Effects of chiral three-nucleon forces on $^4$He-nucleus scattering in a wide range of incident energies

Masakazu Toyokawa,1,* Masanobu Yahiro,1 Takuma Matsumoto,1 and Michio Kohno2

1Department of Physics, Kyushu University, Fukuoka 819-0395, Japan
2Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan

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Background: It is a current important subject to clarify properties of chiral three-nucleon forces (3NFs) not only in nuclear matter but also in scattering between finite-size nuclei. Particularly for the elastic scattering, this study has just started and the properties are not understood in a wide range of incident energies ($E_{in}$).

Aims and approach: We investigate basic properties of chiral 3NFs in nuclear matter with positive energies by using the Brueckner-Hartree-Fock method with chiral two-nucleon forces at NLO and 3NFs at NNLO, and analyze effects of chiral 3NFs on $^4$He elastic scattering from targets $^{208}$Pb, $^{58}$Ni and $^{40}$Ca over a wide range of $30 \lesssim E_{in}/A_P \lesssim 200$ MeV by using the $g$-matrix folding model, where $A_P$ is the mass number of the projectile.

Results: In symmetric nuclear matter with positive energies, chiral 3NFs make the single-particle potential less attractive and more absorptive. The effects mainly come from the Fujita-Miyazawa $2\pi$-exchange 3NF and slightly become larger as $E_{in}$ increases. These effects persist in the optical potentials of $^4$He scattering. As for the differential cross sections of $^4$He scattering, chiral-3NF effects are large in $E_{in}/A_P \gtrsim 60$ MeV and improve the agreement of the theoretical results with the measured ones. Particularly in $E_{in}/A_P \gtrsim 100$ MeV, the folding model reproduces measured differential cross sections pretty well. Cutoff ($\Lambda$) dependence is investigated for both nuclear matter and $^4$He scattering by considering two cases of $\Lambda = 450$ and 550 MeV. The uncertainty coming from the dependence is smaller than chiral-3NF effects even at $E_{in}/A_P = 175$ MeV.

I. INTRODUCTION

How do three-nucleon forces (3NFs) work in nuclear many-body systems? This is an important subject to be answered in nuclear physics. Even if 3NFs do not exist on a fundamental level, they come out in effective theories with a finite momentum cutoff $\Lambda$ by renormalizing the degrees of freedom present above $\Lambda$. The representative example is the $2\pi$-exchange process with intermediate nucleon excited states, typically the $\Delta(1232)$ isobar. It is now called the Fujita-Miyazawa 3NF [1]. As a phenomenological approach, attractive 3NFs were introduced to reproduce the binding energies for light nuclei [2], whereas repulsive 3NFs were used to explain the empirical saturation properties in symmetric nuclear matter [3].

Essential progress on this subject was made by chiral effective field theory (EFT) [4, 5] based on chiral perturbation theory. The theory provides a low-momentum expansion of two-nucleon force (2NF), 3NF and many-nucleon forces, and makes it possible to define the forces systematically. Figure 1 shows chiral 3NFs in the next-to-next-to-leading order (NNLO). Diagram (a) corresponds to the Fujita-Miyazawa $2\pi$-exchange 3NF [1], and diagrams (b) and (c) mean $1\pi$-exchange and contact 3NFs, respectively. The filled-square vertex has a strength $c_D$ in the diagram (b) and $c_E$ in the diagram (c). Quantitative roles of chiral 3NFs were extensively investigated, particularly for light nuclei and nuclear matter [6]; more precisely, see Ref. [7] for light nuclei, Refs. [8, 9] for ab initio nuclear-structure calculations in lighter nuclei and Refs. [10–16] for nuclear matter. In addition, effects of chiral four-nucleon forces were found to be small in nuclear matter [17, 18]. The chiral $g$ matrix, calculated from chiral 2NF+3NF with the Brueckner-Hartree-Fock (BHF) method, yields a reasonable nuclear matter sat-
uration curve for symmetric nuclear matter, when the parameters, $c_D$ and $c_E$, of NNLO 3NFs are tuned [13].

Nuclear scattering is another place to investigate 3NF effects. The theoretical description of $N+d$ scattering has been naturally associated with the necessity of 3NFs [7, 19], when the theory starts with sophisticated 2NFs determined from the experiments. Microscopic evaluation of nuclear optical potentials for nucleon-nucleus (NA) and nucleus-nucleus (AA) elastic scattering has a long history. The $g$-matrix folding model [20–25] is a standard method for deriving the optical potentials of NA and AA elastic scattering microscopically. In fact, the potentials have been used to analyze various kinds of nuclear reactions in many papers. In the model, the optical potentials were obtained by folding the $g$ matrix [20–25] with the projectile (P) density $\rho_P$ and the target (T) one $\rho_T$. This description has been quite successful in explaining many elastic scattering. At first, the effects of 3NFs were phenomenologically investigated in Ref. [23] for NA elastic scattering and in Refs. [22, 26] for NA and AA elastic scattering. The 3NFs reduce differential cross section and improve the agreement with measured vector analyzing powers. However, the role of 3NFs has not been clarified quantitatively, because the folding potential is adjusted to measured cross sections.

In Refs. [27, 28], as the first attempt, we made qualitative discussion for chiral-3NF effects on elastic scattering by using the hybrid method in which the existing local version of Melbourne $g$ matrix [21] was modified on the basis of the chiral $g$ matrix constructed from chiral 2NFs and 3NFs. The work showed that chiral-3NF effects are small for NA elastic scattering, but important for AA elastic scattering. Recently, we directly parameterized the chiral $g$ matrix as a local potential based on chiral 2NF+3NF, as briefly reported in Ref. [25]. In this paper, we present a full understanding of chiral-3NF effects on $^4$He elastic scattering over a wide range of incident energies up to 200 MeV by using the BHF method with chiral 3NFs in NNLO and chiral 3NFs of NNLO. We show that chiral-3NF effects provide density-dependent repulsive and absorptive corrections to the single-particle potential and that the effects slightly become larger as the energy increases. We also point out that the corrections mainly come from the Fujita-Miyazawa $2\pi$-exchange 3NF of diagram (a).

Second, we analyze chiral-3NF effects on $^4$He scattering from various targets in a wide range of incident energies by using the chiral $g$-matrix folding model. In order to make our discussion clear, we take $^4$He scattering as AA scattering, since the $g$-matrix folding model is confirmed to work well for $^4$He scattering in virtue of negligibly small projectile-breakup effects [29, 31]; see Sec. II D for further discussion. In addition, as targets we take heavier nuclei, $^{208}$Pb, $^{58}$Ni and $^{40}$Ca, since the $g$ matrix is evaluated in nuclear matter and is considered to be more suitable for heavier targets. For the targets, the experimental data are available in a wide range of $30 \lesssim E_{\text{in}}/A_P \lesssim 200$ MeV. In the present paper, we mostly consider the case of the cutoff scale $\Lambda = 550$ MeV. As the third subject, $\Lambda$ dependence is investigated for nuclear matter with positive energies and $^4$He elastic scattering by taking two other cases of $\Lambda = 450$ and 550 MeV.

Finally, we provide the local version of chiral $g$ matrix including chiral-3NF effects with a 3-range Gaussian form for the case of $E_{\text{in}}/A_P = 75$ MeV. This may strongly encourage the application of the chiral $g$ matrix for studying various kinds of nuclear reactions. This local version of chiral $g$ matrix is referred to as “Kyushu chiral $g$ matrix” in this paper.

In Sec. II, we present the theoretical framework composed of the BHF method and the folding model, and show some basic results of BHF calculations for chiral 2NF+3NF. In Sec. III, the results of the chiral $g$-matrix folding model are shown for $^4$He elastic scattering. Section IV is devoted to a summary.

II. THEORETICAL FRAMEWORK AND BASIC RESULTS

A. BHF equation for 2NF+3NF

We first recapitulate the BHF method for 2NF+3NF, following Ref. [12]. Because it is not easy to treat a 3NF $V_{123}$ even in nuclear matter, we introduce an effective
2NF $V_{12}^{\text{eff}}$ by applying the mean-field approximation, or the normal ordering prescription, to the 3NF:

$$
\frac{1}{2} \sum_{k_1 k_2} \langle k_1 k_2 | V_{12} | k_1 k_2 \rangle_A + \frac{1}{3} \sum_{k_1 k_2 k_3} \langle k_1 k_2 k_3 | V_{123} | k_1 k_2 k_3 \rangle_A
$$

$$= \frac{1}{2} \sum_{k_1 k_2} \langle k_1 k_2 | V_{12}^{\text{eff}} | k_1 k_2 \rangle_A, \quad (1)
$$

where $A$ means the antisymmetrization and $k_i$ corresponds to quantum numbers of the $i$-th nucleon. Equation (1) leads

$$V_{12}^{\text{eff}} = V_{12} + \frac{1}{3} V_{12(3)}, \quad (2)$$

where $V_{12(3)}$ is defined by summing up 3NF $V_{123}$ over the third nucleon in the Fermi sea:

$$\langle k'_1 k'_2 | V_{12(3)} | k_1 k_2 \rangle_A = \sum_{k_3} \langle k'_1 k'_2 k_3 | V_{123} | k_1 k_2 k_3 \rangle_A \quad (3)$$

with assuming the center-of-mass (c.m.) frame: $k'_1 + k'_2 = k_1 + k_2$. Note the factor $1/3$ in Eq. (2). The $g$ matrix $g_{12}$ is a solution to the BHF equation

$$g_{12} = V_{12}^{\text{eff}} + V_{12}^{\text{eff}} G_0 g_{12}, \quad (4)$$

where $G_0$ is the nucleon propagator with the Pauli exclusion operator in the numerator and with the single-particle energy

$$\epsilon_k = \langle k | T | k \rangle + \text{Re} \langle \mathcal{U}(k) \rangle \quad (5)$$

of the nucleon having a momentum $k$ in the denominator. Here $T$ is the standard kinetic-energy operator of nucleon, and the single-particle potential $\mathcal{U}(k)$ is defined by [12]

$$\mathcal{U}(k) = \sum_{k' \leq k_F} \langle k' k' | \tilde{g}_{12} | k k \rangle_A. \quad (6)$$

with the effective $g$ matrix, so-called $\tilde{g}$ matrix, including additional rearrangement terms of the 3NF origin:

$$\tilde{g}_{12} = g_{12} + \frac{1}{6} V_{12(3)} (1 + G_0 g_{12}). \quad (7)$$

Note that $k$ is related to the incident energy $E_{in}$ as $E_{in} = (hk)^2 / (2m) + \text{Re} \langle \mathcal{U} \rangle$. The present formulation is consistent with the second-order perturbation of Ref. [32], because of the factor 1/6 in Eq. (7). For the symmetric nuclear matter where the proton density $\rho_p$ agrees with the neutron one $\rho_n$, the Fermi momentum $k_F$ is related to the matter density $\rho = \rho_p + \rho_n$ as $k_F^2 = 3 \pi^2 \rho / 2$, so that the normal density $\rho = \rho_0 = 0.17 \text{ fm}^{-3}$ is realized at $k_F = 1.35 \text{ fm}^{-1}$.

**B. Some basic results of BHF calculations**

The $\tilde{g}$ matrix is calculated from chiral 2NF of $N^3LO$ and chiral 3NF of NNLO by using the BHF method. In BHF calculations, the form factor $\exp{[-(q^2 / \Lambda_0) - (q / \Lambda_0)^6]}$ is introduced for both $V_{12}$ and $V_{12(3)}$. We mainly consider the case of $\Lambda = 550$ MeV, and take another case $\Lambda = 450$ MeV where $\Lambda$ dependence of physical quantities is estimated. The low-energy constants relevant for 3NFs are $(c_1, c_3, c_4) = ( -0.81, -3.4, 3.4)$ [33] in units of GeV$^{-1}$.

As noted earlier, some errors were found in nuclear-matter calculations with chiral 3NFs of Ref. [12], after Ref. [25] was published. Although the qualitative importance of chiral 3NFs for improving nuclear matter saturation properties does not change, the saturation curve is changed by the corrections. To restore reasonable nuclear saturation properties, which are basically important for further application for microscopic derivation of nuclear optical potentials, the remaining two parameters $c_D$ and $c_E$ are tuned [30]. In consideration of the uncertainty that the $c_D$ and $c_E$ terms yield almost identical contributions when $c_D \approx 4c_E$, $c_D$ is determined as $-2.5$ by setting $c_E = 0$ for $\Lambda = 450$ MeV and next $c_E$ is fixed as $0.25$ for $\Lambda = 550$ MeV with keeping $c_D = -2.5$. These values are somewhat different from those determined in few-body systems within continuous uncertainties. It has been recognized [9], however, that low-energy-constants fixed solely in few-body systems are not adequate in heavier systems. In this article, we use the corrected version of the chiral $g$ matrix.

It is known that chiral 3NFs make repulsive corrections to the binding energy of symmetric nuclear matter [12]. What happens in positive energy? Figure 2 shows $E_{in}$ dependence of $\mathcal{U}$ for the case of $k_F = 1.2$ fm$^{-1}$ for the cutoff $\Lambda = 550$ MeV. This density is realized in the peripheral region of a target nucleus and hence important for elastic scattering. Filled (open) circles denote the results of BHF calculations with (without) chiral 3NFs. One can see that chiral 3NFs make $\mathcal{U}$ less attractive and more absorptive. The 3NF corrections slightly increase as $E_{in}$ goes up. Our results are consistent with the second-order perturbation calculation by Holt et. al. [32].

Figure 3 shows $\mathcal{U}$ as a function of $E_{in}$ at $k_F = 1.2$ fm$^{-1}$, but two cases of $\Lambda = 450$ and 550 MeV are taken in BHF calculations to see the uncertainty coming from $\Lambda$ dependence on $\mathcal{U}$. The $\Lambda$ dependence is plotted as an error bar. The error bar plotted by a solid (dashed) line denotes the results of BHF calculations with (without) chiral 3NFs; note that panels (a) and (b) correspond to the real and imaginary parts of $\mathcal{U}$. Particularly for BHF calculations with chiral 3NFs, there is a tendency that the uncertainty become larger as $E_{in}$ increases from 80 MeV. Even at $E_{in} = 175$ MeV, however, chiral 3NF effects are larger than the uncertainty. This enables us to make reliable discussion on chiral-3NF effects.

In order to obtain deeper understanding of the properties of chiral 3NFs, we classify $\tilde{g}(k_F, E_{in})$ with the total...
FIG. 2: (Color online) $E_{\text{in}}$ dependence of $\mathcal{U}$ at $k_F = 1.2 \text{ fm}^{-1}$ for the cutoff $\Lambda = 550 \text{ MeV}$. Filled (open) circles stand for the results of BHF calculations with (without) chiral 3NFs. Panels (a) and (b) correspond to the real and imaginary parts of $\mathcal{U}$.

Panels (a) and (b) correspond to the real and imaginary parts of $U^{ST}$ in each $(S, T)$ channel as

$$U = \sum_{ST} (2S + 1)(2T + 1)U^{ST}, \quad (8)$$

where $U^{ST}$ is defined by Eq. (6) with $\tilde{g}$ replaced by $\tilde{g}^{ST}$.

Figure 4 shows $E_{\text{in}}$ dependence of $U^{ST} = (2S+1)(2T+1)U^{ST}$ for the case of $k_F = 1.2 \text{ fm}^{-1}$. Here we do the following three kinds of BHF calculations:

I. All kinds of chiral 3NFs, i.e., diagrams (a)-(c) in Fig. 1, are taken into account.

II. All kinds of chiral 3NFs are switched off. Namely, only chiral 2NF is considered.

III. Diagrams (b) and (c) are ignored by setting $c_D = c_E = 0$ in BHF calculations. Namely, only the Fujita-Miyazawa $2\pi$-exchange 3NF of diagram (a) is considered.

Filled circles (squares) stand for the real (imaginary) part of $U^{ST}$ for calculation I, while open circles (squares) correspond to the real (imaginary) part of $U^{ST}$ for calculation II; note that lines are a guide to the eye. The two calculations show that chiral 3NF effects are significant for $^3\text{O}$ ($S = 1, T = 1$) and $^3\text{E}$ ($S = 1, T = 0$) channel and the real part of $^1\text{E}$ ($S = 0, T = 1$) channel. Small circles (squares) represent the real (imaginary) part of $U^{ST}$ for calculation III. For $^3\text{E}$ and $^3\text{O}$, one can see from calculations II and III that chiral 3NF effects mainly come from the Fujita-Miyazawa $2\pi$-exchange 3NF of diagram (a). For the real part of $^1\text{E}$ ($S = 0, T = 1$) channel, the effect of diagram (a) is sizable, but it is considerably reduced by the effects of diagram (b) and (c). As a net effect of these properties, chiral 3NFs make $\mathcal{U}$ less attractive and more absorptive, and the repulsion mainly stems from diagram (a) in its $^3\text{O}$ component and the absorption does from diagram (a) in its $^3\text{O}$ and $^3\text{E}$ components. The chiral-3NF effects become more significant at larger incident energies. One can easily expect that these properties persist also in the optical potentials of $^4\text{He}$ scattering, since $U$ plays a role of “optical potential” of nucleon scattering in nuclear matter. This point will be discussed later in Sec. III.
C. Local version of chiral $g$ matrix

The $\tilde{g}$ matrix $\tilde{g}(k_F, E_{in})$ of Eq. (7) is a nonlocal potential depending on $k_F$ and $E_{in}$, being calculated in symmetric nuclear matter. In addition, it is obtained numerically. These properties are quite inconvenient in various applications. In order to circumvent the problem, the Melbourne group showed that elastic scattering are determined by the on-shell and near-on-shell components of $g$ matrix [21], and provided a local version of $g$ matrix in which the potential parameters are so determined as to reproduce the relevant components [21, 34, 35]. The Melbourne $g$ matrix thus obtained well accounts for NN scattering in free space that corresponds to the limit of $\rho = 0$, and the Melbourne $g$-matrix folding model reproduces NA scattering, as already mentioned in Sec. I.

In our previous paper [25], following the Melbourne-group procedure [21, 34, 35], we succeeded in parameterizing a local version of chiral $\tilde{g}$ matrix in a 3-range Gaussian form for each of the central, spin-orbit and tensor components. The Gaussian form makes various kinds of numerical calculations efficient. The range and strength parameters were so determined as to reproduce the on-shell and near-on-shell matrix elements of the original $\tilde{g}$ matrix for each spin-isospin channel $\left( S = 0, T = 0 \right)$, $\left( S = 1, T = 0 \right)$, and $\left( S = 1, T = 1 \right)$. Filled circles (squares) represent the real (imaginary) part of original chiral $\tilde{g}$ matrix, whereas the solid (dashed) lines correspond to the real (imaginary) part of original chiral $g$ matrix. For the case of $E_{in}$ = 75 MeV as an example, we present the parameter set of Kyushu chiral $g$ matrix in Appendix A.

Figure 5 shows differential cross sections as a function of center-of-mass scattering angle $\theta_{c.m.}$ for $p+n$ scattering at $E_{in}$ = 150 MeV in free space, i.e., in the limit of $\rho = 0$. The solid and dashed lines denote the results of original and Kyushu chiral $t$ matrices, respectively; note that the $g$ matrix is reduced to the $t$ matrix in the limit of $\rho = 0$. The Kyushu chiral $t$ matrix reproduces the result of original chiral $t$ matrix well.

Figure 6 shows $k_F$ dependence of $U^{ST}$ at $E_{in}$ = 150 MeV. Both 2NF and 3NF are taken into account in BHF calculations. The filled circles (squares) denote the results of the real (imaginary) part of original chiral $g$ matrix, whereas the solid (dashed) lines correspond to the real (imaginary) part of Kyushu chiral $g$ matrix. The range $k_F \lesssim 1.35$ fm$^{-1}$ ($\rho \lesssim \rho_0$) contributes to the optical potentials of $^4\text{He}$ scattering, when the potentials are constructed by the folding model explained in Sec. II.D. In particular, the Fermi momentum $k_F \approx 1.2$ fm$^{-1}$, corresponding to the peripheral region of the optical potentials, is important for the elastic scattering. The Kyushu

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D. Folding model

In this paper, the optical potentials are derived by folding Kyushu chiral $g$ matrix with $p_T$ and $\rho_T$ for $^4$He scattering on $^{208}$Pb, $^{58}$Ni and $^{40}$Ca targets. In general, the folding potential is referred to as a double-folding (DF) model for AA scattering, while it is called a single-folding (SF) model for NA scattering.

In the $g$-matrix SF model for NA elastic scattering, the so-called local-density approximation is taken, that is, the value of $\rho$ in $g(\rho)$ is identified with the value of $\rho_T$ at the midpoint $r_m$ of interacting two nucleons: $\rho = \rho_T(r_m)$. Target-excitation effects on the elastic scattering are well taken into account by this framework. In fact, the Melbourne $g$-matrix SF model succeeded in reproducing NA scattering [21]. In our previous work [25], furthermore, we showed that the Kyushu chiral $g$-matrix SF model also well accounted for proton scattering at $E_{\text{in}} = 65$ MeV and chiral-3NF effects are small there.

The $g$-matrix DF model for AA scattering had a problem to be settled. In order to obtain the $g$ matrix applicable for AA scattering, in principle, we have to consider two Fermi spheres in nuclear-matter calculations and solve a collision between a nucleon in the first Fermi sphere and a nucleon in the second one [37, 38]. However, actual calculations are not feasible. In fact, all the $g$ matrices provided so far were obtained by assuming a single Fermi sphere and solving nucleon scattering on the Fermi sphere. For consistency with the nuclear-matter calculation, we assumed $\rho = \rho_T(r_m)$ in $g(\rho)$ and applied the framework to $^{3,4}$He scattering in a wide energy range of $30 \lesssim E_{\text{in}}/A_P \lesssim 180$ MeV [29, 31]. The Melbourne...
$g$-matrix DF model based on the target-density approximation (TDA) well accounted for $^3$He scattering, particularly for forward differential cross sections where $3NF$ effects are considered to be negligible [22, 23, 26]. In our previous analysis [25], the DF-TDA model based on Kyushu chiral $g$ matrix well explained $^4$He scattering at $E_{\text{in}}/A_P \approx 72$ MeV. We then take the DF-TDA model for $^4$He scattering in this paper throughout all the incident energies $30 \lesssim E_{\text{in}}/A_P \lesssim 180$ MeV where the experimental data are available.

The DF model naturally treats both the direct and knock-on exchange processes [38–40]. In the latter process, interacting two nucleons are exchanged and thereby the potential becomes nonlocal. However, the nonlocality can be localized with high accuracy by the local momentum approximation [20], as proven in Refs. [41, 42]. The folding potential $U(R)$ thus obtained is a function of the distance $R$ between P and T;

$$U(R) = \sum_{\mu \nu} \int dr_P \int dr_T \rho_{\mu}(r_P) \rho_{\nu}(r_T)$$

$$\times \tilde{g}_{\mu \nu}^{\text{DR}}(s, E_{\text{in}}/A_P; \rho)$$

$$- \sum_{\mu \nu} \int dr_P \int dr_T \tilde{\rho}_{\mu}(r_P, s) \tilde{\rho}_{\nu}(r_T, s)$$

$$\times \tilde{g}_{\mu \nu}^{\text{EX}}(s, E_{\text{in}}/A_P; \rho) j_0 \left( \frac{4p+4\Delta}{\sqrt{A}} K(R) s \right),$$

where the indices $\mu$ and $\nu$ are the isospin of corresponding nucleon and $s = r_T - r_P - R$ is the coordinate between interacting two nucleons. The densities $\rho_{\mu}(T)$ and $\tilde{\rho}_{\mu}(T)$ represent the one-body and mixed densities of P (T);

$$\tilde{\rho}_{\mu}(T) = \rho_{\mu}(T) \left( |r_{\mu}(T) + s/2| \right) \frac{3j_1(k_F^{\mu}(T) s)}{k_F^{\mu}(T) s}.$$  \hspace{1cm} (10)

The Fermi momentum $k_F^{\mu}(T)$ is related to the density $\rho_{\mu}(T)$. The direct (exchange) term of $g$-matrix $g_{\mu \nu}^{\text{DR(EX)}}$ is defined by $\tilde{g}_{\mu \nu}^{\text{DR(EX)}}$ as

$$\tilde{g}_{\mu \nu}^{\text{DR(EX)}} = \frac{1}{4} (\tilde{g}_{01}^{01} + 3\tilde{g}_{11}^{11}),$$  \hspace{1cm} (11)

$$\tilde{g}_{\mu \nu}^{\text{DR(EX)}} = \frac{1}{8} (\tilde{g}_{00}^{00} + \tilde{g}_{01}^{01} + 3\tilde{g}_{10}^{10} + 3\tilde{g}_{11}^{11}).$$  \hspace{1cm} (12)

See Refs. [26, 31, 43, 44] for the detail of the formulation of the DF model. The $S$ matrices for $^4$He elastic scattering are obtained by solving the one-body Schrödinger equation with $U(R)$.

For the targets $^{208}$Pb and $^{58}$Ni, the matter densities $\rho_T$ are evaluated by the spherical Hartree-Fock (HF) method based on the Gogny-DIS interaction [45], where the spurious c.m. motions are removed with the standard manner [46]. For the projectile $^4$He and the target $^{40}$Ca, we take the phenomenological proton-density determined from electron scattering [47]; here the finite-size effect of proton charge is unfolded with the standard procedure [48], and the neutron density is assumed to have the same geometry as the proton one, since the difference between the neutron root-mean-square radius and the proton one is only 1% in spherical HF calculations.

### III. RESULTS

Now we analyze $^4$He elastic scattering on nuclei systematically in a wide range $E_{\text{in}}/A_P = 26$–175 MeV. Here heavier targets $^{208}$Pb, $^{58}$Ni and $^{40}$Ca are considered, because the $g$ matrix is calculated in nuclear matter and thereby the $g$-matrix DF model is expected to be more reliable for heavier targets.

Figure 7 shows differential cross sections $d\sigma/d\Omega$ for $^4$He scattering from a $^{208}$Pb target in $E_{\text{in}}/A_P = 26$–175 MeV where the experimental data are available. The solid and dashed lines stand for the results of the Kyushu chiral $g$-matrix DF model with and without $3NF$ effects, respectively. Chiral $3NF$s improve the agreement of the theoretical results with the experimental data. Particularly for $E_{\text{in}}/A_P \gtrsim 100$ MeV, the agreement is pretty good. We can observe the same features also for $^{58}$Ni and $^{40}$Ca targets, as shown in Figs. 8 and 9, although there is a tendency that the agreement becomes better as the target mass increases.

Now we analyze effects of Fujita-Miyazawa $2\pi$-exchange $3NF$ on differential cross sections $d\sigma/d\Omega$ for $^4$He+$^{58}$Ni scattering. In Fig 10, the solid, dashed and dot-dashed lines denote the results of calculations $I$, $II$ and $III$, respectively; see Sec. II B for the definition of $g$-matrix calculations. The difference between calculations $I$ and $II$ means effects of all $3NF$s, and that between calculations II and III corresponds to effects of Fujita-Miyazawa $2\pi$-exchange $3NF$. The resultant cross sections show that the Fujita-Miyazawa $2\pi$-exchange $3NF$ is the main contribution of chiral-$3NF$s effects on $^4$He scattering.

Figure 11 shows the $R$ dependence of the optical potentials $U(R)$ for $^4$He elastic scattering from a $^{58}$Ni target at $E_{\text{in}}/A_P = 26$, 60 and 175 MeV. The solid and dashed lines represent the $U(R)$ with and without chiral-$3NF$s effects; note that only the central potential is generated by the DF-TDA model. As expected, chiral-$3NF$s effects make repulsive and absorptive corrections to the optical potentials, and the corrections slightly increase as $E_{\text{in}}$ goes up; note that the effects hardly depend on $E_{\text{in}}$ in the peripheral region, $R \approx 6$ fm, that is important for the elastic scattering. As already mentioned in Sec. II B, the repulsive correction mainly comes from the Fujita-Miyazawa $2\pi$-exchange $3NF$ in its $^3O$ component, and the absorptive correction stems from the $^3E$ and $^3O$ components of Fujita-Miyazawa $2\pi$-exchange $3NF$.

Figure 12 shows the uncertainty coming from $\Lambda$ dependence of differential cross sections $d\sigma/d\Omega$ for $^4$He+$^{58}$Ni elastic scattering. Here two cases of $\Lambda = 550$ and 450 MeV are considered. $\Lambda$ dependence is shown by a hatching for each of $2NF$s and $2NF+3NF$ calculations; note that the hatching region surrounded by solid (dashed)
FIG. 7: (Color online) Differential cross sections $d\sigma/d\Omega$ as a function of transfer momentum $q$ for $^4$He scattering from a $^{208}$Pb target at $E_{\text{lab}}/A_p = 26–175$ MeV. The solid (dashed) lines denote the results of Kyushu chiral $g$ matrix with (without) 3NF effects. Each cross section is multiplied by the factor shown in the figure. Experimental data are taken from Refs. [49–52].

FIG. 8: (Color online) Same as Fig. 7, but the target nucleus is $^{58}$Ni. Experimental data are taken from Refs. [51, 53–57].
and the dot-dashed line corresponds to the results of calculations III; see Sec. II B for the definition of
Each cross section is multiplied by the factor shown in the figure.

FIG. 9: (Color online) Same as Fig. 7, but the target nucleus is $^{40}$Ca. Experimental data are taken from Refs. [49, 58, 59].

FIG. 10: (Color online) Effects of Fujita-Miyazawa 2π-exchange 3NF on differential cross sections $d\sigma/d\Omega$ for $^4$He+$^{58}$Ni scattering, where $q$ is the transfer momentum. The solid and dashed lines denote the results of calculations I and II, respectively, and the dot-dashed line corresponds to the results of calculations III; see Sec. II B for the definition of $g$-matrix calculations. Each cross section is multiplied by the factor shown in the figure.
FIG. 11: (Color online) Optical potentials $U(R)$ as a function of $R$ for $^4\text{He}+^{58}\text{Ni}$ elastic scattering at $E_{\text{in}}/A_P = 26, 60$ and 175 MeV. The solid (dashed) lines denote the optical potentials with (without) chiral-3NF effects. Panels (a) and (b) represent the real and imaginary parts of $U$, respectively.

lines means the uncertainty coming from $\Lambda$ dependence for 2NF+3NF (2NF) calculations. As expected, $\Lambda$ dependence becomes larger as $E_{\text{in}}$ increases, but the uncertainty coming from $\Lambda$ dependence is still smaller than chiral-3NF effects, even at $E_{\text{in}}/A_P = 175$ MeV.

The scattering amplitude can be decomposed into the near- and far-side components [60]. As illustrated in Fig. 13, these components are well defined, when outgoing waves are generated only in the peripheral region of T. $^4\text{He}$ scattering on a heavier target is a good case. The absorptive correction of chiral-3NF effects makes the decomposition more applicable. The decomposition is a convenient tool for investigating the interplay between differential cross sections $d\sigma/d\Omega$ and the real part of $U(R)$. The near-side (far-side) outgoing waves are mainly induced by repulsive Coulomb (attractive nuclear) force, so that very-forward-angle (middle-angle) scattering are dominated by the near-side (far-side) components. As a consequence of this property, a large interference pattern appears in differential cross sections at the forward angles where the two components become comparable, and the far-side dominance is realized at middle angles after the interference pattern. In the middle angle region, any repulsive correction to $U(R)$ reduces differential cross sections.

Figure 14 shows the near/far decomposition of differential cross sections $d\sigma/d\Omega$ for $^4\text{He}+^{58}\text{Ni}$ scattering at $E_{\text{in}}/A_P = 72$ MeV. The dotted and dashed lines represent the near- and far-side cross sections, respectively, and the solid line denotes differential cross sections before the near/far decomposition; here chiral-3NF effects are taken into account. The solid line shows a large interference pattern at $\theta_{\text{c.m.}} = 5$–15°, and the solid line agrees with the dashed one in 20° $\lesssim \theta_{\text{c.m.}} \lesssim 40°$. The far-side dominance is thus realized in middle angles 20° $\lesssim \theta_{\text{c.m.}} \lesssim 40°$. The far-side dominance in 20° $\lesssim \theta_{\text{c.m.}} \lesssim 40°$ persists, even after chiral 3NFs are switched off. The dot-dashed line is the far-side cross section in which chiral 3NFs are switched off. The repulsive correction coming from chiral 3NFs suppresses differential cross sections in far-side dominant angles 20° $\lesssim \theta_{\text{c.m.}} \lesssim 40°$ from the dot-dashed line to the solid (dashed) line. Thus, chiral-3NF effects become more visible in the far-side dominant angle region.
Finally, we comment on chiral-3NF effects on total reaction cross sections $\sigma_R$ briefly. Radii of stable and unstable nuclei are often determined from measured $\sigma_R$ with the folding model and/or the Glauber model. Figure 15 shows $\sigma_R$ as a function of $E_{\text{in}}/A_P$ for $^4\text{He}$ scattering on $^{58}\text{Ni}$ and $^{208}\text{Pb}$ targets. Closed circles (squares) mean the results of Kyushu chiral $g$ matrix with (without) 3NF effects. The two kinds of results are close to each other, indicating that chiral-3NF effects are negligible for $\sigma_R$. The fact ensures that the determination of nuclear radii from measured $\sigma_R$ is reliable.

FIG. 12: (Color online) Uncertainty coming from $A$ dependence of differential cross sections $d\sigma/d\Omega$ for $^4\text{He}+^{58}\text{Ni}$ elastic scattering. A dependence is drawn by a hatching for each of 2NF and 2NF+3NF calculations, where two cases of $A = 550$ and 450 MeV are taken. Note that the hatching region surrounded by the solid (dashed) lines corresponds to the uncertainty coming from $A$ dependence for 2NF+3NF (2NF) calculations.

FIG. 13: (Color online) Illustration of the near/far decomposition.

FIG. 14: (Color online) Near/far decomposition of differential cross sections $d\sigma/d\Omega$ for $^4\text{He}+^{58}\text{Ni}$ scattering at $E_{\text{in}}/A_P = 72$ MeV. The dotted (dashed) line stands for the near-side (far-side) cross sections, while the solid line denotes differential cross sections before the near/far decomposition; here chiral-3NF effects are taken into account. The dot-dashed line corresponds to the far-side cross section in which chiral 3NFs are switched off.
Properties (1)-(3) persist in the optical potential of $^4\text{He}$ scattering. This is natural, since the single-particle potential plays a role of the optical potential in nuclear matter. However, it should be noted that chiral-3NF effects depend little on $E_{\text{in}}$ in the peripheral region that is important for the elastic scattering.

Chiral-3NF effects are evident for $^4\text{He}$ scattering in $E_{\text{in}}/A_{\text{P}} \gtrsim 60$ MeV at the middle angles where the cross sections are dominated by the far-side component of the scattering amplitude. The repulsive correction of chiral 3NFs reduces the far-side component and thereby yields better agreement with the experimental data. Eventually, the Kyushu chiral $g$-matrix DF model reproduces measured differential cross sections pretty well, particularly for $^4\text{He}$ scattering at $E_{\text{in}}/A_{\text{P}} \gtrsim 100$ MeV.

All the analyses mentioned above were made with $\Lambda = 550$ MeV. In order to investigate $\Lambda$ dependence in nuclear-matter and $^4\text{He}$-scattering calculations, we take $\Lambda = 450$ MeV in addition to $\Lambda = 550$ MeV. The uncertainty coming from $\Lambda$ dependence is smaller than chiral-3NF effects. There is a tendency that the uncertainty becomes larger as $E_{\text{in}}$ increases, but it is still smaller than chiral-3NF effects even at $E_{\text{in}} = 175$ MeV.

Finally, we provide the local version of chiral $g$-matrix with a 3-range Gaussian form for the case of $E_{\text{in}} = 72$ MeV. Numerical numbers are presented in Appendix A. This local version of chiral $g$ matrix strongly encourages us to use it for studying various kinds of nuclear reactions.

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### Appendix A: Parameter set of Kyushu chiral $g$ matrix

In this Appendix, we provide the central part of the Kyushu chiral $g$ matrix, for the case of $E_{\text{in}}/A_{\text{P}} = 75$ MeV, in a 3-range Gaussian form

$$g^{ST}(s, k_F, E_{\text{in}}/A_{\text{P}}) = \sum_{i=1}^{3} g_i^{ST}(k_F, E_{\text{in}}/A_{\text{P}}) e^{-s^2/\lambda_i^2} \quad (A1)$$

in each $(S,T)$ channel. The range parameters are fixed to be $(\lambda_1, \lambda_2, \lambda_3) = (0.4, 0.9, 2.5)$ in units of fm, and the strength parameters $g_i^{ST}(k_F, E_{\text{in}}/A_{\text{P}})$ in units of MeV, which include chiral 3NF effects, are tabulated in Tables I–IV for six cases of the Fermi momentum $k_F$. We will publish parameter sets of other cases on the website [63].
TABLE I: Singlet-even ($S = 0, T = 1$) component of Kyushu chiral $g$ matrix for the incident energy $E_{in}/A_p = 75$ MeV. The range parameters are fixed to be ($\lambda_1, \lambda_2, \lambda_3$) = (0.4, 0.9, 2.5) in units of fm. Entries are in MeV, but $k_F$ is presented in units of fm$^{-1}$.

| $k_F$ | $t = 1$ | $t = 2$ | $t = 3$ | imaginary part |
|-------|---------|---------|---------|----------------|
| 0.00  | 1.78627×10$^4$ | -2.70833×10$^2$ | -4.08777×10$^0$ | 2.16554×10$^4$ |
| 0.60  | 1.47941×10$^3$ | -2.47638×10$^2$ | -3.68242×10$^0$ | 1.13963×10$^3$ |
| 0.80  | 1.36782×10$^3$ | -2.39205×10$^2$ | -3.53502×10$^0$ | 7.66206×10$^2$ |
| 1.10  | 1.20044×10$^3$ | -2.26551×10$^2$ | -3.31392×10$^0$ | 4.05075×10$^2$ |
| 1.20  | 1.09716×10$^3$ | -2.13700×10$^2$ | -3.19454×10$^0$ | 2.04489×10$^2$ |
| 1.30  | 9.54436×10$^2$ | -1.94839×10$^2$ | -3.14638×10$^0$ | 1.57448×10$^2$ |
| 1.40  | 7.65583×10$^2$ | -1.68406×10$^2$ | -3.19684×10$^0$ | 1.05755×10$^2$ |
| 1.50  | 5.88265×10$^2$ | -1.44596×10$^2$ | -3.21499×10$^0$ | 5.29847×10$^2$ |

TABLE II: Triplet-even ($S = 1, T = 0$) component of Kyushu chiral $g$ matrix for the incident energy $E_{in}/A_p = 75$ MeV. See Table I for the detail.

| $k_F$ | $t = 1$ | $t = 2$ | $t = 3$ | imaginary part |
|-------|---------|---------|---------|----------------|
| 0.00  | 1.25135×10$^4$ | -2.14234×10$^2$ | -1.23044×10$^0$ | 2.83745×10$^4$ |
| 0.60  | 1.26094×10$^3$ | -2.62345×10$^2$ | -3.04629×10$^0$ | 1.96817×10$^3$ |
| 0.80  | 1.26443×10$^3$ | -2.79841×10$^2$ | -3.61659×10$^0$ | 1.65218×10$^3$ |
| 1.10  | 1.26966×10$^3$ | -3.06084×10$^2$ | -4.19678×10$^0$ | 1.17821×10$^3$ |
| 1.20  | 1.13634×10$^3$ | -2.90035×10$^2$ | -4.19544×10$^0$ | 9.55010×10$^2$ |
| 1.30  | 9.50006×10$^2$ | -2.63272×10$^2$ | -1.57370×10$^0$ | 8.52716×10$^2$ |
| 1.40  | 5.98320×10$^2$ | -2.11212×10$^2$ | -1.85361×10$^0$ | 8.38254×10$^2$ |
| 1.50  | 4.87230×10$^2$ | -1.95265×10$^2$ | -1.99886×10$^0$ | 8.31225×10$^2$ |

TABLE III: Singlet-odd ($S = 0, T = 1$) component of Kyushu chiral $g$ matrix for the incident energy $E_{in}/A_p = 75$ MeV. See Table I for the detail.

| $k_F$ | $t = 1$ | $t = 2$ | $t = 3$ | imaginary part |
|-------|---------|---------|---------|----------------|
| 0.00  | 1.17797×10$^2$ | 1.54048×10$^1$ | 9.23703×10$^0$ | 6.01921×10$^4$ |
| 0.60  | 2.93289×10$^2$ | 9.73676×10$^1$ | 8.54387×10$^0$ | 4.56013×10$^2$ |
| 0.80  | -2.97222×10$^2$ | 1.27172×10$^2$ | 8.29181×10$^0$ | 4.02955×10$^2$ |
| 1.10  | -5.12600×10$^2$ | 1.71879×10$^2$ | 7.91372×10$^0$ | 3.23369×10$^2$ |
| 1.20  | -6.98327×10$^2$ | 1.94038×10$^2$ | 7.80370×10$^0$ | 3.02061×10$^2$ |
| 1.30  | -8.68560×10$^2$ | 2.13672×10$^2$ | 7.65323×10$^0$ | 3.08332×10$^2$ |
| 1.40  | -1.21630×10$^3$ | 2.43679×10$^2$ | 7.28544×10$^0$ | 3.39578×10$^2$ |
| 1.50  | -1.35278×10$^3$ | 2.58695×10$^2$ | 7.07301×10$^0$ | 3.38596×10$^2$ |

TABLE IV: Triplet-odd ($S = 1, T = 1$) component of Kyushu chiral $g$ matrix for the incident energy $E_{in}/A_p = 75$ MeV. See Table I for the detail.

| $k_F$ | $t = 1$ | $t = 2$ | $t = 3$ | imaginary part |
|-------|---------|---------|---------|----------------|
| 0.00  | 1.48087×10$^2$ | -1.17015×10$^2$ | 4.09818×10$^1$ | 6.20964×10$^4$ |
| 0.60  | 9.89083×10$^2$ | -7.76051×10$^2$ | 3.97949×10$^1$ | 4.90110×10$^2$ |
| 0.80  | 8.11232×10$^2$ | -6.32740×10$^2$ | 3.93633×10$^1$ | 4.03435×10$^2$ |
| 1.10  | 5.43375×10$^2$ | -4.17774×10$^2$ | 3.87159×10$^1$ | 3.65697×10$^2$ |
| 1.20  | 3.30284×10$^2$ | -2.25888×10$^2$ | 3.93880×10$^1$ | 3.57729×10$^2$ |
| 1.30  | 1.31873×10$^2$ | -4.61127×10$^2$ | 3.96357×10$^1$ | 3.94174×10$^2$ |
| 1.40  | -9.73556×10$^1$ | 1.42691×10$^2$ | 3.20404×10$^1$ | 4.67099×10$^2$ |
| 1.50  | -2.46017×10$^2$ | 2.76700×10$^2$ | 3.11786×10$^1$ | 5.04428×10$^2$ |
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