Flower Pollination Algorithm for the Inversion of Schlumberger Sounding Curve

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Abstract. Inversion of schlumberger sounding curve is non-linear, and multi-minimum. All linear inversion strategies can produce local optimum, and depend on the initial model. Meanwhile, the non-linear bionic method for inversion problems does not require an initial model, simple, flexible, derivation-free mechanism and can avoid local optimum. One of the new algorithm of the non-linear bionic method for geophysical inversion problem is the Flower Pollination Algorithm (FPA). The FPA is used for the inversion of schlumberger sounding curve. This algorithm was stimulated by the pollination process for blooming plants. The applicability of the present algorithm was tested on synthetic models A-type and KH-type curve. Numerical tests in MATLAB R2013a for the synthetic data and the observed data show that FPA can find the global minimum. For further study, inverted results using the FPA are contrasted with the damped least-square (DLSQR) inversion program, Particle Swarm Optimization (PSO), and Grey Wolf Optimizer (GWO). The outcomes of the comparison reveal that FPA performs better than the DLSQR inversion program, PSO, and GWO.

1. Introduction

Many inversion algorithms that have been applied in geophysical inverse problems; such as the Occam algorithm, Genetic Algorithm (GA) and PSO [1], and Lavenberg-Marquard using Singular Value Decomposition [2]. In the last decade, much of the work carried out on electrical resistivity has mainly concentrated on 2D, 3D and 4D techniques. Therefore, 1D inversion results are useful in constructing initial models for multidimensional interpretation. Therefore, a uniqueness study of 1D inversion is important. [3] used partial curve matching for the interpretation of Schlumberger sounding curve data. This technique is done manually, so it takes a long time. [4] creates MATLAB code to automate this work so that it is not only fast but also produces more consistent results. However, this result is only suitable for use as an initial model for the inversion process. Recently, the global approach has also been used by some researchers, such as the hybrid monte Carlo-based neural network and the Improved Gray Wolf Optimizer (IGWO).

From the simulations conducted by [5], it shows that FPA is more efficient than GA and PSO. The use of FPA in geophysical inversion problems has been carried out by [6] for self-potential (SP) data. The result is that FPA is robust for SP data interpretation. We demonstrate a MATLAB based inversion...
program that uses FPA for the analysis of schlumberger sounding curve. The applicability of the demonstrate algorithm was proved on synthetic models A-type (three-layered model with \( \rho_1 < \rho_2 < \rho_3 \)) and KH-type (four-layered model with \( \rho_1 < \rho_2 > \rho_3 < \rho_4 \)) curve. Furthermore, a field survey was still operated in Padalarang, Indonesia to analyze the effectiveness of the program on actual data.

2. Methods

2.1. Forward Modeling Formulation

The relationship between apparent resistivity (\( \rho_a \)) and layer parameters (layer thickness and actual resistivity) can be expressed by integral equations that take into account the isotropic and homogeneous earth model. [7] wrote the following equation:

\[
\rho_a(s) = s^2 \int_0^\infty T(\lambda) J_1(\lambda s) \lambda d\lambda
\]  

(1)

Here \( s \) is the half-spaced current electrode \( (AB / 2) \), \( J_1 \) shows the first-order Bessel function of the first form and \( \lambda \) shows the integral variable. The resistivity transform function, \( T(\lambda) \) is written as follows:

\[
T_1(\lambda) = \frac{T_{i+1}(\lambda) + \rho_i \tanh(\lambda h_i)}{[1 + T_{i+1}(\lambda) \tanh(\lambda h_i) / \rho_i], i = n - 1, \ldots, 1}
\]  

(2)

Where \( n \) is the amount of layers, \( \rho_i \) and \( h_i \) are the actual resistivity and thickness of the \( i^{th} \) layer. Forward modelling in this study used code from [8].

2.2. FPA optimization method

The pseudo-code of the FPA is provided in figure 1. FPA was influenced by the pollination technique of flower, which is transfer of pollen grains to the ovule. The pollination technique should be global or local, the global pollination can be written as follows:

\[
s_i^{t+1} = s_i^t + \alpha L(\lambda)(s_i^t - g_{best})
\]  

(3)

Local pollination can be represented mathematically as:
Where, $s_i^f$, $s_j^f$, $s_k^f$ is the pollen of different flowers in the same plant species. This means index $i \neq j \neq k$. $\epsilon$ denotes a uniform distribution between 0 and 1. Probability $p$ describes the change in the two types of pollination. All pollen obtained for each generation is assessed adopting greedy selection to decide which pollen could continue to live until the offspring. The equation used for the greedy selection is as follows:

$$ s_{i}^{f+1} = \begin{cases} s_{i}^{f+1} & \text{if } f(s_{i}^{f+1}) \leq f(s_{i}^{f}) \\ s_{i}^{f} & \text{if } f(s_{i}^{f+1}) > f(s_{i}^{f}) \end{cases} $$

(5)

### 2.3. Objective function

The suitability in the model response and the observed data is stated by a minimum objective function, whose search process is associated with the search for the optimum model. The model is modified in such a way that its response fits the data. In the process, it is clear that inverse modeling can only be performed if the dependence in the data and model parameters (future modeling functions) is known.

The objective function is explained as the total of squares of the data prediction error, which represents the difference in the observed data and the calculated data:

$$ E = \sum_{i=1}^{N} (r \rho_{i}^{cal} - r \rho_{i})^2 $$

(6)

For the objective function to have a clearer meaning in the same unit as the data ($\Omega_m$), it can be represent as the root mean square error (ERMS), with the following equation:

$$ E_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (r \rho_{i}^{cal} - r \rho_{i})^2} $$

(7)

### 3. Results

#### 3.1. Synthetic data

DC data sets over three-layered (A-type model) and four-layered models (KH-type model) are inverted to test the performances of the FPA. Furthermore, wide searching area is designed in the test below to prove the robustness of the FPA. The population $n_{pop}=50$ and the iteration stops for A-type model and KH-type model when $RMS$ smaller than $10^{-10}$ or has converged.

The results of the FPA based on models A-type and KH-types are contrasted with the DLSQR inversion program, PSO, and GWO for verification of the accuracy and efficiency of the FPA. We set inverted parameters (search space and maximum iteration) in the FPA to be the same as those in the DLSQR inversion program, PSO, and GWO. The search space and the experimental results are provided in table 1. Comparisons of RMS errors behaviours between the DLSQR inversion program, PSO, GWO, and FPA are exhibited in figure 2. We can clearly see that the FPA outperform the DLSQR inversion program, PSO and GWO in terms of the accuracy of the final inverted results and their convergence speeds (based on number of iterations).

For the FPA and DLSQR inversion program, the RMS error values are all smaller than the GWO and PSO. For the DLSQR inversion program, the RMS error values always sharply decay at the beginnings of the iterations, but always preced by FPA in the middle iteration. Figure 2 reveal that...
FPA has better performances than the DLSQR inversion program, PSO and GWO in terms of those improved RMS error situation.

**Table 1.** Results of the inversions based on the DLSQR inversion program, PSO, GWO, and FPA.

| Model | Parameters | True value | DLSQR | PSO | GWO | FPA |
|-------|------------|------------|-------|-----|-----|-----|
|       |            | Initial value | Estimated value | Search space |       |     |
| A-type | $\rho_1$ (Ωm) | 30          | 30    | 30  | 30  | 30  |
|        | $\rho_2$ (Ωm) | 100         | 300   | 100.2| 123.4| 25.82| 100  |
|        | $\rho_3$ (Ωm) | 500         | 500.01| 500.01| 500.46| 493.75| 500  |
|        | $h_1$ (m)  | 8           | 10    | 8   | 8.19| 7.32| 8    |
|        | $h_2$ (m)  | 4           | 4     | 4   | 4.23| 1.55| 4    |
| KH-type | $\rho_1$ (Ωm) | 500         | 500   | 500.01| 500.46| 500.31| 500  |
|        | $\rho_2$ (Ωm) | 1000        | 1000  | 1000 | 37.17| 1021.36| 1000 |
|        | $\rho_3$ (Ωm) | 211         | 211   | 211  | 1500| 153.43| 211  |
|        | $\rho_4$ (Ωm) | 1375        | 1375  | 1375 | 1500| 1341.97| 1375 |
|        | $h_1$ (m)  | 4           | 4     | 4   | 30  | 4.07| 4    |
|        | $h_2$ (m)  | 8           | 10    | 8   | 1.22| 8.13| 8    |
|        | $h_3$ (m)  | `12         | `12   | `12 | 1   | 8.42| `12  |

**Figure 2.** Comparisons of RMS error behaviours between the DLSQR inversion program, PSO, GWO, and FPA. (a) RMS error in the iterative process based on A-type model. (b) RMS error in the iterative process based on KH-type model.

**Table 2.** Elapsed time needed in the DLSQR inversion program, PSO, GWO, and FPA inversions.

| Model | Number of iterations | Elapsed time (s) |
|-------|----------------------|------------------|
|       | DLSQR | PSO | GWO | FPA |
| A-type | 10264 | 404.53| 242.17| 270.71| 3173.83 |
| KH-type | 8497 | 399.21| 203.09| 216.31| 2708.82 |

The elapsed time (a Core i5-5200U CPU 2.2GHz) needed for all iteration and the number of iterations to obtain a solution in the DLSQR inversion program, PSO, GWO, and FPA inversions are shown in table 2. Table 2 indicates that the FPA is more time consuming than the DLSQR inversion program,
PSO and GWO. Generally, the FPA requires fewer iterations than the DLSQR inversion program, PSO, and GWO to obtain an acceptable solution.

3.2. Field data

![Field data example](image)

**Figure 3.** A field data example. (a) The calculated and observed apparent resistivity of the FPA. (b) RMS error behaviour of the FPA. (c) Inversion results using the FPA and IPI2Win. (d) Also shown is the borehole data.

The inverted result of the FPA for the observed DC data are illustrated in figure 3. The RMS error between the observed data and the calculated data is 1.9 Ωm (FPA) and 2.56 Ωm (IPI2Win).
illustrates that the calculated apparent resistivity fit the measured observed apparent resistivity quite well. Figure 3b shows the convergence behaviour in the iterative process of the FPA. The RMS error values in figure 3b sharply decay during the first 500 iterations. Figure 3c depicts the inverted solutions from the FPA and IPI2Win. The inverted solution generally conforms to the results obtained using IPI2Win and quite according to the borehole data. Generally, first layer has resistivity value of approximately 27.81 $\Omega$m at depths of 0-2.18 m. The second layer is a low resistivity layer (15.37 $\Omega$m) at depths 2.18 - 10.4 m. The third layer has a higher apparent resistivity 36.21 $\Omega$m in accordance with andesite breccia (igneous rock has higher resistivity than clay at first and second layer).

4. Conclusions
We have shown that the FPA able to inverse problems of Schlumberger sounding curve. In contrast to the DLSQR inversion program, PSO and GWO, the FPA can acquire more accurate results and provides more improved RMS errors. The only downside of FPA is the length of elapsed time it takes to complete the entire iteration. The high performance, efficiency, accuracy, and satisfactory convergence characteristics of the FPA for inverse problems of Schlumberger sounding curve are sufficiently illustrated in this paper.

Acknowledgments
This research received financial support by LPDP (Indonesia Endowment Found for Education), LPPM ITB, and DIKTI (Directorate General of Higher Education of Indonesia).

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