Investigation of dust ion acoustic shock and solitary waves in a viscous dusty plasma

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Abstract
A viscous dusty plasma containing Kappa-\((\kappa -)\) distributed electrons, positive warm viscous ions, and constant negatively charged dust grains with viscosity have been considered to study the modes of dust-ion-acoustic waves (DIAWs) theoretically and numerically. The derivations and basic features of shock and solitary waves with different plasma parameters like Mach number, finite temperature coefficient, unperturbed dust streaming velocity, the kinematic viscosity of dust, etc of this DIAWs mode have been performed. Considering the dynamical equation from Korteweg-de Vries(KdV) equation, a phase portrait has been drawn and the position of the saddle point or col. and center have also been discussed. This type of dusty plasma can be found in celestial bodies. The results of this research work can be applied to study the properties of DIAWs in various astrophysical situations where \(\kappa\)-distributive electrons are present and careful modification of the same model can help us to understand the nature of the DIAWs of laboratory plasma as well.

1. Introduction
In the past few years, dusty plasmas or complex plasmas have become a topic of great research interest around the globe. It is a kind of plasma where nanometer or micrometer-sized particles are suspended. In most of the cases, these nanometer or micrometer-sized particles known as dust are negatively charged but there are various circumstances where these particles may be positively charged. The charge of dust particles varied in size, the greater the size of the dust particles larger their charge will be. Coupling via electrostatic force or any other forces like collision or viscous may happen and this coupling can change from weakly coupled to crystallizing structures in the presence of these dust particles. The dependence of the coupling parameter with the structural parameter has been discussed in detail by Dragan and Kutarov [1]. As we consider the density of dust particles is low, the viscosity is also very low, and no collision is being considered, so the effect of coupling is not taken into account in this present model like many other profound scientists did for dust ion acoustics waves (DIAWs) [2]. Dusty plasma can be found in celestial objects like comets, zodiacal dust clouds, interstellar, circumstellar clouds, etc [3]. Not only astrophysical bodies but there are many examples of laboratory dusty plasma as well [3].

Depending on the motion of the species, dusty plasma can produce various waves, such as dust-acoustic waves (DAWs) [4], and dust ion acoustic waves (DIAWs) [5]. We are going to consider dust ion acoustic waves (DIAWs) as the motion of both ions and dust species and also the distribution of electrons is taken into account. Some authors have mentioned the dust ion-acoustic waves term only by considering the motion of ions [6–13].

Furthermore, the ion-acoustic (IA) [14], electron acoustic (EA) [15] and electron-static (ES) [16] modes can also be present in a dusty plasma environment.

Also, shock waves in plasma have been found to have a greater interest in recent years. Various researchers have found many interesting results in the formation of shock waves in IA, EA, ES cases, and Laser Plasma Interactions (LPI) too [17]. Both theoretical and experimental study of shock waves has been found in recent years [18–21]. In the case of the production of shock waves in dusty plasma, there exists a good number of theoretical and experimental works [21–24]. Shock waves can generally be produced in plasma if there is a...
dissipative force present in the plasma [25]. Two very important dissipative forces arise in the plasma due to collision and viscosity. These two forces cause the damping of plasma because of which shock waves have been formed.

In the low-density astrophysical plasma domain, there is a deviation of the velocity distribution function (VDF) of particles from the Maxwellian distribution due to fewer binary collisions among charges. Lorentzian velocity distribution function describes such superthermal particle distribution. Kappa distribution is one of these kinds of superthermal distribution where the particles have high-energy tails. This distribution can be given as [26, 27]-

$$f_\kappa(\nu) = \frac{n_0 \Gamma(\kappa + 1)}{(\pi \theta^2)^{3/2} \Gamma(\kappa - \frac{1}{2})} \left[ 1 + \frac{\nu^2}{\kappa \theta^2} \right]^{-(\kappa+1)}.$$  

Here, $n_0$ is the unperturbed particle density and $\theta^2 = \frac{(\kappa - \frac{1}{2})^2 k_B T}{m_i}$. The effect of $\kappa$-distributive electrons has been shown by various researchers [28–30]. In the present paper, we have considered collisionless space plasma, and that is why we have taken kappa-distributed electrons. This type of distribution is frequently presented in collisionless space plasma [31].

At the time of the formation of the model of this present work, we have kept mainly two types of space plasma in our minds.

(a) Dusty plasma in an interstellar molecular cloud, a dense plasma, can consist of an H$^+$ ion. In these types of dusty plasma, the normalized charge density of the dust grains is one ($q = 1$). [32, 33]

and

(b) Dusty plasma is produced by the reaction of pickup $H^+$ ions and ambient plasma (i.e. solar wind with $q \sim 1000$) [34–37].

When physical parameters are varied there is a qualitative change in the behavior of the system, these changes will be found in the Bifurcation analysis. Dynamical behaviors of nonlinear IAWs in a magnetized dusty plasma have been reported by Samanta et al [38].

In this work, the motion of ion and dust grains have been considered to construct this model and in both cases kinematic viscosity is present. Also, a finite temperature correction for the ions has also been taken in this model. As the time scale of the electrons is very much less than the other two species so we have taken a distribution for the electrons which is the kappa distribution. The work is performed in the weak non-linearity region and that is why we have used the reductive perturbation technique (RPT). The momentum in the system is majorly supplied by the dust grains as it has heavier mass than ions and on the other hand, the force in the system is majorly supplied by the ions. So keeping these things in mind we have derived the KdV-Burger and KdV equations for the system and most importantly discussed the weak and strong shock in a DIAWs mode for the first time.

This paper is organized in the following way- in section 2 we have formulated the problem for the dusty plasma with a detailed discussion on kinematic viscosity and finite temperature correction in the ion in the following two subsections 2.1 and 2.2. In section 3 we have linearized the dynamic equations given in section 2 and achieved the linear dispersion relation for real and imaginary cases, as two dispersion relations arise here and have the same form that is why we have discussed a general solution for both the equations in subsection 3.1. In the next section 4, we have derived the KdV-Burger(KdVB) equation and in subsection 4.1 we have solved the same. Afterward, under subsection 4.2, without considering the viscosity of both the species (dust grains and the ions), we have derived the KdV equation. The analytical and numerical solutions of the KdV equation have been achieved in subsections 4.2 and 4.3. Next, we have discussed the dynamic character and the stability of the KdVB equation in section 5. Then we have discussed the results and discussion of the solution of the KdVB and KdV equation which eventually gives us the shock fronts and solitary structures in this section 6. Finally, in section 7 we give a brief summary of this paper.

### 2. Basic Formulations

We have considered an unmagnetized dusty plasma in astrophysical space and investigated the dust-ion acoustic (DIA) modes in such a plasma. As we have investigated the DIA mode, the dynamical equations for both the species (ions and negatively charged dust) have been taken into account. The continuity equation for both species is given by:
\[ \frac{\partial n_j}{\partial t} + \frac{\partial (n_j u_j)}{\partial x} = 0, \]

(1)

here \( j = d, i \). Now, the normalized momentum equation for the negative dust species in presence of the viscosity is given by

\[ \frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = g \frac{\partial \phi}{\partial x} + \eta_d \frac{\partial^2 u_d}{\partial x^2}. \]

(2)

The momentum equations for the ions is given by

\[ \frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = - \frac{\partial \phi}{\partial x} - 3 \sigma n_i \frac{\partial u_i}{\partial x} - \eta_i \frac{\partial^2 u_i}{\partial x^2}. \]

(3)

These continuity and momentum equations are bounded by the Poisson’s equation which is given by,

\[ \frac{\partial^2 \phi}{\partial x^2} = n_e - \delta n_i + qn_d. \]

(4)

The electron has a super thermal distribution here in equation (4) and the electron density can be obtained after normalizing and integrating the distribution over velocity space as,

\[ n_e = n_0 e^{-\phi} \left\{ 1 - \frac{\phi}{(\kappa - \frac{3}{2})} \right\}^{-\kappa + \frac{1}{2}}. \]

(5)

All the other quantities have been discussed in the next three subsections 2.1–2.3.

### 2.1. The kinematic viscosity

In the momentum equations for the negative dust and positive ion, we have considered that an effect of kinematic viscosity is present. It is a very special kind of viscosity depending predominately on the velocity of that species. The coefficient of absolute viscosity \( \mu \) is basically the measurement of internal resistance. A force is generated due to this internal resistance and is given by \( F = \mu A \times S \), where \( A \) is the surface area of the plasma and \( S \) is the shear rate in the plasma. This law of force is known as ‘Newton’s law of viscosity’. In equations (2) and (3), the viscosity we use is the kinematic viscosity as we mentioned earlier and this is basically the ratio between absolute viscosity \( \mu \) and fluid mass density \( n \), this is previously discussed by Goswami and Sarkar [39].

### 2.2. The finite temperature correction in the ion

The second term in the right-hand side of the momentum equation of ion (equation (3)) arises due to the finite temperature correction [40, 41]. This finite temperature correction in the system only arises when the ions are going to follow the Fermi–Dirac (FD) statics. As ions are very much lighter than the dust particle then the quantum mechanical tunneling effect comes into act and we have to consider that the chemical potential \( \mu_i \) remains constant due to non-equilibrium dynamics. Considering all these, the non-equilibrium particle density is given by the following equation:

\[ n_i = n_0 e^{-\beta \omega_i} \left\{ 1 - \frac{\omega_i}{(\kappa - \frac{3}{2})} \right\}^{-\kappa + \frac{1}{2}}, \]

where \( m \) is the mass of the ion, \( h \) is the reduced Planck constant, \( n_0 \) is the equilibrium number density, \( \beta = \frac{1}{k_B T_0} \), \( T_0 \) is the back-ground temperature of ion and \( \mu_i \) is the chemical potential. Applying this pressure term without considering the Bohm diffraction term we get the equation (3). Following the same procedure as used by Akbari-Moghanchoughi and Eliasson [42] and Goswami et al [16] we get the \( \sigma \) as

\[ \sigma = \frac{L_{S2J}(-e^{\beta \omega_i})}{L_{S2J}(-e^{\beta \omega_i})} \left( \frac{V_{\text{th}}}{V_{\text{FD}}} \right)^2. \]

Here \( L_{S2J}(x) \) is the polylogarithmic function in \( x \) of the order \( \nu \). Some of the authors [43, 44] consider the plasma as a Fermi gas so the pressure is:

\[ p_i = \frac{1}{3} \frac{m v_i^3}{n_0^2} n_i. \]

And if we consider this pressure in this model it will only be present in the case of the ions because these are lighter than the dust species. Considering this pressure, our momentum equation (3) would look like,

\[ \frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = - \frac{\partial \phi}{\partial x} - \sigma_i n_i \frac{\partial u_i}{\partial x} - \eta_i \frac{\partial^2 u_i}{\partial x^2}, \]

where \( \sigma_i \) is the ratio between ion to electron Fermi temperature.
2.3. Normalization schemes and charge neutrality

From equations (1)–(4), all the equations are normalized and the standard normalization schemes are given by:

\[ \hat{x} \rightarrow \frac{x}{\lambda}, \hat{t} \rightarrow \omega_j t, \hat{\phi} \rightarrow \frac{\phi_0}{\omega}, \hat{u}_j \rightarrow \frac{u_j}{v_{Te}}, \text{ and } \hat{\eta}_j \rightarrow \frac{\eta_j}{\lambda v_{Te}}. \]

Also, at the equilibrium, the charge neutrality condition is \( n_{e0} + qn_{d0} = \delta n_{i0} \). Generally, plasma parameters have effects on the fluctuation of dust charge. But the dust-ion acoustic (DIA) time scale is much shorter than the dust charging time scale. Thus, we can clearly assume that in DIA mode there is no significant effect of dust charge fluctuation. Therefore, the dust charge can be assumed to be constant [45, 46]. In this manuscript, we have all along taken \( \delta = 1 \). The various values of these dust charges in different astrophysical plasmas have been discussed by EL-Labany et al [32].

3. Linear dispersion relation

In order to investigate the linear and nonlinear behavior of dust acoustic wave in this three-component electron-ion-dust plasma, we make the following perturbation expansion for the field quantities \( n_j, u_j, \) and \( \phi \) about their equilibrium values:

\[
\begin{bmatrix} n_j \\ u_j \\ \phi \end{bmatrix} = \begin{bmatrix} 1 \\ u_{j0} \\ \phi_0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_j^{(1)} \\ u_j^{(1)} \\ \phi^{(1)} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_j^{(2)} \\ u_j^{(2)} \\ \phi^{(2)} \end{bmatrix} + \ldots
\]

(6)

We assumed that all field variables vary as \( \exp[i(kx - \omega t)] \) and here \( \omega \) is the normalized wave frequency and \( k \) is the wave number, which contains both real \( (k_r) \) and imaginary \( (k_i) \) parts. Here, the viscous term plays a pivotal role. The dispersion equation has an exponentially decaying complex part in addition to the real dispersion relation. In this case, if we substitute the wave number with real plus imaginary parts \( k = k_r + ik_i \), we obtain the two dispersion relations as given below. The full calculation is given in Appendix A.1, as this type of calculation for the dispersion relations is new for the dusty plasma case.

The real dispersion relation is given by,

\[ P_1\omega'^4 + Q_1\omega'^3 + R_1\omega'^2 + S_1\omega' + T_1 = 0, \]

(7)

where,

\[
\begin{align*}
P_1 &= -\left[ k_i^2 + \frac{1 - 2\kappa}{2\kappa - 3} \right] \\
Q_1 &= -2k_P (u_{j0} + u_{d0}) \\
R_1 &= P_1[4k_r^2u_{j0}u_{d0} + k_r^2u_{d0}^2 + k_r^2u_{j0}^2 + 3\sigma k_r^2] + 2k_r^2(\delta + q^2) \\
S_1 &= -P(2k_r^2u_{j0}u_{d0} + 2k_r^2u_{d0}u_{j0} + 6k_r^2\sigma u_{j0}) + 2k_r^2(\delta + q^2)(u_{d0} + u_{j0}) \\
T_1 &= P_1(k_r^4u_{j0}u_{d0}^2 + 3\sigma k_r^4u_{j0}^2) - k_r^4(\delta + q^2)(u_{d0}^2u_{j0}^2 + 3\sigma)
\end{align*}
\]

(8)

The imaginary dispersion relation is given by,

\[ P_2\omega''^4 + Q_2\omega''^3 + R_2\omega''^2 + S_2\omega'' + T_2 = 0, \]

(9)

where,

\[
\begin{align*}
P_2 &= -\left[ k_i^2 + \frac{1 - 2\kappa}{2\kappa - 3} \right] \\
Q_2 &= P_2k_i^2 \\
R_2 &= -P_2(3\sigma k_i^2 - \eta_{ih}\eta_{ih}k_i^4) - 2k_i^2(\delta + q^2) \\
S_2 &= -3P_2\sigma \eta_{ih}k_i^4 + k_i^4(\delta + q^2)(\eta_{ih} - \eta_{ih}) \\
T_2 &= -3\sigma k_i^4(\delta + q^2)
\end{align*}
\]

(10)

3.1. General solution

These two equations (7) and (9) have the same form the solution of these kinds of equations is given below; The dispersion relations have the form

\[ P\omega^4 + Q\omega^3 + R\omega^2 + S\omega + T = 0, \]

(11)

or,

\[ \omega^4 + \frac{Q}{P}\omega^3 + \frac{R}{P}\omega^2 + \frac{S}{P}\omega + \frac{T}{P} = 0, \]

(12)
or,
\[ \omega^4 + Q_3\omega^3 + R_3\omega^2 + S_3\omega + T_3 = 0, \] (13)

Let, \( \omega = x - \frac{Q_3}{4} \) then the above equation becomes,
\[ x^4 + px^2 + qx + r = 0, \] (14)

where,
\[
\begin{align*}
  p &= -\frac{6Q_3^2}{16} - \frac{3Q_3}{4} + R_3 \\
  q &= \frac{Q_3^3}{8} - \frac{2Q_3R_3}{4} + S_3 \\
  r &= -\frac{3Q_3^4}{256} + \frac{Q_3^2R_3}{16} - \frac{Q_3}{4} + T_3
\end{align*}
\] (15)

From equation (14)
\[
\left( x^2 + \frac{p}{2} \right)^2 = -qx - r + \left( \frac{p}{2} \right)^2
\] (16)

or,
\[
\left( x^2 + \frac{p}{2} + u \right)^2 = -qx - r + \left( \frac{p}{2} \right)^2 + 2ux^2 + pu + u^2
\] (17)

In order to have a perfect square, we have,
\[ 8u^3 + 8pu^2 + (2p^2 - 8r)u - q^2 = 0 \] (18)

or,
\[ u^3 + pu^2 + \left( \frac{2p^2 - 8r}{8} \right)u - \frac{q^2}{8} = 0 \] (19)

or,
\[ u^3 + pu^2 + lu + g = 0, \] (20)

where, \( l = \left( \frac{2p^2 - 8r}{8} \right) \), and \( g = -\frac{q^2}{8} \).

Again, let \( u = t - \frac{p}{3} \) equation yields
\[ t^3 + jt + l = 0, \] (21)

where, \( j = \frac{3l - p^3}{27} \) and \( l = \frac{2p^3 - 9pl + 27g}{27} \). The solution of this equation is,
\[ t = \left( \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{j^3}{27}} \right)^{1/3} + \left( -\frac{1}{2} - \sqrt{\frac{1}{4} + \frac{j^3}{27}} \right)^{1/3} \] (22)

Now, we know \( u = t - \frac{p}{3} \) the equation transforms into
\[ x^2 + \frac{p}{2} + u = \pm \left[ \sqrt{2u} - \frac{2}{2\sqrt{2u}} \right] \] (23)

Solving this equation we get,
\[ x = \pm \left[ \sqrt{-\frac{p}{2} - u} \pm \left( \sqrt{2u} - \frac{2}{2\sqrt{2u}} \right) \right] \] (24)

Therefore we get,
\[ \omega = -\frac{Q_3}{4} + x \] (25)

This equation (25) is the general solution for the equations (7) and (9). Clearly, this general solution has four roots which means the real dispersion relation (Equation (7)) and the imaginary dispersion relation (equation (9)) individually have four roots. Under these four roots in each case, two are physically admissible [47]. Again, in these two roots for each case, we can categorize them as ‘fast’ and ‘slow’ modes.
4. Derivation of KdV-Burgers equation

In order to derive the equation of motion for the nonlinear dust ion acoustic wave, we employ the reductive perturbation technique and define the following stretched variables,

\[
\begin{align*}
\xi &= \varepsilon^{1/2} (x - M t) \\
\tau &= \varepsilon^{3/2} t \\
\eta &= \varepsilon^{1/2} \eta_0
\end{align*}
\]

(26)

where \( \varepsilon \) is a small parameter which characterizes the strength of nonlinearity, and \( M \) is the Mach number i.e. the phase velocity of the wave. The stretching in \( \eta \) is due to the small variations in perpendicular directions.

Now, equations (1)–(4) are written in terms of the stretched coordinates \( \xi, \tau, \) and \( \eta \) and substituting the perturbation expansion given in equation (6). From these equations equating the lowest orders in \( \varepsilon \) i.e. \( \varepsilon^{3/2} \) with the boundary conditions that all variables, that is, \( n_i^{(1)}, u_i^{(1)}, n_d^{(1)}, u_d^{(1)} \) and \( \phi^{(1)} \) tend to zero as \( \xi \to \pm \infty \), we get,

\[
\begin{align*}
\xi &= \varepsilon^{1/2} (x - M t) \\
\tau &= \varepsilon^{3/2} t \\
\eta &= \varepsilon^{1/2} \eta_0
\end{align*}
\]

(27)

Going to next higher order terms in \( \varepsilon \), that is, \( \varepsilon^{5/2} \)-th term, we get,

\[
\begin{align*}
(M - u_{d0}) \frac{\partial u_d^{(2)}}{\partial \xi} &= \frac{\partial u_d^{(2)}}{\partial \xi} + \frac{q}{(M - u_{d0})^2} \frac{\partial \phi^{(1)}}{\partial \tau} + \frac{q^2}{(M - u_{d0})^3} \frac{\partial \phi^{(1)} \phi^{(1)}}{\partial \xi} \\
(M - u_{i0}) \frac{\partial n_i^{(2)}}{\partial \xi} &= \frac{\partial n_i^{(2)}}{\partial \xi} - \frac{1}{(M - u_{i0})^2} \frac{\partial \phi^{(1)} \phi^{(1)}}{\partial \tau} + \frac{1}{(M - u_{i0})^2} \frac{\partial \phi^{(1)} \phi^{(1)}}{\partial \xi}
\end{align*}
\]

(28)

\[
\begin{align*}
\frac{\partial u_d^{(2)}}{\partial \xi} &= -\frac{q}{(M - u_{d0})^2} \frac{\partial \phi^{(1)}}{\partial \tau} + \frac{q^2}{(M - u_{d0})^3} \frac{\partial \phi^{(1)} \phi^{(1)}}{\partial \xi} + \frac{q}{(M - u_{d0})^2} \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{q^2}{(M - u_{d0})^3} \frac{\partial \phi^{(2)} \phi^{(1)}}{\partial \xi} \\
\frac{\partial u_i^{(2)}}{\partial \xi} &= \frac{1}{M - u_{i0}} \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{3 \sigma}{(M - u_{i0})} \frac{\partial \phi^{(1)} \phi^{(1)}}{\partial \xi} + \frac{\eta_{i0}}{(M - u_{i0})^2} \frac{\partial \phi^{(2)} \phi^{(1)}}{\partial \xi}
\end{align*}
\]

(29)

(30)

The second-order perturbation of the Poisson’s equation can be equated as

\[
\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \left[ \frac{1 - 2 \kappa}{2 \kappa - 3} + \frac{4 \kappa^2 - 1}{(2 \kappa - 3)^2} \right] \phi^{(2)} + \frac{(4 \kappa^2 - 1)}{2 (2 \kappa - 3)^2} \phi^{(1)} \phi^{(1)} - \eta_{i0} \phi^{(1)}(2) + q u_d^{(2)}
\]

(32)

Differentiating both sides of equation (32) by \( \xi \) and carrying out a detailed algebraic treatment with the help of the equations from (28) to (31), we get the following equation

\[
\begin{align*}
\frac{\partial \phi^{(1)}}{\partial \tau} + A \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + C \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = 0
\end{align*}
\]

(33)
where, $A = \frac{\Gamma}{\Theta}$, $B = \frac{\Theta}{\Upsilon}$ and $C = \frac{\Sigma}{\Theta}$. The $\Gamma$, $\Theta$ and $\Upsilon$ are given below.

$$\Gamma = \frac{36}{[(M - u_{i0})^2 - 3\sigma^2]} - \frac{32}{(M - u_{i0})^2} - \frac{4\kappa^2}{2(M - u_{i0})^2}$$

$$\Theta = \frac{2(M - u_{i0})}{[(M - u_{i0})^2 - 3\sigma^2]} + \frac{2a^2}{(M - u_{i0})^2}$$

$$\Upsilon = \frac{\eta_{l0}(M - u_{i0})}{[(M - u_{i0})^2 - 3\sigma^2]} - \frac{\eta_{l0}g^2}{(M - u_{i0})^2}$$

(34)

The nonlinear equation (33) is the well known KdV-Burgers(KDVB) equation and the coefficients $A$, $B$ and $C$ are the nonlinear, dispersive and dissipative coefficients, respectively.

4.1. Solution of KdV Burgers equation

The equation (33) is a nonlinear equation in both the variables $\xi$ a space-like variable and $\tau$ a time-like variable.

To solve this equation we have to first join these variables into one wave variable $\psi = \xi - M\tau$ to transform the partial differential equation (PDE) of two variables $\xi$ and $\tau$ into the ordinary differential equation (ODE) of one variable $\psi$ with the application of the boundary conditions when $\psi \to 0$ then $\phi^{(1)} \to 0$ and $\frac{\partial \phi^{(1)}}{\partial \psi} \to 0$ as $[16, 48, 49]$.

$$-M \frac{d\phi^{(1)}}{d\psi} + A\phi^{(1)} \frac{d\phi^{(1)}}{d\psi} + B \frac{d^2\phi^{(1)}}{d\psi^2} + C \frac{d^3\phi^{(1)}}{d\psi^3} = 0$$

(35)

As we are trying to solve this equation with the help of tan-hyperbolic method so now we have to take $s = \tanh \psi$ and assuming a series solution predicted by Wazwaz [50] as $\phi^{(1)}(s) = \sum_{j=0}^{n} a_j s^j$ and we get the following solution,

$$\phi^{(1)} = \frac{12A}{B} \left[1 - \tanh^2(\psi)\right] - \frac{36C}{15A} \tanh(\psi)$$

(36)

This equation (36) is the solution of the KdVB equation (33).

4.2. The KdV equation and the solution

It is very clear that if no viscous force is considered for both the species (ions and dust) then $\eta_{l0} = \eta_{d0} = 0$. So there will be no dissipation and $C = 0$. In this condition, equation (33) is transformed into Kortewg deVries (KdV) equation.

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = 0$$

(37)

The coefficients $A$ and $B$ are already given. Using the same transformation $\psi = \xi - M\tau$ and the boundary condition we have the solution of the KdV equation,

$$\phi^{(1)} = \phi_0 \sech^2 \left( \frac{\psi}{\Delta} \right)$$

(38)

where, $\phi_0 = \frac{3M}{A}$ and $\Delta = 2\sqrt{\frac{M}{A}}$

4.3. Numerical solution of KdV and KdVB

Following the procedure proposed by Soliman, Ali, and Raslan [51], we find out the numerical solution of the KdV equation. Taking $F = A \frac{\phi^{(1)}}{2} + B \frac{\partial \phi^{(1)}}{\partial \xi}$, the equation (37) transforms into the given form

$$\frac{\partial \phi^{(1)}}{\partial \tau} + \frac{\partial F}{\partial \xi} = 0$$

(39)

Using the finite difference method the equation (39) becomes,

$$\frac{\phi^{(1)}_{i+1} - \phi^{(1)}_{i}}{\Delta \tau} + \frac{F_{i+1} - F_{i-1}}{\Delta \xi} = 0$$

(40)

Using the similarity solution procedure we can have the solution of equation (37) as follows (representing the $\phi^{(1)}$ as $\phi$ here),
\[
\phi_i^{n+1} = \phi_i^n - \frac{\Delta \tau}{\Delta \xi} \left[\frac{A}{4} (\phi_i^{n+1} - (\Phi_\xi i + 1 H + (\Phi_\xi i + 1 M_i)_2^2 + (\phi_i + (\Phi_\xi i + 1 H + (\Phi_\xi i + 1 M_i)_2^2 \\
- (\phi_i - (\Phi_\xi i) H + (\Phi_\xi i) M_i)_2^2 - (\phi_i^{n-1} - (\Phi_\xi i^{n-1} H + (\Phi_\xi i^{n-1} M_i)_2^2 \\
+ \frac{B}{2} (b^2 (\Phi_\xi i + 1 H + (\Phi_\xi i + 1 H + (\Phi_\xi i + 1 M_2 - b^2 (\Phi_\xi i) H + (\Phi_\xi i) M_2 - b^2 (\Phi_\xi i) H - (\Phi_\xi i) M_2 \\
+ b^2 (\Phi_\xi i^{n-1} H - (\Phi_\xi i^{n-1} M_2)]
\right) (41)
\]

Here,
\[
H = \frac{1}{4 \cos \alpha} \\
M_i = -\frac{\cos \alpha - 1}{4 \sin \alpha} \\
M_2 = \frac{b^2 \cos \alpha}{4 \sin^2 \alpha}
\]
(42)

where,
\[
\alpha = \frac{1}{2} h \Delta \xi \\
\beta = b M \Delta \tau
\]
(43)

Also
\[
b = \sqrt{\frac{M + AD}{B}}
\]
(44)

and also
\[
\Phi_\xi = \phi_i^{n+1} - 2 \phi_i^n + \phi_i^{n-1} \\
\Phi_\xi = \phi_i^{n+1} - \phi_i^{n-1}
\]
(45)

Here, \(D\) is the local similarity equation. The present scheme shows an amplification factor that is less than one for \(\frac{\Delta \tau}{\Delta \xi} \leq 0.5\), and then the scheme is stable under the condition \(\frac{\Delta \tau}{\Delta \xi} \leq 0.5\) [51].

We can proceed with the same numerical solution for KdVB also. But in the case of KdVB, we have to take a different \(F\) and which is given as \(F = A \frac{\partial^2 \phi}{\partial \xi^2} + B \frac{\partial \phi}{\partial \xi} + C \frac{\partial^2 \phi}{\partial \xi^2}\). Then equation (33) transforms into the same form as equation (39) with a different \(F\). This type of numerical solution for the KdVB equation is quite new. The solution of it following this procedure is given in Appendix A.2.

5. Phase plane analysis

In this section, we transform the KdV equation (37) into a dynamical system with the help of \(\chi = \xi - \nu \tau\) and \(\phi^{(1)}(\xi, \tau) = \phi^{(1)}(\chi)\)

\[
\begin{aligned}
\frac{d\phi^{(1)}}{d\chi} &= z \\
\frac{dz}{d\chi} &= \nu \phi^{(1)} - \frac{A}{B} (\phi^{(1)})^2
\end{aligned}
\]
(46)

To locate critical points by solving the equations \(\dot{\phi} = \dot{z} = 0\), \(-\) sign represents \(\frac{d}{d\chi}\).

Hence, \(\dot{\phi} = 0\) if \(z = 0\) and \(\dot{z} = 0\) if \(\left[\nu \phi - \frac{A}{B} \phi^2\right] = 0\).

If \(z = 0\) then \(\dot{z} = 0\) if \(\left[\nu \phi - \frac{A}{B} \phi^2\right] = 0\) which has two solutions, \(\phi = 0\) and \(\phi = \frac{\nu}{A}\). Therefore, there are two critical points \((0, 0)\) and \((\frac{\nu}{A}, 0)\).

Linearize by finding the Jacobian matrix; hence,
\[
J = \begin{pmatrix}
\nu & 0 & A \\
\frac{A}{B} & \frac{1}{B} & C
\end{pmatrix}
\]
(47)
Linearizing in first critical point that is \((0, 0)\) we get,

\[
J_{(0,0)} = \begin{pmatrix}
0 & 1 \\
\frac{\nu}{B} - \frac{C}{B} & \frac{C}{B}
\end{pmatrix}
\]  

(48)

The Eigenvalues for this Jacobian are 
\[
\lambda = \pm \sqrt{\frac{C}{B} + \left(\frac{C^2}{B^2} - \frac{\nu}{B}\right)}.
\]

Again, linearizing in the second critical point that is \((\frac{C}{\nu}, 0)\) we get,

\[
J_{(\frac{C}{\nu},0)} = \begin{pmatrix}
0 & 1 \\
\frac{\nu}{B} - \frac{C}{B} & \frac{C}{B}
\end{pmatrix}
\]  

(49)

And the eigenvalues for this Jacobian are 
\[
\lambda = \pm \sqrt{\frac{C}{B} + \left(\frac{C^2}{B^2} - \frac{\nu}{B}\right)}.
\]

From the eigenvalues at the two critical points, it is clear that for the critical point \((0, 0)\) the eigenvalues are always real as the dispersive coefficient \(B\) is always positive in this model but there is a possibility that the eigenvalues for the critical point \((\frac{C}{\nu}, 0)\) can be imaginary if \(\frac{\nu}{B} > \frac{C^2}{B^2}\).

The dynamical system 46 is Hamiltonian of the system,

\[
H(\phi, z) = \frac{\phi^2}{2} - \frac{v_0\phi^2}{2B} - \frac{A_0\phi^3}{3B}
\]

(50)

This is obviously a two-dimensional Hamiltonian function.

From the nature of the Jacobian and the eigenvalues, we categorize these critical points into two distinct categories.

(i) At the equilibrium point or critical point \(E_0\)

\[
D = \det J_{(0,0)} = \begin{vmatrix} 0 & 1 \\ \frac{\nu}{B} - \frac{C}{B} & \frac{C}{B} \end{vmatrix} = -\frac{\nu}{B} < 0
\]

which implies \(E_0\) is a saddle point.

(ii) At equilibrium / critical point \(E_1\) i.e. \((\frac{C}{\nu}, 0)\)

\[
D = \det J_{(\frac{C}{\nu},0)} = \begin{vmatrix} 0 & 1 \\ \frac{\nu}{B} - \frac{C}{B} & \frac{C}{B} \end{vmatrix} = \frac{\nu}{B} > 0
\]

(52)

which implies \(E_1\) is a center.

The trace of both the determinate is the same which is \(-\frac{C}{B} > 0\), as \(C\) itself is a negative quantity.

6. Results

In section 4, we have derived and analyzed various linear and nonlinear effects in this plasma model analytically and numerically. We want to interpret the plasma parameter values used here in light of previous observations.

The experimental support of the values we have used for the streaming velocities of ions \((u_{i0})\) & dust \((u_{d0})\) and also the finite temperature coefficients of ions \((\sigma)\) in the figures can be found in the papers by Merlino [52] and Rosenberg [53]. Also, the values of the Mach numbers used here are relatable to different laboratory and astrophysical plasma [54–56]. However, there are possibilities of getting high to very high values of Mach number [57–59]. The explanation for the values of viscous coefficients and kappa-values used to draw the curves has been given later.

We will discuss the variation of the one root of the real dispersion relation with various ion streaming velocities in figure 1. In this contour plot, we can see that the nonlinear damping rate increases as the unperturbed ion velocity \((u_{i0})\) increase. Next, in figure 2 we have discussed the variation of one imaginary root of the imaginary dispersion relation expressed in the equation (9). This is also a contour plot but the nature of it is totally different from the previous one (figure 1). This change in nature has two possible reasons. Firstly, in the calculation of imaginary dispersion relation 9, we do not consider the streaming velocity of ions and the dust grains \(u_{i0} = u_{d0} = 0\). And, secondly, we have considered the viscous coefficients for both species, we got the idea of the numerical value of the normalized viscous coefficient from the experimental paper by Tynan et al [60].
Basically, the kinematic viscosity is the origin of this type of dispersion relation and with the increase of the viscous coefficient, the value of $\omega''$ will increase with the increase of $k_i$. Also from figure 2, we can say that the finite temperature coefficient of plays a pivotal role in the imaginary dispersion relation. The value of $\omega''$ will increase with the increase of the finite temperature coefficient ions ($\sigma$) but will decrease with the increase of $k_i$.

Now, in this section, we will discuss the parametric dependence of the different wave structures. Firstly, in figure 3(a), we discuss the variation of the shock wave for different Mach numbers ($M$) with unperturbed ion streaming velocity ($u_{i0}$) = 0.3, unperturbed dust streaming velocity ($u_{d0}$) = 0.07, the kinematic viscosity of ion ($\eta_{i0}$) = 0.09, the kinematic viscosity of dust ($\eta_{d0}$) = 0.07, finite temperature coefficient ($\sigma$) = 0.65, kappa index ($\kappa$) = 5 and dust charge ($q$) = 1. When the Mach number, i.e. the value of the frame velocity increases the width and the amplitude of the shock fronts increase. It is clear that the nonlinear and dissipative coefficients $A$ and $C$
have negative values, and the values of these decrease with the growth of the Mach number. On the other hand, the dispersive coefficient $B$ is positive, and the value of this coefficient $B$ increases with the increment of the Mach number. This result can also be explained physically, when the wave frame velocity is high that means the dust-ion acoustic waves propagate faster in the medium, and then it has very little time to form the nonlinearity of the system. That is why we have high potential profiles for high Mach number. Also, we want to mention that the medium also has little time to apply the existing dissipation on it which also affects the amplitude and width of the shock profile in a significant manner.

In figure 3(b), we discuss the variation of the shock wave for different finite temperature coefficients ($\sigma$) with unperturbed ion streaming velocity ($u_{i0}$) = 0.3, unperturbed dust streaming velocity ($u_{d0}$) = 0.07, the kinematic viscosity of ion ($\eta_{i0}$) = 0.09, the kinematic viscosity of dust ($\eta_{d0}$) = 0.07, Mach number ($M$) = 1.2, kappa index ($\kappa$) = 5 and dust charge ($q$) = 1. This is basically a degeneracy parameter and discussion of it is a little bit difficult. Here, with the increase of this parameter $\sigma$ the non-linearity of the shock waves decreases but the dispersive and dissipation coefficients both increase. Physically this means that the potential drop across the shock wave enhances with an increase of finite temperature coefficient ($\sigma$), and this would affect the particle acceleration. But it is clearly seen that for the lower value of the degeneracy parameter $\sigma$ has a very significant effect in all these three parameters but with the increase of the value of $\sigma$ the dependence decreases very sharply. For an example, we can say that with the values of the other plasma parameters given in figure 3(b), the change of the parameter $\sigma$ from 0.55 to 0.65, the nonlinear, dispersive, and dissipation coefficients change from $-6.1131$ to $-2.6796$, 0.0539 to 0.1504, and $-0.0721$ to $-0.2818$ but when the parameter $\sigma$ from 0.65 to 0.75 the nonlinear, dispersive, and dissipation coefficients change from $-2.6796$ to $-2.6496$, 0.1504 to 0.1802, and $-0.2818$ to $-0.3464$. This happens as we consider the supersonic shock region (discussed in the discussion of figure 5(b)) and the
unperturbed ion velocity is higher than the unperturbed dust velocity so the effect of degeneracy parameter $\sigma$ becomes less significant when the value of it is sufficiently higher than the critical value i.e. $\sigma_0 = \frac{(M-u_{id})}{u_{id}}$.

Next, we will discuss the change of the shock profile for various unperturbed velocities of the charge-negative dust particles with unperturbed ion streaming velocity ($u_{id} = 0.3$), the kinematic viscosity of dust ($\eta_{id} = 0.07$), the kinematic viscosity of ion ($\eta_{io} = 0.09$), finite temperature coefficient ($\sigma = 0.65$), Mach number ($M = 1.2$), kappa index ($\kappa = 5$) and dust charge ($q = 1$) and it is given in figure 5(c). To our knowledge, this is the first time where the effect of unperturbed dust velocity in the formation of shock waves or solitons is taken into account. Dust particles are comparatively heavier than that of ions. In this whole discussion, we have considered the ion velocity at about ten times greater than the velocity of dust particles. As we said in the previous result that we have discussed most of the results in the supersonic region so the effect of the dust streaming velocity is very less but it is the opposite of the result previously found for electro-static [16] or electron-acoustic [48, 49] cases. In these papers, when the streaming velocities of the constituent particles increase the nonlinearity decrease, and the dispersive effect increases significantly. But in our case, though the nonlinearity decreases with the increase of streaming velocity of dust particles this change is not significant, this effect is contoured by the decrease of the dispersive coefficient and the increase of the dissipative coefficient in the formation of the shock profile. The physical significance of this result may be described as the higher streaming velocity accumulating the particles at the particular place faster and increasing the amplitude of the waveform.

Now we will discuss the change in the shock profile due to the change in the dust kinematic viscosity (DKV) in figure 4. It is very clear from equation (34) that there will be no change in $A$ or $B$ due to changes in DKV the only changes happen in coefficient $C$. The DKV coefficient is the main reason for the shock formation in a DIA wave. The physically acceptable values in the supersonic wave region ($M > 1$) must follow the given relation

$$\frac{\eta_{id}(M-u_{id})^2}{(M-u_{id})^2-3\sigma^2} < \frac{u_{id}q^2}{(M-u_{id})^2}.$$  

From this relation, we can conclude that DKV has a greater impact on the shock profile than the ion kinetic viscosity (IKV). In this figure 4, we have discussed the formations and structures of shock profiles in the supersonic wave region ($M > 1$). The amplitude is increasing and the width is decreasing with the increase of the DKV coefficient ($\eta_{id}$). Physically we can point out that the strength of the shock is directly proportional to DKV, consequently, the strength of the shock is enhanced with increasing values of DKV. A comparison between DKV and IKV is made for the first time in this article.

In figure 5(d) we will see the change in shock profile due to different supra-thermal coefficient ($\kappa$) with unperturbed ion streaming velocity ($u_{io} = 0.3$), unperturbed dust streaming velocity ($u_{id} = 0.07$), the kinematic viscosity of dust ($\eta_{id} = 0.09$), the kinematic viscosity of ion ($\eta_{io} = 0.09$), finite temperature coefficient ($\sigma = 0.65$), Mach number ($M = 1.2$), and dust charge ($q = 1$). For the existence of shock or solitary profile, the value of this supra-thermal coefficient $\kappa$ must be greater than $\frac{3}{7}$, that is why we have taken all the values of $\kappa$ greater than this value. Also, when we are taking the values of $\kappa$ from $2.5 - 7.5$, we keep in mind the Ulysses, Cluster, and Helios observation for slow SW$^-$ [62, 63]. In this case, the dispersive and dissipative coefficient of the shock are remained constant, the only changes can happen in the nonlinear coefficient and this thing is also
clear from equation (34). It is well known that the Kappa parameter is the measure of the discreteness of plasma concentration in space plasma, which is why the increase of the kappa parameter influences the greater shock.

Two fluids can produce shock by propagating alongside or colliding with each other. There are two kinds of shock that can form in the case of fluids the first and mostly found in a viscous plasma is the strong shock when two fluids are propagating or colliding with each other with a velocity greater than the local acoustic waves, i.e. \( M > 1 \) and the second kind of shock when two fluids collide a subsonic speed \( M < 1 \) which is known as weak shock, this type of shock is very well known in case of fluids [64, 65] and for laser-plasma interaction [16] but till now no report has been made in the case of dusty plasma. In figure 5(a), we have discussed the transition of weak to strong shock and in figure 5(b), we have shown two different kinds of shock for subsonic \( M = 0.7 \) and supersonic \( M = 1.2 \) frame velocity.

Now as we have discussed in section 4.2 when there is no viscous term the KdVB transforms into the KdV equation and here we will discuss the analytical solution of the KdV equation, i.e the solitary waves. In figure 6(a), we can see that with an increase in the Mach Number \( M \), the nonlinearity and dispersive coefficient decrease and increase, respectively. This has the same effect as we have seen in case of the figure 3(a). In this case and also in figure 3(a), the Mach number \( M \) also has a critical value i.e. \( M_c = 1.3 \). Greater than this value \( (M > M_c) \) the polarity of the shock waves as well as the solitary waves will change, which means all these rarefactive shock and solitary wave structures become compressive beyond this value of \( M \). As per our knowledge, this is for the first time we have reported this kind of transition in the polarity of the solitons with the help of RPT and the critical value of the Mach number \( (M_c) \). In figure 6(b), we have discussed the variation of solitary profile for different finite temperature degeneracy parameters \( (\sigma) \). Initially, the nonlinear coefficient decreases, and the dispersive & dissipation coefficient increases with the increase of the finite temperature degeneracy parameter. When the degeneracy parameter \( \sigma \) increases greater than 0.661 the nonlinear coefficient starts to increase along with the other two parameters and that is why there is a different tendency of the solitary waves this is the same for cases of the shock waves too as shown in figure 3(b). Now, in figure 6(c), we see that the streaming velocity of dust cannot alter the width but increases the amplitude of the solitary wave. This has happened due to the simultaneous decrease of the nonlinear and dispersive coefficient of the solitary wave with the increase of dust streaming velocity.

In figure 6(d), we have discussed the formation and variation of solitary profiles for various supra-thermal parameters \( (\kappa). \) From equation (34), it is very clear that the only coefficient that is affected by the supra-thermal parameter \( (\kappa) \) is the nonlinear coefficient \( A \) and it has a decreasing tendency with the increasing value of \( \kappa \) that is why the width and amplitude increasing with the increased value of \( \kappa \). Next, we have shown in figure 7 the variation of the solitary profile due to different Mach numbers that are the transition of the solitary profile starting from the subsonic velocity region and ending in the supersonic velocity region for the wave.

In figure 9(a), we have plotted the exact solution of KdV from equation (38) and the numerical solution of the same with the help of a set of equations from 39–45. For the numerical solution, we have considered the space-like steps as \( \Delta \xi = 0.04 \) and time-like steps as \( \Delta \tau = 0.05 \); the maximum error in \( \phi^{(1)} \) from the exact to the numerical solution is 0.009152. Also, in figure 8(b), we have plotted the analytical solution of KdVB from equation (36) and the numerical solution of the same. We have compared the numerical result with the
Figure 6. Solitary for different (a) Mach numbers ($M$) (b) finite temperature coefficients ($\sigma$) (c) unperturbed dust streaming velocities ($u_{d0}$) (d) kappa indices ($\kappa$).

Figure 7. Solitary 3D for different Mach Numbers ($M$) with unperturbed ion streaming velocity ($u_{i0}$) = 0.3, unperturbed dust streaming velocity ($u_{d0}$) = 0.07, finite temperature coefficient ($\sigma$) = 0.65, kappa index ($\kappa$) = 5 and dust charge ($q$) = 1.
analytical one in the weak shock region ($M = 0.35$) with a high value of dust charges ($q$) and DKV coefficient ($\tau_{DKV}$). There is more discrepancy in these two results than the figure 9(a), which may have two reasons - (a) the solution that appeared in equation (36) arises from a series solution, and (b) a dissipative term present in the KdV equation. Also, we want to mention here that a high value of dust charge density helps to decrease the discrepancy between these two treatments.

At last we will discuss the variation of non-linear coefficient with ion density ($\delta$) for different supra-thermal coefficient ($\kappa$) (9(a)) and dissipative coefficient with ion density ($\delta$) for different viscous coefficients of dust ($\eta_{d0}$) (9(b)). It is very much evident from equation (34) that two kinds of shock/soliton may be possible to form. One is compressive shock/soliton and other rarefactive shock/soliton structures in this scenario and for that the non-linear coefficient $A > 0$ and $A < 0$, respectively. But from figure 9(a), we can see the nonlinear coefficient is throughout negative which is why we are getting rarefactive shocks and solitons. Again as $\phi_{0} = \frac{3M}{2}$ so due to increasing the value of $\delta$ will cause a decrease in the amplitude. So, this variation of $A$ with different $\kappa$ is also supported by figures 3(d) and 6(d). In figures 10(a) and 10(b), we have discussed the first-order perturbed density change of dust and ions in the presence and absence of the viscosity with different Mach numbers ($M$). When viscous force is absent in the medium (figure 10(b)), we can see that the density of dust has very little change with various $M$. This is due to the heavy mass of the dust than ion, and also the density of dust and ion has different polarities. But from figure 10(a), we can see that the effect of viscosity has a larger impact than the effect.
of mass as in presence of viscosity there is a sufficient amount of changes in the dust density with the change in the Mach number \( M \) as the viscosity considered in this paper is kinetic in nature.

The Hamiltonian \( H(\phi, z) \) is given in equation (50) defines the trajectory of the phase plot given in figure 11. In this figure 11, we can easily define two equilibrium points \( E_0(0, 0) \) and \( E_1\left(\frac{\kappa}{\sigma}, 0\right) \) of the system equation (46) as saddle point and center. The solitary wave solution 38 of equation (37) represents the homoclinic orbit of system 46. And periodic traveling wave solution of equation (37) represents the periodic orbit of the system 46. In figure 11, the phase portrait of equation (46) is plotted. With the presence of the plasma parameters like \( M = 1, u_{0i} = 0.3, u_{0d} = 0.07, \eta_{i0} = 0.9, \eta_{d0} = 0.07, \sigma = 0.7, \) and \( \kappa = 5 \), we have shown the saddle point \( E_0(0, 0) \) with no separatrix enclosed by the nonlinear homoclinic trajectory \( (NHT_{1,0}) \) which obviously corresponds to the solitary wave solution shown in equation (37). Again around the critical point \( E_1\left(\frac{\kappa}{\sigma}, 0\right) \) corresponding to the periodic solution, there exists the family of nonlinear periodic trajectories \( (NPT_{1,0}) \) [66, 67]. The corresponding analytical solution of the nonlinear homoclinic trajectory \( (NHT_{1,0}) \) has already been shown in figures 6(a)–7.

7. Conclusion

In light of the calculation and study this paper is divided into three parts, firstly with the help of the reductive perturbation technique (RPT) we analytically derived and solved the KdV-Burger and KdV equation and get the
shock and solitary waves as their solution, secondly using finite difference method we have done the numerical solution of KdV and lastly with the help of total energy of the system when there is no dissipative force present in the system we have done the phase plane analysis.

In the model section, we have discussed three things very broadly

(a) The presence of Fermi–Dirac statistics in the case of ion distribution. Various astrophysical and laboratory plasmas are in this category if they follow the condition we have mentioned in section 2.2. $H^+$ plasma is one of these kinds of plasma.

(b) The detailed discussion of kinematic viscosity in both dust and ion species.

(c) The reason behind taking the dust charge as constant in the DIA case has also been discussed in section 2.3.

In the production of shock waves, the dust kinematic viscous coefficient ($\eta_{d0}$) is the most important parameter, so we are cautious in taking the value of this. From the work of Vorona et al [68] we can have some ideas of what the value should be for this parameter and tried to follow this in the manuscript. For figures 3(a)- 3(c), 3(d)- 5(b), and 10(a) we have taken the kinematic viscosity of dust $\eta_{d0} = 0.07$ and for figures 4 and 9(b) it is shown in the legends.

Keeping the above-mentioned values of viscous coefficient and other physically admissible values for different plasma parameters for the numerical study we have found some of the important results that we have mentioned below.

(a) For the first time, we have reported two types of shock which are weak and strong shock in a DIA mode.

(b) Physically accepted values of the plasma parameter for the supersonic wave region ($M > 1$) are discussed.

(c) The critical value of finite temperature degeneracy parameter $\sigma_c$ has also been discussed and again what values would be physically acceptable for the supersonic wave region have been discussed.

(d) The critical value of the Mach number is also discussed in this result beyond which the polarity of shock and solitary waves would changes i.e. rarefactive shock/soliton transform in compressive shock/soliton.

The shock waves in the dusty plasma with Boltzmann-distributed electrons and ions have been experimentally observed by Arora et al [69]. Our present theoretical model will transform into this type of plasma when $\kappa \rightarrow \infty$. In these types of systems, plasma is in thermal equilibrium with the electrostatic Coulomb field. But most of the plasma we encountered in laboratory and astrophysical environments do not follow this condition [70]. So in plasma, realistic velocity distribution functions of electrons are often non-Maxwellian in nature. Particularly in space plasma the particle velocity distributions are well-fitted by the Kappa distribution [71]. So, considering that the electrons follow the Kappa distribution will give us a more realistic result.

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Data availability statement

The data that support the findings of this study are available within the article.

Appendix

Appendix A.1

For the calculation of real imaginary dispersion relations given in equations (7) and (9) we have to consider that the DKV and IKV coefficients have real and imaginary coefficients, i.e. $\eta_d = \eta_{dr} + i\eta_{di}$ and $\eta_i = \eta_{ir} + i\eta_{ii}$. 

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As we are taking the streaming of both the species then from the continuity equation of dust we get,

\[ n_{d}^{(1)} = \frac{k}{(ku_{d0} - \omega)} u_{d}^{(1)} \]  
(A.1)

From the momentum equation of dust we get,

\[ u_{d}^{(1)} = \frac{qk}{(-\omega + ku_{d0} + \eta_{d}k^{2})} \phi^{(1)} \]  
(A.2)

From A.1 and A.2 we can write,

\[ n_{d}^{(1)} = \frac{qk^{2}}{(ku_{d0} - \omega)(-\omega + ku_{d0} + \eta_{d}k^{2})} \phi^{(1)} \]  
(A.3)

Now, from the continuity equation of ions we get,

\[ n_{i}^{(1)} = \frac{k}{(ku_{i0} - \omega)} u_{i}^{(1)} \]  
(A.4)

Again, from the momentum equation of ions we can write,

\[ (-\omega + ku_{i0} - \eta_{i}k^{2})u_{i}^{(1)} + 3\sigma k \phi_{i}^{(1)} = -k \phi^{(1)} \]  
(A.5)

Now with these two equations (A.4) and (A.5) we get,

\[ n_{i}^{(1)} = -\frac{k^{2}}{((-\omega + ku_{i0} - \eta_{i}k^{2})(ku_{i0} - \omega) + 3\sigma k^{2})} \phi^{(1)} \]  
(A.6)

From the Poisson’s equation in 3 with the help of the equations (6), (A.3) and (A.6) we can have the following equation,

\[ -k^{2} = \frac{(1 - 2\kappa)}{(2\kappa - 3)} + (\delta k^{2} + q^{2}k^{2}) \left[ \frac{G + I}{GI} \right] \]  
(A.7)

Here, \( G = (ku_{d0} - \omega)(ku_{d0} - \omega + \eta_{d}k^{2}) \) and \( I = [(\omega + ku_{i0} - \eta_{i}k^{2})(ku_{i0} - \omega) + 3\sigma k^{2}] \).

Obviously, this equation (A.7) is a complicated equation. But we get two comparatively simple solutions in form of equation (7) and equation (9) by considering no viscosity and no streaming velocities respectively.

**Appendix A.2**

To solve the KdVB numerically we have to take \( F \) as given by

\[ F = A \frac{\phi^{(1)}_{i}^{2}}{2} + B \frac{\partial^{2} \phi^{(1)}}{\partial \xi^{2}} + C \frac{\partial \phi^{(1)}}{\partial \xi} \]  
(A.8)

To solve the KdVB equation we use the similarity solution procedure we can have the solution of equation (33) as follows

\[ \phi_{i}^{n+1} = \phi_{i}^{n} - \Delta \xi \left\{ \frac{A}{4} \left[ (\phi_{i+1} - (\Phi_{i})_{i+1}H + (\Phi_{i})_{i+1}M_{i})^{2} + (\phi_{i} + (\Phi_{i})_{i}H + (\Phi_{i})_{i}M_{i})^{2} \right] - (\phi_{i} - (\Phi_{i})_{i}H + (\Phi_{i})_{i}M_{i})^{2} - (\phi_{i-1} - (\Phi_{i})_{i-1}H + (\Phi_{i})_{i-1}M_{i})^{2} \right\} \]

\[ + \frac{B}{2} \left\{ b^{2}(\Phi_{i})_{i+1}H + (\Phi_{i})_{i+1}H + (\Phi_{i})_{i+1}M_{i} - b^{2}(\Phi_{i})_{i}H + (\Phi_{i})_{i}H + (\Phi_{i})_{i}M_{i} - b^{2}(\Phi_{i})_{i-1}H + (\Phi_{i})_{i-1}H + (\Phi_{i})_{i-1}M_{i} \right\} \]

\[ + b^{2}(\Phi_{i})_{i-1}H - (\Phi_{i})_{i-1}H - (\Phi_{i})_{i-1}M_{i}) \right\} + \frac{C}{2} \left\{ b^{2}(\Phi_{i})_{i+1}H + (\Phi_{i})_{i+1}H + (\Phi_{i})_{i+1}M_{i} - b^{2}(\Phi_{i})_{i}H + (\Phi_{i})_{i}H + (\Phi_{i})_{i}M_{i} - b^{2}(\Phi_{i})_{i-1}H + (\Phi_{i})_{i-1}H + (\Phi_{i})_{i-1}M_{i} \right\} \]

(A.9)

All the constants will follow the same form as given by equations (42) and (43) except the constant \( b \), \( b \) will be in the form given by,

\[ b = -\frac{C \pm \sqrt{C^{2} - 4(M + AD)B}}{2B} \]  
(A.10)

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References

[1] Dragan G S and Kutavar V V 2018 Correlation function of the coupling parameter in dusty plasmas AIP Conf. Proc. 1923 1–3020027 https://aip.scitation.org/doi/abs/10.1063/1.5020415

[2] Patric D T, Mohamadou A and Kofane T C 2017 Nonlinear dust ion acoustic waves behaviors analysis in warm viscous dusty plasma with trapped ions Phys. Plasmas 24 12370612 https://aip.scitation.org/doi/10.1063/1.5017505

[3] Shukla P K and Mamun A 2015 Introduction to Dusty Plasma Physics Plasma Phys. Control. Fusion (London: Institute of Physics Publishing) https://iopscience.iop.org/article/10.1088/0741-3335/44/3/7010-7053-0653-X(https://iopscience.iop.org/article/10.1087/0741-3335/44/3/7010)

[4] Rao N, Shukla P and Yu M Y 1990 Dust-acoustic waves in dusty plasmas Planet. Space Sci. 38 543–5464 https://www.sciencedirect.com/science/article/pii/0032063990901471

[5] Shukla P and Silin V 1992 Dust-ion-acoustic wave Phys. Scr. 45 508 https://iopscience.iop.org/article/10.1088/0031-8949/45/5/015

[6] d’Angelo N 1994 Ion-acoustic waves in dusty plasmas Planet. Space Sci. 42 507–11 https://www.sciencedirect.com/science/article/pii/0032063990902225

[7] Bansal S, Agarwal M and Gill T S 2020 Nonplanar ion acoustic waves in dusty plasma with two temperature electrons: application to saturn’s r ring Phys. Plasmas 27 083704 https://aip.scitation.org/doi/10.1063/5.0013015

[8] Ma J X and Yu M 1994 Self-consistent theory of dusty plasma in a dusty plasma Phys. Plasmas 1 3520–2 https://aip.scitation.org/doi/10.1063/1.870887

[9] Vranjes J, Pandey B and Poedts S 2002 Ion-acoustic waves in dusty plasma with charge fluctuations Phys. Plasmas 9 1464–7 https://aip.scitation.org/doi/10.1063/1.1455631

[10] Mamun A A and Tasnim S 2010 Dust-ion-acoustic shock and solitary waves in a dusty electronegative plasma Phys. Plasmas 17 073704 https://iopscience.iop.org/article/10.1063/1.3464224

[11] Hassan M, Biswas S, Habib K and Sultana S 2022 Dust-ion-acoustic waves in a n– nonthermal magnetized collisional dusty plasma with opposite polarity dust Results in Physics 33 105106 https://www.sciencedirect.com/science/article/pii/S2211379721010792

[12] Roy K, Saha T and Chatterjee P 2014 Effect of ion kinematic viscosity on large amplitude dust ion acoustic solitary waves Astrophys. Space Sci. 349 745–51 https://link.springer.com/article/10.1007/s10509-013-1625-9

[13] Adhikary N C 2012 Effect of viscosity on dust-ion acoustic shock wave in dusty plasma with negative ions Phys. Lett. 376 1460–4 https://www.sciencedirect.com/science/article/pii/S0375960112002897

[14] Goswami J, Chandra S and Ghosh B 2018 Study of small amplitude ion-acoustic solitary wave structures and amplitude modulation in e– p–plasma with streaming ions Laser Part. Beams 36 136–43 https://www.cambridge.org/core/journals/laser-and-particle-beams/article/study-of-small-amplitude-ion-acoustic-solitary-wave-structures-and-amplitude-modulation-in-ep-plasma-with-streaming-ions/32DCRDB0BE869F617605F042262031

[15] Devanandan S, Singh S and Lakhina G 2011 Electron acoustic solitary waves with kappa-distributed electrons Phys. Scr. 84 025507 https://iopscience.iop.org/article/10.1088/0031-8949/84/02/025507

[16] Goswami J, Chandra S, Sarkar J, Chaudhuri S and Ghosh B 2020 Collision–less shocks and solitons in dense laser–produced fermi plasma Laser Part. Beams 38 25–38 https://www.cambridge.org/core/journals/laser-and-particle-beams/article/collisionless-shocks-and-solitons-in-dense-laser-produced-fermi-plasma/7FEB0D55193CE681829191B844D10333E

[17] Ghosh A, Goswami J, Chandra S, Das C, Arya Y and Chhabber H 2021 Resonant interactions and chaotic excitation in nonlinear surface waves in dense plasma IEEE Trans. Plasma Sci. 50 611524–13 https://ieeexplore.ieee.org/document/9357918

[18] Barkan A, d’Angelo N and Merlino R 1996 Experiments on ion-acoustic waves in dusty plasmas Planet. Space Sci. 44 239–42 https://www.sciencedirect.com/science/article/pii/S0032063995001093

[19] Nakamura Y, Bailing H and Shukla P 1999 Observation of ion-acoustic shocks in a dusty plasma Phys. Rev. Lett. 83 1602 https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.83.1602

[20] Paul R, Sharma G, Deka K, Adhikari S, Moulick R, Bausik S S and Saikia B K 2022 Experimental study of charging of dust grains in the presence of energetic electrons Plasma Phys. Controlled Fusion 64 035009 https://iopscience.iop.org/article/10.1088/1361-6587/ac4e5

[21] Nakamura Y 2002 Experiments on ion-acoustic shock waves in a dusty plasma Phys. Plasmas 9 440–5 https://aip.scitation.org/doi/10.1063/1.1431974

[22] Kaur B and Saini N 2018 Dust ion-acoustic shock waves in a multicomponent magnetorotating plasma Zeitschrift für Naturforschung A 73 215–23 https://www.degruyter.com/document/doi/10.1515/zna-2017-0397/html

[23] Haider M M and Nahar A 2017 Dust-Ion–Acoustic Solitary and Shock Structures in Multi–Ion Plasmas with Super–Thermal Electrons Zeitschrift für Naturforschung Teil A 72 627–35 https://www.degruyter.com/document/doi/10.1515/zna-2017-0108/html

[24] Denna R et al. 2018 Study of the characteristics of dust acoustic solitary waves and dust acoustic shock waves in electron free dusty space plasma Journal of Modern Physics 9 948 https://www.scirp.org/journal/paperinformation.aspx?paperid=83707

[25] Deka H and Sarma J I 2022 A numerical study of modified burgers’ equation in charged dusty plasmas Global Journal of Pure and Applied Mathematics 18 155–69

[26] Pierrard V and Lazar M 2010 Kappa distributions: Theory and applications in space plasmas Sol. Phys. 267 153–74 https://link.springer.com/article/10.1007/s11107-010-9640-2

[27] Goswami J, Sarkar J, Chandra S and Ghosh B 2021 Amplitude–modulated electron–acoustic waves with bipolar ions and kappa–distributed positrons and warm electrons Pramana 95 1–10 https://www.springer.com/article/10.1007/s12043-021-02085-1

[28] Atteya A, Sultana S and Schlickeiser R 2018 Dust–ion–acoustic solitary waves in magnetized plasmas with positive and negative ions: the role of electrons superthermality Chin. J. Phys. 56 1931–9 https://www.sciencedirect.com/science/article/pii/S0577903180373178

[29] Baluku T and Hellberg M 2008 Dust acoustic solitons in plasmas with kappa–distributed electrons and/or ions Phys. Plasmas 15 123705 https://aip.scitation.org/doi/10.1063/1.3042215

[30] Liu Z and Du J 2009 Dust acoustic instability driven by drifting ions and electrons in the dust plasma with lorentzian kappa distribution Phys. Plasmas 16 123707 https://aip.scitation.org/doi/10.1063/1.3274459

[31] Nicolaou G, Livadiotis G, Owen C J, Verscharen D and Wicks R T 2018 Determining the kappa distributions of space plasmas from observations in a limited energy range Astrophys. J. 865 https://iopscience.iop.org/article/10.3847/1538-4357/aad45d

[32] El-Labany S K, Moslem W M and Safy F M 2006 Effects of temperature–ions, magnetic field, and higher–order nonlinearity on the existence and stability of dust–acoustic solitary waves in saturn’s r ring Phys. Plasmas 13 082903 https://aip.scitation.org/doi/10.1063/1.2336183
[33] Shan S A, Saleem H and Sajid M 2008 Streaming instabilities in multicomponent interstellar clouds Phys. Plasmas 15 072904 https://aip.scitation.org/doi/10.1063/1.2936268

[34] Saleem H and Ali S 2017 Solar wind interaction with dusty plasmas produces instabilities and solitary structures Astrophys. Space Sci. 362 238 https://link.springer.com/article/10.1007/s10509-017-3217-6

[35] Mann I 2008 Interplanetary medium - a dusty plasma Adv. Space Res. 41 160–7 https://www.sciencedirect.com/science/article/abs/pii/S0273117707004217?via%3Dihub

[36] Schwadron N and Gloeckler G 2007 Pickup ions and cosmic rays from dust in the heliosphere Space Sci. Rev. 130 283–91 https://link.springer.com/article/10.1007/s11214-007-9166-6

[37] Holzer T E, Leer E and Zhao XP 1986 Viscosity in the solar wind Journal of Geophysical Research: Space Physics 91 4126–32 https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/JA091iA04p04126

[38] Kumar Samanta U, Saha A and Chatterjee P 2013 Bifurcations of nonlinear ion acoustic travelling waves in the frame of a zakharov-kuznetsov equation in magnetized plasma with a kappa distributed electron Phys. Plasmas 20 052111 https://aip.scitation.org/doi/10.1063/1.4804347

[39] Goswami J and Sarkar J 2021 Kbm approach to electron acoustic envelope soliton in viscous astrophysical plasma Phys. Scr. 96 085601 https://iopscience.iop.org/article/10.1088/1402-4896/ab8551/meta

[40] Amin S M, Pasha A, Choudhury K and Roy Chowdhury A 2007 Saddle-node bifurcation and modulational instability associated with the pulse propagation of dust ion-acoustic waves in a viscous dusty plasma: a complex nonlinear schrodinger equation Phys. Plasmas 14 012110 https://aip.scitation.org/doi/10.1063/1.2409493

[41] Sen B, Das B and Chatterjee P 2008 Effect of electron inertia on large amplitude solitary waves in presence of kinematic viscosity in dusty plasma Eur. Phys. J. D 49 211 https://link.springer.com/article/10.1140/epjd/e2008-00158-3

[42] Akbari-Moghanjoughi M and Ellasson B 2016 Hydrodynamic theory of partially degenerate electron-hole fluids in semiconductors Phys. Scr. 91 105601 https://iopscience.iop.org/article/10.1088/1402-4896/91/10/105601

[43] Lifshitz E M and Pitaevskii L P 2013 Statistical Physics: Theory of the Conquered State (Oxford: Elsevier Science) (Course of Theoretical Physics) https://books.google.co.in/books?id=lggIDAABAQAAJ

[44] Misra A P, Roy Chowdhury K and Roy Chowdhury A 2007 Saddle-node bifurcation and modulational instability associated with the kuznetsov equation in magnetized plasma with a kappa distributed electron Pop. Phys. 14 299–307 https://aip.scitation.org/doi/10.1063/1.2807375

[45] El-Hanbaly A, Sallah M, El-Sheyw E and Darweesh H 2015 Kinematic dust viscosity effect on linear and nonlinear dust-acoustic waves in space plasma with nonthermal ions J. Exp. Theor. Phys. 121 669–79 https://link.springer.com/article/10.1134/S1063776115010074

[46] Shahnamsouri M 2013 Dissipative dust acoustic solitary waves in an electron depleted dusty plasma with superthermal ions IRJANIAN JOURNAL OF SCIENCE AND TECHNOLOGY TRANSACTION A-SCIENCE 37 A 285–291

[47] Chaudhari S and Chowdhury A R 2021 On the effect of electron streaming and existence of quasi-solitary mode in a strongly coupled quantum dusty plasma and near critical nonlinearity Plasma Phys. 4 408–23 https://www.mpiphg.mpg.de/2571-6182/4/3/30

[48] Goswami J, Chandra S and Ghosh B 2019 Shock waves and the formation of solitary structures in electron acoustic wave in inner magnetosphere plasma with relativistically degenerate particles Astrophys. Space Sci. 364 1–7 https://link.springer.com/article/10.1007/s10509-019-3555-7

[49] Goswami J, Chandra S, Sarkar J and Ghosh B 2020 Electronic dusty-acoustic solitary structures and shocks in dense inner magnetosphere finite temperature plasma Radiat. Eff. Defects Solids 175 196–73 https://www.tandfonline.com/doi/abs/10.1080/10421505.2020.1799373?via%3Dihub

[50] Warnau A M 2008 The tanh method for travelling wave solutions to the zhiber-shabat equation and other related equations Commun. Nonlinear Sci. Numer. Simul. 13 384–92 https://www.sciencedirect.com/science/article/abs/pii/S1007570406001420?via%3Dihub

[51] Soliman A, Ali A and Raslan K 2009 Numerical solution for the kdv equation based on similarity reductions Appl. Math. Modell. 33 1107–15 https://www.sciencedirect.com/science/article/pii/S0307904X0800005X?via%3Dihub

[52] Merlino R L 2009 Dust-acoustic waves driven by an ion–dust streaming instability in laboratory discharge dusty plasma experiments Phys. Plasmas 16 124501 https://aip.scitation.org/doi/10.1063/1.3271155

[53] Rosenberg M 1996 Ion-dust streaming instability in processing plasmas Journal of Vacuum Science & Technology A 14 631–3 https://avs.scitation.org/doi/10.1116/1.580157

[54] Nickeler D H and Wiegmann T 2010 Thin current sheets caused by plasma Phys. Plasmas 17 151253–32 https://aip.scitation.org/doi/10.1063/1.371130

[55] Federath C 2016 Magnetic field amplification in turbulent astrophysical plasma J. Plasma Phys. 82 (6) 35582061 https://www.cambridge.org/core/journals/journal-of-plasma-physics/article/magnetic-field-amplification-in-turbulent-astrophysical-plasmas/62F9E801264A975BA355A138493DC36B

[56] Scudder J D 1992 On the causes of temperature change in inhomogeneous low-density astrophysical plasmas Astrophys. J. 398 299–318 https://ui.adsabs.harvard.edu/abs/1992ApJ...398..299S

[57] Takabe H et al 2008 High-mach number collisionless shock and photo-ionized non-lte plasma for laboratory astrophysics with intense lasers Plasma Phys. Controlled Fusion 50 124057 https://aip.scitation.org/doi/10.1088/0741-3335/50/12/124057

[58] Papadopoulos K 1998 Electrons Heating in Superhigh Mach Number Shocks (Netherlands, Dordrecht: Springer) pp 535–47 https://link.springer.com/article/10.1007/BF00793303

[59] Shigemori K et al 2000 Experiments on radiative collapse in laser-produced plasmas relevant to astrophysical jets Phys. Rev. E 62 8838–41 https://journals.aps.org/pre/abstract/10.1103/PhysRevE.62.8838

[60] Tyan G, Holland C, Yu J, James A, Nishijima D, Shimada M and Taheri N 2006 Observation of turbulent-driven shear flow in a cylindrical laboratory plasma device Plasma Phys. Controlled Fusion 48 S51 https://iopscience.iop.org/article/10.1088/0741-3335/48/4/S05

[61] Sharma S K, Boruah A, Nakamura Y and Bailung H 2016 Observation of dust acoustic shock wave in a strongly coupled dusty plasma Phys. Plasmas 23 053702 https://aip.scitation.org/doi/10.1063/1.4950832

[62] Livadiotis G 2015 Statistical background and properties of kappa distributions in space plasmas J. Geophys. Res. 120 1607–19 https://agupubs.onlinelibrary.wiley.com/doi/full/10.1002/2014JA020823

[63] Sverak S, Maksimovic M, Travnicek P M, Marsch E, Fazakerley A N and Scime E E 2009 Radial evolution of nonthermal electron populations in the low-latitude solar wind: Helios, Cluster, and Ulysses Observations Journal of Geophysical Research (Space Physics) 114 A05104 https://agupubs.onlinelibrary.wiley.com/doi/full/10.1002/2009JA013883

[64] Zel’dovich Y B and Raizer Y P 1967 Physics of shock waves and high-temperature hydrodynamic phenomena (Academic Press: Elsevier) https://asmedigitalcollection.asme.org/appliedmechanics/article/34/4/10/1055/425174/Physics-of-Shock-Waves-and-High-Temperature?via%3Dihub
[65] Landau L and Lifshitz E 1959 Fluid Mechanics ed j sykes and w reid (Oxford, England: Pergamon Press) 0-08-033933-6
[66] Dubinin A E and Kolotkov D Y 2012 Ion-acoustic super solitary waves in dusty multispecies plasmas IEEE Trans. Plasma Sci. 40 1429–33 https://ieeexplore.ieee.org/document/6179549
[67] Shome A and Banerjee G 2021 Bifurcation analysis of super nonlinear waves in an electron–positron–ion–dusty plasma having nonthermal distribution of electron and positron Ricerche di Matematica 1 1–15
[68] Vorona N, Gavrikov A, Ivanov A, Petrov O, Fortov V and Shakhova I 2007 Viscosity of a dusty plasma liquid J. Exp. Theor. Phys. 105 824–30 https://link.springer.com/article/10.1134/S1063776107100172
[69] Arora G, Bandyopadhyay P, Hariprasad M G and Sen A 2020 Excitation of dust acoustic shock waves in an inhomogeneous dusty plasma Phys. Plasmas 27 083703 https://aip.scitation.org/doi/10.1063/5.0009397
[70] Hasegawa A, Mima K and Duong-van M 1985 Plasma distribution function in a superthermal radiation field Phys. Rev. Lett. 54 2608–10 https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.54.2608
[71] Pierrard V and Lazar M 2010 Kappa distributions: Theory and applications in space plasmas Sol. Phys. 267 153–74 https://link.springer.com/article/10.1007/s11207-010-9640-2