RADIATIVE CORRECTIONS TO SUPERSYMMETRIC
HIGGS BOSON MASSES

Peter L. White

Physics Department, University of Southampton,
Southampton S09 5NH, UK.

ABSTRACT

We discuss the large radiative corrections to the masses of supersymmetric Higgs bosons in the MSSM and in its simplest extension, the NMSSM, with particular attention paid to the bounds on the lightest CP-even Higgs mass found in both models. In the case of the MSSM, these corrections are found to be primarily associated with the effects top quark and stop squark loops, while for extended models they also include significant contributions from Higgs and Higgsino loops.

1. Introduction - SUSY Higgs Bosons at Tree Level

1.1. MSSM

The Minimal Supersymmetric Standard Model (MSSM) is the simplest possible supersymmetric theory which symmetry breaking can reduce to the Standard Model. It contains not only the usual fermionic sector with SU(3)×SU(2)×U(1) gauge bosons plus their superpartners, but also two supermultiplets of Higgs doublets. This is the minimum Higgs sector which allows for the correct electro-weak breaking structure and mass couplings, because supersymmetry both forces the inclusion of higgsinos, which cause anomalies which cannot be cancelled with only one doublet, and restricts the possible Yukawa couplings between Higgses and fermions.

We shall use the convention that the two doublets are labelled $H_1$ and $H_2$, with $H_2$ coupling to the up type quarks. The superpotential is then given by

$$W = h_u Q H_2 u + h_d Q H_1 d + h_e L H_1 e + \mu H_1 H_2$$

(1)

where we have only shown one generation. When supersymmetry is broken, the resulting scalar potential acquires soft breaking terms of the form

$$V_{soft} = \sum_i m_i^2 |\phi_i^2| + h_u A_u Q H_2 u + h_d A_d Q H_1 d + h_e A_e L H_1 e + \mu B H_1 H_2$$

(2)

where the sum is over all spin 0 particles, and we use the usual convention of using the same label for the scalars as for the full supermultiplet.

*talk presented at SUSY-93, Boston, March 1993
With two doublets we shall have more physical Higgs particles – two CP-even (labelled \(h\) for the lighter and \(H\) for the heavier), one CP-odd (or axial, labelled \(A\)), and one charged (labelled \(H^+\)) – in addition to the usual Goldstone modes.

It is simple to derive the following Higgs potential from the lagrangian:

\[
V_{Higgs} = \frac{\lambda_1}{2} (H_1^+ H_1^-)^2 + \frac{\lambda_2}{2} (H_2^+ H_2^-)^2 + (\lambda_3 + \lambda_4)(H_1^+ H_1^-)(H_2^+ H_2^-) - \lambda_4 |H_1^+ H_1^-|^2 \\
+ m_1^2 (H_1^+ H_1^-) + m_2^2 (H_2^+ H_2^-) - m_3^2 (H_1^+ i \sigma_2 H_2^- + h.c.)
\]

where we may use the requirements of supersymmetry to obtain

\[
\lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{4} \\
\lambda_3 = \frac{g_1^2 - g_2^2}{4} \\
\lambda_4 = \frac{g_2^2}{2}
\]

and the \(m_i\) are functions of the unknown soft parameters of the theory. Thus we may calculate all of the Higgs boson masses in terms of the (known) gauge couplings and three unknown masses. Defining \(<H_1^+> = \nu_1\), \(<H_2^+> = \nu_2\), \(\tan \beta = \nu_2 / \nu_1\), and \(\nu^2 = \nu_1^2 + \nu_2^2\), we may use the relation \(m_c^2 = \frac{1}{2}(g_1^2 + g_2^2)\nu^2\) to remove one of the unknown masses. It is then conventional to reparametrise in terms of \(\tan \beta\) and \(m_A\), the mass of the CP-odd particle. Once we have done this, we find

\[
m_C^2 &= m_A^2 + m_W^2 \\
m_{h,H}^2 &= \frac{1}{2}(m_A^2 + m_Z^2 \pm \sqrt{m_A^4 + m_Z^4 - 2m_A^2 m_Z^2 \cos 4\beta})
\]

where \(m_C\) is the mass of the \(H^+\).

From this analysis, we should note primarily that the \(H^+\) mass is larger than the \(W\) mass; while the \(h\) mass is less than both \(m_A\) and \(m_Z\). This forms an extremely firm prediction about the Higgs sector. We should note, however, that as yet we have not discussed radiative corrections, and are working purely with a low energy effective theory including only Higgs bosons.

### 1.2. NMSSM

The Next-to-Minimal Supersymmetric Model[2] (NMSSM) was discussed in detail in the talk by Terry Elliott at this conference[3]. This model has a Higgs structure identical to that of the MSSM, but with an extra singlet, and an extra term in the superpotential \(\lambda NH_1 H_2 + \frac{k}{4} N^3\) to replace the \(\mu\) parameter of the MSSM.

As was also described earlier at this conference[3, 4], the consideration of the perturbative bound on the coupling \(\lambda\) gives a bound on the mass of the lightest Higgs boson which is rather larger than that found for the MSSM[5].

### 2. Radiative Corrections

So far we have calculated masses using a low energy effective theory from which all heavy particles (including all superpartners) have been removed. The first problem with this analysis is that we have used relations defining the couplings \(\lambda_i\) which are only strictly valid at some high energy scale which we call \(m_{\text{susy}}\), above which supersymmetry is a good approximation. Below this scale, the couplings will
run from their starting values. This suggests a renormalisation group analysis to account for the difference between the values of the parameters at the supersymmetry and Higgs (weak) scales.

Furthermore, in decoupling the superpartners at high energy, we have neglected any threshold effects, which may come both from the fact that the superpartners are not all degenerate with mass $m_{\text{susy}}$ but have some spectrum, and from the finite one-loop diagrams involving these particles which may give non-zero contributions to the Higgs masses. These effects can be best evaluated by either simply calculating the diagrams at some high energy, or by carrying out a one-loop Coleman-Weinberg effective potential calculation.

Lastly, we should also be careful that loops of light Higgs particles do not themselves contribute substantial radiative corrections to the mass matrices.

We shall consider only the top and stop (and, for the charged mass, the sbottom) as coupling strongly to the Higgs sector, as all of the other particles couple through Yukawa couplings which are relatively very small (unless $\tan \beta >> 1$ in which case the bottom quark Yukawa is also large; we shall not consider this case). Further corrections may also be given by loops of Higgses and Higgsinos. We shall neglect the consideration of wave-function renormalisation and of gauge boson loops as relatively small. The former is negligible so long as the masses which we calculate are small relative to those of the particles in the contributing loops.

3. Results - MSSM

The situation in the MSSM is relatively simple, because as we have seen the couplings involving Higgses and Higgsinos are small (of order $g_2^2$ at worst), and so the most important effects are from top and stop loops. If we begin by considering only top effects, we recall that the top mass is given by $m_t = h_t \nu \sin \beta$, where $h_t$ is the top quark Yukawa coupling. The simplest way of finding the top quark contribution is to use the renormalisation group (RG) equations below the supersymmetry scale [6]. These are rather long, but the largest correction [7] is given by

$$16\pi^2 \frac{\partial \lambda_2}{\partial t} = -12 h_t^4 + \cdots$$

where $t$ is the log of the renormalisation point. It is then trivial to solve this reduced form to give the result

$$m_h^2 \leq m_Z^2 + \frac{3}{4\pi^2} h_t^4 \nu_2^2 \log \left( \frac{m_{\text{susy}}^2}{m_t^2} \right)$$

which has also been obtained using effective potential techniques [8]. In fact, solving the full RG equations shows that this estimate is rather conservative so that the bound should be rather lower, because the effects proportional to $h_t^4 g_2^2$ are of the opposite sign. In any case the most important correction here being proportional to $h_t^4$ (and thus to $m_t^4$), it becomes large very rapidly with increasing top mass.

Typical sizes of the increase in the bound found numerically are 2, 11, 22,
35 GeV for a top mass of 120, 140, 160, 180 GeV and \( m_{\text{susy}} = 1 \text{ TeV} \), and for the simplified analytical approximation given above are around 5 to 8 GeV more.

Next we consider the effects of squarks. These are rather messy, and to perform this calculation we may either do an effective potential calculation or evaluate all the appropriate graphs. The results have been given in full by Ellis et al.\[8\], and we shall not present them here, but merely mention the interesting features. Firstly, the terms which are not logarithms involving the renormalisation scale are all proportional to \( A_t + \mu \cot \beta \) to some power. This is the off-diagonal term in the stop mass matrix (to within factors) and so these corrections are zero unless there is mixing between left and right-handed stops. Next, the corrections are all proportional to \( h_t^4 \). Finally, as we would expect by supersymmetry, the logarithmic terms from the divergent diagrams exactly cancel the top corrections if the top and both stops are degenerate. After some algebra, one can derive the following analytic formula for the bound by simply maximising the formulae with respect to all the parameters:

\[
m_h^2 \leq m_Z^2 + \frac{3}{4\pi^2} h_t^4 \nu_2^2 \log \left( \frac{m_{\tilde{t}_2}^2}{m_t^2} \right) + \frac{9}{4\pi^2} h_t^4 \nu_2^2
\]

where we use \( m_{\tilde{t}_2} \) to indicate the heavier of the stop mass eigenstates. Note that the \( m_{\text{susy}} \) dependence has been cancelled; for purposes of calculating top and stop loops, \( m_{\text{susy}} \) is effectively the stop mass.

The finite terms terms are relatively insensitive to the stop masses except indirectly through the constraint that colour must not be broken, and so the stop mass eigenvalues must be positive, which gives \( 2h_t \nu_2 (A_t + \mu \cot \beta) \leq (m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2) \); however they are sensitive to \( A_t \) and \( A_t + \mu \cot \beta \), and so for most arbitrarily chosen values of the parameters the lighter Higgs mass will not approach this bound, and in fact for very large \( A_t \) the corrections to the bound may even be negative.

In summary, Eq. (8) is a pessimistic view of the final bound after radiative corrections; one would not expect it to be realised except in the rather unlikely case that all of the parameters are such as to make the mass as large as possible, but if this bound is covered by the searches at LEP then we can safely rule out the MSSM.

In addition to these corrections to the bound, large corrections also apply to the mass spectrum, particularly in the region where \( \tan \beta \) is small (the bound is saturated as \( \tan \beta \to \infty \)); we shall not discuss these here in detail, except to note that they are particularly affected by contributions from gauge loops. This is because the top quark decouples at a scale \( \relax m_t \) (and similarly for stop squarks), while the Higgs sector continues to contribute to the RG equations down to lower energies.

4. Results - NMSSM

Unfortunately, this rather simple analysis is drastically complicated in the case of the NMSSM. Apart from the obvious point that \( 3 \times 3 \) matrices are not susceptible to simple diagonalisation, the spectrum is much more complicated, and so our RG analysis approach, which implicitly assumed that all Higgs bosons had at
least approximately similar masses, is invalidated. Apart from this, there are now large (order 1) couplings in the Higgs and Higgsino sectors. This will mean that we must carry out an analysis of the effects of loops of these particles.

The effects considered in the MSSM are still of course present [9], and indeed the formulae for the radiative corrections due to top loops is still valid. The stop effects for this model have been derived elsewhere [10], and their effects on the bound have been studied [11], with the result that for the region of high top mass their effects on the bound may be substantial, of order 20GeV; however, this region is not the one where the bound is maximised, and at low top mass, where the bound reaches its highest values, these corrections are negligible. The main effect is thus to lift up the lowest part of the $m_h - m_t$ curve, diluting the strong $m_t$ dependence of the bound.

Now we must also consider effects of Higgs and Higgsino loops. In the event that we make the assumption that we have a region of parameter space in which all of the Higgs states are light (that is there are none which differ from our renormalisation scale by more than, say, a factor of 2 or so), then we may use our RG approach to find the radiative corrections, and thus include all divergent diagrams. This leads to the predicted result that the corrections proportional to $\lambda$ are in fact quite large [12]: the effect on the bound is to raise it by approximately 7GeV (but by rather less for very large and small top mass), and this result is that previously presented by Terry Elliott [3].

In the event that the Higgs spectrum includes heavy mass eigenstates, or if we wish to include Higgsino effects or effects from finite Higgs loops, the situation is much more complicated. The only technique which we can use to deal with this case is that of Coleman-Weinberg analysis, writing out all the tree level mass matrices and then using them to calculate the effective potential. Unfortunately, while for scalars this is quite simple, since the mass Higgs and Higgsino mass matrices reduce to the usual $3 \times 3$ and $2 \times 2$ blocks, there is no such simple result when we consider the charged and CP-odd particles, and we must consider the full $10 \times 10$ mass matrix. This is impossible to do analytically, and so we must carry out the differentiations numerically. This work is still in progress, but it seems that the radiative corrections are typically of order a few GeV and increase the calculated scalar masses for most values of the parameters, reaching larger values if $r >> 1$, where they may also give corrections of opposite sign.

In conclusion, it is relatively simple to find the radiative corrections to the bound from the effects of top and stop loops, the latter being more difficult to do with precision; but a full analysis of the Higgs and Higgsino corrections to the bound awaits the calculation of the analytic effective potential corrections. Furthermore, this depends on arbitrary parameters (such as masses of Higgs bosons which are not directly involved in the bound) to such an extent that it is difficult to find results without making assumptions about which region of parameter space is most interesting. However, for a given set of parameters it is simple to find the full
spectrum including radiative corrections.

5. Conclusions

In conclusion, the effects of radiative corrections on the Higgs mass spectrum are extremely large in both models considered here. In the MSSM they are dominated by the well-known contributions of top and stop loops, while for the NMSSM they also include comparably large and (usually) positive corrections from loops of Higgses and Higgsinos; however in the latter model it is much harder to make concrete predictions with regard to the bound because in general these effects are quite strongly dependent on the spectrum and thus on which region of the very large parameter space is used for the calculations.

One question which we might ask is how dependent the results are on the specific model considered. Clearly, going from the minimal to next-to-minimal models has drastically increased the complexity and size of the radiative corrections, and it seems likely that introducing further singlets, triplets, or other structure would make things even worse. Given the size of the corrections (and difficulty in doing calculations because of the size of the parameter space) in the NMSSM, we should be rather wary of bounds in such models.

6. Acknowledgements

This work was done in collaboration with Terry Elliott and Steve King at the University of Southampton.

I would like to thank Steve Kelley for a very helpful suggestion regarding numerical effective potential techniques.

7. References

1. H. P. Nilles, Phys. Rep. 110 (1984) 1;
   H. E. Haber and G. L. Kane, Phys. Rep. 117 (1985) 75;
   J. F. Gunion, H. E. Haber, G. L. Kane, S. Dawson, “The Higg’s Hunters Guide” (Addison-Wesley, Reading MA, 1990).
2. J. Ellis, J.F. Gunion, H. E. Haber, L. Roszkowski, F. Zwirner, Phys. Rev. D39 (1989) 844.
3. T. Elliott, talk at this conference.
4. G. Kane, talk at this conference.
5. J. R. Espinosa and M. Quiros, Phys. Lett. B279 (1992) 92;
   G. L. Kane, C. Kolda, and G. D. Wells, “Calculable Upper Limit on the Mass of the Lightest Higgs Boson in Perturbatively Valid Supersymmetric Theories with Arbitrary Higgs Sectors”, Michigan preprint UM-TH-93-24, Phys. Rev. Lett., to be published.
6. M. Carena, K. Sasaki, and C. E. M. Wagner, Nucl. Phys. B381 (1992) 66.
7. H. Haber and R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815;
Y. Okada, M. Yamaguchi, and T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1, *Phys. Lett.* **B262** (1991) 54;
R. Barbieri, M. Frigeni, and F. Caravagllos, *Phys. Lett.* **B258** (1991) 167;
J. L. Lopez and D. V. Nanopoulos, *Phys. Lett.* **B266** (1991) 397;
A. Brignole, *Phys. Lett.* **B277** (1992) 313, *Phys. Lett.* **B281** (1992) 284.

8. J. Ellis, G. Ridolfi, and F. Zwirner, *Phys. Lett.* **B257** (1991) 83, *Phys. Lett.* **B262** (1991) 477;
A. Brignole, J. Ellis, G. Ridolfi, and F. Zwirner, *Phys. Lett.* **B271** (1991) 123.

9. P. Binetruy and C. Savoy, *Phys. Lett.* **B277** (1992) 453;
U. Ellwanger and M. Rausch de Trauenberg, *Z. Phys.* **C53** (1992) 521;
U. Ellwanger and M. Lindner, “Constraints on New Physics from the Higgs and Top Masses”, Heidelberg preprint HD-THEP-92-48;
W. ter Veldhuis, “Mass of the Lightest Higgs Boson in the MSSM with an additional Singlet”, Purdue preprint PURD-TH-92-11;
J. R. Espinosa and M. Quiros, “Upper bound on the Lightest Higgs Boson Mass in General Supersymmetric Standard Models”, Madrid preprint IEM-FT-64/92.

10. U. Ellwanger, “Radiative Corrections to the Neutral Higgs Spectrum in Supersymmetry with a Singlet”, Heidelberg Preprint HD-THEP-93-4, *Phys. Lett.* to be published.

11. T. Elliott, S. F. King, and P. L. White, “Squark Contributions to Higgs Boson Masses in the NMSSM”, Southampton Preprint SHEP 92/93-18, in preparation.

12. T. Elliott, S. F. King, and P. L. White, “Supersymmetric Higgs Bosons at the Limit”, Southampton Preprint SHEP 92/93-11, *Phys. Lett.* to be published.