Positive Stage Jerk Feedback Combined with Positive Base Plate Jerk Feedback to Improve the Positioning Speed of a Pneumatic Stage*

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Abstract
In this paper, a unidirectional positioning stage which is supported by four coil-type spring isolators and actuated by four pneumatic cylinders is considered. The stage jerk feedback scheme (SJFB) with positive polarity is proposed to reduce the stage mass, resulting in the improvement of positioning speed. However, reduction of stage mass results in the overshoot that can be controlled by increasing the stage stiffness through the outer feedback loop controller. In addition, to avoid the self-oscillation of stage caused by high gain of SJFB and to improve the repeatability, the conventionally used scheme, namely, base plate jerk feedback (BPJFB) with positive polarity based on its damping role is combined with the SJFB scheme. It is concluded from the experimental results that the proposed control system obeys the related theory and provides high speed positioning with improved repeatability. The amount of the reduced mass through SJFB scheme is also experimentally estimated.

Key words: Pneumatic, Positioning Stage, Base Plate, PDD\(^2\) Controller, Jerk, Positive Feedback, Coil-Type Spring Isolator, Perfect Integrator

1. Introduction

Beside many applications, positioning stages are widely used in the manufacturing process of micro/nano scale semiconductor devices, such as integrated circuits. There are several actuation systems that can drive and control the positioning stages. Compared with other types of actuators, except hydraulic actuators, the pneumatic actuators provide high power-weight-ratio \(^1\). Also, unlike electromagnetic actuators, they are free of magnetic flux leakage and heat generation. In addition, their other advantages can be pointed to their lower cost, clean environment, and easy maintenance. However, the serious problems of pneumatic actuators arise from the nonlinear properties, such as compressibility and friction forces. As long as the dimensions of the semiconductor devices are going to decrease rapidly, precise positioning tools with accurate performance and high speed are highly demanded. In order to meet these demands, control of the horizontal and vertical disturbances is highly essential. Moreover, designing appropriate control strategies is necessary to increase the positioning speed.

There are several control strategies that utilized the positive acceleration feedback for different purposes. For instance, Ref. (2) compared the use of positive position and positive acceleration feedback controllers to compensate the unbalance response and to reduce the resonant peak in a rotor-bearing system. Reference (3) used positive acceleration feedback controller to eliminate the effect of rigid body acceleration in a single flexible link manipulator. Reference (4) proposed the positive acceleration feedback scheme along with a switching controller in a robot manipulator to provide a quick and stable transition force

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response with a nonzero impact velocity. In Ref. (5), the authors proposed the positive acceleration feedback along with a PD controller in an industrial pick-and-place Cartesian robot to validate the tracking trajectories of the motor, load, and their position references. It is shown that the positive acceleration feedback is capable of suppression of the vibration on the load. References (6) and (7) showed through numerical simulation that the positive acceleration feedback of a patient who is supported by a body-weight support treadmill-based locomotion effectively removes the body mass as opposed to the body weight, which results in the easier and smoother training of the patient with the treadmill device.

However, to the best of authors’ knowledge, the use of positive acceleration or positive jerk feedback schemes that aims the improvement of positioning speed is still not reported. Therefore, we firstly propose the positive SJFB scheme to increase the positioning speed in a pneumatically actuated stage. This feedback scheme with a simple structure cannot be easily applied in a real machine tool. The reason is that as much as the feedback gain is increased, the stage mass is decreased. Hence, although the reduction of stage mass results in the increase of positioning speed, the overshoot is also increased which is an issue associated with the SJFB scheme. In addition, it is well known that the positive feedback easily degrades the system stability, especially when the feedback gain is further increased.

In our preceding studies (8, 9), we showed that the BPJFB with positive polarity in a pneumatically actuated stage is the same as the base plate acceleration feedback with positive polarity in XY stages driven by the linear motors (10, 11). The only difference of these feedback controllers is based on the difference in the dynamics of actuation systems, which will be discussed later. These feedback schemes lead the motion of stage as a Slave to synchronize with the motion of base plate as a Master against the stage oscillation caused by the natural vibration of base plate which is conveyed by reaction force. Moreover, in Ref. (12), we demonstrated another function of the BPJFB scheme, which is based on the increase of damping ratio and stiffness of stage. However, although this feature of BPJFB scheme is efficient for control of the oscillation caused by instability, the increase of the stage stiffness results in an offset-like error in the positioning response.

Therefore, the fusion of SJFB control with the BPJFB scheme based on its damping effect is applied to a pneumatic stage to improve positioning speed, stability, and repeatability. Meanwhile, the overshoot and offset-like error respectively generated by the SJFB and BPJFB are controlled by retuning the outer loop gain which corresponds to the stage stiffness such that the rising time achieved by SJFB scheme is almost maintained. In addition, the amount of the mass reduced from the stage mass through SJFB controller is experimentally estimated.

The reminder of this paper is organized as follows: The pneumatic stage, experimental setup, and the proposed control scheme with the related theory are explained in Section 2. The experimental strategy and results are discussed in Section 3. Finally, the summary and remarks are provided in Section 4.

Nomenclatures

- \( M_1 \) : stage mass [kg]
- \( K_1 \) : spring constant of the stage [N/m]
- \( D_1 \) : viscous damping coefficient of the stage [N·s/m]
- \( x_1 \) : stage position [m]
- \( M_2 \) : base plate mass [kg]
- \( K_2 \) : lumped spring constant of the base plate [N/m]
- \( D_2 \) : lumped viscous damping coefficient of the base plate [N·s/m]
- \( x_2 \) : base plate displacement [m]
- \( x_1 - x_2 \) : relative position [m]
- \( K_{pos} \) : detection sensitivity of the optical encoder [count/m]
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$K_{p_d}, K_{d_i}, K_{dd}$: proportional, stiffness, and damping gains of PDD$^2$ cont. [V/count]

$T_d$: time constant of the pseudo differentiators of PDD$^2$ controller [s]

$v$: PDD$^2$ controller output [V]

e: positioning error [counts]

$u$: summation of PDD$^2$ controller, SJFB, and BPJFB outputs [V]

$K_{pi}$: proportional gain of the feedback PI controller [V/V]

$T_{pi}$: time constant of the feedback PI controller [s]

$K_{air}$: proportional gain of the pneumatic driving system [Pa/V]

$T_{air}$: time constant of the pneumatic driving system [s]

d$\theta$: equivalent of the flow disturbance [V]

$p$: pressure difference [Pa]

$S_a$: effective area of actuators [m$^2$]

$f_d$: driving force [N]

$f_{ext}$: external force [N]

$B_1$: SJFB gain [V/(m/s$^2$)]

$T_1$: time constant of the pseudo differentiator used in the SJFB scheme [s]

$T_{sl}$: time constant of low pass filter used in the SJFB scheme [s]

$\alpha_1$: stage acceleration [m/s$^2$]

$B_2$: BPJFB gain [V/(m/s$^2$)]

$T_2$: time constant of the pseudo differentiator used in BPJFB scheme [s]

$\alpha_{bp}$: base plate acceleration [m/s$^2$].

2. Experimental Apparatuses and Proposed Control System

2.1 Pneumatic positioning stage and experimental setup

Figure 1(a) and (b) show a photograph and the experimental setup of pneumatic stage. The system is composed of two main parts, the base plate and the moving stage. The base plate with a very light mass compared with those used in the industries is supported by four coil-type isolators. There is a frame rigidly fixed on the top of base plate with two parallel rolling guides. The stage of 15kg with a resolution of 1µm/count is placed between the rolling guides and guided by the needle-bearings. The stage which can only move towards horizontal axis is actuated by four pneumatic cylinders (M16D300.0S), each with maximum stroke of 10cm corresponding to $10^5$ counts. The dimensions (length × width × thickness) of the stage and base plate are 500 × 260 × 35mm and 995 × 495 × 80mm, respectively.

![Fig. 1 Photograph and experimental setup of the pneumatic positioning stage](image)

From Fig. 1(b), the pressurized air is provided by the air compressor, which is then regulated through a single regulation system. Tow servo valves (EWS3/4) are used to provide controllable pressure to the left and right chambers of actuators. An optical encoder (MercuryTM3500i) is used to measure the relative position as an input signal to the outer
loop controller. An accelerometer (JA-5V) with detection sensitivity of 5V/G is utilized to detect the horizontal acceleration of the base plate caused by the reaction force during positioning. The output of this accelerometer is used as the input signal to the BPJFB scheme. The input signal to the SJFB scheme is detected by a MEMS type accelerometer (SF1500S) with detection sensitivity of 1.2V/G for single output, used in this study. The DSP operating by the real time workshop of the MATLAB/Simulink in discrete-time domain is used as the main controller. The summation of feedback signals from the DSP is fed to the left and right servo valves with opposite polarities to provide a stroke in same direction.

2.2 Proposed control block diagram and related theory

The proposed control block diagram shown in Fig. 2 is composed of two main parts: the model of the plant and the control system. The plant consists of two parts; the pneumatic system and the mechanical system. As usual, the pneumatic system is modeled as a first order lag system with the transfer function shown in the related block in Fig. 2. Where the system and the mechanical system. As usual, the pneumatic system is modeled as a first order lag system with the transfer function shown in the related block in Fig. 2. Where the mechanical system is modeled as a 2-DOF system due to the suspension of base plate on the coil-type spring isolators by neglecting the vertical motion [8-12], for which the equations of motion are defined as follows:

\[ M_s a^2 x_1 = f_d - D_s (x_1 - x_2) - K_p (x_1 - x_2) \]  
\[ (1) \]

\[ M_s a^2 x_2 = f_e + D_s (x_1 - x_2) - D_p a x_2 + K_p (x_1 - x_2) - K_p x_2 \]  
\[ (2) \]

where \( f_e = -f_d \) is the reaction force and \( f_e \) is the external force. Notice that \( f_e \) is zero in the case of positioning, while it is used for estimating the amount of mass reduced from stage mass through the SJFB scheme, which will be discussed in § 3.6.

![Fig. 2 Proposed control scheme of the pneumatic stage](image)

The control system is composed of one outer and two inner feedback loops. The controller of the outer feedback loop is the PDD (Proportional + Derivative + Double Derivative) controller, corresponding to the negative feedback of relative displacement, velocity, and acceleration. The inner loops are referred as to the positive feedback of first pseudo derivatives of stage and base plate acceleration signals, respectively corresponding to the SJFB and BPJFB schemes. It is worth to notice that a first order low pass filter with time constant \( T_{sl} \) is used in SJFB loop to suppress the high frequency noise detected by the MEMS type accelerometer attached to the stage, which will be discussed later.

The time constant of the pneumatic system, \( T_{sl} \), is very large comparing to that of the linear motor. Hence, it is essential to obtain a driving mechanism to operate as a perfect integrator, which can be achieved by employing the feedback PI controller with a time
constant $T_{pi}$ in front of the pneumatic system. In the case $T_{pi} = T_{air}$ is experimentally obtained and it is supposed that $d_{fl} = 0$, the transfer function from $u$ to $f_d$ can be defined as a perfect integrator, as follows:

$$\frac{f_d}{u} = \left[ K_{p_i} \{1 + T_{pi}s\} / T_{pi}s \right] \left[ K_{w_i} S_{ai} / \{1 + T_{air}s\} \right] = \left[ K_{p_i} K_{w_i} S_{ai} \right] / T_{pi}s \quad \text{if} \quad T_{pi} = T_{air} \quad (3)$$

In such a case, the PDD$^2$ controller performs a proper control action comparing to other PID based controllers having an integrator part. For instance, if the PDD$^2$ controller is replaced to a PID controller, then the stability of positioning response degrades due to the double integral action performed in the feedback loop. On the other side, if the driving mechanism is obtained as a perfect integrator, pole and zero cancellation occurs between the driving mechanism and both of the SJFB and BPJFB schemes. To explain it theoretically, let us define $f_d$ from Eq. 2 while the output of the PDD$^2$ controller is zero ($v = 0$).

$$f_d = \left[ B_1 T_s s / \{1 + T_{1}s\} \{1 + T_{air}s\} \right] a_{r_{1}} + \left[ B_1 T_s s / \{1 + T_{1}s\} \right] a_{r_{1}} \quad (4)$$

The values of $T_1$, $T_2$, and $T_{sl}$ are too smaller than 1, so Eq. (4) can be approximated as

$$f_d = \left[ B_1 T_s K_{w_i} K_{w_i} S_{ai} / T_{pi}s \right] a_{r_{1}} + \left[ B_1 T_s K_{w_i} K_{w_i} S_{ai} / T_{pi}s \right] a_{r_{1}} \quad (5)$$

Equation (5) clearly verifies the role of the feedback PI controller for making the driving mechanism as a perfect integrator as defined in Eq. (3), and allowing the cancellation of pole and zero between the driving mechanism and both of the SJFB and BPJFB schemes. Hence, the SJFB and BPJFB controllers are the proper schemes for pneumatically actuated stage, which have the same characteristics as the stage and base plate acceleration feedback schemes have in linear motor driven stages. In fact by applying the feedback PI controller and designing appropriate feedback controllers, the slow dynamics of pneumatic system is compensated.

In order to verify the operating principles of the feedback controllers, let us determine the closed-loop transfer function of the system of Fig. 2 from $r$ to $(x_1 - x_2)$ when all feedback loops are involved. Notice that during calculation, all the pseudo differentiators were assumed as perfect differentiators as well as the low pass filter as unity gain for simplicity, i.e., $T_{sl}/(1 + T_{sl}s) \approx T_{sl}s$, $T_{1}/(1 + T_{1}s) \approx T_{1}s$, $T_{2}/(1 + T_{2}s) \approx T_{2}s$, and $1/(1 + T_{sl}s) \approx 1$. The closed-loop transfer function is given in Eq. (6), in which $N_{cl}(s)$ and $D_{cl}(s)$ are the numerator and denominator, respectively. $N_{cl}$, $N_{cl}$, and $N_{cl}$ are respectively the coefficients of $s^5$, $s^4$, and $s^3$ of the numerator, given in Eqs. (6a) ~ (6c). Due to the page limitation, only the coefficients of $s^5$, $s^4$, and $s^3$ of denominator, i.e., $D_{cl}$, $D_{cl}$, and $D_{cl}$ are given in Eqs. (6d) ~ (6f).

$$\frac{x_1 - x_2}{r} = \frac{N_{cl}(s)}{D_{cl}(s)} = \frac{N_{cl}(s)}{D_{cl}(s)} = \frac{N_{cl}(s)}{D_{cl}(s)} \quad (6)$$

$$N_{cl} = \left[ K_{p_i} K_{w_i} K_{w_i} S_{ai} / T_{pi}s \right] \left[ (M_i - \Delta M_i)_1 + (M_2 + \Delta M_i)_1 + \Delta M_1 - \Delta M_2 \right] \quad (6a)$$

$$N_{cl} = \left[ K_{p_i} K_{w_i} K_{w_i} S_{ai} / T_{pi}s \right] \left[ D_1 + D_2 + \left( K_{w_i} T_{1}s K_{p_i} K_{w_i} S_{ai} / T_{pi}s \right) \right] - \left[ D_1 + \left( K_{w_i} T_{1}s K_{p_i} K_{w_i} S_{ai} / T_{pi}s \right) \right] \quad (6b)$$

$$N_{cl} = \left[ K_{p_i} K_{w_i} K_{w_i} S_{ai} / T_{pi}s \right] \left[ D_1 + D_2 + \left( K_{w_i} T_{1}s K_{p_i} K_{w_i} S_{ai} / T_{pi}s \right) \right] - \left[ D_1 + \left( K_{w_i} T_{1}s K_{p_i} K_{w_i} S_{ai} / T_{pi}s \right) \right] \quad (6c)$$

$$D_{cl} = \left[ (M_1 - \Delta M_1)_1 (M_2 + \Delta M_2)_2 + (\Delta M_1)_2 (\Delta M_2)_1 \right] \quad (6d)$$

$$D_{cl} = \left[ (M_1 - \Delta M_1)_1 (M_2 + \Delta M_2)_2 + (\Delta M_1)_2 (\Delta M_2)_1 \right] \quad (6e)$$
\[ D_{6f} = \left[ (M_1 - \Delta M_1)\left( K_1 + K_2 + (K_3T_2K_{\rho K_\omega S_a}/T_\rho) \right) + D_1 + (K_4T_3K_{\rho K_\omega S_a}/T_\rho) - (\Delta M_2)\left( \frac{K_1 + (K_3T_2K_{\rho K_\omega S_a}/T_\rho)}{T_\rho} \right) \right] \]

where in Eqs. (6a) \~ (6f), \[ \Delta M \] and \[ \Delta M_2 \] are respectively the masses electrically provided by the SJFB and BPJFB, given in Eqs. (6g) and (6h).

\[ \Delta M_1 = \left( B_1T_1K_\rho K_{\omega S_a} \right)/T_\rho \quad (6g) \]

\[ \Delta M_2 = \left( B_2T_2K_\rho K_{\omega S_a} \right)/T_\rho \quad (6h) \]

Notice that to clearly explain the physical phenomena behind the operating principle of each feedback controller individually, the terms of each coefficient in Eqs. (6a) \~ (6f) are not simplified. Before summarizing the physical phenomenon of each controller, it is worth to notify that there is no optimal value for SJFB gain \[ B_1 \], because it is practically impossible to make the stage mass zero through SJFB. However, the value of gain \[ B_1 \] has a significant effect on the optimal value of BPJFB gain \[ B_2 \]. To verify this effect theoretically, the transfer function from the external force \( f_{\text{ext}} \) to \( (x_1 - x_2) \) was derived from the block diagram of Fig. 2 based on superposition theorem by assuming \( v = 0 \).

\[ \frac{x_e - x_0}{f_{\text{ext}}} = \frac{\left( \frac{(\Delta M_1) - (M_1 - \Delta M_1)}{(\Delta M_2) - (M_1 - \Delta M_1)} \right)^2}{\left( \frac{(\Delta M_2) - (M_1 - \Delta M_1)}{(\Delta M_2) - (M_1 - \Delta M_1)} \right)^2 + 1} \quad (7) \]

According to the output zeroing theorem, the numerator of Eq. (7) should be zero to avoid the effect of \( f_{\text{ext}} \). Therefore, substituting \( \Delta M_2 \) from Eq. (6h) into the numerator of Eq. (7) and then setting it to zero gives the optimal gain \( B_{2\text{opt}} \) of the BPJFB scheme which is a function of \( B_1 \), as defined in Eq. (8).

\[ B_{2\text{opt}} = T_\rho \left( M_1 - \Delta M_1 \right)/T_2K_\rho K_{\omega S_a} = T_\rho M_1 - (B_1T_1K_\rho K_{\omega S_a}/T_\rho)T_2K_\rho K_{\omega S_a}B_1 \neq 0 \quad (8) \]

While the optimal value of BPJFB gain without SJFB \( (B_1 = 0) \) can be given as:

\[ B_{2\text{opt}} = T_\rho M_1/T_2K_\rho K_{\omega S_a} \mid B_1 = 0 \quad (9) \]

From the transfer functions given in Eqs. (6) and (7), the following physical properties of the control system can be explained:

1. Referring to Eqs. (6a), (6d), (6e), (6f) and (7), a mass \( \Delta M_1 = B_1T_1K_\rho K_{\omega S_a}/T_\rho \) that is electrically provided by the SJFB scheme is subtracted from the stage mass \( M_1 \), which leads \( M_1 \) to decrease. This phenomenon is the main objective of this study, as the rising time of positioning response can be evidently decreased by reducing stage mass, resulting in the increase of positioning speed.

2. Similarly, referring to Eqs. (6a), (6d), (6e), and (7) a mass \( \Delta M_2 = B_2T_2K_\rho K_{\omega S_a}/T_\rho \) that is electrically provided by the BPJFB scheme is added with the base plate mass \( M_2 \). This phenomenon leads the stage as a Slave to pursue the natural motion of base plate as a Master. Furthermore, the BPJFB scheme has a significant role on the increase of damping coefficient and stiffness of stage. This phenomenon is verified theoretically and experimentally in Ref. (12). Therefore, more details about the both functions of BPJFB scheme are provided in detail in Refs. (8), (9), and (12). In this study, we took the advantage of the damping performance of BPJFB to realize its effectiveness on the stability and repeatability.
3. Referring to Eqs. (6a) ~ (6c), the proportional gain $K_p$ of the PDD$^2$ controller as a common coefficient of the other terms of $N_{cl2}$, $N_{cl1}$, and $N_{cl0}$ shows that this gain is effective for decreasing the rising time of the positioning response.

4. In Eqs. (6c) and (6f), it can be observed that the gain $K_d$ of the PDD$^2$ controller along with some other coefficient results in the increase of the stage stiffness $K_1$. This gain which corresponds to the stiffness controller of stage plays the key role in tuning process. In fact this gain is used to compensate the overshoot caused by the stage mass reduction associated with the SJFB scheme. In addition, this gain is effective to control the offset-like error caused by the increased stiffness associated with the BPJFB scheme while the fusion of SJFB and BPJFB is used.

5. Finally, referring to Eqs. (6b), (6e), and (6f), it is obvious that the gain $K_{dd}$ of the PDD$^2$ controller along with some other parameters result in the increase of stage damping coefficient. Therefore, this gain with having the damping role is efficient to adjust the damping ratio of positioning response.

3. Experimental Strategy, Results, and Discussions

3.1 Experimental strategy

This study is an experimental-based research, in which all the parameters are tuned by using the trial and error method. The parameters shown in Table 1 were kept fixed during the experiments with the same values as tuned in Ref. (9). The experiments were performed through the following steps:

- Precise tuning of PDD$^2$ controller gains without SJFB and BPJFB ($B_1 = B_2 = 0$).
- Using the precise gains of PDD$^2$ controller, applying the SJFB in safe mode while the BPJFB is open ($B_2 = 0$) and compensating the overshoot caused by the stage mass reduction through SJFB scheme.
- Further increasing the SJFB gain $B_1$ until the stage becomes oscillatory and then applying the BPJFB to stabilize the self-oscillation of stage as well as compensating the offset-like error caused by an increase in stage stiffness through BPJFB scheme.
- Experimentally estimating the amount of $\Delta M_1$ reduced by the SJFB controller.

| Parameter   | Value |
|-------------|-------|
| $K_{pi}$    | 10 V/V |
| $T_{pi}$    | 5s    |
| $T_d$       | 7.9577ms |
| $T_2-T_1$   | 3.1831ms |

3.2 Retuning the PDD$^2$ controller gains without SJFB and BPJFB

In Refs. (8) and (9), high gains set to the PDD$^2$ controller to synchronize the error and base plate acceleration signals such that the BPJFB scheme can perform properly. However, based on the basic experiments, the high gains of PDD$^2$ controller easily destabilized the system while the SJFB was applied. This was because of the positive polarity of SJFB that made the scheme very sensitive against high stiffness of stage. For this reason, each gain of the PDD$^2$ controller was retuned individually by positioning the stage with a stroke of 15,000 counts from $r = 35,000 \sim 50,000$ counts at $t = 1$s, while the SJFB and BPJFB loops were kept open ($B_1 = B_2 = 0$). The related results are shown in Fig. 3(a) ~ (c).

The process was started from retuning the proportional gain $K_p$, while the gains $K_d$ and $K_{dd}$ were selectively set to 5 and 10 respectively to maintain the safety. Figure 3(a) shows the effect of $K_p$, in which the rising and settling times are reduced by increasing this gain. On the other hand, the overshoot and transient error are amplified while $K_p$ is increased. However, to have a sufficient speed, $K_p = 0.7$ was selected as the proper value to $K_p$.

Subsequently, to suppress the transient error, it was essential to prioritize tuning of the damping gain $K_{dd}$ while $K_p = 0.7$ was maintained and $K_d$ was still valued 5. Figure 3(b) shows the effect of $K_{dd}$, in which the transient error is damped by increasing the value of
this gain. On the other hand, increasing of $K_{dd}$ significantly results in the reduction of the positioning speed. However, completely suppression of the transient error was highly important, as the SJFB also increases the overshoot due to the stage mass reduction. Hence, as long as the transient error is almost completely suppressed for $K_{dd} = 120$, this value was selected as the appropriate value of $K_{dd}$.

![Fig. 3](image)

**Fig. 3** Experimental results for tuning the gains of PDD² controller while $B_1 = B_2 = 0$:

(a) Tuning $K_p$ while $K_d = 5$ and $K_{dd} = 10$ are selected to maintain safety

(b) Tuning $K_{dd}$ while $K_p = 0.7$ is maintained and $K_d = 5$ is still selective

(c) Tuning $K_d$ while $K_p = 0.7$ and $K_{dd} = 120$ are maintained

However, in Fig. 3(b), an offset-like error is still observed in the positioning response for a long time when $K_p = 0.7$ and $K_{dd} = 120$. This offset-like error is due to the high stiffness, i.e., high value of $K_d$, which does not allow the response to quickly settle on the reference. Therefore, to overcome this issue, the gain $K_d$ was gradually reduced from 5 to 3.8. The related results are shown in Fig. 3(c), in which the offset-like error is removed by setting $K_d = 3.8$. Therefore, the appropriate gains of the PDD² controller were tuned as $K_p = 0.7$, $K_d = 3.8$, and $K_{dd} = 120$. Notice that these values are the nominal values of the mentioned gains, while a separate scale factor gain valued $10^{-5}$ was used in the output of the PDD² controller in the MATLAB diagram to change the units count to volt, which is not shown in Fig. 2 for simplicity.

### 3.3 Applying the SJFB only and control of the overshoot caused by mass reduction

Increasing the positioning speed through PDD² controller is limited, because all three gains of this controller should be adjusted simultaneously to overcome the issue related to a single gain. For instance, referring to Fig. 3(a), the rising time is evidently increased by increasing $K_p$. However, since the transient error is also amplified, we need to increase $K_{dd}$ as well to overcome this issue. In such a case, the speed achieved by increasing $K_p$ is back degraded when $K_{dd}$ is increased (see Fig. 3(b)). If this process is continued, the system will become unstable without a significant improvement in the positioning speed. Hence, a separate controller, e.g., SJFB scheme, is required to be utilized to improve the positioning speed. The theory of SJFB scheme was already explained before in § 2.2. Hereinafter, we are going to present some experimental results to verify whether or not this controller conforms to the corresponding theory. However, let us firstly discuss the reason of using the MEMS type accelerometer and first order low pass filter in the SJFB loop.

Firstly, we used a Servo type accelerometer for the stage, the same as used for the base plate. However, its output was heavily saturated for higher strokes due to its lower range of maximum acceleration detection, as shown in Fig. 4(a). To overcome this problem, we replaced the Servo type accelerometer to MEMS type. However, since the bandwidth of MEMS type accelerometer is from DC to 5kHz, high frequency noise is also detected. To suppress the high frequency noise, a first order low pass filter with a cutoff frequency of 500Hz was employed. Fig. 4(b) shows the waveform of stage acceleration with and without low pass filter detected by the MEMS type sensor for a stroke higher than that used for the Servo type accelerometer. It can be seen that the low pass filter can almost suppress the high
frequency noise and provide an acceptable input to the SJFB scheme. Furthermore, there is no saturation for the MEMS sensor output.

![Fig. 4 Stage acceleration signal detected by different accelerometers:](image)

(a) By the Servo type accelerometer for a stroke of 20,000 counts
(b) By the MEMS type accelerometer for a stroke of 30,000 counts

After tuning the PDD² gains and suppression of the high frequency noise of the MEMS accelerometer through the low pass filter, the SJFB was applied, as the related results are shown in Fig. 5(a) ~ (c). Figure 5(a) shows the effect of $B_1$ on the relative positioning response using single output of MEMS accelerometer and setting appropriate values of PDD² controller’s gains while the BPJFB is open. It can be clearly observed from these results that the rising time is evidently decreased by increasing $B_1$, resulted in the improvement of positioning speed. In addition, the overshoot is amplified when $B_1$ is increased. Decreasing of the rising time and amplification of the overshoot both verify the stage mass reduction achieved by SJFB scheme. However, although positioning speed is increased by applying the SJFB scheme, the overshoot is an unwanted problem that should be solved. This issue might be controlled by retuning any gain of PDD² controller, i.e., $K_p$, $K_d$, and $K_{dd}$. However, referring to Fig. 3(a) and (b), since the gains $K_p$ and $K_{dd}$ have significant effect on the positioning speed, the overshoot caused by stage mass reduction cannot be controlled by decreasing $K_p$ or increasing $K_{dd}$. The reason is that the positioning speed achieved by the SJFB scheme will be reduced as that of without SJFB or even becomes worst.

The only key parameter that can overcome the mentioned overshoot is the gain $K_d$. That is because, this gain has a very small effect on the rising time, as can be seen in Fig. 3(c). Therefore, increasing $K_d$ will not only compensate the overshoot, but also almost maintains the rising time achieved by the SJFB. Figure 5(b) shows retuning of $K_d$ for the purpose of suppression of overshoot caused by stage mass reduction associated with SJFB. From the results, the overshoot is decreased by increasing $K_d$, while it is completely suppressed when $K_d$ is set to 8.7. Fig. 5(c) shows the repeatability without SJFB ($B_1 = 0$, $K_d = 3.8$), with SJFB only when the overshoot is not controlled ($B_1 = 4.5$, $K_d = 3.8$), and with SJFB when the overshoot is compensated ($B_1 = 4.5$, $K_d = 8.7$).

![Fig. 5 Results when only the SJFB is applied and the resulted overshoot is controlled:](image)

(a) Effect of SJFB gain $B_1$ on the stage mass reduction and positioning speed
(b) Compensation of overshoot caused by stage mass reduction by increasing $K_d$
(c) Repeatability without SJFB, with SJFB while the overshoot is not controlled, and with SJFB when the overshoot is compensated
From Fig. 5(c), the overshoot is completely suppressed for $K_d = 8.7$ while the speed is almost the same as that for $K_d = 3.8$. In addition, the settling time is evidently reduced. As a result, comparing the responses with SJFB/without overshoot and without SJFB, an evident improvement can be seen in positioning speed, verifying the successfullness of the SJFB.

3.4 Combining the BPJFB with SJFB and compensation of the related offset-like error

In § 3.3, the experimental results showed the effectiveness of SJFB scheme and $K_d$ on reduction of the rising and settling times of relative positioning response. In this subsection, let us explain the effects of higher values of $B_1$. It is obvious that higher $B_1$ result in the instability of the system because of the positive polarity of SJFB scheme. However, herein by increasing $B_1$, we would like to destabilize the system and check whether or not the BPJFB scheme is efficient to make the system stable and to provide better repeatability. The corresponding results shown in Fig. 6(a) ~ (c) are explained as follows:

First of all, gain $B_1$ was increased until the system became unstable and the stage started self-oscillation, resulted in the excitation of the base plate through the reaction force, as shown in Figs. 6(a) and (b). The results of Fig. 6(a) show that by increasing $B_1$ from 4.5 to 5, although initially the speed is going to slightly improve; the stage becomes unstable which results in its self-oscillation. To maintain the safety, the repeatability was performed only two times for $B_1 = 5$. Notice that for $B_1 = 5$, $K_d$ was set to 8.7, because the stage became strongly oscillatory when $K_d$ was set to 3.8, i.e., due to very low stiffness. This might be the reason that the positioning response for $B_1 = 5$ is shifted down.

Figure 6(b) shows the results without and with BPJFB by setting its optimal gain, $B_2 = 0.1$, including stage stiffness compensation and repeatability. These results show that the self-oscillation of stage caused by the instability due to setting high gain of SJFB is almost completely suppressed when $B_2$ is set to 0.1. However, there is an offset-like error in the positioning response for a while, showing an increase in the stage stiffness. In addition, the initial speed achieved by $B_1 = 5$ is not followed, showing the damping effect of BPJFB scheme. These two features of BPJFB scheme were already demonstrated in Ref. (12), stating that this scheme results in the increase of the damping ratio and stiffness of stage, besides its function as a Master-Slave synchronizing system. However, the increased stiffness of stage associated with the BPJFB scheme can be also compensated by reducing $K_d$. The results of Fig. 6(b) show that by reducing $K_d$ from 8.7 to 6.7, the offset-like error caused by the increased stiffness through the BPJFB scheme is completely removed.

Figure 6(c) shows the relative positioning response including repeatability without SJFB and BPJFB, and with fusion of SJFB and BPJFB while stiffness is also compensated. From these results, an evident improvement in positioning speed for the fusion of SJFB and BPJFB can be clearly observed.

![Fig. 6 Results when SJFB gain is further increased and BPJFB is applied:](image)

(a) Results when the SJFB gain $B_1$ is further increased
(b) Results for application of BPJFB and retuning gain $K_d$ to overcome the offset-like error
(c) Results without and with SJFB and BPJFB schemes including repeatability
3.5 Comparison of experimental results

Referring to the results of Figs. 5(c) and 6(c), the settling time is slightly increased for the fusion of SJFB and BPJFB compared to that for SJFB only. The reason is the damping property of BPJFB scheme, which did not allow the response to follow the initial speed achieved by the increased gain of SJFB, i.e., $B_1 = 5$. However, it is not proper to remove the damping effect of BPJFB scheme by retuning $K_{dd}$, because it results in the loss of stability. Figure 7 shows the closed-up view of the error including repeatability without both SJFB and BPJFB, with SJFB only, and with both SJFB and BPJFB around the settling time. From the results, the repeatability shows smaller error for the case with both SJFB and BPJFB comparing to that for the other cases. In addition, the vibration observed for the SJFB only that is caused by the reaction force is almost disappeared, verifying the Master-Slave property of the BPJFB controller.

Therefore, it is concluded that although the stage becomes unstable by further raising the SJFB gain, combining the BPJFB scheme not only stabilizes the system by suppressing the self-oscillation of stage caused by high value of $B_1$, but also improves the repeatability.

![Fig. 7 Error signals without SJFB and BPJFB, with SJFB only, and with SJFB and BPJFB including repeatability](image)

3.6 Estimation of the amount of reduced mass $\Delta M_1$ associated with SJFB

It might be simply thought that the rate of stage mass reduction $\Delta M_1$ can be easily determined by performing the frequency response and comparing the difference between the resonance frequencies with and without SJFB scheme. However, the frequency response is strongly harmful to the actuators at the neighborhood of resonance frequency, especially for the higher gain of SJFB as well as at high frequency region due to the existence of high nonlinear friction forces that may result in the generation of high temperature. This issue along with the unknown nominal values of plant parameters does not allow to determine the amount of $\Delta M_1$ and to study the system stability through numerical simulation. However, there are three methods that can be used to estimate $\Delta M_1$, which are explained as follows.

**Method 1:** In this method, $\Delta M_1$ can be determined through the ratio of the optimal values of BPJFB gain with and without SJFB, $B_{2\text{opt}}$ and $B'_{2\text{opt}}$, respectively. This ratio can be found by dividing Eq. (8) into Eq. (9) side by side and solving it for $\Delta M_1$, as given in Eq. (10). The values of $B_{2\text{opt}}$ and $B'_{2\text{opt}}$ can be experimentally determined by horizontally applying an impulse type external force $f_{\text{ext}}$ on the base plate while the stage is maintained at servo lock (stopped) condition. $f_{\text{ext}}$ naturally vibrates the base plate which is conveyed to the stage due to the reaction force, resulting in the vibration of error signal $e$. Then for both cases, tuning $B_{2\text{opt}}$ and $B'_{2\text{opt}}$ result in the removal of the error vibration based on the Master-Slave property of BPJFB. Notice that according to Eqs. (8) and (9), for the case with SJFB $B_{2\text{opt}}$ should be smaller than $B'_{2\text{opt}}$ due to the mass $\Delta M_1$ reduced from stage mass $M_1$.

$$\Delta M_1 = \left[\frac{B'_{2\text{opt}} - B_{2\text{opt}}}{B'_{2\text{opt}}}\right]M_1$$

**Method 2:** In this method, $\Delta M_1$ can be estimated through the differences among the natural frequencies of base plate for three different cases, i.e., without both SJFB and BPJFB, with BPJFB only, and with both SJFB and BPJFB. In this method, the base plate
can be assumed as a second order lag system without considering the influence of the dynamics of stage on the base plate characteristics. In addition, it should be supposed that the base plate spring constant $K_z$ does not change for all three mentioned cases. In such a case, we have three different natural frequencies for the base plate, i.e., without both SJFB and BPJFB $f_{2(1)}$, with BPJFB only $f_{2(2)}$, and with both SJFB and BPJFB $f_{2(3)}$, given in Eqs. (11) ~ (13). Notice that in Eq. (12), $\Delta M_2$ is equal to $M_1$ for $B'_{2(\text{opt})}$ while $(M_1 - \Delta M_1)$ in Eq. (13) represents the effect of SJFB gain on reducing the stage mass $M_1$. Solving the set of Eqs. (11) ~ (13) for $\Delta M_1$ gives Eq. (14) for calculation of the reduced mass. The procedure for determining the mentioned natural frequencies of the base plate is the same as explained in method 1, i.e., measuring the natural frequencies for $B_1 = B_2 = 0$ as well as for $B_{2(\text{opt})}$ and $B'_{2(\text{opt})}$ which are tuned against $f_{2(3)}$ horizontally applied on the base plate.

\[
f_{2(1)} = \left(\frac{1}{2}\pi \sqrt{K_z/M_1}\right) \quad B_1 = B_2 = 0 \quad (11)
\]

\[
f_{2(2)} = \left(\frac{1}{2}\pi \sqrt{K_z/(M_2 + M_1)}\right) \quad B_1 = 0, B_2 = B_{2(\text{opt})} \quad (12)
\]

\[
f_{2(3)} = \left(\frac{1}{2}\pi \sqrt{K_z/[M_2 + (M_1 - \Delta M_1)]}\right) \quad B_1 \neq 0, B_2 = B_{2(\text{opt})} \quad (13)
\]

\[
\Delta M_1 = \frac{f_{2(3)}^2}{f_{2(3)}^2 - \left(\frac{f_{2(3)} - f_{2(3)}^2}{f_{2(3)} - f_{2(3)}^2}\right)} M_1 \quad (14)
\]

However, method 2 is very difficult experimentally, because the base plate acceleration signal is not a pure sinusoidal waveform in real application due to the internal uncertainties such as friction, resulting in the measurement error. In addition, the base plate acceleration signal is practically a damped waveform with slightly differences in the frequency and decay ratio for each cycle. Hence, a long calculation is required to measure the averages of damped natural frequencies and decay ratios for several cycles, in order to calculate the undamped natural frequencies. Moreover, this method is further approximated due to the ignorance of stage dynamics and assuming $K_z$ to be constant for all three cases. However, beside the mentioned issues, this method might be efficient for numerical simulation.

**Method 3:** In this method, $\Delta M_1$ can be found by adjusting the amount of a variable mass added to the stage through positioning with SJFB for the same stoke as used without SJFB. The amount of the mentioned mass can be adjusted for a certain value of gain $B_1$ such that the positioning response becomes the same as that for the case without SJFB. As a result, the adjusted mass is in fact equal to $\Delta M_1$. However, this method is time consuming as the amount of the adding mass cannot be primarily estimated. Moreover, it is difficult to rigidly attach the mentioned mass on the stage while it is variable and to fix it at the center of gravity to avoid the mechanical vibrations and geometric errors.

Therefore, in this study, $\Delta M_1$ was estimated by using method 1 through the related procedure explained above. Firstly, $\Delta M_1$ was determined for the $B_1 = 1$ by setting the values of PDD$^2$ controller gains the same as properly tuned for the case with both SJFB and BPJFB ($K_p = 0.7, K_d = 6.7$, and $K_{dd} = 120$). At first $B'_{2(\text{opt})}$ without SJFB and then $B_{2(\text{opt})}$ for $B_1 = 1$ were tuned, as the related results are shown in Fig. 8. From the results, the error becomes almost flat while $B_2$ is set to 0.13, hence, this value is the optimal value of BPJFB gain without SJFB, i.e., $B'_{2(\text{opt})} = 0.13$. However, maintaining $B_2 = 0.13$ for the case with SJFB while $B_1 = 1$, resulted in the increase of the vibration of the error signal. Hence, based on Eq. (8), it was essential to retune $B_2$ by reducing its value until $B_{2(\text{opt})}$ is reached. After several trials, the error became almost flat by setting $B_2 = 0.11$. Hence, the optimal value of BPJFB gain with SJFB for $B_1 = 1$ is found to be $B_{2(\text{opt})} = 0.11$. Therefore, substituting the values of $B'_{2(\text{opt})}$ and $B_{2(\text{opt})}$ into Eq. (10) gives the value of reduced mass for $B_1 = 1$ as to be $\Delta M_1 = 0.153846 M_1$, showing about 15% reduction of the stage mass $M_1$. 


Subsequently, using the mentioned gains of PDD$^2$ controller, it was tried to determine $\Delta M_1$ for $B_1$ higher than 1. However, the stage became unstable after $B_1$ was further increased. To overcome this issue, $K_{dd}$ was increased from 120 to 400 and method 1 was repeated for $B_1 = 2$, as the related results are shown in Fig. 9. From these results, the optimal value of BPJFB gain without SJFB is still the same, i.e., $B^\prime_{2\text{opt}} = 0.13$. However, due to the larger value set to the SJFB gain ($B_1 = 2$), the optimal value of BPJFB gain is reduced to $B^\prime_{2\text{opt}} = 0.09$. Hence, substituting these optimal values in Eq. (10) gives the amount of reduced mass for $B_1 = 2$ as to be $\Delta M_1 = 0.307692 M_f$, showing about 30% reduction of stage mass. Therefore, it is verified that $\Delta M_1$ is linearly and directly proportional to the value of $B_1$, as defined in Eq. (6g). Notice that due to the instability occurred for higher $B_1$, it was difficult to determine $\Delta M_1$ for $B_1$ higher than 2. However, the linearly change in the amount of $\Delta M_1$ calculated for $B_1 = 1$ and 2 shows that the stage mass reduces by 15% for each unity value of $B_1$. Therefore, for $B_1 = 4.5$ for which the stage was safely positioned only with SJFB, the amount of reduced mass can be estimated as $\Delta M_1 = 4.5 \times 15\% M_f = 67.5\% M_f$.

4. Conclusions

The overall results of the proposed control method can be summarized as follows:
1. The SJFB with positive polarity is able to reduce the stage mass by $\Delta M_1$ which is
linearly proportional to gain $B_1$. The stage mass reduction results in the increase of positioning speed. However, the overshoot is also increased, which can be removed by retuning gain $K_d$ such that the rising time attained by SJFB almost maintains.

2. The BPJFB with positive polarity is efficient to be simultaneously combined with SJFB. This scheme is not only capable of suppression of the self-oscillation of stage caused by high gain of SJFB, but also improves the positioning repeatability.

3. There are several methods to estimate the amount of the reduced mass $\Delta M_1$. In this study, the amount of $\Delta M_1$ was estimated through method 1 (the ratio of the optimal values of BPJFB gain with and without SJFB), showing a reduction of 67.5% of stage mass $M_1$ for $B_1 = 4.5$ for which the stage can be stably positioned with SJFB.

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