Goos-Hänchen effect observed for focused x-ray beams under resonant mode excitation

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Abstract: We have coupled a nano-focused synchrotron beam into a planar x-ray waveguide structure through a thinned cladding, using the resonant beam coupling (RBC) geometry, which is well established for coupling of macroscopic x-ray beams into x-ray waveguides. By reducing the beam size and using specially designed waveguide structures with multiple guiding layers, we can observe two reflected beams of similar amplitudes upon resonant mode excitation. At the same time, the second reflected beam is shifted along the surface by several millimeters, constituting a exceptionally large Goos-Hänchen effect. We evidence this effect based on its characteristic far-field patterns resulting from interference of the multiple reflected beams. The experimental results are in perfect agreement with finite-difference simulations.

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References and links

1. E. Spiller, and A. Segmüller, “Propagation of x rays in waveguides,” Appl. Phys. Lett. 24(2), 60–61 (1974).
2. Y. P. Feng, S. K. Sinha, H. W. Deckman, J. B. Hastings, and D. P. Siddons, “X-ray flux enhancement in thin-film waveguides using resonant beam couplers,” Phys. Rev. Lett. 71(4), 537–540 (1993).
3. S. Lagomarsino, A. Cedola, P. Ciofretens, S. Di Fonzo, W. Jark, G. Soulié, and C. Riekle, “Phase contrast hard x-ray microscopy with submicron resolution,” Appl. Phys. Lett. 71(18), 2557–2559 (1997).
4. M. J. Zwanenburg, J. F. Peters, J. H. H. Bongaerts, S. A. de Vries, D. L. Abernathy, and J. F. van der Veen, “Coherent Propagation of X Rays in a Planar Waveguide with a Tunable Air Gap,” Phys. Rev. Lett. 82(8), 1696–1699 (1999).
5. W. Jark, and S. Di Fonzo, “Prediction of the transmission through thin-film waveguides for X-ray microscopy,” J. Synchrotron Radian. 11(5), 386–392 (2004).
6. V. K. Egorov, and E. V. Egorov, “Physics of planar x-ray waveguide,” Proc. SPIE 4502, 148–172 (2001).
7. M. Osterhoff, and T. Salditt, “Coherence filtering of x-ray waveguides: analytical and numerical approach,” Phys. Rev. Lett. 133(10), 103026 (2011).
8. M. Bartels, M. Krenkel, J. Haber, R. N. Wilke, and T. Salditt, “X-Ray Holographic Imaging of Hydrated Biological Cells in Solution,” Phys. Rev. Lett. 114(4), 048103 (2015).
9. F. Pfeiffer, T. Salditt, P. Høghøj, I. Anderson, and N. Schell, “X-ray waveguides with multiple guiding layers,” Phys. Rev. B 62(24), 16939–16943 (2000).
10. Q. Zhong, M. Osterhoff, M. W. Wen, Z. S. Wang, and T. Salditt, “X-ray waveguide arrays: tailored near fields by multi-beam interference,” X-ray Spectrom. 46(2), 107−115 (2017).
11. C. Fuhse, A. Jarre, C. Ollinger, J. Seeger, T. Salditt, and R. Tucoulou, “Front-coupling of a prefocused x-ray beam into a monomodal planar waveguide,” Appl. Phys. Lett. 85(11), 1907–1909 (2004).
12. S. P. Krüger, K. Giewekemeyer, M. Bartels, H. Neubauer, and T. Salditt, “Sub-15 nm beam confinement by twocrossed x-ray waveguides,” Opt. Express 18(13), 13492–13501 (2010).
13. A. Jarre, T. Salditt, T. Panzner, U. Pietsch, and F. Pfeiffer, “White beam x-ray waveguide optics,” Appl. Phys. Lett. 85(2), 161–163 (2004).
14. T. Salditt, F. Pfeiffer, H. Perzl, A. Vix, U. Mennicke, A. Jarre, A. Mazuelas, and T. Metzger, “X-ray waveguides and thin macromolecular films,” Physica B: Phys. Condens. Mat. 336(1), 181–192 (2003).
15. Z. Jiang, D. R. Lee, S. Narayanan, J. Wang, and S. K. Sinha, “Waveguide-enhanced grazing-incidence small-angle x-ray scattering of buried nanostructures in thin films,” Phys. Rev. B 84(7), 075440 (2011).
16. D. Pelliccia, S. Kandasamy, and M. James, “Characterization of thin films for X-ray and neutron waveguiding by X-ray reflectivity and atomic force microscopy,” Physica Sta. Sol. A 210(1), 2416–2422 (2013).
17. J. Wang, M. J. Bedzyk, and M. Caffrey, “Resonance-enhanced x-rays in thin films: a structure probe for membranes and surface layers,” Science 258(5083), 775–778 (1992).
Planar waveguide for hard x-rays are nowadays well established [1–6]. They can be simply realized by a thin film structure consisting of a low density (guiding) layer sandwiched in between layers of high density, the so-called cladding layer. Planar x-ray waveguide can hence be fabricated by a thin film deposition techniques with sub-nm control. Planar x-ray waveguide can be used for the definition of nanometer-sized highly coherent x-ray beams, for mode filtering [7], and for x-ray holographic imaging [8]. More complex structures can be realized by extending and generalizing the sequence of thin film layers. In [9], multiple guiding layers separated by thin claddings were used to exploit coupling between several guided beams. Several coherent beams were extracted at the end of the waveguide, and evidenced by measuring the far-field interference pattern. More recently, waveguide arrays with multiple guiding layers (WGA) where tailored in the guiding layer thickness and position, to achieve a quasi-focusing in the field behind the waveguide exit [10].

Two different geometries are typically used to couple a synchrotron beam into a waveguide. Either the front coupling scheme, where the beam is coupled through the front face directly into...
the guiding layer [11,12], or the resonant beam coupling (RBC) scheme, where the beam enters from the side through a thinned cladding, usually the top face [13–16]. In this case, the guided modes are resonantly excited by shining a parallel beam onto the waveguide under grazing incidence using a precisely controlled incidence angle $\alpha_i$ for each modes. In order to couple into the guiding layer, the evanescent tail of the parallel beam is used, which “reaches through” the thinned top cladding. The illuminated surface area of the planar waveguide (footprint) is typically a few millimeters long. Resonant mode excitation manifests itself in the plateau of total in form of sharp dips (cusps) at a set of $\alpha_i$. At these angles, photons “get trapped” under the resonance conditions in the guiding layer propagating parallel to the surface over an active coupling length [9]. Similar resonant effects - albeit with lower quality factor - can be observed in thin film samples with more general layer sequence (which do not form a waveguide as such) [17]. For infinite samples and beams, the cusp arises since photons are more likely to get absorbed, when photons are coupled into the structure, rather than being reflected at the top. If the footprint reaches the edge of the RBC, the guided beam may also exit at the side. The lateral shift of a waveguided radiation before being reflected can also be regarded as a special manifestation of the Goos-Hänchen effect, well known in the optical regime [18–25]. For visible and near-infrared radiation, it was also shown that this effect can become particularly strong for resonant grating waveguide structures, and can be accompanied by generation of multiple reflected beams [26, 27].

In this work, we study the coupling of finite (sub-μm) x-ray beams into RBCs with three guiding layers in the $[\text{Ni/C}]_3/\text{Ni}$ structure. Using specially designed structures and resonant mode excitation, the Goos-Hänchen effect results in an enormous shift in the reflected beam of several millimeters. Since part of the incident beam is directly reflected without coupling, one can then face a peculiar situation with two (or even more) reflected beams of almost equal amplitude. In other words, optical simulation shows that multi-guide RBCs can be used as coherent beam splitters, with possible future applications in interferometry or holography. Furthermore, such devices could possibly also serve to split and delay ultra-short x-ray pulses. The goal of the present work is hence to shed light on this novel phenomenon, using both optical simulations and an experimental proof-of-concept.

For conventional use of RBCs, the standard simulation tool is a transfer matrix algorithm similar or the well known Parratt formalism [28]. The reflected and transmitted intensity, as well as the internal and external standing electromagnetic field can then be plotted as a function of the structural parameters of the RBCs (layer thickness, composition and density, interface roughness) as a function of $\alpha_i$. Tacitly, such simulations assume infinite beams and samples. For the present purpose, we hence have to turn to alternative techniques, namely finite-difference (FD) simulations of beam propagation, as presented in the next section devoted to optical design, simulation and fabrication of multi-guide RBCs.

2. Simulation, design and fabrication

The IMD software [29] was used to simulate x-ray intensity inside a RBC structure as demonstrated earlier [9,13,14]. Figure 1(a) shows the single layer schematic of a RBC, serving as a reference for the multi-guide RBCs structures. The low density guiding layer is sandwiched in between two high density cladding layers. The top cladding is sufficiently thin to enable coupling of the beam in reflection geometry. Here we use $\text{Ni (5 nm)}/\text{C (50 nm)}/\text{Ni (50 nm)}$ on a GaAs substrate. Figure 1(b) shows the simulated x-ray reflectivity as a function of incident angle $\alpha_i$ in the range from 0.1° to 0.3°, for 13.8 keV photon energy. Note that both the incoming beam size and the RBC length $L$ are treated as infinite. The sharp dips in the x-ray reflectivity between the critical angles $\alpha_i^C$ of C and $\alpha_i^{Ni}$ of Ni evidence the excitation of the $TE_0$, $TE_1$, $TE_2$ and $TE_3$ modes inside the cavities. The corresponding calculated electric field intensity distribution as a function of $\alpha_i$ and the depth in $x$ direction can be conveniently illustrated in form.
Fig. 1. (a) A simple sketch of a RBC, consisting of one C layer (red) and two Ni layers (purple), deposited on the GaAs substrate. (b) Reflectivity as a function of $\alpha_i$ for 13.8 keV photon energy. The critical angles of C ($\alpha_{Cc}$) and Ni ($\alpha_{Ni}$) are shown as black dotted lines. (c) Calculated field intensity in the RBC in logarithmic scaling. The modes $TE_m$ observed at different $\alpha_i$ are labeled. (d-f) The corresponding plots for a multi-guide RBCs structure with three C guiding layers and four Ni cladding layers. In the schematic (d) the incoming beam with a beam size FWHM is coupled into the multi-guide RBCs structure, illuminating the surface over a size $s = \text{FWHM}/\sin(\alpha_i)$. If FWHM and $s$ are sufficiently small, the RBCs structure can exhibit several reflected beams, which is the central phenomenon studied in this work. Important parameters characterizing the 1$^{st}$ and 2$^{nd}$ reflected beams are the path length of 1$^{st}$ reflected beam $l_1$ (red dash line), the path length of 2$^{nd}$ reflected beam $l_2$ (black dash line), the beam offsets on the surface $o$ (green dash lines), and the distance between two beams $p$ (blue dash lines). Note that the simulation shown in (e,f) assume an infinite beam and structure.

of two-dimensional contour plots, as represented in Fig. 1(c). The characteristic antinodes of the x-ray standing waves corresponding to electric modes ($TE_m$, $m = 0, ..., 3$) of RBC structure can be easily located. Figures 1(d-f) show the equivalent plots for the multi-guide RBCs with three guiding layers [Ni (5 nm)/C (50 nm)]$_3$ /Ni (50 nm) on GaAs substrate. The coupling of the modes results in a splitting and lifting of degeneracy, as first discussed in [9].
The simulations shown above have been calculated for an infinite beam. Such simulations can only be expected to describe experiments well, if the beam footprint on the surface $s$ is much larger than the beam offsets $o$ arising from the Goos-Hänchen effect, associated with the multiple reflections and coupling into modes, as schematically illustrated in Fig. 1(d). If the illuminated spot size on the surface $s$ and $o$ become of comparable size, we must expect deviations. In particular, we anticipate the formation of several reflected beams which are well separated in positions. With respect to the schematically sketched geometry, the path length difference $\Delta$ between 1$^{st}$ and 2$^{nd}$ reflected beams is given by $\Delta = l_2 - l_1$, with $l_2 = 2[(o/2)^2 + d_{RBC}^{1/2}]$ and $l_1 = p/\tan(\alpha_i)$, where $d_{RBC}$ denotes the total thickness of the multi-guide RBCs, and $p$ the distance between the two parallel reflected beams. Of course, this geometric optical argument is overly simplistic, but it should certainly give the correct order of magnitude, in particular to estimate the time delay $\Delta t = \Delta / c$ between the two beams, which is on the order of several attoseconds given the grazing incidence angles of mode excitation (a few mrad) and a typical total film thickness of the RBCs (several 100 nm).

![Simulation of mode excitation with finite (sub-μm) beams. The near-field distributions for (a, b, c, d) on-mode conditions at $\alpha_i = 0.133^o$, and (e, g, h, f) off-mode conditions at $\alpha_i = 0.145^o$, simulated for the theoretical RBCs parameters tabulated in Tab. 1, and (the experimental) photon energy 13.8 keV. For the incoming beam a Gaussian profile with beam size FWHM = 600 nm was assumed. (c, g) The 1-D profiles in the near-field, and (d, h) the corresponding normalized far-field patterns.](image)

In order to observe such phenomena, we must collimate or focus the beam down to FWHM values which are on the order of the characteristic RBCs sizes along the vertical direction ($x$). Let’s consider typical values for the experimental design: at the angle of the TE$_0$ mode $\alpha_i = 0.133^o$, a beam size of FWHM = 600 nm would result in $s \approx 260 \, \mu$m, which is smaller than the beam offset $o$, estimated based on $d_{RBC}$ and the fact that multiple-reflections and
coupling in and out of modes will result in a more significant beam offset $\alpha$. In order to make these speculations precise, we need to simulate the propagation, coupling and reflection of finite beams in the RBCs structure. To this end, we use FD calculations of the parabolic (paraxial) wave equation [30].

Figure 2 presents the results of FD simulations, carried out for the multi-guide RBCs sketched in Fig. 1(d), with the theoretical design parameters tabulated in Tab. 1, for (the experimental) photon energy 13.8 keV, and incoming Gaussian beam with beam size $FWHM = 600$ nm. The near-field distributions are shown for two cases: Figs. 2(a-d) on-mode and Figs. 2(e-h) off-mode, where on-mode refers to the excitation of the $TE_0$ mode at $\alpha_i = 0.133^\circ$, and off-mode refers to $\alpha_i = 0.145^\circ$. The total length of the RBCs (and the FD simulation) along $z$ is $L = 10$ mm. Under on-mode conditions, three reflected beams are observed, exiting the RBCs surface at different offsets along $z$. An interference zone [31] is also formed between the primary incoming beam and the $1^{st}$ reflected beam as shown in Fig. 2(b). The guided modes are resonantly excited in the different channels at different positions along $z$. When the guided modes form in the top channel, the $2^{nd}$ or $3^{rd}$ reflected beam comes out from the surface. Figure 2(c) shows a cross-section through the reflected beam in the near-field (red line, orthogonal to the reflected beam). Figure 2(d) shows the corresponding normalized far-field pattern (intensity profiles). For comparison, the off-mode case for $\alpha_i = 0.145^\circ$ is shown in Figs. 2(e) and 2(f). From these simulations, we can conclude that the experimentally observable fingerprint of the multi-reflections is the characteristic lineshape of the reflected beam in the far-field, in particular the cusps visible in Fig. 2(d). Note that these cusps should be observed for constant $\alpha_i$ as a function of the coordinate on the detector, or in a so-called detector-scan along $\theta$ when using a point detector.

Figure 3 presents analogous results for the $TE_2$ mode with 3(a) $FWHM: 600$ nm at $\alpha_i = 0.187^\circ$ and 3(b) $FWHM: 300$ nm at $\alpha_i = 0.185^\circ$, respectively, illustrating the importance of a sufficiently focused incoming beam to clearly separate the reflected beams. Note that in this case, the maximum values of the two reflected beams ($1^{st}$ and $2^{nd}$) are almost equal, which makes this configuration attractive for beam-splitter and delay applications.

To sum up, FD simulations show that multi-guide RBCs structures can result in multiple reflected beams, linked to mode excitation with finite-size (sub-$\mu$m) beams. The phenomenon occurs only at the proper $\alpha_i$ values required for mode excitation. Since the near-field distribution is often not accessible experimentally, it is important to evidence this effect by far-field measurements, which is possible due to the characteristic lineshape associated with the multi-exit beam behavior. Next, we turn to the fabrication of the structures designed according to the FD simulation results.

For the purpose of fabricating the multi-guide RBCs sample, we have used direct-current magnetron sputtering system [32, 33] at the Institute of Precision Optical Engineering at Tongji University, China. The fabricated RBCs consists of three amorphous $C$ layers, and four polycrystalline $Ni$ layers, following the theoretical design parameters as shown in Tab. 1. The in total 7 layers were deposited on GaAs substrates alternately, under the base pressure $3.0 \times 10^{-4}$ Pa. The sputter gas was Ar with purity of 99.999%, and the gas pressure was kept constant at $1.50 \pm 0.2$ mTorr (0.1995 Pa). After the fabrication, the x-ray macroscopic reflectivity (XMR) was obtained with macroscopic beam size, using an in-house Cu $K$ source ($\lambda = 0.154$ nm), equipped with a reflectometer. The 1-D XMR curve [as shown in appendix A] was fitted by using the Genetic Binda algorithm of IMD [29] with individual layer thickness values of the $Ni$ and $C$ layers, as shown in Tab. 1. Afterwards, the 2-D far-field pattern of RBCs sample was carried out at the GINIX (Goettingen Instrument for Nano-Imaging with X-rays) experiment setup, installed at the P10 beamline at the PETRA III synchrotron facility in Hamburg (DESY) [34]. Using the energy of 13.8 keV, the far-field patterns were recorded by a Eiger 4 M pixel detector (Dectris).
Fig. 3. Under excitation of the TE₂ mode, simulated for the theoretical design parameters, the two reflected beams (1st and 2nd) are almost equal amplitude. This is true for a range of beam sizes, here demonstrated for (a) FWHM = 600 nm, and (b) FWHM = 300 nm. Note the small offset in αᵢ appears when changing the beam size.

Table 1. The theoretical designed parameters and XMR fitting structure are compared.

| Layer name | Ni | C | Ni | C | Ni | C | Ni | GaAs sub |
|------------|----|---|----|---|----|---|----|-----------|
| **Theoretical design results** |    |    |    |    |    |    |    |           |
| Layer thickness / nm | 5.0 | 50.0 | 5.0 | 50.0 | 5.0 | 50.0 | 50.0 |           |
| **XMR fitting structure** |    |    |    |    |    |    |    |           |
| Layer thickness / nm | 5.3 | 47.2 | 5.3 | 47.2 | 5.3 | 47.2 | 50.1 |           |

3. Results

Figure 4 presents the schematic of the experimental setup of GINIX at beamline P10, and illustrates the data reduction. The synchrotron is focused onto the RBCs structure by a Kirkpatrick-Baez (KB) mirror system. The illuminating beam size at the focal distance f is determined by the gap of the horizontal slit hₙ (in x direction). With entrance slit larger than the KB acceptance hₙ ≥ 0.4 mm, the beam size at the sample plane SP is around 295 nm in x direction. Decreasing hₙ from 0.4 mm to 0.05 mm, the measurements have been carried out after careful alignment (at hₙ = 0.05 mm), with (successively) larger spot sizes (FWHM), broadened by diffraction. The transmitted primary beam and the desired reflected beam were recorded in the far-field by the Eiger 4 M pixel detector (Dectris), as exemplified in Fig. 4(b), with pixel size 75 μm, placed at D = 5.4 m behind the focal plane. In Fig. 4(c), a close-up of the 2-D lineshape of the reflected beam is shown under off-mode condition at αᵢ = 0.170°, while Fig. 4(d) shows the on-mode lineshape for the TE₂ mode at αᵢ = 0.187°. The 2-D far-field patterns are integrated in y direction to yield the corresponding 1-D profiles, see the Figs. 4(e) and 4(f), respectively. In both 1-D and 2-D representations, the far-field pattern of the on-mode case differs from the reflected beams signal of the off-mode case. Namely, it shows characteristic cusps (minima). Note that due to mirror imperfections, also the off-mode reflected beam exhibits vertical stripes, but under on-mode additional and more pronounced minima are observed. Figure 4(g) presents the intensity distribution, as a function of αᵢ and detector coordinate x, after integration of the intensity along y. In this plot the angular increment is Δαᵢ = 0.001°, while the pixel size (x direction) is 75 μm. Typically, αᵢ was scanned in the range from -0.1° to 0.5°. Except for αᵢ ≈ 0, where the incoming beam can pass above the RBCs surface, the RBCs with its GaAs substrate significantly attenuates the primary beam.
Fig. 4. (a) Schematic of the GINIX experimental setup. The RBCs is positioned in the focal plane at distance \( f \) behind the Kirkpatrick-Baez (KB) mirror system. The reflection plane is horizontal. By closing the slits in front of the KB mirror, the focus is made fully coherent (in the relevant horizontal plane). The diffraction limited spot size is then adjusted by the gap of the horizontal slits \( h_g \). The far-field intensity patterns at 13.8 keV photon energy (selected by the Si(111) monochromator) is then recorded as a function of exit angle \( \alpha_f \) for each finely tuned incidence angle \( \alpha_i \), using a Eiger 4 M pixel detector (Dectris) at distance \( D = 5.4 \) m. The lineshape of the reflected beam is then analyzed as a function of \( \alpha_f - \alpha_i \). (b) An exemplary two-dimensional (2-D) far-field pattern with primary and reflected beams for \( \alpha_i = 0.17^\circ \) and \( h_g = 0.05 \) mm. (c, d) Zoom of the reflected beam, for (c) \( \alpha_i = 0.17^\circ \), and (d) \( \alpha_i = 0.18^\circ \), corresponding to off-mode and on-mode \( (TE_2) \) conditions, respectively. (e, f) The corresponding integrated 1-D far-field curves, obtained after integration along \( y \). (g) The intensity distribution as a function of \( \alpha_i \) and \( x \), integrated over \( y \). The integration domain is indicated by the red dash rectangles in (b). The relevant region \( \alpha_i^C \leq \alpha_i \leq \alpha_i^{Ni} \) containing the characteristic reflectivity cusps of the \( TE_m \) mode is marked by orange lines.

Next, we compare the experimental pattern to the FD simulations. To this end, we start from the integrated intensity patterns as shown in Fig. 4(g), but now transform the detector coordinate along \( x \) to \( \alpha_f - \alpha_i \). In this way the reflected intensity as function of \( \alpha_i \) corresponds to a horizontal stripe, with a width given by the divergence of the reflected beam.

Figure 5(a) shows the experimental result, for the relevant range including in particular the region \( \alpha_i^C \leq \alpha_i \leq \alpha_i^{Ni} \), where four different modes are observed. The angular resolution in \( \alpha_f - \alpha_i \) is about resolution \( 8 \times 10^{-4}^\circ \). The blank gaps due to the inter-module gaps of the detector [see Fig. 4(g)] had been filled with values from neighboring pixels. In order to obtain data of similar structure, we treat the FD results in an analogous manner. First the reflected beam is simulated by near-field propagation, as shown in Fig. 5(b) for the case \( FWHM=600 \) nm and
Fig. 5. Comparison of experimental and simulation results. (a) Experimental results: Close-up of the experimental lineshape in the relevant α_i range, for the data shown in Fig. 4(g), after transformation x → α_f − α_i. (b) FD near-field distribution (amplitude) of the RBCs structure with tabulated XMR fitting parameters, for α_i = 0.133°. (c) Simulated near-field amplitude in the plane indicated by the red line in (b) for all α_i in the relevant range. (d) Simulation results, corresponding to (a), obtained by transforming the FD data shown in (b, c) to the far-field pattern.

α_i = 0.133°. The geometric parameters of the multi-guide RBCs structure were taken from the XMR fitting structure in Tab. 1. For the simulation, the pixel sizes in x and z directions were 1 nm and 0.1 μm, respectively. The amplitude and phase of the reflected beam were then collected at a certain reference plane at about 2.5 mm behind the focus or center of the RBCs structure, indicated by the red line at z = 3 mm in Fig. 5(b). Figure 5(c) exhibits the amplitude distribution along x at z = 3 mm, for all α_i. In this plot, the angular resolution is Δα_f = 10^{-4}°. From this simulated amplitude data as shown in Fig. 5(c) and the corresponding phase (not shown), we then compute the far-field intensity distribution by performing a discrete Fourier transform (FFT algorithm). Note that in order to perform the FFT, we first have to transform the data along the vertical red line in Fig. 5(b) to a plane orthogonal to the reflected beam. This is accomplished by a multiplicative phase factor = exp(ikx sin(α_i)), resulting in a properly tilted coordinate x' for the output field. Figure 5(d) presents the results, i.e. the simulated far-field intensity, again as a function of α_i and α_f − α_i, so that it can be directly compared to the experimental result of Fig. 5(a). The angular resolution in α_f − α_i is 3.03 × 10^{-4}°. Direct observation illustrates that the characteristic stripe patterns observed in the simulated and experimental far-field intensities [Figs. 5(a) and 5(d)] are nearly identical. Note that the observed far-field stripe patterns can be directly attributed to the multi-reflections in the near field for the different on-mode angles α_i. In other words, they indicate the presence of several reflected beams by interference.

Figure 6 shows an extended comparison of the measured far-field patterns and the simulated
far-field patterns, presented for different beam sizes, as controlled by the entrance slits. Namely, results are shown of Figs. 6(a, e) \( hg = 0.1 \) mm, 6(b, f) \( hg = 0.2 \) mm, 6(c, g) \( hg = 0.3 \) mm, and 6(d, h) \( hg = 0.4 \) mm, respectively. For all slits and beam sizes, the experimental results and the simulations are in good agreement. Note that the beam size is an important control parameter, since the multiple reflected beam can only be observed for finite (typically sub-\( \mu \)m) beam sizes.

From the FD simulations, the beam sizes FWHM are 600 nm for the case \( hg = 0.05 \) mm, 300 nm for the case \( hg = 0.1 \) mm, 130 nm for the case \( hg = 0.2 \) mm, 110 nm for the case \( hg = 0.3 \) mm and 90 nm for the case \( hg = 0.4 \) mm, respectively. Note that the relationship between beam size and \( hg \) is further elucidated in appendix B.

4. Discussion and conclusion

The comparison between the simulated and experimental reflectivity clearly shows that the multi-guide RBCs structure can result in two spatially offset reflected beams, when modes are excited (on-mode condition). This requires finite-size illumination (sub-\( \mu \)m) beams. In the far-field the beams interfere due to their divergence, but in the near-field the beams are well separated.

To accentuate these results, the near-field intensity distribution is shown in Fig. 7 for the on-mode condition, both for Fig. 7(a) the theoretical design parameters, and Fig. 7(d) the XMR fitting parameters as tabulated in Tab. 1. In both cases the desired beam-splitting is observed, see Figs. 7(b) and 7(e) the close-ups of the near-field profile, and Figs. 7(c) and 7(f) the corresponding far-field profiles, with the experimental data shown alongside in Fig. 7(f). The experimental 1-D profile (black) and the simulated far field curve (red), shown for the \( TE_2 \) mode at \( hg = 0.1 \) mm [corresponding to Fig. 6(a)], exhibit a similar lineshape with the identical cusps positions. Note that the deviations in the off-mode regions of the far field patterns can be attributed to the fact that the FD simulations assume an idealized Gaussian beam, while the experimental focal lineshape is quite different. Importantly, simulation and experiment give consistent results and both show that the two reflected beams are of nearly equal amplitude, as
desired. Note that the influences of FWHM and the mode number on the respective amplitudes of the two beams are further illustrated in appendix C.

In summary, the multi-guide RBCs with three guiding layers in the [Ni/C]_3/Ni structure, as can be fabricated by state-of-the-art thin film sputter deposition, can exhibit interesting new phenomenon when illuminated with finite-size (sub-μm) beams, which are completely obscured when simulating and measuring with standard macroscopic beams. In particular, multiple reflected beams can be observed exiting the structure at well controlled spatial offset. This effect could be exploited in interferometric applications or for off-axis holographic x-ray imaging. One could, for example, place an object onto the surface at a position illuminated only by the second beam. The associated phase shifts could then be probed by far-field interference with
the first (reference) beam. While the present work is entirely focused on continuous-wave (cw) illumination (both simulation and experiment), one could also explore the effect of such RBCs on ultra-short and focused x-ray pulses, as generated by free-electron laser (FEL) or higher harmonic (HHG) radiation. In particular, such designed multi-guide resonant beam couplers could serve as time-delay beam splitters with attosecond delay, orders of magnitude smaller than current macroscopic pulse delay stages [35, 36]. Multi-guide RBCs in combination with sub-μm beams may further be useful to probe surface and near-surface structure and dynamics. Finally, they offer novel opportunities for x-ray quantum optical experiments, extending the seminal work in this field performed with single guiding layers [37].

Appendix A: X-ray macroscopic reflectivity (XMR)

![Experimental and fitting data graph](image)

Fig. 8. Measured reflectivity (black line) and fitting curve (red line) as a function of incident angle $\alpha_i$ for the RBCs sample.

XMR was performed for the multi-guide RBCs, using an in-house Cu $K_{\alpha}$ source ($\lambda = 0.154$ nm), equipped with collimating multilayer mirrors and a fully motorized reflectometer [38]. The x-ray reflectivity was recorded with 0.1 mm beam as defined by the entrance slits. An angular range of 0° to 8° was scanned. Figure 8 shows the reflectivity as a function of $\alpha_i$ in black line, after subtracting the diffuse (nonspecular) background as measured by an off-set scan (off-set angle of 0.1°), and after performing the illumination correction. The XMR curve was fitted (red line) in the region of 0.05° to 5.5° using the Genetic Binda algorithm of IMD with individual layer thickness values of the Ni/C layers as free parameters. The fitting results are tabulated in Tab. 1.
Fig. 9. (a) Schematic of the experimental setup, and field distribution calculated by free space propagation between the KB mirror and focal plane. The incoming beam, with primary intensity $I_0$ and photon energy 13.8 keV is coupled into the RBCs, which is positioned at $f = 200$ mm in the focal plane of KB mirror system. (b) Close-up of the focal intensity for $h_g = 0.4$ mm, resulting in $FWHM = 40$ nm. (c) Simulated beam profiles (normalized) in the focal plane ($h_g = 0.4$ mm in red line, $h_g = 0.3$ mm in light blue line, $h_g = 0.2$ mm in green line, $h_g = 0.1$ mm in dark blue line, $h_g = 0.05$ mm in purple line). (d) The corresponding 1-D profiles from Fig. 5(a) and Fig. 6 are compared to the reflectivity calculated for infinite beams by IMD (black line), plotted for an angular range of $\alpha_i$ from 0.1 to 0.25 degrees.

Appendix B: The relationship between $h_g$ and beam size

Figures 9(a) and 9(b) present field simulations by free propagation from the KB mirror to the focal (sample) plane $SP$, combined with the schematic of the experimental setup. The focal plane is at position $f = 200$ mm. For $h_g = 0.4$ mm, the spot size in the focal plane is $FWHM = 40$ nm. With the variation of $h_g$ in the simulation, the calculated $FWHM$ are 330 nm for $h_g = 0.05$ mm, 170 nm for $h_g = 0.1$ mm, 90 nm for $h_g = 0.2$ mm, 60 nm for $h_g = 0.3$ mm and 40 nm for $h_g = 0.4$ mm, respectively, as determined from the curves shown in Fig. 9(c). These results deviate from those obtained by matching the lineshape of the FD simulations to the experimental data (where beam size was a free parameter), see values tabulated in Tab. 2. This can be attributed to a number of reasons: Firstly, finite source size is not included. Secondly, the slit size $h_g$ is a nominal value likely to have an offset with respect to the true gap. Finally, the sample may have been positioned slightly out of the focal plane in some scans. Nevertheless, the general trend of increasing $FWHM$ with decreasing $h_g$ is preserved.

Figure 9(d) shows a comparison of the plateau of the total reflection, between the simulations for infinite beam (IMD simulation, black) and the different finite beam sizes, as varied by $h_g$ as...
| FWHM   | Mode-angle | Near-field | Far-field |
|--------|------------|------------|-----------|
| 130 nm | TE₃ mode  | ![Graph a](image) | ![Graph b](image) |
|        | 0.223°     | 1ˢᵗ, 2ⁿᵈ  | 1ˢᵗ, 2ⁿᵈ |
| 190 nm | TE₃ mode  | ![Graph c](image) | ![Graph d](image) |
|        | 0.223°     | 1ˢᵗ, 2ⁿᵈ  | 1ˢᵗ, 2ⁿᵈ |
| 200 nm | TE₇ mode  | ![Graph e](image) | ![Graph f](image) |
|        | 0.187°     | 1ˢᵗ, 2ⁿᵈ  | 1ˢᵗ, 2ⁿᵈ |
| 300 nm | TE₇ mode  | ![Graph g](image) | ![Graph h](image) |
|        | 0.187°     | 1ˢᵗ, 2ⁿᵈ  | 1ˢᵗ, 2ⁿᵈ |
| 700 nm | TE₇ mode  | ![Graph i](image) | ![Graph j](image) |
|        | 0.157°     | 1ˢᵗ, 2ⁿᵈ  | 1ˢᵗ, 2ⁿᵈ |
| 1500 nm| TE₇ mode  | ![Graph i](image) | ![Graph j](image) |
|        | 0.157°     | 1ˢᵗ, 2ⁿᵈ  | 1ˢᵗ, 2ⁿᵈ |

Fig. 10. Using similar simulations as shown in Fig. 7, Figures 10(a, c, e, g, i) near-field and 10(b, d, f, h, j) far-field profiles, calculated for increasing FWHM, from 130 nm to 1800 nm.
the control parameter in FD simulations. The dips (cusps) become much sharper with decreasing $h_g$, i.e. increasing $FWHM$. Compared to the IMD reflectivity, all dips of the simulated results ($h_g = 0.4$ mm in red line, $h_g = 0.3$ mm in light blue line, $h_g = 0.2$ mm in green line, $h_g = 0.1$ mm in dark blue line, $h_g = 0.05$ mm in purple line) keep the antinodes corresponding to the on-mode case at the same positions, but the intensity differences between on-mode and off-mode decreases.

| Conditions | $hg = 0.4$ mm | $hg = 0.3$ mm | $hg = 0.2$ mm | $hg = 0.1$ mm | $hg = 0.05$ mm |
|------------|---------------|---------------|---------------|---------------|---------------|
| $FWHM/\,\text{nm}$ | 40            | 60            | 90            | 170           | 330           |
| $FWHM/\,\text{nm}$ | 90            | 110           | 130           | 300           | 600           |

**Appendix C: Further discussion for the reflected beams**

Figure 10 presents the evolution of the near-field and far-field profiles of the multi-guide RBCs, when the beam size is increased from $FWHM = 130$ nm to $FWHM = 1800$ nm, based on the analogous calculations as shown in Fig. 7. Figures 10(c), 10(e) and 10(g) show that the maximum values of the two reflected beams (1ˢᵗ and 2ⁿᵈ reflected beams), remain roughly equal over the entire range, resulting in the characteristic lineshape of the reflected beam, see Figs. 10(d), 10(f) and 10(h). This phenomenon can be tailored by tuning the parameters of $FWHM$, incident angle $\alpha_i$ and the structure of RBCs.

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