Particle yields and ratios within equilibrium and non-equilibrium statistics

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Abstract – In characterizing the various yields and ratios of well-identified particles in the ALICE experiment, we utilize extensive additive thermal approaches, to which various missing states of the hadron resonances are taken into consideration as well. Despite some non-equilibrium conditions that are slightly driving this statistical approach away from equilibrium, the approaches are and remain additive and extensive. Besides van der Waals repulsive interactions (assuming that the gas constituents are no longer point-like, i.e., finite-volume corrections taken into consideration), finite pion chemical potentials as well as perturbations to the light and strange quark occupation factors are taken into account. When confronting our calculations to the ALICE measurements, we conclude that the proposed conditions for various aspects driving the system out of equilibrium notably improve the reproduction of the experimental results, i.e., improving the statistical fits, especially the finite pion chemical potential. This points out to the great role that the non-equilibrium pion production would play, and the contributions that the hadron resonance missing states come up with, even when the principles of statistical extensivity and additivity are not violated. These results seem to propose revising the conclusions propagated by most of the field, that the produced particles quickly reach a state of local equilibrium leading to a collective expansion often described by fluid dynamics. This situation seems not to remain restrictively valid, at very large collision energies.

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Introduction. – Our understanding of the particle production in the high-energy collisions has been improved, drastically [1]. Currently, we understand that the colliding particles (hadrons) are being smashed, enormously [2]. Before they finally recombine onto hadrons, they form a parton state, in which quarks and gluons are likely created and strongly interacting. Such a non-equilibrium state, e.g., unstable quark-gluon plasma (QGP), rapidly expands and lastly cools down that it evolves to an entirely different state; an equilibrium one, either chemically or later on thermally, for instance, into hadrons again. Thus, the produced particles, which are nothing but hadrons, i.e., we started up with colliding hadrons and at the end we are left with hadrons, are the products out of a non-equilibrium state [1]. The QCD phase transition and the related symmetry breaking and/or restoration greatly contribute to the out-of-equilibrium status of the system of interest.

Due to the absence of an alternative well-functioning approach, overall chemical and thermal equilibria are assumed, for instance, in performing lattice QCD simulations. With this regard, one would recall that the hydrodynamic approaches rely on local thermodynamic equilibrium, while viscous fluid approximation considers a deviation from the local equilibrium. All these approaches are not considered in the present study. Furthermore, when proposing conditions for the chemical freezeout which enable us to study the bulk properties of yields and ratios of the produced particles, chemical equilibria should be adopted. The study of the transverse momentum spectra is almost exclusively based on thermal equilibrium, known as thermal freezeout. These assumptions
seem to work well in characterizing the statistical nature of the bulk properties, at energies up to top RHIC energy. Within this energy regime, the transverse momentum spectra of precisely detected particles, such as pions, kaons and protons, are well described by Boltzmann-Gibbs (BG) statistics. But when moving to the LHC energies, it seems that the picture of global equilibrium faces great challenges. The particle yields and ratios and their transverse momentum spectra are not well characterized. We might highlight what is called anomaly in proton-to-pion ratios at RHIC and LHC, for a recent status we refer to [3], where the inclusion of hadronic cascades after the hypersurface of the chemical freezeout [4] inducing non-equilibrium corrections in a very simple picture contributes to the explanation of the proton-to-pion anomalies [5,6]. This picture was challenged, see, for instance, [7,8], that pions not protons are anomalously created. Also, the present paper aims at an unbiased understanding of the statistical nature of particle production at LHC energies. Otherwise, a price should be paid, e.g., the statistical description of the system of interest turns to have a bias towards (non-) extensivity for instance.

Various proposals have been made to empower the equilibrium statistical approaches to cover RHIC [9] and LHC energies [10–12]. To this end, anomalies in production and annihilation of certain particles (such as proron anomaly) and/or non-extensive statistical approaches (such as the Tsallis types) have been proposed to be applied to LHC energies. Despite the great success of our generic (non-)extensive statistical approach [13–15], which we discuss briefly while concluding the overall message of the present paper, we assume that the pions [7,8], the low-lying Nambu-Goldstone bosons, are the produced particles which significantly affect both bulk and flow properties of the other particles produced in collisions at LHC energies. We recall the assumption, which originally dates back to three decades, that the production of pion particles takes place out of a non-equilibrium process [16].

In the next section, the equilibrium hadron resonance gas (HRG) model shall be outlined, in which the quark occupation factors are also allowed to take values different from the ones characterizing an equilibrium status. The inclusion of the repulsive van der Waals interactions, from the ones characterizing an equilibrium status, implies the existence of Bogoliubov dispersions in the same matter as we do for the PDG particle data group (PDG) [30] can be summed up, additively. Such a mass cutoff defines the temperature validity of the HRG model, setting a natural limitation to the hadronic phase. The quark models predict an excessive number of hadron states [31–34]. Not all of them have found a counterpart yet either experimentally [30] or in the lattice QCD simulations [35]. They are entering our calculations in the same manner as we do for the PDG hadrons and resonances [1]. In other words, this procedure should not violate the additivity and extensivity statistical principles. The missing states are resonances predicted theoretically but not yet confirmed experimentally. Their quantum numbers and physical characteristics are theoretically well known. Basically, they are conjectured to greatly contribute to the fluctuations and the correlations estimated in recent lattice QCD simulations [36]. These were the...
reasons why to add them [37]. Despite the convention that this would not be the case in the present study, as we are strictly focusing on particle yields and ratios: we wanted to add them, as the particle ratios, for instance, are not entirely free of correlations [38]. Another reason for adding the missing states is that they come up with additional degrees of freedom and considerable decay channels to the particles which are subject of this present study.

As given earlier, the constituents of HRG are free (collisionless) particles. Some authors prefer taking into account the repulsive van der Waals interactions in order to partly compensate the strong interactions in the hadronic medium [39] and/or to significantly drift the system towards a non-equilibrium status. Accordingly, each constituent is allowed to have an eigenvolume and the resulting hadron system becomes thermodynamically non-ideal non-equilibrium. The repulsive interactions between hadrons are considered as a phenomenological extension and exclusively based on van der Waals excluded volume [40–43]. Thus, the resulting total volume might be subtracted from the entire volume of the fireball. Considerable modifications in the thermodynamics of the hadron gas such as energy, entropy and number densities are likely expected. The hard-core radius of hadron nuclei can be related to the multiplicity fluctuations and consequently the particle yields and ratios.

The ability of the HRG model with finite-volume constituents without missing states in reproducing the lattice QCD thermodynamics has been confirmed, long ago [39]. But which limits should be set to the proposed eigenvolume? To this end, few remarks are now in order. At radius \( r > 0.2 \text{fm} \), the disagreement with these first-principle calculations, the lattice QCD, is very convincing. Such an agreement becomes more and more excellent with the increase in the particle radii. But, at higher temperatures, the resulting thermodynamic quantities turn to be non-physical. Thus, it was concluded that the excluded volume correction becomes practically irrelevant, as it comes up with a negligible effect at \( r \leq 0.2 \text{fm} \). On the other hand, a remarkable deviation from the lattice QCD calculations appears, especially when relative large values are assigned to the hadron radii. Intensive theoretical works have been devoted to the estimation of the excluded volume and its effects on the particle production and fluctuations, for instance [44]. It is conjectured that the hard-core radius of hadron nuclei can be related to the multiplicity fluctuations and, as a consequence, to the particle yields and ratios [45]. In the present work, we simply assume that all hadrons are spheres and all have the same radius. On other hand, the assumption that the radii would depend on the hadron masses and sizes could come up with a very small improvement. Therefore, we can neglect this.

It is obvious that various types of interactions should be assumed, as well [46,47]. For a possible inclusion of the strong interactions, themselves, we advise the interested readers to consult ref. [21]. Nevertheless, we limit our calculations here to the van der Waals repulsive interactions, which could be estimated by replacing the system volume \( V \) by an actual one \( V_{\text{act}} \).

\[
V_{\text{act}} = V - \sum_h v_h N_h, \tag{2}
\]

where the volume and particle number of each constituent hadron, respectively, read \( v_h = 4(4\pi r_h^3/3) \) and \( N_h \). The eigenvolume \( v_h \) characterized the \( h \)-th hadron particle. From eq. (2), we get \( \mu_\pi = \mu_h - v_p p \), where the thermodynamic pressure \( p \) should be determined in a self-consistent manner \( \sum_h p_{\text{act}}^h(T, \bar{\mu}_h) \) and

\[
\begin{align*}
n &= \frac{\sum_h n_{\text{id}}^h(T, \bar{\mu}_h)}{1 + \sum_h v_h n_{\text{id}}^h(T, \bar{\mu}_h)}, \\
\epsilon &= \frac{\sum_h \epsilon_{\text{id}}^h(T, \bar{\mu}_h)}{1 + \sum_h v_h \epsilon_{\text{id}}^h(T, \bar{\mu}_h)}, \\
s &= \frac{\sum_h s_{\text{id}}^h(T, \bar{\mu}_h)}{1 + \sum_h v_h s_{\text{id}}^h(T, \bar{\mu}_h)}.
\end{align*}
\]

The superscript \( \text{id} \) refers to point-like calculations. In the section entitled “Results”, we discuss on out-of-equilibrium chemical potential that we are proposing to be inserted into the partition function.

- Non-equilibrium approaches: An out-of-equilibrium pion production was successfully implemented in characterizing the \( p_T \)-spectra of the negatively charged bosons from the 200 A GeV O+Au and S+S collisions in the NA35 experiment [16]. To this end, \( \epsilon \) in eq. (1) should be replaced by the azimuthal angle \( \phi \) and the covariant form \( p_\mu w^\mu \) (four-momentum and -velocity) and then we integrate everything over the freeze-out time \( \tau_{\text{fo}} = \tau \); the pion transverse mass reads \( m_\tau = (p_T^2 + m^2)^{1/2} \), the volume element \( d^3p \) is given in the transverse momentum \( p_T \), the rapidity \( y \) and the azimuthal angle \( \phi \); \( d^3p = p_T d\tau d\phi d\gamma \). Thus, the energy can be expressed as \( \epsilon = m_\tau \gamma \). The scalar field \( \phi(x) \) is conjectured to have a unitary transformation by the phase factor exp(\( -i\alpha \)). Therefore, the Bose-Einstein condensation of the lowest-lying Nambu-Goldstone bosons can be studied in \( U(1) \) global symmetry [48]. The single-particle partition function is then given as

\[
\ln z(T, \mu_\pi) = \frac{V}{T} \left( \frac{\mu_\pi^2 - m^2}{T} \right)^{\xi_2} - V \int \frac{d^3p}{(2\pi)^3} \left[ \frac{\epsilon}{T} + \ln(1 + \gamma \gamma' \epsilon' e^{-\frac{\gamma \gamma' \epsilon'}{T}}) \right]. \tag{6}
\]
between results at vanishing measurements (symbols). The top and bottom panels compare between results at vanishing \( \mu_e \) (top panel) and finite \( \mu_e = 0 \) (bottom panel).

The parameter \( \xi \), which can be treated as a variational parameter relating to the charge of condensed particle, carries the infrared characters of the scalar field. At \( |\mu_\pi| < m \), the equilibrium partition function, eq. (1), can be recovered.

**Results.** – The results on the particle yields \( \pi^+, \pi^-, \kappa^-, \kappa^+, \bar{p}, p, \bar{\Lambda}, \bar{\Omega}, \bar{\Xi}, \phi \) calculated from the thermal model with (dashed lines) and without missing states (solid lines) at equilibrium \( \gamma_l = \gamma_s = 1 \) fitted to ALICE measurements (symbols) are depicted in fig. 1. The experimental results are given as symbols with errorbars. The top and bottom panels present a comparison between our calculations at vanishing \( \mu_\pi \) (left) and finite \( \mu_\pi = 0 \) (right). The resulting parameters are listed inside the graphs. From the given \( \chi^2/\text{dof} \), we can draw the conclusion that adding missing states to the PDG compilation considerably improves the statistical fits.

The results at \( \gamma_l \neq 1 \) and \( \gamma_s \neq 1 \) are depicted in fig. 2. When analyzing the resulting fitting parameters, we can draw another conclusion, such as that when \( \gamma_l \) and \( \gamma_s \) are allowed to take values different from unity, the ability of this partial non-equilibrium to reproduce the experimental data increases and this considerably improves the statistical fits. When comparing both figs. 1 and 2 and the resulting fitting parameters we conclude that the pion chemical potential excellently describes the particle yields at the LHC energy.

Similarly to figs. 1 and 2, the results on the particle ratios \( \pi^-/\pi^+, K^-/K^+, p/p, \Lambda/\bar{\Lambda}, \bar{\Omega}/\Omega, \bar{\Xi}/\Xi, K^-/\pi^-, K^+/\pi^+, p/\bar{p}, p/\pi^+, \Lambda/\pi^-, \bar{\Omega}/\pi^+, \Xi/\pi^- \), and \( \bar{\Xi}/\pi^+ \) are depicted in figs. 3 and 4, respectively. The resulting parameters are listed out inside the graphs. This time, the fireball volume is not included. The particle ratios likely
cancel the dependence on $V$, at least to a very large extent. The volume fluctuations, on the other hand, might not be entirely removed. This might be the statistical price to be paid, as long as no other alternative exists so far. Another conclusion can be drawn here as well. $\mu_\pi \neq 0$ helps in improving the reproduction of the different particle ratios by means of the statistical thermal approaches.

With this regard, we might recall a recent study on the production of $\pi$, K, p, and $\Lambda$ and their ratios in Pb+Pb collisions at 2.76 TeV in the blast-wave model with thermal equilibrium mechanism [49]. While the antiparticles-to-particles and the kaons-to-pions ratios were well reproduced, the $p/\pi$ was overestimated by a factor of 1.5. Based on this study [49]. It was found that $p/\pi$ and $K/\pi$ are dominated by the radial flow. In the present study, we have, among other things, avoided constraining the fitting parameters as done in ref. [49]. To solve the $p/\pi$-overestimation, various proposals have been discussed.

When comparing our results with the THERMUS predictions, we conclude that our fitting parameters are smaller. While our strangeness chemical potential $\mu_S$ is determined at $T$ and $\mu_B$ to assure strangeness neutrality, the resulting fitting parameters, at $\gamma \neq 1$, $\gamma_s \neq 1$ with missing hadron states, read $\mu_B = 0.126 \pm 0.01$ MeV (in THERMUS $\mu_B$ is fixed to 1 MeV), $\gamma = 1.19 \pm 0.05$, $\gamma_s = 1.24 \pm 0.07$, $\mu_\pi = 40.12 \pm 0.65$ MeV, and $T = 156.45 \pm 1.75$ MeV.

Conclusions. — In the present paper, we wanted to show whether the non-equilibrium pion production (associated with $\mu_\pi \neq 0$) affects other bulk properties at the LHC energies such as transverse momentum spectra [50]. In reproducing particle number ratios at the LHC energies, we have taken into account $\mu_\pi \neq 0$, i.e., besides the baryon, the strangeness and the electric charge potentials, we imposed $\mu_\pi \neq 0$, as well. We have also taken into consideration various missing states of the hadron resonances. The extensive HRG model is extended to finite-volume constituents and the light and strange quark occupation factors are taken at equilibrium as well as at non-equilibrium.

Our measure for the best statistical fit was the smallest $\chi^2$/dof. In fitting the particle yields, we should/could estimate the fireball volume. We noticed that best fits are accompanied by the smallest volume and the lowest $\mu_\pi$ as well. Accordingly, we were able to draw the conclusions that the various particle yields and ratios produced in the most central (0–5%) Pb+Pb collisions at 2.76 TeV are well reproduced at $\mu_\pi \neq 0$ while both $\gamma_t$ and $\gamma_s$ take values greater than unity.

The present study shows that the particle yields and ratios at the LHC energy would be best reproduced in the extensive thermal models, as is also done at lower energies. This is conditioned to a pion production anomaly, i.e., finite $\mu_\pi$. The reasons why this appears at LHC energy should be subject of future works. The attempt introduced in refs. [7,8], should be intensified.

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