Combining Counterfactual Regret Minimization with Information Gain to Solve Extensive Games with Imperfect Information

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Abstract

Counterfactual regret Minimization (CFR) is an effective algorithm for solving extensive games with imperfect information (IIEG). However, CFR is only allowed to apply in a known environment such as the transition functions of the chance player and reward functions of the terminal nodes are aware in IIEGs. For uncertain scenarios like the cases under Reinforcement Learning (RL), variational information maximizing exploration (VIME) provides a useful framework for exploring environments using information gain. In this paper, we propose a method named VCFR that combines CFR with information gain to calculate Nash Equilibrium (NE) in the scenario of IIEG under RL. By adding information gain to the reward, the average strategy calculated by CFR can be directly used as an interactive strategy, and the exploration efficiency of the algorithm to uncertain environments has been significantly improved. Experimentally, The results demonstrate that this approach can not only effectively reduce the number of interactions with the environment, but also find an approximate NE.

Introduction

Counterfactual regret minimization (CFR) (Zinkevich et al. 2007; Gibson et al. 2012; Brown and Sandholm 2018) is widely used in the benchmark field of the extensive-form game with imperfect information (IIEG) (Osborne and Rubinstein 1994), since it can converge to equilibrium and gain an average strategy through iterations. But in fact, CFR needs much necessary information for calculation to work. In contrast, Reinforcement Learning (RL) (Sutton and Barto 2018) algorithms provide a series of useful frameworks for making decisions in an uncertain environment. Agents constantly interact with the environment to obtain information and improve the game strategies in the end. The advantage of RL methods is that the agents can improve by themselves via setting reward function, but it is easy to make the agents fall into the local optimal state. Therefore, how to use CFR in IIEG under RL for finding a Nash equilibrium (NE) (Nash et al. 1950; Roughgarden 2010), and design an effective exploration strategy to minimize interactions with the environment are important challenges.

More specifically, we focus on the RL problems in two-player zero-sum extensive games with imperfect information (TEGI). Previous work in an algorithm called PSRL-CFR (Zhou, Li, and Zhu 2020) has also investigated the same problem. PSRL-CFR uses Thompson Sampling to model the environment, and achieving the goal of closing the gap between the sampled and real environments via exploring in the direction of the greatest difference between the two sampled environments. However, the efficiency of this exploration method is not good enough. Because the variance of single sampling is large and cannot represent the real environmental distribution.

We use another approach to improve the efficiency of exploration and speed up convergence. Variational information maximizing exploration (VIME) (Houthooft et al. 2016) provides an effective framework for exploring an unknown environment of RL. VIME proposes a curiosity-driven exploration strategy. In the dynamic model, it takes the information gain obtained from the agent’s internal belief as a driving force. The characteristic of this exploration algorithm is that the information owned by the agent is regarded as part of the state, and the agent hopes to obtain new information by traversing new states.

Nevertheless, applying VIME to TEGI missions under unknown environments requires interaction strategies that are to better interact with the environment for collecting data. The goal of the agent is to maximize its reward in a single-agent RL (SARL) (Neto 2005) problem, but the reward of an agent is affected by the environment and the strategies of other agents. In consequence, solving the TEGI is to find an approximate NE where none of the players can increase reward by changing their strategies, and CFR can complete this work when environmental information is available.

In this work, our contributions are summarized as follows:

- We propose a framework named VCFR, which is composed of VIME and CFR. It allows the CFR algorithm to be used rationally for solving the problem of TEGI under RL.
- VCFR indirectly changes the calculation of CFR by adding information gain to the reward. It not only finds approximate Nash equilibria but also obtains strategies that can be used directly in exploration to reduce the number of interactions with the environment.
- VCFR is a plug-and-play framework for solving IIEG in an unknown environment. More specifically, CFR can...
be replaced by other algorithms used to solve strategies, such as CFR+, DCFR, etc.

- Experimental results show that VCFR outperforms other comparison methods, and the number of interactions with environment is fewer than them.

**Notation and Background**

This section briefly introduces the definition of two-player zero-sum imperfect-information extensive game (TEGI) under Reinforcement Learning which is the setting used in our experiment. In addition, we review related techniques, namely variational information maximizing exploration (VIME) and counterfactual regret minimization (CFR), which give us inspiration for a solution.

**Problem Statement of Extensive Game**

The extensive games with two players is a special case of general extensive games with imperfect information which is usually used to model the sequential decision-making game. Therefore, we first introduce the concept of extensive games(for a full treatment, see [Osborne and Rubinstein 1994].)

There is a finite set $A$ finite set $P$ of players in an imperfect-information extensive game, $P = \{1, 2, \ldots, n\}$. The “Nature” of the game is chance player $C$, which chooses actions with unknown probability distribution under Reinforcement Learning. Here, we define $c$ as $C$'s probability of strategies. A history(or state) $h$ denotes all information of each player including chance player. $H$ is a finite set of histories including the empty sequence $\emptyset$, and $h$ can be thought of the finite set of all nodes in the game tree. $Z \subseteq H$ refers to the set of terminal histories. The set of available actions after a non-terminal history $h$ is referred to $A(h) = \{a : (h, a) \in H\}$. $P(h)$ is the player who takes an action after the history $h$. In particular, if $P(h) = C$ then chance player chooses a action with a probability after $h$. $I_i$ of a history $P(h) \neq C$ is an information partition of player $i$, and player $i$ cannot distinguish $h_1, h_2 \in I_i$. The information sets(infosets) of player $i$ is $I_i \in I_i(h)$ when $i$ is at state $h$.

A strategy $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ is a strategy of the set for all players. $\sigma_i(h_1)$ and $\sigma_i(h_2)$ are equal for player $i$ when $h_1, h_2 \in I_i$. For convenience, we let $\sigma_{-i}$ denote all the policies in $\sigma$ except $\sigma_i$. Let $r^*(h, i)$ denotes the reward function at terminal state, i.e., $r^*(h, i)$ is the distribution of the reward of player $i$. The maximal size of available actions for $h$ is referred to $A = \max_{h \in H} |A(h)|$.

$$\pi^\sigma(h) = \Pi_{a \in A(h)\sigma_p(h)}(h', a)$$

denotes the probability of reaching $h'$ when all players choose actions according to $\sigma$. $d^* = (r^*, c^*)$ can be regarded as the unknown environment, where $r^*$ and $c^*$ follow a prior distribution $D$. Since $c^*$ is uncertain under RL, the probability of reaching $h$ depends on $\sigma$ and $c^*$. Defining by formal formula is $\pi_\sigma(h | d^*) = \Pi_{a \in P[h]}(h')$. Similarly, we use $v_i(h | \sigma, (r^*, c^*))$ to refer to the expected payoff for $i$ according to $\sigma$.

**Nash Equilibrium.** Finding a approximate Nash Equilibrium ([Nash et al. 1950] Roughgarden [2010]) is a significant solution to TEGI. The best response means the strategy $BR(\sigma_{-i})$ that maximizes the reward of player $i$ when $\sigma_{-i}$ is given. That is, $v_i(BR(\sigma_{-i}), \sigma_{-i}) = \max_{\sigma_i} v_i(\sigma_i', \sigma_{-i})$. A Nash Equilibrium $\sigma^*$ is a policy where all players play a best response: $\forall p, v_p(\sigma_p^{\sigma^*}, \sigma_{-p}) = \max_{\sigma_p} v_p(\sigma_p', \sigma_{-p})$.

More specifically, Nash Equilibrium has been proven to exist in TEGI. The exploitability is generally used to measure the approximation error of $\sigma = (\sigma_1, \sigma_2)$:

$$\exp(\sigma_i | d^*) = \sum_{\sigma_{-i}} \max_{\sigma_i'} (\sigma_{-i}, \sigma_{-i} | d^*)$$

The total exploitability is denoted by $\sum_{i \in P} \exp(\sigma_i | d^*)$.

**Variational Information Maximizing Exploration**

Variational information maximizing exploration (VIME) is an exploration strategy algorithm based on the maximization of information gain for uncertain environments of Reinforcement Learning problems. VIME makes actions take advantage of the interaction opportunities with the environment to obtain as much information as possible.

VIME adopts Bayesian neural networks (BNNs) to model the environment. The information gain used by VIME refers to the reduction degree of information complexity(uncertainty) in an environment. It is calculated in the dynamic model of environment, $p(s_{t+1} | s_t, a_t; \theta)$. That is, an agent should maximize the decrease degree in uncertain via taking actions. This process can be abstracted as minimizing the entropy over a sequence of actions $a_t$:

$$\sum_{t} (H(\theta | h_t, a_t) - H(\theta | S_{t+1}, h_t, a_t))$$

where $h_t$ is the history of agent and $\theta$ is a set of the random variables $\theta \in \Theta$ about the agent in the environment. The mutual information $\Delta H$ is related to the next state $s_{t+1}$ and the model parameter $\Theta$:

$$\Delta H(\theta, s_{t+1}, a_t; \theta) = H(\theta | h_t) - H(\theta | s_{t+1}, h_t, a_t)$$

which is equal to the KL divergence $D_{KL}$. Furthermore, an agent is encouraged to act towards node with greater $D_{KL}$. So KL divergence can be considered to be consistent with the information gain. If the entropy of $\theta$ can be decreased when the agent is in the state $t+1$, it indicates that the state $s_{t+1}$ is helpful to promote the dynamic belief. For an agent, $D_{KL}$ can be interpreted as an intrinsic reward which is different from rewards in the environment. So the reward of next state is noted that

$$r'(s_{t+1}) = r(s_t) + \eta D_{KL}[p(\theta | h_t, s_{t+1}) || p(\theta | h_t)]$$

where $\eta \in \mathbb{R}^+$ is a discount factor and contributes to exploration.

**Counterfactual Regret Minimization**

The counterfactual regret minimization (CFR) algorithm ([Zinkevich et al. 2007] Brown et al. 2019), which converges to Nash equilibrium by constantly iterating to reduce regrets, has been proved to be successful in two-player zero-sum games with incomplete information through experiments. The core idea of CFR is to apply the regret minimization
algorithms (Littlestone and Warmuth 1994; Chaudhuri, Freund, and Hsu 2009) to each infoset in order to calculate strategies. In other words, it divides the total regret into a number of regrets on infosets. The purpose of optimizing policy and finding NE is achieved by minimizing the cumulative regret. Let $v^t_i(I)$ be the counterfactual value of player $i \in P(I)$ at infoset $I$:

$$
v^t_i(I) = \sum_{z \in z_i} \pi_{-p}^t(z[I]) \pi^t(z[I] \rightarrow z) u_p(z)$$  \hspace{1cm} (5)

The immediate counterfactual regret $r^t(I, a)$ is the counterfactual value difference between taking action $a$ and whole the infoset $I$ on round $t$:

$$r^t(I, a) = v^{t+1}(I, a) - v^t(I)$$  \hspace{1cm} (6)

For infoset $I$, the counterfactual regret of action $a$ after iteration $T$ is:

$$R^T(I, a) = r^T(I, a \mid d^*) + R^{T-1}(I, a \mid d^*)$$ \hspace{1cm} (7)

$$R^T(I, a \mid d^*) = r^T(I, a \mid d^*)$$  \hspace{1cm} when $T = 1$. Formally, The update of strategy $\sigma^{T+1}$ on round $T + 1$ follows as:

$$\sigma^{T+1}(I, a \mid d^*) = \frac{R^T(I, a \mid d^*)}{\sum_{a' \in A(I)} R^T(I, a' \mid d^*)}$$ \hspace{1cm} (8)

where $R^T(I, a \mid d^*)$ is denoted as non-negative. If $\sum_{a' \in A(I)} R^T(I, a' \mid d^*) \leq 0$, a player will choose a strategy uniformly randomly with probability. And the average $\hat{\sigma}^T(I \mid d^*)$ for each infoset $I$ on iteration $T$ is:

$$\hat{\sigma}^T(I \mid d^*) = \frac{\sum_{i=1}^{T} \left( \pi^i_{-p}(I \mid d^*) \sigma^i_p(I \mid d^*) \right)}{\sum_{i=1}^{T} \pi^i_{-p}(I \mid d^*)}.$$

### Method

In this section, we detailedly describe the VCFR algorithm combining Variational Information Maximization Exploration (VIME) and Counterfactual Regret Minimization (CFR) in two parts, which can be used to solve the TEGI problem under RL. In the first part, an exploration method based on information gain is introduced, and applied to game tree within the field of TEGI. The second part describes that the reward with information gain is used by CFR to solve the approximate Nash equilibrium and interactive strategies.

First of all, we give an overview of the algorithm which is presented in detail in Algorithm 1. The posterior distribution of reward corresponding to each action is stored in the data pool $D$. We take the prior distribution as input, and we use Bayesian Neural Network (BNN) (Graves 2011; Blundell et al. 2015) in VIME to obtain the posterior distribution $P$ of the reward corresponding to each action. CFR can calculate the average strategy with the new reward added with information gain, and will explore the environment to collect the data according to the curiosity-driven strategy. The approximate Nash equilibrium will be found after continuous iterations. The whole architecture of our proposed algorithm is shown in Figure 1.

### Modeling Environment with BNN

The construction of the environment model is divided into three steps. Firstly, the posterior distribution of the unknown environment needs to be obtained by interacting with the real environment. Secondly, the unknown environment is modeled by the achieved data. And finally the dynamics model is set up for the environment. More specifically, we adopt BNN as a modeling approach. The BNN is different from other neural networks, and it is a kind of model that can be used to model the posterior distribution to represent weights in the form of distribution. The regularization effect is provided by introducing uncertainty into the weights of neural network. BNN can generate confidence in the prediction results via propagating the uncertainty of the weights into the prediction process. The output of BNN can describe the likelihood of probability distributions, then a posterior distribution can be calculated by sampling or variational inference. BNN has the ability to quantify uncertain information and strong robustness, and it is very suitable for the task of modeling environment.

In this work, BNN maintains an player’s dynamics model $p(r_i | d_i, z, \theta)$, where $d_i$ denotes the environment obtained by sampling. Even if the environment has been modeled, it is
still difficult to find an approximate Nash equilibrium without an effective way to update and explore it. In order to make the modeled unknown environment closer to the real one, and meanwhile make the exploration strategies more efficient, we use information gain for more targeted exploration. The information gain can be defined in this task as the difference between two distributions before and after the environmental update, which is described by the KL divergence of the two distributions:

$$I(r; \Theta |d_t, z) = \mathbb{E}_r [D_{KL} [p(\theta|d_t, z) || p(\theta|d^*, z)]]$$

(9)

We take the calculated KL divergence $D_{KL}$ as a measure of the player’s desire to explore. In other words, the information gain can be considered numerically equal to the KL divergence. The uncertainty of the environment is treated as an intrinsic reward for a player. We set a threshold $\lambda$ for KL divergence. In order to explore the direction of greater curiosity about the environment, we set the threshold $\lambda$ to 1, and use $\lambda$ to periodically update the value of information gain. The update frequency remains unchanged when $D_{KL} > \lambda$. On the contrary, It indicates that the desire for exploration is low when $K_{KL} < \lambda$, and the update frequency is reduced. The computational efficiency of information gain can be greatly improved without affecting the exploration results. The original reward $r_t$ adds to KL divergence for obtaining a new reward $r_t'$ with information gain as follows:

$$r_t' = r_t + \eta D_{KL}$$

(10)

where $r_t'$ will be used later as a processed reward on the interaction strategy. The hyperparameter $\eta$ is set to 0.01, which can reduce the impact on the results calculated from the CFR and also enhance the availability of exploration simultaneously. In the process of learning, maximizing $r'$ will be able to achieve a balance of exploration and exploitation. In this way, the environment can gradually converge to the real one in theory.

**Information Gain Based CFR**

The purpose of information gain based CFR is to find an approximate Nash equilibrium and obtain the average strategy. This average strategy can not only be utilized by the player but also be directly used as an interactive strategy to explore the environment. The traditional CFR algorithm continuously minimize the regret $R^T(I, a)$ by inputting the information of the game tree, such as the strategy combination of each node and the reward of the terminal node. However, the real environment $d'$ and the sampled one are different, so it is not possible to effectively reduce the regret value. In other words, In this way, the approximate Nash equilibrium of sampled environments will not be eventually found by continuously decreasing the exploitability under real environment. Formally, the relationship between exploitability and regret can be expressed by the following formula:

$$\text{expl}(\bar{\sigma}_t|d^*) = \frac{1}{T} \left( \sum_{i \in \{1, 2\}} R_i^T + \sum_{t \leq T} \left( u_i (\sigma^*_T|d^*) - u_i (\sigma^*_T|d_t) \right) \right)$$

(11)

We have also made some improvements to the CFR for situations where some environmental information cannot be known. We use the $r'$, which adds to information gain, obtained in Equation (10) as one of the environmental information sources for CFR. The information gain is added to the terminal node of the game tree. Due to the recursive and iterative characteristics of CFR, the information gain can affect each node from the bottom to top. Different from traditional CFR, which continuously reduces the exploitability to improve the strategy, the addition of information gain is able to keep the reward $r'$ to the direction of environmental exploration to improve the effect. The large cost of time and space is always a difficulty in the problem of extensive games with imperfect information. Inspired by pruning [Brown and Sandholm 2013], judging its arrival probability first for each node in the game tree. When the node is
at a extremely low arrival probability, it will be regarded as a relatively invalid node and not be traversed in this round. For all remaining nodes, the player $i$ makes use of the current strategy $\sigma_i^t$ to calculate the cumulative regret $R_t$ and the counterfactual value (CFV) $v_i^\ast (h)$ (Li et al. 2020). Through the regret matching (RM) (Hart and Mas-Colell 2000) of regret value for each node, the $\sigma_i^t$ can be calculated. In the end, a game tree with $v_i^\ast (h)$, $R_t$ and $\sigma_i^t$ for all valid nodes can be obtained. The significance of the average cumulative regret value $R_T$ and the average strategy $\sigma_T$ is that there is a non-negligible relationship between them and NE. Despite unknown environmental information is used in our method, our goal is still to minimize $R_T$ and improve the player’s reward so that the average strategy approaches an approximate NE.

In our experimental scenario, the interaction strategy is also a vital step, which affects the environmental certainty. Because whether the sampling environment of the model converges to the real one has a significant impact on the calculation results of the availability. When the environment is randomly initialized, the environmental uncertainty is extremely large in the initial state. Therefore, there is a greater variance in environmental distribution. To converge the unknown environment and reduce the variance, the average strategies with information gain are directly used to interact with the real environment for collecting data in our method.

Experiments

This section focuses on the details of the experiment, then we introduce the representative baselines. Finally, we show the experimental results and analyze them.

Experimental Setup

Poker is a game with imperfect information, and is suitable to be used as a platform for evaluating algorithms of equilibrium-finding. In fact, the game techniques related to imperfect information have also been verified with poker games in recent years. In this work, we use a variant of Leduc Hold’em poker. More specifically, we have made some changes to Leduc Hold’em poker (Southey et al. 2012) which has two players with pre-specified bet and raise amounts to serve as an experimental environment for our method. It should be noted that the structure of game tree in changed Leduc Hold’em poker is the same as the previous structure, but the transition probability of $\epsilon$ chance player and the reward function $r$ of terminal node are uncertain.

Each player’s bid is limited to no more than four or five times the big blind. The numbers of nodes in the generated game tree are 9,652 and 34,438 respectively. Initializing randomly $r \in \{-1,1\}$, and the reward function $r(h)$ is a binary distribution.

We take advantage of Bayesian neural network (BNN) to model the environment. The BNN architecture we have adopted is shown in Figure 2. This network has a depth of 3 layers. 6 and 1 are the sizes of the input layer and the output layer. The parameters of the input and hidden layers are set to Gaussian distribution. The size of the hidden layer is different in Leduc(4) and Leduc(5), which are 32 and 64 respectively. The 6-dimensional vector matrix of a single terminal node is encoded as the input to the BNN. We perform 20,000 iterations using a batch size of 500 in Leduc(4) and 1000 in Leduc(5).

Baselines

We choose four kinds of methods as our baselines. The description of baselines are given below:

- **PSRL-CFR**: Posterior sampling for reinforcement learning with counterfactual regret minimization (PSRL-CFR) (Zhou, Li, and Zhu 2020) proposes a framework that combines PSRL and CFR. The environment is transformed into known by Thompson sampling, and the CFR algorithm is used to calculate the approximate Nash equilibria. In order to update data in the environment, the special strategies $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are regarded as interaction strategies.

- **FSP**: Fictitious self-play (Conitzer and Sandholm 2007) is a popular algorithm to find Nash equilibrium (NE) in uncertain setting. FSP needs to make use of Fitted-Q iteration (see PSRL-CFR) with initial hyperparameters to learn the best response to the average strategy of each player’s opponent when the environment is unknown.

- **MCCFR-OS**: Monte Carlo counterfactual regret minimization based on outcome sampling (Lanctot et al. 2009) is a popular variant of CFR that avoids traversing the entire game-tree by sampling only a single playing of the game on each iteration. The $\epsilon$-greedy is the exploration strategy where $\epsilon = 0.1$.

- **Variants of VCFR**: In order to better measure the validity of our experimental methods, we use three additional variants of VCFR: 1) Naive: The reward without information gain is inputted into CFR procedure, and the average strategy by calculating is used as exploration policy to interact with the unknown environment; 2) Naive-DCFR: To prove the generalization of our algorithm framework, we use other variant algorithm Discounted

![Figure 2: The architecture of Bayesian neural network used by this work, where $W_1, W_2 \sim N(\Sigma, \delta)$. The network takes the distribution of original rewards $r$ as input and outputs rewards $r^t$ with information gain for each possible actions.](image-url)
VCFR method performs better than other algorithms in two metric to evaluate the effectiveness of our methods. We iterate the different algorithms on Leduc(4) and Leduc(5) as shown in (c) and (d).

Figure 3: Comparison between VCFR and other algorithms in different environments. (a) and (b) respectively represent the experiment results in Leduc(4) and Leduc(5). Among them, VCFR and naive can be seen as a group of ablation experiments.

Figure 4: The experiment results which can demonstrate the better generalization of our proposed method. (a) and (b) show the ablation experiments’ results of VDCFR in Leduc(4) and Leduc(5). The ablation experiments’ results of VFSP in Leduc(4) and Leduc(5) as shown in (c) and (d).

CFR, DCFR (Brown and Sandholm 2019) is a variant algorithm of CFR, and it has three parameters \( \alpha, \beta, \) and \( \gamma \) as discount factors to improve speed of solution. In every round \( t \), the effects of three parameters are multiplying cumulative regrets \( R \) by \( t^\alpha/(t^\alpha + 1) \) when \( R > 0 \), by \( t^\beta/(t^\beta + 1) \) when \( R < 0 \), and contributions to average policy \( \hat{\sigma} \) by \( \{t/(t+1)\}^\gamma \). The basic process is the same as Navie-CFR, only replacing CFR with DCFR. The previous experiment has an excellent performance when setting \( \alpha = \frac{3}{2}, \beta = 0, \) and \( \gamma = 2; 3) \) Random: The players take random actions in each round.

**Experimental Results**

We iterate the different algorithms on Leduc(4) and Leduc(5) for 20,000 rounds respectively, and each round has to interact with the environment. Exploitability is a popular metric to evaluate the effectiveness of our methods.

Figure 3(a) presents the VCFR and some baselines’ comparison results in Leduc(4). We can see that our proposed VCFR method performs better than other algorithms in two different experimental settings. Specifically, in Leduc(4), VCFR performs the best among all methods, and its exploitability drops to -0.135 at 20,000 rounds. Followed by the PSRLCFR method with exploitability of -0.864. Random and PSRLCFR using a special interaction strategy are also reducing the exploitability, but the speed and the lower bound of convergence are worse than our method. From the comparison result between the Naive CFR and VCFR, it is proved that the average strategy with information gain can not only explore the environment efficiently but also accelerate the convergence. In Leduc(5) with higher space complexity, the results in Figure 3(b) demonstrate that our method still maintains excellent performance. After 20,000 iterations, the exploitability of VCFR, PSRLCFR, Random, Naive and MCCFR-OS are -1.230, -0.980, -0.632, -0.499 and -0.163, respectively. However, the exploitability of VCFR decreases slower than in Leduc(4). It may be because the convergence speed of BNN modeling environment will reduce with the increase of the complexity of the environment. MCCFR-OS and Random perform poorly in both game environments. The reason for this result is probably that their exploration strategies are inefficient.
In addition, to verify the better generalization ability of the proposed method through experiments, we add two additional algorithms: VDCF and VFSP. The algorithm architecture of VDCF and VFSP is similar to VCFR, nonetheless, CFR is replaced by DCFR or FSP. The results of two additional ablation experiments are shown in Figure 3 which can demonstrate the better generalization of our approaches. The comparison of the two ablation experiments, VFSP and FSP, as well as VDCF and Navie-DCFR, shows that the information gain has a significant influence on the average strategy that can be used to explore the environment. Figure 4(a)-4(d) show that the algorithm with information gain can speed up finding approximate Nash equilibrium, and the interaction strategy promotes convergence in an unknown environment.

Related Work

Measure of Uncertainty

There are some previous methods to solve measure of uncertain in unknown environment. The random prior function (Osband, Aslanides, and Cassuret 2018) is originally used to enhance the performance of Bootstrapped DQN. While training bootstrapped function to fit Q with a posterior probability, random network gives each bootstrapped function a fixed prior. Since the prior probabilities are initialized randomly, it is possible to improve the diversity of bootstrapped functions and better fit the distribution of the posterior probabilities. The measurement method of uncertainty obtained by fitting random prior has been proved to be successful in theory and application (Burda et al. 2019; Ciosek et al. 2020). The Deep ensembles (Lakshminarayanan, Pritzel, and Blundell 2016) is a commonly used measurement method of uncertainty. Each model in the ensemble is trained based on bootstrap data, so that the predicted variance between models can be utilized as a measure of epistemic uncertainty. The disadvantage of deep ensembles is that it tends to give overconfident estimates of uncertainty. For obtaining uncertainty estimates, dropout (Gal and Ghahramani 2016) is firstly proposed to model uncertainty in Deep Learning as a practical tool, and it can also be extended to quasi-KL (Gal, Hron, and Kendall 2017). However, in this work, we focus on measuring the uncertainty of the environment with Bayesian Neural network (BNN) (Brosse, Durmus, and Moulines 2018; Rezende, Mohamed, and Wierstra 2014). BNN is a traditional approach to measure uncertainty, which combines probabilistic modeling with neural networks and is able to output the degree of confidence of prediction results.

Exploration under Reinforcement Learning

The exploration methods of Reinforcement Learning (RL) for an unknown environment can be mainly grouped into the following three categories.

The first category related this work is optimistic exploration which is widely used in RL. Upper-Confidence-Bound exploration (Carpentier et al. 2011) used in AlphaGo is similar to greedy selection. They both tend to choose the latest or best actions. The optimistic initial value (Shojaei and Mashhadi 2017) realizes exploration by increasing the initial value of the function, which essentially explores the state with a lower frequency of occurrence. It is worth noting that selection of initial values requires the prior knowledge, and the exploration will be unstable during the initial stage. Agents tend to select actions with higher entropy values in the gradient bandit algorithm (Silver et al. 2014), where the entropy of each action is adjusted by the rewards.

The second category is the posterior sampling (Osband and Van Roy 2017; Chapelle and Li 2011), which incorporates ideas from Bayesian learning and focuses on using posterior probabilities for more targeted exploration. The algorithm based on posterior sampling will modify its probability distribution after each sampling. Through a large number of samples, the variance of each action will be reduced.

The third category is related to our work is exploration based on information gain (Russo and Van Roy 2014). Information gain is generally comprehended as the intrinsic reward of agents, which can measure the contribution of a new state to information. To reach a state where more rewards can be obtained, an agent selects the actions of maximizing empowerment that is calculated by mutual information (Mohamed and Rezende 2015). If a set of states share the same optimal action, then the action can be interpreted as a representation of the states. There is another approach (Still and Precup 2012) also uses mutual information as an exploration reward. If a set of states share the same optimal action, then the action can be considered as a representation of states. The goal is to find the action with the most state information among the strategies which has uniform rewards. That is, it needs to minimize the mutual information of actions and the states.

Conclusion and Discussion

In this paper, we propose a framework named VCFR which combines CFR with information gain. It is able to obtain efficient exploration strategies for solving the problem of finding approximate Nash Equilibrium in the scenario of two-player zero-sum extensive games under unknown environments. Our proposed method is flexible in that two modules are independent of each other, and this means that the CFR can be replaced by any algorithms used to find approximate Nash equilibrium. The results show that our approach outperforms other baselines.

In the future, this approach can be optimized from different perspectives. Our method computes the KL divergence at each round. Although we have tried to set the threshold to shorten the experiment time, the effect is still limited. Therefore, how to design an effective approach to shorten the calculation time of KL divergence is a direction that can improve the results.

Another possible perspective is that reducing the reliance of our method on the structure of two-player zero-sum extended games. The future work needs to be extended to other types of games or extensive form games with three and more players.
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