On the Structure of Low Lying $0^+$ Excited States of Pt isotopes.

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Abstract

The description of the of nuclei energy spectra of $0^+$ states has been done involving the degree of their collectivity as a systematics parameter. Within the framework of this approach the parameter of the collectivity is mainly determined by pairs of particles placed on single "effective" level and coupled to monopole bosons. Holstein-Primakoff transformation of these monopole bosons leads to clear physical explanation of the structure of $0^+$ states in terms of "ideal bosons".

The results may be helpful both for experimentalists and theorists in their investigations of low-lying states structure and transition probabilities.

The great amount of experimental data for energy spectra and transition probabilities evokes the necessity of simplified description that can be easily used by experimentalists for explaining the collective properties of states and their systematics. There are many investigations dedicated to the $E0$ and $E2$ transition probabilities and analyzing the $0^+$ spectra in different nuclei [1]. For instance in the rare - earth region the values of the $B(E2; 2^+_g.s. - >0^+_g.s.)$ and $B(E2; 2^+_K=0^+_g.s. - > 4^+_g.s.)$ transitions as a function of neutron number change drastically for different isotopes [2].

In this paper we study the low-energy $0^+$ spectra of $^{194}$Pt and $^{196}$Pt within the framework of simplified pairing vibrational model using Holstein-Primakoff transformation [3].

As the description of the structure of the $0^+$ nuclear states in terms of pair configurations continues to be of great interest both for theorists and experimentalists we define the Hamiltonian in terms of monopole phonon operators:

\begin{align}
R_+^j &= \frac{1}{2} \sum_m (-1)^{j-m} \alpha_{jm}^\dagger \alpha_{j-m}^\dagger \quad
R_-^j &= \frac{1}{2} \sum_m (-1)^{j-m} \alpha_{j-m} \alpha_{jm} , \\
R_0^j &= \frac{1}{4} \sum_m (\alpha_{jm}^\dagger \alpha_{jm} - \alpha_{j-m} \alpha_{j-m}^\dagger) ,
\end{align}

where $\alpha_{jm}^\dagger, \alpha_{jm}$ are the nucleons creation and annihilation operators.

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Let us consider the Hamiltonian for \( N \) particles placed on "effective" single level \( j \) in terms of the operators \( R_{+}^{j}, R_{-}^{j}, R_{0}^{j} \):

\[
H = \alpha R_{+}^{j} R_{-}^{j} + \beta R_{0}^{j} R_{0}^{j} + \frac{\beta \Omega^{j}}{2} R_{0}^{j}
\]

\[
\Omega^{j} = \frac{2j + 1}{2}
\]

Later in our calculations of the \( 0^{+} \) states energies of \(^{194,196}\text{Pt}\) isotopes we take \( \Omega^{j} = 6 \) that corresponds to \( h\frac{11}{2} \) proton system shell model level.

The operators (3) satisfy the commutation relations:

\[
\left[ R_{0}^{j}, R_{\pm}^{j} \right] = \pm R_{\pm}^{j} \quad \left[ R_{+}^{j}, R_{-}^{j} \right] = 2R_{0}^{j}
\]

In order to simplify the notations further we will omit the indices \( j \).

Let us now present this Hamiltonian in terms of "ideal" boson creation and annihilation operators \( b, b^{\dagger} \): \( \left[ b, b^{\dagger} \right] = 1 \); \( \left[ b, b \right] = \left[ b^{\dagger}, b^{\dagger} \right] = 0 \), using the Holstein-Primakoff transformation [4] for the operators \( R_{+}, R_{-}, R_{0} \):

\[
R_{-} = \sqrt{2\Omega - b^{\dagger} b} b \quad R_{+} = b^{\dagger} \sqrt{2\Omega - b^{\dagger} b} \quad R_{0} = b^{\dagger} b - \Omega
\]

The transformations (4) conserve the commutation relations (3) between \( R_{+}, R_{-}, R_{0} \) operators. Thus for the Hamiltonian (2) in terms of the new boson creation and annihilation operators "ideal bosons" \( b^{\dagger}, b \) we have:

\[
H = A b^{\dagger} b - B b^{\dagger} b b^{\dagger} b
\]

where:

\[
A = \alpha(2\Omega + 1) - \beta \Omega \quad B = \alpha - \beta
\]

The energy of any monopole excited state \( |n\rangle = \frac{1}{\sqrt{n}} (b^{\dagger})^{n} |0\rangle \); where \( b |0\rangle = 0 \) can be written as:

\[
E_{n} = \langle n | Ab^{\dagger} b - B b^{\dagger} b b^{\dagger} b | n \rangle - \langle 0 | Ab^{\dagger} b - B b^{\dagger} b b^{\dagger} b | 0 \rangle = An - Bn^{2}
\]

If we analyze the behavior of the experimental \( 0^{+} \)-state energies using the notation \( n \) - as a systematic parameter for the corresponding \( 0^{+} \)-states after minimizing Chi-square values by permutation of \( n \) we find that the distribution of the \( 0^{+} \) states energies as function of \( n \) can be presented by simple formula:

\[
E_{n} = An - Bn^{2}
\]

We see that the Hamiltonian (2) provides the same energy spectrum as spectrum (5). Also one can check that similar behavior possess all the \( 0^{+} \) state energy spectra in nuclei of rare-earth region.
The parameters of the approach which we have used are presented in figure 1 along with the experimental and calculated $0^+$ state distributions. The calculated energies are distributed in bell form because of the anharmonic terms in the Hamiltonian (5) and often the lowest $0^+$ states have much more collective structure ( bigger $n$ ) than the states with higher energies. In the framework of this simple model we can predict that additional $0^+$ states should exist. We indicate these predicted states by "?" in the figure 1. Thus it may be interesting to measure $E0$ transition probabilities in these nuclei and especially in $^{194}Pt$ nucleus from one phonon $0^+$ state with energy 0.6 MeV to the ground state, and in $^{196}Pt$ nucleus from one phonon $0^+$ state with energy 0.57 MeV to the ground state, in $^{188}Os$ from one phonon $0^+$ state - 0.75 MeV to the ground state and in $^{158}Er$ from one phonon $0^+$ state - 1.2 MeV to the ground state. We point out again that this is only a prediction produced by one simple model.

Let us consider the simplest transition operator that can be written within the framework of our model:

$$\hat{E}_0 = x(R_+ + R_-) = x(b^\dagger \sqrt{2\Omega - bb} + \sqrt{2\Omega - b^\dagger bb})$$

(9)

Then in this approach the non-vanishing transition matrix elements are:

$$\langle n | \hat{E}_0 | n + 1 \rangle = \langle n + 1 | \hat{E}_0 | n \rangle =$$

$$x \frac{1}{\sqrt{(n+1)!n!}} \langle 0 | b^n \sqrt{2\Omega - b^\dagger b} (b^\dagger)^n | 0 \rangle +$$

$$x \frac{1}{\sqrt{(n+1)!n!}} \langle 0 | b^n \sqrt{2\Omega - b^\dagger b} b (b^\dagger)^n | 0 \rangle$$

(10)

Using the commutation relations between the operators $[b^n, (b^\dagger)^m]$:

$$[b^n, (b^\dagger)^m] = \begin{cases} 
\sum_{l=0}^{n-1} \binom{n-1}{m-l} (b^\dagger)^{m-l} b^l & n \leq m \\
\sum_{l=0}^{m-1} \binom{m-1}{n-m+l} (b^\dagger)^l b^{n-m+l} & n \geq m
\end{cases}$$

(11)

for matrix elements (10) we have:

$$\langle n + 1 | \hat{E}_0 | n \rangle = x \frac{1}{\sqrt{(n+1)!n!}} \langle n | \sqrt{2\Omega - b^\dagger b} | n \rangle +$$

$$x \frac{n}{\sqrt{(n+1)!n!}} \langle n | \sqrt{2\Omega - b^\dagger b} | n \rangle =$$

$$x \sqrt{2\Omega - n} \sqrt{n + 1} = \langle n | \hat{E}_0 | n + 1 \rangle \quad n \geq 0$$

(12)

It is suitable to consider the ratio between nuclear matrix elements: $\rho^2 \sim \langle n | \hat{E}_0 | n + 1 \rangle^2$ entering in $E0$ transition probabilities:

$$F(n, k) = \frac{\langle n | \hat{E}_0 | n + 1 \rangle^2}{\langle k | \hat{E}_0 | k + 1 \rangle^2} = \frac{(2\Omega - n)(n + 1)}{(2\Omega - k)(k + 1)}$$

(13)
because it does not depend on any additional parameters. Here we proposed that $x$ does not change with changing the transition energy $\Delta E = E_0(n+1) - E_0(n)$ for isotopes under consideration ($\Omega = 6$) and $n = 1, 2, 3, 4, 5$ is presented in figure 2. The curves in figure 2 indicate that $\langle k|\hat{E}_0|k+1\rangle^2$ in the region of $k = 3 - 8$ are about four times larger than the $\langle 0|\hat{E}_0|1\rangle^2$ values.

Experimental data about the rotational bands in deformed nuclei show that the dependence of the energy on angular momentum $L$ is qualitatively similar for the ground band and the bands constructed on any excited $0^+$ state. So in the first approximation one may consider the rotational bands constructed on different excited $0^+$ states without including the band head structure. Nevertheless the influence of $0^+$ states structure on the rotational spectra must be included in order to explain the small quantitative differences in rotational bands with different $0^+$ band heads as well as transition probabilities, for instance the peculiarities in $B(E2; 2^+_{K^π=0_n^+} \rightarrow 0^+_{g.s.})[5]$. This investigation is in successful progress. Furthermore the results of this paper may be helpful for more sophisticated analysis of the collective structure of the low-lying nuclear states. Having in mind the results of this paper one can estimate directly the degree of collectivity (number of bosons) of any $0^+$ excited state.

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Captions to the figures.

**Figure 1.** The comparison of calculated distribution of $0^+$ state energies as a function of number of phonons $n$ with experiment. Experimental data are taken from tables [5].

**Figure 2.** Ratio $F(n, k)$ (13) for different $n$. 
$E_0(n)$ (MeV)

- **194 Pt**
  - Experiment Hamiltonian (5)
  - $A = 0.64859$
  - $B = 0.04554$
  - $\Omega = 6$

- **196 Pt**
  - Experiment Hamiltonian (5)
  - $A = 0.62249$
  - $B = 0.04547$
  - $\Omega = 6$