Wireless Power Transmission between Two Metamaterial-Inspired Loops at 300 MHz

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Abstract

Based on a provided closed-form wireless power transmission (WPT) efficiency formula, which may be used for any value of load, we have analyzed the WPT efficiencies between two metamaterial-inspired loop antennas in various aspects. Due to the modeling based on low frequency circuit theory, the provided formula at the center resonant frequency has been found to be accurate until when the distance between the two loop antennas increases to 15 cm (about \( \lambda/6 \) at 300 MHz). When the two loops get closer, the resonant frequency has been found to split into two in theory, simulations, and measurements. The EM-simulated and measured efficiencies at new resonant frequencies are 60.9% and 46.3%, respectively, at \( d=15 \) cm. With two extra rings around the loops, the maximum efficiency is enhanced to 93.7% at \( d=15 \) cm. The effect of the additional two rings is about 30%.

**Key words**: Wireless Power Transmission (WPT), Metamaterial, Loop Antenna, Efficiency.

I. Introduction

The concept of wireless power transmission (WPT) was initiated by N. Tesla in 1914 [1], but has not led to any practical application until quite recently, due to its low efficiency. In 2007, an efficient non-radiative power transfer was demonstrated experimentally [2]. This article showed that the efficiency of the power transfer increases to about 40% in 2 meters (\( \lambda/15 \)) at 9.9 MHz employing two single loops and two helical coils for resonance. In this paper, the formulation in [2], based on the coupled-mode theory [3], is recast to employ a more familiar basic circuit theory. The WPT between two metamaterial-inspired loop antennas is then investigated at 300 MHz.

Metamaterial technology has been intensively studied over the last decade [4–7]. The use of the metamaterial-inspired (or capacitively loaded) loop antennas is for a uniform current distribution along the loop, which significantly increases the magnetic field intensity passing through the loop. The reason why we used the term ‘metamaterial-inspired’ is because the phenomenon of uniform current along the loop is just that of the zeroth-order resonance in a series branch of a metamaterial-based transmission line. The WPT efficiencies based on theory, EM simulations, and measurements are compared with each other to validate the provided formulation. The performance of the proposed WPT system is also evaluated with extra rings around the loops.

Fig. 1 shows the equivalent circuit for a wireless power transfer (WPT) system using two metamaterial-inspired loops. Each loop forms a series \( R, L, C \) resonator. \( M \) is the mutual inductance between the loop inductance \( L_1 \) and \( L_2 \). \( V_1 \) is the source voltage at the first loop and \( R_L \) is the load resistance at the second loop. \( R_2 \) is the resistance required to account for the loss in the second loop. \( R_1 \) is the resistance required to account for the loss in the first loop. This may include the possible radiation effects, if any.

Using KVL’s, we obtain:

\[
\left( R + j\omega L_1 + \frac{1}{j\omega C_1} \right) I_1 - j\omega MI_2 = V_1
\]

and

\[
R_2 + R_L + j\omega L_2 + \frac{1}{j\omega C_2} \right) I_2 - j\omega MI_1 = 0.
\]

![Fig. 1. Equivalent circuit for WPT.](image)
When each loop is resonant at an angular frequency \( \omega_0 \), the current on each loop can be expressed as

\[
I_1 = \frac{R_1 + R_2}{R_1 (R_2 + R_1) + \omega_0^2 M^2} V_i
\]

and

\[
I_2 = \frac{j \omega M}{R_1 (R_2 + R_1) + \omega_0^2 M^2} V_i.
\]

The mutual coupling coefficient \( k \) between the two loops is given by:

\[
k = \frac{M}{\sqrt{L_1 L_2}}.
\]

The WPT efficiency \( \eta \) at \( \omega_0 \), which is defined as the ratio of the power \( P_1 \) delivered to the WPT system and the power \( P_2 \) delivered to the load \( R_L \), can be shown to be given by

\[
\eta = \frac{P_2}{P_1} = \frac{1}{1 + \frac{1}{k Q_2} (1 + \beta_e)} \frac{1}{1 + \beta_e}.
\]

where \( \beta_e \) is a ratio of \( R_L \) and \( R_2 \), and \( Q_1 \) and \( Q_2 \) are the unloaded quality factors of the two loop resonators. The coupling coefficient \( k \) in (5) may be obtained from EM-simulated or measured \( S_{21} \) using [7]:

\[
k = \frac{f_0^2 - f_i^2}{f_0^2 + f_i^2}.
\]

The efficiency in (6) can also be shown to be given by:

\[
\eta = \frac{|S_{21}|^2}{1 - |S_{21}|^2}
\]

using EM-simulated or measured S-parameters.

### III. Simulation

A schematic view of the metamaterial-inspired loop antenna is shown in Fig. 2. The presented loop antenna has two advantages. It can be easily constructed on PCBs and it can also be made thin. Even though this loop antenna is not a transmission line because it has only one conducting line, we try to make the current on the loop have the same magnitude and phase using only series-type resonance (by periodically inserting chip capacitors along the loop) [8, 9]. The radius of the loop antenna is 10 cm. The resistance \( R_1 \) and the inductance \( L_1 \) of the loop at resonance have been found to be about 30 \( \Omega \) and 64.6 \( \mu \text{H} \), respectively, using an EM simulation. The width of the loop conductor is 2 mm. The unit cell length has been calculated to be roughly \( \lambda /16 \) with 10 chip capacitors having capacitance 4.35 pF. This arrangement is enough for series resonance to occur as homogeneously as possible over the entire conductor line. Fig. 3 shows a photograph of the fabricated antenna. It has been constructed on a FR-4 substrate with relative permittivity 4.6.

In Fig. 4(a), we show the reflection coefficients \( S_{11} \) based on the EM-simulation and measurement. Their resonant frequencies are shown to be off by about 10 MHz, possibly due to fabrication inaccuracy. The EM-simulated and measured 10dB return loss bandwidths are 1.6% (297 - 302 MHz) at 300 MHz and 2.1% (286 - 292 MHz) at 289 MHz, respectively. The extracted unloaded Q-factor based on [10] is 40.7. Fig. 4(b) shows a relatively uniform current distribution on the loop with about 0.6 \( \lambda \). This uniform current is expected to make strong magnetic fields inside the loop. Fig. 4(c) shows the magnetic field intensity at height \( h \) above center of the loop when the input power is 1 W. The magnetic field intensity generated from the proposed loop antenna is stronger by about 20 dB than that of the conventional one [9].
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Fig. 4. $|S_{11}|$ based on EM-simulation and measurement, current distribution on the loop antenna, and magnetic field intensity at $h$ above the loop center.

Fig. 5 shows the schematic of a wireless power transmission system consisting of two metamaterial-inspired loop antennas. The two identical loop antennas face each other. The symbol $d$ on Fig. 5 represents the distance between the two loop structures.

Fig. 5. Wireless power transmission system.

| $|S_{11}|$ | $|S_{21}|$ |
|----------|----------|
| 5 | 10 |
| 15 | 20 |
| 25 | 30 |

Fig. 6. S-parameters for different distance $d$.

In Fig. 6, the EM-simulated $S_{11}$ and $S_{21}$ of the proposed WPT system have been plotted as a function of frequency for different distances $d$.

The EM-simulated efficiency of the WPT system is shown to be 47.5% using (8) at 300 MHz with $d=5$ cm and $k=0.215$ (using (7)). In Table 1, we summarize the
Table 1. Coupling coefficient and WPT efficiency ($R_1=R_2=30$ Ω, $\beta, \varphi=1$).

| $d$ (cm) | $k$  | $\eta$, efficiency at $f_0$ (%) |
|---------|------|----------------------------------|
|         |      | Theory  | Circuit | EM    |
| 5       | 0.215| 48.8    | 48.6    | 47.5  |
| 10      | 0.104| 45.1    | 45.1    | 44.0  |
| 15      | 0.055| 35.8    | 36.0    | 39.6  |
| 20      | 0.031| 22.2    | 22.2    | 39.9  |
| 25      | 0.023| 15.3    | 14.1    | 28.8  |
| 30      | 0.015| 7.9     | 7.9     | 17.4  |

Table 2. WPT efficiency based on EM-simulation and measurement at new resonant frequency ($f_1$).

| $d$ (cm) | Efficiency (%) | $f_1$ (MHz) |
|---------|----------------|--------------|
|         | EM | Meas. | EM | Meas. |
| 5       | 84.3 | 70.7 | 339.3 | 322.5 |
| 10      | 74.1 | 61.2 | 316.3 | 304.7 |
| 15      | 60.9 | 46.3 | 305.8 | 296.5 |
| 20      | 44.5 | 29.0 | 299.4 | 292.7 |
| 25      | 28.3 | 16.5 | 297.5 | 291.3 |
| 30      | 16.6 | 9.1  | 296.4 | 289.7 |

coupling coefficient ($k$) and WPT efficiency ($\eta$) with different distance ($d$) based on theory and circuit/EM simulations when $\beta, \varphi=1$. The circuit and EM-simulation results agree at distances up to 15 cm, but do not agree well beyond that distance. We observe that the results agree in case $d$ is shorter than $\lambda/6$. This seems to be because our modeling is based on the low frequency circuit theory. The efficiency of the proposed WPT system at 6.7 cm ($\lambda/15$) is 44.3 %, which is 4.3 % higher than that in [2]. In Fig. 6, the resonant frequency is observed to split into two as $d$ becomes smaller. This is due to the strong coupling effect with the approach of the two loops. We analyzed the WPT efficiency at a new resonant frequency ($f_1$). In Table 2, we summarize the WPT efficiency at $f_1$ with different $d$’s. When $d$ is 15 cm, the EM-simulated and measured WPT efficiency are 60.9 % (at 305.8 MHz) and 46.3 % (at 296.5 MHz), respectively. The WPT efficiencies based on EM-simulations and measurements have turned out to be higher at $f_1$ compared with $f_0$ in this work.

Fig. 7 shows the WPT system with two extra rings. The ring is in the shape of a pipe. The thickness of the pipe is 2 mm and its diameter is 2 cm. The radius of the ring is 19.5 cm, which has been optimized for the best wireless power transmission. In Fig. 8, we compare $|S_{21}|$’s among the WPT systems without ring, with one ring, and with two rings when $d=15$ cm ($\lambda/6$).

The maximum efficiency is 93.7 % with two rings. The efficiency of the WPT system with two rings is about 30 % higher than that of the WPT system without ring. It has been found additional currents are induced on the extra rings in the same direction and this leads to a stronger magnetic coupling.

IV. Conclusion

We provided a closed-form formula for WPT system efficiency, which may be used for any load in a receiving loop 2. Then, a thin WPT system consisting of two metamaterial-inspired loop structures, mountable on any flat objects, has been investigated at 300 MHz in its efficiencies. It has been found that due to the formulation based on the low frequency circuit theory, the efficiencies based on theory, EM-simulations, and measurements, agree better with each other when the distance $d$ is shorter than about $\lambda/6$. Due to the significantly enhanced magnetic field intensity of the proposed loop antenna, the EM-simulated and measured efficiencies are
60.9 % and 46.3 %, respectively at \( d=15 \text{ cm} (\lambda/6) \). With two extra rings, the maximum efficiency is 93.7 % at 15 cm (\( \lambda/6 \)). The effect of the additional two rings is about 30%.

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