 Supernova electron capture rates for $^{55}$Co and $^{56}$Ni

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We have calculated the Gamow-Teller strength distributions for the ground states and first excited states in $^{55}$Co and $^{56}$Ni. These calculations have been performed by shell model diagonalization in the $pf$ shell using the KB3 interaction. The Gamow-Teller distributions are used to calculate the electron capture rates for typical presupernova conditions. Our $^{55}$Co rate is noticeably smaller than the presently adopted rate as it is dominated by weak low-lying transitions rather than the strong Gamow-Teller (GT) resonance which is located at a higher excitation energy in the daughter than usually parametrized. Although our $^{56}$Ni rate agrees with the presently adopted rate, we do not confirm the conventional parametrization of the GT centroid. Our results support general trends suggested on the basis of shell model Monte Carlo calculations.

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I. INTRODUCTION

The core of a massive star becomes dynamically unstable when it exhausts its nuclear fuel. If the core mass exceeds the appropriate Chandrasekhar mass, electron degeneracy pressure cannot longer stabilize the center and it collapses. As pointed out by Bethe et al., the collapse is very sensitive to the entropy and to the number of leptons per baryon, $Y_e$. In the early stage of the collapse, $Y_e$ is reduced as electrons are captured by Fe peak nuclei. Knowing the importance of the electron capture process, Fuller et al. (usually called FFN) have systematically estimated the rates for nuclei in the mass range $A = 45 - 60$ putting special emphasis on the importance of capture to the Gamow-Teller (GT) giant resonance. The GT contribution to the rate has been parametrized by FFN on the basis of the independent particle model. To complete the FFN rate estimate, the GT contribution has been supplemented by a contribution simulating low-lying transitions. Recently the FFN rates have been updated and extended to heavier nuclei by Aufderheide et al. These authors also considered the well-known quenching of the Gamow-Teller strength by reducing the independent particle estimate for the GT resonance contribution by a common factor of two.

After experimental (n,p) data clearly indicated that the Gamow-Teller strength is not only quenched (usually by more than a factor 2 compared to the independent particle model), but also fragmented over several states at modest excitation energies in the daughter nucleus, the need for an improved theoretical description has soon been realized. These studies have been performed within the conventional shell model diagonalization approach, however, in strongly restricted model spaces and with residual interactions, which turned out to neither reproduce the quenching nor the position of the GT strength sufficiently well. These model studies therefore had only a limited value, as they required experimental input informations, and they had no predictive power. This situation changed recently as the development of the shell model Monte Carlo technique (SMMC) allows calculations of the GT strength distribution in the complete $pf$ shell. In fact, using the KB3 interaction, it has been demonstrated that both the quenching of the GT strength and its distribution can be well reproduced. Using the SMMC method, Dean et al. have recently calculated electron capture rates for several Fe peak nuclei of importance at the early stage of the presupernova collapse. This calculation indicated systematic differences in the location of the main GT resonance strength compared to the parametrization of FFN. In capture on even-even nuclei the GT strength resides at lower excitation energies in the daughter than assumed by FFN, while in odd-A nuclei the GT strength is centered at higher excitation energies.

The same trend is also seen in the available (n,p) data and has been pointed out for individual cases in .

Ref. demonstrates that, for even-even parent nuclei, the electron capture rates are given by the bulk of the Gamow-Teller strength distribution, which resides at low excitation energies in the daughter, and is well reproduced by the SMMC method. The situation is quite different in odd-A nuclei. Here the bulk of the GT strength is at a too high excitation energy to be of significance for the electron capture rates which are dominated by weak low-lying transitions. Unfortunately the SMMC method is not capable of spectroscopy and does not allow to extract these weak transition strengths. These informations, however, can be obtained from shell model diagonalizations techniques which have made significant progress in the last couple of years to allow now for basically complete $pf$ shell diagonalizations for low-lying states in the $A = 56$ mass range.

Aufderheide et al. have ranked the core nuclei with respect to their importance for the electron capture process in the presupernova . As the two most important nuclei these authors identified $^{55}$Co and $^{56}$Ni for the early
presupernova collapse. Both rates, however, should be strongly affected by the misplacement of the GT resonance position. A new estimate for the capture rate on $^{56}$Ni has already been given in \[20\] based on an SMMC calculation, however, this rate has been corrected by the authors due to possible over-binding effects at the $T = 28$ shell closure in their approach. Due to their importance a calculation of the two rates on the basis of a shell model diagonalization approach seems to be quite useful. We have performed such a calculation for the $^{54}$Ni ground state and the three lowest states in $^{56}$Co using the KB3 interaction $^{[4]}$ and making use of the state-of-the-art diagonalization code ANTOINE $^{[22]}$.

It is well known that $0\hbar \omega$ shell model calculations, i.e. calculations performed in one major shell, overestimate the GT strength by a universal factor $(1.26)^2$ $^{[23,24]}$, often interpreted as a renormalization of the axialvector coupling constant $g_A$ in nuclei. To account for this fact, we have used the renormalized value $g_A = 1$ in the following.

For $^{56}$Ni we have calculated the total GT strength in a full pf shell model calculation, resulting in $B(GT) = g_A^2 |\langle \sigma_{1+} \rangle|^2 = 10.1 g_A^2$ (see also $^{[21]}$), which is in agreement with the SMMC value ($B(GT) = (9.8 \pm 0.4) g_A^2$ $^{[17]}$). The independent particle model yields $B(GT) = 13.7 g_A^2$. The GT strength distribution has been calculated in a model space which allowed a maximum of $6$ particles to be excited from the $f_7/2$ orbital to the rest of the pf-shell in the final nucleus, $^{56}$Co. The $m$-scheme dimension of this calculation is $19831538$. In this truncated calculation we obtain a total GT strength of $10.2 g_A^2$, indicating our calculation is almost converged at this truncation level (see also $^{[23]}$). For $^{55}$Co we have calculated the total GT strength and the distribution in a truncated calculation which fulfills the Ikeda sum rule and in which maximally $5$ particles in the final nucleus are allowed to be excited out of the $f_7/2$ orbital. We obtain a total GT strength of $8.7 g_A^2$ from the ground state of $^{55}$Co, and $8.9 g_A^2$ from both of the excited $J = 3/2$ states. The values are to be compared with the independent particle value of $B(GT) = 12 g_A^2$. We note that for both, $^{55}$Co and $^{56}$Ni, the quenching factor is unusually small due to the shell closure at $^{56}$Ni.

We have performed 33 Lanczos iterations which are usually sufficient to converge in the states at excitation energies below $E = 3$ MeV. At higher excitation energies, $E > 3$ MeV, the calculated GT strengths represent centroids of strengths, which in reality are split over many states. For calculating the electron capture rate, however, a resolution of this strength at higher energies is unimportant.

The GT strength distributions $S_{GT}(E)$ for the lowest states in $^{55}$Co ($J = 7/2$ ground state and the two $J = 3/2$ excited states at $E = 2.165$ MeV and $2.565$ MeV, respectively) and for the $^{56}$Ni ground state are shown in Figs. 1 and 2. The energy scale in these figures has been adjusted such that the lowest calculated state of a given angular momentum agrees with the experimentally known excitation energy; the necessary energy shifts have been less than 300 keV as the shell model calculations reproduce the low-lying spectrum in $^{55}$Fe and $^{56}$Co rather well. We observe that for the $^{55}$Co ground state the GT centroid resides at about $E = 6$ MeV in the daughter $^{55}$Fe, while it is at around $E = 9 - 10$ MeV for the excited states. This result is in agreement with the SMMC study (performed at temperature $T = 0.8$ MeV) which found the centroid of the GT strength at $E = 6.9$ MeV in $^{55}$Fe $^{[9]}$.

To estimate the electron capture rates at finite temperatures, the compilations employed the so-called Brink hypothesis $^{[11]}$, assuming that the GT strength distribution on excited states is the same as for the ground state, only shifted by the excitation energy of the state. For $^{55}$Co this assumption is roughly valid for the bulk of the strength, but it is clearly not justified for the low-lying transitions which are dominated by the individual structures of the states involved. While the $^{55}$Co ground state has rather weak GT transitions to low-lying states in $^{55}$Fe, the first excited $J = 3/2$ state has a strong transition to the $^{55}$Fe ground state (and first excited state).

The quality of our calculation can be tested by calculating the lifetime of $^{55}$Co under terrestrial conditions where it decays by $\beta_+\text{-decay}$. Using the GT matrix elements as calculated in our shell model approach and the experimental energy splittings we calculate a $^{55}$Co lifetime of 16.7 hours, which compares nicely with the experimental value of 17.53 hours.

Under presupernova conditions the electron capture on $^{56}$Ni is dominated by the ground state as the first excited state is too high in excitation energy. The centroid of the GT strength is around $E = 2.5 - 3$ MeV in $^{56}$Co, in agreement with the SMMC estimate given in Ref. $^{[19]}$. For comparison, FFN placed the GT resonance in $^{56}$Co at $E = 3.8$ MeV. For the lifetime of $^{56}$Ni we find 6.7 d, very close to the experimental value of 6.08 d.

The presupernova electron capture rate $\lambda_{ec}$ is given by $^{[34]}$

$$\lambda_{ec} = \frac{\ln 2}{6163 \text{sec}} \sum_{ij} \frac{(2J_i + 1) \exp \left[-E_i/kT\right]}{G} F(Z, E_c) \left(\frac{c^2}{m_e c^2}\right)^3 \left(\int_{E}^{\infty} dp^2 \left(Q_{ij} + E_c\right)^2 \frac{F(Z, E_c)}{1 + \exp \left[\beta_e \left(E_c - \mu_e\right)\right]}\right),$$

(1)

where $E_c$, $p$, and $\mu_e$ are the electron energy, momentum, and chemical potential, and $G = (Q_{ij}^2 - m_e^2 c^4)^{1/2}$ for $Q_{ij} \leq -m_e c^2$, and 0 otherwise. $Q_{ij} = E_i - E_f$ is the nuclear energy difference between the initial and final states, while $S_G$ is their GT transition strength. $G$ is the partition function, $G = \sum_i (2J_i + 1) \exp \left[-E_i/kT\right]$. The Fermi function $F(Z, E_c)$ accounts for the distortion of the electron’s wave function due to the Coulomb field of the nucleus.

The calculated electron capture rates for $^{55}$Co and $^{56}$Ni are shown in Figs. 3 and 4 as function of temperature ($T_\beta$ measures the temperature in $10^9$ K) and for selected
Due to the ranking given in Ref. [4], 55\text{Co} and 56\text{Ni} are the most important electron capture nuclei at temperatures and densities around $T_9 = 3.26$ and $\rho T = 4.32$. Under these conditions the recommended capture rates (the FFN rates are in parenthesis) for 55\text{Co} and 56\text{Ni} are $\lambda = 5.1 \cdot 10^{-2}$ s$^{-1}$ $(8.4 \cdot 10^{-2}$ s$^{-1})$ and $8.6 \cdot 10^{-3}$ s$^{-1}$ $(7.5 \cdot 10^{-3}$ s$^{-1})$, respectively, while our calculation yields $1.6 \cdot 10^{-3}$ s$^{-1}$ for 55\text{Co} and $12.6 \cdot 10^{-3}$ s$^{-1}$ for 56\text{Ni}. For 56\text{Ni} the three rates agree rather well, but this agreement is more or less accidental as a closer inspection shows. In the calculation of [4] the dominant contribution comes from the transition to the low-lying states (simulated for all nuclei by an effective $B(\text{GT})=0.1$ for a transition to a fictitious state at $E=0$), while only 18% originates from the GT resonance placed at $E=3$ MeV. We, however, find that nearly 50% of the capture rate is due to the strong transition to the GT resonance, which in our calculation is located nearly 1 MeV lower in excitation energy than parametrized in Refs. [4][5]. Our 56\text{Ni} rate also approximately agrees with the SMMC estimate of Ref. [19].

As already suggested in [19] the recommended rate for 55\text{Co} is too large (by more than an order of magnitude), as the authors of [4][5] placed the GT resonance at too low an excitation energy and consequently assigned 73\% and 83\%, respectively, to this transition. In contrast, our calculated rate is predominantly given by transitions to the low-lying states. In fact, we recover about 80\% of the rate if we cut the calculated GT spectra at the excitation energy $E=3$ MeV. We also note that the 55\text{Co} rate arises mainly from capture on the ground state at temperatures $T_9 \approx 3.3$ (which are typical for presupernova electron capture on 55\text{Co}), while, due to the rather large excitation energies, contributions from capture on the excited states amount to less than 5\% under the relevant presupernova conditions.

In summary, we have performed state-of-the-art large-scale shell model diagonalization calculations to determine the presupernova electron capture rates on 55\text{Co} and 56\text{Ni}, which are believed to be the most important “electron poisons” at the onset of collapse. Although our calculation approximately agrees with the recommended rate for 56\text{Ni}, it does not confirm the parametrization conventionally used to derive at these rates. Our calculation finds the bulk of the GT strength distribution at around 1 MeV lower in excitation energy than assumed in the parametrization. As has already been noted before this trend seems to be general for even-even parent nuclei, while for odd-A nuclei the parametrization places the GT centroid at too low excitation energies. This suggestion [3] is confirmed in our shell model calculation. In fact we find that the centroid is too high in excitation energy to affect noticeably the electron capture rate under presupernova conditions. Consequently our calculated rate is more than one order of magnitude smaller than the compiled rates [4][5].

What are the consequences for the presupernova collapse? At the onset of collapse ($\rho T \approx 4.3$) the change of $Y_e$ with time, $dY_e/dt$, has been assumed to be predominantly due to electron capture on 55\text{Co} (50\%) and 56\text{Ni} (25\%). With our revised rates, 56\text{Ni} becomes the dominant source for electron capture and $dY_e/dt$ is reduced by nearly a factor of 2. However, for firm conclusions it appears to be reasonable to first update the electron capture rates on all important nuclei and then to perform a simulation of the presupernova collapse. Such a program is in progress.

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FIG. 1. Gamow-Teller strength distributions for the $^{55}$Co ground state (top panel), the $J = 3/2$ excited state at $E = 2.165$ MeV (middle panel) and the $J = 3/2$ excited state at $E = 2.565$ MeV (lower panel). For comparison the parametrized GT spectrum assumed in [4] is indicated by stars in the top panel. This parametrization assumed a fictitious state at $E = 0$ and the GT resonance with a strength half of the independent particle model value. The energy scale refers to excitation energies in the daughter nucleus, where we have shifted the calculated energies as to match the lowest experimentally known for a given angular momentum.

FIG. 2. Gamow-Teller strength distribution for the $^{56}$Ni ground state. For comparison the parametrized GT spectrum assumed in [4] is indicated by stars (see Fig. 1). The energy scale refers to excitation energies in the daughter nucleus, where we have shifted the calculated energies as to match the lowest experimentally known $J = 1$ state in $^{56}$Co.

FIG. 3. Electron capture rates on $^{55}$Co as function of temperature and for selected densities.

FIG. 4. Electron capture rates on $^{56}$Ni as function of temperature and for selected densities.