Phenomenological Aspects of a TeV Scale Alternative left-right Model

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We revisit the alternative left-right symmetric model, motivated by the superstring-inspired $E_6$ model. We systematically analyze the constraints imposed by theoretical and experimental bounds on the parameter space of this class of models. We perform a comprehensive analysis of the Higgs sector and show that three neutral CP-even and two CP-odd Higgs bosons in addition to two charged Higgs bosons can be light, of $\mathcal{O}(100)$ GeV. We emphasize that the predictions of this model for the signal strengths of Higgs decays are consistent with the standard model expectations. We also explore discovery signatures of the exotic down-type quark, which is one of the salient predictions of this model.

I. INTRODUCTION

The discovery of neutrino masses and oscillations confirmed the fact that although the standard model (SM) is extremely accurate, it is still incomplete. The left-right Model (LRM) is the most natural extension of the SM that accounts for the measured neutrino masses and provides an elegant understanding for the origin of the parity violation in low-energy weak interactions \cite{1, 2, 3, 4, 5, 6, 7, 8}. The LRM is based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)/2} \times P$, where $P$ is the discrete parity symmetry. In the LRM, standard model fermions are assigned in the following left- or right-handed doublets:

$$Q_L = \left( \begin{array}{c} u_L \\ d_L \end{array} \right), \quad \psi_L = \left( \begin{array}{c} \nu_L \\ e_L \end{array} \right) \quad \& \quad Q_R = \left( \begin{array}{c} u_R \\ d_R \end{array} \right), \quad \psi_R = \left( \begin{array}{c} \nu_R \\ e_R \end{array} \right). \quad (1)$$

The parity symmetry: $Q_L, \psi_L \leftrightarrow Q_R, \psi_R$ implies that the gauge couplings of left- and right-handed $SU(2)$ are equal, i.e., $g_L = g_R \equiv g$.

The Higgs sector of the LRM consists of (i) bidoublet $\Phi(1, 2, 2^*, 0)$, which is required to construct the SM Yukawa couplings of quarks and leptons. (ii) two scalar triplets $\Delta_L(1, 3, 0, 2)$ and $\Delta_R(1, 0, 3, 2)$ that break $U(1)_{(B-L)/2}$ and generate neutrino Majorana masses. At high-energy scale, well above the electroweak breaking scale, the $SU(2)_R \times U(1)_{(B-L)/2} \times P$ symmetry is broken down to $U(1)_Y$ by the vacuum expectation value (vev) of the neutral component of $\Delta_R$, and hence the right-handed Majorana neutrino mass is generated. In this type of models, the hypercharge $Y$ is defined as $Y = T_{3R} + (B-L)/2$, where $T_{3R}$ is the third component of the right-handed isospin. At lower energy scales, $\Phi$ and $\Delta_L$ acquire vevs that break $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$. It is worth mentioning that in the conventional LRM, one gets the following estimate for the associated vevs: $\langle \Delta_L \rangle = v_L \lesssim \mathcal{O}(1)$ GeV, $\langle \Delta_R \rangle = v_R \gtrsim \mathcal{O}(10^{11})$ GeV, and $\langle \Phi \rangle = diag\{\kappa, \kappa'\}$ with $\kappa' \ll \kappa$ and $\kappa \sim \mathcal{O}(100)$ GeV \textsuperscript{\cite{1, 2, 3, 4}}.

It turns out that the Higgs sector of the LRM, in particular the Higgs triplets, may induce tree-level flavor-violating processes that contradict the current experimental limits. Therefore, it is usually assumed that $SU(2)_R \times U(1)_{(B-L)/2}$ is broken at a very high-energy scale. In this case, it is not possible to detect any residual effect for $SU(2)_R$ gauge symmetry at the TeV scale in the Large Hadron Collider (LHC). This has motivated Ernest Ma, in his pioneering work in 1987 \textsuperscript{\cite{5}}, to study variants of the conventional LRM. He has shown that the superstring-inspired $E_6$ model may lead to two types of left-right models. The first one is the canonical LRM, while the second one is what is known as the alternative left-right Model (ALRM) \textsuperscript{\cite{10, 11}}, where the fermion assignments are different from those in the conventional LRM in the following: (i) an
extra quark, \( d'_R \), instead of \( d_R \), is combined with \( u_R \) and form \( SU(2)_R \) doublet, and (ii) an extra lepton, \( n_R \), instead of \( \nu_R \), is combined with \( e_R \) and forms \( SU(2)_R \) doublet. Therefore, the right-handed neutrino \( \nu_R \) is a true singlet and is no longer a part of the right-handed doublet.

It is remarkable that \( E_6 \) is a complex Lie group of rank 6. It includes the \( SO(10) \) group, so it is a good candidate for grand unification. Some string theories (Heterotic string) predict that the low-energy effective model is symmetric under \( E_6 \). Depending on the string model, \( E_6 \) may be broken to \( SO(10) \) and then to the conventional left-right model or it may have another branch of symmetry breaking that leads to the alternative left-right model that we consider. The particle content of the ALRM, derived from \( E_6 \) model, contains more particles than those in the conventional LRM obtained from \( SO(10) \). This can be simply understood from the fact that the fundamental representation \( 27 \) of \( E_6 \) is equivalent to the fundamental representation \( 16 \) of \( SO(10) \) plus its 10 and singlet representations. In the conventional LRM, all non-SM particles are decoupled and can be quite heavy. However, in the ALRM, they are involved with the SM fermions and will have low-energy consequences. Furthermore, another important difference between the ALRM and the conventional LRM is the fact that tree-level flavor-changing neutral currents are naturally absent so that the \( SU(2)_R \) breaking scale can be of order TeV, allowing to several interesting signatures at the LHC. As the ALRM is a low-energy effective model of the supersymmetric \( E_6 \) model, the gauge couplings are not unified within the ALRM. They are unified in the underlying \( E_6 \) model, similar to the unification of SM gauge couplings in supersymmetric \( SU(5) \).

In this paper, we aim at providing a comprehensive analysis for the phenomenological implications of the ALRM, with emphasis on the possible signatures of this model at the LHC. There are a couple of recent papers \cite{11, 12} that discuss specific phenomenological aspects of the ALRM, namely, the dark matter search and ALRM, with emphasis on the possible signatures of this model at the LHC. There are couple of recent papers \cite{13, 14} that confirm the Higgs discovery with mass around 125 GeV, through Higgs decay channels: \( H \rightarrow \gamma \gamma \), \( H \rightarrow ZZ^{(*)} \rightarrow 4l \), and \( H \rightarrow WW^{(*)} \rightarrow b\bar{b}ll \) at integrated luminosities of 5.1 fb\(^{-1}\) taken at energy \( \sqrt{s} = 7 \) TeV and 19.6 fb\(^{-1}\) taken at \( \sqrt{s} = 8 \) TeV. These results still indicate possible discrepancies between their results for signal strengths in these channels \cite{13, 15, 16, 17, 18}. We show that our ALRM has a rich Higgs sector, and consists of one bidoublet and two left-handed and right-handed doublets. Therefore, one obtains four neutral \( CP \)-even and two \( CP \)-odd Higgs bosons, in addition to two charged Higgs bosons. It turns out that most of these Higgs bosons can be light, of the order the electroweak scale, and can be accessible at the LHC. We also find that the contributions of the charged Higgs bosons to the decay rate of \( H \rightarrow \gamma \gamma \) are not significant. Furthermore, we show that, due to the mixing among the neutral \( CP \)-even Higgs bosons, the couplings of the SM-like Higgs, which is the lightest one, with the top quark and \( W \)-gauge boson are slightly modified respect to the SM ones. Therefore, the ALRM predictions for signal strengths of Higgs decays, in particular, \( H \rightarrow \gamma \gamma \) and \( H \rightarrow W^+W^- \), are consistent with the SM expectation.

Another salient feature of the ALRM is the presence of an extra down-type quark, \( d' \). We analyze the striking signature of this exotic quark at the LHC. We show that the most promising \( d' \)-production channel is \( gg \rightarrow d'd' \), due to the direct coupling of \( d' \) to gluons with a strong coupling constant and color factor. Then, \( d' \) decays to a jet and lepton plus missing energy. We find that the cross section of this process is of \( O(1) \) fb, which can be probed at the LHC with 14 TeV center-of-mass energy.

The paper is organized as follows. In Sec. 2, we briefly review the TeV scale ALRM. Section 3 is devoted to the Higgs sector, in particular, for studying the mixing matrix of the Higgs bosons and investigating the existence of two light charged Higgs bosons. In Sec. 4 we focus on the Higgs decay into a diphoton in the
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with their charge assignments, is presented in Table I \[19\]. As can be seen from this table, extra quarks and

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ALRM. The discovery signatures of extra quark \( d' \) at the LHC is discussed in Sec. 5. Finally, we give our conclusions in Sec. 6.

II. ALTERNATIVE LEFT-RIGHT SYMMETRIC MODEL

We consider an ALRM based on \( SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)/2} \times S \), where \( S \) is a discrete symmetry imposed to distinguish between scalars and their dual scalars. The fermion content of this model, with their charge assignments, is presented in Table I \[19 \]. As can be seen from this table, extra quarks and leptons are predicted as in all \( E_6 \)-based left-right models.

The Higgs sector of our ALRM consists of an \( SU(2)_R \) scalar doublet \( \chi_R \) to break \( SU(2)_R \times U(1)_{(B-L)/2} \) in addition to \( SU(2)_L \) scalar doublet \( \chi_L \) and scalar bidoublet \( \Phi \) that break \( SU(2)_L \times U(1)_Y \). The detailed quantum numbers of these Higgs bosons are presented in Table I \[19 \].

In this case, the most general left-right symmetric Yukawa Lagrangian is given by

\[
\mathcal{L}_Y = \overline{Q}_L Y^d \bar{\Phi} Q_R + \overline{Q}_L Y^d \chi_L d_R + \overline{Q}_R Y^d \chi_R d'_L + \overline{\nu}_L Y^e \Phi \psi_R + \overline{\nu}_R Y^e \bar{\Phi} \psi_R + \overline{\psi}_R Y^e \bar{\chi}_R n_L + \overline{\nu}_R M_R \nu_R + \text{h.c.},
\]

(2)

where \( \bar{\Phi} \) is the dual of the scalar bidoublet \( \Phi \), defined as \( \bar{\Phi} = \tau_2 \Phi^* \tau_2 \), and \( \bar{\chi}_{L,R} \) are the dual of the scalar doublets \( \chi_{L,R} \), defined as \( \bar{\chi}_{L,R} = \tau_2 \chi_{L,R} \). Note that the Yukawa terms like \( \overline{\psi}_L \Phi \nu_R \) and \( \overline{Q}_L \Phi Q_R \) are forbidden by the discrete \( S \) symmetry only. A detailed discussion on the Higgs potential and the associated vevs will be given in the next section. Here, we assume a nonvanishing vev of \( \langle \chi_R \rangle = v_R/\sqrt{2} \) of order TeV with vevs of \( \chi_L \) and \( \Phi \), given by \( \langle \chi_L \rangle = v_L/\sqrt{2} \) and \( \langle \Phi \rangle = \text{diag}(0, k/\sqrt{2}) \). The breaking of \( SU(2)_R \times U(1)_{(B-L)/2} \) down to \( U(1)_Y \) leaves the discrete symmetry: \( L = S + T_{3R} \) unbroken, if the vev of

| Fields | \( SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)/2} \times S \) | \( S \) |
|---|---|---|
| Fermions | \( Q_L \equiv \left( \begin{array}{c} u \\ d \end{array} \right)_L \) | \( (3, 2, 1, +\frac{1}{2}) \) | 0 |
| | \( Q_R \equiv \left( \begin{array}{c} u \\ d' \end{array} \right)_R \) | \( (3, 1, 2, +\frac{1}{2}) \) | 0 |
| | \( d_L \) | \( (3, 1, 1, -\frac{1}{2}) \) | -1 |
| | \( d_R \) | \( (3, 1, 1, -\frac{1}{2}) \) | 0 |
| | \( \psi_L \equiv \left( \begin{array}{c} \nu \\ e \end{array} \right)_L \) | \( (1, 2, 1, -\frac{1}{2}) \) | +\( \frac{1}{2} \) |
| | \( \psi_R \equiv \left( \begin{array}{c} n \\ e \end{array} \right)_R \) | \( (1, 1, 1, 0) \) | +1 |
| | \( \nu_R \) | \( (1, 1, 1, 0) \) | 0 |
| Higgs | \( \Phi \equiv \left( \begin{array}{c} \phi^0_1 \\ \phi^2_1 \\ \phi^0_2 \\ \phi^2_2 \end{array} \right) \) | \( (1, 2, 2^*, 0) \) | -\( \frac{1}{2} \) |
| | \( \chi_L \equiv \left( \begin{array}{c} \chi^+_L \\ \chi^0_L \end{array} \right) \) | \( (1, 2, 1, +\frac{1}{2}) \) | 0 |
| | \( \chi_R \equiv \left( \begin{array}{c} \chi^+_R \\ \chi^0_R \end{array} \right) \) | \( (1, 1, 2, +\frac{1}{2}) \) | +\( \frac{1}{2} \) |
\( \phi^0 \) (which has \( L = -1 \)) is zero, while \( \phi^0 \) (with \( L = 0 \)) could have a nonvanishing vev. In this case, one can easily show that the quarks \( u, d \) and \( d' \) and the charged leptons \( \ell \), in addition to the singlet fermion \( n \), which is called a scotino, acquire the following masses

\[
\begin{align*}
m_u &= \frac{1}{\sqrt{2}} Y_u v \sin \beta, \\
m_d &= \frac{1}{\sqrt{2}} Y_d v \cos \beta, \\
m_{d'} &= \frac{1}{\sqrt{2}} Y_{d'} v R v_R, \\
m_e &= \frac{1}{\sqrt{2}} Y_e v \sin \beta, \\
m_\mu &= \frac{1}{\sqrt{2}} Y_\mu v \cos \beta, \\
m_\nu &= \frac{1}{\sqrt{2}} Y_\nu v R v_R,
\end{align*}
\]

where \( \tan \beta = k/v_L \) and \( \sqrt{v_L^2 + k^2} = v = 246 \text{ GeV} \). Moreover, the neutrino mass matrix is given by

\[
M_\nu = \begin{pmatrix}
\nu_L^T & \nu_R \\
0 & m_{\nu D}
\end{pmatrix},
\]

where \( m_{\nu D} = Y_{\nu L} v_L / \sqrt{2} \). The mass \( M_R \) is not related to the \( SU(2)_R \) symmetry-breaking scale, so it can be quite large. This matrix can be diagonalized by a unitary matrix, \( V_\nu M_\nu V_\nu^T \simeq \text{diag}(m_\nu, m_\nu) \), where \( m_\nu \) and \( m_\nu \) are the well known seesaw eigenvalues of the light and heavy neutrinos, respectively:

\[
m_\nu \simeq m_{\nu D} M_R^{-1} m_{\nu D}^T, \quad m_\nu \simeq M_R.
\]

Now, we turn to the gauge sector of the ALRM; the covariant derivatives of the Higgs bosons are given by

\[
\begin{align*}
D_\mu \Phi &= \partial_\mu \Phi - i \frac{g}{2} \left( \tau^a W_{\mu}^a \right) \Phi - \Phi \tau^a W_{\mu}^a, \\
D_\mu \chi_{L,R} &= \partial_\mu \chi_{L,R} - i \frac{g}{2} \epsilon^a W_{\mu}^a \chi_{L,R} - \frac{g_{BL}}{2} B_\mu \chi_{L,R},
\end{align*}
\]

where \( g_{BL} \) is the gauge coupling of the \( U(1)_{B-L} \) group. After the spontaneous breaking of left-right symmetry down to electroweak and then down to electromagnetism, the associated gauge bosons acquire masses, through the nonvanishing vevs of \( \Phi, \chi_L, \) and \( \chi_R \). Because of the vanishing vev of \( \phi^0 \in \Phi \), the mixing between \( W^\pm_L \) and \( W^\pm_R \) is identically zero. Thus, the physical eigenstates are given by SM gauge bosons \( W^\pm = W^\pm_L \) and \( W^\pm = W^\pm_R \) with masses

\[
\begin{align*}
M_{W^\pm_L} &= \frac{1}{4} g^2 (k^2 + v_L^2) = \frac{1}{4} g^2 v^2, \\
M_{W^\pm_R} &= \frac{1}{4} g^2 (k^2 + v_R^2).
\end{align*}
\]

The experimental searches for \( W' \) at the LHC through their decays to electron/muon and neutrino lead to \( M_{W'} \gtrsim 2.5 \text{ TeV} \). The interactions of our \( W' \) with the SM fermions are given by

\[
L_{W'} = -\frac{ig}{\sqrt{2}} \gamma^\mu W'^{\mu}_{R} V_{ckm}^T \left( \begin{array}{c}
\nu_L \\
p_L \\
1
\end{array} \right) d' - \frac{ig}{\sqrt{2}} \gamma^\mu W'^{\mu}_{L} V_{ckm} \left( \begin{array}{c}
\nu_R \\
p_R \\
1
\end{array} \right) u
\]

\[
- \frac{ig}{\sqrt{2}} \gamma^\mu W'^{\mu}_{R} U_{MNS}^\dagger \left( \begin{array}{c}
\nu_L \\
p_L \\
1
\end{array} \right) e - \frac{ig}{\sqrt{2}} \gamma^\mu W'^{\mu}_{L} U_{MNS}^\dagger \left( \begin{array}{c}
\nu_R \\
p_R \\
1
\end{array} \right) n.
\]

Thus, \( W' \) can decay into an electron and singlet fermion (scotino) \( n \), which appears at the LHC as missing energy. Therefore, the above-mentioned lower bound on \( M_{W'} \) is applicable in our ALRM. This implies that \( v_R \gtrsim O(1) \text{ TeV} \). The situation of the neutral gauge bosons: \( W^3_L, W^3_R \) and \( B \) is more involved. One can show that their mass matrix is given by

\[
\begin{pmatrix}
W^3_L & W^3_R & B \\
\frac{1}{4} g^2 (k^2 + v_L^2) & -\frac{1}{4} g^2 k^2 & -\frac{1}{4} g g_{BL} v_L^2 \\
-\frac{1}{4} g^2 k^2 & \frac{1}{4} g^2 (k^2 + v_R^2) & -\frac{1}{4} g g_{BL} v_R^2 \\
-\frac{1}{4} g g_{BL} v_L^2 & -\frac{1}{4} g g_{BL} v_R^2 & \frac{1}{4} g g_{BL}^2 (v_L^2 + v_R^2)
\end{pmatrix}.
\]
One can define \( s_w \equiv \sin \theta_w = e/g \), and with \( c_w \equiv \cos \theta_w \), then \( g_{BL} = e/\sqrt{c_w^2 - s_w^2} \). It is more convenient to work in the basis \((A, Z_L, Z_R)\), where

\[
\begin{pmatrix}
A \\
Z_L \\
Z_R
\end{pmatrix} = \begin{pmatrix}
s_w & s_w & \frac{\sqrt{c_w^2 - s_w^2}}{c_w} \\
c_w & -s_w^2/c_w & -s_w \sqrt{c_w^2 - s_w^2}/c_w \\
0 & \sqrt{c_w^2 - s_w^2}/c_w & -s_w/c_w
\end{pmatrix} \begin{pmatrix}
W_L^3 \\
W_R^3 \\
B
\end{pmatrix}.
\]

(13)

In this case, one can show that the mass eigenvalue of the gauge boson \( A \) is identically zero. Therefore this gauge boson is the photon that should remain massless after symmetry breaking. The exact eigenstates \( Z, Z' \) are obtained as

\[
\begin{pmatrix}
Z \\
Z'
\end{pmatrix} = \begin{pmatrix}
\cos \vartheta & \sin \vartheta \\
-\sin \vartheta & \cos \vartheta
\end{pmatrix} \begin{pmatrix}
Z_L \\
Z_R
\end{pmatrix}.
\]

(14)

The mixing angle \( \vartheta \) is defined as

\[
\tan 2\vartheta = \frac{2M_{LR}^2}{M_{LL}^2 - M_{RR}^2},
\]

(15)

where

\[
M_{LL}^2 = \frac{g^2 v^2}{4 \cos^2 \theta_w},
\]

(16)

\[
M_{LR}^2 = \frac{g^2 (v^2 \sin^2 \theta_w - k^2 \cos^2 \theta_w)}{4 \cos^2 \theta_w \sqrt{2} \theta_w},
\]

(17)

\[
M_{RR}^2 = \frac{g^2 (2v^2 \sin^4 \theta_w + 2(k^2 + v_R^2) \cos^2 \theta_w - k^2 \sin^2 2\theta_w)}{8 \cos^2 \theta_w \cos 2\theta_w}.
\]

(18)

The eigenvalues \( M_{Z,Z'}^2 \) are given by

\[
M_{Z,Z'}^2 = \frac{1}{2} \left( M_{LL}^2 + M_{RR}^2 + (M_{RR}^2 - M_{LL}^2) \sqrt{1 + \tan^2 2\vartheta} \right).
\]

(19)

It is clear that if \( v_R \gg v \), i.e., \( \vartheta \to 0 \), then \( Z \simeq Z_L \) and \( Z' \simeq Z_R \). The LHC search for the \( Z' \) gauge boson is rather model dependent. However, one may consider \( M_{Z'} \gtrsim 2 \text{ TeV} \) as a conservative lower bound.\(^{22,23}\) In addition, the mixing between \( Z \) and \( Z' \) should be less than \( O(10^{-3}) \).

### III. HIGGS SECTOR IN THE ALRM

#### A. Symmetry breaking

The Higgs sector of our ALRM consists of bidoublet \( \Phi \) with left and right doublets \( \chi_L \) and \( \chi_R \). The charge assignments of these Higgs bosons are shown in Table I. As mentioned in the previous section, the gauge symmetries \( SU(2)_R \times U(1)_{(B-L)/2} \) are spontaneously broken to \( U(1)_Y \) through the vev of \( \chi_R \), and then \( SU(2)_L \times U(1)_Y \) symmetries are broken by vevs of \( \Phi \) and \( \chi_L \). The most general Higgs potential that is invariant under these symmetries is given by

\[
V(\Phi, \chi_{L,R}) = -\mu_2^2 Tr[\Phi^\dagger \Phi] + \lambda_1 (Tr[\Phi^\dagger \Phi])^2 + \lambda_2 Tr[\Phi^\dagger \Phi] Tr[\tilde{\Phi}^\dagger \tilde{\Phi}] \\
-\mu_3^2 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) + \lambda_3 (\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2] + 2\lambda_4 (\chi_L^\dagger \chi_L)(\chi_R^\dagger \chi_R) \\
+2\alpha_1 Tr(\Phi \Phi^\dagger)(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) + 2\alpha_2 (\chi_L^\dagger \Phi^\dagger \Phi \chi_L + \chi_R^\dagger \Phi^\dagger \Phi \chi_R) \\
+2\alpha_3 (\chi_L^\dagger \Phi^\dagger \Phi^\dagger \Phi \chi_L + \chi_R^\dagger \Phi^\dagger \Phi^\dagger \Phi \chi_R) + \mu_5 (\chi_L^\dagger \chi_R^\dagger \Phi^\dagger \Phi^\dagger \Phi \chi_L).
\]
In the Appendix, we provide a detailed study for the conditions that keep the potential (20) bounded from below. It is remarkable that the copositivity conditions \(24, 25\) for this Higgs potential significantly depend on the signs of the following parameters \(\alpha_{12} = \alpha_1 + \alpha_2, \alpha_{13} = \alpha_1 + \alpha_3, \text{and} \lambda_{12} = \lambda_1 + 2\lambda_2\). Here, we present the case with minimal constraints imposed on the potential parameters:

\[
\lambda_1 \geq 0, \lambda_2 \leq 0, \lambda_3 \geq 0, \alpha_2 - \alpha_3 \geq 0, \alpha_{12} \geq 0, \alpha_{13} \geq 0, \lambda_{12} \geq 0. \tag{21}
\]

Also for perturbativity the absolute value of any dimensionless potential parameter is assumed to be less than \(\sqrt{4\pi}\). In addition, from the minimization conditions, one finds that the nonvanishing vevs are given by

\[
v_L v_R = \frac{-\mu_3 k}{\sqrt{2}(\lambda_4 - \lambda_3)^2}, \tag{22}\]

\[
v_L^2 + v_R^2 = \frac{\mu_2^2 - \alpha_{12} k^2}{\lambda_3}, \tag{23}\]

\[
k^2 = \frac{2(\lambda_3 \mu_2^2 - \alpha_{12} \lambda_3^2)(\lambda_4 - \lambda_3) + \lambda_3 \mu_2^4}{2(\lambda_1 \lambda_3 - \alpha_{12}^2)(\lambda_4 - \lambda_3)}. \tag{24}\]

We use these equations to determine three parameters \((\mu_1, \mu_2, \text{and} \lambda_4)\) out of the ten free parameters in the Higgs potential (20) in terms of the vevs: \(k = v \sin \beta, v_L = v \cos \beta, \text{and} v_R \sim O(1) \text{ TeV}\). Note that since the vevs \(k, v_L\) are of the same order and the couplings \(\lambda_{3,4} \lesssim O(1)\), the values of \(\mu_3\) can be smaller than \(v_R\).

**B. Higgs masses and mixing**

We begin by 16 degrees of freedom; 8 of \(\Phi\) and 8 of \(\chi_L\) and \(\chi_R\). After symmetry breaking, two neutral components of these 16 degrees of freedom will be eaten by the neutral gauge bosons \(Z\) and \(Z'\) to acquire their masses. In addition, another four charged components will be eaten by the charged gauge bosons \(W^\pm\) and \(W'^\pm\) to acquire their masses. Therefore, ten scalars remain as physical Higgs bosons in this class of models. As we will explicitly show, four of them give charged Higgs bosons, two lead to pseudoscalar Higgs bosons, and the remaining four give \(CP\)-even neutral Higgs bosons.

1. **Charged Higgs bosons:**

   The mass matrix of the charged Higgs bosons, in the basis \(\begin{pmatrix} \phi_1^+ & \chi_L^+ & \phi_2^+ & \chi_R^+ \end{pmatrix}\), is a block diagonal matrix with the following two matrices, which, respectively, correspond to the bases \(\begin{pmatrix} \phi_1^+ & \chi_L^+ \end{pmatrix}\) and \(\begin{pmatrix} \phi_2^+ & \chi_R^+ \end{pmatrix}\):

\[
M_{1L}^+= \begin{pmatrix}
-(\alpha_2 - \alpha_3)v_L^2 - \frac{\mu_2 v_L \cot \beta}{\sqrt{2}} & (\alpha_2 - \alpha_3)v_L^2 \tan \beta - \frac{\mu_2 v_L}{\sqrt{2}} \\
(\alpha_2 - \alpha_3)v_L^2 \tan \beta - \frac{\mu_2 v_L}{\sqrt{2}} & -(\alpha_2 - \alpha_3)v_L^2 \tan^2 \beta - \frac{\mu_2 v_L}{\sqrt{2}} \tan \beta
\end{pmatrix}, \tag{25}\]

\[
M_{2R}^+= \begin{pmatrix}
-(\alpha_2 - \alpha_3)v_R^2 - \frac{\mu_2 v_R \cot \zeta}{\sqrt{2}} & (\alpha_2 - \alpha_3)v_R^2 \tan \zeta - \frac{\mu_2 v_R}{\sqrt{2}} \\
(\alpha_2 - \alpha_3)v_R^2 \tan \zeta - \frac{\mu_2 v_R}{\sqrt{2}} & -(\alpha_2 - \alpha_3)v_R^2 \tan^2 \zeta - \frac{\mu_2 v_R}{\sqrt{2}} \tan \zeta
\end{pmatrix}, \tag{26}\]

where \(\tan \zeta = k/v_R\), in analogy to \(\tan \beta\) with left-right switch. These matrices can be diagonalized by the unitary transformations: \(V_1^\dagger M_{1L}^+ V_1 = \text{diag} \left(M_{H_1^+}, 0\right)\) and \(V_2^\dagger M_{2R}^+ V_2 = \text{diag} \left(M_{H_2^z}, 0\right)\), where

\[
\begin{pmatrix}
\phi_1^+ \\
\chi_L^+
v_1
\end{pmatrix} = \begin{pmatrix}
\cos \beta & \sin \beta \\
-\sin \beta & \cos \beta\end{pmatrix}\begin{pmatrix}
H_1^+ \\
G_1^+
v_1
\end{pmatrix}, \quad \begin{pmatrix}
\phi_2^+ \\
\chi_R^+
v_2
\end{pmatrix} = \begin{pmatrix}
\cos \zeta & \sin \zeta \\
-\sin \zeta & \cos \zeta\end{pmatrix}\begin{pmatrix}
H_2^z \\
G_2^z
v_2
\end{pmatrix}. \tag{27}\]
The eigenstates $G_{1}^{\pm}$ and $G_{2}^{\pm}$ are the charged Goldstone bosons eaten by the gauge bosons $W^{\pm}$ and $W'^{\pm}$ to acquire their masses. The charged Higgs bosons masses are

$$M_{H_{1}^{\pm}}^{2} = -(\alpha_{2} - \alpha_{3})v_{L}^{2}\sec^{2}\beta - \sqrt{2}\mu_{3}v_{R}\csc 2\beta,$$

(28)

$$M_{H_{2}^{\pm}}^{2} = -(\alpha_{2} - \alpha_{3})v_{R}^{2}\sec^{2}\zeta - \sqrt{2}\mu_{3}v_{L}\csc 2\zeta.$$

(29)

From these expressions, one can show that the mass of the charged Higgs can be of $\mathcal{O}(100)$ GeV.

2. CP-odd Higgs bosons:

We now turn to the neutral Higgs physical fields and their masses. This can be easily obtained if one develops the neutral components of the bidoublets $\Phi$ and the doublets $\chi_{L,R}$ around their vacua into real and imaginary parts, i.e.,

$$\phi_{i}^{0} = \frac{1}{\sqrt{2}}(v_{i} + \phi_{i}^{0R} + i\phi_{i}^{0I}), \quad i = 1, 2, L, R,$$

(30)

where $v_{1} = 0$, $v_{2} = k$, and $\phi_{L,R} = \chi_{L,R}$. In this case, the squared mass matrix of neutral Goldston and CP-odd Higgs bosons is given by

$$M_{ij}^{2} = \frac{\partial^{2}V(\Phi, \chi_{L,R})}{\partial\phi_{i}^{0I}\partial\phi_{j}^{0I}} \bigg|_{(\phi_{i}^{0})=(\phi_{i}^{0I})=0}.$$

(31)

One finds that this mass matrix in the basis of $(\phi_{1}^{0I}, \phi_{2}^{0I}, \chi_{L}^{0I}, \chi_{R}^{0I})$ is factored as a product of the squared mass of $\phi_{1}^{0I}$, which is totally decoupled due to the fact that we have $v_{1} = 0$, times the following $3 \times 3$ squared mass matrix of $(\phi_{2}^{0I}, \chi_{L}^{0I}, \chi_{R}^{0I})$:

$$M_{I}^{2} = -\frac{k\mu_{3}}{2\sqrt{2}}\begin{pmatrix}
\cot\beta\cot\zeta & -\cot\zeta & \cot\beta \\
-\cot\zeta & \tan\beta\cot\zeta & -1 \\
\cot\beta & -1 & \tan\zeta\cot\beta
\end{pmatrix}.$$

(32)

The mass of the first pseudoscalar Higgs boson $\phi_{1}^{0I} \equiv A_{1}$ is given by

$$M_{A_{1}}^{2} = 2k^{2}\lambda_{2} - (\alpha_{2} - \alpha_{3})k(\cot^{2}\beta + \cot^{2}\zeta) - \frac{1}{\sqrt{2}}k\mu_{3}\cot\beta\cot\zeta.$$

(33)

The matrix $M_{I}^{2}$ can be diagonalized by the unitary transformation: $U^\dagger M_{I}^{2}U = \text{diag}(M_{A_{2}^{0}}, 0, 0)$,

$$\begin{pmatrix}
\phi_{2}^{0I} \\
\chi_{L}^{0I} \\
\chi_{R}^{0I}
\end{pmatrix} = U\begin{pmatrix}
1 & \frac{1}{\sqrt{\tan^{2}\beta + \tan^{2}\zeta + 1}} & \frac{1}{\sqrt{\tan^{2}\beta + \tan^{2}\zeta + 1}} \\
\frac{\sqrt{\tan^{2}\beta + \tan^{2}\zeta + 1}}{\tan\beta} & 0 & \frac{\sqrt{\tan^{2}\beta + \tan^{2}\zeta + 1}}{\tan\beta + \tan^{2}\zeta + 1} \\
\frac{\sqrt{\tan^{2}\beta + \tan^{2}\zeta + 1}}{\tan\zeta} & \frac{\sqrt{\tan^{2}\beta + \tan^{2}\zeta + 1}}{\tan\beta + \tan^{2}\zeta + 1} & \frac{\sqrt{\tan^{2}\beta + \tan^{2}\zeta + 1}}{\tan\beta\tan\zeta}
\end{pmatrix}\begin{pmatrix}
A_{2}^{0} \\
G_{1}^{0} \\
G_{2}^{0}
\end{pmatrix},$$

(34)

where $G_{1}^{0}$ and $G_{2}^{0}$ are the neutral Goldstone bosons eaten by the gauge bosons $Z$ and $Z'$ to acquire their masses. The other CP-odd Higgs bosons mass is given by

$$M_{A_{2}}^{2} = \frac{k\mu_{3}(1 + \tan^{2}\beta + \tan^{2}\zeta)}{\sqrt{2}\tan\beta\tan\zeta}.$$

(35)

It is worth mentioning that $M_{A_{2}}^{2}$ constrains the parameter $\mu_{3}$ to be negative. We find that the typical values of CP-odd Higgs masses are of $\mathcal{O}(100)$ GeV.
3. CP-even Higgs bosons:

Finally, we consider the CP-even Higgs bosons. Similar to the CP-odd Higgs, the squared mass matrix of CP-even Higgs bosons is given by

$$M^2_{R_{ij}} = \frac{\partial^2 V(\Phi, \chi_L, R)}{\partial \phi^0_L \partial \phi^0_R} \bigg|_{(\phi^0_L) = (\phi^0_R) = 0}. \quad (36)$$

Again, one finds that $H_1 = \phi^0_R$ is decoupled with mass $M_{H_1} = M_{A_1}$. The remaining squared mass matrix of the CP-even Higgs bosons is given in the basis \((\phi^0_2, \chi^0_L, \chi^0_R)\) by

$$M^2_R = \begin{pmatrix}
  k^2 \lambda_1 - \frac{k_m}{2\sqrt{2}} \cot \beta \cot \zeta & \alpha_{12} k^2 \cot \beta + \frac{k_m}{2\sqrt{2}} \cot \zeta & \alpha_{12} k^2 \cot \zeta + \frac{k_m}{2\sqrt{2}} \cot \beta \\
  \alpha_{12} k^2 \cot \beta + \frac{k_m}{2\sqrt{2}} \cot \zeta & k^2 \lambda_3 \cot^2 \beta - \frac{k_m}{2\sqrt{2}} \tan \beta \cot \zeta & k^2 \lambda_3 \cot \beta \cot \zeta - \frac{k_m}{2\sqrt{2}} \tan \zeta \cot \beta \\
  \alpha_{12} k^2 \cot \zeta + \frac{k_m}{2\sqrt{2}} \cot \beta & k^2 \lambda_3 \cot \beta \cot \zeta - \frac{k_m}{2\sqrt{2}} \tan \zeta \cot \beta & k^2 \lambda_3 \cot^2 \zeta - \frac{k_m}{2\sqrt{2}} \tan \zeta \cot \beta
\end{pmatrix}. \quad (37)$$

This matrix can be diagonalized by a unitary transformation: $T^\dagger M^2_R T = \text{diag} \left( M^2_{H_2}, M^2_{H_3}, M^2_H \right)$. The lightest eigenstate $H$ is the SM-like Higgs, the mass of which we will fix to be 125 GeV. In general, from the numerical checks, we found that three CP-even Higgs bosons ($H$ and $H_{1,3}$) are light [of $\mathcal{O}(100)$ GeV] and the other one $H_2$ is heavy [of $\mathcal{O}(1)$ TeV].

C. Couplings of the SM-like Higgs

From the Yukawa Lagrangian \([2]\), one finds that the SM-like Higgs couplings with fermions in the ALRM are given by

$$Y_{H\bar{u}u} = \frac{m_u}{v} \frac{T_\Phi}{\sin \beta}, \quad Y_{H\bar{d}d} = \frac{m_d}{v} \frac{T_L}{\cos \beta}, \quad Y_{H\bar{d}d'} = \frac{m_d'}{v} \frac{T_R}{}, \quad (38)$$

$$Y_{H\bar{e}e} = \frac{m_e}{v} \frac{T_\Phi}{\sin \beta}, \quad Y_{H\bar{e}e'} = \frac{m_e'}{v} \frac{T_R}{}. \quad (39)$$

where the elements $T_\Phi$, $T_L$, and $T_R$ are the mixing couplings of the gauge eigenstates $\phi^0_L$, $\chi^0_L$, and $\chi^0_R$, respectively, with the lightest Higgs $H$. Similarly, from the kinetic Lagrangian of the scalars, one can derive the following SM-like Higgs couplings with the electroweak gauge bosons,

$$g_{HWW} = g_{M_W} (T_\Phi \sin \beta + T_L \cos \beta), \quad (40)$$

$$g_{HWW'} = g_{M_W} \left( T_\Phi \sin \beta + T_R \frac{v_R}{v} \right), \quad (41)$$

$$g_{HZZ} = g_{LL} \cos^2 \theta + g_{LR} \sin \theta \cos \theta + g_{RR} \sin^2 \theta, \quad (42)$$

$$g_{HZZ'} = g_{LL} \sin^2 \theta - g_{LR} \sin \theta \cos \theta + g_{RR} \cos^2 \theta, \quad (43)$$

where

$$g_{LL} = \frac{g_{M_W}}{\cos^2 \theta_w} (T_\Phi \sin \beta + T_L \cos \beta), \quad (44)$$

$$g_{LR} = \frac{\sqrt{2}g_{M_W}}{\cos^2 \theta_w \sqrt{\cos^2 2\theta_w}} (T_\Phi \cos \beta \cos 2\theta_w - T_L \sin \beta \sin^2 \theta_w), \quad (45)$$

$$g_{RR} = \frac{g_{M_W}}{\sqrt{2} \cos^2 \theta_w \cos 2\theta_w} \left( T_\Phi \cos \beta \cos^2 2\theta_w + T_L \sin \beta \sin^4 \theta_w + T_R \frac{v_R}{v} \cos^4 \theta_w \right). \quad (46)$$

Finally, the SM-like Higgs couplings with the charged Higgs bosons are given by

$$\lambda_{HH_H^\pm H^\mp} = M_{1f} T_\Phi + M_{1L} T_L + M_{1R} T_R, \quad (47)$$

$$\lambda_{HH_H^\pm H^\mp} = M_{2f} T_\Phi + M_{2L} T_L + M_{2R} T_R, \quad (48)$$

where $f = u, d, e, \ell$.
As shown in the previous section, the Higgs couplings $g$ while the ATLAS experiment reported that the signal strength of these decays are given by [13, 17, 18]:

$$
\mu_{\gamma\gamma} = \mu(H \rightarrow \gamma\gamma) = 1.14^{+0.26}_{-0.23},
$$

$$
\mu_{ZZ} = \mu(H \rightarrow ZZ) = 0.91^{+0.3}_{-0.24},
$$

$$
\mu_{WW} = \mu(H \rightarrow WW) = 0.76 \pm 0.21,
$$

while the ATLAS experiment reported that the signal strength of these decays are given by [13, 17, 18]:

$$
\mu_{\gamma\gamma} = \mu(H \rightarrow \gamma\gamma) = 1.17 \pm 0.27,
$$

$$
\mu_{ZZ} = \mu(H \rightarrow ZZ) = 1.7 \pm 0.5,
$$

$$
\mu_{WW} = \mu(H \rightarrow WW) = 1.01 \pm 0.31.
$$

These results indicate enhancement in the diphoton decay channel, with more than $2\sigma$ deviation, which could be a very important signal for possible new physics beyond the SM. Much work has been done to accommodate these results in different extensions of the SM. The Higgs signal strength of the decay channel, $H \rightarrow \gamma\gamma$, relative to the SM expectation is defined as

$$
\mu_{\gamma\gamma} = \frac{\sigma(pp \rightarrow H \rightarrow \gamma\gamma)}{\sigma(pp \rightarrow H \rightarrow \gamma\gamma)_{\text{SM}}} = \frac{\sigma(pp \rightarrow H)}{\sigma(pp \rightarrow H)_{\text{SM}}} \cdot \frac{\text{BR}(H \rightarrow \gamma\gamma)}{\text{BR}(H \rightarrow \gamma\gamma)_{\text{SM}}}
$$

$$
= \frac{\Gamma(H \rightarrow gg)_{\text{SM}}}{\Gamma(H \rightarrow \gamma\gamma)_{\text{SM}}} \cdot \frac{\Gamma(H \rightarrow AA)_{\text{SM}}}{\Gamma(H \rightarrow \gamma\gamma)_{\text{SM}}} = \frac{\kappa_{gg}}{\kappa_{\gamma\gamma}} \cdot \kappa_{\gamma\gamma}^{-1},
$$

where $\sigma(pp \rightarrow H)$ is the total Higgs production cross section and $\text{BR}(H \rightarrow \gamma\gamma)$ is the branching ratio of the corresponding channel. The total Higgs decay width is given by the sum of the dominant Higgs partial decay widths, $\Gamma_{\text{tot}} = \Gamma_{b\bar{b}} + \Gamma_{WW} + \Gamma_{ZZ} + \Gamma_{gg} + \Gamma_{\tau\tau}$. Other partial decay widths are much smaller and can be safely neglected. In the SM with 125 GeV Higgs mass, these partial decay widths are given by: $\Gamma_{b\bar{b}} = 2.3 \times 10^{-3}$ GeV, $\Gamma_{WW} = 8.7 \times 10^{-4}$ GeV, $\Gamma_{ZZ} = 1.1 \times 10^{-4}$ GeV, $\Gamma_{gg} = 3.5 \times 10^{-4}$ and $\Gamma_{\tau\tau} = 2.6 \times 10^{-4}$ GeV.

As shown in the previous section, the Higgs couplings $\delta_{HWW}$ and $Y_{Hb\bar{b}}$ may slightly change from the SM values. Hence, the total decay width of the Higgs boson remains very close to the SM result. This has been confirmed numerically, and to a very good approximation, one can consider $\kappa_{\text{tot}} \approx 1$.

Now, we turn to the SM-like Higgs decay into a diphoton, $W^+W^-$ and $ZZ$ in our ALRM. As shown in the previous section, the low-energy effective theory of the ALRM contains two charged Higgs bosons that can be light, of $\mathcal{O}(100)$ GeV, and may give relevant contributions to the SM-like Higgs decay into a diphoton. In
addition, the couplings of the SM-like Higgs with a top quark and $W$ gauge boson may be suppressed or even flipped, which would lead to significant enhancement/suppression in $\Gamma(H \to \gamma\gamma)$. The Feynman diagrams of the Higgs decay $H \to \gamma\gamma$, mediated by the gauge bosons $W^\pm$, top quark, and light-charged Higgs bosons are shown in Fig. 1. Note that in the conventional LRM there are interaction vertices among charged Higgs, the $W$-boson and neutral Higgs/photons; therefore, another four diagrams with $W^\pm$ and $H^\pm$ running in the loop of triangle diagrams can be generated. In our ALRM, these vertices identically vanish due to the discrete $S$ symmetry. In this case, the one-loop partial decay width of the $H$ decay into two photons is given by [26]

$$\Gamma(H \to \gamma\gamma) = \frac{\alpha^2 m_H^3}{1024\pi^3} \left| \frac{g_{HWW}}{M_W^2} Q_W^2 F_1(x_W) + N_{c,t} Q_t^2 \frac{2 Y_{Htt}}{m_t} F_{1/2}(x_t) + \sum_{i=1}^2 Q_{H_i}^2 \frac{\lambda_{H^\pm H^\mp H^\mp}}{M_{H_i}^2} F_0(x_{H_i}^\pm) \right|^2,$$  

(62)

where $x_t = M_{H_i}^2/4m_t^2$, $x_k = M_{H_1}^2/4M_W^2$, $k = W, H^\pm_1, 2$. The color factor and electric charges are given by $N_{c,t} = 3$, $Q_W = Q_{H^\pm_1} = 1$, and $Q_t = 2/3$. Recall that the relevant Higgs couplings in the ALRM are given by $g_{HWW}$, $Y_{Htt}$, and $\lambda_{H^\pm H^\mp H^\mp}$ in Eqs. (39), (40), (47), and (48), with $T_0 \sim T_L \gg T_R$. Finally, the loop functions $F_i(x)$ are given by [26]

$$F_1(x) = -\left[2x^2 + 3x + 3(2x - 1) \arcsin^2(\sqrt{x}) \right] x^{-2},$$  

(63)

$$F_{1/2}(x) = 2 \left[ x + (x - 1) \arcsin^2(\sqrt{x}) \right] x^{-2},$$  

(64)

$$F_0(x) = -\left[ x - \arcsin^2(\sqrt{x}) \right] x^{-2}.$$  

(65)

For Higgs mass of order 125 GeV and charged Higgs mass of order 200 GeV, the loop functions $F_1(x_W)$, $F_{1/2}(x_t)$, and $F_0(x_{H^\pm})$ are of order $-8.32, +1.38$, and $+0.43$, respectively. Therefore, the partial decay width $\Gamma(H \to \gamma\gamma)$ can be enhanced through one of the following possibilities: (i) large charged Higgs couplings such that $\lambda_{H^\pm H^\mp H^\mp}$ is of order $g_{HWW}/M_W^2$, and with an opposite sign to compensate the difference in sign between $F_1(x_W)$ and $F_0(x_{H^\pm})$; (ii) either the sign of the top Yukawa coupling, $Y_{Htt}$, or the sign of the coupling between the $W$ boson and the SM-like Higgs, $g_{HWW}$, is flipped so that a constructive interference between $W$-gauge boson and top-quark contributions takes place; and (iii) a significant reduction for the top Yukawa coupling, $Y_{Htt}$, to minimize the destructive interference between $W$ and $t$ contributions. In Fig. 2, we display the changes in $g_{HWW}$ and $Y_{Htt}$, normalized to their SM values. As can be seen from this figure, both couplings are slightly changed from their expectations in the SM. In addition, both $g_{HWW}$ and $Y_{Htt}$ may flip their sign simultaneously, and hence the usual destructive interference between $W$-gauge boson and top-quark contributions remains intact. Therefore, one would not expect any enhancement of $\Gamma(H \to \gamma\gamma)$. The sign correlation between the coupling ratios can be understood from the fact that the parameters $T_0$.
and $T_L$ in Eqs. (39) and (40), which lead to the modifications of these couplings, have the same sign in the allowed region of ALRM parameter space, as shown in Fig. [2].

The Higgs boson production at the LHC is dominated by gluon-gluon fusion. As in the SM, this channel is mediated by top quarks via a one-loop triangle diagram. The extra quark $d'$ gives a negligible contribution to $\sigma(gg \to H)$ due to the suppression of its coupling with the SM-like Higgs and also its large mass. As mentioned, the top Yukawa coupling can be slightly different from the SM coupling; therefore, the ratio $\kappa_{gg} = \Gamma(H \to gg)/\Gamma(H \to gg)^{\text{SM}}$ can be slightly deviated from 1.

In Fig. 3, we display the results of $\kappa_{\gamma\gamma} = \Gamma(H \to \gamma\gamma)/\Gamma(H \to \gamma\gamma)^{\text{SM}}$ and $\kappa_{gg}$ as function of $\tan \beta$ for $0 < \lambda_1, \lambda_3, \lambda_4 < \sqrt{4\pi}$, $-\sqrt{4\pi} < \lambda_2 < 0$, $-\sqrt{4\pi} < \alpha_1, \alpha_2, \alpha_3 < \sqrt{4\pi}$, 100 $< M_{H_{1,2}^\pm} < 300$, and $\mu_3 < 0$, to be consistent with the perturbative unitarity and the minimization and boundedness from below conditions (21) - (24). It is worth mentioning that for $\mu_3 < 0$, one finds, from the minimization conditions, that $\lambda_4 - \lambda_3 > 0$, and $\lambda_4 > \lambda_3 \geq 0$. In our numerical analysis, we express the parameters $\mu_2, \mu_3$, and $\alpha_4$ in terms of the three vevs $v_L, v_R$, and $k$ (or $v, \tan \beta$, and $M_W$). We also substitute the parameters $\mu_3$ and $\alpha_3$ in terms of the charged Higgs masses $M_{H_{1,2}^\pm}$ and the parameter $\lambda_1$ in terms of the SM-like Higgs mass $M_H = 125$ GeV. Thus, one can write the matrix $T \equiv T(\tan \beta, M_W, M_{H_{1,2}^\pm}, \lambda_3, \alpha_1, \alpha_2)$. This figure confirms our theoretical expectation and shows that both of $\kappa_{\gamma\gamma}$ and $\kappa_{gg}$ can slightly deviate from 1.

In this case, it is clear that the signal strength $\mu_{\gamma\gamma}$ is also close to the SM expectation and can be still consistent with both ATLAS and CMS experimental results. In Fig. 4 we show the signal strength as a function of $\tan \beta$, where other parameters are scanned in the above-mentioned regions. For completeness, we also present the correlation between $\mu_{\gamma\gamma}$ and $\mu_{ZZ}$, which equals $\mu_{WW}$ in our model. It is remarkable that all signal strengths of Higgs decay channels in the ALRM are slightly less than the SM results.

**V. SIGNATURES AT THE LHC**

In this section we study the interesting signatures of the exotic quark $d'$ associated with our ALRM at the LHC. In particular, we will analyze and compute the cross section for the production of this heavy quark and its subsequent decays into jets, leptons and missing energy. The Lagrangian of $d'$ interactions with the SM quarks can be derived from (2) as

$$\mathcal{L}_Y^{d'} = -\Pi (\cos \zeta Y_3^R P_R + \sin \zeta Y_3^R P_L) H_{1,2}^\pm V_{CKM}^{d'} d' + \text{h.c.},$$

(66)
FIG. 3: \( \kappa_{\gamma\gamma} \) and \( \kappa_{gg} \) as functions of \( \tan\beta \) and the parameters \( \lambda_3, \alpha_1, \alpha_2 \) and \( M_{H^\pm} \).

FIG. 4: (Left panel) The signal strength \( \mu_{\gamma\gamma} \) as a function of \( \tan\beta \) and the parameters \( \lambda_3, \alpha_1, \alpha_2 \) and \( M_{H^\pm} \). (Right panel) Correlation between \( \mu_{\gamma\gamma} \) and \( \mu_{ZZ} \) in the ALRM.

where \( V'_{\text{CKM}} \) is the right-handed quark mixing matrix. In addition, the kinetic Lagrangian of \( d' \) leads to the following interactions with the gauge bosons

\[
\mathcal{L}_{\text{gauge}}^{d'} = -\frac{i g_s}{2} \bar{d}' \gamma^\mu \lambda_a G_{\mu}^{\alpha} d' - \frac{ig}{\sqrt{2}} \bar{d}' P_R \gamma^\mu W^\mu_{\mu} V_{\text{CKM}}^{\alpha} d' - \frac{ig}{\sqrt{2}} \bar{d}' \gamma^\mu P_R W^\mu_{\mu} \bar{V}'_{CKM} d' \\
+ \frac{i}{3} \bar{d}' \gamma^\mu \left[ A_\mu + \left( \hat{P} \sin \vartheta - \frac{1}{2} \tan \vartheta \cos \vartheta \right) Z_\mu + \left( \hat{P} \cos \vartheta + \frac{1}{2} \tan \vartheta \sin \vartheta \right) Z'_\mu \right] d',
\]

where

\[
\hat{P} = \frac{3 \cos 2\theta_w - \sin^2 \theta_w}{6 \sin \theta_w \cos \theta_w \sqrt{\cos 2\theta_w}} P_R + \frac{\sin \theta_w}{\cos \theta_w \sqrt{\cos 2\theta_w}} P_L,
\]

where \( \lambda_a \)'s, \( a = 1, ..., 8 \), are the Gell–Mann matrices and \( \vartheta \) is given in \[13\]. Accordingly, in this case the pair production of \( d' \) at the LHC is dominated by the following channel: \( gg \rightarrow d' d' \). Considering all contributions
where as a function of the invariant mass $M_{d'd'}$. In the left panel, $m_{d'} = 300$ GeV, and in the right panel, $m_{d'} = 500$ GeV.

From s, t, and u-channels, the squared amplitude of this process is given by

$$|\mathcal{M}(gq \to d'd')|^2 = \frac{g_\ast^4}{24s^2} \left(\frac{9m_{d'}^4 - 9m_{d'}^2(\hat{s} + 2\hat{t}) + 4\hat{s}^2 + 9\hat{t}^2}{(m_{d'}^2 - \hat{t})^2} \right)$$

$$\times \left[ \frac{m_{d'}^2 (\hat{s}^3 + 2\hat{s}^2\hat{t} + 8\hat{s}\hat{t}^2 + 8\hat{t}^3) + \hat{t}(\hat{s} + \hat{t}) (\hat{s}^2 + 2\hat{s}\hat{t} + 2\hat{t}^2)}{(-m_{d'}^2 + \hat{s} + \hat{t})^2} \right] - \frac{2m_{d'}^3 - 8m_{d'}^3\hat{t} + m_{d'}^3 (3\hat{s}^2 + 4\hat{s}\hat{t} + 12\hat{t}^2)}{(-m_{d'}^2 + \hat{s} + \hat{t})^2} \right].$$

(69)

In addition the squared amplitude of the pair production of $d'$ through the channel $q\bar{q} \to \gamma/g \to d'd'$ is given by

$$|\mathcal{M}(q\bar{q} \to \gamma/g \to d'd')|^2 = \frac{4(2g_\ast^4 + 9g_\ast^4)}{81s^2} \left(2\hat{t} - 4(m_{d'}^2 + m_{d'}^2) \hat{t} + 2(m_{d'}^2 + m_{d'}^2)^2 + 2\hat{t}^2 + \hat{s}^2 \right),$$

(70)

where $\hat{s}, \hat{t}$ are the partonic Mandelstam variables. The differential cross section is given by

$$\frac{d\hat{\sigma}}{d\cos \theta} = \frac{B}{16\pi s^2} |\mathcal{M}|^2,$$

(71)

where $B = \sqrt{1 - (4m_{d'}^2/s)}$. The cross section of $pp \to d'd'$ is given by

$$\frac{d\sigma}{d\cos \theta} = \sum_{i,j} \int_{x_0}^1 dx f_i(x)f_j \left(\frac{4m_{d'}^2}{sx} \right) \frac{d\hat{\sigma}}{d\cos \theta},$$

(72)

where $i, j$ refer to the partons. The partons energy fractions are given by $x_1x_2 = \hat{s}/s$, so that the minimum parton energy fraction to produce the $d'd'$ pair is $x_0 = 2m_{d'}/\sqrt{s}$. Also, $\hat{t} = -\frac{1}{2}\hat{s}(1 - B \cos \theta) + M_{d'}^2$. Therefore, one finds that the production cross section is given by

$$\frac{d\hat{\sigma}}{dt} = \frac{1}{8\pi s^3} |\mathcal{M}|^2.$$

(73)

In Fig. 5 we display the differential cross section of the $d'$ pair production at the LHC with $\sqrt{s} = 14$ TeV as a function of the invariant mass $M_{d'd'}$ for two choices of $m_{d'}$, namely, $m_{d'} = 300$ and 500 GeV. As can be seen from this figure, the typical value of the $d'$ production cross section is of $\mathcal{O}(1)$ fb, which was quite
accessible at the LHC during its second run. The dominant decay channel of the produced $d'$ quark is given by $d' \rightarrow H_2^± u$, as indicated in (66). One can show that the corresponding decay rate is given by

$$\Gamma (d' \rightarrow uH^-_2 + 2\bar{u}) \approx \frac{|V'_{CKM}|^2}{16\pi\hbar} \left( |Y_\ell|^2 \cos^2 \zeta + |Y_R|^2 \sin^2 \zeta \right) m_{d'} \left( 1 - \frac{M_{H_2^±}^2}{m_{d'}^2} \right)^2. \quad \text{(74)}$$

Here, we assumed that $m_u \ll m_{d'}$. On the other hand, the charged Higgs boson $H_2^±$ decays into a lepton and scotino through the interactions

$$L_{Y_{d'}}^{H_2^±} = \Psi H_2^± U_{MNS}' \left( \cos \zeta Y_L P_L + \sin \zeta Y_R P_R \right) e + \text{h.c.} \quad \text{(75)}$$

Thus, the decay rate of $H_2^- \rightarrow e^- n$, for $m_e = 0$, is given by

$$\Gamma (H_2^- \rightarrow e^- n) = \frac{|U_{MNS}'|^2}{16\pi\hbar} \left( |Y_\ell|^2 \cos^2 \zeta + |Y_R|^2 \sin^2 \zeta \right) M_{H_2^±} \left( 1 - \frac{m_n^2}{M_{H_2^±}^2} \right)^2. \quad \text{(76)}$$

In Fig. 6 we show the total cross section of this process with an opposite-sign dilepton, which is the most striking signature for this exotic quark at the LHC. This cross section can be approximately written as

$$\sigma \left( gg \rightarrow g \rightarrow d'\overline{d}' \rightarrow l^±l'^± + E_T^{\text{miss}} + \text{jets} \right) \approx \sigma \left( gg \rightarrow g \rightarrow d\overline{d}' \right) \times BR \left( d' \rightarrow H_2^± + \text{jets} \right)^2 BR \left( H_2^± \rightarrow l^± + E_T^{\text{miss}} \right)^2. \quad \text{(77)}$$

Since the dominant decay channel of $d'$ is $d' \rightarrow uH_2^-$ and the charged Higgs decays mainly to $l^± + n$, one finds $BR \left( d' \rightarrow uH_2^- \right) \simeq 1$ and $BR \left( H_2^± \rightarrow l^± n \right) \simeq 1$. Therefore, $\sigma \left( gg \rightarrow g \rightarrow d'\overline{d}' \rightarrow l^±l'^± + E_T^{\text{miss}} + \text{jets} \right) \approx \sigma \left( gg \rightarrow g \rightarrow d\overline{d}' \right) \simeq O(1) \text{ fb}$, which can be accessible at the LHC with $\sqrt{s} \simeq 14 \text{ TeV}$. In Fig. 7 we show the reconstructed invariant mass of the extra quark $d'$, which decays into $l + n$ (scotino) + jet, with all possible background. In this figure, we have not imposed any cut yet. Therefore, the background is clearly dominated by the signal. Here, we assume $m_{d'} = 300 \text{ GeV}$, the charged Higgs mass is of order 200 GeV, and the LHC integrated luminosity is of order $200 \text{ fb}^{-1}$.

In Fig. 8 (left panel), we plot the number of reconstructed events per bin of the invariant mass of $d'$ of the above process for signal and SM background at $E_T$ cut $> 200 \text{ GeV}$, where $E_T$ is the missing transverse energy. $E_T = \sum_{\text{visible particles}} p_T$, with $m_{d'} = 300 \text{ GeV}$ and $\sqrt{s} = 14 \text{ TeV}$. This figure shows that it is
FIG. 7: The reconstructed invariant mass of extra quark, $d'$, which decays to $l + jet +$ missing energy and its background for $m_{d'} = 300$ GeV. No cut has been imposed yet.

FIG. 8: The reconstructed invariant mass of the extra quark, $d'$, which decays to $l + jet + missing$ energy for $m_{d'} = 300$ GeV, with $E_T > 200$ GeV cut (left panel) and $H_T < 200$ GeV cut (right panel).

possible to extract a good significance for the extra-quark signal in this channel. In addition, we also impose a cut, $H_T < 200$ GeV, where $H_T$ is the total transverse hadronic energy: $H_T = \sum_{\text{hadronic particles}} ||\vec{p}_T||$. It is remarkable that with $H_T$ cuts the signal can be much larger than the background. We have used Feynrules\[36\] to generate the model files and Calchep\[37\] and MadEvent5\[38,39\] to calculate the numerical values of the cross sections and number of events, respectively.

Finally, we provide in Table III some details for the used cuts on $P_T$ and $H_T$ on the signal and background for the process $pp \rightarrow d'd' \rightarrow (l^- l^+) + (u\bar{u}) + (nn)$. As can be seen from the results in this table, the signal of this process can be much larger than the background if one imposes the proper $H_T$ cuts. It is worth mentioning that the Higgs sector of our model is very similar to the two Higgs doublets in the minimal supersymmetric standard model, where one Higgs doublet couples to up quarks and the second couples to down quarks. Therefore, it does not lead to any flavor-changing neutral current problem, and a light charged Higgs is phenomenologically acceptable. The number of events of exotic quark $d'$ at the LHC may slightly changed if a heavier charged Higgs is considered, but with keeping $m_{H^+} < m_{d'}$, to ensure that
\( \text{BR}(d \to H^+ + \text{jets}) \sim 1. \)

| Cuts (GeV) | Signal (S) | Background (B) | S vs B |
|-----------|------------|----------------|--------|
| Initial (no cut) | 463999 | 9309732 ± 21646 | 0.049840 ± 0.000116 |
| Cut 1 (\(E_T > 200\)) | 72291 ± 247 | 33523 ± 198 | 2.1564 ± 0.0148 |
| Cut 2 (\(H_T < 200\)) | 47977 ± 207 | 1942.7 ± 44.3 | 24.696 ± 0.573 |

TABLE II: Signal vs background for the process \(pp \to d^d' \to (l^- l^+) + (u\bar{u}) + (nn)\) with/without cuts.

VI. CONCLUSIONS

In this paper, we have analyzed some phenomenological aspects of the alternative left-right model, motivated by the superstring-inspired \(E_6\) model. We provided a detailed analysis for the symmetry breaking and Higgs sector of this model, which consists of four neutral CP-even Higgs, two CP-odd Higgs and two charged Higgs bosons. We emphasized that three neutral CP-even Higgs and two CP-odd Higgs in addition to two charged Higgs can be light, of \(\mathcal{O}(100)\) GeV. We also found that the contributions of charged Higgs bosons and the extra exotic quark \(d'\) to \(H \to \gamma\gamma\) are quite negligible. Therefore, our model predicts signal strengths of Higgs decay, in particular, of \(H \to \gamma\gamma\) and \(H \to W^+W^-\) that coincide with the SM expectations.

Finally, we studied the striking signatures of the exotic down-type quark at the LHC. In particular, we computed the cross section of \(d'\)-pair production. We showed that the typical value of this cross section is of \(\mathcal{O}(1)\) fb, which is quite accessible at the LHC. The decay of \(d'\) into a jet, lepton, and missing energy provides an important signature for this class of models at the LHC.

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Appendix

To study the boundedness from below, and hence the stability, of the potential \([20]\) we use the following theorem \([24, 25]\) to ensure that the matrix of the quartic terms, which are dominant at higher values of the fields, is copositive:

**Theorem 1 (Copositivity Criteria)** Let \(a \in \mathbb{R}, b \in \mathbb{R}^{n-1}\) and \(C \in \mathbb{R}^{(n-1) \times (n-1)}\). The symmetric matrix \(M \in \mathbb{R}^{n \times n}\)

\[
M = \begin{pmatrix}
    a & b^T \\
    b & C
\end{pmatrix},
\]

is copositive if and only if

1. \(a \geq 0, C\) is copositive,

2. for any nonzero vector \(y \in \mathbb{R}^{n-1}\), with \(y \geq 0\), if \(b^Ty \leq 0\), it follows that \(y^T(aC - bb^T)y \geq 0\).
The quartic terms of the potential \((20)\) can be written as

\[
4F V \left( \phi_{1,2}^{0,+}, \chi_{L,R}^{0,+} \right) = \lambda_1 \left( |\phi_1^0|^4 + |\phi_1^+|^4 + |\phi_2^0|^4 + |\phi_2^+|^4 \right) + \lambda_3 \left( |\chi_L^0|^4 + |\chi_L^+|^4 + |\chi_R^0|^4 + |\chi_R^+|^4 \right) \\
+ 2 |\phi_1|^2 \left[ \lambda_1 \left( |\phi_1^+|^2 + |\phi_2^+|^2 \right) + \lambda_{12} |\phi_2^0|^2 + \alpha_{13} \left( |\chi_L^0|^2 + |\chi_R^0|^2 \right) \right] + \alpha_{12} \left( |\chi_L^+|^2 + |\chi_R^+|^2 \right) \\
+ 2 |\phi_1|^2 \left[ \lambda_1 |\phi_2|^2 + \lambda_{12} |\phi_2|^2 + \alpha_{13} \left( |\chi_L^0|^2 + |\chi_R^0|^2 \right) + \alpha_{12} \left( |\chi_L^+|^2 + |\chi_R^+|^2 \right) \right] \\
+ 2 |\phi_2|^2 \left[ \lambda_1 |\phi_2|^2 + \alpha_{12} \left( |\chi_L^0|^2 + |\chi_R^0|^2 \right) + \alpha_{13} \left( |\chi_L^+|^2 + |\chi_R^+|^2 \right) \right] \\
+ 2 |\phi_2|^2 \left[ \alpha_{12} \left( |\chi_L^0|^2 + |\chi_R^0|^2 \right) + \alpha_{13} \left( |\chi_L^+|^2 + |\chi_R^+|^2 \right) \right] \\
+ 2 |\chi_L|^2 \left( \lambda_{34} |\chi_L^+|^2 + \lambda_4 |\chi_R^0|^2 + \lambda_4 |\chi_R^+|^2 \right) + 2 |\chi_L^+|^2 \left( \lambda_4 |\chi_R^0|^2 + \lambda_4 |\chi_R^+|^2 \right) \\
+ 2 \lambda_3 |\chi_R^0|^2 |\chi_R^+|^2 - 8 \lambda_2 \left[ \phi_1^0 \phi_2^0 + \phi_1^0 \phi_2^0 \phi_1^+ \phi_2^+ \right] \\
+ 4 (\alpha_2 - \alpha_3) \left[ \left( \phi_1^0 + \phi_2^0 \phi_1^+ \right) \chi_L^0 \chi_L^+ + \left( \phi_2^0 + \phi_1^0 \phi_2^+ \right) \chi_R^0 \chi_R^+ \right],
\]

(78)

where \(\alpha_{12} = \alpha_1 + \alpha_2, \alpha_{13} = \alpha_1 + \alpha_3\) and \(\lambda_{12} = \lambda_1 + 2 \lambda_2\). We have

\[
\phi_{1,2}^{0,+} = |\phi_{1,2}^0|^2 \exp \left[ i \theta_{1,2}^0 \right], \quad \chi_{L,R}^{0,+} = |\chi_{L,R}^0|^2 \exp \left[ i \theta_{L,R}^0 \right].
\]

(79)

By the redefinitions of the fields’ components,

\[
\phi_1^+ \rightarrow \phi_1^+ \exp \left[ i \left( \theta_1^0 - \theta_1^+ \right) \right], \phi_2^0 \rightarrow \phi_2^0 \exp \left[ i \left( \theta_2^0 - \theta_2^+ \right) \right], \phi_2^+ \rightarrow \phi_2^+ \exp \left[ -i \left( \theta_2^0 + \theta_2^+ \right) \right],
\]

(80)

we can write

\[
4F V \left( \phi_{1,2}^{0,+}, \chi_{L,R}^{0,+} \right) = X^T \left[ \begin{array}{c}
4F V X - 8 \lambda_2 |\phi_1^0||\phi_1^+||\phi_2^0||\phi_2^+ | \\
4 (\alpha_2 - \alpha_3) \left[ \left( |\phi_1^0||\phi_2^0 | + |\phi_1^0||\phi_2^+ | + |\phi_1^0||\phi_1^+ \right) \chi_L^0 \chi_L^+ + \left( |\phi_2^0||\phi_2^+ | + |\phi_1^0||\phi_1^+ \right) \chi_R^0 \chi_R^+ \right]
\end{array} \right],
\]

(82)

where

\[
X^T = \left( \begin{array}{ccccccc}
|\phi_1^0|^2 & |\phi_1^+|^2 & |\phi_2^0|^2 & |\phi_2^+|^2 & |\chi_L^0|^2 & |\chi_L^+|^2 & |\chi_R^0|^2 & |\chi_R^+|^2 \\
\lambda_1 & \lambda_1 & \lambda_{12} & \lambda_1 & \alpha_{13} & \alpha_{12} & \alpha_{13} & \alpha_{12} \\
\lambda_1 & \lambda_1 & \lambda_1 & \lambda_{12} & \alpha_{13} & \alpha_{12} & \alpha_{13} & \alpha_{12} \\
\lambda_{12} & \lambda_1 & \lambda_1 & \lambda_{12} & \alpha_{13} & \alpha_{12} & \alpha_{13} & \alpha_{12} \\
\lambda_1 & \lambda_{12} & \lambda_1 & \lambda_{12} & \alpha_{13} & \alpha_{12} & \alpha_{13} & \alpha_{12} \\
\alpha_{13} & \alpha_{13} & \alpha_{12} & \alpha_{13} & \lambda_3 & \lambda_4 & \lambda_4 & \lambda_4 \\
\alpha_{12} & \alpha_{13} & \alpha_{12} & \alpha_{13} & \lambda_3 & \lambda_4 & \lambda_4 & \lambda_4 \\
\alpha_{13} & \alpha_{13} & \alpha_{12} & \alpha_{13} & \lambda_4 & \lambda_3 & \lambda_3 & \lambda_3 \\
\alpha_{12} & \alpha_{13} & \alpha_{12} & \alpha_{13} & \lambda_4 & \lambda_3 & \lambda_3 & \lambda_3 \\
\end{array} \right),
\]

\[
4F V = \left( \begin{array}{cccc}
\lambda_1 & \lambda_1 & \lambda_{12} & \lambda_1 & \alpha_{13} & \alpha_{12} & \alpha_{13} & \alpha_{12} \\
\alpha_{13} & \alpha_{13} & \alpha_{12} & \alpha_{13} & \lambda_3 & \lambda_4 & \lambda_4 & \lambda_4 \\
\alpha_{12} & \alpha_{13} & \alpha_{12} & \alpha_{13} & \lambda_3 & \lambda_4 & \lambda_4 & \lambda_4 \\
\alpha_{13} & \alpha_{13} & \alpha_{12} & \alpha_{13} & \lambda_4 & \lambda_3 & \lambda_3 & \lambda_3 \\
\end{array} \right).
\]

(83)

(84)

For the potential \((78)\) to be bounded from below, it must happen that the matrix \(4F V\) is copositive and \(\lambda_2 \leq 0\) and \(\alpha_2 - \alpha_3 \geq 0\). The pseudoscalar Higgs mass \((35)\) implies that \(\mu_3 < 0\). With the minimization condition \((22)\), both imply that \(\lambda_4 > \lambda_3\). The copositivity implies that the diagonal elements \(\lambda_1, \lambda_3 \geq 0\). Accordingly, \(\lambda_4 > \lambda_3 \geq 0\). It is remarkable that the copositivity of the matrix \(4F V\) significantly depends on the signs of the parameters \(\alpha_{12}, \alpha_{13},\) and \(\lambda_{12}\). Here we present the cases depending on these signs:

1. \(\alpha_{12} \geq 0, \alpha_{13} \geq 0,\) and \(\lambda_{12} \geq 0\): In this case, the matrix \(4F V\) is copositive, and the potential is bounded from below.
2. \( \alpha_{12} \geq 0, \ \alpha_{13} \geq 0, \) and \( \lambda_{12} \leq 0: \) The copositivity conditions are
\[
\lambda_1 + \lambda_2 \geq 0, \ \lambda_1^2 + 8\lambda_1 \lambda_2 + 4\lambda_2^2 \leq 0. \tag{85}
\]
We deduce these conditions in detail considering the case assumptions and using Theorem \( \square \). To make the \( 8 \times 8 \) matrix \( 4F V \) be copositive, we shall make that first with the \( 7 \times 7 \) matrix, \( C \), arising from the matrix \( 4F V \) by eliminating the first row and the first column. In our case, it is sufficient to stop at this level, since the \( 6 \times 6 \) matrix, \( C_1 \), arising from the matrix \( 4F V \) by eliminating the first two rows and the first two columns is already copositive; being a matrix of nonnegative elements. Now,
\[
4F V = \begin{pmatrix} \lambda_1 & b^T \\ b & C \end{pmatrix}, \quad b^T = \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_1 & \alpha_{13} & \alpha_{12} & \alpha_{13} & \alpha_{12} \end{pmatrix}, \tag{86}
\]
\[
C = \begin{pmatrix} \lambda_1 & b_1^T \\ b_1 & C_1 \end{pmatrix}, \quad b_1^T = \begin{pmatrix} \lambda_1 & \lambda_2 & \alpha_{13} & \alpha_{12} & \alpha_{13} \end{pmatrix}. \tag{87}
\]
Let \( y_1^T = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{pmatrix} \) be a vector that satisfies Theorem \( \square \) requests, i.e., a nonzero and a nonnegative vector. Taking \( x_2 \neq 0, \ x_{1,3, \ldots, 6} = 0 \), makes the linear form \( b_1^T y_1 = \lambda_{12} x_2 \leq 0 \) and its corresponding quadratic form
\[
y_1^T (\lambda_1 C_1 - b_1 b_1^T) y_1 = -4\lambda_2 (\lambda_1 + \lambda_2) x_2^2.
\]
Since we have \( \lambda_2 \leq 0 \), we impose the condition
\[
\lambda_1 + \lambda_2 \geq 0 \tag{88}
\]
\]
to make the quadratic form \( y_1^T (\lambda_1 C_1 - b_1 b_1^T) y \geq 0 \) and hence as a necessary condition for the copositivity.
Let us assume that \( x_i \neq 0, \ i = 1, \ldots, 6 \). Then, the linear form
\[
b_1^T y_1 \leq 0 \iff x_2 \geq x_2^{\min} = \frac{1}{-\lambda_{12}} (\lambda_1 x_1 + \alpha_{13} x_3 + \alpha_{12} x_4 + \alpha_{12} x_5 + \alpha_{13} x_6).
\]
\]
The copositivity condition \( \square \) makes the corresponding quadratic form be increasing in \( x_2 \) (for any fixed values of the other \( x_i \)'s), and hence we deduce that
\[
y_1^T (\lambda_1 C_1 - b_1 b_1^T) y_1 \geq y_1^T (\lambda_1 C_1 - b_1 b_1^T) y_1 \bigg|_{x_2 = x_2^{\min}}
\]
\[
= \frac{\lambda_1}{\lambda_{12}^2} \left[ 4\lambda_1 \lambda_2^2 x_1^2 - 2\lambda_2 (\alpha_{13} \lambda_1 - \alpha_{12} \lambda_{12}) x_1 x_3 - 2\lambda_2 (\alpha_{12} \lambda_1 - \alpha_{13} \lambda_{12}) x_1 x_4 + 2x_1 (\alpha_{12} \lambda_1 + \alpha_{13} \lambda_2) (\lambda_1^2 + 2\lambda_1 \lambda_2 + 4\lambda_2^2) - 2(\alpha_{13} \lambda_5 + \alpha_{12} \lambda_{6}) \lambda_1 \lambda_{12} \right]
\]
\[
+ x_3 (\alpha_{13}^2 \lambda_1 - 2\alpha_{12} \alpha_{13} \lambda_{12}) (x_3 + 2x_6) + \lambda_{12}^2 (\lambda_3 x_3 + 2\lambda_4 x_5) + 2x_3 (\alpha_{12} \lambda_1 - \alpha_{13} \lambda_{12}) (x_3 + 2x_6) + \lambda_{12}^2 (\lambda_3 x_3 + 2\lambda_4 x_5) + x_4 (\alpha_{12} \lambda_1 - 2\alpha_{12} \alpha_{13} \lambda_{12}) (x_4 + 2x_5) + \lambda_{12}^2 (\lambda_3 x_4 + 2\lambda_4 x_5)
\]
\[
+ 2x_6 (\alpha_{12} \lambda_1 - \alpha_{12} \alpha_{13} \lambda_{12} + \alpha_{13}^2 \lambda_3) x_6^2 + (\alpha_{13}^2 \lambda_1 - 2\alpha_{12} \alpha_{13} \lambda_{12} + \alpha_{13}^2 \lambda_3) x_6^2.
\tag{89}
\]
By the case assumptions and the copositivity condition \( \square \), the quadratic form \( \square \) is non-negative termwise and the theorem is satisfied.
For the copositivity of the matrix $4FV$, let $y^T = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{pmatrix}$ be a nonzero and a non-negative vector. Let $x_{1,2,3} \neq 0$, $x_{4,...,7} = 0$. Then the linear form

$$b^T y = \lambda_1 (x_1 + x_3) + \lambda_{12} x_2 \leq 0 \iff x_2 \geq x_2^{\text{min}} = \frac{\lambda_1}{-\lambda_{12}} (x_1 + x_3).$$

Condition $[SS]$ makes the corresponding quadratic form

$$y^T (\lambda_1 C - bb^T) y = -4\lambda_2 (x_1 (x_2 - x_3)\lambda_1 + x_2 (x_3\lambda_1 + x_2 (\lambda_1 + \lambda_2)))$$

be increasing in $x_2$ (for any fixed values of $x_{1,3}$), and hence we deduce that

$$y^T (\lambda_1 C - bb^T)|_{x_2 = x_2^{\text{min}}} \geq 0 \iff x_2 \geq x_2^{\text{min}} = \frac{1}{-\lambda_{12}} (\lambda_1 x_1 + \lambda_1 x_3 + \alpha_{13} x_4 + \alpha_{12} x_5 + \alpha_{13} x_6 + \alpha_{12} x_7).$$

As before, conditions $[SS]$ make

$$y^T (\lambda_1 C - bb^T) y \geq y^T (\lambda_1 C - bb^T)|_{x_2 = x_2^{\text{min}}} \geq 0, \forall x_{1,3,...,6}.$$

Hence, the theorem is satisfied, and, finally, the only imposed conditions for the matrix $4FV$ to be copositive in this case are those in $[SS]$. The same procedure is followed to extract the copositivity conditions in the following cases:

3. $\alpha_{12} \geq 0$, $\alpha_{13} \leq 0$, and $\lambda_{12} \geq 0$: The following conditions are necessary for the copositivity:

$$\lambda_1 \lambda_3 - \alpha_{13}^2 \geq 0, \quad \alpha_{13}^2 \geq 0.$$

Since $\lambda_4 - \lambda_3 > 0$, then we must have $\alpha_{13} = 0$. Finally, in this case, the copositivity conditions are

$$\alpha_{12} \geq 0, \quad \alpha_{13} = 0, \quad \lambda_{12} \geq 0 \quad (91).$$

4. $\alpha_{12} \leq 0$, $\alpha_{13} \geq 0$, and $\lambda_{12} \geq 0$: The following conditions are necessary for the copositivity:

$$\lambda_1 \lambda_3 - \alpha_{12}^2 \geq 0, \quad \alpha_{12}^2 \geq 0, \quad \lambda_{12}^2 (\lambda_3 - \lambda_2) \geq 0.$$

Again, either $\alpha_{12} = 0$, $\alpha_{12} = \alpha_{13}$, or $\lambda_1 = 0$. But the minimal copositivity conditions in this case are

$$\alpha_{12} = 0, \quad \alpha_{13} \geq 0, \quad \lambda_{12} \geq 0 \quad (92).$$
5. \( \alpha_{12} \geq 0, \alpha_{13} \leq 0, \) and \( \lambda_{12} \leq 0: \) The copositivity conditions are

\[
\alpha_{12} \geq 0, \quad \alpha_{13} = 0, \quad \lambda_{12} \leq 0, \quad \lambda_1^2 + 8\lambda_1\lambda_2 + 4\lambda_2^2 \leq 0.
\]  

(93)

6. \( \alpha_{12} \leq 0, \alpha_{13} \geq 0, \) and \( \lambda_{12} \leq 0: \) The copositivity conditions are

\[
\alpha_{12} = 0, \quad \alpha_{13} \geq 0, \quad \lambda_{12} \leq 0, \quad \lambda_1^2 + 8\lambda_1\lambda_2 + 4\lambda_2^2 \leq 0.
\]  

(94)

7. \( \alpha_{12} \leq 0, \alpha_{13} \leq 0, \) and \( \lambda_{12} \geq 0: \) The following conditions are necessary for the copositivity:

\[
\lambda_1\lambda_3 - \alpha_{12}^2 \geq 0, \quad \lambda_1\lambda_3 - \alpha_{13}^2 \geq 0, \quad \alpha_{12}(\lambda_3 - \lambda_4) \geq 0, \quad \alpha_{13}(\lambda_3 - \lambda_4) \geq 0.
\]  

(95)

Hence, in this case, the copositivity conditions are

\[
\alpha_{12} = \alpha_{13} = 0, \quad \lambda_{12} \geq 0.
\]  

(96)

8. \( \alpha_{12} \leq 0, \alpha_{13} \leq 0, \) and \( \lambda_{12} \leq 0: \) The copositivity conditions are

\[
\alpha_{12} = \alpha_{13} = 0, \quad \lambda_{12} \leq 0, \quad \lambda_1^2 + 8\lambda_1\lambda_2 + 4\lambda_2^2 \leq 0.
\]  

(97)

[1] R. Mohapatra and J. C. Pati, A Natural Left-Right Symmetry, Phys.Rev. D11 (1975) 2558.
[2] G. Senjanovic and R. N. Mohapatra, Exact Left-Right Symmetry and Spontaneous Violation of Parity, Phys.Rev. D12 (1975) 1502.
[3] R. N. Mohapatra, F. E. Paige, and D. Sidhu, Symmetry Breaking and Naturalness of Parity Conservation in Weak Neutral Currents in Left-Right Symmetric Gauge Theories, Phys.Rev. D17 (1978) 2462.
[4] N. Deshpande, J. Gunion, B. Kayser, and F. I. Olness, Left-right symmetric electroweak models with triplet Higgs, Phys.Rev. D44 (1991) 837–858.
[5] C. S. Aulakh, A. Melfo, and G. Senjanovic, Minimal supersymmetric left-right model, Phys.Rev. D57 (1998) 4174–4178, [hep-ph/9707256].
[6] A. Maiezza, M. Nemevsek, F. Nesti, and G. Senjanovic, Left-Right Symmetry at LHC, Phys.Rev. D82 (2010) 055022, [arXiv:1005.5160].
[7] D. Borah, S. Patra, and U. Sarkar, TeV scale Left Right Symmetry with spontaneous D-parity breaking, Phys.Rev. D83 (2011) 035007, [arXiv:1006.2245].
[8] M. Nemevsek, G. Senjanovic, and V. Tello, Left-Right Symmetry: from Majorana to Dirac, Phys.Rev.Lett. 110 (2013) 151802, [arXiv:1211.2837].
[9] E. Ma, Particle Dichotomy and Left-Right Decomposition of E(6) Superstring Models, Phys.Rev. D36 (1987) 274.
[10] K. Babu, X.-G. He, and E. Ma, New Supersymmetric Left-Right Gauge Model: Higgs Boson Structure and Neutral Current Analysis, Phys.Rev. D36 (1987) 878.
[11] E. Ma, Dark Left-Right Model: CDMS, LHC, etc., J.Phys.Conf.Ser. 315 (2011) 012006, [arXiv:1006.3804].
[12] S. Khalil, H.-S. Lee, and E. Ma, Bound on Z’ Mass from CDMS II in the Dark Left-Right Gauge Model II, Phys.Rev. D81 (2010) 051702, [arXiv:1002.0692].
[13] ATLAS Collaboration Collaboration, G. Aad et al., Measurement of Higgs boson production in the diphoton decay channel in pp collisions at center-of-mass energies of 7 and 8 TeV with the ATLAS detector, arXiv:1408.7084.
[14] CMS Collaboration Collaboration, V. Khachatryan et al., Observation of the diphoton decay of the Higgs boson and measurement of its properties, arXiv:1407.0558.
[15] Talk by Christophe Ochando, on behalf of the CMS collaboration at Rencontres de Moriond, QCD Session March 9-16, 2013: http://moriond.in2p3.fr/QCD/2013/ThursdayMorning/Ochando.pdf.

[16] CMS PAS HIG-13-005, Combination of standard model Higgs boson searches and measurements of the properties of the new boson with a mass near 125 GeV: http://cds.cern.ch/record/1542387/files/HIG-13-005-pas.pdf.

[17] Talk by Eleni Mountricha, on behalf of the ATLAS collaboration at Rencontres de Moriond, QCD Session March 9-16, 2013: http://moriond.in2p3.fr/QCD/2013/ThursdayMorning/Mountricha2.pdf.

[18] ATLAS-CONF-2013-034, Combined coupling measurements of the Higgs-like boson with the ATLAS detector using up to 25 fb$^{-1}$ of proton-proton collision data, March 13, 2013: http://cds.cern.ch/record/1528170/files/ATLAS-CONF-2013-034.pdf.

[19] E. Ma, Dark-matter fermion from left-right symmetry, Phys. Rev. D 85 (May, 2012) 091701.

[20] CMS Collaboration Collaboration, S. Chatrchyan et al., Search for leptonic decays of $W'$ bosons in $pp$ collisions at $\sqrt{s} = 7$ TeV, JHEP 1208 (2012) 023, arXiv:1204.4764.

[21] ATLAS Collaboration Collaboration, G. Aad et al., Search for a heavy gauge boson decaying to a charged lepton and a neutrino in 1 fb$^{-1}$ of $pp$ collisions at $\sqrt{s} = 7$ TeV using the ATLAS detector, Phys.Lett. B705 (2011) 28–46, arXiv:1108.1316.

[22] ATLAS Collaboration Collaboration, G. Aad et al., Search for dilepton resonances in $pp$ collisions at $\sqrt{s} = 7$ TeV with the ATLAS detector, Phys.Rev.Lett. 107 (2011) 272002, arXiv:1108.1582.

[23] CMS Collaboration Collaboration, S. Chatrchyan et al., Search for Resonances in the Dilepton Mass Distribution in $pp$ Collisions at $\sqrt{s} = 7$ TeV, JHEP 1105 (2011) 093, arXiv:1103.0981.

[24] L. Ping and F. Y. Yu, Criteria for copositive matrices of order four, Linear Algebra and its Applications 194 (1993), no. 0109 – 124.

[25] K. Kannike, Vacuum Stability Conditions From Copositivity Criteria, Eur.Phys.J. C72 (2012) 2093, arXiv:1205.3781.

[26] M. Carena, I. Low, and C. E. Wagner, Implications of a Modified Higgs to Diphoton Decay Width, JHEP 1208 (2012) 060, arXiv:1206.1082.

[27] W. Chao, J.-H. Zhang, and Y. Zhang, Vacuum Stability and Higgs Diphoton Decay Rate in the Zee-Babu Model, JHEP 1306 (2013) 039, arXiv:1212.6277.

[28] I. Picek and B. Radovcic, Enhancement of $h \to \gamma \gamma$ by seesaw-motivated exotic scalars, Phys.Lett. B719 (2013) 404–408, arXiv:1210.6449.

[29] W.-F. Chang, J. N. Ng, and J. M. Wu, Constraints on New Scalars from the LHC 125 GeV Higgs Signal, Phys.Rev. D86 (2012) 033003, arXiv:1206.5047.

[30] P. Bhupal Dev, D. K. Ghosh, N. Okada, and I. Saha, 125 GeV gamma ray line and enhanced Higgs di-photon rate from Triplet-Singlet extended MSSM, JHEP 1308 (2013) 020, arXiv:1304.6855.

[31] T. Basak and S. Mohanty, 120 GeV gamma ray line and enhanced Higgs di-photon rate from Triplet-Singlet extended MSSM, JHEP 1308 (2013) 020, arXiv:1304.6855.

[32] J. Cao, P. Wan, J. M. Yang, and J. Zhu, The SM extension with color-octet scalars: diphoton enhancement and global fit of LHC Higgs data, JHEP 1308 (2013) 009, arXiv:1303.2426.

[33] W.-Z. Feng and P. Nath, Higgs diphoton rate and mass enhancement with vector-like leptons and the scale of supersymmetry, Phys.Rev. D87 (2013) 055018, arXiv:1303.0289.

[34] M. Berg, I. Buchberger, D. Ghiленца, and C. Petersson, Higgs diphoton rate enhancement from supersymmetric physics beyond the MSSM, Phys.Rev. D88 (2013) 025017, arXiv:1212.5009.

[35] S. Khalil and S. Salem, Enhancement of in model with -plet, Nuclear Physics B 876 (2013), no. 2 473 – 492.

[36] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr, and B. Fuks, FeynRules 2.0 - A complete toolbox for tree-level phenomenology, arXiv:1310.1921.

[37] A. Belyaev, N. D. Christensen, and A. Pukhov, CalcHEP 3.4 for collider physics within and beyond the Standard Model, Comput.Phys.Commun. 184 (2013) 1729–1769, arXiv:1207.6082.

[38] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer, and T. Stelzer, MadGraph 5 : Going Beyond, JHEP 1106 (2011) 128, arXiv:1106.0522.

[39] E. Conte, B. Fuks, and G. Serret, MadAnalysis 5, A User-Friendly Framework for Collider Phenomenology, Comput.Phys.Commun. 184 (2013) 222–256, arXiv:1206.1599.