Approximating minimum power edge-multi-covers

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Abstract Given an undirected graph with edge costs, the power of a node is the maximum cost of an edge incident to it, and the power of a graph is the sum of the powers of its nodes. Motivated by applications in wireless networks, we consider the following fundamental problem that arises in wireless network design. Given a graph $G = (V, E)$ with edge costs and lower degree bounds $\{r(v) : v \in V\}$, the Min-Power Edge-Multicover problem is to find a minimum-power subgraph $J$ of $G$ such that the degree of every node $v$ in $J$ is at least $r(v)$. Let $k = \max_{v \in V} r(v)$. For $k = \Omega(\log n)$, the previous best approximation ratio for the problem was $O(\log n)$, even for uniform costs (Kortsarz et al. 2011). Our main result improves this ratio to $O(\log k)$ for general costs, and to $O(1)$ for uniform costs. This also implies ratios $O(\log k)$ for the Min-Power $k$-Outconnected Subgraph and $O\left(\log k \log \frac{n}{n-k}\right)$ for the Min-Power $k$-Connected Subgraph problems; the latter is the currently best known ratio for the min-cost version of the problem when $n \leq k(k-1)^2$. In addition, for small values of $k$, we improve the previously best ratio $k+1$ to $k+1/2$.

Keywords Wireless networks · Edge multi-cover · Graph connectivity · Approximation algorithms

A preliminary version of this paper is Cohen and Nutov (2012).

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1 Introduction

1.1 Motivation and problems considered

Wireless networks are studied extensively due to their wide applications. The power consumption of a station determines its transmission range, and thus also the stations it can send messages to; the power typically increases at least quadratically in the transmission range. Assigning power levels to the stations (nodes) determines the resulting communication network. Conversely, given a communication network, the power required at a node $v$ only depends on the farthest node reached directly by $v$. This is in contrast with wired networks, in which every pair of stations that communicate directly incurs a cost. An important network property is fault-tolerance, which is often measured by minimum degree or node-connectivity of the network. Node-connectivity is much more central here than edge-connectivity, as it models stations failures. Such power minimization problems were vastly studied; see for example Althaus et al. (2006), Hajiaghayi et al. (2007), Lando and Nutov (2010), Nutov (2010b, c, 2012), Panigrahi (2011) and the references therein for a small sample of papers in this area. The first problem we consider is the Min-Power Edge-Multicover problem, that seeks to find a low power network with specified lower degree bounds. The second and the third problems are Min-Power $k$-Outconnected Subgraph and Min-Power $k$-Connected Subgraph. We give approximation algorithms for these problems, improving the previously best known ratios.

**Definition 1** Let $(V, J)$ be a graph with edge-costs $\{c_e : e \in J\}$. For a node $v \in V$ let $\delta_J(v)$ denote the set of edges incident to $v$ in $J$. The power $p_J(v)$ of $v$ is the maximum cost of an edge in $J$ incident to $v$, or $0$ if $v$ is an isolated node of $J$; i.e., $p_J(v) = \max_{e \in \delta_J(v)} c_e$ if $\delta_J(v) \neq \emptyset$, and $p_J(v) = 0$ otherwise. For $V' \subseteq V$ the power of $V'$ w.r.t. $J$ is the sum $p_J(V') = \sum_{v \in V'} p_J(v)$ of the powers of the nodes in $V'$.

Unless stated otherwise, all graphs are assumed to be undirected and simple. Let $n = |V|$. Given a graph $G = (V, E)$ with edge-costs $\{c_e : e \in E\}$, we seek to find a low power subgraph $(V, J)$ of $G$ that satisfies some prescribed property. A fundamental problem in Combinatorial Optimization is finding a minimum-cost subgraph that obeys specified degree constraints (sometimes called also “matching problems”) c.f. Schrijver (2004). Another fundamental property is fault-tolerance (connectivity). In fact, these problems are related, and we use our algorithm for the former as a tool for approximating the latter.

**Definition 2** Given degree bounds $r = \{r(v) : v \in V\}$, we say that an edge-set $J$ on $V$ is an $r$-edge cover if $\deg_J(v) \geq r(v)$ for every $v \in V$, where $\deg_J(v) = |\delta_J(v)|$ is the degree of $v$ in the graph $(V, J)$.

| Min-Power Edge-Multicover: |
|---------------------------|
| **Instance:** A graph $G = (V, E)$ with edge-costs $\{c_e : e \in E\}$, degree bounds $r = \{r(v) : v \in V\}$. |
| **Objective:** Find a minimum power $r$-edge cover $J \subseteq E$. |
Given an instance of Min-Power Edge-Multicover, let \( k = \max_{v \in V} r(v) \) denote the maximum requirement.

We now define our connectivity problems. A graph is \( k \)-outconnected from \( s \) if it contains \( k \) internally-disjoint \( sv \)-paths for all \( v \in V \setminus \{s\} \). A graph is \( k \)-connected if it is \( k \)-outconnected from every node, namely, if it contains \( k \) internally-disjoint \( uv \)-paths for all \( u, v \in V \).

**Min-Power \( k \)-Outconnected Subgraph:**

*Instance:* A graph \( G = (V, E) \) with edge-costs \( \{c_e : e \in E\} \), a root \( s \in V \), and an integer \( k \).

*Objective:* Find a minimum-power \( k \)-outconnected from \( s \) spanning subgraph \( J \) of \( G \).

**Min-Power \( k \)-Connected Subgraph:**

*Instance:* A graph \( G = (V, E) \) with edge-costs \( \{c_e : e \in E\} \) and an integer \( k \).

*Objective:* Find a minimum-power \( k \)-connected spanning subgraph \( J \) of \( G \).

### 1.2 Our results

For large values of \( k = \Omega(\log n) \), the previous best approximation ratio for Min-Power Edge-Multicover was \( O(\log n) \), even for uniform costs (Kortsarz et al. 2011) (“uniform costs” means that all the edges in the input graph \( G \) have the same cost). Our main result improves this ratio to \( O(\log k) \) for general costs, and to \( O(1) \) for uniform costs.

**Theorem 1** Min-Power Edge-Multicover admits an \( O(\log k) \)-approximation algorithm. For uniform costs, the problem admits a randomized approximation algorithm with expected approximation ratio \( \rho \), where \( \rho < 2.16851 \) is the real root of the cubic equation \( e(\rho - 1)^3 = 2\rho \).

For small values of \( k \), the problem admits also the ratios \( k + 1 \) for arbitrary \( k \) (Hajiaghayi et al. 2007), while for \( k = 1 \) the best known ratio is \( k + 1/2 = 3/2 \) (Kortsarz and Nutov 2009). Our second result extends the latter ratio to arbitrary \( k \).

**Theorem 2** Min-Power Edge-Multicover admits a \((k + 1/2)\)-approximation algorithm.

For small values of \( k \), say \( k \leq 6 \), the ratio \( k + 1/2 \) is better than \( O(\log k) \) because of the constant hidden in the \( O(\cdot) \) term. And overall, our paper gives the currently best known ratios for all values \( k \geq 2 \).

Theorems 1 and 2 are proved in Sects. 2 and 3, respectively.

Now we discuss some consequences from Theorems 1, and 2. In Lando and Nutov (2010) it is proved that an \( \alpha \)-approximation for Min-Power Edge-Multicover implies an \((\alpha + 4)\)-approximation for Min-Power \( k \)-Outconnected Subgraph. The previous best ratio for Min-Power \( k \)-Outconnected Subgraph was \( O(\log n) + 4 = O(\log n) \) (Lando and Nutov 2010) for large values of \( k = \Omega(\log n) \), and \( k + 1 \) for small values of \( k \) (Nutov 2010c). From Theorem 1 we obtain the following.
Theorem 3 Min-Power $k$-Outconnected Subgraph admits an $O(\log k)$-approximation algorithm.

In Hajiaghayi et al. (2007) it is proved that an $\alpha$-approximation for Min-Power Edge-Multicover and a $\beta$-approximation for Min-Cost $k$-Connected Subgraph implies an $(\alpha + 2\beta)$-approximation for Min-Power $k$-Connected Subgraph. Thus the previous best ratio for Min-Power $k$-Connected Subgraph was $2\beta + O(\log n)$ (Kortsarz et al. 2011), where $\beta$ is the best ratio for Min-Cost $k$-Connected Subgraph (for small values of $k$ better ratios for Min-Power $k$-Connected Subgraph are given in Nutov 2010c). The currently best known value of $\beta$ is 6 for $n > k(k - 1)^2$ (Cheriyan and Végh 2013; Fukunaga et al. 2013), and $O\left(\log k \log \frac{n}{n-k}\right)$ (which is $O(\log k)$, unless $k = n - o(n)$) (Nutov 2010a) otherwise. From Theorem 1 we obtain the following.

Theorem 4 If Min-Cost $k$-Connected Subgraph admits a $\beta$-approximation algorithm then Min-Power $k$-Connected Subgraph admits an $O(\beta + \log k)$-approximation algorithm. In particular, Min-Power $k$-Connected Subgraph admits an $O\left(\log k \log \frac{n}{n-k}\right)$-approximation algorithm.

1.3 Overview of the techniques

Let the trivial solution for Min-Power Edge-Multicover be obtained by picking for every node $v \in V$ the cheapest $r(v)$ edges incident to $v$. It is known and easy to see that this produces an edge set of power at most $(k+1) \cdot \text{opt}$, see Hajiaghayi et al. (2007).

Our $O(\log k)$-approximation algorithm uses the following idea. Extending and generalizing an idea from Kortsarz et al. (2011), we show how to find an edge set $I \subseteq E$ of power $O(\text{opt})$ such that for the residual instance, the trivial solution value is reduced by a constant fraction. We repeatedly find and add such an edge set $I$ to the constructed solution, while updating the degree bounds accordingly to $r(v) \leftarrow \max\{r(v) - \text{deg}_I(v), 0\}$. After $O(\log k)$ steps, the trivial solution value is reduced to $\text{opt}$, and the total power of the edges we picked is $O(\log k) \cdot \text{opt}$. At this point we add to the constructed solution the trivial solution of the residual problem, which at this point has value $\text{opt}$, obtaining an $O(\log k)$-approximate solution.

Our algorithm for uniform costs has two phases. In the first phase we compute an optimal solution $x$ to a certain LP-relaxation for the problem and round it to 1 with probability $\min\{\rho \cdot x, 1\}$. In the second phase we add to the obtained partial solution the trivial solution to the residual problem.

Our $(k+1/2)$-approximation algorithm uses a two-stage reduction. The first reduction reduces Min-Power Edge-Multicover to a constrained version of Min-Power Edge-Multicover with $k = 1$, where we also have lower bounds $\ell_v$ on the power of each node $v \in V$; these lower bounds are determined by the trivial solution to the problem. We will show that a $\rho$-approximation algorithm to this constrained version implies a $(k - 1 + \rho)$-approximation algorithm for Min-Power Edge-Multicover. The second reduction reduces the constrained version to the Minimum-Cost Edge-Cover problem with a loss of $3/2$ in the approximation ratio. As Minimum-Cost
Edge-Cover admits a polynomial time algorithm, we get a ratio $\rho = 3/2$ for the constrained problem, which in turn gives the ratio $k - 1 + \rho = k + 1/2$ for Min-Power Edge-Multicover.

2 Proof of Theorem 1

2.1 Reduction to bipartite graphs

Let Bipartite Min-Power Edge-Multicover be the restriction of Min-Power Edge-Multicover to instances for which the input graph $G = (V, E)$ is a bipartite graph with sides $A, B$, and with $r(a) = 0$ for every $a \in A$ (so, only the nodes in $B$ may have positive degree bounds).

As in Kortsarz et al. (2011), we can reduce Min-Power Edge-Multicover to Bipartite Min-Power Edge-Multicover, by taking two copies $A = \{a_v : v \in V\}$ and $B = \{b_v : v \in V\}$ of $V$, for every edge $e = uv \in E$ adding the two edges $a_v b_v$ and $a_v b_u$ of cost $c_e$ each, and for every $v \in V$ setting $r(b_v) = r(v)$ and $r(a_v) = 0$.

It is proved in Kortsarz et al. (2011) that this reduction invokes a factor of 2 in the approximation ratio, namely, that a $\rho$-approximation for Bipartite Min-Power Edge-Multicover implies a $2\rho$-approximation for general Min-Power Edge-Multicover.

In the case of uniform costs, we can save this factor of 2 using a different reduction.

Proposition 1 Ratio $\rho$ for Bipartite Min-Power Edge-Multicover with unit costs implies ratio $\rho$ for Min-Power Edge-Multicover with uniform costs.

Proof Clearly, the case of uniform costs is equivalent to the case of unit costs. Now we show that for unit costs, Min-Power Edge-Multicover can be reduced to Bipartite Min-Power Edge-Multicover. Let $G = (V, E)$, $r$ be an instance of Min-Power Edge-Multicover with unit costs. If there is an edge $e = uv \in E$ with $r(u), r(v) \geq 1$ or with $r(u) = r(v) = 0$, then we can obtain an equivalent instance by removing $e$ from $G$, and in the case $r(u), r(v) \geq 1$ also decreasing each of $r(u), r(v)$ by 1. Hence we may assume that every $e \in E$ has one end in $A = \{a \in V : r(a) = 0\}$ and the other end in $B = \{b \in V : r(b) \geq 1\}$. The statement follows. 

Consequently, it is sufficient to prove Theorem 1 for Bipartite Min-Power Edge-Multicover, which is done in Sects. 2.2 and 2.3.

2.2 An $O(\log k)$-approximation algorithm for general costs

Let $\text{opt}$ denote the optimal solution value of a problem instance at hand. For $v \in V$, let $w_v$ be the cost of the $r(v)$-th least cost edge incident to $v$ in $E$ if $r(v) \geq 1$, and $w_v = 0$ otherwise. Given a partial solution $J$ to Bipartite Min-Power Edge-Multicover let $r_J(v) = \max\{r(v) - \deg_J(v), 0\}$ be the residual bound of $v$ w.r.t. $J$.

Note that $r_J(v) = r(v)$, since $\deg_J(v) = 0$. Let

$$R_J = \sum_{b \in B} w_b r_J(b).$$
Note that \( R_\emptyset = \sum_{b \in B} w_br_\emptyset(b) = \sum_{b \in B} w_br(b) \). The main step in our algorithm is given in the following lemma, which will be proved later.

**Lemma 1** There exists a polynomial time algorithm that given an edge set \( J \subseteq E \), an integer \( \tau \), and a parameter \( \gamma > 1 \), either correctly establishes that \( \tau < \text{opt} \), or returns an edge set \( I \subseteq E \setminus J \) such that \( p_I(V) \leq (1 + \gamma)\tau \) and \( R_{J \cup I} \leq \theta R_J \), where \( \theta = 1 - \left(1 - \frac{1}{\gamma}\right)(1 - \frac{1}{e}) \).

**Lemma 2** Let \( J \subseteq E \) and let \( F \subseteq E \setminus J \) be an edge set obtained by picking \( r_J(b) \) least cost edges in \( \delta_{E \setminus J}(b) \) for every \( b \in B \). Then \( J \cup F \) is an \( r \)-edge-cover and:

\[
p_F(B) \leq \text{opt}, \quad p_F(A) \leq R_J \leq k \cdot \text{opt}.
\]

**Proof** Since \( F \) is an \( r_J \)-edge-cover, \( J \cup F \) is an \( r \)-edge-cover. By the definition of \( F \), for any \( r \)-edge-cover \( I \), \( p_F(b) \leq w_b \leq p_I(b) \) for all \( b \in B \). In particular, if \( I \) is an optimal \( r \)-edge-cover, then

\[
p_F(B) \leq \sum_{b \in B} w_b \leq \sum_{b \in B} p_I(b) = p_I(B) \leq \text{opt}.
\]

Also,

\[
R_J = \sum_{b \in B} w_br_J(b) \leq k \cdot \sum_{b \in B} w_b \leq k \cdot \text{opt}.
\]

Finally, \( p_F(A) \leq R_J \) since

\[
p_F(A) = \sum_{a \in A} p_F(a) \leq \sum_{a \in A} \sum_{e \in \delta_F(a)} c_e \leq \sum_{e \in F} c_e \leq \sum_{b \in B} w_br_J(b) = R_J.
\]

This concludes the proof of the lemma. \( \square \)

Theorem 1 is deduced from Lemmas 1 and 2 as follows. We set \( \gamma \) to be constant strictly greater than 1, say \( \gamma = 2 \). Then \( \theta = 1 - \frac{1}{2} \left(1 - \frac{1}{e}\right) \). Using binary search, we find the least integer \( \tau \) such that the following procedure computes an edge set \( J \) satisfying \( R_J \leq \tau \).

**Initialization:** \( J \leftarrow \emptyset \).

**Loop:** Repeat \( \lceil \log_{1/\theta} k \rceil \) times:
- If it establishes that \( \tau < \text{opt} \) then return “ERROR” and STOP.
- Else do \( J \leftarrow J \cup I \).

After computing \( J \) as above, we compute an edge set \( F \subseteq E \setminus J \) as in Lemma 2. The edge-set \( J \cup F \) is a feasible solution, by Lemma 2. We claim that for any \( \tau \) \( \text{opt} \) the above procedure returns an edge set \( J \) satisfying \( R_J \leq \tau \); thus binary search indeed applies. To see this, note that \( R_\emptyset \leq k \cdot \text{opt} \) and thus

\[
R_J \leq R_\emptyset \cdot \theta^{\lceil \log_{1/\theta} k \rceil} \leq k \cdot \text{opt} \cdot 1/k = \text{opt} \leq \tau.
\]
Consequently, the least integer $\tau$ for which the above procedure does not return “ERROR” satisfies $\tau \leq \text{opt}$. Thus $p_J(V) \leq \lceil \log_{1/\theta} k \rceil \cdot (1 + \gamma) \cdot \tau = O(\log k) \cdot \text{opt}$. Also, by Lemma 2, $p_F(V) \leq \text{opt} + R_J \leq 2\text{opt}$. Consequently,

$$p_{J \cup F}(V) \leq p_J(V) + p_F(V) = O(\log k) \cdot \text{opt} + 2\text{opt} = O(\log k) \cdot \text{opt}.$$ 

In the rest of this section we prove Lemma 1. It is sufficient to prove the statement in the lemma for the residual instance $((V, E \setminus J), r_J)$ with edge-costs restricted to $E \setminus J$; namely, we may assume that $J = \emptyset$. Let $R = R_\emptyset = \sum_{b \in B} w_br(b)$.

**Definition 3** An edge $e \in E$ incident to a node $b \in B$ is $\tau$-cheap if $c_e \leq \frac{\tau \gamma R}{R} \cdot w_br(b)$.

**Lemma 3** Let $F$ be an $r$-edge-cover, let $\tau \geq p_F(B)$, and let

$$I = \bigcup_{b \in B} \{ e \in \delta_E(b) : c_e \leq \frac{\tau \gamma R}{R} \cdot w_br(b) \}$$

be the set of $\tau$-cheap edges in $E$. Then $R_{I \cap F} \leq R/\gamma$ and $p_I(B) \leq \gamma \tau$.

**Proof** Let $D = \{ b \in B : \delta_{F \setminus I}(b) \neq \emptyset \}$. Since for every $b \in D$ there is an edge $e \in F \setminus I$ incident to $b$ with $c_e > \frac{\tau \gamma R}{R} \cdot w_br(b)$, we have $p_{F \setminus I}(b) \geq \frac{\tau \gamma R}{R} \cdot w_br(b)$ for every $b \in D$. Thus

$$\tau \geq p_F(B) \geq p_{F \setminus I}(B) = \sum_{b \in D} p_{F \setminus I}(b) \geq \tau \cdot \frac{\gamma R}{\sum_{b \in D} w_br(b)} \sum_{b \in D} w_br(b).$$

This implies $\sum_{b \in D} w_br(b) \leq R/\gamma$. Note that for every $b \in B \setminus D$, $\delta_F(b) \subseteq \delta_I(b)$ and hence $r_{I \cap F}(b) = r_F(b) = 0$. Thus we obtain:

$$R_{I \cap F} = \sum_{b \in B} w_br_{I \cap F}(b) = \sum_{b \in D} w_br_{I \cap F}(b) \leq \sum_{b \in D} w_br(b) \leq R/\gamma.$$ 

To see that $p_I(B) \leq \gamma \tau$ note that

$$p_I(B) = \sum_{b \in B} p_I(b) \leq \frac{\tau \gamma R}{R} \sum_{b \in B} w_br(b) = \frac{\tau \gamma R}{R} \cdot R = \tau \gamma.$$

This concludes the proof of the lemma.

In Kortsarz et al. (2011) it is proved that the following problem, which is a particular case of submodular function minimization subject to matroid and knapsack constraint (see Lee et al. 2010), admits a $(1 - \frac{1}{e})$-approximation algorithm.
Bipartite Power-Budgeted Maximum Edge-Multi-Coverage

**Instance:** A bipartite graph $G = (A \cup B, E)$ with edge-costs $\{c_e : e \in E\}$ and node-weights $\{w_v : v \in B\}$, degree bounds $\{r(v) : v \in B\}$, and a budget $\tau$.

**Objective:** Find $I \subseteq E$ with $p_I(A) \leq \tau$ that maximizes

$$\text{val}(I) = \sum_{v \in B} w_v \cdot \min\{\deg_I(v), r(v)\}.$$ 

The following algorithm computes an edge set as in Lemma 1.

1. Among the $\tau$-cheap edges, compute a $(1 - \frac{1}{\gamma})$-approximate solution $I$ to Bipartite Power-Budgeted Maximum Edge-Multi-Coverage.

2. If $R_I \leq \theta R$ then return $I$, where $\theta = 1 - \left(1 - \frac{1}{\gamma}\right) \left(1 - \frac{1}{\rho}\right)$.

   Else declare “$\tau < \text{opt}$”.

   Clearly, $p_I(A) \leq \tau$. By Lemma 3, $p_I(B) \leq \gamma \tau$. Thus $p_I(V) \leq p_I(A) + p_I(B) \leq (1 + \gamma)\tau$.

Now we show that if $\tau \geq \text{opt}$ then $R_I \leq \theta R$. Let $F$ be the set of cheap edges in some optimal solution. Then $p_F(A) \leq \text{opt} \leq \tau$. By Lemma 3 $R_F \leq R/\gamma$, namely, $F$ reduces $R$ by at least $R \left(1 - \frac{1}{\gamma}\right)$. Hence our $(1 - \frac{1}{\gamma})$-approximate solution $I$ to Bipartite Power-Budgeted Maximum Edge-Multi-Coverage reduces $R$ by at least $R \left(1 - \frac{1}{\gamma}\right) \left(1 - \frac{1}{\rho}\right)$. Consequently, $R_I \leq R - R \left(1 - \frac{1}{\gamma}\right) \left(1 - \frac{1}{\rho}\right) = \theta R$, as claimed.

The proof of Theorem 1 for the case of general costs is complete.

2.3 A constant ratio approximation algorithm for unit costs

Bipartite Min-Power Edge-Multicover with unit costs is closely related to the Set-Multicover problem, that can be casted in terms of bipartite graphs as follows.

**Set-Multicover**

**Instance:** A bipartite graph $G = (A \cup B, E)$ and demands $\{r(b) : b \in B\}$.

**Objective:** Find a subgraph $G' = (V', E')$ of $G$ with $\deg_{G'}(b) \geq r(b)$ for every $b \in B$; minimize $|V' \cap A|$.

In fact, it is easy to see that Bipartite Min-Power Edge-Multicover with unit costs is equivalent to the following modification of Set-Multicover, where instead of minimizing $|V' \cap A|$ we seek to minimize $|V' \cap A| + |B|$; namely, the problem we consider is as follows.

**Set-Multicover+**

**Instance:** A bipartite graph $G = (A \cup B, E)$ and demands $\{r(b) : b \in B\}$.

**Objective:** Find a subgraph $G' = (V', E')$ of $G$ with $\deg_{G'}(b) \geq r(b)$ for every $b \in B$; minimize $|V' \cap A| + |B|$.

Clearly, ratio $\rho$ for Set-Multicover implies ratio $\rho$ for Set-Multicover+. As Set-Multicover admits ratio $H(|B|) = \sum_{i=1}^{\lfloor |B| \rfloor} 1/i$ (the harmonic number of $|B|$), so does
Set-Multicover+. On the other hand, Set-Multicover+ is APX-hard even for instances with \( \max_{a \in A} \deg_G(a) = 3 \), by a reduction from 3-Set-Cover. If \(|A| = O(|B|)\) then the problem is clearly approximable within a constant; but we may have \(|A| >> |B|\), if \( k = \max_{b \in B} r(b) \) is large. We prove the following theorem that implies the second part of Theorem 1, and is also of independent interest.

**Theorem 5** Set-Multicover+ admits a randomized approximation algorithm with expected approximation ratio \( \rho \), where \( \rho < 2.16851 \) is the real root of the cubic equation \( e(\rho - 1)^3 = 2 \rho \).

Let \( \Gamma(a) \) denote the set of neighbors of \( a \) in \( G \). Consider the following LP-relaxation for both Set-Multicover and Set-Multicover+

\[
\min \left\{ \sum_{a \in A} x_a : \sum_{a \in \Gamma(b)} x_a \geq r(b) \, \forall b \in B, \, 0 \leq x_a \leq 1 \, \forall a \in A \right\}. \tag{1}
\]

The value of a solution \( x \) to LP (1) is \( x(A) = \sum_{a \in A} x_a \) in the Set-Multicover case, and \( x \, (A) + |B| \) in the Set-Multicover+ case. Given a partial cover \( S \subseteq A \), the residual demand of \( b \in B \) is \( r_S(b) = \max \{ r(b) - |\Gamma(b) \cap S|, 0 \} \). Let \( \rho > 1 \) be a parameter eventually set to be as in Theorem 5. Let \( \gamma = \gamma(\rho) = \frac{(\rho - 1)^3}{2 \rho} \). Note that \( \gamma = 1 \) if, and only if, \( \rho = 2 + \sqrt{3} \), and that the value of \( \rho \) in Theorem 5 is less than \( 2 + \sqrt{3} \).

**Lemma 4** Let \( x \) be a feasible solution to LP (1) such that \( \text{If } x_a < 1/\rho \text{ for all } a \in A \).
Let \( S \subseteq A \) be the random set where probability of choosing every \( a \in A \) is \( \rho \cdot x_a \), independently. Then \( \Pr[r_S(b) \geq 1] \leq e^{-\gamma \cdot r(b)} \) for every \( b \in B \).

**Proof** Let \( C(b) = \Gamma(b) \cap S \) be a random variable that counts the number of times \( b \) is “covered” by \( S \). Clearly, \( r_S(b) \geq 1 \) if, and only if, \( C(b) < r(b) \). The expectation of \( C(b) \) is \( \mu_b = \mathbb{E}[C(b)] = \sum_{a \in \Gamma(b)} \rho \cdot x_a \geq \rho \cdot r(b) \). Since the nodes in \( \Gamma(b) \) are chosen independently, \( C(b) \) is a sum of independent Bernoulli random variables. The statement now follows by applying a variation of the Chernoff bound (c.f. Angluin and Valiant 1979):

\[
\Pr[C(b) < r(b)] = \Pr \left[ C(b) < \left( 1 - \frac{\rho - 1}{\rho} \right) \cdot \rho \cdot r(b) \right] \\
\leq \Pr \left[ C(b) < \left( 1 - \frac{\rho - 1}{\rho} \right) \cdot \mu_b \right] \\
\leq e^{-\frac{1}{2} \left( \frac{\rho - 1}{\rho} \right)^2 \mu_b} \leq e^{-\gamma \cdot r(b)}.
\]

\( \square \)

**Corollary 1** Let \( x \) be a feasible solution to LP (1), and let \( S \subseteq A \) be the random set where probability of choosing every \( a \in A \) is \( \min\{\rho \cdot x_a, 1\} \), independently. Then

\[
\mathbb{E}[r_S(B)] \leq f(\rho)|B|
\]
where \( f(\rho) = \frac{1}{e\gamma} \) if \( 1 < \rho \leq 2 + \sqrt{3} \) and \( f(\rho) = e^{-\gamma} \) if \( \rho \geq 2 + \sqrt{3} \).

**Proof** Let \( A' = \{a \in A : x_a < 1/\rho\} \) be the set of nodes picked with probability \( < 1 \). Let \( r'(b) = r_{A \setminus A'}(b) = \max\{r(b) - |\Gamma(b) \cap (A \setminus A')|, 0\} \), and let \( x' \) be defined by \( x'_a = x_a \) if \( a \in A' \) and \( x'_a = 0 \) otherwise. Let \( S' \subseteq A' \) be the random set where probability of choosing every \( a \in A \) is \( \rho \cdot x'_a \), independently. Since \( Pr\{a \in S\} = Pr\{a \in (A \setminus A') \cup S'\} \) for every \( a \in A \), we have \( \mathbb{E}[r_S(B)] = \mathbb{E}[r_{(A \setminus A') \cup S'}(B)] \). Furthermore, \( \mathbb{E}[r_{(A \setminus A') \cup S'}(B)] = \mathbb{E}[r'_S(B)] \) follows by the definition of \( r' \).

Note that \( x' \) is a feasible solution to LP (1) with the residual requirements \( r' \), and that \( x'_a < 1/\rho \) for all \( a \in A \). By applying Lemma 4 to \( S' \) we have

\[
\mathbb{E}[r'_S(B)] = \sum_{b \in B} \mathbb{E}[r'_S(b)] \leq \sum_{b \in B} Pr\{r'_S(b) \geq 1\} \cdot r'(b) \leq \sum_{b \in B} e^{-\gamma}r'(b) \cdot r'(b).
\]

Let \( z = r'(b) \geq 1 \) and \( f(z) = e^{-\gamma z} \cdot z \). Then \( f'(z) = e^{-\gamma z}(1 - \gamma z) \). Hence in the range \( z \geq 1 \), the function \( f(z) \) has maximum value:

- \( \frac{1}{e\gamma} \) if \( \gamma \leq 1 \) (namely, if \( 1 < \rho \leq 2 + \sqrt{3} \)), attained at \( z = 1/\gamma \).
- \( e^{-\gamma} \) if \( \gamma \geq 1 \) (namely, if \( \rho \geq 2 + \sqrt{3} \)), attained at \( z = 1 \).

Thus \( \mathbb{E}[r_S(B)] = \mathbb{E}[r_{(A \setminus A') \cup S'}(B)] = \mathbb{E}[r'_S(B)] \leq \sum_{b \in B} f(\rho) = f(\rho)|B|. \) The statement follows.

Now we finish the proof of Theorem 5. The algorithm is as follows. We compute an optimal solution \( x \) to LP (1), and then an edge set \( S \) as in Corollary 1. For every \( b \in B \) let \( A_b \) be a set of \( r_S(b) \) neighbors in \( \Gamma(b) \setminus S \), and let \( S' = \bigcup_{b \in B} A_b \). The solution returned is \( S \cup S' \). Note that \( \mathbb{E}[|S'|] \leq \mathbb{E}[r_S(B)] \). Thus by Corollary 1, the expected size of our solution is bounded by

\[
\mathbb{E}[|S|] + \mathbb{E}(r_S(B)) + |B| \leq \rho x(A) + f(\rho)|B| + |B| \leq \max\{\rho, f(\rho) + 1\}(x(A) + |B|).
\]

Consequently, as \( x(A) + |B| \) is a lower bound on the optimal solution value, the approximation ratio is bounded by \( \max\{\rho, f(\rho) + 1\} \). Solving the equation \( \rho = f(\rho) + 1 \) for \( f(\rho) = 1/e\gamma = \frac{2\rho}{e(\rho-1)^2} \) gives the result.

The proof of Theorem 5, and thus also of Theorem 1 for the case of uniform costs, is complete.

### 3 Proof of Theorem 2

We say that an edge set \( F \subseteq E \) covers a node set \( U \subseteq V \), or that \( F \) is a \( U \)-cover, if \( \delta_F(v) \neq \emptyset \) for every \( v \in U \). Consider the following auxiliary problem:

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Restricted Min-Power Edge-Cover

Instance: A graph \( G = (V, E) \) with edge-costs \( \{c_e : e \in E\} \), \( U \subseteq V \), and degree bounds \( \{\ell_v : v \in U\} \).

Objective: Find non-negative numbers \( \{\pi(v) : v \in V\} \) with \( \sum_{v \in V} \pi(v) \) minimum, such that \( \pi(v) \geq \ell_v \) for all \( v \in U \), and such that the edge set \( F = \{e = uv \in E : \pi(u), \pi(v) \geq c_e\} \) covers \( U \).

Later, we will prove the following lemma.

**Lemma 5** Restricted Min-Power Edge-Cover admits a \( 3/2 \)-approximation algorithm.

Theorem 2 is deduced from Lemma 5 and the following statement.

**Lemma 6** If Restricted Min-Power Edge-Cover admits a \( \rho \)-approximation algorithm, then Min-Power Edge-Multicover admits a \( (k - 1 + \rho) \)-approximation algorithm.

**Proof** Let \( G = (V, E), c, r \) be an instance of Min-Power Edge-Multicover. Recall that \( w_v \) denotes the cost of the \( r(v) \)-th least cost edge incident to \( v \) in \( E \) if \( r(v) \geq 1 \), and \( w_v = 0 \) otherwise. Consider the following algorithm.

1. Compute a \( \rho \)-approximate solution \( \{\pi(v) : v \in V\} \) for Restricted Min-Power Edge-Cover with \( U = \{v \in V : r(v) \geq 1\} \) and bounds \( \ell_v = w_v \) for all \( v \in U \).
   Let \( F = \{e = uv \in E : \pi(u), \pi(v) \geq c_e\} \).
2. For every \( v \in V \) let \( I_v \) be the edge-set obtained by picking the least cost \( r_F(v) \) edges in \( \delta_{E \setminus F}(v) \) and let \( I = \bigcup_{v \in V} I_v \).

Clearly, \( F \cup I \) is a feasible solution to Min-Power Edge-Multicover. Let \( \text{opt} \) denote the optimal solution value for Min-Power Edge-Multicover. In what follows note that \( \pi(V) \leq \rho \cdot \text{opt} \) and that \( \sum_{v \in V} w_v \leq \text{opt} \). We claim that

\[
p_{I \cup F}(V) \leq \pi(V) + (k - 1) \cdot \text{opt}.
\]

Here \( p_{I \cup F}(V) \) is the power of \( V \) in the graph \( (V, I \cup F) \), see Definition 1. As \( \pi(V) \leq \rho \cdot \text{opt} \), this implies \( p_{I \cup F}(V) \leq (\rho + k - 1) \cdot \text{opt} \).

For \( v \in V \) let \( \Gamma_v \) be the set of neighbors of \( v \) in the graph \( (V, I_v) \). The contribution of \( I_v \) to the total power is at most \( p_{I_v}(\Gamma_v) + p_{I_v}(v) \). Note that \( \pi(v) \geq p_{I_v}(v) \) and \( \pi(v) \geq p_F(v) \) for every \( v \in V \), hence \( p_{F \cup I_v}(v) \leq \pi(v) \). This implies

\[
p_{F \cup I}(V) \leq \sum_{v \in V} (\pi(v) + p_{I_v}(\Gamma_v)) = \pi(V) + \sum_{v \in V} p_{I_v}(\Gamma_v).
\]

Now observe that \( |\Gamma_v| = |I_v| = r_F(v) \leq k - 1 \) and that \( p_{I_v}(u) \leq w_v \) for every \( u \in \Gamma_v \). Thus

\[
p_{I_v}(\Gamma_v) \leq (k - 1) \cdot w_v \quad \forall v \in V.
\]
Finally, using the fact that \( \sum_{v \in V} w_v \leq \text{opt} \), we obtain

\[
p_{F \cup I}(V) \leq \pi(V) + \sum_{v \in V} p_{I_v}(\Gamma_v) \leq \pi(V) + (k - 1) \sum_{v \in V} w_v \leq \pi(V) + (k - 1) \cdot \text{opt}.
\]

This finishes the proof of the lemma. \( \square \)

To prove Lemma 5, we reduce Restricted Min-Power Edge-Cover to the following problem that admits an exact polynomial time algorithm, c.f. Schrijver (2004).

**Min-Cost Edge-Cover**

**Instance:** A multi-graph (possibly with loops) \( G = (U, E) \) with edge-costs.

**Objective:** Find a minimum cost edge-set \( F \subseteq E \) that covers \( U \).

Our reduction is not approximation ratio preserving, but incurs a loss of 3/2 in the approximation ratio, as stated in the following lemma.

**Lemma 7** There exists a polynomial time algorithm that given an instance \( (G, c, U, \ell) \) of Restricted Min-Power Edge-Cover, constructs in polynomial time an instance \( (G', c') \) of Min-Cost Edge-Cover such that the following holds:

(i) Given a \( U \)-cover \( I' \) in \( G' \), one can find in polynomial time a feasible solution \( \pi \) to the original Restricted Min-Power Edge-Cover instance \( (G, c, U, \ell) \), such that \( \pi(V) \leq c'(I') \).

(ii) \( \text{opt}' \leq \frac{3}{2} \text{opt} \), where \( \text{opt} \) is an optimal solution value to Restricted Min-Power Edge-Cover and \( \text{opt}' \) is the minimum cost of a \( U \)-cover in \( G' \).

Let us show that Lemma 7 implies Lemma 5. Given an instance \( (G, c, U, \ell) \) of Restricted Min-Power Edge-Cover, we construct an instance \( (G', c') \) of Min-Cost Edge-Cover as in Lemma 7, compute an optimal (min-cost) solution \( I' \) to \( (G', c') \), and return a solution \( \pi \) to \( (G, c, U, \ell) \) as in the lemma. Then by properties (i),(ii) in Lemma 7 we have \( \pi(V) \leq c'(I') = \text{opt}' \leq \frac{3}{2} \text{opt} \).

In the rest of this section we prove Lemma 7. Let \( (G, c, U, \ell) \) be an instance of Restricted Min-Power Edge-Cover. Clearly, we may set \( \ell_v = 0 \) for all \( v \in V \setminus U \). For \( I \subseteq E \) let

\[
D(I) = \sum_{v \in V} \max\{p_f(v) - \ell_v, 0\}.
\]

Here is the construction of the instance \( (G', c') \), where \( G' = (U, E') \) and \( E' \) consists of the following three types of edges, where for every edge \( e' \in E' \) corresponds a set \( I(e') \subseteq E \) of one edge or of two edges.

1. For every \( v \in U \), \( E' \) has a loop-edge \( e' = vv \) with \( c'(vv) = \ell_v + D(\{vv\}) \) where \( vv \) is an arbitrary chosen minimum cost edge in \( \delta_E(v) \).

   Here \( I(e') = \{vv\} \).

2. For every \( uv \in E \) such that \( u, v \in U \), \( E' \) has an edge \( e' = uv \) with \( c'(uv) = \ell_u + \ell_v + D(\{uv\}) \).

   Here \( I(e') = \{uv\} \).
3. For every pair of edges $ux, xv \in E$ such that $c(ux) \geq c(xv)$, $E'$ has an edge $e' = uv$ with $c'(uv) = \ell_v + \ell_u + D(\{ux, xv\})$.

Here $I(e') = \{ux, xv\}$.

The following lemma shows that property (i) in Lemma 7 holds.

**Lemma 8** Let $I' \subseteq E'$ be a $U$-cover in $G'$, let $I = \bigcup_{e \in I'} I(e')$, and let $\pi$ be defined by $\pi(v) = \max\{p_I(v), \ell_v\}$. Then $I$ is a $U$-cover in $G$, $\pi$ is a feasible solution to $(G, c, U, \ell)$, and $\pi(V) \leq c'(I')$.

**Proof** We have that $I$ is a $U$-cover in $G$, by the definition of $I$ and since $I(e')$ covers both endnodes of every $e' \in E'$. By the definition of $\pi$, we have that $I \subseteq \{e = uv \in E : \pi(u), \pi(v) \geq c_e\}$. Hence $\pi$ is a feasible solution to $(G, c, U, \ell)$, as claimed.

We prove that $\pi(V) \leq c'(I')$. For $e' = uv \in E'$ let $\ell(e') = \ell_v$ if $e'$ is a type 1 edge, and $\ell(e') = \ell_u + \ell_v$ otherwise. Note that $\pi(v) = \max\{p_I(v), \ell(v)\} = \ell_v + \max\{p_I(v) - \ell(v), 0\}$, hence

$$
\pi(V) = \sum_{v \in U} \ell_v + \sum_{v \in V} \max\{p_I(v) - \ell(v), 0\} = \sum_{v \in U} \ell_v + D(I).
$$

By the definition of $\ell(e')$ and since $I'$ is a $U$-cover $\sum_{e \in U} \ell_v \leq \sum_{e' \in I'} \ell(e')$. Also, $D(I) = D\left(\bigcup_{e' \in I'} I(e')\right)$, by the definition of $I$. Thus we have

$$
\sum_{v \in U} \ell_v + D(I) \leq \sum_{e' \in I'} \ell(e') + D\left(\bigcup_{e' \in I'} I(e')\right).
$$

It is easy to see that $D\left(\bigcup_{e' \in I'} I(e')\right) \leq \sum_{e' \in I'} D(I(e'))$. Finally, note that $\ell(e') + D(I(e')) = c'(e')$ for every $e' \in I'$ (if $e'$ is a type 1 edge, this follows from our assumption that $\ell_v \geq \min\{c_e : e \in \delta_E(v)\}$). Combining we get

$$
\pi(V) = \sum_{v \in U} \ell_v + D(I) \leq \sum_{e' \in I'} \ell(e') + D\left(\bigcup_{e' \in I'} I(e')\right)
\leq \sum_{e' \in I'} \ell(e') + \sum_{e' \in I'} D(I(e')) = \sum_{e' \in I'} (\ell(e') + D(I(e')))
= \sum_{e' \in I'} c'(e') = c'(I').
$$

This finishes the proof of the lemma. □

The following lemma shows that property (ii) in Lemma 7 holds.

**Lemma 9** Let $\{\pi(v) : v \in V\}$ be a feasible solution to an instance $(G, c, U, \ell)$ of Restricted Min-Power Edge-Cover. Then there exists a $U$-cover $I'$ in $G'$ such that $c'(I') \leq 3\pi(V)/2$.

**Proof** Let $I \subseteq \{e = uv \in E : \pi(u), \pi(v) \geq c_e\}$ be an inclusion minimal $U$-cover. We may assume that $\pi(v) = \max\{p_I(v), \ell_v\}$ for every $v \in V$. Since any inclusion
minimal U-cover is a collection of node disjoint stars, it is sufficient to prove the statement for the case when I is a star. Then I has at most one node not in U, and if there is such a node, then it is the center of the star, if |I| ≥ 2; in the case I consists of a single edge e, then we define the center of I to be the endnode of e in V \ U if such exists, or an arbitrary endnode of e otherwise.

We define a U-cover I′ in G′, and show that
\[ c'(I') \leq \frac{3}{2} \sum_{v \in V} \max\{p_I(v), \ell_v\} = \frac{3}{2} \pi(V). \] (2)

Let v0 be the center of I and let \{v_i : 1 \leq i \leq d\} be the leaves of I ordered by descending order of costs \(c(v_0v_i) \geq c(v_0v_{i+1})\). The U-cover I′ ⊆ E′ is defined as follows. We cover each pair \(v_{2i−1}, v_{2i}, i = 1, \ldots, \lfloor d/2 \rfloor\), by a type 3 edge. This covers all the nodes except v0, and maybe v_d if d is odd. We add an additional edge f of type 1 or 2, if there are nodes in U (v_0 and/or v_d) that remain uncovered by the picked type 3 edges. Formally, we have the following 4 cases, see Figure 1.

1. d is even and v_0 \notin U, see Fig. 1a. Then U is covered by type 3 edges.
2. d is odd, and v_0 \notin U, see Fig. 1b. Then we add a type 1 edge f to cover v_d.
3. d is odd and v_0 \in U, see Fig. 1c. Then we add a type 2 edge f to cover v_0, v_d.
4. d is even and v_0 \in U, see Fig. 1d. Then we add a type 1 edge f to cover v_0.

Consider a type 3 edge \(v_{2i−1}v_{2i} \in I'\). Let \(q_i = \max\{c(v_{2i−1}v_0) − \ell_v, 0\}\). Note that \(c'(v_{2i−1}v_{2i}) \leq \pi(v_{2i−1}) + \pi(v_{2i}) + q_i\). The key point is that
\[ q_i \leq \frac{1}{2}(\pi(v_{2i−3}) + \pi(v_{2i−2})) \quad i = 2, \ldots, \lfloor d/2 \rfloor. \]
This is since \( q_i \leq c(v_0v_{2i-1}) \leq \frac{1}{2} (c(v_0v_{2i-3}) + c(v_0v_{2i-2})) \) while \( c(v_0v_j) \leq \pi(v_j) \). Therefore,
\[
\sum_{i=1}^{d/2} c'(v_{2i-1}v_{2i}) \leq \sum_{i=1}^{d/2} [\pi(v_{2i-1}) + \pi(v_{2i}) + q_i] \leq \sum_{i=1}^{2[d/2]} \pi(v_i) + q_1 + \frac{1}{2} \sum_{i=1}^{d-2} \pi(v_i)
\]

Now, we prove that (2) hold in each one of our four cases.

1. \( v_0 \notin U \) and \( d \) is even. Note that \( q_1 \leq c(v_0v_1) \leq \pi(v_0) \). Then:
\[
c'(I') = \sum_{i=1}^{d/2} c'(e_i) + c'(f) \leq \frac{3}{2} \sum_{i=1}^{d} \pi(v_i) + c'(f) + q_1 \leq \frac{3}{2} \sum_{i=0}^{d} \pi(v_i)
\]

2. \( v_0 \notin U \) and \( d \) is odd. In this case \( f = v_dv_d \) is a loop type 1 edge, so \( c'(f) \leq \pi(v_d) + \max(c(v_0v_d) - \ell_{v_0}, 0) \). This implies
\[
q_1 + c'(f) \leq c(v_0v_1) + c(v_0v_d) + \pi(v_d) \leq \pi(v_0) + \frac{1}{2} (\pi(v_0) + \pi(v_d)) + \pi(v_d) = \frac{3}{2} (\pi(v_0) + \pi(v_d)).
\]
Thus
\[
c'(I') = \sum_{i=1}^{d/2} c'(e_i) + c'(f) \leq \frac{3}{2} \sum_{i=1}^{d-1} \pi(v_i) + c'(f) + q_1 \leq \frac{3}{2} \sum_{i=0}^{d} \pi(v_i)
\]

3. \( v_0 \in U \) and \( d \) is odd. In this case \( f = v_0v_d \), so \( c'(f) \leq \max(\ell_{v_0}, c(v_0v_d)) + \pi(v_d) \). This implies \( q_1 + c'(f) \leq c(v_0v_1) + c(v_0v_d) + \pi(v_d) \leq \frac{3}{2} (\pi(v_0) + \pi(v_d)) \). Thus
\[
c'(I') = \sum_{i=1}^{d/2} c'(e_i) + c'(f) \leq \frac{3}{2} \sum_{i=1}^{d-1} \pi(v_i) + c'(f) + q_1 \leq \frac{3}{2} \sum_{i=0}^{d} \pi(v_i)
\]

4. \( v_0 \in U \) and \( d \) is even. In this case \( f = v_0v_0 \) is a loop type 1 edge, so \( c'(f) \leq \ell_{v_0} + c(v_0v_d) \leq \ell_{v_0} + \frac{1}{2} (\pi(v_{d-1}) + \pi(v_d)) \). This implies \( q_1 + c'(f) \leq c(v_0v_1) + \frac{1}{2} (\pi(v_{d-1}) + \pi(v_d)) \). Thus
\[
c'(I') = \sum_{i=1}^{d/2} c'(e_i) + c'(f) \leq \sum_{i=1}^{d} \pi(v_i) + \frac{1}{2} \sum_{i=1}^{d-2} \pi(v_i) + q_1 + c'(f)
\]
\[
\leq \frac{3}{2} \sum_{i=1}^{d} \pi(v_i) + \pi(v_0) \leq \sum_{i=0}^{d} \pi(v_i).
\]
This concludes the proof of the lemma. \( \square \)
As was mentioned, Lemmas 8 and 9 imply Lemma 7, which implies Lemma 5. Lemmas 5 and 6 imply Theorem 2, hence the proof of Theorem 2 is now complete.

4 Conclusions and open problems

The main results of this paper are two new approximation algorithm for Min-Power Edge-Multicover: one with ratio $O(\log k)$ for general costs, and the other with constant ratio for uniform costs. This improves the ratio $O(\log(nk)) = O(\log n)$ of Kortsarz et al. (2011). We also gave a $(k + 1/2)$-approximation algorithm, which is better than our $O(\log k)$-approximation algorithm for small values of $k$ (roughly $k \leq 6$).

The main open problem is whether for general costs, the ratio $O(\log k)$ shown in this paper is tight, or the problem admits a constant ratio approximation algorithm.

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