Shell-model study of single-neutron strength fragmentation in $^{137}$Xe

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Abstract

We have performed shell-model calculations for the nucleus $^{137}$Xe, which was recently studied experimentally using the $^{136}$Xe($d$, $p$) reaction in inverse kinematics. The main aim of our study has been to investigate the single-neutron properties of the observed states, focusing attention on the spectroscopic factors. We have employed a realistic low-momentum two-body effective interaction derived from the CD-Bonn nucleon-nucleon potential that has already proved quite successful in describing the spectroscopic properties of nuclei in the $^{132}$Sn region. Comparison shows that our calculations reproduce very well the experimental excitation energies and yield spectroscopic factors that come close to those extracted from the data.

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In a recent paper [1], the single-neutron structure of the nucleus \(^{137}\text{Xe}\) was investigated at Argonne National Laboratory via the \(^{136}\text{Xe} (d,p)\) reaction in inverse kinematics. In particular, spectroscopic factors for several states in \(^{137}\text{Xe}\) were extracted from the data by DWBA calculations, providing valuable information on the evolution of single-neutron states outside \(^{132}\text{Sn}\), which is a subject of great current interest [2, 3].

Actually, the \((d,p)\) reaction on \(^{136}\text{Xe}\) was first studied more than 40 years ago in normal kinematics [4, 5]. Some twenty years later, a new study of this reaction was performed at GSI in inverse kinematics [6] to test the effectiveness of this method as a tool for the study of nuclei far from stability with transfer reactions. This was also a main motivation for the experiment of [1], which provided a stringent test of the HELIOS spectrometer used to analyze the outgoing protons. In this experiment the beam of \(^{136}\text{Xe}\) was delivered at an energy about twice as high as that used in [6], which implies larger cross sections for transfer to high-\(l\) states.

In other words, the contribution of [1] is two-fold. On the one hand it adds to our understanding of the single-particle structure of \(^{137}\text{Xe}\). On the other hand, it helps pave the way to perform transfer reactions with highly unstable nuclei, which is a major goal of the next generation of radioactive ion beam facilities. This has stimulated the present study, which aims both to interpret the data available for \(^{137}\text{Xe}\) and encourage further experimental efforts.

Over the past several years we have performed various shell-model studies [7, 8] of neutron-rich nuclei beyond \(^{132}\text{Sn}\), which have all yielded results in very good agreement with experiment. In these studies, a unique Hamiltonian has been used with the single-particle energies taken from experiment and the two-body effective interaction derived from the CD-Bonn nucleon-nucleon (\(NN\)) potential [9] without using any adjustable parameter.

The new findings mentioned above have led us us to make use of this Hamiltonian to investigate the single-neutron properties of \(^{137}\text{Xe}\). It should be mentioned here that in a previous work [10] we have performed a study of this nucleus focusing attention on high-spin states produced in the spontaneous fission of \(^{252}\text{Cf}\).

In our calculations, we assume that the four valence protons outside doubly magic \(^{132}\text{Sn}\) occupy the five orbits 0\(g_{7/2}\), 1\(d_{5/2}\), 1\(d_{3/2}\), 0\(h_{11/2}\), and 2\(s_{1/2}\) of the 50-82 shell while the odd neutron has available the six orbits 1\(f_{7/2}\), 2\(p_{3/2}\), 2\(p_{1/2}\), 0\(h_{9/2}\), 1\(f_{5/2}\), and 0\(i_{13/2}\) of the 82-126 shell. We take as single-proton and -neutron energies the experimental energies of the levels
with corresponding spin and parity in $^{133}$Sb and $^{133}$Sn [11], respectively. There are, however, two exceptions since no $1/2^-$ state in $^{133}$Sb and $13/2^+$ state in $^{133}$Sn has yet been observed. Therefore, we take the proton $2s_{1/2}$ and the neutron $0i_{13/2}$ energies from Refs. [12, 13], respectively, where it is discussed how they are determined. It should be noted that we place the neutron $2p_{1/2}$ level at 1.363 MeV excitation energy, which is about 300 keV lower than the previous proposed value [15]. This new value comes from the recent work of Ref. [14], where the short-lived $^{133}$Sn has been studied by performing the $^{132}$Sn($d,p$) reaction in inverse kinematics. It is worth pointing out here that this experiment has provided clear evidence for the purity of the $7/2^-$, $3/2^-$, $1/2^-$ and $5/2^-$ single-neutrons states.

The two-body effective interaction has been derived within the framework of perturbation theory [16, 17] starting, as mentioned before, from the CD-Bonn NN potential renormalized by way of the $V_{\text{low-k}}$ approach [18]. More precisely, we start by deriving $V_{\text{low-k}}$ with a cutoff momentum $\Lambda = 2.2$ fm$^{-1}$. Then, using this potential plus the Coulomb force for protons, we calculate the matrix elements of the effective interaction by means of the $\hat{Q}$-box folded-diagram expansion, with the $\hat{Q}$-box including all diagrams up to second order in the interaction. These diagrams are computed within the harmonic-oscillator basis using intermediate states composed of all possible hole states and particle states restricted to the five proton and neutron shells above the Fermi surface. The oscillator parameter is 7.88 MeV, as obtained from the expression $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$ with $A = 132$. The calculations have been performed by using the OXBASH code [19].

In Table I we report the calculated excitation energies and spectroscopic factors and compare them with those obtained from the $(d,p)$ experiment of Ref. [1]. We only consider experimental states up to the $13/2^+$ one at 1.751 MeV. In the higher-energy region we did not make any attempt to identify the observed states with the calculated ones. In fact, the available experimental information is very scanty while a large number of states with spin and parity corresponding to those of the single-neutron orbits are predicted by the theory. It should be mentioned that all the states reported in Table I were previously known [11], except the second $9/2^-$ state at 1590 keV and the $13/2^+$ state at 1751 keV, the spin-parity assignment to the latter being in accord with a previously unpublished assignment, as mentioned in [1]. Note that the two states at 0.986 and 1.534 MeV reported in [1] with ambiguous spin assignments have been identified as $1/2^-$ and $5/2^-$ states in other experiments [11]. In this regard, it is worth remarking that they are likely to correspond to
the $1/2^-$ and $5/2^-$ states at 0.91 and 1.41 MeV observed in \cite{5}. The discrepancies between the excitation energies measured in the two experiments may be due to the low precision of energy determination in the earlier one \cite{11}.

The experimental energies are well reproduced by the theory, with discrepancies not exceeding 150 keV, the only exception being the $13/2^+$ state, whose energy is overestimated by about 330 keV. This state is found to be essentially of single-particle nature and therefore its energy is quite sensitive to the position of the $0i_{13/2}$ neutron orbit, which, as mentioned above, is not experimentally known. Our calculations confirm the $1/2^-$ assignment for the state at 0.986 MeV, the second $3/2^-$ state being predicted at a much higher energy (about 1.8 MeV). As for the level at 1.534 MeV, we cannot discriminate between $J^\pi = 5/2^-$ and $7/2^-$, the calculated states lying both close to this energy. We will come back to this point later.

Let us now turn to the spectroscopic factors. In our discussion, we find it appropriate to consider also the early results of Ref. \cite{5}. In fact, in that work the spectroscopic factors included in Table I, but those of the $13/2^+$ and the second $9/2^-$ states, are reported and their values are on the whole not very different from those of \cite{1}. The main differences relate to the spectroscopic factors of the ground $7/2^-$ and yrast $9/2^-$ states, the values of \cite{1} (see Table I) being substantially larger than those of \cite{5}, 0.68 and 0.31, respectively.

As can be seen in Table I, an overall good agreement is found between the calculated spectroscopic factors and those obtained in Ref. \cite{1}, most of the theoretical values differing from the experimental ones by no more than 23%. In particular, our value for the ground state, 0.86, gives support to that obtained in \cite{1}. However, the calculated spectroscopic factor of the yrast $9/2^-$ state largely overestimates the value of both \cite{5} and \cite{1}. As a matter of fact, we underestimate the fragmentation of the $9/2^-$ strength. We predict, indeed, a second $9/2^-$ state with essentially no single-particle strength, its spectroscopic factor being 0.01, to be compared with the experimental value of 0.24 \cite{1}. It should be noted, however, that no evidence for this state was found in \cite{5}.

The weak fragmentation of the $9/2^-$ strength resulting from our calculations may be due to a somewhat too strong neutron-proton interaction. In fact, if one ignores the neutron-proton interaction, then the single-particle strength is completely concentrated in the second $9/2^-$ state, lying 250 keV above the yrast one. This may be easily seen when writing the first and second $9/2^-$ unperturbed states as $|^{136}\text{Xe g.s.; } \nu h_{9/2} >$ and $|^{136}\text{Xe } 2^+; \nu f_{7/2} >$, \cite{4}.
respectively.

The spectroscopic factor of the $5/2^-, 7/2^-$ level (identified as $5/2^-$ in \cite{5} with $C^2S = 0.16$) is underestimated by the theory, either if this level is associated with the calculated $7/2^-$ or the $5/2^-$ state. However, based on the small spacing, about 75 keV, predicted between these two states, one cannot exclude that the peak observed at 1.534 MeV, as well as that at 1.41 MeV in \cite{5}, consists of two unresolved levels, the extracted spectroscopic factor corresponding to the sum of their spectroscopic factors.

As for the $13/2^+$ states, only the yrast one has been observed in \cite{1}. However, based on the work of Ref. \cite{20}, the authors of \cite{1} give an estimate of the energy and spectroscopic factor of the second $13/2^+$ state, 3360(110) keV and 0.15(4), respectively, which is consistent with a $13/2^+$ assignment to either of the two observed peaks at 3.310 and 3.470 MeV. We predict a second $13/2^+$ state at 3.452 MeV with a spectroscopic factor of 0.01.

As a general comment, we may say that the predicted single-particle strength, in agreement with experiment, is strongly concentrated in the yrast states only for $J^\pi = 7/2^-$ and $13/2^+$. This is not the case for the $J^\pi = 1/2^-, 3/2^-$ and $5/2^-$ states. However, we find that for the first two angular momenta the spectroscopic factors of the yrast states are still the largest ones with the single-particle strength mainly distributed over the lowest-lying states. In fact, it is sufficient to sum up to an energy of 2 MeV, corresponding to the first three and four lowest-lying states, respectively, to obtain more than 75% of the strength. The situation is quite different for $J^\pi = 5/2^-$. In this case, by summing over the first 20 states, namely up to 3 MeV, we get only 65% of the strength, the largest spectroscopic factor being 0.20 for the 5th excited $5/2^-$ state at 2.039 MeV.

It is now interesting to discuss our results for the centroids of the $0i_{13/2}$ and $0h_{9/2}$ neutron orbits. We find that the energy difference between these two centroids is 0.8 MeV, with a decrease of about 0.3 with respect to the $^{133}$Sn initial value, which is in agreement with the results reported in \cite{1}. To understand the reason for the reduction in the separation of these two orbits, it is convenient to write \cite{21, 22} the energy centroid as

\begin{equation}
\bar{\epsilon}_{j\nu} = \epsilon_{j\nu} + \sum_{j\pi} V^M(j\nu j\pi) N_{j\pi},
\end{equation}

where $\epsilon_{j\nu}$ is the energy of level $j\nu$ in $^{133}$Sn, $N_{j\pi}$ denotes the number of protons occupying the $j\pi$ orbit in the even-even system and $V^M(j\nu j\pi)$ the monopole component of the interaction.
By examining the various terms of Eq. (1), it may be seen that the lowering of the 0\(i_{13/2}\) with respect to the 0\(h_{9/2}\) orbit stems from the fact that the value of \(V^M(\nu i_{13/2} \pi g_{7/2})\), -0.33 MeV, is larger than that of \(V^M(\nu h_{9/2} \pi g_{7/2})\) by a factor of about 2. In fact, the components which play a dominant role in the sum are only those involving the \(\pi g_{7/2}\) orbit, owing to the large occupancy of the latter, 3.1. The remaining proton occupancy, 0.9, is distributed between the 1\(d_{5/2}\) and 0\(h_{11/2}\) orbits and does not make a significant contribution. It is worth mentioning that our calculated proton occupancies for the ground state of \(^{136}\text{Xe}\) are in very good agreement with those obtained from pick-up and stripping reactions \([23]\).

We may therefore say that the monopole components of our realistic effective Hamiltonian account for the reduction in the separation of the 0\(i_{13/2}\) and 0\(h_{9/2}\) orbits. It is relevant to point out that this reduction has been interpreted as an effect of the neutron-proton tensor force \([1, 24]\), which makes us confident in the reliability of the tensor component of our interaction.

In summary, we have given here a shell-model description of \(^{137}\text{Xe}\), focusing attention on the single-neutron structure recently studied through the \(^{136}\text{Xe}(d, p)\) reaction. Our shell-model effective interaction has been derived from the CD-Bonn \(NN\) potential without using any adjustable parameter, in a way completely consistent with our previous studies of other nuclei in the \(^{132}\text{Sn}\) region.

We have obtained a very good agreement for both the measured excitation energies and the spectroscopic factors extracted from the data. In particular, the fragmentation of the single-neutron strength is well reproduced. These results make us confident in the predictive power of our effective interaction and provide motivation for a similar study of other nuclei around \(^{132}\text{Sn}\) which are within reach of \((d, p)\) transfer reactions with radioactive ion beams in inverse kinematics. This study is currently under way.

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TABLE I: Calculated energies and spectroscopic factors for states in $^{137}$Xe compared with those obtained from the experiment of Ref. [1] (see text for details).

| J$^\pi$ | E(MeV) | $C^2 S$ | J$^\pi$ | E(MeV) | $C^2 S$ |
|---------|--------|---------|---------|--------|---------|
| 7/2$^-$ | 0.000  | 0.94    | 7/2$^-$ | 0.000  | 0.86    |
| 3/2$^-$ | 0.601  | 0.52    | 3/2$^-$ | 0.728  | 0.57    |
| 1/2$^-$,3/2$^-$ | 0.986 | 0.35 | 1/2$^-$ | 1.127  | 0.43 |
| 9/2$^{-}$ | 1.218 | 0.43 | 9/2$^{-}$ | 1.327 | 0.72 |
| 5/2$^-$ | 1.303 | 0.22 | 5/2$^-$ | 1.349  | 0.17    |
| 5/2$^{-}$,7/2$^{-}$ | 1.534 | 0.12 | 7/2$^{-}$ | 1.589 | 0.05 |
| (9/2$^-$) | 1.590 | 0.24 | 9/2$^{-}$ | 1.584 | 0.01 |
| (13/2$^+$) | 1.751 | 0.84 | 13/2$^+$ | 2.082 | 0.75 |