Charmonium suppression in QGP at LHC using temperature dependent recombination cross section

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Abstract

We determine the temperature dependent cross section for c ¯c recombination to produce J/ψ. This calculation is carried out in the framework of perturbative QCD (pQCD) and a J/ψ wavefunction is obtained by employing a temperature dependent phenomenological potential between c ¯c. A set of coupled rate equations are established which incorporates color screening, gluon dissociation, collisional damping and recombination in the presence of Quark-Gluon Plasma (QGP) medium expanding under Bjorken scaling law. The final J/ψ suppression thus calculated at mid rapidity for various centrality regions are compared with the experimental data obtained from the Large Hadron Collider (LHC) at \( \sqrt{s_{NN}} = 2.76 \) TeV.

Keywords: Color screening, Recombination, Gluonic dissociation, Collisional damping, Survival probability, pQCD

PACS numbers: 12.38.Mh, 12.38.Gc, 25.75.Nq, 24.10.Pa

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I. INTRODUCTION

Quark Gluon Plasma (QGP) is a state of matter that is supposed to have existed in the first few microseconds after the big bang. There has been a considerable amount of work in studying the QGP state in various $p+p$ and hadron colliders. Charmonium and bottomonium suppression have been two of the key signatures in identifying the existence of QGP. Various experimental [1–5] and theoretical work [6–15] have been done in determining the charmonium and bottomonium suppression. Bottomonium suppression is thought to be cleaner since even at LHC energies, sufficient number of secondary bottom quarks and anti quarks are not produced to form $\Upsilon$. One needs to only model the suppression of the $\Upsilon$ formed due to the initial hard collisions. In the case of charmonia, due to lower rest mass of charm quark, there would be sufficient number of charm and anti charm quarks present even in the latter stages of QGP which can recombine to form more number of $J/\psi$. A more precise estimation of $J/\psi$ suppression needs to incorporate this recombination mechanism with temperature dependent recombination cross-section which is the main aim of the current work. Many authors incorporated the recombination of $c\bar{c}$ pair to quantify charmonium suppression at LHC energy by assuming a fixed value of recombination cross section that seems not very appropriate for heavy ion experiments.

At LHC, in the mid-rapidity region, the produced $J/\psi$ have a transverse momenta in the range 6.5 GeV to 30 GeV and the average transverse momentum is 9.87 GeV [16]. Assuming that the momenta in the longitudinal direction is negligible, it gives an energy in the range of about 10.3 GeV. For the purpose of recombination calculations, this gives an average center of mass energy of 10.3 GeV for the $c\bar{c}$ pair involved in the collisions. On an average, it indicates that the $c\bar{c}$ may have sufficient energy at LHC for pQCD calculations to give reasonable estimate. In the case of a final state gluon being emitted along with $J/\psi$, the initial $c\bar{c}$ pair energy would be even higher, which further justifies pQCD calculations. We use pQCD calculations to determine the temperature dependent recombination time constant, $\Gamma_{\text{recomb}}(T)$ which is based on the temperature dependent cross section $\sigma_{\text{recomb}}^T$ for $c\bar{c} \rightarrow J/\psi$ process at various energies of the $c\bar{c}$ pair. The $J/\psi$ wavefunction used in the calculation of $\sigma_{\text{recomb}}^T$ is determined by using a temperature dependent phenomenological potential. For our pQCD calculation, we mainly consider the process of a quark and anti-quark annihilating to form $J/\psi$ and a gluon in the final state, which is an $\alpha^2$ process. In the $\alpha^2$ process, where there is no final state gluon, the $c\bar{c} \rightarrow J/\psi$ would involve only a very limited phase space domain, in which the sum of energy of incoming $c\bar{c}$ pair needs to be exactly equal to the $J/\psi$ rest mass in the $c\bar{c}$ center of mass frame of reference. Due to this extremely limited phase space domain, it plays negligible role in the recombination process. The other $\alpha^3$ processes involving gluon, quark and antiquark in the initial state have similar limited phase space restrictions. Moreover, they involve a three body collision, and as such these processes are ignored. The $J/\psi$ dissociating into a $c\bar{c}$ pair, and $c\bar{c}$ recombining to produce $J/\psi$ form a system of coupled rate equations, with each feeding the other. In this work, we model this phenomenon by a system of coupled rate equations.

Additional $J/\psi$ and $c\bar{c}$ would be from the interaction of lighter quarks and gluons in the medium. The pQCD framework developed here has used a relativistic modeling of the initial $c\bar{c}$ pair, and hence can be used to calculate the temperature dependent cross section for the lighter quarks to form $J/\psi$. In section II(D), we analyze the impact of the lighter quarks. Additionally some of the $c\bar{c}$ would also decay back to the lighter quarks and gluons. We ignore this decay process. It is to be noted that the process involving the lighter quarks, would not be part of the feedback mechanism happening between $J/\psi$ and $c\bar{c}$, and hence, they would have much lesser impact to the overall suppression or enhancement as compared to the $c\bar{c} \leftrightarrow J/\psi$ process. We also ignore the running of the coupling constant, and instead use a fixed value of the coupling constant.

The charmonium suppression in QGP is not the result of a single mechanism, but is a complex interplay of multiple mechanisms like color screening, gluo-dissociation, collisional damping and recombination. We develop a framework of rate equations, which incorporate all these mechanisms. Color screening is based on Quasi Particle Model (QPM) equation of state (EOS) of QGP described in [17], and the process of gluo-dissociation and collisional damping is based on the equations developed by Wolschin [18]. At the
LHC energies, absorption is expected to be negligible, since the $c\bar{c}$ pair would behave almost as a singlet while traversing the nucleus and hence have negligible interaction with nucleus. $p_t$ broadening due to Cronin effect is inconsequential when expressing $p_t$ integrated suppression at mid rapidity for various centrality bins. That is why, we incorporated only shadowing based on the work done by Vogt [19] as CNM effect in our calculation. In fact in ref. [19], shadowing for Pb+Pb collisions at LHC energy $\sqrt{s_{NN}} = 5.5$ TeV was determined. We use the same framework to calculate shadowing at $\sqrt{s_{NN}} = 2.76$ TeV energy. Our results for $J/\psi$ suppression versus $N_{part}$ are compared with the experimental data in the mid rapidity region [3] obtained from CMS experiment at LHC energy $\sqrt{s_{NN}} = 2.76$ TeV. We find that our predictions show a reasonably good agreement with the above suppression data.

The organization of the rest of the paper is as follows. Section II introduces the system of coupled rate equations, and describes the pQCD calculation of the recombination cross section. Section III briefly describes the mechanism of color screening, gluo dissociation, collisional damping and cold nuclear matter (CNM) effects. The last part of section III, describes the inclusion of all these mechanisms in the rate equations. The section IV gives the results and discussions and finally, conclusion is given in section V.

II. RATE EQUATIONS AND THE RECOMBINATION CROSS SECTION

A. The coupled rate equations

Once the charmonium is formed, we assume that there are two reversible processes which are in play. The first one is the dissociation of charmonium into its constituent charm and anti-charm quark. The second is the recombination of the charm and anti-charm quark to again form charmonium. For a QGP with instantaneous volume $V(t)$, we model these two processes using the following set of rate equations:

$$\frac{dN_{J/\psi}}{dt} = -\Gamma_{diss}(T)N_{J/\psi}(t) + \Gamma_{\text{recomb}}(T)\frac{N_c(t)N_{\bar{c}}(t)}{V(t)}$$

$$\frac{dN_{\bar{c}}}{dt} = \Gamma_{diss}(T)N_{J/\psi}(t) - \Gamma_{\text{recomb}}(T)\frac{N_c(t)N_{\bar{c}}(t)}{V(t)},$$

where $\Gamma_{diss}$ and $\Gamma_{\text{recomb}}$ are gluo dissociation and $c\bar{c}$ recombination rate respectively. The above equations are solved numerically. The initial conditions are given by $N_{J/\psi}(0)$, $N_c(0)$ and $N_{\bar{c}}(0)$ which are derived from $p+p$ collision values, namely $N_{pp}^{J/\psi}$, $N_c^{pp}$ and $N_{\bar{c}}^{pp}$ and then scaled for Pb+Pb collisions. The scaling for Pb+Pb collision is obtained by using the equations [20].

$$N_{J/\psi}(0) = N_{pp}^{J/\psi}(0) \int d^2s \ n_{\text{coll}}(b, s) S(b, s),$$

where the number of collisions is given by

$$n_{\text{coll}}(b, s) = \frac{\sigma_{pp\rightarrow Q\bar{Q}ABTA(s)}T_{AB}(b-s)}{1-\exp(-\sigma_{pp\rightarrow Q\bar{Q}AB})}.$$ 

Woods Saxon distribution [21] is used as a nuclear density function. Here $T_{AB}(b)$ is the nuclear overlap function and following [20], we take $\sigma_{pp\rightarrow Q\bar{Q}} = 0.54$ mb. The shadowing function $S(b, s)$ is determined by the formalism developed by Vogt [19]. It is explained in more detail in the subsection on CNM effect. The value of $N_{pp}^{J/\psi}$ is taken from LHC experimental data [16]. The value of $N_c^{pp}$ and $N_{\bar{c}}^{pp}$ is obtained from the open charm $D$ meson data from LHC [22] and extrapolated till $p_t = 30$ GeV using techniques mentioned in [20, 23]. The open charm $D$ meson distribution is then scaled along the momentum axis according to the momentum fraction of the charm quark present in the $D$ meson to obtain the charm quark momentum distribution. The resultant normalized charm distribution is shown in Fig. 1.

The value of recombination time constant is given by $\Gamma_{\text{recomb}}(T) = \int dp_1 dp_2 f_c(p_1) f_{\bar{c}}(p_2) \sigma_{\text{recomb}}(p_1, p_2) v_{\text{rel}}$, where $v_{\text{rel}}$ is the relative velocity between $c$ and $\bar{c}$ in the laboratory frame. $f_c(p_1)$ and $f_{\bar{c}}(p_2)$ are distribution function of $c$ and $\bar{c}$ with four momenta $p_1$ and $p_2$, respectively. $\sigma_{\text{recomb}}^{T}$ is computed using pQCD calculations. The suppression or enhancement is defined
FIG. 1: Normalized Distribution of charm quark

by

\[ S_{dr} = \frac{N_{J/\psi}(t_{QGP})}{N_{J/\psi}(t_0)}, \quad (2) \]

where \( t_{QGP} \) is the lifetime of QGP, and \( t_0 \) is some initial time, which is described more in section III(E).

B. pQCD calculation of \( \sigma_{recomb} \)

We use pQCD to compute the cross section of charm quark and anti-quark to form a final state gluon and \( J/\psi \). The feynman diagrams corresponding to this process is given in the appendix. We treat the \( J/\psi \) bound state as a linear superposition of definite momentum eigenstates of a free particle. The contribution of each momentum eigenstate is given by the wavefunction \( \psi_T(k, k') \). The bound state is modeled non-relativistically (solution of a Schrödinger equation), with the charm quark and anti-charm quark assumed to have very small momentum spread. From our solution to the Schrödinger equation at the minimum QGP temperature of 170 MeV, i.e. before hadronization, the \( J/\psi \) wavefunction is seen to have a one standard deviation spread of 0.35 GeV (approx.). At higher temperatures, the wavefunction expands spatially leading to a decrease in the momentum spread.

The four momentum conserving delta function \( \delta^4(p_1 + p_2 - k_g - k - k') \) would then imply that the momentum spread of bound state charm quarks and anti-charm quarks needs to be compensated by the momentum spread of input quarks and outgoing gluon. The incoming quarks and anti-quarks may naturally acquire a momenta spread due to confinement in QGP medium. The remaining needs to be compensated by the gluon, which may make the gluon go off shell by a small amount. At higher non zero temperatures of the QGP, where the momentum spread of the bound \( c\bar{c} \) pair is lesser, the gluon needs to go off shell by a much smaller amount than 0.35 GeV. In the current modeling, 0.35 GeV is small compared to an average energy of 10.3 GeV and the \( p_t \) range of 6.5 GeV to 30 GeV. Thus as an approximation we ignore the effect of the momenta spread of the incoming charm quarks and anti-quarks and the outgoing gluon and place the gluon fully on shell. For a bound state consisting of lighter quarks, the momentum spread is expected to be much higher and hence this approximation may not be valid.

With the above approximation of the input quark being in definite momentum eigenstate and the gluon being on shell, the probability amplitude for the various feynman diagrams (given in appendix) is
given by

1. \[ M_1 = \bar{v}(p_2)i\gamma^\mu t^a u(p_1) (\frac{1}{k_2^2} g f^{abc}) \]
   \[ [g^{\mu \nu}(k_1 + k_g)^\sigma + g^{\mu \sigma}(-k_g - k_2)^\nu + g^{\sigma \nu}(k_2 - k_1)^\mu] \]
   \[ \frac{1}{k_2^2} e^a_l(k_g) \left( \int \bar{u}(k)(i\gamma_\alpha t^c)v(k')\psi_T(k, k')dk \right) \]

2. \[ M_2 = \bar{v}(p_2)(i\gamma_\mu t^a)v(\frac{i}{\bar{m} - m}(i\gamma_\mu t^b)\epsilon^{\mu, b}(k_g)u(p_1)(\frac{-ig_{\nu} a}{k_2^2}) \]
   \[ (\int \bar{u}(k)(i\gamma_\alpha t^a)v(k')\psi_T(k, k')dk) \]

3. \[ M_3 = \bar{v}(p_2)(i\gamma_\mu t^a)v(\frac{i}{\bar{m} - m}(i\gamma_\mu t^b)u(p_1)(\frac{-ig_{\nu} a}{k_2^2}) \]
   \[ (\int \bar{u}(k)(i\gamma_\alpha t^a)v(k')\psi_T(k, k')dk) \]

4. \[ M_4 = \bar{v}(p_2)(i\gamma_\mu t^a)v(p_1)(\frac{-ig_{\nu} a}{k_2^2}) \]
   \[ (\int \bar{u}(k)(i\gamma_\mu t^b)v(k')\epsilon^{\mu, b}(k_g) \]
   \[ (\int \bar{u}(k)(i\gamma_\alpha t^a)v(k')\psi_T(k, k')dk) \]

5. \[ M_5 = \bar{v}(p_2)(i\gamma_\mu t^a)v(p_1)(\frac{-ig_{\nu} a}{k_2^2}) \]
   \[ (\int \bar{u}(k)(i\gamma_\mu t^b)v(k')\epsilon^{\mu, b}(k_g) \]
   \[ (\int \bar{u}(k)(i\gamma_\alpha t^a)v(k')\psi_T(k, k')dk) \]

6. \[ M_6 = \int (\bar{v}(p_2)(i\gamma_\mu t^a)v(k')(\frac{-ig_{\nu} a}{k_2^2})) \left( \bar{u}(k)(i\gamma_\alpha t^a)\epsilon^{\mu, b}(k_g) \right) \]
   \[ dk \epsilon^{\mu, b}(k_g) \]

7. \[ M_7 = \int (\bar{v}(p_2)(i\gamma_\mu t^b)\epsilon^{\mu, b}(k_g)u(p_1)(\frac{-ig_{\nu} a}{k_2^2}) \]
   \[ (\bar{u}(k)(i\gamma_\alpha t^a)v(p_1)\psi_T(k, k')) \]

8. \[ M_8 = \int (\bar{v}(p_2)(i\gamma_\mu t^a)v(k')(\frac{-ig_{\nu} a}{k_2^2})) \left( \bar{u}(k)(i\gamma_\mu t^b)\epsilon^{\mu, b}(k_g) \right) \]
   \[ dk \epsilon^{\mu, b}(k_g) \]

9. \[ M_9 = \int (\bar{v}(p_2)(i\gamma_\mu t^a)v(k')(\frac{-ig_{\nu} a}{k_2^2})) \left( \bar{u}(k)(i\gamma_\mu t^b)u(p_1)\psi_T(k, k') \right) \]
   \[ dk \epsilon^{\mu, b}(k_g) \]

10. \[ M_{10} = \int (\bar{v}(p_2)(i\gamma_\mu t^a)v(k')(\frac{-ig_{\nu} a}{k_2^2})) \left( \bar{u}(k)(i\gamma_\mu t^a)\epsilon^{\mu, b}(k_g) \right) \]
    \[ (\frac{-ig_{\nu} a}{k_2^2}) \left( \bar{u}(k)(i\gamma_\mu t^a)u(p_1)\psi_T(k, k') \right) \]
    \[ dk \epsilon^{\mu, b}(k_g). \]

Here \( g \) is the strong coupling constant. The wavefunction \( \psi_T(k, k') \) is obtained numerically, and hence analytical evaluation of the above expressions is not possible. The above amplitudes are evaluated explicitly. \( M^2 \) is then obtained as \( \frac{4 \times g}{(\Sigma_i^{(10)} M_i^*) (\Sigma_i^{(10)} M_i)} \). The factor \( \frac{4 \times g}{(\Sigma_i^{(10)} M_i)} \) comes from averaging over the spins and colors of the incoming charm quark and anti quark. In order to avoid the ghost terms, only the transverse component of the gluon polarization term \( \epsilon^{\mu, b} \) are used.

The charmion wavefunction \( \psi_T(k, k') \) at temperature \( T \) is obtained by solving the Schrödinger equation with a phenomenological potential using the real part of the potential given in Eq.(4). \( m_c \) is the mass of charm quark, while \( u \) and \( v \) are the respective spinors of \( c \) and \( \bar{c} \). We note that \( k = (\sqrt{\bar{k}^2 + m_c^2}, \bar{k}) \) and \( k' = (\sqrt{\bar{k}^2 + m_{c'}^2}, -\bar{k}) \) as 4-momentum of the charm and anti charm quarks. The recombination cross section in the laboratory frame is then computed in cylindrical coordinates.

\[
\sigma(p_1, p_2).v_{rel} = \frac{1}{2\pi} \int_0^{2\pi} d\theta \int_0^{2\pi} d\beta \frac{1}{4\pi^2} \int \frac{1}{d\bar{k}_{com}^2} M^2(p_1, p_2, k_g, k).\bar{K}_{g,1} \frac{2E_1.2E_2.E_{J/q}E_gh_{lab}}{2E_1.2E_2.} \tag{3}
\]

The cylindrical coordinates are chosen so that \( \bar{k}_{com}^2 \) is along the direction of velocity of the center-of-mass \( (\bar{r}_{com}) \) of the \( c\bar{c} \) system. The angle \( \beta \) in the cylindrical coordinates is given by the angle of rotation
around the $\vec{v}^\text{com}$ axis, and $\gamma = \frac{1}{\sqrt{1-|\vec{v}^\text{com}|^2}}$. $k_g$, $k_{J/\psi}$, $p_1$ and $p_2$ are constrained by the 4-momentum conserving delta function $\delta^4(p_1 + p_2 - k_g - k_{J/\psi})$. In general, for all quantities, we use the notation $\vec{k}$ and $\vec{p}$ to denote the spatial component of the 4-momentum $k$ and $p$. $h^{lab}$ is the outcome of integrating the energy conserving delta function $\delta(E_1 + E_2 - E_g - E_{J/\psi})$, and is given by

$$h^{lab} = 1 + \frac{E_g}{(\gamma E_{J/\psi} \cdot (1 + \vec{v}^\text{com}))} - \frac{\vec{v}^\text{com} \cos(\zeta)}{1 + \vec{v}^\text{com}}$$

$\zeta$ is the angle between $\vec{k}_{J/\psi}^\text{com}$ and $\vec{v}^\text{com}$ and $\theta$ is the angle between $c$ and $\bar{c}$ 3-momenta in the laboratory frame. The superscript "com" refers to the $c\bar{c}$ center-of-mass frame.

We take the $\alpha = \frac{g^2}{4\pi} = 0.20$. The value of $\Gamma_{\text{recomb}}$ obtained as a function of temperature is shown in Fig 2. The temperature averaged value of $\Gamma_{\text{recomb}}$ is about 503 $\mu$b. With an average value of $\langle p_t \rangle = 9.87 \text{ GeV}$, we get $v_{rel} = 0.96$, which gives an estimate of $\sigma_{\text{recomb}}^T = \frac{\Gamma_{\text{recomb}}}{v_{rel}} = 524 \mu b$. This value is comparable to value of $\sigma_{\text{recomb}} = 0.65 \text{ mb} = 650 \mu b$ used in [20].

Fig. 3. shows the average value of $\Gamma_{\text{recomb}}$ for the special case of $|p_1| = |p_2|$ at 100 MeV. We see that the value of $\Gamma_{\text{recomb}}(0 \text{MeV})$ first increases a little and then decreases with the center-of-mass energy. The decreasing value with energy is consistent with the fact that the charmonium wavefunction mainly consists of low momentum charm quarks and anti-quarks, which had enabled us to model the charmonium wavefunction non-relativistically in the first place.

C. Volume scaling computation

The volume scaling is based on the quasi particle model (QPM) equation of state (EOS) of QGP and concept of constant entropy condition. The QPM EOS used for computing the volume scaling is described in [17].

In the QPM, the Reynolds number is given by

$$R^{-1} = \frac{(4.0 \eta)}{(3.0 \, T_0 \, \tau_0 \, s)}; \quad \text{where } \eta = \frac{\pi}{4\pi}; \quad s = \text{entropy density density} = 16.41 \text{ GeV}^3 \text{ and } T_0 \text{ is the temperature at } \tau_0 = 0.5 \, fm. \text{ The volume is then given as}$$

$$V(\tau, b) = v_0(b) (\tau_0/\tau)^{R^{-1}-1};$$

FIG. 2: $\Gamma_{\text{recomb}}$ as a function of temperature
where \( v_0(b) \) is the initial volume at \( \tau_0 \), at impact parameter \( b \), and is taken as \( v_0(b) = \pi (R_{Pb} - b/2)^2 \tau_0 \), where \( R_{Pb} = 6.62 \text{ fm} \).

### D. Lighter quarks and gluons

The lighter quarks and gluons would also form \( J/\psi \) and \( c \bar{c} \). After including these processes, the rate equations get modified as

\[
\frac{dN_{J/\psi}}{dt} = -\Gamma_{\text{diss}} N_{J/\psi}(t) + \Gamma_{\text{recomb}} \frac{N_c(t)N_{\bar{c}}(t)}{V(t)} + \Lambda_{q,g \rightarrow J/\psi}
\]

\[
\frac{dN_c}{dt} = \Gamma_{\text{diss}} N_{J/\psi}(t) - \Gamma_{\text{recomb}} \frac{N_c(t)N_{\bar{c}}(t)}{V(t)} + \Lambda_{q,g \rightarrow c \bar{c}},
\]

where

\[
\Lambda_{q,g \rightarrow J/\psi} = \int dp_1 dp_2 f_p(p_1) f_{\bar{p}}(p_2) \sigma_{q,g \rightarrow J/\psi}^{T}(p_1, p_2) v_{\text{rel}}(p_1, p_2)
\]

\[
\Lambda_{q,g \rightarrow c \bar{c}} = \int dp_1 dp_2 f_p(p_1) f_{\bar{p}}(p_2) \sigma_{q,g \rightarrow c \bar{c}}^{T}(p_1, p_2) v_{\text{rel}}(p_1, p_2)
\]

The light quarks and gluons are assumed to be thermalized and hence momentum distribution \( f_p(p_1) \) and \( f_{\bar{p}}(p_2) \) are taken to be Fermi Dirac distribution \( \frac{g_q}{\exp((E-\mu)/T) + 1} \) for light quarks and anti quarks and Bose Einstein \( \frac{g_g}{\exp((E-\mu)/T) - 1} \) for the gluons. The baryonic chemical potential \( \mu \) is taken to be vanishingly small at LHC. The value of degeneracy factor for light quarks \( (g_q) \) and for gluons \( (g_g) \) is taken as 6 and 16, respectively.

For the lighter quarks, only the first five of the ten feynman diagrams are applicable. With the first five diagrams, the value of \( \sigma_{q,g \rightarrow J/\psi}^{T}(p_1, p_2) \) for the lighter quarks are not expected to be negligible when compared to that of \( c \bar{c} \rightarrow J/\psi \) cross section.

However, with vanishing value of \( \mu \), it is seen that most of the quarks and gluons lie in the low energy region. The semilog plot in Fig. 4 shows that at around 1.5 GeV, the particle density reduces to \( 10^{-5} \text{ GeV}^{-4} \). This is not sufficient to form either \( J/\psi \) or \( c \bar{c} \) in any significant quantity, resulting in a negligible...
small value for $\Lambda_{q,g-j/\psi}$ and $\Lambda_{q,g-cc}$. In view of the above facts, with a Fermi Dirac or Bose Einstein distribution for a thermalized quark and gluon, we neglect the contribution arising due to the light quarks and gluons to the rate equation.

III. GLUO-DISSOCIATION AND COLLISIONAL DAMPING

We now briefly describe the gluo-dissociation and collisional damping processes.

A. Collisional Damping

The singlet potential between $c\bar{c}$ pair [18] used in this work is given by

\[ V(r, m_D) = \frac{\sigma_{\text{string}}}{m_D} (1 - e^{-m_D r}) - \alpha_{\text{eff}} \left( m_D + \frac{e^{-m_D r}}{r} \right) - i\alpha_{\text{eff}} T \int_0^\infty \frac{2z dz}{(1 + z^2)^2} \left( 1 - \frac{\sin(m_D rz)}{m_D rz} \right), \]

where $m_D$ is the Debye mass given by $m_D = T \sqrt{\frac{4\pi T}{3}} \left( \frac{N_c^2}{3} + \frac{N_f}{6} \right)$. $N_f = 3$ = number of flavors; $\alpha_s^T = 0.49$; $\sigma_{\text{string}} = 0.192$ GeV$^2$. $\alpha_{\text{eff}} = \frac{4\alpha}{3}$, where we have taken $\alpha = 0.22$. This value of alpha, along with $\alpha_s^T = 0.49$, gives the dissociation temperatures close to the values 381 MeV for $J/\psi$, 190 MeV for $\psi'$, and 197 MeV for $\chi_c$ [24].

Let the singlet wavefunction for the above potential be $g_{nl}(r)$. Collisional damping dissociation time constant = $\Gamma_{\text{damp}} = \int [g_{nl}(r)^\dagger \{Im(V)\} g_{nl}(r)] dr$ [25]. We solve the Schrödinger equation to get the radial wave functions for $1S$, $2S$ and $1P$ states [25].
B. Gluonic dissociation

We model the gluonic dissociation cross section \([18]\) as:

\[
\sigma_{gdiss, nl}(E_g) = \frac{\pi^2 \alpha_u}{N_c^2} \sqrt{\frac{m}{E_g + E_{nl}}} \left( \frac{|J_{nl}^{q-1}|^2 + (l + 1)|J_{nl}^{q+1}|^2}{2l + 1} \right)
\]  

(5)

with \(\alpha_u = 0.59\) and \(J_{nl}^{qd}\) can be expressed using singlet and octet wave functions as \(J_{nl}^{qd} = \int_0^\infty dr \rho_{nl}^d(r) h_q^d(r)\). The octet wave function \(h_{nl}\) is obtained by solving the Schrödinger's equation with potential, \(\alpha_{eff}/(8 r)\) \([18, 25, 26]\). Finally, the cross section is averaged over a Bose-Einstein distribution function of gluons at temperature \(T\) to find the decay rate \(\Gamma_{gdiss}\).

\[
\Gamma_{gdiss, nl} = \frac{g_d}{2\pi^2} \int_0^\infty dp_g \frac{p_g^2 \sigma_{gdiss, nl}(E_g)}{e^{E_g/T} - 1}.
\]

with \(g_d = 16\) for gluons. The net dissociation constant is given by \(\Gamma_{dg} = \Gamma_{damp} + \Gamma_{gdiss}\).

C. Cold Nuclear Matter (CNM) Effect

Vogt had employed the formalism \([19]\) to compute the shadowing effect for various centrality regions at LHC energy \(\sqrt{s_{NN}} = 5.0\) TeV. We use the same formulation to calculate the shadowing effect at center-of-mass energy \(\sqrt{s_{NN}} = 2.76\) TeV. Here we give a brief outline and the details can be obtained from \([19]\). We use the EPS09 \([27]\) parameterization to obtain the shadowing function \(S_p(A, x, \mu)\) at momentum fraction \(x\), and scale \(\mu\). The spatial variation of shadowing is given by

\[
S_p, \rho(A, x, \mu, \vec{r}, z) = 1 + N_p(S_p(A, x, \mu) - 1) \int dz \rho_A(\vec{r}, z) - \int dz \rho_A(0, z)
\]

(6)

where \(N_p\) is determined by the following normalization condition:

\[
\frac{1}{A} \int d^2 r dz \rho_A(s) S_p, S(A, x, \mu, \vec{r}, z) = S_p(A, x, \mu)
\]

(7)

with atomic mass \(A = 208\) for \(Pb\) and \(s = \sqrt{r^2 + z^2}\). The nuclear density \(\rho(s)\) has been taken to be the Woods Saxon distribution: \(\rho_A(s) = \rho_0 \frac{1+\omega(s/R_A)^2}{[1+exp((s-R_A)/d)]}\). The values of \(\rho_0, R_A, d\) and \(\omega\) have been taken from \([21]\) for \(Pb\).

The suppression factor is defined as the ratio:

\[
S_{cnm} = R_{AB}(N_{part}; b) = \frac{d\sigma_{AB}/dy}{T_{AB}(b)d\sigma_{pp}/dy},
\]

(8)

where \(T_{AB}\) is the nuclear overlap function given by

\[
T_{AB}(b) = \int d^2 s d z_1 d z_2 \rho_A(s, z_1) \rho_B(|\vec{b} - \vec{z}|, z_2).
\]

(9)

In this particular case both \(A\) and \(B\) stand for lead, and \(A = B = 208\).

From \([28]\), the color evaporation model (CEM) gives:

\[
\sigma_{AB} = \int d z_1 d z_2 d^2 r d x_1 d x_2 f_g(A, x_1, \mu, r, z_1) f_g(B, x_2, \mu, b - r, z_2) \sigma_{gg,QQ}(x_1, x_2, \mu)
\]

(10)
\[ \sigma_{pp} = \int dx_1 dx_2 f_g(p, x_1, \mu) f_g(p, x_2, \mu) \sigma_{g g Q Q}(x_1, x_2, \mu) \]  \tag{11}

This formalism excludes the explicit modeling of spin and color of the initial and final states. In the above expressions, \( x_1 \) and \( x_2 \) are the momentum fraction of gluons in the two Pb nuclei at \( \sqrt{s_{NN}} = 2.76 \) TeV. \( f_g(A, x, \mu, r, z) \) is determined from \( f_g(p, x, \mu) \) by using the following relation:

\[ f_g(A, x, \mu, r, z) = \rho(s) S(A, x, \mu, r, z) f_g(p, x, \mu) \]  \tag{12}

The gluon distribution function in a proton = \( f_g(p, x, \mu) \) has been estimated using CTEQ6\[29\]. With these relations one can determine \( \frac{d\sigma_{AB}}{dy} \) and \( \frac{d\sigma_{pp}}{dy} \), and finally \( R_{AB} \) is computed.

### D. Color Screening

The color screening model used in the present work is based on pressure profile \[24\] in the transverse plane and cooling law for pressure based on QPM EOS \[17, 25\] for QGP. The cooling law for pressure is given by

\[ p(\tau, r) = A + \frac{B}{\tau^q} + \frac{C}{\tau} + \frac{D}{\tau^{c_2^2}}, \]  \tag{13}

where \( A = -c_1, B = c_2 c_s^2, C = \frac{4\eta q}{3(c_2^2 - 1)} \) and \( D = c_3 \). The constants \( c_1, c_2, c_3 \) are given by

- \( c_1 = -c_2 \tau^q - \frac{4\eta}{3c_2^2 \tau} \);
- \( c_2 = \frac{c_0^{-1} - \frac{4\eta}{3c_2^2 \tau^q}}{\tau} \);
- \( c_3 = (p_0 + c_1) \tau_0^2 c_s^2 - c_2 c_s^2 \tau_0^{-1} - \frac{4\eta}{3} \left( \frac{q}{c_2^2 - \tau} \right)^{c_2^2 - 1} \).

The above constants are determined by using different boundary conditions on pressure and energy density described in \[17, 25\].

Writing the above equations at initial time \( \tau = \tau_0 \) and screening time \( \tau = \tau_s \) and combining with the pressure profile \[17\], we get the following two equations:

\[ p(\tau_0, r) = A + \frac{B}{\tau_0^q} + \frac{C}{\tau_0} + \frac{D}{\tau_0^{c_2^2}} = p(\tau_0, 0) h(r); \]  \tag{14}

\[ p(\tau_s, r) = A + \frac{B}{\tau_s^q} + \frac{C}{\tau_s} + \frac{D}{\tau_s^{c_2^2}} = p_{QGP}, \]  \tag{15}

where \( p_{QGP} \) is the pressure of QGP inside the screening region required to dissociate a particular charmonium state and it is determined by QPM EOS for QGP medium. The above equations are solved numerically to determine radius of screening region \( r_s \) \[17, 25\]. The expression for survival probability due to color screening can be obtained as:

\[ S_c(p_T, N_{part}) = \frac{2(\alpha + 1)}{\pi R_T^2} \int_0^{R_T} dr r \phi_{max}(r) \left( 1 - \frac{r^2}{R_T^2} \right)^\alpha, \]

where \( \alpha = 0.5, R_T \) and \( \phi_{max} \) (which is a function of \( p_T \) and \( r_s \)) are defined in \[17, 25\]. The final suppression due to color screening for \( J/\psi \) is obtained after including the feeddown contribution from higher resonance states of charmonium.
E. Final suppression

The effect of shadowing and the rate equation involving gluo-dissociation, collisional damping and recombination can be put together as: $S_{\text{cnm}}S_{\text{dr}}$. The question arises as to how the suppression due to color screening can be combined. The process of gluo-dissociation and collisional damping can happen even before $J/\psi$ is fully formed. Hence the processes of color screening and gluo-dissociation and collisional damping overlap in time. In Fig. 2, we see that $\Gamma_{\text{recomb}}(T)$ is high at low temperatures. In the latter stages of QGP when temperature is low, color screening is expected to be small. The color screening is dominant in the initial stages when temperature is high. Hence we need to apply color screening only during the initial stages of QGP. In order to incorporate the color screening into the system of rate equations within the framework of the above arguments, we first determine the equivalent $\Gamma_e$ corresponding to $S_c$.

$$\Gamma_e = -\ln(S_c)/(\gamma t_{f0} - t_0).$$

where $t_{f0}$ is the formation time of $J/\psi$, and $\gamma$ is the Lorentz dilation factor corresponding to average $p_t = 9.87$ GeV [12]. Before $t = t_0$, the separation between $c$ and $\bar{c}$ would be very small and the $c$ and $\bar{c}$ pair would act as a color singlet. We take $t_0 = 0.5$ fm as initial start time, before which all processes of dissociation and recombination are assumed to be negligible. The rate equations are then modified as $\Gamma_{\text{diss}} = \Gamma_{\text{dg}} + \Gamma_e$ if time $< (\gamma t_{f0})$ and $\Gamma_{\text{diss}} = \Gamma_{\text{dg}}$ otherwise. The final suppression is then $S_{\text{cnm}}S_{\text{dr}}$, where $S_{\text{dr}}$ now includes color screening.

IV. RESULTS AND DISCUSSIONS

Our calculation of the recombination cross-section shown in Fig. 2, gives expected variation with temperature. As the temperature approaches the dissociation temperature of $J/\psi$, the value of $\Gamma_{\text{recomb}}$ approaches zero in accordance with the expectation. Our predicted $J/\psi$ suppression due to color screening alone is shown in Fig. 5. The experimental data on $J/\psi$ suppression at LHC energy $\sqrt{s_{NN}} = 2.76$ TeV [4] versus $N_{\text{part}}$ is also shown on the same figure for comparison. We see that the suppression is clearly underestimated. This indicates some other processes may be clearly playing a role. Fig. 6 shows the

![Figure 5](image.png)

FIG. 5: Our predicted suppression due to only color screening. Experimental data on $J/\psi$ suppression at LHC are taken from [4].
effect of shadowing and the combination of dissociation and recombination. The curve "only dissociation" is obtained by assigning $\Gamma_{\text{recomb}} = 0$. The difference between the "only dissociation" and "recombination and dissociation" curves is the result of recombination. It can be seen that for peripheral collisions, the dissociation dominates, while for central collisions the effect of recombination is visible. From Fig. 2, one can see that $\Gamma_{\text{recomb}}$ is larger at lower temperature, and this should lead to higher recombination in peripheral collisions where temperature is lower. However we see the contrary. Recombination depends on the product $N_cN_{\bar{c}}$. If we set $N_c = N_{\bar{c}}$, recombination depends quadratically on $N_c$. On the other hand dissociation depends linearly on $N_{J/\psi}$. In the peripheral collisions both $N_{J/\psi}$ and $N_c$ are small, but with the quadratic dependence on $N_c$, the recombination effect is pulled down and dissociation dominates overall.

Finally, Fig. 7 shows the final suppression of $J/\psi$, which is comparable to the experimental data up to the most extent throughout the whole range of $N_{\text{part}}$. Some of the central regions show little under suppression, but it is still just within the boundary of the error bars. The trend of our simulation results is showing reasonably good agreement with the experimental suppression data. One source of uncertainty in suppression might be that strange quarks may not be fully thermalized. If we compare the non-thermalized distribution in Fig. 1 and the thermalized Fermi Dirac or Bose Einstein distribution in Fig. 4, we can see that the distribution is shifted to the right in case the distribution is non-thermalized. A partially non-thermalized strange quark distribution will have more strange quarks with sufficient energy to form $c\bar{c}$ pairs or $J/\psi$. This would lead to a non negligible value of $\Lambda_{q,g\rightarrow c\bar{c}}$ and $\Lambda_{q,g\rightarrow J/\psi}$. This would infuse more $c\bar{c}$ and $J/\psi$ into the system leading to a slightly different value of suppression. The distribution of $c$ and $\bar{c}$ is also taken to be independent of temperature. Any variation in the charm quark distribution with temperature or time, can lead to a change in the total $J/\psi$ production due to recombination. It is also observed that shadowing calculation is a relatively uncertain quantity. Though we have not depicted the uncertainty in this work, EPS09 parameterization gives a range of uncertainty in the shadowing. In spite of the above uncertainties, we see that our model has been able to explain the trend of centrality dependent $J/\psi$ suppression data in the mid rapidity region obtained from LHC experiment with a reasonably agreement.
FIG. 7: Suppression with color screening, gluonic dissociation along with collisional damping, recombination and shadowing. Experimental data on suppression at LHC are taken from [4].

V. CONCLUSIONS

We have analytically calculated the cross section for recombination of $c\bar{c}$ into $J/\psi$ at LHC conditions for various temperatures using pQCD. A temperature dependent phenomenological potential has been utilized for computing the $J/\psi$ wavefunction which is used in the calculation of the recombination cross-section. We have established a set of rate equations which combines color screening, gluodissociation, collisional damping along with the recombination. CNM effects, namely shadowing is also incorporated. We find that the our final results explain the experimental LHC data on $J/\psi$ suppression in the mid rapidity region to a good extent. The difference between the experimental data and the predicted suppression could possibly be attributed due to the limitation in the accuracy of determining the initial conditions for the rate equations, incomplete thermalization of strange quarks and in the limitation in the accuracy of estimating shadowing effect.

Acknowledgment

We thank P. K. Srivastava, et al., for providing the color screening code. One of the authors (S. Ganesh) acknowledges Broadcom India Research Pvt. Ltd. for allowing the use of its computational resources required for this work. M. Mishra is grateful to the Department of Science and Technology (DST), New Delhi for financial assistance from the Fast Track Young Scientist project.

[1] M. C. Abreu et al., (NA50 Collaboration), Phys. Lett. B 477, 28 (2000); B. Alessandro et al., (NA50 Collaboration), Eur. Phys. J. C 39, 335 (2005).
[2] R. Arnaldi et al., (NA60 Collaboration), Phys. Rev. Lett. 99, 132302 (2007); R. Arnaldi (NA60 Collaboration), Presentation at the ECT workshop on "Heavy Quarkonia Production in Heavy-Ion Collisions," Trent (Italy), May 25-29 (2009).
[3] A. Adare et al., (PHENIX Collaboration), Phys. Rev. Lett. 98, 232301 (2007).
[4] The CMS Collaboration, J. High Energy Phys. 05, 063 (2012).
[5] B. Abelev et al., (ALICE Collaboration), Phys. Rev. Lett. 109, 072301 (2012).
[6] J. P. Blaizot and J. Y. Ollitrault, Phys. Rev. Lett. 77, 1703 (1996).
[7] J. P. Blaizot, P. M. Dinh, and J. Y. Ollitrault, Phys. Rev. Lett. 85, 4012 (2000).
[8] A. Capella, E. G. Ferreiro, and A. B. Kaidalov, Phys. Rev. Lett. 85, 2080 (2000).
[9] A. K. Chaudhuri, Phys. Rev. C 64, 054903 (2001); Phys. Lett. B 527, 80 (2002).
[10] A. K. Chaudhuri, Phys. Rev. Lett. 88, 232302 (2002).
[11] A. K. Chaudhuri, Phys. Rev. C 66, 021902 (2002).
[12] A. K. Chaudhuri and P. P. Bhaduri, arXiv:1202.3291[nucl-th], (2012).
[13] Yunpeng Liu et al., J. Phys. G 37, 075110 (2010).
[14] Zhen Qu et al., Nucl. Phys. A 830, 335c (2009).
[15] Rishi Sharma and Ivan Vitev, Phys. Rev. C 87, 044905 (2013).
[16] The CMS Collaboration, arXiv:1201.5069v2 [nucl-ex], (2013)
[17] P. K. Srivastava, M. Mishra and C. P. Singh, Phys. Rev. C 87, 034903 (2013).
[18] F. Nendzig, G. Wolschin, Phys. Rev. C 87, 024911 (2013).
[19] R. Vogt, Phys. Rev. C 81, 044903 (2010).
[20] E. G. Ferreiro, Phys. Lett. B 731, 57-63 (2014).
[21] C. W. deJager, H. deVries, and C. deVries, Atomic Data and Nuclear Data Tables 14 485, (1974).
[22] The ALICE Collaboration, J. High Energy Phys. 07, 191 (2012).
[23] F. Bossu et al., arXiv:1103.2391v3 [nucl-ex], Apr (2012).
[24] M. Mishra, C. P. Singh, V. J. Menon and Ritesh Kumar Dubey, Phys. Lett. B 656, 45 (2007).
[25] S. Ganesh, M. Mishra, Phys. Rev. C 88, 044908 (2013).
[26] C. Y. Wong, Phys. Rev. D 60, 114025 (1999).
[27] K. J. Eskola, H. Paukkunen and C. A. Salgado, J. High Energy Phys. 0904, 065 (2009).
[28] V. Emelyanov, A. Khodinov, S. R. Klein, R. Vogt, Phys. Rev. Lett. 81, 1801-1804 (1998).
[29] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky and W. K. Tung, JHEP 0207, 012 (2002); arXiv:hep-ph/0201195
Appendix A: Feynman Diagrams and Group Theoretic Factors

The Feynman diagrams pertaining to the calculation of $\sigma_{\text{recomb}}$ is shown in the following figures.
FIG. 8: $c\bar{c}$ scattering to emit a gluon and $J/\psi$.

The group theoretic color factors appearing in the calculation of the above diagrams is given below. In the equations written below, $G_{i,j}$ refers to the group theoretic color factor appearing in the calculation of $M_i^* M_j$. $N_c = \text{number of colors} = 3$. the value of $h$ is given by $h^{abc} = d^{abc} + i f^{abc}$ where $f$ is the totally antisymmetric structure constants of SU(3), while $d$ is totally symmetric.

\[
G_{1,1} = N_c/4(N_c^2 - 1); \quad G_{1,2} = i N_c/8(N_c^2 - 1);
\]
\[
G_{1,3} = i N_c/8(1 - N_c^2); \quad G_{1,4} = i N_c/8(N_c^2 - 1);
\]
\[
G_{1,5} = i N_c/8(N_c^2 + 1); \quad G_{1,6} = i /8(N_c^2 + 1);
\]
\[
G_{1,7} = i /8(N_c^2 - 1); \quad G_{1,8} = i /8(N_c^2 + 1);
\]
\[
G_{1,9} = i /8(N_c^2 - 1); \quad G_{1,10} = 2 i f^{bac} f^{bac} h^{abc} + N_c ^2 /8(1 - N_c^2);
\]
\[
G_{2,2} = N_c ^3 /8 - N_c/4 + 1/8 N_c; \quad G_{2,3} = 1/8(N_c + 1/N_c);
\]
\[
G_{2,4} = 4 h^{a'ba} h^{a'a b}; \quad G_{2,5} = 4 h^{ba'a} h^{ba'a};
\]
\[
G_{2,6} = N_c ^3 /8 + 1/4 - 1/8 N_c ^2; \quad G_{2,7} = 1/8(1 - 1/N_c^2);
\]
\[
G_{2,8} = 1/8(1 - 1/N_c^2); \quad G_{2,9} = 1/8(N_c^2 - 1/N_c^2);
\]
\[
G_{2,10} = i /8(N_c^2 - 1);
\]
\[
G_{3,3} = N_c ^3 /8 - N_c/4 + 1/8 N_c ; \quad G_{3,4} = 4 h^{aa'b} h^{aa'b};
\]
\[
G_{3,5} = 4 h^{ba'a} h^{ba'a};
\]
\[
G_{3,6} = 1/8(1 - 1/N_c^2); \quad G_{3,7} = N_c ^3 /8 + 1/4 - 1/8 N_c ^2;
\]
\( G_{3,8} = \frac{1}{8} (N_{c}^2 - 1) \); 
\( G_{3,10} = i/8(N_{c}^2 - 1) \); 
\( G_{4,4} = N_{c}^3/8 - N_{c}/4 + 1/8N_{c} \); 
\( G_{4,5} = \frac{1}{8}(N_{c} + 1) \); 
\( G_{4,6} = \frac{1}{8}(1 - 1/N_{c}^2) \); 
\( G_{4,7} = N_{c}^2/8 + 1/4 - 1/N_{c}^2 \); 
\( G_{4,8} = N_{c}^2/8 + 1/4 - 1/8N_{c}^2 \); 
\( G_{4,9} = 1/8(1 - 1/N_{c}^2) \); 
\( G_{4,10} = i/8(N_{c}^2 + 1) \); 
\( G_{5,5} = N_{c}^3/8 - N_{c}/4 + 1/8N_{c} \); 
\( G_{5,6} = \frac{1}{8}(N_{c}^2 - 1) \); 
\( G_{5,7} = \frac{1}{8}(1 - 1/N_{c}^2) \); 
\( G_{5,8} = \frac{1}{8}(1 - 1/N_{c}^2) \); 
\( G_{5,9} = N_{c}^2/8 + 1/4 - 1/N_{c}^2 \); 
\( G_{5,10} = i/8(N_{c}^2 + 1) \); 
\( G_{6,6} = N_{c}^3/8 - N_{c}/4 + 1/8N_{c} \); 
\( G_{6,7} = 4k^{a'b}k^{a'b} \); 
\( G_{6,8} = 1/8(N_{c} + 1) \); 
\( G_{6,9} = 4k^{ba'a}k^{ba'a} \); 
\( G_{6,10} = iN_{c}/8(N_{c}^2 - 1) \); 
\( G_{7,7} = N_{c}^3/8 - N_{c}/4 + 1/8N_{c} \); 
\( G_{7,8} = 4k^{aa'b}k^{aa'b} \); 
\( G_{7,9} = 1/8(N_{c} + 1) \); 
\( G_{7,10} = iN_{c}/8(N_{c}^2 + 1) \); 
\( G_{8,8} = N_{c}^3/8 - N_{c}/4 + 1/8N_{c} \); 
\( G_{8,9} = 4k^{ba'b}k^{ba'a} \); 
\( G_{8,10} = iN_{c}/8(N_{c}^2 - 1) \); 
\( G_{9,9} = N_{c}^3/8 - N_{c}/4 + 1/8N_{c} \); 
\( G_{9,10} = iN_{c}/8(N_{c}^2 - 1) \); 
\( G_{10,10} = N/4(N^2 - 1) \);