Tachyon cosmology with non-vanishing minimum potential: a unified model

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Abstract

We investigate the tachyon condensation process in the effective theory with non-vanishing minimum potential and its implications to cosmology. It is shown that the tachyon condensation on an unstable three-brane described by this modified tachyon field theory leads to lower-dimensional branes (defects) forming within a stable three-brane. Thus, in the cosmological background, we can get well-behaved tachyon matter after tachyon inflation, (partially) avoiding difficulties encountered in the original tachyon cosmological models. This feature also implies that the tachyon inflated and reheated universe is followed by a Chaplygin gas dark matter and dark energy universe. Hence, such an unstable three-brane behaves quite like our universe, reproducing the key features of the whole evolutionary history of the universe and providing a unified description of inflaton, dark matter and dark energy in a very simple single-scalar field model.

1 Introduction

Rolling tachyon on unstable D-branes is thought of a natural candidate to derive inflation [1, 2, 3, 4, 5, 6, 7]. This tachyon inflation model has some particular merits, e.g., the inflation can end up with a radiation-dominated stage [8, 9] and a “tachyon matter”-dominated stage [10, 11, 12]. All the processes can be described by the Dirac-Born-Infeld (DBI) effective action of a rolling tachyon [13, 14, 11] in the cosmological background:

\[ S = - \int d^4 x \sqrt{-g} V(T) \sqrt{1 + g^{\mu \nu} \partial_\mu T \partial_\nu T}, \]  

(1)
where $V(T)$ is the potential. Besides other forms, the tachyon potential derived in open string field theory in [15] is

$$V(T) = \frac{V_m}{\cosh(\beta T)},$$

where $V_m$ is the tension of the unstable D-brane and $\beta$ is a constant. The potential has the minimum value $V(T = \pm \infty) = 0$ and the maximum value $V(T = 0) = V_m$, with $V_m$ equal to the tension of the unstable D3-brane. At the end of condensation, lower-dimensional defects (stable D-branes and anti-D-branes) form in regions where $T(\vec{x}) = 0$ and pressureless tachyon matter forms in regions where $T(\vec{x}) \to \pm \infty$ (the former can also be viewed as the pressureless tachyon matter).

However, this tachyon cosmological model is examined to be problematic [4, 12, 5]. At early times, the tachyon potential is too steep to produce large enough e-foldings [4, 12, 5]. As tachyon rolls down the potential, we get a tachyon matter dominated universe as the tachyon grows to infinity [10, 11, 4]. The pressureless tachyon matter does not oscillate towards the minimum of the potential so that ordinary tachyonic (p)reheating mechanism does not work. What is more, we also encounter singular caustic formation at late times when considering inhomogeneous condensation around a homogeneously condensing tachyon field [16, 17, 18, 19].

In this paper, we try to improve the original tachyon cosmological model by slightly modifying the tachyon potential. It is known that the effective tachyon potential $V(T)$ should tend to zero as $T \to \pm \infty$ when the closed string vacuum is reached, according to Sen’s conjecture (see review [20]). This conjecture is now taken as a fact and has been verified in string field theory [21, 22, 23, 24, 25, 26, 26, 27, 28, 29] in high accuracy (up to 90% of the conjectured value is achieved in superstring field theory). Nevertheless, we consider the tachyon cosmology in the case when the minimum of the potential is non-zero. A simple way to get this kind of tachyon potential is to adding a positive constant $v$ into the original potential $V$, so that the action for an unstable three-brane becomes

$$S = - \int d^4x \sqrt{-g} [V(T) + v] \sqrt{1 + g^{\mu\nu} \partial_\mu T \partial_\nu T}.$$  

When $T$ is small at the beginning, the potential $V(T) + v \sim V(T)$ (for small $v$). When $T$ grows large at late times, we have $V(T) \sim 0$ so that $V(T) + v \sim v$. Hence, the following features are a natural result of this modified tachyon cosmological model:

- The effective potential is flattened, though the improvement is limited for small $v$.

- The final products of tachyon condensation in the new theory are lower-dimensional branes (defects) within a stable 3-brane. By contrast, in the original theory with $v = 0$, the tachyon condensation leads to defects and closed string vacuum.
• We still have nearly tachyon matter for small $v$. But now it does not generically tend to “freeze” or develop caustics towards the end of condensation, which are a generic result in all regions in the original tachyon field theory.

• The (p)reheating mechanisms adopted in the original theory still work in the new theory. The fluctuations now do not vanish since the descendent 3-brane can fluctuate.

• After undergoing the inflationary, (p)reheating and tachyon matter-dominated stages, the universe on the 3-brane ends with a Chaplygin gas (CG) dark matter and dark energy stage (it does not take infinitely long time to get this stage in the cosmological background, though the minimum potential is at $|T| \to \infty$). The constant $v$ contributes as the cosmological constant in the end. Correspondingly, the “velocity” of the 3-brane grows first to a maximum value at a critical time and then decreases, ultimately to zero.

• All the above evolutionary processes of the universe happen on the same 3-brane, which evolves from an unstable state to a stable state. We do not need to compactify extra dimensions or annihilate extra branes. In previous brane and tachyon inflation models, our universe may reside on the lower-dimensional defects or on a surviving brane from collisions of a stack brane-anti-brane.

Hence, the case with $v \neq 0$ (even with very small $v$) is completely different from the original theory with $v = 0$: they lead to different final products. The $v \neq 0$ case leads to branes within branes, though this kind of theory and potential has not yet been constructed in string theory. This provides almost the simplest scalar field model to date that can give a unified description of the key features of the whole universe history.

The paper is organised as follows. We will examine the properties and dynamics of the modified tachyon field theory (3) in Minkowski spacetime in Section. 2. We further discuss the tachyon cosmology in this modified theory in Section. 3. In Section. 4, the generalised model that can give rise to the generalised Chaplygin gas model is given. Conclusions are made in the last section.

2 Analysis in Minkowski spacetime

The equation of motion from the action (3) is

$$\left[\Box T - f(T)\right] (1 + \partial T \cdot \partial T) = \frac{1}{2} \partial^\mu T \partial_\mu (1 + \partial T \cdot \partial T),$$

(4)
where $\Box = g^{\mu\nu}\partial_\mu\partial_\nu$ and

$$f(T) = \frac{V'}{V + v} = [\ln(V + v)]'.$$

(5)

The primes denote derivative with respect to the field $T$. For the potential (2), the expression is

$$f(T) = -\frac{\beta \tanh(\beta T)}{1 + \frac{v}{V_m} \cosh(\beta T)}.$$

(6)

For $v = 0$, $f(T)$ decreases monotonically from $f(T = 0) = 0$ to $f(T \to \infty) = -\beta$. For $v \neq 0$, the situation is different: $f$ first decreases from $f(T = 0) = 0$ to the minimum value $f_{\text{min}}$ at some finite $T = T_m$, and then increases to $f(T \to \infty) = 0$. For a small $v$, we approximately have

$$T_m \simeq \frac{1}{\beta} \cosh^{-1} \left( \frac{V_m}{v} \right)^{\frac{1}{3}}, \quad f_{\text{min}} \simeq -\frac{\beta}{1 + \left( \frac{v}{V_m} \right)^{\frac{2}{3}}}. \quad (7)$$

In the original theory (1) with $v = 0$, there form lower-dimensional defects in regions where $T = 0$, which are kinks and anti-kinks for the real tachyon field case, representing BPS D2-brane and anti-D2-branes respectively. In the new theory with $v \neq 0$, kinks and anti-kinks should also form at $T = 0$ because the behaviour is similar near the top of the potential $V(T)$ and the modified one $V(T) + v$.

However, in other regions where the tachyon rolls down the potential: $V(T) \to 0$, the action (3) will evolve into the action describing a stable brane with tension $v$, with the tachyon $T$ becoming a massless scalar $Y$:

$$S(T) \longrightarrow S(Y) = -v \int d^4x \sqrt{-g} \sqrt{1 + g^{\mu\nu}\partial_\mu Y \partial_\nu Y}. \quad (8)$$

The scalar $Y$ describes the movement and fluctuations of the stable D3-brane.

Hence, the final products of the condensation of the unstable 3-brane described by the theory (3) are a stable 3-brane plus lower-dimensional stable branes. The latter form inside the 3-brane and may evolve further. We can understand this by taking the unstable 3-brane described by (3) as a non-BPS D3-brane with tension $V_m$ “glued” to another BPS D3-brane with tension $v$. The two branes share the same dynamics and are described the unique action (3). As the tachyon rolls down the potential, the “non-BPS brane” will decay into lower dimensional defects as usual, while the “BPS brane” remain as the final stable 3-brane.

In what follows, we examine the theory in detail in the Minkowski spacetime, which indeed verifies the above speculations.
2.1 Homogeneous solution

In the homogeneous case, the energy density is conserved and is a constant (no energy loss is assumed):

\[ T_{00} = \frac{V + v}{\sqrt{1 - T^2}}. \] (9)

This case is similar to that in the original theory \( v = 0 \). We have the minimum velocity \( \dot{T} \) at \( T = 0 \) and the maximum velocity \( |\dot{T}| = |\dot{T}|_{\text{max}} \) at \( |T| \to \infty \). If the minimum velocity is set to be \( \dot{T}|_{T=0} = 0 \), it is easy to determine \( |\dot{T}|_{\text{max}} \).

\[ |\dot{T}|_{\text{max}} = \left[ 1 - \left( \frac{v}{V_m + v} \right)^2 \right]^{\frac{1}{2}}. \] (10)

As expected, \( |\dot{T}|_{\text{max}} = 1 \) when \( v = 0 \). The brane is driven by the tachyon potential to acquire the maximum velocity \( |\dot{T}|_{\text{max}} < 1 \) for \( v \neq 0 \) at \( T \to \pm \infty \).

2.2 Static solution

In the static and 1-dimensional case (along the \( x \)-direction), the pressure is conserved

\[ T_{11} = -\frac{V + v}{\sqrt{1 + T'^2}} = -(V_0 + v). \] (11)

where \( T' = \partial_x T \) and \( V_0 = V(T_0) \). \( T = T_0 \) is where we get \( T'|_{T=T_0} = 0 \), i.e., where we get the maximum \( |T| \). Thus, we have the solution

\[ T' = \pm \left[ \left( \frac{V + v}{V_0 + v} \right)^2 - 1 \right]^{\frac{1}{2}}. \] (12)

In the \( v = 0 \) case, this is the kink and anti-kink solution. The kink and anti-kink are located at \( T = 0 \). At the end of condensation \( V_0 \to 0 \), the (anti-)kink solution becomes solitonic with the field gradient \( |T'| \to \infty \), representing a BPS (anti-)D2-brane. For the \( v \neq 0 \) case, we also have the maximum field gradient at \( T = 0 \) and

\[ |T'|_{\text{max}} = \left[ \left( \frac{V_m + v}{v} \right)^2 - 1 \right]^{\frac{1}{2}} \text{ as } V_0 \to 0. \] (13)

It looks like that the unstable 3-brane is rotated away from the \( x \)-direction by an angle \( \theta = \tan^{-1} |T'|_{\text{max}} \) at the “kink” position \( T = 0 \) when it evolves into the stable 3-brane.
In the limit \( v = 0 \), the angle is \( \theta = \pi/2 \). This indicates that the descendent BPS (anti-)D2-brane is a point-like object along the \( x \)-dimension at the end of condensation in this case.

The energy of this system is

\[
\mathcal{E} = \int d^3x (V + v) \sqrt{1 + T'^2} = \int d^2x \int dx \frac{(V + v)^2}{V_0 + v}. \tag{14}
\]

It is easy to prove that it is divergent for any \( v \neq 0 \) in the limit \( V_0 \to 0 \). It converges only when \( v = 0 \), giving the tension of the descendent D2-brane: \( \pi V_m/\beta \). That is, in the \( v \neq 0 \) case, we get an extended object, but not a solitonic defect, along the \( x \)-direction.

2.3 Spacetime-dependent condensation

In the original tachyon effective theory (1), both the numerical and theoretical calculation in the spacetime-dependent case indicate that there are singularities either near the core of the defects [30, 31] or around extrema (caustics) [16, 17, 18, 19]. It is suggested that they come from the fact that higher derivative terms are not taken into account in the effective theory [30]. We shall show that, in the new theory with the modified potential, the former singularity still exists while the latter is avoided.

As stated above, the dynamics near the top of the potential with small \( v \neq 0 \) is not altered dramatically compared with the \( v = 0 \) case. In the vicinity of a position where \( T = 0 \), we can expand the field in the following way: \( T(t, x) = a(t)x + (1/6)c(t)x^3 + \cdots \) [30, 19]. Since \( f(T)|_{T=0} \approx -\beta^2 T/[1 + (v/V_m)] \), we have from the equation of motion (4):

\[
\ddot{a} = \frac{2aa'^2 + c}{1 + a^2} + \tilde{\beta}a, \tag{15}
\]

where \( \tilde{\beta} = \beta/(1 + v/V_m)^{1/2} \). Neglecting \( c \), we get the solution:

\[
a(t) = \frac{a_0}{\cos(\beta t)}, \tag{16}
\]

where \( a_0 \) is a constant. Thus, the behaviour of (anti-)kinks is similar to that in the original theory with \( v = 0 \): the gradient grows to infinity in a finite time.

In other regions with no kinks and anti-kinks forming, the tachyon can grow to infinity driven by the potential [16, 17, 18]. Around extrema in these regions, we split the field into a homogeneous field plus small perturbations: \( T(t, \vec{x}) = T_0(t) + \tau(t, \vec{x}) \) [19]. The equation of motion (4) up to the quadratic order thus becomes

\[
[\ddot{\tau} + f(1 - \dot{T}_0^2)] + [\dddot{\tau} - 2\dot{T}_0 \ddot{\tau} - (1 - \dot{T}_0^2)\nabla^2 \tau] + [(f + \ddot{T}_0)\nabla \tau \cdot \nabla \tau - f \dot{\tau}^2 - 2\dot{T}_0(\nabla \tau \cdot \nabla \dot{\tau} - \dot{\tau} \nabla^2 \tau)] = 0. \tag{17}
\]
For $v = 0$, $f(T) \to -\beta$ as $|T| \to \infty$. The leading terms in the above equation indicate that the velocity gets the maximum value $|\dot{T}_0|_{\text{max}} \to 1$, with $\ddot{T}_0 \to 0$, as $|T_0| \to \infty$. The precise solution leads to the relation:

$$1 + \partial T \cdot \partial T \simeq -2\dot{T} + (\nabla\tau)^2 \to 0,$$  \hspace{1cm} (18)

which causes caustic formation: $\tau \to \infty$ in some regions, and causes the perturbations to “freeze”: $\tau \to 0$ in some other regions. The caustic formation is a signal of instability.

Now for the $v \neq 0$ case, $f(T) \to 0$ as $|T| \to \infty$. So we get the maximum velocity $|\dot{T}_0|_{\text{max}} < 1$ at the end of condensation since $\dot{T}_0 \to 0$. If we take $|\dot{T}_0| \simeq |\dot{T}_0|_{\text{max}}$ (a constant) at late times, Eq. (17) becomes

$$[\ddot{T} - (1 - \dot{T}_0^2)\nabla^2\tau] - 2\dot{T}_0(\nabla\tau \cdot \nabla\dot{T} - \dot{T}\nabla^2\tau) = 0.$$  \hspace{1cm} (19)

Thus, it has the plane wave solution of a massless scalar:

$$\tau \sim e^{i\omega t + ik \cdot \vec{x}},$$  \hspace{1cm} (20)

where $\vec{x} = \vec{x}/\sqrt{1 - T_0^2}$ and $\omega^2 = k^2$. Hence, the tachyon now does not generically freeze and develop caustics at the end of condensation in the modified theory. This is because we get a stable 3-brane at late times. However, since a solution satisfying $1 + \partial T \cdot \partial T = 0$ is always a solution to the full equation of motion (4), regardless of what the potential is, caustics may form at some special positions where the spacetime-dependent solution happens to satisfy $1 + \partial Y \cdot \partial Y = 0$. This is also realised in the cosmological background in the Chaplygin gas model [33, 49].

So the preliminary analysis above is verified: the final products of the tachyon condensation on this kind of unstable 3-branes are: a stable 3-brane of tension $v$, containing lower-dimensional defects (stable 2-brane pairs with opposite charges) or their descendent objects.

### 3 Tachyon cosmology with the modified potential

We now consider the tachyon condensation process with the modified potential in the FRW cosmological background. The energy density and the pressure of this system are respectively:

$$\rho = \frac{V + v}{\sqrt{1 - T^2}}, \quad p = -(V + v)\sqrt{1 - T^2}.$$  \hspace{1cm} (21)
The equation of state is
\[ w = \frac{p}{\rho} = -(1 - \dot{T}^2). \] (22)

Thus, we can have pressureless tachyon matter only when \(|\dot{T}| = 1\), which can occur in the \(v = 0\) case.

In a flat universe, the Friedmann equations are
\[ H^2 = \frac{1}{3M_{Pl}^2} \left( \frac{V + v}{\sqrt{1 - \dot{T}^2}} \right), \] (23)
and
\[ \frac{\ddot{a}}{a} = \frac{1}{3M_{Pl}^2} \left( \frac{(V + v)(1 - \frac{3}{2} \dot{T}^2)}{\sqrt{1 - \dot{T}^2}} \right). \] (24)

From the second equation, we learn that the accelerated expansion occurs when \(|\dot{T}| < \sqrt{6}/3\). If the maximum velocity is given by Eq. (10), the condition that we have a decelerated stage after inflation is \(v < \frac{V_m}{(\sqrt{3} - 1)}\).

The equation for \(T(t)\) in the expanding universe is
\[ \frac{\ddot{T}}{1 - \dot{T}^2} + 3H\dot{T} + f(T) = 0, \] (25)
where \(f = \ln(V + v)\)'s. So \(\ln(V + v)\) is the “effective” potential that corresponds to the potential in ordinary scalar field model. It is easy to verify that the addition of the constant \(v \neq 0\) tends to flatten the original effective potential.

In the original tachyon condensation \(v = 0\), \(f(T)\) decreases from zero at \(T = 0\) to the minimum value \(f_{\text{min}}(T) = -\beta\) at \(T \to \infty\). In this situation, we get \(\dot{T} = 0\) and the maximum velocity \(|\dot{T}|_{\text{max}} = 1\) at the end of condensation, as learned from Eq. (25). By contrast, for the \(v \neq 0\) case, \(f(T)\) reaches the minimum value \(f_{\text{min}}(T)\) at a finite field \(T = T_{\text{min}}\) (as given in Eq. (7) for the small \(v\) case) and then turns to grow to zero. So it is possible that we get \(3H\dot{T} + f(T) = 0\) at some critical value \(T = T_c\) (different from \(T_m\) in Minkowski spacetime), where the velocity reaches the maximum value \(|\dot{T}|_{\text{max}}\). In this case, the velocity of the brane constrained by Eq. (25) first grows to \(|\dot{T}|_{\text{max}}\) at \(T = T_c\) and then decreases, ultimately to zero as \(T \to \infty\).

To illustrate this more precisely, we express Eq. (25) as follows, by inserting Eq. (23)
\[ \dot{T} = -(1 - \dot{T}^2)\sqrt{V + v} \left[ \frac{\sqrt{3}}{M_{Pl} (1 - \dot{T}^2)^{\frac{1}{2}}} \dot{T} + g(T) \right], \] (26)
where \(g(T) = f(T)/\sqrt{V + v} \leq 0\). Analogous to \(f(T)\), the function \(g(T)\) equals to zero at \(T = 0, \infty\), and has a negative minimum value at some value of \(T\). This equation can be
further simplified to
\[ \partial_t \left[ \frac{(1 - \dot{T}^2)^{\frac{1}{2}}}{\sqrt{V + v}} \right] = \frac{\sqrt{3}}{2M_{Pl}} \dot{T}^2. \]  
(27)

The first term in the square bracket of Eq. (26) increases monotonically with \( \dot{T} \) as \( T \) grows. At the beginning, the term \( g(T) \) dominates. As the tachyon rolls down the potential, we shall get a critical point where the two terms inside the square bracket cancels out, i.e.,
\[ T = T_c : \quad \ddot{T} = 0. \]  
(28)

That is, we get the maximum velocity \( |\dot{T}|_{\text{max}} \) at this critical point. But this critical value \( T_c \) takes a complicated form generically and cannot be easily determined analytically. For the small \( v \) case, \( |\dot{T}|_{\text{max}} \) is closed to 1 and so it is given by
\[ |\dot{T}|_{\text{max}} = \dot{T}|_{T=T_c} \simeq \sqrt{1 - \frac{9(V + v)^6}{M_{Pl}^4 V^4(r^4)|_{T=T_c}}}. \]  
(29)

Beyond this critical value \( T_c \), the first term in the squared bracket becomes dominant and the velocity \( \dot{T} \) turns to decrease, till to be zero, which can also been seen from Eq. (27).

Still another way to see this evolutionary feature of the velocity is to analyze the action (3) itself. At the beginning, \( \dot{T} \) increases driven by the asymptotic potential \( V + v \). As \( V(T) \) tends to zero at late times, the whole potential \( V + v \) becomes dominant by the constant \( v \) and we get the action (8) for a stable 3-brane, with tachyon \( T \) evolving into a massless scalar \( Y \). It is known that the action (8) in the cosmological background leads to the Chaplygin gas dark matter and dark energy universe [32, 33, 34]. The expansion of the Chaplygin gas universe is driven by the energy released from the deceleration of the stable 3-brane in the bulk, i.e., from the decreasing of the velocity \( \dot{T} \) or \( \dot{Y} \) [35]. Thus, in between the increasing and decreasing stages of the velocity, there must be a critical point \( T = T_c \), at which \( |\dot{T}| \) gets the maximum value \( |\dot{T}|_{\text{max}} \).

In summary, the velocity of the 3-brane changes with the following rule in the FRW metric as time grows
\[ \dot{T} = 0 \rightarrow |\dot{T}| = |\dot{Y}| = |\dot{T}|_{\text{max}} \rightarrow \dot{Y} = 0. \]  
(30)

This evolutionary rule is different from that in the Minkowski spacetime case, in which the brane will never really get the maximum velocity until \( T \rightarrow \pm \infty \).

In terms of Eqs. (22) and (24), the evolutionary feature (30) of the brane velocity implies that the universe will undergo inflation, deceleration (matter-dominated) and
acceleration (dark-energy dominated) stages as time grows if \( v \) is small enough compared with \( V_m \). The evolutionary history of the brane universe is summarised as follows:

\[
\{ \dot{T} = 0 \} \text{ Inflation} \rightarrow \text{radiation dominated} \rightarrow \text{tachyon matter} \n\]

\[\{ |\dot{T}| = |\ddot{T}|_{\text{max}} \} \text{ CG matter} \rightarrow \text{CG dark energy} \{ \dot{T} = \dot{Y} = 0 \}. \tag{31}\]

If \( v \) is large, we may not have the deceleration stage. Then the brane universe always accelerates in the whole evolutionary history. In what follows, we only consider the small \( v \) case.

### 3.1 Inflation

Tachyon inflation occurs on or near the top of the potential. Under the slow-roll conditions: \( T^2 \ll 1 \) and \( \ddot{T} \ll 3H\dot{T} \), we have with the modified potential

\[
H^2 \simeq \frac{1}{3M_{Pl}^2} (V + v), \quad \dot{T} \simeq -\frac{[\ln(V + v)]'}{3H}, \tag{32}\]

The number of e-foldings is given by

\[
N(t) \equiv \int_t^{\tau_0} H(t) dt \simeq -\int_T^{\tau_0} \frac{3H^2(V + v)}{V'} dT. \tag{33}\]

The slow-roll parameters are

\[
\epsilon \simeq \frac{M_{Pl}^2}{3} \frac{V''}{(V + v)^3}, \quad \eta \simeq \frac{M_{Pl}^2}{3} \left[ \frac{3V''}{(V + v)^3} - \frac{2V'''}{(V + v)^2} \right]. \tag{34}\]

Thus, the addition of a constant \( v \) tends to flatten the effective potential and to produce larger e-foldings with the slow-parameters more strongly suppressed. Of course, the improvements are limited in the small \( v \) case. The energy scale difficulty in tachyon inflation pointed out in [5] is not released. Tachyon driven inflation may be only part of the whole inflationary epoch, as suggested in [5, 36].

Since there is very little change in the inflationary equations for the small \( v \) case, the discussion and predictions on inflation in our model should be similar to the original tachyon inflation model [1, 2, 3, 4, 5, 6, 7]. Here, we do not copy the results.

### 3.2 Reheating and “non-freezing” tachyon matter

In the tachyon cosmological model, the universe is quickly dominated by cold tachyon matter after inflation. Therefore, there should be some mechanisms to reheat the universe.
and create particles. As noted in [8], the tachyonic preheating mechanism [37, 38] in hybrid inflation [39] is not applicable in tachyon inflation with $v = 0$ because the tachyon does not oscillate as it rolls down the potential. Therefore, other mechanisms via coupling to gauge field [8, 9] or by introducing the curvaton field [40, 41] are proposed to account for the reheating and particle production in tachyon inflation.

In our case with $v \neq 0$, the tachyon field evolves into a massless scalar towards the end of condensation and the fluctuations do not vanish, since the final product, the stable 3-brane, can fluctuate. So this is more like the tachyonic preheating process in hybrid inflation, compared with the $v = 0$ case.

The reheating mechanism via coupling to gauge field proposed in [8, 9] also applies in our case with small $v$. The DBI effective theory including gauge fields [13, 14, 42]

$$S = - \int d^4x (V + v) \sqrt{- \det (g_{\mu \nu} + \partial_\mu T \partial_\nu T + F_{\mu \nu})}. \quad (35)$$

As we have analyzed, there are also defects forming near $T = 0$ in the modified theory (3). The behaviour is similar to the original case: $T = ax$, with $a$ tending to infinity at the end of condensation. Around extrema, we also have the approximately homogeneous field $T(t)$ plus small perturbations. When the part $V(T)$ dominates the whole potential $V + v$, the discussion of reheating follows that in the original theory made in [8, 9]. At late times when $V \to 0$ and the constant $v$ dominates, the gauge field will decouple from the $T$ field in leading order. Meanwhile, the universe enters into the Chaplygin gas dark universe as $v$ starts to dominate the whole potential.

The velocity $\dot{T}$ reaches the maximum $|\dot{T}|_{\text{max}}$ at a finite critical value $T = T_c$ (i.e., before $V = 0$ is really reached). Approaching this critical value, we get the “tachyon matter” (not really pressureless) if $v$ is small. As analyzed in the previous section, the tachyon matter does not generically “freeze” or develop caustics [17, 18, 19] because it will become the Chaplygin gas dark matter beyond the critical value $T = T_c$ (caustics may form in special case in the Chaplygin gas model [33, 49]). Hence, we get rid (or partially) of the instability problem [4] and the over-abundance problem [12, 43] about tachyon matter found in the original theory: (i) the perturbations $\delta T = \tau$ of the tachyon turn to oscillating modes (since the descendent stable 3-brane can fluctuate) instead of the growing mode $\tau \propto t$ (which causes the instability) in the end; (ii) we get less tachyon matter for a non-vanishing $v$ and even no tachyon matter for a large enough $v$. 

11
3.3 Final stage: Chaplygin gas dark universe

As $V$ tends towards 0 and the constant $v$ dominates the potential, the action (3) of an unstable 3-brane evolves into the one (8) for a stable 3-brane. We enter into the Chaplygin gas-like universe [32, 33, 34] with energy density $\rho = v/\sqrt{1-T^2}$ and pressure $p = -v\sqrt{1-T^2}$ given in Eq. (21), satisfying:

$$p = -\frac{v^2}{\rho}.$$  

(36)

We have discussed the evolutionary process in this stage in detail in our previous work [35].

After the brane acquires this maximum velocity $|\dot{T}|_{\text{max}}$, it turns to slow down due to some mechanisms like gravitational waves linkage into the bulk, fueling the expansion of the universe. The universe will undergo a matter dominated stage and a second-time acceleration stage as the velocity decreases. For very small $v$ (compared with $V_m$), the universe is dominated by the Chaplygin gas dark matter with $\omega \simeq 0$ and with nearly vanishing pressure. However, the perturbations of the field in the Chaplygin gas universe can not account for CMB observations [44, 45] (for a review see [46]). Like the tachyon matter, the perturbations of the Chaplygin gas dark matter behave as cold dark matter. But they disappear in the acceleration stage.

When the velocity decreases further to be lower than $|\dot{T}| = |\dot{Y}| < \sqrt{6}/3$, the expansion of the universe turns to accelerate. As $\dot{Y} \to 0$, the brane universe evolves into a dS universe with the cosmological constant

$$\Lambda = \frac{v}{M_{Pl}^2}.$$  

(37)

Thus, the tension $v$ of the final stable 3-brane is determined by the cosmological constant which should be a very small value.

4 Generalised model

In this section, we give the tachyon field action that can give rise to the generalised Chaplygin gas (GCG) dark universe [33, 47], which agrees better with observations [44, 48, 45, 49]. In the generalised Chaplygin gas model, the energy density and pressure are related via the relation: $p \rho^\alpha = -A$ ($0 < \alpha < 1$). Accordingly, the generalised tachyon field action can be taken as

$$S = -\int d^4x \sqrt{-g} (V + v) \left(1 - |\dot{T}|^{\frac{1+\alpha}{\alpha}}\right)^{\frac{1}{1-\alpha}}.$$  

(38)
Similar generalisation of the DBI action has been discussed in [50]. In the homogeneous case, the maximum speed corresponding to (10) in the previous case is

$$|\dot{T}|_{\text{max}} = \left[ 1 - \left( \frac{v}{V_m + v} \right)^{1+\alpha} \right]^{-\frac{\alpha}{1+\alpha}}. \tag{39}$$

In the FRW cosmology, the energy density and pressure of the system are respectively

$$\rho = (V + v) \left( 1 - |\dot{T}|^{\frac{1+\alpha}{\alpha}} \right)^{-\frac{1}{1+\alpha}}, \quad p = -(V + v) \left( 1 - |\dot{T}|^{\frac{1+\alpha}{\alpha}} \right)^{\frac{\alpha}{1+\alpha}}. \tag{40}$$

The equation of state is

$$\omega = -1 + |\dot{T}|^{\frac{1+\alpha}{\alpha}}. \tag{41}$$

Towards the end of condensation, they satisfy \(^2\):

$$p = -\frac{v^{1+\alpha}}{\rho}, \tag{42}$$

which is just the relation between pressure and energy density in the generalised Chaplygin gas model: \(p\rho^\alpha = -A\) with \(A = v^{1+\alpha}\).

Thus, the Friedmann equations are

$$H^2 = \frac{V + v}{3M_{Pl}^2} \left[ 1 - |\dot{T}|^{\frac{1+\alpha}{\alpha \alpha}} \right]^{-\frac{1}{1+\alpha}}. \tag{43}$$

$$\frac{\ddot{a}}{a} = \frac{V + v}{3M_{Pl}^2} \left( 1 - |\dot{T}|^{\frac{\alpha+1}{\alpha}} \right)^{-\frac{1}{1+\alpha}} \left[ 1 - \frac{3}{2} |\dot{T}|^{\frac{\alpha+1}{\alpha}} \right]. \tag{44}$$

If the maximum velocity is (39), then the condition that we have a deceleration stage in between two acceleration stages is: \(v < V_m/(3^{1/(1+\alpha)} - 1)\).

The equation of the scalar (for non-negative \(\dot{T}\)) can be expressed as

$$\left[ \frac{\ddot{T}}{\alpha(1 - |\dot{T}|^{\frac{1+\alpha}{\alpha}})} + 3H\dot{T} \right] T^{\frac{1-\alpha}{\alpha}} + f(T) = 0, \tag{45}$$

where \(f(T)\) is defined as before. It is easy to know that the velocity \(|\dot{T}|\) increases first and reaches the maximum value at a finite critical value \(T = T_c\). After this point, \(|\dot{T}|\) turns to decrease to zero and we enter into the GCG dark universe.

Under the slow-roll conditions: \(\dot{T}^{(1+\alpha)/\alpha} \ll 1\) and \(\ddot{T} \ll 3H\dot{T}\), we have the similar inflationary equations to Eq. (32): \(H^2 \simeq (V + v)/(3M_{Pl}^2)\) and \(\dot{T}^{1/\alpha} = -[\ln(V + v)]'//(3H)\).

\(^2\)If we want to keep a regular form of the relation: \(p\rho^\alpha = -v^2\), we can express the potential in the action (38) as \((V + v)^{2/(1+\alpha)}\). However, the potential profile in this form is much sharper for \(0 < \alpha < 1\) than in the \(\alpha = 1\) case. But such a choice of the potential is good for the \(\alpha > 1\) case which is suggested in [51] based on supernova data analysis.

13
5 Conclusions

The original tachyon cosmological model is somehow improved by adding a positive constant $v$ into the asymptotic tachyon potential. In particular, some difficulties related to reheating and tachyon matter can be (partially) cured. Moreover, the universe in this modified theory evolves into a Chaplygin gas dark universe at late times. Hence, in this simple single-scalar model, the universe history including the inflationary, radiation-dominated, (dark) matter-dominated and dark energy-dominated stages can be naturally reproduced, at least at the qualitative level. Note that the tachyon matter and Chaplygin gas matter in the model take similarity and can be viewed as one phase. The constant $v$ is the only parameter in this model. It is related to the cosmological constant and controls all the evolutionary processes of the brane universe. An unstable 3-brane described by the action (3) with an appropriate $v$ behaves as our universe automatically in the FRW metric. It is interesting to construct such kind of configuration in string theory in future study.

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