Nonpropagation of tachyon on the BTZ black hole in type 0B string theory

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Abstract

We obtain the BTZ black hole (AdS$_3 \times$S$^3$) as a non-dilatonic solution from type 0B string theory. Analyzing the s-wave perturbation around this black hole background, we show that the tachyon is not a propagating mode.
I. INTRODUCTION

Recently type 0 string theories attract much interest in the study of non-supersymmetric gauge theories [1–3]. Type 0 string theories can be obtained from the worldsheet of type II string theories by performing a non-chiral GSO projection [4]. The resulting theories have world sheet supersymmetry but no spacetime supersymmetry. The crucial difference of type 0 theories with type II theories is to have the doubling of Ramond-Ramond (RR) fields and the tachyon.

One of the simplest way to see the role of tachyon in type 0 theories is to consider the intersecting Dp-branes. The Dp±-brane bound states can be intersected according to the same rule of the type II theories. The D5±-D1± brane black hole was constructed to show that the corresponding near-horizon geometry is AdS$_3$×$S^3$×$T^4$ and it has asymptotically flat space at infinity [7]. This corresponds to the dilatonic solution. It is shown that the tachyon field can be stabilized only in the near-horizon of AdS$_3$×$S^3$×$T^4$. In our previous work [8], we studied extensively the stability by analyzing the potentials surrounding the D5±-D1± brane black hole. In this paper, we find the BTZ black hole as a non-dilatonic solution from type 0B string theory [9]. This is a globally AdS$_3$×$S^3$×$T^4$ solution, which means that it has asymptotically AdS$_3$ spacetime [10]. This is a crucial point that contrasts to the D5±-D1± brane black hole. Here we wish to study whether the tachyon can propagate or not on global AdS$_3$×$S^3$.

The organization of the paper is as follows. In section II, we obtain the BTZ black hole in type 0B string theory. We set up the s-wave perturbation for all fields around this black hole background in Sec. III. Here we choose the harmonic gauge for graviton and use all linearized equations including two Bianchi identities to decouple ($\phi, t$) from the remaining fields. Finally we discuss our results in Sec. IV.
II. BTZ BLACK HOLE

We start with the appropriate truncation of the type 0B string theory in the string frame

\[ S_{10} = \frac{1}{\kappa_{10}^2} \int d^{10} x \sqrt{-g} \left[ e^{-2\phi} \left\{ R + 4(\nabla \Phi)^2 - \frac{1}{4} \left( \nabla T \right)^2 - \frac{m^2}{4} T^2 \right\} \right. \]
\[ \left. - \frac{1}{12} \left\{ f_+(T) F_{++3}^2 + f_-(T) F_{-3}^2 \right\} \right] \]

(1)

where \( f_\pm(T) = 1 \pm \frac{T}{2} \), \( m^2 = -2/\alpha' \), \( \kappa_{10}^2 = (2\pi)^6 \pi g^2 \alpha'^4 \) and \( F_{\pm3} \) are the Ramond-Ramond (RR) three-forms. Comparing with the results of type IIB theory [11,12], the new ingredients are the tachyon \((T)\) and the doubling of the RR fields \((F_{\pm3})\). Here \( \alpha' = 1 \) and \( g \) is the ten-dimensional string coupling constant. We wish to follow the MTW conventions [13].

The equations of motion for action (1) are given by

\[ R_{MN} + \frac{1}{4} g_{MN} \nabla^2 \Phi + 2 \nabla_M \nabla_N \Phi - \frac{1}{2} g_{MN} (\nabla \Phi)^2 \]
\[ - \frac{1}{4} \nabla_M T \nabla_N T - \frac{m^2}{32} g_{MN} T^2 + \frac{1}{48} g_{MN} e^{2\Phi} \left\{ f_+(T) F_{++3}^2 + f_-(T) F_{-3}^2 \right\} \]
\[ - \frac{1}{4} e^{2\Phi} \left\{ f_+(T) F_{++3MPQ} F_{++3NPQ} + f_-(T) F_{-3MPQ} F_{-3NPQ} \right\} = 0, \]

(2)

\[ R + 4 \nabla^2 \Phi - 4 (\nabla \Phi)^2 - \frac{1}{4} (\nabla T)^2 - \frac{m^2}{4} T^2 = 0, \]

(3)

\[ \nabla_M \left\{ f_+(T) F_{++3MNP} \right\} = 0, \]

(4)

\[ \nabla_M \left\{ f_-(T) F_{-3MNP} \right\} = 0, \]

(5)

\[ \nabla^2 T - 2 \nabla \Phi \nabla T - m^2 T - \frac{1}{6} e^{2\Phi} \left\{ f'_+(T) F_{++3}^2 + f'_-(T) F_{-3}^2 \right\} = 0, \]

(6)

where the prime\(('')\) denotes the differentiation with respect to its argument. From Eqs.(2) and (3), one can rewrite the new dilaton equation as

\[ \nabla^2 \Phi - 2 (\nabla \Phi)^2 - \frac{m^2}{8} T^2 - \frac{1}{12} e^{2\Phi} \left\{ f_+(T) F_{++3}^2 + f_-(T) F_{-3}^2 \right\} = 0. \]

(7)

In addition, we need the remaining Maxwell equations as two Bianchi identities [12]

\[ \partial_{\text{[}M} F_{++3NPQ]} = 0. \]

(8)
Here we are interested in non-dilatonic solution. We consider mainly the six-dimensional part by parametrizing $g_{10} = e^{\phi_6}g_6 + e^{2\chi}dx_idx^i$, where $\phi_6 = \Phi - 2\chi$. Note the difference between $\Phi$ and $\phi_6$. The former is the 10D dilaton and the latter is the 6D dilaton. Hereafter we set $\phi_6$ to be zero, which means that

$$\Phi = 2\chi. \quad (9)$$

The black hole solution can be obtained by setting

$$e^{2\Phi} = g^2, \; T = 0, \; F_{\pm 3}^2 = 0, \; \bar{R} = 0. \quad (10)$$

In detail one finds

$$F_{\pm 3} = \frac{\sqrt{2}r_2\epsilon_3}{g} + \sqrt{2}r_1^2ge^{-2\Phi} *_6 \epsilon_3, \quad (11)$$

$$\bar{R}_{\mu\nu} = -\frac{2}{R^2}g_{\mu\nu}^{\text{BTZ}}, \; \bar{R}_{mn} = \frac{2}{R^2}g_{mn}^{S_3}, \quad (12)$$

with $R^2 = r_1r_5$ and $*_6 \epsilon_3$ is the six-dimensional Hodge dual of $\epsilon_3$. In type 0B string theory, the above solution was first suggested in [7]. The ten-dimensional indices $M, N, P, \cdots$ are split into $\mu, \nu, \rho, \cdots$ for the BTZ black hole $(t, \rho, \varphi)$ and $m, n, p, \cdots$ for $S^3(\theta_1, \theta_2, \theta_3)$. The background spacetime $(\text{AdS}_3 \times S^3 \times \text{T}^4)$ is given by

$$ds^2_{10} = ds^2_{\text{BTZ}} + R^2d\Omega_3^2 + dx_i^2 \quad (13)$$

where the BTZ black hole spacetime is given by [14]

$$ds^2_{\text{BTZ}} = -\frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\rho^2R^2}dt^2 + \rho^2(d\varphi - \frac{J}{2\rho^2}dt)^2 + \frac{\rho^2R^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}d\rho^2. \quad (14)$$

Here $M = (\rho_+^2 + \rho_-^2)/R^2, J = 2\rho_+\rho_-/R$ are the mass and angular momentum of the BTZ black hole. In this case, the relevant thermodynamic quantities (Hawking temperature, area of horizon, angular velocity at horizon, left/right temperatures) are given by

$$T^\text{BTZ}_H = \frac{\rho_+^2 - \rho_-^2}{2\pi R^2 \rho_+},$$

$$A^\text{BTZ}_H = 2\pi \rho_+, \; \Omega_H = \frac{J}{2\rho_+^2},$$

$$\frac{1}{T^\text{BTZ}_L/R} = \frac{1}{T^\text{BTZ}_H} \left(1 \pm \frac{\rho_+}{\rho_-}\right). \quad (15)$$
For simplicity, we consider the s-wave perturbation around the 6D background of AdS$_3 \times$S$^3$ as in [12]

\[ F_{\pm 3\mu
\nu} = \bar{F}_{\pm 3\mu
\nu} + F_{\pm 3\mu
\nu} \{ 1 + F_\pm(t, \rho, \varphi, \theta_1, \theta_2, \theta_3) \} , \]  
\[ F_{\pm 3mnp} = \bar{F}_{\pm 3mnp} + F_{\pm 3mnp} \{ 1 + F_\pm(t, \rho, \varphi, \theta_1, \theta_2, \theta_3) \} , \]  
\[ \Phi = \bar{\Phi} + \phi(t, \rho, \varphi, \theta_1, \theta_2, \theta_3) , \]  
\[ g_{MN} = \bar{g}_{MN} + h_{MN}(t, \rho, \varphi, \theta_1, \theta_2, \theta_3) , \]  
\[ T = 0 + t(t, \rho, \varphi, \theta_1, \theta_2, \theta_3) . \]  

We remind the reader that the forms of perturbation should be at least taken to preserve the background symmetry of AdS$_3 \times$S$^3$. In this sense we choose a diagonal form for $F_{\pm 3MNP}$. Here $h_{MN}$ is given by the block diagonal form [10,15]

\[
 h_{MN} = \begin{pmatrix}
 h_{tt} & h_{tp} & h_{t\varphi} \\
 h_{pt} & h_{pp} & h_{p\varphi} & 0 & 0 \\
 h_{\varphi t} & h_{\varphi p} & h_{\varphi \varphi} \\
 0 & h_{\theta_1 \theta_1} & h_{\theta_1 \theta_2} & h_{\theta_1 \theta_3} \\
 0 & 0 & h_{\theta_2 \theta_2} & h_{\theta_2 \theta_3} & h_{\theta_2 \theta_3} \\
 0 & 0 & 0 & h_{ij} 
\end{pmatrix} .
\]  

It is noted that (21) is chosen to preserve the background symmetry of AdS$_3 \times$S$^3 \times$T$^4$. This seems to be simple but it is sufficient for our s-wave calculation with $l = 0$. Here $l$ is given by the relation as $\nabla^2_{S^3} \phi = -l(l + 2)\phi$. This is so because in s-wave calculation the graviton $h_{MN}$ are not propagating modes except $h_{ij}$.

One has to linearize (3), (7) and (6) in order to obtain the equations governing the perturbations as

\[
 \nabla^2(8\phi - h) + \frac{4}{R^2}(h'_{t} + h'_{\rho} + h'_{\varphi} - h'_{\theta_i}) = 0 ,
\]  

(22)
\[ \nabla^2 \phi + \frac{2}{R^2} \left[ (F_+ + F_- - F^\theta_+ - F^\theta_-) - (h^t_\theta + h^\rho_\rho + h^\varphi_\varphi) \right] = 0. \quad (23) \]
\[ (\nabla^2 - m^2) t + \frac{4}{R^2} \left( F_+ - F_- - F^\theta_+ + F^\theta_- \right) = 0, \quad (24) \]

where \( h^\theta_\theta = h^\theta_1 + h^\theta_2 + h^\theta_3 \). Now we attempt to disentangle the mixing between \((\phi, t)\) and other fields by using both the harmonic gauge (\( \nabla_M \hat{h}^{MP} = 0, \hat{h}^{MP} = h^{MP} - \frac{m^2}{2}g^{MP}, h = h^Q_Q \)) and Kalb-Ramond equations from (4) and (5)

\[ \nabla^*_M F^{MNP}_\pm - (\nabla^*_M h^N_Q) F^{MPQ}_\pm + (\nabla^*_M h^P_Q) F^{MQN}_\pm \]
\[ - (\nabla^*_M h^Q_R) F^{QNP}_\pm - h^M_Q (\nabla^*_M F^{QNP}_\pm) \pm (\partial_M t) F^{MNP}_\pm = 0, \quad (25) \]

When \( N = t, P = \varphi \), solving Eq.(25) leads to

\[ \partial_\rho \left( F_\pm \pm t - h^t_\varphi - h^t_\rho + h^\varphi_\rho \right) + \partial_i h^t_\rho + \partial_\varphi h^\rho_\rho = 0. \quad (26) \]

Using the harmonic gauge, the last two terms turn out to be \( \partial_\rho (-h^\rho_\rho + \frac{1}{2} h) \). Then (26) takes the form as

\[ 2 \left( F_\pm \pm t \right) - \left( h^t_\rho + h^\rho_\rho + h^\varphi_\varphi \right) + h^\theta_\theta + h^i_\varphi = 0. \quad (27) \]

The remaining choices for \( N, P \) lead to the same relation as in (27). For \( N = \theta_2, P = \theta_3 \), one obtains the relation

\[ 2 \left( F^\theta_\pm \pm t \right) + \left( h^t_\rho + h^\rho_\rho + h^\varphi_\varphi \right) - h^\theta_\theta + h^i_\varphi = 0. \quad (28) \]

From the Bianchi identities (8) one has

\[ \partial_\theta_1 F_\pm = \partial_\theta_2 F_\pm = \partial_\theta_3 F_\pm = 0, \quad (29) \]
\[ \partial_\rho F^\theta_\pm = \partial_\rho F^\rho_\pm = \partial_\rho F^\varphi_\pm = 0. \quad (30) \]

This implies that \( F_\pm = F_\pm(t, \rho, \varphi) \) are dynamical fields, \( F^\theta_\pm = F^\theta_\pm(\theta_1, \theta_2, \theta_3) \) are non-dynamical fields. Hence we choose \( F^\theta_\pm = 0 \). Then from (28) one obtains an important result as

\[ t = 0, \quad (31) \]
which implies that the tachyon $t$ is a non-propagating mode in the $\text{AdS}_3 \times S^3$ background. On the other hand we find $\mathcal{F}_+ = \mathcal{F}_-$. Plugging this with $t = 0$ into (24) leads to the fact that (24) is trivially satisfied. Now let us consider the simplest case where all but the tachyon $t$ vanish. In this case, we also find $t = 0$ from (25). The non-propagation of the tachyon originates from the coupling of $f_{\pm}(T) F_{\pm 3}^2$ in (1) and the background symmetry (10) of $\text{AdS}_3 \times S^3$. Using (27) and (28), Eqs.(24) and (23) lead to

$$\bar{\nabla}^2 (8\phi - h) - \frac{4}{R^2} h^i_i = 0, \quad (32)$$

$$\bar{\nabla}^2 \phi - \frac{2}{R^2} h^i_i = 0. \quad (33)$$

If $h^i_i = 0$, then one finds

$$h = 8\phi, \quad \bar{\nabla}^2 \phi = 0. \quad (34)$$

However this corresponds to the linearized equation for a minimally coupled scalar. We need to find the linearized equation for the dilaton. If $h^i_i = a\phi$, then Eqs.(32) and (33) lead to the same equation as

$$\bar{\nabla}^2 \phi - \frac{2a}{R^2} \phi = 0 \quad \text{with} \quad h = 6\phi. \quad (35)$$

In order to find $a$, we recall the relation in (9). The original relation comes from the compactification scheme ($g_{10} = e^{\phi_6} g_6 + e^{2\chi} dx_i dx^i$, where $\phi_6 = \Phi - 2\chi$). Here we require the 6D dilaton to be non propagating ($\phi_6 = 0$). Then one finds (9). This means that the dilaton is related to the scale of $T^4$. The linearized version of (9) should be also valid because (9) is an initial constraint. This takes the form

$$\phi = 2\delta \chi. \quad (36)$$

And then using $g_{ij} = \bar{g}_{ij} + h_{ij}, h^i_i = 8\delta \chi = 4\phi$. This implies $a = 4$. We use this relation to derive the correct form of the dilaton equation instead of keeping $h^i_i$ an independent fluctuation. Hence the final equation for the dilaton in the string frame takes the form

$$\bar{\nabla}^2 \phi - \frac{8}{R^2} \phi = 0. \quad (37)$$
Also this corresponds to the equation for a minimally coupled scalar with \( l = 2 \) on AdS\(_3\) × S\(^3\) \([16]\).

**IV. DISCUSSIONS**

A new feature of type 0B string theory is the presence of the tachyon. It is known that while the Minkowski vacuum is unstable in type 0 string theory, the near-horizon geometry of AdS\(_5\) × S\(^5\) should be a stable background for sufficiently small radius \([6]\). The RR fields work to stabilize the tachyon in the near-horizon. It is clear from the D5\(_\pm\)-D1\(_\pm\) brane black hole \([8]\) that the near-horizon of AdS\(_3\) × S\(^3\) × T\(^4\) is stable because \( V_\nu (r) \) and \( V_t (r) \) take the shapes of the potential barrier. If there do not exist the RR fields \((F_\pm, H_\pm)\), one finds a potential well for the tachyon, which induces an instability in the near-horizon \([17]\).

On the other hand, we find that the tachyon cannot propagate on global AdS\(_3\) × S\(^3\). This background corresponds to a non-dilatonic solution. This means that the 10D dilaton(\(\Phi\)) plays no role in setting this background. However, the propagation of \(\Phi\) is alive as shown in Eq.(37) and its absorption cross section was obtained in \([10,16]\). Unfortunately, the tachyon is not a propagating mode. This comes from the Kalb-Ramond equation, the gauge condition and Bianchi identities. In the s-wave\((l = 0)\) calculation the propagation of tachyon is not allowed. This contrasts to our naive expectation such that the tachyon is a propagating mode in the type 0B string theory.

How we do interpret this non-propagation? This mainly due to the spacetime symmetry of global AdS\(_3\) × S\(^3\) background. We remind the reader that the AdS\(_3\) × S\(^3\) background in type IIB theory is maximally supersymmetric \([18]\). Although the type 0B theory is not supersymmetric in spacetime, the AdS\(_3\) × S\(^3\) has the global SL(2,R)\(_L\) × SL(2,R)\(_R\) × SU(2)\(_L\) × SU(2)\(_R\) group of isometries. These are part of an AdS supergroup G=G\(_L\) × G\(_R\), where both G\(_L\) and G\(_R\) contain SU(2) × SL(2,R). This background is indistinguishable from their type IIB cousins \([19]\). It seems that these global symmetries prevent the tachyon from propagating into the global AdS\(_3\) × S\(^3\). This is clear if the global AdS\(_3\) × S\(^3\) (the non-dilatonic solution\([13]\))
is compared with the D5±-D1± brane black hole (the dilatonic solution). The former has AdS3×S3 with asymptotically AdS3 space, while the latter takes AdS3×S3 only in the near-horizon but with asymptotically flat space. This means that the global AdS3×S3 is regarded as the large R limit of the near-horizon AdS3×S3. This global spacetime can be achieved only from the special setting as (10)-(12). As a result the background isometries of global AdS3×S3 may protect the tachyon from propagating on this space.

In conclusion, although the tachyon is propagating and stable in the near-horizon geometry (AdS3×S3) of the D5±-D1± brane black hole, it is not a propagating mode on the global AdS3×S3 background.

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