On the adversarial robustness of DNNs based on error correcting output codes

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Adversarial examples represent a great security threat for deep learning systems, pushing researchers to develop suitable defense mechanisms. The use of networks adopting error-correcting output codes (ECOC) has recently been proposed to deal with white-box attacks. In this paper, we carry out an in-depth investigation of the security achieved by the ECOC approach. In contrast to previous findings, our analysis reveals that, when the attack is in the white-box framework, the ECOC scheme can be attacked by introducing a rather small perturbation. We do so by considering both the popular adversarial attack proposed by Carlini and Wagner (C&W) and a new variant of C&W attack specifically designed for multi-label classification architectures, like the ECOC-based structure. Experimental results regarding different classification tasks demonstrate that ECOC networks can be successfully attacked by both the original C&W attack and the new attack.

Introduction: Deep neural networks can solve complicated computer vision tasks with unprecedented high accuracies. However, they have been shown to be vulnerable to adversarial examples, namely, properly crafted inputs introducing small (often imperceptible) perturbations, that can easily make the networks fail [1][2][3]. As a consequence, several defense mechanisms have been proposed to alleviate this threat. In this work, we demonstrate that ECOC networks can be successfully attacked by adversarial examples.

In general, the activation layer consists of the application of an activation function, that maps the logits into a prescribed range, and a normalization layer, that maps the output of the activation functions into a probability vector, associating a probability value to each class. For simplicity, in [10], the ECOC classifier is built by using a classical one-hot-encoding case, attacking the ECOC scheme introduces a distortion in the image which is just a bit larger (the average PSNR is 1dB or 2dB lower than in the one-hot-encoding case). We also verified that, by increasing the confidence of the attack, adversarial examples can achieve higher probabilities for the predicted target class, similar to those of benign samples, hence making it difficult to use the prediction confidence to detect adversarial samples. Overall, our analysis reveals that the security gain achieved by the ECOC scheme is a minor one, thus calling for more powerful defenses.

In the rest of the paper, we first briefly review the ECOC scheme presented in [10], then we describe the proposed attacks. The setup considered for the experiments and the results we got are reported and discussed in the Experiments section.

ECOC-based classification: Let us first introduce the notation for a general multi-class CNN. Let $x$ be the input of the network and $y$ the class label, $k = 1, 2, ..., M$, where $M$ denotes the number of classes. Let \( f(x) \) indicate the decision function of the network. We denote by $z = (z_1, z_2, ..., z_M)$ the vector with the logit values, that is, as we said, the network values before the final activations and the mapping to class probabilities. For one-hot encoding schemes, $z$ has length $M$ and the logits are directly mapped into probability values through the softmax function $\psi$ as follows:

$$
\psi_k = \psi_k(z) = \frac{\exp(z_k)}{\sum_{i=1}^{M} \exp(z_i)},
$$

for $k = 1, ..., M$. Then, the final prediction is made by letting $f(x) = \text{arg max}_k \psi_k(x)$.

The error-correction-output-coding (ECOC) scheme proposed in [10] assigns a codeword $C_k$ of length $N (N > M)$ to every output class $k = 1, ..., M$. $C_k$ denotes the $M \times N$ codeword matrix. Each element of $C_k$ can take values in $\{-1, 1\}$. In this way, the length of the logit vector $z$ is $N$. The logits are first mapped into the $\{-1, 1\}$ range by means of an activation function $\sigma()$. Then, the probability of class $k$ computed by looking at the correlation with $C_k$, according to the following formula:

$$
\sigma_k = \frac{\text{max} (\sigma(z) \cdot C_k, 0)}{\text{sum}_i^{N} \text{max} (\sigma(z) \cdot C_i, 0)},
$$

where $\cdot$ denotes the inner product and $\sigma()$ is a sigmoid activation function applied element-wise to the logits. Since $C_k$'s take values in $\{-1, 1\}$, the $\max$ is necessary to avoid negative probabilities. A sketch of the ECOC scheme is given in Figure 1. The logits $z$ are first mapped into correlation values, $\psi = \sigma(z) \cdot C$ (mapping step 1), then the vector with the correlations is normalized so to form a probability distribution (mapping step 2) via the softmax-like function in $\psi$. The model's final predicted label is $\arg\max_k \sigma_k$.

The purpose of the ECOC-based approach is to design a classifier which is robust to changes of multiple logits, and then, expectedly, more difficult to attack (with standard one-hot encoding the adversary can succeed by altering a single logit). For the scheme to be effective, codewords characterised by a large minimum Hamming distance must be chosen. For simplicity, in [10], the ECOC classifier is built by using Hadamard codes taking values in $\{-1, 1\}$ (when $C$ is a Hadamard matrix, the Hamming distance for large $M$ approaches the limit value $N/2$). A classical one-hot choice is that, since $C$ is orthogonal, the network outputs a codeword exactly (that is when $\sigma(z) \cdot C_k = 0$), then $\sigma_k = 1$. The tanh function is selected as the activation function $\sigma()$.
The authors also found that, rather than considering a single network with \( N \) outputs, a classifier consisting of an ensemble of several smaller networks, each one outputting a few codeword elements, permits to achieve a larger robustness against attacks. By training separate networks, in fact, the correlation between errors affecting different bits of the code is reduced, thus forcing the attacker to attack all of them independently. In the scheme in Figure 1, every network outputs one codeword element only, then \( N \) ensembl branches are considered.

### Attacking ECOC

As baseline attack to assess the security of the ECOC scheme, we considered the C&W attack [9], due to its popularity and effectiveness. However, C&W attack is originally designed against networks adopting the one-hot (or softmax-like) function. For the one-hot encoding case this corresponds to applying the attack to the logits [9]. In the ECOC case, the attack must be applied to the correlation values \( \rho \), rather than to \( z \). In this way, the optimization problem solved by the attack is

\begin{equation}
\begin{aligned}
\text{minimize } & ||\delta||_2 + \lambda \cdot \max_i (\rho_i(x + \delta) - \rho_i(x + \delta)), \\
\end{aligned}
\end{equation}

where \( \delta \) is the adversarial additive perturbation, \( x + \delta \) is the perturbed input, and \( || \cdot ||_2 \) denotes the \( L_2 \)-norm. \( \lambda \) and \( \epsilon \) are constant parameters ruling, respectively, the tradeoff between the two terms of the optimization problem, and the confidence margin of the attack [9]. Finally, \( t \) denotes the target class corresponding to the codeword \( C_t \), and \( \rho_i(x) = \sigma(z_i \cdot C_t) \).

A key advantage of C&W attack against one-hot encoding networks is that it works directly at the logits level. In fact, logits are more sensitive to modifications of the input than the probability distribution obtained after the softmax activation [10]. When C&W attack is applied against ECOC (by means of \( \sigma(z_i \cdot C_t) \)), it does not work at the output logit level, but after that the correlations are computed (mapping step 1), since this is the layer that precedes the application of the softmax-like function. The correlations between the activations of the logits and the codewords will likely have a reduced sensitivity to input modifications, and this may decrease the effectiveness of the attack.

Based on the above observation, we propose a variant of the C&W attack which works at the logits level, thus maintaining a good sensitivity to input modifications. The new variant, can be formulated as follows:

\begin{equation}
\begin{aligned}
\text{minimize } & ||\delta||_2 - \lambda \cdot \min_i (2t_i - z_i(x + \delta)), \\
\end{aligned}
\end{equation}

where \( (t_1, t_2, \ldots, t_M) = C_t \) is the desired target codeword \( t_\ell \in \{-1, 1\} \). By solving the above optimization, each logit value \( z_i \) of the resulting adversarial image will be highly correlated to \( t_\ell \). Notice that the attack in (4) is generally applicable to any multi-label classification network, regardless of the adopted decoding/decision strategy.

### Table 1: Results of the attack against ECOC for Cifar-10 classification.

| Parameters       | Proposed (ECOC) | C&W (ECOC) | C&W (one-hot) |
|------------------|-----------------|------------|---------------|
| ASR              | 69%             | 44.09      | 46.43         |
| PSNR             | 98%             | 44.35      | 49.78         |
| (1e-1, 2000, 0)  |                 | 95%        | 44.09         |
| (1e-4, 100, 0)   |                 | 96%        | 49.78         |
| (1e-4, 2000, 0)  |                 | 94%        | 44.35         |
| (1e-4, 5000, 0)  |                 | 95%        | 44.35         |
| (1e-4, 20000, 0) |                 | 93%        | 44.35         |

### Table 2: Results of the attack against ECOC for GTSRB classification.

| Parameters       | Proposed (ECOC) | C&W (ECOC) | C&W (one-hot) |
|------------------|-----------------|------------|---------------|
| ASR              | 69%             | 44.09      | 46.43         |
| PSNR             | 98%             | 44.35      | 49.78         |
| (1e-1, 2000, 0)  |                 | 95%        | 44.09         |
| (1e-4, 100, 0)   |                 | 96%        | 49.78         |
| (1e-4, 2000, 0)  |                 | 94%        | 44.35         |
| (1e-4, 5000, 0)  |                 | 95%        | 44.35         |
| (1e-4, 20000, 0) |                 | 93%        | 44.35         |

### Experiments

In [10], the authors tested the robustness of ECOC scheme for various combinations of codeword matrices \( C \), activation functions \( \sigma(\cdot) \) and network structures. The MNIST [14] and Cifar-10 [12] classification tasks were considered (\( M = 10 \) in both cases). In the end, the best performing system was obtained by considering a Hadamard code \( N = 32 \) and the tanh activation function. An ensemble of 4 \( (N/4) \) networks each one outputting 4 bits was considered. The authors argue that using a large number of ensembles increases the performance of the system against attacks (by decreasing the dependency among output bits). Then, in our experiments, we used \( N \) ensembles, with only one output bit each. A diagram of the ECOC scheme with the \( N \) ensemble structure is shown in Figure 1. We used a standard VGG-16 network [15] as the base block of our implementation. Following the ECOC design scheme, the first 6 layers form the so called ‘shared bottom’ part, that is, the layers shared by all the networks of the ensemble. Then, the remaining 10 layers (the last 8 convolutional layers and the 2 fully connected layers) are trained separately for each ensemble branch. In our experiments, we considered the Cifar-10 classification task, with the same setup used in [10]. We also considered the traffic sign classification task (GTSRB database) [13], to test the robustness of ECOC on a different number of classes and a different size of the codewords. To be specific, for traffic sign classification, we set \( M = 32 \), by selecting the classes with more examples among the total number of 44 classes in GTSRB, and chose a Hadamard code with \( N = M = 32 \). For each task, we first trained one \( M \)-class classification network, then we fine-tuned the weights to get the \( N \) ensemble networks. The error rates of the trained models on clean images

1. In fact, in [9], we have \( \delta = 1/2 \tan (\omega) + 1 \) and the minimization is carried out over \( \omega \).
2. Most adversarial attacks work directly on the probability values obtained after the softmax, which makes them less effective than C&W, and prone to gradient-vanishing problems.
are equal to 13.9% for Cifar-10 and 1.28% for traffic sign (GTSRB) classification.

To solve the attacks formalized in eq. (3) and (4), we used the same optimization procedure adopted in [9], by resorting to a binary search to optimize λ, and by performing, for each binary search step, a certain number of gradient descent iterations to build the adversarial image. Finally, we select the adversarial image leading to the minimum distortion. The attack procedure is characterized by four input parameters: the starting point of the binary search, the number of steps of the binary search, the maximum number of iterations for every binary search, and the value of the constant c. For a fair comparison between the two tested attacks, the same input parameters are considered for both of them (with the exception of the value of the confidence margin c, that, as it is evident from the equations describing the attacks, refers to different quantities in the two cases).

We applied C&W and the new attack to 100 images randomly chosen from the test set of each task. The target class of the attack was chosen at random among the remaining M − 1 classes (i.e., all the M classes except the original class of the unperturbed image). The label t of the target class was used to run the C&W attack in eq. [4], while the codeword C_t associated to t is considered in eq. [1] for the new attack.

For both tasks, we run the C&W and the new attack with several settings of the input parameters. The results we got, in terms of Attack Success Rate (ASR) and PSNR (averaged on the successfully attacked images only), are shown in Table 1 and 2 for the Cifar-10 and traffic sign classification respectively. In all the cases, c was set to 0. Results obtained by using the C&W attack against the standard one-hot encoding VGG-16 network with M classes are also reported in the last column. When the parameters are modified to increase the strength of the attack, e.g. by using a larger number of iterations or a larger number of steps during the binary search, C&W attack can easily increase the ASR, at the price of a slightly larger distortion. For instance, for Cifar-10, the ASR of C&W attack increases from 29%, with the setting (1e−4, 1000, 0), to 96%, with the setting (1e−1, 20000, 0), with a decrease in the PSNR of only 5 dB. From the tables, we also see that, for the same parameter setting, the ASR of the new attack variant is higher than that obtain by C&W. For example, for the Cifar-10 case with setting (1e−4, 1000, 0), the ASR is 75% for C&W and 95% for the proposed attack, with a gain of 20% for a virtually identical PSNR (37.9 and 37.6 respectively). Comparing Table 1 and 2, we observe that, for both tasks, in order to attack the ECOC scheme with the new attack proposed in this paper, the attacker must introduce a distortion which is only slightly larger than that necessary to attack the one-hot encoding classifier (the PSNR in the ECOC case is only about 1dB larger for Cifar-10, and about 2 dB larger for GTSRB). For instance, for the Cifar-10, the C&W attack carried out against the one-hot encoding classifier reaches an ASR = 100% with a PSNR of 39.8 with the setting (1e−4, 1000, 0), while the proposed attack can attack the ECOC scheme with an ASR=99% and a PSNR of 38.2 for the same setting. These results confirm that when the knowledge of the system is properly exploited by the attacker, no significant gain of robustness can be achieved by the ECOC scheme.

Conclusion: We have shown that the use of error correction to code the output of a CNN classifier (as proposed in [10]) does not increase significantly the robustness against adversarial examples. In fact the ECOC scheme can be induced to make a wrong decision by introducing a small perturbation into the image, both with the standard C&W attack applied with a proper parameter setting, and with a new variant of the attack, specifically designed for multi-class classification.

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Table 3: Probability values output by the ECOC classifier on Cifar-10 for different confidence margins of the attack. The parameters of the attacks are indicated according to the following format: (starting point, number of steps of binary search, max iterations, confidence). Prob true and target class indicate the probabilities of the original (true) and target classes, before (B) and after (A) the attack.

| C&W attack | Prob true class | Prob target class |
|------------|-----------------|-------------------|
| (1e−4,5,500,0) | (B) 0.934 (A) 0.295 | (B) 0.935 (A) 0.157 |
| (1e−4,5,500,1) | (B) 0.935 (A) 0.157 | (B) 0.935 (A) 0.085 |
| (1e−4,5,500,2) | (B) 0.935 (A) 0.085 | (B) 0.935 (A) 0.046 |
| (1e−4,5,500,3) | (B) 0.935 (A) 0.046 | (B) 0.934 (A) 0.022 |

| Proposed attack | Prob true class | Prob target class |
|-----------------|-----------------|-------------------|
| (1e−4,5,500,0) | (B) 0.965 (A) 0.193 | (B) 0.965 (A) 0.103 |
| (1e−4,5,500,1) | (B) 0.965 (A) 0.103 | (B) 0.965 (A) 0.068 |
| (1e−4,5,500,2) | (B) 0.965 (A) 0.068 | (B) 0.964 (A) 0.043 |
| (1e−4,5,500,3) | (B) 0.964 (A) 0.043 | (B) 0.964 (A) 0.028 |

| ASR | PSNR |
|-----|------|
| 83% | 37.90 |
| 62% | 37.37 |
| 82% | 37.07 |
| 82% | 36.81 |
| 82% | 36.62 |
| 81% | 36.62 |

Note: As it is evident from eq. (3) and (4), a different scale for the value of c has to be considered for the two attacks.
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