Historical aspect of mathematics on Indonesian mathematics textbook

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Abstract. Inviting the students’ activity in reinventing the mathematics concept in the mathematics class, has become more popular in the recent mathematics education curriculum. History and Pedagogical in Mathematics (HPM) research has grown to support reinvention activities. The recent textbook in mathematics based on the Kurikulum 2013 has implemented the HPM perspective as a form of historical snippets. This Paper aimed at drawing the form on how historical perspectives in Indonesian Mathematics Textbook that help students reinvent the mathematics concept. Some parts in Indonesian Mathematics text book have considered the essence of historical perspective as the main idea in reinventing specific concept. The historical aspect is modified and appears with didactica approach that invites students’ active involvement. More examples show factual data of historical concept as motivation on the value of the mathematics.

1. Introduction
Guided reinvention on the mathematical concept has become important perspective in the mathematics education in the recent curriculum. The recent developments on the History and Pedagogical in Mathematics (HPM) research field support the use of the perspective of history in mathematics and its integration in the classroom practice [1][2][3][4]. Some forms of certain history of mathematics in the classroom may be implemented as motivational resources, conceptual development activities resource, or just as alternative fun game resources. In Indonesia, the use of historical perspective has been occurred in the mathematics textbook. However, the researches on how the trends on using historical perspective in Indonesian mathematics textbook have not done thoroughly. The recent curriculum in Indonesia is Kurikulum 2013 which is relying on the use of scientific approach [5]. The major change in the curriculum is the implementation of the Kurikulum 2013 textbook as the main resource in teaching mathematics. The Kurikulum 2013 textbooks consist of students and teachers books by which the teacher could use in initializing the activities, motivating the students, inviting scientific activities, and assessing the students [6]. This opportunity is used for integrating historical perspective of mathematics in the Kurikulum 2013 textbook in various forms.

The issue of integrating history of mathematics in the classroom practice has raised pros and cons in the extend of which historical perspective may help the students’ learning. Argument for integrating history into educational process considered that mathematics teaching may be supported, enriched, and improved in fruitful ways. Jankvist stated that, by integrating history of mathematics in the class, it gains positive impact for students’ motivation factor [7]. Creating excitement and igniting students’ interest
can be produced by having historical perspective in the class. Furthermore, integrating history in the classroom would provide an insight of why certain mathematics subject was developed. This can be used to make sure that the mathematical concept is taught in meaningful ways.

Several objections focus on the material content, the readiness of the teacher, and the students’ perception on history and mathematics [8]. In fact, not all mathematical concept could be delivered using historical perspective boldly. However, lack of achievement and difficulties raised in implementing historical perspective in mathematics classroom may resulted from the teachers’ inability in managing time, finding suitable resources, and improving teacher expertise on didactic and assessment. Tzanakis and Arcavi considered five specific domains in which historical perspective has impact on [8]. They are (1) the process of mathematics’ learning, (2) the development of beliefs on the nature of mathematics and mathematical activity, (3) the teachers’ didactical background and their pedagogical repertoire, (4) the attitude towards mathematics, and (5) the appreciation of mathematics as human activities. The degree of raw content of the history while being implemented in mathematics classroom may limit the potency of HPM. The way on how HPM could be used can be vary as direct historical information, historical inspired instruction, and mathematical awareness [3].

One of possible way in implementing HPM is in the form historical snippets inside mathematics textbook [8]. Furthermore, Tzanakis and Arcavi propose general format of how this historical format appear in most mathematics textbook [8]. The guideline could be used to determine how the historical snippet could invite the mathematics concept reinvention. To be more specific, this Paper aimed at drawing the trend on how historical perspective in Indonesian Mathematics Textbook. The historical perspective may help students in reinventing the mathematics concept. By eliciting the general issues of historical perspective used in the textbook, the aim of implementing historical perspective can be drawn. From the aforementioned facts, improving how integrating history of mathematics in the classroom can be done precisely. Regarding the ‘why’ integrating history in the class is important.

The discussion will be divided into three parts. In the first, we shall describe the selected textbook with historical snippet used for analysis. In the second part, we shall elaborate the conceptual framework for analysing the textbook. In the final part, we shall draw the trend of the historical perspective in Indonesian Mathematics textbook.

2. Conceptual framework
The study will concern in the form of textbook analysis. From several form of how historical aspect may be used in the mathematics classroom, historical snippets are defined as exposition historical information which is incorporated in the mathematics textbook. The conceptual framework (table 1) in analysing and categorizing incorporated historical in the mathematics textbook is using the following format and content proposed by Tzanakis and Arcavi [8]. They are:

2.1. Where the historical exposition is provided
This format categorizes the historical snippet into different places on didactical process. Whether the snippet is used before the mathematics concept introduced, or after the mathematics concept given, or even alongside the teaching of mathematics concept.

2.2. The didactical approach format
This format categorizes the historical snippets whether they are only mean for exposition information or main activity of reinventing mathematics concept.

2.3. The information of historical marker
This format categorizes the historical snippets on how the information in the snippets make the reader notice that that part is integrated with historical perspective.
2.4. The style and design
This format categorizes the snippets into different styles on presentation (informal, easy to read, narrative, and the font and visual format of the snippets).

2.5. The availability of factual data
This content categorizes the snippets whether they include any factual data or not.

2.6. The conceptual issues
This content categorizes the snippets whether they expose any motivation, origin and evolution of an idea, ways of noting and representing idea contrasted with the recent development, certain argument of errors or alternative conceptions, calculation method in former time, etc.

3. Method
The Indonesian textbook used in this analysis is the Kurikulum 2013 official textbook from the centre of curriculum and references which are used in grade VIII. In detail, the revision edition 2014 of this book is used. The Kurikulum 2013 textbook is developed based on the material and competency and framework on international standard such as PISA and TIMSS. The Mathematics content in the textbook consist of data and probability, patterns and number sequences, algebra, and shapes also transformational geometry. The textbook considers the application of conceptual and procedural knowledge with mathematics’ problem solving. In addition, the modelling, proofing and doing estimation are also available within the textbook. The material discussed in the textbook also considered higher order thinking problem with realistic context which close to students’ daily life. Those problems are delivered to lead students from concrete to abstract stage that required problem solving.

4. Result
In general, there are several topics commonly used in textbook which are geometry and algebra. The historical snippets are defined as any exposition of historical aspect of mathematics in the mathematics textbook. In fact, those snippets rarely mention explicitly that this part is using historical aspect of mathematics. Using the format and content categorization from [8], we can conclude that there are categories on historical snippets in Indonesian mathematics textbook. They are historical motivation portion, historical based resources material, and historical based reinvention activity.

4.1. Historical motivation portion
On every chapter in the book, there are historical snippet append in the beginning of the material (A1) (see Figure 1). In this discussed example, the snippet about mathematician Rene Descartes is provided. However, it found that there is a typo in the book related the historical life of Rene Descartes. It should be 1596-1650. In the snippet, we can see that it provides motivation aspect for the students in the chapter by giving information on the famous mathematician related with the topic (B1) (see Figure 2 Section A). The factual data provide information about how important the mathematician, Rene Descartes, in the history of mathematics. The philosophy of Descartes is also embedded in the snippet.

Table 1. Format and content in categorizing historical snippets.

| Format and Content | Category | Details |
|--------------------|----------|---------|
| Where the historical exposition is provided | Introduction of concept (A1) | The historical snippet is delivered before the mathematics concept given. Usually, this part is in a form of motivation phase |
| Alongside the concept(A2) | The historical snippet is delivered alongside the mathematics concept given. This may form as main activity of based on history |
| After the concept(A3) | The historical snippet is delivered after the mathematics concept given as closing concept |

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Table 1. Cont.

| The didactical approach format | The historical snippet is mainly information on the historical background of the concept |
|-------------------------------|-------------------------------------------------------------------------------------|
| Expository information (B1)  |                                                                                     |
| Didactical activity (B2)      | The historical snippet is integrating the historical perspective as the reinventing mathematics concept based on history |
| The information of historical marker |                                                                                      |
| Historical noticeable (C1)    | The historical snippets show the use of historical aspect explicitly                  |
| Historical noticeable (C2)    | The historical aspect in the historical snippet is not appear explicitly              |
| The style and design          |                                                                                     |
| Distinctive from the content of the textbook (D1) | The historical snippets are seen distinctively on the textbook                        |
| Unite with content of the textbook (D2) | The historical snippets are using the textbook general format                        |
| The availability of factual data |                                                                                     |
| Using factual historical information (E1) | The historical snippets use factual historical data in the snippets                     |
| Without any factual historical information (E2) | The historical snippets do not use factual historical data                             |
| The conceptual issues         |                                                                                     |
| Motivation behind mathematics concepts (F1) | The historical snippets provide information about the motive of the appearance of the concept |
| Origin and evolution of an idea (F2) | The historical snippets provide information about the origin of the development of mathematics concept |
| Ways of noting and representing idea contrasted with the recent development (F3) | The historical snippets provide information on the differences of the recent mathematics concept and in the past |
| Certain argument of errors or alternative conceptions (F4) | The historical snippets provide information about the errors and alternative conceptions of mathematics concept in the past |
| Calculation method in former time (F5) | The historical snippets provide information about mathematics method in the former time | etc.

Figure 1. Snapshot of historical motivation portion.
The factual data is also given about the profile of the mathematician and the achievement along with the books (C1) (E1) (see Figure 1 Section B). In this part, a short profile of Rene Descartes is given along with several specific achievements from him who is known as the invention of Cartesian coordinate system and the father of modern mathematics. About the physical appearance of the snippet, it is provided in special part in the chapter (D1) (see Figure 1). The aim of this snippet is to motivate the students before starting the lesson (F1) (see Figure 1 Section C). Several values obtained from the snippet is explicitly provided in the end of the snippet. This aim relating to the core competency defined in the Kurikulum 2013 where the learning activities could invite the students in appraising the favour blessing from God. In addition, the value also invites the students to be grateful and work hard to achieve their good without giving up easily.

This snippet is in the introduction part of the chapter. In the end of the snippet, the textbook provides the aim on why the snippet is given as the motivation portion. The value taken from the history is also provided as the core competency of this chapter.

4.2. Historical based resource material

The historical snippet in the textbook can be seen also as the source of the discussion in the chapter. The activity is adopted from Liu Hiu proof on Pythagorean Theorem [7]. This snippet can be used as the main activity in delivering the topic of Pythagorean Theorem (A2) (see Figure 2). This snippet also provided inside the learning material in the chapter. The resource can be used in the didactical activity in reinventing the genuine proof of the theorem (B2) (see Figure 2). The proof given in the material is similar with Liu Hiu’s proof on Pythagorean theorem as follow:

\[
\begin{align*}
\text{Suppose } AE &= a, AF = b, EF = C \\
ABCD &= EFGH + 4 \times AEF \\
(a + b)^2 &= c^2 + 4 \times \frac{1}{2}ab \\
a^2 + b^2 + 2ab &= c^2 + 2ab \\
a^2 + b^2 &= c^2
\end{align*}
\]

This approach makes use of the right angle triangle as the edge of the biggest square formed by using its both perpendicular sides. The historical snippet is combined through the whole chapter where there is no distinction between the material of the chapter and the snippet (D2) (see also Figure 2).

![Figure 2. Snapshot of historical based resource material.](image)

The historical resources are not explicitly mentioned in this snippet (C2). Furthermore, the historical element is not seen in the snippet as well as the fact of the historical data (E2). Regarding the conceptual issue, this snippet can be considered as how the former proof on Phythagorean Theorem (F2) (see Figure 2). The proof can be seen in the Liu Hiu’s book of the “Nine Chapter”. In addition, this snippet also provides alternative proof on Pythagorean Theorem (F4).

In the textbook, this part is entitled as observation activity. As one of the phase in the scientific approach which is the main framework in Kurikulum 2013, the historical snippet can be used as rich material resources discussion. This activity is aimed for facilitating the students in developing and practicing the ability on collecting mathematics idea.
4.3. The historical based reinvention activity

Further form of historical snippets appeared in the mathematics textbook is in the reinventing activity. The snapshot in Figure 3 provide an example from geometry topic in the area measurement.

This snippet illustrates an activity in reinventing the area of circle (B2) which is considered in the main activity by the time of the concept delivery (A2). The historical snippet is formatted differently inside the material in the chapter (D1) with the historical resources are not explicitly mentioned in this snippet (C2). Furthermore, the historical element cannot be seen in the portion (E2). Regarding the reinventing activity, this snippet can be used in the reinventing activity using the former method of determining the area of circle (F5). In detail, one might consider how the formula of circle area is originated (F2). The following explain the details

\[
A_{\text{Circe}} = A_{\text{Parallelogram}}
\]
\[
= \text{base}_{\text{Parallelogram}} \times \text{height}_{\text{Parallelogram}}
\]
\[
= \frac{1}{2} \text{Circumference}_{\text{Circle}} \times \text{Radius}_{\text{Circle}}
\]
\[
= \frac{1}{2} 2\pi r \times r
\]
\[
A_{\text{Circe}} = \pi r^2
\]

This format categorizes the snippets into different style on presentation (informal, easy to read, narrative, and the font and visual format of the snippets).

![Figure 3. Snapshot of historical based reinventing activity.](image)

5. Conclusion

In summary, the mathematics textbook has integrated the form of historical aspect of mathematics as historical snippets. There are several topics commonly used in textbook as geometry and algebra. This development is a response to the Kurikulum 2013 who demand more on the scientific approach. Certain parts in the textbook consider the historical perspective as the resources in reinventing specific concept. While, other modified historical information as factual data and appear as motivation passage in the beginning of each chapter.

References

[1] U T Jankvist and S M Iversen 2013 Sci. & Edu. 23(1)
[2] M N Fried 2001 Sci. & Edu. 10 391–408
[3] J Fauvel and J Van Maanen 2000 History in mathematics education ICMI study
[4] V J Katz 2000 Using History to teach Mathematics International Perspective MAA 51
[5] Depdikbud 2016 Permentikbud Nomor 21 Tahun 2016 h
[6] Kemendikbud 2013 Matematika Untuk SMP/MTs Kelas VII: Buku Guru
[7] U T Jankvist 2009 Educational Studies in Mathematics 71 235-261
[8] C Tzanakis and A Arcavi History in mathematics education ICMI study pp. 201–24