Scalar dark matter-Higgs coupling in the case of electroweak symmetry breaking driven by unparticle

E. O. Iltan *
Physics Department, Middle East Technical University
Ankara, Turkey

Abstract

We study the possible annihilation cross section of scalar dark matter and its coupling $\lambda_D$ to the standard model Higgs in the case of the electroweak symmetry breaking driven by unparticle. Here the annihilation process occurs with the help of three intermediate scalars which appear after the mixing. By respecting the annihilation rate which is compatible with the current relic density we predict the tree level coupling $\lambda_D$. We observe that the unparticle scaling $d_u$ plays a considerable role on the annihilation process and, therefore, on the coupling $\lambda_D$.

*E-mail address: ciltan@newton.physics.metu.edu.tr
Research aims to understand the nature of the dark matter reaches great interest since it contributes almost 23% of present Universe [1] [2] [3]. The existence of dark matter can not be explained in the framework of the standard model (SM) and, therefore, one needs to go beyond. There exist a number of dark matter candidates in various scenarios such as Supersymmetry, minimal Universal Extra Dimensions and Little Higgs with Tparity. It is believed that a large amount of dark matter is made of cold relics and they are in the class nonrelativistic cold dark matter. Weakly Interacting Massive Particles (WIMPs) are among such dark matter candidates with masses in the range 10 GeV–a few TeV. They are stable, interacting only through weak and gravitational interactions and disappearing by pair annihilation (see for example [4] [5] for further discussion). From the theoretical point of view a chosen discrete symmetry drives the stability in many scenarios mentioned above.

In the present work we introduce an additional scalar SM singlet field $\phi_D$, called as darkon, which was considered first by Silveria [6] and studied by several authors [7] [8] [9] [10] [11] [12], and take the lagrangian obeying the $Z_2$ symmetry $\phi_D \rightarrow -\phi_D$

\[ L_D = \frac{1}{2} \partial_\mu \phi_D \partial^\mu \phi_D - \frac{\lambda}{4} \phi_D^4 - \frac{1}{2} m_0^2 \phi_D^2 - \lambda_D \phi_D^2 (\Phi_1^\dagger \Phi_1), \quad (1) \]

where $\Phi_1$ is the SM Higgs field and $\phi_D$ has no vacuum expectation value. The $Z_2$ symmetry considered ensures that the darkon field can appear as pairs and they are stable in the sense that they do not decay into any other SM particles. They can disappear by annihilating as pairs into SM particles with the help of the exchange particle(s). Now, we will introduce an electroweak symmetry breaking mechanism and study its effect on the Darkon annihilation cross section.

A possible hidden sector beyond the SM can be among the candidates to explain this breaking. Such a hidden sector has been proposed by Georgi [13] [14] as a hypothetical scale invariant one with non-trivial infrared fixed point. It is beyond the SM at high energy level and it comes out as new degrees of freedom, called unparticles, being massless and having non integral scaling dimension $d_u$. The interaction(s) of unparticle with the SM field(s) in the low energy level is defined by the effective lagrangian (see for example [15]). The possibility of the electroweak symmetry breaking due to the mixing between the unparticle and the Higgs boson has been introduced in [16] (see also [17]). In [16] the idea was based on the interaction of the SM scalar sector with the unparticle operator in the form $\lambda (\Phi_1^\dagger \Phi) O_U$, where $\Phi$ is the SM scalar and $O_U$ is the unparticle operator with mass dimension $d_u$ (see [18] [19] [20] [21] [22] [23]). By using the fact that unparticles look like a number of $d_u$ massless particles with mass dimension one and, therefore, the operator $O_u$ can be considered in the form of $(\phi^* \phi)^{d_u \over 2}$, the interaction
term

\[ V \sim \lambda (\Phi^\dagger \Phi) (\phi^* \phi)^2, \] (2)

is induced and it drives the electroweak symmetry breaking in the tree level \[16]\). Recently, in \[25]\, this idea has been applied to the extended scalar sector which was obtained by introducing a shadow Higgs one, the complex scalar \( \phi_2 \) in addition to the SM Higgs \( \phi \). This choice leads to a richer scalar spectrum which appears as three scalars, the SM Higgs \( h_I, h_{II} \) and the heavy \( h_{III} \), after the mixing mechanism (see Appendix for a brief explanation of the toy model used.). These three scalars are the exchange particles in our analysis and we have the annihilation process \( \phi_D \phi_D \rightarrow h_I (h_{II}, h_{III}) \rightarrow X_{SM} \). The total averaging annihilation rate of \( \phi_D \phi_D \) reads

\[< \sigma v_r > = \frac{8 \lambda^2 (n_0^2 \rho_0^2)}{2 m_D} \left[ \frac{c_2^2}{(4 m_D^2 - m_I^2) + i m_I \Gamma_I} + \frac{s_2^2 c_2^2}{(4 m_D^2 - m_{II}^2) + i m_{II} \Gamma_{II}} \right] \]

\[ + \frac{s_2^2 s_2^2}{(4 m_D^2 - m_{III}^2) + i m_{III} \Gamma_{III}} \right] \left( \frac{\Gamma(\tilde{h} \rightarrow X_{SM})}{2} \right), \] (3)

where \( \Gamma(\tilde{h} \rightarrow X_{SM}) = \sum_i \Gamma(\tilde{h} \rightarrow X_{i SM}) \) with virtual Higgs \( \tilde{h} \) having mass \( 2 m_D \) (see \[27, 28]\). Here \( v_r \) is the average relative speed of two darkons \[3\] \( v_r = \frac{2 p_{CM}}{m_D} \) with center of mass momentum \( p_{CM} \). The total annihilation rate can be restricted by using the present dark matter (DM) abundance. The WMAP collaboration \[29]\, provides a precise determination of the present DM abundance (at two sigma level) as

\[ \Omega h^2 = 0.111 \pm 0.018. \] (4)

Finally, by using the expression which connects the annihilation cross section to the relic density

\[ \Omega h^2 = \frac{x_f 10^{-11} GeV^{-2}}{< \sigma v_r >}, \] (5)

with \( x_f \sim 25 \) (see for example \[2, 12, 30, 31, 32]\) one gets the bounds

\[< \sigma v_r > = 0.8 \pm 0.1 \, pb,\]

\[1\) See \[24\] for the necessity of the radiative corrections for the electroweak symmetry breaking from hidden sector due to the interaction in the form \( \lambda (\Phi^\dagger \Phi) \phi^* \phi \).

\[2\) Here the \( U(1)_s \) invariant Lagrangian including the shadow sector and the SM one reads

\[ L = L_{SM} - \frac{1}{4} X^{\mu \nu} X_{\mu \nu} + \left( \partial_\mu - \frac{1}{2} g_s X_\mu \right) \phi_2^2 - V(\Phi_1, \phi_2, \phi) \]

where \( g_s \) is the gauge coupling of \( U(1)_s \) (see \[26\]).

\[3\) Here the assumption is that the speed of the dark matter \( \phi_D \) is small enough to have the approximated result (see for example \[12\]).
which of the order of \((1-2) \times 10^{-9}\) GeV\(^{-2}\). This is the case that s-wave annihilation is dominant (see [33] for details.).

**Discussion**

In the present work we extend the scalar sector by considering a shadow Higgs one with complex scalar and in order to achieve the electroweak symmetry breaking at tree level we assume that the unparticle sector, proposed by Georgi [13], couples to both scalars. Furthermore we introduce so called darkon field, which is a SM singlet with vanishing vacuum expectation value and it couples to the SM Higgs doublet, with coupling \(\lambda_D\). After the symmetry breaking considered in our toy model the tree level interaction \(DDh\) appears with strength \(v_0\lambda_D\) and this coupling is responsible for the annihilation cross section which agrees with the present observed dark matter relic density (eq.(4)).

Here we study the dependence of coupling \(\lambda_D\) to the parameters of the model used, the Darkon mass, the scale dimension \(d_u\) and the parameter \(s_0\). In our calculations we take the Darkon mass in the range of \(10 \leq m_D \leq 80\) and use the central value of the annihilation cross section, namely \(<\sigma v_r> = 0.8\) pb. Notice that in the toy model we consider there exist three intermediate scalars which appear after the mixing and they drive the annihilation process with different couplings.

In Fig.1 we plot \(m_D\) dependence of \(\lambda_D\) for \(d_u = 1.1\). Here the solid-long dashed-dashed-dotted line represents \(\lambda_D\) for \(m_I = 110\) GeV, \(s_0 = 0.1\), \(m_I = 110\) GeV, \(s_0 = 0.5\), \(m_I = 120\) GeV, \(s_0 = 0.1\), \(m_I = 120\) GeV, \(s_0 = 0.5\). \(\lambda_D\) is of the order of 0.1 for the small values of \(m_D\), \(m_D \leq 30\) GeV, and for the range \(m_D \geq 70\) GeV, in the case that the mass \(m_I\) is restricted to \(m_I = 110\) GeV and 120 GeV. In the intermediate region \(\lambda_D\) drops and increases drastically. Since there exist three intermediate particles, at the values of \(m_D\) which satisfies the equalities \(m_D = \frac{m_I}{2}(i = I, II, III)\), \(\lambda_D\) decreases to reach the appropriate annihilation cross section which is compatible with the current relic density. In the figure we have two small values of \(\lambda_D\) for each set of \(m_D\), \(d_u\) and \(s_0\) due to resonant annihilations and the third one is out of the chosen \(m_D\) range which does not include the mass values of the order of \(\frac{m_{III}}{2}\). On the other hand \(\lambda_D\) increases up to the values 0.5 due to the effects of possible interferences of intermediate scalar propagators. The figure shows that increasing mass of \(m_I\) result in a shift of \(\lambda_D\) curve and, for its increasing values, \(\lambda_D\) increases (decreases) for light (heavy) Darkon in order to satisfy the observed relic abundance. Fig.2 is the same as Fig.1 but for \(d_u = 1.5\). It is observed that
there is a considerable enhancement in the coupling $\lambda_D$ for the some intermediate values of $m_D$ and first suppressed value(s) of $\lambda_D$ appears for lighter $m_D$ compared to case of $d_u = 1.1$. This is due to the fact that the mass $m_{II}$ becomes lighter with the increasing values of $d_u$ and the resonant annihilation occurs for a lighter Darkon mass.

Now we study $d_u$ and $s_0$ dependence of the coupling $\lambda_D$ to understand their effect on the annihilation rate more clear.

In Fig.3 we present $d_u$ dependence of $\lambda_D$ for $s_0 = 0.1$. Here the solid-long dashed-dashed-dotted-dash dotted line represents $\lambda_D$ for $m_I = 110 \text{ GeV}$, $m_D = 20 \text{ GeV}$-$m_I = 120 \text{ GeV}$, $m_D = 20 \text{ GeV}$-$m_I = 110 \text{ GeV}$, $m_D = 30 \text{ GeV}$-$m_I = 120 \text{ GeV}$, $m_D = 30 \text{ GeV}$-$m_I = 110 \text{ GeV}$, $m_D = 60 \text{ GeV}$. It is observed that for heavy Darkon $\lambda_D$ is not sensitive to $d_u$. For the light Darkon particle case $\lambda_D$ is sensitive to $d_u$ around the numerical values which the threshold $m_D \sim \frac{m_{II}}{2}$ are reached.

Fig.4 represents $s_0$ dependence of $\lambda_D$ for $d_u = 1.5$. Here the solid-long dashed-dashed-dotted-dash dotted line represents $\lambda_D$ for $m_I = 110 \text{ GeV}$, $m_D = 20 \text{ GeV}$-$m_I = 120 \text{ GeV}$, $m_D = 20 \text{ GeV}$-$m_I = 110 \text{ GeV}$, $m_D = 70 \text{ GeV}$-$m_I = 120 \text{ GeV}$, $m_D = 70 \text{ GeV}$-$m_I = 110 \text{ GeV}$, $m_D = 60 \text{ GeV}$. This figure shows that for light (heavy) Darkon $\lambda_D$ decreases (increases) with increasing values of $s_0$ since the increase in $s_0$ causes that the masses $m_{II}$ and $m_{III}$ become lighter. With the decrease in mass $m_{II}$, $m_D$ reaches $\frac{m_{II}}{2}$ and the resonant annihilation occurs for light Darkon. For the heavy one, $m_D$ becomes far from $\frac{m_{II}}{2}$ as $s_0$ decreases and the annihilation rate becomes small.

In summary, we consider that the electroweak symmetry breaking at tree level occurs with the interaction of the SM Higgs doublet, the hidden scalar and the hidden unparticle sector. Furthermore we introduce an additional scalar SM singlet stable field $\phi_D$, which is a dark matter candidate. This scalar disappears by annihilating as pairs into SM particles with the help of the exchange scalars, appearing after the electroweak symmetry breaking and the mixing. By respecting the annihilation rate which does not contradict with the current relic density, we predict the tree level coupling $\lambda_D$ which drives the annihilation process. We see that the unparticle scaling $d_u$ and the parameter $s_0$ play considerable role on the annihilation and, therefore, on the coupling $\lambda_D$. Once the dark matter mass $m_D$ is fixed by the dark matter search experiments, it would be possible to understand the mechanism behind the possible annihilation process and one could get considerable information about the electroweak symmetry breaking.
Appendix

Here we would like to present briefly (see [25] for details) the possible mechanism of the electroweak symmetry breaking coming from the coupling of unparticle and the scalar sector of the toy model used. The scalar potential which is responsible for the unparticle-neutral scalars mixing reads:

\[
V(\Phi_1, \phi_2, \phi) = \lambda_0(\Phi_1^\dagger \Phi_1)^2 + \lambda'_0(\phi_2^* \phi_2)^2 + \lambda_1(\phi^* \phi)^2 + 2\lambda_2 \mu^{2-d_u} (\Phi_1^\dagger \Phi_1) (\phi^* \phi) \frac{\mu}{\mu} + 2\lambda'_2 \mu^{2-d_u} (\phi_2^* \phi_2) (\phi^* \phi) \frac{\mu}{\mu},
\]  

(6)

where \(\mu\) is the parameter inserted in order to make the couplings \(\lambda_2\) and \(\lambda'_2\) dimensionless. In order to find the minimum of the potential \(V\) along the ray \(\Phi_i = \rho N_i\) with \(\Phi_i = (\Phi_1, \phi_2, \phi)\) (see [24])

\[
\Phi_1 = \frac{\rho}{\sqrt{2}} \left( \begin{array}{c} 0 \\ N_0 \end{array} \right) ; \phi_2 = \frac{\rho}{\sqrt{2}} N'_0 ; \phi = \frac{\rho}{\sqrt{2}} N_1,
\]  

(7)

in unitary gauge, and the potential \(V\)

\[
V(\rho, N_i) = \frac{\rho^4}{4} \left( \lambda_0 N_0^4 + \lambda'_0 N'_0^4 + 2 \left( \frac{\mu}{2} \right)^{-\epsilon} (\lambda_2 N_0^2 + \lambda'_2 N'_0^2) N_1^d + \lambda_1 N_1^4 \right),
\]  

(8)

the stationary condition \(\frac{\partial V}{\partial N_i} |_{\vec{n}}\) along a special \(\vec{n}\) direction should be calculated. Finally, one gets the minimum values of \(N_i\), namely \(n_i\) as

\[
n_0^2 = \frac{\chi}{1 + \chi + \kappa} , \quad n'_0^2 = \frac{1}{1 + \chi + \kappa} , \quad n_1^2 = \frac{\kappa}{1 + \chi + \kappa},
\]  

(9)

where

\[
\chi = \frac{\lambda'_0}{\lambda_0} , \quad \kappa = \sqrt{\frac{d_u}{2} \left( \frac{\lambda_0 + \lambda'_0}{\lambda_0 \lambda_1} \right)},
\]  

(10)

for \(\lambda_2 = \lambda'_2\) which we consider in our calculations. By using eq.(9), the nontrivial minimum value of the potential is obtained as

\[
V(\rho, n_i) = -\frac{\rho^4}{4} \left( \lambda_0 n_0^4 + \lambda'_0 n'_0^4 \right) \epsilon.
\]  

(11)

This is the case that the minimum of the potential is nontrivial, namely \(V(\rho, n_i) \neq 0\) for \(1 < d_u < 2\), without the need for CW mechanism (see [24] for details of CW mechanism). The stationary condition fixes the parameter \(\rho\) as

\[
\rho = \rho_0 = \left( \frac{-2^\epsilon \lambda_2 n_1^4}{\lambda_0 n_0^2} \right)^{\frac{1}{\epsilon}} \mu,
\]  

(12)

\footnote{Here \(\epsilon = \frac{2-d_u}{2}\) and \(\vec{N}\) is taken as the unit vector in the field space as \(N_0^2 + N'_0^2 + N_1^2 = 1\).}
and one gets

\[ \rho_0^2 = \left( \frac{\rho_0}{\mu} \right)^2 = 2 \left( \frac{d}{2} \right)^{1/2} \left( -\frac{\lambda_2}{\lambda_0} \right) \left( 1 - \sqrt{\frac{d}{2} \frac{\lambda_0}{\lambda_2}} + \frac{\lambda_0}{\lambda_2} \right), \]  

(13)

by using the eq. (9) with the help of the restriction

\[ \lambda_2 = -\sqrt{\frac{\lambda_0 \lambda_0 \lambda_1}{\lambda_0 + \lambda_0}}. \]  

(14)

Here the restriction in eq. (14) arises when one chooses \( d_u = 2 \) in the stationary conditions.

Now, we study the mixing matrix of the scalars under consideration. The expansion of the fields \( \Phi_1, \phi_2 \) and \( \phi \) around the vacuum

\[ \Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho_0 n_0 + h \end{pmatrix} ; \phi_2 = \frac{1}{\sqrt{2}} (\rho_0 n'_0 + h') ; \phi = \frac{1}{\sqrt{2}} (\rho_0 n_1 + s), \]  

(15)

results in the potential (eq. (3))

\[ V(h, h', s) = \frac{\lambda_0}{4} (\rho_0 n_0 + h)^4 + \frac{\lambda_0}{4} (\rho_0 n'_0 + h')^4 + \frac{\lambda_1}{4} (\rho_0 n_1 + s)^4 + 2^{-\frac{4}{3}} \lambda_2 \mu^2 \epsilon (\rho_0 n_0 + h)^2 (\rho_0 n_1 + s)^{d_u} + 2^{-\frac{4}{3}} \lambda_2' \mu^{2\epsilon} (\rho_0 n'_0 + h')^2 (\rho_0 n_1 + s)^{d_u}, \]  

(16)

and the mass matrix \((M^2)_{ij}\) = \(\frac{\partial^2 V}{\partial \phi_i \partial \phi_j}\)|\(\phi_i=0\) with \(\phi_i = (h, h', s)\) as

\[
(M^2)_{ij} = 2 \rho_0^2 n_0^2 \begin{pmatrix}
\lambda_0 & 0 & -\left( \frac{d_u \lambda_0}{2} \right)^\frac{1}{3} \left( \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_0} \right)^\frac{1}{3} \\
0 & \lambda_0 & -\left( \frac{d_u \lambda_0}{2} \right)^\frac{1}{3} \left( \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_0} \right)^\frac{1}{3} \\
-\left( \frac{d_u \lambda_0}{2} \right)^\frac{1}{3} \left( \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_0} \right)^\frac{1}{3} & -\left( \frac{d_u \lambda_0}{2} \right)^\frac{1}{3} \left( \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_0} \right)^\frac{1}{3} & \left( 2 - \frac{d_u}{2} \right) \sqrt{\frac{d_u}{2}} \sqrt{\frac{\lambda_0 \lambda_1 (\lambda_0 + \lambda_0)}{\lambda_0}}
\end{pmatrix}
\]  

(17)

with eigenvalues

\[ m_i^2 = 2 \lambda_0 n_0^2 \rho_0^2, \]

\[ m_{11}^2 = \lambda_0 n_0^2 \rho_0^2 \left( 1 + 2 - \frac{d_u}{2} \right) \sqrt{\frac{d_u}{2} \frac{s_10 (1 + s_0)}{2 s_0}} - \sqrt{\Delta} \],

\[ m_{11}^2 = \lambda_0 n_0^2 \rho_0^2 \left( 1 + 2 - \frac{d_u}{2} \right) \sqrt{\frac{d_u}{2} \frac{s_10 (1 + s_0)}{2 s_0}} + \sqrt{\Delta} \],

(18)

where

\[ \Delta = d_u \sqrt{\frac{2d_u s_10 (1 + s_0)}{s_0}} + \left( 1 + \left( \frac{d_u}{2} - 2 \right) \sqrt{\frac{d_u s_10 (1 + s_0)}{2 s_0}} \right)^2. \]  

(19)
Here we used the parametrization
\[ \lambda'_0 = s_0 \lambda_0, \quad \lambda_1 = s_{10} \lambda_0. \]  

(20)

The physical states \( h_I, h_{II}, h_{III} \) are connected to the original states \( h, h', s \) as
\[
\begin{pmatrix}
  h \\
  h' \\
  s
\end{pmatrix}
= 
\begin{pmatrix}
  c_\eta & -c_\eta s_\alpha & s_\eta s_\alpha \\
  s_\eta c_\alpha & -c_\eta c_\alpha & -s_\eta c_\alpha \\
  0 & s_\eta & c_\eta
\end{pmatrix}
\begin{pmatrix}
  h_I \\
  h_{II} \\
  h_{III}
\end{pmatrix},
\]

(21)

where \( c_{\alpha(\eta)} = \cos \alpha(\eta), s_{\alpha(\eta)} = \sin \alpha(\eta) \) and
\[
\tan 2 \alpha = \frac{2 \sqrt{s_0}}{s_0 - 1},
\]
\[
\tan 2 \eta = \left( \frac{d_u}{2} \right)^\frac{1}{2} \frac{2 \left( s_0 s_{10} (1 + s_0) \right)^\frac{1}{2}}{(1 - \frac{d_u}{4}) \sqrt{2 d_u s_{10} (1 + s_0) - \sqrt{s_0}}}. \]

(22)

When \( d_u \to 2 \), the state \( h_{II} \) is massless in the tree level and it has the lightest mass for \( 1 < d_u < 2 \). \( h_I \) and \( h_{III} \) can be identified as the SM Higgs boson and heavy scalar coming from the shadow sector, respectively.

The final restriction is constructed by fixing the vacuum expectation value \( v_0 = n_0 \rho_0 \), by the gauge boson mass \( m_W \) as
\[
v_0^2 = \frac{4 m_W^2}{g_W^2} = \frac{1}{\sqrt{2} G_F},
\]

(23)

where \( G_F \) is the Fermi constant. By using eqs. (9) and (13) we get
\[
\hat{v}_0^2 = c_0 \frac{s_{10} \sqrt{2 s_0 (1 + s_0) + s_0 \sqrt{d_u s_{10}}}}{\sqrt{d s_0 (1 + s_0) + (1 + s_0) \sqrt{2 s_{10}}}},
\]

(24)

with \( c_0 = 2 \left( \frac{d_u}{2} \right)^\frac{1}{2} \). The choice of the parameter \( \mu \) around weak scale as \( \mu = v_0 \) results in the additional restriction which connects parameters \( s_0 \) and \( s_{10} \) (see eq. (21) by considering \( \hat{v}_0^2 = 1 \) as
\[
s_{10} = \frac{1 + s_0}{c_0^2 s_0}.
\]

(25)

When \( d_u \to 2, s_{10} \to \frac{1 + s_0}{4 - s_0} \) and when \( d_u \to 1, s_{10} \to \frac{1 + s_0}{2 s_0} \). It is shown that the ratios are of the order of one and the choice \( \mu = v_0 \) is reasonable (see [16] for the similar discussion.)
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Figure 1: $\lambda_D$ as a function of $m_D$ for $d_u = 1.1$. Here the solid-long dashed-dashed-dotted line represents $\lambda_D$ for $m_I = 110 \, GeV$, $s_0 = 0.1 - m_I = 110 \, GeV$, $s_0 = 0.5 - m_I = 120 \, GeV$, $s_0 = 0.1 - m_I = 120 \, GeV$, $s_0 = 0.5$. 
Figure 2: The same as Fig. I but for $d_u = 1.5$.

Figure 3: $\lambda_D$ as a function of $d_u$ for $s_0 = 0.1$. Here the solid-long dashed-dashed-dotted-dash dotted line represents $\lambda_D$ for $m_I = 110 \text{GeV}$, $m_D = 20 \text{GeV}$-$m_I = 120 \text{GeV}$, $m_D = 20 \text{GeV}$-$m_I = 110 \text{GeV}$, $m_D = 30 \text{GeV}$-$m_I = 120 \text{GeV}$, $m_D = 30 \text{GeV}$-$m_I = 110 \text{GeV}$, $m_D = 60 \text{GeV}$.
Figure 4: $\lambda_D$ as a function of $s_0$ for $d_u = 1.5$. Here the solid-long dashed-dashed-dotted-dash dotted line represents $\lambda_D$ for $m_I = 110\, GeV$, $m_D = 20\, GeV - m_I = 120\, GeV$, $m_D = 20\, GeV - m_I = 110\, GeV$, $m_D = 70\, GeV - m_I = 120\, GeV$, $m_D = 70\, GeV - m_I = 110\, GeV$, $m_D = 60\, GeV$. 