Nature of the vortex-glass order in strongly type-II superconductors

Hikaru Kawamura

Department of Earth and Space Science, Faculty of Science, Osaka University, Toyonaka, 560-0043, Japan

The stability and the critical properties of the three-dimensional vortex-glass order in random type-II superconductors with point disorder is investigated in the unscreened limit based on a lattice XY model with a uniform field. By performing equilibrium Monte Carlo simulations for the system with periodic boundary conditions, the existence of a stable vortex-glass order is established in the unscreened limit. Estimated critical exponents are compared with those of the gauge-glass model.

67.70.+n, 67.57.Lm

In spite of extensive studies for a decade, the question of nature of the thermodynamic phase diagram of random high-$T_c$ superconductors has remained unsettled. In zero field, the possibility of an exotic thermodynamic phase with broken time-reversal symmetry, called the chiral-glass phase, has been discussed [1]. Even more attention has been paid to the in-field properties. For superconductors with point disorder, possible existence of a thermodynamic phase called the vortex-glass (VG) phase, where the vortex is pinned on long length scale by randomly distributed point-pinning centers, was proposed [2]. In such a VG state, the phase of the condensate wavefunction is frozen in time but randomly in space, with a vanishing linear resistivity $\rho_L$. It is a truly superconducting state separated from the vortex-liquid phase with a nonzero $\rho_L$ via a continuous VG transition.

Since cuprate high-$T_c$ superconductors are extremely type-II superconductors where the London penetration depth $\lambda$ is much longer than the coherence length, it is important to clarify first whether the proposed VG state really exists in the type-II, unscreened limit $\lambda \to \infty$. Indeed, stability of the hypothetical VG state has been studied quite extensively by numerical model simulations [3–9]. Many have been based on a highly simplified model called the the gauge-glass model. Previous simulations on the three-dimensional (3D) gauge-glass model gave mutually consistent results that a continuous VG transition occurred at a finite temperature characterized by the critical exponents, $\nu \simeq 1.3, \eta \simeq -0.5, z \simeq 4 - 5$ [3–6].

The gauge-glass model has some drawbacks [3]. It is a spatially isotropic model without a net field threading the system, in contrast to the reality. Furthermore, source of quenched randomness is artificial. The gauge-glass model is a random flux model where the quenched randomness occurs in the phase factor associated with the flux. In reality, the quenched component of the flux is uniform, nothing but the external field, and the quenched randomness occurs in the superconducting coupling or the pinning energy. It remains unclear whether these simplifications underlying the gauge-glass model really unaffec
t the basic physics of the VG ordering in 3D.

Recently, several simulations were performed beyond the gauge-glass model [7–9]. The present author studied the type of the lattice XY model where the above limitations of the gauge-glass model were cured [7]. While the VG state was found to be stable, the estimated critical exponents, particularly $\nu \simeq 2.2$, differed significantly from those of the gauge-glass model, posing a possibility that the gauge-glass model lied in a different universality class. However, due to the effect of employed free boundary conditions, the estimated critical exponents might possibly be subject to large surface effect. Vestergren et al. studied a random pinning model which took care of the above limitations of the gauge-glass model in a different way, to obtain a finite-temperature VG transition characterized by the exponents, $\nu \simeq 0.7, z \simeq 1.5$ [8], which differed significantly from either those of Ref. [7] or from those of the gauge-glass model [4–6]. Olsson and Teitel claimed on the basis of their simulations on the lattice XY model with weak disorder that the VG order was not stable even in the unscreened limit [9]. Thus, once one tries to go beyond the gauge-glass model, the present theoretical situation seems quite confused.

In the present paper, I study the lattice XY model of Ref. [7], but now with applying periodic boundary conditions, to overcome the finite-size effect originated from surface. The Hamiltonian considered is

$$H = - \sum_{ij} J_{ij} \cos(\theta_i - \theta_j - A_{ij}),$$

where $\theta_i$ is the phase of the condensate at the $i$-th site of a simple cubic lattice with $N = L^3$ sites, and the sum is taken over all nearest-neighbor pairs. $A_{ij}$ is a link variable associated with the vector potential due to uniform external magnetic field of intensity $h$ applied in the $z$-direction. In the Landau gauge, it is given by $A_i = (A^x_i, A^y_i, A^z_i) = (0, h i_x, 0)$, where $1 \leq i_x \leq L$ denotes the $x$-coordinate of the site $i$. Quenched randomness occurs in the superconducting coupling $J_{ij}$ which is assumed to be an independent random variable uniformly distributed between $[0, 2J]$, $J > 0$ being a typical coupling strength. I impose periodic boundary conditions in all directions in order to eliminate surface ‘spins’ which might contaminate the bulk critical behavior. The field intensity is chosen to be $h = 2\pi/4 (f = 1/4)$. The lattice sizes are taken to be multiples of four, i.e., $L = 8, 12, 16$ and 20.

Simulation is performed based on the exchange MC method, where the systems at neighboring temperatures
are occasionally exchanged [10]. Equilibration is checked by monitoring the stability of the results against at least three-times longer runs for a subset of samples. Sample average is taken over 980 (L = 8 and 12), 248 (L = 16) and 200 (L = 20) independent bond realizations.

We run two independent sequences of systems (replica 1 and 2) in parallel, and compute a complex overlap q between the local superconducting order parameters of the two replicas \( \psi^{(1,2)}_{i} \equiv \exp(iq_{i}^{(1,2)}) \),

\[
q = \frac{1}{N} \sum_{i} \psi^{(1)*}_{i} \psi^{(2)}_{i},
\]

where the summation is taken over all \( N = L^3 \) sites. In terms of the overlap q, the VG order parameter and the Binder ratio is calculated by

\[
q^{(2)} = \langle |q|^2 \rangle, \quad g = 2 - \frac{\langle |q|^4 \rangle}{\langle |q|^2 \rangle^2},
\]

where \( \langle \cdots \rangle \) represents the thermal average and \( [\cdots] \) represents the average over bond disorder.

By inspecting the spatial pattern of the vortex snapshot and by calculating the Fourier amplitude of the spatial distribution of vortices, it is checked that no periodic vortex-lattice order is formed in the system.

The size and temperature dependence of the calculated Binder ratio is shown in Fig.1(a). As can be seen from Fig.2(a), the transverse mean-square current for \( L \geq 12 \) exhibits a crossing at \( T/J = 0.82 \pm 0.02 \), indicating that the VG transition occurs at a finite temperature. The data show a rather clear splay-out, in contrast to the near marginal merging behavior observed for the case of free boundary conditions [7]. In contrast to the case of the chiral-glass order in the XY spin glass [11], g does not exhibit a negative dip characteristic of one-step-like replica symmetry breaking.

Further evidence of a finite-temperature transition is obtained from the mean-square currents [4]. The current in the \( \mu \) direction \( I_{\mu} (\mu = x, y, z) \) is defined by

\[
I_{\mu} = \sum_{i} J_{ij} \sin(\theta_{i} - \theta_{j} - A_{ij}),
\]

where the sum is taken over all nearest-neighbor bonds along the \( \mu \) direction. We calculate both the longitudinal (along the applied field) and the transverse (perpendicular to the applied field) mean-square currents given by

\[
I^{z}_{L} = \langle I_{z} \rangle, \quad I^{x}_{L} = \langle I_{x} \rangle, \quad I^{y}_{L} = \langle I_{y} \rangle.
\]

As can be seen from Fig.2(a), the transverse mean-square current for \( L \geq 12 \) exhibits a crossing at \( T/J = 0.81 \pm 0.02 \), close to the crossing temperature of the Binder ratio. The longitudinal mean-square current also exhibits a crossing behavior, as is shown in Fig.2(b). Apparently, finite-size effects are severer in \( I^{x}_{L} \) than in \( I^{z}_{L} \), as the crossing for smaller sizes occurs at relative high temperature, which shifts down rapidly to lower temperature with increasing \( L \), i.e., \( T_{cross}/J > 1.1 \) (\( L = 8 \) and 12), \( \simeq 1.00 \) (\( L = 12 \) and 16), and \( \simeq 0.89 \) (\( L = 16 \) and 20). Aside from this strong finite-size correction, the observed behavior seems consistent with the occurrence of a single bulk transition at \( T/J \simeq 0.81 \) as suggested from the transverse mean-square current.

In order to further confirm that both the transverse and longitudinal spatial components order simultaneously, I also compute the transverse VG order parameter and the transverse Binder ratio [7]. These quantities are defined in terms of the layer-overlap \( q_{k}^{(i)} \) for the \( k \)-th \( xy \)-layer of the lattice,

\[
q_{k}^{(i)} = \frac{1}{L^{2}} \sum_{i \in k} \psi^{(1)*}_{i} \psi^{(2)}_{i},
\]

where \( i \in k \) means the sum over all sites belonging to the \( k \)-th layer, by
\[ q_T^{(2)} = \frac{1}{L} \sum_k |<q_k|^2|, \quad g_T = 2 - \frac{\sum_k (|q_k^2|)}{\sum_k (|q_k|^2)^2}. \quad (7) \]

The calculated \( g_T \) is shown in Fig.1(b). As can be seen from the figure, \( g_T \) shows a behavior quite similar to \( g \), exhibiting a clear crossing at \( T/J = 0.81 \pm 0.02 \). This also indicates that the model exhibits a single bulk VG transition where both the transverse and longitudinal components order simultaneously.

Next, we turn to the critical properties of the model. Standard finite-size scaling analysis performed for the bulk Binder ratio, with setting \( T_g/J = 0.82 \), yields \( \nu = 1.2 \pm 0.3 \), as shown in the inset of Fig.3 where \( t \equiv (T - T_g)/T_g \). Then, from the scaling of the order parameter \( q_T^{(2)} \), the critical-point-decay exponent is estimated to be \( \eta = -0.5 \pm 0.1 \); see Fig.3. The estimate of \( \nu \) is also corroborated by a finite-size scaling analysis of the transverse mean-square current \( I_T^2 \), which, with \( T_g/J = 0.81 \), yields \( \nu = 1.0 \pm 0.2 \). The corresponding finite-size scaling plot is given in Fig.4. (Unfortunately, meaningful scaling analysis is not feasible for \( I_L^2 \) due to the rapid shift of the crossing points with \( L \).) In order to examine the possibility of anisotropic scaling, finite-size scaling of \( g_T \) and \( q_T^{(2)} \) (see the inset of Fig.4) is also performed. I get \( \nu = 1.2 \pm 0.3 \) and \( \eta = -0.5 \pm 0.1 \), which agree within the errors with \( \nu \) and \( \eta \) determined from \( g \) and \( q^{(2)} \). No sign of anisotropic scaling is thus found. Combining all these estimates, I finally quote \( \nu = 1.1 \pm 0.2 \), \( \eta = -0.5 \pm 0.1 \).

FIG. 2. Temperature and size dependence of (a) the transverse and (b) the longitudinal mean-square currents.

FIG. 3. Finite-size scaling plot of the VG order parameter (main panel) and of the bulk Binder ratio (inset), with \( T_g/J = 0.82 \), \( \nu = 1.2 \) and \( 1 + \eta = 0.5 \).

FIG. 4. Finite-size scaling plot of the transverse mean-square current (main panel) and of the transverse VG order parameter (inset), with \( T_g/J = 0.81 \). Best values of exponents are taken to be \( \nu = 1.0 \) for \( I_T^2 \), while \( \nu = 1.2 \) and \( 1 + \eta = 0.5 \) for \( q_T^{(2)} \).

On comparing the exponent values obtained here with those obtained for the system with free boundary conditions \([7]\), one sees that they differ considerably. In the range of lattice sizes studied here, the application of either periodic or free boundary significantly influences the
estimates of critical exponents. One naturally expects that periodic boundary conditions give better estimates for the bulk exponents, since no surface 'spin' exists there which might contaminate the bulk critical properties.

If one compares the present estimates of exponents with those of the the gauge-glass model (with periodic boundary), the values of both $\nu$ and $\eta$ are compatible with each other, i.e., $\nu = 1.1 \pm 0.2$ and $\eta = -0.5 \pm 0.1$ for the present model vs. $\nu = 1.39 \pm 0.20$ and $\eta = -0.47 \pm 0.07$ for the gauge-glass model [6]. This coincidence might suggest that the present model belongs to the same universality class as the gauge-glass model. It thus appears that the differences in the form of quenched randomness and in the spatial anisotropy due to external fields are irrelevant to the critical properties of the VG transition.

Finally, I wish to refer to the possible effect of screening in real VG ordering. In real type-II superconductors including high-$T_c$ superconductors, the penetration depth is large, but of course is not infinite. As several calculations on the gauge-glass model and other random XY model have constantly suggested, finite screening effect might destabilize the stable VG state [7,12,13]. Since high-$T_c$ superconductors are strongly type-II superconductors, this screening-induced rounding of a sharp VG transition is a weak effect, visible only at very close to $T_g$ of the unscreened system. With use of the present estimate of $\nu$, a rough estimate of such a rounding (or crossover) temperature $T_x$ may be obtained. Screening effect would be visible when the coherence length $\xi \approx \xi_0 t^{-\nu}$ ($\xi_0$ is the zero-temperature coherence length and $t \equiv (T - T_g)/T_g$) grows comparable to the zero-temperature penetration depth $\lambda_0$. Since the ratio $\lambda_0/\xi_0$ is of order $10^2$ in high-$T_c$ superconductors, the crossover temperature is of order $t_x \sim 10^{-2}$. It means that one has to approach $T_g$ as close as $t \sim 10^{-2}$ in order to see the screening-induced rounding. In other words, in the temperature range outside $t_x$, the critical behavior of the unscreened system is expected to be observed experimentally [14].

In summary, the VG ordering of strongly type-II superconductors with point disorder is investigated based on a lattice XY model with a uniform field. The occurrence of a finite-temperature VG transition is established in the unscreened limit. The estimated critical exponents $\nu$ and $\eta$ are close to the corresponding gauge-glass values, suggesting that the present model belong to the same universality class as the gauge-glass model.

The author is thankful to Dr.S. Teitel for useful discussion. He is also indebted to Dr.J. Lidmar for his comment on the estimate of critical exponents. The numerical calculation was performed on the Hitachi SR8000 at the supercomputer center, ISSP, University of Tokyo.

Note added
In the course of preparing this manuscript, the author learned that Olsson also made a MC simulation of the 3D VG order in the unscreened limit based on a lattice XY model, with a different choice of the random-coupling distribution [cond-mat/0301624]. Finite-temperature VG transition with an exponent $\nu = 1.50 \pm 0.12$ was observed, roughly being consistent with our present result. Critical exponents similar to our present values were also reported very recently by Lidmar based on a different type of VG model [cond-mat/0302577].

[1] H. Kawamura and M. S. Li, Phys. Rev. Letters 78, 1556 (1997); J. Phys. Soc. Jpn. 66, 2110 (1997).
[2] M.P.A. Fisher, Phys. Rev. Letters 62, 1415 (1989); D.S. Fisher, M.P.A. Fisher and D.A. Huse, Phys. Rev. B43, 130 (1991).
[3] D.A. Huse and H.S. Seung, Phys. Rev. B42, R1059 (1990).
[4] J.D. Reger, T.A. Tokuyasu, A.P. Young and M.P.A. Fisher, Phys. Rev. B44, 7147 (1991).
[5] C. Wengel and A.P. Young, Phys. Rev. B56, 5918 (1997).
[6] T. Olson and A.P. Young, Phys. Rev. B61, 12467 (2000).
[7] H. Kawamura, J. Phys. Soc. Jpn. 69, 29 (2000).
[8] A. Vestergren, J. Lidmar and M. Wallin, Phys. Rev. Letters 88, 117004 (2002).
[9] P. Olsson and S. Teitel, Phys. Rev. Letters 87, 137001 (2001).
[10] K. Hukushima and K. Nemoto, J. Phys. Soc. Jpn. 65, 1604 (1996).
[11] H. Kawamura and M. S. Li, Phys. Rev. Letters 87, 187204 (2001).
[12] H.S. Bokil and A.P. Young, Phys. Rev. Letters 74, 3021 (1995); C. Wengel and A.P. Young, Phys. Rev. B54, R8869 (1996).
[13] J. Kisker and H. Rieger, Phys. Rev. B58, R8873 (1998); F. Pfeiffer and H. Rieger, J. Phys. Condens. Matter 14, 2361 (2002).
[14] A.M. Petrean, L.M. Paulius, W.K. Kwok, J.A. Fendrich and G.W. Crabtree, Phys. Rev. Letters 87, 137001 (2001).