Fixed-Time Distributed Secondary Control for Islanded Microgrids With Mobile Emergency Resources Over Switching Communication Topologies

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ABSTRACT Mobile emergency resources (MERs) are critical to the resilience of distribution systems for an emergency response to natural disasters. However, after disasters, the communication network of MERs may be unreliable. For example, the communication topology switches in different modes randomly. The conventional centralized control algorithms may not converge. As a result, the instability of frequency and voltage happened. To alleviate the impacts of the unreliable communication network on the second control performance, this paper regards the distribution system after disasters as multiple microgrids. A distributed secondary control algorithm is designed to regulate frequency and voltage in islanded microgrids over switching communication topologies. The algorithm is guaranteed to converge in a fixed time. Case studies are carried out to demonstrate the effectiveness and robustness of the proposed control algorithm under switching communication topologies.

INDEX TERMS Mobile emergency resources (MERs), resilience, switching topologies, distributed control.

I. INTRODUCTION Mobile emergency resources (MERs), resilience, switching topologies, distributed control.

I. INTRODUCTION Natural disasters and the subsequent disruptive events have caused severe power outages in recent years. For instance, Hurricane Sandy struck the East Coast of America and lead to almost 8.35 million customers without power [1]. This outage introduces tremendous economic loss and significant life risk, which reminds us of the significance to enhance power system resilience, especially for the microgrid-based distribution systems [2]–[4]. Such distribution systems can accommodate different types of distributed energy resources (DERs), including mobile emergency generators (MEGs).

Droop control is widely used to collaboratively manage the DERs. It provides active damping to power systems, whereas it may lead to frequency and voltage deviations when a disturbance occurs. The system frequency depends on the disturbance location and type. When any disturbances occur in an islanded microgrid, system frequency fluctuates among DERs. Such fluctuation impairs the power-sharing among DERs and deteriorates the second control performance which operates in a decentralized manner [5]. To overcome this problem, a master controller is proposed to guide the secondary control in a centralized manner. The master controller needs to collect all the local measurements together [6]. Such centralized schemes, however, bring urgent concerns in high reliable communication networks and computation burden. During natural disasters, the communication network may be broken. It cannot guarantee reliable information update. The centralized control scheme does work in practice.

Many efforts have been made on the distributed secondary control to alleviate the unreliable communication network. Examples include the controller design without requirement on the information of the microgrid topology [7], the sliding mode observer-based controller [8], the event-triggered
controller [9], and the nonlinear controller [10]. These distributed secondary control schemes show the advantages of efficiently lowering the communication and computational burden while remaining the desired frequency and voltage performance. These control schemes, however, do not cope with the difficulties brought by the communication network after natural disasters.

The main hindrance of the communication network during natural disasters is unreliable communication quality, which will impair the DERs to work cooperatively. The system frequency and voltage performance may be degraded. For instance, before Hurricane Sandy struck, four hundred mobile emergency response generators were prepared. However, only a few of them were providing power. One of the reasons is that many communication links were broken down and the wireless communication network becomes unstable [1]. The communication topologies may switch randomly. As a result, the MERs cannot arrive at the designated location to restore the critical load in time. Many works have stressed switching topologies. Examples includes [11]–[15]. Lin et.al proposed controllers based on Lyapunov theory [11]. Dou et.al presented multi-agent based distributed cooperative controllers [12]. However, these controllers suffer from an unknown convergence time. The goal of the secondary controller is to restore frequency and voltage as fast as possible. While in the switching topologies, the distributed secondary controller requires a long time to achieve asymptotic stability convergence. It could not guarantee the fast response performance in this scenario. In another word, compare with asymptotic stability convergence, it is more practical to do some research on the fixed-time control performance.

This paper designs a fixed-time distributed secondary control law for microgrids under switching topologies. The control objective is to restore the frequency and voltage in a fixed time. Compared with certain scenarios with ideal communication conditions, the uncertainty of switching communication topologies makes the controller design more complicated.

The contributions of this paper are

1) Consider the unreliable communication network after disasters, a fixed-time distributed controller is proposed to restore the frequency and voltage of each distributed generator to a reference level.

2) Estimate the convergence time accurately. To achieve better transient performance, the control gains can be adjusted based on the convergence time.

The remainder of this paper is organized as follows. Section II introduces the distributed secondary control problem and describes the preliminaries of graph theory. Section III provides the design of the proposed distributed frequency secondary control law. The guaranteed convergence time is proved to be independent of initial states. Case studies are presented in Section IV to validate the effectiveness of the proposed control law under different operating conditions. Section V concludes this paper.

II. PROBLEM FORMULATION AND PRELIMINARIES

This section declares transportation problem for DG Allocation under the extreme condition, formulates the secondary control problem of islanded microgrids with mobile emergency generators, and introduces the necessary preliminaries on graph theory.

A. TRANSPORTATION PROBLEM FOR DG ALLOCATION

Generally, distribution system after natural disasters can be regarded as a series of islanded microgrids, which is without complete power access to the main grid [16]–[18]. For enhancing the resilience of distribution systems, the mobile emergency generators (MEGs) can be dispatched as distributed generators.

The main hindrance of the real-time allocation of MEGs is the traffic congestion and road damages made by extreme disasters. For instance, in 2012, before Hurricane Sandy coming, the Federal Emergence Management Agency (FEMA) prepared 400 industrial-size truck-mounted MEGs. Nevertheless, only a fraction of MEGs were providing power three days after Hurricane Sandy made landfall [19]. Several scholars propose solutions [4], [16], [20]–[22], which can be summarized as follows:

1) Lei et.al. address that MEGs can be pre-positioned in ideal locations before the natural disasters coming, which is greatly helpful for real-time allocation after the natural disaster strikes [4] [16]. Even though the remote MEGs may be trapped in traffic congestion, the pre-positioned MEGs still can make effective response for electric service restoration.

2) There are many paths to the desired locations and the desired locations are multiple. The dispatched MEGs can optimize their paths and possible locations, which can minimize the travel time and avoid trapping on the traffic flow [20]–[22].

Hence, this paper supposes that the MEGs can arrive the desired locations in time. We concentrate on the secondary control of microgrids rather than the allocation of the MEGs.

B. SECONDARY CONTROL OF MICROGRIDS

Consider an AC islanded microgrid with n controllable inverter-based distributed generators (DGs), including MEGs. The controllable DGs are equipped with distributed controllers and participate in microgrid frequency and voltage regulation to keep the balance between the supply and demand. The uncontrollable one obeys the grid-following mode and they provide the maximum power output.

For the purpose of balancing active and reactive power demands in microgrids, droop control is widely utilized by instantaneously changing the frequency and voltage amplitude. The classic droop control loop are formulated as follows:

\[ \omega_i(t) = \omega_i^*(t) - m_i P_i(t), \]

\[ V_{mi}(t) = V_i^*(t) - \kappa_i Q_i(t), \]
where \( \omega_i(t) \) is the angular frequency of DG \( i \); \( \omega_i^e(t) \) is the primary control reference value of the angular frequency of DG \( i \); \( m_i \) is the frequency droop coefficient of DG \( i \); \( P_i(t) \) is the measured active power of DG \( i \); \( V_m(t) \) is the voltage amplitude of DG \( i \); \( V_i^m(t) \) is the primary control reference value of the voltage output of DG \( i \); \( k_i \) is the voltage droop coefficient of DG \( i \); \( Q_i(t) \) is the measured reactive power of DG \( i \). The frequency secondary control aims to design \( \omega_i^e(t) \) satisfying that \( \omega_i(t) \) is synchronized to a fixed reference value \( \omega_{ref} \).

The primary voltage control strategy given in [23] yields that

\[
V_{di}(t) = V_i^m(t) - k_i Q_i(t), \quad (3) \\
V_{qi}(t) = 0, \quad (4)
\]

where \( V_{di}(t) \) and \( V_{qi}(t) \) voltage setpoints if the DG \( i \) at time \( t \), satisfying that \( V_m(t) \) is the sum of the squares of \( V_{di}(t) \) and \( V_{qi}(t) \). Then we can rewrite (2) as follows:

\[
V_{di}(t) = V_i^m(t) - k_i Q_i(t). \quad (5)
\]

This implies that the voltage secondary control aims to design \( V_i^m(t) \) such that \( V_{di}(t) \) can follow the reference value \( V_{ref} \).

This paper presents a fixed-time distributed secondary control scheme to compensate the frequency and voltage deviations resulted from the primary control. Note that the reactive power sharing accuracy is not strictly required owing to the capacitive compensations implemented by load and lines, the control objectives of this paper are simplified as follows:

1) Eliminate the frequency and voltage deviations within a fixed time \( T_f \), that is,

\[
\lim_{t \to T_f} \omega_i(t) = \omega_i^*, \quad \forall i \in N, \quad (6) \\
\lim_{t \to T_f} V_i(t) \in V_i^*, \quad \forall i \in N. \quad (7)
\]

2) Accomplish the accurate active power sharing, that is,

\[
\lim_{t \to T_f} \left| m_i P_i(t) - m_j P_j(t) \right| = 0, \quad \forall i, j \in N. \quad (8)
\]

C. PRELIMINARIES ON GRAPH THEORY

An islanded microgrid is a typical cyber-physical system and the DGs in the microgrid can contact with their peers through a communication network. Undirected graphs \( G_n(V, \mathcal{E}_n, \mathcal{A}_n) \) are used to declare random switching communication topologies. Suppose that there are \( N \) topologies, and each of them includes \( n \) DGs. Assume that the stochastic process is governed by the signal \( \sigma_i \), \( V = \{v_1, v_2, \ldots, v_n\} \) represents the node set, and \( \mathcal{E}_n \subseteq V \times V \) is the edge set. The adjacency matrix \( \mathcal{A}_n = [a_{ij}(\sigma_i)]_{i,j \in N} \) satisfies \( a_{ii}(\sigma_i) = 0 \) and \( a_{ij}(\sigma_i) > 0 \) if \( (v_i, v_j) \in \mathcal{E}_n \) and \( a_{ij}(\sigma_i) = 0 \) otherwise. The Laplacian matrix \( \mathcal{L}_n = \{L_{ij}(\sigma_i)\}_{i,j \in N} \subseteq \mathbb{R}^{n \times n} \), which contains global interaction information, is defined as \( L_{ij}(\sigma_i) = \sum_{k=1}^n a_{ik}(\sigma_i) \) and \( L_{ii}(\sigma_i) = -\sum_{j \neq i} a_{ij}(\sigma_i) \), \( i \neq j \). If there is one path from any node to the rest of the nodes, this undirected graph is said to be connected [24].

Notice that the frequency and voltage reference level \( \omega_{ref} \) and \( V_{ref} \) can be obtained by several DGs, we can define a virtual node which sends the reference information to some DGs. A diagonal matrix \( B_{\sigma_i} = \text{diag}(b_{n}(\sigma_i)) \subseteq \mathbb{R}^{n \times n} \) is defined to represent the information flow. \( b_{n}(\sigma_i) > 0 \) if DG \( i \) receives the reference value at topology \( \sigma_i \) at time \( t \) and \( b_{n}(\sigma_i) = 0 \) otherwise. The graph including the virtual node is denoted as \( G_n \).

III. DISTRIBUTED FIXED-TIME CONTROL LAW

This section proposes the distributed fixed-time secondary control law and analyses the convergence time of the presented control law.

A. FIXED-TIME FREQUENCY RESTORATION

For frequency and voltage secondary control, differentiating (1) and (2) yields

\[
\dot{\omega}_i(t) = \dot{\omega}_i^e(t) - m_i \dot{P}_i(t) \triangleq u_{col}(t), \quad (9) \\
\dot{V}_d(t) = \dot{V}_i^m(t) - k_i \dot{Q}_i(t) \triangleq u_{vol}(t). \quad (10)
\]

where \( u_{col}(t) \) and \( u_{vol}(t) \) are the control inputs of frequency and voltage regulation, respectively. Besides, let

\[
m_i \dot{P}_i(t) \triangleq u_{pp}(t), \quad (11)
\]

where \( u_{pp}(t) \) is the active power control input. The inputs \( u_{col}(t), u_{vol}(t) \) and \( u_{pp}(t) \) represents the changing rate of the frequency, the changing rate of the voltage, and the changing rate of the active power of DG \( i \), respectively.

Then, we can obtain \( \omega_i^*(t) \) and \( V_i^*(t) \) as follows:

\[
\omega_i^*(t) = \int (u_{col}(t) + u_{pp}(t))dt, \quad (12) \\
V_i^*(t) = \int (k_i \dot{Q}_i(t) + u_{vol}(t))dt. \quad (13)
\]

Based on the distributed finite-time consensus control law presented in [25], [26] and [27], we propose the following controllers:

\[
u_{col}(t) = -\alpha \sum_{j \in N} a_{ij}(\sigma_i) \cdot \text{sign}(\omega_j(t) - \omega_i(t))^{\gamma}, \quad (14) \\
u_{pp}(t) = -\beta \sum_{j \in N} a_{ij}^{\gamma} \cdot \text{sign}(m_j P_j(t) - m_i P_i(t))^{2-\gamma}, \quad (15)
\]

where \( a \) and \( b \) are positive odd integers such that \( a < b \), \( \alpha \), \( \beta \), and \( \gamma \) are given positive control gains, \( \text{sign}(x) \) represents \( \text{sign}(x) \times |x|^p \) and \( \text{sign}(x) \) is the signum function.

Define \( e_{col}(t) \triangleq [e_{col1}(t), e_{col2}(t), \ldots, e_{coln}(t)]^T \) and \( e_{vol} \triangleq \omega_i(t) - \omega_{ref} \) for \( \forall i \in N \). By differentiating \( e_{col}(t) \) along the trajectory of (9), we obtain

\[
\dot{e}_{col}(t) = \dot{\omega}_i(t) = -\alpha \sum_{j \in N} a_{ij}(\sigma_i) \cdot \text{sign}(e_{col}(t) - e_{colj}(t))^{\gamma} \\
-\alpha b_i(\sigma_i) \cdot \text{sign}(e_{col}(t))^{\gamma}, \quad (16)
\]
Conduct a Lyapunov function as follows:
\[ W_1(t) = \frac{1}{2} e_{\omega}^T(t)e_{\omega}(t). \] (17)

Define some new matrices associated with the graph \( \mathcal{G}_\sigma \), as follows:
\[ \tilde{A}_{\sigma} \triangleq [a_{ij}^{\sigma}]_{i,j \in \mathcal{N}}, \]
\[ \tilde{B}_{\sigma} \triangleq \text{diag}\left\{ b_{11}^{\sigma}(\sigma_t), b_{22}^{\sigma}(\sigma_t), \cdots, b_{nn}^{\sigma}(\sigma_t) \right\}, \]
\[ \tilde{D}_{\sigma} \triangleq \tilde{\mathcal{L}}_{\sigma} + \tilde{B}_{\sigma}, \]
where \( \tilde{A}_{\sigma} \) is the adjacency matrix and \( \tilde{\mathcal{L}}_{\sigma} \) is the Laplacian matrix associated with the graph \( \mathcal{G}_\sigma \). Let \( \lambda_{\min}(\tilde{D}) \) be the smallest eigenvalues of matrix \( \tilde{D}_{\sigma} \); that is, \( \min_\sigma \{ \lambda_{\min}(\tilde{D}_{\sigma}) \} \).

Similarly, define some new matrices associated with the graph \( \mathcal{G}_\sigma \), as follows:
\[ \tilde{A}_{\sigma} \triangleq [a_{ij}^{\sigma}]_{i,j \in \mathcal{N}}, \]
\[ \tilde{B}_{\sigma} \triangleq \text{diag}\left\{ b_{11}^{\sigma}(\sigma_t), b_{22}^{\sigma}(\sigma_t), \cdots, b_{nn}^{\sigma}(\sigma_t) \right\}, \]
\[ \tilde{D}_{\sigma} \triangleq \tilde{\mathcal{L}}_{\sigma} + \tilde{B}_{\sigma}, \]
where \( \tilde{A}_{\sigma} \) is the adjacency matrix and \( \tilde{\mathcal{L}}_{\sigma} \) is the Laplacian matrix associated with the graph \( \mathcal{G}_\sigma \). Let \( \lambda_{\min}(\tilde{D}) \) be the smallest eigenvalues of matrix \( \tilde{D}_{\sigma} \).

For simplicity, we introduce the theorems directly.

**Theorem 1**: Suppose each one of the switching topologies \( \mathcal{G}_\sigma \), is connected and the reference values of frequency and voltage are available at least one DG. The system (9) (11) can achieve finite-time frequency restoration and active power restoration under the distributed control law (12, 13). The upper bounds of the settling time can be estimated as follows:
\[ T_\omega = \frac{2b-a}{\pi} bW^{b-a}_1(0) \alpha(b-a)\lambda_{\min}(\tilde{D}), \] (18)
\[ T_P = \frac{\pi bn^{b-a}W^{b-a}_3(0)}{2(b-a)\lambda_2^{b-a}(\tilde{\mathcal{L}})\lambda_2^{b-a}(\tilde{D})\sqrt{\beta}y}. \] (19)

**Proof**: See Appendix A. \( \square \)

**Remark 1**: To deal with the effects caused from randomly switching topologies, the error state \( e_\omega(t) \) is introduced and the associated Lyapunov function is conducted. Based on Lyapunov theory, the sufficient condition of the fixed-time frequency restoration and active power sharing is proposed. Furthermore, the converge time is estimated accurately. According to the estimated convergence time (18) and (19), the control weights \( a \) and \( b \) can be adjusted to achieve robust performance under randomly switching communication topologies.

**B. FIXED-TIME VOLTAGE RESTORATION**

Based on the similar consensus control form given in [25], [26], we propose the following voltage secondary control law:
\[ u_V(t) = \gamma \sum_{j \in \mathcal{N}} a_{ij}(\sigma_t) \text{sgn}(V_{di}(t) - V_{dj}(t))^a_{ij} - \varepsilon b(\sigma_t) \cdot \text{sgn}(V_{di}(t) - V_{ref})^a_{ij}, \] (20)
where \( \varepsilon \) is a given positive scalar.

Let \( e_V(t) = [e_{V_1}(t), e_{V_2}(t), \cdots, e_{V_N}(t)]^T \) with \( e_{V_i}(t) \) such that \( e_{V_i}(t) = V_{di}(t) - V_{ref} \forall i \in \mathcal{N} \). Given a quadratic Lyapunov function as follows:
\[ W_3(t) = \frac{1}{2} e_V^T(t)e_V(t). \] (21)

Then, we jump to the conclusion as follows:

**Theorem 2**: Suppose each one of the switching topologies \( \mathcal{G}_\sigma \), is connected and the reference values voltage are available at least one DG. The system (10) can achieve finite-time voltage restoration the distributed control law (21). The upper bounds of the settling time can be estimated as follows:
\[ T_V = \frac{2b-a}{\pi} bW^{b-a}_3(0) \varepsilon(b-a)\lambda_{\min}(\tilde{D}), \] (22)

**Proof**: See Appendix B. \( \square \)

**Remark 2**: The eigenvalues of topology matrices \( \tilde{\mathcal{L}} \) and \( \tilde{D} \) contains the information of randomly switching communication topologies. Theorems 1 and 2 can explicitly characterize the effects of randomly switching communication topologies on the convergence time of the proposed control laws. It is noted that all the upper bounds of convergence time becomes smaller when the eigenvalue \( \lambda_{\min}(\tilde{D}) \) and \( \lambda_2(\tilde{D}) \) increases.

**C. DESIGN FLOW OF THE PROPOSED METHOD**

The flow chart of the proposed method is presented in FIGURE 1. It can be seen from Figure 1 that the control...
structure contains two parts. The first part is about the frequency and active power control and the second one is about voltage control. Consider the frequency and the active power model (1) and the control laws (14) (15). The control gains $\alpha$, $\beta$ can be determined based on the convergence time (18) and (19). Consider the frequency and the active power model (2) and the control laws (20). The control gains $\gamma$ can be determined based on the convergence time (22). It is found that when the values of $\alpha$, $\beta$, and $\gamma$ increase, the frequency, active power, and voltage can reach their steady-state faster.

### IV. CASE STUDIES
This section gives case studies on a modified 34-bus system introduced in [23]. The modified system is illustrated in FIGURE 2. It is obvious from FIGURE 2 that this system has five DGs and three loads. The communication topology of the five distributed emergency resources (DERs) is switching in three pattern, as shown in FIGURE 3. Intuitively, the frequency and voltage reference value can be obtained by DERs 2 and 5. The case studies are carried out in MATLAB/PSAT environment.

Let $V_{ref} = 380$ V and $\omega_{ref} = 50$ Hz. Set the droop scalars as $m_1 = m_5 = 4.2 \times 10^{-5}$, and $m_2 = m_3 = m_4 = 8.4 \times 10^{-5}$. For simplicity, other parameters are omitted here, which can be seen in [23]. Let $t = 0$ second be the time when secondary control begins. The initial values of system states and controller scalars are given in TABLE 1.

#### TABLE 1. Initial Values of System States and Controller Parameters.

| $\omega_1(0)$ | $\omega_2(0)$ | $\omega_3(0)$ | $\omega_4(0)$ | $\omega_5(0)$ |
|---------------|---------------|---------------|---------------|---------------|
| 49.5 Hz       | 49.6 Hz       | 49.7 Hz       | 49.8 Hz       | 49.9 Hz       |
| $P_1(0)$      | $P_2(0)$      | $P_3(0)$      | $P_4(0)$      | $P_5(0)$      |
| 50 kW         | 20 kW         | 30 kW         | 10 kW         | 50 kW         |
| $V_{G1}(0)$   | $V_{G2}(0)$   | $V_{G3}(0)$   | $V_{G4}(0)$   | $V_{G5}(0)$   |
| 376.9 V       | 377.4 V       | 373.8 V       | 374.3 V       | 379.4 V       |

| $\alpha$      | $\beta$      | $\gamma$     | $\epsilon$   |
|---------------|---------------|---------------|---------------|
| 4             | 0.4           | 8             | 4             |

First, we check the effectiveness of the proposed controllers (14) and (20) for frequency regulation and voltage restoration, respectively. For comparison, the method proposed in [7] is considered. The authors in [7] proposed distributed controllers for secondary frequency and voltage control in islanded microgrids. However, the convergence time did not be estimated and the control gains cannot be selected accurately. The proposed method in this manuscript considers the control gain design problem and gives the convergence time expression. For a fair comparison, the two control methods are applied in the same communication condition.

Letting $a = 10$ and $b = 20$. The communication topology is chosen as the first diagram shown in FIGURE 2. The frequency response and voltage response with our proposed method are illustrated in FIGURE 4 and that with the method proposed in [7] is illustrated in FIGURE 5. It can be seen from FIGURE 4 that the frequency arrives steady value in 1 second while in FIGURE 5, the settling time of frequency is about 2 seconds. Consider the settling time of voltage with
different control methods. Clearly, with the proposed control method, the voltage attains the reference value in 2.2 seconds, but it requires 4.5 seconds with the method [7]. Compared with the method in [7], our proposed control method shows better performance.

The system frequency and voltage performance with the \( \alpha = 10 \) and \( \beta = 20 \) is illustrate in FIGURE 6. The bottom panel of FIGURE 6 illustrate the switching communication topologies, which is numbered as 1, 2, and 3. The frequency deviation and the voltage deviation still can converge to zero in 3 seconds. This shows the robustness of the proposed control law.

Now we analyze the convergence time. According to Theorems 1 and 2, the frequency regulation and voltage restoration can converge to reference level in 10 seconds and 15 seconds, respectively. As shown in FIGUREs 4-6, the frequency and the voltage arrive the reference level in 5 seconds. Case studies verify the correctness of proposed theorems.

### B. PARAMETER ANALYSIS

The impacts of parameters \( \alpha, \beta, \gamma, a \) and \( b \) on the convergence time are analyzed.

Consider the frequency model (1) and the control law (14). The frequency response with different values of the control gain \( \alpha \) is shown in FIGURE 7. It is clear from FIGURE 7 that the settling time is shortened as the control gain \( \alpha \) increases. When \( \alpha = 2 \), the frequency settling time is larger than 2.5 seconds. When \( \alpha = 6 \), the frequency settling time is about 0.5 seconds. However, when \( \alpha = 8 \), more fluctuations exist in the steady-state of the frequency.

Consider the frequency model (1) and the control law (15). The active power response with different values of the control gain \( \beta \) is shown in FIGURE 8. It is clear from FIGURE 8 that the settling time is shortened as the control gain \( \beta \) increases. When \( \beta = 1.8 \), the active power settling time is about 2 seconds. When \( \beta = 2.6 \), the active power settling time is about 1.5 seconds.

Consider the frequency model (2) and the control law (20). The voltage response with different values of the control gain \( \gamma \) is shown in FIGURE 9. It is clear from FIGURE 9 that the settling time is shortened as the control gain \( \gamma \) increases. When \( \gamma = 10 \), the frequency settling time is larger than 0.02 seconds. When \( \gamma = 20 \), the frequency settling time is about 0.01 seconds. However, when \( \gamma = 25 \), more fluctuations exist in the steady-state of the voltage.

Consider the frequency response and voltage response of DG1 with different values of \( a \) and \( b \). As shown in FIGURE 10, when the value of \( \frac{a}{b} \) increases from 0.5 to 0.7, the settling time is shortened from 3 seconds to 1 second. However, when the value of \( \frac{a}{b} \) is larger than 0.7, more fluctuations happen in the frequency response and the voltage response. This implies that the better selection of the values of \( a \) and \( b \) is to guarantee the \( a, b \) such that: \( \frac{a}{b} = 0.7 \).

### C. PLUG-AND-PLAY CAPABILITY

The plug-and-play capability is important for a microgrid due to the intermittent nature of DGs. In other words, when some DGs are plugged into or out of the microgrid, the proposed controllers are still effective to make the whole system operate in a normal mode.
To address the plug-and-play concern, it is assumed that a “DER 6” is added to an suitable node of the microgrid at $t = 4$ s. Besides, it is considered that DER 6 is a neighbor of DER 4 for $4s \leq t \leq 8s$, as is shown in FIGURE 11. The frequency response and voltage response with our proposed method are illustrated in FIGURE 12 and that with the method proposed in [7] is illustrated in FIGURE 13. It can be seen from FIGURE 12 that the frequency arrives steady value in 1 second while in FIGURE 13, the settling time of frequency is about 2 seconds. Consider the settling time of voltage with different control methods. Clearly, with the proposed control method, the voltage attains the reference value in 2.2 seconds, but it requires 4.5 seconds with the method in [7]. Compared with the method in [7], our proposed control method shows better performance.

V. CONCLUSION

This paper considers the secondary control problem of the distribution system after disasters. The unreliable communication network is modeled as randomly switching topologies. A distributed fixed-time secondary control strategy is proposed to regulate the frequency and active power-sharing...
in the distribution system. The convergence time of the proposed algorithm is also been estimated. It is found that the guaranteed convergence time is only determined by control parameters and communication graphs regardless of initial microgrid system states. Thus, the desired control performance in terms of frequency restoration and accurate active power-sharing can be realized within a prescribed time regardless of initial system operation states. Case studies from a distribution system with five mobile emergency generators verify the effectiveness of the proposed design for better control performance. Simulation studies also demonstrate that the proposed control method has better performance than existing approaches.

APPENDIX

A. PROOF OF THEOREM 1

At time $t$, the time derivative of Lyapunov function $W(t)$ along the trajectory of (16) is as follows:

$$W_1(t) = -\alpha \sum_{i \in N} \left( \sum_{j \in N} a_{ij} e_{oij} \cdot \text{sign}(e_{oij} - e_{oij}) \right) + b_i e_{oij} \cdot \text{sign}(e_{oij})$$

$$= -\frac{\alpha}{2} \sum_{i \in N} \left( \sum_{j \in N} a_{ij} \cdot \text{sign}(e_{oij} - e_{oij}) \right) + 2b_i \cdot \text{sign}(e_{oij})$$

$$\leq -\frac{\alpha}{2} \left( \sum_{i \in N} \sum_{j \in N} a_{ij}^2 (e_{oij} - e_{oij})^2 + 2 \sum_{i \in N} b_i^2 e_{oij}^2 \right).$$

(23)

Define

$$\bar{F}(t) = \sum_{i,j \in N} a_{ij}^2 (e_{oij}(t) - e_{oij})^2$$

$$+ 2 \sum_{i \in N} b_i^2 (e_{oij})^2$$

$$= 2e_{oij}^2 (\tilde{D}_{oi} e_{oij}(t)).$$

Note that

$$\tilde{D}_{oi} \geq 2\lambda_{\text{min}}(\tilde{D}_{oi}) e_{oij}(t) e_{oij}(t) = 4\lambda_{\text{min}}(\tilde{D}_{oi}) e_{oij}^2(t) e_{oij}(t),$$

we have

$$W_1(t) \leq -\frac{\alpha}{2} \left( 4\lambda_{\text{min}}(\tilde{D}_{oi}) e_{oij}^2(t) e_{oij}(t) \right)^{\frac{a+b}{2b}}$$

$$= -\frac{2\alpha}{\lambda_{\text{min}}(\tilde{D}_{oi})} \left( \tilde{D}_{oi} \right) W_1^{\frac{a+b}{2b}}(t)$$

$$\leq -\frac{2\alpha}{\lambda_{\text{min}}(\tilde{D})} \left( \tilde{D} \right) W_1^{\frac{a+b}{2b}}(t).$$

(24)

Based on the lemma given in [25], the Lyapunov function $W_1(t)$ can converge to zero within a finite time $T_{\text{o}}$. So $\forall t > T_{\text{o}}$, $\exists \delta > 0$, $|e_{oij}(t)| < \delta$. Based on the definition of $e_{oij}$, namely, $e_{oij}(t) = \omega_i(t) - \omega_{\text{ref}}$, we have $\forall t > T_{\text{o}}$, $|\omega_i(t) - \omega_{\text{ref}}| < 0$. This means that frequency of each DG reach consensus value $\omega_{\text{ref}}$ in a finite-time $T_{\text{o}}$.

Next, we prove the distributed active power sharing is accomplished in finite time with switching communication topologies.

Similarly, let $e_p(t) = [e_{p1}(t), e_{p2}(t), \ldots, e_{pM}(t)]^T$, where $e_{p1}(t) = m_1 P_i(t) = \frac{1}{n} \sum_{i \in N} m_i P_i(t)$, for $\forall i \in N$. The time derivative of $e_p(t)$ along the trajectory of (10) is as follows:

$$\dot{e}_p(t) = -\beta \sum_{j \in N} a_{ij} (\sigma_i(t) - m_i P_i(t) + m_j P_j(t))^T$$

$$-\gamma \sum_{j \in N} a_{ij} (\sigma_i(t) - m_i P_i(t) + m_j P_j(t))^T$$

$$= -\beta \sum_{j \in N} a_{ij} (\sigma_i(t) - m_i P_i(t) + m_j P_j(t))^T$$

$$-\gamma \sum_{j \in N} a_{ij} (\sigma_i(t) - m_i P_i(t) + m_j P_j(t))^T.$$ 

(25)

Conduct a Lyapunov function as $W_2(t) = \frac{1}{2} e_{e_p}^T e(P)$. Then its time derivative along the trajectory is as follows:

$$W_2(t) = -\beta \sum_{i \in N, j \in N} a_{ij} e_{p1}(t) - e_{p2}(t)$$

$$-\gamma \sum_{i \in N, j \in N} a_{ij} e_{p1}(t) - e_{p2}(t)$$

$$= -\frac{\beta}{2} \sum_{i \in N, j \in N} \text{sign} \left( a_{ij}^2 (\sigma_i(t) - e_{p1}(t)) \right)^{\frac{a+b}{2b}}$$

$$-\frac{\gamma}{2} \sum_{i \in N, j \in N} \text{sign} \left( a_{ij}^2 (\sigma_i(t) - e_{p2}(t)) \right)^{\frac{3b-a}{2b}}$$

$$\leq -\frac{\beta}{2} \text{sign} \left( \sum_{i \in N, j \in N} a_{ij}^2 (\sigma_i(t) - e_{p1}(t)) \right)^{\frac{a+b}{2b}}$$

$$-\frac{\gamma}{2} \text{sign} \left( \sum_{i \in N, j \in N} a_{ij}^2 (\sigma_i(t) - e_{p2}(t)) \right)^{\frac{3b-a}{2b}}.$$

(26)

Define

$$F_1(t) = \sum_{i \in N, j \in N} a_{ij}^2 (\sigma_i(t) - e_{p1}(t))^2$$

$$= 2e_{p1}^T \tilde{D}_{oi} e_{p1}(t).$$

(27)

$$F_2(t) = \sum_{i \in N, j \in N} a_{ij}^2 (\sigma_i(t) - e_{p2}(t))^2$$

$$= 2e_{p2}^T \tilde{D}_{oi} e_{p2}(t).$$

(28)

Note that

$$F_1(t) \geq 2\lambda_2(\tilde{D}_{oi}) e_{p1}^2(t) e_{p1}(t),$$

$$F_2(t) \geq 2\lambda_2(\tilde{D}_{oi}) e_{p2}^2(t) e_{p2}(t).$$
we have
\[ F_1(t) \geq 4\lambda_2(\tilde{L}(t))W_2(0), \quad F_1(t) \geq 4\lambda_2(\tilde{L}(t))W_2(0). \]
Thus, it yields that
\[ \dot{W}_2(t) \leq -\frac{\beta}{2} \text{sgn}(4\lambda_2(\tilde{L}(t))W_2(t)), \]
\[ = -2\frac{\sigma}{n-\beta_2} \text{sgn}(4\lambda_2(\tilde{L}(t))W_2(t)), \]
\[ = -2\frac{3\beta_2}{n-\beta_2} \text{sgn}(4\lambda_2(\tilde{L}(t))W_2(t)), \]
\[ \leq -2\frac{3\beta_2}{n-\beta_2} \text{sgn}(4\lambda_2(\tilde{L}(t))W_2(t)), \]
\[ = -2\frac{3\beta_2}{n-\beta_2} \text{sgn}(4\lambda_2(\tilde{L}(t))W_2(t)). \]

Based on the Lemma 4 given in [27], it gives that \( W_2(T) \) converge to zero in a given time \( T_p \). The definition of \( \varepsilon_p(t) \) yields that \( \forall t \geq T_p, \exists \delta > 0 \), inequality \( \left| m_p(t) - \frac{1}{n} \sum_{i \in N} m_p(t) \right| < \delta \) holds. This means the DGs accomplish active power sharing in a finite time \( T_p \) under switching topologies.

B. PROOF OF THEOREM 2

The proof of Theorem 2 can mimic the proof of Theorem 1 given in Appendix A and the detailed is omitted here for simplicity.

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