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Effect of vaccination on the transmission dynamics of COVID-19 in Ethiopia

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A B S T R A C T

Introduction

Coronavirus disease 2019 (COVID-19) is a major public health concern worldwide that has harmed and killed millions of people around the world. It contributes to several political, economic, and social issues. In December 2019, Wuhan, China, reported an unknown-cause viral pneumonia, marking the beginning of a new Coronavirus outbreak [1,2]. The World Health Organization (WHO) declared the disease a pandemic [3] on March 11, 2020. Because of its similarities to SARS-CoV, the virus was given the name Severe Acute Respiratory Syndrome Coronavirus 2. The disease caused by this virus is known as COVID-19. The virus is primarily spread from person to person through contact with the droplets of an infected individual. Droplets are released into the air when an infected person coughs, sneezes, or exhales, and can land in another person’s nose or mouth, where they can be inhaled into the lungs. Someone who is infected yet has no symptoms of disease can spread a virus (asymptomatically infected). This is why keeping a distance of at least 2 meters (6 ft) is necessary. Infected droplets could fall to the ground or settle on objects. A person can contract the virus by touching his or her lips, nose, or eyes. Fever, dry cough, and shortness of breath or difficulty breathing are the most common symptoms. Muscle and joint pain, headaches, sore throats, and a loss of taste or smell are all symptoms.

Social distance, minimizing non-essential trips, case isolation, family quarantine, and school and university shutdowns are all being done to limit the growth of this deadly disease [4–7]. In Ethiopia, the prevention and spread control of COVID-19 is complicated by such a number of variables. The challenges of screening infected individuals, the responsible groups’ weak responses to the outbreak, social conflicts, insufficient human resources in health centers, a lack of active outbreak response, a lack of community awareness, cultural effects, and virus transmission methods are among them. All of these variables allow infectious and noninfectious people to come into contact.

Ethiopia’s Ministry of Health announced the introduction of the COVID-19 vaccination at a high-level national event where frontline health workers were vaccinated to kick off the vaccine campaign. The first shipment of AstraZeneca vaccines from the Serum Institute of India (SII) arrived in Ethiopia on March 6, 2021, thanks to COVAX, which expedited vaccine purchase and delivery. According to the national development and vaccination plan (NDVP) prepared in accordance with the WHO prioritization roadmap, frontline health workers and support personnel, the elderly with serious illnesses, and other high-risk populations would be prioritized for vaccination. By the end of 2021, Ethiopia hopes to have vaccinated 20% of its population [8].

Compartmental dynamic models in epidemiology are viewed as a group of ordinary differential equations and parameters that track the temporal evolution of the number of people in each of the system’s phases. Throughout the last decade, the fractional-order derivative (FOD), which is defined as an extension of the integer derivative to a non-integer order (arbitrary order) operator, has been used to model a range of memory and latency phenomena, including epidemic behavior [9]. In this paper, we investigated mathematical model for COVID-19 that might be used to investigate Coronavirus transmission dynamics with vaccination in Ethiopia. The dynamical transmission mechanism of new Coronavirus is studied using Atangana–Baleanu fractional derivative in the Caputo sense.

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Atangana-Baleanu fractional derivative

**Definition 1.** Let a function \( y(t) \in H^1(0, b) \), \( b > a \). The Atangana-Baleanu fractional derivative in Caputo sense of the order \( \delta \) of \( y(t) \) is given by [10] as:

\[
\frac{AB}{a}D^\delta y(t) = \frac{1 - \delta}{\Gamma(1 - \delta)} \int_a^t (t - \tau)^{-\delta} \frac{\partial}{\partial \tau} y(\tau) d\tau.
\]

(1)

where \( \Gamma(\delta) = \int_0^{\infty} e^{-\xi} \xi^{\delta-1} d\xi \), \( \delta > 0 \), is the one parameter Mittag-Leffler function and \( H^1(0, b) \), \( b > a \) is called the Sobolev space of order one is defined as

\[
H^1(a, b) = \{ y \in L^2(a, b) | y' \in L^2(a, b) \}.
\]

From equation (1), \( \frac{AB}{a}D^\delta y(t) = 0 \) (\( y \) is constant).

**Definition 2.** The AB fractional integral of the function \( y(t) \in H^1(a, b) \), \( b > a \) is given by [10] as:

\[
\frac{AB}{a}I^\delta y(t) = \frac{1 - \delta}{\Gamma(1 - \delta)} \int_a^t (t - \tau)^{\delta-1} y(\tau) d\tau.
\]

(3)

\( \frac{AB}{a}D^\delta y(t) \) is continuous and locally Lipschitz function property. \( \frac{AB}{a}D^\delta \) is a linear operator, and it may be generated as

\[
\frac{AB}{a}D^\delta y(t) = \frac{1 - \delta}{\Gamma(1 - \delta)} \int_a^t (t - \tau)^{\delta-1} y(\tau) d\tau.
\]

(4)

\( \frac{AB}{a}D^\delta y(t) \) is Lipschitz condition:

\[
\| \frac{AB}{a}D^\delta y(t) - \frac{AB}{a}D^\delta y(t) \| \leq \epsilon \| y(t) - y(t) \|.
\]

(5)

where \( \epsilon \) is Lipschitz constant\( ||y(t)|| \leq \max_{0 \leq t \leq T} ||y(t)|| \).

The Laplace transform of AB fractional derivative is [11]

\[
\mathcal{L} \left( \frac{AB}{a}D^\delta f(t) \right)(s) \left| \left( \frac{AB}{a}D^\delta f(t) \right)(s) = \frac{1}{1 - \delta} \frac{\delta}{\Gamma(1 - \delta)} (\delta - 1) f(t) \right.
\]

(6)

**Jacobi Polynomials:**

The Jacobi polynomials \( J_n^{\alpha, \beta}(x) \) of degree \( n \) defined on \([-1, 1]\) can be generated by the recurrence relation [12]:

\[
J_n^{\alpha, \beta}(x) = (a_n x - c_n)J_{n-1}^{\alpha, \beta}(x) - w_n J_{n-2}^{\alpha, \beta}(x), \quad n = 1, 2, 3, ...
\]

(7)

with beginning values \( J_0^{\alpha, \beta}(x) = 1 \) and \( J_1^{\alpha, \beta}(x) = \frac{x - \delta}{\alpha + \beta + 2} \), where

\[
a_n = \frac{(2n + \alpha + \beta + 1)(2n + \alpha + \beta + 2)}{(2n + \alpha + \beta + 1)(2n + \alpha + \beta + 2)}
\]

\[
c_n = \frac{(2n + \alpha + \beta + 1)(2n + \alpha + \beta + 2)}{(2n + \alpha + \beta + 1)(2n + \alpha + \beta + 2)}
\]

\[
w_n = \frac{(2n + \alpha + \beta + 1)(2n + \alpha + \beta + 2)}{(2n + \alpha + \beta + 1)(2n + \alpha + \beta + 2)}
\]

Introducing the variable \( x = 2t-1 \) to defining the so-called shifted Jacobi polynomial (SJP) on the interval \( x \in [0, 1] \). Let \( P_n^{\alpha, \beta}(t) \) denote the shifted Jacobi polynomials \( J_n^{\alpha, \beta}(2t-1) \), and it may be follows as:

\[
P_n^{\alpha, \beta}(t) = (r_n - s_n)P_n^{\alpha, \beta}(t) - a_n P_{n-1}^{\alpha, \beta}(t), \quad n = 1, 2, 3, ...
\]

(8)

with beginning values \( P_0^{\alpha, \beta}(t) = 1 \) and \( P_1^{\alpha, \beta}(t) = \frac{\delta + 2}{2} (2t - 1) + \frac{\delta - \xi}{2} \) where

\[
r_n = \frac{(2n + \alpha + \beta + 1)(2n + \alpha + \beta + 2)}{(2n + \alpha + \beta + 1)(2n + \alpha + \beta + 2)}
\]

\[
s_n = \frac{(2n + \alpha + \beta + 1)(2n + \alpha + \beta + 2)}{(2n + \alpha + \beta + 1)(2n + \alpha + \beta + 2)}
\]

The analytical form of the SJP \( P_n^{\alpha, \beta}(t) \) of degree \( n \):

\[
P_n^{\alpha, \beta}(t) = \sum_{k=0}^{n} \binom{n}{k} \frac{\Gamma(n+1)}{\Gamma(n+k+1)} \frac{\Gamma(n+k+\alpha+\beta+1)}{\Gamma(n+k+\alpha+\beta+1)} t^k.
\]

(9)

A square integrable function \( y(t) \) on \([0, 1]\) can be expressed as follows in terms of the SJP:

\[
y(t) = \sum_{i=0}^{n} a_i P_i^{\alpha, \beta}(t)
\]

(10)

The coefficients \( a_i \) are given by

\[
a_i = (2i+1) \int_0^1 y(t) P_i^{\alpha, \beta}(t) dt, \quad i = 1, 2, 3, ...
\]

(11)

The first \( m + 1 \) terms of shifted Jacobi polynomials are considered in practice, then we have

\[
y_m(t) = \sum_{i=0}^{m} a_i P_i^{\alpha, \beta}(t)
\]

(12)

**Lemma 1.** Let \( y_m(t) \) is an approximate function in terms of the SJP given by in Eq. (12). Suppose that \( \delta \in (0, 1) \) then, we obtain:

\[
\frac{AB}{a}D^\delta y_m(t) \approx \sum_{i=1}^{m} \sum_{j=1}^{m} a_i a_j \psi_{i,j} A'_i(t),
\]

(13)

where

\[
\psi_{i,j} = (1-\delta) \left( \frac{\Gamma(i + \epsilon + \beta + 1) \Gamma(i + j + \epsilon + \beta + 1)}{\Gamma(i + \epsilon + \beta + 1) \Gamma(i + j + \epsilon + \beta + 1)} \right) n^{\epsilon}(\epsilon + 1) (1 - r).
\]

Material and methods

In this paper, we divided Ethiopia’s total population into nine groups: susceptible-S (individuals who are free to the disease can become infected by coming into touch with infected people), vaccinated-V (individuals who have received the COVID-19 vaccine), exposed-E (individuals who have been exposed to the infection but have not yet become infected and cannot transmit the infection to susceptible individuals), asymptomatic infectious-A (having COVID-19 but showing no symptoms at all), symptomatic infectious-I (individuals who are infected with the disease are capable of transmitting the infection to uninfected individuals), quarantined-Q (to limit Coronavirus transmission, infected individuals are quarantined in a certain place and are isolated and cared for at home), hospitalized-H (infected people are being treated in hospitals), recovered-R (those who have recovered from COVID-19), and death-D (COVID-19 caused individuals to die). The whole population in this model defined as

\[
T(t) = S(t) + V(t) + E(t) + A(t) + I(t) + Q(t) + H(t) + R(t) + D(t)
\]

(14)

The model is based on the following assumptions:

(1) All individuals in the population are initially considered as susceptible. i.e. \( S(0) = T(0) \).

(2) People who have been vaccinated may become infected.

(3) Positive values are assumed for all model parameters.

(4) Susceptible individuals have an equal chance of being infected.

(5) Recovered individuals may not go back to the susceptible class (no reinfection).

(6) Individuals become infectious after they have been exposed.

(7) Vertical transmission (mother to her unborn baby) is not considered.

(8) All cases of infection are considered to be among humans.

All new recruited individuals are assumed to be susceptible and are recruited at a rate \( \mathcal{R} \). Susceptible people are vaccinated at a rate \( \mathcal{V} \).
r. Vaccinated people may become infected, but at lower rate than unvaccinated people, because some vaccine does not provide immunity to everyone who receive it http://www.who.ch/. The effective contact rate $\beta$ is multiplied by a factor $\rho(0 \leq \rho \leq 1)$, where $1 - \rho$ describes the vaccine efficiency, $\rho = 0$ represent the vaccine that provide perfect effective protection against COVID-19 infection, while $\rho = 1$ the vaccine provide no protection at all.

The parameter $\sigma$ is the rate at which the exposed individuals become infectious with symptoms of the coronavirus, and $\sigma(1 - \sigma)$ is the rate that the exposed individuals become asymptomatic infectious. The parameter $\eta$ and $\psi$ are the rate constants that the symptomatic and asymptomatic individuals enter to the hospital for more treatment. The home-based isolation program is one of the controlling strategies for the spread of the pandemic. The parameter $\phi(1 - \eta)$ is the rate at which symptomatic infected individuals transfer to the home-based isolation and individuals transfer from HBIC to the treatment center at the rate $\delta$ for better care. Individuals may transfer from treatment centers to HBIC care after improvement at rate $\gamma$ where $\lambda = \frac{\beta(\nu I + A)}{T}$, and $\beta = \frac{\phi(\nu I + A)}{T}$.

Atangana–Baleanu fractional order derivative in Caputo sense is used to described the dynamics of these population as follow:

$$\begin{align*}
ABC D_t^\alpha S(t) &= (\lambda + \tau + \nu) S
ABC D_t^\alpha V(t) &= \lambda S + \delta V - (\sigma + \sigma(1 - \sigma) + \nu) E
ABC D_t^\alpha E(t) &= \sigma(1 - \sigma) E - (\psi + \zeta(1 - \psi) + a + \nu) A
ABC D_t^\alpha A(t) &= \sigma(1 - \sigma) E - (\psi + \zeta(1 - \psi) + a + \nu) A
ABC D_t^\alpha I(t) &= \gamma E - (\eta + \phi(1 - \eta) + b + \nu) I
ABC D_t^\alpha T(t) &= \psi(1 - \eta) I + \gamma H - (\delta + \zeta(1 - \delta) + c + \nu) Q
ABC D_t^\alpha Q(t) &= \psi A + \eta I + \delta Q - (\gamma + \omega(1 - \gamma) + d + \nu) H
ABC D_t^\alpha R(t) &= \zeta(1 - \psi) A + \zeta(1 - \delta) Q + \omega(1 - \gamma) H - \nu R
ABC D_t^\alpha H(t) &= a A + b I + c Q + d H
\end{align*}$$

with positive starting conditions

$$S(0), V(0), E(0), A(0), I(0), Q(0), H(0), R(0), D(0)$$.

The proposed model Eq. (15) in the following form:

$$\begin{cases}
ABC D_t^\alpha S(t) = Y_1(t, S, V, E, A, I, Q, H, R, D), \\
ABC D_t^\alpha V(t) = Y_2(t, S, V, E, A, I, Q, H, R, D), \\
ABC D_t^\alpha E(t) = Y_3(t, S, V, E, A, I, Q, H, R, D), \\
ABC D_t^\alpha A(t) = Y_4(t, S, V, E, A, I, Q, H, R, D), \\
ABC D_t^\alpha I(t) = Y_5(t, S, V, E, A, I, Q, H, R, D), \\
ABC D_t^\alpha T(t) = Y_6(t, S, V, E, A, I, Q, H, R, D), \\
ABC D_t^\alpha Q(t) = Y_7(t, S, V, E, A, I, Q, H, R, D), \\
ABC D_t^\alpha H(t) = Y_8(t, S, V, E, A, I, Q, H, R, D), \\
ABC D_t^\alpha R(t) = Y_9(t, S, V, E, A, I, Q, H, R, D), \\
ABC D_t^\alpha D(t) = Y_{10}(t, S, V, E, A, I, Q, H, R, D),
\end{cases}$$

where

$$\begin{align*}
Y_1 &= 3 - (\lambda + \tau + \nu) S \\
Y_2 &= \lambda S - (\delta + \nu) V \\
Y_3 &= \sigma(1 - \sigma) E - (\psi + \zeta(1 - \psi) + a + \nu) A \\
Y_4 &= \sigma(1 - \sigma) E - (\psi + \zeta(1 - \psi) + a + \nu) A \\
Y_5 &= \gamma E - (\eta + \phi(1 - \eta) + b + \nu) I \\
Y_6 &= \omega(1 - \psi) A + \zeta(1 - \delta) Q + \omega(1 - \gamma) H - \nu R
\end{align*}$$

Thus, the model Eq. (17) takes the form

$$\begin{cases}
ABC D_t^\alpha \Omega(t) = H(t, \Omega(t)), \\
\Omega(0) = \Omega_0 \geq 0
\end{cases}$$

on condition that

$$\begin{align*}
\Omega(t) &= (t, S, V, E, A, I, Q, H, R, D)^T, \\
\Omega(0) &= (t, S_0, V_0, E_0, A_0, I_0, Q_0, H_0, R_0, D_0)^T, \\
H(t, \Omega(t)) &= (Y_1(t, S, V, E, A, I, Q, H, R, D))^T, \\
i &= 1, 2, \ldots, 9
\end{align*}$$

Applying the Atangana–Baleanu fractional integral to model Eq. (19) which is equivalent to model Eq. (15), we obtain

$$\Omega(t) - \Omega(0) = \frac{1}{N(\alpha)} H(t, \Omega(t)) + \frac{\alpha}{N(\alpha)Gamma(\alpha)} \int_0^t H(\xi, \Omega(t))(t - \xi)^{\alpha-1}d\xi$$

Let $S = C([0, s]; R)$ denote the Banach space of all continuous functions from $[0, s]$ to $\mathbb{R}$ endowed with the norm defined by $\|\Omega\|_S = \sup_{t \in T} |\Omega(t)|$. 

![Image: COVID-19 transmission flow of the proposed model.](image-url)
where
\[ |\Delta(t)| = |S(t)| + |V(t)| + |E(t)| + |A(t)| + |I(t)| + |Q(t)| + |H(t)| + |R(t)| + |D(t)| \]
and \( S, V, E, A, I, Q, H, R, D \in \{0, 1\}, R \).

where
\[
\|S\| = \sup_{t \in T} |S(t)|, \quad \|V\| = \sup_{t \in T} |V(t)|, \quad \|E\| = \sup_{t \in T} |E(t)|, \\
\|A\| = \sup_{t \in T} |A(t)|, \quad \|I\| = \sup_{t \in T} |I(t)|, \\
\|Q\| = \sup_{t \in T} |Q(t)|, \quad \|H\| = \sup_{t \in T} |H(t)|, \quad \|R\| = \sup_{t \in T} |R(t)|, \quad \|D\| = \sup_{t \in T} |D(t)|.
\]

Assume that \( S(t), V(t), E(t), A(t), I(t), Q(t), H(t), R(t), D(t) \) are contractions for \( 0 \leq \delta_1 < 1 \), \( i = 1, 2, \ldots, 9 \).

Proof. For the state variable \( S \), we have
\[
\|Y_i(t, S) - Y_i(t, S_i)\| \leq \delta_1 \|S - S_i\| \quad (i = 1, 2, \ldots, 9)
\]
where \( \delta_1 = \frac{\beta}{\gamma + \delta} \).

So, \( Y_i(t, S) \) satisfies the Lipschitz condition with Lipschitz constant \( \delta_1 \). Moreover, if \( 0 \leq \delta_1 < 1 \), then \( Y_i(t, S) \) is a contraction. In the similar way, we can show the existence of Lipschitz constants \( \delta_i \), \( i = 2, 3, \ldots, 9 \) and a contraction principle for \( Y_2(t, V), Y_3(t, E), Y_4(t, A), Y_4(t, I), Y_5(t, Q), Y_6(t, H), Y_7(t, R), Y_8(t, D) \).

Repeating the process recursively leads to
\[
\|Y_n(t, S) - Y_n(t, S_i)\| \leq \delta_1 \|S - S_i\|.
\]

Applying the limit to both sides of Eq. (33) as \( n \to \infty \) we see that \( \|Y_n(t)\| \to 0 \) for
\[
\left( \frac{1 - a}{N(a)} + \frac{\gamma a}{N(a)\Gamma(a + 1)} \right) \delta_1 < 1.
\]

Theorems 1 and 2 give the existence of the solution of model Eq. (15). Let us assume that the following is satisfied:
\[
w(t) - w(0) = w_n(t) - q_n(t)
\]
From Eq. (30) we obtain
\[
\|q_n(t)\| \leq \left( \frac{1 - a}{N(a)} + \frac{\gamma a}{N(a)\Gamma(a + 1)} \right)^{n+1} \delta_1 \|w_n - w_{n-1}\|.
\]

Taking \( t = \gamma \) yields
\[
\|q_n(t)\| \leq \left( \frac{1 - a}{N(a)} + \frac{\gamma a}{N(a)\Gamma(a + 1)} \right)^{n+1} \delta_1 \|w_n - w_{n-1}\|.
\]

Applying the limit to both sides of Eq. (33) as \( n \to \infty \) we see that
\[
\left( \frac{1 - a}{N(a)} + \frac{\gamma a}{N(a)\Gamma(a + 1)} \right) \delta_1 < 1.
\]

Theorem 3. The ABC fractional model Eq. (15) has a unique solution, provided that
\[
\left( \frac{1 - a}{N(a)} + \frac{\gamma a}{N(a)\Gamma(a + 1)} \right) \delta_1 < 1, \quad i = 2, 3, \ldots, 9.
\]

Proof. Let us assume that \( S(t), V(t), E(t), A(t), I(t), Q(t), H(t), R(t), D(t) \) are solutions to Eq. (15). Then
\[
S(t) - S_i(t) = \frac{1 - a}{N(a)} \int_{t}^{r} \left( Y_i(\xi, S) - Y_i(\xi, S_i) \right) d\xi
\]
Taking the norm both sides, we get
\[
\|S(t) - S_i(t)\| \leq \frac{1 - a}{N(a)} \int_{t}^{r} \left( |Y_i(\xi, S) - Y_i(\xi, S_i)| \right) d\xi.
\]

As a result, we have
\[
\|S(t) - S_i(t)\| \leq \frac{1 - a}{N(a)} \int_{t}^{r} \left( |S(t) - S_i(t)| \right) d\xi.
\]

and is contraction for \( 0 \leq \delta_1 < 1 \). The remaining expressions can be reduced to the following expressions:
\[
\|y_{i}(t) - y_{i-1}(t)\| \leq \frac{\delta_1}{N(a)} \left| \gamma \right| \|y_{i-1}(t) - y_{i-2}(t)\|.
\]

\( y = \{V, E, A, I, Q, H, R, D \} \) and are contractions for \( 0 \leq \delta_1 < 1 \), \( i = 2, 3, \ldots, 9 \).
with positive starting conditions is positive invariant for the system Eq. (15).

Proof. Adding each equation of the model Eq. (15), we get
\[ A^0 D^0 T(t) \leq \mathcal{S} - zT. \] 
(37)

Applying the Laplace transform on Eq. (37) and then taking the inverse Laplace transform, we arrive at
\[ T(t) \leq \frac{\mathcal{N}(a)}{\mathcal{N}(a) + (1-a)\alpha} \mathcal{S}_0 + \frac{(1-a)\mathcal{S}}{\mathcal{N}(a) + (1-a)\alpha} E_{a,1}(-z^m) + \frac{a\mathcal{S}}{\mathcal{N}(a) + (1-a)\alpha} E_{a+1}(-z^m) \] 
(38)

where \( z = \frac{\alpha}{\mathcal{N}(a) + (1-a)\alpha} \).

Following the work in [15], we observe that \( T(t) \leq \frac{\mathcal{S}}{P} \) as \( t \to \infty \). Hence \( \Phi \) is the biologically feasible region of the model Eq. (15).

Theorem 4. If the starting solutions satisfy \( S(0) \geq 0, V(0) \geq 0, E(0) \geq 0, I(0) \geq 0, Q(0) \geq 0, H(0) \geq 0, R(0) \geq 0, \) and \( D(0) \geq 0, \) then the solutions \( S(t), V(t), E(t), A(t), I(t), Q(t), H(t), R(t), D(t) \) of the system Eq. (15) are positive for all \( t \geq 0 \).

Proof. From the first equation of Eq. (15),
\[ A^0 D^0 S(t) = \mathcal{S} - (\lambda + r + \psi) S \geq -(\lambda + r + \psi) S \] 
(39)

where \( \lambda = \frac{\beta I}{\tau + \nu} \).

Following Lemma 2, \( \nu I + A \) is bounded by a constant \( \lambda_0 \), we have
\[ A^0 D^0 S(t) \geq -\rho S(t) \] 
(40)

where \( \rho = \frac{\beta I_0}{\tau + \nu} \).

Applying the Laplace transform in the previous inequality (40), we arrive at
\[ S(t) \geq \frac{\mathcal{N}(a)}{\mathcal{N}(a) + (1-a)\rho} S(0) E_{a,1} \left( -\frac{a\rho}{\mathcal{N}(a) + (1-a)\rho} z^m \right) \] 
(41)

Since \( E_{a,1} \left( -\frac{a\rho}{\mathcal{N}(a) + (1-a)\rho} z^m \right) > 0 \) and \( S(0) \geq 0 \), then the solution \( S(t) \geq 0 \). In the same way, from the remaining equations of (15), it is easy to prove that \( V(t), E(t), A(t), I(t), Q(t), H(t), R(t) \) and \( D(t) \) are positive for all \( t \geq 0 \).

The disease-free equilibrium point (dfe) of the model Eq. (15) is:
\[ \mathcal{Z}_0 = \left( \begin{array}{cccc} \frac{\mathcal{S}}{\tau + \nu} & 0 & 0 & 0 \\ \frac{\mathcal{S}}{\tau + \nu} & 0 & 0 & 0 \\ \frac{\mathcal{S}}{\tau + \nu} & 0 & 0 & 0 \\ \frac{\mathcal{S}}{\tau + \nu} & 0 & 0 & 0 \end{array} \right) \] 
(42)

The transmission matrix at \( \mathcal{Z}_0 \)
\[ S = \begin{pmatrix} 0 & x + y & xv + yv & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \]

and the transition matrix at \( \mathcal{Z}_0 \)
\[ T = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ -\sigma(1-x) & m_2 & 0 & 0 \\ -\sigma & 0 & m_3 & 0 \\ 0 & 0 & -\phi(1-\eta) & m_4 - \gamma \end{pmatrix} \]

where
\[ m_1 = x + \sigma(1-x) + \nu, \quad m_2 = \psi + \zeta(1-\psi) + a + \nu, \]
\[ m_3 = \eta + \phi(1-\eta) + b + \nu, \]
\[ m_4 = \delta + \xi(1-\delta) + c + \nu, \quad m_5 = \gamma + \omega(1-\gamma) + d + \nu, \]
\[ x = \frac{\beta_0}{\tau + \nu}, \]
\[ y = \frac{\beta_0}{\tau + \nu}. \]

The required effective reproduction number is the dominant eigenvalue of the next-generation matrix \( ST^{-1} \).

\[ R_0 = \frac{x + y}{m_1} \left( \frac{\sigma(1-x) + \gamma}{m_2} \right) \] 
(43)

Theorem 5. The dfe point (42) is locally asymptotically stable if \( R_0 < 1 \) and unstable if \( R_0 > 1 \).

Proof. We linearize the system of Eqs. (15) as follows:
\[ J_0 = \begin{pmatrix} -\tau - \nu & 0 & 0 & -x & -xv & 0 & 0 & 0 \\ -\tau & -\nu & 0 & -y & -yv & 0 & 0 & 0 \\ 0 & 0 & -m_1 & x + y & xv + yv & 0 & 0 & 0 \\ 0 & 0 & \sigma(1-x) & -m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \pi & -m_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi(1-\eta) & -m_4 & \gamma & 0 \\ 0 & 0 & 0 & 0 & \psi & \eta & \delta & -m_5 \\ 0 & 0 & 0 & \zeta(1-\psi) & 0 & \xi(1-\delta) & \omega(1-\gamma) & -\nu \end{pmatrix} \]
(44)

In this matrix, three of the eigenvalues are negative. That is \( s_1 = s_2 = -\nu, \ s_3 = -\tau - \nu \). The remaining eigenvalues can be obtained from the following characteristic equation:
\[ s^5 + d_1 s^4 + d_2 s^3 + d_3 s^2 + d_4 s + d_5 = 0 \]

where
\[ d_1 = m_1 + m_2 + m_3 + m_4 + m_5 \]
\[ d_2 = (m_1 + m_3 + m_4) m_4 + (x + y) \left( \frac{m_2 \psi}{m_3} + \frac{m_3 \sigma(1-x)}{m_2} \right) + (m_2 + m_3) m_1 (1 - R_0) - \gamma \delta (m_3 + m_4 + m_5) m_2 + (m_1 + m_3) m_4 \]
\[ d_3 = (m_4 + m_5) \] 
\[ + (m_2 + m_3 + m_4) (m_4 m_5 - \gamma \delta) + m_1 + m_2 + m_3 + (1 - R_0) \]
\[ d_4 = m_1 m_2 m_3 (m_4 + m_5) (1 - R_0) + m_1 + m_2 + (m_2 + m_3 + m_4 + m_5) m_4 (1 - R_0) - \gamma \delta \] 
\[ + (m_4 m_5 - \gamma \delta) m_2 m_3 + (x + y) \left( \frac{\gamma \delta (1-x) + \nu \phi}{m_2} \right) + m_1 \left( \frac{m_3 \sigma(1-x)}{m_2} + \frac{m_4 \psi}{m_3} \right) \]
\[ d_5 = m_1 m_2 m_3 (m_4 m_5 - \gamma \delta) (1 - R_0) \]

The coefficients \( d_2, d_3, d_4 \) and \( d_5 \) are positive when \( R_0 < 1 \), and the coefficient \( d_1 \) is positive. Further, the Routh–Hurwitz criteria for fifth-order polynomials are \( d_k > 0, k = 1, 2, 3, 4, 5 \), \( d_1 d_2 d_3 d_4 + d_1 d_2 d_5 + d_1 d_3 d_5 + d_1 d_4 d_5 + d_2 d_3 d_4 + d_2 d_3 d_5 + d_2 d_4 d_5 + d_3 d_4 d_5 \) can be easily satisfied by using the above coefficients. So, \( \mathcal{Z}_0 \) is locally asymptotically stable if \( R_0 < 1 \) and unstable if \( R_0 > 1 \).

To determine the endemic equilibrium point by adjusting the model equations in (15) to zero and solve simultaneously, we get the endemic equilibrium point \( \mathcal{X} = (S^*, V^*, E^*, A^*, I^*, Q^*, H^*, R^*) \).
where
\[
S^* = \frac{3}{\beta \sigma_m \sigma (1 - \pi) \xi \beta \sigma_m \sigma (1 - \pi)} \left( I^* + \tau + \nu \right)
\]
\[
V^* = \frac{m_i m_3 \xi \beta \sigma_m \sigma (1 - \pi)}{3 \xi \beta \sigma_m \sigma (1 - \pi)} \left( I^* + \tau + \nu \right)
\]

The Jacobi spectral collocation method

Approximate \( S(t), V(t), E(t), A(t), I(t), Q(t), H(t), R(t) \) and \( D(t) \) of the model Eq. (15) using shifted Jacobi polynomials with \( m = 3 \) and fractional order \( 0 < \alpha \leq 1 \) as:

\[
S(t) = \sum_{i=0}^{3} a_i P_i^{(1-\alpha)}(t), \quad V(t) = \sum_{i=0}^{3} b_i P_i^{(1-\alpha)}(t), \quad E(t) = \sum_{i=0}^{3} c_i P_i^{(1-\alpha)}(t),
\]
\[
A(t) = \sum_{i=0}^{3} d_i P_i^{(1-\alpha)}(t), \quad I(t) = \sum_{i=0}^{3} e_i P_i^{(1-\alpha)}(t), \quad Q(t) = \sum_{i=0}^{3} f_i P_i^{(1-\alpha)}(t),
\]
\[
H(t) = \sum_{i=0}^{3} g_i P_i^{(1-\alpha)}(t), \quad R(t) = \sum_{i=0}^{3} h_i P_i^{(1-\alpha)}(t), \quad D(t) = \sum_{i=0}^{3} i_i P_i^{(1-\alpha)}(t).
\]

Now, using the idea of Lemma 1 and Eq. (47) on the model Eq. (15) we get:

\[
\sum_{i=0}^{3} \sum_{j=0}^{3} a_i b_j \psi_{i,x} \psi_{j,y}(t) = - 3 \left( \beta \sum_{i=0}^{3} b_j P_i^{(1-\alpha)}(t) + \tau + \nu \right) \sum_{i=0}^{3} a_i P_i^{(1-\alpha)}(t),
\]

\[
\sum_{i=0}^{3} \sum_{j=0}^{3} b_i c_j \psi_{i,x} \psi_{j,y}(t) = 3 \left( \beta \sum_{i=0}^{3} b_j P_i^{(1-\alpha)}(t) + \tau + \nu \right) \sum_{i=0}^{3} a_i P_i^{(1-\alpha)}(t),
\]

\[
\sum_{i=0}^{3} \sum_{j=0}^{3} c_i d_j \psi_{i,x} \psi_{j,y}(t) = 3 \left( \beta \sum_{i=0}^{3} b_j P_i^{(1-\alpha)}(t) + \tau + \nu \right) \sum_{i=0}^{3} a_i P_i^{(1-\alpha)}(t),
\]

\[
\sum_{i=0}^{3} \sum_{j=0}^{3} e_i f_j \psi_{i,x} \psi_{j,y}(t) = 3 \left( \beta \sum_{i=0}^{3} b_j P_i^{(1-\alpha)}(t) + \tau + \nu \right) \sum_{i=0}^{3} a_i P_i^{(1-\alpha)}(t),
\]

\[
\sum_{i=0}^{3} \sum_{j=0}^{3} f_i g_j \psi_{i,x} \psi_{j,y}(t) = 3 \left( \beta \sum_{i=0}^{3} b_j P_i^{(1-\alpha)}(t) + \tau + \nu \right) \sum_{i=0}^{3} a_i P_i^{(1-\alpha)}(t),
\]

\[
\sum_{i=0}^{3} \sum_{j=0}^{3} g_i h_j \psi_{i,x} \psi_{j,y}(t) = 3 \left( \beta \sum_{i=0}^{3} b_j P_i^{(1-\alpha)}(t) + \tau + \nu \right) \sum_{i=0}^{3} a_i P_i^{(1-\alpha)}(t),
\]

\[
\sum_{i=0}^{3} \sum_{j=0}^{3} h_i i_j \psi_{i,x} \psi_{j,y}(t) = 3 \left( \beta \sum_{i=0}^{3} b_j P_i^{(1-\alpha)}(t) + \tau + \nu \right) \sum_{i=0}^{3} a_i P_i^{(1-\alpha)}(t),
\]
Fig. 2. Sensitivity of the fundamental reproduction number $R_0$.

Fig. 3. Impacts of basic reproductive number $R_0$ on the numerical solutions of asymptomatic and symptomatic infected cases.

We use roots of shifted Jacobi polynomials $P^\alpha,\beta_3(t)$ for suitable collocation points.

Also, by substituting Eq. (47) in the starting conditions Eq. (16) we can obtain nine equations:

\[
\begin{align*}
\sum_{i=0}^{3}(-1)^i a_i & = S(0), \\
\sum_{i=0}^{3}(-1)^i b_i & = V(0), \\
\sum_{i=0}^{3}(-1)^i c_i & = E(0), \\
\sum_{i=0}^{3}(-1)^i d_i & = A(0), \\
\sum_{i=0}^{3}(-1)^i e_i & = I(0), \\
\sum_{i=0}^{3}(-1)^i f_i & = Q(0), \\
\sum_{i=0}^{3}(-1)^i g_i & = H(0), \\
\sum_{i=0}^{3}(-1)^i h_i & = R(0), \\
\sum_{i=0}^{3}(-1)^i u_i & = D(0)
\end{align*}
\]

Then, we have 36 equations that can be solved using inexact Newton’s method, for the unknowns $a_i$, $b_i$, $c_i$, $d_i$, $e_i$, $f_i$, $g_i$, $h_i$, and $u_i$ for $i = 0, 1, 2, 3$.

**Numerical results**

The total population of Ethiopia up to June 07, 2021, is approximately 117028692. The initial number of exposed population infectious but not detected by testing has been assumed to be $E(0) = 6659$, vaccination was not started in Ethiopia on February 29, 2021, so the initial vaccinated population was $V(0) = 0$, 935 newly confirmed cases were reported in Ethiopia on February 29, 2021. Of these, 145 were asymptomatic infected cases. That is, $A(0) = 145$ and $I(0) = 790$.

The sensitivity of quantity $R_0$ concerning parameter $\rho$ in Eq. (15) is given by

\[
\rho \frac{\partial R_0}{\partial \rho} = N_{R_0}^\rho
\]
Fig. 4. Real data and model result for asymptomatic and symptomatic infected cases between February 29, 2021 and June 07, 2021 in Ethiopia.

Fig. 5. Exposed and death cases for different values of $\alpha$.

Fig. 6. Asymptomatic and symptomatic cases for different $\alpha$. 
The basic reproduction number $R_0$ could be reduced through the transmission rate $\beta$ and this parameter has a positive impact on $R_0$.

The graphical interpretations of Figures 5 and 6 show the influence of the fractional order $\alpha$ on approximate solutions of the exposed, death, asymptomatic and symptomatic cases.

Numerical results of the behavior of exposed, asymptomatic, symptomatic and death cases in a relation to the vaccination parameter $\tau$ are shown in Figs. 7 and 8 below.

Increasing the vaccine efficiency $1 - \rho$ leads to lowering the number of infected individuals and it is shown in Fig. 9.
Discussion and conclusion

Ethiopia started a first-phase COVID-19 vaccination campaign on March 13, 2021. The first phase focuses on health workers and those who are most exposed to the disease. This dynamics is studied using Atangana–Baleanu fractional order derivative system of equations and the system is converted into an algebraic system of equations using Jacobi spectral collocation method, which is then solved using inexact Newton's method. The simulated results are in good agreement with real data reported in Ethiopia between February 29, 2021 and June 07, 2021. The real data from reproted cases in Ethiopia for asymptomatic and symptomatic confirmed cases, as well as their numerical results, are shown in Figs. 4(a) and 4(b). The simulated results are in good agreement with data reproted in our results. We run the COVID-19 model Eq. (15) for different choices of fractional order \( \alpha \) using the values of the parameters in Table 1. The behavior of approximate solutions of a model Eq. (15) for different values of fractional order \( \alpha \) is shown in Figs. 5 and 6. The fractional order \( \alpha \) has its own impact on a model's numerical solutions. We observed a significant reduction in the number of infected individuals in a fractional order \( \alpha \) model using the values of the parameters in Table 1. Based on Table 1, the basic reproductive number is \( R_0 = 1.2297 > 1 \), indicating that the endemic equilibrium point is locally asymptotically stable and \( E_U \) is locally asymptotically unstable. Fig. 7(a), 7(b), 8(c), and 8(d) indicate that increasing the vaccination parameter \( r \) from 0.000001593 to 0.001593 reduces the number of exposed, asymptomatic, symptomatic, and death cases significantly. For \( r = 0.00001593 \), total number of exposed cases decreased by 8.04%, asymptomatic cases were reduced by at least 19.84%, symptomatically infected cases were reduced by at least 23.09%, and the number of deaths decreased by at least 9.07%. Finally, a notable reduction is observed for \( r = 0.001593 \), total number of exposed cases decreased by 8.04%, asymptomatic cases were reduced by at least 19.84%, symptomatically infected cases were reduced by at least 23.09%, and the number of deaths decreased by at least 9.07%.

Fig. 10. Influence of the effectiveness of vaccine on symptomatic and asymptomatic cases.

is, the disease free equilibrium point becomes stable for \( \nu \leq 0.2017 \), provided that the remaining parameters are kept constant. Also, from Fig. 10, increasing the vaccine efficiency by \( 1 - \rho \) causes a decrease in the numbers of confirmed cases, both symptomatic and asymptomatic. This result shows an increase in the numbers of susceptible population without vaccination and with vaccination, while the number of exposed cases shows a reduction. Vaccination is a powerful health tool, and the Ethiopian government should prioritize importing vaccines from licensed vaccine manufacturing institutes.

References

[1] Zhu H, Wei L, Niu P. The novel coronavirus outbreak in Wuhan, China. Glob Health Res Policy 2020;5(6).
[2] Centers for disease control and prevention, outbreak of 2019 novel coronavirus (2019-nCoV) in Wuhan, China. 2020, Available: www.cdc.gov/ncidod/dbs/sds/ncov. 2020, Available: https://www.who.int/emergencies/diseases/novel-coronavirus-2019/situation-reports.
[3] World Health Organization. WHO Characterizes Covid-19 as a pandemic. 2020, Available: https://www.who.int/emergencies/diseases/novel-coronavirus-2019/situation-reports.
[4] World Health Organization (WHO). Adv Public 2020. Archived from the original on 26 January 2020. Retrieved 10 February 2020.
[5] US Centers for Disease Control and Prevention (CDC). Coronavirus disease 2019 (COVID-19): Prevention and treatment. 2020. Archived from the original on March 11, 2020. Retrieved March 11, 2020.
[6] Mwalili S, Kimathi M, Ojiambo V, Gathungu D, Mbogo R. SEIR Model for COVID-19 dynamics incorporating the environment and social distancing. Mwalili, Others, BMC Notes 2020;13(352). Nairobi, Kenya.
[7] Moore SE, Okyere E. Controlling the transmission dynamics of COVID-19. Univ Cape Coast 2020;1(1):13–21.
[8] WHO. Africa. Ethiopia introduces covid-19 vaccine in a national launching ceremony. Addis Ababa; 2021.
[9] Kermack WO, McKendrick AG. A contribution to the mathematical theory of epidemics. Proc R Soc A 1927;115(772):700–21.
[10] Atangana A, Baleanu D. New fractional derivatives with non-local and non-singular kernel: theory and applications to heat transfer model. Therm Sci 2016;20:763–9.
[11] Alqahtani RT. Atangana-Baleanu derivative with fractional order applied to the model of groundwater within an unconfined aquifer. J Nonlinear Sci Appl 2016;9:3647–54.
[12] Weinstein EW. Jacobi polynomials. From MathWorld-A Wolfram Web Resource. https://mathworld.wolfram.com/JacobiPolynomials.html.
[13] Jelí M, Samet B. On a new generalization of metric spaces. J Fixed Point Theory Appl 2018;20(3):128.
[14] Kirk WA, Vasile I. Fixed point theory: An introduction to metric spaces and fixed point theory. New York: John Wiley–
[15] Baleanu D, Fernandez A. On some new properties of fractional derivatives with Mittag-Leffler kernel. Commun Nonlinear Sci Numer Simul 2018;59:444–62.
[16] Ministry of Health-Ethiopia and Ethiopian Public Health Institute; 2021. www.moh.gov.et and http://www.ephil.gov.et/.

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