q-Plastic deformation

Trinh Van Khoa

Department of physics of polymers and crystal, MGU
E-mail: khoa@polly.phys.msu.ru

1. Let’s consider a continuum with the structure of knots (for example, it is the polymers [18]). It is known that under an action of a force, the media transfers to plastic state. All of the experiments about plasticity had showed that in the plastic state the fields of stress and strains was in strongly fluctuation. Furthermore, the diagram $\gamma = f(e)$ of the stress-strain is similar to the diagram $P = f(V)$ of the pressure-volume in the fluid [1]. In the other side one observes the phase transition in the neighborhood of the end of the crack [2]. So we may consider the process of plastic deformation as critical one in which the formation of the structure was appeared [6]. The density of probability for the transition to the plasticity in the solution of the equation Fokker-Planck. However with the help of potential character of the deformation process we may use Schrodingers equation in the role of Fokker-Planck one. It mean that we may consider the plastic process as one with supersymmetry. Where we take the deformation tensor in the role of order parameter. In paper [17] A.A. Iliushin proposed the conception of ”trajectory of deformation ”in the space $E_5$ (it is the image of the trajectory of plastic wave or of the trajectory of deformation in usual sense). As a matter of fact the space $E_5$ is the one fiber in the deformation bundle [6]. In other word the deformation field is gauge one, in which generalized trajectory Iliushin was their section. Space of deformation $P = \{E_n(x, t)\}$, with most general points of view, is noncommutative. In the paper [6] we constructed the noncommutative model of deformation. In frameworks of this model we give a proof of the Iliusin hypothesis and Koiter hypothesis of existence of singular-
ities on the limiting surface of deformation. Iliusin hypothesis - it is the invariant of the simplectic structure under the diffeomorfizm. The existence of Koiter singularities has a topological reason and number of singularities it is topological number Pontriagin. It is known, that the space of deformation $P = \{E - n\}$ is associative algebra, in frameworks of which can be determined derivative and the differential form. So, we choose universal algebra Connes $\Omega(P)$ as algebra of the differential forms above space of deformation $P$. Thus, differential calculation on $\Omega_{D}(P)$ has a type similar to such in work [9], according to which least differential algebra of algebra of cosicle $C(Der(P), P)$, containing the space $P$, is $\Omega_{D}(P)$, being algebra $C(Der(P), P)$, where $Der(P)$ - algebra of derivative on $P$. In this paper we will construct the model of deformation, in which universal algebra Connes is algebra Hoof. The differential calculus on this algebra has a type of Woronowicz. When we identify the trajectory of deformation (or the line of propagation of plastic wave) with the knot we have archived the density of probability of the to plasticity [6]:

$$Z = \int \exp[-kS_{cs}]de,$$

where $k$— it is elastic coefficient, $S_{cs}$- the action Chern-Sine $S_{cs} = \int \Gamma_{cs}$, and $\Gamma_{cs}$- the form Chern-Simonce for deformation variety.

2. In the study of the exact solved system a new algebraic structure was appeared. It was called by the quantum group. Though quantum group in exact mathematical sense but there are many of its characters that one may to consider as a group. That’s why there is a problem to study and to construct the physical and mechanical models in which quantum group is a symmetry one. As a matter of fact, noncommutative geometry and quantum group are most effective instruments to study the deformation process. Correspond
ing to every process of deformation $E(x, t)$ there is a deformation space $P = \{E(x, t)\}$. In this process there is a reforming of symmetry and a generation of order structure. The coherence state in the block Bobrokov - Revuzenko - Shemiakin was their experimental affirm [6]. In most general view it is the noncommutative space. Furthermore, may be, it is a q-space of deformation. So in the deformation process there are noncommutative bundle and the fluctuation. Now our problem - it is determining functional $Z$ in (*). In the classical case [7] ”secondary calculus” was defined by formalism BRST, in which the space of state $P$ was extended to the space $\tilde{P} = (x, y, \bar{y})$, where $x$- spacetime coordinate, $y, \bar{y}$- noncommutative coordinate. In this space one defined 1-form $\tilde{A}^a$ with the value in the algebra Lie $\mathcal{G}$ of gauge group $G$: $\tilde{P} = (x, y, \bar{y}) = A_{\mu}^a dx^{\mu} + A_y^a dy + A_{\bar{y}d\bar{y}}^a$. So $A_\mu^a(x, o, o)$ was identified with usual gauge field, the ghost and antighost are fields $C^a(x) = A_y^a(x, o, o)dy$, $C^{\bar{a}}(x) = A_{\bar{y}}^a(x, o, o)d\bar{y}$. Thus one may have been generalized tensor gauge field by the 2-form $\tilde{B}^a(x, y, \bar{y}) = \frac{1}{2} B_{\mu\nu} dx^{\mu} \wedge dx^{\nu} + B_{\mu \bar{y}} dx^{\mu} \wedge dy + \frac{1}{2} B_{y \bar{y}} dy \wedge d\bar{y} + \frac{1}{2} B_{\bar{y} \bar{y}} d\bar{y} \wedge \bar{y}$. However in the noncommutative space of deformation it is necessary for me to realize only the first stage of this procedure because the gauge 1-form on $P$ was the differential form with value in matrix algebra of deformation. Thus one can formulate q-BRST cohomology of plastic deformation in the formalism Watamura [8]. It is the comodule algebra upon $\text{Fund}_q(G)$ that was generalized from component comodule $A = C(C^I, A^I, DA^I)/\mathcal{F}$, where $C^I$- is a ghost, $A^I$- a gauge field, $\mathcal{F}$- the commutative covariant correlations. Of course, because of the expansion $P \rightarrow \tilde{A}$ we have to carry out expanding $d \rightarrow \tilde{d} = d + \delta$, where $d-$ differential operator on the spacetime and $\delta$- the BRST operator on the field:

$$\delta^2 = 0, \quad d^2 = 0, \quad d\delta + \delta d = 0$$
\[ d(XY) = (dX)Y + (-1)^{n_x}X(dY) \]
\[ \delta(XY) = (\delta)Y + (-1)^{n_x}X(\delta Y) \]

where \( n_x \)- the index of the degree of the form and the numbers of ghost, \( X, Y \)- are fields. Basic difference between q-BRST deformation and classical BRST [7] is the effect that the differential calculus on the expanded space of deformation \( \tilde{A} \) is bicovariant. In this case we have been used differential calculus in [9] for the group \( SU_q(N) \) and \( SO_q(N) \). It is the differential calculus of the Woronowicz [4] in the limits of the noncommutative geometry Connes [3]. Thus first of all, the bicovariant bimodule \( \Gamma \) on the expanded space of deformation \( \tilde{A} \) was built. It is left and right covariant \( A- \) modul. Letter fundamental bimodul was given. It is the bicovariant bimodul that is linear subspace with right-covariant basic \( \eta^j \) and coaction \( \Delta_L: \Delta_L(\eta^i) = M^j_i \otimes \eta^j \), where \( M^j_i \) are matrix of deformation in space \( \tilde{A} \). At last one defined the bicovariant bimodul \( \Gamma_{Ad} \), right-invariant basic of which was adjoint representation \( \theta^j = \eta^i_+ \eta^-_j, \quad \bar{\eta} \equiv (\eta^i_-)^+, \) where \( \eta^i_\pm \) are right-invariant basic with the coefficient \( f^i_{\pm j} \)[9]. As in the [7] we establish q-BRST equation for the ghost with requiring that BRST-map of the ghost has to satisfy "horizontal condition", e.i. it is the equation Cartan-Mayere with covariant differential calculus. Considering the ghost as right-invariant basic and after separating simple and adjoint representation for the group \( SU(2) \) we obtain

\[ \delta C^a = \frac{-iq}{q^2 + q^{-2}} f^a_{bc} C^b C^c, \quad \delta C^0 = 0, \]  

(1)

Operator \( \delta \) acts on the fields of deformation by the rule [8]

\[ \delta A^I = da^I - ig[C^I, A^I] \]

or

\[ \delta A^a = dC^a - ig(\omega C^0 A^0 + f^a_{bc} C^b A^c), \quad \delta A^0 \]  

(2)

4
As in classical case the strength of deformation field has form

\[ F^{a} = dA^{0} - \frac{-iq}{q^{2} + q^{-2}}f^{a}_{bc}A^{b}A^{c}, \quad F^{0} = dA^{0} \]  \hspace{1cm} (3)

and the equality Bianchi is in the form

\[ dF^{a} = \frac{-iq}{q^{2} + q^{-2}}f^{a}_{bc}[A^{b}F^{c} - F^{b}A^{c}], \quad dF^{0} = 0 \]  \hspace{1cm} (4)

The act of operator \( \delta \) was defined as

\[ \delta F^{I} = i\frac{g}{\omega} [C^{0}, F^{I}] \]

Using this correlation we can build q-class Chern that is similar to [10,11]

\[ Q = F^{I}F^{J}g_{IJ} \]  \hspace{1cm} (5)

where the form Killing \( g_{IJ} \) was defined from the condition

\[ \delta Q = 0, \quad dQ = 0 \]  \hspace{1cm} (6)

Using the permutation operator \( \sigma \) and structure constant \( \Lambda \) in [9] and correlation (2),(3) for \( F \), we obtain the other condition (6) for the form Killing

\[ \delta^{0}_{I}g_{JK} = \Lambda^{SO}T_{JK}^{RT} \Lambda^{RT}_{IJ}g_{RS}, \]

\[ g_{00} = -g^{2}\lambda^{2}[2], \quad q^{-1}g_{+} = qg_{-} = g_{33} = -q^{2}(\lambda + 2) \]

Thus q-class Chern is defined in the form

\[ Q(A) = kP, \]  \hspace{1cm} (7)

where \( k \) is q-homotopy operator [11,12]

\[ k = \int_{0}^{1} l_t \]

and operator \( l_t \) is defined by rule:

\[ l_tA^J_t = 0 \quad l_tF^J_t = \delta_qA^J_t = d_qtA^J, \quad \delta_q \equiv d_qt \frac{\partial}{\partial_q t} \]
\[ l_l\{f(t)g(t)\} = \{l_l f(t)g(t)\} + (-1)^{n_z} f(q^{-1t})\{l_l g(t)\} \]

Parameter \( t \) is defined as

\[
A^J_t = tA^J, \quad F^J_t = tF^J + \frac{ig(t^2 - 1)}{\lambda}(A^0 A^J + A^J A^0), \tag{8}
\]

After placing (8) into (7) and after carrying out integral refer \( t \) we obtain

\[
Q(A) = \frac{t\lambda}{[2]} < A, dA > + \frac{ig(\lambda^2 + 2)}{[3]\lambda} < A, A^0 A^J + A^J A^0 >
\]

3. The main idea in my approach is to identify the trajectory of deformation with a knot. Further, similar to the classical mechanics of continuous medium, using hypothesis in [13] we have to require in order that strength of deformation field along the trajectory of deformation to be maximum. As was showed by Witten in [14] this requirement may be realized when action of deformation field was the action Chern - Simonce. This action was defined by \( Q(A) \).

Thus we obtain the statistical sum describing the probability of the transition elastic-plastic:

\[
Z = N \int [-\beta Q_{cs}(A)] dA \tag{9}
\]

where \( \beta \)- it is elastic coefficient, \( N_1 \)- coefficient depending on the potential at beginning and the end point of trajectory of deformation. If in the medium there is the dislocation line, then generalized matrix Burgers will be presented in the form

\[
b_T = N exp[-\int_C A_{(\mu)}d(x^\mu \otimes 1)] \tag{10}
\]

where \( N \)- it is order exponent part. In this situation we obtain statistical sum reflecting the density of probability of the transition elastic-plastic with the interaction of dislocation

\[
Z = \int \Phi_c(A) exp[-\beta Q_{cs}] dA \tag{11}
\]
\[ \Phi_c = Tr(b_\Gamma) \]

However, it is known that when the medium has been passed on the plastic state the field of strain and stress will be in the fluctuation. As a result the action \( Q \) will be separated on three parts:

\[ Q \rightarrow Q_K + Q_{GF} + Q_{FP} \]

where \( Q_K \)- classical action Chern-Simonce, \( Q_{GF} \)- The action of the fixed gauge and \( Q_{FP} \)- the action of ghost Fadeev-Popov. In the formalism of BRST of Watamura [8] this action has to be the form in order that when \( q \rightarrow 1 \) ones passes to the classical form in [15]. Then we obtain:

\[ Q_{GF} = \frac{\beta}{4\pi} <dA, dA>, \quad Q_{FP} = -dC(DC) \]

where , similar to [16], \( D = d + i[A, A] \). Putting this action into (9) and (11) we obtain the probability of the transition elastic-plastic.

4. In this paper we used the formalism BRST to carry out the procedure of ”secondary calculus”. But, the most interesting for me is the mechanism ”quantization” of quantum group. It is known that the appearance of the order state in continuous medium is the result of the spontaneous breaking of the symmetry. The collective fashion [6] was used in the role of carrier maintaining the order state, may be considered as plastic fashion. In other world, the order state exists because of the concentration of strength in the medium. In this case the symmetry regulates the character of this concentration. At general in the process of plastic deformation may exist as such situation:

Formation of order state = Spontaneous breaking symmetry = Re-construction of symmetry.

However, in this situation the reconstruction of symmetry turns to no reduction of symmetry nut to the q-symmetry. Where we, in secret sense, identity the process of reconstruction of symmetry with the process of ”quantization” of group Lie in the deformation bundle. The
parameters $q$ plays the role of the parameter of regulation. Because of the ghost was considered as the right-invariant basic in the bimodule, then he was one of types of the plastic deformation keeping finite gauge-invariant character of the functional $Q$.

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