EqRank: Theme Evolution in Citation Graphs

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Time evolution of the classification scheme generated by the EqRank algorithm is studied with hep-th citation graph as an example. Intuitive expectations about evolution of an adequate classification scheme for a growing set of objects are formulated. Evolution compliant with these expectations is called natural. It is demonstrated that EqRank yields a naturally evolving classification scheme. We conclude that EqRank can be used as a means to detect new scientific themes, and to track their development.

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I. INTRODUCTION

Detection of "communities" has lately became a popular subject in the studies of complex linked networks [1]. For a citation graph, communities are identified as scientific themes. Most of the papers devoted to this subject deal with separate snapshots of graphs disregarding their time evolution [2].

If we consider the time evolution of a graph evolving nonrandomly, we expect to see regularities in the evolution. Examples are citation graphs or the Web graph (for the latter, vertexes are web pages, and edges are the hyper-links). For citation graphs of scientific papers in a subject field (vertexes of the graph are the papers, edges, the citations), communities are themes forming a classification scheme of the subject field [3, 4]. As time goes, the classification scheme may undergo changes, but the changes should be of a regular character. There should exist a relatively large stable set of core themes that does not change. A few themes may disappear (their contents absorbed by other themes), and new themes may appear.

If we use a classification algorithm, and claim that it yields adequate themes, we should check if its outcome generates a dynamics of graph partitions with the above properties. In the paper, we present a modification of the EqRank algorithm [8] that passes such a check.

The paper is organized as follows: In Section 2, we briefly review previous work on the problem. In Section 3, we formalize the requirements on the dynamics of partitions outlined above. In Section 4, we outline the EqRank algorithm. Section 5 presents results of applications of the algorithm to the hep-th citation graph.

II. RELATED WORK

Despite the forty year history of citation analysis [5], the problems listed above have only recently become a subject of constant interest. In [6], time evolution of themes in computer science has been analysed. The analysis was based on application of a dedicated graph clustering algorithm applied to the data of Citeseer library. In the analysis, a fixed list of themes was used, which is not a consistent approach to the problem. Approach of [7] is closer in spirit to our paper. It was observed in this paper that hierarchical agglomerative clustering yields an unstable partition, and that there exists a small subset of clusters that remains stable against random perturbations of the graph. Namely these stable clusters were called natural communities, and were identified as scientific themes. In our approach, we also consider stability of the partition as a major requirement. Comparing our approach to the one of [7], we point out two distinctions. First, we concentrate not on the stability of particular clusters, but on the stability of the whole partition. As pointed out in [7], there exist many algorithms yielding partitions with a stable core. On the other hand, there must be much less algorithms yielding partitions that are stable as a whole. We describe below a modification of the EqRank algorithm possessing namely this property. Secondly, we require not only the stability of partitions against small random perturbations, but also against large regular perturbations, among which we consider in particular the time evolution of graphs appearing in actual applications. For this latter case, we transform the stability requirement to requirements of natural dynamics. These requirements are naturally expected to be satisfied by the dynamics of evolving partitions.

III. NATURAL THEME DYNAMICS

We consider a graph $G(t)$ growing in time. Take two subsequent moments of time, $t_2 > t_1$. As the graph is
The sets $NT$ partitions satisfying a number of requirements.

Practically, an adequate clustering algorithm applied to an evolving graph should yield a number of partitions of a graph. We expect that $T_2$ is $T_1$ with new clusters (themes) added, and some themes removed:

$$T_2 = T_1 \cup NT\setminus AT_1,$$

where $NT$ is the set of new themes, and $AT_1$ is the set of absorbed themes. To give a meaning to this equality, we have to set a mapping between the elements of $T_2$ and $T_1$:

$$Map_1(T_i) = T'_j,$$

where $T'_j$ is the cluster of the second partition that has a maximal intersection with $T_i$. Analogously, we define $Map_2$ relating a cluster of the second partition to a cluster of the first (notice that generally $Map_1 \neq Map_2$). We can now define the subset of stable themes as a subset of clusters of the first partition satisfying the following requirement:

$$ST_1 = \{T_i : Map_2 \circ Map_1(T_i) = T_i\}.$$  

On the subset of stable themes, we can consider time development of a theme: if $T \in ST$, then $Map_1(T)$ is $T$ at a subsequent moment of time.

Equation (4) can be rewritten as follows:

$$T_2 = Map_1(ST_1) \cup NT,$$

$$T_1 = ST_1 \cup AT_1.$$  

The sets $NT$ and $AT_1$ are respectively complements to the sets $Map_1(ST_1)$ and $ST_1$. For $T' \in NT$, we say that $T'$ has broke away from a cluster $T \in T_1$ if $Map_2(T') = T$. For $T \in AT$, we say that $T$ has been absorbed by $T' \in T_2$ if $Map_1(T) = T'$.

The above characterizations can be applied to any pair of partitions of a graph. Practically, an adequate clustering algorithm applied to an evolving graph should yield partitions satisfying a number of requirements.

The first requirement is that the initial partition $T_1$ would mostly consist of the stable subset $ST$, and the fraction of papers belonging to the stable subset of the partition would be high. We write it as the classification scheme stability requirements:

$$|ST_1|/|T_1| \approx 1,$$

$$| \cup T_j : T_j \in ST_1 | / | \cup T_j \in T_1 | \approx 1.$$  

Notice that we do not require that $|ST_1|/|T_2| \approx 1$, because we want to keep in the consideration the cases than the number of new themes is comparable to the number of stable themes. However, we require that the number of old papers belonging to new themes would not be large (see (6)), i.e., new themes should mostly consist of new papers.

Classification scheme stability requirements (4) restrict evolution of the partition on the level of themes. We also need to restrict the way specific papers move around the partition. We call this set of requirements indexing consistency requirements. They read as follows:

- If a paper belongs to a stable theme in the partition $T_1$, it either remains in the same theme in the partition $T_2$, or moves to a new theme that breaks away from the theme in the first partition.
- If a paper belongs to a new theme in the partition $T_2$, it belongs to the stable theme of $T_1$ from which the new theme has broken away.
- If a paper belongs to a theme of the partition $T_1$ that gets absorbed by a stable theme, it belongs to the stable theme of the partition $T_2$.

The indexing consistency requirements imply the following scenario for forming the new partition from the old one:

- Select the subset $ST_1$ of stable clusters.
- For each cluster $S \in ST_1$, chip off new clusters. The remaining part of $S$ becomes the stable cluster $S'$ of the partition $T_2$.
- Each of the rest of the clusters of $T_1$ is absorbed by a particular stable cluster of $T_2$.

Evidently, this is a very specific scenario. Let us reiterate it less formally: Evolution consists in emanation of new themes from stable themes (several new themes may chip off from a stable theme), and in absorption of unstable themes by stable themes. A separate feature that should be stressed is that a new theme has a unique parent stable theme (in a more general scenario, a new theme could be formed from pieces originating from different stable themes), and that an unstable theme is absorbed by a single stable theme (in a more general scenario, it could be redistributed among several stable themes). Summarizing all the restrictions, we require that evolution of the partition would keep identity of themes (reshuffling of splinters of clusters into new clusters is forbidden).

We call a dynamics of partition the natural dynamics if it satisfies the above classification scheme stability and index consistency requirements.

In practice the above requirements can be satisfied only approximately. We introduce three coefficients to quantify the deviation of the real dynamics from a natural dynamics:

- $CSC_1$ (the first classification stability coefficient) is the percentage of the stable themes with respect to the total number of themes.
The EqRank algorithm was presented in [8]. Here we give an informal description using a metaphor of forming coalitions in a social network, and present a modification of the algorithm yielding partitions with improved stability properties. In particular, the modified algorithm leads to satisfactory numerical values of the above coefficients, characterizing the closeness of the evolution to the natural one.

Consider a social network with vertexes representing persons, and links, the trust or sympathy to the persons to which they are pointing at. The links are weighted, the weight is measuring the extent of trust. Let there be a reason forcing people to form coalitions, and let the decision to join a coalition be taken on the basis of trust to the nearest neighbors.

We define the rule of joining the coalition recursively: a person joins the coalition joined by the person enjoying the maximal trust of the first person. There are many partitions satisfying this rule. In [8] we demonstrated that EqRank yields the maximally detailed partition satisfying this rule. This means that any other partition satisfying the above rule is a coarse-graining of the partition yielded by EqRank.

EqRank follows the practice of forming coalitions: a person joins the person whom she trusts the most, and brings to the coalition the persons trusting her the most. The latter also bring the persons trusting them the most, and so on. Programming this process is simple: The algorithm starts from discarding nonmaximal links. In this way the maximal graph is formed containing only the links expressing the relations of maximal trust. (Note that the maximal graph is directed: If a person trusts the most a person, the latter may trust the most a different person.) Because the maximal link is unique (this is a simplifying assumption), each vertex of the maximal graph defines unambiguously a chain that starts form the vertex, and formed from the vertexes joined by the maximal links. Because the graph is finite, each chain ends on a cycle, which is a nontrivial strongly connected component of the maximal graph. By the definition, all the vertexes of the chain belong to the same cluster (coalition). And the chains that end up on the same cycle should also belong to the same coalition.

The EqRank pseudocode is reduced to the following operations:

- Selection of the maximal subgraph.
- Computation of the strongly connected components of the maximal subgraph, and contraction of the strongly connected components to the vertexes of a factorgraph.
- Selection in the factorgraph of the vertexes without outgoing links (final vertexes). The cycles of the initial graph correspond to the final vertexes.
- Forming a coalition from all the vertexes of the factorgraph from which it is possible to reach one and the same final vertex.

Let us consider again our metaphor of forming coalitions in a social network. We can compute total trust of a person to arbitrary coalition as the sum of the link weights running over all the links joining the person with the members of the coalition. We say that the coalition a person belongs to meets her expectations if the total trust of the person to her own coalition is larger than her total trust to any other coalition. We call a coalition proper if it meets expectations of each of its members. Condition defining proper coalition is much weaker than the one defining community in the strong sense [1] (the latter require that the total trust of a person to her coalition would be greater than the sum of total trusts to all other coalitions). The condition defining the proper coalition allows us to start a process of coalition restructuring. Namely, we let each person to go over to the coalition she trusts the most. After all the persons performe the transition, each of them compares again the trust to the new coalitions. It may happen that the coalition a person finds herself in after the first transition again does not meet her expectations, because it may happen that the most trusted members of the old coalition have left to new coalitions. In this case, the restructuring of the coalitions repeats itself. Assuming that expectations of most of the persons are met with the initial coalitions, we expect that the restructured coalitions meet the expectations of all the persons after finite number of restructuring iterations. We note that the number of improved coalitions is generally reduced with respect to the number of initial coalitions, because initially there may exist coalitions for which all the members leave to another coalitions.

We now give a formal definition of the above iterations. Let \( T = \{T_1, ..., T_n\} \) be a partition of the weighted graph \( G = (V, E, w) \). Partition \( T \) is defined by the function \( f(x) = i, \) where \( x \in T_i \). Let us define the function \( D(V, T): D(x, T_i) = \sum_{y \in T_i} w(x, y) \). This function quantifies the closeness between the vertex \( x \) and the cluster \( T_i \). Let us define the sequence of partitions \( T_n \) as follows:

\[
t_{n+1}(x) = j,
\]

where \( T_j \) belongs to \( T_n \) and gives maximum to the function \( D(x, T_n) \). The process starts from \( t_0 \equiv t \). In words, the partition \( T_{n+1} \) indexes \( x \) with the index \( j \) of the cluster \( T_j \) of the partition \( T_n \) that is the closest one to
We say that the sequence converges at the vertex $x$ if $t_{n+1}(x) = t_n(x)$ starting from some $n$, which may depend on $x$.

Let $V_{\text{max}}(T)$ be the maximal subset of the graph vertexes on which the sequence $T_n$ is converging. Practically, a good clustering algorithm should yield a partition $T$ for which $V_{\text{max}}(T)$ is large enough, and the convergence is fast. We denote the limiting partition of $V_{\text{max}}(T)$ as $\lim(T)$, and call the above restructuring process the reindexing process.

We suggest the following modification of the EqRank algorithm:

- The initial partition is yielded by the standard EqRank algorithm (see the above pseudocode).

- Starting from the initial partition, the limiting partition is constructed by the above reindexing process. The limiting partition is taken as the result yielded by the modified EqRank.

Fig. 1 shows convergence of reindexing for the initial partition yielded by the standard EqRank applied to the hep-th citation graph. As seen, after 10 iterations, the reindexing process converges for 99% of the graph vertexes.

We conclude this section with the following observations:

- Selection of the maximal subgraph is an important stage of the algorithm. It may seem that this selection discards substantial information on the graph encoded in the nonmaximal links. This is not the case if a graph-based proximity measure is used to determine the link weights. In our experiments, we used a linear combination of the co-citation and bibliographic coupling. In this case, even the maximal links encode information on the global network topology. Fig. 2 can be used to support this assertion. It shows the modularity of the partitions yielded by EqRank applied to six snapshots of the hep-th citation graph. The modularity was introduced in [9]. It quantifies both the hidden graph community structure and the quality of an algorithm aimed at approximating this structure. Computation of modularity involves all the links regardless of their weights. The graphs with community structure should have modularity in the range from 0.3 to 0.7. The plot shows that the EqRank partitions have the modularity in this range.

- The reindexing process improves the quality of the community structure implied by the limiting partition $\lim(T)$. It is even more important that evolution of the limiting partition is closer to a natural one (see the above definition of natural evolution) than the one of the initial partition. This is an experimental fact (see below): the values of the coefficients quantifying closeness of the evolution to a natural one are better for the limiting partition than they are for the initial one.

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**FIG. 1:** Iterations decrease reindexing

**FIG. 2:** Modularity from 1993 to 2003

**FIG. 3:** Growth of node and link numbers from 1993 to 2003
V. HEP-TH THEME DYNAMICS

We used hep-th citation graph [10] to study the dynamics of the partitions yielded by the EqRank algorithm. The graph we have studied contained the information on the citations in the papers from the hep-th sector of the electronic archive http://arxiv.org for the years 1992–2003.

We have taken six snapshots of this graph, at the end of 1993, 1995, 1997, 1999, 2001, and 2003. (This means that the first snapshot contains all the papers appeared before 1994, the second, before 1996, etc.) Fig. 3 shows the growth of the number of the graph vertexes (nodes) and links during these years. In contrast to our previous experiment with hep-th [8], we used undirected version of the citation graph. The links of the graph were weighted with a combination of co-citation and bibliographic coupling:

$$W(x, y) = aA^T A + (1 - a)AA^T,$$  \hspace{1cm} (9)

where $a = 0.9$ and $A$ is the adjacency matrix of the graph. For each graph, we applied the modified EqRank. The total number of themes have grown in this period from 6 (in 1993) to 139 (in 2003) (see Fig. 4).

To make an estimate of the closeness of the partition dynamics to a natural one, we computed the first and the second classification stability coefficients (Fig. 5). The plot shows that after 1995 the values of this coefficient are above 80%, which we consider as satisfactory values. Next we computed the theme mixing coefficient, TMC (Fig. 6). Again, after 1995, this coefficient ranges from 10 to 30%. We point out that the number of papers in hep-th was growing fast, and, in view of this, the above values of TMC do not seem to be high. The plot on Fig. 7 demonstrates that TMC grows with the increment in the number of papers. For example, the number of vertexes have grown by 16% from 2001 to 2003, and TMC was about 10%. Apart from TMC defined above, we computed also TMC(cut). In computing it, only the papers whose citation index exceeds the cut were taken into account. The values of TMC(cut) are informative, because in practice it is important to give a correct indexing namely to the well cited papers that can be viewed as specific theme attributes. The higher is the citation
index of a paper, the more important is that its theme index would behave properly. The plots on Figs. 6 and 7 demonstrate the desired behavior of TMC depending on the value of the cut. The higher is the cut, the lower is the value of the corresponding TMC. For example, TMC(40) was about 3% in 2001–2003, while TMC without cut was 10%.

Anomalous values of the coefficients in 1993–1995 can be explained as follows. In this period, the archive was filled fast with the themes already existed outside the archive, and the classification index was changing dramatically. After 1995, the archive became the standard location for the papers in its subject field, most of the themes were represented relatively well, and the coefficients relaxed to satisfactory values.

Tables 1 and 2 of the Appendix represent the themes of the classification obtained for the hep-th citation graph as it was in the mid of 2003. Due to the lack of space, we present only the upper themes ordered by the number of papers in the theme. Most of the themes are easily recognised. For comparison, we used the annual "Review of top cited HEP articles" by Michael Peskin [11] published from 1992 to 2003. The comparison reveals a remarkable correspondence between the scientific themes selected by Peskin as themes of current importance and the themes appearing in the evolution of the classification yielded by EqRank.

VI. CONCLUSIONS

Let us recup. We formulated the notion of natural dynamics of partitions. A reasonable clustering algorithm applied to evolving graphs should yield partitions with natural dynamics. We defined a number of coefficients characterizing the closeness of real partition dynamics to a natural one. We introduced reindexation, which allowed us to transform the initial partition to a new one possessing better structure and dynamics. Lastly, we applied this construction to the partition yielded by EqRank applied to the hep-th citation graph, and demonstrated that the outcome of this procedure is in a good agreement with the description of the themes given by an expert.

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APPENDIX A: THE TABLES

Here we explain the columns of Tables 1 and 2 (The two tables could be joined into a single table; Division into two tables is only to fit each of them into a page).

The first column keeps the number of papers in a theme; In the second column, we keep the pairs of key words characterizing a theme (the pairs were generated automatically by the titles of the papers); The third column contains the titles of the most cited papers of a theme; The fourth column keeps the years a theme was formed; The fifth, the year at which Peskin mentioned the corresponding papers in his "Review...", and the last column keeps either a title of the Section of Peskin's "Review...", or an excerpt from it characterising the theme.

Let us comment on how the years a theme was created (column four) were determined. For a theme, we map it to a theme in the preceding partition with Map 2 (see above for the definition of Map 2). If it is a stable theme, we map it again back in time, and continue until we obtain a new theme in the terminology introduced above. The year when the theme appears as new is the first one of the two years in column four, the second year is the earliest year the theme appears as a stable one.

[1] F. Radicchi, C. Castellano, et al, Defining and identifying communities in networks cond-mat/0309488 (2003).
[2] J. Leskovec, J. Kleinberg, and C. Faloutsos, Graphs over time: densification laws, shrinking diameters and possible explanations In KDD '05 (2005).
[3] Physics and Astronomy Classification Scheme, http://publish.aps.org/PACS/
[4] ACM Computing Classification System, http://www.acm.org/class/
[5] H. Small and B. C. Grith, The Structure of the Scientific Literatures. Identifying and graphing specialties, in Science Studies (1974).
[6] A. Popescul, G. Flake, et al, Clustering and Identifying Temporal Trends in Document Databases, in Proc. Advancements in Digital Libraries, 173-182 (2000).
[7] O. Khan, B. Kulik, et al, Tracking Evolving Communities in Large Linked Networks, in Sackler Colloquium on Mapping Knowledge Domains (2003).
[8] G. Pivovarov and S Trunov, EqRank: a Self-Consistent Equivalence Relation on Graph Vertexes, SIGKDD Explorations 5(2): 185-190 (2003)
[9] M.E.J Newman and M. Girvan, Finding and Evaluating Community Structure in Networks, cond-mat/0308217 (2003)
[10] http://www.cs.cornell.edu/projects/kddcup/datasets.html
[11] http://www.slac.stanford.edu/library/topcites/
| Number of articles | Theme keywords                                              | Authority papers (italic marks the papers mentioned in Peskin’s review)                                                                 | Theme’s years | Peskin’s year | Context                                                                 |
|-------------------|-------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------|---------------|---------------|-------------------------------------------------------------------------|
| 4338              | string theory; heterotic string; type iib; type iia         | *M Theory As A Matrix Model: A Conjecture* [hep-th/9610043]; Dirichlet-Branes and Ramond-Ramond Charges [hep-th/9510017]; String Theory Dynamics In Various Dimensions [hep-th/9503124] | 1993—1995     | 1997          | Mentioned in the Section on M-theory                                   |
| 3008              | black hole; ads/cft correspondence; gauged supergravity; higher spin | *The Large N Limit of Superconformal Field Theories and Supergravity* [hep-th/9711200]; Anti De Sitter Space and Holography [hep-th/9802150]; Gauge Theory Correlators from Non-Critical String Theory [hep-th/9802109]; Anti-de Sitter Space, Thermal Phase Transition, And Confinement In Gauge Theories [hep-th/9803137]; 4d Conformal Field Theories and Strings on Orbifolds [hep-th/9802183] | 1997—1999     | 1998          | “Maldacena’s correspondence...”                                       |
| 1443              | n=2 supersymmetric; integrable system; supersymmetric gauge; lax pair | *Monopole Condensation, And Confinement In N=2 Supersymmetric Yang-Mills Theory* [hep-th/9407087]; Monopoles, Duality and Chiral Symmetry Breaking in N=2 Supersymmetric QCD [hep-th/9408099]; Electric-Magnetic Duality in Supersymmetric Non-Abelian Gauge Theories [hep-th/9411149] | 1993—1995     | 1997          | “Witten uses brane dynamics to give a new derivation of the ... exact solutions of four-dimensional N=2 super-Yang-Mills theory of Seiberg and Witten... Seiberg’s ...picture of N=1 super-Yang-Mills theory can also be understood in this way...” |
| 1398              | noncommutative field; seiberg-witten map; open string; noncommutative geometry | *Noncommutative Geometry and Matrix Theory: Compactification on Tori* [hep-th/9711162]; D-branes and the Noncommutative Torus [hep-th/9711165]; String Theory and Noncommutative Geometry [hep-th/9908149]; Noncommutative Perturbative Dynamics [hep-th/9912072] | 1997—1999     | 2000          | Mentioned in the Section on Noncommutative Geometry                    |
| 1185              | brane world; cosmological constant; extra dimension; randall-sundrum model | *An Alternative to Compactification* [hep-th/9906064]; Modeling the Fifth Dimension with Scalars and Gravity [hep-th/9909134]; On Conventional Cosmology from a Brane Universe [hep-th/9905012] | 1997—1999     | 2000          | Mentioned in the Section on Extra Space Dimensions                     |
| Number of articles | Theme keywords                                                                 | Authority papers (italic marks the papers mentioned in Peskin’s review)                                                                 | Theme’s years | Peskin’s year | Context                                                                 |
|-------------------|--------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------|---------------|---------------|-------------------------------------------------------------------------|
| 437               | extremal black; greybody factor; null model; bekenstein-hawking formula       | Microscopic Origin of the Bekenstein-Hawking Entropy [hep-th/9601029]; D-brane Approach to Black Hole Quantum Mechanics [hep-th/9602043] | 1995—1997     | 1997          | “Strominger and Vafa and Callan and Maldacena have shown that black hole solutions of string theory can be thought to contain D-branes in their compactified dimensions...” |
| 383               | local brst; consistent deformation; nonlocal regularization; brst cohomology;   | Antibracket, Antifields and Gauge Theory Quantization [hep-th/9412228]; Local BRST Cohomology in the Antifield Formalism: I. General Theorems [hep-th/9405109] | 1991—1993     |               |                                                                          |
| 352               | lambda model; wilson loop; vapour phase; two-dimensional yang-mills             | Two Dimensional QCD is a String Theory [hep-th/9301068]; Two Dimensional Gauge Theories Revisited [hep-th/9204083]; Twists and Wilson Loops in the String Theory of Two Dimensional QCD [hep-th/9303046] | 1993—1995     |               |                                                                          |
| 321               | integrable boundary; s matrix; bethe ansatz; affine toda                       | Boundary S-Matrix and Boundary State in Two-Dimensional Integrable Quantum Field Theory [hep-th/9306002]; Factorized Scattering in the Presence of Reflecting Boundaries [hep-th/9304141] | 1995—1997     |               |                                                                          |
| 303               | dilatonic gravity; hawking radiation; hole evaporation; two-dimensional dilaton | The Stretched Horizon and Black Hole Complementarity [hep-th/9306069]; The Endpoint of Hawking Evaporation [hep-th/9206070] | 1991—1993     |               |                                                                          |
| 289               | penrose limit; pp-wave background; plane wave; bmn operator                    | Strings in Flat Space and pp waves from $N = 4$ Super Yang Mills [hep-th/0202021]                                                                 | 2001—2003     | 2003          | "The theory of strings on gravitation wave backgrounds...”               |
| 275               | gangino condensation; heterotic m-theory; supersymmetry breaking               | Type IIB Superstrings, BPS Monopoles, And Three-Dimensional Gauge Dynamics [hep-th/9611230]; Solutions Of Four-Dimensional Field Theories via M Theory [hep-th/9710136] | 1999—2001     | 1997          | "This suggests that there is a connection between the geometry of branes and the exact properties of these gauge theories...” |
| 110               | rolling tachyon; tachyon matter; tachyon field                                | Rolling Tachyon [hep-th/0203211]                                                                                                  | 2001—2003     | 2003          | "...study of the explicit time-dependent evolution of unstable brane configurations...” |