Helioseismic Tests With the FLASH Simulation Code

Pedro A. González-Morales\textsuperscript{1}, Rekha Jain\textsuperscript{2} and Michael J. Thompson\textsuperscript{3}
\textsuperscript{1,2}Dept. Applied Maths at University of Sheffield (UK), \textsuperscript{3}High Altitude Observatory, Boulder, CO (USA).
E-mail: \textsuperscript{1}pedro.gonzalez@sheffield.ac.uk, \textsuperscript{2}r.jain@sheffield.ac.uk, \textsuperscript{3}mjt@ucar.edu

Abstract. We show our first results from local helioseismic simulations using the numerical code FLASH by testing its suitability for simulating subphotospheric wave motions in helioseismology. In order to check the capability of this code for different sources of waves, we have implemented a non-magnetic plane-parallel atmosphere adding a source term to the energy equation. We confirm the capabilities of FLASH code for investigating the propagation of sound waves into a realistically stratified solar interior.

1. Introduction
Helioseismology is the study of pressure and gravity waves which are excited stochastically below the solar surface, these oscillations are a key to understanding the structure and dynamics of the solar interior. Forward modelling using theoretical and simulation studies combined with observational data-analysis are being used for this purpose. In recent years, it has become increasingly obvious that this combined effort is now essential if we are to understand fully the magnetic regions such as sunspots and plages and their influence on these oscillations.

The FLASH code solves the fully compressible, reactive hydrodynamic equations. It was initially developed to model nuclear flashes on the surfaces of neutron stars and white dwarfs, and the interior of white dwarfs; but it has since been applied to modelling a wide variety of astrophysical flows, see also Fryxell et al. (2000) for more details. In this paper we report the first use of FLASH numerical code to understand the behaviour of wave-field propagation in the context of helioseismology (see also Moradi et al. (2010) for a summary of other numerical codes used in local Heliosesimology).

In section 2 we describe the construction of our background model and the form of acoustic source used. In section 3, we describe the numerical model setup. In section 4 we show the time-distance diagrams obtained from our simulations with FLASH. The results are briefly discussed in section 5. We then conclude followed by a short outline of our future work in section 6.

2. The Model
Following the standard procedure, we choose for our non-magnetic plane-parallel stratification the standard Solar Model (the S model) of Christensen-Dalsgaard (1996) and try to make it stable against convection. We consider the adiabatic approximation for an ideal gas, so the adiabatic index, $\Gamma_1 = 5/3$, and the mean molecular weight, $\mu = 1.25$, take constant values.

To eliminate the convective instability in the upper layers the temperature profile is slightly modified according to the convective stability criterion shown below by equation (3) to be less
or equal zero but, keeping the condition of hydrostatic equilibrium as described by eq. (2). We also use the equation of state for an ideal gas given by eq. (1), to close the system of equations.

\[ T = \frac{\mu p}{R \rho} , \]

\[ \frac{dp}{dz} = -g(z)\rho(z) , \]

\[ \left| \frac{dT}{dz} \right| - \left( 1 - \frac{1}{\Gamma_1} \right) \frac{T}{p} \left| \frac{dp}{dz} \right| = s(z) , \]

where \( p, \rho, \) and \( T \) are the pressure, density and temperature respectively and \( R \) is the gas constant. The initial pressure and density are recalculated to maintain the hydrostatic equilibrium and the convective stability (see also, Shelyag et al. (2006) for more details).

We solve Euler’s equations for compressible gas dynamics in two spatial dimensions, adding a single acoustic source \( Q(r, t) \) to the energy equation as described by Rast (1999).

\[ Q(r, t) = Q_0 \left\{ 1 + \tanh \left[ \ln(3) (t - t_0) \right] \right\} \exp \left[ -\frac{(r - r_0)^2}{\sigma_2^2} \right] , \]

where the amplitude is \( Q_0 = -150 \text{ erg s}^{-1} \text{ cm}^{-3} \), the characteristic time length is \( \sigma_1 = 120 \text{ s} \), the full-width at half maximum is \( \sigma_2 = 0.5 \text{ Mm} \), the source shifting time is \( t_0 = 1000 \text{ s} \). This source imposes a cooling event that produces a localised drop in pressure and temperature which then results in an expanding outflow around the source. As a side effect, this source introduces convective instabilities around itself.

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**Figure 1:** Modified S Model profiles for our convectively stable background. The subplot in figure (f) shows, in detail, the relative differences between the sound speeds obtained from our model and the S Model. The rest of the subplots show the details in the upper layers of the corresponding parameter.
3. Numerical Setup

Our numerical domain is represented schematically in figure 2. The simulation box has $D$ Mm of thickness by $L$ Mm of width. The source is located at $L/2$ at a depth $z_s$ below the solar surface; the surface being located at $R_{\odot}$ or $z = 0$. We consider our atmosphere under a vertical gravitational field, constant in time i.e. $g(r, t) = g(z)$.

We carried out three numerical experiments with different depth and grid points as well as different source positions to test the code, see table (1).

![Figure 2: Schematic representation of our numerical domain. The solar surface is represented by the dashed line at $R_{\odot}$. $D$ is the thickness whereas $L$ is the width. The gravitational field is represented by $g(z)$ and the location of the source by an asterisk at $(L/2, z_s)$.](image)

The boundary conditions used in our simulations are outflow for left and right sides in the horizontal direction (ASC FLASH Center 2009). For the vertical direction we keep the hydrostatic equilibrium with outflow condition for the top and bottom boundaries.

4. Results

![Figure 3: Time-distance diagrams for the vertical velocity, $v_z$. The measured level is located at $R_{\odot}$. Figures (a) and (b) correspond to the source depth of 10 Mm and (c) 50 Mm respectively. We can see how the trace due to convective cells around the source is less prominent in model S10.2 than in model S10.1 due to the source position with respect to the measured level.](image)

Table 1: Summary with the model parameters used in our simulations.

| Model Name | S10.1 | S10.2 | S50.1 |
|------------|-------|-------|-------|
| $D$ (Mm)   | 10    | 10    | 50    |
| $L$ (Mm)   | 150   | 150   | 150   |
| $z_s$ (Mm) | -1    | -1.55 | -1    |
| Grid (HxV) | 392x1200 | 392x1200 | 960x6000 |
5. Discussion and Conclusion
In figure 1 we show the profiles for our modified S Model and the position of the source with a vertical dashed line (shown only for experiments S10.1 and S50.1). The ideal gas consideration and the elimination of convection close to the surface have produced a slight increment in the temperature in this region when compared to the unmodified S Model, see subplot of figure 1b.

In figure 1f we show the relative differences of the sound speeds obtained from our model and the S Model, i.e. \( \Delta = (c_s - c_0)/c_s \). The relative difference is greatest close to the upper layers due to the modification of temperature profile, therefore we are modifying the eigenfrequencies and eigenfunctions of the S Model close to the surface in an important way. The difficulties as well as the possibilities of finding a non-convective solar-like model has also been addressed in Schunker et al. (2010).

Figure 3 shows the time-distance diagrams for \( v_z \). In figures 3a and 3b, we can see how the amplitude of \( v_z \) decreases by increasing \( z_s \). Furthermore, we see sharper ridges for S10.1 model. This sharp ridges are due to the convective cells generated by the source. Figure 3c shows the time-distance diagram for model S50.1. We can see similar structure as in models S10.1 and S10.2, but with less numerical noise, due to higher number of points used in horizontal direction.

Comparison with figure 8 of Shelyag et al. (2006), suggests that although there are some differences in the boundary conditions treatment, numerical technique, source position and the atmospheric model, the results obtained with FLASH are similar to their simulation. These preliminary results have shown the capabilities of FLASH code for investigating the helioseismic studies, in particular the propagation of sound waves into a realistically stratified solar interior.

6. Future Work
The most important step will be to implement more accurate boundary conditions to reduce noise and reflections. We also hope to use the adaptive mesh capabilities of FLASH to improve the resolution and reduce the computational time. Clearly we need a more accurate modelling in \( s(z) \) profile to get a more realistic convectively stable S Model. A new source to avoid the insertion of convective instability in our numerical domain and a study with different random sources would be also very interesting. Another important and necessary step is to introduce magnetic fields. Finally it would be interesting to analyse the simulated and the real observational data of wavefields with the same local helioseismic techniques.

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