Anomalous Gauge-Boson Couplings
in High Energy $ep$ Collisions

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Abstract

We investigate the sensitivity of total cross sections of $e + p \rightarrow W, Z$ to CP-conserving non-standard $WW\gamma$ couplings. We include all the important production mechanisms and study the dependence of the total $W$ cross sections on the anomalous $WW\gamma$ couplings, $\kappa$ and $\lambda$. We argue that the ratio of $W$ and $Z$ production cross sections is particularly well suited, being relatively insensitive to uncertainties in the theoretical and experimental parameters.
1. Introduction

Despite impressive experimental confirmation of the correctness of the Standard Model (SM), the most direct consequence of the $SU(2) \times U(1)$ gauge symmetry, the non-abelian self-couplings of $W, Z$, and photon remains poorly measured to date. Furthermore, gauge boson coupling strengths are strongly constrained by gauge invariance, and are sensitive to deviations from the SM. Hence, experimental bounds on these couplings might shed light on new physics beyond the SM.

In order to parametrize non-standard effects, it is important to know what sort of additional couplings can arise once the restrictions due to gauge invariance are lifted. As has been previously shown $[1]$, there can be 14 or more non-standard couplings in the most general case. To keep the analysis manageable, we restrict ourselves to C, P and $U(1)_{\text{em}}$ conserving couplings. This restriction leads to just two anomalous form-factors, traditionally denoted by $\lambda$ and $\kappa$ in the $WW\gamma$ sector of the SM, which can be related to the anomalous electric quadrupole and the anomalous magnetic dipole moment of the $W$ $[2]$. In the SM at tree level, $\lambda = 0$ and $\kappa = 1$. At present the best experimental limits, $-3.6 < \lambda < 3.5$ and $-3.5 < \kappa < 5.9$, are from a recent analysis of the $W\gamma$ production at $sp\bar{p}$s by UA(2) collaboration $[3]$. While these bounds are compatible with the SM, they are still too weak to really be considered as a precision test of the SM. Furthermore, in the absence of beam polarization, it is unlikely that there will be a significant improvement from the study of $W$ pair production at LEP-II $[4]$.

The photoproduction of a single $W$ boson at $ep$ colliders will provide a very precise test of the structure of the Standard Model $WW\gamma$ vertex. The situation there is much cleaner, for example, than in $pp$ or $p\bar{p}$ colliders, where a $W$ and a photon have to be identified in the final state $[3]$. Theoretical studies of the $WW\gamma$ vertex at $ep$ colliders have been performed $[5, 6]$ to investigate the possibility of measuring the anomalous magnetic moment of the $W$. The measurement of $\kappa$ at $ep$ colliders using the shape of the $p_T$ distribution of $W$ production at large $p_T$ has been previously investigated in $[3]$. However, this method suffers from the
disadvantage of being sensitive to uncalculated higher-order QCD corrections, uncertainties in the parton distribution of the photon, experimental systematic uncertainties, etc [7]. We have previously found [6] that a measurement of the anomalous coupling in the $WW\gamma$ vertex at $ep$ colliders can best be achieved by considering the ratio of the $W$ and $Z$ production cross sections. The advantage of using a cross section ratio is that uncertainties from the luminosity, structure functions, higher-order corrections, QCD scale, etc. tend to cancel [6].

In the present study we investigate the possibility of measuring both $\kappa$ and $\lambda$ at the same time by considering the total cross sections of massive gauge bosons $W$ and $Z$ at $ep$ colliders. We include both the lowest-order resolved processes and the dominant direct photoproduction processes. Care must be taken to avoid double counting those phase-space regions of the direct processes which are already included in the resolved processes. In section 2 we first derive the matrix-element-squared averaged over initial state polarizations for the process $\gamma + q \to q^{(l)} + V, V = W, Z$, which is the main production mechanism of $W$ and $Z$ at $ep$ colliders. Also we discuss the structure of the cross sections by including all the possible processes. In section 3 we present our numerical results for the total cross sections as well as the ratio of $W$ and $Z$ cross sections. It is interesting to note that the direct process $\gamma + q \to q^{(l)} + W$ receives contributions from the triple-boson $WW\gamma$ vertex. This then gives the possibility of studying the vertex to check the Standard Model couplings. Section 4 contains our conclusions.

2. Theoretical Details

If we restrict ourselves to C and P even couplings with electromagnetic gauge invariance, the most general $WW\gamma$ vertex can be parametrized in terms of an effective Lagrangian [1]

$$\mathcal{L}_{\text{eff}}^{WW\gamma} = -ie \left[ (W^\mu_{\mu} W^\nu - W^\mu_{\nu} W^\mu_{\nu}) A^\nu + \kappa W^\mu_{\mu} W^\mu_{\nu} F^\nu_{\nu} + \frac{\lambda}{m_W^2} W^\mu_{\mu} W^\mu_{\nu} F^\nu_{\nu} \right],$$

(1)

where $W^\mu$ and $A^\mu$ stand for the $W^-$ and the photon field, respectively. The parameters $\kappa$ and $\lambda$ are related [2] to the magnetic dipole moment ($\mu_W$) and electric quadrupole moment ($Q_W$) of the $W^+$ by
\[ \mu_w = \frac{e}{2m_w}(1 + \kappa + \lambda) \quad \text{and} \quad Q_w = -\frac{e}{m_w^2}(\kappa - \lambda). \] (2)

At the tree level of the Standard Model, the non-abelian gauge structure only allows for \( \kappa = 1 \) and \( \lambda = 0 \).

We begin with a discussion of the matrix elements for the process \( \gamma + q \to q^{(l)} + V, V = W, Z \) which is the dominant mechanism for \( W, Z \) production at high energy \( ep \) colliders. The relevant helicity amplitude may be obtained directly from Ref. [4]. After squaring, and summing over the helicities, and simplifying the resulting expression, we obtain the hard scattering cross sections

\begin{equation}
\left( \frac{d\hat{\sigma}}{dt} \right)^D (\gamma + q \to q^{(l)} + V) = \frac{1}{16\pi s^2} \Sigma |V|^2, \tag{3a}
\end{equation}

with

\begin{align*}
\Sigma |Z|^2 &= -(g_Z^2 e^2 g_q^2) T_0(\hat{u}, \hat{t}, \hat{s}, m_V^2)/2, \\
\Sigma |W|^2 &= -(g^2 e^2 |V_{qq}|^2) T(\hat{u}, \hat{t}, \hat{s}, m_w^2, Q, \kappa, \lambda)/2, \\
\Sigma |W|_{SM}^2 &= -(g^2 e^2 |V_{qq}|^2) T(\hat{u}, \hat{t}, \hat{s}, m_w^2, Q, 1, 0)/2 \\
&= -(g^2 e^2 |V_{qq}|^2) \left( Q - \frac{s}{s + \hat{t}} \right)^2 T_0(\hat{u}, \hat{t}, \hat{s}, m_w^2)/2, \tag{3b}
\end{align*}

and \( Q = |e_q|, \quad g_q^2 = \frac{1}{2}(1 - 4Q x_w + 8Q^2 x_w^2), \quad x_w = 0.23, \)

where the subscript SM denotes the Standard Model parametrization with \( \kappa = 1, \lambda = 0, \)

and where

\begin{align*}
T_0(\hat{s}, \hat{t}, \hat{u}, m_v^2) &= \frac{(\hat{t}^2 + \hat{u}^2 + 2\hat{s} m_v^2)}{\hat{t} \hat{u}}, \\
T(\hat{s}, \hat{t}, \hat{u}, m_w^2, Q, \kappa, \lambda) &= (Q - 1)^2 \frac{\hat{u}}{\hat{t}} + Q^2 \frac{\hat{t}}{\hat{u}} + 2Q(Q - 1)m_w^2 \frac{\hat{s}}{u t} \\
&- \left( (Q - 1) \frac{1}{\hat{t}} - Q \frac{1}{\hat{u}} \right) (2\hat{s} m_w^2 - (1 + \kappa) \hat{u} t) \frac{1}{m_w^2 - \hat{s}} + \frac{\hat{s}}{2m_w^2} \tag{3c}
\end{align*}

\begin{equation}
- \left[ 2\hat{u}(\hat{u} + \hat{s}) \frac{1}{m_w^2} + (1 + \kappa) \left[ \hat{s} - (\hat{u} + \hat{s})^2 \frac{1}{m_w^2} \right] \right] \frac{1}{2(m_w^2 - \hat{s})} \tag{3d}
\end{equation}
\[
+ \left( 8\hat{u}^2 - 16\hat{s}m_w^2 - 4(1 + \kappa)\hat{u}^2 \left[ 1 + \frac{\hat{s}}{m_w^2} \right] \right) \\
+ (1 + \kappa)^2 \left[ 4\hat{u} \hat{t} + (\hat{u}^2 + \hat{p}^2) \frac{\hat{s}}{m_w^2} \right] \frac{1}{8(m_w^2 - \hat{s})^2} \\
- \lambda^2 \frac{\hat{s}\hat{t}\hat{u}}{2m_w^4(m_w^2 - \hat{s})} + \lambda(2\kappa + \lambda - 2) \frac{\hat{s}}{8m_w^2} \left[ 1 + \frac{2\hat{t}\hat{u}}{(m_w^2 - \hat{s})^2} \right].
\]

By setting the quark charge \( Q = |e_q| = 1 \), we can obtain the matrix elements for the processes, \( \gamma + e \rightarrow \nu + W \) and \( \gamma + e \rightarrow e + Z \). With the definitions of \( Y = \hat{s}/4m_w^2, X = (Y - 1/4)(1 + \cos \theta)/2 \) and \( \chi = 1 - \kappa \), the differential cross section with respect to \( \hat{\theta} \), the angle between the outgoing \( W \) and the incoming photon is

\[
\frac{d\sigma}{d\cos \theta} (\gamma + q \rightarrow q' + W) = \frac{\pi \alpha^2(Y - 1/4)}{128m_w^4 Y^2(Y - X)^2 \sin^2 \theta_w} F(Q = |e_q|), \quad (4a)
\]

where

\[
F(Q) = X \left[ 8Y - 4 + (8X^2 + 4X + 1)/Y \right] \\
-8\chi X(Y + X) - 32\lambda(\lambda - \chi)YX(Y - X) + 64\lambda^2 YX(Y - X)^2 \quad (4b)
\]

\[
+ (\lambda - \chi)^2 \left[ (Y^2 + X^2)(4Y - 4X - 1) + 4XY \right] \\
+ 8\xi_Q \left[ -\chi(Y + X) + (\xi_Q + 2X)f \right],
\]

with

\[
\xi_Q = (Y - X)(1 - Q) \quad \text{and} \quad f = [(Y - 1/4)^2 + (X + 1/4)^2] / (XY).
\]

The function \( F(Q = 1) \) represents the matrix element for \( Q = |e_q| = 1 \), i.e. \( \gamma + e \rightarrow \nu + W \). In the Standard Model i.e. \( \lambda = \chi = 0 \), \( F(Q = 1) \) vanishes when the outgoing \( W \) and the incoming photon are antiparallel \( (X = 0) \). And that is the famous radiation zero \[8\]. It is interesting to note that the radiation zero is not a unique feature of the Standard Model. The radiation zero will be present \[9\] whenever

\[
\lambda + \kappa = 1 \quad (\text{or} \quad \lambda = \chi) \quad \text{and} \quad X = 0 \quad (5)
\]

for the process \( \gamma + e \rightarrow \nu + W \).

Next we focus on the total production of \( W \) and \( Z \) in \( ep \) collisions. In the short term these processes will be studied at HERA\((E_e = 30 \text{ GeV}, E_p = 820 \text{ GeV}, \mathcal{L} = 200 \text{ pb}^{-1} \text{ yr}^{-1})\),
while in the long term availability of LEP × LHC (\( E_e = 50 \text{ GeV}, E_p = 8000 \text{ GeV}, \mathcal{L} = 1000 \text{ pb}^{-1} \text{ yr}^{-1} \)) collider will give collision energies in excess of 1 TeV. We first calculate the total cross sections for the five different processes which contribute to single \( W \) and \( Z \) production at \( ep \) colliders. From the sum of these contributions we then calculate the ratio \( \sigma_{\text{total}}(W)/\sigma_{\text{total}}(Z) \) as a function of the anomalous \( WW\gamma \) coupling parameters \( \kappa \) and \( \lambda \). The five processes are

\[
\begin{align*}
e^- + p &\to e^- + W^\pm + X, \\
&\to \nu + W^- + X, \\
&\to e^- + Z + X \text{ (\( Z \) from hadronic vertex),} \\
&\to e^- + Z + X \text{ (\( Z \) from leptonic vertex),} \\
&\to \nu + Z + X.
\end{align*}
\]

The largest contributions for \( W \) and \( Z \) productions come from the processes (6a) and (6c) which are dominated by the real photon exchange Feynman diagrams with a photon emitted from the incoming electron, \( e^- + p \to \gamma/e + p \to V + X \), and have been partly studied as a function of \( \kappa \) in Ref. \[6\]. The dominant subprocesses for \( \gamma + p \to V + X \) would appear to be the lowest order \( \bar{q}^{(l)}_{/\gamma} + q \to V \), where \( q_{/\gamma} \) is a quark inside the photon. However this may not be strictly true, even at very high energies, since quarks inside the photon \( q_{/\gamma} \) exist mainly through the evolution \( \gamma \to q\bar{q} \). Hence the direct process \( \gamma + q \to q^{(l)} + V \) could be competitive with the lowest order contribution \( \bar{q}^{(l)}_{/\gamma} + q \to V \). This raises the subtle question of double counting \[3,10\]. Certain kinematic regions of the direct processes contribute to the evolution of \( q_{/\gamma} \) which is already included in the lowest order process. Both double counting and the mass singularities are removed \[11\] if we subtract the contribution of \( \gamma + q \to q^{(l)} + V \) in which the \( \tilde{t} \)-channel-exchanged quark is on-shell and collinear with the parent photon. Thus the singularity subtracted lowest order contribution from the subprocesses \( \bar{q}^{(l)}_{/\gamma} + q \to V \) is

\[
\sigma^L(e^- + p \to \gamma/e + p \to V + X) = \frac{C_L^2}{s} \int_{m_W^2/s}^{1} \frac{dx_1}{x_1} 
\]
\[ \times \left[ \sum_{qq'} (f_{q/e} - \tilde{f}_{q/e}) (x_1, m_{V}^2) f_{q/p}(x_1 s, m_{V}^2) + (q \leftrightarrow q') \right], \quad (7a) \]

where

\[ C_{LW}^L = \frac{2\pi G_F m_w^2}{3\sqrt{2}} |V_{qq'}|^2, \quad C_{L}^L = \frac{2\pi G_F m_w^2}{3\sqrt{2}} g_{q}^2. \quad (7b) \]

The electron structure functions \( f_{q/e} \) are obtained as usual

\[ f_{q/e}(x, Q^2) = \int_x^1 \frac{dy}{y} f_{q/\gamma}(x y, Q^2) f_{\gamma/e}(y), \quad (8) \]

where \( f_{\gamma/e} \) is the appropriate Weizsäcker-Williams approximation \[12\] of (quasi-real) photon radiation, and \( f_{q/\gamma} \) is the usual photon structure function. The part of photon structure function, \( \tilde{f}_{q/\gamma} \), which results from photon splitting at large \( x \) (with large momentum transfer), has the leading order form as

\[ \tilde{f}_{q/\gamma}^{(0)}(x, Q^2) = \frac{3\alpha e_q^2}{2\pi} (1 - 2x + 2x^2) \log \left( \frac{Q^2}{\Lambda^2} \right), \]

and as before \( \tilde{f}_{q/e}(x, Q^2) = \int_x^1 \frac{dy}{y} \tilde{f}_{q/\gamma}^{(0)}(x y, Q^2) f_{\gamma/e}(y) \). \quad (9)

To obtain the total contribution from the direct subprocess, \( \gamma + q \rightarrow q^{(0)} + V \), we must integrate Eq. (3), regularizing the \( \hat{t} \)-pole of the collinear singularity by cutting at the scale \( \Lambda^2 \) which determines the running of the photon structure functions \( f_{i/\gamma} \). This corresponds to the subtraction used to redefine the photon structure functions in Eq. (7a). Then the hard scattering cross sections from the direct subprocesses are

\[ \hat{\sigma}(\gamma + q \rightarrow q^{(0)} + V) = \frac{C_{V}^D}{\hat{s}} \eta_{V}, \quad (10a) \]

where

\[ \eta_{z}(\hat{s}, m_{z}^2, \Lambda^2) = (1 - 2z + 2z^2) \log \left( \frac{\hat{s} - m_{z}^2}{\Lambda^2} \right) + \frac{1}{2} (1 + 2z - 3z^2), \]

\[ \eta_{W}(\hat{s}, m_{W}^2, \Lambda^2, Q = |e_q|, \kappa, \lambda) = (Q - 1)^2 (1 - 2z + 2z^2) \log \left( \frac{\hat{s} - m_{W}^2}{\Lambda^2} \right) \]

\[ - \left[ (1 - 2z + 2z^2) - 2Q(1 + \kappa + 2z^2) + \frac{(1 - \kappa)^2}{4z} - \frac{(1 + \kappa)^2}{4} \right] \log z \quad (10b) \]

6
\[
+ \left[ \left( 2\kappa + \frac{(1 - \kappa)^2}{16} \right) \frac{1}{z} + \left( \frac{1}{2} + \frac{3(1 + Q^2)}{2} \right) z + (1 + \kappa)Q - \frac{(1 - \kappa)^2}{16} + \frac{Q^2}{2} \right] (1 - z) \\
- \frac{\lambda^2}{4z^2} (z^2 - 2z \log z - 1) + \frac{\lambda}{16z} (2\kappa + \lambda - 2) [(z - 1)(z - 9) + 4(z + 1) \log z],
\]

with
\[
C_W^D = \frac{\alpha G_F m_W^2}{\sqrt{2}} |V_{qq}|^2, \quad C_Z^D = \frac{\alpha G_F m_Z^2}{\sqrt{2}} g_q^2 e_q^2 \quad \text{and} \quad z = \frac{m_V^2}{s}.
\]

The first terms in the \( \eta_{V=W,Z} \) represent the collinear singularity from the \( \hat{t} \)-pole exchange, which is related to the photon structure-function of Eq. (9). This is the singularity that has already been subtracted in Eq. (7), and so we can now add the two contributions, Eqs. (7) and (11), without double counting. The total contribution from the direct subprocess \( \gamma + q \to q^{(l)} + V \) is
\[
\sigma^D(e^- + p \to \gamma/e^- + p \to V + X) = \frac{C_D}{s} \int_{m_{W}^2/s}^{1} \frac{dx_1}{x_1} \int_{m_{Z}^2/x_1s}^{1} \frac{dx_2}{x_2} \times \left[ \sum_q f_{\gamma/e}(x_1, Q^2) f_{q/p}(x_2, Q^2) \right] \eta_V(\hat{s} = x_1 x_2 s).
\]

The processes (6b) and (6d) are dominated by configurations where a (quasi-real) photon is emitted (either elastically or quasi-elastically) from the incoming proton and subsequently scatters off the incoming electron, i.e. \( e^- + p \to e^- + \gamma/p \to e^- (\text{or } \nu) + V \). For the elastic photon, the cross section can be computed using the electrical and magnetic form factors of the proton. For the quasi-elastic scattering photon, the experimental information [13] on electromagnetic structure functions \( W_1 \) and \( W_2 \) can be used, following Ref. [14]. The hard scattering cross section is given by
\[
\dot{\sigma}(e^- + \gamma/p \to e^- (\text{or } \nu) + V) = \frac{C_D}{s} \eta_V(Q = |e_q| = 1).
\]

For process (6e), which is a pure charged current process, we simply use the results of Bauer et. al. [14] to add to the contributions from (6c) and (6d). The contribution from this process to the total \( Z \) production cross section is almost negligible even at LEP \( \times \) LHC \( ep \) collider energies, as can be seen in Table 2.

3. Numerical Results and Discussions
In Table 1 we show the total $W^\pm$ production cross section at HERA and LEP $\times$ LHC $ep$ colliders for a range of values of the anomalous $WW\gamma$ coupling parameters $\kappa$ and $\lambda$. The error range represents the variation in the cross section by varying the theoretical input parameters as follows: $m_V^2/10 \leq Q^2 \leq m_V^2$, photon structure functions $f_{q/\gamma}$ from DG [15] and DO+VMD [14], and proton structure functions $f_{q/p}$ from EHLQ1 [17] and HMRS(B) [18]. It is important to note that once photoproduction experiments at HERA determine $f_{q/\gamma}$ and $f_{q/p}$ more precisely, we will be able to predict the total cross sections for each process with much greater accuracy. The subtraction terms $\tilde{f}_{q/\gamma}$ of Eq. (7) have been calculated using the leading order photon splitting function as in Eq. (9).

We show in Table 2 the cross sections for the various $Z$ production channels at the HERA and LEP $\times$ LHC $ep$ colliders. The errors represent the variation in cross sections obtained by varying the input parameters, as in Table 1. With the anticipated luminosities of $\mathcal{L} = 200 \, \text{pb}^{-1}\text{yr}^{-1}$ (HERA) and $\mathcal{L} = 1000 \, \text{pb}^{-1}\text{yr}^{-1}$ (LEP $\times$ LHC), the total $Z$ production cross section corresponds to 84 events/yr (HERA) and 5400 events/yr (LEP $\times$ LHC). After including a 6.7% leptonic branching ratio (i.e. $Z \rightarrow e^+e^-, \mu^+\mu^-$), the event numbers become about 6 events/yr (HERA) and 360 events/yr (LEP $\times$ LHC).

In Table 3 we show the ratio $\sigma(W^+)/\sigma(Z)$, $\sigma(W^-)/\sigma(Z)$ and $\sigma(W^++W^-)/\sigma(Z)$ for the various values of $\kappa$ and $\lambda$. The input parameters have been varied as in Table 1. Note also that we have not included the uncertainties due to higher order perturbative QCD corrections. While these are expected to have non-negligible effect on the absolute $W$ and $Z$ cross sections - as in $pp$ and $p\bar{p}$ collisions - they are expected to largely cancel in the $W/Z$ cross section ratio, since to a first approximation the gluons are blind to the quark flavor. In Fig. 1, the ratio of $\sigma(W^+)/\sigma(Z)$ and $\sigma(W^-)/\sigma(Z)$ is shown as a function of $\kappa$ and $\lambda$. Rather than vary both parameters simultaneously, we first set $\kappa$ to its Standard Model value and then vary $\lambda$ and vice versa. Assuming $\kappa = 1$ as in Table 1 and Fig 1(a), the $W$ production cross section and the ratio $\sigma(W^+)/\sigma(Z)$ may be used to determine $\lambda$ up to a sign, since when $\kappa = 1$, the dependence on $\lambda$ is quadratic, i.e. $\sigma(W^+)_\kappa=1 \propto \lambda^2$, as can be easily seen in Eqs. (3) and (4). Hence the sign ambiguity can be resolved only by studying a separate
process, such as $W$ pair production in LEP-II $e^+e^-$ collider. Notice also from Fig 1(b) that $\sigma(W^\pm)_{\lambda=0} \propto a\kappa^2 + b\kappa$, and the $W$ cross section has a minimum at $\kappa \approx -0.5$. This means that there is another value $\kappa \approx -2$ which gives the same cross section as the Standard Model value $\kappa = 1$. In Fig. 2, we show three dimensional bar charts of ratio $\sigma(W^\pm)/\sigma(Z)$ at HERA, and $\sigma(W^-)/\sigma(Z)$ at LEP $\times$ LHC. The theoretical input parameters have been fixed: $Q^2 = m_V^2$, $f_{q/\gamma}$ from DG [15], and $f_{q/p}$ from EHLQ1 [17].

To obtain an experimentally measurable ratio $\sigma(ep \rightarrow W^\pm \rightarrow l\nu)/\sigma(ep \rightarrow Z \rightarrow l^+l^-)$ we must multiply the cross section ratio $\sigma(W)/\sigma(Z)$ by the leptonic branching ratio factor

$$R_{BR}(m_t > m_W - m_b, N_\nu = 3) \equiv \frac{BR(W^\pm \rightarrow l\nu)}{BR(Z^\pm \rightarrow l^+l^-)} = 3.23. \quad (13)$$

After 5 years of running, HERA will produce about $30 e+p \rightarrow Z + X \rightarrow l^+ + l^- + X$ events, and this will enable us to determine $\kappa$ and $\lambda$ with a precision of order

$$\Delta \kappa \approx \pm 0.3 \quad \text{for} \quad \lambda = 0,$$

$$\Delta \lambda \approx \pm 0.8 \quad \text{for} \quad \kappa = 1,$$ \quad (14)

which are comparable with the expected constraints from the future LEP-II $e^+e^-$ experiment. At LEP $\times$ LHC, one year’s running will give

$$\Delta \kappa \approx \pm 0.2 \quad \text{for} \quad \lambda = 0,$$

$$\Delta \lambda \approx \pm 0.3 \quad \text{for} \quad \kappa = 1.$$ \quad (15)

4. Conclusion

We have shown how measurements of weak boson production at high energy electron-proton colliders can provide important information on anomalous $WW\gamma$ couplings. We have analyzed the production of massive gauge bosons - $W$ and $Z$. We have included both direct and indirect processes, involving the parton structure of the photon, taking careful account of the double counting problem for the latter. We have also argued that the ratio of $W$ and $Z$ production cross sections is particularly suited to an experimental determination of the
anomalous $WW\gamma$ coupling parameters $\kappa$ and $\lambda$, being relatively insensitive to uncertainties in the theoretical input parameters. In fact, with more precise measurements of these parameters in the next few years - in particular the photon structure functions - the errors in the measured $\kappa$ and $\lambda$ values will ultimately be obtained by the statistical error from the small number of $Z$ events. In this respect, the higher energy LEP × LHC collider offers a significant improvement. Finally we note that our estimated precision on $\kappa$ and $\lambda$ for both $ep$ colliders, Eqs. (14) and (15), is an order of magnitude greater than existing measurements from $W\gamma$ production at $p\bar{p}$ collider [3].

Attempts are at present under way by many authors to constrain the parameter space of $\lambda$ and $\kappa$ by considering various experimental results; production of $W + \gamma$ at $p\bar{p}$ collider [19], process $\gamma e \rightarrow W\nu$ at future $e^+e^-$ and $e\gamma$ colliders [9,20], and also from present low energy data [21]. And those approaches should be regarded as complementary in the efforts to find new physics beyond the Standard Model.

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Table Captions

Table 1. Total $W$-production cross sections (in pb) at HERA and at LEP × LHC, as a function of anomalous $WW\gamma$ coupling parameters $\kappa$ and $\lambda$. The error range represents the uncertainties in the cross sections by varying the theoretical input parameters: $m_V^2/10 \leq Q^2 \leq m_V^2$, photon structure functions $f_{q/\gamma}$ (DG[15] and DO+VMD[16]), and proton structure functions $f_{q/p}$ (EHLQ1[17] and HMRS(B)[18]).

Table 2. The cross sections (in pb) for the various $Z$ production channels at HERA and LEP × LHC $ep$ colliders. The errors represent the variation in cross sections obtained by varying the theoretical input parameters, as in Table 1.

Table 3. Production cross section ratio of $W/Z$ as a function of $\kappa$ and $\lambda$ at HERA and LEP × LHC. We first set $\lambda$ to its Standard Model values ($\lambda = 0$) and then vary $\lambda$ and vice versa.
### HERA W-production Cross-section (in pb)

|        | $ep \to W^+X$ | $ep \to W^-X$ | $ep \to W^\pm X$ |
|--------|----------------|----------------|------------------|
| $\lambda = 0, \kappa = 0.0$ | 0.43 ± 0.08 | 0.47 ± 0.08 | 0.86 ± 0.12 |
| $\lambda = 0, \kappa = 0.5$ | 0.49 ± 0.08 | 0.51 ± 0.07 | 0.97 ± 0.12 |
| $\lambda = 0, \kappa = 1.0$ | 0.59 ± 0.08 | 0.59 ± 0.07 | 1.15 ± 0.12 |
| $\lambda = 0, \kappa = 1.5$ | 0.72 ± 0.08 | 0.70 ± 0.08 | 1.39 ± 0.12 |
| $\lambda = 0, \kappa = 2.0$ | 0.88 ± 0.08 | 0.85 ± 0.09 | 1.69 ± 0.13 |
| $\lambda = 0.0, \kappa = 1$ | 0.59 ± 0.08 | 0.59 ± 0.08 | 1.15 ± 0.12 |
| $\lambda = 0.5, \kappa = 1$ | 0.60 ± 0.08 | 0.61 ± 0.08 | 1.17 ± 0.12 |
| $\lambda = 1.0, \kappa = 1$ | 0.63 ± 0.08 | 0.64 ± 0.08 | 1.23 ± 0.12 |
| $\lambda = 1.5, \kappa = 1$ | 0.68 ± 0.08 | 0.68 ± 0.08 | 1.32 ± 0.13 |
| $\lambda = 2.0, \kappa = 1$ | 0.75 ± 0.08 | 0.74 ± 0.07 | 1.46 ± 0.13 |

### LEP×LHC W-production Cross-section (in pb)

|        | $ep \to W^+X$ | $ep \to W^-X$ | $ep \to W^\pm X$ |
|--------|----------------|----------------|------------------|
| $\lambda = 0, \kappa = 0.0$ | 6.94 ± 2.09 | 8.05 ± 0.82 | 14.99 ± 2.91 |
| $\lambda = 0, \kappa = 0.5$ | 8.65 ± 1.65 | 9.49 ± 0.70 | 18.13 ± 2.36 |
| $\lambda = 0, \kappa = 1.0$ | 11.24 ± 1.46 | 12.12 ± 0.86 | 23.36 ± 2.33 |
| $\lambda = 0, \kappa = 1.5$ | 14.67 ± 1.47 | 16.14 ± 1.02 | 30.81 ± 2.49 |
| $\lambda = 0, \kappa = 2.0$ | 19.06 ± 1.80 | 21.34 ± 1.37 | 40.40 ± 3.17 |
| $\lambda = 0.0, \kappa = 1$ | 11.24 ± 1.46 | 12.13 ± 0.85 | 23.37 ± 2.31 |
| $\lambda = 0.5, \kappa = 1$ | 12.87 ± 1.46 | 14.15 ± 0.95 | 27.02 ± 2.41 |
| $\lambda = 1.0, \kappa = 1$ | 17.79 ± 1.49 | 20.69 ± 0.81 | 38.47 ± 2.29 |
| $\lambda = 1.5, \kappa = 1$ | 25.97 ± 1.51 | 31.12 ± 1.09 | 57.08 ± 2.60 |
| $\lambda = 2.0, \kappa = 1$ | 37.52 ± 1.65 | 45.89 ± 1.26 | 83.40 ± 2.91 |

Table 1.
### Z-production Cross-sections (in pb)

| Process                  | HERA      | LEP×LHC   |
|--------------------------|-----------|-----------|
| $e p \rightarrow e ZX$ (hadronic) | $0.25 \pm 0.05$ | $3.61 \pm 0.59$ |
| $e p \rightarrow e ZX$ (leptonic)  | $0.16$    | $1.17$    |
| $e p \rightarrow \nu ZX$       | $0.004$   | $0.61$    |
| $e p \rightarrow ZX$          | $0.42 \pm 0.05$ | $5.39 \pm 0.59$ |

Table 2.
### HERA $W$-production Ratio

| $\lambda$ | $\kappa$ | $\sigma(W^+)/\sigma(Z)$ | $\sigma(W^-)/\sigma(Z)$ | $\sigma(W^\pm)/\sigma(Z)$ |
|-----------|----------|------------------------|------------------------|--------------------------|
| 0.0       | 0.0      | 1.12 ± 0.11            | 1.29 ± 0.16            | 2.32 ± 0.18              |
| 0.5       | 0.0      | 1.29 ± 0.10            | 1.41 ± 0.14            | 2.60 ± 0.16              |
| 1.0       | 0.0      | 1.55 ± 0.09            | 1.61 ± 0.15            | 3.11 ± 0.16              |
| 1.5       | 0.0      | 1.90 ± 0.07            | 1.91 ± 0.17            | 3.78 ± 0.15              |
| 2.0       | 0.0      | 2.34 ± 0.05            | 2.28 ± 0.21            | 4.61 ± 0.20              |
| 0.0       | 0.5      | 1.55 ± 0.09            | 1.62 ± 0.16            | 3.12 ± 0.16              |
| 0.5       | 0.5      | 1.58 ± 0.08            | 1.67 ± 0.18            | 3.19 ± 0.18              |
| 1.0       | 0.5      | 1.66 ± 0.08            | 1.73 ± 0.17            | 3.34 ± 0.17              |
| 1.5       | 0.5      | 1.79 ± 0.07            | 1.86 ± 0.18            | 3.63 ± 0.19              |
| 2.0       | 0.5      | 1.99 ± 0.06            | 2.00 ± 0.16            | 3.97 ± 0.14              |

### LEP×LHC $W$-production Ratio

| $\lambda$ | $\kappa$ | $\sigma(W^+)/\sigma(Z)$ | $\sigma(W^-)/\sigma(Z)$ | $\sigma(W^\pm)/\sigma(Z)$ |
|-----------|----------|------------------------|------------------------|--------------------------|
| 0.0       | 0.0      | 1.38 ± 0.28            | 1.62 ± 0.06            | 3.00 ± 0.27              |
| 0.5       | 0.0      | 1.73 ± 0.15            | 1.90 ± 0.09            | 3.68 ± 0.11              |
| 1.0       | 0.0      | 2.27 ± 0.06            | 2.43 ± 0.12            | 4.71 ± 0.18              |
| 1.5       | 0.0      | 2.97 ± 0.12            | 3.27 ± 0.19            | 6.24 ± 0.31              |
| 2.0       | 0.0      | 3.89 ± 0.20            | 4.38 ± 0.27            | 8.27 ± 0.47              |
| 0.0       | 0.5      | 2.27 ± 0.06            | 2.43 ± 0.12            | 4.71 ± 0.18              |
| 0.5       | 0.5      | 2.61 ± 0.07            | 2.87 ± 0.15            | 5.48 ± 0.22              |
| 1.0       | 0.5      | 3.62 ± 0.13            | 4.22 ± 0.28            | 7.82 ± 0.37              |
| 1.5       | 0.5      | 5.28 ± 0.25            | 6.35 ± 0.45            | 11.63 ± 0.70             |
| 2.0       | 0.5      | 7.65 ± 0.47            | 9.37 ± 0.74            | 17.01 ± 1.20             |

Table 3.
Figure Captions

Fig. 1. Total production cross section ratios $\sigma(W^-)/\sigma(Z)$ and $\sigma(W^+ + W^-)/\sigma(Z)$ as a function of (a) $\lambda$ ($\kappa = 1$), and (b) $\kappa$ ($\lambda = 0$) at the HERA and LEP × LHC $ep$ colliders. The errors represent the variation in cross sections obtained by varying the theoretical input parameters, as in Table 1. The experimentally measured ratio $\sigma(ep \rightarrow W \rightarrow l\nu)/\sigma(ep \rightarrow Z \rightarrow l^+l^-)$ ($l = e, \mu$) is obtained by multiplying by the leptonic branching ratio factor $BR(W \rightarrow l\nu)/BR(Z \rightarrow l^+l^-) = 3.23$, assuming $m_t > 75$ GeV and three light neutrino species.

Fig. 2. Three dimensional bar chart of ratio (a) $\sigma(W^+ + W^-)/\sigma(Z)$ at the HERA, and (b) $\sigma(W^-)/\sigma(Z)$ at LEP × LHC. Theoretical input parameters have been fixed: $Q^2 = m_V^2$, $f_{q/s}$ from DG [15] and $f_{q/p}$ from EHLQ1 [17].