A Further Study of $CP$ Asymmetries in Pure Penguin-induced $B_{u}^{\pm}$ Decays

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Abstract

We make a further study of $CP$ asymmetries in the pure penguin-induced decay modes $B_{u}^{-} \to K^{(*)0} + K^{(*)-}$ and $B_{u}^{-} \to \bar{K}^{(*)0} + (\pi^{-}, \rho^{-}, a_{1}^{-})$ by using the two-loop renormalization-group-improved effective Hamiltonian and factorization approximation. Different from the previous results obtained without QCD corrections, the $CP$ asymmetries in each of the two groups are enhanced and classified with their respective factorization coefficients. A sum over the pure penguin modes of each group is proposed to obtain an effective branching ratio and a weighted-mean signal of $CP$ violation, which should be statistically significant for experimental observation.

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1 Introduction

Observing \( CP \) violation in the \( B \)-meson system and confronting it with the predictions of the standard model is an important task in particle physics. The basic signal for \( CP \) violation is the partial rate difference between a \( B \) decay mode and its \( CP \)-conjugate process. On the basis of the Cabibbo-Kobayashi-Maskawa (CKM) picture, either \( B^0 - \bar{B}^0 \) mixing [1] or the absorptive parts of QCD-loop-induced (penguin) amplitudes [2] can give rise to significant \( CP \) asymmetries in exclusive nonleptonic \( B \) decays. Among various hadronic decay modes of \( B \) mesons, the pure penguin-induced transitions are of great interest to explore direct \( CP \) violation in the decay amplitude and to test our understanding of the loop effects involved in the quark-level process \( b \to q \) (with \( q = d \) or \( s \)). Recently evidence for the electromagnetic penguin transitions, e.g., \( B^0_d \to K^*0\gamma \) and \( B^-_u \to K^*^-\gamma \), has been obtained by CLEO Collaboration [3]. Accordingly studying \( CP \) violation in the pure strong penguin-induced \( B \) decays becomes a necessary and realistic topic today.

In the literature [4-6], \( CP \) asymmetries in some pure penguin channels have been calculated with the QCD-uncorrected effective Hamiltonian \( H_{\text{pen}} \) (see Eq.(2)) and factorization approximation. For the processes occurring through \( b \to d \) (or \( b \to s \)), those results are approximately independent of the spin properties of final-state mesons. Hence a sum over the relevant modes has been proposed in order to reduce the total number of \( B \) events and to obtain a statistically significant signal of \( CP \) violation. In this paper, we shall use the two-loop renormalization-group-improved effective Hamiltonian \( H_{\text{eff}} \) (see Eq.(4)) to make a further study of \( CP \) asymmetries in the following two handfuls of pure penguin transitions:

\[
\begin{align*}
(\text{I}) & \quad B^-_u \to K^0K^-, K^*0K^-, K^0K^*, K^*0K^*; \\
(\text{II}) & \quad B^-_u \to \bar{K}^0\pi^-, \bar{K}^0\rho^-, \bar{K}^0a_1^-; \quad \bar{K}^*0\pi^-, \bar{K}^*0\rho^-, \bar{K}^*0a_1^-.
\end{align*}
\]

It is worth emphasizing that the above ten decay modes are flavor self-tagging processes, which should be favored for experimental reconstructions. In addition, there are no QCD-loop-induced hairpin diagrams contributing to these transitions\(^3\) and the electroweak penguin effects on them are negligible [8]. Thus application of the factorization approximation to these decays should lead to less uncertainty. Different from the naive results obtained with \( H_{\text{pen}} \), the magnitude of \( CP \) asymmetries in the transitions (I) and (II) are enhanced by the next-to-leading order QCD corrections from \( H_{\text{eff}} \). In each group the asymmetries become non-degenerate and are classified with their respective factorization coefficients, but they

\(^3\)In our sense, some of the QCD-loop-induced transitions such as \( B^-_u \to K^-\phi \) are not the pure penguin decay modes. The reason is that they can also occur through the hairpin diagrams. For a detailed discussion, see Ref. [7].
still keep the same sign. Hence we propose a sum over the modes of each group to obtain an effective branching ratio and a *weighted-mean* signal of CP violation, which should be statistically significant for observation. The feasibility of measuring such weighted-mean CP asymmetries the uncertainties in evaluating them are briefly discussed.

## 2 Effective Hamiltonians and factorization approximation

To calculate decay amplitudes of the aforelisted pure penguin channels, we use the low-energy effective Hamiltonians for $\Delta B = \pm 1$ transitions and the well-known factorization approximation [9]. Neglecting influence of the renormalization group, one used to apply the one-loop penguin Hamiltonian [4-6]

$$H_{\text{eff}}^{\text{pen}}(\Delta B = -1) = -\frac{G_F}{\sqrt{2}} \frac{\alpha_s(m_b)}{8\pi} \left[ \sum_{i=u,c,t} v_i F_i(k^2) \right] \left( -\frac{Q_3}{N_c} + Q_4 - \frac{Q_5}{N_c} + Q_6 \right)$$

(2)

to phenomenological discussions. In $H_{\text{eff}}^{\text{pen}}$, the QCD renormalization effect is assumed to be approximately included by the effective coupling constant $\alpha_s$ at the physical scale $\mu = m_b$; $v_i \equiv V_{ib} V_{iq}^*$ (with $i = u, c, t$ and $q = d, s$) are the CKM factors corresponding to $b \rightarrow q$; $F_i(k^2)$ are the loop integral functions of the time-like penguin diagrams versus the virtual gluon momentum $k^2$ [2,6]; $N_c$ is the number of colors; and $Q_{3,\ldots,6}$ represent the penguin operators. Together with the current-current operators $Q_{1,2}^{u,c}$, $Q_{3,\ldots,6}$ form an operator basis as follows:

$$Q_1^u = (\bar{q}_u \alpha_\beta)_{V-A}(\bar{u}_\beta b_\alpha)_{V-A}, \quad Q_2^u = (\bar{q}_u)_{V-A}(\bar{u}b)_{V-A},$$

$$Q_1^c = (\bar{q}_c \alpha_\beta)_{V-A}(\bar{c}_\beta b_\alpha)_{V-A}, \quad Q_2^c = (\bar{q}_c)_{V-A}(\bar{c}b)_{V-A},$$

$$Q_3 = (\bar{q} b)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A}, \quad Q_4 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}' q'_\beta)_{V-A},$$

$$Q_5 = (\bar{q} b)_{V-A} \sum_{q'} (\bar{q}' q')_{V+A}, \quad Q_6 = (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}' q'_\beta)_{V+A}. $$

(3)

Compared with $H_{\text{eff}}^{\text{pen}}$, the two-loop QCD-corrected Hamiltonian $H_{\text{eff}}$ is of the form [10]

$$H_{\text{eff}}(\Delta B = -1) = \frac{G_F}{\sqrt{2}} \left[ v_u \left( \sum_{i=1}^{2} c_i Q_1^u \right) + v_c \left( \sum_{i=1}^{2} c_i Q_1^c \right) - v_t \left( \sum_{i=3}^{6} c_i Q_1^i \right) \right].$$

(4)

Here the Wilson coefficient functions $c_{i,\ldots,6}$, obtained by applying the renormalization-group-improved perturbation theory, have included the next-to-leading order QCD corrections at the scale $\mu = m_b$. Note that $c_i$ depend on the renormalization scheme used for the four-quark operators $Q_i$. This scheme dependence, however, can be cancelled by certain one-loop matrix elements of the decay mode in question. For a detailed description of the approach to
obtain the renormalization-scheme independent transition amplitudes in two-body $B$ decays, we refer the reader to the article of Fleischer [11].

With the help of $H_{\text{eff}}$, the amplitude of a two-body pure penguin-induced decay mode $B_u^- \to XY$ may be expressed as linear combinations of $<XY|Q_i|B_u^->$ with weighting factors $c_i$ and $v_i$. Following Ref. [11], one can remove the renormalization-scheme dependence of $c_i$ in the physical transition amplitudes by taking one-loop penguin matrix elements of the operators $Q_2^c$ into account (see Fig. 1). The renormalization-scheme independent amplitude of $B_u^- \to XY$ is finally obtained as [11,12]

\[
<XY|H_{\text{eff}}(\Delta B = -1)|B_u^-> = -\frac{G_F}{\sqrt{2}} \left\{ \bar{c}_2 \left[ v_u F_u(k^2) + v_c F_c(k^2) + v_t \left( \frac{10}{9} - \frac{2}{3} \ln \frac{m_b^2}{m_W^2} \right) \right] M_P + v_t M'_P \right\} ,
\]

where $\bar{c}_i$ (with $i = 2, ..., 6$) are the scheme independent Wilson coefficients including the next-to-leading order QCD corrections [10], and $M_P$ and $M'_P$ are the hadronic matrix elements of penguin operators defined by

\[
M_P \equiv \frac{\alpha_s(m_b)}{8\pi} <XY|\left( -\frac{Q_3}{N_c} + Q_4 - \frac{Q_5}{N_c} + Q_6 \right)|B_u^->,
\]

\[
M'_P \equiv \sum_{i=3}^6 \left( \bar{c}_i <XY|Q_i|B_u^-> \right) .
\]

In contrast, the decay amplitude of $B_u^- \to XY$ calculated with $H_{\text{eff}}^{\text{ben}}$ is of the form

\[
<XY|H_{\text{eff}}^{\text{ben}}(\Delta B = -1)|B_u^-> = -\frac{G_F}{\sqrt{2}} \left[ \sum_{i=u,c,t} v_i F_i(k^2) \right] M_P .
\]

The hadronic matrix elements $M_P$ and $M'_P$ are calculated using the naive factorization approximation [9]. Here we only consider contributions from the spectator-type diagrams as illustrated in Fig. 1, while the OZI-forbidden transitions and (or) formfactor-suppressed annihilation topologies are neglected. Consequently $M_P$ and $M'_P$ are both dominated by a single term

\[
M_{qq'q''}^{XY} \equiv <X(q\bar{q}')|(qq')_{V-A}|0> <Y(q''\bar{u})|(q'b)_{V-A}|B_u^-(b\bar{u})> ,
\]

and their factorized expressions are given as

\[
M_P = \left[ \frac{\alpha_s(m_b)}{8\pi} \left( 1 - \frac{1}{N_c^2} \right) (1 + \zeta_P) \right] M_{qq'q''}^{XY},
\]

\[
M'_P = \left[ \left( \frac{\bar{c}_3}{N_c} + \bar{c}_4 \right) + \left( \frac{\bar{c}_5}{N_c} + \bar{c}_6 \right) \zeta_P \right] M_{qq'q''}^{XY}.
\]

In Eq. (9), the factorization coefficient $\zeta_P$ is obtained by transforming the $(V-A)(V+A)$ (or $(S+P)(S-P)$) currents of $Q_{5,6}$ into the $(V-A)(V-A)$ ones. For each specific decay mode,
\( \zeta_P \) depends upon the current masses of quarks as well as the angular momenta and parities of \( X \) and \( Y \) mesons. We list the expressions of \( \zeta_P \) in Table 1 for \( 0^+0^±, 0^±1^±, 1^±0^±, \) and \( 1^±1^± \) final states, respectively. The hadronic matrix elements \( M_{qq'q'}^{XY} \) can be Lorentz-invariantly decomposed with the relevant decay constants and formfactors, as given in Refs. [6,12]. For our purpose, we shall sum over the polarizations of vector or axial vector mesons in the final state and estimate \( |M_{qq'q'}^{XY}|^2 \) by means of the Bauer-Stech-Wirbel (BSW) model [13].

### 3 CP asymmetries

Now let us define the \( CP \)-violating partial decay rate asymmetry for \( B_u^- \to f \) and its \( CP \)-conjugate counterpart \( B_u^+ \to \bar{f} \):

\[
A_{CP}(f) = \frac{\Gamma(B_u^+ \to \bar{f}) - \Gamma(B_u^- \to f)}{\Gamma(B_u^+ \to \bar{f}) + \Gamma(B_u^- \to f)}. \tag{10}
\]

In the CKM mechanism with three families of quarks, every of the aforementioned pure penguin transitions contains two amplitude components that have different \( CP \)-violating weak phases \( (v_u \text{ and } v_c) \) and different \( CP \)-conserving strong phases \( (\text{Im}F_u(k^2) \text{ and } \text{Im}F_c(k^2)) \). The \( CP \) asymmetry is approximately independent of hadronic matrix elements, since \( M_{qq'q'}^{XY} \) can be cancelled in \( A_{CP}(f) \). Thus \( A_{CP}(f) \) is simply expressed as [6]

\[
A_{CP}(f) = \frac{-2\text{Im}(v_u v_c^*) \cdot \text{Im}R_f}{|v_u|^2 + |v_c|^2 \cdot |R_f|^2 + 2\text{Re}(v_u v_c^*) \cdot \text{Re}R_f}, \tag{11}
\]

where \( R_f \) is a ratio of strong-interaction parts of the two amplitude components corresponding to \( v_c \) and \( v_u \). When calculating \( A_{CP}(f) \) with \( H_{\text{eff}}^{\text{pen}} \), we obtain [4-6]

\[
R_f = \frac{F_c(k^2) - F_t(k^2)}{F_u(k^2) - F_t(k^2)}. \tag{12}
\]

This implies that \( CP \) asymmetries in the transitions (I) or (II) will have the same magnitude and the same sign. While applying \( H_{\text{eff}} \) to the analysis, \( R_f \) becomes

\[
R_f' = \frac{F_c(k^2) - S(\zeta_P)}{F_u(k^2) - S(\zeta_P)}, \tag{13}
\]

where

\[
S(\zeta_P) = \frac{10}{9} - \frac{2}{3} \ln \frac{m_b^2}{m_W^2} + \frac{8\pi}{c_2 \cdot \alpha_s(m_b)} \cdot \left( \frac{\bar{c}_3}{N_c} + c_4 \right) + \left( \frac{\bar{c}_5}{N_c} + c_6 \right) \zeta_P \left( 1 - \frac{1}{N_c^2} \right) (1 + \zeta_P). \tag{14}
\]

Clearly \( R_f' \) includes the next-to-leading order QCD corrections and depends upon the factorization parameter \( \zeta_P \). Since the values of \( \zeta_P \) for \( B_u^- \to K^0 K^-, K^0 K^{*-}, \) and \( K^{*-} K^- \) (or
$K^* K^{*0}$ are different from one another, their $CP$ asymmetries will become non-degenerate in contrast with the result obtained from Eq. (12). So is the situation for the pure penguin channels $B_u^- \to \bar{K}^{(*)0} + (\pi^-, \rho^-, \pi^-)$. This feature, arising from the QCD corrections, requires us to reconsider the idea of suming over the pure penguin transitions (I) or (II) to gain a statistically significant $CP$-violating signal.

A comparison between $R_f'$ and $R_f$ shows that the QCD improvement only modifies $\text{Re} F_{u,c}(k^2)$. Hence the asymmetries in each group of decay modes remain the same sign. This ensures that there should be little dilution effect on the $CP$ asymmetries if one sums over the available final states $f$ and $\bar{f}$ in the transitions (I) or (II). In this case, one can obtain an effective branching ratio like

$$B_{\text{eff}} \equiv \sum_f B(f) = \tau_B \sum_f \Gamma(B_u^- \to f),$$

where $\tau_B$ is the lifetime of $B_u^-$ meson. The corresponding $CP$ asymmetry $A_{CP}$ is a weighted-mean value of $A_{CP}(f)$:

$$A_{CP} \equiv \frac{\left[ \sum_f \Gamma(B_u^+ \to \bar{f}) \right] - \left[ \sum_f \Gamma(B_u^- \to f) \right]}{\left[ \sum_f \Gamma(B_u^+ \to \bar{f}) \right] + \left[ \sum_f \Gamma(B_u^- \to f) \right]} = \frac{\sum_f \left\{ A_{CP}(f) \left[ B(\bar{f}) + B(f) \right] \right\}}{\sum_f \left[ B(\bar{f}) + B(f) \right]},$$

Although the value of $A_{CP}$ is impossible to deviate too much from the ones of $A_{CP}(f)$, it is expected that the effective branching ratio $B_{\text{eff}}$ can become several times larger than a single $B(f)$. Thus the total number of $B_u^\pm$ events needed in measuring $A_{CP}$ will be remarkably reduced and could be accessible in the forthcoming $B$-meson factories. Note that the weighted-mean asymmetry depends upon hadronic matrix elements and is therefore difficult to be evaluated in a reliable way. In the long run, however, a great improvement of the present calculations will be possible to yield trustworthy results.

IV Numerical results and discussion

For illustration, we make a numerical estimate of $CP$ asymmetries and branching ratios for the pure penguin-induced decay modes (I) and (II). The CKM parameters are taken as $\lambda = 0.22$, $A = 0.90$, $\rho = -0.50$, and $\eta = 0.30$, which are consistent with the present data on $B^0_d - \bar{B}^0_d$ mixing and $\epsilon_K$ [14]. We choose the values of the current quark masses as $m_u = 5$ MeV, $m_d = 10$ MeV, $m_s = 175$ MeV, $m_c = 1.35$ GeV, $m_b = 4.8$ GeV, and $m_t = 150$ GeV. Fixing $\Lambda_{\overline{MS}}^{(5)} = 200$ MeV, the effective QCD coupling constant and next-to-leading order
Wilson coefficients are obtained [10] as \( \alpha_s(m_b) = 0.18, \bar{c}_1 = -0.273, \bar{c}_2 = 1.123, \bar{c}_3 = 0.014, \bar{c}_4 = -0.032, \bar{c}_5 = 0.009, \) and \( \bar{c}_6 = -0.039. \) With the approximate rule of discarding \( 1/N_c \) corrections in the naive factorization approach [9], here we take \( 1/N_c = 0 \) for the relevant charmless nonleptonic \( B \) transitions. It is not clear what value of \( k^2 \) (the virtual gluon momentum in penguin graphs) should be taken in exclusive \( B \) decays. A simple estimate, involving two-body kinematics and one-gluon exchange to accelerate the spectator quark, yields \( m_b^2/4 \leq k^2 \leq m_b^2/2 \) [4]. To evaluate branching ratios we use \( \tau_B = 1.29 \times 10^{-12} \text{ s} \) [15] and quote values of the relevant decay constants and formfactors from Ref. [13].

Our numerical results of \( CP \) asymmetries \( \mathcal{A}_{CP}(f) \) and branching ratios \( B(f) \) are given in Fig. 2 and Tables 2 and 3, respectively, as functions of \( k^2 \). The following features can be observed:

1. Compared with the previous result without the next-to-leading order QCD corrections, now \( CP \) asymmetries in the transitions (I) or (II) are enhanced to some extent and become non-degenerate for the \( 0^{\pm}0^{\pm}, 0^{\pm}1^{\pm}, \) and \( 1^{\pm}0^{\pm} \) (or \( 1^{\pm}1^{\pm} \)) final states. As expected, \( \mathcal{A}_{CP}(f) \) keep the same sign even though their values may change significantly with \( k^2 \).

2. Among the transitions (I), the \( 0^{\pm}1^{\pm} \)-type decay mode \( B_u^- \rightarrow K^0 K^*- \) has the largest \( CP \) asymmetry but the smallest branching ratio. In contrast, the \( 0^{\pm}0^{\pm} \)-type channel \( B_u^- \rightarrow K^0 K^- \) has a relatively small \( CP \) asymmetry in spite of its largest branching ratio. So is the situation for the \( 0^{\pm}1^{\pm} \)- and \( 0^{\pm}0^{\pm} \)-type decay modes in the transitions (II). One can find that there exists no individual channel which has obvious advantages over the others of the group for measuring \( CP \) violation. For this reason, studying the weighted-mean \( CP \) asymmetries should be interesting in statistics.

3. For either the transitions (I) or (II), the weighted-mean asymmetry \( \mathcal{A}_{CP} \) is almost of the same magnitude as each \( \mathcal{A}_{CP}(f) \), but its corresponding effective branching ratio \( B_{eff} \) is several times larger than the individual \( B(f) \). For example, \( B_{eff}/B(K^0 K^-) \sim 2.5, \) \( B_{eff}/B(K^0 K^*) \sim 7, \) \( B_{eff}/B(K^{*0} K^-) \sim 4, \) and \( B_{eff}/B(K^{*0} K^*) \sim 3.5. \) As a result, measuring \( \mathcal{A}_{CP} \) instead of \( \mathcal{A}_{CP}(f) \) should reduce the needed \( B_u^{\pm} \) events several times and is therefore favored in practice. At the \( 3\sigma \) level, our estimates indicate that about \( 5 \times 10^8 \) and \( 2 \times 10^9 \) \( B_u^{\pm} \) pairs are available to probe the weighted-mean \( CP \) asymmetries of 7\% and 1\%, respectively, in the transitions (I) and (II). In experiments, to detect a handful of pure penguin channels with similar final-state particles (e.g., \( B_u^- \rightarrow K^{(*)0} + K^{(*)-} \)) might not be more difficult than to detect an individual channel among them.

Except for the uncertainty induced by the CKM parameters, there are some other theoretical uncertainties from the assumptions and approximations made in our calculations. It
is worthwhile at this point to give a brief summary of them:

(1) For the pure penguin-induced exclusive $B$ decays in question, $CP$ violation has been treated by postulating that the phases of the penguin amplitudes at the meson level is the same as those of the penguin loops at the quark level. This leads to the problem that the $CP$ asymmetries $A_{CP}(f)$ depend strongly upon the virtual gluon momentum ($k^2$) of the timelike penguin graphs. The large uncertainty arised from $k^2$ cannot be experimentally limited in a meaningful way, since it is not a measurable. To modify this unsatisfactory point, an attempt has been made by calculating the $k^2$ distribution of some exclusive $B$ decays and fold it with the momentum dependence of the loop amplitudes [16].

(2) Neglecting inelastic rescattering effects, pure penguin decay modes have only one isospin component. Hence their $CP$ asymmetries should suffer little from the unknown final-state interactions. Under the same condition, a sum over the transitions (I) or (II) is in principle available without suffering cancellations. Whether the above simplification is reasonable or not can be clarified in the forthcoming experiments of $B$-meson physics.

(3) It is seen that the $CP$ asymmetries $A_{CP}(f)$ are independent of hadronic matrix elements in the factorization approximation. However, the branching ratios $B(f)$ should be sensitive to the model applied to hadronic matrix elements. Although the weighted-mean asymmetries $A_{CP}$ are also affected by hadronic physics, the involved uncertainty should not be drastic in general. The reason is that both $A_{CP}(f)$ and $A_{CP}$ are ratios of the partial decay rates. As an illustration, our numerical estimates of $A_{CP}$ and $B_{eff}$ could give one a feeling of ballpark numbers to be expected.

In conclusion, the pure penguin-induced $B_u^\pm$ decays are of great interest to uncover direct $CP$ violation in the decay amplitude. A weighted-mean signal of $CP$ asymmetries may be more easily observed in experiments, although to evaluate it with reliability remains difficult in theories. In order to test the strong-interaction penguin picture and direct $CP$ violation mechanism at $B$-meson factories, a further study of the dynamics of nonleptonic exclusive $B$ decays is urgent.

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Figure Captions

**Fig. 1** Pure penguin-induced decay modes $B_\mu^- \rightarrow X(qq') + Y(q'\bar{u})$ with $(q, q') = (d, s)$ or $(s, d)$: (a) the tree-level matrix elements of the penguin operators, and (b) the one-loop penguin matrix elements of the current-current operators.

**Fig. 2** $CP$ asymmetries versus $k^2$ (virtual gluon momentum in penguin graphs) for a few pure penguin-induced $B_\mu^\pm$ decay modes: (a) exclusive $b \rightarrow d$, and (b) exclusive $b \rightarrow s$. The circled curve is the degenerate result without QCD corrections; the dashed, dotted, and dot-dashed curves stand for the QCD-corrected results; and the dagged curve is the weighted-mean $CP$-violating signal.
Table 1  Factorization parameters $\zeta_P$ (defined in Eq. (9)) for the decay modes $B_u^- \rightarrow XY$, where $X$ and $Y$ may be $0^\pm$ or $1^\pm$ mesons.

Table 2  Branching ratios $B(f) \times 10^{-7}$ versus $k^2$ (virtual gluon momentum in penguin graphs) for $B_u^- \rightarrow K^{(*)0} + K^{(*)-}$. $B_{\text{eff}} \times 10^{-7}$ is an effective branching ratio obtained by summing over these four pure penguin modes.

Table 3  Branching ratios $B(f) \times 10^{-6}$ versus $k^2$ (virtual gluon momentum in penguin graphs) for $B_u^- \rightarrow \bar{K}^{(*)0} + (\pi^-, \rho^-, a_1^-)$. $B_{\text{eff}} \times 10^{-6}$ is an effective branching ratio obtained by summing over these six pure penguin modes.

### Table 1

| $(J^P_X, J^P_Y)$ | $\zeta_P$ |
|------------------|-----------|
| $(0^\pm, 0^\pm)$ | $\frac{2m_X^2}{(m_q + m_d') m_b}$ |
| $(0^\pm, 1^\pm)$ | $\frac{-2m_X^2}{(m_q + m_d') m_b}$ |
| $(1^\pm, 0^\pm)$ | 0 |
| $(1^\pm, 1^\pm)$ | 0 |
Table 2

| $k^2/m_b^2$ | $B(K^0 K^-)$ | $B(K^0 K^{*-})$ | $B(K^{*0} K^-)$ | $B(K^{*0} K^{*-})$ | $B_{\text{eff}}$ |
|------------|--------------|-----------------|-----------------|-----------------|--------------|
| 0.1        | 12.5         | 0.39            | 7.47            | 9.20            | 29.6         |
| 0.2        | 12.5         | 0.39            | 7.47            | 9.20            | 29.6         |
| 0.3        | 14.2         | 0.46            | 8.54            | 10.5            | 33.7         |
| 0.4        | 15.8         | 0.53            | 9.56            | 11.8            | 37.7         |
| 0.5        | 14.6         | 0.49            | 8.86            | 10.9            | 34.8         |
| 0.6        | 13.8         | 0.46            | 8.32            | 10.2            | 32.8         |
| 0.7        | 13.1         | 0.43            | 7.87            | 9.69            | 31.1         |
| 0.8        | 12.5         | 0.41            | 7.53            | 9.27            | 29.7         |
| 0.9        | 12.0         | 0.39            | 7.22            | 8.89            | 28.5         |
| 1.0        | 11.6         | 0.38            | 6.98            | 8.59            | 27.6         |

Table 3

| $k^2/m_b^2$ | $B(\bar{K}^0 \pi^-)$ | $B(\bar{K}^0 \rho^-)$ | $B(\bar{K}^0 a_1^-)$ | $B(\bar{K}^{*0} \pi^-)$ | $B(\bar{K}^{*0} \rho^-)$ | $B(\bar{K}^{*0} a_1^-)$ | $B_{\text{eff}}$ |
|------------|-----------------------|------------------------|-----------------------|------------------------|------------------------|------------------------|--------------|
| 0.1        | 7.77                  | 0.22                   | 0.19                  | 4.84                   | 5.99                   | 5.39                   | 24.4         |
| 0.2        | 8.77                  | 0.25                   | 0.23                  | 5.50                   | 6.81                   | 6.13                   | 27.7         |
| 0.3        | 11.3                  | 0.36                   | 0.32                  | 7.20                   | 8.92                   | 8.03                   | 36.1         |
| 0.4        | 12.0                  | 0.39                   | 0.35                  | 7.67                   | 9.50                   | 8.55                   | 38.5         |
| 0.5        | 10.7                  | 0.34                   | 0.30                  | 0.81                   | 8.43                   | 7.59                   | 34.2         |
| 0.6        | 9.79                  | 0.31                   | 0.28                  | 6.23                   | 7.71                   | 6.94                   | 31.3         |
| 0.7        | 9.13                  | 0.28                   | 0.25                  | 5.79                   | 7.17                   | 6.45                   | 29.1         |
| 0.8        | 8.65                  | 0.27                   | 0.24                  | 5.47                   | 6.78                   | 6.10                   | 27.5         |
| 0.9        | 8.22                  | 0.25                   | 0.22                  | 5.19                   | 6.43                   | 5.79                   | 26.1         |
| 1.0        | 7.90                  | 0.24                   | 0.21                  | 4.99                   | 6.17                   | 5.55                   | 25.1         |