Quantum features derived from the classical model of a bouncer-walker coupled to a zero-point field

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Abstract. In our bouncer-walker model a quantum is a nonequilibrium steady-state maintained by a permanent throughput of energy. Specifically, we consider a “particle” as a bouncer whose oscillations are phase-locked with those of the energy-momentum reservoir of the zero-point field (ZPF), and we combine this with the random-walk model of the walker, again driven by the ZPF. Starting with this classical toy model of the bouncer-walker we were able to derive fundamental elements of quantum theory [1]. Here this toy model is revisited with special emphasis on the mechanism of emergence. Especially the derivation of the total energy $\hbar \omega_0$ and the coupling to the ZPF are clarified. For this we make use of a sub-quantum equipartition theorem. It can further be shown that the couplings of both bouncer and walker to the ZPF are identical. Then we follow this path in accordance with Ref. [2], expanding the view from the particle in its rest frame to a particle in motion. The basic features of ballistic diffusion are derived, especially the diffusion constant $D$, thus providing a missing link between the different approaches of our previous works [1, 2].

1. Introduction

As explicated already in some of our previous papers [1, 2, 3, 4], we understand the quantum as a well-coordinated emergent system, where particle-like and wave-like phenomena are the result of both stochastic and regular dynamical processes which exchange energy with the surrounding “vacuum”, i.e., the zero-point field (ZPF). Thus, a quantum is modeled as a nonequilibrium steady-state maintained by a permanent throughput of energy. Specifically, we consider a “particle” as a bouncer-walker whose combined movements are coupled to the energy-momentum reservoir of the ZPF. The notion of the bouncer-walker is derived from classical physics, as shown experimentally via the “bouncing droplets” of Couder’s group [5, 6, 7, 8, 9].

The possible existence of such a corresponding, underlying “medium” is a priori independent of quantum theory. For similar views, compare, for example, the works of Hestenes [10], Khrennikov [11], Nelson [12], Nottale [13], and de la Peña and Cetto [14]. We want to underline that by using the expression “classical”, we imply the “updated” present-day status of classical physics, i.e., including present-day statistical physics, nonequilibrium thermodynamics, and the like. Vacuum fluctuations in our terminology thus refer to the statistical mechanics of the ZPF, i.e., a “classical” sub-quantum medium.

In our picture the quantum emerges via phase coupling of the bouncer’s oscillatory frequency $\omega$ with corresponding modes of the ZPF fluctuations, combined with the random-walk model.
of the walker, again driven by the ZPF. Starting with this toy model of the bouncer-walker, we were able to derive fundamental elements of quantum theory from such a classical approach. In Section 2 this toy model is revisited according to [1], but this time with special emphasis on the mechanism of emergence. Especially, the derivation of the total energy $\hbar \omega_0$ and the coupling to the ZPF are clarified. Section 3 follows this path, expanding the view from the particle in its rest frame to a particle in motion [2]. The basic features of ballistic diffusion are derived via a link to the individual bouncer-walker model.

### 2. Modeling the quantum: “bouncer” and “walker”

The bouncer is modeled as a classical oscillator with the following Newtonian equation

$$m\ddot{x} = -m\omega_0^2 x - 2\gamma m\dot{x} + F_0 \cos \omega t.$$  

Eq. 1 describes a forced oscillation of a mass $m$ swinging around a center point along $x(t)$ with amplitude $A$ and resonant frequency $\omega_0$. The damping factor $\gamma$, or friction, allows the oscillator to exchange energy to the ZPF, which in this case is modeled by a locally independent driving force $F(t) = F_0 \cos \omega t$. In the center of mass frame, the system is characterized by a single degree of freedom (DOF).

The stationary solution of Eq. (1), i.e., for $t \gg \gamma^{-1}$, with the ansatz

$$x(t) = A \cos(\omega t + \varphi),$$

yields for the phase shift between the forced oscillation and the forcing oscillation that

$$\tan \varphi = -\frac{2\gamma \omega}{\omega_0^2 - \omega^2},$$

and for the amplitude of the forced oscillation

$$A(\omega) = \frac{F_0 / m}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma \omega)^2}}.$$  

It is well known that the system is stable at the resonant frequency $\omega_0$ of the free undamped oscillator

$$\omega = \omega_0.$$  

With identity [5] we introduce the notations

$$\tau = \frac{2\pi}{\omega_0}, \quad r := A(\omega_0) = \frac{F_0}{2\gamma m \omega_0}.$$  

We assume the net energy balance of the exchange oscillator–ZPF to be zero, i.e., the oscillator–ZPF system is in a steady state. For this, we analyze the energetic balance. By multiplying Eq. (1) with $\dot{x}$ we obtain

$$m\ddot{x} + m\omega_0^2 x \dot{x} = -2\gamma m\dot{x}^2 + F_0 \cos (\omega t) \dot{x}.$$  

Due to the friction term $-2\gamma m\dot{x}^2$ the oscillator loses its energy to the ZPF bath, whereas the oscillator regains its power $F_0 \cos(\omega t) \dot{x}$ from the energy bath via the force $F(t)$. We can rewrite Eq. (7) as

$$\frac{d}{dt} \left( \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m\omega_0^2 x^2 \right) = -2\gamma m\dot{x}^2 + F_0 \cos(\omega t) \dot{x} = 0.$$  

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The term within the brackets is the Hamiltonian of the system. By inserting Eqs. (2) and (5) we see
\[ H = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \omega_0^2 x^2 = \frac{m \omega_0^2 A^2}{2} = \text{const.}, \] thus providing the vanishing of Eq. (8).

One can write down the net work-energy that is taken up by the bouncer during each period \( \tau \) as
\[ W_{\text{bouncer}} = \int_\tau F_0 \cos(\omega t) \dot{x} \, dt = \int_\tau 2 \gamma m \dot{x}^2 \, dt \]
\[ = 2 \gamma m \omega^2 A^2 \int_\tau \sin^2(\omega t + \varphi) \, dt \]
\[ = \gamma m \omega^2 A^2 \tau. \] (10)

With Eqs. (5) and (6) we obtain in the steady state
\[ W_{\text{bouncer}} = W_{\text{bouncer}}(\omega_0) = \gamma m \omega_0^2 r^2 = 2 \pi \gamma m \omega_0 r^2. \] (11)

We now shift to polar coordinates, which allow us to model the system in continuous circular motion. If one introduces the angle \( \theta(t) := \omega_0 t \) and substitutes Eq. (2) into Eq. (9), this yields, as is well known, the two equations
\[ \ddot{r} - r \dot{\theta}^2 + \omega_0^2 r = 0, \] (12)
and
\[ r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 0. \] (13)

From Eq. (13), an invariant quantity is obtained: it is the angular momentum
\[ L(t) = mr^2 \dot{\theta}(t). \] (14)

With \( \theta(t) = \omega_0 t \), and thus \( \dot{\theta} = \omega_0 \), the quantity of Eq. (14) becomes a time-invariant expression, which we denote as
\[ h := mr^2 \omega_0. \] (15)

Note that \( L(t) \) is an invariant even more generally, i.e., for \( \theta(t) := \int \omega(t) \, dt \). Still, for all cases where the time average \( \langle \theta(t) \rangle = \omega_0 t \), one can again write down \( h \) in the form Eq. (15). Thus, we rewrite our result (11) as
\[ W_{\text{bouncer}} = 2 \pi \gamma h. \] (16)

Now let us revisit the energy (9) of the linear harmonic oscillator, which reads, together with Eqs. (6) and (15),
\[ \mathcal{H} = \frac{m \omega_0^2 r^2}{2} = \frac{h \omega_0}{2} = E_{\text{bouncer}}. \] (17)

In a second step, we model another dissipative system. The “walker” is a “particle” driven via a stochastic force, e.g., due to not just one regular, but due to different fluctuating wave-like configurations in the ZPF environment. The particle’s motion, which will generally assume a Brownian-type character, is then described (in any one dimension) by a Langevin stochastic differential equation with velocity \( u = \dot{x} \), driving force \( f(t) \), and friction coefficient \( \zeta \),
\[ m \ddot{u} = -m \zeta u + f(t). \] (18)
Again, “friction”, earlier represented by $\gamma$ and now by $\zeta$, generally describes the coupling between the oscillator (or particle in motion) on the one hand, and the ZPF bath on the other hand. The time-dependent force $f(t)$ is stochastic, i.e., one has as usual for the time-averages

$$\langle f(t) \rangle = 0, \quad \langle f(t)f(t') \rangle = \phi(t-t'),$$

where $\phi(t)$ differs noticeably from zero only for $t < \zeta^{-1}$. The correlation time $\zeta^{-1}$ denotes the time during which the fluctuations of the stochastic force remain correlated.

One usually introduces a coefficient $\lambda$ that measures the strength of the mean square deviation of the stochastic force, such that

$$\phi(t) = \lambda \delta(t).$$

Since friction increases in proportion to the frequency of the stochastic collisions, there must be a connection between $\lambda$ and $\zeta$. One solves the Langevin equation (18) in order to find this connection. Solutions of this equation are well known from the Ornstein-Uhlenbeck theory of Brownian motion [15, 16].

Since the dependence of $f(t)$ is known only statistically, one does not consider the average value of $u(t)$, but instead that of its square,

$$u^2(t) = e^{-2\zeta t} \int_0^t d\tau \int_0^t d\tau' e^{\zeta(t-\tau')} \phi(\tau-\tau') \frac{1}{m} + u_0^2 e^{-2\zeta t}$$

$$= \frac{\lambda}{2\zeta m^2} \left(1 - e^{-2\zeta t}\right) + u_0^2 e^{-2\zeta t} \quad \overset{t \gg \zeta^{-1}}{\longrightarrow} \frac{\lambda}{2\zeta m^2},$$

with $u_0$ being the initial value of the velocity. For $t \gg \zeta^{-1}$, the term with $u_0$ becomes negligible, i.e., $\zeta^{-1}$ then plays the role of a relaxation time. We require that our particle attains thermal equilibrium [17, 18] after long times, so that due to the equipartition theorem on the sub-quantum level the average value of the kinetic energy becomes

$$\frac{1}{2} m u^2(t) = \frac{\lambda}{4\zeta m} = E_{zp} = \frac{1}{2} k T_0.$$ 

where we introduce the average kinetic Energy $E_{zp}$ of the zero-point field, which is the sub-quantum analogon to the thermodynamical expression $k_B T/2$, where $k_B$ is Boltzmann’s constant, and $T$ the classical temperature, whereas $T_0$ in our scenario denotes the vacuum temperature. However, as we today neither know $T_0$ nor the constant $k$ (unless it should turn out as identical to $k_B$), we shall mostly stick to formally using $E_{zp}$. In other words, we shall use the specification “$k T_0$” only occasionally, i.e., in order to point out the close analogy to the usual thermodynamical formalism, and as a reminder that $E_{zp}$ is the “kinetic temperature” of the vacuum’s heat reservoir.

From Eq. (22) one obtains an Einstein-type relation

$$\lambda = 4 \zeta m E_{zp}.$$ 

Similarly, we obtain the mean square displacement of $x(t)$ for $t \gg \zeta^{-1}$. Therefore, one integrates twice to obtain

$$\overline{x^2(t)} = \int_0^t d\tau \int_0^t d\tau' \frac{\lambda}{2\zeta m^2} e^{-\zeta|\tau-\tau'|} \sim \frac{\lambda}{\zeta^2 m^2} t = 2 D t,$$

with the diffusion constant turning out as

$$D = \frac{\lambda}{2\zeta^2 m^2} = \frac{2 E_{zp}}{\zeta m}.$$ 


Now we remind ourselves that we deal here with a steady-state system. Just as with the friction $\zeta$ there exists a flow of (kinetic) energy into the ZPF environment, there must also exist a work-energy flow back into our system of interest. For its calculation, we multiply Eq. (18) by $u = \dot{x}$ and obtain an energy-balance equation. With a natural number $n > 0$ chosen so that $n\tau$ is large enough to make all fluctuating contributions negligible, it yields for the duration of time $n\tau$ the net work-energy of the walker

$$W_{\text{walker}} = \int_{n\tau} m\zeta \dot{x}^2 \, dt = m\zeta \int_{n\tau} u^2(t) \, dt.$$  

(26)

Inserting Eq. (22), we obtain

$$W_{\text{walker}} = n\tau m\zeta u^2(t) = 2n\tau \zeta E_{zp}. \quad \text{(27)}$$

We have so far analyzed two perspectives for our model of a single-particle quantum system:

(i) A harmonic oscillator is driven by the environment via a periodic force $F_0 \cos \omega_0 t$. However, in the $N$-dimensional reference frame of the laboratory, the oscillation is not fixed a priori. Rather, with $\hbar$ as angular momentum, there will be a free rotation in all $N$ dimensions, and possible exchanges of energy will be equally distributed in a stochastic manner.

(ii) Concerning the latter, the flow of energy is on average distributed evenly via the friction $\gamma$ in all $N$ dimensions of the laboratory frame. It can thus also be considered as the stochastic source of the particle moving in $N$ dimensions, each described by the Langevin equation (18).

In order to make the result comparable with Eq. (16), we choose $\tau = 2\pi/\omega_0$ to be identical with the period of Eq. (6). The work-energy for the particle undergoing Brownian motion can thus be written as

$$W_{\text{walker}} = \frac{4\pi}{\omega_0} \zeta E_{zp}, \quad \text{(28)}$$

and, for the general case of $N$ DOF

$$W_{\text{walker}} = \frac{N4\pi}{\omega_0} \zeta E_{zp}. \quad \text{(29)}$$

Accordingly, the walker gains its energy from the heat bath via the oscillations of the bouncer-ZPF bath system in $N$ dimensions: The bouncer, via the coupling $\gamma$, pumps energy to and from the heat bath. There is a continuous flow from the bath to the oscillator, and vice versa. Moreover, and most importantly, during that flow, for long enough times $n\tau$, this coupling of the bouncer can be assumed to be exactly identical with the coupling of the walker. For this reason, we directly compare the results of Eqs. (16) and (29),

$$nW_{\text{bouncer}} = W_{\text{walker}}. \quad \text{(30)}$$

With $n \gg 1$, since we have to take the mean over a large number of stochastic motions, we get

$$n2\pi\gamma\hbar = n\frac{N4\pi}{\omega_0} \zeta E_{zp}. \quad \text{(31)}$$

Now, one generally has that the total energy of a sinusoidal oscillator exactly equals twice its average kinetic energy. With Eq. (17) the bouncer model provided already

$$E_{\text{tot}} = 2E_{\text{bouncer}} = \hbar \omega_0, \quad \text{(32)}$$
and we compare this result with Eq. (31). We achieve the same result for the total energy only if both systems, the bouncer and the walker, are coupled with the same strength to the ZPF bath, i.e., the friction coefficient for both the bouncer and the walker must be identical, $\gamma = \zeta$. We have thus derived the total energy of our model for a quantum “particle”, i.e., a driven steady-state oscillator system, as

$$E_{\text{tot}} = 2NE_{zp} = 2E_{\text{bouncer}} = \hbar \omega_0,$$

(33)

where $\hbar$, as defined in Eq. (15), can now be identified with Planck’s reduced constant. Note that via (15), $\hbar$ is defined as angular momentum in exactly the same manner as it is obtained in an independent earlier derivation by Puthoff [19].

Now, with Boltzmann’s relation $\Delta Q = 2\omega_0 \delta S$ between the heat applied to an oscillating system and a change in the action function $\delta S = \delta \int E_{\text{kin}} \, dt$, respectively, [17, 18] one has

$$\nabla Q = 2\omega_0 \nabla (\delta S).$$

(34)

$\delta S$ relates to the momentum fluctuation via

$$\nabla (\delta S) = \delta p := mu = -\frac{\hbar}{2} \frac{\nabla P}{P},$$

(35)

and therefore, with $P = P_0 e^{-\delta Q/kT_0}$ and (33),

$$mu = \frac{\nabla Q}{2\omega_0}.$$

(36)

As the friction force in Eq. (18) is equal to the gradient of the heat flux,

$$m\zeta u = \nabla Q,$$

(37)

comparison of (36) and (37) provides now a detailed expression for the coupling to the ZPF bath

$$\zeta = \gamma = 2\omega_0.$$

(38)

Note that with Eqs. (33) and (38) one obtains in one dimension the expression for the diffusion constant (25) as

$$D = \frac{2E_{zp}}{\zeta m} = \frac{\hbar}{2m},$$

(39)

which is exactly the usual expression for $D$ in the context of quantum mechanics.

Finally, we can also introduce the recently proposed concept of an “entropic force” [20, 21]. We revisit Eq. (33) and look at the total energy equaling a total work applied to the system. With $S_e$ denoting the entropy one can write

$$E_{\text{tot}} = 2 \langle E_{\text{kin}} \rangle =: F \cdot \Delta x = T_0 \Delta S_e = \frac{1}{2\pi} \int \nabla Q \cdot d\mathbf{r}$$

$$= \Delta Q \text{ (circle)} = 2 \left[ \frac{\hbar \omega_0}{4} - \left( -\frac{\hbar \omega_0}{4} \right) \right] = \hbar \omega_0$$

(40)

which provides an “entropic” view of a harmonic oscillator in its thermal bath.

We know already the total energy of a simple harmonic oscillator $E_{\text{bouncer}}$, Eq. (17). The average kinetic energy of a harmonic oscillator is given by half of its total energy, i.e., by $\langle E_{\text{kin}} \rangle = E_{\text{bouncer}}/2 = \hbar \omega_0/4$, which — because of the local equilibrium — is both the average kinetic energy of the bath and that of the “bouncer” particle. As the latter during one oscillation
varies between 0 and \( \hbar \omega_0/2 \), one has the following entropic scenario. When it is minimal, the tendency towards maximal entropy will provide an entropic force equivalent to the absorption of the heat quantity \( \Delta Q = \hbar \omega_0/4 \). Similarly, when it is maximal, the same tendency will now enforce that the heat \( \Delta Q = \hbar \omega_0/4 \) is given off again to the “thermostat” of the thermal bath. In sum, then, the total energy throughput \( E_{tot} \) along a full circle will equal, according to Eq. (40), \( 2 \langle E_{kin} \rangle (\text{circle}) = \hbar \omega_0/2 = \hbar \omega_0 \). In other words, the formula \( E = \hbar \omega_0 \) does not refer to a classical “object” oscillating with frequency \( \omega_0 \), but rather to a process of a “fleeting constancy”: due to entropic requirements, the energy exchange between bouncer and heat bath will constantly consist of absorbing and emitting heat quantities such that in sum the “total particle energy” emerges as \( \hbar \omega_0 \).

As was shown in [1] one can continue along these classical lines to express further features of quantum mechanics, like the energy spectrum of the harmonic oscillator, or spin, for example.

3. Gaussian particle behaviour

So far, we have described the emerging entity in its rest frame. Now we expand this model and describe the randomly moving walker, characterized by the diffusion constant \( D \), Eq. (39). Here we follow the arguments presented in [2]. As shown above, the nonequilibrium steady-state is characterized by a permanent throughput of energy, or heat flow. First, let us reconsider the Brownian motion of the particle, but from another perspective as compared to the previous chapter. The Brownian motion is a form of kinetic energy provided by the ZPF interaction, and is of course different from the “ordinary” kinetic energy of the particle, which will be introduced later. The total energy of the whole system can be written as

\[
E_{tot} = \hbar \omega + \frac{\langle \delta p \rangle^2}{2m},
\]

where \( \hbar \omega \) is the generalized total energy of the particle and \( \delta p := m \dot{u} \) is said additional, fluctuating momentum of the particle of mass \( m \) [2]. Note that \( \delta p \) can take on an arbitrary value such that \( E_{tot} \) is generally variable.

Every bouncer-walker is a rapidly oscillating object, which itself is guided by the ZPF environment that also contributes some fluctuating momentum to the walker’s propagation. In fact, the walker is the cause of the waves surrounding the particle, and the detailed structure of the wave configurations influences the walker’s path, as the particle both absorbs heat from and emits heat into its environment, both cases of which can be described in terms of momentum fluctuations. Thus, if we imagine the bouncing of a walker in its “fluid” environment, the latter will become “excited” or “heated up” wherever the momentum fluctuations direct the particle to. After some time span (which can be rather short, considering the very rapid oscillations of elementary particles), a whole area of the particle’s environment will be coherently heated up in this way.

Now we expand this further to a source of identical particles, which are prepared in such a way that each one ideally has an initial (classical) velocity \( \mathbf{v} \), which is also called “convective” velocity. Similar arguments are presented by Grössing [in this volume] and [1, 2, 3, 4, 17, 18, 22], but here we want to focus on the interaction with the ZPF. The particles emerge from the source, one at a time only, with a Gaussian probability density \( P \). This comes along with a heat distribution generated by the oscillating (“bouncing”) particle(s), with a maximum at the center of the aperture \( x_0 = vt \). In one dimension the corresponding solution of the heat equation is then

\[
P(x, t) = \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{(x-x_0)^2}{2\sigma^2}},
\]

with the usual variance \( \sigma^2 = \langle \Delta x^2 \rangle = \langle (x - x_0)^2 \rangle \), where we shall choose \( x_0 (t = 0) = 0 \). Note
that from Eq. (41) one has for the averages over particle positions and fluctuations
\[
E_{\text{tot}} = \hbar \omega + \frac{\langle \delta p \rangle^2}{2m} = \text{const.},
\] (43)
with the mean values (generally defined in \(N\)-dimensional configuration space
\[
\langle \delta p \rangle^2 := \int P (\delta p)^2 \, d^N \! x.
\] (44)
Eq. (43) is a statement of total average energy conservation, i.e., holding for all times \(t\). Being a central argument, it is also the starting point for [2]. In Eq. (43), a variation in \(\delta p\) implies a varying “particle energy” \(\hbar \omega\), and \textit{vice versa}, such that each of the summands on the right hand side for itself is not conserved. As can be shown [2], there is an exchange of momentum between the two terms providing a net balance
\[
m \delta v - m \delta u = 0
\] (45)
where \(\delta v\) describes a change in the convective velocity \(v\) paralleled by the “diffusive” momentum fluctuation \(\delta (\delta p) := m \delta u\) in the thermal environment.

Now let us look at the contributions of the diffusive and convective velocities to the total energy. As from Eq. (43) one has that \(\frac{\partial}{\partial t} E_{\text{tot}} = 0\) and thus also \(\delta E_{\text{tot}}(t) - \delta E_{\text{tot}}(0) = 0\), and as only the kinetic energy varies, one obtains \(\delta E_{\text{kin}}(t) = \delta E_{\text{kin}}(0) = \text{const.}\), which yields for any \(t\), with (35) and (42),
\[
\delta E_{\text{kin}}(t) = \frac{m}{2} (\delta v)^2 + \frac{m}{2} u^2 = \frac{m}{2} \delta (\delta p)^2 + \frac{\hbar^2}{8m \sigma^2},
\] (46)
and thus at the initial time, where \(v = 0\):
\[
\delta E_{\text{kin}}(0) = 0 + \frac{m}{2} u_0^2 \bigg|_{t=0} = : \frac{m}{2} u_0^2 = \frac{\hbar^2}{8m \sigma_0^2}.
\] (47)
Now we again make use of the equipartition equation Eq. (22). Together with Eq. (25) we obtain
\[
\frac{m}{2} u_0^2 = \frac{k T_0}{2} = \frac{D \zeta m}{2}.
\] (48)
At the time \(t = 0\) the system is in the prepared state where the fluctuating kinetic energy term is solely determined by the initial value \(\sigma_0\), whereas for later times \(t\) it decomposes into the term representing the particle’s changed kinetic energy and the term including \(\sigma(t)\). At \(t = 0\) the velocity \(u_0\) is determined by the mean displacement \(\sigma_0\) and the relaxation time \(\zeta^{-1}\) of the walker, Eq. (18). By using Eq. (48) we can write
\[
u_0 = \frac{\zeta \sigma_0}{D/\sigma_0}.
\] (49)
With the Gaussian distribution Eq. (42) being a solution of the diffusion equation, one has for the particle’s drift the familiar relation
\[
\overline{x^2(t)} = \overline{x^2} \bigg|_{t=0} + 2Dt.
\] (50)
However, comparison with Eq. (24) indicates that (50) is only a solution for the particle in its rest frame. From Eqs. (46) and (47) we see that in order to account for a Gaussian dispersion as in (46), the diffusivity must become time dependent, too. Thus, one must rewrite Eq. (50) as
\[
\overline{x^2} = \overline{x^2} \bigg|_{t=0} + D(t)t.
\] (51)
In Refs. [2, 3] and [Grössing, this volume] we show that $D(t) = u_0^2 t$. We have thus connected the diffusion processes as described by $D(t)$, which we have found through our model of the bouncer-walker, to the movement of a particle following a Gaussian distribution. With this, we have derived the elements of ballistic diffusion from our classical bouncer-walker model and thus found a missing link between the different approaches of our previous work [1, 2]. We can now expand our thoughts towards interference and look deeper into the emergent properties of the quantum world, and beyond.

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