SALESFORCE CONTRACT DESIGN, JOINT PRICING AND PRODUCTION PLANNING WITH ASYMMETRIC OVERCONFIDENCE SALES AGENT

KEGUI CHEN AND XINYU WANG*

School of Management
China University of Mining and Technology
Xuzhou, China

MIN HUANG

College of Information Science and Engineering
State Key Laboratory of Synthetical Automation for Process Industries
Northeastern University, Shenyang, China

WAI-KI CHING

Advanced Modeling and Applied Computing Laboratory
Department of Mathematics
The University of Hong Kong, Pokfulam Road, Hong Kong, China

(Communicated by Tak-Kuen Siu)

Abstract. We study a supply chain in which a rational manufacturer relies on an overconfident sales agent to sell the products. The actual sales outcome is determined by the sales agent’s selling effort, the product price and a random market condition, and the sales agent is overconfident in their estimation of sales outcome. Apart from them, both of the sales agent’s degree of overconfidence and selling effort are his private information. We consider the salesforce incentive to motivate the sales agent and screen his real degree of overconfidence using a principle-agent method under dual information asymmetry; then the manufacturer uses the information to realize her joint decision on pricing and production. Furthermore, we derive the optimal compensation contract as well as the optimal pricing and production, and compare it to the symmetric overconfidence scenario. Finally, some interesting insights are found: when the manufacturer is uncertain about the sales agent’s the degree of overconfidence, her expected profit decreases while the sales agent with private information exerts less effort but obtains higher income, which implies the value of information; the manufacturer should hire a more overconfident sales agent, while a higher commission rate is not guaranteed. These results suggest that the manufacturer should not only focus on hiring the overconfident sales agent but also on disclosing the degree of overconfidence.

2010 Mathematics Subject Classification. Primary: 90B05; Secondary: 91B42.
Key words and phrases. Overconfidence, asymmetric information, salesforce incentive, optimal pricing and production.

* Corresponding author: Xinyu Wang.
1. **Introduction.** With the development of economic globalization and refined social division of labor, many manufacturers depend heavily on sales agents to sell their products to the customers, the revenue of the sales goes to the manufacturer and the sales agent is compensated according to the sales. The manufacturer expects to sell more products, but more sales need more sales effort which is costly to the sales agent, resulting in conflict goals for the manufacturer and the sales agent. Many firms spend a great amount of resources on salesforce compensation, Zoltners et al. (2008) reported that in the year of 2006 alone, firms in the US spent 800 billion on salesforce compensation, three times the amount spent on advertising. In the sales process, besides the sales agent’s effort, the manufacturer can also improve market demand through certain measures, such as designing of sales contract to give incentive to the sales agent’s participation, and setting the product price. For example, many auto makers determine the same price for one kind of car in different places with different sales agents for a certain period, as well as Sony and its other electronic product (see, Fernando et al. (2009)). In addition, price is an important factor affecting manufacturers, dealers, customer and market future, thus making the sensible pricing policy is a key factor to maintain manufacturer’s interest, mobilizing the enthusiasm of dealers, attract customers, win the competitors, develop and consolidate of the market. The manufacturer makes sales price reflects the fairness of the manufacturer to the end customer. Therefore, the problem of designing compensation plans and setting sensible sales price to maximize manufacturers income is challenging to the manufacturer.

Designing sales compensation plans, pricing and production/order strategies has attracted much analytical study, the existing related research results are mostly based on the complete rationality framework and ignoring the behavior characteristic of the supply chain members. With the development of behavioral economics, more and more doubts regarding the traditional assumption of rationality have emerged. Many experiments have shown that the decision-maker shows all kinds of bias in the decision process, overconfidence is the important psychological and emotional factor leading to the deviations. Behavior research verified people’s overconfidence behavior in the economic, behavioral economists and experimental economists confirmed the same point through a large number of experimental and empirical studies (see, for instance, Kahneman and Tversky (1979), Loch (2007), Bendoly et al. (2010)). Russo and Schoemaker (1992) studied and proposed that most managers overestimated their operating ability and the enterprise’s profitability, for instance, the Royal Dutch Shell Group noticed the newly hired geologist often makes mistakes due to overconfidence in detecting oil, and designed a training program to improve the accuracy of the geologist’s predictions. Eastman Kodak Company overestimated its technical ability and ignored the planning for the future, leading to decline in sales and eventually bankruptcy. Hung and Plott (2001) found in management experiments that when decision-maker involving private or public information, he will give higher weights to his private information. In the sales process, the sales agent has a good knowledge of the market conditions through direct contact with the customers and the market, overestimate his selling ability and show overprecision of market demand, which means he has overconfident tendency. Therefore, how to design effective sales incentive contract with incomplete information and overconfident sales agent, and study the action mechanism of overconfidence on the supply chain system is necessary.
As a kind of typical irrational behavior, overconfidence attracted considerable research attentions in behavioral operation management and behavioral finance, it refers to people’s excessive optimism and confidence on their own ability, knowledge, and the prediction of future. Oskamp (1965) confirmed that psychologists’ confidence in their clinical decisions is really justified. Malmendier and Tate (2012) established the relationship between managerial overconfidence and corporate investment decisions. Moore and Healy (2008) presented a reconciliation of the three distinct ways in which the research literature has defined overconfidence: (1) overestimation of one’s actual performance, (2) overplacement of one’s performance relative to others, and (3) excessive precision in one’s beliefs, the first two stress the overestimate of ability in different perspective, and the third kind means the overestimating of their forecast accuracy. At present, some scholars have incorporated overconfidence behavior with the research field of behavioral finance, the principal-agent theory and supply chain management. The existing research on overconfidence behavior mainly models the overconfidence according to overestimation or overprecision. The overconfidence behavior defined in this paper is the mixture of sales agent’s overestimation of his selling ability and overprecision of the random market demand, and then the degree of overconfidence on the incentive contract, the pricing and production decisions are investigated.

In addition, the sales agent obtains priority information through contacting with the customers in the sales process. In reality, because of knowledge monopoly, trade barriers and conflicts of interest, the sales agent and manufacturer cannot be achieved in the full information sharing, the information asymmetry exists between them. Information asymmetry generally exist in the supply chain and decisions problem, and received considerable research attentions.

The existing research of overconfidence behavior in supply chain and newsvendor models are based on symmetric information, or the degree of overconfidence is the common knowledge. This assumption is limited, in practice, for a variety of purposes, decision makers will hide their true overconfidence information to other decision makers, i.e., the degree of overconfidence is their private information and other participants don’t observe. Pavlov and Katok (2009) built a model of coordinating contracts considering the fairness behavior and find that the reason for supply chains cannot obtain coordination is due to incomplete information, and explains many problems in contract empirical research, such as rejection, and low efficiency, etc. Katok and Pavlov (2013) further investigated the impacts of three factors (inequality aversion, bounded rationality, and incomplete information) on the inefficiency of coordinating a simple supplier-retailer channel. The main result is that incomplete information about the retailer’s degree of inequality aversion plays a more important role than bounded rationality in explaining the suppliers’ behavior. Therefore, in this paper, we consider the sales agent’s the degree of overconfidence as his private information, and measure the degree of overconfidence based on both the overestimation on his selling ability and overprecision on the random market demand’s accuracy.

The contribution of this paper is to formally introduce. Firstly, the present paper, we consider the sales agent’s overconfidence behavior and measure it in two perspective as overestimation and overprecision. Secondly, our model is a combination of moral hazard and adverse selection, besides the sales agent’s selling effort is unobservable, his degree of overconfidence is also his private information, the manufacturer’s compensation plan and joint decision on pricing and production are
based on his subjective probability distribution. Thirdly, our model indicates that
the manufacturer should hire the overconfident sales agent and try to make use of
his overconfidence information, while a higher commission rate is not guaranteed
that is more overconfident. We consider a different setting with joint pricing and
production planning, and salesforce compensation where the sales agent has pri-
vate overconfidence parameter. A linear compensation contract is designed by the
manufacturer for the sales agent to reveal his private degree of overconfidence and
simultaneously maximize their expected profit, obtain the pricing and production
plan, and get a closed-form linear contract to reveal the private degree of overcon-
fidence.

The remainder of this paper is organized as follows. We review in Section 2 the
different streams of literature relevant to our research and describes of the problem
in Section 3. In Section 4, the benchmark model is briefly presented. Section 5
investigates the optimal contract design problem for the asymmetric overconfidence
sales agent. In Section 6, we provide analytical results to show the impact of
the overconfidence, and the result for symmetric scenario is briefly presented for
comparison purpose. We present some numerical examples and provide managerial
observations in Section 7. Finally, we close with concluding remarks and suggestions
for future research in Section 8. All proofs are presented in the Appendix.

2. Literature review. Our work is closely related to the extensive literature on
the sales force, the inventory problem with a joint decision on pricing and produc-
tion/ordering, principal agent theory and asymmetric information, and overconfi-
dence behavior; hence, the brief review of prior works on these related topics.

There is a growing popularity on compensation plan, pricing and production
/ordering decisions in the sales forces in recent years, note that we jointly consider
these two factors in our model. Past theoretical models have devoted considerable
attention to decision on production/ordering and pricing, and sales force issues.
Chao et al.(2012) studied a dynamic inventory and pricing optimization problem in
a periodic review inventory system with setup cost and finite ordering capacity in
each period. Oh et al.(2014) studied coordinated pricing and production decisions
in an assemble-to-order system consider a firm that makes pricing and production
decisions in an ATO system over T time periods. Pal et al.(2014) formulated and
analyzed a joint pricing and ordering policy for two echelon imperfect production
inventory model with two cycle periods. Qin et al.(2014) considered the pricing
and lot-sizing problem for products with quality and physical quantity deteriorat-
ing simultaneously, the deterioration rate of quality and physical quantity is taken
to be time proportional. Sales rebate contract has been vastly studied in sales-
force management (see, for instance, Taylor(2002), Chiu et al.(2011)). Taylor(2002)
investigated channel rebate contract and considered the demand is influenced by
retailer sales effort, a properly designed target rebate and returns contract achieves
coordination and a win-win outcome, but without price dependent demand. Chiu
et al.(2011) show that a policy that combines the use of wholesale price, channel
rebate, and returns can coordinate a channel with both additive and multiplicative
price-dependent demands. These studies are based on full information.

Our model relates to salesforce incentive, the modeling of compensation on sales
forces has been one of the marketing management research issues since the early
work of Basu et al.(1985) and Coughlan(1993) provided the comprehensive reviews.
Salesforce management hamost of the works study the compensation incentives
to induce sales agent to disclose what they know about the hidden cost, market condition, or demand. Gonik (1978) reported a clever scheme under which it is in the salespeople’s interest to forecast accurately and to work hard. Kaya and Ozer (2009) combined the moral hazard, asymmetry cost information, and product pricing dimensions of the quality problem in an OEM-CM relationship. Taylor (2006) characterized the manufacturer’s sale timing preference when the retailer exerts sales effort and when information is asymmetric. Liu et al. (2010) investigated the online dual channel supply chain system and its joint decision on pricing and production under asymmetry cost information. Xu et al. (2010) studied the effects of the presence of a contingent urgent supplier with private cost information on the performance of both the prime supplier and the manufacturer. Ozer and Gal (2011) investigated the information about the supplier’s asymmetric production cost affect the profits and contracting decision. Chen et al. (2012) considered a supply chain in which a CM assembles a product for a large OEM and at the same time produces a different product for a smaller OEM under asymmetric cost information. Cao et al. (2013) studied the incentive contracts that can improve the supply chain performance when the cost information in the dual-channel supply chain is asymmetric. Zhang et al. (2014) studied the problem of designing contracts in a closed-loop supply chain when the cost of collection effort is the retailer’s private information, and analyzed the impact of information on the equilibrium results of supply chain members. Wei et al. (2015) considered the optimal decision problem of a closed-loop supply chain with symmetric and asymmetric information structures using game theory. All of the above literatures are base on the hidden cost. Lee and Yang (2013a) employed a screening model to examine the problem of supply chain contracting involving one retailer and two suppliers under asymmetric demand information. Many studies about compensation plan to induce salespeople to disclose the market condition information, Chen (2005) investigated how a firm can provide incentives to its salesforce so that it is in their interest to truthfully disclose their information about the market and to work hard. Kung and Chen (2011) studied a three-layer supply chain in which a manufacturer relies on a salesperson to sell the products to the consumers, and jointly study the manufacturer’s partner selection problem and the resellers’ salesforce compensation problem. The sales outcome is determined by a random market condition and the salesperson’s service level, both of which are privately observed by the salesperson. Lee and Yang (2013b) investigated the optimal compensation scheme involving one firm and two competing salespersons deployed in different territories under asymmetric market condition information. Saghaian and Chao (2014) studied the dependency of the operational decisions of production/inventory management and the design of salesforce incentives, and consider the problem of joint salesforce incentive design and inventory/production control with both moral hazard and adverse selection due to the sales agent’s private market condition. Different from the above sales force research, Dai and Chao (2013) considered the salesforce incentive and inventory planning problem where the exact value of the sales agent’s risk attitude is the sales agent’s private information, and indicate that the firm should offer a lower commission rate to a salesperson that is more risk averse.

The extant literature has hardly addressed the issue of the decision on production/ordering and pricing, and sales force under incomplete rational, ignore the behavior characteristics. With the development of behavioral economics, more and more doubts regarding the traditional assumption of rationality have emerged.
Many experiments have shown that the decision-maker considers not only the profits but also other behavioral factors such as overconfidence (see, for instance, Kahneman and Tversky (1979), Loch (2007), Bendoly et al. (2010)) and fairness. For the supply chain’s decision making with behavioral factors, most of the research is based on the fairness (see, for instance, Cui et al. (2007), Caliskan-Demirag et al. (2010)). Cui et al. (2007) developed a model in which both parties care about fairness in a dyadic monopoly setting and derive an interesting counter-traditional result that the supplier can coordinate the channel with a constant wholesale price. Caliskan-Demirag et al. (2010) extended the model to non-linear demand functions. Investigations of results demonstrated that a strong preference for fairness can lead to results significantly deviating from traditional theoretical predictions. There are some studies involved behavioral both theoretically and experimentally. Chow et al. (2014) examined the effect of retailers’ minimum profit share concerns on supply chain system performance through a laboratory experimental and analytical modeling approaches. Ho et al. (2014) examined the interaction of distributional and peer-induced fairness in supply chain, and conducted standard economic experiments with subjects motivated by substantial monetary incentives to test the model predictions. So far, most of the behavior research in supply chain is based on full information, fairness behavior is general and member’s overconfidence behavior is not involved. Englmaier and Wambach (2010) analyzed the classic moral hazard problem with the additional assumption that agents are inequity averse, Katok and Pavlov (2013) proposed that incomplete information about the retailer’s degree of inequality aversion plays a more important role than bounded rationality. The above shows that supply chain’s behavioral decision problem under asymmetric information is worth studying, and especially the overconfidence behavior.

For the overconfidence behavior in supply chains, Ren et al. (2013) provided two experiments supporting that underestimating the variance of demand causes orders to deviate from optimal in predictable ways, i.e., the overconfidence on the belief of the demand. Moore and Healy (2008) proposed a model in which the three types of overconfidence arise. Li et al. (2015) studied a decision model with overconfident consumers and analyzed the retailer’s advance selling strategy in a two-period setting. The study of behavior of the supply chain considering overconfidence is rare, and are based on full information. Researching in overconfidence behavior under the asymmetric information, the majority is the effects of overconfidence on incentive contracts in a principal agent framework, there is a growing literature that takes the overconfidence into account. Russo and Schoemaker (1992), and Busenitz and Barney (1997) confirmed overconfidence existing in principal agent scene, Keiber (2002), and Rosa (2011) studied the wage contract design that the principal and the agent with the same level of overconfidence and different levels of overconfidence in the moral hazard framework respectively, and then the the effect of overconfidence on incentive contract were analyzed. Garciet al. (2007) studied financial markets in which both rational and overconfident agents coexist and make endogenous information acquisition decisions, and considers a model in which rational traders coexist with overconfident ones, and show that endogenizing the information acquisition decision generates new predictions on the effects of overconfidence on asset prices, with respect to models with exogenous information distribution. Ludwig et al. (2011) compared the rational and overconfident agents, and showed that moderate overconfidence in a contest can improve the agent’s performance relative to an unbiased opponent and even lead to an advantage in absolute terms. Sandroni and
Squintani (2013) indicated that overconfidence may overturn fundamental relations between observable variables in perfect-competition asymmetric information insurance markets. While in monopolistic insurance markets, overconfidence may be observationally equivalent to variations in the risk composition of the economy. In these models, the degree of overconfidence is usually modeled as a parameter in the utility function and it is assumed to be known information. Katok and Pavlov (2013) further compared inequality aversion, bounded rationality, and incomplete information on the inefficiency of supply chain, and show that incomplete information about the retailer’s degree of inequality aversion plays a more important role than bounded rationality in explaining the suppliers’ behavior. In this paper, the source of asymmetric information comes from the individual decision maker’s degree of overconfidence based on traditional principal-agent theory and newsvendor model. Different from the papers mentioned above, we consider a supply chain with overconfident sales agent who keep the exact value of their degree of overconfidence as private information. Our paper contributes to this line of research by investigating a contract design problem mixed with joint decision on pricing and production when the sales agent’s degree of overconfidence is asymmetric information.

3. Problem formulation. In this paper, we consider a supply chain in which a manufacturer (she) relies on the sales agent (he) to sell her products. The relationship between the manufacturer and the sales agent is that of a principal and an agent. At the beginning of a selling season, a decision about production quantity $Q$ and selling price $p$ are made by the manufacturer, and the production cost per unit is $c$. When production does not match demand, additional costs are incurred. In the case of oversupply, the excess supply is salvaged at $s$ per unit (net salvage value); and in the case of undersupply, the excess demand must be satisfied via an emergency production at a cost of $c'$ per unit. To avoid triviality, we assume that $s < c < c' < p$ (see, for instance, Dai and Chao (2013), Saghafian and Chao (2014)).

Suppose that the sales quantity, or demand, is random and depends on the sales effort $e$ exerted by the sales agent as well as the selling price $p$. We adopt the following linear demand function (as in Ozer, 2011) to model the sales quantity for the product in the following additive form:

$$X = a - bp + e + \theta$$  \hspace{1cm} (1)

where the sales parameter $a > 0$ is the average sales quantity in the case without sales effort, $b > 0$ represents the price elasticity of demand, both are common knowledge. Here $\theta$ is a normally distributed random noise with mean 0 and variance $\sigma^2$. Let $F(\cdot)$ and $f(\cdot)$ represent the cumulative distribution function and probability density function of $\theta$, respectively, rational individuals can correctly recognize the distribution of $\theta$. Moreover, we assume that $\theta$ is sufficiently large such that the probability of $X$ being negative is negligible, $a - bc > 0$ holds, as the expected market demand $a - bp$ is positive only when $a - bc > 0$.

Based on his past experience and local expertise, the sales agent has overconfident tendency, as it has superior information due to her close contact to the customers and market directly through selling the products. For the overconfident sales agent, overestimate their ability of selling abilities and overconfident about the market demand accurate, i.e. his estimate of the variance of consumer demand is biased. For practical purposes, we assume that the demand in the overconfident sales agent’s
mind is
\[ X_0 = a - bp + (1 + k)e + \theta_0 \] (2)
where \( \theta_0 \) is normally distributed with mean 0 and variance \((1 - k)^2 \sigma^2\), here \( k \in [0, 1] \) represents the sales agent’s overconfidence level. As \( k \) increases, overconfidence thus increases, \( k = 0 \) means the sales agent is rational.

The sales agent has superior information, besides the effort level \( e \), the degree of overconfidence \( k \) is also her private information, the manufacturer treat \( k \in [k, \bar{k}] \) as random with distribution \( G(k) \) and density \( g(k) \). We assume that \( G(k) \) satisfies the increasing failure rate property (IFR), that is, the inverse failure rate \( H(k) = \frac{G(k)}{g(k)} \) decreases in \( k \), and \( \overline{G}(k) = 1 - G(k) \), \( G(\bar{k}) = 0 \), \( G(\bar{k}) = 1 \) (see, for instance, Dai and Chao(2013), Kung and Chen(2011)).

The cost of sales effort is assumed to be \( C(e) = b'e^2 / 2 \), which is increasing convex in \( e \), and \( b' > 0 \) is the effort cost parameter. We assume \( 2bb' > 1 \) holds, the assumption ensures that when price sensitivity \( b \) is low, the cost of sales effort should be high enough (high \( b' \)) to prevent the manufacturer from setting an infinite effort level \( e \) and making infinite profit (see, for instance, Kaya and Ozer(2009)).

We use \( s(X_0) \) to denote the compensation sales agent receives from the manufacturer, which depends on his total sales \( X_0 \), we restrict our attention to the class of linear contracts as \( s(X_0) = \alpha + \beta X_0 \), because of the prevalence in practice. Specifically, we use \( (\alpha, \beta) \) to denote the contract signed by the manufacturer and the sales agent, where \( \alpha \) is the base salary and \( \beta \geq 0 \) is the commission rate.

The manufacturer is risk-neutral and maximizes her expected profit. Therefore, her net profit function can be written as follows:
\[ \pi_M = pX - cQ + s(Q - X)^+ - c'(X - Q)^+ - s(X_0) \] (3)
where \( x^+ = \max(0, x) \), and the sales agent’s net income is
\[ \pi_S = s(X_0) - C(e) = \alpha + \beta[a - bp + (1 + k)e + \theta_0] - b'e^2 / 2. \] (4)

The sales agent is risk-averse and maximizes his expected utility, his risk attitude is represented by a negative exponential utility function \( U_S = -e^{-\rho \pi_S} \), where \( \pi_S \) is his net income and \( \rho > 0 \) is the coefficient of absolute risk aversion. Here \( -\bar{U}_0 \) denotes the sales agent’s reservation utility representing the best outside opportunity for the sales agent, and the corresponding certainty equivalence is \( \pi = -\ln \bar{U}_0 / \rho \). Thus, for the sales agent to accept a contract, the contract has to maximize his expected utility among his choices and the expected utility value has to be at least \( -\bar{U}_0 \), i.e. \( E[-e^{-\rho \pi_S}] \geq -\bar{U}_0 \), this is equivalent to
\[ \text{CE}_S = \alpha + \beta[a - bp + (1 + k)e] - b'e^2 / 2 - \rho \sigma^2 \beta^2 (1 - k)^2 / 2 \geq \pi \] (5)
the Individual Rationality (IR) constraint (5) ensures the participation of the sales agent.

The above model assumptions, including the linear payment structure, the negative exponential utility, and the normally distributed randomness, together referred to as the LEN (Linear-Exponential-Normal) assumption, are commonly adopted in the agency literature for tractability. The above assumptions are common knowledge to all the parties concerned. We first investigate the manufacturer’s optimal contract design, joint pricing and production problem under the symmetric overconfidence as the benchmark case, further we consider the sales agent’s degree of overconfidence as his private information, and analyze the impact of overconfidence and asymmetric information on the decisions and income of both sides. Throughout
the paper, for the sake of convenience, the traditional factor $A = b' \rho \sigma^2$ is defined as the product of the effort cost parameter $b'$, the risk aversion level $\rho$ and actual demand variance $\sigma^2$. We use the following notation: we use $E[\cdot]$ to represent the mathematical expectation. The subscripts “R”, “M” and “S” respectively denote the parameters corresponding to the Rational scenario, the Manufacturer and the Sales agent respectively, and the superscript “*” denotes the optimal cases.

4. Benchmark case. To serve as a benchmark, this section studies the selling scheme in which the sales agent’s degree of overconfidence $k$ is common knowledge of both sides. In this case, the manufacturer faces only moral hazard problem (without adverse selection problem). According to the assumptions and Eq. (2) in Section 3, contracting with the symmetric overconfident sales agent, the expected utility of the risk-neutral manufacturer can be written as follows:

$$E(\pi_M) = E[pX - cQ + s(Q - X)^+ - c'(X - Q)^+ - s(X_0)].$$

By the certainty equivalence principle, the sales agent’s certainty equivalence corresponding to the expected utility is given as follows:

$$CE_S = \alpha + \beta[a - bp + (1 + k)e] - \frac{b' e^2}{2} - \frac{\rho \sigma^2 \beta^2 (1 - k)^2}{2}.$$  

Similar to a principal-agent framework, the sales agent acts as the follower, and the manufacturer as a leader designs the incentive contracts and sets the product price and production quantity to maximize her expected utility while satisfying the sales agent’s Individual Rationality (IR) and Incentive Compatibility (IC) constraints. The manufacturer’s decision problem can be expressed as follows:

$$\max_{\alpha, \beta, p, Q} E(\pi_M) = E[pX - cQ + s(Q - X)^+ - c'(X - Q)^+ - s(X_0)]$$

subject to (IR) $CE_S = \alpha + \beta[a - bp + (1 + k)e] - \frac{b' e^2}{2} - \frac{\rho \sigma^2 \beta^2 (1 - k)^2}{2} \geq \pi$

(II) $e^* = \arg \max_{e \geq 0} CE_S$

The IR constraint ensures the participation of the sales agent, because of exceeding the reservation profit. Eq. (9) is IC constraint, assuring that the sales agent does not pretend to choose the other effort level. The sales agent aims to maximize his expected utility, and his incentive compatibility constraints can be replaced by first best effort level as

$$e^* = (1 + k) \beta/b'.$$

Then, we obtain Proposition 1, which characterizes the optimal contract with symmetry overconfidence.

**Proposition 1.** If the manufacturer can observe the sales agent’s degree of overconfidence $k$, the optimal commission rate $\beta^*$, sales agent’s effort $e^*$, joint pricing $p^*$ and production quantity $Q^*$ strategies are given by

$$p^* = \frac{b'(a + bc)[(1+k)^2 + A(1-k)^2] - c(1+k)^2}{2b'[((1+k)^2 + A(1-k)^2) - (1+k)^2]}$$

$$Q^* = a - bp^* + \frac{\beta^* (1+k)}{b'} + F^{-1}\left(\frac{\epsilon' - c}{\sigma - \epsilon}\right) = \frac{a - bc}{2} + \frac{\beta^* (1+k)}{2b'} + F^{-1}\left(\frac{\epsilon' - c}{\sigma - \epsilon}\right)$$

$$\beta^* = \frac{b'(a - bc)(1+k)}{\beta^*(1+k)^2 + A(1-k)^2}$$

$$e^* = \frac{b'}{b'} = \frac{\beta^*(1+k)(a - bc)(1+k)^2}{2b'[((1+k)^2 + A(1-k)^2) - (1+k)^2]}$$
Correspondingly, the optimal base salary $\alpha^*$:

$$
\alpha^* = \pi + \frac{(1 + k)^2}{2b'} + A(1 - k)^2 \beta^2 - \frac{a - bc}{2} \beta^* - \frac{\beta^*}{2} = -\bar{\pi} + \frac{b'(a - bc)^2(1 + k)}{4bb'[1 + A(1 - k)^2] - 2(1 + k)^2} \left[ \frac{(1 + k)(1 + k)^2 + A(1 - k)^2}{2bb'[1 + A(1 - k)^2] - (1 + k)^2} - 1 \right]
$$

The manufacturer’s optimal expected net profit is

$$
E^*(\pi_M) = \frac{b'(a - bc)^2[1 + A(1 - k)^2/(1 + k)^2]}{4bb'[1 + A(1 - k)^2/(1 + k)^2] - 2} - \bar{\pi} - \frac{\sigma(c' - s)}{\sqrt{2\pi}} \exp\left(-\frac{l'(c' - c))}{\sigma^2}\right)
$$

where $a - bc > 0$, $2bb' - 1 > 0$, $A = b'/\rho$, $\Phi(.)$ is the cumulative probability function for the standard normal distribution $N(0,1)$, and $\Phi^{-1}(.)$ is its inverse function.

The proof of Proposition 1 and the other Propositions and Corollaries are given in the Appendix.

From Proposition 1, we know that the sales effort $e^*$, the manufacturer’s optimal expected profit $E^*(\pi_M)$, the commission rate $\beta^*$, the price $p^*$ and the production quantity $Q^*$ are all decreasing in $A = b'/\rho\sigma^2$, i.e. they are all decreasing in the cost parameter $b'$, risk averse $\rho$ and the demand variance $\sigma^2$. Eq.(11) indicates that the unit net salvage value $s$ and unit emergency production cost $c'$ only affect the manufacturer’s production quantity $Q^*$ and her expected profit $E^*(\pi_M)$. It is easy to verify that the manufacturer’s expected profit $E^*(\pi_M)$ are strictly increasing in $c'$ and $s$, which is consistent with the traditional newsvendor model with lost sale penalty cost.

From Proposition 1 (set $k = 0$), we deduce the completely rational case as follows:

**Observation 1.** If the sales agent is rational ($k = 0$), the optimal selling price $p^*_R$, production quantity $Q^*_R$, and the commission rate $\beta^*_R$ as well as the effort $e^*_R$ constitute the unique Bias Nash equilibrium (The subscripts “R” denotes Rational):

$$
\begin{align*}
\begin{cases}
p^*_R &= \frac{b'(a + bc)(1 + A) - c}{2bb'(1 + A) - 1} \\
Q^*_R &= a - bp^*_R + \frac{\beta^*_R}{b'} + \Phi^{-1}\left(\frac{c' - c}{\sigma_s}\right) \\
\beta^*_R &= \frac{b'(a - bc)}{2bb'(1 + A) - 1} \\
e^*_R &= \frac{\beta^*_R}{b'} = \frac{a - bc}{2bb'(1 + A) - 1}
\end{cases}
\end{align*}
$$

Correspondingly, the optimal base salary is

$$
\alpha^*_R = \pi + \frac{A + 1}{2b'} \beta^*_R - \frac{a - bc}{2} \beta^* = \pi + \frac{b'(a - bc)^2}{4bb'(1 + A) - 2} \left[ \frac{1 + A}{2bb'(1 + A) - 1} - 1 \right].
$$

The manufacturer’s optimal expected net profit is

$$
E^*(\pi_{RM}) = \frac{b'(1 + A)(a - bc)^2}{4bb'(1 + A) - 2} - \bar{\pi} - \frac{\sigma(c' - s)}{\sqrt{2\pi}} \exp\left(-\frac{l'(c' - c))}{\sigma^2}\right)
$$

where $a - bc > 0$, $2bb' - 1 > 0$, and $A = b'/\rho\sigma^2$. 

5. A model with asymmetric overconfidence information. This analysis is distinct from previous studies in that, it treats the degree of overconfidence as the private information of the sales agent. We assume that the sales agent has her own private information about her overconfidence $k \in [\underline{k}, \bar{k}]$. The sales agent surely knows the actual $k$, while the manufacturer has only a subjective assessment according to its probability distribution. In this case, the manufacturer faces a mixture of adverse selection and moral hazard. The goal of the manufacturer is to design a menu of incentive contracts and determine the decision on production and pricing so as to maximize her expected profit based on revelation principle.

In this manufacturer-sales agent relationship, the manufacturer designs a menu of compensation contracts and plans the production and pricing decisions, the sequence of events is as follows. 1) The sales agent’s overconfidence information is observed by the manufacturer; 2) The manufacturer offers a menu of compensation contracts $(\alpha(k), \beta(k))$ for the sales agent to self-select; 3) the sales agent decides whether or not to participate and, if so, which contract to sign based on his private information; 4) under a signed contract, the manufacturer determines the joint decision on pricing and the production quantity, and the sales agent makes the effort decision. 5) the sales outcome is realized, and the sales agent is compensated.

We can get the sales agent’s certainty equivalence with Eq.(2), namely

$$ CE_S(k|e) = \alpha(k) + \beta(k)[a - bp(k) + (1 + k)e - b'e^2/2 - \rho \sigma^2 \beta^2(k)(1 - k)^2/2]. $$ (15)

To characterize the optimal compensation design problem, we use backward induction and start with the sales agent’s problem. Suppose the sales agent’s actual overconfidence is $k$ but has chosen the contract $(\alpha(k'), \beta(k'))$ related to $k'$. By choosing effort $e$, the final sales in the overconfident sales agent’s mind is $X_0 = a - bp + (1 + k)e + \theta_0$. Thus, with this, the sales agent’s certainty equivalent (CE) is

$$ CE_S(k, k'|e) = \alpha(k') + \beta(k')[a - bp(k') + (1 + k)e - b'e^2/2 - \rho \sigma^2 \beta^2(k')(1 - k')^2/2]. $$ (16)

Because the exponential function is monotonic, maximizing the expected utility is equivalent to maximizing the CE by choosing

$$ e(k) = \arg \max_{e \geq 0} CE_S(k, k'|e) = (1 + k)\beta(k')/b'. $$ (17)

Substituting Eq.(17) in Eq.(16), the sales agent’s maximum CE is

$$ CE_S(k, k') = \max_{e \geq 0} CE_S(k, k'|e) = \alpha(k') + \beta(k')[a - bp(k')] + \frac{(1 + k)^2 - A(1 - k)^2}{2b'} \beta^2(k') $$ (18)

Let $CE_S(k) \equiv CE_S(k, k)$, thus

$$ CE_S(k) = \alpha(k) + \beta(k)[a - bp(k)] + \frac{(1 + k)^2 - A(1 - k)^2}{2b'} \beta^2(k). $$ (19)

In equilibrium, the manufacturer induces the sales agent to disclose his degree of overconfidence $k$ truthfully by choosing the contract $(\alpha(k), \beta(k))$. It then follows that the expected sales is $a - bp(k) + (1 + k)\beta(k)/b'$ and consequently the manufacturer’s expected profit is $E_k(\Pi_M)$, namely

$$ \int_{\underline{k}}^{\bar{k}} \Pi_M dG(k) $$ (20)
where

$$
\Pi_M = E(\pi_M) = E[pX - cQ + s(Q - X)^{+} - c'(X - Q)^{+} - s(X_0)]
$$

$$
= p[a - bp + (1 + k)\beta(k)/b'] - cQ - \alpha(k) - \beta(k)[a - bp + (1 + k)^2\beta(k)/b'] + E[s(Q - X)^{+} - c'(X - Q)^{+}]
$$

$$
= p[a - bp + (1 + k)\beta(k)/b'] - cQ - \alpha(k) - \beta(k)[a - bp + (1 + k)^2\beta(k)/b'] + s \int_{-\infty}^{Q-a+bp-(1+k)\beta(k)/b'} [Q - a + bp - (1 + k)\beta(k)/b' - x] dF(x)
$$

$$
- c' \int_{Q-a+bp-(1+k)\beta(k)/b'}^{\infty} [x - (Q - a + bp - (1 + k)\beta(k)/b')] dF(x).
$$

Therefore, under asymmetric overconfidence information, the manufacturer’s decision problem can be expressed as follows:

$$
\max_{\alpha(k), \beta(k), p(k), Q(k)} E_k(\Pi_M)
$$

subject to (TC) $CE_S(k) \geq CE_S(k, k')$ \hspace{1cm} (21)

(IR) $CE_S(k) \geq \pi$ \hspace{1cm} (22)

(IC) $e(k) = (1 + k)\beta(k)/b', k, k' \in [k, K]$ \hspace{1cm} (23)

Eq. (21) is Truth-Telling (TC) constraint under adverse selection assuring that the sales agent with overconfidence $k$ selects the contract designed for him; the IR constraint (22) ensures the participation of the sales agent; Eq. (23) is IC constraint under moral hazard to prevent the sales agent from shirking. Under dual information asymmetry of the mixture of adverse selection and moral hazard, the manufacturer safeguards her interests through the menu of contracts. The following proposition characterizes the optimal menu of contracts and decisions.

**Proposition 2.** When the sales agent’s overconfidence information is asymmetric, the manufacturer offers menu of contracts $(\alpha^*(k), \beta^*(k))$, with the optimal menu, it induces the effort level $e^*(k)$, the optimal pricing $p^*(k)$ and production $Q^*(k)$ decisions as

$$
\begin{align*}
    \alpha^*(k) &= \pi + \frac{A(1 - k)^2\beta^2(k) - b'((a - bc) + k(1 + k))\beta^*(k)}{2b'} \\
    Q^*(k) &= a - bp^*(k) + \frac{(1 + k)\beta^*(k)}{b'} + F^{-1}\left(\frac{c' - c}{c' - c}ight) \\
    \beta^*(k) &= \frac{2bA[(1 + k)^2 + A(1 - k)^2 + 2[(1 + k) + A(1 - k)]]H(k) - c(1 + k)^2}{b'(a - bc)(1 + k)} \\
    e^*(k) &= \frac{(1 + k)e^*(k)}{b'} = \frac{2bA[(1 + k)^2 + A(1 - k)^2 + 2[(1 + k) + A(1 - k)]]H(k) - (1 + k)^2}{b'(a - bc)(1 + k)^2}
\end{align*}
$$

Correspondingly, the optimal base salary is

$$
\alpha^*(k) = \pi + \frac{A(1 - k)^2\beta^2(k) - b'((a - bc) + k(1 + k))\beta^*(k)}{2b'} + \int_{k}^{K} \frac{(1 + \tau) + A(1 - \tau)}{b'} \beta^2(\tau) d\tau
$$

The sales agent’s optimal certainty equivalence is

$$
CE^*_S(k) = \pi + \int_{k}^{K} \frac{(1 + \tau) + A(1 - \tau)}{b'} \beta^2(\tau) d\tau
$$

(25)
With the optimal menu contracts, the manufacturer receives an expected profit $E_k^*(\Pi_M)$, where

$$
P_M = \frac{b'(a - bc)^2[(1 + k)^2 + A(1 - k)^2 + 2[(1 + k) + A(1 - k)]H(k)]}{4bb'[(1 + k)^2 + A(1 - k)^2 + 2[(1 + k) + A(1 - k)]H(k)] - 2(1 + k)^2}$$

$$-\pi - \frac{\sigma(c' - s)}{\sqrt{2\pi}} \exp\left(-\frac{(\Phi^{-1}(\frac{c - c'}{\sigma}))^2}{2}\right)$$

where $a - bc > 0$, $2bb' - 1 > 0$, $A = b'\rho\sigma^2$, $H(k) = (1 - G(k))/g(k)$, $E_k(.)$ is the mathematical expectation, $\Phi^{-1}(.)$ is the inverse distribute function for the standard normal distribution $N(0, 1)$.

From Proposition 2, we know that the sales agent’s sales effort $e^*(k)$, the manufacturer’s optimal expected profit $E_k^*(\Pi_M)$, the commission rate $\beta^*(k)$, joint pricing $p^*(k)$ and the production quantity $Q^*(k)$ decisions are all decreasing in $A$, they are all decreasing in the cost parameter $b'$, risk averse $\rho$ and the demand variance $\sigma^2$, which is consistent with the rational case in Proposition 1. The sales agent obtains additional information rent $\int_k^\infty \frac{1}{\sigma^2} \frac{(1 + \tau + A(1 - \tau)\beta^2(\tau))d\tau}{bc}$ due to the private information, rather than the reservation profit $\pi$.

Eq.(24) indicates that the unit net salvage value $s$ and unit emergency production cost $c'$ only affect the manufacturer’s production quantity $Q^*(k)$ and her expected profit $E_k^*(\Pi_M)$. It is easy to verify that the manufacturer’s expected profit $E_k^*(\Pi_M)$ is strictly increasing in $c$ and $s'$, which is consistent with Eq.(11).

6. Analytical results and comparison. Comparing the optimal decisions under asymmetric overconfidence information and symmetric case, we have the following results.

**Corollary 1.** Sales agent’s sales effort $e^*$ and $e^*(k)$, the manufacturer’s price $p^*$ and $p^*(k)$, the production quantity $Q^*$ and $Q^*(k)$, and the corresponding expected profits $E_k^*(\pi_M)$ and $E_k^*(\Pi_M)$, the sales agent’s optimal certainty equivalence $CE_k^*(k)$ are all increasing in $k$.

Corollary 1 shows that, with asymmetric overconfidence information, the sales agent’s sales effort, the manufacturer’s joint decision on production, and pricing and her income all deviate far away from the rational scenarios.

When $k$ is the sales agent’s private information, his optimal certainty equivalence $CE_k^*(k)$ is increasing in $k$. While $k$ is common knowledge of both sides, the sales agent only obtains the reservation profit $\pi$ (not decided by $k$). Corollary 1 implies that the increasing of sales agent’s overconfidence, means Pareto improvement of the income of both sides under asymmetric overconfidence information.

Corollary 1 also indicates that whether the manufacturer knows the sales agent’s overconfidence information, the increasing of the sales agent’s overconfidence means the sales agent is optimistic about job prospects, he is willing to take more risks and exert more sales effort to gain more returns. The increasing effort resulted in the increasing in the overall income, the increased returns are part grabbed by the manufacturer who acts as the game leader, and leads to increasing of the manufacturer’s profit, the increasing of overconfidence means a kind of Pareto improvement to both sides, the results also show that the manufacturer is willing to hire the more overconfident sale agent.
Corollary 2. If $A$ is sufficiently large, the commission rate $\beta^*$ and $\beta^*(k)$ are both increasing in $k$; otherwise, they are decreasing in $k$.

From the proof of Corollary 2 in the Appendix, we know that whether the traditional factor $A = b'\rho\sigma^2$ sufficiently large depends on the sales agent’s overconfidence $k$. Corollary 2 also indicates that the commission rate may not increase monotonically with overconfidence degree.

Observation 2. From Corollary 1, we have the following inequalities hold: $p^* \geq p^*_R$, $Q^* \geq Q^*_R$, $e^* \geq e^*_R$, and $E^*(\pi_M) \geq E^*(\pi_{RM})$. If $A$ is sufficiently large, $\beta^* \geq \beta^*_R$, otherwise, $\beta^* \leq \beta^*_R$ holds.

From Observation 2, we know that the agent’s sales effort, the manufacturer’s expected profit, and the pricing and production decisions more than the rational scenario, while the relation between the commission rate are decided by $A$. Observation 2 implies that the manufacturer prefers to hire the overconfident sales agent.

Corollary 3. When the manufacturer is uncertain about the sales agent’s overconfidence, her expected profit must be less than that in the certain case, which also implies the value of information, i.e.

$$E^*_k(\Pi_M) \leq E^*(\pi_M).$$

Corollary 3 indicates that when the manufacturer faces asymmetric overconfidence information, she screens the hidden information and designs a menu of compensation contracts according to the probability distribution of sales agent’s overconfidence in her mind, there exists bias relative to the incentive contract under the actual overconfidence, the manufacturer’s expected profit declines due to the asymmetric overconfidence information. The conclusion of Corollary 3 implies that the manufacturer should strengthen communication and cooperation with sales agent, to share the overconfidence information in time and improve the incentive strategy according to the obtained information.

Corollary 4. Comparing the optimal decisions of both sides under asymmetric overconfidence information and symmetric scenario, we have the following results hold:

$$\beta^*(k) \leq \beta^*, \quad e^*(k) \leq e^*, \quad Q^*(k) \leq Q^*, \quad p^*(k) \leq p^*.$$

Corollary 4 indicates that if the manufacturer is uncertain about the sales agent’s overconfidence, the commission rate, the sales effort, the production quantity and price will be smaller. Corollary 4 shows that the asymmetric information made the manufacturer relative conservative, and the sales agent with superior information has free riding tendency.

Corollary 5. $CE^*_S(k) \geq CE^*_S = \pi$.

Corollary 5 indicates that, under asymmetric overconfidence information, the sales agent not only obtains the reservation profit $\pi$, but also the additional strictly nonnegative information rent, and the information rent is increasing in $k$ (proved in Corollary 1). When the overconfidence information is common knowledge or rational case, the sales agent only obtains the reservation profit $\pi$.

From the above corollaries, we know that under asymmetric overconfidence information, the manufacturer’s expected profit and the sales agent’s sales effort always
less than the symmetric case, while the sales agent obtain the additional informa-
tion rent. Therefore, mining the sales agent’s overconfidence information is very
valuable.

**Corollary 6.** If $A$ is sufficiently large, facing the menu contracts $(\alpha^*(k), \beta^*(k))$
the manufacturer offered under asymmetric overconfidence information, the sales
agent has no motivation to hide his true overconfidence information, which implies
the menu of contracts are effective.

From the above analysis, we know that contract design problem considering
the sales agent’s private overconfidence information is more consistent with reality
but also more complex, compared to the symmetric overconfidence case and the
rational situation based on traditional principal-agent model, especially considering
the overconfidence as private information exacerbates the complexity among the
decision variables and parameters.

7. **Numerical examples and sensitivity analysis.** In this section, we provide
some numerical examples based on Section 4 and 5. The purpose is two-fold. First,
the examples are used to illustrate the model developed in previous sections to make
further investigation. Secondly, although we obtain the closed-form expression for
solving the optimal decisions, we aim to provide several key managerial insights.
Thus we need to use the numerical examples in investigating the charismatic of
the asymmetric overconfidence, symmetric case is used as the benchmark. In the
following numerical examples, we adopted parameters as follows: $a = 7$, $b = 1$,
$p = 0$, $b' = 1$, $\rho = 1$, $\sigma^2 = 2$, $A = 2$, $s = 1$, $c = 2$, $c' = 3$. In addition, $k$ is uniformly
distributed, and $[k, \bar{k}] = [0, 1]$, thus $H(k) = 1 - k$. 

![Figure 1. The impact of the sales agent’s degree of overconfidence on his optimal effort](image)

From Eq.(13) and Eq.(14), when the sales agent is rational, we obtain $e^*_R = 1$, the
optimal contract is $\alpha^*_R = -1$, $\beta^*_R = 1$; the optimal price and production quantity
are $p^*_R = 5$, and $Q^*_R = 3$, the manufacturer’s corresponding expected profit is
$E^*(\pi_{RM}) = 6.37$.

When the sales agent’s overconfidence information is common knowledge of the
both sides, from Proposition 1, we can derive $e^* = \frac{5}{2[1+2(1-k)^2]/(1+k)^2-1}$, and the
The optimal commission rate versus $k$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{commission_rate.pdf}
\caption{The impact of the sales agent’s degree of overconfidence on the commission rate}
\end{figure}

The optimal pricing decisions versus $k$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{pricing_decisions.pdf}
\caption{The impact of the sales agent’s degree of overconfidence on the price}
\end{figure}

\begin{align*}
\text{When the sales agent’s degree of overconfidence is his private information, from Proposition 2, we obtain the optimal sales effort as } e^*(k) &= \frac{5(1+k)^2}{(1+k)^2+12(1-k)^2+4(1-k)^2}, \\
\text{the optimal menu contracts are } \beta^*(k) &= \frac{5(1+k)}{(1+k)^2+12(1-k)^2+4(1-k)^2}, \\
\text{and } \alpha^*(k) &= (1-k)^2 \beta^*(k) - (k^2 + k + 5) \beta^*(k)/2 + \int_0^{k} (3 - \tau) \beta^*(\tau) d\tau, \\
\text{the optimal price is } p^*(k) &= \frac{5(1+k)^2}{(1+k)^2+12(1-k)^2+4(1-k)^2}. 
\end{align*}
The optimal production decisions versus $k$

\[ Q^*(k) = \frac{5(1+k)^2 + 30(1-k)^2 + 10(1-k^2)}{(1+k)^2 + 12(1-k)^2 + 4(1-k^2)} \]  

The manufacturer’s optimal expected profit $\Pi_M$ satisfies  

\[ \Pi_M = \frac{25(1+k)^2 + 150(1-k)^2 + 50(1-k^2)}{2(1+k)^2 + 24(1-k)^2 + 8(1-k^2)} - 1.128 \]

where $E_k(.)$ is the mathematical expectation, and the sales agent’s certainty equivalence is  

\[ CE_S^*(k) = \int_0^k (3 - \tau) \beta^*2(\tau) d\tau. \]

Other parameters are kept constant, while sales agent’s overconfidence $k$ is in $[0,1)$, we depict the impact of overconfidence $k$ on sales effort $e^*$ and $e^*(k)$, the commission rate $\beta^*$ and $\beta^*(k)$, the price $p^*$ and $p^*(k)$, the production quantity $Q^*$.
and $Q^*(k)$, the manufacturer’s optimal expected profit $E^*(\pi_M)$ and $E^*_k(\Pi_M)$, and the sale agent’s certainty equivalence $CE^*_S(k)$ as summarized in Figs. 1-6 ($k = 0$ implies the sales agent is rational).

From Fig. 1, 3, 4 and 5, we find that the sales effort, joint decision on pricing and production, and the manufacturer’s expected profit are all increasing in $k$, and all relative less under asymmetric overconfidence information. From Fig. 2, we find that in the symmetric case, if $0 \leq k \leq 0.788$, $\beta^*$ is increasing in $k$, and $\beta^*$ is decreasing in $k$ when $0.788 \leq k < 1$, the commission rate could be decreasing in $k$. In other words, when the sales agent becomes more overconfident, he could accept a lower commission rate. From Fig. 6, under asymmetric overconfidence information, the sales agent not only obtains the reservation profit, he also obtains the additional information rent which is increasing in $k$.

8. Conclusions. Firms usually make significant investments in salesforce, according to a Harvard Business Review article (see, Steenburgh and Ahearne (2012)), US companies spend more than 800 billion on salesforce compensation every year. In this article, we study a supply chain with a manufacturer who sells products through an overconfident sales agent, this analysis is distinct from previous studies in that it considers the sales agent’s overconfidence behavior and treats the degree of overconfidence as the private information of the sales agent, and considers two aspects of overconfidence behaviors as overprecision and overplacement simultaneously. The purpose of this paper is to investigate how can a manufacturer make the joint decision on pricing and production quantity and which will affects the market demand, and provide a menu of incentive contract to the sales agent so that they work hard to sell the product and disclose his private overconfidence information. To solve the above problem, the principal-agent model is developed, and the impact of the sales agent’s overconfidence and its asymmetry on the decisions and profit of both sides are depicted, then the results are further compared to the symmetric scenario. Some interesting managerial insights are likewise found, first, the sales

**Figure 6.** The impact of the sales agent’s degree of overconfidence on her expected profits
agent’s effort level, the manufacturer’s joint pricing and production decisions, the corresponding expected profit of both sides are all increasing in the overconfidence level, and deviate far away from the symmetric and rational scenarios, while the commission rate’s monotonicity is not guaranteed. Next, when the manufacturer is uncertain about the sales agent’s overconfidence, her expected profit decreases while the sales agent with private information exerts less effort but obtains additional information rent, which implies the value of information. The results show that the manufacturer should not only focus on hiring the higher overconfidence but also on screening the overconfidence information. Finally a numerical example is provided to demonstrate the validity of the proposed model and results.

Certainly, there are some limitations in this paper. First, we only considered the decision setting in which the sales agent has overconfident behaviors, and ignore the overconfident manufacturer. Second, in our paper, a single parameter is used to depict the overconfident behaviors and asymmetric information, the differences between the measure of overprecision and overplacement is also omitted. Finally, only the traditional linear incentive contract is considered, the quota-based compensation plan should likewise be investigated. As such, in future research, we will investigate quota-based compensation plan and simultaneously consider the overconfident behaviors of both sides, and reasonable overconfidence measure. Future work also includes extending the primal model to other behavioral economics context, behavioral operation research especially the fairness/equity supported strongly by experiments.

Acknowledgments. The authors are grateful to the two anonymous referees and editor-in-chief whose helpful suggestions have led to much improvement of the paper. The research described in this paper was substantially supported by the National Science Foundation for Distinguished Young Scholars of China under Grant no. 71325002; the Fundamental Research Funds for the Central Universities under grant No. 2015QNB05.

Appendix.

Proof of Proposition 1. To substitute the Eq. (2) into the IR constraint (5), the IR condition is binding at optimality

$$CE_S = \alpha + \beta(a - bp) + \frac{(1 + k)^2 - A(1 - k)^2}{2b'} = \pi$$

We can get

$$\alpha = \pi - \beta(a - bp) - \frac{(1+k)^2-A(1-k)^2}{2b'},$$

to substitute the Eq. (2) and the above $\alpha$ into the optimal problem (8), we can get the equivalence problem as

$$\max_{\beta,p,Q} \mathbb{E}(\pi_M) = p[a - bp + (1 + k)\beta/b'] - cQ$$

$$-\pi - \frac{(1 + k)^2 + A(1 - k)^2}{2b'} \beta^2 + \mathbb{E}[s(Q - X)^+ - c'(X - Q)^+]$$

$$= p[a - bp + (1 + k)\beta/b'] - cQ - \pi - \frac{(1 + k)^2 + A(1 - k)^2}{2b'} \beta^2$$

$$+ s \int_{-\infty}^{Q-a+bp-(1+k)\beta/b'} [Q - a + bp - (1 + k)\beta/b' - x]dF(x)$$

$$- c' \int_{Q-a+bp-(1+k)\beta/b'}^{\infty} [x - (Q - a + bp - (1 + k)\beta/b')]dF(x)$$
Taking the second-order partial derivatives of $E(\pi_M)$ with respect to $\beta$, $p$ and $Q$

\[
\frac{\partial^2 E(\pi_M)}{\partial Q^2} = (s - c')f(Q - a + bp - (1 + k)\beta/b')
\]
\[
\frac{\partial^2 E(\pi_M)}{\partial Q \partial p} = \frac{\partial^2 E(\pi_M)}{\partial p \partial Q} = b(s - c')f(Q - a + bp - (1 + k)\beta/b')
\]
\[
\frac{\partial^2 E(\pi_M)}{\partial p^2} = \frac{\partial^2 E(\pi_M)}{\partial p \partial \beta} = \frac{(s - c')f(Q - a + bp - (1 + k)\beta/b')(1 + k)}{b'}
\]
\[
\frac{\partial^2 E(\pi_M)}{\partial \beta^2} = \frac{\partial^2 E(\pi_M)}{\partial p \partial \beta} = -2b + b^2(s - c')f(Q - a + bp - (1 + k)\beta/b')
\]
\[
\frac{\partial^2 E(\pi_M)}{\partial \beta^2} = \frac{\partial^2 E(\pi_M)}{\partial p^2} = \frac{\partial^2 E(\pi_M)}{\partial p \partial \beta} = (1 + k)\frac{b(s - c')f(Q - a + bp - (1 + k)\beta/b') + 1}{b'}
\]
\[
\frac{\partial^2 E(\pi_M)}{\partial p \partial \beta} = \frac{\partial^2 E(\pi_M)}{\partial \beta^2} = \frac{(s - c')f(Q - a + bp - (1 + k)\beta/b')(1 + k)^2}{b'^2}
\]
\[
\frac{\partial^2 E(\pi_M)}{\partial \beta^2} = \frac{\partial^2 E(\pi_M)}{\partial p^2} = (1 + k)^2 + A(1 - k)^2
\]

We obtain the Hessian matrix $H_1$ by second-order derivatives, it is easy to verify that, when $a - bc > 0$ and $2bb' - 1 > 0$, $H_1$ is negative definite concavity, $E(\pi_M)$ is strictly jointly concave in $\beta$, $p$ and $Q$, that is, the first-order derivatives are therefore sufficient. Based on the first order necessary condition

\[
\frac{\partial E(\pi_M)}{\partial \beta} = 0, \quad \frac{\partial E(\pi_M)}{\partial p} = 0, \quad \frac{\partial E(\pi_M)}{\partial Q} = 0
\]

We can obtain

\[
\begin{cases}
(p - c')(1 + k) + c'(s - a + bp - (1 + k)b') - (1 + k)^2 + A(1 - k)^2 \beta = 0 \\
a - 2bp + (1 + k)\beta/b' + c'b + b(s - c')F(Q - a + bp - (1 + k)\beta/b') = 0 \\
(s - c')F(Q - a + bp - (1 + k)\beta/b') + c' - c = 0
\end{cases}
\]

We have

\[
\begin{align*}
\beta^* &= \frac{b'(a - bc)(1 + k)}{2bb'[(1 + k)^2 + A(1 - k)^2] - (1 + k)^2} \\
p^* &= \frac{b'(a + bc)[(1 + k)^2 + A(1 - k)^2] - c(1 + k)^2}{2bb'[(1 + k)^2 + A(1 - k)^2] - (1 + k)^2} \\
Q^* &= a - bp^* + \frac{\beta^*(1 + k)}{b'} + E^{-1}(\frac{c'}{c' - s})
\end{align*}
\]

and the optimal effort strategy is given by

\[
e^* = \frac{\beta^*(1 + k)}{b'} = \frac{(a - bc)(1 + k)^2}{2bb'[(1 + k)^2 + A(1 - k)^2] - (1 + k)^2}
\]

From the binding IR constraint (5), we obtain $\alpha^*$, to substitute the above $e^*$, $\alpha^*$, $\beta^*$, $p^*$ and $Q^*$ into the optimal problem (8), we have

\[
E^*(\pi_M) = p^*[a - bp^* + (1 + k)\beta^*/b'] - cQ^* - \frac{(1 + k)^2 + A(1 - k)^2}{2bb'}\beta^2
\]
\[
+ E[s(Q^* - X)^+ - c'(X - Q^*^+)]
\]
\[
= \frac{b'(a - bc)^2[1 + A(1 - k)^2/(1 + k)^2]}{4bb'[(1 + A(1 - k)^2)/(1 + k)^2] - 2} - \frac{(c' - s)}{E^{-1}(\frac{c'}{c' - s})} \int_{F^{-1}(\frac{c'}{c' - s})}^{\infty} x dF(x)
\]
We can get $F^{-1}\left(\frac{\nu}{\nu - k}\right) = \sigma \Phi^{-1}\left(\frac{\nu}{\nu - k}\right)$, because $\theta$ follows the normal distribution $N(0, \sigma^2)$, then
\[
\int_{F^{-1}\left(\frac{\nu}{\nu - k}\right)}^{\infty} x dF(x) = \int_{\nu \Phi^{-1}\left(\frac{\nu}{\nu - k}\right)}^{\infty} x \frac{1}{\sqrt{2\pi} \sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx = \frac{\sigma}{\sqrt{2\pi}} \exp\left(-\frac{(\Phi^{-1}\left(\frac{\nu}{\nu - k}\right))^2}{2}\right)
\]
thus
\[
E^\ast(\pi_M) = \frac{b'(a - be)^2 [1 + A(1 - k)^2/(1 + k)]}{4b'(1 + A(1 - k)^2/(1 + k)^2) - 2} - \frac{\pi - \sigma(c' - s)}{\sqrt{2\pi}} \exp\left(-\frac{(\Phi^{-1}\left(\frac{\nu}{\nu - k}\right))^2}{2}\right)
\]
where $A = b' \rho \sigma^2$, $\Phi(.)$ is the cumulative probability function for the standard normal distribution $N(0, 1)$, and $\Phi^{-1}(.)$ is its inverse function. Therefore, this Proposition is proved.

Proof of Proposition 2. It follows from the first-order necessary condition of the TC constraint (21) that $\frac{\partial \text{CE}_S(k)}{\partial k} = \frac{(1 + k) + A(1 - k)}{b'} \beta^2(k) \geq 0$ for all $k \in [0, 1)$, which implies that $\text{CE}_S(k)$ is increasing in $k$. The IR constraint (22) implies that $\text{CE}_S(\bar{k}) = \min \text{CE}_S(k) = \bar{\pi}$. Consequently,
\[
\text{CE}_S(k) = \text{CE}_S(\bar{k}) + \int_{\bar{k}}^{k} \frac{(1 + \tau) + A(1 - \tau)}{b} \beta^2(\tau) d\tau
\]
and the binding IR constraint leads to
\[
\alpha(k) = \bar{\pi} - \beta(k)(a - bp) + \int_{\bar{k}}^{k} \frac{(1 + \tau) + A(1 - \tau)}{b} \beta^2(\tau) d\tau - \frac{(1 + k)^2 - A(1 - k)^2}{2b} \beta^2(k)
\]
Replace the $\alpha(k)$ in the manufacturer’s objective function (20), we reduce the problem to max$_{(\beta(k), \rho(k), Q(k))} E_k(\Pi_M)$, where $\Pi_M = p[a - bp + (1 + k) \beta(k)/b'] - cQ + E[s(Q - X)^+ c - c'(X - Q)^+]$, therefore
\[
\Pi_M = p[a - bp + (1 + k) \beta(k)/b'] - cQ - \bar{\pi} - \frac{(1 + k)^2 + A(1 - k)^2}{2b'} \beta^2(k) - \int_{\bar{k}}^{k} \frac{(1 + \tau) + A(1 - \tau)}{b'} \beta^2(\tau) d\tau
\]
\[
+ s \int_{-\infty}^{Q - a + bp - (1 + k) \beta(k)/b'} [Q - a + bp - (1 + k) \beta(k)/b' - x] dF(x)
\]
\[
- c' \int_{Q - a + bp - (1 + k) \beta(k)/b'}^{\infty} [x - (Q - a + bp - (1 + k) \beta(k)/b')] dF(x)
\]
Then, due to $G(\bar{k}) = 0$, $G(\bar{k}) = 1$, and to use the routine approach of change of order of integration:
\[
\int_{\bar{k}}^{k} \frac{(1 + \tau) + A(1 - \tau)}{b'} \beta^2(\tau) d\tau dG(k) = \int_{\bar{k}}^{\bar{k}} \frac{(1 + k) + A(1 - k)}{b'} \beta^2(k) \frac{G(k)}{g(k)} dG(k)
\]
we reduce the problem to \(\max_{(\beta(k), p(k), Q(k))} \int_\mathbb{R} R(\beta, p, Q, k) dG(k)\), where
\[
R(\beta, p, Q, k) = p[a - bp + (1 + k)\beta(k)/b'] - cQ - \mathbb{E}
\]
\[
- \frac{(1 + k)^2 + A(1 - k)^2 + 2[(1 + k) + A(1 - k)]H(k)\beta^2(k)}{2b'}
\]
\[
+ c'(Q - a + bp - (1 + k)\beta(k)/b') - s \int_{-\infty}^{\mathbb{R}} dF(x)
\]
\[
+ c' \int_{Q - a + bp - (1 + k)\beta(k)/b'}^{\infty} x dF(x)
\]
\[
+ (s - c')(Q - a + bp - (1 + k)\beta(k)/b')F(Q - a + bp - (1 + k)\beta(k)/b')
\]

Taking the second-order partial derivatives \(R(\beta, p, Q, k)\) with respect to \(\beta, p\) and \(Q\)
\[
\frac{\partial^2 R}{\partial Q^2} = (s - c')f(Q - a + bp - (1 + k)\beta(k)/b')
\]
\[
\frac{\partial^2 R}{\partial Q \partial p} = \frac{\partial^2 R}{\partial p \partial Q} = b(s - c')f(Q - a + bp - (1 + k)\beta(k)/b')
\]
\[
\frac{\partial^2 R}{\partial Q \partial \beta} = \frac{\partial^2 R}{\partial \beta \partial Q} = \frac{(s - c')f(Q - a + bp - (1 + k)\beta(k)/b')}{b'}(1 + k)
\]
\[
\frac{\partial^2 R}{\partial p^2} = -2b + b^2(s - c')f(Q - a + bp - (1 + k)\beta(k)/b')
\]
\[
\frac{\partial^2 R}{\partial p \partial \beta} = \frac{\partial^2 R}{\partial \beta \partial p} = (1 + k)\frac{b(s - c')f(Q - a + bp - (1 + k)\beta(k)/b')}{b'} + 1
\]
\[
\frac{\partial^2 R}{\partial \beta^2} = \frac{(s - c')f(Q - a + bp - (1 + k)\beta(k)/b')}{b'}(1 + k)^2
\]
\[
- \frac{(1 + k)^2 + A(1 - k)^2 + 2[(1 + k) + A(1 - k)]H(k)}{b'}
\]

We obtain the Hessian matrix \(H_2\) by second-order derivatives. Due to \(a - bc > 0\) and \(2bb' - 1 > 0\), \(H_2\) is negative definite concavity, \(R(\beta, p, Q, k)\) is strictly jointly concave in \(\beta, p\) and \(Q\), that is, the first-order derivatives are therefore sufficient. Based on the first order necessary condition
\[
\frac{\partial R}{\partial \beta} = 0, \quad \frac{\partial R}{\partial p} = 0, \quad \frac{\partial R}{\partial Q} = 0
\]

We can obtain
\[
\begin{align*}
\frac{\partial^2 R}{\partial Q \partial \beta} &= \frac{(s - c')(1 + k) + (c' - s)(1 + k)f(Q - a + bp - (1 + k)\beta/k')}{(1 + k)^2 + A(1 - k)^2 + 2[(1 + k) + A(1 - k)]H(k)} - \frac{b^2}{b'}(1 + k)^2 \beta(k) = 0 \\
\frac{\partial^2 R}{\partial p \partial \beta} &= \frac{a - 2bp + (1 + k)\beta/k' + c'b + (s - c')f(Q - a + bp - (1 + k)\beta/k')}{b'}(1 + k)^2 \beta(k) = 0 \\
\frac{\partial^2 R}{\partial \beta^2} &= \frac{(s - c')f(Q - a + bp - (1 + k)\beta/k') + c' - c}{b'} = 0
\end{align*}
\]

hence, the optimal strategies are
\[
\beta^*(k) = \frac{b'(a - bc)(1 + k)}{2bb'[ (1 + k)^2 + A(1 - k)^2 + 2[(1 + k) + A(1 - k)]H(k)] - (1 + k)^2}
\]
Proof of Corollary 1. Firstly, consider the symmetry case, for simplification, we denote $A$ as $B$ in $k$ and $\sigma k$ in $k$ therefor, and decrease in $k$. Then, with the asymmetry $\alpha k$ in $k$, we have:

$$\begin{align*}
E^* &= - \frac{a - bc}{2bb'B - 1}, \\
p^* &= \frac{b'(a + bc)B - c}{2bb'B - 1}, \\
Q^* &= \frac{a - bc}{2} + \frac{e^*(\frac{c'}{c''})}{2} + F^{-1}(\frac{c' - c''}{\sqrt{2\pi}})
\end{align*}$$

We obtain $\alpha k$ by replacing the $e^*(k), \alpha^*(k), p^*(k)$ and $Q^*(k)$ into the above $R(\beta, p, Q, k)$, the maximum objective value $E_k^*(\Pi_M)$ can be calculated. Therefore, this Proposition is proved.

Owing that $e^*(k)$ is increasing in $A$ and decreasing in $k$ for all $k$ in $[0, 1]$, from Eq. (11), we have:

$$\begin{align*}
e^* &= \frac{a - bc}{2bb'B - 1}, \\
p^* &= \frac{b'(a + bc)B - c}{2bb'B - 1}, \\
Q^* &= \frac{a - bc}{2} + \frac{e^*(\frac{c'}{c''})}{2} + F^{-1}(\frac{c' - c''}{\sqrt{2\pi}})
\end{align*}$$

Therefore, $e^*, p^*$ and $Q^*$ are all decreasing in $B$, that is, they are all increasing in $k$ and decrease in $A$.

Then, with the asymmetry $k$, from Eq. (24), we have:

$$\begin{align*}
e^*(k) &= \frac{a - bc}{2bb'[1 + \frac{A(1-k)^2}{(1+k)^2} + 2(\frac{1}{1+k} + \frac{A(1-k)^2}{(1+k)^2})H(k)] - 1} \\
Q^*(k) &= \frac{a - bc}{2} + \frac{e^*(k)}{2} + F^{-1}(\frac{c' - c''}{\sqrt{2\pi}}), \\
p^*(k) &= \frac{a + bc + e^*(k)}{2b}
\end{align*}$$

From the assumptions in section 3, $H(k)$ is decreasing in $k$, as $A(1-k)^2/(1+k)^2$, $A(1-k)/(1+k)$ are all decreasing in $k$, we have: $1 + A(1-k)^2/(1+k)^2 + 2(1/(1+k) + A(1-k)/H(k)]$ is decreasing in $k$, hence $e^*(k)$ is increasing in $k$. $Q^*(k)$, $p^*(k)$ and $e^*(k)$ have the same monotony, all are increasing in $k$.

Obviously, $E_k^*(\Pi_M) = \frac{b'(a - bc)^2B(k)}{2bb'[B(k) - \frac{A(1-k)^2}{(1+k)^2} + 2(\frac{1}{1+k} + \frac{A(1-k)^2}{(1+k)^2})H(k)] - 1} - \frac{\sigma(c' - s)}{\sqrt{2\pi}} \exp(-\frac{(\Phi^{-1}(\frac{c' - s}{\sqrt{2\pi}}))^2}{2})$, where $B(k) = 1/1 + A(1-k)^2 + 2(1/(1+k) + A(1-k))/H(k)$.

As $C(k) = B(k)/(1+k) = 1 + A(1-k)^2/(1+k)^2 + 2(1/(1+k) + A(1-k)/H(k)) > 1$ is decreasing in $k$, thus $E_k^*(\Pi_M) = \frac{b'(a - bc)^2}{2bb'C(k) - 1} - \frac{\sigma(c' - s)}{\sqrt{2\pi}} \exp(-\frac{(\Phi^{-1}(\frac{c' - s}{\sqrt{2\pi}}))^2}{2})$, and $\frac{\mathrm{d}E_k^*(\Pi_M)}{\mathrm{d}C(k)} = -\frac{b'(a - bc)^2}{2bb'C(k) - 1} < 0$, which implies that $E_k^*(\Pi_M)$ is decreasing in $C(k)$, therefore $E_k^*(\Pi_M)$ is increasing in $k$. 

$$\begin{align*}
p^*(k) &= \frac{b'(a + bc)B - c}{2bb'B - 1}, \\
Q^*(k) &= \frac{a - bc}{2} + \frac{e^*(\frac{c'}{c''})}{2} + F^{-1}(\frac{c' - c''}{\sqrt{2\pi}})
\end{align*}$$
\[
\frac{dCE_S^k(k)}{dk} = \frac{1 + k + A(1 - k)}{b} \beta^2(k) \geq 0, \text{ which implies that } CE_S^k(k) \text{ is increasing in } k.
\]

Therefore, this Corollary is proved.

**Proof of Corollary 2.** Owing that \( \frac{1}{\beta^*} = \frac{2b'[1 + k + A(1 - k)^2/(1 + k)] - (1 + k)}{b'(a - bc)} \), we have

\[
\frac{d(1/\beta^*)}{dk} = \frac{2bb'[1 + A(3 + k)(k - 1)/(1 + k)^2] - 1}{b'(a - bc)}
\]

if \( A > \frac{(1 + k)^2}{3k(1 + k)} \frac{b' - 1}{2bb'} > 0 \), \( \frac{d(1/\beta^*)}{dk} < 0 \), \( 1/\beta^* \) is decreasing in \( k \), i.e. \( \beta^* \) is increasing in \( k \), and vice versa, where \( A = \beta^2 \).

Similarly, \( \frac{1}{\beta^*_R} = \frac{2bb'[1 + k + A(1 - k)^2/(1 + k)] + 2[1 + A(1 - k)/(1 + k)]H(k) - (1 + k)}{b'(a - bc)} \), we have

\[
\frac{d(1/\beta^*_R(k))}{dk} = \frac{2bb'[1 + k + A(3 + k)(k - 1)/(1 + k)^2] + 2H'(k) + \frac{2AH'(k)(1 - k)}{(1 + k)(1 + k)} - \frac{4AH(k)}{(1 + k)^2} - 1}{b'(a - bc)}
\]

if \( A > \frac{(1 + k)^2}{3k(1 + k)} \frac{1 + 2H'(k) - 1/2bb'}{2bb'(1 - k)^2 + (3 + k)(1 - k)}(H'(k) < 0) \), \( \frac{d(1/\beta^*_R(k))}{dk} < 0 \), i.e. \( 1/\beta^*_R(k) \) is decreasing in \( k \), while \( \beta^*_R(k) \) is increasing in \( k \), and vice versa. Therefore, this Corollary is proved.

**Proof of Observation 2.** From the Proof of Corollary 1, \( e^*, p^*, E^*(\pi_M) \) and \( Q^* \) are all increasing in \( k \), if \( k = 0 \), they are reduced to \( e^*_R \), \( p^*_R \), \( E^*(\pi_{RM}) \) and \( Q^*_R \), respectively. Thus \( e^* \geq e^*_R \), \( p^* \geq p^*_R \), \( E^*(\pi_M) \geq E^*(\pi_{RM}) \) and \( Q^* \geq Q^*_R \) hold. Then, we compare \( \beta^* \) and \( \beta^*_R \), due to

\[
\beta^* = \frac{b'(a - bc)}{2bb'[1 + k + A(1 - k)^2/(1 + k)] - (1 + k)}, \quad \beta^*_R = \frac{b'(a - bc)}{2bb'(1 + A) - 1}
\]

so if \( 2bb'[1 + k + A(1 - k)^2/(1 + k)] - (1 + k) \leq 2bb'(1 + A) - 1 \), i.e. \( A \geq \frac{1 + k}{\frac{3k}{3k} - 2bb'} - 1 \), \( \beta^* \geq \beta^*_R \), and vice versa. Therefore, this Observation is proved.

**Proof of Corollary 3.** It is easy to verify that \( \frac{dE^*_S(\Pi_M(k))}{dH(k)} = \frac{-2b'(a - bc)(1 + k)^2}{2bb'(1 + k)^2(1 + k)} < 0 \), and \( 2[(1 + k) + A(1 - k)]H(k) \geq 0 \), holds, when \( [(1 + k) + A(1 - k)]H(k) = 0 \), \( E^*_S(\Pi_M) = E^*(\pi_M) \), therefore \( E^*_S(\Pi_M) \leq E^*(\pi_M) \) holds, where \( H(k) = (1 - G(k))/g(k) \), \( B(k) = (1 + k)^2 + A(1 - k)^2 + 2[(1 + k) + A(1 - k)]H(k) \). Therefore, this Corollary is proved.

**Proof of Corollary 4.** Obviously, if \( H(k) = (1 - G(k))/g(k) = 0 \), \( \beta^*(k) \) degrades into \( \beta^* \), thus \( \beta^*(k) \leq \beta^* \) holds, then \( (1 + k)\beta^*(k)/b' \leq (1 + k)\beta^*/b' \), i.e. \( e^*(k) \leq e^* \) also holds, from

\[
Q^*(k) = \frac{a - bc}{2} + \frac{e^*(k)}{2} + F^{-1}(\frac{c - c^*}{c^* - s}), \quad p^*(k) = \frac{a + bc + e^*(k)}{2b}
\]

we have \( Q^*(k) \leq Q^* \) and \( p^*(k) \leq p^* \). Therefore, this Corollary is proved.

**Proof of Corollary 6.** For the sales agent with private overconfidence \( k \), from the menu contracts in Proposition 2, her certainty equivalence is

\[
CE_S^k(k) = \pi + \int_k^k \frac{(1 + \tau) + A(1 - \tau)}{b} \beta^2(\tau) d\tau
\]
if she chooses contract \((\alpha(k'), \beta(k'))\), her expected utility modified by the certainty equivalence principle is

\[
CE_S(k') = \alpha(k') + \beta(k')(a - bp(k')) + (1 + k)e + b'e^2/2 - \rho \sigma^2\beta^2(k')(1 - k)^2/2
\]

we obtain

\[
\Delta CE = CE_S(k) - CE_S(k') = \int_{k'}^{k} \left( 1 + \tau \right) + A \left( 1 - \tau \right) \frac{(\beta^2(\tau) - \beta^2(k'))}{b} \, d\tau
\]

If \(A\) is sufficiently large, \(\beta^2(k)\) is increasing in \(k\) (see Proof of Proposition 1), thus, either \(k' < \tau < k\) or \(k < \tau < k'\), \(\Delta CE \geq 0\) holds, which implies that the sales agent does not pretend to hidden the actual overconfidence information. Therefore, this Corollary is proved.

REFERENCES

[1] A. Basu, R. Lal, V. Srinivasan and R. Staelin, [Salesforce-compensation plans: An agency theoretic perspective], Marketing Science, 4 (1985), 267–291.
[2] E. Bendoly, R. Croson, P. Goncalves and K. Schultz, [Bodies of knowledge for research in behavioral operations], Production and Operations Management, 19 (2010), 434–452.
[3] L. W. Busenitz and J. B. Barney, Differences between entrepreneurs and managers in large organizations: Biases and heuristics in strategic decision-making, Journal of Business Venturing, 12 (1997), 9–30.
[4] O. Caliskan-Demirag, Y. F. Chen and J. B. Li, Channel coordination under fairness concerns and nonlinear demand, European Journal of Operational Research, 207 (2010), 1321–1326.
[5] E. Cao, Y. Ma, C. Wan and M. Lai, Contracting with asymmetric cost information in a dual-channel supply chain, Operations Research Letters, 41 (2013), 410–414.
[6] X. Chao, B. Yang and Y. Xu, Dynamic inventory and pricing policy in a capacitated stochastic inventory system with fixed ordering cost, Operations Research Letters, 40 (2012), 99–107.
[7] F. Chen, [Salesforce incentives, market information and production/inventory planning], Management Science, 51 (2005), 60–75.
[8] Y. J. Chen, S. Shum and W. Q. Xiao, Should an OEM retain component procurement when the CM produces competing products, Production and Operations Management, 21 (2012), 907–922.
[9] C. H. Chiu, T. M. Choi and C. S. Tang, Price, rebate, and returns supply contracts for coordinating supply chains with price dependent demand, Production and Operations Management, 20 (2011), 81–91.
[10] P. S. Chow, Y. Wang, T. M. Choi and B. Shen, An experimental study on the effects of minimum profit share on supply chains with markdown contracts: risk and profit analysis, Omega, 57 (2015), 85–97.
[11] A. T. Coughlan, [Salesforce Compensation: A Review of MS/OR Advances], In: Eliashberg, J., G.L. Lilien (eds.), Handbook in Operations Research and Management Science, 1993.
[12] T. H. Cui, J. S. Raju and Z. J. Zhang, Fairness and channel coordination, Management Science, 53 (2007), 1303–1314.
[13] Y. Dai and X. L. Chao, [Salesforce contract design and inventory planning with asymmetric risk-averse sales agents], Operations Research Letters, 41 (2013), 86–91.
[14] F. Engelmaier and A. Wambach A, Optimal incentive contracts under inequity aversion, Games and Economic Behavior, 69 (2010), 312–328.
[15] B. Fernando, J. Song and X. Zheng, Free riding in a multi-channel supply chain, Naval Research Logistics, 56 (2009), 745–765.
[16] D. Garcia, F. Sangiorgi and B. Urosevic, Overconfidence and market efficiency with heterogeneous agents, Economic Theory, 30 (2007), 313–336.
[17] J. Gonik, The salesmen’s bonuses to their forecasts, Harvard Business Review, 56 (1978), 116–123.
[18] A. Hung and C. Plott, Information cascades: Replication and an extension to majority rule and conformity rewarding institutions, American Economic Review, 91 (2001), 1508–1520.
[19] T. H. Ho, X. Su and Y. Wu, Distributional and peer-induced fairness in supply chain contract design, *Historical Journal of Film Radio & Television*, 23 (2014), 161–175.

[20] D. Kahneman and A. Tversky, Prospect theory: An analysis of decision under risk, *Econometrica*, 47 (1979), 263–291.

[21] E. Katok and V. Pavlov, Fairness in supply chain contracts: A laboratory study, *Journal of Operations Management*, 31 (2013), 129–137.

[22] (MR2573481) M. Kaya and O. Ozer, Quality risk in outsourcing: Noncontractible product quality and private quality cost information, *Naval Research Logistics*, 56 (2009), 669–685.

[23] K. L. Keiber, *Managerial Compensation Contracts and Overconfidence*, Working paper, WHU Otto Beisheim Graduate School of Management, Vallendar, 2002.

[24] L. C. Kung and Y. J. Chen, Monitoring the market or the salesperson? The value of information in a multilayer supply chain, *Naval Research Logistics*, 58 (2011), 4208–4218.

[25] C. H. Loch, *Behavioral Operations Management*, Hanover: Now Publishers Inc, 2007.

[26] S. Pal, S. S. Sana and K. Chaudhuri, Joint pricing and ordering policy for two echelon imperfect production inventory model with two cycles, *International Journal of Production Economics*, 155 (2014), 229–238.

[27] V. Pavlov and E. Katok, *Fairness and Coordination Failures in Supply Chain Contracts* Working paper, Smeal College of Business, Pennsylvania State University, Pennsylvania, 2009.

[28] Y. Qin, J. Wang and C. Wei, Joint pricing and inventory control for fresh produce and foods with quality and physical quantity deteriorating simultaneously, *Production and Operations Management*, 20 (2011), 92–115.

[29] L. E. De la Rosa, Overconfidence and moral hazard, *Games and Economic Behavior*, 73 (2011), 429–451.

[30] Y. Ren and R. Croson, Overconfidence in newsvendor orders: An experimental study, *Management Science*, 59 (2013), 2502–2517.

[31] J. E. Russo and P. J. H. Schoemaker, Managing Overconfidence, *Sloan Management Review*, 33 (1992), 7–17.

[32] S. Saghaian and X. Chao, The impact of operational decisions on the design of salesforce incentives, *Naval Research Logistics*, 61 (2014), 320–340.

[33] A. Sandroni and F. Squintani, Overconfidence and asymmetric information: The case of insurance, *Journal of Economic Behavior & Organization*, 90 (2013), 149–165.

[34] T. Steenburgh and M. Ahearne, Motivating salespeople: what really works, *Harvard Business Review*, 90 (2012), 70–75.

[35] T. A. Taylor, Supply chain coordination under channel rebates with sales effort effects, *Management Science*, 48 (2002), 992–1007.

[36] T. A. Taylor, Sale Timing in a supply chain: When to sell to the retailer, *Manufacturing & Service Operations Management*, 8 (2006), 23–42.

[37] H. Xu, N. Shi, S. Ma and K. K. Lai, Contracting with an urgent supplier under cost information asymmetry, *European Journal of Operational Research*, 206 (2010), 374–383.
[48] J. Wei, K. Govindan, Y. Li and J. Zhao, Pricing and collecting decisions in a closed-loop supply chain with symmetric and asymmetric information, Computers & Operations Research, 54 (2015), 257–265.

[49] P. Zhang, Y. Xiong, Z. Xiong and W. Yan, Designing contracts for a closed-loop supply chain under information asymmetry, Operations Research Letters, 42 (2014), 150–155.

[50] A. A. Zoltners, P. Sinha and S. E. Lorimer, Sales force effectiveness: A framework for researchers and practitioners, Journal of Personal Selling & Sales Management, 28 (2008), 115–131.

Received September 2015; 1st revision January 2016; final revision June 2016.

E-mail address: chenkegui@cumt.edu.cn
E-mail address: wangxinyu@cumt.edu.cn
E-mail address: mhuang@mail.neu.edu.cn
E-mail address: wching@hku.hk