Testing solar surface flux transport models in the first days after active region emergence

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ABSTRACT

Context. Active regions (ARs) play an important role in the magnetic dynamics of the Sun. Solar surface flux transport models (SFTMs) are used to describe the evolution of the radial magnetic field at the solar surface. The models are kinematic in the sense that the radial component of the magnetic field behaves as passively advected corks. There is, however, uncertainty about using these models in the early stage of AR evolution, where dynamic effects might be important.

Aims. We aim to test the applicability of SFTMs in the first days after the emergence of ARs by comparing them with observations.

Methods. We simulated the evolution of the surface magnetic field of 17 emerging ARs using a local surface flux transport simulation. The models we employ range from passive evolution to models where the inflows around ARs are included.

Results. We find that the simulations using observed surface flows can reproduce the evolution of the observed magnetic flux. The effect of buffeting the field by supergranulation can be described as a diffusion process. The SFTM is applicable after 90% of the peak total unsigned flux of the AR has emerged. Diffusivities in the range between \( D = 250 - 720 \text{ km}^2 \text{ s}^{-1} \) are consistent with the evolution of the AR flux in the first five days after this time. We find that the converging flows around emerging ARs are not important for the evolution of the total flux of the AR in these first five days; their effect of increasing flux cancellation is balanced by the decrease in flux transport away from the AR.

Key words. Sun: activity – Sun: magnetic fields

1. Introduction

Active regions (hereafter ARs) are the surface signature of magnetic flux rising from the interior of the Sun. They are the site of eruptive events such as jets and flares, and they play an important role in the solar dynamo.

During the emergence of ARs, their magnetic polarities move apart and develop a tilt angle, with the leading polarity closer to the equator than the trailing polarity (e.g., Schunker et al. 2020, for a review see van Driel-Gesztelyi & Green 2015). This is consistent with the footpoints of the flux being connected to the subsurface field and separating due to the action of magnetic tension and the drag force (Chen et al. 2017; Schunker et al. 2019). Schunker et al. (2019) calculated separation speeds of AR polarities and suggest that during emergence, the magnetic tension and drag force play a stronger role in transporting the magnetic field than diffusion. They also found that the scatter in polarity positions increases with time consistently with buffeting by supergranulation.

Studying the surface magnetic field of ARs helps to understand their evolution and the buildup of the poloidal field in the solar cycle. In addition to the systematic motions from, for example, magnetic tension, the processes on the solar surface that displace magnetic flux are the random motions of convective motions can be treated as a diffusion process (Leighton 1964), which can be implemented in surface flux transport models (SFTMs) as a random walk. Estimates of the diffusion rate \( D \) from observations typically indicate \( D = 250 \text{ km}^2 \text{ s}^{-1} \) (Jafarzadeh et al. 2014), but higher values up to \( D = 500 \text{ km}^2 \text{ s}^{-1} \) have also been reported (Wang et al. 2002; Yeates 2020).

Inflows on various spatial and temporal scales around evolved ARs have been measured by, for example, Gizon et al. (2001), Haber et al. (2004), Komm et al. (2012), Löptien et al. (2017), and Braun (2019). They span approximately 10° from the AR and have velocities of about 50 m s\(^{-1}\). It is thought that the inflows may be driven by increased cooling in ARs (Spruit 2003). Cameron & Schüssler (2012) propose these inflows as a possible mechanism for a nonlinearity that regulates the solar cycle strengths in the solar dynamo. Recently, Gottschling et al. (2021) measured the evolution of the flows around emerging ARs from before to up to seven days after emergence, finding...
that the time between the AR emergence and the time at which inflows set in after the emergence increases with the total magnetic field of the AR. These inflows have velocities of about 50 m s$^{-1}$ as well, but they appear to be smaller in extent than the inflows around evolved ARs. Gottschling et al. (2021) found no strong dependence of the amplitude of these inflows on the field strength of the ARs. The nature of these observed inflows in the first days after emergence is not clear, and their driving mechanism could be different from that of the inflows around evolved ARs. They could be the result of a passive emergence, in which the rising flux is affected by the supergranulation pattern (Birch et al. 2019). Another scenario is that they are driven by the magnetic tension that moves the polarities apart in the first days after emergence, see Cameron et al. (2010) and Schunker et al. (2019). In a three-dimensional magnetohydrodynamic (MHD) simulation of a rising flux tube in a rotating convection zone, Abbott et al. (2001) also found converging flows.

Several studies have incorporated inflows around ARs in surface flux transport models (De Rosa & Schrijver 2006; Jiang et al. 2010; Cameron & Schüssler 2012; Yeates 2014; Martin-Belda & Cameron 2016, 2017). Martin-Belda & Cameron (2016) found that the inflows enhance flux cancellation and that they can, in conjunction with differential rotation, produce a net tilt angle. This tilt is, however, too small compared to observed tilt angles. Martin-Belda & Cameron (2017) investigated the effect on the large-scale field and found that the inflows can lead to a reduction in the axial dipole moment of 30%. The inclusion of inflows into global SFTMs improves the match to the global dipole for solar cycles 13–21 (Cameron & Schüssler 2012), and it can account for the excess strength of the polar field at activity minimum in simulations by effectively reducing the tilt angle of ARs (Cameron et al. 2010; Yeates 2014). However, Yeates (2020) has recently argued that this excess strength can be a result of the bipolar approximation of the ARs. On the other hand, Yeates (2014) found that the incorporation of the inflows (in form of a perturbation of the meridional flow) delays the dipole reversal times for solar cycle 23 with respect to the observed cycle.

The above studies used simple mathematical descriptions as parametrizations of the inflows. They were included either as a perturbation of the meridional flow at active latitudes (Jiang et al. 2010; Cameron & Schüssler 2012; Yeates 2014), or as the gradient of the magnetic field (De Rosa & Schrijver 2006), with a normalization such that the extent and the amplitude of the inflows are similar to the observed values. The latter, however, raised the problem that the flux of an isolated AR got “trapped” by the inflow field due to the inward-directed flows from all sides, such that the flux cannot escape the AR. The flux is pushed into small, long-lived clumps, which are not observed. Part of this effect might however be caused by the flux-dependent diffusion that was used (Martin-Belda & Cameron 2016).

For this study, we used a local surface flux transport model to simulate the evolution of the magnetic field of ARs, and compared it to the observed evolution. For this, we considered a sample of emerging ARs that take several days to cross the visible disk after the bulk of flux has emerged. The simulations include transport by diffusion and by advection due to surface flows. We tested different models for both. For the advection, we used observed flow maps from correlation tracking of solar granules as well as flow parametrizations from the literature, motivated by the resemblance of the observed flows in the first days after emergence with the inflows around ARs.

This paper is structured as follows. In Sect. 2, we describe the sample of ARs on which we carried out the flux transport simulation. In Sect. 3, we describe the SFTM simulation that we used, as well as the different models of the flow field and the diffusion. Section 4 presents the results, followed by a discussion in Sect. 5.

## 2. Active region sample

We identified ARs that emerge into the quiet Sun and remain on the visible disk for multiple days after their emergence using the Solar Dynamics Observatory Helioseismic Emerging Active Regions (SDO/HEAR) survey (Schunker et al. 2016). The survey consists of 182 emerging ARs that were observed between up to seven days before and after the time of emergence $t_0$, at which the region reaches 10% of the maximum total unsigned flux within the first 36 h after first appearance in the NOAA record.

From the 182 ARs in the HEAR survey, we selected those regions that do not develop a fully fledged sunspot with a clear penumbra over the disk passage. Fully fledged spots show moat flow signatures (Sheeley 1972). At the grid scale that we used in our simulation, which is limited by the observed flows (see Sect. 3.1), the moat flow spatially overlaps with the magnetic field of the spots. In the simulation, this would lead to a disruption of the spots into ring-like structures, which is inconsistent with observations. The sunspot identification was done with the sunspot quality number from Gottschling et al. (2021), where a sunspot quality of 0 indicates a spot with a clear penumbra. After excluding these regions, 92 ARs are left in the sample.

For a comparison of the observed and the simulated magnetic field, the ARs have to remain on the visible disk for several days after the simulations are initialized. To select suitable ARs, we created data cubes of the line-of-sight magnetic field observed by the Helioseismic and Magnetic Imager onboard the Solar Dynamics Observatory (SDO/HMI, Schou et al. 2012), projected to Platte Carree projection and corrected for the viewing angle $\mu$. The cubes have a field of view of $60 \times 60^\circ$ and a grid spacing of 0.4$^\circ$ in both longitude and latitude, centered on the AR centers as defined in the HEAR survey. The grid spacing is the same as that of the observed flow maps (Sect. 3.1). On the temporal axis, the cubes span up to seven days before and after the time of emergence in nonoverlapping twelve-hour averages. We measured the total unsigned flux of each AR as the total unsigned flux in a disk of 5$^\circ$ radius around the center of the AR. From this, we determined the time $t_{90\%}$ at which 90$\%$ of the maximum total unsigned flux of the AR has emerged in the period covered by the HEAR survey. This is in analogy to the definition of the emergence time $t_0$.

The observed flows have data coverage out to only 60$^\circ$ from disk center, which limits the last last time step $t_{\text{last}}$ that can be used in the simulation. We identified $t_{\text{flux}}$ as the last time step where more than half of the field of view of the observed flow field is within a distance of 60$^\circ$ to the disk center.

For the sample selection, we required that the time between $t_{90\%}$ and $t_{\text{last}}$ be at least 5.5 days. This leaves 17 ARs. Appendix A lists all ARs in this sample. Because of the selection criteria (the exclusion of ARs that form fully fledged sunspots, and the requirement of several days of observations after most of the flux has emerged), the selected ARs are relatively weak and short-lived, and they are well into their decaying phase at the end of the observations.

Figure 1 shows a few examples of the evolution of the twelve-hour averaged total unsigned flux over time, relative to $t_{90\%}$. In most cases, the peak flux occurs at the time $t_{90\%} + 0.5$ days. The average total unsigned flux over the sample of
ARs at that time is $1.65 \times 10^{21}$ Mx, with a standard deviation of $0.66 \times 10^{21}$ Mx.

### 3. Cork simulation for local surface flux transport

For the local flux transport simulation, we adapted the cork simulation of Langfellner et al. (2018). The simulation treats the magnetic field from an initial input magnetogram as individual, passive flux elements (“corks”) in $x, y$ coordinates corresponding to the longitudinal and latitudinal axes of the projected input magnetic field map. At each simulation time step $\Delta t$, each cork moves a certain distance from its former position. There are two contributions to this motion: a diffusive part, which is realized as a random walk, and an advective part, which is realized as a flow field that moves each cork according to the velocity vector at its position.

Langfellner et al. (2018) considered only unsigned magnetic field, and included spawning of randomly distributed new field as well as the random removal of existing field. We do not include the random spawning and removal, as we are studying the AR polarities, for which the magnetic field is dominated by the emergence. We have expanded the simulation by considering positive and negative field and incorporating flux cancellation between the two. Corks of opposite polarity that move within 1 Mm of each other were removed from the simulation. The distance threshold is the same as that used by Martin-Belda & Cameron (2016).

We initialized the simulations with individual time steps from the magnetogram cubes described in Sect. 2, recentered to the center of the AR at time $F_{\text{GR}}$. A magnetic flux density of 1 Gauss in the observations corresponds to one cork in the simulation. The output of the simulations are magnetic field maps for each simulation time step $\Delta t$. The (signed) magnetic flux density at each grid element in these maps is the difference between the number of positive and negative corks that have $x, y$ coordinates within that grid element.

We ran the simulations with a simulation time step $\Delta t$ of 30 minutes. We averaged the simulation output to the same twelve-hour averages as the observations for direct comparison between simulations and observations. For each AR, we ran 20 realizations of each simulation model and averaged over them to decrease the realization noise from the random-walk diffusion models. The number of realizations was limited by computation time. The results do, however, not differ from those with less (e.g., five) realizations.

### 3.1. Flow models

We use four different flow models in our simulations: no flow ($u = 0$); observed flows, $u_{\text{obs}}$; parameterized inflows around ARs, $u_{\text{RB}}$; and modified parameterized inflows, $u_{\text{RB}}$, and a diffusion model (no diffusion, $D = 0$; constant diffusivity, $D_c$; or flux-dependent diffusivity, $D_f$). We ran each of these models with two different random-walk diffusion models. In an additional case, we did not add diffusion. Thus, we have nine different simulation setups (see Table 1).

**Table 1. Simulation cases.**

| Flow model $u$ | Diffusion model $D$ |
|----------------|--------------------|
| No flows ('$u = 0$') | $D_c$ |
| No flows ('$u = 0$') | $D_f$ |
| $u_{\text{obs}}$ | No diffusion ('$D = 0$') |
| $u_{\text{RB}}$ | $D_c$ |
| $u_{\text{RB}}$ | $D_f$ |
| $u_{\text{RB}}$ | $D_f$ |

Notes. The simulations were run with a flow model (no flows, $u = 0$; observed flows, $u_{\text{obs}}$; parameterized inflows around ARs, $u_{\text{RB}}$; or modified parameterized inflows, $u_{\text{RB}}$), and a diffusion model (no diffusion, $D = 0$; constant diffusivity, $D_c$; or flux-dependent diffusivity, $D_f$). Figure 3 shows snapshots from each simulation for AR 11137.

**Observed flows ($u = u_{\text{obs}}$).** In this model, we used the flow maps from Gottschling et al. (2021) for the ARs in the sample. The flows stem from local correlation tracking (LCT, November & Simon 1988) of solar continuum intensity images, and they are based on the data processing by Lüptien et al. (2017), who used the Fourier local correlation tracking code (Welsch et al. 2004; Fisher & Welsch 2008) on full-disk continuum intensity images from SDO/HMI. Several changes in the data processing were made by Gottschling et al. (2021) in order to correct for additional systematic effects in the data from Lüptien et al. (2017). Gottschling et al. (2021) describe these changes in detail. The flow maps are in Plate Carree projection with a grid spacing of 0.4′, with the same centering and twelve-hour time steps as the magnetograms (see Sect. 2). In the SFTM, these are Fourier-interpolated to the 30 min simulation time step $\Delta t$. Figure 2 shows the average magnetic field and flows over the sample of ARs at four days after $F_{\text{GR}}$. Converging flows toward the center of the AR are visible, with velocities on the order of 50 m s$^{-1}$. Inflows are more pronounced along the latitudinal axis than along the longitudinal axis.

**Parameterized inflows ($u = u_{\text{RB}}$).** We adopted the parametrization of the inflows around ARs by De Rosa & Schrijver (2006), who used it on the simulation by Schrijver (2001) to study the evolution of the field on large timescales, that is to say multiple rotations. Here, we applied it to the first few days after emergence, motivated by the resemblance of the observed flows to the inflows around ARs (see Fig. 2). The
parametrization is

\[ U = \alpha \nabla (\hat{B}^2), \]

where \( U \) is the flow field \((u_{\text{lon}}, u_{\text{lat}})\), \( \hat{B} \) is the magnetic field, smoothed with a Gaussian with a full width at half maximum (FWHM) of 15°, and \( \alpha \) and \( \beta \) are free parameters. Martin-Belda & Cameron (2017) used Eq. (1) with \( \beta = 1 \) and normalized it such that the peak inflow velocity around an AR of 10° is 50 m s\(^{-1}\), with \( \alpha = 1.8 \times 10^8 \text{ m}^2 \text{ Gauss}^{-1} \text{ s}^{-1} \). We adopted these choices for our model. The choice of \( \beta \) was further motivated by the observed flux density corresponding to the product of the field strength and the filling factor. If the inflows are driven by excess cooling, as suggested by Spruit (2003), they are proportional to the filling factor, and thus \( \beta = 1 \). Because we investigated ARs shortly after their emergence, their extent is considerably smaller than 10°. Therefore, the inflows in this model have extents that are too large and amplitudes that are well below 50 m s\(^{-1}\). In previous studies, these parameterized inflows led to flux clumping. In Appendix B, we therefore compare the inflows from this parametrization to the flow field that would balance the diffusion of a flux concentration. If the inflows were stronger than this flow field, they would lead to flux clumping. We find that the inflows from this parametrization are too weak to cause flux clumping, as was also motivated above.

**Modified parameterized inflows (\( u = u_{\text{FF}} \)).** With this model, we aim for a parametrization of the inflows that more closely resembles the observed flows on our sample of comparatively small (and young) ARs, rather than the evolved large ARs on which the parameters of the above model \((u_{\text{RF}})\) are based. In order to capture both the spatial extent of the flows as well as their amplitude, we compared the extent of the magnetic field, smoothed with different levels of spatial smoothing, to the observed flow field. We found that the magnetic field smoothed with a Gaussian of \( \sigma = 2° \) has a similar extent as the observed flows. The observed average inflow velocities increase from about 10 m s\(^{-1}\) at \( t_{90\%} \) to 40 to 50 m s\(^{-1}\) at \( t_{90\%} + 5 \) days. To capture this evolution, we fit a line between the observed flow velocities and the gradient of the smoothed magnetic field at the same location for each twelve-hour time step. The fit considers the area of \( 20 \times 20° \) around the center of the AR. We further selected only those pixels that lie within \( 2° \) of an absolute magnetic field density above 20 Gauss. We then fit a line to the relation between the flow velocity and the gradient against time. We used the slope and intercept of this fit to calculate the normalization \( \alpha \) in Eq. (1) for each time step.

### 3.2. Diffusion models

We consider three different diffusion models. In the first case, diffusion is the same for all flux elements (\( D_{C} \)), in the second it is flux-dependent (\( D_{F} \)). In the third case, no diffusion is added (\( D = 0 \)).

**Constant diffusivity (\( D_{C} \)).** In this model, the step length of the random walk of each cork along each axis is drawn from a normal distribution, with a standard deviation corresponding to a diffusion rate \( D \) that is constant for all corks. We ran simulations with diffusivities in the range between \( D = 250 \) and 722.5 km\(^2\) s\(^{-1}\).

**Flux-dependent diffusivity (\( D_{F} \)).** This model is based on De Rosa & Schrijver (2006). In their simulation, they treated the magnetic field as flux concentrations that can contain a varying amount of flux and that can merge and break up. The random walk step length \( \Delta r \) of a flux concentration depends on the amount of flux in the concentration as

\[ \Delta r = C(\Phi) \sqrt{4 D \Delta t}, \]

\[ C(\Phi) = 1.7 \exp \left( \frac{-|\Phi|}{3 \times 10^{19} \text{ Mx}} \right), \]

where \( \Phi \) is the absolute flux within a flux concentration and \( D \) is the diffusion rate (Schrijver 2001). This treatment of the magnetic field is different from our simulation, where we considered individual corks that have a constant amount of flux and performed individual random walks. To implement a flux-dependent diffusion rate in our simulation comparable to De Rosa & Schrijver (2006), at each simulation time step we calculated the cork density on a grid in longitude and latitude with grid spacing of 0.4°, oversampled it by \( 4 \times 4 \) pixels, and smoothed it by 0.1°. For each cork, we then calculated the width \( \Delta r \) of the normal distribution from which the random walk step length is drawn using the cork density at the corks’ position, with Eqs. (2) and (3). As reference diffusivity \( D \) in Eq. (2), we used \( D = 250 \text{ km}^2 \text{ s}^{-1} \).

**No diffusion (\( D = 0 \)).** For one simulation using the observed flows, we added no diffusion. In this case, only the flow field displaces the magnetic field.

### 4. Evaluation of the models

We ran simulations using the four different inflow models (Sect. 3.1) and the three different diffusion models (Sect. 3.2) on the sample of 17 ARs (Sect. 2). To evaluate the different simulations in comparison to the observations, for each time step, we measured the total unsigned flux within a disk of the central 5°, as well as the cross correlation between the observations and the simulations. We then studied different simulation start times.
4.1. Active region flux as a function of time

In this section, we initialize the simulations of each AR with the observed magnetic field at the time \( t_{\text{start}} = t_{90\%} + 0.5 \) days. In the simulations that use the observed flow field \( (u_{\text{obs}}) \), stagnation points at which the velocities are zero have an infinitely small width. The magnetic field therefore tends to accumulate in very confined spaces. To reduce the emphasis on these small-scale features, we smoothed the simulated magnetic field maps with a Gaussian with a width of \( \sigma = 0.8^\circ \). A broader smoothing results in higher cross correlations, as the small-scale structures are smeared out.

Figure 3 shows the magnetic field from the observations and the different simulations for AR 11137 at the beginning and at
The parameterized inflow model, \( u \), corresponds to the reference value of \( D \) constant diusivities, from \( D_{c250} = 250 \) km s\(^{-1}\) to \( D_{c722} = 722 \) km s\(^{-1}\). Therefore, the flux-dependent diusivities increase the diusivities to higher magnetic field strengths (typically below 100 Gauss). Figure 2 shows the flux loss due to cancellation and advection, respectively.

In the observations, the leading polarity moves in the prograde direction and toward the equator over time. The trailing polarity moves in the retrograde direction and toward the pole over time. The flux loss in ARs is the result of the cancellation of opposite-polarity field. In our simulation, the flux loss due to flux cancellation is the flux advected out of it, and \( \Phi_{\text{obs}} \) is the total unsigned flux in the area at time \( t \).

\[
\Phi(t) = \Phi(0) + \Phi_{\text{in}}(t) - \Phi_{\text{out}}(t) - \Phi_{\text{obs}}(t)
\]

where \( \Phi(0) \) is the flux inside the AR area at time \( t = 0 \), \( \Phi_{\text{in}}(t) \) is the flux advected into the AR area at time \( t \), \( \Phi_{\text{out}}(t) \) is the flux advected out of the AR area at time \( t \), and \( \Phi_{\text{obs}}(t) \) is the total unsigned flux in the area at time \( t \).

\[
\Phi_{\text{obs}}(t) = \Phi_{\text{obs}}(0) + \Phi_{\text{diff}}(t) - \Phi_{\text{canc}}(t)
\]

where \( \Phi_{\text{obs}}(0) \) is the initial flux in the AR area, \( \Phi_{\text{diff}}(t) \) is the flux diffused due to diffusion processes, and \( \Phi_{\text{canc}}(t) \) is the flux canceled due to cancellation processes.

In the simulations using the observed flows, the motions of the ARs are similar to the observations. In the simulations with a range of ARs in the sample (0.5-1.0), the motions of the ARs are consistent with the observations. However, in smaller ARs (0.5-0.1), the motions of the ARs are consistent with the observations. The flux loss due to cancellation is the flux advected out of the AR area at time \( t \).

The solid and contoured bars indicate flux loss due to cancellation and advection, respectively.
250 to 720 km$^2$ s$^{-1}$. In Appendix F, we derive an analytical solution of the flux loss due to cancellation between two diffusing Gaussian distributions in the case of no flow field and constant diffusion. This is in good agreement with the flux loss due to cancellation in the corresponding simulation. With this small sample of ARs, and a time period of only 5 days, we cannot constrain the diffusivity further.

Figure 4 also shows that the total amount of flux loss is very similar for all models that use the same diffusion model ($D_f$ or $D_c$). Increasing the strength of the inflows (no inflows $u = 0$, weak inflows $u_{D_f}$, and stronger inflows $u_{D_c}$) leads to both more flux loss due to flux cancellation and less flux loss due to advection for both diffusion models. These two effects partially cancel out, such that the net effect of the parameterized inflow models on the evolution of AR flux is small. In conclusion, the inflows are not important in the evolution of the flux budget of the AR in the first five days, but they might play a role in the distribution of the surrounding field.

In all three simulations that use the observed flows, the magnetic flux loss is consistent with that in the observations. The two models that include additional diffusion (blue triangles and filled circles) lose more flux than the simulation that uses the observed flow field and no additional diffusion (blue circles). This is because the additional random walk diffusion adds to the diffusion from the supergranulation and therefore enhances cancellation. We conclude that the bulk transport from supergranulation provides a means by which flux is carried away from the AR, which is consistent with a diffusion process.

### 4.2. Cross correlation as a function of time

As a second evaluation of the simulations, we calculated the cross correlation between the observed and the simulated magnetic field in a window of $10 \times 10^4$ around the center of each map for each of the 17 ARs. The window size was chosen to exclude other ARs in the field of view, which emerge at a later time or are significantly larger than the target AR, such that they exhibit moat flows (see for example the lower right corner in the top left panel of Fig. 3). As in Sect. 4.1, we use the simulations that are initialized at $t_{90\%} + 0.5$ days.

Figure 5 shows the average cross correlation of the simulations with the observations as a function of time. The cross correlation decreases monotonically for all simulations that use no or parameterized flows. The differences between the simulations with the same diffusion model and different flow models indicate that the small-scale distribution of the field is different. Comparing the simulations with the same flow model and different diffusion models, the cases with flux-dependent diffusivity $D_f$ have higher cross correlations to the observations than the $D_{c250}$ cases. This is within the error bars, however. The simulation using the observed flow field and no additional diffusion ($u_{obs}, D = 0$) has a lower correlation to the observations than the other models for most of the four days of simulation time, whereas the two simulations using the observed flow field and additional diffusion remain at a constant cross correlation (within errors) from 2–3 days onward. In the last few time steps, the cross correlation of these is larger than for cases with no or parameterized flows. The low cross correlation in the $u_{obs}, D = 0$ case is a result of the flux being dragged by the flow into a different supergranular downflow lane in the simulation than in the observation. The additional diffusion in the cases of $(u_{obs}, D_{c450})$ and $(u_{obs}, D_f)$ mitigates this and they therefore have a higher cross correlation.

### 4.3. Changing the simulation start time

In Sects. 4.1 and 4.2, we studied simulations initialized at $t_{90\%} + 0.5$ days. Here, we examine the dependence on the initial condition of the simulation by initializing the simulation with the magnetic field at times $t_{90\%} - 1$ day, $t_{90\%} - 0.5$ days, $t_{90\%}$, and $t_{90\%} + 1$ day.

Figure 6 shows the total unsigned flux for the simulations initialized at the five different starting times at 4.5 days after the initial condition of each simulation (that is, the same time has elapsed for all simulations). The simulations starting at $t_{90\%} + 0.5$ days yield the highest fluxes, because this time most often coincides with the time of peak flux of the ARs. Starting at later and earlier times decreases the AR flux in the simulations. In the two cases starting before $t_{90\%}$, the simulated fields differ largely from the observations, as most of the flux has not yet emerged onto the surface.

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**Fig. 5.** Evolution of the average cross correlation between the observed field and the simulations. The error bars indicate the standard error over the sample. The data point at $t_{90\%} + 0.5$ days is the initial condition of the simulations.

**Fig. 6.** Total unsigned flux for different simulation start times relative to $t_{90\%}$ at 4.5 days after each simulation start time. The diffusion in the case of constant diffusivity is 250 km$^2$ s$^{-1}$. 

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5. Discussion

In this work, we compared the observed evolution of 17 emerging ARs with surface flux transport simulations of these regions. We considered nine types of simulations with different models for diffusion and surface flows. We used models where the diffusion is zero, where it is the same for all flux elements, and where it is flux-dependent. For the surface flows, we used observations from local correlation tracking, parameterized models of the inflows around ARs, as well as no flows. We compared the evolution of the magnetic field in the observations and the simulations by calculating the cross correlation as well as the total unsigned flux of the ARs for all time steps. In addition, we tested the validity of the transport simulation for the study of young ARs by varying the starting time of the simulations relative to the time when the bulk of the AR flux has emerged.

We find that simulations using the observed flows can describe the evolution of the total unsigned flux of the ARs starting from the time when 90% of the AR flux has emerged. The supergranular motions act as a random walk process in buffering the magnetic field polarities. This finding from our simulation complements the observations by Schunker et al. (2019), who measured the standard deviation of the positions of AR polarities and draw the same conclusion. However, from our simulation we cannot make a statement as to whether the buffeting is flux-dependent or not. Additional diffusion improves the small-scale structure (measured as the cross correlation). Our findings allow for diffusion rates from the supergranular motions between 250–720 km s\(^{-1}\). The large range is due to the small sample of ARs which was suited for this study, as well as the limitation to about five days for the simulations.

The converging flows around emerging ARs, which we included as parameterized models, increase flux cancellation in the AR in the first five days after 90% of the AR peak total unsigned flux has emerged. The resulting decrease in total unsigned flux is balanced by the decreased advection away from the AR, such that the evolution of the total flux associated with the AR is similar in the different models.

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Appendix A: List of active regions used in the simulation

The NOAA numbers of ARs used in the flux transport simulations are the following: 11088, 11137, 11145, 11146, 11167, 11288, 11437, 11547, 11624, 11626, 11712, 11786, 11789, 11811, 11932, 12064, and 12105.

Appendix B: Flow field balancing diffusion

\[
\frac{\partial B}{\partial t} + \nabla \cdot (uB) = D \nabla^2 B, \tag{B.2}
\]

where \(B\) is the radial magnetic field, \(u\) is the flow field, and \(D\) is the diffusivity. In the situation where the advection and the diffusion balance each other, such that \(\frac{\partial B}{\partial t} = 0\), Eq. (B.2) reduces to

\[
\nabla \cdot (uB) = D \nabla^2 B. \tag{B.3}
\]

Using the vector identity

\[
\nabla \cdot (uB) = \mathbf{B} \cdot (\nabla \cdot \mathbf{u}) + \mathbf{u} \cdot (\nabla \mathbf{B}), \tag{B.4}
\]

Eq. (B.3) then gives

\[
\mathbf{B} \cdot (\nabla \cdot \mathbf{u}) + \mathbf{u} \cdot (\nabla \mathbf{B}) = D \nabla^2 \mathbf{B}. \tag{B.5}
\]

Using Eq. (B.1) and its derivatives gives

\[
\left[ \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right] \frac{(x-x_0)^2}{\sigma^2} + \frac{(y-y_0)^2}{\sigma^2} u_x = -\frac{2D}{\sigma^2} \left( (x-x_0)^2 + (y-y_0)^2 \right)^\frac{1}{2}. \tag{B.6}
\]

From this, we find the solution to be

\[
u = -\frac{D}{\sigma^2} \left( x-x_0 \right)^\frac{1}{2}. \tag{B.7}\]

Figure B.1 shows a latitudinal cut through the center of a 2D Gaussian with a full width at half maximum of 2.4° and a peak field strength of about 100 Gauss, along with the corresponding parameterized inflow models (green and purple lines) and the flow field which balances the effect of diffusion from Eq. (B.7). The figure shows that the \(u_{eq}\) inflow model has velocities that are too low to compensate for the diffusion. The \(u_{eq}\) model is similar to the diffusion within ±2° of the center of the field distribution, and in fact it can overcome the diffusion, leading to flux clumping.

Appendix C: Evolution of observations and example simulations for AR 11137

Figure C.1 shows all time steps of the observations and a few simulations for the AR 11137. The simulations were initialized at \(t_{eq}\) + 0.5 days.
Fig. C.1. Evolution of the magnetic field of AR 11137 in the observations (top row) and four different simulations (second row from the top, no flows and constant diffusivity; third row from the top, flows according to the inflow parametrization based on the gradient of the magnetic field and flux-dependent diffusivity; fourth row from the top, observed flows and constant diffusivity; bottom row, observed flows and flux-dependent diffusivity). The simulations were initialized at $t_{90\%} + 0.5$ days. The times are indicated in the upper left corner of the top panels. The diffusion in the cases of constant diffusivity is $250 \text{ km s}^{-1}$. Reference arrows are given in the lower left corners of the second column. Red (blue) indicates positive (negative) radial magnetic field. All maps have the same saturation at $\pm$ the rounded maximum absolute field strength in the central $10^\circ$ from all simulation time steps. The green (black) contours indicate levels of half and a quarter of the minimum and maximum magnetic field in the central $10^\circ$ of the observation (of each simulation) for each time step individually.
Appendix D: Average simulation

Figure D.1 shows two time steps of the average over the 17 ARs for each simulation.

Fig. D.1. Two example time steps of the observations and the simulations, averaged over the individual ARs. The two columns on the left show the first time step after simulation start, that is, 0.5 days after $t_{\text{start}} = t_{90\%} + 0.5$ days. The two columns on the right show the time step at the end of the simulations at $t_{90\%} + 5.5$ days. The times are indicated in the upper right corner of the top left panels. At each of the two time steps, the top row shows the observed magnetic field and flows (left) and the simulation using observed flows and no additional diffusion (right). For all other rows, the left (right) panels show simulations with constant (flux-dependent) diffusivity $D_c$ ($D_f$). From the second row to the bottom: observed flows, flows according to the parameterized inflow model, flows according to the modified parameterized inflow model, and no flows. The diffusion in the cases of constant diffusivity is $250 \text{ km s}^{-1}$. The arrows indicate the observed and the parameterized flows for the respective observations and simulations. Reference arrows are given in the lower left corners of each panel. Red (blue) indicates positive (negative) radial magnetic field. All maps have the same saturation at ± the rounded maximum absolute field strength in the central 10° from all simulation time steps. The green (black) contours indicate levels of half and a quarter of the minimum and maximum magnetic field in the central 10° of the observation (of each simulation) for each time step individually.
Appendix E: Changing the diffusivity

Figures E.1 and E.2 show the total unsigned flux over the central disk with a radius of 5° for constant diffusivities of 250 and 722.5 km²s⁻¹, respectively. The case $D_{c250}$ corresponds to the diffusivity which serves as a reference for the flux-dependent diffusivity $D_f$. The case $D_{c722.5}$ corresponds to the flux-dependent model $D_f$ in the case that all corks experience no surrounding magnetic flux.

**Fig. E.1.** Left: Evolution of the total unsigned flux over the central disk with a radius of 5°, averaged over the sample of ARs, for the observations and some of the simulations. The error bars indicate the standard error over the sample. Only every sixth error bar is plotted for readability. The data point at $t_{90%} + 0.5$ days is the initial condition of the simulations. The diffusion in the simulation with constant diffusivity and no flows is 250 km²s⁻¹. Right: Amount of flux loss between the last time step of the simulations, at $t_{90%} + 5.5$ days, and the time when the simulations were initialized, at $t_{90%} + 0.5$ days. The black line indicates the flux loss in the observations over the same period, and the gray shaded area indicates the standard error. The solid and contoured bars indicate flux loss due to cancellation and advection, respectively.

**Fig. E.2.** Same as Fig. E.1, but with a diffusion in the cases of constant diffusivity for the parameterized flow models at 722.5 km²s⁻¹.
Appendix F: Analytical solution for constant diffusion and no flows

We derived an analytical description of the evolution of the flux loss. For this, we considered the simplest case, with no flow field acting on the magnetic field \((u = 0)\), and with constant diffusivity \((D_c)\). Because diffusion acts independently per dimension, a 1D setup can be used. For the magnetic field, we considered two Gaussian distributions of opposite sign, centered at positions \(\pm \Delta x\) from the origin:

\[
B_\pm(x, \sigma) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp \left( -\frac{(x \pm \Delta x)^2}{2\sigma^2} \right).
\]  

(F.1)

For small \(\Delta x\), the difference between \(B_+\) and \(B_-\) can be written as

\[
B_+ - B_- = \frac{\Delta x}{\sqrt{2\pi}\sigma^2} \frac{\partial}{\partial x} \left( \exp \left( -\frac{x^2}{2\sigma^2} \right) \right).
\]  

(F.2)

From this, the total unsigned flux can be calculated as

\[
\Phi = \frac{2\Delta x}{\sqrt{2\pi}\sigma^2} \int_{-\infty}^{0} \frac{\partial}{\partial x} \left( \exp \left( -\frac{x^2}{2\sigma^2} \right) \right) \, dx
\]  

(F.3)

\[
= \frac{2\Delta x}{\sqrt{2\pi}\sigma^2}.
\]  

(F.4)

Diffusion broadens the Gaussian distribution over time as

\[
\sigma(t) = \sqrt{2Dt}.
\]  

(F.5)

Therefore,

\[
\Phi(t) = \frac{2\Delta x}{\sqrt{2\pi}\sqrt{2Dt}} = \frac{\Delta x}{\sqrt{\pi Dt}}.
\]  

(F.6)

The total flux loss over time can thus be written as

\[
\Delta \Phi(t) = \Phi(t_{in}) - \Phi(t_{in} + t) = \Phi(t_{in}) \left( 1 - \frac{\Delta x}{\sqrt{\pi D(t_{in} + t)}} \right).
\]  

(F.7)

with a suitable initial condition of \(t_{in} = \frac{\Delta x^2}{4D_c}\).

To compare this to the simulation results, we considered an average \(\Delta x = 1.8^\circ\) (readers can compare this to the distance of the polarities from the center in the x-direction in Fig. D.1) and a time of 5 days. With a total unsigned flux of \(1.65 \times 10^{21}\) Mx at the start time \(t_{in} = t_{90\%} + 0.5\) days, Eq. (F.7) gives, for the three cases of \(D_{c250}\) = 250 km\(^2\)s\(^{-1}\), \(D_{c450}\) = 450 km\(^2\)s\(^{-1}\), and \(D_{c722.5}\) = 722.5 km\(^2\)s\(^{-1}\), flux losses of 0.39 \times 10^{21} Mx, 0.56 \times 10^{21} Mx, and 0.71 \times 10^{21} Mx, respectively. This is in good agreement with the flux losses due to diffusion (solid bars) in the simulations with the corresponding model of \((u = 0), (D_c)\), as shown in Fig. E.1, Fig. 4, and Fig. E.2, respectively. The total flux loss in the observations over this time is 0.66 \times 10^{21} Mx.

From Eq. (F.7), we calculated the minimum and maximum diffusivities \(D_{min}, D_{max}\) that are consistent with the observed flux loss after 5 days to a 2-\(\sigma\) level. These are \(D_{min} = 280\) km\(^2\)s\(^{-1}\) and \(D_{max} = 1350\) km\(^2\)s\(^{-1}\), respectively. We note that this does not include flux loss due to transport away from the AR, which explains why \(D_{min}\) is larger than \(D_{c250}\).