Properties of Massive Rotating Protoneutron Stars with Hyperons: Evolution and Universality

Smruti Smita Lenka\textsuperscript{(a)}, Prasanta Char\textsuperscript{(b)}, Sarmistha Banik\textsuperscript{(a)}

\textsuperscript{(a)}BITS Pilani, Hyderabad Campus, Shameerpet Mandal, Hyderabad 500078, India and
\textsuperscript{(b)}Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune - 411 007, India

Abstract

In this work, we follow several stages of quasi-stationary evolution of a massive and rapidly rotating protoneutron star (PNS) with hyperon content. We use a density dependent (DD) relativistic mean field theory (RMF) model and calculate different quantities such as mass, equatorial radius, moment of inertia, quadrupole moment etc. to get different rotating configurations. We study the effect of the appearance of the lightest of all hyperons, $\Lambda$, on the evolutionary stages of the PNS. We also check its sensitivity to the inclusion of $\phi$ vector meson as a mediator of $\Lambda-\Lambda$ interaction in detail. Finally we investigate the universal relations between moment of inertia and compactness in this context of a hot and young compact object.
I. INTRODUCTION

When a massive star ($M \gtrsim 8M_{\odot}$) reaches the end of its life, its core collapses which leads to a supernova explosion leaving a dense compact remnant at the center. It is called a protoneutron star (PNS). Initially, the PNS is very hot, lepton-rich and rapidly rotating. It deleptonizes releasing the trapped neutrinos. In the process, the neutrinos heat up the PNS while decreasing the net lepton fraction. After that, the PNS enters a steady cooling phase and becomes a cold, catalyzed neutron star (NS) [1, 2]. There have been many detailed studies on the global properties of the PNS with different microphysical inputs [3, 4]. From one of the most comprehensive and systematic studies by Prakash et al., the PNS is believed to have four prominent stages of evolution as it ends up as a cold catalyzed object [2]. These are governed strongly by the nature of matter at very high density as well as the neutrino reaction rates and diffusion timescales.

At the high density core, the Fermi energy of nucleons become sufficiently large; according to Pauli principle, the formation of hyperon becomes energetically favorable. The total energy and also the pressure of the system are lowered by sharing baryon number among several baryon species. The Λ hyperon, being the lightest among other massive baryons are eventually the first to populate the core as the nucleon chemical potential outweighs their in-medium mass [5]. The in-medium mass of hyperon is determined from experimental inputs such as the nature of the nuclear-hyperon interaction. Several studies have been done to investigate the effect of inclusion of hyperon on the structure on PNS [6, 7]. But, they used mostly older equation of state (EOS) parametrizations which are not consistent with the recent observations or the latest experimental data. Therefore, it is important to revisit the problem with more suitable microphysical inputs to find out how sensitive these results are with respect to the parameters of the theory. In this work, we will mainly focus on the dense nuclear matter containing hyperons and study its effect on global properties of massive PNS. Keeping that in mind, we employ a realistic EOS including hyperons for this work. The EOS is constructed using a relativistic hadron field theoretical model with density dependent couplings in the mean field approximation. Specifically, we use the DD2 parameter set for the couplings [8, 9]. It satisfies the constraints on nuclear symmetry energy and its slope parameter as well as the incompressibility from the nuclear physics experiments [10]. On top of that, the astrophysical observations also give us important clues about the EOS of
dense matter. The recent detection of gravitational wave signal from the binary neutron star merger event GW170817 provides the latest information on this regard [11]. Besides, the $\sim 2 \, M_{\text{solar}}$ mass measurement of PSRs J1614-2230 and J0348+0432 has severely constrained the parameter space for the NS EOS [12–14]. Now, the cold NS mass from hyperonic DD2 model is $2.1 \, M_{\text{solar}}$. Thus, it satisfies the observational limit on the maximum mass of the NS. It is also observed that it satisfies the tidal deformability bounds found from GW170817 [15].

Another objective of this work is to study the universal relations recently discovered among various global quantities of compact objects such as normalized moment of inertia ($\bar{I} := I/M^3$) and spin-induced quadrupole moment ($\bar{Q} := QM/J^2$), stellar compactness ($C := M/R$) etc. [16–18]. In the context of the PNS evolution, $\bar{I} - \bar{Q}$ relation has been studied by Martinon et al. [19] for nucleon-only EOS. They have found that the universality is broken at the initial stages after the core bounce, but it is satisfied at later stages. Recently, Marques et al. also investigated the problem for rapidly rotating hot stars with hyperonic EOS [20]. They found that the relation does not change in the presence of hyperons but deviates for high entropy. But, the $\bar{I}$ vs $C$ relations has not been studied for the PNS till now. Previously, we have studied these relations in the presence of exotic components like hyperons and antikaon condensates for cold NS [21]. In this work, we will examine those relations for PNS using the fitting factors provided by Breu and Rezzolla [18]. They studied a large set of nucleonic EOS with different stiffness and showed that universality relation holds for normalized moment of inertia.

The paper is organized in the following way. In section II, we describe the PNS evolution scenarios, followed by section III explaining the equation of state of dense matter. In section IV, we discuss the structure of rotating relativistic star. Finally, in section V we discuss our results and conclude with a summary in section VI.

II. STAGES OF PNS EVOLUTION

We follow a well established evolutionary scenario first proposed by Prakash et al. [2] and subsequently used by others [6, 7]. It suggests that the PNS undergoes roughly four stages of evolution towards becoming a stable, cold catalyzed compact object.

1. Just after its birth, the PNS initially has trapped neutrinos and the electron fraction
is $Y_L = 0.4$. The core of the PNS has an entropy per baryon $s_B = 1$ surrounded by a high entropy, neutrino trapped outer layer, which deleptonizes faster than the core.

2. While the outer layer is being deleptonized, the central object is still neutrino trapped with $Y_L = 0.4$ and heats up to $s_B = 2$.

3. After complete deleptonization, the core becomes neutrino-free and attains high entropy ($Y_\nu = 0, s_B = 2$).

4. Finally, the star settles as a cold stable neutron star (NS) in beta equilibrium.

In the results section, we will denote these four stages with I to IV.

III. EQUATION OF STATE OF PNS MATTER

We follow the formalism used by Banik et al. [22] to construct the EOS relevant to our problem. In this section, we briefly discuss some of the important features of this formalism. For the low-density part, nuclear statistical model of Hempel and Schaffner-Bielich [23] is used to treat the heavy and light nuclei as well as the interacting nucleons, including the excluded volume effects and other in-medium effects. The high density matter is described by a density dependent relativistic mean field model (DDRMF) where the interaction among baryons is mediated by $\sigma, \omega, \rho$ mesons. There is also an additional vector meson ($\phi$) which accounts for the hyperon-hyperon interaction. The nucleon-meson couplings in the present model is density dependent. The functional form for this dependence and the corresponding parameter set (DD2) is taken from Typel et al. [9]. The saturation properties of the DD2 parameter set are in good agreement with the nuclear physics experiments [24]. The hyperon-vector meson couplings are calculated from the SU(6) symmetry relations [25] and hyperon-scalar meson coupling is determined from the hypernuclei data which gives the hypernuclei potential depth in the normal nuclear matter [26].

The high dense matter made up of neutrons, protons and leptons was developed by Hempel and Schaffner-Bielich and is denoted by HS(DD2) [23]. The high density core is expected to consist of hyperons as well. $\Lambda$, the lightest hyperons populate first, the EoS is denoted by BHBA. In the presence of hyperon-hyperon interaction via $\phi$ mesons, the EoS is represented by BHBA$\phi$. Heavier hyperons such as $\Sigma$ and $\Xi$ are not considered due
to limited observational data about their interaction with nucleons or other hyperons. Our PNS models are constructed in a unified way by smoothly matching the high density and low density parts of the EOS following Banik et al. [22].

IV. ROTATING STAR STRUCTURE

We compute the structure of rotating stars using LORENE [27] code which assumes a stationary, axisymmetric spacetime. The line element is given by

\[
g_{\alpha\beta}dx^\alpha dx^\beta = -N^2dt^2 + A^2(dr^2 + r^2d\theta^2) + B^2r^2\sin^2\theta(d\phi - \omega dt)^2, \tag{1}
\]

where, \(N, A, B\) and \(\omega\) are functions of \((r, \theta)\). The energy-momentum tensor for a perfect fluid is related to energy density \(\varepsilon\), pressure \(P\) and four-velocity \(u^\mu\) by, \(T^\mu_\nu = (\varepsilon + P)u^\mu u^\nu + Pg^\mu_\nu\). The equation for stationary motion is given by [28],

\[
\partial_i \left( H + \ln \frac{N}{\Gamma} \right) = Te^{-H} \partial_i s_B, \tag{2}
\]

where, \(s_B\) is the entropy per baryon, \(T\) temperature, \(H = \ln \left( \frac{\varepsilon + P}{n_m n_H} \right)\) is the log-enthalpy, \(\Gamma = (1 - U^2)^{-1/2}\) is the Lorentz factor, \(U\) is the fluid velocity. The quantities \(U, \varepsilon\) and \(P\) are measured by locally non-rotating observer. LORENE is primarily formulated for a barotropic EOS. Still, we get a first integral of motion if we use an EOS with a fixed entropy per baryon. This enables us to use the isentropic EOS that we constructed to study the PNS stages.

The gravitational mass, angular momentum and quadrupole moment are given respectively as [29, 31],

\[
M = \frac{1}{4\pi} \int \sigma_{inN} r^2 \sin^2 \theta dr d\theta d\phi \tag{3}
\]

\[
J = \int A^2B^2(E + P)Ur^3\sin^2\theta dr d\theta d\phi . \tag{4}
\]

\[
Q = -M_2 - \frac{4}{3} \left( b + \frac{1}{4} \right) M^3, \tag{5}
\]

where,

\[
M_2 = -\frac{3}{8\pi} \int \sigma_{inN} \left( \cos^2 \theta - \frac{1}{3} \right) r^4 \sin^2 \theta dr d\theta d\phi \tag{6}
\]

Here, \(\sigma_{inN}\) is given by the RHS of Eq. 3.19 of [28], \(E = \Gamma^2(\varepsilon + P) - P\). The quantity \(b\) is defined by Eq. (3.37) of [31]. Then, the moment of inertia of the rotating star is defined as, \(I = J/\Omega\), where \(\Omega\) is the stellar spin frequency.
V. RESULTS AND DISCUSSIONS

We follow the quasi-stationary evolution of a massive PNS using DD2 EOS. Basically, we want to study how the emergence of Λ hyperons and its interaction affect the PNS stages. We will use mainly three types of EOS discussed in the previous section: i) HS(DD2), ii) BHBA and iii) BHBA\(\phi\) for our calculations with four different configurations (I - IV) relevant to the PNS evolution mentioned in section II.

In Fig. 1, pressure is plotted against number density. The left panel is for nucleons-only HS(DD2) EoS, the middle and the right panels are for EoS with hyperons, namely BHBA and BHBA\(\phi\) respectively. The presence of hyperons softens the EOS. The solid line represents the stage I of evolution i.e. lepton trapped with \(s_B = 1\). As the entropy per baryon increases to \(s_B = 2\), the EOS (dot-dashed) gets stiffer for all three variants. Once the neutrinos leave the system, the EOS is softened again (dashed line). The EOS (dotted line) for the cold catalyzed matter is the softest. This difference in stiffness is pronounced maximum for the BHBA EOS, lesser in BHBA\(\phi\), and least for the HS(DD2). If we compare the EOS for any particular stage, it becomes evident that the HS(DD2) is always the stiffest of the three, followed by BHBA\(\phi\) and BHBA respectively.

Gravitational mass versus radius for the corresponding EoS of Fig. 1 are plotted in Fig. 2. Here also, we follow the same line style as Fig. 1. We find that the stiffer EOS yields higher maximum mass as expected. However, the difference in their corresponding radii is not so prominent. The maximum gravitational mass and their corresponding radii for static configuration are given in table I. We also see the hotter stars with comparatively smaller masses have larger radii than their cold counterparts.

Temperature profile as a function of baryon density is plotted in Fig. 3 for the first three stages of PNS evolution. In each case, the temperature is maximum in the central region of the stars, which falls off rapidly at low density near the surface. In individual panels of Fig. 3 we follow the change in temperature for a particular EOS as the PNS evolves. The entropy per baryon of the PNS increases from \(s_B = 1\) to \(s_B = 2\) leading to a significant increase in the temperature as well as the radius. Inclusion of hyperons lowers the temperature of the central region of the PNS. For example, at baryon density \(1 \text{ fm}^{-3}\), for HS(DD2) EOS, the temperature increases from \(\sim 33\) to \(\sim 68\) MeV from stage I to II, whereas in the presence of Λ hyperons, this rise is from \(\sim 27\) to \(\sim 56\) MeV. This characteristic is consistent with
the argument that the additional degree of freedom, $\Lambda$, lowers the Fermi energy, hence the temperature of the system. When the neutrinos leave the system, the radius decreases as evident from Fig. 3 thereby increasing the temperature by about $\sim 12$ MeV again due to compression. The neutrinos carry away most of the energy, consequently the star becomes cold.

Let us consider a star with baryonic mass $M_B = 1.8 \ M_{\text{solar}}$ and follow the evolution of its different properties such as gravitational mass, radius, moment of inertia and quadrupole moment. In Fig. 4, these quantities are plotted as a function of rotational frequency for stages I-IV, up to the corresponding Kepler limit. All the quantities are observed to increase with rotation. This was noted in earlier work also [19]. The Kepler limit for a cold-catalyzed star of $M_B = 1.8 \ M_{\text{solar}}$ is 938Hz, while for the newly born star it is much less i.e. $\sim 687$ Hz. So initially the PNS is rotating at a lower frequency, only to reach a higher rotation rate as it contracts and cools down to a cold catalyzed $\beta$-equilibrated NS. Interestingly, as the PNS attains a higher entropy in the immediate step of evolution with $s_B = 2$ and $Y_L = 0.4$, it has to slow its rotation rate as this star can only withstand a mass-shedding frequency limit of $\sim 560$Hz. However, it can increase its rotation rate in the later stages of evolution as is evident from Fig. 4. The Kepler limit for all the EoS and different quasi-stationary evolution stages are given in Table II. The gravitational mass remains almost independent of EOS in all the evolutionary stages. But, we notice a EOS-dependent spread in radii of the stars. This spread is maximum for the intermediate stages, but not so explicit for initial and final stages. This is also reflected in the plots for moment of inertia and quadrupole moment vs frequency.

We tabulate the global structural properties of both nonrotating and maximally rotating PNS for a fixed baryonic mass $M_B = 1.8 \ M_{\text{solar}}$ in Table II. We can see the central density decreases from stages I to III for HS(DD2) EOS and then increases after the star attains beta-equilibrium for both static and Keplerian scenarios. But, for the BHBA and BHBA$\phi$ EOS, the central density starts increasing after deleptonization i.e. from stage III. This can be explained as the neutrinos carry away most of the binding energy of the system, after deleptonization, the star contracts and as a result the central density increases for both non rotating and rotating stars and angular momentum increases for the rotating star thereby increasing the Kepler frequency. Emergence of hyperons in these two EOS leads to greater neutrino emission leading to an early contraction. In stage I, the central densities for
HS(DD2) for static and Keplerian cases are almost similar. But, for BHBA and BHBAφ the central densities for Keplerian stars reduce drastically from static stars. Incidentally, we do not see this behavior for other stages. In those cases, the central density for the Keplerian star is always lesser than their static counterparts for all the EOS considered.

The gravitational mass and radius increase from stage I to stage II due to increase in thermal pressure, but decrease for the subsequent stages. This can be attributed to the loss of neutrino pressure from stage II to III and drop of thermal pressure from stage III to IV. This behavior is observed for both nonrotating and Keplerian cases and also for all EOS. The values of masses and radii in the table corroborate to the results of Fig. 4. The angular momentum changes the same way as the Kepler frequency, already discussed in connection with Fig. 4. Finally, we also see the value of the quantity $T/|W|$ steadily increasing from 0.051 to 0.115 during the evolutionary stages. However, these values are rather insensitive to the chosen EOS. This shows the increase in the rotational kinetic energy which leads to rise in ellipticity as is evident from Table III. Interestingly, the change in ellipticity is also independent of EOS. Similarly we have calculated the same set of quantities for a star with fixed baryonic mass $M_B = 2.2 \, M_{solar}$ and they are listed in Table III. The results are qualitatively similar to those of baryonic mass $M_B = 1.8 \, M_{solar}$. These results are consistent with those found in earlier studies [32].

In Figs. 5 and 6 we explore the universality relations for normalized moment of inertia with HS(DD2), BHBA and BHBAφ EOS corresponding to the four stages of PNS with respect to compactness $C$. $I$ is normalized to $M^3$ and $MR^2$ respectively in the two figures. Both the figures have three panels indicating three different spin frequencies from left to right i.e. 100, 300 and 500 Hz. We find the normalized $I$ lines are almost independent of the composition of the star corresponding to each PNS stage. But the lines corresponding to different temperature and lepton fraction are distinctly separated. This pattern is seen for both types of normalizations and also for a particular frequency. Therefore, we might attribute this behavior to the combined effects of temperature and lepton fraction.

Next, we try to quantify the deviations by comparing our calculated values of $\bar{I}$ for a slowly rotating star with the ones we get from the fitting functions and corresponding fitting factors from Breu and Rezzolla [18]. The relation for $I/MR^2$ vs $C$ is given by,

$$\frac{I}{MR^2} = a_0 + a_1 C + a_4 C^4$$

(7)
The values of the constants $a_0, a_1, a_4$ are 0.244, 0.638 and 3.202 respectively [18]. Similarly, the relation for $I/M^3$ vs $C$ is given by,

$$\frac{I}{M^3} = \bar{a}_1 C^{-1} + \bar{a}_2 C^{-2} + \bar{a}_3 C^{-3} + \bar{a}_4 C^{-4}$$  \hspace{1cm} (8)$$

The corresponding values for $\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4$ are taken from Table 2 of Breu and Rezzolla [18]. We plot our results in Fig. 7. We find very high deviations ($\sim 40 - 50\%$) particularly for less compact stars at high entropy and lepton rich EOS for both the normalizations. The deviation becomes smaller i.e. around $\sim 10\%$ for the case of $s_B = 2, Y_\nu = 0$. But, the deviations for the cold EOS are always the least, below $\sim 2 - 3\%$ for every cases. Finally, we consider the stages of a slowly rotating PNS at fixed baryon mass with 1.8 $M_{\odot}$ and 2.2 $M_{\odot}$. Then at each stage, we try to measure the deviations from universality as done by Martinon et al. in the context of $\bar{I}$-Love-$\bar{Q}$ relations. Again we use the aforementioned fitting functions and fitting factors. We plot these results in Figure 8. Here we have used different symbols to distinguish the four evolutionary stages and different color schemes for the three chosen EoS. For the $M_B = 1.8 M_{\odot}$ star, we don’t find the deviations sensitive to the composition of the star. But, the deviations are significant for all stages except the cold catalyzed star. It starts with $\sim 22\%$ for stage I, followed by $\sim 30\%$ and $\sim 11\%$ for stage II and III respectively. Finally, when it reaches the cold catalyzed stage, the deviation falls down to $\sim 2\%$. Thus we conclude the $\bar{I} - C$ relation is broken at the early stages of the life of a PNS, but the universality is restored once the star attains chemical and thermal equilibrium. This result is consistent with the findings of Martinon et al. regarding the $\bar{I}-\bar{Q}$ relation [19]. Although, in our case, we take constant entropy per baryon throughout the star. The deviations can be due to the departure from barotropy introduced as a combined effect of neutrino and thermal pressure. For the $M_B = 2.2 M_{\odot}$ star, the situation is almost similar. Only difference is that there is a deviation for different EOS in the cold NS stage. Nevertheless, they remain below $\sim 2\%$. Thus the conclusion remains the same.

VI. SUMMARY

In the present work, we study the evolutionary stages of a massive PNS containing $\Lambda$ hyperons. We use the results from Prakash et al. to determine the properties of each of these stage [2]. This is done using EOS within the framework of a RMF model with density
dependent couplings. The model uses the parameters which are consistent with several nuclear physics experimental data and astrophysical observational data. We construct the EOS for $s_B = 1$ and $Y_L = 0.4$; $s_B = 2$ and $Y_L = 0.4$; $s_B = 2$ and $Y_\nu = 0$; cold-catalyzed for both nucleonic and hyperonic models. We also calculate the mass-radius sequence for static star with those EOS. We find clear effect of temperature on the size of the stars. The hotter the star, the larger is the radius. Although we don’t find much difference between the hyperonic stars with and without $\phi$ meson, the difference between hyperonic and nucleon-only stars is quite visible in this model. We also note that the properties of less compact stars are governed mostly by their temperature and lepton content. Several global properties of PNS are studied using those EOS in both static and maximally rotating configurations for fixed baryon masses 1.8 and 2.2 $M_{\odot}$. We see qualitative similarities for both cases. Another important finding of our studies is the deviation from $I - C$ universal relations for very hot and neutrino-rich stars. We take a slowly rotating star and measure the deviations for each of the stages. We find that as the star evolves towards the cold catalyzed stage the deviations from universality get smaller and the relations become valid again when the PNS becomes a cold NS. This result is also relevant for studying the remnant of a binary neutron star merger event. In such situation, the matter is also very hot and lepton rich. Therefore, applying any universal relation to make a connection between a quantity measured before merger (e.g. tidal deformability $\kappa_T^T$) and another quantity after merger (e.g. peak frequency $f_2$) requires utmost caution.

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| EoS   | Evolution Stages | \( M_G (M_{\text{solar}}) \) | R(Km) |
|-------|------------------|------------------|-------|
| HS(DD2) | \( s_B=1, Y_L=0.4 \) | 2.369 | 12.713 |
|        | \( s_B=2, Y_L=0.4 \) | 2.386 | 13.475 |
|        | \( s_B=2, Y_\nu=0 \) | 2.437 | 12.889 |
|        | \( T=0 \) | 2.423 | 11.869 |
| BHBA  | \( s_B=1, Y_L=0.4 \) | 2.161 | 13.124 |
|        | \( s_B=2, Y_L=0.4 \) | 2.176 | 13.859 |
|        | \( s_B=2, Y_\nu=0 \) | 2.018 | 12.713 |
|        | \( T=0 \) | 1.955 | 11.737 |
| BHBA\( \phi \) | \( s_B=1, Y_L=0.4 \) | 2.203 | 12.948 |
|        | \( s_B=2, Y_L=0.4 \) | 2.213 | 13.671 |
|        | \( s_B=2, Y_\nu=0 \) | 2.126 | 12.488 |
|        | \( T=0 \) | 2.1 | 11.608 |
TABLE II. Properties of non-rotating and rotating PNSs at the limiting frequency, for a fixed baryonic mass $M_B = 1.8 \, M_{\text{solar}}$. The parameters in the table are: central baryon number density ($n_c$), gravitational mass ($M_G$), circumferential equatorial radius ($R_{eq}$), Kepler frequency ($\nu_K$), angular momentum ($J$), polar to equatorial axis ratio ($r_p/r_{eq}$) and the rotation parameter ($|T/W|$).

| EoS          | PNS stages | $\nu = 0$ | $\nu = \nu_K$ | $\nu = \nu_K$ |
|--------------|------------|------------|----------------|----------------|
|              | $n_c$ [fm$^{-3}$] | $M_G$ [$M_{\text{solar}}$] | $R_{eq}$ [km] | $n_c$ [fm$^{-3}$] | $M_G$ [$M_{\text{solar}}$] | $R_{eq}$ [km] | $\nu_K$ [Hz] | $J$ | $r_p/r_{eq}$ | $|T/W|$ [GM$_{\text{solar}}^2/c$] |
| HS(DD2) $s_B=1$, $Y_L=0.4$ | 0.380 | 1.689 | 16.076 | 0.358 | 1.704 | 22.860 | 687 | 1.335 | 0.618 | 0.051 |
|              | $s_B=2$, $Y_L=0.4$ | 0.348 | 1.715 | 18.291 | 0.314 | 1.725 | 26.280 | 560 | 1.233 | 0.628 | 0.040 |
|              | $s_B=2$, $Y_L=0$ | 0.330 | 1.667 | 15.866 | 0.287 | 1.689 | 22.784 | 694 | 1.661 | 0.597 | 0.075 |
|              | $T= 0$ | 0.386 | 1.619 | 13.252 | 0.331 | 1.652 | 18.554 | 938 | 1.963 | 0.557 | 0.115 |
| BHBA $s_B=1$, $Y_L=0.4$ | 0.392 | 1.689 | 16.060 | 0.355 | 1.702 | 22.858 | 688 | 1.332 | 0.618 | 0.051 |
|              | $s_B=2$, $Y_L=0.4$ | 0.361 | 1.715 | 18.084 | 0.330 | 1.724 | 26.268 | 567 | 1.230 | 0.623 | 0.040 |
|              | $s_B=2$, $Y_L=0$ | 0.408 | 1.665 | 15.222 | 0.336 | 1.683 | 22.124 | 727 | 1.631 | 0.596 | 0.074 |
|              | $T= 0$ | 0.435 | 1.619 | 13.164 | 0.331 | 1.652 | 18.632 | 938 | 1.967 | 0.554 | 0.115 |
| BHBA $\phi$ $s_B=1$, $Y_L=0.4$ | 0.389 | 1.689 | 16.048 | 0.355 | 1.702 | 22.865 | 688 | 1.333 | 0.617 | 0.051 |
|              | $s_B=2$, $Y_L=0.4$ | 0.357 | 1.715 | 18.087 | 0.327 | 1.725 | 26.076 | 566 | 1.229 | 0.629 | 0.040 |
|              | $s_B=2$, $Y_L=0$ | 0.385 | 1.665 | 15.314 | 0.326 | 1.683 | 22.071 | 724 | 1.626 | 0.598 | 0.074 |
|              | $T= 0$ | 0.418 | 1.619 | 13.191 | 0.332 | 1.652 | 18.580 | 938 | 1.967 | 0.556 | 0.115 |
TABLE III. Properties of non-rotating and rotating PNSs at the limiting frequency, for a fixed baryonic mass $M_B = 2.2 M_{\odot}$. The parameters in the table are: central baryon number density ($n_c$), gravitational mass ($M_G$), circumferential equatorial radius ($R_{eq}$), Kepler frequency ($\nu_K$), angular momentum ($J$), polar to equatorial axis ratio ($r_p/r_{eq}$) and the rotation parameter ($|T/W|$).

| EoS          | PNS stages | $\nu = 0$ | $\nu = \nu_K$ | $\nu = 0$ | $\nu = \nu_K$ | $\nu = 0$ | $\nu = \nu_K$ | $\nu = 0$ | $\nu = \nu_K$ |
|--------------|------------|-----------|---------------|-----------|---------------|-----------|---------------|-----------|---------------|
|              |            | $n_c$ [fm$^{-3}$] | $M_G [M_{\odot}]$ | $R_{eq} [\text{km}]$ | $n_c$ [fm$^{-3}$] | $M_G [M_{\odot}]$ | $R_{eq} [\text{km}]$ | $\nu_K$ [Hz] | $J$ $[GM_{\odot}^2/c]$ | $r_p/r_{eq}$ | $|T/W|$ |
| HS(DD2)      | $s_B=1$, $Y_L=0.4$ | 0.466 | 2.011 | 14.997 | 0.413 | 2.041 | 20.433 | 833 | 2.055 | 0.634 | 0.063 |
|              | $s_B=2$, $Y_L=0.4$ | 0.440 | 2.044 | 16.404 | 0.393 | 2.063 | 23.274 | 721 | 1.938 | 0.623 | 0.053 |
|              | $s_B=2$, $Y_L=0$ | 0.409 | 1.99 | 15.152 | 0.338 | 2.013 | 21.395 | 818 | 2.412 | 0.595 | 0.085 |
|              | $T= 0$ | 0.452 | 1.930 | 13.197 | 0.367 | 1.955 | 18.563 | 949 | 2.669 | 0.526 | 0.107 |
| BHBA        | $s_B=1$, $Y_L=0.4$ | 0.526 | 2.01 | 14.779 | 0.433 | 2.039 | 20.231 | 838 | 2.048 | 0.638 | 0.063 |
|              | $s_B=2$, $Y_L=0.4$ | 0.514 | 2.042 | 15.867 | 0.423 | 2.066 | 23.117 | 744 | 1.944 | 0.612 | 0.053 |
|              | $s_B=2$, $Y_L=0$ | 0.664 | 1.978 | 13.613 | 0.439 | 2.022 | 20.197 | 892 | 2.376 | 0.597 | 0.084 |
|              | $T= 0$ | 0.732 | 1.925 | 12.364 | 0.405 | 1.962 | 17.596 | 976 | 2.621 | 0.565 | 0.108 |
| BHBA$\phi$  | $s_B=1$, $Y_L=0.4$ | 0.513 | 2.01 | 14.804 | 0.428 | 2.038 | 21.031 | 844 | 2.073 | 0.608 | 0.064 |
|              | $s_B=2$, $Y_L=0.4$ | 0.496 | 2.042 | 15.926 | 0.416 | 2.066 | 23.085 | 742 | 1.94 | 0.614 | 0.053 |
|              | $s_B=2$, $Y_L=0$ | 0.558 | 1.983 | 14.107 | 0.419 | 2.021 | 20.452 | 880 | 2.387 | 0.594 | 0.085 |
|              | $T= 0$ | 0.573 | 1.928 | 12.812 | 0.375 | 1.973 | 19.265 | 961 | 3.013 | 0.518 | 0.12 |
FIG. 1. Pressure versus number density is plotted for different stages of quasi-stationary evolution of the compact star. The three panels from left to right are for HSDD2, BHBA and BHBA$\phi$ EoS respectively.
FIG. 2. Gravitational mass versus radius for the corresponding EoS of Fig. 1.
FIG. 3. Temperature profile of the compact star as it evolves from $s_B = 1$, $Y_L = 0.4$ to $\beta$-equilibrated neutron star of $s_B = 2$, for different compositions of matter.
FIG. 4. Gravitational mass, radius, moment of Inertia and quadrupole moment of a star evolving according to Sec. II are plotted in (a), (b), (c) and (d) respectively as function of rotation rate. All plots refer to a star with fixed baryonic mass $M_B=1.8M_{\text{solar}}$. 
FIG. 5. Normalized moment of inertia ($I/M^3$) with compactness ($M/R$) for a star rotating at different frequencies as it evolves from PNS to NS.
FIG. 6. Normalized moment of inertia (I/MR²) variation with compactness (M/R) for same cases as in Fig. 5.
FIG. 7. Relative differences $|\bar{I} - \bar{I}_{\text{fit}}|/\bar{I}_{\text{fit}}$ for a slowly rotating PNS at different evolutionary stages; in the left panel $\bar{I} = I/M^3$ and in the right panel $\bar{I} = I/MR^2$. Different color schemes and different symbols are used for different EoS and evolutionary stages respectively.
FIG. 8. Relative differences $|\bar{I} - \bar{I}_{\text{fit}}|/\bar{I}_{\text{fit}}$ for a slowly rotating fixed PNS at different evolutionary stages; in the upper panels $\bar{I} = I/M^3$, whereas in the lower panel $\bar{I} = I/MR^2$. Different color schemes and different symbols are used for different EoS and evolutionary stages respectively. The two columns are for fixed baryon mass $M_B = 1.8M_{\odot}$ and $2.2M_{\odot}$. 