Scaling properties of collective effects at RHIC

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Abstract. Azimuthal anisotropy is one of the key observables to study the properties of matter created in high energy heavy-ion collisions at RHIC and the LHC. The collective behaviour is quantified in terms of anisotropy coefficients $v_n$ measured with respect to their corresponding event planes. Predictions from the viscous hydrodynamics for the scaling of the anisotropic flow coefficients $v_n$ with eccentricity, system size and transverse energy are tested using the recent data from PHENIX Collaboration.

1. Introduction
Full characterization of the transport properties of the strongly interacting matter (the quark-gluon plasma – QGP) produced in heavy ion collisions, is a central goal of the experimental heavy ion programs at both the RHIC and the LHC. One of the key observables sensitive to these properties is collective flow – the anisotropy of the particle emission in the plane transverse to the beam direction [1]. This anisotropy can be quantified as a function of particle transverse momentum $p_T$ and collision centrality (cent) or the number of participant nucleons $N_{\text{part}}$, by the Fourier coefficients $v_n$:

$$v_n = \langle \cos(n(\phi - \psi_n) \rangle,$$

where $\phi$ is the azimuthal angle of an emitted particle, and $\psi_n$ are the azimuths of the estimated participant event planes. The brackets denote averaging over particles and events [2, 3].

Examination of the different scaling properties of the collective flow coefficients can be useful for testing viscous hydrodynamic model compatibility for the QGP matter. It also provides important constraints for distinguishing between different models of initial conditions, as well as a route to constrain the temperature dependence of the ratio of shear viscosity $\eta$ to entropy density $s$: $\eta/s$[4].

2. Acoustic scaling
Collective flow measurements provide a great method to evaluate one of the key transport properties of the QGP – ratio of shear viscosity and entropy density $\eta/s$. There is a qualitative difference between the radial flow and higher angular harmonics. While the former monotonously grows with time, driven by the outward pressure gradient with a fixed sign, the latter are basically sounds, or damped oscillators. One effects of viscosity on sounds is damping of their amplitude. Several works based on the assumption of the acoustic nature of the anisotropic flow suggest that viscous corrections to $v_n/\varepsilon_n$ grow exponentially as $n^2$ and $1/\bar{R}$ [5, 6, 8]:

$$\frac{v_n}{\varepsilon_n} \propto \exp \left[ -C n^2 \frac{\eta}{s TR} \right],$$

where $C$ is some constant, $T$ is the temperature, $\bar{R}$ is the transverse size of the collision zone and $\varepsilon_n$ is the $n$-th order eccentricity moment of the fireball initial state. For a given $n$, this equation indicates a
characteristic linear dependence of $\ln(v_n/\varepsilon_n)$ on $1/\bar{R}$, with a slope proportional to $\eta/s$. This scaling pattern is confirmed by state-of-the-art viscous hydrodynamic calculations [7] shown in figure 1. Figure 1b indicates a clear sensitivity of the linear fit slopes to $\eta/s$. Figure 1c shows dependence of $\ln(v_n/\varepsilon_n)$ on $n^2$ which is also linear [8].

![Figure 1](image)

**Figure 1.** (a) $v_2$ vs. $N_{\text{part}}$ from viscous hydrodynamic calculations [7] for two values of specific shear viscosity as indicated. The results are for $0.15 < p_t < 2.0$ GeV/c for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. (b) $\ln(v_2/\varepsilon_2)$ vs. $1/\bar{R}$ for the $v_2$ values shown in (a). (c) $\ln(v_n/\varepsilon_n)$ vs. $n^2$ for different values of specific shear viscosity. The dashed and dot-dashed curves are linear fits.

The data used for testing the acoustic scaling are taken from measurements by the PHENIX collaboration for Cu+Cu and Au+Au collisions at $\sqrt{s_{NN}} = 62.4$ and 200 GeV. To obtain $v_2$ values the event-plane method was used. Monte Carlo Glauber (MC-Glauber) simulations were used to compute participant eccentricity $\varepsilon_2$(centrality) and $\bar{R}$(centrality) from the two-dimensional profile of the density of sources in the transverse plane [9]; $1/\bar{R} = \sqrt{1/\sigma_x^2 + 1/\sigma_y^2}$, where $\sigma_x$ and $\sigma_y$ are the respective root-mean-square width of the density distribution.

Figures 2a,b show $v_2$ and $v_2/\varepsilon_2$ (respectively) vs. number of the collision participants obtained from the PHENIX collaboration data for $p_t = 1.0 - 2.0$ GeV/c[10]. For ideal fluids one should observe the eccentricity scaling of $v_2$: at a given energy $v_2/\varepsilon_2$ should be independent on the transverse size of the system $\bar{R}$. If the equilibration is incomplete the eccentricity scaling is broken and $v_2/\varepsilon_2$ also depends on the Knudsen number $K = \lambda/\bar{R}$ ($\lambda$ is the mean free path) and should follow the equation (2). Figure 2c shows the dependence of $\ln(v_n/\varepsilon_n)$ on $1/\bar{R}$ for these data, which is well described by linear fits (solid lines). The slopes are found to be the same within errors for Au+Au collisions at $\sqrt{s_{NN}} = 62.4$ and 200 GeV. However, they are different for Au+Au and Cu+Cu at $\sqrt{s_{NN}} = 200$ GeV, which may indicate the larger viscous damping for smaller colliding systems. The slope is larger for Cu+Cu at 62.4 GeV than for 200 GeV. The later observation may suggest that the beam energy scan at RHIC performed for several colliding systems will provide more information which will help to constrain the temperature dependence of $\eta/s$. 


3. Constituent quark scaling

The collective flow coefficients measured for different identified hadrons also shows specific scale properties. Figure 3 shows a comparison of \(v_2(p_T), v_3(p_T)\) and \(v_4(p_T)\) for \(\pi^\pm, K^\pm\) and \(p + \bar{p}\) emerged from 0\%-50\% central Au+Au collisions at \(\sqrt{s_{NN}} = 200\) GeV. The data were obtained by the PHENIX collaboration via the event-plane (solid points) and two-particle correlation (open points) methods. It is noticeable that in the low-\(p_T\) region the anisotropy appears largest for the lightest hadrons whereas in the intermediate-\(p_T\) region (3 < \(p_T\) < 4 GeV/c) this mass dependence partly reverses, such that the anisotropy is greater for the (anti-)baryons (\(n_q = 3\)) than for the mesons (\(n_q = 2\)) at the same \(p_T\) [11].

Figure 3. Fourier coefficients \(v_2, v_3, v_4\) and \(v_4\) with respect to the second harmonic event plane in plots (a) – (d) respectively) for charge combined \(\pi^\pm, K^\pm\) and \(p + \bar{p}\) at midrapidity for 0\%-50\% central Au+Au collisions at \(\sqrt{s_{NN}} = 200\) GeV. The green bands indicate the \(p_T\)-correlated systematic uncertainties of the \(\pi^\pm\) results from the event-plane method. The shaded boxes around the data points are \(p_T\)-uncorrelated systematic uncertainties [11].

The baryon-meson splitting in the intermediate-\(p_T\) region can be taken as an indication that the number of constituent valence quarks \(n_q\) is an important determinant of the final-state hadron flow in this range. “Constituent quark number scaling” has been suggested to test this assumption, i.e. the scaling of \(v_2/n_q\) for identified hadrons to transverse kinetic energy per constituent quark number \(KE_T/n_q\).
[12, 13]. For higher order harmonics following dependence was observed: \( v_n(p_T) \propto (v_2)^{n/2} \) [4, 14], which may stem from the hydrodynamic evolution of the medium [2, 15], the quark coalescence process [16] and the acoustic nature of anisotropic flow [4]. These observations lead to universal (for each harmonic order \( n \)) quark number scaling – \( v_n / (n_q)^{n/2} \) vs. \( K E_T / n_q \). The adherence of the data to this empirical scaling is shown in figure 4 [11]. \( v_4 \{ \Psi_2 \} \) denotes that the fourth harmonic coefficient was evaluated with respect to the second harmonic event plane. The scaled values lie on a single curve for each harmonics for all the hadron species within a ±15% range.

![Figure 4: Quark-number scaling for 0%-50% central Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV. \( n_q \) is the constituent valence quark number of each hadron. Differences between (a) – (d) plots are the same as in figure 3 [11].](image)

4. Conclusion
The test for several scaling relations is presented for anisotropic flow at RHIC using the recent data from the PHENIX Collaboration. The presence of scaling variables provides a very transparent indications of the dynamics underlying a given physical process.

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