Remarks on the Gribov Problem in Direct Maximal Center Gauge

Manfried Faber

Inst. für Kernphysik, Technische Universität Wien, A-1040 Vienna, Austria
E-mail: faber@kph.tuwien.ac.at

Jeff Greensite

Physics and Astronomy Department, San Francisco State University, San Francisco, CA 94117 USA
Theory Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720 USA
E-mail: greensit@quark.sfsu.edu, JPGreensite@lbl.gov

ˇStefan Olejník

Institute of Physics, Slovak Academy of Sciences, SK-842 28 Bratislava, Slovakia
E-mail: fyziolej@savba.sk

ABSTRACT: We review the equivalence of maximal center gauge fixing to the problem of finding the best fit, to a given lattice gauge field, by a thin vortex configuration. This fit is necessarily worst at the location of P-plaquettes. We then compare the fits achieved in Gribov copies generated by (i) over-relaxation; (ii) over-relaxation after Landau gauge preconditioning; and (iii) simulated annealing. Simulated annealing yields the best fit if all links on the lattice are included, but the situation changes if we consider only the lattice volume exterior to P-plaquettes. In this exterior region, the fit is best for Gribov copies generated by over-relaxation, and worst for Gribov copies generated after Landau gauge preconditioning. The two fitting criteria (including or not including the P-plaquettes) yield string tensions differing by $-34\%$ to $+20\%$ respectively, relative to the full string tension. Our usual procedure (“quenched minimization”) seems to be a compromise between these criteria, and yields string tensions at an intermediate value close to the full string tension.

KEYWORDS: Confinement, Lattice Gauge Field Theories, Solitons Monopoles and Instantons
1. Introduction

Direct maximal center gauge is the common name for lattice Landau gauge in the adjoint representation. For the SU(2) group, adjoint links $U_A$ in this gauge satisfy the condition

$$\text{Tr} \sum_{\mu} L_i \left( U_{A\mu}(x) - U_{A\mu}(x - \hat{\mu}) \right) = 0 \quad (1.1)$$

at every point (the $\{L_i\}$ are the SU(2) group generators in the adjoint representation) leaving a residual $Z_2$ symmetry. This gauge is used in studies of confinement, in particular to locate center vortices in thermalized lattice configurations.

As in ordinary lattice Landau gauge, there are many points on the gauge orbit, not related by any residual symmetry, which satisfy the local gauge condition (1.1). These configurations are known as Gribov copies. Difficulties associated with Gribov copies in maximal center gauge have been noted in recent work by Borntakov et al. [1, 2], and previously by Kovács and Tomboulis [3]. In this article we discuss some issues that we believe are relevant to the problem of generating a set of Gribov copies in maximal center gauge, and to choosing the “best” Gribov copy among the set. Before proceeding, we should note that the Gribov problem can be avoided altogether by using the Laplacian center gauge [4]. On the other hand, the Laplacian version does have certain unattractive features, notably the lack of scaling of the $P$-vortex density [5] and the lack of “precocious linearity” [6]. While these features of Laplacian center gauge are by no means fatal, we think it worthwhile to further explore the Gribov copy issue in the older proposal of maximal center gauge.
2. Gauge Fixing as a “Best Fit” Procedure

Gauge fixing always has a flavor of arbitrariness, and claims for a privileged gauge are typically regarded with suspicion. To understand the rationale for maximal center gauge, it is essential to understand its function as a fit of a given lattice configuration by a singular pure gauge (thin vortex) field. This insight, which we will now elaborate, is due to Engelhardt and Reinhardt in ref. [7].

Imagine running a Monte Carlo simulation of lattice SU(2) gauge theory at high $\beta$, and printing out the values of link variables $U_\mu(x)$ in some thermalized configuration. At a glance, these values would look like random numbers, but of course this impression is deceptive. Locally the link variables are only small fluctuations around a pure gauge configuration. Suppose we then ask for the pure gauge configuration $g(x)g^\dagger(x + \hat{\mu})$ which is closest, in lattice configuration space, to the given lattice gauge field $U_\mu(x)$. Using the standard metric on the SU(2) group manifold, this is equivalent to asking for the gauge transformation $g(x)$ which minimizes the square distance in configuration space

$$d^2 = \sum_{x,\mu} \text{Tr} \left[ (U_\mu(x) - g(x)g^\dagger(x + \hat{\mu}))(U_\mu(x) - g(x)g^\dagger(x + \hat{\mu}))^\dagger \right]$$

$$= 2 \sum_{x,\mu} \left( \text{Tr}[I_2] - \text{Tr}[g^\dagger(x)U_\mu(x)g(x + \hat{\mu})] \right)$$

This quantity is minimized by finding the gauge-transformed configuration

$$gU_\mu(x) \equiv g^\dagger(x)U_\mu(x)g(x + \hat{\mu})$$

which maximizes

$$R_{lan} = \sum_{x,\mu} \text{Tr}[g^\dagger U_\mu(x)]$$

From this we conclude that the problem of finding the pure-gauge configuration closest to a given lattice gauge field is completely equivalent to the problem of fixing that lattice gauge field to the Landau gauge.

We next generalize this idea slightly, and allow for $Z_2$ dislocations in the gauge transformation. This means fitting the lattice configuration by a slightly more general form

$$U^\text{vor}_\mu(x) = g(x)Z_\mu(x)g^\dagger(x + \hat{\mu})$$

where $Z_\mu(x) = \pm 1$. This is a thin center vortex configuration, generated by a singular gauge transformation. Note that $U^\text{vor}_\mu$ becomes a continuous pure gauge in the adjoint representation, which is blind to the $Z_\mu(x)$ factor. This motivates a two-step fitting procedure: First, we determine $g(x)$ up to a residual $Z_2$ transformation, by minimizing the square distance $d^2_A$ in configuration space between $U_\mu(x)$ and $U^\text{vor}_\mu(x)$.
in the adjoint representation:
\[
d^2_A = \sum_{x,\mu} \text{Tr} \left[ (U_{A\mu}(x) - g_A(x)g_A^\dagger(x + \hat{\mu}))(U_{A\mu}(x) - g_A(x)g_A^\dagger(x + \hat{\mu}))^\dagger \right] \\
= 2 \sum_{x,\mu} \left( \text{Tr}[I_3] - \text{Tr}[g_A^\dagger(x)U_{A\mu}(x)g_A(x + \hat{\mu})] \right) \\
= 2 \sum_{x,\mu} \left( 4 - \left| \text{Tr}[g^\dagger(x)U_{\mu}(x)g(x + \hat{\mu})] \right|^2 \right) \tag{2.5}
\]

The distance function in this case is determined by the metric on the $SU(2)/Z_2$ group manifold. Minimizing $d^2_A$ is equivalent to fixing to direct maximal center gauge, which seeks the largest value of
\[
R_{dmc} = \sum_{x,\mu} \left| \text{Tr}[gU_{\mu}(x)] \right|^2 \tag{2.6}
\]
The condition that $R$ is stationary leads to the adjoint Landau gauge condition (1.1). Having determined $g(x)$, we then find $Z_{\mu}(x)$ at each link by minimizing
\[
I^2_{\mu}(x) = \text{Tr} \left[ (U_{\mu}(x) - g(x)Z_{\mu}(x)g^\dagger(x + \hat{\mu}))(U_{\mu}(x) - g(x)Z_{\mu}(x)g^\dagger(x + \hat{\mu}))^\dagger \right] \tag{2.7}
\]
which is easily seen to require
\[
Z_{\mu}(x) = \text{sign} \text{Tr}[gU_{\mu}(x)] \tag{2.8}
\]
This is the center projection prescription.

In this way we have shown that maximal center gauge, together with the rule (2.8) for center projection, is equivalent to finding a “best fit” of a given lattice configuration $U_{\mu}(x)$ by a thin vortex configuration $U_{\mu}^{\text{vor}}(x)$. While this (rather trivial) derivation may be novel, the essential point has been made previously in ref. [7].

### 2.1 Center Dominance

The gauge-transformed lattice configuration in maximal center gauge can be written
\[
gU_{\mu}(x) = Z_{\mu}(x)e^{iA_{\mu}(x)} \quad (\text{Tr}e^{iA_{\mu}(x)} \geq 0) \tag{2.9}
\]
with the original lattice configuration
\[
U_{\mu}(x) = g(x)Z_{\mu}(x)e^{iA_{\mu}(x)}g(x + \hat{\mu}) \tag{2.10}
\]
where $g(x), Z_{\mu}(x)$ are determined by the procedure just described. Our claim, elaborated in refs. [6,8], is that the confining properties of $U_{\mu}(x)$ are entirely encoded in the $Z_2$ link variables $Z_{\mu}(x)$; the variables $A_{\mu}(x)$, which are typically of order $1/\beta$, are responsible for short range effects such as the Coulomb force law. Two necessary, but not sufficient, conditions for this claim to hold true are as follows:
1. Set $A_\mu(x) = 0$ in (2.10), which sends $U_\mu(x) \to U_\mu^{\text{vor}}(x)$. The asymptotic string tension in the $U^{\text{vor}}$ configuration should match the asymptotic tension of the full configuration, i.e. “center-projected” Wilson loops

$$Z(C) = \frac{1}{2} \text{Tr}[U^{\text{vor}}(C)]$$

have the same string tension as full Wilson loops (where $Z(C)$ denotes a product of center-projected links around loop $C$). This property is known as “Center Dominance.”

2. Dropping the $Z_\mu(x)$ variables in (2.10), which can be interpreted as removing center vortices from the system, the Wilson loops constructed from the remaining degrees of freedom

$$\tilde{W}(C) \equiv \text{Tr}\left[ \prod_{\text{links} \in C} e^{iA_\mu} \right] = Z(C)\text{Tr}[U(C)]$$

should have zero string tension. This test was carried out by de Forcrand and D’Elia in ref. [9].

The reason that these properties are not sufficient for our purposes is that confining fluctuations might be hidden in the $Z_\mu(x)$ variables along with a lot of short-range physics. If that is the case, then we might not gain much from the truncation of $U_\mu$ to $Z_\mu$. For example, even a naive projection $Z_\mu(x) = \text{sign} \text{Tr}[U_\mu(x)]$ of the original configuration (with no gauge fixing) has the center dominance property, as is easily verified by simple group-theoretic arguments [6, 10]. But this naive projection also reproduces exactly! the Coulomb potential at short distance scales, which is certainly due to gaussian field fluctuations rather than a vortex mechanism. Our objective is to strip away such short-range effects, and to isolate as far as possible the degrees of freedom which are solely responsible for infrared physics.

### 2.2 Where Fitting Fails

The square deviation at each link, away from a pure gauge in the adjoint representation, is proportional to

$$\delta_\mu^2(x) = \frac{1}{8} \text{Tr}\left[ (U_A\mu(x) - g_A(x)g_A^\dagger(x + \hat{\mu}))(U_A\mu(x) - g_A(x)g_A^\dagger(x + \hat{\mu}))^\dagger \right]$$

$$= 1 - \frac{1}{4} \left( \text{Tr}[^gU_\mu(x)] \right)^2$$

This quantity is generally small, of order $1/\beta$, at a local minimum of $d_A^2$. The exception is for some links (at least one) belonging to each P-plaquette. For these links we must have a very poor fit, with $\delta_\mu^2(x) \sim O(1)$ regardless of $\beta$. We recall
that a plaquette $p$ is a P-plaquette iff $Z(p) = -1$, and that P-plaquettes belong to P-vortices.

The fit to a thin vortex has to be bad at P-plaquettes for the following reason: At large $\beta$, we generally have (even at P-plaquettes)

$$\frac{1}{2} \text{Tr}[U(p)] = 1 - O\left(\frac{1}{\beta}\right) \quad (2.14)$$

On the other hand

$$\text{Tr}[U(p)] = Z(p) \text{Tr}\left[ \prod_{\text{links} \in p} e^{iA_\mu(x)} \right] \quad (2.15)$$

But if $Z(p) = -1$, equations (2.14) and (2.15) imply that $A_\mu(x)$ is $O(1)$ rather than $O(1/\beta)$, on one or more links in the plaquette $p$. The magnitude of $A_\mu(x)$ is a measure of the goodness-of-fit, and in fact $\delta^2_\mu$ depends only on this variable. A perfect fit on a link corresponds to $A_\mu = 0$, while $A_\mu \sim O(1)$ is a very bad fit.

In ordinary Landau gauge, $\delta^2_\mu(x)$ can in principle be small on every link. In contrast, in maximal center gauge, we see that this property can only hold if there are no P-vortices found on the lattice.

### 3. Quenched vs. Annealed Gribov Copies

There is no known method for finding the global minimum of the distance function $d^2_A$, but two methods have been employed to generate local minima, i.e. Gribov copies, satisfying the local condition (1.1). These are the methods of simulated quenching, and simulated annealing. The terms derive from the fact that minimizing $d^2_A$ is equivalent to finding the zero-temperature ground state of an analog “spin-glass” system

$$H = - \sum_{x, \mu} \text{Tr}\left[ g_A(x) U_{A\mu}(x) g_A(x + \hat{\mu}) \right]$$

$$= - \sum_{x, \mu} \left( \text{Tr}\left[ g^\dagger(x) U_{\mu}(x) g(x + \hat{\mu}) \right] \right)^2 + \text{const} \quad (3.1)$$

where $g_A(x)$ is the dynamical SO(3) group-valued “spin” variable, and $U_A$ is a set of fixed, stochastic, nearest-neighbor couplings.

An obvious approach to finding the zero-temperature ground state is to cool the system, either gradually (“annealing”) or suddenly (“quenching”). Quenching corresponds to placing the system in contact with a reservoir at zero temperature, and implies that only changes in $g(x)$ which lower the energy density can be accepted. In practice, quenching is implemented by the over-relaxation method, as described in ref. [8]. Simulated annealing is achieved by applying Metropolis updates to the analog spin system, and gradually lowering the temperature variable from some initial value to zero [1]. Neither of these methods obtains the true minimum of $H$; each generates a set of Gribov copies in maximal center gauge.
A variation of the quenching approach was studied by Kovács and Tomboulis [3]. Instead of applying over-relaxation to a random point on the gauge orbit of a thermalized configuration, they first fixed the configuration to ordinary lattice Landau gauge, and then applied over-relaxation to fix to maximal center gauge. We will refer to the quenching procedure via over-relaxation described in [8] as “OR”, the Kovács-Tomboulis variation of this procedure as “KT”, and simulated annealing, studied in maximal center gauge by Bornyakov et al. [1], as “SA”.

In Table 1 we display the average deviation $\delta^2$ per link

$$\delta^2 = \frac{1}{4V} \sum_{x,\mu} \delta^2_{\mu}(x)$$

for Gribov copies generated by the KT, SA, and OR methods ($4V$ is the number of links on the lattice). These methods were applied to thermalized lattice SU(2) configurations generated at the $\beta$ values and lattice sizes shown below. Only one Gribov copy was generated, in each method, for each thermalized lattice; there was no attempt to select the “best” copy out of a set. We also display, in Fig. 1, the fractional deviation

$$f = \frac{\delta^2 - \delta^2(OR)}{\delta^2(OR)}$$

of $\delta^2$ away from the OR result obtained in each method. We see that, according to the criterion of minimizing the average $\delta^2$, simulated annealing gives the best fit to a thin vortex, the KT method is a little worse, and OR is the worst of the three, as already noted in ref. [1].

| $\beta$ | lattice size | Kovács-Tomboulis | sim anneal | pure over-relax |
|---------|--------------|------------------|------------|-----------------|
| 2.1     | $10^4$       | 0.2797(3)        | 0.2736(3)  | 0.2805(4)       |
| 2.2     | $12^4$       | 0.2631(2)        | 0.2591(2)  | 0.2657(2)       |
| 2.3     | $16^4$       | 0.2440(1)        | 0.2406(1)  | 0.2467(1)       |
| 2.4     | $20^4$       | 0.2232(2)        | 0.2206(2)  | 0.2260(2)       |

Table 1: Values for the average $\delta^2$, for Gribov copies generated by three different methods.

There is something a little peculiar about this ordering, however. The OR Gribov copies are known to have reasonably good center dominance properties [8,14]. Gribov copies generated by the KT method result in a vanishing projected string tension [3], while the SA procedure yields projected tensions at roughly 66% of the full string tension [1]. One might have expected that the projected string tension would be correlated directly with $\delta^2$. Instead, what seems to happen is that the projected string tension first drops to zero, as $\delta^2$ falls below the OR result in KT copies, and then rises again to 66% of the full string tension as $\delta^2$ falls further in SA copies.

This raises the question of whether there is some other aspect of the fit to a thin vortex which is better correlated with the projected string tension. We have
already noted that the fit is necessarily very bad at the location of P-plaquettes. This suggests just ignoring the quality of fit at the P-plaquettes, where it is guaranteed to be bad, and concentrating on the fit in the lattice volume exterior to P-plaquettes. We therefore calculate the average square deviation in the exterior region

\[ \delta^2_{ext} = \frac{1}{N_{ext}} \sum_{ext} \delta^2_\mu(x) \]

(3.4)

where \( N_{ext} \) is the number of all links not belonging to P-vortices, and the sum runs over the set of these external links. The quantity \( \delta^2_{ext} \) is a measure of the quality of fit to a thin vortex in this exterior region.

Table 2 displays the values of \( \delta^2_{ext} \), and Fig. 2 the corresponding fractional deviation

\[ f_{ext} = \frac{\delta^2_{ext} - \delta^2_{ext}(OR)}{\delta^2_{ext}(OR)} \]

(3.5)

in the external region. Here the order is quite different than in Table 1 and Fig. 1. This time, the best fit is achieved by Gribov copies generated by quenching (OR), and these copies also have the largest projected string tension. The SA fit is a little worse, with the projected string tension \( \approx 34\% \) lower than the full string tension, while KT copies have by far the worst fit, and also have negligible projected string tension. From this data, it appears that the projected string tension correlates much

---

**Figure 1:** Percent deviation \((f \times 100)\) of \(\delta^2\) from the over-relaxation value, for the simulated annealing and Kovács-Tomboulis methods. Simulated annealing is best at minimizing \(\delta^2\).
better with the fit in the external region, based on $\delta_{\text{ext}}^2$, than with the overall fit $\delta^2$, which includes the P-vortex volume.

| $\beta$ | lattice size | Kovacs-Tomboulis | sim anneal | pure over-relax |
|---------|--------------|------------------|------------|-----------------|
| 2.1     | $10^4$       | 0.2062(4)        | 0.1951(3)  | 0.1936(3)       |
| 2.2     | $12^4$       | 0.2081(4)        | 0.1970(3)  | 0.1944(3)       |
| 2.3     | $16^4$       | 0.2086(1)        | 0.1975(2)  | 0.1948(1)       |
| 2.4     | $20^4$       | 0.2043(1)        | 0.1950(1)  | 0.1927(1)       |

**Table 2:** Values for $\delta_{\text{ext}}^2$.

![Percent Deviation from over-relax: external links](image)

**Figure 2:** Same as Fig. 1 but for the exterior region. Pure over-relaxation is best at minimizing $\delta_{\text{ext}}^2$.

The results just obtained suggest selecting among Gribov copies generated by quenching, on the basis of the smallest average $\delta_{\text{ext}}^2$. But this procedure is also not a great success as regards center dominance. We have found that choosing the lowest $\delta_{\text{ext}}^2$ out of a set of quenched copies leads to projected string tensions which are $\approx 20\%$ higher ($\beta = 2.4$) than the full string tension, as seen, e.g., from the projected Creutz ratio $\chi_{cp}(5,5)$ shown in Fig. 3.

4. Quenched Minimization

If the criterion for the best gauge copy is the best average fit over all links, including
Figure 3: $\chi_{cp}(5,5)$ vs. the number $N_{\text{copy}}$ of Gribov copies generated per configuration, at $\beta = 2.4$. The Creutz ratio is evaluated in the copy with the best exterior fit, minimizing $\delta^2_{\text{ext}}$. Solid and dotted lines are the asymptotic string tension and errorbars, respectively, on the unprojected lattice [11].

P-vortices, then the projected string tension is about 34% lower than the full string tension, as shown by Bornyakov et al. in ref. [1]. On the other hand, if the criterion for best fit is the fit to a thin vortex in the region exterior to P-vortices, then the projected string tension comes out around 20% too high ($\beta = 2.4$). A case can be made for either fitting criterion, but the method which we have used in the past now appears to be something of a compromise between the two. We begin by quenching the analog spin system (3.1), starting from each of a set of random points on the gauge orbit, to generate a set of Gribov copies satisfying (1.1). This procedure, as seen in the previous section, tends to emphasize the exterior fit $\delta^2_{\text{ext}}$ to a thin vortex. Among the gauge copies generated by quenching, we then select the copy with the best overall fit; i.e. the minimum value of $\delta^2$.

In the absence of a compelling reason to prefer the $\delta^2$ or $\delta^2_{\text{ext}}$ criterion, the “quenched minimum” prescription seems a priori as good as any. It is, at least, a perfectly well-defined procedure. We have argued in the past, on the basis of a number of empirical tests, that center projection applied to quenched minima locate physical objects, namely the center vortices. The results of those tests have not changed any, and are worth summarizing once again:

1. Let $W_n(C)$ denote the expectation value of a Wilson loop constructed from unprojected links, evaluated in the subensemble of configurations in which $n$
P-vortices, on the projected lattice, pierce the minimal area of loop $C$. It is found that
\[ \frac{W_n(C)}{W_0(C)} \rightarrow (-1)^n \]  
(4.1)
as expected if P-vortices locate center vortices in the unprojected configurations [8].

2. If a vortex is inserted “by hand” (via a singular gauge transformation) into a thermalized lattice, then the set of P-vortices on the projected lattice includes the inserted vortex [12].

3. Using information about P-vortex location to remove center vortices from the unprojected configuration, one finds that the confining and chiral symmetry breaking properties of the configuration are also removed, and the topological charge goes to zero [9].

4. The density of P-vortices, and therefore the density of vortices identified on the unprojected lattice, scales correctly according to the renormalization group, as first noted in [13] (see also the later results in [6, 8]).

5. Center dominance: The string tension of the projected lattice agrees fairly well with the asymptotic string tension of the unprojected lattice [8, 14].

6. Precocious linearity: Projected Creutz ratios \( \chi_{cp}(I, I) \) vary only a little with \( I \), and there is no Coulombic force at small distances. This indicates that the projected degrees of freedom are not mixed up with short-range physics [6].

Despite these apparent successes, there is still a serious objection that can be raised to the quenched minimization approach, in view of the findings of Bornyakov et al. [1]. The problem is that any local minimum of \( d_A^2 \) can be reached by quenching, if the starting configuration is within the “basin of attraction” of that minimum on the gauge orbit. Thus, if we sample enough random configurations on the gauge orbit by the OR approach, eventually minima obtained by the SA method would be reached, and the projected string tension would drop well below the full string tension.

The answer to this objection is based on the fact that the projected string tension is found to converge rapidly, with the number of Gribov copies generated, to a value in good agreement with the full string tension. Moreover, the convergence improves as the lattice size increases [14]. The implication is that the measure of SA copies must be negligible compared to the measure of OR copies, at least at large volume. Both the volume and copy-number dependence of the projected Creutz ratio \( \chi_{cp}(5, 5) \) at \( \beta = 2.5 \) are illustrated in Fig. 4, taken from ref. [14]. Similar examples of other Creutz ratios, at various \( \beta \) values, can also be found in that reference. The indications
are that on an infinite lattice there is convergence to a result well above the SA value. The probability of a random configuration evolving, under quenching, to an SA minimum seems likely to go to zero in the infinite volume limit (although this conjecture deserves further investigation).

![Graph](image)

**Figure 4:** $\chi_{cp}(5, 5)$ vs. the number $N_{copy}$ of Gribov copies generated per configuration, obtained at various lattice volumes. The quenched minimization procedure is used.

A last point which we would like to make, in connection with quenched Gribov copies, is that the projected lattices of different copies seem to be quite well correlated at distances beyond one fermi, which is the width of a center vortex.\(^1\) To illustrate this fact, we generate two Gribov copies for each thermalized lattice by applying the over-relaxation algorithm both to the original thermalized lattice, and to a random gauge copy of that lattice. The two gauge-fixed lattices are center projected to obtain two projected lattices, denoted $Z_A^\mu(x)$ and $Z_B^\mu(x)$. We then calculate projected Creutz ratios $\chi_{prod}(I, I)$ from the product Wilson loops

$$W_{prod}(C) = \langle Z^A(C)Z^B(C) \rangle$$

(4.2)

If the two projected configurations were perfectly correlated, or if the $A$ and $B$ loops differed only by perimeter effects, then we would find

$$\chi_{prod}(I, I) = 0 \quad \text{(strong correlation)}$$

(4.3)

\(^1\)The width of center vortices can be determined in three different ways: from the falloff of $W_1/W_0$ [14], from the vortex free energy in a finite volume [15], and from the adjoint string-breaking scale [16]. The three determinations are in rough agreement.
At the other extreme, if there were no correlation at all between the projected configurations, we would find

\[ \chi_{\text{prod}}(I, I) = 2\chi_A(I, I) = 2\chi_B(I, I) \quad \text{(no correlation)} \quad (4.4) \]

where \( \chi_A(I, I) = \chi_B(I, I) \) are the projected Creutz ratios obtained from \( Z^A(C) \) or \( Z^B(C) \) loops separately.

![Figure 5: \( \chi_{\text{prod}}(I, I) \) at \( \beta = 2.2 \).](image)

Figures 5 and 6 show our results for \( \chi_{\text{prod}}(I, I) \) at \( \beta = 2.2 \) and \( \beta = 2.3 \), which indicate a tendency towards strong correlation of the two projected Gribov copies in the infrared.

5. Conclusions

We have pointed out that there are at least two reasonable criteria for selecting among Gribov copies in maximal center gauge. One can select the copy which is a best fit to a thin vortex over the entire lattice volume, including links in P-vortices, and this leads to projected string tensions which are some 34% below the full string tension. Alternatively, given that the fit is bound to fail at P-plaquettes, one can select on the basis of the best fit in the lattice volume exterior to P-plaquettes, and in fact the exterior fit is better correlated with the projected string tension. But this second choice leads to projected string tensions which are roughly 20% higher (\( \beta = 2.4 \)) than the full string tension.
A case can be made for either criterion, but the method which we have used in the past, now described as a process of “quenched minimization,” seems to be something of a compromise between the two alternatives: quenching emphasizes the external fit, subsequent minimization the overall fit. The projected string tensions obtained in this way also come out roughly in the middle of the two extremes, and have good center dominance properties. Apart from center dominance, the property of precocious linearity, and the scaling of the vortex density, indicate that center projection has isolated the relevant long-range degrees of freedom, rather than having them mixed in with physics at all scales. Projected lattices are also found to be strongly correlated, in the infrared, among Gribov copies obtained from the quenching procedure.

Nevertheless, the variation in string tension among Gribov copies selected according to different reasonable criteria is much greater than we had expected, and the justification for quenched minimization is empirical rather than theoretical. It is possible that an improved version of maximal center gauge can be devised which retains the appealing “best fit” interpretation, but which softens the contribution to the fitting functional at the location of P-vortices. We consider this to be an interesting direction for further work.

Acknowledgments

Our research is supported in part by Fonds zur Förderung der Wissenschaftlichen
References

[1] V. Bornyakov, D. Komarov, and M. Polikarpov, Phys. Lett. B497 (2001) 151, hep-lat/0009035.

[2] V. Bornyakov, D. Komarov, M. Polikarpov, and A. Veselov, JETP Lett. 71 (2000) 231, hep-lat/0002017.

[3] T. Kovács and E. Tomboulis, Phys. Lett. B463 (1999) 104, hep-lat/9905023.

[4] Ph. de Forcrand and M. Pepe, Nucl. Phys. B598 (2001) 557, hep-lat/0008016.

[5] K. Langfeld, H. Reinhardt, and A. Schäfke, hep-lat/0101010.

[6] M. Faber, J. Greensite, and Š. Olejník, J. High Energy Phys. 01 (1999) 008, hep-lat/9810008.

[7] M. Engelhardt and H. Reinhardt, Nucl. Phys. B567 (2000) 249, hep-th/9907139.

[8] L. Del Debbio, M. Faber, J. Giedt, J. Greensite, and Š. Olejník, Phys. Rev. D58 (1998) 094501, hep-lat/9801027.

[9] Ph. de Forcrand and M. D’Elia, Phys. Rev. Lett. 82 (1999) 4582, hep-lat/9901020.

[10] M. Ogilvie, Phys. Rev. D59 (1999) 074505, hep-lat/9806018.

[11] G. Bali, K. Schilling, and C. Schlichter, Phys. Rev. D51 (1995) 5165, hep-lat/9409005.

[12] M. Faber, J. Greensite, Š. Olejník, and D. Yamada, J. High Energy Phys. 12 (1999) 012, hep-lat/9910033.

[13] K. Langfeld, H. Reinhardt, and O. Tennert, Phys. Lett. B419 (1998) 317, hep-lat/9710068.

[14] R. Bertle, M. Faber, J. Greensite, and Š. Olejník, J. High Energy Phys. 10 (2000) 007, hep-lat/0007043.

[15] T. Kovács and E. Tomboulis, Phys. Rev. Lett. 85 (2000) 704, hep-lat/0002004.

[16] Ph. de Forcrand and O. Philipsen, Phys. Lett. B475 (2000) 280, hep-lat/9912050.