Effect of boundary conditions in turbulent thermal convection

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Abstract – We report an experimental study aiming to clarify the role of boundary conditions (BC) in high Rayleigh number $10^8 < \text{Ra} < 3 \times 10^{12}$ turbulent thermal convection of cryogenic helium gas. We switch between BC closer to constant heat flux (CF) and constant temperature (CT) applied to the highly conducting bottom plate of the aspect-ratio-one cylindrical cell 30 cm in size, leading to dramatic changes in the temperature probability density function and in power spectral density of the temperature fluctuations measured at the bottom plate, while the dynamic thermal behaviour of the top plate and bulk convective flow remain unaffected. Within our experimental accuracy, we find no appreciable changes in Reynolds number $\text{Re}(\text{Ra})$ scaling, in the rate of direction reversals of large scale circulation.

Introduction. – Confined 3D turbulent Rayleigh-Bénard convection (RBC) \cite{1,2} serves as a model system on our way to understand many natural phenomena and represents the most frequently studied thermally driven turbulent flow. The ideal laterally infinite RBC occurs in a fluid layer confined between two horizontal, perfectly conducting plates heated from below in a gravitational field. For an Oberbeck-Boussinesq (OB) fluid it is fully characterized by the Rayleigh (Ra) and the Prandtl (Pr) numbers. The convective heat transfer efficiency is described by the Nusselt number, via the $\text{Nu} = \text{Nu}(\text{Ra}; \text{Pr})$ dependence. Large scale circulation (LSC), also known as “wind” \cite{3} of mean velocity $U$ and dimension of the size of the convective layer, $L$ (or the size $D$ of the RBC cell in the case of laterally confined RBC) is known to exist in RBC and can be characterized by the Reynolds number ($\text{Re}$). The dimensionless numbers describing confined RBC are defined as

$$\text{Nu} = \frac{L q}{\lambda_f \Delta T}; \quad \text{Ra} = \frac{g \alpha_f \Delta T L^3}{\nu_f \kappa_f}; \quad \text{Pr} = \frac{\nu_f}{\kappa_f}; \quad \text{Re} = \frac{U L}{\nu_f}. \quad (1)$$

Here $q$ is the total convective heat flux density, $g$ stands for the acceleration due to gravity, and $\Delta T = T_b - T_i$ is the temperature difference between the parallel top and bottom plates of temperatures $T_i$ and $T_b$ separated by the vertical distance $L$. The properties of the working fluid are characterized by the thermal conductivity, $\lambda_f$, and by the combination $\alpha_f/\nu_f\kappa_f$, where $\alpha_f$ is the isobaric thermal expansion, $\nu_f$ is the kinematic viscosity, and $\kappa_f$ denotes the thermal diffusivity: $\kappa_f = \lambda_f/\rho_f c_p f$, where $\rho_f$ is the density and $c_p f$ specific heat of the working fluid at constant pressure. RBC experiments often take place in cylindrical cells of diameter $D$ and height $L$; the relevant additional parameter is the aspect ratio defined as $\Gamma = D/L$, which might be understood as a first approximation in taking into account the shape of the RBC cell.

From the theoretical/numerical point of view, within the OB approximation and assuming that the flow is incompressible, the 3D RBC is fully described by well-known equations of motion \cite{1,2} which, however, must be complemented with the boundary conditions (BC). While for the velocity field the no-slip BC \cite{4} on all inner surfaces of the RBC cell are taken as justified, the BC for the temperature field can be expressed, for example, in the Dirichlet form —constant temperature (CT) of the solid-fluid boundary ($i.e.$, $T_b = \text{const}; T_i = \text{const}$) or as a constant

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Table 1: Experimental quantities relevant for the generation of cryogenic turbulent RBC flows in the Brno experimental cell [7] in comparison with hypothetical RBC flows of the same Ra and Nu (assuming the same Nu = Nu(Ra) scaling) that would be requested at ambient temperatures using H₂O and SF₆ as working fluids [8] (see footnotes ¹ and ²). The Γ = 1 Brno cell has e = 28 mm thick top and bottom plates 30 cm in diameter made of annealed copper of thermal conductivity λₚ = 2210 and 400 W m⁻¹ K⁻¹ and thermal capacity cₚ = 0.144 and 386 J kg⁻¹ K⁻¹ at T_m = (T₁ + T₂)/2 ≈ 5 K and 300 K, respectively.²

The influence of the vertical wall, made of nominally 0.5 mm thick stainless steel, is neglected. For definition of the displayed physical quantities α_J/ΔT, Qₖ, λₕ/νₕ, h/τ_p, K, τ_p, τ_κ, τ_κ_p, see the text.

| Unit | He | H₂O | SF₆ |
|------|----|-----|-----|
| Ra   | 1  | 2.3×10⁸ | 2.3×10⁸ | 2.3×10⁸ |
| Nu   | 1  | 41.7 | 41.7 | 41.7 |
| Pr   | 0.68 | 5.85 | 0.79 |
| T_m    | K  | 5.009 | 300.0 | 300.0 |
| ΔT     | K  | 1.027 | 0.402 | 8.936 |
| P       | Pa | 1.03×10³ | 1.00×10⁵ | 5.00×10⁵ |
| ν₁      | m²/s | 1.26×10⁻⁵ | 8.57×10⁻⁷ | 5.21×10⁻⁶ |
| ν₉      | m²/s | 1.85×10⁻⁵ | 1.46×10⁻⁶ | 6.62×10⁻⁶ |
| α₁/K   | 2.00×10⁻¹ | 2.75×10⁻₃ | 3.40×10⁻³ |
| λ₁/K   | W/m K | 9.57×10⁻³ | 6.10×10⁻⁻³ | 1.30×10⁻² |
| ρ₁      | kg/m³ | 0.10 | 996.6 | 2.94 |
| cₚ_p   | J/kg K | 5203 | 4181 | 667.9 |
| α_J/ΔT | 1 | 2.10×10⁻¹ | 1.10×10⁻⁴ | 3.00×10⁻² |
| Qₖ_W   | 0.095 | 2.410 | 1.142 |
| λₕ/νₕ_s | 1 | 5539 | 15.7 | 737 |
| τ_p    | s | 27.6 | 1140 | 5344 |
| τₕ    | s | 59.3 | 7372 | 163.1 |
| τ_p    | mm | 1.445 | 4344 | 0.002 |
| K      | 1 | 1.38×10⁴ | 13.0 | 1.29×10⁶ |
| τ_p    | s | 1.8 | 5.3 | 1.9 |
| τₚₚ    | s | 1.2×10⁻³ | 4.1×10⁻¹ | 6.7 |
| τₚₚ    | s | 6.5×10⁻⁴ | 7.7×10⁻² | 3.5 |

¹Properties of H₂O: https://www.nist.gov/srd/refprop.
²Properties of SF₆ evaluated using the computer program kindly provided by G. Ahlers and X. He.
³Properties of Cu: Cryocomp v3.06, Cryodata Inc. (1999).

The thermal conductivity of the electrical leads and, generally, the physical properties of the surrounding medium.

heat flux (CF) supplied via the entire area of the bottom of the RBC cell [5,6].

In experimental studies of RBC, however, the temperature BC at the interface between the fluid and the plate are always a combination of CT and CF. The main motivation of our study is that their relative weight can be, up to some degree, experimentally adjusted in situ. There are many experimental parameters that affect the RBC flow under study; we [9] can loosely divide them into two groups: geometrical and physical. The first group includes the actual size and shape of the cell, thickness of walls and plates, their surface roughness or possible deviation from the horizontal position. The second group includes the actual physical properties of the working fluid as well as of construction materials of the RBC cell, such as thermal conductivity and heat capacity of plates (λₚ; cₚ) and walls,
lent RBC flows, assuming them generated in the same cell at 300 K using frequently used working fluids: H$_2$O and SF$_6$. The numerical value $\alpha_f \Delta T \lesssim 0.2$ is conventionally understood as a satisfactory OB criterion. The thickness of the thermal boundary layer $t_{BL} = L/(2\text{Nu})$ is naturally the same for the same Nu, however, the heat currents $Q_b = qS_p$ ($S_p$ being the plate area) required to be applied to the bottom plate are for complementary RBC flows very different. It would take the time $\tau_p$ to heat just the bottom plate alone (assuming it thermally isolated) by $\Delta T$, and time $\tau_f$ to heat the working fluid by $\Delta T/2$, i.e., to the temperature of turbulent bulk of the RBC flow. An important factor is the heat conductivity, $\lambda_p$, of the plates. Its influence on Nu was thoroughly studied [10,17] and experimentally confirmed by Brown et al. [18], who used H$_2$O in otherwise identical RBC cells with Cu ($\lambda_p \approx 391 \text{ Wm}^{-1} \text{ K}^{-1}$) and Al ($\lambda_p \approx 161 \text{ Wm}^{-1} \text{ K}^{-1}$) plates and concluded that low $\lambda_p$ diminishes the heat transport efficiency, at least in RBC cells of size similar to our own [7]. An important requirement is that the ratio $\lambda_p/(\text{Nu} \lambda_f)$ is high, which for Cu plates and water is low (see table 1) but even lower for Al plates. Note that for SF$_6$ and especially for cryogenic He this ratio is about two (three) orders of magnitude higher.

To ensure a truly constant and uniform plate temperature for any Ra, the plate should possess infinite thermal conductivity and heat capacity, however, any plate falls short of these ideal properties. In experiments, the surface temperature is constant only on the average but not locally and departures from the ideal case are expected, manifested through effects on plume generation [5]. It follows that the key role for establishing the ratio of CT vs. CF BC is played by thermal plumes: two-dimensional sheet-like structures of temperature $\approx T_b$ (hot plume) or $\approx T_l$ (cold plume) and thickness comparable to $t_{BL}$, which initially extend in the vertical direction, eventually to be bent by LSC. If such a plume of area $A$ abruptly detaches, it takes with it (leaves behind) heat $Q_p \approx S\ell_{BL}\rho_f \Delta T/2$, equivalent to a thermal hole in the plate, of thickness $t_p \approx 2Q_p/(S\Delta T \rho_f \rho_p)$ (see table 1); this thermal hole must be refilled using the heat flux delivered by a heater via thermal conduction of the plate. From this point of view, an important parameter is $K = (\rho_p c_p \lambda_p)/(\rho_f c_f \text{Nu} \lambda_p)$ [19]. The characteristic time between two successive plumes has been estimated by Castaing et al. [20] as $\tau_{plm} = (\text{Ra Pr})^{1/2}/(4\text{Nu}^2)$. It decreases with Ra, since Nu increases faster than Ra$^{1/4}$. To assure CT BC the plate should be fast enough to provide consecutive plumes with enough heat by thermal diffusion, which occurs within a characteristic time $\tau_{ph} = (\text{Ra Pr})^{1/2}/(\kappa_f/\kappa_p)$ [17,20]. This means that CT BC will be better achieved if $K$ is big and the ratio $\tau_{ph}/\tau_{plm}$ is small, which is out of the three considered cases best achieved for cryogenic He.

**Experiment.** – In order to appreciate the role of BC on RBC flow, we perform the experiment under nominally the same conditions while switching on and off the

![Fig. 1: The sketch of the Brno RBC cell.](image)

PID-stabilizing scheme of the bottom plate temperature $T_b$. We use the updated version of the Brno experimental cell [7], shown in fig. 1. Essential improvements are: i) the original mid flanges on the sidewall have been gradually
deformed and found prone to leakage; they were therefore replaced and the joints welded together; ii) several small Ge temperature sensors (Ge-on-GaAs film resistance thermometers [21]) attached to tightly stretched thin constantan wires have been installed, via newly made sidewall feedthroughs; their geometrical positions are shown in fig. 1; and iii) in addition to the precisely calibrated stable Ge sensors embedded in the plates, fast DT-670 silicon diodes (Lake Shore) have been attached to plates.

Due to rather complex thermal connection of the top plate with the liquid helium vessel above it, partly via a stainless steel sidewall but mainly via the He gas heat exchange chamber (GHeCH) which itself represents a convection cell, we have focused on changing BC at the bottom plate and compare two distinctly different cases. In both of them, the heat is supplied to the bottom plate via a distributed wire heater. As the distance between heater turns is smaller than the plate thickness, the heat delivered to its upper surface, in the absence of convective flow in the RBC cell, can be thought of as steady and uniformly distributed. Turbulent RBC flow breaks this symmetry both in time and space. Although the total heat flux delivered by the resistive heater to the outer side of the bottom plate remains constant, due to thermal plumes detachment and dynamical thermal properties of the bottom plate, the CF BC is not strictly valid at the bottom solid-fluid boundary of the RBC flow. Despite this caveat, also in view of numerical studies such as [22], hereafter we call this Case 1 as CF BC.

We note that delivering constant heat flux (CF) to the outer side of bottom plate while controlling the mean top plate temperature (via adjusting the pressure in the exchange chamber and, additionally, by fine tuning via uniformly distributed resistive heater glued in the spiral groove on the upper surface of the top plate achieved by using a PID control) is the “standard” way of generating statistically steady turbulent RBC flows studied in our previous experiments, see [9,16,23,24] and references therein.

Case 2 to compare with, hereafter called CT BC, differs in that the bottom plate heater is included in the PID control feedback loop, designed to keep the temperature of the bottom plate stable. The PID scheme uses the reference signal from the fast-responding diode $T_{b3}$. In both cases, the mean temperature difference $\Delta T = \langle T_b \rangle - \langle T_t \rangle$ is kept constant, where the mean temperatures $\langle T_b \rangle$ and $\langle T_t \rangle$ are accurately determined by finely calibrated Ge sensors $T_{b1}$, $T_{b2}$ and $T_{t1}$, $T_{t2}$. The fluctuating values $T_b(t)$ and $T_t(t)$ are monitored by home-calibrated diodes $T_{b3}$ and $T_{t3}$, and the temperature fluctuations $T_Y(t)$ ($N = 1, \ldots, 12$) in various places of the cell interior by small Ge-on-GaAs film sensors [21]. The placement of all sensors is shown in fig. 1.

The probability density functions (PDFs) of the temperature fluctuations of the plates $T_b(t)$ and $T_t(t)$ are evaluated using the signal from the fast-responding diodes $T_{b3}$ and $T_{t3}$ (see fig. 1) with and without the PID control of the bottom plate temperature. All measured PDFs of the fluctuating $T_b(t)$ and $T_t(t)$ about mean temperatures $\langle T_b \rangle$ and $\langle T_t \rangle$, plotted vs. Ra. CF heating (orange squares), while the PDFs measured at the top plate (blue symbols) remain unaffected. The top panel of fig. 4 displays the mean value of the temperature fluctuations $\sigma_b$ measured by a diode $T_{b3}$ normalized by $\Delta T$, plotted vs. Ra. CT: $\Delta T = 0.196 K$; CT: $\Delta T = 1.027 K$. Although the upper one above the crossover to $\gamma = 1/3$ [9,16]. For all investigated $Ra$, the PID control of the bottom plate temperature results in significant narrowing of the bottom plate PDFs, while the PDFs measured at the top plate remain unaffected.

It is instructive to calculate and compare the power spectral density (PSD) of temperature fluctuations for the PID control on and off. As shown in fig. 3, the PID control results in significant depletion of PSDs at low frequencies below about $0.4-0.7$ Hz, while faster temperature fluctuations of the bottom plate are hardly affected. The top plate PSDs remain at all Ra entirely unaffected.

The top panel of fig. 4 displays the mean value of the temperature fluctuations $\sigma_b$ measured by a diode $T_{b3}$ normalized by $\Delta T$, plotted vs. Ra. While $\sigma_b/\Delta T$ slightly increases with increasing $Ra$ (approximately $\propto Ra^{1/4}$), note the growing tendency of $\sigma_b/\Delta T$ with $Ra$, in agreement with [5]) for both CF and CT BC on the bottom plate, the imposed CT BC reduces its numerical values by a factor of about four. The same quantity, $\sigma/\Delta T$ in the top and bottom Cu plates in rectangular RBC cells of various sizes was measured at ambient temperatures under CT and CF BC in a similar study by Huang et al. [25], by using $H_2O$ as the working fluid. It is remarkable that these data, also shown in fig. 4, display the opposite tendency in the $\sigma/\Delta T$ vs. Ra dependence. We speculate that this apparent discrepancy could be explained by very differ-
ent dynamic characteristics of cryogenic He and ambient temperature H₂O turbulent RBC experiments, as some of them differ by orders of magnitude —see table 1.

Let us now discuss the main issue of this study: what changes, if any, are experimentally observed in the bulk of the RBC flow as a consequence of distinctly different BC at the bottom plate. We start with the same quantity, \( \sigma_{\Delta T} \), but measured now not at the plates but in the centre of the RBC cell. The bottom panel of fig. 4 shows that, contrary to the situation at the bottom plate, \( \sigma_{\Delta T} \) in the centre is not appreciably sensitive to the change of BC at the bottom plate and scales \( \propto Ra^{-1/7} \), and behaves in accord with our previous studies [23] performed in the RBC Brno cell as well as with the seminal work of Niemela et al. [26] quoting the best fit \( \sigma_{\Delta T} = 0.37 Ra^{-0.145} \), shown in the bottom panel of fig. 4 as a solid line for comparison. We note in passing that this power law is consistent with theoretical arguments by Pandey et al. [27].

Figure 5 shows examples of PSDs of the temperature fluctuations in the centre of the RBC cell (top) and at the midplane 20 mm from the sidewall (bottom) measured at \( Ra = 2.2 \times 10^{12} \). As is typical for confined high Ra RBC flow, the PSD measured near the sidewall displays the LSC peak, in this case at 0.06 Hz, which is used to calculate the mean velocity of the LSC, the “wind”. In accord with [26], the PSDs are consistent with a roll-off rate of \(-7/5\) for low frequencies where Bolgiano scaling seems

\[
\frac{\sigma}{\Delta T} \propto Ra^{0.145}
\]

In the midplane, the PSDs are measured at 20 mm from the sidewall. The lines represent the slopes of Obukhov-Corrsin and Bolgiano scaling with, respectively, \(-5/3\) and \(-7/5\) roll-off exponents, although Verma et al. argued [28] that the latter is not valid for turbulent RBC flow and the power \(-7/5\) appears due to inapplicability of the Taylor hypothesis [29].
appropriate, whereas for higher frequencies the classical Obukhov-Corrsin scaling with the roll-off exponent $-5/3$ appears more appropriate. The key observation is that at all investigated $Ra$, except for slight depletion at very low frequencies below 0.02 Hz, the bulk PSDs of the temperature fluctuations are unaffected by the imposed change of BC on the bottom plate.

In our previous work [23] we discussed several definitions of Reynolds numbers, evaluated them using the “standard” CF BC and compared them with results published by other authors. Here we utilize temperature fluctuations measured by various single and pairs of Ge sensors in the cell (see fig. 1) and compare Reynolds numbers and their scaling with $Ra$ for CF and CT BC at the bottom plate.

In the case of one probe measurement, the characteristic frequency $f_0$ determined from the peak of the near-wall PSD of the temperature fluctuations (an example shown at the bottom of fig. 5) is used in the definition of the frequency-based Reynolds number $Re_{f0} = 2L^2 f_0/\nu f$. We already discussed that the near-wall PSDs remain unaffected by the change of the BC, so $Re_{f0}$ is unaffected, too.

In the case of two probe measurements, the simplest approach relies on the Taylor frozen flow hypothesis. It uses the time delay between temperature fluctuation records at two nearby sensors spaced by a vertical distance $d$, which determines the mean velocity $U_p$. The corresponding Reynolds number is defined as $Re_p = LU_p/\nu f$. In [23] we claimed observation of a crossover in the slope of $Re_p$ with $Ra$ around $10^{10}$ (complementary to the crossover in $Nu(Ra) \propto Ra^2/7$ scaling from $\gamma \approx 2/7$ to $\gamma \approx 1/3$). We confirm this crossover for both CF and CT BC; it is clearly seen in fig. 6, displaying the compensated plot of $Re_p Pr^{2/3}/Ra^{4/7}$ vs. $Ra$. We have evaluated Reynolds numbers according to all definitions discussed in our previous work [23] and found them hardly affected by the changing CF and CT BC. The $Re_p$ data shown in fig. 6 were evaluated using sensors No 1 and 2 (see fig. 1) which in all experimental runs displayed very rare reversals of the LSC direction: $1.5 \pm 0.2$ (1.8 $\pm$ 0.3) reversals/hour for CF (CT) cases, as the sensors presumably lay near the main LSC plane. The data from the sensor pairs 3, 4 and 5, 6, which lay in the plane perpendicular to the previous pair experienced more frequent reversals of an auxiliary flow: $9.7 \pm 0.4$ (9.9 $\pm$ 0.4) reversals/hour for CF (CT) cases. A similar situation was observed by Sun et al. [30] using PIV combined with thermometry. More detailed statistical study of LSC reversals as well as analysis employing the so-called elliptic approximation in evaluation of $Re$ will be published elsewhere.

Last but not least we now discuss the essential feature of turbulent RBC flow —its ability to transfer heat, usually expressed in dimensionless form, by the Nusselt number. The key question is: Does $Nu$ depend on boundary conditions? Our experimental answer is provided in a graphical form in fig. 7: Changing CF to CT BC does not appreciably change the heat transfer efficiency, at least over the investigated range $10^8 < Ra < 3 \times 10^{12}$. Being fully aware of the fact that accurate determination of the $Nu(Ra)$ dependence involves application of various corrections to the raw data, we do not claim here the absolute accuracy of the displayed compensated $Nu(Ra)$ dependence. We stress, however, that the only difference between the displayed two sets of data is the $in situ$ change of CF (or rather CF-like) and CT (or rather CT-like) BC at the bottom plate as discussed in detail above.

This experimental result can be compared with complimentary numerical studies. Following the earlier simulations of Amati et al. [31], Verzicco and Sreenivasan [5], and 2D simulations of Johnston and Doering [22], Stevens, Lohse and Verzicco [6] performed 3D simulations with improved accuracy, under unconditional validity of the OB approximation for $\Gamma = 1/2$ and $Pr = 0.7$. Figure 7 shows their direct numerical simulation data with CT and CF BC at the bottom plate (with CT BC at the top plate in both cases); only the data obtained on identical grids are displayed in order to rule out resolution effects as much as
possible. In the simulations with CF BC at the bottom plate they calculate Nu from the average $q$ at plates. The error bars are larger in CF than in CT BC because it took more time to reach the statistically steady state. Up to about $10^{10}$ in Ra, CF BC generally results in slightly lower values of Nu (which is not seen experimentally). The difference between CF and CT simulations is, however, rather small, seems to decrease with increasing Ra and disappears within the (statistical) error above $10^{11}$ in Ra.

While simulations [6] are undoubtedly relevant to our experiment, they do not take directly into account the possible influence of physical and geometrical properties of the RBC cell, especially of its plates. This was attempted in the recent study of Foroozani, Krasnov and Schumacher [32], who studied numerically the confined RBC flow bounded by two copper plates from above and below and applied CT and CF BC at the upper surface of the top plate and at the lower surface of the bottom plate (see fig. 1 in ref. [32]), requiring the continuity of temperature and heat flux at the solid-fluid interfaces. This configuration is denoted as the conjugated heat transfer case.

To conclude, we have performed an experimental study aiming to clarify the role of BC on high Ra turbulent RBC flow. Our findings can be summarized as follows. Chang-aiming to clarify the role of BC on high Ra turbulent RBC of different materials of strongly temperature-dependent fluids in RBC cells of the same shape with plates made andV.

To conclude, we have performed an experimental study aiming to clarify the role of BC on high Ra turbulent RBC flow. Our findings can be summarized as follows. Changing the CF-like BC to CT-like BC on the bottom plate by employing the PID control of $T_b$ results in i) significant narrowing of temperature PDF and suppression of the low-frequency part of the PSD evaluated with the use of a fast-responding sensor attached to the bottom plate itself, but hardly affects ii) the dynamic thermal characteristics of the top plate, iii) PSD of temperature fluctuations measured in the centre of the cell as well as in the middle plane near the sidewall, iv) the rate of direction reversals of large scale circulation and finally v) the Nu(Ra) and $\text{Re}(Ra)$ scaling.

**References**

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**Data availability statement:** All data that support the findings of this study are included within the article (and any supplementary files).

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