Wilson Loop Correlator
in the AdS/CFT Correspondence

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Abstract

The AdS/CFT correspondence predicts a phase transition in Wilson loop correlators in the strong coupling $\mathcal{N} = 4$, $D = 4$ SYM theory which arises due to instability of the classical string stretched between the loops. We study this transition in detail by solving equations of motion for the string in the particular case of two circular Wilson loops. The transition is argued to be smoothened at finite 't Hooft coupling by fluctuations of the string world sheet and to be promoted to a sharp crossover. Some general comments about Wilson loop correlators in gauge theories are made.
1 Introduction

According to [1, 2], calculation of Wilson loop correlators in $D = 4$, $\mathcal{N} = 4$ supersymmetric $SU(N)$ Yang-Mills theory at large $N$ and large ’t Hooft coupling amounts in evaluation of the classical string action in anti-de-Sitter space. The string propagates in the bulk of $AdS_5 \times S^5$ with its ends attached to the Wilson loops lying on the boundary. This prescription is a consequence of the AdS/CFT correspondence [3, 4, 5]. Gross and Ooguri pointed out that it implies a kind of phase transition in the correlation function of two Wilson loops [6]. The reason for the Gross-Ooguri phase transition is that the string action, the area of a minimal surface bounded by the loops, generically has two competing saddle points. The minimal surface can have a topology of annulus or can consist of two disconnected pieces spanning individual loops. The annulus evidently has smaller area when the loops are closed enough to one another. But the area of the annulus increases with separation between the loops and eventually disconnected surface becomes energetically more favourable. At large distances, the string behaves classically only in the vicinity of the loops and the connected correlator is saturated by perturbative exchange of lightest supergravity modes between disconnected pieces of the classical world sheet [6, 7]. A jump from one saddle point to the other should lead to the phase transition in the Wilson loop correlator considered as a function of the separation.

Two Wilson loop correlators in the AdS/CFT correspondence were studied at large distances when they are saturated by supergraviton exchange [7]. To our knowledge, the connected solution for the minimal surface has not yet been considered. The arguments of [6] relied upon the pattern of topology change for the surface stretched between two concentric circles in the flat space, fig. 1. For further references, we sketch these arguments here.

When the distance $L$ between the circles is small, the minimal surface is the catenoid: $r(x) = R_m \cosh(x/R_m)$. The area of the catenoid is given by

$$A = \pi R_m L + \pi R_m^2 \sinh\left(\frac{L}{R_m}\right),$$

(1.1)

where $R_m$, the radius of its narrowest section, is determined by boundary conditions:

$$R_m \cosh\left(\frac{L}{2R_m}\right) = R.$$

(1.2)

The left hand side of this equation, as a function of $R_m$, has a minimum at some $R_m = \text{const} \cdot L$, so for large enough $L$ this equation ceases to have any solutions. At the critical distance $L_\ast$, the connected minimal surface becomes unstable. Yet before its area starts to exceed the area of two discs, another minimal surface with the same boundary.

In this paper, we address the issue of how the Gross-Ooguri phase transition is affected by the geometry of $AdS_5$ and by quantum corrections due to fluctuations of the string world sheet. The former problem is much simpler. The solution for two circular Wilson loops is found in sec. 3. The surface with the topology of annulus becomes unstable at certain value of $L$. As a consequence, the Wilson loop correlator in $\mathcal{N} = 4$ SYM undergoes the phase transition at infinite ’t Hooft coupling.

The influence of world-sheet fluctuations is more difficult to estimate. It is still not known how to calculate superstring amplitudes in $AdS_5 \times S^5$. Presence of the background
Ramond-Ramond flux compels one to use the Green-Schwarz formalism \[8, 9, 10, 12, 11, 13\], the covariant quantisation of the superstring in which is notoriously complicated \[14\]. In view of the difficulties with passing to finite $\alpha'$ in AdS geometry, which is equivalent to finite 't Hooft coupling on the SYM side, we pursue the above flat space example to study the world-sheet fluctuations in sec. \[3\]. The string amplitude between two loops can be calculated non-perturbatively in $\alpha'$ in this case \[15, 16\].

## 2 Two Loop Correlator

In the supergravity (strong coupling) limit, the Wilson loop correlator is expressed in terms of an area of the classical string world sheet stretched between the loops \[1, 2\]:

$$\left\langle W(C_1)W(C_2)\right\rangle_{\text{conn}} = \exp \left( -\frac{S}{2\pi \alpha'} \right),$$  \tag{2.1}

$$S = \int d\sigma d\tau \sqrt{\det g_{\mu\nu}} \partial_{a} x^\mu \partial_{b} x^\nu,$$  \tag{2.2}

$$\frac{\delta S}{\delta x^\mu} = 0.$$  \tag{2.3}

We take $C_1$ and $C_2$ to be concentric circles of radius $R$ separated by distance $L$ (fig. 1). This configuration suggests the use of the cylindric coordinates in $\mathbb{R}^4$. The $AdS_5$ metric is then

$$ds^2 = \frac{1}{z^2} \left( dz^2 + dt^2 + dx^2 + dr^2 + r^2 d\varphi^2 \right).$$  \tag{2.4}

The boundary of $AdS$ is at $z = 0$. We chose the units in which the radius of $AdS$ is unity and the string tension is proportional to the Yang-Mills coupling:

$$\alpha' = (2g_{YM}^2N)^{-1/2}.$$
2.1 Minimal Surface

Using symmetries of the problem we take the following ansatz for the minimal surface:

\[ t = 0, \ \varphi = \sigma, \ r = r(\tau), \ x = x(\tau), \ z = z(\tau). \]  (2.5)

Then the equations \( \delta S/\delta t = 0 \) and \( \delta S/\delta \varphi = 0 \) are satisfied identically. The equations for \( r, \ x, \) and \( z \) follow from the action

\[ S = 2\pi \int d\tau \frac{r}{z^2} \sqrt{(r')^2 + (x')^2 + (z')^2}. \]  (2.6)

The variation with respect to \( r \) and \( z \) yields:

\[ \left( \frac{r}{z^2} \frac{r'}{\sqrt{(r')^2 + (x')^2 + (z')^2}} \right)' - \frac{1}{z^2} \frac{1}{\sqrt{(r')^2 + (x')^2 + (z')^2}} = 0, \]  (2.7)

\[ \left( \frac{r}{z^2} \frac{z'}{\sqrt{(r')^2 + (x')^2 + (z')^2}} \right)' + \frac{2r}{z^3} \frac{1}{\sqrt{(r')^2 + (x')^2 + (z')^2}} = 0. \]  (2.8)

The equation which follows from the variation with respect to \( x \) can be integrated once to give

\[ \frac{r}{z^2} \frac{x'}{\sqrt{(r')^2 + (x')^2 + (z')^2}} = k, \]  (2.9)

where \( k \) is an integration constant. If \( k = 0 \), the string does not propagate along the \( x \) direction and sweeps out a surface with the disc topology. The minimal surface in \( AdS_5 \) bounding the single circular Wilson loop was found in \[17, 7\]:

\[ r^2 + z^2 = R^2. \]  (2.10)

If \( k \neq 0 \) (without loss of generality we assume that \( k > 0 \), \( x' \) is always positive and we can choose \( x \) to be one of the coordinates on the string world sheet: \( \tau = x \). With this gauge choice, the equations of motions take the form:

\[ r'' - \frac{r}{k^2 z^4} = 0, \]  (2.11)

\[ z'' + \frac{2r^2}{k^2 z^5} = 0, \]  (2.12)

\[ (z')^2 + (r')^2 + 1 - \frac{r^2}{k^2 z^4} = 0. \]  (2.13)

Boundary conditions for these equations are

\[ r(-L/2) = r(L/2) = R, \]  (2.14)

\[ z(-L/2) = z(L/2) = 0. \]  (2.15)
In principle, it is necessary to regularise the problem by shifting the Wilson loops from the boundary of AdS in order to get the solution with a finite area. Strictly speaking, more appropriate boundary conditions for $z$ are $z(\pm L) = \epsilon$, but the solution itself is not singular in the limit $\epsilon \to 0$ and, since a regularisation does not influence the critical behaviour, we shall first find unregularised solution and shall return to the issue of regularisation later.

Qualitative structure of the solution is clear from eqs. (2.11), (2.12). Closed string emitted by the Wilson loop at $x = -L/2$ falls into the interior of the AdS space, then bounces back, and is absorbed by the Wilson loop at $x = L/2$. The radius of the string decreases till the bouncing point and then starts to increase (fig. 2), like in the flat space.

It is possible to integrate equations (2.11)–(2.13). Adding first two ones multiplied by $r$ and by $u$, respectively, to the third equation, we get:

\[(r^2 + z^2)^{''} + 2 = 0,\]  

or, taking into account boundary conditions:

\[r^2 + z^2 + x^2 = R^2 + \frac{L^2}{4} \equiv a^2.\]  

This equality suggests the substitution:

\[r = \sqrt{a^2 - x^2} \cos \theta, \quad z = \sqrt{a^2 - x^2} \sin \theta.\]  

The equation (2.13) considerably simplifies in the new variables:

\[(a^2 - x^2)(\theta')^2 + \frac{x^2}{a^2 - x^2} + 1 - \frac{\cos^2 \theta}{k^2(a^2 - x^2) \sin^4 \theta} = 0.\]  

The variables separate:

\[\theta' = \pm \frac{a}{a^2 - x^2} \sqrt{\frac{\cos^2 \theta}{k^2 a^2 \sin^4 \theta} - 1},\]
where the upper sign is to be taken for \( x \in [-L/2, 0] \) and the lower sign for \( x \in [0, L/2] \). The solution on the interval \([-L/2, 0]\) is determined by equation

\[
ka \int_0^\theta \frac{d\phi \sin^2 \phi}{\sqrt{\cos^2 \phi - k^2 a^2 \sin^4 \phi}} = \frac{1}{2} \ln \left( \frac{a + \frac{L}{2}}{a - \frac{L}{2}} \right) - \frac{1}{2} \ln \left( \frac{a - x}{a + x} \right)
\]

and should be continued to positive \( x \) by the symmetry \( \theta(x) = \theta(-x) \).

The parameter \( k \) is fixed by requirement of continuity of the solution at \( x = 0 \). Since \( r \) and \( z \) are even functions of \( x \), so is \( \theta(x) \). Hence, the derivative \( \theta'(0) \) vanishes. Equation (2.20) then fixes the value of \( \theta \) at \( x = 0 \):

\[
\theta_0 \equiv \theta(0) = \arccos \left( \frac{\sqrt{4k^2 a^2 + 1} - 1}{2ka} \right).
\]

Substituting \( x = 0 \) in eq. (2.21), we get an equation for \( k \):

\[
F(ka) = \frac{1}{2} \ln \left( \frac{a + \frac{L}{2}}{a - \frac{L}{2}} \right) = \ln \left( \frac{\sqrt{R^2 + \frac{L^2}{4}} + \frac{L}{2}}{R} \right),
\]

where

\[
F(ka) \equiv ka \int_0^{\theta_0} \frac{d\phi \sin^2 \phi}{\sqrt{\cos^2 \phi - k^2 a^2 \sin^4 \phi}}.
\]

The right hand side is the complete elliptic integral, since the upper bound of integration is at the square root branching point: \( \cos^2 \theta_0 - k^2 a^2 \sin^4 \theta_0 = 0 \). The equations (2.18), (2.21), and (2.23) express the solution for the minimal surface in terms of elliptic functions.

### 2.2 Critical behaviour

The existence of the phase transition in the Wilson loop correlator can be inferred from eq. (2.23). The function \( F(\xi) \) turns to zero at \( \xi = 0 \) and at \( \xi = \infty \):

\[
F(\xi) \simeq -\xi \ln \xi \quad (\xi \to 0),
\]

\[
F(\xi) \simeq \frac{\sqrt{2\pi^3}}{\Gamma^2 \left( \frac{1}{4} \right)} \frac{1}{\sqrt{\xi}} \quad (\xi \to \infty).
\]

Consequently, it has a maximum at some \( \xi = \xi_* \). At the same time, the right hand side of equation (2.23) varies from zero to infinity as the separation between loops increases. Continuity arguments show that the branch with larger \( ka \) should be chosen from the two of the solutions of equation (2.23). The two branches meet at \( ka = \xi_* \). If \( L \) exceeds the critical value

\[
L_* = 2R \sinh F(\xi_*),
\]

the equation (2.23) does not have any solutions and the minimal surface with the topology of annulus ceases to exist. Numerically, \( \xi_* = 0.58 \) and

\[
L_* = 1.04R.
\]
The connected minimal surface becomes unstable when Wilson loops are separated by this distance. In fact, the transition from the annulus to the disc topology is of the first order. The critical point is determined by the equality of the free energies in the two phases, the areas of the disconnected and the connected minimal surfaces.

The area of regularised disconnected solution is

\[ S_{\text{disc}} = 2 \left( \frac{2\pi \sqrt{R^2 + \epsilon^2}}{\epsilon} - 2\pi \right) = \frac{4\pi R}{\epsilon} - 4\pi + O(\epsilon). \] (2.27)

The area of the connected surface also requires regularisation. Shift in the boundary conditions for \( z, z(\pm L/2) = \epsilon \) instead of \( z(\pm L/2) = 0 \), renders the area finite. The boundary conditions for \( \theta(x) \) change, accordingly. From (2.18) we find:

\[ \theta(\pm L/2) = \arctan \left( \frac{\epsilon}{R} \right) \simeq \frac{\epsilon}{R}. \] (2.28)

Using equations of motion, we get for the regularised area:

\[
S = 2\pi \int_{-L/2}^{L/2} dx \frac{r}{z^2} \sqrt{1 + \left( \frac{r'}{r} \right)^2 + \left( \frac{z'}{z} \right)^2} = \frac{2\pi}{k} \int_{-L/2}^{L/2} dx \frac{r^2}{z^4} = \frac{4\pi}{k} \int_{-L/2}^{0} dx \frac{\cos^2 \theta}{a^2 - x^2} \sin^4 \theta
\]

This integral can be simplified by the change of variables

\[ \tan \theta = \left( \frac{\sqrt{2} k^2 a^2 + 1 - 1}{2} \right)^{-1/2} \sin \psi. \]

After some algebra, we obtain:

\[
S = \frac{4\pi R}{\epsilon} - 4\pi \frac{\alpha}{\sqrt{\alpha - 1}} \int_{0}^{\pi/2} d\psi \frac{1}{1 + \alpha \sin^2 \psi + \sqrt{1 + \alpha \sin^2 \psi}},
\] (2.30)

where

\[ \alpha = \frac{1 + 2k^2 a^2 + \sqrt{1 + 4k^2 a^2}}{2k^2 a^2}. \] (2.31)

The equations (2.1), (2.23), (2.24), (2.30), and (2.31) determine the correlator of two Wilson loops in the strong-coupling \( \mathcal{N} = 4 \) SYM theory.

The divergent part of the area is the same for the connected and the disconnected solutions. Its origin can be attributed to perimeter divergency of a Wilson loop due to self-energy contributions \([17, 4]\). The renormalised area is always negative. Its short-distance asymptotics can be found with the help of eq. (2.25):

\[
S \simeq \frac{4\pi R}{\epsilon} - 4\pi \frac{\sqrt{2\pi^3}}{\Gamma^2 \left( \frac{1}{4} \right)} k a \simeq \frac{4\pi R}{\epsilon} - \frac{16\pi^4}{\Gamma^4 \left( \frac{1}{4} \right)} \frac{R}{L}.
\] (2.32)

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The area grows with separation between loops and at the point of the phase transition meets the area of the disconnected surface. Solving equations for the critical point numerically, we find:

\[ L_c = 0.91 R. \]  

(2.33)

In spite of evident differences between the classical string propagation in AdS\(_5\) and in the flat space, the qualitative pattern of the Gross-Ooguri phase transition appears to be not much influenced by geometry of the target space. In both cases, the transition is of the first order and takes place at \( L \sim R \). We expect that an influence of the string fluctuations on the phase transition is also universal, at least in some respects. This is our motivation to study the free superstring propagator between two loops in the next section.

3 Two Loop Amplitude in the Free String Theory

The equations (2.1)–(2.3) are expected to be valid at infinite ‘t Hooft coupling and to be replaced by a full sum over random surfaces in AdS\(_5\) \( \times S^5 \) with boundary conditions set by Wilson loops, when the coupling is finite. Presently, fluctuations of the string world sheet in AdS geometry can be accounted for only to the first order in \( \alpha' \) [18, 19]. The superstring amplitude in the flat space is much easier to calculate. It is known exactly and provides a possibility to trace qualitative changes brought in by non-perturbative \( \alpha' \) corrections.

The amplitude for the free type IIB superstring to propagate between two concentric circular loops is a particular case of the boundary state amplitude calculated for the bosonic string in [15] and for NSR string in [16]. After GSO projection, the amplitude takes the form:

\[ A = \text{const} \int_0^\infty ds \frac{\Theta_2(0|is) + \Theta_3(0|is) - \Theta_4(0|is)}{s^5 2\eta^{12}(is)} \exp \left( -\frac{S(s)}{2\alpha'} \right), \]

(3.1)

where

\[ S(s) = \frac{L^2}{s} + 2\pi R^2 \tanh \left( \frac{\pi s}{2} \right). \]

(3.2)

Suppose that \( L^2 \gg \alpha' \) and \( R^2 \gg \alpha' \), then \( S \) is large and the proper time integral (3.1) is saturated by a saddle point, if the latter exists. The saddle point of the action (3.2) is determined by equation

\[ \cosh \left( \frac{\pi s_m}{2} \right) = \frac{R}{L}. \]

(3.3)

By the substitution \( \pi s_m = L/R_m \) we recover the boundary condition (1.2) for the catenoid from this equation. The area of the catenoid (1.4) is also reproduced:

\[ S(s_m) = \frac{L^2}{s_m} + \frac{L^2}{\pi s_m^2} \sinh \pi s_m. \]

(3.4)
Figure 3: The superstring amplitude between two circular loops versus $L^2$ at $R^2 = 20$ (the units are such that $2\pi\alpha' = 1$). The thick solid line is $-\ln A$. Two other curves represent approximations for the amplitude valid in different phases. The thin solid line is the area of the catenoid with $O(\alpha'^0)$ correction added. The line terminates at the point of instability, $L = L^2_*$. The dashed line is $2\pi R^2 - \ln($supergraviton exchange$)$. The estimate for the crossover point based on matching of the classical actions in the two phases is $L_c \simeq 22$ for the chosen value of $R$. Account of the supergraviton exchange and of the first order in the semiclassical expansion shift the crossover point by about 20%. The reason for numerically large deviation from the classical estimate is that the first order corrections are logarithmic in $L^2$.

The saddle point disappears at sufficiently large $L$, because eq. (3.3) has no solutions for $L > L_*$. Beyond the critical point, the main contribution to the integral comes from $s \to \infty$. The action at $s = \infty$ is $2\pi R^2$, the area of two discs. At large distances, the amplitude is saturated by an exchange of massless string modes:

$$A \simeq \text{const} \frac{96}{L^8} e^{-R^2/\alpha'}.$$  

The semiclassical amplitude has a discontinuous first derivative in $L$ at the point of the transition between the two saddle points. The discontinuity persists to any finite order in the $\alpha'$ expansion. The integral (3.1), however, defines an analytic function of $L$. The phase transition is thus replaced by a smooth crossover in the exact amplitude. The larger $R^2$ is, the sharper this crossover will be. The abrupt change in the amplitude is clearly seen in fig. 3 which displays the result of numerical integration of eq. (3.1) at $R^2 = 40\pi\alpha'$. Note that retaining only massless string modes is a good approximation everywhere above the transition, since $L^2$ is much larger than $\alpha'$, whereas below the transition all modes give comparable contribution.

The conclusion from the above consideration is that the world-sheet fluctuations smoothen the Gross-Ooguri phase transition. However, if $\alpha' = (2g_{YM}^2N)^{-1/2}$ is sufficiently small, the transition should show up as a sharp crossover in the Wilson loop correlator.
4 Discussion

Calculations carried out in the previous sections demonstrate that the Wilson loop two point function in $\mathcal{N} = 4$ SYM undergoes the first order phase transition as the distance between loops changes, if the ’t Hooft coupling is infinite. If the coupling is large but finite, tunnelling between the saddle points of the string action smoothens the dependence of the correlator on the distance and the phase transition is replaced by a sharp crossover. The transition seems to be completely washed out in the weak coupling, perturbative regime.

In conclusion, we would like to comment on Wilson loop correlation functions in asymptotically free, confining gauge theories, which are also believed to have some kind of approximate, or even exact string representation. At short distances, $\Lambda_{QCD}^{-1} \ll L \ll R$, the Wilson loop correlator should be dominated by an open string stretched between the loops. This representation suggests that the large-radius limit of the correlator should determine an interaction potential between probe static charges:

$$V(L) = -\lim_{R \to \infty} \frac{\ln \langle W^\dagger(C_1)W(C_2) \rangle_{\text{conn}}}{2\pi R}.$$

The comparison of eq. (2.32) with the potential found in \cite{1} shows that this relation is satisfied by the two loop correlator calculated using the AdS/CFT correspondence.

At large distances between loops, a more appropriate picture is that of the closed string exchange. It is not quite clear whether the open and the closed string regimes are separated by the Gross-Ooguri crossover. General arguments seem to indicate that this is the case. If geometric characteristics of the loops are large in the units of the string tension, a confining string has a large action and hopefully can be described within the semiclassical approximation. The instability of the classical string world sheet will then lead to the phase transition in the semiclassical amplitude. This kind of behaviour certainly holds for the free string.

However, the free string amplitude is rather loose substitute for Wilson loop correlation functions in a gauge theory, basically because of the lack of zig zag invariance \cite{20, 21}. One of the consequences of the violation of zig zag invariance is the Hagedorn transition in the free string case: The amplitude (3.1) diverges at $L^2 \leq L^2_H = 2\pi\alpha'$ due to exponential growth of the closed string density of states. Such kind of divergency of correlation functions at short distances is expected in quantum gravity \cite{22}, but not in gauge theories. In fact, intermediate states in the spectral representation for the Wilson loop correlator,

$$\langle W^\dagger(C_1)W(C_2) \rangle_{\text{conn}} = \int_0^\infty dE \rho_C(E) e^{-EL},$$

$$\rho_C(E) = \sum_{n \neq 0} \delta(E - E_n) \langle 0 | W(C) | n \rangle^2,$$

which generically are glueballs, are expected to have exponentially growing spectrum. But zig zag symmetry suppresses coupling of Wilson loops to glueballs of very high spin, which can be seen expanding the Wilson loop in local operators \cite{23}. Spin $2n$ operators come with coefficients of order $1/n!$. For example, the operator with a minimal number of derivatives is

$$\frac{1}{n!} \text{tr} \left( \oint dx_\mu x_\nu F_{\mu\nu}(x_0) \right)^n.$$
Hence, Wilson loop effectively couples only to states with spin $J < J_0$, where $J_0$ is proportional to the area of the loop. As a result, the spectral density $\rho_C(E)$ should have power-law asymptotics. This property is essentially a consequence of the zig zag invariance, since the latter requires the Wilson loop of zero area to be the unit operator. This is not true for the boundary states in (3.1), which couple to all string modes even at $R = 0$ [15, 16].

The modular transformation converts the Hagedorn divergency of the amplitude to the tachyon singularity in the open string channel [23]. Again, the tachyon instability is not expected to arise in gauge theories.

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