Critical current of layered superconductor with defects in tilted magnetic field

V A Kashurnikov, A N Maksimova, I A Rudnev, D S Odintsov

1National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe sh. 31, 115409 Moscow, Russia

Email: nastymaksimova@yandex.ru

Abstract. The simulation of vortex lattice in layered high-temperature superconductor (HTSC) has been done using Monte Carlo method. The case of highly anisotropic superconductor with crossing lattices of Abrikosov and Josephson vortices under applying of tilted magnetic field has been examined. The critical current as a function of absolute value of dc magnetic field, parallel and perpendicular to the anisotropy axis, has been calculated. The simulations have been done for different anisotropy parameter of HTSC and different concentrations of defects. Configurations of the Josephson vortices in presence of the Abrikosov vortices have been obtained.

1. Introduction

High-temperature superconductors (YBCO, BSCCO) are anisotropic materials with layered structure. The vortex line in HTSC can be represented as a stack of interacting pancake vortices. The vortex lattice of layered superconductor in tilted magnetic field had been widely analyzed both theoretically and experimentally. The effect of tilted field on magnetic and transport properties of HTSC is under significant practical interest [1-3].

Theoretical studies [4-10] show that layered superconductors have a rich phase diagram in tilted magnetic field. The structure of vortex lattice is defined by the parameter \( \alpha = \lambda_J / \gamma s \), which is the ratio of London, \( \lambda \), and Josephson, \( \lambda_J \), lengths [5]. Here \( \gamma \) is an anisotropy parameter and \( s \) is an inter-layer spacing. At large \( \alpha \) the superconductor represents a tilted vortex lattice and at small \( \alpha \) the crossing array of Abrikosov and Josephson vortices is realized. This paper aims to consider the interacting Abrikosov and Josephson vortex lattices in the frame of 3D model of layered HTSC and to obtain numerically the critical current as a function of an angle between the direction of applied magnetic field and an anisotropy axis.

2. Expressions for energy in Monte Carlo simulations

According to theoretical studies [4-10], pancake vortices are generated by \( c \)-axis field and Josephson vortices are generated by in-plane component. Josephson vortices are parallel to superconducting layers (\( ab \)-plane) and located in the inter-layer spacing.

Using the results [11,12] we can obtain the expressions for energy of Josephson vortices which can be used in Monte Carlo simulations. The self-energy of Josephson vortex (per unit length):
\[ \varepsilon_1 = \frac{\Phi_0}{4\pi} H_{cl} = \frac{\Phi_0^2}{16\pi^2\lambda_b(T)\lambda_c(T)} \left[ \ln \frac{\lambda_b(T)}{\lambda_c(T)} + 1.12 \right], \lambda_b = \frac{\lambda_c^2}{d_s}, \lambda_c = (\pi s)^2 + \frac{d_s}{s} \lambda_c^2. \tag{1} \]

\[ d_s = 0.27 \text{ nm} \] is the superconductor layer thickness and the inter-layer spacing \( s = 2.7 \text{ nm}. \)

The interaction energy of two Josephson vortices is defined as a work of Lorenz force (from the supercurrents forming Josephson vortex) when the vortex moves from infinity to the adjusted point \((y_2, s n_1)\) in the vicinity of second vertex (at the position \((y_2, s n_2)\)):

\[ U_{1,2} = \frac{\Phi_0^2}{8\pi^2\lambda_b\lambda_c} K_0 \left( \frac{y_2 - y_1}{\lambda_c} \right)^2 + \frac{s^2(n_2 - n_1)^2}{\lambda_b^2} \left( K_0 \left( \frac{y_2}{\lambda_c} \right) + \frac{z}{\lambda_b} \right)^2, \tag{2} \]

\[ n \] is a number of superconducting layer. Interaction between Abrikosov and Josephson vortex:

\[ U_{1,2} = -\frac{\Phi_0^2}{8\pi^2\lambda_b\lambda_c} \frac{z}{\lambda_b} \frac{x_0 + d/2}{\lambda_b} K_0 \left( \left( \frac{y}{\lambda_c} \right)^2 + \left( \frac{z}{\lambda_b} \right)^2 \right)^2, \]

\[ d \] is a width of a superconductor, \( x_0 \) is the \( x \)-coordinate of Abrikosov vortex and \((x\text{-axis is perpendicular to the transport current)}\), \( y \) and \( z \) are the differences between \( y \)- and \( z \)-coordinates of Abrikosov and Josephson vortices. The origin is in the centre of the superconductor.

Using (1) we can obtain the first critical field for Josephson vortices:

\[ H_{cl} = \frac{\Phi_0}{4\pi\lambda_b(T)\lambda_c(T)} \left[ \ln \frac{\lambda_b(T)}{\lambda_c(T)} + 1.12 \right]. \]

3. Simulation results

The Monte Carlo simulation (see [13-17] for details of the method) for a stack of superconducting layers can be done only for few \((N_L \sim 10)\) layers in the stack. However, at \( \gamma \sim 100 \) (typical for BSCCO) the characteristic distances for Josephson vortex interaction \( \lambda_b \) and \( \lambda_c \) are \( \sim 500 \text{ nm} \) and the triangular lattice of Josephson vortices can be formed only when \( N_L \sim 1000 \). For simulations, we can roughly define a stack of 100 pancakes as an elementary object so that vortex line is still consisted of \( \sim 10 \) elementary objects and Josephson vortices can be placed between any of 1000 layers. This approach is approximately correct since the mutual displacements of pancakes are not numerous in the most part of simulations. The simulations have been done for typical parameters of BSCCO. The size of superconducting slab is \( 5 \times 3 \) \( \mu \text{m} \).

Figure 1 represents the calculated critical current as a function of the magnitude of magnetic field, parallel and perpendicular to superconducting layers. The obtained dependencies are mostly decreasing which is in agreement with experimental results. Measurements on the YBCO samples with columnar defects show that critical current decreases when magnetic field increases and \( j_c \) in \( \mathbf{H} \parallel c \) is lower than in \( \mathbf{H} \parallel ab \). However, if we increase the concentration of columnar defects, the critical current in \( \mathbf{H} \parallel c \) becomes higher. Both these cases, in fact, can be reproduced in simulations. Figure 2 represents the critical current for \( \gamma = 20 \) and both concentrations of defects.

Figure 3 represents the configurations of Josephson vortices. Abrikosov vortices are shown schematically in Figure 3b, in the field of transport current \( H_t = 100 \text{ Gs} \) Abrikosov vortices do not penetrate into the slab. As seen, the triangular lattice of Josephson vortices breaks down in presence of Abrikosov vortices. Moreover, the highest density of Josephson vortices is observed at the upper boundary of the slab. This fact can be explained with features of interaction between Abrikosov and Josephson vortices (2).
**Figure 1.** Critical current as a function of external dc magnetic field. $N_d$ – number of defects in the slab $5 \times 3 \, \mu m$

![Critical current vs. external dc magnetic field](image1)

**Figure 2.** Critical current at $\gamma = 20$

![Critical current for different magnetic field](image2)

**Figure 3.** Configurations of Josephson vortices. a) Self-field of transport current $H_I = 100 \, \text{Gs}$, b) $H_I = 355 \, \text{Gs}$. External dc magnetic field is $H = 320 \, \text{Gs}$ and is tilted to $c$-axis by angle $\theta = 45^\circ$.

![Josephson vortices configurations](image3)
4. Conclusion
The simulation of vortex lattice in HTSC under applying of tilted dc magnetic field has been done by using Monte Carlo method. For this algorithm, the description of interacting Abrikosov and Josephson vortex lattices had been introduced into 3D model of layered HTSC. The critical current as a function of the magnitude of external dc magnetic field, parallel and perpendicular to the superconducting layers has been calculated. The obtained dependences are decreasing and agree qualitatively with experiment. The experimental fact that the critical current of HTSC with columnar defects in field parallel to the anisotropy axis exceeds the critical current in field parallel to the ab plane is qualitatively reproduced in our calculations. The configurations of Josephson vortices in HTSC magnetized by self-field of transport current have been obtained. Our method allows the calculation of magnetization curves, current-voltage characteristics and critical current of layered superconductors with high $\gamma$ in arbitrary magnetic fields, with arbitrary configurations of defects and different anisotropy parameters.

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