Finite element analysis of seepage of earth-rock dams in dry and rainy seasons

R Y Wang\textsuperscript{1,2} and Y L Zhu\textsuperscript{1,2}

\textsuperscript{1}College of Water Conservancy and Hydropower Engineering, Hohai University, Nanjing 210098, China

E-mail: wry6888@126.com; yile1995@foxmail.com

Abstract. The earth-rock dams in Southeast Asia will be in long-term rainfall during the rainy season, and long-term rainfall infiltration will affect the seepage field of earth-rock dams. In this paper, COMSOL software is used to analyze the seepage field of earth-rock dam. Based on the basic differential equations such as Darcy’s law and Rechards’ equation built in software, the numerical simulation of seepage analysis is carried out. On the other hand, taking the stable seepage field as the initial condition and considering the rainfall conditions, further analysis and research on the unsteady seepage field will yield better results.

1. Introduction
Most of earth-rock dams are located in subtropical monsoon regions, with four distinct seasons. The rainfall frequency is uniform compared to Southeast Asia, and there is no long-term rainfall. However, the rainy season and the dry season in Southeast Asia are distinct, and sometimes there is a monthly rainfall event. At this time, the seepage field of the earth-rock dam will be affected by rainfall infiltration. Earth-rock dams, especially in tropical rainforest climates, need to consider the factors of prolonged rainfall.

The finite element calculation and analysis of seepage in earth-rock dams in this paper uses COMSOL Multiphysics software. COMSOL is a large-scale advanced numerical simulation software that is widely used in scientific research and engineering fields, and has excellent performance in various physical processes in the fields of analog science and engineering. Based on the finite element method, COMSOL Multiphysics solves the real physical phenomena by solving partial differential equations or partial differential equations. In the software, the Darcy’s law and the Rechards’ equation are applied to simulate the seepage calculation and finite element calculation of the M island earth-rock dam in M country, and the seepage field of the earth-rock dam under the dry season and rainy season is compared and analyzed. When studying the unsteady seepage field under long-term rainfall conditions, the steady seepage field in the dry season is taken as the initial condition, the flow boundary is used to indicate rainfall infiltration, and the seepage field of earth-rock dam under 30-day rainfall is studied in one day time step.

2. Calculation principle

2.1. Basic theory of seepage calculation
The essence of the finite element calculation of seepage is to solve the partial differential equation and boundary condition formula that the seepage field satisfies. COMSOL Multiphysics contains basic
differential equations and boundary conditions commonly used in seepage calculations in fluid flow modules. In this paper, the Richards’ equation model and the Darcy’s law model are chosen to study the seepage field.

The general seepage partial differential equation based on Darcy's law and water flow continuity equation is:

\[
\frac{\partial}{\partial t} (\rho \varepsilon) + \nabla \cdot \rho \left[ \frac{-K}{\mu} (\nabla p + \rho g \nabla D) \right] = Q_m
\]  

(1)

Where: \( \rho \) is the fluid density; \( K \) is the permeability; \( \mu \) is the fluid viscosity; \( p \) is the pressure; \( D \) is the position head; \( Q_m \) is the source and sink; \( \varepsilon \) is the porosity.

The Richards’ equation describing the fluid flow in saturated unsaturated porous media \([1,2]\) is similar in its application to the Darcy’s law. The difference is that the hydraulic parameters of the material are nonlinear and change with the unsaturated state. The seepage differential equation of the unsteady seepage field based on the Richards’ equation is:

\[
\rho \left( \frac{C_m}{\rho g} S_e S \right) \frac{\partial p}{\partial t} + \nabla \cdot \rho \left[ \frac{-K_S}{\mu} \kappa_r (\nabla p + \rho g \nabla D) \right] = Q_m
\]  

(2)

Among them: \( C_m \) is the water capacity; \( S_e \) is the effective saturation; \( S \) is the water storage coefficient; \( K_S \) is the saturated permeability; \( \kappa_r \) is the relative permeability, \( t \) is the time.

Considering the stable seepage field, \( \rho \left( \frac{C_m}{\rho g} S_e S \right) \frac{\partial p}{\partial t} \) is not considered in the partial differential equation of equation (2).

2.2. Boundary conditions

When calculating the stable seepage field and the unstable seepage field, the boundary conditions of the fixed head, the mixed boundary conditions simulated by the boundary of the permeable layer, and the flow boundary conditions considering the rainfall infiltration are considered in the boundary conditions of COMSOL Multiphysics.

The fixed head boundary conditions below the upstream and downstream water levels can be expressed as:

\[
\text{On the border } \Gamma_1: \quad p = \rho g (H_0 - D)
\]  

(3)

where: \( H_0 \) is the a given head; \( D \) is the elevation.

Mixed boundary conditions can be expressed as:

\[
\text{On the border } \Gamma_2: \quad -n \cdot \rho \mathbf{u} = \rho R_b (H_b - H)
\]  

(4)

where: \( R_b \) is the conductivity; \( H_b \) is the external pressure head.

The flow boundary condition can be expressed as:

\[
\text{On the border } \Gamma_3: \quad -n \cdot \rho \mathbf{u} = N_0
\]  

(5)

where: \( N_0 \) is the inward mass flux.

Since the free surface and the escape point of the seepage are unknown, the escape boundary belongs to the mixed boundary, which is difficult to handle for the complex seepage control system. The escape boundary in the calculation of unsaturated seepage is a special boundary condition. The permeable layer boundary provided in COMSOL Multiphysics is a mixed boundary condition that defines the boundary conditions \([2,3]\).
2.3. Rainfall infiltration
The theory of rainfall infiltration is based on the concept of infiltration rate, which represents the infiltration capacity of soil, and the infiltration rate of rainfall will decrease as the rainfall duration increases [4]. Based on this theory, the rainfall infiltration mode can be divided into two types: 1) Full infiltration mode: When the rainfall intensity is always less than the infiltration capacity, the rainfall will fully infiltrate without runoff. 2) Partial runoff mode: When the rainfall intensity is greater than the infiltration capacity, rainfall will produce runoff.

When calculating the unsteady seepage, the flow boundary and the head boundary are set according to the rainfall conditions and the infiltration capacity of different parts of the dam and dam foundation. In order to better simulate the changes of different stages of rainfall infiltration, this paper considers the characteristics of infiltration rate changing with the infiltration capacity of the dam, and considers the rainfall flow in the form of boundary conditions. The specific treatment method is: when the rainfall intensity is less than the soil permeability, the infiltration amount is the rainfall intensity. At this time, the slope is orthogonally infiltrated and is the flow boundary; when the rainfall intensity is greater than the soil permeability, the infiltration capacity is taken as the permeability of the soil, because part of the rainwater is lost along the slope and a thin layer of water film is formed in the lower part of the dam slope, while the rest of the rainfall is The form is added to the lower part of the dam slope [5,6].

In this paper, the average rainfall intensity of the maximum rainfall of 1483 mm was taken under different working conditions, and the seepage field of rainfall infiltration conditions in the rainy season was analyzed according to the unsteady saturated-unsaturated seepage problem. Firstly, the steady seepage calculation is carried out without considering the rainfall, and then the rainfall factor is added to the model in order to consider the rainfall infiltration, the transient analysis is performed by using the stable seepage field as the initial value of the unsteady seepage field.

3. Engineering case

3.1. Overview of earth-rock dam reservoir project
The total storage capacity of M Island M reservoir project in M country is 810,000 m$^3$, the storage capacity is 550,000 m$^3$, the dead water level is 10.00 m, the normal water storage level is 16.00 m, the design flood level is 17.00 m, and the check flood level is 17.26 m. The retaining earth dam is a compacted earth dam with a dam crest elevation of 17.5 m, a dam axis of 264 m and a maximum dam height of 12.5 m. The top of the dam is 5 m wide. A concrete wave wall is installed on the upstream side of the dam crest, and the top elevation of the wall is 18.7 m. The upstream slope of the dam is 1:3, and the downstream slope of the dam is 1:2.75.

3.2. Establishment of finite element model
In this paper, a two-dimensional finite element model is established for a typical section of a reservoir dam (the riverbed 1-1 section and the right bank section 2-2), and the dam is operated under two different conditions (normal water level and check flood level). The analysis of the stable seepage field and the unsteady seepage field provides a basis for the analysis of the seepage state of the dam.

A two-dimensional finite element model is used to analyze the seepage field at different locations of the dam. According to the condition of the dam, we choose a typical section to establish a planar finite element model:

- The two-dimensional finite element model considers the dam body and part of the dam foundation, and the upper and lower dam foundations take 2 times the dam height range.
- Take the x-axis direction as the water flow direction to be positive toward the downstream; the y-axis direction is the vertical direction, and the upward direction is positive. The coordinate origin x=0.0m corresponds to the position of the dam axis in the upstream and downstream directions, and y=0.0 corresponds to the position where the elevation is zero.
- In the COMSOL Multiphysics, the super-unit CAD model is transferred. According to the structural characteristics and geometric topological properties of the earth dam, the three-node
triangular unit with extremely thinning method is used to mesh the structure. The total number of elements in the 1-1 section and the 2-2 section are 46276 and 35728 respectively. The 1-1 section and the 2-2 section are the sections of the hub plane and the right bank in the plane layout, respectively, see figures 1 and 2.

The working condition 1 of the study is the normal water storage condition, and the upstream water level elevation is 16.00 m. The downstream water level of the 1-1 section takes the ground elevation of the right side of the downstream drainage ditch to be 6.40 m; there is no drainage ditch downstream of the 2-2 section, considering the downstream waterless, the ground elevation at the dam angle is 7.50 m. Working condition 2 is the check of the flood level condition, and the upstream water level elevation is 17.26 m. The downstream water level of the 1-1 section and the 2-2 section is the same as that of the working condition 1.

4. Finite element analysis of dam seepage field

4.1. Finite element analysis of stable seepage field of dam

The flow network diagrams of the 1-1 and 2-2 sections under working conditions 1 and 2 are shown in figures 3-6. The calculation results of the stable seepage flow under different working conditions of different sections are shown in table 1. Under the normal water storage condition of 1-1 section, the downstream water level is lower than the top elevation of the raft head, and the position of the seepage escape point enters the sluice pad drainage point in the immersion line (the first stream line in the flow net of figure 3). The point is the same as the downstream water level. Under the condition of checking the flood position, the middle part of the immersion line has been attached to the downstream slope of the dam body, indicating that the downstream slope seepage also escapes. Because the 2-2 section is waterless in the downstream, the position of the seepage out point of the 2-2 section under the two working conditions is at the elevation of the wetting line into the drainage pad and the downstream ground. Under the condition of checking the flood level, the seepage flow escapes at about 1/2 of the dam height on the downstream slope.

From the point of seepage flow, the seepage flow and total seepage flow of the dam body and dam foundation under the condition of checking the flood level of the riverbed position 1-1 section are the largest (table 1). The allowable osmosis slopes of the dam and dam foundation are 0.5 and 0.45, respectively. From the perspective of the escape slope, the escaping slope of each section of each working condition is smaller than the allowable osmosis slope of the dam foundation (table 1); under the
condition of checking the flood position, the osmotic slope of the 2-2 section dam body entering the
drainage area is larger than the allowable osmosis slope of the dam body. The part of the dam body
exceeding the allowable osmosis slope is in a local position close to the filter layer, so when the
reservoir operates, the focus should be on the seepage of the bank slope, and also, it should both monitor
and ensure the quality of the filter layer and the quality of the joint between the dam and the bank.

Table 1. Comparison table of stable seepage calculation results under different working conditions of
different sections.

| Profile | Single wide seepage flow (m³/d·m⁻¹) | Total seepage flow (m³/year) | Escape slope |
|---------|-----------------------------------|-----------------------------|--------------|
|         | Dam foundation | total | Dam foundation | |
| 1-1     | 0.009 | 0.026 | 0.035 | 3352 | 0.36 | 0.066 |
|         | 0.011 | 0.029 | 0.04  | 3876 | 0.487 | 0.069 |
| 2-2     | 0.007 | 0.01  | 0.017 | 1651 | 0.468 | 0.146 |
|         | 0.009 | 0.011 | 0.02  | 1973 | 0.659 | 0.154 |

4.2. Finite element analysis of stable seepage field of dam

Under different working conditions, the average rainfall intensity of the maximum rainfall of 1483 mm
was taken, and the rainfall factor was added to the model. The unsteady seepage field analysis was
carried out considering the rainfall for 30 days. The flow network diagrams of the 1-1 and 2-2 sections under working conditions 1 and 2 are shown in figures 7-10. The calculation results of the unsteady seepage under different working conditions of different sections are shown in table 2.

Table 2. Comparison table of unsteady seepage calculation results under different working conditions of different sections.

| Section | Working condition | Maximum single-wide seepage flow Dam | Maximum single-wide seepage flow Dam foundation |
|---------|-------------------|--------------------------------------|-----------------------------------------------|
| 1-1     | Normal water level| 0.009                                | 0.026                                          |
|         | Check the flood level | 0.0111                              | 0.029                                          |
| 2-2     | Normal water level| 0.007                                | 0.01                                            |
|         | Check the flood level | 0.0091                              | 0.011                                           |

In the rainy season, when the maximum monthly rainfall intensity lasts for 30 days, the infiltration seepage forms an unsteady seepage. Under the 1-1 section and 2-2 section check flood level, the dam body saturation line escapes on the downstream side, and the escape point is downstream. The 2/3 and 1/2 dam heights of the dam slope are higher than the calculation results of the stable seepage field. Under the various working conditions of each section, the maximum escaping slope of the dam is slightly larger than that of the steady seepage analysis under various working conditions, indicating that the continuous rainfall has an adverse effect on the seepage condition of the dam. Since the permeability coefficient of the dam body and the dam foundation is small, and the single-wide seepage flow is calculated from the dam axis section, the single-wide seepage flow of the dam body and the dam foundation with or without considering the rainfall infiltration is basically the same.

5. Conclusions and prospects

In this paper, COMSOL Multiphysics software is used to analyze the seepage field of earth-rock dams in the dry and rain seasons, combined with the example of M earth-rock dam, to analyze the seepage of earth-rock dams, and to compare and analyze the stable seepage field and the unsteady seepage field. The conclusions are as follows:

- Studying the influence of rainfall infiltration on the seepage field of earth-rock dams, it was found that rainfall infiltration has a great influence on the slope of the embankment dam, which would increase the escape slope and reduce the safety of the project. Long-term rainfall would adversely affect the operation of earth-rock dams. For earth-rock dams with long-term rainfall, this factor needs to be considered in operations, reinforcement and safety assessment.
- Using the Richards equation, the saturated unsaturated seepage model could be used to calculate the saturated unsaturated seepage in the whole domain; using the boundary conditions of the permeable layer to directly establish the overflow boundary condition, and obtain satisfactory results when solving the overflow point and the free surface.
- Considering the calculation of the unsteady seepage field in the rainy season, the flow boundary was applied to consider the rainfall infiltration and solve the calculation. This method had obtained reliable results in the analysis of the unstable seepage field of the earth-rock dam, indicating its reliability and feasibility.

References

[1] Wu M X 2009 Seepage flow in saturated-unsaturated soils finite element algorithm of Richards equation J. Hydraul. Eng-ASCE 40 1274-9
[2] Xu W and Xu Q 2014 Finite element analysis of seepage based on COMSOL multiphysics J. Wuhan Univ. (Eng. Sci.) 2014 165-70
[3] Chui T M and Freyberg D L 2007 The use of COMSOL for integrated hydrological modeling Proc. COMSOL Conf. 2007 (Boston) pp 217-23
[4] Zhou Q 2019 Study on seepage stability of earth-rock dam under rainfall conditions *Water Resources Sci. Technol.* **25** 57-62

[5] Zhu W, Chen X D and Zhong X C 2006 Measurement and analysis of rainfall infiltration law *Rock Soil Mech.* **27** 1873-9

[6] Pan L. Y 2014 Analysis of seepage field of unsaturated soil slope under rainfall conditions (Tianjin, China: Hebei University of Technology)