S-wave Superconductivity due to Orbital and Spin fluctuations in Fe-pnictides: Self-Consistent Vertex Correction with Self-Energy (SC-VCΣ) Analysis

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To understand the amazing variety of the superconducting states of Fe-based superconductors, we analyze the multiorbital Hubbard models for LaFeAsO and LiFeAs going beyond the random-phase approximation (RPA), by calculating the vertex correction (VC) and self-energy correction. Due to the spin+orbital mode coupling described by the VC, both orbital and spin fluctuations mutually develop, consistently with the experimental phase diagram with the orbital and magnetic orders. Due to both fluctuations, the s-wave gap function with sign-reversal (s±-wave), without sign-reversal (s+ -wave), and nodal s-wave states are obtained, compatible with the experimental wide variety of the gap structure. Thus, the present theory provides a microscopic derivation of the normal and superconducting phase diagram based on the realistic Hubbard model.

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One of the main characteristic feature of Fe-based superconductors is their amazing variety of the superconducting gap structure. For example, isotropic fully-superconductors is their amazing variety of the superconducting gap structure. Thus, the present theory provides a microscopic derivation of the normal and superconducting phase diagram based on the realistic Hubbard model.

To clarify the pairing mechanism in strongly correlated systems, its normal state phase diagram should be understood. The coincidence of the structure (or orbital) and magnetic quantum critical points (QCPs) indicates the coexistence of orbital and spin fluctuations in Fe-based superconductors. The large softening of the shear modulus \(C_{66}\) above the structure transition temperature \(T_N\) is a direct evidence of orbital fluctuations \([12,13]\), and the orbital order or polarization had been observed in various compounds by polarized ARPES measurements \([18,19]\). As for the superconductivity, the s-wave state with (without) sign reversal is caused by spin (orbital) fluctuations \([20,21]\). In BaFe\(_2\)As\(_2\), the absence of the horizontal node on the hole-doped Fermi surface (hFS) with 3\(z^2 - r^2\)-orbital character \([6,7]\) indicated the importance of the orbital fluctuations \([9]\), and the loop-nodes on the electron-type Fermi surfaces (e-FSs) \([6,7]\) can be explained by the competition between different spin fluctuations \([11,12]\) or that between spin and orbital fluctuations \([3]\).

The normal state phase diagram with non-magnetic structure transition cannot be explained by the random-phase-approximation (RPA) based on the multiorbital Hubbard model. However, it can be explained by considering the vertex correction (VC), since the spin fluctuations induce the charge quadrupole order \(O_{z^2 - y^2} \equiv n_{xz} - n_{yz} \neq 0\) owing to the spin+orbital mode coupling described by the VC. This mechanism has been demonstrated by the diagrammatic \([20,27]\) as well as the renormalization group \([28]\) methods in several multiorbital models. Similar spin-orbital coupling beyond the RPA had also been discussed in Refs. \([29,30]\). We stress that spin fluctuations could also induce the spin quadrupole \((\phi = s_A \cdot s_B)\) if \(J_1 \sim 2J_2\) is realized \([12]\), so the correct quadrupole order should be elucidated experimentally. The strain-quadrupole order coupling derived from the fitting of \(C_{66}\) is very large, which would be natural for the charge quadrupole scenario \([13]\). Now, the next significant challenge is to elucidate the pairing mechanism of the Fe-based superconductors by taking the VC into account.

In this paper, we develop a theory beyond the RPA, by calculating both the VC and the self-energy \(\Sigma\) self-consistently. By this “self-consistent VC+\(\Sigma\) (SC-VC\(\Sigma\)) method”, we obtain the mutual development of spin and orbital fluctuations. Therefore, both the s-wave state with sign change (s±-wave) and that without sign change (s+ -wave), both of which are promising candidate pairing states, are naturally reproduced based on the two different realistic Hubbard models. No additional interactions such as the quadrupole interaction \([24]\) were introduced to the models. The obtained smooth s± -wave could explain the wide variety of gap structures in Fe-based superconductors.

In the following, we explain the SC-VC\(\Sigma\) method. First, we employ the five-orbital Hubbard model for LaFeAsO introduced in Ref. \([20]\). We denote \(d\)-orbitals 3\(z^2 - r^2\), \(xz\), \(yz\), \(xy\), and \(x^2 - y^2\) as 1, 2, 3, 4 and 5, respectively. Hereafter, \(x, y\)-axes are along the nearest Fe-Fe direction. The Fermi surfaces are mainly composed of orbitals 2, 3 and 4 \([31]\). The susceptibility for the charge (spin) channel is given by the following 25 × 25 matrix form in the orbital basis:

\[
\chi^c(s)(q) = \chi^{irr,c(s)}(q)(1 - \tilde{\Gamma}_c(s)\chi^{irr,c(s)}(q))^{-1},
\]

where \(\chi^{irr,c(s)}(q)\) is given by:
where $q = (q, \omega_l = 2\pi T)$, and $\hat{\chi}^c(s)$ represents the Coulomb interaction for the charge (spin) channel composed of $U$, $U'$ and $J$ given in Refs. [24 25 52]. The irreducible susceptibility in Eq. (1) is given as

$$\hat{\chi}^{irr,c}(q) = \hat{\chi}^0(q) + \hat{\chi}^c(s)(q), \tag{2}$$

where $\hat{\chi}^{0}_{\upsilon',mmm'}(q) = -T\sum_{G}G_{\upsilon m}(p + q)G_{\upsilon' m'}(p)$ is the bare bubble, and the second term is the VC, which is essential to produce the orbital fluctuations as discussed in Ref. [26]. In the SC-VC$_\Sigma$ method, Green’s function $\hat{G}$ is given by Dyson’s equation $\hat{G} = (\hat{G}_0^{-1} - \hat{\Sigma})^{-1}$, where $\hat{G}_0$ is the bare Green’s function and $\hat{\Sigma}$ is the self-energy. (In the SC-VC method, we put $\Sigma = 0$.) In order to measure the distance from the criticality, we introduce the charge (spin) Stoner factor $\alpha^{c(s)}_q(q)$, which is the largest eigenvalue of $\hat{\chi}^{c(s)}_{\upsilon,mmm'}(q)$ at $\omega_l = 0$ [26]. The charge (spin) susceptibility diverges when $\alpha^{c(s)}_q = \max_q(\alpha^{c(s)}_q) = 1$.

Here, we introduce VCs due to the Aslamazov-Larkin (AL) terms, which is the second order term with respect to $\hat{\chi}^{c(s)}$ and becomes important near the QCP [20, 53]. The VCs for charge/spin sector are denoted as $\hat{\chi}^{c(s)}_{\upsilon,mmm'}(q) \equiv \hat{\chi}^{T,T+}(q) + (-)\hat{\chi}^{T+}(q)$. The AL term for the charge sector, $X^{AL,c}_{\upsilon',mmm'}(q)$, is given as

$$T\sum_{k} \sum_{a \sim h} \sum_{k' \sim h} \chi_{\upsilon',mmm'}^{\upsilon,cd}(q;k')V_{ab,cd}(k + q)V_{ef,gh}(q)\Gamma^{\upsilon,gh}_{ef,cd}(q, k) + 3\chi^{\upsilon,gh}_{ab,cd}(q)\Gamma^{\upsilon,gh}_{ef,cd}(q, k), \tag{3}$$

where $\chi^{\upsilon,gh}_{ab,cd}(q) \equiv \chi^{\upsilon,gh}_{ab,cd}(q)$, $\Lambda(q; k)$ and $\Lambda'(q; k)$ are the three-point vertex made of three Green functions [26]. We include all $U^2$-terms without the double counting to obtain reliable results. Note that we neglect $X^{AL,c}$ because the contribution of $X^{AL,c}$ is much smaller than that of $X^{AL,c}$ [26 22].

The $5 \times 5$ self-energy matrix $\hat{\Sigma}$ in the fluctuation-exchange (FLEX) approximation is given by

$$\Sigma_{lm}(k) = T \sum_q \sum_{\upsilon',mm'} \chi_{\upsilon',mmm'}^{\upsilon,cd}(q)G_{\upsilon,mm'}(k - q), \tag{4}$$

where $\hat{\Sigma}(q)$ is the effective interaction for the self-energy; $V^{\upsilon}(q) = \frac{3}{2}V^{\upsilon}(q) + \frac{1}{2}\hat{\Sigma}^{\upsilon}(q)$, $\frac{1}{4}(\hat{\chi}^{\upsilon}(q)\hat{\chi}^{\upsilon}(q) - \hat{\Sigma}^{\upsilon}(q)\hat{\chi}^{\upsilon}(q) - \hat{\chi}^{\upsilon}(q)\hat{\chi}^{\upsilon}(q))$, and $\chi^{\upsilon}(q) = 0$ for others. The third and fourth terms of right hand side in $\hat{\Sigma}(q)$ are required to cancel the double counting in the 2nd order diagrams. By solving above equations, we obtain the susceptibilities and self-energy self-consistently.

In our calculation, we neglect the Maki-Thompson (MT) terms since it is much smaller than the AL term as explained in Ref. [26]. The dominance of the AL term is also verified by recent renormalization group study [28]. We use $\hat{G}_0$ in calculating $\Lambda$ and $\Lambda'$ in Eq. (3) since they are underestimated at high temperatures ($T \sim 0.05$) due to large quasiparticle damping $\text{Im}\Sigma (q, -i\delta) \propto T$.

We use $64 \times 64$-k meshes and 256 Matsubara frequencies at $T = 0.05$ eV, and set the unit of energy as eV. Hereafter, we put the constraint $U = U' + 2J$. The Fermi surfaces in the LaFeAsO model for $n = 6.1$ are shown in Fig. (a), where $\theta$ is denoted by the azimuthal angle from $k_y$ axis. Figure (b), (c) and (d) shows the obtained $\chi^{22,22}(q)$, $\chi^{24,42}(q)$ and $\chi^{34,43}(q)$ for $J/U = 0.15$ and $U = 2.2$ ($\alpha_s = 0.96$ and $\alpha_\alpha = 0.97$) using the SC-VC$_\Sigma$ method, respectively. The development of the ferroorbital fluctuations explains the softening of shear modulus $C_{66}$ and structure transition, and both ferro- and antiferro (AF)-orbital fluctuations are the driving force of the $s_{+\pm}$-wave state. Here, orbital fluctuations can develop for much large $J/U$ compared to the SC-VC method [26], since the value of $U$ for the ordered state increases due to the self-energy, and therefore $X^{AL,c} \propto U^4$ is enlarged. We verified that similar results are obtained for $6.0 \leq n \leq 6.1$, even if the h-FS at $(\pm \pi, \pm \pi)$ appears.

![FIG. 1: (color online) (a)FSs of the LaFeAsO five-orbital model for $n = 6.1$, where $\theta$ is denoted by the azimuthal angle from $k_y$ axis. (b)$\chi^{22,22}(q)$, (c)$\chi^{24,42}(q)$ and (d)$\chi^{34,43}(q)$ given by the SC-VC$_\Sigma$ method for $J/U = 0.15$, $\alpha_s = 0.96$, and $\alpha_\alpha = 0.97$.](image-url)
term in Eq. 6 works to set $\Delta_{FS1,2} \cdot \Delta_{FS3,4} < 0 \ (>)$ 32.

Finally, we obtain a gap function $\Delta(k) = \hat{\phi}(k, \omega_n = \pi T)/\tilde{Z}(k)$ in the band representation, where mass enhancement factor $\tilde{Z}$ is given as $\tilde{Z}_\alpha(k) = 1 - \frac{\ln Z_\alpha(k, \omega_n = \pi T)}{\ln Z_\alpha(k, \omega_n = \pi T)}$ in a band $\alpha$. We obtain $Z \sim 3$ in the present study. We note that the absolute value of $\Delta$ is not important since the Eliashberg Eq. 5 is linearized.

In Eq. 6, $V^{(1)} = \frac{1}{2}T^s - \frac{1}{2}T^c$ represents the first-order terms with respect to the Coulomb interaction. This term gives the Anderson-Morel pseudopotential $\mu^* \approx UN(0)[1 + UN(0) \ln(W_{\text{band}}/\omega_0)]^{-1}$, where $N(0)$ is the density-of-states (DOS) at the Fermi level, $W_{\text{band}}$ is the bandwidth, and $\omega_0$ is the energy-scale of the orbital and spin fluctuations 34: $\omega_0 \sim T$ is expected in optimally-doped compounds close to the orbital and magnetic QCPs. Although $\mu^*$ suppresses the $s_{++}$-wave state, we can expect that this term is approximately canceled out by the weak $e$-$\text{ph}$ interaction $\lambda_{e-ph} \sim 0.2$.

Hereafter, we calculate the superconducting gap functions based on the gap equation in Eq. 6 by fixing the ratio $J/U$ while choosing $U$ so as to satisfy $\alpha_s = 0.97$. Figure 2 (a) and (b) show the obtained gap structures for $J/U = 0.1 \ (U = 2.0; \alpha_s = 0.91)$ with $V^{(1)} (\lambda_E = 0.35)$ and those without $V^{(1)} (\lambda_E = 0.51)$, respectively. The dropping of $V^{(1)}$ will be justified since we expect that $\mu^*$ is as small as $\lambda_{e-ph} \sim 0.2$ due to the retardation. In both cases, fully-gapped $s_{++}$-wave states are obtained. In (b), gap functions of FS1 and FS2 are as large as that of FS3 since $V^{(1)}$ is repulsive interaction and unfavorable to the $s_{++}$-wave state. The $s_{++}$-wave state is derived from the strong developments of $\chi_{22,22}(q)$ and $\chi_{44,42}(q)$ shown in Fig. 4.

Next, we move to $J/U = 0.12$, where the spin fluctuations are stronger than that for $J/U = 0.1$. Figure 2 (c) and (d) show the obtained gap structures for $J/U = 0.12 \ (U = 2.1; \alpha_s = 0.93)$ with $V^{(1)} (\lambda_E = 0.49)$ and those without $V^{(1)} (\lambda_E = 0.57)$, respectively. In both (c) and (d), the obtained states are the nodal $s$-wave states which is intermediate between $s_{++}$-wave and $s_{++}$-wave states due to the competition between orbital and spin fluctuations. We note that the relation $J/U = 0.12 - 0.15$ is estimated by the first principle calculation 35.

We also study the impurity effect on the superconducting state, by introducing the impurity $T$-matrix into Eq. 5 according to Refs. 24 36 37. Here, we discuss the impurity-induced $s_{\pm} \rightarrow s_{++}$ crossover for $J/U = 0.15 \ (U = 2.2; \alpha_s = 0.96; \lambda_E = 0.33)$, in which the spin and orbital fluctuations are comparable. In the absence of impurities ($n_{\text{imp}} = 0$), the $s_{++}$-wave state is realized as shown in Fig. 3 (a). As increasing $n_{\text{imp}}$, the crossover between $s_{++}$-wave and $s_{+}$-wave states is expected, as discussed in Refs. 9 24. In fact, Fig. 3 (b) and (c) shows the obtained gap functions for $n_{\text{imp}} = 5\%$ and $10\%$, respectively, when the impurity potential is $I = 1\text{eV}$. Since the obtained $\lambda_E \sim 0.33$ is almost unchanged for $n_{\text{imp}} = 0 \sim 10\%$, the impurity-induced $s_{\pm} \rightarrow s_{++}$ crossover will be realized with small change in $T_c$. Since the impurity concentration to realize the crossover is scaled with $T$, $n_{\text{imp}} = 10\%$ for $T = 0.05$ would correspond to $n_{\text{imp}} \sim 1\%$ for $T = T_c \sim 0.005$. Therefore, the $s_{++}$-wave state will be realized in various iron-based superconductors with finite randomness.

In the following, we study the superconducting state of LiFeAs, using the tight-binding model given by fitted to the ARPES data of LiFeAs 38. The FSs in the model are shown as Fig. 3 (a) for $n = 6.0$, in which the large $d_{xy}$-orbital h-FS4 around $k = (\pi, \pi)$ is added to the model of LaFeAsO. Within the RPA, the $s_{++}$-wave state is favored by the h-FS4 39. However, as shown in Fig. 4 (b) and
(c) for \( J/U = 0.12, U = 1.2 \) at \( T = 0.03 \) (\( \alpha_s = 0.92 \) and \( \alpha_e = 0.97 \)), the AF-spin and AF-orbital fluctuations are strongly enhanced in the SC-VC\(_2\) method. In contrast, the ferro-orbital fluctuations are relatively small, consistently with the absence of the structure transition in LiFeAs. The gaps obtained for \( J/U = 0.12, n_{\text{imp}} = 0\% \) at \( T = 0.03 \) are \( s_{++} \)-wave mediated by the AF-orbital fluctuation as shown in Fig. 4 (d) with \( \hat{V}^{(1)} \) (\( \lambda_E = 0.40 \)) and (e) without \( \hat{V}^{(1)} \) (\( \lambda_E = 0.53 \)). Thus, the fully-gapped \( s_{++} \)-wave state is realized for \( J/U = 0.12 \) even for \( n_{\text{imp}} = 0 \).

Here, the calculation temperature is much higher than \( T_c \) since the SC-VC\(_2\) method is heavy numerical calculation. For this reason, the obtained gap is nearly isotropic on each FS. It is our important future problem to study the experimental large gap anisotropy at much lower temperatures.

In the present study, we dropped the VC for the electron-boson coupling constant in the gap equation, shown by \( \hat{\Delta}''(q; k) \) in Fig. 4 (f). Recently, we had verified that \( [\hat{\Delta}''(q; k)] \) due to the AL type diagram in Fig. 4 (f) is much larger than unity for the charge sector \( \Lambda'^{\prime\prime} \). This fact means the violation of the Migdal’s theorem. Then, \( \lambda_E \) for the \( s_{++} \)-wave state is enlarged since the charge pairing interaction \( \frac{1}{2} \hat{\Gamma}^{c} \hat{\chi}^{c}(q) \hat{\Gamma}^{c} \), given by the second term in Eq. (9), is multiplied by \( |\hat{\Delta}''(q; k)|^2 \). Therefore, the \( s_{++} \)-wave state is further stabilized by the VC for the electron-orbiton coupling, going beyond the Eliashberg theory.

FIG. 4: (color online) (a) FSs of the LiFeAs model for \( n = 6.0 \). (b) \( \chi^s(q) = \sum_m \chi_{l,m}^s(q) \). (c) \( \chi_{24,42}^{\Delta'}(q) \) given by SC-VC\(_2\) method in the LiFeAs model for \( J/U = 0.12, \alpha_s = 0.92 \), and \( \alpha_e = 0.97 \) at \( T = 0.03 \). (d) \( \theta \) dependences of gap functions for \( J/U = 0.12, n_{\text{imp}} = 0\% \) at \( T = 0.03 \) with \( \hat{V}^{(1)} \). (f) Feynman diagram of \( \hat{\Delta} \) and VC \( \Lambda'' \) for the charge sector \( \hat{\Gamma}^{c} \) in the pairing interaction \( \hat{V}^{E} \).

In summary, we have studied the normal and superconducting states of LaFeAsO and LiFeAs using the SC-VC\(_2\) theory. We obtain both the \( s_{++} \)- and \( s_\pm \)-wave gap, both of which are promising candidate pairing states, based on two very different but realistic tight-binding Hubbard models with \( J/U \lesssim 0.15 \). No additional interactions such as the quadrupole interaction were introduced. We stress that a smooth \( s_{++} \leftrightarrow s_\pm \) crossover could be realized by introducing the impurities or magnetic ordered state, consistently with the robustness of \( T_c \) [41–49] and the wide variety of gap structures in Fe-based superconductors.

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[1] K. Hashimoto, T. Shibauchi, T. Kato, K. Ikada, R. Okazaki, H. Shishido, M. Ishikado, H. Kito, A. Iyo, H. Eisaki, S. Shamoto and Y. Matsuda, Phys. Rev. Lett. 102, 017002 (2009).
[2] H. Ding, P. Richard, K. Nakayama, K. Sugawara, T. Arakane, Y. Sekiba, A. Takayama, S. Souma, T. Sato, T. Takahashi, Z. Wang, X. Dai, Z. Fang, G. F. Chen, J. L. Luo and N. L. Wang, Europhys. Lett. 83, 47001 (2008).
[3] T. Shimojima, F. Sakaguchi, K. Ishizaka, Y. Ishida, T. Kiss, M. Okawa, T. Togashi, C.-T. Chen, S. Watanabe, M. Arita, K. Shimada, H. Namatame, M. Taniguchi, K. Ohgushi, S. Kasahara, T. Terashima, T. Shibauchi, Y. Matsuda, A. Chainani, and S. Shin, Science 332, 564 (2011).
[4] M. A. Tanatar, J.-Ph. Reid, H. Shakeripour, X. G. Luo, N. Doiron-Leyraud, N. Ni, S. L. Budko, P. C. Canfield, R. Prozorov and L. Taillefer, Phys. Rev. Lett. 104, 067002 (2010).
[5] M. Yamashita, Y. Senshu, T. Shibauchi, S. Kasahara, K. Hashimoto, D. Watanabe, H. Ikeda, T. Terashima, I. Vekhter, A. B. Vorontsov, and Y. Matsuda, Phys. Rev. B 84, 060507(R) (2011).
[6] T. Shimojima et al., Solid State Commun. 152, 695 (2012).
[7] T. Yoshida et al., arXiv:1301.4818.
[8] Y. Zhang et al., Nature phys. 8, 371 (2012).
[9] T. Saito, S. Onari, and H. Kontani, Phys. Rev. B 87
In the tight-binding model for BaFe$_2$As$_2$, the nodal structure appears on the e-FSs during the $s^+ \leftrightarrow s^-$ crossover.

[10] S. Maiti et al., Phys. Rev. B 84, 224505 (2011).
[11] M. Khodas and A. V. Chubukov, Phys. Rev. B 86, 144519 (2012).
[12] R.M. Fernandes, L. H. VanBebber, S. Bhattacharya, P. Chandra, V. Keppens, D. Mandrus, M.A. McGuire, B.C. Sales, A.S. Sefat and J. Schmalian, Phys. Rev. Lett. 105, 157003 (2010).
[13] H. Kontani, Y. Inoue, T. Saito, Y. Yamakawa, and S. Onari, Solid State Communications, 152 (2012) 718.
[14] M. Yoshizawa, R. Kamiya, R. Onodera, Y. Nakanishi, K. Kihou, H. Eisaki and C. H. Lee, Phys. Soc. Jpn. 81, 024604 (2012).
[15] T. Goto, R. Kurihara, K. Araki, K. Mitsumoto, M. Akatsu, Y. Nemoto, S. Tatematsu and M. Sato, J. Phys. Soc. Jpn. 80, 073702 (2011).
[16] A. E. Bohmer, P. Burger, F. Hardy, T. Wolf, P. Schweiss, R. Fromknecht, M. Reinecker, W. Schranz, and C. Mein-gast, arXiv:1305.3515.
[17] Y. Gallais, R. M. Fernandes, I. Paul, L. Chauviere, Y.-X. Yang, M.-A. Measson, M. Cazayous, A. Sacuto, D. Colson, and A. Forget, arXiv:1302.6255.
[18] M. Yi, D. H. Lu, J.-H. Chu, J.G. Analytis, A.P. Sorini, A. F. Kemper, S.-K. Mo, R.G. Moore, M. Hashimoto, W.S. Lee, Z. Hussain, T.P. Devereaux, I.R. Fisher and Z.-X. Shen, Proc. Natl. Acad. Sci. 108, 6878.
[19] T. Shimojima et al., arXiv:1305.3875.
[20] K. Kuroki, S. Onari, R. Arita, H. Usui, Y. Tanaka, H. Kontani and H. Aoki, Phys. Rev. Lett. 101, 087004 (2008).
[21] I. I. Mazin, D. J. Singh, M.D. Johannes, and M. H. Du, Phys. Rev. Lett. 101, 057003 (2008).
[22] P.J. Hirschfeld, M.M. Korshunov and I.I. Mazin, Rep. Prog. Phys. 74, 124508 (2011).
[23] A. V. Chubukov, Annu. Rev. Condens. Matter Phys. 3, 57 (2012).
[24] H. Kontani and S. Onari, Phys. Rev. Lett. 104, 157001 (2010).
[25] T. Saito et al., Phys. Rev. B 82, 144510 (2010).
[26] S. Onari and H. Kontani, Phis. Rev. Lett. 109, 137001 (2012).
[27] Y. Ohno, M. Tsuchiizu, S. Onari, and H. Kontani, J. Phys. Soc. Jpn. 82, 013707 (2013).
[28] M. Tsuchiizu, Y. Ohno, S. Onari, and H. Kontani, Phys. Rev. Lett. 111 (2013), (arXiv:1209.3664)
[29] S. Liang, A. Moreo, and E. Dagotto, arXiv:1305.1879.
[30] F. Kruger et al., Phys. Rev. B 79 (2009) 054504.
[31] H. Kontani, T. Saito and S. Onari, Phys. Rev. B 84, 024528 (2011).
[32] T. Takimoto, T. Hotta, T. Maehira and K. Ueda, J. Phys. Condens. Matter 14, L369 (2002).
[33] T. Moriya, *Spin Fluctuations in Itinerant Electron Magnetism* (Springer-Verlag, 1985); A. Kawabata: J. Phys. F 4, 1477 (1974).
[34] S. Onari and H. Kontani, Phys. Rev. Lett. 103 177001 (2009).
[35] Y. Yamakawa, S. Onari, and H. Kontani, Phys. Rev. B 87 195121 (2013).
[36] S.V. Borisenko, V.B. Zabolotnyy, A.A. Kordyuk, D.V. Evtushinsky, T.K. Kim, I.V. Morozov, R. Follath and B.Buchner, Symmetry 4, 251 (2012).
[37] K. Kuroki, H. Usui, S. Onari, R. Arita, H. Aoki, Phys. Rev. B 79, 224511 (2009).
[38] S. Onari and H. Kontani, unpublished.
[39] A. Kawabata, S. C. Lee, T. Moryoshi, Y. Kobayashi and M. Sato, J. Phys. Soc. Jpn. 77, 103704 (2008); M. Sato, Y. Kobayashi, S. C. Lee, H. Takahashi, E. Satomi and Y. Miura, J. Phys. Soc. Jpn. 79, 014710 (2009); S.C. Lee, E. Satomi, Y. Kobayashi, and M. Sato, J. Phys. Soc. Jpn. 79, 023702 (2010).
[40] C. Tarantini, M. Putti, A. Gurevich, Y. Shen, R. K. Singh, J. M. Rowell, N. Newman, D. C. Larbalestier, P. Cheng, Y. Jia, and H.-H. Wen, Phys. Rev. Lett. 104, 087002 (2010).
[41] Y. Nakajima, T. Taen, Y. Tsuchiya, T. Tamegai, H. Kitamura, and T. Murakami, arXiv:1009.2848.
[42] Y. Li, J. Tong, Q. Tao, C. Feng, G. Cao, W. Chen, F.C. Zhang, and Z.A. Xu, New J. Phys. 12 (2010) 083008.