Regenerative memory in time-delayed neuromorphic photonic systems

B. Romeira\(^{(1,2)}\), R. Avó\(^{(2)}\), José M. L. Figueiredo\(^{(2)}\), S. Barland\(^{(3)}\) and J. Javaloyes\(^{(4)}\)

\(^{(1)}\) Centro de electrónica, Optoelectrónica y telecomunicaciones (CEOT), Departamento de Física, Universidade do Algarve, 8005-139, Faro, Portugal
\(^{(2)}\) COBRA Research Institute, Eindhoven University of Technology, P.O. Box 513, NL-5600 MB Eindhoven, Netherlands
\(^{(3)}\) Institut Non-Linéaire de Nice, Université de Nice Sophia Antipolis, CNRS UMR 7335, 06560 Valbonne, France
\(^{(4)}\) Departament de Física, Universitat de les Illes Balears, C/ Valldemossa km 7.5, 07122 Mallorca, Spain

We investigate a regenerative memory based upon a time-delayed neuromorphic photonic oscillator and discuss the link with temporal localized structures. Our experimental implementation is based upon a optoelectronic system composed of a nanoscale nonlinear resonating diode coupled to a laser that we link to the paradigm of neuronal activity, the FitzHugh-Nagumo model with delayed feedback.

Self-feedback connections in neurons are common in the nervous system and are named autapses [1-4]. These synapses between a neuron and a branch of its own axon are ubiquitous and have been found in the neocortex and the hippocampus, to cite but a few [1]. Yet their purpose has remained uncertain. Recent reports suggest that autaptic transmission neurons are involved in the long-lasting response to brief stimulation [3], and that this persistent activity has important implications in local feedback regulation [4], and working memory. On the other hand, recent progresses in multidisciplinary fields including semiconductor physics, Photonics, computing and networking yielded the possibility to emulate some elementary functions of the brain using neuromorphic systems [5-8]. The central goal is to reproduce neuronal synapses by interconnecting thousands of neuron-like elements [9]. While a lot of attention has been dedicated to the network architecture of neuromorphic systems [7, 8], almost no attention has been paid to the self-feedback autaptic connections providing self-localized persistence of neuronal activity.

In this work, we demonstrate temporal localized structures in a bio-inspired time-delayed neuromorphic photonic system that exhibits long-lasting responses and short transients to brief stimulation that can be explored in regenerative memory and information storage. Because we use light at telecommunications wavelengths for the regeneration of the spike-neuron signals, we anticipate the development of fast neuron-inspired optical storage interconnects and signal processing applications. We reduce our physical experimental system to the prototypical FitzHugh-Nagumo (FHN) model, a paradigm of neuronal response, complemented with a self-feedback autaptic connection, see Fig. 1. This analysis bridges our physical photonic system with the biological neuron and extend our findings to other biological, physical and engineering systems.

The global behavior of large-scale neural assemblies [10] and how the brain activity produces higher cognition that can be emulated in physical neuromorphic technologies leads in particular to the question of memory and how information is stored. While, the complexity of the brain is usually ascribed to the gigantic number of axons defining the interconnections, the finite transit time of the electrical information between individual units is also a source of complexity. A lot of interested was devoted during the past decades to the effects of communication time delays [11, 12] and how they can influence the dynamics. For instance, it was shown that dynamical systems mimicking coupled neurons [13-15] exhibit, instead of a steady state, stable periodic pulsating regimes where the two neurons may release energy in anti-phase, the period being related to the delay value. However, the presence of time delays in the dynamics may have a much deeper influence than to induce oscillations. Even a single delayed equation is akin to an infinite dimensional dynamical system and important conceptual links between partial differential equations (PDEs) and delayed differential equations (DDEs) exist [15]. As such, one is lead to wonder if the rich dynamics found in spatially extended systems, and in particular their ability to store information [17], could also exists in the temporal output of a single neuron with a delayed coupling representing an autaptic connection as seen in Fig. 1a).

One of the most interesting aspect of spatially extended out of equilibrium nonlinear systems is their ability to generate complex spatial shapes, e.g. Turing patterns [16], and Localized Structures (LS). Because these LS coexist with the background homogeneous solution, they can be individually addressed and used as bits of information. Localized structures appear in a dissipative environment as attractors, i.e. stable solutions towards which the system will evolve spontaneously from a wide set of initial conditions [18, 19] making them intrinsically robust and allowing for complex interactions. Localized states have been widely observed in nature in systems like granular media [20], gas discharges [21], semiconductor devices [22], reaction-diffusion systems [23], fluids [24, 25], convective systems [26] and optical cavities [27-30]. Neuromorphic responses were achieved in lasers [31] and nanolasers [32] recently and, while in principle fundamentally related to the existence of spatial degrees of freedom, Localized States analogues have recently been observed in delayed dynamical systems [30, 33].
nant tunneling photo-detector (RTD-PD) diode set in an excitable regime and driving an integrated laser diode source (LD). Although electronic neuron-like semiconductor microstructures [34] have been proposed, they operate at rather low-speeds (of the order of 20 kHz) and do not possess optical input/output. The schematic diagram of our experimental neuromorphic photonic memory is depicted in Fig. 1(b). The RTD-PD-LD provides a non-monotonic Current-Voltage (I-V) curve with a region of negative differential resistance, see Fig. 1(c), in which it behaves as an excitable system. Indeed, as demonstrated in [35], it provides an all-or-none response characteristic of neurons. The RTD response drives the laser diode which produces an optical pulse.

Excitability is a concept originally coined to describe the capacity of living organisms e.g. nerves [36–38] or neurons to respond strongly to a weak external stimulus that overcomes a well defined threshold. If the system is perturbed from its rest state, it may relax back toward it steady state in two different ways. If the perturbation remains below a certain threshold, the relaxation is exponential. Above the threshold, the system has to perform a large orbit that involves the whole phase space topology before relaxing again toward the unique fixed point. Such a relaxation is visible for instance in the lower panel of Fig. 2c). These two widely different transitory regimes toward an unique attractor are what defines the so-called excitability phenomenon. During its large excursion in phase space, the system cannot respond to another perturbation which defines incidentally the so-called lethargic time $T_l$ as the temporal extend of the orbit. Well known in physiology, this refractory period corresponds physically to the fact that a large amount of energy is released during the excitable response and may be understood as the time the system needs to recharge before being able to release another response. In neurons, this period of time occurs during the re-polarization and the hyperpolarization of the membrane potential.

In order to operate our neuromorphic photonic system as a regenerative memory, an optical delay line provides the temporal buffer memory that will store the bits of information as light intensity pulses. The excitable response of the RTD ensures the regeneration and the recognition of the signal as a special nonlinear node along the propagation time. An erbium doped fiber amplifier (EDFA) is employed in the loop to control the amount of feedback coupling and compensate for the losses incurred by coupling and decoupling light from the RTD-PD and LD chips (see methods A for details). The light pulse re-injection into the RTD-PD triggers, after the round-trip in the fiber, a new electrical response in the optoelectronic system thereby repeating the cycle with a period close to the optical pulse propagation time, $\tau$, in the fiber.

A precise modeling of the experimental situation can be achieved within the framework of a dynamical model employing a Liénard equation [39] describing the RTD-PD oscillator [40, 41] coupled to the single mode rate equations modeling the laser intensity and its population inversion (see method B). In the absence of feedback, such an approach yields quantitative agreement with the experimental results [35]. However, by assuming that the excitable response is slower than the relaxation oscillation frequency of the laser, one can adiabatically eliminate the laser intensity ($S$) that becomes slaved to the current ($I$) of the RTD. By expanding the nonlinear characteristic of the RTD at the center of the negative differential resistance, denoting $V$ the deviation of the voltage and assuming it is antisymmetric, one obtains exactly the FHN model, making a complete link with our time-delayed neuromorphic photonic system and the paradigm of excitability (see method B for more details). The FHN model [37, 38] represents a simplified version of the theory developed by Hodgkin and Huxley [36, 42] to study how action potentials in neurons are initiated and propagated. In order to allow for memory and complex dynamics, we introduce in the FHN model a delayed perturbation. As such, the equations read in their simplest form

$$\dot{V} = V - \frac{V^3}{3} - I + \eta [I(t - \tau) - I], \quad (1)$$

$$\dot{I} = \epsilon (\beta + V). \quad (2)$$

The stiffness parameters $\epsilon$ denotes the ratio of the time scale governing the slow ($I$) and the fast ($V$) variables while $\beta$ is the bias parameter. We choose $\beta > 0$ without
loss of generality. The amplitude of the delayed feedback is denoted $\eta$. For the sake of convenience we use the so-called form of non invasive feedback [43]. As such, the steady states of the FHN model are unchanged by the presence of feedback. In correspondence with the experimental situation, we re-inject the slow variable after a time delay $\tau$, into the dynamics of the fast one as $I(t-\tau)$, yet very similar results were obtained in other situations, e.g. re-injecting $V(t-\tau)$ instead. If not otherwise stated the parameters are $\varepsilon = 0.05$, $\eta = 0.18$ and $\tau = 500$. We included white Gaussian noise of variable amplitude $\xi$ to model the stochastic processes occurring in our experimental neuromorphic photonic oscillator.

We briefly recall the main properties of Eqs. (1,2) in the absence of delayed feedback. For values of the bias $\beta \gtrsim 1$, the unique steady state $(V_s,I_s) = (-\beta,\beta^3/3-\beta)$ is stable like e.g. in Fig. 2a) yet the system is excitable. For $\beta^* = 1$ an Andronov-Hopf bifurcation occurs at frequency $\omega^* = \sqrt{\xi}$ yielding a weakly nonlinear oscillations of the background solution which rapidly develops into a large amplitude cycle for $\beta < \beta^*$ via the so-called canard phenomenon [44]. In the latter case, the Eqs. (1,2) represents a so-called relaxation oscillator [45, 46].

We are interested in the regimes in which the values of the time delay is large as compared to the lethargic time $T_l$ which in our case is $T_l \sim 3 e^{-1}$, see [47] for more details. We demonstrate in Fig. 2 that Eqs. (1,2) support in this case the storage of information. The presence of a weak delayed perturbation in Eqs. (1,2) induces a multi-stability between an infinity of different temporal patterns that repeat themselves identically with a period close to the time delay $\tau$. For a broad range of values of the feedback rate $\eta$, for which the re-injection of the delayed excitatory orbit after a time delay $\tau$ is sufficient to overcome the excitability threshold, one new, perfectly formed, excitatory orbit gets generated. As such, the excitatory orbit of the solitary FHN system, whose temporal extend is $T_l$, becomes a binary unit of information embedded in a much longer periodic orbit of period $\tau$. Such bits of information get perfectly regenerated after each period and since the mere condition for perfect regeneration is to overcome the excitability threshold, one foresees the signal healing properties and the robustness of this mechanism. We exemplify in Fig. 2a-d) various occurrences of such periodic regimes whose period are close to $\tau$ and that are composed of 0, 1, 3 and 6 bits of information as embedded excitatory responses within the time delay $\tau$. We stress in Figure 2e) that all these regimes coexist between them selves for a wide range of the bias parameter $\beta$. Because these isolated temporal patterns are bistable with the uniform state and are also independent of the boundary conditions, i.e. the time delay value, and are attractor of the dynamics, they can be considered as the equivalent of Localized Structures in time delayed systems.

We also depict in Fig. 2e) the norm of the various solutions as well the maximal number of bits of information that can be stored for a given value of $\tau$. Interestingly, each branch of solution corresponds to a well defined number of temporal LS. However, what is hid-
den in such a projection is that for a given number of bits, i.e., a given branch, an infinity of different arrangement and relative distances exists. The capacity to store information of this regenerative memory can be understood intuitively. Since the temporal extension of the excitable orbit is defined by its lethargic time $T_l$, the maximal amount of elements that can be stored in the time delay is the integer closest to $N \sim \tau/T_l = 8$, in good agreement with numerics where we found $N = 7$ leading to $2^7 = 128$ possible configurations.

A further proof of the mutual independence of the temporal LS can be found in a two dimensional pseudo-spatial representation depicted in Fig. 3 in which the relative motion of the various bits of information over longer time intervals are best observed. In this co-moving reference frame, the horizontal axis is a space-like coordinate that allows to localize the position of the pulses within a given round-trip while the vertical coordinate corresponds to the slow temporal evolution of the system over many round-trips. The mutual independence of these pulses, as demonstrated for instance by their uncorrelated random motion in the presence of noise confirm their nature of Localized Structures. Here, we also demonstrate the nucleation and annihilation process in Fig. 3c) simply by including an extremely large amount of noise. One also notice here a property of the utmost importance: the almost total absence of transients: the localized bits of information can be perfectly written and erased in a single round-trip, at variance with the results of [29, 48] where transients representing tens of round-trips are necessary before a stabilization of the waveform. From an application point of view, our time-delayed neuromorphic photonic memory presents the extraordinary advantage to allow writing and erasing information at a rate comparable to the nominal reading rate. As such, we believe that the possibility to harness the unique properties of the excitable response to be of great importance and to cross boundaries between specific fields. Finally, the existence of the lethargic time induces an effective repulsion between nearest bits of data when they get too close, thereby ensuring signal integrity.

We assessed experimentally the robustness of the writing and storage process by employing several temporal bit patterns, see methods A. Figure 40) shows an example of complete regeneration using a single trigger even composed of one bit (1). Besides the fact that a single triggers is able to generate a perfectly formed train of identical pulses one notices the almost complete absence of transient in Fig. 40). The regeneration of a two bits (11) and a four-bit (1101) pattern in depicted in Fig. 4(a-b). As mentioned previously, the bits must be separated at least by the lethargic time $T_l$ of the excitable system. Moreover, it is worth noticing that triggering an identical bit sequence can be done with an strongly degraded initial pattern, as visible in Fig. 4c). This demonstrate that our system performs single-pass-healing by restoring and self-adjusting the received bits to a fixed amplitude, confirming that the nature of the excitable response renders the regenerative memory almost insensitive (in a certain range) to the exact shape or the amplitude of the addressing pulses. We emphasize that the writing and storage process is extremely robust as shown in Fig. 4 where we show the stable regeneration of a complex pattern of 8-bits (11011101) over a time scale in the ms range.

Finally, we shed some light onto the mechanism of formation of the multiple states of the memory by performing a bifurcation analysis of Eqs. (1,2) We start by considering the extend of the multi-stable region as a function of $\beta \in \{\beta_m(\eta), \beta_M(\eta)\}$ in Fig. (2). In presence of feedback, we notice in Fig. (2) that for $\beta < \beta^h(\eta) = 1.018$ only the fully developed temporal pattern with 7 equi-spaced bits is a stable solution. We identified this lowest limit $\beta^h(\eta)$ as an Andronov-Hopf bifurcation at which the uniform state, i.e., the solution with 0 LS, becomes unstable thereby explaining why for $\beta < \beta^h$ the only possible states are the ones with the largest number of elements since this background oscillation impedes the existence of uniform regions. With $\eta = 0.18$, only the uniform state subsists for values of the bias $\beta > \beta_M(\eta) = 1.3$. The upper limit of the multi-stable region is governed by the following phenomenon. Since the excitability threshold increases with $\beta$, it becomes eventually too large to be overcome for the delayed replica of the excitable orbit scaled by a factor $\eta$, suggesting that the solution disappear as Saddle Node Bifurcations of limit cycles.

The full bifurcation diagram of all the multi-LS solutions was obtained with DDE-BIFTOOL [49] and is depicted in Fig. 5b. In the absence of delayed feedback (left) an Andronov-Hopf bifurcation occurs at $\beta^* = 1$ where weakly nonlinear oscillations rapidly develops in large amplitude relaxation oscillations via the so called canard phenomenon [44]. This explains why the bifurcation seems to be vertical in Fig. 5a. Here, it is also important to notice that the period is a strongly evolving function of $\beta$ (see inset in Fig. 5a). The bifurcation scenario changes dramatically with $\eta \neq 0$. The dominant periodic branch develops a large number of folds as apparent in Fig. 5b. We stress that this folded branch, says at $\beta = 1.1$, corresponds to the solution with a maximal number of bit within the time delay, i.e. the trace depicted in Fig. 2d with $N = 7$, sometimes called the fully developed pattern or the maximal order solution within the context of spatially extended systems. We have been able to identify two different scenarios. Some branches of solution like e.g. the one with $N = 6$ in Fig. 5b) in red, can be disconnected from the rest of the web of solution. As such they form isolae and appear as hardly detectable saddle-node bifurcation of limit cycles. Some other branches, like e.g. the one with $N = 4$ in blue occurs as resonant Neimark-Sacker bifurcations over the principal branch. As a function of the bifurcation parameters $\tau$ and $\eta$, we also found that disconnected branches would reconnect over the maximal order solution branch and disconnect. Finally, the stability analysis of the periodic solutions with 4 LS exhibited $N$ quasi-degenerated
Figure 4. (Color online) Experimental time traces of showing writing and storage of binary-coded data streams in a \( \tau = 100 \mu s \) cavity round-trip time. (Left) data streams input, and (right) laser photo-detected optical output. Sequence of (a-b) 1-bit, (c-d) 4-bit (1101), (e-f) the same 4-bit but with a degraded input which yields the same state in the memory. In panel f) the absence of transient is visible and (g-h) a one byte (11011101) sequence.

Figure 5. (Color online) Left: Amplitude of the periodic solutions and variation of the period along the branches of (inset) for the FHN system without feedback i.e. \( \eta = 0 \). Right: Same diagram with \( \eta = 0.18 \) the colors correspond to the branches of solutions with 1,2,...,7, equi-spaced localized structures.

Floquet multipliers close to unity, demonstrating the independence of the various LS as in [30].

In conclusion, we have demonstrated a regenerative memory based on the paradigm of neuronal dynamics and considering the time delay of propagation of the signal. We disclosed the complex bifurcation diagram from which localized bit of in formations arise from the interplay between the excitable dynamics and the time delay. Our experimental realization comprises a monolithic integrated resonant tunneling diode photo-detector directly modulating a laser diode, and an optical fiber in a delayed feedback loop providing arbitrary large information storage capacity. The applications of the all-or-none response of the excitable RTD to optoelectronic logic are quite direct: by adding a second, external optical beam onto the photo-detector, one can perform an AND oper-
ation between the bit buffer within the fiber and the external modulated beam. This novel system constitute an ideal support for bits in a buffer memory employing standard silica fibers combining several functions like storage, reshaping/healing and XOR operation. Although the present proof of concept memory operates in the MHz range, lethargic times in the few tens of ps range can be achieved by reducing the parasitics of the integrated circuit such as wire bonding connections as demonstrated in [35, 41]. This will strongly reduce the fiber length required to realize the optical buffer and therefore dramatically increase the attainable bit rate, enabling practical applications and future monolithic photonic integration.

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Author Contributions

R. Avó performed the experimental characterization under the supervision of B. Romeira and J. Figueiredo. B. Romeira and J. Figueiredo devised the experiment. J. Javaloyes developed the theory and the numerical analysis and wrote the manuscript together with S. Barland and B. Romeira. All the authors participated to the interpretation of the results.

Materials & Correspondence

Correspondence and requests for materials should be addressed to Julien Javaloyes (email: julien.javaloyes@uib.es).

IV. METHOD

A. Setup details

The experimental realization of the regenerative memory consists in an optoelectronic feedback loop containing a c.w. laser diode, an optical fiber delay line, and an RTD-PD. An erbium doped fiber amplifier (EDFA) is employed in the loop to compensate the losses of coupling and decoupling light from the RTD-PD and LD chips but can be avoided employing RTD-PD layer structures optimized for improved photo-detection response (estimated to be $< 0.2 \text{ A/W}$). The RTD-PD layer structure currently used consists of a 10 nm wide AlAs/InGaAs/AlAs double barrier quantum well (DBQW) structure. The epitaxial layers includes two InGaAlAs regions surrounding the DBQW that act as light absorbing layers for wavelengths around 1.55 $\mu m$. The LD device was an InGaAsP multi-quantum-well active region with an InP:Fe doped uncoated buried heterostructure and ridge mesas, with centre wavelength emission at $\sim 1.55 \mu m$, threshold current $\sim 6 \text{ mA}$ [35], and 3-dB modulation bandwidth in excess of 10 GHz.

In this work, for purposes of demonstration and experimental convenience, the data signals were injected electrically using a bias-T. The binary-coded data streams were generated using an arbitrary function generator (Tektronix AFG3251C). The signals could be also injected optically, taking advantage of the optical input port of the RTD-PD, and therefore buffering can be achieved using either electrical or optical incoming data. By operating the system in the excitable regime close to the NDR region, one would be able to store and regenerate optical bits of information in the fibre, the empty region signaling the "0" bits and the excitable optoelectronic pulses the "1". When these bits are re-injected into the photo-diode they trigger the generation of a new excitabile cycle. This regenerative mechanism occurs after each round trip in the fibre which is extremely robust since even if the bit sequence is strongly deteriorated the all-or-none response of the excitabile RTD allows for a perfect regeneration. We employed a very long optical fiber loop providing 46 $\mu s$ cavity round-trip time, $\tau$. The time $\tau$ was chosen in order that the memory buffer is much larger than the typical excitabile lethargic time, $T_i$, of the RTD-PD-LD excitabile system, where in this case the measured $T_i$ is around 500 $\text{ns}$.

B. Theoretical model

We have analyzed our experimental results in the framework of delayed feedback nonlinear dynamical model systems employing a Liénard equation describing the RTD-PD excitabile oscillator [40, 41] coupled to the single mode rate equations modeling the laser intensity and its population inversion. Regeneration is induced in such dynamical system by a delayed feedback term into the current source of the RTD-PD proportional to the intensity of the laser output. The Liénard oscillator-laser diode dynamical system is given by the following dimensionless coupled delay differential equations (DDEs):

$$\mu \dot{v} = i - f(v) - \eta s(t - \tau_d),$$

$$\mu^{-1} \dot{i} = v_0 - \gamma i - v,$$  

\( i \) and \( v \) are the carrier density and the electric field strength, respectively; \( s(t) \) is the delayed source of the current and \( \tau_d \) is the mode-locking delay; \( \mu \) is a dimensionless parameter which controls the coupling between the laser and the RTD-PD; \( \eta \) is the feedback parameter and \( \gamma \) is the damping coefficient.

The following conditions are assumed to simplify the analysis:

1. The laser is assumed to operate in the NDR regime.
2. The RTD is assumed to operate in the excitable regime.
3. The delay feedback is assumed to be a delayed current source.
4. The laser is assumed to be a single mode laser.

The system of DDEs can be written in a more compact form as:

$$\mu \dot{v} = f(v) - \eta s(t - \tau_d),$$

$$\mu^{-1} \dot{i} = v_0 - \gamma i - v,$$  

where \( f(v) = i - s(t - \tau_d) \) is the Liénard oscillator. The delay feedback term is given by \( s(t - \tau_d) \), and it is the intensity of the laser output delayed by \( \tau_d \).

The analysis of the system involves the study of the Liénard oscillator and the feedback term. The stability of the system can be determined by analyzing the eigenvalues of the Jacobian matrix of the system.

The Liénard oscillator is given by

$$\mu \dot{v} = f(v),$$

where \( f(v) = i - s(t - \tau_d) \).

The feedback term is given by

$$\mu^{-1} \dot{i} = v_0 - \gamma i - v,$$  

where \( v_0 \) is the steady-state solution of the Liénard oscillator.

The system of DDEs can be solved numerically or analytically depending on the complexity of the function \( f(v) \). The analysis of the system involves the study of the Liénard oscillator and the feedback term. The stability of the system can be determined by analyzing the eigenvalues of the Jacobian matrix of the system.

The Liénard oscillator is given by

$$\mu \dot{v} = f(v),$$

where \( f(v) = i - s(t - \tau_d) \).

The feedback term is given by

$$\mu^{-1} \dot{i} = v_0 - \gamma i - v,$$  

where \( v_0 \) is the steady-state solution of the Liénard oscillator.
\begin{align}
\tau_n^{-1} \dot{n} &= \frac{i}{i_{th}} - n - \frac{n - \delta}{1 - \delta} (1 - \epsilon s) s, \\
\tau_p^{-1} \dot{s} &= \frac{n - \delta}{1 - \delta} (1 - \epsilon s) s - s.
\end{align}

Equations (3)-(6) represent the system of equations of the RTD-PD-LD with optical feedback control through the variable \(s(t - \tau_d)\) where \(\tau_d\) is the time-delay with respect to the dimensionless time \(t\); time is normalized to the characteristic\(LC\) resonant tank frequency, \(\omega_0 = (\sqrt{LC})^{-1}\), hence \(\tau = \omega_0 t\). The feedback strength \(\eta\) parameter depends on RTD-PD detection characteristics and the fraction of the laser optical output power reinjected into the delayed feedback loop. Equations (3)-(6) represent the Liénard oscillator where \(v\) and \(i\) are the dimensionless voltage and current variables, respectively. The function \(f(v)\) describes the nonlinear I-V curve, \(R = \gamma(I_0/V_0)\), and \(\mu = V_0/I_0\sqrt{C/L}\) is a dimensionless parameter.

Equations (3)-(6) are the dimensionless rate equations describing LD normalized photon \(s(t)\) and injected carrier \(n(t)\) densities. The charge carrier \(n(t)\) in Eq. (6) is normalized to threshold providing that \(\delta = N_0/N_{th}\), where \(N_0\) is the carrier density for transparency, and \(N_{th}\) is the threshold carrier density; \(\epsilon\) stands for the dimensionless laser gain saturation. The dimensionless laser diode threshold current is \(i_{th}\); The parameters \(\tau_n\) and \(\tau_p\) come from the time rescaling.

The prediction of such a system very good agreement with the experimental results allowing to predict a theoretical limit of operation of several Gb/s for such buffer configuration. In the limit case where the pulses are broad and the dynamics is slow as compared to the relaxation oscillation frequency of the laser, it is possible to adiabatically eliminate the equations for \((s, n)\). As such the delayed intensity \(s(t - \tau)\) becomes proportional to the bias of the laser device \(i(t)\). As a last step, the nonlinear function \(f(v)\) has to be expanded in Taylor series around the center of the NDR. Neglecting the second order term yielding the asymmetry of \(f(v)\) in the expansion and cutting the expansion to third yields the FHN equations presented in the main manuscript, after further trivial rescaling.

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