On the Ground State of QCD inside a Compact Stellar Object

Roberto Casalbuoni

Department of Physics, University of Florence, Florence, Italy and
Sezione INFN, Florence, Italy
E-mail: casalbuoni@fi.infn.it
www.theory.fi.infn.it/casalbuoni/

We describe the effects of the strange quark mass and of the color and electric neutrality on the superconducting phases of QCD.

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1. Introduction

It is now a well established fact that at zero temperature and sufficiently high densities quark matter is a color superconductor\textsuperscript{1,2}. The study starting from first principles was done in Refs.\textsuperscript{3–5}. At baryon chemical potentials much higher than the masses of the quarks $u$, $d$ and $s$, the favored state is the so-called Color-Flavor-Locking (CFL) state, whereas at lower values, when the strange quark decouples, the relevant phase is called two-flavor color superconducting (2SC).

An interesting possibility is that in the interior of compact stellar objects (CSO) some color superconducting phase may exist. In fact the central densities for these stars could be up to $10^{15}$ g/cm$^3$, whereas the temperature is of the order of tens of keV. However the usual assumptions leading to prove that for three flavors the favored state is CFL should now be reviewed. Matter inside a CSO should be electrically neutral and should not carry color. Also conditions for $\beta$-equilibrium should be fulfilled. As far as color is concerned, it is possible to impose a simpler condition, that is color neutrality, since in Ref.\textsuperscript{6} it has been shown that there is no free energy cost in projecting color singlet states out of color neutral ones. Furthermore one has to take into account that at the interesting density the mass of the strange quark is a relevant parameter. All these effects, the mass of the strange quark, $\beta$-equilibrium and color and electric neutrality, imply that
the radii of the Fermi spheres of quarks that would pair are not the same. This difference in radius, as we shall see, is going to create a problem with the usual BCS pairing. Let us start from the mass effects. Suppose to have two fermions of masses \( m_1 = M \) and \( m_2 = 0 \) at the same chemical potential \( \mu \). The corresponding Fermi momenta are \( p_{F_1} = \sqrt{\mu^2 - M^2} \) and \( p_{F_2} = \mu \). We see that the radius of the Fermi sphere of the massive fermion is smaller than the one of the massless particle. If we assume \( M \ll \mu \) the massive particle has an effective chemical potential \( \mu_{\text{eff}} = \sqrt{\mu^2 - M^2} \approx \mu - M^2/2\mu \) and the mismatch between the two Fermi spheres is given by

\[
\delta \mu \approx \frac{M^2}{2\mu}
\]

This shows that the quantity \( M^2/(2\mu) \) behaves as a chemical potential. Therefore for \( M \ll \mu \) the mass effects can be taken into account through the introduction of the mismatch between the chemical potentials of the two fermions given by eq. (1). This is the way that we will follow in our study.

Now let us discuss \( \beta \)-equilibrium. If electrons are present (as generally required by electrical neutrality) chemical potentials of quarks of different electric charge are different. In fact, when at the equilibrium for \( d \rightarrow u e \nu \), we have

\[
\mu_d - \mu_u = \mu_e
\]

From this condition it follows that for a quark of charge \( Q_i \) the chemical potential \( \mu_i \) is given by

\[
\mu_i = \mu + Q_i \mu_Q
\]

where \( \mu_Q \) is the chemical potential associated to the electric charge. Therefore

\[
\mu_e = -\mu_Q
\]

Notice also that \( \mu_e \) is not a free parameter since it is determined by the neutrality condition

\[
Q = -\frac{\partial \Omega}{\partial \mu_e} = 0
\]

At the same time the chemical potentials associated to the color generators \( T_3 \) and \( T_8 \) are determined by the color neutrality conditions

\[
\frac{\partial \Omega}{\partial \mu_3} = \frac{\partial \Omega}{\partial \mu_8} = 0
\]
We see that in general there is a mismatch between the quarks that should pair according to the BCS mechanism at $\delta \mu = 0$. Increasing the mismatch has the effect of destroying the BCS phase and the system either goes into the normal phase or to some different phase. In the next Sections we will explore some of these possible alternatives.

2. Neutrality and $\beta$-equilibrium

Just as a very simple example of the effect of the neutrality and $\beta$-equilibrium conditions, let us consider three non interacting quarks, $u$, $d$ and $s$. The $\beta$-equilibrium requires

$$\mu_{d,s} = \mu_u + \mu_e$$

(7)

The chemical potentials of the single species in term of the baryon chemical potential, $\bar{\mu}$, and of the charge chemical potential, $\mu_Q = -\mu_e$, are therefore

$$\mu_u = \bar{\mu} - \frac{2}{3} \mu_e, \quad \mu_d = \mu_s = \bar{\mu} + \frac{1}{3} \mu_e$$

(8)

The numerical densities of different quarks are given by

$$N_{u,d} = \frac{\mu_{u,d}^3}{\pi^2}, \quad N_s = \frac{(\mu_s^2 - M_s^2)^{3/2}}{\pi^2}, \quad N_e = \frac{\mu_e^3}{3\pi^2}$$

(9)

On the other hand the neutrality condition requires

$$\frac{2}{3} N_u - \frac{1}{3} N_d - \frac{1}{3} N_s - N_e = 0$$

(10)

\[\text{Fig. 1. The Fermi spheres for three non interacting quarks, } u, \text{ } d \text{ and } s \text{ by taking into account the mass of the strange quark (see text).}\]
If the strange quark mass is neglected the previous equation has the simple solution

$$N_u = N_d = N_s, \quad N_e = 0$$ (11)

In this case the Fermi spheres of the three quarks have the same radius (remember that for a single fermion the numerical density is given by $N = p_F^3/(3\pi^2)$). However if we take into account $M_s \neq 0$ at the lowest order in $M_s/\mu$ we get

$$\mu_e \approx M_s^2, \quad p_F^d - p_F^u \approx p_F^u - p_F^s \approx \mu_e$$ (12)

The result is shown in Fig. 1. It can also be shown that in the normal phase the chemical potentials associated to the color charges $T_3$ and $T_8$ vanish. We will make use of these results when we will discuss the LOFF phase.

3. Gapless quasi-fermions

When a mismatch is present, the spectrum of the quasi-particles is modified as follows

$$E_{\delta\mu=0} = \sqrt{(p - \mu)^2 + \Delta^2} \rightarrow E_{\delta\mu} = |\delta\mu \pm \sqrt{(p - \mu)^2 + \Delta^2}|$$ (13)

Therefore for $|\delta\mu| < \Delta$ we have gapped quasi-particles with gaps $\Delta \pm \delta\mu$. However, for $|\delta\mu| = \Delta$ a gapless mode appears and from this point on there are regions of the phase space which do not contribute to the gap equation (blocking region).

The gapless modes are characterized by

$$E(p) = 0 \Rightarrow p = \mu \pm \sqrt{\delta\mu^2 - \Delta^2}$$ (14)

Since the energy cost for pairing two fermions belonging to Fermi spheres with mismatch $\delta\mu$ is $2\delta\mu$ and the energy gained in pairing is $2\Delta$, we see that the fermions begin to unpair for $2\delta\mu \geq 2\Delta$. These considerations will be relevant for the study of the gapless phases when neutrality is required.

4. The gCFL phase

The gCFL phase is a generalization of the CFL phase which has been studied both at $T = 0.8$ and $T \neq 0$. The condensate has now the form

$$\langle 0|\psi_{aL}^\alpha\psi_{bL}^\beta|0 \rangle = \Delta_1 \epsilon^{\alpha\beta1} \epsilon_{a1} + \Delta_2 \epsilon^{\alpha\beta2} \epsilon_{a2} + \Delta_3 \epsilon^{\alpha\beta3} \epsilon_{a3}$$ (15)

The CFL phase corresponds to all the three gaps $\Delta_i$ being equal. Varying the gaps one gets many different phases. In particular we will be interested
to the CFL, to the g2SC characterized by $\Delta_3 \neq 0$ and $\Delta_1 = \Delta_2 = 0$ and to the gCFL phase with $\Delta_3 > \Delta_2 > \Delta_1$. Notice that in the g2SC phase defined here the strange quark is present but unpaired.

In flavor space the gaps $\Delta_i$ correspond to the following pairings in flavor

$$
\Delta_1 \Rightarrow ds, \quad \Delta_2 \Rightarrow us, \quad \Delta_3 \Rightarrow ud
$$

(16)

The mass of the strange quark is taken into account by shifting all the chemical potentials involving the strange quark as follows: $\mu_{as} \rightarrow \mu_{as} - M_s^2/2\mu$. It has also been shown in ref.10 that color and electric neutrality in CFL require

$$
\mu_8 = -\frac{M_s^2}{2\mu}, \quad \mu_e = \mu_3 = 0
$$

(17)

At the same time the various mismatches are given by

$$
\delta\mu_{bd-gs} = \frac{M_s^2}{2\mu}, \quad \delta\mu_{rs-gu} = \mu_e = 0, \quad \delta\mu_{rs-bu} = \mu_e - \frac{M_s^2}{2\mu}
$$

(18)

It turns out that in the gCFL the electron density is different from zero and, as a consequence, the mismatch between the quarks $d$ and $s$ is the first one to give rise to the unpairing of the corresponding quarks. This unpairing is expected to occur for

$$
2\frac{M_s^2}{2\mu} > 2\Delta \Rightarrow \frac{M_s^2}{\mu} > 2\Delta
$$

(19)

This has been substantiated in8 by a calculation in the NJL model based on one gluon-exchange. The authors assume for their calculation a chemical potential, $\mu = 500 \text{ MeV}$ and a CFL gap given by $\Delta = 25 \text{ MeV}$. The transition from the CFL phase, where all gaps are equal, to the gapless phase occurs roughly at $M_s^2/\mu = 2\Delta$. In Fig. 2 we show the free energy of the various phases with reference to the normal phase. The CFL phase is the stable one up to $M_s^2/\mu \approx 2\Delta$. Then the gCFL phase takes over up to about 130 MeV, where the system goes to the normal phase. Notice that except in a very tiny region around this point, the CFL and gCFL phases dominate over the corresponding 2SC and g2SC ones. The thin short-dashed line represents the free energy of the CFL phase up to the point where it becomes equal to the free-energy of the normal phase. This happens for $M_s^2/\mu \approx 4\Delta$.

Although the gCFL phase appears to be energetically favored it cannot be the real ground state. In fact, it has been shown in11,12 that in this phase there is a chromomagnetic instability. This instability manifests itself in the masses of the gluons 1, 2, 3, 8 becoming pure imaginary at the transition
CFL-gCFL. An analogous instability (relative to the gluons 4, 5, 6, 7, 8) occurs in the g2SC phase and it seems to be related to the gapless modes present in homogeneous phases, as conjectured in Ref. 17.

5. Possible solutions of the problem of the chromomagnetic instability

There have been various proposals to solve the problem of the chromomagnetic instability. We will shortly review these attempts before discussing the proposal that at the moment seems to be the favored one, that is the one corresponding to the LOFF phase (see next Section):

- **Gluon condensation.** If one assumes artificially that the expectation values of $A_{\mu3}$ and $A_{\mu8}$ are not zero, and of the order of 10 MeV, the instability goes away. This argument has been done more accurate for the g2SC phase in Refs. 18–21, where it has been considered a model exhibiting chromo-magnetic condensation. It turns out that the rotational symmetry is broken and this makes some connection with the LOFF phase. At the moment these models have not been extended to the three flavor case.

- **CFL-K$^0$ phase.** If the mismatch is not too large (meaning $\delta\mu/\mu \ll 1$) the CFL pattern can be modified by a flavor rotation of the condensate. This is equivalent to have a condensate of kaons. The transition to this phase occurs roughly for a strange quark mass...
satisfying $M_s > m^{1/3} \Delta^{2/3}$, with $m$ the light quark mass and $\Delta$ the CFL condensate. Also this phase exhibits gapless modes and the gluon instability occurs$^{23-25}$. Allowing for a space dependent condensate, a current is generated which eliminates the instability$^{26}$. Also in this case, a space dependent condensation brings a relation to the LOFF phase.

- **Single flavor pairing.** If the stress caused by the mismatch is too big, single flavor pairing could occur. However the gap appears to be too small. It could be important at low chemical potential before the nuclear phase (see, for instance Ref.$^{27}$).

- **Secondary pairing.** The gapless modes could form a secondary gap, but here too the gap is far too small$^{28,29}$.

- **Mixed phases.** The possibility of mixed phases both of nuclear and quark matter$^{30}$ as well as mixed phases of different Color Superconducting$^{31,32}$ phases has been considered. However all these possibilities are either unstable or energetically disfavored.

- **LOFF phase.** In Ref.$^{33}$ it has been shown that the chromomagnetic instability of the g2SC phase is just what is needed in order to make the crystalline, or LOFF phase, energetically favored. Also it turns out that in the LOFF phase there is no chromomagnetic instability although gapless modes are present$^{34}$.

The previous considerations make the LOFF phase worth to be considered and this is what we will do in the next Section.

6. The LOFF Phase

According to the authors of Refs.$^{35,36}$ when fermions belong to different Fermi spheres, they might prefer to pair staying as much as possible close to their own Fermi surface. The total momentum of the pair is not zero, $\vec{p}_1 + \vec{p}_2 = 2\vec{q}$ and, as we shall show, $|\vec{q}|$ is fixed variationally whereas the direction of $\vec{q}$ is chosen spontaneously. Since the total momentum of the pair is not zero the condensate breaks rotational and translational invariance. The simplest form of the condensate compatible with this breaking is just a simple plane wave (more complicated possibilities will be discussed later)

$$\langle \psi(x)\psi(x) \rangle \approx \Delta e^{2i\vec{q} \cdot \vec{x}}$$

(20)

It should also be noticed that the pairs use much less of the Fermi surface than they do in the BCS case. For instance, if both fermions are sitting at their own Fermi surface, they can pair only if they belong to circles fixed
by \( \vec{q} \). More generally there is a quite large region in momentum space (the so called blocking region) which is excluded from pairing. This leads to a condensate generally smaller than the BCS one.

Let us now consider in more detail the LOFF phase (for reviews of this phase see Refs.\textsuperscript{37–40}). For two fermions at different densities we have an extra term in the Hamiltonian which can be written as

\[
H_I = -\delta \mu \sigma_3
\]  

where, in the original LOFF papers\textsuperscript{35,36}, \( \delta \mu \) is proportional to the magnetic field due to the impurities, whereas in the actual case \( \delta \mu = (\mu_1 - \mu_2)/2 \) and \( \sigma_3 \) is a Pauli matrix acting on the two fermion space. According to Refs.\textsuperscript{35,36} this favors the formation of pairs with momenta

\[
\vec{p}_1 = \vec{k} + \vec{q}, \quad \vec{p}_2 = -\vec{k} + \vec{q}
\]  

We will discuss in detail the case of a single plane wave (see eq. (20)). The interaction term of eq. (21) gives rise to a shift in the quasi-particles energy due both to the non-zero momentum of the pair and to the different chemical potentials

\[
E(\vec{p}) - \mu \rightarrow E(\pm \vec{k} + \vec{q}) - \mu \mp \delta \mu \approx E(\vec{p}) \mp \tilde{\mu}
\]  

with

\[
\tilde{\mu} = \delta \mu - \vec{v}_F \cdot \vec{q}
\]  

Notice that the previous dispersion relations show the presence of gapless modes at momenta depending on the angle of \( \vec{v}_F \) with \( \vec{q} \). Here we have assumed \( \delta \mu \ll \mu \) (with \( \mu = (\mu_1 + \mu_2)/2 \)) allowing us to expand \( E \) at the first order in \( \vec{q}/\mu \).

The study of the gap equation shows that increasing \( \delta \mu \) from zero we get first the BCS phase. Then at \( \delta \mu = \delta \mu_1 \) there is a first order transition to the LOFF phase\textsuperscript{35,37}, and at \( \delta \mu = \delta \mu_2 > \delta \mu_1 \) there is a second order phase transition to the normal phase\textsuperscript{35,37}. We start comparing the grand potential in the BCS phase to the one in the normal phase. Their difference is given by (see for example Ref.\textsuperscript{39})

\[
\Omega_{\text{BCS}} - \Omega_{\text{normal}} = -\frac{\vec{v}_F^2}{4\pi^2 v_F} (\Delta_0^2 - 2\delta \mu^2)
\]  

where the first term comes from the energy necessary to the BCS condensation, whereas the last term arises from the grand potential of two free fermions with different chemical potential. We recall also that for massless
fermions $p_F = \mu$ and $v_F = 1$. We have again assumed $\delta \mu \ll \mu$. This implies that there should be a first order phase transition from the BCS to the normal phase at $\delta \mu = \Delta_0/\sqrt{2}$, since the BCS gap does not depend on $\delta \mu$. In order to compare with the LOFF phase one can expand the gap equation around the point $\Delta = 0$ (Ginzburg-Landau expansion) to explore the possibility of a second order phase transition. The result for the free energy is

$$\Omega_{\text{LOFF}} - \Omega_{\text{normal}} \approx -0.44 \rho (\delta \mu - \delta \mu_2)^2$$

At the same time, looking at the minimum in $q$ of the free energy one finds

$$qv_F \approx 1.2 \delta \mu$$

Since we are expanding in $\Delta$, in order to get this result it is enough to minimize the coefficient of $\Delta^2$ in the free-energy (the first term in the Ginzburg-Landau expansion).

We see that in the window between the intersection of the BCS curve and the LOFF curve and $\delta \mu_2$, the LOFF phase is favored. Also at the intersection there is a first order transition between the LOFF and the BCS phase. Furthermore, since $\delta \mu_2$ is very close to $\delta \mu_1$ the intersection point is practically given by $\delta \mu_1$. The window of existence of the LOFF phase $(\delta \mu_1, \delta \mu_2) \approx (0.707, 0.754)\Delta_0$ is rather narrow, but there are indications that considering the realistic case of QCD the window opens up. Such opening occurs also for different crystalline structures than the single plane wave.

7. The LOFF phase with three flavors

In the last Section we would like to illustrate some preliminary result about the LOFF phase with three flavors. This problem has been considered in under various simplifying hypothesis:

- The study has been made in the Ginzburg-Landau approximation.
- Only electrical neutrality has been required and the chemical potentials for the color charges $T_3$ and $T_8$ have been put equal to zero (see later).
- The mass of the strange quark has been introduced as it was done previously for the gCFL phase.
- The study has been restricted to plane waves, assuming the follow-
ing generalization of the gCFL case:

$$\langle \psi^\alpha_{aL}\psi^\beta_{bL} \rangle = \sum_{I=1}^{3} \Delta_I(\vec{x})\epsilon^{\alpha\beta I} \epsilon_{a I b I} , \quad \Delta_I(\vec{x}) = \Delta_I e^{2\vec{q}_I \cdot \vec{x}} \quad (28)$$

- The condensate depends on three momenta, meaning three lengths of the momenta $q_i$ and three angles. In\textsuperscript{44} only four particular geometries have been considered: 1) all the momenta parallel, 2) $\vec{q}_1$ antiparallel to $\vec{q}_2$ and $\vec{q}_3$, 3) $\vec{q}_2$ antiparallel to $\vec{q}_1$ and $\vec{q}_3$, 4) $\vec{q}_3$ antiparallel to $\vec{q}_1$ and $\vec{q}_2$.

The minimization of the free energy with respect to the $|\vec{q}_I|$’s leads to the same result as in eq. (27), $|\vec{q}_I| = 1.2\delta\mu_I$. Let us notice that consistently with the Ginzburg-Landau approximation requiring to be close to the normal phase, we assume $\mu_3 = \mu_8 = 0$ as discussed in Section 2. We remember also that close to the normal phase the Fermi surfaces are given in Fig. 1 and as a consequence at the same order of approximation we expect $\Delta_2 = \Delta_3$ (since $ud$ and $us$ mismatches are equal) and $\Delta_1 = 0$, due to the $sd$ mismatch being the double of the other two. Once we assume $\Delta_1 = 0$ only the two configurations with $q_2$ and $q_3$ parallel or antiparallel remain. However the antiparallel is unlike. In fact, as it can be seen from Fig. 3, in the antiparallel configuration we have two $u$ quarks in the same ring reducing the phase space, and correspondingly the gap, due to the Fermi statistics. This observation is indeed verified by numerical calculations. Then, one has

![Fig. 3. The two Fermi spheres corresponding to $q_2$ (left arrow) and $q_3$ (right arrow) respectively parallel and antiparallel. The pairing rings $du$ and $us$ are shown by thin and thick lines respectively.](image-url)
phase is favored over the normal phase with a gap arriving at almost 0.4 the CFL gap. Of course, it is interesting to compare this result with the gCFL result given in Fig. 2. The comparison is made in Fig. 5. We see that the LOFF phase dominates over gCFL in the interval between 128 MeV and 150 MeV where the transition to the normal phase is located. These results have been confirmed by an exact calculation with respect to the gap (but always at the leading order in the chemical potential), done in Ref. 45. The result found by these authors show that in the range of $M_s$ considered here the Ginzburg-Landau approximation is rather accurate and if any it overestimates the free energy. As a further confirmation of these results, in Ref. 46 we have shown that corrections at the order $1/\mu$ do not modify qualitatively the previous results but rather tend to enlarge the window where LOFF dominates over gCFL.

It has also been shown in Ref. 47 that in the phase studied in this Section the chromo-magnetic instability disappears. Here one has to distinguish the longitudinal and transverse masses of the gluons with respect to the direction of the total momentum of the pair. It results that all these masses are real.

More recently an extension of the simple ansatz of a single plane wave for each gap, as considered in this Section, has been made in Ref. 48. The simple ansatz of eq. (28) has been generalized in the following way

$$
\langle ud \rangle \approx \Delta \sum_a e^{2i\mathbf{q}_a \cdot \mathbf{r}}, \quad \langle us \rangle \approx \Delta \sum_a e^{2i\mathbf{q}_a \cdot \mathbf{r}}, \quad \langle ds \rangle \approx 0
$$

(29)
Fig. 5. Comparison of the free energy of the various phases already considered in Fig. 2 (same notations as here) with the LOFF phase with three flavors.

with the index $a$ running from 1 up to a maximum value of 8. In practice

![Diagram](image)

Fig. 6. Comparison of the free energy of the various phases already considered in Figs. 2 and 5 (same notations as here) with various crystalline structures in the three flavor case.

for any choice of the range of the index $a$ one gets a particular crystalline structure defined by the vectors $\mathbf{q}^a$ pointing at the vertices of the crystal. In ref.\textsuperscript{48} the study has been extended to 11 crystals. The favored structures are the so called CubeX and 2Cube45z. The CubeX is a cube characterized by 4 vectors $\mathbf{q}_2^a$ and 4 $\mathbf{q}_3^a$. Each set of vectors lies in a plane and the two planes
cut at 90 degrees forming a cube. In the 2Cube45z, there are 8 vectors in each set defining two cubes which are rotated one with respect to the other of 45 degrees along the z axis. The free energies for these two crystals are compared with the case of a single plane wave for each pairing (called in this context 2PW) in Fig. 6. We see that the CubeX and the 2Cube45z take over the gCFL phase in almost all the relevant range of $M_2^2/\mu$. Taking into account that this calculation has been made in the Ginzburg-Landau approximation it looks plausible that these two phases are the favorite ones up to the CFL phase.

8. Conclusions

As we have seen there have been numerous attempts in trying to determine the fundamental state of QCD under realistic conditions existing inside a compact stellar objects, that is to say, neutrality in color and electric charge, $\beta$-equilibrium and a non vanishing strange quark mass. Many competing phases have been found. Most of them have fermionic gapless modes. However, gapless modes in presence of a homogeneous condensate seem to lead unavoidably to a chromo-magnetic instability and it seems necessary to consider space dependent condensates. In this respect the LOFF phase, where the space dependence comes about in relation to the non zero total momentum of the pair, seems to be a natural candidate. This phase in the presence of three flavors has been recently considered\cite{44,45,48}. It has been found that there are no chromo-magnetic instabilities\cite{47} and that energetically it is favored almost up to the CFL phase. However, considering the approximations involved in these calculations, before to draw sounded conclusions one should attend for more careful investigations.

References

1. B. Barrois, Nuclear Physics B129, 390 (1977); S. Frautschi, Proceedings of workshop on hadronic matter at extreme density, Erice 1978; D. Bailin and A. Love, Physics Report 107 (1984) 325 .
2. M. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B422(1998) 247 [hep-ph/9711395]; R. Rapp, T. Schafer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. 81, 53 (1998) [hep-ph/9711396].
3. D.T. Son, Phys. Rev. D59 (1999) 094019 [hep-ph/9812287]; T. Schäfer and F. Wilczek, Phys. Rev. D60 (1999) 114033 [hep-ph/9906512]; D.K. Hong, V.A. Miransky, I.A. Shovkovy, and L.C.R. Wijewardhana, Phys. Rev. D61 (2000) 056001 [hep-ph/9906478]; S.D.H. Hsu and M. Schwetz, Nucl. Phys. B572 (2000) 211 [hep-ph/9908310]; W.E. Brown, J.T. Liu, and H.-C. Ren, Phys. Rev. D61 (2000) 114012 [hep-ph/9908248].
4. R.D. Pisarski and D.H. Rischke, *Phys. Rev.* **D61** (2000) 051501 [nucl-th/9907041].
5. I.A. Shovkovy and L.C.R. Wijewardhana, *Phys. Lett.* **B470** (1999) 189 [hep-ph/9910225]; T. Schäfer, *Nucl. Phys.* **B575** (2000) 269 [hep-ph/9909574].
6. P. Amore, M. C. Birse, J. A. McGovern and N. R. Walet, *Phys. Rev.* **D65** (2002) 074005 [hep-ph/0110267].
7. M. Alford, C. Kouvaris and K. Rajagopal, *Phys. Rev. Lett.* **92** (2004) 222001 [hep-ph/0311286].
8. M. Alford, C. Kouvaris and K. Rajagopal, *Phys. Rev.* **D71** (2005) 054009 [hep-ph/0406137].
9. M. Alford, P. Jotwani, C. Kouvaris, J. Kundu and K. Rajagopal, *Phys. Rev.* **D71** (2005) 114011 [astro-ph/0411560].
10. M. Alford and K. Rajagopal, *JHEP* **06** (2002) 031 [hep-ph/0204001].
11. R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli and M. Ruggieri, *Phys. Lett.* **B605** (2005) 362 [hep-ph/0410401].
12. K. Fukushima, *Phys. Rev.* **D70** (2005) 07002 [hep-ph/0506080].
13. M. Huang and I. A. Shovkovy, *Phys. Rev.* **D70** (2004) 051501 [hep-ph/0407049]; *ibidem* *Phys. Rev.* **D70** (2004) 094030 [hep-ph/0408268].
14. M. Hashimoto, *Phys. Lett.* **B642** (2006) 93 [arXiv:hep-ph/0605323].
15. O. Kiriyama, *Phys. Rev.* **D74** (2006) 074019 [arXiv:hep-ph/0608109].
16. O. Kiriyama, arXiv:hep-ph/0609185.
17. M. Alford and Q. Wang, *J. Phys.* **G31** (2005) 719 [hep-ph/0501078].
18. E. V. Gorbar, M. Hashimoto and V. A. Miransky, *Phys. Lett.* **B632** (2006) 305 [arXiv:hep-ph/0507303].
19. E. V. Gorbar, M. Hashimoto and V. A. Miransky, *Phys. Rev. Lett.* **96** (2006) 022005 [arXiv:hep-ph/0509334].
20. O. Kiriyama, D. H. Rischke and I. A. Shovkovy, *Phys. Lett.* **B643** (2006) 331 [arXiv:hep-ph/0606030].
21. L. He, M. Jin and P. Zhuang, arXiv:hep-ph/0610121.
22. P. F. Bedaque and T. Schafer, *Nucl. Phys.* **A697** (2002) 802 [arXiv:hep-ph/0105150].
23. A. Kryjevski and T. Schafer, *Phys. Lett.* **B606** (2005) 52 [arXiv:hep-ph/0407329].
24. A. Kryjevski and D. Yamada, *Phys. Rev.* **D71** (2005) 014011 [arXiv:hep-ph/0407350].
25. A. Kryjevski, arXiv:hep-ph/0508180.
26. A. Gerhold and T. Schafer, *Phys. Rev.* **D73** (2006) 125022 [arXiv:hep-ph/0603257].
27. M. G. Alford, arXiv:hep-lat/0610046.
28. I. Shovkovy and M. Huang, *Prepared for NATO Advanced Study Institute: Structure and Dynamics of Elementary Matter, Kemer, Turkey, 22 Sep - 2 Oct 2003.*
29. M. Alford and Q. h. Wang, *J. Phys.* **G32** (2006) 63 [arXiv:hep-ph/0507269].
30. M. G. Alford, K. Rajagopal, S. Reddy and F. Wilczek, *Phys. Rev.* **D64** (2001) 074017 [arXiv:hep-ph/0105009].
31. M. Buballa, F. Neumann and M. Oertel, *AIP Conf. Proc.* **660** (2003) 196.
32. M. Alford, C. Kouvaris and K. Rajagopal, arXiv:hep-ph/0407257.
33. I. Giannakis and H.C. Ren, Phys. Lett. B611 (2005) 137 [hep-ph/0412015].
34. I. Giannakis and H.C. Ren, Nucl. Phys. B723 (2005) 255 [hep-th/0504053].
35. A. I. Larkin and Yu. N. Ovchinnikov, Sov. Phys. JETP 20 (1965) 762.
36. P. Fulde and R. A. Ferrell, Phys. Rev. 135 (1964) A550.
37. M. G. Alford, J. A. Bowers and K. Rajagopal, Phys. Rev. D63 (2001) 074016 [hep-ph/0008208].
38. J. A. Bowers and K. Rajagopal, Phys. Rev. D66 (2002) 065002 [hep-ph/0204079].
39. R. Casalbuoni and G. Nardulli, Rev. Mod. Phys. 76 (2004) 263 [hep-ph/0305069].
40. J. A. Bowers, hep-ph/0305301.
41. B. S. Chandrasekhar, App. Phys. Lett. 1 (1962) 7.
42. A. K. Leibovich, K. Rajagopal and E. Shuster, Phys. Rev. D64 (2001) 094005 [hep-ph/0104073]; see also I. Giannakis, J. T. Liu and H. C. Ren, Phys. Rev. D66 (2002) 031501 [hep-ph/0202138].
43. R. Casalbuoni, M. Ciminale, M. Mannarelli, G. Nardulli, M. Ruggieri and R. Gatto, Phys. Rev. D70 (2004) 054004 [hep-ph/0404090].
44. R. Casalbuoni, R. Gatto, N. Ippolito, G. Nardulli and M. Ruggieri, Phys. Lett. B627 (2005) 89 [Erratum-ibid. B634 (2006) 565] [arXiv:hep-ph/0507247].
45. M. Mannarelli, K. Rajagopal and R. Sharma, Phys. Rev. D73 (2006) 114012 [arXiv:hep-ph/0603076].
46. R. Casalbuoni, M. Ciminale, R. Gatto, G. Nardulli and M. Ruggieri, Phys. Lett. B642 (2006) 350 [arXiv:hep-ph/0606242].
47. M. Ciminale, G. Nardulli, M. Ruggieri and R. Gatto, Phys. Lett. B636 (2006) 317 [arXiv:hep-ph/0602180].
48. K. Rajagopal and R. Sharma, Phys. Rev. D74 (2006) 094019 [arXiv:hep-ph/0605316].