On Nonlinear Evolution Equation of Second Order in Banach Spaces

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In this work the existence of a solution and the behavior of the existing solution of the nonlinear differential equation of second order in Banach space are studied.

More exactly we study the following nonlinear evolution equation

\[ x_{tt} + A \circ F(x) = g \left( x, A^{-\frac{1}{2}} x_t \right), \quad t \in (0, T), \quad 0 < T < \infty \]  

under the initial conditions

\[ x(0) = x_0, \quad x_t(0) = x_1 \]

here \( A \) is a linear operator in a real Hilbert space \( H \), \( F : X \rightarrow X^* \) and \( g : D(g) \subseteq H \times H \rightarrow H \) are a nonlinear operators, \( X \) is a real Banach space. In particular, if the operator \( A \) is the differential operator, satisfying some condition, then the considered problem become a nonlinear hyperbolic equation with nonlinear main part. Moreover, in the special case, this equation describe of the traffic flow (see, \[1\]). For example, operator \( A \) denotes \(-\Delta\) with Dirichlet boundary condition and \( F(u) = |u|^\rho u \), then we have the nonlinear hyperbolic equation, that one can formulate in the form

\[ u_{tt} - \nabla \cdot (f(u) \nabla u) = g(u), \quad (t, x) \in (0, T) \times \Omega, \quad T \in (0, \infty) \]

\[ u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x), \quad u \big|_{(0,T) \times \partial \Omega} = 0, \]

where \( \Omega \subset \mathbb{R}^n, n \geq 1 \) is a bounded domain with sufficiently smooth boundary \( \partial \Omega \), \( T > 0 \) is arbitrary fixed number.

As it is well known \(-\Delta\) is a self-adjoint, positive operator densely defined in a Hilbert space \( H \equiv L^2(\Omega) \) (and on \( H^1_0(\Omega) \equiv H^1_0 \) with the norm \( \|v\|_{H^1_0} \equiv \|\nabla v\|_{L^2} \equiv \|\nabla v\|_{H^0} \), see, e. g. [2], [3], [4]) and \( f, g : \mathbb{R} \rightarrow \mathbb{R} \) are a continuous functions.

For study we use some different approach (see, [2], [3], [5]) unlike known approach employed for equations with linear main parts (e. g. [2]).

References

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