Method of Wavelet Analysis of Time Series to Assess the Effectiveness of the Country's Energy Security

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Abstract. This article addresses the problems of energy security at the level of Russian regions, which is particularly urgent in modern economic conditions characterized by global digitalization processes. In this case, the problem arises the need to develop mathematical tools for planning and assessing energy security in conditions of high uncertainty and fluctuations of the studied parameters of the external environment.

The paper presents the mathematical justification for the use of wavelet analysis and the construction of spaces of scaling functions in order to obtain the theorem of multifunctional analysis of economic parameters and indicators of regional energy security, based on many intervals and determining the corresponding wavelet spaces.

1. Introduction
Using the wavelet approach, we can successfully determine the trend and eliminate the “interference” from the considered time series. The main idea of wavelet analysis is to present “an indicator in the form of the amplitude of harmonic oscillations of the corresponding frequency” (for example, a time series) in the form of series with respect to a specially constructed basis created from localized functions (wavelets) by means of scale changes and shifts along the time axis.

This development is aimed at applying wavelet analysis to the study of information-time series containing data gaps. The theory proposed for consideration is based on the works of Grossman [8, 9] and Morlaix [11, 12] using the foundations considered in the works of Anderson, Hall, Javert and Peters [1]). Their main idea, proposed for constructing orthonormal wavelet bases for a single interval [1], is to divide the initial information basis into three parts (analysis of the behavior of the study of the “indicator” near control points), the interval and in its middle).

This will examine the study of the basic concepts of wavelet transforms and provide an analysis of indicators and multi-resolution problems on the interval with rational control points. This approach is close to that presented in the works of L. Anderson, Hall N., Javers V., J. Peters, as well as in the works of J. Cheng, V.S. Swan [1], [3], [5]. The technique proposed by the authors is characterized by a more accurate definition of sets of indices and indicators corresponding to wavelets. At the first stage, it begins by constructing the spaces of scaling functions $V_j[a, b]$, which form the multiresolution analysis on the corresponding interval. Then, the analyzed indicators are generalized into a matrix for the situation with the analyzing function given at two disjoint intervals and non-interrelated indicators in order to prepare a study of financial and economic time series with data gaps [2], [4]. The full algorithm for constructing the sample, as well as the use of this design for the analysis of time series with gaps in the data, is quite voluminous and will be presented in the next article.
2. Method
To define the multisolution analysis [5], [12], one should consider the sequence \{P_i\} (i \in \mathbb{Z}) of closed subspaces \(P_i \subseteq L^2(R)\) with the following properties:

1) \(P_{-1} \subseteq P_0 \subseteq P_1 \subseteq P_2 \ldots;\)
2) \(\bigcup_{i \in \mathbb{Z}} P_i = L^2(R)\)
3) \(\bigcup_{i \in \mathbb{Z}} P_i = 0\)

For all \(F(x) \in L^2(R)\), we use the assumption that the following conditions must be satisfied:

4) \(F(x) \in P_i \Leftrightarrow F(2^j x) \in P_{i+1}\);
5) shift invariance: \(F(x) \in P_0 \Leftrightarrow F(x - m) \in P_0\) for all \(m \in M\); 
6) we assume that there exists a function \(\hat{f}(x) \in L^2(R)\) (called a scaling function), that the set of criteria \(\{f(x - k)\} k \in Z\) forms an orthonormal basis for \(P_0\).

Based on the assumption that the function \(\hat{f}\) has the following form:

\(\hat{f}(x) = \sum_{k=0}^{2^M - 1} k \sqrt{2} \hat{f}(2x - k)\),

Moreover, based on the assumption that for any economic indicators in an equal time interval this equation

\(\Theta_{i=-\infty}^{i=1} W_i \theta P_0\)

it is advisable to replace by

\(P_0 \oplus \Theta_{i=0}^{\infty} W_i = L^2(R)\)

3. Results
There are some alternative methods for solving the considered function of approximating the multiresolution of indicators \(L^2([a, b])\), solved in various works of foreign researchers [3], [10]. In this study, we rely on the development of mathematical solutions for multibase constructions presented in the works of W. Meyer and Ya Daubechies [8], [6].

To do this, it is necessary to determine the functions \(\hat{f}_{ik}\) whose support has a nonzero intersection with the interval \([a, b]\). Thus, the following set of indices is presented

\(I_j = \{k: \text{supp} \hat{f}_{ik} \cap (a, b) \neq 0\} = \{k: 2^j a - (2^M - 1) < k < 2^j b\}\).

To ensure the process of orthonormalization, the dependences considered in the work of Meyer [8] will be used, i.e. the square root inverse to the masses of the matrices \(\delta_{i=\infty}^{i=-1} W_i\theta P_0\) and \(\delta_{i=1}^{i=0} W_i\theta P_0\).

Summarizing the conclusions made earlier, we can formulate a theorem.

Theorem: the space \(PP_i, [a, b], i \geq 1\) is the linear span of the functions \(\delta_{i, k, 2^j a M - 1}^{i, k, 2^j a M - 1} \leq k \leq 2^j b - M - a_R / 2\), where the functions \(\delta_{i,k,l}^{\alpha}, 0 \leq \alpha \leq M - 1 - t_l (3)\) and \(\delta_{i,k,R}^{\alpha}, 0 \leq \alpha \leq M - 1 - t_R\).

Considering the missing functions in \(PP_i, [a, b]\) at the left and right control point separately, we can construct the block matrix \(Z\) from the wavelet coefficients [1], [8] as \(\delta_{i} = Z F_{i+1}\).

Therefore, each of the spaces \(P_i, [a, b]\) has an orthonormal basis defined by the extension and translation of some functions \(\hat{f}_{i,k}\):

\(\hat{f}_{i,k}(x) = 2^{i/2} \hat{f}_{i}^i (2^i x - k), i, k \in Z\)

4. Discussion
The considered mathematical model is applicable for all regions of the country, and also allows for the planning, forecasting and analysis of changes in the energy security of a particular territory in the context of globalization and digitalization of the economy.

In the proposed methodology, quantitative information is displayed in the form of a system of indicators of various aspects of the development and functioning of energy production, energy supply, energy consumption, that is, a system of energy security indicators. In addition, it is displayed by comparing the actual values of the indicators in a certain time interval. At the same time, indicators, coefficients and indicators are considered as multidimensional planes, the configuration and structure of which are significantly influenced by time, as well as simultaneous conditions and threats,
representing, in aggregate, the initial information base for substantiating and making decisions on ensuring economic security in the region in the form of a black box.

The problem of optimizing the balances of fuel and energy resources in the Russian regions under the conditions of high volatility of factors affecting the economically efficient functioning of the fuel and energy complex, which is solved with the help of this model, is in the mathematical sense the classical task of modular-functional programming, in the substantial (production and economic) sense - territorial-production model of the fuel and energy complex with blocks of electric power, heat, gas and coal supply, as well as oil refining.

A change in the state of system objects leads to the solution of the problem of the distribution of flows in the system in order to maximize the supply of energy to consumers, i.e. models are formalized as maximum flow problems [23,24].

5. Conclusion
An analysis of nascent or dying trends that determine the degree of energy security, as well as an assessment of the main factors and indicators of the state of the energy industry, both generalized and particular, is carried out using a system of quantitative indicators of energy security, i.e. systems of indicators characterizing the level, structure and probability of occurrence of threats to energy security.

In this article, we considered the construction of the corresponding multiresolution of wavelet spaces corresponding to a finite time interval. The corresponding technique is generalized to the case of two intervals with rational control points. The main difficulties are stated and considered in the study. The application of this technique to study the financial and economic time series is described.

6. References
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