Weak Soft \((1,2)^*\)-\(\tilde{D}_{\omega}\)-Sets and Weak Soft \((1,2)^*\)-\(\tilde{D}_{\omega}\)-Separation Axioms in Soft Bitopological Spaces

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Abstract

In this article we introduce and characterize new types of soft sets in soft bitopological spaces, namely, soft \((1,2)^*\)-omega difference sets (briefly soft \((1,2)^*\)-\(\tilde{D}_{\omega}\)-sets) and weak forms of soft \((1,2)^*\)-omega difference sets. Moreover we use these soft sets to study new types of soft separation axioms, namely, soft \((1,2)^*\)-\(\omega\)-\(\tilde{D}_j\)-spaces, soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-\(\tilde{D}_j\)-spaces, soft \((1,2)^*\)-\(\pre\omega\)-\(\tilde{D}_j\)-spaces, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-\(\tilde{D}_j\)-spaces, and soft \((1,2)^*\)-\(\beta\)-\(\omega\)-\(\tilde{D}_j\)-spaces, for \(j = 0,1,2\). Furthermore we investigate the characterizations and the relations between these types of soft separation axioms and other soft separation axioms. [DOI: 10.22401/ANJS.22.2.07]

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Introduction

Shabir and Naz [1] introduced and studied the concept of soft topological spaces by using the notion of soft sets which is introduced by Molodtsov [2]. Senel and Çağman [3] investigated the concept of soft bitopological spaces over an initial universe set with a fixed set of parameters. Mahmood and Abdul-Hady [4,5] introduced and studied new types of soft sets in soft bitopological spaces called soft \((1,2)^*\)-omega open sets and weak forms of soft \((1,2)^*\)-omega open sets such as soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-open sets, soft \((1,2)^*\)-\(\pre\omega\)-\(\omega\)-open sets, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-open sets and soft \((1,2)^*\)-\(\beta\)-\(\omega\)-open sets and we use them to define and study new classes of soft separation axioms called soft \((1,2)^*\)-omega separation axioms and weak soft \((1,2)^*\)-omega separation axioms in soft bitopological spaces. The purpose of this paper is to define and study new types of soft separation axioms called weak soft \((1,2)^*\)-\(\tilde{D}_{\omega}\)-separation axioms in soft bitopological spaces by using weak soft \((1,2)^*\)-\(\tilde{D}_{\omega}\)-sets such as soft \((1,2)^*\)-\(\omega\)-\(\tilde{D}_j\)-spaces, soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-\(\tilde{D}_j\)-spaces, soft \((1,2)^*\)-\(\pre\omega\)-\(\omega\)-\(\tilde{D}_j\)-spaces, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-\(\tilde{D}_j\)-spaces and soft \((1,2)^*\)-\(\beta\)-\(\omega\)-\(\tilde{D}_j\)-spaces, for \(j = 0,1,2\). Moreover we study the basic properties and the relationships between these types of soft separation axioms and other soft separation axioms.

1. Preliminaries:

Throughout this paper \(X\) is an initial universe set, \(P(X)\) is the power set of \(X\), \(P\) is the set of parameters and \(A \subseteq P\).

Definition (1.1)[2]: A soft set over \(X\) is a pair \((U,A)\), where \(U\) is a function defined by \(U: A \rightarrow P(X)\) and \(A\) is a non-empty subset of \(P\).

Definition (1.2)[6]: A soft set \((U,A)\) over \(X\) is called a soft point if there is exactly \(a \in A\) such that \(U(a) = \{x\}\) for some \(x \in X\) and \(U(a') = \emptyset\). \(\forall\ a' \in A \setminus \{a\}\) and is denoted by \(\tilde{x} = (a,\{x\})\).

Definition (1.3)[6]: A soft point \(\tilde{x} = (a,\{x\})\) is called soft belongs to a soft set \((U,A)\) if \(a \in A\) and \(x \in U(a)\), and is denoted by \(\tilde{x} \in (U,A)\).

Definition (1.4)[6]: A soft set \((U,A)\) over \(X\) is called countable (finite) if the set \(U(a)\) is countable (finite) \(\forall\ a \in A\).
Definition (1.5)[1]: A soft topology on X is a family $\tau$ of soft subsets of $\tilde{X}$ having the following properties:
(i) $\emptyset \in \tau$ and $\tilde{X} \in \tau$.
(ii) If $(U_1, P), (U_2, P) \in \tau$, then $(U_1, P) \cap (U_2, P) \in \tau$.
(iii) If $(U_j, P) \in \tau, \forall j \in \Lambda$, then $\bigcup_{j \in \Lambda} (U_j, P) \in \tau$.

The triple $(X, \tau, P)$ is called a soft topological space over X. The elements of $\tau$ are called soft open sets in $\tilde{X}$. The complement of a soft open set is called soft closed.

Definition (1.6)[3]: Let X be a non-empty set and let $\tau_1$ and $\tau_2$ be soft topologies over X. Then $(X, \tau_1, \tau_2, P)$ is called a soft bitopological space over X.

Definition (1.7)[3]: A soft subset $(U, P)$ of a soft bitopological space $(X, \tau_1, \tau_2, P)$ is called soft $\tau_1\tau_2$-open if $(U, P) = (U_1, P) \cup (U_2, P)$ such that $(U_1, P) \in \tau_1$ and $(U_2, P) \in \tau_2$. The complement of a soft $\tau_1\tau_2$-open set in $\tilde{X}$ is called soft $\tau_1\tau_2$-closed.

Definition (1.8)[4]: A soft subset $(A, P)$ of a soft bitopological space $(X, \tau_1, \tau_2, P)$ is called soft $(1,2)^\omega$-open (briefly soft $(1,2)^\omega$-open) if for each $\tilde{x} \in (A, P)$, there exists a soft $\tau_1\tau_2$-open set $(U, P)$ in $\tilde{X}$ such that $\tilde{x} \in (U, P)$ and $(U, P) - (A, P)$ is countable. The complement of a soft $(1,2)^\omega$-open set is called soft $(1,2)^\omega$-open (briefly soft $(1,2)^\omega$-closed).

Definitions (1.9)[4]: A soft subset $(A, P)$ of a soft bitopological space $(X, \tau_1, \tau_2, P)$ is called:
(i) A soft $(1,2)^\omega$-open set if $(A, P) \subseteq (1,2)^\omega$-oint($\tau_1\tau_2$cl(A,P))
(ii) A soft $(1,2)^\omega$-pre-open set if $(A, P) \subseteq (1,2)^\omega$-oint($\tau_1\tau_2$cl(A,P)).

(iii) A soft $(1,2)^\beta$-$b$-$\omega$-open set if $(A, P) \subseteq (1,2)^\beta$-$b$-$\omega$-oint($\tau_1\tau_2$cl(A,P))
(iv) A soft $(1,2)^\beta$-$\omega$-open set if $(A, P) \subseteq (1,2)^\beta$-$\omega$-oint($\tau_1\tau_2$cl(A,P)).

Proposition (1.10)[4]: If $(X, \tau_1, \tau_2, P)$ is a soft bitopological space. Then:
(i) Every soft $\tau_1\tau_2$-open set is soft $(1,2)^\omega$-open.
(ii) Every soft $(1,2)^\alpha$-$\omega$-open set is soft $(1,2)^\omega$-open.
(iii) Every soft $(1,2)^\alpha$-$\omega$-open set is soft $(1,2)^\omega$-pre-$\omega$-open.
(iv) Every soft $(1,2)^\beta$-$\omega$-open set is soft $(1,2)^\beta$-$\omega$-open.
(v) Every soft $(1,2)^\beta$-$\omega$-open set is soft $(1,2)^\beta$-$\omega$-open.

Definitions (1.11)[5],[7]: A soft bitopological space $(X, \tau_1, \tau_2, P)$ is called a soft $(1,2)^\omega$-$\tilde{T}_0$-space (resp. soft $(1,2)^\alpha$-$\omega$-$\tilde{T}_0$-space, soft $(1,2)^\beta$-$\omega$-$\tilde{T}_0$-space, soft $(1,2)^\beta$-$b$-$\omega$-$\tilde{T}_0$-space, soft $(1,2)^\beta$-$b$-$\omega$-$\tilde{T}_0$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there exists a soft $\tau_1\tau_2$-open (resp. soft $(1,2)^\omega$-open, soft $(1,2)^\alpha$-$\omega$-open, soft $(1,2)^\beta$-$\omega$-open, soft $(1,2)^\beta$-$b$-$\omega$-open, soft $(1,2)^\beta$-$b$-$\omega$-open) set in $\tilde{X}$ containing one of the soft points but not the other.

Definition (1.12)[5],[7]: A soft bitopological space $(X, \tau_1, \tau_2, P)$ is called a soft $(1,2)^\omega$-$\tilde{T}_1$-space (resp. soft $(1,2)^\alpha$-$\omega$-$\tilde{T}_1$-space, soft $(1,2)^\beta$-$\omega$-$\tilde{T}_1$-space, soft $(1,2)^\beta$-$b$-$\omega$-$\tilde{T}_1$-space, soft $(1,2)^\beta$-$b$-$\omega$-$\tilde{T}_1$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there are two soft $\tau_1\tau_2$-open (resp. soft $(1,2)^\omega$-open, soft $(1,2)^\alpha$-$\omega$-open, soft $(1,2)^\beta$-$\omega$-open, soft $(1,2)^\beta$-$b$-$\omega$-open, soft $(1,2)^\beta$-$b$-$\omega$-open) sets $(U, P)$ and $(V, P)$ in $\tilde{X}$ such that $\tilde{x} \in (U, P), \tilde{y} \notin (U, P)$ and $\tilde{y} \in (V, P), \tilde{x} \notin (V, P)$.
Definition (1.13)[5],[7]: A soft bitopological space \((X, \tilde{t}_1, \tilde{t}_2, P)\) is called a soft \((1,2)^*\)-\(T_2\)-space (resp. soft \((1,2)^*\)-\(\omega\)-\(T_2\)-space, soft \((1,2)^*\)-\(\alpha\)-\(T_2\)-space, soft \((1,2)^*\)-\(\beta\)-\(T_2\)-space, soft \((1,2)^*\)-\(\omega\)-\(\beta\)-\(T_2\)-space) if for any two distinct soft points \(\tilde{x}\) and \(\tilde{y}\) of \(\tilde{X}\), there are two soft \(\tilde{t}_1\tilde{t}_2\)-open (resp. soft \((1,2)^*\)-\(\omega\)-open, soft \((1,2)^*\)-\(\alpha\)-open, soft \((1,2)^*\)-\(\beta\)-open, soft \((1,2)^*\)-\(\omega\)-\(\beta\)-open) sets \((U, P)\) and \((V, P)\) in \(\tilde{X}\) such that \(\tilde{x} \in (U, P), \tilde{y} \in (V, P)\) and \((U, P) \cap (V, P) = \emptyset\).

Proposition (1.14)[5]: Every soft bitopological space is a soft \((1,2)^*\)-\(\omega\)-\(T_j\)-space (resp. soft \((1,2)^*\)-\(\alpha\)-\(T_j\)-space, soft \((1,2)^*\)-\(\beta\)-\(T_j\)-space, soft \((1,2)^*\)-\(\omega\)-\(\beta\)-\(T_j\)-space), for \(j = 0, 1\).

Definition (1.15)[5]: A soft function \(f: (X, \tilde{t}_1, \tilde{t}_2, P) \rightarrow (Y, \tilde{f}_1, \tilde{f}_2, P)\) is called strongly soft \((1,2)^*\)-\(\omega\)-continuous (resp. strongly soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-open, strongly soft \((1,2)^*\)-\(\beta\)-\(\omega\)-open) if \(f^{-1}(U, P)\) is a soft \(\tilde{t}_1\tilde{t}_2\)-open set in \(\tilde{X}\) for each soft \((1,2)^*\)-\(\omega\)-open (resp. soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-open, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-open) set \((U, P)\) in \(\tilde{Y}\).

Definition (1.16)[5]: A soft function \(f: (X, \tilde{t}_1, \tilde{t}_2, P) \rightarrow (Y, \tilde{f}_1, \tilde{f}_2, P)\) is called strongly soft \((1,2)^*\)-\(\omega\)-open (resp. strongly soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-open, strongly soft \((1,2)^*\)-\(\beta\)-\(\omega\)-open) if \(f(U, P)\) is a soft \(\tilde{f}_1\tilde{f}_2\)-open set in \(\tilde{Y}\) for each soft \((1,2)^*\)-\(\omega\)-open (resp. soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-open, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-open) set \((U, P)\) in \(\tilde{X}\).

2. Weak Soft \((1,2)^*\)-\(\bar{D}_\omega\)-Sets

In this section, we introduce and study new concepts called soft \((1,2)^*\)-\(\bar{D}_\omega\)-sets, soft \((1,2)^*\)-\(\bar{D}_{\alpha\omega}\)-sets, soft \((1,2)^*\)-\(\bar{D}_{\beta\omega}\)-sets, soft \((1,2)^*\)-\(\bar{D}_{\alpha\beta}\)-sets and soft \((1,2)^*\)-\(\bar{D}_{\beta\alpha}\)-sets in soft bitopological spaces. Further we investigate the relationships between these types of soft sets and other soft sets.

Definition (2.1): A soft subset \((A, P)\) of a soft bitopological space \((X, \tilde{t}_1, \tilde{t}_2, P)\) is called a soft \((1,2)^*\)-\(\bar{D}_\omega\)-set (resp. soft \((1,2)^*\)-\(\bar{D}_{\alpha\omega}\)-set, soft \((1,2)^*\)-\(\bar{D}_{\beta\omega}\)-set, soft \((1,2)^*\)-\(\bar{D}_{\beta\alpha}\)-set) if there exists two soft \((1,2)^*\)-\(\omega\)-open (resp. soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-open, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-open, soft \((1,2)^*\)-\(\alpha\)-\(\beta\)-open) sets \((A_1, P)\) and \((A_2, P)\) in \(\tilde{X}\) such that \((A_1, P) \neq \tilde{X}\) and \((A_2, P) = (A_1, P) \setminus (A_2, P)\).

Example (2.2): Let \(X = \mathcal{R}\), \(P = \{p_1, p_2\}\) and let \(\tilde{t}_1 = (\tilde{X}, \tilde{\phi}, (U, P))\) be soft topologies over \(X\), where \((U, P) = \{(p_1, \{4\}),(p_2, \{4\})\}\). The soft sets in \(\{\tilde{X}, \tilde{\phi}, (U, P)\}\) are soft \(\tilde{t}_1\tilde{t}_2\)-open sets in \(\tilde{X}\). Then \((A, P) = \{(p_1, \{3\}),(p_2, \{3\})\}\) is a soft \((1,2)^*\)-\(\bar{D}_\omega\)-set, since \(\exists (A_1, P) = ((p_1, \mathcal{R} \setminus \{2\}),(p_2, \mathcal{R} \setminus \{2\})\) and \((A_2, P) = ((p_1, \mathcal{R} \setminus \{3\}),(p_2, \mathcal{R} \setminus \{3\})\) are soft \((1,2)^*\)-\(\omega\)-open sets in \(\tilde{X}\) such that \((A_1, P) \neq \mathcal{R}\) and \((A_2, P) = (A_1, P) \setminus (A_2, P)\).

Remark (2.3): In definition (2.1), if \((A_1, P) \neq \tilde{X}\) and \((A_2, P) = \emptyset\), then each proper soft \((1,2)^*\)-\(\omega\)-open (resp. soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-open, soft \((1,2)^*\)-\(\beta\)-\(\omega\)-open, soft \((1,2)^*\)-\(\alpha\)-\(\beta\)-open) subset of \(\tilde{X}\) is a soft \((1,2)^*\)-\(\bar{D}_\omega\)-set (resp. soft \((1,2)^*\)-\(\bar{D}_{\alpha\omega}\)-set, soft \((1,2)^*\)-\(\bar{D}_{\beta\omega}\)-set, soft \((1,2)^*\)-\(\bar{D}_{\beta\alpha}\)-set).

The converse of Remark (2.3) is not true in general as shown in the following examples.

Example (2.4): Let \(X = \mathcal{R}\), \(P = \{p_1, p_2, p_3, p_4\}\) and let \(\tilde{t}_1 = (\tilde{X}, \tilde{\phi}, (U, P))\) and \(\tilde{t}_2 = (\tilde{X}, \tilde{\phi})\) be soft topologies over \(X\), where
(U,P) = ((p_1,{1}),(p_2,{1}),(p_3,{1}),(p_4,{1})). The soft sets in \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} are soft \tilde{\tau}_1\tilde{\tau}_2-open sets in \tilde{\mathcal{X}}. Then (A,P) = ((p_1,\{2\}),(p_2,\{2\}),(p_3,\{2\}),(p_4,\{2\})) is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0}-set and a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0}-set, but is not soft (1,2)*-\alpha_0\omega-open set.

Example (2.5): Let X = \mathcal{R}, P = \{p_1,p_2\} and let \tilde{\tau}_1 = \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} and \tilde{\tau}_2 = \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} be soft topologies over X, where (U,P) = ((p_1,\mathcal{R} - \{1\}),(p_2,\mathcal{R} - \{1\})). The soft sets in \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} are soft \tilde{\tau}_1\tilde{\tau}_2-open sets in \tilde{\mathcal{X}}.

Then (A,P) = ((p_1,\{1\}),(p_2,\{1\})) is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\alpha_0\omega}-set (resp. soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\beta_0\omega}-set, soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set), but is not soft (1,2)*-\beta_0\omega-open set.

Proposition (2.6): If (X,\tilde{\tau}_1,\tilde{\tau}_2,P) is a soft bitopological space. Then:

(i) Every soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\alpha_0\omega}-set.

(ii) Every soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\beta_0\omega}-set is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\alpha_0\omega}-set.

(iii) Every soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\alpha_0\omega}-set.

(iv) Every soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\beta_0\omega}-set is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set.

(v) Every soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\beta_0\omega}-set.

Proof: Follows from proposition (1.10).

Remark (2.7): The converse of proposition (2.6) number (i),(ii) and (iii) in general may not be true. We see that in the following examples:

Example (2.8): Let X = \mathcal{R}, P = \{p_1,p_2\} and let \tilde{\tau}_1 = \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} and \tilde{\tau}_2 = \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} be soft topologies over X, where (U,P) = ((p_1,\{2\}),(p_2,\{2\})). The soft sets in \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} are soft \tilde{\tau}_1\tilde{\tau}_2-open sets in \tilde{\mathcal{X}}. Then (A,P) = ((p_1,\{1\}),(p_2,\{1\})) is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set, since \exists (A_1,P) = ((p_1,\mathcal{R} - \{2\}),(p_2,\mathcal{R} - \{2\})) and (A_2,P) = ((p_1,\mathcal{R} - \{1\}),(p_2,\mathcal{R} - \{1\})) are soft (1,2)*-\omega_0-open sets in \tilde{\mathcal{X}} such that (A_1,P) \neq \tilde{\mathcal{R}} and (A,P) = (A,P)\setminus(A_2,P), but (A,P) is not soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set.

Example (2.9): Let X = \mathcal{R}, P = \{p_1,p_2\} and let \tilde{\tau}_1 = \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} and \tilde{\tau}_2 = \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} be soft topologies over X, where (U,P) = ((p_1,\{3\}),(p_2,\{3\})). The soft sets in \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} are soft \tilde{\tau}_1\tilde{\tau}_2-open sets in \tilde{\mathcal{X}}. Then (A,P) = ((p_1,\{0,3\}),(p_2,\{0,3\})) is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set, but (A,P) is not soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set.

Example (2.10): Let X = \mathcal{R}, P = \{p_1,p_2\} and let \tilde{\tau}_1 = \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} and \tilde{\tau}_2 = \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U,P)\} be soft topologies over X, where (U_1,P) = ((p_1,\{4\}),(p_2,\{4\})) and (U_2,P) = ((p_1,\mathcal{R} - \{4\}),(p_2,\mathcal{R} - \{4\})). The soft sets in \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U_1,P)\} and \{\tilde{\mathcal{X}},\tilde{\mathcal{P}},(U_2,P)\} are soft \tilde{\tau}_1\tilde{\tau}_2-open sets in \tilde{\mathcal{X}}. Then (A,P) = ((p_1,\{0,4\}),(p_2,\{0,4\})) is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set, since (A,P) is a soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set, but (A,P) is not soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-set.

The following diagram shows the relation between the types of soft open sets and each of weak soft (1,2)*-\omega_0-open sets and weak soft (1,2)*-\tilde{\mathcal{D}}_{\omega_0\tau\omega}-sets.
Theorem (2.11):
If \( f : (X, \tilde{\tau}_1, \tilde{\tau}_2, P) \rightarrow (Y, \tilde{\sigma}_1, \tilde{\sigma}_2, P) \) is a strongly soft \((1,2)^*\)-\(\omega\)-continuous (resp. strongly soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-continuous, strongly soft \((1,2)^*\)-pre-\(\omega\)-continuous, strongly soft \((1,2)^*\)-\(b\)-\(\omega\)-continuous) surjective function and \((A, P)\) is a soft \((1,2)^*\)-\(\tilde{D}_{\omega}\)-set (resp. soft \((1,2)^*\)-\(\tilde{D}_{\alpha-\omega}\)-set, soft \((1,2)^*\)-\(\tilde{D}_{pre-\omega}\)-set, soft \((1,2)^*\)-\(\tilde{D}_{b-\omega}\)-set) in \(\tilde{Y}\), then the inverse image of \((A, P)\) is a soft \((1,2)^*\)-\(\tilde{D}\)-set in \(\tilde{X}\).

Proof: Let \((A, P)\) be a soft \((1,2)^*\)-\(\tilde{D}_{\omega}\)-set in \(\tilde{Y}\), then there are two soft \((1,2)^*\)-\(\omega\)-open sets \((A_1, P)\) and \((A_2, P)\) in \(\tilde{Y}\) such that \((A_1, P) \neq \tilde{Y}\) and \((A, P) = (A_1, P) \setminus (A_2, P)\). Since \(f\) is strongly soft \((1,2)^*\)-\(\omega\)-continuous, then by definition (1.15), \(f^{-1}((A_1, P))\) and \(f^{-1}((A_2, P))\) are soft \(\tilde{\tau}_1 \tilde{\tau}_2\)-open sets in \(\tilde{X}\).

Since \((A_1, P) \neq \tilde{Y}\) and \(f\) is surjective, then \(f^{-1}((A_1, P)) \neq \tilde{X}\). Hence \(f^{-1}((A_1, P)) = f^{-1}((A_1, P)) \setminus f^{-1}((A_2, P))\) is a soft \((1,2)^*\)-\(\tilde{D}\)-set in \(\tilde{X}\). By the same way we can prove other cases.

Theorem (2.12):
If \( f : (X, \tilde{\tau}_1, \tilde{\tau}_2, P) \rightarrow (Y, \tilde{\sigma}_1, \tilde{\sigma}_2, P) \) is a strongly soft \((1,2)^*\)-\(\omega\)-open (resp. strongly soft \((1,2)^*\)-\(\alpha\)-\(\omega\)-open, strongly soft \((1,2)^*\)-pre-\(\omega\)-open, strongly soft \((1,2)^*\)-\(b\)-\(\omega\)-open) bijective function and \((A, P)\) is a soft \((1,2)^*\)-\(\tilde{D}_{\omega}\)-set (resp. soft \((1,2)^*\)-\(\tilde{D}_{\alpha-\omega}\)-set, soft \((1,2)^*\)-\(\tilde{D}_{pre-\omega}\)-set, soft \((1,2)^*\)-\(\tilde{D}_{b-\omega}\)-set) in \(\tilde{X}\), then \(f((A, P))\) is a soft \((1,2)^*\)-\(\tilde{D}\)-set in \(\tilde{Y}\).

Proof: Let \((A, P)\) be a soft \((1,2)^*\)-\(\tilde{D}_{\omega}\)-set in \(\tilde{X}\), then there are two soft \((1,2)^*\)-\(\omega\)-open sets \((A_1, P)\) and \((A_2, P)\) in \(\tilde{X}\) such that
(A₁, P) ≠ ꔤX and (A, P) = (A, P) \ (A₂, P). Since f is strongly soft (1,2)*-ω-open, then by definition (1.16), f((A₁, P)) and f((A₂, P)) are soft ꔤ₁ ꔤ₂-open sets in ꔤY. Since (A₁, P) ≠ ꔤX and f is injective, then f((A₁, P)) ≠ ꔤY. Since f is bijective, then f((A₁, P)) = f((A₁, P)) \ f((A₂, P)) is a soft (1,2)*- ꔤD-set in ꔤY. By the same way we can prove other cases.

3. Weak Soft (1,2)*- ꔤD₀-*Separation Axioms

Now, we define and study new types of soft separation axioms in soft bitopological spaces, namely, soft (1,2)*-ω- ꔤD₀-spaces, soft (1,2)*-α-ω- ꔤD₀-spaces, soft (1,2)*-pre-ω- ꔤD₀-spaces, soft (1,2)*-b-ω- ꔤD₀-spaces, soft (1,2)*-β-ω- ꔤD₀-spaces, for j = 0, 1, 2. Further we study the relations between these types of soft separation axioms and other types of soft separation axioms.

Definitions (3.1): A soft bitopological space (X, ꔤ₁, ꔤ₂, P) is called a soft (1,2)*-ω- ꔤD₀-space (resp. soft (1,2)*-α-ω- ꔤD₀-space, soft (1,2)*-pre-ω- ꔤD₀-space, soft (1,2)*-b-ω- ꔤD₀-space, soft (1,2)*-β-ω- ꔤD₀-space) if for any two distinct soft points ꔤx and ꔤy of ꔤX, there exists a soft (1,2)*- ꔤD₀-set (resp. soft (1,2)*- ꔤD₀-set, soft (1,2)*- ꔤD₀-set, soft (1,2)*- ꔤD₀-set, soft (1,2)*- ꔤD₀-set) in ꔤX containing one of the soft points but not the other.

Definitions (3.2): A soft bitopological space (X, ꔤ₁, ꔤ₂, P) is called a soft (1,2)*-ω- ꔤD₁-space (resp. soft (1,2)*-α-ω- ꔤD₁-space, soft (1,2)*-pre-ω- ꔤD₁-space, soft (1,2)*-b-ω- ꔤD₁-space, soft (1,2)*-β-ω- ꔤD₁-space) if for any two distinct soft points ꔤx and ꔤy of ꔤX, there are two soft (1,2)*- ꔤD₀-sets (resp. soft (1,2)*- ꔤD₀-sets, soft (1,2)*- ꔤD₀-sets, soft (1,2)*- ꔤD₀-sets, soft (1,2)*- ꔤD₀-sets), soft (1,2)*- ꔤD₀-sets, soft (1,2)*- ꔤD₀-sets) in ꔤX such that ꔤx ꔤ∈ (U, P) and (V, P) in ꔤX such that ꔤx ꔤ∈ (U, P), ꔤy ꔤ∈ (U, P) and ꔤy ꔤ∈ (V, P), ꔤx ꔤ∈ (V, P).

Definitions (3.3): A soft bitopological space (X, ꔤ₁, ꔤ₂, P) is called a soft (1,2)*-ω- ꔤD₂-space (resp. soft (1,2)*-α-ω- ꔤD₂-space, soft (1,2)*-pre-ω- ꔤD₂-space, soft (1,2)*-b-ω- ꔤD₂-space, soft (1,2)*-β-ω- ꔤD₂-space) if for any two distinct soft points ꔤx and ꔤy of ꔤX, there are two soft (1,2)*- ꔤD₂-sets (resp. soft (1,2)*- ꔤD₂-sets, soft (1,2)*- ꔤD₂-sets, soft (1,2)*- ꔤD₂-sets, soft (1,2)*- ꔤD₂-sets) (U, P) and (V, P) in ꔤX such that ꔤx ꔤ∈ (U, P), ꔤy ꔤ∈ (V, P) and (U, P) ꔤ∈ (V, P) = ꔤφ.

Theorem (3.4):
(i) Every soft (1,2)*- ꔤT₁-space (resp. soft (1,2)*-α- ꔤT₁-space, soft (1,2)*-pre- ꔤT₁-space, soft (1,2)*-b- ꔤT₁-space, soft (1,2)*-β- ꔤT₁-space) is a soft (1,2)*- ꔤD₁-space (resp. soft (1,2)*-α- ꔤD₁-space, soft (1,2)*-pre- ꔤD₁-space, soft (1,2)*-b- ꔤD₁-space, soft (1,2)*-β- ꔤD₁-space), j = 0, 1, 2.
(ii) Every soft (1,2)*-ω- ꔤD_j-space (resp. soft (1,2)*-α- ꔤD_j-space, soft (1,2)*-pre- ꔤD_j-space, soft (1,2)*-b- ꔤD_j-space, soft (1,2)*-β- ꔤD_j-space) is a soft (1,2)*- ꔤD_j⁺₁-space (resp. soft (1,2)*-α- ꔤD_j⁺₁-space, soft (1,2)*-pre- ꔤD_j⁺₁-space, soft (1,2)*-b- ꔤD_j⁺₁-space, soft (1,2)*-β- ꔤD_j⁺₁-space), j = 1, 2.
(iii) Every soft (1,2)*- ꔤD_j-space is a soft (1,2)*-ω- ꔤD_j-space, j = 0, 1, 2.
(iv) Every soft \((1,2)^*-\omega-\tilde{D}_j\)-space is a soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space, \(j = 0,1,2\).

(v) Every soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space is a soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space, \(j = 0,1,2\).

(vi) Every soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space is a soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space, \(j = 0,1,2\).

(vii) Every soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space is a soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space, \(j = 0,1,2\).

**Proof:**

(i) Follows from Remark (2.3).

(ii) It is obvious.

(iii), (iv), (v), (vi), (vii) Follows from proposition (2.6).

**Remark (3.5):** The converse of theorem (3.4), no. (i) in general may not be true. We see that by the following examples:

**Example (3.6):** Let \(X = \{a, b, c\} \) and \(P = \{p\} \) and let \(\tilde{\tau}_1 = \{\tilde{X}, \tilde{\phi}, (U_1, P), (U_2, P)\} \) and \(\tilde{\tau}_2 = \{\tilde{X}, \tilde{\phi}, (U_3, P)\} \) be soft topologies over \(X\), where \((U_1, P) = \{(p, \{a\})\}, \ (U_2, P) = \{(p, \{b\})\} \) and \((U_3, P) = \{(p, \{a, c\})\} \). The soft sets in \(\{\tilde{X}, \tilde{\phi}, (U_1, P), (U_2, P), (U_3, P)\}\) are soft \(\tilde{\tau}_1\) and \(\tilde{\tau}_2\)-open sets. Thus \((X, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*-\tilde{D}_j\)-space, but is not soft \((1,2)^*-\tilde{T}_j\)-space, \(j = 1,2\).

**Example (3.7):** Let \(X = \varnothing \) and \(P = \{p_1, p_2\} \) and let \(\tilde{\tau}_1 = \{(U, P) \in \tilde{X} : (U, P) \subseteq \text{finite}\} \cup \{\emptyset\} \) and \(\tilde{\tau}_2 = \{\tilde{X}, \tilde{\phi}\} \) be soft topologies over \(X\). Thus \((X, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*-\tilde{D}_2\)-space (resp. soft \((1,2)^*-\alpha-\omega-\tilde{D}_2\)-space, soft \((1,2)^*-\alpha-\omega-\tilde{D}_2\)-space, soft \((1,2)^*-\alpha-\omega-\tilde{D}_2\)-space, soft \((1,2)^*-\beta-\omega-\tilde{D}_2\)-space, but is not soft \((1,2)^*-\beta-\omega-\tilde{T}_2\)-space.

**Remark (3.8):** The converse of theorem (3.4), no. (iii) in general may not be true. We see that by the following examples:

**Example (3.9):** Let \(X = \{a, b\} \) and \(P = \{p_1, p_2\} \) and let \(\tilde{\tau}_1 = \{\tilde{X}, \tilde{\phi}, (U_1, P)\} \) and \(\tilde{\tau}_2 = \{\tilde{X}, \tilde{\phi}, (U_2, P)\} \) be soft topologies over \(X\), where \((U_1, P) = \{(p_1, \{a\}), (p_2, \{a\})\} \) and \((U_2, P) = \{(p_1, \{b\}), (p_2, \{b\})\}\). The soft sets in \(\{\tilde{X}, \tilde{\phi}, (U_1, P), (U_2, P)\}\) are soft \(\tilde{\tau}_1\) and \(\tilde{\tau}_2\)-open. Thus \((X, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*-\omega-\tilde{D}_j\)-space, but is not soft \((1,2)^*-\tilde{D}_j\)-space, \(j = 0,1,2\).

**Theorem (3.10):** A soft bitopological space \((X, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*-\omega-\tilde{D}_j\)-space (resp. soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space, soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space, soft \((1,2)^*-\alpha-\omega-\tilde{T}_j\)-space) if and only if it is a soft \((1,2)^*-\omega-\tilde{T}_j\)-space (resp. soft \((1,2)^*-\alpha-\omega-\tilde{T}_j\)-space, soft \((1,2)^*-\alpha-\omega-\tilde{T}_j\)-space, soft \((1,2)^*-\beta-\omega-\tilde{T}_j\)-space), \(j = 0,1\).

**Proof:** Follows from proposition (1.14) and theorem (3.4), no. (i).

**Theorem (3.11):** A soft bitopological space \((X, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*-\omega-\tilde{D}_j\)-space (resp. soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space, soft \((1,2)^*-\alpha-\omega-\tilde{D}_j\)-space, soft \((1,2)^*-\alpha-\omega-\tilde{D}_2\)-space, soft \((1,2)^*-\alpha-\omega-\tilde{D}_2\)-space, soft \((1,2)^*-\beta-\omega-\tilde{D}_2\)-space, soft \((1,2)^*-\beta-\omega-\tilde{D}_2\)-space).

**Proof:** Sufficiency. Follows from theorem (3.4), no. (ii).

Necessity. Let \(\tilde{x}, \tilde{y} \in \tilde{X}\) such that \(\tilde{x} \neq \tilde{y}\). Since \((X, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*-\omega-\tilde{D}_j\)-space, then there exists soft \((1,2)^*-\tilde{D}_\omega\)-sets \((U, P)\) and \((V, P)\) in \(\tilde{X}\) such that \(\tilde{x} \in (U, P)\), \(\tilde{y} \in (V, P)\) and \(\tilde{x} \in (U, P)\) and \(\tilde{y} \in (V, P)\). Let \(U, P) = (U_1, P) \setminus (U_2, P)\) and \((V, P) = \)
(U_3, P) \setminus (U_4, P), \text{ where } (U_1, P), (U_2, P), (U_3, P), (U_4, P) \text{ are soft (1,2)*-o-open sets in } \tilde{X} \text{ and } (U_1, P) \neq \tilde{X}, (U_3, P) \neq \tilde{X}. \text{ By } \tilde{x} \not\in \tilde{x}(V, P) \text{ we have two cases:}

(i) \tilde{x} \not\in \tilde{x}(U_3, P)

(ii) \tilde{x} \not\in \tilde{x}(U_3, P) \text{ and } \tilde{x} \not\in \tilde{x}(U_4, P).

In case (i): \tilde{x} \not\in \tilde{x}(U_3, P). By \tilde{y} \not\in \tilde{y}(U, P) \text{ we have two subcases:}

(a) \tilde{y} \not\in \tilde{y}(U_1, P) \text{ and } \tilde{y} \not\in \tilde{y}(U_2, P)

(b) \tilde{y} \not\in \tilde{y}(U_1, P).

Subcase (a): \tilde{y} \not\in \tilde{y}(U_1, P) \text{ and } \tilde{y} \not\in \tilde{y}(U_2, P). \text{ We have } \tilde{x} \not\in \tilde{x}(U_1, P) \setminus (U_2, P), \tilde{y} \not\in \tilde{y}(U, P), \text{ and } (U_1, P) \setminus (U_2, P) = \emptyset. \text{ Observe that } (U_2, P) \neq \tilde{X} \text{ since } (U, P) \neq \emptyset, \text{ thus by Remarks (2.3), (U_2, P) is a soft (1,2)*-} \tilde{D}_\omega \text{-set.}

Subcase (b): \tilde{y} \not\in \tilde{y}(U_1, P). \text{ Since } \tilde{x} \not\in \tilde{x}(U_1, P) \setminus (U_2, P) \text{ and } \tilde{x} \not\in \tilde{x}(U_3, P), \text{ then } \tilde{x} \not\in \tilde{x}(U_1, P) \setminus ((U_2, P) \cup (U_3, P)). \text{ Since } \tilde{y} \not\in \tilde{y}(U_3, P) \setminus (U_4, P) \text{ and } \tilde{y} \not\in \tilde{y}(U_1, P), \text{ then } \tilde{y} \not\in (U_3, P) \setminus ((U_4, P) \cup (U_1, P)). \text{ Observe that } (U_2, P) \cup (U_3, P) \text{ and } (U_4, P) \cup (U_1, P) \text{ are soft (1,2)*-o-open sets in } \tilde{X}. \text{ Hence } \tilde{x} \not\in (U_1, P) \setminus (U_2, P), \tilde{y} \not\in (U_4, P) \setminus (U_1, P) \text{ and } (U_1, P) \setminus ((U_2, P) \cup (U_3, P)) = \emptyset.

In case (ii): \tilde{x} \not\in \tilde{x}(U_3, P) \text{ and } \tilde{x} \not\in \tilde{x}(U_4, P). \text{ We have } \tilde{y} \not\in \tilde{y}(U_3, P) \setminus (U_4, P), \tilde{x} \not\in \tilde{x}(U_4, P) \text{ and } (U_3, P) \setminus (U_4, P) = \emptyset. \text{ Observe that } (U_4, P) \neq \tilde{X} \text{ since } (V, P) \neq \emptyset, \text{ thus by Remarks (2.3), (U_4, P) is a soft (1,2)*-} \tilde{D}_\omega \text{-set. Hence } (X, \tilde{\tau}_1, \tilde{\tau}_2, P) \text{ is a soft (1,2)*-} \tilde{D}_2 \text{-space. Similarly, we can prove other cases.}

Proposition 3.12:

(i) Every soft (1,2)*-\alpha-o-\tilde{D}_j \text{-space is a soft (1,2)*-o-} \tilde{D}_j \text{-space, } j = 0,1,2.

(ii) Every soft (1,2)*-pre-\omega-\tilde{D}_j \text{-space is a soft (1,2)*-}\alpha-o-\tilde{D}_j \text{-space, } j = 0,1,2.

(iii) Every soft (1,2)*-b-\omega-\tilde{D}_j \text{-space is a soft (1,2)*-pre-}\omega-\tilde{D}_j \text{-space, } j = 0,1,2.

(iv) Every soft (1,2)*-\beta-o-\tilde{D}_j \text{-space is a soft (1,2)*-b-}\omega-\tilde{D}_j \text{-space, } j = 0,1,2.

Proof: (i) If } j = 0 \text{, then let } \tilde{x}, \tilde{y} \not\in \tilde{x} \text{ such that } \tilde{x} \neq \tilde{y}. \text{ Since } \tilde{X} - \{\tilde{x}\} \text{ is a soft (1,2)*-o-open set in } \tilde{X}, \text{ then by Remark (2.3), } \tilde{X} - \{\tilde{x}\} \text{ is a soft (1,2)*-} \tilde{D}_\omega \text{-set in } \tilde{X} \text{ which contains } \tilde{y}, \text{ but not } \tilde{x}. \text{ Hence } (X, \tilde{\tau}_1, \tilde{\tau}_2, P) \text{ is a soft (1,2)*-} \tilde{D}_0 \text{-space. If } j = 1 \text{, then let } \tilde{x}, \tilde{y} \not\in \tilde{X} \text{ such that } \tilde{x} \neq \tilde{y}. \text{ Since } \tilde{X} - \{\tilde{x}\} \text{ and } \tilde{X} - \{\tilde{y}\} \text{ are soft (1,2)*-o-open sets in } \tilde{X}, \text{ then by Remark (2.3), } \tilde{X} - \{\tilde{x}\} \text{ and } \tilde{X} - \{\tilde{y}\} \text{ are soft (1,2)*-} \tilde{D}_\omega \text{-sets in } \tilde{X} \text{ such that } \tilde{X} - \{\tilde{y}\} \text{ containing } \tilde{x}, \text{ but not } \tilde{y} \text{ and } \tilde{X} - \{\tilde{x}\} \text{ containing } \tilde{y}, \text{ but not } \tilde{x}. \text{ Therefore } (X, \tilde{\tau}_1, \tilde{\tau}_2, P) \text{ is a soft (1,2)*-} \tilde{D}_1 \text{-space. If } j = 2 \text{, then let } \tilde{x}, \tilde{y} \not\in \tilde{X} \text{ such that } \tilde{x} \neq \tilde{y}. \text{ Since } \{\tilde{x}\} = (\tilde{X} - \{\tilde{y}\}) \setminus (\tilde{X} - \{\tilde{x}\}) \text{ and } \{\tilde{y}\} = (\tilde{X} - \{\tilde{x}\}) \setminus (\tilde{X} - \{\tilde{y}\}) \text{ are disjoint soft (1,2)*-} \tilde{D}_\omega \text{-sets in } \tilde{X} \text{ containing } \tilde{x} \text{ and } \tilde{y} \text{ respectively, therefore } (X, \tilde{\tau}_1, \tilde{\tau}_2, P) \text{ is a soft (1,2)*-o-} \tilde{D}_2 \text{-space.}

(ii), (iii), and (iv) similar to (i).

The following diagram show the relations between the soft (1,2)*-\tilde{T}_j \text{-spaces and each of soft (1,2)*-} \tilde{D}_j \text{-spaces, soft (1,2)*-o-} \tilde{D}_j \text{-spaces, soft (1,2)*-} \alpha-o-\tilde{D}_j \text{-spaces, soft (1,2)*-} \omega-\tilde{D}_j \text{-spaces, soft (1,2)*-} \beta-o-\tilde{D}_j \text{-spaces, and soft (1,2)*-} \omega-\tilde{D}_j \text{-spaces, for } j = 0,1,2.
**Definition (3.13):** Let \((X, \tau_1, \tau_2, P)\) be a soft bitopological space. A soft point \(\tilde{x} \in \tilde{X}\) which has \(\tilde{X}\) as the only soft \((1,2)^*\)-\(\omega\)-neighborhood (resp. soft \((1,2)^*-\alpha\)-\(\omega\)-neighborhood, soft \((1,2)^*-\text{pre-}\omega\)-neighborhood, soft \((1,2)^*-\beta\)-\(\omega\)-neighborhood) is called a soft \((1,2)^*-\omega\)-neat (resp. soft \((1,2)^*-\alpha\)-neat, soft \((1,2)^*-\text{pre-}\omega\)-neat, soft \((1,2)^*-\beta\)-\(\omega\)-neat) point.

**Theorem (3.14):** Let \((X, \tau_1, \tau_2, P)\) be a soft bitopological space, then the following are equivalent:

\(\text{(i)}\) \((X, \tau_1, \tau_2, P)\) is a soft \((1,2)^*-\omega\)-\(\text{pre-}\omega\)-\(D_1\)-space (resp. soft \((1,2)^*-\alpha\)-\(\omega\)-\(D_1\)-space, soft \((1,2)^*-\text{pre-}\omega\)-\(D_1\)-space, soft \((1,2)^*-\beta\)-\(\omega\)-\(D_1\)-space).

\(\text{(ii)}\) \((X, \tau_1, \tau_2, P)\) has no soft \((1,2)^*-\omega\)-neat (resp. soft \((1,2)^*-\alpha\)-neat, soft \((1,2)^*-\text{pre-}\omega\)-neat, soft \((1,2)^*-\beta\)-\(\omega\)-neat) point.

**Proof:**

(i) \(\Rightarrow\) (ii). Since \((X, \tau_1, \tau_2, P)\) is a soft \((1,2)^*-\omega\)-\(\text{pre-}\omega\)-\(D_1\)-space, then each soft point \(\tilde{x} \in \tilde{X}\) is contained in a soft \((1,2)^*-\omega\)-\(D_0\)-set \((U, P) = (U_1, P) \setminus (U_2, P)\), where \((U_1, P)\) and \((U_2, P)\) are soft \((1,2)^*-\omega\)-open sets and thus in \((U_1, P)\). By definition (2.1), \((U_1, P) \neq \tilde{X}\). This implies that \(\tilde{x}\) is not a soft \((1,2)^*-\omega\)-neat point.

(ii) \(\Rightarrow\) (i). Follows from proposition (1.14) and theorem (3.4), no. (i).

**Theorem (3.15):**

Let \(f: (X, \tau_1, \tau_2, P) \to (Y, \sigma_1, \sigma_2, P)\) be a strongly soft \((1,2)^*-\text{\(\omega\)}\)-continuous (resp. strongly soft \((1,2)^*-\alpha\)-\(\omega\)-continuous, strongly soft \((1,2)^*-\text{pre-}\omega\)-\(\omega\)-continuous, strongly soft \((1,2)^*-\beta\)-\(\omega\)-continuous) bijective function. If \(\tilde{Y}\) is a soft \((1,2)^*-\omega\)-\(\text{pre-}\omega\)-\(D_1\)-space (resp. soft \((1,2)^*-\alpha\)-\(\omega\)-\(D_1\)-space, soft \((1,2)^*-\text{pre-}\omega\)-\(D_1\)-space, soft \((1,2)^*-\beta\)-\(\omega\)-continuous) bijective function. If \(\tilde{Y}\) is a soft \((1,2)^*-\omega\)-\(\text{pre-}\omega\)-\(D_1\)-space (resp. soft \((1,2)^*-\alpha\)-\(\omega\)-\(D_1\)-space, soft \((1,2)^*-\text{pre-}\omega\)-\(D_1\)-space, soft \((1,2)^*-\beta\)-\(\omega\)-continuous) bijective function.
b-\(\tilde{D}_j\)-space, soft \((1,2)^*\)-b-\(\tilde{D}_j\)-space), then \(X\) is a soft \((1,2)^*\)-\(\tilde{D}_j\)-space, \(j=0,1,2\).

**Proof:** Suppose that \((Y, \tilde{\sigma}_1, \tilde{\sigma}_2, P)\) is a soft \((1,2)^*\)-b-\(\tilde{D}_2\)-space. Let \(\tilde{x}_1, \tilde{x}_2 \in \tilde{X}\) such that \(\tilde{x}_1 \neq \tilde{x}_2\). Since \(f\) is injective and \((Y, \tilde{\sigma}_1, \tilde{\sigma}_2, P)\) is a soft \((1,2)^*\)-b-\(\tilde{D}_2\)-space, then there exists disjoint soft \((1,2)^*\)-\(D_\omega\)-sets \((A_1, P)\) and \((A_2, P)\) in \(\tilde{Y}\) such that \(f(\tilde{x}_1) \notin (A_1, P)\) and \(f(\tilde{x}_2) \notin (A_2, P)\). By theorem (2.11), \(f^{-1}((A_1, P))\) and \(f^{-1}((A_2, P))\) are soft \((1,2)^*\)-\(\tilde{D}\)-sets in \(\tilde{X}\). Since \(\tilde{x}_1 \notin f^{-1}((A_1, P))\), \(\tilde{x}_2 \notin f^{-1}((A_2, P))\) and \(f^{-1}(A_1, P) \cap f^{-1}(A_2, P) = \emptyset\). Thus \((X, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*\)-\(\tilde{D}_2\)-space. Similarly, we can prove other cases.

**Theorem (3.16):**

Let \(f: (X, \tilde{\tau}_1, \tilde{\tau}_2, P) \rightarrow (Y, \tilde{\sigma}_1, \tilde{\sigma}_2, P)\) be a strongly soft \((1,2)^*\)-\(\alpha\)-open (resp. strongly soft \((1,2)^*\)-\(\alpha\)-\(\tilde{D}\)-space) function. If \(\tilde{X}\) is a soft \((1,2)^*\)-b-\(\tilde{D}\)-space (resp. soft \((1,2)^*\)-\(\alpha\)-\(\tilde{D}\)-space), \((1,2)^*\)-pre-\(\omega\)-\(\tilde{D}\)-space, \((1,2)^*\)-b-\(\tilde{D}\)-space, then \(\tilde{Y}\) is a soft \((1,2)^*\)-\(\tilde{D}\)-space, \(j=0,1,2\).

**Proof:** Suppose that \((X, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*\)-b-\(\tilde{D}_2\)-space. Let \(\tilde{y}_1, \tilde{y}_2 \in \tilde{Y}\) such that \(\tilde{y}_1 \neq \tilde{y}_2\). Since \(f\) is surjective, then there exists \(\tilde{x}_1, \tilde{x}_2 \in \tilde{X}\) such that \(f(\tilde{x}_1) = \tilde{y}_1\) and \(f(\tilde{x}_2) = \tilde{y}_2\). But \(f\) is a soft function, then \(\tilde{x}_1 \neq \tilde{x}_2\), since \((X, \tilde{\tau}_1, \tilde{\tau}_2, P)\) is a soft \((1,2)^*\)-b-\(\tilde{D}_2\)-space, then there exists disjoint soft \((1,2)^*\)-\(\tilde{D}_\omega\)-sets \((A_1, P)\) and \((A_2, P)\) in \(\tilde{X}\) such that \(\tilde{x}_1 \notin (A_1, P)\) and \(\tilde{x}_2 \notin (A_2, P)\). By theorem (2.12), \(f((A_1, P))\) and \(f((A_2, P))\) are soft \((1,2)^*\)-\(\tilde{D}\)-sets in \(\tilde{Y}\) such that \(f(\tilde{x}_1) = \tilde{y}_1 \in f((A_1, P))\) and \(f(\tilde{x}_2) = \tilde{y}_2 \in f((A_2, P))\). Since \(f\) is injective, then \(f((A_1, P)) \cap f((A_2, P)) = \emptyset\). Hence \((Y, \tilde{\sigma}_1, \tilde{\sigma}_2, P)\) is a soft \((1,2)^*\)-\(\tilde{D}_2\)-space. Similarly, we can prove other cases.

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