The $SU(3)_C \times SU(3)_L \times U(1)_X$ Model from $SU(6)$

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Abstract

We propose the $SU(3)_C \times SU(3)_L \times U(1)_X$ model arising from $SU(6)$ breaking. One family of the Standard Model (SM) fermions arises from two $\bar{6}$ representations and one 15 representation of $SU(6)$ gauge symmetry. To break the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry down to the SM, we introduce three $SU(3)_L$ triplet Higgs fields, where two of them come from $\bar{6}$ representation while the other one from 15 representation. We study the gauge boson masses and Higgs boson masses in details, and find that the Vacuum Expectation Value (VEV) of the Higgs field for $SU(3)_L \times U(1)_X$ gauge symmetry breaking is around 10 TeV. The neutrino masses and mixing can be generated via the littlest inverse seesaw mechanism. In particular, we have normal hierarchy for neutrino masses and the lightest active neutrino is massless. Also, we consider constraints from the charged lepton flavor changing decays as well. Furthermore, introducing two $SU(3)_L$ adjoint fermions, one $SU(3)_C$ adjoint scalar, and one $SU(3)_L$ triplet scalar, we can achieve gauge coupling unification within 1%. These extra particles can provide a dark matter candidate as well.

Keywords:
I. INTRODUCTION

The Standard Model (SM) has made a great achievement in explaining the experimental result. However, many significant problems remain to be answered. One of the most important issues is the fermion generation and the $U(1)_Y$ hypercharge. Since the SM did not explain the origin of the hypercharge, one may expect that the quantum number comes from a bigger group, for example, the Grand Unified Theory (GUT). In the traditional $SU(3)_C \times SU(3)_L \times U(1)_X$ (331) model, it successfully explained why there are three generations by tactfully eliminating $SU(3)_L$ gauge anomalies. However, the $U(1)_X$ number of is given by hand just like $U(1)_Y$ in the SM, which is not satisfying and inspires us to embed the 331 model into a bigger group to understand the $U(1)_X$ number more naturally. In this paper, we shall propose a 331 model generated from a $SU(6)$ model, where the $U(1)_X$ charge is determined from the $SU(6)$ breaking.

In the traditional 331 models [1–20], the left-handed lepton and one left-handed quark triplets are in the 3 fundamental representation of $SU(3)_L$, while two left-handed quark triplets are in the $\bar{3}$ antifundamental representation of $SU(3)_L$. Moreover, to generate all charged fermion masses at tree level. In these models, according to Eq. (1.1), $Q = \pm \text{diag}[^{2\over 3} + X, -{1\over 3} + X, -{1\over 3} + X]$ (there could be a minus sign for 3 multiplets), all the representations must contain two particles with the same charge. For Higgs which contain two zero-charged particles, there must be two of them in the same representation.

For models with $\beta = \sqrt{3}$ [9,11], it is obvious that all the three scalar triplets are all in different representations, because $Q = \pm \text{diag}[1 + X, X, -1 + X]$ for particles in (anti)fundamental representation. Moreover, to generate all charged fermion masses in tree level, we need three scalar triplets and one scalar sextet. Such models also contain exotic charged particles such as double charged Higgs and quarks with charge $\pm{5\over 3}$ and $\pm{4\over 3}$. In particular, there exists the Landau pole problem for $U(1)_X$ not far from the TeV scale.

We for the first time propose the $SU(3)_C \times SU(3)_L \times U(1)_X$ model, which can be obtained from the $SU(6)$ breaking. One family of the SM fermions arises from two $\bar{6}$ representations and one 15 representation of $SU(6)$ gauge symmetry. To break the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry down to the SM gauge symmetry, we introduce three $SU(3)_L$ triplet Higgs fields, where two of them arises from $\bar{6}$ representation while the other one from 15 representation. We discuss the gauge boson masses and Higgs boson masses in details, and show that the Vacuum Expectation Value (VEV) of the Higgs field for $SU(3)_L \times U(1)_X$ gauge symmetry breaking is around 10 TeV. We explain the neutrino masses and mixing via the littlest inverse seesaw mechanism. Especially, the normal hierarchy for neutrino masses is realized and the lightest active neutrino is massless. Moreover, we study constraints from the charged lepton flavor changing decays as well. Furthermore, introducing two $SU(3)_L$ adjoint fermions, one
SU(3)$_C$ adjoint scalar, and one SU(3)$_L$ triplet scalar, we can achieve gauge coupling unification within 1%. These extra particles can give us a dark matter candidate as well.

The paper is organized as follows. In section II, we present the models and Yukawa terms. The gauge sector and Higgs sector are studied in section III and section IV, respectively. We discuss the neutrino masses and mixing, as well as the charged lepton flavor changing decays in section V. In section VI, we consider gauge coupling unification and dark matter candidate. Our conclusion is in section VII.

II. THE SU(3)$_C \times SU(3)$_L $ $\times U(1)_{X}$ MODEL

In our 3-3-1 model, the SU(3)$_C \times SU(3)$_L $ $\times U(1)_{X}$ gauge group arises from a large SU(6) gauge group. With $U(1)_{X}$ charge operator for the 6 representation of the SU(6) group being

$$T_{U(1)_{X}} = \frac{1}{2\sqrt{3}} \text{diag}[-1, -1, -1, 1, 1, 1].$$

The following representations of the SU(6) group can be decomposed into representations of the SU(3)$_C \times SU(3)$_L $ $\times U(1)$_X$ group as below

$$6 \rightarrow (3, 1, \frac{-1}{2\sqrt{3}}) \bigoplus (1, 3, \frac{1}{2\sqrt{3}}),$$
$$\bar{6} \rightarrow (\bar{3}, 1, \frac{1}{2\sqrt{3}}) \bigoplus (1, \bar{3}, \frac{-1}{2\sqrt{3}}),$$
$$15 \rightarrow (\bar{3}, 1, \frac{-1}{\sqrt{3}}) \bigoplus (1, \bar{3}, \frac{1}{\sqrt{3}}) \bigoplus (3, 3, 0).$$

One family of the SM fermions and extra fermions in our model is

$$\bar{6} \rightarrow (1, 3, \frac{-1}{2\sqrt{3}}) \bigoplus (3, 1, \frac{1}{2\sqrt{3}})$$
$$\leftrightarrow f_i = (e_{Li}, -\nu_{Li}, N_i) \bigoplus \bar{d}_{Ri},$$
$$\bar{6}' \rightarrow (1, 3, \frac{-1}{2\sqrt{3}}) \bigoplus (3, 1, \frac{1}{2\sqrt{3}})$$
$$\leftrightarrow f'_i = (e'_{Li}, -\nu'_{Li}, N'_i) \bigoplus D_{Ri}^c,$$
$$15 \rightarrow (3, 3, 0) \bigoplus (1, 3, \frac{1}{\sqrt{3}}) \bigoplus (\bar{3}, 1, \frac{-1}{\sqrt{3}})$$
$$\leftrightarrow f_i = (u_{Li}, d_{Li}, D_{Li}) \bigoplus X f_i^c = (v_{Ri}^c, e_{Ri}^c, \nu_{Ri}^c) \bigoplus \bar{u}_{Ri}^c.$$
In SU(6) model, two $\bar{6}$ anti-fundamental representations and one 15 anti-symmetric representation of the fermions are anomaly free. Thus, our model is anomaly free. To be concrete, we can verify it is easily as well. According to [25, 26], first, for $U(1)_X$, we have
\[ \sum_{\psi_i} X_{\psi_i} = \sum_{\psi_i} X_{\psi_i}^3 = 0, \] (2.11)
which makes $U(1)_X$ gauge structure anomaly free. For gauge structure of $SU(3)_L/SU(3)_C$, since the number of fermion multiplets in 3 representation equals to the number of fermion multiplets in $\bar{3}$ representation for every generation, it is also anomaly free.

Our model has 3 scalar multiplets coming from two $\bar{6}$ and one 15 representations of the SU(6) group, which are
\[ 15 \rightarrow (1, 3, \frac{1}{\sqrt{3}}) : \begin{align*}
T_u &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_u + \rho_1 + i\sigma_1 \\ \sqrt{2} \chi_1^+ \\ \sqrt{2} \chi_2^+ \end{pmatrix} , & < T_u > &= \frac{1}{\sqrt{2}} \begin{pmatrix} v_u \\ 0 \\ 0 \end{pmatrix} , \\
< T > &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_d \\ 0 \end{pmatrix} , \\
T_d &\rightarrow (1, 3, \frac{1}{\sqrt{3}}) : \begin{pmatrix} \sqrt{2} \xi_2^+ \\ v_d + \rho_2 + i\sigma_2 \\ \rho_3 + i\sigma_3 \end{pmatrix} , & < T_d > &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_d \\ 0 \end{pmatrix} , \\
T &\rightarrow (1, 3, \frac{1}{\sqrt{3}}) : \begin{pmatrix} \sqrt{2} \xi_1^- \\ v_t + \rho_4 + i\sigma_4 \\ \rho_5 + i\sigma_5 \end{pmatrix} , & < T > &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_d \\ 0 \end{pmatrix} . \] (2.12, 2.13, 2.14)

We use
\[ \tan \theta = \frac{v_u}{v_d} , \] (2.15)
\[ k = v_t / \sqrt{v_u^2 + v_d^2} , \] (2.16)
to parameterize the 3 VEVs, which break the $SU(3)_L \times U(1)_X$ gauge group down to the $U(1)_{EM}$ gauge group. We write the $U(1)_{EM}$ charge operator as
\[ Q = c_1 T_{8L} + c_2 T_{3L} + c_3 X I . \] (2.17)
Then the condition, which only neutral states of the scalar multiplets can get VEVs, gives
\[ c_1 = \frac{c_2}{\sqrt{3}} = \frac{1}{2} c_3 . \] (2.18)
To make SM particles have the same electric charges as in the SM, we find
\[ c_3 = \frac{2}{\sqrt{3}} , \] (2.19)
leading to
\[ Q = \frac{1}{\sqrt{3}} T_{8L} + T_{3L} + \frac{2}{\sqrt{3}} X I . \] (2.20)
The Yukawa terms and Majorana mass terms of our model are

\[
-L_{\text{qua}} = y_{ij}^q f_i u^c_{Rj} T_u + y_{ij}^d f_i d^c_{Rj} T_d + y_{ij}^D f_i D^c_{Rj} T + H.c,
\]

\[
-L_{\text{lep}} = y_{ij}^l f_i f_j T_u + y_{ij}^f f_i f_j T_d + y_{ij}^N f_i T N_{sij} + y_{ij}^{N'} f_i^T N'_{sij} + H.c,
\]

\[
-L_{\text{neu}}^\text{maj} = \frac{1}{2} \left( N_s \ N'_s \right) \left\{ \begin{array}{c} M_s \\ M_{ss'} \end{array} \right\} \left\{ \begin{array}{c} N_s \\ N'_s \end{array} \right\} + H.c,
\]

\[2.21\]

where \( M_s, M'_s \) and \( M_{ss'} \) are \( 3 \times 3 \) matrix. For simplicity, we do not include all the gauge invariant terms in Eq (2.21).

III. GAUGE BOSONS

We write \( W_a (a = 1, 2, \ldots, 8) \), which is in the adjoint representation of \( SU(3)_L \) in the form of

\[
W_a T_a = \frac{1}{2} \begin{bmatrix}
W_3 + \frac{1}{\sqrt{3}} W_8 & W_1 - i W_2 & W_4 - i W_5 \\
W_1 + i W_2 & -W_3 + \frac{1}{\sqrt{3}} W_8 & W_6 - i W_7 \\
W_4 + i W_5 & W_6 + i W_7 & -\frac{2}{\sqrt{3}} W_8
\end{bmatrix} + H.c.
\]

\[3.1\]

For the adjoint representation of the \( SU(3)_L \) group, the electric charge operator is

\[
Q = \frac{1}{\sqrt{3}} T_{8L} + T_{3L} = \frac{1}{3} \text{diag}[2, -1, 1],
\]

\[3.2\]

\[
[Q, W_a T_a] = \frac{1}{\sqrt{2}} \begin{bmatrix}
0 & \frac{W_1 - i W_2}{\sqrt{2}} & \frac{W_4 - i W_5}{\sqrt{2}} \\
-\frac{W_1 + i W_2}{\sqrt{2}} & 0 & 0 \\
-\frac{W_4 + i W_5}{\sqrt{2}} & 0 & 0
\end{bmatrix}.
\]

\[3.3\]

We thus define \( W^\pm \equiv \frac{W_1 \pm i W_2}{\sqrt{2}}, W'^\pm \equiv \frac{W_4 \pm i W_5}{\sqrt{2}}, V \equiv \frac{W_6 - i W_7}{\sqrt{2}} \) and \( V^* \equiv \frac{W_6 + i W_7}{\sqrt{2}} \). \( W^\pm \) and \( W'^\pm \) are charged, while \( V \) is neutral.

With

\[
D_\mu = \partial_\mu - i g_L W^a_\mu T_a - i g_X X B_\mu,
\]

\[3.4\]

we get

\[
(D^\mu < T >)^+ (D^\mu < T >) + (D^\mu < T_d >)^+ (D^\mu < T_d >) + (D^\mu < T_u >)^+ (D^\mu < T_u >) = \left( \frac{g_L}{2} \sqrt{v_u^2 + v_d^2} \right)^2 W^+ W^- + \left( \frac{g_L}{2} \sqrt{v_u^2 + v_t^2} \right)^2 W'^+ W'^- + \left( \frac{g_L}{2} \sqrt{v_d^2 + v_t^2} \right)^2 V_\mu V'^\mu
\]

\[3.5\]

\[
M^2_{\text{mix}} = \begin{cases}
\frac{g_L^2}{12} \begin{bmatrix}
4 v_u^2 + v_d^2 + v_t^2 \\
2 v_u^2 + v_d^2 + v_t^2 \\
2 v_u^2 - v_d^2 + 2 v_t^2
\end{bmatrix}
- \frac{\sqrt{3} g_L s_{\phi}}{12} \begin{bmatrix}
(2 v_u^2 + v_d^2) \\
(2 v_u^2 - v_d^2) + 2 v_t^2 \\
(2 v_u^2 - v_d^2) + 4 v_t^2
\end{bmatrix}
- \frac{g_L s_{\phi}}{4 \sqrt{3}} \begin{bmatrix}
(v_u^2 + v_d^2) \\
(v_u^2 - v_d^2) + 2 v_t^2 \\
(v_u^2 - v_d^2) + 4 v_t^2
\end{bmatrix}.
\]

\[3.6\]
And we get
\[ M_W = \frac{g_L}{2} \sqrt{v_u^2 + v_d^2}, \quad (3.7) \]
\[ M_{W'} = \frac{g_L}{2} \sqrt{v_u^2 + v_t^2}, \quad (3.8) \]
\[ M_V = \frac{g_L}{2} \sqrt{v_d^2 + v_t^2}. \quad (3.9) \]

To make \( W^\pm \), which is the familiar \( W^\pm \) gauge boson in the SM, have the right mass, we have
\[ \sqrt{v_u^2 + v_d^2} = 246 \text{ GeV}. \quad (3.10) \]

Also, by diagonalizing \( M^2_{\text{mix}} \), we get
\[ M_A = 0, \quad (3.11) \]
\[ M_Z = m_1^2 (1 - \sqrt{1 - \rho}), \quad (3.12) \]
\[ M_{Z'} = m_1^2 (1 + \sqrt{1 - \rho}), \quad (3.13) \]

with
\[ m_1^2 = \frac{1}{6} \left( g_L^2 \left( v_u^2 + v_d^2 + v_t^2 \right) + \frac{g_X^2}{4} (4v_u^2 + v_d^2 + v_t^2) \right), \quad (3.14) \]
\[ \rho = \frac{3g_L^2 (g_L^2 + g_X^2) \left( v_t^2 v_u^2 + v_t^2 v_d^2 + v_t^2 v_t^2 \right) + \frac{g_X^2}{4} (4v_u^2 + v_d^2 + v_t^2)} \left( g_L^2 (v_u^2 + v_d^2 + v_t^2) + \frac{g_X^2}{4} (4v_u^2 + v_d^2 + v_t^2) \right)^2, \quad (3.15) \]

where \( A, Z \) and \( Z' \) are the eigenstates of the mixing of \( B, W_3 \) and \( W_8 \). \( A \) and \( Z \) are the photon and the \( Z \) gauge boson in the SM, respectively.

We also find
\[ B = \frac{g_L}{\sqrt{g_L^2 + g_X^2}} A + \cdots, \quad (3.16) \]

which means \( g_X = \frac{2g_L g_Y}{\sqrt{3g_L^2 - g_Y^2}} \).

With the condition that \( |k| \gg 1 \), we have
\[ M_Z \approx \frac{g_L}{2 \cos \theta_W} \sqrt{v_u^2 + v_d^2}, \quad (3.17) \]
\[ M_{Z'} \approx \frac{g_L}{\sqrt{3 - \tan^2 \theta_W}} |v_t|, \quad (3.18) \]

and
\[ A = \sqrt{\cos^2 \theta_W - \frac{\sin^2 \theta_W}{3} B + \sin \theta_W W_3 + \frac{\sin \theta_W}{\sqrt{3}} W_8}, \quad (3.19) \]
\[ Z \approx -\sin \theta_W \sqrt{1 - \frac{\tan^2 \theta_W}{3} B + \cos \theta_W W_3 - \frac{\sin \theta_W \tan \theta_W}{\sqrt{3}} W_8}, \quad (3.20) \]
\[ Z' \approx \frac{\tan \theta_W}{\sqrt{3}} B - \sqrt{1 - \frac{\tan^2 \theta_W}{3}} W_8, \quad (3.21) \]
where $\theta_W$ is the Weinberg angle.

According to [27] $M_{Z'}$ larger than 4.5 TeV, $|v_t|$ needs to be larger than 10 TeV.

### IV. Higgs Sector

The most general Higgs potential in our model is

$$V_{\text{Higgs}} = -m_1^2 |T|^2 - m_2^2 |T_d|^2 - m_3^2 |T_u|^2 + l_1 |T|^4 + l_2 |T_d|^4 + l_3 |T_u|^4$$

$$+ l_{13} |T|^2 |T_u|^2 + l_{12} |T|^2 |T_d|^2 + l_{23} |T_u|^2 |T_d|^2 + l_{12}' |T^* T_d|^2$$

$$+ \left( y_1 T^* T_d |T|^2 + y_2 T^* T_d |T_d|^2 + y_3 T^* T_d |T_u|^2 + H.c. \right)$$

$$+ \left( -B T^* T_d + A T_u T_d + y_{12} T^* T_d T^* T_d + H.c. \right). \quad (4.1)$$

Since $< \frac{\partial V_{\text{Higgs}}}{\partial \rho_i} > = 0(i = 1, 2, \ldots, 5)$, we get 4 independent relations, which are

$$m_1^2 = \frac{l_{12} v_d^2 v_t + 2l_1 v_l^2 + \sqrt{2} A v_d v_u + l_{13} v_l v_u^2}{2v_l}, \quad (4.2)$$

$$m_2^2 = \frac{l_{12} v_t^2 v_d + 2l_2 v_d^2 + \sqrt{2} A v_l v_u + l_{23} v_d v_u^2}{2v_d}, \quad (4.3)$$

$$m_3^2 = \frac{l_{23} v_u^2 v_d + 2l_3 v_u^3 + \sqrt{2} A v_l v_d + l_{13} v_u v_d^2}{2v_u}, \quad (4.4)$$

$$B = \frac{y_1 v_l^2 + y_2 v_d^2 + y_3 v_u^2}{2}. \quad (4.5)$$

#### A. Mixing of $\bar{\xi}^{\pm}_{1,2}, \chi^{\pm}_{1,2}$

From the Higgs potential $V_{\text{Higgs}}$, we get

$$V_{\text{Higgs}} \ni \left( \chi^+_{1,2}, \bar{\xi}^{+}_{1,2} \right) \left\{ \begin{array}{ccc}
-\frac{A v_l v_u}{\sqrt{2} v_u} & -\frac{A v_d}{\sqrt{2} v_d} & 0 \\
-\frac{A v_d}{\sqrt{2} v_u} & \frac{A v_l v_u}{\sqrt{2} v_d} & 0 \\
0 & 0 & -\frac{A v_l v_d}{\sqrt{2} v_u} \\
0 & 0 & -\frac{A v_d v_l}{\sqrt{2} v_d}
\end{array} \right\} \left( \begin{array}{c}
\chi^-_1 \\
\bar{\xi}^-_2 \\
\chi^+_2 \\
\xi^+_1
\end{array} \right), \quad (4.6)$$
leading to the following eigenstates

\[ \eta_1^\pm = -\frac{v_u}{\sqrt{v_u^2 + v_l^2}} \chi_2^\pm + \frac{v_l}{\sqrt{v_u^2 + v_l^2}} \tilde{\sigma}_1^\pm, \quad m_{\eta_1}^2 = 0, \quad (4.7) \]

\[ \eta_2^\pm = -\frac{v_u}{\sqrt{v_u^2 + v_d^2}} \chi_1^\pm + \frac{v_d}{\sqrt{v_u^2 + v_d^2}} \tilde{\sigma}_2^\pm, \quad m_{\eta_2}^2 = 0, \quad (4.8) \]

\[ \eta_3^\pm = \frac{v_l}{\sqrt{v_u^2 + v_l^2}} \chi_2^\pm + \frac{v_u}{\sqrt{v_u^2 + v_l^2}} \tilde{\sigma}_1^\pm, \quad m_{\eta_3}^2 = -\frac{A v_d (v_l^2 + v_u^2)}{\sqrt{2} v_l v_u}, \quad (4.9) \]

\[ \eta_4^\pm = \frac{v_d}{\sqrt{v_u^2 + v_d^2}} \chi_1^\pm + \frac{v_u}{\sqrt{v_u^2 + v_d^2}} \tilde{\sigma}_2^\pm, \quad m_{\eta_4}^2 = -\frac{A v_l (v_d^2 + v_u^2)}{\sqrt{2} v_d v_u}. \quad (4.10) \]

Apparently \( \eta_1^\pm \) and \( \eta_2^\pm \) are Goldstone bosons.

### B. Mixing of \( \sigma_i \)

We have

\[
V_{\text{Higgs}} \equiv \frac{1}{2} \begin{pmatrix} \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{pmatrix} \begin{pmatrix}
-A v_d v_l & -A v_l & -A v_d & 0 & 0 \\
-A v_l & -A v_l & -A v_u v_l & 0 & 0 \\
-A v_d & -A v_d & -A v_l v_u & 0 & 0 \\
-\frac{A}{\sqrt{2}} v_u & -\frac{A}{\sqrt{2}} v_d & -\frac{A}{\sqrt{2}} v_l & 0 & 0 \\
0 & 0 & 0 & -\frac{v_l}{v_d} m_{34}^2 & m_{34}^2 \\
0 & 0 & 0 & m_{34}^2 & -\frac{v_l}{v_d} m_{34}^2
\end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \end{pmatrix},
\]

(4.11)

where

\[
m_{34}^2 = -\frac{1}{2} \frac{H_{12} v_d v_l + A v_u}{2} + y_{12} v_d v_l.
\]
The eigenstates are

\[
\begin{align*}
a_1 &= \frac{v_u}{\sqrt{v_u^2 + v_t^2}} \sigma_1 - \frac{v_t}{\sqrt{v_u^2 + v_t^2}} \sigma_5, \\
a_2 &= \frac{v_t v_u}{\sqrt{(v_u^2 + v_t^2)(v_u^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2)}} \sigma_1 - \frac{v_d (v_u^2 + v_t^2)}{\sqrt{(v_u^2 + v_t^2)(v_u^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2)}} \sigma_2 + \frac{v_u v_d}{\sqrt{v_u^2 v_d^2 + v_t^2 v_u^2 + v_u^2 v_d^2}} \sigma_5,
\end{align*}
\]

and their masses satisfy

\[
\begin{align*}
m_{a_1}^2 &= m_{a_2}^2 = m_{a_3}^2 = 0, \\
m_{a_4}^2 &= -\frac{v_d^2 + v_t^2}{v_d v_t} m_{34}^2, \\
m_{a_5}^2 &= -\frac{A (v_t^2 v_d^2 + v_u^2 v_u^2 + v_u^2 v_d^2)}{\sqrt{2} v_d v_u v_t}.
\end{align*}
\]

\(a_1, a_2\) and \(a_3\) are Goldstone bosons.

C. Mixing of \(\rho_i\)

From the Higgs potential \(V_{\text{Higgs}}\), we have

\[
V_{\text{Higgs}} \ni \frac{1}{2} \rho_i [M_{\rho}^2] \rho_j, \quad (4.20)
\]

\[
M_{\rho} = \begin{pmatrix}
-\frac{A v_u}{\sqrt{2} v_u} + 2 l_3 v_u^2 & \frac{A v_t}{\sqrt{2} v_u} + l_23 v_d v_u & \frac{A v_t}{\sqrt{2} v_u} + l_13 v_u v_t & y_3 v_u v_t & y_3 v_u v_d \\
\frac{A v_u}{\sqrt{2} v_u} + l_3 v_u v_t & -\frac{A v_t}{\sqrt{2} v_u} + 2 l_2 v_d v_u & \frac{A v_t}{\sqrt{2} v_u} + l_23 v_u v_t & y_3 v_u v_t & y_2 v_d v_t \\
\frac{A v_u}{\sqrt{2} v_u} + l_13 v_u v_t & \frac{A v_t}{\sqrt{2} v_u} + l_3 v_u v_t & -\frac{A v_t}{\sqrt{2} v_u} + 2 l_1 v_d v_u & y_1 v_t^2 & y_1 v_d v_t \\
y_3 v_u v_t & y_2 v_d v_t & y_1 v_t^2 & -\frac{v_t}{v_d} m_{34}^2 & m_{34}^2 \\
y_3 v_u v_d & y_2 v_d^2 & y_1 v_d v_t & m_{34}^2 & -\frac{v_d}{v_t} m_{34}^2
\end{pmatrix}, \quad (4.21)
\]

\[
m_{34}^2 = -\frac{1}{2} l_{12} v_d v_t + \frac{A v_u}{\sqrt{2}} - y_{12} v_d v_t. \quad (4.22)
\]
The lightest eigenstate,

\[ h_1 = -\frac{v_d}{\sqrt{v_d^2 + v_l^2}}\rho_3 + \frac{v_l}{\sqrt{v_d^2 + v_l^2}}\rho_4, \]

is massless, which is a Goldsten boson.

The next to the lightest eigenstate is the SM Higgs boson, whose mass \( M_H \) should be 125 GeV. The independent parameters in the Higgs potential affecting \( M_H \) are \( \tan\theta, k, l_1, l_2, l_3, l_{12}, l_{13}, l_{23}, l'_{12}, y_1, y_2, y_3, y_{12} \) and \( A \). All these parameters except \( A \) are dimensionless. For simplicity, in FIG. 1, we show the dependence of \( M_H \) on \((A, l_3)\) and \((\tan\theta, k)\) respectively while fixing other parameters.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{higgs_mass.png}
\caption{Higgs boson mass. On the \( A - l_3 \) plane, we choose that \( \tan\theta = 6, k = -60 \), and all other dimensionless parameters in \( V_{\text{Higgs}} \) are 0.1. On the \( \tan\theta - k \) plane, we choose that \( A = 1 \text{ TeV}, l_3 = 0.16 \), and all other dimensionless parameters in \( V_{\text{Higgs}} \) are 0.1.}
\end{figure}
V. NEUTRINO MASS, MIXING AND FCNC

From Eq. (2.21), the neutrino mass matrix in the basis \((\nu_L, \nu'_L, \nu'_R, N, N_s, N'_s, N')\) is

\[
M = \begin{pmatrix}
0 & 0 & 0 & \frac{(y^{\nu T}-y^{\nu'})v_u}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & (y^{\nu T}v_l) & \frac{0}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & \frac{y^{\nu T}v_l}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
\frac{(y^{\nu'}-y^{\nu'})v_u}{\sqrt{2}} & 0 & y^{\nu T}v_d & \frac{0}{\sqrt{2}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{y^{\nu T}v_d}{\sqrt{2}} & M_s & M_{ss'} & 0 \\
0 & 0 & 0 & 0 & M'^T_{ss'} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & M_{ss'} & M'_{ss'} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{y^{\nu T}v_l}{\sqrt{2}} & 0
\end{pmatrix} \, .
\] (5.1)

Every element in \(M\) is a 3 × 3 matrix. Because \(\frac{(y^{\nu}-y^{\nu'^T})}{\sqrt{2}}\) is an antisymmetric matrix, we have

\[
\text{det}[M] = 0 \, ,
\] (5.2)

which means the lightest neutrino eigenstate is massless.

We choose \(M_{ss'}\) to be 0 for simplicity. In the limits of \(\tan \theta \gg 1\) and \(|k| \gg 1\), we approximately get that \((\nu_L, N, N_s)\) are only mixing with themselves and the mass matrix is

\[
M' = \begin{pmatrix}
0 & \frac{(y^{\nu'-y^{\nu T})v_u}{\sqrt{2}} & 0 \\
\frac{(y^{\nu'}-y^{\nu T})v_u}{\sqrt{2}} & 0 & y^{\nu T}v_d & \frac{0}{\sqrt{2}} \\
0 & y^{\nu T}v_d & \frac{0}{\sqrt{2}} & M_s
\end{pmatrix} \, .
\] (5.3)

We define \(M_D = \frac{(y^{\nu T}-y^{\nu'})v_u}{\sqrt{2}}\) and \(M_N = \frac{y^{\nu T}v_d}{\sqrt{2}}\). Notice that the situation here looks very similar to the littlest inverse seesaw (LIS) model \([28, 29]\), in which the elements of \(M_s\) is very small to generate the very small neutrino masses. Since \(\text{det}[M_D]\) is zero, the lightest eigenstate of the mixing of \(\nu_L, N\) and \(N_s\) is massless.

The three light eigenvalues of \(M'^T M'\) forms the SM neutrino mass squares, which are constrained by neutrino oscillation experiments. According to \([28]\), in the case that \(M_D, M_s \ll M_N\), the three light neutrino mass squares are eigenvalues of \(M'^T M'\) with

\[
M_v = M_D (M'^T_N)^{-1} M_s M'^{-1}_N M'^T_D \, .
\] (5.4)

For simplicity, we set \(M_N\) and \(M_s\) to be diagonal, which are

\[
M_N = v_t \, \text{diag}[c_N, c_N, c_N] \, ,
\] (5.5)

\[
M_s = \text{diag}[k_1, k_2, k_3] \, ,
\] (5.6)

\[
11
\]
Since $M_D$ is antisymmetric, it can be written as

$$M_D = v_u \begin{bmatrix} 0 & d_1 & d_2 \\ -d_1 & 0 & d_3 \\ -d_2 & -d_3 & 0 \end{bmatrix}.$$  

(5.7)

So we have

$$M_v = \frac{v_u^2}{v_1^2 c_N} \begin{bmatrix} d_1^2 k_2 + d_2^2 k_3 & d_2 d_3 k_3 & -d_1 d_3 k_2 \\ -d_1 d_2 k_2 & d_2^2 d_3 + d_3^2 k_3 & d_1 d_2 k_1 \\ -d_2 d_3 k_2 & -d_1 d_2 k_1 & d_1^2 k_1 + d_2^2 k_3 \end{bmatrix}.$$  

(5.8)

Suppose eigenvalues of $M^t v M_v$ are $m_1^2 = 0$, $m_2^2$, and $m_3^2$. However, we can always rescale $d_i (i = 1, 2, 3)$ and $k_j (j = 1, 2, 3)$ to $10^{-R_D} d_i$ and $R_s k_j$ without changing the neutrino mixing pattern and $\frac{m_3}{m_2}$. But the masses will be changed to $10^{-2R_D} R_s m_i (i = 1, 2, 3)$.

Because the lightest neutrino in our model is massless, we should choose appropriate values of $a_i, k_j, c_N$, tan $\theta$ and $k$ to give

$$U^t v M^t v U_v = \text{diag} \{0, m_2^2 = \Delta m_{21}^2, m_3^2 = \Delta m_{31}^2\}$$

(5.9)

where $U_v$ is parameterized by $\theta_{12}$, $\theta_{13}$, $\theta_{23}$, and $\delta$, i.e., the Normal Hierarchy (NH) for neutrino masses. We choose

$$\begin{align*}
(d_1, d_2, d_3) &= 10^{-R_D} (0.49, 0.29, 0.82), \\
(k_1, k_2, k_3) &= R_s (0.33, 0.038 e^{0.36\pi i}, -0.027 e^{0.13\pi i}),
\end{align*}$$

(5.10) (5.11)

where $R_s$ is determined by $\tan \theta$, $k$, $c_N$, and $R_D$ to give the right neutrino masses. For example, when $\tan \theta = 6$, $k = -60$, $c_N = -1$ and $R_D = 1$, $R_s$ needs to be $1.8 \times 10^{-4} GeV$, giving us the three mixing angles, CP violating phase $\delta$ and neutrino masses in TABLE I.

| Observable      | Model | bpf ± 1σ | bpf ± 1σ |
|-----------------|-------|----------|----------|
| $\Delta m_{21}^2 (10^{-5} eV^2)$ | 7.36  | $7.55^{+0.20}_{-0.16}$ | $7.39^{+0.21}_{-0.20}$ |
| $\Delta m_{31}^2 (10^{-3} eV^2)$ | 2.53  | $2.50 \pm 0.03$ | $2.525^{+0.03}_{-0.031}$ |
| $\theta_{12}^{(\circ)}$ | 33.83 | $34.5^{+1.2}_{-1.0}$ | $33.83^{+0.78}_{-0.76}$ |
| $\theta_{13}^{(\circ)}$ | 8.57  | $8.45^{+0.16}_{-0.14}$ | $8.61^{+0.12}_{-0.13}$ |
| $\theta_{23}^{(\circ)}$ | 49.82 | $47.9^{+1.0}_{-1.7}$ | $49.7^{+0.9}_{-1.1}$ |
| $\delta_{CP}^{(\circ)}$ | $-142.05$ | $-142^{+3.8}_{-2.7}$ | $217^{+4.0}_{-2.8}$ |

TABLE I: Model and experimental values of the light active neutrino masses, leptonic mixing angles and CP violating phase for the scenario of the NH neutrino masses $^{30}$ $^{31}$.

Next, we shall discuss the implication of the 3-3-1 model in the charged lepton flavor changing decays. There are in total three processes, which are $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$. The branch ratio of lepton $\epsilon_i$ decaying to lepton $\epsilon_j$ is

$$BR(\epsilon_i \to \epsilon_j) = \frac{a_\epsilon^3 W m_e s_W^2}{256 \pi^2 T_i} \left| \sum_{k=1}^{k=9} \left( U^t v M^t v U_v \left( \frac{m_k^2}{M^2_W} \right) \frac{1}{M^2_W} + U^t v M^t v U_v \left( \frac{m_k^2}{M^2_{W'}} \right) \frac{1}{M^2_{W'}} \right) \right|^2,$$

(5.12)
where \( U^\dagger M^\dagger M' U = \text{diag}[m_{N_1}^2, m_{N_2}^2, \ldots, m_{N_9}^2] \). Experiment results ask us that the branch ratio of charged lepton decay should satisfy

\[
BR(\mu \to e\gamma) \leq 4.2 \times 10^{-13}, \tag{5.13}
\]

\[
BR(\tau \to \mu\gamma) \leq 4.4 \times 10^{-8}, \tag{5.14}
\]

\[
BR(\tau \to e\gamma) \leq 3.3 \times 10^{-8}. \tag{5.15}
\]

Independent parameters influencing \( BR(e_i \to e_j\gamma) \) are \( \tan \theta, k, R_D, \) and \( c_N \), while \( R_s \) is determined by other parameters to give the right neutrino masses. In FIG. 2 we show the dependence of \( BR(\mu \to e\gamma) \) on these parameters. We find that \( BR(\mu \to e\gamma) \) mainly depends on \( R_D \). To make that \( BR(\mu \to e\gamma) \leq 4.2 \times 10^{-13} \), \( R_D \) needs to be larger than 2.5, which means that \( (d_1, d_2, d_3) < (1.55, 0.92, 2.81) \times 10^{-3} \). In the case that \( R_D \sim 2.5 \), \( BR(\tau \to \mu\gamma) \), and \( BR(\tau \to e\gamma) \) are around \( 10^{-14} \) and \( 10^{-13} \) respectively.

VI. UNIFICATION OF GAUGE COUPLINGS

The Renormalization Group Equation (RGE) for gauge coupling is

\[
\mu \frac{d g_i}{d \mu} = \sum_n \frac{1}{(16\pi^2)^n} \beta_i^{(n)}, \tag{6.1}
\]

where \( i \) stands for the \( i \)-loop correction in RGE running. In this section, we consider two-loop correction. Equations of 1-loop and 2-loop corrections are

\[
\beta_i^{(1)} = b_i g_i^3, \tag{6.2}
\]

\[
\beta_i^{(2)} = B_{ij} g_j^2 + \sum_\alpha d_\alpha \text{Tr}[y_\alpha y_\alpha^\dagger], \tag{6.3}
\]

where \( \alpha = d, u, D, v, e, L', N, N' \).

In our model, we get

\[
b = \left[ \frac{13}{2}, -\frac{9}{2}, -5 \right], \tag{6.4}
\]

\[
B = \begin{bmatrix}
6 & 20 & 12 \\
5 & 23 & 12 \\
\frac{3}{2} & 12 & 12
\end{bmatrix}, \tag{6.5}
\]

\[
d = \begin{bmatrix}
-\frac{3}{4} & -3 & -\frac{3}{4} & -2 & -\frac{5}{2} & -\frac{5}{2} & -\frac{1}{4} & -\frac{1}{4} \\
-\frac{3}{2} & -\frac{3}{2} & -3 & -4 & -2 & -2 & -\frac{1}{2} & -\frac{1}{2} \\
-3 & -3 & -3 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}. \tag{6.6}
\]

To make gauge couplings unify at the GUT scale, we add two fermion multiplets, \( FA \)
FIG. 2: $BR(\mu \to e\gamma)$. On the $R_D - k$ plane, we choose that $c_N = -1$, $\tan\theta = 6$. On the $c_N - \tan\theta$ plane, we choose that $k = -60$, $R_D = 2.5$. The curve of $BR(\mu \to e\gamma)$ is got when $\tan\theta = 6$, $k = -60$, $c_N = -1$. 
and $FA'$, as well as two scalar multiplets, $SA$ and $T'$ in high scale. The details are

\[(1, 8, 0) : FA, \quad \Delta b = (0, 2, 0), \quad \Delta B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 48 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (6.7)\]

\[(1, 8, 0) : FA', \quad \Delta b = (0, 2, 0), \quad \Delta B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 48 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (6.8)\]

\[(8, 1, 0) : SA, \quad \Delta b = (0, 0, 1), \quad \Delta B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 42 \end{bmatrix}, \quad (6.9)\]

\[(1, \bar{3}, \frac{-1}{2\sqrt{3}}) : T', \quad \Delta b = \left( \frac{1}{12}, \frac{1}{6'}, 0 \right), \quad \Delta B = \begin{bmatrix} \frac{1}{12} & 4 & 0 \\ \frac{1}{6} & \frac{11}{3} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (6.10)\]

$FA$ and $FA'$ can decay via the Yukawa coupling terms $FAf_i(T')^*, FA'f_i(T')^*$, $FAf_i(T')^*$, and $FA'f_i(T')^*$. In principle, we can introduce the $Z_2$ symmetry where $FA$, $FA'$, and $(T')^*$ are odd while all the other particles are even. Thus, the lightest particle of $FA$, $FA'$, and $(T')^*$ can be a dark matter candidate. In addition, $SA$ can decay into the SM quarks only at non-renormalizable level, for example, $SAf_iT_u/M_*, SAf_iT_d/M_*$, and $SAf_iD_cR_jT/M_*$. Thus, we have two cases. First, $SA$ can be a dark matter candidate if $Z_2$ symmetry is imposed to forbid $SA$ decaying to quarks. We will leave this part of work in the future. For simplicity, we make all the particles beyond the SM take part in the RGE running at the energy scale of 2 TeV, then the gauge coupling unification can be satisfied with accuracy of 0.65% at the energy scale of $5.2 \times 10^{16}$ GeV, which is shown in FIG. 3.

Alternatively, to make $SA$ decay, we can add two fermion multiplets in $6$ and $\bar{6}$ representation of the $SU(6)$ gauge group respectively, then the gauge coupling unification can be satisfied with accuracy of 0.68% at the energy scale of $6.2 \times 10^{16}$ GeV, which is shown in Fig. 4. Also, we make all the particles beyond the SM take part in the RGE running at the energy scale of 2 TeV.

**VII. CONCLUSIONS**

We have proposed a new $SU(3)_C \times SU(3)_L \times U(1)_X$ model, in which gauge symmetry can be realized from $SU(6)$ breaking. The SM fermions in each of the three generations come from two $\bar{6}$ representations and one $15$ representation of the $SU(6)$ gauge group besides two singlets of the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge group. There are three scalar multiplets, where two come from $\bar{6}$ representations of $SU(6)$ and one from $15$ representation. And their VEVs are $v_u$, $v_d$ and $v_t$, respectively. There are additional gauge bosons, $W^\pm$, $Z'$ and $V/V^*$, in our model besides the SM gauge bosons. $v_t$ needs to be larger than 10 TeV to make the mass of $Z'$ larger than 4 TeV. It is easy to give the 125 GeV Higgs boson mass when we set all the dimensionless parameters in the
Higgs potential to be $\sim 0.1$ and $A$ to be $\sim 1$ TeV. When $M_{s' s'}$ are set to be a zero matrix and in the limits of $\tan \theta \gg 1$ and $|k| \gg 1$, the mixing of $\nu_L$, $N$ and $N_s$ is the same as in the littlest inverse seesaw model. The lightest neutrino in our model is massless. With parameters in $y^\nu$, $y^N$ and $M_s$ set to be appropriate values, we obtained the light active neutrino masses, leptonic mixing angles, and CP violating phase highly consistent with the experimental datas for the scenario of NH neutrino mass. To make $BR(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$, parameters in $y^\nu$ needs to be smaller than $\sim 10^{-3}$, and in
this case \( BR(\tau \rightarrow \mu \gamma) \) and \( BR(\tau \rightarrow e \gamma) \) are around \( 10^{-14} \) and \( 10^{-13} \) respectively. With additional two fermion multiplets, \( FA \) and \( FA' \), as well as two scalar multiplets, \( SA \) and \( T' \), the gauge coupling unification can be realized with accuracy of 0.68\% at the energy scale of \( 6.2 \times 10^{16} \) GeV. \( SA \) can be a dark matter candidate if \( Z_2 \) symmetry is imposed. Alternatively, we can add two fermionic multiplets in 6 and \( \bar{6} \) representations of the \( SU(6) \) gauge group to make \( SA \) decay, then the gauge coupling unification can be satisfied with accuracy of 0.65\% at the energy scale of \( 5.2 \times 10^{16} \) GeV.

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