The optimization of energy storage capacity for distribution networks with the consideration of probability correlation between wind farms based on PSO algorithm

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Abstract. With the rapid development of the energy networks, various forms of renewable energy resources are absorbed into it. Because of the inherent random behaviour of the renewable resources, introducing them into the energy network will destroy the stability of the grids. It is required to use proper energy storages to reduce the uncertain fluctuation from the renewable energy resources. For a concrete model research, this paper presented an explicit method to give suitable capacities of the energy storages in consideration of the economics of the storage, grid losses and the probabilities of the bus voltages violation, for situations of the wind-power generations injected into the power network. Furthermore, the influence of the correlation between the different wind farms on the optimal storage capacity can also be studied by this method.

1. Introduction

The distribution network, widely recognized as a better solution for the energy crisis in the world, has been developing rapidly. However, the more renewable energy resources access into the grids, the more instability factors are injected into the distribution network [1]. For example, with the gradual increasing penetration rate of distributed wind generation in distribution networks, the huge impact of the fluctuations generated from the wind power should be considered seriously. One of the solutions is employing the energy storage system to smooth the output of the renewable generations. A great amount of research effort goes into giving the better the storage energy system technologies from different aspects [2-6]. This paper has a concern on how to give an optimal capacity of the energy storage. For this issue, some explorations have been carried out by using probability theory. The research work in reference [7] presented an optimization for determining the capacity of the storage. Energy storage devices are introduced into the power grid system to balance the fluctuation caused by injection power deviation. However, they do not provide the model for how energy storage device balances the fluctuation caused by injection power deviation, and their model can’t research the case when there are correlations between the wind farms.

In this paper we use an explicit model to show the modifications of the storage device on the power output probability distribution for the wind generations. Based on the probabilistic modification model, we provide a method for discovering the influence of the correlations between the different wind power generations on the optimization of the storage capacity. In the section 2 we describe the storage probability modifications model for the wind power generations. Then, in the section 3 we present the optimization algorithm model based on the probabilistic power flow. Next, in the section 4 we introduce the flowchart of the method based on the particles swarm optimization (PSO) algorithm.
model, and then in section 5 we give a concrete case for getting the optimal storage capacity, and research the influence of the correlations between the wind farms on the optimal storage capacity. Finally, some conclusions are made in the section 6.

2. The storage probability modified model

In this section, we firstly provide a wind power model, and then describe the method for generating samples of the output power of the correlated wind farms. Finally we introduce a model for generating the samples modified by the access of the storages.

2.1. The general wind power model

Many researches show that wind speed do not follow normal distribution, but can be modelled as Weibull distribution with two parameters. This model is applicable for both short term and long term studies. Weibull distribution is formulated as follows [8]:

\[ f(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} \exp \left( -\left( \frac{v}{c} \right)^{k} \right) \]

Where \( v \) is wind speed, \( c \) and \( k \) are the scale and shape parameters of Weibull distribution, respectively. Output power of wind turbine can be transformed from the wind speed via a nonlinear function formulated in the following equation[9],

\[ P_{wind} = \begin{cases} 0 & v \leq v_{c1} \\ k_1 v + k_2 & v_{c1} \leq v \leq v_r \\ P_r & v_r \leq v \leq v_{co} \\ 0 & v \geq v_{co} \end{cases} \]

Where \( P_r \) is the rated capacity of the wind turbine. \( v_{c1}, v_r \) and \( v_{co} \), are the cut-in, rated and cut-out speed of the wind turbine, respectively. The parameters \( k_1 \) and \( k_2 \) of the equation can be expressed by the others parameters mentioned above as \( k_1 = \frac{P_r}{v_r - v_{c1}} \) and \( k_2 = \frac{-k_1 v_{c1}}{} \). In general, the wind farms are considered as PQ buses with a power factor, specially which is 0.95 for the calculations in this paper.

2.2. The method for generating the correlated wind speed samples

In order to research the correlation between the wind farms, we need to obtain the samples of the correlates wind. The general correlation between the normal distributions can be generated by using the orthogonal transformation method. However, wind speeds do not follow normal distribution but rather Weibull distribution. The combination between Cholesky decomposition and Nataf transformation is considered to be better than other methods for generating the correlated samples between the non-normal distributions[10]. Assume that the random vectors in the desired space is \( X(x_1, x_2, \ldots, x_n) \) and its correlation matrix with the non-diagonal elements \( \rho_{ij} \) and the diagonal elements \( I \). By equal probability criterion, the standard normal random vector \( U(u_1, u_2, \ldots, u_n) \) is calculated using the following expression:

\[ u_i = \Phi^{-1}(F_i(x_i)) \]

Where \( \Phi(u_i) \) and \( F_i(x_i) \) are the cumulative distribution functions of the normal distribution and any desired distributions. At the same time, \( \rho_{ij} \) which is the non-diagonal elements of correlation matrix \( C_U \) of the random vector \( U \) can be transformed from the correlation matrix \( C_z \) by the empirical expression \( \rho_{ij} = T \rho_{ij} \), which is proposed in [11]. Specially, for two-parameter Weibull distribution, \( T \) has the following formulation

\[ T = 1.063 - 0.004 \rho_{ij} - 0.2 \left( \frac{\sigma_1}{\mu_1} + \frac{\sigma_2}{\mu_2} \right) - 0.001 \rho_{ij}^2 + 0.337 \left( \frac{\sigma_1}{\mu_1} \right)^2 + 0.007 \rho_{ij} \left( \frac{\sigma_2}{\mu_2} \right)^2 - 0.007 \left( \frac{\sigma_1}{\mu_1} + \frac{\sigma_2}{\mu_2} \right) \]

Since the random vector \( U(u_1, u_2, \ldots, u_n) \) obtained by the above procedure is in the correlated standard normal space, we can get the independent standard normal rand vectors \( Z(z_1, z_2, \ldots, z_n) \) by the equation \( Z = B^{-1}U \), where \( B \) is a lower triangular matrix from the Cholesky decomposition of correlation matrix \( C_U = BB^T \). The above procedure is the Nataf transformation by which the desired random vectors
(RVs) can be transformed from desired input space to independent standard normal space. By the inverse procedure of the Nataf transformation, we can get the desired RVs from the RVs in the standard normal space. The details of the procedure are as follows:
1) Obtaining $C_U$ from the equation $\rho_{ij} = T \rho_{ij}$.
2) Calculating the lower triangular matrix $B$ from $C_U = BB^T$.
3) Generating an $n$-dimensional independent standard normally-distributed vector $Z$ whose elements are independent and identically-distributed standard normal random variables.
4) Getting the normal distribution vector $U = BZ$ where the elements of meet the correlation matrix $C_U$.
5) Generating the wind speed vector using the equation $x_i = F_i^{-1}(\Phi(u_i))$, where $F_i^{-1}$ is the inverse of the cumulative distribution function of the $i$-th wind speed. Each wind speed RV follows Weibull distribution and follows the given correlation matrix $C_X$.

2.3. The probabilistic modification model of the output power by the access of storage
The reference [12] provides a probabilistic correction method which considered the changes of power injection probability model of node consist of wind power and ESS, based on the combination of the operating characteristic of ESS and mechanism of wind-storage output. By this correction method, for getting the probability distribution of the power output of wind-storage combined systems, the storage system parameters are need as follows: the storage capacity $E_s$, the maximum power output $P_{storage\_max}$, the minimum state of the charge $SOC_{min}$, the maximum state of the charge $SOC_{max}$, the number of the charging and discharging in a day $n_d$. The detailed procedure is not introduced here, and can be found in the reference [12].

3. The optimization algorithm of the storages by the probabilistic power flow calculation
In this section, we introduce the probabilistic power flow calculation used in this paper and the objective function for the storage optimization process. By means of the probabilistic power flow of the system with the wind power generations, the node voltage cumulative distribution function and the network losses can be obtained, and then the corresponding probability of the system voltage violations can be evaluated. Based on them, the objective function and control variables in the optimal method for the storage capacity are presented.

3.1. Probabilistic power flow based on the point estimate method and Cornish-fisher expansion
The Probabilistic power flow is necessary in the process of the storage optimization. The most straightforward method for the Probabilistic power flow is the Monte Carlo simulation [13]. But this technique has a poor efficiency for the many cases. We apply the point estimate method and the Cornish-fisher expansion together to calculate the probabilistic model, which can simple greatly the calculation of cumulative distribution function of state variables [14,15].

The set of the general nonlinear equation can be expressed as $y = g(x)$. We use the 2m+1 Point estimate method (PEM) to get the statistical characters of the output random vector $Y$ which can be the voltages of each node in the power network, the network losses for the power flow equation and so on. The basic procedure of the 2m+1 PEM scheme is to use 2m+1 points of uncertain input variables to evaluate the statistical moments of the output variables 2m+1 times. The each input random variable is chosen with three locations and weights which can be written as follows

$$x_{ik} = \mu_i + \xi_{ik} \sigma_i, \quad (k = 1, 2, 3; \ i = 1, 2, ..., m) \tag{5}$$

Where $\mu_i$ and $\sigma_i$ denote the mean and standard deviation of random variable $x_{ik}$. $\xi_{ik}$ denotes the location, which is expressed as $\xi_{ik} = \lambda_3 / 2 + (-1)^{k-1} \frac{\lambda_4}{(3/4) \lambda_3} (k = 1, 2) \xi_{i3} = 0$, where, $\lambda_3$ and $\lambda_4$ are the third and fourth standardized central moments of $x_{ik}$ known as skewness and kurtosis. Weighting factors related to each location $\xi_{ik}$ are calculated by the following equations:

$$\omega_{ik} = \frac{(-1)^{k-1}}{\xi_{ik} (\xi_{i1} - \xi_{i2})}, \quad (k = 1, 2), \quad \omega_{i3} = \frac{1}{m} - \frac{1}{\lambda_4 - \lambda_3^2} \tag{6}$$

After calculation above, 2m+1 vectors of input variables $X$ are expressed as follows:
are the local best known position, and global best known are the acceleration factors, in terms of the . Using the first five cumulants, on all the achieved solutions cumulative distribution and represent are the random and the cumulants of is the cumulant of order n of the cu

\[ \chi(\alpha) = \xi(\alpha) + \frac{1}{6} (\xi^2(\alpha) - \xi(\alpha) - 1) \eta_1 + \frac{1}{24} (\xi^3(\alpha) - 3\xi(\alpha) - 2\xi^2(\alpha) + 1) \eta_2 + \frac{1}{120} (\xi^4(\alpha) - 6\xi^2(\alpha) + 3) \eta_3 - \frac{1}{24} (\xi^4(\alpha) - 5\xi^2(\alpha) + 2) \kappa_3 \eta_1 + \frac{1}{324} (12\xi^4(\alpha) - 3\xi^2(\alpha) + 17) \kappa_4 \eta_3 \]

where \( \chi(\alpha) = F^{-1}(\alpha) \), \( \xi(\alpha) = \Phi^{-1}(\alpha) \) and \( \kappa_n \) is the cumulant of order n of the cumulative distribution function \( F \) which can be obtained from the above moments calculated by the PEM method in this paper.

3.3. The objective function for the probabilistic power flow calculation

By means of the method introduced above, we give the objective function and control variables in the optimal method for the storage capacity as follows [7],

\[ F = K_1 \Delta Q + K_2 \Delta P + \lambda \sum Y \left( \frac{P_{i,Y} - P_{i,Y,lim}}{V_{max} - V_{min}} \right) \]

Where \( \Delta Q \) represents the storage capacity which is the control parameter during the optimal procedure, \( \Delta P \) denotes the active losses, \( P_{i,Y} \) is the probability of the node voltage exceeding limit range from \( V_{min} \) to \( V_{max} \), accordingly \( P_{i,Y,lim} \) is the allowed maximum probability of the node voltage exceeding limit range. \( \kappa_1, \kappa_2 \) and \( \lambda \) are the weight factors respectively for considering the cost of storage, the network losses, and the probability of the violation of the node voltage. The constraints of the objective function are the power flow equations.

4. The particles swarm optimizations algorithm and the flowchart of the method

4.1. The particles swarm optimizations algorithm

The Particle Swarm Optimization (PSO) is an optimal search approach inspired by the simplified social system such as bird flocking or fish schooling [16, 17]. For the general multivariable optimization search case, the swarm consists of a specified number of particles having their own positions and velocities which can be respectively expressed as follows, \( x_i = (x_{i1}, x_{i2}, ..., x_{iM}) \) and \( v_i = (v_{i1}, v_{i2}, ..., v_{iM}) \), where \( i = 1, 2, ..., N \), \( N \) represents the size of the swarm and \( M \) is the dimension of the research space. The fitness value of the particle is related to the objective function. The local best known position is the best solution that achieved by each particle and the global best known position is the best solution among all the achieved solutions during the iteration search process. The main steps for the PSO are summarized below. Firstly PSO also begins with a group of randomly generated solutions, and then updates the position and velocity for each particle in each iteration. For the iteration it can be expressed as follows:

\[ V_i^{k+1} = \omega V_i^k + \varphi_1 R_1 (P_{best}^k - X_i^k) + \varphi_2 R_2 (G_{best}^k - X_i^k) \]

\[ X_i^{k+1} = X_i^k + V_i^{k+1} \]

where \( \omega \) is the inertia weight factor, \( \varphi_1 \) and \( \varphi_2 \) are the acceleration factors, \( R_1 \) and \( R_2 \) are the random numbers between 0 and 1. \( P_{best}^k \) and \( G_{best}^k \) are the local best known position, and global best known
position respectively in the $k$ iteration, which reflects the cooperation and competition mechanism in PSO algorithm.

It is known that the inertia weight of the particles has a large impart on the efficiency of the search algorithm. Some researches show that selecting large value of $\omega$ at the beginning and gradually decreasing it during the iteration process is a better choice. The inertia weight is given by following equation during the iteration process,

$$\omega = \omega_{\text{max}} - \frac{\omega_{\text{max}} - \omega_{\text{min}}}{\text{iter}_{\text{max}}} \times \text{iter}$$

(12)

Where $\omega_{\text{max}}$ and $\omega_{\text{min}}$ are the maximum and minimum values, $\text{iter}$ and $\text{iter}_{\text{max}}$ are the current iteration number and the maximum iteration number, respectively. Finally the research process ends when the idea iteration number is achieved or the locations of all particles converge to the same set.

4.2. The flowchart for the optimal method

The procedure for the proposed optimum scheme in this paper is described as follows:

1. Reading the initial data: including the data of load flow calculation, the parameters of the wind power generations and the correlation matrix between two wind farms, and the PSO parameters which include particle swarm scale, accelerated coefficient and so on.

2. Randomly generating the location of the each particle which represents the storage capacity value and the velocity of the each particle.

3. Generating the speed samples of the two winds farm, and then getting statistic distributions of the power outputs of the wind power generations.

4. Through the Probabilistic power flow calculation, obtaining the value of the initial individual optimal value $P_{\text{best}}$ and the global optimal value $G_{\text{best}}$.

5. Judging termination conditions: To stop the algorithm process and output the results when achieving the maximum iteration times. Otherwise continue to update the particles velocities and the locations and to calculate the next probabilistic calculation.

The procedure above can be showed by the figure 1.

![Figure 1. The flowchart for the optimal method](image)
5. The study case

The algorithm introduced above is used in the modified IEEE33-bus test system. The topology structure is shown in the figure 2. Twelve wind power generators are added to the node 33 of the system. They are divided into two wind farms A and B. Every wind farm consists of six wind power generators. The scale and shape parameters of Weibull distribution of the wind speed in the wind farm A and B is set as $c_A = 2.8$, $k_A = 5.5$ and $c_B = 2.8$, $k_B = 5.5$, respectively. The rated output power of each wind power generations is 0.6 MW. The cut-in, rated and cut-out speed of the wind turbine are set as $v_{ci} = 3\text{ms}^{-1}$, $v_{r} = 14\text{ms}^{-1}$ and $v_{co} = 25\text{ms}^{-1}$ respectively. For generating the combined probabilistic distribution of the power output in the 33 node system, the storage parameters need to be set as follows: $P_{\text{max}_{\text{wind}}} = 0.2\text{MW}$, $\text{SoC}_{\text{max}} = 0.1$, $\text{SoC}_{\text{min}} = 0.8$, $\alpha = 2$. By the wind farm and the storage parameters above, we plot the figure 3 which shows the comparison of the power output in the node 33 with the storage and without storage when the correlation between the two wind farms is 0.6. From figure 3 we find that the increasing capacities of the storage can make the power output focused to the average values, which is the basic reason why storage can reduce the fluctuation in the power grid.

The parameters of the original physical IEEE33-bus system can be found in reference [18]. Since the probabilistic power flow calculation need not only the 33 node power output, but also the probabilistic distributions of the loads in all nodes, the normal distribution is used to represent the probabilistic power flow calculation of certain power flow combined with PSO algorithm, the proper energy storage capacities for the different correlation cases are obtained, which are showed in the table 1.

| Correlation factor | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|--------------------|-----|-----|-----|-----|-----|-----|
| Optimal storage (MW·h) | 5.0 | 5.2 | 5.9 | 6.1 | 6.7 | 7.4 |

Table 1. The optimal storage capacities for different correlations

From the results in table 1, we find that the optimal storage capacity rises with the increase of the correlation between the wind farms.

![Figure 2. The topology structure for the modified IEEE33-bus grid.](image)

![Figure 3. The probabilistic density functions (PDF) for the cases with different storage capacities.](image)

Finally, for showing the effects of the storages on the power grid, the figure 4 and 5 are provided which give the comparisons of between the case with different storage capacities and different correlation factors. From figure 4 and 5, it is found that for the cases with the same correlation factors the active losses and sum of the probability of the node voltage exceeding limit are reduced greatly with the increase of the storage capacities, and that for the cases with the same storage capacities the active power loss and the sum of the probability of the node voltage exceeding limit $P_{\text{sum}_{\text{vel}}}$ is increased
with the correlation factors. With the increase of the correlation factors, the storage capacity should be increased in the system for better economy and stability.

![Figure 4 and 5](image-url)

Figure 4. The active power loss changes with the increase of the storage capacities $E_s$ for different correlation cases.

Figure 5. The sums of the probability of the node voltage exceeding limit decreases with the storage capacities $E_s$ for different correlation cases.

6. Conclusions
This paper introduced a method for obtaining the optimal storage capacity in the power network with the access of the wind power generations. In this method, the objective function for the optimum algorithm considers the cost of storage, the network losses, and the probability of the violation of the node voltage. The Probabilistic power flow based on the point estimate method and Cornish-fisher expansion is carried out during the optimum process based on PSO, which can reflect effects of the corrections between the wind farms. A concrete example in the IEEE33-bus power grid shows that the application of the storage can reduce fluctuations, and that the correlation between the wind farms has a nonnegligible influence on the optimal capacities of the storage. The results show that we should consider the correlations between the different wind power generations for choosing the proper capacities of the energy storage.

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Appendix

Table A1. The standard deviation of the load power, where we use the hypothesis that the deviations of the loads active are the same as their reactive power.

| Node number | Power deviation | Node number | Power deviation |
|-------------|-----------------|-------------|-----------------|
| 2           | 0.0522          | 18          | 0.0820          |
| 3           | 0.1650          | 19          | 0.1050          |
| 4           | 0.0870          | 20          | 0.0706          |
| 5           | 0.0750          | 21          | 0.0396          |
| 6           | 0.0637          | 22          | 0.0336          |
| 7           | 0.0225          | 23          | 0.0961          |
| 8           | 0.0421          | 24          | 0.1150          |
| 9           | 0.2650          | 25          | 0.0852          |
| 10          | 0.0760          | 26          | 0.0241          |
| 11          | 0.0690          | 27          | 0.0233          |
| 12          | 0.0950          | 28          | 0.0537          |
| 13          | 0.0238          | 29          | 0.0786          |
|   |   |   |   |
|---|---|---|---|
| 14 | 0.0405 | 30 | 0.0378 |
| 15 | 0.0492 | 31 | 0.0512 |
| 16 | 0.2150 | 32 | 0.0595 |
| 17 | 0.0910 | 33 | 0.0923 |

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