Chromodynamic Lensing and Transverse Single Spin Asymmetries

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We illustrate how an axial asymmetry in impact parameter dependent parton distributions can give rise to an axial asymmetry for the transverse momentum of the leading quark in the photo-production of hadrons. The effect is related to the asymmetry originating from the Wilson-line phase factor in gauge invariant Sivers distributions. The single spin asymmetry arising from the asymmetry of the impact parameter dependent parton distributions is shown to exhibit a pure \( \sin \phi \) dependence.

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I. INTRODUCTION

Many semi-inclusive hadron production experiments show surprisingly large transverse polarizations or single spin asymmetries (SSA) \(^1\). Moreover, the signs of these polarizations are usually not dependent on the energy. This very stable polarization pattern suggests that there is a simple mechanism that gives rise to these polarization effects.

Recently, it has become clear that phase factors due to final state interactions (FSI) of the struck quark play a crucial role for the leading twist single spin asymmetry (SSA) in semi-inclusive DIS \(^2\). Likewise, the initial state interactions (ISI) of the annihilating antiquark and the spectator quarks is believed to give rise to SSA in the corresponding Drell-Yan process.

According to BHS \(^3\), the SSA depends on the interference of different amplitudes arising from the hadron’s wavefunction and is distinct from probabilistic measures of the target such as transversity.

As has been emphasized in Ref. \(^3\), this mechanism is also consistent with the Sivers mechanism. In Collins’ treatment the FSI of the struck quark are incorporated into Wilson line path-ordered exponentials (see also Ref. \(^4\)). The ISI and FSI of the struck quark with the gluon field produce T-odd spin correlations (BHS), which is why a Sivers asymmetry is allowed.

However, what is not clear from these treatments is why the resulting SSAs are so large and exhibit such stable patterns. The main purpose of this paper is to investigate, in a semi-classical picture, how this phase translates into stable and large SSA.

The physics of the transverse asymmetry can be best seen by focusing on the mean transverse momentum from each quark field operator to infinity

\[
P(x, k_\perp, s_\perp) = \int \frac{dy^- d^2 y_\perp}{16 \pi^3} e^{-i x p^+ y^- + i k_\perp \cdot y_\perp} \langle p | \bar{q}(0, y^-, y_\perp) W^{+\gamma\gamma^{\ast}} W_{0\infty} q(0) | p \rangle.
\]

where \( W_{0\infty} \) reflects the FSI (ISI) of the active quark in an eikonal approximation.

The complex phase in Eq. \(^1\) is reversed under time-reversal and therefore T-odd PDFs may exist \(^3\), which is why a nonzero Sivers asymmetry \(^3\) is possible. However, given the fact that the asymmetry in the quark field operator as well as on the gluon string.

Before proceeding further we would like to switch to light-front gauge \( A^+ = 0 \). Already in an abelian theory

\[
a(z_\perp) = \int_{-\infty}^{\infty} dz^- A^+(z^-, z_\perp)
\]

II. INITIAL (FINAL) STATE INTERACTIONS AND TRANSVERSE SPIN ASYMMETRIES

Ref. \(^3\) explains how ISI and FSI allow the existence of T-odd parton distribution functions. Formally the FSI (ISI) can be incorporated into \( k_\perp \) dependent parton distribution functions (PDFs) by introducing a gauge string
is a gauge invariant quantity, it not entirely possible to accomplish this (in a nonabelian theory, the gauge invariant quantity is \(tr(W_{-\infty,\infty})\), and the argument is similar). In a box of length \(L\) with periodic boundary conditions in the \(z^\perp\)-direction, the closest one can get to LF gauge is \(A^+(z^\perp,z_\perp) = \frac{1}{2}a(z_\perp)\) [3]. If we regulate these zero-modes by working in such a box, using a gauge \(A^+ = const\) and finally taking \(L \to \infty\), what we find is that (see also Appendix [4])

\[
\int_{y^-,y^\perp}^{\infty} dz^- A^+(z^-,y^\perp) \to \frac{1}{2} \int_{-\infty}^{\infty} dz^- A^+(z^-,y^\perp) = \frac{1}{2} \hat{a}(z_\perp)
\]

(4)

and [12]

\[
W_{y^-,y^\perp} \to e^{-\frac{1}{2} \hat{a}(y^\perp)} \equiv w(y^\perp)
\]

(5)

takes the gauge string contribution disappears. Likewise, any light-like gauge string that does not extend to infinity becomes trivial

\[
W_{y^-,y^\perp} \to 1,
\]

(6)

which is why the gauge string can be omitted in the light-like correlations relevant for inclusive DIS.

In such an “almost-LF gauge”, one thus finds that

\[
\mathbf{P}_\perp(x,s_\perp) = i \int \frac{dy^-}{4\pi} e^{-ixp^+y^-} \times \left\langle p \left| \frac{\partial}{\partial y^\perp} \tilde{q}(0,y^-,y^\perp) w^\dagger(y^\perp) \right| y^\perp = 0 \right. \gamma^+ w(0^\perp) \psi(0) \left| p \right\rangle
\]

(7)

In the term where the derivative acts on the quark field the gauge string contribution disappears

\[
\left\langle p \left| \partial_T \left( \tilde{q}(0,y^-,0^\perp) \right) w^\dagger(0^\perp) \gamma^+ w(0^\perp) \psi(0) \right| p \rightangle = \left\langle p \left| \partial_T \left( \tilde{q}(0,y^-,0^\perp) \right) \right. \gamma^+ \psi(0) \left| p \right\rangle
\]

(8)

However, without the gauge string pointing to infinity,

\[
\left\langle p \left| \partial_T \left( \tilde{q}(0,y^-,0^\perp) \right) \right. \gamma^+ \psi(0) \left| p \right\rangle = 0,
\]

(9)

due to time reversal invariance.

Hence the only non-vanishing contribution in Eq. (7) arises when the derivative acts on the gauge string. Upon introducing

\[
\mathbf{I}_\perp(0^\perp) \equiv i\partial_{y^\perp} w^\dagger(y^\perp) \big|_{y^\perp = 0} w(0^\perp)
\]

(10)

\[
= -\frac{q^2}{2} \int_{-\infty}^{\infty} dz^- \partial_{y^\perp} A^+(z^-,0^\perp) = -\frac{q^2}{2} \partial_{y^\perp} a(0^\perp)
\]

we can thus rewrite Eq. (7) in a very compact form

\[
\mathbf{P}_\perp(x,s_\perp) = \int \frac{dy^-}{4\pi} e^{-ixp^+y^-} \times \left\langle p \left| \tilde{q}(0,y^-,0^\perp) \mathbf{I}_\perp(0^\perp) \gamma^+ q(0) \right| p \right\rangle
\]

(11)

which has a very physical interpretation as the correlation between the transverse quark position and the transverse impulse \(\mathbf{I}_\perp(0^\perp)\) acquired by the active quark as it escapes to infinity. This interpretation becomes even more transparent after switching to an impact parameter representation [8, 9, 10, 11]

\[
\left| p^+, \mathbf{R}_\perp, s \right\rangle \equiv \mathcal{N} \int \frac{d^2p_\perp}{2\pi} \left| p^+, p_\perp, s \right\rangle e^{-ip_\perp \cdot \mathbf{R}_\perp}
\]

(12)

where \(\mathcal{N}\) is some normalization. Using the fact that the correlator in Eq. (12) does not change the transverse center of momentum (i.e. it is diagonal in \(\mathbf{R}_\perp\)) this yields

\[
\mathbf{P}_\perp(x, s_\perp) = \int d^2\mathbf{R}_\perp \int \frac{dy^-}{4\pi} e^{-ixp^+y^-} \times \left\langle p^+, \mathbf{R}_\perp \left| \tilde{q}(y^-,0^\perp) \mathbf{I}_\perp(0^\perp) \gamma^+ q(0) \right| p^+, \mathbf{R}_\perp \right\rangle
\]

(13)

\[
= \int d^2\mathbf{R}_\perp \int \frac{dy^-}{4\pi} e^{-ixp^+y^-} \times \left\langle p^+, 0^\perp \left| \tilde{q}(y^-, -\mathbf{R}_\perp) \mathbf{I}_\perp(-\mathbf{R}_\perp) \gamma^+ q(0^+, -\mathbf{R}_\perp) \right| p^+, 0^\perp \right\rangle
\]

\[
= \int d^2\mathbf{b}_\perp \int \frac{dy^-}{4\pi} e^{-ixp^+y^-} \times \left\langle p^+, 0^\perp \left| \tilde{q}(y^-, \mathbf{b}_\perp) \mathbf{I}_\perp(\mathbf{b}_\perp) \gamma^+ q(0^+, \mathbf{b}_\perp) \right| p^+, 0^\perp \right\rangle
\]

(14)

where we used translational invariance and then substituted the (dummy)-integration variable \(\mathbf{R}_\perp\) by the integration variable \(-\mathbf{b}_\perp\). We use \(\left| p^+, 0^\perp \right\rangle\) as a shorthand notation for \(\left| p^+, \mathbf{R}_\perp = 0^\perp \right\rangle\). If we compare this result with the impact parameter dependent parton distributions [8, 9, 11]

\[
q(x, \mathbf{b}_\perp) = \int \frac{dy^-}{4\pi} e^{-ixp^+y^-} \times \left\langle p^+, 0^\perp \left| \tilde{q}(y^-, \mathbf{b}_\perp) \gamma^+ q(0^+, \mathbf{b}_\perp) \right| p^+, 0^\perp \right\rangle
\]

(15)

we realize that Eqs. (13) and (15) differ only by the presence of the operator \(\mathbf{I}_\perp(\mathbf{b}_\perp)\) and an integration over the impact parameter \(d^2\mathbf{b}_\perp\). What we have thus accomplished is to express the mean transverse momentum of the outgoing quarks in terms of a correlation between impact parameter dependent PDFs and the transverse impulse \(\mathbf{I}_\perp(\mathbf{b}_\perp)\) as a function of impact parameter.

The physical interpretation of our result [13] is thus evident: \(\mathbf{I}_\perp(\mathbf{b}_\perp)\) is the net transverse impulse that a quark at \(\perp\) position \(\mathbf{b}_\perp\) receives on its way out. The mean transverse momentum of the outgoing quark can be obtained by correlating the impact parameter dependent parton distribution with the impact parameter dependent impulse for an outgoing quark.

For a transversely polarized target, the impact parameter dependent parton distribution is not axially symmetric [2]. Therefore, even if one assumes to lowest order that \(\mathbf{I}_\perp\) was axially symmetric (which it actually does not have to be), the momentum distribution of the outgoing quark still exhibits an axial asymmetry. And the asymmetry arises from correlating the impact parameter dependent PDFs with the impact parameter dependence
of the impulse due to the FSI. This is one of the main results of this paper and provides a mathematical foundation for the heuristic model for SSAs that was advocated in Ref. [7]:

For a transversely polarized target, the impact parameter dependent parton distribution \( q(x, b_\perp) \) is no longer axially symmetric. In Ref. [7] it was suggested that the final state interactions deflect the outgoing quark in such a way that it receives a transverse momentum that is directed toward the center of the target. As a result, the final state interactions thus translate the axial asymmetry in impact parameter space into an axial asymmetry in the transverse momentum. Eq. (13) demonstrates that this very physical picture for SSA can in fact be related to the Wilson phase contribution discussed in Refs. [2, 3].

III. SIMPLE MODELS FOR THE FINAL STATE INTERACTIONS

In order to illustrate the implications of our results we will in the following adapt a potential model and treat the gluon vector potential as if it was abelian

\[
g A^0(\vec{r}) = V(r) \\
g A(\vec{r}) = 0.
\]  

These are clearly drastic approximations, but what we have in mind is not an exact treatment of the problem but rather a qualitative illustration of the physics that is connected with these phase factors.

Note that the vector potential in Eq. (15) does not satisfy \( A^+ = \text{const.} \). In principle, one could transform the above ansatz into such a gauge. However, this is not necessary since \( I_\perp \) is gauge invariant if we compactify space and we can evaluate \( I_\perp \) it in any gauge.

The specific models that we consider are a logarithmic potential, linear, as well as quadratic confinement.

\[
V_a(r) = c \ln \frac{r}{r_0} \\
V_b(r) = \sigma r \Theta(R - r) + \sigma R \Theta(r - R) \\
V_c(r) = \frac{K}{2} r^2 \Theta(r - R) + \frac{K}{2} R^2 \Theta(R - r). \tag{16}
\]

In the cases \( b \) and \( c \) we have to introduce a long distance cutoff in order to avoid infrared divergences in \( \int dz \partial_\perp A^0 \). The physical mechanism for such a cutoff is provided by pair creation when the active quark has separated far enough from the target. Parameters: \( \sigma = 1 \text{ GeV} \), \( K = 1.4 \frac{\text{GeV}^2}{\text{fm}} \), and \( c = 0.3 \text{ GeV} \). The cutoff radius is somewhat arbitrary and we chose a value of \( R = 1 \text{ fm} \).

The lensing function that assigns a mean transverse momentum for each impact parameter

\[
I_\perp(\vec{b}_\perp) = -\nabla_\perp \frac{1}{2} \int_{-\infty}^{\infty} dz V(\sqrt{b_\perp^2 + z^2}) \tag{17}
\]

in these three models is given by

\[
I_\perp^a(\vec{b}_\perp) = -\frac{c \pi}{2} \frac{\vec{b}_\perp}{|\vec{b}_\perp|} \\
I_\perp^b(\vec{b}_\perp) = -\frac{\sigma \vec{b}_\perp}{2} \ln \left( \frac{R + \sqrt{R^2 - b_\perp^2}}{R - \sqrt{R^2 - b_\perp^2}} \right) \Theta(R^2 - b_\perp^2) \\
I_\perp^c(\vec{b}_\perp) = -2k \vec{b}_\perp \sqrt{R^2 - b_\perp^2} \Theta(R^2 - b_\perp^2). \tag{18}
\]

It is not surprising to find in all 3 models that the momentum is directed opposite in direction to the original transverse position \( \vec{b}_\perp \) since the underlying poten-
and one needs to replace rest frame, relativistic corrections from the boost arise in the finite momentum frame. If one boosts this result into the rest frame of a nucleon that is polarized in the x-direction in the infinite momentum frame, the unpolarized impact parameter behavior when one compares logarithmic, linear, and quadratic potentials.

If the target is polarized in the x-direction in the infinite momentum frame, the unpolarized impact parameter dependent PDFs for flavor $q$ reads [11]

$$q(x, b_\perp) = \int d^2 \Delta_\perp e^{-i \Delta_\perp \cdot b_\perp} \left[ H_q(x, 0, -\Delta_\perp^2) + \frac{i \Delta_\perp}{2M} E_q(x, 0, -\Delta_\perp^2) \right]$$

(19)

where we denoted $E_q$ the Fourier transform of $E_q$, i.e.

$$E_q(x, b_\perp) = \int d^2 \Delta_\perp e^{-i \Delta_\perp \cdot b_\perp} E_q(x, 0, -\Delta_\perp^2).$$

(20)

Several comments are in order: first, Eq. (19) applies to a nucleon that is polarized in the x direction in the infinite momentum frame. If one boosts this result into the rest frame, relativistic corrections from the boost arise and one needs to replace $E_q$ by $E_q + H_q$ in the term that described the asymmetry. Secondly, it should be emphasized that the axial asymmetry in Eq. (19) is described by the $y-$derivative of an axially symmetric function, i.e. the angular dependence is proportional to $\sin(\phi)$, where $\phi$ is the angle relative to the (transverse) spin direction. No higher moments (e.g. $\sin(2\phi)$) are present.

The impact parameter dependent PDFs $H_q(x, 0, -\Delta_\perp^2)$ and $E_q(x, 0, -\Delta_\perp^2)$ are not known yet. In order to proceed, we thus adopt the simple model from [11], where

$$H_q(x, 0, t) = q(x) e^{a t (1-x)}$$

$$E_q(x, 0, t) = \kappa_q H_q(x, 0, t)$$

$$E_u(x, 0, t) = \frac{1}{2} \kappa_u H_u(x, 0, t),$$

(21)

where $\kappa_q$ is the contribution from quark flavor $q$ to the anomalous magnetic moment of the proton. The factor $\frac{1}{2}$ for down quarks reflects the fact that $\int dx H_q(x, 0, t) = 2$. Typical results for the resulting $\perp$ distortion in the model from Ref. [11] are shown in Fig. 1.

The naive model incorporates a number of different features known about GPDs: in the forward limit they reduce to the usual PDFs, at small $x$ the $\perp$ size of the proton grows like $\ln \frac{1}{x}$, and this ansatz also satisfies duality. Although some features, e.g. $\frac{d^2\Delta}{d\perp^2} \sim \text{const.}$ are not correctly described by this model, we still believe that it has some use in illustrating general properties. Furthermore, since the integral of $E_q$ are constrained by known anomalous magnetic moments, it should be clear that the overall scale of any resulting effect is model independent — even if details require a better description for the GPDs.

However, despite all these caveats, the qualitative pic-
ture of large $\perp$ distortions, such as the ones depicted in Fig. 3 should be model independent. As has been emphasized in Ref. 11, the $\perp$ center for each flavor ($\perp$ flavor dipole moment) is related to the anomalous magnetic moment contribution from that flavor and one thus finds that the typical scale for $\perp$ distortions is on the order of 0.2 $fm$. Although these considerations do not constrain the $x$-dependence of the $\perp$ distortion, at least they determine the expected typical size of the effect.

Furthermore, from the fact that the momentum asymmetry arises from correlating the impact parameter space asymmetry (Fig. 3) with the impulse (Fig. 1) it is also clear that the sign of the asymmetry in the nucleon is model independent: if the proton has spin up and if one looks into the direction of the momentum transfer then leading $u$-quark will on average pick up a $\perp$ momentum to the right, while leading $d$-quarks will be deflected to the left.

**IV. ANGULAR AND $x$ DEPENDENCE OF THE ASYMMETRY**

In the above model, the SSA arises when one correlates the angular asymmetry of the impact parameter dependent parton distribution with the lensing function $I_\perp(b_\perp)$. For a transversely polarized target polarized for example in the $x$-direction, the transverse distortion of the impact parameter dependent PDFs is described by the transverse gradient of the Fourier transform of $E(x, 0, -\Delta^2_\perp)$

$$q(x, b_\perp) = \int d^2\Delta_\perp e^{-i\Delta_\perp b_\perp} H(x, 0, -\Delta^2_\perp) \tag{22}$$

$$+ \frac{i}{2M} \frac{\partial}{\partial b_y} \int d^2\Delta_\perp e^{-i\Delta_\perp b_\perp} E(x, 0, -\Delta^2_\perp).$$

Since $E(x, 0, -\Delta^2_\perp)$ is axially symmetric, one thus finds that the transverse distortion of $q(x, b_\perp)$ exhibits a pure sin $\phi$ angular dependence. In general, since the PDFs become transversely distorted, the lensing function $I_\perp(b_\perp)$ may also exhibit an axial asymmetry. However, we expect this effect to be small and in a first approximation we may take $I_\perp(b_\perp)$ to be axially symmetric.

In our simple description for the SSA, the final state interaction is always directed toward the center of the hadron. As a result, the final state transverse momentum always point (anti-) parallel to the transverse position $b_\perp$ that the active quark had before it was struck

$$P(x, k_\perp) = \int d^2b_\perp q(x, b_\perp) \delta(k_\perp - I_\perp(b_\perp)). \tag{23}$$

Therefore the pure pure sin $\phi$ asymmetry of the PDFs in impact parameter space translates into a pure sin $\phi$ asymmetry for the $k_\perp$ distribution of the SSA.

Because of the very large transverse distortion of the impact parameter dependent PDFs entering Eq. (23), it appears that our model predicts asymmetries that are numerically very large ($\sim 0.5$). However, one must keep in mind that our description neglects (among other things) the stochastic nature of the FSI, i.e. even in a semi-classical model one would expect some smearing of $k_\perp$, which would effectively reduce the asymmetry. However, the fact that the asymmetry depends only on sin $\phi$ remains.

In order to determine the actual mean transverse momentum in our model, one needs to correlate the transverse distortion (Fig. 3) with the $\perp$ impulse associated the impact parameter. Because there are significant uncertainties both in the choice of the vector potential (and hence in $I_\perp(b_\perp)$) as well as in the $x$-dependence of $E(x, 0, t)$ (and hence in $q(x, b_\perp)$) the resulting numerical values (in particular their $x$ dependence) would also exhibit a large uncertainties.

Nevertheless, we know that typical $\perp$ distortions for $\perp$ polarized targets are on the order of 0.2 $fm$ (Fig. 3). And we also know that the typical impulse acquired by an outgoing quark for $b_\perp \approx 0.2$ $fm$ is on the order of $I_\perp \approx 0.3 - 0.5$ GeV/$c$. As a result, the natural scale for the transverse momenta that emerges from this picture is also on the order of ($k_\perp$) $\approx 0.2 - 0.4$ GeV/$c$. Generating such a large mean transverse momentum scale in a natural way is one of the main results of this paper.

**V. SUMMARY**

We have provided a physical interpretation of the mechanisms that leads to a transverse single spin asymmetries (SSAs) in semi-inclusive electro-production of mesons. The starting point of our analysis is the BHS/Collins Wilson-line phase $2_{13}$ that describes the final state interaction experienced by the active quark. As the active quark escapes from the target, the chromodynamic field from the remaining spectators provides an impulse that translates the axial asymmetry in transverse position into an axial asymmetry in the transverse momentum of the outgoing quark before it fragments. This is the physics that underlies the observation that a Sivers asymmetry is allowed when one takes final (or initial) state interactions into account. Although the FSI are (to leading order) spin independent, they translate the spin-dependent impact parameter space distributions into a spin dependent transverse momentum of the leading quark.

Because of the attractive nature of the confining interaction in QCD, the mean impulse on the outgoing quark is directed toward the center (of momentum) of the target. The FSI with the spectators thus acts like a convex lens that deflects the active quark toward the center. The transverse impulse acquired by the active quark is a direct consequence of the the Wilson phase factor advocated in Ref. $2_{12}$.

In this simple picture, it is the combination of the transverse position space asymmetry with this chromo-
dynamic lensing effect that gives rise to the transverse momentum asymmetry of the knocked out quarks. Since
the axial asymmetry of impact parameter dependent PDFs for transversely polarized nucleons tends to be
rather large, this simple picture provides a natural mechanism for generating large transverse single-spin asym-
metries.

Of course, our semi-classical picture cannot give an accurate description of the actual dynamics that gives
rise to SSAs, but hopefully it will still help to provide a better understanding of the physics that leads to these
asymmetries.

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APPENDIX A: DETAILED DERIVATION

First we use \[1\] the fact that time-reversal invariance relates transverse momentum distributions with gauge
strings that point forward and backward in time respectively

\[ P(x, k_\perp, s_\perp)|_{\text{future-pointing}} = P(x, -k_\perp, s_\perp)|_{\text{past-pointing}}. \]

After performing a 180° rotation around the \(z\)-axis this implies

\[ P(x, k_\perp, s_\perp)|_{\text{future-pointing}} = P(x, -k_\perp, s_\perp)|_{\text{past-pointing}}. \]  

(this conjecture is consistent with a probabilistic interpretation for the twist-2 parton distributions probed in
deep-inelastic scattering.

However, despite Eq. \[A5\] we may not drop \(A^+\) entirely in Eq. \[A4\] since the integration over \(z^-\) extends to
\(\pm\infty\). In fact, the remaining contribution from the gauge field is exactly the zero-mode advocated in Section III

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