EXTERIOR DIFFERENTIAL FORMS
IN FIELD THEORY

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Abstract

A role of the exterior differential forms in field theory is connected with a fact that they reflect properties of the conservation laws. In field theory a role of the closed exterior forms is well known. A condition of closure of the form means that the closed form is the conservative quantity, and this corresponds to the conservation laws for physical fields. In the present work a role in field theory of the exterior forms, which correspond to the conservation laws for the material systems is clarified. These forms are defined on the accompanying nondifferentiable manifolds, and hence, they are not closed. Transition from the forms, which correspond to the conservation laws for the material systems, to those, which correspond to the conservation laws for physical fields (it is possible under the degenerate transform), describe a mechanism of origin of the physical structures that format physical fields. In the work it is shown that the physical structures are generated by the material systems in the evolutionary process. In Appendices we give an analysis of the principles of thermodynamics and equations of the electromagnetic field. A role of the conservation laws in formation of the pseudometric and metric spaces is also shown.

Introduction. A specific feature of the exterior differential forms is that at the same time they possess algebraic as well as geometric, and topologic, and differential, and integral, and many other properties. It is explained by their complicated internal structure (homogeneity with respect to the basis, skew symmetry, the integration of elements which are composed of two objects with different nature: algebraic (coefficients of the form) and geometric (components of the basis) ones, a structure connection between the forms of different degrees, a dependence on space dimension and on topology of the manifold. Under a conjugation of the form elements, objects of every elements, forms of different degrees, exterior and dual ones, an so on, there realized invariant and structure properties of the exterior differential forms. Just these properties are essential for the invariant field theory. They correspond to the conservation laws and enables one to describe a variety of the physical structures which constitute physical fields.

The closed forms have invariant and structure properties. They correspond to the conservation laws for physical fields. A role of the closed forms has been
described in works by Willer, Shuts and other authors [1-3]. In the present work a role in field theory of the forms, which are not closed because they are defined on the arbitrary nondifferentiable manifolds. Such forms arise under a description of the conservation laws for the physical systems. (About a role of the closed forms in the existing field theories it will be shortly said in the Appendix 1).

Field theories are based on the conservation laws. The exterior differential forms enable one to study properties and specific features of the conservation laws and thus to disclose the basis of field theory.

At present there are many problems associated with the conservation laws. An approach to the conservation laws, their mathematical expression and physical treatment turn out to be different in different branches of science. A concept "the conservation law" in different branches of science carry different sense. In physics the conservation laws reveal themselves as conservative quantities (such conservation laws may be called exact ones); in mechanics of continuous media these conservation laws establish a balance between a change of physical quantities (energy, linear momentum, angular momentum, and mass) and relevant external forcing (such conservation laws may be called the balance conservation laws); in thermodynamics the conservation laws prove to be relevant to the principles of thermodynamics. And what have they in common?

The exact conservation laws relate to **physical fields**. **Physical fields** (electromagnetic, gravitational, nuclear and etc) [4] are special forms of the matter which are carriers of interactions. The balance conservation laws are those for **material systems**. **A material system** is a variety of elements which have internal structure and interact to one another. Examples of elements that constitute the material system are electrons, protons, neutrons, atoms, fluid particles, cosmic objects and others. As examples of material systems it may be thermodynamic, gas dynamical, cosmic systems, systems of elementary particles (pointed above) and others. The physical vacuum in its properties may be regarded as an analogue of the material system that generates some physical fields. As it will be shown in Appendix, the principles of thermodynamics are not special balance conservation laws. They combine the balance conservation laws for energy and linear momentum, and it enables one to understand a nature of their interactions.

In the present work we show that the exact conservation laws follow from the balance conservation laws. And **an interaction of the balance conservation laws that appear to be noncommutative** plays a crucial role. **The principles of thermodynamics may be regarded as an example of accounting for this interaction in the thermodynamic systems**. A transition from the balance conservation laws to the exact ones is accompanied by origin of the physical structures and it forms the basis of the evolutionary processes [5-10]. It is evident that material systems generate the physical structures, and such structures format the physical fields.

In section 1 some properties of the exterior forms utilized in the work are presented. In section 2 it is shown that the closed forms may correspond to the
exact conservation laws. {For the sake of clearness in sections 2-8 we introduce a double notation. By simple lettering and by italic one we respectively designate a name, which has a physical sense, and that, which explains the mathematical sense}. In the next sections we demonstrate a noncommutativity of the balance conservation laws, establish a relation between the balance conservation laws and exact those, and disclose a mechanism of origin of the physical structures that produce the physical fields.

In Appendix 1 we show that the invariant and metric properties of the closed exterior differential forms constitute the basis of existing field theories. In Appendix 2 we give an analysis of the principles of thermodynamics that disclose an interaction of the balance conservation laws for energy and for linear momentum. In Appendix 3 it was shown an influence of the noncommutativity of the balance conservation laws on a development of instability. In Appendix 4 equations of the electromagnetic field were analyzed. In Appendix 5 we present a table of interactions. In Appendix 6 a role of the conservation laws in formation of the pseudometric and metric spaces is shown, and an analysis of the Riemann space and the Einstein equation is presented. In Appendix 7 functional properties of solutions to the differential equations, that are essential for the field theory, are analyzed. These properties reflect those of the conservation laws.

1. Some properties of exterior differential forms. An exterior differential form of degree $p$ ($p$-form) may be presented as $[1-3,11-14]$

$$\theta^p = A_{\alpha_1...\alpha_p} \ dx^\alpha_1 \ dx^\alpha_2 \ ... \ dx^\alpha_p, \quad 0 \leq p \leq n$$

(1)

where the basic $dx^\alpha, \ dx^\alpha dx^\beta, \ dx^\alpha dx^\beta dx^\gamma, \ ...$ obey the condition

$$dx^\alpha dx^\beta = 0$$

$$dx^\alpha dx^\beta = -dx^\beta dx^\alpha \quad \alpha \neq \beta$$

(2)

A differential (exterior) of the form $\theta^p$ is expressed by the formula

$$d\theta^p = dA_{\alpha_1...\alpha_p} \ dx^{\alpha_1} \ dx^{\alpha_2} \ ... \ dx^{\alpha_p}$$

(3)

and it proves to be the differential form of degree $(p + 1)$.

We will point out some properties of the closed forms that correspond to the exact conservation laws.

1) A closure condition of the $p$-form $\theta^p$ (form of degree $p$) is written as

$$d\theta^p = 0$$

(4)

(it is obvious that the closed form is a conservative quantity).

2) If a form is closed only on some structure, i.e. on pseudostructure, then the closure condition may be written in the form

$$dz \theta^p = 0$$

(5)
where the pseudostructure $\pi$ obeys the condition

$$d_\pi \ast \theta^p = 0$$  \hspace{1cm} (6)

here $\ast \theta^p$ is the dual form. (One can see that a form closed on pseudostructure is a preserving object). {To the exterior differential form on the differentiable manifold there corresponds the skew-symmetric tensor. To the dual form that describes the pseudostructure there corresponds the pseudotensor dual to the skew-symmetric tensor. The pseudostructures format cohomologies (cohomologies by De Rham, singular cohomologies [11]), sections of the cotangent bundles and so on. They correspond to eikonals which are on one hand the level surfaces and on the other the cut surfaces}.

3) A form is called exact one if it is equal to total differential:

$$\theta^p = d\theta^{p-1}$$  \hspace{1cm} (7)

The exact forms are closed identically: $d\theta^p = d(d\theta^{p-1}) = 0$

4) Any closed form is a differential. An exact form is a total differential. A closed inexact form is an interior one on the pseudostructure differential.

$$\theta^p = d_\pi \theta^p$$  \hspace{1cm} (8)

5) From Eqs. (7) and (8) it follows that there exist a connection between forms of sequent degrees. There is also a similar integral relation

$$\int_{c^{p+1}} d\theta^p = \int_{\partial c^{p+1}} \theta^p$$

The theorems by Stokes and Gauss are special cases of this relation.

From the definition of the form it follows that elements of differential of the form are equal to components of their commutator. Thus, if the form of first degree is expressed as $\theta = a_\mu d\xi^\mu$, then $d\theta = K_{\alpha\beta} d\xi^\alpha d\xi^\beta$, where components of the form commutator are $K_{\alpha\beta} = a_{\beta;\alpha} - a_{\alpha;\beta}$. Here $a_{\beta;\alpha}$, $a_{\alpha;\beta}$ are the covariant derivatives. In the case of differentiable manifold the covariant derivatives coincide with ordinary ones and the commutator components can be written as

$$K_{\alpha\beta} = \left( \frac{\partial a_\beta}{\partial \xi^\alpha} - \frac{\partial a_\alpha}{\partial \xi^\beta} \right)$$  \hspace{1cm} (9)

If a form is defined on the nondifferentiable manifold, then an additional term will appear in the commutator, this term is a commutator of the metric form of manifold.

2. Conservation laws for physical fields. (A closure condition of exterior forms). As it is seen from the closure conditions (4), (5), a closed form is a conservative quantity, and hence it may correspond to the exact conservation
law. And the closed inexact form is a conservative object, namely, it is a conservative quantity only on some pseudostructure $\pi$. The closure conditions for the exterior form ($d_\pi \theta^p = 0$) and the dual form ($d_\pi^* \theta^p = 0$) are mathematical expression of the exact conservation law.

The exact conservation laws correspond to physical structures that format physical fields. Preserving objects, e.g. conservative quantities (closed exterior forms), on the pseudostructures (dual forms) are the physical structures that format physical fields:

$$
\begin{align*}
\{ d_\pi \theta^p &= 0 \\
 d_\pi^* \theta^p &= 0 
\} \quad \mapsto \quad \{ \theta^p \quad \text{physical structures} \mapsto \quad \text{physical fields}
\end{align*}
$$

Equations for the physical structures ($d_\pi \theta^p = 0$, $d_\pi^* \theta^p = 0$) turn out to coincide with a mathematical expression of the exact conservation law. It is seen that the exact conservation law is that for physical fields.

As any closed form is a differential (either total if the form is exact one: $\theta^p = d\theta^{p-1}$, or interior on the pseudostructure: $\theta^p = d_\pi \theta^{p-1}$ if the form is inexact) of the form of lower degree, then the form of lower degree may correspond to a potential. And the form degree indicates a type of potential. The potential is a scalar if $l = 0$ ($l = p - 1$), it is a vector if $l = 1$, and it is a tensor if $l = 2, 3$. The closed inexact forms of zero, first and second degrees relate respectively to the pseudoscalar, pseudovector and pseudotensor (vortex) fields. {As it was pointed before, the closed form is a conservative quantity. On the other hand, as it is a differential of some form, which in this case plays a role of the potential, the closed form reveals as a potential force. That is, the closed form is dual object. This duality of the closed forms discloses properties of the physical fields described as the carriers of interactions. Below it will be shown, with respect to what the closed form reveals as a potential force.}

Thus, an application of the closed exterior differential forms enable one to see a relation between the conservation laws and the physical structures that produce physical fields. It may be shown that invariant and metric properties of the closed exterior differential forms that correspond to the exact conservation laws constitute the basis of the existing theories which describe physical fields. We are able to verify that all existing theories are complemented by additional conditions of invariance or covariance which are the closure conditions for exterior or dual forms. (In more details this subject is described in Appendix 1).

The existing field theories that are based on the exact conservation laws allow to describe physical fields, find possible physical structures, and understand a variety of physical fields. However, such invariant theories cannot give an answer to the question concerning a mechanism of the genesis of physical structures. Only a theory that does not base on the closure conditions for forms can give an answer. These conditions have to be achieved by themselves spontaneously. A realization of these conditions implies a formation of the closed form, and this
corresponds to the conservation law and gives an indication of a production of physical structure. It is evident that only the evolutionary theory can answer the question about a mechanism of generation of the physical structures. Such a theory is one based on the balance conservation laws for the material systems. (It also bases on the properties of exterior forms, however, in this case the exterior form are defined on the nondifferentiable manifold, and therefore they are nonclosed. These forms are nonintegrable ones. Topological properties of commutators for such forms enable one to understand a role of the balance conservation laws in evolution processes and in formation of physical fields.

3. Noncommutativity of the balance conservation laws for material systems. (Nonidentity of the evolutionary relation obtained from the balance conservation laws). The conservation laws for the material system (continuous medium) express the following: a change of any physical quantity in some volume for a given time interval is balanced out by the flux of this quantity across the boundary of the volume and by the source actions. Under transition to the differential expression the fluxes are substituted by divergences. And the differential or integral equation is supplemented with a dependence of the physical quantities on the state function of the material system. Below a relation (in the differential forms) for the state function will be obtained. Just this relation disclose a specifics of the evolutionary processes. It should be underlined once more that the conservation laws for the material medium are balance ones.

The balance conservation laws (for energy, linear momentum, angular momentum, and mass) establish a balance between a change of physical quantities and an action on the system. And every balance conservation law depends on relevant actions (so the conservation law for energy depends on energetic actions, the conservation law for linear momentum does on force ones, etc). In real processes such actions have a different nature. Therefore the balance conservation laws prove to be noncommutative. {What is the “noncommutativity” of the balance conservation laws? Suppose, that first the energetic and then the force perturbations act on a local domain of the material system (element and its neighborhood). And let at the initial moment the local domain be in some state \( A \). According to the balance conservation law for energy, under an influence of the energetic perturbation the local domain develops from the state \( A \) into any state \( B \). Then according to the balance conservation law for momentum under an influence of the force perturbation it develops from the state \( B \) into any state \( C \). Suppose now that the sequence of the actions changes, namely, first the force perturbation and then the energetic one act, and the system develops first into any state \( B^* \) and then does into the state \( C^* \). If the state \( C^* \) coincides with the state \( C \), that is, the result does not depend on a sequence of perturbation of different type (and on a sequence of performing the relevant balance conservation laws), then it means that the balance conservation laws commute. If the state \( C^* \) does not coincide with the state \( C \), then it means that the balance conservation laws prove to be noncommutative}. This is just of
decisive importance for evolutionary processes and a mechanism of origin of the physical structures. Because of noncommutativity of the balance conservation laws, an external forcing that experience the material system cannot directly convert into physical quantities (energy, linear momentum, angular momentum, mass) of the system, but they convert into some quantity that acts as internal force and is a cause of the evolutionary processes.

To understand how the balance conservation laws interact to one another it is necessary to study a conjunction (self-consistence) of equations governing these laws.

The balance conservation laws may be described by means of differential equations [15-18]. If the material system is not dynamical one (as in the case of thermodynamic system), then the equations of the balance conservation laws may be written in terms of increments of physical quantities and governing variables.

Equations are conjugate ones if they may be contracted into identical relations for differential, i.e. for a closed form. Let analyze the equations that describe the balance conservation laws for energy and linear momentum.

We introduce two frames of reference: the first is inertial one (this system is not connected with the material system), and the second is accompanying one (this system is connected with a manifold created by trajectories of elements of the material system). The energy equation in the inertial frame of reference may be reduced to the form:

$$\frac{D\psi}{Dt} = A$$

where $D/Dt$ is the total derivative with respect to time, $\psi$ is the functional of the state that specifies the material system, $A$ is a quantity, which depends on specific features of the system and on external energy actions on the system. The action functional, entropy, wave function may be regarded as examples of the functional $\psi$. Thus, the equation for energy represented in terms of the action functional $S$ has a similar form: $DS/Dt = L$, where $\psi = S$, $A = L$ is the Lagrange function. In mechanics of continuous media the equation for energy of ideal gases may be presented in the form [16]: $Ds/Dt = 0$, where $s$ is entropy. In this case $\psi = s$, $A = 0$. It is worth note that the examples presented show the mutual relation between the action functional and entropy.

In the accompanying frame of reference a total derivative with respect to time transforms into that along the trajectory. The equation (10) turns out to be written in the form:

$$\frac{\partial \psi}{\partial \xi^1} = A_1$$

here $\xi^1$ is a coordinate along the trajectory. In a similar manner, in the accompanying frame of reference the equation for linear momentum appears to be
reduced to the equation of the form

\[ \frac{\partial \psi}{\partial \xi^\nu} = A_\nu, \quad \nu = 2, \ldots \] (12)

where \( \xi^\nu \) are coordinates in the direction being normal to the trajectory, \( A_\nu \) are the quantities that depend on specific features of the system and external force actions.

The Eqs. (11), (12) may be convoluted into the relation

\[ d\psi = A_\mu d\xi^\mu, \quad (\mu = 1, \nu) \] (13)

where \( d\psi \) is the differential expression \( d\psi = (\partial \psi / \partial \xi^\mu) d\xi^\mu \).

The relation (13) may be written in the form:

\[ d\psi = \omega \] (14)

here \( \omega = A_\mu d\xi^\mu \) is the differential form of the first degree.

As the balance conservation laws are evolutionary ones then the relation obtained is also evolutionary law. The proper evolution relation corresponds to every material system (see Appendices (2)-(4)).

The relation (14) has been obtained from the balance conservation laws for energy and linear momentum. In this context the form \( \omega \) is that of the first degree. If the equations of the balance conservation laws for angular momentum be added to the equations for energy and linear momentum, then in the evolutionary relation this form will be that of the second degree. And in a combination with the equation of the balance conservation law for mass this form will be a form of degree 3. Thus, in the general case the evolutionary relation may be written in the form

\[ d\psi = \omega^p \]

where the degree of the form \( p \) takes the values \( p = 0, 1, 2, 3 \ldots \). (The evolutionary relation for \( p = 0 \) is similar to that in the differential forms, and it has been obtained from interaction of energy and time or momentum and coordinate.)

In the left-hand side of the evolutionary equation there is the functional expression \( d\psi \), which specifies a state of the material system, and in the right-hand side there is the form \( \omega^p \) which coefficients depend on external actions. The meaning of the evolutionary relation lies in a fact that it discloses a specific feature of dependence of the material system state on external actions. As it will be shown below, the evolutionary relation has a specific feature (it may be nonidentic one) which enables one to determine a mechanism of transition of the material system from nonequilibrium state to the equilibrium or locally equilibrium one, and this has a decisive importance for the evolutionary process.

{It should be pointed out the following. If to a state of the material system there correspond a differential of any function, then this indicates that a state
of the material system is the equilibrium or locally equilibrium one. And if the
differential is absent, then this means that the system is in the nonequilibrium
state. As it will be shown below, owing to this specific feature the evolutionary
relation enables one to detect a presence or absence of the differential and as
the result to classify a state of the system.

At this point we show that for real processes the evolutionary relation
appears to be nonidentical one. A relation may be identical one if it re-
lates any measurable metric or invariant objects, i.e. the objects which may be
compared. {A concept “nonidentic relation” may be seemed as contradictive
one. However, this concept carries an in-depth meaning. The identic relation
establishes an exact correspondence between quantities entering into that. The
nonidentic relation can establish an exact correspondence between quantities
entering into that only under some supplementary conditions. If these supple-
mentary conditions do not satisfied, this relation has a physical meaning as well.
If this relation is evolutionary one, then it proves to be a self-varianting relation,
that is, a variation of one object of the relation forces a change of the other
object, and in turn a change of the second object leads to a change of the first
one and so on. As one object is nonmeasurable object, than the other object
cannot be compared with the first one, and hence a process of self-variation can-
not be terminated without additional conditions. Additional conditions can be
realized by themselves under a self-variation of the nonidentic relation owing to
any degrees of freedom. Just the nonidentic relations, which the evolutionary
relations belong to, can describe a self-organization of the material systems.
The principle of self-organization will be clarified later.}

Let as examine the relation (14). For real processes the form \( \omega \) that stands
in the evolutionary relation (14) and depends on the external actions appears
to be nonclosed and hence cannot be an invariant object. For the form to be
closed it needs the differential of the form or its commutator be zero (elements
of a differential of the form are equal to components of its commutator). Let us
consider a commutator of the form \( \omega = A_\mu d\xi^\mu \). Components of the commutator
may be written as follows:

\[
K_{\alpha\beta} = \left( \frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta} \right)
\]  

(15)

(here a term that is connected with a nondifferentiability of the manifold was not
taken into account as yet). Coefficients of the form \( \omega \) have been obtained either
from the balance conservation law for energy or from that for linear momentum.
It means that in the first case the coefficients depend on the energetic action and
in the second case they depend on the force action. In actual processes energetic
and force actions have a different nature and appear to be nonconsistent. The
commutator of the form \( \omega \) constructed from derivatives of such coefficients is
nonzero. This means that a differential of the form \( \omega \) is nonzero as well. Thus,
the form proves to be nonclosed and isn’t an invariant object. Therefore, in
the evolutionary relation there in noninvariant term. Such a relation cannot
be identical one. Hence, without a knowledge of a particular expression for the form, one can argue that for an actual processes the evolutionary relation proves to be nonidentical because of nonconsistency of the external action. To emphasize this fact, it is reasonable to write down the relation (14) in the form

$$d\psi \cong \omega$$

(it was introduced the sign $\cong$ instead of the equality sign $=$). In a similar manner one may prove the nonidentity of the general evolutionary relation and to write it in the form:

$$d\psi \cong \omega^p$$

A nonidentity of the evolutionary relation means that equations of the balance conservation laws turn out to be nonconjugate (thus, if from the energy equation to obtain a derivative of $\psi$ in the direction along the trajectory and from the momentum equation to find a derivative of $\psi$ in the direction normal to the trajectory and next to calculate their mixed derivatives, then from the condition that the commutator of the form $\omega$ is nonzero it follows that mixed derivatives prove to be noncommutative). It points that these balance conservation laws don’t commutate.

4. Nonequilibrum property of material system. Selfvariation of the state of material system. Topologic properties of commutator for nonintegrable form. What is follow from noncommutativity of the balance conservation laws? Let us analyze the evolutionary relation (14). If the balance conservation laws for energy and momentum be commutative, then the relation (14) turns out to be identical, and the differential $d\psi$ can be obtained from that. It may be regarded that $\psi$ is a function of the state. And an existence of the function of state means that the state is locally balanced one. This means that external actions directly transform into physical measurable quantities of the system, namely, energy and momentum of elements of the material system. But because of the evolutionary relation is not identical, one cannot obtain the differential $d\psi$ from that relation, and this means that there is no the function of state. The state of system is nonequilibrum one. Because of the noncommutativity of the balance conservation laws, the external forces cannot directly transform into measurable quantities of the system, and they transform into any nonmeasurable quantity. This nonmeasurable quantity, which is described by the commutator of the form $\omega$, acts as internal force. This means that because of the noncommutativity of the balance conservation laws the state of material system proves to be nonequilibrum one and there is an internal force described by the commutator of the nonintegrable form $\omega$. In the general case this nonmeasurable quantity is described by the commutator of the nonintegrable form $\omega^p$. The nonequilibrum is a moving force of the evolutionary process. A further analysis of the evolutionary relation and the commutator of the form $\omega^p$ allows to understand a mechanism of the evolutionary process.
A nonidentical relation, if it is an evolutionary relation, appears to be self-varying: a change of any object of the relation forces a change of other object and in turn a change of the second object leads to a change of the first one and so on. As one of objects of the nonidentical relation is nonmeasurable object, then the other object cannot be exactly compared with the first object. This property of the nonidentical evolutionary relation makes it possible to describe a self-variation of the nonequilibrium state of the material system. In this case topologic properties of the commutator of nonintegrable form (this form is defined on the nonintegrable manifold) play a deciding role.

If manifold, on which the exterior form is defined, is nondifferentiable one, then an additional term that contains a commutator of the metric form of manifold (this commutator specifies a deformation of the manifold) will enter into a commutator of the exterior form. The form \( \omega^p \), that stands in the evolutionary relation, is defined on the accompanying manifold, which for actual processes turns out to be nondifferentiable one as it is formatted contemporaneously with a change of the state of material system and depends on the physical processes.

Hence, a term containing the characteristics of the manifold will enter into a commutator of the form \( \omega^p \) in addition to a term, which is connected with derivatives of coefficients of the form. An interaction between these terms of different nature just describes a mutual change of the state of material system.

Let us examine this with an example of commutator of the form \( \omega = A_\mu d\xi^\mu \) that enters into the evolutionary relation (14). We assume that at the beginning an associated manifold was differentiable. In this case a commutator of the form \( \omega \) can be written in the form (15). If at the next point in time any action effects on the material system, then this commutator turns out to be nonzero. A state of the material system becomes nonequilibrium and it will appear an internal force whose action will lead to a deformation of the accompanying manifold. Then the accompanying manifold fails to be differentiable. In a commutator of the form \( \omega \) it will appear an additional term, that specifies a deformation of the manifold and is a commutator of the metric form of manifold. If it is possible to define the coefficients of connectivity \( \Gamma_{\alpha\beta}^\sigma \) (for nondifferentiable manifold they are asymmetric ones), then a commutator of the form may be written as

\[
K_{\alpha\beta} = \left( \frac{\partial A_\beta}{\partial \xi^\alpha} - \frac{\partial A_\alpha}{\partial \xi^\beta} \right) + (\Gamma_{\alpha\beta}^\sigma - \Gamma_{\beta\alpha}^\sigma) A_\sigma
\]

where \((\Gamma_{\alpha\beta}^\sigma - \Gamma_{\beta\alpha}^\sigma)\) is a commutator of the metric form (which specifies a torsion of manifold). An emergence of the second term can only change a commutator and cannot make it zero (because terms of the commutator have different nature). In the material system the internal force will continue to act even without external actions. The further deformation (torsion) will go on. This leads to a change of commutator of the metric form, produces a change of the exterior form and its commutator and so on. A process of selfvariation of the commutator that governs by nonidentic evolutionary relation specifies a change
of the external force and selfvariation of nonequilibrum state of the material system. At this point it should be emphasized that such selfvariation of the state of material system proceeds under an action of internal (rather then external) forces and may go on even without action of external forces. And a state of material system remains nonequilibrum, and in actual physical process an internal force could give rise to development of instability in the material system. \{For example, this was pointed out in works by Prigogine [19]. “The excess entropy” in his works is analogous to a commutator of nonintegrable form for thermodynamic system. “A production of excess entropy” leads to development of instability\}. An internal force cannot continuously become equal to zero. The material system cannot continuously transform into an equilibrium (without internal forces) state. However, in the material system an equilibrium state may be locally realized if internal forces transform into potential ones. As an analyses of the evolutionary equation shows, it is possible under additional conditions and it corresponds to emergence of the physical structure.

5. A mechanism of origin of the physical structure. \((Degenerate\ transformation)\). For actual processes a differential of the form \(\omega^p\) that enters into the evolutionary equation (as well as the commutator) is nonzero:

\[
d\omega^p \neq 0
\]

i.e. the form \(\omega^p\) is nonclosed. To the physical structure one has to assign a closed form on pseudostructure, that is to say, that to the physical structure there corresponds some differential \(d_\pi \psi\). Such a differential may be obtained from the evolutionary relation only if the form \(\omega^p\) is closed inexact one, in other words, if the form is subjected to the conditions

\[
\begin{align*}
d_\pi \omega^p &= 0 \\
d_\pi^* \omega^p &= 0
\end{align*}
\]

(in this case the form appears to be an interior differential of any form, namely \(\omega^p = d_\pi \vartheta\). The evolutionary relation becomes identical on the pseudostructure \(\pi\), and a differential \(d_\pi \psi = d_\pi \vartheta\), that corresponds to the physical structure, can be determined from that relation).

A transition from the condition (19) to the condition (20) that corresponds to origin of the physical structure (and a transformation of internal forces into potential ones) is possible only as the degenerate transform, i.e. it is a transition which does not preserve a differential. \{At this point it should be underlined that in this case the degenerate transform is realized as a transition from the accompanying frame of reference to the inertial one\}. Some additional condition has to correspond to the degenerate transform. (A vanishing of such functional expressions as Jacobians, determinants, the Poisson brackets, and others may be regarded as examples of such conditions). Because the evolutionary relation describes the material systems and coefficients of the form depend
on properties of the material system, then it is evident that a condition of the degenerate transform has to be caused by properties of the material system. As an example, the system may have any degrees of freedom. Namely, under realization of any degree of freedom it may take place a redistribution between physical quantities (for example, between energy and linear momentum) in such a way that they become measurable simultaneously. Such degrees of freedom may be translational ones, internal degrees of freedom of elements of the system and so on.

Thus, the physical structure can arise, if, firstly, the material system undergoes nonconsistent external actions and a commutator of the form $\omega_p$ is nonzero. And, secondly, if the material system has degrees of freedom, i.e. there are conjugation conditions for the exterior form $\omega_p$. However, even if these conditions are satisfied, the physical structure appears only in the case, when degrees of freedom of the system are realized (in physical process), namely, the conditions of conjugation are satisfied. It may take place under selfvariation of the state of material system. And it may be realized only spontaneously because it is caused by internal (rather then external) reasons (degrees of freedom are characteristics of the material system and not of external actions).

A creation of the physical structure is a transition of a quantity that acts as internal force into a measurable quantity that acts in the direction normal to the pseudostructure as potential force. {Above it was pointed out the duality of the closed form (as conservative quantity and as potential force). This duality has a physical meaning. The closed form as conservative quantity relates to the physical field. The object, which is conservative quantity on the pseudostructure, is an element of physical field. And the closed form reveals as potential force with respect to the material system. The potential force is an action of the formation originated (see below) on elements of the material system. The potential forces are described, for example, by jumps of derivatives in the direction normal to the characteristics, to the potential surfaces, and so on.}

The physical structure and some measurable quantity that acts as potential force reveal as a new measurable and observable formation that spontaneously arises in the material system. {Fluctuations, pulsations, waves, vortices, massless particles are examples of such formations}. In the physical process this formation spontaneously extracts from the local domain of material system and so it allows the local domain of material system to get rid of internal force and come into locally balanced state. A formation, that has been created in some local domain of the material system and liberated from that, begins to act on the neighboring local domain as a potential force (this forcing was created by system in itself, and therefore this is potential forcing (rather then arbitrary one). The neighboring domain of material system works over this action, which appears to be external for it. If in the process the conditions of conjugation turn out to be satisfied again, then the neighboring domain create a formation of its own. In such a way the formation can move relative to material system. A velocity of moving relative to material system is not a parameter of the system, but this is
some quantity that is realized at every time as the conjugation condition. If the material system is homogeneous one, then a velocity will have the same values (but this is not a constant because it anew arise at every point of time of the evolutionary process). The speed of sound, the speed of electromagnetic waves, the speed of light are formatted in such a way. It is evident, that a velocity of moving relative to material system is defined by internal properties of the system. (As example, the speed of sound $a: a^2 = (dp/d\rho)_s$, where the pressure $p$ and density $\rho$ are characteristics of the material system and the subscript $s$ means a constancy of entropy).

6. Characteristics of a formation originated: intensity, spin, absolute and relative speeds of propagation of the formation. (Characteristics of exterior and dual forms, value of commutator of the nonintegrable form, properties of the material system). As a formation originated is a result of transition of nonmeasurable quantity, which is described by the commutator (and acts like internal force), into measurable quantity (potential force), then it is evident that an intensity of the formation originated is defined by a quantity, which was stored by the commutator of the nonintegrable form at the moment of its appearing. And the first term of the commutator that formatted by mixed derivatives of the form coefficients governs an intensity of the formation, whereas the second term that specifies a deformation of the accompanying manifold (bending, torsion, curvature) is fixed as any internal characteristics of the formation originated. Spin is an example of such a characteristics, and a value of spin depends on degree of the form. An integrating direction, i.e. pseudostructure that is defined by the dual form, determines an absolute speed of propagation of the formation originated (it is a speed in the inertial frame of reference). The speed of propagation relative to the material system (a speed in the accompanying frame of reference), as examples of that are speed of sound, speed of light, speed of electromagnetic waves and so on, is defined by the conjugation conditions, i.e. by degrees of freedom of the material system.

In such a way the following correspondence is established:

1) an intensity of the formation (potential force) is a value of the first term in the commutator of nonintegrable form at the instance of formation creation;

2) spin is the second term in the commutator that is connected with the metric commutator;

3) a preserving quantity (a charge) is the exterior form that has been realized;

4) an absolute speed of propagation of the formation arisen (a speed in the inertial frame of reference) is the integrating direction–pseudostructure–dual form;

5) a speed of propagation relative to the material system (the speed of sound, the speed of light, the speed of electromagnetic wave) is a speed in the accompanying frame of reference–conjugation conditions–additional conditions connected with properties of the material system.
A formation of pseudometric and metric spaces. (An integration of the nonidentical evolutionary relation). An analyses of integrability of the nonidentical evolutionary relation explains a process of formation of pseudostructures and thereby make more evident a mechanism of formation of pseudometric and metric space.

As it is known, the closed form is a differential (exterior or interior) of the form of one less degree. This connection enables one to carry out an integration of the closed form and proceed to the form of one less degree. Such transitions are possible only in identical relations. It may be shown that an integration and transitions with lowering the form degree are allowed in nonidentical relations (nonintegrable forms) as well but only in the case of degenerate transforms. Under a degenerate transform on the pseudostructure it may be obtained the identical relation that can be integrated and it enables one to get a relation with the forms of one less degree. The relation obtained turns out to be nonidentical as well. By integration (under realization of relevant degenerate transform) the nonidentical evolutionary relation with forms of degree $p$, one may successively obtain nonidentical relations with forms of degree $k$, where $k$ takes values from $p$ to 0. At each transition the closed forms on the pseudostructure of sequent degrees $k = p, k = p - 1, ..., k = 0$ are formatted, and this indicates a creation of the physical structures of relevant type. A transition to the exact form of zero degree corresponds to a creation of some element of the material system (massive particle with internal structure). {So called “spontaneous violation of the symmetry” is an example of such a transition}.

To a creation of the physical structures we put into correspondence a formation of the pseudostructures with a dimension which depends on the space dimension. It may be shown that under a generation of closed forms of sequent degrees $k = p, k = p - 1, ..., k = 0$ the pseudostructures of the dimensions $(n + 1 - k): 1, ..., n + 1$ are obtained (here $n$ is a dimension of the initial inertial space). {When deriving the evolutionary relation two frames of reference were used. The first system is inertial one which is connected with the space where the material system situates and is not directly connected with the material system. This is an inertial space, it is the metric space. The second frame of reference is proper one, it is connected with accompanying manifold which is not metric manifold}. When proceeding to the exact closed form of zero degree the metric structure of the dimension $n + 1$ is obtained. As a result we get that under influence of nonconjugate external forcing (and if there are degrees of freedom) the material system can transform the initial inertial space of the dimension $n$ into a space of the dimension $n + 1$. {In the initial space of degree 0 it may be formatted a space of the dimension 1 (such a space may be a time). And in space of the dimension 1 it may be formatted a space of the dimension 2 (time and space coordinate) and so on. Every material system has their proper time. In particular, this approach explains how the proper time is formatted}. In the initial space of the dimension 3 it may be formatted a space of the dimension 4 (time and three space coordinates). Such a space can be convoluted
and a new dimension may happen to be nonrealizable. Thus, the cycle ends and
a new cycle may begin (this corresponds to that one system may be embedded
into the other one). A mechanism of formation of the pseudostructures and
the metric structures can explain, in particular, how the internal construction
of elements of the material system is formatted. So it can be seen that the
inertial spaces are not absolute spaces where actions are developed, but they
are spaces generated by the material systems. A mechanism of formation of the
pseudostructures is at the basis of creation of the pseudometric spaces and of
their transition into the metric spaces. (In Appendix 6 a formation of pseudoriemann and Riemann’s spaces are considered, and conditions of a derivation
of the Einstein equation are analyzed).

It may be shown that equations of the characteristic surfaces, potential sur-
faces (simple or double layer), residue equation and others obtained from the
equations of the mathematical physics are the equations of the pseudostructures.
As the equations for pseudostructure there serve the eikonal equation (the pseu-
dostructure is the level surface, on that the conservative physical quantity is
definded as it follows from properties of the closed inexact form). {In the works
[15, 20] it was shown a connection of equations for single, double, ... eikonals
with the equations of the characteristics, Hamilton equations and so on.}

8. Classification of the physical structures. (Parameters of closed ex-
terior and dual forms). To obtain the physical structures of given physical field
it is necessary to consider the material system which corresponds to this field.
In particular, to obtain the thermodynamic structures (fluctuations, phase tran-
sitions, etc) one has to analyze the evolutionary relation for the thermodynamic
systems, to obtain the gas dynamic ones (waves, jumps, vortices, pulsations) he
has to employ the evolutionary relation for gas dynamic systems, for electro-
magnetic field he must employ a relation obtained from equations for charged
particles. Maybe, the physical vacuum is an analogue to such material system
in the case of elementary particles. {Some concrete relations are presented in
Appendices (2)-(4)}.

The closed forms that correspond to physical structures are generated by the
evolutionary relation having parameter \( p \) which defines a number of interacting
balance conservation laws. Therefore, the physical structures can be classified
by the parameter \( p \). The other parameter is a degree of the closed forms. As it
was shown above, the evolutionary relation of power \( p \) may generate the closed
forms of degree \( 0 \leq k \leq p \). Therefore, the physical structures can be classified
by the parameter \( k \) as well. The closed exterior forms of the same degree
realized in spaces of different dimensions prove to be distinguishable because
a dimension of the pseudostructures, on which the closed forms are defined, depend on the space dimension. As a result, the space dimension also specifies
the physical structures. This parameter determines properties of the physical
structures rather than their type.

Hence, from the analyses of the evolutionary relation one can see that a type
and properties of the physical structures (and accordingly of physical fields) for a given material system depend on a number of interacting balance conservation laws $p$, on the degree of realized closed forms $k$, and on a space dimension. By introduction a classification with respect to $p, k$, and a space dimension we can understand an internal connection of various physical fields and interactions (see Appendix 5).

Conclusions. Thus, the mathematic tool that bases on properties of non-integrable exterior differential forms enables one to understand a role of the conservation laws in the evolutionary processes and disclose a mechanism of formation of the physical fields. Because of nonconsistence of the external effects the balance conservation laws (energy, linear momentum, angular momentum, and mass) for the material systems prove to be noncommutative (it follows from the nonidentity of the evolutionary relation obtained from equations of the balance conservation laws). And then, because of noncommutativity of the balance conservation laws, a state of the material system turns out to be nonequilibrium, and this is the cause of the evolutionary process (a value of the internal force is determined by a commutator of the nonintegrable form that enters into the evolutionary equation). Under some additional conditions that are determined by properties of the material system and realized in the physical processes, it is possible a conjugation of the balance conservation laws (a degenerate transform corresponds to this case), and it is an indicator of a creation of the physical structures (transformation of internal forces into potential ones). And in the material system this reveals as arising some measurable formations: fluctuations, pulsations, waves, vortices, particles and so on (characteristics of formations arisen are determined by ones of the nonintegrable exterior forms, their commutators and conjugation condition as well as by characteristics of the closed exterior forms and dual forms).

It was shown that the physical structures which constitutes the physical fields are generated by the material systems. This process is accompanied with a formation of the pseudometric and metric spaces, and the conservation laws govern this processes.

At this point it worth underline that, in spite of these results are qualitative ones, they can help while studying the evolutionary processes in the concrete material systems and while investigation of concrete physical fields and their formations (see Appendices (2)-(6)).

The mathematical theory that explains the evolutionary process in the material systems and a mechanism of formation of the physical fields may be regarded as the basis of the qualitative evolutionary theory of fields. An investigation carried out allows to conclude that axioms provided the basis of the existing theories are the conjugation conditions of the balance conservation laws for the material systems, which generate the physical fields. It worth emphasize that we used a mathematic tool which bases on properties of the nonintegrable exterior differential forms, i.e. forms defined on the nondifferentiable manifolds. A
specific property of these forms is a presence of nonidentical relations and degenerate transforms (a transition from the accompanying frame of reference to the inertial one).  

Appendix 1.

Exact conservation laws as the basis of existing field theories.  
(Closed exterior forms in field theory).

As the closed exterior differential form corresponds to the conservation laws for physical fields and potentials, they may be exploited for description of physical fields. A dependence of properties of exterior and dual to them forms on a degree of the form, space dimension, complex exterior and interior structure of the forms, various conditions and manners of conjugation provide a diversity of physical structures that format physical fields. It may be shown that essentially all physical theories that describe physical fields are based on operators which are reduced to operators acting on the closed exterior forms. If in addition to the exterior differential d to introduce operators: 1) δ for an operator that takes the form of degree (p + 1) to that of degree p, 2) δ′ for an operator of cotangent transforms, 3) Δ for the transform dδ − δd, 4) Δ′ for the transform dδ′ − δ′d, then the operators of field theory that act on the differential forms can be written in terms of these operators. The operator δ corresponds to Green’s operator, the operator δ′ corresponds to the operator of canonical transform, Δ does to the d’Alembert operator in the 4-dimension space, and Δ corresponds to the Laplace operator [13-14]. Eigenvalues of these operators reveal themselves as conjugation conditions for elements of the differential forms. In the tensor equations the skew symmetric tensors correspond to the closed forms.

All existing field theories are based on the exact conservation laws. The essence of these theories are invariant and metric properties of the closed exterior differential forms that correspond to the exact conservation laws. (In particular, it may be verified that equations of existing field theories are those that have solutions being the closed or dual to them forms). Into all existing field theories there were introduced supplementary conditions of invariance or covariance which appears to be the closure conditions for the exterior or dual forms. Essentially all existing field theories contain elements of noninvariance, i.e. they are based either on functionals that are not identical invariants (such as Lagrangian, action functional, entropy) or on equations (differential, integral, tensor, spinor, matrix and so on) that have no identical invariance (integrability or covariance). Such elements of noninvariance are, for example, nonzero value of the curvature tensor in the Einstein theory [21], the indeterminacy principle in the Heisenberg theory, the torsion in the theory by Weyl [21], the Lorentz force in electromagnetic theory [22], an absence of the general integrability of the Schrödinger equations, nonmetric cross-sections in the Yang-Mills theory, the Lagrange function in the variational methods, an absence of the identical integrability on equations of the mathematical physics and that of identical covariance of the tensor equations, and so on. Only if we assume elements of
noncovariance, we can obtain the closed inexact forms. Precisely such closed forms can correspond to the physical structures that constitute the physical fields.

However, the existing field theories are invariant ones because they are provided with additional conditions, i.e. the conjugation conditions under which the invariance or covariance requirements have to be obeyed. Examples of such conditions may be the identity relations: canonical relations in the Schrödinger equations, gauge invariance in electromagnetic theory, commutator relations in the Schrödinger theory, Christoffel's coefficients of connectivity and identity relations by Bianchi in the Einstein theory, cotangent bundles in the Yang-Mills theory, the Hamilton function in the variational methods, the covariance conditions in the tensor methods, the characteristic relations (integrability conditions) in equations of mathematical physics, etc. It can be shown that these invariance conditions are that of closure for the exterior or dual forms.

As all existing field theories are subjected to the invariance conditions, equations of these theories have only invariant solutions, i.e. those that can be expressed in terms of the closed or dual to them forms and correspond to physical fields. Possible noninvariant solutions, that may be realized if the invariance conditions are not imposed, are not accounted for. From this standpoint it needs to be recognized two types of equations or methods of invariant field theory. A principal distinction of these two types consists in the following. One type has only invariant solutions, the other one, if the invariance conditions to be omitted, may have noninvariant solutions (functionals) that have a physical sense. Equations of the tensor, spinor, matrix or variational forms, the group theory methods, the transformation theory, theory of symmetries, the bundle theory can be assign to the first type. To the second type we may relate the so-called equations of mathematical physics, namely, differential and integral equations that describe physical processes. As it will be shown later, noninvariant solutions to the second type equations play a governing role in description of a mechanism of the physical structure creation. At the same time, just equations and methods of the first type (equations by Maxwell, Schrödinger, Dirac, Einstein, field theory by Yang-Mills, the group theory, theory of symmetries and so on) allow one to discover a great variety of physical structures, describe physical fields, and understand regularities of the physical world.

The field theories that are based on the exact conservation laws allow to describe the physical fields. However, because these theories are invariant ones they cannot answer the question about a mechanism of creating physical structures which format the physical fields. The answer may be done only by theory where the closure conditions of the forms were not introduced. These conditions have to be realized by themselves without external forcing. A realization of such conditions means a creation of the closed form, and it will correspond to the conservation law and point to appearance of the physical structure. It is evident that an answer to a question concerning a mechanism of creation of the physical structures may be given only by the evolutionary theory. As it
was shown above, such a theory is the evolutionary theory which bases on the
balance conservation laws for the material systems.

Appendix 2

Analyses of principles of thermodynamics

The thermodynamics is based on the first and second principles of thermodynamics, which were introduced as postulates [23]. Let us show that the first
principle of thermodynamics follows from the balance conservation laws for en-
ergy and linear momentum and is valid for the case when the heat influx is the
only external action. It appears to be nonidentical relation, and it points out
that the balance conservation laws are noncommutative and a state of thermo-
dynamic system is nonequilibrum one. The second principle of thermodynamics
with the equality sign is obtained from the first that under a realization of the
integrability condition, when a role of the integrating factor plays the tempe-
ration, and it corresponds to a locally equilibrium state of the system while the
temperature is realized. The second principle of thermodynamics with the in-
equality sign takes into account a presence of some actions other than the heat
influx.

As it is well known, the first principle of thermodynamics can be presented
in the form

\[ dE + \delta w = \delta Q \]

where \( dE \) is a change of energy of the thermodynamic system, \( \delta w \) is a work
done by the system (this means that \( \delta w \) is expressed in terms of the system
parameters), \( \delta Q \) is an amount of heat putted into the system (i.e. external
action on the system). As the term \( \delta w \) has to be expressed in terms of the
system parameters and it specifies a real (rather then virtual) change, then it
can be designated by \( dw \), and hence, the first principle of thermodynamics will
take the form

\[ dE + dw = \delta Q \] \hspace{1cm} (1) \]

What is a difference between the first principle of thermodynamics and the
balance conservation laws? The balance conservation law for thermodynamic
system can be written as

\[ dE = \delta Q + \delta G \] \hspace{1cm} (2) \]

where by \( \delta G \) we designate energetic actions with the exception of heat influx.
For thermodynamic system the balance conservation law for linear momentum
(a change of linear momentum of the system in its dependence on the force
mechanical action on the system) can be written as

\[ dw = \delta W \] \hspace{1cm} (3) \]

Here \( \delta W \) signifies a force (mechanical) action on the system (for example, ex-
ternal compression of the system, an influence of boundaries and so on).
If to sum relations (2) and (3), then one obtains the relation

\[ dE + dw = \delta Q + \delta G + \delta W \]  \hspace{1cm} (4)

which is just the evolutionary relation for the thermodynamic system.

By comparison the relation (4), that follows from the balance conservation laws for energy and linear momentum, with the relation (1), one can see that they coincide if the heat influx is the only external action on the thermodynamic system (\( \delta W = 0 \) and \( \delta G = 0 \)).

Thus, the first principle of thermodynamics is obtained from the balance conservation laws for energy and linear momentum. This is analogous to the evolutionary relation.

The significance of the first principle of thermodynamics, as well as of the evolutionary equation, is that it reveals a nature of interactions of two balance conservation laws (rather then it only corresponds to the conservation law for energy). As \( \delta Q \) is not a differential (closed form) then the relation (1), which correspond to the first principle of thermodynamics, as well as the evolutionary equation appear to be the nonidentical nonintegrable relation. This points to the noncommutativity of the balance conservation laws (for energy and linear momentum) and to nonequilibrium state of the thermodynamic system. As it follows from the analyses of the evolutionary relation, a transition to the locally equilibrium state has to correspond to the realization of additional condition (i.e. the integrability condition) under which this relation turns out to be identical one. Such an identical relation is just a relation which corresponds to the second principle of thermodynamics.

Let us consider the case when the work performed by the system is carried out through the compression. Then \( dw = pdV \) (here \( p \) is the pressure and \( V \) is the volume) and \( dE + dw = dE + pdV \). As it is known, the form \( dE + pdV \) can become a differential if there is the integrating factor (a quantity which depends only on the characteristics of the system) \( 1/\theta = pV/R \) which is named as the thermodynamic temperature \( T \) \cite{23}. In this case the form \( (dE + pdV)/T \) turns out to be a differential (interior) of some quantity that referred to as entropy \( S \):

\[ (dE + pdV)/T = dS \]  \hspace{1cm} (5)

(although the form \( dE + pdV \) consists of differentials, in the general case without the integrating factor it is not a differential because of that its terms depend on different variables, namely, the first term is determined by variables that specifies the internal construction of elements, and the second term depends on variables that specify an interaction between elements, for example, a pressure). If the integrating factor \( 1/\theta = T \) to be realized, that is, the relation (5) proves to be satisfied, then from the relation (1), which corresponds to the first principle of thermodynamics, it follows

\[ dS = \delta Q/T \]  \hspace{1cm} (6)
This is just the second principle of thermodynamics for reversible processes. It takes place when a heat influx is the only action on the system. If besides the heat influx the system undergoes any mechanical forcing $\delta W$ (for example, an influence of boundaries), then according to the relation (4) from the relation (5) we obtain

$$dS = (dE + pdV)/T = (\delta Q + \delta W + \delta G)/T$$

(7)

from which it follows, that

$$dS > \delta Q/T$$

(8)

which corresponds to the second principle of thermodynamics for irreversible processes.

The relations (6), (8) which can be written as

$$dS \geq \delta Q/T,$$

(9)

express the second principle of thermodynamics. (It is well to bear in mind that the differentials in the relations (5), (6), (8), (9) are not total ones. They are satisfied only in presence of the integrating factor, namely, the temperature which depends on the system parameters).

Thus, the first principle of thermodynamics is obtained from the balance conservation laws for energy and linear momentum, and the second principle of thermodynamics does from the first one. The second principle of thermodynamics with the equality sign follows from the first principle under the fulfillment of the condition of integrability, i.e. a realization of the integrating factor (temperature). (This transition corresponds to that from the nonequilibrium state to the locally equilibrium state. Phase transitions, the origin of fluctuations, etc are examples of such transitions). And the second principle of thermodynamics with the inequality sign takes into account a presence in the real processes other actions besides the heat influx.

In the case examined above a differential of entropy (rather then entropy itself) becomes the closed form. (In this case entropy manifests itself as the thermodynamic potential, namely, the function of state. To the pseudostructure there corresponds the state equation which determines the temperature dependence on the thermodynamic variables). For entropy to be the closed form itself, a one more condition has to be realized. Such a condition may be a realization of the integrating direction, an example of that is the sound speed: $a^2 = \partial p/\partial \rho = \gamma p/\rho$. In this case it is valid the equality $ds = d(p/\rho^\lambda) = 0$ from which it follows that entropy $s = p/\rho^\lambda = const$ is the closed form (of zero degree). {But it does not mean that a state of the gaseous system is identically isoentropic one. Entropy is constant only along the integrating direction (for example, on the adiabatic curve or on the front of sound wave), whereas in the direction being normal to the integrating direction the normal derivative of entropy has a break}. {It worth underline that both temperature and the sound speed are not continuous thermodynamic variables. They are variables which are realized in
the thermodynamic processes if the thermodynamic system has any degrees of freedom. One may see an analogy between temperature and the sound speed: temperature is the integrating factor and the sound speed is the integrating direction. {Notice that in actual processes a total state of the thermodynamic system is nonequilibrium one and a commutator of the form $dE + pdV$ is nonzero. A quantity that is described by commutator and acts as internal force may grow. Prigogine [19] defined this as the "production of the excess entropy". Just this increase of the internal force is perceived as the growth of entropy in the irreversible processes.}

The closed static system, if left to its own devices, may tend to a state of total thermodynamic equilibrium. This corresponds to tending of the system functional to its asymptotic maximum. In the dynamical system the tending of the system to a state of total thermodynamic equilibrium may be violated by dynamical processes and transitions to a state of local equilibrium.

**Appendix 3**  
**Influence of noncommutativity of the balance conservation laws on a development of instability.**

A noncommutativity of the balance conservation laws, that leads to the emergence of internal forces and an appearance of the nonequilibrium, is a cause of development of instability in the material systems. Hence, to study a cause of development of instability, it is necessary to examine the evolutionary relation obtained from the balance conservation laws and analyze a commutator of the nonintegrable form entered into this relation.

For example, we take the simplest gas dynamical system, namely, a flow of the ideal (nonviscous, heat nonconductive) gas [24]. Suppose, that gas is the thermodynamic system in the local equilibrium (whereas the gas dynamical system may be in the nonequilibrium state). This means that the following relation is satisfied [23]

$$Tds = de + pdV$$

(1)

here $T$, $p$ and $V$ are the temperature, the pressure, and the gas volume respectively, $s$, $e$ are entropy and the internal energy per unit volume of the gas.

Let us introduce two frames of reference: the inertial one, which is not connected with the material system, and the accompanying system that is connected with a manifold formatted by trajectories of elements of the material system. (As examples these may be both Euler’s and Lagrange’s systems of coordinates).

In the inertial system of coordinates the Euler equations are the balance conservation laws for energy, linear momentum and mass of the ideal gas [16]. The balance conservation law for energy of the ideal gas can be written as

$$\frac{Dh}{Dt} - \frac{1}{\rho} \frac{Dp}{Dt} = 0$$
where $D/Dt$ is the total derivative with respect to time (if to designate the spatial coordinates by $x_i$ and velocity components by $u_i$, then $D/Dt = \partial/\partial t + u_i \partial/\partial x_i$), $h$ is the enthalpy of the gas. Expressing the enthalpy in terms of the internal energy $e$ with the help of the formula $h = e + p/\rho$ and using the relation (1) we can reduce the equation of balance conservation law for energy to the form

$$\frac{Ds}{Dt} = 0 \quad (2)$$

And respectively, the equation of balance conservation law for momentum can be presented as [16, 25]

$$\text{grad} s = (\text{grad} h_0 - \mathbf{U} \times \text{rot} \mathbf{U} + \mathbf{U} \times \mathbf{F} + \partial \mathbf{U}/\partial t)/T \quad (3)$$

where $\mathbf{U}$ is the velocity of the gas particle, $h_0 = (\mathbf{U} \cdot \mathbf{U})/2 + h$, $\mathbf{F}$ is the mass force.

As the total derivative with respect to time is that along the trajectory, then in the accompanying frame of reference the equations (2), (3) take the form:

$$\frac{\partial s}{\partial \xi_1} = 0 \quad (4)$$

$$\frac{\partial s}{\partial \xi^\nu} = A_\nu, \quad \nu = 2, ... \quad (5)$$

where $\xi_1$ is the coordinate along the trajectory, $\partial s/\partial \xi^\nu$ is the left-hand side of the equation (3), and $A_\nu$ is obtained from the right-hand side of the relation (3). In the common case when gas is nonideal the equation (2) can be written in the form

$$\frac{\partial s}{\partial \xi^1} = A_1 \quad (6)$$

where $A_1$ is the expression which depends on the energetic actions. In the case of ideal gas $A_1 = 0$ and the equation (6) transforms into (4). In the case of viscous heat-conductive gas, described by a set of the Navier-Stokes equations, in the inertial coordinate system can be written in the form [16]

$$A_1 = \frac{1}{\rho} \frac{\partial}{\partial x_i} \left( - \frac{q_i}{T} \right) - \frac{q_i}{\rho T} \frac{\partial T}{\partial x_i} + \frac{\tau_{ki}}{\rho} \frac{\partial u_i}{\partial x_k} \quad (7)$$

Here $q_i$ is the heat flux, $\tau_{ki}$ is the viscous stress tensor. In the case of reacting gas there are added additional terms connected with the chemical nonequilibrium.

The equations (4) and (5) can be convoluted into the equation

$$ds = \omega \quad (8)$$

where $\omega = A_{\mu} d\xi^\mu$ is the differential form of the first degree (here $\mu = 1, \nu$).

The relation (8) is the evolutionary relation for the gas dynamical system (in the case of the local thermodynamic equilibrium). Here $\psi = s$. (It worth
notice that in the evolutionary relation for the thermodynamic system it is examined a dependence of entropy on the thermodynamic variables (see the relation (1)), but in the evolution relation for the gas dynamical system the entropy dependence on the space-time variables is considered. If the relation (8) be identical one (if the form \( \omega \) be the closed form, i.e. a differential), then one can obtain differential of entropy \( s \) and find entropy as a function of space-time coordinates. Just this entropy will be the gas dynamical function of state, a presence of which will point to that a state of the gas dynamical system is locally equilibrium. And if the relation (8) will not be identical, then the differential of entropy cannot be defined a differential of entropy \( s \). This will point to an absence of the gas dynamical function of the state and nonequilibrium state of the system. Such nonequilibrium is a cause of a development of instability.

Because of that nonequilibrium is produced by internal forces which are described by a commutator of the form \( \omega \), then it becomes evident that a cause of the gas dynamical instability is something that contributes to a commutator of the form \( \omega \). Without an accounting for terms which are connected with a deformation of the manifold formatted by trajectories, the commutator can be written as

\[
K_{1\nu} = \frac{\partial A_\nu}{\partial \xi^1} - \frac{\partial A_1}{\partial \xi^\nu} \quad (9)
\]

From the analyses of the expression \( A_\nu \) and with taking into account that \( A_1 = 0 \), one can see, that into commutator there contribute terms which are related to the multiple connectivity of the flow domain (the second term of the expression for \( A_\nu \)), nonpotentiality of the external forces (the third term) and a nonstationarity of the flow (the forth term). \{In the general case the terms connected with the transport phenomena and the physical and chemical processes contribute to the commutator (9)\}. These factors lead to the origin of internal forces, to nonequilibrium state and to the development of instability of various kinds. And yet for every kind of instability one can find an appropriate term in the commutator of the nonintegrable form, which is responsible for this kind of instability. \{Under this process it is also necessary to consider the evolutionary relations which the balance conservation laws for angular momentum and mass correspond to\}. Thus, there is an unambiguous connection between a kind of instability and the terms that contribute into the commutator of the nonintegrable form in the evolutionary relation.

As it was shown, under the realization of the additional degrees of freedom, a development of instability may lead to origin of the physical structures when the internal forces transform into the potential ones. Such gas dynamical structures are shocks, shock waves, turbulent pulsations and so on. Additional degrees of freedom are realized as the condition of the degenerate transform, namely, vanishing of determinants, Jacobians of transforms, etc. For example, such conditions are justified either on characteristics (determinant of coefficients at the normal derivatives vanishes), or at the singular points (Jacobian is equal to zero).
on the envelop of the characteristics of the Euler equations, or at the singular points of the Navier-Stokes equations (when the viscous gas is considered).

Let us analyze what kinds of instability and what gas dynamical structures may originate under given forcing.

1). *Shock, break of diaphragm and others*. The instability originates because of nonstationarity. The last term in the equation (3) gives a contribution into the commutator. In the case of ideal gas, whose flow is described by equations of the hyperbolic type, a transition to the locally equilibrium state is possible on the characteristics and their envelopes. The corresponding structures are weak shocks and shock waves.

2). *Flow around bodies by ideal (unviscous, thermal unconducting) gas*. Action of the nonpotential body forces. The instability develops because of the multiple connectivity of the flow domain and the nonpotentiality of the body forces. A contribution into the commutator has come from the second and third terms of the right-hand part of the equation (3). As the gas is ideal one and \( \partial s/\partial \xi^1 = A_1 = 0 \), that is there is no contribution into every fluid particle, then the instability of convective type develops. For \( U > a \) (\( U \) is the velocity of the gas particle, \( a \) is the speed of sound) a set of equations of the balance conservation laws belongs to the hyperbolic type and hence a transition to the locally equilibrium state is possible on the characteristics and the envelopes of characteristics as well, and weak shocks and shock waves are the structures of the system. If \( U < a \) when the equations are of elliptic type, such a transition is possible only at singular points. The structures, originated because of the convection, are of the vortex type. Under long forcing the large-scale structures can be produced.

3. *Boundary layer*. The instability originates because of the multiple connectivity of the domain and the transport phenomena (an influence of viscosity and thermal conductivity). Contributions into the commutator produce the second term in the right-hand part of the equation (3) and the second and third terms in the expression (7). A transition to the locally equilibrium state is allowed at the singular points. As in this case \( \partial s/\partial \xi^1 = A_1 \neq 0 \), that is, forcing acts on every gas particle singly, then a development of instability and transitions to the locally equilibrium state are allowed only in single fluid particle. Hence, the structures originated behave as pulsations. These are the turbulent pulsations.

In worth notice that separating of some formation from the local domain is accompanied by a production of the discontinuous surfaces (the contact discontinuities). Unlike shocks and shock waves these discontinuities do not propagate relative to the material system.

**Appendix 4**

**Electromagnetic field**

We will show how it may be obtained the evolutionary relation for the system which generates the electromagnetic field and what are specific features of this relation.
As it is known [11], the electromagnetic field can be presented by some 2-form and its dual one. The Maxwell equations appear to be reduced to that the both forms are closed forms. Let us analyze when the closure conditions are satisfied.

If to utilize the Lorentz force \( \mathbf{F} = \rho (\mathbf{E} + [\mathbf{U} \times \mathbf{H}]/c) \), then a local variation of energy and linear momentum of the charged substance may be written respectively as [17]: \( \rho (\mathbf{U} \cdot \mathbf{E}), \rho (\mathbf{E} + [\mathbf{U} \times \mathbf{H}]/c) \). Here \( \mathbf{E}, \mathbf{H} \) are respectively the electric and magnetic strengths of the field, \( \rho \) is the charge density, \( \mathbf{U} \) is the velocity of the charged substance. These variations of energy and linear momentum are caused by energetic and force actions and are equal to values of these actions. If to denote these actions by \( Q^e, Q^i \), then the balance conservation laws can be written as follows:

\[
\rho (\mathbf{U} \cdot \mathbf{E}) = Q^e \\
\rho (\mathbf{E} + [\mathbf{U} \times \mathbf{H}]/c) = Q^i
\]  

(1)

After elimination of the characteristics of the material system (charged substance) \( \rho \) and \( \mathbf{U} \) by application of the Maxwell-Lorentz equations [17], the left-hand sides of the equations (1) can be expressed only through the strengths of the electromagnetic field and then one can write the equations (1) in the form:

\[
c \text{div} \mathbf{S} = -\frac{\partial}{\partial t} I + Q^e
\]  

(2)

\[
\frac{1}{c} \frac{\partial}{\partial t} \mathbf{S} = \mathbf{G} + Q^i
\]  

(3)

where \( \mathbf{S} = [\mathbf{E} \times \mathbf{H}] \) is the Pointing vector, \( I = (E^2 + H^2)/c, \mathbf{G} = \mathbf{E} \text{div} \mathbf{E} + \text{grad}(\mathbf{E} \cdot \mathbf{E}) - (\mathbf{E} \cdot \text{grad})\mathbf{E} + \text{grad}(\mathbf{H} \cdot \mathbf{H}) - (\mathbf{H} \cdot \text{grad})\mathbf{H} \).

The equation (2) is widely utilized while description of the electromagnetic field and a calculation of energy and the Pointing vector. And the equation (3) does not commonly be taken into account. Actually, the Pointing vector \( \mathbf{S} \) has to obey to two equations, which can be convoluted into the relation

\[
d \psi = \omega^2
\]  

(4)

where 2-form \( d \psi \) corresponds to the vector \( \mathbf{S} \) and coefficients of the form \( \omega^2 \) (the upper subscript shows a degree of the form) are the right-hand parts of the equations (2), (3). It is just the evolutionary relation for the system of charged particles that generates the electromagnetic field.

From the equations (2), (3) or from the evolutionary relation one can find the Pointing vector as some preserveable measured physical quantity only if these equations are conjugated ones, that is, if a commutator formed by mixed derivatives (it is just a commutator of the form \( \omega^2 \)) is equal to zero. And if the commutator is nonzero, then the right-hand side is not a differential and the
Pointing vector is some functional. Under what conditions can the Pointing vector be formatted as a measurable quantity?

Let us choose the local coordinates $l_k$ in such a way that one direction $l_1$ coincides with a direction of the vector $S$. As this chosen direction coincides with a direction of the vector $S = [E \times H]$ and hence is normal to the vectors $E$ and $H$, then one obtains that $\text{div}S = \partial s/\partial l_1$, where $S$ is the module of $S$. And in addition, the projection of the vector $G$ on the chosen direction turns out to be equal to $\partial I/\partial l_1$. As the result, after separating from the vector equation (3) its projection on the chosen direction, the equations (2), (3) can be written as

$$\frac{\partial S}{\partial l_1} = -\frac{1}{c} \frac{\partial I}{\partial t} - \frac{1}{c} Q^e$$  \hspace{1cm} (5)

$$\frac{\partial S}{\partial t} = c \frac{\partial I}{\partial l_1} - cQ^i$$  \hspace{1cm} (6)

$$0 = -G^{''} - cQ^{''}i$$

Here the prime relates to the direction $l_1$, double primes do to the other directions. Under the condition $\partial l_1/\partial t = c$ from the equations (5), (6) it is possible to obtain a relation in the differential forms

$$\frac{\partial S}{\partial l_1} dl_1 + \frac{\partial S}{\partial t} dt = - \left( \frac{\partial I}{\partial l_1} dl_1 + \frac{\partial I}{\partial t} dt \right) - (Q^i dt + Q^{ce} dl_1)$$  \hspace{1cm} (7)

As the second brace in the right-hand side is not a differential (the energetic and force actions have distinguished nature and cannot be conjugated), then one can obtain a closed form only if the right-hand side vanishes:

$$\left( \frac{\partial I}{\partial l_1} dl_1 + \frac{\partial I}{\partial t} dt \right) - (O^i dt + Q^{ce} dl_1) = 0$$  \hspace{1cm} (8)

that is

$$\frac{\partial I}{\partial t} = Q^{ce}, \quad \frac{\partial I}{\partial l_1} = Q^i$$  \hspace{1cm} (9)

In this case $dS = 0$ and the module of the Pointing vector $S$ proves to be the closed form, i.e. a measurable quantity. And the integrating direction (pseudostructure) will be

$$-\frac{\partial S/\partial t}{\partial S/\partial l_1} = \frac{dl_1}{dt} = c$$  \hspace{1cm} (10)

Thus, the constant $c$, that was introduced into the Maxwell equations, is defined as the integrating direction. From the expressions (9) it is evident, that in this case the local energetic and force actions on the material system (charged substance) appear to be transformed into the quantities of the electromagnetic field, namely, energy and linear momentum of the electromagnetic wave that
propagates with the light speed \( c \), which value is defined by the condition (10). One can see, that the constant \( c \) in the Maxwell equation is the speed of the electromagnetic wave and it is defined as the integrating direction.

Appendix 5

On interactions

As it was shown above, a type of the physical structures (and, accordingly, the physical fields) generated by the evolutionary relation depends on a degree of the exterior forms \( p \) and \( k \) (here \( p \) is a degree of the nonintegrable form of the evolutionary relation which is connected with a number of the interacting balance conservation laws, and \( k \) is a degree of the closed form generated by the evolutionary relation) and on a space dimension \( n \). By introducing the classification by numbers \( p, k, n \) one may understand the internal connection of different physical fields and see a connection between interactions. It is reflected in the table presented below.

In the table names of the particles created are given. Numbers placed near particle names correspond to the space dimension. In the curly brackets the sources of interactions are presented. In the lower row we point out the massive particles created by interactions (the exact forms of zero degree obtained by sequent integrating of the forms of degree \( p \) correspond to these particles). From the table one can see a correspondence between the degree \( k \) of the closed forms being realized and a type of interactions. Thus, \( k = 0 \) corresponds to the strong interaction, \( k = 1 \) does to the weak one, \( k = 2 \) corresponds to the electromagnetic interaction, and \( k = 3 \) does to the gravitational interaction.

As a result, we obtain that a type of interaction and a type of the created particles is defined by the degree \( (k) \) of the closed forms realized, and properties of the particles are governed by the degree \( (p) \) of the evolutionary equation, namely, by a number of the interacting balance conservation laws, and the space dimension. The last property is connected with that the closed forms of equal degrees \( k \), but obtained from the evolutionary relations acting in spaces of different dimensions \( n \) (the values of \( p \) are different), are distinguished as they are defined on the pseudostructures of different dimensions (the dimension of the pseudostructure \( (n + 1 − k) \) depends on the dimension of the initial space \( n \).

For this reason the realized physical structures with equal degrees \( k \) of the closed forms are distinguishable. A connection between a type of interactions and the conservation laws may be seen by a comparison of the first and last columns. In the last column the interacting balance conservation laws are pointed out. The arrows show that the conservation laws presented in given cell are added to that from lower cells. Notice that for \( k = 0 \) in the space of zero dimension there is no the momentum. This manifests it self beginning with the space of the dimension 1 (in the forms of degree \( k = 0 \) energy and momentum are formatted independently). For \( k = 0 \) energy and momentum are nonconjugate. The forms dual to them are respectively time and coordinates (time and coordinates have different nature because time is dual to energy and coordinates are dual to linear...


momentum). Energy and time as well as linear momentum and coordinate do not commutate within the framework of the same form because they relate to different forms: exterior and dual ones. (The commutative relations $\hat{q}\hat{p} - \hat{p}\hat{q} = i\hbar$ reflect this fact. The left-hand side of the commutative relations is analogous of the commutator value of the nonintegrable form of zero degree, and the right-hand side is equal to its value at the point in time of the realization of the closed zero degree form, the imaginary unit points to the transverse direction with respect to the pseudostructure). The closed exterior form of degree $k = 1$ corresponds to a conjugation of energy and momentum (for $k = 1$ energy and momentum prove to be the simultaneously measurable quantities).

In the table a single cycle of formation of the physical structures is presented. It contains four levels to every of those it corresponds a proper value $p \ (p = 0, 1, 2, 3)$ and the space dimension $n$. The structures formatted in the preceding cycle serves as a source of interactions for the first level of a new cycle. The sequential cycles reflect properties of the sequentially embedded material systems. And some given level has specific properties which are inherent in the same levels of the other cycles. For example, it may be seen by comparison of the cycle described and the cycle where conductors, semiconductors, dielectrics and neutral elements are sequentially correspond to the exact forms. Properties of elements of the third level, namely, neutrons in one cycle and dielectrics in the other coincide with those of the so-called ”magnetic monopole” [26, 27].
| interaction | \( n \) | i+0 | i+1 | i+2 | i+3 | balance conserv. laws |
|-------------|--------|-----|-----|-----|-----|----------------------|
| k\(p\)     | 0      | 1   | 2   | 3   |     | gravitation          |
|             |        |     |     |     | graviton mass        |
|             |        |     |     |     | ↑ \{ electron \}      |
|             |        |     |     |     | ↑ \{ proton \}        |
|             |        |     |     |     | ↑ \{ neutron \}       |
|             |        |     |     |     | ↑ \{ photon \}        |
|             |        |     |     |     | ↑ \{ neutrino \}      |
|             |        |     |     |     | ↑ \{ quant \}?       |
|             |        |     |     |     |                     |
|             |        |     |     |     | photon 2 angular momentum |
|             |        |     |     |     | ↑ \{ electron \}      |
|             |        |     |     |     | ↑ \{ proton \}        |
|             |        |     |     |     | ↑ \{ neutrino \}      |
|             |        |     |     |     | ↑ { quant }?          |
|             |        |     |     |     |                     |
|             |        |     |     |     | photon 3 linear momentum |
|             |        |     |     |     | ↑ { electron }        |
|             |        |     |     |     | ↑ { proton }          |
|             |        |     |     |     | ↑ { neutrino }        |
|             |        |     |     |     | ↑ { quant }?          |
|             |        |     |     |     |                     |
|             |        |     |     |     | strong energy+ time |
|             |        |     |     |     | or \( p > 0 \) momentum |
|             |        |     |     |     | +coordin.          |
|             |        |     |     |     |                     |
| particles  | electron proton neutron deuteron \( ? \) | material exact forms |

**Appendix 6**

A formation of the metric space

As it was shown above, a noncommutativity of the balance conservation laws and a transition from those to the exact conservation laws explain a mechanism of origin of the physical structures. And as the origin of the physical structures is connected with a formation of the pseudostructures, then by analyzing a mechanism of formation of the physical structures one can understand a mechanism of formation of the pseudometric and metric spaces. {Recall, that the inexact closed forms correspond to the physical structures and pseudostructures. and
the exact forms correspond to elements of the material system and to the metric spaces. It is useful to note some properties of the manifolds.

Assume, that on the manifold one can choose any system of coordinates with the basis \( e_\mu \) and to set up the metric forms of manifold [21]: \( (e_\mu, e_\nu) \), \( (e_\mu, dx^\mu) \), \( (d e_\mu) \). The metric forms and their commutators define metric and differential characteristics of manifold. If the metric forms are closed (commutators are equal to zero), then the metric \( g_{\mu\nu} = (e_\mu e_\nu) \) is defined, and the results of displacement over manifold the point \( d M = (e_\mu dx^\mu) \) and a unit vector \( d A = (de_\mu) \) are independent of the path of displacement (the path of integration). The closed metric forms define the structure of the manifold, and the commutators of the metric forms determine such characteristics of the manifold as bend, torsion, curvature (if the commutators are nonzero, then the results of displacement over manifold of a point or the unit vector depend on the path of displacement). An example of manifold with the closed metric forms is the differentiable manifold whose metric and differential characteristics prove to be consistent [13].

A role of the differential characteristics of the manifold may play the connectivities [21] \( \Gamma^\rho_{\mu\nu} \), \( (\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\nu\mu}) \), \( R^\rho_{\mu\nu\sigma} \), which are components of a commutator of the metric forms.

As it is known [20], these commutators for the Euclidean manifold are equal to zero. In the case of the Riemann manifold the commutator of the metric form of degree two is nonzero: \( R^\rho_{\mu\nu\sigma} \neq 0 \).

If the exterior differential forms are defined on manifolds whose metric forms are nonclosed, then, as it was pointed out, the commutators of the metric forms will enter into the commutators of the exterior differential forms. In particular, components of the commutator of the external form of degree one \( \theta = a_\alpha dx^\alpha \) can be written in terms of connectivities as follows:

\[
K_{\alpha\beta} = \left( \frac{\partial a_\beta}{\partial x^\alpha} - \frac{\partial a_\alpha}{\partial x^\beta} \right) + (\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta}) a_\sigma
\]  

(1)

It is evident, that the commutator of the exterior form consists of two terms. The first one (the first parentheses) depends on coefficients of the exterior form, and the second term does on the differential characteristics of the manifold. It is a typical feature for the commutator of the exterior forms of other degrees as well. Because of this feature the topologic properties of commutators of the exterior forms are manifested: they can realize a mutual relation between the exterior form and the basis, namely, the metric form of manifold. The other specific feature is that terms in the commutator have the different nature, i.e. one term depends on coefficients of the exterior form, and the other depends on the basis. Such terms cannot be equal identically, and hence they cannot make the commutator to be zero. This means that the exterior forms, defined on manifolds with nonclosed metric forms, turns out to be nonclosed (such forms may be called nonintegrable ones in contrast to the nonclosed forms defined on the differentiable manifolds). The above mentioned properties of commutators of the exterior forms enable one to understand a mechanism of formation of the
metric spaces. Note, that the material system is the generator of the formatting metric space, and an accompanying manifold is the base.

When derivation the evolutionary relation there were used two spatial objects: the accompanying manifold (connected with the material system) that has no metric structure for real processes and the inertial space (not connected with the material system) which is the metric space. \{Note, that the metric space formatted becomes the inertial space of one more degree\}

Assume that the initial inertial space has the dimension \( n = 3 \). The material system in such a space is subjected to the balance conservation laws, which equations in the accompanying frame of reference turn out to be convoluted to the evolutionary relation with \( p = n = 3 \):

\[
d\psi \cong \omega^3, \quad d\omega^3 \neq 0
\] (2)

The form \( \omega^3 \) is defined on the accompanying manifold, and therefore this form is nonintegrable, that is, its commutator is nonzero \( (\text{a degree of the form } \psi \text{ equals } 2) \). \{In cosmology and the gravitation theory the equations of ideal fluid are sometimes used \[15, 17, 28\]. In essence, these equations are the balance conservation laws. However, these equations commonly used in the covariant form \[28\], that is, the covariance condition is imposed. To study a process of formation of the pseudometric and metric spaces it is necessary to employ equations that are not subjected to the conditions of invariance or covariance\}\.

A realization of the pseudostructure (an element of the pseudometric space) and an origin of the physical structure, which the closed metric and exterior forms correspond to, is the transition from the nonintegrable form \( \omega^3 \) to the closed form \( \omega^{3*} \) \( (\text{this is connected with the degenerate transform}) \). And it is required the following relations have to be satisfied:

\[
d_\pi \omega^{3*} = 0
\]

\[
d_\pi \omega^{3*} = 0
\] (3)

In the present case a degree of the closed form is \( k = p = 3 \), and a dimension of the pseudostructure is \( m = n + 1 - k = p + 1 - k = 1 \). On the pseudostructure from the evolutionary relation (2) it follows the relation

\[
d_\pi \psi = \omega^3
\] (4)

which is identical one because the closed form \( \omega^3 \) may be expressed through the interior differential. From this relation it can be defined the form \( d_\pi \psi \) which specifies a state of the system and may be referred to as the structural form. \( (\text{In the case under consideration this is the form of degree } 3) \). It corresponds to the conservation law, because a differential of this form \( (\text{interior on the pseudostructure}) \) is equal to zero.

A realization of the physical structure \( (\text{connected with an origin of the physical structure and that the conservation law is fulfilled}) \) is one of the exhibitions
of a mechanism of formatting the metric spaces. It worth to underline that the pseudostructure is realized with respect to the inertial frame of reference. (The degenerate transform corresponds to transition from the frame of reference connected with the accompanying manifold to the inertial coordinate system).

With the aim to be more clear we shall put the tensor expressions into correspondence to the exterior forms. We may put a tensor with $p$ lower (covariant) subscripts into correspondence to the external form of degree $p$ defined on the differentiable manifold. As it is known, a differential of the form degree $p$ on the differentiable manifold is the form of degree $p+1$. We may put a tensor with $p+1$ lower subscripts into correspondence to the differential of the form of degree $p$. By analogy with this we put the tensor expression $K_{\alpha...}$ into correspondence to a differential or to a commutator of the nonintegrable form. With this notation a commutator of the form $\omega^3$ can be written as $K_{\alpha\beta\gamma\chi}$, where three first subscripts correspond to a degree of the form, and the fourth one appears while differentiating the form (from this point and further we shall use the Greek subscripts for the accompanying frame of reference and Latin ones for the inertial that).

A commutator of the basic metric form which can be denoted by $R_{\alpha\beta\gamma\chi}$ enters into a commutator of the nonintegrable form. We may put the covariant tensors of rank 3, $S_{jkl}$ and $T_{jkl}$, (its divergence is equal to zero as they corresponds to the closed forms) into correspondence to the closed forms $d_\pi \psi$ and $\omega^3$ (that are formatted with relevance to the inertial frame of reference). And to the pseudostructure we may put into correspondence the 1-covariant pseudotensor $T^i$ (it corresponds to the closed metric form, i.e. pseudostructure), which is dual to the tensor $T_{jkl}[15]: T^i = *T_{ijk} = \frac{1}{6} \varepsilon^{ijkl} T_{jkl}$ (here $\varepsilon^{ijkl}$ is the completely antisymmetric unit pseudotensor) can be put into correspondence to the pseudostructure. Similarly, by $S^i = *S_{jkl}$ denote the tensor dual to $S_{jkl}$. Now we introduce the tensor expressions:

$$S^i_{jkl} = \{ S_{jkl}^i \}, \quad T^i_{jkl} = \{ T_{jkl}^i \}$$

(5)

{These tensor expressions are not tensors with covariant and contravariant subscripts because, firstly, they combine tensors and pseudotensors, and secondly, in these expressions one cannot rise up and lower subscripts as the metric is not defined as yet}. The tensor expression $S^i_{jkl}$ is a representation of the physical structure (it corresponds to the structure form on pseudostructure), and the tensor expression $T^i_{jkl}$ is a representation of the forms $\omega^3$ and $*\omega^3$ (they correspond to the external actions processed by the system).

With taking into account the relations (3), the relation (4) can be written in terms of the tensor expressions as

$$S^i_{jkl} = T^i_{jkl}$$

(6)

The relation (6) shows that the physical structure (including the pseudstroctures) are produced at the expense of external actions processed by the system.
What is the further mechanism of formation of the metric space?

While origin of the physical structure, a quantity, which is described by a commutator of the nonintegrable form $\omega^3$ and acts as internal force, transforms into potential force, which acts in the direction transverse to the pseudostructure. (If a differential of the form $\omega^3$ be zero, that is, the commutators $R_{\alpha \beta \gamma \chi}$ and $K_{\alpha \beta \gamma \chi}$ be equal to zero, then the potential force will be equal to zero). This potential force becomes a new source of nonequilibrium (even without the extra external actions) and may lead to a further formation of the pseudostructures.

As the relation (4) is the identical one, then it can be integrated. Because the form $\omega^3$ is closed, it is the interior (on the pseudostructure) differential of the form one degree less

$$\omega^3 = d_\pi \omega^2$$

From the relations (4), (7) it follows the relation (below, for the sake of convenience, we shall indicate explicitly a degree of the form $\psi$)

$$d_\pi \psi^2 = d_\pi \omega^2$$

which can be integrated (within the accuracy up to the lower degree forms):

$$\psi^2 = \omega^2$$

This is an integration of the nonidentical evolutionary relation (2) over a single dimensionality which has been formatted.

From the relation (7) one can see that a differential of the form $\omega^2$ is nonzero. The form $\omega^2$ (of degree $p - 1 = 2$) proves to be nonintegrable form (its commutator is nonzero) on the manifold directions remained after integration. To the commutator of the form $\omega^2$ it can be put into correspondence the tensor expression $K^\gamma_{\beta \gamma \chi}$ (three lower subscripts is a degree of the exterior form plus 1, and a single top subscript is the pseudometric dimension formatted). In this case the basic commutator can be written in the form $R^\alpha_{\beta \gamma \chi}$. (If the Bianchi identity [21] is satisfied, then from this tensor it can be obtained the Riemann-Christoffel tensor $G^i_{jkl}$ which corresponds to the Riemann manifold. However it takes place only after the pseudoriemann and Riemann manifolds be completely formatted).

Here it appears some special feature. On the one hand, the form $\omega^2$ obtained turns out to be nonintegrable one, and therefore, it cannot be expressed in terms of differential. And on the other hand, the form $\psi^2$ in the left-hand side of the relation (8), for to become the structural form, has to become the closed form, namely, the differential:

$$\psi^2 = d\psi^1$$

By comparison of the relations (8) and (9), we get

$$d\psi^1 \cong \omega^2$$

which cannot be identity as the form $\omega^2$ is not expressed through differential.
The nonidentical relation (10) is the relation of the type similar to the initial relation (2), however it is the form of one less degree. We may repeat the analysis like for the relation (2) and get the pseudostructure of one more dimension. By sequent integrating of the nonidentical relations we may obtain the pseudometric space. The closed exterior forms of degrees $p, p - 1, ..., 0$, which are inexact, correspond to this space. A transition to the exact form of zero degree will correspond to a transition to the metric space.

With application of the tensor expressions these transitions can be schematically written in the following form:

$$d\psi \cong \omega^3, \quad d\omega^3 \neq 0 \quad (K_{\alpha\beta\gamma\chi} \neq 0, \ R_{\alpha\beta\gamma\chi} \neq 0) \quad (11)$$

$m = 1$

$$S_{ijkl}^i = T_{ijkl}^i \quad (12)$$

$$d\psi \cong \omega^2, \quad \omega^2 \neq 0 : \quad (K_{\alpha\beta\gamma\chi} \neq 0, \ R_{\alpha\beta\gamma\chi} \neq 0) \quad (13)$$

$m = 2$

$$S_{ij}^{ij} = T_{ij}^{ij} \quad (14)$$

$$d\psi \cong \omega^1, \quad \omega^1 \neq 0 : \quad (K_{\alpha\beta\gamma\chi} \neq 0, \ R_{\alpha\beta\gamma\chi} \neq 0) \quad (15)$$

$m = 3$

$$S_{ijk}^{ijk} = T_{ijk}^{ijk} \quad (16)$$

$$d\psi \cong \omega^0, \quad \omega^0 \neq 0 : \quad (K_{\alpha\beta\gamma\chi} \neq 0, \ R_{\alpha\beta\gamma\chi} \neq 0) \quad (17)$$

$m = 4$

$$S_{ijkl}^{ijkl} = T_{ijkl}^{ijkl} \quad (18)$$

$$d\psi \cong \int \omega^0, \quad \omega^0 \neq 0 : \quad (K_{\alpha\beta\gamma\chi} \neq 0, \ R_{\alpha\beta\gamma\chi} \neq 0) \quad (19)$$

$$\psi = 0$$

The line (11) in this scheme corresponds to the nonidentical initial evolutionary relation (with the exterior forms of degree 3). Here the inequality $d\omega^3 \neq 0$ is written in terms of the tensor expressions for the commutators: $(K_{\alpha\beta\gamma\chi} \neq 0, \ R_{\alpha\beta\gamma\chi} \neq 0)$. The dotted line corresponds to the degenerate transform and to the transition from the nonidentical evolutionary relation to the identic relation on the
pseudostructure of the dimension \( m = 1 \) (the line (12)), as well as to the non-
identic relation of one less degree (the line (13)). The line (12) contains the
identic relation in the tensor expressions (see, the relation (6)), which corre-
sponds to the identic relation (4) in the differential forms.

Under the degenerate transform it is once more allowed a transition from the
nonidentic relation in the line (13) to the identic relation on the pseudostructure
of the dimension \( m = 2 \) (the line (14)) and to the new nonidentic relation (the
line (15)). Similar transitions can be realized under the degenerate transforms
up to the closed inexact forms of zero degree. The solid line corresponds to the
transition to the exact form.

A realization of the pseudostructures of dimensions (1, ..., 4) and closed
inexact forms of degrees (3, ..., 0) (an origination of the physical structures
\( S_{ijkl} \), ..., \( S^{ijkl} \)) correspond to formation of the pseudometric manifold. A tran-
sition to the exact form corresponds to a transition to the metric space.

And what can one say concerning the pseudoriemann manifold and the Rie-
mann space?

As it is known, when deriving the Einstein equation [29] it was supposed
that the following conditions to be satisfied: the Bianchi identity is fulfilled, the
connectivity coefficients are symmetric ones (the connectivity coefficients are
the Christoffel symbols), and there is a transformation under which the connectiv-
ity coefficient becomes zero. These conditions are those of realization of the
degenerate transforms for the nonidentical evolutionary relations (13), (15),
(17), (19). If the Bianchi identities are satisfied [21], then from the tensor expres-
sion \( R^{\alpha\beta\gamma\chi}_{\chi} \) the Riemann-Christoffel tensor \( G^{i}_{jkl} \) can be obtained. To the tensor
expression \( R^{\alpha\beta\gamma\chi}_{\chi} \) there corresponds the commutator of the first order metric form
\( (\Gamma_{\mu\nu}^p - \Gamma_{\nu\mu}^p) \), from which under the conditions of symmetry of the connectivity
coefficients \( (\Gamma_{jkl} - \Gamma_{jlk}) = 0 \) the Ricci tensor can be found. To the tensor
expression \( R^{\alpha\beta\gamma\chi}_{\chi} \) there corresponds the connectivity \( \Gamma_{\mu\nu}^p \), from which under the
condition \( \Gamma_{jkl} = \{ j_{kl} \} \) (the connectivity coefficients are equal to the Christoffel
symbols) it can be obtained the tensor expression \( S^{ijkl} \), which corresponds to
the Einstein tensor \( S^{ijkl} = G^{k}_{l} - \frac{1}{2} G \delta^{k}_{l} \) (the tensors \( G^{k}_{l} \) and \( G \) are obtained from
the Riemann-Christoffel tensor with taking into account the symmetry of the
connectivity coefficients). To Einstein’s equation there corresponds the identity
(16) that connects the tensor expression \( S^{ijkl} \) with the tensor expression \( T^{ijkl} \)
which corresponds to the energy-momentum tensor. (It is well to bear in mind
that the metric tensor has not formatted as yet, and therefore the operation of
transfer of low and top subscripts with the help of the metric tensor proves to
be inapplicable). To the tensor expression \( R^{\alpha\beta\gamma\chi} \) there corresponds the connectiv-
ty coefficients, which under a presence of the degenerated transform vanish,
and this corresponds to a formation of the closed (inexact) metric form of zero
degree \( g_{kl} = (e_{k}e_{l}) \). However, at given stage this only corresponds to formation
of the pseudoriemann manifold. A transition from the closed inexact form of
zero degree to the exact form of zero degree corresponds to transition to the
Riemann space.

Appendix 7

Functional properties of solutions to the differential equations.

The field equation. Transformations.

While description of the physical processes by differential equation the following fact is essential. As to the exact conservation laws there have to correspond the closed forms, then to them may correspond only solutions, whose derivatives constitute the closed form, i.e. a differential. What conditions must the differential equations satisfy to, for to have such solutions? Let us trace this by the example of the first order partial differential equation:

$$F(x^i, u, p_i) = 0, \quad p_i = \partial u / \partial x^i$$

(1)

Let us consider the functional relation

$$du = \Theta$$

(2)

where $\Theta = p_i \, dx^i$. This relation is an analog to the evolutionary relation, and in the general case this relation (as well as the evolutionary relation) proves to be nonidentical. For this relation to be identical one, the both parts of the relation have to be differentials, i.e. the closed forms. To obey this condition, the commutator $K_{ij} = \partial p_j / \partial x^i - \partial p_i / \partial x^j$ of the form $\Theta = p_i \, dx^i$ has to be zero. However, from the equation (1) it does not evidently follow, that the derivatives $p_i = \partial u / \partial x^i$, which obey to the equation (and given boundary or initial conditions of the problem), are conjugate, that is, they made a commutator of the form $\Theta$ equal to zero. In the general case without any supplementary conditions a commutator of the form $\Theta$ is nonzero. The form $\Theta = p_i \, dx^i$ proves to be nonclosed and is not a differential unlike the left-hand side of the relation (2). The functional relation (without supplementary conditions) proves to be nonidentical. And as the derivatives of the initial equation do not format a differential, then the corresponding solution to the differential equation $u$ will not be a function of $x^i$. It will depend on the commutator of the form $\Theta$, that is, it will be a functional.

To obtain the solution, which is the function, it is necessary to add a closure condition for the form $\Theta = p_i \, dx^i$ and for the form dual to that (in the present case the functional $F$ plays a role of the form dual to $\Theta$) [3]:

$$\begin{align*}
\{ dF(x^i, u, p_i) &= 0 \\
\quad d(p_i \, dx^i) &= 0
\end{align*}$$

(3)

If expand the differentials, then we get a set of the homogeneous equations with respect to $dx^i$ and $dp_i$ (in space of the dimension $2n$ – initial and tangential):

$$\begin{align*}
\left( \frac{\partial F}{\partial x^i} + \frac{\partial F}{\partial u} p_i \right) \, dx^i + \frac{\partial F}{\partial p_i} \, dp_i &= 0 \\
\quad dp_i \, dx^i - dx^i \, dp_i &= 0
\end{align*}$$

(4)
The solvability conditions for this set (a vanishing of the determinant composed of the coefficients at $dx^i$, $dp_i$) have the form:

$$\frac{dx^i}{\partial F/\partial p_i} = \frac{-dp_i}{\partial F/\partial x^i + p_i \partial F/\partial u} \quad (5)$$

These conditions determine an integrating direction, namely, a pseudostructure, on which the form $\Theta = p_i \, dx^i$ turns out to be closed one, i.e. it becomes a differential, and the relation (2) proves to be identity. If the conditions (5), which may be called the integrability conditions, are fulfilled, the derivatives constitute a differential ($\delta u = p_i \, dx^i = du$) and the solution becomes the function. Just such solutions (functions on the pseudostructures) are so-called generalized solutions [30]. {As the functions, which are the generalized solutions (distributions), are defined only on the pseudostructures, then they have breaks of derivatives in directions being transverse to the pseudostructures. The order of derivatives, which have breaks, is equal to a degree of the exterior form. If the form of zero degree enters into the functional relation, then the function itself will have the breaks.}

If to find the characteristics of the equation, then it appears that the conditions (5) are the equations for characteristics [31]. The characteristics are the pseudostructures, on which the solutions prove to be functions (generalized solutions). {The coordinates of the equations for characteristics are not identical to the independent coordinates in the initial equation (1). A transition from coordinates of the initial space to the characteristic manifold appears to be the degenerate transform, namely, the determinant of the set of equations (4) becomes zero. The derivatives of the equation (1) transfer from the tangent space to cotangent one.}

A partial differential equation of the first order has been analyzed, and the functional relation with the form of the first degree has been considered. (At this point it worth noting that for this equation one has to write down and analyze the additional relation with the zero-order form as well). Similar functional properties have the solutions to all differential equations. And, if the order of the differential equation is $k$, then to this equation there corresponds $(k + 1)$ functional relations, every of which contains the exterior forms of degrees: $k$, $k - 1$, ..., 0. In a similar manner one can investigate the solutions to a set of the differential equations and the ordinary differential equations (for which the nonconjugativity of the unknown functions and initial conditions are examined). {As it is known, the analyses of the unstable solutions and the integrability conditions provides the basis of the qualitative theory of the differential equations. From the functional relation it follows that to the instability it leads a dependence of the solution on the commutator, and as the integrability condition it serves the closure conditions of the form composed of derivatives. It is evident that an analyses of the nonidentical functional relation lies at the basis of the qualitative theory of the differential equations.}
The functional properties of the differential equation play an essential role under description of the physical processes. It is clear, that to the conservation laws, and hence, to the physical structures, there can correspond only generalized solutions, whose derivatives format the closed form. The solutions-functionals have a physical sense as well. The solutions to the equations of the balance conservation laws, which are functionals, describe nonequilibrium states of the material system. And a transition (under the degenerate transformation) from the solution-functional to the generalized solution (a transition from the nonintegrable form to the closed one) corresponds to origin of the physical structure.

Because of that to the physical structures, which format the physical fields, there correspond the closed forms, then only the equations with the additional conditions (integrability conditions) can be the equations of field theory.

Let us return to the equation (1). Suppose, that it does not explicitly depend on \( u \) and it is solved with respect to some variable, for example \( t \), that is, it has the form

\[
\frac{\partial u}{\partial t} + E(t, x^j, p_j) = 0, \quad p_j = \frac{\partial u}{\partial x^j},
\]

(6)

Then the integrability conditions (5) take the form (in this case \( \partial F/\partial p_1 = 1 \))

\[
\frac{dx^j}{dt} = \frac{\partial E}{\partial p_j}, \quad \frac{dp_j}{dt} = -\frac{\partial E}{\partial x^j},
\]

(7)

The conditions (7) are known as the canonical relations. (It can be seen that the canonical relations are the equations for characteristics of the equation (6)). The equation (6) provided with the supplementary conditions, namely, the canonical relations (7), is called the Hamilton-Jacobi equation [31]. The derivatives of this equation format the differential:

\[
\delta u = (\frac{\partial u}{\partial t}) dt + p_j dx^j = -E dt + p_j dx^j = du.
\]

To this type there belongs the equation of field theory

\[
\frac{ds}{dt} + H\left(t, q_j, \frac{\partial s}{\partial q_j}\right) = 0, \quad \frac{\partial s}{\partial q_j} = p_j
\]

(8)

where \( s \) is the field function for the action functional \( S = \int L dt \). Here \( L \) is the Lagrange function, \( H \) is the Hamilton function: \( H(t, q_j, p_j) = p_j \dot{q}_j - L \), \( p_j = \partial L/\partial \dot{q}_j \). To the equation (8) there correspond the closed form \( ds = H dt + p_j dq_j \) (the Poincare invariant). { In the quantum theory an analog to the equation (8) is the Schrödinger equation [32]).

{Here the degenerate transformation is a transition from the Lagrange function to the Hamilton function. An equation for the Lagrange function, that is the Euler variational equation, has been obtained from the condition \( \delta S = 0 \), where \( S \) is the action functional. In the real case, when forces are nonpotential or connections are nonholonomic, the quantity \( \delta S \) is not a closed form, that is, \( d\delta S \neq 0 \). But the Hamilton function is obtained from the condition \( d\delta S = 0 \) which is the closure condition for the form \( \delta S \). A transition from the Lagrange
function $L$ to the Hamilton function $H$ (a transition from variables $q_j$, $\dot{q}_j$ to variables $q_j$, $p_j = \partial L / \partial \dot{q}_j$) is a transition from the tangent space, where the form is nonclosed, to cotangent space with closed form. One can see, that this transition is degenerate one. \{In the invariant field theories there used only nondegenerate transformations, which preserve the differential. By the example of the canonical relations it is possible to show that nondegenerate and degenerate transformations are connected. The canonical relations in the invariant field theory correspond to nondegenerate tangent transformations. At the same time, the canonical relations for the Hamilton-Jacobi equation without supplementary conditions are the equations for characteristics, which the degenerate transformations correspond to. The degenerate transformation is a transition from the tangent space $(q_j, \dot{q}_j)$ to the cotangent (characteristic) manifold $(q_j, p_j)$. It is a transition from manifold that corresponds to the material system, to the physical fields, and it is an origin of the physical structure. On the other hand, the nondegenerate transformation is a transition from one characteristic manifold $(q_j, p_j)$ to the other characteristic manifold $(Q_j, P_j)$, that is, a transition from one physical structure to another physical structure. It is easily shown that it is a specific feature for the relations, which perform such transformations as tangent, gradient, contact, gauge, conformal mapping, and others\}.

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