Elastic-plastic Sliding Asperity Interaction of Machined Multiscale Metal Surface

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Abstract. Sliding process between asperities of machined metal surface is important to analyse wear mechanisms. Elastic-plastic sliding asperity model based on fractal geometry is proposed in this paper to extend the sliding process of single micro-asperity to a macro-asperity. The contact force, the critical interference and residual interference are calculated with fractal dimension $D$ and fractal roughness $G$ in the multiscale views. The influence of fractal parameters on normal and tangential contact force are studied. The results of the simulation show that the plastic deformation and residual interference are the main factors affecting the contacting force and asperity radius. For smaller $G$ and bigger $D$ surfaces, the sliding interaction of asperities is less remarkable and the plastic force accounts for less in the contact force.

Keywords: sliding process; elastic-plastic interaction; fractal geometry; multiscale model.

1. Introduction

Sliding wear is indispensable and considerable for metal surfaces. Most works focused on special metals based on experiments. It is essential to study the mechanism of the sliding process. Asperity-based theory[1], fractal theory[2] and Persson’s theory[3] are classical methods to analyse the contact mechanics of rough metal surfaces. Simplifications and approximations have to be made because different surface machining processes (grinding, milling, turning, etc.), multiple physical characteristics (wear, friction, lubrication, thermal, etc.) and a wide range of factors often affect the contact and sliding between rough surfaces[4,5]. Thus, most previous models are derived from specific condition.

Asperity-based semi-analytical model and finite element model are commonly analysed for sliding asperity interaction. Wang and Schipper[6] extended the Boucly-Nelias-Green (BNG) model[7] of layered semispherical asperity. The factors affecting friction are studied by discrete fast Fourier transform (DC-FFT) technique. Zhao et al.[8] described a frictionless sliding process of parabolic asperity by considering the effect of strain hardening exponents based on the elastic-plastic model proposed by Jackson et al.[9]. Gao et al.[10] presented lateral contact and interaction between shoulder-shoulder asperities and uses the distribution of the contact azimuthal angle to discriminate the contact performances. In addition to asperity-based theory, Persson’s theory is an advanced mechanics way to extrapolate the lateral interaction, adhesion and coalescence between asperities[11]. Mishra et al.[12] studied friction during the Loading and sliding process based on the Material Point Method (MPM). They[13] also analysed the influence of asperity geometric size on sliding forces of elliptical asperities.
However, the contact between real machined metallic surfaces is self-affine as shown in Figure 1. Fractal theory can describe the multi-scale structures of the asperities which are independent of the resolution of the measurement devices\cite{14}. And sliding process of single micro-asperity can be extended to a macro-asperity by fractal geometry. Meng et al.\cite{15} identified Interfacial defect and adhesion wear of $\alpha$-iron asperity-asperity and described the multiscale sliding interaction. Zhang et al.\cite{16} presented an elastic-plastic model of fractal surfaces. Some relevant parameters about oblique asperity contact such as contact angle are introduced to improve the accuracy of load and contact area. Based on this paper, sliding asperity interaction considering the loading and unloading process is depicted in this paper. The surface is assumed to be isotropic and the material is homogeneity. The elastic and plastic contact force is calculated by fractal model. And numerical simulations are applied to acquire the influence of fractal parameters on normal and tangential contact force. With the qualitative analysis of contact force, this work can obtain a more accurate sliding model and might give some helpful results on the sliding asperity interaction in fractal surfaces.

2. Theory and Methodology
Microscopically, the contact points of single asperity contact pairs are not just located at the summits of the asperities according to the real surface topography as shown in Figure 1. And the azimuthal angle along the normal of the contact point is not perpendicular to the substrate of the rough surface generally. Figure 2 describes the asperity sliding interaction process from initial state until removal from contact. The contact angle $\alpha$ is defined as the angle between the tangent plane and the substrate plane and $\tan \alpha = h^{-1}r_a$. Friction, adhesion, coalescence and temperature effects are ignored during the sliding process. And the shape of the asperity is assumed to be sphere. $r_a$ is the tangential deviation of two summits. $R_{upper}$ and $R_{lower}$ are the radii of the upper asperity and lower asperity, respectively. The equivalent radius can be calculated by $R^{-1} = R_{upper}^{-1} + R_{lower}^{-1}$. It can be seen that during the sliding process, the contact angle has persistent changes because of the movement of the contact region and contact point. And permanent deformation is caused by residual interference in the contact area\cite{9}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Simplified contact between rough surfaces with oblique contact asperities.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Schematic of spherical asperity sliding process.}
\end{figure}
Generally, the mating interfaces of an oblique asperity contact pair is treated as asperities condensed by a rigid plane in fractal theory. To exhibit the interaction of the upper and lower asperities, the contact region is extended and exhibited as Figure 2. \( \delta \) is the interferences perpendicular to the surface. 

\[ p = \begin{cases} p_c = 3^{1/4} \left(2\pi \right)^{0.5} G^{D-1} \left(a' \right)^{1.5-0.5D}, & a' > a'_c, \\ p_p = Ha', & a' < a'_c, \end{cases} \]  

(1)

\[ R = \left(a' \right)^{0.5D} \left(2\pi \right)^{-1} G^{1-D}, \]  

(2)

\[ \delta = G^{D-1} \left(a' \right)^{1-0.5D}, \]  

(3)

\[ l = (0.5a')^{0.5}, \]  

(4)

\[ \delta_c = \left(2E' \right)^{-1} \pi H \right)^2 R, \]  

(5)

\[ a'_c = G^2H^{1-D} \left(2E' \right)^{1-D} \left(0.5\pi \right)^{1/2}, \]  

(6)

where \( p_c \) and \( p_p \) are the elastic and plastic components. \( E' \) is the equivalent elastic modulus, defined as \[ \frac{1}{E'} = \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}, \] \( v_1 \) and \( E_1 \), \( v_2 \) and \( E_2 \) are the elastic moduli and Poisson ratios of two contact surfaces, respectively. \( \delta_c \) is the critical interference. \( a' \) is the truncated area and \( a'_c \) is the critical area. \( \sigma_y \) and \( H \) are yield strength and hardness of the softer material. \( D \) is the fractal dimension and \( G \) is the fractal roughness parameter of the surface profile. Due to the instant value of contact force of each sphere asperity pair\[9\], critical interference \( \delta_c \), contact force \( p_c \) and its tangential and normal components, \( p_t \) and \( p_n \) can be written by equations (1)-(6) as:

\[ \delta_c = \frac{2^{0.5D-3}\pi H^2l^D}{E'^2G^{D-1}}, \]  

(7)

\[ p_c = 3^{1/4}2^{4-0.5D} \pi^{0.5} E'G^{D-1}l^{1-D}, \]  

(8)

\[ \begin{align*} p_t &= p_c \sin \alpha, \\ p_n &= p_c \cos \alpha. \end{align*} \]  

(9)

According to method proposed by Zhao et al.\[8\], the continuous sliding process can be approximated as a quasi-static process. In this way, the loading and unloading process can be divided into many static steps and the contact force can be calculated by iterative method. \( i \) is represented as the index of an arbitrary step.

The value of contact angle \( \alpha \) changes with the decrease of \( r_n \) and equal to zero when the contact point moves to the asperity summit. The elastic-plastic contact force during this loading process is continuously increasing. And the asperities begin to rebound elastically during the unloading process. The residual interference \( \delta_{res} \) which would remain indented can be given as\[17\]:

\[ \frac{\delta_{res}}{\delta} = 0.18 \ln \frac{\delta}{\delta_c} - 0.06. \]  

(10)

And the changes on the contact radii in each step can be expressed as:
The elastic-plastic deformation is simulated after $\delta_{\text{res}}$ is calculated. $B$ is the factor of influence of the residual deformation and plastic force.

3. Simulations and Discussions

3.1. Influence of Fractal Dimension and Fractal Roughness on Normal Contact Force

Figure 3 presents the normal contact sliding process in the sliding forces with different fractal parameters. The radii and material of the computed asperities are the same and C45E4 is used to simulate whose material properties are listed in Table 1. The range of asperity overlaps $\delta$ is selected as $0.01R$. The contact force and tangential deviation are effectively normalized by Hertz force and initial tangential position of the contact point.

| Table 1. Material Properties$^{[16]}$. |
|--------------------------------------|
| Elastic modulus (Gpa) | Poisson ratio $\nu$ | Hardness H (HV) |
|-----------------------|---------------------|-----------------|
| 180                   | 0.3                 | 255             |

It can be seen that fractal dimension $D$ and fractal roughness $G$ have great influences on normal contact force. Because of the existence of residual interference and plastic deformation, the spheres are no longer in contact before the tangential deviation arrive the other side. When $x_{r_{a}}^{-1}$ is smaller than zero, the sliding interaction is loading process. And there is no obvious difference in the initial loading state of different fractal parameters. After $x_{r_{a}}^{-1} = -0.6$, the difference becomes evident and bigger fractal parameters has bigger normal contact force until the force is decrease to zero in the unloading process. Furthermore, the plastic deformation is larger for smaller $G$ and bigger $D$.

![Figure 3. The normal contact force in the sliding process, (a) $D=1.5$, (b) $G=1e-6nm$.](image3)

![Figure 4. The tangential contact force in the sliding process, (a) $D=1.5$, (b) $G=1e-6nm$.](image4)
3.2. Influence of Fractal Dimension and Fractal Roughness on Tangential Contact Force

The tangential contact force resembles a sine function and behaves like the dragging effect in Figure 4. However, the negative force in the unloading process is not the same as the loading part. The maximum absolute value of $p_p p^{-1}$ is 0.06 while it is 0.08 in the loading state. The reason is that the plastic deformations produced in the loading process have reduced the radii of the asperities and decrease the contact force in the next iteration. This distinction is also caused by plastic residual interference. And compared to the normal contact force, the tangential contact force begins to differ after $x_a^{-1}$ is equal to zero. It is because sliding occurs at small scales with small the contact angle and asperity overlap. Thus, the value of $p_p p^{-1}$ is much smaller than $p_p p^{-1}$ and it needs a longer loading process to accumulate distinct plastic deformation. The relative tendency of the terminal force is almost the same as the normal contact force. And the tendency of the influence of the fractal parameters is also similar. The increments of the contact force and the decrements of the contact angle lead to this comprehensive curve.

The simulation results described in Figure 3 and Figure 3 basically reflected the qualitative influence of fractal dimension $D$ and fractal roughness $G$ on contact forces. In the multi-scale condition, the sliding interaction is calculated with the different asperities when the length scale $l$ keeps changing with the tangential displacement according to equation (7). And the surface is smoother when the $D$ is bigger and $G$ is smaller. This means more micro-asperities within a macro-asperity region are bearing the load. Hence, the plastic deformation and residual interference play a smaller role. Furthermore, it is effective to select rough surfaces with bigger $D$ and smaller $G$ to decrease elastic-plastic sliding asperity interaction.

4. Conclusion

This paper presented a sliding asperity model based on fractal theory. The contact force and the critical interference are calculated considering the elastic and plastic deformation. The numerical simulation is developed to study the influence of fractal dimension and fractal roughness on the contact normal and tangential force. The results are compared and the effect of plastic deformation and residual interference are analysed. Some conclusions can be drawn as followed.

Fractal dimension $D$ and fractal roughness $G$ have great influences on contact force. For smaller $G$ and bigger $D$ surface, the sliding interaction of asperities is less remarkable and the plastic force account for less in the contact force. The plastic deformation and residual interference are important factors for sliding interaction. In the initial state of loading process, the normal component of the contact force is more obvious than the tangential component. It is because sliding occurs at small scales with small the contact angle and asperity overlap which makes the tangential component much smaller relatively in frictionless surface. It can be seen that sliding interaction model based on fractal theory is an effective method to study the multi-scale characteristics of rough machined surfaces. And sliding interaction with friction, energy loss, material hardening and adhesive should be taken into consideration in further research.

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