Heavy Exotic Molecules

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Abstract

We briefly review the formation of pion-mediated heavy-light exotic molecules with both charm and bottom, under the general strictures of chiral and heavy quark symmetries. The charm isosinglet exotic molecules with $J^{PC} = 1^{++}$ binds, which we identify as the reported neutral $X(3872)$. The bottom isotriplet exotic with $J^{PC} = 1^{+-}$ binds, and is identified as a mixed state of the reported charged exotics $Z_b^+(10610)$ and $Z_b^+(10650)$. The bound bottom isosinglet molecule with $J^{PC} = 1^{++}$ is a possible neutral $X_b(10532)$ to be observed.

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I. INTRODUCTION

A decade ago, the BaBar collaboration [1] and the CLEOII collaboration [2] have reported narrow peaks in the $D_s^+\pi^0$ (2317) and the $D_s^{*+}\pi^0$ (2460) in support of predictions from chiral and heavy-quark symmetry [3–5]. The heavy-light multiplet $(0^-,1^-) = (D, D^*)$ has a chiral partner $(0^+,1^+) = (\bar{D}, \bar{D}^*)$ that is about one constituent mass heavier [3–4]. More recently, the Belle collaboration [6] and the BESIII collaboration [7] have reported new multiquark exotics, outside the standard quark model classification. A key source for these exotics is $\Upsilon(10860)$ and its closeness to the $B\bar{B}^*\pi$ (10744) and $B^*\bar{B}^*\pi$ (10790) tresholds. The smallness of the phase space for the pion decay of $\Upsilon(10860)$ suggests that the decay process is slow, involving a molecular configuration on the way out. Heavy exotic molecules have been reported, such as the neutral $X(3872)$ and the charged $Z_c(3900)^\pm$ and $Z_b(10610)^\pm$. More of these exotics are expected to be unravelled by the DO collaboration at Fermilab [8], and the LHCb collaboration at Cern [9].

Theoretical arguments have predicted the possibility of molecular bound states involving heavy-light charm and bottom mesons through pion exchange [10, 11]. Since, a number of molecular estimates were made by many [11–16]. Non-molecular heavy exotics were also propodsed using constituent quark models [17], heavy solitonic baryons [18, 19], instantons [20] and QCD sum rules [21].

In this contribution, we briefly review our recent analysis of the molecular configurations with heavy-light charm and bottom mesons and their chiral partners, under the general constraints of chiral and heavy-quark symmetry [22]. In section 2, we outline the heavy-light effective action in leading order involving the $(0^\pm,1^\pm)$ multiplets, and formulate the non-relativistic bound state problem in the $J = 1$ channel. The results are summarized in section 3. Our conclusions are in section 4.

II. MOLECULES

The leading part of the heavy-light Lagrangian for the charmed multiplet $(0^-,1^-)$ with pions reads [3, 5]
\[ \mathcal{L} \approx +2i \left( \bar{D} \partial_0 D + \bar{D} \cdot \partial_0 D \right) - \Delta m_D \bar{D} D - \Delta m_D \bar{D} D \\
+ i \frac{g_H}{f_\pi} \text{Tr} \partial_i \pi \left( D_i \bar{D}^\dagger - ID_i + \epsilon_{ijk} D_k \bar{D}_j^\dagger \right) \]  

(1)

with \( \Delta m_i = m_i - m_C \) of the order of a quark constituent mass. The leading part of the heavy-light chiral doublers Lagrangian for the charmed \((0^+, 1^+)\) multiplet with pions reads [3]

\[ \tilde{\mathcal{L}} \approx +2i \left( \tilde{D} \partial_0 \tilde{D} + \tilde{D} \cdot \partial_0 \tilde{D} \right) - \Delta m_{\tilde{D}} \bar{D} \tilde{D} - \Delta m_{\tilde{D}} \bar{D} \tilde{D} \\
+ i \frac{g_H}{f_\pi} \text{Tr} \partial_i \pi \left( i(D_i \tilde{D}^\dagger + \tilde{D} \tilde{D}_i^\dagger) + \epsilon_{ijk} \tilde{D}_k \tilde{D}_j^\dagger \right) \]  

(2)

with again \( \Delta m_i = m_i - m_C \) of the order of a quark constituent mass. The \((0^+, 1^+)\) multiplet mixes with the \((0^-, 1^-)\) by chiral symmetry [3, 4]

\[ \delta \mathcal{L} = \frac{g_{HG}}{f_\pi} \text{Tr} \partial_0 \pi \left( \bar{D}_i^\dagger D_i - i \bar{D}_i^\dagger D + c.c. \right) \]  

(3)

The molecular exotics of the type \( D \bar{D}^* \), follows from (1) through one-pion exchange. The non-relativistic character of the molecules yield naturally to a Hamiltonian description. Let \( D_{00}(r) \) denote the wave function of the molecular scalar, and \( \bar{Y}_{0i}(r) \) and \( Y_{0i}(r) \) denote the wavefunctions of the molecular vectors, and \( T_{ij}(r) \) the wavefunction of the molecular tensors. Using (1-3) for the 2-body interactions, we have

\[ (VT)_{k\tilde{i}} = C \epsilon_{kim} \epsilon_{\tilde{i}jn} \partial_{mn} V(r) T_{ij} \]
\[ (VT)_{0\tilde{0}} = C \partial_0 \tilde{v} T_{ij} \]
\[ (V\tilde{Y})_{k0} = -C \partial_k \tilde{v} V(r) \bar{Y}_{0j} \]
\[ (VT)_{0k} = C \epsilon_{kij} \partial_i \partial_j V(r) T_{ij} \]
\[ (VT)_{\tilde{0}k} = C \epsilon_{kij} \partial_i \partial_j V(r) T_{ij} \]  

(4)

with the isospin factor.
\[ C = I_1 \cdot I_2 = \left( \frac{1}{4} \bigg|_{I=1} , -\frac{3}{4} \bigg|_{I=0} \right) \]  

(5)

Here \( V(r) \) is the regulated one-pion exchange using the standard monopole form factor by analogy with the pion-nucleon form factor \[23\]. It is defined with a core cutoff \( \Lambda \gg m_\pi [11, 23] \)

\[ V(r) = \left( \frac{g_H}{f_\pi} \right)^2 \frac{1}{4\pi} \left( \frac{e^{-m_\pi r}}{r} - \frac{e^{-\Lambda r}}{r} - (\Lambda^2 - m_\pi^2) \frac{e^{-\Lambda r}}{2\Lambda} \right) \]  

(6)

Throughout, we will use \( g_H = 0.6 \) \[3, 4\] and \( \Lambda = 1 \) GeV. The choice of \( \Lambda \) is the major uncertainty in the molecular analysis. The one-pion exchange in \[4\] induces a D-wave admixture much like in the deuteron as a proton-neutron molecule \[23\]. It is very different from one-gluon exchange in heavy quarkonia \[17\].

The pertinent projections onto the higher \( J^{PC} \) channels of the molecular wavefunctions in \[4\] require the use of both vector and higher tensor spherical harmonics \[24, 25\]. For \( J = 1 \), we will use the explicit forms quoted in \[23\] with the \( S L_J \) assignment completly specified. For the \((1^\mp,0^\mp)\) multiplets, there are 4 different \( 1^{PC} \) sectors

\[
1^{++} : T_{ij}^{2,2}(5D_1), Y_i^{0+}(3S_1), Y_i^{2+}(3D_1) \\
1^{--} : T_{ij}^{0,1}(1P_1), T_{ij}^{2,1}(5P_1), T_{ij}^{2,3}(5F_1), Y_i^{1-}(3P_1), D_i^{1}(1P'_1) \\
1^{+-} : T_{ij}^{1,0}(3S_1), T_{ij}^{1,2}(3D_1), Y_i^{0-}(3S'_1), Y_i^{2-}(3D'_1) \\
1^{-+} : T_{ij}^{1,1}(3P_1), Y_i^{1+}(3P'_1) 
\]

(7)

The normalized tensor harmonics are detailed in \[22, 25\].

III. RESULTS

In Fig. 1 we show the the radial components (upper part) and percentage content (lower part) of the bound isosinglet charm wavefunction with energy \( E = 3.867 \) GeV, versus \( r \) in units of \( \Lambda = 1 \) GeV. The intra-coupling between the \((0^-,1^-)\) and \((0^+,1^+)\) multiplet
FIG. 1: $J^{PC} = 1^{++}$: radial wavefunctions (upper plot) and percentage content (lower plot) for the charm isosinglet exotic state ($C = -3/4$).

causes the chiral partners $\bar{D}D^*$ to unbind. The $^S L_J$ assignments referring to the $(0^-, 1^-)$ multiplet, and the $^S \bar{L}_J$ assignments referring to the $(0^+, 1^+)$ multiplet, are those listed in (7). The mixing results in a stronger binding in this channel which is mostly an isosinglet $^1S_3$ contribution in the $(1^-, 0^-)$ multiplet with almost no D-wave admixture. This molecular state carries $J^{PC} = 1^{++}$. It is chiefly an isosinglet $D\bar{D}^*$ molecule, which we identify as the reported exotic $X(3872)$.

FIG. 2: $J^{PC} = 1^{++}$: radial wavefunctions (upper plot) and percentage content (lower plot) for the bottom isosinglet exotic state ($C = -3/4$).

In Fig. 2 we show the radial components (upper part) and percentage content (lower part) of the bound isosinglet bottom wavefunction with energy $E = 10.532$ GeV, versus $r$ in units of $\Lambda = 1$ GeV. Again, the $^S L_J$ and $^S \bar{L}_J$ assignments refer to the $(0^\pm, 1^\pm)$ multiplets respectively, as defined in (7). The $1^{++}$ mixed bound state is mostly a $B\bar{B}^*$ ($^3S_1$) molecule. A comparison of Fig. 1 to Fig. 2 shows that this neutral bottom molecular state is the mirror analogue of the neutral charm molecular state or $X_b(10532)$, yet to be reported.
In Fig. 3 we show the radial components (upper part) and percentage content (lower part) of the bound isosinglet bottom wavefunction with energy \( E = 10.592 \) GeV, versus \( r \) in units of \( \Lambda = 1 \) GeV. From the assignments given in (7), it follows that \( 1^{+-} \) is a mixed isotriplet \( B\bar{B}^* (\bar{3}S'_1) \) molecules, with a small admixture of \( B^*\bar{B}^* (\bar{3}S'_1) \) molecule. This molecule is an admixture of the reported states \( Z_b^+ (10610) \) and \( Z_b^+ (10650) \).

FIG. 3: \( J^{PC} = 1^{+-} \): radial wavefunctions (upper plot) and percentage content (lower plot) for the bottom isotriplet exotic state \((C = +1/4)\).

IV. CONCLUSIONS

We have briefly reported on the molecular states of doubly heavy mesons mediated by one-pion exchange for both the chiral partners \((0^\pm, 1^\pm)\) as a coupled channel problem, recently discussed in [22]. The analysis complements and extends those presented in [11–16] by taking into account the strictures of chiral and heavy quark symmetry, and by retaining most coupled channels between the \((0^-, 1^-)\) multiplet and its chiral partner \((0^+, 1^+)\). The key aspect of this coupling is to cause the molecules in the \((0^-, 1^-)\) multiplet to bind about twice more, and the molecules in the \((0^+, 1^+)\) multiplet to unbind. The charm isosinglet exotic molecules with \( J^{PC} = 1^{++} \) is strictly bound for a pion-exchange cutoff \( \Lambda = 1 \) GeV. This state is identified with the reported isosinglet exotic \( X(3872) \) which in our case is mostly an isosinglet \( D\bar{D}^* \) molecule in the \( ^1S_0 \) channel with no D-wave admixture. The attraction in the isotriplet channel with \( J^{PC} = 1^{+-} \) is too weak to bind the \( D\bar{D}^* \) compound, suggesting that the reported isotriplet \( Z_C(3900)^\pm \) is at best a threshold enhancement. The \( Y(4260), Y(4360) \) and \( Y(4660) \) may point to the possibility of their constituents made of excited \((D_1, D_2)\) heavy mesons and their chiral partners [3, 26]. The isotriplet bottom exotic molecule with
\(J^{PC} = 1^{+-}\) which we have identified with the pair \(Z_b^+(10610)\) and \(Z_b^+(10650)\), which is a mixed state in our analysis. The isosinglet bottom exotic molecule with \(J^{PC} = 1^{++}\) is a potential candidate for \(X_b(10532)\), yet to be measured.

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