Frame-dragging effects on magnetic fields near a rotating black hole

V. Karas, O. Kopáček and D. Kunneriath
Astronomical Institute, Academy of Sciences, Boční II 1401, CZ-14131 Prague, Czech Republic
E-mail: vladimir.karas@ujf.cuni.cz

Abstract. We discuss the role of general relativity frame dragging acting on magnetic field lines near a rotating (Kerr) black hole. Near ergosphere the magnetic structure becomes strongly influenced and magnetic null points can develop. We consider aligned magnetic fields as well as fields inclined with respect to the rotation axis, and the two cases are shown to behave in profoundly different ways. Further, we construct surfaces of equal values of local electric and magnetic intensities, which have not yet been discussed in the full generality of a boosted rotating black hole.

1. Introduction
Various processes can lead to the magnetic topology exhibiting null points, in particular, these can emerge by complex motions of the plasma. In this respect the black hole rotation brings a new situation, as magnetic field lines become twisted in a highly curved spacetime of a rotating black hole, approximately where the ergosphere forms. Here, we examine the resulting structure of the magnetic field, namely, the emergence of critical points in a local frame of a physical observer, resembling the occurrence of X-points.

Previously (Karas & Kopáček, 2009) we considered a special case of a uniform magnetic field in perpendicular orientation with respect to the black hole spin axis, and we demonstrated that magnetic null points can indeed form near a rotating (Kerr) black hole. Here, the embedded magnetic field is allowed a general orientation, i.e., it can be inclined in an arbitrary angle with respect to the rotation axis. The axial symmetry is broken between the gravitational and electromagnetic fields, and this has to result in a truly three-dimensional structure of magnetic field lines along with gravito-magnetically induced electric field. The adopted approach allows us to identify the gravitational effects operating in a magnetically dominated system, where a super-equipartition magnetic field governs the motion of plasma. We further expand the discussion in Karas et al (2012) by constructing surfaces of equal values of local electric and magnetic intensities, even for the case of a general (oblique) orientation of the background magnetic field.

2. Aligned and oblique magnetic fields near black hole horizon
Astrophysical black holes do not support their own intrinsic magnetic field; this has to be generated by external currents and brought down to horizon by accretion. A black hole can also enter a pre-existing magnetic flux tube, and then one asks if the process of magnetic reconnection is influenced by strong gravitational field near horizon. And does the black hole rotation play a significant role in the scenario of this kind?
As we wish to discuss magnetic fields inclined with respect to the spin axis, and we also want to include the fast translatory motion, the following picture appears to be appropriate: Kerr black hole traversing a “magnetic filament”, described as an extended (largely one-dimensional) flux tube. Such an idea can be motivated observationally, by highly ordered and elongated arcs (of about 100µG) that are seen in Galactic Center, within a few parsecs from Sagittarius A* compact radio source (Sgr A*; Yusef-Zadeh et al, 1984). They are thought to represent large-scale magnetic flux tubes that are illuminated by synchrotron emission from relativistic electrons (LaRosa et al, 2004; Morris, 2006).

Given a limited resolution that can be achieved with current imaging techniques, the magnetic filaments cannot be traced down to the characteristic size of the black hole. Therefore, the actual mapping of the magnetic structures near black holes is not directly possible. Nonetheless, their role in accelerating the particles is suggestive, especially because they could help us to understand the origin of particle acceleration and the resulting signatures in the electromagnetic signal.

Naturally, in astrophysically realistic solutions the role of non-ideal plasma will need be included. Currently, neither of the frequently discussed limits (i.e., vacuum vs. force-free approximations) are able to account for both the plasma currents as well as the accelerating electric fields. This task will require resistive MHD, which is, however, beyond the scope of this brief paper.

2.1. The effect of black hole rotation and translatory motion

Starting from King et al (1975) and Bičák & Dvořák (1976), the organised electromagnetic test fields surrounding black holes have been discussed and their astrophysical consequences considered in various papers. The case of oblique geometry, however, has been explored only partially (Bičák and Janiš, 1980). This is caused especially by the fact that the off-equatorial fields are lacking any symmetry, and so are more difficult to visualise. Also, qualitatively new effects on the field structure were not expected.

Nevertheless, in Karas & Kopáček (2009) we were able to demonstrate that the asymptotically perpendicular field lines develop a magnetic neutral point in the equatorial plane. This is interesting because such structures of the magnetic field are relevant for processes of electromagnetic acceleration. The magnetic null point emerges in a local physical frame, and could trigger reconnection. However, the asymptotically perpendicular field is a rather exceptional case. Therefore, here we investigate near-horizon magnetic structures in more detail, also for a general orientation of the magnetic field (see also Karas et al 2012).

The main objective of this discussion is to track the location of magnetic null points and to explore a complex three-dimensional configuration inside the ergosphere. We consider the form of magnetic lines together with the induced electric lines for different values of the model parameters: the inclination angle of the asymptotic magnetic field \( \theta_0 \) (= \( \arctan(B_\parallel/B_\perp) \)), the black hole spin \( a \) (\( a^2 \leq M \)), and the boost velocity \( \beta \) (\( \beta^2 \equiv v_x^2 + v_y^2 + v_z^2 < 1 \)). We observe the layers of alternating magnetic orientation to occur also in the general case, i.e., when the black hole rotates and moves with respect to an oblique magnetic field. However, the three-dimensional structure of the field lines is very complicated as they become highly entangled around the null point.

We specify the gravitational field by Kerr metric. Our starting form of the electromagnetic field is a stationary solution of Maxwell’s test-field electro-vacuum equations in the given curved

\[ r_g = GM/c^2 \approx 1.48 \times 10^{13} M_\odot \text{ cm} \]

where the central black hole mass is expressed in terms of \( M_\odot \equiv M/(10^8 M_\odot) \). The velocity of the Keplerian orbital motion of a particle is then \( v_K \approx 2.1 \times 10^{10} (r/r_g)^{-1/2} \text{ cm s}^{-1} \). The corresponding orbital period is \( T_K \approx 3.1 \times 10^8 (r/r_g)^{3/2} M_\odot \text{ s} \). Hereafter, we use a dimensionless form of geometrized units, where all quantities are scaled with the black hole mass; \( M \) does not appear in the equations explicitly.

\[ 1 \] Gravitational radius \( r_g = GM/c^2 \approx 1.48 \times 10^{13} M_\odot \text{ cm} \), where the central black hole mass is expressed in terms of \( M_\odot = M/(10^8 M_\odot) \). The velocity of the Keplerian orbital motion of a particle is then \( v_K \approx 2.1 \times 10^{10} (r/r_g)^{-1/2} \text{ cm s}^{-1} \). The corresponding orbital period is \( T_K \approx 3.1 \times 10^8 (r/r_g)^{3/2} M_\odot \text{ s} \). Hereafter, we use a dimensionless form of geometrized units, where all quantities are scaled with the black hole mass; \( M \) does not appear in the equations explicitly.
The electromagnetic four-potential $\mathbf{A}$ can be then written as superposition of two contributions: $\mathbf{A} = B_\parallel \mathbf{a}_\parallel + B_\perp \mathbf{a}_\perp$, where $B_\parallel$ and $B_\perp$ define the magnitudes of the two parts.

The aligned field has two non-vanishing components of the normalized electromagnetic four-potential,

$$\begin{align}
a_{t\parallel} &= B_\parallel a \left[ r \Sigma^{-1} \left( 1 + \mu^2 \right) - 1 \right], \\
a_{\phi\parallel} &= B_\parallel \left[ \frac{1}{2} (r^2 + a^2) - a^2 r \Sigma^{-1} (1 + \mu^2) \right] \sigma^2,
\end{align}$$

where we use Boyer-Lindquist $t$, $r$, $\theta$, and $\phi$ dimension-less spheroidal coordinates ($\mu = \cos \theta$, $\sigma = \sin \theta$). Eqs. (1)–(2) represent an asymptotically homogeneous magnetic field (Wald, 1974).

On the other hand, the perpendicular to axis component of the field is given by (Bičák and Janiš, 1980)

$$\begin{align}
a_{t\perp} &= B_\perp a \Sigma^{-1} \Psi \sigma \mu, \\
a_{r\perp} &= -B_\perp (r - 1) \sigma \mu \sin \psi, \\
a_{\theta\perp} &= -B_\perp \left[ \left( r \sigma^2 + \mu^2 \right) a \cos \psi + \left( r^2 \mu^2 + (a^2 - r)(\mu^2 - \sigma^2) \right) \sin \psi \right], \\
a_{\phi\perp} &= -B_\perp \left[ \Delta \cos \psi + (r^2 + a^2) \Sigma^{-1} \Psi \right] \sigma \mu,
\end{align}$$

where $\Delta$ is the Jacobian of the coordinates.

These are astrophysically acceptable approximations which reflect the fact that black holes can only acquire a negligibly small electric charge, while cosmic electromagnetic fields are not strong enough to influence the spacetime metric significantly.
Figure 2. The case of extreme spin, $a = M$, taking into account non-zero velocity of the translatory boost along $x$-direction: $v_x = 0.1$ (left), $v_x = 0.3$ (right).

with $\Sigma(r, \mu)$ and $\Delta(r)$ being the Kerr metric functions, $\psi \equiv \phi + a\delta^{-1}\ln[(r - r_+)/ (r - r_-)]$, $\Psi = r \cos \psi - a \sin \psi$, $\delta = r_+ - r_-$, and $r_{\pm} = 1 \pm \sqrt{1 - a^2}$. Knowing the complete set of four-potential components the magnetic field structure is fully determined: $F_{\mu\nu} \equiv A_{[\mu,\nu]}$.

Equations (3)–(6) describe the field lacking the axial symmetry. Such a situation cannot be strictly stationary, however, the alignment time-scale is very long, and so we can neglect the associated energy losses here. The field line structure depends also on the motion of observers determining the field components. In order to construct and discuss our plots, we employ a free-falling physical frame, evaluate the electromagnetic tensor with respect to the local frame, and use these components to draw the field lines.

In absence of the perpendicular component ($B_\perp = 0$), the field is relatively uncomplicated (see figure 1). Although the frame-dragging acts also on the aligned field lines, their shape can be integrated to find the surfaces of constant magnetic flux in a closed (analytical) form. Previously, the aligned fields were explored especially in context of magnetic field expulsion from a maximally rotating black hole. In our notation, the example in Fig. 1 refers to zero velocity of the translatory boost, i.e. $\beta = 0$.

Once a nonzero boost velocity is included, the structure of the aligned field becomes more complicated (figure 2). This is mainly due to an interplay between the boost of the black hole and its rotation acting jointly on the (originally) aligned field. As $\beta$ increases, the magnetic lines are progressively puffed out of horizon and wound around it (see Kopáček 2011, for more examples and details).
Figure 3. The neighborhood of a magnetic null point of perpendicular magnetic field in the equatorial plane \((x, y) \equiv (r \cos \phi, r \sin \phi)\), i.e., viewed along the rotation axis of an extreme \((a = M)\) black hole. The black circular section in the bottom left corner denotes the horizon, \(r = r_+(a)\). The colour scale indicates the magnetic intensity (in arbitrary units).

2.2. Neutral points of the magnetic field

We explore the case of magnetic field with a non-vanishing component inclined with respect to rotation axis \((B_\perp \neq 0)\). In fact, Karas & Kopáček (2009) explored a strictly perpendicular case. Confining the magnetic lines in the equatorial plane \(\theta = \pi/2\), the nested structure of magnetic layers emerges. These are essential for the magnetic reconnection.

Near-horizon structure of magnetic lines is visualized in figure 3 by using the Line-Integral-Convolution (LIC) method in Matlab. This technique allows us to identify clearly the location of neutral points. It turns out to be particularly useful here with the general orientation of the asymptotic field direction, as the global solution for the field lines is too cumbersome. Further, by introducing \(\xi(r; a) \equiv 1 - r_+(a)/r\) as a new radial coordinate, the horizon surface \(r = r_+\) collapses into the center and the layered structure of magnetic sheets is seen in more detail.

The fully three-dimensional structure of magnetic lines develops outside the equatorial plane. In figures 4–5 we observe a superposition of two essential effects. Firstly, the X-type structure
Figure 4. Left: Magnetic field asymptotically perpendicular to the rotation axis \((z)\) of an \(a = M\) black hole, centered on the origin. Right: Similar example as on the left side, but with the field in a general direction with respect to rotation axis.

Figure 5. Three-dimensional detail around the equatorial plane. Magnetic lines are plotted in blue color. The equatorial lines reside within the plane \(z = 0\), revealing a null point in the origin. The lines adopt some non-zero vertical component outside the equatorial plane. Electric lines are shown in red; they pass through the magnetic null point, crossing the equatorial plane in the vertical direction.
Figure 6. Three-dimensional view of the surfaces of equal values of local electric and magnetic intensities, for the case of an oblique background magnetic field $B_x = B_z$ and the extremal spin $a = M$ of the rotating Kerr black hole. Boost velocity of the linear motion is (from upper left to bottom right panels): $v = 0, 0.6, 0.8, \text{and } 0.9$. Different levels of shading of the surface correspond to the distance from origin (in Boyer-Lindquist coordinates), where the black hole is located. Increasing velocity of the translatory motion causes a growing deformation of the surface.

of the magnetic null point persists also outside the equatorial plane. Secondly, however, the magnetic lines acquire a growing vertical component $B_z$, whereas the electric field passes through the magnetic null point and crosses the equatorial plane vertically. Such a structure suggests that particles can be accelerated by the non-vanishing electric field, and they can stream away from the neutral point.

Finally, another useful way of visualizing the electromagnetic structure is by studying the electromagnetic invariants. These can help us not only to track interesting points of the electromagnetic field but also to identify locations of where charges can be electromagnetically supported or produced via electron-positron pair creation (e.g., Ruffini & Wilson, 1975; Karas & Vokrouhlický, 1991). Recently, Lyutikov (2011) explored the shape of surfaces where the magnitude of the local electric field equates the magnetic field component near a magnetized Schwarzschild black hole. Figure 6 shows these $E = B$ surfaces in our system. Obviously the presence of the black hole as well as its motion and rotation influence the form of these surfaces.
3. Discussion and Conclusions
Rotation is a highly interesting attribute of cosmic black holes. In principle, black holes can be spun up close to extreme $a = M$. In stellar-mass black holes the spin is thought to be chiefly natal (McClintock et al, 2012), whereas supermassive black holes in galactic cores can adjust their angular momentum by accretion, and the outcome of evolution depends largely on the dominant mode of accretion, during their entire life-time. In both cases, the spin is an important characteristic, potentially allowing the efficient acceleration of matter. We showed that it can be also relevant directly for the onset of magnetic reconnection.

We discussed the structure of electromagnetic test-fields and the layered pattern of current sheets that can be induced by the gravito-magnetic action. Neutral points of the magnetic field suggest that magnetic reconnection can occur. The proposed scenario can be astrophysically relevant in circumstances when the black hole rotates and moves across a magnetic flux tube, origin of which is in currents flowing in the cosmic medium far from the black hole. We considered the limit of a magnetically dominated system in which the organised magnetic field dictates the motion to plasma; the opposite limit of a black hole moving through a force-free plasma has been investigated by other authors (recently, Palenzuela et al, 2010; Tchekhovskoy, 2012).

In this paper we considered an idealised situation, starting from an electro-vacuum solution, assuming fast motion and rotation of the black hole, and embedding it in an asymptotically uniform magnetic field. Future simulations should clarify, whether the astrophysically realistic effects of the moving black hole on the surrounding electromagnetic structure in its immediate neighborhood will be similar to those envisaged here. Despite the field-line structure in this paper being induced purely by the action of frame-dragging, the exact choice of the projection tetrad is not very essential for the existence of magnetic layers. The choice of the physical frame does affect the presence and the exact location of the magnetic neutral point, and the associated electric field which accelerates charged particles away from the neutral point. Although the exact location of the neutral point varies with the model parameters, it always occurs very close to the black hole, where the frame-dragging is efficient. Therefore, the point of particle acceleration has to be close to horizon as well.

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