Numerical analysis and Experimental Investigation of Lateral vibration on Drill String under Axial Load Constrained with Horizontal Pipe

Baojin Wang1* Zhongyang Wang2 Anhe Zhu3 Wang Hongda4 Fushen Ren5*

1,2,5.Department Mechanical Science and Engineering, Northeast Petroleum University, China
3. Xuzhou Construction Machinery Group Co.,China
4. Research Institute of Oil Production Engineering,Daqing Oilfield Co Ltd.,China
E-mail: 1.bjwangbaojin@126.com;5.renfushen@126.com

Abstract. Horizontal well technology is an important means to improve drilling efficiency and oil and gas production, but it is easy to generate the lateral vibration of the drill string. This strong lateral vibration of the drill string will further lead to the whirl of the drill string, which will accelerate the wear of the drill string and further lead to the failure of the drill string. According to the rotor dynamics theory and fluid dynamics, the whirling equation of the drill string in horizontal wells is established. The effects of the weight of the drill string, wall friction, internal and external drilling fluid flow velocity and fluid resistance on the whirling of the drill string are studied. The mechanism of inducing the whirling of the horizontal drill string and the boundary conditions for judging the whirling are analyzed. Based on the existing dynamic simulation test equipment of the horizontal drill string in the laboratory, the experimental research on the whirling characteristics of the horizontal drill string is carried out, and the influence laws of the weight on bit and the rotation speed on the whirling of the drill string are studied. The results show that with the development of axial load, the intense of lateral vibration of drill string increases.

1. Introduction
Drill string is located in a very narrow circular space during drilling. Inside and outside of the drill string are filled with drilling fluid. The angle of inclination of horizontal well varies greatly. Due to the force acting on the drill string itself, the drill string in the horizontal section is almost flat against the lower wall. During the rotary drilling process, the friction of the drill string is serious, showing obvious non-linear contact characteristics. At the same time, the force and motion characteristics of the horizontal drill string are very complex due to the interference of ground equipment, formation and fluid viscous resistance. The drill string is subjected to transverse vibration, longitudinal vibration, torsional vibration, whirl and coupling vibration among them. Jansen[1-3] presented the dimensionless whirling motion equation of drill string according to the nonlinear rotor dynamics theory and carried out numerical simulation. A.P.Christoforou[4] analyzed the whirling characteristics of bottom hole assembly by means of Rayleigh beam model. The research results well reflected the whirling characteristics of bottom drill string. Chen et al.[5] discussed the dynamic characteristics of drill bit during whirling of drill string. The research results show that the backward whirl of the drill string will hinder the transmission of the WOB of the bottom hole drill bit by the surface equipment, making the WOB obtained by the drill bit extremely small in actual operation. It is considered that the whirl of the drill string is a severe lateral vibration.
Y.A. Khulief studied the coupling vibration of the lateral vibration and whirl and conducted finite element research. Hayat analyzed and discussed the dynamic characteristics of the bottom hole assembly through combining test and simulation analysis. The influence of different friction coefficients on the whirl trajectory of drill string was discussed during the simulation analysis and simulation test. Gao Deli et al. discussed the causes of different whirl and studied the effect of drill string whirl on borehole deviation prevention. Guan Zhichuan et al. studied the whirl characteristics of drill string at the bottom of vertical well using simulation test equipment, analyzed the influence of different WOB and rotating speed on the whirl trajectory of drill string. Xiao Wensheng analyzed the whirl mechanism of drill string under the action of internal and external drilling fluid and friction. The conditions for determining the whirl instability of the drill string are obtained. Di Qinfeng et al. established the dynamic simulation model of the bottom hole assembly. The bottom hole assembly is simplified to a rotor model. And the whirl characteristics of the drill string under different WOB are studied. Yao Yonghan et al. analyzed the whirling track, whirling speed, radial track and radial speed of drill collar by using the compiled simulation program.

From the research status of drill string lateral vibration and whirl at home and abroad, it can be seen that the research on drill string lateral vibration is mostly focused on vertical well. The research results have been very rich and perfect. Although domestic and foreign scholars have also done a lot of research on horizontal well drill string lateral vibration, these researches are far from enough compared with vertical wells.

In this paper, non-linear factors such as weight of drill string, wall friction, internal and external drilling fluid velocity, density, structural nonlinearity and contact nonlinearity are considered. Based on Hamiltonian principle, the fluid-solid coupling vibration equation of drill string lateral vibration is established. The influence of weight on bit (WOB) and rotation speed on the lateral vibration characteristics of drill string is analyzed and discussed by the research method combining experiment and theoretical analysis.

2. Mathematical Model

Transverse vibration of drill string is a vibration perpendicular to the axis of drill string before deformation. Considering the difference in dynamic characteristics between the horizontal drill string and the vertical drill string, this paper focuses on the analysis of the lateral vibration characteristics of the drill string in the direction of gravity, and simplifies the drill string into an elastic beam with equal cross section hinged at both ends, as shown in figure 1.

\[ m = m_d + m_f = \rho_f \pi (D_{ext}^2 - D_{int}^2) / 4 + \rho \pi D_{int}^2 / 4 \]  

Where, \( m_d \) is the mass of drill string per unit length. \( m_f \) is the mass of drilling fluid per unit length in drill string. \( D_{int} \) is inner diameter of drill string. \( \rho \) is the density of drilling fluid.

Let drilling fluid flow along the axial direction of the drill string. And the speed of the transverse vibration of the drill string in the axial direction can be written as

\[ \gamma = \frac{\partial y}{\partial t} \]
The velocity of drilling fluid in the axial direction is \( V_1 \). The velocity of drilling fluid in the axial direction is the combined velocity of drilling fluid velocity and drill string velocity in the axial direction.

\[
V_{1y} = \frac{\partial y}{\partial t} + V_1 \frac{\partial y}{\partial z}
\]  

(3)

Where, \( V_1 = \frac{4Q}{\pi D_{int}^2} \).

Analyzing the micro-element \( dx \), the total kinetic energy of the micro-element can be written as

\[
\delta T = \int_{z_1}^{z_2} \left[ \frac{1}{2} m_d \frac{\partial y^2}{\partial t^2} + \frac{1}{2} m_s V_1^2 + \frac{\partial y}{\partial t} \frac{\partial y}{\partial z} \right] dz
\]

(4)

Total potential energy is

\[
\delta U = \int_{z_1}^{z_2} \left[ \frac{1}{2} EI (y'')^2 + p_d A y dz \right]
\]

(5)

Work done by non-conservative force is written as

\[
\delta W = -\int_{z_1}^{z_2} \left( F_w - p_{int} A_{int} \right) y'' \delta y dz
\]

(6)

\( E \) is the function of \( y(z,t) \). The equation can be given according to Hamilton principle.

\[
\delta E = 0
\]

(7)

\[
\delta E = \int_{z_1}^{z_2} \left( \frac{1}{2} m_d \frac{\partial y}{\partial t} \frac{\partial y}{\partial t} + \frac{1}{2} m_s V_1^2 + \frac{\partial y}{\partial t} \frac{\partial y}{\partial z} \right) dz dt
\]

(8)

The variational form of equation (8) can be written as

\[
\int_{z_1}^{z_2} \left( m_d \frac{\partial \delta y}{\partial t} \frac{\partial \delta y}{\partial t} + m_s V_1 \delta \frac{\partial y}{\partial t} + \delta \frac{\partial y}{\partial z} \right) dz dt = 0
\]

(9)

Where, \( F = P_w - P_i \cos(\omega t) - F_j \).

The equation can be obtained after integrating equation (9).

\[
\int_{z_1}^{z_2} \left( m_d \frac{\partial \delta y}{\partial t} \frac{\partial \delta y}{\partial t} + 2m_s V_1 \delta \frac{\partial y}{\partial t} + p_d A g \delta y \right) dz dt = 0
\]

(10)

Due to the arbitrariness of \( \delta y \), the differential equation of the coupled vibration of the drill string can be obtained considering the factors of drilling fluid, weight on bit, friction and weight of the drill string.

\[
EI \frac{\partial y^2}{\partial z^2} + \left( m_s V_1^2 + F_j \right) \frac{\partial^2 y}{\partial z \partial t} + 2m_s V_1 \frac{\partial y}{\partial z} \frac{\partial y}{\partial t} + m_d \frac{\partial^2 y}{\partial t^2} + p_d A g = 0
\]

(11)

The micro-element section \( dz \) is taken for analysis. The quality of drilling fluid inside and outside the drill string per unit length is given as follows

\[
m_s = m_d + m_s + m_b
\]

(12)

The mass of drilling fluid in annulus space is given as

\[
m_b = \rho \frac{\pi (D_{int}^2 - D_{ext}^2)}{4}
\]

(13)

Flow speed of drilling fluid in annulus space can be written as
\[ V_2 = \frac{Q}{A_2 - A_1} \]  

(14)

Where, \( V_2 = \frac{Q}{A_2 - A_1} \), \( A_1 = \frac{D_{ext}^2}{4} \), \( A_2 = \frac{D^2}{4} \).

Drilling fluid resistance is

\[ f = C_v \bar{v} \]  

(15)

Where, \( C_v \) is the viscosity coefficient of drilling fluid.

When considering the liquid pressure in the annulus, the additional force exerted by the annular drilling fluid on the drill string is

\[ c_w m_b \frac{\partial^2 y}{\partial t^2} + 2V_2 \frac{\partial^2 y}{\partial z \partial t} + V_2^2 \frac{\partial^2 y}{\partial z^2} + c_v \frac{\partial y}{\partial t} + p_{ext} A_{ext} \frac{\partial^2 y}{\partial z^2} \]  

(16)

Finally, the differential equation can be obtained.

\[ EI \frac{\partial^4 y}{\partial z^4} + 2\left( m_a V_1 + C_m m_b V^2 \right) \frac{\partial^2 y}{\partial z \partial t} + \left( F_r + m_a V_1^2 + C_m m_b V^2 \right) \frac{\partial^2 y}{\partial z^2} + \left( m_a + C_m m_b V^2 \right) \frac{\partial^2 y}{\partial t^2} + \rho_y A_g = 0 \]  

(17)

Where, \( F_r = P_w - P \cos(\omega t) - F_f + p_{ext} A_{ext} - p_{int} A_{int} \).

According to the method of separating variables, the frequency equation of drill string transverse vibration is obtained.

\[ \omega_n = \frac{n \pi^2}{L^2} \sqrt{\frac{EI}{m_b \left( n^2 - \frac{L^4}{\pi^4 EI} \left( m_a V_1^2 + C_m m_b V^2 - \rho t \omega^2 + F + \rho_y A_g \right) \right)}} \]  

(18)

3. Experiment and Simulation Result

The test equipment is designed according to the similarity theory. Thus, there is a similarity ratio between the test parameters and the actual parameters.
As shown in figure 2, red and black lines are experiment and simulation results, respectively. The rotional speeds applied in experiment and simulation are 100r/min and 35.7r/min. The axial load applied in experiment are 0.5kg, 1.5kg and 2.5kg. The axial load applied in simulation are 45kN, 135kN and 225kN. The results show that the intense of whirling of drill string increases with the development of axial load. However, the increase is not significant.

4. Conclusions
In the actual drilling process, larger WOB can increase whirling. Thus, the smaller WOB should be applied. And then, the impact and friction between the drill string and the borehole wall reduce. Moreover, the service life of drill string is prolonged.

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