Wettability Effect on Terminal Velocity of a Single Solid Sphere Falling in a Vertical Pipe

Takuya BATO,1) Tomoya NAKAJIMA,2) Yoshiaki UEDA,3) and Manabu IGUCHI4)*

1) Formerly Faculty of Engineering, Osaka Electro-Communication University. Now at Komei MFG Co. Ltd., 3-10-18 Ikuno-higashi, Ikuno-ku, Osaka, 544-0025 Japan.
2) Institute of Science Research, Osaka Prefecture University, 1-1 Gakuen-cho, Naka-ku, Sakai, Osaka, 599-8531 Japan.
3) Faculty of Science and Engineering, Setsuman University, 17-8 Ikeda-naka-cho, Neyagawa, Osaka, 572-8508 Japan.
4) Faculty of Engineering, Osaka Electro-Communication University, 18-8 Hatsu-cho, Neyagawa, Osaka, 572-8530 Japan.

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Water model experiments were carried out to understand the dynamic behavior of refining agents in a reactor pipe of a continuous refining process. The terminal velocity of a poorly-wetted single solid sphere falling in a vertical pipe was measured as a function of the diameter ratio, \( \lambda = d_s/D \), where \( d_s \) is the sphere diameter and \( D \) is the pipe diameter. The particle Reynolds number, \( Re_p \), was chosen to be greater than 1,000. The results were compared with those for a wetted single sphere to make clear the wettability effect on the terminal velocity.

KEY WORDS: steelmaking; continuous refining; desulfurization; wettability; water model; sphere; drag coefficient.

1. Introduction

The current desulphurization processes are classified into two butch types:1)3) the KR process and injection process. As another process, a continuous refining process using a static mixer is mentioned in this study. The static mixer consists of a circular pipe and some static blades attached to the pipe wall.4) Molten metal and refining agents are introduced into the mixer due to potential energy and mixed passing through the pipe. In this process the dynamic behavior of refining agents is significantly responsible for the refining efficiency, as suggested from the butch types. The refining agents are usually poorly wetted by molten metal. Each poorly-wetted agent is partly covered with the atmospheric gas during its penetration into the bath, while a wetted agent is directly contact with the molten metal. 

The moving velocity characterizing the dynamic behavior of the agents therefore is considered to be affected by the wettability of the agents. Unfortunately, a wettability effect on the velocity is not fully understood yet. Here, the wettability is evaluated in terms of the equilibrium contact angle, \( \theta \). A solid plate is wetted by a liquid droplet for \( 0^\circ \leq \theta < 90^\circ \), while it is poorly wetted for \( 90^\circ \leq \theta \leq 180^\circ \).

As a first step, investigation is carried out in this water model study on the terminal velocity of poorly-wetted refining agents in a vertical pipe. Although the agents are not spherical in shape, they are modeled by spheres of different diameters. Fujita and Watanabe5) focused on a poorly-wetted single sphere moving in a fluid unbounded by vessel walls and concluded on the basis of their own experimental results that the drag coefficient, \( C_D \), becomes smaller than that of a wetted sphere, \( C_{Dg} \), for a particle Reynolds number of \( Re_p \leq 1 \, 000 \) and agrees well with that of a poorly-wetted sphere, \( C_{Dg} \), for a particle Reynolds number of \( Re_p > 1 \, 000 \). The difference between \( C_D \) and \( C_{Dg} \) for \( Re_p \leq 1 \, 000 \) increased with a decrease in \( Re_p \). In other words, the terminal velocity of a poorly-wetted sphere increased compared to that of a wetted sphere for \( Re_p \leq 1 \, 000 \). This tendency can be explained by the fact that the flow separation point shifts towards the backward stagnation point. That is, the wake behind a poorly-wetted sphere becomes smaller than that behind a wetted sphere for \( Re_p \leq 1 \, 000 \).

In this study, the wall effect on the terminal velocity of a poorly-wetted sphere falling in a vertical pipe filled with water was experimentally investigated (see Fig. 1). The Reynolds number range was chosen to be greater than 1,000 because the wettability effect on an unbounded sphere diminishes in this Reynolds number range.6) The wall effect is expected to clearly appear.

2. Estimation Method of Terminal Velocity of a Bounded Sphere

2.1. Empirical Equations Proposed by Hosokawa et al. for a Wetted Sphere

Hosokawa et al.8) classified the falling types of a single wetted sphere in a vertical pipe into rectilinear and rocking types (see, for example, Fig. 2 in run 1 explained later) and proposed an empirical equation of terminal velocity, \( v_t \) [m/s], for each type as:

(1) Rectilinear motion (A sphere descends straight downwards.)

\[
\frac{v_t}{v_{t0}} = \frac{1 - \phi}{\sqrt{1 + 0.16\phi + 1.25\phi^2}} \quad \text{(1)}
\]

(2) Rocking motion (A sphere swings in the lateral direction.)

\[
\frac{v_t}{v_{t0}} = \frac{1 - \phi}{\sqrt{1 + 4.87\phi + 0.81\phi^2}} \quad \text{(2)}
\]

![Fig. 1. A single solid sphere falling in a vertical pipe filled with water.](image-url)
2.2. Estimation Method of Terminal Velocity, \( v_{\infty} \), of an Unbounded Sphere

According to Chemical Engineering Handbook,\(^4\) the terminal velocity, \( v_{\infty} \), of an unbounded single solid sphere of good wettability is given by the following empirical equations.

(1) For \( Re_{\infty} < 1 \)

\[
v_{\infty} = \frac{C_s (\rho_p - \rho_l) d_p^2}{18 \mu_L} \]  

(4)

(2) For \( 1 \leq Re_{\infty} < 10^4 \)

\[
Re_{\infty} = \frac{v_{\infty} d_p}{\nu_L} \]  

(5)

\[
v_{\infty} = \left( \frac{A_1^2 + A_2 - A_1}{1.1} \right)^2 \]  

(6)

\[
A_1 = 4.8 \sqrt{\frac{\mu_L}{(\rho_p \nu_s)}} \]  

(7)

\[
A_2 = 2.54 \sqrt{\frac{\rho_p (\rho_p - \rho_l) g d_p C_s (\rho_p \nu_s)}{\nu_s}} \]  

(8)

where \( C_s \) [-] is Cunningham’s slip correction factor and regarded as unity for liquids, \( \rho_p \) [kg/m\(^3\)] is the particle density, \( \rho_l \) [kg/m\(^3\)] is the liquid density, \( \mu_L \) [Pa·s] is the dynamic viscosity of liquid, \( Re_{\infty} \) is the particle Reynolds number, \( \nu_L \) [m\(^2\)/s] is the kinematic viscosity of liquid, and \( g \) [m/s\(^2\)] is the acceleration due to gravity.

Equations (4) and (6) explicitly give \( v_{\infty} \) for a wide particle Reynolds number range. This is the main reason why these equations were chosen here among many existing equations.

3. Experimental Apparatus and Procedure

Seven combinations of the sphere diameter, \( d_p \), and pipe diameter, \( D \), were chosen, as listed in Table 1 together with the values of \( v_{\infty} \) estimated from Eq. (6). The contact angle of an acrylic resin sphere was 71°. It was changed to 143° due to repellant coating, being the same value as that chosen by Fujita and Watanabe.\(^6\) A single sphere was initially placed just above the water surface in contact with it and then dropped. The falling velocity of the sphere increased monotonically from zero to a certain constant value called the terminal velocity, \( v_t \). The region where the falling velocity increases is called the entrance region. Preliminary experiments revealed that the entrance length is less than 450 [mm] under the experimental conditions listed in Table 1. The test section was divided into three sub-sections of 500 [mm] in length: the upper, middle, and lower ones. These sub-sections are designated as (U), (M), and (L), respectively in Fig. 1.

The time required for a sphere to pass through each sub-section was measured to determine the terminal velocity. Measurements were repeated more than 12 times at every sub-section. Accordingly, a mean terminal velocity was obtained by averaging more than 36 data under every experimental condition. The interval between the successive measurements was longer than 3 [min] to settle the disturbed water flow in the pipe.

4. Experimental Results and Discussion

4.1. Comparison of Measured with Estimated Terminal Velocities of a Single Wetted Sphere in a Vertical Pipe

In order to confirm the adequacy of the measurement method, the terminal velocity values measured in this study were compared with the previous empirical equations, (1) and (2). Table 2 lists the measured and estimated terminal velocities of a single wetted sphere in a vertical pipe. Both the rectilinear and rocking motions were observed in this study (see Fig. 2). The rectilinear motion appeared for a relatively small diameter ratio, while the rocking motion occurred for a relatively large one. The measured and estimated values agreed within a scatter of \(-9\%\) to \(+19\%\).

Equations (1) and (2) are known to approximate previous measured values within a scatter of \(\pm 20\%\) and, hence, the present measurement method is considered to be adequate.

4.2. Comparison of Measured Terminal Velocities between Wetted and Poorly-wetted Spheres

The wettability effect on the terminal velocity is visible in Table 3 and Fig. 3. It should be noted that both the rectilinear and rocking types appeared in run 1. There is no difference between the terminal velocities of wetted and poorly-wetted spheres for a diameter ratio, \( \lambda \), smaller than about 0.75 and a particle Reynolds number greater than 1 000 (runs 1, 2, 3, 5, and 6). When the particle Reynolds number, \( Re_{\infty} \) (= \( \pi d_p v_t \)), is greater than about 1 000, the hydrodynamic drag acting on an unbounded single sphere is mainly governed by the form drag resulting from the vortex
The terminal velocity was estimated from Eq. (6). According to a previous paper of the present authors5), the thickness of the air layer is about 0.80 (see runs 4 and 7). The surface of a poorly-wetted sphere is covered with a thin air layer, as mentioned in the preceding section. In this run, a sphere seems to periodically contact the pipe wall in the course of falling in the pipe. There arises the possibility that a wetted sphere is periodically in direct contact with the pipe wall, while a poorly-wetted sphere contacts the pipe wall via the thin air layer. This difference may cause a difference between the terminal velocities of the two types of spheres. Further experimental investigation is, of course, desirable to fully understand the mechanism of a decrease in the terminal velocity of a poorly-wetted sphere.

In run 4, the particle Reynolds number, Rep, is much smaller than 1 000. Accordingly, the wettability of a sphere and the pipe wall exert their effects on the terminal velocity in a complex manner. Discussion on the terminal velocity for Rep<1 000 must be left for a future study.

### Table 1. Estimated terminal velocity of an unbounded single sphere of good wettability.

| Run | D (×10^-3 m) | d0 (×10^-3 m) | λ (−) | φ (−) | v∞ (est.) (m/s)* | Repp (−) |
|-----|--------------|---------------|-------|-------|-----------------|----------|
| 1   | 17           | 10            | 0.588 | 0.346 | 0.252           | 2.550    |
| 2   | 20           | 10            | 0.500 | 0.250 | 0.252           | 2.500    |
| 3   | 25           | 10            | 0.400 | 0.160 | 0.253           | 2.620    |
| 4   | 17           | 15            | 0.882 | 0.778 | 0.323           | 5.050    |
| 5   | 20           | 15            | 0.750 | 0.563 | 0.323           | 5.130    |
| 6   | 25           | 15            | 0.600 | 0.360 | 0.323           | 5.210    |
| 7   | 25           | 20            | 0.800 | 0.640 | 0.382           | 8.060    |

*: The terminal velocity was estimated from Eq. (6).

### Table 2. Measured and estimated values of the terminal velocity of a bounded single sphere of good wettability.

| Run | Type | v∞ (m/s) (mea.) | v∞ (est.) (m/s)* | v∞ (mea.)/v∞ (est.) (%) | Repp (−) |
|-----|------|-----------------|-----------------|------------------------|----------|
| 1-1 | Rec. | 0.138           | 0.150           | 92                     | 1.041    |
| 1-2 | Rock.* | 0.107           | 0.0987          | 108                    | –        |
| 2   | Rec. | 0.163           | 0.179           | 91                     | 0.162    |
| 3   | Rec. | 0.197           | 0.207           | 95                     | 0.184    |
| 4   | Rock. | 0.0338          | 0.0312          | 108                    | –        |
| 5   | Rock. | 0.0713          | 0.0706          | 101                    | –        |
| 6   | Rock. | 0.144           | 0.122           | 118                    | –        |
| 7   | Rock. | 0.0779          | 0.0652          | 119                    | –        |

*: Estimated from Eq. (1) proposed by Hosokawa et al.6)

**: Estimated from an empirical equation proposed by De-Felice7) (not shown)

*: The rocking type was sometimes observed.

### Table 3. Comparison of terminal velocities between wetted and poorly-wetted spheres.

| Run | Type | v∞ (m/s) (mea.) | v∞ (est.) (m/s) | v∞ (mea.)/v∞ (est.) (%) | Repp (−) | Repp (−) |
|-----|------|-----------------|-----------------|------------------------|----------|----------|
| 1-1 | Rec. | 0.138           | 0.136           | 99                     | 1.400    | 1.380    |
| 1-2 | Rock. | 0.107           | 0.108           | 101                    | 1.080    | 1.090    |
| 2   | Rec. | 0.163           | 0.168           | 103                    | 1.620    | 1.670    |
| 3   | Rec. | 0.197           | 0.193           | 98                     | 2.040    | 2.000    |
| 4   | Rock. | 0.0338          | 0.0306          | 91                     | 530      | 478      |
| 5   | Rock. | 0.0713          | 0.0720          | 101                    | 1.130    | 1.140    |
| 6   | Rock. | 0.144           | 0.146           | 101                    | 2.320    | 2.350    |
| 7   | Rock. | 0.0779          | 0.0697          | 89                     | 1.640    | 1.470    |

(Repp = v∞dp/ν, Repp = v∞dp/ν)

formation behind it. In other words, the friction drag acting on the surface of the sphere is negligibly small compared to the form drag. Accordingly, the total drag is independent of whether the surface of the sphere is wetted or not. This is the main reason why v∞ is independent of the wettability for Rep > 1 000 and λ < 0.75.

A slight difference can be seen between the wetted and poorly-wetted spheres for a diameter ratio, λ, greater than about 0.80 (see runs 4 and 7). The surface of a poorly-wetted sphere is covered with a thin air layer, as mentioned in the preceding section. According to a previous paper of the present authors3) the thickness of the air layer is about 20 [μm]. As the diameter ratio λ increases, the rocking motion of a sphere prevailed. In run 7 both the particle Reynolds numbers, Rep and Rep, were much greater than 1 000 and v∞ was smaller than v∞. Here, the subscripts g and p in v∞ and v∞ denote wetted and poorly-wetted spheres, respectively. The measured values of the terminal velocity of a poorly-wetted sphere significantly scattered compared to that of a wetted sphere. In this run, a sphere seems to periodically contact the pipe wall in the course of falling in the pipe. This difference may cause a difference between the terminal velocities of the two types of spheres.

5. Conclusions

(1) The wettability effect is negligibly small for a sphere Reynolds number, Rep, greater than approximately 1 000 and a diameter ratio, λ, smaller than about 0.75. This is because the contribution of the frictional force on the total drag force of a sphere is negligibly small under this condition.

(2) As the diameter ratio, λ, exceeds about 0.8 and approaches unity, the rocking motion of a sphere prevails. The terminal velocity of a poorly-wetted sphere for Rep>1 000 becomes smaller than that of a wetted sphere.

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