with the orbital focus to within measurement uncertainty of $\pm 80$ AU.
- The infrared emission from Sgr A* is far less luminous than a single star.
- The intrinsic size of Sgr A* at mm-wavelengths is $< 6R_{\text{Sch}}$.
- Sgr A* is intrinsically motionless at the km s$^{-1}$ level at the dynamical center of the Galaxy.

The great impact of these discoveries is their simplicity and elegance. Elliptical orbits for stars provide an absolutely clear and unequivocal proof of a great unseen mass concentration. The discoveries that the compact radio source is at the position of the unseen mass and is motionless provide even more compelling evidence for a
supermassive black hole. Together they form a simple, unique demonstration that the fantastic concept of a supermassive black hole is, with a high degree of certainty, a reality.

Acknowledgements

I thank R. Genzel and A. Ghez for comments and suggestions on a draft of this review.

References

1. W. A. Dent, Science 148 (1965) 1458.
2. P. Maltby & A. T. Moffet, Science 150 (1965) 63.
3. M. J. Rees, Nature 211 (1966) 468.
4. M. H. Cohen, D. L. Jauncey, K. I. Kellermann & Clark, B. G. Science 162 (1968) 88.
5. K. I. Kellermann et al., Ap. J. Lett. 153 (1968) L209.
6. K. I. Kellermann & F. N. Owen, Radio Galaxies and Quasars, in Galactic and Extra-galactic Radio Astronomy, Second Edition, eds. G. L. Verschuur & K. I. Kellermann (Springer-Verlag, New York, 1988), p. 563.
7. A. R. Whitney et al., Science 173 (1971) 225.
8. M. H. Cohen, W. Cannon, G. H. Purcell, D. B. Shaffer, J. J. Broderick, K. I. Kellermann & D. L. Jauncey, Ap. J. 170 (1971) 207.
9. A. Zensus, Ann. Rev. Ast. Astroph. 35 (1997) 603.
10. R. D. Blandford & R. L. Znajek, Mon. Not. Roy. Aston. Soc. 179 (1977) 433.
11. H. C. Ford et al., Ap. J. Lett. 435 (1994) L27.
12. R. J. Harms et al., Ap. J. Lett. 435 (1994) L35.
13. M. J. Claussen, G. M. Heiligman & K. Y. Lo, Nature 310 (1984) 298.
14. N. Nakai, M. Inoue & M. Miyoshi, Nature 361 (1993) 45.
15. M. Miyoshi, J. Moran, J. Herrnstein, L. Greenhill, N. Nakai, P. Diamond & M. Inoue, Nature 373 (1995) 127.
16. L. J. Greenhill, D. R. Jiang, J. M. Moran, M. J. Reid, K. Y. Lo & M. J. Claussen, Ap. J. 440 (1995) 619.
17. J. Herrnstein et al., Nature 400 (1999) 539.
18. E. Maoz Ap. J. Lett. 494 (1998) L181.
19. Y. Tanaka et al., Nature 375 (1995) 659.
20. M. Morris & E. Serabyn Ann. Rev. Ast. Astroph. 34 (1996) 645.
21. B. Balick & R. L. Brown, Ap. J. 194 (1974) 265.
22. K. Y. Lo, D. C. Backer, R. D. Ekers, K. I. Kellermann, M. J. Reid & J. M. Moran, Nature 315 (1985) 124.
23. D. Lynden-Bell & M. J. Rees Mon. Not. Roy. Aston. Soc. 152 (1971) 461.
24. L. M. Ozernoy & R. Genzel, Is Sgr A* Underfed, Underefficient, or Underdone?, in Unsolved Problems of the Milky Way, Proceedings of the 169th Symposium of the International Astronomical Union, eds. L.Blitz and P.Teuben (Kluwer, Dordrecht, 1996), p. 181.
25. J. H. Lacy, C. H. Townes, T. R. Geballe & D. J. Hollenbach, Ap. J. 241 (1980) 132.
26. M. T. McGinn, K. Sellgren, E. E. Becklin & D. N. B. Hall, Ap. J. 338 (1989) 824.
27. K. Sellgren, M. T. McGinn, E. E. Becklin & D. N. Hall, Ap. J. 359 (1990) 112.
28. A. Krabbe, Ap. J. Lett. 447 (1995) L95.
29. J. W. Haller, M. J. Rieke, G. H Rieke, P. Tamblyn, L. Close & F. Melia, Ap. J. 456 (1996) 194.
30. R. Genzel, A. Eckart, T. Ott & F. Eisenhauer Mon. Not. Roy. Ast. Soc. 291 (1997) 219.
31. A. Eckart & R. Genzel, Nature 383 (1996) 415.
32. A. Eckart & R. Genzel, Mon. Not. Roy. Ast. Soc. 284 (1997) 576.
33. A. M. Ghez, B. L. Klein, M. Morris & E. E. Becklin Ap. J. 509 (1998) 678.
34. R. Genzel, C. Pichon, A. Eckart, O. E. Gerhard & T. Ott Mon. Not. Roy. Ast. Soc. 317 (2000) 348.
35. A. M. Ghez, M. Morris, E. E. Becklin, A. Tanner & T. Kremenek Nature 407 (2000) 349.
36. A. Eckart, R. Genzel, T. Ott & R. Schödel, Mon. Not. Roy. Ast. Soc. 331 (2002) 917.
37. R. Schödel et al., Nature 419 (2002) 694.
38. A. M. Ghez, E. Becklin, G. Duchêne, S. Hornstein, M. Morris, S. Salim & A. Tanner, Astronomische Nachrichten 324 (2003) 527.
39. R. Schödel, T. Ott, R. Genzel, A. Eckart, N. Mouawad & T. Alexander, Ap. J. 596 (2003) 1015.
40. A. M. Ghez et al., Ap. J. 620 (2005) 744.
41. J. N. Bahcall & S. Tremaine Ap. J. 244 (1981) 805.
42. J. R. Lu, A. M. Ghez, S. D. Hornstein, M. R. Morris, E. E. Becklin & K. Matthews, to appear in Ap. J..
43. P. J. T. Leonard & D. Merritt Ap. J. 339 (1989) 195.
44. R. Genzel et al., Ap. J. 594 (2003) 812.
45. R. Schödel et al., Astron. Astroph. 469 (2007) 125.
46. N. Mouawad, A. Eckart, S. Pfalzner, R. Schdel, J. Moultaka & R. Spurzem Astronomische Nachrichten 326 (2005) 83.
47. A. Ghez et al., to appear in Ap. J..
48. S. Gillessen, F. Eisenhauer, S. Trippe, T. Alexander, R. Genzel, F. Martins & T. Ott, to appear in Ap. J..
49. K. M. Menten, M. J. Reid, A. Eckart & R. Genzel Ap. J. 475 (1997), L111.
50. M. J. Reid, K. M. Menten, R. Genzel, T. Ott, R. Schödel & Eckart, A. Ap. J. 587 (2003), 208.
51. M. J. Reid, K. M. Menten, S. Trippe, T. Ott & R. Genzel Ap. J. 659 (2007), 378.
52. A. E. E. Rogers et al., Ap. J. Lett. 434 (1994) L59.
53. T. P. Krichbaum et al., Astron. Astroph. Lett. 335 (1998) L106.
54. S. S. Doeleman et al., Astron. J. 121 (2001) 2610.
55. G. C. Bower, H. Falcke, R. M. Herrnstein, J.-H. Zhao, W. M. Goss & D. C. Backer, Ap. J. 524 (2004) 805.
56. Z.-Q. Shen, K. Y. Lo, M.-C. Liang, P. T. P. Ho & J.-H. Zhao, Nature 438 (2005) 62.
57. C. R. Gwinn, R. M. Danen, J. Middleditch, L. M. Ozernoy & T. Kh. Tran Ap. J. Lett. 381 (1991) L43.
58. D. C. Backer & R. A. Sramek, Ap. J. 524 (1999) 805.
59. M. J. Reid, A. C. S. Readhead, R. C. Vermeulen & R. N. Treuhaft, Ap. J. 524 (1999) 816.
60. M. J. Reid & A. Brunthaler, Ap. J. 616 (2004) 872.
61. W. Dehnen & J. Binney, Mon. Not. Roy. Ast. Soc. 298 (1998) 387.
62. P. Chatterjee, L. Hernquist & A. Loeb, Ap. J. 572 (2002) 371.
63. E. N. Dorband, M. Hemsendorf & D. Merritt, J. Comput. Phys. 185 484.
64. D. Merritt, S. Harfst & G. Bertone Astron. J. 6133 (2007) 553.
65. M. Morris, Ap. J. 408 (1993) 496.
66. J. Binney & S. Tremaine, Galactic Dynamics (Princeton Univ. Press, Princeton, 1987).
67. R. D. Viollier, D. Trautmann & G. B. Tupper, Phys. Lett. B 306 (1993) 79.
68. D. Tsiklauri & R. D. Viollier, Mon. Not. Roy. Ast. Soc. 282 (1996) 1299.
69. F. Munynzea, D. Tsiklauri & R. D. Viollier, Ap. J. 509 (1998) 105.
70. N. Bilić, F. Munynzea & R. D. Viollier, Phys. Rev. D 59 4003.
71. F. Munynzea & R. D. Viollier, Ap. J. 564 (2002) 274.
72. D. F. Torres, S. Capozziello & G. Lambiase, Phys. Rev. D 62 (2000) 104012.
73. S. Robertson & D. Leiter, Ap. J. Lett. 596 (2003) L203.
74. M. R. Garcia, J. E. McClintock, R. Narayan, P. Callanan, D. Barret & S. S. Murray, Ap. J. Lett. 553 (2001) L47.
75. J. E. McClintock, R. Narayan & G. B. Rybicki, Ap. J. 615 (2004) 402.
76. R. Narayan & J. E. McClintock, New Astron. Rev. 51 (2007) 733.
77. S. S. Doelman et al., to appear in Nature.
78. A. E. Broderick & R. Narayan, Ap. J. Lett. 638 (2008) L21.
79. R. M. Herrnstein, J.-H. Zhao, G. C. Bower & W. M. Goss, Astron. J. 127 (2004) 3399.
80. J. C. Mauerhan, M. Morris, F. Walter & F. K. Baganoff, Ap. J. Lett. 623 (2005) L25.
81. D. P. Marrone, J. M. Moran, J.-H. Zhao & R. Rao, Ap. J. 640 (2006) 308.
82. R. Genzel et al., Nature 425 (2003) 934.
83. A. M. Ghez et al., Ap. J. Lett. 601 (2004) L159.
84. S. D. Hornstein et al., Ap. J. 667 (2007) 900.
85. T. Do et al., to appear in Ap. J.
86. F. K. Baganoff et al., Nature 413 (2001) 45.
87. W. Misner, K. S. Thorne & J. A. Wheeler, Gravitation, (Freeman, New York, 1973)
88. R. Shafee, J. E. McClintock, R. Narayan, S. W. Davis, L.-X. Li & R. A. Remillard, Ap. J. Lett. 636 (2006) L113.
89. J. E. McClintock, R. Shafee, R. Narayan, R. A. Remillard, S. W. Davis & L.-X. Li, Ap. J. 652 (2006) 518.
90. J. G. Hills, Nature 331 (1988) 687.
91. Q. Yu & S. Tremaine, Ap. J. 599 (2003) 1129.
92. W. R. Brown, M. J. Geller, S. J. Kenyon & M. J. Kurtz, Ap. J. Lett. 622 (2005) L33.
93. R. M. O’Leary & A. Loeb, Mon. Not. Roy. Ast. Soc. 383 (2008) 86.
94. L. Ferrarese & D. Merritt, Physics World 15N6 (2002) 41.
95. X. Fan et al., Astron. J. 122 (2001) 2833.
96. Z. Haiman & A. Loeb, Ap. J. 552 (2005) 459.
97. J. Magorrian et al., Astron. J. 115 (1998) 2285.
98. L. Ferrarese & D. Merritt, Ap. J. Lett. 539 (2000) L9.
99. V. Springel, T. Di Matteo & L. Hernquist Mon. Not. Roy. Ast. Soc. 361 (2005) 776.
100. B. Robertson, L. Hernquist, T. J. Cox, T. Di Matteo, P. F. Hopkins, P. Martini & V. Springel, Ap. J. 641 (2006) 90.
101. http://lisa.nasa.gov/
eds. W. J. Jin, I. Platais and M. A. C. Perryman (Cambridge Univ. Press, Cambridge, 2008), p. 100.

105. N. N. Weinberg, M. Milosavljević & A. M. Ghez, Ap. J. 622 (2005) 878.

106. C. M. Will, Ap. J. Lett. 674 (2008) L25.