The Capacity Load Model of K-Uniform Hyper-Network based on Equal Load Distribution

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Abstract. Cascading failures occur frequently in many network systems. In this paper, we propose a hyper-network Capacity Load Model based on hyper-edge degrees load to explore the influence of cascading failures on some $K$-uniform hyper-networks. In simulation, we attack the hyper-edge which has largest hyper-edge degree. When the hyper-edge fails, its load will be equally distributed to adjacent hyper-edges. Through simulation analysis, we get the relationship between the capacity-load model parameters and the robustness of the $K$-uniform hyper-network. The results show the robustness of the hyper-networks is enhanced with the increase of its hyper-edge capacity. In addition, simulation result show that the robustness of $K$-uniform hyper-networks is strongest at $\alpha=1$.

1. Introduction

With the progress of science and technology, network is becoming more and more important in human life [1,2]. The study of complex network structure and dynamics properties, makes people have a full understanding of complex network. However, in some cases, general network diagrams cannot clearly describe the characteristics of real-world networks. For example, in a network of co-authoring papers [3,4], a simple graph can describe whether there is a cooperative relationship between authors, but it cannot describe whether more authors participate in the writing of an article. Before dealing with this kind of problem, it is usually solved by a bipartite graph method. A set of disjoint points represents the paper, and the other part of the point set that does not want to cross represents the author, but this is easy to cause ambiguity. Using the hyper-graph based hyper-network, the hyper-edge represents the article and the node represents the author, which can clearly indicate the cooperative relationship between scientists and the number of participants in the paper. Hyper-networks theory based on hyper-graph has become an effective tool to study dynamic properties of all kinds of network system. In recent years, for the better show the properties of complex systems, researcher usually use hyper-networks to describe real complex system.

In recent years, more and more researchers have devoted themselves to the study of hyper-networks. [5]. Peng et al. [6] proposed a method based on hyper-network to clearly capture the relationship between various elements in a multi-faceted knowledge representation. Specifically, a knowledge hyper-network model composed of designer network, product network, problem network and knowledge unit network is constructed. Based on Newman and Watts small world network, Li et al [7]. proposed a memritor-based echo state network (MESN), which provided a new method for studying the design of neural morphological computing system.Zhou et al. [8] constructed a new
evolutionary hyper-network, and the analysis results show that the evolutionary hyper-network obeys the generalized power-law distribution. The hyper-network showing “rich get richer” Matthew effect and extensive universality. Wang et al. [9] proposed an improved knowledge diffusion hyper-network (IKDH) model, which is based on the idea of knowledge spreading from the target node to all neighboring nodes along the hyper-edge and knowledge stock. Through experiments, they found that closer or smaller networks have faster knowledge diffusion.

Recently, a series of major emergencies have frequently endangered human society. For example, the blackout in North America, the Indian tsunami, SARS, and the Wenchuan earthquake. Although major emergencies bring limited harm to society, major emergencies Once the event spreads, it will trigger a series of chain reactions, and then cause serious harm to society. The security of complex networks has attracted wide attention. Cascading failure can reduce the stability and security of complex networks, and will bring serious harm. Therefore, many researchers are focusing on cascading failure of complex network [10,11]. In a hyper-network, nodes or hyper-edges are failure due to overloaded. Furthermore, due to the coupling relationship between nodes or hyper-edges, the load of the failure nodes or hyper-edges are redistributed to other normal nodes or hyper-edges, cause new failure nodes and hyper-edges. Eventually, the entire hyper-network could collapse. This process is called cascading failure. Cascading failures often occur in many actual network systems. If a large-scale cascading failure occurs, it will have a huge impact on the entire hyper-network. Therefore, the study of cascading failure of hyper-network becomes more and more important.

Researchers have also proposed many models to analyze cascading failure behaviors of real hyper-network. [12,13]. For instance, a Couple Map Lattice (CML) model [8], OPA model [14] etc.

The Capacity-Load model is an important cascading failure model to analyzed the cascading failure behavior [15,16]. However, there is no the relevant Capacity-Load cascading model to analyzed the cascading failure of hyper-network.

In this paper, we propose a Capacity-Load Model based on equal distribution under two K-uniform hyper-network topological structures. We analyzed the cascading failure process in K-uniform random hyper-network and K-uniform small world hyper-network by the Capacity-Load Model. We obtain the robustness of two type hyper-networks. Besides, we obtain the influence of the load adjustable parameters $α$ on the hyper-network robustness.

2. The Capacity Load Model of Hyper-Network Based on the Equal Load Distribution

In the initial hyper-network, certain load and capacity are assigned to the hyper-edge. When a hyper-edge is damaged due to an external attack, the failed hyper-edge distributes its load to other adjacent hyper-edges in the manner of equal load distribution. The adjacent hyper-edge receives additional load if the total load exceeds it's. When capacity is reached, redistribution of the load will occur, and secondary distribution will cause some of the excess hyper-edges where the load exceeds their capacity will also fail. Over time, failure can spread throughout the hyper-network and cause avalanches [17].

Here we focus on cascading failures caused by the removal of a single hyper-edge. If the load of a hyper-edge is relatively small, the removal of it may not affect the load balance significantly, let alone cause subsequent hyper-network avalanches. On the contrary, when the hyper-edge with a large load is deleted, it may greatly affect the load balance in the hyper-network, and then a series of overload failures will start, eventually making a large hyper-network system disconnected or performance degraded.

2.1. The Initial Hyper-Edge Load

In our hyper-network, the difference from the previous models is that in our hyper-network, the hyper-edge $i$ is assigned by the Hyper-edge Degrees [18] $K_i$, and the initial load is $L_i$: 

$$L_i = \alpha K_i^\beta$$

where $\alpha$ is load parameters and $\beta$ is adjustable parameters, in order to control the initial load strength
of hyper-edge \( i \). This assumption is reasonable. For example, each hyper-edge on a scientist's cooperative network is usually correlated to its hyper-degree.

2.2. The Redistribution of Load
The load on the failures hyper-edge \( i \) can be redistributed to the adjacent hyper-edge \( j \) in the manner of uniform probability. The uniform probability is equal distributed according to the number of adjacent hyper-edges:

\[
\prod_j = \frac{1}{M_i}
\]

(2)

where \( M \) is the number of adjacent hyper-edges of failures hyper-edge \( i \). Similarly, from Equation (2), it can be seen that hyper-edge \( j \) receives extra load \( \Delta L_{ij} \) and is proportional to the initial load, i.e.:

\[
\Delta L_{ij} = \frac{1}{M_i} L_i
\]

(3)

In the actual hyper-network, the capacity of hyper-edge is limited, so we set the threshold \( C_j \) for hyper-edge \( j \) [19,20]:

\[
C_j = (1+T)L_j \quad T \geq 0
\]

(4)

where, the constant \( T \) is a threshold parameter [21,22]. The capacity of the hyper-edge is the maximum load that the hyper-edge can handle, which is the capacity of each hyper-edge to handle the load is limited. We define the capacity parameter in Equation (4). Obviously, the capacity parameter \( T \) represents the capacity of hyper-edge handling load. The higher the value of \( T \) is, the higher the threshold will be. It is well known that the critical value \( T_c \) of \( T \) is an important index to measure hyper-network resistance to cascade failure. When \( T > T_c \), the entire hyper-network will not appear cascading failure, while in \( T < T_c \), the case reflects the occurrence of the crash and cascading phenomenon of the entire hyper-network. Therefore, critical point \( T_c \) is the minimum safe capability to avoid global cascading failure. Obviously, the less the value of \( T_c \) is, the more robust the hyper-network will be the cascade.

\[
L_i + \Delta L_{ij} > C_j
\]

(5)

If Equation (5) is true, the hyper-edge \( j \) will fail and further cause the redistribution of the load \( L_i + \Delta L_{ij} \), and may further cause the failure of other hyper-edges.

In order to measure the effect of robustness on cascading failure of the entire hyper-network, we initially select a hyper-edge \( i \) to delete and distribute its load. The rest of the hyper-edges continue to evolve in the case that Equation (5) is satisfied. We defined a \( FNF_i \) (\( FNF_i \) represents the avalanche size, i.e. the number of disconnected hyper-edges caused by deleting hyper-edge \( i \) in the cascade process), and \( 0 \leq FNF_i \leq N-1 \). In order to quantify the robustness of the entire hyper-network, we adopted the unified avalanche ratio, i.e.

\[
F = \sum_i FNF_i
\]

(6)

\[
FN = \frac{F}{M}
\]

(7)

where \( M \) represents the total number of hyper-edges of the entire hyper-network, and \( F \) represents the total number of failures hyper-edges after the hyper-network evolution is completed. The \( FN \) represents the proportion of the failures of hyper-edges in the whole hyper-network after the
hyper-network evolution is completed and $0 < \text{FN} \leq 1$.

3. The Simulation Results

Based on $K$-uniform random hyper-network and $K$-uniform small world hyper-network, the process of cascading failure is obtained by deliberately attacking the hyper-edge and distributing the load equal. Here we consider how the type of $K$-uniform hyper-network, the number of hyper-edges $M$ and the parameter $\alpha$ affect the robustness of the hyper-network against cascading failure. In order to verify the robustness of different influencing factors to the hyper-network, we designed the following simulation experiment (Table 1). Each datum is averaged over 50 independent realizations.

**Table 1.** Simulation experiment parameters and values.

| Parameter names | Meaning                                      | Value range               |
|-----------------|----------------------------------------------|---------------------------|
| $K$             | Number of nodes in the hyper-edge            | 10                        |
| $M$             | Hyper-edge number                            | $\{1000,1500,2000\}$     |
| $\alpha$        | Load parameter                               | $\{0.8,1,0.1,2\}$        |
| $\beta$         | Adjustable parameter                         | 4                         |
| $L$             | Hyper-edge load                              | --                        |
| $K_{ei}$        | Hyper-edge degrees of hyper-edge $e_i$       | --                        |
| $C$             | Hyper-edge capacity                          | --                        |
| $T$             | Capacity parameter                           | $[1,10]$                  |
| $F$             | Number of failed hyper-edges                 | --                        |
| $\text{FN}$     | Failure hyper-edge ratio                     | $[0,1]$                   |
| $T_c$           | Threshold parameter                          | --                        |

3.1. The Robustness of Two Type $K$-Uniform Hyper-Networks

![Figure 1](image.png)

**Figure 1.** The impact of different types of $K$-uniform hyper-networks on robustness, where the total number of hyper-edges $M=1000$.

When $\text{FN}=1$, the hyper-network crashes globally, and when $\text{FN}<1$, the hyper-network does not crash globally. Figure 1 shows that, the threshold $T_c = 0.2$ for $K$-uniform small world hyper-network and from the small figure we obtain the threshold $T_c = 0.04$ for $K$-uniform random hyper-network. By comparison, we find that when two $K$-uniform hyper-networks have the same number of hyper-edges $M$, the random hyper-network is more robust. In addition, it can be seen that with the increase of capacity parameter $T$, $\text{FN}$ decreases gradually and the robustness is enhanced in the two kinds of hyper-networks.
3.2. Influence of Hyper-Edge Number $M$ on Robustness in Two Type Hyper-Networks

Figure 2. Influence of the number of hyper-edges $M$ in different hyper-networks on the robustness.

From Figure 2, We can see that in $K$-uniform random hyper-network, when $M$ is 1000, 1500, 2000, $T_c$ is 0.07, 0.03, 0.02. Similarly, in $K$-uniform small world hyper-network, when $M$ is 1000, 1500, 2000, $T_c$ is 0.5, 0.4, 0.3. We can conclude that with the increase of the number of hyper-edges $M$, the critical point $T_c$ becomes smaller and smaller, and the robustness of the hyper-network gradually improves.

3.3. Influence of Load Parameter $\alpha$ on Robustness in Two Type Hyper-Networks

Figure 3. Effects of different load parameters $\alpha$ on the robustness of $K$-uniform hyper-networks, where the total number of hyper-edges $M=1000$.

Here we consider the impact of load parameters $\alpha$ on the robustness of the hyper-network. The left panel of Figure 3 show that when the load parameter $\alpha$ is 0.8, 1.0, 1.2, $T_c$ is 0.5, 0.4, 0.3. The right panel of Figure 3 show that when the threshold parameter $\alpha$ is 0.8, 1.0, 1.2, $T_c$ is 0.03, 0.025, 0.035. It can be seen that when $\alpha=1$, the robustness of the two $K$-uniform hyper-networks is the strongest, and $T_c$ is the smallest.

4. Conclusion
To summarize, we propose a capacity-load model based on the $K$-uniform hyper-network topological structure. Based on $K$-uniform random hyper-network and $K$-uniform small world hyper-network, we simulate the process of cascading failure through experiments, and obtain the features of cascade failure by deliberately attacking the hyper-edge. We draw a conclusion that the hyper-network types, number of hyper-edges $M$ and load parameters $\alpha$ have important effects on the robustness of the hyper-network. By comparing the value of the threshold parameter $T_c$, we conclude that under the two $K$-uniform hyper-networks, the random hyper-network is more robust than the small-world hyper-network. With the increase of the number of hyper-edges, the robustness of the hyper-network can be improved. In addition, we find that the robustness of the cascading failure hyper-network is the strongest when the load parameter $\alpha=1$.

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