On stability of toroidal structures in two-tori pulsar wind nebulae

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Abstract. The effect of weakly supersonic flow on two-tori pulsar wind nebula is considered. It is shown that the flow going past the nebula does not destroy its two-tori structure, but, on the contrary, contributes to its stability. Under the effect of the flow, the windward and the leeward polar outflows in the nebula operate at drastically different conditions. In the result, two opposite jets of the nebula which form within these polar outflows may differ in their dynamics and appearance. Our work bears implications for the Vela pulsar wind nebula, which interacts with a supersonic flow of Mach number $\sim 1.3$ produced by the reverse shock of its parent supernova.

1. Introduction
Rotation powered magnetized neutron star (a pulsar) converts most of its rotational energy into a cold relativistic wind. The wind carries along a strong toroidal magnetic field frozen into electron-positron plasma. Because the wind is supersonic, it drives a shock into the ambient medium. Downstream the shock the wind inflates a plasma bubble – a pulsar wind nebula, or PWN, – which emits a bright synchrotron emission. Since the wind power is anisotropic (it scales as $\sin^2 \theta$ \cite{5}, where the colatitude $\theta$ is measured from the rotational axis of the pulsar), the wind terminates at an oblate shock, and inflates an aspherical PWN. The shape, or morphology, of a PWN is a point of our study.

A PWN is shaped both by an ambient medium and by its parent pulsar’s wind. If the nebula expands into a static homogeneous medium, the shape is primarily set by the wind properties. Namely, by an average magnetization $\sigma$ of its plasma, and by an angular extent $2\alpha$ of its low-magnetized region around the rotational equator \cite{6}. This region (sector) emerges due to the inclination $\alpha$ of the rotation axis of the pulsar to its magnetic axis; it embraces the magnetic field stripes of alternating polarity, which, presumably, annihilate. Depending on $\alpha$ and $\sigma$, the PWN may be shaped as a toroid or exhibits a two-tori structure with a one torus set on top of another. In both cases strongly or weakly collimated outflows – jets – are often seen to emerge in opposite directions, along the PWN’s symmetry axis aligned with the rotation axis of the feeding pulsar and perpendicular to the equatorial plane of symmetry of the torus (or the tori).

If the pulsar moves through an ambient medium, or the medium streams past the pulsar, the PWN shape may become affected by this relative motion. Whether the effect is observable depends primarily on the relative velocity. Subsonic relative motion bears no imprints on the
nebula morphology. At fast supersonic motion, a PWN acquires a cometary shape and drives a bow shock; in the result, its morphology loses direct connection with the wind properties. These two extreme cases are beyond the scope of our interest. In this paper we consider the case of a weakly supersonic relative motion. By means of relativistic MHD simulations we study the effects of this motion on the morphology of the two-tori nebulae. Observationally, the objects of this type are exemplified, e.g., by PWNe Vela [20, 15] and Dragonfly [27].

The Vela nebula is powered by the 11 kyr pulsar B0833-45 (the Vela pulsar) located at a distance of \(287^{+10}_{-17}\) pc [11]. The Vela pulsar is moving within its parent supernova remnant (SNR) with the subsonic speed of \(61^{\pm}2\) km/s [11] (the sound speed at the pulsar’s location is \(\sim 560\) km/s [8]). It is believed that the reverse shock from the supernova explosion (which had gave a birth to the pulsar) has recently passed over the Vela nebula from the north direction. The reverse shock has induced behind it a weakly supersonic flow of the Mach number \(\sim 1.3\) [8]. As the result, the weak bow shock ahead of the nebula is formed (see, e.g., in [7]). The multiwavelength observations favour this scenario. The radio observations suggest a presence of the contact discontinuity (that separates the pulsar wind termination shock and the nebula’s bow shock) [12]. The observations in radio, X-rays and TeV \(\gamma\)-rays reveal an extended structure referred as Vela-X, that is located southwest of the Vela pulsar. The structure is thought to be the relic nebula of the Vela pulsar displaced by the reverse shock [15, 13, 9, 2]. The ”new” nebula of the Vela pulsar is estimated to be about \(\sim 17\) yr old [8] in 2003 as follows from the radio observations [12]. The X-ray image of Vela’s (synchrotron) emission (shown in figure 1) exhibits two prominent arcs separated by a region of distinctly suppressed brightness [20, 15]. The arcs and the dark region in-between persist for 10 years of observations [20, 15, 21, 16, 23].

2. Results

The problem setup is as follows. A homogeneous, weakly supersonic flow of Mach number \(\sim 1.3\) is initiated at once in the entire computation domain (in PWN maps in figure 1 it appears to be going downward). Then, a two-tori PWN is inflated within this domain. We coalign the axis of the model nebula with the flow direction, so that the upper jet in the figure is directed against the flow, while the lower jet is co-directed. Other details of the calculations are given in the Appendix.

For the Vela pulsar, the tilt of magnetic to rotation axis (the pulsar inclination \(\alpha\)) is unknown. The value of \(\alpha\) differs vastly in different models aimed at explaining the pulsar emission. The polarization measurements of the pulse profile imply that \(\alpha = 53^\circ\) [14]; models of the pulsed \(\gamma\)-ray emission suggest \(62^\circ-75^\circ\) [1]; the magnetospheric models result in even higher inclinations \(40^\circ-90^\circ\) (see, e.g., in Table 3 in [16]). The model by Rankin (1993) gives \(\alpha = 90^\circ\) and \(\theta_{\text{view}} = 102^\circ\) [20], whereas the model by Pierbattista et al. (2015) results in \(\alpha = 71^\circ\) and \(\theta_{\text{view}} = 83^\circ\) [22]. Here \(\theta_{\text{view}}\) is a viewing angle between the line of sight and the pulsar rotation axis. It also differs between the pulsar models. However, in contrast to \(\alpha\), it can be well constrained from the nebula morphology, since it determines also the tilt of the symmetry axis of the nebula to the line of sight. The Vela PWN morphology suggests that \(\theta_{\text{view}} = 127^\circ\) [15], so we adopt this value below.

The wind magnetization \(\sigma_0\) is not known for either Vela or any other pulsar. For Vela PWN, there have been several attempts to impose restrictions on this parameter (e.g., [9, 10, 6]). However, the numerical models still bear a somewhat distant resemblance to the nebula, as it seems to us. Therefore, as the reference model for the Vela nebula, we prefer to employ a PWN model with a high pulsar inclination and a low-magnetized pulsar wind (\(\alpha = 80^\circ\) and \(\sigma_0 = 0.03\)), suggested in [24] (see this ref. for other models). In our opinion, such a model matches better the Vela morphology, since it develops a wide equatorial region filled with a slow and low-magnetized outflow (which can account for a subluminous equatorial belt seen in the nebula), and two prominent toroidal whirls below and above it.
Figure 1: Right panel: merged Chandra image of the Vela PWN (0.5-8 keV). The brightness is logarithmically scaled in ACIS counts. The yellow arrow depicts the direction of the flow induced by the reverse shock of supernova remnant.

Left panels: A numerical two-tori model of a pulsar wind nebula with $(\alpha, \sigma_0) = (80^\circ, 0.03)$. The nebula is inflated into a stationary ambient medium: up to the age of 6 years – on a coarse spatial grid (at high numerical viscosity), and after that age – on a fine grid with enabled Adaptive Mesh Refinement of the level 2 (at low numerical viscosity). The top and bottom panel stand for the magnetic field and synchrotron emission maps of the model nebula taken by the age of 10 years. The magnetic map shows the (azimuthal) magnetic field $B_\phi$ in $\mu$G; the synthetic synchrotron emission map shows what the model nebula looks like when it is viewed at an angle $\theta_{\text{view}} = 127^\circ$ (the line of sight direction is shown by a black arrow) and rotated in the sky plane to the position angle of the Vela PWN ($130^\circ$).

These whirls appear to be well-formed, fast and highly-magnetized; therefore, at their location the $e^\pm$ particles of multi-TeV energies (accelerated at the termination shock) may emit a bright X-ray synchrotron emission. The emission is Doppler brightened at those parts of the whirls where the instant plasma flow moves toward an observer. In figure 1 the magnetic and the X-ray synchrotron emission maps of this model are given. On the magnetic map, two large scale toroidal vortices of opposite polarity are seen in the northern and southern hemispheres of the nebula; on the synchrotron emission map, one can see two bright arcs (or rings) which resemble the double-torus X-ray morphology observed in the Vela PWN; the latter is shown in the right panel of the figure. The whirls persist as long as small-scale turbulence in fast post-shock outflows is effectively damped and, therefore, does not exert a backreaction onto the shock geometry.
our ideal MHD simulations the damping is achieved through the high numerical viscosity: up to the age $t = 6$ years the nebula is inflated upon a coarse numerical grid (which smears out the small-scale turbulent motions); after that, the fine resolution grid is employed. The fine grid promotes the development of the small-scale turbulence; its accumulation eventually destroys the whirls and erase the region of low magnetization around the equator. Thus, in the absence of dissipation of magnetic and kinetic energy in situ, the reference model loses its two-tori morphology in 3.5 years.

Figure 2 illustrates the effect of a weakly supersonic flow upon the PWN model described above. Three rows in the figure stand for three different ages of the nebula. In each column, magnetic, velocity and synchrotron maps are shown at given age. The maps reveals, at least, seven specific effects of the flow.

First of all, the flow does not erase the two-tori morphology of the nebula; on the contrary, it strongly contributes to the stability of the whirls. Instead of 3.5 years of persistence in a stationary medium, in the supersonic flow the whirls survive at least for $\sim 9$ years (despite the high-resolution grid in use!). During that period the equatorial belt of the nebula remains spectacularly low-magnetized, and appears in the X-ray maps as a region of strongly reduced brightness. The similar “dark equatorial belt” is observed for 11 years in the X-ray map of the Vela nebula.

Second, the windward whirl gradually increases its radial extent relative to the leeward whirl. In the result, the windward torus appears larger in the X-ray maps that the leeward one. Interesting, that the North-West (windward) torus of the Vela nebula, indeed, looks a bit larger.

Third, the windward whirl may be brighter or dimmer than the opposite one; the effect depends on the subtle interplay between the line of sight of the observer, and the Mach number and the direction of the supersonic flow. To some extent, this effect can explain the variations in brightnesses of the Vela PWN.

Fourth, the flow tears off from the whirls’ periphery the light plumes of magnetized plasma and carries them along. These plumes can make up the diffuse wings of the Vela arcs that are observed to be bending southwest (cf. figure 2 in Helfand et al. [15]).

Fifth, the flow results in drastically different conditions at the opposite cusps of the termination shock where the jets form. In the result, the opposite jets of the nebula can differ in their dynamics and appearance, even being of the same origin.

Sixth, under the effect of the flow, the mid-scale vortex begins to develop around the base of the jet in the leeward hemisphere of the nebula. The vortex is fed by a slowly circulating but a highly magnetized plasma, so its developing can be traced only on the magnetic maps. The vortex, in principle, would account for emergence of the bright synchrotron blobs that are observed in the Vela X-ray map near the southern jet’s base (Fig.1, right panel) [24]. These blobs are highly variable but always adhere to the same arch-like pattern as do the knots in the “inner ring” in the Crab nebula.

Seventh, interesting, that the southern Vela’s jet exhibits a bright transverse bar at its base. The similar transverse bar forms at the leeward jet of the model nebula, as can be barely seen in the magnetic and synchrotron maps at $t = 14$ yr.

3. Conclusions

Within the frame of the ideal relativistic MHD model of a pulsar wind nebulae (PWNe), we show that a weakly supersonic flow going past the nebula can contribute, along with the dissipative effects, to the stability of the large-scale structures in the nebula. We built a numerical model of two-tori nebula (that resemble the X-ray morphology of the Vela PWN), and show that a weakly supersonic flow of Mach number $\sim 1.3$ allows the nebula to retain its two-tori morphology for at least 9 years, even in the absence of dissipation of magnetic and kinetic energy in situ.

Note that the Vela PWN actually encounters the supersonic flow that is misaligned with the...
Figure 2: Left to right: velocity (in units of $c$), magnetic field ($B_\phi$, in $\mu$G) and synthetic synchrotron emission maps of the simulated PWN. Top to bottom: maps at $t = 7.5, 10.5$ and 14 yr after the start of simulation. The viewing angle $\theta_{\text{view}} = 127^\circ$ (black arrow shows the line of sight direction) and the position angle is $130^\circ$. The pulsar position is marked by black cross on the synchrotron maps. Parameters of the model: pulsar inclination $\alpha = 80^\circ$, initial wind magnetization $\sigma_0 = 0.03$. The medium with Mach number $\sim 1.3$ is moving in $-z$ direction (shown by the red arrow). The yellow arrow shows the real direction of the flow in the Vela PWN. The run is performed with the coarse grid until $t = 6$ yr and with the AMR afterwards (see in Appendix).

symmetry axis of the nebula. The observations suggest that the flow direction makes $60^\circ$ to the axis. To account properly for the flow effects the full-3D simulations are required. Despite that and many other shortcomings of our simulations, the presented PWN model allows us to gain insights on MHD dynamics in PWNe, and reproduces many specific features of their morphology, as we show in the example of the Vela nebula.
The Vela PWN was taken as an example, because this is the only PWN (after the Crab PWN) whose spatial structures are observationally resolved in great detail, which allows us to make a detailed comparison between predictions of our two-tori models and observables. We emphasize that two-tori models are capable of explaining a wide class of PWNe with a double-torus morphology; for example, the Dragonfly nebula (which seemingly has two-tori structure) [27].

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Appendix
The numerical PWN model is built upon the relativistic MHD module of the PLUTO code [19]. We run the axisymmetric 2.5D simulations (with northern and southern hemispheres simulated independently). The computational domain spans from $r_{\text{min}} = 0.0002$ l.y. to $r_{\text{max}} = 1.4$ l.y. The coarse grid is taken as follows: the computation domain is divided into 32 polar angle bins and into 88 logarithmic intervals in the radial coordinate (so, that a spatial step progressively increases with distance). Until the age of 6 years, the nebula is evolved on the coarse spatial grid (at high numerical viscosity); this helps to quench the small-scale stochastic vorticity in the nebula until it enters self-similar expansion. After 6 years of age, Adaptive Mesh Refinement (AMR) of level 2 and refinement ratio 2 is enabled; it authoritatively sub-splits each cell of the coarse grid into 4 smaller cells, if required, which results in a low numerical viscosity. The synthetic synchrotron emission maps are produced and the pulsar wind is prescribed in the same way as described in the previous works [25, 6, 4].

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