Level statistics in a two-dimensional system with strong spin-orbit coupling

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We study level correlations in a two-dimensional system with a long-range random potential and strong spin-orbit (SO) splitting $\Delta_{so}$ of the spectrum. The level correlations for sufficiently large $\Delta_{so}$ are shown to be described by orthogonal Wigner-Dyson statistics, in contrast to the common point of view that a system with the strong SO coupling belongs to the symplectic statistical ensemble. We demonstrate also that in a wide energy interval the level statistics is completely determined by transitions between two branches of the split spectrum. A sharp resonance is obtained in the two-level correlation function at energy equal to $\Delta_{so}$.

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The advancement of fabrication of nanostructure devices in the last two decades has initiated great interest to properties of small mesoscopic systems (see Ref. 1 for review). One of the most frequently studied characteristics of the mesoscopic samples is the two-level correlation function $R(\epsilon) = \langle \rho(E + \epsilon) \rho(E) \rangle / \rho^2 - 1$ which describes fluctuations of the density of states $\rho(E)$. Here $\rho = \langle \rho(E) \rangle$ and $\langle \ldots \rangle$ denotes averaging over realizations of the random potential. The problem of level correlations in small disordered conductors was first discussed in 2, where it was suggested that the distribution of energy levels is determined by one of the three types of the universal Wigner-Dyson statistics 3. Later on this idea was confirmed by rigorous calculations 4 of $R(\epsilon)$. It has been proved that the universal Wigner-Dyson distribution is realized at small energies $\epsilon \ll E_c$, where $E_c \sim h/\tau_c$ is the Thouless energy (here $\tau_c \sim L^2/D$, $L$ is the size of the system and $D$ is the diffusion coefficient). At larger energies ($\epsilon \gg E_c$) the level statistics is nonuniversal and depends on the dimensionality of the system 3. The most interesting is the 2d case in which the calculation of $R(\epsilon)$ in the standard one-loop approximation gives a zero result 3. In this situation the level statistics is governed by weak localization effects 3 and is sensitive to the boundary conditions and topology of the surface 3.

In this paper we study level correlations in a 2d system with strong SO spectrum splitting $\Delta_{so}$, which originates from an asymmetry of the confining potential of the quantum well 3. The enhanced interest to the systems with strong SO coupling was initiated by recent remarkable experiments of Kravchenko et al. 5 which clearly demonstrated the existence of a metal-insulator transition in 2d systems. The importance of the SO spectrum splitting for the transition was first conjectured in Ref. 6. This idea was strongly supported by very recent experimental data 7,8.

Here, we show that strong SO spectrum splitting leads to unexpected effects in the statistics of levels in a 2d disordered system. A common belief is that the type of the level statistics in a system with the SO coupling depends only on the relation between the spin relaxation time $\tau_{so}$ and $\tau_c$. For the case of very small splitting, when $\tau_{so} > \tau_c$, spin degrees of freedom do not play any role and the system belongs to the orthogonal ensemble - one of the three universal Wigner-Dyson ensembles. With increasing SO coupling, $\tau_{so}$ becomes smaller than $\tau_c$, the spin relaxation comes into play and the system exhibits crossover to symplectic ensemble. This scenario takes place for the case of a short-range potential. As we show below, the system with long range disorder demonstrates reentrance to the orthogonal ensemble at sufficiently large values of $\Delta_{so}$. This surprising result is related to strong suppression of the transitions between two branches of the SO split spectrum in the case $\Delta_{so} \gg \hbar v_F/d$ 3 (here $d$ is the potential correlation length, $v_F$ is the Fermi velocity).

As a consequence, the system is divided into two independent subsystems, the spin degrees of freedom are effectively frozen out, so that in the limit of very large $\Delta_{so}$ we have two independent orthogonal ensembles. We will demonstrate also that long-range nature of the impurity potential is of crucial importance for the level statistics in the nonuniversal region of energies $\epsilon \gg E_c$. We argue that in a wide energy interval in the nonuniversal region the level correlations are completely governed by the rate of transition between two branches of the spectrum and that a narrow resonant peak arises in $R(\epsilon)$ for $\epsilon = \Delta_{so}$.

The term in the Hamiltonian which is responsible for the SO splitting of the energy spectrum 3 is given by $\alpha(\hat{\sigma} \times \hat{n}) \cdot \mathbf{n}$, where $\hat{\sigma}$ are the Pauli matrices, $\hbar \mathbf{k}$ is the electron momentum and $\mathbf{n}$ is the unit vector normal to the 2d layer. A constant $\alpha$ characterizing the strength of the SO coupling is proportional to the electric field arising due to asymmetry of the confining potential of the quantum well. The value of $\alpha$ can be tuned by applying an external electric field perpendicular to the 2d plane 3. The energy spectrum and spin wave-functions of an electron with the effective mass $m_F$ read: $E^\pm(\mathbf{k}) = \hbar^2 k^2 / 2m_F \pm \alpha k$, $\chi_\pm^\mathbf{k} = (1, \pm ie^{i\varphi_\mathbf{k}}) / \sqrt{2}$. The spinors $\chi_\pm^\mathbf{k}$ depend on the polar angle $\varphi_\mathbf{k}$ of the wave vector $\mathbf{k}$ and describe two states with spins polarized along...
the vectors \( \pm (k \times n) \), respectively. Thus the system is divided into two branches, \((+\) and \((-\)) (see Fig. 1a) and the electron spin in each of the branches is rigidly connected to the momentum.

The long-range random impurity potential \( U(r) \) is assumed to be weak enough, so that the inequality \( \tau \gg d/v_F \) holds, where \( \tau \sim \tau_{tr}/(k_Fd)^2 \) is the elastic scattering time, \( k_F \) is the Fermi wave-vector. The presence of this potential leads to transitions both within each branch and between different branches. The respective (intra-branch and interbranch) times of these transitions are given by the following expression

\[
\frac{1}{\tau_{\mu\nu}} = \frac{2\pi}{\hbar} \int \frac{d^2k'}{(2\pi)^2} K_{\mu\nu}(k-k')\delta[E^\nu(k)-E^\nu(k')],
\]

where \( K_{\mu\nu}(k-k') = |\langle \chi^\mu_k|\chi^\nu_{k'} \rangle|^2 K(|k-k'|) \) denotes the type of the branch \((+\) or \(-\)) and the function \( K(k) \) is the Fourier-transform of the potential correlation function \( K(r) = \langle U(r)U(0) \rangle \). The function \( K(r) \) falls off on the scale of \( d \). For the case when the spin-orbit splitting \( \Delta_{so} = 2ak_F \) is not too large \( (\Delta_{so} \ll E_F, \) where \( E_F \) is the Fermi energy), we can set \( \tau_{++} = \tau_{-+} = \tau, \tau_{+-} = \tau_{-+} = \tau_{tr} \). The minimal transferred momentum needed for the transition between the branches is equal to \( 2m\alpha/\hbar \). This momentum should be compared with \( \hbar/d \). If the inequality \( \Delta_{so} \gg \hbar v_F/d \) is fulfilled, then the interbranch transitions are suppressed compared to transitions within one branch (note that this inequality implies also that \( \Delta_{so}\tau \gg \hbar \)). In particular, for the case of the potential created by ionized impurities located at distance \( d \) from the 2d layer, the correlation function \( K(q) \sim \exp(-2qd) \) and the interbranch transition time is given by

\[
\tau_\ast = 4\tau_{tr}(k_Fd)^2 \exp \left( \frac{2\Delta_{so}d}{\hbar v_F} \right) \gg \tau_{tr}.
\]

The factor \((k_Fd)^2\) in this expression is due to the orthogonality of the spinors corresponding to different branches with the same direction of momenta. Note that \( \tau_\ast \sim \tau_{tr} \) for the case of a short-range potential \((K(q) = const)\).

Expressing the density of states in the usual way in terms of exact Green functions \( \rho(E) = (i/2\pi L^2)\text{Tr}(G^R(E) - G^A(E)) \), we can write the two-level correlation function as \( R(\epsilon) = (\Delta^2/8\pi^2)\text{Re}(\text{Tr}G^R(E + \epsilon)\text{Tr}G^A(E)) \), where \( \Delta = 2\pi\hbar^2/mL^2 \) is the mean level spacing and the trace is calculated over spatial and spin variables. The main contribution to the function \( R(\epsilon) \) comes from one-loop diagrams containing two diffuson or cooperon propagators, shown in Fig. 2 in the so-called Hikami representation [14]. In what follows we consider the case of zero magnetic field, so that the cooperon contribution is equal to the diffuson one. This allows us to focus on the calculation of the diffusion contribution only. To separate the two branches of the spectrum, we use the following representation for the averaged Green functions \((G^{R,A}(k,E)) = \sum_{\mu=\pm} g^{R,A}_\mu(k,E)\hat{P}_\mu(k) \), where \( \hat{P}_\mu(k) = |\chi^\mu_k\rangle\langle\chi^\mu_k| \) is the projection operator to the branch \( \mu \). The functions \( g^{R,A}_\mu \) are represented by edges of the "Hikami boxes" in Fig. 2. It is convenient to distinguish two channels: the symmetric channel with \( \mu = \nu, \mu' = \nu' \) and antisymmetric channel with \( \mu = -\nu, \mu' = -\nu' \) (see Fig. 2). We will first concentrate on the region of small energies \((\epsilon \tau_{tr} \ll \hbar) \), where the symmetric contribution dominates. The diffusion propagator in symmetric channel depends on two indices \( \mu \) and \( \mu' \) only: \( D^{\mu\mu'}_\epsilon = D^{\nu\nu'}_\epsilon \)

This propagator corresponds to the contribution of two coherent wave ("particle" and "hole" waves) which belong to the same branch (see Fig. 1b) in any part of the diffusive trajectory. The value \( D^{\mu\mu'}_\epsilon \) is determined from the system of four equations (\( \mu = \pm, \mu' = \pm \)) depicted graphically in Fig. 3, where \( K^{\mu\mu'}_{\nu\nu'}(k,k',q) = K(k-k')\langle \chi^\mu_k|\chi^\nu_{k'}\rangle\langle \chi^\nu_{k'+q}|\chi^\mu_{k+q} \rangle \) (see Fig. 3). These equations should be solved together with the system of equations for \( G^{R,A}_\mu(k,E) \) written in the framework of Self-Consistent Born Approximation. In the diffusion approximation, the solution is given by \( D^{++}_\epsilon(q,\epsilon) = D^{--}_\epsilon(q,\epsilon) = S_\epsilon \hbar/\pi \rho \tau^2 \) and \( D^{+-}_\epsilon(q,\epsilon) = D^{--}_\epsilon(q,\epsilon) = S_\epsilon \hbar/\pi \rho \tau^2 \) [15], where functions

\[
S_\pm(q,\epsilon) = \frac{1}{2} \left( \frac{\hbar}{\hbar Dq^2 - i\epsilon} \pm \frac{1}{\hbar Dq^2 - i\epsilon + \Gamma} \right)
\]

obey the set of equations [13]

\[
\begin{align*}
\frac{\partial S_+}{\partial t} - D\Delta S_+ &= \delta(r)\delta(t) + \frac{S_+ - S_-}{\tau_\ast}, \\
\frac{\partial S_-}{\partial t} - D\Delta S_- &= \frac{S_+ - S_-}{\tau_\ast}.
\end{align*}
\]

Here \( D = v_F\tau_{tr}/2 \) is the diffusion coefficient, \( \Gamma = 2\hbar/\tau^\ast \).

Note that in the case of a short-range potential \( \tau^\ast \sim \tau_{tr} \) and the term \((hDq^2 - i\epsilon + \Gamma)^{\pm1}\) can be neglected in both \( S_+ \) and \( S_- \). The contribution of the symmetric channel to the two-level correlation function is given by

\[
R_\ast(\epsilon) = \frac{\Delta^2}{2\pi^2\hbar^2} \text{Re} \sum_q \left( S_+^2(q,\epsilon) + S_-^2(q,\epsilon) \right) = \frac{\Delta^2}{4\pi^2} \text{Re} \sum_q \left( \frac{1}{(\hbar Dq^2 - i\epsilon)^2} + \frac{1}{(\hbar Dq^2 - i\epsilon + \Gamma)^2} \right).
\]

The first term in the r.h.s. of this expression is the well-known Altshuler-Shklovskii result [14]. The second term is related to the interbranch transitions.

First we discuss the universal region of energies, \( \epsilon \ll E_c \). For \( \Gamma \ll E_c \) one should keep only the term \( q = 0 \) in Eq. (5), which yields

\[
R_\ast(\epsilon) = \frac{\Delta^2}{4\pi^2} \left( -\frac{1}{e^2} + \frac{\Gamma^2 - e^2}{(e^2 + \Gamma^2)^2} \right). \tag{6}
\]

A crossover between two ensembles is clearly seen. For \( \epsilon \ll \Gamma_\ast \) the first term in Eq. (5) dominates. In this
case during the time $\hbar/\epsilon$ many interbranch transitions take place, the system is ergodic and we obtain the standard result \((7)\) for the envelope of the symplectic Wigner-Dyson distribution. For $\Gamma_\star \ll \epsilon \ll E_c$, the time $\sim \hbar/\epsilon$ is not enough for interbranch transition and we have two independent orthogonal ensembles. Thus, we see that the system demonstrates reentrance to the orthogonal ensemble at large $\Delta_{so}$, when $\Gamma_\star$ becomes sufficiently small.

In the nonuniversal region $\epsilon \gg E_c$, where the summation in Eq. \((6)\) can be replaced by the integration, the contribution of the first term in Eq. \((6)\) is equal to zero (for periodic boundary conditions) and the level correlations are governed by the interbranch transitions only:

$$R_s(\epsilon) = \frac{\Delta}{4\pi g} \frac{\Gamma_\star}{\epsilon^2 + \Gamma_\star^2}. \quad (7)$$

Here $g = \pi k_F l_r$ is the conductance in units of $e^2/\pi \hbar$.

Compare now Eq. \((7)\) with the weak localization correction to $R(\epsilon)$ for the symplectic ensemble \((8)\) $R_{sl}(\epsilon) = \Delta/8g^2\epsilon$. The inequality $R_s(\epsilon) \gg R_{sl}(\epsilon)$ is fulfilled in the energy interval $\max(E_c, \Gamma_\star/g) \ll \epsilon \ll g \Gamma_\star$. It can be shown [17] that the boundary-induced and topological contributions \((8)\) are also small compared with Eq. \((7)\). Thus, we see that in the nonuniversal region the interbranch transitions completely govern level correlation in a wide energy interval.

Now we will show that in the region of large energies $|\epsilon| \approx \Delta_{so}$ a narrow resonant peak appears in the two-level correlation function. The physical origin of this peak is this: the exact energy levels lying near the energy $E$ in the lower branch ($-$) are correlated with the exact energy levels lying near the energy $E + \Delta_{so}$ in the upper branch ($+$) (see Fig. 1c). The resonance contribution to $R(\epsilon)$ comes from the diagram in Fig. 2 with $\mu = -\nu$ and $\mu' = -\nu'$ (antisymmetric channel). The diffusions in antisymmetric channel depends on indices $\mu$ and $\mu'$: $D_{\mu\mu'}^{\mu\mu'} = D_{\mu\mu'}^{\mu'\mu}$. The equations for $D_{\mu\mu'}^{\mu\mu'}$ are obtained from equations for $D_{\mu\mu'}^{\mu'\mu}$ by changing the sign of all the indices related to the line corresponding $G^A$ (lower line in Fig. 3). The dashed line represents now the function $K_{\mu\mu'}^{\mu\mu'}(q, k, k') = K(\mathbf{k} - \mathbf{k'})|\langle \chi_{k+q}^{\mu'} | \chi_{k+q}^{\mu'} \rangle |^2$. In the case $\epsilon \approx \Delta_{so}$ the main contribution is represented by $D_{\mu\mu'}^{\mu\mu'}(q, \epsilon) = (\hbar^2/\pi \rho \tau^2)/(\hbar D q^2 + \Gamma - i|\epsilon - \Delta_{so}|)$, where $\Gamma = \hbar/\tilde{\tau}$ and

$$\frac{1}{\tau} = \frac{m}{4\pi \hbar^2 \Delta_{so}^2} \frac{p dp}{(2\pi)^2} K_{\mu}(p) \sim \frac{\nu_F}{d} \left( \frac{\hbar}{\Delta_{so} \tau_r} \right)^2. \quad (8)$$

Here $K_{\mu}(k-k') = \int K(\mathbf{k} - \mathbf{k'}) \cos(\varphi) d\varphi$ and $\varphi$ is the angle between $\mathbf{k}$ and $\mathbf{k'}$. At large spectrum splitting the inequalities $\tau_r \ll \tilde{\tau} \ll \tau^*$ hold. Now we present qualitative arguments clarifying the physical origin of $\tilde{\tau}$ which plays the role of the phase breaking time in the antisymmetric channel. The appearance of a resonance peak in $R(\epsilon)$ is a consequence of coherence of the waves ($+$) and ($-$), which for $\epsilon = \Delta_{so}$ have the same momenta (see Fig. 1c). One might think that the coherence is destroyed by interbranch transitions only. However, in the second order of perturbation theory there exists a virtual process which breaks the coherence. In this process, the wave which in the initial and final state belongs to the branch ($+$), in the intermediate virtual state belongs to the branch ($-$). This process leads to a small correction to the effective random potential which can be estimated as $W \sim \Delta_{so}^2/(\Delta_{so} k_F d)^2$ (here $U$ is the amplitude of random potential and the factor $(k_F d)^2$ is of the same origin as in Eq. \((7)\)). The other wave, in the initial and final state belonging to the branch ($-$), in the intermediate virtual state belongs to the branch ($+$). It is evident that the correction to the potential $W'$ has the same absolute value but the different sign $W' = -W$. The sign difference is related to the fact that the energy of the virtual state lies in first process above, and in the second process below energy of the initial state. As a result, two waves propagate in slightly different effective potentials $U + W$ and $U - W$. On passing the potential correlation length $d$ they acquire the phase difference $\delta \varphi \sim Wd/\nu_F \hbar$. The phase breaking time is then easily estimated as $\tilde{\tau}^{-1} \sim (\delta \varphi)^2/\nu_F d$. Combining these formulas one can obtain the estimate for $\tilde{\tau}$ given in the r.h.s. of Eq. \((8)\). The resonance contribution to $R(\epsilon)$ reads

$$R_{\star}(\epsilon) = \frac{\Delta}{8\pi^2} \frac{\tilde{\Gamma}}{(\epsilon - \Delta_{so})^2 + \tilde{\Gamma}^2}, \quad \tilde{\Gamma} \gg E_c; \quad R_{\star}(\epsilon) = \frac{\Delta^2}{8\pi^2} \frac{\tilde{\Gamma}^2 - (\epsilon - \Delta_{so})^2}{[(\epsilon - \Delta_{so})^2 + \tilde{\Gamma}^2]^2}, \quad \Delta \ll \tilde{\Gamma} \ll E_c.$$

We conclude that a 2$d$ system with a long-range random potential and strong SO splitting of the spectrum exhibits a reentrance to the orthogonal ensemble for sufficiently large values of $\Delta_{so}$. A weak interaction between two sets of random levels corresponding to two branches of the split spectrum leads to the appearance of two effects: (i) in a wide energy interval the level statistics is completely governed by interbranch transitions; (ii) a sharp resonance arises in the two-level correlation function for energies corresponding to the spectrum splitting.

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[15] Note that the diffuson in our case depends on $|k|$ and $|k'|$ (See Fig. 3). Eq. (5) is valid for $|k| = k_\mu$, $|k'| = k'_\mu$, where the values of $k_\mu$ are determined from the conditions $E = E^++(k^+)^2 = E^-(k^-)$.

[16] For $\Gamma_\ast \gg E_c$ the interbranch contribution to $R_\ast(\epsilon)$ in universal region is small and leads only to small correction to the symplectic Wigner-Dyson distribution.

[17] The topological tail in $R(\epsilon)$ (for the case $p = 0$ in notation of [7]) $R_{\text{top}} = -\Delta^2/(12\pi^2\epsilon^2)$ is smaller than $R_\ast(\epsilon)$ if $\epsilon \gg (\Gamma_\ast E_c)^{1/2}$. The boundary-induced contribution $R_b(\epsilon) = -\sqrt{2}\Delta^{3/2}L_b/64\pi g^{1/2}\epsilon^{3/2}L$ (here $L_b$ is the length of the boundary and $L$ is the system size) is negligible for $(\Gamma_\ast^2 E_c L_b^2/L^3)^{1/3} \ll \epsilon \ll \Gamma_\ast^2 L^2/(E_c L^3)$, provided that $\Gamma_\ast \gg E_c L_b^2/L^2$.

**FIGURE CAPTIONS**

Fig. 1 (a) Spin-orbit split energy spectrum; (b) symmetric contribution to the diffuson (two coherent waves are depicted by black dots); (c) antisymmetric contributions to the diffuson.

Fig. 2 One-loop diagram contributing to $R(\epsilon)$.

Fig. 3 Diffuson equation in the symmetric channel.
Fig. 1
Fig. 2
\[ D_s^{\mu \mu'} = K_s^{\mu \mu'} + K_s^{\mu \mu''} D_s^{\mu'' \mu'} \]  

Fig. 3
