Communication Models for Reconfigurable Intelligent Surfaces: From Surface Electromagnetics to Wireless Networks Optimization

This article provides an overview of communication models that are most often employed in wireless communications for analyzing and optimizing reconfigurable intelligent surfaces.

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ABSTRACT | A reconfigurable intelligent surface (RIS) is a planar structure that is engineered to dynamically control the electromagnetic waves. In wireless communications, RISs have recently emerged as a promising technology for realizing programmable and reconfigurable wireless propagation environments through nearly passive signal transformations. With the aid of RISs, a wireless environment becomes part of the network design parameters that are subject to optimization. In this tutorial article, we focus our attention on communication models for RISs. First, we review the communication models that are most often employed in wireless communications and networks for analyzing and optimizing RISs and elaborate on their advantages and limitations. Then, we concentrate on models for RISs that are based on inhomogeneous sheets of surface impedance and offer a step-by-step tutorial on formulating electromagnetically consistent analytical models for optimizing the surface impedance. The differences between local and global designs are discussed and analytically formulated in terms of surface power efficiency and reradiated power flux through the Poynting vector. Finally, with the aid of numerical results, we discuss how approximate global designs can be realized by using locally passive RISs with zero electrical resistance (i.e., inhomogeneous reactance boundaries with no local power amplification) even for large angles of reflection and at high power efficiency.

KEYWORDS | Electromagnetics; metasurfaces; modeling; optimization; reconfigurable intelligent surfaces (RISs); smart radio environments (SREs); wireless communications; wireless networks.

I. INTRODUCTION

The history of wireless communications started with understanding fundamental electric and magnetic phenomena, as well as with related experiments and inventions that were carried out during the last half of the 18th century and the first decades of the 19th century [1]. Wireless communications (often, just wireless) are defined as and are characterized by the transfer of information between two or more points without the need for using an electrical conductor as the medium to perform the transfer. The most common wireless technologies use

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electromagnetic waves. Thanks to the development and wide adoption of five wireless telecommunication standards and the recently started activities on the sixth generation of wireless systems and networks, we do live in a world of electromagnetic waves.

In our daily life, we observe plenty of concrete examples of electromagnetic phenomena, especially in the visible spectrum, for example, the visible light that is specularly reflected when it hits a smooth surface so that we can see ourselves in a mirror, the visible light that changes its route when traveling from one medium to another, which causes, e.g., the virtual distortion of objects in the water, or the visible light that creates complicated rainbow effects formed as a combination of reflection, refraction, and dispersion phenomena. These electromagnetic effects are governed by fundamental laws of physics and are, therefore, ultimately dictated by nature. More precisely, these examples of electromagnetic effects in the visible spectrum are determined by the interactions between the electromagnetic waves and the materials that are hit by them. When an arbitrary electromagnetic wave illuminates a material object, it excites oscillations of the charged particles that constitute the material. These oscillating particles act, in turn, as secondary sources that radiate electromagnetic waves into space, thus producing different wave phenomena. During hundreds of years of research in the field of electromagnetics, today, we do not only understand these phenomena, but we can control the electromagnetic waves, and we can even create new wave effects that go beyond those governed solely by nature [2].

The development of electromagnetics, which is often defined as the theory of electromagnetic fields and waves, has greatly helped us to qualitatively and quantitatively comprehend how the waves propagate and how they interact with material objects [3]. This understanding has inspired researchers to engineer and manufacture artificial electromagnetic materials with controllable material parameters, which, when illuminated by appropriate electromagnetic waves, are capable of realizing wave effects (or transformations) that do not exist in nature. Engineered materials of this kind are referred to as metamaterials, which are often broadly defined as an effective homogeneous material formed by an arrangement of engineered structural elements that are designed to achieve specified and unusual electromagnetic properties [4]. A typical example is constituted by a material that does not reflect the light in agreement with the law of reflection, i.e., the angle of reflection coincides with the angle at which the light illuminates the material but, according to the generalized law of reflection, i.e., the engineered material is capable of bending the light toward specified directions of reradiation, which are different from the angle of incidence [2].

Metamaterials are 3-D artificial (engineered) materials, which are usually bulky, heavy, and often difficult to be fabricated. Due to the inevitable material losses, metamaterials may strongly attenuate the electromagnetic waves that penetrate through them. One possible alternative to overcome the inherent limitations of metamaterials is the use of metasurfaces, which are electrically thin artificial layers with subwavelength inclusions [3]. Metasurfaces are often referred to as the bidimensional version of metamaterials, which, by virtue of the surface equivalence theorem, have the same capabilities of shaping the propagation of the electromagnetic waves that interact with them while being less bulky, lossier, and easier to be fabricated and to be deployed than metamaterials.

A. Programmable Wireless Environments

In current wireless telecommunication standards, different kinds of electromagnetic waves constitute the vehicle for enabling the transmission of information and for allowing users and devices to communicate. Therefore, equipping current wireless telecommunication standards or even designing a new wireless telecommunication standard with the inherent capability of controlling and shaping how the electromagnetic waves propagate in a complex wireless environment and how they interact with material objects (walls, buildings, and so on) would be beneficial [5]. Indeed, the potential application of metasurfaces in the context of wireless communication systems and networks has recently attracted the interest of wireless researchers and engineers. Examples of papers include [6]–[21]. A short technology note that summarizes recent developments and ongoing prestandardization activities is available in [22].

Current wireless systems utilize a variety of transmission technologies, communication protocols, and network deployment strategies. They include millimeter-wave communications, massive multi-input–multi-output (MIMO) systems, and ultradense heterogeneous networks. Currently, available solutions are often based on the deployment, design, and optimization of transmitters, receivers, and network infrastructure elements with power amplification and digital signal processing capabilities, as well as backhaul and power grid availability. Communication engineers usually design transmitters, receivers, network elements, and transmission protocols by assuming not to be able to control how the electromagnetic waves propagate through a wireless environment and how they interact with the material objects that exist in the considered environment. When an electromagnetic wave impinges, for example, upon a metallic wall or upon a glass window, the reflected and refracted waves are not directly controlled by the network operator but are determined by the properties of the electromagnetic waves and the constitutive elements of the material objects that interact with the electromagnetic waves. If the material objects in the wireless environment were coated with or were even made of metamaterials (engineered materials), we could control their interactions with the impinging electromagnetic waves, and we could appropriately shape them as desired. This would enable us to co-design and jointly optimize the electromagnetic waves emitted by the transmitters, how
they interact with the surrounding material objects, and how they are decoded by the receivers.

Metamaterial-coated wireless networks are an emerging design paradigm that is often referred to as programmable wireless environment or smart radio environment (SRE) [11]. An example of SRE is illustrated in Fig. 1. In a conventional wireless environment, the electromagnetic waves that are reflected or refracted by material objects are out of the control of the system designer. As shown in Fig. 1, the reflected electromagnetic waves reach the intended receiver with different phases that may partially cancel out. This phenomenon can be alleviated by equipping, whenever possible, the transmitters and receivers with multiple antennas or by deploying additional infrastructure elements with signal processing units, power amplifiers, and multiple radio frequency chains. In an SRE, on the other hand, the same material objects are coated with metamaterial sheets (i.e., metasurfaces) that shape the reradiated electromagnetic waves so that they reach the intended receiver with approximately the same phase. By codesigning the metamaterial sheets, the transmitters, and the receivers, the performance of wireless networks may be further improved.

B. Reconfigurable Intelligent Surfaces

In the context of wireless communication systems and networks, as exemplified in Fig. 1, the use of planar metamaterial structures or metasurfaces is receiving major attention from the wireless community (see [17] and [21]). The reason lies, as mentioned, in the reduced losses and less complex design of 2-D (either planar or conformal) metasurfaces compared with 3-D metamaterials. Broadly speaking, a metasurface is a metamaterial sheet of subwavelength thickness. Despite their negligible thickness compared with the wavelength of the electromagnetic waves, metasurfaces can be as powerful as metamaterials in terms of wave manipulations while avoiding some of their drawbacks. This is ensured by the surface equivalence theorem, which states that the electromagnetic fields excited by arbitrary sources located in a volumetric material sample can be equivalently created by surface currents enclosing the volume. Therefore, any metamaterial sample can be replaced by electrically thin metasurfaces that are engineered to produce the same scattered electromagnetic waves [3].

In wireless communications, the metasurfaces need to be reconfigurable, so as to ensure that they can shape the electromagnetic waves based on the network conditions. Wireless researchers have adopted different names to refer to a reconfigurable metasurface [22]. In this tutorial article, we adopt the term reconfigurable intelligent surface (RIS) since it is adopted by a recently established industry specification group (ISG) within the European Telecommunications Standards Institute (ETSI) [23]. Broadly speaking, an RIS is an engineered surface that is intelligent (or smart) because it is capable of: 1) applying wave transformations that go beyond those governed solely by nature and 2) being configured any time that the propagation and network conditions require it.

Compared with other technologies, RISs have advantages and limitations, as recently summarized in [22, Table 1]. Within the recently established ETSI-ISG on RISs, an RIS is usually defined as a nearly passive reconfigurable engineered surface that: 1) is implemented by using passive scattering elements; 2) does not require high-cost active components, such as power amplifiers; 3) does not possess sophisticated signal processing capabilities but only the necessary low-power electronic circuits for enabling its reconfigurability; and 4) is not equipped with multiple radio frequency chains for data transmission but requires a simple front end to receive and send control signals. These characteristics suggest that an RIS may be considered a sustainable and environmentally friendly technology solution. The absence of power amplifiers and digital signal processing capabilities naturally poses, how-
ever, important design and deployment challenges to be solved. This includes the impossibility of onboard channel estimation, signal regeneration, and amplification that are currently being tackled by wireless researchers and engineers [17], [21]. Let us consider the problem of channel estimation as an example. The absence of power amplifiers and radio frequency chains makes it not possible for an RIS to transmit pilot signals for channel estimation. Likewise, the absence of signal processing units makes it not possible for an RIS to detect the pilot signals emitted by other devices. Therefore, efficient methods need to be developed for channel estimation and for optimizing RISs based, for example, on partial channel state information, so as to reduce the excessive overhead for channel estimation [24]. Another recently proposed approach is based on developing hybrid RISs that are endowed with integrated communication and sensing capabilities [25].

In wireless communications, an RIS has many potential applications that go beyond its use to turn the environmental objects into digitally controllable smart scatterers, as shown in Fig. 1. Other applications include the design of multistream multiantenna transmitters with a single radio frequency chain (often called holographic surfaces and holographic MIMO) [26] and reconfigurable ambient backscatterers [27]. In general terms, an RIS is a candidate future wireless technology for controlling and shaping the electromagnetic waves in a dynamic and goal-oriented manner, possibly turning the wireless environment into a service [28], and for realizing new transceiver designs and network elements at a lower complexity and power consumption. System-level simulations for evaluating the performance gains offered by the deployment of RISs in a typical urban city served by a fifth-generation cellular network have recently been reported in [29].

The conceptual structure of an RIS is sketched in Fig. 2. As illustrated, an RIS is a planar surface that consists of an array of scattering elements, each of which can independently impose the required phase shift, and possibly an amplitude gain, on the incident electromagnetic waves. By carefully adjusting the phase shifts (and the amplitudes) of all the scattering elements, the reradiated electromagnetic waves can be shaped to propagate toward specified directions. Each RIS element may consist of multiple constitutive elements that are usually referred to as unit cells. The unit cells that constitute each single RIS element have, in general, different shapes and sizes. If the RIS elements are made of the same unit cells and if they are arranged on a spatially periodic array, the resulting RIS is a quasi-periodic structure and the interdistance between the RIS elements is usually referred to as the period of the metasurface. Once the unit cells of each RIS element are designed, the wave transformation that the RIS applies to the incident signals is fixed. The reconfigurability of the RIS is ensured by a network of tuning circuits and a biasing line that control the unit cells. For example, the tuning circuits in Fig. 2 may be positive-intrinsic-negative (PIN) diodes or voltage-controlled varactors. Depending on the control voltage applied throughout the biasing line, the scattering properties of the RIS are adapted to the channel and network conditions, making it a digitally controllable scatterer. The tuning circuit and the biasing line may either control each individual unit cell, or each RIS element individually, or even multiple RIS elements together. Making each unit cell reconfigurable through an independent tuning circuit offers finer control of the electromagnetic waves at the cost of higher implementation complexity and power consumption. Two examples of manufactured engineered metasurfaces are illustrated in Figs. 3 [30] and 4 [31]. The metasurface in Fig. 3 is an RIS made of 196 identical unit cells. Each unit cell is digitally controlled by four varactors, which determines the reflection properties of the unit cell. The metasurface in Fig. 4 is a nonreconfigurable engineered surface, whose elements comprise ten different unit cells. The sizes and arrangements of the ten unit cells are jointly designed to realize a perfect anomalous reflector toward a fixed angle of reradiation with high power efficiency. Interested readers may refer to [32, Sec. 7] for information about the implementation cost and power consumption of deploying RISs instead of multiple access points in millimeter-wave
networks. In [33, Fig. 2], in addition, a general discussion on the fundamental tradeoffs offered by an RIS in terms of scattering performance, power consumption, and area of each unit cell is presented. In general, the implementation cost and power consumption of an RIS are greatly determined by the operating frequency and by the complexity of the circuits that enable the reconfigurability of the RIS elements. Simple unit cell designs are often preferred to reduce the cost and the power consumption at the cost of some inherent performance tradeoffs. A comprehensive study has recently been conducted in [34].

There exist multiple methods for designing an RIS. Interested readers may consult [2] and [35]–[47] for further information. Two typical design methods are the following.

1) The first method is a one-step approach that departs from the design of an individual RIS element, which may or may not be made of multiple unit cells. In this method, the constitutive RIS element is designed in order to realize some predefined phase shifts (and possibly amplitude gains and losses) when a given electromagnetic wave impinges upon it. The RIS element may realize a discrete set of phase shifts, or the phase shift may be continuously controlled (as for the RIS in Fig. 3). The scattering properties, e.g., the phase and the amplitude of the reflection coefficient for reflecting surfaces, are usually characterized with the aid of full-wave numerical simulations. The outcome of this phase consists of defining the size, geometry, thickness, composite material, and control circuitry to realize the desired set of phases and gains. In wireless communications, this is referred to as the RIS alphabet [34]. This characterization is usually performed by applying locally (at the level of the RIS element) periodic boundary conditions, which mimic an infinite homogeneous surface whose constitutive elements are all identical. If the RIS element is made of a single unit cell, the periodic boundary conditions are applied at the unit cell level. If the RIS element is made of multiple unit cells, the scattering response of all the unit cells is jointly characterized. Further information on using periodic boundary conditions for designing an RIS and their inherent advantages and limitations are elaborated on in Section II (see Fig. 6). Once the electromagnetic characterization of the RIS element is complete, the RIS operates by joint optimizing the scattering response of the RIS elements in order to realize the desired wave transformations.

2) The second method is a two-step approach, whose first step consists of engineering the entire RIS surface as a whole. This first phase is a macroscopic design in which the surface position-dependent properties of the RIS are formulated in terms of the specific functionality (e.g., reflection, refraction, and beam splitting) or set of functionalities (e.g., joint reflection and refraction or dynamic switching between reflection and refraction) that the RIS needs to realize. The RIS is usually modeled as a location-dependent continuous sheet of electric surface impedance and magnetic surface admittance. This design method is further elaborated on in Section II, and it is embraced in Section III to illustrate the design and analysis of RISs in a step-by-step and tutorial-like manner. Once the electric surface impedance and magnetic surface admittance are determined, the second step consists of identifying the physical microscopic implementation of the unit cells for the entire RIS and the associated tuning circuits for realizing the electric surface impedance and magnetic surface admittance in practice. During this phase, typically, one departs from a unit cell design of a given shape and optimizes the sizes, interdistances, material, and control circuits of the entire RIS to obtain the target surface impedance and admittance (surface modulation). If the surface impedance and admittance are periodic functions in space, one can jointly optimize only the unit cells that constitute a single period.

The first design approach is inherently local (at the granularity of either the RIS element or the unit cell), while the second design is inherently global (the entire RIS is optimized). Usually, the second method has a higher complexity, but it typically results in superior performance. If correctly implemented, the second method accounts, in a more accurate manner, for the interactions (mutual coupling) among unit cells whose size and interdistance are smaller than half of the wavelength. In Section II, the advantages and limitations of these methods are discussed. Specifically, further details on the design and optimization complexity of local and global designs are given. In the rest of this article, for the avoidance of doubt, we utilize the term local design to refer to designs of RISs in which each unit cell is optimized individually. On the other hand, we utilize the term global design to refer to designs of RISs.
in which groups of unit cells are jointly optimized. With reference to Fig. 2, the local design may correspond to an RIS in which each RIS element comprises a single unit cell. The global design may correspond, on the other hand, to an RIS in which each RIS element comprises several unit cells that are jointly optimized. More in general, several RIS elements may be jointly optimized in a global design.

Before proceeding, it is instructive to discuss the classification of RISs in terms of their power expenditure, i.e., the difference between passive, nearly passive, and hybrid RISs, and the difference between periodic and aperiodic RISs.

1) Passive, Nearly Passive, and Hybrid RISs: In previous text, we have defined an RIS as a nearly passive surface. In this context, the term nearly passive is referred to an RIS whose unit cells cannot amplify the incident electromagnetic waves, but some power is needed for operating the electronic circuits that make an RIS reconfigurable. As a consequence, an RIS cannot be a passive device because, otherwise, it is not reconfigurable. Therefore, the RISs illustrated in Figs. 2 and 3 are not passive surfaces. Engineered surfaces without electronic circuits, and hence not reconfigurable, can, on the other hand, be passive surfaces. This definition of passive and nearly passive RISs is usually referred to each individual unit cell. Stated differently, each unit cell cannot amplify the incident electromagnetic waves, and the only power it needs is that necessary for ensuring its reconfigurability. Therefore, this definition usually applies without ambiguity to a local design. If several unit cells of an RIS are grouped and jointly optimized (e.g., the unit cells of one RIS element in Fig. 2), the notion of passive and nearly passive RISs needs to be refined. In this case, an RIS is defined as nearly passive, in a global sense, if the group of unit cells that are jointly optimized does not amplify the incident electromagnetic waves. This definition does not necessarily imply that each unit cell of the group does not locally amplify the incident electromagnetic waves. In fact, some unit cells may accept power and send it to some other unit cells, which may, in turn, radiate more power than that received locally from the incident electromagnetic wave, so that the total reradiated power is not greater than the total incidence power. From an implementation standpoint, a global nearly passive RIS can be realized either by explicitly deploying small amplifiers on the surface or by realizing passive mechanisms that enable the transfer of power from the unit cells that attenuate the incident electromagnetic waves toward the unit cells that amplify the incident electromagnetic waves. An example of this implementation is the RIS illustrated in Fig. 4, and further information can be found in [31]. Recently, finally, researchers have been investigating the design of hybrid RISs in which some (usually, a relatively small fraction of all the) unit cells amplify the incident electromagnetic waves so that the total reradiated power may be greater than the total incident power. These solutions require active power amplifiers and incur a higher cost and power consumption. However, they may facilitate critical tasks for the deployment of RISs in wireless networks, e.g., channel estimation, and they usually increase the transmission distance (coverage) thanks to the additionally available power [24].

2) Periodic Versus Aperiodic RISs: Why Unit Cells With Sizes and Interdistances Smaller Than Half of the Wavelength?: To better understand the design challenges and tradeoffs for RISs, the notion of periodicity deserves to be elaborated on in detail. For ease of understanding, let us start by comparing the two RISs illustrated in Figs. 3 and 4. In Fig. 3, we see that the surface is characterized by a geometric period that coincides with the size of each unit cell (including the associated electronic circuits) on the surface. The entire RIS can, hence, be viewed as a periodic surface whose geometric period is the size of the unit cell. In Fig. 4, on the other hand, the geometric period of the surface does not coincide with the size of one unit cell. In fact, the unit cells within one RIS element (i.e., ten unit cells in Fig. 4) have different sizes and interdistances. The geometric period of the RIS in Fig. 4 is given by a group of ten unit cells, which are repeated along the rows and columns of the surface. We observe, in addition, that no phase modulation is applied along the columns.
of the surface since the same unit cell is repeated along the columns. In an engineered surface, in fact, the shape and size of each unit cell determine the applied phase modulation. The comparison between Figs. 3 and 4 allows us to understand that the geometric period of an RIS may not necessarily coincide with the size of one unit cell, but it may encompass several unit cells. Also, the geometric period may be different along the two sides of an RIS: in Fig. 4, the geometric period in each column is one unit cell, and the geometric period in each row is ten unit cells.

Besides the geometric period just introduced, the design of an RIS depends on the specific function that it needs to realize, e.g., the reflection of an electromagnetic wave toward an angle different from the angle of incidence. Some wave transformations may result in configuring the unit cells of an RIS according to a repetitive pattern, which leads to a spatially periodic configuration of the unit cells. This concept is illustrated in Fig. 5. More precisely, Fig. 5(a) and (b) shows an example of aperiodic and periodic RISs, respectively. The figure shows an RIS whose geometric period is equal to $p$ along the rows and the columns. Also, the unit cells in Fig. 5(b) are configured according to a period $P$ that comprises several unit cells in each row similar to the RIS in Fig. 4. We refer to the period $P$ as the RIS period, which primarily depends on the function (or wave transformation) that an RIS needs to realize. RISs that realize wave transformations of this kind are referred to as periodic surfaces with period $P$. Otherwise, they are referred to as aperiodic surfaces. Sometimes, the same function (such as anomalous reflection) can be realized by both periodic and aperiodic surfaces.

The optimization of a periodic RIS may be easier than for an aperiodic RIS. The reason is that the optimization complexity of a periodic RIS scales with the number of unit cells in the period $P$, rather than with the total number of unit cells in the RIS. To employ optimization methods for periodic surfaces is, however, necessary that the ratio $P/p$ is a positive integer number. This implies that RISs that are realized as a periodic arrangement of unit cells with geometric period $p$ cannot, in general, be designed as periodic surfaces since it is not possible to obtain, for every wave transformation, a perfect periodic pattern on the surface. In Fig. 4, this issue is avoided since the authors have first identified the correct RIS period $P$ based on the function to realize, and they have then optimized the size and the number of unit cells within the RIS period $P$ to ease the design of the entire RIS (since only ten unit cells need to be tuned). The downside of this approach is that it is difficult to realize reconfigurable structures since the RIS period $P$ needs to be changed according to the specified wave transformation. This is typically not possible, or it is difficult to realize. On the other hand, the reconfigurable surface in Fig. 3 may be optimized under the assumption of being periodic. To do this, however, the RIS can be utilized to realize only the subset of wave transformations for which the ratio $P/p$ is a positive integer number. To better understand, let us consider an RIS that realizes the wave transformation of anomalous reflection, e.g., an electromagnetic wave is steered from an angle of incidence equal to $0^\circ$ toward an angle of reflection ($\theta_\text{r}$) equal to $75^\circ$. Assuming that the electromagnetic waves are plane waves, it is known from [17, eq. 16] that the RIS period of this wave transformation is $P = \lambda/\sin (\theta_\text{r}) = 1.0353\lambda$, where $\lambda$ is the wavelength. Let us assume that $p = \lambda/4$ as for the RIS in Fig. 3. We obtain $P/p = 4.1411$, which is not an integer number, and therefore, the RIS needs to be optimized as an aperiodic surface even though it realizes a periodic wave transformation. This is due to the finite value of the geometric period $P$: the smaller $p$ is, the finer the spatial resolution is, and the more likely an RIS can be treated as a periodic surface. If an RIS is (virtually) a continuous surface with $p \to 0$, then any RIS period $P$ can be, in theory, realized. This possibility of avoiding the need for global optimization can be viewed as a simple justification of the advantages of designing RISs with a geometric period smaller than $\lambda/2$. More generally, the use of subwavelength unit cells allows the control of the near-field distribution that is necessary for highly efficient implementations of RISs. Additional discussions can be found in [34] and [48].

### Table 1: Steering Angles $\theta_\text{r}$ That Can Be Realized by an Anomalous Reflector With the RIS Period $P = \lambda/\sin (\theta_\text{r})$ and the Geometric Period $p = \lambda/N$ With $N \geq 2$ Under the Assumption That the RIS Is Optimized as a Periodic Surface

| $N$ | $n = N + 1$ | $n = N + 2$ | $n = N + 3$ | $n = N + 4$ | $n = N + 10$ |
|-----|-------------|-------------|-------------|-------------|-------------|
| 2   | $\theta_\text{r} \approx 41.81^\circ$ | $\theta_\text{r} \approx 30.00^\circ$ | $\theta_\text{r} \approx 23.58^\circ$ | $\theta_\text{r} \approx 19.47^\circ$ | $\theta_\text{r} \approx 9.59^\circ$ |
| 3   | $\theta_\text{r} \approx 48.59^\circ$ | $\theta_\text{r} \approx 36.87^\circ$ | $\theta_\text{r} \approx 30.00^\circ$ | $\theta_\text{r} \approx 25.37^\circ$ | $\theta_\text{r} \approx 13.34^\circ$ |
| 4   | $\theta_\text{r} \approx 53.13^\circ$ | $\theta_\text{r} \approx 41.81^\circ$ | $\theta_\text{r} \approx 34.85^\circ$ | $\theta_\text{r} \approx 30.00^\circ$ | $\theta_\text{r} \approx 16.60^\circ$ |
| 5   | $\theta_\text{r} \approx 56.44^\circ$ | $\theta_\text{r} \approx 45.58^\circ$ | $\theta_\text{r} \approx 38.68^\circ$ | $\theta_\text{r} \approx 33.75^\circ$ | $\theta_\text{r} \approx 19.47^\circ$ |
| 6   | $\theta_\text{r} \approx 62.73^\circ$ | $\theta_\text{r} \approx 53.13^\circ$ | $\theta_\text{r} \approx 46.66^\circ$ | $\theta_\text{r} \approx 41.81^\circ$ | $\theta_\text{r} \approx 26.39^\circ$ |
| 16  | $\theta_\text{r} \approx 70.25^\circ$ | $\theta_\text{r} \approx 62.73^\circ$ | $\theta_\text{r} \approx 57.36^\circ$ | $\theta_\text{r} \approx 53.13^\circ$ | $\theta_\text{r} \approx 37.98^\circ$ |
| 32  | $\theta_\text{r} \approx 75.86^\circ$ | $\theta_\text{r} \approx 70.25^\circ$ | $\theta_\text{r} \approx 66.10^\circ$ | $\theta_\text{r} \approx 62.73^\circ$ | $\theta_\text{r} \approx 49.63^\circ$ |
angles. Also, the corresponding number of unit cells per RIS period \( P \), i.e., \( n \geq N + 1 \), needs to be large as well.

In conclusion, the design of RISs based on the RIS period \( P \) is a well-consolidated approach in the field of engineered surfaces [31]. This approach may, however, not be readily applicable to the design of reconfigurable surfaces since the sizes and the interdistances of the unit cells need to be adapted according to \( P \), which is, in turn, determined by the specified wave transformation to realize. In wireless communications, it is customary to treat a reconfigurable surface (i.e., an RIS) as a periodic arrangement, with geometric period \( p \), of unit cells and treat it as an aperiodic surface, regardless of whether the RIS needs to realize wave transformations that result in periodic or aperiodic configurations of the unit cells along the entire surface. This leads to the need for developing efficient and scalable optimization algorithms [20], [24], [34]. The impact of the geometric period \( p \), i.e., the spatial discretization of an RIS, has recently been analyzed in [34] with the aid of numerical simulations.

C. Research Opportunities and Challenges

RIS-empowered SREs are an emerging field of research in wireless communications with several open research issues to be tackled in order to quantify the gains that can be expected in realistic wireless network deployments. The major open research challenges have been addressed in many recent papers, e.g., [17], [20], and [21], and they encompass how to efficiently perform channel estimation, how to enable the control of an RIS, where to best deploy RISs, how to efficiently integrate RISs in system-level and ray tracing simulators, and so on. A major open research issue, in addition, consists of developing models for RISs that are electromagnetically consistent and sufficiently tractable for evaluating the performance and optimizing RIS-assisted wireless networks from signal- and system-level perspectives. A summary and comparison of currently available research efforts can be found in [49, Table 1]. The focus of this tutorial article is on this latter open research issue. More precisely, we aim to overview, in a tutorial manner, electromagnetically consistent communication models for RISs that are represented as thin sheets of electromagnetic material. Specifically, the focus of this tutorial article is on the differences and similarities between local and global design criteria for realizing anomalous reflectors, and how, departing from Maxwell’s equations, optimization problems for designing RISs with unitary power efficiency can be formulated and numerically solved.

D. Article Organization

The remainder of this tutorial article is organized as follows. In Section II, we overview the most widely used communication models for RISs. In Section III, we depart from models for RISs that fulfill Maxwell’s equations, and discuss and compare local and global designs for RISs. Also, we formulate optimization problems for designing RISs that are globally optimal and can be realized with purely reactive impedance sheets. In Section IV, numerical results are illustrated in order to quantitatively compare the different designs for RISs that are presented in the previous sections. Finally, Section V concludes this tutorial article.

Disclaimer: Since this article is a tutorial and not a survey paper, we limit ourselves to report only examples of research works that can guide the readers to retrieve further information on modeling, analyzing, and optimizing metasurfaces in general and RISs for wireless applications in particular. A more comprehensive reference list can be found in, e.g., [17] and [20].

II. MODELS FOR RISs WIDELY USED IN WIRELESS COMMUNICATIONS

In this section, we overview three communication models for RISs that have recently been proposed in the literature. The considered communication models are given as examples in order to clarify the modeling assumptions and the conditions under which they can be applied. The third model introduced in this section is further elaborated on in Section III with the aid of step-by-step examples and is utilized to formulate optimization problems for designing RISs. In order to keep the focus on the key aspects of the communication models for ensuring their electromagnetic consistency and validity, we consider an RIS that is deployed in a free-space propagation environment. Multipath propagation can be added to the considered channel model, as described in [50]–[52]. Readers who are interested in comparing the electromagnetically consistent models summarized in this article against conventional models utilized in the vast majority of research works in communications can consult [20] and [34, Sec. 4].

A. Locally Periodic Discrete Model

As mentioned in the previous section, a widely used model for RISs is based on a locally periodic design, in which periodic boundary conditions are applied at the unit cell level (see [38], [39], and [44]). In general, each RIS element is assumed to be comprised of several identical unit cells for reasons that are elaborated on next. To illustrate this communication model, which is widely utilized in wireless communications, we consider the analytical formulation in [33], which has been experimentally validated by the authors with the aid of measurements in an indoor environment. An early version of the same communication model is available in [53]. An in-depth evaluation of this model is presented in [34].

The RIS is modeled as illustrated in Fig. 2. For ease of description, we assume that: 1) each RIS element is constituted by a single unit cell; 2) all the unit cells have the same size and shape; and 3) the interdistance between adjacent unit cells is the same. Therefore, the RIS is modeled as a periodic arrangement of identical
unit cells. The scattering response of each unit cell is configured thanks to the tuning circuit and the biasing line, as illustrated in Fig. 2. We assume that there exist $M$ unit cells in each row and $N$ unit cell in each column of the surface. Therefore, the total number of reconfigurable unit cells is $MN$. The surface area of each unit cell is $d_xd_y$, with $d_x$ and $d_y$ being the horizontal and vertical sizes of each unit cell, respectively.

The RISs considered in [33] operate as reflecting surfaces, and therefore, each unit cell is characterized by a complex reflection coefficient, which is defined as the ratio between the reflected electric field and the incident electric field. We denote the reflection coefficient of the $(m,n)$th unit cell as $\Gamma_{m,n}$. Specifically, the RISs in [33] comprise unit cells that can apply two phase shifts (binary cells) depending on the configuration of the tuning circuit. For illustrative purposes, the values of the reflection coefficients are reported in Table 2. Further details on how these reflection coefficients are computed are given next.

In Tables 2 and 3, for completeness, we report two other examples of RISs that are modeled based on the same principle as the RISs considered in [33]. One of the examples reported in Table 2 considers the RIS introduced in [54], which can simultaneously reflect and refract the incident electromagnetic waves. For this reason, it is characterized by a reflection coefficient and a transmission coefficient, $T_{m,n}$, which is defined as the ratio between the refracted electric field and the incident electric field. Similar to the RISs in [33], the unit cells of the RIS in [54] can be configured in two different states that are characterized by the pairs $(\Gamma_1, T_1)$ and $(\Gamma_2, T_2)$. The other example reported in Table 2 is the RIS introduced in [55], which operates as a reflecting surface, but its unit cells can be configured in four different states. The RIS in Table 3 is modeled as a periodic array of unit cells, similar to the RISs in [33], [54], and [55]. Similar to [33] and [55], in addition, it operates only in reflection mode and is characterized by the reflection coefficient $\Gamma_{m,n}$. However, the reflection coefficient of each unit cell can be varied continuously as a function of a control voltage. Therefore, the phase shift applied by each unit cell can be tuned more finely.

In the five examples of RISs reported in Tables 2 and 3, we note that the amplitude and the phase of the reflection (and transmission) coefficient are not independent of each other. Also, the amplitude of the reflection coefficient is not unitary, and it is not independent of the phase shift. In general, in addition, the reflection and transmission coefficients reported in Tables 2 and 3 depend on the angle of incidence of the electromagnetic waves, as shown in [30, Fig. 2] and [36, Fig. 4]. The examples reported in the two tables are referred to as the canonical case of normal incidence. Interested readers are invited to consult [34] for an in-depth numerical evaluation and comparison of the reflection properties of the RISs in Tables 2 and 3 with focus on the impact of nonideal implementation constraints, i.e., the nonunitary amplitude of the reflection coefficients, the dependence between the amplitude and the phase of the reflection coefficients, and the nonconstant phase differences between the reflection coefficients of the RIS alphabet.

Assuming that the set of possible reflection coefficients (the RIS alphabet), as a function of the tuning circuit, of a single unit cell of the RIS is given, Tang et al. [33] have introduced an analytical model for computing the power observed at a given location of an RIS-assisted communication link. The RIS is assumed to be centered at the origin and to lie in the $xy$ plane (i.e., $z = 0$). The received power can be formulated as follows:

$$P_{\text{RX}} = G_{\text{RX}}^2 \frac{G_{\text{TX}}^2 (d_x d_y)^2}{16 \pi^2} \left( \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{F_{m,n} T_{m,n} G_{\text{TX}} G_{\text{RX}}}{F_{m,n} T_{m,n} G_{\text{TX}} G_{\text{RX}}} e^{-j \frac{2\pi}{\lambda} (r_{m,n} + r_{m,n})} \right)^2$$

where $P_{\text{RX}}$ is the received power at the antenna element of the receiver, $G_{\text{TX}}$ and $G_{\text{RX}}$ are the gains of the transmit and receive antennas, respectively, $d_x d_y$ is the size of the RIS surface, $M$ and $N$ are the number of rows and columns of the RIS, respectively, $\lambda$ is the wavelength of the transmitted electromagnetic waves, and $r_{m,n}$ is the distance between the $m$th row and $n$th column of the RIS and the antenna element of the receiver.
where

\[
F_{m,n} = \left( \frac{\left(\frac{p_{Tx}^m}{r_{mn}}\right)^2 + \left(\frac{p_{Rx}^m}{r_{mn}}\right)^2 - (d_{mn})^2}{2d_{0}^m r_{mn}^*} \right)^{-1+G(Tx)/2} \\
\times \left( \frac{z_{Tx}^m}{r_{mn}} \right) \left( \frac{z_{Rx}^m}{r_{mn}} \right) \\
\times \left( \frac{d_{0}^m}{2d_{0}^m r_{mn}} \right)^2 \left( \frac{r_{mn}}{r_{mn}} \right)^2 - (d_{mn})^2 \right)^{-1+G(Rx)/2}
\]  

(2)

and the following notation is used.

1) \(p_{Tx}^m\) and \(p_{Rx}^m\) are the transmitted and received powers, respectively.
2) \(G(Tx)\) and \(G(Rx)\) are the antenna gains of the transmitter and the receiver, respectively.
3) \(\lambda\) is the wavelength of the electromagnetic wave, and \(j = \sqrt{-1}\) is the imaginary unit.
4) \(r_{mn}\) is the distance between the transmitter and the center point of the \((m,n)\)th unit cell, and \(r_{mn}^*\) is the distance between the center point of the \((m,n)\)th unit cell and the receiver.
5) \(d_{mn}\) is the distance between the center point of the \((m,n)\)th unit cell and the center point of the RIS (i.e., the origin).
6) \(d_{0}^m\) is the distance between the transmitter and the center point of the RIS, and \(d_{0}^{Rx}\) is the distance between the center point of the RIS and the receiver.
7) \(z_{Tx}^m\) and \(z_{Rx}^m\) are the Cartesian coordinates of the transmitter and the receiver on the z-axis, respectively.

The analytical model in (1) has recently been generalized in [34] by capitalizing on scattering theory rather than utilizing antenna theory as in [33]. Interested readers are referred to [34] for further information.

By using (1), it is possible to formulate the received power at any locations of the transmitter and the receiver as a function of the location of the RIS and the configuration of the unit cells. Therefore, the optimal configuration of the MN unit cells of the RIS can be identified in order to, e.g., maximize the received power depending on the location of the receiver. More precisely, let \(\Gamma\) denote the \(M \times N\) matrix of reflection coefficients \(\Gamma_{m,n}\), and let \(\Gamma_{m,n} \in \{\Gamma_1, \Gamma_2, \ldots, \Gamma_\Sigma\}\) be the \(\Sigma\) possible reflection coefficients of each unit cell of the RIS, i.e., the RIS alphabet. In Tables 2 and 3, we have \(\Sigma = 2\) or \(\Sigma = 4\) and \(\Sigma = 14\). With this notation, a typical problem formulation reads as follows:

\[
\begin{align*}
\max_{\Gamma} & \quad F_{Rx}^{(\Gamma)} \\
\text{s.t.} & \quad \Gamma_{m,n} \in \{\Gamma_1, \Gamma_2, \ldots, \Gamma_\Sigma\} \quad \forall m,n.
\end{align*}
\]

(3a)

A simple algorithm for solving (3), which is based on the alternating optimization method, can be found in [34]. The main feature of the algorithm in [34] lies in its applicability to any RIS alphabet without imposing any prior assumptions on the structure of the reflection coefficients \(\Gamma_{m,n}\), e.g., the amplitude and phase of \(\Gamma_{m,n}\) are independent of one another, or the amplitude of \(\Gamma_{m,n}\) is unitary.

As mentioned, the set of \(\Sigma\) states in (3), i.e., the RIS alphabet, is determined by characterizing the electromagnetic response of the constituent unit cell of the RIS by employing a local design. In order to understand the applicability and accuracy of the received power model in (1), based on the solution of the optimization problem in (3), it is instructive to analyze in detail the meaning of local design at the unit cell level and the concept of periodic boundary conditions. To this end, we consider, as an example, a binary unit cell that can take only two states, i.e., \(\Sigma = 2\) and \(\Gamma_{m,n} \in \{\Gamma_1, \Gamma_2\}\).
The reflection coefficients \( \Gamma_1 \) and \( \Gamma_2 \) are obtained by utilizing the procedure sketched in Fig. 6. The reflection coefficient \( \Gamma_1 \) is estimated by considering an infinite-size RIS whose unit cells are all identical and the tuning circuits are set to the same configuration. Therefore, the RIS is effectively turned into an infinite and spatially homogeneous sheet with no phase variation along the entire surface. In this configuration, the surface reflection coefficient is well defined as the ratio between the tangential component of the reflected electric field and the tangential component of the incident electric field [56, Ch. 7]. Since the surface is spatially homogeneous and of an infinite extent, only specular reflection is allowed. The obtained structure is usually analyzed with the aid of full-wave electromagnetic simulators, which model the infinite size of the surface and the periodic repetition of the elementary unit cell by applying the so-called periodic boundary conditions (see [36, Fig. 4]). Thanks to this procedure, the reradiation characteristics of the unit cell in the first possible state are characterized by assuming that it is surrounded by a neighborhood of identical unit cells. This implies that the mutual coupling and the interactions among all the identical unit cells in the considered homogeneous sheet are inherently taken into account when characterizing \( \Gamma_1 \). The same procedure is repeated for estimating \( \Gamma_2 \), with the only difference being that the tuning circuits are set to the configuration that results in the reflection coefficient \( \Gamma_2 \).

Having characterized the reflection coefficients \( \Gamma_1 \) and \( \Gamma_2 \), if necessary as a function of the angle of incidence of the electromagnetic wave, the communication model in (1) stipulates that we may configure the state (either \( \Gamma_1 \) and \( \Gamma_2 \) in the example of Fig. 6) of each unit cell independently of the others and regardless of the states of the neighboring cells. An example is given in the right-hand side illustration of Fig. 6, in which the unit cells are configured to realize beam splitting [34], [38, Fig. 4]. However, caution needs to be paid when (1) is utilized, and the reflection coefficients \( \Gamma_1 \) or \( \Gamma_2 \) are obtained by applying locally periodic boundary conditions at the unit cell level. The reflection coefficients \( \Gamma_1 \) and \( \Gamma_2 \) are, in fact, determined by assuming that a unit cell configured in a given state is surrounded by an infinite (homogeneous) repetition of identical unit cells. When an RIS is configured to operate in practice, as sketched in the right-hand side illustration of Fig. 6, each unit cell is, however, immersed in a spatially inhomogeneous array whose neighboring unit cells can be all different from one another. This implies that the spatial symmetry imposed by the periodic boundary conditions does not hold anymore, and the interactions (mutual coupling) among nearby unit cells are taken into account only in an approximate manner. In addition, a practical RIS is not of infinite extent but it has a finite size even though it can be electrically (very) large. This implies that the notion of the reflection coefficient is only an approximation, and it holds only under the limit of physical optics [57]. For these reasons, the unit cells are not typically optimized individually and independently of one another, but they are optimized in groups (often called macrocells), so as to ensure that the periodic boundary conditions utilized when characterizing each unit cell individually are approximately fulfilled during the normal operation of the RIS [44], [58]. The right-hand side illustration of Fig. 6 is a typical example in which the unit cells are split into groups, each containing 24 unit cells, and the groups are optimized such that the states (\( \Gamma_1 \) or \( \Gamma_2 \)) of all the unit cells in a group are the same. The minimum required size of the group of unit cells for ensuring that (1) is accurate enough for wireless applications is usually characterized with the aid of full-wave simulations.

### B. Mutually Coupled Antenna Elements

The communication model for RISs introduced in Section II-B is widely employed in wireless communications, and several optimization frameworks, under some
simplifying assumptions, have been proposed based on it [17], [20], [34]. The local design at the unit cell level is a widely used method for characterizing the reflection and transmission characteristics of an RIS. As mentioned, however, the mutual coupling among the unit cells is only approximately taken into account since the reflection coefficients of each unit cell (i.e., the RIS alphabet) are typically characterized by applying periodic boundary conditions at the unit cell level [58]. The accuracy of the model in (1) can be improved by, e.g., not characterizing the reradiation properties of each unit cell individually but by analyzing, with full-wave simulations, the reradiation of groups of unit cells as a function of all the possible combinations of their states [59]. In this case, periodic boundary conditions may be applied at the granularity of a group of unit cells in lieu of a single unit cell. The accuracy of this enhanced model is usually improved but at the expense of increasing the modeling and optimization complexity.

Gradoni and Di Renzo [60] have recently introduced a communication model for RISs that explicitly account for the mutual coupling among the RIS elements and for the control circuit of the unit cells. The communication model in [60] is based on the theory of mutually coupled antennas and is directly applicable in multiple antenna communication systems since it resembles a MIMO communication channel. In [51] and [61], it has recently been shown that the model is suitable for formulating optimization problems in general wireless networks, such as the MIMO interference channel, and that it can be utilized to optimize an RIS by explicitly taking into account the mutual coupling among the RIS elements. In this section, we first introduce the communication model in [60], and we then elaborate on the assumptions under which it is developed and, hence, the conditions under which it can be utilized.

The RIS-assisted communication model introduced in [60] is illustrated in Fig. 7. The model resembles a conventional single transmitter–receiver pair MIMO communication link in the presence of an RIS. The transmitter and the receiver are equipped with \( M_0 \) and \( L_0 \leq M_0 \) antenna elements, respectively. The antenna elements are assumed to be thin wire dipoles of perfectly conducting material. The model can be generalized for application to radiating elements different from thin wire dipoles, which are considered in [60] for analytical tractability. Each thin wire dipole at the transmitter is driven by a voltage generator that models the transmit feed line, and each thin wire dipole at the receiver is connected to a load impedance that mimics the receive electric circuit. For simplicity, we assume that the number of symbols (streams) sent by the transmitter is equal to the number of receive antennas. The transmission between the transmitter and the receiver is assisted by an RIS, which comprises \( P \) nearly passive thin wire dipoles that are independently configurable (by an external controller) through tunable impedances. Compared with the illustration of the RIS in Fig. 2, a thin wire dipole in Fig. 7 can be viewed as an approximation for a unit cell. The model can be generalized to different physical structures for the unit cells, e.g., patch antennas. The physical model based on dipoles is considered in [60] because relatively simple analytical or integral expressions for the current distribution of closely spaced thin wire dipoles are available in the literature [56, Ch. 25].

Based on the system model in Fig. 7, an RIS-assisted channel is optimized by appropriately setting the tunable impedances connected to the thin wire dipoles of the RIS. More specifically, Abrardo et al. [60] have introduced an \( L_0 \times M_0 \) end-to-end channel matrix that formulates the voltage measured at the ports of the receive antennas as a function of the voltage generators connected to the ports of the transmit antennas, i.e., \( v_{\text{Rs}} = H v_{\text{Tx}} \), where \( v_{\text{Tx}} \) is the \( M_0 \times 1 \) vector that collects the driving voltages at the transmitter, \( v_{\text{Rs}} \) is the \( L_0 \times 1 \) vector that collects the voltages measured at the ports of the antennas at the receiver, and \( H \) is the \( L_0 \times M_0 \) channel matrix that accounts for the radiating elements (the thin wire dipoles) and the propagation of the electromagnetic waves. From [60, Th. 1], \( H \) can be formulated as

\[
H = \left( I_{L_0} + \Psi_{t,r} Z_{r}^{-1} - \Psi_{t,t} (\Psi_{t,t} + Z_t)^{-1} \Psi_{t,r} Z_r^{-1} \right)^{-1} + \Psi_{t,t} (\Psi_{t,t} + Z_t)^{-1} \tag{4}
\]

where \( I_{L_0} \) is the \( L_0 \times L_0 \) identity matrix, and \( Z_t \) and \( Z_r \) are the \( M_0 \times M_0 \) and \( L_0 \times L_0 \) diagonal matrices that comprise the internal impedances of the transmit generators and the load impedances of the receive antennas, respectively. Furthermore, the following shorthand notation is introduced:

\[
\begin{align*}
\Psi_{t,t} &= Z_{t,t} - Z_{t,s}(Z_{s,s} + Z_{\text{run}})^{-1} Z_{s,t} \tag{5} \\
\Psi_{t,r} &= Z_{t,r} - Z_{t,s}(Z_{s,s} + Z_{\text{run}})^{-1} Z_{s,r} \tag{6} \\
\Psi_{t,s} &= Z_{t,s} - Z_{t,s}(Z_{s,s} + Z_{\text{run}})^{-1} Z_{s,t} \tag{7} \\
\Psi_{r,s} &= Z_{r,s} - Z_{r,s}(Z_{s,s} + Z_{\text{run}})^{-1} Z_{s,r} \tag{8}
\end{align*}
\]

where \( Z_{x,y} \), for \( x,y \in \{t,s,r\} \) with \( t, s, \) and \( r \) identifying the transmitter, the RIS, and the receiver, respectively, is the matrix of mutual (or self if \( x = y \)) impedances between the thin dipoles of \( x \) and the thin dipoles of \( y \), which characterizes the signal propagation and the mutual coupling between \( x \) and \( y \), and \( Z_{\text{run}}^{(k)} \) is the \( P \times P \) diagonal matrix of tunable impedances of the RIS. The matrices \( Z_{x,y} \) for \( x,y \in \{t,s,r\} \) account for the microscopic structure of the RIS and the locations of the transmitter, RIS, and receiver. They can be either computed with the aid of full-wave simulators or can be computed analytically by relying on some approximations. For example, Abrardo et al. [60] have used the induced electromagnetic field method for computing the mutual and self impedances, as well as a sinusoidal approximation for the current distribution on the thin wire dipoles. Under these modeling assumptions,
given the RIS microstructure and the system topology, the impedances in \( Z_{x,y} \) for \( x, y \in \{t, s, r\} \) need to be computed only once and are not usually considered optimization variables in the context of wireless communication systems.

On the other hand, the matrix \( Z_{\text{tun}} \), which ensures the reconfigurability of the RIS, is the matrix to be optimized for steering the electromagnetic wave that is emitted by the transmitter and impinges upon the RIS toward the location of the receiver. Let us consider, for example, that the transmitter and the receiver are equipped with a single antenna, i.e., \( M_0 = L_0 = 1 \), and the objective is to maximize the power at the location of the receiver. Then, the matrix \( H \) is a scalar, i.e., \( v_{rx} = H v_{tx} \), and the optimization problem as a function of \( Z_{\text{tun}} \) can be formulated as follows:

\[
\max_{Z_{\text{tun}}} |H(Z_{\text{tun}})|
\]

s.t. \( Z_{\text{tun},p} \in \{Z_1, Z_2, \ldots, Z_\Xi\} \) \( \forall p = 1, 2, \ldots, P \) \hspace{1cm} (9a)

where \( Z_{\text{tun},p} \) is the \( p \)-th element of \( Z_{\text{tun}} \) and \( \{Z_1, Z_2, \ldots, Z_\Xi\} \) is the set of \( \Xi \) possible discrete values of the tuning impedances that can be implemented.

Qian and Di Renzo [61] have recently solved the optimization problem in (9) under the assumption that the real part of the tunable impedances is fixed and greater than zero, and its imaginary part can be any real value. The constraint that the real part of the impedances is greater than zero ensures that the RIS does not amplify the incident electromagnetic wave, and therefore, no power amplifiers are needed. In fact, a negative resistance is equivalent to the need for using a power amplifier. Abrardo et al. [51] have formulated a more complex optimization problem that maximizes the rate of a MIMO interference channel.

The matrices defined in (5)–(8) have a physical meaning and interpretation. For example, \( \Psi_{t,s} \) represents the transfer matrix (the channel) between the transmitter and the receiver, which accounts for the direct link (\( Z_{t,s} \)) and the RIS-radiated link (\( Z_{s,s} + Z_{\text{tun}} \)). This latter term is the product of three factors: \( Z_{s,s} \) represents the transfer function from the transmitter to the RIS, \( Z_{s,s} \) represents the transfer function from the RIS to the receiver, and \( (Z_{s,s} + Z_{\text{tun}})^{-1} \) models the reradiation from the RIS. The matrix \( Z_{s,s} \) is, in general, a full matrix, which reduces to an almost diagonal matrix, i.e., the amplitudes of the elements in the main diagonal are much larger than the amplitudes of the off-diagonal elements, if the mutual coupling between the thin wire dipoles is negligible.

The channel matrix in (4) can be simplified in several scenarios of practical relevance. If, for example, the transmitter, the receiver, and the RIS are in the far-field of each other, (4) can be simplified without ignoring the mutual coupling among the thin dipoles. The self impedances \( Z_{x,x} \) are, in fact, independent of the transmission distances of the transmitter–receiver, transmitter–RIS, and RIS–receiver links, and they depend only on the interdistances between the thin wire dipoles at the transmitter, the RIS, and the receiver. In the far-field region, thus, the following simplifications can be applied:

\[
\Psi_{t,t} \approx Z_{t,t}
\]
\[
\Psi_{t,r} \approx Z_{t,r}
\]
\[
\Psi_{r,r}Z^{-1}_{r} - \Psi_{r,t}(\Psi_{t,t} + Z_{r})^{-1}\Psi_{t,r}Z^{-1}_{r} \approx \Psi_{r,r}Z^{-1}_{r}
\]

In the far-field region, therefore, \( H \) in (4) can be approximated as follows:

\[
H_{r,t} \approx (I_{t,s} + Z_{t,s}Z_{t,s})^{-1}(Z_{t,t} + Z_{r})^{-1} - (I_{s,s} + Z_{s,s}Z_{s,s})^{-1} \Psi_{s,s}(Z_{s,s} + Z_{\text{tun}})^{-1}Z_{t,t}^{-1} * (Z_{t,t} + Z_{r})^{-1}
\]

In (13), it is not difficult to recognize that the first addend on the right-hand side corresponds to the direct link between the transmitter and the receiver, and the second addend on the right-hand side corresponds to the RIS-radiated link that accounts for the internal impedances of the voltage generator at the transmitter, the load impedances at the receiver, the transfer matrices \( Z_{t,s} \) and \( Z_{s,s} \) that characterize the propagation of the electromagnetic waves from the transmitter to the RIS and from the RIS to the receiver, respectively, and the term \( (Z_{s,s} + Z_{\text{tun}})^{-1} \) that accounts for the mutual coupling and the tuning circuits of the RIS.

By careful inspection of the RIS-radiated link in (13), we can elaborate in deeper detail on the meaning of far-field approximation applied to (4). In (13), the transfer matrices \( Z_{t,s} \) and \( Z_{s,s} \), which characterize the propagation of the electromagnetic waves from the transmitter to the RIS and from the RIS to the receiver, respectively, depend on the distances between the transmitter and each thin wire dipole of the RIS, and between each thin wire dipole of the RIS and the receiver, respectively. As a consequence, the RIS is not viewed as one single point by the transmitter and the receiver, as it is the case, on the other hand, in the Fraunhofer far-field region of the entire surface [62]. Specifically, the far-field approximation considered in (13) is applied to the links between each thin wire dipole of the transmitter and each thin wire dipole of the RIS, as well as to the links between each thin wire dipole of the RIS and each thin wire dipole of the receiver. On the other hand, it is not applied to the links between pairs of thin wire dipoles colocated at the transmitter, colocated at the RIS, and colocated at the receiver, in order to account for the mutual coupling between these elements. Therefore, the approximation in (13) may be sufficiently accurate in several practical wireless scenarios while simplifying the computational complexity of (4). We observe, in fact, that (4) well resembles (1), which can be applied to the near- and far-field regions of the entire RIS.

The communication model in (4), and especially the simplified version in (13), is relatively simple to use in wireless communication systems thanks to the resem-
blance of \( \mathbf{H} \) with a typical MIMO channel model. It is necessary to understand, however, the assumptions and the conditions under which (4) can be applied. Besides the computation of the matrix \( Z_s \), that, if done analytically, usually requires some approximations, the main assumption made to obtain \( \mathbf{H} \) in (4) lies in the considered expressions of the surface currents along the thin wire dipoles. Regardless of whether the wire dipoles operate in transmission, scattering, or reception modes, specifically, it is assumed that their surface current is a sinusoidal function [60, eq. (2)], which fulfills the property to be equal to zero along the entire antenna element if it is open-circuited, i.e., the load impedance connected to the port of the antenna element is equal to infinity. Specifically, the model introduced in [51] assumes that the surface currents in transmission, scattering, and reception modes have a sinusoidal shape whose amplitude evaluated at the port of the antenna element is an unknown variable that is determined by the parameters of the transmitter, RIS, and receiver, e.g., the tunable load impedances of the RIS, upon imposing appropriate boundary conditions. In the case of thin wire dipoles, the tangential component of the electric fields is imposed to be equal to zero on the surface of the dipole. In [51], stated differently, the antenna elements are assumed to be canonical minimum scattering antennas. In simple terms, this assumption implies that a radiating element (a thin wire dipole in Fig. 7) does not radiate if it is open-circuited, and therefore, it is like if it is not present (it is “invisible”) in the network [63]. Concretely, this implies that the \( p \)th thin wire dipole that constitutes the RIS in Fig. 7 does not reradiate in the presence of an electromagnetic wave if \( Z_{\text{run},p} \to \infty \) (i.e., the current flowing in the antenna port of the thin wire dipole is zero), and therefore, it can be removed from the system model. This statement can be readily verified by direct inspection of the simplified channel model in (13): we see, in fact, that the second added on the right-hand side of (13), i.e., the contribution of the RIS, tends to zero if \( Z_{\text{run},p} \to \infty \) for every antenna element. This behavior is, of course, an approximation since the presence of a thin wire dipole in the environment always perturbs (even if it is open-circuited) the electromagnetic field. In general, in fact, the current at the port of a thin wire dipole is equal to zero if \( Z_{\text{run},p} \to \infty \), but it is not necessarily identically equal to zero along the entire thin dipole when it operates in scattering or reception modes. Therefore, caution needs to be paid when applying the communication model in (4) to arbitrary antenna elements since the approximation of canonical minimum scattering antennas needs to be evaluated. In addition, the communication model in (4) is derived under the assumption that the shape (a sinusoidal function) of the surface current density of each thin wire dipole is not influenced by the proximity of the other radiating elements.

C. Inhomogeneous Sheets of Surface Impedance

In this section, we consider models for RISs that abstract their microscopic structure and are focused on the specific wave transformations that the metasurface, as a whole, is intended to realize. More precisely, a metamaterial-based RIS whose unit cells have sizes and interdistances much smaller than the wavelength is homogenizable and can be modeled as a continuous surface sheet through appropriate surface functions, e.g., surface impedances [17], [36], [43], [45]–[47], [49], [58], [59], [62], [64], [65]. This modeling approach is not dissimilar from the characterization of bulk (3-D) metamaterials that are usually represented through effective permittivity and permeability functions that determine the wave phenomena based on Maxwell’s equations. The only difference is that a metasurface is better modeled by effective surface parameters, which manifests themselves in electromagnetic problems that are formulated as effective boundary conditions. These boundary conditions can be expressed in terms of surface polarizabilities, surface susceptibilities, or surface impedances (or admittances) [3]. In this tutorial article, we focus our attention on modeling an RIS through surface impedances.

The adopted modeling approach is, specifically, referred to as macroscopic [17], [49], [64]. Classical wave phenomena in materials or metamaterials are determined by the collective effects of a very large number of atoms that interact with the incident electromagnetic waves. The electromagnetic fields around individual atoms can be described by microscopic Maxwell’s equations. If the sizes of the atoms that constitute the material and the distances between them are much smaller than the wavelength, the electromagnetic fields and the sources in the material can be spatially averaged, thus effectively transforming microscopic Maxwell’s equations into macroscopic Maxwell’s equations. For a metasurface-based RIS, the same principle applies: if an RIS is electrically large and is made of subwavelength reconfigurable scattering elements (unit cells) whose interdistances are much smaller than the wavelength, it is homogenizable and can be modeled through continuous surface averaged (macroscopic) surface impedances. Specifically, two conditions need to be fulfilled to make an RIS homogenizable [46, Sec. 2.1]:
1) the first homogenization condition requires that the incident field varies little over one geometric period (the largest interdistance among the unit cells) of the RIS, i.e., $\max \{d_x, d_y\} \ll \lambda$ and 2) the second homogenization condition requires that the evanescent field scattered by the RIS is negligible at the observation point, i.e., $|z| > \max \{d_x, d_y\}$ for the RIS in Fig. 8, where $\lambda$ is the wavelength of the electromagnetic wave, and $d_x$ and $d_y$ are the horizontal and vertical sizes of its unit cells (assuming that all the unit cells are identical in size). The second condition is typically fulfilled in the far-field of the RIS microstructure [17, Fig. 29], i.e., for observation distances of interest in wireless communications.

Under these assumptions, an RIS can be modeled as an inhomogeneous sheet of polarizable particles (the unit cells) that is characterized by an electric surface impedance and a magnetic surface admittance, which, for general wave transformations, are dyadic tensors. These two dyadic tensors constitute the macroscopic homogenized model of an RIS. The average total electric and magnetic fields that illuminate an RIS induce electric and magnetic currents that introduce a discontinuity between the electromagnetic fields on the two sides of an RIS (below and above the surface), which provides the means for manipulating the wavefront of the incident electromagnetic waves. Once the homogenized and continuous electric surface impedance and magnetic surface admittance are obtained based on the desired wave transformations, the microscopic structure and physical implementation of the RIS in terms of unit cells are obtained by using, e.g., the method described in [17] and [43]. Generally speaking, once the macroscopic surface impedance and admittance are determined, appropriate geometric arrangements of subwavelength unit cells and the associated tuning circuits that exhibit the corresponding electric and magnetic response are characterized by, typically, using full-wave electromagnetic simulations [36].

Based on this modeling approach, an RIS is characterized by a set of algebraic equations that result in boundary conditions for the electromagnetic fields at the two sides of the surface. This set of equations is referred to as generalized sheet transition conditions [66], [67]. Under the assumption that only the tangential components of the electric and magnetic polarization densities are induced in the metasurface, and the RIS lies in the $xy$ plane (i.e., $z = 0$), as illustrated in Fig. 8, the generalized sheet transition conditions can be formulated as follows [36]:

$$
E_{tot} (x, y, z = 0^+) + E_{tot} ^t (x, y, z = 0^-) = 2\vec{Z}_{se} (x, y) (\hat{z} \times H_{tot} (x, y, z = 0^+)) - 2\vec{Z}_{se} (x, y) (\hat{z} \times H_{tot} (x, y, z = 0^-))
$$

$$
H_{tot} (x, y, z = 0^+) + H_{tot} ^t (x, y, z = 0^-) = -2\vec{Y}_{sm} (x, y) (\hat{z} \times E_{tot} (x, y, z = 0^+)) + 2\vec{Y}_{sm} (x, y) (\hat{z} \times E_{tot} (x, y, z = 0^-))
$$

where $\vec{Z}_{se} (x, y)$ and $\vec{Y}_{sm} (x, y)$ are the electric surface impedance and the magnetic surface admittance dyadic tensors that constitute the homogenized macroscopic model of an RIS. In addition, the following definitions for the electric and magnetic fields in (14) and (15) hold:

$$
F_{tot} (x, y, z = 0^+) = F_{inc} (x, y, z = 0^+) + F_{ref} (x, y, z = 0^-)
$$

$$
F_{tot} (x, y, z = 0^-) = F_{tra} (x, y, z = 0^+)
$$

$$
F_{tot} (x, y, z = 0^+) = (\hat{z} \times F_{tot} (x, y, z = 0^+)) \times \hat{z}
$$

where $\hat{z}$ is the unit norm vector that is normal to the RIS as illustrated in Fig. 8, and $F_{inc} (x, y, z = 0^+)$, $F_{ref} (x, y, z = 0^-)$, and $F_{tra} (x, y, z = 0^+)$ with $F = \{E, H\}$ are the incident, reflected, and transmitted (refracted) electric and magnetic fields evaluated on the two sides of the RIS, respectively.

The equations in (14) and (15) completely characterize an RIS in terms of wave transformations, and they can be utilized for the analysis and synthesis of an RIS. As far as the analysis is concerned, it is usually assumed that $\vec{Z}_{se} (x, y)$ and $\vec{Y}_{sm} (x, y)$ are known, and one is interested in solving (14) and (15) for obtaining the surface electric and magnetic fields in the close vicinity of the RIS, but at distances at which the homogenized model can be applied (as detailed in further text). As far as the synthesis is concerned, it is usually assumed that either the surface electric and magnetic fields in the close vicinity of the RIS are explicitly known or an objective function that depends on them is known, and one is interested in identifying the corresponding functions $\vec{Z}_{se} (x, y)$ and $\vec{Y}_{sm} (x, y)$ that provide the desired electromagnetic fields or maximize the objective function of interest. These techniques are referred to as direct and inverse source problems, respectively [68]. In Section III, we present some examples to understand the optimization of RISs as a function of the surface impedance and for different design criteria.

One of the main advantages of modeling an RIS through inhomogeneous sheets of impedance and admittance dyadic tensors lies in the possibility of incorporating them into Maxwell’s equations by leveraging the equivalence principle and the radiation integrals, which allows us to express the electric and magnetic fields anywhere in the volume of interest directly as a function of $\vec{Z}_{se} (x, y)$ and $\vec{Y}_{sm} (x, y)$ [56, Ch. 18]. The main assumption for using this approach consists of resorting to the physical optics approximation [57, Ch. 8]. Even though some approximations are usually needed to obtain the radiated electromagnetic field, the resulting analytical framework is electromagnetically consistent and accounts for the physical implementation of the RIS. This procedure is elaborated on in Section III through some specific examples, and the resulting analytical formulation is utilized for analyzing the radiated electromagnetic field. Another main advantage of the model for RISs based on inhomogeneous
sheets of impedance and admittance dyadic tensors is that the mutual coupling among all the constitutive elements of the RIS (the unit cells) is inherently taken into account since the model inherently abstracts, at the design stage, the physical implementation of the surface. The subsequent discretization of the RIS in unit cells based on the functions $Z_{se}(x,y)$ and $Y_{sm}(x,y)$ implicitly accounts for the local interactions and the mutual coupling among the unit cells [17], [59].

When modeling an RIS as an inhomogeneous sheet of impedance and admittance dyadic tensors, caution needs to be paid, however, to some implicit assumptions that are made. One of these assumptions is that an RIS is modeled as a device with zero thickness. In practice, however, an RIS has a finite thickness and is made of discrete and finite-size unit cells. In order for the homogenized (continuous) version of an RIS, which is utilized at the design stage, to accurately represent the reradiation properties of the manufactured metasurface, it is necessary that the thickness of the surface and the cross section of the unit cells are much smaller than the wavelength of the electromagnetic waves. If these conditions are met, the components of the surface fields that are related to the discretization of the surface in unit cells can be ignored, provided that the observation point is not too close to the surface of the RIS. As a rule of thumb, the observation point should be at least at a distance $|s| > t/2 + \max\{d_x, d_y\}$, where $t$ is the thickness of the surface, and $d_x$ and $d_y$ are the horizontal and vertical sizes of its unit cells (assuming that all the unit cells are identical in size) [36]. In practice, this condition does not pose any constraints in the context of wireless communication systems since we are not usually interested in observation points that are so close to an RIS.

Due to these positive features, representing an RIS as an inhomogeneous sheet of surface impedance constitutes a suitable abstraction model for understanding the achievable performance limits of RISs in wireless networks and for their optimization as a function of different design criteria. In Section II, we utilize this modeling approach and elaborate on how it can be leveraged for obtaining electromagnetically consistent analytical frameworks for RISs that are suitable for performance evaluation and for wireless network optimization. In the next section, more specifically, we focus our attention on an RIS that operates in reflection mode and is impenetrable, i.e., the electric and magnetic fields at $z = 0^-$ are equal to zero in (17). In this case, an RIS is referred to as an inhomogeneous boundary of surface impedance.

### D. Comparison of the Three RIS Models

The three models for RISs described in Sections II-A–II-C are developed under different modeling assumptions, and therefore, their accuracy against full-wave electromagnetic simulations is, in general, different. A summary of the main features and modeling assumptions for each model is available in Table 4 by considering the canonical example of reflecting surfaces for ease of description. The locally periodic discrete model relies on the assumption that the

| RIS model | Main features and assumptions |
|-----------|-------------------------------|
| Locally periodic discrete model | - The incident electromagnetic waves are assumed to be reflected, at any point of the RIS, specularly |
| | - The RIS alphabet can be characterized for an individual unit cell |
| Mutually coupled antenna elements | - The mutual coupling among the RIS elements is explicitly modeled |
| | - The shape of the surface currents of each radiating element is assumed not to be influenced by the proximity of the other radiating elements |
| | - In [60], the shape of the surface currents is assumed to be sinusoidal for transmitting and receiving antennas, and for the RIS elements |
| Inhomogeneous sheets of surface impedance | - The RIS is modeled “as a whole” surface |
| | - Impedance matching between the incident and reflected electromagnetic waves is ensured by design |
| | - The incident electromagnetic waves are not assumed to be reflected, at any point of the RIS, specularly |

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incident electromagnetic waves are reflected, at any point \((x_0, y_0)\) of the RIS, specularly, even though a different reflection amplitude and phase are applied at each point. In other words, the reflection coefficient is defined as if the RIS were an infinite and locally homogeneous surface. For this reason, this model for RISs may not always be accurate. The main advantage of this model is that the RIS alphabet can be characterized for an individual unit cell. The model based on mutually coupled antenna elements has the advantage of explicitly modeling the mutual coupling among the unit cells of the RIS and of being formulated in terms of voltages at the input and output ports of the transmitter and the receiver, respectively. This ease the utilization of the model for wireless network optimization. The model proposed in [60] relies on surface currents that are assumed to have a simple sinusoidal shape when the RIS elements operate in transmission, scattering, and reception modes. The surface currents depend, in general, on the specific radiating/scattering elements, on the electronic circuits that control the radiating/scattering elements, and whether the devices operate in transmission, scattering, or reception modes. In addition, it is assumed that the shape of the surface currents of each radiating element is not influenced by the proximity of the other radiating elements. The RIS model based on inhomogeneous sheets of surface impedance has the advantage of modeling an RIS “as a whole,” and not relying on the assumption that the incident electromagnetic waves are reflected, at any point \((x_0, y_0)\) of the RIS, specularly. Since the RIS is modeled as a whole surface, the reflection at any point of the surface can be nonspecular. This is illustrated in Fig. 9, and it is further elaborated on in Section III. This RIS model ensures that the impedances of the incident and reflected electromagnetic waves are matched since the fulfillment of the boundary conditions in (14) and (15) is imposed by design. In addition, this RIS model can be easily integrated into Maxwell’s equations, under the assumptions of the physical optics approximation, i.e., the currents induced on a finite-size metasurface are the same as those on an infinite-size metasurface (the perturbations of the induced currents near the edges of the metasurface are ignored), and by resorting to vector diffraction theory. This is detailed in Section III. In general terms, the RIS models based on the locally periodic discrete condition and the inhomogeneous sheet of surface impedance characterize the electromagnetic fields on the surface of an RIS, i.e., at the spatial discontinuity created by the metasurface. On the other hand, the RIS model based on mutually coupled antenna elements characterizes the voltages at the input and output ports of the transmitter and the receiver, respectively. Different physical quantities under different assumptions are, therefore, modeled.

E. Reflecting Engineered Surfaces Versus Tilted Homogeneous Surfaces

An RIS is capable of realizing wave transformations that nonengineered surfaces cannot realize. In some cases, however, homogeneous surfaces can be utilized to mimic the behavior of an RIS if some mechanical operations are applied to them. This is the notable case of an anomalous reflector, whose function can be realized by appropriately tilting a homogeneous reflecting surface, i.e., a perfect electric conducting surface. Therefore, it is instructive to elaborate on the advantages of realizing an anomalous reflector through engineered and reconfigurable surfaces, i.e., an RIS, with respect to an appropriately tilted (rotated) perfect electric conductor. Besides the apparent benefit of avoiding the mechanical rotation for realizing anomalous reflection, which cannot be always possible in practice, there are more fundamental benefits in using an RIS instead of a rotated perfect electric conductor. As illustrated in Fig. 10, a perfect electric conductor can steer an electromagnetic wave that impinges upon it from the direction \(\theta_{\text{incident}}\) toward the direction of reflection \(\theta_{\text{reflected}}\) if the homogeneous surface is rotated by an angle \(\theta_{\text{rotation}}\). More precisely, the angle of reflection is \(\theta_{\text{reflected}} = \theta_{\text{incident}} + 2\theta_{\text{rotation}}\) if \(\theta_{\text{incident}}\) and \(\theta_{\text{reflected}}\) are computed with the respect to the normal to the nonrotated perfect electric conductor. In principle, therefore, we can realize anomalous reflection by simply rotating a perfect electric conductor by \(\theta_{\text{rotation}}\), which is chosen to steer the desired reflected electromagnetic wave toward \(\theta_{\text{reflected}}\) for a given \(\theta_{\text{incident}}\), similar to a reflecting RIS. A rotated perfect electric conductor and a reflecting RIS are, however, not equivalent from the power efficiency point of view. The power that impinges upon a rotated perfect electric conductor is equal to \(P_{\text{incident}} = P_0 \cos (\theta_{\text{rotation}})\), where \(P_0\) is the incident power in the absence of rotation and \(\theta_{\text{rotation}}\) is the angle of incidence computed with the respect to the normal to the rotated perfect electric conductor. Since a perfect electric conductor is an electrically large nonresonant homogeneous surface and is not an engineered surface, it is capable of reflecting an amount of power that is at most equal to the incident power, i.e., \(P_{\text{reflected}} = P_0 \cos (\theta_{\text{rotation}})\). The larger the rotation, the smaller the reflected power. An appropriately designed RIS is, on the other hand, capable of overcoming this limitation, and in theory, it is capable of reflecting an amount of power.
equal to $P_0$, i.e., $P_{\text{reflected}} = P_0$, regardless of the angle of incidence and, hence, with 100% of efficiency. This is possible by appropriately modulating the amplitude and the phase of the incident electromagnetic wave. A practical realization of this perfect anomalous reflector is available in [31]. In conclusion, an efficient RIS cannot be replaced, in general, by a titled homogeneous surface. In addition, a perfect electric conductor cannot operate as a focusing lens, which provides optimal performance in the near field.

### III. RIS AS AN INHOMOGENEOUS IMPEDANCE BOUNDARY: ELECTROMAGNETICALLY CONSISTENT MODELING AND OPTIMIZATION

In this section, we concentrate on RISs that are modeled as an impenetrable inhomogeneous sheet of surface impedance, i.e., an inhomogeneous impedance boundary. Thus, an incident electromagnetic wave is only reflected by the surface, while no refracted electromagnetic waves are possible. The objective of this section is to provide the readers with a detailed tutorial-style, step-by-step, example on how to formulate electromagnetically consistent analytical models for RISs and how to use them for evaluating the performance of RISs in wireless networks, as well as for optimizing their surface impedance in order to fulfill specific design requirements. To this end, this section is structured in three interlinked macroparts.

1) First, we introduce the system and signal models departing from Maxwell’s equations and elaborate on the conditions that need to be fulfilled for ensuring that the RIS model is electromagnetically consistent.

2) Then, we overview the concepts of local and global designs by departing from the considered electromagnetically consistent analytical model RISs. We discuss the corresponding design criteria in terms of surface impedance and the implications associated with the practical implementation of specified surface impedances.

3) Finally, we formulate optimization problems for designing the surface impedance of an RIS based on local and global designs, and we discuss the associated design constraints that determine the surface power efficiency, the reradiated power flux at some specified locations, the amount of reradiated power toward spurious directions, and the realization of RISs made of purely reactive surface boundaries that require no local power amplification.

Specifically, the main objective of this section is to introduce a parametric model for RISs, which is conveniently formulated for researchers with a background in wireless communications and is flexible enough for imposing and understanding fundamental tradeoffs between the optimality of the reradiation pattern of an RIS (e.g., whether there exist undesired beams) [34] and the implementation complexity of the corresponding surface (e.g., whether local power amplifications are needed) [31]. As detailed next, this is obtained by departing from Maxwell’s equations and by formulating the tradeoff between performance and complexity in terms of electromagnetic constraints in the optimization problems of interest. The proposed approach is more general than existing methods that are based on codebooks as in [34] since the proposed model for an RIS is sufficient general for wireless applications, and it does not depend on the specific implementation of the RIS: it is only necessary that the wave transformations applied by a general RIS are compliant with Maxwell’s equations subject to the physical optics approximation [57].

### A. Electromagnetically Consistent Modeling of RISs

Throughout this article, we adopt the notation in [56]. In particular, we assume the universal time dependence $e^{j\omega t}$, where $j$ is the imaginary unit and $\omega$ is the angular frequency. Also, $\nabla_\xi$ denotes the gradient computed with respect to the vector $\xi = a\hat{x} + b\hat{y} + cz\hat{z}$, and $\cdot$ and $\times$ denote the scalar and vector products between vectors, respectively. $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts of a complex function, respectively, and $||\xi_1 - \xi_2||$ denotes the Euclidean distance between the points $\xi_1$ and $\xi_2$.

1) **System Model:** We consider a system model in a 3-D space, which includes a transmitter (Tx), a receiver (Rx), and an RIS (a flat surface $S$) that is modeled as an inhomogeneous boundary of surface impedance with negligible thickness with respect to the wavelength of the electromagnetic waves. A sketched representation of the considered system model is given in Fig. 8.

The RIS $S$ is a rectangle that lies in the $xy$ plane (i.e., $z = 0$) whose center is located at the origin. The sides of $S$ are parallel to the x-axis and the y-axis, and they have lengths $2L_x$ and $2L_y$, respectively. Hence, $S$ is defined as follows:

$$S = \{ s = x\hat{x} + y\hat{y} : |x| \leq L_x, |y| \leq L_y \}. \quad (19)$$

We consider a reflecting RIS, i.e., Tx and Rx are located on the same side of $S$. The transmitter, Tx, emits an electromagnetic wave that propagates through the vacuum whose permittivity and permeability are $\varepsilon_0$ and $\mu_0$, respectively. The electromagnetic wave emitted by Tx travels at the speed of light $c = 1/(\varepsilon_0\mu_0)^{1/2}$, and the free-space impedance is defined as $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$. The carrier frequency, the wavelength, and the wavenumber of the electromagnetic wave are denoted by $f$, $\lambda = c/f$, $k = 2\pi/\lambda$, respectively, and $\omega = 2\pi f$.

We denote the center location of Tx and Rx as $r_{Tx} = x_{Tx}\hat{x} + y_{Tx}\hat{y} + z_{Tx}\hat{z}$ and $r_{Rx} = x_{Rx}\hat{x} + y_{Rx}\hat{y} + z_{Rx}\hat{z}$, respectively. The transmitter is characterized by a charge density $\rho(\cdot)$ and a current density $J(\cdot)$ that fulfill the continuity equation [56] and are nonzero only in a small volume.
assumed to be the consistency of the reflected electric and magnetic fields. The
consider only the signal reflected by the RIS and assume
the reflected signal is not necessarily a plane wave. We
consider only the signal reflected by the RIS and assume
that the Tx–Rx direct signal can be ignored due to the
presence of blocking objects.

2) Electric and Magnetic Fields: More precisely, we
assume that the incident plane wave emitted by the
transmitter Tx impinges upon the RIS at an elevation angle
\( \theta_i(r_{Tx}) \) and an azimuth angle \( \varphi_i(r_{Tx}) \), and it is reflected (reradiated), not necessarily as a plane wave, toward the
receiver Rx, whose location is specified by the elevation angle \( \theta_s(r_{Rx}) \) and the azimuth angle \( \varphi_s(r_{Rx}) \). For ease of
analysis, we assume that \( \varphi_i(r_{Tx}) = \varphi_s(r_{Rx}) = \pi/2 \), i.e., the
incident and reflected fields propagate in the \( yz \) plane.

Let \( \mathbf{k}_i(r_{Rx}) \) and \( \mathbf{k}_s(r_{Rx}) \) be the wavevectors of the incident
and reflected electromagnetic waves, respectively, whose
amplitudes are equal to the wavenumber \( k \). In particular,
\( \mathbf{k}_s(r_{Rx}) \) identifies the desired direction of reflection, i.e., the
direction toward which the RIS is intended to maximize
the reradiated power. The wavevectors are defined as follows:

\[
\mathbf{k}_i(r_{Tx}) = k \sin \theta_i(r_{Tx}) \cos \varphi_i(r_{Tx}) \mathbf{x} + k \sin \theta_i(r_{Tx}) \sin \varphi_i(r_{Tx}) \mathbf{y} - k \cos \theta_i(r_{Tx}) \mathbf{z} = (\sin \theta_i(r_{Tx}) \mathbf{y} - \cos \theta_i(r_{Tx}) \mathbf{z})
\]

\[
\mathbf{k}_s(r_{Rx}) = k \sin \varphi_s(r_{Rx}) \cos \varphi_s(r_{Rx}) \mathbf{x} + k \sin \varphi_s(r_{Rx}) \sin \varphi_s(r_{Rx}) \mathbf{y} + k \cos \varphi_s(r_{Rx}) \mathbf{z} = (\sin \varphi_s(r_{Rx}) \mathbf{y} + \cos \varphi_s(r_{Rx}) \mathbf{z}).
\]

The electric field transmitted by the transmitter Tx is
assumed to be \( x \)-polarized and to propagate in free space.
To formulate the incident and reflected fields in the close
proximity of the surface \( S \) located at \( z = 0 \), we introduce
the general vector \( \mathbf{r} = x\mathbf{x} + y\mathbf{y} + z\mathbf{z} \) in the reflection half-
plane (i.e., \( z \geq 0 \)) with \( \tau_m < z < \tau_M \), where \( \tau_m \) and \( \tau_M \)
are positive constant values that are very small, but
\( \tau_m \) is sufficiently large for the impedance boundary model to be
applicable, as elaborated on in Section II-C. The condition
\( z < \tau_M \) is necessary to validate the electromagnetic con-
 sistency of the reflected electric and magnetic fields. The
reradiated field for \( z > \tau_M \), which encompasses the near-
and far-field regions of the RIS, is defined and detailed
next (see (86) in Section III-B2) by utilizing the radiation
integrals. With this notation, the incident electric field is
formulated as follows:

\[
\mathbf{E}_{\text{inc}}(\mathbf{r}, r_{Tx}) = E_{\text{inc}}^0 e^{-jk \cdot (r_{Tx} - \mathbf{R})} \times \mathbf{x} = E_{\text{inc}}^0 e^{-jk \cdot (\sin \theta_i(r_{Tx}) y - \cos \theta_i(r_{Tx}) z)} \mathbf{x}
\]

where \( E_{\text{inc}}^0 \) is a constant (independent of \( \mathbf{r} \)) complex ampli-
tude as dictated by the plane wave assumption.

The corresponding incident magnetic field evaluated at
\( \mathbf{r} \) is obtained from Maxwell’s equations as follows:

\[
\mathbf{H}_{\text{inc}}(\mathbf{r}, r_{Tx}) = -\frac{1}{j\omega \varepsilon_0} \nabla \times \mathbf{E}_{\text{inc}}(\mathbf{r}, r_{Tx}) = -\frac{1}{\tau_0} E_{\text{inc}}^0 e^{-jk \cdot (\sin \theta_i(r_{Tx}) y - \cos \theta_i(r_{Tx}) z)} \times \left[ \cos \theta_i(r_{Tx}) \mathbf{y} + \sin \theta_i(r_{Tx}) \mathbf{z} \right]
\]

where the identity \( k/(\omega \varepsilon_0) = 1/\tau_0 \) is used.

In order to model the field reradiated by the RIS in the
close proximity of \( S \), we utilize the physical optics
approximation method [57], [62]. Accordingly, the field
reflected by the RIS is assumed to be radiated by secondary
currents that are induced on the surface \( S \) by the incident
electromagnetic wave. These currents determine, in turn,
the surface (at \( z = 0^+ \)) electric and magnetic fields. Since
the surface electric and magnetic fields are not known
in advance, the physical optics approximation substitutes
them with their geometrical optics approximation [57]. In
particular, it is assumed that the secondary currents (and
the corresponding surface fields) that are induced on the
finite-size surface \( S \) are the same as those that would be
induced on an infinite-size (i.e., \( L_x \rightarrow \infty \) and \( L_y \rightarrow \infty \))
RIS. This implies that the perturbations of the induced
currents near the edges of the surface \( S \) are neglected.
Even though approximated, the physical optics method is a
suitable approach to guide the design and optimization of
RISs, and shed light on their achievable performance,
without using numerically intensive techniques, such as
the method of moments. In this section, we concentrate
on the reradiated electric and magnetic fields in the close
proximity of the RIS, i.e., evaluated at \( \mathbf{r} \) for \( \tau_m < z < \tau_M \).

Based on the physical optics method and assuming that
the RIS does not change the polarization of the incident
field, the reflected electric field at \( \mathbf{r} \) for \( \tau_m < z < \tau_M \) can be
formulated as

\[
\mathbf{E}_{\text{ref}}(\mathbf{r}, \mathbf{r}_{Rx}, \mathbf{r}_{Tx}) \approx \Gamma_{\text{ref}}(\mathbf{r}, \mathbf{r}_{Rx}, \mathbf{r}_{Tx}) \mathbf{E}_{\text{inc}}(\mathbf{r}, \mathbf{r}_{Tx})
\]

where \( \Gamma_{\text{ref}}(\mathbf{r}, \mathbf{r}_{Rx}, \mathbf{r}_{Tx}) \) is a complex-valued function, which
is usually referred to as the field reflection coefficient.

In general, \( \Gamma_{\text{ref}}(\mathbf{r}, \mathbf{r}_{Rx}, \mathbf{r}_{Tx}) \) is an arbitrary function provided that the reflected field in (24) fulfills Maxwell’s
equations. For ease of analysis, but without loss of generality,
we assume that \( \Gamma_{\text{ref}}(\mathbf{r}, \mathbf{r}_{Rx}, \mathbf{r}_{Tx}) \) is formulated as follows:

\[
\Gamma_{\text{ref}}(\mathbf{r}, \mathbf{r}_{Rx}, \mathbf{r}_{Tx}) = \Gamma(\mathbf{r}_{Rx}, \mathbf{r}_{Tx}) e^{j\phi(\tau_m, \mathbf{r}_{Rx}, \mathbf{r}_{Tx})}
\]

where the following definitions hold:

\[
\Phi(\mathbf{r}, \mathbf{r}_{Rx}, \mathbf{r}_{Tx}) = -k (\sin \theta_i(r_{Rx}) - \sin \theta_i(r_{Tx})) y - k (\cos \theta_i(r_{Rx}) + \cos \theta_i(r_{Tx})) z
\]

\[
\Gamma(\mathbf{r}_{Rx}, \mathbf{r}_{Tx}) = R(\mathbf{r}_{Rx}, \mathbf{r}_{Tx}) e^{j\phi(\tau_m, \mathbf{r}_{Rx}, \mathbf{r}_{Tx})}
\]
where \( R(r_{Rx}, s, r_{Tx}) = |\Gamma(r_{Rx}, s, r_{Tx})| \) and \( \phi(r_{Rx}, s, r_{Tx}) = \angle \Gamma(r_{Rx}, s, r_{Tx}) \) denote the amplitude and the phase of \( \Gamma(r_{Rx}, s, r_{Tx}) \), respectively.

In the considered case study, the incident electric and magnetic fields in (22) and (23), respectively, and the corresponding reradiated electric and magnetic fields are assumed to be independent of \( x \). Hence, we obtain the simplified expressions \( \Phi(r_{Rx}, r, r_{Tx}) = \Phi(r_{Rx}, y, z, r_{Tx}) \), \( \Gamma(r_{Rx}, s, r_{Tx}) = \Gamma(r_{Rx}, y, r_{Tx}) \), \( R(r_{Rx}, s, r_{Tx}) = R(r_{Rx}, y, r_{Tx}) \), and \( \phi(r_{Rx}, s, r_{Tx}) = \phi(r_{Rx}, y, r_{Tx}) \). If more general formulations for the incident and reradiated electromagnetic fields are considered, the approach described in this article applies mutatis mutandis.

The function \( \Gamma_{ref}(r_{Rx}, r = s, r_{Tx}) \) for \( s = xx + yy \in S \) is the field reflection coefficient evaluated on the surface \( S \) (i.e., at \( z = 0^+ \)), which is usually referred to as the surface reflection coefficient and characterizes the reflection properties of the RIS. With a slight abuse of terminology, we refer to \( \Gamma(r_{Rx}, s, r_{Tx}) \) as the surface reflection coefficient as well since \( \Gamma_{ref}(r_{Rx}, r = s, r_{Tx}) \) and \( \Gamma(r_{Rx}, s, r_{Tx}) \) differ by a linear phase shift, i.e., \( \Phi(r_{Rx}, y, z = 0^+, r_{Tx}) \).

The analytical formulation in (25) is not only analytically convenient, as it will be apparent next, but it has a useful physical interpretation. The function \( \Phi(r_{Rx}, r, r_{Tx}) \) corresponds to the phase shift that needs to be applied by the RIS based on the geometrical optics approximation in order to steer (reradiate) an electromagnetic wave the impinging \( s \) from the direction identified by \( k_{Rx}(r_{Tx}) \) toward the direction identified by \( k_{Rx}(r_{Tx}) \) [2]. The complex-valued function \( \Gamma(r_{Rx}, s, r_{Tx}) \) models the amplitude \( R(r_{Rx}, s, r_{Tx}) \) and the phase \( \phi(r_{Rx}, s, r_{Tx}) \) correction terms that are necessary for optimizing an RIS based on more advanced criteria than the canonical geometrical optics method. By setting \( \Gamma(r_{Rx}, s, r_{Tx}) = 1 \), the solution based on the geometrical optics approximation is retrieved. The function \( R(r_{Rx}, s, r_{Tx}) \) accounts for designs of RISs that may need a nonuniform amplitude control along the surface \( S \). In general, as elaborated on next [see, e.g., (67)], the amplitude correction function \( R(r_{Rx}, s, r_{Tx}) \) and the phase correction function \( \phi(r_{Rx}, s, r_{Tx}) \) may not be independent of one another for some optimization criteria for RISs.

As mentioned, \( \Gamma_{ref}(r_{Rx}, r, r_{Tx}) \) is an arbitrary function provided that the reflected field in (24) fulfills Maxwell’s equations. This implies that \( E_{inc}(r_{Rx}, r, r_{Tx}) \) in (24) needs to satisfy Helmholtz’s equation in the source-free region [56]. Since the analytical formulation of the reflected field in (24) is an approximation, it is customary to ensure that Helmholtz’s equation is fulfilled approximately as well. By defining \( E_{ref,x}(r_{Rx}, r, r_{Tx}) = \hat{x} \cdot E_{ref}(r_{Rx}, r, r_{Tx}) \), Helmholtz’s equation on the surface \( S \) (i.e., at \( z = 0^+ \)) reads as follows:

\[
\nabla \cdot (\nabla \cdot (E_{ref,x}(r_{Rx}, r, r_{Tx}))) |_{z = 0^+} \approx -k^2 E_{ref,x}(r_{Rx}, s, r_{Tx}).
\]

(28)

By inserting (24) into (28) and taking into account that \( \Gamma(r_{Rx}, s, r_{Tx}) = \Gamma(r_{Rx}, y, r_{Tx}) \), i.e., it depends only on \( y \), we obtain the following condition:

\[
\left| \frac{d^2}{dy^2} \Gamma(r_{Rx}, y, r_{Tx}) \right| - 2j k \sin \theta_s(r_{Rx}) \frac{d}{dy} \Gamma(r_{Rx}, y, r_{Tx}) \approx k^2 |\Gamma(r_{Rx}, y, r_{Tx})|.
\]

(29)

To obtain an electromagnetically consistent design for an RIS, it is necessary to add the condition in (29) as a constraint when optimizing \( \Gamma_{ref}(r_{Rx}, s, r_{Tx}) \). This is elaborated in Section III-C. It is worth mentioning that the Helmholtz equation in (29) is satisfied with equality if \( \Gamma(r_{Rx}, y, r_{Tx}) \) is independent of \( y \), i.e., it is a constant function along the entire surface \( S \). This is expected because the reflected electric field in (24) would be a perfect plane wave in this case.

In addition to (28), it is necessary that the reflected electric field in (24) fulfills the zero-divergence condition in the source-free region, i.e., \( \nabla \cdot E_{inc}(r_{Rx}, r, r_{Tx}) = 0 \), for it to be a consistent electric field. This imposes an additional constraint on the design and optimization of \( \Gamma_{ref}(r_{Rx}, r, r_{Tx}) \). The zero-divergence condition is always fulfilled, as shown as follows:

\[
\nabla \cdot E_{ref}(r_{Rx}, r, r_{Tx}) \approx \nabla \cdot (\Gamma_{ref}(r_{Rx}, r, r_{Tx}) E_{inc}(r_{Rx}, r, r_{Tx}))
\]

\[
= (\nabla \Gamma_{ref}(r_{Rx}, r, r_{Tx})) \cdot E_{inc}(r_{Rx}, r, r_{Tx})
\]

\[
+ \Gamma_{ref}(r_{Rx}, r, r_{Tx}) \nabla \cdot E_{inc}(r_{Rx}, r, r_{Tx})
\]

\[
= E_{inc,x}((y, z) \cdot r_{Tx}) \left( \frac{\partial}{\partial y} \Gamma_{ref}(r_{Rx}, (y, z) \cdot r_{Tx}) \right) \hat{y} \cdot \hat{x}
\]

\[
+ E_{inc,x}((y, z) \cdot r_{Tx}) \left( \frac{\partial}{\partial z} \Gamma_{ref}(r_{Rx}, (y, z) \cdot r_{Tx}) \right) \hat{z} \cdot \hat{x}
\]

\[
+ \Gamma_{ref}(r_{Rx}, r, r_{Tx}) \left( \frac{\partial}{\partial x} E_{inc,x}((y, z) \cdot r_{Tx}) \right)
\]

\[
= 0
\]

(30)

where \( E_{inc,x}(r_{Rx}, r, r_{Tx}) = \hat{x} \cdot E_{inc}(r_{Rx}, r, r_{Tx}) \), and the identities are obtained by taking into account that neither the incident field nor the field reflection coefficient depend on \( x \).

Given the reflected electric field in (24) that fulfills (29), the final step for ensuring that the reradiated field in the close proximity of the RIS is electromagnetically consistent lies in calculating the reflected magnetic field. Similar to (23), \( H_{ref}(r_{Rx}, r, r_{Tx}) \) is obtained from Maxwell’s equations

\[
H_{ref}(r_{Rx}, r, r_{Tx})
\]

\[
= - \frac{1}{j \omega \mu_0} \nabla \times E_{ref}(r_{Rx}, r, r_{Tx})
\]

\[
= - \frac{E_{ref,0}^x}{j \omega \mu_0} \nabla \times \left[ \Gamma(r_{Rx}, r, r_{Tx}) g(y, z) \hat{x} \right]
\]

\[
= - \frac{E_{ref,0}^x}{j \omega \mu_0} \left( \frac{d}{dy} \Gamma(r_{Rx}, y, r_{Tx}) g(y, z) \hat{y} \right.
\]

\[
-j k \cos \theta_s(r_{Rx}) \Gamma(r_{Rx}, y, r_{Tx}) g(y, z) \hat{y}
\]

\[
\left. + j k \sin \theta_s(r_{Rx}) \Gamma(r_{Rx}, y, r_{Tx}) g(y, z) \hat{z} \right]
\]

(31)
where \( g(y, z) = e^{-jk\sin \theta \cdot (r_{Tx}y + \cos \theta \cdot (r_{Tx}z))} \), \( a \) follows from
\[
\nabla_r \left( \Gamma \left( \Gamma (r_{Rx}, r, r_{Tx}) \right) = y \cdot (d^2 \Gamma (r_{Rx}, y, r_{Tx})/dy) \right), \text{ and } \nabla \times \hat{\mathbf{x}} = -z.
\]

The electromagnetic fields in (22)–(24) and (31) with the constraint (29) constitute a set of electromagnetically consistent equations for analyzing and optimizing RIS-assisted communications. Specifically, the mentioned equations constitute a parametric model for an RIS that is applicable under the assumption that the surface impedance is slowly varying at the wavelength scale, and the physical optics approximation holds. The requirement that the Helmholtz constraint in (29) is fulfilled only approximately, but we can control the amplitude and the phase of the surface reflection coefficient as a function of \( y \), offers a tradeoff between the quality (optimality) of the reradiated electromagnetic field and the implementation complexity of the corresponding RIS. Therefore, the proposed parametric model is flexible enough for finding the best compromise between performance, electromagnetic consistency, and implementation complexity. In Section IV, with the aid of several numerical results, we will see that the quality of the obtained solution corresponds to the presence or absence of undesired radiation modes, while the implementation complexity corresponds to RISs whose surface impedance (which is formally introduced in (41)) [31] is characterized by large variations along the surface and whose real part (i.e., the surface resistance) may be nonzero and negative. Generally speaking, the stricter the requirement on the Helmholtz constraint in (29) is, the better the theoretical quality of the obtained solution, but the more difficult the implementation complexity is.

The proposed parametric model and the modeling assumption that either the amplitude \( |R(r_{Rx}, y, r_{Tx})| \) or the phase \( \phi(r_{Rx}, y, r_{Tx}) \), or, more in general, that both of them depend on \( y \) may be viewed as a model for taking into account possible nuisance parameters, i.e., nonideal effects, in the surface reflection coefficient. This can be understood based on the recent analysis conducted in [34]. Specifically, the impact of nonzero differences between the actual values of \( R(r_{Rx}, y, r_{Tx}) \) and/or \( \phi(r_{Rx}, y, r_{Tx}) \) and their theoretical optimal values can be analyzed as a function of the fulfillment of the Helmholtz constraint in (29) and the reradiated power discussed in Section III-B2. For example, \( R(r_{Rx}, y, r_{Tx}) \) and \( \phi(r_{Rx}, y, r_{Tx}) \) may model the impact of quantizing the amplitude and/or the phase of the surface reflection coefficient, which boils down to analyzing the impact of quantizing the surface impedance. This is not dissimilar from analyzing the impact of phase quantization and the interplay between the amplitude and phase of the locally periodic reflection coefficients recently conducted in [34] based on the RIS model presented in Section II-A. From an optimization point of view, specified constraints on the maximum values of the nuisance parameters may be added to the optimization problem in order to find an acceptable tradeoff between the desired radiation pattern and the implementation complexity of an RIS.

### Plane Wave Spectrum and Floquet’s Expansion:
Before proceeding, it is instructive to note that the electric field reradiated from an RIS may be formulated in different manners. For example: 1) according to the plane-wave representation of an electromagnetic field [56], the reflected electric field may be represented as the sum of plane waves with different wavevectors and 2) according to Floquet’s theorem for periodic structures [69, Sec. 7.1], the reflected electric field may be represented as the sum of multiple diffracted modes with different wavevectors similar to a diffraction grating. Floquet’s theorem, specifically, stipulates the following: if we illuminate an infinite-size and periodic surface with a plane wave, the reflected electromagnetic field can, in general, be formulated as the superposition of plane waves that propagate in different directions. The intensity and the direction of each plane wave depend on how the surface is realized. Further details about Floquet’s theorem are given next [see (48)–(59)].

In general terms, the following expression for the reradiated electric field (in the close proximity of the surface \( S \)) can be considered:

\[
E_{\text{ref}} (r_{Rx}, r, r_{Tx}) \approx \sum_{n} r_{n} (k_0 (r_{Tx})) E_{0, n} e^{-jk_0 (r_{Tx}) r} \hat{x} \quad (32)
\]

where \( r_{n} (k_0 (r_{Tx})) = \Gamma_{\text{ref}, n} (r_{Rx}, r, r_{Tx}; k_0 (r_{Tx})) \) denotes the field reflection coefficient of the nth reradiated mode for a given incident electromagnetic wave [64].

In general, \( r_{n} (k_0 (r_{Tx})) \) depends on the direction of the incident electromagnetic wave, i.e., it depends on \( k_0 (r_{Tx}) \), which is determined by the location of the transmitter. Each term of Floquet's expansion in (32) is a plane wave that propagates toward a specific direction. The corresponding magnetic fields can be calculated by using wave impedances that depend on the propagation angles of each plane wave component, which can be obtained from (31). Based on Floquet's model, the Helmholtz equation is satisfied with equality, and there is no need for imposing (29), which ensures that the surface parameters (i.e., the surface reflection coefficient) are slowly varying.

In this article, we consider the approximated formulation of the reflected electric field in (24) subject to the constraint in (29) since it is the most widely used definition for the reflected electric field in wireless communications. The definition and methods described in this article apply, mutatis mutandis, to (32). Before proceeding, however, it is instructive to study whether the analytical formulation in (32) can be retrieved from (24). To this end, we can leverage Floquet’s theorem under the assumption that the RIS is a periodic structure. This is elaborated on after introducing the notion of surface impedance.

3) **Surface Impedance**: In Section III-A2, the electromagnetic fields are formulated in terms of the field reflection coefficient \( \Gamma_{\text{ref}} (r_{Rx}, r, r_{Tx}) \). We have adopted this formulation because it is widely used in wireless communications.
In order to get insights for system design, however, it is more convenient to design an RIS as a function of the surface impedance, which depends on the tangential components of the incident and reradiated electric and magnetic fields on the surface $S$, i.e., at $z = 0^+$. By direct inspection of the surface impedance, it is relatively simple to identify the structural properties of an RIS and the corresponding options for its practical implementation. For example, it is possible to understand whether an RIS is lossless and whether it can be realized without using active components. This is elaborated on next.

Before introducing the definition of surface impedance, we summarize the tangential components (evaluated at $z = 0^+$) of the incident and reflected electric and magnetic fields. Let $\mathbf{F}(\mathbf{r}_{\text{Rx}}, \mathbf{r}, \mathbf{r}_{\text{Tx}})$ be a generic vector field. The component of $\mathbf{F}(\mathbf{r}_{\text{Rx}}, \mathbf{r}, \mathbf{r}_{\text{Tx}})$ that is tangential to the plane occupied by $S$ and that is evaluated at $z = 0^+$ is defined as follows:

$$\mathbf{F}^t(\mathbf{r}_{\text{Rx}}, \mathbf{r}, \mathbf{r}_{\text{Tx}}) = (\hat{\mathbf{z}} \times \mathbf{F}(\mathbf{r}_{\text{Rx}}, \mathbf{r}, \mathbf{r}_{\text{Tx}})) \times \hat{\mathbf{z}} \big|_{z = 0^+}. \quad (33)$$

From (22)–(24) and (31), the tangential components of the incident and reflected electric and magnetic fields are

$$\mathbf{E}^\text{inc}(y, \mathbf{r}_{\text{Tx}}) = E^x(y, \mathbf{r}_{\text{Tx}}) \hat{x}$$
$$\mathbf{H}^\text{inc}(y, \mathbf{r}_{\text{Tx}}) = -\frac{\cos \theta_i(\mathbf{r}_{\text{Tx}})}{\eta_0} E^y(y, \mathbf{r}_{\text{Tx}}) \hat{y}$$

$$\mathbf{E}^\text{ref}(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}}) = \Gamma_S(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}}) E^x(y, \mathbf{r}_{\text{Tx}}) \hat{x}$$
$$\mathbf{H}^\text{ref}(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}}) = \frac{\cos \theta_i(\mathbf{r}_{\text{Rx}})}{\eta_0} \Gamma_S(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}}) E^y(y, \mathbf{r}_{\text{Tx}}) \hat{y} \quad (37)$$

where $\Gamma_S(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}}) = \Gamma^\text{ref}(\mathbf{r}_{\text{Rx}}, y, z = 0^+, \mathbf{r}_{\text{Tx}})$ and $E^x(y, \mathbf{r}_{\text{Tx}}) = E^x_\text{inc}(y, \mathbf{r}_{\text{Tx}}) \exp(-j k \sin \theta_i(\mathbf{r}_{\text{Tx}}) y)$. Similarly, we use the notation $\Phi_S(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}}) = \Phi(\mathbf{r}_{\text{Rx}}, y, z = 0^+, \mathbf{r}_{\text{Tx}})$. The electromagnetic fields in (34)–(37) are referred to as surface electromagnetic fields, and $\Gamma_S(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}})$ is the surface reflection coefficient.

As mentioned, (36) and (37) are applicable under the assumption that the inequality constraint in (29) is satisfied, i.e., the surface reflection coefficient is slowly varying, at the wavelength scale, along the surface $S$. We note, in addition, that the reflected electric field and the reflected magnetic field are closely related to those of a plane wave that propagates toward the direction $\theta_i(\mathbf{r}_{\text{Rx}})$. This is due to the assumption of plane wave for the incident electromagnetic wave and the definition of the field reflection coefficient in (25), which results in the term $\cos \theta_i(\mathbf{r}_{\text{Rx}})$ in (37). In general, this latter term depends on the specific expression of the function $g(y, z)$ in (31). More precisely, the reflected electric field in (36) can be explicitly written as $E^\text{ref}(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}}) = E^\text{inc}(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}}) e^{-j k \sin \theta_i(\mathbf{r}_{\text{Rx}}) y}$, which is an electromagnetic wave that propagates toward the direction $\theta_i(\mathbf{r}_{\text{Rx}})$. If $R(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}})$ is independent of $y$ and $\phi(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}})$ is a linear function in $y$, $E^\text{ref}(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}})$ is a plane wave [17, eq. (13)]. Locally (i.e., at any point $y$ or, in general, $s$), in addition, the reflected electric and magnetic fields in (36) and (37) are in agreement with the local non specular reflection model illustrated in Fig. 9(b).

The analytical formulation of an RIS as an inhomogeneous sheet given in (14) and (15), which fulfills the generalized sheet transition conditions. The only difference is that the transmitted fields are assumed to be equal to zero in the considered example, i.e., the RIS is assumed to be impenetrable and is a purely reflecting surface. In this case, an RIS can be described only through an electric surface impedance boundary (i.e., (41)), and the magnetic surface admittance is redundant.

By direct inspection and analysis of $Z(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}})$, it is possible to draw important conclusions on the inherent features of an RIS and the associated implementation requirements for realizing specified wave transformations. For example, the following holds.

1) An RIS is locally passive if $\Re(Z(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}})) > 0$.
2) An RIS is locally active if $\Re(Z(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}})) < 0$.
3) An RIS is locally capacitive if $\Im(Z(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}})) < 0$.
4) An RIS is locally inductive if $\Im(Z(\mathbf{r}_{\text{Rx}}, y, \mathbf{r}_{\text{Tx}})) > 0$.

In Section III-B, we formally introduce the criteria for the locally optimum and the globally optimum designs of an RIS as a function of (41) and formulate the corresponding optimization problems in order to obtain the optimal surface impedance. Therefore, it is convenient
to reformulate the Helmholtz constraint in (29) in terms of the surface impedance $Z_{R_S, y, r_T}$). To this end, we first express $Z_{R_S, y, r_T}$ in terms of $\Gamma_S(r_{R_S}, y, r_{r_T})$. By inverting (41), we obtain

$$\Gamma_S(r_{R_S}, y, r_{r_T}) = \frac{Z_{R_S, y, r_T} \cos \theta_S(r_{R_S}) - \eta_0}{Z_{R_S, y, r_T} \cos \theta_S(r_{R_S}) + \eta_0}. \quad (42)$$

From (25), in addition, we have

$$\Gamma(r_{R_S}, y, r_{r_T}) = \Gamma_S(r_{R_S}, y, r_{r_T}) e^{\frac{i}{2} \sin \theta_S(r_{R_S}) - \sin \theta_S(r_{r_T}))} \quad (43)$$

By first inserting (42) in (43) and then plugging the resulting analytical expression in (29), the Helmholtz constraint can be equivalently rewritten, as a function of the surface impedance $Z_{R_S, y, r_T}$, as follows:

$$\left| \frac{d^2}{dy^2} \left( \frac{Z_-^*(y)}{Z_+^*(y)} \right)^r - 2 \eta_0 \sin \theta_S(r_{r_T}) \frac{d}{dy} \left( \frac{Z_-^*(y)}{Z_+^*(y)} \right)^r e^{\frac{i}{2} \sin \theta_S(r_{R_S}) - \sin \theta_S(r_{r_T}))} \right| \leq k^2 \left| \frac{Z_-^*(y)}{Z_+^*(y)} \right| \quad (44)$$

where the following auxiliary functions are defined:

$$Z_-^*(y) = Z_{R_S, y, r_T} \cos \theta_S(r_{R_S}) - \eta_0$$
$$Z_+^*(y) = Z_{R_S, y, r_T} \cos \theta_S(r_{R_S}) + \eta_0. \quad (45)$$

Similar to (29), the inequality in (44) implies that the surface impedance is slowly varying, at the wavelength scale, along the surface $S$.

a) Field versus load reflection coefficient: Before proceeding, it is instructive to analyze the difference between the surface reflection coefficient $\Gamma_S(r_{R_S}, y, r_{r_T})$ and the so-called load reflection coefficient $\Gamma_{load}(r_{R_S}, y, r_{r_T})$, which is it is often considered when designing an RIS [17], [64]. The load reflection coefficient is defined as follows:

$$\Gamma_{load}(r_{R_S}, y, r_{r_T}) = \frac{Z_{R_S, y, r_T} \cos \theta_S(r_{R_S}) - \eta_0}{Z_{R_S, y, r_T} \cos \theta_S(r_{R_S}) + \eta_0} \quad (46)$$

The two reflection coefficients $\Gamma_S(r_{R_S}, y, r_{r_T})$ in (42) and $\Gamma_{load}(r_{R_S}, y, r_{r_T})$ in (46) are not independent of each other, but they are not exactly the same. From the physical point of view, the difference between $\Gamma_{load}(r_{R_S}, y, r_{r_T})$ and $\Gamma_S(r_{R_S}, y, r_{r_T})$ is illustrated in Fig. 9(a) and (b), respectively. Specifically, the load reflection coefficient $\Gamma_{load}(r_{R_S}, y, r_{r_T})$ is defined by assuming that the RIS operates locally, i.e., at every point $y$, as a local specular reflector. The surface reflection coefficient $\Gamma_S(r_{R_S}, y, r_{r_T})$ is, on the other hand, defined by assuming that the RIS operates locally as a local nonspecular reflector. In mathematical terms, the difference is apparent by comparing the denominators of $\Gamma_{load}(r_{R_S}, y, r_{r_T})$ and $\Gamma_S(r_{R_S}, y, r_{r_T})$, and by noting that $\cos \theta_S(r_{R_S})$ is present in (42), and $\cos \theta_S(r_{r_T})$ is present in (46). Notably, the field and load reflection coefficients are identical if specular reflection, i.e., $\theta_S(r_{R_S}) = \theta_S(r_{r_T})$, is considered. The minor difference in the denominators of (42) and (46) results in major differences in the properties of the two reflection coefficients. Let us assume, for example, that the real part of the surface impedance is positive, i.e., $\Re(Z_{R_S, y, r_T}) > 0$. In this case, $|\Gamma_{load}(r_{R_S}, y, r_{r_T})| < 1$. In particular, $|\Gamma_{load}(r_{R_S}, y, r_{r_T})| = 1$ if and only if $\Re(Z_{R_S, y, r_T}) = 0$. In other words, the load reflection coefficient has an amplitude that is locally equal to one if and only if the surface impedance is locally reactive, i.e., the RIS is lossless. This is, however, not necessarily true for $\Gamma_S(r_{R_S}, y, r_{r_T})$ since its amplitude depends on the relation between the angles of incidence and reflection.

By direct inspection of (41), specifically, we obtain that $|\Gamma_S(r_{R_S}, y, r_{r_T})| \leq 1$ if and only if the following condition is fulfilled:

$$|\Gamma_S(r_{R_S}, y, r_{r_T})| \leq 1 \Leftrightarrow \frac{\cos \theta_S(r_{R_S}) - \cos \theta_S(r_{r_T})}{2\eta_0} \leq \frac{\Re(Z_{R_S, y, r_T})}{|Z_{R_S, y, r_T}|^2}. \quad (47)$$

From (47), we evince that the condition $\Re(Z_{R_S, y, r_T}) > 0$ is not sufficient for ensuring that $|\Gamma_S(r_{R_S}, y, r_{r_T})| \leq 1$. The angle of incidence and the desired angle of reflection cannot be ignored. If the angle of incidence is zero, i.e., $\theta_S(r_{R_S}) = 0$, as is often assumed, the condition $\Re(Z_{R_S, y, r_T}) > 0$ is sufficient for ensuring $|\Gamma_S(r_{R_S}, y, r_{r_T})| \leq 1$.

More specifically, let us analyze the design constraints to be imposed on the surface impedance $Z_{R_S, y, r_T}$ for ensuring that $|\Gamma_S(r_{R_S}, y, r_{r_T})| = 1$. In this case, the inequalities in (47) are replaced by equalities. Therefore, by direct inspection of (47), the following design guidelines are drawn.

1) If $\theta_S(r_{R_S}) = \theta_S(r_{r_T})$, we obtain $\Re(Z_{R_S, y, r_T}) = 0$. This implies that the surface impedance is purely reactive, and the RIS is lossless.

2) If $\theta_S(r_{R_S}) > \theta_S(r_{r_T})$, we obtain $\Re(Z_{R_S, y, r_T}) < 0$. This implies that the RIS is locally active.

3) If $\theta_S(r_{R_S}) < \theta_S(r_{r_T})$, we obtain $\Re(Z_{R_S, y, r_T}) > 0$. This implies that the RIS is locally passive.

In order to have a field reflection coefficient with unit amplitude at a given point $y$ on the surface $S$, the obtained findings allow us to conclude that the real part of the surface impedance of the RIS may be negative or positive depending on the angles of incidence and reflection. This implies that the RIS needs to introduce local power amplifications (a negative resistance is equivalent to an amplification) or local power losses along the surface $S$, depending on the desired angle of reflection for a given angle of incidence. By assuming, e.g., normal incidence (i.e., $\theta_S(r_{R_S}) = 0$), we evince that a reflection coefficient with unit amplitude corresponds to an engineered sur-
face $S$ with a positive surface impedance, which implies that no power amplifiers or other sophisticated methods for creating power amplifications through the use of, e.g., surface waves [31], [70], [71], are needed to realize the RIS. The design constraints to be imposed on $Z(r_{Rx},y_1)$ for ensuring that the RIS has a high power efficiency for any pair $(\theta_i(r_{Tx}), \theta_i(r_{Rx}))$ are discussed in Section III-B.

In the rest of this section, we discuss the structural properties of an RIS and optimize its operation as a function of the surface impedance $Z(r_{Rx},y,r_{Tx})$. Whenever necessary to gain engineering insights onto the design of RISs, we will refer to $\Gamma_S(r_{Rx},y,r_{Tx})$ in (42), i.e., the surface reflection coefficient as well since it is directly related to the electric and magnetic fields defined in (24) and, therefore, to Maxwell’s equations. Also, $\Gamma_S(r_{Rx},y,r_{Tx})$ is the most commonly utilized representation for an RIS in wireless communications [17], [20].

b) Surface reflection coefficient and Floquet’s theorem: As mentioned, it is instructive to analyze whether and how the analytical formulation in (32) can be retrieved from (24). We illustrate whether this is possible with an example. Let us assume that the RIS is a periodic structure of infinite size. The periodicity is usually determined by the period of infinite size. The periodicity is usually determined by the period $P$ of the transmitter of interest and determines the period $\Gamma$ of the RIS. In other words, (50) can be written, in its most general formulation, as follows:

$$E^t_{ref,x}(r_{Rx},y,s,r_{Tx}) = \Gamma_S(r_{Rx},y,P,r_{Tx}) E^i_{x,0} e^{-j\theta_i(r_{Tx})}.$$  

which unveils that the reflected electric field is expressed as a summation of plane waves similar to (32). This confirms that the analytical formulation in (24) allows us to retrieve results that are consistent with infinite periodic structures in agreement with Floquet’s theorem. It is interesting to evaluate the special case when $\Gamma(r_{Rx},y,r_{Tx})$ is a constant independent of $y$, i.e., $\Gamma(r_{Rx},y,r_{Tx}) = \Gamma_0$. In this case, we obtain $\Gamma_S(r_{Rx},y,r_{Tx}) = \Gamma_0 e^{-j\theta_i(r_{Tx})}$, where $\theta_i(r_{Tx})$ is the harmonic of the transmitter of interest and determines the period $P$ of the RIS. In other words, (50) can be written, in its most general formulation, as follows:

$$E^t_{ref,x}(r_{Rx},y,s,r_{Tx}) = \Gamma_S(r_{Rx},y,P,r_{Tx}) E^i_{x,0} \sum_{n=1}^{\infty} \mu_n e^{-\frac{j\lambda}{P}n}.$$  

where $\theta_i$ denotes a generic angle of incidence, and for the avoidance of doubt, we have made explicit that the period of the RIS depends on the angles of design $\theta_i(r_{Rx})$ (the specified angle of incidence) and $\theta_i(r_{Tx})$ (the desired angle of reflection), i.e., $P = \frac{\lambda}{\theta_i(r_{Rx})}$. From (48), it follows that the reflected electric field fulfills the condition imposed by Floquet’s theorem. In particular, the difference between the electric field evaluated at two points that differ of one period is equal to $e^{-j\theta_i(r_{Tx})}.$
The general analytical formulation in (51) is useful to characterize the electromagnetic field reradiated by an RIS when it is illuminated by an electromagnetic wave that originates from the direction \( \theta_i (r_{Tx}) \), as well as the electromagnetic field reradiated by the RIS when it is illuminated by an interfering electromagnetic wave (including the multipath generated by scatterers that are not digitally controllable) that originates from any direction \( \theta_i \neq \theta_i (r_{Rx}) \). Therefore, (51) is a relatively general formula in (51). By taking into account that the total reradiated power is distributed among the reradiated modes, a qualitative illustration of typical reradiation conditions for a periodic RIS is given in Fig. 11. Even though multiple reflected modes may exist based on (54), it is important to emphasize that many of them may be weakly excited, i.e., the amount of power carried by them is negligible with respect to the amount of power carried by the other propagating modes. In other words, the reradiation properties of an RIS are jointly determined by the allowed propagating modes and the amount of power that each of them carries, which is quantified by the Fourier coefficients \( |\mu_n| \).

As a concrete case study, let us assume that the surface reflection coefficient is, according to (25), given by

\[
\Gamma_S (r_{Rx}, y, r_{Tx}) = \Gamma (r_{Rx}, y, r_{Tx}) e^{-j k (\sin \theta_i (r_{Rx}) - \sin \theta_i (r_{Tx})) y} \tag{55}
\]

where \( \Gamma (r_{Rx}, y, r_{Tx}) \) is assumed to be a periodic function with period \( \mathcal{P} \), i.e., \( \Gamma (r_{Rx}, y, r_{Tx}) = \Gamma (r_{Rx}, y + \mathcal{P}, r_{Tx}) \) \( \forall y \). Specifically, we assume that the period \( \mathcal{P} \) is equal to

\[
\mathcal{P} = \frac{\lambda}{|\sin \theta_i (r_{Rx}) - \sin \theta_i (r_{Tx})|}. \tag{56}
\]

This implies that \( \Gamma_S (r_{Rx}, y, r_{Tx}) \) is a periodic function with period \( \mathcal{P} \) as well. For simplicity, let us assume again that \( \theta_i (r_{Tx}) = 0 \). Therefore, we obtain

\[
\mathcal{P} = \frac{\lambda}{\sin \theta_i (r_{Rx})} \geq \lambda. \tag{57}
\]

In this case, (54) simplifies to

\[
|n| \leq \frac{1}{\sin \theta_i (r_{Rx})}. \tag{58}
\]

From (58), we evince that at most three modes (i.e., \( n = -1, 0, 1 \)) may be reradiated from the RIS if \( \theta_i (r_{Rx}) \) is close to 90°, which corresponds to the case study in which the desired angle of reradiation is very different from the angle of incidence assumed when designing the RIS. From (53), in particular, the possible angles of reradiation are

\[
\theta_{r,n=1} = -\theta_i (r_{Rx})
\]
\[
\theta_{r,n=0} = 0
\]
\[
\theta_{r,n=+1} = \theta_i (r_{Rx})
\]

and their associated reradiated power is determined by the corresponding three coefficients of the Fourier series expansion of \( \Gamma (r_{Rx}, y, r_{Tx}) \).

From (59), we conclude that, besides the desired angle of reradiation, we may observe two additional radiated modes toward the specular direction and toward the direction that is symmetric to the desired direction of reradiation. The amount of power that is radiated toward these three modes depends, however, on the specific design of
the surface reflection coefficient and surface impedance. In fact, different functions may have the same period, but their corresponding Fourier series are usually different. This point is further clarified in Section IV with the aid of numerical results, where it is shown that surface impedances with almost identical periods result in different reradiated electromagnetic fields. This is because they are characterized by different functions \( \Gamma(r_{\text{Rx}}, y, r_{\text{Tx}}) \).

From (52), we observe, as mentioned, that the possible modes reradiated by an RIS depend on the actual angle of incidence of the electromagnetic waves, regardless of the angles of incidence and reflection for which the RIS is designed, which, however, determines the period \( \mathcal{P} \) of the RIS. If the RIS operates in the presence of other (interfering) electromagnetic waves, this implies that these latter waves are reradiated toward directions that are specified by (53). Therefore, the model for the reflected electric field considered in (32) is not only consistent with Floquet’s theory if the RIS is periodic but it can be used to analyze both intended and interfering electromagnetic waves as well.

**B. Power Efficiency and Reradiated Power Flux**

In this section, we introduce the concepts of power efficiency and reradiated power flux of an RIS as a function of the surface impedance \( Z(r_{\text{Rx}}, y, r_{\text{Tx}}) \). For application to wireless communications and for completeness, we discuss the implications in terms of field reflection coefficient as well. For this analysis, we utilize the notion of Poynting vector [56]. In particular, we consider two designs for an RIS that is referred to as: 1) local design with unitary power efficiency according to a local power conservation principle and 2) global design with unitary power efficiency according to an average or global power conservation principle.

1) **Power Efficiency—Surface Poynting Vector:** The power efficiency characterizes the amount of power that is reradiated toward the desired direction by an RIS, given the amount of incident power. The power efficiency can be deduced by analyzing the net power flow of an RIS in the close vicinity of the surface \( S \) (i.e., at \( z = 0^+ \)). A typical design objective when optimizing an RIS is to engineer the surface impedance \( Z(r_{\text{Rx}}, y, r_{\text{Tx}}) \) so that the power efficiency is unitary [45]. For a lossless RIS, this design criterion implies that the amount of power that is reradiated toward a specified direction of reflection coincides with the amount of incident power, i.e., the net power flow is equal to zero. In this article, we consider only lossless RISs. The net power flow of a lossless RIS can be defined either locally, i.e., for each individual point \( s \in S \), or globally, i.e., for the entire surface \( S \). In this section, we overview the local and global designs for lossless RISs and discuss their properties, advantages, and limitations from the theoretical and implementation points of view.

The departing point to formulate the local and global designs for a lossless RIS is the notion of the surface Poynting vector. By definition, the normal (to the surface \( S \)) component of the surface Poynting vector evaluated at a point \( s \in S \) is defined as follows [56]:

\[
P_S(r_{\text{Rx}}, s, r_{\text{Tx}}) = \frac{1}{2} \Re \left( E_{\text{tot}}^T(r_{\text{Rx}}, s, r_{\text{Tx}}) \times (H_{\text{tot}}(r_{\text{Rx}}, s, r_{\text{Tx}}))^* \right)
\]  

(60)

where \( (\cdot)^* \) denotes the complex conjugate operation. Similar to the preceding text, the total surface electric and magnetic fields in (60) are macroscopic electromagnetic fields that are averaged over the area of a unit cell.

The surface Poynting vector accounts for the interaction between the incident and the reradiated electromagnetic fields in the close vicinity of the surface \( S \) since the total surface electric and magnetic fields defined in (39) and (40) are utilized in (60). This is true regardless of whether the direct link is available or is blocked at the receiver Rx.

a) **Local design:** Based on \( P_S(r_{\text{Rx}}, s, r_{\text{Tx}}) \) in (60), a lossless RIS is defined to be locally passive if the following condition holds true:

\[
P_S(r_{\text{Rx}}, s, r_{\text{Tx}}) = |P_S(r_{\text{Rx}}, s, r_{\text{Tx}})| \leq 0 \quad \forall s \in S.
\]

(61)

By inserting the total surface electric and magnetic fields in (39) and (40) into (61), and by using (34)–(37), the surface Poynting vector evaluated at a point \( y \in [-L_y, L_y]\)
is formulated as follows:

$$P_S(r_{Rx}, y, r_{Tx}) = \frac{1}{2\eta_0} |E_{z,0}^r|^2 S(r_{Rx}, y, r_{Tx})$$  \hspace{1cm} (62)$$

where \(S(r_{Rx}, y, r_{Tx})\) for \(y \in [-L_y, L_y]\) is defined as

$$S(r_{Rx}, y, r_{Tx}) = |\Gamma S(r_{Rx}, y, r_{Tx})|^2 \cos \theta_i(r_{Rx}) - \cos \theta_i(r_{Tx})$$

$$+ \Re \{\Gamma S(r_{Rx}, y, r_{Tx})\} (\cos \theta_i(r_{Rx}) - \cos \theta_i(r_{Tx})) \hspace{1cm} (63)$$

Therefore, \(P_S(r_{Rx}, s, r_{Tx})\) in (61) can be written as follows:

$$P_S(r_{Rx}, y, r_{Tx}) = \frac{1}{2\eta_0} |E_{z,0}^r|^2 S(r_{Rx}, y, r_{Tx}).$$ \hspace{1cm} (64)

By using (42), \(P_S(r_{Rx}, s, r_{Tx})\) can be (equivalently) formulated in terms of the surface impedance \(Z(r_{Rx}, y, r_{Tx})\), as

$$P_S(r_{Rx}, y, r_{Tx}) = \frac{1}{2\eta_0} |E_{z,0}^r|^2 \left(\frac{\cos \theta_i(r_{Tx}) + \cos \theta_i(r_{Rx})}{Z(r_{Rx}, y, r_{Tx}) \cos \theta_i(r_{Rx}) + \eta_0} + \Re \{Z(r_{Rx}, y, r_{Tx})\}\right).\hspace{1cm} (65)$$

By the definition of locally passive design according to (61), we evince from (65) that a lossless RIS is locally passive if and only if \(\Re\{Z(r_{Rx}, y, r_{Tx})\} \geq 0\) \(\forall y \in [-L_y, L_y]\). This is consistent with the engineering insights that can be gained from the surface impedance introduced in Section III-A3 (see (41) and related comments). Notably, a lossless RIS has a locally unitary power efficiency if and only if \(\Re\{Z(r_{Rx}, y, r_{Tx})\} = 0\) \(\forall y \in [-L_y, L_y]\). Accordingly, the surface \(S\) needs to be realized by using only reactive components without using resistive elements. It is worth mentioning that the Helmholtz constraint in (44) needs to be always fulfilled and then taken into consideration. A simple example of local design with unit power efficiency that satisfies the Helmholtz constraint in (44) with equality is when an RIS operates as a simple specular reflector by setting \(\Gamma S(r_{Rx}, y, r_{Tx}) = 1\) \(\forall y \in [-L_y, L_y]\) and \(\cos \theta_i(r_{Rx}) = \cos \theta_i(r_{Tx})\). In general, an infinite number of surface impedances whose real part is equal to zero and fulfills the Helmholtz constraint in (44) can be found, and each of them leads to a different observed power at the receiver \(r_{Rx}\). This is further elaborated on next.

To reframe further engineering insights, especially in the context of wireless communication systems, it is instructive to analyze the design constraints that the surface reflection coefficient \(\Gamma S(r_{Rx}, y, r_{Tx})\) needs to fulfill in order for an RIS to be locally passive and, in particular, for a lossless RIS to have unitary power efficiency. By utilizing the definition of surface reflection coefficient in (27) as a function of the amplitude \(R(r_{Rx}, s, r_{Tx})\), and the phases \(\phi(r_{Rx}, s, r_{Tx})\) and \(\Phi_S(r_{Rx}, y, r_{Tx})\), \(S(r_{Rx}, y, r_{Tx})\) in (63) can be reformulated as

$$S(r_{Rx}, y, r_{Tx}) = (R(r_{Rx}, y, r_{Tx}))^2 \cos \theta_i(r_{Rx}) - \cos \theta_i(r_{Tx})$$

$$+ R(r_{Rx}, y, r_{Tx})(\cos \theta_i(r_{Rx}) - \cos \theta_i(r_{Tx}))$$

$$\ast (\cos(\phi(r_{Rx}, y, r_{Tx}) + \Phi_S(r_{Rx}, y, r_{Tx}))).\hspace{1cm} (66)$$

Equation (66) indicates that \(S(r_{Rx}, y, r_{Tx})\) is a quadratic polynomial in the amplitude \(R(r_{Rx}, s, r_{Tx})\) of the field (and surface) reflection coefficient. Therefore, it can be readily analyzed. By definition, as mentioned, a locally lossless RIS has a locally unitary power efficiency if and only if \(S(r_{Rx}, y, r_{Tx}) = 0\) \(\forall y \in [-L_y, L_y]\). Based on (66), and subject to Helmholtz’s condition in (29), this leads to the design constraint

$$R(r_{Rx}, y, r_{Tx}) = \frac{1}{2} \cos (\Psi(y))(F_{i,r} - 1)$$

$$+ \frac{1}{2} \sqrt{\cos^2(\Psi(y))(F_{i,r} - 1)^2 + 4F_{i,r}} \hspace{1cm} (67)$$

where the following shorthand notation is introduced:

$$F_{i,r} = F(r_{Rx}, r_{Tx}) = \frac{\cos \theta_i(r_{Tx})}{\cos \theta_i(r_{Rx})}$$

$$\Psi(y) = \Psi(r_{Rx}, y, r_{Tx}) = \phi(r_{Rx}, y, r_{Tx}) + \Phi_S(r_{Rx}, y, r_{Tx}).$$

From (67), we conclude that the amplitude \(R(r_{Rx}, y, r_{Tx})\) and the total phase \(\Psi(r_{Rx}, y, r_{Tx})\) of the surface reflection coefficient are intertwined, and they are not, in general, independent of each other if a lossless RIS needs to have a locally unitary power efficiency. By definition, (65) and (67) are equivalent. Therefore, imposing \(\Re\{Z(r_{Rx}, y, r_{Tx})\} = 0\) \(\forall y \in [-L_y, L_y]\) is equivalent to imposing a well-defined relation between the amplitude and the phase of the field reflection coefficient, as obtained in (67). From (47), notably, we know that the condition \(\Re\{Z(r_{Rx}, y, r_{Tx})\} = 0\) does not necessarily imply \(|\Gamma S(r_{Rx}, y, r_{Tx})| = R(r_{Rx}, y, r_{Tx}) \leq 1\). Indeed, whether the amplitude \(R(r_{Rx}, y, r_{Tx})\) of the reflection coefficient is smaller or greater than one depends on the angles of incidence and reflection. If we consider the special case of specular reflection, i.e., \(\theta_i(r_{Rx}) = \theta_i(r_{Tx})\), then \(F_{i,r} = 1\) and we obtain \(R(r_{Rx}, y, r_{Tx}) = 1\) \(\forall y \in [-L_y, L_y]\). If \(\theta_i(r_{Rx}) \neq \theta_i(r_{Tx})\), on the other hand, the amplitude \(R(r_{Rx}, y, r_{Tx})\) of the reflection coefficient is, in general, not unitary.

In wireless communications, the constraint \(R(r_{Rx}, y, r_{Tx}) = 1\) is typically assumed when optimizing an RIS [20]. Also, the total phase \(\Psi(r_{Rx}, y, r_{Tx})\) is often optimized by assuming that it is independent of \(R(r_{Rx}, y, r_{Tx})\). This is not in agreement with the condition obtained in (67) for designing lossless RISs with locally unitary power efficiency. By letting \(R(r_{Rx}, y, r_{Tx}) = |\Gamma S(r_{Rx}, y, r_{Tx})| = 1\) in (47), specifically, the
real part of the surface impedance can only be equal to
\[
\Re(Z(r_{\text{Rx}}, y, r_{\text{Tx}})) = \frac{|Z(r_{\text{Rx}}, y, r_{\text{Tx}})|^2}{2r_{\text{0}}} * (\cos \theta(r_{\text{Rx}}) - \cos \theta(r_{\text{Tx}})). (70)
\]

Therefore, the condition \(\Re(Z(r_{\text{Rx}}, y, r_{\text{Tx}})) = 0\), i.e., locally unitary power efficiency, can be ensured only for specular reflection. If \(\theta_i(r_{\text{Fx}}) \neq \theta_i(r_{\text{Rx}})\), on the other hand, \(\Re(Z(r_{\text{Rx}}, y, r_{\text{Tx}}))\) can be either positive or negative, which results in a locally passive or locally active RIS, respectively, as dictated by (66). Interested readers are invited to consult [31], [45], [47] for examples and discussions.

Let us analyze in further detail the case study for which the constraint \(R(r_{\text{Rx}}, y, r_{\text{Tx}}) = 1\) \(\forall y \in [-L_y, L_y]\) is imposed by design. In this case, \(S(r_{\text{Rx}}, y, r_{\text{Tx}})\) in (63) simplifies to
\[
S(r_{\text{Rx}}, y, r_{\text{Tx}}) = (\cos \theta(r_{\text{Rx}}) - \cos \theta(r_{\text{Tx}}))
* (1 + \cos (\Psi(r_{\text{Rx}}, y, r_{\text{Tx}}))). (71)
\]

From (71), we retrieve, in agreement with (70), that, for phase-gradient reflectors, a unitary power efficiency is obtained only for specular reflection (i.e., \(\theta_i(r_{\text{Fx}}) = \theta_i(r_{\text{Rx}})\)). If the angle of incidence is normal to the surface \(S\) (i.e., \(\theta_i(r_{\text{Fx}}) = 0\)), as is often assumed or implied in the literature, we evince that the net power flow in (71) is negative, i.e., \(S(r_{\text{Rx}}, y, r_{\text{Tx}}) < 0\), for any angle of reflection. This implies that it is possible to steer a normally incident wave toward any directions of reflection while ensuring \(R(r_{\text{Rx}}, y, r_{\text{Tx}}) = 1\) and without necessitating any local power amplification or active components. This is in agreement with (47), which yields \(\Re(Z(r_{\text{Rx}}, y, r_{\text{Tx}})) > 0\) in this considered case study. The power efficiency, however, highly depends on the angle of reflection, and it typically decreases as the angle of reflection increases. If the specified angle of reflection is smaller than the angle of incidence, i.e., \(\cos \theta(r_{\text{Rx}}) < \cos \theta(r_{\text{Fx}})\), (71) unveils that the net power flow is locally positive, i.e., \(S(r_{\text{Rx}}, y, r_{\text{Tx}}) > 0\), which implies that the corresponding wave transformation, with the constraint \(R(r_{\text{Rx}}, y, r_{\text{Tx}}) = 1\), cannot be realized without local power amplification or active components. This is in agreement with (47), i.e., the real part of the surface impedance needs to be negative, and thus, the RIS needs to locally amplify the incident wave. In practice, this implies that, to realize this wave transformation without active components, the amplitude of the reflection coefficient cannot be equal to one, i.e., \(R(r_{\text{Rx}}, y, r_{\text{Tx}}) \neq 1\), and the total phase \(\Psi(r_{\text{Rx}}, y, r_{\text{Tx}})\) of the surface reflection coefficient needs to be carefully engineered. Based on (66), specifically, \(S(r_{\text{Rx}}, y, r_{\text{Tx}}) \leq 0\) if and only if
\[
0 \leq R(r_{\text{Rx}}, y, r_{\text{Tx}}) \leq \frac{1}{2} \cos (\Psi(y)) (F_{i,r} - 1)
+ \frac{1}{2} \sqrt{\cos^2 (\Psi(y)) (F_{i,r} - 1)^2 + 4F_{i,r}}.
(72)
\]

The condition \(S(r_{\text{Fx}}, y, r_{\text{Tx}}) \leq 0\) implies, however, that a smaller amount of power is reradiated toward the specified direction of reflection, and consequently, a smaller amount of power is available at the receiver Rx.

The remarks that originate from (47) and (67) allow us to conclude that the optimization problems typically formulated in wireless communications result in RISs whose power efficiency is not necessarily unitary even though the reflection coefficient is unitary. This is because the condition \(R(r_{\text{Rx}}, y, r_{\text{Tx}}) = 1\) results in \(\Re(Z(r_{\text{Rx}}, y, r_{\text{Tx}})) = 0\) only if \(\theta_i(r_{\text{Rx}}) = \theta_i(r_{\text{Tx}})\). This apparent inconsistency has a simple justification. In wireless communications, the definition of reflection coefficient that is typically utilized is \(\Gamma_{\text{load}}(r_{\text{Fx}}, y, r_{\text{Tx}})\) in (46). We have already noted that \(\Gamma_{\text{load}}(r_{\text{Fx}}, y, r_{\text{Tx}}) = 1\) if and only if \(\Re(Z(r_{\text{Rx}}, y, r_{\text{Tx}})) = 0\). With this definition, as a result, we can, indeed, conclude that a lossless RIS has a local unitary power efficiency if \(\Gamma_{\text{load}}(r_{\text{Fx}}, y, r_{\text{Tx}}) = 1\). Based on \(\Gamma_{\text{load}}(r_{\text{Rx}}, y, r_{\text{Tx}})\), therefore, the conclusions drawn in the context of wireless communications are consistent. Specifically, the load reflection coefficient \(\Gamma_{\text{load}}(r_{\text{Rx}}, y, r_{\text{Tx}})\) consistent with the locally periodic discrete model in Section II-A and with the local specular reflection model illustrated in Fig. 9(a).

b) Global design: According to the local design criterion, a lossless RIS has locally unitary power efficiency if and only if \(S(r_{\text{Rx}}, y, r_{\text{Tx}}) \leq 0\) \(\forall y \in [-L_y, L_y]\). Based on this definition, the real part of the surface impedance needs to be identically equal to zero along the entire surface \(S\), i.e., \(\Re(Z(r_{\text{Rx}}, y, r_{\text{Tx}})) = 0\) \(\forall y \in [-L_y, L_y]\). The main advantage of this design criterion is the relatively simple structure of a lossless RIS, which can be implemented without using resistive components while ensuring a unitary power efficiency. The main disadvantages are, on the other hand, the restricted feasible set of possible implementation options that the constraint \(\Re(Z(r_{\text{Rx}}, y, r_{\text{Tx}})) = 0\) \(\forall y \in [-L_y, L_y]\) offers, the possibly low reradiated power at the location of the receiver Rx, and the impossibility of realizing some wave transformations. To elucidate this latter point, let us analyze the canonical example of anomalous reflection [45].

Let us consider the design of a lossless RIS for which \(R(r_{\text{Rx}}, y, r_{\text{Tx}}) = R_0, \phi(r_{\text{Rx}}, y, r_{\text{Tx}}) = 0, \) and \(\phi(r_{\text{Fx}}, y, r_{\text{Tx}})\) are given in (26) \(\forall y \in [-L_y, L_y]\). This corresponds to an anomalous reflector [45] for which Helmholtz’s condition in (29) is fulfilled with equality. If \(R_0 = 1\) and \(R_0 = (\cos \theta_i(r_{\text{Rx}})/\cos \theta_i(r_{\text{Fx}}))^{1/2}\), we retrieve the geometrical optics solution and the perfect anomalous reflector under the assumption that the RIS is periodic and an integer number of periods is available in \([-L_y, L_y]\), respectively [45], [47]. In this case, the surface impedance \(Z(r_{\text{Rx}}, y, r_{\text{Tx}})\) in (41) simplifies as follows:
\[
Z(r_{\text{Rx}}, y, r_{\text{Tx}}) = \frac{\eta_0}{\cos \theta_i(r_{\text{Rx}})} C_0 - R_0 W(y).
(73)
\]
where the following notation is introduced:

\[ V(y) = e^{j\Phi_S(r_{rx}, y, r_{tx})} \]  \hspace{1cm} (74)

\[ C_0 = \cos \theta_r(r_{tx})/\cos \theta_r(r_{rx}) \]  \hspace{1cm} (75)

Then, the real part of \( Z(r_{rx}, y, r_{tx}) \) in (73) is

\[ \Re(Z(r_{rx}, y, r_{tx})) = \frac{\eta P}{\cos \theta_r(r_{rx})} \frac{N_2(y)}{D_2(y)} \]  \hspace{1cm} (76)

where the following shorthand notation is used:

\[ N_2(y) = \cos \theta_r(r_{rx}) - C_0^2 + (C_0 - 1) R_0 \cos(\Phi_S(r_{rx}, y, r_{tx})) \]  \hspace{1cm} (77)

\[ D_2(y) = C_0^2 + R_0^2 - 2C_0 R_0 \cos(\Phi_S(r_{rx}, y, r_{tx})) \]  \hspace{1cm} (78)

By direct inspection of (76), it follows that \( \Re(Z(r_{rx}, y, r_{tx})) = 0 \) if and only if \( N_2(y) = 0 \) \( \forall y \in [-L_y, L_y] \). However, this condition cannot be ensured \( \forall y \in [-L_y, L_y] \) either for \( R_0 = 1 \) or \( R_0 = (\cos \theta_r(r_{tx})/\cos \theta_r(r_{rx}))^{1/2} = \sqrt{C_0} \). In fact, \( \Re(Z(r_{rx}, y, r_{tx})) \) is, in general, an oscillatory function in \( y \) since \( D_2(y) \geq 0 \) \( \forall y \in [-L_y, L_y] \), and \( N_2(y) \) is positive or negative in \( [-L_y, L_y] \). To gain further insights and for illustrative purposes, let us assume \( C_0 \geq 1 \). This is always true if, e.g., \( \cos \theta_r(r_{rx}) = 1 \), which corresponds to normal incidence. Then, we have the following findings for \( R_0 = 1 \) and \( R_0 = \sqrt{C_0} \), respectively.

1) \( R_0 = 1 \): In this case, \( D_2(y) \geq 0 \) and \( N_2(y) \geq 0 \) \( \forall y \in [-L_y, L_y] \). This implies that a lossless RIS can be implemented without any power amplification, and the surface power flow in (65) is negative and can be very different from zero.

2) \( R_0 = \sqrt{C_0} \): In this case, \( D_2(y) \geq 0 \) \( \forall y \in [-L_y, L_y] \), and \( N_2(y) = C_0(\cos \theta_r(r_{rx}) - C_0 - 1) \cos(\Phi_S(r_{rx}, y, r_{tx})) \), which is an oscillating function in \( [-L_y, L_y] \), i.e., it can take positive and negative values. This implies that this wave transformation cannot be realized without utilizing any sort of power amplification. The power amplification can be virtual, e.g., by using surface waves, or can be realized through actual power amplifiers [31]. The corresponding power flow in (65) is oscillatory in \( [-L_y, L_y] \) as well.

This simple example that corresponds to the design of a canonical anomalous reflector allows us to understand that the constraint \( \Re(Z(r_{rx}, y, r_{tx})) = 0 \) may be too restrictive for enabling the realization of some wave transformations, and other design criteria may be more appropriate to this end. The global design criterion falls in this category since it allows us to relax the inherent constraints of the local design and enlarge the feasible set of wave transformations that can be realized at high power efficiency. The main essence of the global design is to relax the local power efficiency constraint, i.e., for each point of the surface \( S \), with an average power efficiency constraint, i.e., by considering the entire surface \( S \). For completeness, it is worth mentioning that an RIS optimized based on the global design is different from a hybrid or an active RIS [72]. In a hybrid RIS, some RIS elements have the capability of amplifying the incident electromagnetic waves so that the total radiated power is greater than the total incident power. An RIS designed based on the global design does not increase the amount of incident power: the available power budget is just carefully redistributed along the surface [31].

Specifically, based on the definition of the surface Poynting vector \( P_S(r_{rx}, y, r_{tx}) \) in (60), a lossless RIS is defined to be globally passive if the following condition holds true:

\[ P_S(r_{rx}, r_{tx}) = \int_{S \in S} |P_s(r_{rx}, y, r_{tx})| ds \leq 0. \]  \hspace{1cm} (79)

By utilizing the same notation as for the local design, the average power flow \( P_S(r_{rx}, r_{tx}) \) in (79) can be explicitly written as follows:

\[ P_S(r_{rx}, r_{tx}) = \int_{S \in S} P_S(r_{rx}, y, r_{tx}) dy dx = 2L_x \int_{-L_y}^{L_y} P_S(r_{rx}, y, r_{tx}) dy \]  \hspace{1cm} (80)

where \( P_S(r_{rx}, y, r_{tx}) \) is given in (65).

Based on (79), therefore, a lossless RIS has, on average, unitary power efficiency if and only if \( P_S(r_{rx}, r_{tx}) = 0 \). In other words, the power flow does not need to be equal to zero for each point of the surface, but only the power flow integrated along the surface \( S \) needs to be equal to zero. This implies that the global design accounts for solutions that may correspond to practical implementations that require positive and negative values of the surface impedance \( Z(r_{rx}, y, r_{tx}) \). Then, the design constraint is milder: the integral of the local power flow along the surface is equal to zero. In a global design, in particular, the electromagnetic field reradiated by an RIS is not obtained as the local contribution of each RIS element but as the collective action of all the RIS elements as a whole.

Let us consider again the case study of perfect anomalous reflection already analyzed from the point of view of the local design. The wave transformation for which \( R_0 = (\cos \theta_r(r_{tx})/\cos \theta_r(r_{rx}))^{1/2} \) corresponds to the globally optimum design with unitary power efficiency, i.e., \( R_0 \) is obtained by setting \( P_S(r_{rx}, r_{tx}) = 0 \) in (80) under the assumption that the RIS is periodic, and an integer number of periods is available in \( [-L_y, L_y] \). Therefore, a global design offers a greater flexibility than a local design.

c) Local design versus global design: In general, a locally optimal solution with unitary power efficiency is a globally optimal solution with unitary power efficiency as well. The opposite is, however, not true in general. This implies that a global design enlarges the set of feasible solutions that can be found when considering a local design. How-
ever, there exist inherent performance and implementation tradeoffs between local and global designs.

The solutions found according to a local design with unitary power efficiency result in implementations of RISs whose real part of the surface impedance is identically equal to zero. This implies that the corresponding RISs can be realized without utilizing resistive components and by utilizing only capacitive and inductive elements. In general, this simplifies the implementation of the surface $S$. However, the set of feasible solutions that fulfill the local design criterion may be limited, and some wave transformations may not be allowed by the design constraint.

A global design allows, on the other hand, solutions for which the real part of the surface impedance is not necessarily equal to zero. This enlarges the set of feasible solutions. Also, wave transformations that may require positive and negative values of the surface impedance are allowed. However, RISs with resistive elements and with active elements, which correspond to negative values of the surface impedance, are difficult to design and implement. The need for active elements can be avoided by realizing virtual power losses and virtual power gains along the surface $S$ through carefully engineered evanescent Floquet’s harmonics (surface waves) that are excited at the metasurface [31], [70], [71], [73].

Therefore, it would be convenient to identify designs for RIS that have a unitary power efficiency, allow the same feasible set of solutions as the global design, and have the same implementation simplicity as the local design, i.e., they can be realized by using purely reactive components. With this in mind, it is convenient to depart from the global design with unitary power efficiency as the starting point for designing an RIS in order to have a large set of feasible solutions. Once the corresponding optimal solution is found, one can find an approximate solution that corresponds to an implementation of the RIS with a surface impedance whose real part is equal to zero. This would make the implementation of the RIS easier while ensuring high power efficiency. Examples of similar design methods do exist in the recent scientific literature, e.g., [58]. In the next subsection, we illustrate examples of this design paradigm with application to wireless communications and with a focus on how approximated solutions can be found. We anticipate that the approximated solutions can be implemented by utilizing only reactive components, but they may result in slightly lower beam pattern gains and higher side lobes that need to be accurately controlled when formulating the problem.

**Periodic Versus Aperiodic Surfaces:** In Section I, we have discussed the difference between periodic and aperiodic RISs with the help of the example illustrated in Fig. 5. It is instructive to elaborate further on the concept of periodicity of an engineered surface in the context of optimizing an RIS according to the global design criterion. Specifically, we are interested in the RIS period that depends on the specified wave transformation that

$$\chi_2 R_0^2 + \chi_1 R_0 + \chi_0 = 0$$    \hspace{1cm} (81)

where

$$\chi_2 = L_y \cos \theta_l (r_{Rx})$$   \hspace{1cm} (82)

$$\chi_1 = (\cos \theta_l (r_{Rx}) - \cos \theta_l (r_{Tx})) \times \frac{\sin (\Phi (r_{Rx}, y = L_y, r_{Tx}))}{\Phi (r_{Rx}, y = 1, r_{Tx})}$$   \hspace{1cm} (83)

$$\chi_0 = -L_y \cos \theta_l (r_{Tx})$$   \hspace{1cm} (84)

In this case, it is interesting to note that $R_0$ depends on the size of the RIS, i.e., $2L_y$, as well.
An illustration of this case study is given in Fig. 12. Specifically, we plot the average power flow $P_S (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$ in (79) by using $S (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$ in (63), and by setting $R (\mathbf{r}_{Rx}, \mathbf{r}_{Tx}) = R_0 = (1 - \cos \theta_r (\mathbf{r}_{Rx}))^{1/2}$, $\phi (\mathbf{r}_{Rx}, \mathbf{r}_{Tx}) = 0$, and $\Psi (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$, as defined in (26) $\forall \mathbf{y} \in [-L_x, L_x]$. This corresponds to the canonical case study analyzed in the literature [17, 31] when $\theta_r = 0$, and the RIS period is equal to $P = \lambda/\cos \theta_r (\mathbf{r}_{Rx})$. We see that $P_S (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$ is equal to zero only if the length $2L_y$ of the RIS is equal to an integer number of RIS periods. This implies that the solution $R (\mathbf{r}_{Rx}, \mathbf{r}_{Tx}) = R_0 = (1 - \cos \theta_r (\mathbf{r}_{Rx}))^{1/2}$ is, in general, not optimal if the RIS is either aperiodic, or it has an arbitrary size that is not equal to an integer number of RIS periods. This is because, under these conditions, the integration of the addend in the second line of $S (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$ in (63) is not equal to zero. This is thoroughly discussed and formulated in [17, eqs. (17) and (18)]. As mentioned in Section I, however, the RISs studied in wireless communications are often aperiodic surfaces, and their physical size is fixed and is not usually adapted to the RIS period, e.g., to the specified wave transformation to realize. Equation (81) shows an example for which $P_S (\mathbf{r}_{Rx}, \mathbf{r}_{Tx}) = 0$ and $R_0$ can be still formulated in a closed-form expression.

This simple example allows us to further motivate the signal model in (34)–(37), and specifically, the assumption that the surface reflection coefficient $\Gamma (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$ may depend on $y$ (in general, it may depend on $(x, y) \in S$), provided that the Helmholtz constraint in (28) is fulfilled. By allowing $\Gamma (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$ to depend on $y$, i.e., either the amplitude, the phase, or both may depend on $y$ subject to the constraint in (28), it may be possible to design an RIS for which $P_S (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$ is equal to zero (or it is numerically very close to zero), even if the RIS is an aperiodic surface, and its size is arbitrarily chosen, regardless of the RIS period associated with the specified wave transformation to be realized. Equation (81) shows an example for which this is obtained under the assumptions $R (\mathbf{r}_{Rx}, \mathbf{r}_{Tx}) = R_0$, $\phi (\mathbf{r}_{Rx}, \mathbf{r}_{Tx}) = 0$, and $\Psi (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$ given in (26) $\forall \mathbf{y} \in [-L_x, L_x]$. The proposed parametric model can be applied to different case studies, e.g., when the amplitude $R (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$ and the phase $\phi (\mathbf{r}_{Rx}, \mathbf{r}_{Tx})$ may not be optimized independently of each other [34]. More in general, the proposed model allows us to consider different and desirable tradeoffs between design optimality and implementation complexity.

2) Reradiated Power Flux—Poynting Vector: The surface power flow introduced in the preceding text allows us to characterize the efficiency of an RIS as a device that realizes specified wave transformations. It does not offer, however, any information on the amount of power that is available at the receiver Rx, which ultimately determines the performance of a communication link. The surface power efficiency provides information only on the difference between the incident and the reradiated powers in the close vicinity of the surface $S$. If a lossless RIS has a unit power efficiency, the total reradiated power is equal to the total incident power. Power efficiency is, therefore, an important key performance indicator to characterize a communication link in the presence of an RIS. However, it is not sufficient. It is necessary to characterize the power observed at the location of the intended receiver Rx as well. This is possible by introducing the notion of reradiated power flux.

The reradiated power flux characterizes the amount of power that is reradiated by an RIS at an arbitrary point of observation, which can be located in the radiative (Fresnel) near-field and the (Fraunhofer) far-field regions of the surface $S$ [17], [62]. Therefore, the reradiated power flux is not defined only in the close proximity of the surface $S$, i.e., at $z = 0^\circ$. The reradiated power flux provides information on the angular response of the RIS and, in particular, how the incident power is reradiated as a function of the angle of observation. In the far-field region of the RIS, the reradiated power flux is proportional to the radiation pattern (or array factor) of the RIS. The reradiated power flux allows us to characterize the amount of power that is reradiated toward the specified direction of design and toward undesired (spurious) directions of observation. Therefore, both the main lobe and the side lobes of the RIS are characterized. The reradiated power flux is an essential performance indicator in wireless communications since it determines the amount of received power and, therefore, the signal-to-noise and the signal-to-interference ratios. In addition, the recent study reported in [34] highlights the importance of characterizing the reradiation pattern of an RIS for every allowed angle of observation, so as to ensure that the incident power is not directed toward unwanted directions.

The reradiated power flux is defined from the Poynting vector evaluated at any observation point in a 3-D volume, $V$, of interest. Let $\mathbf{r}_{obs} = x_{obs} \hat{x} + y_{obs} \hat{y} + z_{obs} \hat{z}$ be a generic observation point in $V$. For simplicity, we assume that $\mathbf{r}_{obs}$ is located outside the volume $V_{Tx}$ occupied by the transmitter Tx. Therefore, the canonical source-free scenario is considered. For generality, we assume that $\mathbf{r}_{obs}$ is located at a distance $R_{obs}$ from the center of the surface $S$ (the origin of the reference system), and the elevation angle and the azimuth angle are equal to $\theta_\circ$ and $\varphi_\circ$, respectively, with respect to the origin. Thus, $\mathbf{r}_{obs} = R_{obs} (\sin \theta_\circ \cos \varphi_\circ \hat{x} + \sin \theta_\circ \sin \varphi_\circ \hat{y} + \cos \theta_\circ \hat{z})$ with $R_{obs} = |R_{obs}|$.

As mentioned in previous sections, we assume, for simplicity but without loss of generality, that the direct link is blocked at the observation point $\mathbf{r}_{obs}$. This is known to be the most useful case study when deploying an RIS [74], and in addition, the focus of this article is on the electromagnetic field reradiated by the RIS. The direct link can be taken into account, as described in [62]. By definition, the Poynting vector evaluated at $\mathbf{r}_{obs} = x_{obs} \hat{x} + y_{obs} \hat{y} + z_{obs} \hat{z}$ is formulated as follows [56]:

$$\mathbf{P}_{obs} (\mathbf{r}_{obs}) = \frac{1}{2} \mathbf{E}_{obs} (\mathbf{r}_{obs}) \times \mathbf{H}_{obs} (\mathbf{r}_{obs})$$
\[ P_{\text{obs}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) = \frac{1}{2} \Re \big( E_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \times H_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \big) \]  

(86)

where \( E_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \) and \( H_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \) denote the reradiated electric and magnetic fields evaluated at \( r_{\text{obs}} \), respectively.

It is worth emphasizing that \( E_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \) and \( H_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \) in (86) are different from the electric and magnetic fields in the close vicinity of the surface \( S \), as defined in (24), since the latter fields are defined only for \( 7m < z < z_{\text{obs}} < 7m \). The electric and magnetic fields \( E_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \) and \( H_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \) in (86) are, however, uniquely determined by the surface electromagnetic fields in (34)–(37). By using Franz’s formula [56, eq. (18.10.11)], specifically, \( E_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \) and \( H_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \) can be formulated as follows:

\[ E_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) = \frac{1}{j\omega \mu_0} \left( \nabla n_{\text{obs}} \times \left( \nabla n_{\text{obs}} \times A_s(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \right) \right) \]

\[ H_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) = -\frac{1}{j\omega \mu_0} \nabla n_{\text{obs}} \times E_{\text{ref}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) \]

(87)

(88)

where the following shorthand notation is introduced:

\[ A_s(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) = \mu_0 \int_S \left( n_{\text{out}} \times H_{\text{ref}}'(r_{\text{Rx}}, y, r_{\text{Tx}}) \right) G(r_{\text{obs}}, s) \, ds \]

\[ A_{\text{ms}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) = -\epsilon_0 \int_S \left( n_{\text{out}} \times E_{\text{ref}}'(r_{\text{Rx}}, y, r_{\text{Tx}}) \right) G(r_{\text{obs}}, s) \, ds \]

(89)

(90)

and \( n_{\text{out}} = -z \) is the unit norm vector that is perpendicular to the RIS and points toward the transmission side of \( S \), \( E_{\text{ref}}(r_{\text{Rx}}, y, r_{\text{Tx}}) \) and \( H_{\text{ref}}'(r_{\text{Rx}}, y, r_{\text{Tx}}) \) are the surface electric and magnetic fields defined in (36) and (37), respectively, and \( G(r_{\text{obs}}, s) \) is the scalar Green function that corresponds to a point source located at \( s = x\hat{x} + y\hat{y} + z_\text{obs}\hat{z} \in S \) and is observed at \( r_{\text{obs}} \).

\[ G(r_{\text{obs}}, s) = \frac{e^{-jk\|r_{\text{obs}}-s\|}}{4\pi \|r_{\text{obs}}-s\|} \]

(91)

with \( \|r_{\text{obs}}-s\| = \sqrt{(x_{\text{obs}}-x)^2 + (y_{\text{obs}}-y)^2 + z_{\text{obs}}^2} \).

The main assumptions for the validity of (87) and (88) lie in the approximations made for applying (24), i.e., the physical optics approximation [57]. More precisely, \( J(y) = n_{\text{out}} \times H_{\text{ref}}'(r_{\text{Rx}}, y, r_{\text{Tx}}) \) and \( M(y) = -n_{\text{out}} \times E_{\text{ref}}'(r_{\text{Rx}}, y, r_{\text{Tx}}) \) can be interpreted as equivalent electric and magnetic surface currents, respectively, which produce the electromagnetic fields reradiated by the RIS. These equivalent surface currents are obtained under the assumption that the RIS has infinite size, and no edge effects are accounted for in the currents distribution along the surface \( S \). Even though the physical optics method yields an approximated solution, it allows us to obtain analytical expressions that are suitable for performance evaluation, for optimizing RISs based on relevant key performance criteria, and to get engineering insights for network design. The limitations of the physical optics approximation method may be overcome by resorting to numerical methods, e.g., the method of moments, which, however, offers limited design insights and has limited applicability for network optimization in the context of wireless communications.

Under the physical optics approximation, the reradiation integrals in (87) and (88) have general applicability. In this article, for ease of illustration, we focus our attention on networks setups in which \( r_{\text{obs}} \) lies in the Fraunhofer far-field region of the RIS, i.e., \( r_{\text{obs}} \geq z_{\text{obs}} \geq 8L_x^2 + L_y^2 \)/\( \lambda \). In this case, the electric and magnetic fields in (87) and (88), respectively, can be simplified.

More precisely, for ease of writing, let us introduce the following shorthand notation:

\[ E_{\text{ref},x}'(r_{\text{Rx}}, y, r_{\text{Tx}}) = \hat{x} \cdot E_{\text{ref}}(r_{\text{Rx}}, y, r_{\text{Tx}}) \]

\[ H_{\text{ref},y}'(r_{\text{Rx}}, y, r_{\text{Tx}}) = \hat{y} \cdot H_{\text{ref}}'(r_{\text{Rx}}, y, r_{\text{Tx}}) \]

\[ \hat{r}_{\text{obs}}(s) = \frac{(x_{\text{obs}}-x)\hat{x} + (y_{\text{obs}}-y)\hat{y} + z_{\text{obs}}\hat{z} \, \|r_{\text{obs}}-s\|}{\|r_{\text{obs}}-s\|} \]

(92)

(93)

(94)

where \( \hat{r}_{\text{obs}}(s) \) is a unit norm vector that is directed from \( s \) to \( r_{\text{obs}} \). Also, \( E_{\text{ref},x}'(r_{\text{Rx}}, y, r_{\text{Tx}}) \) and \( H_{\text{ref},y}'(r_{\text{Rx}}, y, r_{\text{Tx}}) \) are, by definition, independent of \( r_{\text{obs}} \).

Based on this notation and considerations, we evince that the operator \( \nabla n_{\text{obs}} \) can be moved inside the integrals in (89) and (90), and it operates only on the Green function. Under the mild assumption \( r_{\text{obs}} \geq z_{\text{obs}} \gg \lambda \), the following approximations can be applied to the integrand functions in (89) and (90):

\[ \nabla n_{\text{obs}} \times \left( \nabla n_{\text{obs}} \times \left( E_{\text{ref},x}'(r_{\text{Rx}}, y, r_{\text{Tx}}) G(r_{\text{obs}}, s) \hat{x} \right) \right) \approx -k^2 E_{\text{ref},x}'(r_{\text{Rx}}, y, r_{\text{Tx}}) G(r_{\text{obs}}, s) \times (\hat{r}_{\text{obs}}(s) \times \hat{x}) \]

\[ \nabla n_{\text{obs}} \times \left( H_{\text{ref},y}'(r_{\text{Rx}}, y, r_{\text{Tx}}) G(r_{\text{obs}}, s) \hat{y} \right) \approx -k^2 H_{\text{ref},y}'(r_{\text{Rx}}, y, r_{\text{Tx}}) G(r_{\text{obs}}, s) \times (\hat{r}_{\text{obs}}(s) \times \hat{y}) \]

(95)

(96)
The approximations in (95) and (96) avoid the explicit computation of the derivatives of the electric and magnetic surface fields, and therefore, they make the computation of the power flux relatively simple. In addition, these approximations are applicable to the Fresnel and Fraunhofer regions of the RIS. In the Fraunhofer region of the RIS, (87) and (88) can be further simplified, and for illustrative purposes, we focus our attention on this regime for the rest of this article.

If \( R_{\text{obs}} \geq r_{\text{obs}} \geq 8(L_x^2 + L_y^2)/\lambda \), specifically, the Poynting vector in (86) can be formulated as follows:

\[
P_{\text{obs}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) = \frac{k^2}{2\eta_0} |E_{\text{obs},0}^i|^2 \Theta(\theta, \phi) \cdot |A(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}})|^2 r_{\text{obs},0}
\]

where the following functions are defined:

\[
\begin{align*}
A(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) &= e^{-jkR_{\text{obs}}} \\
\Theta(\theta, \phi) &= \left(1 - \sin^2 \theta \cos^2 \phi_0 \right) \cos^2 \theta \left(1 - \sin^2 \theta \sin^2 \phi_0 \right)
\end{align*}
\]

\[
\Gamma_{\text{obs}}(r_{\text{obs}}) = \frac{\sin \theta_0 \cos \phi_0 x + \sin \theta_0 \sin \phi_0 y + \cos \theta_0 z}{\Gamma_{\text{obs}}}
\]

The analytical expression of the power flux evaluated at \( r_{\text{obs}} \) in (97) explicitly depends on the surface reflection coefficient \( \Gamma_S(r_{\text{Rx}}, y, r_{\text{Tx}}) \), it is electromagnetically consistent provided that the Helmholtz constraint in (28) is fulfilled, and it is simple enough for computing the power reradiated from an RIS as a function of the angle of observation. In Section III-C, (97) is utilized for evaluating the performance of an RIS as a function of the surface impedance.

C. Optimization of the Surface Impedance

In this section, we overview mathematical formulations of canonical optimization problems for RISs that fulfill the design criteria introduced in Section III-B, and we evaluate their performance in terms of power flux, i.e., \( P_{\text{obs}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}}) = |P_{\text{obs}}(r_{\text{Rx}}, r_{\text{obs}}, r_{\text{Tx}})| \), as a function of the observation point. This allows us to characterize the angular response (the reradiation pattern) of an RIS. Notably, we analyze and compare solutions to the optimization problems that fulfill Helmholtz’s condition against solutions that are not subject to Helmholtz’s constraint. As case studies for the optimization criteria, we consider the local design, the global design, and an approximated solution to the global design that is realized by utilizing purely reactive surface impedances. We illustrate how some designs of RISs may lead to a large amount of reradiated power toward directions that are different from the direction of design, i.e., where the receiver Rx is. This is shown to be in agreement with Floquet’s theorem applied to periodic structures. In light of the power conservation principle, these spurious reflections result in a lower amount of reradiated power toward the intended direction. We discuss how these spurious reflections can be kept under control at the design stage as well.

For ease of writing and to facilitate the implementation of the numerical algorithms and the computation of the corresponding numerical solutions, we summarize in Table 5 the main functions utilized in this section. The functions reported in Table 5 are, specifically, discretized into unit cells of length \( \Delta_y \), which denotes the spatial resolution at which the amplitude and phase of the incident wave are controlled and shaped by the RIS. The variable of optimization is the surface impedance, as it provides direct information on how an RIS is implemented. Since the surface impedance has variations only along the \( y \)-axis in the considered case study, it is sampled at the center point of each unit cell, i.e., \( y_n = -L_y - \Delta_y/2 + n\Delta_y \) for \( n = 1, 2, \ldots, N \) and \( \Delta_y = 2L_y/N \). The resulting \( N \) samples are collected in a vector \( \mathbf{Z} \) of size \( N \), as defined in Table 5. For simplicity, similar to the incident and reradiated electromagnetic waves, we assume that \( \phi_0 = \pi/2 \), i.e., the observation point lies in the \( yz \) plane. The RIS is optimized based on specified locations of the transmitter \( \mathbf{T}_{\text{Rx}} \) and the receiver \( \mathbf{R}_{\text{Rx}} \), while \( r_{\text{obs}} \) characterizes the location at which the power flux is observed. In general, \( r_{\text{obs}} \neq r_{\text{Rx}} \neq r_{\text{Tx}} \). For simplicity, similar to Table 5, we use the notation \( \theta_i = \theta(r_{\text{Rx}}) \) and \( \theta_r = \theta(r_{\text{Rx}}) \).

1) Benchmark Solution—Generalized Geometrical Optics: As a benchmark solution for the considered system designs, we consider the canonical linear phase-gradient design, which we refer to as the generalized geometrical optics solution, since it leads to the so-called generalized law of reflection [2]. The generalized geometrical optics solution is a typical local design, which, however, does not necessarily guarantee a locally unitary power efficiency. It corresponds to the following surface reflection coefficient and surface impedance:

\[
\Gamma_{\text{GO}}(r_{\text{Rx}}, y, r_{\text{Tx}}) = \Gamma_{\text{GO}}(y) = e^{-jk(y_\text{obs} - y_\text{obs})} \cos \theta_0 \theta_i - 1 - \Gamma_{\text{GO}}(y) \cos \theta_r
\]

\[
Z_{\text{GO}}(r_{\text{Rx}}, y, r_{\text{Tx}}) = Z_{\text{GO}}(y) = \frac{1 + \Gamma_{\text{GO}}(y)}{\cos \theta_i - \Gamma_{\text{GO}}(y) \cos \theta_r}
\]

The difference between the generalized and the conventional geometrical optics solutions is that, in the former case, the reflected rays are assumed to propagate toward a direction that is different from that dictated by the conventional law of reflection. In conventional geometrical optics, on the other hand, the rays at every point of the surface \( S \) are reflected specularly, i.e., the angle of reflection is equal to the angle of incidence, and the surface impedance
Table 5 Summary of the Functions Utilized for Optimization ($\theta_i - \theta_i(r_{Rx}), \theta_f - \theta_f(r_{Rx})$)

| Original Equation | Discretization |
|-------------------|----------------|
| Surface impedance (vector) in (38) | $Z = [Z_1, Z_2, \ldots, Z_N]$, $Z_n = 2*X_n$, $n = 1, 2, \ldots, N$ |
| Surface reflection coefficient (vector) in (42) | $\Gamma_S(Z) = [\Gamma_1, \Gamma_2, \ldots, \Gamma_N]$, $\Gamma_n(Z_n) = \frac{Z_n}{Z_0}$ |
| $O_{\phi}(Z) = P_{\phi}(r_{Rx}, r_{Rs})$ in (80) | $O_{\phi}(Z) = \frac{|R_{\phi}|}{\eta}I_0 Z_0 \sum_{n=1}^{N} \left[ \left[ \Gamma_n(Z_n) \right]^2 \cos \theta_r + \Re \left\{ \Gamma_n(Z_n) \left( \cos \theta_r - \cos \theta_i \right) \right\} \right]$ |
| Helmholtz’s condition in (44) | $\tilde{H}_n(Z) = \frac{\tilde{f}_n - 2\beta \sin \theta_r f_n}{k_n^2 |Z_n| / \lambda_n}$, $Z_n = Z_0 - \beta y$, $\tilde{f}_n = \frac{f_n + f_{n+1}}{2\Delta y}$, $f_n' = \frac{f_n + f_{n+1}}{2\Delta y}$ |
| Power flow at $r_{obs}$ ($\varphi_0 = \pi/2$) in (97) | $P_{obs}(Z) = \frac{h_{obs}^2}{\pi^2 \eta_c^2 \rho_{Rx}^2} \left| \tilde{A}(Z) \right|^2 \cos^2 \theta_r$, $\tilde{A}(Z) = \Delta_y \sum_{n=1}^{N} \Gamma_n(Z_n) e^{-j(k \sin \theta_i + \sin \theta_f) y_n}$ |

depends only on the angle of incidence. In mathematical terms, the surface impedance that corresponds to the conventional geometrical optics solution is obtained by replacing $\theta_r \rightarrow \theta_i$ in the denominator of (102) while keeping unchanged the definition of the surface reflection coefficient in (101). Under the assumption that $\Gamma_S(r_{Rx}, y, r_{Rs})$ is set as in (101), the difference between the approximations assumed by the conventional and generalized geometrical optics solutions is the same as the local specular reflection illustrated in Fig. 9(a) and the local nonspecular reflection illustrated in Fig. 9(b), respectively.

For completeness, it is instructive to review the analytical steps that allow us to retrieve the geometrical optics solution of the surface reflection coefficient in (101). Geometrical optics, or ray optics, is a model for describing the propagation of electromagnetic waves in terms of rays. In geometrical optics, a ray is an abstraction that is useful for approximating the paths along which the electromagnetic waves propagate under certain circumstances. Generally speaking, the definition of a ray follows Fermat’s principle: the trajectory between two points that are taken by a ray is the path that is traversed in the least time. As far as this article is concerned, the main properties of the rays that we need to consider are that they propagate along straight-line trajectories as they travel in a homogeneous medium and that they bend at the interface between two dissimilar media.

Based on this definition, let us consider the setup illustrated in Fig. 13. According to the geometrical optics approximation, an RIS is modeled as a device that is capable of introducing a phase modulation or phase shift, $\Phi(y)$, to the incident electromagnetic wave. A conventional surface is, on the other hand, characterized by a constant phase modulation along the surface, i.e., $\Phi(y) = \Phi_0 \forall y$. In geometrical optics, an RIS is assumed to be of infinite extent without edges. Also, the geometrical optics approximation does not allow us to model the power of the incident and reflected electromagnetic waves; therefore, the reflection coefficient has a unit amplitude by definition. According to the geometrical optics approximation, the problem formulation consists of finding the phase modulation $\Phi(y)$ so that an electromagnetic wave that impinges upon an RIS from the direction $\theta_i$ is reflected toward the direction $\theta_r$, where the incident and reflected electromagnetic waves are modeled as the two rays illustrated in Fig. 13. This problem formulation amounts to identifying the position $y$ on the RIS according to Fermat’s principle: the trajectory between two points that are taken by a ray is the path that is traversed in the least time. Since the time and the phase shift of a monochromatic electromagnetic wave with carrier frequency $f$ are proportional to each other, i.e., $\varphi(y) = 2\pi f r(y)$, where $c(y)$ and $\tau(y)$ are the phase shift and the time, respectively, Fermat’s principle can be equivalently stated as follows: the trajectory between two points that are taken by a ray is the path that is traversed by minimizing the phase shift.

Based on Fig. 13, the total accumulated phase of the ray that is emitted by the transmitter, is bent by the RIS when it impinges upon it at $y$, and reaches the receiver depends on $y$, and it can be formulated as follows:

\[
\varphi(y) = 2\pi f \left( \frac{m \sqrt{y^2 + h_1^2}}{c} \right) + 2\pi f \left( \frac{m \sqrt{(d-y)^2 + h_2^2}}{c} \right) + \Phi(y) \tag{103}
\]

where $m$ denotes the index of refraction of the medium where the RIS is deployed and $c$ is the speed of light.

The trajectory of the ray according to Fermat’s theorem is obtained by computing the first-order derivative of $\varphi(y)$ and equating it to zero, which yields

\[
mk \sin \theta_i - mk \sin \theta_r + \frac{d\Phi(y)}{dy} = 0 \tag{104}
\]

where $\theta_i$ and $\theta_r$ are the angles of incidence and reflection, respectively. Notably, the expression in (104) is the main
Φ(\(y\))

The phase modulation introduced by an RIS. This is the reason configured by optimizing the first-order derivative of the incidence, the angle of reflection can be appropriately.

\[ \Phi(\Gamma_{1198}) \]

In terms of system implementation, this can be considered an optimization problem in (105) is not necessarily unique. Moreover, the corresponding surface impedance is \(\mathcal{P}_{\text{obs}}(Z_{\text{glob}})\), as defined in Table 5.

\[ \text{PROCEEDINGS OF THE IEEE | Vol. 110, No. 9, September 2022} \]

3) Approximated Global Design—Purely Reactive Impedance Boundary: The surface impedance \(Z_{\text{glob}}\) solution of the optimization problem in (105) is usually characterized by a nonzero real part. As mentioned in the previous subsection, this is not always a suitable design from the implementation point of view since local power gains and local power losses are present along the surface \(S\). A convenient solution from the implementation point of view is, on the other hand, to impose that the surface impedance is purely reactive, i.e., \(\Re(Z_n) = 0\) \(\forall n = 1, 2, \ldots, N\), which is, by definition, both locally and globally optimal and has a unit power efficiency. Inspired by Budhu and Grbic [58], a suitable approach in the context of wireless communications consists of finding a purely reactive surface impedance that provides almost the same power flux evaluated at \(r_{\text{Rx}}\), which is the location of interest, as the power flux obtained with \(Z_{\text{glob}}\). By denoting by \(Z_{\text{reactive}}\) such a purely reactive surface impedance, the mentioned design criterion corresponds to the condition \(\mathcal{P}_{\text{Rx}}(Z_{\text{glob}}) \approx \mathcal{P}_{\text{Rx}}(Z_{\text{reactive}})\).

The corresponding optimization problem is, therefore, formulated as follows:

\[ \begin{align*}
\min_{Z} & \quad |\mathcal{P}_{\text{Rx}}(Z) - \mathcal{P}_{\text{Rx}}(Z_{\text{glob}})| \\
\text{s.t.} & \quad \mathcal{H}_n(Z) \leq \varepsilon \quad \forall n = 1, 2, \ldots, N - 2 \\
& \quad \Re(Z_n) = 0 \quad \forall n = 1, 2, \ldots, N
\end{align*} \] (106a)

where \(\mathcal{P}_{\text{Rx}}(Z_{\text{glob}})\) is the power flux evaluated at the receiver \(r_{\text{Rx}}\) by using the surface impedance that is solution of the optimization problem in (105), and the constraint in (106c) ensures that the real part of the impedance is equal to zero. As mentioned, the corresponding power flux evaluated at \(r_{\text{obs}}\) is equal to \(\mathcal{P}_{\text{obs}}(Z_{\text{reactive}})\).

4) Optimization Constraints on Spurious Reflections: By definition, the optimization problems formulated in (105) and (106) specify the power efficiency and the power flux only in correspondence to the location of the receiver \(r_{\text{Rx}}\), while they do not explicitly account, in the problem formulation, for the power reradiated toward directions different from that of the receiver \(r_{\text{Rx}}\). In general, this implies that a large amount of power may be reradiated toward undesired directions, i.e., directions that are different from the target direction of reflection (where the receiver \(r_{\text{Rx}}\) is). This is apparent by direct inspection of, e.g., (106) and is consistent with Floquet's theory applied to periodic RISs, as discussed in the previous subsections. The problem

criterion for designing conventional reflectarray antennas. In the modern physics literature, (104) is often referred to as the generalized law of reflection [2].

It is apparent from (104) that, given the angle of incidence, the angle of reflection can be appropriately configured by optimizing the first-order derivative of the phase modulation introduced by an RIS. This is the reason why an RIS is often referred to as, according to the (generalized) geometrical optics approximation, a phase-gradient metasurface. If, for example, we set \(\Phi(y) = mk(sin\theta_{\text{desired}} - sin\theta_r)y\), which corresponds to the phase modulation of the reflection coefficient in (101), we obtain, as desired, \(\theta_r = \theta_{\text{desired}}\). Therefore, the geometrical optics solution for the surface reflection coefficient and the corresponding surface impedance in (102) is a direct consequence of Fermat’s principle, under the assumption that an RIS applies a linear phase modulation to the incident electromagnetic wave.

The corresponding discretized versions of the surface reflection coefficient and surface impedance are \(\Gamma_{\text{GO},n} = \Gamma_{\text{GO}}(y_n)\) and \(Z_{\text{GO},n} = Z_{\text{GO}}(y_n)\) for \(n = 1, 2, \ldots, N\), respectively. In this case, no optimization problem needs to be solved, and the power flux evaluated at \(r_{\text{obs}}\) is equal to \(\mathcal{P}_{\text{obs}}(Z_{\text{GO}})\), as defined in Table 5.

2) Global Design—Unit Power Efficiency: As elaborated on in Section III-B, an RIS designed based on a global design with unit power efficiency is a solution to the following constrained optimization problem:

\[ \begin{align*}
\min_{Z} & \quad |\mathcal{O}_S(Z)| \\
\text{s.t.} & \quad H_n(Z) \leq \varepsilon \quad \forall n = 1, 2, \ldots, N - 2
\end{align*} \] (105a)

where \(\varepsilon\) is a small positive constant and the constraint in (105b) ensures that the obtained surface impedance fulfills Helmholtz’s condition. Specifically, the parameter \(\varepsilon\) allows us to control the tradeoff between the optimality of the solution and the corresponding implementation complexity.

Given the problem formulation, the solution of the optimization problem in (105) is not necessarily unique. In terms of system implementation, this can be considered as a convenient solution from the implementation point of view since local power gains and local power losses are present along the surface \(S\). A convenient solution from the implementation point of view is, on the other hand, to impose that the surface impedance is purely reactive, i.e., \(\Re(Z_n) = 0\) \(\forall n = 1, 2, \ldots, N\), which is, by definition, both locally and globally optimal and has a unit power efficiency. Inspired by Budhu and Grbic [58], a suitable approach in the context of wireless communications consists of finding a purely reactive surface impedance that provides almost the same power flux evaluated at \(r_{\text{Rx}}\), which is the location of interest, as the power flux obtained with \(Z_{\text{glob}}\). By denoting by \(Z_{\text{reactive}}\) such a purely reactive surface impedance, the mentioned design criterion corresponds to the condition \(\mathcal{P}_{\text{Rx}}(Z_{\text{glob}}) \approx \mathcal{P}_{\text{Rx}}(Z_{\text{reactive}})\).

The corresponding optimization problem is, therefore, formulated as follows:

\[ \min_{Z} |\mathcal{P}_{\text{Rx}}(Z) - \mathcal{P}_{\text{Rx}}(Z_{\text{glob}})| \]

\[ \text{s.t.} \quad H_n(Z) \leq \varepsilon \quad \forall n = 1, 2, \ldots, N - 2 \]

\[ \Re(Z_n) = 0 \quad \forall n = 1, 2, \ldots, N \] (106c)

where \(\mathcal{P}_{\text{Rx}}(Z_{\text{glob}})\) is the power flux evaluated at the receiver \(r_{\text{Rx}}\) by using the surface impedance that is solution of the optimization problem in (105), and the constraint in (106c) ensures that the real part of the impedance is equal to zero. As mentioned, the corresponding power flux evaluated at \(r_{\text{obs}}\) is equal to \(\mathcal{P}_{\text{obs}}(Z_{\text{reactive}})\).
The formulation imposes that the power fluxes $P_{\text{Rx}}(Z_{\text{glob}})$ and $P_{\text{Rx}}(Z_{\text{glo}0})$ are approximately the same at the receiver Rx, but they may be different at locations $r_{\text{obs}} \neq r_{\text{Rx}}$.

For example, strong reflections toward some directions may emerge since they are not controlled at the design stage. The same comment applies to (105) since a large amount of power may be reradiated toward directions different from $r_{\text{Rx}}$, as no specific constraint is added in the formulation of the optimization problem. Several research works have reported that spurious reflections are often and usually observed [49], [64]. The presence of undesired reflections toward directions that are different from the desired direction of reflection has recently been extensively discussed in [34], in the context of RISs that are optimized based on the locally periodic discrete model. In [34], it is shown that nonideal RIS alphabets may result in a large amount of power that is directed toward undesired directions.

In order to make sure that an optimized RIS does not produce undesired reflections by design, the optimization problems in (105) and (106) need to be modified by adding specific constrains to the radiation pattern of the RIS. Specifically, the optimization problems in (105) and (106) can be reformulated, respectively, as follows:

$$\begin{align*}
\min_{Z} & \quad |O_{\delta}(Z)| \\
\text{s.t.} & \quad H_{n}(Z) \leq \varepsilon \quad \forall n = 1, 2, \ldots, N - 2 \\
& \quad P_{\text{obs}}(Z) \leq \delta \quad \theta_{o} \in [\theta_{11}, \theta_{1u}], [\theta_{21}, \theta_{2u}], \ldots
\end{align*}$$

(107a)

$$\begin{align*}
\min_{Z} & \quad |P_{\text{Rx}}(Z) - P_{\text{Rx}}(Z_{\text{glo}0})| \\
\text{s.t.} & \quad H_{n}(Z) \leq \varepsilon \quad \forall n = 1, 2, \ldots, N - 2 \\
& \quad P_{\text{obs}}(Z) \leq \delta \quad \theta_{o} \in [\theta_{11}, \theta_{1u}], [\theta_{21}, \theta_{2u}], \ldots
\end{align*}$$

(107b)

(108a)

(108b)

(108c)

(108d)

where $[\theta_{i1}, \theta_{iu}]$ for $i = 1, 2, \ldots$ are specified angular sectors where the reradiation of the RIS needs to be kept under some maximum power reradiation constraints and $\delta$ is a small positive constant that quantifies the reradiated power that is allowed toward the specified angular sectors.

**How to Use These Models and Methods in Wireless Communications?** The formulated problems are focused on optimizing the received power at some locations of interest, i.e., where the intended receiver is. They constitute, therefore, the key ingredient for formulating more complex optimization problems that are of interest in wireless communications. As a case study, let us assume a wireless network with one intended link and $I$ interfering links. For simplicity, we analyze single antenna transmitters and receivers. The intended link is identified by the pair of transmitter $Tx_{0}$ and receiver $Rx_{0}$, respectively, and the $i$th interfering link is identified by the pair of transmitter $Tx_{i}$ and receiver $Rx_{0}$. We assume that the data transmission between $Tx_{0}$ and $Rx_{0}$ in the presence of the $I$ interfering transmitters $Tx_{i}$ is assisted by an RIS. For simplicity, we assume no direct links between the transmitters and the receiver, i.e., only the RIS-aided links are available.

An illustration of the considered network model is provided in Fig. 14. The RIS is identified by the surface impedance $Z(r_{\text{Rx}}, y, r_{\text{Rx}})$ [or more, in general, $Z(r_{\text{Rx}, s, r_{\text{Rx}}})$]. The intended link and interfering links are characterized by the received powers $P_{\text{Rx,0}}(Z)$ and $P_{\text{Rx},i}(Z)$, respectively, where $Z$ is the vector of the sampled surface impedance defined in Table 5.

The performance of a wireless system is tightly linked to the statistical distribution of the signal-to-interference + noise ratio (SINR) [75], [76], which is usually defined as follows [77]:

$$\text{SINR}(Z) = \frac{P_{\text{Rx,0}}(Z)}{\sigma_{\text{N}}^{2} + \sum_{i=1}^{I} P_{\text{Rx},i}(Z)}$$

(109)

where $\sigma_{\text{N}}^{2}$ is the noise power at the receiver $Rx_{0}$.

The proposed optimization problems can be reformulated by replacing the received power with the SINR in (109), as the key performance indicator of interest. It is important to mention, however, that the problem formulations considered in this article are applicable to free-space communications since the channel is modeled through the free-space Green’s function. The proposed approach can, however, be generalized for application to fading channels, either by utilizing the stochastic Green’s function [78], [79] or by resorting to the system-level method in [50] and [51], where the RIS-aided links are assumed to be immersed in a statistical multipath channel in which the RIS is viewed as an additional digitally controllable scatterer. The generalization to multiple antenna transmitters and receivers is possible as well, but the impact of the multiple antennas needs to be accounted for starting from (87) and (88). The proposed modeling and optimization framework can, therefore, serve as a starting point to analyze wireless networks and channels that include multiple antenna transceivers and multipath.
propagation. Therefore, it has several applications in wireless communications.

**IV. NUMERICAL EXAMPLES**

In this section, we provide some numerical examples in order to compare and discuss the optimal designs for RISs that are obtained as solutions to the optimization problems formulated in (105)–(108). The aim of this section is to showcase, with the aid of numerical results and illustrations, the properties of the obtained surface impedances, the impact of the Helmholtz constraint, and the reradiated power flux as a function of the angle of observation.

The simulation setup is given in Table 6. As examples, we assume that the incident electromagnetic wave impinges upon the RIS from the normal direction, i.e., \( \theta_i(r_{tx}) = 0 \). Two desired angles of reradiation are considered: \( \theta_r(r_{rx}) = 30^\circ \) and \( \theta_r(r_{rx}) = 75^\circ \). This choice is made in order to highlight the differences, in terms of surface impedance, between a relatively small and a relatively large angle of reflection with respect to the angle of incidence. As far as the nullification of possible spurious reflections is concerned, we focus our attention on the reradiation toward the specular direction since this is one of the most important undesired reradiated modes in the considered case study. This is also in agreement with Floquet's theory that was reviewed in Section III. According to Floquet's theory, we may expect, in general, the existence of more than one spurious reflection, while parasitic specular reflection is always possible. Therefore, we focus our attention on the reradiation toward the specular direction as an example and elaborate on this point in further text. As far as the Helmholtz constraint \( \varepsilon \) is concerned, we consider different case studies in order to analyze the impact of fulfilling Helmholtz's condition on the surface impedance. Specifically, we analyze case studies for which \( \varepsilon = 5 \times 10^{-2} \) and case studies for which \( \varepsilon \) is large, which corresponds to solutions that are not subject to Helmholtz's constraint.

The optimization problems are solved by using the \texttt{fmincon} function in MATLAB, which is a gradient-based algorithm that is designed to work on problems where the objective function and the constraint functions are continuous and have continuous first-order derivatives. The \texttt{fmincon} function is designed to find the minimum of constrained nonlinear multivariable functions. As far the optimization problems in (105) and (107) are concerned, the \texttt{fmincon} function is initialized with the geometrical optics solution \( Z_0 = Z_{GO} \) in (102). As far the optimization problems in (106) and (108) are concerned, the \texttt{fmincon} function is initialized with the imaginary part of the solution of the optimization problems in (105) and (107), i.e., \( Z_0 = j3(Z_{glo}) \).

In Figs. 15 and 16, we illustrate the surface impedance and the reradiated power flux that correspond to the solution of the optimization problems in (105) and (106) for \( \theta_i(r_{tx}) = 30^\circ \) and \( \theta_i(r_{tx}) = 75^\circ \), respectively. For illustrative purposes, the optimization problems for \( \theta_i(r_{tx}) = 30^\circ \) are solved by setting \( \varepsilon = 5 \times 10^{-2} \) (i.e., Helmholtz's constraint is active), and the optimization problems for \( \theta_i(r_{tx}) = 75^\circ \) are solved by setting \( \varepsilon \) to a large value (i.e., Helmholtz's constraint is not active). In this latter case, the impact of Helmholtz's constraint on the surface impedance is analyzed in Fig. 20, where we set \( \varepsilon = 5 \times 10^{-2} \) (i.e., Helmholtz's constraint is active). For ease of visualization, the subfigures that show the surface impedance and the Helmholtz constraint illustrate a single period of the corresponding function. The numerical results confirm the general considerations made in Sections II and III. In both figures, we observe that the main lobe of the reradiation pattern of the RIS is steered toward the desired direction of reflection. The surface impedance obtained as a solution to the optimization problem in (106) is, as desired, purely reactive. By comparing the reradiated power flux (in dB scale and in the polar representation), the performance versus implementation tradeoff between the two surface impedances obtained as solutions of the optimization problems in (105) and (106) is apparent, especially for \( \theta_i(r_{tx}) = 75^\circ \). The surface impedance that is the solution to the optimization problem in (106) offers a good approximation of the main lobe of the reradiation pattern at the cost of slightly higher side lobes. The difference between the main lobe and the side lobes is, however, large and can be controlled by adding additional optimization constraints to the optimization problems, as discussed next for suppressing the specular reflection. As for the case study \( \theta_i(r_{tx}) = 75^\circ \) in Fig. 16, we observe that the real part of the surface impedance obtained by solving the optimization problem in (105) varies along the surface and can take positive and negative values. Finally, the geometrical optics solution results in worse reradiation performance compared with the optimized solutions obtained from the optimization problems in (105) and (106). By direct inspection of Fig. 15(c), we see that the Helmholtz condition is fulfilled according to the imposed optimization constraint, i.e., \( \gamma_h(Z) \leq \varepsilon = 5 \times 10^{-2} \). By direct inspection of Fig. 16(c), we see, on the other hand, that the obtained surface impedance results in a Helmholtz condition that is not significantly smaller than one since no constraint is added to the optimization problem. The impact of

### Table 6 Parameters’ Setup

| Parameter          | Value                      |
|--------------------|----------------------------|
| \( f \)            | 28 GHz                     |
| \( \lambda = c/f \) | 10.7 mm                    |
| \( \theta_i(r_{tx}) \) | 0°                         |
| \( |r_{rx}| = R_{obs} \) | \{30°, 75°\}              |
| \( \theta_r(r_{rx}) \) | 100 m                     |
| \( Z_0 \)          | 377 \Omega                 |
| \( P_0 \)          | 1 Watt/m²                  |
| \( |E^E_{x,0}| = \sqrt{2P_0\nu} \) | 27.45 V/m                |
| \( L_x \)          | 0.5 m                      |
| \( L_y \)          | 0.25 m                     |
| \( \Delta f \)     | \( \lambda/32 \)          |
| \( \delta \)       | \( 10^{-4} \)              |

\( \theta_{1,2} \)  \{0°, 1°\}. step = 0.1°
Helmholtz’s constraint is analyzed in Fig. 20. As far as the geometrical optics solution is concerned, the numerical results confirm that the Helmholtz constraint is fulfilled by definition, as discussed in Sections II and III.

In Figs. 15 and 16, we observe the presence of unwanted reflections toward the specular direction, i.e., $\theta_r = 0$, and toward the direction that is symmetric with respect to the desired direction of reradiation, i.e., $\theta_r = -\theta_r(r_{Rx})$. This is in agreement with Floquet’s theory in (54). In fact, the figures illustrate that the surface impedance is a quasi-periodic function, and the period $P$ is slightly larger than the wavelength $\lambda$. Therefore, three main propagating reradiation modes toward the desired direction of reflection, the specular direction, and the direction that is symmetric with respect to the desired direction of reflection may be present, whose intensity depends on the specific shape of the obtained surface impedance. This is an undesired effect and is particularly pronounced in Fig. 16, in which the difference between the angle of incidence and the angle of reflection is larger, and no constraint on the Helmholtz condition is imposed.

The presence of spurious reflections has two negative consequences: 1) some power that could be directed toward the desired direction of reflection is steered toward other directions and 2) the surface generates interference toward uncontrolled directions, and this may increase the interference toward other devices. Therefore, it is necessary to keep under control these possible spurious reflections by design. The corresponding results are illustrated in Figs. 17 and 18, which are obtained by solving the optimization problems formulated in (107) and (108) for $\theta_r(r_{Rx}) = 30^\circ$ and $\theta_r(r_{Rx}) = 75^\circ$, respectively, under the assumption that only the reradiation mode toward the specular direction is minimized by design, while no optimization constraint is added to the second spurious reradiation mode. In this case, we observe that the specular reflection is below the predefined maximum level, and overall, the side lobes of the reradiation pattern are well below the main lobe. In spite of the additional design constraint that is added in the optimization problems formulated in (107) and (108), a purely reactive solution for the surface impedance exists and can be computed. As far as the Helmholtz constraint is concerned, similar conclusions as for Figs. 15(c) and 16(c) can be drawn. Also, in this case, in fact, $\varepsilon = 5 \cdot 10^{-2}$ is imposed in Fig. 17(c), and a large value of $\varepsilon$ is imposed in Fig. 18(c).

The reradiation efficiency of the considered RIS may be further enhanced by adding a design constraint that accounts for the spurious reradiation toward $\theta_r = -\theta_r(r_{Rx})$ besides the reradiation constraint toward $\theta_r = 0^\circ$. An illustrative example is reported in Fig. 19, which corresponds to the same setup and optimization problem as for Fig. 18 with a large value of $\varepsilon$, with the only addition that the intensity of the reradiated (undesired)

**Fig. 15.** Surface impedance in (102) and solution of the optimization problems in (105) and (106) ($\theta_r(r_{Rx}) = 30^\circ$). (a) Real part of surface impedance $\Re(Z_n)$. (b) Imaginary part of surface impedance $\Im(Z_n)$. (c) Helmholtz’s ratio $\Re(Z_n)$. (d) Power flux versus angle of observation $P_{obs}|Z|$ (dB plot). (e) Power flux versus angle of observation $P_{obs}|Z|$ (polar plot).
Fig. 16. Surface impedance in (102) and solution of the optimization problems in (105) and (106) (θr = 75°). (a) Real part of surface impedance \( \Re(Z_n) \). (b) Imaginary part of surface impedance \( \Im(Z_n) \). (c) Helmholtz's ratio \( \gamma_n(Z) \). (d) Power flux versus angle of observation \( \gamma_{\text{obs}}(Z) \) (dB plot). (e) Power flux versus angle of observation \( \gamma_{\text{obs}}(Z) \) (polar plot).

Fig. 17. Surface impedance in (102) and solution of the optimization problems in (107) and (108) (θr = 30°). (a) Real part of surface impedance \( \Re(Z_n) \). (b) Imaginary part of surface impedance \( \Im(Z_n) \). (c) Helmholtz's ratio \( \gamma_n(Z) \). (d) Power flux versus angle of observation \( \gamma_{\text{obs}}(Z) \) (dB plot). (e) Power flux versus angle of observation \( \gamma_{\text{obs}}(Z) \) (polar plot).
Fig. 18. Surface impedance in (102) and solution of the optimization problems in (107) and (108) \( \theta_r = 75^\circ \). (a) Real part of surface impedance \( \Re(Z_n) \). (b) Imaginary part of surface impedance \( \Im(Z_n) \). (c) Helmholtz’s ratio \( \mathcal{H}_n(Z) \). (d) Power flux versus angle of observation \( \mathcal{P}_{\text{obs}}(Z) \) (dB plot). (e) Power flux versus angle of observation \( \mathcal{P}_{\text{obs}}(Z) \) (polar plot).

Fig. 19. Surface impedance in (102) and solution of the optimization problems in (107) and (108) \( \theta_r = 75^\circ \) An additional constraint on the spurious reflection toward \( \theta_r - 75^\circ \) is added. (a) Real part of surface impedance \( \Re(Z_n) \). (b) Imaginary part of surface impedance \( \Im(Z_n) \). (c) Helmholtz’s ratio \( \mathcal{H}_n(Z) \). (d) Power flux versus angle of observation \( \mathcal{P}_{\text{obs}}(Z) \) (dB plot). (e) Power flux versus angle of observation \( \mathcal{P}_{\text{obs}}(Z) \) (polar plot).
Table 7 Comparison of the Power Flux at the Location of the Receiver Rx

| Without nullification of the specular reflection | With nullification of the specular reflection |
|-----------------------------------------------|-----------------------------------------------|
| $P_{Rx}(Z_{glob}) / P_{Rx}(Z_{GO})$          | $P_{Rx}(Z_{glob}) / P_{Rx}(Z_{reactive})$     |
| $\theta_r(r_{Rx}) = 30^\circ$ (Fig. 15)      | $\theta_r(r_{Rx}) = 30^\circ$ (Fig. 17)       |
| $0.588 \text{ dB}$                           | $0.599 \text{ dB}$                           |
| $\sim 10^{-6} \text{ dB}$                   | $\sim 10^{-2} \text{ dB}$                   |
| $\theta_r(r_{Rx}) = 75^\circ$ (Fig. 16)      | $\theta_r(r_{Rx}) = 75^\circ$ (Fig. 18)       |
| $3.392 \text{ dB}$                           | $4.822 \text{ dB}$                           |
| $\sim 10^{-11} \text{ dB}$                  | $0.269 \text{ dB}$                           |

We observe that a feasible solution for the surface impedance exists, and the intensity of the two spurious directions of reradiation can be made smaller than the predefined intensity threshold (i.e., $\delta$). This example shows that, by adding appropriate optimization constraints to the problem formulation, the reradiation toward the desired direction and the dominant undesired directions can be appropriately engineered through purely reactive surface impedances.

Table 8 Comparison of the Power Flux at the Location of the Receiver Rx and the Peak of the Radiation Pattern

| $\theta_r (r_{Rx}) = 30^\circ$ (Fig. 15 and 17) | $\theta_r (r_{Rx}) = 75^\circ$ (Fig. 16 and 18) |
|-----------------------------------------------|-----------------------------------------------|
| $P_{max}(Z) = \max \{P_{obs}(Z)\}$           | $P_{max}(Z) = \max \{P_{obs}(Z)\}$           |
| $P_{max}(Z) / P_{Rx}(Z)$                      | $P_{max}(Z) / P_{Rx}(Z)$                      |
| $\theta_r (r_{Rx}) = 30^\circ$ (Fig. 15 and 17) | $\theta_r (r_{Rx}) = 75^\circ$ (Fig. 16 and 18) |
| (102) – Geometrical optics                     | (102) – Geometrical optics                     |
| $30^\circ$                                    | $74.8^\circ$                                  |
| 0 dB                                          | 0.0306 dB                                     |
| (105) – Global design                         | (105) – Global design                         |
| $30^\circ$                                    | $74.8^\circ$                                  |
| 0 dB                                          | 0.0325 dB                                     |
| (106) – Reactive boundary                     | (106) – Reactive boundary                     |
| $30^\circ$                                    | $74.8^\circ$                                  |
| 0 dB                                          | 0.0331 dB                                     |
| (107) – Global design with nullification of the specular reflection | (107) – Global design with nullification of the specular reflection |
| $30^\circ$                                    | $74.8^\circ$                                  |
| 0 dB                                          | 0.0508 dB                                     |
| (108) – Reactive boundary with nullification of the specular reflection | (108) – Reactive boundary with nullification of the specular reflection |
| $74.8^\circ$                                  | $74.8^\circ$                                  |
| 0.0142 dB                                     | 0.0142 dB                                     |
Fig. 21. Surface impedance in (102) and solution of the optimization problem in (105) ($\theta_r (\text{Rx}) = 75^\circ$). After obtaining the surface impedance, which is the same as in Fig. 20, its real part is set equal to zero, and the Poynting vector at the receiver is recomputed. (a) Real part of surface impedance $Z_n$. (b) Imaginary part of surface impedance $\Im(Z_n)$. (c) Helmholtz’s ratio $\frac{\Re(Z)}{\Im(Z)}$. (d) Power flux versus angle of observation $P_{\text{obs}}(Z)$ (dB plot). (e) Power flux versus angle of observation $P_{\text{obs}}(Z)$ (polar plot).

Table 9 Comparison of the Power Flux Toward the Direction of the Receiver Rx and the Direction of Specular Reflection

| $\theta_r (\text{Rx})$ | $P_{\text{Rx}}(Z)$ | $P_{\text{Specular}}(Z)$ | $P_{\text{Rx}}(Z) / P_{\text{Specular}}(Z)$ |
|------------------------|-----------------|-----------------|-----------------|
| (102) – Geometrical optics | $-7.871$ dB | $-45.854$ dB | $+37.983$ dB |
| (105) – Global design | $-7.283$ dB | $-33.814$ dB | $+26.532$ dB |
| (106) – Reactive boundary | $-7.283$ dB | $-34.397$ dB | $+27.114$ dB |
| (107) – Global design with nullification of the specular reflection | $-7.271$ dB | $-40.089$ dB | $+32.817$ dB |
| (108) – Reactive boundary with nullification of the specular reflection | $-7.249$ dB | $-40.343$ dB | $+33.094$ dB |

| $\theta_r (\text{Rx})$ | $P_{\text{Rx}}(Z)$ | $P_{\text{Specular}}(Z)$ | $P_{\text{Rx}}(Z) / P_{\text{Specular}}(Z)$ |
|------------------------|-----------------|-----------------|-----------------|
| (102) – Geometrical optics | $-18.362$ dB | $-67.317$ dB | $+48.955$ dB |
| (105) – Global design | $-14.970$ dB | $-17.151$ dB | $+2.181$ dB |
| (107) – Reactive boundary | $-14.970$ dB | $-37.186$ dB | $+22.216$ dB |
| (107) – Global design with nullification of the specular reflection | $-13.540$ dB | $-40.061$ dB | $+26.462$ dB |
| (108) – Reactive boundary with nullification of the specular reflection | $-13.809$ dB | $-40.000$ dB | $+26.191$ dB |

In the obtained numerical results, the surface impedances for the case study $\theta_r (\text{Rx}) = 75^\circ$ are obtained without imposing the fulfillment of Helmholtz’s condition, i.e., $\varepsilon$ is large. It is interesting to analyze the impact of this assumption on the shape of the Poynting vector as a function of the angle of observation. This is illustrated in Fig. 20 by imposing $\varepsilon = 5 \cdot 10^{-2}$. Due to the slow convergence speed of the fmincon function, as an illustrative example, we focus our attention only on the optimization of the problem in (105) without imposing any constraint on the nullification of the undesired directions of reradiation. By comparing Fig. 20 against Fig. 16, we note that adding the Helmholtz constraint in the optimization problem results in a better Poynting vector as a function of the angle of observation. Specifically, we see that a smaller power is steered toward the undesired directions of reradiation even though no constraint is imposed to this end. In addition, we see that the real and imaginary parts of the surface impedance in Fig. 20 are closer to those reported in Fig. 18, where a constraint for the reradiation toward the specular direction is imposed. Even though the reradiation constraint $\delta$ is not fulfilled in Fig. 18, we clearly see that imposing the Helmholtz constraint by design may inherently help in finding a better solution for the surface impedance. These observations that originate from the specific example analyzed in this article require further investigation, along with the development of efficient optimization.
algorithms for solving the considered optimization algorithms for any system setups and optimization constraints.

Besides the impact of the Helmholtz constraint on the surface impedance obtained as a solution to the considered optimization problems, it is interesting to analyze the reradiation properties of an RIS that is modeled as a purely reactive impedance boundary. More specifically, one relevant question is: “given the surface impedance solution of the optimization problem in (105), what would the Poynting vector as a function of the angle of observation be if we set the surface impedance equal to the imaginary part of the surface impedance solution of the optimization problem in (105)? This approach would result in a surface impedance with a real part equal to zero in agreement with the optimization problem in 106. The answer to this question is illustrated in Fig. 21. We see that the resulting surface impedance provides a Poynting vector with strong undesired reflections, and the Helmholtz constraint is not fulfilled anymore. This case study highlights that RIS designs based on reactive boundary sheets cannot be obtained by simply setting equal to zero the real part of the surface impedance that is obtained by solving (105). Therefore, major research efforts need to be put in efficiently solving the optimization problems in (105) and (106) in order to find solutions that offer good reradiation properties, are electromagnetically consistent, and are simple to implement (i.e., the surface impedance is purely reactive).

In Tables 7–9, we analyze, in a more quantitative manner, the solutions obtained by solving the optimization problems in (105), (106), (107), and (108). The data reported in these tables are obtained from Figs. 16 to 18.

In Table 7, we compare the power flux at the location of the receiver Rx. We evince that the geometrical optics solution usually results in a power loss of a few decibels compared with the globally optimum design with unitary power efficiency. In the considered case study, the power difference can be of the order of 4.8 dB for \( \theta_r (\theta_{Rx}) = 75^\circ \). On the other hand, we see that an RIS realized with a purely reactive impedance boundary results in a much smaller power loss compared with the globally optimum design with unitary power efficiency. In the considered case study, the power loss is about 0.27 dB for \( \theta_r (\theta_{Rx}) = 75^\circ \). The results reported in Table 7 provide a quantitative assessment of the implementation complexity versus the achievable performance of RISs that are implemented with purely reactive components.

In Table 8, we analyze the steering accuracy of the considered designs for the surface impedance. More precisely, we compare the peak value of the power flux as a function of the angle of observation with the power flux evaluated at the location of the receiver Rx. If \( \theta_r (\theta_{Rx}) = 30^\circ \), the considered designs result in perfect beamsteering capabilities. In fact, the maximum of the radiation pattern coincides with the desired direction of reradiation. If \( \theta_r (\theta_{Rx}) = 75^\circ \), on the other hand, we observe some pointing errors. In the considered case study, the pointing error is relatively small and is about 0.2\(^\circ\), which results in a power loss of only a fraction of decibels.

In Table 9, we analyze the reradiation properties of the considered designs for RISs in terms of power flux toward the direction of the desired receiver Rx and toward the direction of specular reflection. In general, the received power decreases with the desired angle of reflection. This is due to the so-called obliquity factor in the Poynting vector, i.e., the \( \cos \theta_r (\theta_{Rx}) \) multiplication factor in the power flux in correspondence of the location of the receiver Rx. This is apparent from \( P_{Rx} (Z) \) in Table 9. Due to the obliquity factor, for example, the power loss between \( \theta_r (\theta_{Rx}) = 30^\circ \) and \( \theta_r (\theta_{Rx}) = 75^\circ \) is about 6–11 dB in the considered case studies. In Table 9, in addition, we see that, by taking into account the nullification of the specular reflection at the design stage, we can, at the same time, reduce the amount of power lost toward the specular direction and increase the amount of power steered toward the location of the receiver Rx. This is due to the total power conservation principle.

V. CONCLUSION

In this article, we have overviewed three communication models for RISs that are widely utilized in the context of performance evaluation and optimization of wireless communication systems and networks. We have focused our attention on models for RISs based on inhomogeneous surface impedance boundaries, in light of their ease of integration into Maxwell’s equations and their inherent electromagnetic consistency under typical and practically relevant approximation regimes, e.g., physical optics. With the aid of several numerical examples, we have illustrated design criteria for RISs that are based on local and global optimality criteria, as well as an approximated design criterion that results in purely reactive impedance boundaries. We have discussed their inherent advantages and limitations with the aid of mathematical analysis and numerical simulations.

Specifically, we have introduced a parametric model and an optimization framework for RIS-assisted communications under the assumption that the surface impedance is slowly varying at the wavelength scale, and the physical optics approximation is applicable. The proposed parametric model is intended to offer a tradeoff between the quality (optimality) of the solution (e.g., the absence of undesired beams) and the implementation complexity of an RIS (e.g., physical structures made of purely reactive elements). Based on the proposed model, we have shown that the quality and the implementation complexity of the solution to the formulated optimization problems can be controlled by adding appropriate electromagnetically consistent optimization constraints.

As mentioned, the considered optimization problems and methods are applicable to RISs whose surface impedance is slowly varying at the wavelength scale and the physical optics approximation is applicable.
More advanced designs, which can lead to RISs that realize theoretically perfect anomalous reflections with complete suppression of parasitic scattering, require either an accurate control of the fast-varying surface modes excited at the surface or a careful design of the scattering from diffraction gratings, i.e., metagratings.

As far as the considered optimization problems are concerned, we have focused our attention on problem formulations in which the objective function is given by the surface power efficiency since it characterizes locally and globally optimal designs, and the power flux is utilized as a key performance indicator to showcase the steering capabilities of RISs. Similar optimization problems can be formulated by considering the power flux as the objective function and the surface power efficiency as a design constraint.

Finally, we have discussed the applications of the proposed approach in wireless communications. An important extension of the proposed approach lies in developing efficient numerical algorithms for solving the proposed electromagnetically consistent optimization problems and considering additional optimization constraints, such as imposing feasible (e.g., discrete) values for the surface impedance that can be manufactured in practice.

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