High energy nucleus-nucleus collision and halo radii in different approaches of Glauber theory.

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Abstract

The complete Glauber calculation of the differential cross sections of $^{12}\text{C}$–$^{12}\text{C}$ and halo nuclei on $^{12}\text{C}$ scattering was performed using the previously proposed in Refs.\cite{1, 2} method of generating function. The results are different as compared with the similar calculations in the optical model and the rigid target approximation. The halo nuclei radii extracted from the scattering data via the complete Glauber analysis come out to be larger than those obtained in the approximate approaches.

1 Introduction

In the previous papers\cite{1, 2} we suggest a novel approach to the Glauber theory allowing one to analytically account for all the Glauber diagrams for nucleus–nucleus scattering without additional approximations. It relies on the employment of the generation function that produces the complete Glauber amplitudes. In the present paper we compare it with more simple methods, in particular, in the Section 2 the differential cross section of $^{12}\text{C}$ – $^{12}\text{C}$ scattering evaluated in our approach is compared with those obtained in the optical model and in the rigid target model. We also study the effect due to incorporation of the real part of the strong scattering amplitude apart from the imaginary one and its interference with Coulomb interaction.

We carry out the calculation of the reaction cross sections for the halo nuclei $^8\text{B}$, $^{11}\text{Li}$, $^{11}\text{Be}$, $^{14}\text{Be}$ and using the experimental data on their scattering on $^{12}\text{C}$ we have extracted the halo radii. The results are presented in Section 3. One has to note that
the agreement with the experimental cross sections is achieved in complete Glauber for a bit larger nuclear radii than in the optical model.

2 Elastic $^{12}$C–$^{12}$C scattering in different approaches

The first calculations of the elastic nucleus–nucleus scattering applying the generation function method has been published in our previous paper. Here we briefly recall the main points of the formalism. The generating function $Z(u, v)$ provides the amplitude of the elastic scattering of the incident nucleus $A$ on the fixed target nucleus $B$,

$$F_{AB}^{el}(q) = \frac{ik}{2\pi} \int d^2 b e^{iqb} \left[ 1 - S_{AB}(b) \right],$$

$$S_{AB}(b) = \frac{1}{Z(0, 0)} \frac{\partial^A \partial^B}{\partial u^A \partial v^B} Z(u, v) \bigg|_{u=v=0},$$

(1)

where $q$ is the transferred momentum and $k$ is the mean nucleon momentum in nucleus $A$, the two-dimensional impact vector $b$ lies in the transverse plain to the momentum $k$.

The elastic amplitude is simply related to the total cross section through the optical theorem,

$$\sigma_{AB}^{tot} = \frac{4\pi}{k} \text{Im} F_{AB}^{el}(q = 0) = 2 \int d^2 b \left[ 1 - S_{AB}(b) \right].$$

The difference between the total cross section and the integrated elastic cross section,

$$\sigma_{AB}^{el} = \int d^2 b \left[ 1 - S_{AB}(b) \right]^2,$$

yields the reaction cross section,

$$\sigma_{AB}^r = \sigma_{AB}^{tot} - \sigma_{AB}^{el} = \int d^2 b \left[ 1 - S_{AB}^2(b) \right].$$

Although it is so-called interaction cross section rather than the reaction one that is experimentally measured, the difference between them is estimated to be no more than 2-3%.

The closed expression for the function $Z(u, v)$ has been obtained in Ref.

$$Z(u, v) = e^{W_y(u, v)}, \quad z_y = 1 - \frac{1}{2} \frac{\sigma_{NN}^{tot}}{a^2},$$

(2)

$$W_y(u, v) = \frac{1}{a^2} \int d^2 x \ln \left( \sum_{M \leq A, N \leq B} \frac{z_y^M}{M!N!} \left[ a^2 u \rho_A^1(x - b) \right]^M \left[ a^2 v \rho_B^1(x) \right]^N \right).$$

(3)
The transverse densities entering this formula are expressed through the three dimensional nucleon distributions in the colliding nuclei,

\[ \rho_{A,B}^{\perp}(x_{\perp}) = \int dz \rho_{A,B}(z, x_{\perp}), \quad \int d^2 x_{\perp} \rho_{A,B}^{\perp}(x_{\perp}) = 1, \]

\[ \sigma_{NN}^{\text{tot}} \] is the total nucleon-nucleon cross section, the value \( a^2 = 2\pi\beta \) is related to the slope of the elastic nucleon-nucleon amplitude, see Eq.(8) below.

The function \( W_y(u, v) \) goes as the series built of the overlaps,

\[ t_{m,n}(b) = \frac{1}{a^2} \int d^2 x \left[ a^2 \rho_{A}^{\perp}(x - b) \right]^m \left[ a^2 \rho_{B}^{\perp}(x) \right]^n, \]  

(4)

with \( m \leq A \) and \( n \leq B \). Keeping only the lowest \( m = n = 1 \) term we arrive at the well-known optical approximation

\[ F(A, B) = -\frac{1}{2} \sigma_{NN}^{\text{tot}} T_{AB}(b), \quad T_{AB}(b) = AB t_{1,1}(b). \]

(5)

Another known approximation is the rigid target (or projectile) approximation. It requires one density, say, \( \rho_{A}^{\perp}(x) \), to be kept in the formula (2) only in the linear order, permitting at the same time any powers of \( \rho_{B}^{\perp}(x) \). It yields the generating function

\[ Z(u, v) = e^{v + u T_{rg}(v, b)}, \quad T_{rg}(v, b) = \sum_{n=0}^{\infty} \frac{1}{n!} t_{1,n}(b) v^n, \]

producing for \( B \gg 1 \)

\[ S_{AB}(b) = \left[ T_{rg}(b) \right]^A, \quad T_{rg}(b) = \int d^2 x \rho_{A}^{\perp}(x - b) e^{-\frac{1}{2} a^2 \sigma_{NN}^{\text{tot}} \rho_{B}^{\perp}(x)}. \]

(6)

The complete Glauber amplitude implies all the pieces (4) to be included in the generating function before the derivatives (1) are taken. For relatively light nuclei, \( A, B \lesssim 15 \), it can be done straightforwardly.

In this paper the nucleon density has been taken in a simple Gaussian parameterizations well suited for light nuclei,

\[ \rho(r) = \rho_0 e^{-\frac{r^2}{a^2}}. \]

(7)

This form differs from that used in our previous papers. The change is motivated by exotic halo nuclei analysis in the next section since it is this form that is employed
for the density of the core. The value $a_c$ is expressed through the mean square nuclear radius, $a_c = \sqrt{3/2} R_{rms}$. The nucleon-nucleon elastic scattering amplitude is taken the same as before except for the ratio of the real to imaginary parts, $\varepsilon$, added,

$$f_{NN}^{el}(q) = \frac{ik}{4\pi} \sigma_{NN}^{tot}(1 - i\varepsilon) e^{-\frac{1}{2} \beta q^2}.$$  \hspace{1cm} (8)

For the energy around 1000 MeV per projectile nucleon the total nucleon-nucleon cross section and the slope value (averaged over $pp$ and $pn$ interaction) are\(^{7, 8}\)

$$\sigma_{NN}^{tot} = 43 \text{ mb}, \quad \beta = 0.2 \text{ fm}^2.$$  \hspace{1cm} (9)

With these parameters and the overlap functions\(^{4}\) evaluated for the distribution\(^{7}\) one gets the generating function and the amplitude\(^{1}\). The mean square radius $R_{rms}$ has been adjusted to match the experimental interaction (reaction) cross section $\sigma_{^{12}C-^{12}C} = 853 \pm 6$ mb at the energy about 1 GeV per nucleon\(^{10, 11}\). It turns out to be sufficiently dependent on the approximation, the optical model gives $R_{rms} = 2.19$ fm, in the rigid target approximation $R_{rms} = 2.27$ fm, whereas the complete Glauber calculation results into $R_{rms} = 2.44$ fm. This is a consequence of the fact that for a given radius the optical model cross section would be the largest, the additional screen corrections make it lower.

Taking the last radius value, $R_{rms} = 2.44$ fm, we have calculated the differential cross sections of the elastic $^{12}C-^{12}C$ scattering in the three above approaches for $\varepsilon = 0$. The curves in Fig.1 (left panel) demonstrate a significant difference especially in the diffractive minima positions. The left curve is for the optical model, the next one is for the rigid target approximation and the rightmost curve stands for the complete Glauber calculation.

The situation shown in Fig.1 (right panel) is opposite in the sense that the radius value is separately adjusted for each curve to have the common cross section, $\sigma_{^{12}C-^{12}C} = 853$ mb, for all of them. Now the curves are very close at the interval between $q^2 = 0$ and the first minimum, then the difference increases with the transferred momentum growth.
Figure 1: Left panel: The differential cross sections of the elastic $^{12}$C–$^{12}$C scattering obtained in the optical model (dash-dotted line), rigid target approximations (dotted line) and in the complete Glauber (solid line) for $R_{rms} = 2.44$ fm and $\varepsilon = 0$ for all three curves.

Right panel: The differential cross sections of the elastic $^{12}$C–$^{12}$C scattering obtained in the optical model with $R_{rms} = 2.19$ fm (dash-dotted line), rigid target approximations with $R_{rms} = 2.27$ fm (dotted line) dashed and in the complete Glauber for $R_{rms} = 2.44$ fm (solid line). The cross section $\sigma_{^{12}C-^{12}C} = 853$ mb. is equal for all lines, $\varepsilon = 0$.

The effect coming from the real part of the strong interaction amplitude (8) is presented in Fig.2 for the value $\varepsilon = -0.275$, along with its interference with Coulomb interaction. The differential cross section in one photon exchange approximation reads

$$\frac{d\sigma}{dt} = \frac{\pi}{k^2} |f_C(q) + F_{AB}^{el}(q)|^2,$$

where the Coulomb amplitude is

$$f_C(q) = -M_CE_C^2e^2 \frac{\rho(q)^2}{q^2}, \quad \rho(r) = \int d^3r e^{iqr}\rho(r),$$
$M_C$ and $Z_C$ being the mass and the charge number of $^{12}$C nucleus. The Coulomb amplitude is real and directly interplays with the real part of $F_{AB}^t$.

There are three curves in Fig.2 – the differential $^{12}$C–$^{12}$C cross section evaluated for $\varepsilon = 0$ and without Coulomb corrections (the same as the solid curves in Fig.1), then the curve with the Coulomb corrections, but without the real part of the scattering amplitude and for the real part and Coulomb contribution combined.

As can be seen from the Fig.2 the curves are discernible only in the diffractive minima neighborhoods. The pure Coulomb can be separated out from its mixture with the real part only near the first minimum, where its contribution is several times smaller than that due to the real part, whereas their difference at the second minimum is of three order of magnitude. This is the reason why the first order Coulomb contribution seems to be sufficient in this treatment as the next corrections would be irrelevant.

![Figure 2](image_url)

**Figure 2:** The differential cross sections of the elastic $^{12}$C–$^{12}$C scattering in the complete Glauber approach for $\varepsilon = 0$ without Coulomb contribution (dashed line) with the Coulomb contribution (dash-dotted line) and with $\varepsilon = -0.275$ plus Coulomb contribution (solid line).

## 3 Halo nuclei

The standard treatment of halo nuclei assumes their density to be the sum

\[
\rho(r) = N_c \rho_c(r) + N_v \rho_v(r)
\]
of the core including $N_c$ nucleons and surrounding it halo with $N_v$ valence nucleons. Throughout our analysis, the core density was parameterized as a Gaussian function \( \rho_G(r) = \frac{1}{\pi^{1/2} a_G^3} e^{-\frac{r^2}{a_G^2}} \), where Gaussian width, \( a_G \), is related to the root-mean-square core radius, \( R_c \), as \( a_G = \sqrt{\frac{2}{3}} R_c \). Depending on the shell structure, there are three commonly used parameterizations of the halo density\(^{12}\)

\[
\begin{align*}
\rho_v^G(r) &= \frac{2}{3 \pi^{1/2} a_g^2} r^2 e^{-\frac{r^2}{a_G^2}}, \\
\rho_v^O(r) &= \frac{2}{3 \pi^{1/2} a_o^2} r^2 e^{-\frac{r^2}{a_O^2}}, \\
\rho_v^{2S}(r) &= \frac{2}{3 \pi^{1/2} a_{2s}^2} \left( \frac{r^2}{a_{2s}^2} - \frac{3}{2} \right)^2 e^{-\frac{r^2}{a_{2s}^2}},
\end{align*}
\]

the mean square radius of the halo, \( R_v \), being a single parameter.

Parameter of the core density function, \( a_c \), was determined by the comparison of experimental data on the cross section of the elastic scattering of the corresponding to the core of exotic nucleus on \(^{12}\)C, Refs.\(^{10,11}\) with the reaction (interaction) cross section calculated using the method outlined in previous section. With the known core radii and the densities\(^{10,11}\) (normalized to unity) we make the complete Glauber calculation of the elastic cross sections for the scattering of the exotic nuclei on \(^{12}\)C. It allows to extract the halo radii \( R_v \) from the experimental data\(^{10,11}\) for each particular halo parametrization. The results are shown in the Table 1.

Table 1. Mean square radii of the halo nuclei extracted from the cross sections of their scattering on \(^{12}\)C target. The core radii are chosen to match the cross sections of the would-be core nuclei scattering on \(^{12}\)C. The experimental data are taken from\(^{10,11}\)

| Nuclear structure (core + halo) | Interaction cross section, mb | Mean square radius, fm |
|--------------------------------|-------------------------------|------------------------|
|                                | core | halo | core | G  | O  | 2S  |
| \(^8\)B → \(^7\)Be + p        | 738 ± 9 | 784 ± 14 | 798 ± 6 | 2.47 | 2.95 | 2.95 | 2.93 |
|                               |      |       |      | 3.36 | 3.36 | 3.31 |
| \(^{11}\)Be → \(^{10}\)Be + n | 813 ± 10 | 942 ± 8 | 796 ± 6 | 2.44 | 6.21 | 5.45 | 5.46 |
|                               |      |       |      | 5.51 | 5.35 | 5.36 |
| \(^{11}\)Li → \(^9\)Li + 2n   | 796 ± 6 | 1040 ± 60 | 2.47 | 5.51 | 5.35 | 5.36 |
|                               |      |       |      | 5.42 | 5.53 | 5.55 |

Both the core and the halo radii are about 10% larger if compared with the similar calculations made in the optical model\(^{12}\). It is interesting to note that the necessity to increase the density radius was indicated in Refs.\(^{12,13}\) though because of the different origin – the more complicated halo structure.
4 Conclusion

The results of the Glauber calculations are found to be rather sensitive to the approximation used. A sizable difference between various approaches show up in the differential elastic scattering cross section especially when it is evaluated with the common value of the nuclear density radius. The curves become more close if the reaction cross section is fixed instead of the radius, which value is then separately adjusted within a given approximation. In this case the radius obtained with the complete Glauber calculation turns out to be larger than that in the optical model. This also holds for the exotic nuclei. The complete Glauber yields both the core and the halo radii exceeding the optical model results. Although the difference is not so much it is systematic. The reason is in the screening corrections which reduce the cross section and, therefore, require the radius to be increased.

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