RITORNO RADIATIVO PER LA MISURA DI R: COME E PERCHÉ

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Abstract

The measurement of the pion form factor and, more generally, of the cross section for electron–positron annihilation into hadrons through the radiative return has become an important task for high luminosity colliders such as the Φ- or B-meson factories. This quantity is crucial for predictions of the hadronic contributions to \((g-2)_\mu\), the anomalous magnetic moment of the muon, and to the running of the electromagnetic coupling. But the radiative return opens also the possibility of many other physical applications. The physics potential of this method at high luminosity meson factories is discussed and recent results are reviewed.

Electron–positron annihilation into hadrons is one of the basic reactions of particle physics, crucial for the understanding of hadronic interactions. At high energies, around the \(Z\) resonance, the measurement of the inclusive cross section and its interpretation within perturbative QCD give rise to one of the most precise and theoretically founded determinations of the strong coupling constant \(\alpha_s\). Also, measurements in the intermediate energy region, between 3 GeV and 11 GeV can be used to determine \(\alpha_s\) and at the same time give rise to precise measurements of charm and bottom quark masses. The low energy region is crucial for predictions of the hadronic contributions to \(a_\mu = (g-2)_\mu/2\), the anomalous magnetic moment of the muon, and to

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the running of the electromagnetic coupling from its value at low energy up to $M_Z$. Last, but not least, the investigation of the exclusive final states at large momenta allows for tests of our theoretical understanding of form factors within the framework of perturbative QCD. Beyond the intrinsic interest in this reaction, these studies may provide important clues for the interpretation of exclusive decays of B-mesons, a topic of evident importance for the extraction of CKM matrix elements.

The main uncertainty to $a_\mu$ and $\alpha_{\text{QED}}$ is driven by their respective hadronic contributions, which are not calculable perturbatively, but can be estimated through dispersion integrals

$$a_\mu^{\text{had, LO}} = \left( \frac{\alpha m_\mu^2}{3\pi} \right)^2 \int_{4m_\mu^2}^{\infty} \frac{ds}{s^2} \hat{K}(s) R(s),$$

$$\Delta \alpha_{\text{had}}(m_Z^2) = -\frac{\alpha m_\mu^2}{3\pi} \text{Re} \int_{4m_\mu^2}^{\infty} \frac{ds}{s} \frac{R(s)}{s - m_Z^2 - i\eta},$$

where the spectral function $R(s)$ is obtained from experimental data of the reaction $e^+e^- \rightarrow \text{hadrons}$. The most recent experimental result for $a_\mu$ \cite{4} shows a 2$\sigma$ discrepancy with respect to the SM prediction for this quantity \cite{1,2,3}. Alternatively, one can also use current conservation (CVC) and isospin symmetry to obtain $R(s)$ from $\tau$ decays. In the latter, a 0.7$\sigma$ discrepancy is found \cite{1}, which however is incompatible with the $e^+e^-$ based result. Unaccounted isospin breaking corrections due to the difference of the mass and width of the neutral to the charged $\rho$-meson could explain this discrepancy \cite{3}, leaving the $e^+e^-$ based analysis as the most reliable.

The recent advent of $\Phi$- and $B$-meson factories allows us to exploit the radiative return to explore the hadronic cross section in the whole energy region from threshold up to the nominal energy of the collider in one homogeneous data sample \cite{5,6}. The radiative suppression factor $O(\alpha/\pi)$ is easily compensated at these factories by their enormous luminosity.

In principle, the reaction $e^+e^- \rightarrow \gamma + \text{hadrons}$ receives contributions from both initial- and final-state radiation (Fig. 1), ISR and FSR respectively. Only the former is of interest for the radiative return. A variety of methods to disentangle FSR from the ISR contribution have been described in detail in \cite{5,8,9,10}, among them the employment of suitable kinematical cuts to suppress FSR, or the identification of different distributions, e.g. angular distributions, charge asymmetry, for independent tests of the FSR model amplitude. Notice however that at $B$-factories the $\pi^+\pi^-\gamma$ final state is completely dominated by ISR.
Figure 1: Leading order contributions to the reaction $e^+e^- \rightarrow \pi^+\pi^−\gamma$ from ISR (a) and FSR (b).

The proper analysis requires necessarily the construction of Monte Carlo event generators. The event generators EVA [5] and EVA$\pi$ [7] were based on a leading order treatment of ISR and FSR, supplemented by an approximate inclusion of additional collinear radiation based on structure functions. Subsequently, the event generator PHOKHARA was developed; it is based on a complete next-to-leading order (NLO) treatment of radiative corrections [8, 9, 10]. In its version 2.0 it included ISR at NLO and FSR at LO for $\pi^+\pi^−$ and $\mu^+\mu^−$ final states, and four-pion final states (without FSR) with some improvements with respect to the formulation described in [7].

The most recent version of PHOKHARA, version 3.0 [10], allows for the simultaneous emission of one photon from the initial and one photon from the final state. This includes in particular the radiative return to $\pi^+\pi^−(\gamma)$ and thus the measurement of the (one-photon) inclusive $\pi^+\pi^−$ cross section, an issue closely connected to the question of $\pi^+\pi^−(\gamma)$ contributions to $a_\mu$.

Recently, a new Monte Carlo event generator, EKHARA [11], has been constructed to simulate the reaction $e^+e^- \rightarrow \pi^+\pi^−e^+e^-$, a potential background of the radiative return specially at lower energies. Future developments of PHOKHARA include the simulation of FSR at NLO and the narrow resonances for $\mu^+\mu^−$, as well as many other hadronic channels: $K^+K^−$, $K^0\bar{K}^0$, $3\pi$, $KK\pi$, $p\bar{p}$, and the simulation of the continuum $q\bar{q}$ supplemented by some hadronization model. Encouraging preliminary experimental results from KLOE, BABAR and BELLE [12, 13] demonstrate the power of the method and its physics potential.

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