Numerical simulation of 3D Darcy–Forchheimer fluid flow with the energy and mass transfer over an irregular permeable surface

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The Jeffrey fluid model is capable of accurately characterizing the stress relaxation behavior of non-Newtonian fluids, which a normal viscous fluid model is unable to perform. The primary objective of this paper is to provide a comprehensive investigation into the effects of MHD and thermal radiation on the 3D Jeffery fluid flow over a permeable irregular stretching surface. The consequences of the Darcy effect, variable thickness and chemical reaction are also considered. The phenomena have been modeled as a nonlinear system of PDEs. Using similarity substitution, the modeled equations are reduced to a dimensionless system of ODEs. The parametric continuation method (PCM) is used to determine the numerical solution to the obtained sets of nonlinear differential equations. The impact of physical parameters on temperature, velocity and mass profiles are presented through Figures and Tables. It has been noticed that the energy profile magnifies with the increment of porosity term, thermal radiation and heat source term, while diminishing with the flourishing upshot of power index and Deborah number. Furthermore, the porosity term and wall thickness parameter enhance the skin friction.

The analysis of heat and mass transmission, as well as boundary layer flow across a starching substrate, is an interesting subject of study due to its numerous applications in various industries and metal extraction processes1,2. Sivaraj and Kumar3 evaluated the consequences of mixed convection on time-dependent MHD dusty viscous fluid flow with thermal conduction through the impermeable irregular surface. The streamlines of dusty fluids are found to be greater than the velocity distribution of dust particulates. Alharbi et al.4 reported the three-dimensional ferrofluid flow across an impermeable upright surface, as well as the impacts of slips in a porous medium with hybrid nanoparticles. It's has been discovered that as the volume proportion of nanoparticles (Nps) increases, the rate of heat transport enhances. Ullah et al.5 described the flow variations in the near area of and inside irregular surfaces. Bilal et al.6 investigated the effect of inconsistent 2D & 3D sharpness on instability by simulating fully developed flows over two distinct rough substrates. The two designs are created by superimposing sinusoidal functions with various wavelengths and random amplitudes. Gul et al.7 and Zhou et al.8 established a numerical model that compared the comportment of simple and hybrid NPs moving over an extending sheet. When compared to simple nanofluid, the hybrid nanofluid (HNF) is more efficacious in heat flux due to its excellent thermal properties. Iyyappan and Singh9 examined the flow of a force convective laminar boundary layer on an irregular diverging channel when magnetic field influences were employed. Bilal et al.10 studied the Casson fluid flow under the upshot of magnetic flux over an expanding surface. It has been shown that the variation

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of the magnetic field, Marangoni factor, and unsteadiness component decreases the fluid velocity. Some advanced research have been reported by 11-14.

Non-Newtonian fluids come in a wide variety of types, each with its own set of characteristics. Non-Newtonian fluids are being examined by scientists and researchers due to a broad range of implementations, like drug companies, fiber new tech, cables sealant, food items, crystal growth, psychology and many more. Jeffrey fluid is the most well-known and easiest-to-understand. The Jeffrey fluid parameter and the time retardation parameter elevates this fluid to the top of the non-Newtonian fluids list 15. Ullah et al. 16 evaluated the Jeffrey fluid flow across porous horizontal sheet. The velocity of an unsteady Jeffrey fluid flow over an inestimable plane permeable plate is inspected by Algehyne et al. 17. The results reveal that as the magnetic parameter, the ratio of retardation and relaxation times and Jeffrey fluid parameter increase, the fluid velocity decreases. In the response to an ambient magnetic field, Ali et al. 18 calculated the upshot of energy conduction on the flow of a Jeffrey fluid with immersed NPs through a dynamic flexible substrate. Alrabiah et al. 19 inspected the peristaltic transmission of MHD Jeffrey fluid flow through channel. Saleem et al. evaluated MHD Jeffrey fluid flows with mass and energy transport on an indefinitely circulating inverted cone. 20. Azlina et al. 21 proposed a numerical calculation of the MHD Jeffrey fluid flow through plates in a translucent sheet. Bilal et al. 22 investigate the 2D Jeffrey fluid flow across a continuously extending disc. Kumar et al. 23 discussed the influence of an applied magnets flux on an irregular 2D Jeffrey fluid flow. A theoretical investigation is carried out by Yadav et al. 24 to determine the upshot of a magnetic flux and mixed convection on the Jeffrey fluid flow. The results show that increasing the Jeffrey fluid parameter reduces system stability while increasing magnetic field parameters has the reverse effect. Recently many researchers have worked on this topic 25-28.

The MHD flow plays a vital role in manufacturing heavy machinery, astrophysics, electrical power generation solar power equipment, space vehicle and many other fields. Kumar et al. 29 explore the thermal energy transfer in a hing MHD flow through an extending cylinder while considering magnetic dipoles. Nanoliquid flow across curved stretched sheets is studied numerically by Dhansanaya et al. 30 to determine the effect of magnetic fields on Casson nanoliquid flow. The findings indicated that enhancing the curvature parameter positively affects the velocity profile, but that it has the opposite impact on the thermal gradient. Chu et al. 31 scrutinise Maxwell nano liquid’s radiative flow along with a cylinder by taking into consideration the magnetic effect. The fluid flow and temperature fluctuations of nanofluid flow with the Hall upshot are discussed by Acharya et al. 32. A moving plate with Joule heating is used to demonstrate Magnetohydrodynamic hybrid nanofluid flow with temperature distribution is solved numerically by Lv et al. 33. Kodi et al. 34 presented an analytical assessment of Casson fluid flow with the Hall upshot. The results are obtained through computational strategy PCM.

Here \( u, v, w \) determine the velocity factors in \( x, y \) and \( z \) direction. \( k_p \) is the permeability of the porous medium, \( k_f \) is the thermal conductivity, \( T \) is the temperature of the fluid, \( v \) is the kinematic viscosity, \( Q \) the heat absorption/
generation term, $F$ is the non-uniform inertia factor, where, $C_b$ is the drag coefficient. $D$ is the molecular diffusivity and $\Lambda_1, \Lambda_2$ is the period of relaxation and time retardation respectively.

The boundary conditions are\textsuperscript{14,41}:

\begin{equation}
\begin{align*}
&u - u_w(x) = 0, \quad v - v_w(x) = 0, \quad T - T_w(x) = 0, \quad C - C_w(x) = 0 \text{ as } z = \Lambda_0 \frac{z}{\lambda}, \\
&u \rightarrow 0, \quad v \rightarrow 0, \quad \frac{\partial u}{\partial z} \rightarrow 0, \quad \frac{\partial v}{\partial z} \rightarrow 0, \quad T - T_\infty, \quad C - C_\infty \text{ as } z \rightarrow \infty.
\end{align*}
\end{equation}

where

\begin{equation}
\begin{align*}
\zeta_1 &= \frac{k_B T}{\sqrt{2\pi d^2 p}}, \quad \Omega = x + y + z, \quad \zeta_2 = \left( \frac{2\gamma}{\gamma + 1} \right) \frac{\zeta_1}{Pr}, \quad B = B_0 \Omega^{\frac{n+1}{n}}, \\
&u_w = a \zeta_2^n, \quad v_w = a \Omega^n, \quad T_w - T_\infty = T_0 \Omega^{\frac{n-1}{n}} \quad n \neq 1.
\end{align*}
\end{equation}

In the above equation, we supposed as $n \neq 1$ (i.e., $n = 1$ denotes the surface shape to flat sheet). Where, $n > 1$ and $n < 1$ are yields to surface curviness, inner convex and outer convex due to reduction and increment of wall thicknesses respectively. $T_0, T_\infty$ reference atmospheric liquid temperature $f$ specify the Maxwell coefficient, $b$ shows the thermal adaptation coefficient, $\zeta_1, \zeta_2$ are the constant number, $\lambda$ specific heat ratio.

**Similarity transformation**

The similarity variables are:

\begin{equation}
\begin{align*}
\zeta &= \Omega^{\frac{n+1}{2}} \left( \frac{n+1}{2} U_0 \right) \left( \frac{n}{2} \right)^\frac{1}{2}, \quad \psi = \Omega^{\frac{n+1}{2}} \left( \frac{n}{2} \right)^\frac{1}{2} F(\zeta), \quad \theta(\zeta) = \frac{\theta - \theta_\infty}{\theta_w - \theta_\infty}, \quad \phi(\zeta) = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty}, \\
u &= U_0 \Omega^n f'(\zeta), \quad v = V_0 \Omega^n g'(\zeta), \quad w = \left( \frac{2\nu}{n+1} \right)^\frac{1}{2} U_0 \Omega^n \left[ f(\zeta) + g(\zeta) + \frac{n-1}{2} \Omega^\frac{1}{2} \left( f'(\zeta) + g'(\zeta) \right) \right].
\end{align*}
\end{equation}

By applying the above similarity transformation, Eq. (1) is identically satisfied while Eq. (2–5) take the form as:
\[ \frac{f'''' + (\Lambda_1 + 1)\left((g + f)f'' - \frac{2n}{n+1}(Fr f' + f' g')\right) + D\left(\left(\frac{3n-1}{2}\right)(g'' + f''f'') + (n-1)(f' + g')f''\right)}{\left(\frac{n+1}{2}\right)(g + f)f'''} + \frac{2}{n+1}(1 + \Lambda_1)(M^2_f + P_0)f' = 0, \]

\[ \frac{g'''' + (1 + \Lambda_1)\left((g + f)g'' - \frac{2n}{n+1}(Fr g' + g' f')\right) + D\left(\left(\frac{3n-1}{2}\right)(g'' + f''g'') + (n-1)(f' + g')g''\right)}{\left(\frac{n+1}{2}\right)(g + f)g'''} + \frac{2}{n+1}(1 + \Lambda_1)(M^2_f + P_0)g' = 0, \] (8)

\[ (1 + R)\theta'' + (f + g)Pr\theta' + Hs\theta = 0, \] (9)

\[ \phi'' + \frac{Sc}{n+1} (f + g)\phi - \frac{2}{n+1}C_f\phi = 0. \] (11)

The reduced conditions are:

\[ g(\eta) = f(\eta) = \frac{\sqrt{1 + \eta}}{1 + \eta}, \quad f'(\eta) = \phi'(\eta) = g''(\eta) = \theta'(\eta) = 0 \text{ as } \eta \to 0 \]

\[ f'(<) = g'(\eta) \to 0, \quad f''(\eta) = g''(\eta) = \phi''(\eta) = \theta'(\eta) \to 0 \text{ as } \eta \to \infty. \] (12)

Here, \( D \) and \( M_f \) is the Deborah number and magnetic field, \( Hs \) is the absorption & generation term, \( P_0 \) is the porosity factor, \( R \) is the thermal radiation, \( Pr \) and \( Sc \) is the Prandtl and Schmidt numbers, \( Fr \) is the Darcy Forchheimer term, \( C_r \) is the chemical reaction and \( \Lambda \) wall thickness factor. Mathematically we have

\[ P_0 = \frac{\nu_l}{k_p U_0 \Omega^{n-1}}, \quad Pr = \frac{\mu_f(C_p)_{ref}}{k_f}, \quad \Lambda = \frac{1}{z} \left(\frac{(n+1)U_0}{2v}\right)^\frac{1}{2}, \quad M_f = \frac{\sigma R_0^2}{\rho U_0 \Omega^{n-1}}, \quad Hs = \frac{xQ}{\mu \rho C_p}, \quad D = \Lambda_2 U_0 \Omega^{n-1}, \quad Fr = \frac{xC_k}{k^{1/2}}, \quad C_r = \frac{k_1}{U_0 \Omega^{n-1}}, \quad Sc = \frac{\nu_l}{D}. \] (13)

The friction factor towards \( x \) and \( y \) direction are:

\[ C_{f_x} = \frac{\tau_{xx}}{\rho U_w^2}, \quad C_{f_y} = \frac{\tau_{yy}}{\rho U_w^2}, \] (14)

where

\[ \tau_{xx} = \frac{\mu}{1 + \Lambda_1} \left[ \frac{\partial u}{\partial z} + \left( \frac{u}{\partial x} + \frac{v}{\partial y} \right) \frac{\partial^2 u}{\partial z^2} + \left( \frac{w}{\partial x} + \frac{v}{\partial y} \right) \frac{\partial^2 u}{\partial y \partial z} \right] \Lambda_2, \]

\[ \tau_{yy} = \frac{\mu}{1 + \Lambda_1} \left[ \frac{\partial v}{\partial z} + \left( \frac{u}{\partial x} + \frac{v}{\partial y} \right) \frac{\partial^2 v}{\partial z^2} + \left( \frac{w}{\partial x} + \frac{v}{\partial y} \right) \frac{\partial^2 v}{\partial y \partial z} \right] \Lambda_2. \] (15)

Here the physical quantities are:

\[ N_{ux} = \frac{\Omega q_w}{k_f(T_w - T_\infty)}, \quad \text{where} \quad q_w = -\left(\frac{k_f + 16\sigma^* T_\infty^3}{3k^*}\right) \frac{\partial T}{\partial z}. \] (16)

\[ Sh_x = \frac{\Omega j_w}{D(C_w - C_\infty)}, \quad \text{where} \quad j_w = -D \frac{\partial C}{\partial z}. \] (17)

The skin friction, heat and mass allocation expression are as follows:

\[ Re_x^{0.5} C_{f_x} = \frac{1}{1 + \Lambda_1} \left(\frac{n+1}{2}\right)^{0.5} \left(f''''(0) + D \left(\frac{3n-1}{2} f''''(0)f'(0) + g'(0) - \frac{n+1}{2} f'''(0)(g(0) + f(0))\right)\right), \] (18)

\[ Re_y^{0.5} C_{f_y} = \frac{1}{1 + \Lambda_1} \left(\frac{n+1}{2}\right)^{0.5} \left(g''''(0) + D \left(\frac{3n-1}{2} g''''(0)f'(0) + g'(0) - \frac{n+1}{2} g'''(0)(g(0) + f(0))g''(0)\right)\right), \] (19)

\[ N_{nx} Re_x^{\frac{1}{2}} = -\left(\frac{n+1}{2}\right) \theta'(0), \quad Sh_x Re_x^{\frac{1}{2}} = -\left(\frac{n+1}{2}\right) \phi'(0). \] (20)

Here \( Re = \frac{\Omega U_x}{\nu} \) is the Reynolds number.
**Numerical solution**

The basic methodology steps of PCM approach are as follow\(^{17,42-46}\):

**Step 1: simplification to 1st order ODE.**

\[
\begin{align*}
 h_1 &= f(\eta),
 h_3 &= f''(\eta),
 h_5 &= g'(\eta),
 h_7 &= \theta(\eta),
 h_9 &= \phi(\eta), \\
 h_2 &= f'(\eta),
 h_4 &= g(\eta),
 h_6 &= g''(\eta),
 h_8 &= \theta'(\eta),
 h_{10} &= \phi'(\eta).
\end{align*}
\]

(21)

By putting Eq. (21) in Eqs. (12)–(15) & (16), we get:

\[
\begin{align*}
 (1 - D \cdot \frac{n+1}{2} (h_1 + h_4) + D(n - 1)(h_2 + h_3)) h'_4 &+ (1 + \Lambda_1) \left( (h_1 + h_4)h_3 - \frac{2n}{n+1} (Frh_2^2 + h_2h_3) \right) \\
+ D \cdot \frac{3n-1}{2} (h_3 + h_6)h_3 - \frac{2D}{n+1} (1 + \Lambda_1)(M_p^2 + P_0)h_2 &= 0,
\end{align*}
\]

(22)

\[
\begin{align*}
 (1 - D \cdot \frac{n+1}{2} (h_1 + h_4) + D(n - 1)(h_2 + h_3)) h'_6 &+ (1 + \Lambda_1) \left( (h_4 + h_5)h_6 - \frac{2n}{n+1} (Frh_2^2 + h_2h_3) \right) \\
+ D \cdot \frac{3n-1}{2} (h_2 + h_6)h_6 - D \cdot \frac{2}{n+1} (1 + \Lambda_1)(M_p^2 + P_0)h_5 &= 0,
\end{align*}
\]

(23)

\[
(1 + R)h'_8 + (h_1 + h_4)Pr h_8 + Hsh_7 = 0,
\]

(24)

\[
h_{10}' + Sc \left( (h_1 + h_4)h_{10} - \frac{2}{n+1} C_r h_9 \right) = 0.
\]

(25)

with the corresponding boundary conditions.

\[
\begin{align*}
 h_1(\eta) &= h_4(\eta) = \Lambda \left( \frac{1-n}{1+n} \right),
 h_2(\eta) &= h_5(\eta) = h_7(\eta) = h_9(\eta) = 1 \text{ as } \eta \to 0 \\
 h_2(\eta) &= h_5(\eta) = 0,
 h_3(\eta) &= h_6(\eta) = h_8(\eta) = h_{10}(\eta) = 0 \text{ as } \eta \to \infty.
\end{align*}
\]

(26)

**Step 2: familiarizing the embedding constraint \( p \) in Eqs. (22)–(25).**

\[
\begin{align*}
 (1 - D \cdot \frac{n+1}{2} (h_1 + h_4) + D(n - 1)(h_2 + h_3)) h'_4 &+ \left( (1 + \Lambda_1)(h_1 + h_4) + D \cdot \frac{3n-1}{2} (h_3 + h_6) \right) (h_3 - 1)p \\
- \frac{2n(1 + \Lambda_1)}{n+1} (Frh_2^2 + h_2h_3) - \frac{2D}{n+1} (1 + \Lambda_1)(M_p^2 + P_0)h_2 &= 0,
\end{align*}
\]

(27)

\[
\begin{align*}
 (1 - D \cdot \frac{n+1}{2} (h_1 + h_4) + D(n - 1)(h_2 + h_3)) h'_6 &+ \left( (1 + \Lambda_1)(h_4 + h_5) + D \cdot \frac{3n-1}{2} (h_2 + h_6) \right) (h_6 - 1)p \\
- \frac{2n(1 + \Lambda_1)}{n+1} (Frh_2^2 + h_2h_3) - D \cdot \frac{2}{n+1} (1 + \Lambda_1)(M_p^2 + P_0)h_5 &= 0,
\end{align*}
\]

(28)

\[
(1 + R)h'_8 + (h_1 + h_4)Pr(h_8 - 1)p + Hsh_7 = 0,
\]

(29)

\[
h_{10}' + Sc \left( (h_1 + h_4)(h_{10} - 1)p - \frac{2}{n+1} C_r h_9 \right) = 0.
\]

(30)

**Step 3: solving the Cauchy principal.** Numerical implicit scheme is employed for the above modeled equations, which is defined as below:

\[
\frac{U^{i+1} - U^i}{\Delta \eta} = \Delta U^{i+1}, \quad \frac{W^{i+1} - W^i}{\Delta \eta} = \Delta W^{i+1}.
\]

(31)

Finally, we get:

\[
U^{i+1} = (I - \Delta \Delta \eta)^{-1} U^i, \quad W^{i+1} = (I - \Delta \Delta \eta)^{-1} (W^i + \Delta \eta R).
\]

(32)

**Results and discussion**

The section revealed the physics behind each figure and table. The subsequent trends have been observed:
Velocity profile. Figures 2, 3, 4, 5, 6, 7, 8 explained the presentation of axial $f'(\eta)$ and radial $g'(\eta)$ velocity profile versus the variation of porosity term $P_0$, power-law index $n$, magnetic field constraint $M_F$, Darcy Forchheimer term $Fr$, Deborah number $D$, wall thickness term $\Lambda$ and the ratio of relaxation time to retardation term $\Lambda_1$ respectively. Figures 2 & 3 shows that the velocity curve declines with the growth of porosity term, while augmented with the flourishing upshot of power index constraint. Physically, the number of pours increases...
with the porosity parameter effect, which resists the fluid flow, so causes the reduction in the velocity outline. Figures 4 and 5 reported that the rising frequency of both constraints magnetic field and Darcy Forchhemier effect deduce the velocity distribution. Because the opposing force, which is created due to magnetic effect, resist the flow field, as a result such trend observed. Figures 6 and 7 described that the impact of local Deborah number

Figure 5. Velocity outline versus Darcy Forchhemier term Fr.

Figure 6. Velocity outline versus local Deborah number D.

Figure 7. Velocity profile versus wall thickness term \( \Lambda \).
and wall thickness parameter augmented the velocity field. Figure 8 displays that the upshot of thermal relaxation term, reduce the energy profile.

**Energy profile.** Figures 9, 10, 11, 12, 13, 14 explained the appearance of energy contour $\theta(\eta)$ versus the variation of porosity term $P_0$, power-law index $n$, thermal radiation $R$, heat source term $H_s$, Deborah number...
$D$, thermal relaxation term $\Lambda_1$. Figures 9 and 10 shows that the energy contour amplifies with the increment of porosity term while diminishing with the flourishing upshot of power index constraint. Physically, the number of pours increases with the porosity parameter effect, which resists the fluid flow, so triggers an expansion in the heat. Figures 11 and 12 illustrated that the thermal field boosts with the mounting values of $R$ and $Hs$. The effect of both constraints generates an additional heat inside the fluid, which scores in the advancement of the
Concentration profile. Figures 15, 16 and 17 elaborated the exhibition of mass outline \( \phi(\eta) \) versus \( C_R \), Deborah number \( D \) and Schmidt number \( Sc \) respectively. Figures 15, 16 and 17 assessed that the mass transmis-

Figure 14. Energy profile versus the ratio of relaxation time to retardation term \( \Lambda_1 \).

Figure 15. Mass profile versus the chemical reaction \( C_R \).

Figure 16. Mass profile versus the Deborah number \( D \).

temperature field \( \theta(\eta) \). Figures 13 and 14 presented that Deborah number \( D \) decreases the thermal energy field, while the impact of thermal relaxation term \( \Lambda_1 \) enhances the energy distribution.
The mass profiles reduce with the intensifying upshot of chemical reaction, Deborah number and Schmidt number. Physically, the influence of the Sc enhances the kinetic viscosity of the fluid, while lessen the molecular diffusion, which causes the reduction in mass profile. Similarly, the consequences of chemical reaction and Deborah number also declines the concentration profile $\phi(\eta)$.

**Error analysis.** In Fig. 18, we performed the error analysis, to ensure that our results are accurate up to the lowest residual error scale. Until evaluating and providing physical forecasts, we analyze an error to determine the accuracy of the proposed method.

Tables 1 and 2 illustrated the statistical outcomes for skin friction, Nusselt and Sherwood number versus several physical constraints respectively. Table 3 highlighted the comparative assessment of the present results versus the existing works. The results of Table 3 verify the accuracy of the current analysis.
Conclusion

We have numerically analyzed the energy conveyance through Jeffery fluid flow over an irregular extensible sheet with a porous medium. The consequences of the Darcy effect, variable thickness and chemical reaction are also considered. The phenomena have been modeled as a system of PDEs. Using similarity substitution, the modeled equations are reduced to a dimensionless system of ODEs. The computational technique is used to determine the numerical solution to the obtained sets of nonlinear differential equations. The key conclusions are:

- The velocity profiles \((f'(\eta)), g'(\eta))\) both decline with the increment of porosity term, magnetic field, Darcy Forchheimer and thermal relaxation factor while augmented with the flourishing upshot of power index and local Deborah number.
- The energy profile \(\theta'(\eta)\) magnifies with the increment of porosity term, thermal radiation and heat source term, while diminishing with the flourishing upshot of power index and Deborah number.
- The mass transfer profiles reduce with the rising upshot of \(CR\), Deborah number and Schmidt number.

Table 1. The arithmetical results for skin friction along \(x\) and \(y\) direction.

| \(\Delta_1\) | \(A\) | \(M\) | \(P_0\) | \(D\) | \(C_{f\beta}Re_{\beta}^{1/2}\) | \(C_{f\beta}Re_{\beta}^{1/2}\) |
|---|---|---|---|---|---|---|
| 0.3 | 0.1 | 0.4 | 1.0 | 0.2 | −1.47496 | −1.63760 |
| 0.5 | - | - | - | - | -1.38337 | -2.31511 |
| 0.7 | - | - | - | - | -1.30327 | -2.03290 |
| 0.2 | - | - | - | - | -1.53371 | -2.48535 |
| 0.3 | - | - | - | - | -1.59228 | -2.39394 |
| 0.8 | - | - | - | - | -1.72496 | -3.04651 |
| 1.2 | - | - | - | - | -1.87555 | -3.41097 |
| 1.5 | - | - | - | - | -1.64601 | -3.02635 |
| 2.0 | - | - | - | - | -1.79842 | -3.04401 |
| 0.4 | - | - | - | - | -1.49881 | -3.08712 |
| 0.6 | - | - | - | - | -1.52651 | -3.53664 |

Table 2. The statistical outputs of Sherwood and Nusselt numbers.

| \(\Delta_1\) | \(A\) | \(M\) | \(C_R\) | \(D\) | \((-\frac{n+1}{2})\theta'(0)\) | \((-\frac{n+1}{2})\phi'(0)\) |
|---|---|---|---|---|---|---|
| 0.1 | 1.0 | 0.4 | 1.0 | 0.2 | 0.5807 | 0.8134 |
| 0.3 | - | - | - | - | 0.5730 | 0.8117 |
| 0.5 | - | - | - | - | 0.5657 | 0.8102 |
| 2.0 | - | - | - | - | 0.1523 | 0.8133 |
| 3.0 | - | - | - | - | 0.9234 | 0.8133 |
| 0.8 | - | - | - | - | 0.5552 | 0.8078 |
| 1.2 | - | - | - | - | 0.5225 | 0.2013 |
| 1.5 | - | - | - | - | 0.5807 | 1.2574 |
| 2.0 | - | - | - | - | 0.5807 | 1.2814 |
| 0.4 | - | - | - | - | 0.6013 | 0.9963 |
| 0.6 | - | - | - | - | 0.6177 | 0.9891 |

Table 3. Relative evaluation of current results with the available literature for \(-f''(0)\).

| \(n\) | Reddy et al. | Khader et al. | Khan et al. | Present work |
|---|---|---|---|---|
| 0.0 | 0.95664 | 0.94128 | 0.95136 |
| 0.5 | 0.97894 | 0.9698 | 0.97365 |
| 0.0 | 1.01000 | 0.98364 | 0.99372 |
| 2.0 | 1.02242 | 1.01270 | 1.02281 |
| 3.0 | 1.03488 | 1.02454 | 1.03464 |
| 5.0 | 1.04762 | 1.03758 | 1.04767 |
| 10.0 | 1.06573 | 1.04994 | 1.16995 |
The porosity term and wall thickness parameter enhance the skin friction.

Data availability
All data used in this manuscript have been presented within the article.

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References
1. Elitar, S. et al. Computational assessment of hybrid nanofluid flow with the influence of hall current and chemical reaction over a slender stretching surface. Alex. Eng. J. 61(12), 10319–10331 (2022).
2. Bilal, M. et al. Comparative numerical analysis of Maxwell’s time-dependent thermo-diffusive flow through a stretching cylinder. Case Stud. Therm. Eng. 27, 101301 (2021).
3. Sivaraj, R. & Kumar, B. R. Chemically reacting dusty viscoelastic fluid flow in an irregular channel with convective boundary. Ain Shams Eng. J. 4(1), 93–101 (2013).
4. Alharbi, K. A. M. et al. Computational valuation of darcy ternary-hybrid nanofluid flow across an extending cylinder with induction effects. Micromachines 13(4), 588 (2022).
5. Ullah, I., Ullah, R., Alqarni, M. S., Xia, W. F. & Muhammad, T. Combined heat source and zero mass flux features on magnetized nanofluid flow by radial disk with the applications of Coriolis force and activation energy. Int. Commun. Heat Mass Transfer 126, 105416 (2021).
6. Bilal, M. et al. Parametric simulation of micropolar fluid with thermal radiation across a porous stretching surface. Sci. Rep. 12(1), 1–11 (2022).
7. Gul, T. et al. Magnetic dipole impact on the hybrid nanofluid flow over an extending surface. Sci. Rep. 10(1), 1–13 (2020).
8. Zhou, S. S., Bilal, M., Khan, M. A. & Muhammad, T. Numerical analysis of thermal radiative maxwell nanofluid flow over-stretching porous rotating disk. Micromachines 12(5), 540 (2021).
9. Iyyappan, G. & Singh, A. K. MHD flows on irregular boundary over a diverging channel with viscous dissipation effect. Int. J. Numer. Meth. Heat Fluid Flow 31(12), 2112–2127 (2021).
10. Bilal, M., Saeed, A., Gul, T., Rehman, M. & Khan, A., Thin-film flow of Carreau fluid over a stretching surface including the couple stress and uniform magnetic field. Partial Differ. Equa. Appl. Math. 4, 100162 (2021).
11. Ahmed, N., Khan, U., Mohyud-Din, S. T. & Erturk, V. S. Influence of thermal and concentration gradients on unsteady flow over a stretchable surface. Results Phys. 7, 3153–3162 (2017).
12. Khan, M. & Rasheed, A. Numerical implementation and error analysis of nonlinear coupled fractional viscoelastic fluid model with variable heat flux. Ain Shams Eng. J. 13(3), 101614 (2022).
13. Ahmadian, A., Bilal, M., Khan, M. A. & Asjad, M. I. Numerical analysis of thermal conduction hybrid nanofluid flow over the surface of a wavy spinning disk. Sci. Rep. 10(1), 1–13 (2020).
14. Khan, M. & Rasheed, A. Slip velocity and temperature jump effects on molybdenum disulphide MoS2 and silicon oxide SiO2 hybrid nanofluid near irregular 3D surface. Alex. Eng. J. 60(1), 1689–1701 (2021).
15. Ahmed, N., Khan, U. & Mohyud-Din, S. T. A theoretical investigation of unsteady thermally stratified flow of γAl2O3– H2O and γAl2O3– C2H6O2 nanofluids through a thin slit. J. Phys. Chem. Solids 119, 296–308 (2018).
16. Ullah, Z., Ullah, I., Zaman, G., Khan, H. & Muhammad, T. Mathematical modeling and thermodynamics of Prandtl–Eyring fluid with radiation effect: a numerical approach. Sci. Rep. 11(1), 1–11 (2021).
17. Algehyne, E. A. et al. Numerical simulation of bicoconvective Darcy Forchheimer nanofluid flow with energy transition over a permeable vertical plate. Sci. Rep. 12(1), 1–12 (2022).
18. Ali, A., Masqood, M., Anjum, H. J., Awaits, M. & Sultainuma, M. Analysis of heat transfer on MHD Jeffrey nanofluid flow over nonlinear elongating surface of variable thickness. ZAMM J. Appl. Math. Mechanics 102(2), e202100250 (2022).
19. Alrabiaiah, H. B., Bilal, M., Khan, M. A., Muhammad, T. & Legas, E. Y. Parametric estimation of gyrotactic microorganism hybrid nanofluid flow between the conical gap of spinning disk-cone apparatus. Sci. Rep. 12(1), 1–14 (2022).
20. Saleem, S., Al-Qarni, M. M., Nadeem, S. & Sandeepe, N. Convective heat and mass transfer in magneto Jeffrey fluid flow on a rotating cone with heat source and chemical reaction. Commun. Theor. Phys. 70(5), 534 (2018).
21. Noor, N. A. M., Shafie, S. & Admon, M. A. Unsteady MHD squeezing flow of Jeffrey fluid in a porous medium with thermal radiation, heat generation/absorption and chemical reaction. Phys. Scr. 95(10), 105213 (2020).
22. Bilal, M. et al. Numerical analysis of an unsteady, electroviscous, ternary hybrid nanofluid flow with chemical reaction and activation energy across parallel plates. Micromachines 13(6), 874 (2022).
23. Ojela, O., Raju, A. & Kumar, N. N. Influence of induced magnetic field and radiation on free convective Jeffrey fluid flow between two parallel porous plates with Soret and Dufour effects. J. Mech. 35(5), 657–675 (2019).
24. Yadav, D., Mohamad, A. A. & Awasthi, M. K. The Horton–Rogers–Lapwood problem in a Jeffrey fluid influenced by a vertical magnetic field. Proc. Inst. Mech. Eng. Part F: J. Process Mech. Eng. 235(6), 2119–2128 (2021).
25. Khan, L. M., Lone, S. A., Rasheed, A. & Alam, M. N. Computational simulation of Scott–Blair model to fractional hybrid nanofluid with Darcy medium. Int. Commun. Heat Mass Transfer 130, 105784 (2022).
26. Khan, U., Ahmed, N., Mohyud-Din, S. T., Alsulami, M. D. & Khan, I. A novel analysis of heat transfer in the nanofluid composed by nanodiamond and silver nanomaterials: numerical investigation. Sci. Rep. 12(1), 1–11 (2022).
27. Khan, M. & Rasheed, A. The space–time coupled fractional Cattaneo-Friedrich Maxwell model with Caputo derivatives. Int. J. Appl. Comput. Math. 7(3), 1–23 (2021).
28. Ali, A., Awaits, M., Al-Zubaidi, A., Saleem, S. & Marwat, D. K. Hartmann boundary layer in peristaltic flow for viscoelastic fluid: existence. Ain Shams Eng. J. 13(2), 101555 (2021).
29. Kumar, R. N. et al. Impact of magnetic dipole on ferromagnetic hybrid nanofluid flow over a stretching cylinder. Phys. Scr. 96(4), 045215 (2021).
30. Varun Kumar, R. S., Gunderi Dhananjaya, P., Naveen Kumar, R., Punith Gowda, R. J. & Prasannakumara, B. C. Modeling and theoretical investigation on Casson nanofluid flow over a curved stretching surface with the influence of magnetic field and chemical field and reaction. Int. J. Comput. Methods. Math. Mech. 23(1), 12–19 (2022).
31. Chu, Y. M. et al. Combined impact of Cattaneo-Christov double diffusion and radiative heat flux on bio-convective flow of Maxwell liquid confined by a stretched nano-material surface. Appl. Math. Comput. 419, 126883 (2022).
32. Acharya, N., Mabood, F., Shahzad, S. A. & Badruddin, I. A. Hydrothermal variations of radiative nanofluid flow by the influence of nanoparticles diameter and nanolayer. Int. Commun. Heat Mass Transfer 130, 105781 (2022).
33. Iv, Y. P. et al. Numerical approach towards gyrotactic microorganisms hybrid nanoliquid flow with the hall current and magnetic field over a spinning disk. Sci. Rep. 11(1), 1–13 (2021).
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Competing interests
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