Meson Exchange and Pion Rescattering Contributions to the Cross Section for 

\[ pp \rightarrow pp\pi^0 \]

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Abstract

Short range meson-exchange mechanisms such as \( \rho \rightarrow \omega\pi^0 \) contribute significantly to the amplitude for \( pp \rightarrow pp\pi^0 \) near threshold in addition to pion rescattering. The uncertainty of the latter contribution, which depends on poorly determined parameters, is estimated. Allowing for this uncertainty and taking the short range meson exchange contributions into account makes it possible to reproduce the measured cross section.
The reaction $pp \rightarrow pp\pi^0$ near threshold is peculiarly intriguing because the relatively large value of the momentum transfer ($\approx 360$ MeV/c) implies a very small impulse approximation term (Fig. 1a). Furthermore, the pion rescattering contribution (Fig. 1b) is strongly suppressed because the dominant low energy pion-nucleon ($\pi N$) scattering amplitude is the isovector Weinberg-Tomozawa term, which cannot contribute to this reaction [1]. The isoscalar pion rescattering term can be evaluated using chiral perturbation theory (ChPT), which relates this contribution to the off-shell $\pi N$ scattering amplitude. Previous calculations [2, 3] based on a $O(q^3)$ determination of this amplitude from the isospin-even scattering length and the sigma-term [4] found that the rescattering term leads to a significant destructive interference with the impulse approximation term. Part of this destructive interference arises from intermediate $\Delta (1232)'s$ (Fig. 1c) [2]. The net result is a near cancellation between the pion rescattering contribution based on the chiral Lagrangian and the amplitude for single-nucleon pion production (this cancellation does not appear in off-shell extrapolations unconstrained by ChPT [5]). As a consequence, the empirical cross section [6, 7, 8] has to be understood in terms of non-pionic two-body mechanisms of shorter range than that associated with pion rescattering.

That shorter-ranged mechanisms should play a significant role in the reaction $pp \rightarrow pp\pi^0$ was already implied by the demonstration that the single-nucleon and the pion-exchange amplitude as conventionally calculated with the (now ruled out) on-shell approximation for the $\pi N$ scattering amplitude predict a cross section smaller than the measured one by about a factor of 5 [4, 10]. Subsequently it was found that the effective scalar and vector exchange components of the nucleon-nucleon ($NN$) interaction imply relativistic two-nucleon operators of short range (Fig. 1d) that contribute significantly to the cross section near threshold [11, 12]. The same mechanisms, through PCAC, had earlier been shown to provide a natural explanation [13, 14] of the observed large enhancement in heavy nuclei of the axial charge of the nucleon [15, 16]. The cancellation between the single-nucleon and pion-exchange mechanisms shows, however, that the short-range operators implied by the scalar and vector components of the $NN$ interaction cannot explain all of the measured cross section for $pp \rightarrow pp\pi^0$.

We consider here meson exchange mechanisms in which the pion is pro-
duced at a transition vertex that involves two different heavy mesons (Fig. 1e): most relevant are $\rho\omega$, $\pi'\sigma$ and $\eta a_0$ exchanges. We also consider the effect of the intermediate $N(1440)$ resonance that can be excited by exchanged $\sigma$ and $\omega$ mesons (Fig. 1f). These contributions have a sizeable uncertainty range because of the values (and relative signs) of the various coupling constants are not well known. More dramatic, however, is the theoretical uncertainty associated with the pion rescattering contribution (Fig. 1b), which is estimated by a re-evaluation of the rescattering contribution with another set of parameters: a $O(q^2)$ determination from $\pi N$ sub-threshold parameters [7]. In this case, the interference with the one-body term, while still destructive, is far less complete. Finally, the uncertainty regarding the $NN$ wave functions at small separations is estimated by using two different realistic, modern $NN$ potentials, Argonne V18 [18] and Reid93 [19].

It turns out that the $\rho \omega$ and, to a lesser extent, $\pi'\sigma$ and $\eta a_0$ exchange mechanisms provide sufficiently large matrix elements for the reaction $pp \rightarrow pp\pi^0$ so that, when combined with the short range exchange mechanisms that are associated with the $NN$ interaction, they can provide an – at least qualitative – explanation of the empirical cross section.

Consider first the $\rho \omega$ exchange contribution to the two-nucleon pion production amplitude (Fig. 1e), as derived in Ref. [20]. The key ingredient in this amplitude is the $\pi\rho\omega$ vertex, which was considered by Gell-Mann, Sharp and Wagner [21] in the $\rho$-dominance model for the decay $\omega \rightarrow \pi\pi\pi$:

$$L_{\pi\rho\omega} = -\frac{g_{\pi\rho\omega}}{m_\omega} \epsilon_{\mu\nu\lambda\delta} \partial^\mu \rho^\nu \omega^\lambda \partial^\delta \pi.$$  (1)

Here $\rho_\nu$ and $\omega_\lambda$ are the Proca field operators for the $\rho$ and $\omega$ mesons and $\pi$ is the pion field. The $\pi\rho\omega$ vertex is related to the triangle anomaly, as illustrated explicitly e.g. in Ref. [22], and hence it can be viewed as fairly well established. The numerical value for the coupling constant $g_{\pi\rho\omega}$ is $\simeq -10$ [24] (the phase is fixed as in Ref. [23]).

The discussion of the $\rho \omega$ exchange pion production amplitude is completed by noting that we employ the usual $\omega N \bar{N}$ and $\rho N \bar{N}$ interactions with vector and tensor coupling. For the $\omega$- and $\rho$-nucleon coupling constants we use the values $g_{\omega}^2/4\pi = 1.2$, $\kappa_\omega = 6.1$ and $g_{\rho}^2/4\pi = 25$, $\kappa_\rho = 0$ respectively,
where, as usual, $\kappa_{\rho(\omega)}$ represents the relative strength of the tensor coupling. These values correspond to the Bonn boson-exchange model for the $NN$ interaction \cite{24} and imply that the $\rho$ and $\omega$ exchanges are treated as effective meson exchanges rather than pure meson exchange interactions. We also assume that $g_\rho g_\omega > 0$, in line with the quark model. The $\rho N\bar{N}$ and $\omega N\bar{N}$ vertex form factors are denoted $F_\rho(\vec{k})$ and $F_\omega(\vec{k})$ respectively. Their expressions may be inferred from boson exchange models for the $NN$ interaction.

The complete $\rho\omega$ exchange amplitude for neutral pion production near threshold is evaluated to be

$$
T_{\rho,\omega} = -i g_\rho g_\omega g_{\rho\omega} \frac{\omega_q}{4m_\pi^2} \frac{F_\rho(\vec{k})F_\omega(\vec{k})}{m_\omega (k^2 + m_\rho^2) (k^2 + m_\omega^2)} \left[(2 + \kappa_\rho + \kappa_\omega)(\vec{k}^2\vec{\Sigma} \cdot \vec{P} - \vec{k} \cdot \vec{P} \vec{\Sigma} \cdot \vec{k})
\right.
\left.-2i(1 + \kappa_\rho)(1 + \kappa_\omega)\vec{k}^2\vec{\sigma}^{(1)} \times \vec{\sigma}^{(2)} \cdot \vec{k}\right] \tag{2}
$$

Here we follow the notation of Ref. \cite{24}: $\omega_q^2 = \vec{q}^2 + m_\pi^2$ is the energy of the (on-shell) pion produced with momentum $\vec{q}$ in the center of mass; $\vec{p}$ ($\vec{p}'$) is the center of mass momentum of the incoming (outgoing) proton labelled “1” (those of proton “2” are opposite); $\vec{k} = \vec{p} - \vec{p}'$ is the momentum transferred; $\vec{P} = \vec{p} + \vec{p}'$; $\vec{\sigma}^{(i)}$ is the spin of proton $i$; and $\vec{\Sigma} = \vec{\sigma}^{(1)} - \vec{\sigma}^{(2)}$. In Ref. \cite{20} only the numerically most important last term in this expression was retained.

The $\rho\omega$ exchange operator of Eq. (2) is insignificant for nuclear pion absorption rates in near threshold, but it is of comparable magnitude to the (small) isospin symmetric pion exchange contribution to the pion production (or absorption) amplitude. As a consequence it gives a numerically significant contribution to the matrix element for the reaction $pp \rightarrow pp\pi^0$.

In view of the dominant $\pi\eta$ decay mode of the $a_0(980)$ meson and the important role the effective isospin 1 scalar exchange mechanism plays in the $NN$ interaction, the $\eta a_0$ exchange mechanism (Fig. 1e) could be expected to be of similar significance for nuclear pion production near threshold as the $\rho\omega$ exchange mechanism. The $\pi\eta a_0$ vertex can be described by the Lagrangian

$$
\mathcal{L}_{\pi\eta a_0} = \frac{m_{a_0} g_{\pi\eta a_0}}{m_\pi^2} \{ m_\pi^2 \vec{a}_0 \cdot \vec{\pi} \eta + r_{\pi\eta a_0} \vec{a}_0 \cdot \partial_{\mu} \vec{\pi} \partial^\mu \eta \} \tag{3}
$$
where $\vec{a}_0$ is the isovector $a_0(980)$ meson field and $\eta$ the pseudoscalar $\eta$-meson field. Here $g_{\pi\eta a_0}$ is the non-derivative coupling constant and $r_{\pi\eta a_0}$ is the ratio between two-derivative and non-derivative couplings; higher-derivative interactions could be considered as well. We take the decay width $\Gamma(a_0 \to \pi\eta)$ of the $a_0(980)$ to be 175 MeV, which is the mean of the empirical range 50-300 MeV [23]. We emphasize that the measured value of $\Gamma$ fixes only one combination of the two parameters $g_{\pi\eta a_0}$ and $r_{\pi\eta a_0}$. The derivative term is important in the decay width because of the large three-momenta of the pion and eta, but less important near threshold where the three-momentum of the pion vanishes. Hence the contribution of the $\pi\eta a_0$ vertex to $\pi^0$ production is very sensitive to the parameter $r_{\pi\eta a_0}$. For example, with $r_{\pi\eta a_0} = 0$, one finds $|g_{\pi\eta a_0}| \simeq 3.5$ which would lead to a huge contribution. The values 0 and 3.5 are similar to those found in the linear basis of 't Hooft’s extension of the linear $\sigma$-model, which explains the $U(1)$ anomaly [23], and would generate a large contribution to the pion production amplitude. There is, however, no justification for choosing $r_{\pi\eta a_0} = 0$, as the rest of the calculation employs a pion field that couples via derivatives in the chiral limit. Rather, naturalness implies $|r_{\pi\eta a_0}| \sim 1$; e.g., in the appropriate non–linear basis, 't Hooft’s model gives $r_{\pi\eta a_0} = -2$, while assumed meson saturation of ChPT theory parameters in the meson sector gives $|r_{\pi\eta a_0}| \sim -0.75$. Assuming $|r_{\pi\eta a_0}| \sim 1$, the decay width is controlled by the two-derivative term, and $|r_{\pi\eta a_0}g_{\pi\eta a_0}| \sim 0.21$.

The $\eta N\bar{N}$ and $a_0 N\bar{N}$ coupling Lagrangians are taken to have the usual pseudovector and scalar forms. The coupling constant $f_{\eta NN}$ is determined as $f_{\eta NN} = (m_\eta/2m_N)g_{\eta NN}$, where $g_{\eta NN} = 5$ [24]. Again interpreting the $a_0$ exchange mechanism as an effective representation of the isospin 1 scalar exchange component of the $NN$ interaction, we take the coupling constant $g_{a0NN}$ to have the value 5.9 indicated by the Bonn model [24]. The $\eta N\bar{N}$ and $a_0 N\bar{N}$ vertex form factors are denoted $F_\eta(\vec{k})$ and $F_{a0}(\vec{k})$ respectively, and can be obtained from the same reference.

The complete $\eta a_0$ exchange contribution to the $\pi^0$ production amplitude is then determined to be

$$T^{\eta,a_0} = -i\frac{m_{a_0}}{m_\eta}f_{\eta NN}g_{a0NN}g_{\pi\eta a_0}\frac{F_\eta(\vec{k})F_{a0}(\vec{k})}{(m_\eta^2 + \vec{k}^2)(m_{a_0}^2 + \vec{k}^2)}\left[1 + r_{\pi\eta a_0}\frac{\omega^2}{2m_\pi^2}\right]\vec{\Sigma} \cdot \vec{k},$$

in the same notation as before. Obviously neither the sign nor the magnitude
of this contribution is determined by the decay width alone. In the following, this contribution is estimated by assuming the magnitude of the square bracket to be 1, as suggested by naturalness, and that the overall sign is such that this amplitude interferes constructively with the impulse approximation.

One may proceed in a similar way to incorporate exchange of heavier mesons that can decay into \( \pi^0 \) and another neutral meson. There is, however, only scant information on the coupling of such heavier mesons to nucleons. An exception are the isovector mesons around 1.3 GeV: the \( a_1(1260) \) and the \( \pi'(1300) \). Both have large decay widths, and could be expected to decay predominantly into \( \pi(\pi\pi)_{S-wave} \). Arguments based on chiral symmetry and Regge behaviour can e.g. yield a significant decay width for \( a_1(1260) \rightarrow \pi\epsilon \), where \( \epsilon \) is a broad scalar more or less degenerate with the \( \rho \) \[28\]. An analysis that included both \( a_1(1260) \) and \( \pi'(1300) \) \[29\] has yielded a very small branching ratio for this process, however, while giving \( \Gamma(\pi' \rightarrow \pi\epsilon) = 345 \) MeV. To be definite, we will concentrate on the \( \pi'(1300) \), and use this decay width as input. We consider the Lagrangian

\[ L_{\pi\pi\pi'} = \frac{g_{\pi\pi\pi'}}{f_\pi}[m_\pi^2 \vec{\pi}' \cdot \vec{\pi} \sigma + r_{\pi\pi\pi'} \partial^\mu \vec{\pi}' \cdot \partial_\mu \vec{\pi} \sigma], \]

(5)

where \( \vec{\pi}' \) is the isovector \( \pi'(1300) \) meson field and \( \sigma \) the effective scalar field. Again, here \( g_{\pi\pi\pi'} \) is the non-derivative coupling constant and \( r_{\pi\pi\pi'} \) is the ratio between two-derivative and non-derivative couplings; the pion decay constant \( f_\pi \) was introduced for normalization. And again, assuming \(|r_{\pi\pi\pi'}| \sim 1\), the decay width is controlled by the two-derivative term, and \(|r_{\pi\pi\pi'} g_{\pi\pi\pi'}| \sim 0.7\).

The \( \pi'(1300) \) is not included in most boson-exchange \( NN \) potentials, but these have typically a large \( \pi NN \) form factor parameter \( \Lambda_\pi \simeq 1.3 - 1.5 \) GeV. However, there is a considerable body of evidence, see e.g. \[30\] and references therein, that this form factor is actually much softer. In particular, the use of hard pion-nucleon form factors, typical in nuclear physics, in calculations of deep inelastic structure functions leads to much larger breaking of the SU(3) flavor symmetry of the \( \bar{q} \) sea than is observed \[31, 32\]. A value of \( 730 \pm 100 \) MeV is preferred by those analyses.

The reduction of the effects of one-pion exchange in \( NN \) scattering caused by such a soft form factor may be compensated for by the effects of the ex-
change of a \( \pi'(1300) \) [33]. In particular, a form factor parameter \( \Lambda_{\pi} = 800 \) MeV can be enforced in a one-boson-exchange potential without loss of fitting quality if the \( \pi'(1300) \) is introduced simultaneously, coupling to the nucleon as an ordinary pseudoscalar, with a coupling constant \( g_{\pi'NN}/4\pi = 100 \) [33]. In the same model, the \( \sigma \) has its usual coupling to the nucleon, with a coupling constant \( g_{\sigma NN}/4\pi = 38 \). The respective form factors [33] are denoted here \( F_{\eta}(\vec{k}) \) and \( F_{a_0}(\vec{k}) \).

In the same notation as before, the corresponding amplitude (see Fig. 1e) is

\[
T^{\sigma,\pi'} = -i \frac{m_{\pi}^2}{2m_N f_{\pi}} g_{\sigma NN} g_{\pi'NN} g_{\pi\pi'} \frac{F_{\sigma}(\vec{k}) F_{\pi'}(\vec{k})}{(m_{\pi}^2 + \vec{k}^2)(m_{\pi'}^2 + \vec{k}^2)} [1 + r_{\pi\pi'} \frac{\omega^2}{2m_{\pi}^2}] \vec{\Sigma} \cdot \vec{k}.
\]

Similarly to the \( \eta a_0 \) case, we assume in the following that the square bracket is of \( O(1) \) and indeed is taken as unity in our calculations. We also assume that \( g_{\sigma NN} g_{\pi NN} > 0 \) as in the linear \( \sigma \)-model, and that \( g_{\pi NN} g_{\pi'NN} < 0 \) and \( g_{\pi\pi'} g_{\sigma\pi} < 0 \) as in quark models.

As the \( N(1440) \) is the lowest \( \frac{1}{2}^+ \) nucleon resonance it is expected to contribute to the \( S \)-wave pion production amplitude in a similar way to the \( N\bar{N} \) pair contributions that are implied by the Lorentz-invariant structure of the \( NN \) interaction. As the effective pion rescattering vertices include all \( \pi N \) \( s \)-channel resonances, only amplitudes that arise from excitation of virtual \( N(1440) \) resonances by short range exchange mechanisms should be considered as separate two-nucleon pion production amplitudes. The most important of these are \( \sigma \) - (or \( (\pi\pi)^{I=0}_{S-wave} \)) and \( \omega \)-meson exchange (Fig. 1f).

The appropriate vertices for derivation of the amplitude for the contribution that involves excitation of intermediate \( N(1440) \)'s through \( \sigma \)- and \( \omega \)-exchange are given in Ref. [34]. The \( \sigma N\bar{N} \) coupling constant \( g_\sigma \) is approximately given by the Bonn-model for the \( NN \) interaction [24]. The coupling constant \( g_\sigma^* \) for the excitation of the \( N(1440) \) resonance is estimated from the partial width for the \( N(1440) \rightarrow N(\pi\pi)^{I=0}_{S-wave} \) in Ref. [34] to be \( g_\sigma^* \simeq 1.1 \). The value for the \( \omega NN^* \) coupling constant, \( g_\omega \), is highly uncertain. In Ref. [34] it was estimated to be \( g_\omega \simeq 1.7 \) under the assumption that \( g_\omega^* / g_\omega \approx g_\sigma^* / g_\sigma \). The \( \pi NN(1440) \) coupling is described by a pseudovector Lagrangian. The
πNN* pseudovector coupling constant \( f_\pi^* \) is obtained as \( f_\pi^* = 0.62 \) from the empirical decay width for the \( N(1440) \to N\pi \) decay \[34\]. We assume that the form factors for the \( \sigma N(1440)\bar{N} \) and \( \omega N(1440)\bar{N} \) vertices are the same as the \( \sigma N\bar{N} \) and \( \omega N\bar{N} \) vertices, \( F_\sigma(\bar{k}) \) and \( F_\omega(\bar{k}) \), respectively.

The \( N(1440) \) amplitude is then

\[
T_{R;\sigma,\omega} = -i \frac{f_\pi^*}{m_N(m^* - m_N)} \omega_q \left( \frac{g_\sigma g_\sigma^* F_\sigma^2(\bar{k})}{\bar{k}^2 + m_\sigma^2} - \frac{3m_N - m^* g_\omega g_\omega^* F_\omega^2(\bar{k})}{m^* + m_N} \right) \vec{\Sigma} \cdot \vec{P} \\
-2i \frac{m^* - m_N g_\omega g_\omega^* F_\omega^2(\bar{k})}{m^* + m_N} \frac{1}{\bar{k}^2 + m_\omega^2} \sigma^{(1)} \times \sigma^{(2)} \cdot \bar{k}. \tag{7}
\]

The form of this operator is similar to that of the effective meson-exchange term that involves an intermediate \( N\bar{N} \) pair. The ratio of the \( N(1440) \) scalar meson-exchange part of Eq. (7) to the scalar exchange part \( (g_\sigma^*/g_\sigma)(f_\pi^*/g_A m_\pi) \) \((2m_N/(m^* - m_N)) \) \( \sim 0.25 \). This shows that the \( N(1440) \) contribution enhances the scalar meson-exchange contribution to the \( S \)-wave pion production amplitude by \( \sim 25\% \). The two vector exchange terms in Eq. (7) give contributions of different sign to the matrix element for \( pp \to pp\pi^0 \). Their ratios to the corresponding \( \omega \) pair terms are \( -(g_\omega^*/g_\omega)(f_\pi^*/g_A m_\pi) \) \((2m_N/(m^* - m_N))((3m_N - m^*)/(m^* + m_N)) \) \( \sim -0.15 \) for the first and \( (g_\omega^*/g_\omega)(f_\pi^*/g_A m_\pi) \) \((2m_N/(m^* + m_N)) \) \( \sim 0.05 \) for the second. This partial cancellation between the \( \omega \) exchange terms that involve an intermediate \( N(1440) \) ensures that the \( \omega \) exchange contribution to the cross section for \( pp \to pp\pi^0 \) near threshold will not be very different from the one obtained in Ref. \[1\].

All the above contributions, together with the pair terms of Ref. \[1\], correspond to contact \( \pi NNNN \) interactions in ChPT. According to the modified chiral power counting developed in Ref. \[4\], the pair and \( N(1440) \) terms contribute to the leading order of such contact terms, while the exchange mechanisms contribute to higher orders, i.e. they are down by one power of \( m_\pi \). In that reference, the cross section for the reaction \( pp \to pp\pi^0 \) was calculated on the basis of a reaction amplitude formed primarily of (1) the single nucleon pion production operator, (2) the pion exchange contribution as constrained by chiral perturbation theory, and (3) the short range contributions associated with intermediate \( N\bar{N} \) pair terms. It was shown that the
resulting cross section is too small by a factor $5-10$ because of the remarkable cancellation between the single nucleon and pion rescattering contributions. This cancellation depends crucially on parameters determined by $\pi N$ scattering.

We shall here be mainly concerned with the modifications caused by (i) the additional short-range contributions discussed above and (ii) changes in the numerical values of the parameters that affect the pion rescattering contribution.

The numerical values of the different contributions of the graphs of Fig. 1 are shown in Fig. 2. These are computed using the Reid93 potential \[19\] to provide the initial $^3P_0$ and final $^1S_0$ wave functions. The quantities $J$ refer to the amplitudes after some common kinematic factors and complex phases are removed; see Ref. \[4\] for a precise definition. The label impulse refers to the contribution of Fig. 1a. There are two pion rescattering curves. They both include the same $\Delta$ excitation contribution (Fig. 1c) and a contribution (Fig. 1b) proportional to

$$
4c_1 + \frac{\delta m_N}{2m_\pi^2} - \left(c_2 + c_3 - \frac{g_A^2}{8m_N}\right) = \frac{C}{2m_N},
$$

where $c_i$ are input $\pi N$ parameters, and $\delta m_N \sim 3$ MeV is the strong interaction contribution to the neutron-proton mass difference. The two pion rescattering curves differ in the $c_i$’s and $C$. The one labelled cl (for chiral loops) uses the value of $C = -2.31$ obtained in Ref. \[2\] and corresponds to the $O(q^3)$ parameters $c_i$ as given in Ref. \[1\]. It is obvious that the contribution of the impulse term is almost entirely cancelled by this pion rescattering term. This is why the corresponding cross section in Ref. \[2\] was very close to zero without including the effects of the nucleon-pair terms associated with $\sigma$ and $\omega$ exchange. The other rescattering curve (labelled st for sub-threshold expansion) corresponds to the $O(q^2)$ parameters $c_i$ as given in Ref. \[17\], which give a much smaller $C = -0.29$. (The subthreshold expansion makes use of an analytic description of the $\pi N$ with numerical values determined by data \[35\].) As a consequence the cancellation is less complete in this case. Both of these curves were obtained with a soft pion form factor; employing a hard form factor increases the magnitude of these results by less than 10%.
This is because of the effects of $NN$ repulsion in the final state $^1S_0$ $pp$ wave function and the centrifugal barrier in the initial $^3P_0$ wave function. The integrand for the pion rescattering contribution peaks at about 1.3 fm.

The other curves in Fig. 2 display the shorter-range effects (Fig. 1d,e,f). The effect of the new $\rho\omega\pi^0$ has the same sign as the impulse term and is comparable to the change in the rescattering term; it is also of similar size as the $\sigma$ pair term. The new $\eta\rho\pi^0$ and $\sigma\pi^0\pi^0$ meson exchange terms are smaller under the (conservative) assumptions of this paper. The curve labelled $\omega + \sigma$ includes the effects of the pair diagrams. The effects of the excitation of the $N(1440)$ are also shown. Including the $N(1440)$ enhances the sigma exchange term by about 25%, but the omega exchange term hardly at all because of the cancellation noted above. Therefore the effect of the $N(1440)$ is small. The results in Fig. 2, do have a wide uncertainty margin because of the uncertain meson couplings. In particular, using a value of $r_{\pi\sigma\pi'}$ much smaller than unity would increase the value of the related contribution by a big factor. Likewise, a larger value for $g_\sigma$ [30] would also increase the cross section.

We also note that the magnitude of these shorter-than-pion-range terms is consistent with expectations from the modified chiral power counting of Ref. [2]. Although the convergence of ChPT here is not that impressive, there is no evidence of failure either.

The cross sections, computed using all of the mechanisms of Fig. 1 as well as the very small recoil term of Ref. [2] are shown in Fig. 3, together with experimental data from Refs. [6, 7, 8]. The full curves represent the complete result for the Reid93 potential [19], the lower curve using the $O(q^3)$ rescattering parameters, the higher the $O(q^2)$ parameters. Also shown are the dashed curves with the corresponding results for the Argonne V18 potential [18]. For a given potential, the two different set of rescattering parameters give an estimate of the uncertainty generated by the rescattering mechanism. For a given set of rescattering parameters, the two potentials give an estimate of the uncertainty generated by the nuclear short-range dynamics. The overall theoretical uncertainty from these sources is quite large; it arises because the cross section is basically very small. The inputs (both $pp$ wave functions and various $\pi N$ scattering coefficients) are not known well enough to provide a
more accurate calculation. There is also additional uncertainty in the input coefficients of the meson-exchange Lagrangians of Eqs. (3) and (5). Furthermore, two-pion exchange diagrams can be expected to contribute appreciably, but have so far not been calculated. Within this large inherent theoretical uncertainty, meson exchange can explain the observed cross section.

Further development of ChPT should lead to a reduction of the uncertainties associated with the $\pi N$ scattering coefficients, $pp$ wave functions, and the mesonic coupling constants. A calculation to order $(q^3)$ and comparison with the sub-threshold expansion of Ref. [35] should lead to better description of $\pi N$ scattering data. If the chiral potential of Ordóñez et al [37] were developed to yield a good description of $NN$ data up to the energies relevant here a consistent treatment of the amplitude for $\pi^0$ production and the nuclear wave functions based on the same general chiral Lagrangian could be achieved. Finally an improved treatment of the meson-exchange diagrams is possible by a chiral calculation of the relevant terms as higher order loop diagrams.

The principal conclusion is that the influence of the $\rho\omega\pi^0$ – and to a lesser extent the other shorter-than-pion-range mechanisms considered here – is considerable. This idea can be used to understand independent experiments. In particular, the PCAC relation between the axial charge operator of the nuclear system and the S-wave soft-pion production amplitude [11] and the present results imply that the axial charge operator should have a significant $\rho\omega$ axial exchange current term. As this will contribute to the nuclear enhancement of the effective axial charge of the nucleon [13], this suggests a possibility for further reducing the remaining uncertainties in the magnitude of the short range mechanisms by data on first forbidden nuclear $\beta$-transitions [14, 15].

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**Figure captions**

Fig. 1 Contributions to the amplitude for $pp \rightarrow pp\pi^0$. (a) Impulse approximation, (b) pion rescattering, (c) pion rescattering through an intermediate $\Delta$, (d) pair graphs, (e) meson exchange contributions, (f) excitation of an intermediate $N(1440)$ resonance.

Fig. 2 Individual contributions to the amplitude for $pp \rightarrow pp\pi^0$ as a function of $p'$, the relative momentum between the two final protons for laboratory kinetic energy of 300MeV. The Reid93 potential is used.

Fig. 3 Computed cross sections for two different $NN$ potential models. All of the terms of Fig. 1 are included as well as that of the recoil term of Ref. [2].
Figure 1
R93 potential, $T(\text{LAB}) = 300$ MeV

Fig. 2
