Can Large Extra Dimensions Solve the Proton Radius Puzzle?

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The proton charge radius extracted from the recent muonic hydrogen spectroscopy $^{1,2}$ differs from the CODATA 2010 recommended value $^3$ by more than 4% or 4σ. This discrepancy, dubbed as the “Proton Radius Puzzle”, is a big challenge to the Standard Model of particle physics, and has triggered a number of works on the quantum electrodynamics calculations recently. The proton radius puzzle may indicate the presence of an extra correction which enlarges the 2S-2P energy gap in muonic hydrogen. Here we explore the possibility of large extra dimensions which could modify the Newtonian gravity at small scales and lower the 2S state energy while leaving the 2P state nearly unchanged. We find that such effect could be produced by four or more large extra dimensions which are allowed by the current constraints from low energy physics.

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INTRODUCTION

Recently the charge radius of the proton is precisely measured from the Lamb shift of muonic hydrogen ($\mu p$) $^1$ $^2$, which yields $r_p = 0.84087(39)$ fm. However, this value differs by 4.4σ from the value obtained via electronic hydrogen (H) and deuterium (D) spectroscopic data, which is 0.8758(77) fm. It deviates even more (by 7σ) from the CODADA-2010 recommended value of 0.8775(51) fm, which was obtained from a combination of H and D spectroscopic data and the electron-proton scattering data $^3$. Given the great precision in such measurements, this discrepancy, dubbed as the “Proton Radius Puzzle”, is a severe problem and has received wide attention among physicists. The discrepancy has several possible explanations $^4$.

Of course, there is always the possibility that there is some hidden systematic error and the experiments are not as accurate as claimed. However, many independent electron-proton scattering experiments $^5$ $^{10}$ are in good agreement with each other up to an arbitrary form factor extrapolation to low momentum transfers. The various measurements of transition frequencies in hydrogen $^3$ $^{17}$ $^{20}$ also agree with each other. On the other hand, the muonic hydrogen experiments are even more convincing than these $^1$ $^2$ $^{21}$ $^{23}$.

On the theory side, a large missing term in the various quantum electrodynamics (QED) other than the proton structure effect, is highly unlikely, for the QED calculations in the hydrogen $^3$ $^{24}$ $^{25}$ and the muonic hydrogen $^1$ $^{24}$ $^{30}$ $^{36}$ have been done and checked by many groups over years. There is little room for a missing term or mistake which could contribute as large as 0.3 meV.

Recently, the proton polarizability contribution to Lamb shift in muonic hydrogen $^3$ $^{32}$, which enters in the two-photon exchange term and involves intersection between the QED and QCD has been reanalyzed $^{30}$ $^{40}$ $^{41}$. The uncertainty in this calculation could be large and might be able to account for the proton radius puzzle if there is a lepton-proton interaction caused by high-momentum behavior of the virtual scattering amplitude, which is proportional to the lepton mass to the fourth power. However, this assumption still needs to be confirmed in the future experiments.

The possibility that the proton radius puzzle could imply the existence of a different interaction between ep and $\mu p$ has been extensively explored, since such an interaction goes beyond the Standard Model (SM) of particle physics. The window for such new physics is small because the lepton universality has been rigorously tested $^{44}$ $^{45}$. Several models, which assumed new gauge particles which couple the muon and/or proton with fine-tuned particle coupling or mass, have been proposed to reproduce the experiment results $^{44}$ $^{49}$. The models with new physics are promising but still primitive, and further works are needed to construct a complete gauge theory which can pass through all experiments.

The proton radius puzzle is still a puzzle. It is therefore valuable to search for new sources to explain the discrepancy of the extracted proton radius from hydrogen and muonic hydrogen experiments. In this paper, we explore whether large extra dimensions (LEDs) can solve the proton radius puzzle in the framework of a well-known low energy effective theory proposed by Arkani-Hamed, Dimopoulos and Dvali $^{50}$ (ADD model). LEDs in ADD model can provide a stronger attractive gravitational field in scales much smaller than the size of them, which can lower the energy level of 2S state while leave 2P state nearly unchanged, so it could contribute to the
Lamb shift. More importantly, muon has a mass about 206 times larger than the electron mass, so the Bohr radius of muonic hydrogen is about 186 times smaller than that of the hydrogen. This results in a contribution to the Dirac wave functions.

The ADD model was initially proposed to solve the problem in Standard Model, i.e. the huge difference between the Planck scale of $\sim 10^{19}$ GeV and the electroweak scale ($\sim 1$ TeV), by assuming the existence of $n$ extra spatial dimensions of size $R_n$ which are compactified on an $n$-dimensional torus. The fundamental Planck scale in $(4+n)$-dimensional spacetime ($M_D$) is then related to the 4-dimensional effective Planck scale ($M_P$) by Gauss’s law, $M_D^n = M_P^{n+2} R_n^2$. The fundamental Planck scale can be tuned to $\sim 1$ TeV if the volume $\propto R_n^n$ is large enough, so the hierarchical problem is solved. One consequence of the ADD model is that at scales much smaller than the size of the extra dimensions $R_n$, the gravity force will be strengthened, $V(r) = V_N(r) \times (R_n/r)^n$, where $V_N(r) = G_N m_1 m_2 / r$ is the Newtonian gravity between two particles with mass $m_1$ and $m_2$, and $G_N$ is the Newton gravitational constant.

We calculate the contribution of this modified gravity to the energy levels in hydrogen and muonic hydrogen in Sec. II, and give the size of extra-dimensions which can solve the proton radius puzzle. In Sec. III, we discuss the various constraints on the number and size of the LEDs.

**LAMB SHIFT AND PROTON RADIUS PUZZLE**

If $n$ compactified LEDs are introduced, each with size of $R_n$, the gravity force between particles is modified on small scales as,

$$V(r) = \begin{cases} \frac{G_N m_1 m_2}{G_N m_1 m_2} \times (\frac{R_n}{r})^n, & \text{for } r \ll R_n \\ \frac{\rho}{\rho_B}, & \text{for } r \gg R_n \end{cases}$$

This may be treated as an perturbation on the standard Coulomb potential. The correction on the energy levels is given by

$$\Delta E_{nl} = (n|l|V|nl|),$$

where $n$ is the principal quantum number and $l = 0, 1, \ldots, n-1$ is the angular momentum quantum number. In the calculation of the perturbations on the hydrogen and muonic hydrogen energy levels, either the non-relativistic Schödinger wave function or the relativistic Dirac wave function can be used, and for our purpose the two approaches give negligible small difference. Below we use the simpler Schödinger wave functions to illustrate the calculation, but give the final results using the Dirac wave functions.

Specifically, the LEDs contribute to the 2S state is reduced to

$$\Delta E_{2S} \approx \frac{G_N m_1 m_p}{8 \rho_B} \left(\frac{R_n}{\rho_B}\right)^n \int_{t_0}^{t_n} \frac{(2 - t)^2 e^{-t}}{t^{n-1}} dt$$

$$\approx \frac{E_0}{2(n-2)} \left(\frac{R_n}{\rho_B}\right)^n \left(\frac{r_0}{\rho_B}\right)^2 \frac{1}{n-3}$$

where $E_0 = G_N m_1 m_p / \rho_B$, $\rho_B \equiv h^2/m_r e^2$ is the Bohr radius with $m_r$ the reduced mass, $m_l$ the lepton (electron or muon) mass, and $m_p$ the proton mass. The lower limit of the integral $t_0 = r_0 / \rho_B$ is the small scale cutoff in unit of Bohr radius, and the upper limit is given by $t_n = R_n / \rho_B$. Eq. (3) is valid only if $n \geq 3$. We may neglect the contribution of the Newtonian gravity at large scales. The Bohr radius for hydrogen is $\rho_B = 0.529 \times 10^{-8}$ fm, and for the muonic hydrogen it is $\rho_B pf = 285$ fm, so if the electroweak scale $(l_{EW} \approx 2 \times 10^{-3} \text{ fm})$ or even the proton radius ($r_p \approx 0.8 \text{ fm}$) is taken as the cutoff, $t_0 \sim 0$, and the second line of Eq. (3) results. We see that the correction $\Delta E_{2S} \propto m_l / \rho_B^3$, so for the muonic hydrogen it is more than 9 orders of magnitude larger than for hydrogen.

Similarly, the correction of LEDs to the energy level of 2P state can be calculated as,

$$\Delta E_{2P} \approx E_0 \cdot \left(\frac{R_n}{\rho_B}\right)^n \frac{1}{24} \int_{t_0}^{t_n} e^{-t} \frac{e^{-\frac{t}{l_{EW}}}}{l_{EW}^{n-3}} dt,$$

which will be several orders of magnitude smaller than that of 2S state and can be neglected in the following discussion.

Summing up all the contributions, the Lamb shift in the muonic hydrogen can be written as

$$\Delta E_{LS}(\text{ meV}) = 206.0336(15) - 5.2275(10)r_p^2 + \Delta E_{TPE}; (5)$$

where $r_p$ is in units of fm. The second term (mainly from the one-photon exchange and its radiative corrections) and the third term (two-photon exchange) are dependent on proton structure. The rms charge radius of proton can be extracted by comparing Eq. (3) with the value from experiments. The difference between the proton radius extracted from the hydrogen and muonic hydrogen experiments results in a correction to the Lamb shift in muonic hydrogen,

$$\delta E_{LS,PRP} = 0.329(50) \text{ meV} (6)$$

The error comes mainly from the hydrogen experiments.

Using Eqs. (3,4), we find that in order to explain the proton radius puzzle, the extra dimensions should have

$$R_n = D_n \times (\frac{r_0}{l_{EW}})^{1-2/n}$$

where $D_n$ is the size of the $n$ large extra dimensions (here we take $r_0$ as the electroweak scale $l_{EW}$). For $n = 3, 4, 5, 6$, we have $D_3 = 61 \mu\text{m}$, $D_4 = 31 \text{nm}$, $D_5 = 0.31 \text{nm}$, $D_6 = 0.014 \text{ nm}$ respectively.
The modified gravity of LEDs as given above contributes 0.38 kHz to the Lamb shift in hydrogen. This is compatible with the current measurements \[3\]. It also contributes 2.6 kHz to the 1S-2S transition frequency, which is the best measured one in hydrogen spectroscopy, and this is 77 times larger than the CODATA 2010 suggested value of the experimental uncertainty \[2\]. However, the theory of hydrogen energy level for 1S state has an uncertainty of the order of several kHz, mainly due to two-loop and three-loop calculations (see \[3\] and references therein). Taken into account this theoretical uncertainty, the presence of LEDs with size shown in Eq. \(7\) may still be allowed.

**DISCUSSIONS**

In this work, we calculated the contribution to the Lamb shift of muonic hydrogen from the modified gravity force in models with large extra dimensions. To solve the proton radius puzzle, the LEDs should be as large as shown in Eq. \(7\).

There are many works on the constraint of the ADD model. The model with only one large extra dimension has been ruled out by the experiments over the solar system \[51\]. The model with two or three LEDs are tightly constrained by the tests of the inverse-square law of gravity \[51\]. The model with only one large extra dimension shown in Eq. \(7\).

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Improvements on the measurements of the Lamb shift and the calculations of two-loop and three-loop corrections to the 1S and 2S energy levels in hydrogen are helpful to confirm or exclude the LEDs as the solution to the proton radius puzzle.

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