Dynamic Multiple-Message Broadcast: Bounding Throughput in the Affectance Model

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**Dynamic** Multiple-Message Broadcast (MMB) [1]:

- **Problem:**
  
  packets arrive at some nodes *continuously*, to be delivered to all nodes

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[1] (non-dynamic MMB) Khabbazian-Kowalski PODC 2011
[2] Halldórsson-Wattenhofer, ICALP 2009
[3] Kesselheim, PODC 2012
[4] Kesselheim-Vöcking, DISC 2010
Introduction

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- **metric:** **competitive throughput** of deterministic distributed MMB algorithms

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- **analysis:**
  in the **Affectance model**:
  - Affectance subsumes many interference models, e.g. RN and SINR models
  - conceptual idea: parameterize interference from transmitting **nodes into links**
  - introduced [2,3,4] for link scheduling as link-to-link affectance

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- introduce new model characteristics:
  (based on comm network, affectance function, and a chosen BFS tree)
  - maximum average tree-layer affectance $K$
  - maximum fast-paths affectance $M$

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○ show how these characteristics influence broadcast time complexity:
  if one uses a specific BFS tree (GBST [1]) that minimizes $M(K + M)$
  single broadcast can be done in time $D + O(M(K + M) \log^3 n)$

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- ... also simulations for RN

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Observations:
- throughput measured in the limit ⇒ preprocessing is free ⇒ protocol is distributed
- protocol includes randomized subroutine ⇒ deterministic results are existential
- if movement is slow enough to recompute structure ⇒ can also be applied to mobile networks
- To the best of our knowledge, first work on dynamic MMB under the general Affectance model
The General Affectance Model

Interference:
- 1-hop:
  - Radio Network model without collision detection
- (≥ 1)-hop:
  - value \( a_u(\ell) \leq 1 \) quantifies interference of node \( u \) on link \( \ell \)
  - \( a_v((u, v)) = 1, a_u((u, v)) = 0, \) and \( a_w((u, v)) = 1, w \in N(v) \) and \( w \neq u \)
  - \( a(\cdot) \) is any function s.t. \( a_{\{u,v\}}(\ell) = a_{\{u\}}(\ell) + a_{\{v\}}(\ell) \)
  - affectance degradation parameter \( \alpha \)

Successful transmission:
- transmission from \( u \) is received at \( v \) iff
  - \( u \) transmits
  - \( v \) listens
  - \( a_T((u, v)) < 1 \), where \( T = \{\text{set of nodes transmitting}\} \)
Injection and Performance Metric

**Injection:** *Feasible adversary*: $\exists$ OPT with bounded packet latency.

At most 1 packet may be received by a node in each time slot and all nodes must receive the packet in order to be delivered

$\Rightarrow$ feasible adversarial injection rate **at most 1 packet per time slot.**

**Performance metric:** *competitive throughput in the limit*

\[
\exists f : \lim_{t \to \infty} \frac{d_{ALG}(t)}{d_{OPT}(t)} \in \Omega(f)
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Affectance Characterization

Maximum average tree-layer affectance

Quantifies the difficulty to disseminate from one layer to the next one.

\[ K(T, s) = \max_d \max_{V' \subseteq V_d(T)} \frac{a_{V'}(L(V'))}{|L(V')|} \]
Affectance Characterization

Maximum fast-paths affectance

Quantifies the difficulty for dissemination on a path due to other paths.

\[ M(T, s) = \max_{d,r} \max_{\ell \in F^r_d(T)} a_{F^r_d(T)}(\ell) \]
Low-Affectance Broadcast Spanning Tree (LABST)

Tree construction:

1. $T_{\text{min}} \leftarrow \arg\min_{T \in \text{GBST}(s)} M(T, s)(M(T, s) + K(T, s))$
2. $T_{\text{min}} \Rightarrow \text{LABST } T$

$T$ avoids links between nodes of the same rank with big affectance blowing up GBST ranks by a $M(T)$ multiplicative factor.

Broadcast schedule:

defined using the ranks in $T$
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**Broadcast schedule:**

- defined using the ranks in \( T \)
Low-Affectance Broadcast Spanning Tree (LABST)

Corollary

For any given network of \( n \geq 8 \) nodes and source \( s \), diameter \( D \), and affectance degradation distance \( \lceil \log n \rceil \), there exists a broadcasting schedule of length

\[
D + O(M(T_{\text{min}}, s)(M(T_{\text{min}}, s) + K(T_{\text{min}}, s)) \log^3 n)
\]

For comparison, in Radio Networks:

\[
D + O(\log^3 n) \quad [1]
\]

\[
O(D + \log^2 n) \quad [2]
\]

[1] Gąsieniec-Peleg-Xin, DC 2007
[2] Kowalski-Pelc, DC 2007
MMB Protocol

- define LABST from each source node
- define a MBTF [1] list of source nodes
- assign a token to some source node from list

1. upon receiving the token at node $s$
2. if $queue(s)$ is “empty”:
   1. pass token to next in list
3. else if $queue(s)$ is “small”:
   1. disseminate $\Delta$ packets pipelined with period $\delta$
   2. pass token to next in list
4. else if $queue(s)$ is “big”:
   1. move $s$ to front of list
   2. while $queue(s)$ is “big”: disseminate $\Delta$ packets pipelined with period $\delta$
   3. pass token to next in list

[1] Chlebus-Kowalski-Rokicki 2009
**MMB Protocol Analysis**

**Lemma**

There exists a MMB protocol that achieves a throughput ratio of at least

\[
\lim_{t \to \infty} \frac{1}{1 + \delta} - \frac{2\Delta n^2}{t}
\]

**Corollary**

For any given network of \(n\) nodes, diameter \(D\), affectance degradation distance \(\alpha\), and \(K = \max_{s \in S} K(T_{\text{min}}(s), s)\), there exists a MMB protocol such that the throughput ratio converges to

\[
\frac{1}{O(\alpha K \log n)}
\]

For comparison, in Radio Networks:

- using WEB protocol [1] for propagation converges to \(1/O(\log^2 n)\)
- \(O(1/\log n)\) for any single-instance MMB algorithm [2]

[1] Chlamtac-Weinstein 1987
[2] Ghaffari-Haeupler-Khabbazian 2013
Simulations

![Simulations Graph](image)

- Throughput ratio
- Time slots

**Graph Details:**
- **n=8**
- **n=16**
- **n=32**
- **n=64**
- **n=128**
- **n=256**
- **n=512**
Thank you