Finite temperature $0^{-+}$ glueball spectrum from non-susy D3 brane of Type IIB string theory

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Here, we calculate the pseudo-scalar glueball mass at finite temperature from the holographic QCD in 3 + 1 dimensions. The decoupled geometry of the non-supersymmetric (non-susy) D3 brane at finite temperature, which is the solution of type IIB supergravity, is considered as the dual theory of QCD. We calculate the mass spectrum from the axion fluctuation in this gravity background using WKB approximations. Approximating the WKB equation for various orders of mass, we derive the analytical expressions of the mass spectrum of $0^{-+}$ at finite temperature. Finally, we evaluate the masses of the ground state and the first excited state of the pseudo-scalar glueball numerically, as, $m_{-+} = 2.5\text{GeV}$ and $m_{-+} = 3.8\text{GeV}$ respectively, from the complete WKB equation. The mass of a given state is found to decrease with increasing temperature and becomes zero at confinement-deconfinement transition temperature which is consistent with the idea of confinement and also matches with some recent lattice results. From this temperature variation, the QCD transition point is found to be about 186MeV.

Keywords: Non-susy brane, String/QCD duality, Finite temperature glueball mass, Pseudo-scalar glueball, QCD phase transition

The Relativistic Heavy Ion Collider (RHIC) has already confirmed the plasma state of nucleons. At extremely high energy, the nucleons completely dissolve and produce the plasma state of free quarks and gluons without interactions. As energy decreases we move from the deconfined (plasma) state to confined (bound state of quarks and gluons) state and at sufficiently low energy, we get back the nuclear matters or the hadronic states without any free quarks and gluons. Now when the energy decreases from the plasma state the interactions among the constituents dominates and the interacting gluons form glueballs. Thus we get a mixed phase of free gluons and glueballs. As the energy decreases further the glueball mass increases gradually as a result of increasing coupling. In this transition process, we have a particular range of temperature $175 \pm 15\text{MeV}$, below which we have the glueball dominated phase or the confined phase and above which we have the free gluon dominated phase or the deconfined phase. So, the glueball spectrum is essential to study various important properties of the confined QCD. Till date, no high energy experiment has confirmed the existence of glueball, so there is a lack of phenomenological evidence. However, the MIT bag model, the Skyrme model, chiral perturbation theory, heavy baryon perturbation theory have been modeled to describe the confined phase of QCD; the glueball is not fully explored in that respect. The MIT bag model is a theoretical model of the glueball for various quantum numbers (spin, parity etc). According to this model, depending on the number and the properties of the constituent single-gluon states, we can have glueballs of different spin and parity $J^{PC}$, where $J$ denotes spin of the glueball and $P$ and $C$ denote the spatial parity and the charge conjugation parity respectively. For example, $0^{++}$, $2^{++}$, $0^{-+}$, $2^{-+}$ are made of two gluon modes, whereas, $0^{++}$, $1^{-+}$, $3^{-+}$ consist of three gluon modes. The pseudo-scalar glueball has spin $J = 0$, parity $P = -1$ and $C = +1$ since it is chargeless. $0^{-+}$ consists of a transverse electric and a transverse magnetic gluon modes.

In recent times, the lattice approach is very popular to deal with the confined regime of QCD. The lattice calculations have found the glueball spectrum in various dimensions for both the quenched and the unquenched QCD. They have also calculated various meson spectrum, scattering amplitude etc, although the lattice approach has some technical limitations like the finiteness and discreteness of the lattice and is not suitable to study the dynamics. In the late 90s, after the formulation of the AdS/CFT correspondence, we have seen various strongly coupled gauge theories can be studied from holographic gravity background. In string theory, the AdS/CFT correspondence is a strong/weak duality with respect to the coupling constant of the corresponding theories. The strongly coupled gauge theory
corresponds to weakly coupled supergravity. So when we study holographic QCD in gauge/string duality, we always deal with strongly coupled gauge theory. Thus the asymptotic freedom is not accessible from the holographic method. But other properties of QCD like confinement, RG flow, gluon condensate and various properties of QGP can be studied from dual gravity backgrounds. So a proper holographic approach for low energy QCD is quite significant to study the high energy particle physics and also is a non-trivial application of string theory.

As the effective gauge coupling in gauge/gravity duality is \( \lambda = 4\pi N \Phi \), where \( \Phi \) is the dilaton field in gravity theory, therefore, to get the running coupling in gauge theory, the dual gravity background must have a non-constant dilaton field. Also the \( (d+1) \) dimensional gravity background should not have the \( SO(d,2) \) symmetry, so that the corresponding \( (d-1) + 1 \) dimensional gauge theory is non-conformal which ensures the existence of a \( \Lambda_{\text{QCD}} \)-like fixed point. Considering these two properties of QCD, there are a few phenomenological models to study the glueballs in the holographic QCD. But those models are not directly connected to the ten dimensional compactification. In this article, we consider the finite temperature, non-supersymmetric D3 brane solution of type IIB superstring theory. In the decoupling limit, i.e., in the low energy limit, it gives a non-AdS geometry (however, this geometry is still asymptotically AdS) at finite temperature and contains a non-constant dilaton field. The dilaton field also depends on temperature parameter which justifies the temperature dependence of \( \lambda \). On the gauge theory side, the pseudo-scalar glueball mass is driven by the field operator \( \operatorname{Tr}(F \tilde{F}) \) which is called the pseudo-scalar glueball field operator in the gauge theory. Thus the associated mass can be found from \( \langle F_{\mu \nu} F_{\mu \nu} \rangle \). On the other hand, the pseudo-scalar glueball operator couples to the axion field in the bulk theory. So the fluctuation of the bulk axion field gives the mass spectrum of \( 0^{-+} \). In this article we calculate the spectrum at the finite temperature. Here we take the perturbation of the axion field at finite temperature non-supersymmetric background which gives a Schrödinger-like wave equation. From that equation we evaluate the mass spectrum using semi-classical WKB approximation. The complete analytic solution of the spectrum is not possible due to the complicated form of the WKB equation. We approximate the equations for some particular order of mass and derive the analytic form of the temperature dependence of the spectra. Finally, we numerically evaluate the temperature dependent spectra for the full WKB equation. Here we find the glueballs masses in units of \( u_3/L^2 \) where \( u_3 \) is the fixed mass scale of the theory and \( L^4 = g_N^2 N \) is the gauge coupling. But if we take the ratio of masses of two levels, it becomes a pure number. So it will be more convenient to compare the mass ratios with various lattice calculations rather than comparing the dimensionful mass values. Here we will compare the ratio of the masses of the first excited state to the ground state. On the other hand, by matching the dimensionful mass of a particular level with the lattice result at zero temperature, \( u_3/L^2 \) will be determined and using that value we will evaluate the masses of various levels in units of eV at finite temperature. Thus the full spectrum will be drawn at finite temperature.

We have seen that the gauge/gravity correspondence is applicable for non-supersymmetric D3 brane solutions of type IIB superstring theory, which can be termed as ‘non-supersymmetric AdS/CFT duality’ [24]. The decoupled form of the finite temperature non-supersymmetric D3 brane has been discussed in [23]. The decoupled geometry in string frame is given in eqn.(14) of ref.[18] which has three independent parameters. Now for the following study, we take that decoupled background in the Einstein’s frame with \( \alpha + \beta = 2 \) which can be written as,

\[
d\xi^2 = \frac{u^2}{L^2} G(u)^{\frac{1}{2}} \left\{ (1 - G(u)^{\frac{1}{2}} dt^2 + dx^2) + \frac{L^2}{u^2 G(u)} du^2 + L^2 dt^2 \right\} \]

\[
e^{2\Phi} = e^{2\Phi_0} G(u)^{\frac{1}{2}} \sqrt{6 - 2\Phi} \]

\[
G(u) = 1 + \frac{u_3}{u^4} \]

(1)

where \( -2 \leq \delta \leq 0 \) and \( u_3 \) is a fixed energy scale. Here the radius of the transverse sphere \( S^5 \) is constant. So the dimensional reduction and calculations become simpler. \( \Phi \) is the dilaton field with the vacuum expectation value \( \Phi_0 = \ln g_s \) where the string coupling \( g_s \) is related to Yang-Mills coupling by \( g_M^2 = 4\pi g_s \). The gauge theory temperature \( T \) is related to the gravity theory as

\[
T = \left( -\frac{\delta}{2} \right)^{1/4} \frac{u_3}{\pi L^2} = \left( -\frac{\delta}{T} \right)^{1/4} T_c \]

(2)

We here remark that the above expression for the temperature can be obtained by comparing our solution with eqs.(3.6) and (3.7) of [26]. However, for this purpose we have to make a coordinate transformation from our \( u \) coordinate to their \( z \) coordinate as follows,

\[
u = \frac{L^2}{z} \sqrt{1 - \frac{u_3^2 z^4}{4L^8}} = \frac{L^2}{z} \sqrt{1 - f z^4} \]

(3)

where \( f \equiv u_3^4/(4L^8) \). Note also that the AdS radius \( L \) in our solution is the same as \( R \) in their solution. It is not difficult to check that with the above coordinate transformation our solution [11] precisely matches with eqs.(3.6) and (3.7) of Kim et al. [26] solution if we further identify \( \delta = -2a/f = -\lambda a L^2/u_3^4 \). (Here we like to mention that the solution in [26] has been obtained by solving the
dilaton-gravity in five dimensions, whereas, our solution is obtained as a particular case of decoupled geometry of non-susy D3-brane solution which is a genuine type IIB string theory solution, clarifying the ten dimensional origin of the metric described in \cite{20}. Now since \(a\) is related to the temperature as \(a = (1/4)\pi^4 T^4\) (given in \cite{20}), therefore, we get \(\delta = -2a/f = (-2\pi^4 L^8/a_3^4)T^4\) and hence \(2\) follows. As the two solutions are the same, the background \(1\) has a naked singularity at \(u = 0\) which corresponds to \(z = f^{-1/4}\). One can still define a temperature for this solution and a detailed discussion on this account has been given in \cite{26} which we will not repeat here. However, we mention that there are two mass scales in the corresponding gauge theory denoted by \(m\) and \(\delta\). We notice from (1) that for \(\delta = 0\) (as \(u_3 = 0\) gives pure AdS, we assume it to be non-zero) which also implies \(T = 0\) and so, this is the zero temperature case. In this case \(c = u_3^4/(4L^8)\) is related to the zero temperature gluon condensate. On the other hand, \(c = 0\) corresponds to \(\delta = -2\), there is no gluon condensate and from (2) it follows that in this case \(T = T_c = u_3/(\pi L^2)\). We notice from (1) that for \(\delta = -2\), the metric reduces to AdS\(_5\) black hole (apart from \(S^2\) factor) with no naked singularity and \(T_c\) is the corresponding Hawking temperature. This is a complete deconfined phase (a thermalized state) and \(T_c\) is the critical temperature for the confinement-deconfinement phase transition. Except \(\delta = -2\), the metric has a naked singularity for all other values of this parameter and as argued in \cite{20}, the system in this case is in a non-thermalized state containing free gluons as well as gluon condensate. The singularity implies the limitation to describe such system by gravity configuration. However, as the action itself is finite, there must be a concept of temperature (this is given by the relation \cite{24}) which can also be understood from the corresponding gauge theory point of view that gluon condensate can exist at finite temperature. As the parameter \(\delta\) varies between the value 0 and \(-2\), the quark-gluon plasma goes from the zero temperature gluon condensate phase to the fully deconfined phase at temperature \(T_c\). Here the variation is studied by hand and not as a time dependent process. Also, note that \(\delta\) in the gravity solution \(1\) simply gives an anisotropy in the time direction, whereas, in the gauge theory side this is related to a combination of temperature and the gluon condensate of the quark-gluon plasma.

To proceed, we note that in the background \(1\), the axion field is zero, but its fluctuation can be non-zero. Now we assume \(\chi\) as the fluctuation of axion. Thus the corresponding linearized equation of motion in the string frame is

\[ \nabla^2_{\text{string}} \chi = 0 \]  

(6)

where, \(\nabla^2_{\text{string}}\) is the Laplacian operator in given gravity background \(1\) in string frame. Now for simplicity, we assume that the axion fluctuation is polarized along the world-volume and symmetric on the transverse sphere \(S^5\). So we can take the ansatz \(\chi = \kappa(u)e^{ik_ix^i}\), where \(k^\mu = (E, \vec{p})\) is the four-momentum satisfying \(E = \sqrt{m^2 + p^2}\) and \(x^5 = (t, x^i)\) is the world-volume coordinates. Now taking this ansatz in (5), we get

\[ \partial^2_x^\mu \kappa + \left[ \frac{5}{u} + \left( 1 + \frac{\delta_1}{2} \right) \frac{\partial_u G(u)}{G(u)} \right] \partial_u \kappa \]

\[ + \frac{L^4}{u_3^2} G(u)^{-\frac{4}{5} + \frac{\delta}{5}} \left( E^2 G(u)^{-\frac{4}{5}} - p^2 \right) \kappa = 0 \]

where \(\delta_1 = \sqrt{6 - \frac{3}{2} \delta^2}\). To get a simplified form of the above differential equation, we take the coordinate transformation \(u = u_3 e^y\). Here the coordinate range \(0 < u < \infty\) is transformed to \(-\infty < y < \infty\). Therefore the near-singularity region is magnified in this new radial coordinate. Under this transformation, the above equation becomes,

\[ \partial_y^2 \kappa + \left[ 4 + \left( 1 + \frac{\delta_1}{2} \right) \frac{\partial_y G(y)}{G(y)} \right] \partial_y \kappa \]

\[ + \frac{L^4}{u_3^2} e^{-2y} G(y)^{-\frac{4}{5} + \frac{\delta}{5}} \left( E^2 G(y)^{-\frac{4}{5}} - p^2 \right) \kappa = 0 \]

Here, \(G(y) = 1 + e^{-4y}\). Now we replace \(f(y) = e^{2y} G(y)^{-\frac{4}{5} + \frac{\delta}{5}}\) to get the Schr"{o}dinger-like wave equation,

\[ \frac{d^2 f}{dy^2} - V(y)f = 0 \]  

(7)

where the potential corresponding to the axion fluctuation is

\[ V(y) = 4 \frac{1 + 2e^{-4y}}{(1 + e^{-4y})^2} + \frac{6 - \frac{3}{2} \delta^2}{(1 + e^{4y})^2} \frac{L^4}{u_3^2} e^{-2y} G(y)^{-\frac{4}{5} + \frac{\delta}{5}} \left( (m^2 + p^2)G(y)^{-\frac{4}{5}} - p^2 \right) \]

(8)

In Figure \(1\) we have shown the plot of the potential \(8\) against the dimensionless coordinate \(y\). When \(y\) is very large and negative, the second term of the potential dominates and \(V(y)\) saturates to \(6 - \frac{3}{2} \delta^2\) which is always positive and non-zero for the non-constant dilaton. On
the other hand, at large positive $y$, the first term dominates in $V(y)$ and $V(y)$ merges to a positive constant value which is 4. But in intermediate regime, near $y = 0$, we find a small potential well where $V(y)$ is negative. So the expected glueball state is bounded in this negative potential, i.e., the state is confined within this small potential well. Now to evaluate the energy of such states with WKB approximation, we need to know the turning points in the potential well, i.e., the points between which the associated wave function exists. Here we have already seen two turning points where potential changes sign. Those points can be found analytically from the asymptotic expansions of $V(y)$. The asymptotic forms of this potential at positive and negative infinity give the turning points.

For the positive asymptote, $y > 0$,

$$V(y) \approx 4 - \frac{L^4 m^2}{u_3^2} e^{-2y} \quad (9)$$

So the turning point in the positive segment $y_+ = \ln\left(\frac{L^2 m}{2u_3}\right)$. At the negative asymptote, $y < 0$,

$$V(y) \approx \left(6 - \frac{3}{2} \delta^2\right) - \frac{L^4}{u_3^2} (m^2 + p^2) e^{3(1+\delta/2)y} \quad (10)$$

The negative turning point is $y_- = \frac{2}{3(2+\delta)} \ln\left(\frac{u_3^2 6 - 2 \delta}{L^2 m^2 + p^2}\right)$. Therefore using WKB approximation in (11), we get the following equality,

$$\int_{y_-}^{y_+} dy \sqrt{-V(y)} = \left(n + \frac{1}{2}\right) \pi \quad (11)$$

Here the integer $n = 0, 1, 2, 3, \cdots$ labels the states. It is clear that the right hand side of (11) depends on $n$ only. But the integration on the left hand side of (11) is a function of $u_3, L, \delta, p$ and $m$. So once we evaluate the integration, the mass $m$ of a particular level $n$ can be expressed as a function of temperature $T$ and momentum $p$. But the depth of the potential-well has to be small enough to use the WKB approximation. In Figure 1 the depth of the potential increases with the increasing value of $m$. It shows the depth for $m = 4.0$ is the smallest and the depth for $m = 6.0$ is the largest among the three given values of $p$ and $\delta$. So this method is valid for the lower regime of the spectrum. In this approximation method we can evaluate the glueball mass for first few energy levels like $m_{-\Delta}, m_{+\Delta}, m_{+2\Delta}$ etc., i.e., for the ground state, the first and the second excited states etc.

Due to the complicated form of the potential function in (8) the analytic evaluation of the full integration in (11) is not easy. So here we will expand the potential function as a power series of $m$. According to the parametric condition of the gravity theory, the gauge theory temperature is in the range $0 \leq T \leq T_c$. Therefore we are in the confined state where the glueball dominates in the theory. Due to the interaction among themselves, the three-momentum of the glueball is trivial, i.e., $E \gg p$ for glueball, which gives $m = \sqrt{E^2 - p^2} \approx E$. We scale $E$ and $m$ in such a way that $\frac{L^2}{u_3^2} E^2 = \omega^2 \approx \frac{L^2}{u_3^2} m^2 = M^2$. So, $y_+ = \ln\left(\frac{E}{2}\right)$ and $y_- = \frac{2}{3(2+\delta)} \ln\left(\frac{6 - 2 \delta}{\omega^2}\right)$. Now as $\omega$ is large, i.e., $\omega \gg 1$ we expand the potential function as a power series in $1/\omega$ and solve the equation (11) to find $E$ or $m$. One can easily check that the expansion of $\sqrt{-V}$ has the leading order term of $O(\omega)$ and the other sub-leading terms of $O(\omega^{-1}), O(\omega^{-2}), O(\omega^{-3})$ and so on. Thus considering the terms in (11) upto various powers of $\omega$ the analytic expression of $m$ or $\omega$ can be found.

First we consider only the leading $O(\omega)$ term in $\sqrt{-V}$ and the sub-leading term up to $O(\omega^{-1})$ in (11). Then from the equality (11) the expression of $\omega$ is found to be the following,

$$\omega^{(1)} = \frac{4\Gamma\left[\frac{5}{8} + \frac{3}{16}\delta\right]}{\Gamma\left[1 + \frac{1}{2}\Gamma\left[\frac{5}{8} + \frac{3}{16}\delta\right]\right]} \left[2 + \left(n + \frac{1}{2}\right) \pi + \frac{4}{\sqrt{6}} \sqrt{\frac{2 - \delta}{2 + \delta}}\right] \quad (12)$$

For $\delta = 0$

$$\omega^{(1)} = 3.47437 + 2.09752n$$

Now as we consider all sub-leading terms up to $O(\omega^{-1})$ in $\sqrt{-V}$ and with the terms up to the order of $\omega^{-2}$ in (11), the WKB approximation formula (11) gives the following expression of $\omega$,

$$\omega^{(3)} = \frac{2\Gamma\left[\frac{5}{8} + \frac{3}{16}\delta\right]}{\Gamma\left[1 + \frac{1}{2}\Gamma\left[\frac{5}{8} + \frac{3}{16}\delta\right]\right]} \left\{3 + \sqrt{6} \sqrt{\frac{2 - \delta}{2 + \delta}} + \left(n + \frac{1}{2}\right) \pi\right\}^2 + \left\{3 + \sqrt{6} \sqrt{\frac{2 - \delta}{2 + \delta}} + \left(n + \frac{1}{2}\right) \pi\right\}^2 \cdot \frac{8}{9} \frac{14 - 9 \delta}{14 - 2 \delta} \frac{7}{4} \Gamma\left[\frac{1}{4}\Gamma\left[\frac{5}{8} + \frac{3}{16}\delta\right]\Gamma\left[\frac{3}{8} + \frac{3}{16}\delta\right]\Gamma\left[\frac{3}{8} - \frac{3}{16}\delta\right]\right]^{\frac{1}{2}} \quad (13)$$
TABLE I. Here the pseudo-scalar glueball energies $\omega$ are given in units of $u_3/L^2$ for $\delta = 0$ for various orders analytically and for the whole series numerically.

| $n$ | $\omega^{(1)}$ | $\omega^{(3)}$ | $\omega^{(5)}$ | Numerical |
|-----|----------------|----------------|----------------|-----------|
| 0   | 3.4743         | 4.1273         | 4.20766        | 4.3906    |
| 1   | 5.5719         | 6.4251         | 6.52683        | 6.6177    |
| 2   | 7.6694         | 8.6139         | 8.71688        | 8.7940    |
| 3   | 9.7660         | 10.765         | 10.8678        | 10.942    |
| 4   | 11.864         | 12.898         | 13.0004        | 13.068    |
| 5   | 13.962         | 15.021         | 15.1230        | 15.185    |

For $\delta = 0$

$$\omega^{(3)} = 2.3435 + 1.0487n + \sqrt{3.1818 + 4.9157n + 1.0999n^2}$$

Here we have found the approximate expressions of glueball energies $\omega^{(1)}$ and $\omega^{(3)}$ by considering the sub-leading order terms up to $O(\omega^0)$ and $O(\omega^{-2})$ respectively, starting from the leading order $O(\omega)$ on the left-hand-side of (11). Now the energy or mass ratio of the first excited state to the ground state are $\omega^{(1)}/\omega^{(1)} = 1.60371$ and $\omega^{(3)}/\omega^{(3)} = 1.5567$. The Lattice calculation has found this ratio 1.4602 [21], where $m_{-+} = 2.477\text{GeV}$ and $m_{+} = 3.617\text{GeV}$. For the second approximation $\omega^{(3)}$, the holographic result gets better in accuracy. Now we consider the next sub-leading order of $\omega$ in $\sqrt{V}$ and all of the sub-leading terms with orders higher than $O(\omega^{-4})$ in (11). In this order, the the energy is denoted $\omega^{(5)}$.

For, zero temperature $\delta = 0$, $\omega^{(5)}$ can be found from the following equation.

$$-5.60086 - \frac{0.804218}{\omega^{(5)8/3}} + \frac{14.8333}{\omega^{(5)4}} + \frac{1.28182}{\omega^{(5)5}} + \frac{1.49776\omega^{(5)}}{\omega^{(5)}} = \left(n + \frac{1}{2}\right) \pi$$

(14)

In the Table I we have given the calculated values of $\omega^{(1)}$, $\omega^{(3)}$, $\omega^{(5)}$ respectively at $\delta = 0$. There, in the table, we have also calculated the spectrum numerically. In the numerical estimation, we have considered the whole $\sqrt{V}$ in (11), without any approximation. The turning points have been found by the numerical root finding method and using those roots the numerical integration has been done to solve the equation for $\omega$. Thus, unlike the analytical solutions, the numerical solution of $\omega$ includes no approximation on the value of $\omega$. Therefore we argue that the numerical solution is the exact solution of (11) in our case, as the full analytical solution has not been found. We can see, as the more higher order terms (order of $1/\omega$) are considered in the analytical solutions, the mass value gradually approaches to the exact (numerical) values. In the numerical result $M^*/M = 1.5176$, whereas analytically $\omega^{(5)}/\omega^{(5)} = 1.5511$. So this ratio also approaches to the exact numerical result.

Next, we go to the temperature variation of the spectrum. In our analytic results, We have seen that the glueball energy explicitly depends on the gravity parameters $\delta$ and $u_3$ in (12) and (13). So, using (2), one can write down the glueball energy as a function of $T$. Now if we vary $T/T_c$, where $T_c$ is the transition temperature from confined to deconfined phase, we can see that $\omega^2 \approx M^2$ decreases with increasing temperature and vanishes at $T = T_c$. It is shown in Figure 2. Here we have shown the variations for the first four states of the spectrum.

![FIG. 2. The variation of pseudo-scalar glueball mass$^2$ or energy$^2$ (we have consider momentum is trivial) in the unit of $u_3^2/L^4$ with temperature $T/T_c = \left(\frac{\lambda}{\Lambda^2}\right)^{\frac{1}{4}}$ is shown for different states \(n = 0\) (blue), \(n = 1\) (green), \(n = 2\) (yellow) and \(n = 3\) (orange) for different calculations [dotted line, dashed line and solid line indicate the analytic results $\omega^{(1)}$, $\omega^{(3)}$ and the numerical result respectively].](image-url)
The zero temperature glueball masses from this decoupling limit makes the dual theory tachyon free [24]. The parametric conditions in gravity theory describe the dual gauge theory in a particular range of temperature, 0 ≤ T ≤ Tc, i.e., the confined regime. At Tc the QCD enters completely into deconfined phase. Near this transition point, the glueball melts into the free gluons. The glueball loses its mass with decreasing effective coupling as a result of the increasing temperature. Thus the pseudo-scalar glueball mass is maximum at zero temperature and zero at Tc. Here in our study, we have found the mass in term of u3/L2. So, evaluating the constant u3/L2 from the zero temperature ground state mass given in Lattice data we have found the first excited state mass 3.8GeV at T = 0 which is 0.2GeV larger than the past result [6]. This discrepancy arises mostly due to the approximations which have been used here in our calculational method. After all, we have seen that the mass has a large finite value on the order of GeV at T = 0, whereas it is zero at T = Tc. This type of decrement of the glueball mass with increasing temperature is an important proof of the QCD phase transition, i.e., the confinement-deconfinement transition. Using the same arguments, here we have calculated the transition temperature. We have found it to be 186MeV, which falls in the expected range of Tc. Therefore, along with the holographic estimation of the finite temperature 0−+ spectrum, this work also proves the existence of the QCD confinement from the holographic approach of the string theory. In this method, we are not able to calculate the scalar glueball due to the naked singularity in gravity theory. However, using this holographic approach we can also study the finite temperature spectrum and phase transitions in QCD3.

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| Table II | Here the pseudo-scalar glueball mass (in MeV) are given for first two states at different temperatures, using holographic QCD. Here we have used u3/L2 = 587.07MeV. |
|---------|-----------------------------------------------------------------------------------------------------------------------------|
| T = 0   | T = 0.25Tc | T = 0.50Tc | T = 0.75Tc | T = 0.90Tc |
| m−+    | 2559.95    | 2557.8    | 2523.93   | 2332.18   | 1846.11   |
| m+     | 3885.07    | 3880.73   | 3812.81   | 3440.24   | 2584.08   |

| Table III | Here the pseudo-scalar glueball mass (in MeV) are given for first two states at different temperatures, using holographic QCD. Here we have used Tc = 175MeV. So, u3/L2 = 549.5MeV. |
|-----------|------------------------------------------------------------------------------------------------------------|
| T = 0    | T = 0.25Tc | T = 0.50Tc | T = 0.75Tc | T = 0.90Tc |
| m−      | 2396.12    | 2394.11   | 2362.41   | 2182.93   | 1727.97   |
| m+      | 3636.44    | 3632.38   | 3568.81   | 3220.08   | 2418.71   |

The transition occurs through a range of temperature 150 – 200MeV. Now, using u3/L2 = 587.07MeV, we have written the masses of 0−+ at some finite T values in Table II. So both in the graph and in this table we can see a clear decrement of the glueball masses with increasing temperature. Again, if we take the cross-over point at Tc = 175MeV, u3/L2 = 549.5MeV, the glueball spectrum is given in Table II. The zero temperature glueball masses from this table are m−+ = 2.396GeV and m+ = 3.636GeV which almost exactly match with the recent holographic result at zero temperature [24].

Here in this article we have found the pseudo-scalar glueball spectrum at the finite temperature from the type IIB supergravity background. The dual gauge theory on the non-susy D3 brane is a QCD-like theory, in which the effective gauge coupling changes with the energy and there is a fixed energy scale related to the gravity parameters. Before taking the decoupling limit the non-susy brane may contain tachyonic field but the proper decoupling limit makes the dual theory tachyon free [24].

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