Effective ’t Hooft-Polyakov monopoles from pure SU(3) gauge theory

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The well known topological monopoles originally investigated by ’t Hooft and Polyakov are known to arise in classical Yang-Mills-Higgs theory. With a pure gauge theory it is known that the classical Yang-Mills field equation do not have such finite energy configurations. Here we argue that such configurations may arise in a semi-quantized Yang-Mills theory, where the original gauge group, SU(3), is reduced to a smaller gauge group, SU(2), and with some combination of the coset fields of the SU(3) to SU(2) reduction acting as effective scalar fields. The procedure is called semi-quantized since some of the original gauge fields are treated as quantum degrees of freedom, while others are postulated to be effectively described as classical degrees of freedom. Some speculation is offer on a possible connection between these monopole configurations and the confinement problem, and the nucleon spin puzzle.

I. INTRODUCTION

In two recent papers \cite{1} \cite{2} it was pointed out that quantized SU(2) gauge theory in some approximation is equivalent to a U(1) gauge theory plus a scalar field, i.e. a pure SU(2) gauge theory reduces to a smaller U(1) abelian subgroup plus a symmetry breaking effective Higgs field. The postulate underlying this construction was that in some situations the SU(2) gauge fields can be decomposed into ordered and disordered phases. For the ordered phase the components of the SU(2) field have non-zero quantum average \( \langle a_{a}^{\mu} \rangle \neq 0 \), while the disordered phase has a zero average \( \langle A_{\mu}^{m} \rangle = 0 \). Nevertheless, it is postulated that the condensate of the disordered phase is non-zero (i.e. \( \langle A_{\mu}^{m}A_{\nu}^{n} \rangle \neq 0 \) ) so that it therefore possesses a non-zero energy. Under these conditions the pure SU(2) gauge theory is equivalent to the Ginzburg-Landau theory interacting with the U(1) gauge field.

The aim of this paper is to extend these results to the SU(3) gauge theory. There are interesting differences between the previous SU(2) \( \rightarrow \) U(1) reduction and the present SU(3) \( \rightarrow \) SU(2) reduction. For the SU(2) \( \rightarrow \) U(1) case the gauge field, \( a_{a}^{\mu} \), of the subgroup belongs to an Abelian subgroup U(1), and therefore does not have selfinteraction terms like \( (a_{a}^{\mu}a^{a\nu})^{2} \). In the SU(3) \( \rightarrow \) SU(2) case the gauge fields, \( a_{a}^{\mu} (a = 1, 2, 3) \), belong to a non-Abelian subgroup SU(2), and these gauge field do have selfinteraction terms like \( (a_{a}^{\mu}a^{a\nu})^{2} \). These terms are expected to change some results in comparison with the case investigated in Refs. \cite{2}. In particular in \cite{2} after the reduction from SU(2) \( \rightarrow \) U(1) we obtained an effective Ginzburg-Landau Lagrangian of the form which gives rise to Nielsen-Olesen flux tube solutions \cite{3}. In the present case we will find that the SU(3) \( \rightarrow \) SU(2) reduction yields an effective Georgi-Glashow \cite{4} Lagrangian which has ’t Hooft-Polyakov monopole solutions \cite{5}. The mass of the monopole solutions is inversely proportional to the square of the coupling constant and directly proportional to the mass scale or the mass of the relevant gauge bosons. The monopoles which occur in Grand Unified Theories like SU(5) are therefore extremely massive since the gauge bosons have masses of order \( 10^{15} \) GeV and the couplings constants are perturbatively small so that the monopoles would have masses greater than \( 10^{15} \) GeV. For the effective Yang-Mills-Higgs Lagrangian derived here the coupling constant and mass scale would be that of SU(3) – the coupling constant would be of order 1 and the mass scale would be of order \( \Lambda \approx 200 \) MeV. Thus these monopole solutions would have a mass which would allow them to play a physical role in low energy scale physics. We offer some speculation that such solutions may play a role in the nucleon spin puzzle \cite{6}. The main point to be made is that while pure Yang-Mills theories are proven not to have classical finite energy solutions \cite{7} \cite{8}, in the present case we perform a semi-quantization of the system, and find that finite energy monopole solutions arise not from the classical theory, but from the quantized theory.

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II. SU(3)→SU(2) + COSET DECOMPOSITION

In this section the reduction of SU(3) to SU(2) is defined. We follow the conventions of Ref. 10. Starting with the SU(3) gauge group with generators $T^B$, we define the SU(3) gauge fields, $A_\mu = A_\mu^B T^B$. Let SU(2) be a subgroup of SU(3) and SU(3)/SU(2) is a coset. Then the gauge field $A_\mu$ can be decomposed as

$$A_\mu = A_\mu^B T^B = a_\mu^a T^a + A_\mu^m T^m,$$

(1)

$$a_\mu^a \in SU(2) \quad \text{and} \quad A_\mu^m \in SU(3)/SU(2)$$

(2)

where the indices $a, b, c \ldots$ belongs to the subgroup SU(2) and $m, n, \ldots$ to the coset SU(3)/SU(2); $B$ are SU(3) indices. Based on this the field strength can be decomposed as

$$F^B_{\mu\nu} T^B = F^a_{\mu\nu} T^a + F^m_{\mu\nu} T^m$$

(3)

where

$$F^a_{\mu\nu} = h^a_{\mu\nu} + \Phi^a_{\mu\nu} \in SU(2),$$

(4)

$$h^a_{\mu\nu} = \partial_\mu a_\nu^a - \partial_\nu a_\mu^a + g\epsilon^{abc} a_\mu^b a_\nu^c \in SU(2),$$

(5)

$$\Phi^a_{\mu\nu} = gf_{amn} A^m_\mu A^n_\nu \in SU(2),$$

(6)

$$F^m_{\mu\nu} = F^m_{\mu\nu} + G^m_{\mu\nu} \in SU(3)/SU(2),$$

(7)

$$F^m_{\mu\nu} = \partial_\mu A_\nu^m - \partial_\nu A_\mu^m + g f_{mnp} A^m_\mu A^n_\nu \in SU(3)/SU(2),$$

(8)

$$G^m_{\mu\nu} = gf_{mnb} (A^n_\mu A^b_\nu - A^n_\nu A^b_\mu) \in SU(3)/SU(2)$$

(9)

where $f^{ABC}$ are the structure constants of SU(3), $\epsilon^{abc} = f^{abc}$ are the structure constants of SU(2). The SU(3) Yang-Mills field equations can be decomposed as

$$D_\nu (h^{\mu\nu} + \Phi^{\mu\nu}) = -gf_{amn} A^m_\nu (F^{\mu\nu} + G^{\mu\nu}),$$

(10)

$$D_\nu (F^{m\nu} + G^{m\nu}) = -gf_{mnb} [A^n_\nu (h^{b\nu} + \Phi^{b\nu}) - a^b_\nu (F^{m\nu} + G^{m\nu})]$$

(11)

where $D_\nu[\ldots]^a = \partial_\nu[\ldots]^a + f^{abc} a^b[\ldots]^c$ is the covariant derivative on the subgroup SU(2) and $D_\nu[\ldots]^m = \partial_\nu[\ldots]^m + f^{mnp} A^n_\nu[\ldots]^p$.

III. HEISENBERG QUANTIZATION FOR QCD

In the rest of the paper we will apply a modification of the Heisenberg quantization technique to the system defined in the previous section. In quantizing the classical system given in Eqs. 10 - 11 via Heisenberg’s method 10 one first replaces the classical fields by field operators $a_\mu^a \rightarrow \hat{a}_\mu^a$ and $A_\mu^m \rightarrow \hat{A}_\mu^m$. This yields non-linear, coupled, differential equations for the field operators. One then uses these equations to determine the expectation values for the field operators $\hat{a}_\mu^a$ and $\hat{A}_\mu^m$ (e.g. $\langle \hat{a}_\mu^a \rangle$, where $\langle \cdot \cdot \cdot \rangle = \langle Q | \cdot \cdot \cdot | Q \rangle$ and $| Q \rangle$ is some quantum state). One can also use these equations to determine the expectation values of operators that are built up from the fundamental operators $\hat{a}_\mu^a$ and $\hat{A}_\mu^m$. For example, the “electric” field operator, $\hat{E}_\mu^a = \partial_\mu \hat{a}_\mu^a$, giving the expectation $\langle \hat{E}_\mu^a \rangle$. The simple gauge field expectation values, $\langle A_\mu(x) \rangle$, are obtained by taking the expectation of the operator version of Eqs. 10 - 11 with respect to some quantum state $| Q \rangle$. One problem in using these equations to obtain expectation values like $\langle A_\mu^m \rangle$, is that these equations involve not only powers or derivatives of $\langle A_\mu^m \rangle$ (i.e. terms like $\partial_\alpha (A_\mu^m)$ or $\partial_\alpha \partial_\beta (A_\mu^m)$) but also contain terms like $G_{\mu\nu}^{mn} = \langle A_\mu^m A_\nu^m \rangle$. Starting with the operator version of Eqs. 10 - 11 one can generate an operator differential equation for the product $\hat{A}_\mu^m \hat{A}_\nu^m$ thus allowing the determination of the Green’s function $G_{\mu\nu}^{mn}$. However this equation will in turn contain other, higher order Green’s functions. Repeating these steps leads to an infinite set of equations connecting Green’s functions of ever increasing order. This procedure is very similar to the field correlators approach in QCD (for a review, see 11). In Ref. 12 a set of self coupled equations for such field correlators is given. This construction, leading to an infinite set of coupled, differential equations, does not have an exact, analytical solution and so must be handled using some approximation.

Operators are only well determined if there is a Hilbert space of quantum states. Thus we need to ask about the definition of the quantum states $| Q \rangle$ in the above construction. The resolution to this problem is as follows: There is an one-to-one correspondence between a given quantum state $| Q \rangle$ and the infinite set of quantum expectation values over any product of field operators, $\langle Q | A_\mu^m(x_1) A_\nu^m(x_2) \ldots | Q \rangle$. So if all the Green’s functions –
\[ G_{\mu\nu\cdots}(x_1, x_2 \ldots) \] are known then the quantum states, \(|Q\rangle\) are known, \textit{i.e.} the action of \(|Q\rangle\) on any product of field operators \(\hat{A}_m^a(x_1)\hat{A}_n^a(x_2)\ldots\) is known. The Green’s functions are determined from the above, infinite set of equations (following Heisenberg’s idea).

Another problem associated with products of field operators like \(\hat{A}_m^a(x)\hat{A}_n^a(x)\) is that the two operators occur at the same point. For \textit{non-interacting} field it is well known that such products have a singularity. In this paper we are considering \textit{interacting} fields so it is not known if a singularity would arise for such products of operators evaluated at the same point. Physically it is hypothesized that there are situations in interacting field theories where these singularities do not occur (\textit{e.g.} for flux tubes in Abelian or non-Abelian theory quantities like the “electric” field inside the tube, \(|E_x^2\rangle < \infty\), and energy density \(\varepsilon(x) = \langle(E_x^2)^2\rangle < \infty\) are nonsingular). Here we take as an assumption that such singularities do not occur.

\[ IV. \ \text{BASIC ASSUMPTIONS} \]

It is evident that the full and exact quantization is impossible in this case. Thus we have to look for some simplification in order to obtain equations which can be analyzed. Our basic aim is cut off the infinite equations set using some simplifying assumptions. For this purpose we have to have ansätze for the following 2 and 4-points Green’s functions: \(\langle \hat{A}_m^a(y)\hat{A}_n^a(x) \rangle\), \(\langle a_\mu^a(x) a_\nu^a(y) A_m^a(z) A_n^a(u) \rangle\) and \(\langle A_m^a(x)A_n^a(y) A_p^a(z) A_q^a(u) \rangle\). At first we assume that there are two phases

1. The gauge field components \(a_\mu^a\) \((a=1,2,3\ a_\mu^a \in SU(2))\) belonging to the small subgroup SU(2) are in an ordered phase. Mathematically this means that

\[ \langle a_\mu^a(x) \rangle = (a_\mu^a(x))_{cl}. \]  

The subscript means that this is the classical field. Thus we are treating these components as effectively classical gauge fields in the first approximation.

2. The gauge field components \(A_\mu^m\) \((m=4,5, \ldots, 8\ \text{and} \ A_\mu^m \in SU(3)/SU(2))\) belonging to the coset SU(3)/SU(2) are in a disordered phase (or in other words - a condensate), but have non-zero energy. In mathematical terms this means that

\[ \langle A_\mu^m(x) \rangle = 0, \quad \text{but} \quad \langle A_\mu^m(x)A_n^a(x) \rangle \neq 0. \]  

Later we will postulate a specific, and physically reasonable form for the non-zero term.

3. There is not correlation between ordered (classical) and disordered (quantum) phases

\[ \langle f(a_\mu^a)g(A_m^a) \rangle = f(a_\mu^a) \langle g(A_m^a) \rangle \]  

Later we will give a specific form for this correlation in a 4-point Green’s function.

\[ V. \ \text{DERIVATION OF AN EFFECTIVE LAGRANGIAN} \]

Our quantization procedure will derivate from the Heisenberg method in that we will take the expectation of the Lagrangian rather than for the equations of motions. Thus we will obtain an effective Lagrangian rather than approximate equations of motion. The Lagrangian we obtain from the original SU(3) pure gauge theory is an effective SU(2) Yang-Mills-Higgs system which has monopole solutions. The averaged Lagrangian is

\[ \mathcal{L} = -\frac{1}{4} \langle F_{\mu\nu}^A F_{\mu\nu}^A \rangle = -\frac{1}{4} \left( \langle F_{\mu\nu}^a F_{\mu\nu}^a \rangle + \langle F_{\mu\nu}^m F_{\mu\nu}^m \rangle \right) \]  

here \(F_{\mu\nu}^a\) and \(F_{\mu\nu}^m\) are defined by equations (14)–(19).

\[ A. \ \text{Calculation of} \ \langle F_{\mu\nu}^a F_{\mu\nu}^a \rangle \]

We begin by calculating the first term on the r.h.s. of equation (15)

\[ \langle F_{\mu\nu}^a F_{\mu\nu}^a \rangle = \langle h_{\mu\nu}^a h_{\mu\nu}^a \rangle + 2 \langle h_{\mu\nu}^a \Phi_{\mu\nu}^a \rangle + \langle \Phi_{\mu\nu}^a \Phi_{\mu\nu}^a \rangle. \]  

(16)
Immediately we see that the first term on the r.h.s. of this equation is SU(2) Lagrangian as we assume that $a_{\mu}^a$ and $h_{\mu\nu}^a$ are the classical quantities and consequently

$$\langle h_{\mu\nu}^a h^{a\mu\nu} \rangle \approx h_{\mu\nu}^a h^{a\mu\nu}. \quad (17)$$

The second term in equation (18) is

$$\langle h_{\mu\nu}^a \Phi^{a\mu\nu} \rangle = g f^{amn} \left( (\partial_\mu a_\nu^a - \partial_\nu a_\mu^a) A^{m\nu} A^{n\mu} + g f^{abc} f^{amn} \langle a_\mu^b A_{\nu}^c A_{\mu}^m A_{\nu}^n \rangle \right). \quad (18)$$

Using assumptions 1 and 3 from the previous section these terms become

$$\langle a_{\alpha}^a(x) A_{\mu}^m(y) A_{\nu}^n(z) \rangle = a_{\alpha}^a(x) \langle A_{\mu}^m(y) A_{\nu}^n(z) \rangle = \eta_{\mu\nu} a_{\alpha}^a(x) G^{mn}(y, z) \quad (19)$$

and

$$\langle a_{\alpha}^a(x) a_{\beta}^b(y) A_{\mu}^m(z) A_{\nu}^n(u) \rangle = a_{\alpha}^a(x) a_{\beta}^b(y) \eta_{\mu\nu} G^{mn}(z, u) \quad (20)$$

The function $G^{mn}(x, y)$ is the 2-point correlator (Green’s function) for the disordered phase. Because of the bosonic character of the coset gauge fields $G^{mn}(x, y)$ must be symmetric under exchange of these fields. Also by assumption 2 of the last section this expectation should be non-zero. We take the form for this 2-point correlator to be

$$\langle A_{\mu}^m(y) A_{\nu}^n(x) \rangle = -\frac{1}{3} \eta_{\mu\nu} f^{mp\beta} f^{np\epsilon} \phi^\beta(y) \phi^\epsilon(x) = -\eta_{\mu\nu} G^{mn}(y, x) \quad (21)$$

with

$$G^{mn}(y, x) = \frac{1}{3} f^{mp\beta} f^{np\epsilon} \phi^\beta(y) \phi^\epsilon(x) \quad (22)$$

here $\phi^a$ is a real SU(2) triplet scalar fields. Thus we have replaced the coset gauge fields by an effective scalar field, which will have the scalar field in our effective SU(2)-scalar system. The factor of $-\frac{1}{3}$ is introduced so that the effective scalar field, $\phi$, will have the correct coefficient for the kinetic energy term. If we were to take the scalar field to be constant ($\phi^a(x) \approx \text{const.}$) then (21) and (22) would represent an effective mass-like, condensation term of the coset gauge fields. With this we find that the middle term vanishes

$$\langle h_{\mu\nu}^a \Phi^{a\mu\nu} \rangle = g \eta_{\mu\nu} \left( f^{amn} \left( \partial_\mu a_\nu^a(x) - \partial_\nu a_\mu^a(x) \right) G^{mn}(x, x) + g f^{abc} f^{amn} a_\mu^b(x) a_\nu^c(x) G^{mn}(x, x) \right) = 0 \quad (23)$$

The last term which is quartic in the coset gauge fields will be considered at the end. Up to this point the SU(2) part of the Lagrangian is

$$\langle F_{\mu\nu}^a F^{a\mu\nu} \rangle = \langle h_{\mu\nu}^a h^{a\mu\nu} \rangle + g^2 f^{mp\beta} f^{np\epsilon} \langle A_{\mu}^n A_{\beta}^m A_{\epsilon}^p \rangle \quad (24)$$

### B. Calculation of $\langle F_{\mu\nu}^a F^{a\mu\nu} \rangle$

Next we work on the coset part of the Lagrangian

$$\langle F_{\mu\nu}^a F^{a\mu\nu} \rangle = \langle \left( \left( \partial_\mu A_{\nu}^m - g f^{mbn} A_{\mu}^n a_\nu^b \right) - \left( \partial_\nu A_{\mu}^m - g f^{mnb} A_{\nu}^n a_\mu^b \right) + g f^{mnp} A_{\mu}^n A_{\nu}^p \right]^2 \rangle =$$

$$= 2 \left( \langle \partial_\mu A_{\nu}^m - g f^{mbn} A_{\mu}^n a_\nu^b \rangle^2 \right) - 2 \langle \partial_\mu A_{\nu}^m - g f^{mnb} A_{\nu}^n a_\mu^b \rangle \left( \partial_\nu A_{\mu}^m - g f^{mnb} A_{\nu}^n a_\mu^b \right) +$$

$$+ 4 g \langle \left( \langle \partial_\mu A_{\nu}^m - g f^{mbn} A_{\nu}^n a_\mu^b \rangle f^{mnp} A_{\mu}^n A_{\nu}^p \rangle + g^2 f^{mp\beta} f^{np\epsilon} \langle A_{\mu}^n A_{\beta}^m A_{\epsilon}^p \rangle \rangle \right. \quad (25)$$

First we calculate

$$2 \langle \partial_\mu A_{\nu}^m \rangle^2 = 2 \langle \partial_\mu A_{\nu}^m(y) \partial_\mu A_{\nu}^m(x) \rangle \bigg|_{y=x} = 2 \partial_\mu \partial_\mu \langle A_{\nu}^m(y) A_{\nu}^m(x) \rangle \bigg|_{y=x} =$$

$$= -\frac{2}{3} \eta_{\mu\nu} f^{mp\beta} f^{np\epsilon} \partial_\mu \partial_\mu \phi^\beta \phi^\epsilon = -\frac{8}{3} \partial_\mu \phi^a \partial_\mu \phi^a. \quad (26)$$

Analogously

$$-2 \langle \partial_\mu A_{\nu}^m(y) \partial_\nu A_{\mu}^m(x) \rangle \bigg|_{y=x} = \frac{2}{3} \eta_{\mu\nu} f^{mp\beta} f^{np\epsilon} \partial_\mu \partial_\mu \phi^\beta \phi^\epsilon = -\frac{2}{3} \partial_\mu \phi^a \partial_\mu \phi^a. \quad (27)$$
The term involving the derivative is
\[ -4g \langle (\partial_\mu A_\nu^m) f^{amn} A_\nu^{ap} a_{ap} \rangle = -4g f^{amn} a_\nu^{ap} ((\partial_\mu A_\nu^m) A_\nu^{ap}) = -4g f^{amn} a_\nu^{ap} (x) ((\partial_\mu A_\nu^m) (y)) A_\nu^{ap} (x) \rangle \]
\[ = \frac{4}{3} g \delta^{amn} a_\nu^{ap} f^{fmbn} f^{fpcm} \partial_\mu \phi^c = -\frac{16}{3} g a_\nu^{ap} (f^{amn} f^{fmbn} f^{fpcm}) \partial_\mu \phi^c = \frac{8}{3} g e^{abc} a_\mu \partial_\mu \phi^c. \]

Using \( f^{amn} f^{fmbn} f^{fpcm} = \frac{1}{3}e^{abc} \). Analogously
\[ 4g \langle f^{amn} A_\nu^m (y) a_\nu^{ap} (y) (\partial_\mu A_\nu^p (x)) \rangle \bigg|_{y=x} = \frac{4}{3} g (f^{fmbn} f^{pcm}) a_\mu \phi^c \partial_\mu \phi^c = -\frac{2}{3} g e^{abc} a_\mu \partial_\mu \phi^c. \]

Using (24) the next term is
\[ 2g^2 f^{dmn} A_\nu^m f^{d'mn'} A_\nu^{d'n'} = 2g^2 f^{dmn} f^{d'mn'} f^{pcm} f^{pcm'} \left( a_\mu \phi c + \phi c \right) = \frac{8}{3} g^2 E^{abc} a_\mu \partial_\mu \phi^c. \]

The next term is
\[ -4g \langle (\partial_\mu A_\nu^m) f^{amn} A_\nu^{ap} a_{ap} \rangle = -4g f^{amn} a_\nu^{ap} ((\partial_\mu A_\nu^m) A_\nu^{ap}) = -4g f^{amn} a_\nu^{ap} (x) ((\partial_\mu A_\nu^m) (y)) A_\nu^{ap} (x) \rangle \]
\[ = \frac{4}{3} g \delta^{amn} a_\nu^{ap} f^{fmbn} f^{fpcm} \partial_\mu \phi^c = -\frac{16}{3} g a_\nu^{ap} (f^{amn} f^{fmbn} f^{fpcm}) \partial_\mu \phi^c = \frac{8}{3} g e^{abc} a_\mu \partial_\mu \phi^c. \]

Using (24) the next term is
\[ 2g^2 f^{dmn} A_\nu^m f^{d'mn'} A_\nu^{d'n'} = 2g^2 f^{dmn} f^{d'mn'} f^{pcm} f^{pcm'} \left( a_\mu \phi c + \phi c \right) = \frac{8}{3} g^2 E^{abc} a_\mu \partial_\mu \phi^c. \]

Finally, there are the terms that involve three coset fields (e.g. \( f^{fmbn} (\partial_\mu A_\nu^m A_\nu^{ap} A_\nu^{ap}) \)) and \( f^{fmbn} f^{pcm} a_\mu (A_\nu^m A_\nu^{ap} A_\nu^{ap}) \).

The term involving the derivative is
\[ f^{fmbn} ((\partial_\mu A_\nu^m) A_\nu^{ap} (x) A_\nu^{ap} (z)) \]
\[ \approx \frac{1}{3} f^{fmbn} f^{pcm} a_\mu (A_\nu^m A_\nu^{ap} A_\nu^{ap}) \approx \frac{1}{3} f^{fmbn} f^{pcm} a_\mu (A_\nu^m A_\nu^{ap} A_\nu^{ap}). \]

By the second assumption in the previous section, \( A_\nu^m (x) = 0 \), this term also vanishes. Thus
\[ \langle F_{\mu \nu} F_{\mu \nu} \rangle = -2 \partial_\mu \phi^c \partial_\mu \phi^c + 2g e^{abc} a_\mu \partial_\mu \phi^c - \frac{g^2}{2} e^{abc} a_\mu \partial_\mu \phi^c \]
\[ = \frac{g^2}{2} a_\mu \phi^c a_\mu \phi^c - g^2 f^{fmbn} f^{pcm} \left( a_\mu \phi^c a_\mu \phi^c \right) = \frac{g^2}{2} a_\mu \phi^c a_\mu \phi^c - g^2 f^{fmbn} f^{pcm} \left( A_\mu A_\nu^m A_\nu^{ap} A_\nu^{ap} \right). \]

The full averaged Lagrangian is
\[ -\frac{1}{4} \langle F_{\mu \nu} F_{\mu \nu} \rangle = -\frac{1}{4} h^{mn} h^{apq} + \frac{1}{2} \left( \partial_\mu \partial_\mu + \frac{g}{2} e^{abc} a_\mu \partial_\mu \phi^c \right) \]
\[ + \frac{g^2}{2} a_\mu \phi^c a_\mu \phi^c - \frac{1}{4} g^2 f^{fmbn} f^{pcm} \left( A_\mu A_\nu^m A_\nu^{ap} A_\nu^{ap} \right). \]

where we have collected the quartic terms from eq. (24) and (50) together into \( f^{fmbn} f^{pcm} \left( A_\mu A_\nu^m A_\nu^{ap} A_\nu^{ap} \right).\)
C. The quartic term

In this section we show that the quartic term \(- f^{Anp} f^{A'n'} \left\langle A^p_{\alpha} A^p_{\beta} A^{n'}_{\mu} A^{n'}_{\nu} \right\rangle\) from eq. \((37)\) becomes an effective \(\lambda \phi^4\) interaction term for the effective scalar field introduced in eq. \((21)\). Just as in eq. \((21)\) where a quadratic gauge field expression was replaced by a quadratic effective scalar field expression, here we replace the quartic gauge field term by a quartic scalar field term

\[
\left\langle A^m_\alpha(x) A^n_\beta(y) A^p_\mu(z) A^q_\nu(u) \right\rangle = \left( E^{mpnq}_{1,abcd} \eta_{\alpha \beta} \eta_{\mu \nu} + E^{mpnq}_{2,abcd} \eta_{\alpha \mu} \eta_{\beta \nu} + E^{mpnq}_{3,abcd} \eta_{\alpha \nu} \eta_{\beta \mu} \right) \phi^a(x) \phi^b(y) \phi^c(z) \phi^d(u) \tag{38}
\]

here \(E^{mpnq}_{1,abcd}, E^{mpnq}_{2,abcd}, E^{mpnq}_{3,abcd}\) are constants. Because of the bosonic character of the gauge fields in \((38)\) the indices of these constants in conjunction with the indices of the \(\eta_{\alpha \beta}\)’s must reflect symmetry under exchange of the fields. The simplest choice that satisfies this requirement is

\[
\left\langle A^m_\alpha(x) A^n_\beta(x) A^p_\mu(x) A^q_\nu(x) \right\rangle = (\delta^{mn} \delta^{pq} \eta_{\alpha \beta} \eta_{\mu \nu} + \delta^{mp} \delta^{nq} \eta_{\alpha \mu} \eta_{\beta \nu} + \delta^{mq} \delta^{np} \eta_{\alpha \nu} \eta_{\beta \mu}) e_{abcd} \phi^a(x) \phi^b(y) \phi^c(x) \phi^d(u) \tag{39}
\]

This choice of taking the constants from eq. \((38)\) to be products of Kronecker deltas and fixing \(a = b = c = d\) for the lower indices, satisfies the bosonic character requirement for the gauge fields, and is equivalent to the reduction used for the quartic term in \((2)\). Evaluating eq. \((39)\) at one spacetime point (i.e. \(x = y = z = u\)) and contracting the indices to conform to quartic term in eq. \((38)\) gives

\[
\left\langle A^m_\alpha(x) A^n_\beta(x) A^p_\mu(x) A^q_\nu(x) \right\rangle = (\delta^{mn} \delta^{pq} \eta_{\alpha \beta} \eta_{\mu \nu} + \delta^{mp} \delta^{nq} \eta_{\alpha \mu} \eta_{\beta \nu} + \delta^{mq} \delta^{np} \eta_{\alpha \nu} \eta_{\beta \mu}) (\phi^a(x) \phi^a(x))^2 \tag{40}
\]

where the constants \(e_{abcd}\) are chosen that at the point \(x = y = z = u\)

\[
e_{abcd} = \frac{1}{3} (\delta_{ab} \delta_{cd} + \delta_{ac} \delta_{bd} + \delta_{ad} \delta_{bc})\tag{41}
\]

This expression can be further simplified to

\[
\left\langle A^m_\mu A^n_\nu A^{p}_{\mu} A^{p}_{\nu} \right\rangle = (4 \delta^{mn} \delta^{pq} + 16 \delta^{mp} \delta^{nq} + 4 \delta^{mq} \delta^{np}) (\phi^a(x) \phi^a(x))^2 \tag{42}
\]

Substituting this into the original quartic term of eq. \((37)\) yields

\[
f^{Anp} f^{A'n'} \left\langle A^p_{\alpha} A^p_{\beta} A^{n'}_{\mu} A^{n'}_{\nu} \right\rangle = (4 f^{Amn} f^{A'n'n'} + 16 f^{Anp} f^{A'np} + 4 f^{Anp} f^{A'n'})(\phi^a(x) \phi^a(x))^2 \tag{43}
\]

where the antisymmetry property of the structure constants has been used. Using the explicit expression for the structure constants \((f^{123} = 1; f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2}; f^{458} = f^{678} = \sqrt{3} + 1\) plus those related to these by permutations), and recalling that the index \(A\) runs from 1 to 8 while the indices \(n, p\) run from 4 to 8 one can show that \(f^{Anp} f^{A'n'} = 12 (f^{123} \text{ and related constants do not contribute to this expression})\). Combining these results transforms the quartic term in eq. \((37)\) as

\[
\frac{1}{4} g^2 f^{Anp} f^{A'n'} \left\langle A^p_{\alpha} A^p_{\beta} A^{n'}_{\mu} A^{n'}_{\nu} \right\rangle = 36 g^2 (\phi^a(x) \phi^a(x))^2 \equiv \lambda (\phi^a(x) \phi^a(x))^2 \tag{44}
\]

This has transformed the quartic gauge field term of the coset fields into a quartic interaction term for the effective scalar field. Substituting this result back into the averaged Lagrangian of eq. \((37)\) we find

\[
-\frac{1}{4} \left( F^{A}_{\mu \nu} F^{A}_{\mu \nu} \right) = -\frac{1}{4} h_{\mu \nu} h^{\mu \nu} + \frac{1}{2} \left( \partial_{\mu} \phi^a + \frac{1}{4} \epsilon_{\mu \nu \rho \pi} \phi^b \phi^c \phi^d + \frac{\partial^2}{2} \phi^b \phi^c \phi^d - \frac{1}{2} \lambda (\phi^a) (\phi^a) \right) \tag{45}
\]

The original pure SU(3) gauge theory has been transformed into an SU(2) gauge theory coupled to an effective triplet scalar field. This is similar to the Georgi-Glashow \((4)\) Lagrangian except for the presence of the term \(\frac{1}{2} \phi^b \phi^c \phi^d\) and the absence of a negative mass term for \(\phi^a\) of the form \(m^2 \phi^a \phi^a\).

The Georgi-Glashow Lagrangian is known to have topological monopole solutions \((5)\) which have the form

\[
\phi^a = \frac{a^x f(r)}{gr^2} \ ; \ a^x_0 = 0 \ ; \ a^x_i = \frac{\epsilon_{ijk} a^y_j [1 - h(r)]}{gr^2} \tag{46}
\]
where \( f(r) \) and \( h(r) \) are functions determined by the field equations. For this form of the scalar and SU(2) gauge fields the term, \( \frac{g^2}{2} \epsilon_{abc} \phi^a \phi^b \phi^c \), vanishes from the Lagrangian in eq. \( 41 \) by the antisymmetry of \( \epsilon_{abc} \) and the symmetry of \( \phi^a \phi^b \).

Thus for the monopole ansatz of eq. \( 40 \) the Lagrangian in \( 45 \) becomes equivalent to the Georgi-Glashow Lagrangian minus only the mass term for the scalar field.

In the present work we simply postulate that the effective scalar field develops a negative mass term of the form \( -\frac{m^2}{2} (\phi^a \phi^a) \) which is added by hand to the Lagrangian of \( 45 \) to yield

\[
-\frac{1}{4} h_{\mu\nu} h^{\mu\nu} + \frac{1}{2} \left( \partial_\mu \phi^a + \frac{g}{2} \epsilon^{abc} a^b_{\mu} \phi^c \right)^2 + m^2 \frac{2}{9} (\phi^a \phi^a) - \lambda (\phi^a(x) \phi^a(x))^2 + \frac{g^2}{2} a^a_{\mu} \phi^a \phi^a \phi^a
\]

(47)

The scalar field now has the standard symmetry breaking form and this effective Lagrangian has finite energy ’t Hooft-Polyakov solutions (the last term should not alter the monopole construction since it vanishes under the ansatz in \( 40 \)).

Our final result given in \( 47 \) depends on several crucial assumptions (e.g. the existence of the negative mass term \( -\frac{m^2}{2} (\phi^a \phi^a) \)). In the next section we make some remarks and discuss possible motivations for some of the major assumptions.

VI. REMARKS/SPECULATIONS ABOUT MASS \( m^2 (\phi^a)^2 \) AND \( a^a_{\mu} \phi^a \phi^a \phi^a \) TERMS

First we would like to emphasize that the calculations presented here are nonperturbative in the following sense. It is well known that perturbative techniques do not work in QCD. The degrees of freedom dealt with in this paper have been split into two phases. The first phase is an order phase which is treated as an effectively classical degree of freedom. Perturbations around corresponding solutions probably can be calculated perturbatively using something like Feynman diagram technique. The second phase is a purely quantum, nonperturbative degree of freedom.

In the first approximation these degrees of freedom can be calculated with our simplifications presented above. We can assume that as above in the first case the perturbations around the correlators (Green’s function \( 22 \)) can be calculated using something like Feynman diagram techniques.

In counting the degrees of freedom there is an apparent mismatch between the final and initial degrees of freedom. The averaged Lagrangian \( 17 \) has \( 3 \times 2 = 6 \) (for \( a^a_{\mu} \)) + 3 (for \( \phi^a \)) = 9 degrees of freedom, while the initial SU(3), QCD Lagrangian has \( 8 \times 2 = 16 \) degrees of freedom. Thus there is an apparent shortfall of \( 16 - 9 = 7 \) degrees of freedom. This shortfall occurs in eqs. \( 21, 22 \) where \( 5 \times 2 = 10 \) degrees of freedom from the coset gauge fields, \( A^a_{\mu} \), are put into 3 degrees of freedom, \( \phi^a(x) \). Our postulate is that QCD has nonperturbative and perturbative degrees of freedom. The initial SU(3) Lagrangian contains both kinds of degrees of freedom. The final effective Lagrangian in eq. \( 17 \) contains the nonperturbative degrees of freedom – the SU(2) gauge fields \( a^a_{\mu} \) and the effective scalar fields, \( \phi^a(x) \). The missing degrees of freedom are assumed to be the perturbative ones which remain after the compression from \( A^a_{\mu} \) to \( \phi^a(x) \). These degrees of freedom are handled using standard and perturbative techniques. In this paper our focus has been the nonperturbative degrees of freedom.

We now give a few remarks about the \( m^2 (\phi^a)^2 \) and \( g^2 a^a_{\mu} \phi^a \phi^a \phi^a \) terms. The last term, \( g^2 a^a_{\mu} \phi^a \phi^a \phi^a \), violates the SU(2) gauge invariance of averaged Lagrangian \( 17 \). The initial QCD Lagrangian is SU(3) gauge invariant, and we attempt to reduce this to an SU(2) invariant one (with the presence of the Higgs type scalar field this may be a hidden SU(2) symmetry). But the Lagrangian \( 17 \) contains terms which are not SU(2) invariant. What has become of this desired SU(2) invariance? One possibility is that the averaging, \( \langle Q \ldots | Q \rangle \), must be taken over all different gauge configurations or copies. This is closely connected with the Gribov ambiguity \( 16 \) where a gauge is picked, but different gauge configurations satisfy the chosen condition. In the perturbative regime where one does an expansion in powers of the coupling constant the Gribov ambiguity is not picked up since the different gauge copies are related by a term which is inversely proportional to the coupling constant, which will therefore not be noticed in a perturbative expansion. In the present case since we dealing with the nonperturbative regime we must address this averaging over different gauge copies. We make the assumption that after this averaging over different gauge copies that all SU(2) gauge invariant terms in \( 17 \) remain the same, but gauge non-invariant like \( g^2 a^a_{\mu} \phi^a \phi^a \phi^a \) term will go to zero. This question is very complicated and will be considered more fully in future work.

We conclude with a few comments about the generation of a mass term \( m^2 (\phi^a)^2 \). In Ref. \( 2 \) the tachyonic mass term from the effective scalar field in the SU(2) \( \rightarrow \) U(1) reduction was effected via the condensation of ghost fields that arose from fixing to the Maximal Abelian Gauge \( 14 \). In the present case this mechanism is not directly applicable, since we are reducing from a non-Abelian to a smaller non-Abelian group rather than an Abelian group. It is possible that a similar ghost condensation mechanism occurs in the present non-Abelian to non-Abelian reduction. Another option is that the appropriate mass term could develop via the Coleman-Weinberg mechanism \( 15 \) where radiative corrections to the effective scalar field produce a symmetry breaking mass term for \( \phi^a \). Yet another option for generating the
correct mass term for the scalar field is to assume that the SU(2) gauge field (not the field \( \phi^a \)) develops a positive mass condensate about which the form of the ansatz in eq. (46) is a fluctuation. For example

\[ a_0^a = 0 \, ; \, a_i^a = \frac{m}{g} \delta_i^a + \tilde{\alpha}_i^a \]  

(48)

where \( \tilde{\alpha}_i^a \) is a fluctuation about the first term. If \( \tilde{\alpha}_i^a \) takes the form of the monopole ansatz in eq. (46) then one finds that \( a_0^a a^{ab} = \frac{m^2}{g^2} \delta^{ab} + \text{monopole term} \); the cross term goes away due to the symmetry of \( \delta_{ia} \) and antisymmetry of \( \epsilon_{iab} \). In this way the last term in (44) would give rise to the tachyonic mass term

\[ \frac{g^2}{2} a_\mu^b \phi^a a^c \phi^c - \frac{m^2}{2} \phi^a \phi^a + \frac{g^2}{2} a_\mu^b \tilde{\alpha}_b^a a^c \phi^c. \]  

(49)

This is the most economical method for generating the mass term for \( \phi^a \), since it turns the unwanted last term of (47) into the desired tachyonic mass. The Lagrangian (47) in terms of \( \tilde{\alpha}_i^a \) is almost identical to the Lagrangian in terms of the original \( a_0^a \), since the two gauge fields are related by a constant shift. The only additional, different term comes from the covariant derivative of the effect scalar field, \( \phi^a \). A final possibility is that the symmetry breaking mass term can result as a consequence of a change of the operator description for strongly nonlinear fields as in ref. [19].

In conclusion we would like to emphasize that the problems considered here are nonperturbative problems in QCD and therefore have the same complexity as confinement.

VII. PHYSICAL CONSEQUENCES

The effective Lagrangian arrived at in (47) is of the form that yields finite energy monopole solutions [3]. It has been shown [2] that classical Yang-Mills theory do not have finite energy solutions \( i.e. \) there are no classical glueballs. The scalar field is crucial to having finite energy field configuration. In the present work an effective scalar field is introduced via the quantization of the coset field, \( A_i^a \). From this one can conclude that even though pure, classical Yang-Mills theory does not have finite energy, static field configurations, a quantized Yang-Mills may support such configurations. Other works have given similar conclusions: In [16] finite energy solutions were found for the non-Abelian Born-Infeld system. In [17] it was shown that a modified Yang-Mills Lagrangian (with the modifications speculated to come from quantization) had finite energy solutions. Thus the monopole solutions of the effective Lagrangian (47) can be viewed as a type of color magnetic glueball, since both the SU(2) field, \( a_0^a \), and the effective scalar field, \( \phi^a \), come from a pure SU(3) Yang-Mills theory.

The mass of the monopole solutions is inversely proportional to the square of the coupling constant and directly proportional to the mass scale. If the original SU(3) Lagrangian is associated with the strong interaction then the coupling will be of order 1 and the mass scale will be of order \( \Lambda \approx 200 \text{ MeV} \). Thus unlike the monopoles in Grand Unified Theories, which have masses greater than \( 10^{15} \text{ GeV} \), the monopoles of the effective Lagrangian (47) would have a mass which would allow them to have physical consequences at low energy scales.

One physical use of these low energy scale monopoles would be to explain confinement via the dual superconducting model [20]. In the dual superconductor model of confinement a condensate of color monopoles/antimonopoles is hypothesized to form. This is in analogy to the Cooper pair condensate which consists of electrically charged electrons. The color monopole condensate then exhibits the Meissner effect with respect to color electric fields \( i.e. \) the condensate tries to exclude color electric flux. Two color electric charged particles placed in this condensate would then have their color electric flux squeezed into a thin flux tube or string between the color charges [21]. This would confine the two color charges, since as one tried to separate them the energy density would rise linearly with the distance, rather than falling off with the inverse distance as for a Coulomb potential. Another possible physical application for the monopole solutions would be to contribute to the explanation of the proton spin puzzle. Beginning with the European Muon Collaboration (EMC) [13] experiment it was realized that contrary to the simple quark model, the spin of the proton comes not only from the spins of the valence quarks, but has other contributions. The monopole configurations could provide a possible contribution to the proton spin in the form of field angular momentum. In refs. [18] it was demonstrated that the combination of a monopole solution plus a particle carrying the “electric” charge of the theory gave rise to a field angular momentum for the composite system. If the color monopole solutions of (47) arose inside the proton they would combine with the color electric, valence quarks to give gluonic field angular momentum contributions to the total proton spin. The field angular of a “monopole-electric charge” composite depends on the “magnetic” and “electric” charges. For the case in refs. [18] and also in the present case the field angular momentum would be have a magnitude of \( h \), and would thus be a major contributor to the total spin of the proton.

This approach to scalar fields as a condensate of nonperturbative degrees of freedom of gauge fields may have interesting applications for gravity where scalar fields have various applications: inflation, boson stars, non-Abelian
black holes and so on. Our approach allows us to speculate that these scalar fields are constructed from certain nonperturbative degrees of freedom of non-Abelian gauge fields.

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