The layered perovskite oxide Sr$_2$RuO$_4$ has attracted considerable experimental and theoretical attention since the recent discovery of superconductivity in this compound. Quantitative similarities between the Fermi liquid in $^3$He and that in Sr$_2$RuO$_4$ hint at the possibility of p-wave superconducting pairing. A growing body of experimental evidence, including results obtained from muon spin relaxation, NMR, 1/T$_1$, and Knight shift, neutron scattering, impurity effect, and specific heat measurements, has shown that the pairing symmetry is unconventional, most likely p-wave.

This compound shows also interesting normal state transport properties characterized by multi-bands effects, and provides an ideal opportunity to investigate the crossover from standard to non standard conduction processes. The in plane zero-field resistivity exhibits a Fermi-liquid like behaviour up to $T_{FL}$ $\approx$ 25 K; above this temperature $\rho_{ab}$ rises monotonically with a curvature which is much weaker than $T^2$. Moreover, around 1300 K the in-plane mean free path falls smoothly to less than 1 Å with no sign of resistivity saturation at the Mott-Ioffe-Regel limit, clearly indicating an unconventional high temperature conducting mechanism.

$\rho_c$ reaches a maximum at 120 K then slowly decreases with increasing temperature. Below $\sim T_{FL}$ the out-of-plane resistivity has a quadratic temperature dependence indicating that a crossover to a Fermi-liquid state takes place below 25K prior to the superconducting transition. Thus, below $T_{FL}$ Sr$_2$RuO$_4$ has a resistivity with a quadratic temperature dependence in all directions and it is extremely anisotropic, with $\rho_{ab}/\rho_c \gg 500$, a ratio that is observed in most of the cuprate materials.

The Hall coefficient exhibits a complicated temperature behavior below 1 K it becomes almost temperature independent assuming a value of $-1.15 \times 10^{-10} \text{m}^2/\text{C}$; it has a strong temperature dependence below 25 K then, changes sign at temperatures approximately above 30 K and shows a return to negative values above approximately 130 K.

The $c$–axis magnetoresistance is large and positive and varies linearly with the applied magnetic field; with the increase of the temperature it falls sharply becoming negative above 75 K. The in-plane magnetoresistance is positive and large at low temperatures, then decreases as T is raised up to 80 K.

The aim of the present work is to show, by means of the Boltzmann equation, that an overall agreement for the in-plane magnetotransport quantities can be reproduced in a temperature range up to 150 K, where the relaxation time approximation is applicable, i.e. the mean free path is greater than the lattice spacings. In order to determine the magnetotransport quantities, one has to calculate the conductivity tensor up to second order and this in turn implies the knowledge of the energy band spectrum as well as the explicit expression for the relaxation rate.

The information of the electronic structure is given in Ref. 1 for the three bands crossing the Fermi level and the loss of coherence of the conduction electrons is simulated by assuming an ad hoc temperature behavior for the scattering rates for electrons belonging to different bands. More precisely, the key assumption is that the temperature behaviour of the scattering rate of the carriers is modified by the presence of an applied magnetic field. We argue that the relaxation rate produced by spin fluctuations is suppressed in presence of an external magnetic field implying that the main contribution comes only from the usual electron-electron, impurity, or phonon- mediated scattering, which sum up to be $\sim T^2$ at low temperatures. This effect applies in determining the Hall- and magneto- resistance.

On the other hand, in the absence of magnetic field, according to the Matthiessen rule, the induced spin-fluctuation scattering rate adds to the previous relaxation mechanisms in determining the total resistivity of the system.

The presence of an additional scattering rate is supported by the experimental evidence that the Sr$_2$RuO$_4$ is close to magnetic instabilities which are the source of enhanced spin fluctuations at different $q$ points of the Brillouin zone, thus giving rise to large scattering amplitudes between charge carriers and the spin fluctuations themselves.

$^{17}$O NMR measurements probe spin correlations in Ru $d_{xy}$ and $d_{xz,yz}$ orbitals separately and show that only $\chi_{xy}$ increases monotonically with decreasing temperature down to about 40K, following a Curie-like behaviour, then turns over and tends to level below $T_{FL}$, implying that the spin correlations in the $d_{xy}$ band are predominantly ferromagnetic in origin.

Furthermore, by comparing $^{101}$I/T$_1$T at the Ru site...
and $^{171}/T_1 T$ at the planar O site, due to the different dependence of their hyperfine form factor in k-space, one can probe whether the in-plane spin correlations are ferromagnetic or antiferromagnetic. It turns out that both $^{171}/T_1 T$ and $^{171}/T_1 T$ increase monotonically down to $T_{FL}$, and almost saturate in a Körting-like behaviour.

Moreover, inelastic neutron scattering measurements in the normal state reveal the existence of incommensurate magnetic spin fluctuations located at $q_0 = (\pm 0.6\pi/a, \pm 0.6\pi/a, 0)$ due to the pronounced nesting properties of the almost one-dimensional $d_{x^2-y^2}$ bands. In fact, the 1D sheets can be schematically described by parallel planes separated by $\tilde{q} = \pm 2\pi/3a$, running both in the $x$ and in the $y$ directions which give rise to dynamical nesting effects at the wave vector $k = (\tilde{q}, k_y)$, $k = (k_x, \tilde{q})$, and in particular at $\tilde{q} = (\tilde{q}, \tilde{q})$.

Though not as evident as in the $\chi_{xy}$ and the NMR measurements, due to the presence of few experimental points, the temperature dependence of the imaginary part of the susceptibility $\chi'$, at low energy and at wave vector $q_0$, exhibits a sharp decrease upon temperature increase above a temperature of $\sim T_{FL}$.

Combining the results of NMR, nuclear spin-lattice relaxation rate, and inelastic neutron scattering measurements, it is possible to draw the following physical picture for the Sr$_2$RuO$_4$. There is a strong enhancement of spin fluctuations above $T_{FL}$, mainly due to ferromagnetic correlations between the electrons in the $d_{xy}$ band, as revealed from Knight shifts experiments and nuclear spin relaxation rate, and due to incommensurate contributions from the nesting properties of the almost 1D $d_{x^2-y^2}$ bands. Hence, the main features of the magnetic response turn out to be decoupled for the electrons in the $d_{xy}$ and in the $d_{x^2-y^2}$ bands respectively.

We consider that the same decoupling manifests in the transport properties.

One important experimental observation is that there is a close relation between the change in the magnetic response and the resistivity measurements. Below $\sim T_{FL}$ the in-plane resistivity has a quadratic temperature dependence, while above $\sim T_{FL}$ a superlinear term adds to the $T^2$ contribution.

We assume that above $\sim T_{FL}$ the linear contribution in the in-plane resistivity is mainly determined by small momentum scattering in the $d_{xy}$ band derived by the ferromagnetic spin fluctuations, and we consider also the slow temperature variation of the relaxation rates ($\sim T \ln T$) which comes from the scattering at large momentum transfer due to the incommensurate spin fluctuations for the $d_{x^2-y^2}$ electrons. It is worthwhile pointing out that according to our calculations, the latter does not give any substantial qualitative and quantitative change in the transport properties, mainly due to the limited phase space allowed for the scattering processes.

Specifying these considerations to the multi-band system in question, we assume only for the $\gamma$-band, a scattering rate proportional to $T^2$ in calculating the Hall coefficient and the magnetoresistance, with an additional term reproducing the effects of scattering by ferromagnetic spin fluctuations, to determine the zero-field resistivity.

For a 2D system, the temperature dependence of the scattering rate due to this mechanism is given by:

$$\tau_{sf}^{-1} \sim \int q^2 dq \int \text{Im} \chi(q, \omega) \frac{\partial}{\partial T} n(\omega),$$

where $n(\omega)$ is the Bose distribution and $\chi(q, \omega)$ is the dynamical susceptibility. Within the self-consistent spin fluctuation theory, in a paramagnet close to a ferromagnetic instability, one gets $\tau_{sf}^{-1} \sim T$.

For the other two bands, a quadratic dependence of the scattering rates on the temperature for all the calculated quantities is assumed.

To find the transport coefficients, we must calculate the current defined as:

$$J = \int g(v) v dk. \quad (0.1)$$

where $g(v)$ is the local distribution of electrons.

Confining ourselves to an expansion of order $B^2$, one can easily obtain the following general formula:

$$J_\alpha = \sigma_0^{\alpha \beta} E_\beta + \sigma_1^{\alpha \beta \gamma} E_\beta B_\gamma + \sigma_2^{\alpha \beta \gamma \delta} E_\beta B_\gamma B_\delta, \quad (0.2)$$

where the summation convention over the indices of the cartesian components is assumed and $E_\alpha$ and $B_\beta$ are the components of the external electric and magnetic field, respectively.

To calculate the normal resistivity, the Hall coefficient and the magnetoresistance for a multi-band case, we write the total current as the sum of the contributions coming from the three bands and then we invert the matrix connecting $J$ and $\mathbf{E}$. Neglecting powers above $B^2$, we have:

$$\rho_0 = \frac{1}{\sigma_0^{\text{Tot}}}, \quad (0.3)$$

$$\rho_H = -\rho_0^2 \frac{T_{\text{Tot}}}{\sigma_H^{\text{Tot}}}, \quad (0.4)$$

$$\rho_{MR} = -\rho_0^2 \left[ \sigma_{MR}^{\text{Tot}} + \rho_0 (\sigma_H^{\text{Tot}})^2 \right]; \quad (0.5)$$

where $\sigma_0^{\text{Tot}}$, $\sigma_H^{\text{Tot}}$ and $\sigma_{MR}^{\text{Tot}}$ denote the total conductivity, the total Hall conductivity and the second order total conductivity, respectively.

The explicit computation of the magnetotransport quantities requires the knowledge of the band spectra as well as the relaxation times for describing the collision of the electrons in the bands produced by the $d_{xy}$, $d_{xz}$ and $d_{yz}$ Ru orbitals.

Referring to the energy spectra, we use the electronic energy band structure of Sr$_2$RuO$_4$ recently calculated by using simple methods considering the extended Hückel theory and the tight-binding approximation.
Concerning the relaxation times, in the case of the Hall- and magneto- resistance the following expressions have been considered:

\[(\tau_i)^{-1} = \eta_i + \alpha_i T^2\]

where \(i=(xz, yz, xy)\) indicates the band and \(\eta = (2.75, 2.75, 3.25)\), and \(\alpha = (0.035, 0.04, 0.06)\). The values of \(\eta_i\) have been chosen in a way to get the experimental observed resistivity at \(T=4\) K of \(\sim 0.7\mu\Omega cm^{-1}\). The constraint on the values of \(\alpha_i\) is given by the complicated temperature dependence of \(R_H\) together with the behaviour of the transverse in-plane magnetoresistance. We notice that the behaviour of the \(R_H\) can be reproduced only if \(\alpha_{xz} \sim \alpha_{yz}\) but smaller than \(\alpha_{xy}\). The calculation is very sensitive to the changes in the time collisions of the two hybridized \(z\) bands. Indeed, for small relative variation of \(\alpha_{xz}\) with respect to \(\alpha_{yz}\) the Hall-coefficient does not show sign changing, being always negative.

It is worth pointing out that the explicit expression for the scattering rate is related to several physical mechanisms that give rise to different temperature dependencies of \(\tau_i\). In particular, while \(\eta_i\) could be considered as responsible for the residual resistivity, \(\alpha_i\) could be related to umklapp electron-electron scattering, or inelastic scattering of the electrons by impurities, as well as by phonon mediated interaction between electrons. In the case of electron-electron scattering, an estimation of \(\alpha_i\) can be obtained by means of the relation \((\tau_i^{-1}) \approx (k_BT^2/m_i k_F)\). Using the experimental values for the effective masses as deduced from de Haas-van Alphen experiment, we find that \(\alpha_{xy}\) is greater than \(\alpha_{xz}\) and \(\alpha_{yz}\) and this agrees with our assumption.

Concerning the other mechanisms, though the microscopic expression in the case of the phononic and impurity scattering requires a more accurate analysis, we expect that the considerations above are still valid.

The fit to the experimental data is reported in Fig. 1 for the Hall coefficient and in Fig. 2 for the magnetoresistance. The experimental data are taken from Ref. 8 for \(R_H\) and from Ref. 10 for the magnetoresistance. In both cases, we find a good agreement between the experimental results and the theoretical prediction indicating that the main contribution to scattering rate follows a \(T^2\) power law.

We notice that paramagnetic and ferromagnetic materials can have a large contribution to the Hall effect mainly due to skew scattering. Indeed, in this case moving charge carriers experience a force due to the magnetic field produced by localized magnetic moment and are scattered asymmetrically. Nevertheless, there is no sign of the saturation of the Hall resistivity one expects when there is a large magnetic contribution to the Hall effect. Therefore, we argue that the experimental data for Sr$_2$RuO$_4$ are dominated by the standard orbital Hall resistivity.

We point out that there are other results in literature dealing with the theoretical study of the Hall coefficient in the Sr$_2$RuO$_4$. In Ref. 11 is presented a theoretical fit of the Hall coefficient based on the assumption of two-carrier system and \(R_H\) is calculated within the Drude classical model. While, the authors neglect the contribution of one of the electron pockets and the effective electronic structure of the bands, they reproduce the sign change from negative at low temperature to positive at high temperature but fails to give a quantitative agreement with the experimental data.

In Ref. 12, using methods developed by Ong, an expression for the Hall coefficient in multi-band system is derived. We notice that the main assumption of this derivation is that the mean free path is the same for all the Fermi sheets. This hypothesis of isotropic mean-free path is valid at small temperature where the authors obtain a value for \(R_H\) that compares well with the measured value of the same quantity. Nevertheless, we want to stress that the value of \(R_H\) is extremely sensitive to details of the k-dependent scattering and/or on the energy spectra, and strongly depends on the temperature.

Finally, in Ref. 13 is shown a fit to \(R_H\) similar to the one here presented. However, in Ref. 14, the sign change is reproduced with an accuracy weaker than the one presented in the present paper and no mention to other relevant physical quantities is made.

![FIG. 1. The temperature dependence of the weak-field Hall coefficient. The triangles show data taken from Ref. 8, and the solid line is the theoretical result.](image)

![FIG. 2. The temperature dependence of \(1/\sqrt{\Delta \rho_{ab}/\rho_{ab}}\) (triangles) and the theoretical result (solid line).](image)
rises monotonically with a curvature which is weaker than for the netoresistance. Therefore, we add to the scattering rate to the determination of the Hall coefficient and the magnetoresponse. Its temperature dependence is given by:

\[(\tau^{sf}_{xy})^{-1} = \beta T\]

where \(\beta = 0.6\). The resulting fit is reported in Fig. 3 where the experimental data are taken from\ref{4}. The quite good agreement between the experimental results and the theoretical prediction gives confidence that spin fluctuations affects only the zero field relaxation time for the xy band.

As a final remark, we notice that throughout this paper we have assumed for the relaxation rates an isotropic k-independent form. We have also evaluated the above mentioned quantities assuming for \(\tau_i\) the suitable form in the case of a tetragonal environment. The results are only slightly modified, so that we have confined ourselves to temperature dependent but k-independent \(\tau_i\).

In summary, we have studied the in-plane normal state magnetotransport quantities of the layered compound Sr$_2$RuO$_4$. Using the calculated electron energy band structure, we have computed the temperature dependence of the Hall coefficient, the magnetoresistance and the in-plane resistivity by solving the Boltzmann equation for a multi-orbital system. The reasonably good fit of these physical quantities suggests that the assumption of two contributions in the relaxation rate for the xy-electrons used to quantify the galvanomagnetic transport is essentially correct.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{In plane resistivity vs. temperature: the triangles are the experimental results from\ref{4} while the solid line is the theoretical curve.}
\end{figure}

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