EFFECTIVE LAGRANGIAN FOR HEAVY AND LIGHT MESONS: RADIATIVE B DECAYS.*

R. Casalbuoni
Dipartimento di Fisica, Univ. di Firenze
I.N.F.N., Sezione di Firenze

A. Deandrea, N. Di Bartolomeo and R. Gatto
Département de Physique Théorique, Univ. de Genève

G. Nardulli
Dipartimento di Fisica, Univ. di Bari
I.N.F.N., Sezione di Bari

UGVA-DPT 1993/04-816
BARI-TH/93-140
hep-ph/9304302
April 1993

* Partially supported by the Swiss National Foundation
ABSTRACT

We make use of the information obtained from semileptonic decays of $B$ and $D$ mesons, within an effective lagrangian description based on heavy quark theory and on chiral expansion, to study the radiative decays $B \to K^*\gamma$ and $B_S \to \phi\gamma$, and the pair conversion processes $B \to Ke^+e^-$ and $B \to K^e^+e^-$. We discuss with care the required extrapolations from zero recoils. We obtain a $\text{BR}(B \to K^*\gamma)$ between $1.3 \times 10^{-5}$ and $4.1 \times 10^{-5}$ and $\text{BR}(B_S \to \phi\gamma)$ between $1.4 \times 10^{-5}$ and $4.5 \times 10^{-5}$, with errors mainly arising from the present uncertainty in the Kobayashi-Maskawa element $|V_{ts}|$. For the pair conversion processes we study in particular the lepton effective mass distributions whose measurements would allow for an understanding both long distance and short distance contributions to such processes.
1 Introduction

Radiative $B$ decays into strange final states have received much theoretical attention in the last few years both for their relevance in the framework of the Standard Model and for their possible role in the discovery of new physics. As to the former aspect, it should be observed that the short distance effective hamiltonian describing the decay $b \rightarrow s\gamma$ contains, through loop effects, information on the top quark mass as well as on some poorly known elements of the Cabibbo-Kobayashi-Maskawa matrix. The top quark appears as a virtual state in a penguin-like diagram contributing to $b \rightarrow s\gamma$ and is responsible of a large QCD enhancement in the branching ratio [1], a phenomenon previously noted in $s \rightarrow d\gamma$ decay [2], [3]. Also the latter aspect of interest, i.e. the relevance of radiative $B$ decays for the discovery of new effects beyond the Standard Model, has been deeply studied and in particular we mention here some analyses in the framework of supersymmetry [4].

In this letter we wish to analyze the following exclusive processes

\begin{align}
B & \rightarrow K^*\gamma \\
B_s & \rightarrow \phi\gamma \\
B & \rightarrow Ke^+e^- \\
B & \rightarrow K^*e^+e^-
\end{align}

that are described at the quark level by the above mentioned short distance $b \rightarrow s\gamma$ hamiltonian plus long distance contributions arising from $J/\psi - \gamma$ and $\psi' - \gamma$ conversion (for virtual photons).

As shown in [5], in the limit of infinitely heavy $b$ quark one can relate the short distance hadronic matrix elements for the transitions (1.1)-(1.3) to the matrix elements of the weak currents between a heavy and a light meson, by using the flavour and spin symmetries of the Heavy Quark Effective Theory [HQET] [6]. It should be observed that while the currents appearing in the hadronic matrix elements of (1.1) are computed at $q^2 = 0$, the predictions of HQET are in general reliable for $q^2 \approx q_{max}^2$, i.e. at zero recoil momentum. In [7] it has been shown that such an extrapolation does not generate large corrections and that there exists a special kinematical point in the semileptonic decay $B \rightarrow \rho e\nu$ where one can make predictions that are largely free from hadronic uncertainties; this idea has been subsequently developed in [8].

Our approach to the processes (1.1)-(1.3) is based on an effective chiral theory including mesons containing one heavy quark that has been recently developed in [9], [10]. By this approach we are able to compute processes (1.1)-(1.3) by an effective lagrangian possessing flavour and spin symmetry in the heavy degrees of freedom, and chiral symmetry in the light ones. Due to these symmetries we can evaluate the amplitudes for the processes (1.1)-(1.3) in terms of some constants appearing in the semileptonic decays of $B$ and $D$'s. Using the results obtained in [10] for these decays, we can therefore make predictions for the rare neutral flavour changing processes (1.1)-(1.3).

We conclude this introduction by briefly reviewing some notations of the effective chiral field theory for heavy mesons [9], [10]. The $J^P = 0^-$ and $1^-$ heavy $Q\bar{q}_a$ mesons are represented by a $4 \times 4$ Dirac matrix

\begin{align}
H_a & = \frac{1 + \gamma^5}{2} [P^a_{\mu\nu} \gamma^\mu - P_\mu \gamma_{\nu}] \\
H_a^\dagger & = \gamma_0 H_a^\dagger \gamma_0
\end{align}
where \( a = 1, 2, 3 \) (for \( u, d, s \), \( v \) is the heavy meson velocity, (for \( u, d \) and \( s \) respectively), \( P_{a\mu}^* \) and \( P_a \) are annihilation operators satisfying
\[
\langle 0 | P_a | H_a(0^-) \rangle = \sqrt{M_H} \quad (1.6)
\]
\[
\langle 0 | P_{a\mu}^* | H_a(1^-) \rangle = e^\mu \sqrt{M_H} \quad (1.7)
\]
where \( v_{\mu} P_{a\mu} = 0 \) and \( M_H \) is the heavy meson mass. The light pseudoscalar mesons are described by
\[
\xi = \exp \left( \frac{iM}{f_\pi} \right) \quad (1.8)
\]
where
\[
M = \begin{pmatrix}
\sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & \pi^+ & K^+ \\
\pi^- & \sqrt{\frac{1}{2}} \pi^0 + \sqrt{\frac{1}{6}} \eta & K^0 \\
K^- & \bar{K}^0 & \sqrt{\frac{2}{3}} \eta
\end{pmatrix} \quad (1.9)
\]
with \( f_\pi = 132\text{MeV} \). Under the chiral group \( SU(3) \otimes SU(3) \) the fields transform as follows
\[
\xi \rightarrow g_L \xi U^\dagger = U \xi g_R^\dagger 
\]
\[
\Sigma = \xi^2 \rightarrow g_L \Sigma g_R^\dagger 
\]
\[
H \rightarrow H U^\dagger 
\]
\[
\bar{H} \rightarrow U \bar{H} 
\]
where \( g_L, g_R \) are global \( SU(3) \) transformations and \( U \) depends on the space point \( x \), the fields, \( g_L \) and \( g_R \).

The vector meson resonances belonging to the low lying \( SU(3) \) octet can be introduced by using the hidden gauge symmetry approach, where the \( 1^- \) particles are the gauge bosons of a gauge local subgroup \( SU(3)_{loc} \). Starting from chiral \( U(3) \otimes U(3) \), one gets a nonet of vector fields \( \rho_\mu \) that describe the particles \( \rho, \omega, K^* \) and \( \phi \) (with ideal mixing, i.e. \( \phi = \bar{s}s \)). In this way the HQET chiral lagrangian describing the fields \( H, \xi, \rho_\mu \) as well as their interactions, is given, at the lowest order in light meson derivatives, as follows [10]
\[
\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 
\]
\[
\mathcal{L}_0 = \frac{f_\pi^2}{8} \left< \partial^\mu \Sigma \partial_\mu \Sigma^\dagger \right> \quad (1.15)
\]
\[
\mathcal{L}_2 = -\frac{f_\pi^2}{2} a \left< (\mathcal{V}_\mu - \rho_\mu)^2 \right> + \frac{1}{2g_V^2} \left< F_\mu^{\mu\nu}(\rho) F^{\mu\nu}(\rho) \right> 
\]
\[
+ i\beta \left< H_b v^\mu (\mathcal{V}_\mu - \rho_\mu)_{ba} \bar{H}_a \right> 
\]
\[
+ \frac{\beta^2}{2f_\pi^2 a} \left< \bar{H}_b H_a \bar{H}_a H_b \right> + i\lambda \left< H_b \sigma^{\mu\nu} F_{\mu\nu}(\rho)_{ba} \bar{H}_a \right> 
\]
where \( \langle \cdots \rangle \) means a trace,
\[
D_{\mu ba} = \delta_{ba} \partial_\mu + \mathcal{V}_{\mu ba} = \delta_{ba} \partial_\mu + \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right)_{ba} 
\]
\[
A_{\mu ba} = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right)_{ba} 
\]
\[ F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu] \]  

Moreover
\[ \rho_\mu = \frac{g_\nu}{\sqrt{2}} \hat{\rho}_\mu \]  

where
\[
\hat{\rho} = \begin{pmatrix}
\sqrt{\frac{1}{2}} \rho^0 + \sqrt{\frac{1}{2}} \omega \\
\rho^+ \\
K^{*-} \\
-\sqrt{\frac{1}{2}} \rho^0 + \sqrt{\frac{1}{2}} \omega \\
K^{*0} \\
\phi
\end{pmatrix}
\]  

and
\[ a = 2 \quad g_\nu \approx 5.8 \]  

from the KSRF relations. One can also introduce explicit symmetry breaking terms as illustrated in [9], [10].

We shall also introduce positive parity \(0^+\) and \(1^+\) heavy mesons. As described in [10] we are interested in the states having the total angular momentum of the light degrees of freedom equal to 1/2: \(s_\ell = 1/2\). They are described by a \(4 \times 4\) Dirac matrix analogous to (1.4)
\[ S_a = \frac{1 + \gamma^\ell}{2} [D^\mu_1 \gamma_\mu \gamma_5 - D_0] \]  

where \(D^\mu_1\) and \(D_0\) annihilate a \(1^+\) and \(0^+\) \(Q\bar{q}_a\) states respectively with a normalization analogous to (1.6), (1.7). The complete lagrangian can be found in [10].

## 2 Hadronic matrix elements

As shown in [1], the decay \(B \to K^{*\gamma}\) is mediated at short distances by electromagnetic penguin diagrams that generate an effective hamiltonian given by
\[ H_\gamma = C m_b \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b F_{\mu\nu} + \text{h.c.} \]  

where we have neglected terms of order \(m_s/m_b\). \(F_{\mu\nu}\) is the electromagnetic tensor, \(\sigma_{\mu\nu} = \frac{i}{2} [\gamma_{\mu}, \gamma_{\nu}]\) and \(C\) is given by
\[ C = \frac{G_F}{\sqrt{2}} \frac{e}{16\pi^2} V_{tb} V_{ts}^* F_2 \left( \frac{m_t^2}{m_W^2} \right) \]  

where
\[ F_2(x) = r^{-16/23} \left\{ F_2(x) + \frac{116}{27} \left[ \frac{1}{5} \left( r^{10/23} - 1 \right) + \frac{1}{14} \left( r^{28/23} - 1 \right) \right] \right\} \]  

with \(r = \alpha_s(m_b)/\alpha_s(m_W)\) and \(F_2(x)\) given by
\[ F_2(x) = \frac{x}{(x-1)^3} \left[ \frac{2x^2}{3} + \frac{5x}{12} - \frac{7}{12} - \frac{3x^2 - 2x}{2(x-1)} \log x \right] \]  

\(F_2(x)\) is a smooth function of the top quark mass \(m_t\) with values between 0.55 (at \(m_t = 90\ GeV\)) and 0.68 (for \(m_t = 210\ GeV\)).

The hamiltonian (2.1) describes also the processes (1.2) and (1.3), where a virtual photon is emitted; however in these cases long distance contributions play a major role...
photons, we have the short distance hadronic matrix element

\[ \langle K^*(p', \epsilon)|\bar{s}\sigma^{\mu\nu}(1+\gamma_5)b|\bar{B}(p)\rangle = i\left\{ A(q^2) \left[ p'^\mu\epsilon'^\nu - p'^\nu\epsilon'^\mu - ie^{\mu\nu\lambda\sigma}p_\lambda\epsilon'^\sigma \right] + B(q^2) \left[ p'^\mu\epsilon'^\nu - p'^\nu\epsilon'^\mu - ie^{\mu\nu\lambda\sigma}p_\lambda\epsilon'^\sigma \right] + H(q^2)(\epsilon^* \cdot p) \left[ p'^\mu p'^\nu - p'^\nu p'^\mu - ie^{\mu\nu\lambda\sigma}p_\lambda p'_\sigma \right] \right\} \] (2.5)

On the other hand, for the transition \( B \rightarrow K\gamma \), that can occur only with virtual photons, we have the short distance hadronic matrix element

\[ \langle K(p')|\bar{s}\sigma^{\mu\nu}(1+\gamma_5)b|\bar{B}(p)\rangle = iS(q^2) \left[ p'^\mu p'^\nu - p'^\nu p'^\mu - ie^{\mu\nu\lambda\sigma}p_\lambda p'_\sigma \right] \] (2.6)

Both in (2.5) and in (2.6) we have used the property \( \frac{i}{2}e^{\mu\nu\lambda\sigma}\sigma_{\lambda\sigma}\gamma_5 = -\sigma^{\mu\nu} \) that allows to express matrix elements of \( \bar{s}\sigma^{\mu\nu}b \) in terms of those of \( \bar{s}\sigma^{\mu\nu}\gamma_5b \).

At the lowest order in the derivatives of the pseudoscalar fields, the weak tensor current between light pseudoscalar and negative parity heavy mesons is as follows

\[ L_\mu^a = i\frac{\alpha}{2} \langle\sigma_{\mu\nu}(1+\gamma_5)H_b\xi^\dagger_{ba}\rangle \] (2.7)

that has the same transformation properties of the quark current \( \bar{q}^{\mu}\sigma^{\mu\nu}(1+\gamma_5)Q \). Together with (2.7) we also consider the weak effective current \( \langle H, \langle 1 \rangle \rangle \) corresponding to the quark \( V - A \) current \( \bar{q}^a\gamma^\mu(1-\gamma_5)Q \), i.e.

\[ L_\mu^a = i\frac{\alpha}{2} \langle\gamma^\mu(1-\gamma_5)H_b\xi^\dagger_{ba}\rangle \] (2.8)

We put the same coefficient \( i\alpha/2 \) in (2.7) and (2.8) because, as a consequence of the equations of motion of the heavy quark \( \left( \frac{1+i}{2}b = b \right) \), we have, in the \( b \) rest frame \( \mathbb{F} \),

\[ \gamma^0 b = b \] (2.9)

Therefore

\[ \bar{q}^a\sigma_{0i}(1+\gamma_5)Q = -iq^a\gamma_i(1-\gamma_5)Q \] (2.10)

and the effective currents \( L_\mu^a \) and \( L_\mu^a \) have to satisfy, in the heavy meson rest frame, the relation

\[ L_{0i} = -iL_i \] (2.11)

We also introduce the weak tensor current containing the light vector meson \( \rho^a \) and reproducing \( \bar{q}^a\sigma^{\mu\nu}(1+\gamma_5)Q \)

\[ L_\mu^a = i\alpha_1 \left\{ g^{\mu\alpha}g^{\nu\beta} - \frac{i}{2}e^{\mu\alpha\beta} \right\} \langle\gamma_5H_b[\gamma_\alpha(\rho_\beta - \nabla_\beta)_{bc} - \gamma_\beta(\rho_\alpha - \nabla_\alpha)_{bc}]\xi^\dagger_{ca}\rangle \] (2.12)

\( L_\mu^a \) is related to the vector current \( L_\mu^a \) introduced in \( \mathbb{F} \) to represent \( \bar{q}^a\gamma^\mu(1-\gamma_5)Q \) between light vector particles and heavy mesons:

\[ L_\mu^a = \alpha_1 \langle\gamma_5H_b(\rho^\mu - \nabla^\mu)_{bc}\xi^\dagger_{ca}\rangle \] (2.13)
In order to construct the tensor current we have imposed, similarly to (2.11) the relation
\[ L_{1a}^0 = -i L_{1a}^i. \]

Let us briefly discuss the coupling constants appearing in the previous equations. \( \alpha \) and \( \hat{\alpha} \) are related to the leptonic constants defined by
\[
\langle 0 | \bar{q}^a \gamma^\mu \gamma_5 Q | P_b(p) \rangle = i \frac{\alpha}{\sqrt{m_P}} \delta_{ab}
\]
(2.14)
\[
\langle 0 | \bar{q}^a \gamma^\mu Q | S_b(p) \rangle = i \frac{\hat{\alpha}}{\sqrt{m_S}} \delta_{ab}
\]
(2.15)
where \( P_b \) and \( S_b \) are the \( Q \bar{q}_b \) mesons with \( J^P = 0^- \) and \( 0^+ \) respectively. Both \( \alpha \) and \( \hat{\alpha} \) have a smooth logarithmic dependence on \( m_Q \) that we omit since it leads to negligible numerical effects. From QCD sum rules analysis, as applied to \( f_B \), we obtain \[ f_B = \alpha/\sqrt{m_B} \simeq 200 \text{ MeV}, \]
which implies
\[ \alpha \simeq 0.46 \text{ GeV}^{3/2} \] (2.16)
We assume here that \( m_B \) is large enough so that \( 1/m_B \) is negligible and one could deduce \( \alpha \) by \( \alpha \approx f_B m_B^{1/2} \). QCD sum rules have also been applied to the determination of \( \alpha \) in the \( m_Q \to \infty \) limit; however in this case, large (\( \approx 100\% \)) \( \mathcal{O}(\alpha_s) \) corrections \[ make the results unreliable. As to \( \hat{\alpha} \), we take the results of an analysis \[ based on QCD sum rules:
\[ \hat{\alpha} \simeq \alpha \approx 0.46 \text{ GeV}^{3/2} \] (2.17)
We note that this value of \( \hat{\alpha} \), obtained in the limit \( m_Q \to \infty \) coincides, modulo logarithmic corrections, with the result obtained by extracting \( \hat{\alpha} \) from \( f_{B(0^+)} \) i.e. \( \hat{\alpha} \approx f_{B(0^+)} \sqrt{m_B(0^+)} \).

Finally the constant \( \alpha_1 \) in (2.13) appears in combination with other coupling constants and will be discussed in the next Section.

3 The decays \( B \to K^* \gamma \) and \( B_s \to \phi \gamma \)

To compute \( B \to K^* \gamma \) we consider a polar diagram (with a heavy meson intermediate \( 1^+ \) and \( 1^- \) state between the current and the \( BK^* \) system) and a direct term. We assume that the effective lagrangian and the effective tensor currents of the previous Section provide a reliable way to describe the process only for large values of \( q^2 \), i.e. \( q^2 \approx q^2_{\text{max}} = (m_B - m_{K^*})^2 \), since in writing strong and weak effective couplings we have neglected higher order derivatives of the light fields. Moreover near the zero recoil point the residual heavy meson momentum is small, which is a basic condition of HQET. Following \[ we assume polar dependence in \( q^2 \) (with pole mass suggested by dispersion relations), which represents quite well the data in semileptonic heavy meson decays \[ By using the Feynman rules for the heavy meson chiral lagrangian given in refs. \[ we obtain the results of Table 1 that are valid for any \( q^2 \) and in the limit \( m_Q \to \infty \). We notice that in writing the various contributions in Table 1 we have left the dependence of \( p \cdot p' \) on \( q^2 \), since \( p \cdot p' = \frac{1}{2}(m_B^2 + m_{K^*}^2 - q^2) \), in the term arising from the \( 1^- \) pole and we have assumed that the direct term has a polar dependence with pole mass given by
the $1^+$ pole. These choices can be justified as follows. The results in Table 1 satisfy, for $q^2 \approx q_{\text{max}}^2$ the following relations between form factors of vector and tensor currents:

\[
A(q^2) = i \left\{ \frac{q^2 - m_B^2 - m_K^*}{m_B} \frac{V(q^2)}{m_B + m_K^*} - \frac{m_B + m_K^*}{m_B} A_1(q^2) \right\} \tag{3.1}
\]
\[
B(q^2) = i \frac{2m_B}{m_B + m_K^*} V(q^2) \tag{3.2}
\]
\[
H(q^2) = \frac{2i}{m_B} \left\{ \frac{V(q^2)}{m_B + m_K^*} + \frac{1}{2q^2} \left( \frac{q^2 + m_B^2 - m_K^2}{m_B + m_K^*} A_2(q^2) \right. \right.
\]
\[
\left. \left. + \frac{2m_K^* A_0(q^2) - (m_B + m_K^*) A_1(q^2)}{m_B + m_K^*} \right\} \right\} \tag{3.3}
\]

where $V(q^2), A_j(q^2)$ are the semileptonic form factors in the notations of ref. [18] and they have been obtained in ref. [10] by the same methods employed in the present letter. Eqs. (3.1) and (3.2) coincide with the relations found in ref. [5]; as for (3.3), the result of [5]:

\[
H(q^2) = \frac{2i}{m_B} \left\{ \frac{V(q^2)}{m_B + m_K^*} + \frac{1}{2q^2} \left( \frac{q^2 + m_B^2 - m_K^2}{m_B + m_K^*} A_2(q^2) \right. \right.
\]
\[
\left. \left. + \frac{2m_K^* A_0(q^2) - (m_B + m_K^*) A_1(q^2)}{m_B + m_K^*} \right\} \right\} \tag{3.4}
\]

differs from (3.3) for terms that are subleading in the limit $m_Q \to \infty$ and can be neglected. Following [5] and [7] we assume the results (3.1)-(3.3) hold also for small values of $q^2$, which justifies the above mentioned choices in Table 1.

As a final remark we observe that in computing the form factors from Table 1 one has to consider the coupling constants, $\lambda, \mu$ and

\[
\alpha_{\text{eff}} = \alpha_1(m_B - m_K^*) - \hat{\alpha} \frac{m_B + m_K^*}{2} \tag{3.5}
\]

The constants $\mu$ and $\alpha_{\text{eff}}$ have been determined in [10] by an analysis of the $D$ semileptonic decays, with the result

\[
\alpha_{\text{eff}} = -0.22 \pm 0.02 \text{ GeV}^{3/2}; \quad \mu = -0.13 \pm 0.05 \text{ GeV}^{-1} \tag{3.6}
\]

A major source of uncertainty in the derivation of $\lambda$ comes from the value of the leptonic constant $f_D$ or equivalently $\alpha$; indeed, neglecting $1/m_Q$ corrections (which, incidentally, is consistent with the approximations made in Table 1), one would obtain, from the value of $|V(0)|$ in $D \to K^* \ell \nu\ell$ decay:

\[
|\lambda \alpha| = 0.16 \pm 0.03 \text{ GeV}^{1/2} \tag{3.7}
\]

On the other hand, one could assume a different attitude and take the value $f_D \approx 200 \text{ MeV}$ which is suggested by both QCD sum rules [14] and the lattice calculations [19]. This amounts to include in the final results the set of $1/m_Q$ corrections that contribute to $f_D$ and are known to be large. In this latter case one obtains $|\lambda| = 0.60 \pm 0.11 \text{ GeV}^{-1}$. In [10] both cases have been discussed and the corresponding predictions for $B$ decays were presented. Here we wish to present an argument in favour of the $\text{scaling}$ solution (3.7).

We can consider the process, related to $B \to K^* \ell \nu\ell$,

\[
B \to K^* \psi \tag{3.8}
\]

\[\text{In [10] we used } e^{0123} = +1 \text{ whereas here we use the opposite convention.}\]
for which experimental data are available [21]: \(\text{BR}(B^+ \to K^{*+}\psi) = (1.4 \pm 0.7) \times 10^{-3}\) and \(\text{BR}(B^0 \to K^{*0}\psi) = (1.3 \pm 0.4) \times 10^{-3}\). Assuming factorization and using the Wilson coefficients of [21] and \(|V_{cb}| = 0.045\) we get, in the case of the scaling solution (3.7), the result

\[
\text{BR}(B \to K^*\psi) = 1.5 \times 10^{-3}
\]  

(3.9)

which agrees with the data; whereas the solution obtained with the larger value of \(\lambda\) seems disfavoured, since it leads to \(\text{BR}(B \to K^*\psi) = 2.9 \times 10^{-3}\). For this reason we assume (3.7) as input of our numerical analysis. Using the result (2.16) one would get \(\lambda = 0.35 \pm 0.06\text{GeV}^{-1}\); in any event, for the calculation of \(B \to K^*\gamma\), only (3.7) is actually necessary.

In computing the width for the decay \(B \to K^*\gamma\) the combination of form factors \(A + B\) at \(q^2 = 0\) appears. Using Table I we have the result

\[
|A(0) + B(0)| = 0.53
\]  

(3.10)

Since the radiative width is given by

\[
\Gamma(B \to K^*\gamma) = \left(\frac{m_B^2 - m_{K^*}^2}{2m_B}\right)^3 \frac{2|C|^2m_B^3}{\pi} |A(0) + B(0)|^2
\]  

(3.11)

we get,

\[
\text{BR}(B \to K^*\gamma) = \left[2.5 \times (|V_{ts}|/0.042)^2\right] \times 10^{-5}
\]  

(3.12)

where \(|V_{ts}| = 0.042\) is the central value quoted in [20]. In the previous formula we used \(m_t = 150\text{ GeV}, |V_{tb}| \simeq 1, m_b = 4.7\text{ GeV}\) and \(\tau_{B^+} \simeq \tau_{B^0} \simeq \tau_{B_s} \simeq 1.4\text{ ps}\), both for \(B^0\) and \(B^-\) decays. Our results agree with those of [22] (in our notation they find \(|A(0) + B(0)| \approx 0.46\)), but not with those of [23] that are larger than ours by a factor of 2 in the amplitude.

A similar analysis, with obvious changes, applies to the decay \(B_s \to \phi\gamma\). In this case we obtain \(|A(0) + B(0)| = 0.54\) and

\[
\text{BR}(B_s \to \phi\gamma) = \left[2.7 \times (|V_{ts}|/0.042)^2\right] \times 10^{-5}
\]  

(3.13)

Taking only into account the experimental uncertainty of \(V_{ts}\) gives for the branching ratio \(B \to K^*\gamma\) a range from \(1.3 \times 10^{-5}\) to \(4.1 \times 10^{-5}\), and for \(B_s \to \phi\gamma\) from \(1.4 \times 10^{-5}\) to \(4.5 \times 10^{-5}\).

4 \(B \to Ke^+e^-\) and \(B \to K^*e^+e^-\)

These decays occur dominantly via a quark process \(b \to s\gamma^* \to se^+e^-\) (\(\gamma^* = \text{virtual photon}\)). In the effective lagrangian for \(b \to se^+e^-\) we have to include also the so-called long-distance contributions arising from \(\psi - \gamma\) or \(\psi' - \gamma\) conversion, i.e. from the quark subprocess \(b \to s\psi \to se^+e^-\). The effective lagrangian has been derived in [3, 24] and we shall not report it here for the sake of simplicity.

Let us first discuss \(B \to Ke^+e^-\). The short distance hadronic matrix element relative to this decay has been given in (2.6). We compute it by using the lagrangian and currents of Section 2. The corresponding diagrams are similar to those of \(B \to K^*\gamma\), with \(K^*\)
changed into $K$ and the pole given by the $1^-$ $s\bar{b}$ meson; however it turns out that there is no direct coupling and the only surviving term, in the limit $m_Q \to \infty$, and for $q^2 \approx q^2_{max}$, is the polar contribution.

Assuming, as in previous case, a $q^2$ dependence given by a simple pole (with $m_P = m_{B^*}$), we get the result

$$S(q^2) = \frac{S(0)}{1 - q^2/m^2_{B^*}}$$  \hspace{1cm} (4.1)

with

$$S(0) = \frac{\alpha g}{f_\pi m^2_{B^*} \sqrt{m_B}} (m_{B^*} + m_B - m_K)$$  \hspace{1cm} (4.2)

We can express this result in terms of the form factors which appear in the matrix element of the $V - A$ current ($F_1(0) = F_0(0)$)

$$\langle K(p')|\bar{s}\gamma^\mu(1 - \gamma_5)b|B(p)\rangle = [p^\mu + p'^\mu + \frac{m^2_K - m^2_B}{q^2}q^\mu]F_1(q^2) - \frac{m^2_K - m^2_B}{q^2} q^\mu F_0(q^2)$$  \hspace{1cm} (4.3)

and have been computed in [10]. The following relation between the form factors holds:

$$S(q^2) = -\frac{2F_1(q^2)}{m_B}$$  \hspace{1cm} (4.4)

Two remarks are in order. First (4.4) coincides with the analogous relation found in [5] by Isgur and Wise

$$S(q^2) = 1 - \frac{m^2_K - m^2_B}{q^2} F_1(q^2) - \frac{m^2_K - m^2_B}{q^2} q^\mu F_0(q^2)$$  \hspace{1cm} (4.5)

only at $q^2 \approx q^2_{max}$ and $m_B \to \infty$. As in the case of equation (3.3) we find that some form factors (in this case $F_0$) are subleading when $m_Q \to \infty$, which is expected because the $0^+$ state, contributing to $F_0$, cannot couple to the antisymmetric tensor current $\bar{s}\sigma_{\mu\nu}(1 + \gamma_5)b$.

The second remark is related to the value of the coupling constant $g$ in (4.2). One can obtain it by using data on $D \to K\ell\nu\ell$ or $D \to \pi\ell\nu\ell$, together with flavour symmetry and one meets the same problem found in discussing the constant $\lambda$ in the previous Section. Using the scaling hypothesis we get from semileptonic data [17]

$$|g\alpha| = 0.17 \pm 0.06 \text{ GeV}^{3/2}$$  \hspace{1cm} (4.6)

which therefore gives

$$|S(0)| = 0.18 \text{ GeV}^{-1}, \quad |F_1^{B\to K}(0)| = 0.5$$  \hspace{1cm} (4.7)

Incidentally we note that, using for $\alpha$ the information from QCD sum rules (2.16), one obtains for $g$ in the scaling hypothesis $|g| = 0.37 \pm 0.13$ but again only (4.6) is necessary for $B \to K e^+e^-$. Alternatively one can substitute $\alpha/\sqrt{m_D}$ with $f_D \approx 200 \text{ MeV}$ in $|F_1^{D\to\pi}(0)|$; this would lead to $|F_1^{B\to K}(0)| = 0.85$. As in the previous case one can get a suggestion on how to solve this ambiguity from the non leptonica decay

$$B \to K\psi$$  \hspace{1cm} (4.8)
The experimental data are \[24\]: \( BR(B^+ \rightarrow K^+\psi) = (7.7 \pm 2.0) \times 10^{-4} \) and \( BR(B^0 \rightarrow K^0\psi) = (6.5 \pm 3.1) \times 10^{-4} \); they are compatible with the scaling hypothesis, i.e. with the choice represented by the eqs. (4.6)-(4.7). As a matter of fact (4.7) gives, together with the factorization approximation and \( |V_{cb}| = 0.045 \),

\[
BR(B^+ \rightarrow K^+\psi) = BR(B^0 \rightarrow K^0\psi) = 1.1 \times 10^{-3}
\]  

(4.9)

whereas with the non scaling assumption (\( |F_1^{B \rightarrow K}(0)| = 0.85 \)) one would get for the branching ratio the value \( 3.2 \times 10^{-3} \), which is excluded by the data. Even if this argument is based on additional hypotheses, we take it as a strong indication in favour of the scaling behaviour of the form factors.

Using (2.6) and (4.1)-(4.7), together with the effective lagrangian for the process \( b \rightarrow se^+e^- \), including long distance contributions \[3\], \[24\], we can get the distribution \( d\Gamma(B \rightarrow Ke^+e^-)/dQ^2 \) in the invariant mass squared of the lepton pair \( Q^2 \). We can repeat the same analysis for \( d\Gamma(B \rightarrow K^*e^+e^-)/dQ^2 \), using the form factors reported in Section 3 together with the weak \( V - A \) current given in \[10\]. These results are reported in Figs. 1-2. We confirm the results obtained by previous authors \[11\], \[12\], showing that the widths for the processes \( B \rightarrow Ke^+e^- \) and \( B \rightarrow K^*e^+e^- \) are largely dominated by the long distance contributions \( B \rightarrow K(\ast)\psi, \psi' \rightarrow K(\ast)e^+e^- \). Nevertheless an accurate measurement of the lepton pair spectrum below \( c\bar{c} \) resonances would display the effects of the short distance dynamics arising from the hamiltonian (2.1). This measurement would therefore complement the analysis of the \( B \rightarrow K^*\gamma \) decay process providing further information on the fundamental parameters appearing in (2.1).

We conclude this letter by giving the branching ratios for the decay \( B \rightarrow K\psi(2S) \) and \( B \rightarrow K^*\psi(2S) \) that can be obtained as byproduct of our analysis

\[
BR(B^+ \rightarrow K^+\psi(2S)) = BR(B^0 \rightarrow K^0\psi(2S)) = 3.9 \times 10^{-4}
\]

(4.10)

\[
BR(B^+ \rightarrow K^*\psi(2S)) = BR(B^0 \rightarrow K^*\psi(2S)) = 8.0 \times 10^{-4}
\]

(4.11)

These results are within the experimental upper bounds quoted in \[24\], which are \( 1.5 \times 10^{-3} \) for the branching ratio (4.10) and \( 3.5 \times 10^{-3} \) for (4.11).

5 Conclusions

Radiative decays of the \( B \) mesons possess an important potential for exploring certain elements of the Standard Model and also for discovering possible new physics. We have employed effective chiral theory including mesons with one heavy quark to calculate the decays \( B \rightarrow K^*\gamma \) and \( B_s \rightarrow \phi\gamma \), and the pair production processes \( B \rightarrow Ke^+e^- \) and \( B \rightarrow K^*e^+e^- \). The inherent symmetries have allowed us to calculate these processes in terms of some constants previously determined from the study of \( B \) and \( D \) semileptonic decays within the same model. Extrapolation from the region of zero recoil momentum has required particular attention. Our final result for photon decays is

\[
10^5 \times BR(B \rightarrow K^*\gamma) = 2.5 \times \left( \frac{|V_{ts}|}{0.042} \right)^2
\]

(5.1)

\[
10^5 \times BR(B_s \rightarrow \phi\gamma) = 2.7 \times \left( \frac{|V_{ts}|}{0.042} \right)^2
\]

(5.2)
Present errors in $V_{ts}$ give for $BR(B \rightarrow K^*\gamma)$ values from $1.3 \times 10^{-5}$ to $4.1 \times 10^{-5}$, and for $BR(B_s \rightarrow \phi\gamma)$ from $1.4 \times 10^{-5}$ to $4.5 \times 10^{-5}$. Additional uncertainties are of course implicit in the model chosen, and we have discussed them in detail. Concerning $B \rightarrow Ke^+e^-$ and $B \rightarrow K^*e^+e^-$, long distance contributions have to be included, which we take as dominantly given by $B \rightarrow K^*\psi$ and $B \rightarrow K^*\psi'$, and subsequent $\psi, \psi'$ conversion into $\gamma$. Comparison with the predicted lepton pair mass distributions in their kinematical ranges would allow for verification of both short distance and long distance terms.

**Acknowledgement:** Useful discussions with Dr. P.Colangelo, Dr. F.Feruglio and Prof. N.Paver are gratefully acknowledged. We also thank Prof. S.Stone for kindly communicating us the CLEO result.

**NOTE ADDED:** After having completed this work results from CLEO [25] where made public, giving $BR(B \rightarrow K^*\gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$, which agrees, within the error, with our prediction (3.12).
References

[1] S.Bertolini, F.Borzumati and A.Masiero, Phys. Rev. Lett. 59 (1987) 180; N.G.Deshpande, P.Lo, J.Trampetic, G.Eilam and P.Singer, Phys. Rev. Lett. 59 (1987) 183; B.Grinstein, R.Springer and M.B.Wise, Nucl. Phys. B339 (1990) 269; R.Grigjanis, P.J.O'Donnell, M.Sutherland and H.Navelet, Phys. Lett. B237 (1990) 355; G.Cella, G.Curci, G.Ricciardi and A.Viceré, Phys. Lett. B248 (1990) 181; M.Misiak, Phys. Lett. B269 (1991) 161.

[2] N.Vasanti, Phys. Rev. D13 (1976) 1889; M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Phys. Rev. D18 (1978) 2583; Y.I.Kogan and M.A.Shifman, Sov. J. Nucl. Phys. 38 (1983) 628.

[3] T.Inami and C.S.Lim, Progress Theor. Phys. 65 (1981) 297.

[4] S.Bertolini, F.Borzumati and A.Masiero, Phys. Lett. B192 (1987) 437; R.Barbieri and G.F.Giudice, preprint CERN-TH.6830/93.

[5] N.Isgur and M.B.Wise, Phys. Rev. D42 (1990) 2388.

[6] N.Isgur and M.B.Wise, Phys. Lett. B232 (1989) 113; ibidem B237 (1990) 527; M.B.Voloshin and M.A.Shifman, Sov.J.Nucl.Phys. 45 (1987) 292; ibidem 47 (1988) 511; H.D.Politzer and M.B. Wise, Phys. Lett. 206B (1988) 681; ibidem 208B (1988) 504; E.Eichten and B.Hill, Phys. Lett. 234B (1990) 511; H.Georgi, Phys.Lett. 240B (1990) 447; B.Grinstein, Nucl. Phys. B339 (1990) 253; A.F.Falk, H.Georgi, B.Grinstein and M.B.Wise, Nucl. Phys. B343 (1990) 1.

[7] G.Burdman and J.F.Donoghue, Phys. Lett. B270 (1991) 55.

[8] P.J.O'Donnell and H.K.K.Tung, preprint UTPT-93-02.

[9] M.B.Wise, Phys. Rev. D45 (1992) R2188.

[10] R.Casalbuoni, A.Deandrea, N.Di Bartolomeo, F.Feruglio, R.Gatto and G.Nardulli, Phys. Lett. B299 (1993) 139; ibidem B292 (1992) 371.

[11] C.A.Dominguez, N.Paver and Riazuddin, Z. Phys. C48 (1990) 55.

[12] N.G.Deshpande, J.Trampetic and K.Panose, Phys. Lett. B214 (1988) 467.

[13] P.Colangelo, G.Nardulli, N.Paver and Riazuddin, Z. Phys. C45 (1990) 575.

[14] C.A.Dominguez and N.Paver, Phys. Lett. B197 (1987) 423, E B199 (1987) 596; P.Colangelo, G.Nardulli, A.A.Ovchinnikov and N.Paver, Phys. Lett. B269 (1991) 201; L.J.Reinders, Phys. Rev. D38 (1988) 947.

[15] D.J.Broadhurst and A.G.Grozin, Phys. Lett. B267 (1991) 105 and B274 (1992) 421; M.Neubert, Phys. Rev. D45 (1992) 2451; P.Colangelo, G.Nardulli and N.Paver, preprint BARI-TH 9/132, UTS-UFT-93-3 (1993).

[16] P.Colangelo, G.Nardulli and N.Paver, Phys. Lett B293 (1992) 207.

[17] S.Stone, Syracuse University report HEPSY-1-92, in Heavy Flavours, A.J.Buras and H.Lindner eds. (Singapore 1992).
[18] M. Wirbel, B. Stech and M. Bauer, Z. Phys. C29 (1985) 637.

[19] A. Abada et al., Nucl. Phys. B376 (1992) 172.

[20] Particle Data Group, Review of Particle Properties, Phys. Rev. D45 (1992) S1.

[21] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C34 (1987) 103.

[22] N. G. Deshpande, P. Lo, J. Trampetic, Z. Phys. C40 (1988) 369.

[23] C. A. Dominguez, N. Paver and Riazuddin, Phys. Lett. B214 (1988) 459.

[24] N. G. Deshpande, Bombay HEP Workshop 1989 pgs.538-558, talk given at the workshop on High Energy Phenomenology TIFR, Bombay (1989).

[25] S. Stone, private communication.
| Form Factor | Direct | $1^-$ | $1^+$ |
|-------------|--------|-------|-------|
| $A(q^2)$    | \( \frac{i\sqrt{2}g_V\alpha_1 q_{\text{max}}^2 - m_P^2}{\sqrt{m_B}} \) | \( \frac{i2\sqrt{2}\alpha g_V(p \cdot p')}{\sqrt{m_B}} \) | \( \frac{-i\sqrt{2}m_B\hat{\alpha}g_V(\zeta - 2\mu m_{K^*})}{m_P^2 - q^2} \) |
| $B(q^2)$    | 0      | \( \frac{-i2\sqrt{2}\alpha g_V m_B^{3/2}}{m_P^2 - q^2} \) | 0 |
| $H(q^2)$    | 0      | \( \frac{-i2\sqrt{2}\alpha g_V}{(m_P^2 - q^2)\sqrt{m_B}} \) | \( \frac{-i2\sqrt{2}m_B\hat{\alpha}g_V\mu}{(m_P^2 - q^2)m_B} \) |

**Table I.** Terms contributing to the various form factors of the transition $B \to K^*\gamma$. $m_P$ is the pole mass ($m_P = 5.71 \text{ GeV}$ for the direct and $1^+$ term; and $5.32 \text{ GeV}$ for the $1^-$ contribution). \( p \cdot p' = (m_B^2 + m_{K^*}^2 - q^2)/2 \).
Figure Captions

Fig. 1 Differential distribution in the invariant mass squared of the lepton pair $q^2$ for the process $B \to Ke^+e^-$. 

Fig. 2 Differential distribution in the invariant mass squared of the lepton pair $q^2$ for the process $B \to K^*e^+e^-$. 
