Newtonian gravity from the Higgs field: the sublimation of aether

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Abstract

We illustrate why a space-time structure as in General Relativity is not in contradiction with a dynamical origin of gravity from a scalar field. Further, we argue that the recently discovered gap-less mode of the singlet Higgs field represents the most natural dynamical agent of Newtonian gravity.
1. Following the original induced-gravity models [1], one may attempt to describe gravity as an effective force induced by the vacuum structure, in analogy with the attractive interaction among electrons that can only exist in the presence of an ion lattice. Looking for such a ‘dynamical’ mechanism one should also be able to account for those peculiar ‘geometrical’ properties at the base of the classical space-time description of General Relativity.

The idea that gravitation requires somehow an underlying scalar field arises naturally when considering the non-trivial ambiguity [2] associated with the operative definition of the infinitesimal transformation to the reference frame of a freely falling observer. Indeed, within General Relativity, the Equivalence Principle is used to relate a frame in which a constant force is acting to a frame in a Riemannian space uniformly accelerating with respect to an inertial frame. An alternative and equivalent description [2] is obtained by considering infinitesimal acceleration transformations of the conformal group. In this case, the equivalence is not with a frame in a Riemannian space but with a frame in a more general Weyl space, i.e. a space where the affine connections are not given by the Christoffel symbols [2] since at each point \( x \), besides a symmetric tensor \( g_{\mu\nu}(x) \), there is a vector \( \kappa_\mu(x) \). One usually considers the Weyl space in relation to the original Weyl’s theory including electromagnetism. That theory predicts that the length of a rod depends on its history, which may seem physically untenable. However, the idea of a Weyl space cannot be rejected \textit{a priori} so that one is still faced with the interpretation of the infinitesimal acceleration transformation and as such of the basic ingredient underlying any space-time description in gravitational fields. In this sense, the formal foundation of General Relativity contains an arbitrary assumption, at least beyond the weak-field limit where it has been experimentally tested.

In an alternative approach, one could have argued as follows. \textit{A priori}, each of the two possibilities represents an arbitrary choice. Therefore, it may be natural to solve the ambiguity by adopting the more general conformal-transformation point of view but restricting to that special class of Weyl spaces that are \textit{equivalent} to Riemannian spaces. This equivalence can be expressed as \( |g| \equiv 1 \) and

\[
\frac{\kappa_\mu}{4\pi} = -\partial_\mu \hat{\sigma}
\]

where \( \hat{\sigma} \) is a scalar field. In this case, in fact, the Weyl connections constructed with \( g_{\mu\nu} \) and \( \kappa_\mu \) are the Christoffel symbols of the metric tensor [2]

\[
\hat{g}_{\mu\nu} = e^{-4\pi \hat{\sigma}} g_{\mu\nu}
\]

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To explore the possible implications of this space-time structure, let us consider the case of a static isotropic metric

\[ \hat{g}_{\mu\nu} \equiv (A, -B, -B, -B) \] (3)

for which \( AB^2 = e^{-16\pi \tilde{\sigma}} \). Further, we can require the operative definition of the light velocity to be equivalently obtained as the coordinate velocity \( (dx^2 + dy^2 + dz^2)^{1/2} = \sqrt{A/B} \) obtained from \( ds^2 = 0 \) or the group velocity of a light pulse, solution of the covariant D’Alembert wave equation. In this case, one finds \( AB = \text{const.} \) Finally, with a flat space-time at infinity one gets \( AB = 1 \) and the unique form

\[ ds^2 = e^{8\pi \tilde{\sigma}} dt^2 - e^{-8\pi \tilde{\sigma}} (dx^2 + dy^2 + dz^2) \] (4)

In principle the scalar function \( \tilde{\sigma} \), describing a given gravitational field, can be determined \( \text{without solving any field equation} \) but simply comparing with experiments in the weak-field regime. This would lead to the identification of \( \tilde{\sigma} \) with the Newton potential \( \phi_N \) (‘EXP’=Experimental)

\[ \tilde{\sigma}_{\text{EXP}} = -\frac{\phi_N}{4\pi} \] (5)

i.e. the solution of the Poisson equation in flat space corresponding to a given mass density. In this way Eq.(4) represents the space-time metric of an asymptotically far observer at infinity where the gravitational potential \( \phi_N(\infty) = 0 \).

As an independent argument and to better appreciate the implications of Eqs.(4) and (5), we shall now follow the original idea proposed by Yilmaz long time ago [4]. In this way, Einstein equations for the Yilmaz’s metric

\[ (ds^2)_{\text{Yilmaz}} = e^{-2\phi_N} dt^2 - e^{2\phi_N} (dx^2 + dy^2 + dz^2) \] (6)

become algebraic identities [4] when using Poisson equation. In particular, outside of the sources of \( \phi_N \), namely where \( \Delta \phi_N = 0 \), one can freely interchange \( G^\mu_\nu \equiv R^\mu_\nu - \frac{1}{2} g^\mu_\nu R \) for the metric Eq.(6) with the stress-tensor for the \( \phi_N \) field

\[ t^\mu_\nu(\phi_N) = -\partial^\mu \phi_N \partial_\nu \phi_N + \frac{1}{2} \delta^\mu_\nu \partial^\rho \phi_N \partial_\rho \phi_N \] (7)

In this sense, the scalar field \( \phi_N \) represents the true agent of the gravitational interaction for static fields. Notice that there is no contradiction between a \( scalar \) field, as a dynamical agent, and a \( tensor \) field, namely \( \hat{g}_{\mu\nu} = \hat{g}_{\mu\nu}(\phi_N) \), to account for the relevant space-time effects [4].

Although the overall picture looks similar to General Relativity, there are some non-trivial differences. For instance, in standard General Relativity the free-fall transformation
is understood as a general coordinate transformation. Therefore, differently from $t_\mu^\nu(\phi_N)$, Einstein gravitational stress-tensor is only a pseudotensor $[7]$ that vanishes for some choice of coordinates but becomes non-zero with another set of coordinates.

Also, for weak field (and in the one-body case) the space-time structure $[8]$ becomes equivalent to the Schwarzschild metric

$$(ds^2)_{\text{Schwarzschild}} = \left[1 - \frac{\phi_N}{1 + \phi_N/2}\right]^2 dt^2 - (1 + \phi_N/2)^4 (dx^2 + dy^2 + dz^2)$$

up to higher order terms. However, Eq.(3) applies to a gravitationally interacting N-body system and does not contain black holes. As a consequence, one predicts different stability regimes for compact massive objects as neutron stars $[8]$. At the same time, by exploiting the unique factorization properties of the exponential metric Eq.(6), one can explain the huge quasar red-shifts at moderate distances by purely gravitational (rather than cosmological) effects $[9]$.

Concerning these differences, we emphasize that the metric structure Eq.(4) is of very general nature and it stands by itself regardless of any field equations. On the other hand, in the one-body case, where the two metrics Eqs.(6) and (8) can be compared, there is an interpolating parameterization $[10]$ $(\beta = 1 - \epsilon^2)$

$$ds^2(\beta) = \left[\frac{1 - \epsilon \phi_N/2}{1 + \epsilon \phi_N/2}\right]^{2/\epsilon} dt^2 - \left[\frac{1 + \epsilon \phi_N/2}{1 - \epsilon \phi_N/2}\right]^{2/\epsilon} (1 - \frac{\epsilon^2 \phi_N^2}{4})^2 (dx^2 + dy^2 + dz^2)$$

that reduces to Eq.(6) for $\beta = 1$ and to Eq.(8) for $\beta = 0$. In this case, a sufficiently precise experiment should objectively determine the numerical value of $\beta$. It is surprising, though, that according to refs.$[3, 11]$ one could already use existing measurements, namely the experiments on neutron phase shift in a gravitational field $[12]$ and those on the isotropy of inertia $[13]$, to deduce $|1 - \beta| = \mathcal{O}(10^{-2})$ thus ruling out the Schwarzschild metric Eq.(8).

Beyond the static case, Yilmaz’s original approach can be considered the limit of a framework where the space-time structure is distorted by several fields. Now Eqs.(4) and (2) hold for the general forms

\[ \hat{g}_{\mu\nu} = \hat{g}_{\mu\nu}(\phi_N, A_\mu, B_{\mu\nu}, ...) \]  

\[ \kappa_\mu = \kappa_\mu(\phi_N, A_\mu, B_{\mu\nu}, ...) \]  

where $A_\mu, B_{\mu\nu}, ...$ represent unknown dynamical agents providing possible new contributions (as $A_\mu A^\mu, B_\mu^\mu, B^{\mu\nu}B_{\mu\nu}, ...$) to the effective scalar $\tilde{\sigma}$. Although determining their precise nature would require to control all possible aspects of gravity, one can explore some general
consequence of identifying $\phi, A_\mu, B_{\mu\nu},...$ as long-wavelength excitations (i.e. *disturbances*) of the quantum field theoretical vacuum.

This point of view arises naturally when starting from Minkowski space to describe the elementary particle interactions but taking into account that the physical vacuum of quantum field theory is not trivially ‘empty’. Rather the phenomenon of vacuum condensation [14] should be considered the operative construction of a ‘quantum aether’ [15], i.e. different from the aether of classical physics but also different from the ‘empty’ space-time of special relativity. In this approach, the parametric dependence in Eqs.(10) and (11) is such that any deviation from a flat space-time is due to peculiar field configurations $\phi, A_\mu, B_{\mu\nu},...$ In analogy with the static case and outside of the sources of $\phi, A_\mu, B_{\mu\nu},...$ the metric structure is such to ensure the formal identity between the $G^\mu_\nu$ corresponding to Eq.(11) and a suitable ‘gravitational’ stress-tensor $t^\mu_\nu(\phi, A, B, ...)$. Although one can get agreement with General Relativity to first-order in $\phi, A_\mu, B_{\mu\nu},...$ ($t^\mu_\nu(\phi, A, B, ...) \approx 0$), the problem of the vacuum energy is very different from the standard approach and one can easily understand the absence of any curvature in the unperturbed ground state where, by definition, $t^\mu_\nu(\phi, A, B, ...) = 0$.

However, before attempting to describe the excitation states of the vacuum in the most general frameworks (fastly rotating objects, sources of gravitational waves,...) it is natural to proceed step by step starting from the simplest case. For this reason, in the rest of this Letter, we shall only try to understand the physical meaning of the scalar field $\tilde{\sigma}$ for the well known case of Newtonian gravity. After all, independently of any field equations, the general space-time structure expressed in Eqs.(1)-(5) is consistent with all experiments and requires a single scalar function: the Newton potential. Looking for its dynamical origin, we shall propose the possible interpretation of $\tilde{\sigma}_{\text{EXP}} = -\frac{\phi}{4\pi}$ as a collective excitation of the same scalar condensate at the base of mass generation in the Standard Model of electroweak interactions. In the version Eqs.(1)-(5) of the Equivalence Principle, the density fluctuations of the physical vacuum [18] can be described geometrically as distortions of the flat Minkowski space-time into a particular class of Weyl spaces.

2. Our proposal is motivated by recent theoretical arguments [19, 20] that, quite independently of the Goldstone phenomenon, suggest the existence of a gap-less mode of the singlet Higgs field in the broken phase of a $\lambda\Phi^4$ theory. Its presence is a direct consequence of the quantum nature of the scalar condensate that cannot be treated as a purely classical c-number field. In fact, either considering the re-summation of the one-particle reducible zero-momentum tadpole graphs [19] in a given background field or performing explicitely
the last functional integration over the strength of the zero-momentum mode of the singlet Higgs field \[20\], one finds two possible solutions for the inverse zero-4-momentum propagator in the spontaneously broken phase: a) \( G_a^{-1}(0) = M_h^2 \) and b) \( G_b^{-1}(0) = 0 \).

This result is not totally unexpected and admits a simple geometrical interpretation in terms of the shape of \( V_{LT}(\varphi) \), the effective potential obtained from the Legendre transform (‘LT’) of the generating functional for connected Green’s function. Differently from the conventional non-convex (‘NC’) effective potential \( V_{NC}(\varphi) \), as computed in the loop expansion, \( V_{LT}(\varphi) \) is not an infinitely differentiable function in the presence of spontaneous symmetry breaking \[21\]. Moreover, being rigorously convex downward, \( V_{LT}(\varphi) \) will agree with \( V_{NC}(\varphi) \) everywhere except in the region enclosed by the absolute minima of \( V_{NC}(\varphi) \), say \( \varphi = \pm v \), where it is exactly flat. For this reason, although an exterior second derivative of \( V_{LT}(\varphi) \) will agree with \( V_{NC}'(\pm v) \equiv M_h^2 \), one will also be faced with a vanishing internal second derivative which has no counterpart in \( V_{NC}(\varphi) \).

As anticipated, the main point of Refs.\[19, 20\] is that the ‘Maxwell construction’, i.e. the replacement \( V_{NC}(\varphi) \to V_{LT}(\varphi) \) reflects the quantum nature of the scalar condensate (see Ref.\[22\] for a still further derivation). As such, there are important consequences for the infrared behaviour of the theory. In fact, two possible values for the zero-4-momentum propagator imply two possible types of excitations with different energies when the 3-momentum \( p \to 0 \): a massive one, with \( \tilde{E}_a(p) \to M_h \), and a gap-less one with \( \tilde{E}_b(p) \to 0 \). The latter would clearly dominate the far infrared region \( p \to 0 \) where the massive branch becomes unphysical. Therefore, differently from the most naive perturbative indications, in a (one-component) spontaneously broken \( \lambda \Phi^4 \) theory there is no energy-gap associated with the ‘mass’ \( M_h \) of the shifted field \( h(x) = \Phi(x) - \langle \Phi \rangle \), as it would be for a genuine massive single-particle spectrum where the relation

\[
\tilde{E}(p) = \sqrt{p^2 + M_h^2}
\] (12)

remains true for \( p \to 0 \). Rather, the infrared region is dominated by gap-less collective modes

\[
\tilde{E}(p) \equiv c_s |p|
\] (13)

depending on an unknown parameter \( c_s \) which controls the slope of the spectrum for \( p \to 0 \) and has the meaning of the ‘sound velocity’ for the density fluctuations of the scalar condensate. Indications on its magnitude can be obtained on the base of a semi-classical argument due to Stevenson \[23\] that we shall briefly report below.

Stevenson’s argument starts from a perfect-fluid treatment of the Higgs condensate. In
this approximation, energy-momentum conservation is equivalent to wave propagation with
a squared velocity given by (c is the light velocity)
\[ c_s^2 = c^2 \left( \frac{\partial P}{\partial E} \right) \] (14)
where \( P \) is the pressure and \( E \) the energy density. Introducing the condensate density \( n \),
and using the energy-pressure relation
\[ P = -E + n \frac{\partial E}{\partial n} \] (15)
we obtain
\[ c_s^2 = c^2 \left( \frac{\partial P}{\partial n} \right) \left( \frac{\partial E}{\partial n} \right)^{-1} = c^2 \left( n \frac{\partial^2 E}{\partial n^2} \right) \left( \frac{\partial E}{\partial n} \right)^{-1} \] (16)
For a non-relativistic Bose condensate of neutral particles with mass \( m \) and scattering
length \( a \), where \( \frac{nah^2}{m^2c^2} \ll 1 \) (in this case we explicitly introduce \( \hbar \) and \( c \)) one finds
\[ E = nmc^2 + n^2 \frac{2\pi ah^2}{m} \] (17)
so that
\[ c_s^2 = \frac{4\pi nah^2}{m^2} \] (18)
which is the well known result for the sound velocity in a dilute hard sphere Bose gas [24].

On the other hand, in a fully relativistic case, the additional terms in Eq.(17) are such
that the scalar condensate is spontaneously generated from the ‘empty’ vacuum where \( n = 0 \)
for that particular equilibrium density where [25]
\[ \frac{\partial E}{\partial n} = 0 \] (19)
Therefore, in this approximation, approaching the equilibrium density one finds
\[ c_s^2 \to \infty \] (20)
thus implying that long-wavelength density fluctuations would propagate instantaneously
in the spontaneously broken vacuum.

As Stevenson points out [23], Eq.(20) neglects all possible corrections to the perfect-fluid
approximation, as well as finite-temperature effects (as for instance if the Higgs condensate
were in thermal equilibrium with the microwave background radiation). Still, the
above semi-classical argument suggests that density fluctuations can propagate extremely
fast in the spontaneously broken vacuum, at least in the long-wavelength limit where the
identification \( \text{(13)} \) \( c_s = \frac{d\tilde{E}}{d|\mathbf{p}|} \) applies.
On the other hand, sound waves cannot propagate with too short wavelengths. In our case there is a typical momentum, say \(|p| = \delta\), of the order of the inverse mean free path for the elementary constituents in the scalar condensate, where the collective modes become unphysical and a single-particle spectrum as in Eq.\((12)\) applies. The transition occurs where

\[ \sqrt{\delta^2 + M_h^2} \sim c_s \delta \]  

(21)

so that, for \(c_s \to \infty\), \(\delta\) is naturally infinitesimal in units of \(M_h\). For this reason superluminal wave propagation is restricted to the region \(p \to 0\) and does not necessarily imply violations of causality as for a group velocity \(\frac{dE}{dp} > 1\) when \(|p| \to \infty\) [23].

3. Independent informations on \(c_s\) can be obtained by comparing with phenomenology. Let us ignore, for the moment, the previous indication in Eq.\((20)\) and just explore the consequences of Eq.\((13)\). To this end, let us define \(h(x)\) to be the component of the fluctuation field associated with the long-wavelength modes Eq.\((13)\). Whatever the value of \(c_s\), these dominate the infrared region so that a general yukawa coupling of the Higgs field to fermions will give rise to a long-range attractive potential between any pair of fermion masses \(m_i\) and \(m_j\)

\[ U_\infty(r) = -\frac{1}{4\pi c_s^2 \langle \Phi \rangle^2 \frac{m_i m_j}{r}} \]  

(22)

The above result would have a considerable impact for the Standard Model if we take the value \(\langle \Phi \rangle \sim 246\) GeV related to the Fermi constant. Unless \(c_s\) is an extremely large number (in units of \(c\)) one is faced with strong long-range forces coupled to the inertial masses of the known elementary fermions. Just to have an idea, by assuming \(c_s = 1\) the long-range interaction between two electrons in Eq.\((22)\) would be \(\mathcal{O}(10^{33})\) larger than their purely gravitational attraction. On the other hand, invoking a phenomenologically viable strength, as if \(c_s\langle \Phi \rangle\) would be of the order of the Planck scale, is equivalent to re-obtain nearly instantaneous interactions transmitted by the scalar condensate as in Eq.\((20)\).

On a macroscopic scale, the relevant long-range effects can be evaluated from the effective lagrangian \(\mathcal{L}_{\text{eff}}\) for \(h(x)\). In an expansion in powers of \(h(x)\), the lowest-order interaction term arises from its coupling to the trace of the energy-momentum tensor of ordinary matter \(T^\mu_\mu(x)\) and represents the obvious renormalization of the elementary yukawa couplings

\[ \langle f \mid T^\mu_\mu \mid f \rangle = m_f \bar{\psi}_f \psi_f \]  

(23)

In this way, by defining \(\tilde{\sigma} = \frac{\tilde{h}}{\langle \Phi \rangle}\) we get

\[ \mathcal{L}_{\text{eff}}(\tilde{\sigma}) = \frac{\langle \Phi \rangle^2}{2} \tilde{\sigma} c_s^2 \Delta - \frac{\partial^2}{\partial t^2} \tilde{\sigma} - T^\mu_\mu \tilde{\sigma} + \ldots \]  

(24)
and the linearized equation of motion
\[
[c_s^2 \Delta - \frac{\partial^2}{\partial t^2}]\tilde{\sigma} = \frac{T^\mu_\mu}{\langle \Phi \rangle^2}
\]  
(25)

Now, by assuming very large values of \(c_s\) the \(\tilde{\sigma}\) Green’s function has practically no retardation effects and Eq.(25) describes an instantaneous interaction
\[
\Delta \tilde{\sigma} = \frac{T^\mu_\mu}{c_s^2 \langle \Phi \rangle^2}
\]  
(26)

Finally for slow motions, when the trace of the energy-momentum tensor
\[
T^\mu_\mu(x) \equiv \sum_n \frac{E_n^2 - \mathbf{p}_n \cdot \mathbf{p}_n}{E_n} \delta^3(x - \mathbf{x}_n(t))
\]  
(27)

reduces to the mass density
\[
\rho(x) \equiv \sum_n m_n \delta^3(x - \mathbf{x}_n(t))
\]  
(28)

one gets
\[
\Delta \tilde{\sigma} = \frac{1}{c_s^2 \langle \Phi \rangle^2} \rho(x)
\]  
(29)

Therefore, if the only free parameter of our analysis \(c_s\) would be fixed by the relation
\[
c_s^2 = \frac{1}{G_N \langle \Phi \rangle^2} = \mathcal{O}(10^{33})
\]  
(30)

\(G_N\) being the Newton constant, Eq.(29) would become formally identical to the Poisson equation for the Newton potential allowing for the identification in Eq.(3). As anticipated, Stevenson’s Eq.(20) provides a clue to understand the very large number in Eq.(30) leading to \(c_s \sim 4 \cdot 10^{16}\). Although this may seem an infinitely large value, \(c_s\) is not so fantastically higher than the experimental lower limit \(2 \cdot 10^{10}\) reported by Van Flandern [26] for the ‘speed of gravity’ when gravity is described as a central interaction [27] as in Eq.(29).

Before concluding, we comment on the momentum \(\delta\) Eq.(21) associated with the transition between the two branches of the spectrum Eqs.(13) and (12) for which Eq.(30) implies
\[
\frac{\delta}{M_p} = \mathcal{O}(10^{-16})
\]  
(31)

At distances \(r \sim \delta^{-1}\) the interparticle potential is not given by Eq.(22) but has to be computed from the Fourier transform of the \(h\)–field propagator
\[
D(r) = \int \frac{d^3p}{(2\pi)^3} \frac{e^{ip \cdot r}}{E^2(p)}
\]  
(32)
and depends on the detailed form of the spectrum that interpolates between Eqs.(13) and (12). However in the Standard Model, for \( M_h = \mathcal{O}(\langle \Phi \rangle) \), one predicts in any case a length scale \( R = \delta^{-1} \) in the millimeter range. This length scale would become infinitely large in the limit where the electroweak scale \( M_{EW} \equiv \sqrt{M_h \langle \Phi \rangle} \) is kept fixed and one takes the limit Eq.(20) where \( M_{\text{Planck}} \equiv c s \langle \Phi \rangle \to \infty \). This last phenomenological feature is also found in approaches involving extra ‘large’ space-time dimensions [28] although we have no obvious explanation for this analogy.

4. Summarizing: the Equivalence Principle does not fix uniquely the structure of space-time. In the alternative picture Eqs.(1)-(4), it becomes natural to look for the dynamical interpretation of the scalar field \( \tilde{\sigma}(x) \) associated with Newtonian gravity. Our proposal is to identify \( \tilde{\sigma}_{\text{EXP}} = -\frac{\phi_N}{4\pi} \) as a long-wavelength excitation of the same scalar condensate that induces spontaneous symmetry breaking for electroweak interactions. In this way one gets an alternative, but consistent, space-time description in weak gravitational fields where the differences between Eqs.(6) and (8) should presently be unobservable. We emphasize, however, that our picture would uniquely be singled out by accepting the arguments of refs.[3, 11] for a clear experimental evidence in favour of the Yilmaz metric Eq.(1).

In addition, our proposal represents a natural and ‘economical’ way to account for the gap-less mode of the singlet Higgs field Eq.(13). Its existence reflects the nature of the scalar condensate as a truly physical medium responsible for the deviations from exact Lorentz covariance in the long-wavelength part of the energy spectrum. For this reason, introducing alternative phenomenological frameworks, if possible, would certainly not represent the simplest logical possibility. Moreover, by taking seriously the basic idea that gravity is a long-wavelength excitation of the quantum field theoretical vacuum it will be possible to re-consider long-standing problems (quasar red-shifts, cosmological constant, missing mass, fifth force,...) from a new and unifying perspective. For instance, a hydrodynamical solution [24] of the mass discrepancy in galactic systems is equivalent to introduce deviations from that particular constant-density Poisson flow represented by Newtonian gravity. Such modifications are very natural [30] in an approach where gravity is associated with the density fluctuations of a physical medium.

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