The Mass Distribution of SDSS J1004+4112 Revisited

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(Received 2010 0; accepted 2010 0)

Abstract

We present a strong lens analysis of SDSS J1004+4112, a unique quasar lens produced by a massive cluster of galaxies at $z = 0.68$, using a newly developed software for gravitational lensing. We find that our parametric mass model well reproduces all observations including the positions of quasar images as well as those of multiply imaged galaxies with measured spectroscopic redshifts, time delays between quasar images, and the positions of faint central images. The predicted large total magnification of $\mu \sim 70$ suggests that the lens system is indeed a useful site for studying the fine structure of a distant quasar and its host galaxy. The dark halo component is found to be unimodal centered on the brightest cluster galaxy and the Chandra X-ray surface brightness profile. In addition, the orientation of the halo component is quite consistent with those of the brightest cluster galaxy and member galaxy distribution, implying that the lensing cluster is a relaxed system. The radial profile of the best-fit mass model is in good agreement with a mass profile inferred from the X-ray observation. While the inner radial slope of the dark halo component is consistent with being $-1$, a clear dependence of the predicted A–D time delay on the slope indicates that an additional time delay measurement will improve constraints on the mass model.

Key words: dark matter — galaxies: clusters: general — galaxies: quasars: individual (SDSS J1004+4112) — gravitational lensing

1. Introduction

SDSS J1004+4112 is a unique quasar lens system (Inada et al. 2003; Oguri et al. 2004; Inada et al. 2005; Inada et al. 2008). A quasar at $z = 1.734$ is multiply imaged into five images, with their maximum image separation of $14''7$, produced by a massive cluster of galaxies at $z = 0.68$. It is one of only two known examples of cluster-scale quasar lenses, the other being the triple lens SDSS J1029+2623 with the maximum image separation of $22''5$ (Inada et al. 2006; Oguri et al. 2008). In addition to the quasar images, SDSS J1004+4112 contains spectroscopically confirmed multiply imaged galaxies at $z \sim 3$ (Sharon et al. 2005). Both the quasar images and the lensing cluster have been detected in Chandra X-ray observations (Ota et al. 2006). Moreover, time delays between some of the quasar images have also been measured from long-term optical monitoring observations (Fohlmeister et al. 2007; Fohlmeister et al. 2008).

Such a wealth of observational data available enable detailed investigations of the central mass distribution of the lensing cluster. Indeed, there have been several attempts to model the mass distribution of SDSS J1004+4112, using either parametric or non-parametric method. Oguri et al. (2004) adopted two-component (halo plus central galaxy) model to successfully reproduce the positions of four quasar images, but even the parities and temporal ordering of the quasar images could not be determined because of model degeneracies. Kawano & Oguri (2006) extended mass modeling along this line, and explored how time delay measurements can distinguish different mass profiles. Fohlmeister et al. (2007) pointed out that it is important to include perturbations from member galaxies to reproduce the observed time delay between quasar images A and B. On the other hand, Williams & Saha (2004) and Saha et al. (2007) performed non-parametric mass modeling to show possible substructures in the lensing cluster. From the non-parametric mass modeling, Saha et al. (2006) and Liesenborgs et al. (2009) concluded that the radial mass profile is consistent with that predicted in N-body simulation (Navarro et al. 1997). We summarize previous mass modeling in Table 1.

In this paper, we revisit strong lens modeling of SDSS J1004+4112 adopting a parametric mass model. We include many observational constrains currently available, including central images of lensed quasars and galaxies, flux rations, and time delay between quasar images (see Table 1). In particular this paper represents first parametric mass modeling that includes both the quasar time delays and the positions of multiply imaged galaxies as constraints. We compare our best-fit mass model with the Chandra X-ray observation of this system (Ota et al. 2006).

This paper is organized as follow. We describe our mass model in §2. We show our results in §3, and give conclusion in §2. A new software for gravitational lensing, which is used for the mass modeling, is presented in Appendix. Throughout the paper we adopt the matter density of

\begin{itemize}
\item Constant: $\Omega = 0.3$, $\Omega_{\Lambda} = 0.7$
\item 2006). A quasar at $z = 1.734$ is multiply imaged into five images, with their maximum image separation of $14''7$, produced by a massive cluster of galaxies at $z = 0.68$. It is one of only two known examples of cluster-scale quasar lenses, the other being the triple lens SDSS J1029+2623 with the maximum image separation of $22''5$ (Inada et al. 2006; Oguri et al. 2008). In addition to the quasar images, SDSS J1004+4112 contains spectroscopically confirmed multiply imaged galaxies at $z \sim 3$ (Sharon et al. 2005). Both the quasar images and the lensing cluster have been detected in Chandra X-ray observations (Ota et al. 2006). Moreover, time delays between some of the quasar images have also been measured from long-term optical monitoring observations (Fohlmeister et al. 2007; Fohlmeister et al. 2008).

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The characteristic density is described as $\rho_c = \frac{\Delta(z) \rho(z) c^3}{3 \pi G m_{\text{nfw}}(c)}$, where $\Delta(z)$ is the nonlinear overdensity at redshift $z$ which we adopt values predicted by the spherical collapse model. The lensing deflection angle is related with the projected two-dimensional mass distribution (i.e., convergence $\kappa$) computed by

$$\kappa(r) = \frac{1}{\Sigma_{\text{crit}}} \int_{-\infty}^{\infty} \rho(\sqrt{r^2 + z^2}) dz,$$

with $\Sigma_{\text{crit}} = (c^2/4\pi G)(D_{ls}/D_{ls} D_{ls})$ being the critical surface mass density computed from angular diameter distances between observer, lens, and source.

We then introduce an ellipticity in the projected mass distribution by replacing $r$ in $\kappa(r)$ by the following quantity:

$$\kappa(r) : r \rightarrow v = \sqrt{\frac{x^2}{1-e} + (1-e)y^2},$$

where $e$ is an ellipticity (the axis ratio is $1-e$), and $x$ and $y$ are defined by

$$x = x \cos \theta_e + y \sin \theta_e,$$

$$y = -x \sin \theta_e + y \cos \theta_e.$$
member galaxies to observed values, and assumes that the velocity dispersions and truncation radii scale with the luminosity:

$$\frac{\sigma}{\sigma_*} = \left( \frac{L}{L_*} \right)^{1/4}, \quad (11)$$

$$\frac{a}{a_*} = \left( \frac{L}{L_*} \right)^{1/2}. \quad (12)$$

The mass-to-light ratio becomes constant with this scaling. We include 14 member galaxies within \( < 20'' \) from the center, which are selected from the gri-band Subaru Suprime-cam images (Oguri et al. 2004). We adopt r-band luminosities for the scaling.

To achieve better fit, we also include additional several perturbations. We consider general perturbations whose lens potentials \( \phi \) are described as (see, e.g., Evans \& Witt 2003; Kawano et al. 2004; Congdon \& Keeton 2005; Yoo et al. 2006)

$$\phi = -\frac{c}{m} r^2 \cos m(\theta - \theta_e - \pi/2). \quad (13)$$

In this paper we include four perturbation terms with \( m = 2 \) (external shear; e.g., Keeton et al. 1997), 3, 4, and 5.

2.2. Observational Constraints

We adopt positions of five quasar images measured by Inada et al. (2005) using the Hubble Space Telescope Advanced Camera for Surveys (HST/ACS) F814W image. Considering possible effects of microlensing or small-scale structure, we adopt conservative positional errors of 0''.04, and also relative magnitudes of 0.3 (0.8) for image B-D (E). In addition, we include measured time delays between image A and B (Fohlmeister et al. 2007) and between image A and C (Fohlmeister et al. 2008). When fitting the time delays, we allow the Hubble constant to vary with a Gaussian prior of \( h = 0.72 \pm 0.04 \).

We also include multiply imaged galaxies, identified by Sharon et al. (2005), as constraints. We revisit deep multi-band HST/ACS images (F435W, F555W, and F814W; GO-10509 and GO-10716), and identify several features associated to each lensed images. We use positions of all these features for our mass modeling. We include central images as well (Liesenborgs et al. 2009). We assume larger positional errors of 0''.4 than those of the quasar images, partly because the determination of the centroids of the extended galaxy images are much less accurate.

Figure 1 shows the HST/ACS image of SDSS J1004+4112, together with the positions of multiple images summarized in Tables 2 and 3. A notable feature of this cluster strong lens system, which can easily be seen in the Figure, is that multiple images are distributed in a very wide range in radius, ranging from the central images very near the cluster center to lensed galaxy images at \( \sim 30'' \) from the cluster center. This is apparently good for constraining the density profile of the lensing cluster.

We also add several Gaussian priors to the mass model. Based on the measurement by Inada et al. (2008), we assume the velocity dispersion of the central galaxy G1 to \( \sigma = 352 \pm 13 \text{ kms}^{-1} \). The position of G1 is fixed to the observed position, (7''.114, 4''.409) in our coordinate system whose origin is at the quasar image A. From the observed position and shape, we assume the ellipticity and the position angle to 0.3 \( \pm 0.05 \) and 152 \( \pm 5 \) deg, respectively. Furthermore we add a weak prior to the truncation

Table 2. Constraints from lensed quasar images

| Name | \( \Delta x ['''] \) | \( \Delta y ['''] \) | \( \Delta m \) | \( \Delta t [\text{days}] \) |
|------|-------------|-------------|-------------|-----------------|
| A    | 0.000       | 0.000       | 0           | 0               |
| B    | -1.317      | 3.532       | 0.35 \( \pm 0.3 \) | -40.6 \( \pm 1.8 \) |
| C    | 11.039      | -4.992      | 0.87 \( \pm 0.3 \) | -821.6 \( \pm 2.1 \) |
| D    | 8.399       | 9.707       | 1.50 \( \pm 0.3 \) | ...             |
| E    | 7.197       | 4.603       | 6.30 \( \pm 0.8 \) | ...             |

The quasar redshift is \( z = 1.734 \). The positional error is assumed to 0''.04 for all the lensed quasar images.

Table 3. Constraints from lensed galaxy images

| Name | \( z_a \) | \( \Delta x ['''] \) | \( \Delta y ['''] \) |
|------|-----------|-------------|-------------|
| A1.1 | 3.33      | 3.93        | -2.78       |
| A1.2 | 1.33      | 19.37       |
| A1.3 | 19.23     | 14.67       |
| A1.4 | 18.83     | 15.87       |
| A1.5 | 6.83      | 3.22        |
| A2.1 | 3.33      | 4.13        | -2.68       |
| A2.2 | 1.93      | 19.87       |
| A2.3 | 19.43     | 14.02       |
| A2.4 | 18.33     | 15.72       |
| A2.5 | 6.83      | 3.12        |
| A3.1 | 3.33      | 4.33        | -1.98       |
| A3.2 | 2.73      | 20.37       |
| A3.3 | 19.95     | 13.04       |
| A3.4 | 18.03     | 15.87       |
| A3.5 | 6.83      | 3.02        |
| B1.1 | 2.73      | 8.88        | -2.16       |
| B1.2 | -5.45     | 15.84       |
| B1.3 | 8.33      | 2.57        |
| B2.1 | 8.45      | -2.26       |
| B2.2 | -5.07     | 16.04       |
| B2.3 | 8.33      | 2.57        |
| C1.1 | 3.28      | 10.25       | -3.06       |
| C1.2 | -7.55     | 15.39       |
| C1.3 | 8.49      | 2.72        |
| C2.1 | 9.95      | -3.36       |
| C2.2 | -7.30     | 15.44       |
| C2.3 | 8.49      | 2.72        |

The positional error is assumed to 0''.04 for all the lensed galaxy images. The redshifts are measured spectroscopically.

\(^1\) Strictly speaking, the velocity dispersion \( \sigma_{\text{obs}} \) computed from the density profile can in principle differ from the input parameter \( \sigma \) for the pseudo Jaffe profile. However, from Eliaßdóttir et al. (2007) we find that \( \sigma \approx \sigma_{\text{obs}} \) for values similar to those in the best-fit model (\( s/a \approx 0.05 \)), which suggests that our assumption of \( \sigma = \sigma_{\text{obs}} \) is reasonable.
radius, \( a = 8'' \pm 4'' \), based on the observed correlation between the velocity dispersion and the truncation radius (Natarajan et al. 2009).

2.3. Model Optimization

We use a software named glafic for all the calculations of lens properties and model optimizations (see Appendix 1). We employ a standard \( \chi^2 \) minimization to find the best-fit mass model. Specifically we adopt a downhill simplex method to find a minimum. To speed up the calculations, we estimate \( \chi^2 \) in the source plane, which is found to be sufficiently accurate for our purpose. See Appendix 2 for detailed discussions about the source plane \( \chi^2 \) minimization.

3. Result

3.1. Best-fit NFW Model

First, we fix the inner slope of the dark halo component to \( \alpha = 1 \) (i.e., the NFW profile) and derive the best-fit mass model. Figure 2 shows best-fit critical curves. It is seen that the best-fit model reproduces the observed multiple image families quite well. In addition, it successfully reproduces observed time delays between quasar images. The best-fit model has \( \chi^2 = 31 \) for 39 degree of freedom, suggesting that our choice of errors was reasonable. The contribution from each source is reasonably similar, \( \chi^2 = 4.7 \) from the quasar, 21 from the galaxy A, 0.9 from galaxy B, and 2.6 from galaxy C. The best-fit model predicts magnifications of the five quasar images to \( \mu_A = 29.7 \), \( \mu_B = 19.6 \), \( \mu_C = 11.6 \), \( \mu_D = 5.8 \), and \( \mu_E = 0.16 \). The total magnification for all the quasar images is \( \mu_{\text{tot}} = 67 \).
The model also predicts the time delay between quasar image A and D to be $\Delta t_{AD} = \Delta t_D - \Delta t_A = 1218$ days, and that between quasar image A and E to be $\Delta t_{AE} = \Delta t_E - \Delta t_A = 1674$ days. The AD time delay is slightly smaller than the lower limit reported by Fohlmeister et al. (2008), $\Delta t_{AD} > 1250$ days.

We find that the best-fit centroid of the dark halo (NFW) component is $(6.92^{+0.32}_{-0.32} - 0.32^{+0.25}_{-0.24}, 4.25^{+0.31}_{-0.26})$ at 95% confidence limit, which is quite consistent with the observed position of G1. The result is in marked contrast with Oguri et al. (2004), in which significant offsets between the center of the dark halo component and that of G1 have been reported based on modeling of quasar images A–D. Such large offset is no longer allowed because of additional constraints from multiply imaged galaxies, time delays, and central images. The best-fit center of the dark halo component is also consistent with the X-ray center reported by Ota et al. (2006). Furthermore, the best-fit position angle of $\theta_\text{e} = 152.9$ deg for the dark halo component is quite consistent with the position angle of G1, and also that of the member galaxy distributions studied in Oguri et al. (2004). The concentricity and alignment between dark matter, BCG, and X-ray implies that the lensing cluster is a highly relaxed system (see also Liesenborgs et al. 2009). The best-fit ellipticity of the halo component is $\epsilon = 0.26$.

The best-fit parameters for perturbations terms are $(\epsilon, \theta_\text{e}) = (0.040, 51.8)$ for $m = 2$, $(0.019, 114)$ for $m = 3$, $(0.013, 47)$ for $m = 4$, and $(0.010, 16.5)$ for $m = 5$. Thus the perturbations are rather small, but they are still important for accurate reproduction of lensed images, particularly for those of the quasar (see also Oguri et al. 2004).

One of the most important quantity to characterize strong lensing system is the Einstein radius $r_{\text{Ein}}$. We compute the Einstein radii $r_{\text{Ein}}$ for our best-fit mass model using the following relation:

$$M(< r_{\text{Ein}}) = \pi r_{\text{Ein}}^2 \Sigma_{\text{crit}}.$$  \hspace{1cm} (14)

We find $r_{\text{Ein}} = 8.9''$14 for the quasar redshift $z = 1.734$, and $13.9''$38 for the redshift of the lensed galaxy A, $z = 3.33$. If we compute $r_{\text{Ein}}$ only from the dark halo component excluding any contributions from galaxies, we obtain $r_{\text{Ein}} = 4.9''84$ for $z = 1.734$ and $10''31$ for $z = 3.33$, which are quite different from those compute from the total mass distribution. This suggests that the effect of the BCG G1 on the lens system is quite significant.

3.2. Comparison with X-ray

Next we compare the best-fit radial mass profile derived from strong lensing with that inferred from the Chandra X-ray observation (Ota et al. 2006). In brief, from the Chandra observation the extended X-ray emission from the lensing cluster was detected out to $\sim 1.5'$ from the cluster center, with the temperature of $\sim 6.4$ keV. Assuming the isothermal profiles, Ota et al. (2006) constrained the projected mass profile and argued that the mass within 100 kpc agrees well with the mass expected from strong lensing. Here we compare our result of new mass modeling with the X-ray result.

Figure 3 compares the projected two-dimensional mass profiles from gravitational lensing and X-ray measurements. We confirm that the profiles agree quite well with each other, including radial slopes of the profiles. The agreement suggests that the effect of the halo triaxiality, which affects the apparent two-dimensional lensing masses particularly near the center of the cluster (see, e.g., Oguri et al. 2005; Gavazzi 2005), is not significant.

However, it should be noted that the best-fit halo mass of $M_{\text{vir}} = 1.0 \times 10^{15} h^{-1} M_\odot$ and the concentration of $c_{-2} = 2.8$ are quite different from those inferred from X-ray, $M_{\text{vir}} \sim 4.3 \times 10^{14} h^{-1} M_\odot$ and $c_{-2} \sim 6.1$. One reason for this the strong degeneracy between $M_{\text{vir}}$ and $c_{-2}$ inherent to strong lens mass modeling. Basically strong lenses constrain the central core mass of the cluster, which is a strong function of both $M_{\text{vir}}$ and $c_{-2}$. The determination of $M_{\text{vir}}$ and $c_{-2}$ solely from strong lensing relies on the extrapolation of the subtle change of the radial slope out to much larger radii. The robust determination of these parameters from lensing therefore requires addition constraints from weak gravitational lensing.

3.3. Generalized NFW Profile

Now we allow the inner slope of the dark halo component $\alpha$ to vary to see how well the current strong lens data, including time delays which are quite helpful to break degeneracy in mass models (e.g., Kawano & Oguri 2006), can constrain the inner density profile. Specifically, for each fixed value of $\alpha$ we optimize the other parameters.
Figure 4 shows the $\chi^2$ difference as a function of $\alpha$. We find that our mass modeling is quite consistent with the NFW profile, i.e., $\alpha = 1$. We constrain the range of the slope to $0.76 < \alpha < 1.41$ at 95% confidence limit. The profile as steep as $\alpha = 1.5$ is clearly rejected. There is a clear correlation between $\alpha$ and the velocity dispersion of galaxy G1 such that the best-fit velocity dispersion decreases with increasing $\alpha$, which approximately conserves the central core mass of the total matter distribution.

As discussed in § 3.2, the Chandra X-ray observation suggests that the lensing cluster may have larger value of the concentration parameter than the best-fit NFW model predicts. We include this effect by adding a prior of $c_{-2} = 6 \pm 1.5$ to the mass model to see how the constraint on $\alpha$ is modified. The result shown in Figure 4 indicates that the constraint is basically shifted to the lower $\alpha$, i.e., shallower inner density slope. The resulting range is $0.62 < \alpha < 1.14$ at 95% confidence limit.

In Figure 4, we also show the total magnification factor for the five quasar images, $\mu_{\text{tot}}$, and the time delay between the quasar images A and D, $\Delta t_{\text{AD}}$, predicted by the best-fit model for each fixed $\alpha$. We find that $\mu_{\text{tot}}$ is decreasing and $\Delta t_{\text{AD}}$ is increasing with increasing $\alpha$, which are consistent with well-known dependence of the magnification and time delay on the radial density slope (e.g., Wambsganss & Paczynski 1994; Oguri & Kawano 2003). Thus the reported lower limit of $\Delta t_{\text{AD}} > 1250$ days (Fohlmeister et al. 2008) prefers a steeper inner slope (larger $\alpha$), which appears to be opposed to the effect of the prior from X-ray discussed above. In either case, the strong dependence of $\Delta t_{\text{AD}}$ on $\alpha$ implies that additional A–D time delay measurements will greatly help to constrain the mass model further.

4. Summary

We have revisited parametric mass modeling of SDSS J1004+4112, a unique quasar–cluster lens system, using a newly developed mass modeling software. We include several new constraints, including positions of spectroscopically confirmed multiply imaged galaxies, time delays between some quasar images, and faint central images. Our model comprising of a halo component modeled by the generalized NFW profile, member galaxies including the brightest cluster galaxy G1, and perturbation terms, well successfully reproduced all observations including time delays.

Unlike earlier claims based on parametric mass modeling, we have found that the center and orientation of the dark halo component is in good agreement with those of member galaxies and Chandra X-ray observation, implying that the cluster is highly relaxed. The radial profile from strong lensing is also in excellent agreement with the mass profile inferred from the X-ray observation. Our mass modeling prefers the dark halo component with the inner slope close to $\alpha = 1$, being consistent with so-called NFW density profile. Additional measurement of the time delay between quasar image A and D will be useful to constrain the mass model further. The predicted total
magnification of $\mu_{\text{tot}} = 67$ for the NFW profile is quite large compared with those for typical galaxy-scale lenses, because of the shallower density profiles for clusters. This makes the lens system a good site for studying the fine structure of the quasar through microlensing (Richards et al. 2004; Green 2006; Lamer et al. 2006; Pooley et al. 2007) or for studying host galaxies of distant quasars (Ross et al. 2009). We note that our result based on parametric mass modeling is broadly consistent with recent non-parametric mass modeling by e.g., Liesenborgs et al. (2009).

Our result suggests that the core of the lensing cluster at $z = 0.68$ is highly evolved. Recently, Limousin et al. (2010) showed that the cluster MACSJ1423.8+2404 at $z = 0.54$ is similarly highly relaxed based on the comparison of mass, light, and gas distributions. Therefore our result may point to the fact that relaxed clusters are quite common already at $z \sim 0.6$.

It is clear that the lensing cluster SDSS J1004+4112 is currently one of the best-studied high-redshift clusters whose inner density profile is very tightly constrained by strong lensing. Additional constraints on this cluster with weak lensing, Sunyaev-Zel’dovich effect, and spectroscopic identifications of member galaxies will be important to advance our understanding of high-redshift clusters.

I would like to thank the referee, Marceau Limousin, for useful comments and suggestion.

**Appendix 1.  Lens Modeling Software glafic**

We have developed a comprehensive software package called glafic that can be used for a wide variety of analysis for gravitational lensing. Its features include efficient computations of lensed images for both point and extended sources, handling of multiple sources, a wide range of lens potentials, and the implementation of noble technique for mass modeling. Currently the software can be downloaded from http://www.slac.stanford.edu/~oguri/glafic/.

In this code, we adopt the adaptive mesh refinement algorithm described in Keeton (2001a) for deriving lensed point source images, although the code uses rectangular coordinates rather than polar coordinates. Critical curves are computed using a marching squares technique (Jullo et al. 2007). In simulations of extended sources, one can convolve point spread functions and include a number of galaxies read from a catalog file, which should also be useful for weak lensing work. For more details, readers are referred to a manual available at the URL above.

**Appendix 2.  Source-plane $\chi^2$ Minimization**

Strong lens modeling with the standard $\chi^2$ minimization is sometimes time-consuming, especially when many lens potential components and images are involved. One way to overcome this problem is to evaluate $\chi^2$ in the source plane instead of the image plane. Although the source plane $\chi^2$ involves approximations (given that observational measurements are always made in the image plane) and therefore is less accurate than the image plane $\chi^2$, it allows one to estimate $\chi^2$ without solving the nonlinear lens equation. This technique has been adopted by several authors (e.g., Kayser 1990; Kochanek 1991; Koopmans et al. 1998; Keeton 2001a; Smith et al. 2005; Bradac et al. 2005; Halkola et al. 2006; Jullo et al. 2007; Sand et al. 2008), although the implementations were quite different for different papers. Here we describe our implementation and argue the accuracy of the technique.

For a given source position $\mathbf{u}$, $\chi^2$ is estimated as

$$\chi^2_{\text{pos}} = \chi^2_{\text{pos}} + \chi^2_{\text{flux}} + \chi^2_{\Delta \chi},$$

(A1)

where $\chi^2_{\text{pos}}$, $\chi^2_{\text{flux}}$, and $\chi^2_{\Delta \chi}$ are computed using a marching squares technique (Jullo et al. 2007) for deriving lensed images called glafic for useful comments and suggestion.

Similarly, the magnification and time delay are approximated as

$$\frac{\partial \mu_i}{\partial x_i} = \frac{\partial \mathbf{u}}{\partial x_i},$$

(A7)

Finally the time delay $\Delta t_i$ is

$$\Delta t_i = \frac{1 + z_i}{c} \frac{D_{ls}}{D_{ls}} \left[ \frac{\mathbf{u} - \mathbf{x}_i}{2} - \phi(x_i) \right].$$

(A8)

Computing $\chi^2$ in equation (A1) usually requires the derivation of $\mathbf{x}_i$ for a given $\mathbf{u}$, which is the most time-consuming part because the lens equation is not one-to-one mapping and thus the extensive solution finding in the image plane is needed. In the source plane $\chi^2$ technique, we do not solve the lens equation but just approximate $\chi^2$ as follows. Assuming $\mathbf{x}_i$ and $\mathbf{x}_{i,\text{obs}}$ are close with each other, we can write

$$\mathbf{x}_{i,\text{obs}} - \mathbf{x}_i \approx M_i (\mathbf{u}_{i,\text{obs}} - \mathbf{u}),$$

(A9)

where $\mathbf{u}_{i,\text{obs}}$ is the source position computed from the observed $i$-th image position:

$$\mathbf{u}_{i,\text{obs}} = \mathbf{x}_{i,\text{obs}} - \nabla \phi(x_{i,\text{obs}}),$$

(A10)

and $M_i$ is estimated at $\mathbf{x} = \mathbf{x}_{i,\text{obs}}$. Then equation (A2) becomes

$$\chi^2_{\text{pos}} = \chi_{\text{pos,src}}^2 = \frac{(\mathbf{u}_{i,\text{obs}} - \mathbf{u})^T M_i^2 (\mathbf{u}_{i,\text{obs}} - \mathbf{u})}{\sigma_i^2}.$$  

(A11)
We adopt a four-image system using the same mass model. We consider a mass model consisting of an SIE and external shear. The model parameters are $m$, $\chi_{\text{pos}}^2$, $\chi_{\text{src}}^2$, $\chi_{\Delta \text{src}}^2$, and $\chi_{\Delta \text{img}}^2$, to see the difference of these. The result shown in Figure 5 indicates that $\chi_{\text{src}}^2$ agrees well with $\chi_{\text{img}}^2$ within a few percent level, which are sufficiently accurate to derive the best-fit mass model and errors on best-fit parameters. We note that $\chi_{\text{src}}^2$ is even more accurate when observational constraints are tighter.

We note that Halkola et al. (2006) also studied accuracy of the source plane $\chi^2$ based on modeling of the strong lens cluster A1689. They found a clear correlation between $\chi_{\text{img}}^2$ and $\chi_{\text{src}}^2$, but with different values. This is because they did not take account of the full magnification tensor (eq. [A7]) but simply adopted a magnification factor to approximate $\chi^2$, as has been often done in the literature. Thus our result highlights the quantitative importance of the proper approximation taking the full lens mapping into account.

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