The Breaking of the $SU(3)^3$ Gauge Group

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We discuss why the $SU(3)^3$ supersymmetric model with the most general superpotential can naturally break to the standard model if gauge singlets and a discrete symmetry are included. This mechanism does away with the need for fine-tuning in the form of the assumed absence of certain terms in the superpotential. It also automatically guarantees that any abelian discrete phase symmetry of the GUT will survive the symmetry breaking. Such a discrete symmetry, also known as the matter parity, is needed to suppress both proton decay and the flavor changing neutral current (FCNC), and may help solve the hierarchy problem.
1. Introduction

Previous work \cite{2} has indicated that $SU(3)^3$ has many attractive features as a high-energy supersymmetric gauge group. Besides being a maximal subgroup of $E_6$, which may arise naturally in string theories \cite{3}, it is possible to use discrete symmetries to naturally allow the conventional Higgs doublets to be light. In this paper we will explore this idea further, focusing particularly on the gauge group breaking.

The gauge group in our earlier paper is based on the group $SU(3)_C \times SU(3)_L \times SU(3)_R$, where $SU(3)_C$ is the familiar color $SU(3)$, $SU(3)_L$ contains weak $SU(2)$, and $SU(3)_R$ contains the right-handed analog of weak $SU(2)$. This group is one of the maximal subgroups of $E_6$, with the fundamental 27-dimensional representation of $E_6$ becoming a direct sum of three irreducible representations under $SU(3)^3$: $Ψ_L : (3, \bar{3}, 1)$, $Ψ_R : (\bar{3}, 1, 3)$, $Ψ_ℓ : (1, 3, 3)$, corresponding to the quarks, the anti-quarks, and the leptons respectively. The explicit assignment of left-handed particles is as follows:

\begin{align*}
Ψ_L &= (3, \bar{3}, 1) : \begin{pmatrix} u & d & B \\ u & d & B \\ u & d & B \end{pmatrix}, \\
Ψ_R &= (\bar{3}, 1, 3) : \begin{pmatrix} u^* & u^* & u^* \\ d^* & d^* & d^* \\ B^* & B^* & B^* \end{pmatrix}, \\
Ψ_ℓ &= (1, 3, 3) : \begin{pmatrix} E^0 & E^− & e^− \\ E^+ & E^{o*} & ν \\ e^+ & ν_R & N^o \end{pmatrix},
\end{align*}

where $B$ is an additional superheavy down-type quark, $B^*$ is its anti-particle, and $E$'s and $N^o$ are new superheavy leptons.

The Higgs needed to break $SU(3)^3$ to the standard model can be put into a $(1, 3, \bar{3})$ representation together with those needed to break weak $SU(2)$. In supersymmetrized theories, they are just additional generations of leptons. The VEV’s which break $SU(3)^3$ are usually written as

\begin{align*}
\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & w & 0 \end{pmatrix}.
\end{align*}

We then impose an additional discrete symmetry which helps explain the hierarchy problem \cite{2}. However, two assumptions in Ref. \cite{2} have not been justified: namely, why only two of the superfields develop vacuum expectation values (VEV’s), and why the VEV’s of the mirror superfields do not break the discrete symmetry. Conventional wisdom does not
provide a satisfactory answer to these questions. In fact, they contain other intrinsically unpleasant features. We therefore are forced to take a new, more careful look at the gauge symmetry breaking mechanism.

In Section 2, we first introduce the conventional symmetry breaking mechanism, emphasizing how it fails to explain a few key questions. In Section 3, we detail our mechanism, showing that it is phenomenologically feasible. In Section 4, we summarize our work.

2. The Old Breaking Mechanism of $SU(3)^3$

The conventional method of generating the two necessary VEV’s, as explained in detail by the authors of Ref. [1] is based on one specific string inspired model [8]. As is characteristic of string inspired models, the renormalizable part of the superpotential contains only trilinear terms, $\Psi \Psi \Psi$. Thus the VEV’s in (1.1) are the most general $F$-flat direction of the superpotential after a choice of basis if we assume only two multiplets grow VEV’s. Unfortunately, the $D$-flatness condition is not satisfied by them. To make the symmetry breaking possible, it is necessary to introduce the “mirror particles”, which come from the $\overline{27}$ representation of $E_6$. They transform under $SU(3)^3$ like $\bar{\Psi}_L : (\bar{3}, 3, 1)$, $\bar{\Psi}_R : (3, 1, \bar{3})$, $\bar{\Psi}_\ell : (1, \bar{3}, 3)$. If the $(1, \bar{3}, 3)$ parts gain the following VEV’s,

$\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & v^* \\
0 & 0 & v^*
\end{pmatrix}$ and $\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & w^* \\
0 & 0 & 0
\end{pmatrix}$,

both $F$- and $D$-flatness will be satisfied. The number of light generations $N_g$ will be equal to the difference between the number of supermultiplets and that of their mirror partners, i.e.

$$3 = N_g = (\# \text{ of } \Psi_x) - (\# \text{ of } \bar{\Psi}_x), \quad (2.1)$$

where $x = C, L, R$. Eq. (2.1) automatically guarantees that the model is free of anomalies.

However, these conventional models have several difficulties. One has to do with the magnitudes of $v$ and $w$, which are undetermined until we include non-renormalizable and soft SUSY breaking terms. In order to have phenomenologically acceptable values for the VEV’s, i.e. $v, w \geq 10^{16} GeV$ [2] [3], the first two leading non-renormalizable terms have to vanish [4]. So far, no explanation why this should be so has been suggested. This is not the only unsatisfactory feature in this picture. Notice that since gauge singlets $S$’s are present, we should include terms like $S \Psi \bar{\Psi}$ and $SSS$ in the superpotential. The whole
analysis in Ref. [4] loses its validity as a result. Furthermore, it remains a mystery why a third VEV along the direction of $e^+$ should not develop. Finally, any discrete symmetry will be broken by the four VEV’s in (1.1). Thus this mechanism is not compatible with Ref. [2].

In order to avoid these undesirable features, we will examine carefully the roles the singlets and the discrete symmetry ought to play in the symmetry breaking. We find that although mirror particles are still necessary, other problems can be solved because the superpotential does not have to have flat directions. Instead, it has a few isolated vacua corresponding to various low energy gauge groups, including, of course, the standard model.

3. The New Breaking Mechanism of $SU(3)^3$

Consider the general $SU(3)^3$ model with an additional discrete symmetry $C_N$ (The model in Ref. [3] is thus a special case.). The superpotential obeys

$$W = f^{abc}_{ABC} \Psi_a^A \Psi_b^B \Psi_c^C + f'_{ABC} \Psi_a^A \bar{\Psi}_b^B \bar{\Psi}_c^C + g_{ABC} \Psi_a^A \bar{\Psi}_b^B S_c^C + m_{aB}^A \Psi_a^A \bar{\Psi}_b^B$$

$$+ h_{ABC} S_a^A S_b^B S_c^C + M_{aB}^A S_a^A S_b^B + M'_{a}^A S_0^A + (\text{non-renormalizable terms}) \quad (3.1)$$

where indices $a$ and $b$ indicate the discrete charges of the fields under $C_N$ (For example, $\Psi_a$ is the field which transforms like $i_N^a$, where $i_N$ is the N'th root of 1.), $\Psi$ and $\bar{\Psi}$ stand for $\Psi_\ell$ and $\bar{\Psi}_\ell$ (We have omitted any term containing $\Psi_{L(R)}$ or $\bar{\Psi}_{L(R)}$ because they are not relevant to the symmetry breaking mechanism we are considering.), and indices $A,B,C$ specify different generations of fields with the same quantum numbers. The term $\Psi \bar{\Psi} \Psi$ stands for $\epsilon_{i_1 i_2 i_3} \epsilon_{j_1 j_2 j_3} \Psi_{i_{j_1}} \Psi_{i_{j_2}} \Psi_{i_{j_3}}$, where the $i$’s and $j$’s are $SU(3)_L$ and $SU(3)_R$ indices respectively, and likewise for $\Psi \bar{\Psi} \bar{\Psi}$. Similarly the term $\Psi \bar{\Psi}$ is a shorthand for $\Psi^i \bar{\Psi}^i$. Since in most cases $\bar{M}_A$’s can be set to zero by a shift of $S_0^A$’s, we will drop these terms from now on. Notice that these coefficients are symmetric under permutations of indices. Also, since $C_N$ is a symmetry of the theory, each term in $W$ should carry no discrete charge. Therefore any coefficient vanishes if its $C_N$ indices do not sum to zero.

To obtain the ground state we minimize both the $F$-term and the $D$-term. In order not to break SUSY at this stage, we would like to solve

$$\frac{\partial W}{\partial \phi} \bigg|_{\phi = S, \Psi, \bar{\Psi}} = 0 \quad (3.2)$$
\begin{equation}
\sum_{\phi=\Psi} \phi^\dagger T_L^a \phi - \sum_{\phi=\Psi} \phi T_L^a \phi^\dagger = \sum_{\phi=\Psi} \phi T_R^a \phi^\dagger - \sum_{\phi=\Psi} \phi^\dagger T_R^a \phi = 0,
\end{equation}

where $T_L^a$'s are the generators of $SU(3)_{L(R)}$. Obviously the origin is always a solution for these equations. Since there are as many equations in (3.2) as there are variables, naively we would expect the additional constraints of (3.3) to exclude other solutions. To show that this is not the case, let’s write down those equations in (3.2) more carefully,

\begin{align}
3f_{ABC} \langle \Psi_a^A \rangle \langle \bar{\Psi}_b^B \rangle (g_{AB}^{ab} \langle S^C_0 \rangle + m_{AB}) &= \frac{\partial W}{\partial \bar{\Psi}_b^B} = 0, 
\end{align}

\begin{align}
3f_{ABC} \langle \bar{\Psi}_a^B \rangle \langle \bar{\Psi}_c^C \rangle (g_{AB}^{ab} \langle S^C_0 \rangle + m_{AB}) &= \frac{\partial W}{\partial \Psi_a^A} = 0, 
\end{align}

\begin{align}
g_{ABC} \langle \Psi_a^A \rangle \langle \bar{\Psi}_b^B \rangle (3h_{AB}^{ab} \langle S^C_0 \rangle + 2M^{0c}_{AC}) &= \frac{\partial W}{\partial S^C_0} = 0, 
\end{align}

\begin{align}
g_{ABC} \langle \bar{\Psi}_a^B \rangle \langle \Psi_b^B \rangle (3h_{AB}^{ab} \langle S^C_0 \rangle + 2M^{ac}_{AC}) &= \frac{\partial W}{\partial S^C_{c\neq0}} = 0.
\end{align}

If we set each term in (3.4a, b) to be zero individually, even though the number of constraints seems to increase, many of them may be in fact degenerate, thus there may be less independent constraints than equations in (3.4a, b). So we replace (3.4a, b) with the following,

\begin{align}
3f_{ABC} \langle \Psi_a^A \rangle \langle \bar{\Psi}_b^B \rangle &= 0, 
\end{align}

\begin{align}
3f_{ABC} \langle \bar{\Psi}_a^B \rangle \langle \bar{\Psi}_c^C \rangle &= 0, 
\end{align}

\begin{align}
g_{ABC} \langle \Psi_a^A \rangle \langle S^C_{c\neq0} \rangle &= 0.
\end{align}
\[ g^{abc}_{A} \langle \bar{\psi}^B_b \rangle \langle S^C_c \rangle = 0, \quad (3.5d) \]

\[ g^a_{ABC} \langle S^C_C \rangle + m^a_{AB} = 0. \quad (3.5e) \]

Eq. (3.5e) holds if at least one \( \langle \psi^A_a \rangle \) or \( \langle \bar{\psi}^B_{-a} \rangle \) is nonzero, which is exactly the type of solution we are looking for. Note that eq. (3.5e) is a single non-matrix constraint (in gauge space), even though it is derived from (3.4a,b), two matrix constraints. This can represent a vast decrease in the number of constraints. Also, note that nonzero \( \langle \psi^A_a \rangle \) and \( \langle \bar{\psi}^B_{-a} \rangle \), together with eq. (3.5c,d), generally imply \( \langle S^C_C \rangle \neq 0 \).

Although we have already greatly simplified these equations, the solutions can still be very complicated, especially if the number of generations is large. For example, we have found that solutions which break the hypercharge \( U(1) \) can exist if \( n_{S0} \geq 5 \), where \( n_{S0} \) is the number of \( S_0 \)’s. We will not consider such cases. More interesting is when \( n_{S0} \) is small. For \( n_{S0} \leq 4 \), it is often possible to break \( SU(3)^3 \) to \( SU(3) \times SU(2) \times U(1) \) and also leave an unbroken combination of \( C_N \) with an element of the gauge group. This unbroken symmetry can preserve the lightness of the Higgs doublets[^2]. As an example, we will consider the simplest case, which corresponds to \( n_{S0} = 2 \) and \( n_{\psi_a} = n_{\bar{\psi}_{-a1}} = n_{\psi_{a2}} = n_{\bar{\psi}_{-a2}} = 1 \), where \( n_{\psi_a} \) and \( n_{\bar{\psi}_{-a}} \) are the number of \( \psi_a \)’s and \( \bar{\psi}_{-a} \)’s respectively. Applying the constraints of (3.3), (3.4c,d) and (3.5a,b,e), we find that, after a choice of basis, the solutions are of the following form,

\[ \langle \psi_{a1} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v \end{pmatrix}, \quad \langle \bar{\psi}_{-a1} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v^* \end{pmatrix}, \]

\[ \langle \psi_{a2} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & w & 0 \end{pmatrix}, \quad \langle \bar{\psi}_{-a2} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & w^* \\ 0 & 0 & 0 \end{pmatrix}, \]

Notice that the VEV’s of \( \psi_a \) and \( \bar{\psi}_{-a} \) are hermitian conjugate to each other and that \( \langle \psi_{a1} \rangle \) is perpendicular to \( \langle \psi_{a2} \rangle \). The magnitude of \( v \) and \( w \) is determined and is related to the characteristic mass scale in the original Lagrangian, \( i.e. v, w \sim s \), the supposed string scale, if they are not zero. This is compatible with the result from renormalization group calculation of the running coupling constants in certain versions of the non-minimal SUSY SM[^2].
Notice that the transformation properties of these VEV’s under $C_N$ are exactly such that the product of $C_N$ and a certain element of $SU(3)^3$ remains unbroken. In other words, the VEV’s in the $\Psi$’s do not further break the symmetry. Thus we have justified one of the assumptions made in Ref. [2].

Since there are three degenerate vacua: the origin, the one with $v \neq 0, w = 0$, and the one with both $v, w \neq 0$, we have to determine the true vacuum by soft SUSY breaking terms. Whether or not it favors the vacuum we want over the others depends on the form of the soft breaking term and the coefficients in the superpotential, which are unknown. Nevertheless it seems likely that the standard model is favored.

Compared to the model in Ref. [2], we have introduced new particles $S$’s. After the symmetry breaking, all of them gain masses of the same order as $v$ and $w$. Therefore they are invisible at low energy.

4. Conclusion

We have detailed a supersymmetric gauge model with a gauge symmetry breaking mechanism very similar to SUSY $SU(5)$. In fact, almost everything good about $SU(5)$ can be carried over to our model, while it offers a few additional nice features of its own. To name a few, not only it is a likely product of string theory, but also it offers a natural way to solve the hierarchy problem and, perhaps, the strong $CP$ problem [4]. By natural we mean that neither fine-tuning of continuous parameters nor introduction of exotic particles is necessary. However, a few assumptions have been made in the course of our argument, reflecting our ignorance concerning issues such as what is happening at the Planck scale and what breaks SUSY [4]. It is unlikely that these remaining questions will be completely understood in the near future. Nevertheless, we are encouraged to see that in principle the long-standing hierarchy problem can be solved with the right choice of discrete parameters within the traditional theoretical framework without invoking new revolutionary concepts.

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