Involuntary unemployment in overlapping generations model due to instability of the economy and fiscal policy

Yasuhito Tanaka

Faculty of Economics, Doshisha University, Kyoto, Japan

Abstract

The existence of involuntary unemployment advocated by J. M. Keynes is a very important problem of the modern economic theory. Using a three-generations overlapping generations model, we show that the existence of involuntary unemployment is due to the instability of the economy. Instability of the economy is the instability of the difference equation about the equilibrium price around the full-employment equilibrium, which means that a fall in the nominal wage rate caused by the presence of involuntary unemployment further reduces employment. This instability is due to the negative real balance effect that occurs when consumers’ net savings (the difference between savings and pensions) are smaller than their debt multiplied by the marginal propensity to consume from childhood consumption.

Key Words: overlapping generations model, involuntary unemployment, instability of the economy, negative real balance effect

JEL Classification: E12, E24.

1. Introduction

The existence of involuntary unemployment advocated by J. M. Keynes is a very important problem of the modern economic theory. It is a phenomenon that workers are willing to work at the market wage or just below but are prevented by factors beyond their control, mainly, deficiency of aggregate demand. In traditional Keynesian macroeconomics, the rigidity of the nominal wage rate was thought to be the cause of involuntary unemployment. The efficiency wage hypothesis is the most famous theory that provides a microeconomic basis for the existence of involuntary unemployment due to the rigidity of the nominal wage rate. The main references are Akerlof and Yellen (1986), Katz (1986), Shapiro and Stiglitz (1984) and Yellen (1984). According to the efficiency wage hypothesis, workers may be more diligent if the wage they are earning is higher than the wage determined by the market, fearing that they will be fired for neglecting their work. Companies that are unable to perfectly monitor workers' laziness will pay higher wages than the market price, which will increase their own incentives to work. Firms will not reduce wages to the level where the labor market is in equilibrium. As wages remain high, rationing of jobs occurs and unemployment occurs. At this time, even if the unemployed person offers to work at a lower wage, no company will
accept it for the reasons mentioned above. In addition, since the wage level set by a company is set relative to other companies, no one company will set a wage that is outstanding, and wages will converge to relatively the same level. In other words, under the efficient wage hypothesis, wages and prices will remain relatively stable despite the existence of unemployment in the labor market. Other theories include the insider-outsider theory, which assumes that the labor market is composed of two types of workers: employed workers (insiders) and unemployed workers (outsiders). Please see Blanchard and Summers (1986), (1987) and Lindbeck and Snower (1986), (1987).

Umada (1997), without assuming wage rigidity, derived an upward-sloping labor demand curve from the mark-up principle for firms, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity¹. But his model of firm behavior is ad-hoc. Otaki (2009) assumes indivisibility of labor supply, and has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and R. M. Solow (1981). The arguments of this paper do not depend on bargaining. As discussed by Otaki (2012) and Otaki (2015) (Theorem 2.3), if labor supply is divisible and very small, no unemployment exists. However, we show that even if labor supply is divisible, there may exist involuntary unemployment. Tanaka (2020a) and (2020b) analyzed involuntary unemployment under indivisible labor supply using a model similar to the one in this paper.

If labor supply is indivisible, it may be 1 or 0. On the other hand, in contrast if it is divisible, it takes a real value between 0 and 1. About indivisible labor supply also please see Hansen (1985). Hansen (1985) studies the existence of unemployed workers and fluctuations in the rate of unemployment over the business cycle with indivisible labor supply. To treat an indivisible labor supply in a representative agent model he assumes that people choose lotteries rather than hours worked. Each person chooses a probability of working, then a lottery determines whether or not he actually works. There is a contract between firms and individuals that commits the individual to work the predetermined number of hours with the probability which is chosen by an individual. The contract is being traded, so the individual is paid whether he works or not. The firm provides complete unemployment insurance to the workers.

In this paper we consider consumers’ utility maximization and firms’ profit maximization in an overlapping generations (OLG) model under monopolistic competition according to Otaki (2007), (2009), (2012) and (2015). We extend Otaki’s model to a three-generations OLG model with a childhood period and pay-as-you-go pension system for the older generation consumers. We show that the existence of involuntary unemployment is due to the instability of the economy. Instability of the economy is the instability of the difference equation about the equilibrium price around the full-employment equilibrium, which means that a fall in the nominal wage rate caused by the presence of involuntary unemployment reduces employment. In the next section we explain the model and show the existence of involuntary unemployment when aggregate demand is insufficient. In Section 3 we will show the following results.

¹ Lavoie (2001) presented a similar analysis.
1. If the net savings (the difference between savings and pensions) is greater than debts (due to consumption in childhood period) of consumers, then the positive real balance effect kicks in, and involuntary unemployment will spontaneously dissipate because the decline in nominal wages and prices due to unemployment reduces unemployment.

2. If the net savings is smaller than debts of consumers, then the negative real balance effect kicks in, and involuntary unemployment does not spontaneously dissipate because the decline in the nominal wage and prices due to unemployment further increases unemployment.

In Section 3 we present discussions about fiscal policy in the presence of involuntary unemployment to realize full-employment. We show that the extra government expenditure to realize full-employment should be financed by seigniorage not by government debt.

2. The model and analysis

2.1 Consumers’ utility maximization

We consider a three-periods (0: childhood, 1: younger or working, and 2: older or retired) OLG model under monopolistic competition. It is a re-arrangement and an extension of the model put forth by Otaki (2007), (2009), and (2015). The structure of our model is as follows.

1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by \( z \in [0,1] \). Good \( z \) is monopolistically produced by firm \( z \) with constant returns to scale technology.

2. Consumers consume the goods during the childhood period (Period 0). This consumption is covered by borrowing money from (employed) consumers of the younger generation and/or scholarships. They must repay these debts in their Period 1. However, unemployed consumers cannot repay their own debts. Therefore, we assume that unemployed consumers receive unemployment benefits from the government, which are covered by taxes on employed consumers of the younger generation.

3. During Period 1, consumers supply \( l \) units of labor, repay the debts and save money for their consumption in Period 2. They also pay taxes for the pay-as-you go pension system for the older generation.

4. During Period 2, consumers consume the goods using their savings carried over from their Period 1 earnings, and receive the pay-as-you go pension, which is a lump-sum payment. It is covered by taxes on employed consumers of the younger generation.

5. Consumers determine their consumptions in Periods 1 and 2 and the labor supply at the beginning of Period 1. We assume that their consumption during the childhood period is constant.
Further we make the following assumption.

**Ownership of the firms** Each consumer inherits ownership of the firms from the previous generation. Corporate profits are distributed equally to consumers.

**Zero interest rate** The interest rate will be determined so that the supply of funds from the savings of the younger generation plus government scholarships is equal to the consumption of the childhood generation, but without scholarships there is a large possibility that savings will be insufficient regardless of the interest rate, especially in the presence of a pay-as-you-go pension system. Since it is the scholarship that fills the gap, the interest rate can be controlled by determining the size of the scholarship. If the amount of scholarships is increased or decreased, or if they are made interest-bearing or interest-free, the interest rate will change, and this may change consumption. However, for example, a decline in the interest rate may increase consumption among the younger generation due to the substitution effect, but then consumption among the older generation will decline. The income effect is also ambiguous, since a fall in the interest rate reduces the debt associated with consumption in the childhood period, but lowers the value of savings. Therefore, the possibility that a change in the interest rate will significantly change aggregate demand is small, and it is not an important issue for the existence of involuntary unemployment, which is the theme of this paper. We assume here that the amount of the scholarship is determined so that the interest rate is zero. Repayment of the debts of consumers in their childhood period is assured. Consumers in the younger period are indifferent between lending money to childhood period consumers and savings by money.

**Notation** We use the following notation.

- $C^e_i$: consumption basket of an employed consumer in Period $i$, $i = 1, 2$.
- $C^u_i$: consumption basket of an unemployed consumer in Period $i$, $i = 1, 2$.
- $c^e_i(z)$: consumption of good $z$ of an employed consumer in Period $i$, $i = 1, 2$.
- $c^u_i(z)$: consumption of good $z$ of an unemployed consumer in Period $i$, $i = 1, 2$.
- $D$: consumption basket of an individual in the childhood period, which is constant.
- $P_i$: the price of consumption basket in Period $i$, $i = 1, 2$.
- $p_i(z)$: the price of good $z$ in Period $i$, $i = 1, 2$.
- $\rho = \frac{p_2}{p_1}$: (expected) inflation rate (plus one).
- $W$: nominal wage rate.
- $R$: unemployment benefit for an unemployed individual. $R = D$.
- $\bar{D}$: consumption basket in the childhood period of a next generation consumer.
- $Q$: pay-as-you-go pension for an individual of the older generation.
- $\Theta$: tax payment by an employed individual for the unemployment benefit.
- $\bar{Q}$: pay-as-you-go pension for an individual of the younger generation when he retires.
- $\Psi$: tax payment by an employed individual for the pay-as-you-go pension.

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\[ \Pi: \text{profits of firms which are equally distributed to each consumer.} \]
\[ l: \text{labour supply of an individual.} \]
\[ \Gamma(l): \text{disutility function of labor, which is increasing and convex.} \]
\[ L: \text{total employment.} \]
\[ L_f: \text{population of labor or employment in the full-employment state.} \]
\[ y: \text{labour productivity, which is constant.} \]

We assume that the population \( L_f \) is constant. We also assume that the nominal wage rate is constant in this section. We examine the effects of a change in the nominal wage rate in Section 3.

We consider a two-step method to solve utility maximization of consumers such that:

1. Employed and unemployed consumers maximize their utility by determining consumption baskets in Periods 1 and 2 given their income over two periods:
2. Then, they maximize their consumption baskets given the expenditure in each period.

Since the taxes for unemployed consumers’ unemployment benefits are paid by employed consumers of the same generation, \( D(= R) \) and \( \Theta \) satisfy \( D(L_f - L) = L\Theta \). It means

\[ L(D + \Theta) = L_f D. \]

The price index of the consumption basket in Period 0 is assumed to be 1. Thus, \( D \) is the real value of the consumption in the childhood period of consumers.

Also, since the taxes for the pay-as-you-go pension system are paid by employed consumers of younger generation, \( Q \) and \( \Psi \) satisfy the following relationship:

\[ L\Psi = L_f Q. \]

The utility function of employed consumers of one generation over three periods is

\[ u(C_i^e, C_i^u, D) - \Gamma(l). \]

We assume that \( u(\cdot) \) is a homothetic utility function. The utility function of unemployed consumers is

\[ u(C_i^u, C_i^e, D). \]

The consumption baskets of employed and unemployed consumers in Period \( i \) are

\[ C_i^e = \left( \int_0^1 c_i^e(z) \frac{\sigma - 1}{\sigma} dz \right)^{\frac{\sigma}{\sigma - 1}}, \]

and

\[ C_i^u = \left( \int_0^1 c_i^u(z) \frac{\sigma - 1}{\sigma} dz \right)^{\frac{\sigma}{\sigma - 1}}, \quad i = 1, 2. \]

\( \sigma \) is the elasticity of substitution among the goods, and \( \sigma > 1 \).

The price of consumption basket in Period \( i \) is

\[ p_i = \left( \int_0^1 p_i(z)^{1-\sigma} dz \right)^{\frac{1}{1-\sigma}}, \quad i = 1, 2. \]

The budget constraint for an employed consumer is

\[ P_1 C_i^e + P_2 C_i^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi. \]

The budget constraint for an unemployed consumer is

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Let
\[ P_1 C_1^u + P_2 C_2^u = \Pi - D + R + \hat{Q} = \Pi + \hat{Q} \] (since \( R = D \)).

Let
\[ \alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e}, \quad 1 - \alpha = \frac{P_2 C_2^u}{P_1 C_1^e + P_2 C_2^e}. \]

Since the utility functions \( u(C_1^e, C_2^e, D) \) and \( u(C_1^u, C_2^u, D) \) are homothetic, \( \alpha \) is determined by the relative price \( \frac{P_2}{P_1} \), and do not depend on the income of the consumers. Therefore, we have
\[ \alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_1 C_1^u}{P_1 C_1^e + P_2 C_2^u}. \]

and
\[ 1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_2 C_2^u}{P_1 C_1^e + P_2 C_2^u}. \]

From the first order conditions and the budget constraints for employed and unemployed consumers we obtain the following demand functions for consumption baskets.
\[ C_1^e = \alpha \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_1}, \]
\[ C_2^e = (1 - \alpha) \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_2}, \]
\[ C_1^u = \alpha \frac{\Pi + \hat{Q}}{P_1}, \]

and
\[ C_2^u = (1 - \alpha) \frac{\Pi + \hat{Q}}{P_2}. \]

Solving maximization problems in Step 2 by standard calculations (please see Appendix), the following demand functions of employed and unemployed consumers are derived.
\[ c_1^e(z) = \left( \frac{p_1(z)}{p_1} \right)^{-\sigma} \alpha \left( \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{p_1} \right), \]
\[ c_2^e(z) = \left( \frac{p_2(z)}{p_2} \right)^{-\sigma} (1 - \alpha) \left( \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{p_2} \right), \]
\[ c_1^u(z) = \left( \frac{p_1(z)}{p_1} \right)^{-\sigma} \left( \frac{\Pi + \hat{Q}}{p_1} \right), \]

and
\[ c_2^u(z) = \left( \frac{p_2(z)}{p_2} \right)^{-\sigma} (1 - \alpha) \left( \frac{\Pi + \hat{Q}}{p_2} \right). \]

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:
\[ V^e = u \left( \alpha \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{p_1}, (1 - \alpha) \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{p_2} \right) - \Gamma(t), \]

and
\[ V^u = u \left( \alpha \frac{\Pi + \hat{Q}}{p_1}, (1 - \alpha) \frac{\Pi + \hat{Q}}{p_2} \right). \]
Let \( \omega = \frac{W}{p_1}, \rho = \frac{p_2}{p_1} \). Then, since the real value of \( D \) in the childhood period is constant, we can write
\[
V^e = \varphi \left( \omega l + \frac{\Pi - D - \theta + \bar{Q} - \Psi}{p_1}, \rho \right) - \Gamma(l),
\]
and
\[
V^u = \varphi \left( \frac{\Pi + \bar{Q}}{p_1}, \rho \right).
\]
\( \omega \) is the real wage rate. Denote
\[
I = \omega l + \frac{\Pi - D - \theta + \bar{Q} - \Psi}{p_1}.
\]
The condition for maximization of \( V^e \) with respect to \( l \) given \( \rho \) is
\[
\frac{\partial \varphi}{\partial l} \omega - \Gamma'(l) = 0, \tag{1}
\]
where
\[
\frac{\partial \varphi}{\partial l} = \alpha \frac{\partial u}{\partial c_1^e} + (1 - \alpha) \frac{\partial u}{\partial c_1^u}.
\]
Given \( P_1 \) and \( \rho \) the labor supply is a function of \( \omega \). From (1) we get
\[
\frac{dl}{d\omega} = \frac{\frac{\partial \varphi}{\partial l} \frac{\partial^2 \varphi}{\partial c_1^e \partial c_1^l} \omega}{\Gamma'(l) \frac{\partial^2 \varphi}{\partial l^2}}.
\]
If \( \frac{dl}{d\omega} > 0 \), the labor supply is increasing with respect to the real wage rate \( \omega \). Labor supply \( l \) may depend on the employment \( L \). We assume that \( LL \) is increasing in \( L \).

### 2.2 Firms' profit maximization

Let \( d_1(z) \) be the total demand for good \( z \) by younger generation consumers in Period 1. Then,
\[
d_1(z) = \left( \frac{p_1(z)}{p_1} \right)^{-\sigma} \alpha \left( WL_l + L_f \Pi - L_f D + L_f \bar{Q} - L_f \bar{Q} \right) \frac{p_1}{p_1}. \tag{2}
\]
This is the sum of the demand of employed and unemployed consumers. Note that \( \bar{Q} \) is the pay-as-you-go pension for younger generation consumers in their Period 2. Similarly, their total demand for good \( z \) in Period 2 is written as
\[
d_2(z) = \left( \frac{p_2(z)}{p_2} \right)^{-\sigma} \left( 1 - \alpha \right) \left( WL_l + L_f \Pi - L_f D + L_f \bar{Q} - L_f \bar{Q} \right) \frac{p_1}{p_2}. \tag{3}
\]
Let \( d_2(z) \) be the demand for good \( z \) by the older generation. Then,
\[
d_2(z) = \left( \frac{p_1(z)}{p_1} \right)^{-\sigma} \left( 1 - \alpha \right) \left( WL_l + L_f \Pi - L_f D + L_f \bar{Q} - L_f \bar{Q} \right) \frac{p_1}{p_1}, \tag{4}
\]
where \( W, \Pi, L, \bar{L}, \bar{D} \) and \( \bar{Q} \) are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the debt of an individual, and the pay-as-you-go pension, respectively, during the previous period. \( \bar{\alpha} \) is the value of \( \alpha \) for the older generation. \( \bar{Q} \) is the pay-as-you-go pension for consumers of the older generation themselves. Let
\[
M = \left( 1 - \bar{\alpha} \right) \left( W \bar{L} l + L_f \Pi - L_f \bar{D} + L_f \bar{Q} - L_f \bar{Q} \right).
\]
This is the total savings or the total consumption of the older generation consumers.
including the pay-as-you-go pensions they receive in their Period 2. It is the planned consumption that is determined in Period 1 of the older generation consumers. Net savings is the difference between $M$ and the pay-as-you-go pensions in their Period 2, as follows:

$$\bar{M} = M - L_f Q.$$ 

Their demand for good $z$ is written as $p_{1(z)}^{-\sigma} \frac{M}{p_1}$. Government expenditure constitutes the national income as well as the consumptions of the younger and older generations. It is financed by the tax on the younger generation consumers. Then, the total demand for good $z$ is written as

$$d(z) = \left( \frac{p_{1(z)}}{p_1} \right)^{-\sigma} \frac{Y}{p_1},$$

where $Y$ is the effective demand defined by

$$Y = \alpha (W L_I + L_f \Pi - T - L_f D + L_f \bar{Q} - L_f Q) + G + L_f \bar{D} + M.$$ 

Note that $\bar{D}$ is consumption in the childhood period of a next generation consumer. $G$ is the government expenditure, except for the pay-as-you-go pensions, scholarships and unemployment benefits, and $T$ is the tax revenue for the government expenditure. See Otaki (2007) and Otaki (2015) about this demand function.

Let $L$ and $L_I$ be employment and the “employment × labor supply” of firm $z$. The output of firm $z$ is $L_l y$. At the equilibrium $L_l y = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial (L_l)} = y.$$ 

From (2) $\frac{\partial p_{1(z)}}{\partial (L_l)} = \frac{p_{1(z)}}{\sigma d(z)}$. Thus

$$\frac{\partial p_{1(z)}}{\partial (L_l)} = -\frac{p_{1(z)y}}{\sigma d(z)} = -\frac{p_{1(z)y}}{\sigma l_l y}.$$ 

The profit of firm $z$ is

$$\pi(z) = p_{1(z)} L_l y - LIW.$$ 

The condition for profit maximization is

$$\frac{\partial \pi(z)}{\partial (L_l)} = p_{1(z)} y - L_l y \frac{p_{1(z)y}}{\sigma l_l y} - W = p_{1(z)} y - \frac{p_{1(z)y}}{\sigma} - W = 0.$$ 

Therefore, we obtain

$$p_{1(z)} = \frac{1}{(1-\frac{1}{\sigma})y} W = \frac{1}{(1-\frac{1}{\mu})y} W, \quad \mu = \frac{1}{\sigma}.$$ 

This means that the real wage rate is

$$\omega = (1 - \mu)y.$$ 

Since all firms are symmetric,

$$p_1 = p_{1(z)} = \frac{1}{(1-\mu)y} W.$$ 

(3)

### 2.3 Involuntary unemployment due to instability of the economy

Consider an economy at Period $t$. The (nominal) aggregate supply of the goods is equal to

$$W^t L^t + L_f \Pi^t = P_1^t L^t L^t y.$$ 

The (nominal) aggregate demand is

$$\alpha (W^t L^t + L_f \Pi^t - T^t - L_f D^t + L_f \bar{Q}^t - L_f Q^t) + G^t + L_f \bar{D}^t + M^t.$$
\[ L_t^t = \alpha \left( P_t^t L_t^t y - T_t - L_f D_t^t + L_f \tilde{Q}_t^t - L_f Q_t^t \right) + G^t + L_f \tilde{D}_t^t + M^t. \]

The superscript \( t \) denotes variables at Period \( t \). Since the aggregate demand and supply are equal in the equilibrium,

\[ P_t^t L_t^t y = \alpha \left( P_t^t L_t^t y - T_t - L_f D_t^t + L_f \tilde{Q}_t^t - L_f Q_t^t \right) + G^t + L_f \tilde{D}_t^t + M_t. \]

We obtain \( L_t^t \) as follows:

\[ L_t^t = \frac{\alpha \left( -T_t - L_f D_t^t + L_f \tilde{Q}_t^t - L_f Q_t^t \right) + G^t + L_f \tilde{D}_t^t + M_t}{(1-\alpha) P_t^t y}. \]  

\[ (4) \]

\( L_t^t \) cannot be larger than \( L_f l(L_f) \), where \( l(L_f) \) is the labor supply at full-employment. However, it may be strictly smaller than \( L_f l(L_f) \). Then, we have \( L_t^t < L_f \) and involuntary unemployment exists. We assume balanced budget \( G_t = T_t \). In the full-employment equilibrium without excess demand \( L_t^t = L_f l(L_f) \), \( P_t^t + 1 = P_t^t \), \( \tilde{Q}_t^t = Q_t^t \), \( \tilde{D}_t^t = D_t^t \). Denote the variables in the full-employment equilibrium by a superscript \( * \). Then,

\[ L_f l(L_f) = \frac{\alpha \left( -G^* - L_f D^*_t + L_f \tilde{Q}^*_t - L_f Q^*_t \right) + G^* + L_f \tilde{D}^*_t + M^*}{(1-\alpha) P_t^* y}. \]

Let us denote the real values of \( G^* \), \( \tilde{D}^*_t \) and \( Q^*_t \), respectively, by \( g \), \( d \) and \( q \). We assume that the real values of these variables are maintained even if the prices change. Then,

\[ L_f l(L_f) = \frac{\left(1-\alpha\right) \left( P_t^* g + L_f P_t^* d \right) + M^*}{(1-\alpha) P_t^* y}. \]

This means

\[ M^* = \left(1-\alpha\right) P_t^* \left( L_f l(L_f) y - g - L_f d \right). \]

Suppose that when there exists involuntary unemployment, the nominal wage rate falls. Then, the prices of the goods also fall at the same rate because of the constant returns to scale according to (3). This relation is expressed by the following difference equation.

\[ P_t^{t+1} = \gamma \left( L_t^{t+1} - L_f l(L_f) \right) + P_t^t, \gamma > 0. \]

Let us denote

\[ P_t^{t+1} = f(P_t^t). \]

We assume \( f'(P_t^t) > 0 \). Since \( L_f l(L_f) \) is constant,

\[ f'(P_t^t) = \gamma \frac{\Delta L_t^t}{\Delta P_t^t} + 1. \]  

\[ (5) \]

According to Chap. 4 of Schreiber, Smith and Getz (2014) the stability condition for the full-employment equilibrium is

\[ f'(P_t^t) < 1 \] at \( P_t^t = P_t^* \).

The total savings or total consumption of the older generation \( M^t \) is not constant nor predetermined, but the net savings

\[ \tilde{M}^t = M^t - L_f P_t^t q \]

is predetermined. Also note that \( \tilde{Q}_t^t = P_t^{t+1} q \) in (4). From (4) with \( G_t = T_t = P_t^t g, Q_t = P_t^t q, \tilde{Q}_t^t = P_t^{t+1} q \) and \( \tilde{D}_t^t = P_t^t d \),

\[ f'(P_t^t) = \gamma \frac{L_f P_t^t q - a(-L_f D_t^t + L_f P_t^{t+1} q) - M_t}{(1-\alpha) (P_t^t)^2 y} + \gamma a \frac{L_f P_t^t f(P_t^t) q}{(1-\alpha) (P_t^t)^2 y} + 1, \]

where \( \frac{\Delta M^t}{\Delta P_t^t} = \frac{\Delta L_f P_t^t q}{\Delta P_t^t} = L_f q. \) At \( P_t^t = P_t^* \),

\[ f'(P_t^t) = \gamma \frac{(1-\alpha) L_f P_t^t q + a L_f P_t^t d - M^*}{(1-\alpha) (P_t^t)^2 y} + \gamma a \frac{L_f P_t^t f(P_t^t) q}{(1-\alpha) (P_t^t)^2 y} + 1, \]
Therefore,
\[ f'(P^*_1) = \frac{\gamma[(1-\alpha)L_FP_1q + \alpha_1L_FP_1^d - M^*]}{(1-\alpha)(P^*_1)^2y - \gamma_\alpha FP_1^d} = - \frac{\gamma(M^* - L_FP_1^dq - \alpha L_FP_1^d)}{(1-\alpha)(P_1)^2y - \gamma_\alpha L_FP_1^d} + 1. \]

Then, since \( f'(P^*_1) > 0 \), \( f'(P^*_1) < 1 \) is equivalent to
\[ M^* - L_FP_1^d - L_FP_1^d > 0. \]

On the other hand, in contrast \( f'(P^*_1) > 1 \) is equivalent to
\[ M^* - L_FP_1^d - L_FP_1^d < 0. \]

From (5), if \( f'(P^*_1) > 1 \), then \[ \frac{d^L_t}{dL^*_t} \bigg|_{L^*_1 = P^*_1} > 0, \]
which implies that a fall in the price of the goods decreases the employment, and involuntary unemployment won’t go away naturally. This and (6) mean that due to the fact that consumer debt multiplied by the marginal propensity to consume is greater than net savings, the negative real balance effect works.

We have shown the following results.

**Proposition 1**
1. If the net savings (the difference between savings and pensions) is greater than debts (due to consumption in childhood period) of consumers, then the positive real balance effect kicks in, and involuntary unemployment will spontaneously dissipate because the decline in nominal wages and prices due to unemployment reduces unemployment.
2. If the net savings is smaller than debts of consumers, then the negative real balance effect kicks in, and involuntary unemployment does not spontaneously dissipate because the decline in the nominal wage and prices due to unemployment further increases unemployment.

In this paper, we assume that the production of goods is done only by labor, so there is no capital and no investment, and debt arises only from consumption in the childhood period, but a more general model in which the production of goods is done by capital and labor would allow us to deal with cases in which firms or capitalists have debt. This is an issue for the future.

**3. Fiscal policy by seigniorage to decrease involuntary unemployment**

In this section we consider fiscal policies financed by seigniorage to decrease unemployment in a state with involuntary unemployment.

Assume that in Period \( t \), \( L^tL^t \) expressed by the following equation;
\[ L^tL^t = \frac{(1-\alpha)L_FP_1^d - \alpha L_FP_1^d + M^*}{(1-\alpha)L_FP_1^d}. \]
is smaller than \( L_l(L) \) because \( G^t \) or \( M^t \) is insufficient. The savings of younger generation consumers in this period is
\[ M^{t+1} = (1 - \alpha)(P^*_1L^tL^t - T^t - P^*_1L_d). \]

With \( T^t = T^* \) and \( P^*_1 = P^*_1 \),

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or with \( \frac{T^t}{p_1^t} = \frac{T^*}{p_1^*} \)

\[ M^{t+1} < M^* \]

because \( Lf^{t+1} < L_f L_f \).

Suppose that in Period \( t + 1 \) the full-employment equilibrium is realized by the government expenditure \( G^{t+1} \). We assume \( P_1^{t+1} = P_1^t \). Then,

\[ L_f L_f = \frac{(1-\alpha)L_f P_1^{t+1}d - \alpha G^{t+1} + M^{t+1}}{(1-\alpha)P_1^{t+1}y} \]

We assume

\[ \frac{T^{t+1}}{p_1^{t+1}} = \frac{T^t}{p_1^t}. \]

Then,

\[ L_f L_f = \frac{(1-\alpha)L_f P_1^{t+1}d - \alpha G^{t+1} + M^{t+1}}{(1-\alpha)P_1^{t+1}y} = \frac{(1-\alpha)L_f P_1^{t+1}d - \alpha G^{t+1} + M^{t+1}}{(1-\alpha)P_1^{t+1}y} \]

(1)

In the steady state full-employment equilibrium with balanced budget, we have

\[ L_f L_f = \frac{(1-\alpha)L_f P_1^{t+1}d - \alpha G^{t+1} + M^{t+1}}{(1-\alpha)P_1^{t+1}y} = \frac{(1-\alpha)L_f P_1^{t+1}d - \alpha G^{t+1} + M^{t+1}}{(1-\alpha)P_1^{t+1}y} \]

(2)

If \( P_1^{t+1} = P_1^t = P_1^* \) and \( T^t = T^* = G^* \), from (1), (2) and \( M^{t+1} < M^* \), we obtain \( G^{t+1} > G^* \).

If \( P_1^{t+1} = P_1^t \neq P_1^* \) and \( \frac{T^t}{p_1^t} = \frac{T^*}{p_1^*} = \frac{G^*}{p_1^*} \), then (1), (2) and \( \frac{M^{t+1}}{p_1^t} < \frac{M^*}{p_1^*} \) also mean \( G^{t+1} > G^* \). Therefore, we need budget deficit to realize full-employment in Period \( t + 1 \). The savings of the younger generation consumers in Period \( t + 1 \) is

\[ M^{t+2} = (1 - \alpha)(P_1^{t+1}L_f - T^{t+1} - P_1^{t+1}L_f d) = (1 - \alpha)P_1^{t+1}(L_f L_f - \frac{T^{t+1}}{p_1^{t+1}} - L_f d) \]

This means

\[ \frac{M^{t+2}}{p_1^{t+2}} = \frac{M^*}{p_1^*} \]

(3)

After realizing full-employment in Period \( t + 1 \), to maintain full-employment in Period \( t + 2 \) with \( P_1^{t+2} = P_1^{t+1} = P_1^* \) we need

\[ L_f L_f = \frac{(1-\alpha)L_f P_1^{t+1}d - \alpha G^{t+2} + M^{t+2}}{(1-\alpha)P_1^{t+1}y} \]

From (3) we find \( G^{t+2} = G^* \). Thus, after realizing full-employment in Period \( t + 1 \), we need balanced budget to maintain full-employment in Period 2. Therefore, the extra government expenditure in Period \( t + 1 \) to realize full-employment should be financed by seigniorage not by government debt.

Next, assume that \( P_1^{t+1} < P_1^t \), \( \frac{T^t}{p_1^t} = \frac{T^*}{p_1^*} = \frac{G^*}{p_1^*} \) and \( \frac{T^{t+1}}{p_1^{t+1}} = \frac{T^*}{p_1^*} \). Then, (1) is
\[
L_{lf} = \frac{(1-\alpha)L_{lf}d-\alpha^{\tau^+}\frac{c^{\tau+1}}{p_{\tau+1}^4}M^{\tau+1}}{(1-\alpha)y}
\]

From (2) and (4) we find that

4. Conclusion

In this paper we studied the problem of the existence of involuntary unemployment due to instability of the economy. Instability of the economy is the instability of the difference equation about the equilibrium price around the full-employment equilibrium, which means that a fall in the nominal wage rate caused by the presence of involuntary unemployment further reduces employment. This instability is due to the negative real balance effect.

Appendix: Calculations of Step 2 of consumers’ utility maximization

Lagrange functions in the second step for employed and unemployed consumers are

\[
\mathcal{L}_1^e = \left(\int_0^1 c_1^e(z) \frac{\sigma^{-1}}{\sigma} dz\right)^{\frac{-1}{\sigma-1}} - \lambda_1^e \left[\int_0^1 p_1(z)c_1^e(z)dz - \alpha(Wl + \Pi - D - \Theta + \hat{\gamma} - \Psi)\right],
\]

\[
\mathcal{L}_2^e = \left(\int_0^1 c_2^e(z) \frac{\sigma^{-1}}{\sigma} dz\right)^{\frac{-1}{\sigma-1}} - \lambda_2^e \left[\int_0^1 p_2(z)c_2^e(z)dz - (1 - \alpha)(Wl + \Pi - D - \Theta + \hat{\gamma} - \Psi)\right],
\]

\[
\mathcal{L}_1^u = \left(\int_0^1 c_1^u(z) \frac{\sigma^{-1}}{\sigma} dz\right)^{\frac{-1}{\sigma-1}} - \lambda_1^u \left[\int_0^1 p_1(z)c_1^u(z)dz - \alpha(\Pi + \hat{\gamma})\right],
\]

and

\[
\mathcal{L}_2^u = \left(\int_0^1 c_2^u(z) \frac{\sigma^{-1}}{\sigma} dz\right)^{\frac{-1}{\sigma-1}} - \lambda_2^u \left[\int_0^1 p_2(z)c_2^u(z)dz - \alpha(\Pi + \hat{\gamma})\right],
\]

\(\lambda_1^e, \lambda_2^e, \lambda_1^u, \lambda_2^u\) are Lagrange multipliers.

The first order condition for (A.1) is

\[
\left(\int_0^1 c_1^e(z) \frac{\sigma^{-1}}{\sigma} dz\right)^{\frac{-1}{\sigma-1}} c_1^e(z)^{\frac{1}{\sigma-1}} - \lambda_1^e p_1(z) = 0.
\]

From this

\[
\left(\int_0^1 c_1^e(z) \frac{\sigma^{-1}}{\sigma} dz\right)^{-1} c_1^e(z)^{\frac{-1}{\sigma-1}} = (\lambda_1^e)^{1-\sigma} p_1(z)^{1-\sigma}.
\]

Then,

\[
\left(\int_0^1 c_1^e(z) \frac{\sigma^{-1}}{\sigma} dz\right)^{-1} \int_0^1 c_1^e(z)^{\frac{-1}{\sigma-1}} dz = (\lambda_1^e)^{1-\sigma} \int_0^1 p_1(z)^{1-\sigma} dz = 1,
\]

It means
\[ \lambda_1^e \left( \int_0^1 p_1(z)^{1-\sigma} dz \right)^{1/\sigma} = 1, \]

and so

\[ p_1 = \frac{1}{\lambda_1^e} \]

From (A.2)

\[ \left( \int_0^1 c^e_1(z) \frac{\sigma-1}{\sigma} dz \right)^{\frac{1}{\sigma-1}} c^e_1(z) \frac{\sigma-1}{\sigma} = \lambda_1^e p_1(z)c^e_1(z). \]

Then,

\[ \left( \int_0^1 c^e_1(z) \frac{\sigma-1}{\sigma} dz \right)^{\frac{1}{\sigma-1}} \int_0^1 c^e_1(z) \frac{\sigma-1}{\sigma} dz = \left( \int_0^1 c^e_1(z) \frac{\sigma-1}{\sigma} dz \right)^{\frac{\sigma}{\sigma-1}} \]

\[ = c^e_1 = \lambda_1^e \int_0^1 p_1(z)c^e_1(z)dz = \frac{1}{p_1} \int_0^1 p_1(z)c^e_1(z)dz. \]

Therefore,

\[ \int_0^1 p_1(z)c^e_1(z)dz = p_1 c^e_1. \]

Similarly,

\[ \int_0^1 p_2(z)c^e_2(z)dz = p_2 c^e_2. \]

Thus,

\[ \int_0^1 p_1(z)c^e_1(z)dz + \int_0^1 p_2(z)c^e_2(z)dz = p_1 c^e_1 + p_2 c^e_2 = Wl + \Pi - D - \Theta + \hat{Q} - \Psi, \]

and we obtain

\[ p_1 c^e_1 = \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi). \]

By (A.2)

\[ \left( \int_0^1 c^e_1(z) \frac{\sigma-1}{\sigma} dz \right)^{\frac{\sigma}{\sigma-1}} c^e_1(z) = \lambda_1^e c^e_1(z)^{-1} = \lambda_1^e p_1(z)^{\sigma} = \left( \frac{p_1(z)}{p_1} \right)^{\sigma}. \]

From this we get

\[ c^e_1(z) = \left( \frac{p_1(z)}{p_1} \right)^{-\sigma} \frac{\alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)}{p_1}. \]

\[ c^e_2(z), c^u_1(z) \text{ and } c^u_2(z) \text{ are similarly obtained.} \]

References

G. Akerlof and J. L. Yellen. Efficiency Wage Models of the Labor Market. Cambridge University Press, 1986.

O. J. Blanchard and L. H. Summers. Hysteresis and the european unemployment problem. NBER Macroeconomics Annual, 1986.

O. J. Blanchard and L. H. Summers. Hysteresis in unemployment. European Economic Review, 31: 288–295, 1987.

G. D. Hansen. Indivisible labor and business cycle. Journal of Monetary Economics, 16:309–327, 1985.

L. F. Katz. Efficiency wage theories: A partial evaluation. NBER Macroeconomics Annual, 1986.
M. Lavoie. Efficiency wages in Kaleckian models of employment. *Journal of Post Keynesian Economics*, 23:449–464, 2001.

A. Lindbeck and D. J. Snower. Wage setting, unemployment, and insider-outsider Relations. *American Economic Review*, 76: 235–239, 1986.

A. Lindbeck and D. J. Snower. Efficiency wages versus insiders and outsiders. *European Economic Review*, 31: 407–416, 1987.

I. M. McDonald and R. M. Solow. Wage bargaining and employment. *American Economic Review*, 71:896–908, 1981.

M. Otaki. The dynamically extended Keynesian cross and the welfare-improving fiscal policy. *Economics Letters*, 96:23–29, 2007.

M. Otaki. A welfare economics foundation for the full-employment policy. *Economics Letters*, 102:1–3, 2009.

M. Otaki. The aggregation problem in the employment theory: The representative individual model or individual employees model? (https://www.scirp.org/journal/paperinformation.aspx?paperid=25920). *Theoretical Economics Letters*, 2:530–533, 2012.

M. Otaki. *Keynsian Economics and Price Theory: Re-orientation of a Theory of Monetary Economy*. Springer, 2015.

S. J. Schreiber, K. J. Smith, and W.M. Getz. *Calculus for the Life Sciences*. John Wiley & Sons, 2014.

C. Shapiro and J. E. Stiglitz. Equilibrium unemployment as a worker discipline device. *American Economic Review*, 74:433–444, 1984.

Y. Tanaka. Involuntary unemployment with indivisible labor supply under perfect competition. *Economics Bulletin*, 40:1914–1923, 2020a.

Y. Tanaka. Indivisible labor supply and involuntary unemployment: monopolistic competition model. *Singapore Economic Review*, forthcoming, 2020b.

T. Umada. On the existence of involuntary unemployment (hi-jihatsuteki-shitsugyo no sonzai ni tsuite (in Japanese)). *Yamaguchi Keizaigaku Zasshi*, 45(6):61–73, 1997.

J. L. Yellen. Efficiency wage models of unemployment. *American Economic Review*, 74: 200–205, 1984.