Deep Adaptive Input Normalization for Price Forecasting using Limit Order Book Data
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Abstract—Deep Learning (DL) models can be used to tackle time series analysis tasks with great success. However, the performance of DL models can degenerate rapidly if the data are not appropriately normalized. This issue is even more apparent when DL is used for financial time series forecasting tasks, where the non-stationary and multimodal nature of the data pose significant challenges and severely affect the performance of DL models. In this work, a simple, yet effective, neural layer, that is capable of adaptively normalizing the input time series, while taking into account the distribution of the data, is proposed. The proposed layer is trained in an end-to-end fashion using back-propagation and can lead to significant performance improvements. The effectiveness of the proposed method is demonstrated using a large-scale limit order book dataset.

Index Terms—time series forecasting, data normalization, limit order book data, deep learning

I. INTRODUCTION

Forecasting time series is an increasingly important topic, with several applications to various domains [1]–[7]. Many of these tasks are nowadays tackled using powerful deep learning models [8]–[12], which often lead to state-of-the-art results outperforming the previously used methods. However, applying deep learning models to time series is challenging due to the non-stationary and multimodal nature of the data. This issue is even more apparent for financial time series, since financial data can exhibit significantly different behavior over the time due to a number of reasons, e.g., market volatility.

To allow for training deep learning models with time series data, the data must be first appropriately normalized. Perhaps the most widely used normalization scheme for time series when using DL is the z-score normalization, i.e., subtracting the mean value of the data and dividing by their standard deviation. However, z-score normalization is unable to efficiently handle non-stationary time series, since the statistics used for the normalization are fixed both during the training and inference. Several recent works attempt to tackle this issue either by employing more sophisticated normalization schemes [13]–[15] or by using carefully handcrafted stationary features [16]. Even though these approaches can indeed lead to slightly better performance when used to train deep learning models, they exhibit significant drawbacks, since they are largely based on heuristically-designed normalization/feature extraction schemes, e.g., using price change percentages instead of absolute prices, etc., while there is no actual guarantee that the designed scheme will be indeed be optimal for the task at hand.

To overcome these limitations, we propose a Deep Adaptive Input Normalization (DAIN) layer that is capable of a) learning how the data should be normalized and b) adaptively changing the applied normalization scheme, according to the distribution of the measurements of the current time series, allowing for effectively handling non-stationary and multimodal data. The proposed scheme is straightforward to implement, can be directly trained along with the rest of the parameters of a deep model in an end-to-end fashion using back-propagation and can lead to impressive improvements in the forecasting accuracy. Actually, as we experimentally demonstrate in Section [III] the proposed method allows for directly training deep learning models without applying any form of normalization to the data, since the raw time series is directly fed to the used deep learning model.

The main contribution of this work is the proposal of a deep learning layer that learns how the data should be normalized. To this end, the proposed layer is formulated as a series of two sublayers, as shown in Fig. [I] the first one is responsible for shifting the data into the appropriate region of the feature space, while the second layer is responsible for appropriately scaling the data. Note that the aforementioned process is adaptive, i.e., the applied normalization scheme depends on the actual time series that is fed to the network, and it is also trainable, i.e., the way the proposed layers behave is adapted to the task at hand using back-propagation. The effectiveness of the proposed approach is evaluated using a large-scale limit order book dataset that consists of 4.5 million limit orders [17].

To the best of our knowledge this is the first time that an adaptive and trainable normalization scheme is proposed and effectively used in deep neural networks. The proposed approach is also related to existing normalization approaches for deep neural networks, e.g., batch normalization [18], instance normalization [19], layer normalization [20] and group normalization [21]. However, these approaches are not actually designed for normalizing the input data and, most importantly, they are merely based on the statistics that are calculated during the training/inference, instead of learning how to normalize the data. It is worth noting that it is not straightforward to use non-linear neural layers for adaptively normalizing the data, since these layers usually require normalized data in the first place in order to function correctly. In this work, this issue is addressed by first using a robust and carefully initialized linear layer to estimate how the data should be centered, and
then learning how to scale the data using a non-linear layer that operates on the centered data, effectively overcoming this limitation.

The rest of the paper is structured as follows. First, the proposed method is analytically described in Section II. Then, an extensive experimental evaluation is provided in Section III, while conclusions are drawn in Section IV.

II. DEEP ADAPTIVE INPUT NORMALIZATION

Let \( x = \{x_1, x_2, x_3, \ldots, x_L\} \) be a time series that is composed of \( L \) \( d \)-dimensional measurements. Assuming that each measurement was generated by a Gaussian distribution, then the data can be effectively normalized as:

\[
[x']_l = (x_l - \mu_l)/\sigma_l,
\]

where the notation \( [x]_l \) is used to refer to \( l \)-th element of vector \( x \) and \( \mu_l \) and \( \sigma_l \) are the mean and standard deviation (of each separate variable) of the measurements, respectively. Note that if the previous assumption does not hold, i.e., the measurements were generated by a multimodal distribution, then the sample mean and standard deviation can lead to sub-optimal results, especially if the statistics around each mode differ significantly from each other. In this case, it can be argued that the data should be normalized in an mode-aware fashion, allowing for forming a common representation space.

In this work it is assumed that the data are generated by a Gaussian Mixture Model described by following density function:

\[
p(x) = \sum_{i=1}^{N} \phi_i \mathcal{N}(\mu_i, \Sigma_i),
\]

where \( \phi_i \) is the weight of the \( i \)-th Gaussian with mean \( \mu_i \) and covariance \( \Sigma_i \), which is denoted by \( \mathcal{N}(\mu_i, \Sigma_i) \). Note that even when this assumption does not hold, the data can be similarly modeled using Kernel Density Estimation [22]. Therefore, to normalize the data the Gaussian from which they were generated must be identified and, then, the measurements must be normalized using the estimated mean and co-variance of the corresponding Gaussian. In this work, we also assume that the variables are un-correlated, i.e., the co-variance matrix is diagonal, allowing for simplifying the derivation of the proposed approach.

The goal of the proposed method is to learn how the measurements \( x_i \) should be normalized by appropriately shifting and scaling them:

\[
x' = (x - \alpha) \odot \beta,
\]

where \( \odot \) is the Hadamard (entrywise) multiplication operator. Note that by setting \( \alpha = E[x] \) and \( \beta = E[(x - E[x])^2]^{-1} \) the plain z-score normalization is obtained. However, as it was already discussed, due to the non-stationary nature of the data, the obtained estimations for \( \alpha \) and \( \beta \) might not be the optimal for normalizing every possible measurement vector, since the distribution of the data might significantly drift, invalidating the previous choice for these parameters. This issue becomes even more apparent when the data are multimodal, e.g., when training model using time series data from different stocks that exhibit significantly different behavior (price levers, trading frequency, etc). To overcome these limitations we propose to dynamically estimate these quantities and separately normalize each time series by implicitly estimating the distribution from which each measurement was generated. Therefore, in this work, we propose normalizing each time series using the following scheme:

\[
x' = (x - \alpha(x)) \odot \beta(x),
\]

where both \( \alpha(x) \) and \( \beta(x) \) depend on the current input, instead of being the global averages calculated using the whole dataset.

The proposed architecture is summarized in Fig. 1. First a summary representation of the time series is extracted by averaging all the \( L \) measurements:

\[
s_\alpha = \frac{1}{L} \sum_{i=1}^{L} x_i \in \mathbb{R}^d.
\]

More advanced aggregation methods can also be used for extracting the summary representation, such as the Bag-of-Features representation [23], [24]. However, the averaging approach employed in this paper proved to be quite simple and effective for extracting a summary representation, even though these more advanced methods can possibly further improve the
performance of the proposed method. This summary representation will be used to estimate the distribution from which the current time series was generated, in order to appropriately modify the normalization procedure.

Then, the shifting function $\alpha(x)$ is defined using a linear transformation of the extracted summary representation as:

$$\alpha(x) = W_\alpha s_\alpha \in \mathbb{R}^d,$$

(6)

where $W_\alpha \in \mathbb{R}^{d \times d}$ is the weight matrix of the first neural layer, which is responsible for shifting the measurements across each dimension. Employing a linear transformation layer ensures that the proposed method will be able to handle data that are not appropriately normalized (or even not normalized at all), allowing for training the proposed model in an end-to-end fashion without having to deal with stability issues, such as saturating the activation functions. This layer is called **adaptive shifting layer**, since it estimates how the data must be shifted before feeding them to the network. This process corresponds to estimating the Gaussian from which the data were generated and then subtracting the corresponding mean.

After centering the data using the process described in (6), the data must be appropriately scaled using the scaling function $\beta(x)$. Instead of using the previously calculated summary representation, we propose using the updated centered vectors and updating the summary representation before calculating $\beta(x)$. Therefore, the updated summary representation used for defining the scaling function is calculated as:

$$s_\beta = \frac{1}{L} \sum_{i=1}^{L} (x_i - \alpha(x_i)) \in \mathbb{R}^d.$$

(7)

Then, the scaling function is defined as a non-linear transformation over this summary representation allowing for scaling each of the shifted measurements:

$$\beta(x) = \text{sigm}(W_\beta s_\beta + b_\beta) \in \mathbb{R}^d,$$

(8)

where $\text{sigm}(x) = 1/(1 + \exp(-x))$, while $W_\beta \in \mathbb{R}^{d \times d}$ and $b_\beta \in \mathbb{R}^d$ are the weight matrix and biases of the scaling layer, respectively. This layer is called **adaptive scaling layer**, since it estimates how the data must be scaled before feeding them to the network. Using a non-linear activation function for this layer is not expected to cause any significant issues during the training, since the data are already appropriately centered. However, if such issues exist, then an additional trainable scaling factor can be used to appropriately scale the data. This process corresponds to scaling the data according to their variance, as described in (4), after identifying the distribution from which they were generated.

By plugging (6) and (8) into (4) the output of the proposed normalization layer, which is called Deep Adaptive Input Normalization (DAIN), is obtained:

$$x' = (x - W_\alpha s_\alpha) \odot \text{sigm}(W_\beta s_\beta + b_\beta).$$

(9)

The parameters of the proposed deep adaptive input normalization scheme can be directly learned using gradient descent:

$$\Delta \left( W_\alpha, W_\beta, b_\beta, W \right) = -\eta \left( \eta_\alpha \frac{\partial L}{\partial W_\alpha}, \eta_\beta \frac{\partial L}{\partial W_\beta}, \eta_\beta \frac{\partial L}{\partial b_\beta}, \frac{\partial L}{\partial W} \right),$$

(10)

where $L$ denotes the loss function used for training the network and $W$ denotes the weights of the neural network that follows the proposed layer. Therefore, the proposed scaling scheme can be used on top of every deep learning network and the resulting architecture can be trained using the regular back-propagation algorithm, as also experimentally demonstrated in Section III. Note that separate learning rates are used for the parameters of the first and second sub-layers, i.e., $\eta_\alpha$ and $\eta_\beta$. This was proven essential to ensure the smooth convergence of the proposed method due to the enormous differences in the resulting gradients between the parameters of the shifting layer and the scaling layer. The base learning rate was set to $\eta = 10^{-4}$, while the learning rates for the sub-layers were set to $\eta_\alpha = 10^{-5}$ and $\eta_\beta = 1$ for all the experiments conducted in this paper. The weights of the adaptive shifting layer were initialized to the identity matrix, i.e., $W_\alpha = I_{d \times d}$, while the rest of the parameters were randomly initialized by drawing the weights from a normal distribution. The RMSProp algorithm was used for optimizing the resulting deep architecture in an end-to-end fashion [25].

### III. Experimental Evaluation

For evaluating the proposed method a challenging large-scale dataset (FI-2010), that contains limit order book data, was employed [17]. The data were collected from 5 Finnish companies traded in the Helsinki Exchange (operated by Nasdaq Nordic) and the ten highest and ten lowest ask/bid order prices were measured. The data were gathered over a period of 10 business days from 1st June 2010 to 14th June 2010. Then, the pre-processing and feature extraction pipeline proposed in [26] was employed for processing the 4.5 million limit orders that were collected, leading to a total of 453,975 144-dimensional feature vectors that were extracted.

We also followed the anchored evaluation setup that was proposed in [27]. According to this setup the time series that were extracted from the first day were used to train the model and the data from the second day were used for evaluating the method. Then, the first two days were employed for training the methods, while the data from the next day were used for the evaluation. This process was repeated 9 times, i.e., one time for each of the days available in the dataset (except from the last one, for which no test data are available). The performance of the evaluated methods was measured using the macro-precision, macro-recall, macro-F1 and Cohen’s $\kappa$ [28]. Let $TP_c$, $FP_c$, $TN_c$ and $FN_c$ be the true positives, false positives, true negatives and false negatives of class $c$. The precision of a class is defined as $prec_c = TP_c/(TP_c + FP_c)$, the recall as $recall_c = TP_c/(TP_c + FN_c)$, while the F1 score for a class $c$ is calculated as the harmonic mean of the precision and the recall: $F1_c = 2 \cdot (prec_c \cdot recall_c)/(prec_c + recall_c)$. These metrics are calculated for each class separately and then averaged (macro-averaging). Finally, using the Cohen’s $\kappa$ metric allows for evaluating the agreement between two different sets of annotations, while accounting for the possible random agreements. The mean and standard deviation values over the anchored splits are reported. The trained models were used for predicting the direction of the average mid price (up,
employed MLP. The cross-entropy loss was used for training the first 4 days were employed using the anchored evaluation horizon was set for the next 10 time steps.

Two different neural network architectures were used for the evaluation following the suggestions of the relevant literature: a Multilayer Perceptron (MLP) [29] and a Convolutional Neural Network (CNN) [30, 31]. Both models receive as input the 15 most recent measurement (feature) vectors and predict the future price direction. For the MLP the measurements are flattened into a constant length vector with $15 \times 144 = 2160$ measurements. The MLP is composed of one fully connected hidden layer with 512 neurons (the ReLU activation function is used) followed by a fully connected layer with 3 output neurons (each one corresponding to one of the predicted categories). Dropout with rate of 0.5% is used after the hidden layer [32]. The CNN is composed of a 1-D convolution layer with 256 filters and kernel size of 3, followed by two fully connected layers with the same architectures as in the employed MLP. The cross-entropy loss was used for training both networks.

First, an ablation study was performed to identify the effect of each normalization layer on the performance of the proposed method. The results are reported in Table I. The notation “DAIN (Adaptive Shifting)” is used to refer to applying only (Adaptive Shifting + Adaptive Scaling) refers to using both layers for the normalization process. The optimization ran for 20 epochs over the training data, while for the evaluation the first 4 days were employed using the anchored evaluation scheme that was previously described. The proposed method is also compared to a) not applying any form of normalization to the data (“No normalization”), b) using z-score normalization (as described in (1)), c) subtracting the average measurement from each time series, i.e., $x'_t = x_t - \frac{1}{T} \sum_{t'=1}^{T} x_{t'}$ (called “Sample average normalization” in Table I), d) using the Batch Normalization [18] and e) Instance Normalization layers [19] directly on the input data. Note that Batch Normalization and Instance Normalization were not originally designed for normalizing the input data. However, they can be used for this task, providing an additional baseline of a layer that can be trained for normalizing the data. Both the MLP and CNN were used for the evaluation, while the models were trained for 20 training epochs over the data. Furthermore, the data were sampled with probability inversely proportional to their class frequency, to ensure that each class is equally represented during the training. Thus, data from the less frequent classes were sampled more frequently and vice versa. For all the conducted experiments of the ablation study the prediction horizon was set for the next 10 time steps.

Several conclusions can be drawn from the results reported in Table I. First, using some form of normalization is essential
for ensuring that the models will be successfully trained, since using no normalization leads to $\kappa$ values around 0 (random agreement). Using either z-score normalization or performing sample-based normalization seems to work equally well. Batch Normalization yields performance similar to the z-score normalization, as expected, while the Instance Normalization improve over all the other baseline normalization approaches. When the first layer of the proposed DAIN method is applied (adaptive shifting) the performance of the model increases by over 15% ($\kappa$ values for the MLP and by over 10% ($\kappa$ values) for the CNN for the fixed normalization approaches, highlighting that learning how to adaptively shift each measurement vector, based on the distribution from which the sample was generated, can indeed lead to significant improvements. Note that the adaptive shifting layer directly receives the raw data, without any form of normalization, and yet it manages to learn how they should be normalized in order to successfully train the rest of the network. A key ingredient for this was to a) avoid using any non-linearity in the shifting process (that could possibly lead to saturating the input neurons) and b) appropriately initializing the shifting layer, as previously described. Furthermore, using the additional adaptive scaling layer, that also scales each measurement separately, further improved the performance by more than 25% over using only adaptive shifting, by almost 50% over the evaluated global averaging schemes and by 12-35% over using instance normalization ($\kappa$).

Finally, the models were evaluated using the full training data (10 days) and two different prediction horizons (10 and 20 time steps). The experimental results are reported in Table II using two different models (MLP and CNN). No other form of normalization, e.g., z-score, etc., was employed for the model that uses the proposed DAIN layer (with both adaptive shifting and adaptive scaling) and the Instance Normalization layer. Using Instance Normalization leads to better performance over the plain z-score normalization. However, employing the proposed method again significantly improves the obtained results over the rest of the evaluated methods.

IV. CONCLUSIONS

A deep adaptive normalization method, that can be trained in an end-to-end fashion, was proposed in this paper. The proposed method is easy to implement, while at the same allows for directly using the raw time series data without employing any other explicit form of normalization. The ability of the proposed method to improve the forecasting performance was evaluated using two different deep learning models and a large-scale financial time series dataset. The proposed method consistently outperformed all the other evaluated hand-crafted normalization approaches, as well as other competitive trainable normalization layers.

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