Semiclassical spin transport in LaO/STO system in the presence of multiple Rashba spin-orbit couplings

Anirban Kundu1,2,†, Zhuo Bin Siu1 and Mansoor B A Jalil1,*

1 Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117583, Singapore
2 Department of Physics, Ariel University, Ariel 40700, Israel
* Author to whom any correspondence should be addressed.
† Publisher’s note. Whilst IOP Publishing adheres to and respects UN resolutions regarding the designations of territories (available at: www.un.org/press/en), the policy of IOP Publishing is to use the affiliations provided by its authors on its published articles.

E-mail: elembaj@nus.edu.sg

Keywords: LaO/STO, spin current, spin accumulation, Boltzmann equation

Supplementary material for this article is available online

Abstract

The interaction between linear and cubic spin–orbit couplings with magnetic moments and mobile spin-polarized carriers in the LaAlO3/SrTiO3 (LaO/STO) system provides new avenues for spin transport applications. We study the interplay between linear and cubic Rashba spin orbit coupling (RSOC) on in-plane magnetic moments in the LaO/STO system using the Boltzmann transport theory based on the relaxation time approximation (RTA) and the more refined Schliemann-Loss (SL) delta-potential scattering model. In general, both methods yield a linear (quadratic) relationship in the spin accumulation (spin current) when one of the three RSOC strengths is varied and the other two fixed. The simultaneous presence of multiple types of RSOC with distinct angular dependencies facilitates the breaking of the k-space symmetry of the Fermi surface, thus ensuring a finite spin accumulation upon integration over the entire Fermi surface. While the oft-used RTA method is sufficiently accurate for spin accumulation calculations, the more refined SL model is required for spin current calculations because the RTA method neglects the anisotropy of the Fermi contour arising from the cubic RSOC terms. Based on the refined SL model and under optimal tuning of the RSOC parameters, the spin charge conversion in LaO/STO is predicted to reach a remarkable efficiency of 30%.

1. Introduction

A key requirement in spintronics devices is the efficient conversion of charge currents into spin accumulations or spin currents, which can be used to generate spin transfer torque to induce magnetization switching in magnetic memory systems based on magnetic multilayers, magnetic tunnel junctions, or magnetic skyrmions [1–7]. Previous studies have demonstrated the efficacy of using Rashba spin–orbit coupling (RSOC) to achieve high charge-spin conversion efficiency in systems with broken inversion symmetry [8–20].

Recently, oxide heterostructures have attracted much attention due to their potential applications in spintronics devices [21–26]. Experimentally, oxide heterostructures have been shown to exhibit superconducting [27], and more importantly for our purpose, ferromagnetic phases [28–30]. The surface of SrTiO3 (STO) or its interface with other oxides is also known to host two-dimensional electron gases (2DEGs) with high electron mobility [31]. The individual layers in the oxide heterostructures are insulating, but when stacked together, they provide conduction electrons originating from the d-orbitals of the lattice system [32–34]. The carrier density and related electronic properties can be strongly tuned using a gate voltage. This ability to perform external tuning is another key factor that renders the LaAlO3 (LaO)/STO heterostructure an attractive platform for future spintronics architectures.
Our present work is motivated by the high spin-to-charge conversion efficiency in LaO/STO interfaces under the influence of a strong spin–orbit interaction (SOC) [35,36]. The SOC experienced by conduction electrons in the 2DEG at the LaO/STO interface can be effectively described as a Rashba SOC effect due to the broken inversion symmetry. The RSOC in transition metal oxides [32,37–39] and electron-doped semiconductors [40–42] has been shown to be linear in $k$, i.e. having the generic form of $\alpha (k \times e_z) \cdot \sigma$ where $e_z$ is the unit vector in the out-of-plane direction. However, in hole-doped semiconductors [43–45], rare earth materials [46], and in particular, at the surface of STO [47–49], the cubic RSOC also has a significant presence. In fact, the enhancement of the spin Hall conductivity in heavy-hole semiconductors has been primarily attributed to the cubic RSOC [50]. In the LaO/STO system, the linear RSOC strength is approximately 10 meV Å and that of the cubic RSOC approximately 1–5 eV Å$^3$ [32,48,51,52]. Over the Brillouin zone of the system, the magnitudes of the linear and cubic SOC splittings in the LaO/STO system are comparable.

Owing to the key role of the RSOC in the charge transport of the LaO/STO system, it is important to go beyond the effective low-energy Hamiltonian to study the RSOC effect and its dependence on the finite thickness and confining potential. Such an investigation has been lacking even though there has been numerous studies conducted on spin accumulation and transport in the LaO/STO system [53–57]. A previous study on a LaO/STO 2DEG system with a finite thickness focused on the out-of-plane transmission instead of transport within the 2D plane [58]. To address the shortcomings in earlier derivations of effective low-energy Hamiltonians [34,59] for the LaO/STO 2DEG, Ho et al explicitly accounted for the spatial variation of the confining potential and the resultant quantum confinement [52]. In addition to the usual linear Rashba term in the effective Hamiltonian for the lowest-energy pair of spin-split bands, they found two additional cubic RSOC terms with the forms of $\beta_3(k_x^2 - k_y^2)\left(k_y \sigma_x - k_x \sigma_y\right)$ and $\eta_3k_xk_y\left(k_x \sigma_x - k_y \sigma_y\right)$ where $\beta_3$ and $\eta_3$ are the strengths of the two cubic RSOC terms. The strengths of these cubic RSOC terms can be tuned by varying the thickness of the STO layer and the magnitude of the confining field. Although the effects of the $\beta_3$ term on the spin response in the LaO/STO has been studied previously [57,60], a full study of both the cubic RSOC terms in light of the above study has not been performed. In particular, the $\eta_3$ term has a magnitude that can be comparable or even larger than that of the $\beta_3$ term, but has been neglected hitherto.

In this work, we investigate the interplay between the various linear and cubic RSOC effects on the spin accumulation and spin current in the presence of an applied electric field and magnetization coupling. In our transport calculations, we use two approaches to obtain the non-equilibrium spin accumulation and current, namely, the conventional relaxation time approximation (RTA) and a more refined approach by Schliemann and Loss (SL) [61], which assumes delta-function scattering potentials and takes into account the anisotropy of the Fermi surface of the spin-split bands. (Here, we do not consider spin currents with out-of-plane spin polarizations, which arise from Berry curvature effects [5,62–64] rather than the electric field-induced displacements of the Fermi surfaces and are not captured by the Boltzmann transport equation (BTE).) We show that the simpler RTA approach is sufficient for the spin accumulation calculation but the more accurate SL approach is required to evaluate the spin current. Our results provide a more complete understanding of how the three RSOC terms affect the spin response of the LaO/STO system. We study the interplay between the various RSOC terms and their influence on the spin transport properties of the system. We find that under optimal tuning of the RSOC parameters, the LaO/STO system can reach a high spin-charge conversion efficiency of about 30%.

2. Methods

The LaO/STO system possesses three pairs of conduction bands comprising the $d_{x^2}$, $d_{y^2}$, and $d_{xz}$ bands in which the inversion asymmetry across the interface produces a strong SOC in the system that results in energy splitting within each pair of bands. We adopt the Hamiltonian of Ho et al [52], who considered the lowest-energy quantum well states for each of these bands due to the confining potential in the LaO/STO heterostructure. The six bands in the heterostructure can be then described by a six-by-six Hamiltonian representing two pairs of light electron bands denoted as $\text{le}^\pm$ and a pair of heavy electron bands he (figure 1(a)). Here, we restrict our consideration to low values of Fermi energies in which only the lowest-energy pair of spin-split bands, i.e. the $\text{le}^-$ bands, is physically relevant. In the presence of magnetic dopants, the general form of the effective Hamiltonian for this pair of bands is,

$$H = \frac{k^2}{2m^*} + J_H \sigma \cdot M + \alpha (\sigma \times k) \cdot \hat{z} + \sigma_x \beta_3 \left( k_x k_y^2 - k_y k_x^2 \right) - \sigma_y \eta_3 k_x^2 k_y + \sigma_y \beta_3 \left( k_x^2 k_y - k_y^2 k_x \right) + \sigma_x \eta_3 k_y^2 k_x$$

(1)

where $\alpha$ represents the strength of the linear RSOC, $\beta_3$ and $\eta_3$ are coefficients of the two distinct cubic SOC terms, and $J_H$ is the Heisenberg exchange interaction between conduction electrons and the magnetic moments of the dopants $M$. Here, we consider dopant magnetizations lying in the in-plane direction with
\( \mathbf{M} = M_x \hat{x} + M_y \hat{y} \). Denoting the coefficients of \( \sigma_x \) and \( \sigma_y \) in equation (1) as \( h_x \) and \( h_y \) respectively, \( h_x \) and \( h_y \) are explicitly given by

\[
\begin{align*}
    h_x(k) &= J_1 M_x + \beta_3 k_y \left( k_x^2 - k_y^2 \right) - \eta_3 k_x^2 k_y + \alpha k_y \\
    h_y(k) &= J_1 M_y - \beta_3 k_x \left( k_x^2 - k_y^2 \right) + \eta_3 k_x k_y^2 - \alpha k_x.
\end{align*}
\]

For later convenience, we also denote the spin-independent kinetic energy term as \( h_0(k) = \frac{k^2}{2m^*} \). The eigenenergies of equation (1) are then \( \epsilon_{kb} = h_0(k) + b \sqrt{h_x^2(k) + h_y^2(k)} \) where \( b = \pm 1 \) are the indices for the spin polarized bands and the corresponding eigenstates are given by

\[
\psi_{kb} = \frac{1}{2\sqrt{A}} \left( \begin{array}{c} e^{-i\gamma k} \\ b \end{array} \right) e^{ik \cdot \mathbf{r}},
\]

where

\[
\gamma_k = \tan^{-1} \left( \frac{h_y(k)}{h_x(k)} \right).
\]

The spin \( x \) expectation value of band \( b \) is given by

\[
\langle s_x \rangle_b = b \frac{h_x}{\sqrt{h_x^2 + h_y^2}}
\]

and the \( x \)-component of its group velocity given by

\[
\nu^x_{kb} = \frac{k_x}{m^*} + b \frac{h_x \partial_k h_x(k) + h_y \partial_k h_y(k)}{\sqrt{h_x^2(k) + h_y^2(k)}}.
\]

With the above preliminary results for the spin expectation and group velocity values, we proceed to apply the BTE to calculate the non-equilibrium spin accumulation and spin current induced by an external electric field. The general form of the BTE for each spin-split band is given by

\[
\partial_t f_{kb}(\mathbf{r}, t) + \mathbf{k} \cdot \partial_k f_{kb}(\mathbf{r}, t) + \dot{\mathbf{r}} \cdot \partial_r f_{kb}(\mathbf{r}, t) = I_{\text{coll}} \{ f_{kb}(\mathbf{r}, t) \}
\]

where \( f_{kb}(\mathbf{r}, t) \) is the non-equilibrium distribution function of the \( b \)th band and \( I_{\text{coll}} \{ f_{kb}(\mathbf{r}, t) \} \) is the collision integral. Without loss of generality, we choose the electric field \( \mathbf{E} \) to be along the \( x \) axis such that \( \dot{k} = eE \hat{x} \). At steady state, the explicit time derivative (the first term) is zero. Because we consider a spatially homogenous system, we drop the second term. Under the above two conditions, the BTE reduces to

\[
eE \frac{\partial f_{kb}}{\partial k_x} = I_{\text{coll}} \{ f_{kb} \}.
\]
Because equation (9) is a recursive equation, an exact general solution can only be obtained numerically. However, an analytical solution can be obtained when the change in the distribution function is approximated to the linear order in the external field. To solve equation (9) analytically to the first order in the external field, we first linearize the distribution function as,

\[ f_{kb} = f_{k0}^{(0)} + f_{k0}^{(1)}, \]

where \( f_{k0}^{(0)} \) is the equilibrium distribution, which depends only on the electron energy \( \epsilon_{kb} \), and the second term is the correction due to the external field. We solve for \( f_{k0} \) via two approaches: (a) the RTA and (b) a more refined method that considers the effect of impurity scattering and the highly anisotropic nature of the dispersion using the method proposed by SL [61]. Both approaches will be discussed in detail in the next section. After solving the BTE, we study the effect of the SOC on the transport coefficients comprising the spin accumulation, spin current, and charge-spin conversion efficiency.

The spin accumulation \( \delta S \) is the net spin originating from the two spin-split bands,

\[
\delta S = \sum_k \int \frac{dk}{(2\pi)^2} f_{kb}^{(1)} \langle \psi_{kb} | \sigma | \psi_{kb} \rangle
\]

\[
= \int \frac{dk}{(2\pi)^2} \left( f_{k-}^{(1)} - f_{k+}^{(1)} \right) \left( \hat{x} \cos \gamma_k + \hat{y} \sin \gamma_k \right),
\]

(12)

The spin current density is the tensor product of both the velocity and spin of the electrons. A commonly adopted definition of the spin current is [65]

\[
j_{e,\sigma} = \Re \left( \psi^+ \frac{1}{2} \{ \mathbf{v}, \sigma \} \psi \right).
\]

(13)

This gives, for \( \psi = \psi_{kb} \) and the spin current polarized in the spin \( s = (x, y) \) direction flowing along the \( l \) direction,

\[
j_{lb} = b(\partial_{h0} h_0) \frac{h_z}{\sqrt{h_x^2 + h_y^2}} + \partial_{h_z} h_z,
\]

(14)

which then gives

\[
\delta j_{xx} = \sum_l \int \frac{dk}{(2\pi)^2} \left[ \left( f_{k-}^{(1)} - f_{k+}^{(1)} \right) \left( \partial_{h0} h_0 \right) \cos \gamma_k + \left( f_{k+}^{(1)} + f_{k-}^{(1)} \right) \partial_{h_z} h_z \right],
\]

(15)

\[
\delta j_{xy} = \sum_l \int \frac{dk}{(2\pi)^2} \left[ \left( f_{k-}^{(1)} - f_{k+}^{(1)} \right) \left( \partial_{h0} h_0 \right) \sin \gamma_k + \left( f_{k+}^{(1)} + f_{k-}^{(1)} \right) \partial_{h_z} h_z \right].
\]

(16)

(Here, the lower-case \( j_{lb} \) denotes the expectation value of the spin current for a single state, and the uppercase \( \delta f_{lb} \) the non-equilibrium spin current obtained after integrating over all states.)

2.1. RTA

It is assumed in the RTA that \( I_{coll} \{ f_{kb} \} = f_{k0}^{(1)} / \tau \) where \( \tau \) is the relaxation time. Thus, from equation (9) we obtain,

\[
f_{k0}^{(1)} = -e \tau \left( E \cdot v_{kb} \right) \left( -\partial f_{k0}^{(0)} / \partial \epsilon_{kb} \right).
\]

(17)

At equilibrium, we assume that \( f_{k0}^{(0)} \) is given by the zero-temperature Fermi distribution function. As a consequence, \( -\partial f_{k0}^{(0)} / \partial \epsilon_{kb} = \delta (\epsilon_{kb} - \epsilon_F) \) where \( \epsilon_F \) is the Fermi energy. Thus, only the electrons at the Fermi level contribute to transport. Accordingly, the change in the distribution function is obtained as,

\[
f_{k0}^{(1)} = -eE \tau v_{kb}^0 \delta (\epsilon_{kb} - \epsilon_F).
\]

(18)

With the above distribution function and equation (12), the induced spin density due to the external electric field is given by the following integrals,
\[ \delta S_x = -eE_T \int dk \left( \frac{h_x^2 (\partial_{k_x} h_x) + h_x h_y (\partial_{k_y} h_y)}{h_x^2 + h_y^2} \left( \delta(\epsilon_{k+} - \epsilon_F) + \delta(\epsilon_{k-} - \epsilon_F) \right) + \left( \frac{\partial_{k_x} h_0}{\sqrt{h_x^2 + h_y^2}} \delta(\epsilon_{k+} - \epsilon_F) - \delta(\epsilon_{k-} - \epsilon_F) \right) \right) \right) \] (19)

\[ \delta S_y = -eE_T \int dk \left( \frac{h_x h_y (\partial_{k_x} h_x) + h_y^2 (\partial_{k_y} h_y)}{h_x^2 + h_y^2} \delta(\epsilon_{k+} - \epsilon_F) + \delta(\epsilon_{k-} - \epsilon_F) \right) + \left( \frac{\partial_{k_y} h_0}{\sqrt{h_x^2 + h_y^2}} \delta(\epsilon_{k+} - \epsilon_F) - \delta(\epsilon_{k-} - \epsilon_F) \right) \right) \right) \] (20)

Note that in the above equations, because \( \delta(f(x)) = \sum_{x_0} \delta(x - x_0)/|f'(x_0)| \) for an arbitrary function \( f(x) \) where \( x_0 \) are the roots of \( f(x) \),

\[ \int dk \delta(\epsilon_{k\theta} - \epsilon_F) = \int_{FCs} \frac{d\phi}{\nu_{k\theta}} \] (21)

where \( \int_{FCs} d\phi \) indicates a line integral over the Fermi contour (FC) of band \( b \), and the \( \nu_{k\theta} \) term reflects the density of states in the vicinity of the FC. We parameterize the points lying on the FC of band \( b \) as \( k_{\theta\theta} = k_{\theta\theta}(\cos(\phi)\hat{x} + \sin(\phi)\hat{y}) \) where \( k_{\theta\theta} \) at a particular \( \phi \) is determined from the dispersion relation as the positive real solution to

\[ \epsilon_F = \frac{1}{2m^*} k_{\theta\theta}^2 + b \sqrt{h_x(k_{\theta\theta})^2 + h_y(k_{\theta\theta})^2}. \] (22)

Equation (21) gives

\[ \int dk \delta(\epsilon_{k\theta} - \epsilon_F) = \int d\phi \frac{k_{\theta\theta}}{\nu_{k\theta}}. \] (23)

Similarly, from equations (15) and (16), the spin currents are given by the following integrals,

\[ \delta J_{xx} = -2eE_T \int dk \left[ \left( \frac{\partial_{k_x} h_0}{\sqrt{h_x^2 + h_y^2}} \right) \delta(\epsilon_{k+} - \epsilon_F) + \delta(\epsilon_{k-} - \epsilon_F) \right] \right) \] (24)

\[ \delta J_{xy} = -2eE_T \int dk \left[ \left( \frac{\partial_{k_y} h_0}{\sqrt{h_x^2 + h_y^2}} \right) \delta(\epsilon_{k+} - \epsilon_F) + \delta(\epsilon_{k-} - \epsilon_F) \right] \right) \] (25)

The integrations over the FCs are evaluated numerically to obtain the values of the spin densities and currents at various SOC strengths.

### 2.2. SL approach

In the more refined approach of SL [61], we assume that scattering occurs due to impurities modeled as delta potential functions. In this method, which we shall henceforth refer to as the SL approach for short, the collision integral is given by [66],

\[ I_{coll} \{ f_{k\theta} \} = \int dk' \sum_{k',\theta'} w_{k\theta,k'\theta'} (f_{k\theta} - f'_{k'\theta'}) \] (26)

where \( w_{k\theta,k'\theta'} \) is the transition probability for electrons in the \( (k,\theta) \) state to be scattered into the \( (k',\theta') \) state and only the linear terms in the applied electric field are retained in \( f_{k\theta} \) as per equation (10). We consider electrons to be scattered by impurities. Each impurity is modeled as a delta function potential given by

\[ V_j(r) = a_0 V_0 \delta(r - R_j) \] (27)
where $\mathbf{R}$ is the position of the impurity and $V_0$ is its strength. In the above, $a_0$ is a quantity with the physical dimensions of area introduced for dimensional consistency noting that $\delta(r)$ has the dimensions of inverse area because $\int dr \delta(r) = 1$. The meaning of $a_0$ can be understood intuitively as follows: for an arbitrary potential $V'(r)$ with a finite spatial extent rather than a delta potential spatial variation, $\int dr V'(r)$ quantifies the influence of the potential over the entire system, $a_0 V_0$ in equation (27), which is also the result of integrating equation (27) over the entire system, is the corresponding influence of each of the delta-potential scatterers.

Denoting the number density (i.e. the number per unit area) of such impurities as $n$ and applying the Fermi golden rule, the transition rate due to all the impurities is given by

$$w_{k'b'} = \frac{2\pi}{\hbar} \left| \langle k'b' | \hat{a}_0 V_0 | k'b' \rangle \right|^2 \delta (\epsilon_{k'b'} - \epsilon_{k'b'})$$

(28)

and the transverse relaxation time $\tau_{k'b'}$ has the dimensions of $1/[\text{time}]$, and that of equation (here, we have temporarily restored the $\hbar$ as a check of the dimensional consistency of $w$, and introduced $w_0 \equiv 2\pi n (a_0 V_0)^2/\hbar$, which has the physical dimensions of $[\text{area}]/[\text{energy}]/[\text{time}]$ so that $w$ has the dimensions of $[\text{area}]/[\text{time}]$. This is required by the dimensional consistency of equation (9), where $I_{\text{coll}}$ is required to have the dimensions of $1/[\text{time}]$, and that of equation (26), where $w$ is consequently required to have the dimensions of $[\text{area}]/[\text{time}]$. Note that the factor of area appears to be misplaced in equation (38) in the original SL paper, which corresponds to equation (28) here.)

The solution for $f^{(1)}_{kb}$ in the BTE with the collision integral given in equation (26) can be decomposed into longitudinal and transverse components as follows,

$$f^{(1)}_{kb} = f^{(1)\|}_{kb} + f^{(1)\perp}_{kb}.$$  \hspace{1cm} (30)

Both terms are independent of each other and defined as follows,

$$f^{(1)\|}_{kb} = -e (E \cdot \mathbf{v}_{kb}) \left( -\frac{\partial f^{(0)}_{kb}}{\partial \epsilon_{kb}} \right) \left[ \frac{1}{\tau_{\parallel}} \right]_{\parallel}$$

(31)

$$f^{(1)\perp}_{kb} = -e (\hat{z} \times E \cdot \mathbf{v}_{kb}) \left( -\frac{\partial f^{(0)}_{kb}}{\partial \epsilon_{kb}} \right) \left[ \frac{1}{\tau_{\perp}} \right]_{\perp}$$

(32)

where the (inverses of the) longitudinal relaxation time $\tau_{\parallel}^{-1}$ and the transverse relaxation time $\tau_{\perp}^{-1}$ are,

$$\frac{1}{\tau_{\parallel}} = \sum_{b'} \int \frac{d\mathbf{k}'}{(2\pi)^2} w_{k'b'} \left( 1 - \frac{|\mathbf{v}_{k'b'}|}{|\mathbf{v}_{kb}|} \cos (\vartheta_{kb} - \vartheta_{k'b'}) \right)$$

(33)

$$\frac{1}{\tau_{\perp}} = \sum_{b'} \int \frac{d\mathbf{k}'}{(2\pi)^2} w_{k'b'} \frac{|\mathbf{v}_{k'b'}|}{|\mathbf{v}_{kb}|} \sin (\vartheta_{kb} - \vartheta_{k'b'}).$$

(34)

In the above equations, $\vartheta_{kb}$ is the angle between $\mathbf{v}_{kb} = \partial \epsilon_{kb}/\partial \mathbf{k}$ and $E$, i.e. $\vartheta_{kb} = \angle (\mathbf{v}_{kb}, E)$.

Equations (12), (15) and (16) are then evaluated numerically using $f^{(1)}_{kb}$ in equation (30) to obtain the spin accumulations and currents.

3. Results and discussion

For all our numerical results that follow, we set $\epsilon_F = 30$ meV from the band bottom of the le$^-$ band and $J_\text{SO} M = 10$ meV. These values are chosen so that the Fermi energy falls around the middle of the energy range in which only the le$^-$ spin-split bands exist as propagating states in the bandstructure of figure 1 and the Fermi energy is sufficiently far from the band bottoms of the he and le$^+$ bands for the effects of these bands to be negligible. We treat the SOC strengths $\alpha, \beta_3$, and $\eta_3$ in equation (1) as free parameters and vary $\alpha$ from
−10 to 10 meV Å⁻¹, and β₃ and η₃ from −3 to 3 eV Å⁻³. These ranges for the SOC strengths are based on the values in [52] corresponding to typical thicknesses and out-of-plane electric fields in LaO/STO quantum well structures.

Furthermore, we note that because of the δ(ε₊k_{b}−ε₋k_{b}) term in equation (18) for the RTA approach and in equation (29) for the SL approach and equation (23), the non-equilibrium quantity O ∈ {S, J_b} can be written in the generic form of

\[ O = E \sum_{b=±} \int d\phi \Phi_{b}(\phi)u_{b}(\phi) \]  

where \( u_{b}(\phi) \) is s or \( J_{b} \), for a state at the polar angle \( \phi \) on the FC of band \( b \), and \( \Phi_{b}(\phi) \) is a pre-factor that is proportional to the constant \( \tau \) in the RTA approach and to the constant \( 1/(n(a_{0}V_{0})^{2}) \) in the SL approach. This implies that the RTA and SL results for a given observable and a given set of parameters become identical to each other when \( \tau \) in the former and \( 1/(n(a_{0}V_{0})^{2}) \) in the latter correspond to each other in the following sense: Since the magnitude of \( (\tau_{b}^{[\parallel]})^{-1} \) in the SL approach is much larger than that of \( (\tau_{b}^{[\perp]})^{-1} \) in practice (see supplementary materials figure S1), the latter can be approximated to 0. In this case, the contribution of \( k_{b}^{[\perp]} \) in equation (32) becomes 0 while \( f_{b}^{[\perp]} \) becomes proportional to \( \tau_{b}^{[\parallel]} \), and has a similar form to the RTA \( (\tau_{b}^{[\parallel]})^{-1} \) in equation (17) except for the \( k \) and \( b \) dependence of \( \tau_{b}^{[\parallel]} \) in the former. The value of \( \tau \) in the RTA approach can thus be considered as the average of \( \tau_{b}^{[\parallel]} \) on the FCs of the two spin-split bands for a given value of \( 1/(n(a_{0}V_{0})^{2}) \) in the SL approach.

In the numerical results that follow, we assume this correspondence between the RTA \( \tau \) and SL \( 1/(n(a_{0}V_{0})^{2}) \) approaches for ease of comparison between their results.

3.1. SOC terms

We first introduce the effects of the various SOC terms in the Hamiltonian equation (1). Figures 2(a)–(c) show the respective FCs of the two spin-split bands and the spin expectation value vectors of each state on the FCs when \( J_{11}M = 0 \) and the hypothetical scenarios where only one of \( \alpha \), \( \beta_{3} \), or \( \eta_{3} \) has a finite value.

The system reduces to the familiar linear Rashba 2DEG when only \( \alpha \) has a finite value while \( J_{11}M = 0 \), \( \beta_{3} = 0 \), and \( \eta_{3} = 0 \). The contribution \( H_{\alpha} \) of the linear RSOC term to the Hamiltonian equation (1) in polar coordinates is,

\[ H_{\alpha} = \alpha(\sin\phi)\sigma_{s} - \cos\phi\sigma_{r}. \]  

As shown in figure 2(a), the corresponding FCs take the form of two concentric circles with constant radii. The spin expectation value vectors are tangential to the FCs and rotate in a clockwise (anti-clockwise) manner around the FC for the + (−) band. The variation of \( k_{b} \) and the spin expectation value vectors is more complicated for the cubic SOC terms.

We next consider the terms proportional to \( \beta_{3} \) in equation (1). Figure 2(b) shows that when only \( \beta_{3} \) is non-zero, the spin expectation vectors are approximately tangential to the FC reminiscent of the linear Rashba 2DEG. In contrast to the Rashba 2DEG, the radii of the FCs now vary with \( \phi \). In particular, the FCs for the two bands cross each other at \( \phi = ±\pi/4, \pi ± \pi/4 \), as can be determined from the polar coordinates form of the contribution \( H_{\beta_{3}} \) of the terms proportional to the Hamiltonian \( \beta_{3} \) to equation (1),

\[ H_{\beta_{3}} = \beta_{3}k_{b}(\cos 2\phi)(\sin\phi)\sigma_{s} - \cos\phi\sigma_{r}. \]  

The rotation of the spin expectation vectors in each of the bands switches directions at the values of \( \phi \) where the two bands touch each other.

Figure 2(c) shows the FCs and the spin expectation vectors when only \( \eta_{3} \) has a non-zero value. The contribution \( H_{\eta_{3}} \) of the \( \eta_{3} \) terms to the Hamiltonian equation (1) in polar coordinates form is

\[ H_{\eta_{3}} = \eta_{3}k_{b}(\cos 2\phi)(\sin\phi)\sigma_{s} + \cos\phi\sigma_{r}. \]  

Equation (38) implies that the two bands cross each other at \( \phi = 0, ±\pi/2, \pi \). In contrast to the linear RSOC and \( \beta_{3} \) SOC for which the spin expectation value vectors are tangential to the FCs, the spin expectation vectors are normal to the FCs at these band touching points. Similar to the \( \beta_{3} \) SOC term, the rotation of the spin expectation value vectors in each band reverses direction at the values of \( \phi \) where the two bands touch each other.
Equations (36)–(38) imply that the spin $x$ expectation values are antisymmetric about the $k_x$ axis and symmetric about the $k_y$ axis, whereas the spin $y$ expectation values are symmetric about the $k_x$ axis and antisymmetric about the $k_y$ axis when the linear Rashba and the cubic $\eta_3$ and $\beta_3$ SOCs are individually present. The same symmetries are therefore present when more than one of these SOC terms are simultaneously present in the system, as shown in figure 2(d) for both in-plane spin directions and in figure 2(e) for spin $y$ specifically when all three SOC terms have finite values. In the absence of any external magnetization, these symmetries collectively imply a C2 rotational symmetry of the system. Figure 2(d) shows that the FCs retain the reflection symmetries about the $k_x$ and $k_y$ axes although the band-touching points are now shifted away from $\phi = 0, \pi, \pm \pi/4, \pi$, or $\pm \pi/4$. (The shift can also be seen from the values of $\phi$ at which $s_y = 0$ in figure 2(e) besides $\phi = \pm \pi/2$, at which the spin expectation vector lies along the $\pm x$ directions.) The preserved symmetry of $s_y$ about $\phi = 0$ is clearly evident in figure 2(e).

Figure 2(f) shows the factor $\Phi_b(\phi)$ in equation (35) obtained using the SL and RTA approaches for the parameter set of figure 2(e). When the RTA $\tau$ and SL $1/(n(q_0V_0)^2)$ are in correspondence such that the maximum values of $\Phi_b(\phi)$ for both approaches match, the $\Phi_b(\phi)$ profiles for the two approaches are similar but not exactly identical to each other. (The differences between the RTA and SL profiles cannot be clearly seen at the scale of figure 2(e). However, these differences do lead to visible differences in the variational trends of the spin currents.) Because of the symmetries of the SOC terms and the resultant symmetries of the FCs, $\Phi_b(\phi)$ is symmetrical about $\phi = 0$.

The aforementioned symmetries are broken by the application of the external magnetization, which breaks the time-reversal symmetry of the system. Figure 2(g) shows the FCs and spin expectation value vectors for the same parameter set as figure 2(d) except that a finite magnetization is now applied along the $x$
direction. Unlike the previous cases considered in which the FCs for the two bands are both centered on the $k$-space origin, the centers of the FCs are now slightly displaced with respect to each other. This is because the change in the spin degree of freedom due to the magnetization affects the momentum degree of freedom due to the SOC terms that couple the spin and momentum degrees of freedom together. The broken symmetries are also evident from the spin $y$ expectation values plotted in figure 2(h) where the $s_y$ peaks at negative values of $\phi$ are slightly smaller than those at positive values of $s_y$. $\Phi_b(\phi)$ plotted in figure 2 also has an evident asymmetry about $\phi = 0$. However, the $\Phi_b(\phi)$ profiles obtained using the SL and RTA approaches are still very similar to each other. Recall from equation (35) that the non-equilibrium spin $y$ accumulation $\delta S_y$ is proportional to the product of $s_y(\phi)$ (figure 2(h)) and $\Phi_b(\phi)$ summed over $b = \pm$ and integrated over $\phi$ from $-\pi$ to $\pi$. The asymmetry of $s_y(\phi)$ and $\Phi_b(\phi)$ with respect to both $\phi$ and $b$ reduces the cancellation between the contributions of the two bands to $\delta S_y$ and increases the amplitude of $\delta S_y$ compared to the case where the magnetization is absent. (The contributions of the two bands in figures 2(d)–2(f) without the magnetization cancel each other out to a relatively larger degree but not completely because of the differences in the $\Phi_b(\phi)$ profile between $b = \pm$ in figure 2(f).) It should also be noted that when only the $x$ magnetization is present but not SOC, $\delta S_y$ is zero because $s_y(\phi) = 0$ for all values of $\phi$ and $b$. The SOC results in finite values of $s_y$, which becomes asymmetrical (figure 2(h)) when the $x$ magnetization is applied. This point will be relevant to the subsequent results where we study the effects of varying the three SOC strengths $\alpha$, $\beta_3$, and $\eta_3$ on the spin accumulation and spin current.

3.2. Spin accumulation

Noting that only the spin accumulation polarized perpendicular to the applied magnetization will exert a spin torque on the magnetization, we first study the spin accumulation component in the perpendicular direction. Writing the magnetization vector $\mathbf{M} = M\hat{m}$ with $\hat{m} = \cos(\phi_m)\hat{x} + \sin(\phi_m)\hat{y}$, where $\phi_m$ is the magnetization angle, we denote the spin polarization in the $\hat{z} \times \hat{m}$ direction as $\delta S_\perp$ and study its variation with the SOC strengths.

Figure 3 shows $\delta S_\perp$ plotted as functions of the magnetization angle $\phi_M$ and $\eta_3$ at a fixed value of $\alpha$ while varying the value of $\beta_3$. The variation of $\delta S_\perp$ with $\phi_m$ at the specific values of $\eta_3$ corresponding to the horizontal dotted lines in figure 3(a) are plotted out in the graphs on the lower row of the figure to provide a better visualization of the variation of $\delta S_\perp$. For a given set of $\alpha$, $\beta_3$, and $\eta_3$, $\delta S_\perp$ varies approximately as $\sin(\phi_m)$. (This variation is not exact—for example, a significant deviation from a $\sin(\phi_m)$ behavior can be seen in the plot corresponding to $\eta_3 = -2 \text{ eV Å}^{-3}$ line in figure 3(d) in which there are three local maxima as $\phi_m$ varies from $-\pi$ to $\pi$.) $\delta S_\perp$ has the largest magnitude at a given set of $\alpha$, $\beta_3$, and $\eta_3$ when the magnetization is parallel or anti-parallel to the applied electric field in the $x$ direction ($\phi_m = 0, \pi$) and is
identically 0 when the magnetization is perpendicular to the applied electric field ($\phi_m = \pm \pi/2$). As $\eta_3$ increases from a large negative value to a less negative value, the amplitude of $\delta S_\perp$ decreases until it reaches almost zero across a transitional range of $\eta_3$ (e.g. the $\eta_3 = -2$ eV Å$^{-3}$ line in figure 3(d)). A further increase in $\eta_3$ causes the sign of $\delta S_\perp$ to flip (e.g. the $\eta_3 = 0$ eV Å$^{-3}$ line vs the $\eta_3 = 1$ eV Å$^{-3}$ line in figure 3(e)). The amplitude of $\delta S_\perp$ then continues to increase in the reverse direction as $\eta_3$ increases further. The value of $\eta_3$ at which $\delta S_\perp$ flips sign is dependent on the value of $\beta_3$—it occurs at approximately $\beta_3 = -2.5$ eV Å$^{-3}$ in figure 3(a) where $\beta_3 = -1.5$ eV Å$^{-3}$, $\eta_3 = -0.7$ in figure 3(b) where $\beta_3 = -0.75$ eV Å$^{-3}$, and $\eta_3 = 2.1$ in figure 3(c) where $\beta_3 = 0.75$ eV Å$^{-3}$). A comparison between figures 3(a)–(c) shows that varying the value of $\beta_3$ can also flip the sign of $\delta S_\perp$. Since the largest amplitude of $S_\perp$ is usually obtained when the magnetization is applied along the ±x directions, we now focus on $\delta S_y$ in the subsequent discussion.

Figures 4(a) and (b) show $\delta S_y$ obtained using the SL approach plotted as functions of $\eta_3$ and $\beta_3$ for two different values of $\alpha$. The values of $\delta S_y$ at the specific values of $\beta_3$ indicated by the horizontal dotted lines in figure 4(a) and at the specific values of $\eta_3$ indicated by the vertical dotted lines in figure 4(b) are respectively plotted as functions of $\beta_3$ and $\eta_3$ in figures 4(c) and (d) to provide a clearer visualization of the variational trends along with the corresponding values of $\delta S_y$ obtained using the RTA approach. Figures 4(a) and (b) show that $\delta S_y$ varies monotonically with $\eta_3$ and $\beta_3$. Figures 4(c) and (d) respectively show more clearly that $\delta S_y$ has an approximately linear dependence on $\eta_3$ and $\beta_3$. Figure S2 in the supplementary materials, which shows analogous plots of $\delta S_y$ as functions of $\alpha$ and $\eta_3$ at two values of $\beta_3$, indicates that $\delta S_y$ varies approximately linearly with $\alpha$ as well. This linear dependence of $\delta S_y$ on $\alpha$ is reflected in the fact that although the color plots in figures 4(a) and (b) are visually similar, their color scale bars span different ranges. The approximately linear dependence of $\delta S_y$ on $\alpha$, $\beta_3$, and $\eta_3$ implies that there exist critical values of one of these three parameters at which $\delta S_y$ changes sign when the other two SOC parameters are fixed, as we have seen earlier in the discussion on figure 3 in which $\eta_3$ was varied while $\beta_3$ and $\alpha$ were fixed.
These trends for the variation of one of the three SOC (say, $\eta_3$) while the other two SOC fields ($\alpha, \beta_3$) are fixed can be explained as follows: the spin-dependent parts of the Hamiltonian in equation (1), i.e. $(h_x + h_y)$ given explicitly in equations (2) and (3), consist of the SOC fields and the applied magnetization (which we have set along the x-direction). The energy separation between the FCs of the spin-split bands ($\pm 1$) increases with the magnitude of the spin-dependent terms of the Hamiltonian, i.e. $|h_x + h_y|$. The larger the energy split, the larger would be the k-space separation between the contours of the b=±1 Fermi surfaces on which we perform our integration to obtain the spin accumulation $\delta S_y$ (see figure 2(h)), which translates into a larger net spin accumulation $\delta S_y$. Additionally, $\delta S_y$ is dependent on $h_y$ which in turn arises solely from the SOC terms (noting that $M_y$ is zero), whereas $\delta S_y$ is dependent on $h_x$ which arises from both SOC and the magnetization coupling $M_x$. Hence, the increase in the SOC parameters would lead to a more pronounced increase in $\delta S_y$. The interplay of multiple types of SOC has the important consequence that when we vary one SOC parameter while keeping the other two fixed, the latter (fixed) SOC terms have their own contributions to $\delta S_y$ and thus provide a constant offset. This is evident from the vertical shifts along the $\delta S_y$ axis of the plots shown in figures 4(c) and (d).

For the sign conventions adopted for $H_{\alpha, \beta_3}$, $H_{\eta_3}$ and $H_{\eta_3}$ here, $\delta S_y$ becomes more positive when $\eta_3$ and $\alpha$ become more positive and $\eta_3$ becomes more negative. As expected from the similarity of the $\Phi_3(\phi)$ factors between the RTA and SL approaches shown in figure 2(i), the $\delta S_y$ obtained using the two approaches are also similar, as shown in the comparison between the two approaches in figures 4(c) and (d), and in figures S2c and S2d. This indicates that the RTA is an adequate approximation for investigating the spin accumulation. We now turn our attention to the spin current.

### 3.3. Spin current

In this section, we focus on the spin current flowing parallel to the electric field with a spin polarization parallel to the applied magnetization of $J_{xx} M = 10$ meV in the $x$ direction, i.e. $\delta J_{xx}$, because these currents can flip the sign of the magnetization direction. (Note that $J_{yy}$, i.e. the spin $y$ current flowing in the $x$ direction is zero when both the magnetization and electric field are along the $x$ direction.) We briefly discuss spin currents with other combinations of the current flow and spin polarization directions in the discussion on figure S3 in the supplementary materials.

Figure 5 shows $\delta J_{xx}$ as functions of $\beta_3$ and $\eta_3$ at two values of $\alpha$ obtained using the SL and RTA approaches. The SL and RTA values of $\delta J_{xx}$ at the specific values of $\beta_3$ that are denoted by the horizontal dotted lines in figure 5(a) and the specific values of $\eta_3$ indicated by the vertical dotted lines in figure 5(b) are plotted in figures 5(e) and (f), respectively, to depict their variational trends. Unlike $\delta S_y$, which varies in an approximately linear manner with $\alpha$, $\beta_3$, and $\eta_3$, $\delta J_{xx}$ has an approximately quadratic dependence on $\beta_3$ and $\eta_3$, as shown in figure 5, which results in a non-monotonic variation of $\delta J_{xx}$ with both parameters. The quadratic dependence is most easily seen in figure 5(f) where the $\delta J_{xx}$ minima occur within the range of $\beta_3$ considered. The quadratic dependence of $\delta J_{xx}$ on $\eta_3$ is also evident from the minima corresponding to $\beta_3 = 0$ eV Å$^{-3}$ and $\beta_3 = 0.2$ eV Å$^{-3}$ lines in figure 5(e)). This quadratic dependence may be due to the fact that $\delta J_{xx}$ involves both the velocity as well as the spin of the carriers, the respective operators of which are combined together in an anti-commutator (equation (13)). Extrema in $\delta J_{xx}$ would then occur because of the competition between the opposing trends experienced by the two quantities (velocity and spin accumulation). Comparing figures 5(a) and (b), it is evident that the maximum amplitude of the spin current occurs at relatively small values of $\beta_3$ and $\eta_3$. Furthermore, the signs of $\beta_3$ and $\eta_3$ at which this maximum amplitude occurs (positive $\beta_3$ and negative $\eta_3$ at $\alpha = -0.01$ eV Å$^{-1}$ in figure 5(a) and (vice-versa at $\alpha = 0.01$ eV Å$^{-1}$ in figure 5(b)) can change with the value of $\alpha$. A comparison between figures 5(a) and (c), and between figures 5(b) and (d) shows that unlike the spin $y$ accumulation $\delta S_y$, for which both the SL and RTA approaches give similar results, there are now evident differences between the $\delta J_{xx}$ profiles obtained using the SL approach in figures 5(a) and (b), as opposed to the RTA approach in figures 5(c) and (d). These differences are due to the displacement of the SL and RTA quadratic $\delta J_{xx}$ vs $\beta_3$ (figure 5(e)) and $\delta J_{xx}$ vs $\eta_3$ curves (figure 5(f)) curves with respect to each other, which can be in turn attributed to the anisotropy of the SLA relaxation time on the FCs (see discussion on figure S4 in supplementary materials). The discrepancy between the SL and RTA results for the spin current indicates that the RTA may not be an adequate approximation for the spin current although the main qualitative trends are still captured correctly. We therefore show only the SL values for the subsequent results.

The charge-spin conversion efficiency $[67–69]$, i.e. the ratio of the spin current to the charge current, is an important figure of merit in technological applications. Figure 6 shows $\delta J_{xx}$, the corresponding charge current $\delta J_c$, and the charge-spin efficiency $\xi \equiv \delta J_{xx}/\delta J_c$ plotted as functions of $\alpha$ and $\eta_3$ at two values of $\beta_3$. $\delta J_{xx}$ is plotted as a function of $\alpha$ and $\eta_3$ here to complement the plots of $\delta J_{xx}$ as a function of $\beta_3$ and $\eta_3$ to
Figure 5. (a)–(c) $\delta J_{xx}$ as a function of $\beta_3$ and $\eta_3$ obtained using the (a), (b) SL and (c), (d) RTA approaches at (a), (c) $\alpha = -10 \text{ meV } \AA^{-1}$ and (b), (d) $\alpha = 10 \text{ meV } \AA^{-1}$. (e) $\delta J_{xx}$ as a function of $\eta_3$ for the parameters in (a) and (c) at the values of $\beta_3$ denoted by the dotted lines in (a), and (f) $\delta J_{xx}$ as a function of $\beta_3$ for the parameters in (b) and (d) at the values of $\eta_3$ denoted by the dotted lines in (b).

provide snapshots of the variation of $\delta J_{xx}$ in figure 5, while $\delta J_{xx}$, $\delta J_C$, and $\xi$ are plotted for two values of $\beta_3$ here to show that the qualitative behavior of $\delta J_{xx}$ is similar across different values of $\beta_3$. (The corresponding plots of $\delta J_C$ and $\xi$ for the parameter sets of figure 5 are shown in figure S5 in the supplementary materials.) Similar to its dependence on $\beta_3$ and $\eta_3$, $\delta J_{xx}$ also has a quadratic dependence on $\alpha$, although the quadratic minima do not fall within the ranges of $\alpha$ considered here. Comparing the $\delta J_{xx}$ values in figures 6(a) and (b) against the $J_C$ values in figures 6(c) and (d), it is evident that the regions on the $\alpha - \eta_3$ plane at which $\delta J_{xx}$ has large magnitudes at large positive (negative) values of $\eta_3$ and $\alpha$ at $\beta_3 = -3 \text{ eV } \AA^{-3}$ ($\beta_3 = 3 \text{ eV } \AA^{-3}$) approximately coincide with the regions of small magnitudes of $\delta J_C$ on the $\alpha - \eta_3$ plane. Figure S5 in the supplementary materials shows that this coincidence between the regions of large $\delta J_{xx}$ and small $\delta J_C$ holds on the $\beta_3 - \eta_3$ plane as well. The large values of $J_{xx}$ with small values of charge current translate into a large charge-spin conversion efficiency in excess of 30%.
Figure 6. (a), (b) $\delta J_{xx}$ as a function of $\alpha$ and $\beta_3$ obtained at (a) $\beta_3 = -3$ meV Å$^{-3}$ and (b) $\beta_3 = 3$ meV Å$^{-3}$. (c), (d) The corresponding charge current $\delta J_C$ flowing along the x direction, and (e), (f) charge-spin efficiency $\xi \equiv \delta J_{xx}/\delta J_C$.

4. Conclusion

In this work, we investigated how the interplay between the three types of RSOC in a LaO/STO system affects the spin accumulation and spin current when a magnetization and an electric field are applied. The spin accumulation and spin currents were calculated using both the RTA and the SL approach.

For the spin accumulation, we found that within the parameter ranges investigated, the spin accumulation in the direction perpendicular to the magnetization varies approximately linearly with each of the three RSOC strengths when the other two RSOC strengths are fixed. The latter two RSOC strengths provide an offset to the value of the varied RSOC at which the spin accumulation changes sign. The spin accumulation, in general, increases when the magnitudes of the three RSOC strengths are increased. The RTA results for the spin accumulation agree adequately with those of the more accurate SL approach.
For the spin current, we found that the spin current varies quadratically with each of the three RSOC strengths when the other two RSOC strengths are fixed. The minima of this quadratic dependence depends on the values of the other two RSOC strengths. In contrast to the spin accumulation, there is some variance between the spin current profiles from the RTA and SL approaches because the anisotropy of the FCs were not accounted for in the former. This suggests that the SL approach should be used to calculate the spin currents accurately. A large spin-charge conversion efficiency in excess of 30 % can be achieved in the system.

Our results provide a more complete understanding of how the spin response of the low-energy states in the LaO/STO system is affected by the co-existence of multiple types of RSOC.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

Acknowledgments

This work is supported by the Ministry of Education (MOE) Tier-II Grants MOE2018-T2-2-117 (NUS Grant Nos. A-0005163-00-00) and MOE-T2EP50121-0014 (NUS Grant Nos. A-800086-01-00), and MOE Tier-I FRC Grants (NUS Grant Nos. A-0005110-01-00 and A-8000195-01-00).

ORCID iDs

Aniran Kundu https://orcid.org/0000-0002-9780-9329
Zhuo Bin Siu https://orcid.org/0000-0002-7056-937X
Mansoor B A Jalil https://orcid.org/0000-0002-9513-8680

References

[1] Kiselev S I, Sankey J C, Krivorotov I N, Emley N C, Schoelkopf R J, Buhrman R A and Ralph D C 2003 Microwave oscillations of a nanomagnet driven by a spin-polarized current Nature 425 380–3
[2] Ralph D C and Stiles M D 2008 Spin transfer torques J. Magn. Magn. Mater. 320 1190–216
[3] Mangin S, Ravelosona D, Katine J A, Carey M J, Terris B D and Fullerton E E 2006 Current-induced magnetization reversal in nanopillars with perpendicular anisotropy Nat. Mater. 5 210–5
[4] Jonietz F et al 2010 Spin transfer torques in MnSi at ultralow current densities Science 330 1648–51
[5] Fujita T, Jalil M B A, Tan S G and Murakami S 2011 Gauge fields in spintronics J. Appl. Phys. 110 121301
[6] Tan S G, Jalil M B A, Liu X-J and Fujita T 2011 Spin dynamics under local gauge fields in chiral spin-orbit coupling systems Ann. Phys. 326 207
[7] Tan S G and Jalal M B A 2012 Introduction to the Physics of Nanoelectronics (Amsterdam: Elsevier)
[8] Rojas Sánchez J C, Vila L, Desfonds G, Gambarelli S, Attané J P, De Teresa J M, Magén C and Fert A 2013 Spin-to-charge conversion using Rashba coupling at the interface between non-magnetic materials Nat. Commun. 4 2944
[9] Rojas-Sánchez J-C et al 2016 Spin to charge conversion at room temperature by spin pumping into a new type of topological insulator: α-Sn films Phys. Rev. Lett. 116 096602
[10] Lesne E et al 2016 Highly efficient and tunable spin-to-charge conversion through Rashba coupling at oxide interfaces Nat. Mater. 15 1261–6
[11] Shields B J, Unterreithmeier Q P, de Leon N P, Park H and Lukin M D 2015 Efficient readout of a single spin state in diamond via spin-to-charge conversion Phys. Rev. Lett. 114 136402
[12] Wang H, Kally J, Sue Lee J, Liu T, Chang H, Reifsnyder Hickey D, Andre Mkhoyan K, Wu M, Richardella A and Samarth N 2016 Surface-state-dominated spin-charge current conversion in topological-insulator–ferromagnetic-insulator heterostructures Phys. Rev. Lett. 117 076601
[13] Shi S et al 2018 Efficient charge-spin conversion and magnetization switching through the Rashba effect at topological-insulator/ag interfaces Phys. Rev. B 97 041115
[14] Ghiasi T S, Kaverzin A A, Blah P J and van Wees B J 2019 Charge-to-spin conversion by the Rashba–Edelstein effect in two-dimensional van der Waals heterostructures up to room temperature Nano Lett. 19 5959–66
[15] Shen K, Vignale G and Raimondi R 2014 Microscopic theory of the inverse Edelstein effect Phys. Rev. Lett. 112 096601
[16] Yoda T, Yokoyama T and Murakami S 2018 Orbital Edelstein effect as a condensed-matter analog of solenoids Nano Lett. 18 916–20
[17] Rodriguez-Vega M, Schwiete G, Sinova J and Rossi E 2017 Giant Edelstein effect in topological-insulator–graphene heterostructures Phys. Rev. B 96 235419
[18] Johansson A, Henk Jurgen and Mertig I 2016 Theoretical aspects of the Edelstein effect for anisotropic two-dimensional electron gas and topological insulators Phys. Rev. B 93 195440
[19] Du Y, Gamou H, Takahashi S, Karube S, Kohda M and Nitta J 2020 Disentanglement of spin-orbit torques in Pt/Co bilayers with the presence of spin Hall effect and Rashba-Edelstein effect Phys. Rev. Applied 13 054014
[20] Salemi L, Berritta M, Nandy A K and Oppeneer P M 2019 Orbi tally dominated Rashba-Edelstein effect in noncentrosymmetric antiferromagnets Nat. Commun. 10 5381
[21] Bazaliy Y B 2014 Comment on “metastable state in a shape-anisotropic single-domain nanomagnet subjected to spin-transfer-torque” [appl. phys. lett. 101, 162405 (2012)] Appl. Phys. Lett. 105 116101
[22] Jin Mi-J et al 2017 Nonlocal spin diffusion driven by giant spin Hall effect at oxide heterointerfaces Nano Lett. 17 36–43
[23] Song P et al 2020 Coexistence of large conventional and planar spin Hall effect with long spin diffusion length in a low-symmetry semimetal at room temperature Nat. Mater. 19 292–8
[65] Shi J, Zhang P, Xiao Di and Niu Q 2006 Proper definition of spin current in spin-orbit coupled systems Phys. Rev. Lett. 96 076604
[66] Ashcroft N W and David Mermin N 1976 Solid State Physics (New York: Cengage Learning)
[67] Zhang W, Han W, Jiang X, Yang S-H and Parkin S S P 2015 Role of transparency of platinum-ferromagnet interfaces in determining the intrinsic magnitude of the spin Hall effect Nature Phys. 11 496
[68] Lesne E et al 2016 Highly efficient and tunable spin-to-charge conversion through Rashba coupling at oxide interfaces Nat. Mater. 15 1261–6
[69] Han J, Richardella A, Siddiqui S A, Finley J, Samarth N and Liu L 2017 Room-temperature spin-orbit torque switching induced by a topological insulator Phys. Rev. Lett. 119 077702