Spin resonance in Luttinger liquid with spin-orbit interaction

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Spin-orbit interaction in quantum wires leads to a spin resonance at low temperatures, even in the absence of an external dc magnetic field. We study the effect of electron-electron interaction on the resonance. This interaction is strong in quantum wires. We show that the electron-electron interaction changes the shape of the resonance curve and produces an additional cusp at the plasmon frequency. However, except for very strong electron-electron interaction these changes are weak since this interaction by itself does not break the spin-rotation symmetry that is violated weakly by the spin-orbit interaction and external magnetic field.

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A recent theoretical work [1] has predicted that in one-dimensional (1D) quantum wires with the spin-orbit interaction (SOI) [2] it is possible to observe a relatively sharp electronic spin resonance (ESR) in terahertz range. An external magnetic field oriented along the SOI field separates the Fermi points near the Fermi points from the linear behavior, the charge and spin degrees of freedom do not interact (this is the so-called spin-charge separation). The SOI separates the Fermi points and electron-electron interaction leads to the formation of spin-density wave state when magnetic field is perpendicular to the effective SOI magnetic field [3]. In this Letter we assume that the magnetic field has nonzero component along the SOI field. Such a field terminates the spin-density wave instability [5] and simultaneously separates the spin resonances for left and right movers [1]. The Coulomb interaction is expected to change the shape of the spin resonance line from simple Lorentzian to a power-like one which is characteristic for the Luttinger liquid [3, 4, 6]. The SOI also violates spin-charge separation and thus enables the excitation of the charge waves at spin reversal. It can be seen as a weak resonance at a plasmon frequency instead of the spin-wave frequency. As we show below, both of these effects really take place, though both are weak for not too strong electron-electron interaction.

We consider a nanowire with a cross-section so small that electrons fill partially only the lowest band of the transverse motion (one channel). In this case the Tomonaga-Luttinger model is applied. The standard Luttinger liquid theory starts from the fermionic Hamiltonian with the linearized dispersion [4, 6] to which we add the Rashba SOI, $H_R$:

$$H_0 = -i v_F \sum_\sigma \int dx (\psi_{R,\sigma}^\dagger \partial_x \psi_{R,\sigma} - \psi_{L,\sigma}^\dagger \partial_x \psi_{L,\sigma}) + H_{int} + H_R.$$  

(1)

Here the $x$-axis is taken along the quantum wire; $\partial_x = \partial/\partial x$; $v_F$ is the Fermi velocity; $R, L$ labels the right and left moving fermions; and $\sigma = \uparrow, \downarrow$ are the spin projections. The interaction part of the Hamiltonian, $H_{int}$, contains terms $\rho_{RL}(q)\rho_{RL}(q)$ quadratic in charge densities $\rho_{RL}(q)$ of left and right movers and the terms quadratic in spin densities such that the total spin is conserved, e.g. $\psi_{R,\uparrow}^\dagger(x)\psi_{L,\uparrow}(x)\psi_{L,\downarrow}(x')\psi_{R,\downarrow}(x')$.

With $H_R = 0$ the Hamiltonian (1) has an obvious $SU(2)$ symmetry of rotations in the spin space. The term $H_R = \alpha \int \psi_R^\dagger \sigma_z \psi dx$ in the Hamiltonian $H_R$ represents the Rashba SOI [2] that splits Fermi momenta of up and down spins so that four Fermi points $p_{\rho,\sigma} = \rho v_F - \sigma \alpha m$ appear, but it leaves Fermi velocities unchanged. The Rashba SOI constant $\alpha$ has dimensionalities of velocity and we assume $\alpha \ll v_F$; $\sigma_z$ is the Pauli matrix; $\rho v_F$ is the Fermi momentum at $\alpha = 0$; and $\rho, \sigma = \pm 1$ correspond to right (left) movers and up...
operator, and \( A_z \) denotes the \( x \)-component of the vector potential of the ac field. We use the Coulomb gauge, \( \nabla \cdot A = 0 \), where the scalar potential is zero, and therefore the electric field is \( E = -(1/c)\partial A/\partial t \). The part of the electric current responsible for the spin-flip processes is

\[
j_\sigma(x) = e\alpha\psi^\dagger(x)\sigma_z\psi(x).
\]

The absorption power of electromagnetic field is determined by the real part of the conductivity \( \sigma_\omega \) at the frequency \( \omega \) of the field multiplied by the square of the field’s amplitude \( |E_x(\omega)|^2 \). We employ the Kubo formula for the conductivity:

\[
\sigma_\omega = -\frac{1}{\hbar\omega} \int_0^t \int_0^t dt' \theta(t-t') e^{i\omega(t-t')} \langle [j_\sigma(x,t), j_\sigma(x',t')] \rangle,
\]

where \( l \) is the length of the wire. According to Eq. (3), \( j_\sigma(x) \) is proportional to the density of \( z \)-component of the spin. Therefore, the spin-flip conductivity \( \sigma_\omega \) can be represented as

\[
\sigma_\omega = -\frac{4(\alpha e)^2}{\hbar\omega} \int_{-\infty}^t \langle [S_z(t), S_z(t')] \rangle e^{i\omega(t-t')} dt',
\]

where \( S_z(t) \) is the operator of the total spin projection at the moment of time \( t \). In the absence of magnetic field \( B_\perp \), the \( z \)-component of the total spin is conserved. Therefore, \( \langle [S_z(t), S_z(t')] \rangle = 0 \) and the conductivity associated with the spin flip is zero. The violation of this conservation law at small \( B_\perp \) in the first-order approximation of the time-dependent perturbation theory leads to

\[
\delta S_z(t) = -\frac{i}{\hbar} \int_{-\infty}^t \left[ V_f(t'), S_z(t') \right] dt',
\]

where \( V_f(t) = U_0^{-1}(t)(-g\mu_B B_\perp S_z)U_0(t) \) with \( U_0(t) = \exp(-iH_0t/\hbar) \) being the evolution operator in the absence of magnetic field, and \( S_z \) is the projection of the total spin on the \( z \)-axis. It is convenient to write the Rashba Hamiltonian as a sum over electrons: \( H_\rho = \sum_i \alpha p_i \sigma_z \). The kinetic and interaction energies commute with \( S_z \), and therefore the perturbation operator \( V_f(t) \) becomes

\[
V_f(t) = -\frac{g\mu_B B_\perp}{2} \sum_i \left( \alpha \sigma_z \right) \right),
\]

where \( \omega_i = 2\alpha p_i/\hbar \). Substituting this expression into Eq. (6), we obtain

\[
\delta S_z = \frac{g\mu_B B_\perp}{2\alpha} \sum_i \frac{1}{p_i} \left( \sigma_{+,i} e^{-i\omega_i t} + \sigma_{-,i} e^{i\omega_i t} \right),
\]

where \( \sigma_{\pm,i} = \sigma_{z,i} \pm i\sigma_y \). Condition \( \alpha \ll v_F \) makes it possible to replace the factor \( 1/p_i \) in Eq. (7) by \( \pm 1/p_F \). Then the expression for \( \delta S_z \) becomes proportional to the sum of the operators \( \sigma_{\pm,i} \) for \( i = +1 \) and \( -1 \). In terms of secondary quantized operators it reads (we keep here only right movers):

\[
\delta S_z(t) = \frac{g\mu_B B_\perp}{2\alpha p_F} \int \psi_{R_\uparrow}^\dagger(x,t)\psi_{R_\downarrow}(x,t) dx + \text{h.c.}
\]
The unitary transformation \( U = \exp(-i\sigma_\text{SOI}/\hbar) \) that puts the split Fermi points together, modifies this equation by multiplying the integrand by factor \( \exp(-2i\alpha m x'/\hbar) \). As a result, we find for the conductivity \( \sigma_\text{SOI} \) associated with the spin flip \([14]\),

\[
\sigma_\omega = -\frac{(eg\mu_B B_1)^2}{\hbar \omega |F|} \int_0^l dx \int_0^x \int_{-\infty}^\infty dt' \theta(t-t')e^{i\omega(t-t')}
\times \langle \{\psi_{R,\uparrow}(x,t)\psi_{R,\downarrow}(x,t),\psi_{R,\uparrow}(x',t')\psi_{R,\downarrow}(x',t')\}\rangle
\times e^{-2i\alpha m (x-x')/\hbar}.
\]

An attempt to analyze the spin-flip process in Luttinger liquid has been made in Ref. \[13\]. The authors assumed that Rashba SOI and longitudinal magnetic field only produce the violation of \( SU(2) \) invariance and affect the velocity difference \( \delta v \), but do not separate the Fermi points for spin up and down electrons. The physical origin of such a model where the Fermi points are not separated (as it should be for the realistic Rashba SOI) was not specified in Ref. \[15\].

Since the electrons propagating in one direction with the same velocity strongly interact, the bosonic fields \( \phi_x, \theta_x \) and \( \phi_{\uparrow}, \theta_{\uparrow} \) related to the charge and spin density waves, respectively, give a physically more adequate description of phenomena. The transformation from fermions to bosons (bosonization) reads:

\[
\psi_{\rho,\sigma} = U_{\rho,\sigma} e^{ipk_F x} e^{-i[\rho\phi_x(x) - \theta_x(x) + \rho\phi_{\uparrow}(x) - \theta_{\uparrow}(x)]}/\sqrt{\pi},
\]

where \( U_{\rho,\sigma} \) are the Klein factors which ensure the proper anticommutation relations between the fermion, and \( a_0 \) is the ultraviolet cutoff length. The secondary quantized fermionic wavefunctions \( \psi_{\rho,\sigma} \) can be represented by the linear combinations of right-moving and left-moving fermions \( \psi_{\rho,\sigma} \) with the momenta being close to \( \pm k_F \), i.e., \( \psi_{\rho,\sigma} = \psi_{R,\rho} + i\psi_{L,\rho} \). The advantage of this model is that the interaction energy becomes quadratic in the charge and spin density bosonic operators. The density of fermions becomes linear in bosonic fields \( \phi_{c,s}, \theta_{c,s} \),

\[
\rho_{c,s}(x) = -\frac{\sqrt{2}}{\pi} \partial_x \phi_{c,s}(x).
\]

As we have mentioned, the simplest modification of the fermionic Hamiltonian produced by the Rashba SOI in the absence of magnetic field can be removed by the unitary transformation. At nonzero magnetic field, the fermionic Hamiltonian \( H = H_0 + H_Z \) after this transformation takes the form \[16\]:

\[
H = -i\nu_1 \int dx (\psi_{R,\uparrow}^\dagger \partial_x \psi_{R,\uparrow} - \psi_{L,\downarrow}^\dagger \partial_x \psi_{L,\downarrow})
- i\nu_2 \int dx (\psi_{R,\downarrow}^\dagger \partial_x \psi_{R,\downarrow} - \psi_{L,\uparrow}^\dagger \partial_x \psi_{L,\uparrow}).
\]

A difference of velocities \( \delta v = v_1 - v_2 \) can arise due to the SOI effect in a wire of finite width \[1, 2, 16\], magnetic field, and also quadratic corrections to the electronic dispersion. After bosonization \[10\] Hamiltonian \[12\] takes the form

\[
H = \int \frac{dx}{2\pi} \left[ v_c K_c (\partial_x \theta_c)^2 + \frac{v_c}{K_c} (\partial_x \phi_c)^2 + v_s K_s (\partial_x \theta_s)^2ight)
+ \frac{v_s}{K_s} (\partial_x \phi_s)^2 + \delta v (\partial_x \phi_s \partial_x \theta_s + \partial_x \phi_c \partial_x \theta_c)ight]
\]

where \( v_c \) (\( v_s \)) is the velocity of plasmons (spinons). We have omitted the term \( \int \cos(2\sqrt{2} \phi \omega x)/dx/(2\pi) \) as being irrelevant in the renormalization group procedure for the repulsive interactions \( K_s < 1 \) \[16\].

To find the conductivity \( \sigma_\text{SOI} \) we need to calculate the retarded correlation function \( I_{\uparrow\downarrow}^R(x, t) = -i\theta(t)\langle \psi_{R,\uparrow}(x, t)\psi_{R,\downarrow}(x, t), \psi_{R,\uparrow}(0, 0)\psi_{R,\downarrow}(0, 0) \rangle \) in the ground state of the Hamiltonian \[15\] with fermionic operators \( \psi_{\rho,\sigma} \) given by Eq. \[10\]. Since the perturbation theory is developed for time-ordered averages in the imaginary time \( \tau = -it \), it is instructive to go from \( I(x, t) \) in the Kubo formula to the time-ordered product \( I_{\uparrow\downarrow}^T(x, t) \) by using the Wick theorem, we obtain in terms of bosonic operators:

\[
I_{\uparrow\downarrow}^T(x, t) = \sum_{q, \omega} \left[ 1 - e^{i(\omega \tau - qx)} \right] \langle Y(q, \omega)Y(-q, -\omega) \rangle,
\]

where we introduced \( e^{iX(x)/2(\pi a_0)} = \psi_{R,\uparrow}(x, \tau)\psi_{R,\downarrow}(x, \tau) \) so that \( \langle Y(x, \tau)Y(-x, \tau) \rangle = \delta(x, \tau) \) and \( \tau > 0 \). After obtaining \( I_{\uparrow\downarrow}^T(x, \tau) \), it can be converted into retarded correlator using \( I_{\uparrow\downarrow}^R(t) = \langle I_{\uparrow\downarrow}^T(t) - (I_{\uparrow\downarrow}^T(0))^* \rangle \) \[4\].

To find the correlation functions of fields \( \phi_s \) and \( \theta_s \) in Eq. \[14\] we use the generating functional \( Z[J] \):

\[
Z[J] = \int D\phi D\theta \exp \left[ \int dt dx \left( -\frac{1}{2} \Phi M \Phi + \Phi \Phi \right) \right],
\]

This expression is written in a matrix form with 4-vectors of the field \( \Phi = (\phi_c, \phi_s, \theta_c, \theta_s) \) and "current" \( \Phi \) = \((J_1, J_2, J_3, J_4)\). The 4 x 4 matrix \( M \) describes the system Lagrangian and is presented below. After the standard Gaussian integration we find

\[
Z[J] = (\det M)^{-1/2} \exp \left[ \frac{1}{2} J M^{-1} J \right].
\]

The bosonic correlation functions from Eq. \[14\] are represented in terms of the elements of matrix \( M \) as

\[
\langle \Phi_i(x, \tau)\Phi_j(0, 0) \rangle = \frac{\delta^2 \ln Z}{\delta J_i(x, \tau)\delta J_j(0, 0)}|_{J=0}
= \int \frac{d\omega}{2\pi} \int \frac{dq}{2\pi} e^{i\omega \tau - qx} M^{-1}_{ij}(q, \omega).
\]

The matrix \( M \) is symmetric and has the following nonzero elements \( M_{\phi_c,\phi_c} = v_c q^2/(\pi K_c) \), \( M_{\phi_s,\phi_s} = v_s q^2/(\pi K_s) \).
\[ M_{\omega, s} = v_s K_q q^2 / \pi, \quad M_{\omega, s} = v_s K_q q^2 / \pi, \quad M_{\omega, s} = M_{\omega, s} = v_s K_q q^2 / \pi, \quad M_{\omega, s} = v_s K_q q^2 / \pi, \quad M_{\omega, s} = v_s K_q q^2 / \pi, \quad M_{\omega, s} = v_s K_q q^2 / \pi. \]

With these expressions, \( g(x, t) \) in Eq. (14) takes the form

\[
g(x, \tau) = 2i \int \frac{dq dq}{(2\pi)^2} \left( 1 - e^{i\varphi - \omega \tau} \right) [M^{-1}_{\varphi, \omega}(q, \omega) + M^{-1}_{\varphi, \omega}(q, \omega) - M^{-1}_{\omega, \phi}(q, \omega) - M^{-1}_{\omega, \phi}(q, \omega)]. \tag{18}
\]

At zero SOI (\( \alpha = 0 \)) and magnetic field (\( B_\perp = 0 \)), the system has SU(2) symmetry of spin rotation. This symmetry prevents the renormalization of the interaction constant in the spin channel and therefore \( K_s = 1 \) [13]. A weak SOI (\( \alpha \ll v_F \)) and magnetic field (\( B_\perp \ll \alpha v_F / \mu_B \)) only slightly violate the SU(2) symmetry [17], so that \( K_s \approx 1 - (\alpha / v_F)^2 \) [17]. Therefore, in what follows we put \( K_s = 1 \) up to small corrections of order \( \alpha^2 \). Thus, with this precision up to quadratic in \( \delta v \) terms we find

\[
M^{-1}_{\varphi, \omega}(q, \omega) + M^{-1}_{\omega, \phi}(q, \omega) - M^{-1}_{\omega, \phi}(q, \omega) - M^{-1}_{\omega, \phi}(q, \omega) \approx \frac{2\pi i}{q(\omega + v v q)} - \frac{\delta v}{4K \omega (\omega + \delta v q)^2}. \tag{19}
\]

Performing the integration over frequencies, one finds the correlator as a function of imaginary time. Because of factor \( e^{-i\omega \tau} \) only the poles in the lower half-plane of the complex plane \( \omega \) contribute to the integral. After the integration and analytical continuation, expression (18) turns into a sum of logarithms of the type \( C \ln(x \pm v_c x s t) \), where \( C \) is a constant. Inserting this result in Eq. (14) we find the corresponding time-ordered fermionic correlator but in real time. It can be converted into retarded correlation function as

\[
J_{\varphi, \omega}^{R}(t) = -2\theta(t) \text{Im} J_{\varphi, \omega}^{R}(t) \tag{18}, \quad \text{and using Eq. (9) we obtain}
\]

\[
\sigma_\omega = A \int_{-\infty}^{\infty} \int_{0}^{\infty} e^{i\omega(t-\varphi x)} [K(t + i\delta) - K(t - i\delta)] dt, \tag{19}
\]

\[
K(t) = \frac{1}{(x - v_c t)^x(x + v_c t)^x(x - v_c t)^x}. \tag{20}
\]

where the constant

\[
A = \frac{(e g_\mu_B B_\perp)^2 a^{\lambda + \mu + \nu - 2}}{2\pi^2 B_\perp \alpha}. \tag{21}
\]

We recall that the wavevector \( q \) in the above integral is equal to \( 2\alpha m / \hbar \), cf. Eq. (9). The exact numerical factor \( A \) is obtained here from the comparison with the noninteracting result of Ref. [1] see supplementary material for details. The integrand in the integral over \( x \) has two singularities in the lower half-plane, at \( x = v_c t \) and \( x = v_c t \). The expressions for the exponents \( \lambda, \mu, \) and \( \nu \) are as follows [19]

\[
\lambda = (\delta v)^2 \frac{(1 + K_c)^2}{8K_c(v_c - v_c s)}, \tag{22}
\]

\[
\mu = (\delta v)^2 \frac{(K_c - 1)^2}{8K_c(v_c + v_c s)}, \tag{23}
\]

\[
\nu = 2 - (\delta v)^2 \frac{K_c v_c^2 + (K_c^2 + 1) v_c v_c s + K_c v_c s^2}{2K_c(v_c^2 - v_c s)^2}. \tag{24}
\]

To approximate \( \sigma_\omega \) close to the spin resonance at frequency \( \omega_{res} = v_c q = 2\alpha m v_c / \hbar \), we take \( \gamma \approx |\omega - \omega_{res}| \ll \omega_{res} \) with \( \gamma \) being the width of the resonance which we assume to be small [21]. In the limit \( (v_c - v_c s) \omega_{res} / (v_c s) \gg 1, \) we find

\[
\frac{\Re \sigma_\omega}{\omega^2} \approx A \frac{2\pi \lambda \nu}{(2\nu) - (v_c - v_c s)^2} \frac{(2 - \mu - \nu)}{\gamma} \times \frac{1}{[(\omega - \omega_{res})^2 + \gamma]^21 - \frac{\Delta^2}{\gamma^2}}. \tag{25}
\]

The plasmon singularity has a character of a weak cusp that can be detected only at large enough interaction.

Equations (25) and (26) are obtained under the assumption of well separated spinon and plasmon peaks, \( (v_c - v_c s) q \gg \gamma \) [22]. In the opposite case corresponding to the limit of non-interacting ferms, the peaks at \( \omega = \omega_{res} \) and \( \omega = v_c q \) merge. According to Eq. (19), the combined power of the merged peaks is \( \lambda + \nu = 2 + (\delta v)^2 (1 - K_c) \approx 2/K_c(v_c + v_c s)^2 \). In the limit of non-interacting fermions \( v_c \to v_c s \), and \( K_c \to 1 \), so that the power becomes 2 which corresponds to the Lorentzian shape of the spin resonance [23]. For small interaction \( g_0 \) between fermions, \( K_c \approx 1 - g_0 / \pi, v_c = v_F, \) and \( v_c \approx v_F(1 + g_0 / \pi) \), so that the power deviates from 2 by \( \approx (\delta v)^2 g_0^2 \). Therefore, in the framework of perturbation theory, the shape of the resonance line near \( \omega_{res} \) deviates slightly from Lorentzian. However, generally \( g_0 = (e^2 / \hbar k_F) \ln |q_0| \) can be of the order of 1. For repulsive interactions \( 0 < K_c < 1 \) and for strong fermionic interaction \( K_c \to 0 \). In this case the results (25) and (26) show that the shape of the absorption line may deviate significantly from Lorentzian at sufficiently strong interaction.

In conclusion, we have considered the electron spin-flip resonance caused by internal SOI field in the framework of Luttinger liquid theory. We have shown that the electron interaction incorporated does not destroy the resonance. In this theory it is treated as the excitation of a spin wave with the uniquely specified wavevector. We have found that the Luttinger liquid renormalizations almost do not change the Lorentzian shape of the resonance line at not too strong interaction. It occurs because the SU(2) symmetry in the spin channel is only slightly violated by the spin-orbit interaction and weak dc magnetic field. The same small parameters ensure that the coupling of the spin flip to the charge channel and therefore the excitation of the plasma oscillations is weak at the same conditions. Nevertheless, since the Coulomb interaction in quantum wires is strong, it may be expected that the deviation from Lorentzian shape of the resonance can be
observed experimentally. In this work we considered a simplified model of the SOI whose only effect is the appearance of the difference between Fermi velocities of up and down spins. The exact consideration of the quadratic part of the dispersion is still an open problem.

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[18] See the supplementary material.
[19] They are presented here up to the quadratic order in small $\delta v / v_F$. There is also a contribution $\sim (x + v t)^{-\beta}$ in Eq. (19), but its exponent $\beta \sim (\delta v)^2$ is small and can be neglected to order $(\delta v)^2$.
[20] The resonance width $\gamma$ is mostly due to electron-phonon interaction and can be estimated to be $\sim 10^9 - 10^{10} s^{-1}$. For realistic quantum wires $\gamma \ll \omega_{pe} \sim 10^{12} s^{-1}$.
[21] The details of this calculation are presented in the supplementary material.
Supplementary material for “Spin resonance in Luttinger liquid with spin-orbit interaction”

Relation between $I_{R}^{R}(t)$ and $I_{A}^{T}(t)$

The perturbation theory is valid for time-ordered averages in imaginary time, whereas what we need to calculate is a retarded average $I_{R}^{R}(t)$. Therefore, we need a relationship between $I_{R}^{R}(x, t) = -i\theta(t)\langle B(x, t), A(0, 0) \rangle$ and $I_{A}^{T}(x, \tau) = -\langle T_{\tau}B(x, \tau)A(0, 0) \rangle$ for imaginary time $\tau$ where $B(x, t) = \psi_{R}^{\dagger}(x, t)\psi_{R}(x, t)$ and $A(0, 0) = \psi_{R}^{\dagger}(0, 0)\psi_{R}(0, 0)$ are boson-like operators. These two types of averages are related by equality [4]:

$$I_{R}^{R}(t) = i\theta(t) \left[ I_{A}^{T}(t) - (I_{A}^{T})^{*}\right],$$

which follows from

$$I_{R}^{R}(t) = -i\theta(t) \left[ (B(t)A(0)) - (A(0)B(t)) \right],$$

$$I_{A}^{T}(t) = -\theta(t) \left[ B(t)A(0) + \theta(-t) (A(0)B(t)) \right].$$

For positive time $t > 0$,

$$I_{R}^{R}(t) = -(B(t)A(0)),$$

and $\langle B(t)A(0) \rangle = \langle A(t)B(0) \rangle = \langle A(0)B(t) \rangle = \langle B(t)A(0) \rangle$. In our case, due to the above definitions of $A$ and $B$, $\langle B(t)A(0) \rangle$ differs from $\langle B(t)A(0) \rangle$ by changing the spin components $\sigma \rightarrow -\sigma$. It is equivalent to $Y(x, t) \rightarrow -Y(x, t)$ since we introduced $e^{Y(x, \tau)/2\pi a_{0}} = \psi_{R}^{\dagger}(x, \tau)\psi_{R}(x, \tau)$. However, $Y$ enters in all correlation functions quadratically, see Eq. [13] in the main part, and we conclude that this transformation does not change the correlator. Then $I_{R}^{R}(t) = i\theta(t)\left[ I_{A}^{T}(t) - (I_{A}^{T})^{*}\right]$, and we find

$$I_{R}^{R}(t) = -2\theta(t)\text{Im} I_{R}^{R}(t).$$

Evaluation of conductivity in Eqs. (25) and (26)

To find the absorption power of electromagnetic field we need to calculate $\text{Re}(\sigma_{\omega})$, where the conductivity is given by Eq. [19] in the main text.

The integral in Eq. [19] is

$$\sigma_{\omega} = \mathcal{A} \int_{-\infty}^{\infty} dx \int_{0}^{\infty} e^{i(\omega t - qx)}[K(t + i\delta) - K(t - i\delta)]dt,$$

$$K(t) = \frac{1}{(x - v_{s}t)^{\lambda}(x + v_{s}t)^{\mu}(x - v_{s}t)^{\nu}},$$

where

$$\mathcal{A} = \frac{(eg\mu_{B}B_{z})^{2}a_{0}^{\lambda+\mu+\nu-2}}{\pi^{2}p^{2}c\alpha}.$$
where we introduced new variable $z = -iqu$. As a result of this change of variables, the contour $C_1$ turns into contour $C$ winding around a branch cut going from $z = 0$ to $z = -i\infty$. The contour $C$ can be rotated clockwise together with the branch cut until the latter coincides with the left half of the real axis $z < 0$. Then the contour integral turns into the Hankel’s representation of the inverse Gamma function and gives $2\pi i/\Gamma(\nu)$. After that we can integrate over time with the following final result:

$$I_{\omega,q} \approx A \frac{-2\pi i \Gamma(1 - \lambda - \mu)q^{\nu-1}i^{-\nu-\lambda-\mu}}{(v_s - v_c)^{\lambda} \Gamma(\nu)(\omega - v_s q + i\tau)^{1-\lambda-\mu}}.$$  

(36)

Then the real part of conductivity near $\omega = v_s q$ becomes

$$\text{Re} \sigma_{\omega} \approx A \frac{\Gamma(1 - \lambda - \mu)q^{\nu-1}}{(v_c - v_s)^{\lambda}(v_s + v_c)^{\mu}\Gamma(\nu)} \frac{\gamma}{[(\omega - \omega_{res})^2 + \gamma^2]^{1-\lambda-\mu}},$$

(37)

which for small $\lambda$ and $\mu$ is approximated by Eq. (25). Here we used that $v_c \geq v_s$. The integral $I_2$ over the contour $C_2$ does not contribute to the singularity at $\omega = v_s q$ and therefore can be neglected.

In addition, there is also a small part of $\text{Re} \sigma_{\omega}$ which is independent of $\gamma$:

$$\frac{\pi^2 A \Gamma(1 - \lambda - \mu)q^{\nu-1}(\nu + \lambda + \mu - 2)}{(v_c - v_s)^{\lambda}(v_c + v_s)^{\mu}\Gamma(\nu)|\omega - \omega_{res}|^{1-\lambda-\mu}}.$$  

(38)

It is small in positive parameter $\nu + \lambda + \mu - 2 = (\delta v)^2(K_c - 1)^2/[4K_c(v_c + v_s)^2]$. Nevertheless, $I_2$ contributes to a singularity at plasma frequency $\omega = v_s q$. Next we analyze this singularity. We consider the integral $I_2$ over contour $C_2$ similarly to what we did for $I_1$. We change variable in $I_2$ to $u = x - v_c t$. The mapped contour $C_2$ winds around the branch cut from $u = 0$ to $u = +\infty$. As a result of winding around the branch cut we obtain factor $(1 - e^{-2\pi i(\lambda - 1)}) \approx 2\pi i \lambda$ and the integral over $u$ from 0 to infinity:

$$I_2 \approx 2\pi i \lambda \int_0^\infty du \frac{e^{-iq(u+v_c)\tau}}{|u + (v_c - v_s)\tau|^{\mu}}.$$  

(39)

At small detuning $\delta \omega = |\omega - v_c q|$ from the plasma resonance we expect that similarly to what we observed for $I_1$ the accumulation time for the resonance absorption is large, $t \sim 1/\delta \omega \gg 1/(v_c q)$, and therefore in the factors $u + 2v_c t$, $u + (v_c - v_s) t$ it is possible to neglect $u \sim 1/q$. After this procedure the resulting integral over $t$ diverges at $t = 0$. This divergence however is spurious. It has happened because at small $t < 1/[(v_c - v_s)q]$, the variable $u$ cannot be neglected. It means that the integration over $t$ is effectively cut off at $t_0 \sim 1/[(v_c - v_s)q]$. To estimate the singular part on the background of nonsingular contribution originated from small $t$, we represent the exponent $e^{i(\omega - v_c)\tau}$ as a sum, $e^{i(\omega - v_c)\tau} = [e^{i(\omega - v_c)\tau} - 1] + 1$, and divide the integral over time into two parts:

$$\int_{t_0}^\infty \frac{e^{i(\omega - v_c)\tau} - 1}{t_{\mu+\nu}} dt + \int_{t_0}^\infty \frac{dt}{t_{\mu+\nu}}.$$  

The second integral is approximately equal to $\frac{1}{\nu + \mu - 1}$ and has no singularity. The first integral converges and can be extended to $t = 0$ if $\mu + \nu < 2$. This condition is satisfied in a broad range of not too strong interaction as it can be readily checked from Eqs. (22) – (24). The first integral after the change of variable $\tau = (\omega - v_c)q t$ turns into

$$\int_{t_0}^\infty \frac{e^{i\pi - 1}}{t_{\mu+\nu}} dt.$$  

(40)

The integral in Eq. (40) is a large number $\approx i(2 - \mu - \nu)^{-1}$ proportional to $|\delta v/(v_c - v_s)|^{-2}$. The ratio of the first term to the second has the order of magnitude $|\delta v/(v_c - v_s)|^{-2} |(\omega - v_c q)/(v_c q)|^{\mu+\nu-1}$. Thus, the nonresonant contribution is comparable with the resonant one only in a narrow region close to the resonance $\delta v \approx v_c q |\delta v/(v_c - v_s)|^2$. Combining all the results, we arrive at the expression for the singularity due to spin-flip processes at the plasmon frequency:

$$I_{\omega,q} \approx -A i \lambda^{-1} q^{-1}\frac{2\pi i \lambda \Gamma(1 - \lambda)q^{\nu+1}}{\Gamma(\nu)(\omega - v_c q + i\tau)^{1-\lambda-\mu}}.$$  

(41)

The calculation of the real part gives the following result:

$$\text{Re} I_{\omega,q} \approx A \frac{2\pi i \lambda \Gamma(1 - \lambda)q^{\nu+1}}{(v_c q)^{\mu}(v_c - v_s)^{\mu}(2 - \mu - \nu)} \frac{\gamma}{[(\omega - v_c q)^2 + \gamma^2]^{1-\lambda-\mu}},$$

(42)

c.f. (26) in the main text for $\text{Re} \sigma_{\omega}$. The plasmon singularity has a character of a weak cusp that can be detected only at large enough interaction.

Case of noninteracting electrons

The rate $w$ of the spin flips per one electron found in the work (1) reads:

$$w = \frac{2e\alpha}{2\omega_{res}} \frac{g\mu B}{2\alpha P_F} I_{\omega},$$

(43)

where $I_{\omega}$ is the spectral density of the driving electromagnetic field. It is determined by

$$\mathbb{E}^*(t)\mathbb{E}(t') = \int_{-\infty}^{\infty} e^{i\omega(t-t')} \frac{d\omega}{2\pi}.$$  

(44)

For the monochromatic field with the frequency $\omega_0$ the spectral density is $I_{\omega} = 4\pi \delta(\omega - \omega_0)|E_\omega|^2$, where $E_\omega$ is the complex amplitude of the field. The energy absorption per particle per unit time is equal to $w\hbar\omega$. The Luttinger model neglects
the quadratic part of energy dispersion. From this point of view any particle in the single-occupied range has the same energy of the spin flip or equivalently the same spin-resonance frequency. To calculate the energy losses per unit wire length and time it is necessary to multiply the energy rate per particle by the density of electrons in the single-occupied ranges of the momentum. The latter is equal to

$$n_{sf} = \frac{2\alpha}{\pi \hbar v_F n},$$

where $n = 2p_F/(\pi \hbar)$ is the total one-dimensional density of electrons. As a result we find the absorption power per unit wire length:

$$P = w\hbar \omega n_{sf} = \frac{8e^2 (g\mu_B B)^2 |E_{\omega}|^2}{\hbar p_F^2 v_F} \delta(\omega - \omega_{res}). \quad (45)$$

The real part of conductivity, $\text{Re} \sigma_\omega$, for noninteracting electrons is equal to the power, Eq. (45), divided by $|E_\omega|^2$:

$$\sigma_\omega = \frac{8e^2 (g\mu_B B)^2}{\hbar p_F^2 v_F} \delta(\omega - \omega_{res}). \quad (46)$$

The exact numerical factor for the conductivity of interacting electrons in Luttinger model is extracted by the matching it with the conductivity of noninteracting electrons (46).

In the case of noninteracting electrons $v_s = v_c$, $\mu = 0$, and $\lambda + \nu = 2$. Substituting it into Eqs. (19) – (20) of the main text we find $A \rightarrow A|_{\lambda + \nu = 2}$ and Eq. (20) simplifies to

$$K(t) = \frac{1}{(x - vt)^2}. \quad (47)$$

Since $q$ is positive, the exponent $e^{-iqx}$ in Eq. (19) of the main text vanishes at large $x$ in the lower half-plane of the complex variable $x$. Therefore, the integral over real axis $x$ is equal to the integral in the lower half-plane of $x$. Therefore, only $K(t - i\delta)$ has nonzero contribution, and using the residue theorem,

$$\sigma_\omega = A \int_0^\infty dt e^{i\omega(t-i\delta)}
\times (-2\pi i) \text{Res}_{x=v_s(t-i\delta)} \left[ \frac{e^{-iqx}}{(x - v_s(t - i\delta))^2} \right].$$

At a finite attenuation $\gamma$, the conductivity becomes

$$\sigma_\omega = A \frac{-2\pi i q}{\omega - v_s q + i\gamma}. \quad (48)$$

Taking the real part we find in the limit $\gamma \rightarrow 0$,

$$\text{Re} \sigma_\omega = 2\pi^2 A q \delta(\omega - v_s q). \quad (49)$$

Thus, in the noninteracting limit the conductivity of Luttinger liquid coincides with the conductivity of free electrons provided a correct choice of the factor $A$. Note that the cut-off length $a_0$ disappears from the factor $A$ in this limit. At a finite interaction it enters in a small power $\propto (\delta v)^2$. Therefore it can be defined only by the order of magnitude.