Baryons in the nonperturbative string approach

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Abstract

We present some piloting calculations of masses and short–range correlation coefficients for the ground states of light and heavy baryons in the framework of the simple approximation within the nonperturbative QCD approach.

The purpose of this talk is to discuss the results of the calculation [1] of the masses and wave functions of the heavy baryons in a simple approximation within the nonperturbative QCD (see [2] and references therein). The starting point of the approach is the Feynman–Schwinger representation for the three quark Green function in QCD in which the role of the time parameter along the trajectory of each quark is played by the Fock–Schwinger proper time. The proper and real times for each quark related via a new quantity that eventually plays the role of the dynamical quark mass. The final result is the derivation [2] of the Effective Hamiltonian (EH). For the ground state baryons without radial and orbital excitations in which case tensor and spin-orbit forces do not contribute perturbatively the EH has the following form

\[
H = \sum_{i=1}^{3} \left( \frac{m_i^{(0)}^2}{2m_i} + \frac{m_i}{2} \right) + H_0 + V, \tag{1}
\]

where \(H_0\) is the non-relativistic kinetic energy operator and \(V\) is the sum of the perturbative one gluon exchange potential \(V_c\) and the string potential \(V_{\text{string}}\). The latter has been calculated in [3] as the static energy of the three heavy quarks: \(V_{\text{string}}(r_1, r_2, r_3) = \sigma R_{\text{min}}\), where \(R_{\text{min}}\) is the sum of the three distances \(|r_i|\) from the string junction point, which for simplicity is chosen as coinciding with the center–of–mass coordinate.

In Eq. (1) \(m_i^{(0)}\) are the current quark masses and \(m_i\) are the dynamical quark masses. In contrast to the standard approach of the constituent quark model the dynamical mass \(m_i\) is not a free parameter but it is expressed in terms of the current mass \(m_i^{(0)}\) defined at the appropriate scale of \(\mu \sim 1 \text{ GeV}\) from the condition of the minimum of the baryon mass \(M_B\) as function of \(m_i\): \(\partial M_B(m_i)/\partial m_i = 0\). Technically, this has been done using the einbein (auxiliary fields) approach, which is proven to be rather accurate in various calculations for relativistic systems.

The EH has been already applied to study baryon Regge trajectories [3] and very recently for computation of magnetic moments of light baryons [4]. The essential point of this
talk is that it is very reasonable that the same method should also hold for hadrons containing heavy quarks. In what follows we will concentrate on the masses of double heavy baryons. As in [4] we take as the universal parameter the QCD string tension $\sigma$ fixed in experiment by the meson and baryon Regge slopes. We also include the perturbative Coulomb interaction with the frozen coupling $\alpha_s(1 \text{ GeV}) = 0.4$.

We use the hyper radial approximation (HRA) in the hyper-spherical formalism approach. In the HRA the three quark wave function depends only on the hyper-radius $R^2 = \rho^2 + \lambda^2$, where $\rho$ and $\lambda$ are the appropriate three-body Jacobi variables. Introducing the reduced function $\chi(R) = R^{5/2}\psi(R)$ and averaging $V = V_c + V_{\text{string}}$ over the six-dimensional sphere one obtains the Schrödinger equation

$$\frac{d^2\chi(R)}{dR^2} + 2\mu \left[ E_n + \frac{a}{R} - bR - \frac{15}{8\mu R^2} \right] \chi(R) = 0,$$

where

$$a = \frac{2\alpha_s}{3} \times \frac{16}{3\pi} \sum_{i<j} \sqrt{\frac{\mu_{ij}}{\mu}}, \quad b = \sigma \times \frac{32}{15\pi} \sum_{i<j} \sqrt{\frac{\mu(m_i + m_j)}{m_k(m_1 + m_2 + m_3)}},$$

(3)

$\mu_{ij}$ is the reduced mass of quarks $i$ and $j$ and $\mu$ is an arbitrary parameter with the dimension of mass which drops off in the final expressions. We use the same parameters as in Ref. [5]: $\sigma = 0.17 \text{ GeV}$, $\alpha_s = 0.4$, $m_q^{(0)} = 0.009 \text{ GeV}$, $m_u^{(0)} = 0.17 \text{ GeV}$, $m_c^{(0)} = 1.4 \text{ GeV}$, and $m_b^{(0)} = 4.8 \text{ GeV}$.

The dynamical masses $m_i$ and the ground state eigenvalues $E_0$ calculated using the described above procedure are given for various baryons in Table 1 of Ref. [4]. For the light baryons the values of light quark masses $m_q \sim 450 - 500 \text{ MeV} (q = u, d, s)$ qualitatively agree with the results of Ref. [5] obtained from the analysis of the heavy–light ground meson states, but $\sim 60 \text{ MeV}$ higher than those of Refs. [3, 4]. This difference is due to the different treatment of the Coulomb and spin–spin interactions. The light quark masses are increased by $100 - 150 \text{ MeV}$ when going from the light to heavy baryons. For the heavy quarks ($c$ and $b$) the variation in the values of their dynamical masses in different baryons is marginal. Note that the masses of the light quarks in baryons are slightly smaller than those in the mesons.

For many applications the quantities $R_{ijk} = \langle \psi_{ijk} | 5^{(3)} (r_j - r_i) | \psi_{ijk} \rangle$ are needed. Note that these quantities depend on the third or ‘spectator’ quark through the three–quark wave function. To estimate effects related to the baryon wave function we solve Eq. (2) by the variational method using a simple trial function $\chi(R) \sim R^{5/2} e^{-\mu\beta R^2}$, where $\beta$ is the variational parameter. Then $R_{ijk} = (2\beta^2\mu_{ij} / \pi)^{3/2}$. The results of the variational calculations are given in Table 3 of [4]. Comparing the results with those of Ref. [4] we confirm the inequalities $R_{ijk} < \frac{1}{2} R_{ij}$ and $R_{ijk} > R_{ij}$, if $m_k \leq m_i$, first suggested in Ref. [5] from the observed mass splitting in mesons and baryons. Here $R_{ij}$ is the corresponding quantity for a meson. In particular, we obtain $R_{ijk} / R_{ij} = 0.44, 0.40, 0.37,$ and 0.34 for $ijk = ucd, scu, ubd,$ and $sbd$, respectively. These estimations agree with the results obtained using the non–relativistic quark model or the bag model or QCD sum rules which are typically in the range $0.1 - 0.5$. Note also that if $i, j$ are the light quarks, and the quarks $k$ and $l$ are the heavy then $R_{ijk} \approx R_{ijl}$ (i.e. $R_{qqc} \approx R_{qb}$) in agreement with the limit of the heavy quark effective theory.
Note also that the wave function calculated in HRA show the marginal diquark clustering even in the doubly heavy baryons. E.g. in the \( qcc \) baryon \( \bar{r}_{qc} = 0.61 \text{ fm} \) while \( \bar{r}_{cc} = 0.45 \text{ fm} \). Likewise \( \bar{r}_{qb} = 0.53 \text{ fm} \) and \( \bar{r}_{bb} = 0.25 \text{ fm} \) in the \( qbb \) baryon. This is principally kinematic effect related to the fact that in the HRA the difference between the various \( \bar{r}_{ij} \) in a baryon is due to the factor \( \sqrt{1/\mu_{ij}} \) which varies between \( \sqrt{2/m_i} \) for \( m_i = m_j \) and \( \sqrt{1/m_i} \) for \( m_i \ll m_j \). For the light baryons \( \bar{r}_{qq} \sim 0.7 - 0.8 \text{ fm} \).

To calculate hadron masses we, as in Ref. [3], first renormalize the string potential: 
\[
V_{\text{string}} \rightarrow V_{\text{string}} + \sum C_i,
\]
where the constants \( C_i \) take into account the residual self-energy (RSE) of quarks. In the present work we treat them phenomenologically. We adjust \( C_i \) to reproduce the center-of-gravity for baryons with a given flavor. To this end we consider the spin-averaged masses, such as: 
\[
\left( M_N + M_{\Delta} \right)/2, \quad \left( M_{\Lambda} + M_{\Xi} + 2M_{\Sigma^*} \right)/4
\]
and analogous combinations for \( qqc \) and \( qqb \) states. Then we obtain \( C_q = 0.34, C_s = 0.19, C_c \sim C_b \sim 0 \).

We keep these parameters fixed to calculate the masses given in Table 1, namely the spin–averaged masses (computed without the spin–spin term) of the lowest double heavy baryons. In this Table we also compare our predictions with the results obtained using the additive non–relativistic quark model with the power-law potential [4], relativistic quasipotential quark model [5], the Feynman–Hellmann theorem [6] and with the predictions obtained in the approximation of double heavy diquark [10].

| State  | present work | Ref. [7] | Ref. [5] | Ref. [3] | Ref. [3] | Ref. [10] |
|--------|--------------|---------|----------|----------|----------|----------|
| \( \Xi \{qcc\} \) | 3.69 | 3.70 | 3.71 | 3.66 | 3.48 |
| \( \Omega \{scc\} \) | 3.86 | 3.80 | 3.76 | 3.74 | 3.58 |
| \( \Xi \{qcb\} \) | 6.96 | 6.99 | 6.95 | 7.04 | 6.82 |
| \( \Omega \{scb\} \) | 7.13 | 7.07 | 7.05 | 7.09 | 6.92 |
| \( \Xi \{qbb\} \) | 10.16 | 10.24 | 10.23 | 10.24 | 10.09 |
| \( \Omega \{sbb\} \) | 10.34 | 10.30 | 10.32 | 10.37 | 10.19 |

In conclusion, we have employed the general formalism for the baryons, which is based on nonperturbative QCD and where the only inputs are the string tension \( \sigma \), the strong coupling constant \( \alpha_s \) and two additive constants, \( C_q \) and \( C_s \), the residual self–energies of the light quarks. Using this formalism we have performed the calculations of the spin–averaged masses of baryons with two heavy quarks. One can see from Table 1 that our predictions are especially close to those obtained in Ref. [4] using a variant of the power–law potential adjusted to fit ground state baryons.

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