Universality is a powerful concept in statistical physics that allows the description of critical phenomena on the basis of a few fundamental ingredients. At thermal equilibrium, models such as the Ising model are pivotal in understanding the critical properties of a wide class of physical systems. However, non-equilibrium systems lack a complete classification of their universal properties. In this context, the Kardar–Parisi–Zhang (KPZ) equation appears as a quintessential model to investigate non-equilibrium phenomena and phase transitions. Here we provide an experimental demonstration that one-dimensional (1D) out-of-equilibrium condensates belong to the KPZ universality class.

The KPZ equation is originally proposed to describe the stochastic growth dynamics of an interface height \( h(\mathbf{r}, t) \):

\[
\partial_t h(\mathbf{r}, t) = \nu \nabla^2 h(\mathbf{r}, t) + \frac{\lambda}{2} |\nabla h(\mathbf{r}, t)|^2 + \eta(\mathbf{r}, t),
\]

where \( \mathbf{r} \) is the position vector, \( t \) the time coordinate, \( \nabla \) the gradient operator, while \( \nu \) and \( \lambda \) are model parameters. The first term on the right corresponds to a smoothening diffusion, the second term corresponds to a nonlinear contribution leading to critical roughening, and \( \eta(\mathbf{r}, t) \) is a Gaussian white noise introducing stochasticity. The spatial and temporal correlation functions of \( h(\mathbf{r}, t) \) exhibit power-law behaviours, with critical exponents specific to the KPZ universality class. Currently available observations of KPZ dynamics have mainly focused on growing interfaces in classical systems and lately in quantum magnets. Recent theoretical works have predicted that the spatio-temporal evolution of the phase of a polariton condensate falls in the KPZ universality class. However, unlike an actual interface height, the phase is defined periodically between 0 and 2\( \pi \). This version of the KPZ equation is relevant for out-of-equilibrium systems developing macroscopic spontaneous coherence and also for polar active smectic phases.

The compactness of the phase field results in a rich phase diagram comprising not only the KPZ phase but also other regimes characterized by the proliferation of topological defects. Here we experimentally explore the spatio-temporal dynamics of the first-order coherence in a 1D polariton condensate. We observe the predicted coherence decay, and demonstrate the collapse of the data onto the universal KPZ scaling function. Our theoretical analysis shows how the observed 1D KPZ physics is resilient to the presence of vortex–antivortex (V–AV) pairs.

One-dimensional polariton condensates

Cavity polaritons are hybrid quasiparticles emerging in semiconduc
tor cavities from the strong coupling between electronic excitations (excitons) in a quantum well and cavity photons. Polaritons can be...
created from a gain medium of incoherent excitons (the excitonic reservoir) through bosonic stimulated scattering. Owing to photon leakage through the mirrors, the polariton dynamics is intrinsically out of equilibrium, its steady state being the result of the balance between drive, relaxation and losses. Finally, polaritons can be laterally confined in lattices, enabling band-structure engineering.

The 1D polariton condensates at play consist of the macroscopic occupation of a given state obtained by incoherently pumping the exciton reservoir. Above a critical density, exciton stimulated scattering from the reservoir into this state triggers a spontaneous \( U(1) \) symmetry-breaking of the phase. The condensate and reservoir dynamics are described by two coupled equations:\(^{21}\):

\[
\hbar \frac{\partial}{\partial t} \psi(x, t) = \left[ E(k) - \frac{\hbar}{2} \gamma(k) + g|\psi(x, t)|^2 + 2\gamma k \n n_k(x, t) + \frac{\hbar}{2} \lambda \n \right] \psi(x, t) + \xi(x, t) (2)
\]

\[
\frac{\partial}{\partial t} \n n_k(x, t) = P(x) - (\n n_k + R)|\psi(x, t)|^2 \n (3)
\]

Here, \( x \) and \( t \) are the space and time coordinates, \( k = -i\hbar \partial/\partial x \) is the momentum operator, \( \psi(x, t) = \sqrt[\lambda](\psi(x, t)) e^{i\Lambda(x,t)} \) the polariton condensate field with density \( \rho(x, t) \) and phase \( \theta(x, t) \). \( E(k) \) is the polariton dispersion, \( \gamma(k) \) is the momentum-dependent decay rate, \( g \) is the polariton–polariton interaction strength and \( \hbar \) is the reduced Plank constant.

The exciton reservoir, with density \( \n n_k(x, t) \), is pumped at rate \( P(x) \). Excitons either relax into the polariton condensate by stimulated scattering with rate \( \lambda \) or decay following other channels at rate \( \n n \). The term \( 2\gamma k \n n_k \) describes the polariton repulsive interactions with reservoir excitons (\( \n n_k \) being the exciton–excitation interaction strength). It dominates the polariton blueshift close to the threshold, and induces dephasing through inhomogeneous spectral broadening.\(^{29}\) Finally, \( \xi(x, t) \) describes the Gaussian noise induced by drive and loss.

Ignoring interactions with the reservoir (\( \lambda = 0 \)), previous theoretical studies have shown that the condensate phase \( \theta(x, t) \) follows a KPZ equation\(^ {20, 28}\). The condensate phase profile behaves as a classical interface (Fig. 1a), and develops KPZ spatio-temporal correlations characterized by the phase variance \( \text{Var}\{\delta\theta(x, t)/\delta \theta(x, t)\} \) (where \( \cdot \) stands for statistical averaging, and \( \delta \theta = \theta(x, t) - \theta(x, t_0), t_0 \) being a reference time). Here we derive the mapping to the KPZ equation for \( \lambda = 0 \) and obtain the KPZ parameters in terms of those entering equations (2) and (3) (Supplementary Information).

Experimentally probing KPZ correlations requires extended condensates to avoid finite size effects, a condition that was not fulfilled in early coherence measurements.\(^{30, 31}\) This requirement is demanding owing to the development of a modulation instability, which fragments the condensate into mutually incoherent micrometre-sized puddles.\(^ {32, 33}\) Indeed, repulsive condensate–reservoir interactions result in effective attractive polariton–polariton interactions within the condensate and lead to its destabilization.\(^ {34, 35}\) A solution to tame this instability is to spatially separate the excitonic reservoir from the condensate,\(^ {36}\) or to use negative-mass polaritons in a lattice.\(^ {37}\) The negative mass changes the sign of the effective polariton–polariton interactions, thus restoring the condensate stability. Using this negative-mass technique, we generate stable 1D polariton condensates extending over more than 100\( \mu \)m (Fig. 1b).

The sample consists of a semiconductor microcavity embedding quantum wells (Fig. 1c and Supplementary Information). We use nanotechnology processes to fabricate 1D asymmetric Lieb lattices of coupled micropillars containing three sites per unit cell (Fig. 1c, Methods and Supplementary Information). We incoherently populate the excitonic reservoir using a blue-detuned continuous-wave laser focused on a single lattice, with an elongated flat-top intensity profile.

**Fig. 1** | **KPZ physics in the phase dynamics of a 1D polariton condensate.**\(^ a\) | **Snapshots of the condensate phase evolution obtained by numerically solving equations (2) and (3).**\(^ b\) | **Intensity distribution of the condensate emission measured for** \( P/P_{\text{in}} = 1.1 \).\(^ c\) | **Sketch of the lattice together with the excitation scheme.**

The polariton emission analysed in momentum space below the condensation threshold (Fig. 2a) shows the lattice band structure emerging from the hybridization of the discrete modes confined in each microcavity. Above a power threshold \( P_{\text{in}} = 50 \) mW, the emission peaks at the top of the negative-mass band (Fig. 2b). This feature, together with the nonlinear increase of the emission intensity (Fig. 2c), indicates the onset of polariton condensation. The condensate emission intensity in real space at power \( P = 1.1P_{\text{in}} \) reveals an extended and regular intensity profile envelope (Fig. 1b).

### KPZ scaling in the condensate coherence decay

We define the first-order correlation evaluated between points separated in space by \( \Delta x = 2x \) and delayed by \( \Delta t \):

\[
g^{(1)}(\Delta x, \Delta t) = \frac{\langle \psi^*(x, 0) \psi(-x, 0 + \Delta t) \rangle}{\langle |\psi(x, 0)|^2 \rangle \langle |\psi(-x, 0 + \Delta t)|^2 \rangle}. (4)
\]

Neglecting density–density and density–phase correlations, we show that \( g^{(1)}(\Delta x, \Delta t) = \exp[-\text{Var}\{\delta\theta(x, \Delta t)/\delta \theta(x)\}/2] \) (Supplementary Information). We thus expect KPZ universal scaling to show up as stretched exponentials in the coherence decay: \( g^{(1)}(\Delta x, 0) = \exp(-[\Delta x/\lambda(\beta/2)]^2) \) and \( g^{(1)}(0, \Delta t) = \exp(-[\Delta t/\tau(\beta/2)]^2) \), where \( \gamma \) and \( \beta \) are the universal KPZ critical exponents and \( \lambda \) and \( \tau \) are two non-universal parameters. In one dimension, the ‘roughness’ exponent \( \gamma \) is equal to 1/2 and the ‘growth’ exponent \( \beta \) is equal to 1/3 (refs. 16, 17). Although \( 1/2 \) is common to several universality classes for 1D systems (such as Edwards–Wilkinson), \( \beta = 1/3 \) is an unambiguous signature of KPZ physics.

The condensate coherence \( g^{(1)}(0) \) measured by Michelson interferometry (Fig. 2d and Methods for details) is reported in Fig. 2e. We first focus on the temporal decay of the coherence. To search for the growth exponent, we calculate the temporal derivative \( \partial g^{(1)}(0, \Delta t)/\partial \Delta t \) from our dataset. According to KPZ theory, this derivative scales as a power law with exponent \( 2\beta - 1 = -1/3 \).
In the inset of Fig. 3a, we identify such scaling throughout the temporal window $15 \text{ ps} \leq \Delta t \leq 80 \text{ ps}$. Equivalently, we observe in the main panel a linear increase of $-2 \log (|g|^2 |\Delta t|)$ as a function of $\Delta t^{2/3}$ over the same window (grey shaded area), which demonstrates a key feature of KPZ dynamics. At short timescales, the deviation from KPZ scaling and the saturation of $|g|^2$ are due to an incoherent background (spectrally broad photoluminescence from uncondensed states) that hinders the onset of KPZ fluctuations. For $\Delta t \geq 80 \text{ ps}$, $-2 \log (|g|^2 |\Delta t|)$ also departs from KPZ scaling and follows a super-linear behaviour that we attribute to slow reservoir population fluctuations. We now perform a similar analysis in the spatial domain. The results are shown in Fig. 3b. The spatial derivative $D_x = -2 \partial \log (|g|^2 |\Delta x, 0|))/\partial \Delta x$ exhibits a plateau within the spatial window $30 \mu m \leq \Delta x \leq 60 \mu m$, in agreement with $\chi = 1/2$ (inset). The roughness exponent $\chi = 1/2$ also shows up in the linear increase of $-2 \log (|g|^2 |\Delta x, 0|)$, over the same window (grey shaded area in the main panel). When approaching condensate edges ($\Delta x \geq 60 \mu m$), the coherence decays faster because of enhanced fluctuations at smaller polaron density. Pushing further this data analysis, we fit the coherence decay curves with stretched exponentials and deduce experimental values for the scaling exponents: $\chi_{\exp} = 0.51 \pm 0.08$ and $\beta_{\exp} = 0.36 \pm 0.11$. The uncertainty on $\beta_{\exp}$ allows us to discriminate between the different universality classes relevant for our system, as the KPZ value $\beta = 1/3$ remains the only one lying within the 95% confidence interval on $\beta_{\exp}$ (Supplementary Information).

We now search for KPZ signatures over the whole space–time correlation map. We select all data points where $0.57 \leq |g|^2 \leq 0.75$, the range where we evidence KPZ scaling at $\Delta t = 0$. This space–time window is shown in the bottom inset of Fig. 3c. For this subset of data points, we plot in Fig. 3c the value of $-2 \log (|g|^2 |\Delta x, \Delta t|)/\Delta t^{2/3}$ as a function of the rescaled coordinate $y = \Delta x/\Delta t^{1/3}$, where $y$ is a normalization factor (Supplementary Information). Strikingly, all these data points collapse onto the scaling function $F = C(y \kappa g t^x)/y_{\kappa g t}$, where $F_{\kappa g t}$ is the tabulated dimensionless KPZ universal scaling function. We use the non-universal constants $C_0$ and $y_{\kappa g t}$ as fitting parameters to overlap $F$ with the collapsed data points. This result demonstrates that ID polaron condensates indeed belong to the KPZ universality class. To reinforce the generality of this conclusion, we performed the same measurement and analysis on a different 1D lattice with four sites per unit cell. We also found a KPZ space–time window where all data points collapse onto the universal scaling curve (Supplementary Information).
To complete the picture, we carried out the same analysis at higher excitation powers, and found for both lattices that the spatio-temporal KPZ window shrinks for increasing \(P/P_{in}\) and eventually disappears when \(P/P_{in} > 1.2\) (Supplementary Information).

**Resilience of KPZ physics to space–time vortices**

To numerically reproduce these experimental data, we calculate the phase evolution of the polariton condensate by numerically solving equations (2) and (3). Details about how we set the simulation parameters can be found in Methods. It is noted that we neglect the polariton–polariton interaction energy \(g|\psi|^4\) and take \(g = 0\) in all simulations. As such, this model also applies to spatially extended lasers in the weak coupling regime. The calculated \(|g_{\text{num}}(\Delta_t)|^2\) data are reported in Fig. 3a,b, showing excellent agreement with the experiment. The short- and long-time behaviour is reproduced, together with the shrinking of the KPZ window when the excitation power is increased (Supplementary Information).

We then perform on the numerical data the same analysis as on the experimental data. We plot \(-2 \log(|g_{\text{num}}(\Delta_t)|^2)/\Delta t^2\) as a function of \(\Delta\), selecting the points for which \(0.57 \leq |g_{\text{num}}(\Delta)| \leq 0.75\). The result is shown in Fig. 3c (top inset), together with the KPZ scaling function \(F = C_F \sqrt{\Delta}/y_0\), using for \(C_F\) and \(y_0\) the same values as for the experimental data. The simulated data align to the scaling function, thus fully validating our model.

To deepen our insight into the phase dynamics, we now analyse its stochastic behaviour in the numerical simulations. Figure 4a shows an example of a phase map \(\Delta \phi(x, \Delta t)\) corresponding to a given realization of the noise (others are shown in Supplementary Information). We observe two kinds of phase variation: small-amplitude fluctuations and fast (scarce) phase jumps. These jumps are associated with pairs of close-by spatio-temporal vortices with opposite circulation (see inset), that we name V–AV pairs. To analyse their effect on the phase dynamics, we show in Fig. 4b the unwrapped phase temporal evolution at \(x = 0\) (horizontal line in Fig. 4a). The phase evolution exhibits plateaus with small-amplitude phase fluctuations, separated by phase jumps of approximately 2π, occurring on a fast timescale (about 1 ps) when passing through a V–AV pair. It is noted that for the regime of parameters explored here, almost all vortices appear in V–AV pairs. For higher noise or stronger interactions, activation of single vortices is expected and would lead to other dynamical regimes.\(^{24}\)

We now show that small-amplitude phase fluctuations follow KPZ scaling laws by computing the phase variance \(\langle \Delta \phi \rangle\). As this quantity is extremely sensitive to phase jumps, we filter them out in the calculation (Methods). The result is plotted in Fig. 4c together with the values of \(-2 \log(|g_{\text{num}}(\Delta_t)|^2)\) (calculated without filtering the V–AV pairs). Both quantities exhibit the KPZ power-law scaling over the same time window (grey shaded area), as further illustrated in Fig. 4c (inset) where we plot their time derivative. This result definitely confirms that the first-order coherence is a good observable to probe the KPZ dynamics of the condensate phase, even in the presence of few V–AV pairs.

Another striking signature of KPZ physics lies in the fact that phase fluctuations are governed by a probability distribution, which—unlike the normal distribution—is skewed and exhibits markedly different tails. In Fig. 4d, we show the calculated probability distribution of the unwrapped \(\Delta \phi(0, \Delta t)\), computed over all trajectories (thus including vortices) for \(\Delta t = 50\) ps, that is, in the centre of the KPZ window. All trajectories that have not crossed any V–AV pair contribute to the first peak in the distribution. The second peak corresponds to trajectories that have crossed one V–AV pair before reaching \(\Delta t = 50\) ps and have thus undergone one phase jump close to 2π. Strikingly the first two peaks are skewed and well reproduced by the Tracy–Widom Gaussian orthogonal ensemble (GOE) distribution associated with the flat subclass (Supplementary Information). Cumulating data for various \(\Delta t\), we obtain an agreement with the Tracy–Widom GOE distribution over six decades (see Supplementary Information for details on the analysis). The third peak corresponds to few realizations showing two phase jumps. The lack of statistics prevents precise analysis of its shape. Our simulations highlight that V–AV pairs only modify the probability distribution by adding replicas of the main peak without significantly changing their shape. Moreover, they confirm that in the regime where a low density of V–AV pairs stochastically shows up, KPZ dynamics is not destroyed but occurs piece-wise in the spatio-temporal domain.

To conclude, both our experimental and theoretical analysis prove that KPZ scaling laws are present in the decay of the first-order coherence of 1D driven-dissipative polariton condensates. Our findings apply to any spatially extended driven open systems subject to gain and loss and characterized by a U(1) symmetry-breaking. Our work opens many challenges to be addressed in the future. In 1D, although our results highlight the striking resilience of KPZ physics to V–AV pairs, regimes at higher noise strength or higher nonlinearity remain to be explored.\(^{21}\) Investigating different KPZ universality subclasses, predicted for various geometries of the unwrapped phase profile\(^{22}\), is now also within reach, through engineering the geometry of the condensate environment. Beyond 1D, exciton–polariton lattices offer
exciting perspectives for the exploration of KPZ physics in 2D, where an experimental realization is highly sought after\textsuperscript{11–16} and the role of topological defects still actively debated\textsuperscript{1}. An experimental implementation involving polariton condensates would enable testing of the different models and serve as a general testbed for exploring complex physical systems belonging to the KPZ universality class.

Online content
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Experimental details
The sample used in this paper consists of a high-quality-factor ($Q = 70,000$) /2 Ga$_{0.05}$Al$_{0.95}$As microcavity surrounded by two Al$_{0.20}$Ga$_{0.80}$As/Al$_{0.05}$Ga$_{0.95}$As distributed Bragg reflectors. Three stacks of four 8-nm gallium arsenide (GaAs) quantum wells are embedded in this microstructure, at the anti-nodes of the cavity-mode electromagnetic field, resulting in a 15-meV collective Rabi splitting. The as-grown planar cavity is patterned into 1D lattices of coupled micropillars (3 μm in diameter), using electron beam lithography and dry etching. In this work, we use a 200-μm-long asymmetric Lieb lattice, made of three pillars per unit cell with 2.2-μm centre-to-centre separation distance. The sample is held at cryogenic temperature (4 K) in a closed-cycle cryostation.

We incoherently populate the exciton reservoir using a non-resonant continuous-wave laser at 740 nm (reflectivity minimum of the Bragg mirror). A spatial light modulator enables shaping the excitation spot into a 125-μm-long flat-top beam in the lattice direction. It is noted that our experiment is performed under truly continuous-wave excitation conditions, that is, without any chopper. The polariton emission leaking out through the cavity top mirror is analysed in space, momentum (along the lattice direction) and frequency with a monochromator coupled to a charge-coupled-device camera.

We retrieve the condensate first-order coherence using Michelson interferometry. A two-mirror retro-reflector mounted on a step-motorized translation stage in one of the interferometer arms enables overlapping on a charge-coupled-device camera the field $\mathcal{E}(x, t_0)$ emitted by the condensate at time $t_0$ and position $x$, with $\mathcal{E}(x, t_0 + \Delta t)$, the field emitted at $t_0 + \Delta t$ and position $-x$ ($\Delta t$ is the delay introduced between the interferometer arms by translating the retro-reflector). The fringe contrast gives a direct visualization of the degree of coherence between fields emitted at two points spatially separated by $\Delta x = 2\lambda$ and delayed by $\Delta t$. More specifically, $|g^{(1)}(\Delta x, \Delta t)|^2$ is determined from the fringe visibility $V(\Delta x, \Delta t)$ and from the intensity distributions $|\mathcal{E}(x, t_0)|^2$ and $|\mathcal{E}(x, t_0 + \Delta t)|^2$ measured separately, using

$$2|g^{(1)}(\Delta x, \Delta t)|^2 |\mathcal{E}(x, t_0)|^2 |\mathcal{E}(x, t_0 + \Delta t)|^2 = V(\Delta x, \Delta t) [|\mathcal{E}(x, t_0)|^2 + |\mathcal{E}(x, t_0 + \Delta t)|^2].$$

To probe the temporal scaling of the condensate coherence, we scan the retro-reflector position over $\Delta t = 5 \text{cm}$, corresponding to a maximum time delay of $\Delta t = 2\lambda/c = 330 \text{ ps}$ (being the speed of light in vacuum). During such a scan, we set the camera exposure time to $1 \text{ s}$ and acquire a series of 250 images.

Integration method and parameters for numerical simulations
To compare our experimental findings to theory, we solved numerically the set of equations (2) and (3). The numerical integration of these equations is performed using the interaction picture method. The idea behind this integration scheme is similar to the interaction picture in quantum mechanics. We first split equations (2) and (3) into a linear, exactly solvable part and a remaining nonlinear part. We then solve the linear component in Fourier space and transform it back to real space. We transform equations (2) and (3) by moving into the interaction picture and integrate the resulting nonlinear equation using semi-implicit Runge–Kutta method, with an adaptive time step. We take as the initial condition $\psi(x, t = 0) = 0$, and let the condensate grow under the action of the pumped reservoir. The sampling starts at $t_0$ long after the condensate has reached its stationary density profile. Usually, we perform our simulations using $t_0 = 10 \text{ ns}$.

Some of the parameters entering the numerical simulations are known experimentally. For instance, we use a lattice spacing equal to the experimental lattice period $a = 4.4 \mu m$. The measurement of the polariton dispersion relation $E(k)$ (k being the polariton wave-vector) shown in Fig. 2a provides a good estimate of the polariton mass $m = 3.3 \times 10^{-6} m_e$ ($m_e$ being the electron mass). The k-dependent polariton linewidth $\gamma(k)$ is modelled by the function

$$\gamma(k) = \gamma_0 + (\gamma_c - \gamma_0) \left[ 1 - \exp \left( -\frac{k^2}{\gamma_0} \right) \right],$$

which is compared with the experimental data in Supplementary Fig. 3d (blue solid line). The parameter values are $\gamma_0 = 48.5 \mu$eV and $\gamma_c = 1.6 \times 10^4 \mu$eV $^{-1}$, and $\gamma_0 = 37 \mu$eV. Furthermore, the energy blueshift at threshold is known with good accuracy:

$$2 g_R n_b |g|_th = 0.6 \text{ meV} \equiv \mu_{th}. \quad (7)$$

where $n_b |g|_th = P_{th}/\gamma_{th}$ stands for the reservoir density at condensation threshold and $P_{th} = \gamma_0 \gamma_{th}/\gamma$ for the threshold power. Assuming that $\gamma_0$ does not depend on the pumping power $P$, we find:

$$\gamma_{th} = \frac{2}{\mu_{th}}.$$  

As the reservoir-induced blueshift $2 g_R n_b$ is two orders of magnitude larger than the polariton-induced blueshift $g |\psi|^2$, we take $g = 0$ in our simulations. To qualitatively reproduce the spatial density profile of the condensate in our experiments, we use a spatially dependent flat-top pump profile modelled by

$$P(x) = P \left[ 1 + \tanh((L_0 + x) / \sigma) \right] \left[ 1 + \tanh((L_0 - x) / \sigma) \right] [1 + \tanh(L_0 / \sigma)]^2,$$

where $L_0 = 80 \mu m$ is the length of the pump spot and $\sigma = 9.7 \mu m$ is the width of its decaying edges. The remaining free parameters in our numerical simulations are thus the scattering rate $R$ of excitons into the condensate and the reservoir decay rate $\gamma_{th}$. For those parameters, we choose values within some realistic range yielding the best agreement with the measured $|g|^2$. All the simulations presented in this paper were performed using $R = 8.8 \times 10^{-3} \mu m^2$ s$^{-1}$ and $\gamma_{th} = 0.45 \gamma_0$.

Calculation of the variance of the phase
To calculate $\text{Var}(\Delta \theta)$, we select for each trajectory a 100-ps-wide window where the phase undergoes the smallest amount of jumps. When few phase jumps remain in the selected window, we filter them out by adding at every vortex location another one of opposite charge (black line in Fig. 4b). It is noted that we discard 5% of the realizations, where vortex-free (VF) time windows. To calculate the variance of the phase, we select for each trajectory a 100-ps-wide window where the phase undergoes the smallest amount of jumps. When few phase jumps remain in the selected window, we filter them out by adding at every vortex location another one of opposite charge (black line in Fig. 4b). It is noted that we discard 5% of the realizations, where

$$\text{Var}(\Delta \theta) = \frac{1}{N} \sum_{i=1}^{N} (\Delta \theta_i - \langle \Delta \theta \rangle)^2.$$

Data availability
All datasets generated and analysed during this study are available upon request from the corresponding authors. Source data are provided with this paper.

Code availability
All codes generated during this study are available upon request from the corresponding authors.

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Author contributions Q.F. built the experimental set-up, performed the experiments and analysed the data. D.S. realized the theoretical calculations and numerical simulations. F.B. contributed to the design of the sample structure and initial characterization of the sample. A.L. and M.M. grew the sample by molecular beam epitaxy. I.S., L.L.G. and A.H. fabricated the polariton lattices. Q.F., D.S., I.A., M.W., I.C., A.A., M.R., A.M., L.C., S.R. and J.B. participated in the scientific discussions about all aspects of the work. Q.F., A.M., L.C., S.R. and J.B. wrote the original draft of the paper. Q.F., D.S., I.A., M.W., I.C., A.A., M.R., A.M., L.C., S.R. and J.B. reviewed and edited the paper into its current form. A.M., L.C., S.R. and J.B. supervised the work.

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