Gauge Symmetry, Spontaneous Breaking of Gauge Symmetry: Philosophical Approach

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February 3, 2014

Abstract

This paper deals with the ontology of the vector potential. When the state of the system has the full gauge symmetry of the Hamiltonian, the electromagnetic vector potential may be interpreted as a convenient tool of a mathematical formulation, with no ontological meaning. I argue that this interpretation is in difficulty because the vector potential becomes proportional to the supercurrent in the superfluid phases, which are spontaneously broken gauge symmetry phases, where particle number is not conserved. I suggest that when gauge symmetry is spontaneously broken, the vector potential becomes an emergent material object of nature.

1 Introduction

This paper addresses the question of the ontology of the vector potential. This question has been in debate among physicists ever since Maxwell introduced the vector potential to account for the Faraday effect. The magnetic field is the curl of the vector potential. The latter may be shifted by adding to it a vector field the curl of which vanishes. Thus infinitely many vectors correspond to the vector potential, while the magnetic field is unique. This is what is called gauge symmetry. Theories which are formulated in terms of potentials are “gauge theories”. These acquire growing importance in theoretical physics, in particular in condensed matter, elementary particle physics and cosmology.

Spontaneous symmetry breaking is also a common feature of many fields of physics: the ground state of a many-particle system may be invariant under the
operations of a subgroup of the total symmetry group of the Hamiltonian. This is generally linked to the occurrence of a phase transition from an ordered state, with low symmetry, to a less ordered state with higher symmetry. For example, a ferromagnetic order has axial symmetry around the magnetization vector. A sufficient increase in temperature triggers a transition, at a critical temperature \( T_c \), to a paramagnetic state which has the full rotation symmetry, which is the symmetry group of the Hamiltonian.

I would like to connect those two topics and discuss some of the lessons we can learn about the material world, and knowledge of its laws, by examining various aspects of gauge theories. The latter are relevant in classical physics, and are connected in quantum mechanics through the phase of the wave function. The ontological status of the phase of the wave function may be clarified by discussing some concepts such as the Berry phase and particular cases of spontaneous gauge symmetry breaking, i.e. the phenomena of superfluidity and superconductivity. The relationship of gauge symmetries with electricity conservation is especially clear in the latter phenomena, where the breaking of gauge symmetry is associated with the canonical conjugation of phase and particle number.

## 2 Classical physics

Two topics are of interest in this chapter, that of Maxwell equations for classical electrodynamics, and that of parallel transport, such as is at work in the Foucault pendulum.

### 2.1 Maxwell equations, the Faraday effect, and the vector potential

The electric field \( \vec{E}(x,t) \) and magnetic field \( \vec{B}(x,t) \) obey the four Maxwell equations, which describe how the fields are related to one another and to static or moving charges. Quantities such as the electric potential \( V(x,t) \) and the vector potential \( \vec{A} \) are usually labeled "auxiliary quantities". They determine completely \( \vec{E} \) and \( \vec{B} \) according to:

\[
\vec{B} = \nabla \times \vec{A} \quad \quad \vec{E} = -\frac{\partial}{\partial t} \vec{A} - \nabla V
\]

(1)

On the other hand, \( \vec{B} \) and \( \vec{E} \) do not determine \( V \) and \( \vec{A} \). If \( f \) is a scalar function of space and time, the following transformation on the potentials, a "gauge transformation", does not alter the fields:

\[
\vec{A} \rightarrow \vec{A} + \nabla f \quad \quad V \rightarrow V - \frac{\partial}{\partial t} f.
\]

(2)

This is the essence of gauge invariance, or gauge symmetry. Is this merely an ambiguity of the mathematical representation of physical states? A mere

\(^1\)This is due to equations (1) and to \( \nabla \times \nabla = 0 \).
representation surplus? Reference [1] is an example of philosophical investigation of gauge theories.

David Gross [2] comments on the way Maxwell introduced the vector potential in order to account for the Faraday effect. The latter is the occurrence of an electric current in a closed conducting loop when the magnetic flux threading the loop varies in time. Maxwell did not accept the non locality of the effect: how could an electric current be induced by a magnetic flux far away from the loop, with possibly zero intensity at its locus? Maxwell found a satisfactory solution by inventing the vector potential $\vec{A}$. The time variation of the flux through the loop could now be ascribed to $\partial_t \vec{A}$, together with the relationship of the electric field with the time variation of $\vec{A}$: $\vec{E} = -\partial_t \vec{A}$. The electric field then acts locally on the metallic loop, where the vector potential is non zero: a local description of phenomena is retrieved.

There was no doubt in Maxwell’s mind that $\vec{A}$ was a physical field.

But a problem appears with gauge invariance, as exhibited by equation (2). How can a physical object exist if it can be described by an infinite number of different vector fields? What led Maxwell to think of the vector potential as a physical field was the actual non zero value of $\frac{\partial}{\partial t} \vec{A}$ at the locus of the conductor, and the gauge invariance of the circulation of $\vec{A}$ along a closed loop $\mathcal{C}$. Indeed, $\oint_{\mathcal{C}} \vec{A} \cdot d\vec{s}$ is, through Stokes theorem, equal to the flux $\Phi_{\mathcal{C}}$ of $\vec{B}$ through the loop $\mathcal{C}$, and is gauge invariant.

In the presence of charges and current, Maxwell’s equations impose the conservation of charge. In the classical theory of electromagnetism, the connection between gauge invariance and charge conservation was only realized in 1918 by Emmy Noether’s first theorem [7], as well as Weyl’s attempts to construct a unified theory of gravitation and electromagnetism [8]. This connection will be discussed in another section of this paper.

The positivist attitude towards science (as clearly expressed, for example, by Duhem [3]) prevailed among many of Maxwell’s followers. Hertz, Heaviside, Lorentz, etc., down to Aharonov [4]: just as Cardinal Bellarmin approved Galilei as long as the heliocentric hypothesis allowed to account for phenomena [3] ("sauver les apparences"), many physicists have adopted the view that the vector potential is a practical tool to simplify Maxwell’s equations and to account for phenomena, but has no physical meaning. The rationale behind this view is the gauge dependent nature of $\vec{A}$ which makes it unobservable as a local quantity. As we shall see, a spontaneous gauge symmetry breaking turns this "unobservable" object into a directly measurable one.

The classical Hamilton function $H$ for a single charged particle in the presence of potentials is expressed as:

$$H = \frac{1}{2m} \left( \vec{\rho} - e \vec{A} \right)^2 + eV$$

(3)

where $\vec{\rho}$ is the canonical momentum.

The dynamic equation describing the motion of a charged classical particle
is the Lorentz equation, which can be derived from equation (3):

\[ m \frac{d^2 \vec{r}}{dt^2} = e \vec{E} + e \left( \frac{d\vec{r}}{dt} \right) \wedge \vec{B} \]  
(4)

The hamiltonian (equation (3)) is expressed in terms of the potentials, while equation (4) is expressed in terms of the fields. The choice, in classical physics, is a matter of taste.

2.2 Parallel transport, and the Foucault pendulum

The discovery of the quantum mechanical Berry phase \[5\], discussed later on in this paper, has allowed to re-discover a hitherto little studied gauge invariance connected, in classical physics, to parallel transport.

Two concepts are of interest in this topic: the concept of anholonomy and that of adiabaticity. Quoting Berry \[5\]: "Anholonomy is a geometrical phenomenon in which nonintegrability causes some variables to fail to return to their original values when others, which drive them, are altered through a cycle...Adiabaticity is slow change and denotes phenomena at the borderline between dynamics and statics". I note in passing that adiabaticity is yet another concept which supersedes the kantian antinomy between statics and dynamics.

The simplest example of anholonomy is the change of the direction of the swing of a Foucault pendulum after one rotation of the earth. Visitors to the Panthéon in Paris can check that this is a phenomenon at work in the objective real world.

If a unit vector \( \vec{e} \) is transported in a parallel fashion over the surface of a sphere, its direction is changed by an angle \( \alpha (C) \) after a closed circuit \( C \) on the sphere has been completed. \( \alpha (C) \) is found to be the solid angle subtended by \( C \) at the center of the sphere. It is expressed by the circulation of a certain vector on \( C \), the result of which is independent of the choice of basis vectors\[4\]. The latter freedom of choice is a gauge symmetry, the change of basis vectors being equivalent to a change of gauge. This feature is analogous to what we will find in quantum mechanics, either when discussing adiabatic transport of a quantum state, or the electromagnetic vector potential: an objective phenomenon of nature depends on the circulation of a vector quantity along a closed loop, although that quantity, when gauge invariance prevails, cannot be defined at any point along the circuit.

3 Quantum mechanics

3.1 Electromagnetic gauge symmetry

The ontological question about the vector potential was revived when it became clear that the Schrödinger equation for charged particles in the presence of a

\[ \textit{For details of the derivation see ref. [5].} \]
magnetic field had to be formulated in terms of the potentials $\vec{A}$ and $V$ (apart from the Zeeman term which will be dropped here for simplicity, with no loss of generality). The reason is that there is no such choice as between equations (3) and (4). The only starting point in quantum mechanics is the Hamiltonian which is given by (3) where $\hat{\vec{p}} = -i\hbar \nabla$. Quantum mechanics substitutes the notion of quantum state to that of the classical notion of trajectory. The latter is irrelevant at the microscopic level, as evidenced by the non commutativity of momentum $p$ and coordinate $x$.

Gauge invariance is now expressed by the simultaneous transformation of equation (2) with:

$$\Psi \rightarrow \Psi \exp if(x, y, z, t),$$

where $\Psi$ is a solution of the time dependent Schrödinger equation. Thus the state is described up to a phase factor.

Most quantum mechanics text books, at least up to Berry’s discovery in 1984, state that the overall phase factor of the wave function describing a system has no physical meaning, since $|\Psi|^2$ is unchanged when the overall phase changes. This is a quantum version of the charge conservation described by Maxwell’s equations: a global phase change, which is a global gauge change, conserves the charge. We shall see the limits of that statement.

At first Weyl linked charge conservation to local gauge transformations. The latter are "local" when the gauge shift $f(\vec{r}, t)$ varies in space. In fact Noether showed that global gauge invariance is enough to express charge conservation. Global gauge is the limit of a local gauge when $f$ is a constant.

As regards the 'representation surplus' that gauge freedom represents, it is worth pointing out that this surplus is a blessing for the theorist, because it allows a mathematical treatment of problems which is adapted to the specific geometrical features at hand. The behaviour of the electronic liquid under magnetic field in a long flat ribbon is conveniently expressed in a gauge where $\vec{A}$ is orthogonal to the long dimension of the ribbon. For the physics of a disk, a gauge with rotational symmetry is usually useful. Any gauge choice should yield the same result, but a clumsy choice can make the theory intractable. This is quite analogous to the correct choice of coordinate system – cartesian, polar, cylindrical, spherical, etc. – in a geometry problem. At this stage, considering the vector potential as a mere technical tool – a usefully flexible one at that – for the theory seems rational.

### 3.2 The Aharonov-Bohm effect

The Aharonov-Bohm effect proves that there exist effects of static potentials on microscopic charged particles, even in the region where all fields vanish. The

\[3^3\]This means that we are interested here with orbital degrees of freedom, as if spin degrees of freedom were frozen in a large enough field

\[4^4\]as opposed to the classical case when one could start with the Lorentz force equation.

The lagrangian formulation also deals only with potentials

\[5^5\]see for example ref. [9].

\[6^6\]see ref. [10] for a detailed discussion.
standard experimental set up may be described by the diffraction of electrons by a standard two slits display. An infinitely long solenoid, is placed between the two slits, parallel to them, immediately behind the slit screen; an electric current creates a magnetic flux inside the solenoid, and none outside. The electronic wave function is non zero in regions where no magnetic flux is present. A variation of the flux inside the solenoid causes a displacement of the interference fringes on a second screen placed behind the slits. It is straightforward to relate this displacement to the phase difference $\delta\phi$ of the two electron paths at a given point on the screen. The latter is given by

$$
\delta\phi \propto \frac{e}{\hbar} \oint_C A \cdot ds = e\Phi_B / \hbar = 2\pi\Phi_B / (\hbar/e)
$$

where $\Phi_B$ is the flux threading the solenoid. This flux is gauge invariant. The displacement is periodic when the flux $\Phi_B$ varies in the solenoid, with a period given by the flux quantum $\Phi_B / (\hbar/e)$.

There are various other versions of the same effect. One is the observation of periodic variations – with flux period $2\pi\hbar/e$ – of the resistance of a mesoscopic conducting ring when the flux varies inside a thin solenoid passing through the ring [4]. Yet another variant will be discussed in a later section when I discuss superconductivity. In their 1959 paper [6], Aharonov and Bohm discuss the ontological significance of their findings. They mention that potentials have been regarded as purely mathematical objects. Quoting them: ...

... it would seem natural to propose that, in quantum mechanics, the fundamental natural entities are the potentials, while the fields are defined from them by differentiation. The main objection ...is grounded in the gauge invariance of the theory... As a result the same physical behaviour is obtained from any two potentials $A$ or $A'$ (related by a gauge transformation). This means that insofar as the potentials are richer in properties than the fields, there is no way to reveal this actual richness. It was therefore concluded that the potentials cannot have any meaning, except insofar as they are used mathematically, to calculate the fields. Over the years, this is what Aharonov seems to conclude, since this statement is reproduced in the 2005 book [4]. This is also what is discussed in ref. [1], who asks: is the Aharonov-Bohm effect due to non locality or to a long distance effect of fields? I do not see what kind of long distance action of fields could be invoked except if we admit that Maxwell equations have to be significantly altered. Non locality, on the other hand is now a well accepted feature of quantum mechanics [4]. However, another possibility arises: that the vector potential, the circulation of which on a closed loop leads to a phase factor, has some physical (gauge invariant) reality, although its local value cannot be measured because it is not gauge invariant. The fact that the vector potential becomes a measurable physical object in a superconducting phase lends some credence to this suggestion, as will be discussed in section (4).

\[\text{In this paper the velocity of light, } c, \text{ is put equal to unity throughout.}\]
3.3 Berry phase, Berry connection, Berry curvature

In 1984, Berry [11] discovered the following: a quantum system in an eigenstate, slowly transported along a circuit $C$ by varying parameters $R$ in its Hamiltonian $H(R)$ acquires a geometrical phase factor $\exp(i\gamma(C))$ in addition to the familiar dynamical phase factor. He derived an explicit formula for $\gamma(C)$ in terms of the spectrum and eigenstates of $H(R)$ over a surface spanning $C$. It is a purely geometric object, which does not depend on the adiabatic transport rate around the circuit.

This anholonomy is the quantum analog of the classical one discussed in section (2.2). Although the system is transported around a closed loop, its final state is different from the initial one. The phase choice for the initial state is a gauge degree of freedom, which has no effect on Berry's phase. The latter is thus gauge invariant. A precise definition of adiabaticity is that the motion is slow enough that no finite energy excitation occurs, as is the case when the isolated system is static. This condition in turn is that a finite excitation gap separates the ground state from the first excited state of the system.

One may define the 'Berry connection' $\vec{A}$, the expression of which is given below for completeness:

$$A_\mu(R) \equiv i\langle \Psi_R^{(0)} | \frac{\partial}{\partial R_\mu} \Psi_R^{(0)} \rangle,$$

where $|\Psi_R^{(0)}\rangle$ is the ground state wave function. $\gamma(C)$ is the circulation of $\vec{A}$ along the curve $C$. $\vec{A}$ is not gauge invariant since a gradient of any function $f$ can be added to it with no change for $\gamma(C)$. Various generalizations of equation (7) are described in ref.[5].

The "Berry curvature" is defined as $\vec{B} \equiv \vec{\nabla} \wedge \vec{A}$. The Berry phase is equal to the flux of $\vec{B}$ through the closed curve $C$.

There is a close analogy between the electromagnetic vector potential $\vec{A}$ and the Berry connection $\vec{A}$. If a closed box containing a charged particle is driven slowly around a thin solenoid threaded by a magnetic flux, the Berry phase is shown to be identical to the Aharonov-Bohm phase. $\vec{A}$ is shown to be identical to $\vec{A}$, modulo the charge coefficient. In fact the derivation of the Berry phase is yet another way of demonstrating the Aharonov-Bohm effect [11].

The ontology of the Berry phase is clear from the numerous experimental observations which have followed various predictions, such as the photon polarization phase shift along a coiled optical fiber [12], Nuclear Magnetic Resonance [13], the intrication of charge and spin textures in the Quantum Hall Effects, the quantization of skyrmion charge in a Quantum Hall ferromagnet, etc. What remains mysterious is the ontological status of the Berry connection, which is not gauge invariant. The same questions arise about it that are asked about the electromagnetic vector potential.

Whatever the answer to this question, what emerges from the discovery of the Berry phase, and from the many confirmed predictions and observable
effects, connected directly to it\textsuperscript{9} etc., is that the positivist doubts about the reality of the wave function phase as reflecting a profound fact of nature do not appear to be justified.

4 Superconductivity, a spontaneous breaking of gauge invariance

At low enough temperature, below a "critical temperature" $T_c$, a number of conducting bodies exhibit a thermodynamical phase change wherein the resistance becomes vanishingly small, and the diamagnetic response is perfect, which means that an external magnetic field cannot penetrate inside the body: a superconducting state is stabilized\textsuperscript{15}. This transition is an example of a spontaneous symmetry breaking, whereby the broken symmetry is the electromagnetic gauge symmetry discussed in the preceding sections. When a continuous broken symmetry occurs (as is the case of gauge symmetry), the transition to the new state, which is the appearance of a new quality, is simultaneously characterized by continuity and discontinuity. Indeed, the "order parameter" which describes the new quality grows continuously from zero when the temperature is lowered continuously, but the symmetry is broken discontinuously as soon as the order parameter is non zero. This is easy to understand for a spontaneous breaking of spin rotational symmetry such as ferromagnetism: as soon as an infinitesimal magnetization appears below the critical temperature, the full rotational symmetry of the high temperature phase is broken and rotational symmetry of the state is reduced to an axial rotation symmetry, around the magnetization direction.

The detailed theory of superconductivity is not relevant here, especially as the nature of superconductivity in a whole class of new metallic oxides is still a debated topic. The construction of the theory of the effect discovered by Kamerlingh Onnes in 1911 lasted half a century. One may surmise that one reason for this delay was precisely the elusive nature of the broken symmetry at work, which was clarified some years after the microscopic theory was published\textsuperscript{16} in 1957. The superconducting state is characterized by the pairing of a macroscopic number of electrons in so-called "Cooper pairs". A Cooper pair, formed (at least for a whole class of 'BCS superconductors')\textsuperscript{10} by two electrons of opposite spins, is a zero spin singlet, and, contrary to electrons, is not a fermion, but a boson. For simplicity, the superconducting ground state can be thought of as the condensation of a macroscopic number of such bosons, where they all have the same phase. Thus the superconducting ground state is characterized by a many-particle macroscopic condensate wave function $\Psi_{SC}(r)$, which has amplitude and phase and maintains phase coherence over macroscopic distances. How can one reconcile the appearance of a phase in the ground state, which is,

\textsuperscript{9}see for example chapter 4 in ref. \[5\].

\textsuperscript{10}In the last thirty years, various other classes of superconductors have been discovered and analyzed, with different ways for electrons to assemble in pairs. This does not limit the generality of the discussion in this paper.
as we shall see, a material object with very concrete and measurable properties, with the gauge symmetry of the many-particle Schrödinger equation? This is exactly what spontaneous gauge symmetry breaking is about: below the superconducting critical temperature, the state of the system selects a global phase which is arbitrary between 0 and 2π. This is analogous to the ferromagnetic ground state selecting an arbitrary direction in space if the Hamiltonian is rotationally invariant. In other words the continuous gauge symmetry is broken, and there is no absolute value for the phase of a single piece of superconductor in free space.

What about charge conservation in this state? In order to maintain phase coherence over a macroscopic volume of the superconductor, the total charge fluctuates between macroscopic chunks of the material, by circulation of Cooper pairs, which carry each two electronic charges. In fact, this is of interest for whoever still questions the complementarity of canonically conjugate variables (such as \( p_x \) and \( q_x \) for a single particle with position \( x \) and momentum \( p_x \)). Phase \( \phi \) and particle number \( N \) are conjugate variables, and the Heisenberg relation holds:

\[
\Delta N \Delta \phi \geq 1
\]  

(8)

This limits the accuracy with which \( N \) and \( \phi \) can be simultaneously measured. However, since \( N \approx 10^{23} \), and the fluctuation in \( N \) is of order \( \sqrt{N} \) an accuracy of \( 1/\sqrt{N} \approx 10^{-11} \) on \( \phi \) is highly satisfactory and the phase can be viewed as a semi-classical variable.

The number-phase relationship is expressed in the following relationship\textsuperscript{11}:

\[
|\Psi_N \rangle = \int_0^{2\pi} \exp(-iN\phi/2)|\Psi_\phi \rangle d\phi
\]  

(9)

In this equation, \( |\Psi_N \rangle \) and \( |\Psi_\phi \rangle \) are, respectively, the superconducting states for fixed particle number, or fixed phase. The latter is relevant for macroscopic samples. The former is relevant in small superconducting objects, or in the theoretical description of experiments where single Cooper pairs are manipulated. The factor 1/2 in the exponential under the summation is due to the fact that Cooper pairs carry 2 electronic charges.

The significance of the phase of the superconducting ground state was not immediately perceived by physicists, and it took three years to Josephson\textsuperscript{17}, after the initial BCS paper, to predict that Cooper pairs should be able to tunnel between two superconductors even at zero bias, giving a supercurrent density

\[
J = J_c \sin(\phi_1 - \phi_2)
\]  

(10)

where \( J_c \) is a constant and \( \phi_1, \phi_2 \) are the superconducting phases of the two superconductors. Another spectacular prediction was that in the presence of a finite voltage difference between the two superconductors, the phase difference would increase, following equation (10) with time as \( 2eV/\hbar \), (where \( e \) is the

\textsuperscript{11}The detailed expression for the various states involved is not relevant for this paper and can be found, for instance in ref.\textsuperscript{15}. 

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electron charge and $V_{12}$ the voltage difference) and the current should oscillate with frequency $\omega = 2eV_{12}/\hbar$. As mentioned in ref. [15], "Although originally received with some skepticism, these predictions have been extremely thoroughly verified...Josephson junctions have been utilized in extremely sensitive voltmeters and magnetometers, and in making the most accurate measurements of the ratio of fundamental constants $h/e^{12}$. In fact the standard volt is now defined in terms of the frequency of the Josephson effect". Among the most well known applications of the effect, SQUIDs (Superconducting QUantum Interference Devices) allow unprecedented accuracy in the detection and measurements of very weak magnetic fields. Josephson, aged 33, was awarded the Nobel prize in physics in 1973, for a work done during his PhD. Subsequently he worked on telepathy and paranormal phenomena, with no visible success...

A striking result of the zero resistance property of the superconducting phase was emphasized by London before the BCS microscopic theory was built: the superconducting current is proportional to the vector potential $\vec{A}$:

$$\vec{J}(\vec{r}) \propto \vec{A}(\vec{r}) \quad (11)$$

Since $\vec{J}$ is a measurable material object, $\vec{A}$ has transited from the status of (locally) non measurable object in the normal phase, to that of measurable object of nature in the superconducting phase. At first sight this statement is strange, since $\vec{A}$ appears to be as gauge dependent in the superconducting phase as in the normal one. Then the superconducting current would also be a non physical object, contrary to observations. The answer is that $\vec{J}$ obeys a physical condition, namely $\vec{\nabla} \cdot \vec{J} = 0$ which expresses the fact that there is no point source of superconducting current. Because of equation (11), this condition results in $\vec{\nabla} \cdot \vec{A} = 0$, which is a well known gauge choice, known as the London gauge. Thus, equation (11) expresses in a different way the broken gauge invariance of the superconducting phase. The London gauge does not specify the gauge completely, since all harmonic functions $g$ such that $\vec{\nabla}^2 g = 0$ are possible choices. Here again, the ultimate harmonic gauge choice is dictated by the superconductor geometry.

We are now facing an interesting ontological question: the vector potential appears to be a material object in a broken gauge symmetry phase, such as a superconductor, as evidenced by the proportionality between $\vec{A}$ and the superconducting current density. On the other hand, in the gauge invariant phase, it has measurable effects only through its circulation on a closed curve.

How can we get over (aufheben, in german) this contradiction?

Start with charge conservation, which is a consequence of gauge invariance. It is broken in the broken symmetry phase. Note in passing that this qualifies somewhat the statement that gauge invariance in the normal state is a mere "representation surplus". Charge conservation in the gauge invariant world is

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12 This was written before the discovery of the Quantum Hall Effects.

13 Equation (11) is not valid in all cases and may become an integral relation in case of non local phenomena, which are too technically involved to be relevant in this paper. This has no impact on the discussion in this paper.
a principle of matter, analogous to momentum conservation in a translationally invariant system. A possible way out of the dilemma about the ontological nature of the electromagnetic vector potential would be to view it as an emergent material object in the spontaneously broken gauge invariant phase. The superconducting phase itself is an emergent property (in this case the result of a thermodynamic phase transition) of the many body electron liquid of conductors. The analogy here would be, for instance, with the emergence of ferromagnetism from a macroscopic paramagnetic electronic liquid. But in that case the induction which appears in the ferromagnetic phase is the mere conceptual continuation of the magnetic field in free space. In the gauge invariant phase, however, the vector potential exists only through such anholonomicities as the Aharonov-Bohm phase. What seems to be in trouble, however, is the positivist thesis attributing to the vector potential a mere role of mathematical description of phenomena, with no ontological status. Further work is needed to understand fully the ontological implications of the fact that the vector potential, which is measurable and definitely appears to reflect the properties of a material object in the broken symmetry phase, becomes a sort of ghost in the gauge invariant phase, with only average measurable properties through its circulation on a closed loop.

4.1 Quantization of the fluxoid

Superconductivity exhibits the Meissner effect, which is the expulsion of the external magnetic field from the superconducting material. In fact, at low fields, the field penetrates the superconductor over a skin depth $\lambda_{14}$, called the penetration depth. At a larger distance than $\lambda$ from the surface, the magnetic field vanishes inside the superconductor.

In a multiply connected superconductor in the presence of a magnetic field, an interesting consequence of the Aharonov-Bohm phase -- which in this case is equivalent to a Berry phase --, is the quantization of the flux which threads a hole, or a normal region, passing through the superconductor$^{15}$. The flux quantum has the value $\phi_0 = \frac{2\pi}{e}$, where the factor 2 is due to the two electronic charges carried by each Cooper pair. The quantization is simply due to the superconducting state complex single valued wave function. The phase of the latter must change by an integral multiple of $2\pi$ in making a complete circuit. This flux (or fluxoid) quantization is the analog of the quantization of the angular momentum in an atomic system, and is a well established experimental fact.

In a simply connected superconductor, if the external field intensity increases above a certain critical value $H_{c1}$, a mixed phase arises. In one of the two major

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$^{14}$Typically $\lambda \approx 0.1\mu m$

$^{15}$To be exact, the quantization of the flux occurs if the surface threaded by the magnetic field is bounded by a closed curve located everywhere deep within the superconductor. If the closed curve passes near the surface (at a distance smaller than $\lambda$ ) the quantized quantity is the "fluxoid", which embodies the corrections due to the field within the penetration depth. See ref. for details.
superconducting classes, ("type II superconductors"), the field penetrates the superconductor in the form of flux filaments called vortices. At a distance larger than $\lambda$ from an isolated vortex, the field is reduced to zero; inside the vortex core, superconductivity is suppressed. Here again quantization of the fluxoid threading the vortex was predicted theoretically, and is a well established experimental fact (see ref. [15] for details).

### 4.2 The Anderson-Higgs boson

The recent experimental evidence in favour of the Higgs boson in high energy physics is a major experimental and theoretical achievement. However it came as no big surprise to physicists in condensed matter physics. Indeed, the massless Goldstone bosons, also known as "Nambu-Goldstone" particles, which emerge in any continuous broken symmetry phase acquire a mass in the superconducting phase, because of electromagnetic interactions. This was realized by P. W. Anderson and published in *Physical Review* in April 1963, one year before the papers by Higgs and Brout-Englert [18], although as a qualitative suggestion, with no detailed calculation. The missing ingredients in Anderson’s paper were non Abelian fields and relativity, which do not change qualitatively the mechanism for mass acquisition of the Goldstone boson.

This reflects a large degree of conceptual unity of gauge theories. It is fascinating that our present understanding of nature in such different fields as condensed matter and high energy physics resort to the same basic theoretical ingredients: gauge theories, spontaneous symmetry breaking, acquisition of mass by Goldstone bosons, etc.. It suggests also, if this was necessary, that there is no such thing as a hierarchy of scientific fields of knowledge in physics. The continuous development of knowledge with the related continuous improvement of experimental techniques is as potentially rich in new discoveries and new physical laws in one field as in another. There are as many new surprises at stake for physicists in improving high energy colliders as there are in atomic physics, condensed matter physics, etc., in reaching lower temperatures, larger magnetic fields, larger pressures, etc..

### 5 Conclusions

- If the discussion about the vector potential $\vec{A}$ is limited to normal phases, one may conclude that the potential language – as opposed to the field language – is a mere theoretical tool, and gauge symmetry a "description surplus". The spontaneously broken gauge symmetry phases, such as superconductivity point out the importance of the charge conservation associated to gauge invariance, and suggest that a theory wherein gauge freedom is suppressed, which means non conservation of charge, lead to the emergence of $\vec{A}$ as reflecting the properties of a material object. The understanding and the theoretical description of this object may well still be incomplete, but further advances should not invalidate its connection
with $\vec{A}$.

- The arguments given in this paper for the ontology of $\vec{A}$ within the quantum non relativistic electromagnetic theory give rather strong arguments that relativistic gauge theories, of the Abelian, non Abelian Yang-Mills type, or string theories [2] also reflect, at least in part, objective properties of the material world. The suggestion that, in non relativistic cases, the vector potential is an emergent material object of the spontaneously broken gauge symmetry phase suggests that similar emergent material fields occur in the relativistic broken symmetry phases of the strong or weak interaction world.

- Some concepts have appeared a number of times in this paper. That of emergence is another way of expressing a frequent occurrence in nature: the transformation of quantity in quality. Bind together two spin $1/2$ fermions, they turn into a boson; a large number of microscopic bosons condense in a macroscopic quantum state, the phase of which is measurable; an adiabatic parallel vector transport over a closed circuit on a sphere results in an anholonomy (Berry phase); an adiabatic parallel transport of a quantum system around a flux tube results in an Aharonov-Bohm phase; cool down a metal, it undergoes spontaneous broken symmetries of different types, depending on the interactions between the electrons, or on the crystalline symmetry of the atoms: broken translation, rotation, gauge invariance; cool down the universe some three hundred thousand years after the Big Bang, and a sort of metal insulator transition appears, etc.. The category of emergence encompasses a number of different types of passages from quantity to quality. Phase transitions and broken symmetries is one of them.

“More is different”, wrote P. W. Anderson [19] in a brilliant and devastating attack on reductionism [16]. He wrote: *The ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe. In fact, the more elementary particle physicists tell us about the nature of the fundamental laws, the less relevance they seem to have to the real problems of the rest of science, much less to those of society.*

This paper suggests that the materiality of the vector potential in a quantum broken gauge invariance phase is another example of emergence.

In a second paper [17] I will extend the discussion to the philosophical inferences one can draw from the many wonders of the Quantum Hall Effects.

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[16] It is worth quoting the concluding words of this paper: *...Marx said that quantitative differences become qualitative ones, but a dialogue in Paris in the 1920’s sums it up even more clearly:*  

FITZGERALD: The rich are different from us.  
HEMINGWAY: Yes, they have more money.

[17] In preparation [14].
Acknowledgements. I wish to thank my colleagues of the Laboratoire de Physique des Solides (Université Paris XI-Campus d’Orsay). Discussions with Jean-Noël Fuchs, Mark Goerbig, Gilles Montambaux, Frédéric Piéchon, Marc Gabay, Julien Bobroff have been helpful. Opinions and possible misconceptions expressed in this paper, on the other hand, are under my only responsibility. I would like to thank Professor Maximilian Kistler (Département de Philosophie, IHPST, Université Paris1), for his attention and stimulating suggestions. Special thanks are due to Jean-Noël Fuchs for a careful reading of the manuscript and useful remarks.

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