Mapping out the QCD phase transition in multiparticle production

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Abstract. We analyse multiparticle production in a thermal framework for seven central nucleus + nucleus collisions, $e^+ + e^-$ annihilation into hadrons on the Z resonance and four hadronic reactions ($p + p$ and $p + \bar{p}$ with partial centrality selection), with centre of mass energies ranging from $\sqrt{s} = 2.6$ GeV (per nucleon pair) to 1.8 TeV. Thermodynamic parameters at chemical freeze-out (temperature and baryon and strangeness fugacities) are obtained from appropriate fits, generally improving in quality for reactions subjected to centrality cuts. All systems with non-vanishing fugacities are extrapolated along trajectories of equal energy density, density and entropy density to zero fugacities. The so-obtained temperatures extrapolated to zero fugacities as a function of initial energy density $\varepsilon_{\text{in}}$ universally show a strong rise followed by a saturating limit of $T_{\text{lim}} = 155 \pm 6 \pm 20$ MeV. We interpret this behaviour as mapping out the boundary between quark gluon plasma and hadronic phases. The ratio of strange antiquarks to light ones as a function of the initial energy density $\varepsilon_{\text{in}}$ shows the same behaviour as the temperature, saturating at a value of $0.365 \pm 0.033 \pm 0.07$. No distinctive feature of ‘strangeness enhancement’ is seen for heavy ion collisions relative to hadronic and leptonic reactions, when compared at the same initial energy density.

1. Introduction

Hadronic reactions involving copious production of secondary particles have been associated with an underlying thermodynamic behaviour since the earliest observations in cosmic rays [1]. The observable energy regime ranging up to 140 TeV per nucleon pair corresponding to incident cosmic ray nucleons of $E \leq 10^{10}$ GeV demands elaborate simulations of the observed extended...
air shower development, in order to extract definite multiplicity distributions of the elementary hadronic and nuclear reactions [2].

Thermodynamic models are widely and successfully used to describe particle ratios identified in hadronic and especially heavy ion collisions [3]–[11], but they have also been extended to $e^+e^-$ annihilation into hadronic final states [12, 13]. In heavy ion collisions two main parameters—energy per nucleon pair and centrality—are varied and their influence on thermodynamic variables—temperature and chemical potentials—is studied.

We derive and discuss thermal properties in search of the phase boundary between quark gluon plasma and condensed hadrons. We consider as one characteristic signature of this boundary the critical dependence of kaon number densities on the initial energy density. The kaon multiplicities observed in $\text{Pb} + \text{Pb}$ collisions at $\sqrt{s} = 17$ GeV [14], and in other nucleus + nucleus and $p + p$ collisions at $\sqrt{s} \sim 5–19$ GeV [15, 16], serve as the motivation for our thermal description which extends the previous analysis of one of us [15, 16] (see figure 8).

In section 2 we discuss the dependence of the critical temperature on the vacuum pressure in QCD, limited to the case of vanishing chemical potentials, extending the work of [17]. In section 3 we seek to assign each thermodynamic state established at chemical freeze-out, for any given reaction for which chemical potentials for baryon number and strangeness do not vanish, an equivalent state at zero fugacities. We do this by extrapolating along curves of equal entropy density, energy density and number density. The extrapolation of the temperature of systems with finite fugacities to zero fugacities, and the investigation of their dependence on the initial energy density has been proposed in [16]. We find that a more universal parameter as a measure of positive and negative strangeness production is

$$\lambda_s = 2\langle \bar{s} \rangle / (\langle u \rangle + \langle d \rangle).$$

This parameter is also extrapolated to the zero fugacity systems.

The details of this procedure are worked out in the first four subsections of section 3. The results at zero fugacities are contained in subsection 3.5 and represented in figures 2–8 in relation to the energy density initially achieved in each reaction. Despite large errors, the phase boundary between quark gluon plasma and hadronic phases is clearly mapped out.

In the hadronic phase we use the approximation of non-interacting hadron resonances to describe in this sense the global ratios of hadrons produced in the following reactions:

1. central $\text{Au} + \text{Au}$ collisions at RHIC $\sqrt{s} = 130$ A GeV [18, 19];
2. central $\text{Pb} + \text{Pb}$ collisions at SPS at $\sqrt{s} = 17$ A GeV [20];
   - central $\text{S} + \text{A}$ collisions ($\text{A} = \text{Au}, \text{W}, \text{Pb}$) at SPS $\sqrt{s} = 19$ A GeV [21];
   - central $\text{S} + \text{S}$ collisions at SPS $\sqrt{s} = 19$ A GeV [22, 23];
3. central $\text{Si} + \text{Au}$ collisions at AGS $\sqrt{s} = 5.4$ A GeV [9];
   - central $\text{Au} + \text{Au}$ collisions at AGS $\sqrt{s} = 4.9$ A GeV [24, 25];
4. $\text{Ni} + \text{Ni}$ collisions at GSI $\sqrt{s} = 2.8$ A GeV [26, 27];
5. $e^+e^-$ collisions at LEP $\sqrt{s} = 91.19$ GeV [12, 13];
6. $p + p$ collisions at $\sqrt{s} = 27$ GeV [28];
   - $p + p$ collisions at $\sqrt{s} = 17$ GeV [29, 30];
7. $p + \bar{p}$ collisions at $\sqrt{s} = 900$ GeV [28];
   - peripheral $p + \bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV [31];
   - central $p + \bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV [31].

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2. Outline of basic points

We consider first the thermodynamic potential $\Phi$ of a grand canonical ensemble of hadron resonances without further interactions among them. The thermodynamic variables are

$$V \quad T \quad \chi_b = \mu_b/T \quad \chi_s = \mu_s/T$$

i.e. volume, temperature, baryon and strangeness fugacity. The fugacity of the third component of isospin is set to zero, neglecting isospin asymmetries:

$$\Phi = gV \quad g = g(\beta, \chi_b, \chi_s) = \sum \limits_\alpha g_\alpha + g_0$$

$$\beta = 1/T \quad g = \beta p \quad p$$ is pressure.

In equation (2) the sum extends over hadron resonances denoted by $\alpha$, where we only include the pseudoscalar and vector $u, d, s$ meson nonets, the spin $1/2$ baryon octet and spin $3/2$ decuplet and their antiparticles as well as $f_0(400 - 1200)$ or $\sigma$, interpreted as the scalar glueball [32], for simplicity.

$g_0 = \beta p_0$ takes into account the non-zero vacuum pressure characterizing the hadronic phase in QCD, which we restrict to the three light flavours:

$$p_0(T = 0) = \frac{9}{32\pi^2} B^2 + \frac{1}{4} \Lambda(m_q) = \begin{cases} 0.00302 \text{ GeV}^4 \\ 0.00658 \text{ GeV}^4 \end{cases}$$

$$B^2 = \left\langle 0 \left| \frac{1}{4} F_{\mu\nu}^a F_{a\mu\nu} \right| 0 \right\rangle = \begin{cases} 0.125 \text{ GeV}^4 \\ 0.250 \text{ GeV}^4 \end{cases}$$

$$\Lambda = \sum \limits_q m_q \langle 0 | \bar{q}q | 0 \rangle \sim f_\pi^2 \left( \frac{1}{2} m_\pi^2 + m_K^2 \right) = 0.00217 \text{ GeV}^4.$$  (3)

Converting energy density units to GeV fm$^{-3}$ the range of values for the (positive) vacuum pressure becomes

$$p_0(T = 0) = \begin{cases} 0.377 \text{ GeV fm}^{-3} \\ 0.823 \text{ GeV fm}^{-3} \end{cases}$$

$$p_0(T) \sim p_0(T = 0)(1 - (T/T_{cr})^4).$$  (4)

The temperature variation of the vacuum pressure is an approximation with $T_{cr} = T_{cr}(\chi_b, \chi_s)$.

The quantities $g_\alpha$ in equation (2) for non-interacting resonances are then given by

$$g_\alpha(\beta, \chi_b, \chi_s) = w_\alpha \int_{m_\alpha}^{\infty} dE E p/(2\pi^2) l$$

$$l = \mp \log[1 \mp \exp(-\beta E + \chi_b B_\alpha + \chi_s S_\alpha)]$$

$$w_\alpha = (2I_\alpha + 1)(2S p_\alpha + 1) \quad p = \sqrt{E^2 - m_\alpha^2}.$$  (5)

In equation (5) $(I, S p, B, S)_\alpha$ denote isospin, spin, baryon number and strangeness of the hadron resonance $\alpha$. The $-,+$ signs apply to bosons and fermions, respectively.

If we take into account the variation of the vacuum pressure with temperature then the masses $m_\alpha \rightarrow m_\alpha(T)$ of quasi-excitations become temperature-dependent quantities.

In all subsequent calculations we neglect these temperature-dependent effects, which however do set in dramatically, when the temperature deviates from the critical one by less than 10%:

$$p_0(T) \rightarrow p_0 = p_0(T = 0) \quad m_\alpha(T) \rightarrow m_\alpha = m_\alpha(T = 0).$$  (6)
We state here that a thermal description cannot account for any azimuthal ($\varphi$-) dependence of inclusive cross sections. Thus azimuthal anisotropies provide a measure for non-equilibration.

The potential $\Phi$ in equation (2) and the entropy denoted by $S$ give rise to the differentials

\[
\begin{align*}
\text{d}\Phi &= -\mathcal{E}\text{d}\beta + N_\nu\text{d}\chi_\nu + g\text{d}V \\
\text{d}S &= +\beta\text{d}\mathcal{E} - \chi_\nu\text{d}N_\nu + g\text{d}V \\
S &= \varrho_s V = \Phi + \beta\mathcal{E} - \chi_\nu N_\nu \quad \nu = b, s \\
\varrho_s &= \beta(\varepsilon + p) - \chi_\nu \varrho_\nu.
\end{align*}
\]

In equation (7) $\varrho_s$ denotes the entropy density, $\varepsilon$ the energy density and $\varrho_{b,s}$ the net baryon and strangeness (number) density, respectively.

\[
\begin{align*}
\varrho_\nu &= \sum_\alpha \nu_\alpha \varrho_\alpha \\
\nu_\alpha &= (B_\alpha, S_\alpha)
\end{align*}
\]

As is apparent from equation (7), the entropy density obtains no contribution from the vacuum energy density and pressure, as long as the vacuum pressure is taken as independent of temperature as indicated in equation (6).

Energy density and total number density then take the form

\[
\begin{align*}
\varepsilon &= \varrho_e - p_0 \\
\varrho_e &= \sum_\alpha \varrho_{e\alpha} \quad \varrho_n = \sum_\alpha \varrho_{n\alpha} \\
\left\{ \varrho_{e\alpha}, \varrho_{n\alpha} \right\} &= w_\alpha \int_{m_\alpha}^{\infty} \text{d}E \frac{E \text{d}p/(2\pi^2)}{1} \\
n &= \frac{1}{\exp(\beta E - \chi_B B_\alpha - \chi_S S_\alpha) + 1}.
\end{align*}
\]

The approximation of hadronic interactions by free hadron resonances is of course at best a consistent approximate thermodynamic description. We will use it here to describe in this sense global ratios of hadrons produced in the reactions listed in the introduction.

We will present a detailed analysis of these collisions, as seen at the time of chemical freeze-out, in the next section. The key parameter characterizing not this freeze-out instance but the time of first onset of thermal conditions is the initial energy density $\varepsilon_{in}$. It determines the thermal parameters of the hadronic system as seen at chemical freeze-out and distinguishes those systems which have hadronized after transcurring the quark gluon plasma phase from those which remained throughout in the hadronic phase.

2.1. The phase boundary for vanishing fugacities

In the remainder of this section we analyse the equilibrium condition for both phases to coexist in the case of vanishing fugacities or, equivalently, chemical potentials. This latter choice is just for clarity of argument and also because for limiting centre of mass energies and finite baryon number, strangeness and charge of the initial system this is the limiting case.

This condition corresponds to equating the Gibbs densities of the two phases

\[
\begin{align*}
g^H(T) &= g^{QGP}(T) \\
g^H &= \sum_\alpha g_\alpha + \beta p_0 \\
g^{QGP} &= \sum_{-\alpha} g_{-\alpha} \\
\pm\{\alpha\} &= \begin{cases} + & \text{hadron resonances} \\ - & \text{gluons, quarks and antiquarks (u, d, s)}. \end{cases}
\end{align*}
\]

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In the same approximative vein as applied to the hadronic phase (H) we approximate the various quasi-excitations in the quark gluon phase by free gluons, quarks and antiquarks analogous to the form of $g_\alpha$ in equation (5),

$$g^{-\alpha} = w^{-\alpha} \int_{m^{-\alpha}}^{\infty} dE \frac{E p}{(2\pi^2)l}$$

$$l = \mp \log[1 \mp \exp(-\beta E)] \quad p = \sqrt{E^2 - m^{-\alpha}^2}$$

$$-\alpha = \begin{cases} g_l & w_{gl} = 16, m_{gl} = 0 \\ q & w_q = 6, m_q + \\ \bar{q} & w_{\bar{q}} = 6, m_{\bar{q}} + \\ u, d, s & (5.25, 8.75, 175) \text{ MeV} \end{cases}$$

In equation (11) the colour/spin weights, masses and boson/fermion signs are specified. The quark masses represent a renormalization group invariant 'best' choice. It turns out that near the critical temperature $T_{cr} = O(200 \text{ MeV})$ the (anti)quark contributions to the Gibbs density do not deviate substantially from the limit of vanishing quark masses including $m_s$.

The crossings of $g^H$ and $g^{QGP}$ as displayed in figure 1 seem to suggest a first-order phase transition with respect to energy density.

This is, however, due to the approximation of fixed masses for hadron excitations and of free quark and gluon flavours in the region of $T_{cr} = 194 \pm 18 \text{ MeV}$. We note that the above estimate of the critical temperature at zero fugacities confirms within the theoretical error the previous estimate of Minkowski [17].

Within the approximations and errors we obtain the critical temperature and critical hadronic
energy density on the side of the hadronic phase for vanishing fugacities

\[ T_{cr} = 194 \pm 18 \text{ MeV} \]

\[ \varrho_{ecr} \left( \begin{array}{c} 212 \text{ MeV} \\ 194 \text{ MeV} \\ 176 \text{ MeV} \end{array} \right) = \left( \begin{array}{c} 1.797 \text{ GeV fm}^{-3} \\ 1.018 \text{ GeV fm}^{-3} \\ 0.5406 \text{ GeV fm}^{-3} \end{array} \right). \] (12)

The critical boundary curve could be extended to arbitrary chemical potentials. However, the ‘melting’ of masses of hadronic quasi-excitations modifies the Gibbs density on the side of the hadronic phase, whereas the approximation of free quark and gluon modes does not warrant the extrapolation to arbitrary chemical potentials, especially for low temperature.

3. Reduction of chemical freeze-out parameters to zero fugacities in multiparticle production

In this section we apply the thermal model introduced in the last section (equation (5)) to extract the intensive thermodynamic parameters defined in equation (1). We then extrapolate these parameters to zero fugacities along states with equal entropy density, energy density or density. The goal of this analysis is to compare the temperature at zero fugacities with the initial energy density achieved in the collision in order to reveal a boundary reflecting the QCD phase transition.

One serious well known problem when comparing models to experimental data is due to decays of resonances. This effect is called ‘feeding’. We compare calculated particle ratios to experimental data, taking all (strong, electromagnetic and weak) decays with a branching ratio not below 1% into account. We try to account for experimental acceptance for K\(^+\) and K\(^-\), assuming a 50% feeding to pions, due to their long decay length. The other weak decays (e.g. Λ, K\(_0^s\)) have a much shorter decay length and are assumed to fully feed into secondary particles. The K\(^0\) decay is not considered.

In contrast to [8, 9] we do not take all hadronic resonances below 2 GeV mass into account. The theoretical error inherent to the free resonance approximation allows in our opinion our reduced set. This is supported by the finding that we derive similar thermodynamic parameters with similar accuracy as, for example, the ones found in [20]. Although the high mass resonances are definitely produced at sufficiently high energy, it is not clear that the thermal description becomes more accurate by including them because the quasi-excitations may not correspond to them, especially if these are quark and gluon modes. All similar thermal descriptions suffer from replacing interactions by non-interacting resonances. Furthermore, the isotropic angular distributions characterizing the models are far from reality [35, 36] (e.g. flow phenomena, diffractive phenomena).

We enforce strangeness conservation, to obtain the best fit, except in the case of p+p collisions at \(\sqrt{s} = 17\) GeV. We find the error on the fitted parameters, at \(\pm 1\) unit of the ratio of \(\chi^2/DOF\) away from the best fitted value. Our quoted error reflects the inherent theoretical error, despite the difficulty of quantifying it. Note that other analyses [8, 9] finding a smaller error seem to use as error the fitted parameters at \(\pm 1\) unit of \(\chi^2\) itself.

In the following sections we record whether the data we use are taken with a minimum bias trigger (that means no trigger bias is imposed) or a central respectively a peripheral trigger. Central is understood here to be the lowest impact parameter region. It is usually selected in the experiment by considering only collisions with the largest particle multiplicity (e.g. CERN-WA97, Fermilab-E735) or the largest transverse energy (e.g. CERN-NA52), or
the smallest forward going energy (which reflects the spectator nucleon number) (e.g. CERN-NA49). Correspondingly, peripheral collisions are a selection of collisions with the largest impact parameters using the same means as described above (charged multiplicity, energy).

In the following analysis we also give the quantity \( \lambda_s = \frac{2s}{u + d} \), which we take as a measure of the strangeness suppression factor defined and used in the literature as \( \lambda_s = \frac{2(s + \bar{s})}{(u + \bar{u} + d + \bar{d})} \) [9]. We use antiquarks to consider in a simplified way only the newly produced valence antiquarks. We also give the equivalent \( \lambda_s \) at zero fugacities. From quark counting rules one expects \( \lambda_s \) to be in the range 0.3–0.5 [37]. In principle, one should include the newly produced sea quarks in any definition of \( \lambda_s \), however the relevant proportion of the latter inside hadrons is still the subject of experimental and theoretical investigations.

Generally we find that thermal fits to minimum bias or peripheral nuclear and hadronic reactions are not satisfactory, in contrast to central collisions. We conjecture that this is due to the presence of at least two thermal sources which we attribute to diffractive versus pomeron-induced subprocesses. Diffractive processes at finite \( \sqrt{s} \) feed back into the midrapidity region, whereas pomeron-induced subprocesses result dominantly in particle production at midrapidity. Following this conjecture, we conclude that the relative importance of diffractive to pomeron-induced subprocesses decreases with increasing centrality of the collision. In this respect it would be helpful firstly to increase experimental coverage in the target and/or the projectile fragmentation regions. This study has been proposed by [38, 39]. Secondly, it is important that experiments apply centrality cuts, in nucleus + nucleus as well as in p + nucleus collisions, and in elementary particle collisions.

3.1. Nucleus + nucleus reactions

In the following four subsections we analyse data from nucleus + nucleus collisions at (1) RHIC, (2) SPS, (3) AGS and (4) SIS energies.

3.1.1. Central Au + Au collisions at \( \sqrt{s} = 130 \) GeV. We use three measured ratios at midrapidity from [18, 19], \( \bar{X}/\Lambda, \bar{p}/p \) and \( h^-/N_{ch} \), and we impose strangeness conservation. We exclude weak decay products as is done in the experiment [18]. The data from [19] are preliminary. The predicted and the experimental ratios are shown in table 1. The resulting \( \chi^2/DOF \) is (1.41/1) (confidence level (CL) 23%) for a temperature of 168 ± 40 MeV. When adding a systematic error of 15% linearly to account for the experimental feeding uncertainties and the fact that the ratios are measured at midrapidity only, the resulting \( \chi^2/DOF \) is 0.27/1 (CL 60%). The errors on the temperature and \( \lambda_s \) are not well estimated in this analysis due to the insensitivity of the ratios used to the temperature at fixed fugacities. We estimate the error on the temperature, imposing a variation of the \( \chi^2/DOF \) by 1, while changing the fugacities and fixing \( T \) each time by imposing strangeness conservation. In order to improve the quality of the fit the addition of other ratios more sensitive to temperature variations is needed, such as \( K/\pi \) etc, which will soon be measured by the experiments at RHIC.

After defining the \( (T, \mu_b, \mu_s) \) values describing the particle ratios produced in central Au + Au collisions at \( \sqrt{s} = 130 \) GeV at the chemical freeze-out, we extrapolate to \( T \) at zero fugacities (table 2). Because of the very small value of the fugacities, the extrapolated values do not differ much from those values at finite fugacities.

The values of \( \chi^2/DOF \) of all reactions are shown in table 21 later.
Table 1. RHIC Au + Au at $\sqrt{s} = 130$ GeV. Predicted versus experimental particle ratios for the best fit of our model.

| Ratio    | Model | Data          |
|----------|-------|---------------|
| $\bar{X}/\Lambda$ | 0.718 | 0.70 ± 0.25   |
| $p/p$    | 0.662 | 0.650 ± 0.090 |
| $h^-/N$ charged | 0.491 | 0.432 ± 0.0504 |

Table 2. RHIC Au + Au at $\sqrt{s} = 130$ GeV. Thermodynamic parameters for the best fit with temperatures and $\lambda_s$ extrapolated to zero fugacities.

| $\mu_b$ (GeV) | $\mu_s$ (GeV) | $T$ (GeV) | $\rho_c$ (GeV fm$^{-3}$) | $T_{eq,\rho_c}$ (GeV) | $\lambda_s(eq,\rho_c)$ |
|---------------|---------------|-----------|-----------------|-----------------|-----------------|
| 0.0390        | 0.0084        | 0.168 ± 0.040 | 0.443            | 0.169 ± 0.040   | 0.422$^{+0.105}_{-0.156}$ |
| $\rho_n$ (fm$^3$) | $T_{eq,\rho_n}$ (GeV) | $\lambda_s(eq,\rho_n)$ |
| 0.453         | 0.168 ± 0.040 | 0.443$^{+0.11}_{-0.164}$ |
| $\rho_s$ (fm$^{-3}$) | $T_{eq,\rho_s}$ (GeV) | $\lambda_s(eq,\rho_s)$ |
| 2.83          | 0.168 ± 0.040 | 0.443$^{+0.11}_{-0.164}$ |

3.1.2. Central nucleus + nucleus collisions at $\sim$200 A GeV. Here we show results from central Pb + Pb collisions at 158 A GeV, and from S + S and S + A collisions at 200 A GeV.

Central Pb + Pb collisions at 158 A GeV. For central Pb + Pb collisions at 158 A GeV, we use the particle ratios from table I in [20] to compare with our model predictions. We do not consider a chemical potential for isospin [20], so therefore we do not consider, for example, the $\pi^-/\pi^+$ ratio. We did not use the ratios in [20] for which feeding correction was included and we also did not use the $d/\bar{d}$ ratio, since there is strong experimental evidence that $d$ and $\bar{d}$ are formed in the thermal hadronic freeze-out through coalescence [14,40]. Furthermore, the $p$ may not freeze out in the chemical hadronic freeze-out due to the high cross section for $p + \bar{p}$ annihilation to, for example, $\pi$ as noted in [41].

We introduce a systematic error of 14% (quadratically added) in the ratios including WA97 data, because they are measured at midrapidity only. We also add (quadratically) a 10% systematic error to ratios including $h^-$ due to uncertainty in the feeding correction. The predicted ratios are shown in table 3 together with the experimental data.

The $\chi^2$ at which strangeness is conserved is 16.1 over 12 degrees of freedom (CL = 20%, $\chi^2/DOF = 1.34$) and this corresponds to $T = 162 + 10 - 28$ MeV. The asymmetry of the error (defined as the $T$ deviation 1 unit of $\chi^2/DOF$ away) is due to the fact that the value of 16.1 is not at the minimum. The minimum of $\chi^2$ is 11.9 over 11 degrees of freedom (CL = 50%, $\chi^2/DOF = 1.08$) at $T = 154 + 14 - 16$ MeV. There is a probable physics process which
may induce an imbalance of strangeness in the experiment, if the measurements do not include the target and beam fragmentation regions: in particular, particles with negative $S$ (especially hyperons) may be produced near beam rapidity. In [42] it is conjectured that strangeness is not balanced (with missing particles with $S$ negative). However, we will demand exact strangeness conservation in the following.

After defining the $(T, \mu_b, \mu_s)$ values describing the particle ratios produced in central Pb+Pb collisions at 158 A GeV at the chemical freeze-out we extrapolate to $T$ at zero fugacities. The resulting equivalent temperatures for equal entropy density ($\rho_s$), energy density ($\rho_e$) and number density ($\rho_n$) are shown in table 4.

**Table 3.** SPS Pb + Pb at $\sqrt{s} = 17$ GeV, most central events. Predicted versus experimental particle ratios for the best fit of our model.

| Ratio                  | Model  | Data            |
|------------------------|--------|-----------------|
| $(p - \bar{p})/h^-$   | 0.165  | $0.228 \pm 4.31 \times 10^{-2}$ |
| $\eta/\pi^0$          | $7.35 \times 10^{-2}$ | $8.10 \times 10^{-2} \pm 1.30 \times 10^{-2}$ |
| $K^0_s/\pi^-$         | 0.142  | $0.125 \pm 1.90 \times 10^{-2}$ |
| $K^0_s/h^-$           | 0.124  | $0.123 \pm 2.64 \times 10^{-2}$ |
| $\Lambda/h^-$         | $1.04 \times 10^{-1}$ | $7.70 \times 10^{-2} \pm 1.54 \times 10^{-2}$ |
| $\Lambda/K^0_s$       | 0.835  | $0.630 \pm 1.02 \times 10^{-1}$ |
| $K^+/K^-$             | 1.78   | $1.85 \pm 9.00 \times 10^{-2}$ |
| $K^+/K^-$             | 1.78   | $1.80 \pm 1.00 \times 10^{-1}$ |
| $\Xi^+ / \bar{\Xi}$  | 0.170  | $0.188 \pm 3.90 \times 10^{-2}$ |
| $(\Xi^- + \Xi^+)/(\Lambda + \bar{\Lambda})$ | 0.115 | $0.130 \pm 3.00 \times 10^{-2}$ |
| $\Xi^+ / \Xi^-$       | 0.193  | $0.232 \pm 3.30 \times 10^{-2}$ |
| $\Xi^- / \Xi^-$       | 0.193  | $0.247 \pm 4.30 \times 10^{-2}$ |
| $\Omega^+ / \Omega^-$ | 0.357  | $0.383 \pm 8.10 \times 10^{-2}$ |
| $\Omega / \Xi$        | 0.153  | $0.219 \pm 5.00 \times 10^{-2}$ |

**Table 4.** SPS Pb + Pb at $\sqrt{s} = 17$ GeV, most central events. Thermodynamic parameters for the best fit, and temperatures and $\lambda_s$ extrapolated to zero fugacities.

| $\mu_b$ (GeV) | $\mu_s$ (GeV) | $T$ (GeV) | $\lambda_s$ | $\rho_e$ (GeV fm$^{-3}$) | $T_{eq,\rho_e}$ (GeV) | $\lambda_s(eq, \rho_e)$ | $\rho_n$ (fm$^{-3}$) | $T_{eq,\rho_n}$ (GeV) | $\lambda_s(eq, \rho_n)$ | $\rho_s$ (fm$^{-3}$) | $T_{eq,\rho_s}$ (GeV) | $\lambda_s(eq, \rho_s)$ |
|---------------|---------------|-----------|--------------|--------------------------|-----------------------|--------------------------|----------------------|-----------------------|--------------------------|--------------------|----------------------|--------------------------|
| 0.239         | 0.0517        | 0.162$^{+0.010}_{-0.026}$ | 0.565        | 0.408                    | $0.164^{+0.008}_{-0.030}$ | $0.422^{+0.025}_{-0.118}$ | $0.444$               | $0.164^{+0.008}_{-0.030}$ | $0.418^{+0.025}_{-0.117}$ | 2.8049              | $0.164^{+0.009}_{-0.031}$ | $0.420^{+0.026}_{-0.116}$ |

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be compared with the experimental data 200 A GeV are shown, data are taken from [22, 23]. We use conservation. The resulting predicted ratios are

\[
\begin{array}{cccccc}
\mu_b & \mu_s & T & \lambda_s & \rho_e & T_{eq,\rho_e} \\
0.191 & 0.0426 & 0.166^{+0.010}_{-0.028} & 0.544 & 0.424 & 0.169^{+0.08}_{-0.030} \\
0.464 & & \lambda_s(eq, \rho_e) & & & 0.422^{+0.025}_{-0.118} \\
0.218 & 0.058 & 0.182^{+0.019}_{-0.008} & 0.654^{+0.065}_{-0.003} & 0.96 & 0.187^{+0.021}_{-0.009} \\
0.803 & & \lambda_s(eq, \rho_e) & & & 0.470^{+0.047}_{-0.023} \\
4.98 & & \lambda_s(eq, \rho_e) & & & 0.473^{+0.048}_{-0.023} \\
\end{array}
\]

Table 5. SPS S + A at $\sqrt{s} = 19$ GeV, most central events. Thermodynamic parameters for the best fit, and temperatures and $\lambda_s$ extrapolated to zero fugacities.

Table 6. SPS S + S at $\sqrt{s} = 19$ GeV, most central events. Thermodynamic parameters for the best fit, and temperatures and $\lambda_s$ extrapolated to zero fugacities.

Central S + A collisions at 200 A GeV. In table 5 we give the same quantities for central S + A collisions at 200 A GeV with $A = $ Au, W, Pb. The data used are taken from [21]. Here we mainly use the $\bar{p}/p$ and $K^+/K^-$ ratios to find ($T, \mu_b, \mu_s$) while we also suppose exact strangeness conservation. The resulting predicted ratios are $\bar{p}/p = 0.131$ and $K^+/K^- = 1.59$ which can be compared with the experimental data $\bar{p}/p = 0.12 \pm 0.02$ and $K^+/K^- = 1.59 \pm 0.15$. The resulting temperature is $T = 166 \pm 10 - 28$ MeV. The errors are taken to be percentage wise the same as in the Pb + Pb system. We do not perform a full fit, as the resulting temperature is within the errors in agreement with [21] where a full fit to many ratios is performed. The calculated values in [21] are $T = 165 \pm 5$ MeV, $\mu_b = 175 \pm 5$ MeV, $\mu_s = 42.5 \pm 4.5$ MeV.

Central S + S collisions at 200 A GeV. In table 6 the results for the central S + S collisions at 200 A GeV are shown, data are taken from [22, 23]. We use $K_s^0/\Lambda, (B - \bar{B})/h^-, \bar{K}/\Lambda, K_s^0/h^-$ and $K^+/K^-$ as well as strangeness conservation. The resulting $\chi^2/DOF = 1.95/3 = 0.65$
Table 7. AGS Si + Au at \( \sqrt{s} = 5.4 \) GeV, most central events. Thermodynamic parameters for the best fit, and temperatures and \( \lambda_s \) extrapolated to zero fugacities.

| \( \mu_b \) (GeV) | \( \mu_s \) (GeV) | \( T \) (GeV) | \( \lambda_s \) (GeV fm\(^{-3}\)) | \( T_{\text{eq}, \rho_n} \) (GeV) | \( \lambda_s(\text{eq, } \rho_n) \) |
|------------------|------------------|----------|------------------|------------------|------------------|
| 0.540            | 0.107            | 0.120\(^{-0.008}_{+0.035}\) | 0.548            | 147.4\(^{+0.013}_{-0.056}\) | 0.347\(^{+0.052}_{-0.250}\) |
| \( \rho_n \) (fm\(^{-3}\)) | \( T_{\text{eq}, \rho_n} \) (GeV) | \( \lambda_s(\text{eq, } \rho_n) \) |
| 0.182            | 0.141\(^{+0.013}_{-0.053}\) | 0.322\(^{+0.048}_{-0.23}\) |
| \( \rho_s \) (fm\(^{-3}\)) | \( T_{\text{eq}, \rho_s} \) (GeV) | \( \lambda_s(\text{eq, } \rho_s) \) |
| 1.161            | 0.143\(^{+0.013}_{-0.054}\) | 0.330\(^{+0.049}_{-0.237}\) |

Table 8. AGS Au + Au at \( \sqrt{s} = 4.9 \) GeV, most central events. Predicted versus experimental particle ratios for the best fit of our model.

| Ratio         | Model | Data            |
|---------------|-------|-----------------|
| \( \langle K \rangle / \Lambda \) | 0.562 | 0.675 \( \pm \) 0.144 |
| \( K^+ / K^- \) | 5.65  | 6.303 \( \pm \) 2.55 |
| \( p / \pi^+ \) | 1.088 | 1.098 \( \pm \) 0.127 |
| \( \langle K \rangle / \langle \pi \rangle \) | 7.299 \( \times \) 10\(^{-2}\) | 8.080 \( \times \) 10\(^{-2}\) \( \pm \) 1.420 \( \times \) 10\(^{-2}\) |

(\( \text{CL} \approx 57\%) \) imposing strangeness conservation is at \( T = 182 + 19 - 8 \) MeV. The minimum of \( \chi^2 \) (\( \chi^2 / \text{DOF} = 1.6 / 3 = 0.53, \text{CL} 66\% \)) is at \( T = 186 \pm 12 \) MeV.

3.1.3. Central nucleus + nucleus collisions at \( \sim 10 \) A GeV. Here we show results from central Si + Au collisions at 14.6 A GeV, and from Au + Au collisions at 11.6 A GeV.

Central Si + Au collisions at 14.6 A GeV. In table 7 the results for central Si + Au collisions at 14.6 A GeV are shown. We use the ratios \( K / \pi, K^+ / K^- \), \( \Lambda / \Lambda \) and \( \phi / \pi \) from [9] as well as strangeness conservation. We did not use \( \bar{p} \) in view of the large annihilation expected at low energy. The resulting \( \chi^2 / \text{DOF} = 6.8 / 2 = 3.4 \) (\( \text{CL} \sim 3.3\% \)) imposing strangeness conservation is at \( T = 120 + 8 - 35 \) MeV. We note, however, that the best \( \chi^2 \) ignoring strangeness conservation (\( \chi^2 / \text{DOF} = 0.092 / 2, \text{CL} 63\% \)) is at \( \mu_b / T = 4.05, \mu_s / T = 0.9 \) and \( T = 102 \) MeV.

Central Au + Au collisions at 11.6 A GeV. The results for central Au + Au collisions at 11.6 A GeV are shown in tables 8 and 9. We used \( K^+ / K^- \), \( K / \pi, p / \pi^+ \) and \( K / \Lambda \) ratios from [9]. The resulting \( \chi^2 / \text{DOF} = 0.99 / 2 = 0.496 \) (\( \text{CL} \sim 60\% \)) imposing strangeness conservation is at \( T = 96 + 4 - 5 \) MeV. The minimum of \( \chi^2 \) (\( \chi^2 / \text{DOF} = 0.92 / 2 = 0.46, \text{CL} 63\% \)) is at \( T = 95 \) MeV.

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Table 9. AGS Au + Au at $\sqrt{s} = 4.9$ GeV, most central events. Thermodynamic parameters for the best fit, and temperatures and $\lambda_s$ extrapolated to zero fugacities.

| $\mu_b$ (GeV) | $\mu_s$ (GeV) | $T$ (GeV) | $\lambda_s$ | $\rho_e$ (GeV fm$^{-3}$) | $T_{eq,\rho_e}$ (GeV) | $\lambda_s(eq, \rho_e)$ |
|---------------|---------------|-----------|-------------|------------------------|---------------------|---------------------|
| 0.563         | 0.084         | 0.096 ± 0.005 | 0.281       | 0.484 × 10$^{-1}$    | 0.121 ± 0.008     | 0.233 ± 0.037     |
| $\rho_n$ (fm$^{-3}$) | $T_{eq,\rho_n}$ (GeV) | $\lambda_s(eq, \rho_n)$ |
| 0.602 × 10$^{-1}$ | 0.113 ± 0.0079 | 0.196 ± 0.037 |
| $\rho_s$ (fm$^{-3}$) | $T_{eq,\rho_s}$ (GeV) | $\lambda_s(eq, \rho_s)$ |
| 0.387         | 0.116 ± 0.008 | 0.210 ± 0.037 |

Table 10. GSI Ni + Ni at $\sqrt{s} = 2.6$ GeV, most central events. Predicted versus experimental particle ratios for the best fit of our model.

| Ratio         | Model  | Data                      |
|---------------|--------|---------------------------|
| $\langle K \rangle / \Lambda$ | 0.373  | 0.275 ± 6.72 × 10$^{-2}$  |
| $K^+ / K^-$   | 20.90  | 20.86 ± 6.88              |
| $\langle K \rangle / \langle \pi \rangle$ | 3.16 × 10$^{-3}$ | 3.20 × 10$^{-3} ± 9.10 \times 10^{-4}$ |

3.1.4. Central nucleus + nucleus collisions at $\sim 2$ A GeV.

Central Ni + Ni collisions at 1.9 A GeV. The results for central Ni + Ni collisions at 1.8 A GeV are shown in tables 10 and 11. We use the measured $K^+ / K^-$ and $K / \pi$ ratios from [26, 27], and additionally use the $K / \Lambda$ ratio deduced as $\Lambda \sim \Lambda - \bar{\Lambda} = 2(K^+ - K^-)$ and impose strangeness conservation. We add a systematic error of 15% for the feeding experimental uncertainty linearly. The resulting $\chi^2/DOF = 2.16/1$ (CL $\sim 10\%$) imposing strangeness conservation is at $T = 0.044 \pm 0.0023 - 0.0002$ GeV. The minimum of $\chi^2(\chi^2/DOF = 0.025/1$ with CL better than 90%) is at $T = 0.0448$ GeV.

3.2. Electron + positron reactions

The primary $q\bar{q}$ production in $e^+ + e^-$ collisions is not thermal due to the hard nature of the primary $\gamma$, Z couplings. However, as long as the final particle multiplicity is much higher than two, it is conceivable that this fact does not matter and the subsequent fragmentation of quark–antiquark jets into hadrons is thermal. We use the $K / \pi$, $\rho / \pi$, $\pi / p$ and $\Delta / \pi$ ratios.

Our results for $e^+ + e^-$ collisions at $\sqrt{s} = 91$ GeV are shown in table 12. We use the data from [13], in particular the initially produced particles from the Pei model (table 1 in [13]). We use the difference between the Pei and the Jetset model (also in table 1 in [13]) as the systematic error of the feeding correction performed in this model. The resulting
Table 11. GSI Ni + Ni at $\sqrt{s} = 2.6$ GeV, most central events. Thermodynamic parameters for the best fit, and temperatures and $\lambda_s$ extrapolated to zero fugacities.

| $\mu_b$ (GeV) | $\mu_s$ (GeV) | $T$ (GeV) | $\lambda_s$ | $\rho_c$ (GeV fm$^{-3}$) | $T_{eq,\rho_c}$ (GeV) | $\lambda_s(\text{eq,} \rho_c)$ (GeV fm$^{-3}$) |
|---------------|---------------|-----------|--------------|--------------------------|----------------------|--------------------------------|
| 0.678         | 0.102         | 0.044     | $8.93 \times 10^{-3}$ | 0.987 $\times 10^{-3}$ | 0.602 $\times 10^{-1}$ | 1.55 $\times 10^{-2} +0.0435\pm5.55\times10^{-3}$ |
| $\pm0.0023$   | $\pm0.00335$  |           |              |                          | $\pm0.0114$            |                                                 |

| $\rho_n$ (fm$^{-3}$) | $T_{eq,\rho_n}$ (GeV) | $\lambda_s(\text{eq,} \rho_n)$ |
|----------------------|------------------------|---------------------------------|
| $0.161 \times 10^{-2}$ | $0.504 \times 10^{-1}$ | $5.16 \times 10^{-3} +0.0144\pm1.84\times10^{-3}$ |
| $\pm0.0096$          |                        |                                  |

| $\rho_s$ (fm$^{-3}$) | $T_{eq,\rho_s}$ (GeV) | $\lambda_s(\text{eq,} \rho_s)$ |
|----------------------|------------------------|---------------------------------|
| $1.18 \times 10^{-2}$ | $5.32 \times 10^{-2}$ | $7.38 \times 10^{-3} +0.0207\pm2.64\times10^{-3}$ |
| $\pm0.0104$          |                        |                                  |

Table 12. $e^+ + e^-$ collisions at $\sqrt{s} = 91$ GeV. Thermodynamic parameters for the best fit, and temperatures and $\lambda_s$ extrapolated to zero fugacities.

| $\mu_b$ (GeV) | $\mu_s$ (GeV) | $T$ (GeV) | $\lambda_s$ | $\rho_c$ (GeV fm$^{-3}$) | $\rho_n$ (fm$^{-3}$) | $\rho_s$ (fm$^{-3}$) |
|---------------|---------------|-----------|--------------|--------------------------|----------------------|----------------------|
| 0             | 0             | 0.145$^{+0.039}_{-0.045}$ | 0.338$^{+0.127}_{-0.20}$ | 0.152                    | 0.208                | 1.25                 |

minimum $\chi^2/DOF = 2.52/3 = 0.84$ (CL $\sim 47\%$) is at $T = 145 + 39 - 45$ MeV and $\lambda_s$ is $0.338 + 0.127 - 0.2$.

In [43] a temperature of $160.6 \pm 1.7 \pm 3.1$ is given for $e^+ + e^-$ collisions at 91 GeV, but however with an unacceptable $\chi^2/DOF$ of 60.8/21, having a CL of $10^{-5}$. In this reference, the strangeness has been weighted by a factor $\gamma_s$ which is a free parameter of the fit. In [13] a temperature of $T = 142.4 \pm 0.018$ MeV and $\lambda_s = 0.295 \pm 0.006$ are extracted in $e^+ + e^-$ collisions at 91 GeV. Within our large errors we agree with both [43] and [13]. Our best value of the temperature is, however, nearer to the results of [13].

3.3. Hadronic reactions

3.3.1. Proton + proton reactions. Here we show results from p + p collisions at $\sqrt{s} = 17$ and 27 GeV.

Proton + proton reactions at $\sqrt{s} = 27$ GeV. For p + p collisions at $\sqrt{s} = 27$ GeV we use nine measured ratios from [43], namely $K/\pi$, $p/p$, $\pi/p$, $\eta/\pi^0$, $\Lambda/K^0$, $K^+/K^-$, $\Xi/\Lambda$, $\phi/\pi$ and $\Delta^{++}/p$, and impose strangeness conservation. We add a systematic error of 15% linearly to account for the experimental feeding uncertainties. The predicted and the experimental ratios are shown in

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Table 13. $p+p$ collisions at $\sqrt{s} = 27$ GeV. Predicted versus experimental particle ratios for the best fit of our model. The $\chi^2/DOF$ of this fit is not acceptable.

| Ratio | Model | Data |
|-------|-------|------|
| $K/\pi$ | 0.120 | $7.35 \times 10^{-2} \pm 1.80 \times 10^{-2}$ |
| $p/p$ | $2.65 \times 10^{-2}$ | $5.25 \times 10^{-2} \pm 1.36 \times 10^{-2}$ |
| $\pi/p$ | 5.51 | 3.14 $\pm$ 0.846 |
| $\eta/\pi^0$ | $6.89 \times 10^{-2}$ | $7.75 \times 10^{-2} \pm 3.15 \times 10^{-2}$ |
| $\Lambda/K^0_S$ | 0.533 | 0.539 $\pm$ 0.158 |
| $K^+/K^-$ | 1.52 | 1.48 $\pm$ 0.364 |
| $X/\Lambda$ | $3.88 \times 10^{-2}$ | 0.160 $\pm$ 6.75 $\times 10^{-2}$ |
| $\phi/\pi$ | $1.07 \times 10^{-2}$ | 5.034 $\times 10^{-3} \pm 1.34 \times 10^{-3}$ |
| $\Delta^{++}/p$ | 0.148 | $0.182 \pm 4.97 \times 10^{-2}$ |

Table 14. $p+p$ collisions at $\sqrt{s} = 27$ GeV. Thermodynamic parameters for the best fit, and temperatures and $\lambda_s$ extrapolated to zero fugacities. The $\chi^2/DOF$ of this fit is not acceptable.

| $\mu_b$ (GeV) | $\mu_s$ (GeV) | $T_{eq,\rho_n}$ (fm$^{-3}$) | $\lambda_s(eq,\rho_n)$ |
|----------------|----------------|-----------------------------|------------------------|
| $0.243$ | $0.0282$ | $0.128^{+0.005}_{-0.009}$ | $0.329$ |
| $0.830 \times 10^{-1}$ | $0.132^{+0.005}_{-0.010}$ | $0.283^{+0.0218}_{-0.045}$ |
| $\rho_n$ (fm$^{-3}$) | $T_{eq,\rho_n}$ (GeV) | $\lambda_s(eq,\rho_n)$ |
| $0.123$ | $0.131^{+0.005}_{-0.010}$ | $0.279^{+0.0215}_{-0.044}$ |
| $\rho_s$ (fm$^{-3}$) | $T_{eq,\rho_s}$ (GeV) | $\lambda_s(eq,\rho_s)$ |
| $0.744$ | $0.132^{+0.005}_{-0.010}$ | $0.283^{+0.0218}_{-0.045}$ |

Table 13. We find a temperature of $128 \pm 5 - 9$ MeV, with a $\chi^2/DOF = 71/5 = 14.2$, which has a CL of the order $10^{-14}$, therefore the fit is not acceptable (table 14).

In [28] the authors discuss $p+p$ collisions at four different values of $\sqrt{s}$. For $\sqrt{s} = 27$ GeV considered here, the obtained $\chi^2/DOF$ is 136.4/27 with a CL much less than $10^{-5}$. The obtained temperature in [28] is $169 \pm 2.1 \pm 3.4$.

The bad quality of the fits of $p+p$ collisions at $\sqrt{s} = 27$ GeV renders the resulting temperatures questionable. Nevertheless, we point out that the two analyses yield incompatible temperature values within the errors. The analysis of [28] does not introduce systematic errors as we do. However, it allows for an arbitrary weighting factor (called $\gamma_s$) acting only on strange particles. This factor is similar to an increase of the systematic experimental errors of strange particle multiplicities only.
Table 15. p + p collisions at $\sqrt{s} = 17$ GeV. Predicted versus experimental particle ratios for the worst fit of our model.

| Ratio   | Model  | Data               |
|---------|--------|--------------------|
| $K/\pi$ | 0.136  | $5.97 \times 10^{-2} \pm 9.90 \times 10^{-3}$ |
| $\bar{p}/p$ | $5.97 \times 10^{-2}$ | $4.68 \times 10^{-2} \pm 8.06 \times 10^{-3}$ |
| $\pi/p$ | 4.261  | $3.59 \pm 0.558$   |
| $K^+/K^-$ | 1.55   | $1.61 \pm 0.277$   |
| $\phi/\pi$ | 1.83   | $4.25 \times 10^{-3} \pm 1.17 \times 10^{-3}$ |
| $K/p$   | 0.578  | $0.214 \pm 3.56 \times 10^{-2}$   |

Table 16. p + p collisions at $\sqrt{s} = 17$ GeV. Predicted versus experimental particle ratios for the best fit of our model. Only three ratios are fitted ($\phi/\pi$, $K^+/K^-$, $K/\pi$).

| Ratio   | Model  | Data               |
|---------|--------|--------------------|
| $K/\pi$ | 0.0723 | $5.97 \times 10^{-2} \pm 9.90 \times 10^{-3}$ |
| $\bar{p}/p$ | $5.36 \times 10^{-2}$ | $4.68 \times 10^{-2} \pm 8.06 \times 10^{-3}$ |
| $\pi/p$ | 47.7   | $3.59 \pm 0.558$   |
| $K^+/K^-$ | 1.60   | $1.61 \pm 0.277$   |
| $\phi/\pi$ | $2.06 \times 10^{-3}$ | $4.25 \times 10^{-3} \pm 1.17 \times 10^{-3}$ |
| $K/p$   | 3.44   | $0.214 \pm 3.56 \times 10^{-2}$   |

Proton + proton reactions at $\sqrt{s} = 17$ GeV. We use six measured ratios from [29, 30], namely $K/\pi$, $\bar{p}/p$, $\pi/p$, $K^+/K^-$, $\phi/\pi$ and $K/p$, and impose strangeness conservation. We add a systematic error of 15% linearly to account for the experimental feeding uncertainties. The resulting $\chi^2/DOF$ is $(322/4) = 80.6$ (CL of the order $10^{-66}$) for a temperature of 144 MeV is completely unacceptable.

The predicted ratios in table 15 overestimate the $K^+$, $K^-$ and $\phi$ yields, therefore suggesting that if thermal conditions prevail, then at least two thermal reservoirs are present. This may be due to the influence of diffractive processes in minimum bias triggers.

We therefore repeat the fit with only ratios which are not much influenced by diffractive processes, namely $K/\pi$, $K^+/K^-$ and $\phi/\pi$. Note that the temperature at the minimum of $\chi^2$ does not satisfy strangeness conservation. This is compatible with the hypothesis of the importance of diffractive processes showing up in leading baryons (e.g. p, $\Lambda$). Imposing strangeness conservation, the fit results in a temperature of 144 MeV with a bad $\chi^2/DOF = 169/1$ (CL $\approx 10^{-39}$). At the minimum of $\chi^2$, the temperature is $96 \pm 8 - 9$ MeV with a better $\chi^2/DOF = 5.13/1$ (CL $\approx 2\%$). These results are shown in tables 16 and 17. The very small errors resulting for $p + p$ collisions at $\sqrt{s} = 17$ and 27 GeV should not be trusted because of the bad CL of the fit.

It would be important to impose a centrality cut on the $p + p$ data and give the particle yields and, in particular, the net strangeness in the full phase space acceptance and/or in a region near midrapidity.
Table 17. p + p collisions at $\sqrt{s} = 17$ GeV. Thermodynamic parameters for the best fit, and temperatures and $\lambda_s$ extrapolated to zero fugacities. Only three ratios are fitted ($\phi/\pi$, $K^+/K^-$, $K/\pi$).

| $\rho_b$ (GeV) | $\rho_s$ (GeV) | $T$ (GeV) | $\lambda_s$ | $T_{eq,\rho_b}$ (GeV) | $\lambda_s(eq, \rho_b)$ | $\rho_n$ (fm$^{-3}$) | $T_{eq,\rho_n}$ (GeV) | $\lambda_s(eq, \rho_n)$ |
|----------------|----------------|-----------|-------------|----------------------|---------------------|----------------|------------------|----------------|
| 0.222          | 0.0277         | 0.096$^{+0.008}_{-0.009}$ | 0.151       | 0.0124              | 0.121$^{+0.034}_{-0.039}$ | 0.0293         | 0.096$^{+0.008}_{-0.010}$ | 0.121$^{+0.034}_{-0.039}$ |
|                |                |           |             |                      |                     |                |                  |                  |

Table 18. p + $\bar{p}$ collisions at $\sqrt{s} = 900$ GeV. Predicted versus experimental particle ratios for the best fit of our model.

| Ratio          | Model     | Data                          |
|----------------|-----------|-------------------------------|
| $K^0_s$/charged| $5.73 \times 10^{-2}$ | $3.85 \times 10^{-2} \pm 1.06 \times 10^{-2}$ |
| $n$/charged    | $2.17 \times 10^{-2}$ | $2.81 \times 10^{-2} \pm 1.25 \times 10^{-2}$ |
| $\Lambda/K^0_s$| 0.181     | 0.277 $\pm$ 0.114           |
| $\Lambda/\Xi^-$| 7.96      | 10.86 $\pm$ 8.344         |
| $\Xi^-$/charged| $1.305 \times 10^{-3}$ | $9.83 \times 10^{-4} \pm 7.12 \times 10^{-4}$ |

3.3.2. Proton + antiproton reactions. Here we show results from p + $\bar{p}$ collisions at $\sqrt{s} = 900$ GeV and central and peripheral p + $\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV.

Central proton + antiproton reactions at $\sqrt{s} = 900$ GeV. For p + $\bar{p}$ collisions at $\sqrt{s} = 900$ GeV we use five measured ratios out of the multiplicities from [28]. In particular, $K^0_s$/charged and $N$/charged, $\Lambda/K^0_s$, $\Lambda/\Xi^-$, $\Xi^-$/charged. The predicted and the experimental ratios are shown in table 18. The temperature from the fit is:

$$T(p + \bar{p}, \sqrt{s} = 900 \text{ GeV}) = 143 + 21 - 37 \text{ MeV}$$

with a $\chi^2/DOF$ of 4.48/4 having a CL of ~34%, while $\lambda_s$ is

$$\lambda_s(p + \bar{p}, \sqrt{s} = 900 \text{ GeV}) = 0.330 + 0.077 - 0.166.$$  

We add linearly a systematic error of 15% to account for experimental feeding uncertainties. The experimental errors are very large (e.g. 57% for $\Xi^-$), so therefore the resulting temperature also has a large error.

Peripheral proton + antiproton reactions at $\sqrt{s} = 1.8$ GeV. For the most peripheral p + $\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV we use two measured ratios from [31], namely $K/\pi$ and $p/\pi$.
Table 19. Most peripheral $p + \bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV. Predicted versus experimental particle ratios for the best fit of our model.

| Ratio | Model          | Data               |
|-------|----------------|--------------------|
| $K/\pi$ | 0.144          | $0.115 \pm 1.20 \times 10^{-2}$ |
| $\bar{p}/\pi$ | $6.87 \times 10^{-2}$ | $7.40 \times 10^{-2} \pm 9.00 \times 10^{-3}$ |

Table 20. Most central $p + \bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV. Predicted versus experimental particle ratios for the best fit of our model.

| Ratio | Model          | Data               |
|-------|----------------|--------------------|
| $K/\pi$ | 0.1424         | $0.1420 \pm 2.00 \times 10^{-2}$ |
| $\bar{p}/\pi$ | $6.24 \times 10^{-2}$ | $6.20 \times 10^{-2} \pm 9.00 \times 10^{-3}$ |

We use the only two available ratios from experiment, which with zero fugacities and temperature as the only parameter leaves one degree of freedom. The most peripheral collisions are defined as those with the lowest measured charge multiplicity. The temperature from the fit is

$$T(\text{peripheral } p + \bar{p}, \sqrt{s} = 1.8 \text{ TeV}) = 140 \pm 8 \text{ MeV}$$

with a $\chi^2/DOF$ of 18.04/1 having a CL of $\sim 2.0 \times 10^{-4}$, so therefore the fit is very bad. We add a systematic error of 0.004 linearly to account for the experimental feeding uncertainties. This error has been estimated from deviations between the ratios found in different experimental runs with $p + \bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV [44]. The predicted and the experimental ratios are shown in table 19. Then the resulting temperature is $154 \pm 9$ MeV and the $\chi^2/DOF$ is 6.36/1 having a CL of $\sim 96\%$. The quality of the fit is low as in the minimum bias $p + p$ data at lower energies. $\lambda_s$ is

$$\lambda_s(\text{peripheral } p + \bar{p}, \sqrt{s} = 1.8 \text{ TeV}) = 0.372 + 0.031 - 0.034.$$ 

Central proton + antiproton reactions at $\sqrt{s} = 1.8$ TeV. The results for the most central $p + \bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV are shown in table 20. The most central collisions are defined as those with the highest measured charge multiplicity. For the most central $p + \bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV we use again the two measured ratios from [31], namely $K/\pi$ and $\bar{p}/\pi$. The predicted and the experimental ratios are shown in table 20. The temperature from the fit is

$$T(\text{central } p + \bar{p}, \sqrt{s} = 1.8 \text{ TeV}) = 150 \pm 9 \text{ MeV}$$

with a $\chi^2/DOF$ of $2.28 \times 10^{-3}/1$ having a CL of $\sim 96\%$. $\lambda_s$ is

$$\lambda_s(\text{central } p + \bar{p}, \sqrt{s} = 1.8 \text{ TeV}) = 0.357 + 0.035 - 0.033.$$ 

We did not add a systematic error.

We think that the dramatic change in the quality of the fit between peripheral and central $p + \bar{p}$ collisions cannot be explained by overestimated experimental errors in the central collisions, respectively by underestimated experimental errors in the peripheral collisions, alone. Barring the obvious possibility that peripheral $p + \bar{p}$ collisions are non-thermal, the situation supports...
Table 21. $\chi^2/DOF$ and the confidence level for each collision system analysed. The reactions are taken with central trigger unless stated otherwise. Strangeness conservation (SC) is taken into account in the second row while it is not in the third row.

| Collision       | $\chi^2/DOF$ (CL)       | $\chi^2/DOF$ (CL) no SC |
|-----------------|-------------------------|-------------------------|
| Au + Au $\sqrt{s} = 130$ GeV |
| with no systematic error | 1.41/1 (23%) | 11.9/11 = 1.08 (50%) |
| with systematic error 15% | 0.27/1 (60%) | |
| Pb + Pb $\sqrt{s} = 17$ GeV |
| No fit performed | 1.95/3 = 0.65 (57%) | 1.6/3 = 0.53 (66%) |
| S + A $\sqrt{s} = 19$ GeV |
| | 6.8/2 = 3.4 (3.3%) | 0.092/2 = 0.046 (63%) |
| Au + Au $\sqrt{s} = 4.9$ GeV |
| | 0.99/2 = 0.50 (60%) | 0.92/2 = 0.46 (63%) |
| Si + Au $\sqrt{s} = 5.4$ GeV |
| Ni + Ni $\sqrt{s} = 2.6$ GeV |
| p + p $\sqrt{s} = 17$ GeV |
| minimum bias (3 ratios) | 169/1 ($10^{-39}$) | 5.13/1 (2%) |
| p + p $\sqrt{s} = 27$ GeV |
| minimum bias | 71/5 = 14.2 ($10^{-14}$) | 68.6/5 = 13.7 ($10^{-13}$) |
| p + $\bar{p}$ $\sqrt{s} = 900$ GeV |
| minimum bias | 4.48/4 (34%) | |
| p + $\bar{p}$ $\sqrt{s} = 1.8$ TeV |
| peripheral, no systematic error | 2.3 × 10^{-3}/1 (96%) | |
| peripheral, with systematic error | 18/1 (2 × 10^{-4}) | |
| e^+ + e^- $\sqrt{s} = 91$ GeV |
| minimum bias | 6.36/1 (1.1%) | |

our previous conjecture on the important role of diffractive processes in peripheral and minimum bias hadronic collisions.

A summary of the values of $\chi^2/DOF$ and the confidence levels for each collision system analysed is shown in Table 21.

3.4. The initial energy density estimation

To estimate the initial energy density achieved in each collision we use several methods. The first method (α) is based on Bjorken’s formula [45]:

$$\epsilon_{in} = \frac{(dE_T/dy)_{ycm}}{\pi R_T^2 \tau}$$  (13)

where $(dE_T/dy)_{ycm}$ is the transverse energy $(E_T = \sqrt{p_T^2 + m^2})$ per unit rapidity at midrapidity and $R_T$ is the transverse radius of the particle source after a formation time $\tau$ taken to be 1 fm/c.
We estimate \((dE_T/dy)_{ycm}\) when not available from experiment as follows. We first estimate the total \(E_T\) as
\[
(dE_T/dy)_{ycm}^{tot} = \sum_{\alpha} \left[ (dN/dy)_{\alpha}(ycm) \sqrt{\langle p_T \rangle_{\alpha}^2 + m_{\alpha}^2} \right] (\text{method } (\alpha))
\]
where the sum runs over particles of type \(\alpha\) with multiplicity per rapidity unit at midrapidity \((ycm)\) \((dN/dy)_{\alpha}\), mean transverse momentum \(\langle p_T \rangle_{\alpha}\) and mass \(m_{\alpha}\). The mean transverse momentum is also taken at midrapidity, when available. We take three types of particles, \(\pi\), \(K\) and nucleons, as well as antinucleons when available. To calculate the differential \((dN/dy)_{ycm}\), when not available, we first find the mean \((dN/dy)\), dividing the total particle multiplicity by the total rapidity interval, and then we use an extrapolation factor \(A\) to extract the value at midrapidity. This factor is found from measured rapidity distributions of particles at or as near as possible to the \(\sqrt{s}\) considered. This may vary from nuclear to hadronic reactions and with \(\sqrt{s}\). In the cases where \(dN/dy\) of the particles is measured at midrapidity we use these values.

The formation time is taken in the literature usually in all reactions as 1 fm/c, and this is what we use here. In the following we give some examples of our \(\epsilon_{in}\) estimation with method \((\alpha)\).

1. For the \(e^+ + e^-\) collisions at \(\sqrt{s} = 91\) GeV we use the multiplicities from [13].

For two jet events in \(e^+ + e^-\) collisions the jet axis is the longitudinal axis defining rapidity and \(p_T\) refers to it. We use \(dN/dy\) at midrapidity for \(\pi, K\) and \(p\) in quark jets produced in \(e^+ + e^-\) collisions at \(\sqrt{s} = 91\) GeV by the DELPHI collaboration [46].

We use the mean transverse momenta from hadron + hadron collisions at high \(\sqrt{s}\). In particular, we use the mean transverse momenta from \(p + \bar{p}\) collisions at 1.8 TeV from [31] at a charged multiplicity \(N \sim 45\) (to represent minimum bias values), namely
\[
\langle p_T \rangle_{\pi} = 0.34 \text{ GeV} \quad \langle p_T \rangle_{K} = 0.5 \text{ GeV} \quad \langle p_T \rangle_{p} = 0.59 \text{ GeV}.
\]

In hadron collisions the mean transverse momenta do not change much with \(\sqrt{s}\) above 10 GeV (see, e.g., figure 2.7 in [27]). The transverse size of the initial hadronic system of quarks and antiquarks produced is given by the uncertainty principle as \(1/\langle p_T \rangle\) over the average \(p_T\). For the transverse radius of \(e^+ + e^-\) collisions we take, therefore, the inverse mean \(p_T\) of 0.34 GeV, giving \(R_T = 0.6\) fm. The resulting initial energy density is
\[
\epsilon_{in}(e^+ + e^-, \sqrt{s} = 91 \text{ GeV}) = 1.84 \text{ GeV fm}^{-3}.
\]

The systematic error (which in this case is dominated by the uncertainty in the \(R_T\) determination) is about 50%.

2. For \(p + \bar{p}\) at \(\sqrt{s} = 1.8\) TeV. For \(p + \bar{p}\) we use the same method, while we use as transverse radius the radius of the nucleon of 0.8 fm. We use again pions, kaons, nucleons and antinucleons, and the mean transverse momenta measured in [31]. We derive \(dE_T/dy(y_{cm})\) from the measured charged total multiplicity per unit rapidity at midrapidity, using the ratios and mean transverse momenta measured as a function of charged multiplicity in [31]. No extrapolation factor to midrapidity is needed here. We find
\[
\epsilon_{in}(dN_c/d\eta = 19) = 7.61 \text{ GeV fm}^{-3} \quad \epsilon_{in}(dN_c/d\eta = 15) = 5.82 \text{ GeV fm}^{-3}
\]
\[\epsilon_{in}(dN_c/d\eta = 11.5) = 4.31 \text{ GeV fm}^{-3}\]
\[\epsilon_{in}(dN_c/d\eta = 5.4) = 2.03 \text{ GeV fm}^{-3}\]
\[\epsilon_{in}(dN_c/d\eta = 3) = 0.772 \text{ GeV fm}^{-3}\]

In [31] the \(\epsilon_{in}\) for \(dN_c/d\eta = 15\) of 3 GeV fm\(^{-3}\) is calculated using pions only, and therefore is lower than our estimate\(^\dagger\).

(3) \(p + \bar{p}\) at \(\sqrt{s} = 900\) GeV. For \(p + \bar{p}\) at \(\sqrt{s} = 900\) GeV we use the multiplicities from [43] and the mean transverse momenta for \(p + \bar{p}\) at \(\sqrt{s} = 1.8\) TeV from [31]. We take the mean multiplicity per unit rapidity without extrapolation factor \(A\) to midrapidity since we do not know the shape of the rapidity distributions. We find

\[\epsilon_{in}(p + \bar{p}, \sqrt{s} = 900 \text{ GeV}) = 1.23 \text{ GeV fm}^{-3}\]

The pseudorapidity distribution of charged particles produced in \(p + \bar{p}\) collisions at \(\sqrt{s} = 1.8\) TeV is relatively flat over six rapidity units as measured by CDF [54]. However, this may differ at the lower \(\sqrt{s}\) of 900 GeV. We use the extrapolation factor \(A = 1.49\) (= \(dN/dy(ycm)/dN/dy(\text{mean})\)) from the measured \(dN/dy\) distribution in [47]. The resulting \(\epsilon_{in}\) has an uncertainty of about 50%.

(4) \(Au + Au\) at \(\sqrt{s} = 130\) GeV. For \(Au + Au\) at \(\sqrt{s} = 130\) GeV we use the mean \(p_T\) from \(p + \bar{p}\) collisions at \(\sqrt{s} = 1.8\) TeV and the particle multiplicities at midrapidity (negative hadrons, total charged multiplicity, nucleons estimated from \(p - \bar{p} \sim N_{\text{charged}} - N_{\text{negatives}}\), as well as antinucleons using the \(\bar{p}/p\) ratio) from [18, 19].

With the above method (\(\alpha\)) (equations (13), (14)) we calculate the \(\epsilon_{in}\) for several reactions. We summarize the results here:

\[\epsilon_{in}(Au + Au, \sqrt{s} = 4.9 \text{ GeV}) = 0.86 \text{ GeV fm}^{-3}\]

with \(m_T - m_0\) instead of \(m_T\) for nucleons,

\[\epsilon_{in}(Si + Au, \sqrt{s} = 5.4 \text{ GeV}) = 0.53 \text{ GeV fm}^{-3}\]

with \(m_T - m_0\) instead of \(m_T\) for nucleons.

In the following we use \(m_T\) for nucleons too,

\[\epsilon_{in}(Au + Au, \sqrt{s} = 4.9 \text{ GeV}) = 1.70 \text{ GeV fm}^{-3}\]
\[\epsilon_{in}(Si + Au, \sqrt{s} = 5.4 \text{ GeV}) = 1.53 \text{ GeV fm}^{-3}\]
\[\epsilon_{in}(S + S, \sqrt{s} = 19 \text{ GeV}) = 1.21 \text{ GeV fm}^{-3}\]
\[\epsilon_{in}(Pb + Pb, \sqrt{s} = 17 \text{ GeV}) = 2.4 \text{ GeV fm}^{-3}\]
\[\epsilon_{in}(Au + Au, \sqrt{s} = 130 \text{ GeV}) = 6.34 \text{ GeV fm}^{-3}\]
\[\epsilon_{in}(p + \bar{p}, \sqrt{s} = 900 \text{ GeV}) = 1.23 \text{ GeV fm}^{-3}\]
\[\epsilon_{in}(p + \bar{p}, \sqrt{s} = 1.8 \text{ TeV most central}) = 7.61 \text{ GeV fm}^{-3}\]
\[\epsilon_{in}(p + \bar{p}, \sqrt{s} = 1.8 \text{ TeV most peripheral}) = 0.77 \text{ GeV fm}^{-3}\]

\(^\dagger\) We find the same \(\epsilon_{in}\) as [31], when we use only pions.

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\[ \epsilon_{in}(p + p, \sqrt{s} = 17 \text{ GeV}) = 0.42 \text{ GeV fm}^{-3} \]
\[ \epsilon_{in}(e^+ + e^-, \sqrt{s} = 91 \text{ GeV}) = 1.84 \text{ GeV fm}^{-3}. \]

The initial energy density has been estimated by experimenters for some reactions shown here, using another method (\(\beta\)), namely the Bjorken formula (equation (13)) and the measured transverse energy:
\[ E_T(\text{lab}) = (E \sin \theta)_\text{lab} \quad (\text{method (\(\beta\))}) \quad (15) \]
where \(E\) is the total energy measured with, for example, calorimeters and \(\theta\) is the angle to the incident beam direction. The resulting values (used in figures 1, 2 and 5 in the following section) are:
\[ \epsilon_{in}(\text{Pb} + \text{Pb}, \sqrt{s} = 17 \text{ GeV}) = 3.2 \text{ GeV fm}^{-3} \text{ [48]} \]
\[ \epsilon_{in}(\text{S} + \text{Au}, \sqrt{s} = 19 \text{ GeV}) = 2.6 \text{ GeV fm}^{-3} \text{ [48, 49]} \]
\[ \epsilon_{in}(\text{S} + \text{S}, \sqrt{s} = 19 \text{ GeV}) = 1.3 \text{ GeV fm}^{-3} \text{ [48, 49]} \]
\[ \epsilon_{in}(\text{Au} + \text{Au}, \sqrt{s} = 4.9 \text{ GeV}) = 1.3 \text{ GeV fm}^{-3} \text{ [50]} \]
\[ \epsilon_{in}(\text{Si} + \text{Au}, \sqrt{s} = 5.4 \text{ GeV}) = 0.9 \text{ GeV fm}^{-3} \text{ (see [16] and references therein).} \]

We estimate the maximal initial energy density with a third method (\(\gamma\)), taking the nuclear energy density of two overlapping nuclei \(2\epsilon_A\) times the \(\gamma\) factor of the colliding particles in the centre of mass minus one:
\[ \epsilon_{\gamma} = 2\epsilon_A(\gamma - 1) \quad (\text{method (\(\gamma\))}) \quad (16) \]
with \(\gamma = (\sqrt{s}/2)/m_{\text{nucleon}},\) and \(\epsilon_{A,small} = 0.179 \text{ GeV fm}^{-3}\) for small nuclei and \(\epsilon_{A,big} = 0.138 \text{ GeV fm}^{-3}\) for large nuclei is the normal nuclear matter density. The value in equation (16) multiplied by the stopping power gives an estimate of the initial energy density available for heating.

We apply this method to the \(\text{Si} + \text{Au}\) and \(\text{Au} + \text{Au}\) collisions at \(\sqrt{s} \sim 5 \text{ GeV},\) \(\text{Ni} + \text{Ni}\) collisions at \(\sqrt{s} = 2.6 \text{ GeV}\) and \(\text{Pb} + \text{Pb}\) collisions at \(\sqrt{s} = 17 \text{ GeV},\) to calculate the maximal achieved initial energy density, yielding:
\[ \epsilon_{in}(\text{Au} + \text{Au}, \sqrt{s} = 4.9 \text{ GeV}) = 0.44 \text{ GeV fm}^{-3} \]
\[ \epsilon_{in}(\text{Si} + \text{Au}, \sqrt{s} = 5.4 \text{ GeV}) = 0.67 \text{ GeV fm}^{-3} \]
\[ \epsilon_{in}(\text{Ni} + \text{Ni}, \sqrt{s} = 2.6 \text{ GeV}) = 0.138 \text{ GeV fm}^{-3} \]
\[ \epsilon_{in}(\text{Pb} + \text{Pb}, \sqrt{s} = 17 \text{ GeV}) = 2.25 \text{ GeV fm}^{-3}. \]

A few comments are in order.

(1) A problem with method (\(\alpha\)) is that, in general, it underestimates \(E_T\) slightly, since not all particles are taken into account.

(2) A general problem arises with equation (15) of method (\(\beta\)): for particles produced near midrapidity \(E \times \sin \theta\) is approximately \(p_T\) not \(E_T\), for \(\gamma_{\text{cm}} \gg 1\), whereas \(E \times \sin \theta\) is approximately \(m_T\) for particles moving non-relativistically in the laboratory frame.

(3) A problem with method (\(\gamma\)) is that it assumes geometrical and not dynamical compression, possibly slightly underestimating \(\epsilon_{in}\).

† The first three values here are valid for head-on collisions [48].

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Figure 2. Temperature at chemical freeze-out extrapolated to zero fugacities along an isentropic path as a function of the initial energy density for several nucleus + nucleus, hadron + hadron and lepton + lepton collisions. For the fits we demand a confidence level greater than 1%.

(4) The methods (α) and (β) should in principle give similar results. Our $\epsilon_{in}$ (method (α)) agrees well with the calorimetric estimation of NA35 for $S + S$ at $\sqrt{s} = 19$ GeV and of NA49 for $Pb + Pb$ at $\sqrt{s} = 17$ GeV [48, 49], for the 5% $\sigma_{tot}$ centrality trigger. The calorimetric estimation is taken to be $\epsilon_{in}(Pb + Pb) \sim 0.77 \times 3.2$ GeV fm$^{-3}$ = 2.46 GeV fm$^{-3}$ where $0.77 = (dE_T/d\eta)(5%\sigma_{tot})/(dE_T/d\eta)$ (head-on collisions) from [48]. This correction is needed because $\epsilon_{in} = 3.2$ GeV fm$^{-3}$ is estimated for head-on collisions, while our calculation uses $m_T$ and $d/dy$ taken with the 5% $\sigma_{tot}$ centrality trigger. Additionally, method (γ)—and therefore all three methods (α), (β) and (γ)—give similar results for $Pb + Pb$ collisions at $\sqrt{s} = 17$ GeV.

(5) We find, however, different $\epsilon_{in}$ values for the low $\sqrt{s}$ reactions $Si + Au$ and $Au + Au$ at 5.4 and 4.9 GeV, for each one of the above methods with a scatter of a factor of two or more. Also the maximal estimated $\epsilon_{in}$ with method (γ) is lower than the results from methods (α) and (β). We therefore conclude that the Bjorken estimate may not be adequate for low-energy reactions. This estimate was not meant to be used in the non-relativistic regime. The method (γ) seems more adequate for these systems. Nevertheless, we show two distinct cases in the following figures. Using method (γ) $\epsilon_{in}$ is slightly higher for $Si + Au$ at 5.4 GeV than for $Au + Au$ at 4.9 GeV $\sqrt{s}$, unlike the results of methods (α) and (β). The stopping power which is missing in equation (16) can, however, hardly be very different for these two reactions, but may have an influence on the ordering of $\epsilon_{in}$ in the above two reactions.

3.4.1. Systematic error estimation summary. Comparing results from methods (α) and (γ) we estimate the systematic error in $Si + Au$ and $Au + Au$ collisions at $\sqrt{s} = 5.4$ and 4.9 GeV to be about 50%, mainly resulting from the difficulty in applying the Bjorken formula. For
Figure 3. The number of strange antiquarks over the mean number of light ones at chemical freeze-out and for zero fugacities as a function of the initial energy density for several nucleus + nucleus, hadron + hadron and lepton + lepton collisions. For the fits we demand a confidence level greater than 1%.

Figure 4. Temperature at chemical freeze-out and for zero fugacities as a function of the initial energy density for several nucleus + nucleus, hadron + hadron and lepton + lepton collisions. The initial energy density has been estimated in a different way from that in figures 2 and 3 (see text). For the fits we demand a confidence level greater than 1%.
Figure 5. The number of strange antiquarks over the mean number of light ones at chemical freeze-out and for zero fugacities as a function of the initial energy density for several nucleus+nucleus, hadron+hadron and lepton+lepton collisions. The initial energy density has been estimated in a different way from that in figures 2 and 3 (see text). For the fits we demand a confidence level greater than 1%.

Figure 6. Temperature at chemical freeze-out and for zero fugacities as a function of the initial energy density for several nucleus+nucleus, hadron+hadron and lepton+lepton collisions. The initial energy density has been estimated in a different way as from that in figures 2 and 3 (see text). For the fits we demand a confidence level greater than 10%.

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Figure 7. The number of strange antiquarks over the mean number of light ones at chemical freeze-out and for zero fugacities as a function of the initial energy density for several nucleus + nucleus, hadron + hadron and lepton + lepton collisions. The initial energy density has been estimated in a different way from that in figures 2 and 3 (see text). For the fits we demand a confidence level greater than 10%.

the higher energy nucleus + nucleus collisions the systematic error is smaller at \( \sim 30\% \) [15] resulting from the estimation of \( E_T \). The systematic error for \( e^+ + e^- \) is about 50%, resulting from uncertainties in the transverse area definition. The systematic error for \( p + p \) collisions at \( \sqrt{s} = 900 \) GeV results from the \( E_T \) definition and the extrapolation to midrapidity, and is estimated to be about 50%. For \( p + \bar{p} \) collisions at the Tevatron, the uncertainty comes mainly from not taking all particles into account with method (\( \alpha \)) and is about 30%. The systematic error on the \( \epsilon_{in} \) estimate for \( \text{Ni} + \text{Ni} \) at \( \sqrt{s} = 2.6 \) GeV is 50%, resulting from comparison of our \( \epsilon_{in} \) value to model calculations [51].

3.5. The combined results: \( T, \lambda_s \) at zero fugacities

Figures 2 and 3 show the resulting temperature and the \( \lambda_s \) factor extrapolated to zero fugacities along isentropic paths, as a function of the initial energy density. The latter is taken from the experimentally measured transverse energy, when available, that is: for \( \text{Si} + \text{Au} \), \( \sqrt{s} = 5.4 \) GeV; for \( \text{Au} + \text{Au}, \sqrt{s} = 4.9 \) GeV; for \( \text{S} + \text{S}, \sqrt{s} = 19 \) GeV; \( \text{S} + \text{A}, \sqrt{s} = 19 \) GeV; and for \( \text{Pb} + \text{Pb}, \sqrt{s} = 17 \) GeV. For the remaining colliding systems, where no initial energy density estimation is available, we use our method (\( \alpha \)) based on equation (14), except for the \( \text{Ni} + \text{Ni} \) system where we use the method (\( \gamma \)). The fact that the initial energy density estimation in figures 2 and 3 was not done with the same method introduces an additional systematic error on the \( \epsilon_{in} \) scale. To reduce this uncertainty and show the systematic error of the initial energy density estimation we use in the following the \( \epsilon_{in} \) estimated for (1) high \( \sqrt{s} (>10 \) GeV) by equations (13) and (14), and for (2) low \( \sqrt{s} (<10 \) GeV) by equation (16).
Figure 8. (a) Kaon yield per interaction over the source volume at the thermal freeze-out as a function of the initial energy density ($\epsilon_{in}$) from [15, 16]. (b) Temperature at chemical freeze-out and for zero fugacities as a function of the initial energy density for several nucleus + nucleus, hadron + hadron and lepton + lepton collisions. All collision systems shown in (a) are displayed in figure (b) using the same initial energy density as defined in [15, 16]. For the fits we demand a confidence level greater than 1%.

Figures 4 and 5 therefore show the temperature and the $\lambda_s$ factor again, extrapolated to zero fugacities along isentropic paths, as a function of the initial energy density, with a different estimation of the latter. In particular, the initial energy density is not taken from the experimentally measured transverse energy, but is estimated using our method ($\alpha$) based on equation (14), with the exception of the data at low $\sqrt{s}$ ($<10$ GeV), that is, for Ni + Ni at $\sqrt{s} = 2.6$ GeV, Si + Au at $\sqrt{s} = 5.4$ GeV and Au + Au at $\sqrt{s} = 4.9$ GeV, where we used equation (16), which, as discussed in the previous section, seems more adequate for the low energies than the Bjorken estimate.

Figure 8(a) shows the number density of kaons as a function of the initial energy density from [16], while in figure 8(b) the temperature extrapolated to zero fugacities along isentropic
paths is shown as a function of the initial energy density. In figure 8(a), the collision systems p + p at \( \sqrt{s} = 17 \) GeV, S + S at \( \sqrt{s} = 19 \) GeV, Pb + Pb at \( \sqrt{s} = 17 \) GeV, Au + Au at \( \sqrt{s} = 4.9 \) GeV and Si + Au at \( \sqrt{s} = 5.4 \) GeV are shown. In figure 8(b) all the above reactions are shown and additionally the remaining analysed heavy ion collisions in the last section, namely S + A at \( \sqrt{s} = 19 \) GeV, Au + Au at \( \sqrt{s} = 130 \) GeV and Ni + Ni at \( \sqrt{s} = 2.6 \) GeV. The initial energy density of the common reactions displayed in figures 8(a) and 8(b) (that is, of all reactions shown in 8(a)) is defined in the same way, to allow their direct comparison. In particular, the initial energy densities for all heavy ion systems is taken from the experimental calorimetric measurements (method (\( \beta \)) above) when these are available. For Au + Au at \( \sqrt{s} = 130 \) GeV we use our estimate with method (\( \alpha \)). For p + p at \( \sqrt{s} = 17 \) GeV we used the estimate from [16] of \( \epsilon_{in}(p + p, \sqrt{s} = 17 \) GeV) = 0.85 GeV fm\(^{-3}\).

This is found in [16] by (a) estimating the dependence of \( \epsilon_{in} \) for Pb + Pb collisions at \( \sqrt{s} = 17 \) GeV on the number of participant nucleons \( N \) and (b) extrapolating this function to \( N = 2 \).

The rise and subsequent saturation seen in the kaon number density below and above \( \epsilon = 1.3 \) GeV fm\(^{-3}\) (figure 8(a)) shows a clear relation to the same behaviour seen in the temperature (figure 8(b)) as a function of the initial energy density.

It is apparent from figures 2–5 and 8 and the present discussion that a precise calculation of the critical energy density affecting all figures 2–8, needs more experimental data in the \( \epsilon \) region around 1 GeV fm\(^{-3}\). The errors in the temperature (figures 2 and 4) are large around this region. Another uncertainty arises from the determination of the initial energy density, especially at low \( \sqrt{s} \) where the Bjorken estimate may not be adequate. Our best estimate of the initial energy density is shown in figures 4–7. In figures 2–5 and 8 we show results demanding a confidence level greater than 1%. In figures 6 and 7 we show the results demanding a confidence level greater than 10%.

4. Conclusions

We have discussed in section 2 the dependence of the critical temperature in the QCD phase transition on the vacuum pressure including, in addition, modes of non-interacting hadron resonances for u, d, s flavours of light quarks as defining the hadronic phase. Equating the pressure of this hadronic phase to the pressure of the plasma phase, represented by non-interacting u, d, s quarks and antiquarks and gluons as shown in figure 1, we obtain for zero fugacities \( T_{cr} = 194 \pm 18 \) MeV.

We have performed a thermal analysis of yields in multiparticle production for 13 reactions summarized in table 21 and discussed in detail in section 3. The intensive thermal parameters (temperature and baryon as well as strangeness fugacity) together with an error estimate are used to extrapolate the state associated with the chemical freeze-out of each reaction, studied along curves of equal entropy, energy and number density, to zero fugacities.

We represent the states so-obtained by two intensive parameters, \( T \) and \( \lambda_s = 2\langle \bar{s} \rangle / (\langle \bar{u} \rangle + \langle \bar{d} \rangle) \), i.e. temperature and the ratio of antistrange to non-strange (valence) antiquark abundances. These two quantities as functions of the initial hadronic energy density achieved in each reaction are displayed in figures 2–5. The initial energy densities are estimated in subsection 3.4. The resulting error (horizontal error in figures 2–8) is of the order of 50% around \( \epsilon_{crit} \), for the \( e^+ + e^- \) reactions.
collisions at $\sqrt{s} = 91$ GeV and for $p + \bar{p}$ collisions at $\sqrt{s} = 900$ GeV, and approximately 30% at higher $\epsilon_{in}$.

The reactions with an estimated confidence level above 10% were retained, and they fall, within the errors, into two groups:

Group I with $\epsilon_{in} \gtrsim 1$ GeV fm$^{-3}$

1. Central Au + Au collisions at RHIC $\sqrt{s} = 130$ A GeV
2. Central Pb + Pb collisions at $\sqrt{s} = 17$ A GeV
3. Central S + A (A = Pb, Au, W) collisions at $\sqrt{s} = 19$ A GeV
4. Central S + S collisions at $\sqrt{s} = 19$ A GeV
5. $e^+ + e^-$ collisions at LEP $\sqrt{s} = 91.19$ GeV
6. $p + \bar{p}$ collisions at $\sqrt{s} = 900$ GeV
7. Central $p + \bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV

Group II with $\epsilon_{in} \lesssim 1$ GeV fm$^{-3}$

8. Central Au + Au collisions at $\sqrt{s} = 4.9$ GeV
9. Ni + Ni collisions at GSI $\sqrt{s} = 2.8$ A GeV.

Both quantities $T$ and $\lambda_s$ saturate to constant values

$$T_{lim} = 155 \pm 6 \pm 20 \text{ MeV (syst)} \quad \lambda_{lim} = 0.365 \pm 0.033 \pm 0.07 \text{ (syst)}$$

above the dividing energy density $\epsilon_{in} \sim 1$ GeV fm$^{-3}$. Within the errors this is compatible with the critical energy density $\epsilon_{crit} \sim 1$ GeV fm$^{-3}$ obtained in lattice QCD [52] as well as with the results on the critical parameters obtained in section 2 (equation (12)). This latter agreement and the extension of the work in [17] to include excited hadronic degrees of freedom is a new result.

This limiting temperature is expected to be somewhat below the critical one, because hadronization is not an instantaneous process. The mean values show a difference in temperature of 40 MeV, but the errors mainly due to the approximations of free quasi-excitations and experimental errors are large, and when added linearly also amount to $\sim 40$ MeV.

We estimate the systematic error on the limiting $T$ and $\lambda_s$ values, fitting the high $\epsilon_{in}$ region of figures 4 and 5 using a line fit and a horizontal line fit, varying in both cases the number of bins taken and estimating the deviations of the mean values resulting from each of the two fits. The error is $\sim 20$ MeV for the temperature and 0.07 for $\lambda_s$.

We interpret this saturation as characteristic for all reactions which before chemical freeze-out have passed through the quark gluon plasma phase, as opposed to those which remained throughout in the hadronic phase.

We note that, due to the large errors, the dividing line as drawn at $\epsilon_{in} \sim 1$ GeV fm$^{-3}$ is subject to a corresponding uncertainty.

The saturation phenomenon observed here is, in our interpretation, confirmed by the same phenomenon derived for the kaon number density as a function of the initial energy density by Kabana [15, 16] (figure 8).

The agreement of the theoretical thermodynamic model with experimental ratios of identified particle multiplicities is quantified in the errors as discussed in section 3. We emphasize that agreement cannot be perfect due to several obvious and other less obvious facts.

(a) The rapidity and transverse momentum distributions do not agree with the spherically symmetric distributions in the thermodynamic description.
(b) There may well exist kinematically distinct thermal systems, as is indicated by the phenomenon of longitudinal and transverse flow [36].

(c) The target and projectile diffractive regions may form further distinct thermal subsystems characterized by a different temperature than the particles produced near midrapidity [53].

The results derived here do indicate, in our interpretation, a high degree of thermalization of the systems studied. This implies a clear indication that the quark gluon plasma phase is part of the hadronization process characteristic for hadronic, leptonic and nuclear reactions above the critical energy density. The universal excitation of the quark gluon plasma phase in nuclear and hadronic as well as leptonic multiparticle production is a new result.

The behaviour of strangeness production as reflected by the parameter λ_s also reveals a new aspect: the saturation phenomenon corresponds to a strangeness enhancement relative to states with low initial energy density. No ‘strangeness enhancement’ is seen for heavy ion collisions relative to hadronic and leptonic reactions, when compared at the same initial energy density and zero fugacities. This enhancement arises only if systems with very different thermal properties, in particular different fugacities, are compared to one another at the same √s, therefore at different energy densities when nucleus+nucleus collisions are compared to elementary particle collisions.

LHC results for central p+p and Au+Au collisions at √s = 14, 5.5 TeV will be important for the confirmation of the above picture.

We conclude by proposing further experimentation in the following areas:

(i) study of p + π collisions at the Tevatron, with centrality selection on transverse energy, similar to nuclear collisions, and possibly with an improved particle identification option including the fragmentation regions;
(ii) in conjunction with (i) an extension to compare open and closed c + c̄ production.
(iii) within the program of heavy ion collisions at RHIC and at the SPS, already under way, a dedicated study in the neighbourhood of ϵ_{in,crit} = 1 GeV fm^{-3}, for example for ϵ_{in} = (0.6–1.5) GeV fm^{-3}, into nucleus+nucleus, p+Au and p+p collisions, using centrality selection.

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