On calculation method for dangerously hydrated loess soil with consideration for elastic-plastic soil properties

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Abstract. On account of the flow theory of plastically compressible media formulas based on moisture-elasticity and plasticity model for strain analysis of dangerously hydrated loess ground are presented. For derivation of the constitutive equations of moisture transfer P. Ya. Polubarinova-Kochina's formula, Darcy's law for filtration rate and S.F. Averyanov to calculate the parameters at partial water saturation were considered. To estimate stress strain behaviour in subsiding building foundations, a finite element method and iterative methods are applied. The constitutive equation for plastic straining of dilating media is given. At each iterative step a linear problem is solved for soil ground with variable stress-strain properties that depend on a stress-strain state from the previous step. The parameters of matrix for stress-strain properties can be calculated during stabilometrical field tests or by processing the statistical relationship between strain and strength properties. The results of numerical studies and large-scale field tests, performed on experimental area of collapsible loess soil are compared. It is indicated that nonlinearly strained loess foundations of buildings must be regarded under partial saturation conditions, during dangerous hydration process and during stress field and humidity interaction in the course of moisture transfer.

1. Introduction
A Silty-clayed loess soils, often found in Rostov oblast and other areas of North Caucasus, are characterized by hydrocon-solidation. This phenomenon leads to deterioration of soil structural properties, causing deformations and damaging buildings.

Process patterns of moisture transfer in loess soils were studied by G.M. Lomize, A.A. Mustafayev, A.P. Pshenichkin, O.Ye. Prikhodchenko and others [1-4].

A substantial contribution to methods of foundation analysis was made by M.Yu. Abelev, S.S. Vyalov, Yu.N. Murzenko, A.A. Mustafayev, V.P. Ananyev and others [5-9].

Models for foundation analysis were developed in the works of Yu.K. Zaretskiy, G.V. Vasilkov and others [10-23].

Foundation analysis of collapsible loess soils is divided into two limit state groups (based on bearing capability and strain) [24,25]. This being said, strain based analysis is considered determinant.

Results of the in situ plate load tests based on "two curves" performed on hydrated loess soils show that pressure influences foundation settlement distinctly nonlinearly. At the same time, strain analysis of loess soils is generally performed according to linearly strained elastic half-space model, which is fairly relative.
Based on the flow theory of plastically compressible media, the author develops and implements into practice a calculation method for stress strain behavior of subsiding building foundations at partial saturation with the account of elastic properties of soil.

Practical problems were solved in the settings of natural water content of the soil, prehydrated soil and dangerously hydrated soil.

Calculation results are compared with large-scale field tests.

2. Main resolving equations for non-stationary problem of filtration in soils at partial saturation

It is considered that pressure, permeability coefficient, and diffusion coefficient are preset functions of saturation.

For derivation of moisture transfer equation P.Ya. Polubarinova-Kochina's formula for partial saturation is considered [26]. Coefficient of permeability at partial saturation and fluid pressure

\[
K(w) = K_0 \left( \frac{w - w_0}{n - w} \right)^{3.56}
\]

\[
P(w) = -\frac{P_0 w_0 - w_{sat}^3 - w^3}{w - w_{sat}^3 - w_0^3}
\]

where \(K_0\) - coefficient of permeability at full water saturation, \(n\) - soil porosity, \(w_0\) - amount of bound water per soil unit volume (volume humidity of bound water), \(P_0\) - pressure at bound water humidity; \(w_{sat}\) - humidity of full water saturation.

Amount of bound water equals the sum of humidity indexes of tightly bound water (roughly corresponds with humidity of air dry soil) and friable water; for some types of loess soils this number varies from 0.012 (aeolian) to 0.128 (loam soils).

Initial boundary value problem of moisture transfer is given as

\[
\begin{align*}
-\nabla \cdot \mathbf{B} \cdot \mathbf{D} \mathbf{w} - \mathbf{B} \cdot \mathbf{J} \cdot \mathbf{w} &= 0; \quad \mathbf{w} \in V_1 \\
-\nabla \cdot \mathbf{D} \mathbf{w} - \mathbf{D} \cdot \mathbf{J} \cdot \mathbf{v}_s &= 0; \quad \mathbf{w} \in S_1 \\
w - w_s &= 0; \quad \mathbf{w} \in S_2 \\
w - w_0 &= 0; \quad \mathbf{w} \in V_1, S_1, t
\end{align*}
\]

where \(\mathbf{B}\) - differencing matrix, \(\mathbf{D}\) - matrix of diffusion coefficients, \(\mathbf{v}_s\) - filtration rate, \(\mathbf{w}\) - moisture level change rate. At partial saturation Darcy's law is observed.

\[
\mathbf{v} = -\mathbf{D} \mathbf{w} - \mathbf{J} \cdot \mathbf{w}, \quad \mathbf{w} \in V_1
\]

where \(\mathbf{J}\) - permeability coefficient.

To find solutions at relatively small time intervals convolved finite element method and direct integration method are used. A set of equations is linearized within random time frame.

Constitutive equation for moisture transfer in matrix form within a time gap is given as:

\[
\left(\theta \Delta t K'' + 3C\right) \cdot q^{n+1} + \left[(3 - \theta) \Delta t K'' - 3C\right] \cdot q^n + \left[P' (3 - \theta) + \theta P^{n+1}\right] = 0
\]

3. Application of elastic flow with hardening theory to estimate stress strain behaviour during static testing figures

To estimate stress strain behaviour in subsiding building foundations a finite element method and iterative methods are applied.

Constitutive equation for plastic straining of dilating media is given as ([27]):
\[ d\sigma_{ij} = 2G_0 \left( d\varepsilon_{ij} + \frac{v_0}{1-v_0} \delta_{ij} \sigma_{km} d\varepsilon_{km} \right) - \left( \frac{\beta_1}{9} \delta_{ij} \sigma_{km} + \frac{2\beta_2}{9G_1} (\varepsilon_{ij} \delta_{km} + \delta_{ij} \varepsilon_{km}) + \frac{4\beta_3}{9G_1} \varepsilon_{ij} \varepsilon_{km} \right) d\varepsilon_{km} \]  

or \( \Delta\sigma = H_1 \Delta \varepsilon \), where \( \Delta\sigma^T = \{d\sigma_{11}...d\tau_{23}\} \), \( \Delta \varepsilon^T = \{d\varepsilon_{11}...d\gamma_{23}\} \).

Dependence matrix

\[ H_1 = D_0 + (K_k - K_0) \cdot D_1 + \frac{2g}{9G_1} \cdot D_2 + \frac{4(G_k - G_0)}{3G_1^2} D_3 \]  

where \( K_0, K_k \) are correspondingly, initial and tangent moduli of volume strain; \( G_0, G_k \) are, correspondingly, initial and tangent moduli of shear strain; \( g \) - dilatancy modulus; \( D_0, D_1, D_2, D_3 \) - matrixes registering elastic and plastic properties of materials.

For the soils, it is assumed that the medium is isotropic, plastically compressible, that incremental strains consist of recoverable and plastic strains, stress deviators and incremental plastic strain deviators are similar and coaxial.

Volume plastic strain and Odquist parameter are continuously differentiable functions of mean stress and stress intensity, which meet the reciprocation requirement.

Constitutive equation for strains (6) implies there two loading surfaces.

The limit state corresponds with Mises-Schleicher-Botkin theory.

Initial boundary value problem for moisture-elasticity and plasticity is taken up as (8):

\[ A_T^T (\sigma^n + H_1 A \Delta u) + \rho + BP = 0; \quad \in V_1 \]

\[ A_S^T (\sigma^n + H_1 A \Delta u) + \rho_S = 0; \quad \in S_1 \]

\[ u - u_0 = 0; \quad \in S_2 \]

\[ -B_T^T DBW - B_T^T kJ + \tilde{w} = 0; \quad \in V_1 \]

\[ -B_S^T DBW - B_S^T kJ - V_5 = 0; \quad \in S_1 \]

\[ w - w_0 = 0; \quad \in S_2 \]

\[ w - w_0 = 0; \quad \in V_1, S_1 \]

where \( A_T, B_T \) - direction cosine matrixes; \( A_T^T, B_T^T \) - differencing matrixes; \( P \) - pore pressure; \( \rho_S \) - volume forces; \( u_0 \) - preset displacement, \( w_0 \) - preset humidity.

At each iterative step a linear problem is solved for soil ground with variable stress-strain properties that depend on stress-strain state from the previous step. Parameters of matrix for stress-strain properties \( H_1 \) can be calculated during stabilometrical field tests or by processing the statistical relationship between strain and strength properties.

First stage of solution algorithm for moisture-elasticity and plasticity includes estimation of stress strain behaviour of the foundation with natural water content. Then the pressure from building weight onto the foundation is added. Then stress strain behaviour components are calculated. In case of sudden hydration (dangerous hydration) the problem of moisture transfer is solved. Strength and stress-related properties depending on the moisture index are corrected and new stress strain behaviour of the soil is calculated up to the specified time gap.

Taking into account that calculation methods are based on iterative processes, loess soil is regarded as homogeneous and isotropic within each time gap and strain process as consisting of successive states of equilibrium during dynamic load.

To solve the problem of moisture-elasticity and plasticity the author uses linear-fractional dependences.
4. Results of numerical study in the test area in Rostov-on-Don

Finite element and step-by-step methods are used based on the results of field plate load test according to "two curves" scheme.

Estimation of stress-strain state of nonlinear loess foundation includes the following calculation.

First, dead load subsidence value and type of ground subsidence conditions are calculated based on the results of laboratory research. The results of field plate load test according to "two curves" scheme are processed and the modulus of total soil strain at each load step is defined. A mathematical expression for subsidence-load curve \( S=f(P) \) for natural water content soil and prehydrated soil are found separately. The building load is added to soil foundation. In case of sudden hydration the problem of moisture-elasticity and plasticity is solved. Stress-strain properties of the soil are corrected at each time gap depending on the stress-strain state of the soil at previous step. Calculation is made up to the specified time frame.

Field plate load tests using strain-gauge installations were performed in the western part of Rostov-on-Don [28]. Stress-strain properties of test area soil:

\[
\begin{align*}
H = & -0.15 - 15m, \\
\rho = & 1.59 - 1.76 t / m^3, \\
\rho_s = & 2.68 - 2.7 t / m^3, \\
\rho_d = & 1.43 - 1.5 t / m^3, \\
W = & 0.108 - 0.165, \\
W_p = & 0.203 - 0.2, \\
W_{sat} = & 0.33 - 0.28, \\
W_{t} = & 0.307 - 0.3, \\
\epsilon = & 0.33 - 0.59, \\
\varphi_{nat} = & 22 - 24 \text{deg.}, \\
\varphi_{sat} = & 20 - 21 \text{deg.}, \\
C_{nat} = & 65 - 62 kPa, \\
C_{sat} = & 12kPa, \\
E_{nat} = & 9.8 - 10.7 mPa, \\
E_{sat} = & 2.1 - 2.4 mPa .
\end{align*}
\]

Here, loess soil depositions are found, which display subsidence properties up to 4.0 m deep (I type of ground subsidence conditions).

In the course of experiment round plates of 0.788 m in diameter and 5000 cm² in size were used. To install the plates a pit was excavated, diameter \( D \), where \( D \) is a plate diameter at 1 m in the ground.

During field tests according to "one curve" scheme, the plate received the load of 0.025-0.05 mPa in steps finally reaching 0.3 mPa. At every load step the author waited for relative strain stabilization. Afterwards the foundation was hydrated.

The author selected the same algorithm for mathematical representation of the model during the research of stress strain behaviour in plate foundation applying finite element method.

Further on, the case when plate stiffness and soil ratio \( E_{sat} = E_{gr} \) equals 1000.0 (rigid plate) is reviewed.

It was specified that all points of the plate at its foot must equally subside, however, mean pressure to the soil must remain at 300 kPa (at 1000.0 mm). Modulus of total soil strain was calculated considering the pressure under the foundation foot.

After soil hydration the checkpoints were at 3, 6, 16, 24, 48, 72, 96, 120 and 144 hours.

During dangerous hydration the border between hydrated and dry displays stress concentration, especially at the top of the mass during the first hours after hydration was initiated. After hydration front passage, isobar position gradually stabilized. According to the results of field tests, at \( P=300 \) kPa plate settlement on test area was 26.6 mm, plate subsidence reached 206.1 mm at total vertical strain of 228.2 mm after its stabilization. When applying finite element method to calculate vertical displacement of rigid plate in dangerous hydration conditions with the load of \( P=300 \) kPa, settlement was 32.76 mm, subsidence was 172.3 mm at total vertical displacement of 205.06 mm. Calculations show that within the first two days of hydration 87% (in 24 hours) to 94% (in 48 hours) of subsiding strain value was reached, confirming the results of the field tests.

That being said, dangerous hydration is assumed to be one of the most hazardous conditions of the foundation.

Calculation results are compared with large-scale field tests.
5. Summary

Having compared numerical studies based on finite element method and large-scale field tests, the author justifies the applicability of the flow theory of plastically compressible dilating media to calculate stress-strain state of collapsible loess soils under conditions of hydration.

Nonlinear initial boundary value problem was solved using finite element method and step-by-step algorithm; it is linearized within small time gaps. Elastic and plastic properties of soil medium are registered through moduli of volume and shear strain at initial and final time gap. Associated flow rule and Mises-Schleicher-Botkin strength condition are involved. It is suggested that there are nonlinear dependences of stress intensity on mean strain and strain intensity and of mean stress on mean strain and strain intensity. Model parameters can be calculated following standard testing.

It is indicated that nonlinearly strained loess foundations of buildings must be regarded under partial saturation conditions, during dangerous hydration process and during stress field and humidity interaction in the course of moisture transfer.

For derivation of the constitutive equations of moisture transfer P. Ya. Polubarinova-Kochina's formula, Darcy's law for filtration rate and S.F. Averyanov to calculate parameters at partial water saturation were considered.

In solving practical problems using the models of moisture-elasticity and plasticity a number of peculiarities in stress strain behaviour change in the foundation during hydration were determined. During hydration of axially symmetrical plate foundation from the round pit (I type of ground subsidence conditions) within the first two days of hydration 87% (in 24 hours) to 94% (in 48 hours) of full subsiding strain value was reached, confirming the results of the large-scale field tests in this area.

It is considered that pressure, permeability coefficient, and diffusion coefficient are preset functions of saturation.

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