Deep Learning Explicit Differentiable Predictive Control Laws for Buildings

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Abstract: We present a differentiable predictive control (DPC) methodology for learning constrained control laws for unknown nonlinear systems. DPC poses an approximate solution to multiparametric programming problems emerging from explicit nonlinear model predictive control (MPC). Contrary to approximate MPC, DPC does not require supervision by an expert controller. Instead, a system dynamics model is learned from the observed system’s dynamics, and the neural control law is optimized offline by leveraging the differentiable closed-loop system model. The combination of a differentiable closed-loop system and penalty methods for constraint handling of system outputs and inputs allows us to optimize the control law’s parameters directly by backpropagating economic MPC loss through the learned system model. The control performance of the proposed DPC method is demonstrated in simulation using learned model of multi-zone building thermal dynamics.

Keywords: constrained deep learning; neural state space models; system identification; differentiable predictive control; explicit nonlinear MPC

1. INTRODUCTION

Model predictive control (MPC) (Mayne, 2014) is a successful control method that found its way into building control applications including thermal energy storage (Tang and Wang, 2019), thermal comfort (Park and Nagy, 2018) and optimization of energy management systems (Hilliard et al., 2015). MPC with linear predictive models and constraints has been extensively studied and successfully applied over several decades for set-point tracking and economic optimization (Tang and Wang, 2019). However, building models are generally nonlinear and more complex than the Jacobian linearizations (Liu et al., 2018). Expanding research on numerical methods and tools (Andersson et al., 2019) for optimal control problems has paved the way for wider use of nonlinear MPC (NMPC) in building control applications (Jorissen et al., 2018). However, despite the advances in computational tools, the complexity of the online optimization is preventing low-cost deployment and maintenance of MPC in building control applications.

Explicit MPC (Bemporad et al., 2002; Alessio and Bemporad, 2009) aims to overcome the online computational drawbacks by using off-line optimization based on multiparametric programming to obtain an explicit control law, whose online evaluation is reduced to simple function evaluations (Parisio et al., 2014). However, despite its appealing features, current multiparametric solvers (Herczeg et al., 2013) unfortunately do not scale to the complexity required by building control applications. Addressing the scalability issues, approximate MPC relies on learning-based methods to obtain explicit control laws by imitating MPC from observational data (Drgoňa et al., 2018; Gángo et al., 2019). Karg and Lucia (2018) show that deep neural networks with ReLU activations can exactly represent piecewise affine explicit MPC control laws. While Hertneck et al. (2018) synthesize a neural network-based approximate robust MPC controller with statistical guarantees for closed-loop stability and constraint satisfaction.

A promising research direction emerged in recent years, combining constrained optimization with deep learning in so-called differentiable optimization (Agrawal et al., 2019). Amos et al. (2018) learn the controller by differentiating through the KKT conditions of underlying linear MPC problem, while East et al. (2020) propose an infinite-horizon differentiable MPC framework based on discrete-time algebraic Riccati equation. Chen et al. (2019) introduced Gnu-RL, a method that adopts the differentiable MPC (Amos et al., 2018) by combining it with policy gradient algorithm for online learning. However, differentiable MPC methods described thus far require the supervision of a pre-computed expert controller to generate the training data, similarly as in the case of approximate MPC.

In this paper, we focus on differentiable predictive control (DPC) (Drgoňa et al., 2020a), a recently proposed learning-based method for synthesizing explicit control laws for unknown dynamical systems. Even though similar in spirit to differentiable MPC approaches, DPC can provide scalable optimization of explicit control laws directly without the supervision of an expert controller. Previous studies of DPC have been performed considering unknown linear (Drgoňa et al., 2020b), and nonlinear (Drgoňa et al., 2020a) systems in single-input single-output (SISO) set-
ttings. In this paper, we expand the capabilities of DPC
with the following contributions:

(1) Use of novel system identification method for the
unknown nonlinear dynamical systems, combining prin-
ciples of constrained optimization with deep neural
networks and state-space models.
(2) Differentiable parametrization of the closed-loop dy-
namics models composed of neural controllers and the
identified neural state-space models.
(3) Scalable constrained deep learning of explicit control
laws for multi-input multi-output nonlinear systems
with economic objectives and dynamic input and
output constraints enforced during learning using
penalty methods.
(4) Empirical demonstration of the closed-loop control
and scalability of DPC in simulations using a white-
box multi-zone building model as a controlled system.

The presented case study represents an empirical eval-
uation of the DPC method for learning constrained con-
rol policies for multi-input multi-output systems. Dealing
with the plant-model mismatch and lack of theoretical
stability guarantees represent the main limitations of the
proposed study and will be addressed in future work.

2. DIFFERENTIABLE PREDICTIVE CONTROL

In this section, we introduce Differentiable Predictive Con-
trol (DPC), a new deep learning-based constrained control
method inspired by MPC. The conceptual methodology
shown in Fig. 1 consists of two main steps. Assuming
time series dataset obtained from the measurements of
the system dynamics, in the first step, we perform sys-
tem identification using a physics-constrained neural state-
space model (SSM) introduced in section 2.1. In the second
step, we close the loop by combining the neural SSM (1)
with neural control law obtaining a differentiable closed-
loop dynamics model parametrized by neural networks.
This allows us to leverage standard tools in deep learning
to optimize the neural control law parameters by back-
propagating the MPC-inspired loss function through the
learned system dynamics model.

2.1 Neural State Space Models

As a inherent part of the DPC methodology, we consider
the approximation of the controlled system dynamics by
means of neural state space models (SSM) (Masti and
Bemporad, 2018; Drogoña et al., 2020c). In this paper, we
consider a neural SSM architecture given as follows:

\[
\begin{align*}
    x_t &= f_s(y_{t-N}; \ldots; y_t) \quad (1a) \\
    x_{t+1} &= f_s(x_t) + f_r(u_t) + f_d(d_t) \quad (1b) \\
    y_{t+1} &= f_p(x_{t+1}) \quad (1c)
\end{align*}
\]

where \( f_s \) and \( f_p \) represent the state and output dynamics,
respectively. The dynamics of control actions \( u_t \), and
turbulence \( d_t \) is modeled by nonlinear sub-modules \( f_s \),
and \( f_p \). All components, \( f_s, f_y, f_r, \) and \( f_d \) can be either
parametrized by deep neural networks or linear maps.
To handle partially observable systems, we use additional
neural network \( f_o \), representing state observer estimating
the hidden states \( x_t \) from the past time series of the output
measurements. Then combining all equations in (1) gives
us nonlinear autoregressive model mapping measurements
of the system outputs over the past \( N \)-steps \([y_{t-N}; \ldots; y_t]\)
to future time step prediction \( y_{t+1} \).

2.2 Neural Parametrization of the Closed-Loop Dynamics

Constructing a differentiable model of the closed-loop dyna-
mical system represents a core conceptual idea behind the
DPC methodology. In this paper, we use the neural state
space model (1) to represent the open-loop system
dynamics and a fully connected neural network control law
\( \pi_\theta(\xi) : \mathbb{R}^m \to \mathbb{R}^n \) given as:

\[
\begin{align*}
    \pi_\theta(\xi) &= W_L h_L + b_L \quad (2a) \\
    h_t &= \sigma(W_{t-1} h_{t-1} + b_{t-1}) \quad (2b) \\
    h_0 &= \xi \quad (2c)
\end{align*}
\]

where \( \pi_\theta(\xi) \) parametrized by \( \theta = \{ W_1, \ldots, W_L, b_1, \ldots, b_L \} \)
with \( W_l \) and \( b_l \) representing weights and biases for hidden
layers \( l \in \mathbb{N}_L^1 \), respectively. Then \( \sigma : \mathbb{R}^m \to \mathbb{R}^m \) repre-
sents the elementwise application of a univariate activation
function \( \sigma : \mathbb{R} \to \mathbb{R} \) to linearly transformed hidden layer
states \( h_l \) as given in (2b).

In this paper, the control law \( U_f = \pi_\theta(\xi) \) is mapping
the features \( \xi \) to future control actions trajectories
\( U_f = [u_{t-1}; \ldots; u_{t+N}] \) where \( N \) defines the length of the
prediction horizon. The computed control trajectories \( U_f \)
are used to rollout the system dynamics model (1) over
\( N \)-steps ahead into the future. Then we close the loop
by using past output trajectories of the system dynamics
\( y_p = [y_{t-N}; \ldots; y_t] \) as features \( \xi = y_p \) for the the neural
control law \( \pi_\theta(\xi) \). For more expressive control laws, the
features \( \xi \) can contain additional vectors such as distur-
bance forecasts \( D_f \), or previews of dynamic references
and constraints imposed on states, inputs or outputs. For
the sake of brevity, we compactly represent the closed-loop
dynamics as follows:

\[
\begin{align*}
    U_f &= \pi_\theta(\xi) \quad (3a) \\
    Y_f &= f_{SSM}(y_p, U_f, D_f) \quad (3b)
\end{align*}
\]

where \( \pi_\theta \) gives the control law (2) and \( f_{SSM} \) represents
\( N \)-step ahead rollout of the system dynamics model (1).
The parametrization of the closed-loop model (3) by deep
neural networks (DNN) is motivated by the expressivity
and scalability of DNNs.

Remark 1. To decrease the dimensionality of the feature
space, we could leverage the latent space \( x_t \) of the pre-
diction model \( f_{SSM} \) as policy features. Similarly, we could
apply the principle of receding horizon control (RHC) to
the DPC policy \( \pi_\theta \), computing only the first time step of
the control action \( u_t \) instead of a control trajectory \( U_f \).

2.3 Constraints Satisfaction via Penalty Methods

Constraints satisfaction is a premier feature of MPC facil-
itated by system dynamics equations. We argue that in-
corporating principles of MPC in the parametrized closed-
loop model represents a systematic way of handling con-
straints in learning-based control synthesis. In particular,
having explicit parametrizations of the system dynam-
ics (1) and control law (2) allow us to simulate the in-
fluence of the computed control actions on future state
trajectories. In this work, we enforce the time-varying
Fig. 1. Conceptual overview of Differentiable Predictive Control (DPC) methodology. Step 1: physics-constrained system identification. Step 2: Learning neural control law by backpropagation of the MPC loss through the system model.

The second term weighted with $Q_{du}$ penalizes the control rate of change and represents a simple strategy for smoothing the control trajectories. Third and fourth terms weighted with $Q_{y}$ are constraints penalties on controlled outputs, while the last two terms weighted with $Q_{u}$ are penalties on control action constraints. Using economic MPC objective with soft constraints penalties offers a principled way of optimizing the parametrized closed-loop dynamics (3) with complex performance criteria.

2.5 Optimization of Differentiable Predictive Control Laws

In this section, we present DPC as a two-step algorithm (Algorithm 1) for optimizing explicit control laws using parametrized differentiable closed-loop dynamics model (3).

Algorithm 1 Differentiable Predictive Control (DPC)

1. **Inputs:** datasets for modeling, $D_{sysID}$, and control $D_{ctrl}$.
2. **Step 1:** System identification of the model $f_{SSM}$ (1) using the dataset $D_{sysID}$ of observed system dynamics.
3. **Step 2:** Learning explicit control law $\pi_{\theta}$ (2) by optimizing economic MPC loss function (5) subject to closed-loop dynamics model (3) using synthetic dataset $D_{ctrl}$.
4. **Outputs:** Learned system dynamics model $f_{SSM}$ (1) and explicit control law $\pi_{\theta}$ (2).

The first step of the DPC algorithm (1) performs system identification of the neural state-space model (1). In the second step of Algorithm 1, we optimize the parameters $\theta$ of neural control law $\pi_{\theta}$ (2) by minimizing the loss function $L$ (5) subject to closed-loop dynamics model (3) and penalty constraints on states and outputs (4), respectively. As part of the closed-loop model (3), we use the system dynamics model $M$ (1) with parameters obtained from the system identification in step one.

For decoupling the system identification from control optimization, the system model parameters in (1) remain fixed during the optimization of the control law parameters (2) using the backpropagation algorithm. In the forward pass, we sample the features $\xi$ of the control law (2) to generate...
In the second step of DPC algorithm 1, we train the control constrained autoregressive state-space model (1). Then we compute the derivatives of the loss function (5) and backpropagate them through the closed-loop dynamics model (3) to optimize the parameters of the neural control law (2) using stochastic gradient descent updates.

From a control-theoretic perspective, the structure of the unrolled closed-loop system dynamics model (3) in DPC resembles the structure of the constraints in the dense MPC formulation, also called the single shooting approach. Then the optimization of the DPC control policy via automatic differentiation of differentiable closed-loop system model (3) can be interpreted as an offline model-based iterative learning approach. As a result, DPC is capable of synthesizing highly complex constrained control laws by optimizing the simulated closed-loop behavior without solving supervisory MPC as in the case of imitation learning (Hertneck et al., 2018; Zhang et al., 2019).

3. SIMULATION CASE STUDIES

3.1 Emulator of Building Physics

As a ground truth plant model, we use a white-box building thermal model developed using Modelica’s IDEAS library (Baetens et al., 2015) and envelope linearization method (Picard et al., 2015). The building of interest represents a six-zone residential house located in Belgium. The heating ventilation and air conditioning (HVAC) system consists of a central gas-boiler and a single radiator per zone of the building. From the control perspective, the system has six outputs representing zone operative temperatures, seven control actions corresponding to one supply temperature of the central heating unit, and six mass flow rates, one per zone. The building envelope is modeled by 286 unmeasured states, and the dynamics are additively affected by 41 environmental disturbances such as solar irradiation and ambient temperature. For system identification and control, we assume to have the forecast of only the ambient temperature. This model was previously used in the MPC context in Drgoňa et al. (2018). We refer to this reference for further technical details on the emulator’s building physics.

3.2 Datasets and Optimization

For the system identification step of DPC algorithm 1, we use the building physics emulator described in section 3.1 to generate the time series dataset \( D_{\text{sysID}} \) sampled with \( T_s = 15 \) min rate, in the form of tuples of control inputs \( u \), disturbance signals \( d \), and system outputs \( y \) given as:

\[
D_{\text{sysID}} = \{(u_t^i, d_t^i, y_t^i), (u_{t+T_s}^i, d_{t+T_s}^i, y_{t+T_s}^i), \ldots \}, \quad i \in \mathbb{N}_{n},
\]

with \( n \) batches indexed by \( i \) = \( \mathbb{N}_{n} \) of time series trajectories, each \( N \)-steps long. We use this dataset that carries the building operation data to train the parameters of physics-constrained autoregressive state-space model (1).

In the second step of DPC algorithm 1, we train the control law \( \pi_{\theta}(\xi) \) using following synthetically generated training dataset:

\[
D_{\text{ctrl}} = \{(\hat{y}_{t-Ns}^i, \sum_{t}^i u_t^i, \hat{y}_{t}^i, \hat{y}_{t+T_s}^i, \hat{y}_{t+T_s}^i, \hat{y}_{t+NT_s}^i, d_t^i, \ldots) \}
\]

where \( \hat{y} \) are sampled past output trajectories, which represent perturbation of the initial conditions for the closed-loop system dynamics (3). Similarly we sample \( y, \hat{y}, u, \) and \( \xi \) representing time-varying lower and upper bounds on controlled outputs and control actions, respectively. For measured disturbances \( d \) we assume to have access to their \( N \)-step ahead forecast. We select a subset of trajectories from dataset \( D_{\text{ctrl}} \) to act as features \( \xi \) for the neural control law (2) with their compact notation \( \xi_p = \{\hat{y}_{t-Ns}, \hat{y}_t, \hat{y}_{t+T_s}, \hat{y}_{t+NT_s}, d_t\} \), \( D_f = [d_t^1, \ldots, d_{t+N}^1] \). Both datasets \( D_{\text{sysID}} \) and \( D_{\text{ctrl}} \) contain 8640 time samples corresponding to 90 days of the building operation data. We split each to training, development, and test sets of equal length with 2858 time samples.

We implement the DPC algorithm 1 using Pytorch library (Paszke et al., 2019). In the first step, we train the system model (1) with 25266 parameters. Subsequently, we construct the closed-loop dynamics model (3) and train the neural control law (2) over the prediction horizon of \( N = 32 \) steps by optimizing the loss function (5) using the Adam optimizer (Kingma and Ba, 2014), with 1000 gradient descent updates, randomly initialized weights, and learning rate of 0.001. The resulting control law \( \pi_{\theta}(\xi) : \mathbb{R}^{141} \rightarrow \mathbb{R}^{224} \) with 4-layers with 100 hidden units and GELU activation functions maps 416 features \( \xi = \{\hat{y}_p, \sum_{t}^i u_t; D_f\} \) to 224 control action points \( U_f \) what corresponds to 188624 parameters in total. Optimizing this control law using DPC algorithm 1 and the above-specified setup took under 100 seconds on a desktop machine with 64-bit 2.60 GHz Intel(R) i7-8850H CPU and 16 GB RAM.

3.3 Physics-constrained System Identification

To improve the accuracy and generalization of the proposed neural state space model (1) we constrain the model based on known building physics. The white-box building models typically use thermal resistance-capacitance (RC) networks and convective heat flow equations as summarized in Drgoňa et al. (2020c). These type of building models can be accurately approximated by Hammerstein architecture, with state \( f_c x_c \) = \( Ax_c \), and output \( f_d x_d \) = \( Cx_d \) dynamics represented by linear maps. Then the system dynamics matrix \( A \) approximates the RC network describing the thermal dynamics of the building envelope structure, \( f_u \) is the nonlinear HVAC dynamics approximating the convective heat flow equation, and \( f_d \) is the nonlinear effect generated by the ambient conditions. The system outputs \( y_r \) represent zone operative temperatures, the hidden states \( x_r \) represent lumped temperatures of the building envelope, control actions \( u \) denote the supply temperature and mass flows for each zone, while disturbance signal \( d \) represents the ambient temperature. This interpretation allows us to enforce physically realistic bounds on the dynamic behavior of individual components. For instance, we know that building envelopes are dissipative systems. Thus we impose constraints on the eigenvalues of \( A \) to learn a strictly stable system (Tuor et al., 2020). Additionally, we can use the penalty method to constrain the input dynamics influence to remain within a realistic range.
3.4 Closed-Loop Control

We demonstrate the control performance of the explicit control laws obtained by means of DPC algorithm 1. Fig. 2 plots the closed-loop control trajectories evaluated on the learned state-space model (1), hence considering no plant-model mismatch. The left column represents controlled outputs (zone temperatures) that need to stay within time-varying bounds. The right column shows constrained control actions (mass flows) for each output computed by explicit DPC control laws. These trajectories demonstrate the capability to synthesize control laws with near-optimal performance in terms of economic objectives and constraints satisfaction using DPC algorithm 1. However, when deployed to control the emulator, the DPC policy learned on the nominal model was not able to compensate for the plant model mismatch. Thus these initial results indicate that due to close-to-optimal performance on the nominal model (Fig. 2), the field performance of the DPC boils down to a problem of accurately identifying the system dynamics with coupling.

4. LIMITATIONS AND FUTURE WORK

The lack of theoretical stability guarantees and the ability to deal with the plant model are the main limitations of the DPC methodology presented in this study. In future work, we plan to expand the DPC methodology for the systematic handling of plant-model mismatch. To introduce real-time feedback capabilities into DPC, we aim to incorporate the principles of robust and off-set free MPC, as well as adaptive control updates. The authors are also working on theoretical closed-loop stability guarantees for DPC. Our future empirical work will include a systematic comparison with classical MPC methods on experiments including a wide range of nonlinear systems.

5. CONCLUSIONS

This paper applied a recently proposed differentiable predictive control (DPC) methodology to a multi-zone building control problem. We have expanded the prior capabilities of DPC in learning complex explicit control laws for unknown nonlinear dynamical systems. In particular, we have shown that DPC can systematically handle economic objectives subject to dynamic constraints imposed on nonlinear system dynamics with multiple inputs and outputs. The DPC closed-loop control capabilities are empirically demonstrated in a simulation case study using a multi-zone building thermal dynamics model. We have shown that it is possible to learn explicit control laws for constrained nonlinear optimal control problems with a large number of states and long prediction horizons. By doing so, we tackle the complexity limitations of classical explicit MPC based on multiparametric programming. Simultaneously, DPC overcomes the main limitation of imitation learning-based approaches such as approximate MPC by alleviating the need for supervision from the model predictive controller (MPC). Based on the presented features, we believe that the proposed DPC methodology has long-term potential, not only for research but also for practical applications, such as building control that requires fast development and low-cost deployment and maintenance on hardware with limited computational resources. However, several practical challenges, such as dealing with the plant model mismatch, need to be addressed in future work.

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