\[ \alpha \simeq \frac{\pi}{2} \text{ from} \]

supersymmetric spontaneous flavor breaking

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We propose a flavor model where both CP and flavor symmetries are broken at the supersymmetric level. The model is an effective SU(5) theory based on a U(2) horizontal symmetry. The minimum of the supersymmetric scalar potential can be exactly solved to yield a realistic pattern of charged fermion masses. The Higgs sector contains a symmetric, an antisymmetric and two vector fields, plus their U(2) conjugates. These Higgs fields are the only fields strictly required to break the flavor and CP symmetries and generate masses for all charged fermions including the up-quark. The model predicts the existence of an absolute minimum in the space of CP-phases. The value \( \alpha \simeq \pi/2 \) is predicted in a particular limit of the parameter space of the model.

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I. INTRODUCTION

The flavor problem is a persistent challenge of modern particle physics. Despite the numerous and diverse ideas have been proposed to explain the fermion mass hierarchies there are only a few models that can be considered realistic, i.e. that fit the data with precision. Some of these models are afflicted by serious limitations: they are effective field theories, have to resort to contrived flavor symmetries, or introduce the breaking of the flavor symmetry ad-hoc (i.e., hierarchies are generated through Higgs vevs that are postulated rather than obtained from the minimization of the scalar potential). Underlying these difficulties is the fact that the unknown flavor symmetry, active at some high energy scale, is completely broken at low energies, with the result that more than one Higgs doublet becomes necessary. Furthermore, to generate a realistic spectra of fermion masses, Higgs fields playing no role in the breaking of the flavor symmetry usually have to be introduced. In many cases, once the Higgs sector has been postulated, the analysis of the minimum becomes intractable, since the number of couplings allowed by the flavor symmetry rapidly increases with the number of Higgses.

One would like to simplify the analysis of the flavor vacuum by reducing, as much as possible, the number of couplings relevant in the breaking of the flavor symmetry. In this paper, we propose a realistic three-generation supersymmetric flavor model where the spontaneous breaking of CP and flavor symmetries occur at the supersymmetric level, obviating supersymmetry breaking terms. We have introduced the minimum number of Higgs fields required to break flavor and CP symmetries and generate a realistic pattern of charged fermion masses. In this model, supersymmetry allow us not only to reduce the number of parameters in the flavor sector, but also to solve the flavor vacuum analytically. The result is a more predictive flavor model.

II. THE MODEL

Let us consider a supersymmetric SU(5) model based on a horizontal U(2) flavor symmetry. We will assume that left and right handed third generation matter fields unify in the usual representations 10 and 5 of SU(5)

\[ \psi^3_{10}, \quad \psi^3_5, \]  

(1)
which transform as singlets of U(2). We will assume that first and second generations transform as fundamental representations of U(2), which we will denote as
\[ \Psi_{10} = \begin{pmatrix} \psi_{10}^1 \\ \psi_{10}^2 \end{pmatrix}, \quad \Psi_5 = \begin{pmatrix} \psi_5^1 \\ \psi_5^2 \end{pmatrix}. \] (2)

We will assume that there are two U(2) singlet Higgs fields transforming under the representations 5 and \( \bar{5} \) of SU(5), \( \mathcal{H}_5 \) and \( \mathcal{H}_{\bar{5}} \), which contain the usual electroweak symmetry breaking Higgs fields.

Finally we will assume that there are four flavor breaking chiral superfields
\[ S^{a\bar{b}}, \quad A^{a\bar{b}}, \quad F_1^a, \quad F_2^a, \] (3)
and their U(2) conjugates, \( \bar{S}, \bar{A}, \bar{F}_1, \bar{F}_2 \). These fields transform as a symmetric, antisymmetric and vector fields under U(2). Therefore at the renormalizable level only two Yukawa interactions are allowed by the symmetry
\[ \frac{1}{4} h_t \psi_{10} \psi_{10} \mathcal{H}_5 + \sqrt{2} h_b \psi_{10} \psi_{5} \mathcal{H}_{\bar{5}}. \] (4)

These terms will generate masses for the third generation fermions predicting \( m_b = m_{\tau} \) at the GUT scale. The supersymmetry conserving scalar potential is determined by the superpotential of the model. The most general renormalizable superpotential invariant under U(2) can be separated in three parts, \( W = W_A + W_S + W_F \). \( W_S \) contains all the interactions which include the symmetric tensors \( S \) and \( \bar{S} \)
\[ W_S = \mu_S S \bar{S} + \lambda_1 (F_1 \bar{S} F_1 + \bar{F}_1 S \bar{F}_1) + \lambda_2 (F_2 \bar{S} F_2 + \bar{F}_2 S \bar{F}_2) + \lambda_s (F_1 \bar{S} F_2 + \bar{F}_1 S \bar{F}_2). \] (5)

\( W_A \) contains all the interactions which include the antisymmetric tensors \( A \) and \( \bar{A} \),
\[ W_A = \mu_A A \bar{A} + \lambda_a (F_1 \bar{A} F_2 + \bar{F}_1 A \bar{F}_2), \] (6)
and \( W_F \) contains all the interactions which include only the vector fields \( F_1, F_2 \) and \( \bar{F}_1, \bar{F}_2 \),
\[ W_F = \rho_1 F_1 \bar{F}_1 + \rho_2 F_2 \bar{F}_2 + \rho_3 F_1 \bar{F}_2 + \rho_4 F_2 \bar{F}_1. \] (7)

We note that the fields \( F_1 \) and \( F_2 \) have the same quantum numbers. We could rewrite the superpotential as a function of two alternative vector fields, mixings of the original vector fields. Therefore we will assume from now on, without any loss of generality, that the fields fields \( F_1 \) and \( F_2 \) have been conveniently defined to cancel the couplings \( \lambda_1 \) and \( \lambda_2 \). This choice will considerably simplify the forecoming analysis. In a supersymmetric theory the vacuum energy is bound to
vanish. Therefore the supersymmetric scalar potential associated to $W$, $V = V_S + V_A + V_F$, must be zero at the minimum. If the real couplings $\mu_S$ and $\mu_A$ are not zero there is a solution satisfying $V_S = 0 = V_A$. This can be given by

$$\langle S^{ab} \rangle = -\frac{\lambda_s}{\mu_S} \left( V_1^a V_2^b + V_2^a V_1^b \right),$$

and analogously for $\langle \bar{S}_{ab} \rangle$ and $\langle \bar{A}_{ab} \rangle$. Here $V_1^a = \langle F_1^a \rangle$ and $V_2^b = \langle F_2^b \rangle$. We can always give an explicit expression for the minimum in a particular U(2) basis where the vevs $V_1$ and $V_2$ adopt the form,

$$V_1 = \begin{bmatrix} v e^{i\psi} \\ -v_1 e^{i\phi} \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0 \\ v_2 e^{-i\phi'} \end{bmatrix}.$$ (10)

Here the vevs $v, v_1$ and $v_2$ are defined positive and the sign in the entry $V_1^2$ has been introduced for convenience in the rest of the analysis. Since the solutions given by Eqs. 8 and 9 cancel the $V_S$ and $V_A$ components of the scalar potential, we must require the remaining component of the scalar potential, $V_F$, to cancel if the solution is to be a minimum. For instance, using Eqs. 8 and 9, the component $|\partial W/\partial F_1|^2$ of $V_F$ can be written as,

$$\left| \frac{\partial W}{\partial F_1} \right|^2 = \rho_s^2 F_1 \bar{F}_1 + \rho_s^2 F_2 \bar{F}_2 + \rho_1 \rho_3 (F_1 \bar{F}_2 + F_2 \bar{F}_1) - \rho_1 \rho_3 (F_1 \bar{F}_2 + F_2 \bar{F}_1) + \cdots$$ (11)

We are interested in finding at least one solution consistent with the data. Let us assume that the bilinear terms in $V_F$ dominate. Later we will see that this assumption is consistent with $\mu_S \ll \rho_i$. In the basis given by Eqs. 10 this condition translates into the equation,

$$\mu_1 v^2 + (\sqrt{\mu_1} v_1 - \sqrt{\mu_2} v_2)^2 + 2(\sqrt{\mu_1} \mu_2 - \mu \cos(\phi + \phi')) v_1 v_2 = 0.$$ (12)

where $\mu_1 = (\rho_1^2 + \rho_2^2), \mu_2 = (\rho_2^2 + \rho_3^2)$ and $\mu = (\rho_1 \rho_3 + \rho_2 \rho_4)$. This equation admits a non-trivial vacuum given by

$$v_2 = \sqrt{\frac{\mu_1}{\mu_2}} v_1,$$

$$\cos(\phi + \phi') = \frac{\sqrt{\mu_1 \mu_2}}{\mu} \left( 1 - \frac{v^2}{2 v_1^2} \right).$$ (14)

We note that the existence of a non-trivial vacuum requires the presence of both vectorial Higgs fields, $F_1$ and $F_2$. If one of these fields were not present, it would not be possible to break the flavor symmetry at the supersymmetric level, which would force us to resort to supersymmetry breaking terms. [11]
III. SU(5) REPRESENTATIONS OF THE HIGGS FIELDS

Yukawa couplings for the first and second generations are generated at higher order by non-renormalizable interactions that are generically of the form

$$\frac{1}{M} (\Psi_{10}S\Psi_{10}H_5 + \Psi_{10}A\Psi_{10}H_5 + \Psi_5S\Psi_{10}H_5 + \Psi_5A\Psi_{10}H_5),$$

(15)

where flavor indices have been omitted. Yukawa couplings mixing the first and second generations with the third generation are also generated at higher order by non-renormalizable interactions,

$$\frac{1}{M} (\sigma_u\psi_{10}F_j\Psi_{10}H_5 + \psi_{10}F_j\Psi_5H_5 + \psi_5F_j\Psi_{10}H_5)$$

(16)

where \(j = 1, 2\) and again flavor indices have been omitted. Here we find it convenient to introduce an additional real coupling, \(\sigma_u\), to differentiate the relative size and sign of the contributions to the (13) and (23) mixing in the down and up-type quark sectors. If the flavor breaking Higgs fields were SU(5) singlets, then, after flavor symmetry breaking, 3 × 3 symmetric Yukawa matrices would be generated for the charged leptons and down-type quark fields, of the form,

$$Y = \begin{bmatrix} \frac{1}{M}(\langle S \rangle + \langle A \rangle) & \frac{1}{M}(\langle F_1 \rangle + \langle F_2 \rangle) \\ \frac{1}{M}(\langle F_1 \rangle + \langle F_2 \rangle) & h \end{bmatrix},$$

(17)

where \(h\) represents generically a third generation Yukawa coupling. This Yukawa matrix, nevertheless, would predict wrong mass relations between the charged lepton and the down-type quark sector, \(m_e/m_\mu = m_d/m_s\) and \(m_\mu/m_\tau = m_s/m_b\).

It was pointed out some time ago that it may be possible to explain some of the observed differences between the charged lepton and down-type quark sectors by promoting the SM vertical symmetry to a GUT symmetry as the SU(5) symmetry of Georgi and Glashow \[12\]. In the context of SU(5), the observed empirical relations find one on their simplest explanations, the so-called Georgi-Jarlskog mechanism \[13\], which in the case of a U(2) flavor symmetry adopts a particular implementation proposed in Ref. \[2\]. If the flavor-symmetric field \(S\) transforms as a 75 representation of SU(5) the tensor product, \(S\mathcal{H}_5\) transforms effectively as a \(\overline{45}\) representation of SU(5). It is known that this would account perfectly for the approximate empirical factor 3 that connects the muon-tau and strange-bottom mass ratios (at the GUT scale), \(m_\mu/m_\tau \approx 3m_s/m_b\). Furthermore if the flavor-antisymmetric tensor \(A\) transforms as a SU(5) singlet, the operator \(\Psi_5A\Psi_{10}H_5\) will generate the same contributions to the (12) and (21) entries in both the charged lepton and down-type quark Yukawa matrices. This, together with the representation properties of \(S\) postulated above,
could help, as we will see later, to explain a second well known empirical relation connecting the
down-strange and the electron-muon mass ratios, \((m_d/m_s)^{1/2} \approx 3(m_e/m_\mu)^{1/2}\). Finally the vector
fields \(F_1\) and \(F_2\) could transform as a singlet or alternatively under the representation \(24\) of SU(5).
In both cases the operators in Eq. 16 would generate the entries \((i3)\) and \((3i)\), \((i=1,2)\) of the Yukawa
matrices.

IV. THE DOWN-TYPE QUARK YUKAWA MATRIX

We note that the Yukawa matrices generated by the model could in principle contain complex
phases in all the entries. Some of these phases do not generate observable effects in the fermion
sector. In this model one can always redefine the phases of the matter fields to make the upper
left submatrix real, which is just a convenient basis to study CP-violating effects. This phase
redefinition, which does not change the predictions of the model, can be simply enforced assuming
that the entries \((12),(21)\) and \((22)\) of the vevs \(\langle S\rangle\) and \(\langle A\rangle\) are real. In doing so we obtain relations
between the phases of Higgs fields that allow us to see more clearly the predictions that the model
makes for the CKM phase. In the U(2) basis given by Eq. 18 the vev \(\langle A\rangle\) takes the form,

\[
\langle A \rangle = \frac{\lambda_a e^{i(\psi - \phi')}}{\mu A} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.
\]

The condition \(\text{Im} \langle A \rangle = 0\) implies the relation \(\psi = \phi'\). In the same basis, and assuming \(\psi = \phi'\),\n\(\text{Im} \langle S \rangle\) is given by,

\[
-\frac{2}{\mu S} \begin{bmatrix} 0 & 0 \\ 0 & \lambda_a v_1 v_2 \sin(\phi - \phi') \end{bmatrix}.
\]

The condition \(\text{Im} \langle S \rangle = 0\) implies that the phases \(\phi\) and \(\phi'\) are equal, \(\phi' = \phi\), with \(\phi\) undetermined.
Taking this into account we can adopt, for convenience, an equivalent alternative notation for the
vevs \(V_1\) and \(V_2\),

\[
V_1 = \begin{bmatrix} v e^{i \omega' / 2} \\ -v_1 e^{i \omega' / 2} \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0 \\ v_2 e^{-i \omega' / 2} \end{bmatrix}.
\]

The phase \(\alpha'\) will be the only phase that manifests in the Yukawa matrices. A non-zero value
would signal the appearance of CP-violation. Using this notation Eq. 14 can be rewritten as,

\[
\cos(\alpha') = \frac{\sqrt{\mu_1 \mu_2}}{\mu} \left(1 - \frac{v^2}{2v_1^2}\right).
\]
The down-type quark Yukawa matrix can be conveniently rewritten in the form,

\[ Y_D = h_b \begin{bmatrix} 0 & 2a\zeta\eta e^{i\alpha'} & 2a\eta \lambda^+ \\ av\lambda^- & 2a v_1\tilde{\lambda}_a & a\lambda_{cb} \\ 2a\zeta\eta e^{i\alpha'} & a\lambda_{cb} & 1 \end{bmatrix} . \]  

(22)

where,

\[ \lambda^\pm = -\left(\tilde{\lambda}_s \mp \tilde{\lambda}_a\right) , \]

\[ \tilde{\lambda}_s = \frac{\lambda_s}{\mu_S}, \quad \tilde{\lambda}_a = \frac{\lambda_a}{\mu_A} , \]

\[ \lambda_{cb} = (e^{-i\frac{\alpha'}{2}} - \zeta e^{i\frac{\alpha'}{2}}) \]

and

\[ \zeta = \frac{v_1}{v_2}, \quad a = \frac{v_2}{M}, \quad \eta = \frac{v}{2v_1} . \]  

(26)

The parameters \( a, \eta \) and \( \zeta \), as we will analyze later, are directly correlated with the absolute values of the CKM elements. Therefore we naively expect them to be smaller than 1. In Secs. VI and VII we will study the precision predictions of the model. For the moment, let us assume that \( 2\eta\zeta \lesssim v\lambda^\pm \). In that case the down-strange and electron-muon mass ratios predicted by the model are related. These are given by,

\[ \frac{m_d}{m_s} \approx \eta^2 \left(\frac{\lambda_s^2 - \tilde{\lambda}_a^2}{\tilde{\lambda}_s^2}\right), \quad \frac{m_e}{m_\mu} \approx \eta^2 \left(\frac{9\tilde{\lambda}_s^2 - \tilde{\lambda}_a^2}{9\lambda_s^2}\right) . \]  

(27)

V. THE UP-TYPE QUARK YUKAWA MATRIX

Assuming that the U(2) flavor breaking fields \( S, A \) and \( F_i \) transform under the representations 75, 1 and 1, respectively, implies that two of the associated higher order operators in the up-type quark sector are exactly zero,

\[ \Psi_{10} S \Psi_{10} H_5 = 0, \quad \Psi_{10} A \Psi_{10} H_5 = 0. \]  

(28)

If this were the case the up-type quark Yukawa matrix would have the form,

\[ Y_U = h_t \begin{bmatrix} 0 & 0 & 2\theta_u a\zeta\eta e^{i\alpha'} \\ 0 & 0 & \theta_u a\lambda_{cb} \\ 2\theta_u a\zeta\eta e^{i\alpha'} & \theta_u a\lambda_{cb} & 1 \end{bmatrix} . \]  

(29)

where \( \theta_u = \sigma_u/h_t \). This matrix is afflicted by a serious phenomenological problem: it predicts that the up-quark is massless. Although the possibility of a massless up-quark has been considered in
the past as a solution to the strong CP-problem, more recent studies of pseudoscalar masses and decay constants, along with other arguments, suggest that the up-quark mass is nonzero. Other flavor models proposed in the literature, when faced with the problem of the smallness of the up-quark mass, have resorted to introducing additional Higgs fields. Interestingly in the current model one can generate a mass for the up quark at second order in the effective operator expansion without resorting to additional Higgses. At second order in powers of $1/M$ there are only three terms that can contribute to the up-type quark Yukawa matrix. These terms are generically of the form,

$$\frac{1}{M^2} \Psi_{10} \Psi_{10} H_5 (\lambda' F_i F_i + \lambda'' F_1 F_2),$$  \hspace{1cm} (30)

where $i, j = 1, 2$ and flavor indices have been omitted. The two terms proportional to $\lambda'$ will generate an additional contribution to the charm quark mass but they will not contribute to the up-quark mass. The up-quark mass will be generated by the operator proportional to $\lambda''$. Taking all three contributions into account the up-type quark Yukawa matrix takes the form,

$$Y_U = h_t \begin{bmatrix}
a^2 \frac{\lambda'' v^2}{v_2^2} & a^2 \lambda_{uc} \frac{v}{v_2} & 2\theta_u a \zeta \eta e^{i \theta'} \\
a^2 \frac{\lambda_{uc} v}{v_2} & a^2 \lambda_{ct} & \theta_u a \lambda_{cb} \\
2\theta_u a \zeta \eta e^{i \theta'} & \theta_u a \lambda_{cb} & 1
\end{bmatrix},$$  \hspace{1cm} (31)

where,

$$\lambda_{ct} = (\lambda'' \zeta + \lambda' \zeta^2 + \lambda'),$$  \hspace{1cm} (32)

$$\lambda_{uc} = (\lambda'' + \lambda' \zeta).$$  \hspace{1cm} (33)

The parameters $a$, $\eta$ and $\zeta$ were defined in Eq. [26].

VI. DETERMINATION OF THE PARAMETERS OF THE MODEL

In this section we will analyze how to determine the parameters of the model using some of the quark mass ratios and mixing angles. In the next section we will study the predictions the model makes once its parameters have been determined. We will make our point by proving that there is at least one particular limit of the parameter space that can reproduce observations. We will see that at this point of the parameter space the fundamental parameters are easily calculable. Let us assume that the following hierarchies in the lagrangian parameters hold,

$$\theta_u \ll 1,$$  \hspace{1cm} (34)
\[
\frac{\lambda_a}{\mu_A} \approx -2 \frac{\lambda_s}{\mu_S}, \quad (35)
\]
\[
\sqrt{\mu_1 \mu_2} \ll \mu, \quad (36)
\]
\[
\lambda'' \ll \lambda', \quad (37)
\]

The down and up-type quark Yukawa matrices can be brought to diagonal form by a biunitary diagonalization, \((V_D^L)^\dagger Y_D V_D^R = (h_d, h_s, h_b)\) and \((V_U^L)^\dagger Y_U V_U^R = (h_u, h_c, h_t)\). The CKM matrix is defined by \(V_{CKM} = (V_U^L)^\dagger V_D^L\). If \(\theta_u \ll 1\) the CKM elements are to leading order determined from the mixing in the down-type quark Yukawa matrix, i.e. \(V_{CKM} \approx V_D^L\), while the mixing arising from the up-type quark sector can be neglected. Therefore the unitary matrix \(V_{CKM}\) can be calculated from Eq. 22. To leading order it is a function of the four real parameters \(\zeta, a, \lambda, \eta\) and the complex phase \(\alpha'\)

\[
V_{CKM} \approx \begin{bmatrix}
1 - \frac{\lambda^2}{2} & \lambda & 2\eta \zeta a e^{i\alpha'/2} \\
\lambda & \frac{\lambda^2}{2} + a^2 r^2 - 1 & (e^{i\frac{\alpha'}{2}} - \zeta e^{i\frac{\alpha'}{2}}) a \\
-a\lambda(e^{i\frac{\alpha'}{2}} - \zeta(1 - 2\eta/\lambda)e^{-i\frac{\alpha'}{2}}) & (e^{i\frac{\alpha'}{2}} - \zeta e^{-i\frac{\alpha'}{2}}) a & 1 - a^2 r^2
\end{bmatrix}. \quad (38)
\]

where \(r^2 = (1 - 4\zeta \cos \alpha' + 4\zeta^2)\). Using the second assumption in our parameter space, \(\tilde{\lambda}_a \approx -2\tilde{\lambda}_s\), we obtain from Eq. 22

\[
\lambda = -\eta \frac{(\tilde{\lambda}_s + \tilde{\lambda}_a)}{\lambda_s} \approx \eta. \quad (39)
\]

The parameters \(\lambda, a\) and \(\zeta\) can be determined using experimental data. The parameter \(\lambda\) is to leading order given by \(\lambda = |V_{us}|\). The parameters \(a\) and \(\zeta\) can be determined using the absolute values of \(|V_{us}|, |V_{ub}|\) and \(|V_{cb}|\) by,

\[
a = \frac{|V_{cb}|}{r}, \quad \frac{\zeta}{r} = \frac{|V_{ub}|}{2|V_{cb}| |V_{us}|}. \quad (40)
\]

It is a trivial check to prove that the angle \(\alpha'\) introduced in the parametrization of the CKM matrix given in Eq. 38 coincides to leading order in powers of \(\zeta\) with the standard definition of \(\alpha\),

\[
\alpha = \text{Arg} \left[ -\frac{V_{td} V_{tb}^*}{V_{us} V_{ub}^*} \right] = \alpha' - \zeta.
\]

Furthermore, with our third assumption on the parameter space of the model, \(\sqrt{\mu_1 \mu_2} \ll \mu\), Eq. 21 predicts the angle \(\alpha'\) to be \(\pi/2\) since

\[
\cos \alpha' \propto \frac{\sqrt{\mu_1 \mu_2}}{\mu} \ll 1. \quad (41)
\]

Therefore the phase \(\alpha\) in that limit of the parameter space is given by

\[
\alpha \approx \frac{\pi}{2} - \zeta. \quad (42)
\]
Assuming that $\alpha' \approx \pi/2$ and using Eqs. the parameters $a$ and $\zeta$, as a good approximation, are given by,

$$a \approx |V_{cb}|, \quad \zeta \approx \frac{|V_{ub}|}{2|V_{cb}| |V_{us}|} \quad (43)$$

Using the values of the CKM elements given by the PDG collaboration $|V_{us}| = 0.220 \pm 0.0026$, $|V_{ub}| = 0.00367 \pm 0.00047$ and $|V_{cb}| = 0.0413 \pm 0.0015$ and the ratio $|V_{ub}| / |V_{cb}| = 0.086 \pm 0.008$, we obtain approximately $\zeta \approx 0.19 \approx \lambda$ and $a \approx \lambda^2$. The couplings $\lambda'$ and $\lambda''$ can be determined from the up-charm and charm-top quark mass ratios. Taking into account that $a, \lambda, \zeta \ll 1$ and making use of our fourth assumption on the parameter space of the model, $\lambda'' < \lambda'$, these are given to leading order by,

$$\frac{m_c}{m_t} = a^2 \lambda'(1 + \zeta \rho + \zeta^2), \quad (44)$$

$$\frac{m_u}{m_c} = \frac{8 \lambda^2 \zeta^3 \rho}{(\zeta + \rho)}, \quad (45)$$

Here $\rho = \lambda'' / \lambda'$. Numerically, as a good approximation, $m_c/m_t \approx \lambda^3/2$ and $m_u/m_c \approx \lambda^4$ (at low energy). Therefore, taking into account that $a \approx \lambda^2$, a good estimation of the couplings $\lambda'$ and $\lambda''$ is $\lambda' \approx 3$ and $\lambda'' \approx \lambda \lambda'$. Finally there is one more dimensionless model parameter, $v_1 \hat{\lambda}_s$, that appears in the down-quark type Yukawa matrix, $Y_D$, and that has to be determined from the strange-bottom mass ratio,

$$\frac{m_s}{m_b} = 2av_1 \hat{\lambda}_s. \quad (46)$$

Numerically the ratio $m_s/m_b$ is know to be approximately $\lambda^2/2$ (at low energy). Therefore

$$v_1 \hat{\lambda}_s \approx \lambda.$$

Taking this into account we are now ready to understand that neglecting quartic terms in Eq. was perfectly consistent with the assumption $\mu_s \ll \rho_i$. We note that the numerical estimations of the parameters of the model have been implemented at the scale of flavor breaking, which we assumed to be close to the GUT scale. The parameters $\lambda$, $\zeta$ and $\rho$ are very approximately renormalization scale independent. On the other hand the parameters $a$, $\lambda'$ and $v_1 \hat{\lambda}_s$, which are determined from $|V_{cb}|$, $m_c/m_t$ and $m_s/m_b$ respectively, depend on the location of the flavor breaking scale. As a consequence, renormalization corrections to these observables, whose expressions are available in the literature, must be implemented if a precision calculation of the fundamental parameters of the model is required.
VII. PREDICTIONS

Once the fundamental parameters of the model have been determined as in the previous section, we are able to make three real predictions for fermion mass ratios. First we note that the model predicts \((Y_D)_{11} = 0\), while \((Y_D)_{13}\), which is determined from \(|V_{ub}|\), plays a minor role in the determination of the down quark mass. We thus obtain the prediction,

\[
\frac{m_d}{m_s} \approx |V_{us}|^2. \tag{47}
\]

This is a prediction that is well confirmed by the data. It is also an empirical relation that has been known for 37 years [19]. Using the sum rules to extract the masses of the lighter quarks we obtain, \((m_d/m_s)^{1/2} = 0.209 \pm 0.019\) [16].

In the charged-lepton sector, given the proposed SU(5) representation assignments of the Higgs fields, our model predicts that the charged lepton Yukawa matrix, \(Y_L\), is the same as the down-type quark Yukawa matrix, except that the contributions from the symmetric tensor \(S\) contain an additional factor 3. We thus obtain two predictions for the charged lepton mass ratios (see Eqs. 27),

\[
\frac{(m_e/m_{\mu})^{1/2}}{(m_e/m_{\tau})^{1/2}} \approx \frac{1}{3} \left( \frac{5}{3} \right)^{1/2}, \frac{(m_d/m_s)^{1/2}}{(m_d/m_b)^{1/2}} \approx \frac{1}{2.5}, \tag{48}
\]

\[
\frac{m_{\mu}}{m_{\tau}} \approx 3 \left( \frac{m_s}{m_b} \right), \tag{49}
\]

The first of these predictions, which is approximately renormalization scale independent, is fully consistent at 1σ with the experimental data [16],

\[
\frac{(m_d/m_s)^{1/2}}{(m_e/m_{\mu})^{1/2}}_{\exp} = 3.06 \pm 0.48. \tag{50}
\]

The second prediction is renormalization scale dependent and holds at the flavor breaking scale. Using experimental values we obtain at the electroweak scale [16],

\[
\frac{m_{\mu}}{m_{\tau}} = \frac{m_s}{m_b} \exp, \tag{51}
\]

Extrapolating to the GUT scale, 10^{16} GeV, in a supersymmetric scenario (with low \(\tan \beta\)) we obtain, \((m_{\mu}/m_{\tau})_{\text{GUT}} = (3.14 \pm 0.4) \times (m_s/m_b)_{\text{GUT}}\).

Finally we will show that the prediction that the model makes in the limit \(\sqrt{\mu_1 \mu_2} \ll \mu\) for the phase \(\alpha'\) successfully reproduces the measured value of CP violation in the quark sector. We have shown in the previous section that the model contains a particular limit, \(\sqrt{\mu_1 \mu_2} \ll \mu\), where the
CP phase $\alpha$ is predicted to be close to $\pi/2$. It is convenient to define the variables,

$$\delta' = \frac{\pi}{2} - \alpha', \quad \delta = \frac{\pi}{2} - \alpha,$$

and solve the Eq. 10 for $\zeta$ expanding around $\delta' = 0$. We obtain,

$$\zeta = \frac{1}{\sqrt{3}}(1 - \frac{4}{3}z) + \frac{\delta'}{3}(1 - \frac{8}{3}z),$$

where $z$ is a perturbative parameter defined as,

$$z = 1 - \frac{2|V_{ub}|}{|V_{cb}| |V_{us}|}.$$

$z$ can be determined from the measured absolute values of the CKM elements. Using the value of $|V_{us}|$ given by the PDG collaboration and the ratio $|V_{ub}|/|V_{cb}| = 0.086 \pm 0.008$ [15], we obtain $z = 0.22 \pm 0.08$. Finally using our parametrization of the CKM matrix we obtain a simple expression for the leading-order relation between the angles $\beta$ and $\alpha$,

$$\beta = \text{Arg} \left[ \frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right] = \text{Arg} \left[ 1 - \zeta e^{-i\alpha} \right].$$

Using the formula for $\zeta$ given in Eq. 53 and expanding around $z, \zeta, \delta = 0$ we find that in the limit $\sqrt{\mu_1 \mu_2} \ll \mu$, the model predicts the phase $\beta$ to be,

$$\beta = \frac{\pi}{6} + \frac{\delta}{2} - \frac{z}{\sqrt{3}} + O(\delta^2).$$

For instance, using the numerical values of $z$ and taking into account that $\delta \approx \zeta$, as calculated above from the absolute values of the CKM elements, we find that, $\beta = 28.5^\circ \pm 5^\circ$, while $\gamma \approx \pi/2 - \beta + \delta = 76.5^\circ \pm 5^\circ$. These values are within the 1 sigma windows for the measured value for $\beta$, $\beta_{\text{exp}} = 23.3^\circ \pm 1.6^\circ$ [20, 21], and the indirect determination through CKM global fits for $\gamma$, $\gamma_{\text{fit}} \approx 61^\circ \pm 11^\circ$ at 95% C.L..

Some comments must be added regarding the size of the indirect effects of new physics in flavor changing and CP processes. All these processes appear usually in many susy models as a consequence of the presence of flavor and CP violating terms in the soft supersymmetry breaking sector. In many susy flavor models the breaking of flavor symmetries directly generates those terms in the soft susy breaking sector. In this model this is not so because the flavor symmetry is broken "before" supersymmetry is broken. Although the mechanism of susy breaking has not been specified this is not so relevant since, for instance, two different alternative mechanisms could explain the suppression of those terms: A) flavor is broken at the supersymmetric level of the theory at the GUT scale. The supersymmetry is broken at a much lower scale that could go from around
The radiative transmission of flavor violation to the susy breaking sector would be suppressed in this case because it would require the presence in the loops of particles of very disparate scales. If for instance the flavor symmetry were a local gauge symmetry it would require the presence in the loops of flavored Higgses or flavored gauge bosons, whose masses are of the order of the GUT scale, together with susy particles whose masses are much lower, B) alternatively supersymmetry could be broken through a flavor blind mechanism as in for instance gauge mediated models and this problem would not be an issue.

VIII. CONCLUSIONS

We have proposed a supersymmetric grand unified model for the breaking of flavor and CP symmetries. The model fits all the data in the charged lepton sector with precision and makes three successful predictions for fermion mass ratios. We believe that this model has the following features worth emphasizing,

- the model allows us to solve the flavor scalar potential analytically,
- supersymmetry is the key ingredient that makes the model predictive and solvable,
- the presence of two vector flavored Higgses is necessary for the existence of a non-trivial supersymmetric vacuum,
- all the Higgses introduced play a role in the breaking of the flavor symmetry,
- no additional Higgses have to be postulated to generate a mass for the up-quark,
- and, most importantly, in this model CP symmetry is broken in a predictive way.

We should point out that many models for the spontaneous breaking of CP are more descriptive than predictive. Certain parameters must be tuned so as to be able to reproduce the data. On the other hand, the model here proposed contains a simple limit of its parameter space where the measured $\beta$ phase is successfully reproduced. In this regard, a precise measurement of $\gamma$ and $\alpha$, which is the next challenge for B factories, is of paramount importance. It is expected that if the luminosity in the upgraded B factories is high enough, $\gamma$ could be determined with approximately 5% percent uncertainty; a measurement with an uncertainty at the level of 1% will require a superB factory. A determination of $\alpha$ at less than the 5% level may also have to wait for the superB factories.
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