Holography and the scale invariance of density fluctuations

João Magueijo\textsuperscript{1,2,3}, Lee Smolin\textsuperscript{1,4} and Carlo R Contaldi\textsuperscript{3}

\textsuperscript{1} Perimeter Institute for Theoretical Physics, 31 Caroline St N, Waterloo N2 L 2Y5, Canada
\textsuperscript{2} Canadian Institute for Theoretical Astrophysics, 60 St George St, Toronto M5S 3H8, Canada
\textsuperscript{3} Theoretical Physics Group, Imperial College, Prince Consort Road, London SW7 2BZ, UK
\textsuperscript{4} Department of Physics, University of Waterloo, Waterloo, Ontario N2 L 3G1, Canada

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Abstract
We study a scenario for the very early universe in which there is a fast phase transition from a non-geometric, high temperature phase to a low temperature, geometric phase described by a classical solution to the Einstein equations. In spite of the absence of a classical metric, the thermodynamics of the high temperature phase may be described by making use of the holographic principle. The thermal spectrum of fluctuations in the high temperature phase manifests itself after the phase transition as a scale-invariant spectrum of fluctuations. A simple model of the phase transition confirms that the near scale invariance of the fluctuations is natural, but the model also withstands a detailed comparison with the data.

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In this paper, we propose a new hypothesis about the very early universe in which the notion of holography plays a key role. We will see that it gives rise to a scale-invariant distribution of density fluctuations, without invoking inflation. This hypothesis is inspired by a scenario that has been proposed recently [1] in which the universe begins in a high temperature phase (phase I) which is not described in terms of fields on classical spacetime manifolds; rather it has a non-geometrical, purely quantum mechanical description, which can be expressed simply in terms of the holographic principle. The spacetime geometry is created in a phase transition, into phase II, in which it is appropriate to describe the universe to a decent approximation in terms of fields or other degrees of freedom (particles, strings, etc) moving in a classical background geometry. This may be called \textit{geometrogenesis}.

The phase transition happens at temperature $T_c$ and can be more or less abrupt (in a sense to be made precise later) and imprints on phase II a spectrum of thermal fluctuations which arise in phase I. Specifically, in the model of [1], in phase I the Universe is represented by a disordered graph, where each node has a very large valence; during the phase transition the average valence drops dramatically and locality, as well as other standard geometric concepts \textit{emerge}. The high interconnectedness of phase I makes the whole thermalize, thereby solving...
the horizon and homogeneity problems; the next challenge is to generate an almost scale-invariant spectrum of fluctuations.

The innovation in this paper is to use the holographic principle to characterize the two phases. This principle, as enunciated by 't Hooft [3], asserts that any region of space bounded by a surface \( S \) of area \( A \) can be described by a finite number of degrees of freedom given by \( N = A/(4G\bar{h}) \). These evolve according to fundamental dynamics, given by a Hamiltonian \( H \). The term 'holography' has also acquired other different meanings [4], but we stress that we use it in its original sense [5].

To describe classical physics there must be non-local correlations among the degrees of freedom on the surface, so as to make it appear that the dynamics are local in a volume \( V \) described by a classical geometry in a region \( R \) that \( S \) bounds. Close to the ground state, we expect that \( V \approx A^2 \) whenever the curvature can be neglected. But in phase I, there is not yet a classical spacetime geometry. We propose that this phase be then characterized as a disordered phase, which means that the non-local correlations which are needed on the surface to construct the illusion of local three dimensional physics are not yet established, and are peculiar to phase II. At the same time, the degrees of freedom in phase I are highly interconnected, as in the model of [1]; this makes the whole Universe thermalize. This process homogenizes the Universe, but the question is now to work out the remaining (thermal) fluctuations in this system.

Our idea is very simple, and even though we will derive it step by step later, we sketch it first. If phase I is in thermal equilibrium, we expect that the \( N \) (local) fundamental degrees of freedom are excited to energy \( T \). Thus, in this disordered phase the holographic principle implies that \( E = NT = \frac{A}{4\pi\bar{G}}T \), that is, for \( T > T_c \) the specific heat at fixed \( R \) is \( c_R = \frac{\partial E}{\partial T} = N \), which scales like the area. A well known result tells us that the thermal fluctuations in the energy contained in a fixed region are given by \( \sigma_E^2 \approx \frac{T_c^4}{4\pi\bar{G}}A \). Therefore in phase I we have \( \sigma_E^2 \approx \frac{T_c^4}{4\pi\bar{G}}A \), and because this scales like \( A \) (instead of \( V \), as usual) we have a scale-invariant spectrum of fluctuations, rather than the usual white noise. If the phase transition is 'fast', when the classical metric emerges in phase II these fluctuations propagate to the potential. Most modes are now outside the Hubble radius, so we end up with a scale-invariant spectrum of adiabatic density fluctuations, with amplitude fixed by the ratio \( T_c/T_{Pl} \).

We shall proceed as follows. We first present a description of the two phases, justified by basic facts of the quantum theory of gravity, and compute the associated density fluctuations. We next propose a scenario for the generation of classical geometry during the transition. This results in a scale-invariant distribution of fluctuations outside the horizon of the classical geometry that emerges at the end of the transition. We then devote some time to weakening our hypotheses, showing how the essential results apply to a large class of models of this type. Finally we quantify departures from scale invariance, predicting the amplitude, spectral index, and its running in terms of \( T_c \) and \( \gamma \), the critical exponent characterizing the phase transition. A comparison with other models and a word on tensor modes closes this paper.

To characterize the two phases in more detail, we must make a few general assumptions. Firstly, the universe can be described at all times as having fixed three-dimensional spatial topology, but not necessarily a fixed classical metric. Secondly, physics in any region \( R \) can be described in a Hamiltonian formulation in terms of a Hilbert space \( \mathcal{H}_R \). \( R \) has a boundary \( \partial R = S \). Finally among the operators in \( \mathcal{H}_R \) are \( a \), the area of \( S \), the Hamiltonian constraint \( \mathcal{C} \) and diffeomorphism constraints \( \mathcal{D} \) and a boundary contribution to the Hamiltonian \( \hat{h} = \int_S \hat{\mu} \), where \( \hat{\mu} \) is a local operator on the boundary. The total Hamiltonian for quantum spacetime and matter in \( R \) is then

\[
\hat{H} = \int_S \hat{\mu} + \int_R NC + v^a \mathcal{D}_a.
\]
The quantum states of interest are physical states that are in the kernel of $C$ and $D$. These assumptions are common to many approaches to quantum gravity as they follow just from diffeomorphism invariance.

We then hypothesize that there are two different kinds of solutions to the quantum constraints which each characterize one of the phases. We characterize phase II as being ordered three dimensionally. This means that there is a classical non-degenerate 3-metric $q_{ab}$ on $\mathcal{R}$ such that the physics can be described to a good approximation by a semi-classical state built from $q_{ab}$. We characterize phase I as being disordered. This means that there is no such three-dimensional classical metric $q_{ab}$.

In phase II there is no mass gap, so that, in the limit of an infinite area, there is a continuum of states above a ground state $|\text{II}\rangle$ where $\hat{H}|\text{II}\rangle=0$. These correspond to gravitons and other massless excitations. In this phase, correlations develop on the boundary $S$ corresponding to the fact that the lowest energy excitations have a long wavelength.

In phase I there is a mass gap, of the order of the Planck mass, $M_{Pl}$. This is a property of one form of the ADM energy in LQG given by Thiemann [2], and it is believed to be a general feature of quantum gravity. Absence of a continuum above the ground state negates the existence of gravitons and of a semi-classical state. Thus, this is a high temperature phase, with $T \approx M_{Pl}$, corresponding to pre-geometry. Unusual as this phase may be, it is still ruled by Hamiltonian dynamics, and so a canonical ensemble (and temperature) can be defined.

We hypothesize that in this phase, there are no correlations on the boundary. At the same time, quantum geometry is quantized on the boundary. Thus, let $\sigma$ and $\sigma'$ be regions of the boundary $S$ and let $\hat{h}(\sigma) = \int_\sigma \hat{\phi}$, where $\hat{\phi}$ is the degree of freedom on the boundary. Thus,

$$\langle E \rangle = \text{Tr}[\rho_T^I \hat{H}] = b M_{Pl} T \langle A \rangle,$$

(2)

where $b$ is some dimensionless constant and $\langle A \rangle = \text{Tr}[\rho_T^I \hat{A}]$, so that in phase I the energy is proportional to the energy on the boundary. This implies, in particular, that the specific heat at a fixed area is proportional to the area:

$$c_A = \left( \frac{\partial \langle E \rangle}{\partial T} \right)_{\langle A \rangle} = b \frac{\langle A \rangle}{hG}.$$

(3)

Since we have a Hamiltonian on the surfaces in phase I, we can study its thermodynamics. Here, we are allowed to use only the following quantities which are assumed to exist for the region $\mathcal{R}$: (i) a Hilbert space of spatially diffeomorphism invariant states, (ii) an area operator $\hat{A}$ and (iii) a Hamiltonian operator $\hat{H}$ as described above. The thermal physics is defined by the partition function

$$Z = \sum_i e^{-\beta E_i},$$

(4)

where $\beta = T^{-1}$. In what follows the exact nature of the microphysical states $i$ does not matter, but only the form of the partition function. The total energy $U$ inside the region $\mathcal{R}$ is

$$U = \langle E \rangle = \frac{\sum_i E_i e^{-\beta E_i}}{\sum_i e^{-\beta E_i}} = -\frac{d \log Z}{d\beta},$$

(5)

and its variance is denoted by

$$\sigma_E^2 = \langle E^2 \rangle - \langle E \rangle^2 = \frac{d^2 \log Z}{d\beta^2} = -\frac{dU}{d\beta} = T^2 c_{\langle A \rangle},$$

(6)

where $c_{\langle A \rangle}$ is the specific heat at a constant expectation value of area $\langle A \rangle$. We use (3) to write

$$\sigma_E^2 = b T^2 \langle A \rangle,$$

(7)

and this is all we need from phase I.
In phase II we have, in addition to the observables of phase I, the metric in the interior of the region $\mathcal{R}$, as well as all other usual quantities. Regarding the transition from phase I to II, we make the following hypotheses.

(i) The phase transition begins when the temperature falls to a critical temperature $T_c$. This is something that is found by means of numerical simulations. In principle the temperature $T_c$ depends only on model parameters (and on the Planck scale), but in practice simulations are required to find it.

(ii) The phase transition proceeds from large scales down to a small scale $l_0$. At any $T < T_c$ the geometry is characterized by a length scale $R(T)$ such that the geometry appears classical to all modes of the field with wavelength $\lambda \geq R(T)$. The geometry that these modes probe should be as simple as possible; hence, up to small fluctuations imprinted on it by the density fluctuations created in phase I, it is homogeneous and isotropic. $l_0$ is not zero because if we probe small enough even in the ground state, the continuum dissolves into the quantum geometry.

(iii) Since $R(T)$ is infinite at $T = T_c$ and then falls to $l_0$, the dependence of $R$ on $T$ during the transition is modelled by the function

$$R(T) = \left(\frac{T_c}{T_c - T}\right)^\gamma,$$

valid for $T \leq T_c$. This introduces a parameter which is the critical exponent $\gamma$.

Our purpose is now to compute the spectrum of fluctuations left over in phase II. We recall that in phase I there is a Hamiltonian but not a stress–energy tensor; therefore, we can only talk about energy fluctuations. Also there is no sense of Fourier modes, so only the variance $\sigma_E^2$ can be defined. We must now work out onto which phase II structures we should map these energy fluctuations, as they emerge out of phase I. Firstly it is clear that as the concept of length and volume is created we will have $\langle A \rangle = A = 4\pi R^2$, and $V(R) = \frac{4}{3}\pi R^3$. This defines energy density perturbations $\delta\rho = \delta E/V$. We then have $\sigma^2_\rho(R) = \frac{1}{R^6} \sigma^2_E(R)$, where $\sigma^2_\rho(R)$ is the mean square perturbation in the region. We also have $c_A = c_V$ (fixed $R$ now means fixed $\langle A \rangle$ and $\langle V \rangle$), so using (3) and (6) we have

$$\sigma^2_\rho(R) = \frac{\sigma^2_E(R)}{R^6} = \frac{T^2}{R^6} c_V = \frac{4\pi b T^2}{G R^2}.$$

These fluctuations emerge in phase II outside the horizon defined by the inverse Hubble parameter $H^{-1} \sim T_c^{-2}$; we assume that they are mapped into $\delta\rho$ defined in the longitudinal/comoving gauge (see [6, 7]). Also, other components of the stress–energy tensor are now well defined: we set the anisotropic stress to zero and assume a pressure fluctuation so as to make the fluctuations adiabatic (this is convenient, but not strictly necessary). Then the metric in phase II may be written (with $a$ the expansion factor and $\eta$ the conformal time) as

$$ds^2 = a^2(\eta)[-d\eta^2(1 - 2\Phi) + (1 + 2\Phi) dx^2],$$

where $\Phi$ labels small fluctuations in the metric, and these are related to the comoving $\delta\rho$ by the Poisson equation

$$k^2\Phi = 4\pi Ga^2\delta\rho$$

for all modes, large and small (again see [6, 7]). If we relate the density fluctuations defined by (9) to their dimensionless power spectrum by the formula $\sigma^2_\rho(R) \sim \mathcal{P}_{\delta\rho}(k = a/R)$ (see [6]), we finally get

$$\mathcal{P}_{\delta\rho}(k) = \frac{16\pi^2 G^2 a^4}{k^4} \mathcal{P}_{\delta\rho}(k) \sim \frac{G^2 a^4}{k^4} \sigma^2_\rho(R = a/k).$$

(11)
If we neglect the variation in $T$ during the transition, we find, using (9),

$$P_\Phi = \frac{k^2}{a^2} T^2 c V \sim \frac{G}{T} T_c^2 = \frac{T_c^2}{T_{Pl}^2}$$

(12)

that is a scale-invariant spectrum, with the amplitude $A \sim T_c/T_{Pl} \sim 10^{-5}$. Now, in fact, given (8), the spectrum in phase II cannot be exactly scale invariant, because different scales freeze-in at slightly different temperatures and therefore with slightly different amplitudes. Using (8), we find that more precisely

$$P_\Phi(k) = 2 \left( \frac{T}{T_{Pl}} \right)^2 \left[ 1 - \left( \frac{l_0 k}{a_c} \right)^\frac{1}{\gamma} \right]^2$$

(13)

We have neglected the variation in $a$ during the phase transition—so that $k \approx a_c/R(T)$—because it varies very little as the physically relevant $k$ emerge into phase II (a slight improvement could be obtained by numerically taking this into account). Since the spectrum is slightly red, we believe that we are not affected by the concerns voiced in [16, 17], questioning the standard formula for mapping position space variances and Fourier space power spectra. These only affect spectra with $n_S \geq 1$.

This interesting result does not depend on all the assumptions on phase I made above. Absence of metric or vanishing of $V$ in phase I, for example, is not strictly needed. Indeed, the only requirement for scale invariance is that the specific heat for a fixed region be proportional to the area. This requirement is realized in any other model where energy behaves like a ‘surface tension’, that is, $E \propto R^2 e(T)$, with general $e = C T^\zeta$. For our model $\zeta = 1$ and $C \sim 1$, but if there were no scale (like $G$) in the problem we would expect $e \propto T^3$, just like $E = VT^4$ usually. In these alternative scenarios, our results only change in detail. For example, we would get

$$P_\Phi(k) = C \zeta \left( \frac{T_c}{T_{Pl}} \right)^{\zeta + 1} \left[ 1 - \left( \frac{l_0 k}{a_c} \right)^\frac{1}{\gamma} \right]^{\zeta + 1}$$

(14)

so $T_c \approx 10^{-\frac{m}{\zeta} + \frac{2n}{\zeta} + 1}$. If $C$ is not order 1, and if $\zeta$ is not 1, $T_c$ may be quite different from our estimate, but everything else only changes in detail. In what follows we shall explore $\zeta = 1, 3$ but set $C = 1$.

Our results are also valid if the energy remains extensive, but the thermal correlations are string-like infinite tubes (rather than volumes with diameter $\sim 1/T$), with a section $\Sigma = \Sigma(T)$. Thermal fluctuations may be seen as a Poisson process (with variance $\sigma^2 \sim N$) for these uncorrelated regions. Usually, their number $N$ scales like the volume ($N = V T^4$); thus a white noise spectrum is formed. But for filamentary correlated regions, $N$ scales like the area ($N \sim V/(R \Sigma) \sim A/\Sigma(T)$), i.e. scale invariance. This is another way of understanding our derivations. It is also the reason why the Hagedorn phase scenario of [9] leads to scale invariance, and it may connect with the work of [10].

Finally, note that our assumption that the comoving gauge $\delta \rho$ is to be matched across the phase transition can be substantiated in models where there is a metric structure even in phase I. Then one may write the usual continuity and Euler equations in both phases and make an ansatz where all variables are potentially step functions at the transition time. By examining which variables have got time derivatives in the continuity equation (say), we find which of them result in delta functions proportional to their discontinuity; this leads to matching conditions. If we use the Newtonian gauge, the matching always involves a combination of $\delta \rho$ and $\Phi$ (see equation (14.141) of [6], where both these variables have a time
Use of the comoving gauge (see equation (14.149) of [6]) leads to the conclusion that $\delta \rho$ should be the same on either side of the transition.

We now relate the breaking of scale invariance to observation. Referring the non-scale-invariant bit of the spectrum to pivot $k_* = 0.002 \text{Mpc}^{-1}$ as usual [8], we thus get

$$P_\phi(k) = \zeta \left( \frac{T_c}{T_{Pl}} \right)^{\gamma+1} \left[ 1 - \left( \frac{\alpha k}{k_*} \right)^{\gamma} \right]^2$$

(15)

with

$$\alpha = \frac{l_0}{a_c} k_* \approx 5.5 \times 10^{-29} \frac{l_0}{l_P} T_c T_{Pl}$$

(16)

(we have ignored variations in the $g$ factor relating $T$ and $1/a$). The term in $\alpha$ is therefore small and so (15) can be expanded into a scale-invariant term plus a negative component. The total is therefore a red spectrum with an effective tilt

$$n - 1 = \frac{d \ln P}{d \ln k} = -\frac{\zeta + 1}{\gamma} \left( \frac{\alpha k}{k_*} \right)^{\gamma}.$$  

(17)

Note that we get a red tilt to the spectrum because we postulated that the phase transition is outside-in.

Observations probe at most $\ln(k/k_*) \sim 10$, so (17) can also be expanded, leading to a series $n = n_0 + n_1 + n_2 + \cdots$, with $n_0 = 1$,

$$n_1 = -\frac{\zeta + 1}{\gamma} \alpha^{1/\gamma}$$  

(18)

and most crucially the second-order prediction

$$\frac{dn_2}{d \ln k} = \frac{n_1}{\gamma}, \quad \text{i.e.} \quad \frac{dn}{d \ln k} \approx \frac{n - 1}{\gamma}$$

(19)

which can be seen as a ‘consistency condition’, similar to that found in inflation regarding scalars and tensors, but involving only scalar observables. Here two theoretical parameters ($T_c$ and $\gamma$) fix three observables (including the amplitude) leading to a universal constraint between the deviation $n_S - 1$ on a given scale and the running of $n_S$. This cannot be converted into a more compact formula because eliminating $\gamma$ from (19) using (18) can only be done by solving a transcendental equation.

Given the smallness of $\alpha$, departures from scale invariance are usually very small. If $l_0 \sim T_c^{-1}$ they are maximized (at scale $k_*$) for $\gamma = -\ln \alpha \approx 65$, with $n - 1 \approx 0.005(\zeta + 1)$, of the order of a few per cent. However, if $\gamma \sim 1$ the deviations would be of the order $10^{-30}$. For $l_0 \gg T_c^{-1}$ we can generate larger deviations, but still only for $\gamma \sim -\ln \alpha \approx 65 - \ln(l_0 T_c)$, with $\gamma \sim 1$ again leading to infinitesimal deviations.

Using a modified version of the CAMB code [11], we calculate the predicted CMB and large scale structure (LSS) power spectra for our model. We then use a Monte Carlo Markov chain (MCMC) for sampling the likelihood as a function of model parameters [12]. We fit a conventional combination of CMB and LSS data sets [13]. We adopt the same number (six) of parameters as conventional (flat, power law) models, but trade $n$ and amplitude $A$ parameters for $\gamma$ and $T_c$. The fits obtained are therefore directly comparable to the standard power law $P_\phi(k)$ based fits since the number of model parameters are the same. The results are shown in figure 1 for $\zeta = 1, 3$ and $l_0^{-1} = T_c, 10^{11} \text{GeV}$. We find that all runs give good fits to the data although the choice of $\zeta = 1$ requires very large values of $\gamma$ to achieve a sufficient amount of red tilt. However for $\zeta = 3$ we obtain a slightly better fit that the conventional power law models, as detailed in table 1.
Figure 1. Marginalized confidence contours for $\gamma$ and $\ln(\tau_c/\tau_0)$. The contours represent the 68% and 95% integrals. The top and bottom panels show results for $\zeta = 1$ and 3, respectively. The underlying (yellow) region is for $l_0 = T_c^{-1}$ while the overlayed (green) region is for $l_0^{-1} = 10^{11}$ GeV.

(This figure is in colour only in the electronic version)

Table 1. Median values for $\gamma$ and $T_c$. The errors are obtained from the 68% confidence integrals. $\Delta \ln L$ is with respect to the best conventional power law spectrum.

| $l_0$         | $\zeta$ | $T_c^{-1}(\times 10^4)$ | $\gamma$ | $\Delta \ln L$ |
|--------------|---------|-------------------------|----------|----------------|
| $T_c^{-1}$   | $\zeta = 1$ | 1.04^{+0.13}_{-0.14}    | 103.7^{+46.2}_{-42.4} | $-0.75$      |
| $T_c^{-1}$   | $\zeta = 3$ | 80.3^{+6.4}_{-6.4}      | 59.9^{+8.8}_{-8.8}   | $+0.04$      |
| $(10^{11}$ GeV)$^{-1}$ | $\zeta = 1$ | 1.18^{+0.16}_{-0.16}    | 104.2^{+45.6}_{-39.5} | $-0.32$      |
| $(10^{11}$ GeV)$^{-1}$ | $\zeta = 3$ | 76.4^{+6.6}_{-6.2}      | 43.0^{+6.6}_{-6.4}   | $+0.07$      |

It is interesting to compare our model with other alternatives. Slow-roll inflation also predicts a near scale-invariant spectrum, i.e. $n_0 = 1$, with correction $n_1 = -6\epsilon + 2\eta$, in terms of slow-roll parameters $\epsilon = \frac{1}{2} \left( V'/V \right)^2$ and $\eta = V''/V$ (where $V(\Phi)$ is the inflaton potential). There is also a second-order logarithmic running of $n$, but for a general $V$ this is independent of $n_1$, since it depends on $\xi^2 = V'V''/V^2$. More generally inflation can produce any spectrum of scalar fluctuations if one carefully designs $V(\phi)$, and it is not a falsifiable theory until we consider the tensor modes, which do impose a consistency condition. By contrast we predict a scalar ‘consistency condition’ defined implicitly by (18) and (19). We stress that this is only true within a fixed model, e.g. one in which $\zeta$ is kept fixed.

Our work has obvious parallels with that of [9] on the Hagedorn phase. However, our phase transition is a transition in the description of spacetime rather than in the matter content. This profound conceptual difference has a very practical implication: function (8) controlling...
the progress of the phase transition is entirely different from that (equation (12)) controlling
the amplitude of the fluctuations, thus giving rise to the naturalness of scale invariance. This
is not the case with [9] where both scales are controlled by \( T_c/(T_c - T) \), and a degree of
(debatable) fine tuning is required. It would be interesting to study how those models fare
with deviations from exact scale invariance. Our scenario can also be seen as a variant of a
varying speed of light cosmology [14] in that in phase I, all degrees of freedom are assumed
to be in causal contact and thermal equilibrium.

To conclude and summarize, we have presented a model for the emergence of classical
spacetime from a quantum, non-geometric pre-era informed by a version of the holographic
principle. Our model develops the general idea in [1] in that we make particular hypotheses
about the phase transition, which we labelled as (i) to (iii). These hypotheses could be checked
in the context of different models of quantum gravity. What is remarkable is that there are
specific consequences of non-perturbative quantum gravity models that may be calculable
which, given our picture, map on to quantities which are measurable in the CMB. In particular
the phase transition temperature is mapped to the amplitudes of fluctuations by (12), the
direction of the transition-large to small scales rather than the reverse maps to the tilt of the
spectrum being red or blue, while the speed of the transition, parameterized by \( \gamma \), is measurable
in the tilt (17). Note that the scenario also implies the consistency condition (19) which, given
\( \zeta \), is a precise prediction. Alternatively, the two parameters \( \gamma \) and \( \zeta \) are together determined
by the tilt and running of the spectrum. Our results are applicable to a large class of similar
models.

We can point out that exact scale invariance is a natural prediction of this model, without
any fine tuning. This is, in fact, not completely ruled out by the data. However, the data
presently prefer the model’s ability to produce a slightly red spectrum, with a very characteristic
running, encoded in consistency condition (19). This requires large critical exponents (of order
50); whether this is reasonable or not might be investigated in explicit models of phase I. It has
also been suggested that geometrogenesis might be a cross-over rather than a phase transition,
in which case slight deviations from scale invariance would be more natural.

Finally, we note that fluctuations are very nearly Gaussian [15] and tensor modes are
expected to be negligible in our preliminary estimates. However more work needs to be done.
An issue that has also been entirely overlooked by the literature is whether these models solve
the flatness problem. Clearly thermalization and locality imply a homogeneous geometry in
phase II, but why this should be spatially flat (an unstable fixed point in the new dynamics)
remains to be proved.

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