Observational Aspects of Symmetries of the Neutral B Meson System

Maria Fidecaro\textsuperscript{1}, Hans-Jürg Gerber\textsuperscript{2}, Thomas Ruf\textsuperscript{1}

\textsuperscript{1}CERN, CH-1211 Geneva 23, Switzerland
\textsuperscript{2}ETHZ, IPP, CH-8093 Zürich, Switzerland

Abstract

We revisit various results, which have been obtained by the BABAR and Belle Collaborations over the last thirteen years, concerning symmetry properties of the Hamiltonian, which governs the time evolution and the decay of neutral $B$ mesons. We find that those measurements, which established $CP$ violation in $B$ meson decay, 13 years ago, had as well established $T$ (time-reversal) symmetry violation. They also confirmed $CPT$ symmetry in the decay ($T_{CPT} = 0$) and symmetry with respect to time-reversal ($\epsilon = 0$) and to $CPT$ ($\delta = 0$) in the $B^0\bar{B}^0$ oscillation. Motion-reversal symmetry vs. time-reversal symmetry is discussed.
1 Introduction

A system of neutral mesons such as $B^0, \bar{B}^0$ or $K^0, \bar{K}^0$ is a privileged laboratory for the study of weak-interaction’s symmetries. Even though the phenomenological framework is well understood since long time [1][4], recent discussions in the physics community [8] show that it may be useful to revisit a few points, in order to fully (and correctly) exploit the experimental results. This process is then at the origin of the present note.

We focus on the $B^0 \bar{B}^0$ system, and refer to experimental results [6][10] that have been achieved by measurements of the decay products of $B^0 \bar{B}^0$ pairs created in the entangled antisymmetric state

$$|\Psi\rangle = \left((|B^0\rangle |\bar{B}^0\rangle - |\bar{B}^0\rangle |B^0\rangle) / \sqrt{2} \right) \tag{1}$$

where the first $B$ in this notation moves in direction $\vec{p}$ and the second in direction $-\vec{p}$.

Within the Weisskopf-Wigner approximation [1] the time evolution of a neutral $B$-meson, and its decay into a state $f$ is described by the amplitude $A_{bf}$,

$$A_{bf} = \langle f | T | e^{-i\Lambda t} | B \rangle \tag{2}$$

where $T$ and $\Lambda$ are represented by constant, complex $2 \times 2$ matrices $T = (T^{ij}) = (f^i | T | f^j)$ and $\Lambda = (\Lambda^{ij}) = (B^i | \Lambda | B^j)$, $i, j = 1(2)$. We consider experiments with final states $f^i = J/\psi K^0$ or $f^i = \mu^+ \nu_\mu (\bar{\nu}_\mu) X$. Here $B^{1(2)}$, $K^{1(2)}$ and $\mu^+ (\mu^-)$ or $e^+ (e^-)$, respectively.

We recall that a symmetry is a property of the hermitian Hamiltonian ($H = \hat{H}_0 + H_{weak}$) of the Schrödinger equation which is defined in a space sufficiently complete to include all the particle states under consideration, also the decay products [1]. Thus the aim of the experiments is to establish properties of the weak interaction Hamiltonian $H_{weak}$ by measuring observable combinations of the elements of $\Lambda$ and of $T$, which represent these properties.

As $CP$ violation implies $T$ and/or $CPT$ violation, we specifically consider the classical aim posed by the discoverers of $T$ violation [11] “to express quantitatively the fraction of the observed $CP$ violation due to $T$ violation and $CPT$ violation separately”.

In passing, we show that a more recent treatment, which attempts to define $CP$ violation due to $T$ violation, results in a division of the symmetry violations in the matrix $\Lambda$ and/or $T$ for the order $\mathcal{O}(\epsilon)$ into $CP$-violating and $CPT$-violating parts.

2 Observables of Symmetries

Together with a parametrization of the matrices $\Lambda$ and $T$, the equations (1) and (2) are a sufficient basis for the description of the symmetry properties of the experimental results [6][10]. Symmetry properties of the Hamiltonian often manifest themselves in an especially simple and direct way in relations between measured quantities. Here, Table 1 gives a summary, with definitions and derivations as found in [1][4], and the phase conventions of [2]. Our approach is analogous to [13].

| Symmetry of $H_{weak}$ | requires for the matrix $\Lambda$ | requires for the matrix $T$ |
|------------------------|---------------------------------|-----------------------------|
| $T$                    | $\Lambda_T = |\Lambda^{11}|^2 - |\Lambda^{22}|^2 = 0$ | $T_T = \text{Im}(T^{11} \cdot T^{22}) = 0$ |
| $CPT$                  | $\Lambda_{CPT} = |\Lambda^{12}| = 0$ | $T_{CPT} = |T^{11}|^2 - |T^{22}|^2 = 0$ |
| $CP$                   | $\Lambda_T = 0$ and $\Lambda_{CPT} = 0$ | $T_T = 0$ and $T_{CPT} = 0$ |

Let us pose

$$\Lambda^{11} = m - i\gamma/2 - \delta \Delta m, \quad \Lambda^{22} = m - i\gamma/2 + \delta \Delta m, \tag{3}$$

$$\Lambda^{12} = (1 - 2\epsilon) \Delta m/2, \quad \Lambda^{21} = (1 + 2\epsilon) \Delta m/2 \tag{4}$$

with real $m, \gamma, \Delta m, \epsilon$, and complex $\delta$. For the observables of the symmetry violations in the matrix $\Lambda$, i.e. in the $B^0 \bar{B}^0$ oscillation, we deduce from eqs. (3), (4), and Table 1

$$\Lambda_T = 2 \epsilon (\Delta m)^2 + \mathcal{O}(\epsilon^2), \tag{5}$$

$$\Lambda_{CPT} = 2 \delta \Delta m. \tag{6}$$

We note that, with eqs. (3), (4), and (5), the difference of the widths of the eigenstates of $\Lambda$ becomes

$$\Delta \Gamma = 2 \Delta m \cdot \text{Im}(\sqrt{1 - 4\epsilon^2 + 4\delta^2}).$$

This lets us recognize that, if $\Delta \Gamma = 0$, our matrix $\Lambda$ still allows for a finite $\epsilon$ ($|\epsilon| < 1/2$), in accordance with [14]. This is in contrast to widely repeated affirmations [15], that $\Delta \Gamma = 0$ would imply time-reversal symmetry of $\Lambda$, i.e. $\epsilon = \Lambda_T = 0$.

In terms of $\Lambda = \mathcal{M} - \frac{1}{2} \Gamma$ ($\mathcal{M} = M^1, \Gamma = \Gamma^1), \Lambda^{12} = |M^{12}| \cdot \cos(\phi_T), \Delta m = 2 |M^{12}|, \phi_M = 0$ and $\Delta \Gamma \approx -2 |\Gamma^{12}| \cdot \cos(\phi_T).$ We admit $|\Gamma^{12}| \ll |M^{12}|.$
In order to calculate the amplitude $A_{\ell f}$ in eq.(2), we need to evaluate the exponential in terms of $\Lambda$. We do this by summing up the power series (as explained in [13]). Let $U = (U^{ij}) = e^{-i\Lambda t}$ and find

$$
\begin{align*}
U^{11} &= U_0(\cos(\omega t) + i 2\delta \sin(\omega t)), \\
U^{12} &= U_0(-i(1 - 2\epsilon) \sin(\omega t)), \\
U^{21} &= U_0(-i(1 + 2\epsilon) \sin(\omega t)), \\
|U_0|^2 &= e^{-\epsilon t}, \\
\omega &= \Delta m/2 + O(\delta^2, \epsilon^2).
\end{align*}
$$

For the matrix $(T^{ij}) = \langle (|\psi/K^\dagger|T|B^i\rangle)$, we assume

$$
T^{12} = T^{21} = 0,
$$

with complex $T^{11}$, $T^{22}$, corresponding to the "$\Delta b = \Delta S$ rule". From Table 1, and with the (arbitrary) normalization $|T^{11}|^2 + |T^{22}|^2 = 2$, we deduce the useful identity among the (diagonal) elements of $T$,

$$
T^2_T + T^2_{CPT}/4 + (\text{Re}(T^{11} T^{22}))^2 \equiv (|T^{11}|^2 + |T^{22}|^2)^2/4 = 1.
$$

Results based on eqs. (1) to (11) will turn out to be sensitive to all the four symmetry parameters in Table 1.

Throughout this work, we assume that channels have one single amplitude. Two interfering amplitudes may fake non-CP symmetry of the $T$ matrix as with $T^{11}$, $T^{22}$ which are exactly associated each with its own proper time dependence:

$$
T^{11}_T \pm T^{22}_{CPT}/\sqrt{2},
$$

In rewriting (12), we can explicitly derive the formula for the state $|S_{f_1}\rangle$, which survived the decay to $f_1$, and its (single particle) time evolution and decay to $f_2$ as

$$
A_{f_1, f_2}(t) = \tilde{A}_{f_1, f_2} = \langle f_2|T e^{-i\Lambda t}|S_{f_1}\rangle
$$

with

$$
|S_{f_1}\rangle = \frac{b|B^0\rangle + \bar{b}^\dagger|\bar{B}^0\rangle}{\sqrt{2}},
$$

The variety of expected frequency distributions $|A_{f_1, f_2}(t)|^2$ is displayed in Table 2. We find that the parameters of the data analysis are the $T$ and $CPT$ violation parameters of the $T$ matrix, $T_T$ and $T_{CPT}$, concerning the decay, and those, $p_i$, $q_i$ ($i = 1, 2, 5, 6$), concerning mainly the $B^0/\bar{B}^0$ oscillation matrix $\Lambda$. In the limit of $CP$ symmetry of $\Lambda$ the $p_i$, $q_i$ all vanish. Then $T_T$ and $T_{CPT}$ are exactly associated each with its own proper time dependence: $T_T$ with $\pm \sin(\Delta m t)$, and $T_{CPT}$ with $\pm \cos(\Delta m t)$.

Table 2 also allows one to read off the relations of the measured distributions to the symmetry violating parameters of $\Lambda$ and $T$, as demonstrated below, and also to construct combinations of data which are true signatures for specific violations.

3 Experiments

3.1 General description

Call $A_{f_1, f_2}(t)$ the amplitude for the decay of an entangled, antisymmetric $B^0/\bar{B}^0$ pair into a final state with the two observed particles $f_1$ (at time $t_0$) and $f_2$ (at later time $t > 0$). With specific choices of the two final states $f_1, f_2$, we can uniquely represent the complete set of results of the $CP$, $T$- and $CPT$-symmetry violation studies listed in Table 2 and performed by [6-10] through [10], by making use of eq. (12) below [10], whose derivation we sketch here. We note with (13), section 2.7, that the time evolution acts on the two-particle state $|\Psi\rangle$ of eq. (1) solely by a multiplicative factor, which is independent of the symmetry violations under consideration, and which does not influence the decay properties of $|\Psi\rangle$. We may thus, without loss of generality, arbitrarily choose $t_0 = 0$, $t > 0$, and apply eq. (2) to the single-particle components in $|\Psi\rangle$, to obtain

$$
A_{f_1, f_2}(t) = \langle f_1|T|B^0\rangle \langle f_2|T e^{-i\Lambda t}|B^0\rangle - \langle f_1|T|\bar{B}^0\rangle \langle f_2|T e^{-i\Lambda t}|\bar{B}^0\rangle /\sqrt{2}.
$$

In rewriting (12), we can explicitly derive the formula for the state $|S_{f_1}\rangle$, which survived the decay to $f_1$, and its (single particle) time evolution and decay to $f_2$ as

$$
A_{f_1, f_2}(t) = \tilde{A}_{f_1, f_2} = \langle f_2|T e^{-i\Lambda t}|S_{f_1}\rangle
$$

with

$$
|S_{f_1}\rangle = \frac{b|B^0\rangle + \bar{b}^\dagger|\bar{B}^0\rangle}{\sqrt{2}},
$$

The experiments [6-10] have measured in 2001/2 all the data sets listed in Table 2, and thereby discovered $CP$ violation in the matrix $T$. We show now that these data furthermore establish time-reversal symmetry violation in $H_{weak}$, and are compatible as well with $CPT$ symmetry of the $T$ matrix as with $\epsilon = 0, \delta = 0$, i.e. $CP$ symmetry of $\Lambda$.

To this purpose we consult Table 2 and calculate

$$
(1) - (2) = (p_1 - p_2) + (T_{CPT} - (p_1 - p_2)) \cos(\Delta m t) + (2T_T + (q_1 - q_2)) \sin(\Delta m t).
$$
Table 2: The measurements, classified according to eq. (12).

General expressions for the expected frequency distributions in terms of $T_{CPT}$, $T_T$, $\epsilon$, $\delta$. In the limit $\epsilon = \delta = 0$, they are all of the form $(1 \pm \frac{1}{2} T_{CPT} \cos(\Delta m t) \pm T_T \sin(\Delta m t)) e^{-\gamma t}$. $\mu^+$ is a shorthand for $\mu^-\bar{\nu}_e X$ or $e^-\nu_e X$. $\mu^+$ for $\mu^+\bar{\nu}_e X$, etc. By the "$\Delta b = \Delta Q$ rule", a $B^0(\bar{B}^0)$ decays semileptonically always into $\mu^+ + \ldots (\mu^- + \ldots)$. $|K_{S(L)} > = ([|K^0 > | K^0 > ]/\sqrt{2}$ has been used. All 10 measurements have been performed.

| Name of measurement | 1st decay | 2nd decay | $|A_{f_1,f_2}(t)|^2$ $\propto$ $a + b \cos(\Delta m t) + c \sin(\Delta m t)$ |
|---------------------|-----------|-----------|-------------------------------------------------|
| $B^0 \to K^0_S$    | $\{1\}$  | $\mu^-$   | $J/\psi K^0_S$ $1 + p_1$ $+ \frac{1}{2} T_{CPT} - p_1$ $+ T_T + q_1$ |
| $\bar{B}^0 \to K^0_S$ | $\{2\}$  | $\mu^+$   | $J/\psi K^0_S$ $1 + p_2$ $- \frac{1}{2} T_{CPT} - p_2$ $- T_T + q_2$ |
| $K^0_L \to B^0$    | $\{3\}$  | $\mu^-$   | $J/\psi K^0_S$ $1 + p_1$ $+ \frac{1}{2} T_{CPT} - p_1$ $- T_T - q_1$ |
| $K^0_L \to B^0$    | $\{4\}$  | $\mu^+$   | $J/\psi K^0_S$ $1 + p_2$ $- \frac{1}{2} T_{CPT} - p_2$ $+ T_T - q_2$ |
| $B^0 \to K^0_L$    | $\{5\}$  | $\mu^-$   | $J/\psi K^0_L$ $1 + p_5$ $+ \frac{1}{2} T_{CPT} - p_5$ $- T_T + q_5$ |
| $\bar{B}^0 \to K^0_L$ | $\{6\}$  | $\mu^+$   | $J/\psi K^0_L$ $1 + p_6$ $- \frac{1}{2} T_{CPT} - p_6$ $+ T_T + q_6$ |
| $K^0_S \to B^0$    | $\{7\}$  | $\mu^+$   | $J/\psi K^0_S$ $1 + p_5$ $+ \frac{1}{2} T_{CPT} - p_5$ $+ T_T - q_5$ |
| $K^0_S \to B^0$    | $\{8\}$  | $\mu^+$   | $J/\psi K^0_S$ $1 + p_6$ $- \frac{1}{2} T_{CPT} - p_6$ $- T_T - q_6$ |
| $B^0 \to B^0$      | $\{9\}$  | $\mu^+$   | $\frac{1}{2} (1 - 4\epsilon)$ $- \frac{1}{2} (1 + 4\epsilon)$ $0$ |
| $B^0 \to B^0$      | $\{10\}$ | $\mu^-$   | $\frac{1}{2} (1 + 4\epsilon)$ $- \frac{1}{2} (1 + 4\epsilon)$ $0$ |

The terms with $\epsilon$ and $\delta$ (upper signs for $p_1, p_5, q_1, q_5$).

$p_1(p_2) = \epsilon (\pm 2 - T_{CPT}) + 2\text{Re}(\delta) \cdot \text{Re}(\text{T}^{11+*T^{22}}) - 2\text{Im}(\delta) \cdot T_T$

$p_5(p_6) = \epsilon (\pm 2 - T_{CPT}) \pm 2\text{Re}(\delta) \cdot \text{Re}(\text{T}^{11+*T^{22}}) + 2\text{Im}(\delta) \cdot T_T$

$q_1(q_2) = \epsilon \cdot 2 T_T - \text{Im}(\delta)(\pm 2 + T_{CPT})$

$q_5(q_6) = -\epsilon \cdot 2 T_T - \text{Im}(\delta)(\pm 2 + T_{CPT})$

Identity: $q_1 + q_6 - (q_2 + q_5) = 0$

Similarly, we calculate $\{5\} - \{6\}$ and summarize the results as follows.

\[ C_{CP_{S(L)}} \equiv \left| A_{\mu^-,J/\psi K^0_S(S(L))} (t) \right|^2 - \left| A_{\mu^+,J/\psi K^0_S(S(L))} (t) \right|^2 \]

\[ \propto 4 \epsilon \pm 4\text{Re}(\delta) \cdot \text{Re}(\text{T}^{11+*T^{22}}) \]

\[ + \{T_{CPT} - 4 \epsilon \pm 4\text{Re}(\delta) \cdot \text{Re}(\text{T}^{11+*T^{22}}) \} \cos(\Delta m t) \]

\[ + \{\pm 2 T_T - 4\text{Im}(\delta)\} \sin(\Delta m t) \].

The experimental results for $C_{CP_{S}}$ and $C_{CP_{L}}$ show no time independent terms, $4\epsilon \pm 4\text{Re}(\delta) \cdot \text{Re}(\text{T}^{11+*T^{22}}) \approx 0$, and no $\cos(\Delta m t)$ signals. $\{T_{CPT} - 4 \epsilon \pm 4\text{Re}(\delta) \cdot \text{Re}(\text{T}^{11+*T^{22}}) \} \approx 0$. From this we conclude $\epsilon \approx 0$, $4\text{Re}(\delta) \cdot \text{Re}(\text{T}^{11+*T^{22}}) \approx 0$, and $T_{CPT} \approx 0$. The $\sin(\Delta m t)$ amplitudes are equal but with opposite signs, and, in absolute value, $< 2$, implying $\text{Im}(\delta) \approx 0$ and $|T_T|^2 \leq 1$. From (11) now follows $\text{Re}(\text{T}^{11+*T^{22}}) \not= 0$ and thus $\text{Re}(\delta) \approx 0$. The $p_i$ and $q_i$ defined in Table 2 are thus all compatible with zero.

Quantitative results for $T_T$ and $T_{CPT}$ may be read off from $\{6\}$ and $\{7,8\}$, who analyze their data also with two free parameters $\{9,10\}$, corresponding to $T_T$ and $T_{CPT}$.

The experiment [3] has set a stringent limit on $T$-symmetry violation in the $\Lambda$ matrix of the $B^0\bar{B}^0$ system with a direct measurement of $\epsilon$. See Table 2 (entries $\{9\}$ and $\{10\}$) and Table 3. The method is analogous to the one of the CPLEAR experiment [17,18] for the $K^0\bar{K}^0$ system, where also a signature for $T$-violation ("Kabir asymmetry") has been directly measured. The experiments of the general identity, valid in two dimensions (see $\{13\}$), $\Lambda^{21}/\Lambda^{12} \equiv (e^{-i\Lambda})^{12}/(e^{-i\Lambda})^{21} = U^{21}/U^{12}$ from which

\[ \epsilon \approx \frac{1}{4} \left| \Lambda^{12} \right|^2 - \frac{1}{4} \left| \Lambda^{21} \right|^2 - \frac{1}{4} \left| U^{21} \right|^2 - \frac{1}{4} \left| U^{12} \right|^2 \]

\[ \frac{1}{4} \left| A_{\mu^-} \right|^2 - \frac{1}{4} \left| A_{\mu^+} \right|^2 \]

\[ \frac{1}{4} \left| A_{\mu^-} \right|^2 - \frac{1}{4} \left| A_{\mu^+} \right|^2 \]

\[ \times \frac{1}{4} \left| A_{\mu^-} \right|^2 - \frac{1}{4} \left| A_{\mu^+} \right|^2 \]

the connection from the data to the $T$-symmetry violation signal $\epsilon$, follows - without any assumptions on $CPT$ symmetry or on the value of $\Delta t$ of the $\Lambda$ matrix.

A reanalysis of the results in 2007 of the BABAR and Belle collaborations by [19] has shown that the data contradict motion-reversal symmetry (see $\{5\}$ in the $B^0\bar{B}^0$ system).

In summary, the discovered $CP$ violation in the $B^0\bar{B}^0$ system is $T$-symmetry violation in the decay-amplitude matrix.
Due to the presence of $T_{\text{CPT}}$, of the $p_i$, and $q_i$, our results contradict the attempt [12,20] to define the differences $\{2a\}$ to $\{2d\}$, each as a signature for $T$ violation. In the lower part, signatures for $T$- and $CPT$- symmetry violations are indicated.

| Display in [10] Rates compared | Expected $a \propto a + b \cos(\Delta m t) + c \sin(\Delta m t)$ |
|--------------------------------|---------------------------------------------------------------|
| **Figure** 2a $\{2\} - \{7\} \equiv \{2a\}$ | $p_2 - p_5 - T_{\text{CPT}} - (p_2 - p_5) - 2T_T + q_2 + q_5$ |
| 2b $\{4\} - \{5\} \equiv \{2b\}$ | $p_2 - p_5 - T_{\text{CPT}} - (p_2 - p_5) + 2T_T - q_2 - q_5$ |
| 2c $\{6\} - \{3\} \equiv \{2c\}$ | $p_6 - p_1 - T_{\text{CPT}} - (p_6 - p_1) + 2T_T + q_1 + q_6$ |
| 2d $\{8\} - \{1\} \equiv \{2d\}$ | $p_6 - p_1 - T_{\text{CPT}} - (p_6 - p_1) - 2T_T - q_1 - q_6$ |

Signatures are for $T_T$: $-8T_T \sin(\Delta m t)$ for $\{2a\} - \{2b\} - \{2c\} + \{2d\}$

$T_{\text{CPT}}$: $-4T_{\text{CPT}}$, $\Lambda_T = 4\epsilon$, $\approx (\{10\} - \{9\}) / (\{10\} + \{9\})$

$T, T_T \neq 0$ with $T_{\text{CPT}} \approx 0$. In the $K^0\bar{K}^0$ system, however, the $CP$-violation is $T$-symmetry violation in oscillations, $\Lambda_T \neq 0$ with $\Lambda_{\text{CPT}} \approx 0$.

### 3.3 Recent results

The analysis by [10] is based on [12] with novel notions of $CPT$-, $CP$-, and $T$-symmetry, which, in contrast to the classical definitions [1], are not related to properties of the weak interaction Hamiltonian, but to comparisons of surviving states $|S_{i'}\rangle$ with suitably motion-reversal transformed ones of type $|S_{j'}\rangle$. The novel definitions are less general than the classical ones as they need the assumption of $T_{\text{CPT}} = 0$. This new analysis then becomes a special case of our present work, and in turn looses the possibility to address the "classical aim", mentioned in our Introduction. (Details below).

To prove that the phenomenology of [12] uses $T_{\text{CPT}} = 0$, it is sufficient to express their eq.(A.5 of [12]) in terms of the elements of the matrix $T$, $T^{11}$ and $T^{22}$, to find

$$a\beta^* = -1 = - |T^{11}|^2 / |T^{22}|^2$$

$T_{\text{CPT}} = 0$.

The work of [12] specifies 3 sets of 4 pairs of measurements, whose comparisons are supposed to indicate the violations of the 3 symmetries mentioned above. (See Tables 1, 2, 3 of [12].) Each of the 24 measurements is completely determined by the products of the first and the second decay of the antisymmetric, entangled $B^0\bar{B}^0$ pair. Their amplitudes are thus uniquely given by our eq. (12). The corresponding rates are listed in our Table 2, labeled $\{1\}$ to $\{8\}$.

The envisaged $T$-violating comparisons, labelled $\{2a\}$ to $\{2d\}$ in Table 3, depend also on $T_{\text{CPT}}$, and thus contradict the affirmation in [10], that "Any difference in these two rates is evidence for $T$-symmetry violation", since a $T$-symmetric, $CPT$-violating Hamiltonian $H_{\text{weak}} (T_T = 0, T_{\text{CPT}} \neq 0)$ would just also create such rate differences.

The $CP$-violating comparisons in Table 2 of [12] also depend on $T_{\text{CPT}} \cos(\Delta mt)$ and on $1/T_T \sin(\Delta mt)$. This confirms that $T$- and/or $CPT$-violation imply $CP$-violation. $T$-violation in the (2 by 2 dimensional) $B^0\bar{B}^0$ system is thus never independent of $CP$ violation. See also [14].

The $CPT$-violation comparisons in Table 3 of [12] neither depend on $T_{\text{CPT}}$ nor on $T_T$, and are thus, contrary to the authors' intentions, unable to detect $CPT$ symmetry violation in the matrix $T$.

Nevertheless, the measured frequency distributions $\{2a\}$ to $\{2d\}$ show a dominant $\sin(\Delta m t)$ time-dependence, meaning, for this reason, that $T_{\text{CPT}} \approx 0$, and with the previous knowledge about the vanishing of the $q_i$, that $T_T \neq 0$, i.e. $T$-symmetry violation is confirmed. (More combinations are discussed in [5]). In the lower part of Table 3, we indicate rate combinations which are true signatures of $T$- or $CPT$- symmetry violations.

### 4 Conclusion

The experiments [6] and [7] have discovered $CP$ violation in the $B^0\bar{B}^0$ system. Our analysis shows that this $CP$ violation is dominantly $T$ violation, with the same statistical significance. Furthermore, their data sets contain the information which allows for the estimation of all symmetry-violating parameters indicated in Table 1. $CP$ symmetry of the matrix $A$, which governs the $B^0\bar{B}^0$ oscillation, is confirmed.

The novel definitions of the symmetries ($CP, T, CPT$) used by [12,20] are more restrictive than the classical ones [1].

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