A NEW TYPE OF COMPACT STELLAR POPULATION: DARK STAR CLUSTERS

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ABSTRACT

Among the most explored directions in the study of dense stellar systems is the investigation of the effects of the retention of supernova remnants, especially of the massive stellar remnant black holes (BHs), in star clusters. By virtue of their eventual high central concentration, these stellar mass BHs potentially invoke a wide variety of physical phenomena, the most important ones being emission of gravitational waves (GWs), formation of X-ray binaries, and modification of the dynamical evolution of the cluster. Here we propose, for the first time, that rapid removal of stars from the outer parts of a cluster by the strong tidal field in the inner region of our Galaxy can unveil its BH sub-cluster, which appears as a star cluster that is gravitationally bound by an invisible mass. We study the formation and properties of such systems through direct N-body computations and estimate that they can be present in significant numbers in the inner region of the Milky Way. We call such objects “dark star clusters” (DSCs) as they appear dimmer than normal star clusters of similar mass and they comprise a predicted, new class of entities. The finding of DSCs will robustly cross-check BH retention; they will not only constrain the uncertain natal kicks of BHs, thereby the widely debated theoretical models of BH formation, but will also pinpoint star clusters as potential sites for GW emission for forthcoming ground-based detectors such as the Advanced LIGO.

Finally, we also discuss the relevance of DSCs for the nature of IRS 13E.

Key words: black hole physics – Galaxy: center – gravitational waves – methods: numerical – open clusters and associations: individual (IRS 13E) – stars: kinematics and dynamics

Online-only material: color figures

1. INTRODUCTION

Compact remnants of massive stars in star clusters, which are neutron stars (NSs) and black holes (BHs), form a dynamically interesting sub-population due to their tendency of segregating toward the cluster’s center and augmenting their population density therein. In this respect, the BHs are special in that they undergo a “runaway” mass segregation. These remnant BHs are typically several 10s of M⊙ heavy, enough to form a Spitzer-unstable sub-system, provided a significant number of them are retained in their parent cluster. Due to this instability (also called the mass-stratification instability; Spitzer 1987), the continually sinking BHs cannot come to an energy equipartition with the local surrounding stars and finally end up in a central, highly concentrated sub-cluster made purely of BHs, which is self-gravitating and dynamically nearly isolated from the rest of the stellar cluster (Merritt et al. 2004; Mackey et al. 2008; Banerjee et al. 2010).

Such a dense environment of BHs is dynamically very active due to the formation of BH–BH binaries via three-body encounters (Heggie & Hut 2003) and their hardening by super-elastic encounters (Heggie 1975) with their surrounding BHs. Studies of the dynamics of pure BH sub-clusters using Monte Carlo and direct N-body integration methods indicate that the dynamical BH–BH merger events they generate are likely to contribute a significant gravitational wave (GW) detection rate to the future Advanced LIGO (AdLIGO) and LISA GW observatories (Portegies Zwart & McMillan 2000; Benacquista 2002; O’Leary et al. 2006; Moody & Sigurdsson 2009; Banerjee et al. 2010; Downing et al. 2011). Such studies show that a BH sub-cluster is typically self-depleted in a few Gyr due to the super-elastic dynamical encounters and the resulting escape of the BHs (O’Leary et al. 2006; Banerjee et al. 2010). The energy extracted from the tight BH–BH binaries heats up and expands the cluster’s core (Merritt et al. 2004; Mackey et al. 2008), which can be detectable by future optical missions such as Gaia.

Furthermore, the BHs can be important for dynamically formed BH X-ray sources due to their encounters with the surrounding stars (Ivanova et al. 2010). X-ray observations have indicated the presence of BH X-ray binary candidates in GCs (Maccarone et al. 2007; Brassington et al. 2010). The presence of a BH sub-cluster within a star cluster, therefore, has the potential to give rise to a plethora of physical phenomena, all of which have significance to upcoming prime missions such as the Gaia, AdLIGO, and the present and future X-ray missions.

Is it possible to obtain any direct observational signature of the presence of a BH sub-cluster with a star cluster? We predict here, for the first time, that within a few kpc from the Galactic center, rapid tidal stripping of star clusters by the strong tidal field can expose its BH sub-cluster. This would happen when the timescale of the preferential removal of stars from the outer regions of the cluster is shorter or comparable to the encounter-driven self-depletion timescale of its central BH sub-cluster (see above). Such a dissolved phase of the cluster would consist of a few stars orbiting around a cluster of BHs and would observationally appear as a highly super-virial star cluster with a large mass-to-light ratio.

As we discuss here, a number of direct N-body computations of model star clusters indeed support the formation of such systems. These objects comprise a predicted, new class of compact stellar populations which we name “dark star clusters” (hereafter DSCs). The importance of DSCs is twofold: on one hand, if they are found to exist, then they guarantee that star clusters are potential sites for GW emission and formation of BH X-ray binaries and, on the other hand, they naturally constrain the uncertain natal kicks of BHs (Willems et al. 2005), as DSCs can form only if a significant number of BHs retain in the cluster following their progenitor supernovae. This, in turn, restricts
2. COMPUTATIONS

We compute the evolution of model star clusters subjected to the Galactic tidal field using the direct N-body integration method. For our purposes, we use the state-of-the-art N-body integration code “NBODY6” (Aarseth 2003), which, apart from utilizing a highly sophisticated numerical integration scheme (Makino & Aarsth 1992), also follows the evolution of the individual stars until their remnant phases, using an analytic but well-tested stellar evolution recipe (Hurley et al. 2000). A unique feature of NBODY6 is its use of highly accurate regularization methods in resolving close encounters (Kustaanheimo & Stiefel 1965; Aarseth 2003). Furthermore, the code exploits the remarkable hardware-accelerated computing capacity of Graphical Processing Units in integrating the centers of masses. NBODY6 currently incorporates general relativistic effects only through analytic prescriptions of GW energy loss.

2.1. Dark Star Clusters

We follow the evolution of initial Plummer clusters (Kroupa 2008) of single stars, having masses between $10^3 M_\odot < M_{c}(0) < 7.5 \times 10^4 M_\odot$ and half-mass radii between 1.0 pc $< r_h(0) < 3.5$ pc. All the clusters initially consist of zero-age main-sequence (ZAMS) stars with their masses $m$ chosen from the canonical initial mass function (IMF; Kroupa 2001) $\rho(m) \propto m^{\alpha}$, where $\alpha = -1.3$ for $0.07 M_\odot < m < 0.5 M_\odot$ and $\alpha = -2.3$ (Salpeter index) for $m > 0.5 M_\odot$. Their metallicities are chosen to be solar, as suitable for our Galaxy’s disk. We assume for now that all the supernova remnants (i.e., BHs and NSs) receive low natal kicks in general such that they remain bound to their parent clusters at their formation. Such models follow circular orbit around a point mass of $M_c = 2 \times 10^{10}$ $M_\odot$ representing the Milky Way bulge.

Figure 1 (top panel) shows an example of the evolution of the virial coefficient for one of our computed model clusters with initially $N(0) = 6.5 \times 10^4$ stars and $r_h(0) = 3.5$ pc, located at $R_G = 2.0$ kpc Galactocentric distance. The orange curve...
shows the time evolution of the virial coefficient, $Q$, for all the members bound to the cluster including the BHs and the NSs, which mostly remains constant at $Q \approx 0.5$, as it should be for the quasi-static relaxation of a self-gravitating system through two-body encounters (Heggie & Hut 2003).

The green curve in Figure 1 (top) shows the time evolution of the virial coefficient taking into account only the luminous objects, i.e., the nuclear burning stars and the white dwarfs, which are those an observer sees. The BHs formed have masses $\approx 10 M_\odot$ and the NSs are typically of $\approx 2 M_\odot$, which are significantly more massive than the majority of the remaining luminous members and hence are segregated to the cluster’s center. As the lower-mass luminous stars in the outer regions are stripped by the external field, the gravitational potential of these central invisible remnants becomes increasingly important. Therefore, the kinetic energy of the luminous sub-system increasingly exceeds the corresponding self-equilibrium (or quasi-static) value as the constituents perceive a potential that becomes increasingly deeper than their self-potential. This correspondingly raises their exclusive virial coefficient, $Q_*$, above 0.5 as in Figure 1 (top panel, green curve). The cluster thus evolves to a DSC state (see Section 2.1.1). Note that while $Q_*$ reaches a very high value, the system as a whole remains bound since $Q < 1$ throughout (except at the final dissolved state, not shown in the figure).

Notably, NSs form a few factors more often than the BHs and contribute significantly in elevating $Q_*$. However, the NSs being lighter than the BHs, their sub-population occupies a more extended zone in the cluster’s center. This causes them to get stripped earlier than the BHs (cf. Figure 1, bottom panel) so that in the late evolutionary phase, typically when $Q_* > 1$, it is mostly the BH population that contributes to the augmented $Q_*$, thereby determining the lifetime of the DSC phase (see Section 2.1.1). Nevertheless, at large enough distances from the Galactic center, where slower tidal stripping causes the DSC phase to appear later than the self-depletion of the BH sub-cluster (see Section 1; Sigurdsson & Hernquist 1993; Merritt et al. 2004; O’Leary et al. 2006; Moody & Sigurdsson 2009; Downing et al. 2010; Banerjee et al. 2010), the NSs constitute the primary dark component of the corresponding DSC state (see Section 2.1.1). This self-depletion process, although operative for both the NS and the BH sub-populations, is more efficient for the latter as it is significantly more concentrated.

### 2.1.1. Galactic Population of DSCs

Figure 2 (top) shows the expected increasing trend in the lifetime of the DSCs with initial cluster mass $M_{cl}(0)$ ($R_G = 2$ kpc, $r_h(0) = 3.5$ pc). The DSC phase can be defined when the cluster appears unbound, i.e., $Q_*>1.0$ or when it appears significantly super-virial, which we take when $Q_*>0.75$, and denote the corresponding lifetimes by $\tau_{DSC}$, with $Q_* = 1.0$ and 0.75, respectively. Figure 2 (bottom) shows $\tau_{DSC,1.0}$ against $R_G$ ($M_{cl}(0) = 3 \times 10^5 M_\odot$, $r_h(0) = 3.5$ pc). $\tau_{DSC,0.75}$ also increases with increasing $R_G$ since the DSC takes a longer time to get depleted in a weaker external field. Notably, for $R_G \gtrsim 4$ kpc, the DSC state becomes NS dominated. Beyond $R_G \gtrsim 5.5$ kpc, it takes more than the age of our Galaxy ($\approx 10$ Gyr) for the representative cluster to evolve to its DSC phase (i.e., $Q_*>0.75$).

It then follows that for the present-day Galaxy the DSCs are formed from clusters with $M_{cl}(0) \gtrsim 10^5 M_\odot$ within $R_G \lesssim 5$ kpc, to be taken as representative numbers, and have lifetimes $\tau_{DSC,1.0} \approx 150$ Myr and $\tau_{DSC,0.75} \approx 250$ Myr as conservative estimates. Although these estimates are based on a point-mass tidal field, note that the DSCs’ progenitor clusters form in the Galactic disk and orbit on nearly circular paths (in the equatorial plane) and hence would experience the same external field with an axisymmetric disk-like distribution of the same mass. The $\tau_{DSC, Q_* - R_G}$ dependence can however be moderately modified as the clusters see a mass increasing moderately with $R_G$.

To estimate the Galactic population of DSCs, we take the average star cluster formation rate over the entire Galactic disk ($R_G < 10$ kpc) to be $0.16 M_\odot \text{yr}^{-1}$ (Larsen 2009) following a Schechter initial cluster mass function (Schechter 1976) over the mass range $100 M_\odot \leq M_{cl}(0) \leq 2 \times 10^5 M_\odot$ (Larsen 2009) which is assumed to remain invariant over the last few hundred Myr. This implies $\approx 0.5$ clusters form per Myr which have properties ($M_{cl}(0) > 10^4 M_\odot$ and $R_G < 5$ kpc) that must have them evolve to DSCs if a sufficiently high fraction of BHs are retained. Assuming a steady state conversion to the DSC phase with lifetime $\tau_{DSC,1.0} \approx 150$ Myr (see above), the expected number of $Q_* > 1$ DSCs in the Galaxy within $R_G < 5$ kpc is $N_{DSC,1.0} \approx 75$. For DSCs with $Q_* > 0.75$, the corresponding number is $N_{DSC,0.75} \approx 125$. Hence, a significant number of DSCs can be expected in the inner Galactic zone.

### 2.2. Can IRS 13E Be a DSC?

There has been recent concern with the widely debated IRS 13E (hereafter IRS13E); an extremely compact stellar association of a few young, massive stars at a close projection to the Galactic center that apparently survives the extreme tidal field by being bound by an invisible mass (Maillard et al. 2004). While a $\approx 1300 M_\odot$ intermediate-mass black hole (IMBH) was widely believed to be this invisible mass (Portegies Zwart & McMillan 2002; Maillard et al. 2004; Portegies Zwart et al. 2006), this possibility has recently been ruled out with a significant confidence by Fritz et al. (2010) through their newer proper-motion measurements, leaving the nature of the invisible component of IRS13E currently ambiguous. This status of IRS13E prompts us to consider whether its dark component can be an ensemble of stellar-mass BHs instead. To that end, we perform preliminary $N$-body calculations to determine whether a DSC configuration resembling IRS13E can be a possible fate of a star cluster very close to the Galactic center.

These computed clusters follow circular orbits around the Galactic supermassive black hole (SMBH; a central mass of $\approx 5.4 \times 10^6 M_\odot$) which is a combination of the mass of the SMBH and that of the nuclear star cluster within $R_G \lesssim 1$ pc) within $R_G < \text{few pc}$, where star formation has been shown to possibly lead to a top-heavy stellar IMF (Bartko et al. 2010; Nayakshin & Sunyaev 2005; Nayakshin et al. 2006; Bonnell & Rice 2008). Since we are primarily interested in the final state, we begin the $N$-body calculations from an evolved phase of the cluster, for computational ease. Therefore, we initiate the computations with Plummer clusters made of stars which are pre-evolved until $t_0 \approx 3.5$ Myr age. This age is slightly earlier than when the most massive star (of $\approx 150 M_\odot$) evolves to a BH. All the stars are thus still on their main sequence.

Observations indicate a very flat IMF ($\alpha = -0.45 \pm 0.3$; see Bartko et al. 2010) for stars close to the Galactic center and also a dearth of low-mass stars. The latter is evident from a declining density of B-stars away from the central SMBH (Bartko et al. 2010) and a significant lack of coronal X-ray emission (Nayakshin & Sunyaev 2005) from the SgrA* field. It is currently unclear from the literature what would be the
Figure 2. Top: the lifetimes, $t_{DSC,Q^*}$, of the DSCs with increasing initial Plummer cluster mass $M_{cl}(0)$ ($R_G = 2$ kpc, $r_h(0) = 3.5$ pc). Bottom: $t_{DSC,Q^*}$ ($Q^* > 1$) as a function of $R_G$ for a Plummer cluster with $M_{cl}(0) = 3 \times 10^4 M_\odot$ and $r_h(0) = 3.5$ pc. (A color version of this figure is available in the online journal.)

lower-mass limit of such an IMF, which, in turn, determines the $\alpha$ of the mass function (MF) and its lower limit $M_l$ at cluster age $\tau_0$. In this preliminary study, we simply take the total number of stars in the cluster $N$, $\alpha$, and $M_l$ at age $\tau_0$ as free parameters. The upper MF limit is chosen to be the canonical $150 M_\odot$ and the stellar metallicity is solar. These limits are the ZAMS values and they are appropriately reduced during the $\tau_0 \approx 3.5$ Myr pre-evolution by the NBODY6’s built-in stellar evolution prescription (see above). Like the computations in Section 2.1, we retain all the BHs (of $\sim 10 M_\odot$ as obtained from within NBODY6) in the cluster. At the beginning of the computations, the models are taken compact to a similar extent as IRS13E (Maillard et al. 2004), with half-mass radii between $0.02$ pc $< r_h(0) < 0.04$ pc and they orbit within $2$ pc $< R_G < 4$ pc.

We find that to reach an IRS13E-like state, i.e., a state where typically $\approx 10$ luminous, young stars are very tightly bound to a cluster of $\approx 100$ BHs, it takes $N \approx 6.5 \times 10^3$ and a rather extreme lower cutoff of $M_l \approx 35 M_\odot$ for MF index $\alpha \approx 0$ (at age $\tau_0$), this $\alpha$ being close to the upper limit of the Bartko et al. (2010) index. A steeper MF requires even higher $M_l$. As a demonstration, it can be seen in Figure 3 (top panel) that this cluster eventually evolves to a configuration consisting of $\approx 130$ BHs, comprising a $\approx 1300 M_\odot$ dark component, orbited by $< 10$ young stars. Although the half-mass radius of the system expands at the beginning of the evolution, primarily driven by the mass loss through the massive stars’ winds and their supernovae, it finally collapses to $\approx 0.02$ pc, as shown in Figure 3 (middle panel), by the time the system arrives at the above configuration. The luminous members in this state include $1–2$ O-stars (those still on the main sequence) and a few helium and Wolf-Rayet stars, thereby being of similar variety as observed in IRS13E. The final state of the system, therefore, resembles IRS13E in terms of compactness and stellar content (Maillard et al. 2004).

Figure 3 (lower panel) shows that the luminous sub-system becomes highly super-virial, i.e., a DSC, as the IRS13E-like state occurs.

The calculations presented in this section are preliminary which suggest an intermediate state of a cluster, close to the
Figure 3. Evolution of the numbers of the luminous members, $N_*$, and of the BHs, $N_{BH}$, for a model star cluster computation, leading to an IRS13E solution. The direct $N$-body computation starts with a Plummer cluster of pre-evolved stars having $N(0) = 6.5 \times 10^3$, $r_s(0) = 0.025$ pc, and $R_G = 4$ pc, where the stars are taken from an $\alpha = 0$ MF within $35 \, M_\odot < M < 150 \, M_\odot$ ZAMS mass (see Section 2.2). The evolution of the Lagrangian radii of the system is shown in the middle panel. The final state of the cluster resembles that of IRS13E. The evolution of the viral coefficient $Q$, for the whole system, and $Q_*$, for the luminous sub-system is shown in the bottom panel.

(A color version of this figure is available in the online journal.)

Galactic center, that would evolve to an IRS13E-like configuration. It remains an open question whether any reasonable initial cluster evolves to such an intermediate phase, which depends on the low-mass limit of the Galactic-central IMF, its index and the initial mass and compactness of the cluster, at a given $R_G$. A scan over these parameters, beginning with much larger and more compact clusters, is necessary to determine such possible initial configuration(s), which is much more compute intensive.
Although the above $M_l$ appears too high for a lower cutoff, its progenitor cluster would have a smaller limit taking into account the rapid tidal stripping over the 3.5 Myr pre-evolution. Tidal stripping also implies a steeper initial index, in better agreement with the observed one.

Although we oversimplify by effectively ignoring the dynamical evolution of the cluster during the stellar pre-evolution, the dynamical history is not instrumental in determining the occurrence of an IRS13E-type state which happens merely due to the competition between stellar evolution and tidal dissolution of the cluster. It is enough that the cluster remains bound by the time most of the stars become BHs quenching the wind mass loss; the system would then core-collapse to a compact configuration irrespective of its history. This is in contrast with the “classical” DSCs discussed in Section 2.1 whose formation depends crucially on mass segregation.

A potential drawback of this “in-situ” model is that it is likely to significantly overproduce the number of massive stars seen in the Galactic center.

Given these drawbacks, the calculations in this section are only to suggest that IRS13E might be a DSC but are by no means conclusive.

3. DISCUSSION

Our calculations (Section 2.1) signify that a gravitationally bound star cluster naturally evolves to an apparent super-virial state, while remaining bound as a whole, as a consequence of the interplay between the dynamics and the evolution of stars, provided a significant number of stellar remnants survive in the bound system after their formation via supernovae. The existence of such dark star clusters is a first-time prediction and serves as an excellent cross-check of the retention of supernova remnants in star clusters, the effects of which is widely explored. The very presence of DSCs would kill two birds with one stone by having consequences on the widely debated theoretical stellar collapse models (Janka et al. 2007) due to the implied direct constraints on natal kicks, and by securing star clusters as potential sources for the forthcoming...
AdLIGO (Harry et al. 2010) GW detector (Portegies Zwart & McMillan 2000; Banerjee et al. 2010). Given that the DSCs must have a significant population in the inner region of our Galaxy, it is tempting to conduct a survey of intermediate-aged (see below) stellar assemblies to identify them.

DSCs can be observationally distinguished from actually dissolving clusters (which are also super-virial and hence expanding and are generally young) through their compact sizes in spite of their intermediate ages, as shown in Figure 4. That the apparent super-virial state of a DSC is not due to an IMBH, would be indicated by the absence of a central cusp in its velocity-dispersion profile, which would be there otherwise (Noyola et al. 2008).

The present studies justify DSCs as predicted, new type of compact stellar populations. An immediate improvement over this study is to explore any effects of varying orbital eccentricity and different initial profile types (e.g., using King instead of Plummer profiles). Also, the effect of a varying star (and hence cluster) formation rate over the Galactic disk needs to be incorporated to determine the predicted Galactic DSC population, as an improvement over our assumption of a uniform average cluster formation. Another development would be to consider a disk-like mass distribution instead of a central point mass (but see Section 2.1.1).

The preliminary computations in Section 2.2 suggest that the dark component of IRS13E can be an ensemble of stellar mass BHs and IRS13E may therefore perhaps be a DSC. However, this conclusion should, for now, be taken as being suggestive rather than conclusive due to the drawbacks discussed (Section 2.2). We postpone a more detailed and self-consistent study on this issue to a future paper.

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