Observational constraints on Rastall’s cosmology

C. E. M. Batista

Physics Department, Universidade Estadual de Feira de Santana, Brazil

J. C. Fabris, O. F. Piattella, and A. M. Velasquez-Toribio

Physics Department, Universidade Federal do Espírito Santo, Brazil

Rastall’s theory is a modification of General Relativity, based on non-conservation of the stress-energy tensor. The latter is encoded in a parameter $\gamma$ such that $\gamma = 1$ restores the usual $T^\mu{}_{\nu} = 0$ law. We test Rastall’s theory in cosmology, investigating a two-fluid model and using data from type Ia supernovae and large scale structure observation. In our model, one of the fluids possesses zero pressure and obeys the usual conservation law, whereas the other is described by an equation of state $p_x = \omega_x \rho_x$, with $\omega_x$ constant. Confrontation against observational data favours a scenario with $\gamma \sim 2$ (which curiously corresponds to the original formulation of General Relativity, based on the Ricci tensor) and $\omega_x \sim -1$. 

*cedumagalhaes@hotmail.com
†fabris@pq.cnpq.br
‡oliver.piattella@ufes.br
§alan.toribio@ufes.br
I. INTRODUCTION

The nature of dark matter and dark energy is one of the most important issues today in physics. There are strong observational evidences in astrophysics and cosmology for the existence of these two exotic components of the cosmic energy budget, indicating that about 95% of the universe is composed of dark matter (about 25%) and dark energy (about 70%), but no direct detection has been reported until now. The usual candidates to dark matter (neutralinos and axions, for example) and dark energy (cosmological constant, quintessence, etc.) lead to very robust scenarios, but at same time they must face theoretical and observational issues. For recent reviews on the subject, see for example [1–3].

Among the alternatives to the standard description of the dark sector there is the possibility of a modification of gravity theory on large scales, see [4] for a recent review on the subject. An example are the so-called $f(R)$ theories, which are based on the inclusion of non-linear curvature terms in the Einstein-Hilbert action [5]. Another example, the one we pursue in this paper, is to touch one of the cornerstone of the gravity theory: the usual conservation laws for matter components. This kind of formulation was introduced by Rastall some 40 years ago [6, 7], and has been recently investigated in a cosmological context, giving some interesting results concerning the dynamics of the dark sector [8–12].

Rastall’s motivation for modifying the usual conservation laws is based on the fact that the latter have been directly tested only locally and in a weak-field regime. On the other hand, the introduction of covariant derivatives imply, in some sense, an exchange of energy between matter and the gravitational field. Hence, in general, non-trivial generalizations of the conservation law are in principle possible. Besides, particle production in curved space-times is a central issue of quantum field theory on such spaces. Therefore, one may regard modifications of the usual (classical) conservation laws as effective, semi-classical approaches to such phenomenon. Rastall’s proposal is the following:

$$T_{\mu \nu ; \mu} = \kappa R^{\nu}, \quad (1)$$

where the semicolon denotes the covariant derivative with respect to a generic metric and $\kappa$ is a (dimension-full) free parameter. The above relation can be rewritten as

$$T_{\mu \nu ; \mu} = \gamma - \frac{1}{2}T^{\nu}, \quad (2)$$

where $T$ is the trace of the stress-energy tensor and $\gamma$ is now a dimensionless free parameter. When $\kappa = 0$, then $\gamma = 1$ and the usual conservation law (and thus General Relativity) is recovered.
In a one-fluid model, it is possible to redefine the energy-momentum tensor in order to recover the usual conservation law \[13\]. In this sense, Rastall’s theory is just a redefinition of the fluid equation of state. However, in a multi-fluid case the modification introduced by Rastall opens possibilities of non-trivial interaction between the different components. In \[9\], this quality is used to investigate the consequences of the modified conservation law for a model of the dark sector of the universe. The authors investigate a two-fluid model, one of them being pressure-less matter \(p_m = 0\), whereas the other obeying the vacuum energy equation of state \(p_x = -\rho_x\). Assuming that the matter component obeys the usual conservation law, then the vacuum energy conservation law is affected by the presence of matter, via Eq. (2). The main result of \[9\] indicate that the model is completely equivalent to the ΛCDM at the background and linear perturbations levels. There is just one striking difference: dark energy may now agglomerate. This fact could have important consequences at the non-linear level, which is a regime where the ΛCDM faces some difficulties \[14\].

In the model studied in \[9\], the equivalence with the ΛCDM at background and linear perturbations levels implies that no constraints on the parameter \(\gamma\) can be established using the corresponding observational tests. Such constraints are, on the other hand, possible using non-linear data. In the present paper our goal is to verify to what extent the model studied in reference \[9\] is a favourable configuration. To do this, we repeat the analysis made there but with the \(x\) component now being described by a more general equation state, i.e. \(p_x = \omega_x \rho_x\), with \(\omega_x\) constant.

We show that in this case the parameter \(\gamma\) appears explicitly in the background and linear perturbation equations. Therefore, using type Ia supernovae and matter power spectrum observational data, we calculate its probability distribution function (PDF) along with the one for the matter density parameter \(\Omega_{m0}\), and for the equation of state parameter \(\omega_x\). We obtain that the PDF’s are are peaked about \(\omega_x \sim -1\), \(\Omega_{m0} \sim 0.3\) and \(\gamma \sim 2\). Remarkably, the case \(\gamma = 2\) corresponds to the original formulation of General Relativity (based on the Ricci tensor) and also to a zero speed of sound for a self-interacting scalar field obeying the modified conservation equation proposed by Rastall’s theory \[10\].

The paper is organized as follows. In Sec. II we present Rastall’s theory, deriving some cosmological considerations. In Sec. III the fields equation are presented and in Sec. IV the confrontation with observation is carried out. In Sec. V we present our conclusions.
II. RASTALL’S THEORY

According to Eq. (2), Einstein equations must be modified as follows:

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G \left( T_{\mu\nu} - \frac{\gamma - 1}{2}g_{\mu\nu}T \right), \]  

in order to be compatible with Bianchi identities (we use \( c = 1 \) units hereafter).

We consider a two-fluid model. The first component mimics baryons and dark matter, i.e. has negligible effective pressure, whereas the second describes an exotic dark component responsible for the acceleration of the universe, and has an equation of state of the form \( p_x = \omega_x \rho_x \), with \( \omega_x \) constant. We assume the matter component to conserve as usual. This is important in order to have matter agglomeration required to form the local structures. The field equations become

\[ R_{\mu\nu} = 8\pi G \left[ T_{x\mu\nu} + T_{m\mu\nu} - \frac{1}{2}(2 - \gamma)g_{\mu\nu}(T_x + T_m) \right], \]

where subscripts or superscripts \( x \) and \( m \) denote the dark energy and the matter component, respectively. On large scales we assume the universe to be described by the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric,

\[ ds^2 = dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right], \]

where \( a(t) \) is the scale factor and \( k \) is the curvature of the spatial sections. The perfect fluid energy-momentum tensor has the following form:

\[ T_{j}^{\mu\nu} = (\rho_j + p_j)u^\mu u^\nu - p_j g^{\mu\nu}, \]

where \( j = x, m \). We focus on the flat case \((k = 0)\), where the field equations take on the following form:

\[ \frac{\ddot{a}}{a} = \frac{4\pi G}{3} \left\{ [3(\gamma - 2)w_x - \gamma] \rho_x - \gamma \rho_m \right\}, \]

\[ \left( \frac{\ddot{a}}{a} \right)^2 = \frac{4\pi G}{3} \rho_x \left[ 3 - 3(1 - \gamma)w_x + (3 - \gamma)\rho_m \right], \]

\[ \dot{\rho}_x + 3\frac{\dot{a}}{a}(1 + w_x)\rho_x = \frac{\gamma - 1}{2} \left\{ (1 - 3w_x)\dot{\rho}_x + \dot{\rho}_m \right\}, \]

\[ \dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = 0, \]

where the dot denotes derivative with respect to the cosmic time \( t \). Note how the expansion history, therefore the accelerated phase, of the universe depends of the parameters \( \gamma \) and \( \omega_x \). The evolution
profiles of \( \rho_m \) and \( \rho_x \) are easily determined by solving the last two equations, yielding
\[
\rho_x = \rho_{x0}a^{-\frac{6(1+w_x)}{2(\gamma-1)(1-3w_x)}} + \frac{3(1-\gamma)\rho_{m0}}{6w_x + 3(\gamma-1)(1-3w_x)}a^{-3},
\]
\[
\rho_m = \frac{\rho_{m0}}{a^3},
\]
where \( \rho_{m0} \) and \( \rho_{x0} \) are their present values (i.e. for \( a = 1 \)). The present-time density parameters of the exotic fluid \( x \) and of matter are given by
\[
\Omega_{x0} = \frac{8\pi G\rho_{x0}}{3H_0^2}, \quad \Omega_{m0} = \frac{8\pi G\rho_{m0}}{3H_0^2},
\]
and in a flat universe are related as follows:
\[
\Omega_{x0} = \frac{2 - \Omega_{m0}}{3} \left\{ \frac{3(1-\gamma)}{6w_x + 3(\gamma-1)(1-3w_x)} \left[ 3 - \gamma - 3(1-\gamma)w_x \right] + (3-\gamma) \right\}
\frac{3 - \gamma - 3(1-\gamma)w_x}{3}. \quad (15)
\]
In the special case where \( \gamma = 1 \) the above relation recovers the corresponding General Relativity one \( \Omega_{x0} + \Omega_{m0} = 1 \). We now turn to the perturbation analysis of our two-fluid model.

**III. PERTURBATIVE ANALYSIS**

We write the perturbed metric as \( g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu} \), where \( g^{(0)}_{\mu\nu} \) indicates the background flat FLRW metric and \( h_{\mu\nu} \) is a small fluctuation. We choose the synchronous gauge condition, i.e. \( h_{\mu0} = 0 \) and introduce the perturbations in the matter functions as follows:
\[
\rho_x = \rho_x^{(0)} + \delta \rho_x, \quad \rho_m = \rho_m^{(0)} + \delta \rho_m, \quad u_m = u_m^{(0)} + \delta u_m,
\]
\[
p_x = p_x^{(0)} + \delta p_x, \quad p_m = p_m^{(0)} + \delta p_m, \quad u_x = u_x^{(0)} + \delta u_x,
\]
In the expressions (16)-(17), the superscript \( (0) \) indicates the background functions; \( \delta \rho_x, \delta \rho_m, \delta u_m, \delta u_x, \delta p_x, \delta p_m \), represent the perturbed quantities in density, four-velocity and pressure. We also introduce the following customary definitions:
\[
\delta_x \equiv \frac{\delta \rho_x}{\rho_x}, \quad \delta_m \equiv \frac{\delta \rho_m}{\rho_m}, \quad \Theta \equiv \partial_k \delta u_k, \quad h \equiv \frac{\sum_{k=1}^3 h_{kk}}{a^2}.
\]
For deeper detail, see for example [15]. After standard calculations, the perturbed conservation equation for the matter component can be cast in the following very simple form:
\[
\dot{\delta}_m = \frac{\dot{h}}{2}.
\]
Introducing the functions
\[
\tilde{g}(a) = \ddot{a} = -a \left\{ \left[ \gamma + 3(2-\gamma)\omega_x \right] \frac{\Omega_x}{2} + \gamma \frac{\Omega_m}{2} \right\},
\]
\[
\tilde{f}(a) = \dot{a} = a \sqrt{\frac{\Omega_x}{2} \left[ 3 - \gamma - 3(1-\gamma)\omega_x \right] + \frac{\Omega_m}{2}(3-\gamma)},
\]
where
\[
\Omega_x = \Omega_{x0} a^{-\frac{3(1 + w_x)}{2(\gamma - 1)(1 - 3w_x)}} + \frac{3(1 - \gamma)\Omega_{m0}}{6w_x + 3(\gamma - 1)(1 - 3w_x)} a^{-3},
\]  
\[
\Omega_m = \frac{\Omega_{m0}}{a^3},
\]
and calculating the perturbed component \(R_{00}\), we get the following equation:
\[
\ddot{\delta}_m + 2\frac{f(a)}{a} \dot{\delta}_m - \frac{3\gamma}{2} \Omega_m \delta_m = \frac{3}{2}\left[\gamma + 3(2 - \gamma)\omega_x\right] \dot{\delta}_x \Omega_x.
\]  
Calculating the perturbation of (5) we obtain for \(\nu = 0\)
\[
\delta \rho_x + 3\frac{f(a)}{a} (1 + \omega_x) \dot{\delta} \rho_x + (1 + \omega_x) \rho_x \left(\Theta_x - \frac{\dot{h}}{2}\right) = \left(\gamma - \frac{1}{2}\right) \left[(1 - 3\omega_x) \dot{\delta} \rho_x + \dot{\delta} \rho_m\right].
\]  
and for \(\nu = i\)
\[
(1 + \omega_x) \dot{\rho}_x \Theta_x + (1 + \omega_x) \rho_x \dot{\Theta}_x + (1 + \omega_x) \rho_x \left(\frac{5f(a)}{a}\right) \Theta_x = \frac{\nabla^2 \delta \rho_x}{a^2} \left[\frac{1 - \gamma}{2} + \left(\frac{3\gamma - 5}{2}\right) \omega_x\right] + \frac{\nabla^2 \delta \rho_m}{a^2} \left(\frac{1 - \gamma}{2}\right).
\]
It is useful to rewrite the above equations in terms of the derivative of the scale factor \(a\) instead of the cosmic time \(t\) since the former is directly connected with the dimensionless redshift quantity through \(z = -1 + 1/a\). In this way, Eq. (24) becomes
\[
\delta^\prime_m + \delta^\prime_m \left(\frac{g(a)}{f^2(a)} + \frac{2}{a}\right) - 3\frac{\gamma}{2} \Omega_m \delta_m = \frac{3}{2} \left[\gamma + 3(2 - \gamma)\omega_x\right] \delta_x \Omega_x,
\]  
Equation (25) turns into
\[
\Theta^\prime_x + \left[\frac{\Omega_x}{\Omega_x} + \frac{1}{1 + \frac{1}{2}(1 - 3\omega_x)} \frac{3f(a)}{a} (1 + \omega_x)\right] \frac{\delta_x}{f(a)} + \frac{1 + \omega_x}{1 + \frac{1}{2}(1 - 3\omega_x)} \dot{\Theta}_x = \delta^\prime_m \left[\frac{(1 + \omega_x) + \frac{\Omega_m}{2f^2(a)}(\gamma - 1)}{1 + \frac{(\gamma - 1)(1 - 3\omega_x)}{2}}\right] + \frac{\delta_m (\gamma - 1) \dot{\Omega}_m}{2f(a) \Omega_x} \left[1 + \frac{(1 - \gamma)(1 - 3\omega_x)}{2}\right],
\]  
and finally Eq. (26) reads
\[
\Theta^\prime_x + \left[\frac{\rho^\prime_x + \frac{5}{a}}{\rho_x}\right] \Theta_x = \frac{1}{1 + \omega_x \frac{f(a)}{a^2}} \left[\frac{1 - \gamma}{2} + \left(\frac{3\gamma - 5}{2}\right) \omega_x\right] + \frac{1}{1 + \omega_x \Omega_x} \frac{\Omega_m}{f(a) a^2} \left(\frac{1 - \gamma}{2}\right),
\]
where the prime denotes derivation with respect to the scale factor.
IV. OBSERVATIONAL CONSTRAINTS

We submit our model to two tests: the first focuses on the background evolution and is based on type Ia supernova data (the Constitution set [16]); the second is a perturbation test and we employ matter power spectrum data from the 2dFGRS [17].

We perform a Bayesian analysis, calculating first the $\chi^2$ function, defined as follows:

$$\chi^2 = \sum_{i=1}^{N} \frac{(x_i^{th} - x_i^{ob})^2}{\sigma_i^2},$$

(31)

where $N$ is the total number of observational data, $x_i^{th}$ is the theoretical prediction of some physical quantity and $x_i^{ob}$ is its value observed within an error bar $\sigma_i$. Assuming the data to be independent Gaussian random variables, see for example [18], the likelihood distribution function is constructed from the $\chi^2$ function as follows

$$L(p^j) = e^{-\chi^2(p^j)/2},$$

(32)

where $p^j$ represent the free parameters of the theory. For our model, we have four free parameters, i.e. $h$, $\Omega_m$, $\gamma$ and $\omega_x$. However, we choose to fix $h = 0.7$, on the basis of the current consensus on the value of the Hubble constant, see also [19], hereby reducing the dimension of the free-parameter space to three.

A. Supernovae Ia

Type Ia supernovae data consist in the distance modulus $\mu$, i.e.

$$\mu = m_{obs}(z_i) - M = 5 \log \left( \frac{d_L}{\text{Mpc}} \right) + 25,$$

(33)

where $m_{obs}$ is the apparent magnitude, $M$ is the absolute magnitude and $d_L$ is the luminosity distance. It can also be written as

$$\mu = 5 \log_{10} D_L(z) + \mu_0,$$

(34)

where $D_L = (H_0d_L)/c$ is the Hubble-free luminosity distance and $\mu_0$ is the zero point offset, defined by

$$\mu_0 = 5 \log_{10} \left( \frac{cH^{-1}_0}{\text{Mpc}} \right) + 25 = 42.38 - 5 \log_{10} h.$$

(35)

We employ data from the so-called Constitution set [16], which includes 397 distance moduli, out of which 100 come from the new low-z CfA3 sample and the rest from the Union set. Both samples
have a redshift range of $0.015 \leq z \leq 1.55$. The main improvement of the Constitution sample is the inclusion of a larger number of nearby ($z < 0.2$) supernovae, thus reducing the statistical uncertainty.

The chi-squared we employ for the type Ia supernovae test is then

$$\chi^2_{SNIa}(p) = \sum_{i=1}^{397} \frac{[\mu_{th}(p, z_i) - \mu_{obs,i}(z_i)]^2}{\sigma_{ob,i}^2}, \quad (36)$$

where $p = (\Omega_m, \omega_x, \gamma, \mu_0)$. The chi-squared can be minimized with respect to $\mu_0$, since the latter is an independent parameter. Expanding Eq. (36) with respect to $\mu_0$, we obtain

$$\chi^2(p)_{SNIa} = A(p) - 2\mu_0B(p) + \mu_0^2C(p), \quad (37)$$

which has a minimum for $\mu_0 = B(p)/C(p)$, giving thus

$$\chi^2_{SNIa, min} = \bar{\chi}^2_{SNIa} = A(p) - \frac{B^2(p)}{C(p)}, \quad (38)$$

where

$$A(p_i) = \sum_{i}^n \frac{[\mu_{th}(p, \mu_0 = 0) - \mu_{obs,i}]^2}{\sigma_i^2}, \quad (39)$$

$$B(p_i) = \sum_{i}^n \frac{\mu_{th}(p, \mu_0 = 0) - \mu_{obs}}{\sigma_i}, \quad (40)$$

$$C(p_i) = \frac{1}{\sigma_i^2}. \quad (41)$$

As a side remark, for the single-fluid model Rastall’s theory has been confronted against supernova data (Union sample) leading to results that can be competitive to the $\Lambda$CDM data under special conditions [20].

### B. Power Spectrum

We use the 2dFGRS data [17] to compare the predictions on the matter power spectrum of our model with observation and obtain constraints on the parameters. The chi-squared, in this case, is

$$\chi^2_{P(k)}(\Omega_m, \omega_x, \gamma) = \sum_{i=1}^{39} \frac{[P(k, \Omega_m, \omega_x, \gamma) - P_{obs}(k, \Omega_m, \omega_x, \gamma)]^2}{\sigma_i^2}. \quad (42)$$

The number of free parameters is three ($\omega_x$, $\gamma$, and $\Omega_m$) because $h$ is absorbed in the units of the wave-number $k$ (h Mpc$^{-1}$).
In order to compute the matter power spectrum, we choose the BBKS fitting function as initial condition after equality for the transfer function \([21]\):

\[
T_{\text{BBKS}}(k) = \frac{\ln(1 + 2.34q)}{2.34q[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{1/4}},
\]

(43)

where \(q = k/(\Sigma h\text{Mpc}^{-1})\), with the shape parameter \(\Sigma = \Omega_{m0}h\gamma\Omega_{b0}\) properly taking into account the baryon contribution, which we assume \(\Omega_{b0} = 0.04\). The matter power spectrum can be finally written in the form

\[
P(k) = Ak^nT^2(k)T^2_{\text{BBKS}}(k),
\]

(44)

where \(n\) measures the slope of the primordial power spectrum (we assume \(n = 1\) \([19, 22]\)), and \(A\) is a normalization constant. To obtain power spectra for our model, we numerically solve the relevant equations presented in the previous section from \(z = 500\) to \(z = 0\) and calculate \(T^2\) of Eq. (44). For the normalization constant, we follow the methodology of \([23]\).

Note that in \([24]\), the matter power spectrum is used to constrain the parameter \(\gamma\), but with a decomposition of the matter components different from that employed here and in \([9]\). This results in \(\gamma = 1\), i.e. General Relativity, as the preferred configuration.

C. Joint analysis

Considering the observational data to be independent, we define the total chi-squared as

\[
\chi^2 = \bar{\chi}^2_{\text{SNIa}} + \chi^2_P(k),
\]

(45)

and the likelihood as

\[
L(\Omega_{m0}, \omega_x, \gamma) = \exp\left(-\chi^2/2\right).
\]

(46)

We assume flat priors for the parameters, therefore, up to a normalization constant, the likelihood is also the posterior probability. In FIG. \([4, 7]\) we show the results for the individual and joint analysis. The principal result is the low probability for the \(\Omega_{m0}\) parameter using the 2dFGRS data only and a peak of probability near \(\gamma \sim 2\) and \(\omega_x \sim -1\).
TABLE I. Observational constraints on the model parameters (1σ errors).

| Data                          | $\chi^2_{\text{min}}$ | $\Omega_{m0}$ | $\omega_x$ | $\gamma$ |
|-------------------------------|------------------------|----------------|------------|----------|
| 2dFGRS ($\gamma = 2.0$)      | 18.1400 $\pm$ 0.150   | $0.200^{+0.056}_{-0.150}$ | $-0.955^{+0.120}_{-0.031}$ | 2.0 |
| SNIA ($\gamma = 2.0$)        | 465.508 $\pm$ 0.545   | $0.310^{+0.207}_{-0.545}$ | $-0.985^{+0.150}_{-0.129}$ | 2.0 |
| 2dFGRS+SNIA ($\gamma = 2.0$) | 483.657 $\pm$ 0.429   | $0.301^{+0.120}_{-0.429}$ | $-1.005^{+0.130}_{-0.129}$ | 2.0 |
| 2dFGRS                        | 18.1497 $\pm$ 0.052   | $0.239^{+0.052}_{-0.207}$ | $-1.350^{+0.107}_{-0.205}$ | 1.810 $^{+1.507}_{-0.207}$ |
| SNIA                         | 465.294 $\pm$ 0.306   | $0.306^{+0.150}_{-0.030}$ | $-0.965^{+0.201}_{-0.051}$ | 1.831 $^{+0.305}_{-1.507}$ |
| 2dFGRS+SNIA                   | 482.649 $\pm$ 0.21     | $0.21^{+0.051}_{-0.045}$  | $-0.962^{+0.107}_{-0.204}$ | 1.814 $^{+1.205}_{-1.502}$ |

V. RESULTS AND DISCUSSION

We compared the predictions of a cosmological model based on Rastall’s theory against type Ia supernovae and matter power spectrum data. We determined a probability distribution function for each of the free parameters ($\Omega_{m0}, \omega_x, \gamma$).

In Table 1 we show the best fit values (with 1σ errors). They are very close to the ΛCDM results, but with a quite important dispersion. A clear feature is that supernovae test favours a higher value for $\Omega_{m0}$ with respect to the power spectrum estimations. The joint analysis, for this parameter, depends on fixing or not $\gamma$: it is higher when $\gamma = 2$ and smaller when $\gamma$ is left free. It is clear also from table 1 that the PDF for $\gamma$ is not highly dependent on which test is used. In general, $\gamma$ is peaked around 1.8 but with a quite large dispersion, which leads to the value $\gamma = 2$ quite probable. Remarkably, the General Relativity case $\gamma = 1$ is not statistically favoured from the analysis performed here. For the equation of state parameter in almost all cases, the most probable configuration is near the cosmological constant case $\omega_x = -1$, except when only the power spectrum test is used, with $\gamma$ free: for this case, the equation of state parameter extends considerably to the phantom region ($\omega_x < -1$).

The results reported here indicate a scenario that fits the requirements for the structure formation process, since $\gamma = 2$ implies a sound speed around zero [10], at least for the scalar field case. Moreover, the vacuum energy equation of state is somehow favoured, supporting the configuration studied in [9]. In this sense, a natural extension is to consider the non-linear regime for structure formation. Moreover, Rastall’s theory has been investigated until now on a cosmological ground only. It is important to verify its properties on smaller scales, for example investigating the stability properties of objects such as stars and the weak-field dynamics.
FIG. 1. Type Ia supernovae and matter power spectrum data [16, 17] with the best fit curves.

FIG. 2. Marginalized probability density functions for the parameters $\omega_x$ and $\Omega_{m0}$ for fixed $\gamma = 2$ and using type Ia supernovae data only.

FIG. 3. Marginalized probability density functions for the parameters $\omega_x$ and $\Omega_{m0}$ and $\gamma$, using type Ia supernovae data only.
FIG. 4. Marginalized probability density functions for the parameters $\omega_x$ and $\Omega_{m0}$ for fixed $\gamma = 2$ and using matter power spectrum data only.

FIG. 5. Marginalized probability density functions for the parameters $\omega_x$ and $\Omega_{m0}$ and $\gamma$, using matter power spectrum data only.

FIG. 6. Confidence contours at 1$\sigma$ and 2$\sigma$ levels on the $(\Omega_{m0}, \omega_{x0})$ plane and marginalized probability density functions for the parameters $\omega_x$ and $\Omega_{m0}$. In these plots we fix $\gamma = 2$ and use both type Ia supernovae and matter power spectrum data.
FIG. 7. Confidence contours at 1σ and 2σ levels on the \((\Omega_m, \omega_x)\) plane and marginalized probability density functions for the parameters \(\omega_x, \Omega_m\) and \(\gamma = 2\). We use both type Ia supernovae and matter power spectrum data.
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