Holographic Smart EM Skins for Advanced Beam Power Shaping in Next Generation Wireless Environments

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G. Oliveri, P. Rocca, M. Salucci, and A. Massa

Abstract

An innovative approach for the synthesis of inexpensive holographic smart electromagnetic (EM) skins with advanced beamforming features is proposed. The complex multiscale smart skin design is formulated within the Generalized Sheet Transition Condition (GSTC) framework as a combination of a mask-constrained isophoric inverse source problem and a micro-scale susceptibility dyadic optimization. The solution strategy integrates a local search procedure based on the iterative projection technique (IPT) and a System-by-Design (SbD)-based optimization loop for the identification of optimal metasurface descriptors matching the desired surface currents. The performance and the efficiency of the proposed approach are assessed in a set of representative test cases concerned with different smart skin apertures and target pattern masks.

Key words: Smart Skins; EM Holography; Next-Generation Communications; Iterative Projection Method; System-by-Design; Metasurfaces; Metamaterials.
1 Introduction and Rationale

The next generation of wireless cellular systems is envisaged to fulfil unprecedented requirements in terms of data transfer speed, flexibility, coverage, reliability, and quality of service [1]-[5]. The need to meet such ambitious expectations, while still relying on cost-effective and efficient technologies, is motivating a deep re-visitation of the paradigms currently adopted in the design and the deployment of wireless communication systems [2]-[5]. As a matter of fact, the transition between subsequent wireless communication generations has traditionally consisted in upgrading the technological solutions of the user terminals as well as of the provider base stations and network [2]-[5]. On the contrary, the propagation environment has been considered as a fundamental, but essentially uncontrollable, element/actor of the wireless scenario [2]-[5]. This viewpoint is being completely overrun by the emerging paradigm of the Smart Electromagnetic Environment (SEE) [1]-[8]. The transformative SEE vision originates from the key idea that the wireless propagation can be partially controlled by properly “tailoring” the reflection by buildings and urban structures [2]-[5][7][8]. In the SEE scenario, the environment is no longer an uncontrollable part of a wireless system, but rather it can cooperatively support the propagation to improve the coverage, the data rate, and the network reliability without the need to install additional base stations [3]-[5][8][9].

The revolutionary potentialities of the SEE are based on the exploitation of thin metasurfaces operating as smart electromagnetic (EM) skins [3][6][8][10][11]. In short, such a technology enables the meta-atomic manipulation of the reflected/transmitted wavefronts to overcome the traditional Snell’s laws [3][10]-[12]. This implies a wide set of unconventional phenomena such as anomalous reflections, focusing/lensing effects, polarization control, perfect absorption, holography, non-reciprocity, extreme energy accumulation, and enhanced security [3][10]-[12]. Depending on manipulation properties and technological constraints, different classes of smart skins have been considered. On the one hand, dynamically adjustable artificial materials operating as Reconfigurable Intelligent Surfaces (RIS) give the control of the reflected wave properties in real time [2]-[5][10], but at the cost of non-negligible implementation complexity, costs, and power consumption. On the other hand, static passive smart EM skins (SPSSs) virtually imply no running costs after installation and they potentially have advanced beamforming capabilities.
However, the design of SPSSs is very challenging because of the reduced set of degrees-of-freedom (DoFs) more severely constrained in terms of final layout complexity (e.g., passive instead of active, static instead of reconfigurable). As a matter of fact, let us notice that the arising wave manipulation device must be simple, light, and inexpensive to manufacture despite its wide EM size comprising hundreds of thousands of unit cells. Moreover, it is required to have a careful and robust macro-scale beam control for enabling beam focusing since no adjustment (e.g., no calibration or real-time control) to the reflection properties is possible after the prototyping. Furthermore, the SPSSs must be large enough to guarantee that an adequate level of power is reflected towards the whole coverage area since no “per-user” beam is allowed. This results in a huge number of micro-scale design descriptors to be optimized during the SPSS synthesis. Finally, unlike reflectarray engineering, the SPSSs performance must be yielded with a limited control on the surface orientation with respect to the incident direction of the illuminating beam.

The objective of this paper is to give some indications on the feasibility of simple and inexpensive holographic SPSSs suitable for advanced wave manipulations and beamforming. Towards this end, the complex multi-scale EM design problem at hand is firstly formulated within the Generalized Sheet Transition Condition (GSTC) theoretical framework [12][14]-[16]. Then, a phase-only inverse source (IS) approach is adopted to generalize the concepts introduced in [17] for reflectarray engineering to the synthesis of a holographic metasurface working in the SEE scenario. The footprint coverage capabilities of the smart EM skin are successively optimized by combining (a) a local search approach, based on the Iterative Projection Technique (IPT) [18] and (b) a customized version of the System-by-Design (SbD) paradigm. More in detail, the former (a) is aimed at deducing the reference/ideal surface currents affording the user-defined footprint pattern, while the other (b) is devoted to set the descriptors (i.e., the DoFs) of the SPSS for matching those reference currents. Such methodological choices, to implement a synthesis method for SPSSs, are driven by (i) the accuracy of the GSTC theory in accounting for the complex EM response of smart skins in the SEE framework [4][5][12], (ii) the effectiveness of the SbD in handling complex multi-scale design problems [19]-[23], and (iii) the intrinsic advantages of exploiting an IS formulation when determining surface currents [17] (e.g., the
possibility to introduce non-radiating components for fitting further user-requirements in terms of manufacturing, as well). Consequently, the main innovative contributions of this work lie in (i) the customization of the SbD paradigm within the GSTC framework, (ii) the combination of the SbD-based technique and of an IPT-based source synthesis process to afford complex pattern footprints with simple and inexpensive SPSS layouts, and (iii) the numerical assessment of the effectiveness of the proposed approach as well as of the feasibility of holographic SPSSs able to generate complex footprints.

The outline of the paper is as follows. The problem of designing a holographic smart passive EM skin fitting user-defined requirements is formulated in Sect. 2. Section 3 details the proposed synthesis approach. Selected numerical results, drawn from an extensive numerical validation, are illustrated in Sect. 4. Finally, some concluding remarks are reported (Sect. 5).

2 Problem Formulation

With reference to the scenario in Fig. 1 and without loss of generality, let us consider a SPSS composed by \( P \times Q \) meta-film unit cells located at the positions \( \{ r_{pq} \in \Omega; \ p = 1, \ldots, \ P; \ q = 1, \ldots, \ Q \} \), \( \Omega \) being the smart skin aperture/support, and illuminated by an incident plane wave impinging from the angular direction \( (\varphi^{inc}, \vartheta^{inc}) \) whose associated electric and magnetic fields are \[24\][25]

\[
E^{inc}(r) \triangleq \left( E^{inc}_{\perp} \hat{e}_{\perp} + E^{inc}_{\parallel} \hat{e}_{\parallel} \right) \exp \left( -j k^{inc} \cdot r \right) \tag{1}
\]

and \( H^{inc}(r) \triangleq \frac{1}{\eta_0 k^{inc}} k^{inc} \times E^{inc}(r) \), respectively, \( k^{inc} \) being the incident wave vector

\[
k^{inc} \triangleq -k_0 \left[ \sin (\vartheta^{inc}) \cos (\varphi^{inc}) \hat{x} + \sin (\varphi^{inc}) \sin (\varphi^{inc}) \hat{y} + \cos (\varphi^{inc}) \hat{z} \right], \tag{2}
\]

while \( r = (x, y, z) \) is the metasurface local coordinate, \( k_0 \) and \( \eta_0 \) being the free-space wavenumber and intrinsic impedance, respectively. Moreover, \( \hat{e}_{\perp} = \frac{k^{inc} \times \hat{n}}{|k^{inc} \times \hat{n}|} \) and \( \hat{e}_{\parallel} = \frac{\hat{e}_{\perp} \times k^{inc}}{|\hat{e}_{\perp} \times k^{inc}|} \) are the “perpendicular” and “parallel” unit vectors (i.e., TE and TM mode), respectively, while \( E^{inc}_{\perp} \) and \( E^{inc}_{\parallel} \) are the corresponding complex-valued coefficients, \( \hat{n} \) is the normal to the smart skin surface, and \( |\cdot| \) is the vector magnitude operator. In far-field, the electric field reflected by the
SPSS is given by \[ 25 \]
\[
E^{FF}(\mathbf{r}) \approx \frac{j k_0 \exp (-j k_0 |\mathbf{r}|)}{4\pi |\mathbf{r}|} \int_{\Omega} \{ \mathbf{r} \times [n_0 \mathbf{r} \times \mathbf{J}^e(\mathbf{r}) + \mathbf{J}^m(\mathbf{r})] \exp (j k_0 \mathbf{r} \cdot \mathbf{r}) \} d\mathbf{r}
\]
(3)
where \( \mathbf{\hat{r}} = \frac{\mathbf{r}}{|\mathbf{r}|} \). Moreover, the effective equivalent electric/magnetic surface current \[12\] \[25\], \( \mathbf{J}^e(\mathbf{r})/\mathbf{J}^m(\mathbf{r}) \), is computed according to the GSTC as follows \[12\] \[16\]:
\[
\mathbf{J}^e(\mathbf{r}) = j \omega B^e_t(\mathbf{r}) - \mathbf{\hat{n}} \times \nabla_t B^m_n(\mathbf{r}) \quad \mathbf{r} \in \Omega
\]
(4)
\[
\mathbf{J}^m(\mathbf{r}) = j \omega \mu_0 B^m_t(\mathbf{r}) + \mathbf{\hat{n}} \times \nabla_t B^e_n(\mathbf{r}) \quad \mathbf{r} \in \Omega
\]
(5)
where \( \varepsilon_0 \) and \( \mu_0 \) are the free-space permittivity and permeability, respectively, while \( \mathbf{B}^e(\mathbf{r}) = \mathbf{B}^e_t(\mathbf{r}) + \mathbf{B}^e_n(\mathbf{r}) \mathbf{\hat{n}} \) and \( \mathbf{B}^m(\mathbf{r}) = \mathbf{B}^m_t(\mathbf{r}) + \mathbf{B}^m_n(\mathbf{r}) \mathbf{\hat{n}} \) are the electric and the magnetic polarization surface densities whose expressions, under the local periodicity assumption and considering (sufficiently) symmetric unit cells, are \[12\] \[16\]
\[
\mathbf{B}^e(\mathbf{r}) \approx \sum_{p=1}^P \sum_{q=1}^Q \left[ \varepsilon_0 \mathbf{\chi}(\mathbf{d}_{pq}) \cdot \mathbf{E}^{ave}_{pq} \right] \Pi_{pq}^{pq}(\mathbf{r}) \quad \mathbf{r} \in \Omega
\]
(6)
\[
\mathbf{B}^m(\mathbf{r}) \approx \sum_{p=1}^P \sum_{q=1}^Q \left[ \xi(\mathbf{d}_{pq}) \cdot \mathbf{H}^{ave}_{pq} \right] \Pi_{pq}^{pq}(\mathbf{r}) \quad \mathbf{r} \in \Omega.
\]
(7)
where \( \mathbf{\chi}(\mathbf{d}_{pq}) \triangleq \sum_{i=x,y,z} \chi_{ii} \mathbf{d}_{pq} \mathbf{\hat{i}} \mathbf{\hat{i}} \) and \( \xi(\mathbf{d}_{pq}) \triangleq \sum_{i=x,y,z} \xi_{ii} \mathbf{d}_{pq} \mathbf{\hat{i}} \mathbf{\hat{i}} \) are the diagonal tensors of the electric and the magnetic local surface susceptibilities of the \((p, q)-th \) unit cell described by the \( L \)-size set \( \mathbf{d}_{pq} \triangleq \{ d_{pq}^{(l)}, l = 1, ..., L \} \), while \( \Pi_{pq}^{pq}(\mathbf{r}) \triangleq \{ 1 \text{ if } \mathbf{r} \in \Omega_{pq}, 0 \text{ if } \mathbf{r} \notin \Omega_{pq} \} \) is the basis function defined on the \((p, q)-th \) cell support \( \Omega_{pq} \) \((\sum_{p=1}^P \sum_{q=1}^Q \Omega_{pq} = \Omega) \). Moreover, \( \Psi^{ave}_{pq}(\Psi = \{ \mathbf{E}, \mathbf{H} \}) \) is the surface averaged field defined as \[12\]
\[
\Psi^{ave}_{pq} \triangleq \frac{\int_{\Omega_{pq}} [\Psi^{inc}(\mathbf{r}) + \Psi^{ref}(\mathbf{r})] d\mathbf{r}}{2 \times \int_{\Omega_{pq}} d\mathbf{r}}.
\]
(8)
where the local reflected electric/magnetic field \( \Psi^{ref} \) is given by
\[
\Psi^{ref}(\mathbf{r}) = \mathbf{\Gamma} \left[ \mathbf{\chi}(\mathbf{d}_{pq}), \xi(\mathbf{d}_{pq}) \right] \cdot \Psi^{inc}(\mathbf{r})
\]
(9)
where $\Gamma$ is the local reflection tensor \([12]\)

$$
\Gamma[\chi, \xi] \triangleq \begin{bmatrix}
\Gamma_{\perp\perp} & \Gamma_{\parallel\perp} \\
\Gamma_{\perp\parallel} & \Gamma_{\parallel\parallel}
\end{bmatrix}.
$$

(10)

According to the above derivation, the design of the holographic SPSS able to generate a desired footprint mask in a Coverage Region $\Xi$ can be carried out by solving the following two subproblems:

**Sub-Problem 1** - The synthesis of the ideal/reference surface currents, $\{[J^w(r)]^*; w = \{e, m\}\}$, that radiate a far-field pattern (3) fitting in $\Xi$ (i.e., $r \in \Xi$) the pattern requirements expressed in terms of lower, $L(r)$, and upper, $U(r)$, user-defined footprint power masks

$$
L(r) \leq \left|\left|E_{FF}(r)\right|\right|^2 \leq U(r);
$$

(11)

**Sub-Problem 2** - The retrieval of the optimal setup of the SPSS descriptors, $D^{opt} = \{d_{pq}^{opt}; p = 1, ..., P; q = 1, ..., Q\}$ so that the target surface currents computed by substituting (6) and (7) in (4) and (5) are as close as possible to the ideal ones, $\{[J^w(r)]^*; w = \{e, m\}\}$, derived in the Sub-Problem 1

$$
D^{opt} = \arg\left\{\min_{\mathcal{D}} \psi \left(J^w(r); [J^w(r)]^*\right)\right\}
$$

(12)

where \(\psi(J^w(r); [J^w(r)]^*) \triangleq \frac{\sum_{w=\{e,m\}} \|J^w(r)^* - J^w(r)\|}{\sum_{w=\{e,m\}} \|J^w(r)\|}\) is the surface currents fidelity index, while $\mathcal{D} \triangleq \{d_{pq}; p = 1, ..., P; q = 1, ..., Q\}$ and $\|\cdot\|$ stands for the $\ell_2$-norm operator.

It is worth to point out the multi-scale nature of the overall SPSS synthesis, which is aimed at fulfilling macro-scale objectives [i.e., footprint pattern features according to (11)], while acting at the unit-cell level by optimizing the small-scale descriptors of the SPSS unit cells, $\{d_{pq}^{(l)}; l = 1, ..., L; p = 1, ..., P; q = 1, ..., Q\}$. Moreover, it is very important to take into account, when defining the synthesis strategy, that the computational complexity of the problem at hand

\[7\]
is very high since the total number of descriptors, \( N_D \triangleq P \times Q \times L \), quickly grows with the smart skin aperture and the complexity of the shape of the SPSS unit cell.

### 3 Synthesis Procedure

To solve the synthesis problem formulated in Sect. 2 in terms of two sub-problems, a combination of ad-hoc customized techniques is considered and detailed in the following. As for the IS concerned with the synthesis of the ideal surface currents, \( \{ J^w (r) \}^* \), \( w = \{ e, m \} \), according to (11) (Sub-Problem 1), it suffers from ill-posedness and non-uniqueness as outlined in [17] when dealing with the design of reflectarray surface currents [26]. Moreover, it is worth pointing out that the design method used in [17] cannot be directly translated to the SPSS case since it fits a pattern matching objective instead of a “footprint pattern mask constrained” one (11). Thus, a different solution strategy inspired by the IPT [18] is proposed hereinafter. Towards this purpose, the “pattern” feasible space

\[
\mathcal{F} \{ [E^{FF} (r)]^* \} \triangleq \{ [E^{FF} (r)]^* : \mathcal{L} (r) \leq |[E^{FF} (r)]^*|^2 \leq \mathcal{U} (r) ; r \in \Xi \} \tag{13}
\]

and the “current” feasible space

\[
\mathcal{F} \{ [J^w (r)]^* \} \triangleq \{ [J^w (r)]^* : [J^w (r)]^* = C^w \exp [j\psi^w (r)] ; r \in \Omega \} \tag{14}
\]

are firstly defined, where \( C^w \) and \( \psi^w (r) \) are the constant magnitude and the profile of the locally-controlled phase of the \( w \)-th (\( w = \{ e, m \} \)) current component, respectively. While different scenarios and assumptions can be accounted for defining the feasibility spaces (13) and (14), one should notice that the statement in (14) implies that the SPSS unit cells do not to allow a control of the local magnitude of the electric/magnetic currents. Subject to these assumptions, the IPT-based design of the SPSS currents design (Fig. 2) is implemented according to the following iterative procedure (\( h = 1, ..., H \) being the iteration index) [18].
• **Initialization** \((h = 0)\) - The \(w\)-th \((w = \{e, m\})\) surface current is discretized

\[
J^w_h (r) = \sum_{p=1}^{P} \sum_{q=1}^{Q} \left[ (J^w_h)_{pq}^x \hat{x} + (J^w_h)_{pq}^y \hat{y} \right] \Pi^{pq} (r),
\]  

the expansion coefficients, \(\{(J^w_h)_{pq}^x; p = 1, \ldots, P; q = 1, \ldots, Q\}\) and \(\{(J^w_h)_{pq}^y; p = 1, \ldots, P; q = 1, \ldots, Q\}\) being set to random values such that the condition \(\|(J^w_h)_{pq}^x \hat{x} + (J^w_h)_{pq}^y \hat{y}\| = C^w \) holds true;

• **Pattern Computation** - The far-field pattern \(\mathbf{E}^{FF}_h (r)\) is evaluated in the coverage region \(\Xi\) by substituting (15) in (3);

• **Projection to Pattern Feasibility Space** - The projected pattern \(\tilde{\mathbf{E}}^{FF}_h (r)\) is obtained by setting

\[
\tilde{\mathbf{E}}^{FF}_h (r) = \begin{cases} 
\sqrt{U (r)} & \text{if } |\mathbf{E}^{FF}_h (r)|^2 > U (r) \\
\sqrt{L (r)} & \text{if } |\mathbf{E}^{FF}_h (r)|^2 < L (r) \\
\mathbf{E}^{FF}_h (r) & \text{otherwise};
\end{cases}
\]  

\(16\)

• **Convergence Check** - The algorithm is terminated by returning the ideal/reference \(w\)-th \((w = \{e, m\})\) surface current, \([\mathbf{J}^w (r)]^* = \mathbf{J}^w_N (r)\), if \(h = H\) or the pattern matching index, \(\chi_h\),

\[
\chi_h = \frac{\int_{\Xi} \left| \tilde{\mathbf{E}}^{FF}_h (r) - \mathbf{E}^{FF}_h (r) \right|^2 \, dr}{\int_{\Xi} |\mathbf{E}^{FF}_h (r)|^2 \, dr}
\]  

satisfies the condition \(\chi_h \leq \chi^*\), \(\chi^*\) being a user-chosen convergence threshold;

• **Computation of Minimum Norm Currents** - Compute the minimum norm component of the \(w\)-th \((w = \{e, m\})\) surface current, \([\mathbf{J}^w_h (r)]^{MN}\), by solving (3) with respect to the currents. Towards this end, the method based on the truncated singular value decomposition, detailed in [17], is applied;

• **Projection to Current Feasibility Space** - Update the iteration index \((h \leftarrow h + 1)\) and evaluate the \(w\)-th \((w = \{e, m\})\) projected surface current \(\mathbf{J}^w_h (r)\)
\[ J_{h}^{w}(r) = C_{w}^{u} \sum_{p=1}^{P} \sum_{q=1}^{Q} \frac{\left( (J_{h-1}^{w})_{pq}^{y} \right)^{MN} \tilde{x} + \left( (J_{h-1}^{w})_{y}^{pq} \right)^{MN} \tilde{y}}{\left( (J_{h-1}^{w})_{x}^{pq} \right)^{MN} \tilde{x} + \left( (J_{h-1}^{w})_{y}^{pq} \right)^{MN} \tilde{y}} \sqrt{\Pi_{pq}^{I}(r)}. \] (18)

Restart process from the “Pattern Computation” step.

Once the reference surface currents, \{[J^{w}(r)]^s; w = \{e, m\}\}, have been found, the Subproblem 2 is then addressed by solving (12). More in detail, an iterative SbD-based strategy inspired by [22] is customized to the problem at hand by implementing the following blocks of the functional flowchart in Fig. 2: (i) the “Solution Space Exploration (SSE)” block aimed at optimizing the SPSS descriptors by defining a succession of \(S\) iterations \(s = 1, \ldots, S\) where \(G\) trial solutions, \{\(D^{(s)}_g\); \(g = 1, \ldots, G\)\} (i.e., the population \(D^{(s)}\)), \(D^{(s)}_g \triangleq \{d_{pq|_g}^{(s)}; p = 1, \ldots, P; q = 1, \ldots, Q\}\) being the \(g\)-th one at the \(s\)-th iteration, evolve towards the global solution \(D^{opt}\) \[(12)\]. Because of the non-linear function to be optimized and the ill-posed nature of the problem at hand, a global search mechanism based on the Particle Swarm Optimizer [27] has been chosen to update/evolve the population of trial solutions at each \(s\)-th \((s = 1, \ldots, S)\) step, \(D^{(s)}\); (ii) the “Cost Function (CF)” evaluation block that implements the discretized version of \((12)\); (iii) the “Surface Current Evaluation (SCE)” block that implements \((4)\) and \((5)\) to determine \(J^{w}(r)\) starting from \(B^{w}(r), w = \{e, m\}\); (iv) the “Polarization Surface Densities Evaluation (PSDE)” block that implements \((6)\) and \((7)\) to yield the \(w\)-th \((w = \{e, m\}\) polarization surface density, \(B^{w}(r)\); (v) the “Local Susceptibility Dyadics Digital Twin (LS-DDT)” block devoted to determine \(\tilde{\chi}(d_{pq|_g}^{(s)})\) and \(\tilde{\xi}(d_{pq|_g}^{(s)})\) to be used in the PSDE block to compute the polarization surface densities at each \((s = 1, \ldots, S)\) iteration for each \(g\)-th \((g = 1, \ldots, G)\) guess solution in each \((p, q)\)-th \((p = 1, \ldots, P; q = 1, \ldots, Q)\) unit cell of the SPSS.

As for this latter and analogously to the unit cells of reflectarrays [22] [28], the full-wave evaluation of each susceptibility tensor set, \(\tilde{\chi}(d_{pq|_g}^{(s)})\) and \(\tilde{\xi}(d_{pq|_g}^{(s)})\), generated in the SbD iterative process turns out computationally unfeasible since this would require the numerical modelling and the full-wave solution of \(P \times Q \times G \times S\) SPSSs. Therefore, the dyadics \(\tilde{\chi}(d)\) and \(\tilde{\xi}(d)\) are approximated with their surrogates \(\tilde{\chi}^{\prime}(d)\) and \(\tilde{\xi}^{\prime}(d)\) defined by a trained Digital Twin (DT), which is implemented according to a statistical learning approach based on the Ordinary Kriging (OK) method [22] [28]. This choice is related to the effectiveness of the OK in defining
accurate and reliable surrogate models of wave manipulating devices \cite{22,28}. On the other hand, the reader should consider that here, unlike the reflectarray case \cite{22,28}, the DT has to predict the local susceptibility tensors rather than the local reflection coefficient. This means that 6 complex coefficients (i.e., diagonal entries of $\chi (d)$ and $\xi (d)$) must be taken into account instead of 4 terms (i.e., the $2 \times 2$ entries of the reflection matrix \cite{22,28}), but also that the DT of a SPSS can neglect the incidence angle of the illuminating field since the susceptibility tensor, unlike the reflection coefficients \cite{12}, does not depend on it.

4 Numerical Results

This section is aimed at illustrating the IPT-SbD design process and at numerically assessing its effectiveness in synthesizing holographic SPSSs suitable for footprint pattern shaping. Besides the value of the pattern matching index, $\mathcal{X}$ \cite{17}, of the SPSS final layout (i.e., $\mathcal{X}^{SPSS}$), the accuracy of each step of the synthesis process has been also “quantified” by computing the reference pattern matching $\mathcal{X}^{IPT}$ ($\mathcal{X}^{IPT} \triangleq \mathcal{X}_H$ - Sub-Problem 1) and the surface current fidelity index $\nu^{SbD}$ ($\nu^{SbD} \triangleq \nu \left( J^w (r) \right)_{s=S}; \left[ J^w (r) \right]^*$ - Sub-Problem 2). In the numerical analysis, different SPSS apertures and target footprint masks have been considered by assuming a benchmark SEE scenario where a base station illuminates from $(\theta^{inc}, \varphi^{inc}) = (20, 105)$ [deg] the smart skin with a linearly-polarized plane wave having a slant +45 [deg] polarization at $f = 30$ [GHz].

As for the metasurface unit cell, a square metallic patch with periodicity $\delta_x = \delta_y = 5.0 \times 10^{-3}$ [m] printed on a single-layer substrate (Rogers 3003 dielectric with thickness $\tau = 5.08 \times 10^{-4}$ [m]) has been used ($L = 1$) and modeled in HFSS \cite{29} for generating/training the LSDDT block \cite{22,28}. Such a simple structure has been chosen to highlight the potentials of the IPT-SbD strategy even when dealing with elementary unit cells. As for the IPT-SbD parametric configuration, the following setup has been chosen according to the guidelines in \cite{22}: $H = 10^3$, $\mathcal{X}^* = 10^{-4}$, $S = 10^4$, and $G = 10$.

The first numerical experiment deals with a $P \times Q = 200 \times 200$ holographic SPSS with an $1 \times 1$ [m] support $\Omega$ (Fig. 1) located at the position $(x', y', z') = (0, 0, 15)$ [m] in the global coordinate system, $r' = (x', y', z')$. Moreover, the upper and the lower masks have been defined so that the skin reflects a constant-power square footprint in the coverage region $\Xi$ of lateral
size 10 [m] centered at \((x', y', z') = (-25, 25, 0) [m]\) [“Square Footprint” - Fig. 3(a)], while a -30 [dB] footprint power reduction has been enforced outside \(\Xi\) in the observation region \(\Theta\) of extension \(120 \times 60 [m^2]\). According to the proposed design approach (Fig. 2), the synthesis of the \(w\)-th \((w = \{e, m\})\) ideal surface current, \([\mathbf{J}^w(\mathbf{r})]^{*}\), has been carried out by solving the associated \(IS\) problem through the \(IPT\)-based iterative procedure. The evolution of the \(IPT\) cost function during the iterative process, \(X_h (h = 1, \ldots, H) [\text{Fig. 3(b)}]\), shows that there is a quick minimization \([i.e., \frac{X_h}{X_0} < 10^{-3} \text{ when } h \geq 25 - \text{Fig. 3(b)}]\) and a convergence to a solution with a very small mismatch from the target footprint pattern, \(X_H = 6.44 \times 10^{-4}\) (Tab. I) in less than 4 minutes\(^{(1)}\) (Tab. I) thanks to the exploitation of a fast Fourier transform within the \(IPT\) loop despite the huge number of unknowns \([i.e., N_D = 4.0 \times 10^4]\). For illustrative purposes, the phase of the dominant component \([i.e., \text{slant } +45 [\text{deg}] \text{ polarization}]\) of the synthesized ideal current is reported in Fig. 4(a). Concerning the second step \((Sub-Problem 2)\) aimed at determining the \(SPSS\) layout that supports the \(IPT\)-computed reference currents, the \(SbD\) optimization process quickly \((\Delta t^{SbD} < 10 [s] - \text{Tab. I})\) yields, thanks to an accurate matching \([i.e., v^{SbD} = 2.05 \times 10^{-1} - \text{Tab. I}]\) with the reference current \([\text{Fig. 4(b) vs. Fig. 4(a)}]\), a final layout \([\text{Fig. 4(c)}]\) that faithfully fulfils the mask requirements \([i.e., X^{SPSS} = 1.08 \times 10^{-3}]\) as pictorially confirmed by the plot of the radiated footprint pattern within the observation region \([\text{Fig. 5(a) vs. Fig. 3(a)}]\). For completeness, the angular power distribution is reported in Fig. 5(b) to point out the “focusing” skills of the synthesized \(SPSS\) or, in other words, the ability of such a holographic metasurface to compensate the angular beam distortion caused by the position and the orientation of the coverage region with respect to the smart skin and the incident wave.

The feasibility of the \(SPSS\) synthesis is checked next against the more challenging “Checkerboard” footprint mask \([\text{Fig. 6(a)}]\). Although the problem at hand features a more complex target footprint, the arising holographic arrangement \([\text{Fig. 6(b)}]\) fits the radiation constraints \([\text{Fig. 7(a) vs. Fig. 6(a)}]\) with an effective angular control of the radiated power \([\text{Fig. 7(b)}]\). It is also interesting to note that the greater complexity of the pattern mask \([\text{Fig. 6(a) vs. Fig. 3(a)}]\) impacts neither on the \(CPU\) time for synthesis process \([\Delta t^{IPT}]_{\text{Checkerboard}} = 2.37 \times 10^2 [s]\)

\(^{(1)}\)For the sake of fairness, all the computation times refer to non-optimized MATLAB implementations executed on a single-core laptop running at 1.60 GHz.
vs. $\Delta t_{\text{IP T}}^{\text{Square}} = 2.31 \times 10^2$ [s] and $\Delta t_{\text{SbD}}^{\text{Square}} = 9.08$ [s] vs. $\Delta t_{\text{SbD}}^{\text{Checkerboard}} = 9.30$ [s] - Tab. I] nor on the convergence of the two-step synthesis as quantitatively confirmed by the values of the currents and pattern matching indexes in Tab. I (e.g., $\chi_{\text{SPS S}}^{\text{Checkerboard}} = 7.78 \times 10^{-4}$ vs. $\chi_{\text{SP S S}}^{\text{Square}} = 1.08 \times 10^{-3}$ and $\upsilon_{\text{SbD}}^{\text{Checkerboard}} = 2.04 \times 10^{-1}$ vs. $\upsilon_{\text{SbD}}^{\text{Square}} = 2.05 \times 10^{-1}$).

The possibility to simultaneously cover a wider region (i.e., $\Xi$ is a rectangle of $20 \times 80$ [m] modeling a short street in front of the smart skin) with a locally-complex footprint is addressed next by dealing with the “IEEE” shape in Fig. 8(a). Figure 8(b) shows the behavior of the IPT cost function $\chi_h$ during the iterative optimization of the currents distribution towards the reference one ($\chi_{\text{IP T}}^{\text{IP T}} = 3.65 \times 10^{-3}$) then approximated ($\upsilon_{\text{SbD}} = 2.06 \times 10^{-1}$) by the SbD layout in Fig. 9(a) that radiates the well controlled footprint in Fig. 9(b) ($\chi_{\text{SP S S}}^{\text{SP S S}} = 4.84 \times 10^{-3}$ vs. $\chi_{\text{IP T}}^{\text{IP T}} = 3.65 \times 10^{-3}$). As it can be observed, the synthesized SPSS not only fits the footprint mask, but also compensates the path loss to generate uniform levels of power over within the observation region $\Theta$ at considerably different distances from the smart skin [Fig. 9(b)].

Finally, the last experiment is devoted to assess the proposed design approach as well as its dependence on the SPSS aperture when dealing with advanced beamforming tasks involving detailed footprint shapes. Towards this end, the “ELEDIA” mask [Fig. 8(c)] has been considered and the SPSS design has been carried out by varying its support $\Omega$ from $P \times Q = 25 \times 25$ [i.e., $N_D = 625$ - Fig. 10(a)] up to $P \times Q = 400 \times 400$ unit cells [i.e., $N_D = 1.6 \times 10^5$ - Fig. 10(f)] and the corresponding footprints are shown in Figs. 11(a)-11(f). For completeness, Figure 12 gives the plots of the matching indexes. From these results, one can infer the following outcomes: (a) unless the smallest apertures (i.e., $P \times Q = 25 \times 25$), the proposed IPT-SbD approach can handle complex footprints [see Figs. 11(c)-11(f) vs. Fig. 8(c)]; (b) as expected, it profitably leverages the increased number of descriptors of wider apertures to improve the beamforming accuracy as quantitatively confirmed by the behavior of $\chi_{\text{SP S S}}^{\text{SPS S}} = \chi_{\text{SbD}}^{\text{SbD}}, \upsilon_{\text{SbD}}^{\text{SbD}}$ and $\chi_{\text{IP T}}^{\text{IP T}}$ in Fig. 12(b) and Tab. I as well as by the evolution of the IPT process versus the iteration number $h$ ($h = 1, ..., H$) [Fig. 12(a)]; (c) the entire synthesis process turns out to be extremely efficient whatever the number of DoFs and pattern footprint. As a representative example, the reader can consider that when $P \times Q = 400 \times 400$, the whole CPU-time is $\Delta t_{\text{IP T}} + \Delta t_{\text{SbD}} < 18$ [min].
5 Conclusions

The possibility to efficiently and effectively synthesize inexpensive smart EM skins supporting advanced beamforming capabilities has been addressed. More specifically, the design of passive/static smart skins with enhanced wave manipulation capabilities has been formulated within the GSTC theoretical framework by exploiting an IS formulation. An integrated synthesis procedure has been then proposed that combines a mask-constrained isophoric source design based on the IPT and a SbD-driven optimization for determining the SPSS layout fitting user-defined beam-pattern requirements. The feasibility of suitable cheap and passive wave manipulation holographic metasurfaces has been assessed as well as the effectiveness of the proposed synthesis approach against different footprint targets, skin dimensions, and coverage regions. The outcomes from such a numerical validation have confirmed that structurally simple yet high-performance holographic metasurfaces can be yielded [e.g., Fig. 10] with the proposed SPSS design process that efficiently handles large apertures, as well (Tab. 1). Future works, beyond the scope of this paper, will be devoted to extend such a design technique to multifunction and reconfigurable smart EM skins as well as to analyze its potentials when exploiting more complex unit cells and non-uniform/unconventional arrangements of the “re-radiating” elements [30].

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FIGURE CAPTIONS

- **Figure 1.** *Problem geometry*. Sketch of the smart EM skin scenario.

- **Figure 2.** *SPSS Design Approach*. Flowchart of the *IPT-SbD* holographic metasurface synthesis process.

- **Figure 3.** *Numerical Validation* (“Square” Footprint, $P = Q = 200$) - Plot of (a) the footprint pattern mask $[U(r'); r' \in \Theta]$ and (b) evolution of the *IPT* cost function versus the iteration index, $h (h = 1, \ldots, H)$.

- **Figure 4.** *Numerical Validation* (“Square” Footprint, $P = Q = 200$) - Plot of the phase distribution of (a) the *IPT* reference/ideal current along with (b) that generated by the synthesized *SPSS* layout (c).

- **Figure 5.** *Numerical Validation* (“Square” Footprint, $P = Q = 200$) - Plots of the radiated (a) footprint pattern within the observation region $\Theta$ and (b) angular power distribution.

- **Figure 6.** *Numerical Validation* (“Checkerboard” Footprint, $P = Q = 200$) - Plot of (a) the footprint pattern mask $[U(r'); r' \in \Theta]$ and (b) layout of the synthesized *SPSS*.

- **Figure 7.** *Numerical Validation* (“Checkerboard” Footprint, $P = Q = 200$) - Plots of the radiated (a) footprint pattern within the observation region $\Theta$ and (b) angular power distribution.

- **Figure 8.** *Numerical Validation* ($P = Q = 200$) - Plots of (a) the “IEEE” and (c) the “ELEDIA” footprint pattern mask $[U(r'); r' \in \Theta]$ along with the (b) evolution of the *IPT* cost function versus the iteration index, $h (h = 1, \ldots, H)$.

- **Figure 9.** *Numerical Validation* (“IEEE” Footprint, $P = Q = 200$) - Plots of (b) the *SPSS* layout and of (a) the corresponding footprint pattern within the observation region $\Theta$. 

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• **Figure 10.** *Numerical Validation ("ELEDIA" Footprint)* - Layouts of the synthesized SPSS when (a) $P = Q = 25$, (b) $P = Q = 50$, (c) $P = Q = 100$, (d) $P = Q = 150$, (e) $P = Q = 200$, and (f) $P = Q = 400$.

• **Figure 11.** *Numerical Validation ("ELEDIA" Footprint)* - Footprint patterns radiated in the observation region $\Theta$ by the SPSS with (a) $P = Q = 25$ [Fig. 10(a)], (b) $P = Q = 50$ [Fig. 10(b)], (c) $P = Q = 100$ [Fig. 10(c)], (d) $P = Q = 150$ [Fig. 10(d)], (e) $P = Q = 200$ [Fig. 10(e)], and (f) $P = Q = 400$ [Fig. 10(f)] unit cells.

• **Figure 12.** *Numerical Validation ("ELEDIA" Footprint)* - Plot of (a) the evolution of the $IPT$ cost function versus the iteration index, $h$ ($h = 1, ..., H$) and of (b) the matching indexes ($X^{SPSS} = X^{SbD}$, $\nu^{SbD}$, and $X^{IPT}$) versus the SPSS size.

**TABLE CAPTIONS**

• Table I. *Numerical Validation.* Matching and computational indexes.
Fig. 1 - G. Oliveri et al., “Holographic Smart EM Skins ...”
Fig. 2 - G. Oliveri et al., “Holographic Smart EM Skins ...”
Fig. 3 - G. Oliveri et al., “Holographic Smart EM Skins ...”
Fig. 4 - G. Oliveri et al., “Holographic Smart EM Skins ...”
Fig. 5 - G. Oliveri et al., “Holographic Smart EM Skins ...”
(a) 

Checkerboard Footprint, $z' = 0$

$P=Q=200$, 'Checkerboard Footprint'

(b)

Fig. 6 - G. Oliveri et al., “Holographic Smart EM Skins ...”
P=Q=200, Checkerboard Footprint, z'=0

(a)

|E_{\text{FF}}(x',y',z')|^2 [\text{dB}] (Normalized value)

(b)

Fig. 7 - G. Oliveri et al., “Holographic Smart EM Skins ...”
Fig. 8 - G. Oliveri et al., “Holographic Smart EM Skins ...”
Fig. 9 - G. Oliveri et al., “Holographic Smart EM Skins ...”
Fig. 10 - G. Oliveri et al., “Holographic Smart EM Skins ...”
Fig. 11 - G. Oliveri et al., “Holographic Smart EM Skins ...”
Fig. 12 - G. Oliveri et al., “Holographic Smart EM Skins ...”
| Footprint Name | Footprint Mask | $P \times Q$ | $\Delta t^{IPT}$ [s] | $\Delta t^{SbD}$ [s] | $\chi^{IPT}$ | $\alpha^{SbD}$ | $\chi^{SPSS}$ |
|----------------|---------------|-------------|-----------------|-----------------|-------------|--------------|--------------|
| **Square**     | Fig. 3(a)     | 200 × 200   | 2.31 × 10^2     | 9.30            | 6.44 × 10^-4 | 2.05 × 10^-1 | 1.08 × 10^-3 |
| **Checkerboard**| Fig. 6(a)     | 200 × 200   | 2.37 × 10^2     | 9.08            | 5.27 × 10^-4 | 2.04 × 10^-1 | 7.78 × 10^-4 |
| **IEEE**       | Fig. 8(b)     | 200 × 200   | 1.89 × 10^2     | 8.82            | 3.65 × 10^-3 | 2.06 × 10^-1 | 4.84 × 10^-3 |
| **ELEDIA**     | Fig. 8(c)     | 25 × 25     | 1.22 × 10^2     | 1.64 × 10^-1    | 5.70 × 10^-3 | 2.11 × 10^-1 | 6.55 × 10^-3 |
| **ELEDIA**     | Fig. 8(c)     | 50 × 50     | 1.46 × 10^2     | 6.20 × 10^-1    | 2.21 × 10^-3 | 2.07 × 10^-1 | 2.72 × 10^-3 |
| **ELEDIA**     | Fig. 8(c)     | 100 × 100   | 1.53 × 10^2     | 2.45            | 1.22 × 10^-3 | 2.03 × 10^-1 | 1.55 × 10^-3 |
| **ELEDIA**     | Fig. 8(c)     | 150 × 150   | 1.64 × 10^2     | 6.35            | 5.97 × 10^-4 | 2.04 × 10^-1 | 8.15 × 10^-4 |
| **ELEDIA**     | Fig. 8(c)     | 200 × 200   | 2.66 × 10^2     | 9.31            | 3.70 × 10^-4 | 2.07 × 10^-1 | 5.65 × 10^-4 |
| **ELEDIA**     | Fig. 8(c)     | 400 × 400   | 1.03 × 10^3     | 3.61 × 10^1     | 1.92 × 10^-4 | 2.05 × 10^-1 | 3.38 × 10^-4 |