Model updating of Pescara Benchmark: interval vs. traditional method

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Abstract. In this work FE model updating is used to detect parameter variations from representative models of the Pescara benchmark beams. For a considered series of the beams experimental modal data are available at a certain beam stage. For this stage model updating results are obtained and different updating techniques are compared. Standard techniques based on parameters sensitivity calculation are applied and, in this case, commercial robust software and self-implemented are used. A new interval model updating approach is also applied to possibly cope with experimental uncertainties and parameters variability. The interval algorithm automatically gives a bound of the results; on the contrary sensitivity approach gives a crisp result. Hence, to have a proper comparison, a mid term comparison is also given from the calculated interval solution.

1. Introduction
This work is a part of a research project named Brividi, coordinated by the Politecnico di Torino, that concerns the development of new strategies to detect damage in bridges under vibration response. Among many techniques for damage detection, finite model updating [1] is one of most often used [2]. During the Brividi project a series of beams have been tested and their vibration response has been measured. The tests have been made in the Pescara Testing Lab (from this the benchmark name) and they are both static and dynamic, made for beams in sound and damaged state. The beams are precast concrete constituted and the damage is given by corrosion induced in the steel ropes.

In this work a few set of the collected experimental data is considered. The data refers to a sound stage of a particular beam series, named T7.

The goal of this first model updating step is not to detect damage, but to provide an initial validated FE model of the tested beams. An a priori model of the beams is initially implemented, having some uncertain parameters, in particular flexural stiffness and boundary conditions at physical supports. Then damage will be evaluated from the sound updated model, by considering a new set of experimental data from the damaged state tests.

To ensure the robustness in obtaining a sound updated model, two different methods are applied and compared. The first method is a sensitivity based approach [3], that is applied using two different software packages for comparison. The second method is a novel interval analysis approach [4] to

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model updating. The updating parameters are hence considered as uncertain variables bounded in intervals, and the solution is searched inside the parameters intervals. The updating method is based on the optimisation strategies presented in the book of Hansen and Walster [5]. This kind of optimisation methods are hence attractive for model updating purposes, in fact they allow to find more than one solution when it is not unique, as it is the general case of engineering inverse problems. A preliminary error analysis is shown in section 3 to present the problem of multiple solutions (non-uniqueness), even in the case of an apparently simple example. In section 4 the obtained solutions are compared to define the initial engineering sound model of the T7 beams.

2. Analysis of the experimental data
The measured response follows from dynamic tests performed during the Brividi Research Project, on the so called Pescara benchmark beams, as the tests were done in the Chieti-Pescara University Testing Lab (S.C.A.M.).

![Figure 1. Nominal section properties of the T7 series of Pescara beams, as given by the producer.](image1)

![Figure 2. Sensor positions and one of the tested beam](image2)

Here only a small set of the available data is considered. In particular the data regarding the tests on sound beams series named T7. Different beam series have the same nominal section, but different areas and positions of precast ropes. In this case the nominal section is depicted in Fig. 1.

The considered experimental data consist in the first three free vibration modes (eigenfrequencies and eigenvectors), obtained by a frequency domain identification technique [6]. The identification
technique is applied to signals recorded in free vibration conditions, obtained by applying an impulse excitation in different beam positions. Nine sensors distributed along the beam length give the recorded signals. (see Fig. 2).

The obtained experimental results are summarized in Table 1.

### Table 1. Experimental data.

| Mode | Frequency [Hz] | Sensor channel | Mode shape 1 | Mode shape 2 | Mode shape 3 |
|------|----------------|----------------|--------------|--------------|--------------|
| 1    | 19.25          | 1              | 0.01         | -0.06        | 0.32         |
| 2    | 74.74          | 2              | 0.47         | -0.80        | 0.97         |
| 3    | 161.56         | 3              | 0.72         | -0.96        | 0.58         |
| 4    | 0.90           | 4              | 0.90         | -0.76        | -0.22        |
| 5    | 1.00           | 5              | 1.00         | 0.03         | -0.83        |
| 6    | 0.94           | 6              | 0.94         | 0.84         | 0.04         |
| 7    | 0.70           | 7              | 0.70         | 1.00         | 0.69         |
| 8    | 0.47           | 8              | 0.47         | 0.82         | 1.00         |
| 9    | 0.01           | 9              | 0.01         | 0.02         | 0.23         |

### 3. Model updating

#### 3.1. Sensitivity approach implemented in FEMUP tool

The implemented model updating technique is based on a sensitivity approach [3]. Defining $\mathbf{z}_s$ as the vector of experimental measures ($\mathbf{F}_s$, $\mathbf{U}_s$), eigenfrequencies $\mathbf{F}_s$ and eigenvectors $\mathbf{U}_s$ respectively, an equivalent vector of numerical measures $\mathbf{z}(\mathbf{p}) = (\mathbf{F}, \mathbf{U})$ can be defined, which is calculated from an FEM of the structure, and that depends on the vector $\mathbf{p}$ of the updating parameters to be updated. In the actual case $\mathbf{p}$ is the vector of initial values of the vertical springs ($K_{v1}, K_{v2}$).

The distance between the a priori model and the experimental one is defined by the error $\mathbf{e}$:

$$\mathbf{e} = \mathbf{z}_s - \mathbf{z}(\mathbf{p}).$$

This error can be minimized to find the optimum set of model parameters $\mathbf{p}$ that better match the experimental results. An objective function to be minimized can be defined on purpose, which is given by

$$l(\mathbf{p}) = \langle \mathbf{e}, \mathbf{e} \rangle = \left\| \mathbf{z}_s - \mathbf{z}(\mathbf{p}) \right\|_2^2$$

the operator $\left\| \cdot \right\|_2$ being the 2-Euclidean norm. Details and stability of the procedure are considered according to [7].

#### 3.2. Interval approach implemented in INTIM

The used method has been developed by one of the authors. It is called Interval intersection method (INTIM) and develops from a basic Branch and Bound optimisation approach as described in the book of Hansen and Walster [5]. Details of the method can be found in [4]. Here a very concise rationale of the method is given to allow interpreting the obtained results. We denote intervals by including in square brackets [], we call mid point the central value of the interval and upper and lower bounds the interval limits. The interval radius defines the distance between the mid point to the upper or the lower bound and represents the uncertainty amount associated with the interval.

In INTIM we suppose to start from an initial search parameters interval $[\mathbf{p}_i]$ such that $\mathbf{p} = (p_1, \ldots, p_n) \in [\mathbf{p}_i]$. Two steps mainly constitute the algorithm. A branching step is performed to
subdivide \([p_0]\) in smaller sub-intervals, such that \([p_i] \subset [p_0]\). A bounding step is properly formulated to decide whether \([p_i]\) should be retained or rejected. In this work a sub-interval \([p_i]\) is retained if the calculated interval frequencies \([F_i] = F([p_i])\) include the experimental frequencies \(F_s\), or if the mean value of the calculated \(MAC([U_i], U_s)\) is \(\geq 0.95\) (see reference [1] for \(MAC\) index); \([U_i]\) being the calculated interval eigenvectors by the sub-interval \([p_i]\). Branching and bounding steps are iteratively applied and the procedure stops when for some \([p_i]\) the acceptance criteria hold together with a stop tolerance so that \(K_i\) cannot be further branched.

The above procedure has been implemented in the Matlab® through the INTLAB toolbox [8]. INTIM makes use of the scheme suggested in Hansen and Walster (2004) where the search domain is iteratively branched according to the bisection rule. The selected criterion to stop the iterations, in the \(n\)-dimensional case, \(p: p_1 \times \ldots \times p_n\), is as following

\[
\min \{\Delta p_1, \ldots, \Delta p_n\} \leq \Delta_{tol} \tag{2}
\]

That is when the radius \(\Delta p\) of at least one of the \(n\) parameters is smaller than a user selectable tolerance \(\Delta_{tol}\). The tolerance \(\Delta_{tol}\) is an important parameter, but it is very difficult to choose and it could be considered as a heuristic parameter. A discussion about it could be found in [4]. In our experience a good initial choice could range between 1/10 and 1/100 of the initial uncertainty radius \(\Delta p_0\).

4. Error function analysis

We consider now the modal analysis problem of the beam depicted in the following Figure 3.

![Figure 3. Beam with vertical elastic boundary conditions](image)

In our problem the elastic stiffnesses \(K_{v1}\) and \(K_{v2}\) are considered as uncertain parameters, varying in the same interval \(K_{v1} = K_{v2} = [1, 500] \times 10^3\) kN/m. In this case rotational springs are omitted, as evident vertical displacements have been experimentally observed at supports, both in statics and dynamic tests.

The interval limits are assumed by engineering skill as, in present case, values smaller than the lower bound \(1.0 \times 10^3\) kN/m could be considered as absence of supports, and values bigger than the upper bound could be considered as clamps.

The model parameters \(K_{v1}\) and \(K_{v2}\) are updated by considering three different tools:

- the commercial software FEMTOOLS [9];
- an independent software developed at University “Roma Tre” and named FEMUP;
- an independent interval updating tool implemented in Matlab® and named INTIM.
The first two tools share the same method that is a sensitivity approach. On the contrary the INTIM tool implements a completely different method based on interval admissibility (inclusion) test as described in the previous section.

Before presenting the updating results a preliminary analysis is made to build up the error function (1) surfaces. This can be done here as we have only two uncertain parameters. But contrary to what could be thought, choosing the solution straight by the error function surfaces it is difficult, due to the presence of multiple solutions.

The error functions are calculated by considering two different cases:
(a) Frequency comparison,
(b) Frequency and Eigenvectors comparison,

The two considered cases are reported in figures 4 and 5 respectively. They clearly show that both functions are very difficult to minimise, due to their topology. In case (a) the function presents very flat zones, with a valley along the borders. Using the sensitivity approach is very difficult to locate the solution in the flat zone, as the sensitivity is a sort of gradient of the function, and as the solution path come up in the valley again a very flat zone is found. In any case it is difficult to try to select a solution by hand, as there are many points that approximately share the same function minimum value. In case (b) a lot of close local minima appear, especially along the flat valley of case (a). It follows a very complicated problem from a minimisation point of view, in particular if a gradient method is used.

The error analysis shows that the present case, although could be considered a very simple one, due to the presence of only two parameters, it is not so simple from a mathematical point of view. The problem is also very bad conditioned from an engineering point if we have as goal to find, among all the possible mathematical solutions (local and global minima), a solution that is also physically admissible.

5. Results

5.1. FEMUP results

Initial parameters \( K_{v10} = K_{v20} = 7.5 \times 10^4 \) kN/m

Updated parameters \( K_{v1u} = 2.08 \times 10^4 \) kN/m, \( K_{v2u} = 3.64 \times 10^4 \) kN/m

Parameter variations \( \Delta K_{v1\%} = 72.32\% \), \( \Delta K_{v2\%} = 51.51\% \)

\( \Delta K_{v1\%}, \Delta K_{v2\%} \) being the parameter percentage variations between the initial values \( K_{v10}, K_{v20} \) and the updated values \( K_{v1u}, K_{v2u} \).

| Table 2. Frequency comparison of FEMUP updating. |
|---|---|---|---|---|
| Experimental | Mode | Initial [Hz] | Updated [Hz] | Initial (%error) | Updated (%error) |
| 1 | 19.25 | 19.32 | 19.25 | 0.37% | 0.03% |
| 2 | 74.74 | 76.56 | 75.35 | 2.43% | 0.82% |
| 3 | 161.56 | 169.65 | 162.99 | 5.00% | 0.89% |

| Table 3. MAC comparison of FEMUP updating. |
|---|---|---|
| Mode | Initial | Updated |
| 1 | 1.00 | 1.00 |
| 2 | 1.00 | 1.00 |
| 3 | 0.96 | 0.99 |
In Fig. 5 a graphical comparison of the third mode shape is given. The selected FEMUP solution had converged in 26 iterations. Hence the figure legend reports the experimental mode shape (sper) at each of the nine sensors, the initial model mode shape (0) and the updated model mode shape (26). As in the MAC Table 2, we can note that the solution improve significantly only for the third mode, as the first two mode shapes are very well correlated in principle, hence they are not depicted in the following. In Fig. 5 can also be observed the considerable vertical component of the displacement at supports (1 and 9 abscissa points), which are modelled as vertical springs in that direction.

**Table 4.** Frequency comparison of FEMTOOLS updating.

| Mode | Experimental Frequency [Hz] | Model Initial Frequency [Hz] | Model Updated Frequency [Hz] | % Error | % Error |
|------|-----------------------------|------------------------------|-------------------------------|---------|---------|
| 1    | 19.25                       | 19.33                        | 19.25                         | 0.41%   | 0.02%   |
| 2    | 74.74                       | 76.64                        | 75.13                         | 2.54%   | 0.52%   |
| 3    | 161.56                      | 169.78                       | 161.33                        | 5.09%   | 0.21%   |

**Table 5.** MAC comparison of FEMTOOLS updating.

| Mode | MAC Initial | MAC Updated |
|------|-------------|-------------|
| 1    | 1.00        | 1.00        |
| 2    | 1.00        | 1.00        |
| 3    | 0.96        | 0.99        |

5.2. **FEMTOOLS results**

Initial parameters $K_{v10} = K_{v20} = 7.5 \times 10^4 \text{ kN/m}$

Updated parameters $K_{v1u} = 1.80 \times 10^4 \text{ kN/m}, K_{v2u} = 3.12 \times 10^4 \text{ kN/m}$

Parameter variations $\Delta K_{v1} = 76.03\%$, $\Delta K_{v2} = 58.43\%$
5.3. INTIM results

Based on the experience of the previous deterministic analyses, we chose the following starting intervals (that are the counterpart of the FEMUP and FEMTOOLS starting points or initial conditions):
\[ [K_{v10}] = [1.0, 5.5] \times 10^4 \text{kN/m}, [K_{v20}] = [1.0, 6.2] \times 10^4 \text{kN/m} \]

These intervals define the physical bound of interest. Even if in the interval case we do not minimise the deterministic error function of Equation (1), we equally expect more than one solution interval, as stated by the error function analysis. The solution is also found according to the radius tolerance that is here fixed as \( \Delta_{tol} = \max(\Delta K_{vi0})/16 \), with \( i = 1,2 \) and \( \Delta K_{vi0} \) being the radius of the initial interval box \([K_{vi0}]\).

At the end of the INTIM updating process the solution is constituted by four intervals whose radii fall inside the defined tolerance \( \Delta_{tol} \). Each of these intervals is considered as a possible solution, because it is characterised by a complete inclusion of the calculated interval frequencies in the experimental one, and the mean MAC (ref. to section 3.2) of the interval eigenvectors is greater than 0.95. This solution is depicted in Figure 7, with respect to the initial search box \([K_{vi0}]\).

We finally take the interval with the biggest mean MAC value as solution, obtaining the following result:

**Solution interval**

\[ [K_{vi,sol}] = [(1.8911, 2.0469), (2.5108, 2.6952)] \times 10^4 \text{kN/m} \]

**Mid values**

\( K_{vi,sol} = (1.9690, 2.6030) \times 10^4 \text{kN/m} \)

| Experimental | Model |
|--------------|-------|
| Mode | Frequency [Hz] | Interval solution [Hz] | Mid value [Hz] | Mid (%error) | Interval (%error) |
| 1 | 19.25 | [18.49, 19.96] | 19.23 | 0.14% | [-3.66, 3.94]% |
| 2 | 74.74 | [74.34, 76.33] | 75.33 | -0.80% | [-2.12, 0.53]% |
| 3 | 161.56 | [161.27, 163.37] | 162.32 | -0.47% | [-1.12, 0.18]% |

| Mode | Single mode | Mean |
|------|-------------|------|
| 1    | 1.000       |      |
| 2    | 0.998       | 0.994|
| 3    | 0.985       |      |

**Table 6. Frequency comparison of INTIM updating.**

**Table 7. MAC comparison of INTIM updating.**

**6. Discussion of the results**

In this work the same updating problem solution is obtained in different ways. We showed first (Section 4) that the error functions to be minimised are very bad shaped for the inverse problem solution. This is the first notable result, as we can expect in principle a simple updating problem for a simple mechanical system (Fig. 3).

The sensitivity based model updating we state that could have difficulties in the actual case, due to the error function gradient dependency. But if we compare the obtained solutions by FEMUP and FEMTOOLS, we can assert that they are very near each other. The FEMUP result is stiffer than the FEMTOOLS one, and if we look at the error function in Fig. 5 we could interpret that, the two solutions are different local minima around the same portion of the function. The solution gap could be given by differences in the algorithmic implementation. Of course we have to take into consideration that FEMTOOLS is a very tested tool and that its solution is very robust. For this reason
we should be happy about the self implemented FEMUP result, as the FEMUP algorithm is able to catch the same solution attractors. The modal comparison is again a little preferable that from FEMTOOLS if we consider the eigenfrequencies. No difference indeed if we look at the MAC values of Tables 3 and 5. In Figures 5 and 6 appears evident the updating contribution of the springs (selected as updating parameters), if we look at the first and last sensor positions.

The INTIM result has also to be considered a very good one. The main reason is that INTIM is a very new method, having few applications with experimental data. At this stage of development INTIM is a gradient free algorithm and is not affected by the function topology. For this reason it is computationally less efficient, if compared with a sensitivity method. INTIM simply select solutions by means of inclusion. For this reason the four solutions found by INTIM are those for which we have a complete inclusion of the eigenfrequencies with best MAC values. In that way the solution is very robust and we can note that the best interval \([K_{\text{vi}}]\) is again not far from the sensitivity solutions. The same can be deduced if we consider the mid point comparison of the interval solution in Tables 6 and 7.

In conclusion the INTIM solution \([K_{\text{vi}}]\) could be considered as the preferable for physical reasons. In fact the guaranteed inclusion of interval analysis is also interpreted as the impossibility to find solution outside \([K_{\text{vi}}]\). And because the found sensitivity solutions are not or partially included in it, they lie in intervals for which interval inclusion does not hold. This is a personal interpretation of this work author’s that is based on the inclusion property of interval analysis and that is already presented in [4].

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8. References
[1] Friswell MI and Mottershead JE 1995 *Finite Element Model Updating in Structural Dynamics* (Kluwer Academic Publishers: Dordrecht)
[2] Teughels A and De Roeck G 2005 *Arch. Comput. Meth. Engng.* 12-2 123
[3] Mottershead JE, Link M, Friswell MI 2011 *Mech. Syst. Signal. Pr.* in Press
[4] Gabriele S and Valente C 2009 *Int. J. Reliability and Safety* 3 1-3 79
[5] Hansen ER and Walster G 2004 *Global Optimization Using Interval Analysis* (Marcel Dekker: New York)
[6] Valente C, Spina D, Gabriele S 2007 *Key Eng. Mat.* 347 259
[7] Camillacci R and Gabriele S 2005 *Mech. Syst. Signal. Pr.* 19-3 597
[8] Rump SM 1999 *INTLAB - INTerval LABoratory. In Tibor Csendes, editor, Developments in Reliable Computing* (Kluwer Academic Publishers: Dordrecht) 77
[9] FEMTOOLS [www.femtools.com](http://www.femtools.com)