Ghost Aperture Synthesis Imaging with Computational Aberration Cancellation

Shuai Sun\textsuperscript{1,2,3}, Zhen-Wu Nie\textsuperscript{1,2}, Yue-Gang Li\textsuperscript{2,4},
Hui-Zu Lin\textsuperscript{1,2,3}, Wei-Tao Liu\textsuperscript{1,2,3,*}, Ping-Xing Chen\textsuperscript{1,2,3}

\textsuperscript{1}Institute for Quantum Science and Technology, 
College of Science, National University of Defense Technology, 
Changsha, Hunan, 410073, People’s Republic of China
\textsuperscript{2}Interdisciplinary Center of Quantum Information, 
College of Science, National University of Defense Technology, 
Changsha, Hunan, 410073, People’s Republic of China
\textsuperscript{3}Hunan Key Laboratory of Mechanism and Technology of Quantum Information, 
Changsha, Hunan, 410073, People’s Republic of China
\textsuperscript{4}State Key Laboratory of Advanced Optical Communication Systems and Networks, 
Institute for Quantum Sensing and Information Processing, 
School of Sensing Science and Engineering, 
Shanghai Jiao Tong University, Shanghai, 200240, People’s Republic of China and
*wtliu@nudt.edu.cn

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Abstract

Although optical aperture synthesis has been generally regarded as the only access to very large imager for over a century, the problem of phasing all the giant sub-apertures on the scale of wavelength is still prohibitive. Besides, the accompanied adaptive optics combatting the atmospheric turbulence is also bulky and complicated. We here propose a new paradigm aperture synthesis imager through turbulence, based on computational ghost imaging method. The complex aberrations on the signal path are computationally cancelled by introducing an optimum compensation phase on the reference path. With the advanced aberration cancellation, our imager is free from phasing and aberrations problem. The image degradation due to turbulence is also suppressed and even eliminated without any guide star or wavefront shaping device. Experimentally, diffraction-limited imaging is achieved under turbulence featuring transverse coherence length far less than the optical aperture of the system.

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Around 1890, H. Fizeau and A. A. Michelson successively came up with stellar interferometry\cite{1–3}, hence reveal that widely separated but coherently combined sub-apertures can realize the resolving ability far surpassing individual one. Since manufacturing a whole aperture with diameter of meters is much beyond the reach of current technologies, the technique of optical aperture synthesis (OAS) is recognized as the only approach to huge interferometers and imagers. The concept of OAS subsequently encouraged the birth of intensity interferometer\cite{4–6}, dominated the design of the hypertelescopes\cite{7}, and inspired the blossoming field of radio astronomy\cite{8}.

To achieve the diffraction limit of OAS systems, all the giant, separated sub-apertures need to be phased to an accuracy of optical wavelength, which requires various of specific alignment techniques including extremely accurate clocks and time markers\cite{1–3, 9}. In addition, bulky adaptive optics is also indispensable to actively eliminate the aberrations of OAS system caused by the gravity and temperature variation\cite{2, 10}, also to correct the wavefront distortion induced by atmospheric turbulence\cite{10, 11}. Either phasing problem or adaptive optics is a formidable task currently. Therefore, the OAS technique is usually of astronomical cost. Essentially, both of the phasing operation and adaptive optics are to preserve the first-order coherence of the photons from the sub-apertures. Utilizing the second-order coherence\cite{12}, intensity interferometer can work without phasing operation. However, its sampling result only represents the squared magnitude of the Fourier transform of the image intensity. The loss of phase information severely obstructs the restoration of images, especially for complicated or faint objects\cite{13, 14}.

Ghost Imaging (GI) is an unconventional method to acquire images from the second-order coherence of light field\cite{15, 16, 18, 21, 52}. As a direct descendant, computational GI (CGI)\cite{22} also offers the advantage to perform standoff sensing lenslessly\cite{23}, makes neither the first-order coherence of light field nor huge telescope necessary for diffraction-limited imaging\cite{24}. The wavefront distortion caused by atmospheric turbulence will apparently degenerate the performance of GI\cite{23, 25–28}. This is accompanied with a good news that nonlocal wavefront tailoring is demonstrated recently. Exploiting the correlations displayed by the two-photon state, the dispersion\cite{29, 33}, distortion\cite{34, 35} and aberration\cite{36, 38} imposed on the local wave packet of one photon can be corrected by modulating the other counterpart photon. This therefore allows to perform (the analog of) two-photon aberration cancellation scheme based on GI system, and thereby construct a profoundly different OAS
Here we propose a new physical paradigm of OAS imager based on CGI and nonlocal aberration cancellation. The unknown, intricate aberrations of the dephased sub-apertures and the wavefront deformation induced by turbulence on the signal path are both computationally suppressed and even cancelled, by introducing an optimum compensation phase on the reference path. This ghost OAS (GOAS) offers significant advantages, which are not only free from the phasing problem, but also capable to image through turbulence, without any guide star or wavefront sensing/shaping devices. Compared with conventional OAS, the phasing-free, sensor-less GOAS imager is far easier to construct and to conduct. The way of active illumination is also promising in the applications of standoff sensing.

The GOAS imager is shown as Fig.1. A pulsed laser is divided into dozens of distributed, far separated sub-sources with parallel emitting direction. Each sub-source gets a relative phase due to the different beam length, which is random and unknown without any phasing operation. The phase of each sub-source is also modulated individually, thus temporally varying speckle patterns can be produced in the far field, which propagate through the atmospheric turbulence to a rough-surfaced target. The light reflected from the target is collected by a bucket detector. The sampling process of GOAS means projecting a sequence of random speckle patterns and recording the resultant bucket signals. When phase modulators and bucket detector with high bandwidth are used, the sampling can be finished within the coherence time of turbulence, which is typically milliseconds. Then, the relative phase can be treated as fixed within the sampling, under uncomplicated stabilization procedures for the whole source. In the presence of random relative phase and the turbulence, diffraction-limited images of the target can be recovered from the cross correlation between the bucket signals and the algorithmically optimized reference speckle patterns.

Taking the length difference among the divided beams less than the coherence length of the pulse, the $m^{th}$ $\{m = 1, 2, ...M\}$ in all $M$ illumination pattern emitted from the synthetic source can be written as

$$E_0(\vec{\rho}_0, m) = \sum_{n=1}^{N} \xi(\vec{\rho}_0 - \vec{\rho}_n)e^{i[\phi(n,m) + \phi_d(n)]}.$$  \hspace{1cm} (1)

Here $N$ is the number of sub-sources. $\vec{\rho}_0$ is the coordinates of the sources plane and $\vec{\rho}_n$ is the coordinates of the center of $n^{th}$ $\{n = 1, 2..., N\}$ sub-source. $\phi(n, m)$ and $\phi_d(n)$ represents
FIG. 1: Diagrammatic sketch of GOAS. A laser is emitted from dozens of distributed, phase-modulated but dephased sub-sources. Temporally varying speckles undergoes the turbulence and illuminates the target. The reflected light is implemented bucket detection. Due to the relative phase among the sub-sources and the turbulence, the illumination patterns imprinted on the target are unknown. However, images with diffraction limit can be recovered after computational aberration cancellation.

the modulation phase and relative phase, respectively. The modulation phase temporally varies with the index $m$ while the relative phase keeps temporal constant during the sampling process. Besides, the modulation phase is known while the random relative phase is unknown. The Gaussian beam collimated by each sub-source is

$$\xi(\vec{\rho}_0 - \vec{\rho}_n) = \sqrt{\frac{2P}{\pi\omega_0^2}} e^{-\frac{|\vec{\rho}_0 - \vec{\rho}_n|^2}{2\omega_0^2}},$$  \hspace{1cm} (2)$$

where $P$ represents the laser intensity from each sub-source with units Watts. $\omega_0$ is the beam waist of the sub-source, which is several millimeters and far less than the coherence length of the turbulence. Since GOAS is operated within a single atmospheric coherence time, the behavior of the light field $L$-m propagating through atmospheric turbulence can be characterized via extended Huygens-Fresnel principle$^{[23, 28, 40]}$, and the $m^{th}$ illumination
pattern imprinted on the target and reflected back to the detector can be expressed by,

\[
E_t(\vec{ρ}_t, m) = \frac{k_0}{i2\pi L} \int_{a_0} d\vec{ρ}_0 E_0(\vec{ρ}_0, m) \times e^{i k_0 (L + |\vec{ρ}_t - \vec{ρ}_0|^2/2L)} e^{\Psi_t(\vec{ρ}_t, \vec{ρ}_0)}
\]

(3)

and

\[
E_d(\vec{ρ}_d, m) = \frac{k_0}{i2\pi L} \int d\vec{ρ}_t E_t(\vec{ρ}_t, m) T(\vec{ρ}_t) \times e^{i k_0 (L + |\vec{ρ}_d - \vec{ρ}_t|^2/2L)} e^{\Psi_s(\vec{ρ}_d, \vec{ρ}_t)}
\]

(4)

respectively, where \(a_0\) is the baseline length of the synthetic source and \(a_0 \gg \omega_0\). \(k_0\) is the wave number. The target is a quasi-Lambertian reflector \[23, 28\] and its field-reflection coefficient is \(T(\vec{ρ}_t)\). The complex function \(\Psi_t(\vec{ρ}_t, \vec{ρ}_0)\) and \(\Psi_s(\vec{ρ}_d, \vec{ρ}_t)\) specifies a frozen Kolmogorov-spectrum turbulence on the source-to-target and target-to-detector path, respectively. For instance, the real and imaginary parts of \(\Psi_t(\vec{ρ}_t, \vec{ρ}_0)\) represent the log amplitude and phase fluctuations of the field from a point-like sub-source at \(\vec{ρ}_0\), undergoing turbulence to receiver at \(\vec{ρ}_t\).

The arriving field from the target is collected by bucket detector and the produced current signal is

\[
i_b(m) = \frac{q\eta}{hf_0} \int d\vec{ρ}_d A^2_b(\vec{ρ}_d) |E_d(\vec{ρ}_d, m)|^2
\]

(5)

where \(h\) is Planck’s constant and \(f_0\) is the laser’s optical frequency in order that \(i_b(m)\) have the correct units for photocurrent. \(q\) is the charge of electron. \(\eta\) and \(A_b(\vec{ρ}_d)\) represents the quantum efficiency and field-transmission pupil of the bucket detector, respectively.

Then an initial result of GOAS can be obtained from the cross-correlation between the bucket signal and reference intensity patterns as

\[
G(\vec{ρ}_r) = \langle i_b(t)I(\vec{ρ}_r, t) \rangle - \langle i_b(t) \rangle \langle I(\vec{ρ}_r, t) \rangle
\]

(6)

with \(\langle \cdot \rangle\) being the temporal average of all \(M\) samplings within a single atmospheric coherence time. \(I(\vec{ρ}_r, m)\) is the \(m^{th}\) reference intensity pattern (not shown in Fig.1), which is computed via a suppositional vacuum propagation of the synthetic source yields

\[
I(\vec{ρ}_r, m) = \left| \int_{a_0} d\vec{ρ}_0' E_0'(\vec{ρ}_0', m) \frac{k_0}{i2\pi L} e^{i k_0 (L + |\vec{ρ}_r - \vec{ρ}_0'|^2/2L)} \right|^2
\]

(7)
If all the sub-sources are phased, which means $E'_0(\vec{\rho}_0, m) = E_0(\vec{\rho}_0, m)$, then CGI can be performed and the image of the target can be directly reconstructed via Eq. (6)\textsuperscript{[22]}. However, in GOAS, the sub-sources are dephased and the relative phase is random and unknown. The $m^{th}$ emitted field on the source takes the form of

$$E'_0(\vec{\rho}'_0, t) = \sum_{n=1}^{N} \xi(\vec{\rho}'_0 - \vec{\rho}_n)e^{i[\phi(n,m) + \phi_c(n)]}. \quad (8)$$

Here $\phi_c(n)$ is an artificially introduced phase, whose significant role will act in the following. Since the introduced phase in Eq. (8) is different from the relative phase in Eq. (1), the calculated reference patterns will be significantly different from that imprinted on the target, which is a case far beyond the reach of CGI. Consequently, subjecting Eq. (1)-(5) and Eq. (7)-(8) into Eq. (6), the image of the target cannot be extracted directly, but the spatial information of the target will be involved as (see supplements for details)

$$G(\vec{\rho}_r) = \alpha T(\vec{\rho}_t)e^{-|\vec{\rho}_t|^2/a^2_L} * \left|\mathcal{S}(\vec{\rho}_r - \vec{\rho}_t)\right|^2, \quad (9)$$

where $\alpha$ is constant and $*$ denotes the convolution operation. The GOAS imager of Eq. (9) can be viewed as an active, incoherent imaging system. $T(\vec{\rho}_t)$ is the average intensity-reflection coefficient of the target to be imaged. $e^{-|\vec{\rho}_t|^2/a^2_L}$ is a Gaussian function with a width of $a_L = L/\omega_0 k_0$, corresponding to the field of view. The shift-invariant point-spread function (PSF) of GOAS is dominated by $\mathcal{S}(\vec{\rho}_r - \vec{\rho}_t)$, which is the Fourier transform with a representation as,

$$\mathcal{S}(\vec{\rho}_r - \vec{\rho}_t) = \int_{a_0} d\vec{\rho}_0 S(\vec{\rho}_0)e^{i[\phi_c(n) - \phi_d(n) - \psi(n) + i\omega_0 (\vec{\rho}_r - \vec{\rho}_t)\vec{\rho}_0/L]}, \quad (10)$$

in which $S(\vec{\rho}_0)$ characterizes the distribution of the sub-sources. $\psi (n)$ represents the phase distortion across the source plane induced by turbulence on the source-to-target path. Since both of $\psi (n)$ and $\phi_d (n)$ are random, when the phase $\phi_c (n)$ is arbitrary initially, PSF of GOAS is a random speckle pattern with degeneration caused by turbulence. Hence the initial result $G(\vec{\rho}_r)$ is featureless and low-contrast. But notably, the PSF possesses an adjustable phase $\phi_c (n)$ thus can work just like deformable mirrors\textsuperscript{[41]}. 


In GOAS, neither the mode number of relative phase or that of the phase distortion induced by turbulence can be larger than the number of the sub-sources, this therefore allows the adjustable phase $\phi_c(n)$ to computationally compensate the random phase factor in Eq.(10) to near a plane wave. Then the PSF will turn to be a diffraction-limited spot, the width of which is $\rho_L = 2L/k_0a_0$. By achieving the computational aberration cancellation, the resolution of GOAS will reach the diffraction limit determined by the baseline, thus we state GOAS is an OAS imager.

Here we propose to find the optimum compensation phase $\phi_c(n)$ to implement aberration cancellation by maximizing the gradient of $G(\tilde{\rho}_r)$, which is defined as

$$g = \int d\tilde{r}' |G(\tilde{\rho}_r) * \Lambda(\tilde{\rho}_r - \tilde{r}')|^2.$$  

Eq.(11)

Here $\Lambda$ is two-dimensional Prewitt operators. In GOAS, a given $\phi_c(n)$ will produce an initial result $G(\tilde{\rho}_r)$ with a certain gradient. Besides, the gradient of $G(\tilde{\rho}_r)$ will increase and reach its maximum when the power of the PSF concentrates from random speckle to a diffraction-limited spot, which corresponds to that the diffraction-limited image is achieved (see supplements for details). Therefore, maximizing the gradient of $G(\tilde{\rho}_r)$ can guide the optimal $\phi_c(n)$ to be found via an iterative process.

To demonstrate GOAS imager, a 532nm laser is modulated by 64 isolated macro-pixels (bin 4×4 pixels) with preloaded unknown random phase on SLM1, to achieve 64 sub-sources with random relative phase, as depicted in Fig.2. The size of each macro-pixel is 0.08mm and the distance between the farthest two macro-pixels is about 2mm, thus the baseline length is 25 times larger than the beam waist of sub-sources. The light from SLM1 forms speckles, undergoes SLM2 and illuminates the object. Phase screen is induced by SLM2 to simulate the turbulence on the source-to-target path. The distance between the fresh surface of SLM1 and the object is 3m, thus the spatial resolution determined by the baseline is about 0.80mm on the object plane. The light reflected back from the object is collected by a CCD, which works as a bucket detector. The reference intensity patterns are calculated via Eq.(7).

Firstly, GOAS is verified without turbulence. In this case SLM2 work as a Fourier lens with a focus of 1.5m to meet the far field propagation. The results are shown in Fig.3. When the introduced phase $\phi_c(n)$ is arbitrary, the relative phase cannot be compensated and the
FIG. 2: Setup used to demonstrate GOAS. HWP: half wavelength plate. SLM1/SLM2: spatial light modulator (LCOS-SLM X10468-01). The intensity recorded by CCD (Allied Vision Stingray F-125 B) is integrated spatially as bucket signal. The sub-sources with relative phase are preloaded on SLM1. A Fourier lens or a turbulent phase screen is achieved by SLM2.

PSF is random speckle, causing the initial results featureless as shown in the second row. After the optimum compensation phase is found via the image-guide iterative process, the calculated sequential reference patterns from Eq.(6) will feature the same intensity distribution with the ones imprinted on the object, thus the final images are retrieved as depicted by the third row. The line width of the strings in Fig. 3d is about 0.89\,mm, which is close to the diffraction limit of the synthetic source. In Fig.3n, the lines can be distinguished clearly, implying that the diffraction limit of the GOAS is achieved.

Then the validity of GOAS under turbulence is verified. Fig.4 exhibits the imaging results when SLM2 is loaded a phase screen of turbulence, the strength of which is indicated by the refractive-index structure parameter $C_n^2 (m^{-2/3})$\cite{48}. The phase screen, which is produced under the a Monte-Carlo method\cite{49}, represents a turbulence volume with thickness of 300 meters. Since the actual optical path between the synthetic source and the object is 3m, the geometric size of the experimental setup needs to be spatially scaled up 100 times in
FIG. 3: Imaging results without turbulence. a-e: objects from USAF1951. f-g: initial results under the dephased sub-sources. k-o: final results obtained by GOAS after the computational aberration cancellation. All the results are reconstructed with $4 \times 10^4$ samplings.

the digital production the phase screen. This operation can be treated as that the tabletop experiment equivalently demonstrate a scene in which the geometric size is expanded by 100 times. Considering the scaling factor, the baseline length of the synthetic source becomes $a_0 = 0.2\text{m}$ and the width of the three-slit in Fig.4 is about $0.45\text{m}$. In contrast, the transverse coherence length of the turbulence will be $\rho_T = 0.08\text{m}$ or smaller when $C_n^2 > 10^{-14}(m^{-2/3})$, which is calculated as $\rho_T = \left[0.423k_0^2C_n^2 L \right]^{-3/5}$ [49] with $L$ being the thickness of the turbulence and $L = 300\text{m}$. That is, regardless of the baseline length of GOAS, the resolution of the system will be degenerated largely when $C_n^2 > 10^{-14}(m^{-2/3})$. The first and the third rows are the results of CGI[22] with the same distributed sources as GOAS. With the increase of the turbulent strength, deformation and degeneration appear in the images. Especially when $C_n^2 > 10^{-14}(m^{-2/3})$, the image apparently suffers resolution reduction. As a contrary, original images are still obtained by our method, since the wavefront deformations caused by turbulence are computationally corrected, which is far beyond the reach of CGI.

In practice, the installation or measurement of the baselines with limited accuracy will
cause errors between the actual coordinates of the sub-sources and the one used to calculate the reference intensity pattern, especially when the diameter of whole source reaches meters or larger. Therefore, the robust of GOAS one coordinate error need to be verified. To verify that, coordinate errors are introduced. The actual source realized on SLM1 is depicted as Fig.5(a), while the one used to calculate the reference intensity pattern are shown in the first row of Fig.5. The second and third rows show the retrieved images under different normalized coordinate error, which is quantitated by the ratio between average deviation of the coordinate error and the size of the sub-source as \( \sigma_c = \frac{1}{N} \sum_{n=1}^{N} |\vec{\rho}_0 - \vec{\rho}_n'| / N\omega_0 \). With the
increase of the coordinate error, the mutual coherence degree between the distribution of the actual sub-sources and that of the reference one is reduced, causing the visibility reduction of the retrieved image. However, the reduced mutual coherence is still a peak function when $\sigma_c \sim 1$, thus images with considerable quality can still be reconstructed. These results show that installation accuracy that approximates to the beam waist of single sub-source is enough for GOAS. Considering the beam waist of sub-source is several millimeters or larger, this accuracy is not a challenge for current technology.

![Figure 5](image)

**FIG. 5:** Imaging results under coordinate error. a; the actual distribution of the sub-sources;a-d: the distribution of the sub-sources used to calculate the reference intensity pattern. e-l: the reconstructed images from GOSA under different coordinate errors. All the results are reconstructed with $4 \times 10^4$ samplings.

Although GOAS is capable to computationally correct the wavefront distortion caused by the weak-to-moderate Kolmogorov-spectrum turbulence, we believe that optical imaging through turbulence with too intense fluctuation is still a challenge. Compared with the turbulence fluctuating in milliseconds, the relative phase among the sub-sources usually drifts much slower, especially after the synthetic source being stabilized by means such as vibration isolation, heat insulation and so on. Therefore, once an image of the target has
been recovered, the optimum compensation phase $\phi_c(n)$ can be used directly in the following imaging process as the first parents to significantly improve the convergence speed of the iterative process. Compared with the costly calibration technique and devices-based adaptive optics, this new aberration cancellation in GOAS is easy to conduct, and unrestricted by the limited bandwidth, spectral width, dynamic range or other physical parameters of the optical modulation devices.

I. SUPPLEMENTS

In this supplement, we present the additional details on the theory and experiment of GOAS. The procedure of the image-guide iterative process is also presented in detail.

A. the theory of GOAS

1. The synthetic source

Considering that the length difference among the divided beams is less than the coherence length of the laser pulse but far longer than the wavelength, speckle pattern can be produced after the laser emitted from the distributed, phase-modulated sub-sources. Since the modulation phase of the sub-sources is independent from each other, the temporally varying speckle patterns is pseudo-thermal light when the number of the sub-sources is modest or more\cite{[50]}. Then the cross correlation of the synthetic source can be expressed as

$$\langle E_0(\tilde{\rho}_0, t_1)E_0(\tilde{\rho}_0', t_2) \rangle = \langle E'_0(\tilde{\rho}_0, t_1)E'_0(\tilde{\rho}_0', t_2) \rangle$$

$$= \langle E_0(\tilde{\rho}_0, t_1)E'_0(\tilde{\rho}_0', t_2) \rangle$$

$$= 0,$$

$$\langle E_0(\tilde{\rho}_0, t_1)E_0^*(\tilde{\rho}_0', t_2) \rangle = \langle E'_0^*(\tilde{\rho}_0, t_1)E'_0(\tilde{\rho}_0', t_2) \rangle$$

$$= \langle E_0^*(\tilde{\rho}_0, t_1)E'_0(\tilde{\rho}_0', t_2) \rangle$$

$$= \frac{2NP}{\pi \omega_0^2} S(\tilde{\rho}_0) e^{-\frac{|	ilde{\rho}_0-\tilde{\rho}_0'|^2}{2\delta^2}} e^{-\frac{|t_1-t_2|^2}{2\sigma^2}}, \tag{S1}$$
Here $\langle \cdot \rangle$ represents the ensemble average for classical correlation. $S(\vec{\rho}_0) = \sum_{n=1}^{N} e^{-\frac{|\vec{\rho}_0 - \vec{\rho}_n|^2}{2\omega_0^2}}$ characterizes the intensity distribution of the sub-sources. $S(\vec{\rho}_0)$ is a two-dimensional, broaden (the width is determined by the waist of the sub-source) comb function with random distribution, which accounts for the dozens of randomly, widely distributed sub-sources. To ensure that the sub-apertures are approximately nonoverlapping, the center coordinate and the beam waist of the sub-source subject to $\min_{k \neq j} |\vec{\rho}_k - \vec{\rho}_j| > 3\omega_0$. $P$ is the intensity of each sub-source with unit Watts. $T_0$ is the coherence time of the light field. In Fig.1, the temporally varying speckle pattern is produced by randomly modulating the phase of each sub-source, thus $T_0$ is the reciprocal of the bandwidth of the phase modulators. If a bucket detector with a higher bandwidth is used, which means the detector can easily record the signal within $T_0$, then the factor of coherence time in the correlation results can be ignored. In GOAS, each sub-source also gets a relative phase $\phi_d(n)$ on the signal path while an artificially introduced phase $\phi_c(n)$ when calculate the reference pattern. Since both of the phases are treated as constant and independent of the ensemble average, the phase-insensitive cross correlation of the source can be written as

$$\langle E_0^*(\vec{\rho}_0, m) E_0'(\vec{\rho}_0', m) \rangle = \frac{2NP}{\pi\omega_0^2} S(\vec{\rho}_0) e^{-\frac{|\vec{\rho}_0 - \vec{\rho}_0'|^2}{2\omega_0^2}} e^{i[\phi_c(n) - \phi_d(n)]}$$

where $m$ is the index of emitted illumination pattern and the complex phase factor $e^{i[\phi_c(n) - \phi_d(n)]}$ accounts for its invariance over time. Considering the profile of $S(\vec{\rho}_0)$, the synthetic source can also be seen as that a pseudo-thermal source with diameter $a_0$ is masked except for $N$ small sub-sources with waist of $\omega_0$, satisfying $\omega_0 \ll a_0$. Besides, the relative phase among the sub-sources is random and unkown.

2. propagation through turbulence

The phase-modulated laser emitted from the synthetic source propagates through $L$-m turbulence and illuminates the object. The light reflected by the target undergoes $L$-m turbulence and arrives the plane of bucket detector. Because the turbulence is random, they can only be described statistically by using statistical estimates such as variances, or covariances. As an example, the mutual coherence function of the turbulence on the
source-to-target and target-to-detector path follows\cite{28}

\[
\langle e^{\Psi_t^*(\vec{\rho}_t, \vec{\rho}_0)} e^{\Psi_t^*(\vec{\rho}_t', \vec{\rho}_0')} \rangle
= e^{-\langle |\vec{\rho}_t - \vec{\rho}_t'|^2 + (\vec{\rho}_t - \vec{\rho}_t') \cdot (\vec{\rho}_0 - \vec{\rho}_0') + |\vec{\rho}_0 - \vec{\rho}_0'|^2 \rangle / 2 \rho_T^2}
\]  (S3)

and

\[
\langle e^{\Psi_d^*(\vec{\rho}_d, \vec{\rho}_0)} e^{\Psi_d^*(\vec{\rho}_d', \vec{\rho}_0')} \rangle
= e^{-\langle |\vec{\rho}_d - \vec{\rho}_d'|^2 + (\vec{\rho}_d - \vec{\rho}_d') \cdot (\vec{\rho}_0 - \vec{\rho}_0') + |\vec{\rho}_0 - \vec{\rho}_0'|^2 \rangle / 2 \rho_T^2}
\]  (S4)

respectively. Here $\rho_T$ is the spatial coherence length of the turbulence and $\rho_T = [0.423 k_0^2 C_n^2 L]^{-3/5}$ for plane waves\cite{49}. $C_n^2$ is the refractive index structure parameter of the turbulence in the propagation direction\cite{48}, which can be treated approximately as a constant when the angle between the propagation direction and the horizontal is small.

3. target and bucket signal

In the natural world, almost all the objects possess a rough surface with random depths substantially exceeding the optical wavelength\cite{50}. Under the illumination of speckle with a small transverse coherence length, the rough-surfaced target can be treated as a quasi-Lambertian reflector with a random field-reflection coefficient $T(\vec{\rho}_t)$\cite{51}. Following Lidar theory\cite{23, 28, 51}, the target possesses the autocorrelation function,

\[
\langle T^*(\vec{\rho}_t) T(\vec{\rho}_t') \rangle = \lambda^2 T(\vec{\rho}_t) \delta(\vec{\rho}_t - \vec{\rho}_t')
\]  (S5)

where $\delta(\cdot)$ is a Drac function, $T(\vec{\rho}_t)$ is the average intensity-reflection coefficient and $T(\vec{\rho}_t) \leq 1$.

Under the illumination of $M$ temporally varying speckle patterns, the $m^{th}$ reflected field from the target is collected by the bucket detector. The bucket signal can be evolved from Eq. (5) as

\[
i_b(m) = \frac{q\eta A_b}{h f_0} \langle E_d^*(\vec{\rho}_d, m) E_d(\vec{\rho}_d, m) \rangle
\]  (S6)

in which $A_b \equiv \int d\vec{\rho}_d A_b^2(\vec{\rho}_d)$ represents the area of the photosensitive region of the bucket detector.
4. *initial result in GOAS*

In GOAS, the initial result $G(\vec{p}_r)$ is calculated from the cross correlation between the bucket signals and the reference intensity pattern shown by Eq.(6). In the following, we will conduct the derivation of the initial results.

According to Eq.(S6), the first term in the right of Eq.(6) can be written as

$$\langle i_b(m)I(\vec{p}_r, m) \rangle = \frac{q\eta A_b}{hf_0} \langle E_d^*(\vec{p}_d, m)E_r^*(\vec{p}_r, m)E_r(\vec{p}_r, m)E_d(\vec{p}_d, m) \rangle$$

where the second-order moment of intensity is evolved into the fourth-order moment of the field arriving the bucket detector and the reference one. When the back-propagate from the bucket detector to the target\cite{23,28} is used,

$$\langle E_d^*(\vec{p}_d, m)E_r^*(\vec{p}_r, m)E_r(\vec{p}_r, m)E_d(\vec{p}_d, m) \rangle = \int d\vec{p}_t \int d\vec{p}_t' \langle T^*(\vec{p}_t)T(\vec{p}_t') \rangle \langle e^{i\Psi_d(\vec{p}_d,\vec{p}_t')} e^{i\Psi_d(\vec{p}_d,\vec{p}_t')} \rangle$$

$$\times \langle E_t^*(\vec{p}_t, m)E^*_r(\vec{p}_r, m)E_r(\vec{p}_r, m)E_d(\vec{p}_t', m) \rangle$$

$$\times \frac{k_0^2 e^{i k_0 |\vec{p}_t'|^2 - |\vec{p}_t|^2|/2L + ik_0 \vec{p}_d \cdot (\vec{p}_t - \vec{p}_t')/L}}{4\pi^2 L^2}$$

(S8)

Considering the statistical property of the turbulence shown by Eq.(S4) and that of the target’s reflection coefficient expressed as Eq.(S5), Eq.(S8) can be simplified as

$$\langle E_d^*(\vec{p}_d, m)E_r^*(\vec{p}_r, m)E_r(\vec{p}_r, m)E_d(\vec{p}_d, m) \rangle = \frac{1}{L^2} \int d\vec{p}_t T(\vec{p}_t) \times \langle E_t^*(\vec{p}_t, m)E^*_r(\vec{p}_r, m)E_r(\vec{p}_r, t)E_t(\vec{p}_t, m) \rangle$$

(S9)

which exhibits that the fourth-order moment of the field arriving on the bucket detector and the reference one is proportional to the fourth-order moment of the field imprinted on the target and the reference one. That is, the turbulence on the target-to-detector path will not affect the cross correlation results. This arises from the statistically-independent randomness of the turbulence and the that of the quasi-Lambertian target’s surface. In addition, when the target is not quasi-Lambertian, Eq.(S9) can be still reasonable under the condition that the diameter of the lens in front of the bucket detector is large enough that
the intensity fluctuation of the reflected light from the target induced by the turbulence is negligible. This condition is not difficult to achieve, since the lens in front of the bucket detector is to concentrate the energy rather than imaging. Energy-concentration lens is less unaffected by aberrations, thus is much easier to obtain than manufacturing an imaging lens with the same size.

According to the isotropy of the Kolmogorov-spectrum turbulence, the light field imprinted on the target plane \( E_t(\tilde{\rho}_t, m) \) can still be treated as obeying Gaussian random distribution, so does \( E_r(\tilde{\rho}_r, m) \). Then the Complex Gaussian moment theorem can be used and the fourth-order field moment in Eq. (S9) can be expressed as

\[
\langle E_t^\ast(\tilde{\rho}_t, m)E_r(\tilde{\rho}_r, m)E_r(\tilde{\rho}_r, m)E_d(\tilde{\rho}_t, m) \rangle = \langle E_t^\ast(\tilde{\rho}_t, m)E_t(\tilde{\rho}_t, m) \rangle \langle E_r^\ast(\tilde{\rho}_r, m)E_r(\tilde{\rho}_r, m) \rangle + |\langle E_t^\ast(\tilde{\rho}_t, m)E_r(\tilde{\rho}_r, m) \rangle|^2
\]

(S10)

where the first term of the right hand is the product between the average intensity of the reference pattern and that of the pattern imprinted on the target. The second term is the square of the first-order mutual coherence of the reference field and the field imprinted on the target. Substituting Eq.(S7), (S9) and (S10) to Eq.(6), we have

\[
G(\tilde{\rho}_r) = \frac{q\eta A_b}{\hbar f_0 L^2} \int d\tilde{\rho}_t T(\tilde{\rho}_t) \left[ |\langle I(\tilde{\rho}_r, m) \rangle|^2 - \langle i_b(m) \rangle \right] 
\]

\[
= \frac{q\eta A_b}{\hbar f_0 L^2} \int d\tilde{\rho}_t T(\tilde{\rho}_t) |\langle E_t^\ast(\tilde{\rho}_t, m)E_r(\tilde{\rho}_r, m) \rangle|^2 \]

(S11)

in which the product of the average intensity is eliminated and the square of the first-order mutual coherence is left in the integral. As a integral kernel of the image formation, the square of the first-order mutual coherence plays an important role to characterize the features of GOAS (we will detail below). Here we discuss the case that the path length \( L \) satisfies far field propagation of the synthetic pseudo-thermal source, which means \( k_0 a_0 \omega_0 / 2L \ll 1 \) \[18, 52, 55\]. Then the second-order moments of cross correlation in Eq.(S11) can be written as the coherence propagation of the mutual coherence of the source, as
\[
\langle E_t^* (\tilde{\rho}_t, m) E_r (\tilde{\rho}_r, m) \rangle = \int_{a_0} d\tilde{\rho}_0 \int_{a_0} d\tilde{\rho}_0' \langle E_0^* (\tilde{\rho}_0, m) E_0' (\tilde{\rho}_0', m) \rangle e^{i(\chi(\tilde{\rho}_t, \tilde{\rho}_0) - i\psi(\tilde{\rho}_t, \tilde{\rho}_0))} \frac{k_0^2 e^{ik_0(|\tilde{\rho}|^2 - |\tilde{\rho}|^2)}}{4\pi^2 L^2}
\]

\[
\approx \int_{a_0} d\tilde{\rho}_0 \int_{a_0} d\tilde{\rho}_0' \langle E_0^* (\tilde{\rho}_0, m) E_0' (\tilde{\rho}_0', m) \rangle e^{-i\psi(\tilde{\rho}_t, \tilde{\rho}_0)} \frac{k_0^2 e^{ik_0(|\tilde{\rho}|^2 - |\tilde{\rho}|^2)}}{4\pi^2 L^2} \left( L + (\tilde{\rho}_t \cdot \tilde{\rho}_0') - \tilde{\rho}_t \cdot \tilde{\rho}_0 \right)
\]

(S12)

where \(\chi(\tilde{\rho}_t, \tilde{\rho}_0)\) and \(\psi(\tilde{\rho}_t, \tilde{\rho}_0)\) represents the log amplitude and phase fluctuations of the field caused by the turbulence on source-to-target path respectively, and \(\Psi_t^* (\tilde{\rho}_t, \tilde{\rho}_0) = \chi(\tilde{\rho}_t, \tilde{\rho}_0) - i\psi(\tilde{\rho}_t, \tilde{\rho}_0)\). The approximation arises from that image quality can be degraded by both phase and amplitude distortions of the optical wavefront but the effect of phase fluctuations is predominant [26, 27, 41]. Substituting Eq. (S2) into Eq. (S12), one can obtain

\[
\langle E_t^* (\tilde{\rho}_t, m) E_r (\tilde{\rho}_r, m) \rangle = \frac{NP k_0^2 e^{ik_0(|\tilde{\rho}_t|^2 - |\tilde{\rho}_r|^2)}}{2\pi^3 L^2 a_L^2} \int_{a_0} d\tilde{\rho}_0 \int_{a_0} d\tilde{\rho}_0' S(\tilde{\rho}_0) S(\tilde{\rho}_0') e^{i(\phi_e(n) - \phi_d(n))} e^{-i\psi(\tilde{\rho}_t, \tilde{\rho}_0)} e^{-\frac{|\tilde{\rho}_0 - \tilde{\rho}_0'|^2}{2a_L^2}} e^{ik_0(\tilde{\rho}_0 - \tilde{\rho}_r)}
\]

\[
= \frac{NP k_0^2 e^{ik_0(|\tilde{\rho}_t|^2 - |\tilde{\rho}_r|^2)}}{2\pi^3 L^2 a_L^2} \int_{a_0} d\tilde{\rho}_0 S(\tilde{\rho}_0) e^{i(\phi_e(n) - \phi_d(n) - \psi(\tilde{\rho}_t, \tilde{\rho}_0))} S(\tilde{\rho}_0) e^{ik_0(\tilde{\rho}_0 - \tilde{\rho}_r)} / L
\]

\[
= \frac{NP k_0^2 e^{ik_0(|\tilde{\rho}_t|^2 - |\tilde{\rho}_r|^2)}}{2\pi^3 L^2 a_L^2} e^{-\frac{|\tilde{\rho}_0|^2}{2a_L^2}} \tilde{S}(\tilde{\rho}_r - \tilde{\rho}_t)
\]

(S13)

in which \(\Delta \tilde{\rho} = \tilde{\rho}_0 - \tilde{\rho}_0', a_L = L / \omega_0 k_0\), is the beam waist of the sub-source on the target plane, which is also the field of view (FOV) illuminated by the source. \(\tilde{S}(\tilde{\rho}_r - \tilde{\rho}_t)\) is a Fourier transform with a representation as,

\[
\tilde{S}(\tilde{\rho}_r - \tilde{\rho}_t) = \int_{a_0} d\tilde{\rho}_0 S(\tilde{\rho}_0) e^{i(\phi_e(n) - \phi_d(n) - \psi(n))} e^{ik_0(\tilde{\rho}_0 - \tilde{\rho}_r)} / L,
\]

in which the transform kernel is a complex-valued function. The real part of the kernel is the distribution of the synthetic source, which determines the spatial frequency of the PSF. The imaginary part is three phase factors, which includes the adjustable compensation phase \(\phi_e(n)\), the unknown relative phase \(\phi_d(n)\) and phase fluctuation \(\psi(n)\) induced by turbulence. Notably, the spatial arguments of phase fluctuation \(\psi(\tilde{\rho}_t, \tilde{\rho}_0)\) is replaced by \(\psi(n)\) in Eq.
This means the wavefront distortion caused by the turbulence on source-to-target path is represented by a phase screen $\psi(n)$ and be corrected on the source plane\[11\,54\]. In our method, the spatial frequency of the PSF is optimized by designing both the number and the spatial distribution of the sub-sources thus wavefront distortion can be computationally corrected well. In principle, except for source plane, the wavefront distortion can also be computationally corrected by inserting a phase screen on another plane away from the source when calculating the reference pattern. The degree of the inserted phase screen and the distance between the phase screen and the source will the influence on the finally FOV of imaging, due to the anisoplanicity of turbulence\[41\,53\].

Substituting Eq.(S13) into Eq.(S11), the initial results from the cross correlation between the bucket signal and the reference pattern becomes

$$
G(\tilde{\rho}_r) = \frac{4q\eta A_b N^2 P^2}{h f_0 \omega_0^4 \lambda^4 L^6} \int d\tilde{\rho}_t |T(\tilde{\rho}_t) e^{-|\tilde{\rho}_t|^2/a_L^2} S(\tilde{\rho}_r - \tilde{\rho}_t)|^2
$$

$$
= \alpha T(\tilde{\rho}_t) e^{-|\tilde{\rho}_t|^2/a_L^2} \star \tilde{S}(\tilde{\rho}_r - \tilde{\rho}_t)
$$

in which $\alpha = \frac{4q\eta A_b N^2 P^2}{h f_0 \omega_0^4 \lambda^4 L^6}$ is constant. Eq. (S15) expresses an incoherent imaging system. The $T(\tilde{\rho}_t)$ is the target to be imaged. $e^{-|\tilde{\rho}_t|^2/a_L^2}$ is a Gaussian function with a width of $a_L = L/\omega_0 k_0$, which actually determines the largest FOV. The shift-invariant PSF of GOAS is characterized by $|\tilde{S}(\tilde{\rho}_r - \tilde{\rho}_t)|^2$. As shown by Eq. (S14), the PSF with adjustable phase $\phi_c(n)$ can work just like deformable mirrors to compensate the relative phase and the turbulent distortion. Significantly different from traditional adaptive optics, the wavefront correction is performed computationally in GOAS rather than in sensor-dependent way. The challenge is to find out the optimum $\phi_c(n)$, which will explained in the next part.

**B. The details of the GOAS experiment**

1. *image-guide iterative process*

In GOAS, image sharpening can be used to guide the the optimization of the compensation phase $\phi_c(n)$ to be found adaptively\[42\]. To achieve that, the sharpness of the initial
result can be quantified by the gradient as

$$g = \sum_x \sum_y \left[ (G(x, y) \ast \Lambda_x)^2 + (G(x, y) \ast \Lambda_y)^2 \right].$$  \hspace{1cm} (S16)

where \( \ast \) represents the convolution operation, \( x \) and \( y \) are two orthogonal component of the vector \( \vec{\rho}_r \), \( \Lambda_x \) and \( \Lambda_y \) are Prewitt operators at two directions and

$$\Lambda_x = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad \Lambda_y = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}. \hspace{1cm} (S17)$$

FIG. 6: Flow diagram of the image-guide iterative process to figure out the optimum compensation phase.

Eq.(S16) is the discretization of Eq.(11) for the convenient of data processing.

Fig. S1 shows the flow diagram of the image-guide iterative process. Initially, when an arbitrary \( \phi^k_c(n) \) is used, the contrast of \( G(\vec{\rho}_r) \) is low and the gradient calculated via Eq. (S16) is small. Then the algorithm starts to iterative, each cycle of which include three
steps. Firstly, the parent population of $\phi_k^c(n)$ is generated and the corresponding $G_k(\vec{\rho}_r)$ can be obtained. Secondly, mutate the subpopulation of $\phi_k^c(n)$ which can produce $G_k(\vec{\rho}_r)$ with larger gradient. Thirdly hybridize the mutated subpopulation with another new random population of $\phi_k^c(n)$ to give birth to the child population $\phi_{k+1}^c(n)$. The child population is also the parent in the next cycle, until the gradient of $G_k(\vec{\rho}_r)$ is large enough. In this kind of image-guide iterative process, the guide functions are usually independent with the position of the image, thus the lateral position of the object will be discarded in the final image.

FIG. 7: The largest gradient of $G_k(\vec{\rho}_r)$ of every generation. Y-axis is the largest gradient among all the $G_k(\vec{\rho}_r)$ produced by $k$th population of $\phi_k^c(n)$.

In our experiment, $4 \times 10^4$ samplings are performed to retrieve the image. That means with each $\phi_k^c(n)$, $4 \times 10^4$ reference pattern and the subsequent result $G_k(\vec{\rho}_r)$ are calculated. The number of the sub-sources is 64. The population size in each cycle is 128 and the initial mutation rate is 30%, which will be reduced with the increase of the generation index. Once the mean error of the compensated phase is smaller than $\pi/2$, an image $G_k(\vec{\rho}_r)$ with considerate contrast can be obtained. Consequently, the gradient of the image converges quickly under the iterative process. Fig. S2(a) and S2(b) shows the largest gradient of every population in the iterative process to obtain Fig. 3(o) and Fig.4(g), respectively. The both gradient of the $G_k(\vec{\rho}_r)$ start to converge when $k \sim 300$. If more sub-sources are used, the phase distortion caused by the turbulence can be compensated more accurately and an image with higher contrast can be obtained. In that case, the converges speed of the image-guide iterative process will not be reduced largely if more RAM can be used and the
parallelism of data processing can be improved.

2. The turbulence phase screen

The turbulent phase screen preloaded on SLM2 is produced via a Monte-Carlo method \cite{49}. A phase screen represents an extended turbulent volume, and the parameter of the produced phase screen is closely related to the thickness of the turbulence, which is usually 100\( m \) or thicker. However, the distance between the source and the target in our table-top experiment is only 3\( m \), which is too short for a practical phase screen. To meet the geometry required by phase screen model, the actual size of the experimental setup is spatially scaled up 100 times in the digital production the phase screen. That is, the lab-table experiment equivalently demonstrate a scene where the distance between the source and the object is 300\( m \). Consequently, considering the scaling factor, the baseline length of the synthetic source is enlarged as \( a_0 = 0.2 \text{m} \), the beam waist of the sub-source is scaled up to \( 0.008 \text{m} \) and the size of the object becomes about 0.45\( \text{m} \).

In traditional adaptive optics, the compensated FOV is limited after wavefront correction. It occurs because of the differences between wavefront coming from different directions, which is called the anisoplanicity of the turbulence\cite{53}. If a guide source is used to sense the wavefront, the compensation will be good only for objects close enough to the guide source. As the angular distance between the object and the guide source increases, image quality decreases. When one can accept Mean Square Error (MSE) of the compensation less than 1 radian, the FOV \( \theta \) subjects to \( \theta \propto \frac{\rho_T}{d} \), in which \( \rho_T \) is the coherence length of the turbulence and \( d \) is the distance between the turbulence layer and the wavefront compensation devices. In GOAS, an image with considerate contrast can be obtained once the MSE of the compensation is less than \( \pi/2 \), thus the FOV of GOAS can be larger than traditional adaptive optics. Better yet, GOAS allows to compensate the turbulence in the source-to-object path by inserting a phase screen on the conjugate plane in the reference path, which means the distance between the turbulence-layer and the source is almost equal to the distance between the correction plane and the source. Then \( d \) can next to zero and the FOV \( \theta \) of GOAS will be large enough.
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