Two-phonon background for the double giant resonance

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One of the most exciting features of the recently discovered double giant dipole resonance (DGDR) is the absolute value of its excitation cross section in relativistic heavy ion collisions. These values were extracted from total cross sections by separating a contribution arising from the excitation of single giant dipole (GDR) and quadrupole (GQR) resonances and were found to be enhanced by factors of order of 2-3 for $^{136}\text{Xe}$ and $^{197}\text{Au}$ as compared to any theoretical calculations available. Although for $^{208}\text{Pb}$ the experiment-theory correspondence is much better, theoretical calculations still underestimate the DGDR cross section by about 30%.

At the present time we have an experimental proof that Coulomb excitation of the DGDR in relativistic heavy ion collisions occurs in a two-step process. The DGDR states are embedded in a sea of other two-phonon states. In this paper we consider the sea contribution to the total cross sections. It will be concluded that numerous two-phonon states, other than [GDR $\otimes$ GDR] ones, excited in one-step processes form a physical background in the DGDR energy region which has to be taken into account in the analysis of experimental data.

The first investigation of the direct excitation of two-phonon states at high energies from the ground state was done in refs. 2,8. The main purpose of these papers was to look for resonance structures in photoexcitation cross sections related to the excitation of rather selected two-phonon configurations. Although in ref. 2 it was pointed out that [GDR $\otimes$ GQR] $^+_1$ and [GDR $\otimes$ $2^+_1$] $^+_1$ configurations were not the only ones (among many different two-phonon $[1^+ \otimes 2^+]^+_1$ states) to determine the total cross section, two-phonon $^+_1$ configurations made of phonons of other multipolarities were omitted in calculation.

While dealing with electromagnetic, or with Coulomb excitation from a $0^+$ ground state, the priority attention has to be paid to the final states with the total angular momentum and parity $J^\pi = 1^-$. Making use of the formalism presented in ref. 2 we have calculated firstly the cross section of photoexcitation of two-phonon states $[\lambda_1^{\pi_1} \otimes \lambda_2^{\pi_2}]_1^-$, where $\lambda_1^{\pi_1}$ and $\lambda_2^{\pi_2}$ are both natural $\lambda^{\pi}$ ($\pi^\pi = (-1)^{\lambda}$) and unnatural $\lambda^{\pi}$ ($\pi^\pi = (-1)^{\lambda+1}$) parity phonons with multipolarity $\lambda$ from 0 to 9. A phonon basis is obtained by solving quasiparticle-RPA equations for each multipolarity $\lambda^{\pi}$ within the quasiparticle-phonon model (QPM). These equations provide a set of one-phonon states $\lambda^{\pi}(i)$ with the same spin and parity, but with different excitation energies $E(i)$ and internal fermion structure of phonons; the index $i$ is introduced to distinguish between them.

The results of the calculation for $^{136}\text{Xe}$ and $^{208}\text{Pb}$ integrated over the energy interval from 20 to 35 MeV are presented in table I. Each configuration $[\lambda_1^{\pi_1} \otimes \lambda_2^{\pi_2}]$ in the table means a sum over a plenty of two-phonon states made of phonons with a given spin and parity $\lambda_1^{\pi_1}, \lambda_2^{\pi_2}$, but different RPA root numbers $i_1, i_2$ of its constituents

$$
\sigma([\lambda_1^{\pi_1} \otimes \lambda_2^{\pi_2}]) = \sum_{i_1, i_2} \sigma([\lambda_1^{\pi_1}(i_1) \otimes \lambda_2^{\pi_2}(i_2)])
$$

The total number of two-phonon $1^-$ states included in this calculation for each nucleus is about $10^5$. The absolute value of the photoexcitation of any two-phonon state under consideration is negligibly small because of the lower density of such states. As a rule, unnatural parity phonons play less important role than natural parity ones. For these reasons we presented in the table only the sums for [natural$\otimes$unnatural] and [unnatural$\otimes$unnatural] two-phonon configurations.

The cross section of the photoexcitation of all two-phonon $1^-$ states in the energy region 20-35 MeV from the ground state equals in our calculation to 511 mb and 423 mb for $^{136}\text{Xe}$ and $^{208}\text{Pb}$, respectively. It is not surprising that we got a larger value for $^{136}\text{Xe}$ than for $^{208}\text{Pb}$. This is because the phonon states in Xe are composed of a larger number of two-quasiparticle configurations due to the pairing. The same values for two-phonon states with angular momentum and parity $J^\pi = 2^+$ are an order of magnitude smaller. We point out that the direct excitation of $[1^+ \otimes 1^-]_2^+$ configuration or [GDR $\otimes$ GDR]$_2^+$ is negligibly weak. The calculated values should be compared to the cross section of the photoexcitation of the single-phonon GDR which in our calculation equals to 2006 mb and 2790 mb, respectively. A contribution of two-phonon $1^-$ states to the total cross section at GDR energies is weaker than at higher energies because of the lower density of two-phonon states.
and lower excitation energy and can be neglected considering the GDR itself.

For 208Pb photoexcitation cross sections are known from experimental studies in \((\gamma, n)\) reactions up to the excitation energy about 25 MeV \(^8\). It was shown that QPM provides a very good description of the experimental data in the GDR region \(^8\), while theoretical calculations at higher excitation energies which account for contributions from the single-phonon GDR and GQR essentially underestimated the experimental cross section \(^8\). The experimental cross sections above 17 MeV are shown in Fig. 1 together with theoretical predictions. The results of the calculations are presented as strength functions obtained with averaging parameter equal to 1 MeV. The contribution to the total cross section of the GQR \(^8\) (short-dashed curve), the high energy tail of GDR (long-dashed curve), and their sum (squared curve), are taken from ref. \(^8\). The curve with triangles represents the contribution of the direct excitation of the two-phonon states from our present studies. The two-phonon states form practically a flat background in the whole energy region under consideration. Summing together the photoexcitation cross sections of all one- and two-phonon states we get a solid curve which is in a very good agreement with the experimental data.

From our investigation of photoexcitation cross sections we conclude that in this reaction very many different two-phonon states above the GDR contribute on a comparable level, forming altogether a flat physical background which should be taken into account in the description of experimental data. On the other hand, Coulomb excitation in relativistic heavy ion collisions provides a unique opportunity to excite a very selected number of two-phonon states by the absorption of two virtual \(\gamma\)’s in a single process of projectile-target interaction \(^8\). Theoretically this process is described using the second order perturbation theory of the semi-classical approach of A. Winther and K. Alder \(^8\). Since excitation cross sections to second order are much weaker than to first order of the theory, only two-phonon states connected to the ground states by two E1-transitions can be observed. These two-phonon states have the structure \(1^- (i) \otimes 1^- (i')|_{0^+, 2^+}\) and form the DGDR.

To describe the properties of the DGDR in 208Pb we applied the technique developed in refs. \(^1^1\)\(^2\). Within this technique we couple two-phonon \(|1^- (i) \otimes 1^- (i')|_{0^+, 2^+}\) states with many three-phonon ones. Two-phonon states are built from the same six \(1^- (i)\) RPA phonons as in ref. \(^13\) which have the largest B(E1) values and exhaust 90.6% of the EWSR. Only \(0^+\) and \(2^+\) components of the DGDR were considered (see ref. \(^13\) for the quenching of the \(1^+\) component). We included in the calculation about 7000 three-phonon states which have the largest matrix elements for the interaction with the selected 21 two-phonon states. Diagonalization of the QPM Hamiltonian on the basis of these two- and three-phonon states yields a set of \(0^+\) (and \(2^+\)) states; the wave function of each state includes all two- and three-phonon terms with different weights for different states. To distinguish between these states we introduce the index \(\nu\). To obtain the Coulomb excitation cross section in second order perturbation theory we also need to have the structure of the GDR as an intermediate state. For that we calculated the GDR fine structure by coupling the same six strongest one-phonon \(1^- (i)\) states to about 1200 two-phonon \(1^- \) states in the GDR region.

The cross section of the DGDR\((\nu_{0^+, 2^+})\) states excitation via the GDR\((\nu_{1^-})\) states in this reaction equals to

\[
\sigma_{\nu_{0^+, 2^+}} = \left| \sum_{\nu_{1^-}} A(E_{\nu_{1^-}, E_{\nu_{0^+, 2^+}}}) < 1^- (\nu_{1^-})|E1||0^+_{g.s.}> \times < [1^- \otimes 1^- (\nu_{0^+, 2^+})||E1||1^- (\nu_{1^-})]> \right|^2
\]

where \(A(E_1, E_2)\) is the energy dependent reaction amplitude. A straightforward calculation of the two-step process for the excitation of 7000 DGDR states via 1200 intermediate GDR states is very time consuming. Making use of a very smooth dependence of the function \(A(E_1, E_2)\) on both arguments we tabulated this function and used it in the final calculation of the DGDR Coulomb excitation cross section in relativistic heavy ion collisions. We considered the excitation of the DGDR in the projectile for a 208Pb (640 A-MeV) \(+208\)Pb collision, according to the experiment in ref. \(^3\), and used the minimum value of the impact parameter, \(b= 15.54\) fm, corresponding to the parameterization of ref. \(^14\).

The cross section for Coulomb excitation of the DGDR is presented in Fig. 2 by the short-dashed curve as a strength function calculated with an averaging parameter equal to 1 MeV. The width of the DGDR for 208Pb is

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\(\text{The cross section for Coulomb excitation of the DGDR is presented in Fig. 2 by the short-dashed curve as a strength function calculated with an averaging parameter equal to 1 MeV. The width of the DGDR for } 208\text{Pb is}\)
very close to $\sqrt{2}$ times the width of the single GDR, as for the case of $^{136}$Xe [12]. The contribution of the background of the two-phonon $1^-$ states to the total cross section is shown by a long-dashed curve in the same figure. It was calculated in first order perturbation theory. The role of the background in this reaction is much less important than in photoexcitation studies. First, it is because in heavy ion collisions we have a special mechanism to excite selected two-phonon states in the two-step process. Second, the Coulomb excitation amplitude is exponentially decreasing with the excitation energy, while the E1 photoexcitation amplitude is linearly increasing. Nonetheless, Fig. 2 shows that the direct excitation of two-phonon $1^-$ states cannot be completely excluded from consideration of this reaction. Integrated over the energy interval from 20 to 35 MeV these states give a cross section of 50.3 mb which should be compared to the experimental cross section in the DGDR region for the $^{208}$Pb $(640$ A-MeV) + $^{208}$Pb reaction which is equal to 380 mb [3]. Taking into account that the reported experiment/theory enhancement is of about 30\% for this reaction, the two-phonon $1^-$ states omitted in extracting the DGDR strength from experimental cross section maybe responsible for an appreciable part of the effect. Apprceciably values of one-step processes in DGDR excitation is not in contradiction with experimental findings. It is known that Coulomb excitation of a projectile in an n-step process has the following dependence on the target charge: $Z_T^{(2-n)}$. The reported value $n = 1.8(3)$ in the DGDR region for $^{208}$Pb [3] allows for some contribution of one-step transitions.

The solid line in Fig. 2 is the sum of DGDR and two-phonon background excitations in relativistic heavy ion collisions. As mentioned above, this curve has a closer correspondence to the experimental cross sections reported for the DGDR than the short-dashed one. Centroids and widths calculated for the short-dashed and solid curves of this figure display other interesting features. Taking into account the background of two-phonon $1^-$ states results in a visual shift of the DGDR centroid by -200 keV and a 16\% increase of the “DGDR” width. The same effect, although with large experimental uncertainties, was reported in an experiment with respect to the harmonic picture of the DGDR excitation [3]. We point out that the effect of the visual shift of the DGDR centroid is even a bit larger than anharmonicity effects examined for this nucleus in ref. [13].

The effect of the direct excitation of two-phonon states from the ground state was considered in the present paper under the assumption that these states do not interact between each other and with other one- and three-phonon configurations. The first type of interaction, unharmonicity effect, is rather weak as discussed in the refs. [2, 15] and cannot change our conclusions as far as the wide energy region is considered. But one may argue that a weak coupling to one-phonon $1^-$ states with much larger matrix elements of photoexcitation from the ground state may reduce the calculated cross sections due to the destructive interference with our two-phonon $1^-$ states. Of course, the total contribution of this interference term integrated from 0 MeV to infinity equals to zero because of orthogonality of wave functions but it cannot be excluded from a consideration for a limited energy interval. To estimate its influence on our results we have performed a calculation in which $1^-$ states in $^{208}$Pb where described by the wave function consisting one- and two-phonon components. The interaction between these components has been calculated microscopically [3]. The full calculation with including all $10^5$ two-phonon $1^-$ configurations is not possible and we have done several tests with different truncations of the two-phonon basis ending up in total $10^3$ components for each test. All one-phonon configurations with the energy below 35 MeV have been included in calculations. Selecting important two-phonon configurations we have chosen the ones which have the largest matrix elements of the electromagnetic excitation $< |\lambda_T^{i_1}(i_1) \otimes \lambda_T^{i_2}(i_2)|_{1-}\sqrt{|\lambda_T^{i_1}(i_1) \otimes \lambda_T^{i_2}(i_2)|_{1-}|E1||g.s.}>$ and the largest matrix elements of interaction with one-phonon $1^-$ states. The results of our test calculations are the following.

We do observe the effect of interference between transitions to one- and two-phonon components of the wave functions of $1^-$ states. In all runs the interference is constructive in the whole energy region under consideration leading to an additional enhancement of the calculated cross sections. It becomes destructive only below 14.5 MeV reducing a little the total E1-strength for the GDR. In the DGDR energy region we estimate from our calculations the size of the interference effect
roughly as 30-50% of what we get from the direct excitation of non-interacting two-phonon 1− states. We think that this estimate is an upper limit for the 208Pb because many other two-phonon configurations will block somehow a coupling of selected two-phonon configurations to one-phonon ones. On the other hand, the interference between transitions to one- and two-phonon configurations may play much important role for the 136Xe in which the interaction between these configurations is sufficiently stronger as compared to 208Pb. It requires an extra study.

In any case, the influence of a sea of two-phonon 1− states above 20 MeV on calculated cross sections considered in the present paper in the approximation of the pure two-phonon configurations can be taken as a lower limit of the real effect.

In conclusion, we investigated the contribution of the direct excitation of a sea of two-phonon states above the GDR to photon-neutron cross sections and to cross sections in relativistic heavy ion Coulomb excitation. These states form a flat structureless background which is very important for the correct description of the photon-neutron data. Due to the peculiarities of Coulomb excitation in heavy ion collisions, its role is less important in this case, but its consideration appreciably removes the disagreement between experiment and theory.

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| Configuration | 136Xe, mb | 208Pb, mb |
|---------------|---------|---------|
| [0+ ⊗ 1−]1−  | 4.4     | 3.9     |
| [1− ⊗ 2+]1−  | 36.6    | 44.8    |
| [2+ ⊗ 3−]1−  | 82.8    | 33.1    |
| [3− ⊗ 4+]1−  | 101.0   | 56.7    |
| [4+ ⊗ 5−]1−  | 68.9    | 37.3    |
| [5− ⊗ 6+]1−  | 49.2    | 46.2    |
| [6+ ⊗ 7−]1−  | 31.9    | 49.8    |
| [7− ⊗ 8+]1−  | 13.6    | 12.5    |
| [8+ ⊗ 9−]1−  | 4.9     | 9.0     |
| \[ \sum_{\lambda_1,\lambda_2=1}^2 [\lambda_1^{n1} \otimes \lambda_2^{n2}]_{1−} \] | 71.4 | 58.5 |
| \[ \sum_{\lambda_1,\lambda_2=1}^2 [\lambda_1^{n1} \otimes \lambda_2^{n2}]_{1−} \] | 46.7 | 71.1 |
| \[ \sum_{\lambda_1,\lambda_2=0}^9 [\lambda_1^{n1} \otimes \lambda_2^{n2}]_{1−} \] | 511.4 | 422.9 |
| \[ \text{GDR} \otimes \text{GDR} \]2+ | 0.33 | 0.22 |
| \[ \sum_{\lambda_1,\lambda_2=1}^2 [\lambda_1^{n1} \otimes \lambda_2^{n2}]_{2+} \] | 38.1 | 21.7 |
FIG. 1. Photo-neutron cross section for $^{208}$Pb. Experimental data (dots with experimental errors) are from ref. [8]. The long-dashed curve is the high energy tail of the GDR, the short-dashed curve is the GQR, and the curve with squares is their sum. The contribution of two-phonon states is plotted by a curve with triangles. The solid curve is the total calculated cross section.

FIG. 2. The contribution for the excitation of two-phonon $1^-$ states (long-dashed curve) in first order perturbation theory, and for two-phonon $0^+$ and $2^+$ DGDR states in second order (short-dashed curve). The total cross section (for $^{208}$Pb (640 A·MeV) + $^{208}$Pb) is shown by the solid curve.
$^{208}\text{Pb} + ^{208}\text{Pb}$

$(640 \text{ A}\cdot\text{MeV})$