**INTRODUCTION**

Capillary, rotary, and vibrating wire viscometers, and Marsh funnels are commonly used to measure the rheology of drilling fluids. The hydraulic principle of capillary viscometers is based on the plastic fluid flow law in tubes under structural flow. Then, the operation must meet specific experimental conditions. Rotary and vibrating wire viscometers have complex structures, require a power supply for measurements, and have limited use in field operations. By contrast, the Marsh funnel has a simple structure, convenient operation, and easily accessible data; thus, they are widely used to measure the rheology of drilling fluids and cement pastes in laboratory
and field operations. The development of the Marsh funnel is credited to Hallan N. Marsh, who regulated the design and usage of his funnel viscometer. The funnel consists of a cone equipped with a removable tube attached to its bottom. Although the dimensions and measurement standards of the funnel vary from one country to another, its principle is the same. It is used to measure the time, in seconds, required to fill a measuring cup of a specific volume. In the United States, the measuring cup volume of the Marsh funnel is one quart. This measured flow time is referred to as the Marsh funnel viscosity (MFV). Then, the longer the flow time of the fluid, the lower the fluidity, and the higher the MFV.

Fluids in the drilling industry are non-Newtonian expect for a few well-understood Newtonian fluids, such as water, air, and some oil products. Many rheological models have been proposed to describe the flow characteristics of non-Newtonian fluids. The Bingham model indicates that flow occurs when the driving force on non-Newtonian fluids is greater than the yield stress. The power-law model describes the flow characteristics of non-Newtonian fluids at low shear rates. The rheological curve of the Carson model is a straight line, and the rheological properties of non-Newtonian fluids at high shear rates can be predicted using this model measurement at low and medium shear rates. The three-parameter Herschel-Bulkley model accurately describes the rheological properties of non-Newtonian fluids over a wide range of shear rates. Compared with the Herschel-Bulkley model, the Nasiri-Ashrafizadeh model adds a fourth parameter to describe non-Newtonian fluids.

Even though the Marsh funnel has been widely used over the last century, its measurement is considered purely empirical and of no fundamental significance. The reason is that the MFV is only a dynamic viscosity and cannot characterize actual rheology. Thus, measuring rheology with the Marsh funnel is becoming increasingly important. One goal of this research is to expand the function of the Marsh funnel, given that it is a simple device that can reliably quantify the rheology of all relevant fluids. Previous research is reviewed below.

Pitt was the first to research this area. Using the Skelland formula, experimental data, and the program Fortran, he mathematically modeled apparent viscosity in terms of the MFV and the density of power-law fluids. However, his numerical simulation using water assumed that the flow inside the Marsh funnel is laminar and underestimated the actual shear rate.

Le Roy et al. presented a model between the MFV and the viscosity of purely Newtonian fluids using the Poiseuille law. Experimental data from glycerol-water mixtures verified that the model accuracy increases with viscosity. However, the model does not apply to fluids with yield stress.

Roussel et al. demonstrated that for Bingham fluids, the MFV could be calculated from the plastic viscosity and yield stress. The model was presented using the Buckingham-Reiner equation and was validated using several cement pastes. In addition, a method using two different sizes of tubes in which one fluid is measured by two sets of parameters was proposed. However, this work had low calculation accuracy.

Nguyen et al. found that not all flows in tubes are of the Poiseuille type. Thus, a corrective coefficient of the rate based on numerical simulation was introduced. The authors established a semianalytical approach that allows one to determine the MFV as a function of the rheological parameters of Herschel-Bulkley fluids. In this study, the rheological parameters of Herschel-Bulkley fluids were measured using a Haake rheometer. As a result, the calculated MFV was close to the measured data. However, the calculation conditions were strict, and the calculation process was complicated.

Balhoff et al. used mass balance to derive the relationship between liquid height and flow time inside the Marsh funnel and combined the pressure drop to determine the mathematical expression of the wall shear stress. For Newtonian fluids, such as mineral oil, the ordinary differential equation was solved by the Hagen-Poiseuille equation; for non-Newtonian fluids, such as bentonite slurry, the Runge-Kutta numerical method was applied to obtain the rheological properties. They plotted the liquid height vs the flow time curves and compared them with the funnel and prediction data. Furthermore, they used the ultimate static height in the Marsh funnel to calculate the yield stress. However, the liquid height in this research excluded the height of the tube section, which leads to the calculation values of wall shear stress is small than the actual data.

According to the hydrostatics and the geometric relationship inside the Marsh funnel, Guria et al. used volume flow rate and liquid height to estimate wall shear rate and wall shear stress, respectively. The actual yield stress was calculated from the liquid height under no-flow conditions. Additionally, the apparent viscosity and plastic viscosity were measured using a Fann viscometer. The apparent viscosity, plastic viscosity, and yield stress of Newtonian fluids (such as synthetic crude oil) and some non-Newtonian fluids (such as PEG + NaCl + bentonite) could be computed from the Marsh funnel readings. However, the authors did not consider the flow factor, which varies across fluids and directly affects the fluid flow through the Marsh funnel.

A recent work by Sedaghat used the Bernoulli equation to optimize the mathematical calculation model for the wall shear stress and added different geometrical correction factors for the wall shear rate of Newtonian and non-Newtonian fluids. A Newtonian fluid, that is, mineral
oil, was used to validate the new model. For non-Newtonian fluids, the measurements of eight drilling fluids were tested. The flow factor was proposed to estimate the actual flow rate. Although this model is accurate, the calculation process was cumbersome, which is not suitable for field applications.

With growing computational power, the computational fluid dynamics (CFD) modeling approach is becoming increasingly sophisticated in the petroleum industry. It can model actual work conditions in fully three-dimensional (3D) models without creating extensively simplified assumptions. Yan et al. studied the effects of the helical angle and rotational speed of the blade of a hole-cleaning device on the swirl strength of a decaying swirl flow using CFD modeling techniques. Wang et al. adopted the CFD modeling approach to study the working principle of a good washing device and its parameter optimization in practical applications. Cao et al. used the CFD code ANSYS Fluent to simulate the velocity and pressure distribution inside a foam breaker, and the optimum distance between the two annular slits was determined through simulation. In the case of the Marsh funnel, only Sadrizadeh et al. applied the CFD technique. They monitored changes in the fluid volume fraction inside the Marsh funnel and the velocity distribution and pressure distribution on the section. However, their research focused only on Newtonian fluids, and the application of the CFD technique to study the flow field inside the Marsh funnel was insufficient.

A series of mathematical models has been proposed to characterize fluid rheology using the Marsh funnel, but the use of these models has shortcomings. Using these models must follow many strict conditions. Furthermore, the accuracy of simple models is inferior, and the calculation error is always more than 30%. The calculation process of high-accuracy models is complicated and often needs the aid of computer calculation software. Moreover, the proposed mathematical models have not been applied to field applications. Lastly, accurately obtaining rheological parameters inside the Marsh funnel, such as the wall shear rate, at any time and any location is challenging due to the defaults of theoretical analysis. In the present study, we developed simplified mathematical models for the wall shear rate, wall shear stress, and their consistency plots using a Marsh funnel. We propose mathematical models for the rheological parameters of Newtonian and non-Newtonian fluids. These new mathematical models are not only accurate in calculation (with errors of less than 10%) but also concise in calculation (with only flow factor and density measurements). An experimental setup of the Marsh funnel measuring system is established, and five Newtonian fluids, four nonweighted non-Newtonian fluids, and eight weighted non-Newtonian fluids are tested by this system.

Afterward, the rheological parameters measured by the present model are compared with those of other analytical models and ZNN-6 viscometer measurements. Moreover, the CFD modeling approach is implemented, and a two-phase flow 3D model is created. The wall shear rate and wall shear stress at different observation points are investigated using the CFD model, and the data are compared with those of the present model and other analytical models. Lastly, we explore the fluid viscosity contours, velocity contours, and y-axis-direction velocity contours inside the Marsh funnel.

2 | METHODOLOGY

2.1 | Experimental method

In this test, we prepared the experimental fluids and the experimental setup of the Marsh funnel measuring system.

2.1.1 | Fluid preparation

Except for a few well-understood Newtonian fluids, such as water, air, and some oil products, fluids in the drilling industry are non-Newtonian. Five kinds of Newtonian fluids were used in this work, as shown in Table 1.

For reference, we used the experimental design from Abdulrahman et al. and Sedaghat and selected 12 drilling fluids as non-Newtonian fluids. The drilling fluids included four nonweighted non-Newtonian fluids and eight weighted non-Newtonian fluids; their compositions are presented in Table 2.

The drilling fluid additives used in this research were all commercial products. Mineral, heavy, light, fuel, and engine oils, glycerin, polyamionic cellulose low viscosity (PAC LV), polyethylene glycol (PEG), and xanthan gum (XC) were provided by China Oilfield Services. Sodium silicate, sodium hydroxide, calcium carbonate, and sodium chloride were provided by Sinopharm Chemical Reagent Co., Ltd. Bentonite was obtained from Weifang Huawei Bentonite Group Co., Ltd. in accordance with the American Petroleum Institute (API) standard. Barite was obtained from Hubei Sand Technology Co., Ltd. These drilling fluid additives were used in accordance with the mass fraction in this work.

2.1.2 | Rheology tests

The rheological properties of the drilling fluids were tested using a ZNN-6 six-speed rotating viscometer (Qingdao
Haitongda Special Instrument Co., Ltd.). Moreover, based on the API-recommended standard, the Herschel-Bulkley model for the drilling fluids as non-Newtonian fluids are suggested, and the expression is given by3,9:

$$\tau_w = \tau_0 + K \cdot \dot{\gamma}_w^n$$

where $\tau_w$ is the wall shear stress, $\dot{\gamma}_w$ is the wall shear rate, $\tau_0$ is the yield stress, $K$ is the consistency index, and $n$ is the flow index. For the sake of accuracy, the Herschel-Bulkley parameters were fitted by Equation (1). The apparent viscosity, $\mu_a$, and plastic viscosity, $\mu_p$, are widely used rheological parameters based on the Bingham model and calculated by the following equation29,30:

\begin{align*}
\text{apparent viscosity: } \mu_a \text{ (mPa s)} &= \frac{\theta_{600}}{2}, \\
\text{plastic viscosity: } \mu_p \text{ (mPa s)} &= \theta_{600} - \theta_{300},
\end{align*}

where $\theta_{600}$ and $\theta_{300}$ are the readings at 600 and 300 rpm from the ZNN-6 viscometer, respectively.

To collect the data automatically and continuously, we established the experimental setup of the Marsh funnel measuring system, and its schematic is presented in Figure 1. The Marsh funnel (Qingdao Haitongda Special Instrument Co., Ltd.) was affixed to an experimental bench, and the outlet of the tube was equipped with a solenoid switch (Jiaxing Hengyi New Energy Co., Ltd.) connected to the proportion integration differentiation (PID) controller (Hangzhou Meacon Automation Technology Co., Ltd.). A measuring cup was placed on top of an electronic balance (Qingdao Haitongda Special Instrument Co., Ltd.) to which a data collection system (DAS, Hangtu Technology Co., Ltd.) was connected. The workstation (Dell Inc) processed the data collected from the PID controller and the DAS. Then, the experimental setup can directly export the discharge volume-vs-discharge time curves and the liquid height-vs-discharge time curves.

Guria et al22 and Sedaghat23 proposed the following equations to calculate rheological parameters from Marsh funnel measurements, and we compared these findings with values measured by the ZNN-6 viscometer:

| TABLE 1 | Newtonian fluids and their composition |
|---------|--------------------------------------|
| Fluid   | Fluid composition                     |
| Mineral oil (Balhoff et al 2011) | -                                   |
| Synthetic oil (Guria et al 2013)  | Mineral oil: heavy oil: light oil = 1.0:3.2:0.6 by volume ratio |
| Fuel oil | -                                   |
| Engine oil | -                               |
| Glycerin | -                                   |

| TABLE 2 | Non-Newtonian fluids and their composition |
|---------|------------------------------------------|
| Fluid no. | Fluid composition                        |
| Fluid 1 | Fluids 1-6: Guria et al (2013) |
| Fluid 2 | Fluids 1-6: Guria et al (2013) |
| Fluid 3 | Fluids 1-6: Guria et al (2013) |
| Fluid 4 | Fluids 1-6: Guria et al (2013) |
| Fluid 5 | Fluids 1-6: Guria et al (2013) |
| Fluid 6 | Fluids 1-6: Guria et al (2013) |
| Fluid 7 | Fluids 7-12: Abdulrahman et al (2015); Sedaghat (2017) |
| Fluid 8 | Fluids 7-12: Abdulrahman et al (2015); Sedaghat (2017) |
| Fluid 9 | Fluids 7-12: Abdulrahman et al (2015); Sedaghat (2017) |
| Fluid 10 | Fluids 7-12: Abdulrahman et al (2015); Sedaghat (2017) |
| Fluid 11 | Fluids 7-12: Abdulrahman et al (2015); Sedaghat (2017) |
| Fluid 12 | Fluids 7-12: Abdulrahman et al (2015); Sedaghat (2017) |

FIGURE 1 Schematic of experimental setup of the Marsh funnel measuring system: experimental bench, Marsh funnel, electronic balance, DAS, PID controller, and workstation
apparent viscosity: \( \mu_a \) (mPa s) = \( \frac{\tau_{1020}}{1020} \times 1000 \), \( \text{(4)} \)

plastic viscosity: \( \mu_p \) (mPa s) = \( \frac{\tau_{1020} - \tau_{510}}{1020 - 510} \times 1000 \), \( \text{(5)} \)

yield stress: \( \tau_0 \) (Pa) = \( \mu_a - \mu_p \), \( \text{(6)} \)

where \( \tau_{1020} \) and \( \tau_{510} \) are the wall shear stress values at wall shear rate of 1020 and 510 seconds \(^{-1} \).

### 2.2 CFD method

A two-phase flow 3D model was established to study the flow field inside the Marsh funnel on the basis of the CFD approach. In this model, the fluid phase is considered an incompressible fluid on the basis of Eulerian approach. In the following sections, the governing equations of motions and simulation strategy of the CFD code are presented.

#### 2.2.1 Governing equations

**Continuity equation**

The continuity equation for the fluid phase is as follows:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = S_m,
\]

where \( \mathbf{v} \) is the fluid velocity vector, \( \rho \) is fluid density, and \( S_m \) is the source term added to the continuous phase from the dispersed second phase and any user-defined source.

**Momentum conservation equation**

The momentum conservation equation can be expressed as follows:

\[
\frac{\partial ((\rho \mathbf{v}))}{\partial t} + \nabla \cdot ((\rho \mathbf{v} \mathbf{v})) = -\nabla P + \nabla \cdot \left[ \mu \left( \nabla \mathbf{v} + \nabla \mathbf{v}^T \right) \right] + \rho \mathbf{g} + \mathbf{F},
\]

where \( P \) is the static pressure, \( \mathbf{g} \) is the acceleration of gravity, and \( \mathbf{F} \) is the other model-dependent source term.

**Volume-of-fluid method**

The volume-of-fluid formulation relies on the fact that two phases are not interpenetrating. This method poses the following volumetric relation:

\[
V_{cell} = V_g + V_l,
\]

where the indexes \( g \) and \( l \) refer to gas and liquid, respectively. The volume fraction of the fluid phase \( \alpha \) in the computational cell is as follows:

\[
\alpha = \frac{V_l}{V_l + V_g} = \frac{V_l}{V_{cell}},
\]

where the function \( \alpha \) indicates the volume fraction of liquid in the two-phase flow. Its value is 1 in the complete liquid situation, 0 in the complete gas situation, and \( 0 < \alpha < 1 \) when an interface exists.

The interfaces between the phases are tracked by the solution of a continuity equation for the volume fraction of one (or more) of the phases. For the \( i^{th} \) phase, this equation can be obtained as follows:

\[
\sum_{i=1}^{n} \alpha_i = 1,
\]

**Shear stress transport k-\( \omega \) method**

The drilling fluids flowed through the Marsh funnel with a low Reynolds number, and we selected the shear stress transport \( k-\omega \) method. The turbulence kinetic energy, \( k \), and specific dissipation rate, \( \omega \), can be obtained from the following equations:

\[
\frac{\partial (k \rho)}{\partial t} + \nabla \cdot (\rho \mathbf{v} k) = P_k - \frac{\rho k^{3/2}}{l_{k-\omega}} + \frac{\partial}{\partial x} \left[ \left( \mu + \frac{\mu_t}{\sigma} \right) \frac{\partial \omega}{\partial x} \right],
\]

\[
\frac{\partial (\omega \rho)}{\partial t} + \nabla \cdot (\rho \mathbf{v} \omega) = \omega \alpha E_k - \beta \rho \omega^2 + \frac{\partial}{\partial x} \left[ \left( \frac{\mu_t}{\sigma} \right) \frac{\partial k}{\partial x} \right] + 2 \rho (1 - F_1) \sigma \frac{1}{\omega} \frac{\partial k}{\partial x},
\]

\[
\mu_t = \frac{\rho \alpha k}{\max(a_0, S F_2)},
\]

where \( P_k \) and \( P_\omega \) are turbulent generation terms; \( F_1 \) and \( F_2 \) are blending functions; \( S \) is the constant term of the shear stress tensor; \( a, \alpha, \beta, \) and \( \sigma \) represent any constant in the model; and \( \mu_t \) represents the coefficient of eddy viscosity.

### 2.2.2 Simulation strategy

**Model geometry and boundary conditions**

The software GAMBIT 2.4.6 was used to model the computational mesh of the flow field inside the Marsh funnel. A
structural mesh was applied to reduce the computational time and improve accuracy. The 3D meshes were assembled using hexahedral cells, forming a structured mesh. The geometry domain and computational mesh of the Marsh funnel are sketched in Figure 2. The dimensions, which were specified from the ASTM and used in the simulations, are listed in Table 3.

The volume of fluid of the multiphase model was selected. Here, gas was adopted as the secondary phase, and liquid was adopted as the primary phase. The pressure-outlet boundary type and the proper volume fractions of the secondary phases were imposed at the outlet, and the pressure-inlet boundary type was conducted for the inlet. The value of the outlet pressure was set to be equal to atmospheric pressure. The wall was subjected to no-slip boundary conditions (zero velocity on the wall).

Discretization and numerical solution
In this work, the governing equations were discretized via the finite volume approach on each cell. The CFD model for the flow field was solved by the semi-implicit method for pressure-linked equation algorithm. The spatial discretization of the gradient equation was conducted using the least-squares cell-based method. The pressure equation was solved by the PRESTO! method, and the georeconstruct method was chosen for the volume fraction equation. For the momentum, turbulent kinetic energy, and specific dissipation rate equations, the second-order upwind method was applied. The convergence of unsteady gas-liquid calculations was confirmed by negligible values (10e−4) at each time step of the global mass and momentum imbalances in the computational domain to avoid the divergence of the numerical solution and nonphysical flow patterns. This strategy was simulated with commercial CFD code FLUENT 18.2. The selected time step for convergence was 5 × 10−3 seconds. Convergence was attained in less than 10 iterations of each time step.

The difference between the averaged flow properties at this time and the successive time steps was negligible (statistical steady-state condition). Computations were performed on a computing facility with a 32-core Intel® CPU processor (3.1 GHz Xenon® CPU) and 64 GB of RAM.

3 | MATHEMATICAL MODELS FOR FLOW FIELD

We developed simplified mathematical models for the wall shear rate, wall shear stress, and their consistency plots. We further propose mathematical models for the rheological parameters of Newtonian and non-Newtonian fluids.

3.1 | Flow factor

For viscous flows, Sedaghat et al. proposed that the flow rate at the outlet can be expressed as 
\[
v = f \sqrt{2gh},
\]
where \( f \) is a dimensionless coefficient called the flow factor (rate factor). The flow factor, which is the ratio of the actual flow rate to the ideal flow rate, is determined by:
\[
f = \frac{1}{\sqrt{1 + \xi_{\text{Com}}}},
\]
where \( \xi_{\text{Com}} \) is the complex resistance factor, which is obtained by:
\[
\xi_{\text{Com}} = \xi_T + \lambda \frac{H_T}{2R_T},
\]
where \( \xi_T \) is the local resistance factor, and \( \lambda \) is the on-way resistance factor. The local resistance factor inside the Marsh funnel is caused by several reasons. First, velocity distribution changes considerably as fluid flows near the junction of the cone and the tube or the outlet. Second, friction head loss is caused by the viscosity of the fluid because the inner surface of the Marsh funnel is not perfectly slippery. Lastly, the changes in momentum are due to the mixing of viscous fluid molecules. The last reason has a decisive role. The flow factor can generally be a variable function of the fluid’s properties, but it is assumed constant in this study for simplicity. On the basis of the mass

| TABLE 3 Dimensions of the Marsh funnel |
|----------------------------------------|
| Dimension                              | SI unit       |
| Radius of cone at mesh \((R_c)\)       | 0.06985 m     |
| Height of cone at mesh \((H_c)\)       | 0.2794 m      |
| Radius of copper tube \((R_T)\)        | 0.00238125 m  |
| Height of copper tube \((H_T)\)        | 0.0508 m      |
balance, the mathematical model for the flow factor was proposed. The continuity equation for the deforming control volume is given by:

\[ dQ = A_T f \sqrt{2gh} dt = \pi r^2 dh. \]  

Substituting and separating variables is given by:

\[ \sqrt{2gR_T^2} dt = \frac{r^2}{\sqrt{h}} dh. \]  

From the geometric analysis, the mathematical relationship between liquid radius, \( r \), and liquid height, \( h \), is obtained by:

\[ r = \begin{cases} 0.2422 \cdot h - 0.00992632, & H_T + H_C \geq h > H_T \\ R_T, & h \leq H_T \end{cases} \]  

For convenience in writing, we used \( a_1 \) and \( a_2 \) to denote 0.2422 and 0.00992632, respectively. Here, the liquid volume in the tube is negligible compared with that in the cone, as \( V_T = 0.905 \text{ cm}^3 \) and \( V_C = 1486.704 \text{ cm}^3 \). In addition, the time when the free top surface flows through the tube section is transient, which can be verified by the CFD approach, as shown in Figure 3. This observation is one among those obtained from the methodology used in this study. Figure 3 demonstrates that the time the free top surface consumes as it flows through the tube section lasts 0.08 seconds.

Substituting Equation (20) into Equation (19) yield the following:

\[ \frac{a_1^2 h^2 - 2a_1a_2h + a_2^2}{\sqrt{h}} \frac{dh}{dh} = \sqrt{2gR_T^2} dt. \]  

FIGURE 3  Liquid height-vs-discharge time of Fluid 1: The liquid height drops from 0.0508 to 0 m at a discharge time of 0.08 s.

The volume flow rate is expressed as:

\[ Q = A_T f \sqrt{2gh}. \]  

For the wall shear rate of Newtonian fluids, Sedaghat\textsuperscript{23} recommended a geometrical correction factor, \( \frac{1}{\eta'} \left( \frac{h}{H_T} \right)^\frac{1}{2} \), as follows:

\[ \frac{0.3302}{f_0^{0.0508}} \int \frac{a_1^2 h^2 - 2a_1a_2h + a_2^2}{\sqrt{h}} \frac{dh}{dh} = \int \frac{2gR_T^2}{\sqrt{h}} dt, \]

\[ f \cdot T_F = 38.0. \]  

From Equation (23), \( f \) can be calculated directly by measuring the final discharge time, \( T_F \).

### 3.2 Wall shear stress and wall shear rate

The wall shear stress and shear rate are the research emphasis of the flow field inside the Marsh funnel, and the apparent viscosity, plastic viscosity, and yield stress can be computed from them. Sedaghat\textsuperscript{23} obtained the equations for the wall shear stress at the outlet:

\[ \tau_w = \begin{cases} \frac{1}{2} \rho g h \left( 1 - f^2 \right)^{\frac{R_T}{H_T}}, & H_T + H_C \geq h > H_T \\ \frac{1}{2} \rho g \left( 1 - f^2 \right)^{R_T}, & h \leq H_T \end{cases} \]  

Brydson\textsuperscript{35} and his followers reported a developed method to determine the shear rate in a pipe for fully developed and steady flows:

\[ -\dot{\gamma}_w \tau_w = \frac{1}{\pi R_T^2} \left( \dot{\tau}_w \frac{dQ}{d\tau_w} + 3 \dot{\tau}_w Q \right). \]

By simplifying and reorganizing both sides of the equation, we can obtain:

\[ -\dot{\gamma}_w = \frac{\tau_w}{\pi R_T^3} \frac{dQ}{d\tau_w} + \frac{3Q}{\pi R_T^3}. \]  

In previous research,\textsuperscript{22,23,35} the flow behavior index, \( \eta' \), was proposed as \( \frac{1}{\eta'} = \frac{d \log (4Q/\pi R_T^3)}{d \log (\tau_w)} \) to obtain wall shear stress-vs-wall shear rate curves. The interim parameter is determined from the curve of \( d \log (4Q/\pi R_T^3) \)-vs-\( d \log (\tau_w) \), which has no precise physical significance. In the present work, we propose simplified mathematical methods for calculating the wall shear stress and wall shear rate without using the flow behavior index.

The volume flow rate is expressed as:

\[ Q = A_T f \sqrt{2gh}. \]  

For the wall shear rate of Newtonian fluids, Sedaghat\textsuperscript{23} recommended a geometrical correction factor, \( \frac{1}{\eta'} \left( \frac{h}{H_T} \right)^\frac{1}{2} \), as follows:
\[ \dot{\gamma}_w = \frac{1}{2} \left( \frac{h}{H_T} \right)^{\frac{2}{5}} \left( \frac{\tau_w}{\pi R_T^2} \cdot \frac{dQ}{d\tau_w} + \frac{3Q}{\pi R_T^3} \right). \]  

Equations (24) and (27) are substituted into Equation (28), and the equation for the wall shear rate of Newtonian fluids can be represented by:

\[ \dot{\gamma}_w = \frac{1}{2} \left( \frac{h}{H_T} \right)^{\frac{2}{5}} \left( \frac{3 \cdot A_{f} \sqrt{2gh}}{\pi R_T^2} + \frac{\frac{1}{2} \rho gh (1 - f^2)}{\pi R_T^3} \cdot \frac{d(A_{f} \sqrt{2gh})}{d\tau_w} \right) \]  

The wall shear stress vs the wall shear rate curves of non-weighted non-Newtonian fluids are defined as:

\[ \frac{\tau_w}{\gamma_w} = \frac{1}{2} \rho gh (1 - f^2) \frac{B_f}{H_T} = 3.58625 \times 10^{-6} \cdot \rho \cdot \frac{1 - f^2}{f} \cdot h^{-\frac{1}{2}}. \]  

For the wall shear rate of weighted non-Newtonian fluids, we propose a new correction factor, \( \frac{1}{4 \sqrt{2}} \left( \frac{h}{H_T} \right) \), as follows:

\[ \dot{\gamma}_w = \frac{1}{4 \sqrt{2}} \left( \frac{h}{H_T} \right) \left( \frac{\tau_w}{\pi R_T^2} \cdot \frac{dQ}{d\tau_w} + \frac{3Q}{\pi R_T^3} \right). \]  

Equations (24) and (27) are substituted into Equation (36), and the equation for the wall shear rate of weighted non-Newtonian fluids is expressed as:

\[ \dot{\gamma}_w = \frac{1}{4 \sqrt{2}} \left( \frac{h}{H_T} \right) \left( \frac{3 \cdot A_{f} \sqrt{2gh} + \frac{\frac{1}{2} \rho gh (1 - f^2)}{\pi R_T^3} \cdot \frac{d(A_{f} \sqrt{2gh})}{d\tau_w}}{\frac{\pi R_T^3}{8B_f H_T f h} \cdot \frac{dQ}{d\tau_w} + \frac{3Q}{\pi R_T^3}} \right). \]  

The wall shear stress vs-wall shear rate curves of weighted non-Newtonian fluids are given by:

\[ \tau_w = \frac{1}{2} \rho gh (1 - f^2) \frac{B_f}{H_T} = 22643.96034 \cdot f \cdot h^{-\frac{1}{2}}. \]  

Equations (35) and (39) show that the value of \( \frac{\tau_w}{\gamma_w} \) is related to the density, flow factor, and liquid height of non-Newtonian fluids. Here, the liquid height can be calculated from the residual volume of liquid inside the Marsh funnel, and the geometrical relationship is calculated as:

\[ V_R = \left\{ \begin{array}{ll} \frac{1}{2} \cdot \pi \cdot [R_T^2 + R_T \cdot (a_h - a_z) + (a_h - a_z)^2], & H_T + H_C \geq H_T \\ \pi \cdot R_T^2 \cdot h, & h \leq H_T \end{array} \right. \]  

where \( V_R \) is the liquid residual volume. The data of liquid height can be output directly from the workstation of the experimental setup.
### 3.3 Rheological parameters

The wall shear rate of nonweighted non-Newtonian fluids at the outlet is 1020 seconds\(^{-1}\). Thus, we can obtain:

\[
1020 = 64046.79164 \cdot f \cdot h_{1020}^2, 
\]

\[
h_{1020} = 0.0632997 \cdot f^{-\frac{1}{2}}. 
\]

Substituting Equation (42) into Equation (24) yields:

\[
\tau_{1020} = \frac{1}{2} \rho g h_{1020} (1 - f^2) \frac{R_T}{H_T}, 
\]

\[
\tau_{1020} = 0.0145392 \cdot \rho \cdot (1 - f^2) \cdot f^{-\frac{1}{2}}. 
\]

Equation (44) is substituted into Equation (4), and the apparent viscosity of nonweighted non-Newtonian fluids is as follows:

\[
\mu_a = \frac{0.0145392 \cdot \rho \cdot (1 - f^2) \cdot f^{-\frac{1}{2}}}{1020} \times 1000 
= 0.0142541 \cdot \rho \cdot (1 - f^2) \cdot f^{-\frac{1}{2}}. 
\]

The wall shear stress of nonweighted non-Newtonian fluids at wall shear rate of 510 s\(^{-1}\) is given by:

\[
h_{510} = 0.0398763 \cdot f^{-\frac{1}{2}}, 
\]

\[
\tau_{510} = 0.00915908 \cdot \rho \cdot (1 - f^2) \cdot f^{-\frac{1}{2}}. 
\]

Equations (45) and (47) are substituted into Equation (5), and the plastic viscosity of nonweighted non-Newtonian fluids is as follows:

\[
\mu_p = \frac{0.0145392 \cdot \rho \cdot (1 - f^2) \cdot f^{-\frac{1}{2}} - 0.00915908 \cdot \rho \cdot (1 - f^2) \cdot f^{-\frac{1}{2}}}{1020 - 510} \times 1000 
= 0.0105493 \cdot \rho \cdot (1 - f^2) \cdot f^{-\frac{1}{2}}. 
\]

### 4 RESULTS AND DISCUSSIONS

The following sections present the quantitative and qualitative analysis of the experimental and numerical results, respectively.

#### 4.1 Experimental results of the rheological properties

The viscosity of Newtonian fluids could be calculated from Equation (31). Now, the values are compared with the data using the ZNN-6 viscometer, Guria et al (2013) model, and Sedaghat (2017) model in Table 4. The systematic error between the ZNN-6 viscometer measurements and present model, the systematic error between the ZNN-6 viscometer measurements and Guria et al (2013) model, and that between the ZNN-6 viscometer measurements and Sedaghat (2017) model are listed in Table 5 to compare the Newtonian viscosities obtained by the different models.

Tables 4 and 5 show that the systematic errors of Newtonian fluids between the present model and the ZNN-6 viscometer measurements are small, that is, 1.52%, 1.64%,
1.03%, 1.08%, and 1.46% for mineral, synthetic, fuel, and engine oils, and glycerin, respectively; the average value is 1.35%. All systematic errors are less than 2.00%. Therefore, the present model closely matches with ZNN-6 viscometer measurements for Newtonian fluids. The same results are obtained between the ZNN-6 viscometer measurements and Sedaghat (2017) model. By contrast, the average systematic error between the Guria et al (2013) model and ZNN-6 viscometer measurements is approximately 21.56%, and the minimum value reaches up to 9.25% (glycerin). Lastly, the calculation accuracy of the Guria et al (2013) model for Newtonian fluids is low.

For the four nonweighted non-Newtonian fluids, the apparent viscosity, plastic viscosity, and yield stress can be determined by Equations (45), (48), and (49), respectively. For the eight weighted non-Newtonian fluids, the apparent viscosity, plastic viscosity, and yield stress can be determined by Equations (50), (51), and (52), respectively. The values are compared with the data measured by the ZNN-6 viscometer, Guria et al (2013) model, and Sedaghat (2017) model in Table 6. The systematic error between the ZNN-6 viscometer measurements and present model and that between the ZNN-6 viscometer measurements and Sedaghat (2017) model are listed in Table 7 for comparison.

Tables 6 and 7 show that the average systematic error of the apparent viscosity between the ZNN-6 viscometer measurements and present model is 8.18%, the maximum value is 11.82% (Fluid 5), and the minimum value is 1.54% (Fluid 2). Moreover, the plastic viscosity ranges between 0.17% and 12.80%, with an average of 5.42%. Therefore, the apparent and plastic viscosities of the present model for non-Newtonian fluids match the ZNN-6 viscometer measurements, and the average systematic error is less than 10.00%. Although the average systematic error of yield stress is 21.43%, which indicates that some error exists in the present model for the yield stress, this model’s results do not remarkably differ from the ZNN-6 viscometer measurements.

Despite the considerable change in the systematic error of the apparent viscosity between the ZNN-6 viscometer and other models, the present model closely matches with ZNN-6 viscometer measurements for Newtonian fluids. The same results are obtained between the ZNN-6 viscometer measurements and Sedaghat (2017) model. By contrast, the average systematic error between the Guria et al (2013) model and ZNN-6 viscometer measurements is approximately 21.56%, and the minimum value reaches up to 9.25% (glycerin). Lastly, the calculation accuracy of the Guria et al (2013) model for Newtonian fluids is low.

### TABLE 5 Details of the systematic error of Newtonian viscosity among different models

| Fluid       | M1 & M2 | M1 & M3 | M1 & M4 |
|-------------|---------|---------|---------|
| Mineral oil | 1.52%   | 23.69%  | 1.52%   |
| Synthetic oil | 1.64%   | 29.18%  | 1.64%   |
| Fuel oil    | 1.03%   | 34.64%  | 1.03%   |
| Engine oil  | 1.08%   | 11.05%  | 1.08%   |
| Glycerin    | 1.46%   | 9.25%   | 1.46%   |
| Average     | 1.35%   | 21.56%  | 1.35%   |

Note: M1, M2, M3, and M4 represent the ZNN-6 viscometer measurements, present model, Guria et al (2013) model, and Sedaghat (2017) model, respectively.

### TABLE 6 Rheological properties of 12 non-Newtonian fluids obtained by the ZNN-6 viscometer, present model, Guria et al (2013) model, and Sedaghat (2017) model

| Fluid no. | Density (kg/cm³) | Flow factor /u1D741 | μa (mPa s) | ϕ (mPa s) | T0 (Pa) |
|-----------|-----------------|----------------------|------------|-----------|--------|
| M1        | M2               | M3                   | M4         | M1        | M2     | M3     | M4     | M1       | M2     | M3     | M4     | M1       | M2     | M3     | M4     |
| Fluid 1   | 1040.0           | 0.418                | 20.00      | 21.88     | 32.76   | 21.88  | 14.00  | 15.35   | 38.92    | 15.35  | 6.53   | 4.90   | 6.53    | 4.90   |
| Fluid 2   | 1125.0           | 0.501                | 37.50      | 36.09     | 43.80   | 36.09  | 19.05  | 19.05   | 39.28    | 19.05  | 6.60   | 5.99   | 9.83    | 5.99   |
| Fluid 3   | 1125.0           | 0.521                | 35.97      | 36.09     | 38.92   | 35.97  | 19.05  | 19.05   | 39.28    | 19.05  | 6.60   | 5.99   | 9.83    | 5.99   |
| Fluid 4   | 1035.0           | 0.729                | 16.50      | 14.29     | 8.53    | 14.29  | 7.28   | 7.28    | 12.00    | 7.28   | 4.60   | 3.61   | 2.53    | 3.61   |
| Fluid 5   | 1129.0           | 0.795                | 14.50      | 12.00     | 6.60    | 12.00  | 10.77  | 10.77   | 16.08    | 10.77  | 3.43   | 2.31   | 1.48    | 2.31   |
| Fluid 6   | 1129.0           | 0.813                | 16.50      | 14.29     | 8.53    | 14.29  | 7.28   | 7.28    | 12.00    | 7.28   | 4.60   | 3.61   | 2.53    | 3.61   |
| Fluid 7   | 1060.0           | 0.795                | 14.50      | 12.00     | 6.60    | 12.00  | 10.77  | 10.77   | 16.08    | 10.77  | 3.43   | 2.31   | 1.48    | 2.31   |
| Fluid 8   | 1135.0           | 0.538                | 15.00      | 16.08     | 30.34   | 16.08  | 10.00  | 10.00   | 16.08    | 10.00  | 4.60   | 3.43   | 2.31    | 3.43   |
| Fluid 9   | 1135.0           | 0.704                | 22.50      | 20.62     | 28.21   | 20.62  | 15.00  | 15.00   | 23.89    | 15.00  | 4.60   | 3.43   | 2.31    | 3.43   |
| Fluid 10  | 1065.0           | 0.571                | 21.00      | 14.50     | 13.85   | 14.50  | 9.36   | 9.36    | 13.85    | 9.36   | 3.43   | 2.31   | 3.43    | 2.31   |
| Fluid 11  | 1155.0           | 0.590                | 36.09      | 30.34     | 49.70   | 30.34  | 18.71  | 18.71   | 30.34    | 18.71  | 5.99   | 4.90   | 3.95    | 4.90   |
| Fluid 12  | 1155.0           | 0.804                | 38.00      | 31.50     | 52.75   | 31.50  | 18.71  | 18.71   | 30.34    | 18.71  | 5.99   | 4.90   | 3.95    | 4.90   |

Note: The M1, M2, M3, and M4 represent the ZNN-6 viscometer measurements, present model, Guria et al (2013) model, and Sedaghat (2017) model, respectively.
measurements and Sedaghat (2017) model, common points are found in the same kinds of fluids. The systematic error is small for the nonweighted fluids but large for the weighted ones. Similar results are obtained for plastic viscosity. The rheological properties obtained by the Sedaghat (2017) model for nonweighted non-Newtonian fluids can improve by following the ZNN-6 viscometer results, but the values of the weighted non-Newtonian fluids for the Sedaghat (2017) model have a substantial error. Moreover, the results of the Guria et al (2013) model deviate markedly from the actual values.

These results show consistency between the present model and the data measured by the ZNN-6 viscometer for the Newtonian and non-Newtonian fluids. As a whole, the proposed mathematical models for the Newtonian viscosity, apparent viscosity, plastic viscosity, and yield stress are concise and accurate, and we need to measure only the density and flow factor. Hence, the present model is available for field applications.

4.2 Validation of the CFD model

The numerical simulation of the flow field inside the Marsh funnel is an approximate solution to the partial differential equation. To ensure data accuracy, the number of cells should be large enough that the calculation results no longer change significantly as the number of cells increase. However, given the need to satisfy the mesh density with limited computer operating efficiency, the computation amount should be reduced, and the stability and convergence speed of the calculation convergence should be improved. The cell independence problem is tested by increasing the number of cells via trial and error to obtain an adequate convergence of the computations. According to ASTM,9 the flow duration for one quart (946 cm$^3$) of water is 26 ± 0.5 seconds at a temperature of 21 ± 3°C. First, the cell number independence verified on the basis of the MFV of water. The numerical simulation results of fluid flow inside the Marsh funnel with different cell numbers are shown in Figure 4 to show the independence of computational cells.

The observations from the results indicate that the MFV of the CFD model decreases as the number of cells increases from 200 000 to 300 000. As the number of cells reaches 250 000, the MFV of the CFD model becomes 26.1 seconds. Afterward, as the number of cells increases, the MFV of Fluid no. & $\mu_a$ & $\mu_p$ & $\tau_0$ & $\mu_a$ & $\mu_p$ & $\tau_0$
Fluid 1 & 9.40% & 9.64% & 4.81% & 9.40% & 9.64% & 28.57%
Fluid 2 & 1.54% & 4.37% & 16.67% & 49.23% & 73.07% & 25.67%
Fluid 3 & 5.64% & 2.87% & 17.21% & 50.20% & 53.96% & 57.72%
Fluid 4 & 10.21% & 0.17% & 29.92% & 10.21% & 0.17% & 47.09%
Fluid 5 & 11.82% & 10.25% & 17.83% & 55.88% & 57.50% & 64.35%
Fluid 6 & 8.97% & 6.95% & 25.92% & 54.88% & 55.90% & 68.03%
Fluid 7 & 7.20% & 12.80% & 17.67% & 7.20% & 12.80% & 38.25%
Fluid 8 & 8.36% & 1.73% & 29.57% & 54.18% & 51.80% & 69.65%
Fluid 9 & 10.90% & 4.48% & 27.13% & 55.43% & 54.76% & 68.52%
Fluid 10 & 10.52% & 5.83% & 30.07% & 10.52% & 5.83% & 47.52%
Fluid 11 & 7.13% & 1.43% & 23.71% & 53.55% & 53.28% & 67.14%
Fluid 12 & 6.45% & 4.52% & 16.70% & 53.23% & 54.76% & 64.13%
Average & 8.18% & 5.42% & 21.43% & 38.62% & 40.29% & 53.89%

| Fluid no. | $\mu_a$ | $\mu_p$ | $\tau_0$ | $\mu_a$ | $\mu_p$ | $\tau_0$
|----------|---------|---------|--------|---------|---------|--------|
| Fluid 1  | 9.40%   | 9.64%   | 4.81%  | 9.40%   | 9.64%   | 28.57% |
| Fluid 2  | 1.54%   | 4.37%   | 16.67% | 49.23%  | 73.07%  | 25.67% |
| Fluid 3  | 5.64%   | 2.87%   | 17.21% | 50.20%  | 53.96%  | 57.72% |
| Fluid 4  | 10.21%  | 0.17%   | 29.92% | 10.21%  | 0.17%   | 47.09% |
| Fluid 5  | 11.82%  | 10.25%  | 17.83% | 55.88%  | 57.50%  | 64.35% |
| Fluid 6  | 8.97%   | 6.95%   | 25.92% | 54.88%  | 55.90%  | 68.03% |
| Fluid 7  | 7.20%   | 12.80%  | 17.67% | 7.20%   | 12.80%  | 38.25% |
| Fluid 8  | 8.36%   | 1.73%   | 29.57% | 54.18%  | 51.80%  | 69.65% |
| Fluid 9  | 10.90%  | 4.48%   | 27.13% | 55.43%  | 54.76%  | 68.52% |
| Fluid 10 | 10.52%  | 5.83%   | 30.07% | 10.52%  | 5.83%   | 47.52% |
| Fluid 11 | 7.13%   | 1.43%   | 23.71% | 53.55%  | 53.28%  | 67.14% |
| Fluid 12 | 6.45%   | 4.52%   | 16.70% | 53.23%  | 54.76%  | 64.13% |
| Average  | 8.18%   | 5.42%   | 21.43% | 38.62%  | 40.29%  | 53.89% |

Note: The M1, M2, and M4 represent the ZNN-6 viscometer measurements, present model, and Sedaghat (2017) model.
the CFD model no longer changes significantly. Therefore, in this work, the number of cells of the two-phase flow 3D model is set to 250,000 to ensure calculation accuracy and adequate computation speed.

Second, the CFD simulation results for each fluid are compared with the experimental data. By obtaining examples, we select mineral oil from Newtonian fluids, Fluid 1 from non-weighted non-Newtonian fluids, and Fluid 8 from weighted non-Newtonian fluids. The others are given in the Appendix S1. The discharge volume-vs-discharge time curves of the different fluids are presented in Figure 5.

Figure 5 shows excellent consistency between the CFD model and the experimental data. The average systematic errors between them for mineral oil, Fluid 1, and Fluid 8 are merely 3.02%, 3.16%, and 4.11%, respectively. The above experiments indicate that the two-phase flow 3D model established in this work can accurately characterize the flow field of Newtonian and non-Newtonian fluids inside the Marsh funnel.

4.3 | Numerical results of wall shear rate and wall shear stress

The above detailed theoretical analysis focuses on the wall shear rate and wall shear stress at the outlet of the Marsh funnel. Directly solving the laminar flow of Newtonian fluids and turbulent flow of non-Newtonian fluids by theoretical analysis is difficult. Restricted by the solving ability of the computer and the structure of the Marsh funnel, we could not investigate the flow field inside the Marsh funnel. Furthermore, the assumptions added to simplify the flow field model affect the authenticity of the solution results. With the aid of the CFD approach, we could investigate the wall shear rate and shear stress at any time and any location. Given that the Marsh funnel consists of a cone and a tube, we select three observation points, namely, the outlet of the tube (Circle A), the junction of the cone and the tube (Circle B), and the cone (Circle C/\(z = 0.10\) m), to characterize the values comprehensively. A schematic is shown in Figure 6.

The wall shear rate-vs-discharge time curves of mineral oil, Fluid 1, and Fluid 8 and the data of the CFD model compared with those of the present, Guria et al (2013), and Sedaghat (2017) models are presented in Figure 7.
Figure 7A,C,E shows that the nature and scale of the wall shear rate vary considerably due to the different observation points and drilling fluids. The wall shear rate at Circle C is minimal and constant because the flow velocity here is minimal. The wall shear rate at Circle B is the largest. According to the Bernoulli equation, the flow rate of the fluid from the
cone to the tube increases sharply. Moreover, the entire variation trend of the wall shear rate at Circle B presents a rapid increase at the beginning, a rapid decrease at the end, and a steady decrease in the middle stage. The wall shear rate at Circle A is less than that at Circle B, and the variation tendency at Circle A is consistent with that at Circle B. Overall,
the whole change trends of the wall shear rate of mineral oil, Fluid 1, and Fluid 8 are similar, but the values for different fluids are not equal. As shown in the figure, the wall shear rate of mineral oil varies between 50 and 300 seconds$^{-1}$, the variation range of Fluid 1 is 900-6000 seconds$^{-1}$, and the changing scope of Fluid 8 is 900-7800 seconds$^{-1}$.

Figure 7B shows that the values of the present model almost coincide with those of the CFD model for the Newtonian fluids. In Figure 7D,F, for the non-Newtonian fluids, the values of the present model are higher than those of the CFD model at first, then the difference between the two gradually decreases as discharge time increases, and finally coincides. Similarly, the Sedaghat (2017) model has high accuracy in measuring the Newtonian and non-weighted non-Newtonian fluids, but the accuracy for calculating weighted non-Newtonian fluids is low. For the Newtonian and non-Newtonian fluids, the Guria et al (2013) model cannot accurately calculate the wall shear rate.

The wall shear stress-vs-discharge volume curves of the drilling fluids and the data measured by the CFD approach compared with those of the present, Balhoff et al (2011), Guria et al (2013), and Sedaghat (2017) models are presented in Figure 8.

Figure 8A,C,E exhibits a marked variation in the wall shear stress due to the different observation points and drilling fluids. A high value at Circle A is observed, and entire variation trend presents a rapid increase at the beginning, a rapid decrease at the end, and a steady decrease in the middle stage. The wall shear stress at Circle B is the largest, and the variation tendency here is consistent with that at Circle A. The values of the wall shear stress at Circle C are minimal (nearly zero). In summary, the entire change trend of the wall shear stress of mineral oil, Fluid 1, and Fluid 8 is similar, but the values for different fluids are not equal. As shown in the figure, the wall shear stress of mineral oil varies between 20 and 90 Pa, the variation range of Fluid 1 is 10-70 Pa, and the changing scope of Fluid 8 is 10-55 Pa.

Figure 8B displays an excellent data consistency between the present, CFD, and Sedaghat (2017) models for the Newtonian fluids. In Figure 8D,F, for the non-Newtonian fluids, the values of the present and Sedaghat (2017) models are smaller than those of the CFD model at first, with an increase of discharge volume the difference among them gradually decreases, and finally coincides. The data calculated by the Balhoff et al (2011) and Guria et al (2013) models are always higher than the actual values.

The consistency plots ($\tau_w/\dot{\gamma}$) of mineral oil, Fluid 1, and Fluid 8 for the CFD, present, Guria et al (2013), and Sedaghat (2017) models are always higher than the actual values.

The consistency plots ($\tau_w/\dot{\gamma}$) of mineral oil, Fluid 1, and Fluid 8 for the CFD, present, Guria et al (2013), and Sedaghat (2017) models are displayed in Figure 9. A consistency between the present and CFD models is observed for the Newtonian and non-Newtonian fluids. Furthermore, the calculation accuracy of the Newtonian and nonweighted non-Newtonian fluids in the Sedaghta (2017) model is high, but this model has a poor calculation accuracy for weighted non-Newtonian fluids. By contrast, the Guria et al (2013) model has a significant calculation error for Newtonian and non-Newtonian fluids. The rest of the consistency plots...
The CFD approach can be used to analyze the wall shear rate and wall shear stress at any time and any location inside the Marsh funnel. Here, the entire variation trend of the two rheological parameters presents a rapid increase at the beginning, a rapid decrease at the end, and a steady decrease in the middle stage. Overall, an excellent consistency between the present and CFD models is achieved. Moreover, the calculation accuracy of the Sedaghat (2017) model for the Newtonian and non-weighted non-Newtonian fluids is high, but this model has a poor calculation accuracy for the weighted non-Newtonian fluids. By contrast, the Balhoff et al (2011) and Guria et al (2013) models have a significant error in calculation.

4.4 Analysis of the flow field inside the Marsh funnel

The viscosity, velocity, and y-axis-direction velocity contours of the flow field inside the Marsh funnel are explored using the CFD approach.

The viscosity contours in the vertical transverse of Fluid 1 at different times are exhibited in Figure 10. A wide variation is observed in terms of viscosity inside the Marsh funnel. At
the cone section, viscosity is higher close to the free top surface; for example, the value near the free top surface reaches up to 400 mPa s at 2 seconds. By contrast, the viscosity at the tube section is small. Moreover, the flow field inside the Marsh funnel has a good axial symmetry for the center z-axis.

The velocity contours in the vertical transverse of Fluid 1 at different times are shown in Figure 11. The velocity within this study considerably varies. The velocity at the cone section is small (always less than 0.2 m/s), but that at the outlet of the tube is higher than 2.0 m/s. The velocity at the tube section changes significantly with flow time. Furthermore, the velocity of Fluid 1 close to the outlet and the tube center is high.

For precision, the velocity in Figure 11 should be called the z-axis-direction velocity. In a previous study, we assumed that the flow inside the Marsh funnel is one dimensional (z-axis-direction), and we ignored the x-axis- and y-axis-direction velocities. Given the axisymmetric flow inside the Marsh funnel, the x-axis-direction velocity contours are equal to those of the y-axis. The y-axis-direction velocity contours of Fluid 1 at different times and horizontal transverses are displayed in Figure 12. The actual y-axis-direction velocity is observed. Although the values at the cone and tube sections are less than 0.05 m/s, the y-axis-direction velocity exists. Moreover, the y-axis velocity at the junction is remarkable, but it is less than 0.20 m/s.

5 | CONCLUSION

In our work, we developed simplified mathematical models for the wall shear rate, wall shear stress, and their consistency plots and proposed mathematical models for the rheological parameters of Newtonian and non-Newtonian fluids. The flow factor was introduced to characterize the flow behavior of 17 drilling fluids flowing through a Marsh funnel. The rheological parameters of these drilling fluids were measured using a Marsh funnel and a ZNN-6 viscometer, and quantitative analysis was performed to compare the data with those of the present and other analytical models. Moreover, the CFD modeling approach was implemented, and a two-phase flow 3D model was established. The CFD outcomes of the wall shear rate and wall shear stress were compared with those of the present and other analytical models by qualitative analysis.

According to the experimental results, the average systematic errors of the Newtonian, apparent, and plastic viscosities between the present model and the ZNN-6 viscometer measurements were 1.35%, 8.18%, and 5.42%, respectively. The average systematic error of the yield stress was 21.43%, which was under the controllable scope. At the construction site, we only needed to measure the flow factor and density, and the rheological parameters could be calculated directly and accurately using the present model, which is beneficial for a successful and safe drilling operation.

The numerical results revealed that the present model for the wall shear rate and wall shear stress is consistent with the CFD model. Moreover, the flow field inside the Marsh funnel is axisymmetric and has a component velocity in the horizontal direction. Hence, the CFD approach is a necessary complement for exploring the flow field inside the Marsh funnel.

Results also demonstrated that the rheological parameters can be accurately and conveniently calculated by the present model using a Marsh funnel. However, the findings presented in this study have some deficiencies; for instance,
the flow factor was assumed constant for simplicity, and the geometrical correction factor for the wall shear rate of the weighted drilling fluids was a semiempirical coefficient obtained from a series of numerical results. In a future study, the effects of different weighting materials (eg, potassium formate, barite, manganese ore) on the geometrical correction factor for the wall shear rate of drilling fluids will be investigated.

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CONFLICT OF INTEREST
None declared.

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REFERENCES
1. Lin H, Che J, Zhang JT, Feng X. Measurements of the viscosities of Kr and Xe by the two-capillary viscometry. Fluid Phase Equilib. 2016;418:198-203.
2. Cannon MR, Manning RE, Bell JD. Viscosity measurement. Kinetic energy correction and new viscometer. Anal Chem. 1960;32(3):355-358.
3. Fernandes RR, Turezo G, Andrade DEV, Franco AT, Negrão COR. Are the rheological properties of water-based and synthetic drilling fluids obtained by the Fann 35A viscometer reliable? J Petrol Sci Eng. 2019;177:872-879.
4. Zambrano JR, Sobrino M, Martín MC, Villamañán MA, Chamorro CR, Segovia JJ. Contributing to accurate high pressure viscosity measurements: vibrating wire viscometer and falling body viscometer techniques. J Chem Thermodyn. 2016;96:104-116.
5. Marsh HN. Properties and treatment of rotary mud. Trans AIME. 1931;92(01):234-251.
6. NF P 18-507. Additions pour béton hydraulique — Besoin en eau, contrôle de la régularité — Méthode par mesure de la fluidité par écoulement au cône de Marsh. CEN; 1992.
7. EN 12 715. Execution of Special Geotechnical Work—Grouting. BSI; 2000.
8. EN 445. Goutr for Prestressing Tendons-Test Methods. BSI; 2007.
9. ASTM DE6910/D6910M. Standard Test Method for Marsh Funnel Viscosity of Clay Construction Slurries. ASTM; 2009.
10. DIN 4127. Earthworks and Foundation Engineering—Test Methods for Supporting Fluids Used in the Construction of Diaphragm Walls and their Constituent Products. DIN Deutsches Institut für Normung e.V.; 2014.
11. Metzner AB, Reed JC. Flow of non-Newtonian fluids—correlation of the laminar, transition, and turbulent-flow regions. AIChE J. 1955;1(4):434-440.
12. Bingham EC. An investigation of the laws of plastic flow. Bull Bureau Stand. 1917;13(278):309-353. Washington: US Government Printing Office.
13. Wallick GC, Savins JG. A comparison of differential and integral descriptions of the annular flow of a power-law fluid. SPE J. 1969;9(03):311-315.
14. Casson N. A flow equation for pigment-oil suspensions of the printing ink type. Rheology of Disperse Systems. (pp. 84-104). New York, NY: Pergamon Press; 1959.
15. Herschel WH, Bulky U. Measurement of consistency as applied to rubber-benzene solutions. Am Soc Test Proc. 1926;26(2):621-633.
16. Andaverde JA, Wong-Loya JA, Vargas-Tabares Y. A practical method for determining the rheology of drilling fluid. J Petrol Sci Eng. 2019;180:150-158.
17. Pitt MJ. The Marsh funnel and drilling fluid viscosity: a new equation for field use. SPE Drill Complet. 2000;15(01):3-6.
18. Le Roy R, Roussel N. The Marsh Cone as a viscometer: theoretical analysis and practical limits. Mater Struct. 2005;38(1):25-30.
19. Roussel N, Le Roy R. The Marsh cone: a test or a rheological apparatus? Cement Concrete Res. 2005;35(5):823-830.
20. Nguyen VH, Rémond S, Gallias JL, Bigas JP, Muller P. Flow of Herschel-Bulkley fluids through the Marsh cone. J Non-Newton Fluid. 2006;139(1-2):128-134.
21. Balhoff MT, Lake LW, Bommer PM, Lewis RE, Weber MJ, Calderin JM. Rheological and yield stress measurements of non-Newtonian fluids using a Marsh Funnel. J Petrol Sci Eng. 2011;77(3-4):393-402.
22. Guria C, Kumar R, Mishra P. Rheological analysis of drilling fluid using Marsh Funnel. J Petrol Sci Eng. 2013;105:62-69.
23. Sedaghat A. A novel and robust model for determining rheological properties of Newtonian and non-Newtonian fluids in a marsh funnel. J Petrol Sci Eng. 2017;156:896-916.
24. Yan T, Qu J, Sun X, Li Z, Li W. Investigation on horizontal and deviated wellbore cleanout by hole cleaning device using CFD approach. Energy Sci Eng. 2019;7:1292-1305.
25. Wang CS, Zhang L. Fluid simulation in a cyclone reverse circulation well washing device based on computational fluid dynamics. Energy Sci Eng. 2019;7:1306-1314.
26. Cao P, Chen Z, Liu M, Cao H, Chen B. Numerical and experimental study of a novel aerodynamic foam breaker for foam drilling fluid. Energy Sci Eng. 2019;7:1544-1556.
27. Sadrizadeh S, Nejad Ghafar A, Halilovic A, Håkansson U. Numerical, experimental and analytical studies on fluid flow through a Marsh funnel. J Appl Fluid Mech. 2017;10(6):1501-1507.
28. Abdulrahman HA, Jouda AS, Mohammed MM, Mohammed MM, Elfadil MO. Calculation Rheological Properties of Water Base Mud Marsh Funnel Using, Khartoum, Sudan: Sudan University of Science and Technology; 2015.
29. Jia H, Huang P, Wang Q, et al. Investigation of inhibition mechanism of three deep eutectic solvents as potential shale inhibitors in water-based drilling fluids. Fuel. 2019;244:403-411.
30. Zhao X, Qiu Z, Huang W, Wang M. Mechanism and method for controlling low-temperature rheology of water-based drilling fluids in deepwater drilling. J Petrol Sci Eng. 2017;154:405-416.
31. Sedaghat A, Omar MAA, Damrah S, Gaith M. Mathematical modelling of the flow rate in a marsh funnel. JETR. 2017;1(1):1-12.
32. Moramarco T, Dingman SL. On the theoretical velocity distribution and flow resistance in natural channels. J Hydrol. 2017;555:777-785.
33. Valizadeh K, Farahbakhsh S, Bateni A, et al. A parametric study to simulate the non-Newtonian turbulent flow in spiral tubes. *Energy Sci Eng*. 2020;8(1):134-139.

34. Gheshlaghi B, Nazaripoor H, Kumar A, Sadrzadeh M. Effect of intrinsic angular momentum in the capillary filling dynamics of viscous fluids. *J Colloid Interf Sci*. 2016;479:80-86.

35. Brydson JA. *Flow Properties of Polymer Melts*, 2nd ed. London, UK: George Goodwin Limited; 1981.

**SUPPORTING INFORMATION**

Additional supporting information may be found online in the Supporting Information section.

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