

Model in-cognizant control of residential HVAC units with limited sensing and actuation

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Abstract—In this paper, we consider the problem of controlling residential heating, ventilation and air conditioning (HVAC) units in response to changes in grid-side electrical power imbalances causing unacceptable frequency. We derive a novel energy-based model that relates the HVAC physics-based dynamics to both real and reactive power balance at the point of interconnection with the grid. Based on this modeling, we propose a composite control comprising of a robust sliding mode controller in tandem with a slower model predictive controller that can achieve near-optimal physical and economic performance. In contrast to several other approaches in the literature, we analyze whether a limited number of HVAC units can meet the stringent performance metrics set by the ARPA-E/NODES program on following the regulation signal, while maintaining consumer comfort. Theoretical and simulation-based evidence is provided to show that the proposed approach to control a single HVAC unit results in a provable response simultaneously satisfying NODES program performance metrics and consumer comfort constraints. The use of this model overcomes fundamental issues concerning limited sensor measurements and model uncertainties.

Index Terms—Demand response, Load flexibility, Thermostatically controlled loads, Residential HVAC loads, Energy-based modeling and control

I. INTRODUCTION

Renewable Portfolio standards being adopted by several states in the US lead to large-scale deployment of solar panels and wind farms [1]. These technologies are often not controlled by the grid operator and thus result in hard-to-predict disturbances in the grid. Shown in Fig. 1 is the projected net load change in California over the next several years [2], leading to increased reliance on ancillary services. Ancillary service is a general term used to refer to a variety of operations beyond generation and transmission that are required to maintain grid stability and security. These services generally include frequency control, spinning reserves, and operating reserves, all related to additional power adjustments needed to offset the mismatch between the net demand and scheduled generation. Conventionally generation units have been utilized for providing ancillary services. A major operational challenge is that these units have significant inertia and thus incur increased wear and tear, as they follow the net load patterns, such as the one indicated in Fig. 1.

Therefore, utilities are exploring the potential of the flexibility provided by the residential consumer-end resources distributed throughout the grid, such as the electric vehicles, batteries and thermostatically controlled loads (TCLs) like electric water heaters, air conditioners and refrigeration units [3], [4], [5], [6]. This paper’s particular interest is the flexibility offered by the heating, ventilation, and air conditioning (HVAC) units. They comprise 40% of the total residential power demand and thus present an enormous potential for providing various types of ancillary services to the grid [7]. Furthermore, their inherent thermal storage can be leveraged to precisely modify the power consumption to provide ancillary services while still meeting the desired temperature requirements of the end-user.

![Fig. 1. The duck curve showing steep ramping needs and over generation of wind farms in California](Image)

Different control methods broadly categorized as direct and indirect load control can enable HVACs to participate in ancillary services. In the direct load control method, the utility or the aggregator models many HVAC units and interrupts the power signals of several HVAC units to obtain desired aggregate performance. This method requires accurate prediction of the thermal flexibility of each HVAC unit to have a reliable supply of ancillary services at an aggregate level [3], [8], [9]. Through the indirect load control method, the consumer or the appliance automatically adjusts its switching cycles in response to real-time electricity prices or frequency deviations in power systems, thereby providing ancillary services indirectly [10], [11], [12]. The relationship between the HVAC temperature setpoints and the real-time pricing is often complicated. Thus indirect load control method results

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in deviations from the power schedules, especially when the cluster of loads is not diverse enough or if a significant fraction of them are operating at their limits [13], [14]. Some other control methods combining these different methods include passivity-based distributed optimization [15], [16], [17], multi-stage robust optimization considering detailed HVAC models [18], two-layered-control distributing the decision-making between local and global controllers [19]. However, these approaches do not consider the practical limitations of the controllers or the availability of sensor measurements. For example, the HVAC needs to remain in the OFF state for a specific amount of time before switching it back ON. These are called short-cycling constraints, which have been considered to some extent in [8], [13], [20]. More importantly, to allow for the participation of a cluster of loads, the NODES program of ARPA-E has indicated the performance metrics in terms of the response time, ramp time, and availability of units explained in Section 2 [21]. However, most methods proposed in the literature can meet these metrics only upon considering a large number of units that are diverse enough [13], [14], [22]. Thus in this paper, we consider effects of a limited number of HVAC units and assess their capability in following the scaled-down regulation signal.

In Section II, we review conventional HVAC models, and identify sub-objectives needed to pose the control problem. In Section III, we propose a novel energy-space model for the HVAC system building upon our recent work [23], [24], [25], [26]. Utilizing this model, we next summarize our proposed multi-layered energy-based control of HVAC in Section IV. It comprises a device-level sliding mode control tracking an output variable in energy-space to a reference value, explained in Section V. Novel closed-loop droop relations mapping the output variable in energy-space to a reference value, explained in Section V. These are further utilized in a slower time-scale MPC-based controller to maximize the efficiency and meet the regulation signals. We finally conclude by summarizing contributions of this paper and suggest directions for future research in Section VII.

II. PROBLEM FORMULATION

In this section, we review some fundamentals starting from the simplified model of a residential HVAC unit along with numerous constraints imposed by the consumer, HVAC manufacturer and the operator for it to qualify as a reliable source of regulation.

A. Conventional models of residential HVAC system

HVAC system comprises the space or zone to be heated/cooled and the auxiliary electrical equipment that injects hot/cool air into the space. As a result, we can characterize two types of state variables, as shown in Fig. 2. First set of them characterize the temperatures of multiple zones, representing the different rooms, walls and are denoted as $x_T$ [27]. The electrical equipment comprises a fan that circulates air into the zones, while the heater/cooling unit maintains the temperature of the supply air. Let us denote the respective state variables using $x_a$. The models of these sub-systems are not exactly known.

Objective 1: The control design should be robust to model and parameter uncertainties and be simple enough to utilize limited available sensor measurements.

In residential HVAC systems, often single zone temperature is utilized to characterize the thermal load [20]. Furthermore, the power rating of the fan is orders of magnitude smaller than that of the compression system or arc furnace for cooling or heating, respectively. Therefore, in the rest of the draft, we consider the dynamical model in Eqns. (1a) and only consider single control input, which is the switch position of the cooling unit, denoted as $u$. Here, $P_u$ is the instantaneous power entering the zone, further expanded in Eqn. (1b).

$$C \dot{T} = -\frac{1}{R} (T - T_0) + \frac{P_{rated}}{P_u} u \tag{1a}$$

$$P_{rated} = \dot{m}_a C_p (T_{sup} - T) \tag{1b}$$

$T$, $T_0$ and $T_{sup}$ respectively denote the zonal temperature, ambient temperature and the supply temperature that is maintained by the compression system in heater/cooler block. $C, R$ represents the thermal capacitance and resistance, respectively. $\dot{m}_a$ is the airflow rate, $C_p$ is the specific heat capacity of the air. $T_{sup}, \dot{m}_a$ form the auxiliary state variables in Fig. 2 and are typically assumed to be constant.

Objective 2: It is required to ensure that the temperature reaches the setpoint $T_{ref}$ as set by the consumer with a tolerable temperature deviations of $T_{db}$, i.e. $T(t) \in [T_{ref} - T_{db}, T_{ref} + T_{db}]$ ∀t.

The pre-programmed embedded automation is based on PID controllers and satisfies Objective 2 under all circumstances. In the context of Eqn. (1a), it is interpreted as a bang-bang control, turning OFF for temperature within the limits and back ON when the temperature is off the limits. For the simplified model in Eqn. (1a), the logic can be interpreted.
as follows:
\[ u = u_{em} = 1 \quad \text{if} \quad T < T^{ref} - T_{db} \quad (2a) \]
\[ = 0 \quad \text{if} \quad T > T^{ref} + T_{db} \quad (2b) \]

**Objective 3:** Several considerations from the perspective of the control designer, manufacturer and service provider respectively are stated as follows:
- The control needs to be implemented using the knowledge of temperature and electrical power consumption alone. This is because sensors for other internal variables are unavailable and are expensive.
- The compressor needs to be left on at least for 5 mins and it can remain in that state at most for 15 minutes.
- To have reliable services, typical demand response program testing requires the curtailment lengths to be at the maximum of 5 minutes every day, 2 hours per week.

**B. Ancillary services**

The aggregator or service provider acts as the middle man between the individual HVAC unit and the grid operator. The aggregator’s responsibility is to provide the regulation service as promised to the grid operator a priori, which further requires each HVAC unit to adjust its electricity consumption accordingly while satisfying objectives 1-3. In order to understand the distinction between the baseline energy signals and the regulation signals from the perspective of the grid operator, we next summarize the decomposition of net inflexible demand (shown in black in Fig. 3) over different timescales. Slow reserve or regulation reserve signal. A few Independent System Operators (ISOs) predict also the intra-hour load variations with a slope, instead of sample and hold signals shown in Fig. 3 and the corresponding ancillary service product is referred to as ramping reserve. The difference between the black colored and red-colored signals are impossible to predict and are instead sensed through frequency mismatch. These imbalances are assumed to be zero-mean and are offset through frequency regulation reserves.

In this paper, we specifically focus on the provision of forecast-able regulation reserves, although the proposed methods in the paper can be utilized towards supplying other ancillary services. The mechanism involved in the controllable units submitting the bids and subsequent clearing of bids for each of the market products is quite complex and is out of the scope of this paper. In this paper, we assume that there is an energy dispatch signal communicated to controllable units every \( kT_t \) time, and faster adjustments based on improved inflexible demand predictions, provided every \( nT_s \) time.

**Objective 4:** From the grid perspective when analysing a single HVAC unit, over \( T_s \) rate, it is desired for the average power consumption every \( nT_s \) instant equal to the sum of energy dispatch \( P(kT_t) \) and commanded regulation signal \( P^{reg}(nT_s) \) for all samples over the tertiary and secondary timescale given by \( nT_s \in [kT_t, (k + 1)T_t] \). Furthermore, the HVAC unit (or its aggregator) must mention the limit on power adjustments they can achieve over \( T_s \) rate, ahead of time, preferably at \( T_t \) rate, referred to as reserve capacity \( B[k] \).

\[
P(nT_s) = P(kT_t) + P^{reg}(nT_s) \quad (3a)
\]
\[
|P^{reg}[n]| \leq B[k] \quad \forall nT_s \in [kT_t, (k + 1)T_t] \quad (3b)
\]

By doing so, the unit makes a commitment and possibly gets paid for allocating the reserve capacity even if it gets utilized or not. Here \( n \) and \( k \) are used to represent sample number evolving at \( T_s \) and \( T_t \) rate respectively. Note also that the timescale involved in objective 1 and that in objective 3 are quite different, which could still be satisfied simultaneously in an average sense.

**C. ARPA-E performance metrics**

The utility performs a few tests to see if the units willing to participate in ancillary services meet specific performance metrics before being deemed a reliable provider. There is no clear rationale for setting these performance metrics. These metrics are not standard even for current grid operations where controllable generation units primarily provide ancillary services. On similar lines, ARPA-E has proposed performance metrics for different types of ancillary services as needed to be met by a group of small-capacity flexible units coordinated by and collectively referred to as Network Optimized Distributed Energy System (NODES) [21]. We interpret these metrics in the context of a single HVAC unit of interest in this paper.

**Objective 5:** The averaged power consumption over \( T_s \) rate \( P[n] \) of each HVAC unit should observe the following specifications as it is commanded to follow step change in regulation signal at time \( t_0 \), which is also pictured in Fig. 4.
- Response time (\(t_r\)): The time it takes for the actuators to respond to the commanded signals - less than 5 seconds.
- Reserve magnitude target (RMT): The maximum step change in regulation signal that the HVAC unit can track - 7% of rated capacity \(^2\)
- Reserve magnitude variability tolerance (RMVT): Maximum deviation between the actual power adjustments and the reserve magnitude target - 5% of 'RMT’
- Ramp time (\(t_a\)): The maximum time it takes for the HVAC unit to reach the 'RMT’ and stay within 'RMVT’ deviations - 5 seconds
- Availability time (\(t_a\)): The period of time for which the reserves can be supplied - 3 hours

We ensure satisfaction of these performance metrics by explicitly considering some of these metrics in the control design as will be explained in Section IV. For the rest of the draft, we assume that the commanded regulation \(P_{\text{reg}}[t]\) is within the reserve capacity \(B(kT_i)\) that varies over tertiary control timescale, which is out of the scope of this paper.

### III. ENERGY-BASED MODELING OF HVAC

The schematic of energy interaction for the detailed HVAC model is shown in Fig. 5. The compressor system can operate either in heating/cooling mode that dictates the supply temperature \(T_s\) into the zone, used in the simplified model in Eqn. (1a). The AHU injects air at desired temperature into the zone, which circulates back through the fan. In Fig. 5 each bi-directional arrow indicates the shared instantaneous power and generalized rate of reactive power that will be defined later in this section. The zone is associated with thermal energy \(U\) controlled by an air handling unit (AHU). It comprises a blower fan, a heat exchanged and a compression unit. The compressor and the fan can individually be controlled, each of which are associated with a pair of instantaneous and generalized rate of reactive power.

All the interactions between subsystems can get overly complex if modeled in conventional state space. We instead assess the overall energy conversion dynamics, and strive to design the control without exactly knowing the internal physical models.

\(^2\) It is stated 7% of the peak inflexible demand in the original specifications

#### A. Preliminary definitions of variables in energy space

In any energy domain, power variable are defined using the port variables called effort flow variables. These pairs are (voltage, current) in electric domain, (pressure, volume flow rate) in fluid domain and similarly based on Eqn. (4), they are (Temperature, entropy flow) in thermal domain. The time integral of this quantity is defined as the energy injected into the component. We start by defining thermal energy.

**Definition 1.** The internal or thermal energy of a thermodynamic fluid (air in the case of HVAC) can be expressed as follows:

\[
\dot{U} = \int_0^t TdS = \int_0^t T \frac{dS}{dt} dt = \int_0^t TS_f dt \tag{4}
\]

Here, \(dS\) is the incremental entropy of the medium, \(S_f = \dot{S}\) is the net entropy flow into finite volume at a temperature \(T\).

**Definition 2.** The energy in tangent space is defined by replacing the effort-flow variables in definition of stored energy with that of its derivatives.

\[
U_t = \int_0^t dS_f \frac{dT}{dT} dt \tag{5}
\]

Irrespective of energy domain, each arrow in Fig. 5 models the power variables defined next.

**Definition 3.** Given the generalized effort \(e\) and flow variables \(f\) at a port, the instantaneous power and rate of change of generalized reactive power absorbed by the port is defined as in Eqns. (5) \cite{24}, \cite{28}.

\[
P = ef \tag{6a}
\]

\[
\dot{Q} = e \frac{df}{dt} - f \frac{de}{dt} \tag{6b}
\]

We have defined here the thermal energy and thermal energy in tangent space. The similar characterization for electromechanical energy domains is explained in \cite{24}, \cite{24} and other energy domains in \cite{25}. The total energy of the HVAC system \(E\) is the sum of the stored energy of the sub-systems in AHU \(E_a\) and thermal energy \(E\). We have shown in \cite{24} that the dynamics of energy exchanges of a component with the rest of the system can be captured by modeling the dynamics of aggregate variables energy \(E\) and its rate of change \(p = \dot{E}\). The model was derived for electrical circuits

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Fig. 4. Performance metrics to be satisfied for participating in ancillary services

Fig. 5. Schematic of HVAC system in energy space. The overall system is associated with stored energy \(E\) and its rate of change \(p\) which can be controlled by the switch shown in green, resulting in net controlled power variables \(P_a, Q_a\) entering the grid.
and was proven to hold for complex electromechanical systems by extending the definitions of energy space variables through effort-flow analogy for multiple energy domains. In the same way, extending the analogy to the hydraulic energy domain with a continuum of space, we have derived models for turbo-machinery in this energy space in \cite{23}. We now extend the modeling approach to also capture thermal interactions within the HVAC system.

B. Stand-alone HVAC model in energy space

We now utilize the definitions of energy space variables to show that the previously proposed energy space model in \cite{24} also holds for thermal interactions of HVAC. In order to keep the derivations simple, we assume negligible stored energy in AHU when compared to that of the zone. We assume \( E \equiv U \) and thus model the corresponding internal thermal energy conversion alone as in Definition 1 as the aggregate energy variable. Note however that the general energy space model holds even if the assumption is relaxed.

Proposition 1. A general interaction model of a stand-alone component is given as

\[
\begin{align*}
\dot{E} &= -\frac{\dot{E}}{\tau} + P_u = p \\
\dot{p} &= 4E_t + \dot{Q}_T = 4E_t + 2\dot{Q}_T - \dot{Q}_u
\end{align*}
\]  

\[ (7a) \quad (7b) \]

Here, \( \dot{E} \) \( \dot{p} \) represents the total damping losses. \( E_t \) is the total stored energy in tangent space. \( \dot{Q}_T \) is the reactive power absorbed by the thermal processes in the zone.

Proof. Starting from the model in Eqn. (1a), we show that the aggregate energy space model holds in Appendix C.

Remark 1. Here \( p = P_u \) for when the damping losses \( \dot{E} \) = 0. In such cases \( p \) is the supplied power and \( \int_0^t E_{dt} \) is the power corresponding to the potential to do useful work \( \int_0^t E_{dt} \). Therefore, the difference \( (Q_u - 2Q_T) \) represents component of power corresponding to wasted work. It represents the dynamic inefficiencies associated with the energy conversion processes at instantaneous time. It must be noted that this inefficiency is different from the damping losses from linear frictions, viscosity etc., which is captured in the term \( \dot{E} \). This dynamic inefficiency arises due to the difference in the rate at which thermal power gets absorbed by the zone and the rate at which the grid injects power.

Notably, the energy space model is linear, thereby warranting provable control design. The model in Eqn. (7) is proven to hold for simplified thermal model of HVAC but it holds for any complex energy conversion processes, for example even if the detailed AHU sub-system dynamics were to be included. The model only utilizes the overall stored energy of the system and the interaction variables seen at the interface. The thermal reactive power \( \dot{Q}_T \) might seem like a complex function of internal states as in Eqn. (20b). However, only its upper bounds are needed to be known for provable control as will be shown later in Section V. Furthermore, the stored energy in tangent space can either be estimated as in \cite{29} or upper bounded for use in control design.

C. Interactive interconnected model

There exists a unique transformation of state and state derivatives called interaction variable \( z_{r,\text{out}} \) that has important structural properties. The rate of change of interaction variable is zero when disconnected from the grid. For details, see \cite{30}. These outgoing interaction variables are defined as:

\[
\begin{align*}
\dot{z}_{r,\text{out}} &= \left[ \begin{array}{c}
P_{r,\text{out}} \\
\dot{Q}_{r,\text{out}}
\end{array} \right] = \left[ \begin{array}{c}
p + \frac{E}{4E_t + 2\dot{Q}_T - \dot{p}} \\
\end{array} \right] = \phi(\hat{x}, \hat{\dot{x}})
\end{align*}
\]

The definitions for \( p, E, \dot{Q}_T \) for the simple dynamical model in Eqn. (1a) is derived in Appendix C. As a result the outgoing interaction variable is abstracted as a function of extended states and its derivatives which is defined as \( \hat{x} = [x, \dot{x}]^T \).

When the AHU dynamics are ignored, the only state variable of consideration here is the zone temperature and \( u \) is the switch that can cut off the electric power supply.

On the other hand, there is also incoming interactions into the HVAC that is controllable and is denoted as \( z_{r,\text{in}} = z_u = [P_u, \dot{Q}_u]^T \). From Eqn. (1a), it may appear that only \( P_u \) enters the physical model. However during transients, the change in effort and flow variables at electrical terminals result in reactive power \( \dot{Q}_u \) which can further be controlled. \( z_{r,\text{out}} \) is created through thermal processes, while \( z_{r,\text{in}} \) is the result of electric control, both of which need to be same for system interconnection to be stable and feasible \cite{23, 30}. During transients this difference is not zero and this difference thereby is utilized as the control objective in the rest of the paper.

The problem of designing electrical control in Fig. 2 using detailed internal model can quickly become cumbersome. With this aggregate model postulation, we have been able to reduce the model complexity whereby the problem is now that to design the electrical power injections to achieve desired performance mapped into energy and power space.

IV. PROPOSED ENERGY-BASED CONTROL

We propose to utilize the dynamical model in energy space that sufficiently captures the energy/power interactions of the device with the rest of the system to control the switching cycles. Notably, the usage of simple enough models leads to minimal sensor measurement requirements. The overall control architecture, as seen by the device, is shown in Fig. 6 Here, the primary control samples are represented by \( [p] \), typically evolving at the rate of seconds. The secondary house level control signals are exchanged at the rates of minutes, identified by the sample number \( [n] \). Finally, the tertiary control signals are exchanged and implemented at the rate of market-clearing timescales, typically of order 1 hour, identified by the sample number \( [k] \).

We describe in this paper the primary and secondary layers of control design that result in provable performance while tracking a regulation signal. The proposed primary control design accomplishes the objectives 1-3. The sliding mode design results in robust performance insensitive to ambient temperature variations and internal heat gains, model uncertainties, etc. Furthermore, the sliding surface has a direct relation to the temperature requirements. Further the discrete switching logic in Eqn. (12) accounts for the temperature
Fig. 6. Proposed multi-layered information exchange for control of HVAC

This quantity represents the device’s desired incremental energy consumption. We consider incremental energy consumption since the temperature dynamics of HVAC systems change slowly and never settle. We set the reference value to be the incoming interaction variable which from a grid perspective, may include additional regulation signals as follows:

$$y_z^{ref}(t) = P_u(t) + P^{reg}[n] \forall t \in [(n-1)T_s, nT_s]$$  \hspace{1cm} (9b)$$

Here, $P^{reg}[n]$ is the feed-forward regulation signal that enters every $T_s$ timestep. The control is expected to ensure $y_z \rightarrow y_z^{ref}$ before $t = nT_s$. To this end, we design a robust sliding mode control as in Eqn. (9c) that ensures finite settling time.

$$\dot{Q}_u = \alpha \text{sign}(\sigma)$$  \hspace{1cm} (9c)$$

where $\sigma = y_z - y_z^{ref}$ represents the sliding surface, and $\alpha$ is the sliding mode gain.

In the conventional space model in Eqn. (1a), the control input is $P_u$. We thus re-express $\dot{Q}_u$ in terms of $P_u$ based on definitions in Eqn. (5)

$$\dot{P}_u = 2e \frac{d}{dt} - \dot{Q}_u = 2e \frac{d}{dt} \left(\frac{P_u}{e}\right) - \dot{Q}_u$$

$$\Rightarrow \dot{P}_u = 2\frac{P_u}{T - T_0} \frac{d}{dt} + \dot{Q}_u$$

(10)$$

As explained in Appendix C, the effort-flow variables upon ignoring AHU dynamics of HVAC system are the relative temperature $T - T_0$ and the entropy flow $S_f$ which can further be re-expressed in terms of instantaneous power $P_u$.

**Theorem 1.** The closed loop model in conventional state space (Eqn. (1a), (10) observes following properties

1. It is diffeomorphic to the model in energy space Eqn. (7) both with respect to the virtual control input $\dot{Q}_u$.
2. The virtual control design in Eqn. (9c) ensures finite settling time if the sliding mode gain is set equal to $\alpha = T + K$ where $4E_t + 2QT \leq T$, $K > 0$
3. The objective of $y_z \rightarrow y_z^{ref}$ is achieved within a finite reaching time $t_r \leq \frac{K}{2 |\sigma(0)|}$ where $|\sigma(0)|$ represents the distance of operating point from the sliding surface at initial time.

**Proof.** The proof is provided in the Appendix D.

The finite reaching time result above facilitates provable tracking of regulation signals every $T_s$ time step. In particular, the sliding mode gain needs to be chosen so that $t_r$ is at least ten times smaller than secondary control timescale $T_s$.

**A. Implementation-aware design**

In the residential units, there is not enough flexibility to change the digital control logic embedded. Thus the internally computed signals can only be blocked, which is implemented on a Raspberry PI for blocking ON/OFF signals at the interface. From Eqn. (10), assuming negligible effect of the first term due to rate of change of temperature, we have the following simplification

$$\dot{P}_u = \dot{Q}_u = \alpha \text{sign}(\sigma)$$

(11)$$

V. LOWER LAYER PROVABLE PRIMARY CONTROL

HVAC systems already have an embedded proprietary digital control. We propose to overwrite these signals through a robust primary control based on sliding mode control. Consider the energy space model in Eqn. (7) and let $\dot{Q}_u$ be the available degree of control and let us select the output variable of interest as in Eqn. (9a).

$$y_z = P^{ref}_{\text{out}} = \frac{E^{ref}}{\tau} = \frac{1}{R} (T^{ref} - T)$$

(9a)$$
The discrete time implementation is then proposed as

\[ P_u[p] = P_u[p-1] + T_p \alpha \left( T^{ref}-T[p] - y_{z}^{ref}[n] \right) \leq 0 \]

\[ P_u[p] = P_u[p-1] - T_p \alpha \left( T^{ref}-T[p] - y_{z}^{ref}[n] \right) > 0 \]

\[ = P_u[p-1] \text{ otherwise} \]  

(12)

Since the implementation only lets the ON/OFF control the electrical power input is modeled as \( P_u = P^{rated} \) where \( P^{rated} \) represents the name plate capacity of the HVAC unit while \( u \in \{0,1\} \) represents the switching position. The logic for the switching position is finally based on dynamical controlled computed in Eqn. (12) which is then utilized in the following digital logic for HVAC operating in cooling mode (The inequalities are to be reversed if it were operating as a heating unit).

\[ u[p] = 1 \quad P_u[p] > P^{rated} \]

\[ = 0 \quad P_u[p] < 0 \]

\[ = u[p-1] \text{ otherwise} \]  

(13)

The discrete-time implementation in Eqns. (12) and (13) corresponds to an equivalent continuous time signal of virtual control \( Q_u \) in Eqn. (12) over slightly longer timescale than \( T_p \), but shorter than \( T_s \).

B. Simulation study

Next, in order to validate the ARPA-E performance metrics for the provision of reserves, a power adjustment of 0.2 kW at 0.5 hours is initiated. The secondary control action as will be described next in Section VI computes the desired reference in energy space. Here for constant step change that is being studied, we set a constant regulation signal \( P^{reg}[n] = +0.2kW \) for use in Eqn. (9). The primary control given by Eqns. (12) and (13) then tracks this reference as shown in Fig. 7(b).

Notice that the response time of less than 5 seconds is achieved because of the implementation timestep chosen as \( T_p = 0.36 \) seconds. The ramp time is clearly less than 5 minutes, which can further be improved with selection of sliding mode gain \( \alpha \) utilized in Eqn. (9). The reserve magnitude variability tolerance of less than 5% of the target is achieved because of the implementation timestep chosen as \( T_p \). The duration of 30 minutes of reserves availability has also been validated.

The electrical power is also shown in Fig. 7(b). Here, the base signal utilized for comparison is the one when the embedded automation does not respond to the regulation signal, i.e., the step-change in \( y_{z}^{ref}[n] \) at 0.5 hours. After 0.5 hours, notice that there is a difference of approximately 0.2 kW as required by the regulation reserve signal.

The temperature is also ensured to be within the permissible range of 60 – 65°F as shown in Figure 7(c). Finally, the imposition of the deadband in Eqn. (12) of the control also ensures that the switching actions are not too frequent, as shown in Figure 7(d).

We have thereby reassessed objective 5 in context of the unit testing of a single device for step change in regulation signal. Here we have assumed that consistent reference signal \( y_{z}^{ref} \) is provided so that the comfort metrics are not violated. In order to ensure that, we pose a secondary control problem formulation over relatively slower timescales.

VI. EFFICIENT SECONDARY LAYER CONTROL

With the primary control explained in Section V we have ensured that the tracking of \( y_{z} \) to \( y_{z}^{ref} \) happens within finite time. Once the primary dynamics settle, we can derive a quasi-static droop relation as stated next.

**Proposition 2.** Given the closed loop model in Eqn. (9) there exists a three-way incremental droop relation between the outputs in energy space \( \Delta y_z[n] \), the power consumption \( \Delta P_u[n] \) and the outputs of interest \( \Delta y_{z}^{ref}[n] \) where the coefficients \( \alpha \) and \( \beta \) can possibly be operating conditions dependent.

\[ \Delta y_{z}[n] = (1 - \sigma(\Delta)) \Delta y_{z}^{ref}[n] - \sigma(\Delta) \Delta P^{reg}[n] \]  

(14)

These droop relations are utilized to solve an MPC problem to obtain references for outputs in energy space \( y_{z}^{ref}[n] \) over slower timescales for a horizon length of one or several market-clearing time intervals of length \( T_1 = 1 \) hour. The secondary layer control problem is posed as follows:

\[ \min_{\Delta y_{z}^{ref}[n]} \sum_{n_1 T_S = (k+1) T_S}^{n_2 T_S = k T_S} \mu^{e} \left| P_u[n] \right| + \mu^{reg} \left| \Delta P_u[n] - \Delta P^{reg}[n] \right| \]  

subject to

\[ \Delta y_{z}[n] = (1 - \sigma(\Delta)) \Delta y_{z}^{ref}[n] - \sigma(\Delta) \Delta P^{reg}[n] \]  

(15a)

\[ y_{z}[n_1] = y_{z,0} \]  

(15b)

\[ \Delta P_u[n] = \Delta y_{z}^{ref}[n] - \Delta P^{reg}[n] \]  

(15c)

\[ y_{z,min} \left( T^{ref}, T_{db}, P_u[n] \right) \leq y_{z}[n] \leq y_{z,max} \left( T^{ref}, T_{db}, P_u[n] \right) \]  

\[ \forall n \in [n_1, n_2] \]
In the droop model in Eqn. (15b), we assume the droop \( \sigma \) although is operating conditions dependent, is changing much slower than \( T_s \). The droop equation forms the secondary control discrete time model with state \( y_z \), control \( \Delta y_z^{ef} \) and the exogenous inputs \( \Delta P^{ref} \). Eqn. (15c) is utilized as an additional constraint to model the power consumption that is to be considered in the objective function. \( \mu^{reg} \) is the penalty of not following the regulation signal, while \( \mu^e \) is the fixed energy cost being paid by the device. As a result, the objective of secondary control is to optimize the trade-offs between energy consumption and provably supply of reserves. \( y_z^{min}, y_z^{max} \) are the limits on the outputs of interest, which can be computed based on the internal constraints on temperature limits. The number of switching cycles in a given time frame can be adjusted by selecting the \( T_{db} \) in these constraints. The result of this optimization is the sequence of reference signals \( y_z^{ref}[n] \) to which the primary control responds to.

### A. Simulation study

The interactive primary and secondary controllers are simulated for a house with HVAC with permissible temperature of with secondary control timestep of \( T_s = 5 \) minutes and a horizon length of \( 1 \) step to obtain secondary control actions as the regulation signal arrives. The assumed costs are \( \mu^{reg} = 100\$/kW \) and \( \mu^e = 10\$/kWh \). The trajectories obtained with this approach is compared with that when the horizon length is 12 steps, amounting to 1 hour of planning.

The overlaid plots of temperature are for the cases with and without MPC are shown in Fig. 8. Notice that the temperature crosses the limits at few time instants because of the constraint on \( y_z \) with upper and lower bounds serving as proxies to temperature constraints were softened by plugging them in objective function with a penalty factor of 1e2.

Notice that the secondary control MPC problem computes the reference signals that would result in the temperatures to be within pre-specified limits. As a result, the primary control implementation would not lead to saturation.

![Fig. 8. Tracking of regulation signal with interactive primary and secondary control of HVAC system](image)

Fig. 8(a) and 8(b) respectively show the result of the control action for the case with and without MPC respectively. In these plots, the primary control action results in fast-changing values of \( y_z \) shown in blue perfectly chasing the reference signal \( y_z^{ref}[n] \) in red computed every 5 minutes. Overlay is also the regulation signal, which is approximately same as the secondary control action \( y_z^{ef}[n] \) since \( \Delta y_z[n] \) is ensured to remain close to zero to minimize temperature deviations. Notice from Figure 8(a) that the case with MPC results in perfect tracking of the regulation signal while the case without MPC emphasizes more on minimizing objectives at current time instant without any foresight of future temperature trajectories.

### VII. Conclusion

The contributions of this paper are multi-fold. First, a novel energy-based approach is introduced to facilitate model-free control design utilizing minimal sensor measurement data for implementation. Using this modeling, a robust sliding mode control of energy dynamics is derived, along with theoretical conditions under which proposed control design results in a provable three-way droop relation between output variable of interest, secondary control signals and the desired electrical power adjustments over slower time scales. It is then shown how a secondary layer model predictive controller accommodates provable droop relations, performance metrics and implementation constraints. Finally, simulation-based evidence is presented for backing up claims on efficiency and provability of the proposed design method using real data and practical consideration of a HVAC unit in one community in Texas. In particular, it is shown that the controlled real power consumed by the HVAC comprises two components: one that does real work and the other corresponding to wasted work. The former compensates for the effect of energy imbalances created while participating in the provision of regulation reserves. The latter corresponds to the wasted energy required to compensate for the effect of thermal consumption created due to the temperature set-point adjustment for tracking the regulation reserve signal. Notably, this first-of-its-kind relation between electrical and thermal processes is viewed as being fundamental to having sufficient granularity of the model. Further work is needed toward deploying these controllers and fully utilizing their potential.

### Appendix A

#### DIRECT LOAD CONTROL

We review in this section one of the commonly utilized demand response schemes called direct load control. When the HVAC is not participating in ancillary services, the applied
control is a result of temperature deadband logic in Eqn. (2), resulting in the electrical power schedules, that is referred to as the baseline power consumption $P(kT_i)$.

The HVAC unit under consideration utilizes Honeywell’s thermostats, which has a follower board that allows for the interruption of the electrical power injections $u$ to be equal to the external interruption signal $u_{ext}$, through an API call. The objective considered while producing $u_{ext}$ signals is to typically make maximum arbitrage profit out of several HVAC units. In context of single HVAC unit though, the logic can be generalized as follows: Until time $t = pT_p$, if the average power consumed is less than the the required regulation signal, the control needs to be switched ON at the next time step. Otherwise, it needs to be switched OFF. Mathematically,

$$u_{ext}(pT_p) = 1 \text{ if } \sum_{p=1}^{p} \frac{P_u(pT_p)}{T_p} < \Delta P^{reg}[n]T_s$$

$$= 0 \text{ otherwise} \quad (16a)$$

Here, $\Delta P^{reg}[n]$ is the change in the commanded regulation signals from $(n - 1)T_s$ to $nT_s$ time. Often, the cycling constraints stated in Objective 3 are also incorporated in the logic given in Eqn. (16).

The external interruption signal $u_{ext}$ when activated by the grid operator, can lead to violation of consumer preferences. To avoid such mishaps, the embedded automation always acts as a bypass control when temperature violations occur. Thus, electrical flexibility of HVAC can be harnessed only as long as the temperature is within the bounds specified in Objective 2 as shown below

$$u = u_{ext} \text{ if } T \in [T - T_{db}, T + T_{db}]$$

$$= u_{em} \text{ otherwise} \quad (17)$$

The schematic of such architecture is shown in Fig. 10. Through such switching logic, the applied control when utilizing $u_{ext}$ implicitly depends on $P^{reg}[n]$ and on temperature bounds when $u_{em}$ is active. However, the exact analytic relations for the control design of single HVAC unit may not exactly be known since $u_{ext}$ is a result of aggregate performance of thousands of HVAC units and $u_{em}$ is a result of internal designer specific models and controls of fans and compression units.

**APPENDIX B**

**PARAMETER ESTIMATION OF THE SIMPLIFIED HVAC MODEL**

Since the actual HVAC model in Fig. 5 can be quite complex, we utilize the model in Eqn. (1a) as a simplified model for the simulations in this paper. To allow for this model to represent an actual HVAC unit to the best possible extent, we utilize the seconds-granularity data on electrical consumption of HVAC unit in a lab at PSI, to perform parameter estimation [34]. In this section, we show that the simplicity of the model in Eqn. (1a) allows for analytical derivation of the unknown parameters. We claim that the model is sufficiently rich for us to test the performance of our controller which is the thesis of this paper.

Notice from Eqn. (1a) that it is a first order model and analytical expressions of temperature variations can be obtained. Based on simple calculus, we can write

$$T(t) = T(0)e^{-t/(RC)}$$

$$= T(0)e^{-t/(RC)} + \left( \frac{T_0 + R P_{rated}}{1 - e^{-t/(RC)}} \right) \cdot 0\text{ w}$$

Here, $T(0)$ denotes the initial value of the temperature.

For the control in Eqn. (2), assuming the ambient temperature changes are negligible, the duration of ON time and OFF time can be analytically computed as [20]

$$\tau_{ON} = RC \log \left( \frac{T_{ref} + T_{db} - T_0 + R P_{rated}}{T_{ref} - T_{db} - T_0 + R P_{rated}} \right)$$

$$\tau_{OFF} = RC \log \left( \frac{T_{ref} - T_{db} - T_0}{T_{ref} + T_{db} - T_0} \right)$$

For the considered HVAC unit, measurements are available for a 3 hour duration from 12 am to 3 am, during which the average ambient temperature was $T_0 = 75^0 F$. As a result, the lab unit’s reference temperature $T_{ref}$ was set to a low $61^0$ with a tolerable deviation of $T_{db} = 2^0 F$. The internal embedded automation based on PID control of compressors and fans results in the switch positions shown in Fig. 11 in blue. The OFF and ON duration of the switching on an average was found to be 35 minutes and 5 minutes respectively.

This information along with the nominal power consumption of HVAC unit assumed to be $P_{rated} = 1.7KW$ can be plugged in Eqn. (19) to solve for the unknown parameter. The resulting values are found to be $R = 7.9926K/J$ and $C = 0.1242J/K$.

Utilizing these estimated parameters, the model in Eqn. (1a) and switching logic in Eqn. (2) over the same three hour duration for when the ambient temperature changes from $71^0 F$ to $75^0 F$ results in switch positions and temperature trajectories overlaid on the measured values as shown in red in Fig. 11 and 12 respectively. We next assess the effect of considering slightly more granular HVAC model by considering the electrical power supply model based on
Fig. 11. Switch positions: Actual control actions taken in HVAC in blue; estimated actions based on switching logic in red and yellow for different models of controllable power

Eqn. (1b) with the parameters \( m_a = 1 \text{kg/h}, T_a = 23^\circ F \). The corresponding trajectories are shown in yellow in Figs. 11 and 12.

Fig. 12. Temperature trajectories: Temperature sensor measurements in blue; Simulations using model in Eqn. (1a) and switching logic in Eqn. (2) in red; model with time-varying power input in yellow

From the Fig. 12, notice that both the types of models are almost close to each other, but they are both considerably different from the measurements. This is because the models of compressors and fans are not taken into consideration. However, important is to note from Fig. 11 that the number of times the HVAC units switch can be predicted reasonably well with these simplified models, which is of more importance for the phenomena of interest in this paper.

Appendix C

Proof of Proposition 1

Proof. We first derive the expressions for energy variables in context of the simplified dynamical models in use in Eqn. (1a). In Definition 1 the entropy is a complex function of temperature given by \( S = K(T) \), which leads to the simplification for stored energy as in Eqn. (20a) Since entropy and entropy flow are abstract concepts and are not physically measurable, empirical relations are often utilized. The term \( T - T_0 \frac{\partial K(T)}{\partial T} \) is assumed constant and is called the thermal capacitance \( C_w \) used in the simplified model in Eqn. (1a), thereby resulting in following commonly used relation

\[
E \approx U = \int_0^t T \frac{\partial K(T)}{\partial T} \frac{dT}{dt} dt = \frac{\partial K(T)}{\partial T} (T - T_0) (T - T_0)
\]  

(20a)

Here, we have assumed initial temperature is the ambient temperature \( T_0 \). Next, the stored energy in tangent space in Eqn. (3) can similarly be simplified as

\[
E_i = \int_0^t \frac{dK(T)}{dT} \frac{dT}{dt} \frac{dT}{dt} dt = \int_0^t 2 \frac{dK(T)}{dT} (\frac{dT}{dt})^2 dt = \frac{1}{2} \frac{dK(T)}{dT} (\frac{dT}{dt})^2 = \frac{C_w}{2(T-T_0)} (\frac{dT}{dt})^2
\]  

(20b)

In the second equation above, we have assumed the term \( \frac{\partial K(T)}{\partial T} \) is time independent. We have further assumed the time derivative of temperature at initial time is zero. For the simplified model, the injected power as viewed from the perspective of thermal energy domain is given as in Eqn. (1b). However it also must satisfy the definition of instantaneous power in Eqn. (5), thereby resulting in:

\[
S_f (T - T_0) = m_a C_p (T_{sup} - T) \Rightarrow S_f = (m_a C_p (T_{sup} - T) / (T - T_0) )
\]  

(20c)

Here, we assume the term \( S_f \) is time-independent. With this effort flow decoupling for the thermal processes involved, we can derive the rate of reactive power based on Eqn. (3) as:

\[
\dot{Q}_T = (T - T_0) \frac{d}{dt} \left( \frac{m_a C_p}{T-T_0} (T_{sup} - T) \right) - \left( \frac{m_a C_p}{T-T_0} (T_{sup} - T) \right) \frac{d}{dt} (T - T_0)
\]  

(20d)

Defining the damping losses \( \dot{E} \) with the time constant \( \tau = \frac{R}{C_w} \), we can derive the the first equation in Eqn. (7) starting from the thermal model in Eqn. (1a) as follows

\[
\dot{E} = C_w \frac{dT}{dt} = -\frac{1}{R} (T - T_0) + P_u = -\frac{E}{\tau} + P_u
\]  

(21a)

Now let us denote the integrand in Eqn. (20a) as rate of change of stored energy \( p \). Taking its time derivative, we obtain the following simplification

\[
\dot{p} = \frac{\partial K(T)}{\partial T} (\frac{dT}{dt})^2 + \frac{\partial K(T)}{\partial T} (T - T_0) \frac{dT}{dt} = 2E_i + C_w \frac{dT}{dt}^2
\]  

(21b)
Here we used the expression for stored energy in tangent space from Eqn. (20a). Expanding the second term using the model in Eqn. (1a), we obtain
\[ \frac{d}{dt} \left( \frac{1}{2} \sigma^T \dot{\xi} \right) = \frac{1}{2} \sigma^T \frac{d}{dt} \dot{\xi} \]

Finally, since \( \dot{Q}_u = \dot{Q}_T \) for the simplified representation without AHU dynamics, we obtain the desired result. \( \square \)

APPENDIX D
PROOF OF THEOREM 1

Proof. (1): From the expression for stored energy in Eqn. (20a), \( E = C_w T \) and \( p = \dot{E} = -\frac{1}{\tau} (T - T_0) + \dot{P}_w = -\frac{1}{\tau} \left( \frac{E}{C_w} - T_0 \right) + \dot{P}_w \). Clearly, there is a one-one mapping from \([T, P_w]\) to \([E, p]\). We can thereby utilize energy space model in place of the conventional state space model for the rest of analysis and control design. Even if the AHU dynamics were modeled, we could establish diffeomorphism between states appearing at the interfaces and the energy space variables. For details, refer to [30].

(2) & (3): Let \( \sigma = y_z - y_z^{ref} \). Starting from Eqn. (9a) and (9b) and taking the time derivative we have
\[ \frac{d}{dt} = \frac{d}{dt} (y_z - y_z^{ref}) = -\frac{1}{\tau} \frac{d}{dt} \dot{\xi} - \dot{\xi}_p = -C_w \dot{\xi} = -\left( 4E_t + 2\dot{Q}_T \right) + \dot{Q}_u \]

Plugging in the control design in Eqn. (9c), we obtain
\[ \sigma = (L - \alpha) \text{sign}(\sigma) \leq -K \text{sign}(\sigma) \]
Consider next a storage function \( V = \frac{1}{2} \sigma^2 \). We obtain
\[ \dot{V} = \sigma \dot{\sigma} \leq -K \left| \sigma \right| \leq -K \left( \frac{1}{2} \sigma \right)^{1/2} \]

From this expression, clearly \( \dot{V} \leq 0 \) for positive gain \( K \) and by LaSalle’s invariance theorem, we obtain the desired result. Furthermore, taking the time integral of Eqn. (24), we obtain
\[ V^{1/2}(t) - V^{1/2}(0) \leq -\sqrt{2} K t \]
\[ \Rightarrow \left| \sigma(t) \right| - \left| \sigma(0) \right| \leq -\sqrt{2} K t \]

For \( \sigma(t) \to 0 \), rearranging terms, we see that the maximum reaching time is \( t_r \leq \frac{\sigma(0)}{\sqrt{2} K} \)

APPENDIX E
PROOF OF PROPPOSITION 2

Consider the closed loop dynamical model in Eqn. (1a) and (10) with the control in Eqn. (11).
\[ \ddot{T} = -\frac{1}{\tau T} (T - T_0) + \frac{1}{\tau T} \dot{P}_w \]
\[ \dot{P}_w = 2 \frac{P_w}{T - T_0} \frac{d}{dt} \dot{x} + \alpha \text{sign}(\sigma) \]

Simplifying the second equation further, we have
\[ \dot{P}_w = \left( \frac{2}{R C_w} \right) P_w + \left( \frac{\alpha}{\sigma} \right) \sigma + 2 \frac{1}{C_w (T - T_0)} P_w^2 \]

At time \( t = n T_s \) taking the time derivative to zero. Repeating the same at \( t = (n - 1) T_s \) and then taking the difference between the two equations, we obtain
\[ a \Delta P_u[n] = b(\dot{x}) \Delta \sigma[n] \]

Here, \( \Delta \sigma[n] = (\cdot) (n T_s) - (\cdot) ((n - 1) T_s) \). We have ignored the incremental effects of the quadratic term in Eqn. (26e) to obtain Eqn. (26c). We have also assumed the coefficient \( b(\dot{x}) \) is almost constant between subsequent \( n T_s \) timesteps. Since \( \sigma = y_z - y_z^{ref} \), we can establish the following relation
\[ \Delta \sigma[n] = \Delta y_z[n] - \Delta y_z^{ref}[n] \]

Furthermore, from Eqn. (9b), we can write an incremental relation as
\[ \Delta P_u[n] = \Delta y_z^{ref}[n] - \Delta P^\text{reg}[n] \]

Substituting Eqn. (26e) and (26d) into Eqn. (26c), we obtain
\[ a(\Delta y_z^{ref}[n] - \Delta P^\text{reg}[n]) = b(\dot{x}) (\Delta y_z[n] - \Delta y_z^{ref}[n]) \]

Rearranging the terms, we obtain
\[ \Delta y_z[n] = \left( 1 - b(\dot{x})^{-1} a \right) \Delta y_z^{ref}[n] - \left( b(\dot{x})^{-1} a \right) \Delta P^\text{reg}[n] \]

Remark 2. Notice the similarity of HVAC droop relation in Eqn. (25) to the one typically utilized for generation resources [35, 36].
\[ \Delta \omega[n] = (1 - \sigma G D_G) \Delta \omega^{ref}[n] - \sigma G \Delta P^\text{reg}[n] \]

Here \( \sigma G \) is the generator droop and \( D \) is the rotor damping.

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REFERENCES

[1] R. Wiser, R. Wiser, G. Barbose, L. Bird, S. Churchill, J. Deyette, and E. Holt, “Renewable portfolio standards in the United States: a status report with data through 2007,” Lawrence Berkeley National Lab.(LBNL), Berkeley, CA (United States), Tech. Rep., 2008.

[2] J. Goodin, “California independent system operator demand response & proxy demand resources,” in 2012 IEEE PES Innovative Smart Grid Technologies (ISGT). IEEE, 2012, pp. 1–3.

[3] D. S. Callaway, “Tapping the energy storage potential in electric loads to deliver load following and regulation, with application to wind energy,” Energy Conversion and Management, vol. 50, no. 5, pp. 1389–1400, 2009.

[4] S. Koch, J. L. Mathieu, and D. S. Callaway, “Modeling and control of aggregated heterogeneous thermostatically controlled loads for ancillary services,” in Proc. PSCC. Citeseer, 2011, pp. 1–7.

[5] J. Qin, Y. Chow, J. Yang, and R. Rajagopal, “Modeling and online control of generalized energy storage networks,” in Proceedings of the 5th international conference on Future energy systems, 2014, pp. 27–38.

[6] M. Alizadeh, A. Scaglione, A. Applebaum, G. Kesidis, and K. Levitt, “Scalable and anonymous modeling of large populations of flexible appliances,” arXiv preprint arXiv:1404.1938, 2014.

[7] U. DOE, “Quantitative technology review: An assessment of energy technologies and research opportunities,” 2015.

[8] W. Zhang, J. Lian, C.-Y. Chang, and K. Kalsi, “Aggregated modeling and control of air conditioning loads for demand response,” IEEE transactions on power systems, vol. 28, no. 4, pp. 4655–4664, 2013.

[9] M. Almassalkhi, J. Frolik, and P. Hines, “Packetized energy management: asynchronous and anonymous coordination of thermostatically controlled loads,” in 2017 American Control Conference (ACC). IEEE, 2017, pp. 1431–1437.

[10] N. Lu and D. P. Chassin, “A state-queueing model of thermostatically controlled appliances,” IEEE Transactions on Power Systems, vol. 19, no. 3, pp. 1666–1673, 2004.

[11] K. Subbarao, J. C. Fuller, K. Kalsi, A. Somani, R. G. Pratt, S. E. Widergren, and D. P. Chassin, “Transactional control and coordination of distributed assets for ancillary services,” Pacific Northwest National Lab.(PNNL), Richland, WA (United States), Tech. Rep., 2013.

[12] H. Shu, R. Yu, and S. Rahardja, “Dynamic incentive strategy for voluntary demand response based on tcp scheme,” in Proceedings of The 2012 Asia Pacific Signal and Information Processing Association Annual Summit and Conference. IEEE, 2012, pp. 1–6.

[13] H. Hao, B. M. Sanandaji, K. Poolla, and T. L. Vincent, “Aggregate flexibility of thermostatically controlled loads,” IEEE Transactions on Power Systems, vol. 30, no. 1, pp. 189–198, 2014.

[14] N. Lu, “An evaluation of the hvac load potential for providing load balancing service,” IEEE Transactions on Smart Grid, vol. 3, no. 3, pp. 1263–1270, 2012.

[15] V. Chinde, K. C. Kosaraju, A. Kelkar, R. Pasumarty, S. Sarkar, and N. M. Singh, “A passivity-based power-shaping control of building hvac systems,” Journal of Dynamic Systems, Measurement, and Control, vol. 139, no. 11, 2017.

[16] S. Mukherjee, S. Mishra, and J. T. Wen, “Building temperature control: A passivity-based approach,” in 2012 IEEE 51st IEEE Conference on Decision and Control (CDC). IEEE, 2012, pp. 6902–6907.

[17] T. Hatanaka, X. Zhang, W. Shi, M. Zhu, and N. Li, “An integrated design of optimization and physical dynamics for energy efficient buildings: A passivity approach,” in 2017 IEEE Conference on Control Technology and Applications (CCTA). IEEE, 2017, pp. 1050–1057.

[18] F. A. Qureshi and C. N. Jones, “Hierarchical control of building hvac system for ancillary services provision,” Energy and Buildings, vol. 169, pp. 216–227, 2018.

[19] T. Navidi, A. El Gamal, and R. Rajagopal, “A two-layer decentralized control architecture for der coordination,” in 2018 IEEE Conference on Decision and Control (CDC). IEEE, 2018, pp. 6019–6024.

[20] B. M. Sanandaji, T. L. Vincent, and K. Poolla, “Ramping rate flexibility of residential hvac loads,” IEEE Transactions on Sustainable Energy, vol. 7, no. 2, pp. 865–874, 2015.

[21] “Network optimized distributed energy systems (nodes) funding opportunity no. de-foa-0001289,” US DOE Advanced Research Projects Agency-Energy (ARPA-E), Tech. Rep., 2015.

[22] K. Subbarao, J. C. Fuller, K. Kalsi, A. Somani, R. G. Pratt, S. E. Widergren, and D. P. Chassin, “Transactive control and coordination of generalized energy storage networks,” in Proceedings of the Annual Conference of the IEEE Industrial Electronics Society, 2015.

[23] R. Jaddivada, “A unified modeling for control of reactive power dynamics in electrical energy systems,” Ph.D. dissertation, Massachusetts Institute of Technology, 2020.

[24] R. Jaddivada, “Dynamic monitoring and decision systems for enabling sustainable energy services,” Proceedings of the IEEE, vol. 99, no. 1, pp. 58–79, 2010.

[25] R. Jaddivada, R. Jaddivada, and M. Korpas, “Interactive protocols for distributed energy resource management systems (derms),” IET Generation, Transmission & Distribution, vol. 14, no. 11, pp. 2065–2081, 2020.

[26] J.-Y. Joo and M. D. Illic, “A multi-layered adaptive load management (alm) system: Information exchange between market participants for efficient and reliable energy use,” in IEEE PES T&D 2010. IEEE, 2010, pp. 1–7.

[27] “Pecan street inc. dataport.” [Online]. Available: https://www.pecanstreet.org/dataport/

[28] M. D. Illic, N. Popli, J.-Y. Joo, and Y. Hou, “A possible engineering and economic framework for implementing demand side participation in frequency regulation at value,” in 2011 IEEE Power and Energy Society General Meeting. IEEE, 2011, pp. 1–7.

[29] M. D. Illic and J. Zaborszky, Dynamics and control of large electric power systems. Wiley New York, 2000.