QUASIFREE PROCESSES FROM NUCLEI: MESON PHOTOPRODUCTION AND ELECTRON SCATTERING

L.J. ABU-RADDAD AND J. PIEKAREWICZ

1 Theory Group, Research Center for Nuclear Physics, Osaka University, 10-1 Mihogaoka, Ibaraki City, Osaka 567-0047, Japan

2 Department of Physics, Florida State University, Tallahassee, FL 32306, USA

We have developed a relativistic formalism for studying quasi-free processes from nuclei. The formalism can be applied with ease to a variety of processes and renders transparent analytical expressions for all observables. We have applied it to kaon photoproduction and to electron scattering. For the case of the kaon, we compute the recoil polarization of the lambda-hyperon and the photon asymmetry. Our results indicate that polarization observables are insensitive to relativistic, nuclear target, and distortion effects. Yet, they are sensitive to the reactive content, making them ideal tools for the study of modifications to the elementary amplitude — such as in the production, propagation, and decay of nucleon resonances — in the nuclear medium. For the case of the electron, we have calculated the spectral function of $^4$He. An observable is identified for the clean and model-independent extraction of the spectral function. Our calculations provide baseline predictions for the recently measured, but not yet fully analyzed, momentum distribution of $^4$He by the A1-collaboration from Mainz. Our approach predicts momentum distributions for $^4$He that rival some of the best non-relativistic calculations to date.

1 Introduction

Faced by an increasing demand for studying quasi-free processes from nuclei, we have developed a general fully relativistic treatment for studying such interactions. The power of the theoretical approach employed here lies in its simplicity. Analytic expressions for the response of a mean-field ground state may be provided in the plane-wave limit. The added computational demands placed on such a formalism, relative to that from a free on-shell proton, are minimal. The formalism owes its simplicity to an algebraic trick, first introduced by Gardner and Piekarewicz, that enables one to define a “bound” (in direct analogy to the free) nucleon propagator. Indeed, the Dirac structure of the bound nucleon propagator is identical to that of the free Feynman propagator. As a consequence, the power of Feynman’s trace

*ELECTRONIC ADDRESS: LAITH@RCNP.OSAKA-U.AC.JP
†ELECTRONIC ADDRESS: JORGEP@CSIT.FSU.EDU
techniques may be employed throughout the formalism.

We have applied this formalism to two kinds of processes: kaon photoproduction and electron scattering. Further, there is a promising potential of applying it to many processes being studied experimentally at various laboratories. We will give here a brief introduction to this formalism and we will discuss some of the results of using it.

The investigation of the quasifree kaon photoproduction process is impelled by recent experimental advances and the increasing interest in the study of strangeness-production reactions from nuclei. These reactions form our gate to the relatively unexplored territory of hypernuclear physics. Moreover, these reactions constitute the basis for studying novel physical phenomena, such as the existence of a kaon condensate in the interior of neutron stars.

As for electron scattering, the appeal of this reaction is due to the perceived sensitivity of the process to the nucleon momentum distribution. Interest in this reaction has stimulated a tremendous amount of experimental work at electron facilities such as NIKHEF, MIT/Bates, and Saclay, who have championed this effort for several decades. Our motivation for studying this process is twofold: First, we use this formalism to compute the spectral function of $^4$He in anticipation of the recently measured, but not yet fully analyzed, A1-collaboration data from Mainz. Second, we take advantage of the L/T separation at Mainz to introduce what we regard as the cleanest physical observable from which to extract the nucleon spectral function.

## 2 Formalism

We provide here a brief discussion of our formalism. We use a plane-wave formalism and incorporate no distortions. Our rationale for this is that we concentrate on polarization observable which are typically insensitive to distortions. Moreover, in some occasions the effect of distortions is determined from other treatments and thus we are able to concentrate on the fundamental physics with no diversions. Notably, there is a definite appeal in terms of practicality: we can use now the Gardner’s and Piekarewicz’s trick which renders transparent analytical results for all observables.

The Gardner and Piekarewicz trick enables us to introduce the concept of a “bound-state propagator”:

\[
S_{\alpha}(p) = \frac{1}{2j + 1} \sum_{m} U_{\alpha,m}(p) U_{\alpha,m}^*(p) = (\not{p} + M_{\alpha}), \quad (\alpha = \{E, \kappa\}). \tag{1}
\]
The mass-, energy-, and momentum-like quantities in this expression are defined in terms of the upper component of the Dirac spinor $g_\alpha(p)$ and the lower component of the Dirac spinor $f_\alpha(p)$.

The evident similarity in structure between the free and bound propagators for the direct product of spinors results in an enormous simplification; we can now employ the powerful trace techniques developed by Feynman to evaluate all observables — irrespective if the nucleon is free or bound to a nucleus. It is important to note, however, that this enormous simplification would have been lost if distortion effects would have been incorporated.

In order to automate the straightforward but lengthy procedure of calculating these Feynman traces, we rely on the *FeynCalc 1.0* package with *Mathematica 2.0* to calculate all traces involving $\gamma$-matrices.

### 3 Results

#### 3.1 kaon quasifree process

We start the discussion of our results by examining the nuclear dependence of the polarization observables. Fig. 3 displays the recoil polarization ($P$) of the $\Lambda$—hyperon and the photon asymmetry ($\Sigma$) as a function of the kaon scattering angle for the knockout of a valence proton for a variety of nuclei, ranging from $^4\text{He}$ to $^{208}\text{Pb}$.
from \(^4\text{He}\) all the way to \(^{208}\text{Pb}\). These observables were evaluated at a photon energy of \(E_\gamma = 1400\ \text{MeV}\) and at a missing momentum of \(p_m = 120\ \text{MeV}\).

We have used the Saclay-Lyon model for the elementary amplitude \(^1\). We have included also polarization observables from a single proton to establish a baseline for comparison against our bound–nucleon calculations. The sensitivity of the polarization observables to the nuclear target is rather small. Moreover, the deviations from the free value are significant. This indicates important modifications to the elementary process in the nuclear medium. Although not shown, we have studied the importance of relativity and found that these observables are insensitive to relativistic dynamics.

Having established the independence of polarization observables to relativistic effects and to a large extent to the nuclear target we are now in a good position to discuss the sensitivity of these observables to the elementary amplitude (note that an insensitivity of polarization observables to final-state interaction has been shown in Ref. \(^2\)). We display in Fig. 2 the differential cross section as a function of the kaon scattering angle for the knockout of a

\begin{figure}
\centering
\includegraphics[width=0.7\textwidth]{fig2.png}
\caption{The differential cross section for the knockout of a proton from \(^{12}\text{C}\) using various models for the elementary amplitude.}
\end{figure}

proton from the \(p^{3/2}\) orbital in \(^{12}\text{C}\) using four different models for the elementary amplitude \(^3\). Although there are noticeable differences between the models, primarily at small angles, these differences are relatively small. Much more significant, however, are the differences between the various sets for the
case of the polarization observables displayed in Fig. 3. The added sensitivity
to the choice of amplitude exhibited by the polarization observables should not come as a surprise; unraveling subtle details about the dynamics is the hallmark of polarization observables. In particular, polarization observables show a strong sensitivity to the inclusion of the off-shell treatment for the various high-spin resonances, as suggested in Ref. [15].

3.2 electron quasifree process

There is a vast amount of literature on \((e, e'p)\) reaction in the quasifree region. Most relevant to our present discussion is the one pertaining to fully relativistic calculations such as the extensive set of studies conducted by the “Spanish” group of Udias and collaborators [16, 17, 18, 19, 20, 21]. These studies have shown that the many subtleties intrinsic to the relativistic approach challenge much of the “conventional wisdom” developed within the non-relativistic framework and that, as a result, a radical revision of ideas may be required.

The experimental extraction of the spectral function is based on a non-relativistic plane-wave result that is typically referred to as the factorization [4].

\[
S(E, p) = \frac{1}{p'E_p'\sigma_{eN}} \frac{d^6\sigma}{dE_{e'}d\Omega_{e'}dE_p' d\Omega_p}. \tag{2}
\]
However, this procedure is problematic. First, the quasifree cross section [the
numerator in Eq. (2)] suffers from the off-shell ambiguity; different on-shell
equivalent forms for the single-nucleon current yield different results. Second,
the problem gets compounded by the use of an elementary electron-proton
cross section ($\sigma_{eN}$) evaluated at off-shell kinematics. Finally, the projection
of the bound-state wave-function into the negative-energy sector as well as
other relativistic effects spoil this assumed cross section factorization.

To be noted here that the projection of the bound-state spinor into the
negative-energy states dominate at large missing momenta and may mimic
effects perceived as “exotic” from the non-relativistic point of view, such as
an asymmetry in the missing-momentum distribution or short-range corre-
lations. Indeed, Caballero and collaborators have confirmed that these
contributions can have significant effect on various observables, especially at
large missing momenta.

While a consistent relativistic treatment seems to have spoiled the fa-
torization picture obtained from a non-relativistic analysis, and with it the simple
relation between the cross-section ratio and the spectral function [Eq. (2)],
the situation is not without remedy. Having evaluated all matrix elements
analytically in the plane-wave limit, the source of the problem can be readily
identified in the form of several ambiguous kinematical factors when evaluated
off-shell. Thus we search for an observable that exhibits a weak depen-
dence on these quantities and we find, perhaps not surprisingly, that the longitudi-
nal component of the hadronic tensor could be such an observable which is
given (in parallel kinematics) by

$$R_L \equiv W^{00} \simeq F_1^2(E_p' + M) \rho(p),$$

where $\rho(p)$ is nothing but the momentum distribution of the bound nucleon.

This expression depends on unambiguous kinematics quantities and is
valid up to small (1-3 %) second-order corrections. It is also independent of
the small components of the Dirac spinors and of the negative-energy states.
Moreover, it is free from of off-shell ambiguities.

The momentum distribution for $^4$He is displayed in Fig. 4 using various
methods for its extraction. The solid line gives the “canonical” momentum
distribution, obtained from the Fourier transform of the $1S_{1/2}$ proton wave-
function. The momentum distribution extracted from the longitudinal re-
sponse (dot-dashed line) is practically indistinguishable from the canonical
momentum distribution. To be noted here that the contribution from the
anomalous form factor $F_2$ to the longitudinal response (see the dotted line in
the figure) is small because it appears multiplied by two out of three “small”
quantities in the problem.
Figure 4. The proton momentum distribution $\rho_2$ for $^4$He as a function of the missing momentum extracted using various methods. The inset shows the corresponding integrand from which the shell occupancy may be extracted.

The last calculation displayed in Fig. 4 corresponds to a momentum distribution extracted from the factorization approximation (long dashed line). The momentum distribution extracted in this manner overestimates the canonical momentum distribution over the whole range of missing momenta and integrates to 2.9 rather than 2; this represents a discrepancy of 45 percent.

In Fig. 5 a comparison is made between our results and non-relativistic state-of-the-art calculations of the momentum distribution of $^4$He. The solid line displays, exactly as in Fig. 4, the canonical momentum distribution. We see no need to include the momentum distribution extracted from the longitudinal response as it has been shown to give identical results.

In addition to our own calculation, we have also included the variational results of Schiavilla and collaborators [26] for both the Urbana [27] (dashed line) and the Argonne [28] (long-dashed line) potentials, with both of them using Model VII for the three-nucleon interaction. The variational calculation of Wiringa and collaborators [29], [30], [31] (dashed-dotted) has also been included; this uses the Argonne v18 potential [32] supplemented with the Urbana IX three-nucleon interaction [33]. Figure 5 also shows NIKHEF data by van den Brand and collaborators [34], [35], as well as preliminary data from MAINZ by Florizone and collaborators [6], [7]. Comparisons to the preliminary Mainz data of Kozlov and collaborators [8], [9], [10] have also been made (although the data
is not shown). Thus, high-quality data for the momentum distribution of $^4$He is now available up to a missing momentum of about 200 MeV. We find the results of Fig. 5 quite remarkable. It appears that a simple relativistic mean-field calculation of the momentum distribution rivals — and in some cases surpasses — some of the most sophisticated non-relativistic predictions. Still, theoretical predictions of the momentum distribution overestimate the experimental data by up to 50-60%. Part of the discrepancy is attributed to distortion effects which are estimated at about 12%. However, distortions are not able to account for the full discrepancy. We have argued earlier that an additional source of error may arise from the factorization approximation used to extract the spectral function from the experimental cross section. We are confident that the approach suggested here, based on the extraction of the spectral function from the longitudinal response, is robust. While the method adds further experimental demands, as a Rosenbluth separation of the cross section is now required, the extracted spectral function appears to be weakly dependent on off-shell extrapolations and relativistic effects.

4 Conclusions

We have developed a relativistic formalism for studying quasi-free processes from nuclei. The formalism can be applied with ease to a variety of processes
and renders transparent analytical expressions for all observables. We have applied it to the processes of kaon photoproduction and electron scattering.

For the kaon quasifree process, we have found that the polarization observables are very sensitive to the fundamental physics in this process, but at the same time mostly insensitive to distortion effects, relativistic effects, and nuclear target effects. We conclude that the polarization observables are one of the cleanest tools for probing both the elementary amplitude ($\gamma p \rightarrow K^+ \Lambda$) and nuclear medium modifications.

For the electron quasifree process, we have derived a robust procedure for extracting the momentum distribution using the longitudinal response. Furthermore, we found that the relativistic mean-field calculation of the momentum distribution in $^4$He rivals — and in some cases surpasses — some of the most sophisticated non-relativistic predictions to date.

Acknowledgments

This work was supported in part by the United States Department of Energy under Contract No. DE-FG05-92ER40750 and in part by a joint fellowship from the Japan Society for the Promotion of Science and the United States National Science Foundation.

References

1. L.J. Abu-Raddad and J. Piekarewicz, Phys. Rev. C 61, 014604 (2000).
2. L.J. Abu-Raddad, “Photoproduction of pseudoscalar mesons from nuclei”, Ph.D. thesis, Florida State University (unpublished). Available at: nucl-th/0005068 (2000).
3. L.J. Abu-Raddad and J. Piekarewicz, Phys. Rev. C 64, 064902 (2001).
4. S. Gardner, and J. Piekarewicz, Phys. Rev. C 50, 2822 (1994).
5. D.B. Kaplan and A.E. Nelson, Phys. Lett. B 175, (1986), 57; B 179, (1986), 409(E).
6. R.E.J. Florizone, Ph.D. thesis, Massachusetts Institute of Technology (unpublished). Available at: http://www.lphys.uni-mainz.de/A1/publications/doctor/.
7. R.E.J. Florizone et al., to be published.
8. A. Kozlov, Ph.D. thesis, University of Melbourne (unpublished). Available at: http://www.lphys.uni-mainz.de/A1/publications/doctor/.
9. A. Kozlov et al., Nucl. Phys. A684, 460 (2001).
10. A. Kozlov et al., to be published.
11. R. Mertig and A. Hubland, *Guide to FeynCalc 1.0*, downloaded from the internet, 1992; R. Mertig, Comp. Phys. Comm. 60, 165 (1991); [http://www.feyncalc.org/](http://www.feyncalc.org/).

12. J.C. David, C. Fayard, G.H. Lamot, and B. Saghai, Phys. Rev. C 53, 2613 (1996).

13. C. Bennhold, F.X. Lee, T. Mart, and L.E. Wright, Nucl. Phys. A639, 227c (1998).

14. R. Williams, C.R. Ji, and S. R. Cotanch, Phys. Rev. D 41, 1449 (1990).

15. T. Mizutani, C. Fayard, G.-H. Lamot, and B. Saghai, Phys. Rev. C 58, 75 (1998).

16. J.M. Udias, P. Sarriguren, E. Moya de Guerra, E. Garrido, and J.A. Caballero, Phys. Rev. C 48, 2731 (1993).

17. J.M. Udias, P. Sarriguren, E. Moya de Guerra, and J.A. Caballero, Phys. Rev. C 53, 1488 (1996).

18. K. Amir-Azimi-Nili, J.M. Udias, H. Muther, L.D. Skouras, and A. Polls, Nucl. Phys. A625, 633 (1997).

19. J.A. Caballero, T.W. Donnelly, E. Moya de Guerra, and J.M. Udias, Nucl. Phys. A632, 323 (1998).

20. J.A. Caballero, T.W. Donnelly, E. Moya de Guerra, and Nucl. Phys. A643, 189 (1998).

21. J.M. Udias, J.A. Caballero, E. Moya de Guerra, J.E. Amaro, and T.W. Donnelly, Phys. Rev. Lett. 83, 5451 (1999).

22. J.M. Udias, J.A. Caballero, E. Moya de Guerra, Javier R. Vignote, and A. Escuderos, [nucl-th/0101038](http://arxiv.org/abs/nucl-th/0101038).

23. S. Frullani and J. Mougey, Adv. Nucl. Phys. 14, 1 (1984).

24. T. de Forest Jr., Nucl. Phys. A392, 232 (1983).

25. J. Piekarewicz and R.A. Rego Phys. Rev. C 45, 1654 (1992).

26. R. Schiavilla, V.R. Pandharipande, and R.B. Wiringa, Nucl. Phys. A449, 219, (1986).

27. I.E. Lagaris and V.R. Pandharipande, Nucl. Phys. A359, 331, (1981).

28. R.B. Wiringa *et al.*, Phys. Rev. C 29, 1207, (1984).

29. R. B. Wiringa, Phys. Rev. C 43, 1585 (1991).

30. R. B. Wiringa, private communication.

31. J.L. Forest *et al.*, Phys. Rev. C 54 646 (1996).

32. R.B. Wiringa *et al.*, Phys. Rev. C 51, 38, (1995).

33. B.S. Publiner *et al.*, Phys. Rev. Lett. 74, 4396, (1995).

34. J.F.J. van den Brand *et al.*, Nucl. Phys. A534, 637, (1991).

35. J.F.J. van den Brand *et al.*, Phys. Rev. Lett. 60, 2006, (1988).

36. R. Schiavilla *et al.*, Phys. Rev. Lett. 65, 835 (1990).