THE DECAYS OF \( B_c \) MESON TO \( P \)-WAVE CHARMONIUM \( \chi_c(h_c) \) AND TO LEPTONS WITH GLUON BREMSSTRAHLUNG\(^2\)

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Theoretical computations on the decays of \( B_c \) meson to \( P \)-wave charmonium \( \chi_c \) or \( h_c \) with some particle(s) else and on the decays of \( B_c \) meson to leptons with gluon bremsstrahlung, and the results as well are outlined.

1 Introduction

The meson \( B_c \) was observed successfully first by CDF collaboration in RUN-I at Tevatron\(^2\). CDF observation was through the cascade decay \( B_c \to J/\psi + \bar{l} + \nu_l \) with \( J/\psi \to \mu + \bar{\mu} \) and the values of \( B_c \) mass, lifetime and a combined ratio of the cross sections and branching ratios as below

\[
m_{B_c} = 6.40 \pm 0.39 \pm 0.13\text{GeV} , \quad \tau_{B_c} = 0.46^{+0.18}_{-0.10} \pm 0.03\text{ps}
\]

are obtained.

Before the observation, in literature there were many theoretical estimates on \( B_c \) properties. Considering the theoretical uncertainties and experimental errors, the previous theoretical estimates are consistent with the CDF observation. Moreover, according to the theoretical estimates, experimental study of \( B_c \) in LHC, even in Tevatron RUN-II and RUN-III where numerous \( B_c \) mesons can be produced, is particularly interesting not only in understanding \( B_c \) meson itself but also for new physics studies, such as observing \( CP \) violation in \( B_s, B_s - \bar{B}_s \) mixing, and setting the more stringent constraint for multi Higgs doublet model (MHDM, an extension of standard model with Higgs doublets more than two) etc.

The branching ratio of the decays \( B_c \to B_l \cdots \) is very great and the lifetime \( \tau_{B_c} \approx 0.5 \) ps is very suitable for modern vertex detector, so study of the \( CP \) violation in \( B_s \) and \( B_s - \bar{B}_s \) mixing in terms of the \( B_s \) mesons from \( B_c \) decays has great advantages: the \( B_s \) is tagged precisely (\( B_c \) is charged) and many backgrounds may be well rejected by the vertex of the \( B_c \) decay. In MHDM the Feynman vertices of charged Higgses are proportional to the relevant Fermion masses, so the decay \( B_c \to \tau + \nu_\tau + \gamma \), especially, to measure \( \tau \) transverse polarization, will give much more tighter constraint than before on MHDM parameters.

Recently we completed computation on the decays of \( B_c \) meson to a \( P \)-wave charmonium \( \chi_c \) or \( h_c \) (semileptonic decays \( B_c \to \chi_c(h_c)\bar{l}\nu_l \) and nonleptonic decays \( B_c \to \chi_c(h_c)h, h \) means a hadron) and obtained interesting results: the concerned decays are sizable (accessible in RUN-II, RUN-III and in LHC). Especially, the decays to \( h_c \) may be used as a new ‘window’ to observe \( h_c \) state. The decays \( B_c \to \chi_c \bar{l}\nu_l \) with \( \chi_c \to J/\psi\gamma \) followed, being a background, affect the observation of CDF substantially etc.

A few years ago NRQCD made achievement in solving the puzzle of \( J/\psi \) and \( \psi' \) production in hadron collision by the so-called color octet mechanism, but it still has open problems. An outstanding problem is that the experimental measurements on the polarization of the produced \( J/\psi \) and \( \psi' \) in hadron collision are deviated from NRQCD predictions. Therefore to explore the...
roles caused by the color octet components of the other ‘double heavy’ mesons such as $B_c$ meson, in addition to the charmonia, certainly is interesting.

According to NRQCD, $B_c$ meson state, similar to charmonia, should be an expansion in Fock space:

$$|B_c> = O(v^0)|(c\bar{b})|1^1 S_0> + O(v)|(c\bar{b})|8^3 S_1|g> + O(v)|(c\bar{b})|8^1 P_1|g> + \ldots . \quad (1)$$

To pursue the roles of the color octet components, we have computed the charged lepton spectra and the width of the inclusive decay $B_c \rightarrow l + \nu + \ldots$ (here ‘\ldots’ means hadrons), and found that, indeed through measuring the spectra, the roles of the octet components of $B_c$ may be verified experimentally.

Here I only outline the topics mentioned above, whereas the details can be found in Ref. [3,4].

2 The Decays of $B_c$ Meson to $P$-wave Charmonium

To compute the semileptonic decays $B_c \rightarrow \chi_c(h_c)\ell\nu$, all of the form factors, related to the weak current matrix elements sandwiched by the states of $B_c$ and the $P$-wave charmonium, should be calculated precisely. In the present case special care on the recoil effects should be paid, because in the decays the recoil momentum can be great, even relativistic. Thus the method, the so-called generalized instantaneous approximation (GIA) is adopted so as to treat the recoil effects properly. As a result of the momentum recoil effects being treated properly, all of the form factors (for the decays of $B_c$ to a $P$-wave charmonium $\chi_c$ or $h_c$) depend on two ‘general functions’ $\xi_1$ and $\xi_2$:

$$(\epsilon^\lambda \cdot \epsilon_0)\xi_1 \equiv \int \frac{d^3 q'_\parallel}{(2\pi)^3} \psi^*_{n1\lambda}(q'_\mu T)\psi_{n00}(q_{pT}), \quad (\lambda^0 \cdot \epsilon_0)\xi_2 \equiv \int \frac{d^3 q'_\perp}{(2\pi)^3} \psi^*_{n1\lambda}(q'_\mu T)\psi_{n00}(q_{pT})q'_{\perp\perp}, \quad (2)$$

where

$$d^3 p_T = k^2_{pT}dk_{pT}dsd\phi, \quad q'^\mu = q_{p\parallel} + q_{p\perp}, \quad q''_{p\parallel} \equiv (p \cdot q/M_p^2)p^\mu, \quad q''_{p\perp} \equiv q^\mu - q_{p\parallel}^\mu,$$

$$q_p = \frac{p \cdot q}{M_p}, \quad q_{pT} = \sqrt{q_{p\parallel}^2 - q'^2_{p\perp}}, \quad \epsilon_{0\mu} \equiv \frac{p_{\mu} - q_{p\parallel}^\mu}{\sqrt{(p \cdot q)^2 - M^2}},$$

$\phi$ is the azimuthal angle and $s = (k_p q_p - k \cdot q)/(k_p q_{pT})$. $\epsilon^\lambda(p)$ is the $P$-wave ($L=1$) orbital ‘polarization’ in moving $p$ ($p^2 = m^2$).

Based on GIA, the functions $\xi_1$ and $\xi_2$ may be calculated numerically, as long as the wave functions of $B_c$ meson and the $P$-wave charmonium $\chi_c$ and $h_c$ are calculated with potential model. The result is shown in Fig.1. We may see from Fig.1 that $\xi_1$ may be negligible in the cases when momentum recoil is tiny, i.e. it approaches to zero as the recoil approaches zero, and becomes comparable with $\xi_2$ when the momentum recoil turns great. Here $\xi_2$ is ‘essential’ in a sense because it cannot turn to zero no matter how small (or great) the momentum recoil is. Note that the present situation is different from the decays of $B_c$ to an $S$-wave charmonium, where the one, being ‘essential’ and similar to $\xi_1$, is always dominant over the other (similar to $\xi_2$). Hence in leading order calculations, in the case of $B_c$ to an $S$-wave charmonium, all of the form factors depend on $\xi_1$ only (all the dependence on $\xi_2$ is ignored). The reason, why there is the difference, is that, in the case of $B_c$ to a $P$-wave charmonium, the ‘essential’ one ($\xi_2$) is suppressed, if comparing with the normalization of the wave functions for $B_c$ or the $P$-wave charmonium, so $\xi_1$ can become comparable with $\xi_2$ as the momentum recoil approaches great. Whereas in the case of $B_c$ to an $S$-wave charmonium, there is no such suppression on the ‘essential’ one (the one similar to $\xi_1$) at all.

\footnote{The method of the so-called generalized instantaneous approximation was suggested first in Ref.[5].}
Figure 1: The functions $\xi_1$ and $\xi_2$ vs. $t_m - t$. The solid line is of $\xi_1$ and the dashed one is of $\xi_2$.

Figure 2: The energy spectra of the charged lepton for the decays $B_c \rightarrow \chi_c(h_c) + e(\mu) + \nu_{e(\mu)}$ (the left figure) and for the decays $B_c \rightarrow \chi_c(h_c) + \tau + \nu_\tau$ (the right figure). The solid line is of $h_c[1^1P_1]$. The dotted-blank-dashed line is of $\chi_c[3^3P_1]$. The dashed line is of $\chi_c[3^3P_2]$. The dotted-dashed line is of $\chi_c[3^1P_2]$.

To give you some general feature, here I present the results of the semileptonic decays only. The decay widths are put in Tab.1 and the spectra of the charged leptons for the decays are put in Fig.2 ($\tau$ lepton is massive, so its mass cannot be ignored as done in the cases of $e, \mu$, thus Fig.2 contains two figures as described in the caption.)

|                | $\Gamma(B_c \rightarrow h_c\ell\nu_\ell)$ | $\Gamma(B_c \rightarrow \chi_c\ell\nu_\ell)$ | $\Gamma(B_c \rightarrow \chi_c\ell\nu_\ell)$ | $\Gamma(B_c \rightarrow \chi_c\ell\nu_\ell)$ |
|----------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| $e(\mu)$       | 2.509                                    | 1.686                                    | 2.206                                    | 2.732                                    |
| $\tau$         | 0.356                                    | 0.249                                    | 0.346                                    | 0.422                                    |

One may see clearly from Fig.2 and the table that the branching ratios of the above semileptonic decays (a lot of nonleptonic decays to) are sizable if bearing $B_c$ lifetime $\tau_{B_c} \simeq 0.5$ ps in mind, especially, $B_c \rightarrow h_c\nu_\ell$, can be used as a new window to observe $h_c$.

3 The Decays of $B_c$ Meson to Leptons $l^+\nu_l$ with Gluon Bremsstrahlung

There are two kinds of $B_c$ decays to leptons with gluon bremsstrahlung. One is those induced by color singlet components of $B_c$ and the other is those induced by color octet components of $B_c$. In QCD the lowest order, the one induced by the color singlet components is a decay with two gluon bremsstrahlung $B_c[1^1S_0] \rightarrow \bar{l} + \nu_l + g + g$, whereas the ones induced by color octet components are the decays with one gluon bremsstrahlung $B_c[8^3S_1] \rightarrow \bar{l} + \nu_l + g$ and $B_c[8^1S_1] \rightarrow \bar{l} + \nu_l + g$.

The color octet ones are of one order lower than that of the color singlet one, but based on NRQCD velocity scale rule, the situation for the two kinds decays is inverse, so these two kinds of processes may be comparable based on magnitude order consideration. Therefore we computed them together and tried to see the possibility for experimental verification of the color octet components.

The Feynman diagrams relevant to color singlet one and the imaginary part of Feynman diagrams relevant to the widths of the color octet ones are shown in Fig.3. The final results of the calculated decay widths are proportional to relevant matrix elements accordingly.

According to the velocity scale rule of NRQCD, the relative magnitude order of the matrix elements for the color octet and color singlet components can be estimated. Moreover the wave

\footnote{Calculations on the nonleptonic decays are straightforward as long as the effective Lagrangian including QCD corrections and ‘factorization assumption’ are adopted. The results can be found in Ref.[3].}
The Feynman diagrams: left three Feynman diagrams are typical ones for the decay $B_c \rightarrow \bar{l}\nu g g$ (there are six diagrams in total for the decay and the other three can be obtained with the two gluons ‘crossed’).

The right four diagrams are those related to the widths for the decays $B_c[8^3S_1] \rightarrow \bar{l}\nu g$ and $B_c[8^1P_1] \rightarrow \bar{l}\nu g$ respectively (Each of them has a cut line and a dotted long gluon line. The former indicates to take imaginary part of the diagrams and the later indicates a very soft gluon.)

The energy spectra of the charged lepton for $B_c$ decays to leptons with gluon bremsstrahlung. The left figure is when color octet matrix elements are taken to be ten percent of color singlet one. The right figure is when the color octet matrix elements are taken to be thirty percent of the color singlet one. The dashed line in the figures stands is of the color singlet decay $B_c \rightarrow \bar{l}\nu g g$. The dotted and the solid lines are of the color octet decay $B_c \rightarrow \bar{l}\nu l g$ with the $(c\bar{b}[8^3S_1])$ and $(c\bar{b}[8^1P_1])$ components respectively.

It is interesting to see from Fig.4 that, in the region around the end point of the spectra of the charged lepton, the contributions from color octet components becomes greater than those of color singlet, so that potentially it is possible to verify the color octet components in $B_c$ meson through measuring the spectra carefully.

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