Towards Explainable Inference about Object Motion using Qualitative Reasoning

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Abstract

The capability of making explainable inferences regarding physical processes has long been desired. One fundamental physical process is object motion. Inferring what causes the motion of a group of objects can even be a challenging task for experts, e.g., in forensics science. Most of the work in the literature relies on physics simulation to draw such inferences. The simulation requires a precise model of the underlying domain to work well and is essentially a black-box from which one can hardly obtain any useful explanation.

By contrast, qualitative reasoning methods have the advantage in making transparent inferences with ambiguous information, which makes it suitable for this task. However, there has been no suitable qualitative theory proposed for object motion in three-dimensional space. In this paper, we take this challenge and develop a qualitative theory for the motion of rigid objects. Based on this theory, we develop a reasoning method to solve a very interesting problem: Assuming there are several objects that were initially at rest and now have started to move. We want to infer what action causes the movement of these objects.

1 Introduction

We are living in an era where an increasing number of AI agents entering into our daily lives and helping us with daily tasks such as household chores. To successfully perform these tasks, an AI agent needs to understand its surrounding environment and to be able to draw useful inferences based on its perceptual input. Living in a physical world requires AI be capable of inferring physical behaviours of everyday objects. This capability not only involves predicting what behaviours an object can have but also being able to figure out what causes their behaviours. In this paper, we focus on reasoning about object motion which is a most common physical behaviour of an object. Making an inference about object motion can be a challenging task for AI.

For example, Fig. 1a shows a scene where a set of blocks were initially at rest. At a certain time, there was an action made to exert an impulse at one of the blocks, which caused the movement of the blocks as depicted in Fig. 1b. When we observe this change, one natural question to ask is where and in which direction the impulse has been made. We humans can make such inference rapidly given only the information obtained from our visual perception. The knowledge we have about the scene is ambiguous, in the sense that we do not know exact physical parameters of the blocks, precise shapes or coordinates of their locations etc. However, we can still draw useful inferences based on this piece of knowledge and we can provide clear explanations of how the inference is derived. What human does in making the spatial or physical inference is conceptually similar to the methodology adopted by the qualitative reasoning community where the entities in that problem domain are characterised by a spatial representation, and the inference is drawn by reasoning about the constraints or relations between the entities.

As object motion in 3D space can be complex, it is critical to ensure that the qualitative representation is expressive enough and can capture all possible motions of an object in the space. Hence, we develop our theory according to a well-established physics modelling approach that is also widely used in nowadays physics engines. We devise a qualitative representation for spatial entities and constraints that the modelling approach uses for motion prediction. We show that our theory can cover all the possibilities of the motion of objects in a system that can be described by the modelling approach. The key contribution of this paper is
that we provide a qualitative theory for modelling rigid body motion in three-dimensional space. The theory is flexible as one can specify constraints in both qualitative or quantitative formulas based on their prior knowledge, and the reasoning method can be straightforwardly integrated into visual perception module. We demonstrate its usefulness by using it to solve a class of problems as illustrated in the above example.

2 Background and Related Work

There are two main research streams of modelling and reasoning about physical systems, namely qualitative physics and simulation-based reasoning. Qualitative physics (for a survey, see Davis 2008) emerged in the early 1980s, which uses symbolic approach to describe behaviours of physical systems. For example, Forbus (1984) proposed the qualitative process theory for modeling physical processes. Forbus (1981) provided a reasoning method for the motion of a ball in 2D space. (Nielsen 1990; Stahovich, Davis, and Shrobe 2000) formalised different qualitative theories to describe constrained mechanical systems such as a clock. Kuipers (1990) developed a qualitative simulation framework that predicts physical behaviours based on qualitative differential equation models. However, this framework lacks a way to model object motion and forces in three-dimensional space. Physical interactions (e.g., surface contact) between extended objects are not considered in this framework either. Recently, there has been some qualitative spatial calculi developed for three-dimensional space such as the spatial representation of three-dimensional rotation (Asl and Davis 2014) and trajectory (Mavridis et al. 2015). As far as we know, none of the above methods addresses motion of extended objects in three-dimensional space.

As an object movement can also be viewed as a spatial change of an object, the topic of our paper is broadly related to the area of qualitative spatial change and actions. One well-established work in this area is Galton (2000) that models qualitative state space based on the sign representation. De Kleer and Brown (1984) proposed a qualitative theory for representing and reasoning about motion of rigid objects. The formalisation of the theory is inspired by works from the two different fields: simulation and qualitative reasoning. Specifically, we develop a qualitative representation to describe forces and their effects on object motion. The qualitative representation is based on sign calculus which has been proved as a versatile tool in modelling physical process. (De Kleer and Brown 1984). To model contact forces between objects and the physical constraints between the forces, we refer to the theory of rigid body dynamics that is widely applied in many state-of-the-art physics engines. The goal of our theory is to capture all the possible motions of a group of rigid objects when the qualitative representation of their forces is known. Given an observed change in the motion of an object, the formalisation should also allow inferring what forces have caused the change.

In the below sections, we start by introducing a standard routine of rigid body simulation. We then present a qualitative representation and reasoning schema for object motion, which is in reminiscent of the simulation routine. Based on the proposed qualitative representation, we provide a formal definition of the action inference problem mentioned in the introduction.

3 A Qualitative Theory of Object Motion

We consider the domain of rigid object dynamics and propose a qualitative theory for representing and reasoning about motion of rigid objects. The formalisation of the theory is inspired by works from the two different fields: simulation and qualitative reasoning. Specifically, we develop a qualitative representation to describe forces and their effects on object motion. The qualitative representation is based on sign calculus which has been proved as a versatile tool in modelling physical process. (De Kleer and Brown 1984). To model contact forces between objects and the physical constraints between the forces, we refer to the theory of rigid body dynamics that is widely applied in many state-of-the-art physics engines. The goal of our theory is to capture all the possible motions of a group of rigid objects when the qualitative representation of their forces is known. Given an observed change in the motion of an object, the formalisation should also allow inferring what forces have caused the change.

In the below sections, we start by introducing a standard routine of rigid body simulation. We then present a qualitative representation and reasoning schema for object motion, which is in reminiscent of the simulation routine. Based on the proposed qualitative representation, we provide a formal definition of the action inference problem mentioned in the introduction.

Rigid Body Dynamics in Simulation

In a typical simulation of rigid body systems, the behaviour of the system is modelled as an ordinary differential equation that depends on time, given by \( f(x, t) = \dot{x} \) where \( x \) is the state vector of the system and \( \dot{x} \) is the time derivative of the state. Time in the simulation is often discretised into time points. Given a system state \( x_t \) at time point \( t \), to predict the state at a future time point \( t + \Delta t \) the simulation runs a numerical method, e.g., using Euler’s method the future state is calculated as

\[
\begin{align*}
\Delta x &= f(x_t, t) \\
x_{t+\Delta t} &= x_t + \Delta x
\end{align*}
\]
\[ x_{t+\Delta t} = x_t + \Delta t \cdot \dot{x}_t \] (1)

The key step in the simulation is to calculate the time derivative \( \dot{x} \) at each time point. The simulation first performs collision detection to identify objects that are in contact with each other. The region of a contact area is approximated by a set of contact points that are the corner points of the region. The simulation then computes normal and friction forces at the contact points according to certain physical constraints. Given that force is the time derivative of momentum, the momentum of the objects can be calculated based on the obtained forces according to Eq. (1).

Hence, as long as we know what forces are acted upon an object at a given time point, we can predict the motion of the object at a next time point. On the other hand, as long as we know the difference between the object motion at two time points, we can infer what forces are contributing to these changes. Given that we do not know the precise parameters of the underlying system, the calculation above is likely to provide inaccurate results, and this inaccuracy will be accumulated during numerical integration, which makes it less likely to find real explanations. This problem can be solved by reformulating the rigid dynamics theory into a qualitative theory that has the advantage of dealing with ambiguous and imprecise knowledge.

### Qualitative Representation of Object Motion

As force and momentum are vector quantities, we begin by introducing a standard qualitative representation for vectors. Specifically, we use sign calculus [De Kleer and Brown 1984] to represent a vector of numbers with each component of the vector is replaced by a sign that indicates whether the component is positive (+), negative (−), or zero (0). Hence, we make 27 distinctions of a three dimensional vector. We denote the set of all the distinct sign vectors as \( \mathfrak{S} = \{(a, b, c) | a, b, c \in \{+, −, 0\}\} \). The inverse of a sign or sign vector \( s \) is written as \( s^{-1} \). E.g., \( (+, −, 0)^{-1} = (−, +, 0) \).

In this paper, we adopt a fixed reference frame with the \( x\)-axis in the table representing the ground plane and the \( z \) axis in the opposite direction of the gravity.

Table 1 shows the table of three basic arithmetic operations between signs, namely, addition(\( \oplus \)), subtraction(\( \ominus \)), multiplication(\( \otimes \)). The asterisk sign * in the tables refers to an indefinite result with * = \{+, −, 0\}. The addition and subtraction between sign vectors are defined similarly, simply applying the corresponding sign operation between their components pair-wise. A sign vector that has an indefinite component is treated as a set of sign vectors that has only definite components. E.g., the sign vector \( * = \{+, +, +\} \) refers to a set \( \{0, +, +, (−, +, +), (+, +, +)\} \). We use a big addition symbol \( \bigoplus \) in the same way as the symbol \( \Sigma \) used in the mathematical summation, which will generate a set of sign vectors as a result. Another two fundamental operations of 3D vectors are inner product (\( \cdot \)) and outer product (\( \times \)), we define their sign-vector version in the same way as they are defined for numerical vectors:

\[
(u_1, u_2, u_3) \otimes (v_1, v_2, v_3) = (u_1 \odot v_1) \oplus (u_2 \odot v_2),
(u_3 \odot v_1) \oplus (u_1 \odot v_3),
(u_3 \odot v_2) \oplus (u_2 \odot v_1)
\]

(2)

Based on this formalism, we now propose a qualitative representation for the forces in our domain.

**Definition 1 (Qualitative Force).** A qualitative force on an object \( O \) is a 3-tuple \( \langle \mathbf{qd}, \mathbf{qr}, O \rangle \) where \( \mathbf{qd} \) is a sign vector representing the qualitative direction of the force, \( \mathbf{qr} \) is a sign vector of the direction pointed from the mass centre of \( O \) to the point where the force is acted upon. The qualitative force of gravity on \( O \) is \( \langle \mathbf{qd} = (0, 0, −), \mathbf{qr} = (0, 0, 0), O \rangle \).

Given a qualitative force (see Fig. 2), its components \( \langle \mathbf{qr}, O \rangle \) refer to a qualitative location where the actual force is acted upon. We can obtain a more accurate region when the shape of an object is given. For notational convenience, we define a procedure \( Q \) that will convert a 3D vector of real numbers to their sign counterparts or convert an actual force to its qualitative form.

We characterise the state of an object at a time point by the qualitative direction of its motion.

**Definition 2 (Object State).** The state of an object \( O \) at time \( t \), denoted \( O_t \), is a tuple \( \langle \mathbf{qv}_t, \mathbf{qω}_t \rangle \) where \( \mathbf{qv}_t, \mathbf{qω}_t \) are the sign vectors of the object’s linear and angular velocity, respectively. There are \( 27 \times 27 = 729 \) possible qualitative states.
Lemma 1. Given a state change $\Delta(O_{t_1}, O_{t_2})$ resulted from a set of actual forces $\{f_1, f_2, \cdots, f_n\}$ acted upon $O$ between $t_1$ and $t_2$, let $QF$ be a set of qualitative forces obtained by converting each actual force to its qualitative form, and let $D$ be another set of qualitative forces. If $QF \subset D$, then $\Delta(O_{t_1}, O_{t_2}) \in D$.

Proof. Let $v_{t_1}$ be the linear velocity of an object at time $t_1$ and $m$ the mass of the object, let $\{f_1, f_2, \cdots, f_n\}$ be the set of the actual non-negligible forces on the object between time $t_1$ and $t_2$. By Newton’s second law of motion, we have:

$$v_{t_2} - v_{t_1} = \frac{1}{m} \sum_{i}^{n} F_i, F_i = \int_{t_i}^{t_j} f_i dt, t_i \leq t_j \leq t_2$$

We can safely discard the constant $\frac{1}{m}$ and convert the equation to the sign representation. By definition of sign summation we have

$$Q(v_{t_2} - v_{t_1}) = Q\left(\sum_{i}^{n} F_i\right) \in \bigoplus_i Q(F_i) \quad (4)$$

By definition of sign addition, we have $Q(F_i) = Q(f_i)$ and replace it in Eq. 4 gives

$$Q(v_{t_2} - v_{t_1}) \in \bigoplus_{i} Q(F_i) = \bigoplus_{q, q \rho, \rho} qd \quad (5)$$

We can obtain the equation for the angular velocity in the similar way:

$$Q(\omega_{t_2} - \omega_{t_1}) \in \bigoplus_{q, q \rho, \rho} qr \otimes qd \quad (6)$$

Because $QF \subset D$, we have $QF \in P(D)$ and by Eq. 3 we have $\langle Q(v_{t_2} - v_{t_1}), Q(\omega_{t_2} - \omega_{t_1}) \rangle = \Delta(O_{t_1}, O_{t_2}) \in D$.

Lemma 1 says that given the set of qualitative forces $D$, as long as $D$ contains all the qualitative forces of the actual non-negligible forces, one can always find the observed state change in $D$. This lemma is crucial for the search of qualitative forces that cause the state change as the lemma guarantees that the algorithm will never miss a candidate set that contains solutions.

Ideally, for every object, we want to obtain a small-sized $D$ that contains all the actual qualitative forces on the object. The size of $D$ is up to $27 \times 27 = 729$, which is the number of possible combinations of two sign vectors. Now we introduce several rules that can help to reduce the size of $D$ without discarding any solutions. Each rule specifies a condition that has to be met by the assignment of the qualitative forces. We first specify the conditions in numerical formulas (can be used when precise numbers are known) and then define their qualitative versions by replacing the numerical operations with the sign operations. Let $p$ be the contact point between two objects $O_i$ and $O_j$.

Rule 1 (Vanishing Point). A contact point $p$ is a vanishing point when $O_i$ and $O_j$ are moving away at $p$. There is no contact force at any vanishing point.
moving away, the contact force will disappear at the point forces between objects. Therefore, when the two objects are started to move away at the points indicated by the green arrow) is applied, the green object will denote the contact points. After a sufficiently large force (in-blue objects are in contact initially (a), and the red dots indicate the contact normal and the contact force on the object. The qualitative version of the rule is derived by replacing each numerical operation in the original rule with the corresponding qualitative operation, therefore, the qualitative rule will cover all the cases where the original rule is satisfied (the rule condition is met).

The concept of vanishing point is introduced in Baraff [1989] based on the assumption that there is no attraction forces between objects. Therefore, when the two objects are moving away, the contact force will disappear at the point (see Fig. 3). The condition of not “moving away” is given by\
\[
\hat{n} \cdot (x_i - x_j) \leq 0, x = v + \omega \times r
\]
where \(\hat{n}\) is the contact normal and \(x\) is the linear velocity of the point \(p\) on the object. The qualitative version of the condition can be defined in the same way:
\[
\emptyset \neq \{-,0\} \cap \bigcup_{q\in{x},\{q\}} Q(\hat{n}) \odot Q(q) : qv \oplus (q\omega \odot qr)
\]

**Rule 2 (No Attraction Force).** The direction \(\hat{f}\) of the contact force on an object should be pointing inwards to the mass centre, given by \(\hat{f} : \hat{n} \geq 0\).

This rule is also based on the no-attraction-force assumption, which requires contact forces always push the two objects away from each other. The qualitative version requires \(\{+,0\} \cap \{q\} \odot Q(\hat{n}) \neq \emptyset\).

**Rule 3 (Newton’s Third Law of motion).** Given a contact point between \(O_i\) and \(O_j\), and let \(qd_i\) and \(qd_j\) be the qualitative directions of the two contact forces on \(O_i\) and \(O_j\), respectively. \(qd_i^{-1} = qd_j\).

This rule is given by Newton’s third law of motion: When two object are in contact, the contact force on one object should be in the opposite direction of the contact force on the other object.

**Lemma 2.** Given a rule that is satisfied by a set of actual forces \(\{f_1, f_2 \cdots f_n\}\), the qualitative version of the rule is also satisfied by \(\{Q(f_1), Q(f_2) \cdots Q(f_n)\}\).

**Proof.** The rule applies a series of arithmetic operations to the force vectors and is satisfied when the result is contained in the range specified by the rule condition. By the definition of sign calculus, each sign corresponds to a certain interval of real numbers, and the result of a sign operation covers all the possibilities of applying the corresponding numerical operation between numbers. Given that a qualitative rule is derived by replacing each numerical operation in the original rule with the corresponding qualitative operation, therefore, the qualitative rule will cover all the cases where the original rule is satisfied (the rule condition is met).

### 4 Solving Action Inference Problem

#### AIP-SAT

We solve AIP by formalising it as a constraint satisfaction problem AIP-SAT \((X, D, C)\) where \(X\) is the set of variables with each variable can be assigned with a value from its non-empty domain \(D_i \in D\). \(C\) is set of constraints with each constraint specifies some relations that must be held between a subset of variables. The goal is to find an assignment of qualitative forces that can cause the observed change from \(O_{t_1}\) to \(O_{t_2}\).

**Definition 6 (AIP-SAT).** Given an AIP problem \(AIP(O_{t_1}, O_{t_2})\), and let \(n\) be the number of force variables, we obtain the following AIP-SAT problem:

- **\(X\):** Each variable \(x_{1:n}\) corresponds to a force at a contact point or the force of gravity at the mass centre, and \(x_{action}\) is the variable of the action. Given an object \(O_i\), \(X_i\) denotes a subset of variables whose forces are on \(O_i\).
- **\(D\):** Each domain \(D_{1:n} = \langle qd_{1:n}, qr_{1:n}, O \rangle : qd \in \mathbb{S}\) contains a set of qualitative forces that \(x_{1:n}\) can be assigned with; \(qr\) and \(O\) are fixed except for the action variable \(D_{n} = \langle qd_n, qr_n, O \rangle : qd_n, qr_n \in \mathbb{S}, O \in \mathbb{O} \rangle\) as we need to infer the location upon which the action is exerted.
- **\(C\):** There are two constraints, namely,
  - \(C_1 : \forall O_i \in \mathbb{O}, \Delta(O_{t_1}, O_{t_2}) \in D_{X_i}\) where \(D_{X_i}\) is a set of assigned values of the variables in \(X_i\).
  - \(C_2 : \forall x \in X, value\ of\ x\ is\ consistent\ with\ Rule\ Rule\ 3\)

Constraint \(C_1\) requires that for each object \(O\), there is a combination of forces that can change the qualitative state from \(O_{t_1}\) to \(O_{t_2}\). Constraint \(C_2\) can be viewed as a set of unary constraints that restrict the domain of the force directions so that the assignment does not break any physical rule introduced in Sec.3.

#### AIP-Solver

In this section, we introduce a complete AIP-SAT algorithm based on graph-based tree search.
Structure Graph We use a directed multi-graph $G$ to represent a set $O$ of objects and their spatial relations at time $t$. Each vertex $v$ refers to an object $O \in O$ and is labeled by the state tuple $\Delta t$. The vertex also maintains a flag that can be set to a status of either “checked” or “to-check”. There is an edge from $v_i$ to $v_j$ if there is a contact point between $O_i$ and $O_j$. Each edge represents a variable. During the search, the algorithm will label an edge with a qualitative force tuple, which can be viewed as we assign a value to a variable.

The algorithm employs depth-first tree search to find solutions to a given AIP-SAT($O_{t_1}, O_{t_2}$) problem. Each node in the search tree maintains a structure graph, and each vertex of the graph is labelled with the corresponding object state $O_{t_2}$ at time $t_2$. In the beginning, all the vertices are marked as “to-check”. When branching a node, the algorithm finds a partial assignment of the variables belonging to the object $O_i$ represented by the “to-check” vertex. If a partial assignment is found, the algorithm will label the vertex with the corresponding object state $O_{t_2}$. After the branching, the flag of the vertex is set to “checked”. The algorithm keeps branching nodes until there is no more vertex “to-check”. An assignment is found when every vertex $v_i$ has been checked and labelled with $O_{t_2}$.

Let $D_{kwn}$ be a set of variables of $O_i$ that have already been assigned values.

To branch the root node (see Alg. 1), for each object $O_i \in O$, the algorithm obtains a set $QF$ of qualitative forces that are complying with Rule 2 (Line 2). It creates a new node by adding each qualitative force $qf \in QF$ to the set $D_{kwn}$ of $O_i$ (Line 3-4). The algorithm will start searching first from a set of objects that have different states at time $t_1$ and $t_2$, i.e., $\Delta (O_{t_1}, O_{t_2}) \neq 0$.

To branch an intermediate node (see Alg. 2), the algorithm selects an arbitrary “to-check” vertex $v_i$ in the graph, where preferences are given to the ones that have nonempty $D_{kwn}$. If it assigns qualitative forces to variables so that $C_1$ and $C_2$ are satisfied. A backtrack happens when there is no such assignment.

To make an assignment, the algorithm first obtains the set of incoming edges $E_{in}$ that are not yet labeled and are not connected to any vertex that has already been checked, which provides a set $X_i$ of unassigned variables (Line 11). The next task is to find a valid assignment to those variables in $X_i$ so that the constraints hold with $D_{kwn} \cup D_{X_i}$ (Line 12).

Having obtained the assignment, the algorithm adds the assigned variables to $D_{kwn}$ (Line 13), and then creates a child node by updating its graph in the following steps (Line 14):

- Label $v_i$ with $O_{t_2}$ and set the flag to “checked”.
- Label the edges in $E_{in}$ according to the assignment and label their corresponding outgoing edges $E_{out}$ with the qualitative forces in the opposite direction by Rule 2.
- For each vertex $v_j$ whose incoming edges labeled in the previous step, add the corresponding qualitative forces to the set $D_{kwn}$ of $v_j$.

Once a solution is detected, the assignment can be straightforwardly obtained from the set $D_{kwn}$ of each vertex $v_i$.

### Algorithm 1: Branch the root node

**Input:** $O$ : the set of objects in a scene

```plaintext
1 for $O_i \in O$ do
2 \hspace{1em} $QF_i \leftarrow \{qf, qr, O_i\}$ satisfies Rule 2; $q_d, qr \in E$;
3 \hspace{1em} for $qf \in QF_i$ do
4 \hspace{2em} $D_{kwn} \leftarrow \{qf\}$;
5 \hspace{1em} Branch Intermediate Node($O_i, D_{kwn}$);
6 end
7 end
```

**Algorithm 2:** Branch an intermediate node

**Input:** $O_i, D_{kwn}^i$ : the object at an intermediate node and the variables that have already been assigned.

```plaintext
1 $E_{in} \leftarrow \{e(v_j, v_i), v_j \text{ is labeled with “to-check”}\} 
2 \hspace{1em} e(v_j, v_i) \text{ is an edge from } v_j \text{ to } v_i; 
3 \hspace{1em} \Xi_i \leftarrow \text{variables given by } E_{in}; 
4 \hspace{1em} \Xi_i \leftarrow \text{variables given by } E_{in}; 
5 \hspace{1em} Assign each variable $x_i \in \Xi_i$ so that $C_1$ and $C_2$ is satisfied; 
6 \hspace{1em} $D_{kwn}^i \leftarrow D_{kwn}^i \cup D_{X_i};$ 
7 \hspace{1em} Label $v_i$ with $O_{t_2}$ and set the flag to “checked”;
8 \hspace{1em} for $e(v_j, v_i) \in E_{in}$ do
9 \hspace{2em} label $e(v_j, v_i)$ with the assignment of the corresponding variable $x_i$;
10 \hspace{2em} label $e(v_i, v_j)$ with the qualitative force $qf$ given by Rule 3;
11 \hspace{2em} $D_{kwn}^i \leftarrow D_{kwn}^i \cup \{qf\};$
12 end
13 end
```

**Theorem 1 (AIP-Solver is complete).** Given an AIP($O_{t_1}, O_{t_2}$) problem, the algorithm always finds an assignment that contains the actual qualitative forces leading to the state changes.

**Proof.** Let $\{f_{\text{action}}, f_1, f_2 \ldots f_n\}$ be the set of the actual forces that contribute to the state change and let $A$ be the corresponding assignment with $\{x_{\text{action}} \leftarrow Q(f_{\text{action}}), x_1 \leftarrow Q(f_1) \ldots x_n \leftarrow Q(f_n)\}$. We prove the theorem by contradiction. Assuming the assignment $A$ is not found by the algorithm. Since the algorithm will check each vertex in the graph, the partial assignment of $A$ must be pruned by the algorithm at a branching stage of the search.

Without loss of generality, assuming the partial assignment is pruned at a node with a “to-check” vertex $v_i$, and let $D_A$, be the set of qualitative forces on $O_i$ according to the assignment $A$. Given the fact that the node is pruned, by the construction of the algorithm, we know $\Delta (O_{t_1}, O_{t_2}) \neq \Delta_{D_A}$. When $D_A \subseteq D$, the pruning contradicts Lemma 1. When $D_A \not\subseteq D$, it means that some of the qualitative forces in $D_A$ have been discarded by the algorithm. As the algorithm only discards assignments if they violate Rule 1, 3 given the actual forces do not violate any rule, the pruning contradicts Lemma 2.

The branching factor of each intermediate node is equal
to the number of possible partial assignments of the variables in $\mathcal{X}_i$. This number can be huge when there are many variables that can be assigned to multiple values. To avoid the expensive branch factor, in practice, we can group some assignments of a variable into a set $A_i$ and the set is considered as a single assignment. A constraint is satisfied by $A_i$ as long as it is satisfied by any individual assignment in $A_i$. By this, we can substantially reduce the branching factor while it can be proven that the completeness of the algorithm is still guaranteed.

The algorithm can be made even more efficient by using heuristics that capture certain knowledge of the underlying domain. Here we give two example heuristics:

Given a variable of a contact force and its assigned qualitative force, if the assignment is not made by Rule 3. we call this qualitative force resistant force.

**Heuristic 1.** If a qualitative force is a resistant force, it can only cancel, but not overwhelm, the effects of other forces.

For example, given a resistant force of direction $(+, -, 0)$ and another non-resistant force $(-, -, 0)$. Combining the two forces by this heuristic gives $(-, 0)$, $(-, 0)$. We create this heuristic according to the standard simulation routine (Baraff 1997a) of computing contact forces. Once the simulation detects a potential inner-penetration (collision) between objects, it will calculate a contact force (resistant force) to resist the inner-penetration. When the collision is inelastic, the effect of this resistant force on object’s momentum can hardly overwhelm the effect of the force that causes the inner-penetration.

**Heuristic 2.** The action that is made to an object will cause the movement of the object.

This heuristic assumes that an object will change its state after an action is made to it. Under this assumption, the algorithm can start the search from only the objects whose states have changed.

## 5 Evaluation

We implemented the algorithm and evaluated it in both simulated and the real-world environments.

We use Mujoco(www.mujoco.org) that is a state-of-the-art physics engine used in the robotics research. We create several scenes with each scene contains a set of blocks of different sizes and physical properties. In the beginning, the blocks are forming a stable structure, and we use the state of the objects as the before state. Since we use simulation, we obtain the locations of contact points and mass centres directly from the simulator, and use them to compute the sign vectors. One thing to emphasise is that we do not have to know the exact numbers as it does not make any difference to the result if two different numbers give the same sign vector. We make an action that exerts an impulse on one of the objects and the impulse is chosen to cause a significant movement of objects. We simulate the scene for a number of time steps and then take the state of the objects as the after state. We run the algorithm on the two states to infer the qualitative representation of the action. As we proved, the algorithm found correct actions in all the experiments.

Given a scene, the total number of candidate qualitative actions is equal to the number of all the possible combinations of qualitative force directions (except the zero sign vector) and their possible qualitative locations. In the example given in Fig. 1, where there are 15 objects, there are $26 \times 27 \times 15 = 10530$ possible qualitative actions. The algorithm finds 48 different qualitative forces on two objects using Heuristic 1-2 while using only Heuristic 2 it finds 772 different qualitative forces on the same objects (see Fig. 2).

At first glance, there are lots of possibilities, however, these possibilities can be further eliminated if we could infer an range of possible action force directions or locations. We also construct a scene that has a simple structure (see Fig. 5a-c) and manually verify each identified qualitative forces to see whether they can cause the observed movement as the actual force does. Surprisingly, most of the reported solutions can make it, and some solution (see Fig. 5a) requires very strict conditions to be successfully instantiated in the simulation.

In the real world experiment, we obtain visual inputs of a scene using Kinect 2 that generates RGBD images. The time gap between the before and after scenes is around 1 second. We use a segmentation algorithm provided in (Silberman et al. 2012) to detect boxes and their spatial properties. We
obtain the linear and angular velocity of each object from the differences in their positions and orientations at two different time points. To further demonstrate the capability of dealing with ambiguous information, we only use the contact points that are visible from the image. An example scene is shown in Fig. 6. Our method detected the real solution and found 80 possible qualitative forces on either the blue or red box.

**Discussion, Generalisation and Future Work**

The algorithm we present relies on very general assumptions. Given a specific problem domain, we can keep adding realistic assumptions about the domain for better performance. As hinted above, the algorithm can benefit from spatial reasoning algorithms that help to restrict the range of possible qualitative actions further. For example, in a robotics manipulation scenario, we could infer the range of the space that a robotic arm can reach and use this knowledge to prune those unreachable actions. This reasoning could be done either numerically such as trajectory planning or qualitatively such as reasoning about qualitative rotations of arms. The explanation generated by the algorithm is in the form of a set of the assigned qualitative forces which are readily be labelled with causal roles according to the theory proposed in [Wolff and Barbey 2015].

Our theory can work with qualitative and quantitative constraints in the same way as we specified the rules. One future direction can be extending the theory to cover magnitude quantities. By this we could infer whether the mass or inertial tensor of an object would prevent any potential movement caused by a force of certain magnitude. Another promising direction is to use the qualitative inference to guide the search of solutions for simulation-based methods. The hierarchical optimisation framework proposed in [Toussaint 2015] could be one possibility.

The completeness proof only holds when we know the qualitative locations, i.e., the tuple \(qr, O\), of all the contact points beforehand. However, it could be possible that some of the contact points are missing due to imperfect perception or unobserved collisions between \(t_1\) and \(t_2\). To deal with this problem, we can extend the formulation of AIP-SAT by adding free variables representing the forces on unknown contact points. Each free variable has the same form as the action variable of which the qualitative location is not fixed. Hence, it would be desirable to have a method to tell if the qualitative location of each assignment is actually reachable by the involved objects. A simple heuristic can be to check whether there is an open path between the two collided objects given their motion state. As inferring potential collisions between multiple moving objects itself is a challenging problem in both simulation (collision detection [Kockara et al. 2007]) and qualitative reasoning ([Ge et al. 2016]) area, we left the investigation of this problem as future work.

**6 Conclusion**

This paper proposed a qualitative theory for the motion of rigid objects based on the modelling approaches from qualitative reasoning and physics simulation areas. Based on the formulation we solved an interesting action inference problem. We proved the completeness of the algorithm and applied it to both simulated and real-world environments. We hope this work opens up a new direction of using qualitative reasoning for drawing explainable physical inferences in daily scenarios, which could contribute to achieving the goal of building intelligent physical systems.

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