Artificial high effective permittivity medium in a SIW filled with metallic cylinders

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Abstract. A new topology of step-impedance band-pass filters in Substrate Integrated Waveguide (SIW) technology has been recently demonstrated in which low effective permittivity regions have been achieved by removing part of the substrate material and then shielding the perforated structure. Alternatively, in this work a new way to obtain an increased relative permittivity in the guiding region is proposed by periodically inserting metallic inclusions. This paper shows the results of a systematic study of the effective permittivity obtained in this way in a SIW in order to synthesize a higher effective permittivity, which can be used in the filter design.

1. Introduction
Recently, a new topology of step-impedance bandpass filters in SIW technology has been theoretically and experimentally demonstrated [1],[2]. The high impedance regions, i.e., those with low effective permittivity, have been achieved by inserting air holes to lower the substrate permittivity. However, there is a natural limit in the low effective permittivity obtained in the perforated regions, given by the maximum volume of dielectric material that can be removed. In [1], a reduction of the relative permittivity in a perforated SIW waveguide of 42% was achieved, which was theoretically and experimentally demonstrated. Alternatively, in this work we propose a way to obtain an increased relative permittivity in the guiding region by inserting in the dielectric an array of metallic inclusions, as demonstrated in previous works [3],[4]. This paper presents the numerical results of a study of the effective permittivity in a SIW in which metallic cylinders have been periodically inserted to synthesize a higher effective permittivity. The study has been performed by using the commercial software tool Ansys High Frequency Structural Simulator (HFSS). Several insertion densities have been simulated to study the influence of the metal/dielectric volume relation in the effective permittivity. Finally, our preliminary results have been compared to the predictions of a theoretical model of this artificial propagation medium [5] to provide a helpful insight for designing new devices.

2. Study of an artificial high effective permittivity medium in a SIW filled with metallic cylinders
Fig. 1(a) shows a SIW section, with two rows of parallel metallic posts (or via holes) characterized by their separation \( s_v \) and diameter \( d_v \), delimiting the area of propagation of the waveguide. In our case, the value of the waveguide height is \( b = 0.63 \text{ mm} \), corresponding to
the thickness of the substrate employed. In this SIW, metallic cylinders characterized by their height $h$, diameter $d_c$ and separation $s_c$ -in this particular triangular pattern configuration- have been added along the propagation region. Thus, the propagation constant of this guide is determined by the width $a_{SIW}$ (see Fig. 1(a)) of the SIW, and also by the effective permittivity obtained after the insertion of the metallic cylinders, whose height $h$ must be lower than, but not far from, the waveguide height $b$ to achieve a high effective permittivity in the waveguide.

Using the equivalence between a SIW and a rectangular waveguide of a slightly lower width $a$ [6], we have characterized the electromagnetic field propagation in this equivalent waveguide with periodic metallic cylinders. In our study, we have obtained the dispersion curve of the waveguide modes with the commercial software tool Ansys HFSS, which also yields the cutoff frequency of the modes. For designing purposes, we can restrict our study to the monomode regime of the waveguide. In this case, it is interesting to define an equivalent dielectric homogeneous waveguide whose effective permittivity is related to the cutoff frequency of its TE$_{10}$ mode through the following expression:

\[ \varepsilon_{ref} = \frac{c^2}{4a_{eff}f_c^2} \]  

where $c$ is the speed of light in free space and $f_c$ is the cutoff frequency of the first waveguide mode provided by the simulation software.

Using eq. (1), we have obtained the effective relative permittivity of a rectangular waveguide of dimensions $a = 8.5$ mm and $b = 0.63$ $\mu$m (being $b$ the height of the substrate employed – Taconic CER-10–, with $\varepsilon_r = 10$ and $\tan\delta = 0.0035$), periodically filled with a triangular pattern of metallic cylinders along the waveguide, with alternating columns of 5 and 4 cylinders of height $h = 4/5b$ along the waveguide for this particular configuration (see Fig. 1(b)).

Firstly, we have obtained the effective permittivity of this partially filled rectangular waveguide with metallic cylinders as a function of their diameter $d_c$, with a fixed distance between their centers of $s_c = 1.6$ mm. In Fig. 2 it is represented the effective permittivity of this waveguide as a function of $d_c$, where it can be seen that $\varepsilon_{ref}$ varies from $\varepsilon_{ref} = 10$ (when $d_c = 0$, i.e., for the case without metallic cylinders) up to 22.8 (for $d_c = 1.1$ mm). This graph can be employed in order to synthesize a desired $\varepsilon_{ref}$ for a filter design.

Next, we have analyzed the variation of the effective permittivity with the distance between cylinders, for a fixed diameter $d_c = 1.1$ mm. In Fig. 3 it is represented the effective permittivity of this waveguide as a function of $s_c$ from 1.3 mm (corresponding to a minimum distance between the edges of adjacent cylinders of 200 $\mu$m, in order to ensure a certain mechanical strength) to
Figure 2. $\varepsilon_{\text{ref}}$ as a function of $d_c$, for a fixed separation $s_c = 1.6$ mm.

Figure 3. $\varepsilon_{\text{ref}}$ as a function of $s_c$, for a fixed diameter $d_c = 1$ mm.

1.7 mm (distance between the edges of 600 $\mu$m). Again, this graph allows to synthesize a desired $\varepsilon_{\text{ref}}$ for designing purposes.

At the sight of the previous results, we have chosen an implementable configuration of $d_c = 1.1$ mm and $s_c = 1.6$ mm (with a distance between the edges of 0.5 mm), which yields an effective permittivity of $\varepsilon_{\text{ref}} = 22.8$. For this configuration, we have computed the dispersion diagram of the periodically filled waveguide with metallic cylinders. In Fig. 4 it is compared the dispersion curve of the periodically filled waveguide with metallic cylinders for $d_c = 1.1$ mm and $s_c = 1.6$ mm (crosses) with that of its equivalent dielectric homogeneous waveguide (solid line) of $\varepsilon_{\text{ref}} = 22.8$. In this figure, it can be checked that the effective permittivity equivalence is only strictly satisfied at cutoff, given the frequency dependent nature of the metallic inclusions. However, the equivalence can be used as a first approximation in the filter design.

In order to check the validity of the previous results, we have analyzed the dispersion behaviour of a periodically filled SIW with metallic cylinders. To this end, we have obtained the equivalent rectangular width in SIW technology $a_{\text{SIW}}$, with the following parameters for the via holes: $d_v = 0.7$ mm, $s_v = 0.95$ mm, which provide a width of SIW of $a_{\text{SIW}} = 9.2$ mm. On the other hand, we have designed microstrip to SIW transitions consisting of microstrip tapers [7] (see Fig. 5), whose dimensions $W_t$ and $L_t$ have been optimized for a maximum matching of the waveguide transitions, while the width of the microstrip line is 0.6 mm (50 $\Omega$). In Fig. 6 there are shown the scattering parameters of a 46 mm length section of the periodically filled SIW with metallic cylinders under study (including dielectric and conductor losses -copper-) obtained with Ansys HFSS, showing a cutoff frequency of $f_c = 3.68$ GHz, which is very similar to the value previously obtained of 3.70 GHz in the dispersion curve with Ansys HFSS for the same waveguide configuration.
3. Theoretical modeling and comparison with simulations

A theoretical model of the artificial propagation medium is convenient to provide a better understanding of the electromagnetic problem and helpful insight for designing new devices. The theoretical study of artificial dielectrics began early in the history of electromagnetism [5]. Some particular problems have been solved in the context of electrostatics, as the one that

Finally, the dispersion curve of the SIW waveguide periodically filled with metallic cylinders whose electrical response is shown in Fig. 6 has been computed from its scattering parameters obtained for two different lengths using the same approach as that presented in [8]. The simulated dispersion curve obtained by this method has been presented in Fig. 4 with dashed line, showing a good agreement in the represented operation band. All the theoretical analysis performed in this work is going to be experimentally verified shortly.
we have chosen to model the propagation in the cylinder-loaded SIW: a tetragonal array of conducting cylinders oriented along one of the array axis and with the external electric field parallel to the cylinders. The effective permittivity relative to the host medium permittivity \( \tilde{\varepsilon}_{\text{ref}} \) is given by [9]

\[
\tilde{\varepsilon}_{\text{ref}} = 1 + \frac{N \alpha/\varepsilon_0}{1 - C \alpha/\varepsilon_0},
\]

(2)

where \( \alpha \) is the polarizability of the inclusions, \( N \) the number of inclusions per unit volume, and \( C \) is an interaction constant providing the correction to the external electric field due to the dipoles surrounding the dipole under consideration. For an infinite tetragonal network of dipoles, the interaction constant is given by [9]

\[
C = \frac{1}{l_y^3} \left\{ \frac{1.202}{\pi} - 16\pi K_0 \left( \frac{2\pi l_t}{l_y} \right) \right\}
\]

(3)

In this expression, \( Y \) is the axis parallel to the external electric field, \( l_t \) are the unit cell dimensions along the \( X \) and \( Z \) axes, while \( l_y \) is the unit cell dimension along the \( Y \) axis, and \( K_0 \) is the modified Bessel function of second kind and zero order.

Applying the image theory to the waveguide walls allows us to represent the waveguide as an infinite dipole array. We will place just a layer of cylindrical insertions parallel to the wider side of the waveguide. Therefore, \( l_y = b \) in eq. (3), being \( b \) the height of the waveguide.

Referring the polarizability of a perfectly conducting cylinder, we have followed the paper of Taylor [10]. The polarizability for the field oriented along the cylinder axis is given by

\[
\frac{\alpha}{\varepsilon_0 V_0} = \frac{2h}{D} \left[ \frac{r_b}{3} + r_0 \frac{\Gamma(3/2)}{2^{2/3} \Gamma(13/6)} \right] + t_b + t_0 \frac{2^{1/3}}{\Gamma(5/3)}
\]

(4)

where \( h \) is the height of the cylinder, \( D \) its diameter and \( V_0 \) its volume, and where the parameters \( r_b, r_0, t_b \) and \( t_0 \) depend on the aspect ratio \( h/D \) [10]. For the substrate presented in the preceding sections, with \( b = 0.63 \) mm, we have chosen as feasible dimensions \( h = D = 0.5 \) mm. We have simulated with Ansys HFSS a waveguide with \( a = 10 \) mm and several densities of inclusions: 4 to 9 cylinders along the wider side of the waveguide, corresponding to \( l_t \) between 1.1 and 2.5 mm. We have left a distance \( l_t/2 \) from the outermost cylinders to the vertical walls of the waveguide to get the same spacing up to the first image dipole formed by these walls. The effective permittivity of this transmission media has been obtained from the simulation by the method explained in the last section.

The results are compared to the predictions of the theoretical model expressed by formulas 2-4 in Fig. 7. It is clear that the model predictions scale much faster with the density of inclusions. This divergence could be due to several causes: 1) The dipole approximation may be inadequate for high inclusion densities; 2) The inhomogeneity of the electric field for the guided mode is not considered in the model; 3) The theoretical model is electrostatic, while the propagating mode is electromagnetic. Those aspects of the problem are now under consideration to elaborate a more sophisticated model for the SIW effective permittivity.

4. Conclusion
In this work it is proposed a new way to increase the permittivity in a SIW by inserting a periodic array of metallic cylinders along the waveguide. A systematic study of the resulting effective permittivity has been performed as a function of the cylinders diameter and separation. The results of this study have been numerically checked through simulations of the electrical response of a particular design of a SIW filled with metallic cylinders.
Figure 7. $\varepsilon_{\text{ref}}$ of a waveguide loaded with conducting cylinders: results of Ansys HFSS simulation and theoretical model.

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