Twisted Sector Yukawa Couplings For The $Z_3 \times Z_3$ Orbifold

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ABSTRACT

The moduli dependent Yukawa couplings between twisted sectors of the $Z_3 \times Z_3$ orbifold are studied.
A comparison of orbifold compactified string theory models [1,2] with observation will require amongst other things a knowledge of the Yukawa couplings. Particularly important is the moduli dependence of Yukawa couplings, which can arise amongst twisted sector states [3, 4] because their exponential dependence on orbifold “moduli” may have a bearing on hierarchies [5]. Twisted sector Yukawa couplings have been investigated for the $Z_3$ orbifold [3-6], for the $Z_7$ orbifold [7] and, more recently, for all the $Z_N$ orbifolds [8-10]. It is our purpose here to study twisted sector Yukawa couplings for the $Z_3 \times Z_3$ orbifold, which has also been used [2] in the computation of potentially realistic models. In the first instance the $Z_3 \times Z_3$ orbifold may be realised (see, for example, Ref.[2]) by first constructing an $[SU(3)]^3$ torus using the identifications under translations

$$X_i \sim X_i + e_i, \quad i = 1, 2, 3$$

(1)

and

$$X_i \sim X_i + \tilde{e}_i, \quad i = 1, 2, 3$$

(2)

with

$$e_i = 1, \quad \tilde{e}_i = e^{2\pi i/3},$$

(3)

for all values of $i$, where the 3-complex coordinates $X_i$ define the six-dimensional compact manifold. The orbifold with point group $Z_3 \times Z_3$ is then obtained by identifying points on the torus under the rotations

$$\theta : X_1 \rightarrow e^{2\pi i/3}X_1, \quad X_2 \rightarrow X_2, \quad X_1 \rightarrow e^{4\pi i/3}X_3$$

(4)

and

$$\omega : X_1 \rightarrow X_1, \quad X_2 \rightarrow e^{2\pi i/3}X_2, \quad X_3 \rightarrow e^{4\pi i/3}X_3.$$  

(5)

The action of the point group on the lattice is then

$$\theta e_1 = \tilde{e}_1, \quad \theta e_2 = e_2, \quad \theta e_3 = -e_3 - \tilde{e}_3, \quad \theta \tilde{e}_1 = -e_1 - \tilde{e}_1, \quad \theta \tilde{e}_2 = \tilde{e}_2, \quad \theta \tilde{e}_3 = e_3$$

(6)
\( \omega e_1 = e_1, \ \omega e_2 = \tilde{e}_2, \ \omega e_3 = -e_3 - \tilde{e}_3, \ \omega \tilde{e}_1 = \tilde{e}_1, \ \omega \tilde{e}_2 = -e_2 - \tilde{e}_2, \ \omega \tilde{e}_3 = e_3 \)

(7)

More generally, the original (rigid) lattice (3) may be deformed in ways which preserve the action (6) and (7) of the point group. To obtain the most general choice of the lattice compatible with the point group we must require that all the scalar products \( e_i.e_j, \ e_i.\tilde{e}_j \) and \( \tilde{e}_i.\tilde{e}_j \) are preserved by the transformations (6) and (7). If we write

\[
e_i.e_j = |e_i||e_j| \cos \theta_{ij} \tag{8}
\]

\[
e_i.\tilde{e}_j = |e_i||\tilde{e}_j| \cos \theta_{ij} \tag{9}
\]

and

\[
\tilde{e}_i.\tilde{e}_j = |\tilde{e}_i||\tilde{e}_j| \cos \theta_{ij} \tag{10}
\]

we find that

\[
|\tilde{e}_i| = |e_i|, \quad i = 1, 2, 3 \tag{11}
\]

and that all angles between distinct basis vectors are fixed to be \( \pi/2 \) with the exception of \( \theta_{11}, \theta_{22} \) and \( \theta_{33} \) which are fixed to be \( 2\pi/3 \)

\[
\cos \theta_{11} = \cos \theta_{22} = \cos \theta_{33} = -\frac{1}{2} \tag{12}
\]

Thus we may take the 3 independent deformations of the lattice compatible with the point group to be

\[
R_i \equiv |e_i|, \quad i = 1, 2, 3 \tag{13}
\]

In general, left chiral massless states occur in the \( \theta, \theta^2, \omega, \omega^2, \theta \omega^2, \theta^2 \omega \) and \( \theta \omega \) twisted sectors, and the point group selection rules and \( H \)-momentum selection rules restrict [2] the allowed couplings amongst these sectors to the forms

\[
DDD, \ \bar{A}BC, \ \bar{A}B\bar{C}, \ ACD, \ B\bar{C}D \text{ and } \bar{A}\bar{B}D \tag{14}
\]

where we use \( A, \ \bar{A}, \ B, \ \bar{B}, \ C, \ \bar{C} \) and \( D \), respectively, to denote the twisted sectors listed above. The fixed tori and fixed points for the twisted sectors are readily
obtained using the action (6) and (7) of the generators of the point group on the deformed lattice. The $\theta$ twisted sector ($A$) states are associated with the 9 inequivalent fixed tori given by

$$f_\theta = \frac{m_1}{3}(2e_1 + \tilde{e}_1) + \frac{m_3}{3}(e_3 - \tilde{e}_3) + a_2 e_2 + b_2 \tilde{e}_2, \quad m_1, m_2 = 0, \pm 1$$

(15)

where $a_2$ and $b_2$ are arbitrary. These are fixed tori of the space group elements $(\theta, l(\theta))$ where

$$l(\theta) = (I - \theta)f_\theta + (I - \theta)\Lambda = m_1e_1 + m_3e_3 + (I - \theta)\Lambda,$$

(16)

where $\Lambda$ signifies an arbitrary lattice vector. The $\theta^2$—twisted sector ($\bar{A}$) states are associated with the same fixed tori $f_\theta$ but with the space group elements $(\theta^2, l(\theta^2))$ where

$$l(\theta^2) = (I - \theta^2)f_\theta + (I - \theta^2)\Lambda = -m_1e_1 - m_3e_3 + (I - \theta^2)\Lambda.$$  

(17)

Similarly, the $\omega$ and $\omega^2$—twisted sector states ($B$ and $\bar{B}$) are associated with 9 inequivalent fixed tori

$$f_\omega = \frac{n_2}{3}(2e_2 + \tilde{e}_2) + \frac{n_3}{3}(e_3 - \tilde{e}_3) + c_1 e_1 + d_1 \tilde{e}_1, \quad n_2, n_3 = 0, \pm 1,$$

(18)

where $c_1$ and $d_1$ are arbitrary. These are fixed tori of the space group elements $(\omega, l(\omega))$ with

$$l(\omega) = (I - \omega)f_\omega + (I - \omega)\Lambda = n_2e_2 + n_3e_3 + (I - \omega)\Lambda$$

(19)

and $(\omega^2, l(\omega^2))$ with

$$l(\omega^2) = (I - \omega^2)f_\omega + (I - \omega^2)\Lambda = -n_2e_2 - n_3e_3 + (I - \omega^2)\Lambda.$$  

(20)

Also the $\theta \omega$ and $\theta^2 \omega$—twisted sector states ($C$ and $\bar{C}$) are associated with the 9
inequivalent fixed tori

\[ f_{\theta \omega^2} = \frac{p_1}{3}(2e_1 + \bar{e}_1) + \frac{p_2}{3}(e_2 - \bar{e}_2) + g_3 e_3 + h_3 \bar{e}_3, \quad p_1, p_2 = 0, \pm 1 \]  

where \( g_3 \) and \( h_3 \) are arbitrary. These are fixed tori of the space group elements \( (\theta \omega^2, l(\theta \omega^2)) \) with

\[ l(\theta \omega^2) = (I - \theta \omega^2)f_{\theta \omega^2} + (I - \theta \omega^2)\Lambda = p_1 e_1 + p_2 e_2 + (I - \theta \omega^2)\Lambda \]  

and \( (\theta^2 \omega, l(\theta^2 \omega)) \) with

\[ l(\theta^2 \omega) = (I - \theta^2 \omega)f_{\theta^2 \omega} + (I - \theta^2 \omega)\Lambda = -p_1 e_1 - p_2 e_2 + (I - \theta^2 \omega)\Lambda \]  

Finally, the \( \theta \omega \) twisted sector states \( (D) \) are associated with the 27 inequivalent fixed points

\[ f_{\theta \omega} = \frac{1}{3} \sum_{i=1}^{3} r_i(2e_i + \bar{e}_i), \quad r_1, r_2, r_3 = 0, \pm 1 \]  

with corresponding space group elements \( (\theta \omega, l(\theta \omega)) \) where

\[ l(\theta \omega) = \sum_{i=1}^{3} r_i e_i + (I - \theta \omega)\Lambda. \]  

All of these fixed tori and fixed points reduce to those of [11] in the case of the rigid lattice of (3). The space group selection rules for couplings amongst these twisted sectors are identical in form to those given in Ref. [11] for the rigid lattice case.

The values of the Yukawa couplings amongst the twisted sectors just discussed are determined in detail by three-point functions involving fermionic and bosonic string degrees of freedom. However, the crucial dependence on the deformation
parameters (moduli) and the particular fixed points and fixed tori is entirely con-
tained in (bosonic) twist field correlation functions [3, 4] of the type

\[ Z = \prod_{i=1}^{3} < \sigma^i_\alpha(z_1, \bar{z}_1) \sigma^i_\beta(z_2, \bar{z}_2) \sigma^i_\gamma(z_3, \bar{z}_3) > \] (26)

where \( \alpha, \beta \) and \( \gamma \) label the twisted sectors at the particular fixed points and
fixed tori involved, and the index \( i \) distinguishes the twist fields associated with
the complex coordinates \( X_i, i = 1, 2, 3 \). In the case of the \( DDD \) coupling, the
discussion is identical up to a point to that of the \( Z_3 \) orbifold [4-6] because \( \theta \omega \)
is identical to the point group element generating \( Z_3 \) and the same rigid lattice
is involved. However, whereas the deformation parameters (moduli) for the \( Z_3 \)
orbifold contains angles as well as moduli, the deformation paramters which enter
the Yukawa couplings for the \( Z_3 \times Z_3 \) orbifold are restricted to \( R_1, R_2 \) and \( R_3 \) of
(13).

Let the three fixed points involved be labelled by \( r_1^i, r_2^i, \) and \( r_3^i, i = 1, 2, 3, \)
respectively, in the notation of (24). The space group selection rules [3, 4, 11]
requires

\[ \sum_{J=1}^{3} r^J_i = 0 \text{ (mod 3)}, \quad i = 1, 2, 3, \] (27)

If we define

\[ d_i = r_1^i - r_2^i \] (28)

then we can write the leading exponential in the \( DDD \) Yukawa couplings as

\[ Z_{DDD} \sim \exp \left( -\frac{1}{2\pi \sqrt{3}} \sum_i \Delta_i^2 R_i^2 \right) \] (29)

where

\[ \Delta_i = d_i, \quad \text{for } d_i = 0, \pm 1 \] (30)
and
\[ \Delta_i = d_i \mp 3, \quad \text{for } d_i = \pm 2. \tag{31} \]

For the $AB\bar{C}$ Yukawa coupling we need to evaluate
\[ Z_{AB\bar{C}} = \prod_{i=1}^{3} < \sigma_i^1(z_1, \bar{z}_1) \sigma_i^1(\omega^2 z_2, \bar{z}_2) \sigma_i^1(\omega^2 z_3, \bar{z}_3) > \tag{32} \]
where the index labelling the fixed torus for each twist field has been suppressed. Because $\theta$ only rotates $X_1$ and $X_3$, $\omega^2$ only rotates $X_2$ and $X_3$, and $\theta^2 \omega$ only rotates $X_1$ and $X_2$, some of the twist fields are the identity and $Z_{AB\bar{C}}$ reduces to
\[ Z_{AB\bar{C}} = < \sigma_1^1(z_1, \bar{z}_1) \sigma_1^1(\omega^2 z_3, \bar{z}_3) > < \sigma_2^2(\omega^2 z_2, \bar{z}_2) \sigma_2^2(\omega^2 z_3, \bar{z}_3) > \times < \sigma_3^3(z_1, \bar{z}_1) \sigma_3^3(\omega^2 z_2, \bar{z}_2) >. \tag{33} \]
Thus, only 2-point functions for twist fields are involved (which can be normalized to 1) and no dependence on moduli or the specific fixed tori involved arises. A similar argument applies for the $\bar{A}BC$ Yukawa coupling.

For the $ACD$ coupling the situation is more interesting. In that case, we have to evaluate
\[ Z_{ACD} = \prod_{i=1}^{3} < \sigma_i^1(z_1, \bar{z}_1) \sigma_{\theta, \omega^2}^1(z_2, \bar{z}_2) \sigma_{\theta, \omega^2}^1(z_3, \bar{z}_3) >. \tag{34} \]
In view of the fact that $\theta$ only rotates $X_1$ and $X_3$ and $\theta \omega^2$ only rotates $X_1$ and $X_2$ this reduces to
\[ Z_{ACD} = < \sigma_1^1(z_1, \bar{z}_1) \sigma_{\theta, \omega^2}^1(z_2, \bar{z}_2) \sigma_{\theta, \omega^2}^1(z_3, \bar{z}_3) > < \sigma_2^2(\omega^2 z_2, \bar{z}_2) \sigma_{\theta, \omega^2}^2(z_3, \bar{z}_3) > \times < \sigma_3^3(z_1, \bar{z}_1) \sigma_{\theta, \omega^2}^3(z_3, \bar{z}_3) >. \tag{35} \]
The last two factors can be normalized to 1, but the first factor is non-trivial and can be calculated using the methods of Ref. [3]. The three twist field correlation
function

\[ Z_1 = \sigma_\theta^1(z_1, \bar{z}_1) \sigma_{\theta_2}^1(z_2, \bar{z}_2) \sigma_{\theta_3}^1(z_3, \bar{z}_3) > \]  \tag{36}

factors into a quantum piece \( Z_{qu} \) and a classical piece with all the dependence on the moduli and the particular fixed points and fixed tori involved contained in the classical piece.

\[ Z_1 = Z_{qu} \sum_{X_{cl}} e^{-S_{cl}}, \]  \tag{37}

where the classical action is

\[ S_{cl} = \frac{1}{\pi} \int d^2 z \left( \frac{\partial X_1}{\partial z} \frac{\partial X_1}{\partial \bar{z}} + \frac{\partial X_1}{\partial \bar{z}} \frac{\partial X_1}{\partial z} \right). \]  \tag{38}

Because of the string equations of motion

\[ \frac{\partial^2 X_1}{\partial z \partial \bar{z}} = 0, \]  \tag{39}

\( \partial X_1/\partial z \) and \( \partial X_1/\partial \bar{z} \) are functions of \( z \) and \( \bar{z} \) alone, respectively, which have to be chosen to respect the boundary conditions at \( z_1, z_2 \) and \( z_3 \) implicit in the operator product expansions with the twist fields. Here, the relevant operator product expansions are

\[ \frac{\partial X_1}{\partial z} \sigma_\theta^1(z_1, \bar{z}_1) \sim (z - z_1)^{-2/3}, \]
\[ \frac{\partial X_1}{\partial z} \sigma_{\theta_2}^1(z_2, \bar{z}_2) \sim (z - z_2)^{-2/3}, \]  \tag{40}
\[ \frac{\partial X_1}{\partial z} \sigma_{\theta_3}^1(z_3, \bar{z}_3) \sim (z - z_3)^{-2/3}, \]

and

\[ \frac{\partial X_1}{\partial \bar{z}} \sigma_\theta^1(z_1, \bar{z}_1) \sim (\bar{z} - \bar{z}_1)^{-1/3}, \]
\[ \frac{\partial X_1}{\partial \bar{z}} \sigma_{\theta_2}^1(z_2, \bar{z}_2) \sim (\bar{z} - \bar{z}_2)^{-1/3}, \]  \tag{41}
\[ \frac{\partial X_1}{\partial \bar{z}} \sigma_{\theta_3}^1(z_3, \bar{z}_3) \sim (\bar{z} - \bar{z}_3)^{-1/3}. \]
Correspondingly, $\partial X_1/\partial z$ and $\partial X_1/\partial \bar{z}$ are of the form

$$\frac{\partial X_1}{\partial z} = a_1(z - z_1)^{-2/3}(z - z_2)^{-2/3}(z - z_3)^{-2/3}$$  \hspace{1cm} (42)

and

$$\frac{\partial X_1}{\partial \bar{z}} = b_1(\bar{z} - \bar{z}_1)^{-1/3}(\bar{z} - \bar{z}_2)^{-1/3}(\bar{z} - \bar{z}_3)^{-1/3}.$$  \hspace{1cm} (43)

Only the holomorphic field $\partial X_1/\partial z$ is an acceptable classical solution, because $\partial X_1/\partial \bar{z}$ gives a divergent contribution to the classical action. The contribution of $\partial X_1/\partial z$ is of the form

$$S_{cl} = \left[ \frac{\Gamma(1/3)}{\Gamma(2/3)} \right]^3 |a_1|^2 |z_3|^{-4/3}$$  \hspace{1cm} (44)

where we have used $SL(2, C)$ invariance to set

$$z_1 = 0, \quad z_2 = 1, \quad z_3 = \infty$$  \hspace{1cm} (45)

and the integral has been evaluated with the aid of Appendix A of ref. [12]. The allowed values of $a_1$ are determined by the global monodromy condition [3]

$$\oint_C dz \frac{\partial X_1}{\partial z} = v_1$$  \hspace{1cm} (46)

where $C$ is a closed contour around which $X_1$ is shifted by $v_1$ but not rotated. In the present case, the contour may be chosen to be the contour of fig. 1, where the point $z_1 = 0$ is encircled once anti-clockwise and the point $z_2 = 1$ is encircled once clockwise. The relevant integral is an integral of the type [9]

$$\oint_C dz z^{-1-k_1 w}(z - 1)^{-1-k_2 w} = -2i \sin(k_1 k_2 \pi w) \frac{\Gamma(k_1 w) \Gamma(k_2 w)}{\Gamma(k_1 w + k_2 w)}$$  \hspace{1cm} (47)

where the contour $C$ is such that it encircles $z = 0$ $k_2$ times anti-clockwise and
\[ z = 1 \ k_1 \text{ times clockwise. Thus,} \]

\[ a_1 = \frac{i}{\sqrt{3}} \frac{\Gamma(2/3)}{[\Gamma(1/3)]^2} (-z_3)^{2/3} v_1 \]

Moreover, the shift \( v_1 \) on \( X_1 \) for the contour of fig. 1 is obtained as the component in the \( X_1 \)-plane (corresponding to \( \mathbf{e}_1 \) and \( \bar{\mathbf{e}}_1 \)) of the product of space group elements

\[
\left( \theta, (I - \theta) f_\theta + (I - \theta) \Lambda \right) \left( \theta^2 \omega, (I - \theta^2 \omega) f_{\theta^2 \omega} + (I - \theta^2 \omega) \Lambda' \right)
\]

where \( \theta^2 \omega \) is \((\theta \omega)^{-1}\).

Consequently \( v_1 \) is of the form

\[ v_1 = (m_1 - p_1)e_1 + (I - \theta)\Lambda + (I - \theta^2 \omega)\Lambda' \]

where the notations of (15) and (21) for the fixed tori are being used, and \( \Lambda \) and \( \Lambda' \) denote arbitrary lattice vectors. Combining (37), (38), (44), (48) and (49) and recalling that \( |e_1| = R_1 \), we find the leading order behaviour for the \( ACD \) Yukawa coupling

\[ Z_{ACD} \sim 1, \quad \text{for } m_1 - p_1 = 0, \]

\[ \sim \exp \left( -\frac{R_1^2}{2\pi \sqrt{3}} \right), \quad \text{for } m_1 - p_1 = \pm 1, \pm 2. \]

A similar discussion applies to the \( B\bar{C}D \) and \( \bar{A}\bar{B}D \) Yukawa couplings. For the \( B\bar{C}D \) coupling, the relevant quantity is

\[ Z_{B\bar{C}D} = \langle \sigma_\omega^2(z_1, \bar{z}_1)\sigma_{\theta^2 \omega}^2(z_2, \bar{z}_2)\sigma_{\bar{\theta}\omega}^2(z_3, \bar{z}_3) \rangle \]

and the leading behaviour is

\[ Z_{B\bar{C}D} \sim 1, \quad \text{for } n_2 + p_2 = 0, \]

\[ \sim \exp \left( -\frac{R_2^2}{2\pi \sqrt{3}} \right), \quad \text{for } n_2 + p_2 = \pm 1, \pm 2 \]

with the notation of (18) and (21) for the fixed tori. For the \( \bar{A}\bar{B}D \) coupling the
relevant quantity is

\[ Z_{\bar{A}\bar{B}D} = <\sigma^3_{\theta^2}(z_1, \bar{z}_1)\sigma^3_{\omega^2}(z_2, \bar{z}_2)\sigma^3_{\theta^2}(z_3, \bar{z}_3)> \]  

(53)

and the leading behaviour is

\[ Z_{\bar{A}\bar{B}D} \sim 1, \quad \text{for } n_3 - m_3 = 0, \]
\[ \sim \exp\left(-\frac{R^2_3}{2\pi\sqrt{3}}\right), \quad \text{for } n_3 - m_3 = \pm 1, \pm 2 \]  

(54)

with the notation of (15) and (18) for the fixed tori. It is of some interest to compare the behavior of the \( Z_3 \times Z_3 \) orbifold Yukawa couplings just obtained with those of the 1\(^9\) Gepner model [13] which is generally believed to correspond to the \( Z_3 \times Z_3 \) orbifold (without Wilson lines) at an enhanced symmetry point where \( R_1 = R_2 = R_3 \). Elsewhere [11] we have already made identifications between the massless states of these two models and checked that the space group selection rules of the orbifold and the various selection rules of the Gepner model lead to the same Yukawa couplings being non-zero. Using these identifications and the connection [15, 16] between \( N = 2 \) superconformal models and \( SU(2) \) WZNW models, it is not difficult to show that the couplings consistent with the selection rules obtained from the 1\(^9\) Gepner model are of the following form. For the \( DDD \) couplings, and with \( d_i \) as in (28), the non-zero couplings are given by

\[ b^9 \quad \text{for } d_1 = \pm 1, \pm 2, \quad d_2 = \pm 1, \pm 2, \quad d_3 = \pm 1, \pm 2 \]  

etc

(55)

Other choices of the \( d_i \) [11] are not consistent with the \( U(1)^6 \) selection rule which is present at the enhanced symmetry point. For the \( A\bar{B}\bar{C} \) couplings, all couplings are given by \( b^9 \), and the non-zero \( ACD \) couplings are given by

\[ b^9 \quad \text{for } m_1 - p_1 = \pm 1, \pm 2 \]  

(56)

Other choices of the \( m_1 - p_1 \) are not consistent with the \( U(1)^6 \) selection rule at the enhanced symmetry point. In these expressions, and in terms of the quantum
numbers \( \left( \begin{array}{c} j \\ m \\ \bar{m} \end{array} \right) \) of \( SU(2) \) WZNW primary fields, we have used the following notation

\[
b = \langle \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \rangle^2 \left( \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \right), \tag{57}
\]

(The couplings \( \bar{A}BC, B\bar{C}D \) and \( \bar{A}\bar{B}D \) are similar) It can be seen that the above expressions for the \( DDD \), \( \bar{A}\bar{B}\bar{C} \), and \( ACD \) couplings are consistent with (29) at the enhanced symmetry point \( R_1 = R_2 = R_3 \), with the independence of the non-zero \( \bar{A}\bar{B}\bar{C} \) couplings from the particular fixed tori involved, and with (50).

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**FIGURE CAPTION**

Fig.1 The contour $C$ used in the global monodromy condition.