A STURDY NON-NEGATIVE MATRIX FACTORIZATION FOR NONLINEAR HYPERSPECTRAL UNMIXING

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Abstract

To depict the hyperspectral data, here a sturdy mixing model is implemented by employing various perfect spectral signatures mixture, which enhances the generally utilized linear mixture model (LMM) by inserting an extra term that describes the potential nonlinear effects (NEs), which are addressed as additive nonlinearities (NLs) those are disseminated without dense. Accompanying the traditional nonnegativity and sum-to-one restraints underlying to the spectral mixing, this proposed model heads to a novel pattern of sturdy nonnegative matrix factorization (S-NMF) with a term named group sparse outlier. The factorization is presented as an issue of optimization which is later dealt by an iterative block-coordinated descent algorithm (IB-CDA) regarding the updates with maximization-minimization. Moreover, distinctive hyperspectral mixture models also presented by adopting the considerations like NEs, mismodelling effects (MEs) and endmember variability (EV). The extensive simulation analysis by the implementation of proposed models with their estimation approaches tested on synthetic images. Further, it is also shown that the comparative analysis with the conventional approaches.

Keywords : Hyperspectral images, spectral unmixing, linear mixture models, nonlinear mixture models, nonlinear spectral unmixing.

I. Introduction

Hyperspectral image investigation, which renders significant and comprehensive gathered measurements description in several areas of application like spectro-microscopy [XXXIV], remote sensing [XXI], food monitoring [II] and planetology [XXX] is done by a prefaced concern problem named as spectral unmixing (SU),
which was an area of intensive interest over the last two decades. SU is an issue of separating source comprising of reconstructing the material’s endmember spectrum which is there in the scene and measuring their symmetries or abundances inside every pixel of HIS. It consists in decomposing $P$ multi-band observations $Y = [y_1, \ldots, y_P]$ into a collection of $K$ individual spectra $R = [r_1, \ldots, r_K]$, called endmembers, and estimating their relative abundances $A = [a_1, \ldots, a_P]$ for each observation [XXXVII], [XXV]. Numerous SU approaches presented in the literatures of geoscience, signal and image processing depends on LMM, $Y \approx RA$. In truth, a good estimation of the physical procedure is rendered by LMM which inherent the observation and has outcome in practicable solvent for numerous applications. Be that as it may, LMM is not suitable for several settings like the models with volumetric dispersing or suggest materials mixture or terrain alleviation. For example, when assuming the scenes like arenaceous, light is mattered to the development of multiple dispersing and assimilation, that led largely NEs. Quite difficult optical modelling is necessitated to deal with the thorough examination of such models and further it is required to recourse to estimation models to make the issue amenable. For those effects [XLII], [XXXVI], which suffers LMM, an alternate approach is adopted named as nonlinear mixture models (NLMM), which consists of couple of categorizations like first is based on procedure of signal and attempts to build flexible models that can constitutes NLs with a wide range. Second is based on physical and admits the multiple scattered familiar models like polynomial [XLIV] or bilinear [VI, VII, XXVI, XLI, XXII]. Conversely, in an image of remotely sensed those are compiled of vegetation (e.g., trees), photons interaction with several scene elements results in NEs that can be assumed employing bilinear models [XI, XIII]. As explained in [XLVI], many of these models only differ by the restraints enforced on the bilinear term. Furthermore, to estimate second order NLs with wide range, a polynomial post NLMM is implemented in [XLIV], which has established its capability to depict numerous NEs, in vegetated areas [XXXV]. A general characteristic of such models is that they all integrate an additional term that accounts for NLs to the traditional LMM. However, these models are not without flaws that they necessitate to select a particular pattern of NL, which is highly restricted in practice. A complete overview of NL models with their supported SU algorithms are presented in [XXXVI].

Usually, endmember matrix ($M$) is unchanged for entire image in both LMM and NLMM, which seems as a enlighten alteration and the spectra of $M$ can be changed across the making of an image. This is referred as EV or spectral variability [X, XII], which has been established as an applicable origin of error in reckon of abundance. In the community of hyperspectral imaging, this has drawing attention of development pursuit [V, X, XII].

In the literature, numerous approaches were implemented to depict this EV and they can be accumulated into couple of major categories where the first one assumes every physical material as known endmember set [XV, XVI] or reckoned from the
data [XIV, XXXII]. In addition, numerous $M$ parametric representation is discovered like the every $M$ is multiplied by a picture element subordinate invariant to explicit a variation in illumination in the scene that is noticed [XX, XXVII, XXXII]. The latter category trusts on a numerous $M$ statistical representation which are considered as random vectors with rendered probability dispersions. There are couple of statistical models like normal compositional model (NCM) [IX, XVIII, XXXVIII] and beta compositional model (BCM) [XLIII] where the first one considers Gaussian dispersions for numerous $M$ and the latter feats the physically naturalistic $M$ range reflectance by attributing them a beta distribution.

Processing of HSI also relate to the MEs, which introduced due to the presence of several physical development like NEs or EV. Be that as it may, these MEs can also be due to the dispersed mistakes in the processing chain of signal. In truth, three stages are employed for operating SU:

- $M$ estimation.
- Employment of endmember extraction approaches (EEA) like vertex component analysis (VCA) [XXVIII] and N-FINDR [XXXIII] to distinguish $M$.
- Abundances approximation underneath physical nonnegativity and sum-to-one restraints employing algorithms like entirely restrained least squares [XVII].

A scenario of supervised unmixing is assumed in most of works which targets at abundance estimation if the couple of primary levels of unmixing is developed successfully [XXIII, XXIV, XLV]. However, an error on the estimated number of $M$ or in their spectra might results in worse estimation of abundance, which is assumed by several refreshing sturdy SU algorithms those targeted at mitigating the outliers of MEs influence [XLVII]. In a like manner to the NMMs explained above, our proposed model is constructed based on traditional LMM, with the comprehension of an auxiliary linear term that accounts for the impacts of NEs. Hence, the proposed model can be assumed to study any mixture that outcome from the compounding of an additive part $MA$ and the residual term [XL]. Indeed, the first stage investigates NEs which can be modelled by assuming a variation to the polynomial term described in literature, that relies only on the spectra of endmember. The second stage assumes the impact of EV by innovating a smooth linear divergence of every known spectra of endmember. Thus, reverse to the NCM explained in some of related articles, the proposed model considers the endmembers in presence of EV with smooth spatial and spectral fluctuation. In addition, the third stage innovated for the residual term which accounted for the MEs with the smooth spectral and spatial properties of corrupting term. Grounded on the residual component analysis (RCA) model [I], global formulation considers the LMM to be profaned by a linear term whose formula can be accommodated to describe for the analyzed development. This residual term is formulated as a compounding of $M$ or abundances contingent the analyzed NEs or EV. The unknown parameter vector joint posterior dispersion is then derived for NEs,
EV and MEs by employing the likelihood and the assumed prior distributions. However, it is quite difficult to calculate the minimum mean square error (MMSE) and maximum a posteriori (MAP) estimators of these parameters from the prevailed joint posteriors, which estimates the MAP by assuming the coordinate descent algorithm (CDA) [IV,XXIX]. Innovating a pliable approach of unmixing which is capable to examine a significant sort of remotely sensed scenes is the major thought of implementing our proposed unmixing methodology. Therefore, a reasonable model to specify the maximum extent of pixels in a remotely sensed images referred to the LMM.

Conversely, as exemplified in earlier publications in the literature, consideration of LMM won’t assist for the areas those are particularly localized, primarily placed at the heterogeneous areas interface. For this restricted number of picture elements, the SU approaches based on LMM failed to recover the materials and their abundances. Of such assumption, our objective is to implement a novel SU model which suits for the both scenarios. However, the work addressed in [III], doesn’t operates well with the larger NEs since CDA algorithm updation is quite difficult and it fails to produce good abundance estimate with nonlinear model that was corrupted by additive noise. Therefore, this work aimed at solving these issues and proposed a new CDA algorithm that is named as iterative block coordinate descent algorithm (IB-CDA) that consecutively updates the abundances, the noise variances and the residual terms iteratively. Moreover, S-NMF is employed to minimize the issue where the LMM fails to assume the pixels with nonlinearities. As such, we implemented to decompose the \( L \times P \) matrix of the observations with multi-band as \( Y \approx RA + N \), where \( N \) is a not thick (and non-negative) residual term explicating NEs. The implemented SU models and their reckon algorithms are experimented with both synthetic and real HSI. The incurred outcome is extremely identical and disclose the proposed SU model’s possibility with their supported illation algorithms.

The rest of this article is summarized as follows. Section II innovates the derivations for the proposed mixture model and its versions to handle with NEs, EV and MEs. Section III explains the proposed methodology. Section IV discusses the simulated results of proposed approaches using synthetic images with known ground truth. and real HSIs. Section V concludes the article and future work also discussed in the same finally reported the references.

II. Mixing Model: NL, EV and MEs

The SU expression is grounded on RCA model that is formulated as the summation of LMM and residual term. In practice, the general observation model for the \((L \times 1)\) pixel spectrum \( y_{i,j} \) is given by

\[
y_{i,j} = \sum_{r=1}^{R} a_{r,i,j} s_{r,i,j} + \varphi_{i,j}(S_{i,j}, a_{i,j}) + e_{i,j} = S_{i,j} a_{i,j} + \varphi_{i,j}(S_{i,j}, a_{i,j}) + e_{i,j}
\]

where \( a_{i,j} \) is an \((R \times 1)\) abundances vector affiliated with the picture element \((i,j)\), number of endmembers are denoted as \( R \), \( e_{i,j} \sim \mathcal{N}(0, \Sigma) \) denotes the linear focused

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Gaussian noise with a matrix of diagonal covariance \( \Sigma = \text{diag}(\sigma^2) \), where \( \sigma^2 = [\sigma_1^2, \ldots, \sigma_L^2]^T \) is an \((L \times 1)\) vector comprising the variances of noise with \( L \) spectral bands, \( S_{i,j}(M) = S_{i,j} \) is the matrix of endmember that relies on every picture element to innovate the impact of EV, the known matrix of endmember that is unchanged denoted as \( M \) and residual term is denoted as \( \varphi_{i,j}(S_{i,j}, a_{i,j}) \), which might rely on the abundances or endmembers to model the impacts of NL or EV. Because of physical restraints, the vector of abundance \( a_{i,j} = [a_{r,i,j}, \ldots, a_{R,i,j}]^T \) gratifies the adopting positivity and sum-to-one restraints.

\[
a_{r,i,j} \geq 0, \forall r \in \{1, \ldots, R\} \quad \text{and} \quad \sum_{r=1}^{R} a_{r,i,j} = 1 \tag{2}
\]

A usual model which can be adjusted to describe for unlike physical development is demonstrated in Equation (1). The implementation of NEs model is done to handle with the numerous impacts caused by the scattering which seems in the presence of tress or terrain relief and even both. The model of EV describes for the endmember’s deviation which is usually noticed in bearing of shadow and vegetation like grass or trees. In practice, noticing the impacts of both NL and EV at the same time. A usual model is shown by Equation (1) which can be adjusted to consideration for various physical development. Designing of NEs model is done for handling the impacts of multiple scattering which come into sight in presence of tress or/and terrain relief. Similarly, EV model is designed for endmembers deviation which is usually noticed in presence of shadow and vegetation like grass or trees. While analyzing a scene, a simultaneous impact of NL and EV can be noticed. Hence, to account for both impacts, ME model is designed.

**Impact of NL:**

Underlying or inbuilt restrictions of LMM can be overcome by the procedure of NLMM since LMM is not suitable for few HSI, which consists of vegetation, trees or urban areas. The models associated to bilinear or polynomial have disclosed meaningful outcome for these HSI by dealing with the effects of double scattering issue. Moreover, these models assume second order interactions between \( M \) and ignores the impact of higher order conditions. The following polynomial/bilinear nonlinear model is considered in this article

\[
y_{i,j} = c_{i,j} M a_{i,j} + \varphi_{i,j}^{NL}(M) + e_{i,j} \tag{3}
\]

where there is a similar residual component as in [28]:

\[
\varphi_{i,j}^{NL}(M) = c_{i,j}^2 \left( \sum_{k=1}^{R} \sum_{k'=k+1}^{R} \gamma_{i,j}^{(k,k')} \sqrt{\mathcal{T} m_k \mathcal{O} m_{k'}} + \sum_{k'=1}^{R} \gamma_{i,j}^{(k,k')} \mathcal{T} m_k \mathcal{O} m_{k'} \right) \tag{4}
\]

With \( \gamma_{i,j} = \left[ y_{i,j}^{(1)}, \ldots, y_{i,j}^{(R)}, y_{i,j}^{(1,2)}, \ldots, y_{i,j}^{(R-1,R)} \right]^T \mathcal{O} \) is the \((D \times 1)\) vector of positive nonlinearity coefficients, \( D = \frac{R(R+1)}{2} \), \( \mathcal{O} \) denotes the Hadamard (term wise) product, \( k \) denoted as endmember spectra, \( s_{r,i,j} = c_{i,j} m_r \), \( \forall i,j \), with \( c_{i,j} \) an illumination coefficient which relies on pixel. The model in Equation (3) extrapolates the Somers
et al. model with inclusion of $c_{i,j}$, which is an illumination argument of EV and invoice for the main endmember spectral changeability. Adverse to the model of RCA (which can be attained with free $\gamma_{i,j}$ marginalization), Equation (3) assumes a physical positive $\gamma_{i,j}$. Also remark that Equation (3) extrapolates the LMM which can be attained at $\gamma_{i,j} = 0$, and $c_{i,j} = 1, \forall i,j$ and consists a polynomial such as configuration as for the models of bilinear. At last, remark that Equation (3) was studied by Altmann et al. in absence of illumination variation and rendered good performance for HSIs unmixing when assuming an approach of Markov chain Monte-Carlo (MCMC). But, MCMC is an expensive approach in terms of computational complexity, thus it is required to assume a quicker approach for enhanced performance of HSIs unmixing, which is referred as coordinate descent algorithm (CDA) [III]. But still, it was unsuitable and failed to consider the pixel NLs, which motivated us to extend the CDA to IB-CDA.

Impact of EV:

Usually, HSIs symbolize very larger scenes due to their lower spatial resolution. Hence, it makes conscious awareness to consider that similar material (like vegetation) might dissent w.r.t. the regions of image leading in what is cited as EV, which innovates the alteration in the scale and shape of the spectrum of the $M$ in every pixel, i.e., $s_{i,j}$ reckons on the position of pixel. Regardless of the truth that these spectra are associated with the homogeneous material, they disclose few

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**Algorithm 1: CDA**

1. Approximate $M$ by employing EEA (like VCA, N-FINDR, etc.)
2. Parameter initialization of $A^{(0)}$ with SUNSAL-FCLS, $r^{(0)}$, $\Gamma^{(0)}$, $D^{(0)}$, $c^{(0)}$, $s^{(0)}$, $\Sigma^{(0)}$, and $t$
3. $conv = 0$
4. **while** $conv = 0$ **do**
5. Utilize SUNSAL-FCLS for updating $A^{(t)}$
6. CDA-NL: Utilize SUNSAL-FCLS to update $\Gamma^{(t)}$
7. CDA-EV: Use standard least squares (SLS) for updating $K^{(t)}$
8. CDA-ME: $D^{(t)}$ update with SLS
9. CDA-NL and CDA-ME: Employ SLS to update $(s, \omega)^{(t)}$
10. Update $\Sigma^{(t)}$
11. CDA-ME: Use SLS to update $c^{(t)}$
12. CDA-NL: Utilize the result of third order polynomial to update $c^{(t)}$
13. Adjust $conv = 1$, when the criterion of the convergence is met
14. $t = t + 1$
15. **end while**

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dissimilarities which is referred as the impact of EV. To spotlight this impact, mean of spectrum is calculated in every band of spectrum and let that EV is incurred by calculating the conflict between the spectrum mean and the spectra. Thus, to describe for the shape variability of every endmember, it can be estimated with the summation of unchanged spectrum and a smooth spectral operation constituting EV, that can be designed by an approach of parametric like spline or an approach like Gaussian. Here, following EV model considered

$$s_{r,i,j} = m_r + k_{r,i,j}$$  \hspace{1cm} (5)

where $k_{r,i,j}$ is a smooth spectral function, which leads to

$$y_{i,j} = Ma_{i,j} + \psi_{i,j}^{EV}(a_{i,j}) + e_{i,j}$$  \hspace{1cm} (6)

Where

$$\psi_{i,j}^{EV}(a_{i,j}) = \sum_{r=1}^{R} a_{r,i,j} k_{r,i,j}$$  \hspace{1cm} (7)

Note that there is no illumination parameter $c_{i,j}$, in Equation (7) since the smooth function $k_{r,i,j}$ comprises its impact. The model in Equation (7) associates to the models in state-of-art as succeeds:

- This can extrapolate the LMM which can be recovered for $k_{r,i,j} = 0_L, \forall i,j$.
- The model in [XXXI] can be extrapolated with the inclusion of shape variability impact.
- It consists the homogeneous formulation as in [XXXIX] while answering for the spectral smoothness of residuals.

Remark that, in the special mode where $k_{r,i,j}$ is Gaussian processed, the model of Equation (6) will enhance the GNCM by admitting the unruffled nature of EV.

Outliers or Impacts of Mismodelling (MEs):

Due to its simplicity, there was a wide usage of LMM in most of the applications. However, as disclosed in earlier sections, LMM was not suitable in many scenarios where there is a presence of NL, variability or MEs because of the chain errors in signal processing. This section answers for MEs or the outliers existed currently by reckoning a residual condition which discloses both spatial and spectral correlations. The reflexion model of ME is as follows:

$$y_{i,j} = c_{i,j} Ma_{i,j} + \psi_{i,j}^{ME}(M) + e_{i,j}$$ with $\psi_{i,j}^{ME} = d_{i,j}$  \hspace{1cm} (8)

Where a function of smooth spectral is denoted as $d_{i,j}$. Likewise, the earlier models, Equation (8) mitigates to LMM when the values of $d_{i,j} = 0_L$, and $c_{i,j} = 1, \forall i,j$. Remember that, other models in the literature are innovated to answer for the outlier’s impact which proposed both spatial and spectral correlated outliers with the consideration of discrete Markov random fields (D-MRF). Equation (8) is a special mode of Equation (7) when $k_{r,i,j} = k_{r,i,j}, \forall r,r'$ and $c_{i,j} = 1$, i.e., unlike $M$. 

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influenced by the similar variability. Similarly, NL model in Equation (4) diminishes to Equation (8) when $y(k,k') = y, \forall k, k'$ due to that spectra $m_k \bigcap m_{k'}, \forall, k, k'$ are usually unruffled. Be that as it may, Equation (8) is more capable being change due to that it doesn’t consider the positivity restraint (primarily for accounting EV).

III. Proposed Methodology

Model design:

$$y_{i,j} \approx \sum_{r=1}^{R} a_{r,i,j} m_r + n_{i,j}$$

(9)

The matrix formulation for above equation is given by

$$Y \approx RA + N$$

(10)

Majority of bilinear mixing models are generalized by Equation (9), those have been utilized widely for HSI analysis attained or captured over multi-layered regions with remote sensors. An extensive examination of this prescribed context is done in the literature for vegetation or urban canopies characterization. The estimation arbitrary sign in Equation (9) and Equation (10) lies the understate of a unsimilarity measure $D(Y|RA + N)$. The unsimilarity assess is like $D(A|B) = \sum_{ij} d(a_{ij}|b_{ij})$, where $d(x|y)$ is either the squared Euclidean distance (SED) or the Kullback-Leibler divergence (KLD), which are accosted in this since these are widely utilized in non-negative matrix factorization (NMF). The SED is very usual in hyperspectral unmixing (HSU) but KLD also guided the advantages in recent publications. This approach of NMF also acceptable to other fit assesses like $\beta$-divergence. The matrices $Y, R$ and $A$ are non-negative with universe and let the coefficients of abundance to sum to one, i.e.,

$$a_{i,j} \in \mathbb{S}^{R} \equiv \{ a \in \mathbb{N}^{R}|a_r \geq 0, \sum_{r=1}^{R} a_r = 1 \}$$

(11)

Like the majority models of HS data, this article also considers the nonlinear element $n_{i,j}$ as nonnegative, which permits a reasonable equivalence with the latter assignments, which motivated us to implement this approach, and is is physically enough inspired for the models of multi-layer.

Usually, it is expected that $n_{i,j}$ is to be frequently zero, i.e., in general, traditional LMM is adopted by the pixels and when the LMM consideration is failed then NLs become active and results in nonzero values for $n_{i,j}$. This measure to conclude that energy sparse vector.

$$e = \left[ \| n_{r,1} \|_2, \ldots, \| n_{r,i,j} \|_2 \right]$$

(12)

In eq. (12), $\| \cdot \|_2$ denotes the Euclidean norm. Sparsity can be routinely enforced by $\ell_1$-regularization, as done next. In light of earlier section, our objective is to solve the minimization problem defined by

$$\min_{M,A,N} J(R,A,N) = D(Y|RA + N) + \lambda N \|_{2,1} \quad s. t. R \geq 0, A \geq 0, N \geq 0$$
\[
\| a_{ij} \|_1 = 1 \tag{13}
\]

Where \( \lambda \) is the nonnegative penalty weight, \( A \geq 0 \) denotes coefficients nonnegativity of \( A \) and \( \| \cdot \|_{2,1} \) is the so called \( \ell_{2,1} \)-norm defined by
\[
\| N \|_{2,1} = \| e \|_1 = \sum_{i,j=1}^{P} \| n_{ij} \|_2 \tag{14}
\]

The issue of sturdy-NMF (S-NMF) is referred by Equation (13), which is a nonnegative version of robust PCA [XXXIX] and seemed as various forms in the literature. Eq. (13) defines a sturdy NMF (S-NMF) problem. S-NMF is a nonnegative variant of robust PCA [XXXIX] which has appeared in different forms in the literature. To resolve the S-NMF understate issue cited in Equation (5), IB-CDA is proposed which updates the parameters \( M, A \) and \( N \) iteratively. Every parameter is modified subject to a condition supported by the current worth of another parameters.

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**Algorithm 2: S-NMF with IB-CDA**

1: Initialize \( R, A \) and \( N \)
2: Set \( \beta = 1 \ or \ 2, \lambda \) and convergence tolerance \( tol \)
3: \( S = RA \)
4: \( \hat{S} = S + N \)
5: while \( \sigma \alpha > tol \), do
6: \( R = R [Y^T (Y^T + \lambda A \text{ diag}[\| e_{r,i} \|_1^\lambda] | n_{r,j} | \|_1]^{-1} \]
7: \( \hat{S} = S + N \)
8: Update abundance \( A \)
9: \( A = A \text{ diag}[\| e_{r,i} \|_1^\lambda, \ldots, \| e_{r,i} \|_1^\lambda]^{-1} \)
10: \( S = RA \)
11: \( Y = S + N \)
12: \( R = R \text{ [\| e_{r,i} \|_1^\lambda]^{\lambda \alpha} \] } \)
13: \( S = RA \)
14: \( Y = S + N \)
15: Calculate the decrease error associated to the objective function.
16: end while

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and such that there is a diminishment in the objective function. In Algorithm 2, all operators preceded by a dot ‘·’ are entry wise MATLAB-like operations and fraction bars shall be taken term-to-term as well. Additionally, $1_{M,N}$ denotes the $M \times N$ matrix with coefficients equal to 1.

**Setting $\lambda$ Value:**

The trade-off among the data-fitting term $D(Y|RA + N)$ and the penalty term $\|N\|_{2,1}$ is controlled by the hyperparameter called $\lambda$, which is quite complicate to set the accurate value such as in any other supposed approach of variational which involves a term called regularization. This section specifies the apparently reasonable values for the hyperparameter $\lambda$. Our proposed methodology is depending on the moments approach and comprises in translating the Equation (5) i.e., an objective function as a combined likeliness and in coping with the empiric average of the HIS data with its approximation earlier in time in the statistical model. The SED and the KLD are “pseudo-likelihood” for probabilistic models (Gaussian and Poisson, respectively) such that $E[Y|RA + N] = RA + N$. In the same analogy, the term $\lambda\|N\|_{2,1}$ can be interpreted as a log-prior term. Using some results from [VIII], the corresponding prior distribution $p(n_{r,i,j} | \lambda)$ for each column of $R$ can be obtained as a scale mixture of conditionally independent half-Normal distributions, with a Gamma distribution assigned to the scale parameter.

**IV. Results and Discussion**

This part describes the performance assessment of proposed HSU using S-NMF with synthetic data. Primarily it is given the evaluation criterion for the quality of HSU. Later, the comparative evaluation with existing HSU and proposed HSU using S-NMF is provided. For synthetic images, the abundances are known and the unmixing quality can be evaluated by using the root mean square error (RMSE):

$$\text{RMSE}(A) = \sqrt{\frac{1}{X} \sum_{x=1}^{X} \|a_x - \hat{a}_x\|^2}$$

(15)

The unmixing performance can also be evaluated by considering the reconstruction error (RE) and spectral angular mapper (SAM):

$$\text{RE} = \sqrt{\frac{1}{X} \sum_{x=1}^{X} \|P_x - Y_x\|^2}$$

(16)

$$\text{SAM} = \frac{1}{X} \sum_{x=1}^{X} \arccos \left( \frac{P_x^T Y_x}{\|P_x\|\|Y_x\|} \right)$$

(17)

Where $\arccos(\cdot)$ is the function of inverse cosine and $Y_x, P_x$ are the measured and estimated $x^{th}$ pixel spectra.

**Evaluation:**

This section evaluates the performance of the proposed unmixing algorithms when considering different mixture models. This describes the performance assessment of...
proposed HSU using S-NMF when assuming dissimilar mixing models. With the size of $100 \times 100$ pixels and $L = 207$ spectral bands, four synthetic images are produced by employing $R = 3$ endmembers representing to the available spectral signatures in the ENVI software library [XIX]. All images have been corrupted by i.i.d. Gaussian noise (with SNR=25 dB) for a fair comparison with SU algorithms using this assumption. Distinctive unmixing schemes are employed for processing of these images that are equate to the proposed models. For every model, it is considered that there are known endmembers and had assumed spectra of ENVI is employed to innovate the images.

Table 1. Results on synthetic data

| Method       | RMSE ($\times 10^{-2}$) | RE ($\times 10^{-2}$) | SAM($\times 10^{-2}$) |
|--------------|-------------------------|-----------------------|------------------------|
| FCLS [XVII]  | 24.75                   | 15.74                 | 10.64                  |
| SU-SAL-CLS [XXIII] | 16.55                   | 4.17                  | 7.57                   |
| AEB [X]      | 45.72                   | 3.05                  | 6.46                   |
| CDA-EV [III] | 16.59                   | 3.34                  | 6.64                   |
| CDA-ME [III] | 6.61                    | 2.89                  | 6.17                   |
| CDA-NL [III] | 3.86                    | 2.86                  | 6.16                   |
| Proposed method | 1.57                   | 1.20                  | 4.90                   |

Table I shows the obtained results when considering synthetic data image. The superior performance is obtained by proposed SU model and it is sturdier against various NLs since it achieved the best value of RMSE compared to the conventional SU models. Further, it also achieved the best outcome for the values of RE and SAM also which indicates that the proposed SU model is suitable for the different physical developments like NEs, EV or MEs.

V. Conclusions

This article presented an innovative unmixing model to specify the hyperspectral data. Unlike the existing or conventional non-linear HSU models, proposed HSU model doesn’t need the NL specification. Further, this article derived an innovative form of sturdy NMF issue by implementing IB-CDA which associated with increasing updates. Our proposed HSU model tested with synthetic data and shown superiority when compared to existing HSU models. In addition, parameters like RE, RMSE and SAM also calculated to disclose the effectiveness of proposed unmixing algorithm over the existing approaches. Based on these values only the proposed HSU model concluded that it is sturdy against the NLs, EV and MEs. Future work includes that denoising of hyperspectral images for an effective unmixing of hyperspectral images with more efficacious mixing models.

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