Geometric Dispersion in Corrugated Coaxial Cables

Brian R. Poole, Life Senior Member, IEEE, and Natalie B. Kostinski, Member, IEEE

Abstract—Geometric dispersion of a periodic coaxial transmission line (TL) with outer conductor sinusoidal corrugations and uniform inner conductor is examined. This is of practical importance when the TL is long and/or used for wideband pulse propagation. Eigenfrequency simulations with a finite element code, and an analytic approach based on a modal decomposition of the electromagnetic field and Floquet theory, are used to determine the dispersion relation. To gain insight into the behavior of the coaxial TL dispersion relation for the TEM-like mode in the long-wavelength limit, approximations are developed for the dispersion relation of a simpler planar TL with rectangular corrugations. Finally, comparisons between coaxial TLs with other corrugation profiles, including rectangular and triangular, are made.

Index Terms—Coaxial cables, dispersion, eigenmode, guided-wave structures, periodic structures.

I. INTRODUCTION

PERIODICALLY corrugated structures are commonly used in the microwave regime for transmission of wideband signals in accelerator facilities [1], and as slow wave structures in microwave sources [2]–[5], accelerating components, mode converters, microwave filters [6]–[8], and antennae [9]. Such structures also find application in the optical regime [10], [11], for example as photonic crystals and distributed Bragg reflectors (DBRs) [12].

In particle accelerator facilities, wideband signals may be transmitted through ~cm-radius and ~100-m-long corrugated coaxial cables. As recently as 2009 [1], it was noted that the outer conductor sinusoidal corrugations produce significant higher-order dispersion that cause ringing in the transmitted pulses. Here we take a novel analytic approach to developing the dispersion relation of corrugated cables, which is more general than asymptotic and perturbative methods taken in the literature [10], [11], and compare it to finite element simulation results.

The dispersion relation for a guided wave structure relates the longitudinal wavenumber $k_0$ to the frequency $\omega$. This relation allows any potential distortion of the input signal to be characterized, such as pulse rise time erosion, broadening, and ringing. Generally, both dispersion (material and/or geometric) and attenuation (due to dielectric and conductor losses) can contribute to distortion of the input signal. This article will focus on the unique aspect of geometric dispersion present in corrugated cables, ignoring all sources of loss and material dispersion.

To determine the dispersion relation for the lowest order TM modes, a fully electromagnetic approach is taken via two methods. The first method uses frequency domain solvers in the multiphysics software COMSOL [17]. In particular, an eigenmode solver is employed with perfect electric conductor (PEC) boundary conditions on the radial boundaries, and periodic boundary conditions on the axial boundaries, to determine the eigenvalues for one period of a corrugated cable.

The second method is analytic, based on the modal decomposition of the electromagnetic field using Floquet theory, to develop an exact representation of the dispersion relation as an infinite set of algebraic equations. By appropriately truncating the size of the associated matrix, it is possible to develop a highly accurate solution for the lower order modes in closed form. This contrasts to the existing literature of asymptotic and perturbative methods as applied to planar and circular waveguides [13]–[16], but with a different inner boundary condition due to the presence of the inner conductor. Specifically, our method based on modal expansion is advantageous over these other methods due to its exact and more general nature, with approximations completed only at the end with truncation. It should be pointed out that there are also conventional optical analysis techniques that lend themselves to the microwave regime that may be applicable to this problem, e.g., spatial Fourier decomposition of the corrugation profile [10], [11].

Using the derived dispersion relation, we construct a model in which a transmission line (TL) approach is adapted for a sinusoidal perturbation. The 3-D Maxwell equations are reduced to 1-D in the TL approach, in order to define a transfer function for any given length of cable, assuming a linear time-invariant system. To gain insight into the behavior of the coaxial TL dispersion relation for the TEM-like mode in the long-wavelength limit, approximations are developed for the dispersion relation of a planar TL with rectangular corrugations. It was found that the dispersion relation $k_0(\omega)$ is captured with only odd-order terms in $\omega$. Finally, coaxial TLs with other geometric corrugation profiles are considered, including rectangular and triangular, which were found to have similar dispersion relations but significant cumulative effects.

Accelerator facilities are a potential source of high energy X-rays. Corrugated cables with a solid metal outer shield provide an advantage over conventional braided shield cables as they offer improved immunity to both X-ray fluence and electromagnetic interference (EMI). These effects can lead to
spurious signals due to X-ray generated photoelectrons, as well as dielectric degradation due to radiation-induced conductivity. Moreover, corrugated cables still provide some degree of flexibility that conventional rigid coaxial cable does not. The cable dispersion relation gives the essential argument of the cable transfer function and therefore allows one to characterize the input signal properties via inverse Fourier transforms.

**II. Dispersion Analysis**

The periodic coaxial TL consists of a sinusoidally corrugated outer conductor and uniform inner conductor as shown in Fig. 1. The TL has outer conductor with periodicity \( d \), corrugation depth \( 2R_1 \), constant inner conductor radius \( a \), and a dielectric medium with dielectric constant \( \epsilon \). The outer conductor radius is defined by the following equation where \( h = 2\pi/d \) and \( \alpha = R_1/R_0 \):

\[
R_n(z) = R_0 + R_1 \cos(hz) = R_0(1 + \alpha \cos(hz)).
\]  
\( n \) is an integer. The outer conductor radius is defined by the following equation where \( h = \frac{2\pi}{d} \) and \( \alpha = \frac{R_1}{R_0} \):

\[
R_n(z) = R_0 + R_1 \cos(hz) = R_0(1 + \alpha \cos(hz)).
\]  
\( n \) is an integer.

It is assumed that the conductors are perfectly conducting. This geometry constitutes a periodic structure with azimuthal symmetry. For the purposes of this article only azimuthally symmetric \( T_{M0n} \) modes will be considered. Only the lowest order \( T_{M0n} \) mode, which is TEM-like, allows propagation of wideband signals that have frequency content from dc to the microwave regime. All the TE modes have a nonzero cutoff frequency which excludes low frequency propagation. The \( T_{M0n} \) fields can be represented as a sum of Floquet harmonics with azimuthal mode number \( m = 0 \). Assuming harmonic fields with an \( e^{-j\omega t} \) dependence, the axial electric field is expressed as

\[
E_z(r, z) = \sum_{n=-\infty}^{\infty} E_{zn}(r)e^{j\alpha z}
\]

where

\[
k_n = k_0 + nh = k_0 + \frac{2\pi n}{d}, \quad n = 0, \pm 1, \pm 2, \ldots
\]

**A. Eigenmode Solutions**

The dispersion properties of the structure are determined using the eigenmode solver in the COMSOL multiphysics [17] software. For this calculation, only a single unit cell is modeled over axial length \( d \) using periodic boundary conditions at the two axial boundaries. The properties of the cable are: \( d = 7 \text{ mm}, \ a = 4.4 \text{ mm}, \ R_0 = 11.125 \text{ mm}, \) and \( R_1 = 0.625 \text{ mm} \) yielding \( \alpha = 0.0562 \). The dielectric constant filling the cable is \( \epsilon = 1.3 \). These parameters correspond to RF-19 cable. For all eigenfrequency simulations, the finite element mesh is refined until an accurate and converged set of eigenmodes are found.

Fig. 2 shows the dispersion relation for the first three axisymmetric \( T_{M0n} \) modes over one Brillouin zone. The first lowest order mode is a TEM-like mode in the long-wavelength regime, while the next two modes are higher-order modes (HOM’s).

Figs. 3–5 show the electric and magnetic field distributions at low frequency \((k_0d = \pi/100, \ f = 186.4 \text{ MHz})\), mid-frequency \((k_0d = \pi/2, \ f = 9.31 \text{ GHz})\), and high frequency \((k_0d = \pi, \ f = 17.96 \text{ GHz})\), respectively, for the lowest order TEM-like mode. These are snapshots at one point in phase. To generate a four-period structure, a single unit cell was replicated as adjacent cells with the appropriate phase advance. The contours of constant amplitude are used to show the curvature of the fields. The field amplitudes shown in the color bars are only relevant within a given figure for the three field components since the amplitudes of any of the eigenmodes can be scaled by an arbitrary complex constant. However, the relative amplitudes between the field components for each frequency case shown in Figs. 3–5 are preserved since the electric and magnetic fields are associated with the same eigenmode. Therefore, the color bars are only meaningful within a single figure.

In the low frequency, long-wavelength regime as shown in Fig. 3, the field distribution looks quite similar to an uncorrugated coaxial TL near the center conductor, where the transverse fields have a \( 1/r \) dependence. Since the wavelength is long compared to the four periods of the structure, the phase advance along it is not apparent. The spatial periodicity of the fields seen closer to the corrugations are due to higher-order space harmonics and are not associated with the phase advance of the \( n = 0 \) space harmonic along the structure. At shorter wavelengths, Figs. 4 and 5 clearly show the phase advance of the \( n = 0 \) space harmonic as well as the stronger influence of the higher-order space harmonics, as they start to perturb the field closer to the inner conductor.
B. Analytic Modal Expansion Method to Determine Dispersion Relation

The modal expansion method was used to determine the dispersion relation for corrugated cylindrical waveguides by [2] and [3] in the design of backward wave oscillators (BWOs). In this article, we expand that technique to periodic coaxial structures, similar to [6]. The defining equation for the axial electric field for $\text{TM}_{0n}$ modes is given by

$$\left[\frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} \right) + \Gamma_n^2 \right] E_{zn}(r) = 0, \quad a \leq r \leq R_w,$$

where

$$\Gamma_n^2 = \frac{\epsilon \alpha^2}{c^2} - k_0^2.$$

The solution to (4) is given by

$$E_{zn} = A_n J_0(\Gamma_n r) + B_n Y_0(\Gamma_n r), \quad a \leq r \leq R_w,$$

where $J_0$ is the zeroth order Bessel function of the first kind and $Y_0$ is the zeroth order Neumann function, or Bessel function of the second kind. Determining the dispersion relation for the periodic structure requires the application of boundary conditions for the tangential electric field at the inner conductor boundary, $r = a$, and along the outer conductor boundary, $r = R_w(z)$. Fig. 6 shows the relation between the total vector electric field and the tangential electric field at the outer conductor boundary.

Here, the total electric field $\mathbf{E}$ is given by

$$\mathbf{E} = \hat{r}_0 E_r + \hat{z}_0 E_z,$$

and $\hat{t}$ is a tangent vector to the outer conductor given by

$$\hat{t} = \hat{r}_0 \frac{dR_w}{dz} + \hat{z}_0.$$
The boundary conditions for the tangential electric field on the inner and outer boundaries are given by

$$E_r |_{r = a} = 0$$  \hspace{1cm} (9)

and

$$\left. \left( E_r \frac{dR_w}{dz} + E_z \right) \right|_{r = R_w} = 0.$$  \hspace{1cm} (10)

Using (2) and (6), the inner conductor boundary condition (9) becomes

$$\sum_{n = -\infty}^{\infty} [A_n J_0(\Gamma_n a) + B_n Y_0(\Gamma_n a)] e^{i k_n z} = 0.$$  \hspace{1cm} (11)

Similarly, the outer conductor boundary condition (10) becomes

$$\left( \sum_{n = -\infty}^{\infty} E_r e^{i k_n z} \frac{dR_w}{dz} + \sum_{n = -\infty}^{\infty} E_z e^{i k_n z} \right) |_{r = R_w} = 0.$$  \hspace{1cm} (12)

The radial and axial electric fields are related by

$$E_r = \frac{\text{i} k_n}{\Gamma_n^2} \frac{dE_z}{dr}.$$  \hspace{1cm} (13)

Substituting (6) into (13) gives

$$\frac{dE_z}{dr} |_{r = R_w} = -\Gamma_n A_n J_1(\Gamma_n R_w) - \Gamma_n B_n Y_1(\Gamma_n R_w).$$  \hspace{1cm} (14)

Using the chain rule to write the first term of (14) in terms of the outer wall function $R_w(z)$ gives

$$A_n \frac{d}{dz} J_0(\Gamma_n R_w) = -\Gamma_n A_n \frac{dR_w}{dz} J_1(\Gamma_n R_w).$$  \hspace{1cm} (15)

Similarly, using the chain rule to write the second term of (14) in terms of the outer wall function $R_w(z)$ gives

$$B_n \frac{d}{dz} Y_0(\Gamma_n R_w) = -\Gamma_n B_n \frac{dR_w}{dz} Y_1(\Gamma_n R_w).$$  \hspace{1cm} (16)

Using (6), (15), and (16), (12) becomes

$$\sum_{n = -\infty}^{\infty} \frac{\text{i} k_n}{\Gamma_n^2} \left[ -A_n \Gamma_n J_1(\Gamma_n R_w) - B_n \Gamma_n Y_1(\Gamma_n R_w) \right] \frac{dR_w}{dz} e^{i k_n z}$$

$$+ \sum_{n = -\infty}^{\infty} [A_n J_0(\Gamma_n R_w) + B_n Y_0(\Gamma_n R_w)] e^{i k_n z} = 0.$$  \hspace{1cm} (17)

Equations (11) and (17) can be cast into a matrix equation by factoring out $e^{i k_n z}$, and multiplying by $e^{-i m} h z$ and integrating the equations in $z$ from $-d/2$ to $d/2$. The following integrals are defined after a change of variables, $u = h z$:

$$P_{mn}(\omega, k_0) = \frac{1}{h} \int_{-\pi}^{\pi} e^{i(n-m)u} J_0(\Gamma_n a) du$$  \hspace{1cm} (18)

$$Q_{mn}(\omega, k_0) = \frac{1}{h} \int_{-\pi}^{\pi} e^{i(n-m)u} Y_0(\Gamma_n a) du$$  \hspace{1cm} (19)

$$R_{mn}(\omega, k_0) = \int_{-\pi/2}^{\pi/2} e^{i(n-m)z} \left[ 1 + \frac{\text{i} k_n d}{\Gamma_n^2} \right] J_0(\Gamma_n R_w) dz$$  \hspace{1cm} (20)

$$S_{mn}(\omega, k_0) = \int_{-\pi/2}^{\pi/2} e^{i(n-m)z} \left[ 1 + \frac{\text{i} k_n d}{\Gamma_n^2} \right] Y_0(\Gamma_n R_w) dz.$$  \hspace{1cm} (21)

$P_{mn}$ and $Q_{mn}$ simplify as

$$P_{mn}(\omega, k_0) = \frac{2\pi}{h} J_0(\Gamma_n a) \delta_{mn} = d J_0(\Gamma_n a) \delta_{mn}$$  \hspace{1cm} (22)

$$Q_{mn}(\omega, k_0) = \frac{2\pi}{h} Y_0(\Gamma_n a) \delta_{mn} = d Y_0(\Gamma_n a) \delta_{mn}$$  \hspace{1cm} (23)

where $\delta_{mn}$ is the Kronecker delta. The $R_{mn}$ and $S_{mn}$ integrals can be simplified by integration by parts as

$$R_{mn}(\omega, k_0) = \frac{d}{h} \int_{\pi/2}^{\pi/2} \left( \frac{\text{e}^{i n u}}{\text{e}^{i m u}} \right) J_0(\Gamma_n R_w) du$$  \hspace{1cm} (24)

and

$$S_{mn}(\omega, k_0) = \frac{d}{h} \int_{\pi/2}^{\pi/2} \left( \frac{\text{e}^{i n u}}{\text{e}^{i m u}} \right) Y_0(\Gamma_n R_w) du.$$  \hspace{1cm} (25)

where

$$I_{mn}^x(\omega, k_0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(n-m)u} J_0(\Gamma_n R_0(1 + \alpha \text{cos} u)) du$$

$$= \frac{1}{\pi} \int_{0}^{\pi} \text{cos}((n-m)u) J_0(\Gamma_n R_0(1 + \alpha \text{cos} u)) du$$  \hspace{1cm} (26)

and

$$I_{mn}^y(\omega, k_0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(n-m)u} Y_0(\Gamma_n R_0(1 + \alpha \text{cos} u)) du$$

$$= \frac{1}{\pi} \int_{0}^{\pi} \text{cos}((n-m)u) Y_0(\Gamma_n R_0(1 + \alpha \text{cos} u)) du.$$  \hspace{1cm} (27)

The integrals in (26) and (27) were evaluated numerically using global adaptive quadrature routines in MATLAB [18]. Equations (11) and (17) can be cast into a matrix equation

$$[D(\omega, k_0)] \cdot C = 0$$  \hspace{1cm} (28)

where the elements of the matrix $D$ are defined by $P_{mn}$, $Q_{mn}$, $R_{mn}$, $S_{mn}$, and $C$ constitute the various electric field coefficients, $A_n$ and $B_n$. Equation (28) has a nontrivial solution if

$$\det[D(\omega, k_0)] = 0.$$  \hspace{1cm} (29)

Equation (29) constitutes the dispersion relation for the system.

Fig. 7 shows a comparison of the COMSOL eigenmode study and the results from the $10 \times 10$ matrix using space harmonics of order $m = -2, -1, 0, 1, 2$ as well as using only two space harmonics of order $m = -1, 0$. The maximum relative errors using the truncated matrix for five space harmonics were 0.005%, 0.005%, and 0.013% for the first three lowest order
modes across the entire Brillouin while the maximum relative errors for the truncated matrix for two space harmonics were still quite accurate with maximum relative errors of 0.36%, 0.45%, and 2.3% for the first three lowest order modes across the entire Brillouin zone.

C. Planar Transmission Line With Rectangular Corrugations

To develop a framework for the corrugated coaxial TL dispersion relation in the long wavelength limit, the 2-D planar TL with rectangular corrugations is considered. This simpler geometry establishes the presence of only odd-order terms in the group velocity is negative is not captured using (34). The dispersion relation in the long wavelength limit, the 2-D planar TL with rectangular corrugations is considered. This simpler geometry establishes the presence of only odd-order terms in the entire Brillouin zone.

The field solutions for the TM modes are given by the following equations:

\[ E_z = \sum_{n=-\infty}^{\infty} A_n \sin(\Gamma_n(b-x)) e^{i k_n z} \]  \( (30) \)

\[ E_x = \sum_{n=-\infty}^{\infty} -\frac{i k_n}{\Gamma_n} A_n \cos(\Gamma_n(b-x)) e^{i k_n z} \]  \( (31) \)

\[ H_y = \sum_{n=-\infty}^{\infty} -\frac{i \omega \epsilon_0 k_0}{\Gamma_n} A_n \cos(\Gamma_n(b-x)) e^{i k_n z} \]  \( (32) \)

where \( \Gamma_n \) was defined in (5). For the planar geometry, the dispersion relation is given by

\[ \frac{\sqrt{\epsilon}}{Z_0} \cot \left( \frac{\omega \sqrt{\epsilon}}{c} s \right) = -\sum_{n=-\infty}^{\infty} \frac{\omega \epsilon_0 \pi d}{n \pi \Gamma_n \sin \left( \frac{n \pi w}{d} \right)} \cot (\Gamma_n b) \]  \( (33) \)

and \( Z_0 = (\mu_0/\epsilon_0)^{1/2} \) is the impedance of free space.

Equations (30)–(33) use the boundary condition \( E_z(x=b,z)=0 \) and assume the axial electric field at \( x=0 \) is uniform across each corrugation gap, with the field across each corrugation gap delayed by a phase factor, \( \exp(i k_0 z) \). Neglecting higher-order space harmonics (only considering the \( n=0 \) term), the dispersion relation reduces to

\[ \tan \left( \frac{\omega \sqrt{\epsilon}}{c} s \right) + \frac{\Gamma_0 d c}{\omega \sqrt{\epsilon} \omega} \tan(\Gamma_0 b) = 0 \]  \( (34) \)

where

\[ \Gamma_0^2 = \frac{\epsilon \omega^2}{c^2} - k_0^2. \]  \( (35) \)

The dispersion properties of this structure with parameters comparable to the coaxial case analyzed in Section II-A were again determined using COMSOL and compared with the solution of (34). The parameters used for the planar TL were chosen for comparison to RF-19 cable: \( d = 7 \text{ mm}, \omega = d/2 = 3.5 \text{ mm}, b = 6.1 \text{ mm}, s = 2R_1 = 1.25 \text{ mm}, \) and dielectric constant \( \epsilon = 1.3. \)

Fig. 9 shows a comparison of the eigenmode study and the results from (34). The TEM-like mode as calculated from (34) deviates from the eigenmode solution above \( \sim 4.3 \text{ GHz} \) and the solution near the \( \pi \)-mode is not predicted accurately. This is primarily due to neglecting the \( n = -1 \) space harmonic in (33). Similarly, the 1st HOM is not captured very well from (34), again due to the exclusion of the \( n = -1 \) space harmonic. This is readily apparent from Fig. 9 since the region where the group velocity is negative is not captured using (34).

To obtain an estimate of the linear phase advance per unit length correction and higher order dispersion, (34) can be reduced, using the series expansion of \( \tan(\sqrt{\epsilon \cos/c}) \) to...
The expression for $k_0(\omega)$ can be expanded as a power series in odd powers of $\delta$ as

$$k_0(\omega) \approx \left(\frac{\sqrt{\varepsilon_0}c}{\omega}\right)\sqrt{1 + ws/bd}$$

$$+ \frac{\sqrt{\varepsilon_0}c}{\omega} \left(\frac{ws^3}{6bd}\right)\left(\frac{1}{\sqrt{1 + ws/bd}}\right) + O(\omega^5).$$ (40)

With the substitution $w = d/2$ and the condition $s/2b \ll 1$, (40) reduces to the result of Kotzian et al. [1]. One can see (40) is of the form

$$k_0(\omega) = \sqrt{\varepsilon_0}c + \gamma_1\omega + \gamma_3\omega^3$$ (41)

where $\gamma_1$ and $\gamma_3$ are defined in subsequent (42) and (43). A series of COMSOL simulations were done varying the corrugation depth $s = 0.125$ mm to $s = 3$ mm with the parameters described previously. The dispersion data $k_0(\omega)$ was used from the simulations and the baseline phase term of $(k_0(\omega) - \sqrt{\varepsilon_0}c)$ was subtracted out to fit the phase advance per unit length correction to a polynomial in $\omega$. Using (40) and (41) the linear and cubic phase terms are given by the following equations:

$$\gamma_1 = \frac{\sqrt{\varepsilon_0}c}{d} \left(\sqrt{1 + ws/bd} - 1\right)$$ (42)

$$\gamma_3 = \left(\frac{\sqrt{\varepsilon_0}}{c}\right)^3 \left(\frac{ws^3}{6bd}\right)\left(\frac{1}{\sqrt{1 + ws/bd}}\right).$$ (43)

These terms are valid when $ks \ll 1$, where $k = \sqrt{\varepsilon_0}c$. It should be noted that $\gamma_1$ and $\gamma_3$ are dependent only on geometric parameters and the dielectric constant of the cable, and are independent of frequency $\omega$ (in contrast to material dispersion). Figs. 10 and 11 show a comparison of $\gamma_1$ and $\gamma_3$ as found from COMSOL simulations and (42) and (43) for a wide corrugation slot ($w/d = 1/2$ with $w = 3.5$ mm) and a narrow corrugation slot ($w/d = 1/7$ with $w = 1$ mm), respectively. Also shown on the plots are fits using effective corrugation depths that were determined empirically. It was found that an effective depth $s$ was needed to properly fit (42) and (43) to the more accurate simulation results.

There are two geometric ratios that are relevant to the terms $\gamma_1$ and $\gamma_3$: the corrugation width to depth ratio, $w/s$, and the corrugation width to period ratio, $w/d$. In the long-wavelength regime, $\gamma_1$ and $\gamma_3$ are primarily due to the additional inductance per unit length from the slots. When $w/s$ is large, the corrugation slot itself does not look like a simple planar TL, which explains the deviation from linear behavior of $\gamma_1$ for small $s$ as shown in Figs. 10(a) and 11(a). Similarly, when $w/d$ is large, the effective corrugation slot depth is smaller than the actual corrugation slot depth. Moreover, there is a contribution from the fringe fields at the interface of the corrugation slots with the primary TL. This contribution is strongly influenced by the electric field curvature, which makes the effective depth of the corrugations slots larger than the actual depth.

D. Comparison of Coaxial Transmission Lines With Different Corrugation Profiles

The influence of corrugation shape on the coaxial TL dispersion relation was investigated using COMSOL for sinusoidal,
Fig. 11. Terms $\gamma_1$ and $\gamma_3$ for the planar TL with rectangular corrugations $w = 1 \text{ mm}$, $w/d = 1/7$, showing (a) $\gamma_1$ and (b) $\gamma_3$ from (42) to (43) respectively compared with COMSOL simulations. Due to the interface between the TL and the corrugations, the effective depth is 0.16 mm smaller than the actual corrugation depth for $\gamma_1$ and 0.55 mm larger than the actual corrugation depth for $\gamma_3$.

Fig. 12. Coaxial TL dispersion relation comparing the lowest order TEM-like mode for sinusoidal, rectangular, and triangular corrugation profiles. Inset shows detail near the $\pi$ mode.

rectangular, and triangular corrugation profiles, with the same periodicity, minimum outer wall radius, and corrugation depth. Fig. 12 is an eigenmode comparison for the three corrugation profiles under consideration, showing nearly identical dispersion plots.

It is evident that the rectangular corrugation will introduce more dispersion in the cable than the sinusoidal and triangular corrugations. However, it is easy to reach an incorrect conclusion about the effect of the corrugation profiles on wave propagation in the structure. Slight changes in the dispersion relation which produce only small changes in the phase advance along the TL can have profound effects if the TL is long or the pulse bandwidth is large. Thus there are limitations on the accuracy of utilizing the dispersion relation to determine the effect of pulse propagation in long cables. Inaccuracies in the finite element eigenmode solver due to meshing or numerical issues may lead to similar problems if the dispersion relation is used to determine effect of pulse propagation in long cables. It should be noted that finite element solutions for all the simulations were run with different mesh densities to ensure that each simulation was converged.

Fig. 13 shows the linear and cubic phase terms as a function of corrugation depth. These exhibit similar behavior to the planar TL with rectangular corrugations discussed in the previous
An exploration of different corrugation profiles revealed that one can fine-tune a geometric dispersion at the edge of the passband/Brillouin zone, near the $\pi$ mode. From Fig. 13, even in the low-frequency regime, there are significant differences in the phase advance corrections between three corrugation profiles with the same base parameters. This ability to fine-tune the geometric dispersion will have potentially broad applicability across microwave and photonic fields.

III. CONCLUSION

Eigenfrequency simulations using the finite element code, COMSOL, were used to determine the geometric dispersion relation of a coaxial TL (baseline case: RF-19) with sinusoidal corrugations on the outer wall, assuming no conductor loss or material dispersion.

An analytic technique using a modal expansion method with Floquet theory was developed to calculate the geometric dispersion relation and compare with the finite element solutions. It was found that reasonably accurate solutions for the geometric dispersion can be determined using only two space harmonics for the cases examined.

The dispersion relation of the simpler planar TL with rectangular corrugations with comparable RF-19 parameters was examined to establish a functional form for the coaxial corrugated TL with different geometric profiles. We found that the phase advance corrections for all geometries (planar, coaxial, and profiles: rectangular, triangular, and sinusoidal) could be expressed as an odd power series in $\omega$ whose coefficients were purely determined by geometric parameters and the dielectric constant, in the long-wavelength limit.

While the dispersive properties are similar for the different geometric profiles, for high bandwidth signals in long TLs, there may be substantial differences in the output pulse shape. Since dispersion is a cumulative effect, by carefully selecting the corrugation profile, one can potentially minimize deleterious effects in long-distance TLs. Another benefit of micromanagement of the geometric dispersion is increased bandwidth for high power sources that rely on periodic structures. For RF-19 cable there is substantial additional phase advance caused by the corrugations and for higher bandwidth pulses in long cables the geometric dispersion will distort the pulse. This is particularly relevant if reconstruction of the input signal to the TL is of critical importance. A future article will address reconstruction of the input pulse via direct time domain simulations and analytic methods that utilize approximations of the transfer function.

REFERENCES

[1] G. Kotzian, F. Caspers, S. Federmann, W. Höfle, and R. de Maria, “Ring-ing in the pulse response of long and wideband coaxial transmission lines due to group delay dispersion,” in Proc. Part. Accel. Conf., Vancouver, BC, Canada, 2009, pp. 3519–3521.
[2] J. A. Swegle, J. W. Poukey, and G. T. Leifeste, “Backward wave oscillators with rippled wall resonators: Analytic theory and numerical simulation,” Phys. Fluids, vol. 28, pp. 2682–2894, Sep. 1985.
[3] J. A. Swegle et al., “Scaling studies and time-resolved microwave measurements on a relativistic backward-wave oscillator,” IEEE Trans. Plasma Sci., vol. 21, no. 6, pp. 714–724, Dec. 1993.
[4] J. J. Barroso and J. P. L. Neto, “Coaxial Bragg reflector,” in Proc. SBMO/IEEE MTT-S Int. Conf. Microw. Optoelectron., Jul. 2005, pp. 137–140.
[5] Y. X. Lai, S. C. Zhang, and H. B. Zhang, “A coaxial Bragg reflector for cyclotron autoresonance maser oscillators,” IEEE Microw. Wireless Compon. Lett., vol. 17, no. 5, pp. 328–330, May 2007.
[6] S. Chen, J. Zhang, J. Zhang, D. Zhang, and H. Wang, “Numerical computation of dispersion curves for both symmetric and asymmetric modes in metal coaxial slow wave structures,” IEEE Trans. Electron Devices, vol. 67, no. 1, pp. 322–327, Jan. 2020.
[7] M. del Mar Sánchez-López, J. A. Davis, and K. Crabtree, “Coaxial cable analogs of multilayer dielectric optical coatings,” Amer. J. Phys., vol. 71, no. 12, pp. 1314–1319, Dec. 2003.

[8] A. Haché and A. Slimani, “A model coaxial photonic crystal for studying band structures, dispersion, field localization, and superluminal effects,” Amer. J. Phys., vol. 72, no. 7, pp. 916–921, Jul. 2004.

[9] J. H. Wang and K. K. Mei, “Theory and analysis of leaky coaxial cables with periodic slots,” IEEE Trans. Antennas Propag., vol. 49, no. 12, pp. 1723–1732, Dec. 2001.

[10] A. Yariv and P. Yeh, Optical Waves in Crystals: Propagation and Control of Laser Radiation. Hoboken, NJ, USA: Wiley, 1984, pp. 155–219 and 405–503.

[11] H. Kogelnik, Theory of Dielectric Waveguides. Berlin, Germany: Springer, 1975, pp. 13–81.

[12] A. Grieco and Y. Fainman, “Characterization of distributed Bragg reflectors,” IEEE J. Quantum Electron., vol. 50, no. 6, pp. 453–457, Jun. 2014.

[13] A. E. Sanderson, “Effect of surface roughness on propagation of the TEM mode,” in Advances in Microwaves, vol. 7, L. Young, Ed. Amsterdam, The Netherlands: Elsevier, 1971, pp. 1–57.

[14] S. A. Kheifets, “Electromagnetic fields in an axial symmetric waveguide with variable cross section,” IEEE Trans. Microw. Theory Techn., vol. MTT-29, no. 3, pp. 222–229, Mar. 1981.

[15] O. R. Asfar and A. H. Nayfeh, “Circular waveguide with sinusoidally perturbed walls,” IEEE Trans. Microw. Theory Techn., vol. MTT-23, no. 9, pp. 728–734, Sep. 1975.

[16] A. Nayfeh and O. Asfar, “Parallel plate waveguide with sinusoidally perturbed boundaries,” J. Appl. Phys., vol. 45, no. 11, pp. 728–734, Nov. 1974.

[17] (2019). COMSOL MultiPhysics V.5.5 is a Product, COMSOL, Burlington, MA, USA. [Online]. Available: http://www.comsol.com

[18] (2021). MATLAB Release 2021b is a Product, Mathworks, Natick, MA, USA. [Online]. Available: http://www.mathworks.com

[19] S. Ramo, J. R. Whinnery, and T. V. Duzer, Fields and Waves in Communication Electronics, 3rd ed. New York, NY, USA: Wiley, 1994, pp. 482–486.

Brian R. Poole (Life Senior Member, IEEE) received the B.S. degree in electrical engineering and the M.S. and Ph.D. degrees in electrophysics from the Polytechnic Institute of New York (now the New York University Tandon School of Engineering and Applied Science), Brooklyn, NY, USA, in 1977, 1979, and 1984 respectively.

He is currently an Applied Physicist and an Electrical Engineer with the Lawrence Livermore National Laboratory (LLNL), Livermore, CA, USA. He is the author or coauthor of over 60 journal articles and conference papers. He has four U.S. patents. While at LLNL, his research was focused on high power microwave transmission systems for magnetic fusion experiments, high power microwaves sources, including BWO’s, Cerenkov sources, and beam-plasma interactions. He was also responsible for the design of the fast kicker system and beam transport simulations for the DARHT-II accelerator at the Los Alamos National Laboratory, Los Alamos, NM, USA. His current research interests include microwave sources using novel engineered structures, such as photonic crystals and meta-materials, and beam transport in high current accelerators.

Dr. Poole is also a member of the American Physical Society Division of Beams. He was a recipient of the Ernst Weber Scholarship for the 1979–1980 academic year.

Natalie B. Kostinski (Member, IEEE) received the B.S.E. degree in electrical engineering from the University of Michigan, Ann Arbor, MI, USA, in 2006, and the M.A. and Ph.D. degrees in electrical engineering from Princeton University, Princeton, NJ, USA, in 2008 and 2011, respectively.

She is currently an Experimental Physicist with the Lawrence Livermore National Laboratory, Livermore, CA, USA.