Numerical Simulation of the Fracture Characteristics of Copper EFP with Different Constitutive Models

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Abstract. With three different constitutive model, LS-DYNA hydro code is applied to simulate the forming and fracture characteristics of Copper EFP. By comparison of numerical results of different models, the appropriate constitutive model to simulate the complete and fracture EFPs can be obtained. By comparison of experimental and numerical results, it can be concluded that Steinberg-Guinan constitutive model gives the best result to predict the complete EFP’s forming shape with minimum error, while Johnson-Cook constitutive model with failure model and h-adaptive algorithm is the most suitable method to predict the fracture of EFP in axial direction. The research results are important and significant in the design of shaped charge, which could provide a guideline to predict the fracture of EFP in numerical simulation.

Keywords: Explosively Formed Projectile; Constitutive model; Johnson-Cook failure model; h-adaptive algorithm; Numerical simulation.

1. Introduction

Shock waves from the detonation of explosive structure can be used to deform and flip a liner of typical ductile metal into an explosively formed projectile (EFP), also known as explosively formed penetrator or self-forging fragment (SFF). With an aerodynamically stable long-rod shape and a rather high velocity about 1500 to 3000 m/s, EFP can be used as an effective way to defeat armored targets from a long distance[1-4]. For certain liner material, when the length-to-diameter ration reaches the maximum value, fracture in axial direction occurs. In general, for copper, the maximum feasible length-to-diameter could be about 3 in experiment[5]. Fong[6], Thompson[7], Pappu[8] and Borkowski[9] have all observed the fracture of Copper EFP, but whether there is a feasible constitutive model or a new kind of numerical method to simulate the forming and fracture process have not been studied yet.

In this paper, three different kinds of constitutive model (Johnson-Cook, Zerilli-Armstrong and Steinberg-Guinan) are used to study the forming performance of Copper EFP. By comparison of numerical results, the appropriate constitutive model to simulate the complete and fracture EFPs can be obtained. By Johnson-Cook constitutive model with failure model and h-adaptive algorithm, the fracture phenomenon of Copper EFP can be simulated by LS-DYNA, which is significant in providing a new kind of numerical method to predict whether EFP would fracture or not in the forming stage in the EFP design area.
2. Numerical Modeling and Simulation Scheme

Fig.1 shows the structure of a typical EFP charge structure. Generally, an EFP charge structure is consist of an high-explosive (HE) charge, liner and casing, as shown in fig.1(a). The material of the HE charge is explosive 8701, which is a kind of RDX based explosive. The length to diameter ration of the charge is 0.8, with radius of curvature noted as $R_o$, diameter noted as $D_c$ and length noted as $l$. The liner is made of OFHC Copper, with a constant wall thickness of $h$. The radius of outer curvature is defined as $R_o$, and the radius of inner curvature is $R_i$. For the hemispherical liner, the wall thickness $h$ equals to $R_o-R_i$, which is $0.06D_c$. The material of casing is steel #45, with a thickness $\delta$ which equals to $0.045D_c$. The numerical simulation model is presented in fig.1(b), and the nonliner dynamics software LS-DYNA is used to modeling and simulate the forming and fracture characteristics of Copper EFP. The 3D assembly model and components model are presented in fig.1(c) and fig.1(d). Center detonation point is used to ignite the explosive charge.

![Figure 1](image-url)

**Figure 1.** Geometry and numerical simulation model of EFP charge structure.

| Components | Material | Density $\rho$(g/cm$^3$) | Equation of State | Constitutive Model | Failure Model |
|------------|----------|--------------------------|-------------------|-------------------|--------------|
| Charge     | 8701     | 1.71                     | JWL               | High-Explosive-Burn | /            |
| Casing     | Steel #45| 7.83                     | Grüneisen         | Johnson-Cook       | /            |

**Table 1.** Material models used in numerical simulation

| Explosive | $D$(m/s) | $A$(GPa) | $B$(GPa) | $R_1$ | $R_2$ | $\omega$ | $E_0$(J/mm$^3$) |
|-----------|----------|----------|----------|-------|-------|----------|------------------|
| 8701      | 8315     | 881.45   | 10.459   | 4.80  | 1.0   | 0.32     | 0.087            |

**Table 2.** JWL EOS parameters of explosive 8701.

| Material | Grüneisen Coefficient | $C$(m/s) | $S_1$ | $S_2$ | $a$ |
|----------|-----------------------|----------|-------|-------|-----|
| OFHC     | 2.02                  | 3490     | 1.489 | 0     | 0.47|

**Table 3.** EOS parameters of OFHC liner.

As shown in fig.1(b), all of the components of EFP charge structure are modelled with 2D Lagrange algorithm. The elements are generated with axisymmetric solid-area weighted shell. The mesh size is around 0.5mm per grid, and half model symmetric about y axis is carried out.
The material models of charge and casing are listed in Table 1. The behavior of the high-explosive charge is characterized by the Jones-Wilkins-Lee (JWL) equation of state which is widely used to describe the pressure-volume relationship of explosive. The EOS parameters of explosive 8701 are listed in Table 2.

The Grüneisen equation of state\cite{10} is used in conjunction with constitutive model. The EOS can be used to describe how the materials interact with the shock wave and is base on Hugoniot’s relation between the \( v_s \) and the \( v_p \), as \( v_s=c_0+sv_p \), where \( v_s \) is the shock wave velocity, \( v_p \) is the material particle velocity, \( c_0 \) is the wave speed and \( s \) is a material-related coefficient. The expression of equation of state of Grüneisen for compressed state is

\[
p = \rho S_1 \gamma_0 \mu (\gamma_0 + a \mu) E \quad (1)
\]

In the expanded state,

\[
p = \rho_0 C^2 \gamma_0 (\gamma_0 + a \mu) E \quad (2)
\]

where \( C \) is the intercept of velocity curve between shock wave and particle, \( S_1, S_2 \) and \( S_3 \) represent the slope of the \( v_s-v_p \) curve, \( \gamma_0 \) is the coefficient of Grüneisen and \( a \) is one-order correction of \( \gamma_0 \). \( \mu = \rho / \rho_0 - 1 \) is an non-dimensional coefficient based on initial and instantaneous material densities. The parameters are listed in Table 3.

Johnson-Cook, Zerilli-Armstrong, and Steinberg-Guinan are three typical kinds of constitutive models for metal materials currently implemented in finite element codes, which can be used to describe the dynamic response behavior appropriately in the explosion process. A brief description of the three material models are presented below.

2.1. Johnson-Cook Model

Table 4. Material parameters of Johnson-Cook model for OFHC Copper liner\cite{11, 12}

| \( \rho (\text{g/cm}^3) \) | \( \sigma_0 (\text{MPa}) \) | \( B (\text{MPa}) \) | \( n \) | \( C \) | \( m \) | \( D_1 \) | \( D_2 \) | \( D_3 \) | \( D_4 \) | \( D_5 \) |
|-----------------|-----------------|-----------------|-----|-----|-----|------|------|------|------|------|
| 8.96            | 90              | 292             | 0.31| 0.025| 1.09| 0.54 | 4.89 | -3.03| 0.014| 1.12 |

The Johnson-Cook model\cite{11, 12} is a widely used constitutive model which incorporates the effect of strain rate dependent work hardening and thermal softening. The Johnson-Cook constitutive relation is given by

\[
\sigma = \left( \sigma_0 + Be^\epsilon \right) \left( 1 + C \ln \frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right) \left( 1 - T^m \right) \quad (3)
\]

where \( \epsilon \) is the plastic strain and the temperature factor is expressed as

\[
T^* = \frac{T - T_r}{T_m - T_r} \quad (4)
\]

Where \( T_r \) is the room temperature, \( T_m \) is melt temperature of the material. \( \sigma_0, B, n, C \) and \( m \) are material-related parameters. The failure model is also included in the Johnson-Cook constitutive model.

The strain at fracture is given by

\[
\epsilon^f = D_1 + D_2 e^{D_3 \sigma^*} \left[ 1 + D_4 \ln \epsilon^* \right] \left[ 1 + D_5 T^* \right] \quad (5)
\]

where \( \sigma^* = \sigma_m/\sigma_0 \) is the ration of pressure divided by effective stress, where
Fracture occurs when the damage parameter reaches the value of 1.

\[ p = \sum \frac{\Delta \varepsilon^d}{\varepsilon^d} \]  

### 2.2. Zerilli-Armstrong Model

For Fcc (Face Center Cubic) metals, the Zerilli-Armstrong model \[13\] expresses the flow stress as follows,

\[ \sigma = C_0 + C_2 \varepsilon^{1/3} \exp \left( -C_3 T + C_4 T \log \left( \dot{\varepsilon}^* \right) \right) \]  

where \( \varepsilon \) is effective plastic strain, \( \dot{\varepsilon}^* = \dot{\varepsilon}/\dot{\varepsilon}_0 \) is the effective plastic strain rate. \( C_0, C_2, C_3, C_4 \) are material-related coefficients, and \( T \) is the absolute temperature. As typical FCC metal, the material parameters of Zerilli-Armstrong model for OFHC Copper liner are listed in Table 5.

| Material | \( \rho \) (g/cm\(^3\)) | \( C_0 \) (MPa) | \( C_2 \) (MPa) | \( C_3 \) (K\(^{-1}\)) | \( C_4 \) (K\(^{-1}\)) |
|----------|-------------------|-----------------|-----------------|-----------------|-----------------|
| OFHC     | 8.96              | 65              | 890             | 0.0028          | 0.000115        |

### 2.3. Steinberg-Guinan Model

The Steinberg-Guinan model \[14\] is available for modelling materials at very high strain rate (>10\(^5\) s\(^{-1}\)). The yield stress is a function of temperature and pressure. In the Steinberg-Guinan constitutive relation, the shear modulus, \( G \), before the material melts, can be expressed as

\[ G = G_0 \left[ 1 + b p V^{\frac{1}{3}} - h \left( \frac{E_c - E_m}{3R'} - 300 \right) \right] e^{-\frac{E_m}{E_c}} \]  

where \( p \) is the pressure, \( V \) is the relative volume, \( E_c \) is the cold compression energy:

\[ E_c (x) = \int_0^x p dx - \frac{900 R' \exp (ax)}{(1-x)^{2(1-\alpha)\frac{a}{2}}} \]  

\[ x = 1 - V \]  

and \( E_m \) is the melting energy:

\[ E_m (x) = E_c (x) + 3 R' T_m (x) \]  

which is in terms of the melting temperature \( T_m (x) \):

\[ T_m (x) = \frac{T_m \exp(2ax)}{V^{\frac{4(1-\alpha)\frac{a}{2}}{}})} \]  

and the melting temperature at \( p = p_0, T_{m0} \).

The yield stress \( \sigma_y \) is given by:

\[ \sigma_y = \sigma_0' \left[ 1 + b' p V^{\frac{1}{3}} - h \left( \frac{E_c - E_m}{3R'} - 300 \right) \right] e^{-\frac{E_m}{E_c}} \]  

when \( E_m \) exceeds \( E_c \), here \( \sigma_0' \) is given by:
\[ \sigma'_0 = \sigma_0 \left[ 1 + \beta \left( \gamma_i + \varepsilon^p \right) \right]^n \]  

(16)

Where \( \sigma_0 \) is the initial yield stress and \( \gamma_i \) is the initial plastic strain. If the work-hardened yield stress \( \sigma'_0 \) exceeds \( \sigma_m \), \( \sigma'_0 \) is set to \( \sigma_m \). After the melting point, \( \sigma_y \) and \( G \) are set to one half their initial value. The material parameters of Steinberg-Guinan model for OFHC Copper liner are presented in Table 6.

| \( \rho \) (g/cm\(^3\)) | \( G_0 \) (MPa) | \( \sigma_0 \) (MPa) | \( \beta \) | \( n \) | \( \gamma_i \) | \( \sigma_m \) | \( b \) |
|-----------------|----------------|----------------|--------|-----|-------|--------|-----|
| 8.96            | 47700          | 120            | 36     | 0.45| 0     | 640    | 36  |
| \( b' \)         | \( h' \)       | \( f' \)       | \( T_{mol} \) (K) | \( \gamma_0 \) | \( a \) | PC     | SPALL|
| 0.45            | 0.000377       | 0.001          | 1790   | 2.02| 1.5   | -9     | 3   |

3. Numerical Results and Discussions

As constitutive modes can be used to simulate the dynamic response of metal materials with or without the failure models, the Johnson-Cook constitutive model is used to simulate the forming and fracture characteristics with or without its failure model. The formation of an typical EFP is shown in Fig.2. Two typical radius of outer curvature of 1.071\( D_c \) and 0.893\( D_c \) are chosen. The numerical simulation results are presented below.

![Figure 2. Formation of an explosively formed projectile.](image)

3.1. Constitutive Models with no Failure

The forming characteristics of Copper EFP with radius of outer curvature 1.071\( D_c \) and 0.893\( D_c \) are listed in Table 7 and Table 8, respectively. \( L \) and \( D \) are the length and diameter of EFP. By comparison, when \( R_o \) is set to 1.071\( D_c \), it could get EFPs with similar shape by using all the three kinds of constitutive model. The Johnson-Cook model gives longer EFP with smaller diameter, while the Steinberg-Guinan model gives shorter EFP with larger diameter. When \( R_o \) is set to 0.893\( D_c \), the Steinberg-Guinan model gives the shortest EFP with the largest diameter, while Johnson-Cook model gives the longest EFP with a tendency of necking with the longest length-to-diameter 5.18.

| \( R_o \) | Constitutive Model | Forming State | \( L/D_c \) | \( D/D_c \) |
|----------|--------------------|---------------|------------|------------|
| 1.071\( D_c \) | Johnson-Cook | 0.897 | 0.302 |
| 1.071\( D_c \) | Zerilli-Armstrong | 0.779 | 0.334 |
| 1.071\( D_c \) | Steinberg-Guinan | 0.672 | 0.372 |
### Table 8. Forming characteristics of Copper EFP with $R_o=0.893D_c$.  

| $R_o$   | Constitutive Model     | Forming State | $L/D_c$ | $D/D_c$ |
|---------|------------------------|---------------|---------|---------|
| $0.893D_c$ | Johnson-Cook          |               | 1.28    | 0.247   |
| $0.893D_c$ | Zerilli-Armstrong      |               | 0.944   | 0.270   |
| $0.893D_c$ | Steinberg-Guinan       |               | 0.796   | 0.290   |

3.2. Constitutive Models with Failure  
As the necking tendency observed in Tab.8, the Johnson-Cook failure model is used in conjunction with the Johnson-Cook constitutive model to predict whether the formed EFP would fracture due to the excessive aspect ration. The numerical results are presented in Tab.9. The plastic strain and deformed mesh of EFP with $R_o=0.893D_c$ are presented in Fig.3.

### Table 9. Forming characteristics of Copper EFP with $R_o$ equals to $1.071D_c$ and $0.893D_c$.  

| $R_o$   | Constitutive Model     | Forming State | $L/D_c$ | $D/D_c$ |
|---------|------------------------|---------------|---------|---------|
| $1.071D_c$ | Johnson-Cook          |               | 0.890   | 0.302   |
| $0.893D_c$ | Johnson-Cook          |               | 1.263   | 0.248   |

![Figure 3](image)  
**Figure 3.** Plastic strain and deformed mesh of EFP with $R_o=0.893D_c$.  

From the Tab.8, it can be conclude that Johnson-Cook constitutive model with failure model still cannot be used to predict whether the EFP with radius of liner’s outer curvature of $0.893D_c$. But the contour of the effective plastic strain and the severely deformed mesh of the EFP indicate that fracture in the axial direction could much likely to occur.

3.3. Constitutive Models with Failure Model and h-adaptive Algorithm  
Thanks to the h-adaptive method for the shell elements is included in LS-DYNA, it is possible to carry out the numerical simulation combining the strength of the adaptive algorithm. As shown in Fig.4, in the h-adaptive method, the elements are subdivided into smaller elements whenever an error indicator shows that subdivision of the elements will provide improved accuracy and avoid the severe deformation in the mesh\cite{15}. The Forming characteristics of Copper EFP with Johnson-Cook failure...
model and h-adaptive algorithm are listed in Tab.10.

![Figure 4](image)

**Figure 4.** Quadrilateral element fissioned to fourth level in h-adaptive algorithm\(^{[15]}\).

### Table 10. Forming characteristics of Copper EFP with Johnson-Cook failure model and h-adaptive algorithm.

| \(R_o\)  | Constitutive Model     | Forming State | \(L/\text{D}_C\) | \(D/\text{D}_C\) |
|---------|------------------------|---------------|-------------------|-------------------|
| 1.071\(D_c\) | Johnson-Cook           |               | 0.877            | 0.306             |
| 0.893\(D_c\) | Johnson-Cook           |               | 1.211            | 0.251             |

As showed in Fig.5, \(L_{\text{head}}\) and \(L_{\text{tail}}\) are the length of the head and tail segment of EFP, thus the whole length of the fractured EFP \(L = L_{\text{head}} + L_{\text{tail}}\). As shown in Tab.8, with the failure model and h-adaptive algorithm, it is practicable to obtain the fracture phenomenon in the forming of EFP.

![Figure 5](image)

**Figure 5.** The EFP’s parameters after fracture.

### 4. Experimental Validation

To validate the simulation results, flash X-ray experiment was carried out to observe the forming and fracture state of Copper EFP. Scandiflash-450 system is used for the flash X-ray experiment, which is designed by Scandiflash AB Company in Sweden and widely used in ballistics and hypervelocity impact studies. Tab.9 compares the parameters of EFP’s forming state in experiment and simulation.

### Table 11. Comparison of Copper EFP’s forming states in experiment and simulation.

| \(R_o\)  | Forming State          | Constitutive model             | \(L/\text{D}_C\) | \(D/\text{D}_C\) |
|---------|------------------------|--------------------------------|-------------------|-------------------|
| 1.071\(D_c\) | [Image](image)        | Steinberg-Guinan               | 0.672             | 0.372             |
| 0.893\(D_c\) | [Image](image)        | Johnson-Cook failure model and h-adaptive algorithm | 1.211             | 0.251             |

/
From the Tab.11, it can be concluded that when the Copper EFP is complete, the Steinberg-Guinan constitutive model gives the best result to predict EFP’s forming shape with minimum error, while when it comes to the fracture phenomenon, Johnson-Cook constitutive model with failure model and h-adaptive algorithm is most suitable method to predict the fracture of EFP in axial direction.

5. Conclusions

(1) Three different kinds of constitutive model (Johnson-Cook, Zerilli-Armstrong and Steinberg-Guinan) are chosen to study the forming characteristics of Copper EFP. By the comparison of numerical results, it can be concluded that the Steinberg-Guinan model gives the shortest EFP with the largest diameter, while Johnson-Cook model gives the longest EFP with smallest diameter;

(2) By comparing with the experimental results, it can be inferred that: when the Copper EFP is complete, the Steinberg-Guinan constitutive model gives the best result to predict EFP’s forming shape with minimum error, while when it comes to the fracture phenomenon, Johnson-Cook constitutive model with failure model and h-adaptive algorithm is most suitable method to predict the fracture of EFP in axial direction.

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