Chiral Symmetry and Vertex Symmetry in the extended Moeller-Rosenfeld Model

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Abstract

Vertex symmetry for interacting fermions will be shown to lead to a Lagrangian exhibiting $SU(2N)_W$ invariance associated with the subgroup $SU(2N)_q \times SU(2N)_{\bar{q}}$ generated by C-odd and C-even spin operators. Approximate $SU(6)_W$ vertex symmetry as well as chiral invariance will then be shown to follow from a principle of maximum smoothness (Möller-Rosenfeld) of the bound state quark wave function.

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Introduction

In this paper we shall connect some approximate symmetries of nuclear forces to what is believed to be the fundamental theory of nuclear forces: Quantum Chromodynamics (QCD). Ideally we should be able to calculate all hadronic constants like the meson-baryon, meson-meson coupling constants (more generally the invariant form factors) and baryon and meson masses to give the strength and the range of nuclear forces, starting from the dynamics of quarks and gluons. We are still far from this goal. However, we can set ourselves more modest objectives. As an intermediate step we might concentrate on the properties of the coupling of mesons to quarks, then try to infer general properties of hadronic interactions from the assumed properties of the former. If we study the nucleon-nucleon interaction, say from the nucleon-nucleon, meson-nucleon scattering, or the spectra of light nuclei, two kinds of approximate symmetries emerge: (a) A Wigner $SU(4)$ symmetry combining isospin with spin and broken by spin dependent terms; (b) An approximate chiral symmetry broken by the pion mass. Recently more evidence has accumulated in nuclear physics extending the domain of validity to more hadronic processes\[1\]. The important role played by the $\Delta$ exchange along nucleon exchange in phenomena exhibiting chiral symmetry points out to a possible link of the $SU(4)$ symmetry (which puts $N$ and $\Delta$ in the same multiplet) with chiral symmetry\[2\]. In turn this relation can only arise from the combination of the $SU(4)$ and chiral symmetries at the deeper quark level as a consequence of QCD. Another piece of evidence comes from the role the scalar mesons $\epsilon(f)$ and $\delta(a_0)$ play in decay processes. Scalar meson couplings calculated by this method will be shown to be consistent with chiral and $SU(4)$ symmetries at the quark level. This fundamental $SU(4) \times SU(4)$ symmetry leads to a universal coupling of quarks to pseudoscalar, vector mesons as well as to scalar and pseudovector mesons. In turn the exchange of all these mesons between nucleons yield nuclear forces exhibiting an approximate chiral $SU(4) \times SU(4)$ symmetry. Arguments supporting this view will be given in this paper. It will be shown that for relativistic quarks the potential is invariant under chiral transformations and the $SU(4)$ group except for definite spin dependent terms and mass differences between mesons.

Starting from field theory of free and interacting quarks we will first examine quark spin and solve a quantum mechanical bound state problem as an approximation to field theory bound state problem by extracting a potential $V$. We will then study the general properties of the confining (long range) part of $V$ and its short range part and address models leading to a spin independent or $SU(N \times 2)$
invariant potentials (where $N$ is the number of flavors). For relativistic quarks we will show that the potential is invariant under chiral transformations and the $SU(4)$ group except for definite spin dependent terms and mass differences between mesons, and show invariance of the potential using a larger group structure.

**Quark-meson couplings**

The fundamental couplings in QCD are the self coupling of the gluons and the gluon-quark couplings which are determined uniquely by the gauge principle of local color invariance ($SU(3)_c$). Through this interaction quarks are assumed to form $q\bar{q}$ (meson) and $(qqq)$ (baryon) bound states. Because of asymptotic freedom, quarks behave like quasi-free particles at small separation. There is a vertex function between the $q\bar{q}$ meson bound state and the quasi free $q$ and $\bar{q}$. For each quantum state of the $q\bar{q}$ system this vertex function can be approximated by a constant which we shall call the quark-meson coupling constant $g_{q\bar{q}M}$. Then, under the additivity hypothesis the meson-nucleon coupling constant is

$$\Gamma_{NNM} = 3 g_{q\bar{q}M}. \quad (1)$$

The relativistic coupling constants are defined in terms of an effective quark meson interaction Lagrangian

$$\mathcal{L}_{\text{int}} = g_s \bar{q}\tau_a q\chi^a + h_s \bar{q}\tau_a \sigma_{\mu\nu} q\partial_\mu \chi^a + g_{Psv} \bar{q}\gamma_5 \gamma_\mu qW^a_{\mu}\nu$$

$$+ h_{Psv} \bar{q}\tau_a \sigma_{\mu\nu} q\tilde{W}^a_{\mu}\nu + g_{Psv} \bar{q}\gamma_5 \tau_a \phi^a + h_{Psv} \bar{q}\gamma_5 \gamma_\mu q\partial_\mu \phi^a$$

$$+ g_v \bar{q}\tau_a \gamma_\mu qV^a_\mu + h_v \bar{q}\tau_a \sigma_{\mu\nu} qV^a_{\mu\nu} \quad (2)$$

where $\alpha = 0, 1, 2, 3$, $\tau_0 = 1$ and

$$V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu, \quad W^a_\mu = \partial_\mu W^a_\nu = \partial_\nu W^a_\mu, \quad (3)$$

$$\tilde{W}^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\kappa} W^a_{\rho\kappa}, \quad (4)$$

$$\partial_\mu V^a_\mu = 0, \quad \partial_\mu W^a_\mu = 0. \quad (5)$$

Also note that the $h_s$ term in Eq. (2) is a total divergence and can be omitted. The derivative (momentum dependent) interactions are important since $\mathcal{L}_{\text{int}}$ is
an effective Lagrangian and there is no renormalizability constraint in the Lagrangian. The constants $g$ and $h$ can be calculated from QCD following a definite scheme. For example at short distances coupling is similar to the probability of dissociation of positronium into its constituents, the massless photon field playing the role of the gluon field. For group theoretical treatment of this problem see Barut and Aydin. Rather than embarking on such a calculation we can review the available phenomenological information on these couplings, starting from the nucleon-meson vertex.

Also note that because $SU(4) \times SU(4)$ is embedded in $SU(6) \times SU(6)$, there are two pseudoscalar fields $\phi^0$, namely $\eta$ and $\eta'$ and two vector fields $V^0_\mu$, namely $\omega_\mu$ and $\phi_\mu$, with $\eta'$ being approximately $SU(4)$ singlet. There is a similar doubling of isosinglets for scalars and pseudovectors. We have the following identifications:

$$
\begin{align*}
\phi^0 & : \eta, \eta' \\
\phi^i & : \pi^i (i = 1, 2, 3) \\
V^0_\mu & : \omega_\mu, \phi_\mu \\
V^i_\mu & : \rho^i_\mu \\
\chi^0 & : \epsilon(f_0) \\
\chi^i & : \delta^{i}(a^i_0) \\
W^0_\mu & : E_\mu \\
W^i_\mu & : A^{(1)i}_\mu (a^{(1)i}_0)
\end{align*}
$$

To determine the representation we start from the quarks $q^\alpha_L$ and $q^\alpha_R$ ($\alpha = 1, 2$), with $\alpha$ the isospin index, and with further identifications

$$
\begin{align*}
\bar{q}^\alpha_L & : (4, 1), \quad \bar{q}^\alpha_R : (1, 4) \\
\bar{q}^\alpha_L = i\sigma_2 q^\alpha_L & : (4, 1), \quad \bar{q}^\alpha_R = i\sigma_2 q^\alpha_R : (1, 4)
\end{align*}
$$

$$
\bar{q}^\tau a q = q^\dagger_L \tau^a q_R + q^\dagger_R \tau^a q_L, \quad \bar{q}^\gamma_5 \tau^a q = q^\dagger_L \tau^a q_R - q^\dagger_R \tau^a q_L
$$

so that
\[ \frac{1}{2} \tilde{q} \left( 1 + \frac{\gamma_5}{2} \tau^a q \right) \longleftrightarrow \frac{1}{2} q_L \tau^a q_R : (\bar{4}, 4) \quad \pi, \quad \delta(\alpha_0) \]
\[ \frac{1}{2} \tilde{q} \left( 1 - \frac{\gamma_5}{2} \tau^a q \right) \longleftrightarrow \frac{1}{2} q_R^\dagger \tau^a q_L : (4, \bar{4}) \quad \epsilon(\eta, f_0) \]
\[ \frac{1}{2} \tilde{q} \tau^a \gamma_5 \gamma_5 \sigma_{\mu\nu} q \longleftrightarrow q_L^\dagger \tau^a \sigma^i q_R : (\bar{4}, 4) \quad \rho^a_{\mu\nu}, \quad B_{\mu\nu}^a, \quad A^a_{\mu\nu} \]
\[ \frac{1}{2} \tilde{q} \tau^a \gamma_5 \gamma_5 \sigma_{\mu\nu} q \longleftrightarrow q_R^\dagger \tau^a \sigma^i q_L : (4, \bar{4}) \quad \omega_{\mu\nu}, \quad B_{\mu\nu}^0, \quad a_{\mu\nu}^0 \]
\[ \tilde{q} \tau^a \frac{1 + \gamma_5}{2} \gamma_\mu q \longleftrightarrow q_L^\dagger \tau^a \sigma^i q_L : (15, 1) + (1, 1) \]
\[ \tilde{q} \tau^a \frac{1 - \gamma_5}{2} \gamma_\mu q \longleftrightarrow q_R^\dagger \tau^a \sigma^i q_R : (1, 15) + (1, 1) \]  \hspace{1cm} (9)

Thus, each meson is described by its generalized coordinates and generalized momenta (derivative field). The combination of these fields in the interaction Hamiltonian generate the \((\bar{4}, 4), (4, \bar{4}), (15, 1), (1, 15)\) and \((1, 1)\) representation of the chiral \(SU(4) \times SU(4)\).

The Dirac \(q^a\) quark Lagrangian can be regarded as being \(SU(4) \times SU(4)\) invariant since the \(u\) and \(d\) quarks can be taken to have negligible masses in the first approximation.

A mass term for the quarks that can be induced by a non-zero vacuum expectation value of the isosinglet scalar field will behave like the neutral component of \((4, \bar{4}) + (\bar{4}, 4)\), breaking the symmetry down to \(SU(4)\) and isospin \(SU(2)\).

Under the \(SU(4)\) subgroup of \(SU(4) \times SU(4)\), the nucleon \(N\) and the \(\Delta\) resonance form the 20-dimensional representation contained in the symmetric part of the product \(4 \otimes 4 \otimes 4\). The various derivative couplings of \(N\) and \(\Delta\) are contained in \([(4, 1) + (1, 4)]^3\) product representations of \(SU(4) \times SU(4)\).

Let us now briefly look at the free quark approximation to QCD and vertex symmetry for interacting fermions, then buildup the symmetry in the potential approximation. We shall later return also to embedding these structures in larger groups and look at group decompositions.
The Free Quark Approximation to QCD

First consider the simplified case of colored quarks with no flavor, interacting through the exchange of color gluons and obeying the Dirac equation

\[ \gamma_\mu D_\mu \psi^i = m \psi^i \quad (i = 1, 2, 3) \]

(10)

where \( m \) is the quark mass, \( i \) the color index and \( D_\mu \) the covariant derivative

\[ D_\mu = \partial_\mu + \frac{i}{2} g \lambda^a B^a_\mu, \quad (a = 1, \ldots, 8) \]

(11)

where \( g \) is the quark-gluon coupling constant and the \( \lambda \)’s are the eight \( 3 \times 3 \) Gell-Mann matrices associated with \( SU(3) \). When we consider a quark and an antiquark localized at very short distance separation, then, from asymptotic freedom \([4]\) we know that the renormalized strong fine structure constant \( g^2/4\pi = \alpha_s \) tends to zero at high momentum transfers (short spatial separation). In such a situation the covariant derivative \( D_\mu \) can be replaced by \( \partial_\mu \) and in the zeroth approximation we can treat the quark inside the hadron as obeying the free Dirac equation for small \( x \), the origin being the center of mass of the quark system in the hadron. Then we can write

\[ \psi(x) = \int \frac{d^3p}{\sqrt{2p_0}} e^{\frac{i}{2} \gamma_5 \sigma \cdot \lambda(p)} e^{i \gamma_4 p \cdot x} (a_p + \gamma_5 \hat{b}_p) \]

(12)

where

\[ a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \hat{b} = i \sigma_2 b^* = \begin{pmatrix} b^*_2 \\ -b^*_1 \end{pmatrix} \]

(13)

\[ \lambda(p) = \frac{P}{|P|} \tanh^{-1} \frac{|P|}{p_0} \]

(14)

\[ a_p = \begin{pmatrix} a_p \\ \gamma_4 a_p \end{pmatrix}, \quad \text{if } \gamma_5 \text{ is diagonal} \]

and

\[ a_p = \begin{pmatrix} a_p \\ 0 \end{pmatrix}, \quad \text{if } \gamma_4 \text{ is diagonal} \]

(15)
\(a^\dagger_{p,s_z}, b^\dagger_{p,s_z}\ (s_z = 1, 2)\) are respectively creation operators for particle and antiparticle Wigner states for the Poincaré group corresponding to momentum \(p\) and spin component \(s_z\), i.e.

\[
a^\dagger_{p,s_z}|0> = |m, s = \frac{1}{2}\cdot p, s_z>
\]

with

\[
p_0^2 - |p|^2 = m^2
\]

The exponential can be evaluated as

\[
ex^\left\{\frac{1}{2}\gamma_5\sigma\cdot\lambda(p)\right\} = \sqrt{p_0 + \gamma_5\sigma\cdot p} = \frac{m + p_0 + \gamma_5\sigma\cdot p}{\sqrt{2m(m + p_0)}}
\]

We also have the useful identity

\[
ex^\left\{\frac{1}{2}\gamma_5\sigma\cdot\lambda(p)\right\} \frac{1 + \gamma_4}{2} = \sqrt{\frac{p_0 + i\gamma_\cdot p}{p_0}} \frac{1 + \gamma_4}{2}
\]

The projection operators \(\frac{1}{2}(1 \pm \gamma_4)\) project the large (+) or small (−) components of the free wave function. To simplify the formulae we have also omitted the color index. For a more general description in terms of boost operators and Heisenberg algebras satisfied by these operators we refer to our earlier paper \[5\].

We have the following conserved quantities

(a) Quark Number:

\[
B = \int j_\mu d\sigma = \int j_0 d^3 x
\]

where

\[
j_\mu = \bar{\psi}_i \gamma_\mu \psi_i, \quad (i = 1, 2, 3); \quad \partial_\mu j_\mu = 0
\]

This is the number of quarks minus the number of antiquarks.

(b) Charges

Putting back the flavor index \(r\) in the case of 3 quarks

\[
Q^a = \frac{1}{2} \int \bar{\psi}^r \gamma_\mu \lambda^a_{rs} \psi^s d\sigma^\mu
\]

(c) Energy Momentum

\[
P_\mu = \int T_{\mu\nu} d\sigma^\nu, \quad T_{\mu\nu} = \bar{\psi}^r \gamma_\mu \partial_\nu \psi^r
\]
The Hamiltonian $P^0$ is given by:

$$H = P^0 = \int p^0 (a^+_p a_p + b^+_p b_p) d^3 p$$  \hspace{1cm} (24)

(d) Conserved Spin Operators

We now make the Tani\[6\]-Foldy-Wouthuysen\[7\] transformation which is the same as the Wigner transformation on particle states. Let us rewrite the Dirac equation in the form

$$iH\psi = (im\gamma_4 - i\gamma_4 \gamma_5 \partial_n)\psi$$  \hspace{1cm} (25)

$(i\partial_n = p_n$ on a momentum eigenstate).

The covariant states are obtained by applying on the vacuum the operator $\psi^c = \gamma_2 \psi^\ast$, the charge conjugate operator obtained by exchanging $a$’s and $b$’s.

$$\psi^c|0\rangle = \int d^3 p \exp\{\frac{i}{2}\gamma \cdot \lambda(p)\}|p\rangle e^{ip \cdot x}$$  \hspace{1cm} (26)

where we have used

$$\gamma_5 \sigma = i \gamma_4 \gamma, \quad \gamma_4 \hat{a}_p|0\rangle = \hat{a}_p|0\rangle$$  \hspace{1cm} (27)

and Eq.(19). To obtain the covariant state from the Wigner state we must then apply a non-local operator on $|p\rangle \exp(ip \cdot x)$. We find

$$\psi^c|0\rangle = W(-i\nabla) \int \hat{a}_p e^{ip \cdot x} (d^3 p)|0\rangle$$  \hspace{1cm} (28)

where

$$W(-i\nabla) = \sqrt{\frac{m + \gamma \cdot \nabla}{(m^2 - \nabla^2)^{\frac{1}{2}}}}$$  \hspace{1cm} (29)

so that the operator

$$\varsigma(x) = W(-i\nabla)\psi(x)$$  \hspace{1cm} (30)

creates Wigner antiquark states and $\varsigma^c(x)$ creates quark states. The non-local operator $W(-i\nabla)$ is a unitary operator that connects covariant states with Wigner states.
We can now transform the Hamiltonian with $W$. Transforming Eq. (25) we find
\begin{equation}
H' = WHW^{-1} = \gamma_4 E_p = \gamma_4 \sqrt{m^2 + |p|^2} = \gamma_4 (m^2 - \nabla^2)^{1/2}
\end{equation}
(31)

Thus, the transformation diagonalizes $H$ in a representation for which $\gamma_4$ is diagonal. The expansion of Eq. (31) in $m^{-2} \nabla^2$ gives the Schrödinger operator for the upper and lower components separately, plus relativistic correction terms. Note that $H'$, unlike $H$, commutes with the spin operators $\sigma_n$ and with $\gamma_4 \sigma_n$

\begin{equation}
[\sigma_n, WHW^{-1}] = 0
\end{equation}
(32)

or

\begin{equation}
[W^{-1} \sigma_n W, H] = 0
\end{equation}
(33)

Hence, the non-local objects
\begin{equation}
\frac{1}{2} \sigma_n' = \frac{1}{2} W^{-1} \sigma_n W, \quad \frac{1}{2} (1 \pm \gamma_4) \sigma_n'
\end{equation}
(34)

are constants of the motion. These are the non-covariant spin operators that are conserved for free quarks and approximately conserved in QCD. Thus the conserved generators of the relativistic but non-covariant $SU(6) \times SU(6)$ are

\begin{equation}
\frac{1}{2} (1 \pm \gamma_4) \{ \frac{1}{2} \sigma_n', \frac{1}{2} \lambda^a, \frac{1}{4} \sigma_n' \lambda^a \}
\end{equation}
(35)

If we work with the set of generators $\frac{1}{2} \sigma_n, \frac{1}{2} \lambda^a, \frac{1}{4} \sigma_n \lambda^a$ where $n = 1, 2, 3$ and $a = 1, \cdots, 8$ we must use the transformed Hamiltonian $H'$. We now turn to another transformation of the Hamiltonian that singles out a direction and therefore is valid on the light cone. For extremely relativistic quarks the expansion of $H$ in $m^{-2} \nabla^2$ or the velocity square is not a good one. In this case we can remove transverse components of $p$, say $p_\perp$ ($p_1$ and $p_2$ if the direction singled out is the third axis) by a unitary transformation. This is the Cini-Touschek\cite{8} transformation rediscovered by Melosh\cite{9}. It is given by

\begin{equation}
W_\perp = \exp \left\{ \frac{i}{2} \tan^{-1} \frac{p_\perp}{m} \gamma_\perp \cdot p_\perp \right\}
\end{equation}
(36)

where

\begin{equation}
\gamma_\perp \cdot p_\perp = \gamma_1 p_1 + \gamma_2 p_2, \quad p_\perp = \sqrt{p_1^2 + p_2^2}
\end{equation}
(37)
\[ p_n = -i \partial_n \]  

Then the transformed Hamiltonian takes the form

\[ H'' = W^{-1}_\perp H W = \gamma_5 \sigma_3 p_3 + \gamma_4 \sqrt{m^2 + p_\perp^2} \]  

Note that \( H'' \) no longer commutes with \( \sigma_n \) and \( \gamma_4 \sigma_n \) but only with the subset

\[ \frac{1}{2} \gamma_4 \sigma_1, \quad \frac{1}{2} \gamma_4 \sigma_2 \quad \text{and} \quad \frac{1}{2} \sigma_3 \]  

that also generates an \( SU(2) \) group. Then

\[ \frac{1}{2} \sigma''_1 = \frac{1}{2} W^{-1}_\perp \gamma_4 \sigma_1 W_\perp, \quad \frac{1}{2} \sigma''_2 = \frac{1}{2} W^{-1}_\perp \gamma_4 \sigma_2 W_\perp \]

\[ \frac{1}{2} \sigma''_3 = \frac{1}{2} W^{-1}_\perp \sigma_3 W_\perp \]  

also generate rotations and the generators of the \( SU(6)_W \) group

\[ \frac{1}{2} \sigma''_n, \quad \frac{1}{2} \lambda^a, \quad \frac{1}{4} \sigma''_n \lambda^a \]  

 commute with the original Hamiltonian \( H \).

The operators Eq. (40) are the Stech\[10\] spin operators. They were rediscovered and called \( W \)-spin operators by Lipkin and Meshkov\[11\]. We have seen that they are not conserved quantities, but their Cini-Touschek transforms \( \sigma''_n \) are conserved in the free quark limit. This property was discovered by Melosh\[9\].

The importance of the spin operators \( \sigma''_n \) is that they also occur in the symmetry of the vertex in which the singled out direction is given by the momentum transfer.

The meaning of \( \gamma_4 \) can be elucidated by exhibiting the transformation properties of the conserved quantities\[5,12\]. We distinguish between C-odd and C-even conserved covariant spin tensors. The C-even tensor is given by

\[ \Omega_{\mu\nu} = -i \int H_{\mu\nu\lambda} d^3x \]  

where

\[ H_{\mu\nu\lambda} = -\frac{1}{4} \bar{\psi} \sigma_{\mu\nu} \partial_\lambda \psi = -\frac{i}{2} \epsilon_{\mu\nu\lambda\rho} \bar{\psi} \gamma_5 \gamma_\rho \psi + \text{additional terms} \]
is conserved:

\[ \partial_\lambda H_{\mu\nu\lambda} = 0 \]  \hspace{1cm} (45)

so that the \( \Omega_{\mu\nu} \) are time independent. These are related to the Wigner spin as

\[ W^{-1} \Omega_0 W = 0 \]

\[ \frac{1}{2} \epsilon_{ijk} W^{-1} \Omega_{jk} W = \Omega_i = \frac{1}{2} \int (a_p^\dagger \sigma_i a_p + b_p^\dagger \sigma_i b_p) d^3 p \]  \hspace{1cm} (46)

Hence they can be interpreted as quark spin plus antiquark spin and are associated with the \( \sigma_i' \).

Note that \( \Omega_i \) generate \( SU(2) \) but not the covariant quantities \( \Omega_{\mu\nu} \). The \( C \)-odd conserved covariant tensor is

\[ \Omega_5 = -i \int H_{0\mu} d^3 x \]  \hspace{1cm} (47)

where

\[ H_{\mu\nu} = -i \frac{\bar{\psi} \gamma_5 \gamma_\mu \gamma_\nu \psi}{4m} = \frac{1}{2} \bar{\psi} \tilde{\sigma}_{\mu\nu} \psi + \text{add. terms} \]  \hspace{1cm} (48)

and

\[ \partial_\nu H_{\mu\nu} = 0 \]  \hspace{1cm} (49)

Making the unitary \( W \) transformation we find the non-covariant objects

\[ \Omega'_i = W^{-1} \Omega_5 W = \frac{1}{2} \int (a_p^\dagger \sigma_i a_p - b_p^\dagger \sigma_i b_p) d^3 p \]  \hspace{1cm} (50)

\[ W^{-1} \Omega_{50} W = O \]  \hspace{1cm} (51)

These conserved quantities are the quark spin minus the antiquark spin and correspond to \( \gamma_4 \sigma_i' \). The \( \frac{1}{2}(\Omega_i \pm \Omega'_i) \) generate the \( SU(2) \times SU(2) \) group of quark and antiquark spin. They correspond to \( \frac{1}{2}(1 \pm \gamma_4)\sigma_i' \). These are sometimes called the \( f \) spin and the \( \bar{f} \) spin.

When we do the Cini-Touschek-Melosh transformation on \( \Omega_{\mu\nu} \) and \( \Omega_{5\mu} \), the \( C \)-odd operators \( \Omega'_1, \Omega'_2 \) and the \( C \)-even \( \Omega_3 \) commute with the Hamiltonian \( H'' \).
If now we let $p_3 \to \infty$ then
\[ \Omega_3 \to \frac{1}{2} \sigma_3 \to \frac{1}{2} \gamma_5 \tag{52} \]
and the $C$-even subgroup of $SU(6)$ becomes the chiral $SU(3) \times SU(3)$ group whose generators commute with massless Dirac Hamiltonian, so that
\[ \lim_{p_3 \to \infty} [SU(3) \times SU(3)]_{\sigma_3} = [SU(3) \times SU(3)]_{\gamma_5} \tag{53} \]

To sum up, the free Dirac Hamiltonian in the Foldy-Wouthuysen form admits a $SU(2N)_q \times SU(2N)_{\bar{q}}$ symmetry where $N$ corresponds to the dimension of the internal space and $q, \bar{q}$ refer respectively to quarks and antiquarks. If a longitudinal direction is singled out by an additional term in the Lagrangian, then the Dirac Hamiltonian can be transformed by a Cini-Touschek transformation relative to the longitudinal direction and the Hamiltonian has a $SU(2N)_W$ invariance associated with the subgroup of $SU(2N)_q \times SU(2N)_{\bar{q}}$ involving the $C$-odd transverse spin and the $C$-even longitudinal spin.

**Vertex Symmetry for Interacting Fermions**

Consider the coupling of the fermion $\psi$ to a boson $\phi$. If the virtual boson with momentum $q$ is emitted by the fermion $\psi(p)$ in momentum space so that the fermion wave function changes from $\psi(p)$ to $\psi(p')$, then $p - p' = q$ is the momentum transfer. $\psi(p)$ and $\psi(p')$ may refer to states with different internal quantum numbers such as $u$ and $d$ quarks. In the approximation that these states are degenerate in mass the momentum transfer $q$ is space-like and singles out a space direction. Thus a Yukawa interaction term will involve a direction in momentum space (the $q$ direction) which can be taken as the direction singled out by the Cini-Touschek transformation.

Then the interacting Hamiltonian can have a non-local ($q$-dependent) $SU(6)_W$ symmetry, provided the free boson Lagrangian also exhibits an $SU(6)_q \times SU(6)_{\bar{q}}$ symmetry generated by C-even and C-odd spin operators. Hence we need two kinds of bosons to achieve vertex symmetry: C-even bosons $\phi$ and C-odd bosons $\omega_{\mu}$. Those which have the same parity can be grouped in the same multiplet. Negative parity bosons will transform like the $S$-states of the $(\bar{q}q)$ system, so that we can choose multiplets of $J^{PC}$ bosons which transform like $\psi \gamma_5 \psi (0^{-+})$ and $\bar{\psi} \gamma_{\mu} \psi (1^{-+})$. 

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Let us write such a Yukawa coupling in the case of no internal symmetry

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} \]

\[ \mathcal{L}_0 = \mathcal{L}_0(\psi) + \mathcal{L}_0(\phi) + \mathcal{L}_0(\omega_\mu) \]

\[ \mathcal{L}_{int} = ig\bar{\psi}(\gamma_5\phi + \gamma_\mu\omega_\mu)\psi \quad (54) \]

where \( \mathcal{L}_0(\psi) \), \( \mathcal{L}_0(\phi) \) and \( \mathcal{L}_0(\omega_\mu) \) are respectively free lagrangians for the fermion, the pseudoscalar and the vector bosons, with \( \mathcal{L}_0(\omega_\mu) \) chosen so that the equations of motion also yield the subsidiary condition

\[ \partial_\mu\omega_\mu = 0 \quad (55) \]

essential for associating \( \omega_\mu \) with a \( 1^{--} \) object, and \( \mathcal{L}_{int} \) is the simplest non-derivative Yukawa coupling for the fermion-boson system.

We could now find the Fourier transform of \( \mathcal{L} \) by introducing the Fourier transforms \( \tilde{\psi}(p') \), \( \tilde{\psi}(p) \) and \( \tilde{\phi}(q) \), \( \tilde{\omega}_\mu(q) \), and then do a Cini-Touschek transformation with respect to \( q \). Instead we shall consider a simplified situation corresponding to quasi-free heavy fermions that will justify the static limit. Then

\[ \xi_\pm = \frac{1}{2}(1 \pm \gamma_4)\psi \quad (56) \]

correspond approximately to the particles and antiparticles. Writing \( M \) for the large quark mass, we can consider an effective interaction of the quarks with mesons (\( \phi \) and \( \omega_\mu \)) that are in fact \( q\bar{q} \) bound states bound by gluon exchange forces. If \( \psi \) is quasi-free we can write

\[ \bar{\psi}\gamma_5\psi = \frac{1}{2M}\partial_\mu(\bar{\psi}\gamma_5\gamma_\mu\psi) \]

\[ ig\phi\bar{\psi}\gamma_5\psi = \frac{-ig}{2M}\bar{\psi}\gamma_5\gamma_\mu\psi\partial_\mu\phi + \text{total divergence} \]

\[ = \frac{-ig}{2M}\bar{\psi}\gamma_5\gamma_4\psi\partial_0\phi - \frac{ig}{2M}\bar{\psi}\gamma_5\gamma \cdot (\nabla \phi)\psi \quad (57) \]

The first term is of the order \( \frac{\mu^2}{2M} \) if \( \mu \) is the meson mass and negligible in the static limit. In the second term the operator \( \gamma_4\gamma_5\gamma = -\sigma \) commutes with \( \gamma_4 \) and as
a result does not mix particle states with antiparticle states, so that putting $\xi_+ = \xi$ (particle wave function) we can make the replacement

$$ig\phi \bar{\psi} \gamma_5 \psi \rightarrow \frac{ig}{2M} \xi^\dagger \sigma \cdot (\nabla \phi) \xi$$  \hspace{1cm} (58)

For the vector meson $\omega_0$ is not independent of the independent variables $\omega_i$ ($i = 1, 2, 3$) and we can write $ig\bar{\psi} \gamma \cdot \omega \psi = g\psi^\dagger \gamma_5 \sigma \cdot \omega \psi$. Expressing small components in terms of the large components in the free-Dirac approximation, and neglecting total divergence terms we can make the replacement

$$g\psi^\dagger \gamma_5 \sigma \cdot \omega \psi \rightarrow \frac{ig}{2M} \xi^\dagger \left[\left(\sigma \times \nabla \right) \cdot \omega \right] \xi + \frac{ig}{2M} \xi^\dagger \left(\nabla \cdot \omega \right) \xi$$  \hspace{1cm} (59)

In momentum space $i\nabla$ gives the virtual space-like momentum $k$ of the mesons, so that if we call $\sigma_\parallel$ and $\sigma_\perp$ the longitudinal and transverse spin operators we have

$$i\sigma \cdot \nabla = |k| \frac{\sigma \cdot k}{|k|} = |k| \sigma_\parallel$$  \hspace{1cm} (60)

$$i\sigma \times \nabla = |k| \frac{\sigma \times k}{|k|} = |k| \sigma_\perp$$  \hspace{1cm} (61)

and the interaction term takes the approximate form

$$\mathcal{L}_{int} = \frac{g|k|}{2M} \psi^\dagger \left(\sigma_\parallel \phi + \gamma_4 \sigma_\perp \cdot \omega_\perp + \omega_\parallel \right) \psi$$

$$\sim \frac{g|k|}{2M} \xi^\dagger \omega_\parallel \xi + \frac{g|k|}{2M} \xi^\dagger \left(\sigma_\parallel \phi + \sigma_\perp \cdot \omega_\perp \right) \xi$$  \hspace{1cm} (62)

with $\phi$ and $\omega_\perp$ forming an $SU(2)_W$ triplet. $\omega_\parallel = \frac{k \omega}{|k|}$ is a singlet. If we interpret $\mathcal{L}_{int}$ in $x$ space, then

$$|k| = \sqrt{-\nabla^2}$$  \hspace{1cm} (63)

$$\sigma_\parallel = W^\dagger (-i\nabla) \sigma_3 W (-i\nabla) = -\frac{i\sigma \cdot \nabla}{\sqrt{-\nabla^2}}$$

$$\sigma_\perp = W^\dagger (-i\nabla) \sigma_1 W (-i\nabla)$$

$$\sigma_\perp = W^\dagger (-i\nabla) \sigma_2 W (-i\nabla)$$  \hspace{1cm} (64)
\[ W = \sqrt{\frac{M + \sigma \cdot \nabla}{(M^2 - \nabla^2)^{\frac{1}{2}}}} \tag{65} \]

with
\[ W^\dagger (-i \nabla) \ W (-i \nabla) = 1 \tag{66} \]

Both \( \sigma_\parallel \) and \( \sigma_\perp \) generate \( SU(2)_W \) and combine with an internal \( SU(N) \) to give \( SU(2N)_W \). Performing a Cini-Touschek-Melosh transormation in the \( k \) direction for the kinetic term we see that the generators of this \( SU(2N)_W \) commute with the transformed Hamiltonian including the effective approximate interaction term.

Here it is important to note that the Yukawa coupling Eq.\((54)\) is invariant under the local symmetry group \( SO(4, 1) \) acting on the five dimensional representation \((\phi, \omega_\mu)\) of \( SO(4, 1) \). But this group does not leave \( L_0 \) invariant since Eq.\((55)\) that is a consequence of the choice of \( L_0 \) is not covariant under \( SO(4, 1) \). Only the non-local \( SU(2)_W \) is the symmetry of \( L = L_0 + L_{\text{int}} \). If we include the internal symmetry group \( SU(3) \), then we can write a Yukawa interaction term that has \( SU(6, 6) \) invariance instead of \( SO(4, 1) \) in our simplified example. This is the group introduced by Salam et. al. As is well known, \( SU(6, 6) \) cannot be a symmetry of the Lagrangian. But as far as we can separate positive and negative frequency parts of \( \psi \) we have an approximate \( SU(6)_W \) symmetry for the whole Lagrangian. In OCD where the quarks are quasi-free inside the hadron, such an approximation is justifiable because of asymptotic freedom. In that sense we may say that the approximate vertex symmetry mixing spin and internal degrees of freedom like isospin is better understood within the QCD framework. The covariant generalization involves the spin current densities Eq.\((44)\) and Eq.\((48)\). The corresponding covariant spin operators \( \Omega_\mu \) and \( \Omega_5 \) that are conserved in the free quark approximation do not lead to a close algebra in general. However it can be shown\([12]\) that the matrix elements of the transverse part of \( \Omega_5 \) and the longitudinal part of \( \Omega_\mu \) between one particle states do lead to closure and to \( SU(6)_W \) symmetry in a special frame.

Since mesons are extended objects, a quark-meson coupling such as Eq.\((54)\) cannot be a local interaction. In general there will be a form factor \( F(q) \), where \( q \) is the momentum transfer at the vertex. Expanding the form factor in powers of \( q \) is equivalent to writing effective interactions involving higher derivatives of the meson fields. As an example we can write an effective Lagrangian with pseudovector coupling for the pseudoscalar meson and a Pauli coupling for the vector
meson $\omega_\mu$, so that

$$L'_\text{int} = \frac{f}{2M} \bar{\psi} \left[ i\gamma_5 \gamma_\mu \partial_\mu \phi + \frac{1}{2} \sigma_{\mu\nu} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu) \right] \psi$$  \hspace{1cm} (67)$$

where $f$ is a new coupling constant. We can also write

$$L'_\text{int} = \frac{f}{2M} \bar{\psi} \left[ \gamma_5 \partial_0 \phi + \sigma \cdot \nabla \phi + i\gamma_5 \gamma_4 \sigma \cdot (\partial_0 \omega - \nabla \omega_0) + \gamma_4 (\sigma \times \nabla) \cdot \omega \right] \psi$$  \hspace{1cm} (68)$$

Taking the momentum $k$ of the exchanged mesons to be space-like and along the third (longitudinal) direction, we find

$$L'_\text{int} = \frac{f}{2M} \bar{k} \psi \left[ \gamma_5 \partial_0 \phi + \gamma_4 \sigma \perp \cdot \omega \perp \right] \psi + O(k^2)$$  \hspace{1cm} (69)$$

again exhibiting the W-spin invariance of the vertex.

### Symmetry in the Potential Approximation

In the one boson exchange approximation between heavy quarks we consider the case when the quarks emit two gluons that can turn into a color singlet, C-even, $q\bar{q}$ bound state (effectively a pseudoscalar meson $\phi$) or the alternative case when they emit three gluons that go into a color-singlet, C-odd, $q\bar{q}$ bound state (effectively a vector meson $\omega_\mu$). Then the mesons $\phi$ and $\omega$ will have effective coupling constants ($g$ for non derivative and $f$ for derivative couplings) arising from the expansion of their form factors. We can use these couplings also to describe the baryon baryon potential due to meson exchange. Indeed the baryons host three quasi-free quarks each. Following the usual quark additivity assumption we shall let one quark from each baryon interact at a time, while the other (spectator) quarks remain idle. When the active quarks emit several gluons in a colored state, that state cannot propagate due to color confinement. The active quarks can make contact only through emission of two or three gluon color singlet states which, in turn can create mesons in intermediate states. Hence $g$ and $f$ will also be effective coupling constants for the baryon-meson interactions and will enter in the calculation of effective baryon baryon potentials due to meson exchange. Thus, starting from a scenario in QCD we are able to justify the Moeller-Rosenfeld model in which baryons exchange vector and pseudoscalar mesons with symmetrical coupling constants. Accordingly we consider in the following the effective quark-meson vertex with no renormalizability restriction on the form of the Yukawa...
interaction. The φ exchange between quarks will give for the Fourier transform of the potential

$$v(k) = \frac{g}{2M^2} (\sigma^{(1)} \cdot \mathbf{k})(\sigma^{(2)} \cdot \mathbf{k})(|k|^2 + m^2)^{-1}$$  \hspace{1cm} (70)$$

where \(k\) is the space-like momentum transfer (in a frame with \(k_0 = 0\), \(m\) is the meson mass, \(M\) the quark mass (that becomes the baryon mass when we add the masses of the spectator quarks) and \(f\) the pseudoscalar coupling constant. Its Fourier transform gives the potential due to φ, namely

$$V^{(\phi)} \approx \frac{m g^2 m^2}{4\pi} \left[ \frac{1}{3} \sigma^{(1)} \cdot \sigma^{(2)} \phi + S_{12} \chi \right]$$  \hspace{1cm} (71)$$

where φ and \(\chi\) are functions of \(x = mr\), and \(S_{12}\) is the tensor operator

$$S_{12} = \frac{3(\sigma^{(1)} \cdot \mathbf{r})(\sigma^{(2)} \cdot \mathbf{r})}{r^2} - \sigma^{(1)} \cdot \sigma^{(2)}$$  \hspace{1cm} (72)$$

with

$$r = |\mathbf{r}_1 - \mathbf{r}_2|$$  \hspace{1cm} (73)$$

and

$$\phi(x) = \frac{e^{-x}}{x}, \quad \chi(x) = \left( \frac{1}{3} + \frac{1}{x} + \frac{1}{x^2} \right) \frac{e^{-x}}{x}$$  \hspace{1cm} (74)$$

Similary, the exchange of the vector meson \(\omega\) gives in the same approximation of neglecting recoil

$$V^{(\omega)} \approx \frac{m}{4\pi} \left[ g^2 (1 + \frac{m^2}{8M^2}) \phi + g^2 \frac{m^2}{4M^2} \left[ \frac{2}{3} \sigma^{(1)} \cdot \sigma^{(2)} \phi - S_{12} \chi \right] \right]$$  \hspace{1cm} (75)$$

In this limit we can also put

$$f = \frac{g m}{2M}$$  \hspace{1cm} (76)$$

In this approximation the total potential \(V\) is

$$V \approx \frac{m}{4\pi} g^2 \left\{ (1 + \frac{m^2}{8M^2}) \phi + \frac{m^2}{4M^2} \sigma^{(1)} \cdot \sigma^{(2)} \phi \right\}$$  \hspace{1cm} (77)$$

Introducing the projection operator

$$P^S = \frac{1}{2} (1 + \sigma^{(1)} \cdot \sigma^{(2)}), \quad (P^S)^2 = P^S$$  \hspace{1cm} (78)$$
with eigenvalue 1 on spin singlet states, we have

\[ V = \frac{m}{4\pi} g^2 \left\{ (1 - \frac{m^2}{8M^2})\phi + \frac{m^2}{2M^2} P^S \phi \right\} \]  \hspace{1cm} (79)

Hence the potential is a superposition of two spin independent potentials namely \( \phi \) and \( P^S \phi \). The tensor terms \( S_{12} \) in \( V^\phi \) and \( V^\omega \) are spin dependent and highly singular at \( x = 0 \). If \( \phi \) and \( \omega \) have the same mass and the same coupling constant these terms cancel, resulting in a smooth wave function for the solution of the Schrödinger equation with \( V \) as potential. Rarita and Schwinger \(^{15}\) observed that even in the case \( m_\phi \neq m_\omega \) the potential is smooth enough for the Schrödinger equation to be soluble provided that \( g_\phi = g_\omega \).

In order to generalize this model to include isospin we note that diagrams considered so far violate the Zweig rule \(^{16}\) according to which disconnected quark diagrams are suppressed. The \( \phi \) and \( \omega \) that are exchanged in these Zweig-forbidden diagrams are not only color singlets but also flavor singlets, i.e. they have zero isospin or unitary spin. Another effective interaction between quarks will occur through the exchange of color singlet but flavor carrying \( q\bar{q} \) bound states, such that \( q \) and \( \bar{q} \) are associated with different flavor quantum numbers. For example we can have

\[ d + u \rightarrow d + (u + \bar{d}) + d \rightarrow d + u \]  \hspace{1cm} (80)

where the bracketed \( u \) and \( \bar{d} \) form an intermediate bound state (\( \pi^+ \) or \( \rho^+ \)) that is exchanged between a \( u \) quark and a \( d \) quark. Again we can call \( g \) the effective coupling constant (\( u\bar{d}\pi^+ \)) or (\( u\bar{d}\rho^+ \)). If \( m \) is the mass of the exchanged meson and \( M \) the common mass of \( u \) and \( d \) assumed to be degenerate, the potentials resulting from \( \pi^+ \) and \( \rho^+ \) exchange will have respectively the forms Eq.(71) and Eq.(75), so that for the equal coupling constant and equal mass case the total potential will have the form Eq.(79). Including all the charged and neutral states, the total \( u - d \) potential results from the exchange of \( 0^- \) mesons \( \eta, \pi^i \) and \( 1^- \) mesons \( \omega_\mu \) and \( \rho_\mu \), where \( i = 1, 2, 3 \) are isospin labels. Denoting the degenerate \( (ud) \) pair by \( \psi(I = 1/2) \) we have the effective interaction term

\[ \mathcal{L}_{\text{int}} = \frac{i}{2} g\bar{\psi} (\gamma_5 \eta + \gamma_\mu \omega_\mu + \gamma_5 \vec{\pi} \cdot \vec{\pi} + \gamma_\mu \vec{\tau} \cdot \vec{\rho}_\mu) \psi \]  \hspace{1cm} (81)

Including the spin states, the quark doublet (particles) form the 4 dimensional representation of \( SU(4) \) while the bound state \( \eta \) is an \( SU(4) \) singlet. The \( \pi, \omega \) and \( \rho \) form the adjoint 15 dimensional representation of \( SU(4) \). With respect to
the $SU(4)_W$ symmetry of the vertex, $\omega_\parallel$ is the $SU(4)_W$ singlet, while the $SU(4)_W$ (15) consists of

$$\eta, \omega_\perp, \rho_\parallel^i, \pi_i^\parallel, (\pi_i^\perp, \rho_\perp^i)$$

(82)

The exchange of these mesons leads to the potential of the form

$$\alpha \phi + \frac{1}{4} \beta (1 + \sigma^{(1)} \cdot \sigma^{(2)})(1 + \tau^{(1)} \cdot \tau^{(2)}) \phi$$

(83)

that commutes with the $SU(4)$ generators found from $\frac{1}{4}(1 + \sigma^{(1)} \cdot \sigma^{(2)})$ and $\frac{1}{2}(1 + \tau^{(1)} \cdot \tau^{(2)})$. This potential is therefore spin and isospin independent in the limit of degenerate masses and equal coupling.

If the mass degeneracy is lifted by allowing $\pi_i$ and $\rho_i^i$ to have different masses, a tensor force $S_{12}$ occurs in the potential between $u$ and $d$, hence, between the proton and the neutron, so that the deuteron acquires an electric quadrupole moment. The right sign and magnitude are found if the $\rho$ is three to four times heavier than the pion, in accordance with experiment.

The $SU(4)$ symmetry we have found is broken when we add to the potential recoil terms associated with the one boson exchange. These are proportional to $L \cdot S$ (spin-orbit) and $Q_{12}$ (quadrupole) given by

$$L = r \times p, \quad S = \frac{1}{2} (\sigma^{(1)} + \sigma^{(2)})$$

$$Q_{12} = \frac{1}{2} [ (\sigma^{(1)} \cdot L)(\sigma^{(2)} \cdot L) + (\sigma^{(2)} \cdot L)(\sigma^{(1)} \cdot L) ]$$

(84)

Using $\phi$ and $\chi$ defined by Eq. (74) we can write the additional terms associated with $\omega$ exchange as

$$V_{add}^\omega = -\frac{m^4}{4\pi^2} \frac{3m^2}{2M^2} \left( \frac{1}{x} + \frac{1}{x^2} \right) \phi L \cdot S + \frac{m^4}{4\pi^2} \frac{3m^4}{16M^4} \frac{1}{x^2} \chi Q_{12}$$

(85)

The exchange of the pseudoscalar $\phi$ contributes no $L \cdot S$ or $Q_{12}$ terms that may cancel the highly singular additional terms due to $\omega$ exchange. A $Q_{12}$ term that can cancel the $Q_{12}$ term can only come from the exchange of a scalar meson $\sigma$. Accordingly we can enlarge our scheme by adding a scalar field $\sigma$ with the Yukawa interaction

$$\mathcal{L}_{\text{int}}^\sigma = g \bar{\psi}_1 \psi \sigma$$

(86)
to the interaction term Eq.\((55)\). Then we have an additional potential due to \(\sigma\) exchange that reads
\[
V^\sigma = -\frac{m^4}{4\pi g^2} \{ \frac{m^2}{8M^2} \phi + \frac{m^2}{2M^2} \left( \frac{1}{x} + \frac{1}{x^2} \right) \phi L \cdot S + \frac{3m^4}{16M^4} \frac{1}{x^2} \chi Q_{12} \} \tag{87}
\]

In the total potential
\[
V = V^\phi + V^\omega + V^\sigma \tag{88}
\]
the \(Q_{12}\) terms cancel, leaving
\[
V = \frac{m^4}{4\pi g^2} \frac{m^2}{2M^2} \{ P^S - 4 \left( \frac{1}{x} + \frac{1}{x^2} \right) L \cdot S \} \tag{89}
\]

The term with \(P^S\) (where \(P^S\) is given by Eq.\((78)\)) is spin independent while the term in \(L \cdot S\) breaks this symmetry.

### Chiral Symmetry in the Extended Moeller-Rosenfeld (M-R) Model.

The M-R model without isospin is based on \(\omega\) and \(\phi\) exchange. We have seen that, neglecting recoil, the model gives a spin-independent potential between the fermions. Including recoil we get a highly singular potential with spin dependent, spin orbit and quadrupole terms. The extended M-R model, with the inclusion of a scalar \(\sigma\)-meson gives a spin independent potential broken only by relativistic spin-orbit terms. But now, for vanishing fermion mass, the Lagrangian exhibits a higher symmetry, namely invariance under \(U(1) \times SU(2)_W\) where \(U(1)\) which rotates \(\phi\) and \(\sigma\) is a chiral group. It follows that the extended model has both \(SU(2)_W\) and chiral invariance. If we introduce isovector fields \(\pi^i\) (pseudoscalar), \(\rho_i^\mu\) (vector), \(\delta^i\) (scalar), the potential becomes
\[
W = (1 + \tau_i^{(1)} \tau_i^{(2)}) V \tag{90}
\]
where \(V\) is given by Eq.\((89)\). The new potential is both spin and isospin independent except for spin dependent \(L \cdot S\) terms. The generators of \(SU(4)_W\) are
\[
S = \frac{1}{2}(\sigma^{(1)} + \sigma^{(2)}), \quad T^i = \frac{1}{2}(\tau_i^{(1)} + \tau_i^{(2)}), \quad S T^i \tag{91}
\]
$SU(4)_W$ becomes $SU(6)_W$ if the isospin is replaced by unitary spin. But the Lagrangian also has an additional chiral symmetry in the zero quark mass limit. If the masses of the $u$ and $d$ quarks are neglected we have an $SU(2)_L \times SU(2)_R$ invariance in chromodynamics. For free quarks, combination with spin leads to $SU(4)_L \times SU(4)_R$ invariance. The $q\bar{q}$ states belonging to the adjoint representation will have both positive and negative parities. Adding singlets $\sigma$ and $\phi$ we consider the 32 states

$$E_n, \quad \sigma, \quad \phi, \quad \omega_n \quad (n = 1, 2, 3)$$

$$A^i_n, \quad \delta^i, \quad \pi^i, \quad \rho^i_n \quad (i = 1, 2, 3)$$

(92)

where $i$ is the isospin index and $n$ labels the independent spatial components of mesons with spin 1. Here $\phi$, $\pi$, $\omega_n$, $\rho^i_n$ have negative parity while the scalar mesons ($\sigma$, $\delta^i$) and the pseudovector mesons ($E_n$, $A^i_n$) have positive parity. The scalar mesons $\sigma$ and $\delta^i$ are necessary to cancel the $Q_{12}$ terms due to $\omega$ and $\rho$ exchange respectively. Thus the smoothing out of the wave functions for bound states of light quarks (for which the recoil corrections are important) enlarges the $SU(4)_W$ vertex symmetry that combines spin and isospin to include also the chiral symmetry $SU(2)_L \times SU(2)_R$.

As indicated before, using the additivity principle for the quarks inside the baryons and letting one quark in each baryon exchange a meson at a time, both the $SU(4)_W$ symmetry and the chiral symmetry are reflected in the potential between two baryons.

Note that the cancellation of the $Q_{12}$ terms requires the degeneracy of $\sigma$ with $\omega_n$ and $\delta^i$ with $\rho^i_n$. This corresponds to an $O(4)$ subgroup of $SU(4) \times SU(4)$ which seems to be broken less than $SU(4)$.

**Larger group structure**

Now $U(4, 4)$ has 64 generators, whereas $SU(4, 4)$ has 63. Writing

$$SU(4, 4) \supset SU(4) \times SU(4) \supset SU(4) \quad \text{or} \quad SU(4)_W$$

(93)

and

$$SU(4, 4) \supset SO(4, 4) \supset SO(4) \times SO(4) \supset SO(4).$$

(94)
Then part of the $\mathcal{L}$ that is in (1)-rep is

$$\bar{\psi} \epsilon \psi = \psi^\dagger_L \psi_R \epsilon + \text{h.c.} \tag{95}$$

In (28)-rep of $SO(4,4)$ part of $\mathcal{L}$ is

$$\bar{\psi}[\sigma_{\mu\nu}\{(\partial_{\mu}\omega_{\nu} - \partial_{\nu}\omega_{\mu}) + i\gamma_5(\partial_{\mu}E_{\nu} - \partial_{\nu}E_{\mu}) + \frac{\tau}{2}(i\gamma_5\pi + \delta)\}]\psi +$$

$$\bar{\psi}i\gamma_5\gamma_\mu(E_{\mu} + \partial_{\mu}\tilde{\eta})\psi + \bar{\psi}i\gamma_\mu\frac{\tau}{2}\psi_R =$$

$$\psi^\dagger_L \sigma_{\chi}(\nabla_\chi \mathbf{\omega} + i\nabla_\chi \mathbf{E}) + (\partial_\theta \mathbf{E} - \nabla E_0) + i(\partial_0 \omega - \nabla \omega_0)\} \psi_R + \text{h.c.} + \psi^\dagger_L i\tau \cdot \pi \psi_R + \psi^\dagger_L i\tau \cdot \delta \psi_R + \text{h.c.} +$$

$$+ (\psi^\dagger_L \pi_R \psi_R)(E_0 + \partial_0 \tilde{\eta}) + (\psi^\dagger_L \sigma_\mu \pi_R + \psi^\dagger_R \sigma_\mu \psi_R)(E_0 + \partial_0 \tilde{\eta}) \tag{96}$$

and in (35) of $SO(4,4)$ it is

$$\bar{\psi}[i\gamma_5 \eta + \frac{1}{2}\sigma_{\mu\nu} \frac{\tau}{2}\{(\partial_{\mu}\rho_{\nu} - \partial_{\nu}\rho_{\mu}) + i\gamma_5(\partial_{\mu}\alpha_{\nu} - \partial_{\nu}\alpha_{\mu})\}]\psi =$$

$$\bar{\psi}\{i\gamma_\mu\omega_\mu + i\gamma_5\gamma_\mu \frac{\tau}{2} \cdot (\alpha_\mu + \partial_\mu\pi)\} \psi \tag{97}$$

We now look at decomposition under $SU(4)_L \times SU(4)_R$ subgroup of $SU(4,4)$. Parts containing

$$\psi^\dagger_L \Omega \psi_R - \psi^\dagger_R \Omega \psi_L$$

terms have negative parity, while the parts with

$$\psi^\dagger_L \Omega \psi_R + \psi^\dagger_R \Omega \psi_L$$

have positive parity. These lead to combined parts in $\mathcal{L}$
\[ \psi_L^\dagger \{ \epsilon + \sigma_n \left[ (\partial_0 E_n - \partial_n E_0) + (\nabla \times \omega) \right] + \sigma \cdot \tau \} \psi_R + h.c. \]

where we put the positive parity parts in the first half, and the negative parity parts in the second half of this equation. Now that \( \tilde{\eta} \) is a mixture of \( \eta \) and \( \eta' \) we can now consider the 32 states (16 negative and 16 positive parity parts) which transform like \((4, \bar{4}) + (\bar{4}, 4)\) under \( SU(4)_L \times SU(4)_R \) with \( \partial_\mu E_\mu = 0, \partial_\mu \omega_\mu = 0, \partial_\mu a_\mu = 0, \) and \( \partial_\mu \rho_\mu = 0 \). For time-like meson momentum, \( E_0, \omega_0, a_0, \rho_0 = 0 \), and for space-like meson momentum in \( O_z \) direction \( E_3, \omega_3, a_3, \rho_3 = 0 \) so that \( E_\parallel, \omega_\parallel, a_\parallel, \) and \( \rho_\parallel \) vanish. The \( 4 \times 4 = 1 + 15 \) occur in \( \psi_L^\dagger \psi_L, \psi_R^\dagger \psi_R \) parts. \( LL, RR \) parts give the 31 generators

\[ (1, 1) + (15, 1) + (1, 15) \]

put together with above 32 giving the 63 generators of \( SU(4, 4) \). One boson exchange of above \((4, \bar{4}) + (\bar{4}, 4)\) gives on cancellation due to equality of \( q - \bar{q} \) meson coupling constant a potential of the form

\[ \frac{1}{2} (1 + \tau^{(1)} \cdot \tau^{(2)}) \frac{1}{2} (1 + \sigma^{(1)} \cdot \sigma^{(2)}) V(r) \]

while the \( L \cdot S \) term breaks the symmetry.

It is also possible to use non-compact form of \( Sp(4) \supset SU(4, 4) \).

Making the \( J^{PC} \) identifications \( \tilde{\eta} (0^{-+}), \pi (0^{-+}), \omega \mu (1^{-+}), \epsilon (f_0) (0^{++}), \delta (a_0) (0^{++}), E_\mu (f_{1\mu}) (1^{-+}), A_\mu (b_\mu) (1^{-+}) \), under \( SU(2)_L \times SU(2)_R \) exchanging particles \( \tilde{\eta} \) with \( \delta (a_0), \pi \) with \( \epsilon (f_0), \omega \mu \) with \( A_\mu (b_\mu), \) and \( \rho_\mu \) with \( E_\mu (f_{1\mu}) \) also leads to the same potential including relativistic corrections (recoil).

**Remarks on Symmetries Involving Spin and Internal Symmetries**

We have seen that an approximate combination of spin and internal degrees of freedom arises in effective Lagrangians where the couplings of the \( q\bar{q} \) bound states
to quarks ($q$) or baryons ($qqq$) are almost equal and the masses are almost degen-
erate. The $SU(4)$, $SU(6)$ or $SU(8)$ generators that close are not in general covari-
ant. They are related to the covariant operators $\sigma_n$, $\sigma_n \lambda^\alpha$ by a unitary transformation that is momentum dependent, hence non-local. This unitary transformation can be worked out for the free quark case. It serves as a model for the interacting case and works very well in phenomenological applications\cite{18,19,20}. The success of this procedure is now partially understood by the asymptotic freedom of quarks interacting through gluon exchange. At short spatial separation quarks are quasi free, so that their positive and negative frequency parts can be separated and they can be subjected to Foldy-Wouthuysen-Tani or Cini-Touschek-Melosh type transformations. The vertex then exhibits an $SU(6)_W$ type symmetry for a quark emitting a meson with space-like momentum. If the meson momentum is time-like, the meson can only be coupled to a quark-antiquark pair, each with a time-like momentum (in general not on the mass shell). In this case the effective interaction Lagrangians of the form Eq.(54) and Eq.(67) have an $SU(2)$ (no flavor), $SU(4)$ (isospin flavor) or $SU(6)$ (unitary spin flavor) symmetry with the correspondences (in the $SU(4)$ case)

$$\tau^i \leftrightarrow \pi^i, \quad \sigma \leftrightarrow \omega, \quad \tau^i \sigma \leftrightarrow \rho^i \quad (101)$$

which give the usual $SU(4)$ classification for the bound states $q\bar{q}$. The spin dependence of the one-gluon exchange between $q$ and $\bar{q}$ introduces a $\sigma^{(1)} \cdot \sigma^{(2)}$ term that splits $\pi$ from $\omega$ and $\rho$ and leads to the mass formula for $SU(6)$

$$M = M_0 + M_1 Y + M_2 [I(I + 1) - \frac{1}{4} Y^2] - M_3 S(S + 1) \quad (102)$$

that was proposed\cite{21} in 1964. The $M_0$ part comes from the exact $SU(6)$ limit valid for the spin independent confining potential\cite{22}. This spin independence is only seen in the lattice approximation in QCD. We have found a partial justification in the Moeller-Rosenfeld model with quarks exchanging almost degenerate $q\bar{q}$ bound states. The $S(S + 1)$ part comes from short range one-gluon exchange forces as shown by de Rújula, Georgi and Glashow\cite{23} who find the value of $M_3$ to be proportional to $\alpha_s$, the QCD fine structure constant and inversely proportional to the masses of the constituent quarks. Finally the $M_1$ and $M_2$ terms come from quark mass difference.

The spin independent confining force can be largely simulated by an effective scalar field as shown by Schnitzer\cite{24}.
A phenomenological bag model description of the hadron is obtained by R. Friedberg and T. D. Lee\cite{25} if the phenomenological scalar field (regarded as a function of invariants constructed out of two-gluon fields and three-gluon fields) is assumed to have different vacuum expectation values inside and outside the hadron. As shown by the same authors the large mass of the color and flavor singlet can be understood by the contribution of the 2-gluon annihilation diagram for $q\bar{q}$.

Once we have this phenomenological picture for hadrons we can get non-trivial decay amplitude relations for mesons going into 2 mesons or baryons decaying into baryons (by emission of mesons or photons) by using the unitary transformation between the vertex spin operators and the covariant spin operators. The non-covariant spin belongs to the 35-dimensional adjoint representation of SU(6)$_W$. It is a function of the covariant spin operators of current algebra and the momentum. The momentum dependence can be taken into account by consideration\cite{26} of the group SU(6)$_W \times SO(3)$ where the SO(3) is associated with the orbital angular momentum exhibiting the momentum dependence. The SO(3) part can be described by the quantum number $L = 0, 1, 2, \ldots$. We have seen that in the infinite momentum frame $\gamma_5$ converges to the longitudinal spin $\sigma_\parallel$ or $\sigma_3$ (with $O_2$ being chosen as the longitudinal direction). The pion operator $\gamma_5 \tau$ in current algebra becomes $\sigma_3 \tau$. The conserved spin is obtained by a unitary transformation of this operator. It will belong in general to a superposition of $(35, L = 0, L = 1, L = 2, \ldots)$. For a given matrix element of the pion operator between two states (meson states for pionic decay of mesons, baryonic state for pionic decays of baryons) only a limited number of these representations $(35, L)$ will contribute. That will immediately give non-trivial relations between meson-meson-pion amplitudes or between baryon-baryon-pion amplitudes. $\gamma$-decays or vector meson decays are handled in similar fashion. The details of this analysis can be found in the reviews of Meshkov\cite{27} for meson decays, Hey\cite{28} for baryon decays or in the numerous original articles\cite{18,19,20} exploring the unitary transformation between the SU(6) operators of the non-covariant constituent quarks and the SU(6) operators of current algebra generated by covariant current quarks. More model dependent and detailed descriptions of the bound states and decay processes are obtained in a semi-classical field theory of quarks bound by a confining (harmonic or linear) potential that is from the outset taken to be spin independent\cite{22}.

The chiral structure is related to the approximate zero mass of the $u$ and $d$ quarks and the negligibly small mass of the pion\cite{29}. In quark phenomenology
the $u$ and $d$ quark masses are usually taken to be one third of the proton or neutron masses. On the other hand, if they are evaluated from the deviations from exact chiral symmetry in current algebra, they are found to be of the order of a few MeV. The two pictures can be reconciled by putting massless quarks in a bag (resulting from an effective scalar field) of radius $R$, assuming that the sphere of radius $R$ is the boundary of regions where the scalar field assumes different vacuum expectation values. Then, quarks behave as if they had a mass proportional to $R^{-1}$ which can be adjusted to one third of the proton mass. For an attempt to understand the zero pion mass within this phenomenological bag model based on QCD, see Friedberg and Lee’s article\textsuperscript{[25]}.
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