Is the stop mass below the top mass?

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Abstract

It is shown that a top mass of $174 \pm 17$ GeV, as quoted recently by the CDF Collaboration, constrains the mixing angle between the Higgs doublets in the Minimal Supersymmetric extension of the Standard Model (MSSM) to: $1.2 < \tan \beta < 5.5$ at the 90\% C.L.. The most probable value corresponds to $\tan \beta = 1.56$; such a small value causes a large mixing in the stop sector and the lightest stop is likely to be below the top mass. In this case the stop production in $p \bar{p}$ collisions would contribute to the top signature, thus providing a possible explanation for the large effective $t \bar{t}$ cross section observed by CDF.
1 Introduction

The Grand Unification idea has been subjected recently to a new test using the new precise LEP data [1, 2, 3]. The result clearly indicates that the minimal Standard Model (SM) does not lead to unification of the coupling constants, if they are extrapolated to high energies [3]. On the contrary, within the Minimal Supersymmetric extension of the Standard Model (MSSM) unification is achieved. Supersymmetry [4] presupposes a symmetry between fermions and bosons, thus doubling the particle spectrum of the SM. The predicted particles are indicated by a tilde above the usual SM symbol. Since these supersymmetric particles (“sparticles”) have not yet been observed, supersymmetry must be broken. From the unification condition a first estimate of the SUSY breaking scale could be made: it was found to be of the order of 1000 GeV, or more precisely $10^{3\pm1}$ GeV [2].

Assuming soft symmetry breaking at the Grand Unification (GUT) scale, all sparticle masses can be expressed in terms of 5 parameters and the masses at low energies are then determined by the well known Renormalization Group (RG) equations. The parameters are: $m_0$, the common mass of the spin 0 squarks and sleptons; $m_{1/2}$, the common mass of the spin 1/2 gauginos; $\mu$, the mixing parameter between the Higgs doublets; $\tan \beta$, the ratio of the vacuum expectation values of the two Higgs doublets; and $A$, the trilinear coupling in the Higgs sector. So many parameters cannot be derived from the unification condition alone. Further constraints can be considered:

- $M_Z$ predicted from electroweak symmetry breaking [4 – 13].
- Constraints from the unification of Yukawa couplings [10, 11, 14 – 23].
- Constraints from the lower limit on the proton lifetime [24, 25, 26].
- Experimental lower limits on SUSY masses [27, 28].
- Constraints from the top mass suggested by CDF [29].

We perform a statistical analysis, in which all constraints are implemented in a $\chi^2$ definition and try to find the most probable region of the parameter space by minimizing the $\chi^2$ function. The results will be presented after a short description of the experimental input values and it is shown that a likely solution has the lightest stop mass below the top mass.

2 Unification of the Couplings

In the SM based on the group $SU(3) \times SU(2) \times U(1)$ the couplings are defined as:

$$\begin{align*}
\alpha_1 &= (5/3)g'^2/(4\pi) = 5\alpha/(3\cos^2 \theta_W) \\
\alpha_2 &= g^2/(4\pi) = \alpha/\sin^2 \theta_W \\
\alpha_3 &= g_s^2/(4\pi)
\end{align*}$$

(1)

where $g'$, $g$ and $g_s$ are the $U(1)$, $SU(2)$ and $SU(3)$ coupling constants; the first two coupling constants are related to the fine structure constant by: $e = \sqrt{4\pi\alpha} = g \sin \theta_W = g' \cos \theta_W$.

In the $\overline{MS}$ renormalization scheme the world averaged values of the coupling constants at the $Z^0$ energy are

$$\begin{align*}
\alpha^{-1}(M_Z) &= 127.9 \pm 0.1 \\
\sin^2 \theta_{W,\overline{MS}} &= 0.2324 \pm 0.0005 \\
\alpha_3 &= 0.123 \pm 0.006.
\end{align*}$$

(2) – (4)
The value of $\alpha^{-1}$ is given in ref. [30] and the value of $\sin^2 \theta_{\text{MS}}$ has been taken from a detailed analysis of all available data by Langacker and Polonsky [31], which agrees with the latest analysis of the LEP data [32]. The error includes the uncertainty from the top quark. We have not used the smaller error of 0.0003 for a given value of $m_t$, since the fit was only done within the SM, not the MSSM, so we prefer to use the more conservative error including the uncertainty from $m_t$.

The $\alpha_3$ value corresponds to the value at $M_Z$ as determined from quantities calculated in the “Next to Leading Log Approximation” [33]. These quantities are less sensitive to the renormalization scale, which is an indicator of the unknown higher order corrections; they are the dominant uncertainties in quantities relying on second order QCD calculations [34]. This $\alpha_s$ value is in excellent agreement with a preliminary value of $0.120 \pm 0.006$ from a fit to the $Z^0$ cross sections and asymmetries measured at LEP [32], for which the third order QCD corrections have been calculated too; the renormalization scale uncertainty is correspondingly small.

The top quark mass was simultaneously fitted to all electroweak data and found to be [32]:

$$M_{\text{top}} = 166^{+17}_{-19} +^{19}_{-22} \text{ GeV},$$

(5)

where the first error is statistical and the second error corresponds to a variation of the Higgs mass between 60 and 1000 GeV. The central value corresponds to a Higgs mass of 300 GeV. Preliminary analysis including the LEP 1993 data find $M_{\text{top}} = 165 \pm 12 \pm 18$ GeV and $M_{\text{top}} = 174 \pm 11^{+17}_{-19} \text{ GeV}$, if the new SLD data from SLAC is included [35]. These values are in good agreement with recent results quoted by the CDF Collaboration [29]:

$$M_{\text{top}} = 174^{+10}_{-10} +^{13}_{-12} \text{ GeV},$$

(6)

where the first error is statistical and the second error systematic.

For SUSY models, the dimensional reduction $\overline{DR}$ scheme is a more appropriate renormalization scheme [36]. This scheme also has the advantage that all thresholds can be treated by simple step approximations. Thus unification occurs in the $\overline{DR}$ scheme if all three $\alpha_i^{-1}(\mu)$ meet exactly at one point. This crossing point then gives the mass of the heavy gauge bosons. The $\overline{MS}$ and $\overline{DR}$ couplings differ by a small offset

$$\frac{1}{\alpha_i^{\overline{DR}}} = \frac{1}{\alpha_i^{\overline{MS}}} - \frac{C_i}{12\pi},$$

(7)

where the $C_i$ are the quadratic Casimir coefficients of the group ($C_i = N$ for SU($N$) and 0 for U(1) so $\alpha_1$ stays the same). Throughout the following, we use the $\overline{DR}$ scheme for the MSSM.

3 $M_Z$ from Electroweak Symmetry Breaking

In the MSSM at least two Higgs doublets have to be introduced. Radiative corrections from the heavy top and stop quarks can drive one of the Higgs masses negative, thus causing spontaneous symmetry breaking in the electroweak sector. In this case the Higgs potential does not have its minimum for all fields equal zero, but the minimum is obtained for non-zero vacuum expectation values of the fields. The scale, where symmetry breaking occurs depends on the starting values of the mass parameters at the GUT scale, the top mass and the evolution of the couplings and masses. This gives strong constraints between the known $Z^0$ mass and the SUSY mass parameters, as demonstrated e.g. in ref. [10].
After including the one-loop corrections to the potential \[37, 38, 39, 40, 41, 20\], the \(M_Z\) mass becomes dependent on the top- and stop quark masses too. The corrections are zero if the top- and stop quark masses are identical, i.e. if supersymmetry would be exact. They grow with the difference \(\tilde{m}_t^2 - m_t^2\), so these corrections become unnaturally large for large values of the stop masses \[42, 43\].

4 \(m_b/m_\tau\) Mass Ratio

Unification of the Yukawa couplings for a given generation at the GUT scale predicts relations for quark and lepton masses within a given family. Unfortunately, for the light quarks the masses are uncertain, but the ratio of b-quark and \(\tau\)-lepton masses can be correctly predicted by the radiative mass corrections \[44, 9, 18, 19, 16, 17\].

Assuming the simplest possible GUT model based on SU(5) gauge group, one has at the GUT scale: \(m_b = m_\tau\). To calculate the experimentally observed mass ratio the RG equations for the running masses have to be used. By a physical mass we understand the value of the running mass at the energy scale equal to the mass itself. This definition of the mass is used throughout this paper.

From the RG equations for the Yukawa couplings one can easily obtain the RGE for the ratio \[12, 13\]

\[
\frac{R_{b\tau}}{m_\tau} = \frac{m_b}{m_\tau} = \sqrt{\frac{Y_b}{Y_\tau}}.
\]

For the running mass of the b-quark we used \[13\]:

\[
m_b = 4.25 \pm 0.3 \text{ GeV}.\tag{8}
\]

This mass depends on the choice of scale and the value of \(\alpha_s(m_b)\). Consequently, we have assigned a rather conservative error of 0.3 GeV instead of the proposed value of 0.1 GeV \[15\]. Note that the running mass (in the \(\overline{MS}\) scheme) is related to the physical (pole) mass \(m_b^{\text{pole}}\) by \[15\]:

\[
m_b = m_b^{\text{pole}} \left( 1 - \frac{4 \alpha_s}{3 \pi} - 12.4 \left(\frac{\alpha_s}{\pi}\right)^2 \right) \approx 0.825 \; m_b^{\text{pole}},\tag{9}
\]

so \(m_b = 4.25\) corresponds to \(m_b^{\text{pole}} \approx 5\) GeV. We ignore the running of \(m_\tau\) below \(m_b\) and use for the \(\tau\) mass: \(m_\tau = 1.7771 \pm 0.0005\) GeV \[16\].

5 Top Mass Constraints

The top mass can be expressed as:

\[
m_t^2 = (4\pi)^2 Y_t(t) \; v^2 \; \sin^2(\beta),\tag{10}
\]

where the running of the Yukawa coupling \(Y_t\) as function of \(t = \log\left(\frac{M_{\text{GUT}}^2}{Q^2}\right)\) in first order\(^4\) is given by \[9\]:

\[
Y_t(t) = \frac{Y_t(0) E(t)}{1 + 6Y_t(0) F(t)},\tag{11}
\]

\(^4\) Throughout the analysis we have used the second order RG equations, for which no analytical solution exists, but this will not change the following arguments dramatically.
where $E$ and $F$ are functions of the couplings only (see appendix). One observes that $Y_t(t)$ becomes independent of $Y_t(0)$ for large values of $Y_t(0)$, implying an upper limit on the top mass. Requiring electroweak symmetry breaking implies a minimal value of the top Yukawa coupling, typically $Y_t(0) \geq O(10^{-2})$. In this case the term $6Y_t(0)F(t)$ in the denominator of eq. (11) is much larger than one, since $F(t) \approx 290$ at the weak scale, where $t \approx 66$. In this case $Y_t(t) = E(t)/6F(t)$, so from eq. (10) it follows:

$$m_t^2 = \frac{(4\pi)^2 E(t)}{6F(t)} v^2 \sin^2(\beta) \approx (190 \text{ GeV})^2 \sin^2(\beta),$$

(12)

The physical (pole) mass is about 6% larger than the running mass [15]:

$$M_t^{pole} = m_t \left(1 + \frac{4}{3} \frac{\alpha_s}{\pi}\right) \approx (200 \text{ GeV}) \sin \beta,$$

(13)

The electroweak breaking conditions require $\pi/4 < \beta < \pi/2$ ; hence the equation above implies for the MSSM approximately:

$$145 < M_t^{pole} < 200 \text{ GeV},$$

(14)

which is consistent with the experimental values given in eqns. (5) and (6).

For large top masses, the b-quark mass becomes a sensitive function of $m_t$ and of the starting values of the gauge couplings at $M_{GUT}$ [11].

6 Experimental Lower Limits on SUSY Masses

SUSY particles have not been found so far and from the searches at LEP one knows that the lower limit on the charged leptons and charginos is about half the $Z^0$ mass (45 GeV) [28] and the Higgs mass has to be above 62 GeV [27]. The lower limit on the lightest neutralino is 18.4 GeV [28], while the sneutrinos have to be above 41 GeV [28]. These limits require minimal values for the SUSY mass parameters.

There exist also limits on squark and gluino masses from the hadron colliders [28], but these limits depend on the assumed decay modes. Furthermore, if one takes the limits given above into account, the constraints from the limits of all other particles are usually fulfilled, so they do not provide additional reductions of the parameter space in case of the minimal SUSY model.

7 Proton Lifetime Limits

GUT’s predict proton decay and the present lower limits on the proton lifetime yield quite strong constraints on the GUT scale and the SUSY parameters. The direct decay $p \rightarrow e^+\pi^0$ via s-channel exchange requires the GUT scale to be above $10^{15}$ GeV. This is not fulfilled in the SM, but always fulfilled in the MSSM. Therefore we do not consider this constraint. However, the decay via box diagrams with winos and Higgsinos predict much shorter lifetimes, especially in the preferred mode $p \rightarrow \pi K^+$. From the present experimental lower limit of $10^{32}$ yr for this decay mode Arnowitt and Nath [23] deduce an upper limit on the parameter B:

$$B < (293 \pm 42) \frac{M_{H_0}}{3M_{GUT}} \text{ GeV}^{-1}$$

(15)
Here $M_{H_3}$ is the Higgs triplet mass, which is expected to be of the order of $M_{GUT}$. To obtain a conservative upper limit on $B$, we allow $M_{H_3}$ to become an order of magnitude heavier than $M_{GUT}$, so we require

$$B < 977 \pm 140 \text{ GeV}^{-1}.$$  

(16)

The uncertainties from the unknown heavy Higgs mass are large compared with the contributions from the first and third generation, which contribute through the mixing in the CKM matrix. Therefore we only consider the second order generation contribution, which can be written as [25]:

$$B = \frac{-2\alpha_2}{\alpha_3 \sin(2\beta)} \frac{m_{\tilde{g}}}{m_{\tilde{g}}^2} 10^6$$  

(17)

where $\alpha_2$ and $\alpha_3$ are the coupling constant of the $SU(2)$ and $SU(3)$ groups at the SUSY scale, respectively. One observes that the upper limit on $B$ favours small gluino masses $m_{\tilde{g}}$, large squark masses $m_{\tilde{q}}$, and small values of $\tan \beta$. To fulfill this constraint requires

$$\tan \beta < 10$$  

(18)

for practically the whole parameter space [23, 13].

### 8 Fit Strategy

From the five parameters in the MSSM plus the common coupling $\alpha_{GUT}$ at the unification scale $M_{GUT}$ one can determine all other SUSY masses, the $b$-quark mass, and $M_Z$ by performing the complete evolution of the couplings and masses including all thresholds. Details can be found in [12, 43].

The most probable parameter values were obtained by minimizing the following $\chi^2$ function [5]:

$$\chi^2 = \sum_{i=1}^{3} \frac{(\alpha_i^{-1}(M_Z) - \alpha_{MSSM_i}(M_Z))^2}{\sigma_i^2}$$  

$$+ \frac{(M_Z - 91.18)^2}{\sigma_Z^2}$$  

$$+ \frac{(m_b - 4.25)^2}{\sigma_b^2}$$  

$$+ \frac{(B - 997)^2}{\sigma_B^2} (for \ B > 997)$$  

$$+ \frac{(D(m1m2m3))^2}{\sigma_D^2} (for \ D > 0)$$  

$$+ \frac{(\tilde{M} - \tilde{M}_{exp})^2}{\sigma_{\tilde{M}}^2} (for \ \tilde{M} > \tilde{M}_{exp}).$$  

(19)

We use the MINUIT program from F. James and M. Roos, *MINUIT Function Minimization and Error Analysis*, CERN Program Library Long Writeup D506; Release 92.1, from March 1992. Our $\chi^2$ has discontinuities due to the experimental bounds on various quantities, which become “active” only for specific regions of the parameter space. Consequently the derivatives are not everywhere defined. The option SIMPLEX, which does not rely on derivatives, can be used to find the monotonous region and the option MIGRAD to optimize inside this region.
The first term is the contribution of the difference between the three calculated and measured gauge coupling constants at $M_Z$ and the following two terms are the contributions from the $M_Z$-mass and $m_b$-mass constraints. The last three terms impose constraints from the proton lifetime limits, from electroweak symmetry breaking, i.e. $D = V_H(v_1,v_2) - V_H(0,0) < 0$, and from experimental lower limits on the SUSY masses. The top mass, or equivalently, the top Yukawa coupling enters sensitively into the calculation of $m_b$ and $M_Z$. Instead of the top Yukawa coupling one could have taken the top mass as a parameter. However, if the couplings are evolved from $M_{GUT}$ downwards, it is more convenient to run also the Yukawa coupling downward, since the RG equations of the gauge and Yukawa couplings form a set of coupled differential equations in second order. Once the Yukawa coupling is known at $M_{GUT}$, the top mass can be calculated at any scale.

The following errors were attributed: $\sigma_i$ are the experimental errors in the coupling constants, as given above, $\sigma_b = 0.3$ GeV, $\sigma_B = 140$ GeV$^{-1}$, while $\sigma_D$ and $\sigma_{\tilde{M}}$ were set to 10 GeV. The values of the latter errors are not critical, since the corresponding terms in the numerator are zero in case of a good fit and even for the 90% C.L. limits these constraints could be fulfilled and the $\chi^2$ was determined by the other terms, for which one knows the errors.

The light thresholds are taken into account in the evolution of the coupling constants by changing the coefficients of the RGE at the value $Q = m_i$, where the threshold masses $m_i$ are obtained from the analytical solutions of the corresponding RGE. These solutions depend on the integration range, which was chosen between $m_i$ and $M_{GUT}$. However, since one does not know $m_i$ at the beginning, an iterative procedure has to be used: one first uses $M_Z$ as a lower integration limit, calculates $m_i$, and uses this as lower limit in the next iteration. Of course, since the coupling constants are running, the latter have to be iterated too, so the values of $\alpha_i(m_i)$ have to be used for calculating the mass at the scale $m_i$ [10, 47]. Usually three to five iterations are enough to find a stable solution.

Following Ellis, Kelley and Nanopoulos [48] the possible effects from heavy thresholds are set to zero, since the proton lifetime limits forbid the Higgs triplet masses to be below $M_{GUT}$. These heavy thresholds have been considered by other authors for different assumptions [49, 31, 50].

9 Results

We first consider fits without proton lifetime constraints, since they are only important in the determination of lower limits, as will be discussed below. The upper part of fig. 1 shows the evolution of the coupling constants in the MSSM for two cases: one for the minimum value of the $\chi^2$ function given in eq. 19 (solid lines) and one corresponding to the 90% C.L. upper limit of the thresholds of the light SUSY particles (dashed lines). The position of the light thresholds is shown in the bottom part as jumps in the first order $\beta$ coefficients, which are increased as soon as a new threshold is passed. Also the second order coefficients are changed correspondingly, but their effect on the evolution is not visible in the top figure in contrast to the first order effects, which change the slope of the lines considerably in the top figure. One observes that the changes in the coupling constants occur in a rather narrow energy regime, so qualitatively this picture is very similar to the case, in which all sparticles were assumed to be degenerate at an effective SUSY mass scale $M_{SUSY}$ [2]. Since the running of the couplings depends only logarithmically on the sparticle masses, the 90% C.L. upper limits are as large as several TeV, as shown by the dashed lines in fig. 1 and more quantitatively in table 1.
With the fitted SUSY parameters given at the top of the table, the corresponding masses of the SUSY particles can be calculated. Their values are given in the lower part of the table. 

The 90% C.L. upper and lower limits on the masses are obtained by scanning $m_0$ and $m_{1/2}$ till the $\chi^2$ value increases by 1.64, while optimizing the values of $\tan \beta$, $\mu$, $\alpha_{\text{GUT}}$, $Y_t(0)$ and $M_{\text{GUT}}$. The lower limits on the SUSY parameters are shown in the left column of table 1. The lowest values of $m_0$ and $m_{1/2}$ are required to have simultaneously a sneutrino mass above 42 GeV and a wino mass above 45 GeV. If the proton lifetime limit is included, either $m_0$ or $m_{1/2}$ have to be above a certain limit (see eq. 10). Since the squarks and gauginos are much more sensitive to $m_{1/2}$ than $m_0$, one obtains the lower limits by increasing $m_0$. The minimum value for $m_0$ is about 400 GeV in this case. But in both cases the $\chi^2$ increase for the lower limits is due to the b-mass, which is predicted to be 4.6 GeV from the parameters determining the lower limits, so it gives a contributions to the $\chi^2$ function, which requires $m_0=4.25$ GeV (see eq. 8). The 90% C.L. upper limits can be several TeV. If one requires that only solutions are allowed for which the corrections to $M_Z$ are not large compared to $M_Z$ itself, one has to limit the mass of the heaviest stop quark to about one TeV. The corresponding 90% C.L. upper limits of the individual sparticle masses are given in the right hand column of table 1. The correction to $M_Z$ is 6 times $M_Z$ in this case.

The mass of the lightest Higgs particle, called $h$ in table 1, is a rather strong function of $m_t$, as shown in fig. 2 for various choices of $\tan \beta$, $m_0$ and $m_{1/2}$. All other parameters were optimized for these inputs and after the fit the values of the Higgs and top mass were calculated and plotted. One observes that the mass of the lightest Higgs particle varies between 60 and 150 GeV and the top mass between 134 and 190 GeV. Furthermore, it is evident that $\tan \beta$ almost uniquely determines the value of $m_t$ (through eq. 11), since even if $m_{1/2}$ and $m_0$ are varied between 100 and 1000 GeV, one finds practically the same $m_t$ for a given $\tan \beta$. The value of $m_t$ varies between 134 and 190 GeV, if $\tan \beta$ is varied between 1.2 and 10. This range is in excellent agreement with the estimates given in eq. 14, if one takes into account that $M_t^{\text{pole}} \approx 1.06 m_t$ (see eq. 13). The shaded area in fig. 2 indicates the results on the top mass quoted by the CDF Collaboration [29]. It clearly favours low values of $\tan \beta$. Adding to the $\chi^2$ a term $(M_t-174)^2/17^2$ yields after minimization:

$$1.2 < \tan \beta < 5.5$$

at the 90% C.L. (20)

The most probable value corresponds to $\tan \beta = 1.56$ as indicated by the star in the figure.

Such a low value leads to a large mixing in the stop sector, in which case a likely value of the lightest stop is below the top mass (see the typical fit in table 1), although stop masses above the top mass are not excluded, as shown by the upper limits in table 1. Also a change in sign of the Higgs mixing parameter $\mu$ leads to stop masses above the top mass, but the $\chi^2$ value is hardly worse in that case, so this cannot be excluded either. Varying $A_t(0)$ between $+3 m_0$ and $-3 m_0$ does not influence the results very much, since it is usually compensated by a change in $\mu$, so $A_t(0)$ was kept zero, but its non-zero value at lower energies was taken into account.

If the stop mass is below the top mass, it cannot decay into the top, but can decay as follows:

$$\tilde{t}_1 \to \tilde{\chi}_1^\pm + b \to \chi_1^0 + W + b \to \chi_1^0 + \text{lepton} + \nu + b,$$

which is experimentally very similar to the normal top decay signature [51]. Additional stop production could be an explanation for the excess of events seen by the CDF Collaboration: they observe an effective cross section for top pair production of $13.9^{+6.1}_{-4.8}$ pb, while the calculated $t\bar{t}$ cross section is only $5.8^{+0.8}_{-0.4}$ pb [28].
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| Symbol           | Lower limits | Typical fit | 90% C.L. Upper limits |
|------------------|--------------|-------------|----------------------|
| Constraints      | GEY          | GEY+P       | GEY+(PF)             | GEY+ (P)     | GEY+(P)+F   |
| $m_0$            | 65           | 400         | 400                  | 400          | 400         |
| $m_{1/2}$        | 37           | 80          | 111                  | 1600         | 475         |
| $\mu$            | -117         | 330         | 870                  | 1842         | 1101        |
| $\tan \beta$    | 3.0          | 3.0         | 1.56                 | 8.5          | 2.9         |
| $Y_t(0)$         | 0.0158       | 0.0035      | 0.0150               | 0.0023       | 0.0084      |
| $M_t^{pole}$     | -            | -           | 175                  | 178          | 189         |
| $m_t$            | -            | -           | 165                  | 168          | 178         |
| $1/\alpha_{GUT}$| 23.8         | 24.3        | 24.5                 | 25.9         | 25.2        |
| $M_{GUT}$        | $2.3 \times 10^{16}$ | $2.0 \times 10^{16}$ | $2.0 \times 10^{16}$ | $0.8 \times 10^{16}$ | $1.3 \times 10^{16}$ |
| SUSY masses in [GeV] |
| $\chi_0^0(\tilde{\chi})$ | 18 | 25 | 41 | 720 | 202 |
| $\chi_2^0(\tilde{Z})$ | 39 | 52 | 80 | 1346 | 386 |
| $\chi_1^\pm(\tilde{W})$ | 46 | 48 | 79 | 1347 | 386 |
| $\tilde{g}$      | 109          | 217         | 293                  | 3377         | 1105        |
| $\tilde{e}_L$    | 82           | 406         | 409                  | 1160         | 521         |
| $\tilde{e}_R$    | 67           | 401         | 402                  | 729          | 440         |
| $\tilde{\nu}_L$  | 41           | 400         | 406                  | 1157         | 516         |
| $\tilde{q}_L$    | 120          | 443         | 477                  | 3030         | 1071        |
| $\tilde{q}_R$    | 115          | 440         | 471                  | 2872         | 1030        |
| $\tilde{b}_L$    | 112          | 352         | 369                  | 2610         | 903         |
| $\tilde{b}_R$    | 119          | 440         | 471                  | 2862         | 1027        |
| $\tilde{t}_1$    | -            | -           | 144                  | 2333         | 725         |
| $\tilde{t}_2$    | -            | -           | 467                  | 2817         | 1008        |
| $\chi_3^0(\tilde{H}_1)$ | 109 | 292 | 540 | 1771 | 799 |
| $\chi_4^0(\tilde{H}_2)$ | 120 | 313 | 556 | 1780 | 812 |
| $\chi_2^\pm(\tilde{H}^\pm)$ | 129 | 315 | 566 | 1816 | 831 |
| $h$              | -            | -           | 87                   | 146          | 127         |
| $H$              | 118          | 523         | 812                  | 2218         | 1033        |
| $A$              | 92           | 521         | 810                  | 2217         | 1031        |
| $H^\pm$          | 121          | 527         | 813                  | 2219         | 1034        |

Table 1: Values of SUSY masses and parameters for various constraints: G=gauge coupling unification; E=electroweak symmetry breaking; Y=Yukawa coupling unification; P=Proton lifetime constraint; F=finetuning constraint. Constraints in brackets indicate that they are fulfilled but not required. The minimum values of the lightest Higgs mass, the stop mass and the top mass can't be reached for the parameters minimizing the squarks and slepton masses. One needs smaller values of $\tan \beta$ in that case.
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Figure 1: Evolution of the inverse of the three couplings in the MSSM. The line above $M_{\text{GUT}}$ follows the prediction from the supersymmetric SU(5) model. The SUSY thresholds have been indicated in the lower part of the curve: they are treated as step functions in the first order $\beta$ coefficients in the renormalization group equations, which correspond to a change in slope in the evolution of the couplings in the top figure. The dashed lines correspond to the 90% C.L. upper limit for the SUSY thresholds.
Figure 2: The mass of the lightest Higgs particle as function of the top quark mass for values of \( \tan \beta \) between 1.2 and 10 and values of \( m_0 \) and \( m_{1/2} \) between 100 and 1000 GeV. The parameters of \( \mu, M_{\text{GUT}}, \alpha_{\text{GUT}} \) and \( Y_t(0) \) are optimized for each choice of these parameters; the corresponding values of the top and lightest Higgs mass are shown as symbols. For small values of \( m_{1/2} \) the Higgs mass increases with \( m_0 \), as shown for a “string” of points, each representing a step of 100 GeV in \( m_0 \) for a given value of \( m_{1/2} \), which is increasing in steps of 100 GeV, starting with the low values for the lowest strings. At high values of \( m_{1/2} \) the value of \( m_0 \) becomes irrelevant and the “string” shrinks to a point. Note the strong positive correlation between \( m_{\text{higgs}} \) and all other parameters: the highest value of the Higgs mass corresponds to the maximum values of the input parameters, i.e. \( \tan \beta = 10, m_0 = m_{1/2} = 1000 \) GeV; this value does not correspond to the minimum \( \chi^2 \). More likely values are: \( m_{\text{higgs}} \approx 87 \) GeV for \( m_{1/2} = 100 \) GeV, \( m_0 = 400 \) GeV, \( \mu = 822 \) GeV and \( \tan \beta = 1.6 \), as indicated by the star. The hatched area corresponds to the top mass range measured by [29].
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