Spiky CMB distortions from primordial bubbles

Heling Deng

Physics Department, Arizona State University, Tempe, AZ 85287, U.S.A.
E-mail: heling.deng@asu.edu

Received March 28, 2020
Accepted April 29, 2020
Published May 20, 2020

Abstract. Primordial bubbles that possibly nucleate through quantum tunneling during inflation in a multi-dimensional potential might have left some relic detectable at the present time. These bubbles turn into black holes during the radiation era, which may account for the LIGO black holes, supermassive black holes, and may play an important role in dark matter. Typically, these black holes are surrounded by an energy deficit in the form of a spherical sound wave packet propagating outwards. In this work we study how this perturbation of the cosmic plasma dissipates before the time of recombination, leading to spectral distortions in CMB. We find that there may exist some rare regions on the last scattering surface containing huge black holes, which have produced potentially detectable point-like signals of $\mu$-type distortion.

Keywords: primordial black holes, Cosmic strings, domain walls, monopoles, CMBR theory, dark matter theory

ArXiv ePrint: 2003.02485
1 Introduction

Primordial black holes (PBHs) are hypothetical black holes formed in the early universe before large scale structures and galaxies. Depending on the mechanism, a PBH could have a mass that ranges from the Planck mass ($M_{Pl} \sim 10^{5} \text{g}$) to many orders of magnitude larger than the solar mass ($M_\odot \sim 10^{33} \text{g}$). PBHs with mass $M \lesssim 10^{15} \text{g}$ would have evaporated by now due to Hawking radiation. The lack of detected $\gamma$-rays from PBHs imposes stringent constraints on their present density (see, e.g., ref. [1]), hence attempts to constrain PBHs have mainly been focused on nonevaporating ones.\(^1\)

Recent interest in PBHs was largely stimulated by LIGO, which detected gravitational waves from inspiraling and merging black holes of masses $O(10–100) M_\odot$ (e.g., [6–9]). These masses are slightly larger than what one would expect for ordinary stellar black holes, so LIGO black holes might have a primordial origin [10–13].

Another motivation to consider PBHs comes from supermassive black holes (SMBHs) at the center of many galaxies [14, 15]. Their masses range from around $10^6 M_\odot$ to $10^{10} M_\odot$, and observations indicate that many of them are already in place at high redshifts (see ref. [16] for a review). Standard accretion models are unable to explain such large masses due to insufficient cosmic time [17]. One is then led to consider other possibilities, and PBHs is a natural candidate [18–22] for they could have large masses by birth.

In addition, it has also long been suggested that dark matter observed in galaxies and clusters may be (partly) made up of PBHs. At the moment there are stringent constraints on the dark matter fraction in PBHs within the mass range $10^{15} \text{g}–10^4 M_\odot$ based on dynamical, microlensing and astrophysical effects (see, e.g., refs. [1, 13, 23] and references therein). This possibility has been excluded for most black hole masses, but some windows still remain.

PBHs can be formed in a variety of mechanisms. The most popular scenario is PBH formation during the radiation era from inflationary density perturbations [24–29]. After inflation, a large overdensity ($\delta \equiv \delta \rho/\rho = O(1)$) of superhorizontal scales can overcome pressure and collapse into a black hole at horizon reentry. As is well known, density perturbations on scales $10^{-4} \text{Mpc}^{-1} \lesssim k \lesssim 1 \text{Mpc}^{-1}$ are the seeds of large scale structures observed today,\(^1\)

Some recent works [2–5] show that constraints from Galactic positrons could put slightly more stringent bounds near $M \sim 10^{15} \text{g}$ than the extragalactic $\gamma$-ray background.
and the rms amplitude $\delta_{\text{rms}}$ is as small as $10^{-5}$. On the other hand, the formation of PBHs requires $\delta_{\text{rms}} > 0.1$ on small scales, provided the primordial perturbations are Gaussian. One then has to construct inflationary potentials with this feature. However, there is still a serious problem: SMBH formation is subject to a stringent observational constraint. During the radiation era, density perturbations oscillate as sound waves dissipated by Silk damping on scales smaller than the photon diffusion length [30]. During the time $6 \times 10^6 \lesssim t \lesssim 10^{10}$ s, the released energy cannot be completely thermalized, leading to so-called $\mu$-distortion in the CMB spectrum [31–34]. The current upper bound on $\mu$-distortion is $|\mu| < 9 \times 10^{-5}$, from the COBE/FIRAS satellite in the 1990s [35].\(^2\) The scales of perturbations dissipated during the $\mu$-era roughly correspond to PBH masses $10^5 M_\odot < M < 10^{12} M_\odot$. Based on refs. [39, 40], it was shown in refs. [41, 42] that Gaussian perturbations are unable to form black holes within this mass range in any appreciable numbers due to the strong constraint on $\mu$. This problem can be avoided if primordial perturbations are non-Gaussian (e.g., [43]) or if PBHs are formed in other mechanisms (e.g., collapse of cosmic strings [44–47], bubble collisions [48], and collapse of closed domain walls [49, 50]).

In this work, we will turn to the second possibility and study the model developed in refs. [51–53], in which PBHs are formed by primordial bubbles (spherical domain walls or vacuum bubbles) that nucleate during inflation via quantum tunneling. This model predicts distinctive PBH mass spectra ranging over many orders of magnitude, and can account for LIGO black holes, SMBHs and/or dark matter.\(^3\)

Since quantum tunneling conserves total energy, each black hole should be compensated by an energy deficit. In this paper, we will focus on the scenario of domain wall bubbles\(^4\) where each black holes is surrounded by an underdense shell with lower temperature propagating outwards at the speed of sound. After time $t \sim 6 \times 10^6$ s, when the complete thermodynamic equilibrium between photons and the electromagnetic plasma can no longer be established, the dissipation of the shell by photon diffusion injects heat into the plasma, potentially leading to detectable spectral distortions in CMB. As we will see, this model satisfies the constraints imposed by observations, and predicts spiky distortions in some rare spots on the CMB sky, which is a unique feature of this model.\(^5\)

We also note that besides being able to avoid the $\mu$-distortion constraint on primordial density perturbations, another attractive feature of our model is that these PBHs have zero spin, which is not expected in most mechanisms proposed before. It was shown in ref. [55] that the super-radiance of rotating black holes would release energy into the plasma in the early universe, resulting in spectral distortions. This enforces further constraints on the mass range $10^{-8}–0.2 M_\odot$. Our model is apparently not subject to this bound.

The rest of the paper is organized as follows. The mechanism of PBH formation from spherical domain walls is reviewed in section 2, along with the current observational constraints. CMB spectral distortions in this model are estimated in section 3. Conclusions are summarized and discussed in section 4. We set $c = \hbar = G = k_B = 1$ throughout the paper.

\(^2\)Hypothetical future measurements being discussed (beyond the previously proposed mission PIXIE [36]) are expected to significantly improve this limit or even successfully detect CMB distortions [37, 38].

\(^3\)Ref. [54] extended the work in ref. [53] and constructed a PBH spectrum that is able to simultaneously explain all these three.

\(^4\)Much of the analysis in this paper also applies to the scenario of vacuum bubbles. See section 4.

\(^5\)The anisotropy of spectral distortions are usually considered in the scenario of local-type primordial non-Gaussianity at small scales.
2 The scenario and observational bounds

Inflation is usually pictured as being driven by the evolution of a scalar field called the inflaton, which slowly rolls down its potential. The energy scale of the potential remains almost a constant during inflation, which causes a nearly exponential expansion. The field eventually ends up at a minimum of the potential corresponding to our vacuum. In general, the inflaton runs in a multi-dimensional potential, including a number of other minima. In a natural and simple scenario, the inflaton field is coupled to a double-well potential with two degenerate vacua. As the field slowly rolls down in one of the two wells along the direction that leads to our vacuum, it may tunnel to the other well, resulting in the nucleation of a spherical domain wall bubble [56].

The formation of such a bubble can be thought of as a quantum process in a de Sitter background, where a spherical wall tunnels from a vanishing size to the Hubble size \( H^{-1} \). It then gets stretched by the inflationary expansion. Bubbles formed at earlier times expand to larger sizes, and at the end of inflation have a wide distribution [51, 52],

\[
n(R_i) \approx \Gamma R_i^{-3},
\]

where \( n(R_i) = R_i dn/dR_i \) is the number density of bubbles of radius \( \sim R_i \) when inflation ends, and \( \Gamma \) is the dimensionless bubble nucleation rate per Hubble volume per Hubble time. We assume that \( \Gamma \) is constant during inflation, which is usually the case for small-field inflation.

Domain walls can be characterized by a finite energy density \( \sigma \) per unit surface, and a tension of equal magnitude. In the thin wall limit, the metric for a planar domain wall in the \( yz \)-plane (in an otherwise empty space) is given by [57, 58]

\[
ds^2 = - (1 - H_\sigma |x|)^2 dt^2 + dx^2 + (1 - H_\sigma |x|)^2 e^{2H_\sigma t} (dy^2 + dz^2),
\]

where

\[
H_\sigma \equiv 2\pi \sigma. \tag{2.3}
\]

The \((x, t)\)-part of the metric is a \((1 + 1)\)-dimensional Rindler space. A geodesic observer near the wall would be “pushed away” with acceleration \( H_\sigma \). This means gravity produced by the wall is repulsive. In addition, we can see from the \((y, z)\)-part of the metric that the hypersurface \( x = 0 \), corresponding to the worldsheet of the wall, expands exponentially at a constant rate \( H_\sigma \).

When inflation ends, matter is created both inside and outside the wall due to the symmetry of the potential. Then the wall can be thought of as a comovingly static bubble living in an FRW universe dominated by radiation. Let \( t_H \) be the time when the bubble comes back within the Hubble horizon. If \( t_H \ll H^{-1}_\sigma \), the gravitational effect of the domain wall can be neglected. The bubble grows with the Hubble flow for a while, and at \( t_H \) realizes it’s actually a sphere rather than a planar wall. Then the surface tension forces it to shrink to a black hole. Such bubbles are called “subcritical”. On the other hand, a wall with \( t_H \gg H^{-1}_\sigma \) is called “supercritical”, and its gravitational effect becomes significant at time \( H^{-1}_\sigma \). Due to its repulsive nature, the wall pushes the ambient radiation away, leaving two almost empty shells. The exterior FRW region continues its power law expansion, but the wall grows exponentially. This is possible only if a wormhole is created outside the domain wall bubble. The bubble grows without bound into a baby universe, and the wormhole eventually pinches off, turning into a black hole.
In ref. [52], we found that the black hole masses in this scenario can be approximated by

\[
M \sim \begin{cases} 
H_i H_i^2 R_i^4, & M_{\text{min}} < M < M_* \\
H_i R_i^2, & M > M_*
\end{cases},
\]

(2.4)

where \( H_i \) is the Hubble expansion rate during inflation, \( M_* \sim O(10) H_i^{-1} \) is the transition mass that connects the subcritical and supercritical regimes, and \( M_{\text{min}} \sim H_i H_i^{-2} \) is the mass of the smallest black hole in this scenario (because the bubble forms with the Hubble size \( H_i^{-1} \)). Depending on the microphysics, the two characteristic masses \( M_* \) and \( M_{\text{min}} \) can have values within a wide range.

The mass distribution of black holes is conveniently characterized by the mass function

\[
f(M) = \frac{M^2}{\rho_{\text{CDM}}} \frac{dn(t)}{dM},
\]

(2.5)

where \( \rho_{\text{CDM}} \) is the mass density of cold dark matter (CDM). Here \( M^2 dn/dM \) can be interpreted as the mass density of black holes in the mass range \( \Delta M \sim M \). Since the black hole density and \( \rho_{\text{CDM}} \) are diluted by the cosmic expansion in the same way, \( f(M) \) remains constant in time.

During the radiation era \( (t < t_{\text{eq}}) \), the dark matter density is of the order

\[
\rho_{\text{CDM}}(t) \sim \left( B t^{3/2} M_{\text{eq}}^{1/2} \right)^{-1},
\]

(2.6)

where \( B \sim 10 \) is a constant and \( M_{\text{eq}} \sim 10^{17} \) is the dark matter mass within a Hubble radius at \( t_{\text{eq}} \). The mass function in our model is then given by

\[
f(M) \sim B \Gamma M_{\text{eq}}^{1/2} \begin{cases} 
M^{1/4} M_*^{-3/4}, & M_{\text{min}} < M < M_* \\
M^{-1/2}, & M > M_*
\end{cases}.
\]

(2.7)

Observational constraints on \( f(M) \) within the mass range \( 10^{15} \text{g} - 10^4 \) have been extensively studied (see, e.g., ref. [1] for a recent review). Some conservative bounds are summarized in figure 1. As an illustration, we also show the distribution of the form (2.7) with a set of parameters, which can account for the LIGO observations (see below). Strictly speaking, the bounds in figure 1 only apply to a monochromatic spectrum, where all black holes are of the same mass. The method of using these bounds on an extended spectrum was investigated in ref. [59] and applied in, e.g., ref. [53], but the difference it brings is not significant.

LIGO black holes have masses around \( 30 M_\odot \). In order to have the PBH merger rate suggested by the LIGO detectors, one requires [11]

\[
f(M \sim 30 M_\odot) \sim 10^{-3},
\]

(2.8)

which can be consistent with the constraint from the Planck satellite (the rightmost bound in figure 1) [60, 61] only if

\[
M_* \lesssim 10^2 M_\odot, \quad \Gamma \sim 10^{-12}.
\]

(2.9)
The number density of PBH of mass $\sim M > M_*$ at the present time is

$$n_M^{(e)} \sim B \left( \frac{M_{\text{eq}}}{M} \right)^{1/2} \frac{\rho_{\text{CMB}}}{M} \sim 10^{20} \Gamma \left( \frac{M}{M_\odot} \right)^{-3/2} \text{Mpc}^{-3}. \quad (2.10)$$

The seeds for SMBHs should have density $n_M^{(e)} \sim 0.1 \text{Mpc}^{-3}$. From eq. (2.10), the mass of such black holes in our scenario is $M_{\text{seed}} \sim 10^{14} \Gamma^{2/3} M_\odot$ (assuming that $M_{\text{seed}} > M_*$). Requiring $M_{\text{seed}} > 10^3 M_\odot \ [20]$ yields $\Gamma > 10^{-17}$. The largest black hole we can expect to find in the observable universe (of radius $\sim 10 \text{ Gpc}$) has mass $M \sim 10^{22} \Gamma^{2/3} M_\odot$. For $\Gamma \sim 10^{-12}$, this gives $M \sim 10^{14} M_\odot$. 

Let $\eta_\sigma$ be the energy scale of the wall tension, and $\eta_i$ be the inflationary scale, then we have $H_i \sim \eta_i^2 / M_{\text{Pl}}$ and $H_\sigma \sim \eta_\sigma^2 / M_{\text{Pl}}^2$. An example of a set of parameters that may account for both SMBHs and LIGO black holes is $\eta_\sigma \sim 10^7 \text{ GeV}$ and $\Gamma \sim 10^{-12}$. In this case, $M_\sigma \sim M_\odot$, $f(M \sim 30 M_\odot) \sim 10^{-3}$, and $M_{\text{seed}} \sim 10^6 M_\odot$. The corresponding mass distribution is shown in figure 1. With these parameters, the black holes can also account for $\sim 10\%$ of the dark matter.

Another interesting set of parameters is $\Gamma \sim 10^{-17}$, $\eta_i \sim 10^7 \text{ GeV}$ and $\eta_\sigma \sim 10^{11} \text{ GeV}$, which give $M_\sigma \sim M_{\text{min}} \sim 10^{19} \text{ g}$. In this case $f(M_\sigma) \sim 1$, which means these black holes can constitute all of the dark matter. Meanwhile, we have $M_{\text{seed}} \sim 10^3 M_\odot$, so they may also serve as seeds for SMBHs.

In the rest of the paper, we will show that, if the LIGO black holes were indeed formed by this mechanism, then as a consequence of the perturbation from the domain walls, there might exist some huge black holes on the boundary of the observable universe that are accompanied by detectable point-like distortions in the CMB spectrum.

Figure 1. Constraints from different observations on the fraction of dark matter in PBHs as a function of the PBH mass for a monochromatic spectrum. More detail can be found in, e.g., ref. [1] and references therein. As an illustration, we also show the PBH mass function $f(M)$ for our domain wall model with $\Gamma = 10^{-12}$, $M_* = M_\odot \approx 10^{33} \text{ g}$ and $M_{\text{min}} = 10^{-12} M_\odot \approx 10^{21} \text{ g}$.
3 Spectral distortions from domain wall bubbles

CMB photons began their journey roughly $10^{13}$ s after the end of inflation, coming to us from the so-called last scattering surface (LSS) with comoving radius $\sim 10$ Gpc and thickness $\sim 20$ Mpc. During the last scattering, photons decoupled from electrons, which combined with protons and formed atoms. Photons then travelled across the transparent universe in all directions, hardly interacting with other particles. Several generations of observations have revealed that the energy spectrum of the CMB is extremely close to a perfect black-body of temperature $T \approx 2.7$ K. Departures from the black-body spectrum, caused by the deviation from the equilibrium of photons and electrons, are commonly referred to as spectral distortions, which would encode important information from the early universe. Although no primordial distortions have been discovered, they could arise in plenty of processes. One important mechanism is the dissipation of sound waves.

PBH formation in our scenario stems from quantum tunneling, which conserves the global energy. For an outside observer not affected by the domain wall, the central object has such a mass as if it is made of normal FRW radiation. Roughly speaking, this implies that the resulting black hole should be compensated by some energy deficit. In our setting, it was shown by our simulations that, when the black hole is formed, it is surrounded by a shell of low density, and the outer edge of the shell propagates outwards at the speed of sound $c_s = 1/\sqrt{3}$ \cite{52}. Simulations also showed that the black hole formation time $t_M$ is related to its mass $M$ by $t_M \sim 3M$. Hence the wave front is at $\sim 2c_st_M \sim 3M$ at $t_M$. Considering that the black hole horizon is of size $\sim 2M$, the thickness of the shell at $t_M$ is

$$s(t_M) \sim M. \tag{3.1}$$

As the wave front propagates outwards, radiation near the black hole falls in and comes back to the FRW density. At late times, after it leaves the black hole, the shell behaves like a spherical underdense sound wave packet (with a tiny density contrast). Figure 2 shows a sketch of this picture. In an FRW universe, the shell’s thickness gets redshifted by the cosmic expansion,

$$s(t) \sim (Mt)^{1/2}. \tag{3.2}$$

Besides, it is well known that sound waves in a radiation dominated plasma get dissipated by photon diffusion (also known as the Silk damping) with a characteristic physical length scale

$$\lambda(t) \sim \sqrt{\frac{t}{\sigma_T n_e}} \propto t^{5/4}, \tag{3.3}$$

where $\sigma_T$ is the Thomson cross-section and $n_e$ is the electron density. As the underdense shell propagates, its thickness is smeared to a larger size. Effectively, the photon-electron fluid can be approximately described by an imperfect fluid with a shear viscosity \cite{62}. The amplitude of the Fourier mode of a sound wave with physical wave number $k$ is approximately damped by $e^{-(k\lambda)^2}$, which is of the Gaussian form. It can be shown that, for a 1-D plane wave packet with a Gaussian profile (whose Fourier modes are also Gaussian), its thickness grows like \cite{63}

$$d(t) = \sqrt{\lambda^2(t) + s^2(t)}, \tag{3.4}$$

and its amplitude gets damped by a factor $d(t)$. As a result, the area of the 1-D packet stays constant. In our setting, this implies the energy deficit of the shell is unchanged ($\sim M$).
density contrast in the shell is then given by

\[ \delta_M(t) \sim \frac{M}{\rho_r(t) \cdot 4\pi (2c_st)^2 d(t)} = \frac{2M}{d(t)}, \]

(3.5)

where \(4\pi (2c_st)^2 d(t)\) is the volume of the shell, and \(\rho_r(t) = 3/32\pi t^2\) is the background density.

The sound wave energy density is given by \(\rho_s = \rho_r \delta_M^2/4\), hence the total sound wave energy in a spherical shell can be estimated as

\[ E_{Ms}(t) \sim \frac{1}{4} \rho_r(t) \delta_M^2(t) \cdot 4\pi (2c_st)^2 d(t) \sim \frac{M^2}{2d(t)}, \]

(3.6)

which is smaller than \(M\) because \(d(t) > M\).

Although the linearized energy deficit of the underdense shell is a constant \((M \propto \delta_M)\), the sound wave energy \((E_{Ms} \propto \delta_M^2)\) gets dissipated by photon diffusion and turns into heat released into the background plasma, leading to spectral distortions. Depending on the cosmic age when this happens, distortions have different forms. These shall be the topics of the upcoming subsections.

### 3.1 \(\mu\)-distortion

\(\mu\)-distortion in CMB is produced by the mixture of photons of different temperatures during the period known as the \(\mu\)-era, when \(t_{th} \lesssim t \lesssim t_{\mu}\). At time \(t < t_{th} \sim 6 \times 10^6\) s (the corresponding redshift being \(z > 2 \times 10^6\)) the mixture is completely thermalized and we have a perfect black-body spectrum. At \(t_{th} \lesssim t \lesssim t_{\mu} \sim 10^{10}\) s \((5 \times 10^4 < z < 2 \times 10^6)\), photon number changing processes (like bremsstrahlung and double Compton scattering) become ineffective, while the Compton process still allows photons to reach equilibrium with the plasma, resulting in a spectrum with nonzero chemical potential where there is energy release [31–34].

In our scenario, the comoving radius of the shell’s wave front (which is also approximately the comoving sound horizon) \(r(t) \sim 2c_st/a(t)\) (where \(a(t)\) is the scale factor) ranges.
from $r(t_{th}) \sim 0.1$ Mpc to $r(t_{\mu}) \sim 5$ Mpc during the $\mu$-era; on the other hand, the photon diffusion scale grows till the time of recombination, reaching the comoving value $\lambda^{(c)}_{\text{rec}} \equiv \lambda^{(c)}(t_{\text{rec}}) \sim 50$ Mpc. (As a comparison, $\lambda^{(c)}(t_{th}) \sim 0.55$ kpc and $\lambda^{(c)}(t_{\mu}) \sim 0.14$ Mpc [40, 64].) We shall refer to a spherical region of comoving radius $\lambda^{(c)}_{\text{rec}}$ as a Silk region. Since $\lambda^{(c)}_{\text{rec}} \gg r(t_{\mu})$, the damping of the shell acts effectively as a point-like energy release, and photons with distorted spectra are eventually mixed within a Silk region containing black holes. Let us now estimate the $\mu$-distortion from a single black hole.

Consider that a shell sweeps through a comoving point at the wave front within the time interval $(t, t + d/c_s)$. The sound wave energy density is given by

$$\rho_s(t) = \frac{1}{4} \rho_r(t) \delta_M^2(t) = \rho_r \left( \frac{M}{d(t)} \right)^2. \quad (3.7)$$

Then the energy density injected in the neighborhood of the point after the shell passes by is\(^6\)

$$\delta \rho_s(t) \sim \rho_r(t) M^2 \left[ d^{-2}(t) - d^{-2} \left( t + \frac{d}{c_s} \right) \right] \approx \frac{2 \rho_r(t) M^2 \dot{d}(t)}{c_s d^2(t)}, \quad (3.8)$$

where the overdot represents the first derivative with respect to time. So the dimensionless chemical potential $\mu \equiv -\mu_{th}/T$, where $\mu_{th}$ is the thermodynamic chemical potential) at this point is [31–34]

$$\mu(t) \approx 1.4 \frac{\delta \rho_s(t)}{\rho_r(t)} \approx \frac{2.8 M^2 \dot{d}(t)}{c_s d^2(t)}. \quad (3.9)$$

The underdense wave packet propagates outwards, consecutively leaving behind photons with different chemical potentials. These photons then mix together. Consider a chemical potential $\mu$ produced at comoving radius $r$. Noting that the cosmic expansion doesn’t change the value of $\mu$ at a certain point, the contribution from the comoving sphere with an infinitesimal thickness $dr$ to the photon mixture is $\propto \mu \cdot 4\pi r^2 dr$. Therefore, the $\mu$-distortion averaged over a Silk region (with comoving volume centered at a single black hole of mass $\sim M$ is

$$\mu_{sM} = \frac{4\pi \int_{r_{th}}^{r_{th}} \mu r^2 dr}{4\pi \lambda^{(c)}_{\text{rec}}^3/3} = \frac{3 \int_{t_{th}}^{t_{th}} \mu r^2 \dot{r} dt}{\lambda^{(c)}_{\text{rec}}^3}, \quad (3.10)$$

where $r(t) = 2c_s t/a(t)$, with $a(t) \propto t^{1/2}$. With the expression of $\mu(t)$ in eq. (3.9), this gives

$$\mu_{sM} = \mathcal{O}(10) \left( \frac{M}{M_{\text{rec}}} \right)^{3/2} \int_{t_{th}}^{t_{th}} \frac{\dot{d}(t)}{d^2(t)} t^{1/2} dt \equiv \mathcal{O}(10) \left( \frac{M}{M_{\text{rec}}} \right)^{3/2} I, \quad (3.11)$$

where $M_{\text{rec}} \equiv \lambda^{2}(t_{\text{rec}})/t_{\text{rec}} \sim 10^{17} M_\odot$, and we have defined $I \equiv \int_{t_{th}}^{t_{th}} \left[ \frac{\dot{d}(t) t^{1/2}}{d^2(t)} \right] dt$.

If there are many ($> 1$) of these black holes within a Silk region, the resulting $\mu$-distortion is the sum of their contributions. In our model, the number density of supercritical black holes is given by

$$n_M(t) \sim \rho_{\text{CMB}}(t) \frac{f(M)}{M} \sim \Gamma (Mt)^{-3/2}, \quad (3.12)$$

\(^6\)Here we assume $d \ll t$, i.e., the cosmic expansion can be neglected when the shell passes through the point. Later we will see the case when this is not satisfied.
then the total $\mu$-distortion from the contribution of black holes of mass $\sim M$ is

$$
\mu_M = \mu_{sM} \cdot n_M(t_{rec}) \frac{4\pi}{3} \lambda^3(t_{rec}) = \mathcal{O}(10) \Gamma I.
$$

(3.13)

Considering that $d(t) = \sqrt{\lambda^2(t) + s^2(t)}$, with $\lambda(t) \propto t^{5/4}$ and $s(t) = (M t)^{1/2}$, the integral $I$ in eqs. (3.11) and (3.13) can be found analytically, with the result

$$
I = \ln \left[ t^{1/2} \left( \left( \frac{M_I}{M} + 1 \right)^{1/2} + 1 \right) \right]^{2/3} \left( \frac{M_I}{M} + 1 \right)^{-1/2} \bigg|_{t_{th}}^t, \quad (3.14)
$$

where $M_I \equiv \lambda^2(t)/t$. We have $M_{th} \equiv \lambda^2(t_{th})/t_{th} \sim 10^7 M_\odot$, and $M_\mu \equiv \lambda^2(t_\mu)/t_\mu \sim 10^{12} M_\odot$. The function $I(M)$ is plotted in figure 3. The physical meanings of $M_{th}$ and $M_\mu$ as well as the behavior of $I(M)$ will be clear in the following two limits.

### 3.1.1 $\lambda(t) \gg s(t)$ during $\mu$-era

Although the photon diffusion length $\lambda \propto t^{5/4}$ is initially small compared to the shell thickness $s \propto t^{1/2}$, it grows faster and may at some point catch up with $s$. If this happens before $t_{th}$, we have $d(t) \sim \lambda(t)$ during the $\mu$-era. Then since $\lambda(t_{th}) \gtrsim s(t_{th}) \sim (M t_{th})^{1/2}$, the black hole mass should satisfy $M \lesssim M_{th} \ll M_\mu$. In this limit, only the second term on the right hand side of eq. (3.14) is important, and we have

$$
I \sim \left( \frac{M}{M_{th}} \right)^{1/2}. \quad (3.15)
$$

Therefore, by eq. (3.11), the $\mu$-distortion from a single black hole is

$$
\mu_{sM} \sim 10^{-28} \left( \frac{M}{M_\odot} \right)^2. \quad (3.16)
$$
If there are many of these black holes within a Silk region, then by eq. (3.13), the total 
\(\mu\)-distortion is
\[
\mu_M = \mathcal{O}(10) \Gamma \left( \frac{M}{M_{\text{th}}} \right)^{1/2}.
\] (3.17)

We can see that the largest \(\mu_M\) is from the largest black hole in this limit.

### 3.1.2 \(\lambda(t) \ll s(t)\) during \(\mu\)-era

On the other hand, if the photon diffusion length catches up with the shell thickness after 
\(t_{\mu}\), then we have \(d(t) \sim s(t)\) during the \(\mu\)-era. Since \(\lambda(t_{\mu}) \lesssim s(t_{\mu})\), the black hole mass should 
satisfy \(M \gtrsim M_\mu \gg M_{\text{th}}\). In this limit, the right hand side of eq. (3.14) is dominated by 
\(\ln t^{1/2}\), and we thus obtain
\[
I \sim \frac{1}{2} \ln \left( \frac{t_{\mu}}{t_{\text{th}}} \right) \approx 3.
\] (3.18)

This is independent of the black hole mass, so \(I\) approaches a constant \(\sim 3\) when \(M \gtrsim 10^{12} M_\odot\). 
Then by eq. (3.11), the \(\mu\)-distortion from a single black hole is
\[
\mu_M \sim 10^{-24} \left( \frac{M}{M_{\odot}} \right)^{3/2}.
\] (3.19)

The mass of the largest black hole in a Silk region \(\bar{M}\) can be found from eq. (2.10) by setting 
\(n_{(c)}^M \sim \left( 4\pi \lambda_{\text{rec}}^3 / 3 \right)^{-1}\), which gives \(\bar{M} \sim 10^{17} \Gamma^{2/3} M_\odot\). In order to account for the 
LIGO black holes in our model, we need \(\Gamma \sim 10^{-12}\), which gives \(\bar{M} \sim 10^9 M_\odot\). By equalizing eqs. (3.15) and (3.18), one finds \(M \sim 10^8 M_\odot < \bar{M}\). Therefore, a rough estimate of the 
\(\mu\)-distortion in a typical Silk region (which is also an all-sky \(\mu\)) can be given by eq. (3.19), 
which yields
\[
\bar{\mu} \sim 10^{-24} \left( \frac{\bar{M}}{M_\odot} \right)^{3/2} \sim 10^{-11}.
\] (3.20)

This is much smaller than the value predicted from the Silk damping of sound waves in the 
standard \(\Lambda\)CDM model (\(\bar{\mu} \sim 10^{-8}\)) [65], and is thus too small to be detected.

However, a Silk region is but a tiny patch on the CMB sky. There will be regions that 
contain black holes with \(M \gg \bar{M}\). Typically, the mass of the largest black hole \(M_{\text{max}}\) one 
can find on the LSS can be estimated by setting 
\(n_{(c)}^M \sim \left[ 4\pi (10 \text{ Gpc})^2 \times 50 \text{ Mpc} \right]^{-1}\), where 
10 Gpc is the radius of the LSS. With \(\Gamma \sim 10^{-12}\), this gives \(M_{\text{max}} \sim 3 \cdot 10^{12} M_\odot\), which, by 
eq (3.19), gives the largest spiky \(\mu\)-distortion on the CMB sky,
\[
\mu_{\text{max}} \sim 10^{-24} \left( \frac{M_{\text{max}}}{M_\odot} \right)^{3/2} \sim 10^{-6} - 10^{-5}.
\] (3.21)

Unlike \(\bar{\mu}\), this is larger than \(10^{-8}\) and hence potentially detectable.

A caveat of the above estimate is the following. For a mass as large as \(M_{\text{max}}\), the black 
hole happens to be formed at \(t_M \sim t_{\text{th}}\), that is, the beginning of the \(\mu\)-era. With such a 
large black hole, the thickness of the underdense shell is larger than the photon diffusion 
length during the \(\mu\)-era, and is given by \((Mt)^{1/2}\), which at \(t_{\text{th}}\) is about the Hubble size. In 
the above analysis we assume that the shell sweeps through a comoving point within a time 
much smaller than the Hubble time, but this is no longer true when the shell’s thickness is
comparable to the Hubble size, as is the case here. This means the estimate in the second limit above should be checked more carefully. Now we show that the result in eq. (3.19) should be enhanced by one order of magnitude.

In fact, the shell ranges from \(2c_s t - (Mt)^{1/2}\) (the inner edge of the shell) to \(2c_s t\) (the wave front) and it sweeps through a point within the time interval \((t, t + \Delta t)\). We then have

\[
\frac{2c_s t}{a(t)} = \frac{2c_s(t + \Delta t) - \sqrt{M(t + \Delta t)}}{a(t + \Delta t)},
\]

where the cosmic expansion has been taken into account. This gives

\[
t + \Delta t \sim \left( M^{1/2} + t^{1/2} \right)^2.
\]

Therefore, instead of eq. (3.8), the sound wave energy density injected into the point should be given by

\[
\delta \rho_s \sim \rho_r(t) \left[ \frac{M}{(Mt)^{1/2}} \right]^{2t} \left[ \frac{Mt}{32\pi} \right]^{t-3 - (t + \Delta t)^{-3}}.
\]

At \(t \sim t_{\text{th}}, \Delta t \sim t_{\text{th}}\). As the shell passes through the point, the background density \(\rho_s(t)\) decreases only by a factor of order \(O(1-10)\). Therefore the chemical potential at that point can be approximated by

\[
\mu(t) \sim \frac{\delta \rho_s}{\rho_r(t)} \sim \frac{Mt}{I} \left[ 1 - \left( \frac{\sqrt{M}}{t} + 1 \right)^{-6} \right].
\]

Therefore, by eq. (3.10), the \(\mu\)-distortion averaged over a Silk region is

\[
\mu_{\text{SM}} = \mathcal{O}(10) \int_{t_{\text{th}}}^{t_{\text{th}}} \mu(t)t^{1/2}dt = \mathcal{O}(10) \left( \frac{M}{M_{\text{rec}}} \right)^{3/2} J,
\]

where the integral \(J\) is defined by considering the expression of \(\mu(t)\) in eq. (3.25). Numerically evaluating it for \(M \sim M_{\text{max}}\) gives \(J \approx 30\), which is \(\sim 10\) times larger than the value of \(I\) in eq. (3.18). Therefore the value of \(\mu\) within such a Silk region is

\[
\mu_{\text{max}} \sim 10^{-5} - 10^{-4},
\]

which is an order of magnitude larger than the result in eq. (3.21).

The current observational upper bound on the all-sky distortion \(\tilde{\mu}\) is \(9 \times 10^{-5}\), from COBE/FIRAS [35]. Although \(\tilde{\mu}\) from domain wall bubbles \((\sim 10^{-11})\) is smaller than that predicted from \(\Lambda\)CDM \((\sim 10^{-8})\), point-like signals with \(\mu_{\text{max}} \sim 10^{-5}\) in rare Silk patches (whose angular size in CMB is \(\sim 0.2^\circ\))\(^7\) marginally satisfy the current bound \((\sim 10^{-4})\) [64], and can possibly be detected in the near future. If the signal from \(M_{\text{max}}\) gets contaminated, black holes with, say, \(M \sim M_{\text{max}}/3\) would give a weaker distortion \(\mu_{\text{max}} \sim 10^{-6}\).

\(^7\)It was shown in refs. [66, 67] that the \(\mu\)-dissipation scale is 1.4 times smaller than the photon diffusion scale at the time of recombination.
3.2 \( y \)-distortion

After \( t_\mu \), even the Compton process ceases to be extremely efficient, so that photons can no longer reach equilibrium with the heated plasma. As a result, the photon spectrum gets Compton-distorted, similar to the Sunyaev-Zeldovich effect observed in galaxy clusters, characterized by the \( y \)-type spectrum with the magnitude of distortion given by the so-called \( y \)-parameter.

In our setting, as in the case of \( \mu \)-distortion, large black holes are accompanied with a \( y \)-distortion produced by the scattering of photons off the heated electrons as the underdense shell dissipates. For the largest black hole on the LSS, its mass \( M_{\text{max}} \sim M_\mu \), which means the thickness of the shell is approximately the photon diffusion scale at \( t > t_\mu \). This lies in the first limit discussed in subsection 3.1. Similar to eq. (3.11), and by considering eq. (3.15), the \( y \)-parameter from a single black hole of mass \( \sim M \) within a Silk region is given by

\[
y_{sM} \sim O(10) \left( \frac{M}{M_{\text{rec}}} \right)^{3/2} \left( \frac{M}{M_\mu} \right)^{1/2}.
\]

For \( M \sim M_{\text{max}} \), this gives \( \mu_{\text{max}} \sim 10^{-6} \). However, it was shown in ref. [68] that the ΛCDM model predicts an all-sky \( \bar{y} \sim 10^{-6} \) (only one order of magnitude below the current upper bound from COBE/FIRAS). Therefore, the \( y \)-distortion from the domain wall bubbles are probably too weak to be detected.

3.3 Temperature fluctuations

We finally consider the temperature fluctuation in the underdense shell on the CMB sky. For a black hole with mass \( M \sim 10^{12} M_\odot \), the (physical) thickness of the shell at recombination is \( \lambda(t_{\text{rec}}) \sim 10^{-3} \lambda^{(c)}_{\text{rec}} \sim 0.05 \text{ Mpc} \) (angular scale \( \sim 0.2^\circ \)). The temperature fluctuation in this region is

\[
\frac{\delta T}{T} \sim \frac{1}{4} \frac{\delta M(t_{\text{rec}})}{\lambda(t_{\text{rec}})} \sim \frac{M}{\lambda(t_{\text{rec}})} \sim 10^{-6}.
\]

The size of a shell on the LSS at \( t_{\text{rec}} \) corresponds to an angular scale of \( \sim 1^\circ \). Since this is several times larger than the thickness of the LSS, the energy deficit in the shell could induce ring-like temperature patterns on the CMB sky, with diameter \( \sim 1^\circ \) and thickness \( \sim 0.2^\circ \). The spiky \( \mu \)-distortion can be found at the center of the ring. It is also possible that, since the shell itself is also thicker than the LSS, the temperature fluctuation would be a round spot instead of a ring, with an angular size \( \lesssim 1^\circ \).

4 Conclusions and discussion

In this paper we have estimated the spectral distortions in CMB as a relic of primordial bubbles (spherical domain walls, in particular) that nucleate during inflation through quantum tunneling. These bubbles turn into black holes after Hubble reentry during the radiation era. Since quantum tunneling conserves the global energy, each black hole comes with an energy deficit that compensates the black hole mass. The energy deficit propagates outwards in the form of a spherical, underdense sound wave packet, and the dissipation of the wave due to photon diffusion releases energy into the background plasma. This would lead to both \( \mu \)-type and \( y \)-type distortions in CMB. In order not to violate the constraints from the LIGO observations, the bubble nucleation rate during inflation has an upper bound \( \Gamma \lesssim 10^{-12} \), which,
along with the observational constraints on the dark matter fraction in PBHs, roughly fixes the mass spectrum in this model. Our calculations show that the average magnitude of distortions is much smaller than what was predicted in the standard ΛCDM mode and so is unlikely to be observed.

However, bubbles formed earlier during inflation would have larger sizes and thus turn into huge black holes. On the last scattering surface, there could be some rare (∼1) Silk patches that contain a black hole of mass $M \sim 10^{12} M_\odot$, leading to localized peaks of distortions with $\mu_{\max} \sim 10^{-5}$. Such a spiky distribution of spectral distortions would be a unique signal of our PBH model, and is potentially detectable in the near future.

Similar conclusions can as well be drawn for the scenario of vacuum bubbles. Such bubbles can be formed when the tunneling takes the field to a well with an energy density smaller than the inflationary scale. One of the main differences is that vacuum bubbles tend to grow at the speed of light, and thus run into the ambient radiation fluid with a large Lorentz factor after inflation ends. In ref. [63] we studied in detail the case of “solid” bubbles, which remains a vacuum inside, and completely reflect the fluid outside. It was shown by simulations that the black hole spectrum in this scenario is the same as that for domain wall bubbles in the supercritical regime. As a bubble runs into the radiation, a shock (overdense) wave would be formed due to energy transfer, followed by an underdense shell. Spectral distortions can be produced in the same manner, except that there’s additional dissipation of an overdense thin shell with energy excess $\sim M$. In ref. [63], we argued that, for black holes with mass $M < M_{\text{th}}$, the effect comes mainly from the underdense shell; but for larger black holes, distortions are dominated by the dissipation of the overdense region, because the sound wave energy in the underdense shell is relatively negligible. Considering that (i) the thickness of the overdense shell is smeared to the photon diffusion length before the $\mu$-era (corresponding to the first limit in subsection 3.1), (ii) the PBH mass function for supercritical black holes is the same as that in eq. (2.7), and (iii) the overdense shell gets dissipated in the same way as the underdense shell, we can use eqs. (3.16) and (3.17) to estimate the $\mu$-distortion from vacuum bubbles. By eq. (3.20), we come to the same conclusion as in ref. [63] that the average $\mu$-distortion is $\sim 10^{-11}$ and would be negligible.

However, for the largest black hole on the LSS, the resulting $\mu_{\max}$ was overestimated in ref. [63], which has the result $\mu_{\max} = \mathcal{O}(0.1-1)$ for $M_{\max} \sim 10^{12} M_\odot$. This apparently violates the observational bound $\mu_{\max} \lesssim 10^{-4}$. On the other hand, by eq. (3.16) in this paper, we have $\mu_{\max} \sim 10^{-3}$\footnote{As mentioned in subsection 3.1, if the largest possible signal gets contaminated, black holes with $M \sim M_{\max}/3$ would give a distortion weaker by a factor of 10.}. The difference comes from the fact that, in ref. [63], we used $k_D^4$ to characterize the photon diffusion scale, where $k_D$ is the comoving wave number that enters the Silk damping term $e^{-\left(k/k_D\right)^2}$ for a fluctuation mode with wave number $k$; while in this paper, we use $\lambda_{\text{rec}}^{(c)} = 2\pi/k_D$. For instance, the comoving photon diffusion scale at recombination $\lambda_{\text{rec}}^{(c)}$ is taken to be 50 Mpc here, but in ref. [63] we used 10 Mpc. A change this leads to is in the ratio $M_{\max}/\bar{M} \propto \left(\lambda_{\text{rec}}^{(c)}\right)^{-4/3}$, which we used in the relation $\mu_{\max} = \bar{\mu}(M_{\max}/\bar{M})^2$ to find the distortion from $M_{\max}$ in ref. [63]. Consequently, the change of $\lambda_{\text{rec}}^{(c)}$ decreases $\mu_{\max}$ by a factor of $5^{8/3} = \mathcal{O}(100)$.

Yet another scenario of PBH formation is from vacuum bubbles that interact with standard model particles only gravitationally after inflation. In this case, the radiation fluid is able to penetrate the wall and freely flows into the bubble interior. As a result, after inflation ends, the bubble can grow into a much larger size before coming to a stop with...
respect to the Hubble flow. We expect the energy deficit created afterwards outside the bubble would behave in a similar way, as an underdense sound wave packet, and hence a spiky spectral distortion is expected in some rare regions. However, in this scenario the PBH mass function might have a different form, which will be left as future research.

Lastly we note that, a recent work [69] studies the disk and spherical accretion of a halo around PBHs, strongly constraining the dark matter fraction in PBHs in the mass range around $1-10^4 M_\odot$, reaching $f_{PBH} < 3 \times 10^{-9}$ at $\sim 10^4 M_\odot$. This is much more stringent than the bound shown in figure 1. Although it may be less secure than constraints from dynamical effects [1], if such a bound were to be taken into account, the following two possibilities in our model may be considered. On the one hand, we could impose an upper cut-off on the black hole mass, which can be achieved by relaxing the assumption that the bubble nucleation rate is a constant during inflation [63]. In this case, our PBHs could account for LIGO black holes but not SMBHs, and the resulting CMB spectral distortions would be too weak to be detected. On the other hand, we could shift the transition mass $M_*$ to a value much larger than $10^4 M_\odot$. Then our PBHs could still be the seeds for SMBHs at the galactic centers, and spiky signals of CMB distortions could potentially be observed.

Acknowledgments

I am grateful to Alex Vilenkin and Masaki Yamada for useful comments and discussion. This work is supported by the U.S. Department of Energy, Office of High Energy Physics, under Award No. DE-SC0019470 at Arizona State University.

References

[1] B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Constraints on Primordial Black Holes, arXiv:2002.12778 [INSPIRE].

[2] M. Boudaud and M. Cirelli, Voyager 1 e± Further Constrain Primordial Black Holes as Dark Matter, Phys. Rev. Lett. 122 (2019) 041104 [arXiv:1807.03075] [INSPIRE].

[3] W. DeRocco and P.W. Graham, Constraining Primordial Black Hole Abundance with the Galactic 511 keV Line, Phys. Rev. Lett. 123 (2019) 251102 [arXiv:1906.07740] [INSPIRE].

[4] R. Laha, Primordial Black Holes as a Dark Matter Candidate Are Severely Constrained by the Galactic Center 511 keV γ-Ray Line, Phys. Rev. Lett. 123 (2019) 251101 [arXiv:1906.09994] [INSPIRE].

[5] B. Dasgupta, R. Laha and A. Ray, Neutrino and positron constraints on spinning primordial black hole dark matter, arXiv:1912.01014 [INSPIRE].

[6] LIGO Scientific and VIRGO collaborations, Observation of Gravitational Waves from a Binary Black Hole Merger, Phys. Rev. Lett. 116 (2016) 061102 [arXiv:1602.03837] [INSPIRE].

[7] LIGO Scientific and VIRGO collaborations, GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence, Phys. Rev. Lett. 116 (2016) 241103 [arXiv:1606.04855] [INSPIRE].

[8] LIGO Scientific and VIRGO collaborations, GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2, Phys. Rev. Lett. 118 (2017) 221101 [Erratum ibid. 121 (2018) 129901] [arXiv:1706.01812] [INSPIRE].

[9] LIGO Scientific and VIRGO collaborations, GW170814: A Three-Detector Observation of Gravitational Waves from a Binary Black Hole Coalescence, Phys. Rev. Lett. 119 (2017) 141101 [arXiv:1709.09660] [INSPIRE].
[10] S. Bird et al., Did LIGO detect dark matter?, *Phys. Rev. Lett.* **116** (2016) 201301 [arXiv:1603.00464] [SPIRE].

[11] M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama, Primordial Black Hole Scenario for the Gravitational-Wave Event GW150914, *Phys. Rev. Lett.* **117** (2016) 061101 [arXiv:1603.08338] [SPIRE].

[12] S. Clesse and J. García-Bellido, The clustering of massive Primordial Black Holes as Dark Matter: measuring their mass distribution with Advanced LIGO, *Phys. Dark Univ.* **15** (2017) 142 [arXiv:1603.05234] [SPIRE].

[13] B. Carr, F. Kuhnel and M. Sandstad, Primordial Black Holes as Dark Matter, *Phys. Rev.* **D 94** (2016) 083504 [arXiv:1607.06077] [SPIRE].

[14] D. Lynden-Bell, Galactic nuclei as collapsed old quasars, *Nature* **223** (1969) 690 [SPIRE].

[15] J. Kormendy and D. Richstone, Inward bound: The Search for supermassive black holes in galactic nuclei, *Ann. Rev. Astron. Astrophys.* **33** (1995) 581 [SPIRE].

[16] J. Kormendy and L.C. Ho, Coevolution (Or Not) of Supermassive Black Holes and Host Galaxies, *Ann. Rev. Astron. Astrophys.* **51** (2013) 511 [arXiv:1304.7762] [SPIRE].

[17] Z. Haiman, Constraints from gravitational recoil on the growth of supermassive black holes at high redshift, *Astrophys. J.* **613** (2004) 36 [astro-ph/0404196] [SPIRE].

[18] S.G. Rubin, A.S. Sakharov and M.Y. Khlopov, The Formation of primary galactic nuclei during phase transitions in the early universe, *J. Exp. Theor. Phys.* **91** (2004) 921 [SPIRE].

[19] R. Bean and J. Magueijo, Could supermassive black holes be quintessential primordial black holes?, *Phys. Rev.* **D 66** (2002) 063505 [astro-ph/0204486] [SPIRE].

[20] N. Duechting, Supermassive black holes from primordial black hole seeds, *Phys. Rev.* **D 70** (2004) 064015 [astro-ph/0406260] [SPIRE].

[21] S. Clesse and J. García-Bellido, Massive Primordial Black Holes from Hybrid Inflation as Dark Matter and the seeds of Galaxies, *Phys. Rev.* **D 92** (2015) 023524 [arXiv:1501.07565] [SPIRE].

[22] B. Carr and J. Silk, Primordial Black Holes as Generators of Cosmic Structures, *Mon. Not. Roy. Astron. Soc.* **478** (2018) 3756 [arXiv:1801.00672] [SPIRE].

[23] K. Inomata, M. Kawasaki, K. Mukaida and T.T. Yanagida, Double inflation as a single origin of primordial black holes for all dark matter and LIGO observations, *Phys. Rev.* **D 97** (2018) 043514 [arXiv:1711.06129] [SPIRE].

[24] P. Ivanov, P. Naselsky and I. Novikov, Inflation and primordial black holes as dark matter, *Phys. Rev.* **D 50** (1994) 7173 [SPIRE].

[25] J. García-Bellido, A.D. Linde and D. Wands, Density perturbations and black hole formation in hybrid inflation, *Phys. Rev.* **D 54** (1996) 6040 [astro-ph/9605094] [SPIRE].

[26] M. Kawasaki, N. Sugiyama and T. Yanagida, Primordial black hole formation in a double inflation model in supergravity, *Phys. Rev.* **D 57** (1998) 6050 [hep-th/9710259] [SPIRE].

[27] J. Yokoyama, Chaotic new inflation and formation of primordial black holes, *Phys. Rev.* **D 58** (1998) 083510 [astro-ph/9802357] [SPIRE].

[28] J. García-Bellido and E. Ruiz Morales, Primordial black holes from single field models of inflation, *Phys. Dark Univ.* **18** (2017) 47 [arXiv:1702.03901] [SPIRE].

[29] M.P. Hertzberg and M. Yamada, Primordial Black Holes from Polynomial Potentials in Single Field Inflation, *Phys. Rev.* **D 97** (2018) 083509 [arXiv:1712.09750] [SPIRE].
[30] J. Silk, *Cosmic black body radiation and galaxy formation*, Astrophys. J. **151** (1968) 459 [inSPIRE].

[31] R.A. Sunyaev and Y.B. Zeldovich, *The Interaction of matter and radiation in the hot model of the universe*, Astrophys. Space Sci. **7** (1970) 20 [inSPIRE].

[32] R. Daly, *Spectral distortions of the microwave background radiation resulting from the damping of pressure waves*, Astrophys. J. **371** (1991) 14.

[33] J.D. Barrow and P. Coles, *Primordial density fluctuations and the microwave background spectrum*, Mon. Not. Roy. Astron. Soc. **248** (1991) 52.

[34] W. Hu and J. Silk, *Thermalization and spectral distortions of the cosmic background radiation*, Phys. Rev. **D 48** (1993) 485 [inSPIRE].

[35] D.J. Fixsen, E.S. Cheng, J.M. Gales, J.C. Mather, R.A. Shafer and E.L. Wright, *The Cosmic Microwave Background spectrum from the full COBE FIRAS data set*, Astrophys. J. **473** (1996) 576 [arXiv:astro-ph/9605054] [inSPIRE].

[36] A. Kogut et al., *The Primordial Inflation Explorer (PIXIE): A Nulling Polarimeter for Cosmic Microwave Background Observations*, JCAP **07** (2011) 025 [arXiv:1105.2044] [inSPIRE].

[37] A. Kogut et al., *CMB Spectral Distortions: Status and Prospects*, arXiv:1907.13195 [inSPIRE].

[38] J. Chluba et al., *New Horizons in Cosmology with Spectral Distortions of the Cosmic Microwave Background*, arXiv:1909.01593 [inSPIRE].

[39] J. Chluba, R. Khatri and R.A. Sunyaev, *CMB at $2 \times 2$ order: The dissipation of primordial acoustic waves and the observable part of the associated energy release*, Mon. Not. Roy. Astron. Soc. **425** (2012) 1129 [arXiv:1202.0057] [inSPIRE].

[40] J. Chluba, A.L. Erickcek and I. Ben-Dayan, *Probing the inflaton: Small-scale power spectrum constraints from measurements of the CMB energy spectrum*, Astrophys. J. **758** (2012) 76 [arXiv:1203.2681] [inSPIRE].

[41] K. Kohri, T. Nakama and T. Suyama, *Testing scenarios of primordial black holes being the seeds of supermassive black holes by ultracompact minihalos and CMB $\mu$-distortions*, Phys. Rev. **D 90** (2014) 083514 [arXiv:1405.5999] [inSPIRE].

[42] T. Nakama, B. Carr and J. Silk, *Limits on primordial black holes from $\mu$ distortions in cosmic microwave background*, Phys. Rev. **D 97** (2018) 043525 [arXiv:1710.06945] [inSPIRE].

[43] T. Nakama, T. Suyama and J. Yokoyama, *Supermassive black holes formed by direct collapse of inflationary perturbations*, Phys. Rev. **D 94** (2016) 103522 [arXiv:1609.02245] [inSPIRE].

[44] S.W. Hawking, *Black Holes From Cosmic Strings*, Phys. Lett. B **231** (1989) 237 [inSPIRE].

[45] A. Polnarev and R. Zembowicz, *Formation of Primordial Black Holes by Cosmic Strings*, Phys. Rev. **D 43** (1991) 112 [astro-ph/9208212] [inSPIRE].

[46] J. Garriga and A. Vilenkin, *Black holes from nucleating strings*, Phys. Rev. **D 47** (1993) 3265 [hep-ph/9208212] [inSPIRE].

[47] R.R. Caldwell and P. Casper, *Formation of black holes from collapsed cosmic string loops*, Phys. Rev. **D 53** (1996) 3002 [gr-qc/9509012] [inSPIRE].

[48] S.W. Hawking, I.G. Moss and J.M. Stewart, *Bubble Collisions in the Very Early Universe*, Phys. Rev. **D 26** (1982) 2681 [inSPIRE].

[49] S.G. Rubin, M.Y. Khlopov and A.S. Sakharov, *Primordial black holes from nonequilibrium second order phase transition*, Grav. Cosmol. **6** (2000) 51 [hep-ph/0005271] [inSPIRE].

[50] M.Y. Khlopov, S.G. Rubin and A.S. Sakharov, *Primordial structure of massive black hole clusters*, Astropart. Phys. **23** (2005) 265 [astro-ph/0401532] [inSPIRE].
[51] J. Garriga, A. Vilenkin and J. Zhang, *Black holes and the multiverse*, JCAP **02** (2016) 064 [arXiv:1512.01819] [inSPIRE].

[52] H. Deng, J. Garriga and A. Vilenkin, *Primordial black hole and wormhole formation by domain walls*, JCAP **04** (2017) 050 [arXiv:1612.03753] [inSPIRE].

[53] H. Deng and A. Vilenkin, *Primordial black hole formation by vacuum bubbles*, JCAP **12** (2017) 044 [arXiv:1710.02865] [inSPIRE].

[54] A. Kusenko, M. Sasaki, S. Sugiyama, M. Takada, V. Takhistov and E. Vitagliano, *Exploring Primordial Black Holes from Multiverse with Optical Telescopes*, arXiv:2001.09160 [inSPIRE].

[55] P. Pani and A. Loeb, *Constraining Primordial Black-Hole Bombs through Spectral Distortions of the Cosmic Microwave Background*, Phys. Rev. D **88** (2013) 041301 [arXiv:1307.5176] [inSPIRE].

[56] R. Basu, A.H. Guth and A. Vilenkin, *Quantum creation of topological defects during inflation*, JCAP **05** (2020) 037 [arXiv:1512.01819] [inSPIRE].

[57] A. Vilenkin, *Gravitational Field of Vacuum Domain Walls*, Phys. Lett. **133B** (1983) 177 [inSPIRE].

[58] J. Ipser and P. Sikivie, *The Gravitationally Repulsive Domain Wall*, Phys. Rev. D **30** (1984) 712 [inSPIRE].

[59] B. Carr, M. Raidal, T. Tenkanen, V. Vaskonen and H. Veermäe, *Primordial black hole constraints for extended mass functions*, Phys. Rev. D **96** (2017) 023514 [arXiv:1705.05567] [inSPIRE].

[60] M. Ricotti, J.P. Ostriker and K.J. Mack, *Effect of Primordial Black Holes on the Cosmic Microwave Background and Cosmological Parameter Estimates*, Astrophys. J. **680** (2008) 829 [arXiv:0709.0524] [inSPIRE].

[61] Y. Ali-Haïmoud and M. Kamionkowski, *Cosmic microwave background limits on accreting primordial black holes*, Phys. Rev. D **95** (2017) 043534 [arXiv:1612.05644] [inSPIRE].

[62] S. Weinberg, *Entropy generation and the survival of protogalaxies in an expanding universe*, Astrophys. J. **168** (1971) 175 [inSPIRE].

[63] H. Deng, A. Vilenkin and M. Yamada, *CMB spectral distortions from black holes formed by vacuum bubbles*, JCAP **07** (2018) 059 [arXiv:1804.10059] [inSPIRE].

[64] R. Khatri and R. Sunyaev, *Constraints on \( \mu \)-distortion fluctuations and primordial non-Gaussianity from Planck data*, JCAP **09** (2015) 026 [arXiv:1507.05615] [inSPIRE].

[65] J. Chluba, *Which spectral distortions does \( \Lambda \)CDM actually predict?*, Mon. Not. Roy. Astron. Soc. **460** (2016) 227 [arXiv:1603.02496] [inSPIRE].

[66] E. Pajer and M. Zaldarriaga, *A hydrodynamical approach to CMB \( \mu \)-distortion from primordial perturbations*, JCAP **02** (2013) 036 [arXiv:1206.4479] [inSPIRE].

[67] J. Chluba, E. Dimastrogiovanni, M.A. Amin and M. Kamionkowski, *Evolution of CMB spectral distortion anisotropies and tests of primordial non-Gaussianity*, Mon. Not. Roy. Astron. Soc. **466** (2017) 2390 [arXiv:1610.08711] [inSPIRE].

[68] J.C. Hill, N. Battaglia, J. Chluba, S. Ferraro, E. Schaan and D.N. Spergel, *Taking the Universe’s Temperature with Spectral Distortions of the Cosmic Microwave Background*, Phys. Rev. Lett. **115** (2015) 261301 [arXiv:1507.01583] [inSPIRE].

[69] P.D. Serpico, V. Poulin, D. Inman and K. Kohri, *CMB bounds on primordial black holes including dark matter halo accretion*, arXiv:2002.10771 [inSPIRE].