The age of water and carbon in lake-catchments: A simple dynamical model

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Abstract

The rate at which water and carbon move through lake-catchments influences carbon metabolism, storage, and transport to down-stream ecosystems. This article focuses on conceptual-mathematical modeling of the “age” of water and carbon in lake-catchment systems, where “age” is defined as the time since the water parcel and environmental tracer entered the system. We test a framework for implementing models and data for the Lake Mendota lake-catchment system. We show that the lake-catchment system provides the basis for predicting time varying age of carbon along flow paths, and the residence times of dissolved organic carbon (DOC) leaving each compartment, which further helps to provide a deeper understanding of fundamental biogeochemical dynamic processes of carbon and water cycling within the lake-catchment system. Three scenarios illustrate how the age of DOC might evolve from climate or land cover change that alters the hydrologic water balance through increases or decreases in lake-catchment precipitation and evapotranspiration.

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The rate at which water and carbon move through lake-catchments strongly influences the extent to which carbon is metabolized, stored, or transported to the next downstream ecosystem (Hanson et al. 2014b). However, lake-catchments can have complex hydrologic flow-paths, residence times, and multiple sources and reservoirs of carbon. Representing this complexity in spatially explicit dynamical models presents serious challenges, both in terms of model development and the variety and amount of data required (Leonard and Duffy 2013). A conceptually simpler approach to characterizing coupled hydrology and carbon transport through the full catchment would provide for a synthetic measure of the hydrologic and chemical residence times that has lower data requirements and is more easily applied to a variety of catchments but one that still preserves the important dynamic characteristics of the lake-catchment ecosystem. Our goal is to develop a “lower dimensional” modeling strategy that can answer questions such as: What are the fundamental time scales of change in a lake-catchment system? Will climate change affect the level, water quality, and carbon balance of lakes and rivers? What is the role of ecological disturbance from changing land-cover or changing climate on the lake-basin age or residence time? How long does it take for nutrient pulses in the watershed to reach and pass through the catchment and the lake? Can a model for the “age” of water and carbon species support this understanding and lead to better strategies for detection and attribution of carbon cycling? We address these questions through hypothetical scenarios based on a real lake-catchment setting in Wisconsin, U.S.A.

The theory for residence time and age of dissolved species in lake-basins has a long and diverse history. The usual definitions for age is the average time a solute spends within a defined reservoir volume or volumes, while residence time is the average age of the solute leaving the same reservoir(s). Note that here we extend these definitions to include multiple interacting reservoirs and utilize the notion that for well-mixed systems the residence time and age are approximately equivalent quantities. One of the earliest references to age in lake basins is from Edmund Halley’s 1715 paper to the Royal Society of London (Halley 1714–1716) where he proposed an equation to estimate the age of dissolved salts in topographically closed lake-basins. By taking the ratio of the accumulated mass of dissolved salts in the lake to the mass-flow of salts carried into the lake from the drainage basin, one could in principal estimate the potential age of salts in the lake (Age=m/m=f=mass/massflux). He went on to discuss the driving hydrologic processes of precipitation and evaporation that explain the accumulation of salinity. However, at the time no one had carried out the required measurements necessary to make the age calculation. Halley also discussed how his method, applied to lake-basins over the earth, could serve as an analogy to the mass flux of chemical denudation from major rivers entering the ocean and potentially a way to estimate the age of the earth. Later, many authors followed up on Halley’s proposal to measure the mass flux of rivers and to arrive at a measure of geological age. In 1895, Joly (1899) extended Halley’s age concept for closed lake-basins to sodium accumulation in the oceans. He also framed the problem in terms of birth and death processes of biological systems by analogy to population models of the time (Joly 1911). Developments in the field of population biology in the early 1800s and the new theory of probability offered important extensions to the age model for particles and populations that included not just the mean age proposed by Halley and Joly but allowed a calculation of the distribution of ages within a population. Important papers by McKendrick (McKendrick 1926) and von Foerster (von Foerster 1959b), independently put age theory on a firmer footing by illustrating how time and age could be rendered as independent variables in a differential equation. The idea was extended by Rotenberg (1972) in a formal probabilistic framework.

The concept of age has been applied to the age and residence time of environmental systems in ocean, atmospheric, and paleo-sciences. In hydrologic studies, stable and radioactive isotopes, organic, and inorganic solutes have long been used as environmental tracers to estimate the age and residence time of hydrologic systems (Nauman 1969; Allison and Holmes 1973; Phillip 1995; IHP-V 2001; Kazemi et al. 2006), and recently there has been an emergence of research focused on spatial isotopic approaches for biogeographical studies across landscapes, referred to as isoscapes (Bowen 2010).

“Age” as a system property

For the purposes of this article, the “age” of a dissolved species such as dissolved organic carbon (DOC), of interest here, is an extensive property, which is defined as the elapsed time since the solute or tracer of interest entered the system. The initial age is assumed to be zero or some other initial value, and in this sense the age or residence time within the system is a relative quantity. Many authors have noted that the interpretation of age or residence time is complicated by the fact that age depends on the fluid pathway, the physical and chemical interactions along the path, and the forcing or inputs to the catchment (Duffy 2010).

In this article, the goal is to explore a strategy for estimating the relative age of DOC in lake-catchments and to formulate the model based on what we refer to as the “Essential Terrestrial Variables” (ETVs) (e.g., catchment and lake areas, depth, soil properties, drainage density, land use and cover, climate, etc.), which represent the essential information required to implement the modeling scheme.

Lake-catchment setting

We first consider the general problem of partitioning the important physical connections in the lake-catchment...
system. Figure 1 illustrates the partitioning based on area-edge connections that puts the spatial complexity of the model in the hands of the user. The catchment storage elements (Fig. 1a) are hillslopes connected to lake edges and/or hillslopes connected to stream segments. The hillslopes represent stores of water and solutes integrated over the relevant contributing sub-areas, and then routed from the sub-area to the lake or streams. The strategy is similar to the hydrologic response unit (HRU) approach used in the SWAT model (Gassman et al. 2007). The hillslope allows estimation of surface runoff and groundwater baseflow to supply each hillslope-stream or hillslope-lake element. Our goal in this article is to develop a simple approach, while still representing the important features of the water and carbon problem described above, and which could be easily extended for more complex cases. The general point is that the age modeling approach based on hillslopes represents an extensible model element that can be adapted to existing codes with multiple interacting compartments, and where each hillslope represents the average properties contributing to a sub-domain in the catchment. Figure 1b,c illustrates the lake and hillslope elements, and Table 1 provides the required data needed to implement the model. Lake Mendota, located in south-central Wisconsin, and its catchment are our study site.

A model for solute age

Given rudimentary data for the catchment and lake water balance and DOC, the distribution of DOC concentration through time and the distribution of DOC age through time can be evaluated in the same way that McKendrick (1926) and von Foerster (1959b) approached population theory. First, assume the concentration distribution \( C(t, \tau) \) depends on both time and age. von Foerster showed that the underlying model depends on time, age, sources, and sinks:

\[
\frac{\partial C}{\partial t} + \frac{\partial C}{\partial \tau} = \sum I_i - \sum O_j
\]

where \( I \) and \( O \) are the various solute inputs and outputs including chemical reactions. Delhez et al. (1999) made an important advance by showing that the joint distribution function \( C(t, \tau) \) in Eq. 1 could be integrated over the age variable \( \tau \) to form the model in terms of the classical moments of the distribution \( C(t, \tau) \), that is:

\[
\begin{align*}
\frac{\partial \mu_0}{\partial t} & = k_1 \mu_0^1 - k_2 \mu_0 \\
\frac{\partial \mu_1}{\partial t} & = \mu_0 - k_2 \mu_1 \\
\text{Population Age} & = A(t) = \frac{\mu_1(t)}{\mu_0(t)}
\end{align*}
\]

where the \( 0^{th} \) and \( 1^{st} \) moments of \( C(t, \tau) \) with respect to \( \tau \) are \( \mu_0 \) and \( \mu_1 \), and \( k_1 \mu_0 \) and \( k_2 \mu_0 \) are first-order production rates (sources) and losses (sinks) with rate constants \( k_1 \) and \( k_2 \). The \( 0^{th} \) moment is the usual concentration transport equation and the \( 1^{st} \) moment is an auxiliary equation that depends on the concentration. The last term in Eq. 2 formally defines the age as the ratio of the moments \( \mu_1 \) and \( \mu_0 \) (see Delhez et al. 1999; Duffy 2010 for details). The important point to consider here is that Eq. 2 does not require that we know the explicit form of the function \( C(t, \tau) \), only that it exists and has moment equations (Eq. 2) representing the physical system to be solved. An example is given next.
Table 1. The ETVs for the Mendota lake-catchment. Mapping to variables in 1.8 provided in the 2nd column. Unless specified otherwise, data are from the North Temperate Lakes Long Term Ecological Research Program.

| Variable | Application to Eq. 8 | Scenario 1 | Scenario 2 | Scenario 3 |
|----------|-----------------------|------------|------------|------------|
| **Climate and hydrology** | | | | |
| Mean annual precipitation (MaP) | | 0.00262 m d^{-1} | 0.00385 | 0.00159 |
| Mean annual potential evaporation (MaPE) | | 0.00239 m d^{-1} | 0.00239 | 0.00239 |
| Mean annual evaporation | | 0.00177 m d^{-1} | 0.00204 | 0.00133 |
| Outflow | | Calculated | Calculated | Calculated |
| Mean annual temperature (MaT) | Used for potential evapotranspiration (PET) | 6.7°C | N/A | N/A |
| Min-max daily temperatures (MaR) | Used for PET | -20.6°C, 30.6°C | N/A | N/A |
| Mean annual runoff (MaO) | | 0.000418 m d^{-1} | — | — |
| Runoff ratio (Rr) | | — | 0.16 | — |
| Volumetric outflow | | — | 2 | 2 |
| Precipitation concentration of organic carbon | | 2 g m^{-3} | 2 | 2 |
| Evaporation concentration of organic carbon | | 0 g m^{-3} | 0 | 0 |
| **Drainage basin characteristics** | | | | |
| Stream type | General context | Perennial | Perennial | Perennial |
| Location | General context | 43.0878, -89.4301 | 43.0878, -89.4301 | 43.0878, -89.4301 |
| Drainage area | A1 | 604 km² | 604 | 604 |
| Lake area | A2 | 39.6 km² | 39.6 | 39.6 |
| Lake mean depth | d2 | 12.8 m | 12.8 | 12.8 |
| **Soil, regolith, and bedrock properties** | | | | |
| Soil depth | Used as part of the overall regolith thickness | 2 m | N/A | N/A |
| Depth to bedrock | d1 | 5 m | 5 | 5 |
| Regolith-soil hydraulic conductivity (Ks) | 1.6-6.0 m d^{-1} | N/A | N/A |
| Soil macro-porosity (ns) | n = ns | 0.1-0.4 m m^{-1} | 0.1 | 0.1 |
| Soil type (So) | General context | Silt-loam, sandy soil | N/A | N/A |
| Labile organic carbon soil pool | s | 100 g m^{-3} (assumed) | 100 | 100 |
| Rate constant for desorption of soil labile carbon | ks | 0.001 d^{-1} | 0.001 | 0.001 |
| Constant of recalcitrant carbon fraction | m | 0.1 | 0.1 | 0.1 |
| **Ecology** | | | | |
| Biome | General context | Oak Savannah | Oak Savannah | Oak Savannah |
| NLCD land cover | General context | Agricultural crop/developed | — | — |
| Rate constant for soil carbon respiration | k1 | 0.0004 d^{-1} | 0.0004 | 0.0004 |
| Rate constant for lake carbon respiration | k2 | 0.0008 d^{-1} | 0.0008 | 0.0008 |
| Rate constant for lake carbon burial | kb | 0.0003 d^{-1} (Hanson et al. 2014a) | 0.0003 | 0.0003 |

N/A = Not Applicable.
The age modeling scheme: Steady-state flow in a well-mixed lake example

In order to demonstrate how “age” is simulated, consider a simple well-mixed lake setting with the volumetric water balance

$$\frac{dV}{dt} = Q^p - Q^e + Q^I - Q$$

and steady-state or time-average form:

$$Q^p - Q^e + Q^I - Q = 0$$

where the flows are partitioned into volumetric precipitation $Q^p$, net evaporation $Q^e$, volumetric surface inflow $Q^I$, and outflow $Q$. The lake outflow $Q$ is further defined as a function of the volume of lake storage $[Q = f(V)]$.

We define the “age” as the elapsed time since the tracer, isotope, or solute has entered the system as input. In other words, the sample is assumed to have zero age upon entering the lake. Now consider an experimental study where we wish to establish the age and/or the residence time of a conservative tracer in a single well-mixed reservoir. Assuming the water balance is steady (Eq. 4), the concentration-age conservative tracer in a single well-mixed reservoir. Assuming the water balance is steady (Eq. 4), the concentration-age model following (Eq. 2) has the form:

$$\frac{dC}{dt} = k(C^2 - C)$$
$$\frac{ds}{dt} = C - kx$$
$$k = \frac{Q}{V}$$
$$\text{Age}(t) = \frac{x(t)}{C(t)}$$

where the input is $C^i$, the output is $C$ and $k^{-1}$ is a time constant. A linear relation is assumed for lake outflow-excess storage relation, $Q = k(V - V_0)$, where $V_0$ is the residual pool storage. Following Eq. 2, $z$ is the 1st moment of the system and the Age is determined by the ratio of the solutions for $z$ and $C$. The steady-state, or the time averaged flows, are given by Eq. 4, and defined as volumetric lake precipitation, catchment inflows to the lake, and lake evapotranspiration, respectively. It is useful to examine the steady-state water and solute balance for Eqs. 2–5:

$$Q = Q^p - Q^e + Q^I$$
$$\bar{C} = \frac{Q^p}{Q} \bar{C}^p - \frac{Q^e}{Q} \bar{C}^e + \frac{Q^I}{Q} \bar{C}^I$$
$$\bar{x} = \frac{\bar{C}}{k}$$
$$\text{Age} = \frac{\bar{x}}{\bar{C}} = \frac{V}{Q^p - Q^e + Q^I} = k^{-1}$$

From Eq. 6, we can see that for a nonreactive tracer the steady-state age computed from the water balance ($\text{Age} = V/Q$) and age computed from the ratio of solute moments are identical ($\text{Age} = \bar{x}/\bar{C}$). In addition, Eq. 5 shows that there is no need to know the form of the age or residence time distribution as is usually done (Nauman 1969; Kazemi et al. 2006), since we can calculate it directly from the model results, $C(t)$ and $x(t)$ or $\bar{C}$ and $\bar{x}$. Next, we examine the dynamic solution of the model for constant inputs from a given initial condition.

Figure 2a illustrates the concentration-time history for a range of initial conditions and input concentrations. Figure 2b illustrates the time evolution of solute Age(t) for a range of initial concentrations, $C_0$. Figure 2c illustrates the effect of varying the initial Age$(t = 0)$ itself of the solute with other variables held constant. In Fig. 2d, we see the impact of the rate constant $k$ on the time evolution of Age(t) in the lake. The important point in Fig. 2d is that as $Q \rightarrow 0$, the Age(t) tends to a linear increasing function of time, or when the hydrology slows down (e.g., long drought intervals) the age of the solute becomes a simple clock. We explore the implications of these results for DOC in a lake-catchment system next.

The lake-catchment system for DOC

A general approach for the lake-catchment illustrated in Fig. 1a would include multiple sub-catchment hillslopes connected to streams and other hillslopes that contribute directly to the lake. However, for simplicity, we will develop the strategy for a single representative hillslope, or catchment mean hillslope, contributing to the lake. Extensions to more complex partitions are straightforward. The flow is assumed to be steady-state. The model, allowing for production and loss terms, has the form:

$$Q^p - Q^e - Q_1 = 0$$
$$Q^e - Q^e + Q^I + Q_2 = 0$$
$$\frac{dc_1}{dt} = \frac{Q^p}{nV_1} (c^i_1 - c_1) + \frac{Q^e}{nV_1} (c_1 - c_1 - m \cdot s) - k_1 c_1$$
$$\frac{dc_2}{dt} = k_1 (c_1 - m \cdot s)$$
$$\frac{dx_1}{dt} = c_1 - \frac{Q^p}{nV_1} x_1 + \frac{Q^e}{nV_1} (x_1 - k_1 x_1 - m \cdot \delta)$$
$$\frac{dx_2}{dt} = c_2 - \frac{Q^e}{V_2} x_2 + \frac{Q^p}{V_2} (x_2 - x_1) - k_2 x_2 - k_2 x_2$$
$$\frac{db}{dt} = m \cdot s + k_1 (x_1 - m \cdot \delta)$$

$$\text{Age}_1(t) = \frac{x_1(t)}{c_1(t)}$$
$$\text{Age}_2(t) = \frac{x_2(t)}{c_2(t)}$$
$$\text{Age}_s(t) = \frac{b(t)}{s(t)}$$
The variables are defined as follows, where concentration refers to a solute, such as DOC:

\[ Q_{p}^1 \] = volumetric precipitation = \([L^3T^{-1}]\)
\[ Q_{e}^1 \] = volumetric evapotranspiration = \([L^3T^{-1}]\)
\[ Q_{p}^{ec} \] = volumetric potential evapotranspiration = \([L^3T^{-1}]\)
\[ Q_{t}^1 \] = volumetric lake/catchment outflow = \([L^3T^{-1}]\)
\[ c_{l}^1 = \text{lake/catchment concentration} = [ML^{-3}] \]
\[ s = \text{equivalent concentration of labile carbon pool} = [ML^{-3}] \]
\[ c_{l}^{12} = \text{lake/catchment precipitation concentration} = [ML^{-3}] \]
\[ c_{f}^{12} = \text{lake/catchment evapotranspiration concentration} = [ML^{-3}] \]
\[ x_{l}^1 = \text{lake/catchment age concentration} = [MTL^{-3}] \]
\[ \beta = \text{catchment carbon pool age concentration} = [MTL^{-3}] \]
\[ n = \text{soil porosity} = [L^3L^{-3}] \]
\[ V_1 = n \cdot d_1 \cdot A_1 = \text{catchment storage volume} = [L^3] \]
\[ V_2 = d_2 \cdot A_2 = \text{lake storage volume} = [L^3] \]
\[ d_1 = \text{catchment average storage depth} = [L] \]
\[ d_2 = \text{lake average depth} = [L] \]
\[ A_1 = \text{catchment area} = ([L]^2) \]
\[ A_2 = \text{lake area} = [L^2] \]
\[ k_l = \text{rate constant for desorption of labile carbon} = [T^{-1}] \]
\[ k_{r,2} = \text{rate constant for lake/catchment carbon respiration} = [T^{-1}] \]
\[ k_{b,2} = \text{rate constant for lake carbon burial} = [T^{-1}] \]
\[ m = \text{constant for recalcitrant carbon fraction} = [MT^{-1}] \]

The above system is solved in Mathematica and R.

**Application: The age of DOC in the Mendota lake-catchment**

We develop three scenarios of the Lake Mendota, Wisconsin, catchment to demonstrate how changes in climate and land use might alter the age structure of DOC in the catchment and in the lake. Lake Mendota (39.6 km² and its watershed (drainage area 604 km²), are well described by Lathrop and Carpenter (2013), and the lake and catchment characteristics directly relevant to the model are summarized in Table 1. Lake Mendota is a relatively large eutrophic lake, with a mean hydrologic residence time estimated to be \( \sim 4 \) yr, and moderate total organic carbon concentration of about \( 6 \) mg L\(^{-1}\). Once predominately oak savannah, Lake Mendota’s catchment is now 67% agricultural lands and 22% developed (2011 National Land Cover Database). The lake is fed by a network of streams that flow into the western and northern ends of the lake.

Three hypothetical scenarios for the evolution of DOC concentration and DOC age in the Mendota lake-catchment are defined in Table 1 and summarized in Fig. 3. Each scenario represents an abrupt transition from undeveloped historical land cover (e.g., oak Savanna biome) that is converted to agricultural land use at \( t = 0 \) (140 yr BP). Scenario 1 (Fig. 3a) represents the evolution of DOC concentration and DOC age under historical hydroclimatic conditions and an abrupt change in land use \( (t = 0) \) where the initial conditions represent predevelopment. Scenario 2 (Fig. 3b) represents the same land transition but with an increased precipitation climate regime. Scenario 3 (Fig. 3c) represents the same land transition but with a decreased precipitation climate regime. We assume the initial soil pool of labile DOC is subject to a desorption process. The model allows for 1st order DOC respiration in the catchment and the lake and 1st order DOC burial in the lake. Typical published values are used for the rate constants (see Table 1). A limited calibration was made by trial and error to estimate the initial DOC pool, while catchment parameters were provided by the North Temperate Lakes Long Term Ecological Research Program. It is important to note that the model does not estimate the mass and mass flux changes in the recalcitrant carbon pool, which typically occur in such disturbances (Hobbie and Likens 1973). However, this component can be included where details of the carbon pool and the DOC leaching coefficients are available. Here, we focus on the hydrologic components of the system.

In Scenario 1 (Fig. 3a), recent past climatic conditions for the lake and catchment lead to estimates of the DOC and DOC age for the lake inflow loading (catchment outflow \( Q_{e1} \)) that roughly match the observed data (DOC \( \sim 8 \) mg L\(^{-1}\) and \( Q_{e1} = 0.000418 \) md\(^{-1}\)). The steady-state concentration of lake DOC approximates the observed lake concentration (DOC \( \sim 4.5 \) mg L\(^{-1}\)), and the age of lake DOC in the scenario also is similar to what has been estimated in the literature (Hanson et al. 2014b). In Scenario 2 (accelerated hydrology, Fig. 3b), the precipitation is increased, evapotranspiration increased only slightly, and the mean runoff increased significantly. All other parameters remain the same as in Scenario 1. This scenario represents conditions, for example, where deforestation and agricultural development has led to an increase in the effective precipitation and runoff consistent with observed deforestation impacts in the upper mid-west. In the case of deforestation, canopy interception decreases dramatically, and increased ground heating (increased albedo) can lead to local convective thunderstorms. The resulting increase in effective precipitation has the effect of accelerating the hydrologic cycle relative to Scenario 1. In Scenario 3 (decelerated hydrology), the precipitation is decreased as compared to Scenario 1, and actual evapotranspiration is decreased over Scenario 1 such that \( Q_{p}^1 = Q_{e1} \). This scenario illustrates the effect of drought or long term dry conditions on the carbon pool.
**Results/discussion**

The three scenarios in Fig. 3 provide a plausible story of how historical land use change may be influenced by changing climate conditions. The steady-state age equations for the catchment and lake DOC, \(s_1\), \(s_2\) and \(s_s\), are also useful functions for interpretation and are shown in Eq. 9:

\[
\begin{align*}
\bar{\tau}_1 &= \frac{2}{\frac{Q_1}{nV_1} - \frac{Q_i}{nV_1} + k_{11}} \\
\bar{\tau}_2 &= \frac{1}{\frac{Q_2}{V_2} - \frac{Q_i}{V_2} + k_{12} + k_{02}} + \frac{2\frac{Q_1}{V_2}}{n \cdot d_1 \left( \frac{\frac{Q_1}{nV_1} + \frac{Q_i}{nV_1} + k_{11}}{nV_1} \right) + \frac{Q_1}{V_2} \left( \frac{\frac{Q_1}{nV_1} + \frac{Q_i}{nV_1} + k_{11}}{nV_1} \right)} \\
\bar{\tau}_s &= \frac{1}{k_s} + \frac{2}{\frac{Q_1}{nV_1} - \frac{Q_i}{nV_1} + k_{11}}
\end{align*}
\]

The effects of abrupt land use change in Scenario 1 with historical precipitation observations (Fig. 3a) clearly leads to a reduction in the DOC pool, which we can expect may evolve over many decades. The age of DOC in the catchment and lake outflow respond quickly to the land use change but then gradually adjust over the 140 yr simulation. The wet conditions of Scenario 2 (Fig. 3b), with the same land use change, but with increased effective precipitation, has the effect of significantly reducing DOC in the catchment, along with a rapid reduction in the labile DOC pool, although the lake DOC increases only slightly above Scenario 1. The ages of catchment and lake DOC are reduced (48% and 44%, respectively) in proportion to the increase in runoff, \(\bar{\tau} = k^{-1} = V/Q\). In Scenario 3, drier conditions led to increased DOC concentrations in the catchment and decreased DOC in the lake, relative to Scenario 1. The age of the catchment and lake outflows showed significant increases (56% and 49%, respectively). The large increase in the residence time in the lake had the effect of reducing DOC in the lake through respiration and burial.

Figure 4 illustrates the water balance for the three scenarios by putting the results in a global framework based on the...
Budyko water-climate paradigm (Dooge 1992). The so-called Budyko curve has been a very successful model of long term mean catchment hydrologic conditions, since it captures the effects of water limited arid and semi-arid catchments on a continuum with energy limited humid catchments, as well as the relation to major biomes. Figure 4 and Eq. 10 show one form of the Budyko curve from Pike and Turc (Dooge 1992).

\[
\frac{Q^e}{Q^{pe}} = \frac{Q^p/Q^{pe}}{[1+(Q^p/Q^{pe})^n]^{1/n}} \tag{10}
\]

In Fig. 4, the exponent is \( n = 2 \) as suggested by Dooge (1992) while the dashed lines provide a measure of uncertainty \( (n = 1.5 \text{ and } 2.5) \). The ratio of precipitation to potential evapotranspiration \( Q^p/Q^{pe} \) is the humidity index, and \( Q^p/Q^{pe} \) is the ratio of actual evapotranspiration to potential evapotranspiration. The Lake Mendota catchment (Scenario 1) is located slightly to the right of the water limited–energy limited divide, with sub-humid somewhat dry conditions, and where precipitation slightly exceeds potential evapotranspiration, and typical of a dry savanna-forest biome. Equation 10 is sometimes written in terms of the aridity index \( Q^{pe}/Q^p \) or the inverse of the humidity index.

Scenario 2, with increasing precipitation relative to \( Q^p \), shows how a change in precipitation would tend to favor a more humid forest biome, while Scenario 3 with \( Q^p \sim Q^e \) would tend to move the biome toward a more semi-arid...
setting. We also note that relative to Scenario 1, the wetter Scenario 2 has the effect of greatly decreasing the residence time and age of DOC in the Mendota lake-catchment; while Scenario 3 has the effect of greatly increasing the residence time. Clearly, age is a diagnostic property of lake-catchment systems.

The use of simplified models for lake-catchment DOC that include the concept of DOC age can provide a quantitative measure of land use and climate change and seems to be a promising tool for improving our understanding of lake-catchment ecohydrologic systems, while offering new possibilities for managing ecosystem services.

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