The Blind Estimation of Carrier Frequency Offset of Non-cooperative Burst Signal

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Abstract—In order to address the challenge of carrier blind synchronization and blind demodulation for non-cooperative burst signals, a modification scheme of carrier frequency offset estimation is proposed in this paper. Due to the existence of carrier frequency offset and phase offset, the received signal constellation is circular. In the absence of prior information, a non-data-aided method is adopted to estimate carrier frequency offset of the received signal. Then the loop processing method is used to compensate and correct the carrier frequency offset and phase offset to complete the carrier recovery. Simulation results show that the proposed method has high accuracy in carrier frequency offset and phase offset correction when the signal to noise ratio is close to or higher than that of demodulation. Moreover, it has wide extension in engineering practice.

1. INTRODUCTION
As a new non-real-time communication method, burst communication is to reduce the exposure time of the signal in the air from the time domain [1]. Due to its characteristics of anti-interception and no prior information, it brings many difficulties to the signal demodulation and recovery of the original data information of the non-partner in the communication reconnaissance [2].

In this paper, a single signal segment burst MPSK signal with extremely short dwell time is taken as the research object. The reconnaissance module has provided the corresponding carrier frequency parameter estimation. Furthermore, frequency offset estimation is used to eliminate the influence of parameter estimation error and Doppler frequency shift, and the blind demodulation of the signal is accurately completed. In this paper, a joint carrier recovery algorithm based on phase difference and loop processing is proposed. The non-data-assisted forward estimation algorithm is used to estimate the carrier frequency offset to complete fast synchronization, and then the carrier loop tracking algorithm is used to accurately estimate and correct.

2. MATHEMATICAL MODEL OF RECEIVED SIGNAL
The signals received by the receiver are [3]:

\[ y_l = A a_l e^{i(2\pi f_s l T + \phi_l)} + n_l, \quad l = 0, 1, \ldots, L - 1 \]  

(1)

\( A \) represents the signal amplitude; \( a_l \) represents the transmitted MPSK symbol; \( L \) is the data length; \( f_s \) is the carrier frequency offset; \( \phi_l \) is the carrier phase offset; \( T \) is the symbol period; \( n_l \) is
the additive white Gaussian noise, and its variance is $\sigma^2$. Using the MPSK modulation method, and the Non-Data-Aided (NDA) algorithm uses the $M$ power calculation processing method, the carrier frequency offset estimation of the MPSK signal based on the maximum likelihood NDA method [4]:

$$\hat{f}_c = \frac{1}{M} \arg \max_j \left\{ \sum_{n=0}^{L-1} z_i^* e^{-j2\pi mf_n} \right\}, \quad z_i = y_i^M$$  \hspace{1cm} (2)

2.1 Carrier Frequency Offset Estimation Based on Signal Autocorrelation

In order to obtain good performance, the simplified algorithm can approximate the maximum likelihood estimation. The maximum likelihood function can be deformed and constructed [5].

From (2), the essence of maximum likelihood estimation is to find the maximum value of the following function:

$$H(f) = \left| \sum_{n=0}^{L-1} z_i^* e^{-j2\pi mf_n} \right|^2, \quad z_i = y_i^m$$  \hspace{1cm} (3)

The autocorrelation function contains carrier frequency offset information. The autocorrelation function is expressed as:

$$R(m) = \frac{1}{L-m} \sum_{n=0}^{L-1} z_i^* z_{i-m}^*, 1 \leq m \leq N$$  \hspace{1cm} (4)

In the formula: $N$ is the design parameter, the value range is generally $N \leq L/2$. Substituting $z_i = y_i^m$ and (1) into (4) gives:

$$R(m) = \frac{1}{L-m} \sum_{n=0}^{L-1} e^{M(2\pi mf_n + \gamma_n)}, 1 \leq m \leq N$$  \hspace{1cm} (5)

In the formula: $\gamma_n$ is zero mean noise, which has the same statistical characteristics as $n_i$. From formula (5), we can get:

$$\arg \left\{ R(m) \right\} = [2\pi Mf_m T + \gamma_n]_{2\pi}$$  \hspace{1cm} (6)

The estimation range of the carrier frequency offset estimation algorithm based on signal autocorrelation is negatively correlated with the design parameter $N$ [6]. The larger the value of $N$ is, the smaller the unbiased estimation range of the algorithm is. In order to solve this problem, NDA M&M algorithm uses the difference operation of autocorrelation function to obtain a new estimated parameter with unbiased estimation range independent of design parameters $N$ [7].

Let $\phi(m) = [\arg \left\{ R(m) \right\} - \arg \left\{ R(m-1) \right\}]_{2\pi}$ to get the carrier frequency offset calculation formula of NDA M&M algorithm:

$$\hat{f}_c = \frac{1}{2\pi M} \sum_{m=1}^{L} w(m) \left[ \arg \left\{ R(m) \right\} - \arg \left\{ R(m-1) \right\} \right]_{2\pi}$$  \hspace{1cm} (7)

Where $w(m)$ is a smooth window function:

$$w(m) = \frac{3 \left[ (L-m)(L-m+1) - N(L-N) \right]}{N(4N^2 - 6NL + 3L^2 - 1)}$$  \hspace{1cm} (8)

The NDA M&M algorithm uses auto-correlation difference calculation to expand the phase, solves the phase folding problem, and expands the unbiased estimation range of the algorithm. Theoretically,
the unbiased estimation range of this algorithm is $|f| < 1/(2MT)$, and it is not limited to the design parameter $N$.

The calculation complexity of the NDA M&M algorithm is the number of complex times: $\frac{N}{2} (2L - N - 1) + N + 1$; the number of complex additions: $\frac{N}{2} (2L - N - 1) + N$; the number of phase angles: $N$. In the carrier synchronization process of non-cooperative burst signals, the estimation range, estimation accuracy, signal-to-noise ratio threshold and calculation complexity are mutually restricted, and must be comprehensively considered according to the applicable occasions and requirements to meet the actual application requirements.

2.2 Carrier Frequency Offset Estimation Based on Signal Phase

In order to reduce the amount of calculation, Tretter proposed a carrier frequency offset estimation algorithm based on the signal phase, and changed it into an NDA estimation parameter [8].

Rewrite $y_j$ as:

$$y_j = Ae^{j(\theta - \phi + \mu)} + n_i = Ae^{j(\theta + \phi)}(1 + \mu_i), \quad l = 0, 1, ..., L - 1$$  \hspace{1cm} (9)

In the formula, MPSK sends the symbol, $\theta_i = l/M, \ i = 0, 1, ..., M - 1$, $\mu_i = n_i \exp(-j(2\pi f_i T + \theta_i + \phi_i)) / A$, $\mu_i = R(\mu_i) + jI(\mu_i)$. When $SNR \gg 1$, $z_i$ can be expressed as:

$$z_i = y_j^M = Ae^{j(2\pi f_i T + \phi + I(\mu_i))}, \quad l = 0, 1, ..., L - 1$$  \hspace{1cm} (10)

It can be seen from equation (10) that the phase contains all the information of the carrier frequency offset and phase offset to be estimated [9]:

$$\arg\{z_i\} = M(2\pi f_i T + \phi + I(\mu_i))$$  \hspace{1cm} (11)

A carrier frequency offset estimation algorithm based on phase difference uses differential calculation processing to expand the phase and performs phase expansion through differential operation to obtain an algorithm that can reach the theoretical maximum unbiased estimation range of the NDA algorithm. From (11), we can get:

$$\arg\{z_i\} - \arg\{z_{i-1}\} = \arg\{z_i z_{i-1}^*\} = M(2\pi f_i T + I(\mu_i) - I(\mu_{i-1}))$$  \hspace{1cm} (12)

In the formula: $\xi_i = \mu_i - \mu_{i-1}$ is zero mean noise, which is the same as the statistical performance of $\mu_i$. From this, the mathematical expression of carrier frequency offset estimation of NDA Kay algorithm can be derived:

$$\hat{f}_i = \frac{1}{2\pi MT} \sum_{l=0}^{L-1} w(l) \arg\{z_i z_{i-1}^*\}, \quad z_i = y_j^M$$  \hspace{1cm} (13)

In the formula: $w(l)$ is the smooth window function, the expression is as follows:

$$w(l) = \frac{3L}{2(L^2 - 1)} \left\{ 1 - \left( \frac{l - (L/2 - 1)}{L/2} \right)^2 \right\}$$  \hspace{1cm} (14)

Simulation 1: Compare the relationship between the unbiased estimation range of NDA M&M algorithm and NDA Kay algorithm with the signal-to-noise ratio.

The length of the burst signal is 150 symbols. The design parameter $N$ is 25. The signal modulation is QPSK modulation and the signal-to-noise ratio takes three values: 5dB, 15dB, and 20dB respectively. The average value of different normalized carrier frequency offset estimation parameters obtained by simulation is shown in Fig. 1.
According to the content discussed above, the theoretical maximum unbiased estimation range of NDA algorithm is \( |f/T| < 0.125 \). As shown in Fig. 1, when the signal-to-noise ratio \( E_b/N_0 \) is large, the actual unbiased estimation parameter range of the two algorithms approximates the theoretical value. In this case of \( E_b/N_0 \geq 15\text{dB} \), the unbiased estimation range of the NDA M&M algorithm agrees well with the theoretical value, and the change is basically small. The NDA Kay algorithm is significantly affected by the signal-to-noise ratio, its performance is better under high signal-to-noise ratio.

Simulation 2: Compare the MSE performance of NDA Kay algorithm under different carrier frequency offsets.

The length of the burst signal is 150 symbols. The design parameter \( N \) is 25. The signal modulation is QPSK modulation and the normalized frequency offset \( f/T \) takes three values of \( 1e^{-3} \), \( 5e^{-3} \), \( 5e^{-2} \) respectively, which are all within the unbiased estimation range of the NDA Kay algorithm. The three selected frequency offsets are simulated separately, and the MSE performance of the estimated parameters is shown in Fig. 2.

Figure 2 The estimation performance of NDA Kay algorithm with different frequency offset

3. JOINT CARRIER RECOVERY ALGORITHM BASED ON PHASE DIFFERENCE AND LOOP PROCESSING

According to the research background and requirements of this paper, a joint algorithm is proposed to achieve carrier synchronization of non-cooperative burst signals. This algorithm is based on the combination of coarse carrier frequency offset estimation and fine loop processing. Through the algorithm simulation and performance analysis, it can be concluded that the NDA Kay algorithm has a small calculation amount and no phase folding problem. The implementation of coarse frequency offset estimation uses the NDA Kay algorithm. This section mainly explains the loop processing for fine estimation compensation algorithm. The carrier frequency offset is estimated by the NDA Kay
algorithm for coarse synchronization, and then the phase offset is compensated and corrected by the loop processing module. The block diagram of the joint correction algorithm for phase difference and loop processing is as follows:

![Block diagram of joint carrier recovery algorithm based on phase difference division and loop processing](image)

Figure 3 The block diagram of joint carrier recovery algorithm based on phase difference division and loop processing

The loop filter design of the joint algorithm is mainly for the design of the filter coefficient, which is the key to the loop processing module.

In a high-dynamic environment, it is necessary to meet the requirement that the capture band is large enough and the capture time is short, and the requirement of high tracking accuracy is required [10]. They are mutually restrictive. Therefore, it is necessary to balance the two points and design the filter parameters reasonably to meet the demand. The design of this paper uses a second-order proportional integral filter [11], whose structure is shown in Fig. 4.

![Second-order proportional integral filter structure](image)

Figure 4 Second-order proportional integral filter structure

The loop filter coefficients $C_1$ and $C_2$ can be calculated by equation (15):

$$C_1 = \frac{1}{K} \frac{4(\omega_n T)^2 + 8\xi \omega_n T}{4 + 4\xi \omega_n T + (\omega_n T)^2}$$

$$C_2 = \frac{1}{K} \frac{4(\omega_n T)^2}{4 + 4\xi \omega_n T + (\omega_n T)^2}$$

(15)

In the formula: $T$ is the sampling time; $\omega_n$ is the natural angular frequency of the loop; $\xi$ is the damping coefficient; $K$ is the total gain of the loop.

When $\omega_n T \ll 1$, there is:

$$C_1 = \frac{2\xi \omega_n T}{K}$$

$$C_2 = \frac{(\omega_n T)^2}{K}$$

(16)

Simulation 3: Since the loop tracking compensation method has certain requirements on the data length, it cannot be too short. The input data length processed by the loop is set to 400, $E_s / N_0 = 15$ dB, and the modulation method is QPSK. The normalized carrier frequency offset is $f_c T = 0.05$, and the random initial phase offset is $3\pi/5$, as shown in Fig. 5.
The simulation results show that the joint algorithm can accurately realize carrier frequency offset estimation and correction. The joint algorithm takes into account real-time performance while taking into account accuracy. After correcting the deviation, the constellation of the signal is not circular and has no deflection angle, which can accurately recover the data information.

4. CONCLUSION
This paper proposes a joint carrier recovery algorithm based on phase difference and loop processing for burst communication signals. This method can estimate the carrier frequency offset through the NDA Kay algorithm, which has a wide frequency offset estimation range. The loop processing method is used to accurately compensate and correct the estimated frequency offset to complete carrier recovery. The simulation shows that the scheme has good performance and solves the problem that the existence of carrier frequency offset causes the received signal constellation to deflect by an angle and rotate continuously. And Lay a good foundation for subsequent demodulation and decoding.

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