The polarization tensor of neutral gluons in external fields at high temperature

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Abstract

The one-loop polarization operator of neutral gluons in the background constant Abelian isotopic, $H_3$, and hypercharge, $H_8$, chromomagnetic fields combined with $A_0$ electrostatic potential at high temperature is calculated. The case when $A_0 = 0$ is investigated separately. The proper time method is applied. It is found that neutral gluons do not acquire magnetic masses in the background fields, in contrast to the charged ones. The application of the results are discussed.

1 Introduction

Investigation of the deconfinement phase of QCD remains of considerable interest for high-energy physics and cosmology. Among the most important objects here is a gluon polarization tensor (PT) containing information on the excitation spectrum of quark-gluon plasma. First the QCD PT was calculated and investigated in one-loop order of perturbation theory at $T \neq 0$ by Kalashnikov and Klimov \cite{1}, \cite{2} (see also surveys \cite{3} and \cite{4}, \cite{5} where the results on higher order contributions are discussed). As it has been shown, the space components of the one-loop gluon propagator calculated within a standard perturbation theory possesses a fictitious infrared pole at $k_4 = 0, \bar{k} \sim g^2 T$ which could not be removed by any further resummations. These infrared divergencies of the thermal Green functions provide the most challenging difficulties in understanding the internal structure of perturbative finite temperature QCD. It is believed, however, that formation of some condensate fields, such as a uniform ”colour” magnetic field ($H_c = \text{const.}$) or electrostatic potential (so-called $A_0$ condensate), can improve the infrared properties of the theory. These condensate fields may arise in the deconfinement phase of QCD due to a peculiar dynamics of non-Abelian gauge fields, as it was argued by several authors \cite{6}-\cite{14}. In the paper by Kalashnikov \cite{15} it was demonstrated, in particular,
that the $A_0$ condensate shifts the fictitious pole and introduces the gluon magnetic mass of the order $m^2 \sim g^4 T^2$. At the same time, in Ref. [16] it was discovered that in the presence of the external Abelian chromomagnetic fields $H$ the transversal charged gluons acquire a magnetic mass $m^2_{\text{magn.}} \sim g^2 \sqrt{gH \tau}$. At the same time, in Ref. [16] it was discovered that in the presence of the external Abelian chromomagnetic fields $H$, the transversal charged gluons acquire a magnetic mass $m^2_{\text{magn.}} \sim g^2 \sqrt{gH \tau}$. It acts to stabilize the external field. In Ref. [12] it was found within the SU(3) gluodynamics that at high temperature a specific combination of the Abelian hypercharge, $H_8$, and isotopic spin, $H_3$, fields is generated and is stable due to this magnetic mass. It is also of the order $\sim g^4 T^2$. The tachyonic (unstable) modes of the transversal charged gluons, which appear in the energy spectrum of the charged vector particles when the homogenous magnetic field is applied to the system, are removed by these high-temperature radiative corrections. Moreover, an imaginary part of the effective potential (EP) of the background fields is cancelled if the contribution of the daisy diagrams with this magnetic mass is taken into consideration. Hence, one has to believe that the non-trivial configuration of the classical magnetic fields $H_3$ and $H_8$ is generated in the deconfinement phase.

It is interesting to see in actual calculations whether or not the magnetic mass of the neutral gluons is generated in the external field at high temperature. Actually, this is not expected because on general theoretical grounds the fields belonging to the Abelian projection of the non-Abelian groups remain massless. It is also important to know whether or not the fictitious pole of the neutral gluons is preserved when a magnetic field and $A_0$ is present in the system.

The aim of the present paper is to calculate the one-loop polarization operator of the neutral gluons in SU(3) gluodynamics in the external fields $H_3$ and $H_8$ and $A_0$ (or $A_4$) electrostatic potential at high temperature and check whether the full propagator of neutral gluons $Q_3$ and $Q_8$ contains the fictitious pole leading to the infrared instability. If this is not the case, one is able to conclude that the formation of the condensate fields play the role of an infrared regulator and the transversal components of neutral gluons are unscreened. It is necessary to note that at zero temperature this problem was investigated in Ref. [20]. We will begin with the case when both the chromomagnetic fields and the electrostatic potentials are present in the system. Then the case of $gA_0 = 0$ will be separately analysed. We will restrict our consideration to the one-loop approximation. To evaluate integrals over a three-dimantion momentum the Fock-Schwinger proper time method will be applied. The most essential steps of calculation are given in the Appendix 1.

2 Calculation of the polarization operator

We start our analysis with the expression of the Lagrangian of neutral gluons in (Euclidean) SU(3) gluodynamics:

$$L_{\text{neut.gl.}} = -\frac{1}{4} Q_3^{\mu \nu} Q_3^{\mu \nu} - \frac{1}{4} Q_8^{\mu \nu} Q_8^{\mu \nu} - \frac{1}{2} (\partial_\mu Q_3^\mu)(\partial_\nu Q_3^\nu) - \frac{1}{2} (\partial_\mu Q_8^\mu)(\partial_\nu Q_8^\nu) + ig Q_3^{\mu \nu} W_1^{\mu \nu} + ig Q_3^{\mu \nu} (W_1^+ (\partial_\mu W_1^- - \partial_\nu W_1^- - h.c.) + i \sqrt{\frac{2}{3}} g ((Q_8^{\mu \nu} + \frac{1}{\sqrt{6}} Q_3^{\mu \nu}) W_2^+ W_2^- + (Q_8^{\mu \nu} - \frac{1}{\sqrt{6}} Q_3^{\mu \nu}) W_3^+ W_3^-) + i \sqrt{\frac{3}{2}} g (Q_8^\mu + \frac{1}{\sqrt{6}} Q_3^\mu) (W_2^+ \partial_\mu W_2^- - \partial_\nu W_2^- - h.c.)$$
charged ghost fields. Thin wavy line corresponds to the neutral glu on fields.

A set of diagrams in Fig. 1, where double wavy lines represent the Green function describing propagation of the neutral gluons in the background fields.

gauge) the operator form the above Green functions are given by the expressions (in Feynman’s space and chromomagnetic fields are chosen to be directed along the third axis of the Euclidean $A$ and $B$ is introduced. The external potential is chosen in the form $B^a_{\mu} = \delta^{a3} B_{3\mu} + \delta^{a8} B_{8\mu}$, where $B_{3\mu} = H_3 \delta_{\mu2} x_1 + \delta_{\mu4} g A_3$ and $B_{8\mu} = H_8 \delta_{\mu2} x_1 + \delta_{\mu4} g A_8$. In these formalae the notations $A_3$ and $A_8$ correspond accordingly to $A_0^{a=3}$ and $A_0^{a=8}$ electrostatic potentials. The constant chromomagnetic fields are chosen to be directed along the third axis of the Euclidean space and $a = 3$ and $a = 8$ of the colour $SU_c(3)$-space: $F_{\mu\nu}^{a \text{ ext}} = \delta^{a3} F_{\mu\nu}^{a=3} + \delta^{a8} F_{\mu\nu}^{a=8}$, $F_{12}^{a} = - F_{21}^{a} = H_a$, $a = 3, 8$. From the Lagrangian one can easily derive the diagrams describing propagation of the neutral gluons in the background fields.

In the one-loop approximation the PO of neural gluons is determined by the standard set of diagrams in Fig. 1, where double wavy lines represent the Green function $G_{\Gamma \mu \alpha}(x, y)$ for the charged gluons, dashed double lines represent the Green function $D(x, y)$ for the charged ghost fields. Thin wavy line corresponds to the neutral gluon fields $Q^{\Gamma}_{\mu}$. In the operator form the above Green functions are given by the expressions (in Feynman’s gauge)

$$G_{r=1 \mu \nu}(P) = -[P^2 + 2igF_{3\mu\nu}]^{-1},$$

$$G_{r=2,3 \mu \nu}(P) = -[P^2 + \sqrt{6}i\lambda_{+} gF_{3\mu\nu}]^{-1},$$

$$D(P) = -\frac{1}{P^2},$$

$$\lambda_{+} = 1 \pm \frac{1}{\sqrt{6}} \frac{H_3}{H_8}.$$ 

To calculate the PO we make use of the proper time representation and the Schwinger operator formalism $[17]$. The PO of the neutral gluons in the background fields at $T \neq 0$ can be written as

$$\Pi_{\mu\nu}^{a=3} = - g^2 T \sum_{P_4} \int \frac{d^3 P}{(2\pi)^3} \left( \Pi_{\mu\nu}(k, P) + \frac{1}{4} \tilde{\Pi}_{\mu\nu}(k, P) \right),$$

$$\Pi_{\mu\nu}^{a=8} = - \frac{3}{2} g^2 T \sum_{P_4} \int \frac{d^3 P}{(2\pi)^3} \tilde{\Pi}_{\mu\nu}(k, P),$$

where

$$\Pi_{\mu\nu}(k, P) = \Pi_{\mu\nu}^{r=1}(k, P),$$

$$\tilde{\Pi}_{\mu\nu}(k, P) = \sum_{r=2,3} \Pi_{\mu\nu}(k, P),$$

$$\Pi^{\Gamma}_{\mu\nu}(k, P) = \left\{ \Gamma_{r, \mu\alpha,\beta}(P, k) G_{r \beta\lambda}(P) \Gamma_{r, \nu\sigma,\lambda}(P, k) G_{r \sigma\alpha}(P - k) - 2\delta_{\mu\nu} G_{r \alpha\alpha}(P) + 2(2P - k)_{\mu} D(P)(2P - k)_{\nu} D(P - k) - 2\delta_{\mu\nu} D(P) \right\},$$

$$\Gamma_{\mu\alpha,\beta} = (2P - k)_{\mu} \delta_{\alpha\beta} - 2(k_{\alpha} \delta_{\beta\mu} - k_{\beta} \delta_{\alpha\mu}),$$

$$\Gamma_{\mu\alpha,\beta} = (2P - k)_{\mu} \delta_{\alpha\beta} - 2(k_{\alpha} \delta_{\beta\mu} - k_{\beta} \delta_{\alpha\mu}).$$
\[P_4 = 2\pi lT + gA_3, \quad P_i = i\partial_i + gB_{3i} \text{ for } r = 1 \text{ and } P_4 = 2\pi lT + \sqrt{\frac{3}{2}} \mu_+ gA_8, \]
\[P_i = i\partial_i + \sqrt{\frac{3}{2}} \lambda_\pm gB_{3i} \text{ for } r = 2, 3, \text{ respectively}; \quad l = 0, \pm 1, \pm 2, \ldots, \text{ and} \]
\[\mu_\pm = 1 \pm \frac{1}{\sqrt{6}} A_3. \]

Assume now that the values of potentials \(A_3\) and \(A_8\) satisfy the following conditions: \(gA_3 \ll T\) and \(gA_8 \ll T\). This is natural because the quantities \(gA_{3,8}\) are expected to be of order \(g^2 T\), as it is pointed out in Refs. 10-11 for \(SU(2)\) case. To investigate the high temperature limit of (3) and (4) one can take the \(l = 0\) term only in the sum over \(P_4\) 3.

To evaluate the expression for the PO let us apply the Schwinger proper-time method modified for the case of high temperature. From technical point of view, this case is similar to the zero temperature one, so one may consult for more details, for example, to Refs. 18-22, where the PO of photon as well as neutral gluon in the external (chromo)magnetic field were calculated at \(T = 0\). The basic steps of the calculating procedure are noted in the Appendix. For simplicity it is convenient to introduce the following notations:

\[H_\pm = \sqrt{\frac{3}{2}} \lambda_\pm H_3; \quad A_\pm = \sqrt{\frac{3}{2}} \mu_\pm A_3; \quad m = \frac{(gA_3)^2}{gH_3}, \quad m_\pm = \frac{(gA_\pm)^2}{gH_\pm}. \tag{5}\]

Then the final result of evaluation (3) and (4) reads:

\[\Pi_{ij}^{a=3,8} = (\delta_{ij} - \frac{k_i k_j}{k^2})k^2 \Pi_{a=3,8}^{(1)} + (B-k)(B-k)J \Pi_{a=3,8}^{(2)}, \tag{6}\]
\[\Pi_{i4}^{a=3,8} = -\Pi_{i4}^{a=3,8} = i(B-k)J \Pi_{a=3,8}^{(3)}, \quad \Pi_{i4}^{a=3,8} = \Pi_{a=3,8}^{(4)} - \Psi_{a=3,8}. \tag{7}\]

Here the quantities \(\Pi_{a=3,8}^{(i)}, \quad i = 1, \ldots, 4, \text{ and } \Psi_{a=3,8}\) are

\[\Pi_{a=3}^{(i)} = \Pi^{(i)} + \frac{1}{6} \Pi_{a=8}^{(i)}, \quad \Psi_{a=3} = \Psi + \frac{1}{6} \Psi_{a=8}, \tag{8}\]

\[\Pi^{(i)} = -\frac{g^2}{8\pi^3/2} T \frac{1}{\sqrt{gH_3}} \int_0^1 du \int_0^\infty \frac{dx}{sh(x)} \sqrt{x} \exp[-\Phi - x]J^{(i)}(x, u), \tag{9}\]

\[\Pi_{a=8}^{(i)} = -\frac{3g^2 T}{16\pi^3/2} \int_0^1 du \int_0^\infty \frac{dx}{sh(x)} \sqrt{x} \left\{\frac{J^{(i)}(x, u)}{\sqrt{gH_+}} e^{-\Phi_+ - x} + \frac{J^{(i)}(x, u)}{\sqrt{gH_-}} e^{-\Phi_- - x}\right\}, \tag{10}\]

\[\Psi = \frac{g^2}{4\pi^3/2} \sqrt{gH_3} T \int_0^\infty \frac{dx}{\sqrt{x}} \left[\frac{2}{sh(x)} + 4sh(x)\right] e^{-x}, \tag{11}\]

\[\Psi_{a=8} = \frac{3g^2}{8\pi^3/2} T \int_0^\infty \frac{dx}{\sqrt{x}} \left[\frac{2}{sh(x)} + 4sh(x)\right] \left\{\sqrt{gH_+} e^{-x} + \sqrt{gH_-} e^{-x}\right\}. \tag{12}\]
Here
\[
\Phi = xu(1 - u) \frac{k_3^2}{gH_3} + k_1^2 \frac{\zeta}{2gH_3}, \quad \Phi_\pm = xu(1 - u) \frac{k_3^2}{gH_\pm} + k_1^2 \frac{\zeta}{2gH_\pm}
\]
and
\[
\zeta = \frac{ch(x) - ch(x(1 - 2u))}{sh(x)}, \quad k_1^2 = k_1^2 + k_2^2
\]

Exact expressions for the functions \( f^{(i)} \) and \( l_\pm^{(i)} \) are adduced in the Appendix 2. The matrix \( B_{ij} \) is a usual two dimension antisymmetric tensor,
\[
B_{ij} = \epsilon_{ij} = \delta_{i2}\delta_{1j} - \delta_{i1}\delta_{2j}.
\]

The spatial part of the PO is transversal manifestly, as it is required by gauge invariance. Note that \( \Pi^{(i=3,4)} = 0 \) for \( A_3 = A_\pm = 0 \).

Now let us consider the high-temperature expansion, \( gH_{3,8} \ll T^2, (gA_{3,8})^2 \ll T^2 \), of the expressions in Eqs. (8)-(12). Assuming that the quantities \( gH_{3,8} \) and \( (gA_{3,8})^2 \) are of the same order of magnitude, we investigate the two separate regimes: \( |\vec{k}| \ll g^2 T \) and \( |\vec{k}| \gtrsim gT \). In the former case, with the additional condition \( k_1^2 \ll gH_{3,8} \) and \( k_3^2 < gH_{3,8} \), the main contributions to integrals come from the integration domain where \( x \gg 1 \). Carrying out integrations we obtain
\[
\Pi^{(i)}_{a=3} = \Pi^{(i)}(gH_3; \nu; m) + \frac{1}{6} \Pi^{(i)}_{a=8}, \quad (13)
\]

\[
\Pi^{(i=1,2)}_{a=8} = \frac{3}{2} \left\{ \Pi^{(i=1,2)}(gH_+; \nu_+; m_) + \Pi^{(i=1,2)}(gH_+; \nu_-; m_-) \right\}, \quad (14)
\]

\[
\Pi^{(3)}_{a=8} = \frac{3}{2} \left\{ \frac{A_+}{A_3} \Pi^{(3)}(gH_+; \nu_+; m_+) + \frac{A_-}{A_3} \Pi^{(3)}(gH_-; \nu_-; m_-) \right\}, \quad (15)
\]

\[
\Pi^{(4)}_{a=8} = \frac{3}{2} \left\{ \left( \frac{A_+}{A_3} \right)^2 \Pi^{(4)}(gH_+; \nu_+; m_+) + \left( \frac{A_-}{A_3} \right)^2 \Pi^{(4)}(gH_-; \nu_-; m_-) \right\}. \quad (16)
\]

Here \( \nu = \frac{k_3^2}{4gH_3}, \nu_\pm = \frac{k_3^2}{gH_\pm} \) and functions \( \Pi^{(i)}(\alpha; \beta; \gamma) \) are represented by the following expressions:
\[
\Pi^{(1)}(\alpha; \beta; \gamma) = -\frac{g^2 T}{2\pi \sqrt{\alpha}} \frac{1}{4(\gamma \beta + 1) + \beta^2} \left[ \frac{3}{2} - \gamma + \beta \right] + \frac{1}{\sqrt{\gamma + \beta}}
\]
\[
\Pi^{(2)}(\alpha; \beta; \gamma) = -\frac{g^2 T}{8\pi \sqrt{\alpha}} \left[ \frac{2}{(\gamma - 1)(\gamma + \beta - 1)} - \frac{1}{1 + \beta(\gamma + \beta + 1)} \left( \frac{1 + \beta}{\sqrt{\gamma - 1}} - \frac{1}{\sqrt{\gamma + \beta + 1}} \right) \right] - \Pi^{(1)},
\]
\[
\Pi^{(3)}(\alpha; \beta; \gamma) = -\frac{g^2 T}{\pi \sqrt{\alpha}} \frac{gA_3 \sqrt{\gamma - 1}}{4(\gamma - 1) + \beta}.
\]
\[ \Pi^{(4)}(\alpha; \beta \to 0; \gamma) = -\frac{g^2 T}{2\pi \sqrt{\alpha}} \frac{(gA_3)^2}{(\gamma - 1)^{3/2}}, \]

where according to (13)-(16) instead of variables \( \alpha, \beta \) and \( \gamma \) one has to substitute \( gH_3, \nu, m \) or \( gHh_\pm, \nu_\pm, m_\pm \), respectively. For \( \Psi_{a=3} \) and \( \Psi_{a=8} \) we have:

\[ \Psi_{a=3} = \Psi(gH_3; m) + \frac{1}{6} \Psi_{a=8}, \quad (17) \]

\[ \Psi_{a=8} = \frac{3}{2} (\Psi(gH_+; m_+) + \Psi(gH_-; m_-)), \quad (18) \]

where

\[ \Psi(\alpha; \gamma) = \frac{g^2 T}{\pi} \sqrt{\alpha} \left[ \frac{1}{\sqrt{\gamma} + 1} + \sqrt{\gamma - 1} \right]. \]

For the values \( m = 1 \) and/or \( m_\pm = 1 \) the functions \( \Pi^{(i=1,2)} \) and \( \Pi^{(4)} \) become divergent whereas \( \Pi^{(3)} \) is equal to zero.

In the case of \( |\bar{k}| \geq gT \) and \( k_\perp^2 \gg (gA_{3,8})^2 \) (but \( m > 1 \) and \( m_\pm > 1 \)), the main contributions to integrals come from the region \( x \sim 0 \). Expanding the integrand functions into the power series over the variable \( x \), one can obtain for the spatial components (19):

\[ \Pi_{ij}^{a=3,8} \sim (\delta_{ij} - \frac{k_i k_j}{k^2}) \bar{k}^2 \Pi_{a=3,8}^{(1)} + (Bk) \Pi_{a=3,8}^{(2)}. \]

Here

\[ \Pi_{a=3}^{(1)} = -\frac{21}{16} C, \quad \Pi_{a=3}^{(2)} = \frac{3}{4} C, \quad \Pi_{a=8}^{(1)} = -\frac{21}{8} C, \quad \Pi_{a=8}^{(2)} = \frac{3}{2} C, \quad C = \frac{g^2 T}{k_\perp}. \]

It is remarkable that the quantities (20), which are, of course, only the leading terms of perturbative expansion, do not depend upon the condensate fields. For the momentum scale \( k_\perp \sim T \) the constant \( C \) is of order \( g^2 \) and, therefore, perturbative theory is actually governed by the parameter \( g^2 \). However, for the scale \( k_\perp \sim gT \ll T \) the effective expansion parameter becomes \( g \). Hence one can see that perturbative features of the model are aggravated with the decreasing \( k_\perp \).

### 3 Polarization operator in the external magnetic fields

In this section we consider the PO in the external chromomagnetic fields \( H_{3,8} \) (but \( gA_{3,8} = 0 \)). We merely put the parameters \( A_3, A_\pm \), in the Eqs. (9)-(12) equal to zero. In this case the integrands in the r.h.s. of Eqs. (9)-(12) are nonanalytical for large \( x \). To ensure the convergence of integrals with respect to \( x \) one has to rotate the integration contour by the standard rule: \( x \to ix \). Then, assuming again that \( k_\perp^2 \ll gH_{3,8} \) and \( k_3^2 \ll gH_{3,8} \), the main contributions come from large \( x \) and we obtain:

\[ \Pi_{a=3}^{(i)} = \Pi^{(i)}(gH_3; \nu) + \frac{1}{6} \Pi_{a=8}^{(i)}, \quad (21) \]
\[ \Pi^{(i=1,2)}_{a=8} = \frac{3}{2} \left\{ \Pi^{(i=1,2)}(gH_+;\nu_+) + \Pi^{(i=1,2)}(gH_-;\nu_-) \right\}, \]

where

\[ \Pi^{(1)}(\alpha;\beta) = -\frac{g^2 T}{2\pi \sqrt{\alpha(1 + 4\beta^2)}} \left[ 3\beta^2 - \frac{1}{2} + i(3\beta^2 + \frac{1}{2}) \right], \]

\[ \Pi^{(2)}(\alpha;\beta) = -\frac{g^2 T}{2\pi \sqrt{\alpha}} \left[ \frac{1}{4\sqrt{\beta + 1}} - \frac{2(1 - 2\beta)}{1 + 4\beta} + i\left( \frac{1}{2(1 - \beta)} + \frac{1 + 2\beta}{1 + 4\beta} - \frac{1 + \beta}{4(1 + \beta(\beta + 1))} \right) \right] - \Pi^{(1)}. \]

The Debye masses of neutral gluons are

\[ \text{Re}(\Pi^{a=3}_4) = \frac{g^2}{\pi} T \left[ \sqrt{gH_3} + \sqrt{gH_8}\frac{1}{4}(\sqrt{\lambda_+} + \sqrt{\lambda_-}) \right] \]

and

\[ \text{Re}(\Pi^{a=8}_4) = \frac{3g^2}{2\pi} T \sqrt{gH_8}(\sqrt{\lambda_+} + \sqrt{\lambda_-}). \]

Quantities (21) and (22), as well as \( \Pi_4 \), include imaginary parts reflecting the existence of the tachyonic mode in the tree-level spectrum of charged gluons. It should be noted, that expressions for \( \Pi_4 \) represent the next-to-leading terms. To calculate the leading terms one has to perform summation over the discrete frequencies \( P_4 \).

### 4 Discussion

To discuss the results obtained, let us consider the full propagator of the neutral gluons \( Q^{a=3,8}_\mu \). To one-loop order the transversal part of the propagator spatial components has the following structure

\[ G_{ij}^{\text{tr}} = \left( \delta_{ij} - \frac{k_i k_j}{k^2} \right) \frac{1}{k^2(1 + \Pi^{(1)})} - \frac{(B k)_i (B k)_j}{k^2} \frac{\Pi^{(2)}}{[k^2(1 + \Pi^{(1)}) + k^2 \Pi^{(2)}(1 + \Pi^{(1)})^2]}, \]

where the functions \( \Pi^{(1,2)} \) are given by Eqs. (13), (14), (20), (21) and (22). In the case of \( gA_{3,8} \neq 0 \) (see Eqs. (13), (14) and (20)), the full propagator does not contain a non-trivial pole. Hence, one has to conclude that the neutral gluons do not acquire magnetic masses in the presence of the background fields \( A_{3,8} \) and \( H_{3,8} \).

Here a more serious problem arises. Namely, if the condensate fields are of the order \( g^2 T \), as it was argued in Refs. [6]-[11], then, for the case of \( k^2 \ll \sqrt{gH_{3,8}} \) (see [13]-[16]), the factors \( \frac{g^2 T}{\sqrt{gH_{3,8}}} \) appearing in (13)-(16) turn out to be of order \( O(1) \) and the perturbative expansion breaks down for the momentum scale \( k \ll g^2 T \). Therefore one cannot explore the infrared region \( (\vec{k} \to 0) \) by usual perturbative methods and our conclusion is valid for
the scale $k_\perp \geq gT$, only. In this region perturbation theory is reliable (see Eqs. (19) - (20) and the text below).

In the case of the chromomagnetic fields been taken into consideration the quantities $\Pi^{(1,2)}$ were found to be complex, and Eq. (25) can be rewritten as

$$G_{tr}^{ij} = (\delta_{ij} - \frac{k_i k_j}{k^2}) \frac{1 + Re\Pi^{(1)} - i Im\Pi^{(1)}}{k^2[(1 + Re\Pi^{(1)})^2 + (Im\Pi^{(1)})^2]} - (B\bar{k})_i (B\bar{k})_j \frac{(1 + Re\Pi^{(1)} - i Im\Pi^{(1)})\Pi^{(2)}}{k^2[(1 + Re\Pi^{(1)})^2 + (Im\Pi^{(1)})^2]} \times \frac{(\bar{k}^2(1 + Re\Pi^{(1)}) + k^2 Re\Pi^{(2)}) - i (\bar{k}^2 Im\Pi^{(1)} + k^2 Im\Pi^{(2)}))}{[(\bar{k}^2(1 + Re\Pi^{(1)}) + k^2 Im\Pi^{(2)})^2 + (\bar{k}^2 Im\Pi^{(1)} + k^2 Im\Pi^{(2)})^2]}.$$  (26)

This expression has also a pole at $\bar{k}^2 = 0$, only. However, the imaginary part that arises in Eq. (26) has a "tachyonic" origin, as it was mentioned above. Really, the calculation of $\Pi_{ij}$ (as well as $\Pi_{44}$) has been carried out with the bare propagators of the charged gluons substituted into internal lines of diagrams. This results in a non-analyticity of integrands with respect to the variable $x$ in the $\Pi_{\mu\nu}$. In this sense the carried out one-loop calculation of the PO appears to be insufficient: to obtain a correct independent of the imaginary part expressions for (26), the charged gluon propagators accounting for the magnetic mass derived in the paper [12] must be used. But now, when we know the origin of the imaginary part, it does not matter when the problem on the magnetic mass of the neutral gluons is investigated.

In the present paper it was straightforwardly demonstrated that the transversal neutral gluon fields are not screened by thermal fluctuations if the non-trivial condensates present in the QCD deconfinement phase. We arrived at the following picture when the assumed formation of the condensate fields $gA_{3,8}$ and $gH_{3,8}$ determine the effective masses of the charged gluons while the neutral spatial components do not acquire magnetic masses in the fields. It is resonable to suppose that this picture will be also valid when only chromomagnetic fields $gH_{3,8}$ are generated in the system although higher-order contributions to the neutral gluon PT must be taken into account in this case. It is worth to emphasize that in the infrared region, $k \to 0$, the full propagator (25) does not contain the "fictitious" pole. This is in contrast to the case of trivial vacuum [1], [3].

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6 Appendix 1

To illustrate the basic stages of evaluating the PO (3-4) let us consider the integral:

\[ I_{ij} = \frac{g^2}{\beta} \int \frac{d^3P}{(2\pi)^3} \Pi_{ij}(\vec{k}, P), \]

which represents the contribution of the charged fields \( W_{r=1}^\pm \) (and the corresponding ghosts) to the \( \Pi_{ij}^{\alpha=3} \) at high temperature. The rest components of the tensor \( I_{\mu\nu} \) are calculated analogously. Following the standard procedure we introduce a proper time \( \tau \) for each propagator appearing in \( \Pi_{ij}(\vec{k}, \vec{P}) \):

\[ D(\vec{P}) = -\int_0^\infty ds e^{-s\vec{P}^2}, \]

\[ G_{r=1\mu\nu}(\vec{P}) = -\int_0^\infty ds e^{-s\vec{P}^2 - 2igF_{\mu\nu}s}, \]

Then, the whole expression for \( I \) can be rewritten in the form:

\[ I_{ij} = \frac{g^2}{\beta} \int_0^\infty ds_1 ds_2 \int \frac{d^3P}{(2\pi)^3} \exp[-(s_1 + s_2)(gA_3)^2] \times \]

\[ \{ \left[ \Gamma_{1/m}(P, \vec{k}) \Lambda_{mn}(\sigma_1) \Gamma_{1/s,n}(P', \vec{k}) \Lambda_{il}(\sigma'_1) - 2(2P - \vec{k})_i(2P' - \vec{k})_l \right] \theta_{r=1} \}

\[ -2g^2/\beta \delta_{ij} \int_0^\infty ds \int \frac{d^3P}{(2\pi)^3} [Tr \Lambda(\sigma''_1) - 2] e^{-s(P^2 + (gA_3)^2)}, \]

where \( \Gamma_{1/m} = (2P - \vec{k})_i \delta_{im} - 2(k_i \delta_{mi} - k_m \delta_{ii}) \) is the vertex factor,

\[ \theta_{r=1} = e^{-s_1\vec{P}^2} e^{-s_2(\vec{P} - \vec{k})^2}, \quad \Lambda_{ij}(x) = R_{ij} - B_{ij}^2 \partial_x(x) - iB_{ij} s x \]

and variables \( \sigma_1, \sigma'_1, \sigma''_1 \) are

\[ \sigma_{r=1} = 2igH_3s_1, \quad \sigma'_{r=1} = 2igH_3s_2, \quad \sigma''_{r=1} = 2igH_3s. \]

We introduced the following designation: \( P' = (exp[-2igF_3s_1]P)_i, P_i = i\delta_i + gB_{3i} \). The matrices \( R_{ij}, B_{ij} \) and \( B_{ij}^2 \) are:

\[ R_{ij} = \delta_{i3}\delta_{3j}, \quad B_{ij} = \delta_{i2}\delta_{1j} - \delta_{i1}\delta_{2j}, \quad B_{ij}^2 = B_{ij}B_{ij}. \]

Next, three-dimensional integration with respect to \( \vec{P} \) in \( I_{ij} \) is carried out by means of the transition to the conjugate variable \( X'_i \):

\[ [X_i, P_j] = i\delta_{ij}. \]

By using the eigenstates of the operator \( X_i \) as determined by the condition \( X'_i = 0 \), the integral over \( \vec{P} \) can be represented as

\[ \int \frac{d^3P}{(2\pi)^3} f(\vec{P}) = \langle \vec{X}' = 0 | f(\vec{P}) | \vec{X}' = 0 \rangle. \]

Hence, performing the following transformation of variables \( s_1 \) and \( s_2 \): \( s_1 = s(1 - u), s_2 = su \), we have for \( I_{ij} \)
\[ I_{ij} = \frac{g^2}{\beta} \int_0^1 du \int_0^\infty ds s \exp[-s(gA_3)^2] \]

\[ \langle \Gamma_{it,m}(\vec{P}, \vec{k}) \Lambda_{mn}(\sigma_1) \Gamma_{j,n}(\vec{P}', \vec{k}) \Lambda_{nl}(\sigma_1') - 2(2\vec{P} - \vec{k})_i(2\vec{P}' - \vec{k})_j \theta_{r=1} \rangle \]

\[ -2\frac{g^2}{\beta} \delta_{ij} \int_0^\infty ds \left[ Tr\Lambda(\sigma_1'') - 2 \right] \exp[-s(gA_3)^2] \langle e^{-sp^2} \rangle, \]

For convenience we use a notation \( \langle \vec{X}' = 0 \mid ... \mid \vec{X}' = 0 \rangle = \langle \ldots \rangle \). Now one needs to calculate the quantities \( \langle \theta_{r=1} \rangle, \langle P_i \theta_{r=1} \rangle \) and \( \langle P_i P_j \theta_{r=1} \rangle \) according to the procedure described in Ref. [17]. The result reads

\[ \langle P_i \theta_{r=1} \rangle = \left( \frac{A}{D} \right)_i \theta_{r=1}, \]

\[ \langle P_i P_j \theta_{r=1} \rangle = \left[ \left( \frac{A}{D} \vec{k} \right)_i \left( \frac{A}{D} \vec{k} \right)_j - ig \left( \frac{F}{D} \right)_ij \right] \theta_{r=1}, \]

\[ \langle \theta_{r=1} \rangle = \frac{1}{(4\pi s)^{3/2} \text{sh}(gH_3s)} e^{-\Phi_3}, \]

\[ \Phi_3 = k_3^2 s u (1 - u) + k_1^2 \frac{\zeta}{2gH_3}, \]

where \( A = e^{-2iguFs} - 1, D = e^{-2iguFs} - 1, k_1 = k_1^2 + k_2^2 \) and

\[ \zeta = \frac{\text{ch}(gH_3s) - \text{ch}(gH_3(1 - 2u)s)}{\text{sh}(gH_3s)}. \]

Note that

\[ \langle e^{-sp^2} \rangle = \frac{1}{(4\pi s)^{3/2} \text{sh}(gH_3s)}. \]

Finally, after integrating by parts, we arrive at:

\[ I_{ij} = \frac{g^2}{8\pi^{3/2}} \frac{T}{gH_3} \int_0^1 du \int_0^\infty dx \frac{dx}{\text{sh}(x)} \sqrt{x} \exp[-\Phi - xm] M_{ij}(x, u). \]

Here designations [14] are used and \( x = gH_3s \). The matrix \( M_{ij} \) is defined by

\[ M_{ij} = \left\{ 2\text{ch}(2x) \left[ (\rho \vec{k})_i (\rho \vec{k})_j - \rho_{ij}(\vec{k} \cdot \vec{k}) + (\lambda \vec{k})_i(\lambda \vec{k})_j \right] + 8(B\vec{k})_i(B\vec{k})_j \zeta \text{sh}(2x) \right. \]

\[ + 4 \left[ -\left( \Lambda(\sigma') \vec{k} \right)_i(\Lambda(\sigma') \vec{k})_j - \left( \Lambda(-\sigma') \vec{k} \right)_i(\Lambda(-\sigma') \vec{k})_j \right] \]

\[ + \Lambda_{ij}(\sigma')(\vec{k} \Lambda(\sigma') \vec{k}) + \Lambda_{ij}(-\sigma')(\vec{k} \Lambda(-\sigma') \vec{k}) \left\}, \right. \]

where \( \sigma = 2x(1 - u), \sigma' = 2xu, \rho = (1 - 2u)R - \xi B^2, \lambda = \zeta B, \xi = \frac{\text{sh}(x)(1 - 2u)}{\text{sh}(x)}. \) It can be easily verified that quantity \( I_{ij} \) is manifestly transversal, \( k_i I_{ij} = k_j I_{ij} = 0, \) as it should be due to the gauge invariance.
Now we can apply described above procedure to evaluate the $\Pi_{ij}^{a=3,8}$. The result is given by:

$$\Pi_{ij}^{a=3} = I_{ij} + \frac{1}{6} \Pi_{ij}^{a=8},$$

$$\Pi_{ij}^{(i)} = \frac{3g^2T}{16\pi^{3/2}} \int_0^1 du \int_0^\infty dx \frac{1}{sh(x)} \sqrt{x} \left\{ \frac{1}{\sqrt{gH_+}} e^{-\Phi_+-xm_+} + \frac{1}{\sqrt{gH_-}} e^{-\Phi_--xm_-} \right\} M_{ij}(x,u).$$

It is convenient to rewrite the operators $\Pi_{ij}^{a=3,8}$, using their eigenvectors, $b_i^\rho$, and eigenvalues, $\kappa_{a=3,8}^\rho$, as:

$$\Pi_{ij}^{a=3,8} = \sum_{\rho=1}^3 \kappa_{a=3,8}^\rho \frac{b_i^\rho b_j^\rho}{|b^\rho|^2},$$

$$\Pi_{ij}^{a=3,8} b_j^\rho = \kappa_{a=3,8}^\rho b_i^\rho,$$

where $b_i^{\rho=1} = (B\bar{k})_i$, $b_i^{\rho=2} = (R\bar{k})_i + \frac{k_i^2}{k_+^2} (B^2\bar{k})_i$, and $b_i^{\rho=3} = k_i$. The eigenvectors $b_i^\rho$ satisfy the condition of completeness:

$$\sum_{\rho=1}^3 \frac{b_i^\rho b_j^\rho}{|b^\rho|^2} = \delta_{ij}.$$

Hence, since $\kappa^{\rho=3} = 0$ because of transversality of the $\Pi_{ij}^{a=3,8}$, we obtain (6).

7 Appendix 2

The functions $f^{(i)}(x,u)$ and $l^{(i)}_\pm(x,u)$ are:

$$f^{(1)} = 4 \left[ ch(x)ch(x[1-2u]) - \frac{1}{2} (1-2u) \xi ch(2x) \right],$$

$$f^{(2)} = 4 \left[ ch(2x) - 2sh(2x)\zeta - \frac{1}{2} ch(2x)(\xi^2 - \zeta^2) \right] - f^{(1)},$$

$$f^{(3)} = 4gA_3 \left[ 2sh(2x) - ch(2x)\zeta \right], \quad f^{(4)} = 8(gA_3)^2 ch(2x),$$

$$l^{(i=1,2)}_{\pm} = f^{(i=1,2)}_{\pm}, \quad l^{(3)}_{\pm} = \frac{A_{\pm}}{A_3} f^{(3)}_{\pm}, \quad l^{(4)}_{\pm} = \frac{(A_{\pm})^2}{(A_3)^2} f^{(4)}_{\pm}$$

and

$$\xi = \frac{sh(x(1-2u))}{sh(x)}, \quad \zeta = \frac{ch(x) - ch(x(1-2u))}{sh(x)}.$$
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Figure 1: Polarization operator of neutral gluons in the one-loop approximation.