A Soluble Theory of Massless Scalar $QED_2$

F. T. Brandt†, Ashok Das‡ and J. Frenkel†
†Instituto de Física, Universidade de São Paulo
São Paulo, SP 05315-970, BRAZIL
‡Department of Physics and Astronomy, University of Rochester
Rochester, NY 14627, USA

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Abstract

In this brief report, we analyze a generalized theory of massless scalar $QED_2$ and show that, unlike the conventional scalar $QED_2$, it is free from infrared divergence problems. The model is exactly soluble and may describe, in an 1+1 dimensional space-time, noninteracting spin-one tachyons.
It is well known that massless particles give rise, in general, to infrared divergence problems in quantum field theories [1]. For example, let us consider the theory of scalar $QED$ in $1 + 1$ dimensions ($QED_2$), described by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu\phi)^* D^{\mu}\phi - m^2 \phi^* \phi$$  \hspace{1cm} (1)$$

where

$$D_\mu\phi = (\partial_\mu + ieA_\mu)\phi$$  \hspace{1cm} (2)$$

In this theory, the photon self-energy, at one loop, is given by the contributions shown in Fig. 1. These diagrams are easy to evaluate and give (We would use a gauge invariant regularization throughout.)

$$\Gamma^{\mu\nu}(p) = -ie^2 \int \frac{d^2k}{(2\pi)^2} \left[ -\frac{2\eta^{\mu\nu}}{k^2 - m^2 + i\epsilon} + \frac{(2k + p)^\mu(2k + p)^\nu}{(k^2 - m^2 + i\epsilon)((k + p)^2 - m^2 + i\epsilon)} \right]$$

$$= \frac{e^2}{\pi} \left( \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \left[ \frac{R}{2} \ln \left( \frac{1 + R}{1 - R} \right) - 1 \right]$$  \hspace{1cm} (3)$$

where

$$R = \sqrt{1 - 4m^2/p^2}$$  \hspace{1cm} (4)$$

It is clear now that the photon self-energy in Eq. (3) is ultraviolet finite, but diverges like $\ln (1/m)$ as $m \to 0$. Furthermore, it can be checked that the higher order contributions do not lead to an improved infrared behavior of the photon self-energy. This is the problem with massless scalar fields in $1 + 1$ dimensions and it is well known that even supersymmetric theories involving massless scalar fields suffer from this problem [2]. In this brief report, we present a generalized scalar $QED_2$ which is free from infrared problems and bring out various other interesting properties associated with this system.
Let us consider two flavors of complex scalar fields, $\phi_a, a = 1, 2$, and the theory described by the Lagrangian density
\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_{\mu} \phi_a)^* D^\mu \phi_a - m^2 \phi_a^* \phi_a - \frac{ie}{2} \epsilon^{\mu\nu} F_{\mu\nu} \phi_a^* (\sigma_3)_{ab} \phi_b,
\]
where $\epsilon^{\mu\nu}$ is the antisymmetric Levi-Civita tensor in 1+1 dimensions, with $\epsilon^{01} = 1$. This theory differs from the conventional scalar $QED_2$ of Eq. (1) (for two flavors) because of the last term in Eq. (5) which has the form of a “Pauli interaction”. It is again straightforward to calculate the photon self-energy in this theory where one has to evaluate, in addition, only the contribution coming from the new interaction term. Instead of the result given in Eq. (3), one finds in this case that
\[
\Gamma^{\mu\nu}(p) = -\frac{2e^2}{\pi} \left( \eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \left[ 1 + \frac{2m^2}{p^2} \frac{1}{R} \ln \left( \frac{1 + R}{1 - R} \right) \right]
\]
It is interesting that the second term in the bracket on the right hand side of Eq. (6) vanishes when $m \to 0$. Thus, unlike the conventional scalar $QED_2$ of Eq. (1), this theory is infrared finite, at least to one loop order, in the vanishing mass limit. Consequently, it may be worth analyzing the properties of this model further.

First, let us note that the extra coupling in Eq. (5) is purely imaginary. This implies that the generalized theory in Eq. (5), is not Hermitian in Minkowski space. However, it is worth noting that this theory is invariant under the combined operations of PT. Such theories are of interest in recent literature [3]. Although non-Hermitian interactions may lead, in general, to problems of unitarity, their effect in this theory is not so problematic. To understand this, we first remark that the model is actually Hermitian in Euclidean space. It is well known [4], that the path integral which defines the generating functional is best evaluated by rotating to Euclidean space, where path integrals are well behaved. Then, the massless scalar fields can be integrated out in the path integral, leading to an exact action which is Hermitian. After rotating back to Minkowski space, we obtain
\[
\Gamma_S = -i \ln \left[ \frac{\det(-D_{\mu} D^\mu - \frac{ie}{2} \epsilon^{\mu\nu} F_{\mu\nu} \sigma_3)}{\det(-\partial_{\mu} \partial^{\mu})} \right]^{-1}
\]
where $\Gamma_S$ represents the contribution of the scalar loops to the effective action. Now, choosing
\[
\gamma^0 = \sigma_2; \quad \gamma^1 = i\sigma_1; \quad \text{and} \quad \gamma_5 = \gamma^0 \gamma^1 = \sigma_3
\]
and using the two-dimensional identity
\[ \gamma^\mu \gamma^\nu = \eta^{\mu\nu} + \epsilon^{\mu\nu\gamma_5} \]

it is easy to see that
\[ \Gamma_S = -i \ln \left[ \begin{array}{c} \det(-\partial^2) \\ \det(-\partial^2) \end{array} \right]^{-1} = 2i \ln \left[ \begin{array}{c} \det(i\not\!\partial) \\ \det(i\not\!\partial) \end{array} \right] \]

The quantity in the bracket, on the other hand, is the determinant for a massless fermion interacting minimally with the photon field in 1+1 dimensions (Schwinger model). It is exactly soluble and is free from infrared problems leading to
\[ \Gamma_S = -e^2 \pi \int d^2 x A_\mu \left( \eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right) A_\nu \]

Thus, we see that the generalized scalar QED\(_2\) (Eq. (5)) is exactly soluble in the massless limit and has the form similar to that obtained in the case of the massless Schwinger model. Although the additional interaction in (5) is non-Hermitian, the effective action for the theory is Hermitian,
\[ \mathcal{L}_{eff} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{e^2}{\pi} A_\mu \left( \eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right) A_\nu \]

Even though the effective action in (12) has the same form as that obtained in the case of the massless Schwinger model, there is one crucial difference. Namely, the mass term for the photon has the wrong sign. We would like to emphasize that this is not a consequence of the additional non-Hermitian interaction in Eq. (5). Rather, it is a reflection of the bosonic nature of the fundamental fields being integrated out. (Fermions have an extra negative sign coming from the loops, or equivalently the powers of the determinant for bosons is inverse of those for fermions\[4\].) The additional interaction merely cancels the infrared divergence of the theory, as can be seen from (3) and (6). As a result of this, the effective photon theory (12) is tachyonic and we can think of the tachyonic photon as a bound state of scalar fields. Tachyonic quantum field theories have, of course, been discussed in the literature\[4\], but we would like to emphasize that, unlike earlier discussions, this is a spin-1 theory of tachyons.
It is also interesting to note here that in a theory of scalars and fermions interacting with photons described by the Lagrangian density \((a = 1, 2)\)

\[
\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}_a D^\mu \phi_a + (D_\mu \phi_a)^* D^\mu \phi_a - \frac{i\epsilon}{2} \epsilon^{\mu\nu} F_{\mu\nu} \phi_a^* (\sigma_3)_{ab} \phi_b,
\]

the radiative corrections due to bosons and fermions cancel, leading to an effective Lagrangian density

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}
\]

Namely, in such a theory, the photon would remain massless. This is quite interesting since radiative corrections due to fermions and bosons generally cancel in supersymmetric theories whereas the theory of Eq. (13) does not appear to have any supersymmetry. Indeed, we have not been able to find any conventional supersymmetry transformation which leaves the theory of Eq. (13) invariant. Rather, we find several equivalent, BRST-like nilpotent fermionic symmetries of this theory. For example, it is easy to check that the theory of Eq. (13) is invariant under the symmetry transformations

\[
\begin{align*}
\delta \phi_a &= \sigma_{ab}^+ \bar{\epsilon} (1 + \gamma_5) \psi_b \quad ; \quad \delta \phi_a^* = \sigma_{ab}^+ \bar{\psi}_b (1 + \gamma_5) \epsilon \\
\delta \bar{\psi}_a &= -\sigma_{ab}^+ \epsilon (1 + \gamma_5) \gamma^\mu (i D_\mu \phi_b)^* \quad ; \quad \delta \psi_a = -\sigma_{ab}^+ \gamma^\mu (i D_\mu \phi_b) (1 + \gamma_5) \epsilon \\
\delta A_\mu &= 0
\end{align*}
\]

where \(\sigma^+ \equiv (1 + \sigma_3)/2\) and \(\epsilon\) is a constant Majorana spinor. The fact that these transformations do not form a self-adjoint set may be a reflection of the non-Hermitian nature of the Lagrangian density (13).

Finally, we note that, instead of (5), if we consider a fermionic theory of the form

\[
\mathcal{L}_f = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \psi)(D^\mu \psi) - \frac{i\epsilon}{2} \epsilon^{\mu\nu} F_{\mu\nu} \bar{\psi} \gamma_5 \psi
\]

then, the additional interaction, in this case, would exactly represent the Pauli interaction and this Lagrangian density would be Hermitian. This is a second order fermion Lagrangian density. Normally, higher derivative theories lead to problems of ghosts. However, in this case, it can be easily checked following the steps (7)-(10) (with appropriate identifications) that the effective action is identical to that obtained in the case of the massless Schwinger model. There is no problem of ghosts and that one can think of Eq. (16) as a second order representation of the Schwinger model.
In conclusion, we have analyzed a generalized, massless scalar $QED_2$ and shown that it is free from infrared divergences. We have tried to bring out various other interesting properties associated with this model.

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