Application of Monte Carlo Simulations to Improve Basketball Shooting Strategy

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The underlying physics of basketball shooting seems to be a straightforward example of Newtonian mechanics that can easily be traced by using numerical methods. However, a human basketball player does not make use of all the possible basketball trajectories. Instead, a basketball player will build up a database of successful shots and select the trajectory that has the greatest tolerance to the small variations of the real world. We simulate the basketball player’s shooting training as a Monte Carlo sequence to build optimal shooting strategies, such as the launch speed and angle of the basketball, and whether to take a direct shot or a bank shot, as a function of the player’s court position and height. The phase-space volume \( \Omega \) that belongs to the successful launch velocities generated by Monte Carlo simulations is then used as the criterion to optimize a shooting strategy that incorporates not only mechanical, but also human, factors.

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I. INTRODUCTION

The underlying physics of basketball may seem to be too well understood to merit another survey. That may well be the reason so little research into the physics of basketball has been done, despite the huge popularity of the game. Furthermore, we possess enough computational power to realistically trace the motion of a basketball. These circumstances seem to suggest that we only need to calculate all the physical trajectories of a basketball and run the appropriate statistical analysis. This has been the approach of earlier researchers who studied the physics of basketball [1–7].

However, a human player should better not make use of all the possible basketball trajectories, even if it was possible, on account of underlying physics. High launch angle shots occupy less phase-space volume because the velocity space volume element is proportional to the cosine of the launch angle measured from the horizontal plane. A smaller phase-space volume means less tolerance for small inaccuracies in aim. Such an attempt would have great entertainment value, but nobody would try it under pressure to win.

We will introduce the phase-space volume associated with the launch velocity as an important criterion in the optimization of the shooting strategy. We consider a basketball player shooting from a position in the court given by \((x, y)\), where \(x, y = 1, 2, 3 \text{ } m\), defined relative to the center of the hoop (Fig. 1). We consider two shooters with different heights. Successful throws have usually been analyzed in the form of launch angle \(\theta\) vs. launch speed \(v\). The launch angle is measured from the horizontal plane, following the ordinary usage of the word and the conventions in the previous studies [1–7].

We analyze the bank shots in Fig. 2 by replacing the each bouncing spot on the backboard with a Gaussian function. Then, the bouncing spots will become a continuous distribution on the backboard. After a normalization, it should correspond to the probability density of bankshots on the backboard. We observe that the distribution is elongated upward (Fig. 3). In Fig. 3, only the right half of the backboard above the rim is shown. The

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We consider 9 positions near the hoop given by \((x, y)\), where \(x, y = 1, 2, 3 \text{ m}\), as marked in the diagram.

Tran and Silverberg \cite{4} introduce standard deviations \(\sigma_\theta\) and \(\sigma_v\) about the optimal values of \(\theta\) and \(v\) and keep the successful shooting ratio at a value typical of professional players. The skill level of the player is reflected in the successful shooting ratio. This scheme would be equivalent to selecting a rectangular region in the \(\theta - v\) diagram centered at \(\theta_0\) and \(v_0\). Arbitrariness exists in setting the optimal values \((\theta_0\) and \(v_0\)) and the standard deviations \((\sigma_\theta\) and \(\sigma_v\)).

We propose to view the basketball player’s shooting training process as a Monte Carlo sequence. A basketball player will judge his shootings by how far the basketball is located from the center of the rim when it passes the horizontal plane located at the height of the hoop. The skill of the basketball player will be described by the temperature parameter of the Monte Carlo simulation. Additional parameters are not needed in this approach.

As Silverberg \textit{et al.} \cite{5} point out, the players tend to shoot more accurately when they are closer to the hoop. The ability of the shooter to control the launch parameters does not change as a function of his position, but the nearer to the hoop the shooter is positioned, the more launch angle and speed (thus, the more phase-space volume) is available to the shooter. Therefore, we choose to calculate the phase-space volume occupied by successful launch velocities from the Monte Carlo sequence and take it as the criterion for successful shots.

**II. NUMERICAL MODEL**

The dynamics of the basketball is governed by the gravitational force and air resistance. The effect of the Magnus force on a basketball is ignored because of its small magnitude. However, we assume that the basket-
ball is shot with a back spin and that the back spin results in a gain in the downward velocity as it bounces off the backboard, given by \( \Delta v = \hat{n} \times (\vec{v} \times \hat{z}) \), where \( \hat{n} \) is the unit vector normal to the backboard and \( \Delta v = 0.2 \text{ m/s} \).

The basketball is assumed to be a rigid thin shell with a restitution coefficient of 0.75. The air resistance force is expressed by Okubo and Hubbard [6,7] as

\[
\vec{F}_{\text{air}} = -\frac{1}{2} C_d \rho A \vec{v} \hat{v},
\]

where \( \rho \) is the density of air, \( C_d = 0.54 \) is the drag coefficient of the basketball in air, \( A \) is the cross-sectional area of the basketball, and \( \vec{v} \) is the velocity of the basketball.

The equations were integrated using the fourth-order Runge-Kutta method with a time step size of \( 1 \times 10^{-3} \text{ s} \).

In the Monte Carlo simulation, the key variable is the horizontal deviation of the basketball from the center of the hoop \( R \). A new launch velocity will be accepted if it passes nearer to the center of the hoop. If \( R \) is larger, the new launch velocity will be accepted with a probability of \( \exp(-\Delta R/\kappa) \). We used \( \kappa = 0.5 \text{ m} \) throughout the Monte Carlo simulations. A smaller value will correspond to a more accurate shooter.

To calculate the phase-space volume \( \Omega \) occupied by the successful shots within the Monte Carlo sequence, we set up a grid in \( \vec{v} \)-space at intervals of \( 0.01 \text{ m/s} \) and counted the number of cubes that belonged to the successful shots. The smaller the phase-space volume \( \Omega \), the more difficult the shot will be.

We compared bank shots and direct shots made from rectangular grid positions on the court. We define the \( z \)-axis as the upward direction, the \( x \)-axis as the direction starting at the center of the hoop and stretching toward the sideline parallel to the backboard, and the \( y \)-axis stretching toward the center line perpendicular to the backboard. We considered two release heights, \( z = 1.9 \text{ m} \) and \( z = 2.1 \text{ m} \), to investigate how the player’s height influences the output.

### III. RESULTS AND DISCUSSION

Figures 4 and 5 depict the \( \theta - v \) diagram and the bank shot distributions on the backboard, respectively, when the shooter is located at \((3 \text{ m}, 3 \text{ m})\) from the center of the hoop with a release height of 1.9 m. In this case, the Monte Carlo simulation produces a range of shooting parameters consistent with our intuition. The \( \theta - v \) diagram does not extend to extreme values, but remain in the neighborhood of intuitive values. We can see that the backboard profile is now more symmetrical and does not unduly extend upwards. However, its center has moved slightly upward, which means that the undue contribution from the lower extreme part of the \( \theta - v \) diagram (Fig. 2) is now removed by the Monte Carlo simulation.

To our surprise, in some cases, the optimal \( \theta_0 \) and \( v_0 \) lie in an unexpected part of the \( \theta - v \) diagram. For a direct shooting from \((2 \text{ m}, 3 \text{ m})\) with a release height of 1.9 m, the optimal launch condition obtained from the Monte Carlo simulation is located far down the horse-shoe shape (Fig. 6). We also observe that there exists a competition between high launch angle trajectories and low launch angle trajectories. At some point in the court, the competition may be a close one, increasing the variety and complexity of the basketball shooting. Noteworthy is that the points in the \( \theta - v \) diagram may occupy different phase-space volumes. Thus, we need to trace all successful shots in \( \vec{v} \)-space. We opted to divide the phase space into cubic cells of side \( 1.0 \times 10^{-2} \text{ m/s} \) and count the number of occupied cells to estimate \( \Omega \).

Although no reliable data exist, we have an impression that smaller basketball players are more likely to utilize...
Table 1. Phase-space volume difference $\Omega_{z=2.1} - \Omega_{z=1.9}$ for bank shots by a shooter at $(x, y)$ in units of $1 \times 10^{-3} \text{ (m/s)}^3$.

| y       | x = 1 | x = 2 | x = 3 |
|---------|-------|-------|-------|
| y = 1   | 7.3   | -5.1  | -0.1  |
| y = 2   | -14.3 | -16.0 | 3.6   |
| y = 3   | -25.3 | -0.9  | -10.3 |

Table 2. Phase-space volume difference $\Omega_{z=2.1} - \Omega_{z=1.9}$ for direct shots by a shooter at $(x, y)$ in units of $1 \times 10^{-3} \text{ (m/s)}^3$.

| y       | x = 1 | x = 2 | x = 3 |
|---------|-------|-------|-------|
| y = 1   | 6.6   | -4.4  | 0.4   |
| y = 2   | -1.9  | 3.7   | 0.2   |
| y = 3   | -1.4  | -0.8  | -0.2  |

bank shots. Because deliberate bank shots are attempted closer to the hoop, we considered the court positions of the players $(x, y)$, where $x$ and $y$ are $1 \text{ m}$, $2 \text{ m}$, and $3 \text{ m}$, and calculated the phase-space volume at each court positions for the taller player with $z = 2.1 \text{ m}$ and the smaller player with $z = 1.9 \text{ m}$. The results for the bank shots are summarized in Table 1, and the results for the direct shots in Table 2. The value of $\Omega_{z=2.1} - \Omega_{z=1.9}$ for a shooter at each grid point in the court is given. The taller player does better, whether in bank shots or in direct shots, when he shoots from $(1 \text{ m}, 1 \text{ m})$, that is, very near to the hoop. However, from approximately the $1 \text{ m}$ to $3 \text{ m}$ range, the shorter player has an overall advantage in bank shots and the taller in direct shots. These findings are consistent with our previous impression.

Also of interest is to see if bank shots can be strategically better choices, and if so, under what conditions. The phase-space volume difference between the bank shots and the direct shots are summarized in Table 3 and Table 4. For the shorter player with $z = 1.9 \text{ m}$ (Table 3), the bank shots had more phase-space volume than the direct shots when the shooter was closer to the center line. However, the direct shots were favorable if the shooter was located near the backboard and/or the sideline. This result is in agreement with those of Silverberg et al. [5]. The taller player $(z = 2.1 \text{ m})$ again does not benefit from bank shots as much as the shorter player (Table 4). The difference in the phase-space volume is significantly reduced even at those court positions where the bank shots are favored over the direct shots. These findings probably explain why tall professional basketball players do not choose bank shots as often as shorter players do.

IV. CONCLUSIONS

We propose viewing the basketball shoots as a Monte Carlo sequence in which a player judges his own shooting by how far he has missed the hoop. In our Monte Carlo Simulations, the difficulty of a shot can be measured by using the associated phase-space volume. Our results offer some explanations for tall players less frequently opting for bank shots than shorter players and for the more favorable positions to try bank shots without introducing 	extit{ad hoc} assumptions on the launch velocity distributions.

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