Experimental study of the initial growth of a localized turbulent patch in a stably stratified fluid

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Abstract

We present a laboratory experiment of the initial growth of a turbulent patch in a stably stratified fluid. The patch is created due to a localized source of turbulence, generated by a horizontally oriented and vertically oscillating grid much smaller than the tank size and far from solid boundaries. Synchronized and overlapping particle image velocimetry (PIV) and planar laser induced fluorescence (PLIF) measurements capture the evolution of the patch through its initial growth until it reached a maximum size. The simultaneous measurements of density and velocity fields allow for a direct quantification of the distribution of kinetic energy, buoyancy and degree of mixing within the patch. We can also relate the propagation speed of the turbulent/non-turbulent interface and its thickness to the properties of the turbulent fluid inside the evolving patch. The velocity measurements in this setup indicate significant transient effects inside the patch during its growth. A local analysis of the turbulent/non-turbulent interface provides direct measurements of the entrainment velocity $w_e$ as compared to the local vertical velocity and turbulent intensity at the proximity of the interface. The detailed information about the growth of localized sources of turbulence in stratified environment might be of use in stealth design of autonomous underwater vehicles.

Keywords: turbulent patch, local entrainment and stable stratification

1. Introduction

Strong localized turbulent perturbations, referred to as “turbulent patches”, are ubiquitous processes in stratified fluids in nature and engineering applications. Evidence of their existence in oceanic flows was found in horizontal and vertical small scales through flow measurement \cite{14, 15, 18, 21}. In addition, the evolution of localized turbulent sources appears to be critical in the context of

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wakes of drag or self-propelled bodies in stratified environment [4, 16, 28, 20]. It
was shown that the far wake evolution depends on its initial growth to the max-
imum size and mixing in the near wake region prior to its vertical collapse [20].

To study the growth of localized turbulent sources in the laboratory, van de
Watering [28] mechanically created an impulsively disturbed patch in a quiescent
linearly stratified flow and measured its vertical growth rate in time. Wu [30]
measured the evolution of flow intrusion formed by the injection of fluid with
uniform density into a stably stratified environment, which is somewhat similar
to the growth of a patch. Fernando and co-authors [8, 10] studied the growth of
turbulent patches using vertically oscillating horizontal grids, either spanning
the full extent of the tank [10], or the extent of the tank in one direction [8].
Another work considered injected mixed volumes of fluid [13].

Although the localized sources of turbulence in stratified environment were
created differently (impulsive or continuous forcing, with or without mean flow)
they exhibit a common behavior: they grow vertically at a rate initially un-
affected by the stratification and their maximum vertical extent is reached
on a time scale proportional to the buoyancy frequency of the ambient fluid,
\[ N = \left( -g/\rho_0 \frac{d\rho}{dz} \right)^{1/2} \]
(\(g\) is the acceleration due to gravity, \(\rho\) is the
density and \(\rho_0\) is a reference density). Unfortunately there are no overlapping
measurements of turbulence and density fields inside these sources that can help
to understand the key mechanisms during the initial growth phase.

The goal of this work is to better understand the flow and mixing within
a localized turbulent patch and entrainment region through the turbulent/non-
turbulent interface during its initial growth. We consider a patch that is free
to grow in all directions. It is created mechanically by a horizontally oriented
finite grid, oscillating vertically. The grid is small relative to the tank size and
located in its center such that the patch growth is unaffected by the free surface
or the tank boundaries.

We measure velocity and density fields simultaneously, inside and outside
the patch, using particle image velocimetry (PIV) and planar laser induced
fluorescence (PLIF) methods. We implement the refractive index matching
method [19] to create linear stable stratification. The results provide insight
into the fluid dynamics and buoyancy distribution within the patch and enables
a local analysis of the turbulent/non-turbulent interface.

The paper is organized as follows. Section 2 presents the experimental setup
and measurement techniques, including detailed description of the PIV and
PLIF calibration. Section 3 demonstrates the results in terms of the initial
growth of the patch and its spatially averaged characteristics, as well as the
local analysis near the turbulent/non-turbulent interface (TNTI), aiming at
processes responsible for the TNTI propagation. Discussion and concluding
remarks appear in Section 4.

2. Experimental setup

The problem studied here is a localized mixed turbulent region in a linearly
stable density profile, \(\rho(z)\) shown schematically in Fig. 1a. In our experiments
this corresponds to a buoyancy frequency of $N = 1 \text{s}^{-1}$, measured by PLIF as well as by a pycnometer with small volumes of liquid extracted at different depths.

In order to measure the key parameters of the phenomenon, we create a localized source of turbulence in a stably stratified environment, and measure the turbulent flow as it evolves in time. We follow the patch using optical measurement methods, starting from rest ($t = 0$) through the growth phase up to its collapse. We maintain the refractive index matching in order to allow for accurate density and velocity measurements at high spatial resolution as described in Section 2.2.

The experiments are performed in a glass tank with a $200 \times 500 \text{mm}^2$ cross-section and a depth of 200 mm. Experiments are carried out using different solutions of an index-of-refraction matched stratified mixture of sugar, water and Epsom salts following the method of McDougall [19] described in detail below. A stable linear density gradient is established in the tank using the free-flow two-tank method [9].

### 2.1. Turbulent agitation

The patch is created using a vertically oscillating horizontally-oriented grid located at the mid-depth in the center of the tank. The grid size is $60 \times 60 \text{mm}^2$ constructed of plastic square bars of 2 mm and a mesh size of 10 mm resulting with solidity of 30.6%. The oscillations are formed via a slide on a linear ball-bearing rail connected to a motor with an eccentric linkage. The stroke length (peak-to-peak) is 10 mm. Over the first two seconds of the oscillations, the motor is provided a ramp input to minimize any impulse from the start-up procedure, followed by a steady current input. The forcing is stopped after 8 seconds in total.

The experiments were performed in two sets: a) 50 repetitions for a single frequency of 5 Hz (not shown for the sake of brevity), and b) repeating the patch growth at various frequencies, from 3 to 8 Hz, with increments of 1 Hz. We used the two sets of experiments to verify that the statistics converged and that ensemble averaged statistics from the first set correspond to the spatially averaged statistics in the second set. In both sets we measure only the initial
Table 1: Experimental parameters of different experimental runs for increasing oscillation frequency of the grid.

| $f$ [Hz] | $\delta_{\text{max}}$ [m] | $\sigma_u(\delta_{\text{max}})$ [m/s] | $Re$ | $Ri$ |
|----------|----------------|-----------------|---|---|
| 3        | 0.026          | $2.1 \times 10^{-3}$ | 55 | 146 |
| 4        | 0.034          | $3.6 \times 10^{-3}$ | 122 | 87  |
| 5        | 0.038          | $4.6 \times 10^{-3}$ | 179 | 69  |
| 6        | 0.043          | $5.2 \times 10^{-3}$ | 224 | 68  |
| 7        | 0.048          | $6.3 \times 10^{-3}$ | 308 | 57  |
| 8        | 0.053          | $7.4 \times 10^{-3}$ | 396 | 50  |

growth, from $N t = 0$ to $N t = 8$, such that the background stratification is preserved. Density measurements were repeated to ensure that the background stratification profiles were the same at the beginning and at the end of each test.

The patch initially grows as a result of the agitation until it reaches a maximum. The vertical patch size will be denoted $\delta$ and the typical turbulent velocity scale associated with the turbulence inside the patch will be denoted $\sigma_u$. Both $\delta$ as $\sigma_u$ will be defined in Section 3. In Table 1, the measurements of the maximum patch size, $\delta_{\text{max}}$ and the corresponding level of turbulence, $\sigma_u(\delta_{\text{max}})$ are presented for the 6 different frequencies.

The key dimensionless parameters characterizing the patch dynamics are the bulk Reynolds number $Re = \sigma_u \delta / \nu$, representative of turbulence strength relative to viscosity, and the bulk Richardson number $Ri = N^2 \delta^2 / \sigma_u^2$, representative of the strength of the stratification relative to turbulence levels inside the patch. In Table 1 we report that $Re$ ranges from 50 to 400 whilst $Ri$ ranges from 50 to 150, indicating that the stratification is relatively strong in our case as compared to the turbulence levels. In the results and discussion sections we will refer to the experiments by their forcing frequency in the range $f = [3, 8]$ Hz with increments of 1 Hz.

2.2. Refractive index matching

Obtaining a linear stable stratification using variations in salt concentration is a commonly used experimental technique [10, 25, 8]. However, when using optical methods there are some concerns about the distortion of the light path of the laser [19] in addition to the deviation of the light paths towards the camera [1].

Following McDougall [19] we use two liquids: a sugar–water solution and an Epsom salt (MgSO$_4$)–water solution. The refractive index, $n$ and density, $\rho$ (at 20°C, measured in g/mL) can be expressed by a Taylor series [19] as:

$$n(z) = 1.3330 + 0.20286 C_A(z) + 0.14445 C_B(z) + \mathcal{O}(z^2) \quad (1)$$

$$\rho(z) = 0.9982 + 1.0054 C_A(z) + 0.38407 C_B(z) + \mathcal{O}(z^2)$$

where $C_A$ and $C_B$ are the concentrations (weight of solute per unit weight of solution) of Epsom-salt and sugar, respectively. The desired density difference
of 20 g/L between the bottom of the tank and the free surface is obtained with a maximum difference in the index of refraction which compares favorably with previous studies [7]. The density difference across the patch did not exceed 1%, thus the Boussinesq approximation can be considered valid. The maximum difference in the index of refraction of $\Delta n < 0.00001$ is obtained in each solution separately and for the two solutions being mixed in agreement with the results of McDougall [19]. The kinematic viscosity of the solution is measured at various depths by using a viscometer. Across the maximum extent of the patch height (10 cm), the viscosity change is less than 2.3%.

2.3. Particle Image Velocimetry (PIV)

Two-dimensional, two-component PIV measurements are performed in a vertical plane passing through the horizontal center of the tank as shown in Fig. 1b. The PIV system in this study is composed of a Nd:YAG laser (120 mJ/pulse, wavelength of 532 nm) and a 11 MP double-exposure CCD camera with a 12-bit sensor. The imaging set-up yields a physical magnification of 56.2 $\mu$m/pixel. The large spatial resolution and the dynamic range of this camera limited sampling rate to 2 Hz. PIV measurements are performed such that tracers (polyamide spheres, mean diameter of 50 $\mu$m, $\rho_p = 1.03$ g/cm$^3$, Dantec Inc.) have an average displacement of approximately 5 pixels for each experiment. The timing is set from time interval $\Delta t = 0.018$ s for the $f = 3$ Hz experiment down to $\Delta t = 0.006$ s for the $f = 8$ Hz run. External triggering system initiates the motion of the grid and the first PIV image pair at the same time. In addition, the PIV measurements are synchronized with the overlapping PLIF measurements to obtain simultaneous measurements of density and velocity.

The PIV image pairs are processed using the open source OpenPIV software, described in Taylor et al. [26] with interrogation windows of $32 \times 32$ pixels$^2$ and 50% overlap. Spurious vectors are identified first by a global filter and followed by a local median filter. The rejected vectors are replaced by interpolation using the local mean and the data are smoothed using a Gaussian filter. The PIV uncertainty was estimated through the so-called particle disparity method, as described in Sciacchitano et al. [24]. The average uncertainty values of the horizontal ($\varepsilon_{\Delta x}$) and vertical ($\varepsilon_{\Delta z}$) displacements, measured both $\sim 0.1$ pixels. These correspond to instantaneous velocities errors of the order of $7 \times 10^{-4}$ m/s.

2.4. Planar laser induced fluorescence (PLIF)

The PLIF camera collects images simultaneously with the PIV measurements using the setup shown in Fig. 1b. The PLIF method enables the measurement of the spatial density distribution. Rhodamine 6G is added to the mixing tank of the heavier liquid (the Epsom salt solution) at a concentration of 40 $\mu$g/L. The density can be obtained through measurements of the Rhodamine 6G concentration. This technique was previously used in the study of interfacial mixing in stratified environments [2], wave breaking [27], jets [23], and in stratified gravity currents [22]. The common assumption that the Rhodamine 6G follows the density field relies on the notion that the diffusivity of
turbulence ($\kappa_T \approx \sigma_u \times l = 1 \text{ cm}^2\text{s}^{-1}$) is significantly greater than the molecular diffusivity of the Rhodamine 6G ($\kappa_{R6G} = 0.12 \times 10^{-5} \text{ cm}^2\text{s}^{-1}$), Epsom salt ($\kappa_{MgSO_4} = 0.61 \times 10^{-5} \text{ cm}^2\text{s}^{-1}$), and sucrose ($\kappa_s = 0.45 \times 10^{-5} \text{ cm}^2\text{s}^{-1}$). Thus, the Batchelor scale $\lambda_B(=\eta/Sc^{1/2})$, of the Epsom salt, which we estimate using the Kolmogorov length scale ($\eta \approx 200 \mu\text{m}$ based on the turbulent velocity and length scales) and the Schmidt number of the Epsom salt, $Sc (= \nu/\kappa)$, is found to be of the order of 10 $\mu\text{m}$. This scale is much smaller than the size covered by one pixel (56 $\mu\text{m}$). The under-resolution of the density field is due to the need to accommodate the large field-of-view of the experiment, following the motion of the growing patch.

Figure 2: (a) Raw PLIF image with the laser light entering the field of view from the right. The three control points A, B, and C are used for the calibration procedure. (b) Calibration of concentration $C$ to PLIF image intensity $I_F$ obtained at the points A, B and C. Note that only the values in the linear range $C = [20, 110]$ $\mu\text{g/L}$ are used for the calibration.

The CCD camera for the PLIF measurements is identical to the PIV camera (11 MP, 12 bit camera). Both cameras are equipped with 60 mm Nikkor lenses. A bandpass optical filter (Newport 20BPF10-550) with an admittance band of 550 ± 10 nm is fitted to the PLIF camera to transfer the light at 555 nm that Rhodamine 6G emits when excited by the Nd:YAG laser and blocks the laser light at 532 nm diffracted by the PIV tracers.

The PLIF calibration procedure follows the method of Crimaldi [6]. The solutions and the setup were carefully prepared for the calibration to avoid errors related to pH and temperature, as suggested by Vanderwel and Tavoularis [29]. The intensity of light fluoresced by the dye substance $I_F$ is directly proportional to its concentration $C$ and the intensity of light excitation, $I$, and it is inversely proportional to the ratio of the laser light intensity and the saturated intensity of the particular dye, $I_s$:  

$$I_F \propto C \frac{I}{1 + I/I_s} \quad (2)$$

For carefully constructed experiment the intensity in PLIF images $I \ll I_s$ and consequently the Eq. 3 can be formulated as $I_F \propto CI$ [6]. Thus, the
fluorescent light captured on the imaging sensor at pixel $i,j$ is a function of the pixel location (due to imperfections of imaging optics), local dye concentration $C(i,j)$ and the light intensity $I(i,j)$:

$$I_F(i,j) = \alpha(i,j) C(i,j) I(i,j)$$

(3)

where $\alpha(i,j)$ is a point-wise calibration coefficient.

The procedure yields a the calibration coefficient $\alpha(i,j)$. We use uniform solutions of seven different concentrations of Rhodamine 6G with a homogeneous mixture of Epsom salts, sucrose and water to replicate the experimental conditions. The pixel-by-pixel, concentration-independent, calibration coefficient, $\alpha(i,j)$ is obtained using an image processing procedure that is based on: i) a background image obtained as an average of the five different uniform calibration concentration fields, $I(i,j)$ ii) the dark response of the camera obtained using the images with the lens cap covered, $B(i,j)$ and iii) the step-wise correction scheme of the attenuation coefficient, $a(i,j)$ that integrates attenuation along the light rays originating from the laser source:

$$\alpha(i,j) = \frac{1}{N_c} \sum_c \frac{I(i,j) - B(i,j)}{a(i,j)C}$$

(4)

The laser light sheet is aligned to pass the tank horizontally from one side to another. The most significant correction is calculated in a step-wise manner from the uniform concentration calibration images, from one side of the image, to the other following the light beam paths of the laser sheet, modeled using a radial coordinate system with the origin at the laser source (at the back focal position of the cylindrical lens). The radially and azimuthally varying attenuation coefficient (upon transformation of $r, \theta$ to $i,j$ for each pixel) is computed assuming a constant absorption coefficient $\beta$ of Rhodamine 6G, obtained from the work of Ferrier [12], $\beta = 1.1 \times 10^5$ (cm M)$^{-1}$. Constant absorption coefficient assumption was shown to be valid for concentrations below 50 $\mu$g/L:

$$a(i,j) = \exp \left[ -\beta \int_{r_0}^{r} C(r', \theta) dr' \right]$$

(5)

An example raw PLIF calibration image of the uniform concentration fluid illuminated by the laser sheet is shown in Fig. 2(a). The values at three different points (A, B, and C), used for the calibration procedure example according to Eqs. 3 and 4 are shown in Fig. 2(b). Note that we performed seven calibration runs with monotonically increased concentrations, but used only five that refer to the linear part of the fit, in the range of $C = [20, 110]$ $\mu$g/L. The variation of $\alpha(i,j)$ across the image is smaller than 2% for this range, as shown in Fig. 2(b). Vanderwel & Tavoularis [29] quantified the uncertainty due to the secondary fluorescence due to ambient light, light scattered by PIV seeding particles, reflections and concentration dependent primary fluorescence at higher concentrations of 75 $\mu$g/L in a similar setup. In the experiments, we used the same dye but at lower concentration (40 $\mu$g/L), a thinner laser sheet, and a smaller...
tank. All the experiments performed without ambient light and our PIV seeding density was lower and uniformly distributed. Moreover, we worked at concentrations that are below the limit that was shown by Baj et al. [3] to induce non-linear dye effects (approximately 300 µg/L). In the experiment, the flow type and the uniform spread of the Rhodamine 6G into the tank reduced the possibility of additional sources of uncertainty such as a high concentration gradients similar to injected plumes [29]. Vanderwel & Tavoularis [29] stated that the ratio between the apparent concentration $C_{ap}$ and the actual peak of the concentration $C_p$ can be expressed as a function of the laser sheet thickness $\sigma_l$ and the instantaneous half depth of the patch $\sigma_p$. The ratio $C_{ap}/C_p$ decreases as an error function of the ratio $\sigma_l/\sigma_p$ [29]. If $\sigma_l/\sigma_p \leq 0.3$, the error can be neglected. In our case $\sigma_l/\sigma_p \sim 0.03$ and the secondary illumination effect was therefore negligible during the calibration process.

The combination of PIV and PLIF experiments allow for simultaneous measurements of density and velocity at high spatial resolution. An example of these overlapped data is shown in Fig. 3. In this figure we also indicate the region of horizontally homogeneous part of the patch which is used later for the quantitative spatially averaged measurements of the patch size, interface thickness, turbulence kinetic energy, vorticity and the buoyancy distributions.

3. Results

3.1. Evolution of the patch interface

The patch evolution in time is shown from PLIF images in Fig. 4 for different time instants normalised by the buoyancy frequency, $Nt$, and for the representative forcing frequencies $f = 3$, 5, and 8 Hz ($Nt = 0$ is the time when the grid starts from rest). For the sake of clarity, these are not the raw PLIF images. The first image (corresponding to the initially undisturbed concentration field of a linear stable stratified fluid at the dimensionless time $Nt=0$) was subtracted to the raw images. The resulting images are also processed with the noise low pass filter, and adjusted for brightness and contrast enhancement. Between $Nt = 0$ and $Nt = 2$ the patch develops close to the oscillating grid, within
the region covered by the stroke of the grid. We mask this region during PIV and PLIF processing to avoid large errors related to the oscillating grid motion. The results from this interval will be excluded from the following quantitative analysis.

As depicted in Fig. 4, at early times, individual vortices emanating from the grid (particularly visible for the $f = 8$ Hz) rapidly merge and form a uniformly mixed fluid which moves away from the grid. As time progresses, counter-rotating structures are formed, pulling fluid inward from the sides at the level of the grid, and ejecting fluid outwards away from the grid due to baroclinic torque.

When the grid is switched off at $Nt = 8$, the formation of an intrusive gravity current is observed. Note that the wave-like features that seem to be radiating away from the turbulent region are artifacts associated with the imaging and not related to internal gravity waves. Indeed, the artifacts are visible in the thoroughly mixed and uniform density calibration images as well and are most likely caused by an optical filter, which was slightly smaller than the camera lens.

Raw images similar to those shown in Fig. 4 are converted into density fields $\rho(x, z, t)$ through the PLIF calibration procedure (described in Section 2.4) and transformed into the buoyancy fields according to the following relation:

$$ b = g \frac{\rho_0 - \rho}{\rho_0} \quad (6) $$

Here it should be noted that density variations are small such that $\rho_0 - \rho$ can be interpreted as a perturbation buoyancy consistent with the Boussinesq approximation.

The results from Fig. 5 onwards will be presented as if the measurements were made above the grid instead of below it. The governing equations are invariant under the transformation $z \rightarrow -z$, $w \rightarrow -w$, $b \rightarrow -b$, and this transformation avoids unnecessary complications of discussing negative values in the buoyancy, velocity and $z$. It is done only for convenience as it makes easier to interpret data. For consistency we will denote the transformed vertical coordinate as $z'$ in the following figures.

Figure 5 shows the zoom in view on buoyancy $b(x, z', t)$ in the region of interest (indicated in Fig. 3) for the experiment performed at $f = 5$ Hz. As shown in Figs. 4-5, the patch is a relatively well-mixed region. The local thickness of the interface (at different $x$), intended as the vertical size of the region separating the background stratification and the well mixed region of the patch, decreases with time during the initial growth. We analyze these properties using the spatially averaged profiles in the following.

3.2. Spatially averaged description of the patch

In this section we present spatially (horizontally) averaged statistics of the flow in the quasi-homogeneous region previously shown in Fig. 4. The spatial averages are denoted by angular brackets $\langle \cdot \rangle$ (without a subscript) and depend on $z$ and $t$ only.
Figure 4: PLIF images showing the development of the turbulent patch at different time instants $Nt = 1.5, 4.5, 7.5, 11.5$ for the grid frequencies $f = 3, 5, 8$ Hz.
Figure 5: Time evolution of the buoyancy $b(z',t)$ [m/s$^2$] for forcing frequencies of 3, 5, and 8 Hz. Note the positive direction of $z'$ upon an invariant transformation is pointing upwards.
Figure 6: Buoyancy profiles $\langle b(z') \rangle$ at different time instants, $Nt = 2, 3, 4, \text{ and } 5.5$. Note the three slopes: the well-mixed patch, across the interface and the background buoyancy.

The spatially averaged buoyancy profiles $\langle b \rangle$ as a function of time and $z'$ are depicted in Fig. 6, using the data of all frequencies for the representative time instants. The initial condition is the background stratification given by $\langle b \rangle = N^2 z'$ where $N^2 = 1 \text{ s}^{-2}$.

The buoyancy profiles $\langle b \rangle$ in Fig. 6 can be decomposed into three distinct regions following to the slopes trends: i) inside the patch where the turbulent mixing decreases the stratification; ii) the TNTI that develops starting from $Nt = 2$ (dash-dot line) in which the local values for $N^2$ are much larger than the background stratification, and iii) the background ambient with the slope close to $N^2 \approx 1 \text{ s}^{-2}$. The TNTI appears to move away from the grid until $Nt \approx 5.5$ (thick line), indicative of the turbulent entrainment that is taking place. Note that the values of $N^2$ inferred from the spatially averaged statistics likely underestimate the local values, due to the large-scale deformation of the flow near the interface, visible in Fig. 4. Local analysis across the interface is discussed in Section 3.5 and it provides an estimate of the local (as opposed to spatially averaged) TNTI interface thickness.

We define the patch size, $\delta$ and the interface thickness $\Delta$, using the three slopes. Well mixed fluid inside the patch is characterized by $N^2 \approx 0.3 \text{ s}^{-2}$ for all forcing frequencies, shown as the slope of a dashed line in Fig. 7. The background stratification is defined by $N^2 = 1 \text{ s}^{-2}$. The slope across the interface is approximately $0.65 \text{ s}^{-2}$ as emphasized by a middle dashed line in Fig. 7. The intersection of each averaged profile $\langle b \rangle$ with this line (empty circles) is defined here as the mid-point of the interface and defines the patch size $\delta(f, t)$. Following Craske et al. [5], we decide to use 25% and 75% of the value at the intersection...
to mark the upper and the lower limits of the layer (full symbols) that we call hereinafter the “interface layer”. The distance between these two limits on the profile serves as an estimate of the interface layer thickness, $\langle \Delta(f,t) \rangle$. This approach to measure the position and the thickness of the interfacial layer is relatively robust and less sensitive to experimental noise than other equivalent interface detection techniques (for instance using fixed thresholds or marking the maximum buoyancy gradient $d\langle b \rangle/dz'$).

Figure 8 presents the results of the patch growth in terms of $\delta(t)$, estimated using the aforementioned method (see Fig. 7) applied to the spatially averaged buoyancy profiles, shown in Fig. 6. The results are shown versus the non-dimensional time $Nt$, for each frequency. In the cases of the actuation frequencies higher than $f = 5$ Hz, the patch grows rapidly until $Nt \approx 4$, after which the growth rate decreases drastically to half of its original value, followed by a patch growth until $Nt \approx 5.5$, where it reaches its maximum size. At lower frequencies of $f = 3, 4$ Hz, the maximum vertical extension $\delta_{\text{max}}$ does not exhibit a clear peak and the maximum size is not easily defined. We infer that is due to a relatively low level of turbulent intensity. For consistency, we define the maximum size of all the cases at $Nt = 5.5$. It can be seen that the patch size is larger for higher frequencies, which is consistent with earlier observations [8, 16, 28, 20].

After that, the patch size declines until $Nt \approx 8$, the time when the grid stops. Note that the patch collapses before we stop the motor and the evolution
cycle (transient, initial growth and the maximum size) occurs roughly at the same time independent of the grid frequency. The dashed lines are parabolic best fit curves used to emphasize the maximum and the following collapse. The error bars are due to the uncertainty in determining the position of the interface. The inset of Fig. 8 shows the relation between the maximum patch size, \( \delta_{\text{max}}(Nt = 5.5) \) and the forcing frequency, \( f \). A linear trend is discernible, which is consistent with earlier observations [28].

3.3. Spatially averaged and turbulent flow statistics

The horizontally averaged vertical velocity \( \langle w \rangle \) is shown in Fig. 9 (a-c) at three instants in time for the forcing frequencies 3, 5 and 8 Hz. Positive vertical velocities for the first two time instants are indicative of a mean flow away from the grid which is consistent with the observed counter-rotating rolls at the edges of the grid discernible in Fig. 4. At \( Nt = 5.5 \), the mean velocity has changed sign for both the \( f = 3 \) and 5 Hz cases, and partly for \( f = 8 \) Hz. The change of sign is due to the presence of well-mixed fluid inside the patch which creates hydrostatic pressure larger than the ambient which in turn will cause the formation of an intrusive gravity current. Next we quantify the spatially averaged turbulent kinetic energy (TKE), defined as \( k = \frac{1}{2} \langle 2u'^2 + w'^2 \rangle \) where \( u' = u - \langle u \rangle \), \( w' = w - \langle w \rangle \), and \( v' = u' \). The TKE is shown in Fig. 9 (d-f). The TNTI is emphasized in these figures by the sharp drop-off of TKE near the interface. However, the TKE at the TNTI location varies strongly in time, indicating that the vertical distribution of TKE varies strongly in time but may also be an artefact from the spatial averaging. Note that the slope of TKE across the TNTI increases with the intensity of the patch forcing which is in our

![Graphs showing growth of patch size and maximum patch size as a function of time and frequency.](image-url)
Figure 9: Time-evolution of spatially averaged profiles at $Nt = 3, 4, 5.5$ (dashed, thin solid and the thick solid lines, respectively) of (a-c) spatially averaged vertical velocity $\langle w \rangle$, (d-f) turbulent kinetic energy $k = \frac{1}{2} \left( \langle 2u'^2 + w'^2 \rangle \right)$ and (g-i) grid action parameter $K = u_{rms} z'$. Line types are identical to Fig. 7, (left) 3 Hz (center) 5 Hz and (right) 8 Hz, respectively. The empty dots mark the interface position $\delta(t)$ at the specific time.
Figure 9(d-f) also show that the shape of the turbulence profiles varies strongly with time.

### 3.4. Turbulent velocity parameterization

In the works that created turbulent patches using an horizontal grid spanning all the width of the tank (at least in one direction), the turbulent velocity was parameterized using the so-called “grid action” [17, 10]. This parameterization relates root mean square of the horizontal fluctuations $u_{\text{rms}}$ to the distance from the planar source, $z'$ as follows:

$$u_{\text{rms}} = K z'^{-1}. \quad (7)$$

where $K$ is the so-called grid action parameter, which can be related to specific grid parameter (stroke length, mesh size etc.). The parameterization is known to work well sufficiently far away from the grid and is valid in steady-state situations. Neither of these conditions is met for the case we are considering here, which is confirmed in Fig. 9(g-i) where we plot a compensated plot $u_{\text{rms}} z'$, which should be constant in $z'$ (although potentially not constant in time). As can be seen from the figure, the profiles change significantly, both in space in time.

Here, we parameterize the level of turbulence according to the integral value of turbulent fluctuations inside the patch, $\sigma_u$, defined as:

$$\sigma_u = \left[ \frac{1}{5} \int_{\delta_0}^{\delta} k(z') dz' \right]^{1/2} \quad (8)$$

Dependence of $\sigma_u$ on the forcing frequency $f$ at $Nt = 5.5$ is shown in Fig. 10. The results show clearly that the level of velocity fluctuations $\sigma_u$ is linearly dependent on the forcing frequency. Note that the linear relation between $\sigma_u$ and frequency is also consistent with the observed linear dependence of $\delta_{\text{max}}$ and $f$. Indeed, based on dimensional arguments, we expect that $\delta \propto \sigma_u / N$, which upon substituting $\sigma_u \propto f$ results in $\delta_{\text{max}} \propto f / N$ (as shown in Fig. 8). This can be also explained based on the fact that the maximum patch size corresponds to the point where the the potential energy is equal to the kinetic energy, thus $\delta_{\text{max}}^2 \propto \sigma_u^2 / g N^2$ [10, 20] independent of velocity or frequency of the grid.

### 3.5. Local analysis of entrainment across the interface

It is noteworthy that the interface thickness, as observed in Fig. 5 and emphasized in the spatially averaged sense in Fig. 7 is clearly reduced with frequency increased. This is different from the previously reported results at lower $N$ in which the thickness of the interface is typically proportional to the size of the patch [e.g. 11, and references therein]. In order to analyze the thickness of the interface, one has to measure it locally. This is because the spatially averaged profiles include also the effect of the large scale undulations (of the patch scale). Utilizing the interface detection method used before, but now on the instantaneous data we estimate the position of the interface, $\delta(x, t)$ as well as the local...
thickness of the interface, $\Delta(x,t)$. In Fig. 11(a) we plot the buoyancy field (as a color map) on which we mark the location of the interface (solid curve), and the corresponding 25% and 75% of the interface value (dashed lines). The distance between the two dashed lines is the local thickness of the interface, $\Delta(x,t)$.

The values of the interface thickness $\Delta(x,t)$, measured at different $x$ locations and different time instants, are then averaged along $x$ and plotted in Fig. 11(b). It appears that the thickness of the interface is decreasing with increasing frequency and the interface sharpens with time, at least until the $Nt \approx 5.5$ that corresponds to the critical moment when the growth stops and the patch collapses. It is noteworthy that at this moment, the size of the patch is maximal and the thickness is minimal. Since the patch is well-mixed, the largest gradient of buoyancy occurs in proximity of the interface at this time instant of $Nt \approx 5.5$.

3.6. Turbulent entrainment

We show in the following how the turbulent entrainment is associated with the evolution of the patch vertical size with time. In the previous section it was shown that there is a substantial component of the spatially averaged vertical flow $\langle w \rangle$ inside the patch, implying that the propagation of the interface due to turbulent mixing is:

$$w_e(t) = \left( \frac{d\delta}{dt} \right) - \langle w \rangle_{\delta}$$

(9)
Figure 11: (a) example buoyancy map for 8Hz, at time $Nt = 5.5$ with the marked interface position and thickness of the interface, $\Delta(x)$ defined according to the $[25\%, 75\%]$. (b) thickness of the interface $\langle \Delta \rangle(f, t)$ for different frequencies versus dimensionless time. Color coding is identical to Fig. 7.

Subscript $\delta$ implies averaging along the interface defined by $\delta(x, t)$, according to $\langle \chi \rangle_{\delta} \equiv L^{-1} \int_{-L/2}^{L/2} \chi(x, \delta(x), t) dx$ where $\chi$ is the variable that is being averaged.

The quantities $\langle d\delta/dt \rangle_{\delta}$, $\langle w \rangle_{\delta}$ and the entrainment velocity $w_e$ are plotted in Fig. 12a, c, d for all the experiments.

It can be seen that $w_e$ is largely positive as expected: the turbulence mixing is pushing the interface outwards as long as the grid is oscillating. In addition, as $w_e$ remains positive for all time, the patch collapse should occur following mean flow effects.

For the sake of local balance, we use a local turbulent velocity scale $\langle q \rangle_{\delta}$, where $q = k^{1/2}$ is the square root of the TKE. The local turbulent velocity scale is shown in Fig. 12b and its comparison with the entrainment velocity $w_e$ (Fig. 12d) emphasize some degree of correlation between these key parameters.

The outstanding feature of Figure 12 is that the interface position, the mean flow component $\langle w \rangle_{\delta}$, the turbulence kinetic energy $\langle k^{1/2} \rangle_{\delta}$ and the entrainment velocity $w_e$ vary strongly in time and are all of similar magnitude. These transients in the presence of turbulence and stratification make it unlikely to find an entrainment “law” that captures the entire phenomena.

Through the paper we consistently use dimensional units. It is arguable that figure axes and conclusions could be made relative to non-dimensional numbers so that the results could be interpreted in a more general context (and used in other experiments). For instance, the frequency $f$ corresponds to a change in both $Re$ and $Ri$ for all cases. However the problem is such that it we found it difficult to assign a consistent set of characteristic scales that works satisfactorily throughout the different phases of the patch evolution. We attempted to present Fig. 10 and fig. 12 in a dimensionless form using $Sf$ as a characteristic velocity scale. However, since the entrainment $w_e$ was found to scale on the local Richardson number, it did not scale properly with this non-
Figure 12: Locally measured quantities: (a) rate of growth of a patch, $\langle d\delta /dt\rangle_\delta$, (b) square root of the available kinetic energy at the interface, $\langle k \rangle_\delta$, (c) average vertical velocity $\langle w \rangle_\delta$, and (d) entrainment velocity, $w_e$, all in units of [m/s]. Color coding is identical to Fig. 7.
...dimensionalisation. Furthermore, the Reynolds and Richardson numbers used in the work are based on internal (measured) scales rather than scale provided by the parameters; these scales consequently do not offer information beyond replacing time with either $Ri(t)$ or $Re(t)$. Therefore, the results are presented in the most optimal form, given the information that the experiments have provided us with.

4. Concluding remarks

We have performed simultaneous PIV and PLIF experiments in a localized turbulent patch, freely developing far from the walls in a stably stratified environment. Consistent with previous studies we observe that the patch vertical size, $\delta(t)$, initially grows rapidly, then reaches a maximum, followed by a collapse and subsequent equilibrium through the formation of a gravity current. The time it takes to reach the maximum size is $Nt \approx 5.5$, independent of frequency, except for the cases at $f = 3, 4$ Hz, where the maximum occurs earlier or is not well defined.

This paper focused on the early transient stages of the patch i.e. $Nt < 8$. The measurements show significant large-scale flow features in the form of counter-rotating vortices on the lateral patch edges after the grid was switched on, which are both visible directly in the buoyancy field (Fig. 4) and indirectly in the form of a mean velocity away from the grid in the central area of the patch (Fig. 12c). The turbulence intensity inside the turbulent patch is high, exceeding the mean flow for all frequencies. Near the turbulent/non-turbulent interface, turbulent levels drop off rapidly, whilst the large scale flow components extend beyond the turbulent region.

The data demonstrated strong transients both for the mean flow and the turbulence. The distributions of the root-mean-square of turbulent velocity fluctuations do not show a decay with the distance from the grid. It means that in such localized turbulent patches, continuously forced by finite grids far from the boundaries we cannot use the parameterization of grid action.

Instead we present another robust measure for the turbulence level inside the patch, $\sigma_u$, defined as a spatial average inside the well-mixed region. It is found that $\sigma_u$ at $Nt \approx 5.5$ and the size $\delta_{\text{max}}$ are both proportional to the grid forcing frequency, $f$.

The local analysis at the interface is carried out, depicting to what extent the TNTI propagates by advection due to the mean flow and to what extent by turbulence and turbulent entrainment across the interface. It is observed that advection by the mean flow is the process responsible for the sign reversal in $d\delta/dt$ after $Nt \approx 5.5$ and the following collapse. Importantly, the entrainment velocity remains positive as turbulence consistently causes outward propagation of the interface due to turbulent mixing.

This study addresses the internal distributions of turbulent kinetic energy and mixing within localized regions if forced continuously far from the boundaries. This information and the proposed parameterizations could be useful...
in the design of propulsion of future small unmanned underwater vehicles and mixing impellers in fluidized bed types of bio-reactors.

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