Multicomponent fractional quantum Hall states with subband and spin degrees of freedom

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In wide GaAs quantum wells where two electric subbands are occupied we apply a parallel magnetic field or increase the electron density to cause a crossing of the two \( N = 0 \) Landau levels of these subbands and with opposite spins. Near the crossing, the fractional quantum Hall states in the filling factor range \( 1 < \nu < 3 \) exhibit a remarkable sequence of pseudospin polarization transitions resulting from the interplay between the spin and subband degrees of freedom. The field positions of the transitions yield a new and quantitative measure of the composite Fermions’ discrete energy level separations. Surprisingly, the separations are smaller when the electrons have higher spin-polarization.

An interacting two-dimensional electron system (2DES) exhibits numerous fractional quantum Hall states (FQHs) when it is subjected to a large perpendicular magnetic field \( (B_\perp) \) and electrons occupy the lowest \( (N = 0) \) Landau levels (LLs) \([1, 2]\). These incompressible many-body states can be explained by composite Fermions (CFs), quasiparticles formed by attaching an effective many-body states can be explained by composite Fermions (CFs), quasiparticles formed by attaching an even number \((2p)\) of quantized flux quanta to each electron \([2,4]\). When \( B_\perp \) deviates from half-integer fillings, the CFs formed their own discrete energy levels, the so-called \( \Lambda \) levels, and the Fourier transform of the Shubnikov-de Haas oscillations \( \nu \) are separated by \( \Delta_{\text{SAS}} \) or \( \Delta_{\text{SAS}} \) and \( \Delta_{\text{CF}} \) levels – are close in energy \((\epsilon_{\text{LL}} \approx 7\,\text{meV})\), so the lowest four LLs – the \( S_0^\uparrow \), \( S_0^\downarrow \), \( A_0^\uparrow \) and \( A_0^\downarrow \) levels – are close in energy \((\epsilon_{\text{LL}} = \sqrt{\hbar/eB_\perp}) \) is the magnetic length and \( \epsilon \) is the dielectric constant; \( \uparrow \) and \( \downarrow \) refer to up- and down-spin). When we apply \( B_\parallel \), \( \Delta_{\text{SAS}} \), which is now the energy separation between the \( S_0^\uparrow \) and \( A_0^\uparrow \) levels, is reduced while \( E_Z \) increases, and a crossing of the \( S_0^\downarrow \) and \( A_0^\uparrow \) levels occurs at \( \nu = 2 \) when \( \Delta_{\text{SAS}} = E_Z \); see Fig. 1(a). Near this crossing, we observe a remarkable pattern of appearing and disappearing FQHs in the filling range \( 1 < \nu < 3 \), revealing the formation of FQHs with both spin and subband degrees of freedom simultaneously. We describe our results in a simplified two-component picture with SU(2) symmetry, where the two pseudospins are the \( S_0^\downarrow \) and \( A_0^\uparrow \) levels. The observed transitions allow us to quantitatively determine \( h\omega_{\text{CF}} \) in terms of \( \Delta_{\text{SAS}} - E_Z \). Our data reveal several puzzling features; e.g., we find that at a fixed \( \nu \), \( h\omega_{\text{CF}} \) is about twice larger when \( \Delta_{\text{SAS}} > E_Z \) compared to when \( \Delta_{\text{SAS}} < E_Z \), indicating that \( h\omega_{\text{CF}} \) for the 2DES is more spin-polarized.

Our wafers, grown by molecular beam epitaxy, contain a 65- or 55-nm-wide GaAs QW bounded on each side by undoped \( \text{Al}_{0.23}\text{Ga}_{0.76}\text{As} \) spacer layers and Si \( \delta \)-doped layers. We cut 4 mm \( \times \) 4 mm square samples from the wafers, put alloyed InSn contacts at four corners, and then fit the samples with an evaporated Ti/Au front-gate and an In back-gate to change the 2DES density \( n \) while keeping the charge distribution symmetric [Fig. 1(b)]. We measure the transport coefficients in a dilution refrigerator with base temperature \( T \approx 30 \,\text{mK} \), using low-frequency \((< 30 \,\text{Hz})\) lock-in technique and a rotatable sample platform to induce \( B_\parallel \). At \( B_\parallel = 0 \), the Fourier transform of the Shubnikov-de Haas oscillations exhibits two peaks that correspond to the electron densities in the two subbands. The difference between these densities yields values for \( \Delta_{\text{SAS}} \) which are in excellent agreement with the results of our \( B = 0 \) self-consistent calculations. To calculate \( \Delta_{\text{SAS}} \) at finite \( B_\parallel \) and \( B_\perp \) [Fig. 1(a)], we employ a perturbative simulation introduced in Ref. \([3]\), where we assume \( B_\parallel \) only mixes LLs from different subbands but does not change the QW potential.
Figure 1 (c) shows longitudinal magnetoresistance $R_{xx}$ traces for the 65-nm-wide QW sample for $1 < \nu < 3$, measured at $n = 2.12 \times 10^{11} \text{ cm}^{-2}$, $T \simeq 135 \text{ mK}$, and different tilt angles ($\theta$). As we increase $\theta$, the $\nu = 2$ $R_{xx}$ plateau narrows near $\theta \simeq 37^\circ$ and then widens again at larger $\theta$, suggesting a weakening of the integer QHE at $\nu = 2$. This weakening, but not disappearing, is similar to what is seen at certain LL crossings at other integral $\nu \gtrsim 3$ when the electron interaction preserves the energy gap through QHE ferromagnetism.

At $B_{||} = 0$, $\Delta_{\text{SAS}}$ is larger than $E_Z$. As we increase $B_{||}$, $\Delta_{\text{SAS}}$ decreases and $E_Z$ increases, so that the $S_0\uparrow$ and $A_0\uparrow$ levels cross when $\Delta_{\text{SAS}} \simeq E_Z$. In Fig. 1(a), we show the calculated energies of the four $N = 0$ LLs relative to the $S_0\uparrow$ level as a function of $B_{||}$ at $\nu = 2$. We use a three-fold enhanced $E_Z$ to match the experimental observation that the $\nu = 2$ crossing occurs near $\theta \simeq 37^\circ$ ($B_\parallel \simeq 3.5 \text{ T}$). This enhancement of $E_Z$ is not surprising and has also been reported in previous studies; see, e.g., Ref. 28.

More interestingly, near $\theta \simeq 37^\circ$, the FQHSs on both sides of $\nu = 2$ show a rich series of transitions, which are marked with black arrows in Figs. 1(c)-(e). The $\nu = 4/3$ FQHS is strong at both small and large $\theta$, but becomes weak at $\theta \simeq 30^\circ$ [Figs. 1(c) and (e)]. The $\nu = 8/3$ FQHS also experiences one transition, at $\theta \simeq 45^\circ$ [Figs. 1(c) and (d)]. Meanwhile, the $\nu = 5/3$ and $7/3$ FQHSs become weak twice: at $\theta \simeq 30^\circ$ and $40^\circ$ for $\nu = 5/3$, and at $\theta \simeq 37^\circ$ and $45^\circ$ for $\nu = 7/3$. Data taken at lower temperature $T \simeq 65 \text{ mK}$, shown in Figs. 1(d) and (e), reveal a more remarkable pattern of higher-order FQHS transitions. On the left side of $\nu = 2$ [Fig. 1(d)], the $\nu = 13/5$ FQHS weakens twice at $\theta \simeq 40.5^\circ$ and $46.1^\circ$. The $\nu = 12/5$ FQHS, on the other hand, becomes weak three times, at $\theta \simeq 36.9^\circ$, $42.4^\circ$ and $46.1^\circ$. On the high-field side of $\nu = 2$, as seen in Fig. 1(e), the $\nu = 7/5$ FQHS weakens twice, at $\theta \simeq 27.5^\circ$ and $35^\circ$, and the $\nu = 8/5$ FQHS thrice, at $\theta \simeq 27.5^\circ$, $36.2^\circ$ and $39.4^\circ$. We also observe in Fig. 2(e) three transitions at $\nu = 10/7$ and four transitions at $\nu = 11/7$, which can be better seen at $T \simeq 30 \text{ mK}$ (data not shown here). We summarize in Fig. 2(a) the values of $B_{||}$ and $B_\perp$ where all the transitions are observed.

The fact that these transitions occur near the crossing of the $S_0\downarrow$ and $A_0\downarrow$ levels when $\Delta_{\text{SAS}} \simeq E_Z$, suggests that both spin and subband degrees of freedom are playing a role. Since the other two energy levels, $S_0\uparrow$ and $A_0\downarrow$, are reasonably far in energy, we neglect them.
FQHS which has \(2(b)\). By invoking particle-hole symmetry, which links |\(\nu\)| = 3 and four configurations [Fig. 2(b)], also in agreement with Fig. 2(a), data. Using similar logic, we can explain our data in a simple picture where the electron S0↑ levels at \(\nu = 5/3\), on the other hand, has two filled \(\Lambda\) levels (\(\nu_{\text{CF}} = 2\)), three pseudospin configurations [(\(\nu_{\text{CF}1}, \nu_{\text{CF}2}\)) = (2,0), (1,1) and (0,2)], and two transitions when \(\nu_{\text{CF}} = 3\) and four configurations [Fig. 2(b)]. By invoking particle-hole symmetry, which links the FQHSs at \(\nu = 4/3\) to its system (e.g., 4/3 to 8/3), and also utilizing negative CF fillings, e.g., \(\nu_{\text{CF}} = -2\) for the \(\nu = 5/3\) state, we can explain all the transitions summarized in Fig. 2(a). In general, a FQHS with \(\nu_{\text{CF}}\) has \(|\nu_{\text{CF}}| + 1\) types of pseudospin configurations, and \(|\nu_{\text{CF}}|\) pseudospin polarization transitions which occur whenever \(|\Delta|\) equals a multiple integer of \(\hbar \omega_{\text{CF}}\) [see Fig. 2(b)].

Next we proceed to quantitatively determine the energies for pseudospin polarization energies. Note in Fig. 2(a) that, when \(\nu_{\text{CF}}\) is odd, one transition occurs exactly at \(\Delta = 0\). In Fig. 2(a), we first fit the dashed blue line through these transition points. We then focus on a particular \(\nu\), e.g. 8/5, and calculate \(\Delta_{\text{SAS}}\) as a function of \(B_{\parallel}\) at this filling, as shown in Fig. 3(a). Using the value of \(B_{\parallel}\) at which the \(\Delta = 0\) transition for \(\nu = 8/5\) occurs (\(B_{\parallel} = 3.96 \, \text{T}\)), we determine a value for \(g^*\) (\(\approx 0.134\)) so that \(\Delta_{\text{SAS}} = E_Z\) at \(B_{\parallel} = 3.96 \, \text{T}\). We then plot \(E_Z\) (= \(g^* \mu_B B\)) at \(\nu = 8/5\) as a function of \(B_{\parallel}\) in Fig. 3(a), and determine \(\Delta\) for the other two 8/5 transitions. Since these transitions are expected to occur when \(\Delta = \pm 2\hbar \omega_{\text{CF}}\) [see Fig. 2(b)], we find \(\hbar \omega_{\text{CF}} = 5.5\) K for \(\Delta > 0\) and \(\hbar \omega_{\text{CF}} = 2.5\) K for \(\Delta < 0\). Using this procedure we can deduce \(\hbar \omega_{\text{CF}}\) for FQHSs at \(\nu = 8/5, 12/5,\) and 10/7 which have a \(\Delta = 0\) transition on the blue line in Fig. 3(a). For the FQHSs at \(\nu = 13/5, 7/3, 5/3\) and 7/5, we assume \(\Delta = 0\) at the intersection of the blue line and the vertical lines that mark these fillings in Fig. 3(a) and, following a similar procedure, find \(\hbar \omega_{\text{CF}}\) from the transitions’ \(B_{\parallel}\) values.

In Fig. 3(c) we plot as a function of \(1/(2\nu_{\text{CF}} + 1)\) all the deduced values of \(\hbar \omega_{\text{CF}}\), normalized to the Coulomb interaction \(V_C\), for different FQHSs. Figure 3(b) data allow us to make a quantitative comparison of the deduced \(\hbar \omega_{\text{CF}}/V_C\) to the theoretically expected values. Concentrating on \(\Delta > 0\) and \(1 < \nu < 2\), we find that \(\hbar \omega_{\text{CF}}/V_C \approx 0.11/(2\nu_{\text{CF}} + 1)\). This is in very good agreement with the results of theoretical calculations which predict \(\hbar \omega_{\text{CF}}/V_C \approx 0.10/(2\nu_{\text{CF}} + 1)\) for an ideal 2DES with zero layer-thickness [2, 4]. Such good agreement is likely fortuitous as we expect \(\hbar \omega_{\text{CF}}/V_C\) to be somewhat reduced (by \(\approx 2\) times) compared to its ideal value because of the finite layer thickness of the 2DES in our samples [2, 20, 31]. It is possible that the \(\hbar \omega_{\text{CF}}\) values we report are exaggerated because of the inaccuracy of the perturbative calculations we use to determine the \(B_{\parallel}\)-dependence of \(\Delta_{\text{SAS}}\). In Fig. 3(b) we also include data (open squares) taken in a symmetric, 55-nm-wide GaAs QW in which we measured \(\hbar \omega_{\text{CF}}\) at \(B_{\parallel} = 0\). In this sample, instead of applying \(B_{\parallel}\), we induced the crossing of the S0↑ and A0↑ levels at \(1 < \nu < 3\) by increasing the density to reduce \(\Delta_{\text{SAS}}\) and increase \(E_Z\) [10]. As seen in Fig. 3(c), the measured \(\hbar \omega_{\text{CF}}\) are indeed smaller than those deduced for the 65-nm-wide QW from the \(B_{\parallel}\)-dependent measurements. However, they are still larger than the values measured in 2DESs with only spin degree of freedom; e.g., \(\hbar \omega_{\text{CF}}/V_C \approx 0.005\) at \(\nu = 7/5\) in a 65-nm-wide QW [20]. We conclude that \(\hbar \omega_{\text{CF}}\) in our 2DES with both spin and subband degrees of freedom is larger than in a 2DES with a spin degree of freedom, even though it might be somewhat exaggerated because of the inaccu-
FIG. 3. (color online) (a) Calculated $\Delta_{SAS}$ and $E_Z$ as a function of $B_T$ at $\nu = 8/5$. (b) $\Lambda$ level diagram showing the four different pseudospin configurations for $\nu = 8/5$ ($\nu_{CF} = -3$) in our two-component picture. The pseudospin polarization transitions are expected when $\Delta = 0, \pm 2\hbar \omega_{CF}$ [see Fig. 2(b)]. (c) The $\Lambda$ level separation in units of $V_C = e^2/(4\pi\epsilon d)$ as a function of $1/(2\nu_{CF} + 1)$, deduced from $\Delta$ at which the FQHS transitions are observed in Fig. 2(a). Data are shown both $1 < \nu < 2$ (solid black squares) and $2 < \nu < 3$ (solid red circles). The open back squares are $\hbar \omega_{CF}$ measured for $1 < \nu < 2$ in a symmetric 55-nm-wide GaAs QW at purely perpendicular field (see text).

racy of our perturbative calculations. We hope that our data will stimulate more precise future calculations [32].

Finally, we highlight several additional noteworthy features of Fig. 3(c) [33]. First, the results for $1 < \nu < 2$ (black solid symbols) approximately match those for $2 < \nu < 3$ (red symbols), suggesting the particle-hole symmetry [$\nu \leftrightarrow (4 - \nu)$] is preserved. This is surprising because previous studies report broken particle-hole symmetry in systems with only spin or valley degrees of freedom [20–23]. Second, for $\Delta > 0$ (or $\nu < 2$), $\hbar \omega_{CF}$ for the FQHSs at $\nu = 5/3$ and $7/3$ are similar. This suggests that $\hbar \omega_{CF}$ does not depend on which LL ($S_0$ or $A0\uparrow$) hosts the CFs. However, when the system is subband-polarized ($\Delta > 0$), $\hbar \omega_{CF}/V_C$ is about twice larger compared to the case when the system is spin-polarized ($\Delta < 0$). This asymmetry is surprising, but it is qualitatively consistent with the experimental observation that $\hbar \omega_{CF}/V_C$ in GaAs 2DEGs are larger than in AlAs 2DEGs in which the electrons are always fully spin-polarized [23].

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[29] At very large $B_\parallel \gtrsim 5$ T, which we do not reach here, the separation between the S0↑ and A0↑ levels becomes very small and the electrons in the QW split into a bilayer system which supports other FQHS configurations, e.g., a $\nu = 4/3$ FQHS which is composed of FQHSs at 2/3 layer filling. In the $B_\parallel$ range of our study, the 2DES exhibits the normal sequence of FQHSs seen in single-layer systems.

[30] Note that, because in our system the lowest (S0↑) level is always fully occupied in the range of interest here, a FQHS at $\nu$ is equivalent to the $\nu' = \nu - 1$ FQHS in a system such as 2D electrons in an AlAs QW where the crossing levels are the lowest two (valley) LLs; see, e.g., Refs. [21–24].

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