Beating pattern in radiation-induced oscillatory magnetoresistance in 2DES: coupling of plasmon-like and acoustic phonon modes.

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We present a microscopic theory on the observation of a beating pattern in the radiation-induced magnetoresistance oscillations at very low magnetic field. We consider that such a beating pattern develops as a result of the coupling between two oscillatory components: the first is a system of electron Landau states being harmonically driven by radiation. The second is a lattice oscillation, i.e., an acoustic phonon mode. We analyze the dependence of the beating pattern on temperature, radiation frequency and power. We conclude that the beating pattern is an evidence of the radiation-driven nature of the irradiated Landau states that makes them behave as a collective plasma oscillation at the radiation frequency. Thus, the frequency of such plasmons could be tuned from microwave to terahertz in the same nanodevice with an apparent technological application.

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Beating patterns show up when there are two oscillatory contributions coexisting and coupled in the same physical system. For instance, beating patterns can be observed in magnetoresistance ($R_{xx}$) of two-dimensional electron systems (2DES) when there are two populated conduction electron subbands involved in the transport. This situation gives rise to the well-known magneto-intersubband scattering oscillations (MISO)\textsuperscript{1,2}. They can also be obtained in $R_{xx}$ of 2DES with strong Rashba spin-orbit coupling\textsuperscript{3,4}. For both cases the two oscillatory subsystems are the two sets of broadened Landau levels of slightly different energies. On the other hand, two important physical effects in magnetotransport of 2DES
FIG. 1: Calculated $R_{xx}$ for a radiation frequency of 105 GHz. In panel a) $R_{xx}$ vs $B$ and in b) $R_{xx}$ vs $w/w_c$. Apart from the usual RIRO we observe a beat at low $B$ with a node around $B = 0.02T$ in panel a). In panel b) we observe a change in phase difference of $\pi$ when crossing the node.

were discovered more than a decade ago by Mani et al.\textsuperscript{5}: the radiation-induced magnetoresistance oscillations (RIRO) and the even more striking of zero resistance states (ZRS)\textsuperscript{5,6}. To date, there is not a clear consensus on the physical origin of such remarkable effects. After a huge number of experiments\textsuperscript{7–23} and proposed theories\textsuperscript{24–47} to explain them, we have to admit that they are still under debate.

Soon after the discovery of RIRO and ZRS, another surprising experimental result regarding RIRO was published\textsuperscript{49}. It consisted in an unexpected beating pattern at very low $B$ superimposed to RIRO. As with RIRO this beating pattern was radiation-induced. This subtle effect was overlooked by the scientific community and very little attention was paid. Nevertheless, there has recently been shown new experimental evidence presenting a similar beating pattern profile on RIRO at very low $B$\textsuperscript{50} too. As explained above, this radiation-induced beating pattern indicates the presence of two comparable oscillatory contributions. However, possible physical scenarios giving rise to beating patterns, such as two populated electron subbands or Rashba spin-orbit coupling, cannot easily explain the obtained experimental results\textsuperscript{49,50}.

In this letter we develop a microscopic theory to explain the beating pattern in RIRO
FIG. 2: Dependence of the beating pattern on the radiation frequency ranging from 30 GHz to 105 GHz. The node position does not change irrespective of the radiation frequency.

based, on the one hand, on the physical effect of plasmon-phonon coupling in polar semiconductors.\textsuperscript{51–53} According to it, in polar semiconductors like GaAs, collective oscillations of electrical charges (plasmons) and lattice ions oscillations (phonons) can couple via Coulomb interaction. As a result, the initially individual (plasmon and phonon) modes re-adjust their frequencies to give rise to new hybrid plasmon-phonon coupled modes. On the other hand, our microscopic theory is based on the previous model for RIRO, the radiation-driven electron orbit model. This model, in turn, is based on the exact solution of the electronic wave function in the presence of a static magnetic field and radiation. In this model the electrons orbits or Landau States (LS) move back and forth harmonically driven by radiation, (driven-LS), at the radiation frequency ($\omega$). Thus, the guiding centers of the LS perform harmonic and \textit{classical} trajectories making the system of driven-LS behave like a collective oscillation of electric charge, i.e. a plasmon-like mode. Now, this plasmon-like mode, with acoustic frequency, can couple with a collective lattice ions oscillation of similar amplitude and frequency, an acoustic phonon mode. Thus, we can observe the rise of a beating pattern, for instance, in $R_{xx}$. Therefore, the observation of beats in the RIRO profile would be a clear evidence of the spatial swinging nature of the irradiated LS. This provides a source of excitation of acoustic plasmon-like modes in 2DES with a frequency ranging from the microwave (MW) to the terahertz (THz) part of the spectrum.
FIG. 3: Dependence of the beating pattern on temperature. In the upper panel we exhibit irradiated $R_{xx}$ vs $B$ and in the lower panel the same vs $w/w_c$. In the lower panel the node position moves to lower $w/w_c$ for decreasing $T$. In the upper panel we observe the opposite trend, the node moves to higher $B$ for decreasing $T$.

As we said above, the radiation-driven electron orbits mode$^{32,33}$ was developed to explain the striking effects of RIRO and ZRS. One of the main conclusion of this theory is that under radiation the LS oscillate, with their guiding centers, at the radiation frequency according to $X(t) = X_0 + A \sin wt$, where $X(t)$ is the time dependent LS guiding center position, $X_0$ is the same without radiation and $A = \frac{eE_0}{m^*\sqrt{(w^2 - w_c^2)^2 + \gamma^4}}$ where, in turn, $E_0$ is the radiation electric field and $w_c$ the cyclotron frequency. $\gamma$ is a phenomenologically introduced damping factor for the electron scattering with the lattice ions. Following the physics of plasmon-phonon coupling we consider that the system of driven-LS, behaving like a "plasmon-like" mode, can couple with an acoustic phonon mode of similar frequency and amplitude. We derive classically and similarly to a system of coupled harmonic oscillators, the new frequencies of the hybrid modes$^{54}$ and the guiding center position of the driven-LS mode. Firstly, and after some algebra, these frequencies are given by$^{55,57}$:

$$2w^2_\pm = (w^2 + w^2_{ac}) \pm \sqrt{(w^2 - w^2_{ac}) + 16\lambda^2 w_{ac}w}$$ (1)

where $w$ is the frequency of both, radiation and the driven-LS mode. $w_{ac}$ is the frequency of the acoustic phonon mode and $\lambda$ the plasmon-phonon coupling constant. If $w$ is only slightly
different from \(w_{ac}\), i.e., \(w \simeq w_{ac}\), then it is straightforward to finally obtain that \(w_{+} \simeq w_{-} \pm \lambda\), where \(\lambda \ll w\). Secondly, the position of the guiding center of the hybrid driven-LS mode is now \(X(t) = X_{0} + A \sin w_{+} t + B \sin w_{-} t\).

With similar algebra as before, we introduce the damping that the hybrid driven-LS mode undergoes due to scattering with the lattice ions. The obtained expression for \(w_{+}\) and \(X(t)\) are now:

\[
2w_{\pm}^2 = (w_{1}^2 + w_{2}^2) \pm \sqrt{(w_{1}^2 - w_{2}^2)^2 + 16\lambda^2 w_{2} w_{1}}
\]

\[
X(t) = X_{0} + e^{-\frac{2}{\tau}} [A \sin w_{+} t + B \sin w_{-} t]
\]

where \(w_{1}^2 = w^2 - \frac{\gamma^2}{4} \simeq w^2\) and \(w_{2}^2 = w_{ac}^2 - \frac{\gamma^2}{4} \simeq w_{ac}^2\), considering that \(\gamma^2 \ll w^2, w_{ac}^2\). With the new obtained expression for \(X(t)\) (Eq. 3) and according to the radiation-driven electron orbit model, we obtain for the average distance advanced by the electron in a scattering event,

\[
\Delta X(t) = \Delta X_{0} - 2A e^{-\frac{2}{\tau}} \sin \left( \frac{2\pi}{w_{c}} \right) \cos \left( \frac{2\pi}{w_{c}} \lambda \right)
\]

where we have considered that the amplitudes of both modes are similar, i.e., \(A \simeq B\) and that \(w_{\pm} \simeq w \pm \lambda\). The time, \(\tau\), according to the radiation driven electron orbit model\(^{58,59}\), is the ”flight time”, the time it takes the electron to jump due to scattering from one orbit to another and its value is given by \(\tau = \frac{2\pi}{w_{c}}\). Following the same RIRO model and using the obtained \(\Delta X(t)\), we end up with an expression for \(R_{xx}\):\(^{58,59}\)

\[
R_{xx} \propto \Delta X_{0} - 2A e^{-\frac{2}{\tau}} \sin \left( \frac{2\pi}{w_{c}} \right) \cos \left( \frac{2\pi}{w_{c}} \lambda \right)
\]

where we want to stand out the essential part that explains the appearance of the beating pattern in RIRO.

In Fig. 1 we exhibit calculated \(R_{xx}\) for a radiation frequency of 105 GHz. In Fig. 1a, \(R_{xx}\) vs \(B\) and in Fig. 1b, \(R_{xx}\) vs \(w/w_{c}\). Apart from the usual RIRO we observe a beat at very low \(B\) with a node around \(B = 0.02T\) in the upper panel. In the lower panel the vertical lines for integer values of the abscissa indicate a phase change of \(\pi\) in the \(R_{xx}\) oscillations when crossing the node. Eq. 5 readily explains the rise of a beat when \(w_{+}\) is just slightly different from \(w_{-}\) or in other words, when \(\lambda \ll w\); the change of phase in \(\pi\) is due to the modulation of the slower function, i.e., the cosine function. In Fig. 2 we present the dependence of the
beating pattern on the frequency, ranging from 30 GHz to 105 GHz. We observe a constant $B$-position for the node irrespective of the frequency. We find again the explanation in Eq. 5. We can tell that the node position depends on the cosine function where $w$ does not show up. Thus, the node position is immune to $w$. However any variation of the coupling constant $\lambda$ that shows up in the cosine will clearly affect the node position and even the number of beats that can be observed.

In Fig. 3 we present the calculated results of the dependence of the beating pattern on temperature ($T$) for a radiation frequency of 40 GHz and $T$ from 0.5K to 1.0K. In Fig. 3a, we exhibit $R_{xx}$ vs $B$ and in Fig. 3b $R_{xx}$ vs $w/w_c$. For both panels the curves are shifted for clarity. Interestingly enough, as exhibited in the lower panel, the node position is not constant and moves to lower $w/w_c$ for decreasing $T$, and in the same way the beat gets more intense. In the upper panel we observe the opposite trend, the node moves to higher $B$ for decreasing $T$ but the intensity increase keeps the same as in the lower one. The displacement of the node and the intensity variation indicate that a changing temperature affects the driven-LS-phonon coupling and, in turn, $\lambda$. In Fig. 4 we study the beating pattern in function of the radiation power ($P$). $P$ runs from 0.7 mW to 2 mW. Similarly as in Fig.3, in panel a) we exhibit irradiated $R_{xx}$ vs $B$ and in panel b) the same vs $w/w_c$. As with $T$, we observe a similar node displacement and beat intensity variation for decreasing $P$, proving that $\lambda$ depends on $P$ too. Thus, both quantities, $T$ and $P$, affect the beat in
FIG. 5: Dependence of the beating pattern on radiation power and temperature for a radiation frequency of 105 GHz. In the upper panel we exhibit irradiated $R_{xx}$ vs $B$ and the lower one $R_{xx}$ vs $w/w_c$. As expected, we observe an extra beat and node when going from $P = 1\, mW$ and $T = 0.5K$ to $P = 0.1\, mW$ and $T = 0.1K$.

the same way: a decrease of any of them makes the coupling stronger and $\lambda$ bigger. And, on the other hand, an increase gives rise to a progressive destruction of the beat. The physical explanation can be readily obtained from our model. A higher $T$ triggers a more intense scattering between the electrons in their orbits and the lattice ions. In the case of an increasing $P$, the amplitude of the driven-LS oscillations gets also bigger, and in turn, the probability for the electrons in their orbits to be scattered is also higher too. Thus, for both increasing quantities we obtain a similar damping effect on the driven-LS-acoustic phonon coupling that gets progressively destroyed. To study the dependence of $\lambda$, i.e., of the beating pattern on $T$ and $P$, we have developed a phenomenological equation consisting in adding a first order (linear) correction to $\lambda$ in the variation of $T$ and $P$. Thus, for $T$, $\lambda = \lambda_0 - \lambda_1 (T - T_0)$ where $\lambda_0$ and $\lambda_1$ are constants and $T_0 = 0.5K$ in agreement with experimental values. And a similar equation for $P$, $\lambda = \lambda_0 - \lambda_1 (P - P_0)$ and $P_0 = 0.7mW$. The calculated results are in qualitatively good agreement with experiments.

In Fig. 5 we exhibit the calculated results when simultaneously changing $T$ and $P$. Ac-
According to our model, for instance in the case of decreasing, this will lead to a strengthening of the beating pattern effect. Thus, we would expect the rise of more nodes and beats. This is presented in Fig. 5 for a radiation frequency of 105 GHz; in the upper panel $R_{xx}$ vs $B$ and the lower one $R_{xx}$ vs $w/w_c$. As expected, we observe the appearance of an extra beat and node when going from $P = 1 mW$ and $T = 0.5 K$ to $P = 0.1 mW$ and $T = 0.1 K$. We observe the jump of $\pi$ when crossing a node. Thus, in this figure we observe a total phase change of $2\pi$ when crossing two nodes in a row. Again this is explained by the effect of modulation of the cosine function on RIRO.

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