Research Article

Delay-Partitioning Approach to Stability of Linear Discrete-Time Systems with Interval-Like Time-Varying Delay

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1. Introduction

A source of instability for discrete-time systems is time delay which inevitably exists in various engineering systems. In many applications, time delays are unavoidable and must be taken into account in a realistic system design, for instance, chemical processes, echo cancellation, local loop equalization, multipath propagation in mobile communication, array signal processing, congestion analysis and control in high speed networks, neural networks, and long transmission line in pneumatic systems [1–6]. The stability analysis of time-delay systems has received considerable attention during the last two decades [3–7]. According to dependence of delay, the existing stability criteria are generally classified into two categories: delay-dependent criteria and delay-independent criteria. It is well known that delay-independent stability criteria are usually more conservative than the delay-dependent ones, especially if the size of time delay is small [6–10]. Therefore, much attention has been paid in recent years to the study of delay-dependent stability criteria.

A number of publications relating to the delay-dependent stability of continuous time-delay systems have appeared (see, e.g., [10–17] and the references cited therein). In contrast, less attention has been paid to studying the problem of stability of discrete time-delay systems. Several delay-dependent criteria for the stability of discrete-time systems have appeared [3, 6, 7, 18, 19]. Reference [7] (see [6] also) presents a novel delay-dependent linear-matrix-inequality-(LMI-) based condition for the global asymptotic stability of linear discrete-time systems with interval-like time-varying delay. The criteria proposed in [6, 7] are less conservative with smaller numerical complexity than [19–23]. The delay partitioning approach has been efficiently applied in [24–30] to the stability analysis of systems with time-varying delays. In the context of stability analysis of linear discrete systems with time-varying delay, the delay partitioning concept is first utilized in [30]. The approaches in [29, 30] divide the lower bound of the time-varying delay into a number of partitions. The criteria in [29, 30] are not only delay dependent but also dependent on the partitioning size. Though the approaches in [29, 30] provide less conservative stability results than [19, 20], these approaches would lead to heavier computational burden and more complicated synthesis procedure.

This paper studies the problem of stability analysis of linear discrete-time system with interval-like time-varying delay in the state. In this paper, inspired by the work of [6, 7, 30], an alternative to the approach presented in [29] for the stability analysis of linear discrete-time systems with...
interval-like time-varying delay in the state is brought out. The proposed method exploits the delay partitioning idea and does not introduce any free weighting matrices. The paper is organized as follows. Section 2 presents a description of the system under consideration. A novel LMI-based criterion for the global asymptotic stability of discrete-time state-delayed systems is proposed in Section 3. The proposed criterion depends on the size of delay as well as partition size. In Section 4, the approach is extended to derive global asymptotic stable conditions for delayed discrete-time systems with norm-bounded uncertainties. Finally, in Section 5, numerical examples are given to illustrate the effectiveness of the presented results.

2. System Description

The following notations are used throughout the paper:

- $\mathbb{R}^{p \times q}$: Set of $p \times q$ real matrices,
- $\mathbb{R}^n$: Set of $p \times 1$ real vectors,
- $I$: Identity matrix of appropriate dimension; the order is specified in subscript as the need arises,
- $0$: Null matrix or null vector of appropriate dimension; the orders are specified in subscripts as the need arises,
- $B^T$: Transpose of the matrix (or vector) $B$,
- $B > 0$: $B$ is positive-definite symmetric matrix,
- $B < 0$: $B$ is negative-definite symmetric matrix.

The system under consideration is given by

$$x(k+1) = Ax(k) + A_1 x(k-d(k)), \quad (1)$$

$$x(k) = \phi(k), \quad k = -h_2, -h_2+1, \ldots, 0, \quad (2)$$

where $x(k) \in \mathbb{R}^n$ is the system state vector, $A$ and $A_1$ are constant matrices with appropriate dimensions, $d(k)$ is a positive integer representing interval-like time-varying delay satisfying

$$1 \leq h_1 \leq d(k) \leq h_2, (3)$$

where $h_1$ and $h_2$ are known positive integers representing the lower and upper delay bounds, respectively, and $\phi(k)$ is an initial value at time $k$. Let the lower bound of the delay $h_1$ be divided into $m$ number of partitions such that

$$h_1 = \tau m, (4)$$

where $\tau$ is an integer representing partition size.

3. Proposed Criterion

In this section, inspired by the work of [6, 7, 30], an LMI-based criterion for the global asymptotic stability of system (1)–(4) is established.

The main result may be stated as follows.

**Theorem 1.** For given positive integers $\tau, m$, and $h_2$, the system in (1)–(4) is asymptotically stable if there exist real matrices $P = P^T > 0, Q_i = Q_i^T > 0 (i=1,2,3)$, and $Z_i = Z_i^T > 0 (i=1,2)$ such that

$$\Psi_1 = \Psi - [0_{n \times n} I_n - I_n 0_n]Z_2^T \times [0_{n \times n} I_n - I_n 0_n] < 0, \quad (5)$$

$$\Psi_2 = \Psi - [0_{n \times (m+1)n} I_n - I_n]Z_2^T \times [0_{n \times (m+1)n} I_n - I_n] < 0, \quad (6)$$

where

$$\Psi = A_1^T \Phi_1 A_1 + A_2^T \Phi_2 A_2 + A_3^T \Phi_3 A_3 - A_1^T \Phi_4 A_1 + A_1^T \Phi_2 A_3$$

$$+ A_3^T \Phi_5 A_1 - A_5^T Z_1 A_5 + A_5^T Z_2 A_5 + A_5^T Z_3 A_3 + A_5^T Z_4 A_0,$n

$$A_1 = [I_n 0_{n \times (m+2)n}], \quad A_2 = \begin{bmatrix} I_{mn} & 0_{mn \times n} & 0_{mn \times 2n}^T \end{bmatrix}, \quad (7)$$

$$A_3 = [0_{n \times (m+1)n} I_n 0_n], \quad A_4 = [0_{n \times (m+2)n} I_n 0_n], \quad (8)$$

$$A_5 = [0_n I_n 0_{n \times (m+1)n}], \quad A_6 = [0_{n \times (m+2)n} I_n 0_{n \times 2n} 0_{n \times 2n}^T], \quad (9)$$

$$\Phi_1 = A^T P A - P + Q_2 + (h_2 - \tau m + 1) Q_3 - Z_1$$

$$+ (A - I_n)^T (\tau Z_1 + (h_2 - \tau m)^2 Z_2) (A - I_n),$$

$$\Phi_2 = \begin{bmatrix} Q_2 & 0 \\ 0 & -Q_2 \end{bmatrix}, \quad (10)$$

$$\Phi_3 = A_1^T (P + \tau Z_1 + (h_2 - \tau m)^2 Z_2) A_1 - Q_3 - 2Z_2,$n

$$\Phi_4 = Q_2 + Z_2,$n

$$\Phi_5 = A_1^T P A_1 + (A - I_n)^T (\tau Z_1 + (h_2 - \tau m)^2 Z_2) A_1, \quad (11)$$

Proof. Consider a Lyapunov function [29, 30]

$$V(k) = x^T(k) P x(k) + \sum_{i=k-\tau}^{k-1} Y^T(i) Q_i Y(i)$$

$$+ \sum_{i=k-h_2}^{k-1} x^T(i) Q_2 x(i) + \sum_{j=h_2+1}^{\tau} \sum_{i=k+j}^{k-1} x^T(i) Q_3 x(i)$$

$$+ \sum_{j=\tau}^{\tau-1} \sum_{i=k+j}^{k-1} \tau \Delta x^T(i) Z_3 \Delta x(i)$$

$$- \sum_{i=k-h_2}^{k-1} \sum_{j=h_2+1}^{\tau} (h_2 - \tau m) \Delta x^T(i) Z_2 \Delta x(i), \quad (12)$$

$$+ \sum_{j=\tau}^{\tau-1} \sum_{i=k+j}^{k-1} \tau \Delta x^T(i) Z_3 \Delta x(i)$$

$$+ \sum_{i=k-h_2}^{k-1} \sum_{j=h_2+1}^{\tau} (h_2 - \tau m) \Delta x^T(i) Z_2 \Delta x(i), \quad (15)$$
where
\[
\mathbf{Y}(i) = \left[ x^T(i) \ x^T(i - \tau) \ \cdots \ x^T(i - (m - 1) \tau) \right]^T,
\]
\[
\Delta x(i) = x(i + 1) - x(i).
\]  \hfill (16)  \hfill (17)

Taking the forward difference of (15) along the solution of the system (1)-(2), we have
\[
\Delta V(k) = V(k + 1) - V(k)
\]
\[
= \left[ A\mathbf{x}(k) + A_1\mathbf{x}(k - d(k)) \right]^T \mathbf{P} \left[ A\mathbf{x}(k) + A_1\mathbf{x}(k - d(k)) \right]
\]
\[
- x^T(k) \mathbf{P}x(k) + Y^T(k) \mathbf{Q}_1\mathbf{Y}(k) - Y^T(k - \tau) \mathbf{Q}_1\mathbf{Y}(k - \tau)
\]
\[
+ x^T(k) \mathbf{Q}_1\mathbf{x}(k - \tau) - x^T(k - h_2) \mathbf{Q}_2\mathbf{x}(k - h_2)
\]
\[
+ (h_2 - \tau m + 1) x^T(k) \mathbf{Q}_3\mathbf{x}(k) + \Delta x^T(k) \tau^2 \mathbf{Z}_1 \Delta \mathbf{x}(k)
\]
\[
+ \Delta x^T(k) (h_2 - \tau m)^2 \mathbf{Z}_2 \Delta \mathbf{x}(k) - \sum_{i=k-h_2}^{k-\tau} x^T(i) \mathbf{Q}_2\mathbf{x}(i)
\]
\[
- \sum_{i=k-\tau}^{k-m-1} \tau \Delta x^T(i) \mathbf{Q}_1\Delta \mathbf{x}(i)
\]
\[
- \sum_{i=k-h_2}^{k-d(k)-1} (h_2 - \tau m) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i).
\]

Now, we have the following relation [6, 7]:
\[
- \sum_{i=k-h_2}^{k-m-1} (h_2 - \tau m) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i)
\]
\[
= - \sum_{i=k-h_2}^{k-d(k)-1} (h_2 - \tau m) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i)
\]
\[
- \sum_{i=k-d(k)}^{k-m-1} (h_2 - d(k)) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i)
\]
\[
- \sum_{i=k-d(k)}^{k-m-1} (d(k) - \tau m) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i).
\]

Letting \( \beta = (d(k) - \tau m)/(h_2 - \tau m) \), one obtains [6]
\[
- \sum_{i=k-h_2}^{k-d(k)-1} (d(k) - \tau m) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i)
\]
\[
= - \beta \sum_{i=k-h_2}^{k-d(k)-1} (h_2 - \tau m) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i)
\]
\[
\leq - \beta \sum_{i=k-h_2}^{k-d(k)-1} (h_2 - d(k)) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i),
\]
\[
- \sum_{i=k-d(k)}^{k-m-1} (h_2 - d(k)) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i)
\]
\[
= -(1 - \beta) \sum_{i=k-d(k)}^{k-m-1} (h_2 - \tau m) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i)
\]
\[
\leq -(1 - \beta) \sum_{i=k-d(k)}^{k-m-1} (d(k) - \tau m) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i).
\]

Applying Lemma 1 in [22] and using (23)-(24), we have [6, 7]
\[
- \sum_{i=k-h_2}^{k-m-1} (h_2 - \tau m) \Delta x^T(i) \mathbf{Z}_2 \Delta \mathbf{x}(i)
\]
\[
\leq -(x(k - d(k)) - x(k - h_2))^T \mathbf{Z}_2 (x(k - d(k)) - x(k - h_2))
\]
\[
= (A - I_n)\mathbf{x}(k) + A_1\mathbf{x}(k - d(k)).
\]  \hfill (22)
Employing (18)–(22) and (25), we have the following inequality:

\[
\Delta V(k) \leq \zeta^T(k) [(1 - \beta) \Psi_1 + \beta \Psi_2] \zeta(k). \tag{26}
\]

Since \(0 \leq \beta \leq 1\), \(\Delta V(k) < 0\) for all nonzero \(\zeta(k)\) if (5) hold true. This completes the proof of Theorem 1.

\(\square\)

Remark 2. To prove Theorem 1, a methodology similar to [6, 7] has been adopted.

Remark 3. It may be noted that the inequalities (5) are LMIs and can be effectively solved by using MATLAB LMI toolbox [31, 32].

Remark 4. For a given \(\tau\) and \(m\), the allowable maximum value of \(h_2\) for guaranteeing the global asymptotic stability of system (1)–(4) can be obtained by iteratively solving (5).

Remark 5. For \(m = 1\), after some algebraic manipulations, it can be shown that Theorem 1 is equivalent to Theorem 1 in [7]. The parameters \(m\) and \(\tau\) (subject to (4)) in Theorem 1 represent some additional degrees of freedom, that is, in comparison with Theorem 1 in [7] which is free of these parameters.

Remark 6. A comparison of the number of the decision variables involved in several recent stability results is summarized in Table 1. It may be observed that the size of complexity in [6, 7, 18, 19, 23] is only related to state dimension \(n\), whereas the complexity of [29, Theorem 3.1], [30, Theorem 2], and Theorem 1 depends on both \(n\) and \(m\).

The total number of scalar decision variables of Theorem 1 is \(D_1 = (n/2)[n(m^2 + 5) + (m + 5)]\), and the total row size of the LMIs is \(L_1 = n(3m + 11)\). The numerical complexity of Theorem 1 is proportional to \(L_1D_1^2\) [14]. The total number of scalar decision variables of Theorem 2 in [30] is \(D_2 = (n/2)[n(3m^2 + 18m + 41) + (3m + 11)]\), the total row size of the LMIs is \(L_2 = n(7m + 31)\), and the numerical complexity is proportional to \(L_2D_2^2\). Therefore, Theorem 1 has much smaller numerical complexity than Theorem 2 in [30]. Since no free weighting matrix has been introduced in the presented method, Theorem 1 involves less number of decision variables as compared to Theorem 3.1 in [29] (see Table 1). Thus, in general, Theorem 1 is numerically less complex than Theorem 3.1 in [29].

\[\begin{array}{l}
\text{Table 1: Comparison of the number of decision variables involved in various methods.} \\
\hline
\text{Methods} & \text{Number of the decision variables} \\
\hline
\text{Theorem 1 in [19]} & (23n^2 + 5n)/2 \\
\text{Theorem 3 in [19]} & (67n^2 + 9n)/2 \\
\text{Theorem 1 in [23]} & 9n^2 + 3n \\
\text{Theorem 1 in [18]} & 13n^2 + 3n \\
\text{Proposition 2 in [6]} & 3n^2 + 3n \\
\text{Theorem 1 in [7]} & 3n^2 + 3n \\
\text{Theorem 2 in [30]} & (n/2)[n(3m^2 + 18m + 41) + (3m + 11)] \\
\text{Theorem 3.1 in [29]} & (n/2)[n(m^2 + 4m + 19) + (m + 5)] \\
\text{Theorem 1} & (n/2)[n(m^2 + 5) + (m + 5)] \\
\hline
\end{array}\]

Remark 7. Using similar steps as in the proof of Proposition 8 in [30], it is easy to establish that the conservatism of the stability result obtained via Theorem 1 is nonincreasing as the number of partitions increases.

4. Extensions to Uncertain State-Delayed Discrete-Time Systems

In this section, we extend the previously discussed approach to derive global asymptotically stable conditions for delayed discrete-time systems with norm-bounded uncertainties.

Consider the system given by

\[
x(k + 1) = (A + \Delta A)x(k) + (A_1 + \Delta A_1)x(k - d(k)).
\tag{27}
\]

The parameter uncertainties \(\Delta A\) and \(\Delta A_1\) are assumed to be norm-bounded and of the following form:

\[
[\Delta A \quad \Delta A_1] = HF\begin{bmatrix} E_0 & E_1 \end{bmatrix},
\tag{28}
\]

where \(H \in \mathbb{R}^{m \times q}\) and \(E_i \in \mathbb{R}^{p \times q}\) \((i = 0, 1)\) are known constant matrices and \(F \in \mathbb{R}^{N \times p}\) is an unknown matrix which satisfies

\[
F^TF \leq I.
\tag{29}
\]

The parameter uncertainties \(\Delta A\) and \(\Delta A_1\) are said to be admissible if both (28) and (29) are satisfied.

The following lemma is needed in the proof of our next result.

Lemma 8 (see [5, 25]). Let \(\Sigma, \Gamma, F,\) and \(M\) be the real matrices of appropriate dimensions with \(M\) satisfying \(M = M^T\); then

\[
M + \Sigma \Gamma^T + \Gamma^T F^T \Sigma^T < 0,
\tag{30}
\]

for all \(F^TF \leq I\), if and only if there exists a scalar \(\epsilon > 0\) such that

\[
M + \epsilon^{-1} \Sigma \Sigma^T + \epsilon \Gamma^T \Gamma < 0.
\tag{31}
\]

Theorem 9. For given scalars \(h_1\) and \(h_2\), system described by (27)–(29) and (2)–(4) is globally asymptotically stable for all the admissible uncertainties, if there exist real matrices
Table 2: Comparison of the maximum values of $h_2$ for given $h_1$.

| Methods            | $h_1 = 4$ | $h_1 = 6$ | $h_1 = 10$ | $h_1 = 12$ | $h_1 = 20$ |
|--------------------|-----------|-----------|------------|------------|------------|
| Theorem 1 in [20]  | 8         | 9         | 12         | 13         | 20         |
| Theorem 1 in [19]  | 13        | 14        | 15         | 16         | 22         |
| Proposition 2 in [6]| 15        | 16        | 18         | 19         | 25         |
| Theorem 1 in [7]   | 15        | 16        | 18         | 19         | 25         |
| Theorem 1          | 15 ($m = 2$, $\tau = 4$) | 16 ($m = 1$, $\tau = 6$) | 18 ($m = 1$, $\tau = 10$) | 19 ($m = 1$, $\tau = 12$) | 25 ($m = 1$, $\tau = 20$) |
|                    | 15 ($m = 2$, $\tau = 2$) | 16 ($m = 2$, $\tau = 3$) | 18 ($m = 2$, $\tau = 5$) | 20 ($m = 2$, $\tau = 6$) | 26 ($m = 2$, $\tau = 10$) |

Table 3: Comparison of upper bound $\bar{z}$ for given $h_1$ and $h_2$.

| Methods            | $2 \leq d(k) \leq 7$ | $3 \leq d(k) \leq 9$ | $5 \leq d(k) \leq 10$ | $6 \leq d(k) \leq 12$ | $10 \leq d(k) \leq 15$ | $20 \leq d(k) \leq 26$ |
|--------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| Theorem 5 in [20]  | 0.0830                 | Infeasible             | Infeasible             | Infeasible             | Infeasible             | Infeasible             |
| Corollary 2 in [19]| 0.1901                 | 0.1457                 | 0.1313                 | 0.0906                 | 0.0655                 | Infeasible             |
| Theorem 4 in [7]   | 0.1920                 | 0.1548                 | 0.1425                 | 0.1146                 | 0.1023                 | Infeasible             |
| Theorem 9          | 0.1927 ($m = 2$, $\tau = 1$) | 0.1560 ($m = 3$, $\tau = 1$) | 0.1476 ($m = 5$, $\tau = 1$) | 0.1194 ($m = 2$, $\tau = 3$) | 0.1080 ($m = 2$, $\tau = 5$) | 0.0837 ($m = 5$, $\tau = 4$) |

$P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1, 2, 3)$, and $Z_i = Z_i^T > 0$ $(i = 1, 2)$ and a scalar $\varepsilon > 0$ such that the following LMIs hold:

$$
\begin{align*}
\mathbf{\Psi}_1 &= \mathbf{\Psi} - \Lambda_7^T Z_7 \Lambda_7, \\
\mathbf{\Psi}_2 &= \mathbf{\Psi} - \Lambda_8^T Z_8 \Lambda_8, \\
\mathbf{\Psi} &= \Lambda_1^T \Phi_1 \Lambda_1 + \Lambda_2^T \Phi_2 \Lambda_2 + \Lambda_3^T \Phi_3 \Lambda_3 - \Lambda_4 \Phi_4 \Lambda_4 - \Lambda_5 \Phi_5 \Lambda_5 + \Lambda_7^T Z_7 \Lambda_7 + \Lambda_8^T Z_8 \Lambda_8 + \Lambda_9^T Z_9 \Lambda_9 + \Lambda_1^T Z_1 \Lambda_1, \\
\Phi_1 &= -P + Q_2 + (h_2 - \tau m + 1) Q_3 - Z_1,
\end{align*}
$$

where

$$
\begin{align*}
\Lambda_7 &= \begin{bmatrix} I_n & 0_n & 0_n \end{bmatrix}, \\
\Lambda_8 &= \begin{bmatrix} I_n \end{bmatrix} - I_n, \\
\Phi_1 &= -P + Q_2 + (h_2 - \tau m + 1) Q_3 - Z_1,
\end{align*}
$$

$$
\begin{align*}
\mathbf{\Psi}_1 &= \begin{bmatrix} Q_1 & 0 \\ 0 & -Q_1 \end{bmatrix}, \\
\Phi_2 &= \begin{bmatrix} Q_1 & 0 \\ 0 & -Q_1 \end{bmatrix}, \\
\mathbf{\Phi}_3 &= -Q_3 - 2Z_2, \\
\mathbf{\Phi}_4 &= Q_2 + Z_2.
\end{align*}
$$

Proof. Applying Theorem 1, the sufficient conditions for the global asymptotic stability of system described by (27)–(29) and (2)–(4) are obtained as follows:

$$
\begin{align*}
\mathbf{\Psi}_1 &= \mathbf{\Psi} - \Lambda_1 \mathbf{\Phi}_1 \Lambda_1 + \Lambda_2 \mathbf{\Phi}_2 \Lambda_2 + \Lambda_3 \mathbf{\Phi}_3 \Lambda_3 - \Lambda_4 \mathbf{\Phi}_4 \Lambda_4 - \Lambda_5 \mathbf{\Phi}_5 \Lambda_5 + \Lambda_7 \mathbf{\Phi}_7 \Lambda_7 + \Lambda_8 \mathbf{\Phi}_8 \Lambda_8 + \Lambda_9 \mathbf{\Phi}_9 \Lambda_9 + \Lambda_1 \mathbf{\Phi}_1 \Lambda_1, \\
\mathbf{\Phi}_1 &= -P + Q_2 + (h_2 - \tau m + 1) Q_3 - Z_1,
\end{align*}
$$

and $\Lambda_i$ $(i = 1, \ldots, 6)$ in (35) are given by (7)–(9).
Example 2 (see [7, 19]). Consider the following delayed discrete-time system with parameter uncertainty:

\[
\begin{bmatrix}
\begin{array}{c}
\mathbf{x}(k+1) = \\
\alpha(k) + \mathbf{H}^T \mathbf{P} \mathbf{H}^T \mathbf{Z} \\
\end{array}
\end{bmatrix} < 0.
\]

(41)

where \(\mathbf{x}(k) = [x_1(k), x_2(k)]^T\) and \(|\alpha(k)| \leq \bar{\alpha}\). System (44) can be represented in the form of (27) with

\[
A = \begin{bmatrix}
0.8 & 0 \\
0 & 0.9
\end{bmatrix}, \quad A_1 = \begin{bmatrix}
-0.1 & 0 \\
-0.1 & -0.1
\end{bmatrix}, \quad H = \begin{bmatrix}
\bar{\alpha}& \\
0
\end{bmatrix},
\]

\[
E_0 = \begin{bmatrix}
1 & 0
\end{bmatrix}, \quad E_1 = \begin{bmatrix}
0 & 0
\end{bmatrix}, \quad F = \frac{\alpha(k)}{\bar{\alpha}}.
\]

(45)

For given \(h_1\) and \(h_2\), we wish to find \(\bar{\alpha}\) such that the uncertain system (44) is asymptotically stable for any \(|\alpha(k)| \leq \bar{\alpha}\). Table 3 presents a comparison of the values of \(\bar{\alpha}\) obtained by various methods. From Table 3, it is clear that Theorem 9 can provide less conservative results than [7, 19, 20]. In the situation where \(20 \leq d(k) \leq 26\), the stability criteria in [7, 19, 20] are invalid whereas the value of \(\bar{\alpha}\) obtained by Theorem 9 is 0.0837 for \(m = 5\), \(\tau = 4\).

6. Conclusion

In this paper, we have considered the problem of global asymptotic stability of linear discrete-time systems with interval-like time-varying delay in the state. By utilizing the concept of delay partitioning, an LMI-based criterion for the global asymptotic stability of such systems has been established. The proposed criterion depends on both the size of delay and partition size. With the help of numerical examples, it has been illustrated that the proposed stability condition may provide less conservative result than most of the existing results [6, 7, 19, 20] due to the delay partitioning, and it may become even less conservative when the partitioning goes finer. Since no free weighting matrix has been introduced in the presented method, Theorem 1 involves less number of decision variables as compared to Theorem 3.1 in [29].
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