Quantum reservoir computing implementation on a Josephson mixer

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Quantum reservoir computing is a promising approach to quantum neural networks capable of solving hard learning tasks on both classical and quantum input data. However, current approaches with qubits are limited by low connectivity. We propose an implementation for quantum reservoir that obtains a large number of densely connected neurons by using parametrically coupled quantum oscillators instead of physically coupled qubits. We analyse a specific hardware implementation based on superconducting circuits: with just two coupled quantum oscillators, we obtain a quantum reservoir with up to 64 neurons. We obtain state-of-the-art accuracy of 99 % on benchmark tasks that otherwise require at least 24 classical oscillators to be solved. Our results give the coupling and dissipation requirements in the system and show how they affect the performance of the quantum reservoir. Beyond quantum reservoir computing, the use of parametrically coupled bosonic modes holds promise for realizing large quantum neural network architectures, with billions of neurons implemented on 10 coupled quantum oscillators.

INTRODUCTION

Quantum neural networks are the subject of intensive research today. They emulate a large number of neurons with only a small number of physical components, which facilitates scaling up compared to classical approaches. Indeed, by encoding the responses of the neurons in the populations of base states, a system of $N$ qubits provides up to $2^N$ neurons. Moreover, such quantum neural networks could automatically transform complex quantum data into simple outputs representing the class of the input, that could then be measured with just a few samples compared to the millions needed today. The first experimentally realized quantum neural networks are made of 1 to 40 qubits, each connected to two or four nearest neighbors.

The major challenge of the field is now to experimentally realize neural networks capable of real-world classification tasks, and thus containing millions of neurons each connected by thousands of connections. The use of qubits poses a conceptual and technical problem for this purpose. Indeed, when connectivity is obtained with pairwise couplers between qubits, distant qubits cannot be interconnected without very cumbersome classical circuitry in existing 2D architectures.

Here we develop an alternative approach to quantum neural networks that is both scalable and compatible with experimental implementations. We propose to leverage the complex dynamics of coherently coupled quantum oscillators, combined with their infinite number of basis states, to populate a large number of neurons much more efficiently than with qubits. We describe a concrete implementation of such a system on a superconducting circuit that has been experimentally-tested and can be accurately modelled: the Josephson mixer, which consists of two superconducting quantum oscillators coherently coupled with a ring of Josephson junctions. We then show through simulations with experimentally-validated models that this system classifies and predicts time-series data with high efficiency through the approach of reservoir computing.

With just two quantum oscillators, up to 8 states can be populated in each oscillator with significant probability amplitudes, which yields a quantum reservoir with up to 64 neurons.

We evaluate its performance on two benchmark tasks in the field of reservoir computing, sine-square waveform classification and Mackey-Glass chaotic time series prediction, that test the ability of the reservoir to memorize input data and transform it in a non-linear way. We obtain a state-of-the-art accuracy of 99 % with our system of two coherently coupled oscillators, which otherwise requires 24 classical oscillators to achieve. With 10 oscillators we could have 3 billions of neurons, comparable to the most impressive neural networks capable of hard tasks such as natural language processing or generating images from text descriptions.

The results show that the Josephson mixer circuit implements a high quality reservoir computer capable of complex tasks, and opens the path to experimental implementations of quantum reservoirs based on a large number of basis state neurons, and provides a quantum neural network platform compatible with numerous algorithms exploiting physics and dynamics for computing.

QUANTUM RESERVOIR COMPUTING

Reservoir computing is a machine learning paradigm that uses nonlinear dynamical systems for temporal information processing. Its principle is illustrated in FIG. 1. The reservoir (blue area) is a dynamical system with arbitrary but fixed recurrent connections. It takes as input data that is not easily separable in different classes. The role of the reservoir is to project these inputs into a highly dimensional state space in which the data becomes linearly separable. The reservoir outputs are then classified by a linear, fully connected layer (shown in red dashed arrows) that can be trained by a simple linear regression. Physical implementations of reservoir comput-
ing commonly perform this projection of input data to a high dimensional space through complex non-linear dynamics and the outputs are obtained by measuring some variable on that system.\footnote{\textsuperscript{12,20}}

The fully connected layer is usually realized in software, and multiplies the measurement outputs $F(X)$ by a weight matrix $W$, such that

$$W F(X) = Y.$$ (1)

The weight matrix is trained to make the neural network output $Y$ match the target vector $\tilde{Y}$. The particularity of reservoir computing compared to deep neural networks is that training is performed in a single step by matrix inversion

$$W = \tilde{Y}_{\text{train}} F^\dagger(X_{\text{train}}),$$ (2)

where $X_{\text{train}}$ is the training data, $\tilde{Y}_{\text{train}}$ is the training target, and $F^\dagger$ is the Moore-Penrose pseudo-inverse of the matrix $F$ containing the outputs $f(x_i)$ of the reservoir neurons for all the training examples.\footnote{\textsuperscript{13,21}} The learned weight matrix is applied on the test data contained in the vector $X_{\text{test}}$, in order to find the neural network prediction

$$Y_{\text{test}} = W F(X_{\text{test}}).$$ (3)

Comparing the prediction to the test target $\tilde{Y}_{\text{test}}$ allows to evaluate the prediction accuracy, i.e. the fraction of times the data point is correctly classified, as well as the root mean square error

$$\text{RMSE} = \frac{1}{N} \sqrt{\sum_{i=1}^{N} (y_i - \tilde{y}_i)^2}.$$ (4)

We will use these measures to evaluate the performance of the reservoir, and thus the capacity of the chosen dynamical system to efficiently implement reservoir computing.

Classical reservoir computing was implemented on different dynamical physical systems, ranging from silicon photonics\footnote{\textsuperscript{10}} and optoelectronic\footnote{\textsuperscript{11,12}}, to spintronic nano-oscillators.\footnote{\textsuperscript{20}} Quantum reservoir computing was first proposed in 2017, with a reservoir whose neurons correspond to the basis states of a set of qubits, and computational capabilities are identical to 100-500 classical neurons with only 5-7 qubit.\footnote{\textsuperscript{22}} Experimentally, quantum reservoir was implemented on 4 static spins and 8 neurons and a dissipative reservoir was implemented on up to 10 qubits.\footnote{\textsuperscript{23}} We have recently highlighted that dynamical systems of coherently coupled quantum oscillators possess all the required features for quantum reservoir computing.\footnote{\textsuperscript{4}}

JOSEPHSON MIXER

We consider the implementation of quantum reservoir computing on two coupled quantum oscillators $a$ and $b$ (FIG. 2(a)). In this case, the reservoir neurons are given by the basis states $|n_a, n_b\rangle$, and the reservoir outputs by their occupation probabilities. In the view of an experimental realization, we propose to use a superconducting circuit called the Josephson mixer. The Josephson mixer has the advantage of being a well-known circuit that has been used for years in the field of quantum information for quantum limited amplification,\footnote{\textsuperscript{13}} transduction,\footnote{\textsuperscript{26}} circulation\footnote{\textsuperscript{27}} and qubit readout.\footnote{\textsuperscript{25}} It can thus be used for an immediate implementation of quantum reservoir computing with the already existing hardware. It consists of two superconducting resonators, with a ring of four Josephson junctions inserted in the middle (FIG. 2(b)). We use the fundamental electromagnetic modes of the resonators as quantum oscillators $a$ and $b$. The Josephson ring implements three-wave mixing between these two modes and a common mode,\footnote{\textsuperscript{40}} such that the Hamiltonian of the circuit writes

$$H = \hbar \omega_a \hat{a}^\dagger \hat{a} + \hbar \omega_b \hat{b}^\dagger \hat{b} + \hbar \chi (\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{c} + \hat{c}^\dagger \hat{c}),$$ (5)

where $\omega_a$ and $\omega_b$ are the resonance frequencies of the two quantum oscillators, and $\hat{a}$, $\hat{b}$ and $\hat{c}$ are photon annihilation operators in the modes $a$, $b$ and $c$. When the common mode is off-resonantly driven with a pump tone at the difference of the oscillator frequencies, $\omega_p = \omega_a - \omega_b$, the interaction Hamiltonian in the rotating wave approximation reduces to photon conversion between the modes $a$ and $b$,

$$\hat{H}_{\text{conv}} = g \left( \hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger \right),$$ (6)

where $g = \chi p$ is the conversion coupling rate that can be controlled by the pump amplitude $p$. Such tunable coupling allows us to study the performance of the quantum reservoir as a function of the coupling strength.

We drive each oscillator on resonance with an amplitude that encodes the input data, such that the population of the basis states depends on the input value, the duration for which the drive signal is applied, and the previous input values, as long as each input is sent
FIG. 2. (a) Schematic of two quantum oscillators with different resonance frequencies $\omega_a$ and $\omega_b$ and energy levels separated by $\hbar \omega_a$ and $\hbar \omega_b$. The oscillators are resonantly driven at amplitudes $\epsilon_a$ and $\epsilon_b$, with dissipation rates $\kappa_a$ and $\kappa_b$. The coherent coupling at a rate $g$ results in the exchange of excitations between oscillators. (b) Schematic of the Josephson mixer with two resonators $a$ and $b$ (yellow and blue), with a ring of four Josephson junctions (boxed crosses) in their crossing. Resonators fundamental modes are resonantly driven at amplitudes $\epsilon_a$ and $\epsilon_b$ and parametrically coupled by a pump tone at an amplitude $p$.

for a time shorter than the lifetime of the oscillators. The dynamics of the system is driven by three main processes: resonant drives, dissipation and conversion of photons between the oscillators at a rate $g$. We use typical experimental parameters for the Josephson mixer with resonators frequencies $\omega_a = 2\pi \times 10$ GHz and $\omega_b = 2\pi \times 9$ GHz and dissipation rates $\kappa_a = 2\pi \times 17$ MHz, $\kappa_b = 2\pi \times 21$ MHz.

**LEARNING TASKS WITH THE QUANTUM RESERVOIR**

In order to evaluate the capacity of the quantum reservoir with oscillators, we address two standard benchmark tasks of reservoir computing, i.e. a classification task that requires a lot of nonlinearity and short-term memory, and a prediction task that requires both short- and long-term memory. In order to assess the advantage brought by the quantum nature of the reservoir, we compare it to the performance of classical reservoirs on the same tasks. We distinguish the contribution of dynamics and of specifically quantum properties, by comparing with both static and dynamic classical reservoirs. For the static reservoir we perform software simulations of reservoirs with neurons that apply a nonlinear ReLu function (typically used in machine learning). For the dynamical reservoir we simulate spin-torque nano-oscillators as neurons, such as they were used in [20,26], and also compare the different simulation results with the experimental performance obtained in [26]. The methodology and parameters used for simulations are described in Methods.

**A. Sine and square waveform classification**

The first learning task that we address is the classification of points belonging to sine and square waveforms. The input data is sent as a time-series, consisting of randomly arranged sine and square waveforms, each discretized in 8 points, as shown in FIG. 3(a). The neural network gives a binary output, equal to 0 if it estimates that the input point belongs to a square, and to 1 if it estimates that it belongs to a sine. This task was specifically conceived to test the nonlinearity and the memory of a neural network as the input data points cannot be linearly separated and the extremal points require memory to be distinguished (input points equal to $\pm 1$ can both belong to a sine and to a square). At least 24 classical neurons are needed to solve this task with an accuracy $>99\%$.

We send the input drives to the oscillators for 100 ns, one immediately after the other, and we measure the occupation probabilities at the end of each drive. We investigate the performance of the quantum reservoir as a function of the number of measured basis states. We first measure the states $|00\rangle$ to $|33\rangle$, yielding 16 output...
neurons. The reservoir prediction, obtained with Eq. (6), is shown in FIG. 3(b). The prediction matches the target with 99.5% accuracy and RMSE = 0.003. This is a very good performance — indeed, it requires at least 40 static classical neurons and 24 dynamical classical neurons (see FIG. 4 for simulations and 22 for experiments) to achieve it. The fact that it is obtained with only 16 measured quantum neurons points to the first aspect of quantum advantage: all the 49 basis states that are populated participate to data processing and transformation even tough they are not measured. We push this even further and perform learning while only measuring states up to (22), which yields 9 neurons. Strikingly, we still obtain an accuracy of 99.5% (FIG. 3(c)). The noise in the prediction is a little higher, which translates by a higher RMSE = 0.017. Nevertheless, the task that requires at least 24 classical dynamical neurons is perfectly solved by measuring only 9 quantum neurons.

In essence, this advantage comes from the fact that oscillators are in a quantum superposition of states. It is very interesting from the experimental point of view: even tough quantum measurements need to be repeated multiple times in order to reconstruct probability amplitudes to find a system in a given state, a much smaller number of states needs to be measured compared to the classical case. Furthermore, all the measurements are performed on the same device, which simplifies the experimental setup, as they can be measured simultaneously using frequency multiplexing.22

B. Mackey-Glass chaotic time-series prediction

The second benchmark task that we address is the prediction of Mackey-Glass chaotic time-series. Compared to classification, time-series prediction requires the reservoir to have more memory. It also allows us to investigate the impact of the reservoir temporal dynamics on its prediction capacity; in particular we study how the coupling between the oscillators and their dissipation rates impact the reservoir performance.

The input data is obtained from the equation

$$\frac{\partial x(t)}{\partial t} = \frac{\beta x(t - \tau_M) - \gamma x(t)}{1 + x^{10}(t - \tau_M)},$$  \hspace{1cm} (7)$$

and is chaotic for parameters β = 0.2, γ = 0.1 and τM = 1.23. A subset of the input data is shown in FIG. 5(a). Each point is sent for 100 ns, such that delay = 20 corresponds to 2 µs. In all the simulations we measure 16 basis state neurons, from |00⟩ to |33⟩.

We train the reservoir on 1000 points and test it on another set of previously unseen 1000 points. The results are shown in FIG. 5(b-c). We plot the average logarithmic error on 1000 test points as a function of the delay for different reservoir parameters such as dissipation rates and oscillator couplings. In all the cases, we observe an overall logarithmic increase of the error as a function of the delay, which corresponds to the memory of the reservoir - points further in future are harder to predict because the memory is lost. Nevertheless, the error saturates for large delays. This saturation is due to the fact that the reservoir learns the range in which the points are situated, and in particular the region where the minima and the maxima of the time-series, that contain a lot of points, are concentrated. Another common feature that can be noticed in all the figures are the oscillations in the error signal. Their periodicity is twice the periodicity of the input data, and corresponds to the fact that data is easier to predict at the minima and maxima of the time-series, than at the slopes where the gradient is large.

We first study the impact of the oscillator dissipation rates κa and κb on the reservoir performance (FIG. 5(b)). We observe that for high dissipation, the error is globally larger, and most importantly, increases faster — meaning that the memory of the neural network is shorter. It is thus important to have high quality factor oscillators to solve tasks that require a lot of memory. Second, we study the impact of the oscillator coupling rate g on the reservoir performance (FIG. 5(c)). For a lower coupling the error increases; indeed, strong coupling gives rise to multiple data transformations between different basis states which is essential for learning. It also leads to a significant population of a larger number of basis state neurons that contribute to computing.

This task was solved in simulations with similar performance with 50 classical dynamical neurons such as skyrmions22 and experimentally with a classical RC oscillator time-multiplexed 400 times to obtain 400 virtual neurons21. Here we solve it with just two physical devices and 16 measured basis state neurons.

DISCUSSION

We have shown that a simple superconducting circuit, composed of two coherently coupled quantum oscillators,
can successfully implement quantum reservoir computing. This circuit has been exploited for quantum computing for years, and can be readily used to realize experimentally larger scale quantum reservoir computing.

Compared to classical reservoir, quantum reservoir allows to encode neurons as basis states and obtain number of neurons exponential in the number of physical devices. Furthermore, even though it was not the focus of this paper, where we processed classical data, numerical simulations of different quantum reservoirs have shown that quantum reservoirs can process input quantum states and simultaneously estimate their different properties, as well as perform quantum tomography. This is particularly interesting in the age where quantum computing encodes information in quantum states and begins to produce more and more quantum data.

Quantum reservoirs can be implemented on different quantum systems. First works have naturally focused on qubits, as the most common quantum hardware. Nevertheless, quantum oscillators compared to qubits have a net advantage for scaling - they have an infinite number of basis states, compared to qubits that only have two - and they can be much more efficiently populated using resonant drives and coherent coupling. With just two quantum oscillators, we can populate up to 8 states in each oscillator with significant probability amplitudes, which yields a quantum reservoir with up to 64 neurons. By measuring only 16 basis states, we obtain a performance equivalent to 24 classical oscillators. There is an advantage in the number of physical devices, which simplifies experimental implementation, and also in the number of neurons that need to be measured, which simplifies the measurement procedure.

Reservoir computing was already simulated on a single nonlinear quantum oscillator and on a system of coupled nonlinear parametric oscillators. Our work significantly differs from these approaches. In these works, quantum oscillators were operating in the semi-classical regime, where a strong input signal with a large number of photons induces Kerr nonlinearity. In that regime, each oscillator yields two output neurons, i.e. the two field quadratures, that can be sampled in time in order to increase the number of effective neurons. Our approach fully exploits the quantum nature of the system by using the basis states as neurons, which allows to increase the memory of the system as there is no need for sampling, and reduce both the number of physical devices in the system and the number of necessary measurements.

We believe that this solution is very promising for the implementation of quantum neural networks as it is scalable. Indeed, it has recently been shown that multiple oscillators can be parametrically coupled all-to-all through a common waveguide. This new paradigm that we propose would thus allow to realize larger scale quantum neural networks with readily available devices.

METHODS

C. Quantum simulations

We simulate the dynamics of the Josephson Mixer using the library QuantumOptics.jl for simulating open quantum systems in Julia. The dynamics can be captured by the quantum master equation

$$\dot{\rho} = -i[\hat{H} + \hat{H}_{\text{drive}}, \rho] + \hat{C} \rho \hat{C}^\dagger - \frac{1}{2} \hat{C}^\dagger \hat{C} \rho - \frac{1}{2} \rho \hat{C}^\dagger \hat{C}, \quad (8)$$

where $\rho$ is the density matrix of the system. $\hat{H}_{\text{drive}}$ is the resonant drive Hamiltonian

$$\hat{H}_{\text{drive}} = i\epsilon_a \sqrt{2\kappa_a} (\hat{a} - \hat{a}^\dagger) + i\epsilon_b \sqrt{2\kappa_b} (\hat{b} - \hat{b}^\dagger) \quad (9)$$
are modeled as that of a nonlinear auto-oscillator neuron, as in E. Dynamic reservoir simulations shown in FIG. 4. The simulations of the static reservoir, the size of the memory of a single step in the time, which is sufficient for the sine and square waveform classification task. In the simulations of the basis states occupations

\[ p(n_a, n_b) = \langle n_a n_b | \rho | n_a n_b \rangle. \]  

D. Static reservoir simulations

The simulations of the classical reservoirs, both static and dynamic, were performed in the library pytorch for training neural networks in Python. The state of the reservoir at time \( t \) is

\[ y(t) = f(W_{in}x(t) + W_{res}x(t - 1)) \]  

where \( f \) is the ReLu function, \( W_{in} \) is the vector that has the length of the size of the reservoir, and maps the input data on the reservoir, \( W_{res} \) is a square matrix that has the dimension of the size of the reservoir and which gives the memory to the reservoir. Here the reservoir has the memory of a single step in the time, which is sufficient for the sine and square waveform classification task. In the simulations of the static reservoir, the size of the reservoir is equal to the number of measured neurons, shown in FIG. 4.

E. Dynamic reservoir simulations

The simulations of dynamical classical reservoir were realized considering a spin-torque nano-oscillator as a neuron, as in. The dynamics of the nano-oscillator can be modeled as that of a nonlinear auto-oscillator

\[ \frac{dp}{dt} = 2(-\Gamma(1 + Qp) + W_{in}I\sigma(1 - p))p \]  

where \( p \) is the power of the oscillator, \( \Gamma \) is the damping rate, \( Q \) is the non-linearity, \( I \) is the current that drives the oscillator and \( \sigma \) is a factor related to the geometry of the oscillator. Input data is encoded in the current \( I \) and mapped by the randomly generated vector \( W_{in} \) on the reservoir. Reservoir outputs are obtained from the oscillator power \( p \) by numerically integrating the Eq. (13).

DATA AVAILABILITY

Data are available from the authors on reasonable request.

REFERENCES

1. J. Herrmann, S. M. Lloba, A. Remm, P. Kapteijis, N. M. McMahon, C. Scarato, F. Sliwa, C. K. Andersen, C. Hollings, S. Kienlen, N. Lacroix, S. Lazar, M. Kerschbaum, D. C. Zamzum, G. J. Norris, M. J. Hartmann, A. Waltz, and C. Eichler, arXiv:2109.05099v1 (2021)
2. H.-Y. Huang, M. Broughton, J. Cotler, S. Chen, J. Li, M. Mohseni, H. Neven, R. Babbush, R. Kuen, JohnPreskill, and J. R. McClean, Science 376, 6598 (2022).
3. J. Dudas, J. Grollier, and D. Marković, arXiv:2204.11270 (2022)
4. N. Bergel, F. Schackert, M. Metcalfe, R. Vijay, V. E. Manucharyan, L. Xiong, D. E. Prober, J. R. Schoelkopf, S. M. Girvin, and M. H. Devoret, Nature 465, 64 (2010)
5. N. Roch, E. Furin, C. Nguyen, P. Morin, P. Campagne-Ibarcq, M. H. Devoret, and B. Huard, Physical review letters 108, 147701 (2012)
6. B. Abdo, K. Sliwa, F. Schackert, N. Berger, M. Hatridge, L. Frazin, A. D. Stone, and M. Devoret, Phys. Rev. Lett. 110, 173902 (2013)
7. B. Abdo, K. Sliwa, S. Shankar, M. Hatridge, L. Frazin, R. Schoelkopf, and M. Devoret, Physical Review Letters 112, 167701 (2014)
8. D. Marković, S. Jezoine, Q. Ficheux, S. Fedorchenko, S. Flicicett, T. Coudreau, P. Milman, Z. Leghtas, and B. Huard, Physical Review Letters 121, 040505 (2018)
9. D. Marković, J. D. Pillot, E. Furin, N. Roch, and B. Huard, Physical Review Applied 12, 1 (2019)
10. B. Abdo, A. Kamal, and M. Devoret, Physical Review B 87, 014508 (2013)
11. H. Haas and H. Jaeger, Science 304, 78 (2004)
12. A. Ramesh, P. Dharhiwal, A. Nichol, C. Chu, and M. Chen, arXiv:2204.06125 (2022)
13. M. H. Amin, E. Andriyash, J. Rolfe, B. Kulchytzsky, and R. Melko, Physical Review X 8, 21050 (2018)
14. T. D. Devitt, B. McIver, E. Bengio, Frontiers in Computational Neuroscience 11, 24 (2017)
15. R. T. Q. Chen, M. Bab孫ska, J. Bettencourt, and D. Duvenda, NeurIPS 278, 191 (2019)
16. T. Onodera, E. Ng, and P. L. McMahon, npj Quantum Information 6, 1 (2020)
17. Y. Paoquet, F. Duport, A. Smerieri, J. Dambre, B. Schrauwen, M. Haelterman, and S. Massar, Scientific Reports 2, 468 (2012)
18. D. Brunner, M. C. Soriano, C. R. Mirasso, and I. Fischer, Nature Communications 4, 1364 (2013)
19. P. Vandoorne, P. Mechet, T. Van Vaerenbergh, M. Fiers, G. Moreth, D. Verstraeten, B. Schrauwen, J. Dambre, and P. Bienstman, Nature Communications 5, 1 (2014)
20. J. Torrejon, M. Rion, F. A. Araujo, S. Tsunegi, G. Kansa, D. Quelroz, P. Bortolotti, V. Cros, K. Yakushiji, A. Fukushina, H. Kubota, S. Yuasa, M. D. Stiles, and J. Grollier, Nature 547, 428 (2017)
21. L. Appeltant, M. C. Soriano, G. Van Der Sande, J. Danckaert, S. Massar, J. Dambre, B. Schrauwen, C. R. Mirasso, and I. Fischer, Nature Communications 2, 468 (2011).
22. K. Fujii and K. Nakajima, Physical Review Applied 8, 024030 (2017)
23. M. Negoro, K. Mitarai, K. Fujii, K. Nakajima, and M. Kitagawa, arXiv:1806.10910 (2018)
24. J. Chen, H. I. Nurdin, and N. Yamamoto, Physical Review Applied 14, 024065 (2020)
25. P. Campagne-Ibarcq, E. Furin, N. Roch, D. Darson, P. Morfin, M. Mirrahimi, M. H. Devoret, F. Mallet, and B. Huard, Physical Review X 3, 021008 (2013)
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AUTHOR CONTRIBUTIONS

D.M. and J.G. conceived the project. D.M. performed the calculations. J.D. performed the quantum simulations. E.P. and A.M. performed the classical dynamical simulations. D.M. performed the classical static simulations. D.M. and J.G. wrote the paper.

COMPETING INTERESTS

The Authors declare no Competing Financial or Non-Financial Interests.