Scheme for teleporting an arbitrary superposition of atomic Dicke states via multi-fold coincidence detection

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Abstract. We propose a novel scheme for teleporting an arbitrary superposition of two-atom Dicke states with atoms trapped in optical cavities. The scheme requires a multi-fold coincidence detection and is insensitive to the imperfection of the photon detectors, and the fidelity is unity for any superposition of Dicke states. Further, we also point out that scheme can be extended to teleport an arbitrary superposition of $N$-atom Dicke states with unit fidelity.

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1. Introduction

Quantum teleportation is not only a fundamental phenomenon of the quantum world, but also one of the key procedures in the area of quantum information processing [1]. By teleportation, an unknown quantum state can be transported from one place to another without moving through the intervening space. Since the pioneering contribution of Bennett et al [1], quantum teleportation was first demonstrated experimentally by using spontaneous parametric down-conversions [2].
More recently, teleportation of a single atomic state was reported in a trapped-ion system [3]. However, in realizing quantum information processing, it is expected to use atoms as stationary qubits to store quantum information in long-lived internal states, and photons as flying qubits to transport quantum information over long distances. Cavity QED provides a promising candidate for such a scheme. Since Bose et al [4] proposed a scheme to teleport a single atomic state from one cavity to another and generate the EPR state between two distant atoms, there have been many proposals for generating quantum entanglement and implementing conditional quantum logic gate between distant atoms [5]–[7]. Recently, quantum entanglement between a single trapped atom and a single photon was observed experimentally [8, 9], which is a crucial step towards long-range quantum networks.

On the other hand, the set of the multi-atom Dicke states is an important class of multi-qubit states [10]. The Dicke states and their superpositions, including the multi-atom GHZ states [11], the W states [12] and spin-squeezed states [13]–[16] as special cases, have applications in the demonstration of quantum nonlocality [11, 17], high-precision spectroscopy [14, 15], and quantum information processing [18]. In [6], a conditional scheme has been proposed for generating arbitrary superposition of N-atom Dicke states. Further, Di et al [19] considered the teleportation of an arbitrary superposition of atomic Dicke states between two distant cavities. However, the scheme requires a photon detector to distinguish zero photons, one photon, two photons and more than two photons, so that the inefficiency of the photon detector influences the fidelity of the teleportation. Moreover, the fidelity is not unity for superposition of Dicke states.

In this paper, we propose an alternative scheme for teleporting an arbitrary superposition of atomic Dicke states from one cavity to another. In contrast to scheme [19], our scheme is based on multiphoton coincidence detection and has the following advantages: the fidelity is unity for any superposition of Dicke states, and the scheme is insensitive to the imperfection of the photon detectors, i.e. the scheme does not require distinguishing between zero, one and two photons. Imperfection of the photon detectors decreases the success probability, but has no influence on the fidelity of the teleportation.

The paper is organized as follows. In section 2, we present the scheme for teleporting an arbitrary superposition of two-atom Dicke states and give a brief discussion on the influence of quantum noise on the scheme. In section 3, we extend the scheme to implement teleportation of N-atom Dicke-states.

2. Teleportation of two-atom Dicke states

In this section, we explain the scheme by teleporting an arbitrary superposition of two-atom Dicke states. The schematic representation of the scheme is shown in figure 1, which consists of two optical cavities and each contains two trapped atoms. The photons leaking out from the cavities A and B interfere at a polarization beam splitter (PBS), with the outputs detected by four photon detectors \(D_{AH}, D_{AV}, D_{BH}, D_{BV}\) after two polarization-rotators and two PBSs. For the photons decaying from the cavities A and B, quarter wave plates (QWP) are inserted before the first PBS, which are used to map the left-circular photons (right-circular photons) out of the cavity on the horizontal polarization photons (vertical polarization photons). The scheme consists of two steps. In the preparation stage, by using adiabatic passage technique, Alice maps the unknown Dicke-state superposition into the two-mode cavity state, and Bob generates a
maximally entangled state between atoms and two-mode cavity fields. In the detection stage, Alice uses photon detectors to measure the photons leaking out from two cavities. If each of the four detectors registers a single click during this stage, the scheme is successful, superposition of Dicke states is transferred from cavity A to B with unit fidelity. In the scheme, imperfection of the photon detectors decreases the success probability, but has no influence on the fidelity of the teleportation.

We now analyse the scheme in detail. The level structure of atoms is shown in figure 2, which has four ground states and three excited states. For concreteness, we consider a possible implementation using $^{87}$Rb, whose usefulness in the quantum information context has been demonstrated in recent experiments [8]. The ground states $|g_L\rangle$, $|g_0\rangle$, $|g_R\rangle$ correspond to $|F=1, m=-1\rangle$, $|F=1, m=0\rangle$ and $|F=1, m=1\rangle$ of $S_{1/2}$, respectively, and one could use $|F=2, m=0\rangle$ of $S_{1/2}$ as the ground state $|g_a\rangle$. The excited states $|e_L\rangle$, $|e_0\rangle$ and $|e_R\rangle$ correspond to $|F=1, m=-1\rangle$, $|F=1, m=0\rangle$ and $|F=1, m=1\rangle$ of $P_{3/2}$, respectively. The lifetimes of the atomic levels $|g_L\rangle$, $|g_R\rangle$, $|g_0\rangle$, $|g_a\rangle$ are comparatively long so that spontaneous decay of these states can be neglected. We encode the ground states $|g_L\rangle$ and $|g_R\rangle$ as logic zero and one states, i.e. $|g_L\rangle = |0\rangle$ and $|g_R\rangle = |1\rangle$. The transitions $|e_0\rangle \leftrightarrow |g_0\rangle$ and $|e_L\rangle \leftrightarrow |g_0\rangle$ are coupled to cavity mode $a_L$ ($a_R$) with the left-circular (right-circular) polarization. On Alice’s side, the transition $|e_L\rangle \leftrightarrow |g_L\rangle$ and $|e_R\rangle \leftrightarrow |g_R\rangle$ are driven by $\pi$-polarized classical fields with the Rabi frequency $\Omega_A(t)$. The transition between $|e_0\rangle$ and $|g_0\rangle$ is electric dipole forbidden [20]. On Bob’s side, one use $\pi$-polarized classical fields to drive the transition $|e_0\rangle \leftrightarrow |g_a\rangle$ with the Rabi frequency $\Omega_B(t)$.
Figure 2. (a) The involved atomic levels and transitions for each atom of Alice. Alice’s qubit is encoded in the two Zeeman sublevels $|g_L\rangle$ and $|g_R\rangle$. For the initial state equation (2), only four transitions are included. $|e_L\rangle_A \rightarrow |g_0\rangle_A$ ($|e_R\rangle_A \rightarrow |g_0\rangle_A$) is coupled to the left-circularly (right-circularly) polarized mode of the cavity. $|e_L\rangle_A \leftrightarrow |g_L\rangle_A$ and $|e_R\rangle_A \leftrightarrow |g_R\rangle_A$ are driven by $\pi$-polarized classical fields, and the transition between $|e_0\rangle_A$ and $|g_0\rangle_A$ is electric dipole forbidden. (b) The involved atomic levels and transitions for each atom of Bob. Bob’s qubit is also encoded in the two Zeeman sublevels $|g_L\rangle$ and $|g_R\rangle$. For initial state $|g_a, g_a\rangle |0, 0\rangle$, only three transitions are included. $|e_0\rangle_B \rightarrow |g_R\rangle_B$ ($|e_0\rangle_B \rightarrow |g_L\rangle_B$) is coupled to the left-circularly (right-circularly) polarized mode of the cavity. $|e_0\rangle_B \leftrightarrow |g_a\rangle_B$ is driven by $\pi$-polarized classical fields.

Initially the atomic state in cavity A is prepared in a Dicke-state superposition of the form

$$C_0|g_L, g_L\rangle + C_1 \frac{|g_L, g_R\rangle + |g_R, g_L\rangle}{\sqrt{2}} + C_2|g_R, g_R\rangle,$$

where coefficients $C_i$ are arbitrary and satisfy $|C_1|^2 + |C_2|^2 + |C_3|^2 = 1$. We also assume that two cavity modes are in the vacuum state $|0, 0\rangle_A$. Thus, the initial state of the system is

$$|\Psi(0)\rangle_A = \left( C_0|g_L, g_L\rangle + C_1 \frac{|g_L, g_R\rangle + |g_R, g_L\rangle}{\sqrt{2}} + C_2|g_R, g_R\rangle \right) |0, 0\rangle_A.$$
In the limit of $\Omega_1/\omega_1$, where $a^\dagger_1$ and $a^\dagger_2$ denote the creation operators for the corresponding polarized mode of the cavity. $g_A$ is the atom-cavity coupling, and the time dependence of the $\Omega_A(t)$ can be controlled by the external laser field. The Hamiltonian (3) has the following orthogonal dark states

$$H_A = \sum_{i=1,2} \Omega_A(t)|e_L, i\rangle\langle e_L| + |e_R, i\rangle\langle e_R| + \sum_{i=1,2} [g_A(a_L|e_L, i\rangle\langle g_0| + a_R|e_R, i\rangle\langle g_0|) + H.C.],$$

where $a^\dagger_i$ and $a^\dagger_A$ are the creation operators for the corresponding polarized mode of the cavity. $g_A$ is the atom-cavity coupling, and the time dependence of the $\Omega_A(t)$ can be controlled by the external laser field. The Hamiltonian (3) has the following orthogonal dark states

$$D(t)_{LL} = \frac{1}{\sqrt{g_A^2 + 2g_A^2\Omega_A^2 + \Omega_A^2/2}} \left[ g_A^2|g_L\rangle\langle g_L|_2|0\rangle\langle 0| - g_A\Omega_A(|g_0\rangle_1|g_L|_2 + |g_L\rangle_1|g_0\rangle_2|L\rangle\langle 0| + \frac{\Omega_A^2}{\sqrt{2}}|g_0\rangle_1|g_0\rangle_2|2L\rangle\langle 0| \right].$$

$$D(t)_{RR} = \frac{1}{\sqrt{g_A^2 + 2g_A^2\Omega_A^2 + \Omega_A^2/2}} \left[ g_A^2|g_R\rangle\langle g_R|_2|0\rangle\langle 0| - g_A\Omega_A(|g_0\rangle_1|g_R|_2 + |g_R\rangle_1|g_0\rangle_2|R\rangle\langle 0| + \frac{\Omega_A^2}{\sqrt{2}}|g_0\rangle_1|g_0\rangle_2|2R\rangle\langle 0| \right].$$

$$D(t)_{LR} = \frac{1}{\sqrt{g_A^2 + 2g_A^2\Omega_A^2 + \Omega_A^2/2}} \left[ g_A^2(|g_0\rangle_1|g_L|_2 + |g_L\rangle_1|g_0\rangle_2|0\rangle\langle 0| + \frac{\Omega_A^2}{\sqrt{2}}|g_0\rangle_1|g_0\rangle_2|L\rangle\langle 0| \right].$$

Based on the dark states equations (4)–(6), one can map the two-atom Dicke states equation (1) into the two-photon cavity states. Initially, we choose the parameters to satisfy $\Omega_A \ll g_A$. In the limit of $\Omega_A(t) \ll g_A$, the dark states $D(t)_{LL}$, $D(t)_{LR}$ and $D(t)_{RR}$ coincide with the states $|g_L, g_L\rangle_0, 0\rangle_A, (|g_L, g_R\rangle_0 + |g_R, g_L\rangle_0)\sqrt{2}$ and $|g_R, g_R\rangle_0, 0\rangle_A$, respectively. Therefore, by adiabatically increasing the coupling $\Omega_A(t)$, the initial state (2) of the system evolves in the dark space into the following state at the time $t$

$$\Psi(t) = |\Psi(t)\rangle_A = |D(t)_{LL}\rangle + |D(t)_{LR}\rangle + |D(t)_{RR}\rangle.$$
For Bob, two atoms are initially prepared in the ground states $|g_a, g_a\rangle$ and cavity fields are in the vacuum states. For such an initial state, only three transitions are involved as depicted in figure 2(b), and the Hamiltonian describing the system is given by

$$H_B = \sum_{i=1,2} \Omega_B(t)e_0_i(g_a) + \sum_{i=1,2} [g_B(a_L|e_0\rangle_i(g_R|a_R\rangle_i(g_L)|e_0\rangle_i(g_L)) + H.C.]. \quad (8)$$

By adiabatically tuning $\Omega_B(t)$ to go from $\Omega_B \ll g_B$, the initial state $|g_a, g_a\rangle|0, 0\rangle_B$ evolves into the following dark state at the time $t$

$$|\Psi(t)\rangle_B = \frac{1}{\sqrt{4g_B^4 + 4g_B^2\Omega_B^2 + 2\Omega_B^4/3}} \left\{ 2g_B^2|g_a\rangle_1|g_a\rangle_2|0\rangle - g_B\Omega_B(|g_a\rangle_1|g_a\rangle_2|0\rangle - g_B\Omega_B(|g_a\rangle_1|g_a\rangle_2|0\rangle \right\} + \frac{\sqrt{2\Omega_B^3(t)}}{3}$$

$$\times \left[ |g_{L}1|g_{L}2|0\rangle|2R\rangle + \frac{1}{\sqrt{2}}(|g_{R1}|g_{R2}|L\rangle|0\rangle + |g_{R1}|g_{R2}|L\rangle|R\rangle + |g_{R1}|g_{R2}|2L\rangle|0\rangle \right]. \quad (9)$$

In order to consider the effect of the cavity decay and photon observation on the state evolution, it is convenient to follow a quantum trajectory description [21]. The evolution of the system’s wave function is governed by a non-Hermitian Hamiltonian

$$H_j' = H_j - i\kappa(a_L^+a_L + a_R^+a_R), \quad j = A, B, \quad (10)$$

as long as no photon decays from the cavity. In this case, the state of the atom–cavity system $j(j = A, B)$ at the time $t$ evolves into

$$|\Psi(t)\rangle_j' = \exp(-iH_j't)|\Psi(t)\rangle_j, \quad j = A, B. \quad (11)$$

Here we assume that two optical cavities have the same loss rate $\kappa$ for all modes, and $H_A$ and $H_B$ correspond to the equations (3) and (8). Following [19, 21], a direct calculation gives the following results for Alice’s state at the time $t$

$$|\Psi(t)\rangle_A' = \frac{g_A^2}{\sqrt{g_A^4 + 2g_A^2\Omega_A^2 + \Omega_A^4/2}} \left( C_0|g_{L}, g_{L}\rangle + C_1 \frac{|g_{L}, g_{R}\rangle + |g_{R}, g_{L}\rangle}{\sqrt{2}} + C_2|g_{R}, g_{R}\rangle \right) |0, 0\rangle_A$$

$$- \frac{g_A\Omega_A e^{-\kappa t}}{\sqrt{g_A^4 + 2g_A^2\Omega_A^2 + \Omega_A^4/2}} \left. \right\}_{C_0(|g_{01}|g_{L}2 + |g_{L}1|g_{02}|L\rangle|0\rangle} + C_1 \frac{(|g_{01}|g_{L}2 + |g_{L}1|g_{02}|L\rangle|0\rangle + |g_{01}|g_{R}2 + |g_{R}1|g_{02}|L\rangle|0\rangle}{\sqrt{2}}$$

$$+ C_2(|g_{01}|g_{R}2 + |g_{R}1|g_{02}|L\rangle|0\rangle + |g_{01}|g_{R}2 + |g_{R}1|g_{02}|L\rangle|0\rangle)$$

$$+ \frac{\Omega_A^2 e^{-2\kappa t}}{\sqrt{2(g_A^4 + 2g_A^2\Omega_A^2 + \Omega_A^4/2)}} \times |g_{01}|g_{R}2|2L\rangle|0\rangle + C_1|L\rangle|R\rangle + C_2|0\rangle|2R\rangle, \quad (12)$$
This implements the preparation stage of the protocol. The success probability is given by

\[ P_A = \frac{g_A^4 + 2g_A^2\Omega_A^2 \Omega_B^2 e^{-2\kappa t} + \Omega_A^4/2e^{-4\kappa t}}{g_A^4 + 2g_A^2\Omega_A^2 + \Omega_A^4/2}. \]

and Bob’s states

\[ |\Psi(t)\rangle_B' = \frac{1}{\sqrt{4g_B^4 + 4g_B^2\Omega_B^2 + 2\Omega_B^4/3}} \left\{ \begin{array}{l} 2g_B^2|g_a\rangle_1|g_a\rangle_2|0\rangle_0 - g_B\Omega_B e^{-\kappa t}|(|g_a\rangle_1|g_L\rangle_2) \\
+ |g_L\rangle_1|g_a\rangle_2|0\rangle_0|1R\rangle + (|g_a\rangle_1|g_R\rangle_2 + |g_R\rangle_1|g_a\rangle_2)|1L\rangle|0\rangle_0|1L\rangle|0\rangle_0 \\
+ \sqrt{2}\Omega_B^4(t)e^{-2\kappa t} \left[ |g_L\rangle_1|g_L\rangle_2|2R\rangle + \frac{1}{\sqrt{2}}(|g_R\rangle_1|g_L\rangle_2 + |g_L\rangle_1|g_R\rangle_2)|L\rangle|R\rangle \\
+ |g_R\rangle_1|g_R\rangle_2|2L\rangle|0\rangle_0 \right] \right\}. \] (13)

with the success probability

\[ P_B = \frac{4g_B^4 + 2g_B^2\Omega_B^2 e^{-2\kappa t} + 2e^{-4\kappa t}\Omega_B^4/3}{4g_B^4 + 4g_B^2\Omega_B^2 + 2\Omega_B^4/3}. \]

If we assume that the interaction Hamiltonian (3) and (8) are applied to the atom–cavity system A and B simultaneously, so that the preparation of the atom–cavity states |\Psi(t)\rangle'_j (j = A, B) ends at the same time. The joint state of two atom–cavity systems A and B is given by

\[ |\Phi\rangle_{AB} = |\Psi(t)\rangle'_A \otimes |\Psi(t)\rangle'_B \] (14)

This implements the preparation stage of the protocol. The success probability is given by \( P_{\text{su}} = P_A P_B \), i.e. the probability that no photon decays from either atom–cavity system during the preparation. If the conditions \( \Omega_j e^{-\kappa t} \gg g_j \) are satisfied, the last terms in equations (12) and (13) are much larger than other terms, then |\Psi(t)\rangle'_A and |\Psi(t)\rangle'_B are reduced into the forms

\[ |\Psi(t)\rangle'_A = C_0|2L\rangle|0\rangle + C_1|L\rangle|R\rangle + C_2|0\rangle|2R\rangle \] (15)

and

\[ |\Psi(t)\rangle'_B = \frac{1}{\sqrt{3}} \left[ |g_L\rangle_1|g_L\rangle_2|2R\rangle + \frac{1}{\sqrt{2}}(|g_R\rangle_1|g_L\rangle_2 + |g_L\rangle_1|g_R\rangle_2)|L\rangle|R\rangle + |g_R\rangle_1|g_R\rangle_2|2L\rangle|0\rangle \right]. \] (16)

which demonstrates that Alice maps the unknown Dicke-state superposition equation (1) into the two-mode state (15) and Bob generates a maximally entangled state between atoms and two-mode cavity fields (16).

Now we consider the detection stage, in which we make a photon number measurement with four photon detectors \( D_j \) \( (j = \text{AH, AV, BH, BV}) \) on the output modes of the set-up. We assume that photons are detected at the time \( \tau \). This assumption is posed to calculate the system’s time evolution during this time interval in a consistent way with the ‘no-photon-emission-Hamiltonian’ (10). The detection of one photon with the detector \( D_j \) \( (j = \text{AH, AV, BH, BV}) \) can
be formulated with the operator $b_j$ on the joint state $|\Psi(\tau)\rangle_A^j |\Psi(\tau)\rangle_B^j$. As shown in figure 1, the photons leaking out from the cavities A and B are mixed at the PBS, whose action is to transmit the horizontal polarization and reflect vertical polarization. Since the photons coming from cavities are circularly polarized, two QWPs are inserted before PBS to map the left-circular photons (right-circular photons) into the horizontal polarization photons (vertical polarization photons).

After leaving the PBS, photon polarizations are rotated by polarization-rotations, whose actions are given by transformation $a_H \rightarrow \cos \theta a_H + \sin \theta a_V$ and $a_V \rightarrow \cos \theta a_V - \sin \theta a_H$, where $\theta$ is rotation angle and will be determined later. Thus the operators of the four detectors have the following forms

$$
\begin{align*}
    b_{AH} & = \cos \theta a_{AL} + \sin \theta a_{BR}, \\
    b_{AH} & = \cos \theta a_{BL} + \sin \theta a_{AR}, \\
    b_{AV} & = \cos \theta a_{AR} - \sin \theta a_{BL}.
\end{align*}
$$

(17)

If each of the four detectors detects one photon, the state of the total system is projected into

$$
|\Phi_f\rangle = b_{AH} b_{AV} b_{BH} b_{BV} |\Psi(\tau)\rangle_A^j |\Psi(\tau)\rangle_B^j
= C_0 \sin^2 2\theta |g_R, g_R\rangle + C_1 \cos^2 2\theta |g_L, g_R\rangle + |g_R, g_L\rangle + C_2 \sin^2 2\theta |g_L, g_L\rangle.
$$

(18)

If we choose parameter $\theta = \pi/8$, equation (18) becomes

$$
|\Phi_f\rangle = C_0 |g_R, g_R\rangle + C_1 |g_L, g_R\rangle + |g_R, g_L\rangle + C_2 |g_L, g_L\rangle.
$$

(19)

Based on the four-photon coincidence, Bob performs local unitary operations\(^1\) to his atoms to transform state(19) into equation (1), the teleportation is thus finished. The success probability to achieving four-photon coincidence is $P_{\text{succ}} = (1 - e^{-2\kappa \tau})^4/12$.

We now give a brief discussion on the influence of the quantum noise on the scheme. Firstly, it is evident that the scheme is inherently robust to photon loss, which includes the contribution from channel attenuation, and the inefficiency of the photon detectors. All these kinds of noise can be considered by an overall photon loss probability $\eta$ [4]. It is noticed that the present scheme is based on the four-photon coincidence detection. If one photon is lost, a click from each of the detectors is never recorded. In this case, the scheme fails to teleport superposition of Dicke-states. Therefore the imperfection of photon detectors only decreases the success probability $P_{\text{succ}}$ by a factor of $(1 - \eta)^4$, but have no influence on the fidelity of the expected operation.

Next we show that the scheme is insensitive to the phase accumulated by the photons on their way from the ions to the place where they are detected. The phases $\varphi_1 = kL_1$ and $\varphi_2 = kL_2$, where $k$ is the wavenumber and $L_j$ are the optical lengths which photons travel from the $j$th cavity towards the photon detectors, lead only to a multiplicative factor $e^{i2(\varphi_1 + \varphi_2)}$ in equation (19). This result demonstrates that phase accumulated by the photons has no effect on the conditional implementation of the quantum operation. Thirdly, since the scheme is based on adiabatic passage technology, atomic excited states can be decoupled from the evolution, and the influence of atomic spontaneous emission on the teleportation could be suppressed.

\(^1\) The local unitary operations are $|g_L\rangle_i \leftrightarrow |g_R\rangle_i$, $i = 1, 2$, which can be realized by using classical fields along the cavity axis and the atoms need not be addressed individually.
3. Teleportation of $N$-atom Dicke states

In this section, we turn to the problem of teleportation of $N$-atom Dicke states from one cavity to another. We use the notation $|g_L^{\otimes m} g_R^{\otimes N-m}\rangle$ to denote a normalized Dicke state where $m$ atoms are in the level $g_L$ and $N_a - m$ atoms are in the level $g_R$. The atomic state in cavity A is initially prepared in Dicke-state superposition of the form

$$|\Psi_N\rangle = \sum_{m=0}^{N} C_m |g_L^{\otimes m} g_R^{\otimes N-m}\rangle,$$

and cavity fields are in the vacuum states. For such an initial state, the Hamiltonian describing the system is still given by equation (3), which has the following dark states

$$|D_m(t)\rangle \propto \left(\Omega_A^{\dagger} a_L - g_A \sum_{i=1}^{N} |g_L\rangle_i \langle g_0| \right)^m \left(\Omega_A a_R^{\dagger} - g_A \sum_{i=1}^{N} |g_R\rangle_i \langle g_0| \right)^{N-m} |g_0\rangle^{\otimes N} |0,0\rangle_A. \quad (21)$$

Thus, by adiabatically adjusting the coupling $\Omega_A(t)$ from $\Omega_A \ll g_A$ to $\Omega_A \gg g_A$, based on the dark states, one can map the $N$-atom Dicke superposition states into the two-mode $N$-photon states

$$|\Psi\rangle_A = \sum_{m=0}^{N} C_m |mL\rangle |(N - m)R\rangle. \quad (22)$$

For Bob, $N$ atoms are initially prepared in the state $|g_a\rangle$ and cavity fields are in the vacuum states. For such initial state, the Hamiltonian describing the system is given by equation (8). In the adiabatic limit, the initial state $|g_a\rangle^{\otimes N} |0,0\rangle_B$ evolves into the following dark state

$$|D(t)\rangle_B \propto \sum_{m=0}^{N} (-1)^m \left(\frac{\Omega_B}{g_B}\right)^m \frac{1}{m! (m+1)!} \left(e_R^{\dagger} \sum_{i=0}^{N} |g_L\rangle_i \langle g_a| + a_L^{\dagger} \sum_{i=0}^{N} |g_R\rangle_i \langle g_a| \right)^m |g_a\rangle^{\otimes N} |0,0\rangle_B. \quad (23)$$

By adiabatically tuning $\Omega_B(t)$ to go from $\Omega_B \ll g_B$ to $\Omega_B \gg g_B$, achieves the maximally entangled states of $N$-atom Dicke states and $N$-photon polarization states

$$|\Psi\rangle_B = \frac{1}{\sqrt{N}} \sum_{m=0}^{N} |g_L^{\otimes m} g_R^{\otimes N-m}\rangle |mR\rangle |(N - m)R\rangle \rangle. \quad (24)$$

In order to realize teleportation, photons which originate from the state $|\Psi\rangle_A$ and $|\Psi\rangle_B$ are injected into the set-up shown in figure 3. The operators of the detectors $D_i$ in device $M_A$ and $M_B$ have the following forms

$$b_{Ai} = \cos \theta_i a_{AL} - \sin \theta_i e^{i\phi_i} a_{BR}$$
$$b_{Bi} = \cos \theta_i a_{BL} - \sin \theta_i e^{i\phi_i} a_{AR}, \quad (i = 1, 2, \ldots, N), \quad (25)$$
where $\theta_i$ and $\varphi_i$ are determined by the polarization rotator $i$. If the each of photon detectors $D_i$ ($i = 1, 2, \ldots, N$) in $M_A$ and $M_B$ detects one photon, and parameters $\tan \theta_1 e^{-i\varphi_1}, \ldots, \tan \theta_N e^{-i\varphi_N}$ are chosen to be the $N$ complex roots of the characteristic polynomial\(^2\) [22]

$$
\sum_{n=0}^{N+1} \frac{1}{\sqrt{n!(N-n)!}} (\tan \theta e^{i\varphi})^n = 0,
$$

(26)

the state of the total system is projected into

$$
|\Phi_1\rangle = \bigotimes_{j=1}^{N} b_{A_j} b_{B_j} |\Psi\rangle_A |\psi\rangle_B = \sum_{j=1}^{N} C_m |g_R^{\otimes m} g_L^{\otimes N-m} \rangle.
$$

(27)

Based on the measurement result, Bob performs local operation to transform equation (27) to equation (20). This demonstrates that the proposed set-up, which is shown in figure 3, definitely implements the teleportation of $N$-atom Dick-state superposition with unit fidelity. The success probability of conditional realization is $P_{\text{meas}} = N^{1-2N} \prod_{i=1}^{N} \cos \theta_i^d$, which decreases exponentially with increasing $N$. The quantum noise has no influence on the fidelity of the teleportation, but decreases the success probability $(1-\eta)^2 N$.

In summary, we have presented schemes to teleport an arbitrary superposition of two-atom Dicke states from one cavity to another. In contrast to the scheme [19], our scheme requires

\(^2\) The action of measurement device $M$ is to project on the two-mode $N$-photon entangled states $|\Psi_N\rangle = \sum_{m=0}^{N} |m_H, (N-m)V\rangle_{LM}$. In terms of creation operators, such an $N$-photon state can be written in the form $|\Psi_N\rangle = (a_{H1}^\dagger \cos \theta_1 + a_{V1}^\dagger \sin \theta_1 e^{-i\varphi_1}) \cdots (a_{H1}^\dagger \cos \theta_N + a_{V1}^\dagger \sin \theta_N e^{-i\varphi_N}) |0, 0\rangle_{LM}$, where parameters $\tan \theta_1 e^{-i\varphi_1}, \ldots, \tan \theta_N e^{-i\varphi_N}$ are chosen to be the $N$ complex roots of equation (26).
multi-photon coincidence detection, and is insensitive to quantum noise, which has no influence on the fidelity of the teleportation, but decreases the success probability. The fidelity is unity for any superposition of Dicke states. Finally, it is briefly pointed out that scheme can be modified to teleport a superposition of $N$-atom Dicke states.

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