The shallow slope of the $z \sim 6$ quasar luminosity function: limits from the lack of multiple-image gravitational lenses

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ABSTRACT

We place a limit on the logarithmic slope of the luminous quasar luminosity function at $z \sim 6$ of $\beta \gtrsim -3.0$ (90 per cent) using gravitational lensing constraints to build on the limit of $\beta \gtrsim -3.3$ (90 per cent) derived from an analysis of the luminosity distribution by Fan et al. This tight constraint is obtained by noting that, of the two quasars that are lensed by foreground galaxies, neither is multiply imaged. These observations are surprising if the luminosity function is steep because magnification bias results in an overabundance of multiply imaged relative to singly imaged lensed quasars. Our Bayesian analysis uses the a posteriori information regarding alignments with foreground galaxies of the two lensed quasars, and provides a constraint on $\beta$ that is nearly independent of the uncertain evolution in the lens population. The results suggest that the bright end of the quasar luminosity function continues to flatten out to $z \sim 6$, as is observed between $z \sim 3$ and $\sim 5$. Provided that SDSS J1148+5251 at $z = 6.37$ is magnified by an intervening lens galaxy at $z \sim 5$, we also show that the high lens redshift in this system (if it is real) implies a comoving density of massive galaxies that may be close to constant out to high redshift. The combination of constraints on the quasar luminosity function and lens galaxy evolution is used to compute an improved estimate for the $z \sim 6$ multiple-image lens fraction of $\sim 1$–3 per cent.

Key words: gravitational lensing – quasars: general.

1 INTRODUCTION

The quasar luminosity function (LF) is the most basic property of the quasar population. At low redshifts several decades of study have yielded a well-defined optical quasar LF with power-law slopes at both the faint and bright ends that do not evolve out to $z \sim 3$ (e.g. Boyle, Shanks & Peterson 1988; Hartwick & Schade 1990; Pei 1995a; Boyle et al. 2000). At higher redshifts only very bright quasars are currently observable at the magnitude limit of large surveys, and recent evidence suggests that the slope of their LF is significantly shallower than observed at $z \lesssim 3$ (Schmidt, Schneider & Gunn 1995; Fan et al. 2001a). There are now six quasars known with redshifts $z \gtrsim 5.8$ (Fan et al. 2001b, 2003). These very high-redshift quasars provide important constraints for studies of structure formation (Turner 1991; Haiman & Loeb 2001) and reionization (e.g. Madau, Haardt & Rees 1999; Wyithe & Loeb). Determination of their LF (Fan et al. 2003) is critical if we are to address these issues.

As a population of sources for gravitational lensing the $z \sim 6$ quasars are unique, being the only sample where the gravitational lensing probability may be of the order of unity (Wyithe & Loeb 2002a,b; Comerford, Haiman & Schaye 2002). The high expected lensing rate arises through a large magnification bias which increases the fraction of gravitational lenses at a given flux level by drawing sources from the fainter, more numerous population into a flux-limited sample. As a result, the fraction of quasars in a sample that are multiply imaged by gravitational lenses is sensitive to the slope of the LF. Conversely, the observed multiple-image lens fraction may be used to limit the unknown slope of a LF. This exercise was undertaken by Fan et al. (2003). They presented likelihood functions for $\beta$ given the absence of multiply imaged quasars in the $z \sim 6$ sample, and found the lack of lenses to be surprising at the $\sim 90$ per cent level if $\beta \lesssim -3.5$. This lensing constraint is consistent with their findings for $\beta$ through direct analysis of the luminosity distribution in the sample which yielded $\beta \gtrsim -3.35$ (90 per cent). However, there is a complication. The lens fraction is linearly related (nearly) to the efficiency of the lens population, which is proportional to the expectation value of the velocity dispersion to the fourth power and to the space density of galaxies. Moreover, the lensing rate requires extrapolation of the local galaxy population to higher redshifts, and is also sensitive to cosmology (e.g. Kochanek 1996). Thus the unbiased lensing cross-section is quite uncertain.

While high-resolution imaging data (Fan et al. 2001b, 2003; Richards et al. 2004) shows none of the six $z \sim 6$ quasars to be multiply imaged, galaxies have been detected near the line of sight to two quasars (Shioya et al. 2002; White et al. 2003). This is
puzzling because magnification bias should result in highly magnified multiply imaged sources being over-represented among a population of quasars the images of which are located near foreground galaxies. The effect becomes larger as the quasar LF becomes steeper. In this paper we present a Bayesian analysis that employs information on a posteriori alignments of the two lensed quasars. This statistic is much less sensitive to the uncertainties in the lens cross-section than is the fraction of multiply imaged quasars, and we show that it produces a tighter limit on $\beta$.

The paper is set out as follows. In Section 2 we compute the gravitational lens cross-section in light of the recently measured velocity function of galaxies, the probability of multiple imaging for different LFs and the limits on the quasar LF that result from the observed lack of multiply imaged quasars. We then discuss a Bayesian approach to computing the lens fraction in Section 3. In Section 4 we describe the two high-redshift quasars thought to be magnified by gravitational lensing. The probability of getting a multiply imaged quasar within a subsample of quasars observed to be near a lens galaxy is discussed in Section 5. These probabilities are used to compute likelihood functions for the fraction of lensed singly imaged quasars, and to derive limits on $\beta$. In Section 6 we use the redshifts of the lens galaxies to constrain simple parametric models for the evolution of the lens galaxy population. We then combine these results with the limits on $\beta$ to estimate the expected multiple-image fraction for $z \sim 6$ quasars in Section 7. Finally in Section 8 we discuss the implications of possible multiple imaging in SDSS J1148+5251 for $\beta$ before presenting our conclusions in Section 9. Where required, we assume the most recent cosmological parameters obtained through fits to WMAP data (Spergel et al. 2003). These include density parameters of $\Omega_m = 0.27$ in matter, $\Omega_b = 0.044$ in baryons and $\Omega_{\Lambda} = 0.73$ in a cosmological constant, and a Hubble constant of $H_0 = 71$ km s$^{-1}$ Mpc$^{-1}$.

2 THE GRAVITATIONAL LENS CROSS-SECTION AND MULTIPLE-IMAGING RATE

The probability that a singular isothermal sphere (SIS) galaxy will lens a background source is proportional to the fourth power of its velocity dispersion $\sigma$. Thus what is required to compute the cross-section for gravitational lensing is the velocity function. Until recently the velocity function for early-type galaxies (the dominant lens population; Kochanek 1996) had to be computed through combination of a galaxy LF with the Faber & Jackson (1976) relation. However, this procedure ignores the intrinsic scatter in the Faber & Jackson relation and is an unreliable method (e.g. Kochanek 1993). A more reliable representation is now possible using the measured velocity dispersion function of early-type galaxies. Sheth et al. (2003) presented the measured velocity dispersion function for early-type galaxies. They suggested an analytic fit of the form

$$\phi(\sigma) = \phi_0 \left(\frac{\sigma}{\sigma_*}\right)^\alpha \exp\left[\frac{-\left(\sigma/\sigma_*\right)^\alpha}{\Gamma(\alpha/\beta)}\right] \frac{d\sigma}{\sigma},$$

(1)

where $\phi_0$ is the number density of galaxies and $\sigma_*$ is a characteristic velocity dispersion. Sheth et al. (2003) found that the parameters $\sigma_*$, $\alpha$ and $\beta$ are strongly correlated with one another. From Sheth et al. we take $\phi_0 = (2.0 \pm 0.1) \times 10^{-5}(h_0)^{-2}$ Mpc$^{-3}$, $\alpha = 6.5 \pm 1.0$, $\beta = (14.75/\alpha)^{1/\alpha}$ and $\sigma_* = 1611(\alpha/\beta)/\Gamma[(\alpha + 1)/\beta]$ km s$^{-1}$.

Given the Einstein radius (ER) for an SIS

$$\xi_0 = 4\pi c \left(\frac{\sigma}{c}\right)^2 \frac{D_s D_b}{D_L},$$

(2)

(where $D_s$, $D_a$ and $D_b$ are the angular diameter distances of the lens and source, and from the source to the lens), and a constant comoving density of galaxies, the multiply imaged gravitational lens cross-section for sources at $z_i$ is

$$\tau(z_i) = \int_0^{z_i} \int_0^{\infty} dz \frac{1}{D_s} \frac{c}{dz} \pi \xi_0^2.$$  

(3)

We found the mean and twice the variance for a set of $\tau$ computed assuming the parameters $\phi_\star$ and $\alpha$ to be distributed as Gaussian within their quoted uncertainties. This procedure yields $\tau(z_i = 6) = (2.5 \pm 0.25) \times 10^{-3}$. The statistical uncertainty of 10 per cent is significantly smaller than is obtained through use of the Faber & Jackson (1976) relation and a LF. The value of $\tau(z_i = 6)$ obtained from the velocity function is a factor of $\sim 3$ smaller than obtained in estimates of the lens fraction in the very high-redshift quasar samples (Wyithe & Loeb 2002a,b; Comerford et al. 2002). While the implied lens fraction for these samples is still expected to be an order of magnitude higher than in lower redshift samples, it may result in fewer than one lens being expected in the current sample, hence limiting the use of the lens fraction for constraining the slope of the LF (Wyithe & Loeb 2002b; Comerford et al. 2002; Fan et al. 2003).

As shown above, the use of a measured velocity function reduces the statistical uncertainty in $\tau$ to 10 per cent. However, the remaining systematic error due to the uncertain evolution in the lens population (e.g. Keeton 2002) makes limits based on the lens fraction uncertain. For example, if the comoving density of galaxies drops in proportion to $(1 + z)^{-\gamma}$ (density evolution), then for $\gamma = 1$ we find $\tau(z_i = 6) = (0.9 \pm 0.09) \times 10^{-3}$, while for $\gamma = 2$ the value drops to $\tau(z_i = 6) = (0.4 \pm 0.04) \times 10^{-3}$. Unless otherwise specified, we assume a constant comoving density of galaxies ($\gamma = 0$). We consider only spherical lenses in this paper. Previous studies (Kochanek & Blandford 1987; Blandford & Kochanek 1987) have found that the introduction of ellipticities $\leq 0.2$ into nearly singular profiles has little effect on the lensing cross-section and image magnification. However, the strong magnification bias will favour a high fraction of four-image lenses (Rusin & Tegmark 2001). Finally, we note that the $i$-band dropout quasars are selected independently of morphology, and so do not select against lenses, although the possibility that the lens galaxy itself will prevent detection should be accounted for (Wyithe & Loeb 2002b).

With these points in mind, we compute the a priori probability of multiple imaging in a sample of six quasars with $z \sim 6$ for different LFs. The six quasars with which we compare our calculations form the complete sample described by Fan et al. (2003). A similar calculation has already been performed by Fan et al. (2003), although we add the improvement of a more accurate $\tau$ computed from the velocity function, extend the calculation to account for constraints from the luminosity distribution (Fan et al. 2003), and provide a posteriori limits on $\beta$. We also modify $\beta$ to include only those lens galaxies that would not have contaminated the $i$-band dropout selection of the quasars as discussed by Wyithe & Loeb (2002b). We compute the fraction of multiply imaged sources without imposing selection criteria, which could result in an overestimate of the lens fraction. Nevertheless, these calculations may be compared with real data which have detection limits on both the minimum flux ratio and the image separation. This is because magnification bias results in most lens systems having similar flux ratios, while there is only a small probability for image separations that lie below the detection limit (0.1–0.4 arcsec) of the observations described by Fan et al. (2003) and Richards et al. (2004). The probability that a quasar with luminosity $L$ and magnification bias $B$ will be multiply imaged by a foreground galaxy is

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The probability, given a LF $\Phi_1$, is

$$p_{\text{len}}(L) \sim \frac{B(L)\tau}{B(L)\tau + (1-\tau)},$$

where $p_{\text{len}}$ is slightly overestimated because we have not included any magnification bias for single-image quasars. The sum of magnifications of multiple images ($\mu$) formed by an SIS has a probability distribution of the form

$$\frac{dp_{\text{len}}}{d\mu} = \frac{\delta}{\mu^3} \quad \text{for} \quad \mu \geq 2,$$

resulting in a magnification bias $B(L)$ for an SIS and a LF $\Phi(L)$ of

$$B(L) = \int_2^\infty \frac{d\mu}{\mu^3} \frac{\Phi(L/\mu)}{\Phi(L)}.$$

The probability, given a LF $\Phi$, that in a sample of six quasars at $z \sim 6$ none will be lensed is

$$p_{\text{nolens}} = \prod_{i=1}^{N_q} [1 - p_{\text{len}}(L_i)],$$

where $N_q = 6$ and the $L_i$ are the luminosities of the $N_q$ quasars.

### 2.1 Lens fractions for double-power-law luminosity functions

A successful fit to the low-redshift ($z \lesssim 2$) quasar LF is the double power law (e.g. Boyle et al. 2000)

$$\Phi(L) = \frac{\Phi_0}{L^{-\alpha} + L^{-\beta}} ,$$

where $\alpha$ and $\beta$ (note that we have defined $\alpha$ and $\beta$ to be negative) are the slopes of the faint and bright ends of the quasar LF respectively, and we have expressed the luminosity $L$ in units of the characteristic break luminosity. Use of the double-power-law form for $\Phi(L)$ implies that we must specify two additional parameters before deriving limits on $\beta$. First, since high magnifications will draw quasars that are fainter than the break into the sample, the magnification bias will depend on $\alpha$. Throughout the paper we present results for three different values of $\alpha = -1.0, -1.75$ and $-2.5$. These values are arbitrary; however, they cover a range of slopes from one that is shallower than is observed at low redshifts, through to one that is equivalent to the slope of the luminous quasar luminosity function at $z \sim 4.3$. In addition, the use of the double power law requires specification of the quasar luminosity with respect to the break. The $z \sim 6$ quasars range in luminosity from $M_{1450} = -27.15$ to $-27.90$ (Fan et al. 2001b, 2003). In this paper we specify the luminosity of the quasars relative to the LF in terms of the difference between the magnitude of the faintest quasar and that of the break ($\Delta M = 2.5 \log_{10}(L)$. For example, if the break were at $M_{1450} = -26.15$ then $\Delta M = 1$.

The resulting probability $p_{\text{nolens}}$ that, in the sample of six $z \sim 6$ quasars, none will be multiply imaged (given $\alpha = -1.75, \Delta M = 4$) is plotted as a function of $\beta$ in the left hand panels of Fig. 1 (thick dashed grey line). The lack of multiply imaged quasars is only surprising at the 50 per cent level for values of $\beta < -3.3$. The probability $p_{\text{nolens}}$ is also tabulated in Table 1 for various values of $\alpha, \beta$ and $\Delta M$. We find that the lack of multiply imaged quasars is only surprising at the 10 per cent level if $\beta \lesssim -4$ and $\Delta M \gtrsim 3$ or if $\beta \lesssim -3.75$ and $\Delta M \gtrsim 4$. We note that these probabilities (and those in fig. 9 of Fan et al. 2003) represent a likelihood function for $\beta$ rather than direct limits on $\beta$. We now turn to computation of these limits.

The posterior probability for $\beta$ is

$$\frac{dP}{d\beta}\bigg|_{\beta=0} = N p_{\text{nolens}} \frac{dP_{\text{prior}}}{d\beta} ,$$

where $dP_{\text{prior}}/d\beta$ is the prior probability for $\beta$, and $N$ is a normalizing constant. We assume that the prior probability for the slope is flat

![Figure 1](https://academic.oup.com/mnras/article-abstract/351/4/1266/1133061)

**Figure 1.** Left: likelihood functions for $\beta$. Centre: differential probability distributions for $\beta$. Right: cumulative probability distributions for $\beta$. The lensing constraints based on the fraction of multiply imaged quasars in the sample are shown by the dashed grey curves. The lensing constraints that include information on the alignments between quasars and foreground galaxies are given for different values of $\alpha$ (dark lines). The solid grey curves in each panel correspond to the likelihood and probability functions for $\beta$ based on the luminosity distribution alone (Fan et al. 2003). Results are shown for a double-power-law LF with $\alpha = -1.75$ and $\Delta M = 4$. A constant comoving density of lens galaxies was assumed.

**Table 1.** The probability, given $\alpha$, that in a sample of six $z \sim 6$ quasars none will be lensed. The values are tabulated assuming a constant comoving density of lens galaxies and various values of $\alpha$ and $\Delta M$.

| $\alpha$ | $\Delta M$ | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
|----------|-----------|---|---|---|---|---|---|---|---|---|---|---|---|
| $\beta = -4.0$ | 0.77 | 0.42 | 0.11 | 0.01 | 0.70 | 0.35 | 0.08 | 0.006 | 0.47 | 0.18 | 0.03 | 0.002 |
| $\beta = -3.75$ | 0.82 | 0.57 | 0.28 | 0.08 | 0.08 | 0.77 | 0.51 | 0.24 | 0.07 | 0.56 | 0.33 | 0.13 | 0.03 |
| $\beta = -3.5$ | 0.87 | 0.70 | 0.50 | 0.32 | 0.63 | 0.47 | 0.29 | 0.06 | 0.66 | 0.51 | 0.37 | 0.23 |
| $\beta = -3.0$ | 0.92 | 0.86 | 0.78 | 0.72 | 0.89 | 0.84 | 0.78 | 0.72 | 0.78 | 0.75 | 0.72 | 0.70 |
| $\beta = -2.5$ | 0.96 | 0.93 | 0.91 | 0.90 | 0.94 | 0.93 | 0.91 | 0.90 | 0.87 | 0.87 | 0.87 | 0.87 |
| $\beta = -2.0$ | 0.97 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.92 | 0.91 | 0.90 | 0.89 |

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between two bounds $\beta_{\text{min}}$ and $\beta_{\text{max}}$, hence
\begin{equation}
\frac{dP_{\text{prior}}}{d\beta} = (\beta_{\text{max}} - \beta_{\text{min}})^{-1}.
\end{equation}

The absence of multiply imaged quasars in the sample is surprising if the slope is steep ($\beta$ is small) because the multiple-image magnification bias is large in that case, but is less surprising as $\beta$ is increased (shallower slopes). Hence the likelihood function $p_{\text{a lens}}$ is increasing with $\beta$ so that the fraction of multiply imaged quasars carries no information on its upper limit. As a result, the confidence with which a steep slope can be excluded given the lensing observations alone is sensitive to $\beta_{\text{max}}$. This dependence implies that a small probability $p$ given some $\beta$ that a sample of quasars would contain no lenses does not translate to a limit on $\beta$ at the $100(1-p)$ per cent level. A second, independent constraint that limits the upper bound on $\beta$ and hence lowers the dependence on $dP_{\text{prior}}/d\beta$ is required.

2.2 The addition of constraints from the luminosity distribution

Fan et al. (2003) have derived the constraint $\beta \pm \Delta \beta$ where $\beta = -2.3$ and $\Delta \beta = 0.8$ from their analysis of the luminosity distribution of the $z \sim 6$ quasars. We take this bound to be the $1\sigma$ level of a Gaussian likelihood function for $\beta$. We have plotted this likelihood function for $\beta$ (normalized to a maximum of 1), as well as the differential and cumulative probability distributions for $\beta$, in the in the left-hand, central and right-hand panels of Fig. 1 (solid grey lines).

We now have two constraints on $\beta$, one from the fraction of multiply imaged quasars and one from the distribution of luminosities. Assuming these constraints to be independent (this assumption is discussed below), we find a joint likelihood function, and hence posterior probability distribution for $\beta$:
\begin{equation}
\frac{dP}{d\beta}|_{\beta_{\text{max}}=0} = Np_{\text{a lens}} \exp \left[ -\frac{1}{2} \left( \frac{\bar{\beta} - \beta}{\Delta \beta} \right)^2 \right] \frac{dP_{\text{prior}}}{d\beta}.
\end{equation}

The likelihood function is now normalizable because the distribution of luminosities constrains shallow slopes with large values of $\beta$ (Fan et al. 2003), and is quite insensitive to the prior $dP/d\beta|_{\beta_{\text{max}}=0}$ as a result. We plot the posterior differential and cumulative probability distributions in the central and right-hand panels of Fig. 1. These are shown by the dashed light lines, and should be compared to the posterior probability distributions based on the distribution of luminosities alone (solid light lines). The lensing constraint disfavours steeper slopes (smaller values of $\beta$), resulting in a narrower probability distribution for $\beta$. The most likely value for the slope is $\sim -2.2$. The addition of the constraint from the fraction of multiply imaged lensed quasars improves on the limits obtained by Fan et al. (2003) from the distribution of luminosities alone. For this choice of $\Delta M$ and $\alpha$, the lack of multiply imaged quasars rules out $\beta \lesssim -3.0$ at the 90 per cent level. We also construct posterior cumulative probability distributions for various values of $\alpha$ and $\Delta M$ and plot the results in Fig. 2 (thick dashed light lines). The addition of lensing constraints significantly improves the LF limits provided that the quasars are not too close to the LF break. For a constant comoving density of lens galaxies (upper two rows) we find $\beta \gtrsim -3.2 \rightarrow -3.0$ (90 per cent) for $\Delta M \gtrsim 2$.

It should be noted that the two constraints are not quite independent. In general gravitational lensing tends to flatten the slope of the quasar LF by drawing populous faint quasars into a bright quasar sample (e.g. Pei 1995b). However, there are two reasons to think that this is not a problem within the very high-redshift quasar sample. First, the objects that are lensed are not multiply imaged and so have magnifications smaller than $\sim 2$. Secondly, the average change of slope is $\Delta \beta \sim 0.2$ even in the most optimistic lensing scenario (where $\beta \sim -3.5$) for the $z \sim 6$ quasars (Wyithe & Loeb 2002b).

3 BAYESIAN APPROACH TO COMPUTING THE MULTIPLE-IMAGE FRACTION

We may also use a Bayesian approach to compute the multiple-image fraction. This will provide us with a natural framework within which to add additional a posteriori information on alignments of quasars with foreground galaxies in Section 5. Consider sources with unlensed impact parameters (in units of the ER) $y = x - 1$ with associated magnifications $\mu$. We write the likelihood per logarithm of $x$ of observing a singly imaged lensed quasar (including magnification bias)
\begin{equation}
L_{\text{single}} = x(x-1) \frac{1}{\mu_{\text{single}}} \frac{\Phi(L/\mu_{\text{single}})}{\Phi(L)},
\end{equation}

where the factor $(x-1)$ accounts for the additional solid angle available at large $y$, and $\mu_{\text{single}} = x/(x-1)$. This likelihood may be compared to the corresponding average likelihood of observing a multiply imaged quasar
\begin{equation}
L_{\text{mult}} = \int_0^1 dy \left( L_{\text{single}} + L_{\text{mult}} \right).
\end{equation}

The likelihood that a quasar will be singly imaged at $x$ rather than multiply imaged is therefore
\begin{equation}
p_{\text{single}}(\beta | x) = \frac{L_{\text{single}}}{L_{\text{single}} + L_{\text{mult}}},
\end{equation}

We may also calculate the posterior probability that a quasar will not be lensed:
\begin{equation}
p_{\text{lens, bayes}} = \int_2^{\infty} dx p_{\text{single}}(\beta | x) \frac{dP_{\text{prior}}}{dx},
\end{equation}

where $dP_{\text{prior}}/dx$ is the prior probability for the $x$. This quantity is 1 minus the lens fraction and may be approximated using the usual formula for the lens fraction $\tau B$ or more accurate forms such as equation (4).

For the sample of $z \sim 6$ quasars, the posterior probability distribution for $\beta$ is therefore
\begin{equation}
\frac{dP}{d\beta} = N \exp \left[ -\frac{1}{2} \left( \frac{\bar{\beta} - \beta}{\Delta \beta} \right)^2 \right] \frac{dP_{\text{prior}}}{d\beta} \times \left\{ \prod_{i=1}^{M} \left[ \int_2^{\infty} dx p_{\text{single}}(\beta | x, L_i) \frac{dP_{\text{prior}}}{dx} \right] \right\},
\end{equation}

where the prior probability distribution for $x$ can be computed from the derivative of the Poisson probability that a source lies within a circle of radius $x - 1$ around a randomly positioned galaxy:
\begin{equation}
\frac{dP_{\text{prior}}}{dx} = 2\pi(x-1)\exp[-(\pi(x-1)^2)].
\end{equation}

Equation (17) yields identical limits to those based on the multiple-image fraction (dashed light lines in Figs 1 and 2) as computed in the usual way from equation (4). Moreover, the magnification distribution for singly imaged sources is naturally normalized within
Figure 2. Cumulative probability distributions for $\beta$. The lensing constraints based on the fraction of multiply imaged quasars in the sample are shown by the dashed grey curves. The lensing constraints that include information on the alignments between quasars and foreground galaxies are denoted by the dark lines. The solid grey curves correspond to the probability functions for $\beta$ based on the luminosity distribution alone (Fan et al. 2003). Results are shown for a double-power-law LF for various values of $\alpha$ and $\Delta M$. The upper and lower two rows show results assuming density evolution with $\gamma = 0$ and 2 respectively.

The formalism, and hence the magnification bias of singly imaged sources is directly included in the calculation.

4 TWO LENSED $z \sim 6$ QUASARS

While none of the six $z \sim 6$ quasars discovered by Fan et al. (2001b, 2003) in the Sloan Digital Sky Survey data is multiply imaged, two have close alignment with a foreground galaxy, implying that they are moderately magnified. We therefore refer to these quasars as lensed, although neither is multiply imaged.

4.1 SDSS J1044−0125 at $z = 5.74$

Shioya et al. (2002) have reported a faint foreground galaxy with $z \sim 1.5–2.5$ at a separation of $\theta = 1.9$ arcsec. Shioya et al. (2002) estimate the velocity dispersion to be $\sigma \sim 140–280$ km s$^{-1}$ for this redshift range. A second image would be detectable if $\sigma \gtrsim 220$ km s$^{-1}$. For an SIS galaxy this implies that the magnification of the image could be as high as $\mu = 2$, and that the image is located at $x \sim 2–10$ ER (where $x \sim 10$ ER corresponds to a $\sigma \sim 140$ km s$^{-1}$ SIS at $z \sim 2.5$).
4.2 SDSS J1148+5251 at $z = 6.42$

This is one of two quasars known to exhibit Gunn–Peterson absorption troughs. White et al. (2003) present spectra showing emission features in the Ly$\beta$ trough which they interpret as being Ly$\alpha$ emission from a foreground galaxy at $z \sim 4.9$. This is a likely scenario if gravitational lensing is important and was predicted by Wyithe & Loeb (2002b). An updated redshift ($z = 6.42$) was provided by Walter et al. (2003). While the alignment is presumed high, we do not know the degree of alignment with the foreground galaxy, and hence we cannot know $x$, although if the quasar is not multiply imaged it must be larger than 2, and is probably comparable to the case of SDSS J1044–0125. Walter et al. (2003) do, however, provide an estimate of $\sigma \sim 250 \text{ km s}^{-1}$ for the foreground galaxy from the velocity structure seen in the C IV absorption system. For an SIS at $z = 4.9$ lensing a source at $z = 6.37$ with a separation $\theta$ we have

$$x = 4 \left( \frac{\theta}{1 \text{arcsec}} \right)^2 \left( \frac{\sigma}{250 \text{ km s}^{-1}} \right)^{-2} \text{ER.} \quad (19)$$

5 IMPROVED LIMITS ON $\beta$ FROM CLOSE ALIGNMENTS WITH FOREGROUND GALAXIES

The Bayesian approach (Section 3) to computing limits on $\beta$ from the multiple-image fraction allows us to include the a posteriori information on the alignments of the two lensed $z \sim 6$ quasars.

For illustration, we begin with a hypothetical sample of six quasars with $\Delta M = 4, \sigma = -1.75$ and (lensed) impact parameter $x$. The relative likelihoods for different values of $\beta$ given the lack of multiple images are $p_{\text{angle}}(\beta | x) \propto \sigma$ as specified in equation (15) where $N_{\text{q}} = 6$. These likelihoods are shown in the left-hand panel of Fig. 1 for values of $x$ ranging from 5 to 20 (dark lines). Smaller values of $\beta$ (steeper slopes) are strongly disfavoured, particularly if $x$ is not too large. Next we find the joint likelihood function and hence a posterior probability distribution for $\beta$ given a common impact parameter $x$ for six quasar pairs that are not multiply imaged:

$$\frac{dP}{d\beta} = N \left[p_{\text{angle}} \right]^{6} \exp \left[-\frac{1}{2} \left( \frac{\beta - \bar{\beta}}{\Delta \beta} \right)^{2} \right] \frac{dP_{\text{prior}}}{d\beta}. \quad (20)$$

The resulting posterior and cumulative probability distributions are shown in the central and right-hand panels of Fig. 1 (dark lines). The most likely value is near $\beta \sim -2$ and the cumulative distributions suggest that $\beta \geq -3.1 \rightarrow -2.7$ at the 90 per cent level where the systematic dependence is on $x$. Thus any additional information about close alignments produces constraints that may significantly tighten the lower limits on the slope of the $z \sim 6$ quasar LF, with improvements in the limit that are greater than 0.5 units in $\beta$ for cases where the alignment is high.

The above example suggests strong dependence of the limits derived for $\beta$ on the value of $x$. For the two lensed SDSS quasars discussed in Section 4, there are observational limits on $x$ in the form

$$\frac{dP}{dx} \propto L_{\sigma} \frac{dP_{\text{prior}}}{dx}, \quad (21)$$

where $L_{\sigma}$ is the likelihood for $x$ given the observations of $\sigma$ and $z$ for the lens galaxy, and $dP_{\text{prior}}/dx$ is the prior probability for $x$. Given the relation $x = x(\sigma, z)$, the likelihood $L_{\sigma}$ is

$$L_{\sigma} = \frac{dx}{d\sigma} L_{\sigma} \frac{dP_{\text{prior}}}{dx} \frac{dL_{\sigma}}{dz} \frac{dP_{\text{prior}}}{dz}. \quad (22)$$

For SDSS J1044 – 0215 Shioya et al. (2002) find the majority of the dependence in the likelihoods for $\sigma$ and $z$ to be systematic, while there is no information in this regard for SDSS J1044 – 0215 (White et al. 2003). We assume flat distributions $dP/\sigma$ with limits of $2 < x < 10$ for the two lensed quasars. For the other four quasars we assume the prior probability distribution for $x$ (equation 18).

The posterior differential probability distribution for $\beta$ then becomes

$$\frac{dP}{d\beta} = N \exp \left[-\frac{1}{2} \left( \frac{\beta - \bar{\beta}}{\Delta \beta} \right)^{2} \right] \frac{dP_{\text{prior}}}{d\beta} \times \left\{ \prod_{i=1,2} \left[ \int_{-\infty}^{\infty} dx_{i} p_{\text{angle}}(\beta | x_{i}, L_{i}) \right] \right\} \times \left\{ \prod_{i=3,6} \left[ \int_{-\infty}^{\infty} dx_{i} p_{\text{angle}}(\beta | x_{i}, L_{i}) dP_{\text{prior}} \right] \right\}. \quad (23)$$

The resulting cumulative probability distributions are shown in Fig. 2. The limits on $\beta$ are significantly tighter than those obtained from the distribution of luminosities alone except in cases where $\alpha$ is large (shallow faint-end slope) and $\Delta M$ is small (so that magnifications associated with multiple images tend to draw quasars with $\Delta M < 0$ into the sample). In addition, the limits are tighter than those obtained through consideration of the luminosity distribution and multiple-image fraction (dashed light lines). For a constant comoving density of lens galaxies and $\Delta M \geq 2$ we find $\beta \geq -3.1 \rightarrow -3.0$, while for $\Delta M \geq 4$ we obtain $\beta \geq -3.0 \rightarrow -2.9$ (both with 90 per cent confidence). Thus the tightest limits come from the inclusion of the a posteriori information that two of the quasars have close alignment with foreground galaxies. At this point we note that if the possible lens galaxy at $z = 4.96$ were not real, then the constraints would be relaxed. However, a single lens galaxy in the sample would still result in improved limits on $\beta$.

A second important point regarding the limit on $\beta$ provided by equation (23) is that, unlike the limit from the multiple-image fraction, it is nearly independent of the value of $\tau$. To demonstrate this independence we have computed constraints on $\beta$ (shown in the lower two panels of Fig. 2) that assume a dependence in the comoving density of galaxies of $(1 + z)^{-\gamma}$ where $\gamma = 2$ (resulting in $\tau = 0.0004$). These limits may be compared with results that assume a constant comoving density of lens galaxies (in the upper two panels of Fig. 2). The limits obtained from the multiple-image fraction are much weaker if $\gamma = 2$. On the other hand, the limits that use a posteriori observations of the quasar–lens galaxy alignment are quite insensitive to $\gamma$. The reason is that the role of $\tau$ is replaced by $dP/d\sigma$ for the two quasars that provide the largest contribution to the likelihood change between large and small values of $\beta$.

5.1 A posteriori choice of statistic

We have computed limits on the value of $\beta$ using two lensing-based constraints, and a posteriori chosen the better one. This practice becomes unfair if a large number of different constraints are available where each produces a different limit. In the situation described we have two different lensing constraints. However, the second constraint utilizes additional rather than different information. Thus we are justified in choosing it a posteriori.
Figure 3. Left: the differential lens cross-section for a source at $z = 6.37$ for different values of $\gamma$. Centre: the corresponding fraction of lens galaxies at redshifts larger than $z$. Right: the posterior cumulative probability for $\gamma$. The upper and lower rows correspond to velocity and density evolution respectively.

6 LIMITS ON GALAXY EVOLUTION FROM LENS GALAXY REDSHIFTS

As noted by White et al. (2003), the candidate lens galaxy in the system SDSS J1148 + 5251 is found at an improbably high redshift, which could provide an argument against the lens hypothesis in this system. This is quantified in Fig. 3, where the upper curves show the differential cross-section for a source at $z = 6.37$ lensed by a constant comoving density of galaxies (left-hand panel), and the fraction of the total cross-section that is found at a redshift larger than $z$ (right-hand panel). We see that the prior probability of finding a lens at $z \gtrsim 4.94$ among two lensed quasars is only $\sim 0.01$. However, the probability of finding a lens at high redshift is an a posteriori statistic, i.e. we have chosen one of a possible number of a priori unlikely events after the observation has already been made. Moreover, the selection of lenses within the sample is not uniform in redshift. In particular, since the galaxy in front of SDSS J1148 + 5251 was identified spectroscopically via its Ly$\alpha$ emission line, it would be more easily identified at high redshift. In addition, the cross-sections plotted in Fig. 3 refer to multiple imaging so that the low probability for a high-redshift lens results from the small size of the Einstein ring radius at high redshift. In contrast, one can compute the redshift distribution of galaxies (with velocity dispersions larger than $\sigma$) that lie within some specified angular separation along the line of sight to a background quasar. This distribution implies that there would be one chance in $\sim 3$ of finding a galaxy with the observed alignment at $z > 4.96$. Magnification bias increases the likelihood of the lensing scenario, and probably results in an a posteriori probability of finding a high-redshift foreground galaxy that lies somewhere between these two extremes.

The redshift distribution of gravitational lenses may be used to constrain evolution in the lens galaxy population (Kochanek 1992; Ofek, Rix & Maoz 2003). While the absolute probability of having observed a high-redshift lens is difficult to quantify, we may more easily discuss the relative likelihoods of observing a high-redshift lens as a function of lens galaxy population. For definiteness we consider two parametrizations for the evolution of the lens galaxy population: first, evolution of the characteristic velocity $\sigma_*(z) = \sigma_* (1 + z)^{-\gamma}$; secondly, evolution of the characteristic density $\phi_*(z) = \phi_* (1 + z)^{-\gamma}$, which we term velocity and density evolution respectively. The left-hand panels of Fig. 3 demonstrate the effect on the lens cross-section of varying $\gamma$. Values of $\gamma$ that differ from 0 (constant comoving evolution) result in a lens population that is truncated at high redshift. This effect may also be seen in the central panels of Fig. 3 where we have plotted the fraction of cross-section at redshifts larger than $z$. Values of $\gamma > 0$ are disfavoured by the existence of a lensing galaxy at $z = 4.96$ among a sample of only two lens galaxies.

To quantify this statement we construct a likelihood function for $\gamma$ from the product of the normalized probabilities for the lens redshifts. The likelihood should include constants ($s_i$) to account for the relative detectabilities of the two lenses (the two galaxies were discovered separately via different techniques), although the limits on galaxy evolution are independent of these since the constants are
independent of the evolution. The likelihood function is

\[ L_{\gamma} = \prod_{i=1}^{2} \int_{z}^{\infty} \frac{dP_{i}}{dx}(x-1)^{2} \frac{dr_{i}}{dz} B_{i} \propto \prod_{i=1}^{2} \frac{dr_{i}}{dz} \]  

where the \( dr/dz \) are differential cross-sections evaluated at the lens and source redshifts, the integrals over the distributions \((dP_{i}/dx)(x-1)^{2}\) account for the relative alignments of the quasar and galaxy, and the \( B_{i} \) are the magnification biases. The relative likelihood is dependent only on the product of the differential cross-sections. Note that the likelihood (equation 24) is only applicable if the magnification bias has aided in selection of the quasar, so that the source may be considered lensed. This is a caveat to the constraints imposed on \( \gamma \) in this section.

In the right-hand panels of Fig. 3 we plot the posterior cumulative probability for \( \gamma \):

\[ P(< \gamma) = \int_{\gamma}^{\infty} N L_{\gamma} \frac{dP_{\text{prior}}}{dy} \]  

assuming a flat prior probability distribution for \( \gamma \) at values greater than 0. By excluding the possibility of \( \gamma < 0 \) we are assuming that the lens galaxy population increases monotonically in time as expected in hierarchical merging scenarios. This choice also leads to more conservative limits on \( \gamma \). We find \( \gamma \lesssim 0.4 \) and \( \lesssim 1.6 \) at the 90 per cent level assuming velocity and density evolution respectively. The possible presence of a lens galaxy at such a high redshift therefore offers an opportunity to constrain the (mass selected) comoving density of massive galaxies to be close to constant out to high redshifts. This result is consistent with the study of Ofek et al. (2003), who performed a detailed study on a large sample (15) of multiple-image lenses at \( z \sim 1 - 2 \).

The result that the (mass selected) comoving density of massive galaxies should be close to constant out to high redshifts relies very heavily on the existence of the possible lens galaxy at \( z \sim 4.96 \). Should this galaxy not be real, then no meaningful constraints can be imposed on \( \gamma \). On the other hand, a comoving density of lens galaxies that is close to constant out to \( z \sim 5 \) may not be too surprising in light of the Press & Schechter (1974) prediction for the velocity function of dark matter haloes (number per cubic comoving Mpc per unit velocity). Taking the circular velocity \( v_{\text{vir}} \) to equal the virial velocity of an SIS dark matter halo with mass \( M \) (Barkana & Loeb 2001), and a velocity dispersion \( \sigma = v_{\text{vir}}/\sqrt{2} \), we find

\[ \frac{dn}{d\sigma} = M \frac{dn}{dM} \frac{\sqrt{2}}{3v_{\text{vir}}} \]  

where \( dn/dM \) is the Press & Schechter (1974) mass function. The resulting velocity function of dark matter haloes is plotted in Fig. 4 at a series of redshifts. Note that near velocity dispersions of \( \sigma \sim 200 \text{ km s}^{-1} \), which dominate the lens cross-section, there is less than an or-

on \( \beta \) and \( \gamma \) in Sections 5 and 6 are independent. In Fig. 5 we have plotted the resulting joint probability function (dark contours); the contours shown are at 61, 26, 14 and 3.6 per cent of the peak value, corresponding to the 1σ, 2σ, 3σ and 4σ levels of a Gaussian distribution. The upper two and lower two rows of Fig. 5 correspond to velocity and density evolution respectively, and in each case results are shown for a double-power-law LF with various values of \( \alpha \) and \( \Delta M \). The figure shows that the preferred values are found near \( \gamma = 0 \) and \( \beta = -2.1 \). We also show contours of the multiple-image fraction (light contours); the solid, dashed, dot–dashed and dotted contours correspond to lens fractions of 0.1, 0.03, 0.01 and 0.003. We find that the multiple-image fraction should be \( \sim 1-3 \) per cent. This value is lower than previous estimates because of constraints on the shallow luminosity function, and unfortunately implies that a \( z \sim 6 \) lens may not be found among the complete sample of SDSS \( z \sim 6 \) quasars.

8 WHAT IF SDSS J1148+5251 WERE MULTIPLE IMAGED?

While current high-resolution imaging suggests that all of the \( z \sim 6 \) quasars are point sources, White et al. (2003) note that the possibility that SDSS J1148+5251 is multiply imaged cannot be ruled out by current observations owing to the small angular diameter of the Einstein ring in this system: \( \Delta \theta \sim 0.3 \text{ arcsec} \) for a \( \sigma = 250 \text{ km s}^{-1} \) galaxy at \( z = 4.94 \) lensing a quasar at \( z = 6.37 \). For comparison, the \( z \sim 6 \) quasars have been imaged at a resolution of \( \sim 0.1-0.4 \text{ arcsec} \) (Fan et al. 2003; Richards et al. 2004). It is therefore possible that SDSS J1148+5251 is multiply imaged but appears as a point source, although we note that this is an unlikely scenario since, with \( \lesssim 0.4 \text{ arcsec} \) seeing, a double with a 0.3-arcsec separation should be recognizable if the flux ratio is smaller than 10:1 (Kochanek, private communication). Multiple imaging of SDSS J1148+5251 would have important implications for the study of the \( z \sim 6 \) LF, invalidating the constraints on \( \beta \) obtained in Section 5. We have therefore computed the limits imposed on \( \beta \) by the observation of one multiply imaged source (SDSS J1148+5251) and one singly imaged source with high alignment (SDSS J1044+0125) among a sample of six quasars at \( z \sim 6 \).

The likelihood function for this scenario may be written as the product of the probability that a source is lensed with the likelihood

\[ P(< \gamma) = \int_{\gamma}^{\infty} N L_{\gamma} \frac{dP_{\text{prior}}}{dy} \]  

assuming a flat prior probability distribution for \( \gamma \) at values greater than 0. By excluding the possibility of \( \gamma < 0 \) we are assuming that the lens galaxy population increases monotonically in time as expected in hierarchical merging scenarios. This choice also leads to more conservative limits on \( \gamma \). We find \( \gamma \lesssim 0.4 \) and \( \lesssim 1.6 \) at the 90 per cent level assuming velocity and density evolution respectively. The possible presence of a lens galaxy at such a high redshift therefore offers an opportunity to constrain the (mass selected) comoving density of massive galaxies to be close to constant out to high redshifts. This result is consistent with the study of Ofek et al. (2003), who performed a detailed study on a large sample (15) of multiple-image lenses at \( z \sim 1 - 2 \).

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\[ \frac{dn}{d\sigma} = M \frac{dn}{dM} \frac{\sqrt{2}}{3v_{\text{vir}}} \]  

where \( dn/dM \) is the Press & Schechter (1974) mass function. The resulting velocity function of dark matter haloes is plotted in Fig. 4 at a series of redshifts. Note that near velocity dispersions of \( \sigma \sim 200 \text{ km s}^{-1} \), which dominate the lens cross-section, there is less than an order of magnitude of evolution in \( dn/d\sigma \) from \( z \sim 1 \) to \( \sim 5 \). If massive galaxies occupied dark matter haloes in the past as they do today, we would therefore expect little evolution in the lens population, even out to large redshifts.

7 WHAT IS THE MULTIPLE-IMAGE LENSING RATE FOR \( z \sim 6 \) QUASARS?

We may combine the information obtained for \( \beta \) and \( \gamma \) and estimate the expected multiple-imaging rate for \( z \sim 6 \) quasars. Since our earlier limits on \( \beta \) are independent of \( \tau \), while the limits derived for \( \gamma \) are insensitive to the magnification bias, the constraints placed

\[ \frac{dn}{d\sigma} = M \frac{dn}{dM} \frac{\sqrt{2}}{3v_{\text{vir}}} \]  

where \( dn/dM \) is the Press & Schechter (1974) mass function. The resulting velocity function of dark matter haloes is plotted in Fig. 4 at a series of redshifts. Note that near velocity dispersions of \( \sigma \sim 200 \text{ km s}^{-1} \), which dominate the lens cross-section, there is less than an order of magnitude of evolution in \( dn/d\sigma \) from \( z \sim 1 \) to \( \sim 5 \). If massive galaxies occupied dark matter haloes in the past as they do today, we would therefore expect little evolution in the lens population, even out to large redshifts.

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that the remainder are singly imaged. The observation that one of the quasars is multiply imaged constrains large values of $\beta$ (shallow slopes), while the observation of high alignment without multiple imaging limits small values of $\beta$ (steep slopes) as discussed in previous sections. The maximum of the combined likelihood function lies in the lower end of the range specified by the luminosity distribution (Fan et al. 2003). The posterior probability function for the sample of $z \sim 6$ quasars may be written

$$\frac{dP}{d\beta} = N \exp \left[ -\frac{1}{2} \left( \frac{\beta - \bar{\beta}}{\Delta \beta} \right)^2 \right] \frac{dP_{\text{prior}}}{d\beta} \times p_{\text{lens}}(\beta, L_1) \int_{x_2}^{10} dx \ p_{\text{single}}(\beta|x, L_2) \times \left\{ \prod_{i=3}^{6} \int_{x_2}^{\infty} dx \ p_{\text{single}}(\beta|x, L_i) \frac{dP_{\text{prior}}}{dx} \right\} \tag{27}$$

Figure 5. Dark contours: the joint probability function for $\beta$ and $\gamma$. The contours show 61, 26, 14 and 3.6 per cent of the peak value, corresponding to the $1\sigma$, $2\sigma$, $3\sigma$ and $4\sigma$ levels. Grey contours: the predicted lensing rate. The solid, dashed, dot-dashed and dotted contours correspond to lens fractions of 0.1, 0.03, 0.01 and 0.003. The upper and lower rows correspond to velocity and density evolution respectively and results are shown for a double-power-law LF with various values of $\alpha$ and $\Delta M$. 

The shallow slope of the $z \sim 6$ quasar luminosity function

Figure 6. Differential probability distributions for $\beta$. The lensing constraints that include information on the alignments between singly imaged quasars and foreground galaxies as well as a single multiply imaged quasar are denoted by the dark lines. The solid grey curves correspond to the probability functions for $\beta$ based on the luminosity distribution alone (Fan et al. 2003). Results are shown for a double-power-law LF for various values of $\alpha$ and $\Delta M$. The upper and lower two rows show results assuming density evolution with $\gamma = 0$ and 2 respectively.

and is plotted in Fig. 6 for various values of $\Delta M$ and $\alpha$. We find the preferred value in this case to be $\beta \sim -3$. As noted in Section 2.1, it is surprising that one quasar in the sample would be lensed, but not at a highly significant level. The observation of one lensed quasar therefore prefers steeper slopes (smaller values of $\beta$), for which the magnification bias is larger, and also slightly tightens the allowed range for $\beta$. In summary, if SDSS J1148+5251 were multiply imaged the preferred value for the slope would be $\beta \sim -3$, which is ruled out at the 90 per cent level if the quasar is singly imaged but with a high alignment. From Fig. 5 we see that $\beta \sim -3$ implies a lens fraction of $\sim 3\%$–10 per cent rather than $\sim 0.3\%$–1 per cent, which is more consistent with previous estimates. This underlines the importance of determining whether SDSS J1148+5251 is multiply imaged or merely lensed.

9 CONCLUSION

From their analysis of the luminosity distribution of quasars at $z \sim 6$, Fan et al. (2003) determined a slope for the quasar LF of $\beta \gtrsim -3.3$ (90 per cent). This slope may be consistent with the value found for the slope of the LF at $z \sim 4.3$, but is not consistent with the slope of the LF of bright quasars at $z \lesssim 3$. It is also possible to constrain
the slope of the LF using the fraction of multiply imaged lensed quasars. Fan et al. (2003) computed the probability of obtaining a lens fraction of zero as a function of $\beta$ and found that the constraints were similar to those of the LF. We have performed a Bayesian analysis including both of these (nearly) independent constraints, yielding the result that at 90 per cent confidence $\beta \gtrsim -3.3 \rightarrow -3.0$ provided that the quasars are at least 2 mag brighter than the unknown position of a break in a double-power-law LF. The systematic dependence in the constraint is due to the unknown slope of the LF at fainter luminosities, the luminosity of the break and the uncertain evolution in the lens galaxy population.

While inclusion of constraints from the multiple-image fraction somewhat improves the limits on $\beta$, we have shown that the additional information from observations that neither of the two quasars that lie near to the line of sight to foreground galaxies (and which are therefore lensed) is multiply imaged provides a stronger lensing-based constraint on the slope $\beta$. We find that for a double-power-law LF $\beta \gtrsim -3.1 \rightarrow -2.9$ with 90 per cent confidence. Unlike the constraint that uses only the multiple-image fraction, this limit is nearly independent of evolution in the lens population, and adds further evidence of a trend to shallower LF slopes at large redshifts.

We also find that the existence of a lens galaxy at $z \sim 5$ in a sample of two lenses constrains the evolution in the massive galaxy population to be close to that of constant comoving density (provided that the quasar behind the $z \sim 5$ galaxy is subject to magnification bias). This lack of evolution in the lens population is consistent with the lack of redshift evolution in the velocity function of dark matter haloes (for velocity dispersions near 200 km s$^{-1}$) as predicted by the Press–Schechter formalism.

Finally, the constraints on the quasar luminosity function and lens population have been used to compute an improved estimate for the expected $z \sim 6$ multiple-image lens fraction of $\sim 1–3$ per cent. This value is lower than previous estimates owing to the tight constraints on the slope of the LF.

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