The collapse distance of femtosecond pulses in air

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(Dated: July 24, 2015)

Abstract

The conventional semi-empirical formula for collapse distance [Phys. Rev. 179, 862 (1969); Prog. Quant. Electr. 4, 35 (1975)] has been widely used in many applications. However, it is not applicable when the dispersion length is smaller than or has similar order-of-magnitude as the collapse distance. For the “enough short” pulses, there exists a threshold for the initial peak power, with which the collapse distance has a maximum value due to the competition between the Kerr self-focusing and the group velocity dispersion. New semi-empirical formulas are obtained for the collapse distance of the pulse with the initial power being less or larger than the threshold, and they can match the numerical simulations gracefully.

PACS numbers: 42.65.Jx, 42.65.Tg
I. INTRODUCTION

The ultra-short high intensity laser pulses have many applications such as laser-guided electric discharge [1], terahertz generation [2] and remote sensing [3]. Such applications need extremely high light powers which can be obtained via the collapse of pulses [4–7].

For a Gaussian beam, if the initial peak power exceeds the critical power for self-focusing, the beam will undergo collapse until the higher-order processes such as plasma or high-order Kerr effects halt the collapse [8–10]. In most cases, the collapse distance $L_c$ (the propagation length of the self-focusing beam until collapse) can be well approximated by a semi-empirical formula [8, 10–15]

$$L_c \approx L_{semi}^c = \frac{0.367 \pi n_0 r_0^2 / \lambda_0}{\sqrt{[(P_{in}/P_{cr})^{1/2} - 0.852]^2 - 0.0219}} ,$$

where $n_0$ is the refractive index, $r_0$ is the beam width which is at $1/e^2$ level of intensity, $\lambda_0$ is the laser wavelength in vacuum, $P_{in}$ is the initial pulse’s peak power, and $P_{cr}$ is the critical power for self-focusing which can be written as $P_{cr} = 3.77 \lambda_0^2 / 8 \pi n_0 n_2$ with $n_2$ being the Kerr index. However, for the very high powers such as $P_{in} = 100 P_{cr}$, experiments and numerical simulations show that the collapse distance cannot be described by $L_{semi}^c$ and a transition from a $1/\sqrt{P_{in}}$ to a $1/P_{in}$ scaling was observed [16]. Recently, simulations show that the group-velocity-dispersion (GVD) has a great influence on $L_c$ when the pressure is relatively high, e.g., 10 atm [17].

In this paper, we investigate the collapse distance of femtosecond laser pulses in air for different temporal durations. We find that for the “very short” pulses, there exists a threshold for the initial power, with which the collapse distance has a maximum value. More importantly, new semi-empirical formulas are obtained for the collapse distance of the pulse with the initial power being less than or larger than the threshold. The new formulas take into account of the influence of GVD and can match the numerical simulations perfectly, including the cases of high pressures at which the GVD becomes more important.

II. THEORETICAL MODEL

The physical processes which halt beam collapse is still controversial. Some groups think the plasma prevents collapse (Kerr-plasma model) [18–24], while other groups believe the collapse is stopped by the high-order Kerr effects (HOKE model) [25–31]. Fortunately,
the collapse distances are almost the same for Kerr-plasma model and HOKE model \cite{25,32}, since the HOKE and the plasma effect becomes important only after the pulse has collapsed \cite{33}, and this phenomena has also been confirmed in our numerical simulations (not shown here). In this work we study the collapse distance of the pulse for different temporal durations via HOKE model, which can be described by an extended non-linear Schrödinger equation (NLSE) \cite{25,27}

\[
\frac{\partial A}{\partial z} = \frac{i}{2k_0} \Delta_\perp A - \frac{ik''}{2} \frac{\partial^2 A}{\partial \tau^2} + \frac{ik_0}{n_0} \left( \sum_{j=1}^{4} n_{2s,j} |A|^{2s,j} \right) A - \frac{ik_0 \omega_{pe}^2}{2 \omega_0^2} A - \frac{A}{2} \sum_{l=O_2,N_2} \left( \frac{W_l(I)U_l}{|A|^2} (\rho_{at,l} - \rho_{e,l}) \right),
\]

where \( A \) represents the envelope of the electric field, and \( z \) denotes the propagation distance. \( k_0 = 2\pi/\lambda_0 \) (\( \lambda_0 = 800 \) nm) is the central wave number. The Laplacian operator \( \Delta_\perp = \partial^2_r + 1/r \partial_r \) denotes the beam transverse diffraction. The remaining terms in the right-hand-side of Eq.(1) account for the group velocity dispersion with the second order dispersion coefficient \( k'' = 0.2 \) fs\(^2\)/cm, Kerr and high-order Kerr effect with nonlinear refractive index \( n_2 = 1.2 \times 10^{-19} \) cm\(^2\)/W, \( n_3 = -1.5 \times 10^{-33} \) cm\(^4\)/W\(^2\), \( n_6 = 2.1 \times 10^{-46} \) cm\(^6\)/W\(^4\), and \( n_8 = -0.8 \times 10^{-59} \) cm\(^8\)/W\(^4\) \cite{34,35}, plasma defocusing with the plasma oscillation frequency \( \omega_{pe} = \sqrt{q_e^2 \rho/m_e \epsilon_0} \) (\( q_e \) is the electron charge, \( m_e \) is the electron mass and \( \rho \) is the free electron density.), the energy loss caused by MPA. \( W_{N_2}(I) \) and \( W_{O_2}(I) \) are the photoionization rate of \( N_2 \) and \( O_2 \). \( \rho_{at,N_2} = 2.1 \times 10^{25} \) m\(^{-3}\) and \( \rho_{at,O_2} = 5.7 \times 10^{24} \) m\(^{-3}\) are the density of \( N_2 \) and \( O_2 \) molecular at 1 atm. \( \rho_{e,N_2} \) and \( \rho_{e,O_2} \) are free electron density ionized by \( N_2 \) and \( O_2 \) \((\rho = \rho_{e,N_2} + \rho_{e,O_2})\) which can be calculated by the following equations

\[
\frac{\partial \rho_{e,N_2}}{\partial \tau} = W_{N_2}(I) (\rho_{at,N_2} - \rho_{e,N_2}) \quad \text{(3)}
\]

\[
\frac{\partial \rho_{e,O_2}}{\partial \tau} = W_{O_2}(I) (\rho_{at,O_2} - \rho_{e,O_2}) \quad \text{(4)}
\]

The photoionization rate of \( N_2 \) and \( O_2 \) is obtained by the Keldysh-PPT (Perelomov-Popov-Terent’ev) formula \cite{36}.

The initial pulse investigated in this paper is Gaussian beam which can be written as:

\[
E(r, \tau, 0) = \sqrt{\frac{2P_m}{\pi r_0^2}} \exp \left( -\frac{r^2}{r_0^2} - \frac{\tau^2}{\tau_0^2} \right),
\]

where \( \tau_0 \) is the temporal duration.
III. Results and Discussions

Follow Li et al. [17], we define the collapse distance as the distance between the light source and the position where the laser beam has the smallest radius. Fig. 1 shows the evolution of the beam radius of pulses which have different temporal durations (80 fs and 300 fs). It can be seen that the collapse distances of the two pulses have big difference (207 m and 135 m). In contrast, the semi-empirical formula in Eq. (1) gives $L_{\text{semi}} = 129$ m since it does not take into account of the pulse’s duration. Therefore, the temporal duration of pulse may have a large influence on the collapse distance in some circumstances.

![Graph showing the variation of the collapse distances with different initial powers and temporal durations.](image)

FIG. 1. Beam radius of the pulse at different propagation distances. The input beam are Gaussian pulses with $r_0 = 18$ mm and $P_{in}/P_{cr} = 20$.

From now on, we focus on the influence of the temporal duration to the collapse distance. In the simulations, we consider the Gaussian pulse with two different beam radius ($r_0 = 2$ mm and $r_0 = 9$ mm).

Fig. 2 shows the variation of the collapse distances with different initial powers and temporal durations. Here $r_0 = 2$ mm, $P_{in}/P_{cr}$ varies from 1.5 to 6, and $\tau_0$ varies from 10 to 100 fs. For comparisons, the collapse distances calculated by $L_{\text{semi}}$ is also given and shown in solid line. We can observe that $L_{\text{semi}}$ can approximate $L_c$ very well for the long durations such as $\tau_0 = 60$ fs, 80 fs and 100 fs, but it fails for the short ones, e.g., $\tau_0 = 10$ fs, 15 fs and 30 fs. In order to see more clearly how the collapse distances of the short-temporal-duration
pulses change with the input power, we show the results in Fig. 3 for \( \tau_0 = 10 \text{ fs}, 15 \text{ fs}, 30 \text{ fs} \) and \( P_{in}/P_{cr} \) being in the range between 1.5 and 12. It can be seen from this figure that the collapse distances of pulses with these short durations have a maxima and tend to \( L_{c semi} \) in the limit of \( P_{in}/P_{cr} \) going to infinity. We define the input peak power which leads to the maximum collapse distance as \( P_{MC} \). If \( P_{in} < P_{MC} \), the collapse distance increases with \( P_{in} \), and if \( P_{in} > P_{MC} \), the collapse distance decreases with \( P_{in} \).

![Graph showing variation of collapse distance with input peak power](image)

FIG. 2. Variation of the collapse distance of pulses with the initial peak power. The input beam are Gaussian pulses with \( r_0 = 2 \text{ mm} \).

Fig. 4 shows the collapse distances for the pulses with \( r_0 = 9 \text{ mm} \). \( P_{in}/P_{cr} \) varies from 2 to 30, and \( \tau_0 = 40 \text{ fs}, 80 \text{ fs}, 200 \text{ fs}, 450 \text{ fs} \), respectively. From Fig. 4 we can get the same conclusion as shown in Fig. 2 and 3 that \( L_{c} \) has a maximal value when the temporal duration is very short. It is worth pointing out that “very short” is relative to the radius of pulses. For example, \( \tau_0 = 80 \text{ fs} \) is “very short” when \( r_0 = 9 \text{ mm} \), whereas it is not “very short” when \( r_0 = 2 \text{ mm} \).

The phenomena that \( L_{c semi} \) can not fit \( L_{c} \) well for the “very short” pulses is due to GVD. The normal GVD enables power exchanges between different time slices of the pulse and disperses the latter in time, and this may contribute to maintaining the pulse self-guiding at smaller intensity levels \(^{37,39}\). If the pulse’s temporal duration is long enough, the effects of GVD can be neglected when the propagation distance is less than the collapse...
distance, and thus $L_{c}^{semi}$ can fit the collapse distance very well. However, if the pulse’s
temporal duration is very short, the GVD effect will have a strong competition to the Kerr
self-focusing effect when the initial power is small. Let’s consider an extremely short pulse
with small initial peak power. In this extreme situation, the GVD effect overwhelms the
Kerr self-focusing, and the collapse distance equals the propagation distance at which the
peak intensity decreases to $P_{cr}$. At the same time, the temporal duration varies with $z$ as
$\tau_{1}(z) = \tau_{0}[1 + (z/2L_{GVD})^{2}]^{1/2}$ \[40\], here $L_{GVD} = \tau_{0}^{2}/2k''$ denotes the dispersion length \[12\],
and thus the peak power can be written as $P(z) = P_{in}/\tau_{1}(z)$. The collapse distance in this
case can be approximated by

$$L_{c}^{extreme} = 2L_{GVD}\sqrt{(P_{in}/P_{cr})^{2} - 1}.$$  \[(6)\]

This equation may account for the phenomena that the collapse distances of short pulses
increase with $P_{in}/P_{cr}$ when $P_{in} < P_{MC}$ (Figs. 2-4), though Eq. 6 can not be used to estimate
$L_{c}$ accurately in general cases.

With closer inspections for Figs. 2-4 we find that $L_{c}$ is proportional to $P_{in}/P_{cr}$ when
$P_{in} < P_{MC}$, and the slope is dependent on the temporal duration and the radius of pulse.
Taking into account the GVD effects, we finally obtain the new semi-empirical formulas as
FIG. 4. Variation of the collapse distance with the initial peak power \( P_{\text{in}}/P_{\text{cr}} \) range from 2 to 30. The input beam are Gaussian pulses with \( r_0 = 9 \) mm. The temporal durations of the pulses are 40 fs, 80 fs, 200 fs, 450 fs.

follow

\[
L_{\text{semi,new}} = \begin{cases} 
0.225 L_{\text{GVD}} \frac{P_{\text{in}}}{P_{\text{cr}}} + 0.320 L_{\text{DF}}, & \text{if } P_{\text{in}} < P_{\text{MC}}, \\
L_{\text{semi}} \times 1.878^N, & \text{otherwise,} 
\end{cases}
\tag{7a}
\]

where \( L_{\text{DF}} = \pi n_0 r_0^2 / \lambda_0 \) is the Rayleigh length, which account for the contribution from the diffraction. \( N \) is a dimensionless parameter and

\[
N = \frac{L_{\text{semi}}}{L_{\text{GVD}}},
\tag{8}
\]

which characterizes the relative importance of the Kerr self-focusing and the GVD. When \( N \gg 1 \) GVD dominates, while for \( N \ll 1 \) Kerr self-focusing dominates.

From Eq. (7), we can see that when \( P_{\text{in}} < P_{\text{MC}} \) the collapse distance is mainly determined by the GVD and the diffraction, otherwise, all the GVD, the diffraction and the Kerr self-focusing plays important roles. The input power threshold \( P_{\text{MC}} \) can be obtained via equaling Eq. (7a) and Eq. (7b). The relations between \( P_{\text{MC}} \), the duration \( \tau_0 \) and the radius \( r_0 \) of the laser pulse are shown in Fig. 5. It can be seen that \( P_{\text{MC}} \) increases with \( r_0 \). At the same time, \( P_{\text{MC}} \) decreases with \( \tau_0 \), and it reduces to \( P_{\text{cr}} \) in the the limit of \( \tau_0 \to \infty \).

In real applications of the new semi-empirical formula, we do not need the value of \( P_{\text{MC}} \).
Notice that Eq. (7a) is a monotonically increasing function of $P_m$, while Eq. (7b) is a monotonically decreasing function of $P_m$, and the value of input power corresponding to the intersection of these two functions is $P_{MC}$. Therefore, we can directly use $\min\{\text{Eq. (7a), Eq. (7b)}\}$ to obtain the collapse distance $L_c$ for a given $P_m$, without knowing the value of $P_{MC}$.

Fig. 6 shows the comparisons of the collapse distances between the new semi-empirical formula, the direct numerical simulations, and the conventional semi-empirical formula. It can be seen that the new semi-empirical formula agree with the simulation results very well, for both the long and short pulses.

Now we turn to the case shown in Fig. 1, in which the collapse distances are obtained by numerical simulations. Basing on the new semi-empirical formula given in Eq. (7), we obtain the collapse distance of 214 m (207 m in simulation) for 80 fs and 134 m (135 m in simulation) for 300 fs.

Eq. (7) can also be used to calculate the collapse distance at high pressures. $L_{GV\,D}$ is inversely proportional to the pressure, and play more important roles at high pressure. For example, in the work of [17], the authors obtained the collapse distance of the pulse with $\tau_0 = 50$ fs and $r_0 = 1.2$ mm, $P_m = 4P_{cr}$ is 2.3 m at 10 atm, while the new semi-empirical formula gives $L_c = 2.2$ m, perfectly matched.
FIG. 6. The collapse distance predicted by the new semi-empirical formula. The input beam are Gaussian pulses with (a) $r_0=2\ \text{mm}$, (b) $r_0=9\ \text{mm}$.

IV. CONCLUSION

GVD plays important roles in the evolution of the laser pulses in air. For the short pulses, due to the competition between GVD and Kerr self-focusing, there exists a threshold $P_{MC}$ for the initial power, with which the collapse distance has a maximum value. If the
initial power is less than the threshold, the collapse distance is proportional to the input power. If the initial power is larger than the threshold, the collapse distance decreases with the input power, and is larger than that obtained by the conventional semi-empirical formula. Taking into account the effects of GVD, we present new semi-empirical formulas, which match the numerical simulations very well. The formulas can also be applicable to the cases for high pressures in which the GVD effects become more important.

V. ACKNOWLEDGMENTS

This work was supported in part by the Ph.D. Programs Foundation of Ministry of Education of China Grant No. 20110184110016, the National Basic Research Program of China (973 Program) Grant No. 2013CB328904, and the Fundamental Research Funds for the Central Universities.

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