Jets of SS 433 on scales of dozens parsecs

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Abstract

The radio nebula W50 harbours the relativistic binary system SS 433, which is a source of the powerful wind and jets. The origin of W50 is wrapped in the interplay of the wind, supernova remnant and jets. The evolution of the jets on the scales of the nebula is a Rosetta stone for its origin.

To disentangle the roles of these components, we study physical conditions of the jets propagation inside W50, and determine deceleration of the jets.

The morphology and parameters of the interior of W50 are analyzed using the available observations of the eastern X-ray lobe, which traces the jet.

In order to estimate deceleration of this jet, we devised a simplistic model of the viscous interaction, via turbulence, of a jet with the ambient medium, which would fit mass entrainment from the ambient medium into the jets of the radio galaxy 3C31, the well studied case of continuously decelerating jets.

X-ray observations suggest that the eastern jet persists through W50 as hollow one, and is recollimated to the opening $\sim 30^\circ$. From the thermal emission of the eastern X-ray lobe, we determine a pressure of $P \sim 3 \cdot 10^{-11}$ erg/cm$^3$ inside W50. In the frame of a theory of the dynamics of radiative supernova remnants and stellar wind bubbles, this pressure in combination with other known parameters restricts W50's origin to a supernova happened $\sim 100,000$ yr ago. Also, this pressure in our entrainment model gives a deceleration of the jet by $\sim 60\%$ in the bounds of W50's spherical component, of radius $\sim 40$ pc. In this case, the age of the jet should be $\ll 27,000$ yr so as to satisfy the sphericity of W50.

The entrainment model comes to the viscous stress in a jet of a form $\sigma = \alpha P$, where the viscosity parameter $\alpha$ is predefined by the model.

1 Introduction

Astrophysical jets derive a significant part of energy released in accretion disks, and influence radically their environment. The powerful jets of SS 433 (Abell & Margon 1979) are so intimately interconnected with the surrounding shell-type radio nebula W50, that the nebula is thought to be not entirely of a supernova’s origin. These jets are radiatively inefficient, that suggests their huge kinetic energy, of flux $L_{k0} \sim 10^{39}$ erg/s, transforms into the thermal and mechanical energies of W50, and the latter is inflated partly by them. The revealed by Bordas et al. (2015) gamma-ray emission, located within the W50 area, points one more item of the jets expenses: up to $10\%$ of percentages of the jets energy might transform into relativistic particles. Moreover, it may indicate a region where the jets decelerate and transmit energy to the nebula: in the interior of W50, not at the nebula shell.

The role of the jets of SS 433 in formation of the peculiar W50 is unknown (see Farnes et al. (2016) for a comprehensive review of the origin of W50). The morphology of W50 in radio emission closely resembles an achatina, the giant African snail (Dubner et al. 1998; see also Fig. 1). W50 looks like consisting of coils, narrowing to the tips of the nebula like a pyramid. The torque and elongated appearance of W50
might be due to the jets, which, however, are not explicitly resolved at the scales of W50 ($1' \times 2'$). By a convention, one discerns in W50 a spherical component, of radius $\sim 29'$ (Dubner et al. 1998), and two protrusions, the so called ears.

Figure 1: The 1–2 keV image of the bright knot region (the XMM-Newton observatory), laid over W50’s radio image (Dubner et al. 1998), depicts the geometry of the eastern X-ray lobe in the hard emission. The images are given at the epoch J2000, north is up, east is left, SS433 (RA = 19°11′49.57, Dec. = 04°58′57″9) is on the right. There are indicated the spherical component of W50, the precession cone and its axis, ticked every 15′ from the beginning at SS433. The borders of Watson et al. ’s (1983) cut, for the radial profile of brightness, are delineated by arrows.

W50 is elongated along the jets, which are observed closely to SS433 as outflows in X-ray, optical and radio bands. At distances from the jets source up to ~ 6'', or at distances $r \leq 0.13\,\text{pc}^{1}$ the jets show a regular precession with period $162.250\,\text{days}$, under angle $19.75\,\text{°}$, around an axis whose inclination to the line of sight is $78.8\,\text{°}$ (Davydov et al. 2008) and position angle on the plane of the sky is $98.2\,\text{°}$ (Stirling et al. 2002). At these distances the jets are ballistic, unchanging (e.g. Fig. 2 of Roberts et al. 2010), except may be the predicted 10% deceleration and twist (the shift of the precession phase by $\sim -0.1$) in the innermost $\sim 0.5''$ (Panferov 2014; see also Stirling et al. 2004). At larger distances the jets become unobservable, possibly because of weakening of the interaction of the jet with the ambient medium. The jet there would

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1$'' = 0.67 \cdot 10^{17}$ cm at the observer distance of $D = 4.5\,\text{kpc}$, accepted hereinafter (Stirling et al. 2002; Panferov 2010; Marshall et al. 2013; see also Lockman et al. 2007, and Panferov 2014 for discussion on the distance); data from the literature are compiled to this distance.
look like a hollow cone of tightly wound coils, probably blending. The cone of the precession is indicated on the radio image (Dubner et al. 1998) of the eastern part of W50 in Fig. 1—evidently, the cone fits the orientation and transverse size of the ear. The overlaid X-ray image in Fig. 1 suggests that the lobe of the X-ray emission, observed at distances $> 15'$ from SS 433 (Watson et al. 1983; Brinkmann et al. 1996), shapes the jet. However, the angular extension of the lobe in the hard X-ray band, $> 1$ keV, is much smaller than the opening 40° of the precession cone. Are the jets recollimated? On the contrary, the optical filaments at the contact between the ears and sphere of W 50 (Boumis et al. 2007, Fig. 1 and 2), which possibly trace the interaction of the jet and spherical shell of W 50, subtend an angle a little more than 40° at SS 433. The eastern lobe, which is more exposed in observations, has sharp edges in the hard X-ray band and is almost perfectly axisymmetrical and smooth, except the bright knot (Fig. 1 Brinkmann et al. 2007, Fig. 3 and 8). The latter is probably the segment of the nearly spherical shell, heated by the jet (Watson et al. 1983).

More than 30 years as the problem of the recollimation of the SS 433 jets inspires its investigation. Eichler (1983) has proposed that a precessing jet merges in a smooth hollow cone and inevitably undergoes focusing by the ambient pressure. Kochanek & Hawley (1990) have ascertained the problems staying before the hydrodynamical simulations of Eichler’s mechanism and of the pointed form of the W 50 ears for the case of a hollow conical jet. Later, from a series of hydrodynamical simulations targeted at the ears geometry, Goodall et al. (2011b) have devised a history of the jet evolution intermittent in the speed and precession cone opening; moreover, they got the recollimation mechanism at work, although inefficient for the formation of the ears of the observed narrowness. However, there are not seen the traits of the intermittency, known for intermittent astrophysical jets: in W 50 the X-ray lobes are rather smooth at large scales.

In particular, Goodall et al. (2011b) resume that in the case of a permanency the SS 433 jet should decelerate only at the terminal shock, i.e. in the ear. On other hand, from the non-observation of the proper motion of the radio filaments in the ears, Goodall et al. (2011a) received that the jets velocity in the ears is at least eight times smaller (for the distance $D = 4.5$ kpc) than the optical jets velocity $v_{e0} = 0.2581c$, $c$ being the speed of light (Davydov et al. 2008). Besides, the brightness of the X-ray ring-like structure at distances $\sim 60'$, thought to be a terminal shock, coincident with the radio filaments in the eastern ear, is much smaller than one of the X-ray lobe at 15–45' from SS 433 (Brinkmann et al. 2007, Fig. 1 and 2). It is hardly to explain unless the jets decelerate in the interior of W 50.

Thus, the questions about the evolution of the jets of SS 433 at W 50’s scale and their role in the inflation of W 50 remain. This paper solves the key question: Could the jets decelerate in the interior of W 50, before the termination in the ears? Firstly, in Sect. 2 we characterize the physical conditions of the jets propagation in the eastern X-ray lobe using the available X-ray observations, namely the density and pressure in the surroundings of the jet. On the basis of the found pressure, the age of the nebula W 50 is evaluated, and the roles of a supernova remnant (SNR), SS 433 system’s wind, and the jets in the nebula formation are clarified in Sect. 3. Encouraged by the studies of the decelerating relativistic jets of the Fanaroff-Riley class I radio galaxies, in Sect. 4.1 we construct the model which would fit mass entrainment from the environment into the jets of the radio galaxy 3C 31, the well known case of continuously decelerating jets. The entrainment model is applied to the SS 433 jet, and results on the jet behaviour at scales of dozens parsecs are presented in Sect. 4.2. The conclusions are given in Sect. 5.

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2 The HEASARC data archive, http://heasarc.gsfc.nasa.gov/FTP/xmm//data/rev0//0075140401/PPS/, the EPIC PN camera of the XMM-Newton observatory.
Deceleration of the SS 433 jets issues from the interaction of the jets with the ambient medium. Physical conditions of the jets propagation are little known. From the Einstein observatory’s observations, Watson et al. (1983) estimated temperature $kT \sim 2$ keV and average electron number density $n_e \sim 0.1$ cm$^{-3}$ between 15° and 42° from SS 433, in the cone of half-angle $\sim 25°$, with an apex at SS 433, aligned with the eastern jet axis, and the density range $\sim 0.16-0.7$ cm$^{-3}$ between the faintest, at $r > 45°$, and brightest, at $r \sim 35°$, regions; their estimation $kT \lesssim 0.17$ keV for the W 50 shell from the non-detection implies an age of W 50 of $\gtrsim 50000$ yr as determined from the Sedov solution for a SNR. Things have changed with the better performances of the XMM-Newton observations of the eastern region of W 50 (Brinkmann et al. 2007; the following parameters are only for the eastern X-ray lobe): in the bright knot at $r \sim 35°$ and in the sampled region closer to SS 433, the temperature turns out rather $\approx 0.22$ keV, ten times smaller, and the filamentary X-ray emission in the radio ear, supposedly of the jet terminal shock, has a temperature of $\approx 0.28$ keV; besides, spectra of these regions are consistent roughly with the solar abundances, have a hard power law component, and all W 50 in the field of view of the observations shines faintly in the soft X-ray, $< 2$ keV. Luminosity, $\gtrsim 6 \cdot 10^{34}$ erg/s for the power law plus thermal components, and number density, $n_e \sim 0.2-0.7$ cm$^{-3}$, were estimated only for the bright knot. Unfortunately, the detail distribution of physical parameters over the lobe was not determined, and those parameters that are found, are not proper parameters of the jet surroundings, being extracted from the radiation projected over different regions of W 50.

The eastern X-ray lobe has the very contrasty narrow conical core of hard emission, $> 1$ keV, mainly of the power law nature (Brinkmann et al. 2007, Fig. 8). This core is more or less uniform, has a flat transverse profile, its opening is $\sim 30°$, although it subtends an angle of 18° at SS 433. The apex of the hard emission core falls slightly above the precession axis, at a distance from SS 433 a little larger than 15°. It is seen also on Fig. 1 where the most contrasty image of the lobe, in the band 1.0–2.0 keV, is overlaid on a radio image of W 50. At the energies $< 1$ keV, this core is not seen against a diffuse background (Brinkmann et al. 2007, Fig. 8), though a hint of the separation between the core and the surroundings could be in the two small troughs in the cut of the ROSAT image in the band $< 1$ keV (Brinkmann et al. 1996, Fig. 8): they are separated by 18° in azimuth relatively SS 433, nearly symmetrically relatively the maximum in the cuts at higher energies. Such X-ray core is thought to map the hollow conical jet per se, which is recollimated and separates an interior spine from an exterior cocoon, and thus the hard emission of the lobe is localized to the jet and its neighborhood. The hollow morphology of the jet is supported also by the ring-like X-ray structure in the ear (Brinkmann et al. 2007). The more wide spread soft emission of the lobe increases barely to the jet axis. We accept therefore that this soft emission has a thermal component, as argued by Brinkmann et al. (2007) for the different regions of the lobe. The indistinguishability of the spin at energies $< 1$ keV means that temperatures of the cocoon and spin are nearly the same, and equal the temperature $\approx 0.22$ keV of the two region of Brinkmann et al.’s (2007) spectral analysis.

Now, we are going to estimate density of the thermal gas of W 50 in the jet surroundings. For this purpose we make use of the normalization of the thermal component of the knot flux, found by Brinkmann et al. (2007) (actually an intricate problem even for the most bright knot) – an unabsorbed flux of $F_0 = 9.9 \cdot 10^{-12}$ erg/cm$^2$ s in the energy band 0.2–2 keV at a distance of 5 kpc, – and scale $F_0$ to the thermal flux of the core using the radial profile of the lobe brightness in the 0.5–4.5 keV energy band, obtained by Watson et al. (1983, Fig. 2) from the cut over the azimuth range $2\phi_0 = 16°$, indicated in Fig. 2 (note that this cut overlays the hard emission core not exactly). From this profile, a ratio of the absorbed count fluxes of the core, taken between the distances 15° and 33°, and the knot, between 33° and 42°, is $a \equiv f^c/f^k = 0.62$, where the indexes c and k mean the core and knot, and the prime means an absorbed value. We note that $f'$ is the background subtracted count flux in the 0.5–4.5 keV energy
band, integrated over an area of the cut. The hardness ratio $f' (>1.5\text{ keV})/f' (<1.5\text{ keV})$, over the ROSAT energy range 0.1–2.4 keV, is nearly constant through the core and knot (Brinkmann et al. 1996, Fig. 9). Besides, the latter two are similar in a spectrum (Brinkmann et al. 2007). Hence, they are nearly equal in a ratio $h \equiv f_{pl}/f_k$ of fluxes of the power law and thermal components, i.e., $h_k \approx h_k$. Then a ratio of the unabsorbed count fluxes of the core and knot equals approximately one of the absorbed fluxes: $f_c/f_k \approx a$. With substitution of the decomposition $f_c = f_{\text{ch}} + f_{\text{pl}} = f_{\text{ch}}(1 + h_k)$, this equation reduces to the proportion

$$a \approx \frac{f_c}{f_k} \approx \frac{f_{\text{ch}}}{f_k} = \frac{F_{\text{ch}}}{F_0}$$

(1)

where one accounts for the parity of the core and knot in $h$ and temperature. What else should be done is to translate the 0.2–2 keV energy flux $F_{\text{ch}}$, emitted by the chosen core region, into the total thermal flux. The translation coefficient of $\approx 3.1$ was determined from the APEC model spectrum (V. Doroshenko, private communication) of an optically-thin coronal plasma of the solar abundances, and of temperature 0.22 keV. Upon these the corresponding total thermal luminosity of the core is $L_{\text{ch}} \approx 3.8 \cdot 10^{34} \text{ erg/s} D_{4.5}^2$, where $D_{4.5} \equiv D/4.5\text{ kpc}$.

The region of the clearly discerned soft emission is symmetrical about the jet axis and subtends an angle of $2\theta_c \sim 60^\circ$ at SS 433 (Watson et al. 1983, Fig. 3; Brinkmann et al. 2007, Fig. 8). A volume of the segment of this region, which contributes into the radial profile of (Watson et al.) (1983) between distances $r_1 = 15$ and $r_2 = 33$, can be approximated by the formula for a cone truncated on the top and the two opposite sides, neglecting the inclination of the jet axis to the plane of the sky, as follows:

$$V_c = \frac{\pi - 2\phi + \sin 2\phi}{3} \tan^2 \theta_c (r_3^2 - r_1^2) = 4.1 \cdot 10^{58} \text{ cm}^3,$$

(2)

where $\phi = \arccos(\tan \theta_e/\tan \theta_c)$. A homogeneous number density of the jet surroundings in the eastern X-ray lobe we estimate as $n_e = \sqrt{\chi L_{\text{ch}}/\Lambda(T)} V_c \approx 0.15 \text{ cm}^{-3} D_{4.5}^{1/2}$. Here, the equilibrium cooling rate $\Lambda = 10^{-22.32} \text{ erg cm}^3/s$ from (Sutherland & Dopita 1993) and the electron to ion number density ratio $\chi \equiv n_e/n_i = 1.1$ for the solar abundances are accepted. For a consistency check, our estimation of the knot density is $n_k \approx 0.56 \text{ cm}^{-3}$ vs. Brinkmann et al.’s (2007) 0.2–0.7 cm$^{-3}$. Fortuitously, nearly the same estimation have been given by Watson et al. (1983) for the lobe in whole, $n_e \sim 0.1 \text{ cm}^{-3}$ for the distance 5.5 kpc, although they used the ten times larger temperature. The ready density estimation $n_e \sim \sqrt{L_{\text{ch}}/V_c}$ gives only an upper limit of the average density in a heterogeneous medium, a case anticipated in the jet vicinity. Here, we used the minuscule information contained in the X-ray observational data. In principle, an exceptional geometry of the X-ray lobe – axisymmetric one, with the axis lying closely to the plane of the sky – allows to restore tridimensional axisymmetric distribution of the lobe emissivity and, therefore, of the gas density.

The temperature and density of the X-ray lobe bound tightly those of the W 50 interior, which we consider here only beyond the first ~15’ from the center. W 50 is rather of the same density as the lobe, $n_e \sim 0.1 \text{ cm}^{-3}$: the jet ejected mass is nothing but $2L_{\text{ch}}t_j/e_g \approx 5 \cdot 10^{-3} \text{ M}_\odot$ over the tentative lifetime of $t_j = 10^4$ yr. And W 50 would have a temperature of ~0.1 keV to be yet observed in whole in the soft X-ray emission (Brinkmann et al. 2007). It can be seen also on the particular cut of W 50, shown by Brinkmann et al. (1996, Fig. 8), that in the energy band <1 keV, dominated by the thermal emission, the X-ray lobe is hardly discernible against W 50, while the latter is noticeable against the background. Then pressure within W 50 is $P_{\text{WSO}} = (1 + 1/\chi) n_e kT \sim 3 \cdot 10^{-11} \text{ erg/cm}^3$. Its increase only several times will make W 50 overall as bright as the X-ray lobes. Surprisingly, this is nearly the pressure claimed by Eichler (1983) for the recollimation of SS 433’s jets. We note that a non-thermal pressure in W 50 of $0.56 \cdot 10^{-11} \text{ erg/cm}^3$ is significantly smaller than $P_{\text{WSO}}$, by our estimation from the data of Dubner et al. (1998) on the synchrotron radio emission of the spherical component; and the thermal pressure found in the X-ray lobe is inconsistent with the gas density ~0.01–1 cm$^{-3}$ in the eastern optical filaments, reported by Abolmasov et al. (2010).
3 Origin of the radio nebula W50

Given the W50 inner pressure $P_{W50}$, we could elucidate the age of W 50, its origin and the role of SS 433’s jets in the nebula evolution. The spherical component of the nebula W 50 could be inflated by a supernova or a powerful wind. In both cases, while the bubble has passed the first short stage, of free expansion into the interstellar medium, the radius of its forward shock develops for long as

$$R(t) = \xi (E_0/\rho_0)^{1/3}t,$$

where $\xi = 1.15$ and 0.88 for the SNR and wind, respectively, $\rho_0$ the density of the interstellar medium, which is assumed henceforth to be homogeneous, $b = 2/3$, $E_0$ the initial energy of the SNR, and $E_0 = L_{\text{int}} t$ in the case of the wind with kinetic luminosity $L_{\text{w}}$ (see Weaver et al. (1977) on a wind bubble, and e.g. Kim & Ostriker (2015) on a SNR). This is so called adiabatic phase, or the Sedov phase in the case of a SNR. Further, after the formation of a dense, thin and cold shell behind the forward shock, as a result of radiative cooling of the swept-up and compressed interstellar gas, the bubble expansion changes: in the case of a wind bubble, only in constant $\xi$, 0.76, and a SNR obeys a new law, $R \propto t^{2/7}$, as is prescribed by the snowplow model of the shell, driven by pressure of the bubble hot interior through the interstellar medium.

W50 has likely passed the adiabatic phase, as it is evidenced by a faint optical emission on the periphery of W 50 (Boumis et al. 2007). The interior of so mature bubble is suggested to be almost isobaric and to have nearly flat profiles of temperature and density due to thermal conduction (Weaver et al. 1977; Chiu & Cox 1992), not to mention the possible turbulent mixing, driven by the jets. Moreover, in any scenario of the W 50 origin, the powerful hypersonic wind from the super-Eddington accretion disc in SS 433 system, of mass flow rate $\dot{M}_{\text{w}} \sim 0.6 \cdot 10^{-4} D_{4.5}^3 M_0/\text{yr}$ (Fuchs et al. 2006; see also Perez & Blundell 2009), would extend only to an inner shock in the bubble interior, before which the gas is cool, $\sim 2 \cdot 10^3 \text{K}$, and tenuous, $n_w \propto r^{-2}$, and beyond the shock the gas becomes hot and dense (Weaver et al. 1977). The shock radius would be of the order of $\sqrt{\dot{M}_{\text{w}}/4\pi P_{W50}} \sim 13 \text{ pc}$, or $\sim 10^4 \text{ yr}$, as determined from continuity of momentum flux at the shock jump, where $v_w = 1500 \text{ km/s}$ is the wind maximal velocity (e.g. Perez & Blundell 2009). However, we should not rely on the isotropy assumed in this estimation. Such inner structure of W50 conforms with the nearly flat appearance of the X-ray lobe, and with the ignition of the X-ray lobe at the radius $\sim 15^\circ$, supposedly the wind inner shock radius (Königl 1983).

A mean pressure of a bubble is determined as $P = (\Gamma - 1) \epsilon_{\text{tot}}/V$, where $\Gamma = 5/3$ is the adiabatic index for thermal plasma, $E_0$ the thermal energy of the bubble interior of volume $V = 4\pi R^3/3$. In the Sedov phase (the adiabatic phase), this energy is constant, $E_{\text{sh}} = E_{\text{sh}0} \equiv 0.717 E_0$, and in the snowplow phase the SNR energy decreases due to adiabatic and radiative losses as $E_{\text{sh}} = 0.8 E_{\text{sh}0}(r_{\text{sh}}/R)^2(t_{\text{sh}}/t)$, the function fitted by Kim & Ostriker (2015) to their numerical simulations. Here

$$t_{\text{sh}} = 4.0 \cdot 10^4 E_{51}^{0.22} n_0^{-0.59} \text{ yr}, \quad r_{\text{sh}} = 22.1 E_{51}^{0.29} n_0^{-0.43} \text{ pc}$$

are the time and radius at the shell formation epoch, $E_{51} \equiv E_0/10^{51} \text{ erg}$, $n_0 \equiv n_H/1 \text{ cm}^{-3}$, and $n_H$ the interstellar hydrogen density. In the case of a wind bubble, $E_{\text{sh}} = (5/11) E_0$ in both the adiabatic and the idealized snowplow phases, that however doesn’t account for the radiative cooling of the hot interior gas, which can be significant at large times, $> 100000 \text{ yr}$ in the particular case of $L_{\text{w}} \sim 10^{36} \text{ erg/s}$ (Weaver et al. 1977). Using the above formulated laws $R(t)$ and $E_{\text{sh}}(R, t)$, we have plotted the contours of $E_0$, $\rho_0$ and $L_w$ on the plane $(t, P)$ in Fig. 2 where $P$ is the mean pressure inside a spherical bubble of the radius 38 pc of W50’s spherical component (at the distance 4.5 kpc). The interstellar density contours for both models are $n_H = 0.4$ and 1.6 cm$^{-3}$, that overlaps the initial density range of the HI gas in W 50’s neighbourhood, found by Lockman et al. (2007) and scaled to the distance 4.5 kpc. The SNR energy contours $E_{\text{sh}} = 0.4$ and 1.6 times $10^{51} \text{ erg}$ just overlap the typical energy $10^{51} \text{ erg}$ of SNRs. The wind power contours $L_w = 4 \cdot 10^{37}$
Figure 2: Contour map of the parameters of interstellar bubbles on the plane (age, pressure). The red solid and long dashed lines on the left – for SNRs. The blue short dashed lines – for wind blown bubbles. The red long dashed line – the line of the shell formation in SNRs: the Sedov phase (adiabatic) lies on the left of this line. The thick lines – the interstellar hydrogen density contours, in units of cm\(^{-3}\). The thin lines – the SNR initial energy contours, in units of 10\(^{51}\) erg, and the wind power contours, in units of 10\(^{37}\) erg/s. The line of circles – the observed pressure of the W50 nebula.

and 2 \cdot 10^{39} \text{erg/s} demarcate a region between the kinetic luminosities of the supercritical accretion disk wind and of the hypothetical wind of the jets: as if they energize W50 isotropically.

In view of the observed pressure \(P_{W50} \sim 3 \cdot 10^{11} \text{erg/cm}^3\), the line of shell formation in Fig. 2 determined by Eqs. [4], restricts the age of W50 as a SNR by \(t_{W50} \geq 97 \text{000 yr}\), and an intersection of the admitted \(n_H\) and \(E_0\) ranges corresponds to the ages just over 100 000 yr – cf. the most probable lifetimes 10 000–100 000 yr of SS 433 as a binary system undergoing a super-Eddington accretion, estimated by King et al. (2000). Further, the disk wind scenario matches well the observed pressure, but at the cost of the uncomfortable large ages \(\sim 300 000–700 000\) yr. However, the unaccounted radiative cooling of the hot interior gas of a wind bubble would shift the bottom parts of the wind contours to the lower ages in the same manner as it bends the SNR contours while they pass the shell formation line. With respect to the jets as a source of the wind, without a preexisted SNR, they would overwhelm W50 in the energy in \(~ 10\) times during a lifetime of \(>100 000\) yr. So, the wind scenario with the admitted \(n_H\) range issues in either the improbable large age or exaggerated pressure. This is due to the fact that a continuous injection of energy is less effective way to blow a bubble than an initial blast, as it is seen from the difference between both models in the constant \(\xi\) in Eq. [3]. The exaggeration of pressure seems to be proper to the mix scenario as well, when the jets are fully slowed down in the interior of a preexisted bubble. In reality, the decelerated jets would distort sphericity of the bubble. All this suggests that the radio nebula W 50 was initiated by a supernova explosion well near 100 000 yr ago, and the SS 433 jets propagate in the preexisted SNR and have to deposit most of their huge kinetic energy beyond the SNR radius, unless their lifetime is small enough:

\[ t_j \ll \frac{E_0}{2\xi L_{40}} \sim 16 000 (E_{51}/\xi) \text{yr} \]

where \(\xi\) is the ratio of the jets energy transfered to the SNR to the total energy of the jets. This SNR would have an initial energy in the range 1.0–3.7 \cdot 10^{51} \text{erg}, bounded by the interstellar density range \(n_H = 0.4–1.6 \text{cm}^{-3}\).

The above picture on the origin of W 50 is built on the assumption that the interstellar medium is homogeneous. In the case of the inhomogeneity, the bubble expands faster, therefore its age for a given radius is smaller, that depends on the volume filling (e.g. Korolev et al. 2015, Fig. 4). At that, the relative
roles of the SNR, disk wind and jets in the origin of W50, discussed above, are thought not to change. We note that just the inhomogeneity allows so small age as \( \sim 100,000 \) yr in the wind blown bubble model devised by Königl (1983).

4 Deceleration of the SS 433 jets in the X-ray lobes

To link the jets of SS 433 observed downstream of the source to only \( \sim 6'' \) with the jet signatures at parsec scales is a challenging problem. In the innermost regions of W 50, the jet travels through the low density cavity, evacuated by the hypersonic wind from SS 433. Just before the wind shock, at the suggested radius \( r_1 \sim 15' \), the isotropic wind hydrogen density \( n_w(r_1) = \dot{M}_w/(4\pi r_1^2 v_w) \sim 3 \cdot 10^{-4} \) cm\(^{-3} \) is much smaller than the observed density of the X-ray lobe, \( m_H \) being the mass of hydrogen. Going through this shock into the significantly more denser medium, some hundreds times, the jet becomes strongly underpressured, and possibly realigns to the state of pressure equilibrium with the surroundings. There are not seen separate strong shock waves, but abrupt appearance of the X-ray lobe. The conspicuous conical core of the lobe, of half-angle \( \theta_c \sim 15^\circ \), suggests that the jet is recollimated to the core opening. Moreover, the jet should be hollow: if all mass within the cone, of mass density \( \rho_a = (1 + 1/\chi) \bar{\mu} n_e \), were involved into the jet, the latter would be as slow as

\[
\frac{1}{r_1 \tan \theta_c} \sqrt{\frac{\Pi_j}{\pi \rho_a}} \sim 700 \text{ km/s,}
\]

at the X-ray lobe beginning due to conservation of the jet momentum flux \( \Pi_j \), that is \( \sim \) a hundredth of the initial velocity, and means almost full dissipation of the jet kinetic energy. This does not stand out in brightness and, therefore, is improbable; besides, the jet should yet thrust the ear and provide the ring like morphology of the terminal shock at distances \( r \sim 60'' \) (Brinkmann et al. 2007). Hereafter, we accept this geometry of the jet, with the tentative half-angle \( \theta_j \sim 4^\circ \) of the filled surface layer of the hollow conical jet, that is ready by the jet nutation at the origin (Borisov & Fabrika 1987), and can be substantiated by width of the brightness depression at the boundary of the hard X-ray core, observed in the soft X-ray cut, that was discussed in Sect. 2.

The fact of observation of the more or less smooth X-ray lobe suggests continuous deceleration of the jet of SS 433. In general, a jet spends momentum via viscous stresses at the interface with the ambient medium. The viscosity probably is dominated by turbulence and magnetic fields. Indeed, in two shearing flows, the boundary between them is inevitably unstable, that gives rise to turbulent mixing of the fluid in the boundary layer (Landau & Lifshitz 1987, §§29, 36). Thereafter, this layer ingests mass from both flows: between vortex-free and vortex layers fluid flows only from the first to the second (Landau & Lifshitz 1987, §35). As a result, in the case of a jet, the latter entrains the ambient medium. This picture is supported by the observed evolution of a transverse velocity profile of the gradually decelerating jets of Fanaroff-Riley class I (FRI) radio galaxies, and by a conservation-law analysis of these jets (see Laing & Bridle (2014) for a comprehensive study of the kinematics and dynamics of FRI jets). However, deceleration of relativistic jets is not predictable by existing models. We note that the jets of SS 433 similarly to FRI jets have not a prominent hot spot at the end, contrary to FR class II jets, which are essentially relativistic until the hot spot.

4.1 Mechanism of mass entrainment in relativistic jets

For the jets of the FRI radio galaxy 3C 31, Laing & Bridle (2002) derived the semiempirical function of the rate of mass injection into the jet (or, rather, the entrainment rate) in dependence on the jet distance \( r \) from the galaxy nucleus on the basis of the laws of conservation of mass, energy and momentum. Further, Wang et al. (2009) derived the entrainment rate for the case of equilibrium in pressure of these jets with
their surroundings. Fig. 3 shows the longitudinal profile of mass flux $\dot{q}_e$ of the entrained surrounding matter per unit area of the jet surface, henceforth the external entrainment, for the 3C31 jets, obtained from Wang et al.’s (2009) boundary-layer entrainment $g$ and internal entrainment $g_s$, which could be from the supposed jet-contained stars, as follows:

$$\dot{q}_e = \frac{1}{2\pi R_j} \frac{d}{dr} (g - g_s), \quad (7)$$

where $R_j$ is the jet radius, given by the jet geometry from Wang et al. (2009). However, it is not clear how much correct is the external entrainment, defined in this way, just subtracting the internal entrainment, because Wang et al. (2009) didn’t account for the internal entrainment in their model of the boundary-layer entrainment, while the latter is the total entrainment required by the jet kinematics. The internal entrainment rate normalized to unit area of the jet surface is also shown in Fig. 3. As Wang et al. (2009) have noted, uncertainties of the internal entrainment defined by their Eq. (37) are large.

Looking for a formalism of the entrainment, which would hopefully fit any relativistic jet, we have confronted the profile $\dot{q}_e(r)$ in Fig. 3 with a simplistic model. Let in a frame comoving with the mean local flow in the medium adjacent to a jet, the velocity of turbulent irregular pulsations in this flow is $v_t$, and the velocity of the jet $v_j$. The pulsating component of a movement we restrict for the sake of simplification only to six reciprocally orthogonal directions, e.g. along the Cartesian coordinate axes, with X axis aligned with the vector $v_j$, so that in the comoving frame at any point of the average jet boundary turbulent fluid moves into the jet only one sixth of time. Hence, the density of mass flux into the jet is $\rho_a v_t / 6$, $\rho_a$ being the mass density of the ambient medium. A part of the mass flux is swallowed by the jet, and the remainder continues the turbulent dancing with the adjacent flow. Namely, we impose that the jet absorbs only the component of momentum $p$ of the turbulence pulse in the jet rest frame, which is perpendicular to the jet boundary, i.e. a proportion of the absorbed part of the momentum is

$$\sigma_e \equiv \frac{p_\perp}{p} = \frac{v_\perp}{\sqrt{v^2 + v^2_\parallel}} = \frac{\eta}{\sqrt{\gamma^2 + \eta^2}}, \quad (8)$$

under constancy of mass density of the turbulent fluid, where $v_\perp = v / \gamma$ is the perpendicular component of the velocity of the turbulence pulse in the jet rest frame, $v_\parallel = v_j$ the parallel component, $\eta = v_t / v_j$, and $\gamma = (1 - v_j/c)^{-1/2}$ is the Lorentz factor (for relativistic transformations of velocity see e.g. Landau & Lifshitz 1971, §5). The parameter $\sigma_e$ is in essence the cross-section of the entrainment process. Note that
\( \sigma = \sin i \), where \( i \) is the inclination of the velocity of the turbulence pulse to the jet boundary in the jet rest frame. Then, the flux density of the entrainment is

\[
\dot{q}_e = \sigma \beta \rho_a v_j = \frac{P}{v_j},
\]

where \( \beta = 1/6 \), \( P = \rho_a c_s^2 / \Gamma \) the external pressure, \( c_s \) being the sound speed in the external medium,

\[
\alpha = \beta \Gamma M_t^2 \frac{1}{\sqrt{\gamma^2 + \eta^2}}
\]

the dimensionless viscosity parameter, and \( M_t = \alpha / c_s \) the Mach number of the turbulence. The matter entrained from the surroundings is mixing with the jet matter, and accelerating to the jet velocity \( v_j \), that should be provided by the flux of the jet momentum

\[
\sigma = \dot{q}_e v_j = \alpha P
\]

per unit area of the jet surface, transferred normally to the jet boundary. That is just definition of a viscous stress. It appears that the viscous stress at the jet boundary has the functional form just as in Shakura & Sunyaev’s (1973) \( \alpha \)-model for accretion disks, and as have been derived by Begelman (1982) for extragalactic jets to explain their heating via viscous stresses.

To plot the entrainment function, we used the profiles of pressure \( P(r) \), density \( \rho_a(r) \) and velocity \( v_j(r) \) from Wang et al. (2009). A Mach number of the turbulence of \( M_t = 1 \) was accepted as the best guess for supersonic jets. In Fig. 3 this function fits the overall data on the 3C31 jet, the magnitude and slope of the external entrainment profile, and does it excellently for the jet region beyond \( \sim 6 \) kpc if \( 1/5 \) is used instead of the coefficient \( \beta = 1/6 \) in Eq. (9). The worse match before \( \sim 6 \) kpc can be explained partly by a difference between the flaring region at 1.1–3.5 kpc, where the jet realigns (Wang et al. 2009), and the more quiescent outer jet, and, on the other hand, by the increase of the internal entrainment upstream of the jet (Fig. 3), which is crudely estimated and subtracted to get the external entrainment. Such successful fit encourages us to accept the model and apply it to the SS433 jets, under the coefficient \( \beta = 1/5 \).

4.2 Entrainment model of deceleration of the SS 433 jets

How much the velocity of the SS 433 jet changes with the distance due to the injection of matter from the ambient medium we can derive straightforwardly from the relationship \( \dot{v} \propto \Pi / M \) between momentum flux \( \Pi \), which is constant, and mass flux \( M \). The derivation is following. For the jet flow of the rest-frame mass density \( \rho_j \) and enthalpy \( \omega = \rho_j c_s^2 + \epsilon + P \), where \( \epsilon \) is the internal energy density, and \( P \) is the pressure, with velocity vector \( v_j \) and Lorentz factor \( \gamma \), the flux densities of mass and x-component of momentum through the surface piece, which outward unit normal vector is \( n \), are respectively

\[
\dot{\rho} = \gamma \rho_j (nv_j), \quad \dot{p}_x = \gamma^2 \frac{\omega}{c^2} v_{jx} (nv_j) + P n_x \approx \gamma \dot{v}_{jx} + P n_x,
\]

where index \( x \) means the component of a vector along the jet axis. The right part of Eq. (13) is derived neglecting the terms \( \epsilon \) and \( P \) in the enthalpy \( \omega \) of the mild relativistic jets of SS 433.

Let the closed surface of an integration of the flux densities consists of the two cuts of the jet by the spheres of radii \( r_0 \) and \( r \), centered at the jet source, and the jet boundary surfaces between them, so that the velocity vector \( v_j \) is normal to the cuts and tangential to the boundary surface. The jet density and velocity are supposedly functions of only the distance \( r \), i.e. have flat profiles over the spherical cuts, that could be provided by turbulent mixing. Then, for the hollow conical jet, the mass fluxes in absolute value are

\[
M_j(r) = \gamma \rho_j v_j S,
\]

10
Figure 4: The mass flux along the SS 443 jet provided by the entrainment, $\dot{M}_e$, in units of initial mass flux, $\dot{M}_{j0}$.

through the jet cut of an area $S$, and

$$\dot{M}_e = 2\pi (\sin(\theta_c - \theta_j) + \sin(\theta_c + \theta_j)) \int_{r_0}^{r} \dot{q}_e(\zeta) \zeta d\zeta, \quad (15)$$

through the jet surfaces between the cuts, where the entrainment flux density is assumed the same on the both sides of the hollow jet. Over the chosen above closed surface, the integrations of the densities \(9, 12\) for the flux of mass, and \(13\) for the flux of momentum give zeros:

$$-\dot{M}_j(r_0) - \dot{M}_e + \dot{M}_j(r) = 0, \quad (16)$$

$$-f(\gamma \dot{M}_j v_j + PS)_r - f(\gamma \dot{M}_j v_j + PS)_r = 0, \quad (17)$$

in the case of the jet in a steady state, one more idealization of the model. Here, the first and third parts in both equations are the fluxes through the spherical cuts at radii $r_0$ and $r$, respectively, $f$ the modulus $|n_x|$ averaged over a cut, and the momentum injected through the jet boundary surface by the entrained matter is neglected, because it is miniscule. The buoyancy term

$$F = 2\pi (\sin(\theta_c + \theta_j) - \sin(\theta_c - \theta_j)) \int_{r_0}^{r} P d\zeta \quad (18)$$

summed with other members of Eq. \(17\), containing pressure $P$, results in zero in the case of pressure equilibrium of the SS 433 jet with the ambient medium, assumed to be isobaric. Then, Eq. \(17\) is reducing to the anticipated ratio of the jet velocities at radii $r$ and $r_0$

$$\frac{v_j(r)}{v_j(r_0)} = \frac{\gamma(r_0) \dot{M}_j(r_0)}{\gamma(r) \dot{M}_j(r)} \approx \frac{\dot{M}_j(r_0)}{\dot{M}_j(r)}, \quad (19)$$

where the approximation by the right part has a maximal error of 3.6%. Eqs. \((19, 16, 15, 9)\) have a solution

$$v_j(r) = v_j(r_0) \exp \left\{ -\frac{\alpha A P (r^2 - r_0^2)}{2\dot{M}_j(r_0)v_j(r_0)} \right\}, \quad (20)$$

under the conditions $\eta \ll \gamma = 1$, and constancy of the viscosity parameter $\alpha$ and the pressure $P$, where $A = 2\pi (\sin(\theta_c - \theta_j) + \sin(\theta_c + \theta_j))$.

The profile along the jet of the mass flux $\dot{M}_e$ in Fig. 4 which follows from the system of Eqs. \((19, 16, 15, 9)\), shows a rate of the loading of mass from the surroundings into the SS 433 jet as much as the initial jet mass flux $\dot{M}_{j0} = 2L_{j0}/v_{j0}^2$. There were used the parameters $\beta = 1/5$ and $M_t = 1$, the jet
geometry with \( \theta_j = 4^\circ \) and \( \theta_c = 15^\circ \), and the uniform physical conditions \( P = 3 \cdot 10^{-11} \text{erg/cm}^3 \) and \( n_0 = 0.1 \text{cm}^{-3} \) in the region \( r > 20 \text{pc} \) (Sect. 2). This loading issues in a relative decrement of the jet speed of 
\[ \delta V_j = 1 - V_j(r_0) = \kappa/(1 + \kappa) \approx 63\% \]
 at the distance of the X-ray bright knot ~ 35\' (~ 46 pc), i.e. yet before the entry into the ear, where \( \kappa = M_j/M_\nu(r_0) \) and the radius \( r_0 \) refers to the parameters at the jet beginning. The jet kinetic luminosity is sinking by the same part, \( 1 - L_j/L_\nu(r_0) = \delta V_j \). The mass loading in the wind cavity region, at \( r < 20 \text{pc} \), is insignificant, because of low pressure. In contrast, the density and pressure in the region of the shell, marked by the X-ray bright knot, should only rise, and the mass loading should do so.

5 Concluding remarks

The morphology of the eastern X-ray lobe in W50 evidences a continuous proceeding of SS433’s jet through the nebula on scales of dozens parsecs. At that, the jet is found to be recollimated from the opening 40\° to the ~ 30\°. The X-ray brightness distributions in the soft and hard energy bands of the XMM-Newton observations are consistent with the hollow structure of the jet. This picture suggests that the jet bypasses the X-ray brightest knot, which resides at the place of the shell existed even before the jet.

From the X-ray observations presented in (Watson et al. 1983; Brinkmann et al. 1996; Brinkmann et al. 2007), we derived an electron density of \( \sim 0.1 \text{cm}^{-3} \) and a pressure of \( \sim 3 \cdot 10^{-11} \text{erg/cm}^3 \) for the thermal gas in the spherical component of W50. We note that only the XMM-Newton observations have allowed Brinkmann et al. (2007) to separate surely the thermal components of the X-ray emission of the bright knot and, less surely, of the region to the west of the knot. However, they have not characterized the physical conditions in the X-ray lobe in whole. Solely a model of SNR dynamics predicts the observed pressure in conjunction with the reliable ages of SS433 10,000–100,000 yr (King et al. 2000), not a model of stellar wind bubble. Though, this is not conclusive in the case of an inhomogeneous ambient medium, when the wind origin of W50 is possible too (Konigl 1983). Such small observed pressure pinpoints W50 as an old SNR of age \(~ 100,000 \text{yr} \), which is obtained accounting for radiative losses of the SNR in the pressure driven phase, unlike all previous estimations of W50’s age, based on the Sedov model. In the case of an inhomogeneous interstellar medium, the age would be smaller. An influence of the jets on the spherical component would be restricted to prevent a large distortion of the sphericity, therefore the jets exhaust most of their energy into the ears or are much younger than the SNR. The thermal energy content in the W50 nebula is \( \sim (\Gamma - 1)P_{50}(V_{\text{SNR}} + V_E + V_W) = 4.4 \cdot 10^{50} \text{erg,} \) where \( V_{\text{SNR}} = 6.6 \cdot 10^{50} \text{cm}^3, \) \( V_E = 2.4 \cdot 10^{60} \text{cm}^3, \) and \( V_W = 6.9 \cdot 10^{59} \text{cm}^3 \).

The simulations of Goodall et al. (2011b) provide an important view that the jet-SNR interaction doesn’t influence the expansion of W50’s spherical shell, and the jets are a relatively young phenomenon, \( \lesssim 20,000 \text{yr} \), the age they give to the nebula W50. The latter issues naturally from an amount of the jets momentum imparted to the ears, and hence from the ears size. Besides clarification of the role of the jets in the origin of W50, this justifies the application of theories of the spherical wind bubbles and SNRs to the evolution of W50. However, so small age of W50’s spherical component, obtained in the simulations, contradicts not only to our result, based on a theory for the shock dynamics of SNRs, but also to the simulations on SNRs elsewhere (e.g. Kim & Ostriker 2015; Korolev et al. 2015).

Goodall et al. ’s (2011b) simulations invoke the intermittent activity of the jets for the observed shape of the ears. At that, the continuous activity, coupled with the recollimation and deceleration, appeals more urgently as the observed non-unique properties of the broad class of jets, of radio galaxies of Fanaroff-Riley Class I (Laing & Bridle 2014). There seems the sameness between them and SS 433’s
jets: the pressure jump in the surroundings (afforded possibly by the wind in the case of W50), beyond which the jets brighten, recollimate and decelerate (the latter two are obvious only for FRI jets), and the faintness of terminal shock at the jet head (contrary to the prominent hotspots of FRII jets) – the characteristic of decelerated jets.

Indeed, the derived pressure of W50's interior turns out to be so large as to cause deceleration of the jet via the viscous jet-surroundings interaction. There is as yet no theory of this interaction for relativistic astrophysical jets. The studies of FRI jets suggest that these jets decelerate continuously via entrainment of the ambient medium. We devised the model of the entrainment, which successfully fits the semiempirical profile of entrainment rate for the jets of the radio galaxy 3C31, found by Wang et al. (2009). By this model, the entrainment results from turbulence at the jet surface, that exerts in effect a viscous tension in the jet $\sigma = \alpha P$, of the well known form in the theory of accretion disks, where the viscosity parameter $\alpha$ is defined by the model. This model predicts that the eastern jet of SS433 decelerates by $\sim 60\%$ in bounds of the circular shell of W50.

Thus, the decelerated jet would inject $\sim 1/2$ of its energy into W50's spherical component. For the sphericity, the age of the jets, Eq. (5), should be much smaller than 27,000 yr, that is consistent with the age delimitation in Goodall et al. (2011b). Also, there seems to be a balance between the jet power and the luminosity of the X-ray lobe. During tentative lifetime $t_J \sim 10^4$ yr the jet has put approximately a half the ejected energy $W_J = L_{\text{bol}} t_J \sim 3 \cdot 10^{50}$ erg in the expansion of W50, another part has been deposited into the thermal energy of the interior gas. While an estimation $W_{\text{th}} = L_{\text{bol}} t_{\text{th}} \sim 3 \cdot 10^{49}$ erg, where $L_{\text{th}} \sim 10^{35}$ erg/s is the thermal luminosity of the X-ray lobe (Sect. 2), and $t_{\text{th}} \sim kT/n_e \Lambda \sim 10^7$ yr the time of the radiative cooling of the lobe, gives for the thermal energy $\sim 1/10$ of the $W_J$ vs. the above proportion $\sim 1/2$. The discrepancy by a factor of 5 is possibly attributable to roughness of our estimations.

Further studies of the distribution of physical parameters in the nebula W50, in particular on the basis of the available X-ray data, would improve the above picture of the evolution of SS433’s jets on scales of dozens parsecs.

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