PRINCIPLE OF RELATIVITY, 24 POSSIBLE KINEMATICAL ALGEBRAS AND NEW GEOMETRIES WITH POINCARÉ SYMMETRY

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From the principle of relativity with two universal invariant parameters $c$ and $l$, 24 possible kinematical (including geometrical and static) algebras can be obtained. Each algebra is of 10 dimensional, generating the symmetry of a 4 dimensional homogeneous space-time or a pure space. In addition to the ordinary Poincaré algebra, there is another Poincaré algebra among the 24 algebras. New 4d geometries with the new Poincaré symmetry are presented. The motion of free particles on one of the new space-times is discussed.

Keywords: Kinematical algebras, Poincaré symmetry; 4d degenerate geometry.

1. Introduction

The principle of relativity, laws of non-gravitational physics having the same form in all inertial frames, is valid not only in the Minkowski space-time but also in the de Sitter (dS) space-time. Based on the principle of relativity and the postulate of the two universal invariant parameters ($c, l$), dS invariant special relativity can be established in a dS space-time, where $c$ is the vacuum speed of light at the origin and $l$ is the dS radius. For brevity, the principle of relativity and the postulate of the two universal invariant parameters are known as the principle of relativity with two universal invariant parameters, denoted by $PoR_{c,l}$. Very recently, in the study of the principle of relativity with two invariant parameters, we construct 24 kinematical algebras, including purely geometrical ones and static one. Each algebra is of 10 dimensional. The 11 of them are the algebras in Bacry-Lévy-Leblond theorem. They are (Anti-)dS ($d_{±}$), Poincaré ($p$), (Anti-)Newton-Hooke ($n_{±}$), Inhomogeneous $SO(4)$ and para-Poincaré ($p'_{±}$), Galilei ($g$), Carroll ($c$), para-Galilei ($g'$) and
static (s) algebras. The 3 of them correspond to the Euclid geometry (e), Riemann geometry (r), and Lobachevski geometry (l), which can be obtained by relaxing the third assumption in Bacry-Lévy-Leblond theorem. It is remarkable that among the 10 new kinematical or purely geometrical algebras, there is another Poincaré algebra.

It is well known that the Poincaré symmetry is the foundation of Einstein’s special relativity, relativistic field theories in Minkowski space-time, particle physics, as well as the Poincaré gauge theories of gravity, etc. Conventionally, only the Minkowski space-time is invariant under global Poincaré transformations. It is natural to ask: what is the role played by the new Poincaré symmetry.

The aim of the present talk is twofold. One is to exhibit 24 kinematical algebras, including purely geometrical ones and static one. The other is to first present the nontrivial 4d geometries which are invariant under the new Poincaré transformations. The structure of the new nontrivial 4d geometries will be explored briefly. The motion of free particles on the one of the new space-times is also discussed.

The talk is divided into 6 parts. After the introduction, I shall review the inertial motion and Umov-Weyl-Fock-Hua (UWFH) transformations, show all possible kinematical and geometrical algebras, present the new nontrivial geometries, and study the motion of free particles, successively. Finally, I shall end my talk with the summary.

2. Inertial Motions and UWFH transformations

In a Cartesian coordinate system in a flat space-time (no matter whether it is relativistic one or not), the motions satisfying

\[
\begin{align*}
    x^i &= x_0^i + v^i(t - t_0), \\
    v^i &= \frac{dx^i}{dt} = \text{consts.} \quad i = 1, 2, 3
\end{align*}
\]

or

\[
\frac{d^2 x^i}{dt^2} = 0
\]

\footnote{An algebra is said to be Poincaré one if (1) it is isomorphic to \(\mathfrak{iso}(1,3)\) algebra, (2) the unique Abelean ideal of the \(\mathfrak{iso}(1,3)\) algebra is regarded as a translation sub-algebra and is divided into the time translation and space translations as a 1d and a 3d representation, respectively, of \(\mathfrak{so}(3)\) sub-algebra of the \(\mathfrak{so}(1,3)\) sub-algebra and (3) the algebra is invariant under the suitably defined parity and time-reversal operation.}
are called the uniform rectilinear motions or inertial motions. The set of observers in the space-time, moving according to Eq. (1) or Eq. (2), make up an inertial frame, denoted by $\mathcal{F}$. In a given flat space-time, the forms of Eq. (1) and Eq. (2) are unchanged under the linear coordinate transformation with 10 parameters.

It has been shown that the forms of Eq. (1) and Eq. (2) are unchanged under the linear fractional transformations with a common denominator in the Beltrami coordinate system in a(n) (A)dS space-time.\cite{1,2} In other words, the above concept of inertial motions and inertial frame can be generalized to the (A)dS space-times.\cite{1,2} Then, the principle of relativity can be generalized to the two space-times. Obviously, in the (A)dS space-time, the (A)dS radius $l$ is an invariant parameter in addition to the invariant speed of light $c$. Therefore, the postulate of invariant speed of light $c$ in Einstein’s special relativity should be replaced by the postulate of two invariant parameters. The principle of relativity and the postulate of an invariant parameter should be replaced by the principle of relativity with two universal invariant parameters, $PoR_{c,l}$.

Furthermore, the forms of Eq. (1) and Eq. (2) are also unchanged under the linear fractional transformations with a common denominator in (Anti-)Newton-Hooke (A)NH space-times.\cite{6} Then, a simple question appears: what is the most general transformation $T$ s.t.

$$ T : \quad x'^{\mu} = f^{\mu}(x), x^0 = ct, \mu = 0, \cdots, 3, $$

preserving the form of Eq. (1) and Eq. (2)?

The following theorem answers the question.

**Theorem 2.1.** The most general transformations are the linear fractional transformations:

$$ T : \quad l^{-1}x'^{\mu} = \frac{A^{\mu}_{\nu}}{b'^{\nu}} x^{\nu} + a^{\mu}, $$

and

$$ det \ T = \left| \begin{array}{cc} A & a \\ b' & d \end{array} \right| = 1, $$

where $A = \{ A^{\mu}_{\nu} \}$ a $4 \times 4$ matrix, $a, b$ $4 \times 1$ matrices, $d \in R$ and $b'^{\nu} = \eta b$ with $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$.

The proof of the theorem can be found in Ref. 7–9. The question was first raised and answered by Umov and Weyl.\cite{10} Fock and Hua also studied the
question in details in their books.\(^7,^8\) Therefore, we name the transformations by Umov-Weyl-Fock-Hua (UWFH) transformations.

Clearly, all UWFH transformations form a group. The number of generators of the group is 24. It implies that more possible space-times admit Eq.(1) and Eq.(2), as expected.

3. Possible Kinematical Groups

In order to clarify how many space-times admit Eq.(1) and Eq.(2), we begin with the Beltrami model of (A)dS spacetime

\[
\begin{align*}
    ds^2_{\pm} &= \left( \frac{\eta_{\mu\nu}}{\sigma_{\pm}(x)} \pm \frac{x_\mu x_\nu}{l^2 \sigma^2_{\pm}(x)} \right) dx^\mu dx^\nu,
\end{align*}
\]

where \(x_\mu = \eta_{\mu\lambda} x^\lambda\) and

\[
\sigma_{\pm}(x) = \sigma_{\pm}(x,x) = 1 \mp l^{-2} x_\mu x_\mu > 0.
\]

In the above equations, the upper sign corresponds to dS space-time and the lower sign to AdS space-time. The (A)dS space-time is invariant under the (A)dS transformations, respectively. The generators of (A)dS group are

\[
\begin{align*}
P_{\pm}^\mu &= (\delta_{\mu}^\nu \mp l^{-2} x_\mu x^\nu) \partial_{\nu}, \\
L_{\mu\nu} &= x_\mu P_\nu - x_\nu P_\mu = x_\mu \partial_{\nu} - x_\nu \partial_{\mu} \in \mathfrak{so}(1,3),
\end{align*}
\]

or

\[
\begin{align*}
H^\pm &= t \partial_t \mp \nu^2 t x^\nu \partial_\nu, \\
P_i^\pm &= \partial_i \mp l^{-2} x_i x^\nu \partial_\nu, \quad \nu = c/l, \\
K_i &= t \partial_i - c^{-2} x_i \partial_t, \\
J_i &= \frac{1}{2} \epsilon^{ijk} L_{jk} = \frac{1}{2} \epsilon^{ijk} (x_j \partial_k - x_k \partial_j),
\end{align*}
\]

where \(H^\pm\) are called the Beltrami-time-translation generators, \(P_i^\pm\) are known as the Beltrami-space-translation generators, \(K_i\) and \(J_i\) are the Lorentz boost generators and the space-rotation generators, as usual. \(H^\pm\) are the scalar representation of the \(\mathfrak{so}(3)\) spanned by \(J_i\). \(P_i^\pm\) and \(K_i\) are vector representations of the \(\mathfrak{so}(3)\).

Now, we can write down the 24 generators for the group which keeps Eq.(1) and Eq.(2). They are: 4 kinds of generators for time translation

\[
H^\pm = \partial_t \mp \nu^{-2} t x^\nu \partial_\mu, \quad H := \partial_t, \quad H' = -\nu^{-2} t x^\nu \partial_\mu;
\]

4 kinds of generators for space translation

\[
P_i^\pm = \partial_i \mp l^{-2} x_i x^\mu \partial_\mu, \quad P_i := \partial_i, \quad P_i' = -l^{-2} x_i x^\mu \partial_\mu;
\]

4 kinds of generators for boost

\[
\begin{align*}
K_t &= t \partial_t - c^{-2} x_i \partial_i, \\
K_t^\delta &= t \partial_t, \\
K_i^\delta &= -c^{-2} x_i \partial_t, \\
N_t &= t \partial_t + c^{-2} x_i \partial_i.
\end{align*}
\]
3 generators of rotation $J_i$, as shown in Eq.(9) and

$$R_{ij} = R_{ji} = x_i \partial_j + x_j \partial_i, \quad (i < j)$$
$$M_0 = t \partial_t, \quad M_1 = x^1 \partial_t, \quad M_2 = x^2 \partial_2, \quad M_3 = x^3 \partial_3.$$ (13)

Among them, two time-translation generators, two sets of space-translation generators, and two sets of boost generators are independent, respectively. $H = \frac{1}{2} (H^+ + H^-)$ and $P^\pm_i = \frac{1}{2} (P^+_i + P^-_i)$ are ordinary time- and space-translation generators, respectively. $H' = \frac{1}{2} (H^+ - H^-)$ and $P'_i = \frac{1}{2} (P^+_i - P^-_i)$ are known as pseudo-time- and pseudo-space-translation generators, respectively. $K^q_i$, $K^q_i$, and $K^q_i$ are the Galilei-boost generators, $K^q_i$ and $K^q_i$ are the geometrical-boost generators. $T := (H^\pm, P^\pm_i, J_i, K_i, N_i, M_0, M_i, R_{ij})$ spans a closed algebra,

$$[P^+_i, \ J^\pm_j] = (1 - \delta_i\delta_j) l^\pm_2 R_{ij} - 2l^{-2} \delta_i\delta_j (M_{ij} + \Sigma_k M_{kij}),$$
$$[P^\pm_i, M_j] = \delta_i\delta_j P^\pm_j,$$
$$[H^+, H^-] = 2\nu^2 (M_0 + \Sigma_k M_k),$$
$$[H^\pm, M_0] = H^\pm,$$
$$[K_i, M_0] = -N_i,$$
$$[K_i, R_{jk}] = -\delta_{ij} N_k - \delta_{ik} N_j,$$
$$[N_i, M_j] = \delta_{ij} K_j,$$
$$[N_i, R_{jk}] = -\delta_{ij} K_k - \delta_{ik} K_j,$$ (14)

where no summation is taken for the repeated indexes in brackets.

It has been shown that there are 24 possible kinematical (including geometrical) algebras, in which $J_i$ serve as the space-rotation generators. They include 4 pure geometrical algebras and 1 static algebra. The algebras, the sets of generators, and the commutators are listed in Table 1. In Table 1, $H$, $P$, $K$ are the shorthands for the time-translation, space-translation, and boost generators, respectively. $[P, P] = l^{-2} J$ implies $[P^+_i, P^+_j] = -l^{-2} \epsilon_{ijk} J_k$, etc. ($\epsilon_{123} = -\epsilon_{12} = 1$; $\eta_{ij} = -\delta_{ij}$.) The commutators between $J$s and the commutators between $J$ and $H$, $P$, $K$ are not included in Table 1 because they have the same form for different algebras. Apart from the three classical geometrical algebras, there are 10 more possible algebras than those in BLL paper.

In particular, both the set of generators $(H, P_i, K_i, J_i)$ and the set of
generators \((H', P_i', K_i, J_i)\) satisfy the Poincaré algebra
\[
[H, P_i] = 0, \quad [K_i, K_j] = -c^{-2} \epsilon_{ijk} J_k, \quad [P_i, K_j] = c^{-2} H,
\]
\[
I : \quad [H, P_i] = 0, \quad [H, K_i] = P_i, \quad [J_i, H] = 0, \quad [J_i, P_j] = \epsilon_{ijk} P_k, \quad [J_i, K_j] = \epsilon_{ijk} K_k, \quad [J_i, J_j] = \epsilon_{ijk} J_k,
\]
\[
[H', P_i'] = 0, \quad [K_i, K_j] = -c^{-2} \epsilon_{ijk} J_k, \quad [P_i', K_j] = c^{-2} H',
\]
\[
II : \quad [H', P_i'] = 0, \quad [H', K_i] = P_i', \quad [J_i, H'] = 0, \quad [J_i, P_j'] = \epsilon_{ijk} P_k', \quad [J_i, K_j] = \epsilon_{ijk} K_k, \quad [J_i, J_j] = \epsilon_{ijk} J_k,
\]
respectively. The former set of generators are the generators of the ordinary Poincaré transformations:
\[
x'^\mu = L_\mu^\rho x^\rho + L_\alpha^\mu a^\alpha, \quad L \in SO(1, 3)
\]
which can be realized by a 5x5 matrix
\[
\begin{pmatrix}
L_\alpha^\mu \\
0 & 1
\end{pmatrix}.
\]
The ordinary Poincaré transformation preserves the metric of the Minkowski space-time. The latter set of generators are the generators of transformations:

\[ x'_\mu = \frac{L^\mu_\nu x^\nu}{1 + b^\mu_\nu x^\nu}, \]  

which can be expressed in terms of matrices,

\[ \begin{pmatrix} L & 0 \\ b^t & 1 \end{pmatrix}. \]  

The set of all matrices of type (20) are the transpose of the set of all matrices of type (18) in the Minkowski space-time. Therefore, the set of all matrix (20) also form a Poincaré group. It should be noted that the new Poincaré group does not preserves the metric of the Minkowski space-time. Instead, it preserves the light cone at origin in the Minkowski space-time.3,4

4. New Geometries with Poincaré Symmetry

Now that the new Poincaré group does not preserve the metric of the Minkowski space-time, what geometry does the new Poincaré group preserve?

It can be checked that \((M^\pm, g^\pm, h^\pm, \nabla^\pm)\) are invariant under the new Poincaré transformations, where \(g^\pm\) is a 4d type-(0,2) degenerate symmetric tensor field

\[ g^\pm = g^{\pm}_{\mu\nu} dx^\mu \otimes dx^\nu = \pm \frac{l^2}{(x \cdot x)^2} (\eta_{\mu\rho} \eta_{\nu\tau} - \eta_{\mu\nu} \eta_{\rho\tau}) x^\rho x^\tau dx^\mu dx^\nu, \]  

\(h^\pm\) is a 4d type-(2,0) degenerate symmetric tensor field

\[ h^\pm = h^{\pm}_{\mu\nu} \partial_\mu \otimes \partial_\nu = l^{-4} (x \cdot x) x^\mu x^\nu \partial_\mu \partial_\nu, \]  

and \(\nabla^\pm\) is a connection compatible to \(g^\pm\) and \(h^\pm\), i.e.

\[ \nabla^\pm_\lambda g^{\pm}_{\mu\nu} = \partial_\lambda g^{\pm}_{\mu\nu} - \Gamma^\pm_{\lambda\rho} g^{\pm}_{\mu\nu} - \Gamma^\pm_{\lambda\nu} g^{\pm}_{\mu\rho} = 0 \]  

and

\[ \nabla^\pm_\lambda h^{\pm}_{\mu\nu} = \partial_\lambda h^{\pm}_{\mu\nu} + \Gamma^\pm_{\lambda\rho} h^{\pm}_{\mu\nu} + \Gamma^\pm_{\lambda\nu} h^{\pm}_{\rho\mu} = 0, \]  

respectively, with connection coefficients,

\[ \Gamma^\mu_{\pm \nu\lambda} = \frac{(x_\nu \delta^\mu_{\lambda} + \delta^\nu_{\lambda} x_\mu)}{x \cdot x}. \]
In the above equations,
\[
x \cdot x = \eta_{\mu\nu} x^\mu x^\nu \begin{cases} < 0 & \text{for upper sign,} \\ > 0 & \text{for lower sign.} \end{cases}
\]
Clearly, \(|g| = |h| = 0\). The ranks of \(g\) and \(h\) are 3 and 1, respectively. It can be shown that when and only when \(\forall \xi \in p_2 \subset TM^\pm\),
\[
\begin{align*}
\mathcal{L}_\xi g^\pm_{\mu\nu} &= g^\pm_{\mu\nu,\lambda} \xi^\lambda + g^\pm_{\mu\lambda} \partial_\nu \xi^\lambda + g^\pm_{\nu\lambda} \partial_\mu \xi^\lambda = 0, \\
\mathcal{L}_\xi h^\pm_{\mu\nu} &= h^\pm_{\mu\nu,\lambda} \xi^\lambda - h^\pm_{\mu\lambda} \partial_\nu \xi^\lambda - h^\pm_{\nu\lambda} \partial_\mu \xi^\lambda = 0, \\
[\mathcal{L}_\xi, \nabla^\pm] &= 0
\end{align*}
\]
are valid simultaneously. By definition, the curvature is
\[
R^\mu_{\pm\nu\lambda\sigma} = l^{-2}(g^\pm_{\nu\lambda,\sigma} - g^\pm_{\nu\sigma,\lambda})
\]
and
\[
R^\lambda_{\pm\mu\nu} = R^\lambda_{\pm\mu\nu\lambda} = 3l^{-2} g^\pm_{\mu\nu}.
\]
In order to see the structures of the manifolds more transparently, we consider the coordinate transformations,
\[
\begin{align*}
x^0 &= l^2 \rho^{-1} \sinh(\psi/l) \\
x^1 &= l^2 \rho^{-1} \cosh(\psi/l) \sin \theta \cos \phi \\
x^2 &= l^2 \rho^{-1} \cosh(\psi/l) \sin \theta \sin \phi \\
x^3 &= l^2 \rho^{-1} \cosh(\psi/l) \cos \theta
\end{align*}
\]
for \(x \cdot x < 0\),
\[
\begin{align*}
x^0 &= l^2 \eta^{-1} \cosh(r/l) \\
x^1 &= l^2 \eta^{-1} \sinh(r/l) \sin \theta \cos \phi \\
x^2 &= l^2 \eta^{-1} \sinh(r/l) \sin \theta \sin \phi \\
x^3 &= l^2 \eta^{-1} \sinh(r/l) \cos \theta
\end{align*}
\]
for \(x \cdot x > 0\).

Under the coordinate transformations, Eqs.(21), (22), and (25) become, respectively
\[
g^\pm = \begin{cases} d\psi^2 - l^2 \cosh^2(\psi/l)d\Omega_2^2 & \text{for } x \cdot x < 0 \\ -dr^2 - l^2 \sinh^2(r/l)d\Omega_2^2 & \text{for } x \cdot x > 0, \end{cases}
\]
\[
h^\pm = \begin{cases} -\left(\frac{\partial}{\partial \rho}\right)^2 & \text{for } x \cdot x < 0 \\ \left(\frac{\partial}{\partial \rho}\right)^2 & \text{for } x \cdot x < 0, \end{cases}
\]
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\[ \Gamma_{\psi\theta} = t \sinh(\psi/l) \cosh(\psi/l), \quad \Gamma_{\psi\phi} = \overline{\Gamma}_{\psi\phi} \sin^2 \theta \]
\[ \Gamma_{\theta\psi} = \Gamma_{\phi\theta} = \Gamma_{\phi\psi} = \overline{\Gamma}_{\phi\psi} = t^{-1} \tanh(\psi/l) \quad \text{for } x \cdot x < 0, \quad (34) \]
\[ \Gamma_{\theta\phi} = -\sin \theta \cos \theta, \quad \Gamma_{\phi\theta} = \overline{\Gamma}_{\phi\theta} = \cot \theta \]
\[ \Gamma_{\alpha\beta} = -l^{-2} g_{\alpha\beta}, \quad \text{others vanish,} \]

where \((\overline{x}^\alpha;\overline{x}^3) = (\psi,\theta,\phi,\rho)\), and
\[ \Gamma_{\eta\theta} = -l^{-2} \eta_{ij} \]
\[ \Gamma_{r\theta} = -l \sinh(r/l) \cosh(r/l), \quad \Gamma_{\phi\theta} = \Gamma_{\phi\theta} = l^{-1} \tanh(r/l) \quad \text{for } x \cdot x > 0, \quad (35) \]
\[ \Gamma_{\theta\phi} = -\sin \theta \cos \theta, \quad \Gamma_{\phi\theta} = \overline{\Gamma}_{\phi\theta} = \cot \theta \]
\[ \text{others vanish,} \]

They show that the space-times are the homogeneous spaces and that
\[ M^+ = ISO(1,3)/ISO(1,2) = \mathbb{R} \times dS^3 \quad \text{for } x \cdot x < 0, \quad (37) \]
\[ M^- = ISO(1,3)/ISO(3) = \mathbb{R} \times H^3 \quad \text{for } x \cdot x > 0. \quad (38) \]

It is obvious that on \(M^+\) there is no spatial \(SO(3)\) isotropy at each point though there exists the algebraic \(SO(3)\) isotropy, but on \(M^-\) there exists the spatial \(SO(3)\) isotropy at each point.

Further studies show that \((M^-,g^-,h_-,\nabla^-)\) satisfies the three basic assumptions in the Theorem in Ref. 5. The kinematics on it will have better behaviors than on \((M^+,g^+,h_+,\nabla^+)\). Therefore, we shall study the kinematics briefly.

5. Motion of a Free Particle on \((M^-,g^-,h_-,\nabla^-)\)

The motion for a free particle is still supposed to be determined by the geodesic equation
\[ \frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\mu\nu\lambda}^\lambda \frac{dx^\nu}{d\lambda} \frac{dx^\lambda}{d\lambda} = 0, \quad (39) \]
as usual. It gives rise to the ‘uniform rectilinear’ motion
\[ x^i = a^i x^0 + lb^i \quad (40) \]
if $x^0$ and $x^i$ are regarded as the ‘temporal’ and ‘spatial’ coordinates, respectively, where $a^i$ and $b^i$ are two dimensionless constants. The result is consistent with Eq.(1) and Eq.(2), which are the start points of our work.

The ‘uniform rectilinear’ motion (40) can also be obtained from the Lagrangian

$$L = \frac{mlc}{x \cdot x} \sqrt{(\eta_{\mu\nu} \eta_{\tau\sigma} - \eta_{\mu\tau} \eta_{\nu\sigma}) x^\eta x^\tau x^\mu x^\nu}. \quad (41)$$

The Euler-Lagrangian equation is equivalent to

$$[(x \cdot x)(\dot{x} \cdot \dot{x}) - (x \cdot x)^2] \ddot{x}_\kappa + [(x \cdot x)(\dot{x} \cdot \dot{x}) - (x \cdot x) x_\kappa - (x \cdot x) \dot{x}_\kappa] \dot{x}_\kappa = 0.$$

The nonzero determinant of its coefficients for $\ddot{x}$ requires $\ddot{x}_\kappa = 0$.

It should be noted that the coordinate system $x^0/c$ and $x^i$ are not the intrinsic coordinates of the time and space, respectively. Therefore, Eq.(40) cannot be interpreted as the uniform rectilinear motion or inertial motion in the space-time $(M^-, g^-, h^-, \nabla^-)$ in the usual sense.

6. Summary

There are 24 different possible kinematical groups, including geometrical ones and static one. Each has 10 parameters. Among the 24 possible kinematical groups, there exists a new Poincaré symmetry in addition to the ordinary Poincaré symmetry.

The new Poincaré symmetry does not preserve the metric of the Minkowski space-time. Instead, it preserves the degenerate geometries $(M^\pm, g^\pm, h^\pm, \nabla^\pm)$ presented in the talk. The degenerate geometries and their topology are dramatically different from those of the Minkowski space-time. But, they are still homogeneous spaces. The physical applications of the degenerate space-times need to be explored.

From the study on the degenerate geometries with Poincaré symmetry, we can see that algebraic SO(3) isotropy does not always imply the geometrical space isotropy. Whether a kinematics possesses the space isotropy or space-time isotropy should be determined by the underlying geometry.

The Lagrangian for a free particle can be defined on the new geometry. In the coordinate system $x^\mu$, the motion takes the form of the uniform rectilinear motion. Unfortunately, the coordinate system $x^i$ and $x^0/c$ do not the intrinsic coordinates of the space and time. Therefore, ‘the uniform rectilinear motion’ is not in the usual sense.
Acknowledgments

I am grateful to H.-Y. Guo, Y. Tian, H.-t. Wu, X.-N. Wu, Z. Xu, and B. Zhou for the cooperation in the works related to the talk. I would like to thank Z.-N. Hu, W. T. Ni, J. Xu, and H.-X. Yang for helpful discussion. The work is supported in part by NSFC under Grant Nos. 10775140, 10975141, and KIFCAS (KJCX3-SYW-S03).

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