Application of improved wavelet denoising method in low-frequency oscillation analysis of power system

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Abstract. The Prony algorithm has been widely used in low-frequency oscillation analysis of power system due to its good mathematical characteristics. However, the Prony algorithm is very sensitive to noise and requires a high level of input signal. This paper overcomes the shortcomings of traditional soft and hard threshold denoising methods by improving the threshold and threshold function of wavelet denoising. Using the improved wavelet threshold denoising method to preprocess the sampled signal can effectively improve the anti-interference ability of the Prony algorithm. The simulation experiment is performed with MATLAB and the experimental results show the effectiveness of the improved method.

1. Introduction

The Low-frequency oscillation of power system refers to the phenomenon that power, voltage and other electric quantity oscillate in the system due to the relative swing between the rotors of synchronous generators running in parallel after the system is disturbed. Once the low-frequency oscillation occurs, it will last for a long time, which will seriously threaten the safe and stable operation of power grid and may cause large-scale power blackout [1]. The purpose of studying low-frequency oscillation of power system is to analyze whether there are weakly damped oscillation modes in the system, and to take measures to enhance the damping of these modes when there are weakly damped oscillation modes in the system, so as to reduce the possibility of oscillation, or to make the oscillation subside as soon as possible.

The signal processing methods used for Identification of low frequency oscillation modes include Fourier transform method, Kalman filter method [2], Prony method [3] and HHT method [4]. The Fourier transform has a higher requirement on the signal it processes. When the signal does not meet the condition of absolute integrability, the Fourier transform is powerless. Meanwhile, the time-frequency window of the Fourier transform is fixed, which is not conducive to the analysis of different frequency components of the signal and cannot reflect the damping characteristics of low-frequency oscillation modes. The Kalman filter method has the shortcoming of not reflecting the attenuation characteristics of low-frequency oscillation damping, which greatly limits its development in the field of low-frequency oscillation mode identification. HHT method is an adaptive time-frequency analysis method, which can carry out adaptive decomposition according to the local time-varying characteristics of the signal itself. It can not only get a high time-frequency resolution, but also has a good time-frequency aggregation, which is very suitable for the analysis of non-stationary and nonlinear signals. However, the HHT method lacks a solid theoretical basis, and there are many uncertain factors in the EMD decomposition process, such as endpoint delay deviation, mode mixing and interpolation distortion, which seriously affect the reliability of the method in low-frequency oscillation mode identification.
As a signal processing method, Prony algorithm uses linear combination of exponential functions to fit equal-interval sampling data, and can fit the oscillation amplitude, frequency, attenuation factor and phase of the data, so the identification results have high accuracy. However, the accuracy of Prony algorithm is greatly affected by signal noise [5]. Therefore, researchers have carried out a lot of research work to find a method with good anti-noise effect.

Denoising pre-processing of the input signal can obtain better recognition results. In [6], the smoothing filtering algorithm is combined with Prony algorithm, and interpolation algorithm is adopted to improve the sampling rate. This algorithm improves the anti-interference ability and accuracy of Prony algorithm to a certain extent, but it is of low sensitivity and has a poor suppression effect on occasional impulsive interference; In [7], wavelet threshold denoising is adopted to suppress noise, but the threshold adopted in this paper is a fixed threshold, and the denoising effect needs to be further improved; In [8], a method for low-frequency oscillation pattern recognition of power systems based on fuzzy filtering and Prony algorithm is proposed. This method firstly used simple fuzzy rules to filter discrete measured signals, and then used the improved Prony algorithm to analyze the filtered signals, but the algorithm lacks adaptive ability; In [9], it is proposed to classify the input signals, analyze the low-frequency oscillation of the system by using the same kind and different kinds of multi-signal methods, and denoise the data to suppress the influence of noise on the identification accuracy of algorithm. If the same kind of signals cannot be correctly identified, the identification error will occur; In [10], the threshold based on wavelet denoising is improved, so that the threshold changes with the number of wavelet decomposition layers, thus achieving a better filtering effect on the input signal. However, the threshold function in this paper adopts the hard threshold function, which still needs to be improved to improve the filtering effect; In [11], An on-line denoising algorithm is proposed to detect unavoidable measured noise in the acquired data. The proposed denoising procedure is based upon the discrete wavelet transform (DWT) and works without thresholds for the localization of noise, as well for the stop criterium of the algorithm; In [12], a novel approach based on nonconvex sparse regularization denoising and adaptive sparse decomposition is proposed and it can effectively extract the weak fault frequency and its harmonics from raw vibration signals.

Aiming at the shortcomings of the above several denoising methods, this paper proposes a wavelet denoising method based on improved threshold and threshold function, when the input signal is pre-processed, the influence of noise signal can be reduced more effectively, so as to improve the accuracy of the identification results of Prony algorithm. Finally, the simulation results prove the superiority of the improved wavelet threshold algorithm proposed in this paper.

2. Introduction of Prony algorithm

The Prony algorithm is a common algorithm for extracting stationary oscillation modes. It is for equal-interval sampling data, if the model is a linear combination of a set of p exponential functions with arbitrary amplitude, phase, frequency and attenuation factor, and its discrete time function form is [13]:

\[
\hat{y}(n) = \sum_{i=1}^{p} A_i \exp(j\theta_i) \exp[(\alpha_i + j2\pi f_i)\Delta t]n \quad (n = 0, 1, ..., N - 1)
\]

(1)

Where \(A_i\) is the amplitude; \(\theta_i\) is the phase; \(\alpha_i\) is the attenuation factor; \(f_i\) is the oscillation frequency; \(\Delta t\) is the sampling interval; \(N\) is the number of samples.

Let \(\hat{y}(n)\) approximate the actual sampled data \(y(n)\). Construct the cost function:

\[
\varepsilon = \sum_{n=0}^{N-1} |y(n) - \hat{y}(n)|^2
\]

(2)

To achieve minimum cost function and to obtain the parameters in \(\hat{y}(n)\). Define characteristic polynomial:
\[ \psi(z) = \prod_{i=1}^{p} (z - z_i) = \sum_{i=0}^{p} a_i z^{p-i} \] (3)

Thus \( \hat{y}(n) \) meet the recursive difference equation is:

\[ \hat{y}(n) = - \sum_{i=1}^{p} a_i \hat{y}(n-i), \quad (n = 0, 1, N - 1) \] (4)

In order to establish Prony method, the error between the actual measured data \( y(n) \) and its approximation \( \hat{y}(n) \) is defined as \( e(n) \):

\[ y(n) - \hat{y}(n) = e(n), \quad (n = 0, 1, ..., N - 1) \] (5)

Substitute Equation (5) into Equation (4) to get:

\[ y(n) = - \sum_{i=1}^{p} a_i y(n-i) + \sum_{i=0}^{p} a_i e(n-i) \] (6)

Make the following definition:

\[ e(n) = \sum_{i=0}^{p} a_i e(n-i), \quad (n = p, ..., N - 1) \] (7)

If \( \sum_{n=p}^{N-1} |e(n)|^2 \) is minimized, a set of linear matrix equations can be obtained:

\[
\begin{bmatrix}
  y(p) & y(p-1) & \cdots & y(0) \\
  y(p+1) & y(p) & \cdots & y(1) \\
  \vdots & \vdots & \ddots & \vdots \\
  y(N-1) & y(N-2) & \cdots & y(N-p-1)
\end{bmatrix}
\begin{bmatrix}
  1 \\
  a_1 \\
  \vdots \\
  a_p
\end{bmatrix}
= \begin{bmatrix}
  \epsilon(p) \\
  \epsilon(p+1) \\
  \vdots \\
  \epsilon(N-1)
\end{bmatrix}
\] (8)

In order to minimize the function \( J(a) = \sum_{n=p}^{N-1} |e(n)|^2 = \sum_{n=p}^{N-1} \left| \sum_{j=0}^{p} a_j y(n-j) \right|^2 \), let \( \frac{\partial J(a)}{\partial a_i} = 0 \), then:

\[
\sum_{j=0}^{p} a_j \left| \sum_{n=p}^{N-1} y(n-j) y^*(n-i) \right| = 0 (i = 1, ..., p)
\] (9)

The main steps of the extended Prony method can be described as follows:

(a) Define the sample function \( r(i, j) = \sum_{n=p}^{N-1} y(n-j) y^*(n-i) \) and construct a matrix of extended orders:

\[
R = \begin{bmatrix}
  r(1,0) & r(1,1) & \cdots & r(1,p_e) \\
  r(2,0) & r(2,1) & \cdots & r(2,p_e) \\
  \vdots & \vdots & \ddots & \vdots \\
  r(p_e,0) & r(p_e,1) & \cdots & r(p_e,p_e)
\end{bmatrix}
\] (10)

The effective rank \( P \) of the matrix \( R \) and the overall least squares estimate of the coefficients \( a_1, ..., a_p \) are determined by the singular value decomposition method.

(b) To solve the root \( z_1, ..., z_p \) of the characteristic polynomial \( \psi(z) = 0 \) and use Equation (4) to calculate \( \hat{y}(n) \), \( n = 1, ..., N - 1 \), where \( \hat{y}(0) = y(0) \).

(c) The exponential model Equation (1) is reduced to a linear equation of the unknown parameter \( b_i \), which is expressed in matrix form as:
\[
\begin{bmatrix}
1 & 1 & \ldots & 1 \\
z_1 & z_2 & \ldots & z_p \\
\vdots & \vdots & \ddots & \vdots \\
z_1^{N-1} & z_2^{N-1} & \ldots & z_p^{N-1}
\end{bmatrix}
\begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_p
\end{bmatrix}
= 
\begin{bmatrix}
\hat{y}(0) \\
\hat{y}(1) \\
\vdots \\
\hat{y}(N-1)
\end{bmatrix}
\tag{11}
\]

Using the above equation to calculate the parameter \(b_1, \ldots, b_p\).

(d) When the coefficients \(b_1, \ldots, b_p\) and \(z_1, \ldots, z_p\) are obtained, amplitude \(A_i\), phase \(\theta_i\), frequency \(f_i\) and attenuation factor \(\alpha_i\) can be calculated by the following equation:

\[
\begin{aligned}
A_i &= |b_i| \\
\theta_i &= \arctan(\text{Im}(b_i)/\text{Re}(b_i)) \\
\alpha_i &= \ln|z_i|/\Delta t \\
f_i &= \arctan(\text{Im}(z_i)/\text{Re}(z_i))/(2\pi\Delta t)
\end{aligned}
\tag{12}
\]

3. Signal processing based on wavelet transform

Wavelet signals are a class of fast-attenuating wave signals with limited energy and are relatively concentrated in local regions. \(\psi_{(j,k)}(t)\), family of wavelet functions, is composed of the wavelet function \(\psi(t)\) by expansion, contraction and translation. \(\phi_{(j,k)}(t)\), family of Scaling functions, is composed of the Scaling function \(\phi(t)\) by expansion, contraction and translation. Using the family of wavelet functions and the family of scaling functions, the wavelet expansion of the signal can be expressed as \([14]\):

\[
y(t) = \sum_k c_{j0,k} \phi_{j0,k}(t) + \sum_{j=j_0}^{\infty} \sum_k d_{j,k} \psi_{j,k}(t)
\tag{13}
\]

The expansion coefficient \(c_{j0,k}\) reflects the distribution of the low-frequency components in the signal \(y(t)\), and the series of expansion coefficients \(d_{j,k}\) reflects the distribution of the high-frequency components in the signal \(y(t)\). These expansion coefficients are the discrete wavelet transforms of the signal.

Wavelet transform of signal is an effective method for time-frequency analysis of signal, and its linear time-frequency analysis of non-stationary signal has different resolution at different positions of time-frequency plane, that is, wavelet transform is a multi-resolution analysis method.

3.1. Wavelet threshold denoising

The mathematical model of the noise-containing signal \(y(t)\) is generally:

\[
y(t) = x(t) + e(t)
\tag{14}
\]

Where \(x(t)\) is the useful signal and \(e(t)\) is the noise signal.

The purpose of denoising the signal \(y(t)\) is to suppress the noise signal component \(e(t)\), thereby restoring the signal \(x(t)\).

When the signal is filtered in the frequency domain, the useful signal and the noise signal are separated according to the different position of spectrum distribution in the frequency domain. It is generally believed that the power system mainly has white noise, and the white noise is in a wide frequency range, and the noise energy contained in the frequency band of the same bandwidth is the same, the spectrum of the signal and the spectrum of the noise overlap in the frequency domain, and the filtering based on the frequency domain is not effective.

In the wavelet transform of signals, the expansion coefficients of many actual signals are mostly concentrated on fewer coefficients, which creates favorable conditions for data processing. In the signal processing based on wavelet transform, different components of the signal can be separated.
according to the amplitude of the expansion coefficient of the useful signal and the unwanted signal. The amplitude of the minority expansion coefficient corresponding to the useful component is necessarily larger, while the amplitude of the majority expansion coefficient corresponding to the unwanted component is necessarily smaller.

Selecting a wavelet basis function, and sample the signals at equal intervals to obtain the sequence of samples c_{j+1,k} corresponding to the signal, and then perform N-level discrete wavelet transform based on c_{j+1,k} to obtain wavelet expansion coefficients d_{j,k}, d_{j-1,k}, …, d_{j-N+1,k} of different scales of N levels and a first-order approximate expansion coefficient c_{j,N+1,k}. The corresponding wavelet threshold and threshold function are selected for processing the wavelet expansion coefficients at each level to obtain the processed wavelet expansion coefficients \( \hat{d}_{j,k}, \hat{d}_{j-1,k}, …, \hat{d}_{j-N+1,k} \), and then the wavelet reconstruction is performed with the new wavelet coefficients to obtain the denoised signal.

### 3.2. Selection of threshold

Selecting the appropriate threshold is the key to denoising by wavelet threshold. If the threshold is selected to be small, then the wavelet coefficients processed by the threshold function will still contain more noise components, resulting in insufficient signal denoising; if the threshold is selected to be large, the wavelet coefficients processed by the threshold function will lose more useful components, resulting in signal distortion after reconstruction [13]. The selected threshold is preferably just above the maximum level of noise. The maximum amplitude of the general noise has a very high probability just below \( \lambda = \sigma \sqrt{\ln(N)} \), so the threshold selected is usually:

\[
\lambda = \sigma \sqrt{\ln(N)} \tag{15}
\]

Where \( \sigma \) is the noise intensity, \( N \) is the signal length, \( \sigma = \text{median}[d_{j,k}]/0.6745 \), and \( \text{median}[d_{j,k}] \) represents the median of the wavelet coefficients on the scale j.

However, the above threshold is not optimal, because it is constant across all scales. In general, the wavelet coefficients corresponding to the noise components are evenly distributed on each scale, and as the scale j increases, the amplitude of the wavelet coefficients decreases gradually, so the threshold is taken as \( \lambda = \sigma \sqrt{\ln(N)/\ln(2e + j - 1)} \) in [15], but the threshold does not take the actual distribution of noise in different layers of different types of signals into account, it remains to be improved.

\[
W_j = \left( \max(|d_{j,k}|) \sum_k |d_{j,k}| \right) L_j \tag{16}
\]

Where \( L_j \) represents the length of the wavelet coefficient at the j scale.

Take the new threshold as:

\[
\lambda_j = \sigma \sqrt{\ln(N)/\ln(j + W_j)} \tag{17}
\]

If there are more useful components in the wavelet coefficients of the jth layer, the value of \( W_j \) will be larger, and the value of \( \lambda_j \) will be relatively smaller; If there are more noise components in the wavelet coefficients of the jth layer, the value of \( W_j \) will be smaller. The value of \( \lambda_j \) will be relatively larger, so that the selection of the threshold is more in line with the noise distribution in the actual disturbance signal [16]. The new threshold not only considers that the amplitude of the wavelet coefficients is gradually reduced with the increase of the scale j under normal conditions, but also takes the actual distribution of noise in each layer of different types of signals into account, making up for the deficiency of threshold in [10].

### 3.3. Selection of threshold function

Traditional threshold functions have two types: hard threshold and soft threshold function. Soft threshold processing is to set the coefficient below the threshold value to zero, and the coefficient
above the threshold value to reduce correspondingly. Hard threshold processing is only to set the coefficients below the threshold value to zero. Let $\lambda$ be the threshold. The two threshold functions are defined as follows:

Hard threshold function:

$$\hat{d}_{j,k} = \begin{cases} d_{j,k}, |d_{j,k}| \geq \lambda; \\ 0, |d_{j,k}| < \lambda. \end{cases}$$

(18)

Soft threshold function:

$$\hat{d}_{j,k} = \begin{cases} \text{sgn}(d_{j,k})(|d_{j,k}| - \lambda), |d_{j,k}| \geq \lambda; \\ 0, |d_{j,k}| < \lambda. \end{cases}$$

(19)

These two methods are small in calculation, simple in implementation, and widely used. But both methods have some limitations. The wavelet coefficient $\hat{d}_{j,k}$ obtained by using soft threshold function has good continuity, but when $|d_{j,k}| \geq \lambda$, there is always a fixed deviation of $\lambda$ between the processed wavelet coefficient value and the actual value, which directly affects the degree of approximation between the reconstructed signal and the original input signal. The hard threshold function is a discontinuous function. After thresholding with a hard threshold function, the obtained new wavelet coefficients have poor continuity, and the reconstructed signal tends to have oscillations that are not present in the original signal, resulting in reconstructed signal distortion [13].

In order to make up for the deficiency of the traditional threshold function, this paper proposes an improved algorithm:

$$\hat{d}_{j,k} = \begin{cases} \text{sgn}(d_{j,k})\left(|d_{j,k}| - \lambda \sin^2\left(\frac{\pi}{2} \times \frac{\lambda}{|d_{j,k}|}\right)\right), \\ 0, |d_{j,k}| < \lambda; \end{cases}$$

(20)

When $|d_{j,k}| \geq \lambda$, the value of $\frac{\lambda}{|d_{j,k}|}$ is between 0 and 1, so that the value of $\sin^2\left(\frac{\pi}{2} \times \frac{\lambda}{|d_{j,k}|}\right)$ is also between 0 and 1. When $|d_{j,k}|$ gradually increases, the value of $\frac{\lambda}{|d_{j,k}|}$ decreases continuously, and the value of $\sin^2\left(\frac{\pi}{2} \times \frac{\lambda}{|d_{j,k}|}\right)$ decreases continuously, thus reducing the fixed deviation of the size $\lambda$ existing in the soft threshold function, which makes up for the deficiency of the soft threshold function. At the same time, $\lim_{d_{j,k}\rightarrow \lambda} \hat{d}_{j,k} = 0$ and when $|d_{j,k}| = \lambda$, $\hat{d}_{j,k} = 0$, indicating that the algorithm is continuous at $\lambda$, which makes up for the deficiency of the hard threshold function.

4. Construct signal simulation experiment

In order to facilitate comparison, the same SNR as that in Reference [10] is selected. The SNR adopted in this paper is:

$$\text{SNR} = 10 \log \left[ \frac{\sum y^2(n)}{\sum (\hat{y}(n) - y(n))^2} \right]$$

(21)

In order to facilitate comparison, the same mean square error as in Literature [12] is selected. The mean square error adopted in this paper is:

$$\text{MSE} = \frac{1}{N} \sum_{n=1}^{N} (\hat{y}(n) - y(n))^2$$

(22)
Where \( y(n) \) is the original signal, \( \hat{y}(n) \) is the reconstructed signal after wavelet denoising, and \( N \) is the signal length.

In order to prove the superiority of the improved denoising method in this paper, the signal is constructed as follows:

\[
y(t) = 3e^{-0.25t} \cos(2\pi \times 1.2t + 180^\circ) + 8.5e^{-0.15t} \cos(2\pi \times 0.6t + 60^\circ)
\]  

The sampling frequency of the signal is 100Hz, and the sampling points are 1000.

The signal is superimposed with 10dB white noise perturbation, and then the simulation experiment is carried out in MATLAB by soft threshold method, hard threshold method, improved threshold denoising method in [10] and improved method in this paper. The wavelet basis used in the simulation is sym6 wavelet, and the decomposition layer number is 5. The results of the simulation experiment are shown in the following table and figure.

| Denoising method        | SNR   | MSE  |
|-------------------------|-------|------|
| Soft threshold method   | 20.22 | 0.15 |
| Hard threshold method   | 20.43 | 0.14 |
| Methods in Literature [12] | 21.1  | 0.12 |
| Improved method of this paper | 22.53 | 0.09 |

It can be seen from the results in Table 1 and Figure 1 that the denoising effect of the improved method proposed in this paper is superior to other methods in terms of signal to noise ratio and error of mean square.
The signal denoised by the method in [10] and the improved method in this paper is identified by the extended Prony algorithm, and the identification results are shown in the following table.

It can be seen from the results of Table 2 that for the noise-containing signal, after denoising with the improved method in this paper, the extended Prony algorithm can extract the oscillation mode parameters more accurately.

### Table 2. Identification results of two methods.

| Mode | Identification parameter | Theoretical value | Methods in Literature [12] | Improved method of this paper |
|------|--------------------------|-------------------|----------------------------|------------------------------|
| 1    | frequency                | 0.6               | 0.62                       | 0.6                          |
|      | Attenuation factor       | -0.15             | -0.18                      | -0.16                        |
|      | Damping ratio            | 0.04              | 0.046                      | 0.042                        |
| 2    | frequency                | 1.2               | 1.21                       | 1.21                         |
|      | Attenuation factor       | -0.25             | -0.27                      | -0.24                        |
|      | Damping ratio            | 0.033             | 0.036                      | 0.032                        |

### 5. Conclusions

After analyzing the advantages and disadvantages of traditional soft and hard threshold functions and fixed thresholds, this paper proposes a wavelet threshold denoising algorithm based on improved threshold and threshold function, which can effectively overcome the influence of noise on the identification accuracy of Prony algorithm. The MATLAB simulation experiment proves that the improved wavelet threshold denoising algorithm proposed in this paper can improve the recognition accuracy of the extended Prony algorithm.

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