We present the results of a novel model-independent fit to elastic proton-proton differential cross section data at $\sqrt{s} = 52.8$ GeV. Taking into account the error propagation from the fit parameters, we determine the scattering amplitude in the impact parameter space (the proton profile function) and its statistical uncertainty region. We show that both the real and imaginary parts of the profile are consistent with two dynamical contributions, one from a central dense region, up to roughly 1 fm and another from a peripheral evanescent region from 1 to 3 fm.

1. Introduction

In the absence of a pure QCD description of high-energy soft diffractive processes, elastic hadron scattering constitutes one of the topical unsolved problems in Particle Physics. At this stage, empirical information, extracted from the experimental data, play a fundamental role in the construction of phenomenological models and to establish connections between experimental data and first principles and theorems in the underlying quantum field theory. With this strategy, we have investigated proton-proton ($pp$) and antiproton-proton ($\bar{p}p$) elastic scattering by means of model independent approaches (the inverse problem) and have extracted several properties of these processes in the impact parameter space. In these works we made use of an analytical parametrization for the scattering amplitude as a sum of exponentials, but with constrained real and imaginary parts. In this communication, we make use of a novel analytical parametrization for the amplitude without the above constraint and we also develop a detailed data reduction procedure which allows a better fit result on statistical grounds. We treat here only $pp$ scattering at $\sqrt{s} = 52.8$ GeV together with data in the large momentum transfer region from $pp$ scattering at 27.5...
From this data reduction, we extract the complex Profile Function (impact parameter space) and show that both the real and imaginary parts of the profile indicate two distinct contributions, one central and one peripheral.

2. Parametrization and Fit Results

We consider here the following physical quantities that characterize the high-energy elastic hadron scattering:

\[
\frac{d\sigma}{dq^2}(s, q^2) = \pi |F(s, q^2)|^2, \tag{1}
\]

the total cross section (Optical Theorem) and the \(\rho\) parameter,

\[
\sigma_{\text{tot}}(s) = 4\pi \text{Im} F(s, q^2 = 0), \quad \rho(s) = \frac{\text{Re} F(s, q^2 = 0)}{\text{Im} F(s, q^2 = 0)}, \tag{2}
\]

where \(F\) is the scattering amplitude, \(s\) and \(q^2 = -t\) are the Mandelstam variables.

The largest set of data presently available on differential cross sections, as a function of the momentum transfer, concerns proton-proton scattering at \(\sqrt{s} = 52.8\) GeV. The data was carefully compiled and normalized by Amaldi and Schubert\(^4\) and include the optical point, \(d\sigma/dq^2(q^2 = 0) = \sigma_{\text{tot}}^2/(1 + \rho^2)/16\pi\), with the average experimental values \(\sigma_{\text{tot}} = 42.67 \pm 0.19\) mb and \(\rho = 0.078 \pm 0.010\). Based on the independence of the experimental data with the energy at large momentum transfer (ISR region)\(^2\) we also include in the analysis the data from \(pp\) scattering at \(\sqrt{s} = 27.5\) GeV in the region \(5.5 \leq q^2 \leq 14\) GeV\(^2\).

We consider the parametrization for the amplitude as a sum of exponentials in \(q^2\), with both real and imaginary parts connected with the forward experimental data on \(\sigma_{\text{tot}}\) and \(\rho\) through Eq. (2). Denoting \(a_i, b_i, i = 1, 2, ..., m\) and \(c_j, d_j, j = 1, 2, ..., n\) the real free parameters associated with the real and imaginary parts of the amplitude, respectively, the parametrization can be expressed by

\[
F(s, q^2) = \left\{ \frac{\rho \sigma_{\text{tot}}}{4\pi} - \sum_{i=2}^{m} a_i \right\} e^{-b_i q^2} + \sum_{i=2}^{m} a_ie^{-b_i q^2} + i \left\{ \frac{\sigma_{\text{tot}}}{4\pi} - \sum_{j=2}^{n} c_j \right\} e^{-d_j q^2} + \sum_{j=2}^{n} c_j e^{-d_j q^2} \right\}, \tag{3}
\]

allowing the elimination of two free parameters, which we choose above to be \(a_1\) (real part) and \(c_1\) (imaginary part). With this we have \(2(m + n) - 2\) free fit parameters, with \(\sigma_{\text{tot}}\) and \(\rho\) given by the corresponding experimental values quoted above.

With parametrization (3) we fit the \(pp\) differential cross section data at 52.8 GeV through Eq. (1) and the CERN-Minuit code. Since we have no information on the contribution from the real part of the amplitude beyond the forward region, we started the fit by looking for the best result with all possible choices of contributions, namely, \((m, n) = (0, 1), (1, 1), (1, 2), (2, 1), (2, 2), \text{etc...}\), until reaching the smallest
$\chi^2$/DOF. In a second step we added the data from 27.5 GeV, testing in the same way different contributions from the real and imaginary parts.

In Fig. 1 we display the fit results to $pp$ data at $\sqrt{s} = 52.8$ GeV ($q_{\text{max}}^2 = 9.8$ GeV$^2$, left panel) and adding data at 27.5 GeV ($q_{\text{max}}^2 = 14$ GeV$^2$, right panel), together with the corresponding experimental data and uncertainty regions obtained by error propagation from the fit parameters. In the first case we obtained $\chi^2$/DOF = 1.43 and in the second $\chi^2$/DOF = 1.55, which are below our previous results, $\chi^2$/DOF = 1.65 and $\chi^2$/DOF = 2.07, respectively.

3. Proton Profile Function

In the Impact Parameter Representation (Fraunhofer diffraction), the elastic scattering amplitude at fixed energy is expressed by $\Gamma(q) = i \int_0^\infty b dB J_0(bq) \Gamma(b)$, where $b$ is the impact parameter, $J_0$ is the zero order Bessel function (azimuthal symmetry assumed) and $\Gamma(b)$ is the Profile Function. By inverting the symmetrical Fourier transform we can determine Re $\Gamma(b)$ and Im $\Gamma(b)$ in terms of Im $F(q)$ and Re $F(q)$, respectively. The results obtained by means of parametrization (3) and the fit up to $q_{\text{max}}^2 = 14$ GeV$^2$ are shown in Fig. 2, together with the uncertainty region determined by standard error propagation from the fit parameters.

4. Conclusions

From Fig. 2 we see that, within the uncertainty regions, both the real and imaginary parts of the profile present a change of curvature around 1 fm, with the imaginary
part becoming negative above this point (1 - 3 fm). We can interpret the result as two different (dynamical) contributions, one associated with a dense central region and another with an evanescent peripheral region. This naive interpretation, however, seems not in disagreement with the picture of a hadron as a valence quark core surrounded by a gluon cloud.

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