Unlocking Energy Neutrality in Energy Harvesting Wireless Sensor Networks: An Approach Based on Distributed Compressed Sensing

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Abstract—This paper advocates the use of the emerging distributed compressed sensing (DCS) paradigm to deploy energy harvesting (EH) wireless sensor networks (WSN) with practical network lifetime and data gathering rates that are substantially higher than the state-of-the-art. The basis of our work is a centralized EH WSN architecture where the sensors convey data to a fusion center, using stylized models that capture the fact that the signals collected by different nodes can exhibit correlation and that the energy harvested by different nodes can also exhibit some degree of correlation. Via the probability of incorrect data reconstruction, we characterize the performance of a compressive sensing (CS) based data acquisition and reconstruction scheme and the proposed DCS based approach. These performance characteristics, which are performed both analytically and numerically, embody the effect of various system phenomena and parameters such as signal correlation, EH correlation, network size, and energy availability level. As an illustrative example, our results unveil that, for an EH WSN consisting of five SNs with our simple signal correlation and EH model, a target probability of incorrect reconstruction of $10^{-2}$, and under the same EH capability as CS, the proposed approach allows for a ten-fold increase in data gathering capability.

I. INTRODUCTION

FUTURE deployments of wireless sensor network (WSN) infrastructures are expected to be equipped with energy harvesters (e.g., piezoelectric, thermal or photovoltaic) to substantially increase their autonomy and lifetime [1]–[4]. The use of energy harvesting (EH) sensor nodes (SNs) has thus been emerging in various sensing applications. For example, a WSN for a greenhouse monitoring system using solar energy and super capacitor storage is proposed in [5]: self-autonomous wireless SNs with wind EH for remote sensing of wind-driven wildfire spread is proposed in [6]: radio frequency (RF) EH for a low-power wireless impedance SN for structural health monitoring is proposed in [7]: various other EH WSN applications appear in [3]. The International Society of Automation (ISA) has also been formulating standards in automation applications for EH [5].

However, it is also recognized that the gap that exists between the sensors’ EH supply and the sensors’ energy demand is not likely to close in the near future due to limitations in current EH technology, together with the surge in demand for more data-intensive applications [8]. For example, the typical power derived from EH, which ranges from hundreds of $\mu$W to tens of $mW$ [8], is not always sufficient to power SNs for data-intensive applications, e.g., image sensors. Consequently, the realization of energy neutral (or nearly energy neutral) WSNs for data-intensive applications remains a very challenging problem that calls for advances not only in EH capability but also in energy management (EM) capability for EH WSNs.

These considerations have motivated the design of general energy efficient data acquisition and transmission schemes for WSNs [9]–[13]. In particular, physical-layer based energy efficient designs have been proposed to reduce the impact of the transmission of redundant data on energy efficiency, including low-complexity low-memory source compression schemes for uncorrelated sensor signals [9] and distributed source compression (DSC) schemes for correlated sensor signals [10]. Compressive sensing (CS), which is a new sampling paradigm [14], [15], has also been proposed to reduce energy consumption associated with data acquisition and transmission in a WSN [16]–[19]. For example, by exploiting the CS principle, a reduced number of weighted sums of sensor readings, instead of individual readings, are delivered to the fusion center to reduce both communication and computation costs [16], an adaptive and nonuniform compressive sampling approach is applied to improve the energy efficiency of SNs in [17], and a group gossip scheme with an improved averaging time that exploits sparse recovery techniques is proposed in [18] to address the distributed averaging problem. An CS-based data gathering scheme for EH WSNs has been proposed in [18], which is formulated as an CS problem by exploiting the correlation across different sensor signals. Medium access control (MAC) layer based energy efficient designs have also been proposed to reduce the impact of other factors on energy efficiency, such as packet collisions, overhearing irrelevant signals, transmitting and receiving control overhead, and idle listening [11], [12]. Energy consumption can also be reduced via efficient EM [13].

Most existing works on data transmission and acquisition
schemes focus on the design of an intelligent point-to-point wireless communication system with EH capability [4], [20]–[23], while network-level EM is required for WSNs with multiple sensors and bases stations (BSs) [24]–[27]. Gatzianas et al. propose a downlink cross-layer resource allocation policy that maximizes a specified total system utility subject to energy and power constraints with a single node with EH capability and multiple sinks [24]; Iannello et al. investigate the performance of various MAC protocols for collecting data from EH-capable sensors with uncertain energy availability [25]. Besbes et al. derive the optimal number of nodes leading to the minimum EH requirement for each tier of a WSN with a cluster-tree topology [26]. Huang analyzes the maximum spatial throughput of a mobile ad hoc network powered by EH using a stochastic-geometry model [27]. Gurakan et al. propose energy cooperation schemes among different nodes with wireless energy transfer [28], e.g., electromagnetic energy transfer, which incurs energy loss owing to wireless energy transportation and an increase in the hardware cost.

However, the existing physical-layer, MAC-layer or EM schemes do not explicitly integrate two fundamental mechanisms associated with the EH and the sensing processes in an EH WSN: energy diversity and sensing diversity.

This article—at the core of its contribution—advocates the use of distributed compressive sensing (DCS) in order to deploy WSNs with practical network lifetime and data gathering rates that are substantially higher than the state-of-the-art. As a generalization of CS [14], [15], DCS [29], [30] exploits both intra-signal correlation, i.e., correlation within a sensor signal, and inter-signal correlation, i.e., correlation across different sensor signals, in order to sense and reconstruct more efficiently a collection of signals. The key attributes of the proposed approach that lead to efficient EM are associated with the following:

- The number of measurements (data projections) at the various sensors can be substantially lower than the data dimensionality without compromising data recovery [14], [15].
- The number of measurements (data projections) at the various sensors can be traded-off, subject to certain restrictions, without compromising data recovery [29], [30].

Since the number of projections acts as a proxy to energy efficiency—in view of the fact that transmission energy tends to be orders of magnitude higher than sensing/computational energy in various WSN applications [31], [32]—then the proposed approach provides for: i) a substantial improvement in energy efficiency in relation to other approaches, such as methods that do not exploit any form of source compression; and ii) adapting energy consumption to the random nature of energy availability in EH systems. That is, this paper argues that, due to the energy diversity associated with the EH process (where the harvested energy can vary from sensor to sensor) and the sensing diversity associated with the DCS process (where the number of projections can also vary from sensor to sensor), one should be able to closely match the energy supply to the energy demand in order to unlock the possibility for energy neutral operation in EH WSNs.

It is interesting to point out that one could also argue that other instantiations of distributed compression, such as distributed source coding (DSC), would also provide a means to match the energy supply to the energy demand in an EH WSN. However, DCS is generally accepted to be less complex than DSC, and it also requires fewer signal modelling assumptions than DSC [33]–[36].

Our contributions can be summarized as follows:

- We propose a DCS-based sensing approach to unlock energy neutrality in EH WSNs by matching the energy demand to the profile of energy supply. In contrast to using wireless energy transfer that attempts to match energy supply to the energy demand, the proposed approach adjusts the energy demand for different sensors to match energy supply. Our approach is fundamentally different from other CS/DCS approaches [14], [15] for WSNs that focus purely on the gain of energy efficiency by using CS principles;
- We derive a lower bound to the probability of incorrect data reconstruction for both a CS-based data acquisition scheme, which only exploits correlation within sensor signals, and the proposed DCS-based data acquisition scheme, which further exploits correlation across different sensor signals, by analyzing the fundamental limits of CS and DCS to uniquely identify the sparse signal ensemble.
- We characterize the performance of the proposed approach analytically (via bounds), as well as numerically via simulations that embody the effect of various system phenomena and parameters such as signal correlation, energy harvesting correlation, network size, and energy availability level. In particular, we show that there exist an optimal number of signals to be joint reconstructed.

The basic principles associated with the proposed DCS approach for EH WSNs are unveiled in the sequel. Section II describes in detail the proposed sensing approach including the data acquisition, transmission and reconstruction schemes, the energy consumption model, and the EH model adopted by the EH WSN. Section III derives a lower bound to the probability of incorrect data reconstruction for various data acquisition schemes. Section IV provides simulation results both with synthetic and real data that support the potential of the approach to considerably increase the lifetime of a network. General concluding remarks are drawn in Section V. The more technical aspects of the work, including the proofs, are relegated to the Appendices.

The following notational conventions are adopted throughout this paper. Lower-case letters denote scalars; boldface upper-case letters denote matrices; boldface lower-case letters denote column vectors; calligraphic upper-case letters denote support sets and 0 denotes a vector or a matrix with all zeros. The superscript \((\cdot)^T\) denotes matrix transpose. The \(\ell_0\) norm, the \(\ell_1\) norm, and the \(\ell_2\) norm of vectors, are denoted by \(\| \cdot \|_0\), \(\| \cdot \|_1\), and \(\| \cdot \|_2\), respectively. \(\Pr(\cdot)\) and \(P_x(\cdot)\) denote the probability and the probability density function (PDF) of \(x\) respectively. Finally, \(\mathcal{N}(\mu, \Sigma)\) denotes the multivariate normal distribution with mean vector \(\mu\) and covariance matrix \(\Sigma\).
A. DCS Based Data Acquisition and Transmission

The SNs capture low-dimensional projections of the original high-dimensional data during each activation time \(iT - T_{\text{act}} \leq t \leq iT\), which are given by:

\[
y_k(i) = \Phi_k(i)f_k(i),
\]

where \(y_k(i) \in \mathbb{R}^{m_k(i)}\) is the projections vector at the \(k\)th SN corresponding to the \(i\)th time interval, \(f_k(i) \in \mathbb{R}^{n(i)}\) is the original (Nyquist-sampled) data vector at the \(k\)th SN corresponding to the \(i\)th time interval, and \(\Phi_k(i) \in \mathbb{R}^{m_k(i) \times n(i)}\) is the projections matrix where \(m_k(i) \ll n(i)\) for any time interval \(i\) and SN \(k\). In practice, one may obtain the projections vector from the original data signal using analogue CS encoders \([37, 38]\), whereby the projections vector is obtained directly from the analogue continuous-time data, or using digital CS encoders \([31]\), whereby the projections vector is obtained from the Nyquist sampled discrete-time data via \((?)\). Recent studies suggest that digital CS encoders are more energy efficient than analogue CS encoders for WSNs \([31]\).

The SNs then convey the low-dimensional projections of the original high-dimensional data to the fusion center, i.e., the BS.

B. DCS Based Data Reconstruction

We take the signals \(f_k(i) \in \mathbb{R}^{n(i)}\) to admit a sparse representation \(x_k(i) \in \mathbb{R}^{n(i)}\) in some orthonormal basis \(\Psi(i) \in \mathbb{R}^{n(i) \times n(i)}\), i.e.,

\[
f_k(i) = \Psi(i)x_k(i),
\]

where \(\|x_k(i)\|_0 = s(i) \ll m_k(i) \ll n(i)\). In addition, we also take the sparse representations to obey the sparse common component and innovations (SCCI) model that has been frequently used to capture intra- and inter-signal correlation typical of physical signals (e.g., temperature, humidity) in WSNs \([29, 30]\). i.e., we write

\[
x_k(i) = z_c(i) + z_k(i),
\]

where \(z_c(i) \in \mathbb{R}^{n(i)}\) with \(\|z_c(i)\|_0 = s'(i) \ll n(i)\) denotes the common component of the sparse representation \(x_k(i) \in \mathbb{R}^{n(i)}\), which is common to the signals captured by the various SNs, and \(z_k(i) \in \mathbb{R}^{n(i)}\) with \(\|z_k(i)\|_0 = s''(i) \ll n(i)\) denotes the innovations component of the sparse representation \(x_k(i) \in \mathbb{R}^{n(i)}\), which is specific to the signals captured by each SN. This model applies to scenarios where a WSN is monitoring specific physical phenomena such as temperature or humidity where the common component models global factors, e.g., the sun and prevailing winds, and the innovations component models local factors, e.g., the terrain and shade.

\footnote{Note that the dimensionality of the projections can vary in different activation times and different SNs.}
Note that $s'_s(i) + s'_s(i) \geq s(i)$. Note also that the signal sparsity $s'_s(i), s'_s(i)$, and $s(i)$, the signal dimensionality $n(i)$, and the orthonormal dictionary $\Psi(i)$ are in general independent of the activation interval $i$.

In view of the signal model in (49) and (42), it is possible to reconstruct the original signal from the signal projections using either standard CS recovery algorithms or DCS recovery algorithms. CS recovery only considers intra-signal correlation; in contrast, DCS considers both inter- and intra-signal correlation [29].

1) CS Reconstruction Algorithms: CS signal reconstruction only assumes that the signals admit a sparse representation in some orthonormal basis or frame, e.g., the discrete Fourier basis, or the cosine or wavelet basis. Therefore, the typical signal reconstruction process behind conventional CS approaches involves solving the following optimization problem to recover individually the original signals captured by the various sensors in each activation interval [14], [15]:

$$\min_{x_k(i)} \|x_k(i)\|_1 \quad \text{s.t.} \quad A_k(i)x_k(i) = y_k(i),$$

where $A_k(i) = \Phi_k(i)\Psi(i) \in \mathbb{R}^{m_k(i) \times n(i)}$. Other common reconstruction approaches include greedy algorithms such as orthogonal matching pursuit (OMP) [39], [40] and compressive sampling matching pursuit (CoSaMP) [41] (see also [42]).

2) DCS Reconstruction Algorithms: In contrast, the signal reconstruction process behind the adopted DCS approach—which exploits the SCCI model in (49) together with (42)—involves solving the following optimization problem to recover jointly the original signals captured by various sensors in each activation interval [29]:

$$\min_{\tilde{x}(i)} \|\tilde{x}(i)\|_1 \quad \text{s.t.} \quad \tilde{A}(i)\tilde{x}(i) = \tilde{y}(i),$$

where $\tilde{x}(i) = [z_1(i)^T z_1(i)^T \cdots z_K(i)^T]^T \in \mathbb{R}^{(K+1)n(i)}$ is the extended sparse signal vector, $\tilde{y}(i) = [y_1(i)^T \cdots y_K(i)^T]^T \in \mathbb{R}^{\sum_{k=1}^K m_k(i)}$ is the extended measurements vector, and $\tilde{A}(i) \in \mathbb{R}^{(\sum_{k=1}^K m_k(i)) \times (K+1)n(i)}$ is the extended sensing matrix given by

$$\tilde{A}(i) = \begin{bmatrix} A_1(i) & A_1(i) & 0 & 0 & \cdots & 0 \\ A_2(i) & 0 & A_2(i) & 0 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ A_K(i) & 0 & 0 & 0 & \cdots & A_K(i) \end{bmatrix}.$$ 

C. Energy Consumption and Harvesting Models

We assume that the SNs use all the available energy in their local battery during each activation interval [3], which is given by:

$$\xi_C^E(i) = \xi_H^E(i),$$

where $\xi_C^E(i)$ is the energy harvested by SN $k$ in the interval $(i-1)T \leq t < iT$, and $\xi_C^E(i)$ is the energy consumed by SN $k$ in the interval $iT - T_{act} \leq t < iT$.

1) Energy Consumption Model: We assume that the energy consumed for sensing, computing and transmitting one measurement (projection) is essentially a constant $\tau > 0$. Hence, the energy consumed by SN $k$ during activation interval $i$ is modelled as follows:

$$\xi_k^E(i) = \tau m_k(i),$$

where $\xi_k^E(i)$ denotes a component of the harvested energy that is common to all SNs and $\xi_k^E(i)$ denotes a component of the harvested energy that is specific to the $i$th SN. We assume that $i)$ $\xi_k^E(i)$ follows an exponential distribution with parameter $\lambda_c > 0$ and that $\xi_k^E(i), k = 1, \ldots, K$ follows an exponential distribution with parameter $\lambda_k > 0, k = 1, \ldots, K$; ii) $\xi_k^E(i)$ and $\xi_k^E(i), k = 1, \ldots, K$ are independent; and iii) EH across time slots is independent.

It is clear that this correlated EH model is akin to the signal correlation model. The motivation for using such a model relates to the fact that SNs that are close together are also likely to—in addition to sense correlated signals—harvest correlated amounts of energy. Further, these assumptions are also motivated by the following: i) many energy sources, e.g., radio frequency (RF) energy [44], [45] and vibration energy [45], are known to exhibit exponential decay depending on the distance from the source; therefore, under the assumption of an RF source (or vibration source) and sensors located at various distances around it, both $\xi_k^E(i)$ and $\xi_k^E(i)$ would be exponentially decaying; ii) the instantaneous operational state of the physical energy converter circuitry of every sensor is independent from the one of other sensors [44], [45]; iii) the energy source can be modeled as a memoryless process as, for instance, both RF [44] and vibration harvesting [45] fluctuate randomly across time. Overall, our modelling approach is expected to capture key elements of the EH process, in addition to retaining some degree of analytical tractability.

3This assumption is motivated by the fact that if the computation is regular (which is the case in CS-based and and DCS-based data gathering) and the PHY/MAC layers are not adapting the modulation, coding and retransmission strategies during the active time (which is the case under low-energy IEEE 802.15.4 PHY and MAC-layer processing under SN-oriented operating systems—e.g., nullMAC in the Contiki OS) then computing and transmitting one measurement will come at quasi-constant energy consumption. Furthermore, the energy consumption is approximately linear as in (77) since the energy consumed for sensing and computing is much smaller than the energy consumed for transmitting [43].

4We assume that $\sum_{k=1}^K \lambda_k \neq \lambda_c$. This mathematical technicality does not result in a substantial loss of generality, but is required in order to simplify the ensuing analysis.
III. ANALYSIS: LOWER BOUNDS TO THE PROBABILITY OF INCORRECT DATA RECONSTRUCTION

Via lower bounds\(^4\) to the probability of incorrect data reconstruction (i.e., the probability of failure to reconstruct the data captured by all the SNs at the BS), we compare the performance of the proposed DCS scheme to that of conventional CS data acquisition schemes for EH WSNs. The probability of incorrect data reconstruction associated with the data gathering approaches can be lower bounded by the probability that the energy availability at the SNs is not sufficient to fit the energy consumption requirements at the various SNs. These energy consumption requirements are in turn dictated by the set of conditions on the number of measurements at the various SNs necessary for successful CS or DCS data reconstruction at the BS as shown in Appendix A.

Theorem 1: The probability of incorrect data reconstruction under the proposed signal and EH models for CS data acquisition and DCS data acquisition in an EH WSN can be lower bounded in any activation interval as follows:\(^5\)

\[
\Pr_{\text{CS}} \geq 1 - \frac{\sum_{k=1}^{K} \lambda_k e^{-\lambda_s \tau}}{\sum_1^{K} \lambda_k - \lambda_c} + \lambda_c - \frac{\sum_{k=1}^{K} \lambda_k s^s \tau}{\sum_1^{K} \lambda_k - \lambda_c},
\]

and

\[
\Pr_{\text{DCS}} \geq 1 - \min \{ \frac{\sum_{k=1}^{K} \lambda_k e^{-\lambda_s \tau}}{\sum_1^{K} \lambda_k - \lambda_c} - \frac{\sum_{k=1}^{K} \lambda_k s^s \tau}{\sum_1^{K} \lambda_k - \lambda_c}, \frac{e^{-\lambda_c (\frac{s^s}{K} + s^s \tau)} + \sum_{k=1}^{K} \lambda_c e^{-\lambda_c (\frac{s^s}{K} + s^s \tau) - \lambda_c s^s \tau}}{\prod_{k=1}^{K} (K \lambda_k - \lambda_c) (1 - \lambda_k/\lambda_j)} \}.
\]

Proof: See Appendix B.

The lower bounds to the probability of incorrect data reconstruction embody various attributes associated with the performance of the various data gathering schemes. One can immediately infer from the lower bound in (\ref{prcs}) that the performance of CS based data acquisition tends to deteriorate with the increase in the number of sensor nodes \(K\), the increase in the signal sparsity \(s\), and the decrease in mean energy availability \(\lambda_c\) or \(\lambda_k\) \((k = 1, \ldots, K)\). One can also infer additional behavior associated with the lower bounds by conducting an asymptotic analysis—using Taylor series expansions—in the regime where the EH process is highly correlated across the SNs \((\lambda_k \to \infty)\) \((k = 1, \ldots, K)\) and in the regime where the EH process is highly uncorrelated across the SNs \((\lambda_c \to \infty)\).

When the EH process is highly correlated, i.e., \(\lambda_k \to \infty\) \((k = 1, \ldots, K)\) and \(\lambda_c\) is finite, the lower bounds to the probability of incorrect data reconstruction can be expanded as follows:

\[
\Pr_{\text{CS}} \geq 1 - e^{-\lambda_c \tau} + \mathcal{O}(1/K).
\]

We can thus conclude via (\ref{prcs}) and (\ref{prcs5}) that:

- The mean available energy per SN, which is now given by \(1/\lambda_c\), dramatically affects the performance of both data acquisition methods. In particular, the lower bounds to the probability of incorrect data reconstruction in (\ref{prcs}) and (\ref{prcs5}) now increase exponentially to unity with the increase in \(\lambda_c\).
- The signal sparsity also affects the performance of CS and DCS data acquisition considerably. Since \(s \approx s^s + s^s \geq s^s/K + s^s\) one concludes that the lower bound in (\ref{prcs}) is higher than the lower bound in (\ref{prcs5}).
- The network size, as expected, does not affect the lower bounds associated with CS data acquisition (since the signals are reconstructed independently); in contrast, the network size affects the lower bound associated with DCS data acquisition via the common signal component (since the signals are reconstructed simultaneously). In view of the fact that \(s \approx s^s + s^s \geq s^s/K + s^s\) one can immediately conclude that the lower bound in (\ref{prcs5}) can be much higher than the lower bound in (\ref{prcs}) for a network with a large number of nodes (particularly when \(s^s > s^s\)).

In contrast, when the EH process is highly uncorrelated, i.e., \(\lambda_k \to \infty\) and \(\lambda_k\) \((k = 1, \ldots, K)\) are finite, the lower bounds to the probability of incorrect data reconstruction can be expanded as follows:

\[
\Pr_{\text{DCS}} \geq 1 - e^{-\lambda_c s^s \tau} + \mathcal{O}(1/\lambda_c),
\]

\[
\Pr_{\text{DCS}} \geq \max \{ 1 - e^{-\lambda_c (s^s \tau + K s^s \tau)} - \lambda_c e^{-\lambda_c (s^s \tau + K s^s \tau)} \}
\]

\[
+ \mathcal{O}(1/\lambda_c).
\]

We can also conclude via (\ref{prdc}) and (\ref{prldc}) that:

- The mean available energy per SN, which is now given by \(1/\lambda_k\) \((k = 1, \ldots, K)\), also dramatically affects the performance of both data acquisition methods. In particular, the lower bounds to the probability of incorrect data reconstruction in (\ref{prdc}) and (\ref{prldc}) now increase rapidly to unity with the increase in \(\lambda_k\) \((k = 1, \ldots, K)\).
- The signal sparsity affects the performance of CS and DCS data acquisition. As \(s > s^s\), the lower bound associated with CS data acquisition is higher than the first term of the lower bound associated with DCS. In addition, as \(K s \approx K (s^s + s^s) \geq s^s + K s^s\) the lower bound associated with CS data acquisition, which results from \(1 - \Pr(\xi_1 \geq s^s, \ldots, \xi_K \geq s^s \tau)\), is also higher than the second term of the lower bound associated with DCS, which results from \(1 - \Pr\left( \prod_{k=1}^{K} \xi_k \geq s^s \tau + K s^s \tau \right)\).
- The behavior of the performance of CS and DCS data acquisition as a function of the network size is more interesting in the highly uncorrelated than in the correlated EH scenario. In particular, the lower bound associated with CS data acquisition in (\ref{prdc}) rapidly tends to unity with increasing network size. In contrast, the behavior of
the lower bound associated with DCS data acquisition in (??) depends on the interplay between the two terms in the argument of the max(·, ·) function: the first term tends to increase with the increase in \( K \), but the second term, which coincides with the cumulative distribution function (CDF) of a generalized Erlang distributed random variable with mean value \( \sum_{k=1}^{K} \lambda_k/\lambda \cdot s' \), could decrease with the increase in \( K \). One then infers that there may be an optimal network size for DCS based data acquisition in the highly uncorrelated EH scenario.

Finally in Fig. 3 we give a comparison of the lower bounds with the performance achieved using practical CS and DCS algorithm\(^7\), and also with the optimal achievable DCS performance. The optimal achievable DCS performance is obtained directly from the sufficient conditions for DCS successful reconstruction in Appendix A by using Monte Carlo simulations. We note that the bounds – both our lower bound derived from the necessary conditions in Appendix A and the upper bound derived from the sufficient conditions in Appendix A – tend to be lower by about an order of magnitude than the actual performance associated with the reconstruction algorithms in (??) and (??). The reason is due to the fact that the conditions in Theorem 2 in Appendix A apply to an algorithm that leverages enumerate search over all possible sparse patterns \([30]\), rather than \( \ell_1 \) based algorithms.

In addition, numerical results both with synthetic and real data in the sequel reveal that our lower bounds also embody the main performance trends, hence can be used to gauge core issues surrounding the effect of various system phenomena and parameters. In particular, they show the fact that the DCS acquisition and reconstruction approach, in view of its ability to strike a trade-off between the number of measurements taken at different sensors without compromising data reconstruction quality, offers the means to match the energy demand to the random nature of the energy supply in order to increase the lifetime and/or the data gathering capability of the network.

\(^7\)The generation of the data and reconstruction algorithms used are the same as the synthetic experiments given in Section IV.

IV. NUMERICAL RESULTS

We now illustrate the potential of the approach both with synthetic data and real data collected by the WSN located in the Intel Berkeley Research Lab [46]. Note that we retain the previous synthetic EH model in both instances. In addition to the greedy EM which acts as the baseline for our proposed DCS-based approach, we also consider the usage of a conservative EM, where the SNs use only a fraction \( \delta \) of the available energy in their local battery in the activation interval. Concretely, the energy consumption corresponding to the number of projections used to convey the data from the SN to the BS is chosen to match the fraction \( \delta \) of the entire available energy budget.

A. Synthetic Experiments

For the synthetic experiments, we generate the sparse signal representations \( \mathbf{x}_k \) \( (k = 1, \ldots, K) \) randomly with non-zero components drawn independent identically distributed (i.i.d.) from a Gaussian distribution with zero mean and unit variance. These sparse signal representations obey the SCI model where the common component of the signals exhibits a pre-specified support size and the innovation components of various signals exhibit the same support size. We also generate the equivalent sensing matrices \( \mathbf{A}_k \) \( (k = 1, \ldots, K) \) randomly with elements drawn i.i.d. from zero mean and unit variance Gaussian distribution. The EH process obeys the proposed correlated EH model where the common component of the harvested energy across the SNs follows an exponential distribution with a pre-specified mean \( 1/\lambda_c \) and the innovation component of the harvested energy per SN are drawn from i.i.d. exponential distributions with the same mean \( 1/\lambda = 1/\lambda_1 = \ldots = 1/\lambda_K \). We use CVX, a package for specifying and solving convex programs [47], to reconstruct the signals for the CS case in (??) and the DCS case in (??). The results for greedy EM apply to any activation interval. In turn, the results for conservative EM apply for a large activation interval (steady-state). This can be achieved by initializing the remaining energy per SN to be equal to \( \delta(1/\lambda_c + 1/\lambda) \).
different ratios between average value of the common energy component and average value of the innovation energy component. As expected, the performance improves with the increase in average harvested energy per SN for all schemes. For the less correlated EH scenario (λ/λc = 5), as predicted by analysis, we observe that DCS performs better than CS under any EM approach and that conservative EM also performs better than greedy EM. For the more correlated EH scenario, we also observe that, in line with the analysis, DCS is better than CS under any EM approach and that conservative EM is better than the greedy EM. These trends are due to the fact that DCS is able to adapt to the energy variability across the SNs whereas CS cannot perform such adaptation. It is also interesting to note that—even though Fig. 4—appears to suggest that the performance in the more EH correlated scenario tends to be better than that in the less EH correlated scenario—there appears to be a λ/λc value that leads to the best performance as shown in Table I.

Table I

| Scheme                    | s′/s | s′/s = 1/2 | s′/s = 2 | s′/s = 6 | s′/s = 10 |
|---------------------------|------|-----------|----------|---------|----------|
| CS greedy EM              | 0.6436 | 0.4768 | 0.4434 | 0.4534  |          |
| CS conservative EM        | 0.3736 | 0.2008 | 0.1936 | 0.2584  |          |
| DCS greedy EM             | 0.4218 | 0.2374 | 0.2164 | 0.3422  |          |
| DCS conservative EM       | 0.0590 | 0.0182 | 0.0252 | 0.1062  |          |

Fig. 4 shows the probability of incorrect data reconstruction vs. average harvested energy per SN (i.e., 1/λc + 1/λ) for different ratios between the average energy of the common component and the average energy of the innovation component (i.e., s′/s). As expected, the performance improves with the increase in average harvested energy per SN for all schemes. For the less correlated EH scenario (λ/λc = 5), as predicted by analysis, we observe that DCS performs better than CS under any EM approach and that conservative EM also performs better than greedy EM. For the more correlated EH scenario, we also observe that, in line with the analysis, DCS is better than CS under any EM approach and that conservative EM is better than the greedy EM. These trends are due to the fact that DCS is able to adapt to the energy variability across the SNs whereas CS cannot perform such adaptation. It is also interesting to note that—even though Fig. 4—appears to suggest that the performance in the more EH correlated scenario tends to be better than that in the less EH correlated scenario—there appears to be a λ/λc value that leads to the best performance as shown in Table I.

Fig. 5 shows the probability of incorrect data reconstruction vs. total signal sparsity level s′ + s′c (K = 2, λc = 1/20, 1/λ = 1/20, τ = 1, n = 50 and δ = 0.5).

Fig. 6 shows the probability of incorrect reconstruction vs. number of SNs K (τ = 1, n = 50 and δ = 0.5). The left sub-figure corresponds to s′ = 1, s′c = 4 and 1/λc + 1/λ = 40; the right sub-figure corresponds to 1/λ = 20, 1/λc = 20 and s′ + s′c = 6.
TABLE II
THE AVERAGE HARVESTED ENERGY PER SN REQUIRED FOR A TARGET PROBABILITY OF INCORRECT RECONSTRUCTION OF $10^{-3}$ ($\gamma = 1$, $s' = 1$, $s_c = 4$, $\delta = 0.5$, $n = 50$ AND $\lambda = \lambda_c$).

|                | CS     | CS   | DCS  | DCS  |
|----------------|--------|------|------|------|
| $K = 2$        | 330    | 140  | 50   | 30   |
| $K = 5$        | 560    | 170  | 80   | 40   |

As the amount of harvested energy is a function of the duty cycle of sensors, in comparison to the CS scheme, the duty cycle of sensors could be increased by approximately 4 times by using the proposed approach.

B. Experiments With Real Data

For the real data experiment\(^8\), we use temperature data collected by a WSN located in the Intel Berkeley Research lab [46]. In particular, we only use the contiguous temperature data that was available from 8 SNs, i.e., SN 2, 3, 4, 7, 8, 9, 10 and 11. We assume the use of a typical 250kbps 62.64mV/17.4mAmA $\times$ 3.6V ZigBee RF transceiver\(^9\) and the use of a solar panel with an average harvesting capability of 10$\mu$W/cm$^2$ for the indoor environment given in [2], in order to carry out EH and energy consumption calculations. We also assume that harvested power is exponentially distributed with $\frac{1}{\lambda_c} = \frac{1}{\lambda_c} = 5\mu$W/cm$^2$ ($k = 1, \ldots, K$). The SNs independently and randomly collect a small portion of the original samples and transmit them to the BS based on the available energy. The temperature signals have length $n = 256$. Note that the temperature signals monitored by the WSN are compressible rather than exactly sparse via the discrete cosine transform (DCT). Thus, we assume that the reconstruction is successful if relative recovery error for a single SN satisfies

$$\frac{\|f_k - \hat{f}_k\|^2}{\|f_k\|^2} < 10^{-3},$$

where $f_k$ and $\hat{f}_k$ denote the original signal and the reconstructed signal of the $k$th SN respectively.

Fig. 7 shows the probability of incorrect reconstruction for $K = 2$ SNs, i.e., SN 2 and SN 3, achieved by the DCS and the CS data gathering schemes for various solar panel sizes. It is clear that the DCS scheme requires much lower energy levels in relation to the CS scheme for a certain target probability of incorrect reconstruction. For example, for a probability of incorrect reconstruction equal to $10^{-2}$, the use of DCS and CS with greedy EM require solar panels larger than 36cm$^2$, while with conservative EM this falls to 14cm$^2$ and 25cm$^2$ respectively, so that one can considerably ease the EH capability requirements.

Fig. 8 shows the probability of incorrect reconstruction with a solar panel of fixed size achieved by the DCS and the CS schemes for various numbers of SNs. We note once again that the CS scheme fails as the number of SNs increases, but the DCS scheme does not.

To conclude, the settings behind ???????????? are such that the WSN is powered only via the energy harvested from the environment: the fact that DCS based data gathering enables one to collect data without considerable penalties on the data reconstruction error forms the basis of the energy neutrality claims.

V. CONCLUSIONS

It has been established that DCS based data acquisition and reconstruction offers the means to match the energy demand to the energy supply in EH WSNs. Therefore, this paradigm provides substantial gains in energy efficiency for a certain target data reconstruction quality in comparison to CS based data acquisition and reconstruction—such energy efficiency gains then translate immediately into gains in network lifetime and/or gains in network data gathering capability.

The potential of DCS based data acquisition and reconstruction to unlock energy neutrality has been unveiled in a setting associated with a simple centralized EH WSN architecture, two simple EM schemes, and two relatively primitive models: i) a signal model that aims to capture the fact that the signals collected by different nodes exhibit some correlation; and
ii) a EH model that also aims to capture the fact that the energy harvested by different nodes also exhibit some degree of correlation. It is clear that the generalization of the work to more complex EH WSN architectures, more complex EM schemes, and more realistic signal and EH models will also lead to additional insight. In addition, the generalization of the work to systems that encompass various other phenomena such as data quantization, data loss, additive noise, fading, limited SN battery capacity and SN battery leakage will also cast further light on the potential of the approach.

**APPENDIX A**

**THE NECESSARY AND SUFFICIENT CONDITIONS FOR DCS RECONSTRUCTION**

The basis of our analysis are necessary conditions for the successful reconstruction of compressively sensed signals that obey the SCCI model. These necessary conditions along with sufficient conditions, which have been put forth in [30], are reviewed here.

Let us write

\[ \mathbf{x}(i) = \mathbf{P}(i)\theta(i), \]

where \( \mathbf{x}(i) = [x_1(i)^T \ldots x_K(i)^T]^T \in \mathbb{R}^{Kn(i)} \) is the extended sparse signal representation vector, \( \theta(i) = [\theta_1(i)^T \theta_2(i)^T \ldots \theta_K(i)^T]^T \in \mathbb{R}^{s(i)_1} + K s(i) \) is a vector with no zero values, \( \theta_c(i) \in \mathbb{R}^{s(i)_c}, \theta_k(i) \in \mathbb{R}^{s(i)_k} (k = 1, \ldots, K), \) and \( \mathbf{P}(i) \in \mathbb{R}^{K n(i) \times s(i)_1 + K s(i)} \) denotes a location that admits the form:

\[
\mathbf{P}(i) = \begin{bmatrix}
    \mathbf{P}_c(i) & \mathbf{P}_1(i) & 0 & 0 & \cdots & 0 \\
    \mathbf{P}_c(i) & 0 & \mathbf{P}_2(i) & 0 & \cdots & 0 \\
    \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
    \mathbf{P}_c(i) & 0 & 0 & 0 & \cdots & \mathbf{P}_K(i)
\end{bmatrix},
\]

where \( \mathbf{P}_c(i) \in \mathbb{R}^{n(i) \times s(i)_c} \) and \( \mathbf{P}_k(i) \in \mathbb{R}^{n(i) \times s(i)_k} (k = 1, \ldots, K) \) are different submatrices of \( n(i) \times n(i) \) identity matrices.

There can be overlap between the support of the common component associated with \( \mathbf{P}_c(i) \) and the supports of innovation components associated with \( \mathbf{P}_k(i) (k = 1, \ldots, K) \). The overlap size is defined as follows:

**Definition 1 ([30 Definition 4]):** Let \( \mathcal{J} \subseteq \{1, \ldots, K\} \). Consider also \( K \) correlated signals with sparse representations \( x_k(i) (k = 1, \ldots, K) \) that follow the SCCI model with a given location matrix \( \mathbf{P}(i) \). The overlap size between the common component support and innovation component supports for all signals \( k \in \{\mathcal{J}^C\} \) is defined as

\[
q(\mathcal{J}, \mathbf{P}(i)) := |\{j \in \{1, \ldots, n(i)\} : \text{row } j \text{ of } \mathbf{P}_c(i) \text{ has nonzero components and } \forall k \notin \mathcal{J}, \text{ row } j \text{ of } \mathbf{P}_k(i) \text{ has nonzero components}\}|.
\]

Note that \( q\{1, \ldots, K\}, \mathbf{P}(i)\} = s_c(i). \)

The following Theorem gives the sufficient condition for the joint successful reconstruction of the \( K \) correlated sparse signals with an algorithm based on an enumerative search over all possible sparse patterns [30].

**Theorem 2 ([30 Theorem 1]):** Let the equivalent sensing matrices \( \mathbf{A}_k(i) \in \mathbb{R}^{m_k(i) \times n(i)} (k = 1, \ldots, K) \) be populated with i.i.d. Gaussian entries. Let also the \( K \) correlated signals with sparse representations \( x_k(i) (k = 1, \ldots, K) \) follow the SCCI model with a full-rank location matrix \( \mathbf{P}(i) \) if

\[
\sum_{k \in \mathcal{J}} m_k(i) \geq |\mathcal{J}| s_c(i) + q(\mathcal{J}, \mathbf{P}(i)) + |\mathcal{J}| \quad (17)
\]

for all subsets \( \mathcal{J} \subseteq \{1, \ldots, K\} \), then the \( K \) correlated signals can be successfully recovered.

The following Theorem now gives a necessary condition for the joint successful reconstruction of the \( K \) correlated sparse signals [30].

**Theorem 3 ([30 Theorem 2]):** Let the equivalent sensing matrices \( \mathbf{A}_k(i) \in \mathbb{R}^{m_k(i) \times n(i)} (k = 1, \ldots, K) \). Let also the \( K \) correlated signals with sparse representations \( x_k(i) (k = 1, \ldots, K) \) follow the SCCI model with a full-rank location matrix \( \mathbf{P}(i) \). If

\[
\sum_{k \in \mathcal{J}} m_k(i) < |\mathcal{J}| s_c(i) + q(\mathcal{J}, \mathbf{P}(i)) \quad (18)
\]

for any subset \( \mathcal{J} \subseteq \{1, \ldots, K\} \), there exists a different set of correlated signals with sparse representations that also follow the SCCI model with signal measurements that are identical to those of the original desired signal.

The necessary conditions for successful DCS reconstruction in Theorem 3 also specialize to the conventional necessary condition for successful CS reconstruction that entails that the number of projections ought to be greater than or equal to the signal sparsity, by taking \( K = 1 \) and removing the common component.

In addition to the sufficient and necessary conditions for DCS reconstruction embodied in Theorems 2 and 3 respectively, it has been observed that the number of measurements of various signals for DCS can be substantially lower than the number of measurements in CS. In addition, it has also been observed, as hinted at in Theorems 2 and 3, that the number of measurements of various signals for DCS can also be adjusted without compromising data recovery in practice owing to the inter-signal correlation [29, 30, 48].

**APPENDIX B**

**PROOF OF THEOREM 1**

The analysis requires the distribution of a sum of independent exponential random variables with distinct parameters in some calculations [49].

**Lemma 1:** Let \( \beta_1, \ldots, \beta_K \) be \( K \) independent exponential random variables with distinct parameters \( \lambda_1, \ldots, \lambda_K \) respectively. Then, the probability density function of the random variable \( \beta = \sum_{k=1}^{K} \beta_k \) is given by:

\[
P_{\beta}(t) = \sum_{k=1}^{K} \prod_{j=1,j\neq k}^{K} (\lambda_j - \lambda_k) e^{-\lambda_k t} H(t), \quad (19)
\]

where \( H(t) = 1 \) if \( t \geq 0 \) and \( H(t) = 0 \) otherwise.
A. CS Based Data Acquisition and Reconstruction

By using the assumptions about the EH process in Section II, the probability of incorrect data collection due to energy depletion for a CS data acquisition scheme can be lower bounded as follows:

\[
Pr_{\text{CS}} \geq 1 - Pr \left( \xi_1 \geq s \tau, \ldots, \xi_K \geq s \tau \right) \\
= 1 - Pr \left( \xi_1^H \geq s \tau - \xi_1^c, \ldots, \xi_K^H \geq s \tau - \xi_K^c \right) \\
= 1 - \int_{s \tau - \xi_1^c}^{\infty} \cdots \int_{s \tau - \xi_K^c}^{\infty} \sum_{k=1}^{K} P_{\xi_k}(t) dt \cdot \cdots \cdot dt K dt \\
= 1 - \sum_{k=1}^{K} \frac{\lambda_k}{\lambda_k - \lambda_c} e^{-\lambda_c s \tau} + \sum_{k=1}^{K} \frac{\lambda_k}{\lambda_k - \lambda_c} e^{-\sum_{k=1}^{K} \lambda_k s \tau}, \tag{20}
\]

Note that we use the specialization of the necessary condition in Theorem 3 from the DCS to the CS case that states that the number of measurements per sensor has to be greater than or equal to the signal sparsity for successful reconstruction.

B. DCS Based Data Acquisition and Reconstruction

By using the assumptions associated with the EH process in Section II together with the necessary conditions for successful reconstruction in Appendix A, the probability of incorrect data collection due to energy depletion for a DCS data acquisition scheme can be lower bounded as follows:

\[
Pr_{\text{DCS}} \geq 1 - \min \left\{ \sum_{k=1}^{K} \frac{\lambda_k e^{-\lambda_c s \tau}}{\lambda_k - \lambda_c} - \frac{\lambda_c e^{-\sum_{k=1}^{K} \lambda_k s \tau}}{\sum_{k=1}^{K} \lambda_k - \lambda_c} \right\} \\
\geq 1 - \min \left\{ \Pr \left( \xi_1 \geq s \tau, \ldots, \xi_K \geq s \tau \right), \Pr \left( \sum_{k=1}^{K} \xi_k \geq s \tau + K s \tau \right) \right\}, \tag{21b}
\]

where in (21a) we loosen the bound in order to reduce the number of conditions on the harvested energy (where equality holds for \( K = 2 \)), and in (21b) we loosen the bound further in order to drop the dependency on the location matrix. As

\[
Pr \left( \xi_1 \geq s \tau, \ldots, \xi_K \geq s \tau \right) = \frac{\sum_{k=1}^{K} \lambda_k e^{-\lambda_c s \tau}}{\sum_{k=1}^{K} \lambda_k - \lambda_c} - \frac{\lambda_c e^{-\sum_{k=1}^{K} \lambda_k s \tau}}{\sum_{k=1}^{K} \lambda_k - \lambda_c}, \tag{22}
\]

and

\[
Pr \left( \sum_{k=1}^{K} \xi_k \geq s \tau + K s \tau \right) = \frac{\sum_{k=1}^{K} \xi_k^H}{\sum_{k=1}^{K} \xi_k^c} \geq s \tau + K s \tau \\
= \int_{0}^{\infty} P_{\xi_k^H} \left( t \right) dt + \int_{s \tau + \xi_k^c}^{\infty} P_{\xi_k^H} \left( t \right) \Pr \left( \sum_{k=1}^{K} \xi_k^H \geq s \tau + K s \tau - \xi_k^c \right) dt \\
= e^{-\lambda_c \left( s \tau + s \tau \right)} + \int_{0}^{\infty} P_{\xi_k^H} \left( t \right) e^{-\lambda_c \left( \sum_{k=1}^{K} \xi_k^H \right)} \prod_{j=1}^{K} \lambda_j dt \\
= e^{-\lambda_c \left( s \tau + s \tau \right)} + \int_{0}^{\infty} P_{\xi_k^H} \left( t \right) \prod_{j=1}^{K} \lambda_j dt \\
= e^{-\lambda_c \left( s \tau + s \tau \right)} + \prod_{j=1}^{K} \lambda_j \left( 1 - \lambda_k / \lambda_j \right), \tag{23a}
\]

where Lemma 1 is used in deriving (23a), then we have

\[
Pr_{\text{DCS}} \geq 1 - \min \left\{ \sum_{k=1}^{K} \frac{\lambda_k e^{-\lambda_c s \tau}}{\lambda_k - \lambda_c} - \frac{\lambda_c e^{-\sum_{k=1}^{K} \lambda_k s \tau}}{\sum_{k=1}^{K} \lambda_k - \lambda_c} \right\} \\
\geq 1 - \min \left\{ \Pr \left( \xi_1 \geq s \tau, \ldots, \xi_K \geq s \tau \right), \Pr \left( \sum_{k=1}^{K} \xi_k \geq s \tau + K s \tau \right) \right\}, \tag{24}
\]

REFERENCES

[1] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava, “Power management in energy harvesting sensor networks,” ACM Transactions on Embedded Computing Systems, vol. 6, no. 4, p. 32, 2007.

[2] S. Roundy, D. Steingart, L. Frechette, P. Wright, and J. Rabaey, “Power sources for wireless sensor networks,” Wireless Sensor Networks, pp. 1–17, 2004.

[3] S. Sudevalayam and P. Kulkarni, “Energy harvesting sensor nodes: Survey and implications,” Communications Surveys Tutorials, IEEE, vol. 13, no. 3, pp. 443–461, 2011.

[4] P. Blasco, D. Gunduz, and M. Dohler, “A learning theoretic approach to energy harvesting communication system optimization,” Wireless Communications, IEEE Transactions on, vol. 12, no. 4, pp. 1872–1882, 2013.

[5] R. Vullers, R. Schaijk, H. Visser, J. Penders, and C. Hoof, “Energy harvesting for autonomous wireless sensor networks,” Solid-State Circuits Magazine, IEEE, vol. 2, no. 2, pp. 29–38, 2010.

[6] Y. K. Tan and S. Panda, “Self-autonomous wireless sensor nodes with wind energy harvesting for remote sensing of wind-driven wildfire spread,” Instrumentation and Measurement, IEEE Transactions on, vol. 60, no. 4, pp. 1367–1377, 2011.

[7] K. M. Farinholt, G. Park, and C. Farrar, “Rf energy transmission for a low-power wireless impedance sensor node,” Sensors Journal, IEEE, vol. 9, no. 7, pp. 793–800, 2009.

[8] N. C. Gungor, G. P. Hancke et al., Industrial Wireless Sensor Networks: Applications, Protocols, and Standards. CRC Press, Taylor & Francis Group, 2013.

[9] F. Marcelloni and M. Vecchio, “A simple algorithm for data compression in wireless sensor networks,” Communications Letters, IEEE, vol. 12, no. 6, pp. 411–413, 2008.

[10] R. Cristescu, B. Beferull-Lozano, and M. Vetterli, “Networked slepian-wolf: theory, algorithms, and scaling laws,” Information Theory, IEEE Transactions on, vol. 51, no. 12, pp. 4057–4073, 2005.

[11] A. Bachir, M. Dohler, T. Watteyne, and K. K. Leung, “Mac essentials for wireless sensor networks,” Communications Surveys & Tutorials, IEEE, vol. 12, no. 2, pp. 222–248, 2010.

[12] Y. Tay, K. Jamieson, and H. Balakrishnan, “Collision-minimizing csma and its applications to wireless sensor networks,” Selected Areas in Communications, IEEE Journal on, vol. 22, no. 6, pp. 1048–1057, 2004.
