Chapter 10
Phase Transition and the City

10.1 Introduction

Phase transition (PT) and the city’ is a very wide issue. At a grand macro scale it might refer to the PT of the urban revolution—the first appearance of urban society some 5500 years ago, while at a micro scale to the PTs in the life path of a single citizen: living with parents, at the city and at the suburb. In between, at the mezzo scale, there is still a wide range of scales such as PT processes of the emergence of cities and urban society out of Middle Ages feudalism, or still at a smaller scale, nowadays suburbanization and gentrification, or more specific case studies such as the balconies, lofts,.. and much more. So a chapter (not a book) on PT and the city must start with some definition of boundaries.

One way to set boundaries is to choose scale and specific case studies and start from there. This is what we’ll do in the present chapter. We’ll start (Sect. 10.2) with several case studies: the case of Tel Aviv balconies (10.2.1) which we’ve already described in brief in Chap. 4 (Sect. 4.4.2). Next (Sects. 10.2.2–10.2.5) we portray a scenario of the evolution of a metropolitan area. The scenario roughly follows the evolution of the Tel Aviv metropolitan area; yet with some modifications it can typify similar processes in other parts of the world. Central to our synergetics’ interpretation of the above case studies is an interplay between control parameter (CP) and order parameter (OP). Thus, to generalize the case studies discussed in Sect. 10.2, we elaborate in Sect. 10.3 this issue by discussing the CP-OP-PT interplay in the context of potential urban scenarios. Based on the latter generalization we develop two mathematical models that illustrate the relations between CP,OP, PT: A model of ‘growing cities’ which represents the most typical urban process (Sect. 10.4), and a model referring to the information production process (Sect. 10.5). We conclude the chapter (Sect. 10.6) by an overview on the descriptive and mathematical parts of this chapter and their implications.
10.2 Case Studies from Tel Aviv and Its Metropolitan Area

10.2.1 Tel Aviv Balconies

We’ve referred to the story of TA balconies in brief already in Chap. 4; here is “the full story”. Open balconies were a dominant feature in the urban landscape of Tel Aviv from its day one at the beginning of the twentieth century. Influenced specifically by the modernist Bauhaus school and the local climate, almost all residential buildings were built with open balconies. During the 1930s and 1940s there were here and there attempts to close balconies and transform them to half rooms. These attempts, which implied a violation of the Tel Aviv planning law (and OP), were prevented by the city’s planning authorities (“enslaved” in the language of synergetics). Yet the closed balcony possibility was not forgotten.

Following independence and massive immigration waves in the 1950 and the resulting shortage of residential space (CP), the closing balcony solution (OP) once again appeared (as “fluctuations”) and in building after building balconies were closed (slaving diffusion). This time however, the municipality could not stop the process despite the fact that it was as before a violation of the planning law.

As noted in Chap. 4 (Sect. 4.4.2), the process started probably at end 1950s, when an unknown resident of Tel Aviv enlarged his/her apartment by closing the balcony and making it a “half-room” (Fig. 10.1, Balconies 1 and 2a). Some neighbors liked the idea and did the same. An interpersonal sequential SIRNIA design process gave rise to a space-time diffusion of closing balconies. As more and more balconies were closed, the form of the closed balconies gradually changed (Fig. 10.1). This process is still evolving as the remaining open balconies are being closed and balconies

![Fig. 10.1 “The butterfly effect of Tel Aviv balconies”: 1 Open balcony. 2a Closed by asbestos shutters. 2b Closed by plastic shutters. 2c Closed by glass windows. 3a From the outside it looks as a balcony; from the inside part of the living room or a kitchen. 3b No balconies. 4 “Jumping balconies”. Source: Portugali and Stolk (2014)
previously closed in the older style are being renovated in line with the new more fashionable style.

The above, however, is not the end of the story. At a certain stage, the municipal planning authorities decided to intervene and started to tax all balconies, open and closed, as if they are a regular room. The response was another phase transition and a bifurcation: On the one hand, in old buildings, inhabitants continued closing balconies as above; on the other, developers with their designers started to design and build new buildings with already closed balconies, that is, with no balconies at all (Fig. 10.1).

Yet another small-scale phase transition took place at the end of the twentieth century, with the arrival of postmodern architecture: “Quoting” past patterns became fashionable and architects started to apply for permits to build balconies. Remembering their past planning experience, but wishing not to lag behind the advancing (post)modern style, the city planning authorities gave architects and developers permits to build open balconies, but in a way that, technically, would not enable them to be closed as in the past. The result was the “jumping balconies” which are typical to Israel’s urban landscape only (Fig. 10.1). The very final stage and phase transition that happened some 15 years ago, was a return to square one: open balconies are once again allowed; closing them is prohibited.

Note’ firstly, the role of memory, namely, that the closed balcony possibility was not forgotten. Secondly, that the phase transition of the first closed balconies was followed by a subsequent sequence of phase transitions reminiscent “aftershocks” that follow a major earth quake.

**Fig. 10.2** Changes in the city size distribution of Israel 1922–1959. Source: Bell (1962)
10.2.2 From Primate to Power Law Distribution

The above PT of the balconies and its subsequent events was an urban landscape expression to a mega-scale PT that following the war of independence and has dramatically shaken the Israeli urban system. While there is no room here to describe the process in details (see Portugali 1993), the following can be said: Following independence (in 1948), in less than 10 years the rank-size distribution of the Israeli urban system was transformed from a non-hierarchical *primate city* structure,1 into a hierarchical power-law structure (Fig. 10.2).

The above transformation can be further observed in the sequence of maps of Fig. 10.3 that concentrate on the evolution of the Tel Aviv metropolitan area. Here too we observe a major structural change (PT) between the map of 1942 to that of 1952. The force behind this change (CP or OP) is the massive immigration waves that started in the late 1940s, before the establishment of the state of Israel, of Jews

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1One/few big cities and many small settlements with no intermediate-size cities in between (Jefferson 1939).
from Europe who immigrated (illegally) to Israel that was then under the British mandate. After independence, the waves intensified, with immigration waves of Jews from Europe and from Islamic countries, specifically from Yemen (1948–1950), Iraq (1950–52) and North Africa (mid 1950s). In less than 10 years, these waves have tripled the population of Israel as a whole and specifically its central area, north and south of Tel Aviv. Temporary immigrants tents camps were transformed into towns and previously relatively independent small towns and agricultural settlements, experienced a sudden and fast population growth.

### 10.2.3 Suburbanization and Gentrification

The growth of the urban system continued from the 1970s onwards, but this time not due to immigration waves, but as a consequence of the “classical” suburbanization-metropolization internal migration processes. Since end 1960s and in the 1970s, Tel Aviv was once again subject to housing shortage, this time, however, due to “classical” urban processes that were typical of many cities around the world: natural demographic growth, rising standards of living, high rate of car ownership, people in
old residential neighborhood get older, young families move out to nearby towns but continue to work in Tel Aviv, the number of Tel Aviv residents is declining, previous residential neighborhood are transformed to CBD (central business districts) … Tel Aviv becomes the metropolitan core, firstly to suburban towns around it and a bit later to a larger area—to agricultural rural settlements thus transforming them to suburbs of the big city (a process often termed as \textit{rurbanization}). The process as a whole is well represented by the set of maps in Fig. 10.3.

In the end 1970s and during the 1980s a new PT and OP—\textit{Gentrification}: “Yuppie” (“young urban professional”) and then “regular” young middle-class families moved from the suburbs back to the city centers, pushing out the poor and occupying their neighborhoods, as well as previously residential areas that have been transformed to CBD or semi-industrial areas at the city center (e.g. lofts) and now once again became residential. The latter processes of gentrification entailed, firstly, \textit{The Rise of the Creative Class} (Florida 2002) that plays a key role in the socio-economy of today’s post-industrial cities—specifically in global cities that are connected to the global socio-economy. Secondly, to a socio-economic and cultural gap between the central global cities and the peripheral towns and cities that “were left behind”—a gap that shows itself in the Brexit in England, the Yellow jacket in France and more. In both suburbanization and gentrification, the PT gave rise to a new OP and to a long period of steady state (e.g. commuting in the first and walking/cycling etc. in the second).

\subsection*{10.2.4 The Evolving Fractal Structure}

It is interesting to note that the above geo-historical description of the Tel Aviv metropolitan area shows up in the evolution of the fractal dimension of that area. Based on the set of maps in Fig. 10.3, Benguigui et al. (2000) have studied the time evolution of the fractal dimension of the region that now forms the metropolitan area of Tel Aviv. The study started by identifying, by means of visual inspection of the maps in Fig. 10.3, three study regions in the metropolitan area as presented in Fig. 10.4.

The study suggests that the central part of the Tel Aviv metropolitan area, that included Tel Aviv and its adjacent towns (region 1 in Fig. 10.4), was from the start fractal, when its fractal dimension $D$ increased from 1.533 in the year 1935, to 1.809 in 1991, with a rapid increase between 1971 and 1978; apparently due to the suburbanization process noted above. Region 2 too was from the start (1935) fractal, however, with $D$ values ranging from 1.387 in 1935 to 1.733 in 1991, with a pronounced “jump” between the years 1941 and 1952, reflecting the immigration waves before and after 1950 noted above. The gap in the fractal dimensions between regions 1 and 2 indicates that the Tel Aviv metropolitan area is not yet fully integrated and unified. Finally, judging from the evolving $D$ of region 3 one can say that the entire metropolitan area of Tel Aviv became fractal only after 1985.
10.2.5 Intermediate Discussion

As in the story of the balconies, so in the process of migration to, and urbanization of, the agricultural villages into suburb towns, the process started as SIRNIA (Chap. 4) derived fluctuations in the form of violation of the planning law. In the case of the agricultural villages, the planning law (OP) prohibited to transform agricultural land into built-up area. In fact, what we had here, was a competition between two OPs: an urban OP driven by demographic growth, housing shortage, increasing demand for, and prices of, urban land; versus an agricultural OP driven by The Governmental Committee for the Preservation of Agricultural Land, itself driven by the Zionist ideology “to return to the land”. As implied from the above, the urban OP (“market forces”) won the competition: many farmers/land owners found it beneficial to sell parts of their land to urban private developers. And again, as in the case of balconies, after a decade or two the authorities had no choice but to legalize the transformation.

In the case studies described above, we can identify three major PTs: the case of the Tel Aviv balconies, the process of suburbanization and the counter process of gentrification. In all three the PT took the form of a space-time diffusion process composed of a sequence of smaller-scale PTs. In the case of the closed balconies PT, the closed balconies were followed by buildings with no balconies, then by “jumping balconies” and finally, once again, by open balconies. The suburbanization PT was followed, or was associated with, a sequence of smaller scale PTs: the transformation of Tel Aviv into a CBD and in parallel, the urbanization of the rural-agricultural settlements (“rurbanization”). In each individual agricultural settlement, such a change implied a local PT. Finally, in the case of the gentrification PT, the subsequent sequence of smaller scale PTs included the transformation of Tel Aviv into a mixed uses residential (“creative class”) + “smart” CBD.

Thus, unlike PTs in physics (laser, ice to water,…) where due to the slaving principle an emerging OP is followed by StS, in cities, the PTs are followed by slaving that takes the form of a sequence of small-scale PTs and further events and developments. In this respect the OP is similar to what Bohm and Peat (1987) have termed “generative order”, an order that has the potential to generate other orders or events.

10.3 The Interplay Between OPs and CPs in PT

As we’ve seen in the case studies just described, central to them is a play between population (e.g. numbers of immigrations and/or natural increase) and housing (e.g. availability of flats). As we’ve seen in Chap. 3 above and in subsequent chapters, central to Synergetics is the interplay between control parameter (CP) and order parameter (OP). This raises the question ‘who is how?’—which of the two is OP and which CP?
An answer to this question comes from Synergetics 1st Foundation, cf. e.g. Chap. 3. There is a unique criterion based on time-scale separation. CPs are constant or change at least much more slowly than OPs. Thus housing facilities that change at least in general much more slowly than population size are CPs and population size OP. On the other hand, a technical innovation that spreads/grows much more rapidly than population size is now the OP. The last decades events in Detroit US present yet another example: here, a drastic drop in a city’s economy, entailed massive outmigration\(^2\) and as a consequence, a “wave” of surplus flats and houses. As a result, the relations between population and housing were transformed: the number of available flats and houses became OP that was “controlled” by population as CP, as is evident by the fact that many houses were blocked. A PT thus occurs when a CP is changed beyond a critical value and the OP has to respond to this change or even a new OP is generated because of some innovation. Thus PTs are singular situations.

The relations between CP, OP, and by implication PT, depends as we’ve just seen on the specific circumstances that take place in a city or system of cities. As noted above, in all cases the decisive distinction between OPs and CPs rests on the criterion of time-scale separation. In Table 10.1 we provide a list of typical urban scenarios and their entailed CP, OP relations. We describe each scenario by four variables that we use in the mathematical models we develop in Sect. 10.4 below: OP (\(q\)), CP (\(\lambda\)), saturation rate (\(\gamma\)), and immigration rate (\(Q\)). As will transpire below, these variables are associated with the Verhulst equation, also known as “logistic equation” which has found a number of applications in various disciplines. In the model developed below we use this equation in a novel way by adding to it a random “force” \(F\).

**Growing cities.** This is the most typical urban development in most parts of the world: urban population and thus cities are growing very fast. In Europe, due to immigration, in the Far East due to massive migration processes from the rural countryside to the urban and metropolitan centers: In India and other countries, as

\(^2\)The city population declined from 1,850,000 in 1950 to 680,000 in 2015 and 672,662 in 2020.
a consequence of spontaneous, self-organized processes, while in China as part of centralized governmental policy. The result of these processes is an unprecedented reality in which for the first time in human history more than half of world population lives in cities.

**Shrinking cities.** Also called *counter-urbanization*, referring to cities who are losing population for various reasons ranging from economic crisis to “regular” migration from peripheral rural regions to metropolitan centers (Pallagst et al. 2014). In some cases shrinking is associated with socio-economic crises, as recently in Detroit, while in others shrinking cities become even more prosperous than before (Hartt 2018).

**New products.** This refers to the “classical” spatial diffusion processes due to new products, innovations, and so on (see detailed discussion in Chap. 11 that follows). Just consider the effect the invention of a new product such as the ‘automobile’ had on cities—on their structure, urban landscape, daily life and much more. Following what Schwab (2016) has termed *The 4th Industrial Revolution*—the so called Industry 4.0 with its smart artifacts—there are currently *smart cities* studies attempting to appreciate the effect of smart devices on cities. In Chap. 14 we deal with this issue in some details.

**Coronavirus.** Similar to the above, with virus instead of innovation. The coronavirus has all the ingredients of a complex SO system: unpredictability, uncertainty, abruptly emerging PT and so on. A specifically interesting question concerns its future effects on several socio-economic and cultural trends: increasing economic globalization, on the one hand, versus increasing nationalist feelings on the other (e.g. Brexit). One lesson from Synergetics is the effect and role of fluctuations (Chaps. 3, 4) is that when they occur in instable periods their effect might be dramatic. See further notes in the concluding chapter.

**Information production (IP).** As noted in Chap. 4, “Urban dynamics is a kind of production process—producing artifacts of all kinds … These artifacts convey data from which urban agents extract SI and PI with their entailed SHI; …. We term this SIRN-IA process the City’s *information production* (IP). This IP process can be seen as, or gives rise to, the city’s order parameter …”.

Of the above five scenarios, the first—growing cities—is the most typical urban process, while the fifth—IP is the more general and can thus be applied to each of the other scenarios as well as to specific economic, social or cultural events that take place in a city. In what follows we thus develop two models: one regarding the case of growing cities and one regarding IP.

### 10.4 Growing Cities

In this section we deal with population dynamics, where we assume that the number of citizens $q$ plays the role of OP. Later we will discuss other interpretations of $q$. We base our approach on the results of Chap. 5 where we derived a prototypical OP equation for numbers, i.e. Equation (5.28). By an obvious change of notations, we
write Eq. (5.28) in the form

\[
\frac{dq}{dt} = \lambda q - \gamma q^2 + F(q, t)
\]  

(10.1)

When we put \( F = 0 \), (10.1) becomes the familiar Verhulst equation. \( \lambda \) and \( \gamma \) play the role of control parameters (CP) which we consider as given and time-independent.

The important new feature of (10.1) as compared to the Verhulst equation is the occurrence of fluctuations/random force \( F(q, t) \). As we have seen in Chap. 9, such forces play an important role in PTs and NPTs. The specific choice of the form of \( F(q, t) \) must be based on the situation considered. Here we consider the case that the number of citizens may randomly change due to a change of residence—people moving into or out of the city as described in Sects. 10.1 and 10.2 above. As we’ve further seen above, these moves may happen at a low rate or a high one—an immigration wave. In order not to load the formalism, we treat only moves into the city. Since the mathematical details are somewhat involved (and not essential for our presentation) we sketch only the essential steps. We require that the fluctuating force \( F(q, t) \) obeys a relation of the form

\[
\langle F(t)F(t') \rangle = Q \delta(t - t')
\]  

(10.2)

The brackets \( \langle ... \rangle \) indicate the average over the random process. \( Q \) is the fluctuation strength and \( \delta(t - t') \) Dirac’s function. Note the occurrence of \( q(t) \) in (10.2) which means that the size of the random fluctuations is proportional to the population size. This implies mathematical intricacy. Since the population size \( q \) changes with each move abruptly, the value of \( q \) to be used remains undefined.

Here we use the Stratonovich calculus according to which we use the average value of \( q \) before and after the move. Under this assumption we may derive a Fokker-Planck equation for the distribution function \( f(q; t) \) (for some more details cf. 10.4.1 below).

\[
\dot{f} = -\frac{\partial (Kf)}{\partial q} + \frac{Q}{4} \frac{\partial f}{\partial q} + \frac{Q}{2} \frac{\partial}{\partial q} \left( q \frac{\partial f}{2q} \right)
\]  

(10.3)

where

\[
K = \lambda q - \gamma q^2
\]  

(10.4)

In the following we want to calculate the time evolution of the mean value of \( q \) and its variance \( S = \langle q^2 \rangle - \langle q \rangle^2 \) as well as their steady state values.

1. **Time evolution of \( \langle q \rangle \)**

   We multiply (10.3) by \( q \) and integrate both sides of \( q \) from \( q = 0 \) till \( q = \infty \); using the definition
10.4 Growing Cities

\[ \int_0^{\infty} q f(q,t) dq = \langle q(t) \rangle \equiv \langle q \rangle \]  
(10.5)

and the normalization of \( f \), \( \int_0^{\infty} f dq = 1 \) for all times we obtain

\[ \frac{d}{dt} \langle q \rangle = \lambda \langle q \rangle - \gamma \langle q^2 \rangle + \frac{Q}{4} \]  
(10.6)

2. Time evolution of \( S \)

In an analogous way we obtain

\[ \frac{dS}{dt} = 2\lambda S - 2\gamma (\langle q^3 \rangle - \langle q \rangle \langle q^2 \rangle) + 2Q \langle q \rangle \]  
(10.7)

The last term in (10.6), \( Q/4 \), represents an average increase of population due to taking residence/immigration. This increase entails an increase of variance according to the last term in (10.7), \( 2Q \langle q \rangle \).

Because of nonlinear terms, such as \( \langle q^2 \rangle \) in (10.6) and (10.7), these equations can only approximately be solved (see below). On the other hand, we obtain an explicit solution of the Fokker-Planck equation (10.3) in the case of steady state where

\[ \frac{df}{dt} = 0 \]  
(10.8)

\[ f(q) = N q^{-1/2} \exp(\alpha q - \beta q^2) \]

\[ \lambda = \frac{2\alpha}{Q}, \beta = \frac{\gamma}{Q} \]  
(10.9)

Most importantly, the normalization constant \( N \) and all moments \( \langle q^n \rangle, n = 1, 2, \ldots \) can be expressed by well-known integrals (see below). For our purpose, we can cast them in a handy form for important limited cases. In the spirit of PT theories we consider the transition from one steady state to a new steady state. In the following we consider several scenarios. We begin with the following situation. In the beginning, at time \( t = 0 \), we assume \( q = 0 \), i.e. no population. This is, of course, a very artificial assumption. But this model allows us to familiarize us with the general approach. In addition, it allows us to deal with realistic cases if we identify \( q \) with the number of a specific industrial product, e.g. electric cars.

We assume that the parameters \( \lambda, \gamma, Q \) are fixed. To solve (10.6) approximately, we approximate

\[ \langle q^2 \rangle b y \langle q \rangle^2 \]  
(10.10)
which is a good approximation if the distribution function is sharply peaked which we are not sure of, however. Therefore it is important to be able to compare the approximate solution to (10.6) with an exact result.

Under the assumption (10.10), the steady state solution to (10.6) reads

\[ \langle q \rangle = \frac{\lambda}{2\gamma} + \left( \frac{\lambda^2}{4\gamma^2} + \frac{Q}{4\gamma} \right)^{1/2} \]  

(10.11)

We compare this with the exact values based on the exact solution of the Fokker-Planck equation (10.9) for the two limiting cases “\( \lambda \) small” and “\( \lambda \) large”. In the case “\( \lambda \) small” we assume

\[ \lambda^2 \ll Q\gamma \]  

(10.12)

Then

\[ \langle q \rangle \approx \frac{1}{2} \left( \frac{Q}{\gamma} \right)^2 \]  

(10.13)

whereas the “exact” result reads (cf. below)

\[ \langle q \rangle = 0.3 \left( \frac{Q}{\gamma} \right)^{1/2} + \text{small corrections}. \]  

(10.14)

When “\( \lambda \) large”, we neglect the term containing \( Q \) so that (10.11) becomes

\[ \langle q \rangle \approx \frac{\lambda}{\gamma} \]

The exact result reads

\[ \langle q \rangle = \frac{\lambda}{\gamma} + \text{small corrections} \]  

(10.15)

This remarkable coincidence allows us also to study the time-dependence of \( \langle q \rangle \) and \( S \) based on Eqs. (10.6) and (10.7) under the approximation (10.10).

While the solutions can be found without any further approximations, we confine our analysis to the initial phase where the nonlinearities can be ignored. From (10.6) we obtain

\[ \langle q(t) \rangle = \frac{Q}{4\lambda} (e^{\lambda t} - 1) \]  

(10.16)

An interesting question is, at which time \( t \) the behavior (10.16) stops because of the effect of saturation. Since we may assume that the exact \( \langle q(t) \rangle \) follows approximately
a sigmoidal, i.e. S-shaped curve, we postulate that (10.16) comes close to a fraction, say $\frac{1}{2}$, of the steady state value, be it approximate or exact. In the case “$\lambda$ small” [cf. (10.12)] we find because of (10.16) and (10.13)

$$e^{\lambda t} - 1 \approx 2 \frac{\lambda}{Q} \left( \frac{Q}{4\gamma} \right)^{\frac{1}{2}} = \frac{\lambda}{(\gamma Q)^{1/2}} \ll 1$$  

(10.17)

or

$$e^{\lambda t} \ll 2$$  

(10.18)

which means a comparatively slow increase of $q(t)$.

On the other hand, if “$\lambda$ large” we arrive at

$$e^{\lambda t} \gg 1$$  

(10.19)

i.e. an exponential increase of $q(t)$.

The solution to (10.7) allows us to discuss the behavior of the variance during the initial phase that corresponds to that of $\langle q(t) \rangle$ (10.16). Again we neglect the nonlinear terms. An elementary calculation, without any further approximation, leads us to

$$S(t) = \frac{Q^2}{4\lambda^2} (e^{\lambda t} - 1)^2$$  

(10.20)

or, in terms of $\langle q(t) \rangle$

$$S(t) = 4\langle q(t) \rangle^2$$  

(10.21)

We turn to our second scenario where we start from the steady state of a city with $q_0 > 0$ inhabitants. We study what happens to the population size $q$, if either the growth parameter $\lambda$ or the fluctuation parameter $Q$ are suddenly and appreciably increased. In the first case, we deal with a sudden improvement of living conditions, e.g. after the end of a war, end of an economic crisis etc. A case in point is the situation in Israeli cities after the 1967 Six Days War that was followed with an economic boom. In the second case, we may think of immigration waves, that as described above, in the early 1950s followed the independence of Israel. In principle, we may also think of a sudden decrease of the saturation parameter $\gamma$, e.g. by a sudden building boom, though this won’t be treated here.

Our previous treatment of the scenario, where initially $q = 0$, allows us to deal with the present scenarios in an elegant fashion. Namely, the equations for $\langle q(t) \rangle$ and $S$ (10.6) and (10.7), and the steady state $f(q)$ hold also now, provided we insert the new CPs and solve (10.6) and (10.7) under the new initial conditions where we use steady state values of $\langle q \rangle$, $S$ evaluated with the old parameter values. Again we
consider an initial transition phase where we may neglect the quadratic terms in (10.6) and (10.7).

We begin with scenario (2) where $\lambda$ is increased $\lambda > \lambda_0$, where $\lambda_0$ former value, but $Q$ is kept fixed. Since the expression $\langle q \rangle$ is somewhat clumsy, we introduce the denotation

$$X = \langle q \rangle$$

(10.22)

that allows us to simply indicate the needed attributes of $X$, namely initial state: $X_i$, final state: $X_f$, time dependence and parameter $\lambda$ used: $X(t, \lambda)$. By use of these denotations, we may write the time-dependent solution to (10.6) (quadratic terms ignored)

$$X(t; \lambda) = \frac{Q}{4\lambda}(e^{\lambda t} - 1) + X_i(\lambda_0)e^{\lambda t}$$

(10.23)

In analogy to scenario (1) above, we estimate the time $t$ after which $X$ equals a fraction, say $\frac{1}{2}$, of the final value $X_f(\lambda)$. We may either use the exact or the approximate values of $X_i, X_f$. We obtain

$$e^{\lambda t} \approx \left( \frac{Q}{4\lambda} + X_i(\lambda_0) \right)^{-1} \left( \frac{1}{2} X_f(\lambda) + \frac{Q}{4\lambda} \right)$$

(10.24)

We evaluate (10.24) in the case “$\lambda_0$ small”, “$\lambda$ large”, which implies a considerable increase of $\lambda_0$. In this case

$$X_i(\lambda_0) \approx \frac{1}{2} \left( \frac{Q}{\gamma} \right)^{1/2} \text{ or "exact": } \frac{3}{2} \left( \frac{Q}{\gamma} \right)$$

(10.25)

$$X_f(\lambda) \approx \frac{\lambda}{\gamma} \text{ also"exact"}$$

(10.26)

Using the inequality “$\lambda$ large”, $\lambda^2 \gg Q \gamma$ we may readily evaluate the leading term on the r.h.s. of (10.24), so that

$$e^{\lambda t} \approx \frac{\lambda}{(Q \gamma)^{1/2}} \gg 1$$

(10.27)

which implies a rapid increase of $X(t)$, i.e. $\langle q(t) \rangle$. As we may show, $S$ increases similarly rapidly. To conclude scenario (2), i.e. increase of $\lambda$, we consider the case that both $\lambda_0$ and $\lambda$ are “large” so that

$$X_i(\lambda_0) = \frac{\lambda_0}{\gamma}, X_f(\lambda) = \frac{\lambda}{\gamma}$$

(10.28)
Under the realistic assumption that

\[ Q \ll \lambda \quad (10.29) \]

(possibly excluding the case of an immigration wave, see below), the relation (10.24) reduces to

\[ e^{\lambda t} \approx \frac{1}{2} \left( \frac{\lambda}{\lambda_0} \right) \quad (10.30) \]

Putting \( \lambda = \lambda_0 + \Delta \lambda \) and assuming

\[ \frac{\Delta \lambda}{\lambda_0} < 1 \quad (10.31) \]

we may replace (10.30) by

\[ \lambda t \approx \Delta \lambda_0 \quad (10.32) \]

which implies a moderate growth of \( e^{\lambda t} \).

Our 3rd scenario deals with the impact of an immigration wave where \( Q \) is suddenly and considerably increased. Having the inequality (10.12) in mind, which we interpret in terms of \( Q \), we consider the transition from \( Q_0 \) small, corresponding to “\( \lambda \) large” to \( Q \) large, corresponding to “\( \lambda \) small”. We adopt the notation (10.22) and use (10.24) where \( X_i(Q_0) \) replaces \( X_i(\lambda_0) \). If “\( \lambda \) large”, then

\[ X_i(Q_0) = \frac{\lambda}{\gamma} \quad (10.33) \]

whereas now, “\( \lambda \) small”

\[ X_f(Q) \approx \frac{1}{2} \left( \frac{Q}{\gamma} \right)^{1/2} \quad (10.34) \]

Under the provision that we use (10.33) and (10.34) in (10.24) we may evaluate (10.24)

\[ e^{\lambda t} \approx 1 + \frac{\lambda}{(\gamma Q)^{1/2}} + \ldots \quad (10.35) \]

or equivalently
\[ t \approx \frac{\lambda}{(\gamma Q)^{1/2}} + \ldots \] 

This result is remarkable because the time \( t \) after which saturation sets in, does not depend on the replication rate \( \lambda \), but only on the saturation coefficient \( \gamma \) (representing e.g. accommodation facilities) and the immigration influx proportional to \( Q \). Note that this “observation” is relevant to the situation in Israel immediately following independence, as described above: the first response to the influx of new immigrants was to build “tent settlements” (Ma’abarot in Hebrew, meaning transition settlements). At a later stage they were transformed to “development towns”. Note further that nowadays such “tent towns” are widespread phenomenon in many parts of the world to overcome “saturation limits”.

### 10.4.1 Some Mathematical Details

Using a precise mathematical formulation we write Eq. (10.1) as stochastic differential equation

\[ dq(t) = K(q(t))dt + g(q(t))dv(t) \quad (10.37) \]

where \( \langle dv \rangle = 0, \quad \langle dv(t)dv(t) \rangle = dt \).

By use of Stratonovich calculus the Fokker-Planck-equation belonging to (10.37) reads

\[ \frac{df}{dt} = -\frac{\partial (Kf)}{\partial q} + \frac{1}{2} \frac{\partial}{\partial q} \left( \left( g \frac{\partial g}{\partial q} \right) f + g^2 \frac{\partial f}{\partial q} \right) \quad (10.38) \]

In the steady state

\[ \frac{df}{dt} = 0 \quad (10.39) \]

This allows us to integrate (10.38) which yields, observing the integrability condition

\[ \lim_{q \to \infty} f(q) = 0 \]

\[ -K(q)f + \frac{1}{2} \left( g \frac{\partial g}{\partial q} \right) f + \frac{1}{2} g^2 \frac{\partial f}{\partial q} = 0 \quad (10.40) \]

To make contact with (10.1) with specify

\[ K = \lambda q - \gamma q^2, \quad g = \sqrt{Q} \sqrt{q}, \quad q \geq 0 \quad (10.41) \]
Inserting (10.41) in (10.40) and a slight rearrangement of terms leads us to

\[
\frac{\partial f}{\partial q} = \left( 2Q^{-1}(\lambda - \gamma q) - \frac{1}{2q} \right)f
\]

(10.42)

The solution to (10.42) reads

\[
f(q) = Nq^{-1/2} \exp(\alpha q - \beta q^2)
\]

(10.43)

with \(N\) normalization constant, and

\[
\alpha = \frac{2\lambda}{Q}, \quad \beta = \frac{\gamma}{Q}
\]

(10.44)

The moments \(\langle q^n \rangle, \quad n = 1, 2, \ldots\) can be obtained by derivatives of the partition function

\[
Z = \int_0^\infty q^{-1/2} \exp(\alpha q - \beta q^2) dq
\]

(10.45)

e.g.

\[
\langle q \rangle = \frac{\partial}{\partial \alpha} \ln Z
\]

(10.46)

By means of the transformation, \(q = \xi^2\), \(Z\) can be cast into the form

\[
Z = \int_{-\infty}^\infty \exp(\alpha \xi^2 - \beta \xi^4) d\xi
\]

(10.47)

which has been exactly evaluated. In fact, (10.47) is a special case of the standard integral

\[
\int_{-\infty}^\infty \xi^n \exp(\alpha \xi^{2m} - \beta \xi^{4m}) d\xi = (2m)^{-1}(2\beta)^{-(n+1)/4m} \Gamma\left(\frac{n + 1}{2m}\right) D_{-(n+1/2m)}(K) \exp(K^2/4),
\]

where \(K = -\alpha/\sqrt{2\beta}\), \(\Gamma\) : gamma function, \(D\) parabolic cylinder function, \(n, m\) integers > 0.
Equations (10.14) and (10.15) of the main text can be derived from (10.47) and (10.37) as follows. If $\lambda$ or $\alpha$ [cf. (10.44)] is small, we may expand part of the exponential function in (10.47) in terms of powers of $\alpha$ (Taylor series). If $\lambda$ is large, the integral in (10.47) can be evaluated by the method of steepest descent.

### 10.5 Information Production (IP)

#### 10.5.1 Basic Model

In this Sect. we cast the results of Chap. 4 on SIRNIA, in particular on IP, in a quantitative form. We focus our attention on the production of Shannsonian information (SHI) and pragmatic information (PI) and their interplay. We denote the quantity of SHI, averaged over 24 h, by $s$, and that of PI by $p$. Both $s$ and $p$ are measured in bits and refer to the whole city with $n$ citizens. On average, each citizen has a certain capability of converting information into action. We denote the daily conversion rate by $r$. Then the total amount of PI measured by $p$ increases per day (24 h) by

$$ns \quad (10.48)$$

On the other hand, there is a daily decay of possibilities to perform actions for a variety of reasons. This decay is proportional to $p$ and a rate $\gamma$. We assume that $\gamma$ is a constant, i.e., independent of $p$ and $s$.

Taking (10.48) and the losses $-\gamma p$ together, we arrive at the net production eq. for $p$

$$\frac{dp}{dt} = nrs - \gamma p \quad (10.49)$$

We turn to the production of SHI, i.e., $s$. Each citizen contributes by his/her action to an increase of $s$ at a rate $r$, so that the total increase of $s$ (per day) is given by

$$nr \quad (10.50)$$

Due to “collective forgetting” and other reasons there will be a loss of $s$ at an average rate $r$ that we assume to be constant.

Finally, there are events beyond the control of citizens. These events are of a stochastic nature and contribute to an increase of $s$ (i.e. of uncertainty). We take their effect into account by a random force $F(t)$. Thus our 2nd equation reads

$$\frac{ds}{dt} = n\gamma s + F(t) \quad (10.51)$$
Equations (10.49) and (10.51) form our basic model. In the following we explore its most interesting implications step by step.

10.5.2 Information Explosion

Since this effect exists whether or not there is a fluctuation force, in (10.51) we put $F = 0$. To solve (10.49) and (10.51) we put

$$p = p_0 e^{\lambda t}, \ s = s_0 e^{\lambda t}$$

(10.52)

and obtain, in the standard way,

$$\lambda = -\frac{\Gamma + \gamma}{2} \pm \frac{1}{2} \left( (\Gamma - \gamma)^2 + 4n^2 r r' \right)^{1/2}$$

(10.53)

An instability occurs, if one characteristic value $\lambda > 0$. This happens if

$$n^2 r r' > \gamma \Gamma$$

(10.54)

Provided $r r'$ and $\gamma \Gamma$ are constant, such an instability may happen if the number of citizens exceeds a critical value. One effect might even be the breakdown of the whole information system.

As Eq. (10.54) reveals, the instability can be avoided if we decrease the conversion rates $r$ or/and $r'$, or if we increase the loss rates $\gamma$ or/and $\Gamma$. Below we will treat a realistic example in more detail.

10.5.3 What is the OP?

Since our model contains the two variables $p$ and $s$, we have to invoke a criterion that allows us to identify the OP. To this end, we apply the time-scale separation criterion: The “long living” variable enslaves the “short living” one. But what means “long/short living”? In the context of information it is the memory. Clearly that of a society is much longer than that of an individual. Since SHI refers to the city in total, whereas PI refers to the action of individuals, clearly $\Gamma' \ll \gamma$ so that $s$ must be the relevant OP.

Within the formalism of synergetics, this allows us—in a good approximation to put in (10.49)

$$\frac{dp}{dt} \approx 0$$

(10.55)
so that

\[ p \approx \frac{nrs}{\gamma} \quad (10.56) \]

Inserting (10.56) into (10.51) we obtain

\[ \frac{ds}{dt} = Rs + F(t) \quad (10.57) \]

where

\[ R = \frac{n^2rr'}{\gamma} - \Gamma \quad (10.58) \]

is the net replication rate.

\( R > 0 \) is equivalent to the instability condition (10.54). If \( R < 0 \), a stable, but stochastic, i.e. fluctuating, on average finite state of \( s \) is reached. In view of the actual, realistic situation with ever increasing values of \( p \) and \( s \), this situation is rather unlikely, however. So we deal with the more interesting and important case of \( R > 0 \).

### 10.5.4 What May Curb the Unlimited Growth of \( s \) (and \( p \))? 

From a theoretical point of view this goal can be reached by a lowering of \( R > 0 \) to \( R = 0 \). To achieve this we may think of governmental regulations or “natural” mechanism. In the spirit of our book that deals with selforganization, we disregard governmental regulations (e.g. taxation of SHI production or transmission). Rather our focus is the behavior of citizens who convert SHI into action and is captured by the conversion coefficient \( r \). It is most plausible that a citizen’s capability of converting \( s \) into \( p \) decreases with increasing \( s \). We model this effect by putting

\[ r = r_0(1 + as)^{-1} \approx r_0 - bs \quad (10.59) \]

where \( b > 0 \) is constant.

By inserting our hypothesis into \( R(10.58) \) and thus in (10.59), we obtain after rearrangement of terms

\[ \frac{ds}{dt} = as - Cs^2 + F(t) \quad (10.60) \]

where
\[ a = \frac{n^2 r r'}{\gamma} - \Gamma, \quad C = \frac{n^2 b r'}{\gamma} \] (10.61)

Equation (10.60) becomes identical with the generalized Verhulst equation (10.1) of Sect. 10.4, provided we subject the fluctuating force to the same conditions as \( F \) in (10.2). This is a very natural condition because it gives rise to a constant influx rate as we have shown in Sect. 10.4. Now, we may apply the whole analysis of Sect. 10.3 to the present case of IP provided we identify \( \lambda \rightarrow a, \gamma \rightarrow C, Q \) as before.

### 10.5.5 Concluding Remark on Information Production

Based on plausibility arguments, we have derived equations for the production of Shannon and pragmatic information. Surely, in the future, more detailed theories will be required. Nevertheless, as an important and realistic result we obtained an unlimited growth of SHI and PI, an effect that many decades ago had been termed by Rolf Landauer of IBM “information pollution”. We have considered one possible mechanism that may curb the unlimited growth, namely the increasing inability of people to convert SHI into PI. This situation may change, however, with an increasing “smartification” of cities including homes, offices, etc. We will return to this issue in Chap. 14.

### 10.6 Conclusions

‘PT and the city’, as noted at the outset of this chapter, ‘is a very wide issue’, referring on the one hand, to grand-scale events such as the first emergence of cities, while on the other, to smaller scale urban phenomena such as suburbanization or gentrification noted above. As we’ve seen, in the various scales, the PT processes start bottom-up out of an interplay between two kinds of parameters: CP and OP. Our aim in this chapter was to study the intricate relations between the two. We did so descriptively by reference to case studies and formally by two mathematical models: demographically ‘growing cities’ and urban dynamics as IP. While the mathematization and quantification of the two urban processes gave us a useful tool to explore these complex urban processes, we are fully aware of the existence of other non-quantifiable forces and parameters that play similar roles in PT processes and the city. For example, the planning laws and regulations that stand at the center of the case studies in Sect. 10.2, do not lend themselves to quantification, yet they function as CPs and OPs. The insight from the mathematical models guides us in discussing them qualitatively as will further transpire from Chap. 15 on the planning implications of our study.

A basic element in our synergetic cities view is that the emergence of an urban OP is just ‘the first half of the story’. The second half is that once an OP emerges in
a city, it not only describes the system city but also prescribes, that is “enslaves”, the behavior and action of the parts—the urban agents, and so on in circular causality. While this statement holds quite generally true, it needs further specifications of the kind of city OP we are dealing with. For instance the OP “number of citizens” that we use in our ‘growing cities’ model (Sect. 10.4), steers/regulates the behavior of citizens responding to housing, working, living conditions, i.e. it enslaves a certain “behavioral pattern”. In turn, this OP is brought about by this specific collective behavior. However, there are (many) other behavioral patterns/features of individuals that are influenced by the individuals’ interaction with a social group. An explicit example is the feature “urban regulatory focus” that we will deal with in Chap. 11. In this case, the OP “city size” is not sufficient to capture the whole process, but rather we have to introduce some kind of OP hierarchy. Our approach may also shed light on the phenomenon of radicalization. These relations between the slaving principle, circular causality and urban regulatory focus form the content of the next chapter.

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