Compact integrated optical sensors and electromagnetic actuators for vibration isolation systems in the gravitational-wave detector KAGRA

Tomotada Akutsu,\textsuperscript{1,2,}\textsuperscript{*} Fabián Erasmo Peña Arellano,\textsuperscript{1,3} Ayaka Shoda,\textsuperscript{1} Yoshinori Fujii,\textsuperscript{4} Koki Okutomi,\textsuperscript{3} Mark Andrew Barton,\textsuperscript{1,5} Ryutaro Takahashi,\textsuperscript{1} Kentaro Komori,\textsuperscript{6} Naoki Aritomi,\textsuperscript{6} Tomomichi Shimoda,\textsuperscript{6} Satoru Takano,\textsuperscript{6} Hiroki Takeda,\textsuperscript{6} Enzo Nicolas Tapia San Martin,\textsuperscript{1,7} Ryohei Kozu,\textsuperscript{3} Bungo Ikenoue,\textsuperscript{1} Yoshiyuki Obuchi,\textsuperscript{1} Mitsuhiro Fukushima,\textsuperscript{3} Yoichi Aso,\textsuperscript{1,2} Yuta Michimura,\textsuperscript{6} Osamu Miyakawa,\textsuperscript{3} and Masahiro Kamiizumi\textsuperscript{3}

\textsuperscript{1}National Astronomical Observatory of Japan (NAOJ), Mitaka, Tokyo 181-8588, Japan
\textsuperscript{2}The Graduate University for Advanced Studies (Sokendai), Mitaka, Tokyo 181-8588, Japan
\textsuperscript{3}KAGRA Observatory, Institute for Cosmic Ray Research (ICRR), University of Tokyo, Hida, Gifu 506-1205, Japan
\textsuperscript{4}Department of Astronomy, University of Tokyo, Bunkyo, Tokyo 113-0033, Japan
\textsuperscript{5}Institute for Gravitational Research, University of Glasgow, Glasgow G12 8QQ, UK
\textsuperscript{6}Department of Physics, University of Tokyo, Bunkyo, Tokyo 113-0033, Japan
\textsuperscript{7}Nikhef, Science Park 105, 1098 XG Amsterdam, The Netherlands

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This paper reports on the design and characteristics of a compact module integrating an optical displacement sensor and an electromagnetic actuator for use with vibration-isolation systems installed in KAGRA, the 3-km baseline gravitational-wave detector in Japan. In technical concept, the module belongs to a family tree of similar modules (called OSEM) used in other interferometric gravitational-wave detector projects. After the initial test run of KAGRA in 2016, the sensor part, which is a type of fork sensor (or slot sensor), was modified by increasing the spacing of the slot from 5 mm to 15 mm to avoid the risk of mechanical interference with the sensor flag in operation. We confirmed the sensor performance is comparable to that of the previous design despite the modification. We also confirmed the sensor noise is consistent with the theoretical noise budget. The noise level is 0.5 nm/Hz\textsuperscript{1/2} at 1 Hz and 0.1 nm/Hz\textsuperscript{1/2} at 10 Hz, and the linear range of the sensor is 0.7 mm or more. We measured the response of the actuator to be 1 N/A, and also measured the resistances and inductances of coils of the actuators to confirm their consistency. Coupling coefficients among the different degrees of freedom were also measured and evaluated. A potential concern about thermal noise contribution due to eddy current loss is discussed. In addition, for reference in the future, we summarize theoretical formulae that will be useful for designing similar actuators. As of 2020, 42 of the modules are in operation at the site.

I. INTRODUCTION

Vibration isolation is an essential technique for state of the art instruments including gravitational-wave (GW) detectors to achieve accurate measurements. The terrestrial GW detectors such as LIGO \textsuperscript{[6]}, Virgo \textsuperscript{[2]}, and KAGRA \textsuperscript{[3,4]} are large-scale laser interferometers with baseline lengths of 3-4 km, and mirrors used in the interferometers are suspended by multi-stage pendulums in ultra-high vacuum for vibration isolation. The suspensions filter out seismic motion transferring to the mirrors above the mechanical resonant frequencies of the pendulums. For accurate observation of GWs with space-time strain of $\sim 10^{-21}$, the vibration of the ground must be attenuated $10^{10}$ times or more above 10 Hz \textsuperscript{[5]}. 

In practice, we must also damp the oscillations of the pendulums, which amplify the mirror vibrations at the resonant frequencies. Kinetic energy stored in the resonant modes can be dissipated with active or passive damping control, but the damping control should not degrade vibration-isolation performance of the pendulums in the frequency range for the GW observation above 10 Hz. In this paper, we report on a compact module for such sophisticated active damping of the pendulums, especially used in the KAGRA interferometer.

Technically, our module belongs to a family tree of designs of OSEM, optical sensor and electromagnetic actuator. In the late 1990s, one of the first generation of OSEMs was designed for LIGO \textsuperscript{[6]}. Since then, several types of OSEMs have been developed and incorporated into laser interferometric GW detectors \textsuperscript{[7,11]}. Like previous designs, our OSEM integrates a contactless sensor and a contactless actuator into a fist-sized body that can be attached to a suspended damper.

An advantage of such a compact sensor-actuator is that it can push or pull the same point at which it exactly looks. This feature is helpful for designing control filters for damping. Regardless of the number of the OSEMs attached to a suspended body, the basis of sensors and actuators are identical and so trivial; one can start with a set of simple feedback filters on the trivial basis, and learn the actual behavior of the suspension before moving on to develop a more sophisticated control topology \textsuperscript{[12]}. The damping could be achieved even if their bases are different, but that is not necessarily possible without aligning the bases.

As an example of the suspended damper, let us consider a single pendulum with a recoil mass (Fig. 1). In this case, the “suspended mass” represents an item such as a mirror to be isolated from seismic motion, and the OSEM is attached to the recoil mass. Then, one can detect the displacement of the mirror with respect to the recoil mass using the sensor, and also damp the mirror motion using the actuator so that the mirror comes to be stationary with respect to the recoil mass. If the resonant frequency of the recoil mass pendulum, $f_1$, is appropriately shifted from that of the mirror pendulum, $f_2$, this system isolates the mirror from seismic motion without...
isolation ratio (m/m) of the suspension. This can be shown by simple calculations: the equations of motion of the system are

\begin{align}
    m_1 \ddot{x}_1 + c(x_1 - x_2) + k_1(x_1 - x_0) &= 0, \quad (1a) \\
    m_2 \ddot{x}_2 + c(x_2 - \dot{x}_1) + k_2(x_2 - x_0) &= 0, \quad (1b)
\end{align}

where \(x_0\) represents the seismic motion; \(x_1\) and \(x_2\) are the displacement of the mirror and the recoil mass, respectively; \(m_1\) and \(m_2\) are masses of the mirror and the recoil mass; \(k_1\) and \(k_2\) are spring constants of the respective suspensions; \(c\) is the damping coefficient for the damper.

In the frequency domain, the equations of motion become

\begin{align}
    -\omega^2 m_1 \dot{x}_1 + ic\omega(\dot{x}_1 - x_2) + k_1 \dot{x}_1 &= k_1 \dot{x}_0, \quad (2a) \\
    -\omega^2 m_2 \dot{x}_2 + ic\omega(\dot{x}_2 - \dot{x}_1) + k_2 \dot{x}_2 &= k_2 \dot{x}_0, \quad (2b)
\end{align}

where \(\omega\) is angular frequency, \(\dot{x}_p\) represents frequency components corresponding to \(x_p\) for \(p = 0, 1, 2,\) and \(i\) is the imaginary unit. Solving the equations, one can obtain the ratio

\[
\frac{\dot{x}_1}{\dot{x}_0} = \frac{1 + 2i(\xi_1 \frac{\omega}{\omega_1} + \xi_2 \frac{\omega}{\omega_2} - \frac{\omega_1}{\omega_2}) + 4\xi_1 \xi_2 \frac{\omega_1}{\omega_2} \frac{\omega}{\omega_1}}{1 + 2i(\xi_1 \frac{\omega_2}{\omega_1} - \xi_2 \frac{\omega_1}{\omega_2}) (1 + 2i\xi_1 \frac{\omega_1}{\omega_2} - \frac{\omega_1}{\omega_2}) + 4\xi_1 \xi_2 \frac{\omega_2}{\omega_1} \frac{\omega_1}{\omega_2}}, \quad (3)
\]

where \(\omega_p \equiv \sqrt{k_p/m_p} = 2\pi f_p\) and \(\xi_p \equiv c/(2\sqrt{m_p k_p})\) for \(p = 1\) and 2, respectively.

Fig. 2 shows some examples of the form of Eq. (3) as a function of temporal frequency for different parameters; here \(m_1 = 1\) kg, \(m_2 = 0.5\) kg, and \(f_1 = 1\) Hz are fixed, while either \(\xi_1\) or \(\xi_2\) is varied. The thick green line is a reference case of \(c = 0\) or \(\xi_1 = 0\), which corresponds to a simple suspended mirror without damping; the isolation ratio is proportional to \(f^{-2}\) above \(f_1\), while the mirror oscillates considerably at \(f_1\). In contrast, the red line comes from a typical application of the suspended damper; it engages damping control of \(\xi_1 = 0.2\) with the pendulum for the recoil-mass suspension, which is more rigid (\(f_2 = 3\) Hz) than that for the mirror. Compared to the reference line, the damping suppresses the peak around \(f_1\) without degrading the frequency response above 10 Hz, although it makes oscillation acceptably small around \(f_2\).

The frequency response above 10 Hz will degrade with increasing rigidity of the recoil-mass suspension. The dashed yellow line in Fig. 2 shows an extreme case of where the damper is fixed to the ground. This is why we prefer the compact local sensor-actuator attachable to the suspended recoil mass. In reality, sensors fixed to the ground also take part in the damping control along with frequency-dependent filters.

Another extreme case is of \(f_1 = f_2\), where the denominator of Eq. (3) becomes 0. As shown by the dash-dotted blue line in Fig. 2, the resonant peak diverges, and so the oscillation seen from the ground will continue forever in this model, which should be avoided.

In this paper, we will report on the latest version of our OSEMs. As of 2020, 42 OSEMs, fabricated within KAGRA, are used in the KAGRA interferometer. There are seven suspensions for room-temperature mirrors, in each of which six OSEMs are attached to the recoil mass for the intermediate mass. The intermediate mass is located at the second bottom level of the multi-stage pendulum system, and suspends the mirror at the lowest level \([11]\). The original concept was for there to be an additional four OSEMs on the recoil mass for the mirror, and this was tried with the first generation of our OSEM in an off-site test \([10]\). For the initial test run of KAGRA \([3]\), we created a second-generation OSEM reflecting lessons we learned during the initial test run and to reduce risk and initially used ten (six+four) of these in each suspension. Later, to further reduce risk, we decided to eliminate flags from the mirrors and instead use OSEM bodies with just the coil parts.
in conjunction with much smaller magnets on the mirror; this variant is not discussed further here.

In the remainder of this paper, we show the overall design of our OSEMs in Section II, and describe the characteristics of the sensor and actuator parts in Section III and IV, respectively. Section V gives further detail of the design and issues with it. Section VI is a summary. Some useful theoretical formulae are summarized in appendices.

II. DESIGN

This section gives an overview of the design of the OSEMs currently installed at KAGRA, and more detailed descriptions of the sensor and actuator parts.

A. Overview of the design

Fig. 3 shows a schematic view of our OSEM. An optical sensor and an electromagnetic coil are integrated into a single module. The main body of the module is made of polyether ether ketone (PEEK), and the outer diameter is 62 mm. A bobbin structure protrudes from the main body to be a coil former, and has an outer diameter of 38 mm with a central through hole. The central through-hole is 15.5 mm in diameter and the main body is 42 mm long.

The optical sensor for displacement measurement is a shadow sensor consisting of a transmitter (TX) and a receiver (RX); see also Figs. 4 and 5. A light-emitting diode (LED) in the TX emits a light beam, which crosses the central hole, and illuminates a photodiode (PD) in the RX. The tip of the sensor flag (see Fig. 3) protrudes into the central hole, and partly shades the light beam. The light power reaching the PD varies monotonically with respect to the insertion depth, and so does the photocurrent from the PD. As a whole, this setup works as a displacement sensor for the relative position between the flag tip and the OSEM body.

The electromagnetic actuator is a coreless voice coil actuator without yoke, consisting of a cylindrical permanent magnet and the coil (Fig. 5). The electric current carried in the coil wire generates an electromagnetic force between the OSEM body and the magnet. The magnet is integrated into the sensor flag. The assembly of the sensor flag and the magnet is attached to the intermediate mass, while its counterpart, the OSEM body, is attached to the recoil mass for the intermediate mass [10] [11].

The electronic interface of the sensor and the actuator is a micro-D connector (plug/male; Glenair, GMR7590-9P-1BPN). The connector, the LED, the PD, and both ends of the coil wire are all soldered to a flexible printed circuit made of polyimide film for low outgassing rate and for a reasonable production cost. The thickness of the film is 50 μm, and electronic lines are printed with copper foil of 35 μm in thickness and 0.4 mm in width.

In operation, the maximum rating of current for the coil is set at 100 mA [5] to prevent damage of the coil from Joule heating (taking into account the experimental investigations in LIGO [9] [13] [14]). Note that the Joule heat cannot escape via convection in vacuum, where the OSEMs are used. Of the parts for the OSEM, the LED and PD are the weakest against heat; their maximum operating temperature is 100°C. Aluminum alloy used for the holders of the LED or the PD has a recrystallization temperature of ~150°C, and so we usually avoid leaving the material above ~120°C for safety margin, even during baking. A tiny amount of epoxy adhesive, Loctite Ablestik 2116 by Henkel, is used to fix the coil wire to the body, and has a maximum operating temperature of 130°C. As seen later, the coil resistance is ~11Ω, and the current of 100 mA on the resistance will cause Joule heat of ~0.1 W. Although no critical damage has been observed for the OSEMs in operation so far, the thermal design combined with the outgassing rate should be revisited elsewhere.

B. Sensor part

The transmitter (TX) and the receiver (RX) work as a shadow sensor together with the sensor flag (Figs. 3 and 5). The fundamental design of the TX and RX follow those of the Birmingham OSEMs (BOSEMs) of LIGO [7] [9] [15].

In our current design, the major difference from the previous prototype [10] is that the separation between the TX and RX has been widened. In the prototype, the opposing surfaces of the ends of the TX and RX stuck out into the central hole of the OSEM body, and the separation of the surfaces was as narrow as 5 mm. As the sensor flag had a 2-mm thickness (Fig. 5), there were only 1.5-mm gaps on each side of the flag when it was located at the nominal position. With this setup, there was a risk that the flag would touch and be knocked off by the TX or RX. This was a particular problem for the mirror and the intermediate mass, where the flags were attached with EP30 adhesive. Such accidents occurred several times during installation and commissioning activities before the initial test run of KAGRA and also in the off-site test. To mitigate the risk, the TX and RX were retracted in a redesign such that their faces are almost along the inner surface of the central hole in the current design; see the cross sectional view in Fig. 5. Currently, the separation is 15 mm (note that the inner diameter of the central hole itself is 15.5 mm), and the nominal gap around the flag is widened from 1.5 mm to 6.5 mm. As a drawback, the wider gap reduces light power arriving at the PD by about half. The effects are discussed later in this paper. As a further precaution, the flags for the intermediate recoil mass were changed to a magnetically self-assembling design; see the next section.

The TX encapsulates an LED in an aluminum structure, the LED carrier (Fig. 4), in which the LED is held by a sleeve made of machinable ceramic, Macor by Corning, for electric insulation. The LED is a TSTS7100 by Vishay in a TO-18 package, which emits light at 950 nm. An uncoated planoconvex lens made of BK7, 08PGQ6 by Comar Optics, is put in front of the LED for collimating the emitting light beam (Fig. 5). The LED and the lenses are fixed to the carrier with PEEK retainers. A mask with a slit of width 1.4 mm and length 4.6 mm is attached in front of the lens to limit the light
C. Actuator part

An electromagnetic coil serves as an actuator to exert a force between the OSEM body and a magnet on the sensor flag (Fig. 5). The permanent magnet, KE110 by Niroku, has a beam to the PD. The width is determined so that it gives a suitable linear range for the shadow sensor (Fig. 6), while the length is determined to be slightly larger than the PD photosensitive surface, which is a 3.4-mm square.

The RX encapsulates a PD in an aluminum structure, the PD carrier (Fig. 4), in which the PD is held by a sleeve made of Macor (Fig. 5). The PD is a S1223-01 by Hamamatsu Photonics in a TO-5 package, which has a response of about 0.58 A/W at the working wavelength of the LED, 950 nm. The PD is fixed to the carrier with a PEEK retainer. The PD carrier has an aperture 6 mm in diameter, which corresponds to the input window of the PD (5.9 mm in diameter). In the specification, the photosensitive surface of the PD is off-center by 0.3 mm; we tried to change the PDs used in the first generation to more suitable ones (such as BPX65 by Centronic), but unfortunately we were not able to. There are no apparent problems in operating the KAGRA interferometer so far, but if the need/opportunity for another round of redesign occurs, we will revisit the choice of PD.
cylindrical shape 10 mm in diameter and 10 mm in thickness, and is made of samarium-cobalt (SmCo) YKG28 with nickel plating [5]. The magnet is attached on the bottom of the sensor flag, which is made of aluminum alloy for non-magnetization.

The electromagnetic coil is a multilayer coreless solenoid formed with a single copper wire with polyimide coating, of magnet wire type NW16-C of the National Electrical Manufacturers Association (NEMA). A bare conductor in the wire has a diameter of 0.32 mm, corresponding to AWG-28, while the total outer diameter including the coating is 0.36 mm (B1282803 by MWS Wire Industries). The wire is wound on a bobbin structure with ∼ 600 turns in total, 22 turns per layer and 28 layers in design. The inner diameter and axial length of the coil are 18 mm and 8 mm, respectively.

In practice, fabrication errors are unavoidable for coil winding. For example, 8 mm/0.36 mm ≈ 22.2 turns, so the number of turns per layer in design is an approximation. Similarly, the outer diameter of the coil differs from a naively calculated value of 38.16 mm (18 mm plus twice 28 layers of 0.36 mm circles). The circular cross sections can be more closely packed within a limited area in the manner that the distance of each layer will be $\sqrt{3}/2$ of each circle diameter. Then, the minimum outer diameter of the coil is 35.46 mm in theory. In fact, the actual measured value is about 36.5 mm, which is close to midway between the two extreme cases. In addition, the outermost layer of the winding sometimes finishes in the middle or so.

From the design, the resulting inductance and resistance of the coil can be computed. Here, we assume 27.5 layers for the coil winding, taking such fabrication errors into account. For
the inductance, 8.9 mH is obtained by the method described in Appendix A. The total length of the wire for coil winding is expected to be ~ 53.0 m from summing up circumferences of every wire loop. From this number, 11.3 Ω is obtained for the resistance, where we use a value of resistance per meter 0.214 Ω/m at 20°C found in a specification document of the wire. The estimated inductance and resistance agree with our measurements as discussed later.

The coil former or the bobbin structure is made of carbon-loaded PEEK, Ketron CA30 (the old name is PK-450CA) by Mitsubishi Chemical Advanced Materials, with conductivity chosen as a trade-off to prevent unwanted charge-up by static electricity in the body while avoiding unwanted loss of kinetic energy from eddy currents in the bulk. Unwanted magnetisation of the coil former is also avoided. The specification document of this material, however, does not clearly show the lower limit of the electrical resistivity, one of the authors of this paper already mentioned that it was too conductive [13]. We look into this point further in the discussion section. We did not have a chance to change the coil formers, which were left unchanged or diverted from the ones used in the early days [10]. In contrast, the coil wires were changed from the thinner ones (0.20 mm diameter) used in the prototypes to optimize the control scheme [5][16].

While redesigning the OSEM, we also modified the flag to be magnetically self-assembling, so as to reduce the risk of damage in the event of them being bumped. The detail is out of the scope of this paper, and partly discussed in our internal document [17], so here we touch on it briefly. Instead of gluing the sensor flag directly to magnet, a thin SS400 steel disk (“magnet plate” in Fig. 5) is glued on the bottom of the flag. The steel disk then magnetically adheres to the magnet; the same treatment is done for the base between the magnet and the intermediate mass. Before the modification, during the installation and operation of the suspensions, the magnets could sometimes knock against the surrounding structures and be detached at the bonding areas. Once that happened, the recovery would take a long time, typically about one week, for cleaning the remnant of the bond and re-gluing. Thanks to the modification, we can easily reattach the flag tips and magnets even when they are accidentally knocked.

III. CHARACTERISING THE SENSORS

This section reports on the gain and the noise of the sensor part of the OSEMs. Note that sensitivity is a word widely used with different meanings; one is a ratio of the output with respect to the input, and another is like a minimum threshold value of the input above which the input apparently changes the output. Therefore, we will use simply gain and noise, respectively, to avoid the confusion.

A. Sensor gain

The gain of the sensor can be derived from the response of the sensor as a function of flag position. Before shipping OS-EMs to the KAGRA site, we measured the responses one by one with a testbench in a clean booth (Fig. 6). The testbench has a dummy sensor flag (Fig. 6(b)), with the same shape as the actual sensor flag shown in Fig. 5. The dummy flag is attached onto an XY translation stage with micrometers. We can insert or retract the dummy flag into or out of the OSEM under test by adjusting the micrometers.

Typical responses are shown in Fig. 6(a) for six samples out of the 43 units in total. The measurements are shown by empty or filled circles. The dashed lines indicate linear fits to the measurements, where the data subsets used for the fitting are shown by the filled circles. The estimated slopes

| S/N | Gain (V/mm) | R (Ω) | L (mH) |
|-----|-------------|-------|--------|
| 22  | 4.133 ± 0.034 | 11.33 | 8.40  |
| 23  | 3.800 ± 0.029 | 11.58 | 8.40  |
| 24  | 4.340 ± 0.021 | 11.30 | 8.40  |
| 25  | 4.876 ± 0.015 | 11.68 | 8.40  |
| 26  | 3.833 ± 0.060 | 11.66 | 8.44  |
| 27  | 3.983 ± 0.031 | 11.33 | 8.40  |
| 28  | 3.937 ± 0.053 | 11.38 | 8.45  |
| 29  | 3.867 ± 0.032 | 11.22 | 8.47  |
| 30  | 4.173 ± 0.032 | 11.34 | 8.45  |
| 31  | 4.007 ± 0.029 | 11.25 | 8.36  |
| 32  | 4.247 ± 0.056 | (no data) | (no data) |
| 33  | 4.420 ± 0.027 | 11.37 | 8.45  |
| 34  | 4.097 ± 0.053 | 11.45 | 8.37  |
| 35  | 4.510 ± 0.032 | 11.20 | 8.43  |
| 36  | 4.100 ± 0.029 | 11.17 | 8.39  |
| 37  | 4.330 ± 0.051 | 11.52 | 8.43  |
| 38  | 4.098 ± 0.046 | 11.23 | 8.47  |
| 39  | 4.029 ± 0.033 | 11.33 | 8.41  |
| 40  | 4.005 ± 0.048 | 11.31 | 8.41  |
| 41  | 4.085 ± 0.029 | 11.31 | 8.42  |
| 42  | 4.363 ± 0.019 | 11.29 | 8.45  |
| 43  | 4.027 ± 0.026 | 11.36 | 8.46  |
| 44  | 3.957 ± 0.048 | 11.31 | 8.48  |
| 45  | 4.078 ± 0.040 | 11.21 | 8.50  |
| 46  | 4.390 ± 0.023 | 11.30 | 8.10  |
| 47  | 4.288 ± 0.034 | 11.60 | 8.50  |
| 48  | 3.811 ± 0.099 | 11.35 | 8.40  |
| 49  | 4.225 ± 0.051 | 11.61 | 8.50  |
| 50  | 4.218 ± 0.033 | 11.28 | 8.50  |
| 51  | 3.965 ± 0.051 | 11.33 | 8.50  |
| 52  | 3.438 ± 0.016 | 11.63 | 8.40  |
| 53  | 4.071 ± 0.031 | 11.28 | 8.37  |
| 54  | 4.008 ± 0.082 | 11.62 | 8.39  |
| 55  | 3.766 ± 0.042 | 11.63 | 8.38  |
| 56  | 4.301 ± 0.055 | 11.31 | 8.38  |
| 57  | 4.659 ± 0.036 | 11.54 | 8.42  |
| 58  | 3.723 ± 0.060 | 11.58 | 8.42  |
| 59  | 4.040 ± 0.087 | 11.58 | 8.35  |
| 60  | 3.730 ± 0.028 | 11.51 | 8.32  |
| 61  | 3.732 ± 0.035 | 11.57 | 8.36  |
| 62  | 3.433 ± 0.025 | 11.57 | 8.37  |
| 63  | 3.799 ± 0.046 | 11.56 | 8.27  |
| 64  | 4.321 ± 0.008 | 11.60 | 8.35  |
FIG. 6. (a) Typical measured responses of the sensors with respect to the relative displacement of the tip of the dummy sensor flag. The vertical axis on the left indicates single-ended output voltage from a transimpedance amplifier in the driver circuit, while it is converted (divided by 38.3 kΩ) to the equivalent photocurrent into the circuit on the right. The measurements are shown by circles; the data subsets shown by filled circles are used for linear fit to estimate the sensor gains (V/mm in this figure); the dashed lines are the fit curves. The inset histogram shows the distribution of the gains of all the fabricated sensors. All the estimated gains are listed in Table I. (b) Schematic view of a clean-room compatible testbench for the OSEMs. The dummy sensor flag is attached onto the XY translation stage and movable with two micrometers.

or the gains are also shown in the legend along with the serial numbers assigned to each OSEM for identification. The vertical axis in the left indicates single-ended output voltage from a driver circuit for the sensors. The driver circuit of the sensor for the measurements was the same type as used for KAGRA. Note that the driver circuits are usually used in differential signaling at the site, but our measurements were done with single-ended for simplicity. For reference, equivalent photocurrent input to a transimpedance stage in the driver circuit is shown in the right vertical axis. The transimpedance is 38.3 kΩ, so the photocurrent can be obtained by dividing the measured voltages by this resistance.

The overall shape of the response curve is similar for all the sensors. The gain depends on the maximum photocurrent, which is obtained when the sensor flag does not occult the light beam at all. On the other hand, the linear ranges are independent from the maximum photocurrent values; every sample shows about 0.7 mm or more for the linear range.

All the estimated gains are listed in Table I and a distribution histogram of them is shown as an inset in Fig. 6(a). For the gains, the mean and the standard deviation are 4.09 and 0.288 V/mm, respectively, while the median is 4.08 V/mm. Note that the mean or median corresponds to ∼ 0.106 A/m in terms of the photocurrent. The distribution has a peak around the mean or median. The six data sets plotted as typical responses in Fig. 5 are randomly sampled from each bin of the histogram. Currently, 42 OSEMs out of them are installed in KAGRA.

B. Sensor noise

To estimate the sensor noise, we measured and analyzed the noise of the sensor built in a spare OSEM left off site, which was not installed at KAGRA. Fig. 7 shows one-sided amplitude spectra of several relevant measurements and some theoretical curves for the analysis. The vertical axis on the left of the figure indicates equivalent current input at the transimpedance opamp of the driver circuit, while it is converted to displacement noise on the right. The conversion factor (or gain) was measured as ∼ 0.106 A/m in the same manner as described in the previous subsection. All measurements in the figure were done in air, but the testbench was covered with an opaque shield to isolate it from ambient light and sound.

The solid orange curve “nominal” in Fig. 7 is a measurement corresponding to the noise output of the sensor when the sensor flag is fixed at the middle of the linear range of the response function, or at the nominal position. The spectrum indicates 0.48 nm/Hz$^{1/2}$ at 1 Hz, and 0.13 nm/Hz$^{1/2}$ at 10 Hz. Even though the separation between the TX and the RX is widened, and so the light power received by the PD decreases by half, the noise level is comparable to that of the LIGO BOSEMs [7, 15].

The nominal noise level can be explained by contributions from the intensity noise of the light source (LED) and the shot noise for the photocurrent. The solid red curve “full open” in Fig. 7 was measured when the sensor flag was extracted out of the OSEM central hole, so the photocurrent in the sensor out-
FIG. 7. One-sided amplitude spectral density of the sensor noise, and the relevant several measurements and theoretical lines. The vertical axis on the left is for showing the data in terms of the equivalent current input noise at the transimpedance stage of the sensor driver circuit, while they are converted to the output noise of the displacement sensor on the right.

The sensor output is dominated by the shot noise above 100 Hz. Below that frequency, it is dominated by the intensity noise of the LED down to ~0.1 Hz. In the interferometer, these sensors will only be used for detecting mechanical resonances in the low-frequency region. Because both the intensity noise and the sensor gain will increase linearly with increasing light power on the PD, the resultant calibrated sensor noise will not be improved in the low-frequency region by simply increasing the light power.

The solid blue curve “full closed” in Fig. 7 was measured when the sensor flag was inserted into the OSEM central hole well past the working point so that the PD was totally shaded by the sensor flag. The measurement corresponds to the noise contribution of the driver circuit. The dashed blue line shows a theoretical estimate of the circuit noise. The calculation includes noise contributions from the thermal noise of the transimpedance (38.3 kΩ and 100 nF in parallel), and the voltage and current noise of the opamp, which is the OP2177 by Analog Devices. Although the theoretical line is slightly lower than the measurement, it can explain the approximate magnitude and frequency dependence of the measurement above 10 Hz. Below that frequency, the measurement is contaminated by the noise from the measuring instrument, an Agilent 35670A (shown in the grey line). Above 100 Hz, the theoretical noise is dominated by the opamp noise, while thermal noise of the transimpedance dominates below the frequency.

For comparison and confirmation, we also ran a circuit simulator, LTspice by Linear Technology, to estimate the noise contribution of the driver circuit; see the dashed red line in Fig. 7. Unfortunately, the corner frequency of the flicker noise appears to be different between the spice model of the opamp provided by the company and its specification document. The discrepancy between the simulator result and the measurement (the solid blue curve) would happen due to this issue. According to the specification document, the corner frequency should be around 3 Hz, while the opamp Spice model shows it around 50 Hz. Note that, for simplicity, the flicker-noise behavior was not taken into account when calculating the theoretical noise shown in the dashed blue line.

If one would like to improve the noise level more at 1 Hz, where the contribution from the LED intensity noise is dominant, the intensity noise should be reduced. One way would be...
As mentioned in Section II, the design gives a resistance of 11.3 Ω and an inductance of 8.9 mH. For resistance, the designed value falls within the standard deviation of the measurements, and is close to the median as well. For inductance, the measurements agree with the designed value within 10%, but tend to be lower.

The reduction of inductance in the measurements is likely due to imperfections in the coil fabrication, apart from systematic errors in the measurements. In visual inspection, the wire winding of the coils is sometimes random. Moreover, the outermost layer of the winding finishes in the middle, as already mentioned.

So far, we have not found critical troubles related to these issues, and so keep things as they are. The current-carrying wire will expand or shrink in diameter under a magnetic field, and so the random winding would cause non-uniform stress to slip the wire loops, and that could become a noise source; the worst case would be breakage of the wire. For risk mitigation, the coil fabrication must be improved in any redesign; using square wires may be helpful.

### B. Electromagnetic forces

We measured the electromagnetic forces with respect to the current carried in the coil, and to the displacement of the magnet in the axial and radial directions (Fig. 9). For the measurements, we dedicated a spare OSEM left in our storage (but not included in Table I), together with three spare magnets of the same type as installed in KAGRA (see Section II).

Fig. 9 (d) shows a schematic view of the measurement setup. The coil in the OSEM under test is located over the cylindrical permanent magnet under test. The coil is supported by a bar from the XZ translation stage with micrometers on a rigid structure. By adjusting the micrometers, the relative position of the coil and the magnet can be adjusted in the radial and axial directions. The ends of the coil wire are connected to a DC power supply, P4K-80L by Matsusada Precision. The electromagnetic force arising on the coil and the magnet was measured by an electronic balance, BL-220H by Shimadzu. The magnet was raised on a handmade plastic stage to keep it well separated from the top surface of the balance. Without that distance, the magnet and the coil came too close to the electronic balance, and the magnetic field from the coil affected the reading of the balance, making the measurements of the electromagnetic force inaccurate.

A user-friendly actuator must have linearity from input to output. In Fig. 9 (a), the filled circles show the measurements of the electromagnetic forces arising from the current carried into the coil, while the dashed lines are the fitted curves. For every combination, the measured points can be fit by a linear function, and the coefficient is ~ 1N/A; see Table I. As seen below, the force arising also depends on the displacement of the magnet relative to the coil in the axial and radial directions. Before measuring each combination, each time we adjusted the gap between the coil and the magnet in the axial and radial directions to locate the magnet at a “sweet spot”, where the coefficient becomes insensitive to those displace-
FIG. 9. Variations of the electromagnetic forces arising between a spare OSEM and three spare magnets in the same shape and material (SmCo); the forces with respect to (a) the current carried in the coil, (b) the displacement of the magnet in the axial direction, and (c) that in the radial. The filled circles show the measurements. In (a) and (c), fitted curves are drawn by the dashed lines; see also Table II. The solid blue curves in (a) and (b) are theoretical estimates from the design but not fitted. A schematic view of the measurement system is shown in (d).

TABLE II. Fit parameters for each curve drawn in Fig. 9(a) and (c). Note that each set of measured points in (c) is offset along the horizontal axis to minimize the fitted quadratic curve at $x = 0$.

| Coefficients | #1          | #2          | #3          |
|--------------|-------------|-------------|-------------|
|              | Force/Current |             |             |
| Fit curve: $y = ax + b$ | 1.01, −0.38 | 1.02, −0.37 | 1.01, −0.35 |
| Fit curve: $y = ax^2 + b$ | (a: mN/mm$^2$, b: mN) |             |             |
| Force/Radial disp. | 0.82, 90.1  | 0.78, 91.3  | 0.80, 90.9  |

The solid blue line in Fig. 9(a) shows an estimate from the design of the coil and a magnetic flux density at the surface on the end of the cylindrical magnet, $B_s = 409$ mT, which was measured with a teslameter, TM-801 by Kanetec. From this number, we can assume a point magnetic dipole moment equivalent to the cylindrical magnet; $m_z = 0.57$ Am$^2$ being coaxial with the cylinder; see Appendix B. By following the method written there, the theoretical estimate of the current-to-force coefficient is $\sim 1.102$ N/A. As in Table II the coefficients obtained from the measurements are consistent with the theoretical estimate.

In order to find the sweet spot for each combination, we measured the electromagnetic forces while varying the distance between the coil and the magnet being on-axis in the axial direction; see Fig. 9(b). The current carried into the coil was fixed at about 90 mA; here, 1 V was applied to the coil with a power supply, and the resistance of the coil was measured as 11.1 $\Omega$ for this OSEM under test. In this figure, the horizontal axis shows the distance between the center points of the cylindrical magnet and the coil; for example, 0 mm means the magnet is located at the center of the coil. For every combination, the sweet spot, where the maximum force arises, is found at 6.6-6.7 mm.

The solid blue curve in Fig. 9(b) shows an estimate from the dipole equivalent to the cylindrical magnet discussed already. By a close look at the curve, the sweet spot can be found at $\sim 6.8$ mm without relying on numerical search algorithms. As in Appendix B, an analytic form of the curve can be derived, but it is difficult for us to find the sweet spot as an analytic solution. Despite a number of degenerated parameters in the analytic form of the curve, the measurements show a good agreement with the theoretical estimate without curve fitting.

Similar measurements in the radial direction are shown in Fig. 9(c). We used a quadratic curve (empirically) for fitting every combination of the coil and the magnets, as the system is rotationally symmetric; see also Table II. In the figure, each set of measured points is offset in the horizontal axis so that the fitted curve is centered at 0 mm. In fact, we observed that the electromagnetic force was minimized when the magnet came onto the axis of the coil (i.e. at 0 mm). The cylindrical magnet has a diameter of 10 mm, while the central hole of the coil is 15.5 mm in inner diameter (Figs. 3 and 5), so the magnet can only move 2.75 mm at most in the radial di-
discovered thanks to the narrow separation. Unwanted off-centering of the magnet can be easily discovered thanks to the narrow separation.

V. DISCUSSIONS

A. In comparison with the previous design

This subsection compares the responses of the previous and current design of the optical sensors in the OSEMs. As mentioned already, we widened the separation between the TX and the RX from 5 mm to 15 mm after the initial test run of KAGRA, mainly for risk mitigation, and so the nominal gaps between the sensor flag and each of those units increased from 1.5 mm to 6.5 mm (Fig. 5).

As such, the modification was done without changing the OSEM body because we wanted to reuse as many parts as possible from the previous OSEMs [10]. If we had a chance to design the OSEM body from scratch, we will revisit the whole mechanical design including the diameters of the holes for the TX and RX.

Before mass production of the current OSEMs, we prepared prototypes, and compared them with the previous design. Fig. 10 shows the responses of the current prototype and the previous sensor (labeled “wide” and “narrow”, respectively) in the same manner as in Fig. 6 (a). For the measurements, we used a handmade test circuit (single-ended signaling) rather than the driver circuit used in KAGRA. In the test circuit, the LED driving current is adjustable with a variable resistor, and so we can vary the brightness of the LED. Note that the gain-setting resistors in the KAGRA driver circuits are non-variable and would need to be replaced to adjust the gain. For normalizing the responses of the two designs, we adjusted the LED driving current in each measurement so that the readout voltage became equal with each other at the fully open location of the sensor flag (about 9.6 V at around 3 mm in Fig. 10). The full-open readout from the “wide” prototype was half that of the “narrow” one without the normalization.

The dashed lines in Fig. 10 are linear fits to the data from the two measurements in this normalized condition, and the slopes are $8.90 \pm 0.13 \text{ V/mm}$ and $8.76 \pm 0.14 \text{ V/mm}$ for the “wide” prototype and the “narrow” one, respectively. They are within the margin of error from each other.

The 50% reduction is due to the increased separation between the TX and RX. In contrast, the normalized responses do not vary. This invariance is partly due to the tight collimation of the light beam from the LED (discussed later). So far, we have driven the LEDs in the current OSEMs at the site without compensating for the reduction. If compensation is needed in the future, we will modify the driver circuits.

B. Insensitivity to motion in other directions

We had a concern that the wider separation of the TX and RX in the sensor would increase couplings from the motions in unwanted other degrees of freedom. Thus, we evaluated the coupling coefficients with a prototype of the current OSEM sensor before the mass production and then confirmed they were negligible.

Fig. 11 shows the measured variation of the output voltages of the sensor prototype “wide” in response to motions in the other degrees of freedom; their fitted curves are also drawn. Estimates of the coupling coefficients from the fitted curves are summarized in Table III. As in the case of Fig. 10, we measured them with one of the previous “narrow” OSEMs as well, and the measurements are also shown in Fig. 11. For the measurements, the handmade test circuit introduced in the previous subsection was used, and so the “wide” curves are normalized to those of “narrow”. In addition, the measurements shown in Fig. 11 can be directly compared with those in Fig. 10.

For the measurements, we prepared a dedicated testbench (Fig. 12), with which we can move the dummy sensor flag in the transverse, vertical, roll, pitch, or yaw directions, as well as the (nominal) longitudinal direction; see the figure for the definition of the directions.

The measurements were done while fixing the flag tip at three different longitudinal locations, A, B, and C in Fig. 10. The data set collected at each of the locations is indicated with the corresponding letter, A, B, or C in Fig. 11. B is the nominal location of the sensor. A and C correspond to the upper and lower outer edges of the linear range, respectively.

As summarized in Table III, the responses to transverse, vertical, and roll motions were evaluated with linear fits, while quadratic curves were chosen for pitch and yaw; these were judged by eye to be the minimum degrees of polynomial to give a good fit.

For the vertical motion, every data set shows almost no slope. It is at most 0.02 V/mm in absolute value, which is
TABLE III. Fit parameters for each curve drawn in Fig. 11. The responses to transverse, vertical, and roll motions are fitted to a linear function \( y = ax + \text{Const.} \), and estimates of the slopes are shown. For pitch and yaw, quadratic curves \( y = ax^2 + bx + \text{Const.} \) are chosen for fit; note that the slopes at \( x = 0 \) correspond to \( b \) for such quadratic curves.

| Degrees of freedom | Set A | Set B | Set C |
|--------------------|-------|-------|-------|
|                    | Fit curve: \( y = ax + \text{Const.} \) (\( a \): V/mm or V/deg) | Fit curve: \( y = ax^2 + bx + \text{Const.} \) (\( a \): V/deg\(^2\), \( b \): V/deg) |
| Transverse         | Narrow: -0.125(23) | -0.066(15) | 0.108(6) |
|                    | Wide: -0.337(20) | 0.032(11) | 0.240(15) |
| Vertical           | Narrow: 0.0000(0) | -0.022(36) | 0.0111(31) |
|                    | Wide: -0.0057(33) | 0.0200(61) | 0.0149(14) |
| Roll               | Narrow: -0.0016(9) | 0.0013(3) | 0.0024(6) |
|                    | Wide: -0.0016(6) | 0.0010(9) | 0.0026(2) |

For the transverse motion, the estimated slopes vary greatly depending on the longitudinal locations of the flag tip. The slope at A is 10 times larger (in absolute value) than that of negligible in comparison with the response in the longitudinal direction of \( \sim 8 \) V/mm. In other words, the coupling coefficient from vertical to longitudinal can be calculated as \( 0.02/8 \approx 2 \times 10^{-3} \) mm/mm. The flag tip is 9 mm in vertical width, which is larger than the vertical width of the intensity profile of the light beam (about 7 mm; see (e) or (f) in Fig. 13), so it is unsurprising that vertical shift of the sensor flag does not change the shading of the light beam.

The same explanation can be applied to the roll motion. Even a 20-deg rolling of the sensor flag would change the vertical tip width seen from the RX by \( \cos 20^\circ \), which corresponds to 8.5 mm, and is still larger than 7 mm. For the pitch motion, whether the flag rotates in positive or negative, the tip shades the light beam in the same manner, and so the responses show even-function behaviors like quadratic. In the case of the quadratic form like \( y = ax^2 + bx + c \), the slope at \( x = 0 \) is \( b \). Thus, the pitch response around the nominal setup is about or less than 0.03 V/deg according to Table III, and the coupling coefficient from pitch to longitudinal can be calculated as \( \sim 3 \times 10^{-3} \) mm/deg or 0.2 mm/deg. Note that the possible rotation in pitch or yaw would be less than 0.2 rad due to mechanical limiting of the OSEM central hole.

A similar explanation can be applied to yaw motion. One exception is that the minimum point of each of the quadratic curves slightly varies with respect to the longitudinal locations of the sensor flag tip (A, B, and C). We are still not sure of the reason but those variations would be related to the responses in the transverse motion.

For the transverse motion, the estimated slopes vary greatly depending on the longitudinal locations of the flag tip. The slope at A is 10 times larger (in absolute value) than that of
B for the “wide” design. By comparing the behaviors of the “wide” and “narrow” OSEMs at A and B, the slopes are two or more times larger (in absolute value) for the “wide” than that for the “narrow”. We are still not sure what is behind these strange behaviors. They might be related to non-uniform cross-sectional profile of the light beam from the LED (see Fig. 13 and the related discussion there). So far, the couplings are so small that they are not practically problematic for KAGRA, but further investigation could be required in the future.

C. Lens and slit in the TX

We need the lens in the TX of the sensor to make the emission profile of light from the LED smooth and uniform, as in the LIGO OSEMs [7, 18]. A non-uniform profile will distort the linear response of the sensor. We measured the emission profiles of the LEDs in the TX prototypes under various conditions (Fig. 13). For the measurements, we used a beam profiler, CinCam CMOS-1202 by Cinogy, which has an active area of 6.8 mm × 5.4 mm.

In Fig. 13 (a), (b), and (c) show the emission profiles measured at distance of 12 mm, 22 mm, and 26 mm, respectively, from the LED; (a) and (b) were obtained from the identical LED, but (c) was not. For the measurements, we detached the built-in lens and the slit lid from the TX (see Figs. 4 and 5). The spatial profile departs from a uniform shape to a spotted square-like shape with increasing distance. Looking closely at the center of (a), one can find a seed of such a spotted square pattern. The spotted square indicates an image of the emitting chip and the shadows of wires to it inside the LED package. In the current design, the nominal separation from the LED to the sensitive area of the PD is 28.7 mm. If we let the profile propagate, the non-uniform light illuminates the PD, and cut out unnecessary light. The drawback of the slit aperture is apparent in (e); the light profile is unexpectedly distorted. So far, it has not distorted the linear response of the sensors (see Fig. 6), but further investigation could be required in the future.

D. Eddy current loss

Eddy current loss in the actuator body can be a severe noise source for the suspensions [19]. As was mentioned above, the OSEM body is made of carbon-loaded PEEK, Ketron CA30. Although this type of material usually shows very high electrical resistivity $\rho \sim 10^3 \Omega \cdot m$, our measurements show a variety. In LIGO, they also found this material was too conductive [13], and changed to a more suitable one, ESD 480 by Semitron, which has a clear description of the lower limit of the resistivity in the specification. Thus, let us quantify the energy loss and evaluate them.

Already in the early days of the GW detectors [20], one could not neglect the eddy current loss of coil-magnet actuators in vibration-isolation systems for optics. The loss in the actuator will cause unwanted viscous damping, and so reduce the vibration-isolation ratio. In addition, the loss will introduce thermal noise into the system. The noise process has been usually explained by the fluctuation-dissipation theorem [21, 22]. Hereafter we will take the dynamic model in Eqs. (1) as an example for discussing the eddy current loss.

Like the other resonant systems, the loss in a pendulum can be characterized in terms of $Q$-factor at the resonance or loss angle $\phi$ [23, 24].

$$Q_1 = \frac{m_1 \omega_1}{c} = \frac{1}{2 \xi_1},$$

$$\phi = \frac{\omega}{Q_1 \omega_1},$$

where the notation follows Eqs. (1). Usually, we aim to retain $Q \sim 10^5$ or more for our pendulums when freely swinging. By assuming $m_1 \sim 1 \text{ kg}$ and $\omega_1 \sim 2 \pi \times 1 \text{ Hz}$, the net amount of the...
viscous damping coefficient $c$ should be $\sim 6 \times 10^{-5} \text{N/(m/s)}$ or less. One of the components of $c$ comes from the eddy current loss, and the theoretical estimate can be calculated with Eq. (C5).

For the calculation, let us assume the magnetic dipole $m_z$ to be the value we discussed in the previous section, and the electric resistivity of the bobbin material to be $\rho \sim 10^3 \Omega \cdot \text{m}$ from the specification. In Eq. (C5), taking the integral limits as $(r_1, r_2) = (7.75, 9.0) \text{mm}$ and $(z_1, z_2) = (-0.3, 41.7) \text{mm}$, we obtain $c \sim 3 \times 10^{-12} \text{N/(m/s)}$, which is well below the target value. In fact, the integration range is not so obvious especially for $r_2$, as the actual bobbin has a complicated structure. The integral, however, converges rapidly as increasing $r_2$, and the resultant $c$ will be at most $\sim 8 \times 10^{-12} \text{N/(m/s)}$; the order of magnitude does not change so much.

The OSEM body had almost no conductivity as expected, but we found out that low resistances were sometimes observed between small screw holes on the OSEM body, while relatively wider surfaces were well insulated. We do not fully understand the origin of the issue yet.

Suppose $\sim 90 \Omega$ was measured between such two screw holes at 1 cm distance. Estimating an effective cross section of the current path is difficult but let us assume a $\sim 1$ cm diameter circle. This leads $\rho \sim 0.7 \Omega \cdot \text{m}$. In fact, the specification for the material just mentions the upper limit of the resistivity such as $< 10^3 \Omega \cdot \text{m}$, but not for the lower limit, and so literally the measurement here satisfies the specification. As in Eq. (C5), however, $c$ is proportional to $\rho^{-1}$, and so it increases to $c \sim 10^{-9} \text{N/(m/s)}$. From the viewpoint of thermal noise due to the eddy current loss, the large $c$ might be problematic as discussed below.

In the dynamic model in Eqs. (1), the thermal noise contribution at the suspended mass can be calculated as (21):

$$S_z = \frac{4k_B T}{\omega^2} \Re [Z^{-1}], \quad \text{(6)}$$

where $k_B$ is the Boltzmann constant, $T$ is the temperature of the system, and $Z$ is the mechanical impedance of the suspended mass. Note that $S_z$ is a one-sided amplitude spectrum density of the suspended-mass fluctuations due to thermal noise. To derive $Z$, assuming a fluctuation force $F_1$ on the suspended mass, we can rewrite Eqs. (1) as

$$m_1 \ddot{x}_1 + c(\dot{x}_1 - \dot{x}_2) + k_1 x_1 = F_1, \quad \text{(7a)}$$

$$m_2 \ddot{x}_2 + c(\dot{x}_2 - \dot{x}_1) + k_2 x_2 = 0. \quad \text{(7b)}$$

In the same way as from Eqs. (1) to (3), we can obtain

$$\frac{\dot{x}_1}{F_1} = \frac{1}{k_1} \frac{1 + 2i \xi_1 \frac{\omega}{\omega_1} - \frac{\omega^2}{\omega_1^2}}{(1 + 2i \xi_2 \frac{\omega}{\omega_2} - \frac{\omega^2}{\omega_2^2})(1 + 2i \xi_1 \frac{\omega}{\omega_1} - \frac{\omega^2}{\omega_1^2}) + 4 \xi_1 \xi_2 \frac{\omega}{\omega_1 \omega_2}} \quad \text{(8)}$$

in the frequency domain. From this relation, the mechanical impedance is derived as $Z = F_1 / (i \omega \dot{x}_1)$.

In Fig. 14, the solid curves show $S_z$ for two different $\rho$; we assume the temperature $T = 300 \text{K}$, the masses $(m_1, m_2) = (1, 0.5) \text{kg}$, and the resonant frequencies $(f_1, f_2) = (13) \text{Hz}$, respectively. The noise spectra are proportional to $\sim f^{-2}$ above $f_1$. The dashed lines show the asymptotes $S_z \sim (4k_B T \omega_1^2 / (m_1 \omega_1^4 \omega^4))^{1/2}$.

As already mentioned, full OSEMs are only installed at the penultimate stage of the pendulums at KAGRA today, so their thermal noise will not significantly affect fluctuations of the mirror at the lowest stage. However we also need to consider the solenoid actuators attached to the recoil mass for the (room-temperature) mirrors, which are made of the same material; this point will be discussed in detail elsewhere.

VI. CONCLUSIONS

In this paper, we have developed a compact module integrating an optical sensor and an electromagnetic actuator (OSEM). The module is especially for vibration-isolation systems in the GW detector KAGRA, where 42 are in operation without apparent issues as of 2020. For risk mitigation, the sensor part was modified after the initial test run of KAGRA in 2016, according to the lessons we learned during the run. The modification is to widen the spacing of the sensor slot from 5 mm to 15 mm to avoid the risk of mechanical interference with the sensor flag in operation. In order to investigate the effect on the module performance due to this modification, we characterized the modified module. The sensor noise is about 0.5 nm/Hz$^{1/2}$ at 1 Hz, and 0.1 nm/Hz$^{1/2}$ at 10 Hz. The actuation coefficient is 1 N/A. The responses of the sensor and actuator in the extraneous degrees of freedom are negligible. Some potential concerns in the design including eddy current loss of the module body are also discussed.
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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Appendix A: Inductance of a multilayer solenoid

To calculate the inductance of a coil, let us assume each turn of the coil to be an individual closed loop of conductor. The coil can be approximated as an accumulation of such loops.

In practice, self-inductance $L$ of the coil can be estimated by a summation:

$$L = \sum_{i=1}^{N} L_i + \sum_{i,j=1}^{N} M_{ij}, \quad (A1)$$

where $N$ is the total number of the loops, $L_i$ is the self-inductance of each loop for $i = 1 \ldots N$, and $M_{ij}$ is the mutual inductance between the loops $i$ and $j$. This is a summation of the self-inductance of every loop and the mutual inductance of all the possible combinations among the loops.

The mutual inductance is calculated by Neumann’s formula. According to the formula, mutual inductance of a pair of loops in a space with magnetic permeability $\mu$ is given by

$$M_{ij} = \frac{\mu}{4\pi} \oint_{C_i} \oint_{C_j} \cos \theta ds_i ds_j l \quad (A2)$$

where $ds_i$ and $ds_j$ are line elements along the closed loops $C_i$ and $C_j$, respectively; $l$ and $\theta$ are the distance and angle between $ds_i$ and $ds_j$, respectively. Note that $M_{ij} = M_{ji}$.

In the case of the multilayer solenoid for our purpose, every loop is circular and aligned along the same axis, and so Eq. (A2) reduces to

$$M_{ij} = \mu \sqrt{r_i r_j} \left[ \left( \frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right], \quad (A3)$$

where $r_i$ and $r_j$ are the radii of the loops $i$ and $j$, respectively. Note that $K(k)$ is the complete elliptic integrals of the first kind and $E(k)$ is the complete elliptic integrals of the second kind in the following forms:

$$K(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \phi)^{-1/2} d\phi, \quad (A4a)$$

$$E(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \phi)^{1/2} d\phi. \quad (A4b)$$

In the same manner, the self-inductance of each loop, $L_i$, can be calculated by Eq. (A3) with $i = j$, but $d$ should be replaced with a geometric mean distance $d = r_w \exp(-1/4)$, where $r_w$ is a cross-sectional radius of the conductor in the electric wire.

We wrote a simple Python script to compute $L$ in Eq. (A1). For computation of $M_{ij}$, Eq. (A3) is transformed to

$$M_{ij} = \mu \sqrt{r_i r_j} \frac{2}{\sqrt{k_1}} (K(k_1) - E(k_1)), \quad (A5)$$

where the new modulus $k_1 \equiv (1 - k')/(1 + k')$ and $k' \equiv \sqrt{1 - k^2}$, otherwise the integrand in Eq. (A4a) would not converge when $d \to 0$. Our script computed $K(k_1) - E(k_1)$ by the arithmetic-geometric mean for quick convergence.

Appendix B: Electromagnetic force by a solenoid actuator

In this section, we summarize useful formulae for designing a solenoid actuator, especially for estimating the electromagnetic forces arising between a multilayer solenoid and a cylindrical magnet. The detail is out of the scope of this paper and to be reported somewhere.

Suppose a point magnetic dipole moment $\vec{m}$ at rest is in an external field of magnetic flux density $\vec{B}_{\text{ext}}$. Then the field exerts an electromagnetic force $\vec{F}$ on the dipole moment as

$$\vec{F} = \nabla (\vec{m} \cdot \vec{B}_{\text{ext}}) - \frac{1}{c_0^2} \frac{d}{dt} (\vec{m} \times \vec{E}_{\text{ext}}) \quad (B1)$$

$$= (\vec{m} \cdot \nabla) \vec{B}_{\text{ext}}, \quad (B2)$$

where $c_0$ is the speed of light in vacuum, and $\vec{E}_{\text{ext}}$ is an external electric field that can be taken to be zero for our application; the complicated argument on “hidden momentum” in the formulae for force is out of the scope of this paper.

Let us take the cylindrical coordinates $(r, \theta, z)$ with basis vectors $\{\vec{e}_r, \vec{e}_\theta, \vec{e}_z\}$, as the actuator, consisting of the coil and magnet, is cylindrically symmetric in the nominal setup; let us align the $z$-axis onto the common axis of the coil and the
magnet. Suppose the magnetic dipole is a constant: \( \vec{m} = m_e \epsilon_z \). Then the force can be calculated as \( \vec{F} = \nabla (\vec{m} \cdot \vec{B}_{ext}) = \nabla (m_e B_z) \), and so

\[
\vec{F} = m_e \left( e_r \frac{\partial B_z}{\partial r} + e_\theta \frac{\partial B_z}{\partial \theta} + e_z \frac{\partial B_z}{\partial z} \right),
\]

where \( B_z \) is the \( z \)-component of \( \vec{B}_{ext} \). Our main concern in this system is the \( z \)-component of \( \vec{F} \). Specifically,

\[
F_z = m_e \frac{\partial B_z}{\partial z}.
\]

Note that the force can be also calculated as a reaction force, which is exerted on the current carried in the coil under an magnetic field made by the cylindrical magnet.

The on-axis field of a solenoid coil can be calculated by integrating all the contributions from every circular current loop in the coil, \( B_z = \mu_0 I_a (a^2 + z^2)^{-3/2} / r \), where \( \mu_0 \) is the magnetic permeability in vacuum, \( a \) is a radius of each loop, and \( I \) is the current carried in the wire. The location \( z \), where \( B_z \) is evaluated, is measured from the center point of the loop.

The on-axis field \( B_z \) of a single-layer \( N \)-turn solenoid having a radius of \( a \) and a length of \( L \) can be found in some textbooks as \[33\,37\]

\[
B_z = \mu_0 \frac{IN}{2L} \left( C(z + \frac{L}{2}, a) - C(z - \frac{L}{2}, a) \right),
\]

where \( C(x,a) = x / (x^2 + a^2)^{1/2} \), and the origin of \( z \) is at the center point of the coil.

In the same manner, the on-axis field \( B_z \) of a multilayer solenoid can be calculated as \[37\,38\]

\[
B_z = \mu_0 \frac{NI}{2L} \left( C(z + \frac{L}{2}, a) - C(z - \frac{L}{2}, a) \right),
\]

where \( a_1 \) and \( a_2 \) are the inner and outer radii of the coil, respectively. Note that \( N \) still represents the total number of turns of the wire; we consider a tiny area element \( dN \) in the winding-wire region, where the number density of the wire turns was \( dN = N dr dz / (L(a_2 - a_1)) \), and so the corresponding current was \( I dN \) in the area element.

Deriving the field gradient \( \frac{\partial B_z}{\partial z} \) is trivial, but the resultant analytic form is complicated, so we do not show it here; recent computers can draw a graph of the analytic form quickly. Note that \( B_z \) becomes almost flat within the coil \((|z| < L/2)\), and so \( \partial B_z / \partial z \sim 0 \). In addition, \( \partial B_z / \partial z \) has two peaks of opposite sign around ends of the coil, \( z \sim \pm L/2 \); because \( B_z \) is even, the derivative is odd. We could obtain \( z \)-locations of these peaks by solving \( \partial^2 B_z / \partial z^2 = 0 \), but that is difficult in analytic form.

A cylindrical magnet having a radius of \( R \) and a height of \( h \) provides an on-axis field \( B_z \) at a location \( z \) away from the center point of the magnet as \[39\]

\[
B_z = \frac{1}{2} \mu_0 M_z (C(z + \frac{L}{2}, R) - C(z - \frac{L}{2}, R)),
\]

where \( M_z \) is the magnitude of a uniform magnetization \( \vec{M} = M_z \epsilon_z \) of the magnet. The net magnetic flux density near the magnet is \( \vec{B} = \mu_0 (\vec{H} + \vec{M}) \), where \( \vec{H} \) is an external magnetic field, if any. Let \( \vec{H} \) be 0 hereafter. Then the magnet provides remanence, or residual magnetic flux density: \( \vec{B}_r = \mu_0 \vec{M} \). We can measure the magnetic flux density of the surface on one end of the cylindrical magnet, \( B_z \), with a teslameter. Equating the measured \( B_z \) to Eq. \( \text{(B7)} \) at \( z = h/2 \), we can obtain an estimate of \( M_z \):

\[
B_z = \mu_0 M_z = 2B_s (1 + (R/h)^2)^{1/2}.
\]

One can compare the estimate with a specification for the consistency provided by the magnet company. Finally, using the definition \( M_z = m_e / V \), where \( V = \pi R^2 h \) is the volume of the magnet, we can obtain an estimate of \( m_e \) as

\[
m_e = 2B_s \pi R^2 (h^2 + R^2)^{1/2} / \mu_0.
\]

The force in Eq. \( \text{(B5)} \) can be calculated by combining \( \partial B_z / \partial z \) from Eq. \( \text{(B6)} \) and the estimate of \( m_e \) in Eq. \( \text{(B9)} \).

Note that this estimation method uses Eq. \( \text{(B7)} \) to involve the measured \( B_z \). If we use the field formula for a magnetic dipole at the origin \[33\], or Eq. \( \text{(C5)} \), instead of Eq. \( \text{(B7)} \), then, instead of Eq. \( \text{(B9)} \), the estimate would be \( m_e = 2B_s \pi (h/2)^3 / \mu_0 \) by substituting \( z = h/2 \) and \( r = 0 \) into the dipole field; this loses the information of the magnet radius \( R \). Eq. \( \text{(B7)} \) reduces to the dipole field only when \(|z| \gg h \) and \(|z| \gg R \). In fact, the estimates with Eq. \( \text{(B9)} \) matched better with the measured forces in our case.

**Appendix C: Eddy current loss in a coil bobbin**

In this section, we will review how to estimate a viscous damping coefficient due to eddy current loss in the coil bobbin of a solenoid actuator. A similar discussion can be found in several documents \[24\,25\,26\,40\], but we want to summarize them into a short article.

Suppose a magnet and an electric conductor move slowly relative to each other. A magnetic flux \( \Phi \) from the magnet induces electromotive force \( \mathcal{E} \) in the conductor: \( \mathcal{E} = -d\Phi / dt \). Then let us consider magnetic flux across an area \( S \) in the conductor, \( \mathcal{F} = \int_S \vec{B} \cdot ndS \), where \( \vec{B} \) is the magnetic flux density, and \( n \) is the unit normal vector at the surface on \( S \). The elec-
The electromotive force induced in $S$ is written as [33]

$$\varepsilon = -\frac{d}{dt} \int_S \vec{B} \cdot \vec{n} dS = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{n} dS - \int_C (\vec{B} \times \vec{v}) \cdot d\vec{l}, \quad (C1)$$

where $C$ represents a contour of $S$, $d\vec{l}$ is a line element along $C$, and $\vec{v}$ is the velocity of $C$. Hereafter we will look at the system from the rest frame of the magnet, and so the first term of the right-hand side of Eq. (C1) becomes 0; only the second term, the contour integral along $C$, is left.

Let us consider the system consisting of the cylindrical magnet and the coil bobbin of the solenoid actuator and take the cylindrical coordinate in the same manner as in Appendix [B]. The coil bobbin moves in the $z$ direction, and so $\vec{v} = v \hat{e}_z$. The line element can be written as $d\vec{l} = dl \hat{e}_l$. Assuming $\vec{B} = B_r \hat{e}_r + B_\theta \hat{e}_\theta + B_z \hat{e}_z$ comes from a magnetic dipole, we obtain $B_\theta = 0$ and the other components as [33]

$$B_r = \mu_0 \frac{3m}{4\pi} \frac{r}{(r^2 + z^2)^{5/2}}, \quad (C2)$$

$$B_z = \mu_0 \frac{m}{4\pi} \frac{r^2 - z^2}{(r^2 + z^2)^{5/2}}, \quad (C3)$$

where the notation is the same as in Appendix [B], the origin of the coordinate is at the center point of the magnetic dipole. Substituting these into Eq. (C1) and noting that $e_r \times e_z = -e_\theta$, the electromotive force is $\varepsilon = 2\pi r B_r v_z$, where $r$ is a radius of the circular path for the contour integral. The electromotive force arises when a circular eddy current is induced around the central hole in the coil bobbin.

Consider a thin toroidal structure with a rectangular cross section in the coil bobbin at a radius of $r$ from the $z$-axis; the cross section is $dr dz$ and the circumference of the center of the toroid is $2\pi r$. The resistance of the toroid is $R = \rho \pi r^2/(dr dz)$, where $\rho$ is the volume resistivity of the material of the bobbin. The Joule heating by the eddy current carried in the toroid is $dP = \varepsilon^2/2R$. Integrating $dP$ over the effective region of $r$ and $z$, the net energy loss by the eddy current is $P = 9\mu_0^2 m_r^2 \pi^2 D/(8\pi \rho)$, where

$$D = \int_{r_1}^{r_2} \int_{z_1}^{z_2} \frac{r^2 z^2}{(r^2 + z^2)^2} dz dr. \quad (C4)$$

Here $z_1 < z_2$ and $0 < r_1 < r_2$. Note that the integrand goes to zero rapidly if $r, z \to \pm \infty$; the lower limits $r_1$ and $z_1$ would mostly determine the value of $D$. Deriving an analytic form of $D$ is trivial [41], but the form is complicated, so we do not show it here. Today, a numerical integral is sufficiently fast and easily coded using suitable libraries such as `scipy`, and coding the numerical integral is likely less error-prone than coding the complicated analytic form. In practice, the coil bobbin and the surrounding structures are not exactly cylindrical, so the calculation shown here is merely an approximation.

From Eq. (1), the total mechanical energy of the two-body system is $E = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} k (x_1 - x_2)^2$. Then, the rate of change of the total energy with respect to time is calculated as $\frac{dE}{dt} = -c(\dot{x}_1 - \dot{x}_2)^2$. Equating the dissipated power $-dE/dt$ corresponding to $P$, we obtain a viscous damping coefficient $c = P/\dot{L}^2$, or

$$c = \frac{9\mu_0^2 m_r^2 \pi^2 D}{8\pi \rho} \quad (C5)$$

due to the eddy current loss. If needed, one would include a term of viscous damping force with this coefficient into the equations of motion under consideration [42].

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