Mathematical Model of the One-stage Magneto-optical Sensor Based on Faraday Effect

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Abstract. The aim of this work is to refine a model of magneto-optical sensors based on Faraday’s longitudinal magneto-optical effect. The tasks of the study include computer modeling and analysis of the transfer characteristic of a single-stage magneto-optical sensor for various polarization of the input beam and non-ideal optical components. The proposed mathematical model and software make it possible to take into account the non-ideal characteristics of film polaroids observed in operation in the near infrared region and at increased temperatures. On the basis of the results of the model analysis it was found that the dependence of normalized transmission \( T(\gamma_2) \) has periodic nature. Choosing the angle \( (\gamma_3 - \gamma_1) \) makes it possible to shift the initial operation point and change the sensitivity \( dT/d\gamma_2 \). The influence of the input beam polarization increases with the increase of polaroid parameter deviation from ideal and shows itself as reduction of modulation depth and angular shift of the sensor conversion response.

1. Introduction
Magneto-optical sensors are often used for measuring electric and magnetic fields. The action of the sensors is based on Faraday longitudinal magneto-optical effect [1-5]. The magnetic field effect in sensors of this kind causes magnetic rotary polarization of transmitted light. This rotation is transformed into intensity modulation at the sensor output with the help of an analyzer.

Fiber-optic sensors of magnetic and electric fields are developed mostly for remote control in direct-current and alternating-current networks [3]. The need for these sensors is caused by the necessity of electric separation of high and low voltage of measuring tools. Stringent requirements to the dynamic range width, temperature interval (-20 …+80 °C), and low measurement error (no more than 1%) are imposed on such sensors.

Three types of fiber-optic sensors are distinguished [4]: 1) interferometric single-mode fiber sensors with a magnetostrictive cladding; 2) sensors based on Faraday longitudinal magneto-optical effect in single-mode fiber; 3) sensors based on Faraday longitudinal magneto-optical effect in a bulk element – a magneto-optical rotator coupled with a multi-mode fiber waveguide. Sensors of the third type are discussed in this paper.

The Jones matrix method [6-7] applicable for polarized radiation analysis is one of the most efficient methods of calculating polarization optical circuits. The authors of [2] showed the use of Jones matrix method to describe various states of beam polarization and distribution of its resultant
intensity in passing the optical system “polarizer – magneto-optical rotator – analyzer”. However, the proposed model does not take into account the non-ideal analyzers, nor is it investigated.

The aim of this work is to refine a model of magneto-optical sensors based on Faraday’s longitudinal magneto-optical effect. The tasks of the study include computer modeling and analysis of the transfer characteristic of a single-stage magneto-optical sensor for various polarization of the input beam and non-ideal optical components.

2. Mathematical modeling

The scheme of a magneto-optical sensor with a single-pass sensing element in the form of a magneto-optical film is shown in Figure 1. Passing through the polarizer P the light from the light source becomes linearly polarized with an azimuth angle $\gamma_1$. An yttrium-iron garnet (YIG) magneto-optical film turns the plane of polarization about an angle $\gamma_2$. An analyzer with an azimuth angle $\gamma_3$ is installed at the output of the circuit. The analyzer transforms the rotation of the polarization plane into intensity modulation.

The difference between the azimuth angles of the polarizer and analyzer $(\gamma_3 - \gamma_1)$ determines the initial operation point of the transfer characteristic of the circuit. Thus, it is possible to optimize the sensor’s sensitivity to variations of low or medium values of the magnetic field.

![Figure 1. Figure with short caption (caption centred).](image)

The Jones matrix method applicable for completely polarized input light is used to calculate such a system. Unlike the calculation technique based on Malus law the Jones matrix method takes into account the state of polarization of the input beam, elliptical in the general case, and makes it possible to calculate the transmission of an optical scheme in the case of using non-ideal polarizers. The Maxwell column is operated on within the framework of the Jones method. The elements of the column determine the amplitudes and phases of transverse components of the electric vector of the electromagnetic field:

$$
\bar{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} A_x \exp(i\varphi_x) \\ A_y \exp(i\varphi_y) \end{bmatrix},
$$

where $A_x$, $A_y$ are amplitudes of $x$ - and $y$ - components of the oscillating electric field, respectively, [V/m]; $\varphi_x$, $\varphi_y$ are phases of $x$ - and $y$ - components of the electric field [rad].

To calculate the beam intensity the Maxwell vector is multiplied from left by its complex conjugated transpose [6]. In so doing we get:

$$
\bar{E}^* \bar{E} = \begin{bmatrix} E_x^* \\ E_y^* \end{bmatrix} \begin{bmatrix} E_x & E_y \end{bmatrix} = A_x^2 + A_y^2 \exp(2i\varphi_x - 2i\varphi_y) + \text{Re}(A_x A_y \exp(2i\varphi_x - 2i\varphi_y)),
$$

where $\text{Re}$ is the real part of a complex number.
\[ I = \overline{E^* E} = \begin{bmatrix} A_x e^{i\phi_x} & A_y e^{i\phi_y} \end{bmatrix} \begin{bmatrix} A_x e^{i\phi_x} \\ A_y e^{i\phi_y} \end{bmatrix} = A_x^2 + A_y^2. \]  

(2)

If the response of the medium to the electromagnetic field effect is linear it can be expected that if the vector:

\[ \overline{E^m} = \begin{bmatrix} A_x^m \exp(i\phi_x^m) \\ A_y^m \exp(i\phi_y^m) \end{bmatrix} \]

is a Maxwell column of incident radiation the output beam Maxwell column can be written as:

\[ \overline{E^{ou}} = \begin{bmatrix} A_x^{ou} \exp(i\phi_x^{ou}) \\ A_y^{ou} \exp(i\phi_y^{ou}) \end{bmatrix}, \]

(3)

with

\[ \begin{bmatrix} A_x^{ou} \exp(i\phi_x^{ou}) \\ A_y^{ou} \exp(i\phi_y^{ou}) \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} A_x^m \exp(i\phi_x^m) \\ A_y^m \exp(i\phi_y^m) \end{bmatrix}, \]

(4)

where the matrix \( J \) consisting of four elements \( J_{ij} \), is called the Jones matrix of the optical device (magneto-optical sensor).

Thus, the matrix equation governing the polarization of an optical system according to the Jones method in concise form is as follows:

\[ \overline{E^{ou}} = J \overline{E^m}. \]

(6)

An non-ideal polarizer with an azimuth angle \( \gamma_1 \) to the axis \( x \), with transmission amplitudes \( t_x \) and \( t_y \) along the axes \( x \) and \( y \), respectively, can be represented as the product of matrices [6,7]:

\[ M_1 = \begin{bmatrix} \cos \gamma_1 & -\sin \gamma_1 \\ \sin \gamma_1 & \cos \gamma_1 \end{bmatrix} \begin{bmatrix} t_x \exp(i\phi_{x1}) & 0 \\ 0 & t_y \exp(i\phi_{y1}) \end{bmatrix} \begin{bmatrix} \cos \gamma_1 & \sin \gamma_1 \\ -\sin \gamma_1 & \cos \gamma_1 \end{bmatrix}. \]

(7)

An analyzer at the circuit output with the azimuth angle \( \gamma_3 \), is described similarly:

\[ M_3 = \begin{bmatrix} \cos \gamma_3 & -\sin \gamma_3 \\ \sin \gamma_3 & \cos \gamma_3 \end{bmatrix} \begin{bmatrix} t_x \exp(i\phi_{x3}) & 0 \\ 0 & t_y \exp(i\phi_{y3}) \end{bmatrix} \begin{bmatrix} \cos \gamma_3 & \sin \gamma_3 \\ -\sin \gamma_3 & \cos \gamma_3 \end{bmatrix}. \]

(8)

A magneto-optical Faraday rotator with the rotation angle \( \gamma_2 \) and the transmission coefficient \( t_2 \) is described with the following Jones matrix [8]:

\[ M_2 = t_2 \begin{bmatrix} \cos \gamma_2 & \sin \gamma_2 \\ -\sin \gamma_2 & \cos \gamma_2 \end{bmatrix}. \]

(9)

The Maxwell column of the output beam for a single-stage circuit is determined:

\[ \overline{E^{ou}} = M_3 M_2 M_1 \overline{E^m}. \]

(10)
The intensities of input and output radiation, according to (2) are written as follows:

\[ I_{in} = \frac{\overline{E_{in}^*}}{E_{in}} \]  
\[ I_{out} = \frac{\overline{E_{out}^*}}{E_{out}} \]  

and the expression for the intensity transmission coefficient (the transfer characteristic):

\[ T = \frac{I_{out}}{I_{in}}. \]  

Figures 2, 3 present the results of calculated transmission \( T \) for circular (\( \varphi_{in} - \varphi_{in} = \pi / 2 \)) and elliptical (\( \varphi_{y} - \varphi_{x} = \pi / 6 \)) polarization of incident beam on condition that \( \varphi_{x1} = \varphi_{y1} \), \( \varphi_{x3} = \varphi_{y3} \), \( t_2 = \sqrt{0.15} \).

**Figure 2.** Results of modeling: (a) circular polarization of the input beam; (b) dependence of normalized intensity transmission on the Faraday rotation angle \( \gamma \) for ideal (\( t_x = 1, t_y = 0 \), solid line) and non-ideal (\( t_x = \sqrt{0.95}, t_y = \sqrt{0.05} \), dotted line) polarizer and analyzer, \( \gamma_1 = \gamma_2 = 0^\circ \).
Figure 3. Results of modeling: (a) elliptical polarization of the input beam; (b) dependence of normalized intensity transmission on the Faraday rotation angle $\gamma_2$ for ideal ($t_x=1$, $t_y=0$, solid line) and non-ideal ($t_x=\sqrt{0.95}$, $t_y=\sqrt{0.05}$, dotted line) polarizer and analyzer, $\gamma_1=\gamma_2=0^\circ$.

Figures 4, 5 present the results of calculated transmission $T$ for elliptical ($\varphi_{\text{in}}^x-\varphi_{\text{in}}^y=\pi/6$) polarization of incident beam on condition that $\varphi_{\text{in}}^x=\varphi_{\text{in}}^y$, $\varphi_{\text{in}}^x=\varphi_{\text{in}}^y$, $t_2=\sqrt{0.15}$. These figures allow to estimate the effect of the alignment accuracy on the modulation depth of the transmission intensity.

Figure 4. Results of modeling: (a) elliptical polarization of the input beam; (b) dependence of normalized intensity transmission on the Faraday rotation angle $\gamma_2$ for ideal ($t_x=1$, $t_y=0$, solid line) and non-ideal ($t_x=\sqrt{0.95}$, $t_y=\sqrt{0.05}$, dotted line) polarizer and analyzer, $\gamma_1=\gamma_2=0^\circ$.

Figure 5. Results of modeling: (a) elliptical polarization of the input beam; (b) dependence of normalized intensity transmission on the Faraday rotation angle $\gamma_2$ for ideal ($t_x=1$, $t_y=0$, solid line) and non-ideal ($t_x=\sqrt{0.95}$, $t_y=\sqrt{0.05}$, dotted line) polarizer and analyzer, $\gamma_1=\gamma_2=0^\circ$.\"
As can be seen from the figures, at a small angle of polarization rotation, a change in the modulation depth of the transmission intensity is observed. It can be used to select the optimal incidence angle of polarized beam. By varying the incidence angle of the polarized beam it is possible to achieve the required sensitivity. But it imposes certain limitations on the alignment accuracy of the entire chain.

It follows from the calculations that the sensor transmission for the ideal polarizer and analyzer does not exceed $0.5T_{\gamma}$ of the input beam intensity. In the case of non-ideal polarizer and analyzer the output modulation depth intensity is reduced. The instability of the state of polarization of the input beam leads to a shift of the transmission characteristic on the scale of angles $\gamma$. Thus, it is necessary to stabilize both the intensity of incident radiation and the state of its polarization for magneto-optical sensors.

3. Conclusion
The proposed mathematical model and software make it possible to take into account the non-ideal characteristics of film polaroids observed in operation in the near infrared region and at increased temperatures. On the basis of the results of the model analysis it was found that the dependence of normalized transmission $T(\gamma)$ has periodic nature. Choosing the angle $(\gamma - \gamma_1)$ makes it possible to shift the initial operation point and change the sensitivity $dT/d\gamma$. The influence of the input beam polarization increases with the increase of polaroid parameter deviation from ideal and shows itself as reduction of modulation depth and angular shift of the sensor conversion response.

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