Chiral symmetry breaking and electromagnetic structure of the nucleon

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Abstract.
The electromagnetic form factors are the most fundamental quantities to describe the internal structure of the nucleon and are related to the charge radii of the baryons. We have calculated the charge radii of octet baryons in the framework of chiral constituent quark model with the inclusion of the spin-spin generated configuration mixing. This model is quite successful in predicting the low energy hadron matrix elements. The results obtained in the case of charge radii are also comparable to the latest experimental studies and show improvement over some theoretical interpretations.

Keywords: Electromagnetic form factors, chiral symmetry breaking, charge radii, chiral constituent quark model

PACS: 13.40.Gp, 12.39.Fe

INTRODUCTION

The electromagnetic form factors are the fundamental quantities of theoretical and experimental interest to investigate the internal structure of nucleon. The knowledge of internal structure of nucleon in terms of quark and gluon degrees of freedom in QCD provide a basis for understanding more complex, strongly interacting matter. Recently, a wide variety of accurately measured data have been accumulated for the static properties of baryons, for example, masses, electromagnetic moments, charge radii, and low energy dynamical properties such as scattering lengths and decay rates etc.. The charge radii and magnetic moments, as measured for the distribution of charge and magnetization, represent important observables in hadronic physics as they lie in the nonperturbative range of QCD and give valuable information on the internal structure of hadrons. While QCD is accepted as the fundamental theory of strong interactions, it cannot be solved accurately in the nonperturbative regime. A coherent understanding of the hadron structure in this energy regime is necessary to describe the strong interactions as they are sensitive to the pion cloud and provide a test for the QCD inspired effective field theories based on the chiral symmetry. A promising approach is offered by constituent-quark models (CQMs). Modern CQMs can be constructed so as to include the relevant properties of QCD in the nonperturbative regime, notably the consequences of the spontaneous breaking of chiral symmetry ($\chi$SB).

ELECTROMAGNETIC FORM FACTORS

The internal structure of nucleon is determined in terms of electromagnetic Dirac and Pauli from factors $F_1(Q^2)$ and $F_2(Q^2)$ or equivalently in terms of the electric and magnetic Sachs form factors $G_E(Q^2)$ and $G_M(Q^2)$ [1]. The issue of determination of the form factors has been revisited in the recent past with several new experiments measuring the form factors with precision at MAMI [2] and JLAB [3]. It has been shown that the proton form factors determined from the measurements of polarization transfer [3] were in significant disagreement with those obtained from the Rosenbluth separation [4]. This inconsistency leads to a large uncertainty in our knowledge of the proton electromagnetic form factors and urge the necessity for the new parameterizations and analysis [5].

The most general form of the hadronic current for a spin $\frac{1}{2}$-nucleon with internal structure is given as

$$\langle B|J_{\text{had}}^\mu(0)|B'\rangle = \bar{u}(p') \left( \gamma^\mu F_1(Q^2) + i \frac{\sigma_{\mu\nu} q_\nu}{2M} F_2(Q^2) \right) u(p),$$

(1)

where $u(p)$ and $u(p')$ are the 4-spinors of the nucleon in the initial and final states respectively. The Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ are the only two form factors allowed by relativistic invariance. These form factors
are normalized in such a way that at \( Q^2 = 0 \), they reduces to electric charge and the anomalous magnetic moment in units of the elementary charge and the nuclear magneton \( \mu_N \), for example,

\[
F_1^p(0) = 1, \quad F_2^p(0) = 1.793, \quad F_1^n(0) = 0, \quad F_2^n(0) = -1.913.
\]

(2)

In analogy with the non-relativistic physics, we can associate the form factors with the Fourier transforms of the charge and magnetization densities. However, the charge distribution \( \rho(r) \) has to be calculated by a 3-dimensional Fourier transform of the form factor as function of \( q \), whereas the form factors are generally functions of \( Q^2 = q^2 - \omega^2 \). It would be important to mention here that there exists a special Lorentz frame, the Breit or brick-wall frame, in which the energy of the (space-like) virtual photon vanishes. This can be realized by choosing \( p_1 = -\frac{1}{2}q \) and \( p_2 = +\frac{1}{2}q \) leading to \( E_1 = E_2, \omega = 0 \) and \( Q^2 = q^2 \). Thus, in the Breit frame, Eq. (1) takes the following form [1]

\[
J_\mu = \left( G_E(Q^2), \frac{i}{2M} \sigma \times q Q_M(Q^2) \right),
\]

(3)

where \( G_E(Q^2) \) stands for the time-like component of \( J_\mu \) hence identified with the Fourier transform of the electric charge distribution, whereas \( G_M(Q^2) \) is interpreted as the Fourier transform of the electric mean square charge radius. A charge radius is important as it particularly suggests the possibility of measuring the charge radii of other long-lived strange baryons such as \( \Lambda, \Sigma, \Xi \) in the near future.

The Fourier transform of the Sachs form factors can be expressed as

\[
G_E(q^2) = \int \rho(r) e^{iq \cdot r} d^3 r = \int \rho(r) d^3 r - \frac{q^2}{6} \int \rho(r) r^2 d^3 r + ... ,
\]

(5)

where the first integral yields the total charge in units of \( e \), i.e., 1 for the proton and 0 for the neutron, and the second integral defines the square of the electric mean square charge radius.

**CHARGE RADIUS OF THE NUCLEON**

The mean square charge radius of a given baryon \( r_B^2 \) is one of the important low energy characteristic giving its possible “size” and its precise determination give information about the internal structure of the baryons. In general, \( r_B^2 \), which is a scalar under spatial rotation is defined as \( r_B^2 = \int d^3 \rho(r) r^2 \), where \( \rho(r) \) is the charge density. A charge radius is the first nontrivial moment of a Coulomb monopole \( G_C(q^2) \) transition amplitude.

In the recent past, with the advent of new facilities at JLAB, SELEX Collaborations, the baryons charge radii are being investigated. The results are available for the charge radii of \( p, n \), and very recently for the strange baryon \( \Sigma^- \). For the case of proton we have \( r_p = 0.877 \pm 0.007 \) fm\(^2 \) ( \( r_\Sigma^- = 0.779 \pm 0.025 \) fm\(^2 \) [6]), for neutron we have \( r_n = -0.1161 \pm 0.0022 \) fm\(^2 \), [7], and for the case of \( \Sigma^- \) we have \( r_\Sigma^- = 0.61 \pm 0.21 \) fm\(^2 \) [8]. The measurement of the \( \Sigma^- \) charge radii is important as it particularly suggests the possibility of measuring the charge radii of other long-lived strange baryons such as \( \Lambda, \Sigma, \Xi \) in the near future.

In the general parameterization (GP) method [9], the charge radii operator can be expressed in terms of the sum of one-, two-, and three-quark contributions

\[
r_B^2 = A \sum_{i=1}^{3} e_i \sigma_i + B \sum_{ij \neq j} e_i \sigma_i \cdot \sigma_j + C \sum_{i \neq j \neq k} e_i \sigma_i \cdot \sigma_j \cdot \sigma_k ,
\]

(6)

where \( e_i \), and \( \sigma_i \) are the charge and spin of the i-th quark. The constants \( A, B, \) and \( C \) can be determined from the experimental observations on charge radii and quadrupole moments of the baryons.

The charge radii for the octet baryons can be calculated by evaluating matrix elements of the operator in Eq. (6) between spin-flavor wave functions \( |B \rangle \) as \( \langle B | r^2 | B \rangle \). It is straightforward to verify that, for the octet baryons, the operators involving two- and three-quark terms in Eq. (6) can be simplified as

\[
\sum_{i \neq j} e_i (\sigma_i \cdot \sigma_j) = 2J \sum_i e_i \sigma_i - 3 \sum_i e_i , \quad \sum_{i \neq j \neq k} e_i (\sigma_i \cdot \sigma_j \cdot \sigma_k) = -3 \sum_i e_i - \sum_{i \neq j} e_i (\sigma_i \cdot \sigma_j).
\]

(7)
Using the expectation value of operator $2J \cdot \sum_{i,j} \epsilon_i \sigma_i$ between the baryon wavefunctions $|B\rangle$ in the initial and final state baryons, the operators in Eq. (7) become

$$\sum_{i \neq j} \epsilon_i (\sigma_i \cdot \sigma_j) = 3 \sum_i \epsilon_i \sigma_{i\bar{k}} - 3 \sum_i \epsilon_i, \quad \sum_{i \neq j \neq k} \epsilon_i (\sigma_j \cdot \sigma_k) = -3 \sum_i \epsilon_i \sigma_{i\bar{k}}.$$  \hspace{1cm} (8)

The charge radii in Eq. (6) for the octet baryons can now be expressed as

$$r_B^2 = (A - 3B) \sum_i \epsilon_i + 3(B - C) \sum_i \epsilon_i \sigma_{i\bar{e}},$$  \hspace{1cm} (9)

It is clear from this expression that the determination of charge radii reduces to the calculation of the flavor and spin structure of given octet baryon ($\bar{e}_i \equiv \langle B | \sum_i \epsilon_i | B \rangle$, and $e_i \sigma_{i\bar{e}} \equiv \langle B | \sum_i \epsilon_i \sigma_{i\bar{e}} | B \rangle$). Here $|B\rangle$ is the baryon wave function and ($\sum_i \epsilon_i$) and ($\sum_i \epsilon_i \sigma_{i\bar{e}}$) are the charge and spin operators defined as

$$\sum_i \epsilon_i = \sum_{q' = u,d,s} n^B_{q'} q + \sum_{q' = u,d,s} n^B_{q'} \bar{q} = n^B_u u + n^B_d d + n^B_s s + n^B_{\bar{u}} \bar{u} + n^B_{\bar{d}} \bar{d} + n^B_{\bar{s}} \bar{s},$$  \hspace{1cm} (10)

$$\sum_i \epsilon_i \sigma_{i\bar{e}} = \sum_{q' = u,d,s} (n^B_{q'} q_+ + n^B_{q'} q_-) = n^B_u u_+ + n^B_d d_+ + n^B_s s_+ + n^B_{\bar{u}} \bar{u}_- + n^B_{\bar{d}} \bar{d}_- + n^B_{\bar{s}} \bar{s}_-, \hspace{1cm} (11)$$

where $n^B_q$ ($n^B_{\bar{q}}$) is the number of quarks with charge $q$ ($\bar{q}$), $n^B_{q'} q_+$ ($n^B_{q'} q_-)$ being the number of quarks with spin $q_+$ ($q_-$) quarks.

Using the SU(6) spin-flavor symmetry of the wave functions in the naive quark model (NQM), the charge radii of proton and neutron become

$$r_p^2 = (A - 3B)(2u + d) + 3(B - C) \left( \frac{4}{3} u_+ - \frac{1}{3} d_+ \right) = A - 3B, \hspace{1cm} (12)$$

$$r_n^2 = (A - 3B)(u + 2d) + 3(B - C) \left( -\frac{1}{3} u_+ + \frac{4}{3} d_+ \right) = -2B + 2C. \hspace{1cm} (13)$$

The naive quark model (NQM) calculations show that the results are in disagreement with the available experimental data. In this context, it therefore becomes desirable to extend this model to understand the role played by chiral symmetry breaking.

**CHIRAL SYMMETRY BREAKING**

The global symmetry which arises in the QCD lagrangian, if we neglect the small quark masses and consider the light quarks as massless particles, is the chiral symmetry of SU(3)$_L \times$ SU(3)$_R$ group. Since the spectrum of the hadrons in the known sector, does not display parity doublets, we believe that the chiral symmetry is spontaneously broken around a scale of 1 GeV as

$$SU(3)_L \times SU(3)_R \rightarrow SU(3)_{L+R} \rightarrow SU(3)_V.$$  \hspace{1cm} (14)

As a consequence there exist a set of massless particles called the Goldstone bosons (GBs) which are further identified with the observed ($\pi$, $K$, $\eta$). Although these are massive but are interpreted as the GBs of the spontaneously broken chiral symmetry as their masses are small compared to the nucleon mass. The QCD Lagrangian is also invariant under the axial $U(1)$ symmetry, this breaking symmetry picks the $\eta'$ as the ninth GBs.

If QCD leads to quark confinement, the mass parameters $m_q$, are not directly observable quantities. However, they can be determined in terms of observable hadronic masses through current algebra methods. These quark masses are called current quark masses in order to distinguish them from constituent quark masses. Constituent quark masses, also called effective quark masses, are parameters used in phenomenological quark models of hadronic structures and are in general larger than the current quark masses.
CHIRAL CONSTITUENT QUARK MODEL

One of the most successful model in the nonperturbative regime of QCD which incorporates χSB is the chiral constituent quark model (χCQM) [10]. The basic process in the χCQM is the emission of a GB by a constituent quark which further splits into a q̅q pair as \( q_+ \to GB^0 + q_+ \to (q̅q') + q_+ \), where q̅q' + q' constitute the “quark sea” [11, 12, 13]. The effective Lagrangian describing interaction between quarks and a nonet of GBs is the splittings of mixing and details of the spin, isospin and spatial parts of the wavefunction, can be found in Refs. [14, 15, 16]. Using generated by the chromodynamic spin-spin forces and it improves the low energy dynamics of hadrons. Configuration recently it has been observed that the baryon wavefunction is modified due to the minimal configuration mixing process and chiral symmetry breaking in the present the quadrupole moment. The best fit values obtained are \( A = 0.921, B = 0.105, \) and \( C = 0.032, \) respectively. We have presented the χCQM configuration results for the charge radii of octet baryons in Table 1. For the sake of comparison, we have

\[
q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad \Phi = \begin{pmatrix} \frac{\pi^u}{\sqrt{2}} + \beta \eta \frac{\pi^s}{\sqrt{6}} + \zeta \frac{\pi^d}{\sqrt{3}} \\ -\frac{\pi^u}{\sqrt{2}} + \beta \eta \frac{\pi^s}{\sqrt{6}} + \zeta \frac{\pi^d}{\sqrt{3}} \\ \alpha K^+ \\ \alpha K^- \end{pmatrix},
\]

where \( g_8 \) and \( \zeta \) are the coupling constants for the singlet and octet GBs. SU(3) symmetry breaking is introduced by considering \( M_\pi > M_{\alpha d} \) as well as by considering the masses of GBs to be nondegenerate (\( M_{K,\eta} > M_{\pi} \) and \( M_{\eta'} > M_{K,\eta} \) [10, 11, 12, 13]. The parameter \( a = (g_8)^2 \) denotes the transition probability of chiral fluctuation of the splittings \( u(d) \to d(u) + \pi^+(-) \), whereas \( \alpha^2 a, \beta^2 a \) and \( \zeta^2 a \) respectively denote the probabilities of transitions of \( u(d) \to s + K^{-(s)}, u(d,s) \to u(d,s) + \eta, \) and \( u(d,s) \to u(d,s) + \eta' \).

The charge radii operator in Eq. (9), involves the knowledge of spin and flavor structure of baryons in χCQM. A redistribution of flavor and spin takes place among the “sea quarks” in the interior of hadron due to the fluctuation process and chiral symmetry breaking in the χCQM. The flavor and spin content can now be calculated by substituting for every constituent quark by

\[
q \to P_q q + |\psi(q)|^2, \quad q_\pm \to P_q q_\pm + |\psi(q_\pm)|^2,
\]

where \( P_q = 1 - \sum P_q \) is the transition probability of no emission of GB from any of the \( q \) quark. \( |\psi(q)|^2 \) is the transition probability of the \( q \) quark, and \( |\psi(q_\pm)|^2 \) is the probability of transforming a \( q_\pm \) quark [10].

RESULTS AND DISCUSSION

Recently it has been observed that the baryon wavefunction is modified due to the minimal configuration mixing generated by the chromodynamic spin-spin forces and it improves the low energy dynamics of hadrons. Configuration mixing and details of the spin, isospin and spatial parts of the wavefunction, can be found in Refs. [14, 15, 16]. Using the mixed wavefunction in GP method, the charge radii for the proton and neutron can now be expressed as

\[
r_p^2 = (A - 3B)(2u + d) + 3(B - C) \left[ \cos^2 \phi \left( \frac{4}{3}u + \frac{1}{3}d \right) + \sin^2 \phi \left( \frac{2}{3}u + \frac{2}{3}d \right) \right],
\]

\[
r_n^2 = (A - 3B)(u + 2d) + 3(B - C) \left[ \cos^2 \phi \left( \frac{1}{3}u + \frac{4}{3}d \right) + \sin^2 \phi \left( \frac{1}{3}u + \frac{2}{3}d \right) \right].
\]

The modified charge radii in the χCQM, after the modified quark content from Eq. (16), is expressed as

\[
r_p^2 = (A - 3B)(1 - a - 2a^2) + 3(B - C) \left[ \cos^2 \phi \left( 1 - \frac{a}{3} \right) + \sin^2 \phi \left( \frac{1}{3} - \frac{a}{9} \right) \right],
\]

\[
r_n^2 = (A - 3B)(1 - a^2) + 3(B - C) \left[ \cos^2 \phi \left( \frac{2}{3} + \frac{a}{9} \right) + \sin^2 \phi \left( \frac{1}{3} + \frac{2}{9} \right) \right].
\]

The charge radii of the other octet baryons can be calculated in a similar manner. For the χCQM parameters, we have used the same set of symmetry breaking parameters as in Ref. [10] \( a = 0.12, \alpha = 0.45, \beta = 0.45, \zeta = -0.15 \). In addition to the χCQM parameters, the GP parameters corresponding to the one-, two- and three-quark contributions (A, B, and C, respectively), have to be fitted to the numerical values of charge radii of proton, neutron and their quadrupole moment. The best fit values obtained are \( A = 0.921, B = 0.105, \) and \( C = 0.032, \) respectively. We have presented the χCQM config results for the charge radii of octet baryons in Table 1. For the sake of comparison, we have
TABLE 1. Charge radii of the octet baryons.

| Baryon | Data [7, 8] | NQM   | $\chi$CQM | $\chi$CQM$_{\text{conf}}$ |
|--------|-------------|-------|-----------|--------------------------|
| $r^2_n$ | $r_n = 0.877 \pm 0.007$ | 0.809 | 0.784 | 0.769 |
| $r^2_p$ | $-0.1161 \pm 0.0022$ | 0.809 | 0.784 | 0.769 |
| $r^2_{\Sigma^+}$ | ... | 0.61 $\pm 0.21$ | 0.679 | 0.675 | 0.675 |
| $r^2_{\Sigma^-}$ | ... | $-0.130$ | $-0.131$ | $-0.116$ |
| $r^2_{\Sigma^0}$ | ... | 0.679 | 0.675 | 0.675 |
| $r^2_{\Xi^0}$ | ... | $-0.065$ | $-0.054$ | $-0.047$ |
| $r^2_{\Xi^-}$ | ... | $-0.130$ | $-0.131$ | $-0.116$ |
| $r^2_{\Lambda}$ | ... | $-0.065$ | $-0.069$ | $-0.062$ |

also presented the results for NQM and $\chi$CQM without configuration mixing. A cursory look at the results reveal that the NQM and $\chi$CQM predictions for the charge radii are on the higher side as compared to the results with configuration mixing. As for the case of other low energy hadronic matrix elements, the results with configuration mixing in this case also are of the right order of magnitude when compared with the available experimental data. This gives a strong impetus to the spin-spin generated configuration mixing in the baryon wavefunctions. This can be particularly observed in the case of the charge radii of strange baryon $\Sigma^-$ which matches well with experimental data. It is interesting to observe that in this case there is no effect of configuration mixing which can be easily understood when we look into its detailed structure. Further, our model successfully predicts the charge radii for the other octet baryons where there is no experimental data available. Therefore, refinement of data for the charge radii of other octet baryons would have important implications for understanding the basic tenets of $\chi$CQM and configuration mixing.

ACKNOWLEDGMENTS

H.D. would like to thank Department of Science and Technology, Government of India and the organizers of CHIRAL2010: International Workshop on Chiral Symmetry in Hadrons and Nuclei for financial support.

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