The influence of vertical load to the natural vibration of series isolation system

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Abstract. The influence of axial load to the natural vibration of series isolation system is analyzed. The natural frequency of series isolation system is solved by differential quadrature method. According to the vertical load which is the main factor of natural vibration characteristic on the series isolation system, the parameter analysis is carried out. It should provide the basis for the vibration characteristic analysis for the structure of bearing on the top of first story column, and it can also provide evidence for the overall stability analysis of series isolation structure.

1. Introduction
The natural vibration performance is the basic characteristic of the structure under a certain boundary condition. It is an intrinsic characteristic that can not be changed by external factors. It determines the dynamic response of the structure. In solid mechanics, the natural vibration analysis of the structure is indispensable[1-2]. The method can grasp the frequency and mode of the structure more comprehensively, and it may provide an prospective overview on dynamic structural response under a single dynamic loading or a complex coupled loading.

Figure 1. Series isolation system

The stability and the collapse resistant of the structure depend on the instability of the vertical component. The weak layer of structure of bearing on the top of first story column is the location of...
the isolation layer\cite{3-5}. Therefore, the study of the natural vibration characteristics of series isolation system is significant to understand the collapse performance of the structure of bearing on the top of first story column under the component. In this paper, the differential quadrature method\cite{6-9} is used to solve the natural vibration control equation of the series isolation system. According to the vertical load which is the main factor affecting the natural vibration characteristic on the series isolation system, so the parameter analysis is analyzed.

2. Natural vibration equation

By the finite element method, the series isolation system is divided into two units, and $\mathbf{e}$ represents the unit $e$ in the equation. Under the vertical load $P$, $v^e(y,t)$, $\varphi^e(y,t)$ are the horizontal displacement and the cross section angle caused by bending respectively.

There is no lose of generality, the lateral natural vibration of series isolation system is set as harmonic vibration\cite{10-14}. So,

$$v^e = V^e(y)e^{i\omega t}, \quad \varphi^e = \Phi^e(y)e^{i\omega t}$$  \hfill (1)

In the formula, $\omega$ is the natural frequency of the system. Considering the influence of axial force to the lateral vibration of series isolation system, the control equation of series isolation system is:

$$\left\{ \begin{array}{l}
(k^e A^e_s G^e + P) \frac{\partial^2 \Phi^e}{\partial y^2} - k^e A^e_s G^e \frac{\partial^2 \Phi^e}{\partial y^2} - \omega^2 \rho^e A^e V^e = 0 \\
E^e I^e \frac{\partial^2 \Phi^e}{\partial y^2} + (k^e A^e_s G^e + P) \Phi^e - (k^e A^e_s G^e + P) \frac{\partial V^e}{\partial y} - \omega^2 \rho^e I^e \Phi^e = 0 
\end{array} \right. \hfill (2)$$

The boundary conditions:

$$\begin{cases}
y = 0, \quad u^T = 0; \quad v^T = 0; \quad \varphi^T = 0; \\
y = H, \quad \varphi^T = 0; \quad Q^T = 0; \quad M^T = 0;
\end{cases} \hfill (3)$$

Internal boundary conditions:

Internal boundary conditions (coordination conditions) are supported at the connection position with reinforced concrete column and rubber bearing including displacement coordination condition and internal force balance condition:

$$\begin{cases}
w^T = w^z, \quad \varphi^T = \varphi^z; \\
Q^T = Q^z, \quad M^T = M^z; \quad y = h;
\end{cases} \hfill (4)$$

3. Mode calculation

In practical engineering, the concrete strength is 30 MPa, the elastic modulus $E = 3.0 \times 10^6$ Pa, shear shape factor $\kappa = 0.845$, section size of the basement cantilever column is $1050 \text{ mm} \times 1050 \text{ mm}$, and the total height of series isolation system is 3 m including rubber bearing and column. Shear shape factor of rubber bearing $\kappa = 0.899$, other parameters of rubber bearing are shown in table 1.

| Table 1. Rubber bearing parameters |
|-----------------------------------|
| Model   | height (mm) | rubber layer thickness (mm) | shear modulus (GPa) |
|---------|-------------|-----------------------------|---------------------|
| LRB500  | 164         | 4.87                        | 0.6                 |
In the numerical calculation, the number of nodes\textsuperscript{[15-16]} in each segment is 11, and the first ten order mode of the series isolation system is obtained by using the differential quadrature method under 10MPa vertical load, in figure 2. The first 11 nodes from the horizontal axis are the cantilever columns, and the next 11 nodes from the 11th node are the rubber bearing units. It shows that the tenth order mode is dominated by the cantilever column, the third order mode is dominated by both rubber bearing and cantilever column, other modes are dominated by the rubber bearing.

\textbf{Figure 2.} The first ten order modes of the series isolation system under 10MPa

4. Analysis of vertical load parameter
There are many factors influencing the vibration of the series isolation system, such as vertical load, bearing stiffness, slenderness ratio, modulus of elasticity and moment of inertia, and so on. But in this paper, it only discusses the impact of vertical force. In the parametric analysis, the height of the cantilever column is divided into four levels such as 3 m, 3.6 m, 4.2 m, 4.8 m. And the cross sectional dimensions of the square cantilever column are 0.8 m, 0.9 m, 1.0 m, 1.2m. LRB500 is chosen to be used in the analysis, whose specific parameters are shown in table 1. The vertical loads are 10 MPa, 12 MPa, and 15 MPa, based on the rules of the design specification for seismic design of buildings (GB50011-2010).

It shows the relationship between the dimensionless frequency parameter and vertical load under three vertical loads in Figure 3. The figures show that the frequency is decreased with the increase of vertical load, and the frequency parameter under 12 MPa to 15 MPa is changed violently more than 10 MPa to 12 MPa, it can be reflected from the straight slope in the each figures.

\[ P \text{ (Mpa)} \]
\[ \frac{\text{Dimensionless Parameter}}{\text{P (Mpa)}} \]

5. Conclusion
The main work of the paper is to analyse the influence to the natural frequency of the series isolation system under different vertical load. The differential quadrature method is used to solve the control equation and boundary conditions of the discrete series isolation system. The series isolation system is similar to the characteristic of the variable-stiffness and non-uniform beam in solid mechanics. The influence of the vertical load to the frequency parameters of the series isolation system has been reflected the general rule: the natural frequency of the series isolation system is decreased with the increase of vertical load, as the vertical load closing to the critical load of the system, the natural frequency tend to zero and the system tends to be unstable.
This paper only discusses the impact of the vertical force to the mode of series isolation system, but there are many factors that can affect series isolation system, such as vertical load, bearing stiffness, slenderness ratio, modulus of elasticity and moment of inertia, and so on. Further discussions will consider on other factors, and get the comprehensive evaluation index. Furthermore, it also should be study the stability and collapse resistance of series isolation system comprehensively, based on the seismic response analysis.

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