Conjugate impedance matched metamaterials: 
Physical modeling and wave interaction effects

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Abstract. Conjugate impedance matched metamaterials are shown to be effective traps for 
electromagnetic waves. Objects made of such materials are able to receive radiation even when 
it is not directly incident on their surface. Here, we develop methods of physical modeling of 
such objects and investigate interactions of conjugate impedance matched superabsorbers with 
passing electromagnetic radiation. We study realizations of such metamaterials with meshes 
of loaded transmission lines and develop a theory of electromagnetic wave propagation and 
absorption in such media. Peculiar wave propagation, wave trapping and absorption effects 
in metamaterial black holes and wormholes are demonstrated. Possible modifications under 
the goal of optimizing absorption while minimizing complexity of the involved metamaterials 
are discussed. Conjugate-impedance matched superabsorbers may find applications as efficient 
harvesters of electromagnetic radiation, and as novel antennas and sensors.

1. Introduction
From wave optics, we know that the scattering and absorption cross sections of resonant particles 
can be much greater than those of non-resonant bodies with the same dimensions [1, 2]. For 
instance, absorption in subwavelength metallic particles exhibiting plasmonic resonance can be 
orders of magnitude higher than the same for a black-body type absorber of a comparable 
physical size. Effectively, such resonant particles are able to collect the incident wave power 
from an area much bigger than the physical cross section of the particles.

The same physical principle of optimal resonant absorption is used when designing compact 
receiving antennas. From the theory of wire antennas it is known that a short wire dipole (with 
length much smaller than half wavelength) is a rather ineffective receiver unless it is loaded 
with complex impedance $Z_{\text{load}}(\omega) = Z_{\text{dip}}^*(\omega)$, where $Z_{\text{dip}}^*(\omega)$ is the complex-conjugate of the 
input impedance of the dipole antenna at the frequency $\omega$. Such a conjugate matched load 
compensates for the excess reactance of the short dipole antenna, tunes it in resonance with the 
incident field, and provides for the maximum of the received power.
Figure 1. (a) Circuit diagram of the loaded 2D transmission line (TL) mesh. The unit cell of the structure is indicated with a dashed square. (b) Schematic representation of a TL-based wormhole structure formed by two electrically connected two-dimensional DPS and DNG domains. The two separate domains can be realized with unloaded (DPS) and loaded (DNG) TL meshes. The two TL meshes are electrically connected at the circumference $r = a$ of the wormhole neck.

If an object supports higher order multipolar resonances (in addition to the main electric dipolar mode) at the same frequency $\omega$, its absorption cross section can be made higher than that of a resonant dipole. In fact, it can be shown [3]–[5] that there is no ultimate upper limit on the effective absorption cross section of a resonant object when more and more multipolar modes of the object pile up at the same resonant wavelength. With this principle, an exotic object — metamaterial black hole — formed by a medium with simultaneously negative permittivity and permeability can be constructed to possess arbitrarily large absorption cross section, theoretically, independently of the physical size of the object. The required condition is that double-negative (DNG) metamaterials with arbitrarily small loss and arbitrarily large ranges of permittivity and permeability values are attainable. Such metamaterial superabsorbers can be used as efficient harvesters of electromagnetic radiation at microwave frequencies or as super-Planckian radiative heat emitters at infrared and optical frequencies.

2. Physical modeling of superabsorption effect in two dimensions

From our previous study [3] we know that a spherical metamaterial body with radius $a$ and radially dependent isotropic complex permittivity $\varepsilon(r) = \varepsilon'(r) - j\varepsilon''(r)$ and permeability $\mu(r) = \mu'(r) - j\mu''(r)$ satisfying $\varepsilon'(r)/\varepsilon_0 = \mu'(r)/\mu_0 = -a^2/r^2$, $|\varepsilon''(r)/\varepsilon'(r)| = |\mu''(r)/\mu'(r)| = \tan\delta \to 0$, has (theoretically) infinite effective absorption cross-section $\sigma_{\text{abs}}(\omega) \to \infty$ at the frequency $\omega$, independently of the physical radius of the body $a$. Here, $\varepsilon_0$ and $\mu_0$ stand for the permittivity and the permeability of the surrounding space, e.g., free space. For objects made of the materials with finite values of the loss tangent: $\tan\delta > 0$, and a limited variation range of the relative material parameters when $r \to 0$, the absorption cross section is finite, but still it can be large as compared to the physical dimensions of the body, even when $a \gg \lambda$, where $\lambda$ is the radiation wavelength. An analogous result can be obtained as well in the case of a cylindrical body with the radius $a$ and the anisotropic radially dependent material parameters [4].

Unfortunately, with the facilities that are currently at our hands a practical demonstration of the superabsorption effect in three dimensions appears to be rather difficult. Therefore, here
Figure 2. Panel (a) shows a realization of the wormhole structure shown in Figure 1 (b) with the DPS and DNG unit cells depicted in panels (b) and (c), respectively. The DPS unit cell is formed by an unloaded crossing of two symmetric strip lines. The DNG unit cell is formed by two crossed stripline segments loaded at the crossing by a lumped inductor connected to the ground (which realizes $Y$), and by four lumped capacitors at the edges of the unit cell (which realize $Z/2$). The lumped elements are shown in blue. (d) Distributions of the nodal voltage for the wormhole structure under plane wave incidence as functions of the geometrical coordinates $x < 120$ (the DPS domain) and $x > 120$ (the DNG domain) and $y$ expressed in units of the unit cell size $d$. (e) The field distribution for the ring object confined to the region $r < R_{\text{obj}} = 30d$ (where the region $x < 90$ represents the DPS domain with the object and $x > 90$ corresponds to the DNG domain) with optimized values of the characteristic impedance and the propagation factor and with a greatly reduced number of DNG cells. In all these plots, there are no unit cells in the white colored regions, and, respectively, there is no wave propagation in these regions.

we aim at physical modeling of this effect in a setup with reduced dimensionality, namely in two dimensions. Many limitations of volumetric metamaterials can be overcome when using two-dimensional metamaterials (2D) realized with meshes of loaded transmission lines (TL). The theory of such 2D metamaterials has been developed in a number of well-known works. There is also a possibility to extend such concepts to 3D (e.g., Ref. [6]). The unit cell of a generic 2D TL-based metamaterial is shown in Figure 1 (a). The electromagnetic waves in such structures are represented by waves of electric currents and voltages in the transmission line segments. By selecting proper loads, a TL-based metamaterial can be made to support forward or backward waves, and thus operate effectively as a double positive (DPS) medium with $\varepsilon'_{\text{eff}} > 0$ and $\mu'_{\text{eff}} > 0$, or as a DNG medium with $\varepsilon'_{\text{eff}} < 0$ and $\mu'_{\text{eff}} < 0$.

In this work we show that in the 2D case the superabsorption effect can be modeled by a wormhole structure [7] composed of two separate TL meshes: The unloaded DPS mesh and the loaded DNG mesh, electrically connected at the circumference of the wormhole. This structure is schematically shown in Figure 1 (b). The DPS and DNG domains in the top and bottom halves of this structure have to be conjugate impedance matched for all spatial harmonics of the incident field. In the practical realization of such a structure we employ meshes of strip lines [Figure 2 (a)] with the unit cells shown in Figure 2 (b,c). These unit cells fill the DPS and the
DNG domains shown in Figure 1 (b) by the golden and the blue surfaces, respectively. These DPS and DNG networks are laid atop one another and are separated by the common ground plane. A wave propagating in the DPS network towards the wormhole, after passing through the connections at the circumference of the wormhole, propagates in the DNG region in the outward direction and is absorbed at the edge of the DNG mesh by matched loads.

Simulation results obtained with the frequency-domain transmission line matrix (FDTLM) method are shown in Figure 2 (d). In this case, the wormhole structure is formed by the DPS and DNG domains occupying an area of $120 \times 120$ cells with each cell being a square of size $d \times d$ and a wormhole of radius $R_{WH} = 30d$, with $d = 5\text{ mm}$. There are no cells at the middle of the domains where $r < R_{WH}$. Respectively, in this middle region (the white area in the figure) there is no propagation. In this case, the numerically calculated value of of the absorption cross section is $\sigma_{\text{abs}}/(2R_{WH}) = 1.49$, i.e., this metamaterial object performs about 50% better than a simple black body type absorber of the same radius. Note that this result is achieved for an object with a rather large electrical size $\beta_0 R_{WH} = 30\beta_0 d = 9.45$, which means that the circumference of the object is on the order of 10 wavelength.

In order to reduce the number of DNG unit cells in the structure, we perform a numerical optimization to confine all DNG cells within the region $r < R_{obj}$. The optimization establishes optimal variation profiles for $\beta_0 d$ (the electric length) and $Z_0$ (the TL impedance) inside the object. We consider an object with $R_{obj} = 30d$, which is formed by 14 concentric rings of DNG cells. The inner radius of the smallest ring is $r = R_{WH} = 16d$, at which point the wormhole starts. In the DNG domain the region $R_{WH} < r < R_{obj}$ is filled with the cells whose electric parameters match to the parameters of the last ring. The field distribution for the optimized ring structure is depicted in Figure 2 (e).

3. Conclusions

In this work, physical modeling of the superabsorption effect in the conjugate impedance matched metamaterials has been considered. The effect has been modeled with 2D TL-based metamaterial structures. We have shown that, in this model, a finite-size conjugate impedance matched superabsorbing object can be equivalently represented with the wormhole structure formed by two electrically connected DNG and DPS domains. We have also found that the superabsorption effect can be observed in nonuniform structures comprising fewer DNG cells. Namely, such an effect can be observed for a metamaterial ring object that fits entirely within a region of finite radius $r < R_{obj}$, while $\beta_0 R_{obj} \gg 1$. This is especially interesting for applications of the metamaterial superabsorbers as efficient harvesters of electromagnetic radiation which absorb more energy from an incoming wave than what is incident directly on their surface.

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