Modeling and estimation of cutting forces in ball helical milling process

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1 Introduction

Helical milling has attracted much attention as typical hole-making technology for difficult-to-cut materials, such as titanium alloy and CFRP; during the machining process, the rotating cutter traverses a helical trajectory to machine a hole with a diameter larger than that of the tool. As one of the new hole-making technique, helical milling offers many advantages compared to conventional drilling. For example, one tool can be used to produce different borehole geometries by changing the processing parameters, so it offers the possibility for a dynamic correction of the bore diameter during the hole-making process with lower cutting forces and better chip transportation.

Abstract

Milling forces play an important role in the milling process and are generally calculated by the mechanistic or numerical methods; reliable model of cutting forces is very important for the simulation of milling process, which has big scientific significance to further improve machining quality. Ball helical milling technology is used to make holes based on the cutting principle of helical milling using ball end cutter, and due to the influence of spherical surface machining characteristic, the modeling of cutting force in ball helical milling is difficult. Therefore, the main purpose of this paper is to establish an analytical cutting force model in the ball helical milling process. Considering cutting characteristics in the axial feed, the kinematics of ball helical milling is first presented, then the chip thickness distribution in different directions along the cutting edges is predicted. Furthermore, based on the characteristics of helical milling technology and geometry shape of ball end cutter and the classical mechanical cutting force model, through the study on the ball-end milling mechanics, a new relatively accurate theoretical cutting force model is established. At the same time, cutting force coefficients are identified through instantaneous force method according to the Ti-alloy experimental research result. Finally, higher simulation precision of cutting force model in ball helical milling process is received.

Keywords Ball helical milling · Cutting forces · Cutting coefficients · Instantaneous force method

Ball end cutter can be used to complete helical milling process, but the dynamic milling process of ball end cutter is relatively complex, which is not only related to the basic machining parameters and tool geometry, but also has a complex relationship with the dynamic change of cutting zone.

Cutting forces are very important parameters in the cutting process; appropriate prediction of the force components collaborates with the correct choice of the cutting parameters can avoid machine tool vibrations, improve workpiece surface quality, and guarantee process stability. Through modeling and analysis of cutting force, the cutting state with tool wear and machining quality can be further understood and discussed. Wang et al. [1, 2] first considered the processing of periphery edges and frontal edges respectively in the helical milling of titanium alloy and carbon fiber composites, established a relatively accurate analytical model of cutting force, identified the cutting force coefficients through the average-based force method, and modified cutting force coefficients considering the effect of cutting parameters. Liu et al. [3] used a combination of simulation and experiment to study the cutting force modeling method for the helical milling process.
process. Ventura et al. [4] described an approach for modeling the cutting forces in the helical milling process based on the analysis of tool contact angle and depth of cut. The model could be used to predict the behavior and magnitude of the force acting on the tool. Zhou et al. [5] analyzed and described the undeformed chip geometries produced by each tooth and the instantaneous chip geometries, developed non-linear cutting force model. The results showed that the model could be used to predict the cutting force generated in the steady cutting stage. Shang et al. [6] developed a cutting force model and built a new calibration method of cutting force coefficients in the helical milling process, but the results showed that average coefficients must be modified according to cutting experiments. Rey et al. [7] set up cutting forces model depending on the tool geometry and cutting conditions, the influence of the side edges and bottom edges was considered in detail, and the radial and axial effects of the three-way cutting forces were considered, the corresponding unique coefficient identification experiment was carried out by analyzing the basic cutting process of each cutting edge respectively. In addition, Li et al. [8] simulated the cutting force in the helical milling process, and described the surface topology and surface quality of hole-making. Through the study on the helical milling of aviation aluminum alloy, Li et al. [9] calculated the instantaneous uncut chip thickness (IUCT) with high precision considering the cutter runout effect; the dual mechanism force model was adopted to predict cutting forces in the helical milling process. Araujo et al. [10] mainly considered the geometric shape of the tool and its tridimensional tool trajectory, analyzed the change of cutting force in thread milling process, which was similar to helical milling process.

In order to improve hole-making quality, the tilted helical milling technology was proposed using ball end cutter [11], the kinematic and cutting principle were discussed respectively, but sound mechanics model had not been developed. The cutting force modeling of ball-end milling is more complicated, the main reason lies in that the spherical surface of ball-end cutter participates in the cutting process and is not equivalent to end-milling cutter. Yucesan [12] and Altintas [13] carried out an in-depth analysis of the mechanical dynamics in the milling process performed by ball-end cutters, analyzed the change of cutting force in the time domain, considered the instantaneous regenerated chip load and force coefficients, and established cutting force model. Jia et al. [14] introduced interaction between material properties and machining conditions, presented a novel ball-end milling force prediction model. Wojciechowski et al. [15] proposed an accurate cutting force model applied for the finishing ball end milling, including the influence of surface inclination and cutter’s run out. The result showed that the relative error was reduced from 16% to 7%. Ball end cutter is mainly used in surface machining; the focus of the research lies in that the analysis of the non-deformable chip thickness in the surface machining process and the change of the cutter’s contact area, thus for the ball helical milling process, it is also important to discuss the change of the contact area.

Axial feed is simultaneously completed in the ball helical milling process; axial force is far greater than cutting force in other directions. In addition, due to the special characteristics of the ball-end cutter, the main contact area between the tool and the workpiece material is mainly the spherical cutting edges, so during the ball helical milling process, along with the axial feed, the instantaneous tool cut-in angle, cut-out angle, chip thickness, and contact area change from time to time. Therefore, the construction of the cutting force model in the ball helical milling process is particularly difficult. It is necessary to first calculate the relative position parameters of the tool and workpiece, and then determine the instantaneous chip thickness, which are summed up to obtain the instantaneous milling load of the ball end cutter. Based on the basic machining principles of helical milling and the geometry of ball end cutter, this paper considers the tool movement and material removal during cutting process, integrates mechanical principle to establish a relatively accurate cutting force model for ball helical milling process.

2 Kinematics of helical milling

2.1 Kinematic analysis

In order to analyze the tool movement in detail in the helical milling process, three coordinate systems are set up; there are respectively the cutting edge coordinate system (CECS) O_2xyz, tool coordinate system (TCS) O_2x_2y_2z_2, and workpiece coordinate system (WCS) O_1x_1y_1z_1, as shown in Fig. 1. The tool rotation axis is taken as the z_2 axis in the TCS, the vertical upward direction is the positive direction, at the same time,
tool tip is taken as the starting point of the $z_2$-axis. The cutting tool begins to cut in the CECS, in which rotational angle of the reference flute $\theta$ is positive in clockwise direction and measured from $x$-axis, the cutter rotates counterclockwise to complete the orbital revolution with $\phi$, as shown in Fig. 1.

In the helical milling process, due to the complex movement of the ball end cutter, the track of tool center is helical line, the ramp angle is set to $\alpha$, as shown in Fig. 2. Then, in the plane $x_1O_1y_1$, the related parameters will meet follow relationships.

\[
\begin{align*}
\theta_j(t) &= \theta_{j0} + \frac{2\pi nt}{60} & \text{(1)} \\
\phi(t) &= \phi_0 + \frac{2\pi nt}{60} & \text{(2)}
\end{align*}
\]

The axial feed rate per tooth of the tool $s_a$ is

\[
s_a = \frac{a_p \cdot n_r \cdot \pi N}{n} \tag{3}
\]

Feed rate per tooth in the tool center $s_t$ is

\[
s_t = \frac{\pi n_r (D_h - D_t)}{n \cdot N} \tag{4}
\]

In addition, as shown in Fig. 2, although the tool is perpendicular to plane $x_1O_1y_1$, the tool cutting edge is not perpendicular to the plane due to the existence of the ramp angle, the material will be removed by spherical cutting edge, unlike general end mill, the ball end cutter has no obvious side edge and bottom edges, but in the ball helical milling process, spherical cutting edges need to complete main cutting process.

That is to say, it is characterized by a simultaneous movement of the tool on a circular path and a superimposed movement in the axial direction, the superposition of circular and axial feed is actually helical feed, which is different to the only peripheral or frontal feed.

### 2.2 Geometry of ball end cutter

There are tri-directional micro-elements cutting forces at point $P$, as shown in Fig. 3.

Assuming that the micro-elements cutting forces are tangential direction $dF_t$, radial direction $dF_r$ and axial direction $dF_a$ respectively.

The generalized geometric model for ball-end cutter was presented by Altintas et al. [16, 17]. The ball end cutter has both spherical and cylindrical body, and the cutting edges are usually helical, axial position angle $\kappa$ within the arc zone varies along the cutter axis. The flutes of the ball end cutter meet at the tip of the sphere, and are ground with a constant helix lead. The position angle at the hemispherical section varies due to the increasing diameter along its axis. The cutter with constant lead is taken as a base model, then

\[
x^2 + y^2 + (R_t z)^2 = R_t^2 \tag{5}
\]

The cutter radius $R$ in $x_2O_2z_2$ plane at axial location $z$ is

\[
R(z) = R_t \sqrt{1 - \left(\frac{1 - \frac{z}{R_t}}{1 - \frac{D_t}{R_t}}\right)^2} \tag{6}
\]

Due to the influence of helix angle, the points on a flute do not contact with the workpiece at the same time. Then, the position angle $\kappa$ of a point on the flute can be expressed as

\[
\kappa(z) = \sin^{-1}\left(\frac{R(z)}{R_t}\right) \tag{7}
\]
2.3 Variation of undeformed chip thickness and contact area

In the mechanistic model, the cutting forces are usually assumed to be proportional to the undeformed chip thickness. In the ball helical milling process, the numbers of cutting edge involved in the processing gradually increase; cutting forces are considered as the function of axial position angle. In order to observe the cutting state of the ball helical milling process, especially in the initial cutting stage, the contact part of the tool and the workpiece starts from the tip of the cutting head, the spherical cutting edges at different heights gradually participate in the cutting, and the total cutting depth gradually increases with the increase of the cutting width, as shown in Fig. 4. The state of tool cutting into the workpiece is illustrated in Fig. 4a; it can be seen that contact zones increase gradually, until spherical cutting edges completely enter the cutting state. The state in Fig. 4b is just reverse, spherical cutting edges are gradually cutting out of the cutting state.

Whole cutting state in Fig. 4 shows that spherical cutting edges participate in and out of the cutting process gradually, the direct reflection is the cutting forces increase gradually. For pure horizontal feed, the contribution of points near the tool tip to chip thickness generation is almost negligible; however, for pure vertical feed, the contribution from those points is significant [14]. In order to obtain a steady cut while plunging with a ball end cutter, spherical part of the tool has to engage with the workpiece from tool tip to ball cylindrical part transition level; once the entire ball part engages with...
the workpiece, engagement conditions do not change in the following feed movements of the cutter, and cylindrical part does not contribute to cutting.

3 Modeling of cutting forces

The ball-end cutter completes a certain amount of axial feed for each helical movement around the center of the pre-machined hole. According to the kinematic analysis of the contact area, it is known that the material removal process is more complicated, which is obviously different from that of the end mill. Therefore, the cutting force modeling process needs to be adjusted according to the actual situation, and it must be considered separately for the circumferential feed and the axial feed of spherical cutting edges.

3.1 Modeling of microelement cutting forces

In order to calculate the relatively accurate cutting force during ball helical milling process, the cutter is divided into axial disc elements of height \( dz \) where orthogonal to oblique transformation is implemented, and the surface cut by the helical ball end mill slice at each axial level is digitized by a number of points. According to the principle of orthogonal cutting, assuming that the point is located on the first cutting edge and its axial height is \( z \), the differential geometry method is used to discretize the tool into several axially distributed micro-elements; the differential forces in radial, tangential, and axial directions [18, 19] are expressed by:

\[
\begin{align*}
\left\{ \begin{array}{l}
\text{\text{d}F_r(\theta, z)} = K_w h_1(\theta, z) db + K_w dS \\
\text{\text{d}F_t(\theta, z)} = K_c h_1(\theta, z) db + K_t dS \\
\text{\text{d}F_n(\theta, z)} = K_w h_1(\theta, z) db + K_n dS
\end{array} \right.
\]

For ball end cutter, the uncut chip thickness normal to the cutting edge will vary with the position of the cutting point according to regenerated surface. The differential width is the projected length of an infinitesimal cutting flute in the direction along the cutting speed, while chip width can be found using differential cutting edge height and axial position angle, the un-deformed chip width can be expressed as follows:

\[
d b = dz/\sin\kappa
\]

Length of the edge discrete element \( dS \) can be known from the geometry of the cutting tool

\[
dS(z) = dz \sqrt{\left(R(z)\psi(z)\right)^2 + \left(R'(z)\right)^2 + 1}
\]

For ball end cutter, the effective radius involved in the cutting process is

\[
R(z) = R_0\sin\kappa(z)
\]

Since there is a spherical helix, the following formula exists for the arc length of the cutting

\[
dS(\kappa) = R\sqrt{\left(1 + \sin^4\kappa\tan^2\beta\right)} \cdot d\kappa
\]

In addition, the instantaneous cross angle at the axial depth \( z \) between the cutter tooth and the workpiece can be expressed as

\[
\theta_f(t) = \theta_f(t) + (N - 1) \frac{2\pi}{N} - \psi(z)
\]

Due to the influence of helix angle, a point on the cutting edge will lag behind the upper end of the cutting edge, the lag angle \( \psi \) can be calculated as

\[
\psi(z) = \frac{z \cdot \tan\beta}{R_t}
\]

Namely, when \( z = 0 \), \( \psi(z) = 0 \), helix angle does not affect the rotation angle \( \theta \).

3.2 Analysis of the cutting process

In the ball end milling process, when the cutter completes the plane feed, the cutting depth does not change, and the position angle remains constant within a certain range. Only the three-way cutting forces caused by the periphery feed in the spherical area is considered [12]. But during the ball helical milling process, the tool completes a helical feed, then the scope of position angles changes at each moment. The position angles vary from 0 to 90 degrees, and the total cutting depth keeps changing, that is, the tool position is changing. Since the axial depth of cut per revolution is constant, two typical moments in the cutting process are considered, as shown in Fig. 5.

It can be seen that at different times, the size of the removed material is different, but the biggest thickness at the bottom end does not change, which is equal to the axial feed per tooth, and the chip width is equal to the effective cutting radius of the tool involved. When the part of the spherical surface is cut in the cutting state, as shown in Fig. 5 a, effective radius is smaller than tool radius, when the spherical surface of tool is fully involved in the cutting state, as shown in Fig. 5 b, the axial spherical chip is the largest, then it will maintain until the tool penetrates the bottom surface of the workpiece, and the thickness of the axial chip gradually decreases to zero. Namely, the cross-sectional area of the removed material is different with same axial feed per tooth, which is related to the axial position angle. In addition, due to the axial feed of the tool, the undeformed chip thickness is a function of the position angle.
3.3 Undeformed chip thickness

During the ball helical milling process, while the tool completes the transversal feed in the $x_2O_2y_2$ plane, it maintains a certain feed rate in the axial direction. The feed of the cutting edge in different directions will cause different undeformed chips, as shown in Fig. 6.

When the tool feeds in the transversal direction, the feed of the tool can be referred to the general ball end mill [18], as shown in Fig. 6 a, the chip thickness $h_{jr}$ can be expressed as follows

$$h_{jr}(\theta, \kappa) = s_r\sin(\theta_j(t))\sin(\kappa(z)) \tag{15}$$

When the tool feeds in the axial direction, the undeformed chip shape is related to the position angle, as shown in Fig. 6 b, the undeformed chip thickness in the $x_2O_2z_2$ plane $h_{jc}$ is

$$h_{jc}(\kappa) = s_a\cos(\kappa(z)) \tag{16}$$

That is, when the axial position angle is equal to $90^\circ$, the undeformed chip thickness in the axial direction is zero, and when the axial position angle is equal to $0^\circ$, the axial undeformed chip thickness is the largest. It can be seen that the chip shape and thickness in both directions will change with the change of axial position angle. The instantaneous undeformed chip thickness will be influenced by the transversal and axial feed motion. As shown in Fig. 7, the axial feed motion along with the reverse $z_2$-direction will increase the undeformed chip thickness; it is assumed that the initial position of the spherical element of the tool is $C$, and the final position is $C_2$, which is actually the combination of axial feed and peripheral feed, and it is assumed that $M_1N_1 = M_2N_2$, then the final chip thickness can be expressed in terms of $M_2P$, which is expressed as

$$M_2P = M_2N_2 + N_2P \tag{17}$$

Due to ball end cutter complete circumferential and axial feeds in the helical milling process, so total chip thickness $h_j$ can be calculated as

$$h_j(\theta, \kappa) = s_r\sin(\theta_j(t))\sin(\kappa(z)) + s_a\cos(\kappa(z)) \tag{18}$$

When the processing depth is more than $R_t$, the cutting area is the largest with maximum cutting force, therefore, stable cutting stage will be chosen to next analysis process. The changeable scope of axial position angle $\kappa$ lies in

$$0 \leq \kappa \leq \pi/2 \tag{19}$$
When \( \kappa = 0^\circ \) and \( \kappa = 90^\circ \), the undeformed chip thickness reflects only one part of the circumferential feed or axial feed respectively.

### 3.4 Transformation of cutting force

The initial analysis of the cutting force is performed in CECS, and then through the coordinate transformation to TCS, and to complete the corresponding calculation, the cutting forces need to be converted into WCS. Generally, cutting force in \( x \) and \( y \) direction will be related to tangential, radial, and axial directions, cutting force in \( z \) direction is only related to radial and axial directions in the ball helical milling process.

In the stable stage, the cutting edge participating in the cutting is not the same in different cutting position; there will be cut-in and cut-out process, namely, the axial cutting zone is a circular area, and in the \( x_1O_1y_1 \) plane, the axial force needs to be considered separately. Then considering the helix angle, cutting forces in TCS can be calculated as

\[
\begin{bmatrix}
\frac{dF_{x\theta}}{d\theta} \\
\frac{dF_{y\theta}}{d\theta} \\
\frac{dF_{z\theta}}{d\theta}
\end{bmatrix} = \begin{bmatrix}
\cos \alpha & 0 & 0 \\
0 & \cos \alpha & 0 \\
-\sin \alpha & -\sin \alpha & 1
\end{bmatrix}
\begin{bmatrix}
\frac{dF_{x\phi}}{d\phi} \\
\frac{dF_{y\phi}}{d\phi} \\
\frac{dF_{z\phi}}{d\phi}
\end{bmatrix}
\]

The cutting force on the \( \theta \)th cutting edge can be obtained by integrating the axial height of the tool as

\[
\begin{align*}
\frac{dF_{x\theta}}{d\theta} &= \int_{z_1}^{z_2} dF_{x\theta}(	heta, z) \\
\frac{dF_{y\theta}}{d\theta} &= \int_{z_1}^{z_2} dF_{y\theta}(	heta, z) \\
\frac{dF_{z\theta}}{d\theta} &= \int_{z_1}^{z_2} dF_{z\theta}(	heta, z)
\end{align*}
\]

Fig. 7 Determination of undeformed chip thickness

The uncut chip thickness normal to the cutting edge will vary with the position of the cutting point. The cutting force can be calculated separately for each infinitesimal element; total cutting forces are the sum of all infinitesimal element force as follows:

\[
\begin{bmatrix}
F_{x\theta} \\
F_{y\theta} \\
F_{z\theta}
\end{bmatrix} = \int_{\theta_{min}}^{\theta_{max}} \begin{bmatrix}
\frac{dF_{x\theta}}{d\theta} \\
\frac{dF_{y\theta}}{d\theta} \\
\frac{dF_{z\theta}}{d\theta}
\end{bmatrix} d\theta d\kappa
\]

In the ball helical milling process, the engagement conditions between the cutter and the workpiece determine the chip thickness. These geometric parameters are used to calculate cutting forces at each tool rotation angle, where \( \theta_{st} = 0, \theta_{st} = \pi \) and \( \kappa_{min} = 0, \kappa_{max} = \pi/2 \).

In the ball helical milling process, the cutting force of the tool is not only related to the rotation angle of the tool, but also related to the orbital angle of the tool and the axial position angle. Because the evaluation and measurement of the cutting force ultimately need to be performed in WCS, the instantaneous cutting forces can be expressed to the WCS as follows:

\[
\begin{align*}
F_x(\theta, \phi) &= \sum_{i=1}^{N} g(\theta_i) [F_{x\phi}\cos \phi + F_{y\phi}\sin \phi] \\
F_y(\theta, \phi) &= \sum_{i=1}^{N} g(\theta_i) [-F_{x\phi}\sin \phi + F_{y\phi}\cos \phi] \\
F_z(\theta, \phi) &= \sum_{i=1}^{N} g(\theta_i) F_{z\phi}
\end{align*}
\]

Since milling is an intermittent operation, angular position of flutes needs to be known to check whether the flute is in or out of the cutting at any instant, \( sog(\theta_i) \) is a special parameter used to show if the tool is in the cut or not, when the tool is in cut, \( g(\theta_i) = 1 \), otherwise \( g(\theta_i) = 0 \). Where \( \phi \) changes from 0 to 2\( \pi \).

In order to remove material, there must be a relative motion between the tool and the workpiece. Since the cutting speed at the tool tip is zero, and the workpiece is fixed, cutting does not take place. On the other hand, the cutting tool is forced to provide plastic deformation even at the tip point; therefore, the desired deformation is realized by indentation of the tool instead of cutting [20]. However, the indentation effect is not
included in the existing cutting force model, the indentation force depends on axial feed speed.

\[ F_2(z) = K_z s_a R_h \]  

(24)

Namely, the relationship between axial feed rate per tooth and indentation force is established, due to the hole-making diameter is fixed, the indentation force is only related to the axial feed rate per tooth, axial cutting coefficients \( K_z \) can be received through the linear fitted to the average axial forces. Thus, the final cutting forces model in ball helical milling process is illustrated as follows:

\[
\begin{align*}
F_x(\theta, \phi) &= \sum_{i=1}^{N} g(\theta_i) [F_{x_{ri}} \cos \phi + F_{y_{ri}} \sin \phi] \\
F_y(\theta, \phi) &= \sum_{i=1}^{N} g(\theta_i) [-F_{x_{ri}} \sin \phi + F_{y_{ri}} \cos \phi] \\
F_z(\theta, \phi) &= \sum_{i=1}^{N} g(\theta_i) F_{z_{ri}} + F_2(z)
\end{align*}
\]  

(25)

## 4 Cutting experiments setup

### 4.1 Experiment conditions

The material used in the test was Ti6Al4V plate with 120 mm × 120 mm × 10 mm. Three-factor and three-level experiments were carried out in the machining center; the two edges ball end cutter with TiAlN coating based on cemented carbide was used, and the diameter was 6 mm with 35° helix angle and normal rake angle of 0° on ball part; cutting forces were measured in the \( x, y, \) and \( z \) directions using 3-direction Kistler 9119AA2 table type dynamometer, as shown in Fig. 8.

The related cutting parameters are illustrated in Table 1.

### 4.2 Analysis of experiment results

#### 4.2.1 General trend of cutting forces

When the spindle speed is 3200 rpm, the feed rate per tooth is 0.04 mm/t, and depth of cut per revolution is 0.1 mm/rev; three-axis cutting forces are illustrated in Fig. 9 a, and the filtered values (after 50 Hz) are showed in Fig. 9 b.

It can be seen from Fig. 9 a that the \( xy \)-direction cutting forces show clear change. The reason maybe lies in the cutting edges of ball end cutter gradually enter the cutting state, the effective cutting radius of the participating cutting tools gradually increase, so the cutting forces also show the same change. When the cutting edges are fully involved into cutting, the cutting forces will keep the state until cutting out of the process. When the tool cuts near the through hole, the remaining material thickness is lower than a certain value, and the give up phenomenon of the tool will occur. The manifestation is that the support rigidity of the bottom material to the milling cutter becomes smaller, and the cutting forces gradually reduce until the milling is completed. It also can be seen that the forces in \( x \) and \( y \) directions are far lower than the axial direction. The axial cutting force fluctuates greatly. The main reason maybe lies in that the selected ball-end cutter is a two-edge cutting tool, the \( x \) and \( y \) direction cutting forces are mainly related to the radial feed rate. When rotation speeds are same with same feed rate per tooths in the helical milling.

| Factors                      | Levels | 1   | 2   | 3   |
|------------------------------|--------|-----|-----|-----|
| Rotation speed (rpm)         |        | 3200| 3700| 4200|
| Feed rate per tooth (mm/t)   |        | 0.04| 0.05| 0.06|
| Axial depth of cut per rev (mm/rev) |    | 0.1 | 0.15| 0.2 |

Fig. 8 Ball helical milling experiment
process with four-edges end mill [2], the revolution speed is half for the two-edge compared to the four-edge tools, then the cutting forces in the x and y directions decrease into the half, while because the feed rate in the axial direction has not changed, the cutting force value in the z direction does not change. For a two-edge ball end mill, the fluctuation of the cutting force in the z direction is big due to the influence of the position angle, which is very different to the end mill, but the degree of tool wear will be significantly reduced.

4.2.2 Axial cutting forces

As can be seen from the experimental results, when the ball end cutter is used to complete the milling process, the axial cutting forces present different variation rules, as shown in Fig. 10, it can be seen that, on the whole, axial cutting forces present obvious fluctuation phenomenon; the corresponding cutting parameters and results are illustrated in Table 2.

It can be seen from Fig. 10 that the amplitude fluctuations of axial cutting forces are about 30 mm with no significant difference. While for fluctuation cycle or frequency, it can be seen from Fig. 10 a and b, Fig. 10 e and f, cutting depths per revolution are obviously different, but the frequencies of the wave are not affected, and according to the Fig. 10 c and d, only feed rates per tooth are different, but the cycles have changed, and according to the Fig. 10 a and d, the cycles are different, the impact of feed rate per tooth is bigger, and then according to Fig. 10 d and e, rotation speed has the influence to the changeable cycle. That is to say, the effect of the axial cutting depth per revolution on the fluctuation of the axial force can be ignored, and the effect of feed rate per tooth and rotation speed on the axial cutting force must be considered. Then, the main analysis should be placed on these two cutting parameters.

According to Eq. 2 and Eq. 4, when the tool and hole-making diameter are determined, change in rotation speed and feed rate can be directly converted to revolution angle of the tool, in addition, Rey et al.[7] discussed that axial force signals were periodic with different frequencies, the forces applied on the axial part of the tool were variable, with a frequency equal to the tool revolution frequency in the helical milling of titanium alloy. Thus the effect of tool revolution angle on axial cutting force must be considered in the ball helical milling process.

5 Identification of cutting force coefficients

The key in the modeling of cutting forces is to identify the cutting force coefficients accurately. Cutting force coefficients for ball end cutter are generally considered to be the function of the position angle when the average force coefficient identification method is used. While in the helical milling process, although the tool must complete the axial feed on the basis of rotation and revolution, the axial depth of cut per revolution is relatively small, but in the single-factor slot test, the usual cutting depth is relatively large. Gradisek [21] discussed when the depth of cut for the ball end cutter was lower than 0.3 mm, smaller radial immersions yielded larger coefficients than larger immersions, while for the depth of cut was bigger than

| Table 2 | Cutting parameters corresponding to Fig. 10 |
|---------|--------------------------------------------|
| Fig. 10a | Fig. 10b | Fig. 10c | Fig. 10d | Fig. 10e | Fig. 10f |
| Rotation speed (rpm) | 3200 | 3200 | 3200 | 3200 | 3700 | 3700 |
| Feed rate per tooth (mm/t) | 0.04 | 0.04 | 0.05 | 0.06 | 0.06 | 0.06 |
| Axial depth of cut per revolution (mm/rev) | 0.1 | 0.15 | 0.1 | 0.1 | 0.1 | 0.15 |
| Change of axial direction force(N) | 55–80 | 50–75 | 55–80 | 67–90 | 50–80 | 22–52 |
| Number of cycles | 3.5 | 3.5 | 4.5 | 5.5 | 6 | 6 |

Fig. 9 Cutting force data obtained from experiments. a Original forces. b Filtered forces.
0.3 mm, the situation was reversed. In the helical milling process, axial depths of cut per revolution lie in 0.1–0.3 mm/rev, and the tool always keeps axial feed state; the slot test is not completely suitable for the identification of the cutting force coefficient, and the change of cutting forces coefficients needs to be considered in detail.

There are six coefficients including the cutting force coefficients $K_{tc}$, $K_{rc}$, and $K_{ac}$, rubbing force coefficients $K_{te}$, $K_{re}$ and $K_{ae}$ in Eq. (8); the identification of cutting force coefficients is the key problem in modeling of the cutting forces. But due to the different action of two kinds of cutting force coefficients, they need to be analyzed respectively.

Feed rate per tooth in both circumferential and axial directions in the ball helical milling process is very small, in fact, the cutting force coefficient will decrease with the increase of undeformed chip thickness, when the chip thickness increases to a certain range, the cutting force coefficients are constant; the undeformed chip thickness can be expressed as the function of feed rate, namely, undeformed chip thickness is directly represented feed rate, and cutting force coefficients are the nonlinear function of cutting parameters [22].

In order to understand the effects of cutting parameters on the cutting force coefficients, the instantaneous force method is used to identify cutting force coefficients according to each kind of cutting condition; the cutting force coefficients under different cutting conditions are calculated based on the recognition of simulated and measured forces in the time domain. The simulated and experimental forces can be represented as the function between cutting coefficients and relevant cutting parameters: cutting time $t$, orbital angle $\phi$, rotational angle $\theta$, and undeformed chip thickness $h$ and cutting depth per revolution $a_p$.

The indentation force coefficients can be identified through linear fitted to the average axial forces. Namely, supposed that corresponding experimental and simulated cutting forces are same, the rest of cutting force coefficients can be solved from Eq. (26).

$$\begin{align*}
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}
_{\text{Simulated}} &= \begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix}
_{\text{Experimental}} = [A(t, \theta, \phi, h, a)]
\begin{bmatrix}
K_{tc} \\
K_{re} \\
K_{te} \\
K_{rc} \\
K_{ae} \\
K_{ac}
\end{bmatrix} \\
\sum_{n} ||y_i - f_i(t, K_{tc}, K_{rc}, K_{ac}, K_{te}, K_{re}, K_{ae})||_D^2 + ||m-m_p||_M^2
\end{align*}$$ (26)

There will be three-direction cutting forces measured through experiments at time $t_1$, at next time $t_2$, there will also be three-direction cutting forces, corresponding different time will have the corresponding cutting forces; thus, a time period is selected as the basis for analysis. In this way, supposed that the simulated force and the experimental force values are equal in several discrete times, a constrained least square fitting method can be used, simultaneously in order to improve the results, different weights and constraints could be selected for analysis to eliminate the influence of random errors. The objective is to minimize the following equation:

$$\sum_{n} ||y_i - f_i(t, K_{tc}, K_{rc}, K_{ac}, K_{te}, K_{re}, K_{ae})||_D^2 + ||m-m_p||_M^2$$ (27)
Where \( y_i \) is the measured force, \( f_i \) represents the simulated force, \( m \) and \( m_p \) are the iterative solution vector and initial estimation of cutting coefficients values, \( D \) and \( M \) are weighting factors. The discrete time and its corresponding measured force are substituted into Eq. (27), the constrained least square method is used to solve the instantaneous cutting coefficients, and the cutting force coefficients under different cutting conditions can be identified.

### 6 Analysis and discussion

Substituting the identified cutting force coefficients into the cutting force model, the simulated three-way forces under different cutting parameters can be obtained, and the measured forces after being filtered are drawn in the same figure with the simulated ones; the related cutting parameters corresponding to the Fig. 11 are shown in Table 3.

The theoretical model can be used to predict the change of the cutting forces in the ball helical milling of titanium alloy when the cutting parameters change. It can be seen from Fig. 11 that the simulated cutting forces are basically consistent with the measured values, the force predictions adequately capture the general trends of the cutting forces. The presented approach is generally valid and may be extended to complex cutting state, although the predicted cutting forces agree well with the measured ones both in shape and magnitude, there are approximately 17% errors. The largest difference between predictions and experimental results may be due to the following reasons: first, actual measurement values have some noise interference despite filtering being taken to reduce part of the interference, which will affect the waveform of the actual measured values. Second, the analytical model assumes that the tool and workpiece are rigid and there is no tool deflection and vibration, and the effect of the tool run-out is not considered in the modeling process, which will affect not the instantaneous cutting force waveform, but the size of the cutting forces.

### 7 Conclusions

The contact characteristics between ball end cutter and the workpiece were analyzed first, then based on the kinematic characteristics of helical milling technology, specific calculation methods were given for the motion parameters during ball helical milling process.
The indentation force coefficients were identified through linear fitted method based on cutting experiment first; the main six cutting force coefficients were identified using instantaneous force method, and these coefficients were identified with different cutting parameters. The research results could be used to analyze and simulate the cutting forces under different cutting parameters.

Cutting forces in the ball helical milling of titanium alloy under different cutting conditions were simulated, and the changing trend and the size were basically consistent, which verified the correctness of the analytical model of cutting force.

Nomenclature

\( \theta (^\circ) \), Rotational angle; \( \phi (^\circ) \), Orbital angle; \( n (rpm) \), Rotation speed; \( \dot{n} (rpm) \), Orbit speed; \( s_{a} (mm/t) \), Axial feed rate per tooth; \( s_{f} (mm/t) \), Feed rate per tooth at tool center; \( a_{r} (mm/rev) \), Axial depth of cut per revolution; \( N \), Number of cutting edges; \( D_{h} (R_{h})(mm) \), Tool diameter (radius); \( \beta (^\circ) \), Helix angle of tool; \( R (mm) \), Local cutter radius; \( \alpha (^\circ) \), Ramp angle; \( \epsilon (mm) \), Total cutting depth; \( \kappa (mm) \), Axial position angle; \( dF_{x} \), \( dF_{y} \), and \( dF_{z}(N) \), Micro cutting force in tangential, radial and axial directions; \( h (mm) \), Undefomed chip thickness; \( d_{b} (mm) \), Micro cutting width; \( d_{c} (mm) \), Micro cutting depth; \( dS (mm) \), Length of the edge discrete element; \( \phi_{z}(^\circ) \), Radial lag angle; \( e (mm) \), Eccentricity; \( H (mm) \), Thickness of workpiece; \( F_{x}(N) \), Cutting forces in the \( x \) direction; \( F_{y}(N) \), Cutting forces in the \( y \) direction; \( F_{z}(N) \), Cutting forces in the \( z \) direction; \( K_{\alpha_{x}} \), \( K_{\alpha_{r}} \), and \( K_{\alpha_{z}}(N/mm^{2}) \), Cutting force coefficients in tangential, radial and axial directions; \( K_{\tau_{x}} \), \( K_{\tau_{r}} \) and \( K_{\tau_{z}}(N/mm) \), Rubbing force coefficients in tangential, radial and axial directions; \( K_{c}(N/mm2) \), Axial cutting coefficients

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Declarations

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