Magnetoplasmons in quantum rings

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Abstract

We have studied the structure and dipole charge density response of nanorings as a function of the magnetic field using local-spin density functional theory. Two small rings consisting of 12 and 22 electrons confined by a positively charged background are used to represent the cases of a narrow and a wide ring. The results are qualitatively compared with experimental data existing on microrings and on antidots. A smaller ring containing 5 electrons is also analyzed to allow for a closer comparison with a recent experiment on a two electron quantum ring.

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I. INTRODUCTION

The study of collective excitations in bounded two-dimensional electron systems (2dES) is a subject of current interest, especially for the particular geometry called quantum dot in which a number of electrons is confined into a rather small, almost two-dimensional region produced by present available etching technologies, and for the quasi-one-dimensional structures called quantum wires (see for example Refs. 1, 2 for a comprehensive description of quantum dots and wires). Less effort has been put in the investigation of these excitations in quantum antidots, i.e., the reversed structure of dots made in the 2dES.

Recently, magnetoplasmons arising in ring confining geometry have also attracted some interest. The first experimental studies concerned structures in the micron scale, etched into a molecular-beam-epitaxy-grown $\delta$-doped GaAs-Ga$_x$Al$_{1-x}$As heterostructure, of outer diameter $\sim 50$ $\mu$m and inner diameter in the 12-30 $\mu$m range. The observed magnetoplasmon resonances bear some of the properties of the dynamical response of a classical 2dES. Later on, a hydrodynamic theory based on the Thomas-Fermi-Dirac-von Weizsäcker approximation has been used to describe $N = 400$ electron rings which yields a good account of the experimental data after an appropriate scaling of them at zero magnetic field ($B$). Plasmon modes in very narrow rings have been described within a Hartree-random phase approximation, and the charge density response of a dot with a repulsive impurity in its center has also been worked out. The optical absorption and inelastic scattering of a two electron quantum ring of a rather large radius (480 nm) and width (20 nm) has been discussed in detail, and single electron properties of quantum rings with parabolic confinement have been discussed, with the aim of determining the effect of electron-electron interactions on the energy spectrum and magnetic moment associated with the persistent current in a quantum ring.

The far-infrared (FIR) charge density excitation (CDE) appears to depend on the ring width. The measured CDE's are bundled into a high energy group and a low energy group, which in contradistinction with the case of dots do not merge at $B = 0$. The low energy peaks arrange into two distinct branches. For narrow (NR) rings, both have a negative $B$ dispersion, whereas for broad (BR) rings one branch displays a positive $B$ dispersion at small magnetic fields. The high energy peaks arrange into one (narrow rings) or several (broad rings) branches. The high energy branches display a negative $B$ dispersion at small magnetic fields.

The low energy peaks have been explained as edge magnetoplasmons excited at the inner and outer boundaries of the ring, whereas the high energy peaks are bulk magnetoplasmons. It is worth to recall that in the case of antidots, only one edge magnetoplasmon is detected whose energy goes to zero with $B$. It thus seems that the observed ring plasmons exhibit features of either dots or antidots depending on the ring widthness and $B$ value.

Very recently, nanorings in InAs-GaAs heterostructures have been fabricated in the 15-40 nm radius range, and the FIR response has been measured for a two electron ring. Two sets of peaks appear in the response, as in the case of microrings. Depending on the $B$ value, one to three main peaks have been detected and arranged into four energy branches with $B$. 
dispersions which seem to differ from the microring systematics\textsuperscript{19}. The two branches starting from the $B = 0$ high energy peak are similar to those of quantum dots, and according to the analysis of the experimental data presented in Ref.\textsuperscript{19}, the two branches corresponding to the low energy peaks seem to display both a positive $B$ dispersion. It is worth to notice that the experimental results on microrings cover a low $B$ range (up to 2 T), whereas the ones on nanorings extend up to 14 T, but no data on the low energy nanoring peaks have been recorded below 4 T.

The studied nanorings present an elongation in the [1,-1,0] direction. Likely, it is not distorting much the electrons from being circularly distributed. Otherwise, one would have that at $B = 0$ the two high energy branches do not merge at all, as they seem to do. A similar situation, namely, a non circularly symmetric dot hosting quite a circularly symmetric electronic density is also found for few electron quantum dots\textsuperscript{17}. Besides, during their manufacture nanorings had to be further covered to complete the necessary layer structure\textsuperscript{19}. All that might result in nontrivial changes with respect the CDE’s of a ‘clean’, circularly symmetric ring, and it calls for a microscopic investigation in which the basic ingredients for a proper description of such nanostructures are taken into account and might guide the experimental analysis as a kind of ‘reference spectrum’ obtained under controled geometrical conditions.

We present here three such spectra obtained within time-dependent local-spin density functional theory (TDLSDFT). The first two correspond to circularly symmetric nanorings made of 12 and 22 electrons embedded into a GaAs-Ga$_x$Al$_{1-x}$As heterostructure. Although the method can handle a smaller number of electrons, the possibility of describing the two electron structure\textsuperscript{19} is beyond its reach and for that reason we have renounced to it from the start, presenting only results obtained for a 5 electron nanoring of equal size as a third example. It is dobtless that the natural evolution of the field will make it possible a quantitative comparison between TDLSDT and experimental results still to come.

This work is organized as follows. We discuss in Sect. II the results we have obtained for the ring ground states (gs), which are the starting point for the study of their charge density excitations presented in Sec. III. Finally, the concluding remarks are given in Sec. IV.

## II. THE GROUND STATE OF QUANTUM RINGS

We consider a circularly symmetric quantum ring made of $N$ electrons moving in the $z = 0$ plane where they are confined by the potential $V^+(r)$ created by $N^+$ positive charges uniformly distributed between an outer $R_o$ and inner $R_i$ radius in the presence of a constant magnetic field $B$ in the positive $z$ direction. In the local-spin density approximation (LSDA), the single electron wave functions are given by the solution of the Kohn-Sham (KS) equations

$$\left[ -\frac{1}{2} \nabla^2 + \frac{1}{2} \omega_c^2 \ell_z^2 + \frac{1}{8} \omega_c^2 r^2 - V^+(r) \\
+ V^H + V^{xc} + (W^{xc} + \frac{1}{2} g^* \mu_B B) s_z \right] \varphi_\alpha(r, \theta) = \epsilon_\alpha \varphi_\alpha(r, \theta),$$

where $V^H = \int d\vec{r}' \rho(\vec{r}')/|\vec{r} - \vec{r}'|$ is the Hartree potential. $V^{xc} = \partial E_{xc}(\rho, m)/\partial \rho|_{gs}$ and $W^{xc} = \partial E_{xc}(\rho, m)/\partial m|_{gs}$ are the variations of the exchange-correlation energy density $E_{xc}(\rho, m)$ in
the local approximation taken at the ground state, and \( \rho(r) \) and \( m(r) \) are the electron and spin magnetization densities. The exchange-correlation energy density \( E_{xc} \) has been constructed from the results on the nonpolarized and fully polarized two dimensional electron gas using the two dimensional von Barth and Hedin prescription to interpolate between both regimes.

We have used effective atomic units (\( \hbar = e^2/\epsilon = m = 1 \)), where \( \epsilon \) is the dielectric constant of the semiconductor and \( m \) is the electron effective mass. In units of the bare electron mass \( m_e \) one has \( m = m^*m_e \). In this system of units, the length unit is the effective Bohr radius \( a_0^* = a_0\epsilon/m^* \), and the energy unit is the effective Hartree \( H^* = Hm^*/\epsilon^2 \). For GaAs we have taken \( \epsilon = 12.4 \), \( m^* = 0.067 \), and \( g^* = -0.44 \), which yields \( a_0^* = 97.9 \) Å and \( H^* \sim 11.9 \) meV. In Eq. (1) \( \omega_c = eB/(mc) \) is the cyclotron frequency and \( \mu_B = e\hbar/(2m_ec) \) is the Bohr magneton.

As a consequence of circular symmetry the \( \varphi_\alpha \)'s are eigenstates of the orbital angular momentum \( \ell_z \), i.e., \( \varphi_\alpha(r, \theta) = u_{n\ell\sigma}(r)e^{-i\ell\theta} \), with \( \ell = 0, \pm 1, \pm 2, \ldots \). The gs electron density is given by \( \rho(r) = \sum \alpha n_\alpha |u_\alpha(r)|^2 \), while the gs spin magnetization density is expressed in terms of the spin of orbital \( \alpha \), \( \langle \sigma_z \rangle_\alpha \), as \( m(r) = \sum \alpha n_\alpha \langle \sigma_z \rangle_\alpha |u_\alpha(r)|^2 \). The numerical calculations reported in the following have been performed at a small but finite temperature \( T \leq 0.1 \) K, and the KS equations have been solved by integration in \( r \) space. The thermal occupation probabilities \( n_\alpha \) are determined by the normalization condition \( N = \sum \alpha n_\alpha = \sum \alpha 1/\{1 + \exp[(\epsilon_\alpha - \mu)/k_B T]\} \) which fixes the chemical potential \( \mu \).

The \( V^+(r) \) potential is analytical and can be expressed in terms of the elliptic \( E \) and \( K \) functions:

\[
V^+(r) = \frac{4N^+}{\pi(R_a^2 - R_i^2)} \times \begin{cases} 
\left[ R_o E \left( \frac{r}{R_o} \right) - R_i E \left( \frac{r}{R_i} \right) \right] & \text{if } r < R_i \\
\left[ R_o E \left( \frac{r}{R_o} \right) - r E \left( \frac{R_o}{r} \right) \right] + r \left[ 1 - \left( \frac{R_o}{r} \right)^2 \right] K \left( \frac{R_o}{r} \right) & \text{if } R_o > r > R_i \\
r \left\{ E \left( \frac{R_o}{r} \right) - E \left( \frac{R_i}{r} \right) + \left[ 1 - \left( \frac{R_o}{r} \right)^2 \right] K \left( \frac{R_o}{r} \right) - \left[ 1 - \left( \frac{R_i}{r} \right)^2 \right] K \left( \frac{R_i}{r} \right) \right\} & \text{if } r > R_o.
\end{cases}
\]

As previously indicated, we have considered two nanorings. The narrow one has \( R_o = 100 \) nm, \( R_i = 70 \) nm, \( N = 12 \) and \( N^+ = 14 \), and the broad one has \( R_o = 100 \) nm, \( R_i = 37.5 \) nm, \( N = 22 \), and \( N^+ = 24 \). These values have been selected to roughly have in both rings the same average surface densities as in the \( N = 25 \) quantum dot described in Refs. 23, 24, as well as the same outer radius. That would allow to make a comparison between FIR modes arising in somehow similar dot and ring geometries. The radii ratio in the broad ring is similar to that of Ref. 19.

Figure 1 represents several electron densities for selected \( B \) values as a function of the radial distance in the case of the NR ring, and Fig. 2 in the case of the BR ring. In the latter case, at \( B = 0 \) the central electron density is not zero, but it is around two orders of magnitude smaller than its maximum value. At present, it is unclear to us whether a different confining potential that prevents the electrons from having a sizeable probability of being inside the ring ‘hole’, as the parabolic confinement of Ref. 15, would be more realistic.

In the case of the NR ring, the electronic density has no structure, presenting a gaussian-like shape whose width decreases with increasing \( B \). In contradistinction, in the BR ring
an incipient bulk density region appears as well as the characteristic ‘bump’ at the edges clearly visible in dots confined by a disk geometry.

Figures 3 and 4 represent the single particle (sp) energies as a function of the orbital angular momentum $\ell$ and different $B$ values. The $N = 12$ ring becomes fully polarized between $B = 2$ and 3 T, and the $N = 22$ ring between $B = 3$ and 4 T (in the $N = 25$ quantum dot it happens at $B \sim 3.6$ T). It can be seen from the corresponding panels in these figures that at $B = 0$ both rings have a z component of the total spin different from zero, $S_z = 1$. Since the $N = 10$ and 20 rings are close shell systems, this means that Hund’s first rule is obeyed by these small rings, as it is in small dots.

The sp energies are arranged into bands which are bent upwards at both ends not only at low $B$. This is a peculiarity of the ring geometry, which bears simultaneously the characteristics of dot and antidot bands, the former ones bending upwards at high $\ell$, and the later ones at small $\ell$. The existence of two bendings when a magnetic field is applied is the microscopic origin of the two edge magnetoplasmons, as we shall discuss in the next Section.

When the ring becomes fully polarized, increasing $B$ further produces the displacement as a whole of the set of occupied sp levels to higher $\ell$’s. We have found that this is the mechanism rings have to keep its total orbital angular momentum $L_z$ increasing with $B$. That can be seen for example, in the high $B$ panels corresponding to the NR ring (see also Fig. 11). We have plotted in Fig. 5 the evolution of $L_z$ and $2S_z$ with $B$ for the NR ring, and in Fig. 6 for the BR ring.

The shifting upwards in $\ell$ of the whole sp spectrum with increasing $B$ is a distinct characteristic of rings that deserves further investigation. In quantum dots, the stability region in the $N - B$ phase plane of the fully polarized configuration, called maximum density droplet (MDD) state, built from sp orbitals having $\ell = 0,1,2 \ldots N - 1$ is limited from the left by a line $B_f$ representing, for a given number number of electrons, the magnetic field at which $2S_z = N$, and from the right by a line $B_r$ at which edge reconstruction starts. This is a rather narrow region, a few tenths of tesla wide because after fully polarization the magnetic field is very effective in promoting electrons from high to higher sp levels, reconstructing the dot edge. In rings this is quite not so because the existence of an electron depletion at the center and the consequent upwards bending of the sp bands allows for an alternative mechanism to keep increasing $L_z$ while retaining the simplicity of the gs wave function, namely, a Slater determinant made of the lowest possible $\ell$ sp states from a minimum $\ell_m$ to a maximum $\ell_M$ such that $N = \ell_M - \ell_m + 1$. Taking as an example the situation of the NR ring, at $B = 11$ T we have found that $\ell_m = 54$ and $\ell_M = 65$. It might well happen that for quantum rings, no equivalent of a kind of edge reconstruction mechanism exists, but addressing this point is beyond the capabilities of the density functional we are using.

III. CHARGE DENSITY EXCITATIONS OF A QUANTUM RING.

A. Longitudinal response within TDLSDT

Once the gs has been obtained, we determine the induced densities originated by an external field employing linear-response theory. Following Refs. 33–34, we can write the
variation $\delta \rho_{\sigma}$ induced in the spin density $\rho_{\sigma}$ ($\sigma \equiv \uparrow, \downarrow$) by an external spin-dependent field $F$, whose non-temporal dependence we denote as $F = \sum_{\sigma} f_{\sigma}(\vec{r}) |\sigma\rangle\langle\sigma|$:  

$$
\delta \rho_{\sigma}(\vec{r}, \omega) = \sum_{\sigma'} \int d\vec{r}' \chi_{\sigma\sigma'}(\vec{r}, \vec{r}'; \omega) f_{\sigma'}(\vec{r}') ,
$$  

(3)

where $\chi_{\sigma\sigma'}$ is the spin-density correlation function. In this limit, the frequency $\omega$ corresponds to the harmonic time dependence of the external field $F$ and of the induced $\delta \rho_{\sigma}$. Eq. (3) is a $2 \times 2$ matrix equation in the two-component Pauli space. In longitudinal response theory, $F$ is diagonal in this space, and we write its diagonal components as a vector $F \equiv \left( f_{\uparrow} f_{\downarrow} \right)$. For the dipole operator we then have

$$
D_{\rho} \equiv \left( \begin{array}{c} x \\ x \end{array} \right) \quad \text{and} \quad D_{m} \equiv \left( \begin{array}{c} x \\ -x \end{array} \right) ,
$$

(4)

where the field $D_{m}$ will cause longitudinal spin excitations not quite studied here because of the lack of experimental information on them, but introduced at this point for the sake of clearness.

TDLSDT assumes that electrons respond as free particles to the perturbing effective field, which consists of the external plus the induced field arising from the changes produced by the perturbation in the gs mean field. This condition defines the TDLSDT correlation function $\chi_{\sigma\sigma'}$ in terms of the free particle spin-density correlation function $\chi^{(0)}_{\sigma\sigma'}$ through a Dyson-type integral equation:

$$
\chi_{\sigma\sigma'}(\vec{r}, \vec{r}'; \omega) = \chi^{(0)}_{\sigma\sigma'}(\vec{r}, \vec{r}'; \omega) + \sum_{\sigma_1\sigma_2} \int d\vec{r}_1 d\vec{r}_2 \chi^{(0)}_{\sigma_1\sigma_1}(\vec{r}, \vec{r}_1; \omega) K_{\sigma_1\sigma_2}(\vec{r}_1, \vec{r}_2) \chi_{\sigma_2\sigma'}(\vec{r}_2, \vec{r}'; \omega) .
$$

(5)

The free particle spin-correlation function at finite temperature is obtained from the KS sp wave functions, energies and occupation probabilities:

$$
\chi^{(0)}_{\sigma\sigma'}(\vec{r}, \vec{r}'; \omega) = -\delta_{\sigma,\sigma'} \sum_{\alpha\beta} \varphi^*_\alpha(\vec{r}) \varphi_\beta(\vec{r}) \frac{n_\alpha - n_\beta}{\epsilon_\alpha - \epsilon_\beta + \omega + i\eta} \varphi^*_\beta(\vec{r}') \varphi_\alpha(\vec{r}') .
$$

(6)

The label $\alpha$ ($\beta$) refers to a sp level with spin $\sigma$ ($\sigma'$) and occupation probability $n_\alpha$ ($n_\beta$). To simplify the analysis of the results, we have added a small but finite imaginary part $\eta$ to the energy $\omega$. This will make an average of the strength function by transforming the $\delta$-peaks into Lorentzians of width $2\eta$.

The kernel $K_{\sigma\sigma'}(\vec{r}_1, \vec{r}_2)$ is the residual two-body interaction

$$
K_{\sigma\sigma'}(\vec{r}_1, \vec{r}_2) = \frac{1}{|\vec{r}_1 - \vec{r}_2|} + \left. \frac{\partial^2 \mathcal{E}_{xc}(\rho, m)}{\partial \rho_{\sigma} \partial \rho_{\sigma'}} \right|_{gs} \delta(\vec{r}_1 - \vec{r}_2) ,
$$

(7)

where

$$
\left. \frac{\partial^2 \mathcal{E}_{xc}}{\partial \rho_{\sigma} \partial \rho_{\sigma'}} \right|_{gs} = \left. \frac{\partial^2 \mathcal{E}_{xc}}{\partial \rho^2} \right|_{gs} + \left. \eta_\sigma + \eta_\sigma' \right|_{gs} \left. \frac{\partial^2 \mathcal{E}_{xc}}{\partial \rho \partial m} \right|_{gs} + \left. \eta_\sigma \eta_\sigma' \right|_{gs} \left. \frac{\partial^2 \mathcal{E}_{xc}}{\partial m^2} \right|_{gs} \equiv K(r) + \left( \eta_\sigma + \eta_\sigma' \right) L(r) + \eta_\sigma \eta_\sigma' I(r) ,
$$

(8)
with \(\eta^\uparrow = 1, \eta^\downarrow = -1\). The last expression is the definition of the \(K\), \(L\), and \(I\) functions.

When the system is not polarized, there are only two independent correlation functions. These are \(\chi_{\rho\rho}\) and \(\chi_{mm}\) describing, respectively, the density response to \(D_\rho\) and the spin response to \(D_m\). They are given by

\[
\chi_{\rho\rho} = \chi_{\uparrow\uparrow} + \chi_{\downarrow\downarrow} + \chi_{\uparrow\downarrow} + \chi_{\downarrow\uparrow},
\]

\[
\chi_{mm} = \chi_{\uparrow\uparrow} + \chi_{\downarrow\downarrow} - \chi_{\uparrow\downarrow} - \chi_{\downarrow\uparrow},
\]

and the four equations (9) reduce to two uncoupled equations for \(\chi_{\rho\rho}\) and \(\chi_{mm}\) whose kernels are given by \(1/r_{12} + K\delta(r_{12})\) and \(I\delta(r_{12})\), respectively, and the free particle correlation function \(\chi^{(0)} = \chi^{(0)}_{\uparrow\uparrow} + \chi^{(0)}_{\downarrow\downarrow} = 2\chi^{(0)}_{\uparrow\uparrow}\) is the same in both channels because \(\chi^{(0)}_{\uparrow\uparrow} = \chi^{(0)}_{\downarrow\downarrow}\). This constitutes the paramagnetic limit of the longitudinal response with uncoupled density and spin channels, in which the residual interaction consists of a Coulomb direct plus an exchange-correlation terms in one case, and only of an exchange-correlation term in the other.

When the system is polarized one no longer has \(\chi^{(0)}_{\uparrow\uparrow} = \chi^{(0)}_{\downarrow\downarrow}\), and there are two more independent correlation functions

\[
\chi_{\rho m} = \chi_{\uparrow\uparrow} - \chi_{\downarrow\downarrow} - \chi_{\uparrow\downarrow} + \chi_{\downarrow\uparrow},
\]

\[
\chi_{m\rho} = \chi_{\uparrow\uparrow} - \chi_{\downarrow\downarrow} + \chi_{\uparrow\downarrow} - \chi_{\downarrow\uparrow},
\]

which produce the density response to \(D_m\) and the spin response to \(D_\rho\), respectively. Since we are interested only in the charge density response, from now on we shall restrict ourselves to the discussion of the electron response to \(D_\rho\), apart from presenting as an example how the longitudinal spin response looks like in two selected cases. We refer the reader to Ref. 25 for a thorough discussion of the longitudinal response in quantum dots, of direct applicability to quantum rings.

Equations (9) have been solved as a generalized matrix equation in coordinate space after performing an angular decomposition of \(\chi_{\sigma\sigma'}\) and \(K_{\sigma\sigma'}\) of the kind

\[
K_{\sigma\sigma'}(\mathbf{r}, \mathbf{r}') = \sum_\ell K_{\sigma\sigma'}^{(\ell)}(r, r') e^{i\ell(\theta - \theta')}.
\]

Only modes with \(\ell = \pm 1\) couple to the external dipole field \(D_\rho\). This can be readily seen performing the angular integral in Eq. (9). In practice, we have considered the multipole expansion of the external field, using the dipole vectors

\[
D_\rho^{(\pm 1)} = \frac{1}{2}re^{\pm i\theta} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.
\]

(12)

For a polarized system having a non zero magnetization in the gs, the \(\ell = \pm 1\) modes are not degenerate and may give rise to two excitation branches with \(\Delta L_z = \pm 1\), where \(L_z\) is the gs orbital angular momentum.

The charge density response function for the dipole field has been obtained from the \(\ell = \pm 1\) components of the correlation functions \(\chi_{\rho\rho}^{(\pm 1)}(r, r'; \omega)\) as:

\[
\alpha_{\rho\rho}(\omega) = \pi^2 \int d\mathbf{r}_1 \, d\mathbf{r}_2 \, r_{12}^2 \left( \chi_{\rho\rho}^{(+1)}(r_1, r_2; \omega) + \chi_{\rho\rho}^{(-1)}(r_1, r_2; \omega) \right)
\]

\[
\equiv \alpha_{\rho\rho}^{(+1)}(\omega) + \alpha_{\rho\rho}^{(-1)}(\omega).
\]

(13)
Its imaginary part is related to the strength function as

\[ S_{\rho \rho}(\omega) = \frac{1}{\pi} \text{Im}[\alpha_{\rho \rho}(\omega)]. \]

To check the numerical accuracy of the calculations we have used the f-sum rule for the dipole operator, which can be expressed in terms of gs quantities:

\[ m^{(\rho \rho)}_1 = \int S_{\rho \rho}(\omega) \omega d\omega = \frac{1}{2} \langle 0 | [x, [H, x]] | 0 \rangle = \frac{N}{2}. \] (14)

We have checked that in our calculations the f-sum rule is fulfilled within a 95% or better.

**B. Numerical results**

Figures 7 and 8 show the charge density strength function for the \( N = 12 \) and the \( N = 22 \) electron rings, respectively, and several \( B \) values. The plus or minus sign close to the more intense peaks indicates that they are originated either by \( D_{\rho}^{(+1)} \) or by \( D_{\rho}^{(-1)} \). Obviously, at zero magnetic field, (+) and (−) excitations are degenerated.

The CDE’s displayed in these figures are easier to understand starting from the high \( B \) results and having in mind the sp levels drawn in Figs. 3 and 4. First notice that as in dots, the (+) low energy modes are intraband CDE’s from the outer ring boundary. The (−) low energy modes are intraband CDE’s of the inner ring boundary, obviously absent in dots. However, they are the only edge modes in antidots. The (+) modes arise when the dipole field changes by +1 the total \( L_z \) of the ring, and the (−) ones when this change is −1. Figs. 3 and 4 show than indeed, both kind of edge modes are possible in rings.

The higher energy peaks are bulk modes arising from interband transitions. At moderate \( B \) values, both positive and negative high energy peaks are present in the strength, but at high \( B \) values only modes excited by \( D_{\rho}^{(-1)} \) have an appreciable intensity: as in the dot case, the (+) low energy edge mode is taking all the strength corresponding to the \( D_{\rho}^{(+1)} \) operator.

Fig. 4 shows that for some \( B \) values, the sp energies are distributed following a very symmetric pattern as a function of \( \ell \). This is the reason why sometimes (+) and (−) edge modes are nearly degenerated. Their splitting is not regular as a function of \( B \), indicating a kind of ‘shell structure’ effect that only a microscopic model can reveal. Still, the gross features of the three energy branches displayed in Fig. 4 is very similar to that of narrow microring: two low energy edge modes with a negative \( B \) dispersion, and a high energy mode which at low \( B \) has a negative dispersion, and eventually a positive \( B \) dispersion at high magnetic fields.

At zero magnetic field, or generally speaking, at low \( B \), the CDE’s are delocalized as in the case of quantum dots. Notice for instance that (+) and (−) excitations are degenerated, and it has no sense to associate any of them to excitations coming from the inner or outer ring boundary. Besides, in the case of BR rings the low \( B \) strength in rather fragmented, rendering more complex the analysis. It is worth to recall that a similar fragmentation occurs in the case of dots if one uses a positively charged disk to model the confining potential. Still, two quite distinct structures, one at high and another at low excitation energies are present at \( B = 0 \) in the case of rings, whose origin can be traced back from the results at high \( B \), and one may associate the low energy peaks to intraband and the high energy peaks to interband transitions.

In the BR ring case, the \( B \) dispersion of the (−) edge mode is firstly positive, reaches a maximum at around \( B = 1 \) T, and then becomes negative. The high energy peaks with
appreciable strength are now only (−) modes. Again, these features are those displayed by broad microrings.

For quantum dots and rings of similar size and electron number, one expects that the energy of the \( B = 0 \) mode is lower for the ring than for the dot. Actually, this is an experimental fact\(^\text{19}\) that we can qualitatively explain using a sum rule method. We have found that at \( B = 0 \) the average frequency of the dipole mode can be written as\(^\text{24}\)

\[
\Omega^2 = \frac{1}{2N} \int d\mathbf{r} \Delta V^+(r) \rho(r) .
\] (15)

Taking a parabola \( \omega_0^2 r^2 / 2 \) as confining potential \( V^+(r) \) for a dot, and \( \omega_0^2 (r - R)^2 / 2 \) for a ring of mean radius \( R \) having the same number of electrons, one can easily check that \( \Omega = \omega_0 \) for the dot, and \( \Omega \sim \omega_0 / \sqrt{2} \) for a narrow ring, or for a ring broad enough so that the electronic density can be considered as being constant.

The FIR response of BR rings have also some features in common with antidots, which we recall that at \( B \neq 0 \) basically comes from the \( D_{\theta}^{(-1)} \) component of the dipole operator. One is the \( B \) dispersion of the inner edge mode. The other one is the transfer of strength from the low to the high energy \((−)\) peak\(^\text{3,4}\).

In all cases we have studied, CDE’s emerge as collective peaks. The residual electron-hole (e-h) interaction shifts CDE’s to higher energies from the sp excitations (SPE) which constitute the free response (see Fig. 10 below). In the longitudinal spin case, the residual interaction is attractive but weak, as it is only due to the exchange-correlation potential. As an example, we show in Fig. 9 the three responses for the NR and BR rings at \( B = 1 \) T.

Finally, we discuss the results we have obtained for a nanoring more similar to that experimentally studied\(^\text{19}\). In this case, \( R_o = 40 \) nm, \( R_i = 15 \) nm, and \( N = N^+ = 5 \). Figure 10 shows the charge density strength function at several \( B \) values, and Fig. 11 the sp energy levels.

Basically, the results are qualitatively similar to those of the broad nanoring already discussed (they have the same \( R_o/R_i \) ratio). When a magnetic field is applied, it can be clearly seen the transfer of strength between the edge and bulk \((−)\) branches as \( B \) increases, quite similar to the antidot case as we have already pointed out. The transfer is possible because both branches have the same polarization. The coupling is very inefficient in narrow rings, and the \((−)\) high and low energy peaks keep their own strength. This is the situation displayed in Fig. 7 for the \( N = 12 \) ring.

It is worth to notice the evolution with \( B \) of the \((−)\) edge mode, which is a rather high energy mode with a positive \( B \) dispersion from \( B = 1 \) to \( 4 \) T, and whose energy abruptly falls between \( 4.5 \) and \( 5 \) T. This decreasing is due to a change in the occupied sp levels which illustrates the relevance of shell effects especially in the case of a small number of electrons. A look at the panels corresponding to \( B = 2 \) and \( 5 \) T in Fig. 11 explains the effect. It can be seen how asymmetrically are distributed the sp levels, with a much large energy difference for the e-h pairs contributing to the edge excitation of the inner ring boundary than for those building the edge excitation of the outer boundary. This explains the large energy of the \((−)\) edge excitation up to \( B \sim 4.5 \) T. Of course, this is a qualitative argument since the residual e-h interaction has a sizeable effect in the charge density channel. On the contrary, and \( B = 5 \) T and above, the sp levels are distributed more symmetrically, the e-h energy differences are smaller and the \((+)\) and \((−)\) edge modes follow the BR ring systematics.
IV. SUMMARY AND OUTLOOK

In this work we have studied in some detail CDE’s in quantum rings. We have confirmed the expectations put forward by Dahl et al that plasmon resonances in quantum rings are dominated to a large extent by geometric effects, although shell effects may cause, in the case of few electron nanorings, effects that cannot be systematized. Apart from an example, we have restricted our analysis to CDE’s. It would be as simple to describe SDE’s and SPE’s within TDLSDT, much along the case of quantum dots, if experimental information becomes available.

Our work complements the theoretical description of microrings made by Zaremba. Even if a kind of characteristic pattern can be established for narrow or broad nanorings, this confining geometry allows to study much richer spectra than in dots or antidots. It might then offer the possibility of testing theoretical descriptions that are equally well describing plasmon modes in quantum dots, even if their complexity is quite different.

The lack of experimental results for nanorings hosting several electrons has not allowed us to make a quantitative comparison of our calculations with experiments. A qualitative comparison between the calculated $N = 5$ and the measured $N = 2$ FIR spectrum is inconclusive. To unambiguously arrange the peaks into branches and disentangle the $B$ dispersion of the plasmon modes, it would be essential to experimentally assign the polarization state to the main energy peaks. This has been paramount in the analysis of the theoretical FIR response, which otherwise would have not allowed us to distinguish between peak fragmentation and different plasmon branches in some cases. Alternatively, calculations for rings with as many electrons as in the experiments might guide to distribute the experimental data into branches. TDLSDT may be a useful tool for doing so in nanorings with a few more electrons than those studied so far. Other more microscopic methods are better suited for a two electron ring provided the geometry is adjusted to the experimental situation.

Finally, we have also determined that Hund’s first rule is fulfilled in the quantum rings we have studied, and have elucidated a possible mechanism by which a fully polarized quantum ring may have a rather simple gs structure in a wide range of magnetic fields.

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In one-electron operators such as $F$, $D_\rho$ and $D_m$, a sum on the electron coordinates is implicit.
FIGURES

FIG. 1. Electronic densities (10^{10} \text{ cm}^{-2}) as a function of the radial distance (nm) for the narrow ring.

FIG. 2. Same as Fig. 1 for the broad ring.

FIG. 3. Single particle energies as a function of orbital angular momentum $\ell$ corresponding to the narrow ring. The horizontal lines represent the electron chemical potential. The full, upright triangles represent $\sigma = \uparrow$ bands, and the empty, downright triangles represent $\sigma = \downarrow$ bands.

FIG. 4. Same as Fig. 3 for the broad ring.

FIG. 5. Total orbital and spin angular momenta $L_z$ and $2S_z$ as a function of $B$ for the narrow ring. The dashed lines are drawn to guide the eye.

FIG. 6. Same as Fig. 5 for the broad ring.

FIG. 7. Strength function (arbitrary units) as a function of the excitation energy (meV) for the narrow ring at several $B$ values. The arrows indicate the value of the cyclotron frequency. The $(−)$ or $(+)$ symbol close to the more intense peaks denotes the character of the dipole polarization.

FIG. 8. Same as Fig. 7 for the broad ring.

FIG. 9. Strength function (arbitrary units) at $B = 1 \text{ T}$ as a function of the excitation energy (meV) for the narrow ring (top panel) and the broad ring (bottom panel). The solid line is the charge density response, the dotted line the longitudinal spin density response, and the dashed line the free electron response.

FIG. 10. Same as Fig. 7 for the $N = N^+ = 5$ nanoring with $R_o = 40 \text{ nm}$ and $R_i = 15 \text{ nm}$. The free strength function is also plotted (thin lines).

FIG. 11. Same as Fig. 3 for the $N = N^+ = 5$ nanoring with $R_o = 40 \text{ nm}$ and $R_i = 15 \text{ nm}$. 

13
\[ \rho (10^{10} \text{ cm}^{-2}) \]

- \( B = 0 \text{T} \)
- \( B = 1 \text{T} \)
- \( B = 2 \text{T} \)
- \( B = 3 \text{T} \)
- \( B = 4 \text{T} \)

\[ r (\text{nm}) \]
The graph shows the relationship between magnetic field strength (B(T)) and the components of angular momentum: \( L_z \) and \( 2S_z \). The data points are represented by black circles for \( L_z \) and triangles for \( 2S_z \) with a solid line connecting the points for \( L_z \) and a dashed line for \( 2S_z \).
