Probing Gluon Bose Correlations in Deep Inelastic Scatterings

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We study correlations originating from the quantum nature of gluons in a hadronic wave function. Bose-Einstein correlation between identical particles lead to the enhancement in the number of pairs of gluons with the same quantum numbers and small relative momentum. We show that these preexisting correlations can be probed in Deep Inelastic Scattering experiments at high energy. Specifically, we consider diffractive dijet plus a third jet production. The azimuthal dependence displays a peak at the zero relative angle between the transverse momentum imbalance of the photon-going dijet and the transverse momentum of the hadron-going jet. Our calculations explicitly show that the peak originates from Bose enhancement. Comparing electron-proton to electron-nucleus collisions, we demonstrate that the nuclear target enhances the relative strength of the peak. With the future high luminosity Electron-Ion Collider the proposed measurements of gluon Bose enhancement become experimentally feasible.

**Introduction.** The future Electron-Ion Collider will provide a unique opportunity to explore the multidimensional structure of protons and nuclei [1,2]. Although most of the experimental measurements and theoretical studies are focused on single-parton distributions [3,6], complete theoretical and experimental understanding of a hadron wave function is not possible without observables sensitive to multi-parton correlations. The simplest objects that probe such correlations are multiparton distribution functions [7,11] and generalized parton distributions [12,13]. Among these, gluon distributions play the most important role in high energy collisions. Indeed, the measurements at HERA established that the small x [14] tail of the hadronic wave function is dominated by gluons [15,18]. Thus it is imperative to identify observables sensitive to multi-gluon correlations. The universal source of correlations between (identical) gluons is Bose enhancement due to quantum statistics. In the hadron wave function this induces the enhancement in the number of pairs of gluons with the same quantum numbers and small relative momentum [19,21]. These Bose-Einstein correlations have been suggested as a possible mechanism for producing ridge correlations in p-p scattering [20,22], however strong final state interactions make their effect difficult to isolate. Here we show that these correlations can be probed in a clean straightforward way in Deep Inelastic Scattering (DIS) experiments. We also observe that the effect due to these correlations is enhanced by saturation effects in the target hadron.

The general idea is as follows. At high energy, the intuitive picture of DIS in the infinite momentum frame is that of the virtual photon fluctuating into quark-antiquark pair (dipole) which scatters on the gluon field of the fast moving hadron target. As a result two jets with the transverse momenta \(p_1\) and \(p_2\) are produced. The transverse momentum imbalance \(\Delta = p_1 + p_2\) is acquired due to the interaction of the dipole with the hadron. Consider now a final state which, in addition to the \(q\bar{q}\) dijet, contains a gluon jet with transverse momentum \(p_3\) originating from the hadron. In the hadronic wave function prior to the scattering this gluon is Bose correlated with an identical gluon (the two have momenta \(k_1 \approx k_2\)). The exchange of the gluon \(k_1\) between the hadron and the dijet leads to non-zero momentum imbalance \(|\Delta| = |k_1|\) when \(k' \approx 0\). In this situation, the momentum of the produced gluon does not change significantly (\(p_3 \approx k_2\)) and the primordial Bose-Einstein correlations should lead to the increase in the cross-section of the trijet production, when the transverse momenta \(p_3 \approx \pm (p_1 + p_2)\).

**FIG. 1.** Schematic diagram showing the trijet production in \(\gamma^*N\) collisions. Bose-Einstein correlations in the hadron wave function lead to the increase in the cross-section of the trijet production, when the transverse momenta \(p_3 \approx \pm (p_1 + p_2)\).
kinematical window the Bose-Einstein induced correlation is clearly seen in the trijet momentum distribution and may be sufficiently strong to be experimentally measurable. Moreover, we argue that the unique features of this correlation facilitate its experimental identification and separation from the background. To minimize the effects due to Sudakov radiation [23, 24] we consider here a trijet configuration where the $q\bar{q}$ dijet is in the color singlet state. This still leaves room for Sudakov radiation from the gluon $p_3$, however if the transverse momentum is not too large we don’t expect this to qualitatively change the picture [25].

**Trijet production in high energy DIS.** We focus on final states containing a color singlet $q\bar{q}$ dijet and the third jet originating from the nucleus, i.e. there is a large rapidity gap between the dijet and the third jet (as opposed to Refs. [26, 27] where the rapidity gap is between proton and the trijet). We work in a frame where both the virtual photon and the hadronic target carry zero initial longitudinal momentum, while the hadronic target — large component of light cone momentum, while the hadronic dijet and the third jet originating from the nucleus, i.e. there is a large rapidity gap. We expand the $S$-matrix operator to second order in the eikonal coupling. This corresponds to two gluon exchange constants, also includes the field emitted by valence partons, also includes the field emitted by the virtual photon and the hadronic target carry zero initial longitudinal momentum.

The expectation value of quark, anti-quark, and gluon dipole, respectively. Here $p_1$, $p_2$, and $p_3$ are the momenta for the quark, antiquark and gluons, respectively. The expectation value of quark, anti-quark, and gluon ($d$, $b$, and $\bar{a}$) number operators are evaluated over the final state

$$|\psi_F\rangle = \hat{C}^\dagger \hat{S} |\gamma^*\rangle \otimes |N\rangle.$$ (2)

Following the CGC framework, the initial hadron state $|N\rangle$ is represented in terms of the state of the valence degrees of freedom $|v\rangle$ and the vacuum of the soft gluons in the presence of the valence sources $|s\rangle = \hat{C}|0\rangle$ as $|N\rangle = |v\rangle \otimes |s\rangle$, with the coherent state operator $\hat{C}$.

$$\hat{C} = \exp \left\{ i \int d^2x b_i^a(x) \int_{\Lambda^{-\infty}}^\Lambda \frac{dk^\perp}{2\pi |k^\perp|} \left( \hat{a}^\dagger_i (k^\perp, x) \right. \right.$$  

$$\left. + \hat{a}_i^\dagger (k^\perp, x) \right) \right\}.$$  

Here $b_i^a(x)$ is the classical Weizsäcker-Williams (WW) field generated by the valence degrees of freedom. The initial virtual photon state can be approximated by the quark-antiquark pairs $|\gamma^*\rangle \approx \sum_q \Psi_q \bar{q} \Psi_{q\bar{q}}$ with $\Psi_{q\bar{q}}$ the dipole wavefunction whose explicit expression will be given below (see also [38, 39]).

To arrive at Eq. (2), two effects are taken into account. The first effect is due to eikonal $S$-matrix interaction between the dipole and the hadron:

$$\hat{S} = \exp \left\{ i \int d^2x j_D(x) \frac{\partial \hat{a}_i}{\partial x^2} \hat{A}_i^\dagger (x) \right\}.$$ (3)

Here the color current operator of the target dipole is

$$j_D^a(y) = g \sum_s \int_0^\infty \frac{dk^+}{2k^+(2\pi)} \left[ b_{h_1,s}^i (k^+, y) t^a_{h_1,h_2} \hat{b}_{h_2,s} (k^+, y) \right.$$  

$$\left. + \hat{b}_{h_1,s}^i (k^+, y) t^a_{h_1,h_2} \hat{b}_{h_2,s}^\dagger (k^+, y) \right]$$

and the gluon field operator $\hat{A}_i^\dagger (x)$ has the conventional mode expansion in terms of gluon creation and annihilation operators in the light-cone gauge [40, 41]. The hadron gluon field can be separated into the low and high longitudinal momentum modes. The latter can be treated as a classical field (the WW field), while the former, the quantum part, in addition to the field produced by valence partons, also includes the field emitted by the higher longitudinal momentum gluonic modes [42].

$$A_i^\dagger (x) = b_i^\dagger (x) + \delta A_i (x).$$

At the leading order in the coupling constant, $\delta A_i = \frac{\alpha_s}{2\pi} j_G^a (x)$ with the gluon density operator

$$j_G^a (x) = ig f^{abc} \int_{k^- < \Lambda^-} \frac{dk^-}{2k^- (2\pi)} \hat{a}_i^\dagger (k^-, x) \hat{a}_c^\dagger (k^-, x).$$

The calculation is simplest in the dilute limit and here we expand the $S$-matrix operator to second order in the eikonal coupling. This corresponds to two gluon exchange in the amplitude. One of the four possible two gluon exchange diagrams is shown in Fig. 1.

The second effect is due to final state radiations. It is accounted for by the “dressing operator” $\hat{C}^\dagger$. The reason this additional factor is necessary is that the eikonal
\[ S(\omega, \pm) \sim \frac{1}{4\pi} \frac{1}{\omega} \left( \frac{\omega^2 - m^2}{\omega^2} \right) \]

The trijet observable is then readily computed; we present it in the following semi-factorizable form

\[
\frac{d^3N}{d^3p_1 d^3p_2 d^3p_3} = \int \frac{d^2k}{(2\pi)^2} \frac{d^2l}{(2\pi)^2} O_{\text{dipole}}^{abcd}(\{p_i\}; \{k, l\}) \times O_{\text{hadron}}^{abcd}(\{p_i\}; \{k, l\}).
\]  

The part involving the hadron reads

\[
O_{\text{hadron}}^{abcd}(\{p_i\}; \{k, l\}) = \frac{L_j(p + 1, p_3)}{L_j(p + k, p_1)} \times \int f^{gh} f^{de} (v) \rho^a(l) \rho^b(-k) \rho^c(p + k|v). \quad (5)
\]

The part involving the dipole, the terms contribut-
polarized photons are ing to the diffractive production are of the saturation scales.

FIG. 4. The normalized tri-jet correlation for different values of the rapidity difference becomes too large [52, 53].

Here the normalization factor is computed by integrating over the angle $N = \int d\theta d^3N_{l,T}/d^3p_1d^3p_2d^3p_3$. We also need to specify the momentum magnitudes of the quark-antiquark jets $p_1, p_2$. In principle, one can integrate over all possible $p_1, p_2$ subject to the constraint $|\Delta| = |p_1 + p_2|$. However integrating over a large phase space masks the Bose enhancement signal. Instead, we further select events with particular values of $p_1, p_2$ for which the Bose enhancement signal is more prominent. For illustrative purposes we choose $p_3 = (p_3, 0)$ along the x-axis and $z_1 = z_2 = 1/2$ without loss of generality. We focus on transversely polarized virtual photon.

In Fig. 2 we present numerical results for the dependence of $C(p_3, \Delta, \theta)$ on the azimuthal angle $\theta$ with $p_3 = \Delta = 10$ GeV at $p_1 = 10$ GeV for the transverse polarization of the virtual photon with $Q = 1$ GeV, and the nuclear target with the saturation momentum $Q_s = 2$ GeV. The figure demonstrates a peak at zero azimuthal angle when $p_2 \lesssim 4$ GeV. The zero-angle peak at $p_3 = \Delta$ is the salient feature of the Bose-Einstein correlation. The plots show the angular region $-\pi/4 \leq \theta \leq \pi/4$. For angles close to $\theta = \pm \pi$ we observe, as expected a very large enhancement which arises due to the low momentum gluons in the target with $|k_1|$ or $|k_2| \approx \Lambda_{QCD}$, which is not related to Bose enhancement.

We have also examined the values $p_1 = 11-15$ GeV (at fixed $p_3 = \Delta$), and found that the zero-angle peak is also present although the range of the relevant values of $p_2$ is smaller. We have explored other momentum regions by scaling $p_3 = \Delta$ between 5 and 15 GeV, and have observed that the zero-angle peak persist. Furthermore, we checked the Bose enhancement signals for longitudinal polarization of virtual photon. The enhancement signal here exists as well, albeit it is less prominent compared than for the transverse polarization in the same kinematics.

Note that the kinematics we explore is different from the frequently considered correlation limit $Q \gg \Delta$. In that regime, we analytically demonstrated the absence of the zero-angle peak due to inability of the exchanged gluon with momentum $< Q$ to resolve the structure of the dipole. The color neutrality then results in sizable suppression through Eq. (7).

Fig. 3 demonstrates that the zero-angle peak in the correlation function is the direct reflection of the Bose-Einstein correlation in the hadronic wave function. Varying the UV cutoff we see that the peak comes from the momentum integration region $k = -\Delta$. This corresponds to $k' \approx 0$ in Fig. 1 which means $k_2 \approx -p_3$ and $k_1 = k = -(p_1 + p_2)$, thus in this region of phase space the trijet directly probes the momenta of the two gluons in the hadronic wavefunction. For $p_2 \geq 4$ GeV, the phase space region $k \approx -\Delta$ is overwhelmed by contributions from the rest of the phase space $k \gtrsim |\Delta|$; this leads to the suppression of the zero-angle peak in Fig. 2 for $p_2 \geq 4$ GeV.

Finally, to utilize the unique characteristics of EIC to
accelerate both protons and nuclei, we performed the calculations for different values of the gluon saturation scale $Q_s$. Fig. 4 demonstrates that Bose enhancement peak becomes more pronounced with increasing the gluon saturation scale. Although the trijet momentum imbalance is mainly due to gluons with momentum larger than the gluon saturation scale, increasing the gluon saturation scale enhances the probability of larger momentum transfer between the dipole and the nuclear target. Thus the correlation displays a clear dependence on the nuclear number and can serve as a sensitive probe of the saturation scale. It is interesting that the main effect of saturation is to enhance the relative importance of the Bose-Einstein correlations in the trijet spectrum.

Conclusions. We demonstrated that the measurement of gluonic Bose-Einstein correlations in hadronic wave function is accessible in DIS. The observable we propose is production of three jets at high energy with one of the jets separated in rapidity from the other two (dijet) produced in the photon-going direction [54]. We showed the presence of zero angle correlation between the momentum imbalance of the dijet and the transverse momentum of the third jet. Its origin is due to preexisting Bose-Einstein correlations and the transverse momentum of the third jet. Its transverse momentum becomes more pronounced with increasing the gluon saturation scale. Although the trijet momentum imbalance is mainly due to gluons with momentum larger than the gluon saturation scale, increasing the gluon saturation scale enhances the probability of larger momentum transfer between the dipole and the nuclear target. Thus the correlation displays a clear dependence on the nuclear number and can serve as a sensitive probe of the saturation scale.

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For a configuration when $q\bar{q}$ dijet is not in a color singlet state, the leading contribution would originate from single gluon exchange, which does not probe BE correlations and is not expected to contribute to near side correlations.