Constraints on Weakly Mixed Sterile Neutrinos in the Light of SNO Salt Phase and 766.3 Ty KamLAND Data

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Abstract

The possibility of flavor transitions into sterile neutrinos (accompanying the dominant LMA transitions) in the solar boron neutrino flux has been examined in a scenario proposed by Hollanda and Smirnov to overcome some generic problems of the pure LMA scenario. It is found that the most recent SNO salt phase solar neutrino data and the KamLAND 766.3 Ty spectral data, allow for a significant sterile presence in the solar boron neutrino flux reaching the earth.

1 Introduction

The SNO [1, 2] and KamLAND [3] experiments have played a crucial role in resolving the longstanding solar neutrino problem in terms of large mixing angle (LMA) MSW oscillations and are expected to play an important role in the refinement of the LMA solution which is undergoing a deeper scrutiny. Does the LMA solution explain all the solar neutrino data satisfactorily? There are, at least, two generic predictions of LMA indicating new physics beyond LMA. One of these is the prediction of the high argon production rate for Homestake experiment which is about $2\sigma$ above the observed rate. Another generic prediction of the LMA scenario is the ‘spectral upturn’ at low energies. Within the LMA parameter space, the survival probability should increase with decrease in energy and for the best fit parameters, the upturn could be as large as 10-15% between 8 MeV and 5 MeV [4]. In fact, the spectral upturn at low energies is expected to increase further with the KamLAND 766.3 Ty spectral data [5] favoring a larger value of $\Delta m^2$ [6]. However, neither SuperKamiokande nor SNO has reported any statistically significant ‘rise-up’ in the observed neutrino survival probability. Both these predictions of the LMA solution can, only, be tested in the forthcoming phase of high precision measurements [7] in the solar neutrino experiments and are crucial for the final confirmation of the LMA solution.

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Another unresolved issue is whether the solar neutrinos oscillate into the sterile component. The main motivation for postulating the existence of the sterile neutrino species comes from the LSND experiment which reported a significant $\nu_\mu \rightarrow \nu_e$ oscillation probability $^8$ which requires a new mass scale. Since, the Z-decay width constrains the number of weakly interacting light neutrino species to be very close to three $^9$, one is forced to postulate a sterile neutrino. While the purely sterile oscillation solution is excluded at $7.6 \sigma$ $^{10}$, the solar electron neutrinos could still oscillate into both active and sterile neutrinos, a scenario which is, largely, unconstrained at present. In fact, a combined analysis $^{11}$ of solar and atmospheric neutrino data has shown that the active-sterile admixture can take any value between zero and one. While the SNO charged current data excluded the maximal mixing to sterile neutrinos at $5.4 \sigma$ $^2$, arbitrary active-sterile admixtures were not considered. Consequently, a significant sterile fraction in the solar neutrino flux reaching the earth is, still, possible. The discovery of the sterile neutrinos would be of great importance for particle physics, even though, it is, still, not clear how these hypothetical ‘exotic’ degrees of freedom would fit into elementary particle theory.

The possibility of subdominant transitions into sterile neutrino states accompanying the dominant LMA flavor transitions has been examined earlier $^{12}$ and upper bounds on the sterile neutrino fraction in the non-electronic boron neutrino flux have been derived. However, the subdominant transitions into sterile states have neither been confirmed nor ruled out at a statistically significant level. In the present work, the possibility of flavor transitions into sterile component in the solar boron neutrino flux has been examined in a model presented by Hollanda and Smirnov $^4$ to lower the abnormally high argon production rate in the Homestake experiment and, also, to lower the ‘spectral upturn’ in the low energy boron neutrino spectrum predicted in the pure LMA scenario.

2 Weakly Mixed Sterile Neutrinos

We introduce a sterile neutrino $\nu_s$ which mixes weakly with the active flavors $\nu_e$ and $\nu_\mu$ to form the mass eigenstates $\nu_0$, $\nu_1$ and $\nu_2$ given by

$$
\begin{align*}
\nu_0 &= (\cos \alpha) \nu_s + \sin \alpha \{ (\cos \theta) \nu_e - (\sin \theta) \nu_\mu \}, \\
\nu_1 &= (\cos \alpha) \{ (\cos \theta) \nu_e - (\sin \theta) \nu_\mu \} - (\sin \alpha) \nu_s, \\
\nu_2 &= (\sin \theta) \nu_e + (\cos \theta) \nu_\mu,
\end{align*}
$$

with masses $m_0$, $m_1$ and $m_2$, respectively. It is assumed that $\sin^2 \alpha \ll 1$ (weak mixing) so that $\nu_s$ is mainly present in the mass eigenstate $\nu_0$ only. Following Hollanda and Smirnov $^4$, we assume the mass hierarchy $m_1 < m_0 < m_2$, and define the following mass squared differences:

$$
\begin{align*}
\Delta m^2_{01} &= m_0^2 - m_1^2, \\
\Delta m^2_{12} &= m_2^2 - m_1^2.
\end{align*}
$$

The energy eigenlevels for the above neutrino system ($\nu_0$, $\nu_1$ and $\nu_2$) are denoted by $\lambda_0$, $\lambda_1$ and $\lambda_2$, respectively. For the mass hierarchy assumed above, the level $\lambda_0$ crosses the level $\lambda_1$.
only and $\lambda_2$ is, approximately, the same as it would be in the pure LMA two flavor scenario in the absence of any sterile mixing. Neglecting the small admixture of $\nu_e$ in $\nu_0$, one obtains

\begin{align}
P_{ee} &= P_{LMA} - P_{es} \cos^2 \theta, \tag{3} \\
P_{e\mu} &= 1 - P_{LMA} - P_{es} \sin^2 \theta, \tag{4} \\
P_{es} &= \cos^2 \theta \sin^2 \alpha_m (1 + P_c \cos 2\alpha_m), \tag{5}
\end{align}

where, $P_{LMA}$, given by

\[ P_{LMA} = \frac{1}{2} + \frac{1}{2} \cos 2\theta \cos 2\theta, \tag{6} \]

is the survival probability for electron neutrinos in the pure LMA scenario. $P_c$ is the crossing probability at the point where $\lambda_0$ and $\lambda_1$ cross while $\theta_m$ and $\alpha_m$ are the mixing angles in the matter. The symbols $P_{ee}$, $P_{e\mu}$ and $P_{es}$ have their usual meaning. Hollanda and Smirnov have shown that the introduction of sterile admixture leads to a decrease in the ‘rise-up’ in the boron neutrino spectrum at lower energies and, also, reduces the argon production rate at Homestake to a phenomenological acceptable level for

\[
\Delta m_{01}^2 \sim (2 - 20) \times 10^{-5} \text{eV}^2, \\
\sin^2 2\alpha \sim (10^{-5} - 10^{-3}). \tag{7}
\]

Apart from some rather ‘exotic’ scenarios proposed in literature, it happens to be the simplest and the most plausible scenario to overcome the generic problems of the pure LMA solution mentioned earlier. Therefore, it is important to constrain the sterile component in this scenario (referred to as the (LMA+sterile) scenario, henceforth) in the light of the SNO solar neutrino data.

In this (LMA+ sterile) scenario,

\begin{align}
P_{ee} &= \frac{\phi_{cc}^{SNO}}{\phi_B}, \tag{8} \\
P_{e\mu} &= \frac{\phi_{nc}^{SNO} - \phi_{cc}^{SNO}}{\phi_B}, \tag{9} \\
P_{es} &= 1 - \frac{\phi_{nc}^{SNO}}{\phi_B}, \tag{10}
\end{align}

where $\phi_{cc}$ and $\phi_{nc}$ are the fluxes measured at SNO through CC and NC reactions, respectively, and $\phi_B$ is the total boron neutrino flux.

From equations (8-10), one obtains

\[ P_{e\mu} = \frac{1 - x}{x} P_{ee} \tag{11} \]

and

\[ P_{es} = 1 - \frac{P_{ee}}{x} \tag{12} \]

where

\[ x = \frac{\phi_{cc}^{SNO}}{\phi_{nc}^{SNO}}. \tag{13} \]
The ratio of nonelectronic active neutrino flux to total nonelectronic (active+sterile) neutrino flux, denoted by \( \sin^2 \varphi \), is given by

\[
\sin^2 \varphi = \frac{\phi_{NC}^{SNO} - \phi_{CC}^{SNO}}{\phi_B - \phi_{CC}^{SNO}}.
\] (14)

In the LMA scenario

\[
P_{LMA}(\nu_e \to \nu_e) = P_{LMA} = x,
\] (15)

where \( x \) is given by equation (13). However, in the (LMA +sterile) scenario, \( \phi_B \) and \( \phi_{NC}^{SNO} \) are not equal and one has to use the relation (8)

\[
P_{ee} = \frac{\phi_{CC}^{SNO}}{\phi_B}.
\]

instead, where \( \phi_B \) is, now, an independent quantity which can not be determined from the SNO CC and NC fluxes. One can use the boron neutrino flux given by the standard solar model (SSM) for \( \phi_B \) to calculate \( \sin^2 \varphi \) and \( P_{ee} \). However, because of large errors in the SSM boron neutrino flux (\( \phi_{SSM} \)), only a lower bound on \( \sin^2 \varphi \) can be obtained while the upper bound becomes larger than unity [12].

Without assuming \( P_{ee} \), one cannot calculate \( \sin^2 \varphi \) and \( \phi_B \), uniquely and only a family of solutions corresponding to different values of \( P_{ee} \) is obtained. Equations (8) and (14) can be rewritten as a set of coupled equations as follows:

\[
\sin^2 \varphi = 1 - x \frac{P_{ee}}{1 - P_{ee}},
\] (16)

and

\[
\phi_B = \frac{\phi_{CC}^{SNO}}{P_{ee}}.
\] (17)

This degeneracy is well known as the \((f_B - \sin^2 \varphi)\) degeneracy in the literature [12]. The value of \( \phi_B \) is, usually, given in the units of the central value of \( \phi_{SSM} \), so that \( f_B = \phi_B / \phi_{SSM} \).

To gain further insight into this degeneracy, we rewrite equations (11,12) as follows

\[
(1 - x)P_{ee} - x P_{e\mu} = 0
\] (18)

and

\[
P_{ee} + x P_{es} = x.
\] (19)

Since, we have only two equations relating three unknowns viz. \( P_{ee}, P_{e\mu} \) and \( P_{es} \), a unique solution is not possible and one obtains a family of solutions, instead, corresponding to different values of \( P_{ee} \). Balentekin et al. [13] identify \( P_{ee} \) with \( P_{LMA} \) (electron neutrino survival probability in the pure LMA scenario in the absence of any sterile transitions) and use equations (8), (14) and (15) to constrain the sterile component. However, equation (15) cannot be used to derive meaningful constraints on the sterile component since one obtains \( \sin^2 \varphi = 1 \) on substitution of equation (15) in equation (16). To overcome this problem, we use equation (3) alongwith equations (18,19) to constrain the sterile component. We collect all these equations below:
$P_{ee} + P_{es} \cos^2 \theta = P_{LMA},$

$(1 - x) P_{ee} - x P_{e\mu} = 0,$

$P_{ee} + x P_{es} = x. \tag{20}$

This set of coupled equations has the following simultaneous solution

$$P_{ee} = x \frac{\cos^2 \theta - P_{LMA}}{\cos^2 \theta - x}, \tag{21}$$

$$P_{e\mu} = (1 - x) \frac{\cos^2 \theta - P_{LMA}}{\cos^2 \theta - x}, \tag{22}$$

$$P_{es} = \frac{P_{LMA} - x}{\cos^2 \theta - x}. \tag{23}$$

3 Results and Discussion

In order to examine the possibility of transitions into sterile neutrinos, we plot the $1\sigma$ allowed upper and lower values of $P_{ee} = \frac{\phi_{SNO}}{\phi_B}$ [equation (8)] allowed by the salt phase SNO data \cite{2} and BP04 \cite{15} in Figure 1 which, also, depicts $P_{ee}$ as a function of $P_{es}$ as given by equation (12). It is clear from Figure 1 that significant transitions into sterile neutrinos are allowed by the SNO salt phase data and, in fact, the $1\sigma$ upper bound on $P_{es}$ could be as large as 0.4. More precise bounds on the sterile fraction in the boron neutrino flux can, only, be obtained with more precise measurements of CC and NC rates at SNO in the future. It may be pertinent to mention here that the boron neutrino flux estimates in the SSM have, rather, large uncertainties. Consequently, considerable improvements in the boron neutrino flux estimates in the SSM are required for obtaining meaningful constraints on the possible sterile neutrino fraction in the boron neutrino flux.

One can obtain the electron neutrino survival probability $P_{ee}$, the transition probability into muon neutrinos $P_{e\mu}$ and the transition probability into the sterile neutrinos $P_{es}$ from equations (21-23) by substituting the flux-averaged value of $P_{LMA}$ (calculated for the values of $\Delta m^2$ and $\theta$ and $1\sigma$ errors therein taken from \cite{16}) and the value of $x$ reported by SNO \cite{2}. It is important to realize here that the SNO CC flux is the actual electron neutrino flux $\phi_{e\nu}$ if the boron energy spectrum is assumed to be undistorted. Since, the SuperKamiokande \cite{17} (with a better precision) has not reported any, statistically significant, spectral distortions in the boron neutrino spectrum, we assume an undistorted boron neutrino spectrum and identify $P_{LMA}$ with $x$ at the high energy end. However, since the LMA scenario predicts a significant spectral upturn at lower energies \cite{9}, $P_{LMA}$ is expected to be, significantly, different from $x$ at lower energies. For the flux averaged value of $P_{LMA}$, $P_{ee} = 0.239^{+0.063}_{-0.055}$, $P_{e\mu} = 0.543^{+0.042}_{-0.048}$ and $P_{es} = 0.218^{+0.103}_{-0.105}$, which is non-zero at about $2.1\sigma$. The probabilities $P_{ee}, P_{e\mu}$ and $P_{es}$ have been plotted in Figure 2 as functions of $x$ where the $1\sigma$ upper and lower bounds have, also, been shown. It is clear from Figure 2 that the $1\sigma$ values of $P_{ee}$ and $P_{es}$ are of comparable magnitude. In fact, for the smaller values of $x$, the transition probability into the sterile neutrinos can, even, be larger than the electron neutrino survival probability.
It is, also, clear from Figure 2, that the errors in the value of \( x \) are the most significant sources of error in the values of the probabilities and once the value of \( x \) is settled by the experiments, the errors in the probabilities will become much smaller. For example, for a most conservative choice of \( x = 0.341 \), the transition probability into the sterile neutrinos is found to be \( 0.146^{+0.040}_{-0.033} \), which is non-zero at 4.4\( \sigma \)C.L. If the value of \( x \) is settled below this value (i.e. \( x \leq 0.341 \), as is most likely) by the experiments, the transition probability into the sterile neutrinos as well as the corresponding confidence level will be larger than the above values (at \( x = 0.341 \)). Thus, the main source of error in the transition probability into the sterile neutrinos being the uncertainty in the value of \( x \), a more precise determination of the value of \( x \) (below the value 0.341) will give a non-zero value of \( P_{es} \) at more than 4.4 standard deviations.

From equations (16) and (17), one obtains

\[
\sin^2 \varphi = \frac{(1-x)(\cos^2 \theta - P_{LMA})}{\cos^2 \theta - x(1+\cos^2 \theta - P_{LMA})},
\]

\[
f_B = R_{NC} \frac{\cos^2 \theta - x}{\cos^2 \theta - P_{LMA}},
\]

where \( f_B \) and \( R_{NC} \) are the boron neutrino flux and SNO NC flux normalized to the central SSM bornon neutrino flux, respectively. The \( (f_B - \sin^2 \varphi) \) degeneracy is, thus, lifted by the use of equation (3).

In Figure 3, we plot \( \sin^2 \varphi \) (equation (24)) as a function of \( x \). The active neutrino fraction, \( \sin^2 \varphi \), increases with the increase in the SNO CC flux and decrease in the SNO NC flux. Thus, the sterile fraction in the active solar boron neutrino flux, \( \cos^2 \varphi \), will be larger if the forthcoming measurements at SNO favor smaller values of CC flux and larger values of NC flux. For \( x = 0.341 \), \( \sin^2 \varphi = 0.713^{+0.125}_{-0.107} \) and \( f_B = 1.14^{+0.29}_{-0.23} \). Thus, the sterile fraction in the boron neutrino flux is non-zero at about 2.3\( \sigma \)C.L. indicating oscillation of boron neutrinos into sterile states in the current SNO data. The behavior of \( \sin^2 \varphi \) with \( x \) in the present work is different from that reported by Balentekin et al. [14] by identifying \( P_{ee} \) with \( P_{LMA} \) and substituting the numerical value of \( P_{LMA} \) obtained in the pure LMA scenario. However, as discussed earlier, such an approach can not be used to derive meaningful constraints on the sterile fraction.

The boron neutrino flux obtained from equation (25) has been plotted as a function of \( x \) in Figure 4. Most of the 1\( \sigma \) region of the boron neutrino flux lies above its central value in the SSM in contradiction with the value of boron neutrino flux in the pure LMA scenario.

4 Conclusions

In conclusion, the prospects for constraining the sterile neutrino fraction in the born neutrino flux reaching the earth have been examined in a scenario discussed by Hollanda and Smirnov [1] to overcome some generic problems of the LMA scenario. The indications for the presence of sterile component in the boron neutrino flux in the light of the latest SNO salt phase data
and the 766.3 Ty KamLAND data are found to be strong enough to be taken seriously. A precise determination of the CC and NC fluxes at SNO within the present 1σ range gives a transition probability into sterile states which is non-zero at more than 4.4σ. It is found that the sterile component in the boron neutrino flux could be as large as the electron neutrino flux for the present central values of SNO CC and NC fluxes. If the future measurements at SNO yield smaller values of CC/NC flux ratio, the sterile fraction will be, further, enhanced. Thus, there are strong indications of a sterile presence in the boron neutrino flux reaching the detectors on the earth but it will require a precise determination of CC and NC fluxes to pass a final judgment.

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Figure 1: The $1\sigma$ allowed region in the $P_{ee} - P_{es}$ space.
Figure 2: $P_{ee}$, $P_{e\mu}$ and $P_{es}$ as functions of $x$. 
Figure 3: $\sin^2 \varphi$ as a function of $x$. 
Figure 4: $f_B$ as a function of $x$. 