Soliton response to transient trap variations

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Abstract

The response of bright and dark solitons to rapid variations in an expulsive longitudinal trap is investigated. We concentrate on the effect of transient changes in the trap frequency in the form of temporal delta kicks and the hyperbolic cotangent functions. Exact expressions are obtained for the soliton profiles. This is accomplished using the fact that a suitable linear Schrödinger stationary state solution in time can be effectively combined with the solutions of the nonlinear Schrödinger equation, for obtaining solutions of the Gross–Pitaevskii equation with a time-dependent scattering length in a harmonic trap. Interestingly, there is rapid pulse amplification in certain scenarios.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Bose–Einstein condensates (BECs) \cite{1–3} can be confined to one and two dimensions in suitable traps \cite{4–6}, which has opened up the possibility for their technological applications. BEC on a chip is a two-dimensional configuration \cite{7}, whereas cigar-shaped BECs refer to a quasi-one-dimensional scenario \cite{8–10}. In the latter case, the transverse confinement is taken to be strong, with the longitudinal trap frequency having comparatively much smaller value. In this condition, dark \cite{11–13} and bright \cite{14–19} solitons have been observed in repulsive and attractive coupling regimes, respectively. In the context of the BEC, bright and dark solitons respectively, correspond to density lumps and rarefactions compared to the background. The corresponding mean field Gross–Pitaevskii (GP) equation being the familiar nonlinear Schrödinger equation (NLSE) in quasi-one dimensions \cite{20}, the observation of localized solitons has demonstrated the ubiquity of solitons in diverse branches of physics, which share common nonlinear behaviour.

Very interestingly, in the cigar-shaped BECs, a number of parameters such as the scattering length and trap frequencies can be changed in a controlled manner, leading to the possibility of coherent control of solitons. The former can have a temporal variation due to Feshbach resonance, a scenario which has been extensively studied in the literature \cite{21–23}. In particular, the discovery of bright solitons and soliton trains has led to a systematic investigation of the impact of the change in the scattering length on the BEC profile. Recently, the temporal modulation of the nonlinearity in one dimension arising due to the corresponding variations in the transverse trap frequency has been systematically studied in the context of generation of Faraday modes \cite{24, 25}.

In a recent paper \cite{26}, a method to obtain analytical solutions of the GP equation for a range of temporal variations of parameters such as the scattering length, the longitudinal trap frequency etc was demonstrated. It was shown that in the process of reducing the GP equation to a known nonlinear equation, having Jacobi elliptic functions as solutions, the consistency conditions obtained mapped onto the linear Schrödinger equation. This mapping enables us to obtain exact solutions of the GP equation for a wide range of temporal modulations of the control parameters. These solutions exhibit soliton trains and localized solitons (both bright and dark) for different regimes of the coupling parameter. Interestingly, exact solutions of the GP equation have been obtained for other kinds of trap potentials, for example, linear traps \cite{27} using Darboux transformations. The effect of the variations in the time-dependent coupling on the soliton profiles has also received a lot of attention \cite{27–29}.
The goal of the present paper is to investigate the soliton dynamics in a BEC in an expulsive harmonic trap, whose frequency is time dependent. The expulsive trap accelerates the soliton. Very interestingly it is found that for certain conditions it can be compressed, particularly under a sudden transient variation in the trap frequency. As we have mentioned earlier, a temporal modulation in the transverse frequency leads to the generation of Faraday modes in the longitudinal direction. Hence, it is of interest to study the effect of transient variations in the longitudinal trap on the BEC profiles. We concentrate here on transient variations, and for explicitness we use the delta functions and the hyperbolic cotangent functions to model rapid temporal variations in the trap frequency. We obtain exact solutions and study the spatio-temporal dynamics of the solitons, and their response to the changes in the trap frequency. We point out that harmonic traps decorated with delta functions and a dimple potential have been recently studied numerically [30], and the properties such as atomic density, chemical potential, critical temperature etc of the condensate have been obtained.

In the following section, we obtain the exact solutions of the GP equation and illustrate how we can modulate the expulsive trap frequency using suitable quantum mechanical models [26]. In section 3, the trap frequency is transiently changed by hyperbolic cotangent functions. In section 4, we study the response of the solitons to sudden changes in the trap frequency in the form of delta kicks. For all the cases studied we obtain profiles of both dark and bright soliton trains, and localized soliton solutions. The spatio-temporal dynamics of the soliton solutions and the behaviour of the nonlinearity with time in the attractive and repulsive coupling regimes are plotted. It is found that for certain examples there is an amplification in the soliton amplitude resulting in its compression while in other examples the solitons spread with a decaying amplitude.

2. The dynamics of the Bose–Einstein condensate

For a BEC at zero temperature, confined in a cylindrical harmonic trap, \( V_0(x, y) = m'\omega_\perp^2(x^2 + y^2)/2 \), and time-dependent harmonic confinement along the \( z \) direction \( V_1(z, t) = m'\omega_0^2(t)z^2/2 \), the dynamics are governed by the GP equation:

\[
\frac{\partial}{\partial t} \psi(r, t) = -\frac{\hbar^2}{2m'} \nabla^2 \psi + U|\psi(r, t)|^2 + V \psi(r, t),
\]

where \( V = V_0 + V_1, m' \) is the mass of the particle, and \( U = 4\pi\hbar^2 a_\perp(t)/m' \) gives the nonlinear coupling with \( a_\perp(t) \) denoting the scattering length. The nonlinearity can be either attractive \( (a_\perp < 0) \) or repulsive \( (a_\perp > 0) \). In order to reduce the 3D GP equation to quasi-one dimensions, it is assumed that the interaction energy of atoms is much less than the kinetic energy in the transverse direction [9]. Substituting the following trial wavefunction in dimensionless units,

\[
\psi(r, t) = \frac{1}{\sqrt{2\pi a_B a_\perp}} \exp\left( \frac{z}{a_\perp} \right) \exp\left( -i\omega_\perp t - \frac{x^2 + y^2}{2a_\perp^2} \right),
\]

we obtain the quasi-1D NLSE:

\[
i\partial_t \psi = -\frac{i}{2} \partial_z^2 \psi + |\psi|^2 \psi + \frac{1}{2} M(t)z^2 \psi.
\]

Here, \( \gamma(t) = 2\alpha_\perp(t)/a_B, M(t) = \omega_0^2(t)/a_\perp^2, a_\perp = (\hbar/m'\omega_\perp)^{1/2} \) with \( a_B \) being the Bohr’s radius. We have kept \( M(t) \), related to the trap frequency, time dependent. This enables us to modulate the trap frequency, allowing us to study the effect of different temporal profiles of the trapping potential on the solitons. The specific case of \( M(t) \), being a constant, describes the scenario where the oscillator potential is either confining \( (M > 0) \) or repulsive \( (M < 0) \). Analytic solutions to the above NLSE are obtained using the ansatz solution

\[
\psi(z, t) = \sqrt{\Phi(z)}F[A(t)(z - l(t))\exp\{i\Phi(z, t)\}],
\]

where \( A(t) \) describes the amplitude, and \( l(t) = \int_0^t v(t') \, dt' \) determines the location of the centre of mass, with \( v(t) \) being its velocity. We assume the phase to be of the form

\[
\Phi(z, t) = a(t) - \frac{1}{2} c(t)z^2.
\]

Substitution of \( \psi(z, t) \) in the NLSE gives us the consistency conditions:

\[
A(t) = A_0 \exp\left( \int_0^t c(t') \, dt' \right), \quad \frac{dl(t)}{dt} + c(t)l(t) = 0
\]

and

\[
\gamma(t) = \frac{A(t)}{A_0},
\]

where \( A_0, \gamma_0 \) and \( l_0 \) are constants. We also obtain \( a(t) = a_0 + \frac{1}{2} \int_0^t A^2(t') \, dt' \) along with the Riccati equation for \( c(t) \):

\[
\frac{dc(t)}{dt} = -c^2(t) = M(t).
\]

From the above equations, it can be seen that \( A(t), \gamma(t) \) and \( l(t) \) are all non-trivially related to the phase component \( c(t) \). Added to this, the fact that the Riccati equation can be mapped onto the linear Schrödinger eigenvalue problem with potential \( V(t) \),

\[
-\phi''(t) - M(t)\phi(t) = 0
\]

with \( M(t) = M_0 - V(t) \), using the change of variable,

\[
c(t) = -\frac{d}{dt} \ln[\phi(t)],
\]

gives us control over the dynamics of the BEC. Hence, knowing \( c(t) \) or the solution \( \phi(t) \) of (9), allows us to obtain analytical expressions of the control parameters through equations (10) and (6)–(7). It is a well-known fact that for many potential models, the Schrödinger equation can be solved exactly. In the following sections, we exploit this and the connection between \( c(t) \) and \( \phi(t) \) to study the dynamics of the BEC analytically, when the oscillator frequency undergoes a rapid change.

The use of the above consistency conditions along with the ansatz solution in the NLSE gives us the following differential equation for \( F \), in terms of the new variable \( T = A(t)(z - l(t)) \):

\[
F'''(T) - \lambda F(T) + 2\epsilon F^3(T) = 0,
\]
with \( \kappa = -\frac{m}{h^2} \), and the differentiation is with respect to \( T \). The general solution of this differential equation can be written in terms of the 12 Jacobi elliptic functions \( \text{sn}(T, m), \text{cn}(T, m), \text{dn}(T, m) \) etc where \( m \) is the elliptic modulus [31]. The solitons will arise in the limit \( m \to 1 \), where \( \text{dn}(T, m) = \text{cn}(T, m) \to \text{sech}(T) \) and \( \text{sn}(T, m) \to \tanh(T) \).

Thus for \( \gamma_0 \leq 0 \), bright soliton trains of the form

\[
\psi(z, t) = \sqrt{\Lambda(t)} \text{cn}(T/\tau_0, m) \exp[i\Phi(z, t)]
\]  

exist, where \( \tau_0^2 = -mA_0/\gamma_0 \) and \( \lambda = (2m - 1)/\tau_0^2 \) (these coefficients are obtained using equations (6)–(9)). In the limiting case \( m = 1 \), equation (12) corresponds to a bright soliton solution. It has been seen that for \( \gamma_0 > 0 \), there exist dark soliton trains of the form

\[
\psi(z, t) = \sqrt{\Lambda(t)} \text{sn}(T/\tau_0, m) \exp[i\Phi(z, t)],
\]

with \( \tau_0^2 = mA_0/\gamma_0 \), \( \lambda = -(m + 1)/\tau_0^2 \) and in the limit \( m \to 1 \), the above solution corresponds to a dark soliton.

In the following sections, we obtain the profiles of bright and dark soliton trains, and solitons in a BEC in an expulsive oscillator potential, with experimentally achievable temporal variations of the trap frequency. As seen in (8), the oscillator frequency \( M(t) \) depends on \( c(t) \). We first concentrate on the response of the soliton profiles to the variations in the trap frequency when \( c(t) \) is equal to the hyperbolic cotangent functions.

### 3. Dynamics of the BEC with \( c(t) = \pm B \coth(t) \)

In this section, we look at the examples where the trap frequency of the confining trap is modulated using \( c(t) = \pm B \coth(t) \), \( B > 0 \). Using the consistency conditions in (6) and (7) we obtain the analytical solutions of the NLSE as shown below.

**Case (a).** We take \( c(t) = B \coth(t), \ B > 0 \) in (8) and obtain

\[
M(t) = -B^2 - B(B + 1) \coth^2(t).
\]

Since \( M_0 = -B^2 \), we are looking at an expulsive oscillator scenario. We point out that \( V(x) = B(B + 1) \coth^2(x) \) corresponds to the supersymmetric Rosen–Morse potential with superpotential \( W = B \coth(x) \) [32]. The function \( V(t) \) has a singularity at \( t = 0 \) which causes a sudden change in the trap frequency. Substituting for \( c(t) \) in equations (6) and (7) one obtains

\[
a(t) = a_0 - \frac{(\lambda - 1)A_0^2}{2} \left( \sinh(t) - 4t \right),
\]

\[
A(t) = A_0 \left( \frac{\sinh(t)}{\sinh(\lambda t)} \right)^B,
\]

\[
I(t) = I_0 \left( \frac{\sinh(\lambda t)}{\sinh(t)} \right)^B,
\]

which when substituted in (12) and (13), give the following solutions describing the bright and dark soliton trains:

\[
\psi(z, t) = \sqrt{A_0 \sinh(t)} \text{cn} \left( \frac{A_0 \sinh(t)}{\tau_0} \frac{\sinh(\lambda t)}{m} \right) \exp(i\Phi(z, t)),
\]

(17)

\[
\psi(z, t) = \sqrt{A_0 \sinh(t)} \text{sn} \left( \frac{A_0 \sinh(t)}{\tau_0} \frac{\sinh(\lambda t)}{m} \right) \exp(i\Phi(z, t)),
\]

(18)

respectively. The dynamics of these solutions are plotted in figures 1(a) and (b), where the matter wave density \( |\psi(z, t)|^2 \) is plotted for varying \( t \) and \( z \). We can see that the soliton trains get amplified with time as we move away from the origin. In the limit \( m \to 1 \), from (17) and (18) we get the bright and dark solitons, plotted in figures 1(c) and (d), respectively. These solitons are highly localized and are seen to be diverging from the \( t = 0 \) line in the \( t-z \) plane. By changing the initial time to some \( t_0 \), instead of \( t = 0 \) in the consistency conditions and varying \( l_0 \), the location of the centre of mass can be changed. The nonlinearity \( \gamma(t) \) is plotted against varying \( t \), for the attractive \( (\gamma_0 < 0) \) and the repulsive \( (\gamma_0 > 0) \) coupling regimes in figures 1(e) and (f) respectively, and we see that for a given \( \gamma_0 \), the nonlinearity does not change sign implying that we are either in an attractive or repulsive coupling regimes. By changing the constants \( B, A_0, \gamma_0, l_0 \) the amplification and the location of the solitons can be changed. We would like to point out here that in the weak interacting limit \( \frac{\partial^2 \Phi}{\partial \lambda^2} \big|_{\lambda=0} |\psi(z, t)|^2 \lesssim 1 \) [9], where \( \psi(z, t) = \sqrt{\Lambda} \psi(z, t) \). Hence, for the given trap parameters, the soliton amplification is restricted by the above condition.

**Case (b).** For \( c(t) = -B \coth(t), \ B > 0, \ M(t) = -B^2 - B(B + 1) \coth^2(t) \). Here again we are looking at the expulsive oscillator scenario, and the potential \( B(B - 1) \coth^2(x) \) corresponds to the supersymmetric Poschl–Teller potential [32]. Moreover, this case is interesting because for \( B = 1 \), this potential corresponds to the free particle problem which has been studied in [26]. Substituting \( c(t) \) in equations (6) and (7), we obtain

\[
A(t) = A_0 \left( \frac{\sinh(t)}{\sinh(\lambda t)} \right)^B, \quad \gamma(t) = \gamma_0 \left( \frac{\sinh(t)}{\sinh(\lambda t)} \right)^B,
\]

(19)

\[
l(t) = l_0 \left( \frac{\sinh(\lambda t)}{\sinh(t)} \right)^B.
\]

As in the above case, we obtain the profiles of both bright and dark soliton trains, and localized solitons which are plotted in figures 2(a)–(f). Although the function \( V(t) \) has a singularity at \( t = 0 \), as in case (a), the response of the soliton solutions is entirely different. We see that the soliton trains have a very high amplitude around \( t = 0 \), and they decay as we move away from the origin i.e. the soliton profile spreads with time. The decay and the amplitude can be controlled by changing \( A_0 \). It should be noted that the amplitude of the solitons cannot increase indefinitely, but is constrained by the condition stated in the previous case. The solitons plotted in figures 2(c) and (d) are again highly localized, but in this case converge towards the \( t = 0 \) line in the \( t-z \) plane. From figures 2(e) and (f), we see that though the nonlinearity does not change sign for a given \( \gamma_0 \), it does increase rapidly in magnitude around \( t = 0 \), unlike the previous case where \( \gamma(t) \) varied slowly with time.
Figure 1. $c(t) = \coth(t)$ in the expulsive oscillator regime. Plots of the bright (a), (c) and dark (b), (d) soliton solutions, $ψ(z, t) = \sqrt{A(t)}F\{A(t)[z - l(t)]/τ_0, m}\exp[i\lambda(t) - \frac{1}{2}c(t)z^2]$ with $F = \text{cn}(T/τ_0, m)$ and $F = \text{sn}(T/τ_0, m)$ respectively, are shown. The plots (a)–(d) show $|ψ(z, t)|^2$ versus $t$ and $z$, and the plots (e) and (f) show the variation of the nonlinearity $γ(t)$ with $t$. The values of the constant parameters are $A_0 = 0.5$, $l_0 = 5$, $τ_0 = 0.9$ and $B = 2$. (a) Bright soliton train: $m = 0.5$, $τ_0 = 0.375$. (b) Dark soliton train: $m = 0.5$, $τ_0 = 0.375$. (c) Bright soliton: $m = 1$, $τ_0 = 1$. (d) Dark soliton: $m = 1$, $τ_0 = 1$. (e) $γ(t)$ versus $t$: $γ_0 = -0.5$. (f) $γ(t)$ versus $t$: $γ_0 = 0.5$.

4. Response of the BEC to delta kicks in the trap frequency

In this example, we vary the trap frequency with an attractive delta potential located at the origin and see how the solitons respond to a sudden kick. Here,

$V(t) = -V_0 δ(t), \quad V_0 > 0,$

and the solutions of (9) are

$φ_1(t) = \sqrt{\frac{V_0}{2}} \exp\left(-\frac{V_0}{2}t\right) \quad (t > 0)$

$\phi_2(t) = \sqrt{\frac{V_0}{2}} \exp\left(\frac{V_0}{2}t\right) \quad (t > 0)$

with $M_0 = -\frac{V_0}{2}$. In order to obtain $φ_2(t)$, we have made use of the fact that the second solution of a second-order differential equation can be obtained from the first solution using the relation $φ_2(t) = φ_1(t) \int [dφ(t')]/(φ_1(t')^2)$. We can write $c(t)$ in terms of $φ(t)$ using (10) and obtain the consistency conditions in terms of $φ(t)$ as

$A(t) = A_0 \frac{φ(0)}{φ(t)}, \quad γ(t) = γ_0 \frac{φ(0)}{φ(t)}, \quad l(t) = l_0 \frac{φ(t)}{φ(0)}.$

Now we proceed to study the soliton profiles using the two solutions given in (21) and (22) separately.

Case (a). First we consider the solutions in (21), and from equation (10) we obtain $c(t) = V_0/2$ for $t > 0$ and $c(t) =$
Figure 2. \( c(t) = -\coth(t) \) in the expulsive oscillator regime. Plots of the bright (a), (c) and dark (b), (d) soliton solutions, \( \psi(z,t) = \sqrt{A(t)} \{ \frac{1}{\tau_0} |z - l(t)| \}^m \exp[i(a(t) - \frac{1}{2} (t)^2)] \) with \( F = \cn(T/\tau_0, m) \) and \( F = \sn(T/\tau_0, m) \) respectively, are shown. The plots (a)–(d) show \( |\psi(z,t)|^2 \) versus \( t \) and \( z \), and the plots (e) and (f) show the variation of the nonlinearity \( \gamma(t) \) with \( t \). The values of the constant parameters are \( A_0 = 0.5 \), \( l_0 = 5 \), \( t_0 = 0.8812 \) and \( B = 2 \). (a) Bright soliton train: \( m = 0.5 \), \( \tau_0 = 0.375 \). (b) Dark soliton train: \( m = 0.5 \), \( \tau_0 = 0.375 \). (c) Bright soliton: \( m = 1 \), \( \tau_0 = 1 \). (d) Dark soliton: \( m = 1 \), \( \tau_0 = 1 \). (e) \( \gamma(t) \) versus \( t \): \( \gamma_0 = -0.5 \). (f) \( \gamma(t) \) versus \( t \): \( \gamma_0 = 0.5 \).
Figure 3. Attractive δ potential in the expulsive oscillator regime, \( \phi(t) = \sqrt{V_0/2} \exp(-V_0|t|/2) \). Plots of the bright (a), (c) and dark (b), (d) soliton solutions, \( \psi(z,t) = \sqrt{A_0} F\{A(t)[z-l(t)]/\tau_0, m\} \exp[i\alpha(t) - \frac{1}{2}c(t)z^2] \) with \( F = \text{cn}(T/\tau_0, m) \) and \( F = \text{sn}(T/\tau_0, m) \) respectively, are shown. The plots (a)–(d) show \( |\psi(z,t)|^2 \) versus \( t \) and \( z \), and the plots (e) and (f) show the variation of the nonlinearity \( \gamma(t) \) with \( t \). The values of the constant parameters are \( V_0 = 10, A_0 = 0.5, l_0 = 5 \). (a) Bright soliton train: \( m = 0.5, \tau_0 = 0.375 \). (b) Dark soliton train: \( m = 0.5, \tau_0 = 0.375 \). (c) Bright soliton: \( m = 1, \tau_0 = 1 \). (d) Dark soliton: \( m = 1, \tau_0 = 1 \). (e) \( \gamma(t) \) versus \( t \): \( \gamma_0 = -0.5 \). (f) \( \gamma(t) \) versus \( t \): \( \gamma_0 = 0.5 \).

In conclusion, we have analytically treated the effect of variations of the longitudinal trap frequency on the BEC profile. We have looked at the cases where the trap frequency is modulated using hyperbolic cotangent functions and delta kicks. It is observed that in the gain/loss less scenario, these variations in the trap frequency resulted in the soliton profiles either amplifying or spreading. We have also shown that we can manipulate the amplification and the location of the solitons by varying the control parameters.

The soliton profiles are plotted in figure 4. We observe that the soliton train amplitude decays rapidly as \( t \) increases unlike the previous case. For smaller values of \( V_0 \), the trains exist for longer time. Thus the decay of the solutions can be controlled by varying \( V_0 \).
Figure 4. Attractive $\delta$ potential in the expulsive oscillator regime, $\phi(t) = \sqrt{2V_0} \exp(V_0 t/2)$. Plots of the bright (a), (c) and dark (b), (d) soliton solutions, $\psi(z,t) = \sqrt{A(t)} F\{[A(t)][z - l(t)]\}/\tau_0, m \} \exp[i\alpha(t) - \frac{i}{2} c(t) z^2]$ with $F = cn(T/\tau_0, m)$ and $F = sn(T/\tau_0, m)$ respectively, are shown. The plots (a)–(d) show $|\psi(z,t)|^2$ versus $t$, and $z$ and the plots (e) and (f) show the variation of the nonlinearity $\gamma(t)$ with $t$. The values of the constant parameters are $V_0 = 10, A_0 = 0.5, l_0 = 5$. (a) Bright soliton train: $m = 0.5, \tau_0 = 0.375$. (b) Dark soliton train: $m = 0.5, \tau_0 = 0.375$. (c) Bright soliton: $m = 1, \tau_0 = 1$. (d) Dark soliton: $m = 1, \tau_0 = 1$. (e) $\gamma(t)$ versus $t$: $\gamma_0 = -0.5$. (f) $\gamma(t)$ versus $t$: $\gamma_0 = 0.5$.

The fact that a stationary Schrödinger eigenvalue problem can be combined with solutions of the nonlinear Schrödinger equation to obtain the solutions of the Gross–Pitaevskii equation with time-dependent parameters, enabled us to obtain the exact dynamical behaviour. As mentioned in the introduction, there are other methods of obtaining exact solutions of the GP equation. The method used in this paper is simpler, and the fact that the Schrödinger equation can be solved exactly for many potential models offers us a huge choice of temporal variations of the trap frequency from the class of exactly solvable models. We can choose models which can be realized experimentally. Along with this wide choice, the other advantage is that we have total control over the soliton dynamics. It would be interesting to see how solitons evolve in space and time for other choices of variations in the trap frequency. A detailed study keeping in mind the experimental scenarios is under progress and will be reported elsewhere. Another interesting problem will be the study of two-soliton dynamics under similar variations in the trap frequency [29].

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