Comment on “Inferring Statistical Complexity”

Nearly 30 years ago, in [1] the authors proposed some supposedly novel measures of time series complexity, and their relations to existing concepts in nonlinear dynamical systems. At that time it seemed that the multiple faults of this paper would make it obsolete soon. Since this has not happened, and these faults still infest the literature on what is now called “computational mechanics” (CM) [2], I want here to rectify the situation.

(i) In [1] a Rényi graph complexity $C_\alpha$ was defined, and it was proposed that it is related to the well known Rényi entropies $h_\alpha$. Unfortunately, no such connection exists for general $\alpha$, since the Rényi index $\alpha$ has completely different meanings in both. Indeed, $C_\alpha$ with $\alpha \neq 0, 1$ has not found any application so far.

(ii) Both the complexity measure $C_1$ (called “statistical complexity” in [1]) and “$c$-machines” had been introduced previously in [3]. $C_1$ is just what was called “forecasting complexity” (FC) in [4][7], and “$c$-machines” had been called minimal deterministic automata. In [3] it had also been proven that FC is bounded from below by what is called “excess entropy” $E$ in [1] (the mutual information between past and future [3]), and that this bound is in general not saturated. In contrast, in [3] it was claimed that FC and $E$ are “simply proportional”.

(iii) While it was not pointed out explicitly, the way how the nodes of the “$c$-machine” were constructed (as equivalence classes of elements of a partitioning in trajectory space) implies that they also are elements of a partitioning – as stated explicitly, e.g., in [2]. This is true in simple cases, but not in general. There exist very simple models [6][8], where they are elements of a covering in which trajectories are multiply covered. This is presumably the most serious mistake, as the claim that these nodes (called “causal states” in [2]) are elements of a partitioning is repeated until now in the CM literature, and makes e.g. several proofs in [2] obsolete.

(iv) Figure 1a in [1] shows supposedly the “$c$-machine” that corresponds to the length 16 cylinder set of the critical Feigenbaum attractor. Unfortunately, it was not said whether it correspond to the trajectories on the attractor or in its basin of attraction. The latter had been given in [6], while the former is shown in Fig. 1. They are both different. Indeed, all algorithms for constructing $c$-machines from finite data used in the CM literature up to $\sim 2005$ are wrong, while the correct algorithm had been given in [7]. In that paper, also an efficient algorithm for computing $E$ had been given – the supposed unavailability of such an algorithm was considered a major problem in the CM literature until $\sim 2005$.

(v) The marked single-peaked structure of Fig. 2 in [1] results from the fact that probabilities and entropies were simply estimated from fixed length trajectory pieces. A much more careful graph of a similar quantity (called “set complexity” in [3][4]) had been given already in [6], and it displays a much richer and more complex structure.

(vi) In contrast to what its title says, no attempt was made in [1] to actually infer FC – nor was made such an attempt in any later paper on CM. As discussed in [7][8], this is not by chance, as inferring FC from imprecise or measured data (as opposed to computing it for a precisely given model) is very difficult and so far unsolved. That is also why only set complexity was estimated in [6].

More details on CM, “$c$-machines”, and complexity measures are given in [8].

Peter Grassberger
JSC, FZ Jülich, D-52425 Jülich, Germany

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