Linear Track Underwater Carrier SINS Correction Method Based on Hydroacoustic Single Beacon

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ABSTRACT
Precise positioning of underwater vehicles using only a single beacon for range measurement is one of the toughest challenges for underwater navigation, especially when the underwater vehicle travels along a straight path resulting in the system unobservable. In this paper, acoustic range measurements are combined with inertial navigation to determine the position of an underwater vehicle using two consecutive measurement points. The contributions of the work presented here are twofold: First, the underwater single beacon localization algorithm proposed in this paper is able to operate under a linear track, and give the solution of the method to reject multi-valuedness. Second, considering the actual work of underwater carriers, extending the strategy to any trajectory can be used. The results obtained by the algorithm are fed back to the inertial navigation system, which can suppress the continued dispersion of inertial navigation errors. The experimental results show that the proposed “Linear track underwater carrier Strapdown Inertial Navigation System (SINS) correction method based on hydroacoustic single beacon” can solve the positioning problem of underwater long-range carriers and suppress further dispersion of their inertial navigation system errors.

INDEX TERMS
Underwater navigation, strapdown inertial navigation system, acoustic positioning systems, error correction.

I. INTRODUCTION
Inertial navigation technology and hydroacoustic navigation technology are currently considered by the industry as a good means of navigation underwater, each has its own advantages and disadvantages. How to make the two technologies complement each other and together provide a space-time reference for underwater carriers is one of the toughest challenges for underwater navigation [1]–[2]. A trend and research focus in underwater navigation is positioning method based on single beacon range measurement assistance [3]–[5].

A least squares based single transponder acquisition ranging and localization method is proposed in reference [6]. In reference [7], the concept of Virtual Long Baseline (VLBL) was introduced and had a profound impact on the subsequent research of scholars [8]–[10]. However, in this single-beacon acoustic localization model based on a virtual long baseline, the geometric distribution of the virtual beacons is affected by the carrier trajectory, which can affect the observability of the whole system. It has been shown that when the track of the underwater carrier is straight, it leads to the system unobservable [11], [12]. In order to solve this problem, many scholars have conducted research: In reference [13], the observability of single-beacon localization has been analyzed and a filter design has been performed. In reference [14], observability analysis of 3D underwater carriers trimming trajectories in the presence of ocean currents using range and depth measurements was studied. The reference [15] not only analyzes and discusses the observability problem, but also discusses the robustness of the algorithm. The above scholars have conducted a lot of research, which provides a good reference for algorithm engineering, but none of them solved the problem of unobservability under linear trajectory at the root.

Reference [16] proposes an algorithm for processing information characteristics based on the Kalman filter algorithm, which effectively improves the efficiency of the algorithm data. Reference [17] studied the Unscented Kalman Filter (UKF) algorithm for underwater carrier localization based on single-beacon ranging, and the algorithm directly uses the beacon propagation time and carrier-to-water velocity as the observation into nonlinear filtering, and the results show that...
the UKF algorithm is more accurate than extended kalman filter (EKF). Reference [18] proposes a concentric circle matching localization algorithm based on single scale ranging, and the method proposes matching based on the relative position of the ranging points on the trajectory circle. The above methods are still based on dead reckoning, or directly using range information. None of them combine the state measurement of the inertial navigation system, no state estimation of the sensor and system errors, so the algorithm accuracy is limited.

In order to solve the above problems, in this paper, a method which acoustic range measurements are combined with Strapdown Inertial Navigation System (SINS) to determine the position of an underwater vehicle using two consecutive measurement points is proposed. This approach is extended to any trajectory that can be used, the positioning results back to the inertial navigation system can effectively suppress the inertial navigation error dispersion.

The remaining of the paper is organized as follows. Section 2 derives SINS-related coordinate system definition, mechanical arrangement and error equation. Section 3 gives the algorithmic description of Linear track underwater carrier SINS correction method based on hydroacoustic single beacon. Simulation tests are performed alignment in Section 4, and discusses multi-valuedness and its solutions in this section. Experiment tests are performed alignment in Section 5, the problem of algorithm extension and error suppression is presented in this section. Discussions and conclusions are made in Section 6.

II. SINS-RELATED REFERENCE FRAMES, MECHANICAL ARRANGEMENT AND ERROR EQUATION

Strapdown Inertial Navigation System (SINS) has become the mainstream of modern inertial navigation technology research and development due to its strong autonomy, high stealth, simple structure, small size and other advantages that make it widely concerned and applied in aviation, aerospace, navigation and other fields [23]. SINS is essentially an integral derivative system based on Newton’s second law, requiring integration of the gyroscope and accelerometer outputs [24].

A. REFERENCE FRAMES

The main reference frames involved in alignment are firstly presented as follows.

1) INERTIAL FRAME \((O − x_iy_iZ_i, \ i\)-FRAME\)

Inertial frame (i-frame). Earth-centered-fixed orthogonal reference frame, with the origin locating in the geocentric, the \(x_i\) axis pointing to the vernal equinox, the \(z_i\) axis pointing to the North Pole, and the \(y_i\) axis completing the right-handed system. \(O − x_iy_iZ_i\) is fixed to the Earth and does not rotate with the Earth.

2) EARTH FRAME \((O − x_ey_ez_e, \ e\)-FRAME\)

Earth frame (e-frame). Earth-centered Earth-fixed (ECEF) orthogonal reference frame, with the origin locating in the center of the earth, the \(x_e\) axis pointing the Earth’s prime meridian, the \(z_e\) axis pointing in the direction of the North Pole, and the \(y_e\) axis completing the right-handed system. \(O − x_ey_ez_e\) is fixed to the Earth and rotates with the Earth. Angular velocity of \(O − x_ey_ez_e\) with respect to \(O − x_iy_iZ_i\) is \(\omega_{ie}\).

3) NAVIGATION FRAME \((O − x_ny_nZ_n, \ n\)-FRAME\)

Navigation frame (n-frame). Orthogonal reference frame, the origin is located at the center of mass of the carrier, \(x_n, y_n\) and \(z_n\) point to the east, north and upward vertical.

4) BODY FRAME \((O − x_by_bz_b, \ b\)-FRAME\)

Body frame (b-frame). Orthogonal reference frame aligned with inertial measurement unit (IMU) axes, the origin is located at the center of mass of the carrier, \(x_b, y_b\) and \(z_b\) point to the right, front and upward vertical. \(O − x_by_bz_b\) is directly fixed to the carrier, \(x_b\) axis is the pitch axis, \(y_b\) axis is the roll axis, and \(z_b\) axis is the yaw axis. Rotation of the carrier about the \(xb\) axis, \(yb\) axis and \(zb\) axis will get the pitch, roll and heading angles, respectively.

![Schematic diagram of common reference frames.](image)

FIGURE 1. Schematic diagram of common reference frames.

B. SINS MECHANICAL ORCHESTRATION

Ideally, the SINS mechanical choreography equation is shown in equation (1) (3) (4) [25]–[27]

The attitude equation is given by

\[
\dot{C}_b^n = C_b^n(\omega_{nb}^b \times) \tag{1}
\]

where \(C_b^n\) encodes the directional cosine matrix from \(b\)-frame to \(n\)-frame, \(\omega_{nb}^b\) represents the \(n\)-frame angular rate with respect to the \(b\)-frame, \(\omega_{nb}^b \times\) is The antisymmetric matrix of matrix \(\omega_{nb}^b\).
Suppose the vector \( A = [a_1 a_2 a_3]^T \), then \([A \times]\) the formula is shown in equation (2) [28].

\[
[A \times] = \begin{bmatrix}
0 & -a_3 & a_2 \\
a_3 & 0 & -a_1 \\
a_2 & a_1 & 0
\end{bmatrix}
\]  

(2)

The velocity kinematics equation is given by

\[
\dot{v}^n = C_n^b f^b - (2\omega_{ie} + \omega_{en}) \times v^n + g^n
\]

(3)

where \( v^n = [v_E v_N v_U]^T \) encodes the velocity relative to the Earth, \( f^b \) represents the specific force measured by accelerometers in the b-frame, \( \omega_{en} \) is the Earth rotation rate with respect to the i-frame, \( \omega_{en} \) is the angular rate of the n-frame with respect to the e-frame, \( g^n \) is the gravity vector in the n-frame.

The position equation is given by

\[
\begin{aligned}
\dot{L} &= \frac{-v_N}{R_M + h} \\
\dot{\lambda} &= \frac{v_E \sec L}{R_N + h}
\end{aligned}
\]

(4)

where \( L \) is the local geographic latitude, \( \lambda \) is the local geographic longitude, \( R_M \) represents radius of the Earth’s meridian circle, \( R_N \) represents radius of Prime Vertical.

When it comes to underwater navigation, since the radius of the Earth is much larger than the height (depth), we can approximate \( R_M + h \) and \( R_N + h \) as the radius of the Earth \( R_e \).

The relationship between \( \omega_{nb} \), \( \omega_{ie} \) and \( \omega_{en} \) is as follows:

\[
\begin{aligned}
\omega_{nb} &= \omega_{ie} - C_{b}^n \omega_{en} \\
\omega_{en} &= \omega_{ie} + \omega_{en}
\end{aligned}
\]

(5)

\[ \omega_{en} \] represents the n-frame angular rate with respect to the i-frame, \( \omega_{ie} \) is the body angular rate measured by gyroscopes in the body frame. \( \omega_{ie} \) and \( \omega_{en} \) is given by equations (7) and (8).

\[
\begin{aligned}
\omega_{ie} &= \begin{bmatrix}
0 \\
\omega_{ie} \cos L \\
\omega_{ie} \sin L
\end{bmatrix} \\
\omega_{en} &= \begin{bmatrix}
\frac{-v_N}{R_M + h} \\
\frac{v_E}{R_N + h} \\
\frac{v_E}{R_N + h} \tan L
\end{bmatrix}
\end{aligned}
\]

(7)

\[ \omega_{en} \]

In fact, there is a rotation error between the mathematical platform frame \( n' \) (computational coordinate system) and the navigation frame \( n \) of the SINS simulation. So in the SINS simulation. Therefore, the deviations between \( C_{b}^n \) and \( C_{b}^{n'} \) reflect the deviations between \( n' \) and \( n \). These deviations are given by (9)-(11) [23],[29].

\[ \text{C. SINS ERROR EQUATION} \]

\[
\begin{aligned}
\dot{\phi}_E &= -\frac{1}{R_e} \delta_{VN} + (\omega_{ie} \sin L + \frac{v_E}{R_e} \tan L) \phi_N \\
-\omega_{ie} \cos L \frac{v_E}{R_e} \tan L \phi_U + \epsilon_E \\
-\omega_{ie} \sin L \frac{v_E}{R_e} \tan L \phi_E - \frac{v_N}{R_e} \phi_U + \epsilon_N \\
\phi_U &= (\omega_{ie} \cos L + \frac{v_E}{R_e} \sec^2 L) \delta L + \frac{\tan L}{R_e} \delta_{VE} \\
+ (\omega_{ie} \cos L + \frac{v_E}{R_e} \phi_E + \frac{v_N}{R_e} \phi_U + \epsilon_U).
\end{aligned}
\]

(9)

Equation (9) is called the strapdown inertial navigation attitude error (differential) equation, which reflects the variation law of the misalignment angle \( \phi \) of the computed navigation frame (\( n' \)-frame) with respect to the ideal navigation frame (\( n \)-frame).

\[
\begin{aligned}
\delta v_E &= (2\omega_{ie} \frac{v_U}{R_e} \sin L + 2\omega_{ie} \frac{v_N}{R_e} \cos L + \frac{v_E}{R_e} \sec^2 L) \delta L \\
+ (\frac{v_N}{R_e} \tan L - \frac{v_N}{R_e} \delta_{VE} + 2\omega_{ie} \sin L + \frac{v_E}{R_e} \tan L) \delta v_N \\
- (2\omega_{ie} \cos L + \frac{v_E}{R_e} \tan L) \delta v_N - (2\omega_{ie} \cos L + \frac{v_E}{R_e} \delta v_U) \\
-f_U \phi_N + f_N \phi_U + \delta v_E \\
\delta v_N &= -(2\omega_{ie} \cos L \frac{v_E}{R_e} \sin L + \frac{v_E}{R_e} \sec L) \delta L \\
-2(\omega_{ie} \sin L + \frac{v_E}{R_e} \tan L) \delta v_N \\
- \frac{v_U}{R_e} \delta v_N - \frac{v_N}{R_e} \delta v_U + f_U \phi_E - f_E \phi_U + \delta v_N \\
\delta v_U &= -2\omega_{ie} \frac{v_N}{R_e} \sin L \delta L + 2(\omega_{ie} \cos L + \frac{v_E}{R_e}) \delta v_E \\
+ \frac{2v_N}{R_e} \delta v_N - f_N \phi_E + f_E \phi_N + \delta v_U
\end{aligned}
\]

(10)

\[
\begin{aligned}
\delta \lambda &= \frac{v_E \sec L \tan L}{R_e} \delta L + \frac{\sec L}{R_e} \delta v_E \\
\delta L &= \frac{\delta v_N}{R_e}
\end{aligned}
\]

(11)

Similarly, the velocity (position) error is the deviation between the calculated velocity (position) and the ideal velocity (position) in the navigation computer. Equation (10) and Equation (11) refer to the variation law of the inertial navigation frame describing this deviation, which becomes the velocity (position) error (differential) equation.

In the above three equations, \( \delta L \) and \( \delta \lambda \) denote latitude error, longitude error, respectively. Note that the inertial navigation velocity component is \( v^n = [v_E v_N v_U]^T \), and the velocity error component is \( \delta v^n = [\delta v_E \delta v_N \delta v_U]^T \). The attitude component of the inertial navigation is \( \phi = [\phi_E \phi_N \phi_U]^T \).

\[ [e_E \ e_N \ e_U]^T = C_{b}^n [e_x^b e_y^b e_z^b]^T \] denotes equivalent gyroscope drift. \[ [v_E \ v_N \ v_U]^T = C_{b}^n [v^x_b v^y_b v^z_b]^T \] denotes equivalent the bias of the accelerometer, where \[ [e_x^b e_y^b e_z^b]^T \] denotes gyroscope constant drift and \[ [v^x_b v^y_b v^z_b]^T \] denotes the bias stability of the accelerometer.
The above three equations reflect the variation law of the misalignment angle of the computed navigation frame relative to the ideal navigation frame, the deviation between the computed and ideal velocities, and the longitude-latitude altitude error, which can accurately reflect the motion parameters of the carrier at each moment through the mechanical choreography and error equations of the inertial navigation.

III. HYDROACOUSTIC SINGLE-BEACON RANGING-ASSISTED SINS CORRECTION METHOD UNDER STRAIGHT TRAJECTORY

Since the depth of the underwater carrier can be accurately given by the depth gauge and the depth of the hydroacoustic beacon is a known quantity. The oblique distance between the underwater carrier and the hydroacoustic beacon is projected according to equation (12). In this case, the navigation of the underwater carrier can be considered as a straight-line navigation trajectory at a constant depth, which can be solved by geometric relations for two-dimensional coordinates [30], [31].

Therefore, the three parameters \( l_i, l_{i+1}\) and \( S_i \) obtained during the measurement from \( i\) seconds to \( i+1\) seconds can form a triangle.

The area of a triangle can be given by Heron’s formula.

\[
A = \sqrt{K(K - l_i)(K - l_{i+1})(K - S_i)}
\]  

(14)

where \( K = (l_i + l_{i+1} + S_i)/2 \).

The linear distance \( d_i \) from the hydroacoustic beacon to the route is given as

\[
d_i = \frac{2A_i}{S_i}
\]  

(15)

Substituting (14) into (15), we have

\[
d_i = \frac{2A_i}{S_i} = \frac{2\sqrt{K(K - l_i)(K - l_{i+1})(K - S_i)}}{S_i}
\]  

(16)

After obtaining the linear distance \( d_i \) from the beacon to the route, the angle \( \theta_{i+1} \) can be obtained from the geometric relationship in Figure 2.

\[
\theta_{i+1} = \arccos\left(d_i/l_{i+1}\right)
\]  

(17)

If \( l_{i+1} > l_i \), indicates that the underwater carrier tends to move away from the beacon on its route.

\[
\beta_{i+1} = 90^\circ - \theta_{i+1}
\]  

(18)

If \( l_{i+1} < l_i \), indicates that the underwater carrier is trending closer to the beacon on its route.

\[
\beta_{i+1} = 90^\circ + \theta_{i+1}
\]  

(19)

Therefore, when the acoustic beacon coordinates are \((L_0, \lambda_0, h_0)\), according to the \( l_{i+1}\) position and heading angle (The heading angle is defined as positive from north to west, and the range is \(0^\circ \sim 360^\circ\)) relationship, the position of the underwater carrier at this time can be expressed as

\[
L_{i+1} = L_0 + [l_{i+1}\cos(\psi + \beta_{i+1})]/(RM + h_1)
\]

\[
\lambda_{i+1} = \lambda_0 + [l_{i+1}\sin(\psi + \beta_{i+1})]/[(RN + h_1)\cos L_{i+1}]
\]  

(20)

Thus, the correction of the position at \( l_{i+1}\) is completed by the combined positioning method of single beacon assisted inertial navigation system.

In fact, this combined positioning method is performed by using geometric relations to solve triangles. Since in the plane, any three points that are not co-linear can form a triangle. As long as the line between the two points at any point of the underwater carrier travels is not over the beacon, and both have range information, the distance can be approximated by ignoring the trajectory between the two points. The inertial navigation information output between two points and the two range information can form a triangle, using this method for SINS system calibration (If the track passes the beacon point with a range value of 0 or a smaller value relative to the track, the beacon position can be used directly to calibrate the SINS). However, it should be noted that the inertial system

As shown in Figure 2, the hydroacoustic beacon is located at point \((L_0, \lambda_0, h_0)\) and is stationary. Underwater carriers get the distance to the acoustic beacon by hydroacoustic ranging after each period of time. The distance of each measurement is respectively \( l_1, l_2, l_3, \ldots, l_i \).

In the two-dimensional plane, the plane distance between the underwater carrier and the acoustic beacon is given by

\[
l_i' = \sqrt{l_i^2 - (h_1 - h_0)^2}
\]  

(12)

The time interval of each measurement is \( t_1, t_2, t_3, \ldots, t_i \). The distance travelled underwater during a measurement period can be calculated from the output velocity of the inertial navigation system.

The velocity of the inertial guide output is \( v \). The distance \((S_1, S_2, S_3, \ldots, S_i)\) traveled during the measurement period is given by

\[
S_i = \int V_i dt_i
\]  

(13)
will accumulate errors, so the “any two points” is generally taken as two adjacent points with short time interval.

The algorithm can use the heading, velocity, and position output from the SINS system combined with the range information from the hydroacoustic ranging system to correct the position through a combined single-beacon assisted inertial navigation system positioning method, and the obtained position can also be fed back into the inertial navigation system for inertial calibration, or back into the hydroacoustic ranging system for acoustic correction to provide more accurate information subsequently with a positive feedback mechanism [35].

IV. SIMULATION TEST RESEARCH AND ANALYSIS

0.02 gyroscope accuracy IMU represents the medium device level of inertial navigation, suitable for small AUV, UUV and ordinary surface ships; while 0.005 gyroscope accuracy IMU represents the medium to high level device level of inertial navigation, to a certain extent can be used for long-duration underwater carriers; and the accelerometer in IMU is generally matched to its gyroscope accuracy. The main beacon ranging error on the market from low level to high level generally varies between 1% to 1 milliunit [21]–[24], [27]. Therefore, the simulation parameters are set to the following two IMU conditions and three ranging error conditions.

Simulation of combined positioning of a single beacon assisted inertial navigation system using a computer. Let the gyroscope bias stability of the inertial navigation systems be $0.02^\circ/h$, the random walk coefficient be $0.002^\circ/\sqrt{h}$, the bias stability of the accelerometer be $100\mu g$, and the random noise be $10\mu g$, assuming that the inertial navigation system does not have initial velocity error and attitude error. Setting the beacon position is 114.475N, 31.117E, and the carrier departure position is 114.4N, 30.8E, with 0 pitch and 0 roll, speed of 5m/s uniform speed to the north direction. Calibration starts after 2 hours of operation of the inertial guidance system and is performed at 1-minute intervals on the route. Ranging system error were set to two thousandths, five thousandths, and one hundredth of a percent of the slope (the errors were Gaussian distributed), and the combined correction was performed after two ranging sessions [31].

The simulated ranging equation is given by

$$L'(i) = k \cdot L(i) \cdot X + L(i)$$

In Eq. (21), $L(i)$ is the true distance to the beacon per minute, $k$ represents the error factor based on the slope distance ($2\%$, $5\%$ and $1\%$), $X$ is the standard Gaussian distribution, and $X \sim N(0, 1)$.

**FIGURE 3. Schematic diagram of the algorithm model.**

**FIGURE 4. Distance to beacon.**

The blue curve in Figure 4 reflects the trend of carrier-to-beacon distance with time.

**FIGURE 5. Probability density curve of ranging error.**

Figure 5 shows the $L'(i) - L(i)$ probability density curve of the ranging error, which is Gaussian distribution. The yellow curve represents the ranging error of two thousandths, the red curve represents the ranging error of five thousandths, and the yellow curve represents the ranging error of one hundredths, which can simulate the real ranging error to a certain extent.

Figure 6 and Figure 7 show the calibration curves of the eastward and northward position errors under the above test conditions for a period of 120 min starting from 2 h. The blue curve is the pure inertia error curve, the red curve is the solution value under 0.002 times the ranging error, the yellow curve is the solution value under 0.005 times the
ranging error, and the purple curve is the solution value under 0.01 times the ranging error.

![Figures 6 and 7](image)

**FIGURE 6.** Eastward position error under simulation conditions.

**FIGURE 7.** Northward position error under simulation conditions.

The average improvement of positioning accuracy for the eastward error is 91.10%, 86.69% and 82.94% under the three ranging error conditions, respectively, and the average improvement of positioning accuracy for the northward error is 94.72%, 91.17% and 73.74% under the three ranging error conditions, respectively. From the second minute onward, the algorithm starts to take effect in the sliding time window mode for the localization solution. The algorithm is affected by the range error, and when the inertial navigation error is large or the range error is large, the algorithm cannot produce correct results or wrong results, and deviates from the true value, but in general, as shown in Figures 6 and 7 and the above analysis, the algorithm can effectively reduce the positioning error under the simulation conditions.

Figure 8(a), (b), and (c) show the two-dimensional planar trajectory plots under two thousandths, five thousandths, and one hundredths range errors, respectively. The solid blue line is the real course, the solid red line is the inertially indicated track, and the red circle on the course represents the first
point at the start of the calibration (corresponding to the point at 0 minutes in Figure 6 and Figure 7). The blue asterisk is the beacon position, the yellow circle is the correct algorithm solution, and the red circle are incremental roots due to multi-valuedness.

In fact, the algorithm will result in at least four real solutions at each time, which are distributed in four “quadrants” in two axisymmetric and centrosymmetric directions with respect to the horizontal and vertical coordinates where the beacon is located, as shown in Figure 9. The problem of “front and back” has been solved, but the problem of “left and right” has not been solved. For this problem, the true solution can be finally determined by detecting the distance between the ship and the beacon to determine whether it is close to or far from the beacon after the ship has changed its course in the actual operation.

In practical applications, the accuracy of shipboard inertial navigation will be higher, so an additional set of control tests to simulate the navigation of shipboard high-precision inertial navigation, the gyroscope bias stability is \(0.005^\circ/h\), the random walk coefficient is \(0.0005^\circ/\sqrt{h}\), the bias stability of the accelerometer is \(20\mu g\), the random noise is \(1\mu g\), assume that the inertial navigation system does not exist initial velocity and attitude error.

Setting the beacon position is 114.475N, 31.117E, and the carrier departure position is 114.4N, 30.8E, with 0 pitch and 0 roll, speed of 5m / s uniform speed to the north direction. Calibration starts after 2 hours of operation of the inertial guidance system and is performed at 1-minute intervals on the route. Ranging system error were set to two thousandths, five thousandths, and one hundredth of a percent of the slope (the errors were Gaussian distributed), and the combined correction was performed after two ranging sessions. The distance to the beacon and the probability density error curve of the ranging error are the same as Figure 4 and Figure 5, In this paper, the error condition is called condition 2.

Figure 10 and Figure 11 show the error curves of the eastward and northward positions under condition 2.

The blue curve is the inertia error curve, the red curve is the solved value under 0.002 times of ranging error, the yellow curve is the solved value under 0.005 times of ranging error, the purple curve is the solved value under 0.01 times of ranging error. From Fig. 10 the analysis shows that the eastward error improves the positioning accuracy by 87.41%, 64.17% and 62.32% respectively under the three ranging error conditions, and the northward error improves the positioning accuracy by 90.19%, 85.56% and 56.54% respectively under the three ranging error conditions.

In general, after the gyroscope accuracy is improved, the positioning accuracy is improved than before. For example, the trajectory solved by the algorithm in Figure 12 is closer to the real trajectory than Figure 8. However, there is some error divergence situations, because the range error is not improved but the gyroscope accuracy is improved. This mismatch is mainly a partial misunderstanding caused by the fact that the triangle formed by the algorithm cannot be closed correctly.
In order to verify the effectiveness of the algorithm, a shipboard lake test based on the fiber-optic SINS was designed, as shown in Figure 13, the shipboard test platform consists of the fiber-optic SINS, the data acquisition system (IPC), the NovAtel OEM628 board and its Global Navigation Satellite System (GNSS) receiver, GNSS antenna and hydroacoustic communication ranging system. In the experiment, the output frequency of the inertial navigation system data update rate is 100 Hz, the GNSS position data update rate is 10 Hz, and the main performance indexes of the fiber optic IMU are shown in Table 1. It should be noted that, because this marine inertial navigation has not been calibrated for a long time, the actual accuracy indexes cannot reach those shown in Table 1, and the actual accuracy index of the gyroscope is about equivalent to 0.05°/h gyroscope [32]–[35].

| Sensor  | Characteristic          | Value               |
|---------|-------------------------|---------------------|
| Gyroscope | Bias stability       | < 0.02 °/h            |
|         | Angular random walk   | < 0.005 °/√h         |
|         | Measurement range     | ± 0.05 °/s           |
|         | Update rate           | 100 Hz               |
| Accelerometer | Bias                 | < 50 μg              |
|         | Random walk           | < 50 μg/√h           |
|         | Measurement range     | ± 20 g               |
|         | Update rate           | 100 Hz               |

The shipboard test was conducted in Mulan Lake in Wuhan, using the scheme of “1 master station + 2 slave
stations”, with the master station as the reference station for Real-time kinematic (RTK) (shown in Figure 14), which was set up on the balcony on the fourth floor of the Mulan Lake test and training base facing the lake. Slave station 1 was set up on a ship moored to a small island in the center of the lake to simulate a beacon station with an acoustic beacon communication ranging system (shown in Figure 15). Slave station 2 was set up on another boat to simulate an underwater carrier driving into the beacon service area as a mobile carrier platform for combined inertial navigation and hydroacoustic verification.

During the test verification, the master station (RTK reference station) sends correction information to the slave stations on the lake. The two slave stations on the lake are equipped with GNSS receivers, which receive the correction information and complete their own high-precision positioning. The positioning information can be used as the reference position information of the two slave stations. Slave 2 is a mobile station, simulating the movement of the carrier, and the distance between slave 1 and 2 is obtained using a hydroacoustic communication distance measurement system.

In figure 16 the black color represents the ship’s trajectory, $L(i)$ represents the first $i$ measurement value, and the gray dashed line corresponding to each two adjacent measurements is the position difference between the two points. The position difference between the two points, which is derived by the inertial system, forms a triangle, and can be calculated using the algorithm in combination with the heading information between the two points (in the actual navigation there is usually no large sudden heading turn).

B. EXPERIMENTAL RESULTS AND ANALYSIS

The shipboard experiment process is: the ship is first stopped at the dock mooring state for about 30 minutes. Initial alignment is performed during this time and simulates the errors accumulated by the inertial navigation device. After that, the beacon is sunk into the water along the IMU position, and the carrier starts to enter the acoustic beacon range to start the communication, simulating the underwater carrier to enter the acoustic beacon range after working for a long time, and conducting multiple hydroacoustic range measurements. The real map and the test vessel trajectory as shown in Figure 17. The red curve is the track of the experimental vessel, the blue track is the track of the beacon vessel, which set off before the experimental vessel and moored at the small island in the center of the lake.

Thirteen range measurements were conducted in the test, and 13 range values were obtained using acoustic positioning ranging. A comparison of the hydroacoustic ranging and RTK results showed that the mean hydroacoustic ranging error was 9.13 m and the root mean square of the ranging error was 8.55 m. The schematic diagram of the test tracks and ranging positions is shown in Figure 18, which shows that the tracks between each two points can almost be regarded as straight tracks.
The algorithm is solved once for every two adjacent ranging points (*) on the counterclockwise route in Figure 18, and there are 12 solutions. The solution position is the position at the 2nd to 13th ranging point (*) respectively.

The comparison between the position after the algorithm solution and the pure inertial navigation position error is shown in Figure 19 and Figure 20, the blue bar indicates the position error after the algorithm iteration, and the red bar indicates the position error of SINS pure inertial solution. The average improvement of latitude accuracy is 69.27%, and the average improvement of longitude accuracy is 84.05%. Due to the low accuracy of inertial navigation and ranging system in this test, the result of algorithm is still in the order of 100 meters, and the correction effect will be further improved if the marine high accuracy inertial navigation and ranging system is installed.

From the mechanical choreography of SINS, it is known that the navigation parameters of each moment are determined by the navigation parameters of the previous moment and the gyroscope and accelerometer outputs of this moment together. So the navigation parameters obtained from the iteration of the algorithm are fed back to SINS to update the equations, which can suppress the continued dispersion of errors and realize the calibration of the inertial navigation system. The obtained navigation parameters for the 12 ranging points are fed back to the SINS navigation settlement [36], [37], and the obtained results are compared with the SINS pure inertial solution as shown in Figures 21-24.

The position obtained is re-substituted into the SINS pure inertial solution to make it calibrate the SINS. In Figure 23 and Figure 24, the stepped points are the sudden change in position due to the effect of feeding the correction results from the algorithm iteration into the SINS update equation. The phenomenon of “stepped planes” in the graph is due to the property of inertia can maintain a certain level of accuracy over a short period of time. The same time point in the velocity error curve also accurately reflects this property of the inertial navigation. Between the 2000s and 3000s, the inertial navigation shows a tendency to diverge due to the longer time interval between the two corrections for the
inertial navigation compared to the other points. After about 4800s, no more ranging is performed, and the submerged carrier can be considered as moving out of the working area of the hydroacoustic beacon, and the navigation accuracy is maintained by the inertial navigation. The results of the algorithm can be fed back to the SINS system to effectively suppress the dispersion of the error in velocity and position of the SINS, and it is obvious from the figure that the error curve after calibration is still consistent with the variation of the inertial navigation error to some extent.

VI. CONCLUSION

Research on the navigation and positioning of underwater carriers is quit important, especially for military applications. However, when the trajectory of the underwater carrier is straight, it can cause the system to be unobservable. In order to solve the problems existing in traditional single-beacon acoustic localization algorithm fails under straight trajectory, in this paper, a method which acoustic range measurements are combined with inertial navigation to determine the position of an underwater vehicle using two consecutive measurement points is proposed. The multivaluedness problem of its solution is analyzed and the solution is given.

According to the analysis of the simulation experiment results, it can be seen that the positioning accuracy of the algorithm is affected not only by the accuracy of the inertial device, but also by the accuracy of the acoustic ranging system. Under the simulation conditions of the shipboard high-precision inertial navigation system, effective navigation parameters can be output while avoiding the unobservable problem under linear trajectory.

Considering the actual working needs of underwater carriers and the practical proof of the algorithm, lake trial tests were carried out. The experimental results show that the solved latitude accuracy is improved by 69.27% and longitude accuracy is improved by 84.05% on average compared with the pure inertial navigation solution. The results obtained by the algorithm are fed back into the SINS system, which can effectively suppress the error dispersion of the speed and position of SINS.
The results of simulations and experimental measurements demonstrate that: The problem of navigation and positioning of underwater carriers under linear trajectories is effectively solved, which can make the carrier be calibrated for inertial navigation devices without surfacing, and its error dispersion is suppressed.

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