About moving of the orthogonal drive of the underwater mobile robot to a new position

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Abstract. The solution of one of the problems arising in the design of mobile walking robots: the problem of achievement of minimum of energy costs at overcoming obstacles, increasing the efficiency of a multi-legged mechanism and a reduction of the developed power due to a change in the mode of motion of the multi-legged walking apparatus is presented.

Introduction.
A feature of mobile robots with walking motors is the ability to contactlessly overcome obstacles. This, along with the potential ideal maneuverability, is their main advantage over robots with traditional types of propulsors, since they are inferior to them in other important indicators, such as speed of movement, energy consumption per unit path. This is explained by the fact that the walking mover is an unbalanced mechanism and therefore its motion characteristics when moving from one position to another significantly affect the discussed parameters. The known law of transfer of walking movers [1], which ensures the comfort of movement of a mobile robot (figure 1). Indeed, in the absence of information about the soil profile, trajectory 2, in the absolute movement of the foot of the walking robot from position A to new position B, provides movement without “tripping” and “sliding” for any soil profile 1. The task is only to establish the law of motion along the trajectory [2].

This type of trajectory is also explained by the fact that it was proposed in the absence of a developed information-measuring system. But if there is reliable information about the geometric characteristics of the profile of the supporting surface, the trajectory and, in general, the laws of motion of the transported foot can be different, for example, the trajectory can be as shown in figure 1 (curve 3).
Figure 1. The trajectory of the movement of the foot of the walking mover
1 – profile of the support surface; 2 – absolute trajectory according to N. Umnov;
3 – possible trajectory; S – the distance to the obstacle; H – the height of the obstacle to be
overcome; h – the height of the foot.

This depends on the criterion of transport optimality, in the general case consisting of several
indicators, for example, the minimum heat loss in drive motors, the minimum rms power developed by
the engines, the minimum distance traveled, etc. [3, 4]. From these indicators, an optimality criterion
can be constructed and the boundaries of the Pareto-optimal motion modes can be determined [5].
Known methods for controlling the transfer of stop cyclic walking movers of the machine "Octopus-
M" when overcoming forbidden zones due to changes in gait [6].

The laws of transferring stop movers to a new position also affect the energy efficiency of mobile
robots with walking movers [7]. This is also confirmed for biped robots [8]. For walking robots with
orthogonal-rotary propulsors, for example, for the Ortonog machine, the law of vertical movement of
the propeller foot is established in accordance with the optimality criterion, which consists of several
indicators [9]. Also established the laws of separation of the foot from the ground for an underwater
mobile robot [10], confirmed experimentally [11].

However, the problem of determining the optimal programmed foot movement of the propulsion of
an underwater mobile robot simultaneously in the horizontal and vertical directions in the presence of
obstacles was not considered. The relevance of this task is due to both the much greater forces of
resistance to the movement of the feet in a denser water compared to air, and the unevenness of the
bottom, which makes it necessary to step over fairly large obstacles.

Formulation of the problem.
The movement of the foot of the walking propulsion of an underwater mobile robot in the vertical
plane as a material point is considered. In the process of moving the foot overcomes a ledge of height
H, located at a distance S from point A of the beginning of the movement (figure 1). In the process of
movement, forces act on the foot (figure 2): P is the difference between gravity and buoyancy forces,
resistance forces to movement Rx, Ry, proportional forces to the speeds Vx = \dot{x}, Vy = \dot{y} and developed
by vertical T drives and horizontal F displacement.

Thus, the differential equations of motion have the form
\[
\begin{align*}
\begin{cases}
    m\ddot{x} = F - \mu_x \dot{x} \\
    m\ddot{y} = T - P - \mu_y \dot{y}
\end{cases}
\end{align*}
\]  \hspace{1cm} (1)

where \(\mu_x, \mu_y\) are the known coefficients of the viscous resistance of the liquid, \(m\) is the reduced foot mass.

The boundary conditions have the form
\[
\begin{align*}
    &\text{at } t = 0, \quad x_0 = 0, \quad y_0 = 0, \quad \dot{x}_0 = 0, \quad \dot{y}_0 = 0; \\
    &\text{at } t = \tau, \quad x_\tau = L, \quad y_\tau = h, \quad \dot{x}_\tau = 0, \quad \dot{y}_\tau = 0. 
\end{align*}
\]  \hspace{1cm} (2)

**Figure 2.** Scheme for settlement foot transfer

The dimensionless indicators of the quality of movement \(I_j\) are introduced, together, forming the efficiency criterion \(I\)
\[
I = \sum k_j I_j 
\]  \hspace{1cm} (3)

where \(k_j\) are weight coefficients determined up to a constant factor and selected in accordance with the significance of each indicator.

\[
I_1 = \frac{1}{A} \left( \alpha_1 \int_0^\tau F^2 \, dt + \alpha_2 \int_0^\tau T^2 \, dt \right) 
\]  \hspace{1cm} (4)

where \(I_1\) is the dimensionless level of heat loss in the drive motors, \(\alpha_1, \alpha_2\) are the known constants characterizing the drive motors, \(A\) is the arbitrarily set reference work, for example, \(A = mgH\).

\[
I_2 = \frac{1}{g^2} \int_0^\tau x^2 \, dt; \quad I_3 = \frac{1}{g^2} \int_0^\tau y^2 \, dt 
\]  \hspace{1cm} (5)

\(I_2, I_3\) – indicators characterizing the dimensionless square of rms acceleration, \(g\) – gravitational acceleration.
\[ I_4 = \frac{1}{2} \int_0^T [(F \dot{x})^2 + (T \dot{y})^2] \, dt \]  

(6)

\( I_4 \) is an indicator characterizing the dimensionless square of the rms power of the drives.

The task is to determine such laws of motion \( x = x(t) \) and \( y = y(t) \) that provide overcoming obstacles (figure 1), satisfy boundary conditions (2) and deliver a minimum to functional (3).

Solution method. The method for solving the problem is based on the "splitting" of the movement into two stages.

The first stage \( 0 < t < \tau_1 \):

\[ x(0) = 0, \quad \dot{x}(0) = 0, \quad y(0) = 0, \quad \dot{y}(0) = 0, \]

\[ x(\tau_1) = S, \quad \dot{x}(\tau_1) = U, \quad y(\tau_1) = H, \quad \dot{y}(\tau_1) = 0. \]

(7)

The second stage \( 0 < t < \tau - \tau_1 \):

\[ x(0) = S, \quad \dot{x}(0) = U, \quad y(0) = H, \quad \dot{y}(0) = 0, \]

\[ x(\tau - \tau_1) = L - S, \quad \dot{x}(\tau - \tau_1) = 0, \quad y(\tau - \tau_1) = h, \quad \dot{y}(\tau - \tau_1) = 0. \]

(8)

Each of the stages is characterized by two control parameters \( U, \tau_1 \), and at each of them the task of the minimum of functional (3) is posed, which at each stage leads to the Euler-Poisson equations [12]

\[ \frac{d^2}{dt^2} \left( \frac{\partial \Phi}{\partial \dot{x}} \right) - \frac{d}{dt} \left( \frac{\partial \Phi}{\partial \dot{y}} \right) + \frac{\partial \Phi}{\partial x} = 0 \]

\[ \frac{d^2}{dt^2} \left( \frac{\partial \Phi}{\partial \dot{y}} \right) - \frac{d}{dt} \left( \frac{\partial \Phi}{\partial \dot{x}} \right) + \frac{\partial \Phi}{\partial y} = 0 \]

(9)

where \( \Phi = \frac{k_4}{A} [\alpha_x (m \ddot{x} + \mu_x \dot{x})^2 + \alpha_y (m \ddot{y} + P + \mu_y \dot{y})^2] + \frac{k_2}{\tau g^2} \ddot{x}^2 + \frac{k_3}{\tau g^2} \ddot{y}^2 + \frac{k_5}{A^2} [(m \ddot{x} + \mu_x \dot{x})^2 \ddot{x}^2 + (m \ddot{y} + P + \mu_y \dot{y})^2 \ddot{y}^2] \).

For \( k_4 = 0 \), equations (9) have the form

\[ \begin{cases} x^{IV} - \lambda_x \dot{y}^2 = 0 \\ y^{IV} - \lambda_y \dot{y}^2 = 0 \end{cases} \]

(10)

where \( \lambda_x = \mu_x / m, \lambda_y = \mu_y / m \).

Equations (10) are solved at each stage under the introduced boundary conditions (7), (8).

Moreover, the optimality according to the selected criterion (3) at each of the stages, together with the choice of control parameters \( U, \tau_1 \) ensures optimality in the entire driving mode.

Indeed, if the functional defined on the interval \([0, \tau]\) is represented as the sum of two functionals defined on the interval \([0, \tau_1]\) and \([\tau_1, \tau]\), then this sum will depend on the boundary conditions at \( t = \tau_1 \). Equality of the sum of the functionals to the original will be ensured if the boundary conditions lie on the extremals of the original functional.

The solution of differential equations (10) at each stage has the form

\[ x(t) = C_1 + C_2 t + C_3 e^{\lambda_x t} + C_4 e^{-\lambda_x t} \]

\[ y(t) = B_1 + B_2 t + B_3 e^{\lambda_y t} + B_4 e^{-\lambda_y t} \]

(11)

At the first stage of the boundary conditions imply

\[ \begin{cases} C_1 + C_3 + C_4 = 0 \\ C_2 + C_3 \lambda_x - C_4 \lambda_x = 0 \\ C_1 + C_2 \tau_1 + C_3 e^{\lambda_x \tau_1} + C_4 e^{-\lambda_x \tau_1} = S \\ C_2 + C_3 \lambda_x e^{\lambda_x \tau_1} - C_4 \lambda_x e^{-\lambda_x \tau_1} = U \end{cases} \]
Where from

\[
C_3 = \left( e^{\lambda x (t - t_1)} - e^{\lambda x (t_1 - 1)} \right) \frac{d_y}{dx};
\]

\[
C_4 = \left( e^{\lambda y (t_1 - 1)} - e^{\lambda y (t_1 - 1)} \right) \frac{d_y}{dx};
\]

\[
C_1 = -(C_4 + C_3); C_2 = (C_4 - C_3) \lambda_x
\]

\[
\begin{cases}
B_1 + B_3 + B_4 = 0 \\
B_2 + B_3 \lambda_y - B_4 \lambda_y = 0 \\
B_1 + B_2 \lambda_x + B_3 e^{\lambda y (t_1 - 1)} + B_4 e^{-\lambda y (t_1 - 1)} = H \\
B_2 + B_3 \lambda_y e^{\lambda y (t_1 - 1)} - B_4 \lambda_y e^{-\lambda y (t_1 - 1)} = 0
\end{cases}
\]

\[
B_3 = \frac{H e^{\lambda y (t_1 - 1)}}{d_y}; B_4 = \frac{H e^{\lambda y (t_1 - 1)}}{d_y};
\]

\[
B_1 = -(B_4 + B_3); B_2 = (B_4 - B_3) \lambda_y.
\]

In the second stage, respectively

\[
\begin{cases}
C_1 + C_3 + C_4 = 0 \\
C_2 + C_3 \lambda_x - C_4 \lambda_x = U \\
C_1 + C_2 \lambda_x \left( e^{\lambda x (t - t_1)} - 1 \right) + C_4 \lambda_x \left( e^{-\lambda x (t_1 - 1)} - 1 \right) = L - S \\
C_2 + C_3 \lambda_x e^{\lambda x (t - t_1)} - C_4 \lambda_x e^{-\lambda x (t_1 - 1)} = 0
\end{cases}
\]

\[
C_3 = \left( e^{\lambda x (t - t_1)} - e^{\lambda x (t_1 - 1)} \right) \frac{d_y}{dx};
\]

\[
C_4 = \left( e^{\lambda y (t_1 - 1)} - e^{\lambda y (t_1 - 1)} \right) \frac{d_y}{dx};
\]

\[
C_1 = -(C_3 + C_4); C_2 = U - \lambda_x (C_3 - C_4).
\]

\[
\begin{cases}
B_1 + B_3 + B_4 = H \\
B_2 + B_3 \lambda_y - B_4 \lambda_y = 0 \\
B_1 + B_2 \lambda_x + B_3 e^{\lambda y (t_1 - 1)} + B_4 e^{-\lambda y (t_1 - 1)} = -h \\
B_2 + B_3 \lambda_y e^{\lambda y (t_1 - 1)} - B_4 \lambda_y e^{-\lambda y (t_1 - 1)} = 0
\end{cases}
\]

\[
B_3 = \frac{(H + h) e^{\lambda y (t_1 - 1)}}{d_y}; B_4 = \frac{(H + h) e^{\lambda y (t_1 - 1)}}{d_y};
\]

\[
B_1 = H - (B_3 - B_4); B_2 = (B_4 - B_3) \lambda_y.
\]

Thus, after determining in accordance with expression (11) and the equations of motion (1) of the transferred mover, all particular indicators and the value of the optimality criterion are determined

\[
I = \int_0^T \Phi dt
\]

The model task of determining the optimal driving conditions and analysis of the results. A complete solution of the problem, in addition to determining the optimal modes of motion at each stage, requires the determination of two additional control parameters $U$ and $t_1$ that provide a minimum of (16), in the general case, and some indicator $I_0$, for example, the level of heat loss, in particular.
The method for determining such control parameters is described by the example of solving a model problem. An Ortonog-type walking machine with dual walking mechanisms is considered. The reduced mass of each mechanism is \( m = 70 \) kg, the step length is \( L = 0.91 \) m, and the constants characterizing the drive motors are \( \alpha_1 = \alpha_2 = 0.0001 \). The speed of the machine \( V \) is related to the step length and the transfer time of the foot \( \tau \) by the ratio

\[
V = \frac{L}{2\tau}
\]  

(17)

The soil profile and the position of the machine in front of the obstacle are described by the parameters \( H = 0.3 \) m, \( h = 0 \), \( S = 0.3L = 0.273 \) m.

Thus, the variable parameters are the constructive \( \lambda_x, \lambda_y \), which characterize the propulsion streamlining by the external environment - water, and the desired control actions \( U = \gamma V, \tau_1 = \varepsilon \tau = \varepsilon \frac{L}{2V} \)

which are convenient to set with the dimensionless parameters \( \gamma \) and \( \varepsilon \).

As a result of mathematical modeling, the dependences of dimensionless indicators ((4) - (5)) included in the optimality criterion (3) are obtained.

In the graphs of figure 3 shows the dependence of the level of heat loss in drive motors on the speed of motion with known geometric characteristics of the bottom profile. Their analysis shows that there is an optimal mode of foot transfer through an obstacle, characterized by the parameters \( \gamma \) and \( \varepsilon \), depending on the soil profile.

**Figure 3.** The dependence of the dimensionless value of the level of heat loss in drive motors on the speed of movement:

a. at \( \varepsilon = 0.35; 1 - \gamma = 2; \quad 2 - \gamma = 3; \quad 3 - \gamma = 4; \)

b. at \( \gamma = 3; \quad 1 - \varepsilon = 0.2; \quad 2 - \varepsilon = 0.35; \quad 3 - \varepsilon = 0.5. \)

In the graphs of figures 4 and 5 show, respectively, the dependences of the dimensionless squares of mean-square accelerations, which also confirm the presence of optimal modes characterized by the same parameters \( \gamma \) and \( \varepsilon \).
Conclusion.
A design scheme is proposed and a mathematical model is developed that describes the dynamics of the movement of the transported foot of the walking mover of an underwater mobile robot. The indicators characterizing the optimality of the programmed mode of movement are formulated and methods for their determination are developed. The dependences of quality indicators on the speed of the robot and the control parameters $\gamma$, $\varepsilon$ are established.
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