Scaling Violation and Inelasticity of Very High Energy Proton-Proton Interactions.

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Abstract

The pseudorapidity measurements at LHC, although in the central region only, allows to perform preliminary tests of the multiparticle production extrapolation formula inspired by the recent cosmic ray data analysis. Feynman scaling violation in the form proposed originally by Wdowczyk and Wolfendale in 70s has been applied to the Pierre Auger Observatory and the Hi-Res group measurements. The consistency of the Extensive Air Shower development and anisotropy data was found for smoothly rise of the scaling violation parameter. We have shown that the longitudinal momenta of produced particles determined inclusively as rapidity (pseudorapidity) distributions measured by LHC experiments follow the some universal high energy distribution scaled respectively. The high degree of Feynman scaling violation is confirmed. The decrease of the very high energy interaction inelasticity suggested by cosmic ray data analysis is found to be consistent with LHC measurements up to 7 TeV.

Keywords: high-energy interactions, cosmic rays, Feynman scaling, inelasticity, extensive air showers.

1. Introduction

The inclusive description of minimum bias LHC events is not as spectacular as, e.g., Higgs hunting, but is essential for other very important scientific endeavours. One of them is the Ultra High-Energy Cosmic Ray (UHECR) problem and the answer to the question of an existence of Greizen-Zatsepin-Kuzmin (GZK) cut-off [1]. The origin and nature of cosmic rays is studied for almost exactly 100 years. The great experimental effort has been taken recently by two groups: the Pierre Auger Observatory [2] and the Hi-Res experiment [3]. The progress is observed, but the answers are still not decisive. The cosmic rays of energies of about $10^{20}$ eV, if they are protons, should not reach us from cosmological distances. On the other hand anisotropy measurements show that they probably actually do. Our knowledge about the nature of UHECR is based on observation of giant Extensive Air Showers (EAS) - cascades of secondary particles created in the atmosphere when the single atomic nucleus (proton in a simplest case) enters from above. It is expected that the EAS initiated by protons and iron nuclei should differ. This difference is determined by the rate of energy dissipation. Thus it depends strongly on the distribution of secondaries produced in the forward direction and on the nature of primary particle: its atomic mass. The long-lasting discussions on the primary cosmic ray mass composition at the very end of the cosmic ray energy spectrum, in the so-called "ankle" region ($E_{\text{lab}} > 10^{18}$ eV), could not be conclusive also because of the lack on the more exact knowledge of the very high energy interaction physics, what makes the importance of the high energy proton fragmentation even greater for cosmic ray physicist, astronomers and cosmologists.

Searching for regularities and phenomenological description of the multiparticle production model is as old as the modeling in high-energy physics itself. Starting from simple Fermi thermodynamical model, to the first parton (quark) model propositions by Feynman, the model extrapolation to much higher, cosmic ray energies was one of the most important and most wanted model predictions. It is usually in the form of a kind of scaling. The idea of limited fragmentation [4] applied to the quark-jet hadronization led to introduction of the Feynman scaling variable of $x_F$ and the universal fragmentation function $f(x_F, s) = f_F(x_F)$ [5]. This brilliant idea works well for the first collider.
experiments up to $\sqrt{s} \sim 60$ GeV. However, when applied to cosmic ray EAS development, it was questioned already at the "knee" energies of $E_{lab} \sim 10^{15}$ eV. The SPS ($\sqrt{s} \sim 200 - 900$ GeV) experiments allow to quantify the scaling violation. The scale-breaking model of Wdowczyk and Wolfendale has been proposed to described the CR data at the beginning of ’70 [2]. It is, in a sense, a generalization of the Feynman scaling idea introducing the one scaling violation parameter.

In Ref. [7] we have shown that the light composition suggested by the studies of the anisotropy and the average depth of the shower maximum ($x_{\text{max}}$) does not contradict other results, mainly the width of the $x_{\text{max}}$ distribution, only if one assume strong Feynman scaling violation.

The rapidity (pseudorapidity) distributions were measured by LHC experiments: ALICE[8], CMS[9, 10] and ATLAS [11] (the last for $p_{\perp} > 0.5$ GeV only) in the central rapidity region $|\eta| \lesssim 2.5$ for c.m.s. energies of 900 GeV, 2.3 TeV and 7 TeV. Narrow range of a rapidity (pseudorapidity) at first sight does not allow to study important characteristics of very forward particle production. To study the fragmentation region new measurements, specially if one assume strong Feynman scaling violation.

2. Rapidity distribution

Rapidity distributions measured in LHC experiments cover the central region where the produced particles are dynamically separated from the valence quarks of colliding hadrons. The central rapidity density $\rho(0) = 1/\sigma \left( \frac{d\sigma}{dy} \right)_{\text{max}}$ is the variable describing the particle production there. The original Feynman scaling preserves the value of the central rapidity density. The plateau in rapidity is characteristic feature of independent jet fragmentation model as well as statistical models with limited transverse momentum phase space. Unfortunately, it is known for long, that such simple picture does not work.

The phenomenological fit of the $\rho(0)$ rise made more than twenty years ago in Ref. [12] is still valid. The 900 GeV LHC measurements match well SPS UA5 result. The systematic discrepancy seen by CMS detector [9] does not change this general opinion.

2.1. Feynman scaling

Feynman scaling [3] can be expressed introducing one universal function $f_F$ of the variable $x = p_{\parallel}/p_{\text{max}}$ which describes the invariant momentum (longitudinal $p_\parallel$) distribution of particles crated in the high-energy inelastic (and non single diffractive) interaction

$$\frac{E}{\sqrt{s}/2} \frac{1}{\sigma} \frac{d^3\sigma}{dx_{\perp} dp_{\perp}} = f_F(x, p_{\perp}, s) = f_F(x, p_{\perp})$$

where $\sqrt{s}$ is the interaction c.m.s. energy, $E$, $p_\parallel$ and $p_{\perp}$ are energy, and longitudinal and transverse momenta of outgoing particles ($p_{\text{max}} \approx \sqrt{s}/2$). Change of variable from Feynman $x$ to rapidity $y$ gives

$$\frac{1}{\sigma} \frac{d^3\sigma}{dy_{\perp} dp_{\perp}} = f_F(x(y), p_{\perp})$$

where $x(y) = \sqrt{p_{\parallel}^2 + m^2}/(\sqrt{s}/2) \sinh(y)$. Using an approximate relation $\sqrt{p_{\parallel}^2 + m^2} \sinh(y) \approx p_{\perp} \sinh(\eta)$ and introducing the very convenient variable: pseudorapidity $\eta = -\ln \tan(\Theta/2)$ we have

$$\frac{1}{\sigma} \frac{d^3\sigma}{d\eta_{\perp} dp_{\perp}} = f_F(\frac{2p_{\perp}}{\sqrt{s}} \sinh(\eta), p_{\perp})$$

The integration over all $p_{\perp}$ is obvious with uncorrelated $p_{\perp}$ and $p_\parallel$ and the universality of the $p_{\perp}$ distribution

$$\frac{1}{\sigma} \frac{d\sigma}{d\eta} = F_F\left(\frac{2(p_{\perp})}{\sqrt{s}} \sinh(\eta)\right)$$

The factor $\langle p_{\perp} \rangle$ is a constant related to the transverse momentum scale.
We are interested of the extremely forward part of the (pseudo)rapidity distribution – projectile fragmentation region. It is convenient to move the longitudinal momentum distribution to the anti-laboratory frame ($\eta \rightarrow \eta'$) where the projectile is at rest prior to the collision. This is done shifting the c.m.s. (pseudo)rapidity distribution by $\Delta \eta = \ln (\sqrt{s}/m)$

$$\sinh(\eta') = \sinh(\eta - \Delta \eta) = \sinh(\eta - \ln(\sqrt{s}/m)) \approx e^{\eta - \ln(\sqrt{s}/m)/2} = e^{\eta} \frac{m}{\sqrt{s}} \sinh(\eta).$$  \hspace{1cm} (5)

After such transformation the direct comparison of particle production at different values of interaction c.m.s. energy is possible

$$\frac{1}{\sigma} \frac{d\sigma}{d\eta} \approx F_F \left( \frac{2\langle p_\perp \rangle}{m} \sinh(\eta') \right) = F_\eta(\eta') \hspace{1cm} \text{(6)}.$$

This form of Feynman scaling was tested e.g. in ref. [12] and it is found that it is valid only very approximately. We can see this in Fig. 1a, where previous millennium data are plotted as a function of the anti-laboratory pseudorapidity. The recent data from CMS [9, 10] and ALICE [8] are shown in Fig. 1b.

It is known that Feynman scaling is violated at least by the continuous increase of the central rapidity density what is easily seen in Fig. 1.

2.2. Feynman scaling violation

The original Feynman scaling implies that the inelasticity of proton-proton interaction, defined as a fraction of incoming energy carried by newly created particle, is universal, the same for all interaction energies. The first observations suggested an attractive value of 0.5. The rise of some characteristics of the interactions (like, e.g., average $p_\perp$ or central rapidity density we mentioned above) makes the assumption about the constancy of the inelasticity not quite well justified. Introducing the multiplicative factor proportional to the observed rise of the rapidity plateau to the right-hand side of Eq. (5) we can try to recover a form of scaling. Applying this procedure the simplicity of the original Feynman idea is lost and the next correction for the rise of the average transverse momentum could be introduced here as well. We have used in the present work the average transverse momentum rise of the form $\langle p_\perp \rangle = 0.413 - 0.017 \ln(s) + .00143 \ln^2(s)$ shown in Fig. 4 of Ref. [10]. The additional inelasticity control parameter...
is an index in a power law multiplicative factor. These two modifications lead according to Eq. (4) to only slightly more complicated scaling formula

\[ \frac{1}{\sigma} \frac{d\sigma}{d\eta} = \left( \frac{s}{s_0} \right)^{\alpha_F} F_F \left( \frac{2(p_\perp)}{\sqrt{s}} \sinh(\eta) \right) \]  

(7)

We have used the UA5 data measured at \( \sqrt{s_0} = 546 \text{ GeV} \) c.m.s. energy \(^{[12]}\) as a datum. The very accurate measured NSD pseudorapidity distribution have been used as a definition of the universal \( F_F \) function. We adjusted the \( \alpha_F \) parameter value to minimize the discrepancy between Eq. (7) scaling prediction and the distributions of pseudorapidity measured at different energies: from ISR to 7 TeV of LHC. The results are given in Fig. 2.

Values of \( \alpha_F \) increase from \(~\sim 0.05\) found for ISR 53 GeV to \(~\sim 0.11\) at LHC 7 TeV. The increase is statistically not very significant, at least for the overall inelasticity, what will be discussed later. The accuracy of the data scaling according to Eq. (7) can be estimated with the help of statistical tests. The \( \chi^2 \) values for the ISR and SPS are of about \( \chi^2/NDF \approx 40/20 \). The systematic uncertainties of the Tevatron and LHC results makes the \( \chi^2/NDF \) smaller but the overall tendency seen in Fig. 2 suggests strongly that proposed modification of the Feynman scaling is not a right solution for the extrapolation of interaction properties to the very high interaction energies.

2.3. Wdowczyk and Wolfendale scaling

It was shown in Ref. \(^{[7]}\) that the almost forty years old modification known as Wdowczyk and Wolfendale (WW) scaling \(^{[6]}\) could be still satisfactorily used to scale the interaction properties to the ultra high (> 10\(^{19}\) eV) cosmic ray energies.

The original idea of the WW scaling

\[ f(x, p_\perp, s) = (s/s_0)^\alpha \, f_{WW}(x (s/s_0)^\alpha, p_\perp) \]  

(8)

is an extension of the Feynman fragmentation formula of Eq. (11) (the limit for \( \alpha = 0 \)) with the possibility to get the 'thermodynamical limit' of \( n \sim s^{1/4} \) with \( \alpha = 0.25 \).

The WW model in its version of mid ’80 has been successfully used for the EAS studies around ‘the knee’. Its extension introducing partial inelasticities (energy fraction carried by specific types of particles), and the transverse momentum rise with interaction energy dependencies, as discussed above, gave better description of the production of different kinds of secondaries. As a result of this improvements the first power-law factor index was released and
gave an extra model parameter. This more flexible formula was applied, e.g., in Ref. [12] where the agreement of the WW model predictions and the UA5 measured rapidity distributions was shown. It should be mentioned that original Wdowczyk and Wolfendale model gave a complete description of the multiparticle production process to be used mainly in EAS studies, so it contains such details as partial inelasticities, transverse momenta, semiinclusive properties etc. The fit shown in Ref. [12] is the effective, average description of inclusive data of rapidity (pseudorapidity) only.

In the present work we explore the WW scaling of the form

\[
\frac{d\sigma}{d\eta} = \left( \frac{s}{s_0} \right)^{\alpha'} F_{WW} \left( \frac{\langle p_T \rangle}{\langle p_T^0 \rangle} \sinh(\eta) \left( \frac{s}{s_0} \right)^{-1/2} \right),
\]

where \( \langle p_T^0 \rangle \) is the average transverse momentum at the datum interaction energy (\( \sqrt{s_0} = 546 \text{ GeV} \)).

We have adjusted first both \( \alpha \) and \( \alpha' \) parameters independently to get the best scaling performance. Results are given in Fig. 4.

Obtained values of \( \alpha \) and \( \alpha' \) are shown in Fig. 4a. Horizontal lines show results from Ref. [12] (solid for \( \alpha \) and dashed for \( \alpha' \), respectively). The thick solid broken line is the result for \( \alpha \) of our UHECR analysis [7]. It is seen that the predictions from Ref. [7] and the LHC data are consistent. Although the large uncertainties, which are result of limited rapidity range as well as possible systematics, do not allow for any stronger conclusions.

We can, however, use the UHECR data analysis predictions for the values of \( \alpha \) and test if results of the fit, with such reduced free parameter space, remains in agreement with the WW scaling. It can be seen in Fig. 5.

The data description is not much worse than the one presented in Fig. 3. The constancy of the \( \alpha' \) suggested by WW original papers and seen in Fig. 4a, still holds as presented as in Fig. 4b.

### 3. Inelasticity

In Ref. [7] it is found quite unexpected high energy behaviour of interaction inelasticity coefficient. It was obtained as a result of the experimental suggestion that the composition of the UHECR is quite light, contains a significant proton fraction. The WW model with the strong Feynman scaling violation leads to continuous decrease of the energy fraction released to the secondaries produced in very high energy interactions. Eq. (9) gives the inelasticity energy dependence

\[
K(s) = K_0 \left( \frac{s}{s_0} \right)^{\alpha' - \alpha},
\]
Figure 4: W&W scaling parameters predictions for $\alpha$ (solid symbols and solid lines) and for $\alpha'$ (open symbols and dashed line) adjusted to the data (a), and values of $\alpha$ taken from the UHECR analysis [7] and only $\alpha'$ used as a free parameter of the fit (b).

Figure 5: Wdowczyk and Wolfendale scaling results with $\alpha$ set to the UHECR analysis data and $\alpha'$ adjusted to each experimental data set shown as in the Fig. 3.
while for the modified Feynman scaling formula Eq. (7) it is

\[ K(s) = K_0 \left( \frac{s}{s_0} \right)^{\alpha_F} . \]  

(11)

In the Fig. 6 we have shown results of our analysis. Open symbols show the fast rise of the inelasticity for modified Feynman scaling formula. Even if the \( \alpha_F \) follow the lower energy, smaller value, in the UHECR domain the saturation is expected. Filled symbols were obtained for WW scaling. The solid line gives the predictions from Ref. [7] obtained using UHECR data. The dashed line is the fit from Ref. [12] of the WW scaling parameters to SPS data. The value of 0.5 is also shown.

The open symbols are for the modified Feynman scaling with \( \alpha_F \) parameter. Solid line shows the UHECR data analysis prediction from Ref. [7]. Dashed line is the inelasticity fit from Ref. [12]. The ‘canonical’ value 0.5 is shown by short dashed line.

4. Summary

We have shown that the minimum bias pseudorapidity distributions measured by LHC experiments can be very well described with the scale-breaking Wdowczyk and Wolfendale formula.

The scaling violation observed for the energies up to SPS \( \sqrt{s} = 900 \text{ GeV} \) and 1800 GeV in Tevatron was upheld recently in the analysis of new UHECR data.

The phenomenological model of Wdowczyk and Wolfendale introduces two model parameters. The value of one of them: \( \alpha \), was originally found to be equal to 0.13 using interpolation of the \( x_F = p_\parallel / p_{\text{max}} \) distributions between \( \sqrt{s} \approx 10 \text{ GeV} \) and ISR energies. Later interpolations including SPS data gave the value of 0.18 and finally the effective value of 0.25 was found in Ref. [12]. The increase of the central rapidity density reported also in Ref. [12] suggests \( \alpha = 2 \times 0.105 = 0.21 \). This value gives the Extensive Air Showers development maximum position \( x_{\text{max}} \) for proton initiated showers not far from measured [2,8] as it is shown in Ref. [7].
The UHECR data suggests further smooth rise of the scale-breaking parameter. The first measurements at LHC up to 7 TeV c.m.s. energy agree with the trend observed at lower energies and seems to smoothly bridge accelerator results and these on very high energy interaction of cosmic ray protons. The limited range of measured pseudorapidities does not allow for a stronger statement. The more forward particle production data is highly welcome.

The rising inelasticity for (modified) Feynman scaling is obviously in contrary to the Wdowczyk and Wolfendale scaling and cosmic ray data. Comparing the pseudorapidity distributions in Figs. 3b and 5b we can say that the LHC pseudorapidity data analysis favours the second possibility.

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