Fast LLMMSE filter for low-dose CT imaging

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Low-dose X-ray CT technology is one of important directions of current research and development of medical imaging equipment. A fast algorithm of blockwise sinogram filtering is presented for real-time low-dose CT imaging. A nonstationary Gaussian noise model of low-dose sinogram data is proposed in the low-mA (tube current) CT protocol. Then, according to the linear minimum mean square error principle, an adaptive blockwise algorithm is built to filter contaminated sinogram data caused by photon starvation. A moving sum technique is used to speed the algorithm into a linear time one, regardless of the block size and the data range. The proposed fast filtering gives a better performance in noise reduction and detail preservation in the reconstructed images, which is verified in experiments on simulated and real data compared with some related filtering methods.

Introduction: As an important medical imaging mode, X-ray computed tomography (CT) is one of common clinical diagnosis methods. With wider applications of X-ray CT scan in examining human body, its radiation hazard to public health has received more and more attention [2]. In face of increasingly severe radiation threat, the International Organization for Medical Physics (IOMP) has made quality control standards in medical radiation, advocating that the X-ray diagnosis should follow the principle of legitimacy in practice and protection optimization (As Low As Reasonably Achievable, ALARA) [3]. Aiming to obtain best diagnostic effect at a minimum cost and radiation dose, low-dose CT technology has become one of main directions of current research and development of medical imaging equipment.

Low-dose CT scanning poses a huge challenge for CT reconstruction. Under the low-dose condition the photon starvation appears at detector bins of CT system [1], which leads to heavily noisy projection data and consequent degraded CT images full of heavy streak artifacts and noise when reconstructed from traditional algorithms. Thus, researchers have made great efforts from two aspects of low-dose CT imaging. A nonstationary Gaussian noise model of low-dose sinogram data is proposed in the low-mA (tube current) CT protocol. Then, according to the linear minimum mean square error principle, an adaptive blockwise algorithm is built to filter contaminated sinogram data caused by photon starvation. A moving sum technique is used to speed the algorithm into a linear time one, regardless of the block size and the data range. The proposed fast filtering gives a better performance in noise reduction and detail preservation in the reconstructed images, which is verified in experiments on simulated and real data compared with some related filtering methods.

Blockwise filtering with LLMMSE estimation: Considering the sinogram data model (1), if we impose a linear constraint on the estimation of the original data, we have the linear minimum mean square error (LLMMSE) estimator

$$\hat{p}_{LLMMSE} = E(p) + C_{pq}C_{q}^{-1}(q - E(q)),$$

(2)

where \(E(p)\) and \(E(q)\) are the ensemble means of \(p\) and \(q\), respectively; \(C_{pq}\) and \(C_{q}\) are the cross-covariance of \(p\) and \(q\); \(C_{q}^{-1}\) is the inversion of the covariance of \(q\).

In order to compute nonstationary ensemble statistics, we propose a blockwise filtering of noisy sinogram data, and refer to it as a blockwise local linear mean square error (LLMMSE-B) filter. Assuming uncorrelated noise and the nonstationary mean and nonstationary variance (NMVN) image model (3), for a point \((i, j)\) belonging to a block \(w_k\) centered at the point \(k\), the LLMMSE filter is defined as

$$\hat{p}_{LLMMSE}(i, j) = \tilde{\eta}(i, j) + \frac{v_n(i, j) - \sigma_n^2(i, j)}{v_q(i, j)}(q(i, j) - \tilde{\eta}(i, j)),$$

(3)

where \(\tilde{\eta}\) and \(v_q\) are the local spatial mean and variance of \(\eta\); \(\sigma_n^2\) is local spatial variance of nonstationary noise \(n\).

However, a point \((i, j)\) is involved in all the overlapping blocks \(w_k\) that covers \((i, j)\). For different values of \(\hat{p}_{LLMMSE}(i, j)\) computed in different windows, we average all the possible values of \(\hat{p}_{LLMMSE}(i, j)\).

Let \(a(i, j) = \frac{v_q(i, j) - \sigma_n^2(i, j)}{v_q(i, j)}\) and \(b(i, j) = (1 - a(i, j))\tilde{\eta}(i, j)\).

Thus, the blockwise filtering with LLMMSE estimation is computed by

$$\hat{p}_{LLMMSE-B}(i, j) = \frac{1}{|w_k|} \sum_{k(s, t) \in w_k} (a(s, t))q(i, j) + b(s, t),$$

(4)

where \(|w_k|\) is the number of points in \(w_k\). Due to the symmetry of blockwise computing, (5) can be rewritten as

$$\hat{p}_{LLMMSE-B}(i, j) = \tilde{\eta}(i, j) + b(i, j),$$

(5)

where \(\tilde{\eta}\) and \(b\) are the local spatial means of \(a\) and \(b\), respectively.

In order to efficiently compute local spatial means in the proposed method, the box filter is exploited using a moving sum technique, which speeds the algorithm into a linear time one, regardless of the block size and the data range.

Based on what has been discussed above, we summarize our proposed algorithm as follows. The noisy sinogram data \(q\) is filtered by LLMMSE-B to obtain a better estimate \(\hat{p}_{LLMMSE-B}\). Then, a low-dose CT image is reconstructed from filtered sinogram data \(\hat{p}_{LLMMSE-B}\) with the classic FBP method.

Experiments and analyses: First, we perform computer simulations to verify our method. The simulated sinogram data are produced by projecting a 2-D modified Shepp-Logan head phantom (256 × 256) using the fan-beam ray-driven algorithm [14]. As described in [5], the size of sinogram is 888 × 984, where 888 and 984 are numbers of detector bin and angle sample, respectively. The noisy sinogram data for low-dose CT are simulated by adding nonstationary Gaussian noise to noise-free sinogram, where the variance of the nonstationary Gaussian noise is determined by the exponential relationship following the formula (1). In this study we take \(f = 22500\) and \(\eta = 22000\) to simulate a case of lower tube current (mAs).

In the proposed filtering, the parameters are chosen as follows: \(3 \times 3\) mask for calculating both median and mean values; local noise variance is 0.8 times the noise estimated according to the formula (1) using the means filter; a Hanning filter with default settings is employed in the fan-beam FBP reconstruction [14].

As shown in Fig[1] the FBP reconstructed image from noisy sinogram data is obviously full of noise and streak artifacts, compared with the image from the filtered sinogram by our method, which produces fewer annoying artifacts. Furthermore, observing local profiles (203-209 rows, 126th column) in reconstructed images from filtered sinograms by both the median and our processings, one can see that our result displays a better approximation to the original one and sharper edges with smaller edge width than the result by the median filter.

To further validate our method, we calculate the signal to noise ratios (SNR) of reconstructed images. We also record the running time of our
proposed filtering, where the time of FBP reconstruction is not included. All methods are implemented using the MATLAB programming on a desktop computer with Intel Core(TM)2 Quad 2.83 GHz CPU and 4.00 GB Memory. In Table 1, we compare the reconstructed images using such filtering methods as the median filter (MED), the LLMMSE estimator (LLMMSE) and our method. One can see that, our proposed method has highest SNR with a little bigger time consumption.

Table 1: SNR results and Running time of sinogram filtering for low-dose FBP reconstruction by related methods.

| SNR       | Noisy | MED | LLMMSE | Ours |
|-----------|-------|-----|--------|------|
| Time (sec.) | 23.5726 | 24.3442 | 24.6771 | 23.8756 |
|           | 0.0783 | 0.1045 | 0.1219 |

Finally, we examine our filtering method on real sinogram data by scanning a head phantom in the protocols of both high tube current (400mA) and low one (120mA). In Fig.1, fan-beam FBP reconstructed images are shown from original sinograms with different tube currents and the filtered sinogram by our method, respectively. One can observe that, even if in the protocol of a low tube current the FBP reconstructed images from original sinograms with 120mA and 400mA, and the filtered sinogram by our method, respectively.

In a word, based on above experiments and data analyses, it is verified that the sinogram filtering is an effective method in low-dose CT imaging, and our proposed filtering performs better than related methods in removing noise and artifacts and preserving important details of sinogram data. More important, as a fast data filter our method can be used in the realtime imaging process.

Conclusions: A fast sinogram filtering based on the LLMMSE estimation is proposed for realtime low-dose X-ray CT imaging in the framework of classic filtered backprojection reconstruction. Developing the nonstationary Gaussian noise model of low-dose sinogram data, and the moving sum technique to speed the algorithm, the proposed fast filtering performs better in noise-resolution tradeoff for reconstructed images, which is verified in the experiments on simulated and real data compared with related filtering methods.

For future research, we will further try to optimize the algorithm in the process of adaptive filtering according to the statistics of noise.

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