Time-dependent proportional limit stress of carbon fiber-reinforced silicon carbide ceramic-matrix composites considering interface oxidation

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This paper reports on an investigation of time-dependent proportional limit stress of carbon fiber-reinforced silicon carbide ceramic-matrix composites (C/SiC CMCs). The temperature-dependent material parameters are considered to determine the proportional limit stress. The effects of the fiber volume fraction, interface properties and matrix fracture energy on the time-dependent proportional limit stress and interface debonding of C/SiC composite are discussed. The temperature-dependent proportional limit stress increases with the fiber volume fraction, interface shear stress, interface debonded energy and matrix fracture energy. The fiber/matrix interface debonded length increases as the oxidation time increases, leading to a decrease in the proportional limit stress. The experimental time-dependent proportional limit stresses of C/SiC composites corresponding to different oxidation times are predicted.

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Key-words : Ceramic-matrix composites (CMCs), Proportional limit stress, Matrix cracking, Interface debonding, Interface oxidation

1. Introduction

Oil prices have risen recently creating a need for fuel-efficient aero-engines. New materials with low density and the ability to withstand high temperatures will contribute to improving fuel-efficiency. Ceramics have such properties, but their brittleness limits their fields of application. Ceramic matrix composites (CMCs) with a fiber-reinforced brittle matrix can solve this problem, and they are being developed as next-generation materials for aero engines.1)–4) The NASA Environmentally Responsible Aviation project is evaluating the ability of CMCs to decrease noise, emissions and fuel burn, including a 42 dB reduction in noise, a 75% reduction in landing and take-off NOx emissions (below CAEP 6), a 70% reduction in cruise NOx emissions, and a 50% decrease in aircraft fuel burn/energy consumption.5) When a CMC is subjected to stress, however, many microscopic failure mechanisms are generated inside the composite, i.e., matrix cracking, fiber/matrix interface debonding and fiber fracture, leading to quasi-ductile behavior in tensile stress-strain curves.6)–8) Notably, in CMCs the onset of nonlinearity, i.e., the proportional limit, does not represent the yield point and onset of work hardening as it does in metals.9) Instead, the proportional limit in CMCs is often associated with the macroscopic manifestation of first matrix cracking. Proportional limit stress (PLS) is a more important property than fracture strength in structural component designed.10) The safety design factor is obtained by comparing the PLS with the applied stress state (σa), and the value of safe design should be greater than one, i.e., PLS/σa > 1.

Many researchers have performed experimental and theoretical investigations of matrix cracking in fiber-reinforced CMCs. The energy balance approach can be used to determine the steady-state matrix cracking stress, including the ACK model,11) AK model,12) BHE model,13) Kuo-Chou model,14) Sutcu-Hilling model,15) Chiang model,16) and Li model,17) and the stress intensity factor method is adopted to determine the short matrix cracking stress, including the MCE model,18) MC model,19) McCartney model,20) Chiang-Wang-Chou model,21) Danchaivijit-Shetty model22) and Thouless-Evans model.23) Kim and Pagano,24) and Dutton et al.25) conducted the first investigation of matrix cracking in CMCs using the acoustic emission, an optical microscope and a scanning electronic microscope. It was found that the experimental first matrix cracking stress is much lower than the theoretical results predicted by ACK model.11) The micro matrix cracking appears first in the matrix rich region, and with increasing applied stress, micro matrix cracks propagate until stopping at the fiber/matrix interface. In fact, these micro matrix cracks do not affect the macro strain and stiffness of CMCs;26) at higher applied stress, however, these cracks
evolve first into the short matrix cracking defined by MCE model,\(^{18}\) and then into the steady-state matrix cracking defined by ACK model.\(^{11}\) The steady-state matrix cracking model can be used to predict the PLS. In the studies mentioned above, however, the time-dependent proportional limit stress of fiber-reinforced CMCs has not been investigated.

In this paper, the time-dependent proportional limit stress of C/\textit{SiC} composites is investigated in consideration of fiber/matrix interface debonding and oxidation. The temperature-dependent fiber and matrix elastic modulus, fiber/matrix interface shear stress, fiber and matrix thermal expansion coefficient, fiber/matrix interface debonded energy and matrix fracture energy are considered to determine the time-dependent micro stress field, the time-dependent fiber/matrix interface oxidation and debonded length and the proportional limit stress. The effects of the fiber volume fraction, fiber/matrix interface shear stress, fiber/matrix interface debonded energy and matrix fracture energy on the time-dependent proportional limit stress and fiber/matrix interface debonded length of C/\textit{SiC} composites are discussed. The experimental time-dependent proportional limit stress of C/\textit{SiC} composites corresponding to different oxidation times are predicted using the present analysis.

2. Theoretical analysis

Due to a mismatch between the axial thermal expansion coefficients of the carbon fiber and silicon carbide matrix, unavoidable micro-cracks were created in the matrix when the composite was cooled down from a high fabricated temperature to ambient temperature. These processing-induced micro-cracks, which were present on the surface of the material, served as avenues for the ingress of environmental atmosphere into the composite at elevated temperature, as shown in Fig. 1. The time-dependent fiber/matrix interface oxidation length of \(\xi\) can be determined using the following equation:\(^{27}\)

\[
\zeta(t, T) = \varphi_1(T) \left\{ 1 - \exp \left[ -\frac{\varphi_2(T) t}{b} \right] \right\} \tag{1}
\]

where \(b\) is the delay factor considering the deceleration of reduced oxygen activity; \(t\) denotes the oxidation time; and \(\varphi_1(T)\) and \(\varphi_2(T)\) are temperature-dependent parameters described using Arrhenius law:\(^{27}\)

\[
\varphi_1(T) = 7.021 \times 10^{-3} \times \exp \left( \frac{8231}{T} \right) \tag{2}
\]

\[
\varphi_2(T) = 227.1 \times \exp \left( -\frac{17090}{T} \right) \tag{3}
\]

The temperature-dependent fiber/matrix interface shear stress in the oxidation region and debonded region can be determined using the following equations:\(^{28}\)

\[
\tau_\text{f}(T) = \tau_0 + \mu \frac{[\alpha_{\text{rf}}(T) - \alpha_{\text{rm}}(T)](T_m - T)}{A} \tag{4}
\]

\[
\tau_\text{m}(T) = \tau_s + \mu \frac{[\alpha_{\text{rf}}(T) - \alpha_{\text{rm}}(T)](T_m - T)}{A} \tag{5}
\]

where \(\tau_0\) and \(\tau_s\) denote the initial interface shear stress; \(\mu\) denotes the interface frictional coefficient; \(\alpha_{\text{rf}}\) and \(\alpha_{\text{rm}}\) denote the fiber and matrix radial thermal expansion coefficient, respectively; \(T_m\) and \(T\) denote the processing temperature and testing temperature; and \(A\) is a constant dependent on the elastic properties of the matrix and fibers.

2.1 Time-dependent stress analysis

The composite with fiber volume fraction \(V_f\) is loaded by a remote uniform stress \(\sigma\) normal to a long crack plane, as shown in Fig. 1. The unit cell in the downstream region I is extracted from the ceramic composite system containing a single fiber surrounded by a hollow matrix cylinder, as shown in Fig. 2. The fiber radius is \(r_f\), and the matrix radius is \(R\) \((R = r_f/V_f^{1/2})\). The length of the unit cell is half matrix crack spacing \(l_d/2\), and the interface oxidation length and interface debonded length are \(\zeta\) and \(l_d\).
respectively. The fiber and matrix axial stress distributions in the fiber/matrix interface debonded areas of region I can be determined using the following equations:

$$\sigma_f(z) = \begin{cases} \sigma & \text{for } z \in [0, \xi(t, T)] \\ \sigma - \frac{2\tau(T)}{r_f}z & \text{for } z \in [\xi(t, T), l_d(t, T)] \end{cases}$$

(6)

$$\sigma_m(z) = \begin{cases} 2\frac{V_f}{V_m} \frac{\tau(T)}{r_f}z & \text{for } z \in [0, \xi(t, T)] \\ 2\frac{V_f}{V_m} \frac{\tau(t, T)}{r_f}z + 2\frac{V_f}{V_m} \frac{\tau(T)}{r_f} & \text{for } z \in [\xi(t, T), l_d(t, T)] \end{cases}$$

(7)

The fiber and matrix axial stresses in the interface bonded region ($l_d < z$) can be determined using the following equations:

$$\sigma_f^b(z, T) = \frac{E_f(T)}{E_c(T)} \sigma$$

(8)

$$\sigma_m^b(z, T) = \frac{E_m(T)}{E_c(T)} \sigma$$

(9)

As shown in Fig. 1, the upstream region III is so far away from the crack tip that the stress and strain fields are also uniform. The fiber and matrix have the same displacements and the fiber and matrix stresses can be determined using the following equations:

$$\sigma_f^b(z, T) = \frac{E_f(T)}{E_c(T)} \sigma$$

(10)

$$\sigma_m^b(z, T) = \frac{E_m(T)}{E_c(T)} \sigma$$

(11)

2.2 Time-dependent fiber/matrix interface debonding

The fracture mechanics approach is adopted in the present analysis to determine the fiber/matrix interface debonded length.\(^\text{29}\)

$$\gamma_d(T) = -\frac{F}{4\pi r_f} \frac{\partial \omega_f(z = 0)}{\partial l_d} - \frac{1}{2} \int_0^{\omega_d(T)} \frac{\partial \omega(z)}{\partial l_d} \, dz$$

(12)

where $\gamma_d(T)$ denotes the temperature-dependent fiber/matrix interface debonded energy; $F(= \pi r_f^2 \sigma / V_f)$ denotes the fiber load at the matrix cracking plane; $\omega_f(z = 0)$ denotes the fiber axial displacement at the matrix cracking plane; and $\omega(z)$ denotes the relative displacement between the fiber and the matrix. Based on the fiber and matrix axial stress distributions in Eqs. (4) and (5), the axial displacements of the fiber and matrix of $w_f(z)$ and $w_m(z)$ can be determined using the following equations:

$$w_f(z, t, T) = \int_z^{\xi} \frac{\sigma_f(z, t, T)}{E_f(T)} \, dz = \int_{l_d}^{l_d} \frac{\sigma}{E_f(T)} \, dz$$

$$w_m(z, t, T) = \int_z^{\xi} \frac{\sigma_m(z, t, T)}{E_m(T)} \, dz = \int_{l_d}^{l_d} \frac{\sigma}{E_m(T)} \, dz$$

(13)

(14)

The relative displacement between the fiber and the matrix of $v(z)$ can be determined using the following equation:

$$v(z) = |w_f(z) - w_m(z)|$$

$$v(z) = \frac{\sigma}{V_f E_f(T)} [l_d(t, T) - z]$$

(15)

Substituting $w_f(z = 0)$ and $v(z)$ into Eq. (12) leads to formation of the following equation:

$$\frac{E_f(T) \tau_f^2(T)}{r_f V_m E_c(T) E_m(T)} [l_d(t, T) - \xi(t, T)]^2$$

$$\frac{V_f E_f(T)}{r_f V_c E(T) E_m(T)} [l_d(t, T) - \xi(t, T)]^2$$

$$\frac{2 E_c(T) \tau_c^2(T)}{r_f V_m E(T) E_m(T)} [l_d(t, T) - \xi(t, T)] \xi(t, T)$$

$$\frac{V_f E_f(T)}{r_f V_m E(T) E_m(T)} \xi^2(t, T) + \frac{E_c(T) \tau_c^2(T)}{4 V_c^2}$$

(16)
Solving Eq. (16), the time-dependent fiber/matrix interface debonded length of $l_d$ can be determined using the following equation:

$$l_d(t, T) = \left[ 1 - \frac{\tau(T)}{\tau(T)} \right] \zeta(t, T) + \frac{\tau(T)}{\tau(T)} \frac{V_m E_m(T) \sigma}{2 V_f E_f(T) \tau(T)} - \sqrt{\frac{\tau(T)}{\tau(T)} \frac{V_m E_m(T) E_f(T)}{E_m(T) \tau(T)}} Y_d(T)$$ (17)

### 2.3 Time-dependent proportional limit stress

The energy balance relationship for evaluating the proportional limit stress of fiber-reinforced CMCs can be determined using the following equation:31)

$$\frac{1}{2} \int_{-\infty}^{\infty} \left\{ \frac{V_f}{E_f(T)} (\sigma_m^{\text{II}}(T) - \sigma_m^{\text{I}}(T))^2 + \frac{V_m}{E_m(T)} (\sigma_m^{\text{III}}(T) - \sigma_m^{\text{II}}(T))^2 \right\} dx$$

$$+ \frac{1}{2\pi R} G_m(T) \int_{-\infty}^{R} \int_{\eta}^{R} \left[ \tau(r, T) \right] 2\pi r dr dx = V_m Y_m(T) + \frac{4V_d l_d(T)}{r_f} Y_d(T)$$ (18)

where $Y_m(T)$ denotes the temperature-dependent matrix fracture energy.

Substituting the upstream and downstream temperature-dependent fiber and matrix axial stresses of Eqs. (6)–(11) and the temperature-dependent fiber/matrix interface debonded length of Eq. (17) into Eq. (18), the energy balance equation leads to the following equation:

$$\alpha \sigma^2 + \beta \sigma + \delta = 0$$ (19)

where

$$\alpha = \frac{V_m E_m(T) l_d(t, T)}{V_f E_f(T) E_f(T)}$$ (20)

$$\beta = -\frac{2 \tau(T)}{r_f E_f(T)} \left\{ l_d(t, T) - \zeta(t, T) \right\}$$

$$+ \frac{\tau(T)}{\tau(T)} \cdot \frac{4V_d l_d(T)}{r_f} \left\{ l_d(t, T) - \zeta(t, T) \right\}$$ (21)

$$\delta = \frac{4}{3} \frac{V_i E_i(T)}{V_m E_m(T)} \left\{ \frac{\tau(T)}{\tau(T)} \right\}^2$$

$$\times \left\{ l_d(t, T) - \zeta(t, T) \right\}^3 + \frac{\tau(T)}{\tau(T)} \left\{ \frac{\tau(T)}{\tau(T)} \right\}^2 \zeta(t, T)$$

$$+ \frac{\tau(T)}{\tau(T)} \frac{4V_d l_d(T)}{r_f} \left\{ l_d(t, T) - \zeta(t, T) \right\}$$

$$\times \left\{ l_d(t, T) - \left\{ 1 - \frac{\tau(T)}{\tau(T)} \right\} \zeta(t, T) - V_m Y_m(T) \right\} = \frac{4V_d l_d(t, T)}{r_f} Y_d(T)$$ (22)

### 3. Results and discussion

The ceramic composite system of C/SiC is used as a case study and its material properties are given by $V_i = 30\%$, $r_i = 3.5 \mu$m, $\gamma_m = 25$ J/m² (at room temperature) and $\gamma_d = 0.13$ J/m² (at room temperature).

The temperature-dependent carbon fiber elastic modulus of $E_f(T)$ can be described using the following equation:30)

$$E_f(T) = 230 \left[ 1 - 2.86 \times 10^{-4} \exp \left( \frac{T}{324} \right) \right]$$

$$T < 2273 \text{ K}$$ (23)

The temperature-dependent SiC matrix elastic modulus of $E_m(T)$ can be described using the following equation:31)

$$E_m(T) = \frac{350}{530} \left[ 460 - 0.04T \exp \left( \frac{-962}{T} \right) \right]$$

$$T \in [300 \text{ K} \text{ 1773 K}]$$ (24)

The temperature-dependent carbon fiber axial and radial thermal expansion coefficient of $\alpha_{f(0)}(T)$ and $\alpha_{d(0)}(T)$ can be described using the following equations:32)

$$\alpha_{f(0)}(T) = 2.529 \times 10^{-3} - 1.569 \times 10^{-4}T$$

$$+ 2.228 \times 10^{-7}T^2 - 1.877 \times 10^{-11}T^3$$

$$- 1.288 \times 10^{-14}T^4, \quad T \in [300 \text{ K} \text{ 2500 K}]$$ (25)

$$\alpha_{f(0)}(T) = -1.86 \times 10^{-1} + 5.85 \times 10^{-4}T$$

$$- 1.36 \times 10^{-8}T^2 + 1.06 \times 10^{-22}T^3, \quad T \in [300 \text{ K} \text{ 2500 K}]$$ (26)

The temperature-dependent SiC matrix axial and radial thermal expansion coefficient of $\alpha_{m(0)}(T)$ and $\alpha_{m(0)}(T)$ can be described using the following equations:33)

$$\alpha_{m(0)}(T) = \alpha_{m(0)}(T)$$

$$= \left\{ \begin{array}{ll}
-1.8276 + 0.0178T - 1.5544 \times 10^{-5}T^2 & 5.0 \times 10^{-6}/K, \; T > 1273 \text{ K}
+ 4.5246 \times 10^{-9}T^3, \quad T \in [125 \text{ K} \text{ 1273 K}]
\end{array} \right. $$ (27)

The temperature-dependent fiber/matrix interface debonded energy of $\gamma_d(T)$ and the matrix fracture energy of $\gamma_m(T)$ can be described using the following equations:33)

$$\gamma_d(T) = \gamma_d \left[ 1 - \int_{T_0}^{T} \frac{C_p(T) dT}{\int_{T_0}^{T} C_p(T) dT} \right]$$ (28)

$$\gamma_m(T) = \gamma_m \left[ 1 - \int_{T_0}^{T} \frac{C_p(T) dT}{\int_{T_0}^{T} C_p(T) dT} \right]$$ (29)

where $T_0$ denotes the reference temperature; $T_m$ denotes the fabricated temperature; $\gamma_d$ and $\gamma_m$ denote the interface debonded energy and matrix fracture energy at the reference temperature $T_0$. $C_p(T)$ can be described using the following equation:
\[ C_p(T) = 76.337 + 109.039 \times 10^{-3}T - 6.535 \times 10^5T^{-2} - 27.083 \times 10^{-6}T^2 \] (30)

When the oxidation temperature is below 1000°C, the oxidation of C/SiC composite is controlled by the carbon/oxygen reaction, as shown in Eq. (31); and when the temperature is higher than 1000°C, the oxidation of C/SiC composite is controlled by the SiC/oxygen reaction, as shown in Eq. (32). \(^34\), \(^35\) In the present analysis, an oxidation temperature below 1000°C is considered, but the effects of the physical properties of SiO\(_2\) on the fiber/matrix interface shear stress and energy and matrix fracture energy have not been considered.

\[
\begin{align*}
C + O_2 &\rightarrow 2CO \\
\text{SiC} + 2O_2 &\rightarrow \text{SiO}_2 + CO_2
\end{align*}
\] (31) (32)

The effects of the fiber volume fraction, fiber/matrix interface shear stress, fiber/matrix interface debonded energy and matrix fracture energy on the temperature/time-dependent proportional limit stress and fiber/matrix interface debonding are discussed below.

3.1 Effect of fiber volume fraction on proportional limit stress and interface debonded length

The time-dependent proportional limit stress (\(\sigma_{\text{PLS}}\)) and fiber/matrix interface debonded length normalized by the fiber radius (\(l_d/r_f\)) of C/SiC composite are shown as functions of temperature for \(V_f = 25\%\) and \(30\%\) with oxidation times of 0, 1 and 2 h, respectively (Fig. 3). The increase in \(\sigma_{\text{PLS}}\) with increases in \(V_f\) shown in Fig. 3 is attributed to an increase in the stress transfer capacity between the fiber and matrix and the stress carried by the matrix. The decrease in \(l_d/r_f\) with increases in \(V_f\) shown in Fig. 3 is owing to the decrease in the fiber bridging stress at the matrix cracking plane. \(\sigma_{\text{PLS}}\) increases with increases in temperature up to a certain temperature of around 1200–1250°C and decreases slightly above the temperature range for the oxidized C/SiC composites although \(\sigma_{\text{PLS}}\) increases monotonically with increases in temperature for the composite without oxidation. The value of \(l_d/r_f\) increases monotonically with increases in temperature regardless of oxidation. \(\sigma_{\text{PLS}}\) decreases with increases in the oxidation time due to the increase in \(l_d/r_f\).

3.2 Effect of fiber/matrix interface shear stress on proportional limit stress and interface debonded length

The temperature-dependent proportional limit stress (\(\sigma_{\text{PLS}}\)) and fiber/matrix interface debonded length, normalized by the fiber radius (\(l_d/r_f\)) of C/SiC composites are shown as functions of temperature for \(\tau_0 = 30\) and 35 MPa with oxidation times of 0, 1 and 2 h, respectively (Fig. 4). The increase in \(\sigma_{\text{PLS}}\) with increases in \(\tau_0\) shown in Fig. 4
is attributed to increases in the stress transfer capacity between the fiber and matrix and the stress carried by the matrix. The decrease of $l_d/r_f$ with increases in $\tau_0$ shown in Fig. 4 is owing to the higher energy dissipated during the propagation of fiber/matrix interface debonding. $\sigma_{PLS}$ increases with increases in temperature up to a certain temperature of around 1100–1200°C and decreases slightly at above the temperature range for the oxidized C/SiC composite, although $\sigma_{PLS}$ increases monotonically with increases in temperature for the composite without oxidation. The value of $l_d/r_f$ increases monotonically with increases in temperature regardless of oxidation. $\sigma_{PLS}$ decreases with increases in the oxidation time due to the increase in $l_d/r_f$.

3.3 Effect of fiber/matrix interface debonded energy on proportional limit stress and interface debonded length

The temperature-dependent proportional limit stress ($\sigma_{PLS}$) and fiber/matrix interface debonded length normalized by the fiber radius ($l_d/r_f$) of C/SiC composite are shown as functions of temperature for $\gamma_0 = 0.3$ and 0.5 J/m² with oxidation times of 0, 1 and 2 h, respectively (Fig. 5). The increase in $\sigma_{PLS}$ with increases in $\gamma_0$ shown in Fig. 5 is attributed to the increase in the stress carried by the matrix. The decrease in $l_d/r_f$ with increases in $\gamma_0$ shown in Fig. 5 is owing to the more energy dissipated during the process of fiber/matrix interface debonding. $\sigma_{PLS}$ increases with increases in temperature up to a certain level of around 1100–1250°C and decreases slightly at above the temperature range for the oxidized C/SiC composite, although $\sigma_{PLS}$ increases monotonically with increases in temperature for the composite without oxidation. The value of $l_d/r_f$ increases monotonically with increases in temperature regardless of oxidation. $\sigma_{PLS}$ decreases with increases in the oxidation time due to the increase in $l_d/r_f$.

3.4 Effect of matrix fracture energy on proportional limit stress and interface debonded length

The temperature-dependent proportional limit stress ($\sigma_{PLS}$) and fiber/matrix interface debonded length normalized by the fiber radius ($l_d/r_f$) of C/SiC composite are shown as functions of temperature for $\gamma_m = 20$ and 25 J/m² with oxidation times of 0, 1 and 2 h, respectively (Fig. 6). The increase of $\sigma_{PLS}$ with an increase in $\gamma_m$ shown in Fig. 6 is attributed to the increase in the energy dissipated during matrix cracking. The increase in $l_d/r_f$ with increases in $\gamma_m$ shown in Fig. 6 is owing to the high proportional limit stress. $\sigma_{PLS}$ increases with increases in temperature up to a certain temperature of around 1150–1250°C and decreases slightly at above the temperature range for the oxidized C/SiC composite, although $\sigma_{PLS}$ increases monotonically with increases in temperature for
Fig. 5. (a) The proportional limit stress versus temperature curves when $\gamma_d = 0.3 \text{ J/m}^2$ and $t = 0, 1$ and 2 h; (b) the interface debonded length versus temperature curves when $\gamma_d = 0.3 \text{ J/m}^2$ and $t = 0, 1$ and 2 h; (c) the proportional limit stress versus temperature curves when $\gamma_d = 0.5 \text{ J/m}^2$ and $t = 0, 1$ and 2 h; and (d) the interface debonded length versus temperature curves when $\gamma_d = 0.5 \text{ J/m}^2$ and $t = 0, 1$ and 2 h.

Fig. 6. (a) The proportional limit stress versus temperature curves when $\gamma_m = 20 \text{ J/m}^2$ and $t = 0, 1$ and 2 h; (b) the interface debonded length versus temperature curves when $\gamma_m = 20 \text{ J/m}^2$ and $t = 0, 1$ and 2 h; (c) the proportional limit stress versus temperature curves when $\gamma_m = 25 \text{ J/m}^2$ and $t = 0, 1$ and 2 h; and (d) the interface debonded length versus temperature curves when $\gamma_m = 25 \text{ J/m}^2$ and $t = 0, 1$ and 2 h.
the composite without oxidation. The value of \( \frac{l_d}{r_f} \) increases monotonically with increases in temperature regardless of oxidation. \( \sigma_{PLS} \) decreases with increases in the oxidation time due to the increase in \( \frac{l_d}{r_f} \).

3.5 Experimental comparisons

Yang et al.\(^{36}\) investigated the tensile behavior of 2D C/SiC composite at elevated temperatures. The T300-C/SiC composite was fabricated using the chemical vapor infiltration method with a pyrolytic carbon interphase of 1.5–2.0 µm. The fiber volume was 40%, and the composite density was about 2.0 g/cm³. The tensile tests were performed under displacement control at a loading speed of 0.3 mm/min. The tensile stress–strain curves of the 2D C/SiC composite at \( T = 973 \) and 1273 K are shown in Fig. 7. There are five methods of evaluating the proportional limit stress of 2D C/SiC composite, i.e., the 0.05% strain offset method, “eyeball” method, deviation from linearity method (stress-based), deviation from linearity method (elastic modulus-based) and 0.05% extension under load method. In this study, the 0.05% strain offset method was used to determine the proportional limit stress of 2D C/SiC composite. At \( T = 973 \) K, the proportional limit stress was \( \sigma_{PLS} = 50 \) MPa and the tensile strength was \( \sigma_{uts} = 232 \) MPa with a failure strain of 0.25%. And at \( T = 1273 \) K, the proportional limit stress was \( \sigma_{PLS} = 80 \) MPa and the tensile strength was \( \sigma_{uts} = 271 \) MPa with a failure strain of 0.33%. The experimental and theoretical time-dependent proportional limit stress \( \sigma_{PLS} \) and fiber/matrix interface debonded length normalized by the fiber radius \( \frac{l_d}{r_f} \) of 2D C/SiC composite are shown as functions of temperature with oxidation times of 0, 1 and 2 h, respectively (Fig. 8). \( \sigma_{PLS} \) increased with increases in temperature up to a certain temperature of around 1150–1250°C and decreased slightly at above the temperature range for oxidized C/SiC composites although \( \sigma_{PLS} \) increased monotonically with increases in the temperature of the composite without oxidation. The value of \( \frac{l_d}{r_f} \) increased monotonically with increases in temperature regardless of oxidation. \( \sigma_{PLS} \) decreased with increases in the oxidation time due to the increase in \( \frac{l_d}{r_f} \).

4. Conclusions

In this study, the time-dependent proportional limit stress of C/SiC composite was investigated with a focus on fiber/matrix interface debonding and oxidation. The
temperature-dependent fiber and matrix elastic modulus, fiber/matrix interface shear stress, fiber and matrix thermal expansion coefficient, fiber/matrix interface debonded energy and matrix fracture energy were considered to determine the time-dependent micro stress field, the time-dependent fiber/matrix interface oxidation and debonded length and the proportional limit stress. The effects of the fiber volume fraction, fiber/matrix interface shear stress, fiber/matrix interface debonded energy and matrix fracture energy on the time-dependent proportional limit stress and fiber/matrix interface debonded length of C/SiC composite were discussed. The experimental time-dependent proportional limit stress of C/SiC composite corresponding to different oxidation times were predicted.

1) When the fiber volume fraction, fiber/matrix interface shear stress and fiber/matrix interface debonded energy increased, the proportional limit stress increased, and the fiber/matrix interface debonded length decreased.

2) When the matrix fracture energy increased, the proportional limit stress and the fiber/matrix interface debonded length increased.

3) When the oxidation time increased, the fiber/matrix interface debonded length increased, due to the decrease in the fiber/matrix interface shear stress in the oxidation region, leading to a decrease in the proportional limit stress corresponding to different temperatures.

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