Centralized Coded Caching with User Cooperation

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Abstract

In this paper, we consider the coded-caching broadcast network with user cooperation, where a server connects with multiple users and the users can cooperate with each other through a cooperation network. We propose a centralized coded caching scheme based on a new deterministic placement strategy and a parallel delivery strategy. It is shown that the new scheme optimally allocate the communication loads on the server and users, obtaining cooperation gain and parallel gain that greatly reduces the transmission delay. Furthermore, we show that the number of users who parallelly send information should decrease when the users’ caching size increases. In other words, letting more users parallelly send information could be harmful. Finally, we derive a constant multiplicative gap between the lower bound and upper bound on the transmission delay, which proves that our scheme is order optimal.

Index Terms

Cache, cooperation, delay

I. INTRODUCTION

Due to the continuous growth of traffic in the network and the growing needs for higher Internet speed from users, it’s imperative to improve the performance of the network. One of the promising directions to improve the quality of service is to utilize the cache memories in the network. In [1] Maddah-Ali and Niesen proposed a novel scheme, namely coded caching scheme, to improve the transmission efficiency of each transmission. It obtains a global caching gain by creating multicasting opportunities for different users. This new class of caching system has attracted significant interests [2]–[7].

To further improve the quality of service, one can combine caching with user cooperation. It is particularly common and useful in fog network [8], where the edge users can carry out some amount of communication. This can include, for example, device-to-device (D2D) networks and ad-hoc networks. In [9], coded caching schemes were proposed for a D2D noiseless network with the absence of server. In [10], the caching problem on a two-user D2D wireless network with the presence of a server was studied. In [12], a maximum distance separable (MDS) coded caching scheme was proposed to reduce the communication load in highly-dense wireless networks considering device mobility.

In this paper, we study a $K$-user ($K \geq 2$) coded-caching broadcast network with user cooperation. In this network, a server connects with all users through a noiseless shared link, and the users can communicate with each other through a noiseless cooperation network. The cooperation network is parameterized by a positive integer $\alpha_{\text{max}} \in \{1, \ldots, \lfloor K/2 \rfloor \}$, denoting the maximum number of users allowed to parallelly send data in the cooperation network. For example, when $\alpha_{\text{max}}$ equals to 1, the cooperation network operates as a simple shared link connecting all users, which is easy and low-cost to implement in fog network. The main contributions are summarized below.

- We propose a novel coded caching scheme that fully exploits user cooperation and optimally allocates communication loads between the server and users. The scheme achieves a cooperation gain (offered by the cooperation among the users) and a parallel gain (offered by the parallel transmission of the server and multiple users) that greatly reduces the transmission delay.
- We show that the number of users parallelly sending information should decrease with the increase of the users’ caching size. In other words, letting more users parallelly send information could be harmful. When users’ caching size is sufficiently large, only one user should be allowed to send information, indicating that the cooperation network can be just a simple shared link connecting all users. These insights could guide the implementation of practical caching-aided communication networks with user cooperation.
- Lower bounds on the transmission delay are established. Moreover, we show that our scheme achieves the optimal transmission delay within a constant multiplicative gap.

The remainder of this paper is as follows. We first present the system model in Section II and summarize our main results in Section III. Followed are detailed descriptions of the centralized coded caching schemes with user cooperation in Section IV. Section V concludes this paper.

II. SYSTEM MODEL AND PROBLEM DEFINITION

Consider a caching network consisting of a single server and $K$ users as depicted in Fig. 1. The server connects with all users through a noiseless shared link, and the users can communicate with each other through a cooperation network.
Fig. 1. Caching system considered in this paper. A server connects with $K$ cache-enabled users and the users can cooperate through a flexible network.

The server has a database of $N$ independent files $W_1, \ldots, W_N$. Each $W_n$, $n = 1, \ldots, N$, is uniformly distributed over 
\[ [2^F] \triangleq \{1, \ldots, 2^F\}, \]
for some positive integer $F$. Each user $k \in [K]$ is equipped with a cache memory of size $M_F$ bits, where $M \in [0, N]$.

The cooperation network is parameterized by a positive integer $\alpha_{\text{max}} \in \mathbb{Z}^+$, denoting the maximum number of users allowed to send data parallelly in the cooperation network. In this work, we made the following assumptions:

- The users can receive signals interference-free from the server and other users simultaneously. This is reasonable since the server and the users could operate in two separate bands or layers.
- Each user can receive data from at most one user, and if one user sends data, it cannot simultaneously receive data transmitted by other users. This makes the cooperation network easy to implement and indicates $\alpha_{\text{max}} \leq \lfloor \frac{K}{2} \rfloor$.
- The cooperation network is flexible, in the sense that each user can flexibly select a subset of users to cooperate with, and the subset can be changed during the transmission. This is a similar assumption in [3] and can account for some high-flexibility network such as fog network.

Given $\alpha_{\text{max}}$ and the assumptions above, the cooperation network can be characterized as below: Let parameter $\alpha \in \mathbb{Z}^+$, $\alpha \leq \alpha_{\text{max}}$, denote the number of users that exactly send information parallelly during the data transmission. There exists a routing strategy at network nodes such that the $K$ users can be partitioned into $\alpha$ groups: $G_1, \ldots, G_\alpha$, where 
\[ G_j \subseteq [K], \ G_j \cap G_i = \emptyset, \ G_1 \cup \cdots \cup G_\alpha = [K]. \] (1)

In each group $G_j, j = 1, \ldots, \alpha$, a user $u_j$ is allowed to send data to all other users in $G_j$. The group $\{G_j\}$ and the user $u_j$ can be changed during the delivery phase. Notice that when $\alpha_{\text{max}} = 1$, the flexible network degenerates into a fixed shared link, allowing only one user to send data to all other users, which is more common and easier to implement.

The system works in two phases: a placement phase and a delivery phase. In the placement phase, all users will access the entire library $W_1, \ldots, W_N$ and fill the content to their caches. More specifically, each user $k$ maps $W_1, \ldots, W_N$ to its cache contents:
\[ Z_k \triangleq \phi_k(W_1, \ldots, W_N), \] (2)
for some caching function
\[ \phi_k : [2^F]^N \rightarrow [2^{MF}]. \] (3)

In the delivery phase, each user requests one of the $N$ files from the library. We denote the demand of user $k$ by $d_k \in [N]$, and the corresponding file by $W_{d_k}$. Let $d \triangleq (d_1, \ldots, d_k)$ denote the users’ request vector. After the users’ requests $d$ are informed to the server and all users, the server produces symbol $X \triangleq f_d(W_1, \ldots, W_N)$, and user $k \in \{1, \ldots, K\}$ produces symbol $X_k \triangleq f_k,d(Z_k)$,
\[ X_k \triangleq f_k,d(Z_k), \] (4)

Each user $k$ can produce $X_k$ as a function of $Z_k$ and the received signals from the server, but since the transmitter’s signal is broadcasted to all users, it’s equivalent to send $X_k = f(Z_k)$. 

\[ 1 \] Each user $k$ can produce $X_k$ as a function of $Z_k$ and the received signals from the server, but since the transmitter’s signal is broadcasted to all users, it’s equivalent to send $X_k = f(Z_k)$. 

\[ 2 \]
for some encoding functions

\[ f_d : [2^F]^N \rightarrow \left[ [2^{R_1 F}] \right], \]
\[ f_{k,d} : [2^{MF}] \rightarrow \left[ [2^{R_2 F}] \right] \]

(5a) \hspace{1cm} (5b)

where \( R_1 \) and \( R_2 \) denote the rate transmitted by the server each user, respectively.

User \( k \) perfectly observes the signals sent by the server and other users, and decodes its desired message as

\[ \hat{W}_{dk} = \psi_{k,d}(X, Y_k, Z_k) \]

where \( Y_k \in \{X_1, \ldots, X_K\} \) denotes user \( k \)'s received signal sent from other users, and \( \psi_{k,d} \) is some decoding function

\[ \psi_{k,d} : \left[ [2^{R_1 F}] \right] \times \left[ [2^{R_2 F}] \right] \times [2^{MF}] \rightarrow [2^F]. \]

(6)

We define the worst-case probability of error as

\[ P_e \triangleq \max_{d \in F} \max_{k \in [K]} \Pr(\hat{W}_{dk} \neq W_{dk}). \]

(7)

A caching scheme \((M_1, R_1, R_2)\) consists of caching functions \( (3) \), encoding functions \( (5) \) and decoding functions \( (6) \). We say that the rate region \((M, R_1, R_2)\) is achievable if for every \( \epsilon > 0 \) and every large enough file size \( F \), there exists a caching scheme such that \( P_e \) is less than \( \epsilon \).

Given achievable region \((M, R_1, R_2)\), we define the transmission delay

\[ R \triangleq \max\{R_1, R_2\}, \]

(8)

and the optimal transmission delay \( R^* \triangleq \inf\{R\} \).

Our goal is to design the coded caching scheme such that the transmission delay is minimized. Finally, in this paper we assume \( K \leq N \). Extending the results to the scenario \( K \geq N \) is straightforward, referred to [1].

III. MAIN RESULTS

Consider the cache-aided network with user cooperation described in Section II

**Theorem 1.** Let \( t \triangleq KM/N \in \mathbb{Z}^+ \). For memory size \( M \in \{0, N/K, 2N/K, \ldots, N\} \), the optimal transmission delay \( R^* \) is upper bounded by \( R^* \leq R_C \), where

\[ R_C \triangleq \min_{1 \leq \alpha \leq \alpha_{\max}} K \left( 1 - \frac{M}{N} \right) \frac{1}{1 + t + \alpha \min\{\frac{K}{\alpha} - 1, t\}}. \]

(9)

For general \( 0 \leq M \leq N \), the lower convex envelope of these points is achievable.

**Proof.** See the scheme in Section [IV].

Recall that parameter \( \alpha \) denotes the number of users who exactly send information parallelly in the delivery phase. From [9], it's easy to obtain the optimal value of \( \alpha \), denoted by \( \alpha^* \):

\[ \alpha^* = \begin{cases} 1, & t \geq K - 1, \\ \max\{\alpha : \left\lfloor \frac{K}{\alpha} \right\rfloor - 1 = t\}, & 1 < t \leq K - 1, \\ \alpha_{\max}, & t \leq \left\lfloor \frac{K}{\alpha_{\max}} \right\rfloor - 1. \end{cases} \]

(10)

It’s interesting to find that the number of users parallelly sending information should decrease as the users’ caching size \( M \) increases for given \((K, N, \alpha_{\max})\). To simplify the explanation, we assume \( \alpha_{\max} = \left\lfloor \frac{K}{2} \right\rfloor \) and \( KM/N \in \mathbb{Z}^+ \). When \( M \leq N(\left\lfloor \frac{K}{\alpha_{\max}} \right\rfloor - 1)/K \), we have \( \alpha^* = \alpha_{\max} \) and thus it’s beneficial to let the most users parallelly send information. As \( M \) increases, \( \alpha^* \) decreases and \( \alpha^* \leq \alpha_{\max} \), which indicates that letting more users parallelly send information could be harmful. For example, when \( N = 100, K = 10, \alpha_{\max} = 5, M = 40 \), then \( \alpha^* = 2 < \alpha_{\max} \), and thus only two users rather than 5 users should parallelly send data. In the extreme case when \( M \geq (K - 1)N/K \), only one user should be allowed to send information, implying that when users’ caching size is sufficiently large, the cooperation network can be just a simple share link connecting all users.

Comparing \( R_C \) with the delay achieved by the scheme without user cooperation in [1], i.e., \( K(1 - \frac{M}{N}) \frac{1}{1+t} \), \( R_C \) consists of an additional factor

\[ G_c \triangleq \frac{1}{1 + \frac{\alpha^*}{1+t} \min\{\left\lfloor \frac{K}{\alpha^*} \right\rfloor - 1, t\}}. \]
which we call the *cooperation gain*, since it arises from the user cooperation. Comparing $R_C$ with the delay achieved by the scheme for D2D network without server [9], i.e., $\frac{N}{M}(1 - \frac{M}{N})$, $R_C$ consists of an additional factor

$$G_p \equiv \frac{1}{1 + \frac{1}{t} + \alpha_{\max} \min\{|\frac{K}{N}| - 1, t|}$$

which we call the *parallel gain*, since it arises from the parallel transmission of the server and users. Both gains depend on $K$, $M/N$ and $\alpha_{\max}$. When fixing $(K, N, \alpha_{\max})$, $G_c$ in general is not a monotonic function of $M$. It is monotonic decreasing when $M \leq \lfloor \frac{K}{N} \rfloor - 1$, and monotonic decreasing when $M \geq \lfloor \frac{K}{N} \rfloor - 1$. $G_p$ is a monotonic increasing function of $M$. The cooperation gain and parallel gain are plotted in Fig. 2 for given $(K, N, \alpha_{\max})$.

**Theorem 2.** For memory size $0 \leq M \leq N$, the optimal transmission delay is lower bounded by

$$R^* \geq \max\left\{ \frac{1}{2} \left( 1 \frac{M}{N} \right), \max_{s \in [K]} \left( s - \frac{K}{N/s} \right), \max_{s \in [K]} \left( s - \frac{sM}{N/s} \right) \right\}$$

$$1 + \frac{1}{t} + \alpha_{\max} \min\{|\frac{K}{N}| - 1, t|} \right\} \right\} \right\} \right\}$$

(11)

**Proof.** See the proof in Appendix A.

**Theorem 3.** For memory size $0 \leq M \leq N$,

$$R_C/R^* \leq 31.$$

(12)

**Proof.** See the proof in Appendix B.

The numerical result is presented in Fig. 3. It shows that the delay of the proposed scheme is close to the lower bound and much tighter than the upper bound achieved by the schemes in [1] and [9].

IV. PROOF OF THEOREM

In this section, we present a caching scheme achieving an upper bound on the transmission delay for the network depicted in Fig. 1. One may come up with the idea to use the scheme introduced in [9] considering caching-aided D2D network without server, but there are two main problems:

- When each user sends data to other users in its partition group, in order to achieve the maximum multicast gain, user $u_j$ in the group $G_j$ should broadcast a coded data consisting of $|G_j| - 1$ useful subfiles required by the remaining users in group $G_j$. Also, the amount of subfiles should support the server and $\alpha$ users to simultaneously send data in every transmission slot. These can not be guaranteed by the file-splitting process and caching placement phase introduced in [9].
- In [9], the users are fixed in a mesh network, leading to an unchanging group partition during the delivery phase, which is not the same case in our model. Moreover, our model can have the server share some communication loads with the users. These two facts result in great difference in the delivery phase compared to that in [9]. To achieve the optimal delay, we need to fully exploit the dynamic strategy and optimally allocate the communicate loads at the server and users.
We describe a novel coded caching scheme for any $K$, $N$ and $M$ such that $t = \frac{KM}{N}$ is a positive integer. When $t$ is not an integer, we can use a resource sharing scheme as in [1].

Introduce integers $\alpha, L_1, L_2$ and $L \triangleq L_1 + L_2$, where $1 \leq \alpha \leq \alpha_{\text{max}}$, $L_2 > 0$, and $L_1 \geq 0$ such that

$$\frac{K \cdot \left(\binom{K-1}{t}\cdot L_1}{t}}{\alpha \min\left\{\left\lfloor\frac{N}{\alpha}\right\rfloor - 1, t\right\}} \in \mathbb{Z}^+,$$

and

$$\frac{L_1}{L_2} = \frac{\alpha \min\left\{\left\lfloor\frac{N}{\alpha}\right\rfloor - 1, t\right\}}{1 + t}. \tag{13b}$$

Here $L_1/L$ and $L_2/L$ denote the proportions of communication loads assigned to the users and the server respectively. Condition \(13a\) ensures that the number of subfiles can support a maximum multicast gain when user sending data, and \(13b\) ensures that communication loads can be optimally allocated at the server and the users.

In the placement phase, each file is split into $L \binom{K}{t}$ subfiles of equal size. We index the subfiles of $W_n$ by the superscript $l \in [L]$ and subscript $\mathcal{T} \subset [K]$:

$$W_n = \left(W_{n,T}^l : l \in [L], \mathcal{T} \subset [K], |\mathcal{T}| = t\right). \tag{14}$$

User $k$ caches all the subfiles when $k \in \mathcal{T}$ for all $n = 1, \ldots, N$ and $l = 1, \ldots, L$, so it requires

$$N \cdot \frac{F}{L \binom{K}{t}} \cdot L \left(\frac{K-1}{t} - 1\right) = F \cdot \frac{Nt}{K} = MF$$

bits of cache, satisfying the cache size constraint.

In the delivery phase, each user $k$ requests file $W_{dk}$. The requests vector $d$ are informed by the server and all the users. Note that different parts of the file $W_{dk}$ has been stored in the users’ caches, and thus the uncached parts of $W_{dk}$ can be sent by the server and users. Divide the uncached subfiles of $W_{dk}$ into two parts: one that is sent by the server and the other that is sent by the users in the network. Subfiles

$$\left(W_{dk,T}^1, \ldots, W_{dk,T}^{L_1} : \mathcal{T} \subset [K], |\mathcal{T}| = t, k \notin \mathcal{T}\right)$$

are requested by user $k$ and will be sent by the users, thus $\frac{L_1}{t}$ represents the fraction of the subfiles sent by the users. Subfiles

$$\left(W_{dk,T}^{L_1+1}, \ldots, W_{dk,T}^{L} : \mathcal{T} \subset [K], |\mathcal{T}| = t, k \notin \mathcal{T}\right)$$

are requested by user $k$ and will be sent by the server, thus $\frac{L_2}{t}$ represents the the fraction of the subfiles sent by the server.

Our objective is to get the upper bound of transmission delay in the worst request case, so we assume that each of the users makes unique requests. Thus, the total number of requested subfiles for $K$ users is $K \cdot \frac{(K-1)}{t} \cdot L$.

First consider the subfiles sent by the users. In order to create multicast opportunities among users, we partition the $K$ users into $\alpha$ groups of equal size:

$$\mathcal{G}_1, \ldots, \mathcal{G}_\alpha$$
where for $i, j = 1, \ldots, \alpha$, $\mathcal{G}_i \subseteq [K]: |\mathcal{G}_i| = \lfloor K/\alpha \rfloor$, and $\mathcal{G}_i \cap \mathcal{G}_j = \emptyset$, if $i \neq j$. In each group $\mathcal{G}_i$, one of $\lfloor K/\alpha \rfloor$ users plays the role of server and sends symbols based on its cached contents to the remaining $(\lfloor K/\alpha \rfloor - 1)$ users in the group.

Focus on a group $\mathcal{G}_i$ and a set $S \subset [K]: |S| = t + 1$. If $|\mathcal{G}_i| \leq |S|$, then all nodes in $\mathcal{G}_i$ share subfiles ($W_{i,T}^l : l \in [L_1], n \in [N], \mathcal{G}_i \subseteq T, |T| = t$). For this case, user $k \in \mathcal{G}_i$ sends a XOR symbol that contains the requested subfiles for all remaining $\lfloor K/\alpha \rfloor - 1$ users in $\mathcal{G}_i$. If $|\mathcal{G}_i| > |S|$, then the nodes in $S$ share subfiles ($W_{i,T}^l : l \in [L_1], n \in [N], T \subset S, |T| = t$). For this case, user $k \in S$ sends a XOR symbol that contains the requested subfiles for all remaining $t$ users in $S$. Other groups perform the similar steps and concurrently deliver the remaining requested subfiles to other users.

By changing the partition and performing the delivery strategy described above, we can finally send all the requested subfiles $(W_{d_k,T}^1, \ldots, W_{d_k,T}^L : T \subset [K], |T| = t, k \notin T)^{K}_{k=1}$ (15) to the users. Since $\alpha$ groups work in a parallel manner ($\alpha$ users can concurrently deliver contents), and each user in a group delivers a symbol containing $\min\{\lfloor K/\alpha \rfloor - 1, t\}$ subfiles requested by the users in its group, to send all requested subfiles in (15), we need

$$\frac{K \cdot (K-1) \cdot L_1}{\alpha \min\{\lfloor K/\alpha \rfloor - 1, t\}}$$

(16) times of transmission each of rate $\frac{1}{L(t)}$. Notice that $L_1$ is chosen according to (15a), ensuring that (10) equals to an integer. Thus, the rate sent by the users is

$$R_2 = \frac{K \cdot (K-1) \cdot L_1}{\alpha \min\{\lfloor K/\alpha \rfloor - 1, t\}} \cdot \frac{1}{L(t)}$$

$$= \frac{L_1}{L} \cdot K \left(1 - \frac{M}{N}\right) \cdot \frac{1}{\alpha \min\{\lfloor K/\alpha \rfloor - 1, t\}}.$$ (17)

Now we describe the delivery of the subfiles sent by the server. For each $l = L_1 + 1, \ldots, L$, apply the delivery strategy as in [1]. Specifically, the server sends $\oplus_{k \in S} W_{d_k,T \setminus \{k\}}^l$ for all $S \subset [K]: |S| = t + 1$ and $l = L_1 + 1, \ldots, L$. We obtain the rate sent by the server

$$R_1 = \frac{L_2}{L} \cdot K \left(1 - \frac{M}{N}\right) \cdot \frac{1}{1 + KM/N}.$$ (18)

Since the server and users transmit the signals simultaneously, the transmission delay of the whole network is the maximum between $R_1$ and $R_2$, i.e., $R = \max\{R_1, R_2\}$.

Given (13b), (17) and (18), we find that $R_1 = R_2$, which means that the transmission delay reaches the optimal point. The transmission delay can thus be rewritten as

$$R = K \left(1 - \frac{M}{N}\right) \frac{1}{1 + t + \alpha \min\{\lfloor K/\alpha \rfloor - 1, t\}}.$$ (19)

In order to explain the key steps in caching scheme described above, we will then present a simple example.

**Example 1:** Consider a network consisting of $K = 6$ users with cache size $M = 4$, and a library of $N = 6$ files. Thus $t = KM/N = 4$. Let $\alpha = 2$, that is to say we separate the 6 users into 2 groups of equal size. The choice of the groups is not unique. We choose $L_2 = 2$ and $L_1 = 1$ such that $\frac{K(K-1)L_1}{\min\{\alpha([K/\alpha]-1), t\}} = 15$ is an integer. Split each file $W_n$, for $n = 1, \ldots, N$, into $3(6) = 45$ subfiles:

$$W_n = (W_{n,T}^l : l \in [3], T \subset [6], |T| = 4).$$

We list all the requested subfiles by the users as follows: for $l = 1, 2, 3$,

$$W_{d_1}^{(2345)}, W_{d_1}^{(2346)}, W_{d_1}^{(2356)}, W_{d_1}^{(2456)}, W_{d_1}^{(3456)}$$

$$W_{d_2}^{(1345)}, W_{d_2}^{(1346)}, W_{d_2}^{(1356)}, W_{d_2}^{(1456)}, W_{d_2}^{(2456)}$$

$$W_{d_3}^{(1245)}, W_{d_3}^{(1246)}, W_{d_3}^{(1256)}, W_{d_3}^{(1456)}, W_{d_3}^{(2456)}$$

$$W_{d_4}^{(1235)}, W_{d_4}^{(1236)}, W_{d_4}^{(1356)}, W_{d_4}^{(1356)}, W_{d_4}^{(2356)}$$

$$W_{d_5}^{(1234)}, W_{d_5}^{(1236)}, W_{d_5}^{(1246)}, W_{d_5}^{(1346)}, W_{d_5}^{(2346)}$$

$$W_{d_6}^{(1234)}, W_{d_6}^{(1235)}, W_{d_6}^{(1245)}, W_{d_6}^{(1345)}, W_{d_6}^{(2345)}.$$ (20)

The users can finish the transmission in different partitions. Table 1 shows one kind of the partition for example and explains how the users send the requested subfiles for $l = 1, 2$. In Table 1 all the users send an XOR symbol of subfiles with superscript $l = 1$ at the beginning. Note that the subfiles $W_{d_1}^{(1345)}$ and $W_{d_1}^{(1234)}$ are left since $\frac{K(K-1)}{\alpha([K/\alpha]-1)}$ is not an integer. Similarly, for subfiles with $l = 2$, $W_{d_2}^{(2345)}$ and $W_{d_2}^{(2456)}$ are not sent to user 3 and 4. In the last transmission, user 1 delivers the XOR
message $W^2_{d_1,\{1256\}} \oplus W^1_{d_2,\{1245\}}$ to user 2 and 3, and user 6 delivers $W^1_{d_3,\{1235\}} \oplus W^2_{d_4,\{2356\}}$ to user 5 and 6. The rate transmitted by the users is $R_2 = \frac{1}{3}$.

The server delivers subfiles for $l = 3$ in the same way as in [1]. Specifically, it sends symbols $\oplus_{k \in S} W^l_{d_k, S\backslash \{k\}}$, for all $S \subseteq [K] : |S| = 5$. Thus the rate sent by the server is $R_1 = \frac{2}{3}$, and the transmission delay $R = \max\{R_1, R_2\} = \frac{1}{3}$, which is less than the delay achieved by the centralized coded caching scheme without user cooperation.

V. CONCLUSIONS

In this paper, we consider a coded-caching broadcast network with user cooperation. An order optimal scheme is proposed which achieves a cooperation gain and a parallel gain, by exploiting user cooperation and parallel transmission at the server and users. Furthermore, we show that letting more users parallely send information could be harmful. The directions of future research could be on the decentralized caching problem and wireless network with caching and cooperation.

APPENDIX A

CONVERSE PROOF

Due to the flexibility of cooperation network, the connection and partitioning status between users can change during the delivery phase, we can’t drive our converse directly like [1]. Moreover, the parallel transmission of the server and many users results in abundant transmitting signals, making the scenario more sophisticated.

Let $R^*_1$ and $R^*_2$ denote the optimal rate sent by the server and each user. We first consider an ideal case where every user is served by an exclusive server and user, which both store full files in the database, then we easy to obtain $R^* \geq \frac{1}{2}(1 - \frac{M}{N})$.

Next, consider the first $s$ users with cache contents $Z_1, ..., Z_s$. Denote $X_{1,0}$ to be the signal sent by the server, and $X_{1,1}, ..., X_{1,s_{\max}}$ to be the signals sent by the $s_{\max}$ users, respectively, where $X_{j,i} \in \{2^{|H_{s_{\max}}|}\}$ for $j \in [s]$ and $i \in [s_{\max}]$. Assume that $W_1, ..., W_s$ is determined by $X_{1,0}, X_{1,1}, ..., X_{1,s_{\max}}$ and $Z_1, ..., Z_s$. Also, define $X_{2,s_{\max}+1}, X_{2,s_{\max}+2}, ..., X_{2,s}$ to be the signals which enable the users to decode $W_{s+1}, ..., W_{s_r}$. Continue the same process such that $X_{[N/s],0}, X_{[N/s],1}, ..., X_{[N/s],s_{\max}}$ are the signals which enable the users to decode $W_{[N/s]-s+1}, ..., W_{[N/s]}$. We then have $Z_1, ..., Z_s, X_{1,0}, ..., X_{[N/s],0},$ and $X_{1,1}, ..., X_{1,s_{\max}}, ..., X_{[N/s],1}, ..., X_{[N/s],s_{\max}}$ determine $W_1, ..., W_{[N/s]}$. Let

$$\mathbf{X}_{1:s_{\max}} \triangleq (X_{1,1}, ..., X_{1,s_{\max}}, ..., X_{[N/s],1}, ..., X_{[N/s],s_{\max}}).$$

By the definitions of $R^*_1$, $R^*_2$ and the encoding function (5b), we have

$$H(X_1,0, ..., X_{[N/s],0}) \leq |N/s|R^*_1 F,$$

$$H(X_{1:s_{\max}}) \leq |N/s|s_{\max}R^*_2 F,$$

$$H(X_{1:s_{\max}}, Z_1, ..., Z_s) \leq KMF.$$
Consider then the cut separating \(X_1,0,\ldots,X_{\lfloor N/s \rfloor},0, X_1:0\max, \text{ and } Z_1,\ldots,Z_s\) from the corresponding \(s\) users. By the cut-set bound and (19), we have
\[
\left\lfloor \frac{N}{s} \right\rfloor sF \leq \left\lfloor \frac{N}{s} \right\rfloor R_1^*F + KMF, 
\]
(20)
\[
\left\lfloor \frac{N}{s} \right\rfloor sF \leq \left\lfloor \frac{N}{s} \right\rfloor R_1^*F + sMF + \left\lfloor \frac{N}{s} \right\rfloor \alpha_{\max} R_2^*F. 
\]
(21)
Since we have \(R^* \geq R_1^*\) and \(R^* \geq \max\{R_1^*,R_2^*\}\) from the above definition, solving for \(R^*\) and optimizing over all possible choices of \(s\), we obtain
\[
R^* \geq \max_{s \in [K]} \left( s - \frac{KM}{\left\lceil N/s \right\rceil} \right), 
\]
(22a)
\[
R^* \geq \max_{s \in [K]} \left( s - \frac{sM}{\left\lceil N/s \right\rceil} \right) \cdot \frac{1}{1 + \alpha_{\max}}. 
\]
(22b)

**APPENDIX B**

**PROOF OF THEOREM 3**

We prove that \(R_C\) is within a constant multiplicative gap of the minimum feasible delay \(R^*\) for all values of \(M\). To prove the result, we compare them in the following regimes.

- If \(0.6393 < t < \lfloor K/\alpha \rfloor - 1\), from Theorem 2, we have
  \[
  R^* \geq \frac{1}{12} \cdot \frac{K}{1 - \frac{M}{N}} \cdot \frac{1}{1 + \alpha_{\max}} \cdot \frac{1}{1 + \alpha_{\max}}, 
  \]
  where (a) follows from [1, Theorem 3]. Then we have
  \[
  \frac{R_C}{R^*} \leq 12 \cdot \frac{(1 + \alpha_{\max})(1 + t)}{1 + t + \alpha t} \cdot \frac{(1 + \alpha_{\max})}{1 + \alpha t/1 + t} \leq 12 \cdot \frac{(1 + \alpha_{\max})}{1 + \alpha \cdot 0.6393/(1 + 0.6393)} \leq 31 
  \]
  (24)
  where the last inequality holds since we can choose \(\alpha = \alpha_{\max}\).

- If \(t > \lfloor K/\alpha \rfloor - 1\), we have
  \[
  \frac{R_C}{R^*} \leq \frac{K(1 - \frac{M}{N})}{1 + \alpha \cdot \frac{1}{\lfloor K/\alpha \rfloor - 1}} \cdot \frac{1}{2K} \leq \frac{2K}{K + KM/N} \leq 2 
  \]
  (25)
  where (a) follows from that we can choose \(\alpha = 1\).

- If \(t \leq 0.6393\), setting \(s = 0.275N\), we have
  \[
  R^* \geq s - \frac{KM}{\left\lceil N/s \right\rceil} \geq s - \frac{KM}{N/s - 1} = 0.275N - t \cdot 0.3793N \geq 0.0325N > \frac{1}{31} \cdot N 
  \]
  (26)
  where (a) holds since \(\lfloor x \rfloor \geq x - 1\) for any \(x \geq 1\). Note that for all values of \(M\), the transmission delay
  \[
  R_C \leq \min\{K,N\}. 
  \]
  (27)

Combining with (26) and (27), we have \(R_C/R^* \leq 31\).
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