Cosmic censor of shock-wave singularities

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Abstract: We construct a fluid-dynamical analogue of the cosmic censorship hypothesis of Penrose, wherein this naked nondispersive shock-wave singularity is censored (prohibited) due to the microscopic structure of the underlying ether and the resulting effective trans-Planckian dispersion. We find that including quantum pressure in Bose-Einstein condensates provides such a censor: Approaching the instant of shock $t_{\text{shock}}$, rapid spatial oscillations of density and velocity develop, which begin to emerge already slightly before $t_{\text{shock}}$. These oscillations render the spacetime structure completely regular, and therefore lead to a removal (censoring) of the spacetime singularity.

I. INTRODUCTION

In Einsteinian gravity, singularities are ubiquitous [1–3]. However, the physical spacetime nature of these singularities is still under debate. The singularity theorems by Stephen Hawking and Roger Penrose state that if there either exists a trapped surface due to gravitational collapse or the Universe is assumed to be spatially closed, spacetime singularities are formed with the following conditions being satisfied: We have Einstein gravity at zero or negative cosmological constant, the weak energy condition is maintained, closed timelike curves are absent, and every timelike or null geodesic enters a region where the curvature is not specially aligned with the geodesic [4–7]. As these theorems guarantee that if there exists a trapped surface in spacetime, a singularity must form, one may ask the question if the reverse holds true, and whether a singularity may form without a horizon enclosing it (naked singularity). The cosmic censorship hypothesis (CCH), then, in its weak form, states that generic gravitational collapse, starting from a nonsingular initial state, can not create a naked singularity in spacetime [2, 8, 9].

However, explicit counterexamples to the CCH for the physical spacetime nature of these singularities are indeed hidden were developed, among which backreaction is a prominent example [15–18]. It is thus fair to say that the CCH is still widely debated, as regards the possible mechanisms for either violating or preserving it, and whether these mechanisms are of quantum or classical origin, also cf. Ref. [19]. This is largely due to the fact that there is no applicable quantum theory of gravity, in particular complete in the ultraviolet, with which to ascertain whether a given argument for (or against) the CCH is true.

The seminal paper of Unruh [20] triggered, especially recently, with a substantial improvement of experimental capabilities, on a broad front a field which was coined analogue gravity [21]. Its essence is that it models the propagation of classical and quantum fields on curved spacetime backgrounds, exploring various phenomena inaccessible at present in the realm of gravity proper, see, e.g., Refs. [22–39]. A particularly promising arena are Bose-Einstein condensates (BECs) due to the atomic precision control and accurate correlation function resolution they offer [40–61].

Acoustic black holes (“dumb” holes [62]) or cosmological horizons are thus well established and experimentally realized within the analogue gravity realm. On the other hand, distinct from Einstein gravity, where singularities are ubiquitous, singularities in quantum fluids, and with particular regard to their acoustic spacetime properties, have not been experimentally investigated yet, to the best of our knowledge. It is important here to pause, and to clearly state at the outset the most important differences of analogue gravity and Einstein gravity: In analogue gravity, the acoustic spacetime metric is governed by nonlinear fluid dynamics and not by a solution of the Einstein equations. In Einstein gravity, black holes (and, as a result, also singularities in spacetime due to the theorems by Hawking and Penrose) are formed from gravitational collapse of matter. In fluids, it is the transition of subsonic to supersonic flow which creates an effective dumb hole horizon for linear sound in the medium. Distinct from Einstein gravity, this analogue gravitational field, providing a background effective spacetime for linear perturbations on top of it, is governed by a velocity scalar [63], in a comparable way to a nonlinear self-interacting scalar field theory of gravity [64]. In the present work, we establish a highly nonlinear process creating a naked singularity in the acoustic spacetime metric, physically represented by a shock wave in a BEC without dispersion included (that is in the so-called Thomas-Fermi limit). For this nondispersive shock, the nonlinearity causes a stepwise discontinuity in the acoustic metric components, and as a result a naked timelike Ricci curvature singularity of the effective spacetime emerges.

In the real BEC quantum fluid, dispersive effects can however not be neglected, due to the quantum pressure,
II. FLUID DYNAMICS OF DILUTE BOSE-EINSTEIN CONDENSATES

A. Fluid perturbations

Dilute BECs represent inviscid, barotropic, and irrotational fluids, where, importantly, the quantum pressure term is added to the Euler equation. Setting the atomic mass \( m = 1 \), we have to solve the following set [66]:

\[
\begin{align*}
\partial_t \rho + \nabla \cdot (\rho v) &= 0, \\
\partial_t v + v \cdot \nabla v &= -\nabla p + \frac{\hbar^2}{2m} \left( \frac{\nabla^2 \rho}{\rho} \right) - \nabla V_{\text{ext}}, \\
p &= p(\rho) = \frac{1}{2} g \rho^2, \\
\nabla \times v &= 0 \Rightarrow v = \nabla \Phi.
\end{align*}
\]

These equations are the only field equations occurring in our problem for condensate density \( \rho(r,t) \) and condensate velocity \( v(r,t) \), and the spacetime metric for sound is then a derived and not fundamental (also see below). In the above relation (4), \( \Phi \) is a velocity potential due to the cold quantum gas experimentalist to create certain classes of effective spacetimes (see for an overview [21]), while the condensate pressure \( p \) arises from the two-body repulsive contact interaction between atoms, where the coefficient \( g \) is proportional to the \( s \)-wave scattering length in the dilute gas [66]. Finally, the term \( \frac{\hbar^2}{2m} \frac{\nabla^2 \rho}{\rho} \) in the Euler equation (2), is the so-called quantum pressure term [66]. From the barotropic equation of state (3), the sound speed \( c_s = \sqrt{\frac{\partial p}{\partial \rho}} = \sqrt{g \rho}; \) stability implies that \( g > 0 \).

We linearize the fluid equations over the background of a dispersive shock wave in a BEC [67]. The quantum pressure term is negligible until the shock is closely approached. Due to the quantum pressure term, the discontinuity in the flow, which were expected to be present in the nondispersive post-shock phase [65], is regularized. One observes instead an oscillation pattern in the density profile upon approaching the shock (Fig. 5 in Appendix B). To physically distinguish classical sound wave from the background, one works with a linear perturbation with different space and time scale than the background flow, as discussed in the literature for linear sound propagation over background [21], and for nonlinear sound as well [63]. We denote background quantities with subscript \( (0) \) and the linear perturbations with subscript \( (1) \). We write \( v = v^{(0)} + \nabla \Phi^{(1)} \) by following the conventions of Ref. [63]. For example, with a dispersive nonlinear wave as the background, initially, when \( t \) is much less than the shock time \( t_{\text{shock}} \), the wave is linear and nondispersive. For \( t \ll t_{\text{shock}} \), such a linear wave satisfies the massless Klein-Gordon (KG) field equation over the analogue Minkowski spacetime of a uniform static medium as background. We call this the initial background, and denote it with subscript \( 0 \). According to the Riemann wave equation for travelling one-dimensional (1D) waves, see Eq. (12) below, the intrinsic nonlinearity of the fluid-dynamical equations becomes significant in the course of time as the wave approaches the shock [65]. The KG analogy then does not hold anymore. In Ref. [63], we have described the classical backreaction of the nonlinear perturbation onto the acoustic metric, and defined a new background by absorbing these nonlinear perturbations into it.

Here, we go near and beyond the shock time, with now in addition the quantum pressure, which originates from the spatial stiffness of the macroscopic BEC wavefunction against deformations, becoming significant. Linearizing (1) gives

\[
\frac{\partial \rho^{(1)}}{\partial t} + \nabla \cdot (\rho^{(0)} \nabla \Phi^{(1)} + \rho^{(1)} v^{(0)}) = 0.
\]

The linearized Euler equation follows from the Eq. (2):

\[
\begin{align*}
\frac{\partial \Phi^{(1)}}{\partial t} + \frac{c_s^{(0)}}{\rho^{(0)}} \rho^{(1)} + v^{(0)} \cdot \nabla \Phi^{(1)} \\
+ \frac{\hbar^2}{2 \rho^{(0)} (1 + \rho^{(1)} \rho^{(0)})} - \frac{1}{\rho^{(0)}} (\nabla \rho^{(0)})^2 \\
+ \frac{\hbar^2}{2 \rho^{(0)}} \left( 1 + \frac{\rho^{(1)}}{\rho^{(0)}} \right) - \nabla^2 \rho^{(1)} = 0.
\end{align*}
\]

Incorporating only the gradient terms from the background, thus neglecting \( \nabla \rho^{(1)}, \) and \( \nabla^2 \rho^{(1)} \), we get

\[
\rho^{(1)} \frac{1 + \hbar^2 \alpha}{4 c_s^{(0)}} \frac{\rho^{(0)}}{\rho^{(0)}} = -\frac{\nabla \rho^{(0)}}{\rho^{(0)}}.
\]

Here, we introduced a parameter \( \alpha \) via

\[
\alpha = \frac{1}{4 c_s^{(0)} \rho^{(0)}} \cdot \nabla \frac{\rho^{(0)}}{\rho^{(0)}}.
\]

We can then define a new length scale \( l = l(x,t) \) via

\[
l^{-2} = \hbar^2 |\alpha| / \xi^2 \]

which characterizes the background spatial variation, and where the spatiotemporally local healing length is given by \( \xi(x,t) = |\xi(\rho^{(0)})| = \frac{\hbar}{\sqrt{g \rho^{(0)}}}. \)

The competition of the “microscopic” structure dictated by \( \xi \) and the “background” scale \( l \) is expressed by \( \alpha(x,t) \) which thus appears in the metric \( g_{\mu\nu} \) in Eq. (9) below.
B. Spacetime metric in the dispersive fluid

Now, we substitute \( \rho_{(1)} \) from Eq. (7) into Eq. (5), dropping the terms in the last closed bracket of Eq. (6). This is the limit where the linear perturbation of all physical quantities such as \( \rho_{(1)}, p_{(1)} \) can be written in terms of partial derivatives in \( \Phi_{(1)} \), and the full solution can be obtained when \( \Phi_{(1)} \) over a known background has been solved for. Going beyond this limit requires to solve for \( \rho_{(1)} \) also, and the equation of motion for \( \Phi_{(1)} \) becomes an integro-differential equation [41]. As a result, the acoustic spacetime metric is not local in space and time anymore. Here, we restrict ourselves to small wave number \( k \) excitations, i.e., perturbations with wavelength larger than the coherence length \( \xi(\rho_{(0)}) \). In this limit, we can construct an acoustic metric local in spacetime.

Linearizing in the perturbation amplitude now proceeds still as conventionally carried out in the analogue gravity literature [20, 21]. The difference is found in the dispersive nature of the background. The latter is controlled by well-posed initial (and/or boundary) conditions by the experimentalist. Over such an externally fixed, albeit nonlinear and dispersive background, any excitation to linear order is called a perturbation. In our particular case, the highly nonlinear and dispersive background flow is clearly distinct from the linear nondispersive perturbations which experience the effective spacetime produced from such a background medium. We then compare the equation of the scalar field \( \Phi_{(1)} \) to that of a minimally coupled massless KG field equation, and find the following effective spacetime metric in 3+1D,

\[
q_{\mu\nu} = \rho_{(0)} \left[ \begin{array}{cccc}
-(c_{(0)}^2 - v_{(0)}^2) & -v_{(0)}^T \\
\cdots & \cdots & \cdots & \cdots \\
-v_{(0)} & \cdots & \text{I}_{3\times3} \\
\end{array} \right],
\]

where \( \sigma(\rho) \gg \xi(\rho) \). Here, at the center of our quasi-1D BEC set up, we produce a source of gravitational wave (GW) with density being almost uniform towards the boundary, mimicking asymptotically flat effective spacetime with a GW source. This longitudinal GW is different from its counterpart in Einstein gravity, in that the spacetime lacks general covariance, and the GW cannot be represented in its usual transverse and traceless form, cf. the discussion in [61].

The Thomas-Fermi profile in Eq. (11) (neglecting the impact of quantum pressure on the initial state) can be created by focusing a laser detuned from atomic resonance onto the center of the one dimensional condensate, with a size \( \gg \sigma \) [67]. Switching off the laser creates a nonlinear dispersive propagating wave with high frequency oscillations when the shock occurs, as previously described in [67], see for a detailed description Appendix B. Shock waves in quasi-1D BECs have been experimentally observed [70], and also in nonlinear photon fluids.

### III. DISPERSIVE SHOCK WAVES

We consider the propagation of a wave, initially created as a Gaussian distribution, in the condensate. We consider a realistic situation, with the effect of quantum pressure included, i.e., a highly nonlinear dispersive wave [67]. The acoustic metric of such nonlinear dispersive pulse wave in our quasi-1D BEC set up, is given by the Eq. (9) with \( v_{(0)} \) having only one component along \( x \) axis, \( v_{(0)}(x, t) \).

We choose the initial wave profile [67] as the Gaussian

\[
\rho_{(0)}(x, t = 0) = \rho_{\infty} \left( 1 + 2\eta \exp \left[ -\frac{x^2}{2\sigma^2} \right] \right),
\]

where \( \sigma(\rho) \gg \xi(\rho) \). Here, at the center of our quasi-1D BEC set up, we produce a source of gravitational wave (GW) with density being almost uniform towards the boundary, mimicking asymptotically flat effective spacetime with a GW source. This longitudinal GW is different from its counterpart in Einstein gravity, in that the spacetime lacks general covariance, and the GW cannot be represented in its usual transverse and traceless form, cf. the discussion in [61].

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| Table I. Defining background flows from nonlinearity and dispersion and their associated metrics, where \( l \) is the length scale defined below (8). For Background (i), \( \rho_0, v_0 \) represent a solution of the nondispersive fluid equations without quantum pressure, and are initially chosen as the background before the shock develops, with perturbations treated to linear order. This initial Background (i) corresponds to the conventional analogue gravity metric and may or may not derive from nonlinear fluid equations; for example a uniform static medium does not represent a nonlinear background. For the Background (ii), \( \rho_{(0)}, v_{(0)} \), are found from the fully nonlinear, coupled fluid equations for both background and perturbations, however without quantum pressure included. cf. Ref. [63]. Finally, for the Background (iii), \( \rho_{(0)}, v_{(0)} \) are found from the nonlinear fluid equations applied to the background motion alone, but now with quantum pressure included. |
|---|---|---|
| Background (i) | Background (ii) | Background (iii) |
| \( \rho_0, v_0 \) | \( \rho_{(0)}, v_{(0)} \) | \( \rho_{(0)}, v_{(0)} \) |
| \( l \gg \xi \) | \( l \gg \xi \) | \( l \sim \xi \) |
| \( g_{\mu\nu} \) | \( g_{\mu\nu} \) | \( q_{\mu\nu} \) |
The second identity directly relating density to flow speed perturbations is valid for a simple wave [65]. The left-moving travelling wave comes with a ‘$-$’ sign in front of $c_{d0}$ in the above equations; $\rho(0) = \rho_0$ for $v = 0$, $\rho_0 \simeq 0$ of Eq. (11) since $\sigma \ll l$. This first-order quasi linear partial differential equation leads to multivalued valued solution by the method of characteristics [74].

By obeying momentum and mass conservation across the discontinuity, one is led to the equal area rule $\oint (x - x_s) dv = 0$ where $x_s$ is the shock location (location of discontinuity), to avoid such a multivalued solution from the shock time ($\approx t_{\text{shock}}$) onward [65]. We discuss this issue further in Appendix A. In the presence of quantum pressure, the solution (density, velocity etc) becomes oscillatory around the discontinuity, in comparison in the Fig. 2. Therefore, the solution becomes a well behaved function of $x$ and $t$ [67], see Appendix B. Numerical solution of Eq. (1) together with Eq. (4) produces $\eta_{tt}$ in Fig. 1. As expected, $\alpha$ is practically zero in the nonoscillatory region. The $\alpha$ correction term in the metric $q_{tt}$, which is usually hidden in a slowly varying background, is amplified in a region where quantum pressure is important: It is a significant contribution relative to the other forces in the Euler-type evolution of momentum (2) in the oscillatory region (cf. Fig. 7 in the Appendix B). Remarkably, the oscillations in the solution starts just slightly before the shock time $t_{\text{shock}}$ (see Fig. 6 Appendix B), whereas $t_{\text{shock}}$ is computed in the zero quantum pressure limit. Therefore, $t_{\text{shock}}$ maintains its importance as a time scale even with quantum pressure, signifying the time of initiation of oscillation. A linear travelling 1D wave can not stay linear forever, after a certain time nonlinearity makes the $v$ profile steeper, with negative $\frac{\partial v}{\partial t}$ This renders, in turn, the quantum pressure significant. Thus nonlinearity invites dispersion due to quantum pressure to play a significant role, also see the Appendices A and B.

IV. CENSORING THE NAKED SINGULARITY

We now aim to find what a discontinuity in the solution means for the effective spacetime. We denote the acoustic metric for the nondispersive metric as $g_{\mu\nu}$, cf. Table I. We stress that, while the metric is derived nondispersively, it is still taking the nonlinearity of the fluid into account [63]. It reads [75]

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{\rho(0)}{c_{s0}} \left[ -c_{s0}^2 (v^2_{(0)} - v^2_{(0)}) dt^2 - 2v_{(0)} dx^i dt + \sum_{i=1,2,3} (dx^i)^2 \right].$$ (14)

In the above metric, $\rho(0), v_{(0)}$ are found from the solution of the fluid equations without quantum pressure. This is background (ii) in Table I. For nonlinear dispersive shock wave, background (ii) and background (iii) coincide very
the region around shock location $x = x_s$, in the asymptotic region, i.e., near the condensate wall, Background (ii) and Background (iii) coincide with the Background (i) which is uniform and static, i.e., an acoustic analogue of Minkowski spacetime.

Evidently the acoustic metric is discontinuous at $x = x_s$ after the shock has occurred. We compute the Ricci scalar, $R$ [76] for $g_{\mu\nu}$ for the right moving travelling wave satisfying Eq. (12). We perform the calculations in Mathematica, replacing $\partial_t$ by $\partial_x$ derivatives, employing the Riemann wave equation (12). This procedure leads to the surprisingly simple relation

$$R = \frac{(1+\gamma)}{\rho(0)} \frac{\partial^2 v(0)(t,x)}{\partial x^2}. \quad (15)$$

expressing the curvature scalar solely by the second spatial derivative of the background flow field. At $x = x_s$, $v(0) = v_1$ and $\rho(0) = \rho_1$ which are the pre-shock values of velocity and density respectively, related to each other by the Eq. (13). Since in this case, the wave is propagating from left to right, at $x = x_s$, $v(0)$ first has $v_1$ then it jumps to post-shock value $v_2$ ($< v_1$), thus unrealistic multivalued $v(0)$ is avoided. At $x = x_s$, $x$ limit of $v(0)$ doesn’t exist, but it has a definite value which is $v_1$, and as a consequence; this discontinuity can be written mathematically in terms of a Heaviside step function, see Appendix A. $\frac{\partial v(0)(x,t)}{\partial x} = -\infty$ at $x = x_s$, and $\frac{\partial^2 v(0)(x,t)}{\partial x^2}$ at $x = x_s$ can be expressed as a summation of $\delta(0)$ and $\delta'(0)$ (with definite coefficients) type of infinities (in Appendix A); where $'$ denotes a $x$ derivative. We discuss the visualization of Dirac delta distributions through a delta-sequence function in Fig. 4 of Appendix A.

We plot in Fig. 3 the Ricci scalar of the nondispersive wave as it approaches the curvature singularity in the pre-shock phase $t < t_{\text{shock}}$. As can be seen, the expression (15) implies the existence of a (strong) curvature singularity at $x = x_s$, where $x_s$ is the position of discontinuity at $t \geq t_{\text{shock}}$. Since the velocity at any $x$ remains always very much less than the minimum value of sound speed $c_0$ ($= g\rho_0$), there is no event horizon present in the acoustic metric. Since at $x = x_s$, $v(0) = v_1$; sound speed $c_{s,0} = c_{s,1} = c_0 + \left(\frac{1}{2}\right) v_1$, and the travel speed of the discontinuity is $u = c_{s,0} + \left(\frac{1}{2}\right) (v_1 + v_2)$, $v_2 (< v_1)$ is the post-shock value of $v(0)$ [65]. Hence $c_{s,1} > u$. In Eq. (14), by putting $dx = udt$, $dy = dz = 0$, we find $ds^2 = \frac{u}{v_1} \left(-c_{s,1}^2 + (u-v_1)^2\right) dt^2$, from the above discussion, we notice that $c_{s,1} > |u-v_1|$. Therefore, at $x = x_s$, the discontinuity follows a timelike trajectory, representing a naked singularity. When we, on the other hand, solve the fluid equations with quantum pressure, the solution oscillates instead of discontinuity, we render the curvature for the metric $g_{\mu\nu}$ ∀ $x$ and $t$ finite, thus removing the singularity, cf. Fig. 2. However, for nondispersive waves, the discontinuity does not persist for $t \to \infty$, and $(v_1 - v_2)$ then falls to zero [65].

V. CONCLUSION

We demonstrated that the quantum pressure term leads to a regular oscillatory numerical solution for travelling waves in a quasi-1D BEC, thus prohibiting the otherwise naked singularity. Analogue gravity is effectively an aether theory, for which we have shown, using a particular initial condition, that the occurrence of a naked singularity is forbidden. Whether singularities in the dispersive aether of the BEC arise for any given nonsingular initial condition is an open question.

We have thus provided, for a BEC laboratory analogue simulating curved spacetimes, a sensor prototype operating in the trans-Planckian sector of the dispersion relation, which is based on microscopic quantum many-body physics of the system, and is thus naturally complete in the ultraviolet. To ultimately resolve the question of whether the CCH holds true, this latter property is crucial also for any proper quantum gravity.
In a cosmological context, the Trans-Planckian Censorship Conjecture (TCC) can be viewed as a momentum space version of the CCH, replacing the timelike singularity by the set of trans-Planckian wavelengths and the event horizon by the Hubble horizon \([77]\), to avoid (any variant of) the so-called trans-Planckian problem \([78]\).

An observer with access to infrared (sub-Hubble) modes must, for the TCC to hold, be shielded by the Hubble horizon from the trans-Planckian modes. Extending the line of argument presented, we expect that a cold atom analogue of the TCC can also be established, cf. \([47]\).

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**Appendix A: Nondispersive shock waves and the curvature singularity**

In this Appendix, first we briefly introduce the equal area principle introduced in \([65]\) for nondispersive shock waves, and then we proceed to calculating the Ricci scalar curvature for such a nondispersive shock wave.

The Riemann wave Eq. (12) can be solved by the analytical techniques for partial differential equations, i.e., the method of characteristics. This analytical solution \([61]\) gives rise to multivalued solution after a certain time, \(t_{\text{shock}}\). At \(t = t_{\text{shock}}\), \(\frac{\partial v}{\partial x}\) reaches infinity \([65]\). If we follow the method of characteristics \([63, 65]\) to solve Eq. (12) for the case without quantum pressure to avoid multivalued solution of density and velocity after \(t_{\text{shock}}\), the solution has to become discontinuous. This jump in velocity (and density) approximately satisfies the equal area rule \([65]\):

\[
\int_{v_1}^{v_2} (x - x_s) dv = 0,
\]

where \(v_1\) and \(v_2\) \((v_1 > v_2)\) are the pre-shock and post-shock values of discontinuous velocity \(v(0)\) across the position of discontinuity (shock) at \(x = x_s\). As a result, \(\rho_1\) and \(\rho_2\) are pre-shock and post-shock values of density \(\rho(0)\) related to \(v_1\) and \(v_2\) by

\[
\rho_{1,2} = \rho_0 \left[ 1 + \left( \frac{\gamma - 1}{2} \right) \frac{v_{1,2}}{c_{so}} \right]^{\frac{1}{\gamma - 1}}.
\]

With this discontinuity, velocity and density profiles are not multivalued anymore, which is discussed in detail by the classic textbook \([65]\). The expression of Ricci scalar (Eq. (15)) in the nondispersive limit is proportional to the second derivative in \(v(0)\); here we discuss an analytical way to calculate the second derivative of \(v(0)\) with a discontinuity at \(x = x_s\). This discontinuous velocity profile \(v(x, t)\) at fixed time \(t > t_{\text{shock}}\) can be written in a compact approximate way,

\[
v(0)(x, t) = (1 - \Theta(x - x_s)) f_1(x) + \Theta(x - x_s) f_2(x),
\]

where \(\Theta\) is the Heaviside step function, defined by \(\Theta(x - x_s) = 1\) for \(x > x_s\) and \(\Theta(x - x_s) = 0\) for \(x \leq x_s\). Furthermore, \(f_1(x)\), \(f_2(x)\) are two polynomials with finite coefficients in a compact way.

Then, the slopes of \(f_1(x)\) and \(f_2(x)\), at \(x = x_s\) smoothly fits into the pre-shock curve segment and post-shock curve segment, respectively. We find

\[
\frac{\partial v(0)}{\partial x} = \frac{df_1}{dx} + \frac{df_2}{dx} + \delta(x - x_s)(f_2(x) - f_1(x)),
\]

where \(\delta(x - x_s)\) is the Dirac delta distribution. The first two finite terms of the equation has a similar pattern to the Eq. (A3) for obvious reasons. Therefore,

\[
\left. \frac{\partial v(0)}{\partial x} \right|_{x=x_s} = \delta(0)(v_2 - v_1) + \left. \frac{df_1}{dx} \right|_{x=x_s}.
\]

Evidently, the first term on the right hand side dominates over the second term, rendering \(\frac{dv}{dx} \mid_{x=x_s} \) to be \(-\infty\), since \(v_2 < v_1\).

\[
\frac{\partial^2 v(0)}{\partial x^2} = (1 - \Theta(x - x_s)) \frac{d^2 f_1}{dx^2} + \Theta(x - x_s) \frac{d^2 f_2}{dx^2} + 2\delta(x - x_s) \left( \frac{df_2}{dx} - \frac{df_1}{dx} \right) + \delta'(x - x_s)(f_2(x) - f_1(x)).
\]

Therefore, at \(x = x_s\), ignoring the finite term \(\frac{d^2 f_1}{dx^2} \mid_{x=x_s}\), we write down the infinite terms as follows,

\[
\left. \frac{\partial^2 v(0)}{\partial x^2} \right|_{x=x_s} = 2\delta(0) \left( \frac{df_2}{dx} \mid_{x=x_s} - \frac{df_1}{dx} \mid_{x=x_s} \right) + \delta'(0)(v_2 - v_1).
\]

According to Fig. 2, \(\frac{df_1}{dx} \mid_{x=x_s}\) is negative; it always stays negative in the post-shock phase, and \(\frac{df_2}{dx} \mid_{x=x_s}\) is positive. Numerics in fact shows that, initially after \(t_{\text{shock}}\), \(\frac{df_1}{dx} \mid_{x=x_s}\) is negative, but eventually it becomes positive over time. The quantity \(\frac{\partial^2 v(0)}{\partial x^2} \mid_{x=x_s}\) above consists of two
In fact, the system behaves.

In this Appendix, we collect our numerical findings on dispersive shock waves with initial conditions \((11)\), as described in the main text. Some of these results have been presented already in Ref. [67], but for the convenience of the reader, we reproduce here these results together with a few additional observations, where our overall aim is to inspect closely the initiation of the oscillation of the dispersive shock waves, which is due to the quantum pressure term.

Specifically, in Fig. 5, we observe how the oscillation region is slowly spreading with progressing time. In Fig. 6, we display how the shock wave enters the oscillation phase, just prior to the shock time \(t_{\text{shock}}\). Finally, in Fig. 7, we display in some detail the onset of oscillations due to the quantum pressure becoming significant.

In Appendix B: Initiation of oscillations in dispersive shock waves

In this Appendix, we collect our numerical findings on dispersive shock waves with initial conditions \((11)\), as described in the main text. Some of these results have been presented already in Ref. [67], but for the convenience of the reader, we reproduce here these results together with a few additional observations, where our overall aim is to inspect closely the initiation of the oscillation of the dispersive shock waves, which is due to the quantum pressure term.

Specifically, in Fig. 5, we observe how the oscillation region is slowly spreading with progressing time. In Fig. 6, we display how the shock wave enters the oscillation phase, just prior to the shock time \(t_{\text{shock}}\). Finally, in Fig. 7, we display in some detail the onset of oscillations due to the quantum pressure becoming significant.
FIG. 7. (Top) We compare the contribution of the terms involving derivatives in $x$ in the 1D momentum equation Eq. (2) at $t = 27$. We see that the quantum pressure term becomes significant only in the oscillatory region. (Bottom) The quantum pressure is very much smaller, by several orders of magnitudes, in the nonoscillatory region. The parameters are identical to those in Fig. 1.

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