Acoustic phonons mediated non-equilibrium spin current in the presence of Rashba and Dresselhaus spin-orbit couplings

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Abstract

Influence of electrons interaction with longitudinal acoustic phonons on magnetoelectric and spin-related transport effects are investigated. The considered system is a two dimensional electron gas system with both Rashba and Dresselhaus spin-orbit couplings. The works which have previously been performed in this field, have revealed that the Rashba and Dresselhaus couplings cannot be responsible for spin-current in the non-equilibrium regime. In the current Letter, a semi-classical method was employed using the Boltzmann approach and it was shown that the spin-current of the system, in general, does not go all the way to zero when the electron-phonon coupling is taken into account. It was also shown that spin accumulation of the system could be influenced by electron-phonon coupling.

Keywords: spintronics- spin polarized transport- electron-phonon coupling- spin-orbit coupling- non-equilibrium spin current.

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1. Introduction

Spintronics has attracted more and more attention, from both theoretical and experimental sides, during the last several years. Effective control of spin polarized transport is very important, especially for practical applications; for example, in multilayers. Therefore, many studies have been conducted to explain this phenomenon [1, 2]. According to the results of these studies, manipulation of spin can be realized by applying magnetic fields or Rashba interaction. The Rashba interaction arises from inversion asymmetry in the system and can be effectively controlled by applying a gate voltage [3, 4, 5]. Due to the tunable nature of Rashba coupling, it is the most popular method for manipulation of electron spin. This kind of spin-orbit interaction (SOI) plays a central role in Datta and Das spin field-effect transistor (SFET) [6]. Generally, SOI has a significant role in magnetoresistance effects, known as weak localization [7]. In the field of spin-transport, many interesting features have been demonstrated for this type of spin-orbit coupling (SOC) [8, 9, 10]. For example, it has been verified that spin-orbit scattering can induce localization/antilocalization transition in a two-dimensional electron gas (2DEG) system [11, 12]. Meanwhile, Rashba interaction has been also suggested for spin interference devices and spin-filters [13, 14, 15].

Another spin-orbit coupling that provides a new parameter which should be considered in designing spin-dependent devices, is the Dresselhaus coupling. Dresselhaus coupling is induced by the bulk inversion asymmetry [16]. Effects of spin-orbit couplings (SOCs) in semiconductors have attracted growing interest due to their roles in semiconductor spintronics.
Manipulation of spin makes new functionality in electronic devices. Control of spin accumulation by spin-orbit interactions has a great potential in the field of spintronics [17, 18, 19, 20]. In the presence of these two different spin-orbit interactions, i.e. the Rashba and Dresselhaus couplings, in a two-dimensional electron gas system, one can effectively control both magnitude and direction of non-equilibrium spin accumulation [19]. Meanwhile, spin-current vanishes exactly in the non-equilibrium regime induced by an in-plane driving electric field [19]. This implies that there is no spin-polarized current accompanied by the spin accumulation of the system [19]. Rashba obtained non-vanishing spin-current in equilibrium state. Therefore, this spin-current can not describe any real transport of spins in non-equilibrium regime induced by an in-plane driving electric field [21]. For a non-equilibrium system, a highly anisotropic spin response to an in-plane electric field has been discovered [22]. However, as mentioned before, based on the semiclassical approach, Huang and Hu showed that the non-equilibrium spin current vanishes exactly in two-dimensional electron gases, in the presence of both Rashba and Dresselhaus couplings. Meanwhile non-equilibrium spin accumulation can be obtained in this case [19]. In addition, Inoue et al. obtained similar results based on the Green’s function approach for two-dimensional electron gases [23]. It should be noted that the work described by Huang and Hu [19], has been based on a semi-classical approach developed by Schliemann and Loss [24]. They formulated the anisotropic effects of energy dispersion relation and scattering matrixes in the presence of spin-orbit couplings. Meanwhile, the exact solution to the Boltzmann equation for two-dimensional anisotropic
systems was provided by Výborný et al. [25], in which it was shown that, for a Rashba type two-band model, discrepancy between exact and approximate Schliemann and Loss approach, remains only at the level of higher order corrections [25]. Therefore, the Schliemann and Loss approach was applied so that the results of the current letter could be compared with the results of Huang and Hu [19] in the same theoretical framework. It can be easily shown that, in the presence of both spin-orbit couplings i.e. Rashba and Dresselhaus interactions, Schliemann and Loss method is still a good approximation in comparison with the exact Výborný approach, which could be due to the fact that anisotropic term of the energy dispersion relation (which was induced by Dresselhaus coupling) is negligible and the anisotropic effects can be entered only through the scattering matrixes.

The polaronic properties have been studied theoretically in the presence of the Rashba and Dresselhaus spin-orbit interactions and weak electron-phonon coupling in a pioneering work [26]. It was demonstrated that in the presence of both spin-orbit couplings, self-energy correction of the electron energy and polaron effective mass show an angular anisotropy [26]. In this letter, based on the mentioned semiclassical approach, influence of the electron-phonon scattering has been considered on spin-transport quantities in a two-dimensional electron gas when the spin-orbit interactions are present. It was verified that the electron-phonon scattering results in non-vanishing spin-current that could be controlled by spin-orbit interactions. It has also been found that the spin-current is influenced by the electron-phonon coupling. The details of the numerical results have briefly been addressed in the present letter.
2. Model and approach

The total Hamiltonian is given by

\[ \hat{H} = \hat{H}_0 + \hat{V}_{im} + \hat{H}_{el-ph}, \]  

(1)

in which, \( \hat{H}_0 \) is the kinetic energy and spin-orbit interactions (including both Rashba and Dresselhaus spin-orbit couplings), for a 2DEG namely

\[ \hat{H}_0 = \frac{\hbar^2 k^2}{2m} + \alpha (\hat{\sigma}_x k_y - \hat{\sigma}_y k_x) + \beta (\hat{\sigma}_x k_x - \hat{\sigma}_y k_y), \]  

(2)

where \( \mathbf{k} \) is the wave vector of conduction electrons, \( \sigma_i (i = x, y) \) are Pauli matrices, \( \alpha \) and \( \beta \) denote the strengths of the Rashba and Dresselhaus interaction, respectively.

For a given wave vector \( \mathbf{k} \),

\[ | \mathbf{k} \lambda > = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i \frac{\phi_k}{2}} \\ \lambda e^{-i \frac{\phi_k}{2}} \end{pmatrix}, \]  

(3)

are the eigenfunctions of \( \hat{H}_0 \) where \( \lambda = \pm 1 \) and \( \phi_k \) defined as

\[ \tan \phi_k = \frac{\alpha k_x + \beta k_y}{\alpha k_y - \beta k_x}. \]  

(4)

The corresponding eigenvalues of \( \hat{H}_0 \) are

\[ \epsilon_{\mathbf{k}\lambda} = \frac{\hbar^2 k^2}{2m} + \lambda \sqrt{(\alpha^2 + \beta^2)k^2 + 4\alpha\beta k_x k_y}. \]  

(5)

The expectation value of spin of electrons along the x and y directions in a given state \( | \mathbf{k} \lambda > \) can easily be found as follows

\[ S_{\lambda,x}^{(0)}(\mathbf{k}) = \frac{\hbar}{2} \lambda \cos(\phi_k), \quad S_{\lambda,y}^{(0)}(\mathbf{k}) = \frac{-\hbar}{2} \lambda \sin(\phi_k). \]  

(6)
The second term of Hamiltonian, \( V_{im}(r) \) is potential of impurities given as

\[
V_{im}(r) = \sum_i (J\sigma.\vec{m}(r))\delta(r - r_i),
\]

where the sum is performed over all of the randomly distributed impurities, \( J \) is the exchange interaction strength of magnetic impurities with conduction electrons and \( \vec{m}(r) \) is the unit vector along the local magnetization.

\[
<k'\lambda'q'|V_{im}(r)|k\lambda q> = C_{k',k}\delta_{n_q,n_{q'}}
\left(
\begin{array}{cc}
+J_z & J_x - iJ_y \\
J_x + iJ_y & -J_z
\end{array}
\right).
\]

In which

\[
J_z = m_zJ, \quad J_y = m_yJ, \quad J_x = m_xJ,
\]

and

\[
C_{k',k} = (1/\sqrt{L_xL_y})\sum_j\exp(i(\vec{k'} - \vec{k}).\vec{r}_j).
\]

For long range magnetic interactions, because of the shape anisotropy, we take \( m_z = 0 \) and for randomly oriented magnetic moments of impurities, one can assume

\[
<m_x> = <m_y> = 0, \quad <m_x^2> = <m_y^2> = 1/2.
\]

Since it was assumed that the magnetic moments of the impurities have been randomly oriented, therefore they can not be responsible for spin-polarized effects and spin current.

The last term of the Hamiltonian, \( \hat{H}_{el-ph} \) is electron-phonon interaction and can be expressed as \( [27] \),

\[
\hat{H}_{el-ph} = D_{ac}\nabla.\vec{u}(r),
\]
here, $D_{ac}$ is defined as deformation potential for electron scattering by acoustic phonons and $\vec{u}(r)$ is a small displacement vector of an ion from its equilibrium position, $\vec{R}$.

For a two-dimensional system, the displacement is determined as

$$\vec{u}(r) = \sum_q \sqrt{\frac{\hbar}{2MN\omega_q}} e_{q} \left[ a_q e^{i\vec{q}.\vec{r}} + a_q^\dagger e^{-i\vec{q}.\vec{r}} \right],$$

(13)

where, $M$ and $N$ are mass and number of the ions, respectively. $\hat{e}_q$ is a unit vector in displacement direction and $\omega_q = V_s q$ in which $V_s$ is the sound velocity and $q$ is wave vector of phonon.

Using from eq. (13), the electron-phonon interaction can be written as

$$\hat{H}_{el-ph} = D_{ac} \sum_q \sqrt{\frac{\hbar}{2MNW_q}} (i\hat{e}_q \cdot \vec{q}) \left[ a_q e^{i\vec{q}.\vec{r}} - a_q^\dagger e^{-i\vec{q}.\vec{r}} \right].$$

(14)

by defining

$$c(q) = D_{ac} \sqrt{\frac{\hbar}{2MNW_q}} (i\hat{e}_q \cdot \vec{q}),$$

(15)

we obtain the following result, directly

$$\hat{H}_{el-ph} = \sum_q \left[ c(q) a_q e^{i\vec{q}.\vec{r}} + c^*(q) a_q^\dagger e^{-i\vec{q}.\vec{r}} \right].$$

(16)

The eigenstate of the phonon Hamiltonian in harmonic approximation is defined by $|n_q\rangle$, where $n_q$ is the phonon occupation number, so we can define a new basis as follows, $| k\lambda n_q \rangle = | k\lambda \rangle \otimes | n_q \rangle$. Scattering matrix of electron-phonon interaction is given as follows

$$< k'\lambda' n'_q | \hat{H}_{el-ph} | k\lambda n_q > = \begin{cases} \delta_{n'_q, n_q - 1} \delta_{\lambda', \lambda} c(q) \sqrt{n_q}, & \text{if } k' = k + q, \\ \delta_{n'_q, n_q + 1} \delta_{\lambda', \lambda} c^*(q) \sqrt{n_q + 1}, & \text{if } k' = k - q. \end{cases}$$

(17)
As mentioned, in the current letter we have employed the procedure which have been used in \[19\]. The two last terms of the Hamiltonian are responsible for both spin-dependent and spin-independent relaxation mechanisms. If we rename these two terms as
\[
V = \hat{V}_{im} + \hat{H}_{el-ph}
\] (18)

The Lippman-Schwinger scattering state of a conduction electron reads,
\[
| k\lambda n >_{scat} = | k\lambda n > + \sum_{k',q',\lambda'} V_{k'\lambda'n',k\lambda n} \frac{\epsilon_{k\lambda} - \epsilon_{k'\lambda'} + i\eta}{\epsilon_{k\lambda} - \epsilon_{k'\lambda'} + i\eta} | k'q' >_{\lambda'},
\] (19)

where $\eta$ is a small positive quantity. Then the spin expectation value in a given scattering state is
\[
S_{\lambda,i}(k) = S^{(0)}_{\lambda,i}(k) + \hbar \sum_{k',q',\lambda'} \text{Re} \left[ \frac{V_{k'\lambda'n',k\lambda n} < \sigma_i >_{k'\lambda'n',k\lambda n} \epsilon_{k\lambda} - \epsilon_{k'\lambda'} + i\eta}{\epsilon_{k\lambda} - \epsilon_{k'\lambda'} + i\eta} \right].
\] (20)

Here we have defined $S_{\lambda,i}(k) = \langle k\lambda n | \hat{S}_i | k\lambda n >_{scat}$. Therefore one can obtain spin expectation value as seen in eq. (21).
\[
S_{\lambda,i}(k) = S^{(0)}_{\lambda,i}(k) + \hbar \sum_{k',q',\lambda'} \text{Re} \left[ V_{k'\lambda'n',k\lambda n} < \sigma_i >_{k'\lambda'n',k\lambda n} \right] Pr \left[ \epsilon_{k\lambda} - \epsilon_{k'\lambda'} \right] i\pi \delta(\epsilon_{k\lambda} - \epsilon_{k'\lambda'}).
\] (21)

Here $< \sigma_i >_{k'\lambda',k\lambda}$ is the expectation value of the Pauli Matrix for a Lippman-Schwinger scattering state. Then, net spin density can be giving by
\[
< S_i > = \sum_{k,q,\lambda} S_{\lambda,i}(k)f_\lambda(k,q),
\] (22)

in which $f_\lambda(k,q)$ is non-equilibrium distribution function of conduction electrons. Where in the absence of external electric field, this can be reduced to
the equilibrium Fermi-Dirac distribution,
\[ f_{\lambda}(k, q) = f_0(\epsilon_{k\lambda}) = \frac{1}{1 + e^{(\epsilon_{k\lambda} - \epsilon_F)/k_BT}}. \]  
\( (23) \)

We have used the Debye model, so the summation over \( q \) is easily calculated by replacing it with an integral. This integral can be considered to be evaluated in an interval starting from \( q = 0 \) to the Debye wave vector, \( q_D \). This wave vector is directly related to the free-electron Fermi wave vector. In two-dimensional metals, \( q_D = \sqrt{2} z k_F \), where \( k_F \) is the free-electron Fermi wave vector, and \( z \) is the nominal valence \[28\].

In the presence of scatterings, the non-equilibrium distribution function will be derived by solving the Boltzmann equation (in steady state for a homogeneous system),
\[ \dot{k} \frac{\partial f_{\lambda}}{\partial k} = (\frac{\partial f_{\lambda}}{\partial t})_{\text{coll}}, \]  
\( (24) \)

where \( \dot{k} = -\frac{eE}{\hbar} \) and \( (\frac{\partial f_{\lambda}}{\partial t})_{\text{coll}} \) is called the collision integral, that in elastic scattering approximation reads \[29\]
\[ (\frac{\partial f_{\lambda}}{\partial t})_{\text{coll}} = - \sum_{k'q'\lambda'} W_{k'\lambda'q', k\lambda q} f_{\lambda'}(k, q)(1 - f_{\lambda'}(k', q')) \delta(\epsilon_{k\lambda} - \epsilon_{k'\lambda'}) + \sum_{k'q'\lambda'} W_{k'\lambda'q', k\lambda q} f_{\lambda'}(k', q')(1 - f_{\lambda}(k, q)) \delta(\epsilon_{k\lambda} - \epsilon_{k'\lambda'}). \]  
\( (25) \)

In this equation, \( W_{k'\lambda'q', k\lambda q} \) are the transition probabilities that are given by the Fermi’s golden rule, \( W_{k'\lambda'q', k\lambda q} = \frac{2\pi}{\hbar} |V_{k'\lambda'q', k\lambda q}|^2 \).

Since \( \delta_{n'q, nq} \) selects only the diagonal elements of \( |V_{k'\lambda'q', k\lambda q}|^2 \) while \( \delta_{n'q-1, nq} \delta_{k'k+q} \) and \( \delta_{n'q+1, nq} \delta_{k'k-q} \) select some of the non-diagonal elements of \( |V_{k'\lambda'q', k\lambda q}|^2 \), therefore one can easily obtain
\[ |V_{k'\lambda'q', k\lambda q}|^2 = |< k'\lambda' q' | \hat{H}_{el-ph} | k\lambda q >|^2 + |< k'\lambda' q' | V_{imm} | k\lambda q >|^2. \]  
\( (26) \)
and accordingly

\[ W_{k'\chi'q',k\lambda q} = W_{k'\chi'q',k\lambda q}^{(1)} + W_{k'\chi'q',k\lambda q}^{(2)} + W_{k'\chi'q',k\lambda q}^{(3)}, \tag{27} \]

Where \( W_{k'\chi'q',k\lambda q}^{(1)} \) comes from the impurity potential,

\[ W_{k'\chi'q',k\lambda q}^{(1)} = \begin{pmatrix} 0 & J^2 \\ J^2 & 0 \end{pmatrix} n_s \delta_{n_q,n_q}. \tag{28} \]

In which we have used the following approximation, \( \frac{1}{L_x L_y} \sum_j \sum_{j'} \exp(i(k' - k).(\mathbf{r}_j - \mathbf{r}_{j'})) = n_i \), where \( n_i \) is impurity density, \( L_x, L_y \) are system dimensions and it should be noted that the summations have been performed over the random position of impurities.

Meanwhile \( W_{k'\chi'q',k\lambda q}^{(2)} \) and \( W_{k'\chi'q',k\lambda q}^{(3)} \) are representing the electron-phonon interaction, where \( W_{k'\chi'q',k\lambda q}^{(2)} \) presents phonon absorption contribution,

\[ W_{k'\chi'q',k\lambda q}^{(2)} = \delta_{n_q,n_q} - 1 \delta_{k',k+q} \delta_{\chi',\lambda} \begin{pmatrix} c_q \sqrt{n_q} & 0 \\ 0 & c_q \sqrt{n_q} \end{pmatrix}, \tag{29} \]

and \( W_{k'\chi'q',k\lambda q}^{(3)} \) should be considered for the case of emission,

\[ W_{k'\chi'q',k\lambda q}^{(3)} = \delta_{n_q,n_q+1} \delta_{k',k-q} \delta_{\chi',\lambda} \begin{pmatrix} c_q^* \sqrt{n_q+1} & 0 \\ 0 & c_q^* \sqrt{n_q+1} \end{pmatrix}. \tag{30} \]

The energy dispersion of conduction electrons becomes anisotropic in the presence of the spin-orbit interactions. This anisotropy manifests itself in the scattering process and one can choose an anisotropic solution to the Boltzmann equation as follows \[19, 24\],

\[ \delta f_{\lambda}(k,q) = e \frac{\partial f_0(c_{k\lambda})}{\partial e_{k\lambda}} [a_{kq\lambda}(E,v_{k\lambda}) + b_{kq\lambda}(E \times \mathbf{e}_z)\cdot v_{k\lambda}]. \tag{31} \]
\[ \mathbf{v}_{k\lambda} = \frac{1}{\hbar} \nabla_k \epsilon_{k\lambda}, \quad (32) \]

where \( \mathbf{v}_{k\lambda} \) is velocity of conduction electrons, \( \delta f_\lambda = f_\lambda - f_0 \), \( \hat{e}_z \) is a unit vector perpendicular to the two-dimensional plane, \( a_{kq\lambda} \) and \( b_{kq\lambda} \) are two unknown coefficients that can be determined self-consistently by using eq. (24) and eq. (31) in which \( b_{kq\lambda}(\mathbf{E} \times \hat{e}_z).\mathbf{v}_{k\lambda} \) arises due to the anisotropic nature of the system. Then one can find that the unknown coefficients \( a_{kq\lambda} \) and \( b_{kq\lambda} \) should satisfy following equations [19]

\[
\frac{a_{kq\lambda}}{\tau^{(1)}_{kq\lambda}} + \frac{b_{kq\lambda}}{\tau^{(2)}_{kq\lambda}} = 1, \quad (33)
\]

\[
\frac{a_{kq\lambda}}{\tau^{(2)}_{kq\lambda}} - \frac{b_{kq\lambda}}{\tau^{(1)}_{kq\lambda}} = 0, \quad (34)
\]

From Eqs. (33) and (34), one can easily obtain

\[
a_{kq\lambda} = \frac{\tau^{(1)}_{kq\lambda}}{1 + \left[\frac{\tau^{(2)}_{kq\lambda}}{\tau^{(1)}_{kq\lambda}}\right]^2}, \quad b_{kq\lambda} = \frac{\tau^{(2)}_{kq\lambda}}{1 + \left[\frac{\tau^{(2)}_{kq\lambda}}{\tau^{(1)}_{kq\lambda}}\right]^2}, \quad (35)
\]

in which \( \tau^{(1)}_{kq\lambda} \) and \( \tau^{(2)}_{kq\lambda} \) are two relaxation times, defined by

\[
\frac{1}{\tau^{(1)}_{kq\lambda}} = \sum_{k'q',\lambda'} W_{k'\lambda'q,k\lambda_nq} \left\{ 1 - \left| \frac{\mathbf{v}_{k'\lambda'}}{\mathbf{v}_{k\lambda}} \right| \cos[\theta(\mathbf{v}_{k\lambda} \wedge \mathbf{v}_{k'\lambda'})] \right\}, \quad (36)
\]

\[
\frac{1}{\tau^{(2)}_{kq\lambda}} = \sum_{k'q',\lambda'} W_{k'\lambda'q,k\lambda_nq} \left| \frac{\mathbf{v}_{k'\lambda'}}{\mathbf{v}_{k\lambda}} \right| \sin[\theta(\mathbf{v}_{k\lambda} \wedge \mathbf{v}_{k'\lambda'})], \quad (37)
\]

where \( \theta(\mathbf{v}_{k\lambda} \wedge \mathbf{v}_{k'\lambda'}) \) is the angle between \( \mathbf{v}_{k\lambda} \) and \( \mathbf{v}_{k'\lambda'} \).

The spin current operator is defined as [21]

\[
\hat{j}_z^i = \frac{\hbar}{2} \{ \sigma_i, \hat{v}_x \}, \quad (38)
\]
where \( \hat{v}_x = \hbar^{-1}\left( \frac{\partial \hat{H}}{\partial k_x} \right) \) is known as velocity operator. Expectation value of spin-current in a given scattering state i.e. \( J^i_x(\mathbf{k}, \lambda) = \langle \mathbf{k}\lambda n_q | \hat{J}^i_x | \mathbf{k}\lambda n_q \rangle_{\text{scat}} \) is as follows

\[
J^i_x(\mathbf{k}, \lambda) = J^{i(0)}_x(\mathbf{k}, \lambda) + \hbar \sum_{\mathbf{k}'q',\lambda'} \text{Re}[V_{\mathbf{k}'\lambda'\mathbf{n}_q, k\lambda n_q} < J^i_x >_{\mathbf{k}'\lambda'\mathbf{n}_q, k\lambda n_q} \frac{1}{\epsilon_{k\lambda} - \epsilon_{k'\lambda'}}] \\
- V_{\mathbf{k}'\lambda'\mathbf{n}_q, k\lambda n_q} < J^i_x >_{\mathbf{k}'\lambda'\mathbf{n}_q, k\lambda n_q} i\pi \delta(\epsilon_{k\lambda} - \epsilon_{k'\lambda'})].
\]

In which we have defined \( J^{i(0)}_x = \langle \mathbf{k}\lambda n_q | \hat{J}^i_x | \mathbf{k}\lambda n_q \rangle \). Then the transport spin-current in x direction with spin parallel to the x or y axes, is given by

\[
J^i_x = \sum_{\mathbf{k}q,\lambda} \hat{J}^i_x(\mathbf{k}, \lambda) \delta f_\lambda(\mathbf{k}, \mathbf{q}), \quad (i = s_x, s_y)
\]

3. Results

In the current letter, the spin accumulation and spin-current of a two dimensional electron gas have been obtained in the presence of the Rashba, Dresselhaus and electron-phonon interactions. This was accomplished by utilizing a semi-classical model developed for anisotropic systems. This anisotropy is induced by spin-orbit couplings in the scattering matrix or in the energy dispersion. As mentioned before, the Rashba and Dresselhaus couplings cannot be responsible for generation of spin-current in non-equilibrium regime [19, 23]. Meanwhile, non-equilibrium spin accumulation is effectively controlled by these spin-orbit interactions [19]. Results of the present letter show that the electron-phonon interaction has a considerable role in the generation of spin-current which was expected to be obtained by spin-orbit couplings. Spin accumulation of the system is also controlled by the strength
of electron-phonon coupling.

In the present system, electric field was assumed to be applied along the $x$ direction and the numerical parameters have been chosen as follows $\epsilon_f = 10\,\text{eV}$ is the Fermi energy, $J = 0.1\,\text{eV}$, $n_i = 10^{10}\,\text{cm}^{-2}$ is the density of impurities, $T = 1\,\text{K}$ and $V_s = 4950\,\text{m/s}$. In addition Rashba and Dresselhaus couplings have been denoted by $\epsilon_\alpha = m_\alpha^2/\hbar^2$, $\epsilon_\beta = m_\beta^2/\hbar^2$, respectively. The Rashba coupling can reach high values up to $0.2\,\text{eV}$ in epitaxial graphene grown on a Ni(111) substrate [30]. However, in the present letter, a typically lower range has been chosen for SOC as reported for other materials.

It has been shown that the electron-phonon interaction could not be considered as an underestimating effect on spin-dependent mechanisms. It was also demonstrated that, at low electron-phonon coupling strengths, the lattice vibrations are more effective.

In Fig. 1 longitudinal and Fig. 2 transverse spin-current has been depicted as a function of the deformation potential. Spin-current has been induced due to the lattice-electron interactions. These figures clearly show that spin-current of the system has a accountable value, in which its sign and magnitude could be controlled by the SOCs. At the same time, as shown in Figs. 3 and 4, longitudinal and transverse spin accumulations can be effectively changed by the Rashba and Dresselhaus couplings.

Therefore these results show that when effect of electron-phonon interaction is taken into account, in the semi-classical regime, it turns out that both components of the spin-current could take non-zero values. It seems that details of the scattering potential has an important role in the generation of spin-current in the presence of the Rashba and Dresselhaus couplings. As
reported in [19], Huang and Hu found that short-range delta function impurity scatterings (which actually have spherical symmetry) result in zero spin-polarized current in the system.

An important feature which can be inferred from the results is the fact that the absolute value of spin-current and spin accumulation decreases for high electron-phonon couplings, as depicted in Fig. 1- Fig. 4. Spin-current induced by the lattice longitudinal vibrations disappears in the limit of high deformation potential and rapidly increases for low electron-phonon couplings. Unlike the spin-orbit couplings, electron-phonon interaction could change the order of magnitude of the spin-current. However, it should be noted that numerical results reveals that, in the limit of $D_{ac} \rightarrow 0$ spin current vanishes abruptly, and it was a numerical discontinuity (not included in the figures) at $D_{ac} = 0$. If identical conditions are chosen from [19] i.e. when $D_{ac} = 0$ and in the case of nonmagnetic impurities, $J = 0$, numerical results show that spin-current identically vanishes which is in agreement with the results that have been pointed out in [19] for identical conditions.

As mentioned before, anisotropic effects can be induced by two different sources: the energy dispersion relation and the scattering matrix of the anisotropic relaxations. In the current case, according to the present calculations, the first source of anisotropy is small and could be neglected. For isotropic spin-independent relaxations, the scattering matrix should be an isotropic function, even for anisotropic eigen-state spinors. However, in the current case of electron-phonon scatterings, it seems that redistribution of the carriers’ population by this relaxation mechanism could change the ensemble average of Rashba and Dresselhaus k-dependent effective magnetic
field; i.e., in the presence of this q-dependent relaxation, where \( q \) is the phonon wave number, \( \delta f_\lambda(k, q) \) could no longer exhibit odd function properties. It should be noted that this property \( (\delta f_\lambda(-k) = -\delta f_\lambda(k)) \) was shown to be responsible for zero spin current in the system \([19]\). Meanwhile, this new distribution function, \( \delta f_\lambda(k, q) \), can produce non-vanishing spin current in the system since the effective field in the present case has been modified. However, increasing the electron-phonon coupling strength ultimately decreases both spin current and spin accumulation of the system, as shown in Fig. \( \square \) Fig. \( \square \). Unlike the intermediate range of deformation potential at high electron-phonon couplings, the momentum of electrons is effectively randomized by the electron-phonon interaction since the relaxation time of the states decreases. Therefore in this case, population of the carriers approaches to the limit of isotropic scatterings, in which the spin current of the system vanishes.

4. Conclusion

In this letter, a semi-classical approach have been implemented for studying magnetoelectric effects of a 2DEG system. The primary focus of this letter was on showing that the electron-phonon coupling has an important role in generation of the spin-current in non-equilibrium regime since it was verified that the Rashba and Dresselhaus couplings (when the electron-phonon coupling is absent) cannot be responsible for spin-current in this regime. It was numerically verified that, even at low electron-phonon couplings, the lattice vibrations can mediate in the spin-transport process, modulated by spin-orbit interactions.
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Fig. 1: Longitudinal spin current as a function of the deformation potential for different SO couplings.

Fig. 2: Transverse spin current as a function of the deformation potential for different SO couplings.

Fig. 3: Longitudinal spin accumulation as a function of the deformation potential for different SO couplings.

Fig. 4: Transverse spin accumulation as a function of the deformation potential for different SO couplings.
Figure 1:
$J^{v}_{x}/(\epsilon_{F}/k_{F})$

Figure 2:

$S_{x}/\hbar$

Figure 3:
Figure 4: