Neutron Monitors and muon detectors for solar modulation studies: Interstellar flux, yield function, and assessment of critical parameters in count rate calculations

D. Maurin\textsuperscript{a,\,*}, A. Cheminet\textsuperscript{b,c}, L. Derome\textsuperscript{a}, A. Ghelfi\textsuperscript{a}, G. Hubert\textsuperscript{b}

\textsuperscript{a}LPSC, Université Grenoble-Alpes, CNRS/IN2P3, 53 avenue des Martyrs, 38026 Grenoble, France
\textsuperscript{b}ONERA (French Aerospace Lab), 2 avenue Edouard Belin, 31055 Toulouse Cedex 4, France
\textsuperscript{c}IRSN (Institute for Radiological Protection and Nuclear Safety), CE Cadarache, Bât. 159, 13115 Saint-Paul-Lez-Durance Cedex, France

Abstract

Particles count rates at given Earth location and altitude result from the convolution of (i) the interstellar (IS) cosmic-ray fluxes outside the solar cavity, (ii) the time-dependent modulation of IS into Top-of-Atmosphere (TOA) fluxes, (iii) the rigidity cut-off (or geomagnetic transmission function) and grammage at the counter location, (iv) the atmosphere response to incoming TOA cosmic rays (shower development), and (v) the counter response to the various particles/energies in the shower. Count rates from neutron monitors or muon counters are therefore a proxy to solar activity. In this paper, we review all ingredients, discuss how their uncertainties impact count rate calculations, and how they translate into variation/uncertainties on the level of solar modulation \(\phi\) (in the simple Force-Field approximation). The main uncertainty for neutron monitors is related to the yield function. However, many other effects have a significant impact, at the 5\(-10\% level on \(\phi\) values. We find no clear ranking of the dominant effects, as some depend on the station position and/or the weather and/or the season. An abacus to translate any variation of count rates (for neutron and \(\mu\) detectors) to a variation of the solar modulation \(\phi\) is provided.

Keywords: Cosmic rays, Solar modulation, Yield function, Geomagnetic cutoff, Neutron monitor, Muon detector

1. Introduction

After the discovery of cosmic rays (CR) by Hess in 1912, ground-based CR detectors located at various latitudes, longitudes and altitudes, played a major role to determine the CR composition and spectrum (see Stoker 2009 for a historical perspective). From the 50’s, networks of Neutron monitors (Simpson, 2000) and muon telescopes (Duldig, 2000) were developed. They provide today one of the most valuable data to inspect time variations of the integrated CR flux in the 10 – 100 GeV/n range.

The formal link between these variations and the Sun activity was established in the mid-fifties, by means of a transport equation of CR fluxes in the solar cavity (Parker, 1965, Jokipii 1966). In the 80’s, the effect of particle drift was shown to be responsible to a charge-sign dependent modulation (Potgieter, 2013), following the Sun polarity cycle\textsuperscript{1}. However, the Force-Field approximation (Gleeson and Axford, 1967, 1968) has remained widely used thanks to its simplicity: this approximation, used in this work, has only one parameter \(\phi(t)\).

Several strategies have been developed for time series reconstruction of the modulation level \(\phi(t)\), and/or CR TOA fluxes at any time (of interest for many applications):

- Using CR data (e.g., Davis et al., 2001; Buchvarova et al., 2011; Buchvarova and Draganov, 2013): it is the most direct approach, but the time coverage is limited to a few decades with a poor sampling;
- Comparison of calculated and measured count rates in ground-based detectors (Usoskin et al., 1999, 2002, 2005, 2011): it covers a larger period (60 yrs), with a very good time resolution (a few minutes)\textsuperscript{2};
- Extracting relationships between the modulation level and solar activity proxies, based on empirical (Badhwar and O’Neill 1994, 1996; O’Neill 2006; O’Neill 2010) or semi-empirical (Nymmik et al. 1992, Nymmik et al. 1994, 1996; Tylka et al. 1997; Nymmik 2007; Ahluwalia 2013) approaches.

All these strategies provide a satisfactory description of CR fluxes, though some fare better than others (for comparisons, see Buchvarova and Velinov 2010; Mrigakshi et al. 2012; Zhao and Qin 2013; Matthiä et al. 2013). Note

\textsuperscript{1}A 11-yr average periodicity was established ∼ 250 yrs ago from sunspot series (see Vaquero, 2007; Usoskin, 2013, for a review). The now well observed 22-yr cycle (polarity reversal every 11 yrs) was first hinted at from magnetograph observations by Babcock (1961).

\textsuperscript{2}In the same spirit, the concentration of the cosmogenic radionuclide \(^{10}\)Be in ice cores (Webber and Higbie, 2003; Herbst et al., 2010) covers several thousands of years, but with a poor time resolution.
also that empirical methods are expected to provide effective and less meaningful values for $\phi$ (O’Neill, 2006).

In this paper, we focus on the second strategy, for a systematic study of all uncertainties affecting the calculation of expected count rates in NM and muon detectors. This requires the description of the atmosphere and of the ground-based detector responses to incoming CRs (e.g., Clem and Dorman, 2000). Many uncertainties have been discussed separately in the literature ( uncertainty on the yield function, geomagnetic rigidity cutoff, seasonal effects...). We believe it is useful to recap and gather them in a single study, re-assess which ones are the most important, and link these uncertainties to the expected level of variation/uncertainty they imply on the modulation level $\phi(t)$. The complementarity (different uncertainties and time coverage) of NM count rates and TOA CR flux measurements to obtain time-series of the solar modulation parameters is left to a second study\(^3\).

The paper is organised as follows: we start with a general presentation of the ingredients involved in the count rate calculations (Sect. 2), and discuss a new fit for the IS fluxes (Sect. 3). We then detail the calculation of the propagation in the atmosphere, providing a new yield function parametrisation (Sect. 4). Combining these inputs allows us to link the count rate variation with the solar modulation parameter, and to study the various sources of uncertainties (Sect. 5). The final ranking of the uncertainties in terms of both count rates and $\phi$ concludes this study (Sect. 6).

2. From IS fluxes to ground-based detector count rates

A ground-based detector $D$ at geographical coordinate $\vec{r} = (\varphi, \lambda, h)$ measures, at time $t$, a count rate per unit interval $N^D(\vec{r},t)$, from the production (from CRs) of secondary particles in the atmosphere (atmospheric shower):

$$N^D(\vec{r},t) = \int_0^\infty T(\vec{r},R,t) \times \sum_{i=CRs} Y_i^D(R,h) \frac{dJ^\text{TOA}}{dR}(R,t) dR,$$

with $R = pc/Ze$, and $i$ running on CR species:

- $T(\vec{r},R,t)$ is the transmission function in the geomagnetic field, which depends on the detector location and can vary with time (Sect. 5.2.2);
- $Y_i^D(R,h)$ is the yield function at altitude $h$, i.e. the detector response (in count m$^{-2}$ sr) to a unit intensity of primary CR species $i$ at rigidity $R$ (Sect. 4);
- $dJ^\text{TOA}/dR$ is the top-of-atmosphere (TOA) modulated differential flux (or intensity) per rigidity interval $dR$, at rigidity $R$, time $t$, and for the CR species $i$ (in m$^{-2}$ s$^{-1}$ sr$^{-1}$ GV$^{-1}$). It is obtained from the interstellar flux $J^\text{IS}$ (Sects. 2.1 and 3) modulated by a solar modulation model (Sect. 2.2).

In the most general case, $T$ and $Y$ above are entangled, due to the complex structure of the geomagnetic field, and the dependence of the transmission factor and the yield function on the primary particle incident angle (see Sect. 5.2.2). A common practice is to consider the two terms independently, average the yield function over a few incident angles, and take a simple rigidity (or equivalently energy) effective vertical cutoff for the transmission function (see, e.g., Cooke et al., 1991, for definitions). In this paper, unless stated otherwise, this is what we assume, and the effective vertical cutoff rigidity $R^\text{eff}$ is referred to as the rigidity cut-off $R_c$ for short.

2.1. Interstellar flux

At high energy ($\gtrsim 50$ GeV/n), the effect of solar modulation is negligible, and the IS spectra is directly obtained from CR data measurements. The recent PAMELA (PAMELA Collaboration et al., 2011) and CREAM (Ahn et al., 2010) data hint at a hardening of the spectrum above a few hundreds of GeV/n. However, preliminary AMS-02 results (shown at ICRC 2013 in Rio) seem to indicate otherwise. In any case, the CR contribution to ground-based detector count rates above 10 TeV/n is negligible, whereas CRs above 100 GeV/n contribute to $\lesssim 10\%$ of the total (see Fig. 10). Hence, the results in this paper do not depend on the exact high energy dependence of the IS fluxes. Waiting for a clarification, we assume that a pure power law prevails up to the highest energies.

At lower energy, fluxes are modulated by the solar activity (Sect. 2.2). Measurements at different times and/or different positions in the solar cavity (e.g., Webber et al., 2008; Webber and Higbie 2009) allow to get the IS spectrum down to several hundreds of GeV/n, whereas other proxies can push this limit down to a few tens of MeV/n: actually (i) indirect measurements from CR ionisation in the ISM (Webber, 1987; Nath and Biermann, 1994; Webber, 1998); (ii) the impact of CR on molecules formation (Padovani et al., 2009; Indriolo and McCall, 2012; Nath et al., 2012), and (iii) $\gamma$-ray emissions in molecular clouds Neronov et al. (2012), seem to favour a low-energy flattening/break. A recent and exciting development is provided by the Voyager 1 spacecraft, which is witnessing what is believed to be the first direct measurement of the local interstellar spectrum in the $10 - 100$ MeV/n energy range (Webber and Higbie, 2013; Webber et al., 2013a,b).

2.2. Force-Field approximation for solar modulation

The force-field approximation was first derived by Gleeson and Axford (1967, 1968). A simpler derivation is provided, e.g., in Perko (1987) and Boella et al. (1998), and the force-field approach limitation is discussed in Caballero-Lopez and Moraal (2004). It provides an analytical one-to-one correspondence between TOA and IS

\(^3\)Recent CR instruments such as PAMELA and the AMS-02 on the International Space Station are or will be providing high-statistics fluxes on an unprecedented time frequency, which renders this comparison even more appealing.
energies, and also fluxes. For a given species (mass number $A$ and charge $Z$), at any given time, we have ($E$ is the total energy, $p$ the momentum, $T/n$ the kinetic energy per nucleon, and $J \equiv dJ/dT/n$ is the CR intensity with respect to $T/n$):

$$\frac{E^{\text{TOA}}}{A} = \frac{E^{\text{IS}}}{A} - \frac{|Z|}{A} \phi,$$

$$J^{\text{TOA}} (E^{\text{TOA}}) = \left( \frac{p^{\text{TOA}}}{p^{\text{IS}}} \right)^2 \times J^{\text{IS}} (E^{\text{IS}}),$$

where the solar modulation parameter $\phi(t)$ has the dimension of a rigidity (or an electric potential). Equation (2) amounts to both an energy and flux shift of the IS quantities (toward smaller values) to get TOA ones. We recall that $\Phi = |Z|/A \times \phi$ is sometimes used instead of $\phi$ (used throughout the paper).

3. Determination of IS fluxes: from H to Ni

Due to the interplay between the CR relative abundances and the yield function, the most important primary CR contributors to the count rates are protons, heliums, and heavier nuclei (in a small but non negligible fraction). In recent studies, in addition to proton and helium, the contribution of species heavier than He is accounted for (see Sect. 3.2). The best-fit parameters and their error are gathered in Table 1, and the value $\phi$ for each epoch are given in Table 2. Note that the uncertainties for heavy nuclei are probably underestimated since it is based on a single set of data (HEAO3-C2) for energies above a few GeV/n. The next to last column represents (at 10 GV) the fraction of a given CR flux to the sum of all CR fluxes: due to the scarcity of data, we have no choice here, but to rely on many experiments, i.e. AMS01 (AMS Collaboration et al., 2002, 2011), BESS93 (Wang et al., 2002), BESS94, 95, 97, and 98 (Myers et al., 2005), BESS00 (Kim et al., 2013), CAPRICE94 (Boezio et al., 1999), CAPRICE98 (Papini et al., 2004), IMAX92 (de Nolfo et al., 2000; Menn et al., 2000), and PAMELA (PAMELA Collaboration et al., 2013b) measurements.

Best-fit values. The parameters $c_1$, $c_2$, and $c_3$ of Eq. (3) are simultaneously fitted to H and He data, and then up to Fe data, having a single modulation level for each data taking period. This is necessary to reduce the degeneracy between the chosen IS flux parametrisation and the modulation parameter (see Sect. 3.2). The best-fit parameters and their error are gathered in Table 1, and the value $\phi$ for each epoch are given in Table 2. Note that the uncertainties for heavy nuclei are probably underestimated since it is based on a single set of data (HEAO3-C2) for energies above a few GeV/n. The next to last column represents (at 10 GV) the fraction of a given CR flux to the sum of all contributions. The last column gives, at the same rigidity, a gross estimate of the yield weighted contribution $\langle f_j \rangle_Y$ of any species $j$ to NM count rates:

$$\langle f_j \rangle_Y = \sum_i \frac{\mathcal{Y}_i(R) \times J_{i,\text{TOA}}(R)}{\mathcal{Y}_i(R) \times J_{i,\text{TOA}}(R)},$$

$$\langle f_j \rangle_Y \approx \langle f_j \rangle_A = \sum_i \frac{A_i J_{i,\text{TOA}}}{\langle A_i J_{i,\text{TOA}} \rangle}.$$
Table 1: Best-fit parameters \( C_0, C_1, \) and \( C_2 \) in Eq. (4) for all IS CR flux elements \( j \) from H to Fe. The last two columns (calculated at 10 GV) are the fraction of the flux, and of the contribution of each species to NM count rates Eq. (6). See text for details.

| CR | \( C_0 \) (m² s sr GV⁻¹) | \( C_1 \) | \( C_2 \) | \( f_j = \frac{J_j}{\sum_i J_i} \) | \( \langle f_j \rangle_A = \frac{A_i J_j}{\sum_i A_i J_i} \) |
|----|-------------------|-------|-------|----------------|-----------------
| H \(^1\) | 23350 ± 184 | 2.10 ± 0.10 | 2.839 ± 0.003 | 83.7 | 49.1 |
| \(^2\)H | 835.5 ± 29.5 | 3.62 ± 0.08 | 2.950 ± 0.060 | 2.21 | 2.59 |
| \(^3\)He | 512.7 ± 2.0 | 6.70 ± 0.01 | 3.045 ± 0.003 | 1.08 | 1.90 |
| \(^6\)He \(^\dagger\) | 3657.3 ± 38.5 | 1.77 ± 0.04 | 2.782 ± 0.003 | 14.6 | 34.4 |
| Li | 18.86 ± 0.86 | 4.58 ± 0.07 | 3.200 ± 0.400 | 0.027 | 0.11 |
| Be | 22.09 ± 0.18 | 6.57 ± 0.08 | 2.948 ± 0.003 | 0.054 | 0.29 |
| B | 72.77 ± 0.32 | 5.77 ± 0.01 | 3.086 ± 0.002 | 0.132 | 0.85 |
| C | 116.5 ± 0.50 | 4.26 ± 0.01 | 2.791 ± 0.002 | 0.438 | 3.08 |
| N | 45.7 ± 0.45 | 5.19 ± 0.02 | 2.971 ± 0.004 | 0.112 | 0.92 |
| O | 95.5 ± 0.35 | 3.87 ± 0.01 | 2.733 ± 0.002 | 0.413 | 3.88 |
| F | 35.73 ± 0.03 | 5.63 ± 0.02 | 2.979 ± 0.005 | 0.084 | 0.94 |
| Ne | 16.75 ± 0.10 | 4.29 ± 0.02 | 2.779 ± 0.003 | 0.065 | 0.76 |
| Na | 4.945 ± 0.034 | 5.02 ± 0.02 | 2.922 ± 0.003 | 0.013 | 0.18 |
| Mg | 20.25 ± 0.10 | 4.03 ± 0.02 | 2.755 ± 0.003 | 0.083 | 1.17 |
| Al | 4.165 ± 0.029 | 4.65 ± 0.02 | 2.812 ± 0.004 | 0.015 | 0.23 |
| Si | 13.5 ± 0.10 | 3.86 ± 0.02 | 2.681 ± 0.003 | 0.066 | 1.08 |
| P | 1.084 ± 0.014 | 5.99 ± 0.04 | 2.938 ± 0.007 | 0.003 | 0.05 |
| S | 3.445 ± 0.025 | 4.87 ± 0.02 | 2.785 ± 0.004 | 0.013 | 0.24 |
| Cl | 1.428 ± 0.015 | 6.65 ± 0.03 | 3.052 ± 0.007 | 0.003 | 0.06 |
| Ar | 2.64 ± 0.04 | 6.24 ± 0.04 | 3.075 ± 0.007 | 0.005 | 0.11 |
| K | 2.192 ± 0.005 | 6.37 ± 0.06 | 3.110 ± 0.010 | 0.004 | 0.09 |
| Ca | 3.70 ± 0.03 | 5.24 ± 0.05 | 2.991 ± 0.005 | 0.009 | 0.20 |
| Sc | 1.106 ± 0.019 | 5.68 ± 0.04 | 3.120 ± 0.008 | 0.002 | 0.05 |
| Ti | 3.126 ± 0.032 | 4.96 ± 0.03 | 3.062 ± 0.005 | 0.006 | 0.17 |
| V | 1.357 ± 0.014 | 4.82 ± 0.03 | 2.995 ± 0.006 | 0.003 | 0.09 |
| Cr | 2.271 ± 0.019 | 4.51 ± 0.03 | 2.919 ± 0.005 | 0.006 | 0.19 |
| Mn | 1.132 ± 0.014 | 4.11 ± 0.03 | 2.776 ± 0.005 | 0.004 | 0.14 |
| Fe | 8.032 ± 0.046 | 3.37 ± 0.03 | 2.600 ± 0.001 | 0.047 | 1.55 |
| Co | 0.0055 ± 0.0037 | 3.54 ± 0.03 | 2.610 ± 0.010 | 0.002 < 10⁻¹ | 0.002 < 10⁻² |
| Ni | 8.405 ± 0.019 | 4.50 ± 0.10 | 2.600 ± 0.020 | 0.002 | 0.08 |

\(^\dagger\) \( H = \ ^1\)H+\(^2\)H, and \( He = \ ^3\)He+\(^4\)He.

\( C_2 \approx 2.8 \), or \( C_2 \approx 3.0 \). They correspond respectively to the so-called primary species (CRs accelerated in sources and propagated in the Galaxy) and secondaries species (spallative products of primary species). Furthermore, heavier species suffer more inelastic interactions than lighter ones during propagation, providing a flatter IS flux at low energy. The last two columns on Table 1 show that:

(i) the contribution to count rates of heavy nuclei are significant up to Ni: low fluxes for heavy species are redeemed by the number of nucleons available in the yield function;

(ii) primary species contribute more than secondary ones: the contribution of heavier species \( Z \geq 3 \) w.r.t. to He, in this simple estimate, is \( (Z \geq 3)/\text{He} = 0.480 \), in agreement with the value 0.428 used in Usoskin et al. (2011);

(iii) secondary species \( Z \geq 3 \) contribute up to \( \sim 4\% \) of the total, but due to a steeper slope w.r.t. primary species, their contribution decreases with rigidity;

(iv) the \(^2\)H and \(^3\)He isotopes also contribute to \( 4\% \) of the total, and they should be dealt with separately from the rest because of their different A/Z value (they are not similarly modulated).

Anticipating on the description of a realistic yield function (presented in Sects. 4.2 and 4.3), Fig. 1 shows the result of the full calculation (without approximation) Eq. (6), as a function of rigidity (for TOA fluxes modulated at \( \phi = 600 \) MV). The top panel zooms in on the fractional contributions of secondary species, and primary species heavier than He. The numbers are in fair agreement with those given in the last column of Table 1, but with two noteworthy features:
Table 2: Best-fit modulation parameters for each experiment considered, along with the published values (if exists) for illustration.

| Experiment | Period         | $\phi_{\text{fit}}$ (MV) | $\phi_{\text{publ.}}$ (MV) |
|------------|----------------|---------------------------|-----------------------------|
| HEAO3-C2   | 1979/10-1980/06| 648 ± 13                  | 600                         |
| IMAX92     | 1992/07        | 698 ± 9                   | 750                         |
| CAPRICE94  | 1994/08        | 671 ± 12                  | 710                         |
| ACE-CRIS   | 1997/08-1998/04| 227 ± 10                  | 325                         |
| ACE-CRIS   | 1998/01-1999/01| 396 ± 9                   | 550                         |
| CAPRICE98  | 1998/05        | 502 ± 17                  | 600                         |
| AMS-01     | 1998/06        | 648 ± 9                   | 650                         |
| BESS00     | 2000/08        | 1339 ± 14                 | 1300                        |
| BESS-TeV   | 2002/08        | 1004 ± 11                 | 1109                        |
| ACE-CRIS   | 2001/05-2003/09| 796 ± 18                  | 900                         |
| PAMELA     | 2006/11-2006/12| 391 ± 4                   | -                           |
| PAMELA     | 2007/11-2007/12| 426 ± 4                   | -                           |
| PAMELA     | 2006/07-2006/12| 461 ± 6                   | 500                         |
| PAMELA     | 2006/07-2009/12| 522 ± 5                   | -                           |
| PAMELA     | 2008/11-2012/12| 399 ± 5                   | -                           |
| ACE-CRIS   | 2009/03-2010/01| 188 ± 15                 | 250                         |
| PAMELA     | 2009/12-2010/01| 248 ± 6                   | -                           |

(i) the contributions are not constant with energy, peaking between 10 – 50 GV for secondary species, while constantly increasing for primary species. The heavier the species, the larger the increase. This is explained by the increase of the ratio of heavy to light primary species with energy, due to spallation effects at low energy (see Fig. 14 of Putze et al., 2011). As a result, at 100 GV, the Fe contribution is almost at the level of the C one;

(ii) $^2$H has a different A/Z ratio than all other species shown in the top panel, hence its contribution is shifted to lower rigidity.

The bottom panel of Fig. 1 shows the contributions of $^1$H=H−$^2$H (50%), $^4$He=He−$^3$He (30%), and the sum of all other contributions (20%). The latter differs significantly from previous results:

(i) the full calculation of the contribution of species heavier than helium gives

$$s_{Z>2} = 0.611^{+0.016}_{-0.009},$$

instead of the value 0.428 obtained in the simple estimate and used in the literature;

(ii) we check that this approximation is better than 1% for all modulation levels, as illustrated by the difference between ‘true’ (all species) and ‘scaled’ $^4$He contributions, shown in grey. The uncertainty $^{0.016}_{-0.009}$, i.e. $\sim 1.5 – 2.5\%$ on this factor, is obtained by propagating the errors on the CR IS flux parameters given in Table 1.

3.2. Degeneracy between $J_{\text{IS}}$ and $\phi$

It has been shown that taking different parametrisations for the IS fluxes provides similar TOA fluxes and count rates, but with a shifted modulation parameters in time series (Usoskin et al., 2005; Herbst et al., 2010). Indeed, unless either strong assumptions are made on the transport coefficients in the solar cavity, or IS data are available, or sufficient data covering all modulation periods with a good precision exist, the degeneracy is difficult to lift.

To illustrate this point, Fig. 2 shows the IS proton flux (symbols) for several parametrisations behaving very differently at low energy. The reference IS flux (star) is the one fitted in the previous section. We then modulate protons for this reference flux (solid lines) at three different modulation levels ($\phi_{\text{ref}} = 0.4$ GV in red, 0.9 GV in blue,
The parameter \( \Delta \phi \) gives the mean modulation shift in order to apply to \( \phi_{\text{ref}} \) in order to minimise the difference between (i) the reference modulation level and (ii) a given IS parametrisation \( \phi \) for all IS flux parametrisations. As can be seen on the figure, there always exists a value for which all the fluxes are very close to one another: this is what is meant by a degeneracy between the IS flux parametrisation and the modulation parameter value. The shift to apply slightly depends on \( \phi_{\text{ref}} \) itself (Fig. 3 in Herbst et al. 2010). It is illustrated in Fig. 4 showing \( \phi_{\text{ref}} \) for all IS flux parametrisations. Except for WH09 (Webber and Higbie, 2009), the most recent parametrisations are in better agreement than the older ones. The BO11 model (O’Neill, 2010) is the most compatible with the present study.

He fluxes \((\times 0.1)\) are also shown in Fig. 2: the solid line is our best-fit IS flux, others He fluxes being obtained by scaling protons by 0.05 (Usoskin et al., 2005). For a better view, \( p/He \) ratios for all parametrisations are plotted in Fig. 4 (for \( \phi_{\text{ref}} = 400 \) MV), against the recent PAMELA data (PAMELA Collaboration et al., 2011). We find that the following scaling values give less scatter w.r.t. the data:

\[
J_{\text{He}/p} = \frac{dJ_{\text{He}}/dR}{dJ_{p}/dR} = \begin{cases} 
0.0525 & (\text{GC75}), \\
0.0475 & (B00+U05), \\
0.0550 & (L03), \\
0.0560 & (WH03), \\
0.0525 & (S07), \\
0.0605 & (WH09).
\end{cases}
\]

3.3. Dealing with IS flux uncertainties in count rate and \( \phi \) time series

The above degeneracy prevents us from obtaining substantial constraints on the IS flux. Nevertheless, it is still...
possible to provide useful uncertainties, as TOA fluxes rather than IS fluxes are relevant to count rate calculations.

**TOA flux uncertainty.** It can be estimated in two different approaches:

1. ‘data’ uncertainty: we assume that the reference IS flux model is the correct one, so that TOA uncertainties are directly obtained from the uncertainty on the fitted flux parameters (see Table 1). This gives a \( \sim 2.5\% \) relative uncertainty, as shown in Fig. 5 for H (magenta dashed line) and He (red dotted line);

2. ‘model scatter’ uncertainty: lacking conclusive evidence to favour a particular IS flux shape, we can alternatively assume that all parametrisations are equally valid to provide similar (but not equal) effective TOA flux values. Plugging the appropriate effective modulation level \( \phi_i \) to get, for each IS flux model \( i \), its effective TOA flux (see Fig. 3), we form the quantity

\[
S_i(R) = \frac{J_i^{\text{TOA}}(\phi_i + \Delta \phi_i) - J_i^{\text{TOA}}(\phi_i)}{J_i^{\text{TOA}}(\phi_i)}.
\]

The “model scatter” uncertainty is obtained by keeping minimal and maximal values of \( S_i(R) \) over several \( \phi_i \) and IS flux parametrisations \( i \) (we discard L03 which is too far away from the data). In Fig. 5, the corresponding curves are shown for He (blue stars) and H (cyan circles). Note that some of the IS spectra used in this approach are probably already excluded by current data, so that the uncertainty range \( \sim 10 - 30\% \) for H and He is (certainly too) conservative.

\*A more consistent analysis, i.e. fitting the different IS flux parametrisations on the same data to evaluate a more realistic “model scatter” uncertainty, is left for a future study. The benefit of keeping the IS fluxes as used in the literature so far, is to give a flavour of systematic differences related to their use.

The uncertainty related to the contribution of heavier species should also be taken into account: the ‘scaling approximation’ factor Eq. (6) to account for CR heavier than He gives \( s_{Z>2} = 0.611 \pm 2\% \pm 1\% \).

**Impact on \( \phi \) reconstruction.** Our ignorance of the real IS flux shape has a strong impact on the determination of the modulation level \( \phi \). However, it can be absorbed as a global (i.e., time-independent) shift \( \Delta \phi \) once a given IS flux model is chosen. As seen from Fig. 3, the shift can be quite large (from -250 MV to 200 MV). To obtain times series (and their uncertainties) and compare different results given in the literature, the procedure is as follows: (i) calculate \( N_{\text{ref}}(t) \) and \( \phi_{\text{ref}}(t) \) from a reference \( J_\text{IS}^{\text{ref}}(R) \) and \((1+z_{Z>2}) \times J_\text{He,ref}^{\text{IS}}(R)\); (ii) the modulation level \( \phi_i(t) \)

\[
\begin{align*}
\{ N_{\text{calc}}(t) & = N_{\text{ref}}(t) \pm \Delta N_{\text{ref}}^{\text{TOA}}(t), \\
\phi_i(t) & = \phi_{\text{ref}}(t) ± \Delta \phi_{\text{ref}}^{\text{TOA}}(t) + \Delta \phi_i^{\text{IS}}; \}
\end{align*}
\]

for any given IS flux parametrisation \( J_i^{\text{IS}} \) is then simply related to the reference one:

\[
\begin{align*}
J_i^{\text{TOA}}(\phi_i + \Delta \phi_i) - J_i^{\text{TOA}}(\phi_i) \\
\end{align*}
\]

In the above equations, \( \Delta N_{\text{ref}}^{\text{TOA}} \) and \( \Delta \phi_{\text{ref}}^{\text{TOA}} \) are evaluated by propagating uncertainties of TOA flux quantities (see Fig. 5).

**4. Atmospheric propagation, yield function, and detectors**

When entering the Earth atmosphere, CRs initiate cascades of nuclear reactions involving primary energetic particles (mainly hydrogen and helium but also heavier nuclei) and atmospheric nuclei such as oxygen or nitrogen. The so-called Extensive Air Showers (EAS) generate secondary particles along their path, to be detected by ground-based instruments.

In this section, we discuss the generation of secondary particles (Sect. 4.1) as an input to provide a new yield function parametrisation (Sect. 4.2) for NMs (Sect. 4.3) and muon detectors (Sect. 4.4). We also discuss neutron spectrometers (Sect. 4.5) as a mean to study seasonal effects in NMs.

**4.1. Atmospheric propagation of secondaries (n, p, μ)**

The secondary atmospheric radiation field is composed of various hadronic components (mostly neutrons, protons,
Table 3: AtmoRad main features.

| Parameter | Qty | Bins | Range | Unit |
|-----------|-----|------|-------|------|
| Primary i | H   | 18   | [0.1–251.2] | GeV/n |
|           | He  |      |        |      |
| Secondary k | n | 70 | [10^{-3}–10^{11}] | eV |
|           | p, μ± | 25 | [10^6–10^{11}] |      |
| Incidence | [θ_l, θ_{l+1}] | 3 | {0, π/6, π/3, π/2} | rad |
| Altitude h | 36 | [0–30] | km asl |

and pions). Charged pions undergo leptonic decays producing positive and negative muons. Key quantities are^9^:

- $\varphi_k(T_k, r, t)$: spectral fluence rate (cm$^{-2}$ s$^{-1}$ MeV$^{-1}$) of the $k$-type secondary particle at kinetic energy $T_k$, coordinates $r = (\varphi, \lambda, h)$, and time $t$;
- $\varphi'_i(h, T_i \rightarrow T_k)$: spectral fluence (MeV$^{-1}$) of the $k$-type secondary induced at altitude $h$ by a $i$-type primary of kinetic energy $T_i$, and incidence within the zenith angle range $[\theta_l, \theta_{l+1}]$.

Several works were dedicated to numerically estimate the spectral fluence rate $\varphi_k$ during the solar activity cycle (e.g., Roesler et al. 1998; Roesler et al. 2002; Sato and Nishita 2006; Nesterenok 2013). We rely here on the database of spectral fluence values $\varphi'_i$ presented in Cheminet et al. (2013c), in which Monte Carlo (MC) calculations were performed with GEANT4 (Geant4 Collaboration et al., 2003). For any coordinates $r$ and solar modulation potential $\phi(t)$, the spectral fluence rate $\varphi_k$ is obtained from $\varphi'_i$ read from the database as follows:

$$\varphi_k(T_k, r, t) = S_T \sum_{l=1}^{3} \Omega(\theta_l, \theta_{l+1}) \sum_i \Omega_i(\theta_l, \theta_{l+1}) \sum_{T_k}^{T_{max}} \Delta T_i \times J_i^{\text{TOA}}(T_i, \phi(t)) \times \varphi'_i(h, T_i \rightarrow T_k),$$

with $S_T = \pi \cdot (R_E + h_{\text{max}})^2$, $R_E = 6.378.14$ km (Earth radius), and $h_{\text{max}} = 85$ km (highest atmospheric altitude).

A C++ routine named AtmoRad (ATMOSpheric Radiation) developed at ONERA implements the various ingredients entering Eq. (9). It handles both QGSP\_BERT\_HP and QGSP\_BIC\_HP reference users’ physics lists. Table 3 lists the quantities (primary and secondary species) and bins (energy and altitude range, incidence angles) used. The calculations were validated by extensive comparison with measurements, especially for the neutron component (Cheminet et al., 2013b). In the following we use QGSP\_BERT\_HP physic’s list. Figure 6 is an illustration of the neutron, proton, and muon spectral fluence rates obtained at sea level with a cut-off rigidity $R_c = 0.8$ GV (similar to conditions at the Oulu NM station). The solid and dotted lines correspond to a period of minimum and maximum solar modulation potential $\phi(t)$ equal to 0.4 GV and 1.5 GV, respectively. Muons are the most numerous particles above a few hundreds of MeV, but the relative contribution of various secondaries to count rates in a detector depends on its efficiency to each species.

### 4.2. Yield function calculation and parametrisation

As given in Eq. (1), the yield function $Y^D_{i/k}(T_i, h)$ of a ground-based detector $D$ at altitude $h$ is its response (in count m$^{-2}$ sr$^{-1}$) to the unit intensity of primary CR $i$ at kinetic energy $T_i$. It can be described in terms of

- $Y^D_{i/k}(T_i, h)$: partial yield function from $i$-type primary species into $k$-type secondary species (in count m$^{-2}$ sr$^{-1}$);
- $\mathcal{E}^D_{k}$: detector efficiency to $k$-type secondary species.

The yield and partial yield functions are then given by

$$Y^D_i(T_i, h) = \sum_{k=n, p, \mu,} Y^D_{i/k}(T_i, h),$$

$$Y^D_{i/k}(T_i, h) = S_T \sum_{l=1}^{3} \Omega(l, \theta_{l+1}) \times \int_0^{\infty} \mathcal{E}^D_k(T_k) \times \varphi'_i(h, T_i \rightarrow T_k) dT_k.$$

Note that only the primary CR ions $^1$H or $^4$He are evaluated below. For further usage, the resulting $Y^D_{He \rightarrow p}$ and $Y^D_{He \rightarrow \mu}$ are parametrised with a universal form $(T_i/n)$ is the kinetic energy per nucleon of $i$)

$$Y^D_i(T_i, h) = \exp(h f_k) I_{\alpha, i} \left( \frac{a_{i,k}}{T_{\text{max}}} - e_{i,k} \right) I = \log(T_i/n) + b_{i,k}$$

---

^9^Below we use the altitude $h$, but the atmospheric depth or gramage or pressure could have been equally used (conversion is made using the barometric formula).
where the best-fit coefficients \((a_{ik}, b_{ik}, c_{ik}, d_{ik}, e_{ik}, \text{ and } f_k)\) are calculated for the various detectors \(D\) considered (the fit is appropriate for altitudes up to \(h = 5,000 \text{ m}\)). The yield function for any other primary of atomic mass \(A\) at rigidity \(R\) is rescaled from the \(^4\text{He}\) yield (at the same rigidity), namely

\[
y^D_A(R,h) = \frac{A}{4} \times y^\text{He}_D(R,h).
\] (13)

This assumption was tested with nitrogen, oxygen, and iron in Mishev and Velinov (2011), and was found to work well in the lower atmosphere (below 15 km).

4.3. Response and yield function for 6-NM64

Standardised Neutron Monitors (NM64 model) are widely used across the world to monitor CRs since the 1950s (Simpson, 2000). They provide count rates with very interesting time intervals (typically one minute) thanks to the high efficiency of the detectors. An elementary unit of a Neutron Monitor (6-NM64) consists of six BF\(_3\) proportional counter tubes which are mounted in raw and surrounded by a cylindrical polyethylene moderator. The tubes and the inner moderator are inserted in a large volume of lead (the producer). The outer walls of the NM64, the so-called reflector are again made of polyethylene or wood. A more detailed description of the standard NM64-type Neutron Monitor can be found elsewhere (Hatton and Carmichael, 1964).

**NM response function.** NMs are optimised to measure the high-energy hadronic component of ground level secondaries above 100 MeV (Simpson, 2000). However, in spite of their name, they are also sensitive to other secondary radiations (protons, pions, and muons). The efficiency of NM64 to various species have been calculated in the literature from MC simulations with FLUKA (Clem and Dorman, 2000) or GEANT4 (Pioch et al., 2011). A detailed comparison of the efficiencies obtained in the literature is carried out in Clem and Dorman (2000) and Pioch et al. (2011): a very good agreement was found, be it for incident protons and neutrons (the calculation for the latter were also compared to the only existing beam calibration data from accelerator of Shibata et al. 1997, 1999). Differences up to a factor of two (above GeV energies) nevertheless exist depending on which of the GEANT4 physics model or event generator is selected. As our fluence is calculated with GEANT4, we choose to directly use the efficiency given in Pioch et al. (2011), also calculated with GEANT4, and which is in very good agreement with the results of Clem and Dorman (2000).

**Relative contribution of secondary species (n, p, and \(\mu\)).** Although muons are the most numerous terrestrial particles (see Fig. 6), the efficiency of the 6-NM64 to muons is very low (3.5 order of magnitude below the hadrons at 1 GeV, see Fig. 5 of Clem and Dorman 2000). Hence they do not contribute much to the total count rate in a NM (Clem and Dorman, 2000). To back up this comment, we calculate the fraction of count rates from the secondary k particles. Using Eqs. (9,10,11) in

\[
N^\text{NM}(\vec{r},t) = \sum_{k=n,p,\mu...} N^\text{NM}_k(\vec{r},t),
\] (14)

the contribution \(N^\text{D}_k\) can be expressed to be

\[
N^\text{NM}_k(\vec{r},t) = \sum_{i=\text{CRs}} \int_0^\infty \mathcal{E}^\text{NM}_k(T_k) \times \hat{\phi}_k(T_k,\vec{r},t) \, dT_k.
\] (15)

Folding the fluence rate (calculated with ATMO RAD) with the 6-NM64 efficiency (from Clem and Dorman 2000), we gather in Table 4 the total count rate and the fraction due to the k-th particle, at Oulu location. Note that the total count rate calculated in the table is slightly lower than the observed one\(^\text{10}\); this difference amounts to an extra normalisation of the yield function that will be addressed in our next study (see also Usoskin et al. 2011). The variation between a minimum and maximum modulation is almost a factor of two. The main contributions come from secondary neutrons (87%), protons (8%) and \(\mu^-\) (5%), the \(\mu^\text{+}\) contribution being negligible (0.2%).

**Results for our NM yield function.** The partial yield functions \(y^\text{NM}_k\) data points (symbols) calculated in this study from Eq. (11) are shown in Fig. 7 for different primary i and secondary k particles. Also shown are the best fits (lines) to these data relying on Eq. (12), whereas the best-fit parameters are gathered in Table 5. As can be seen in this table, the altitude dependence is steeper for nucleons than for muons. The energy dependence is similar to the yield function obtained in previous studies (see below), with a sharp cutoff at low energy, and a shallow power-law dependence at high energy.

**Comparison to other yield functions.** The total yield function \(y^\text{6-NM64}\) (summed over all k secondary particles) is compared to previous calculations in Fig. 8. The figure shows the ratio of any given parametrisation to ours, for protons (left panels), and helium (right panels). In the top panel, the error bands correspond to the range between calculations at sea level altitude and at 2 km.

| \(\phi\) [GV] | \(N^{6-\text{NM64}}\) [s\(^{-1}\)] | \(N_k/N\) [%] |
|---|---|---|
| n | p | \(\mu^+\) | \(\mu^-\) |
| 0.4 | 91 | 87.2 | 7.9 | 0.2 | 4.7 |
| 1.5 | 57 | 87.4 | 8.0 | 0.2 | 4.4 |

\(\text{http://www.nmdb.eu/nest/search.php}\)
AtmoRad

Figure 7: Levenberg-Marquardt fit of the yield function (in count m^2 sr) for a single tube with AtmoRad (see Fig. 7), relying on the parametrisation Eq. (12).

Table 5: Best-fit parameters for a 6-NM64 yield function (in count m^2 sr) for a single tube with AtmoRad (see Fig. 7), relying on the parametrisation Eq. (12).

\[
\begin{array}{ccccccc}
  i \rightarrow k & a_{ik} & b_{ik} & c_{ik} & d_{ik} & e_{ik} & f_k [\text{m}^{-1}] \\
  p \rightarrow n & -0.105 & 2.862 & 66.98 & 2.648 & -5.432 & 0.00067 \\
  \alpha \rightarrow n & -2.442 & 5.484 & 138.9 & 0.834 & -48.71 \\
  p \rightarrow p & 0.5281 & 1.588 & 142.0 & 6.295 & -1.367 & 0.00069 \\
  \alpha \rightarrow p & 0.2219 & 1.803 & 132.6 & 4.753 & -3.219 \\
  p \rightarrow \mu^- & 0.722 & 0.686 & 4.104 & 4.742 & -0.802 & 0.00025 \\
  \alpha \rightarrow \mu^- & 0.626 & 1.276 & 70.48 & 5.824 & -1.340 & 0.00025 \\
\end{array}
\]

The different parametrisations rely on several MC generators/atmospheric models/NM responses (Clem, 1999; Flückiger et al., 2008; Matthiä et al., 2009; Mishev et al., 2013) or latitude and altitude surveys (Nagashima et al., 1989; Caballero-Lopez and Moraal, 2012). In that respect, the various yield functions can be considered to be in fair agreement. Because of the overall uncertainties in the modelling, the results are usually taken to be up to a global normalisation factor. This absorbs parts of the difference if one is interested in count rate studies (e.g., Usoskin et al., 2011). The bottom panel shows the same quantity as in the top panel, but rescaled to 1 at 10 GV. Actually, Mishev et al. (2013) recently proposed a correction factor \(G\) to account for a hitherto forgotten geometrical factor (related to the NM effective size). These authors find that this correction is necessary to match existing latitude NM surveys (see also Sect. 5.1.2). In principle, this correction (see Eq. B.1) must be applied to any MC-based calculations, i.e. this work, CD00, F08, and M09 (it is by construction included in M13). These G-corrected yields at sea level altitude (renormalised at 10 GV) are shown in the bottom panel of Fig. 8: a quite good agreement is now found in the 5 – 50 GeV range, where most of the counts come from (see below). This scatter (of the yield functions) is propagated to calculate count rate uncertainties in Sect. 5.2.1.

4.4. Cosmic-ray muon intensity

At sea level, muons are the most abundant charged particles, and they can be used in principle to monitor solar activity. Experimental aspects related to the detection of atmospheric muons are discussed, e.g., in Cecchini and Spurio (2012). Muon telescopes generally consist of layers of charged particle detectors and absorbing material, with the capability to determine the direction of \(\mu\) arrival. The quantity of material crossed by the \(\mu\) sets the detector threshold, which increases with the zenith.
Table 6: Best-fit parameters for the yield function (in $m^2 sr$) of a generic $\mu$ detector with AtmORad (see Fig. 9), relying on the parametrisation Eq. (12). Positive and negative muons are counted together.

| $i \to \mu$ | $a_{ik}$ | $b_{ik}$ | $c_{ik}$ | $d_{ik}$ | $e_{ik}$ | $f_k$ [m$^{-1}$] |
|-------------|---------|---------|---------|---------|---------|----------|
| $p \to \mu$ | 0.9116  | 2.068   | 66.41   | 5.818   | 2.755   | 0.00025  |
| $\alpha \to \mu$ | -0.1315 | 1.789   | 49.75   | 2.495   | -3.702  |          |

Figure 9: Levenberg-Marquardt fit of the AtmORad yield function for muons at $+1400$ m (Auger altitude). The corresponding fit parameters are gathered in Table 6.

angle for multi-directional telescopes. Some astroparticle physics detectors have also shown exquisite sensitivities to muons, as exemplified, on the one hand, by the measurements by L3 magnetic muon spectrometer at the LEP collider at CERN (L3 Collaboration et al., 2004), or on the other hand, by the huge array of surface detectors at the Pierre Auger Observatory (Pierre Auger Collaboration et al., 2011). In particular, The Pierre Auger Observatory, thanks to its 3,000 km$^2$ collection area, provides interesting data in the context of solar activity monitoring. The Auger scaler data (corrected for pressure), publicly available$^{11}$, are 15 minutes averages of the scaler rates, recorded since 2005. The threshold of the scaler mode is very low with a very high efficiency, so that in practice, it allows a muon counter equivalent mode (the scaler data variability were found to be well correlated with NM variations, Pierre Auger Collaboration et al., 2011).

In order to compare the behaviours of NMs and muon detectors, we calculate from AtmORad the yield function of a perfect muon detector of 1 m$^2$. In AtmORad, muons were validated by a cross-comparison with the EXPACS code, itself validated on CAPRICE 97 data (Kremer et al., 1999). The best-fit parameters relying on Eq. (12) are gathered in Table 6. This parametrisation should provide a fair estimate of the expected variability, e.g., for the Auger scaler data. Above 10 GV, we checked that it is in very good agreement with the results of Poirier and D’Andrea (2002) for protons. It is used in the rest of the paper to illustrate the results to expect from a generic muon detector.

4.5. Neutron spectrometers to study NM count rates

Recently, Bonner Sphere Spectrometers (BSS) were deployed at ground level and mountain altitudes in order to characterise the CR-induced neutron spectrum over long-term periods for dosimetry or microelectronics reliability purposes (Rühm et al. 2009a; Hubert et al. 2013). Unlike NMs, BSS are only sensitive to the neutron component. However, BSS are far less efficient than NMs and dynamics of one spectrum per hour can be reached at best (in high altitude stations). A BSS designed to cover a wide range of energies (from $10^{-2}$ meV to GeV) generally consists of a set of homogeneous polyethylene (PE) spheres with increasing diameters $d$. A high pressure $^3$He spherical proportional counter placed in the centre allows high detection efficiency. Additionally, spectrometers include some PE spheres with inner tungsten or lead shells in order to increase the response to neutrons above 20 MeV. These extended spheres (HE) behave like small NMs.

After an unfolding procedure (Cheminet et al., 2012b), the neutron spectral fluence rate $\varphi_n^{BSS}(T_n, \vec{r}, t)$ can be derived from BSS data (i.e., count rates $M_d(\vec{r}, t)$ for each of the $d$-Bonner sphere). The neutron component is very sensitive to local changes induced by meteorological and seasonal effects. BSS measurements allow to quantify such variations and to correlate them with variations expected/observed in NM count rates: we recall that neutrons amount to $\sim 87\%$ of the total count rate in NMs (see Table 4). Considering the local neutron count rate $N_n^{X-NM64}(\vec{r}, t)$ of a X-NM64 at the BSS coordinates, we have

$$N_n^{NM}(\vec{r}, t) = \frac{X}{6} \int_0^\infty E_n^{NM}(T_n) \varphi_n^{BSS}(T_n, \vec{r}, t) dT_n. \quad (16)$$

BSS measurements are used to study the seasonal snow effects of NMs in Sect. 5.3.

5. Count rates: variations and uncertainties

In this section, count rates are calculated from Eq. (1), which involves the yield function $Y_\mathcal{P}(h, R)$, the modulated fluxes $J_\mathcal{P}^{TOA}(t)$ for all CR species $i$, and the geomagnetic transmission $T(R, \vec{r}, t)$. To validate our code, we compare count rate variations (vs $R_\ast$) to existing latitude surveys (Sect. 5.1). We then propagate, on count rates, IS flux and yield function uncertainties (Sect. 5.2.1), and geomagnetic transmission function uncertainties (Sect. 5.2.2). We conclude the section with time dependent effects (on count rates) unrelated to solar modulation (Sect. 5.3). Note that the altitude dependence of the yield function is mostly independent of energy and weakly dependent on species (see Table 5), hence we fix it to $h = 0$ m below.

$^{11}$http://auger.colostate.edu/ED/scaler.php
5.1. Count rate variation $N(R_c, \phi)$ vs $R_c$

5.1.1. Relative contribution per rigidity bin

The top panel of Fig. 10 shows (considering the contribution of all CR species) the fractional contribution per rigidity bin of the integrand $Y \times J^{TOA}$. The two shaded areas correspond to a period of minimal (blue shaded area) and maximal (red hatched area) modulation level. First, CRs below 1 GV and above ~ 50 TV contribute to less than 1% of the total: this mitigates the impact of having large differences at low and high energy between various yield function parameterisations (see Fig. 8). Second, increasing the modulation level shifts the CR rigidities contributing most to count rates to higher values. Third, detectors are located at different rigidity cutoff values $R_c$: the higher $R_c$, the larger the fraction of events that cannot contribute to the total. To illustrate this, a sample of NM stations are shown as vertical segments on the plot, along with their names (the description of many NM stations can be retrieved from the Neutron Monitor Data Base).

The bottom panel of Fig. 10 proposes a complementary view, that is the cumulative of the count rates with $R$. For MC-based yield function parametrisations (this paper, CD00, F08, M09), we take into account the $G$-correction of Mishev et al. (2013). All parametrisations give quite similar results, where 50%, (resp. 80% and 90%) of the count rates are reached when integrating up to 10 GV (resp. 30 and 70 GV). The value of the highest energy contributing is very sensitive to the high-energy slope of the yield function. For instance, we chose for CD00 and F08 (see App. B) a high energy extrapolation $Y \propto E^{0.5}$. Would a $Y \propto E$ chosen instead, 80% of the total count rate would be shifted from $R = 30$ GV to 80 GV. It is thus important for future MC-based calculations to push the rigidity range up to 1 TV.

On both the top and bottom panels of Fig. 10, the result for a muon detector is shown in grey lines. The solid and dashed lines correspond respectively to periods of minimal and maximal modulation level. With respect to NMs, the mean energy contributing to count rates is shifted to higher energy, in a region where the impact of the solar modulation is smaller. Hence, the relative count rate variation $\Delta N/N$ to a change of the modulation level $\Delta \phi$ is smaller for $\mu$ detectors than for NMs.

5.1.2. Comparison to latitude survey data

Latitude survey experiments onboard planes, trucks, or ships cruising between equatorial and polar regions is another tool to derive yield functions and/or to compare with direct count rate calculations (Dorman, 2004). Monthly long ship surveys are generally performed during solar minimum periods—the most stable in terms of modulation changes—, in order to be only sensitive to rigidity cutoff effects (Moraal et al., 2000).

Data available. The solar cycle has an 11-year periodicity, and several surveys were carried out at minimum activity since the 50’s: 1954 (Rose et al., 1956; Keith et al., 1968), 1965 (Carmichael et al., 1965; Keith et al., 1968), 1976 (Potgieter et al., 1979; Stoker et al., 1980), 1986 (Moraal et al., 1989), 1997 (Iucci et al., 2000), but none that we are aware of in the last solar minimum period. The data from 1965 are discarded since they were found to differ from the similar 1954 and 1976 survey data (Potgieter et al., 1979). Data from 1976, 1986, and 1997, are also found to be in agreement (see, e.g., Fig. 4 of Mishev et al. 2013), with 1997 data thoroughly corrected from meteorological and geomagnetic effects (Iucci et al., 2000; Villaresi et al., 2000; Dorman et al., 2000).

To compare these data with calculations based on CR fluxes and yield functions, the knowledge of the modulation level to apply is critical. With no CR data available in

\[^{12}\text{NMDB: http://www.nmdb.eu}\]
Tables and figures might be missing here, but the text is generally about analyzing CR data and understanding the modulation level at solar minimum.

1954, we have to base our calculation on 1976, 1986, and 1997 CR measurements. We list in Table 7 the epoch of these surveys and the closest (in time) CR data available (retrieved from CRDB13).

### Modulation level at solar minimum

The CR data listed in Table 7 allow us to determine consistently (i.e., given our IS flux parametrisation) the modulation level \(< \phi_{\text{ref}}^\text{min} >\). This level applies for epochs of minimal modulation. Figure 11 shows the best-fit \(\phi\) required to match CR TOA fluxes. A simultaneous fit of H and He data is performed using the force-field approximation (Sect. 2.2) and our reference IS flux parametrization—Eq. (3) and Table 1—. Note that the value of \(\phi\) for PAMELA is directly reproduced from Table 2. We find that all \(\phi\) values are consistent with one another, and we take in the following \(< \phi_{\text{ref}}^\text{min} >\) = 470 ± 20 MV. This value is slightly higher than the one used in Mishev et al. (2013)14.

### NM and \(\mu\)-detector latitude dependence

Figure 11 shows, as a function of \(R_c\), a comparison of the normalised (at \(R_c = 0\) GV) count rate variations (for various yield functions) to survey data (only the 1986-1987 survey is shown for clarity). The solar modulation level is set to \(\phi = 470\) MV, appropriate for a period of minimal activity (see above). We find, in agreement with the conclusions of Mishev et al. (2013), that taking into account the \(G\)-correction factor (see Eq. (B.1)) gives a much better match to latitude survey data (the curves without this correction are not shown). NM-survey based yield functions (N89 and CL12) give a similar albeit slightly less good match. Note that the scatter observed from the use of the various yield functions is larger than the variation obtained by shifting the modulation \(\phi_{\text{ref}}^\text{min}\) by ±20 MV.

Concerning the variation of count rates with \(R_c\), as already underlined, count rates decrease when \(R_c\) increase—\(R_c\) is the lower boundary of the integral Eq. (1).

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13http://lpsc.in2p3.fr/crdb

14The former (see bottom panel of Fig. 10). Vertical segments show the rigidity cut-off for a sample of NMs.

Over the whole \(R_c\) range, the variation is less marked for a \(\mu\)-like detector (≤ 20%, grey line) than for NMs (≤ 50%). This is understood as the mean rigidity of CRs contributing to the count rate is higher for the latter than for the former (see bottom panel of Fig. 10).

### 5.2. Count rate relative uncertainty \(\frac{\Delta N}{N}(R_c, \phi)\)

#### 5.2.1. Error from IS flux and yield function modelling

The two panels in Fig. 12 show the errors on the count rate calculation as a function of \(R_c\) (for NM64 and \(\mu\) detectors) propagated from the uncertainties on CR fluxes (top) and yield functions (bottom). For simplicity, the errors are symmetrised, i.e. we consider \(\text{err}_{\text{min}} + \text{err}_{\text{max}}) / 2\). For NM64 detectors, the solid blue line (resp. dashed red line) corresponds to the propagation of errors on the IS flux ‘data’ (resp. ‘model’) as discussed in Sect. 3.3 and shown in Fig. 5. These uncertainties are, to a very good approximation, independent of the solar modulation level.
and of the rigidity cut-off. It means that they contribute only to a global shift in count rate times series (no time dependence, and no detector location dependence). This uncertainty is at the level of 2–6% for NMs, with a slightly larger range 2–8% for \( \mu \) detectors (dashed grey line). Future CR data (e.g. AMS-02) will likely shrink these uncertainties at the percent level.

On the same figure, the lines with symbols show the uncertainty related to the existing dispersion among the proposed NM64 yield functions in the literature (see Sect. 4.3, Fig. 8, and App. B). We recall that yield functions are generally considered up to a normalisation. To get a meaningful result, we re-normalised all count rates to a reference value (set arbitrarily to \( R_c = 6.3 \text{ GV} \)), leading to a pinch in the curves. There is a mild dependence on the modulation level, but the overall uncertainty is estimated to be below 8% over the whole \( R_c \) range, and more particularly, at the 2 – 5% level for NMs located at \( R_c < 10 \text{ GV} \). The dispersion is much smaller for \( \mu \) detector (< 0.2%, grey lines and symbols): the latter is probably not conservative and may reflect the fact the only two parameterisations of their yield function are used for this study.

### 5.2.2. Uncertainties from transmission function \( \mathcal{T} \)

A key parameter for calculations is the transmission function of charged particles in the geomagnetic field. Several factors can be taken into account to have an estimate of the associated uncertainties on the count rates. Indeed, the transmission depends on the geographical longitude and latitude \((\varphi, \lambda)\), which can be calculated for a given state of the Earth magnetosphere (Smart et al., 2000). The latter varies in time, and its full description requires both the long-term evolution of the geomagnetic field (International Geomagnetic Reference Field\(^{15}\)) and the short-term magnetospheric field model (Tsyganenko and Sitnov, 2005; Kubyshkina et al., 2009; Tsyganenko, 2013). A good summary of the past studies and findings is given in Smart and Shea (2003, 2009).

The complicated structure of the geomagnetic field leads to a quasi-random structure of allowed and forbidden orbits, denoted ‘penumbra’. The effective vertical rigidity cut-off (see Cooke et al. 1991), used so far in this analysis (\( R_{c}^{\text{eff}} \equiv R_c \)), consists in a weighted average value accounting for the allowed bands (between the upper and lower cut-off values). With the assumption that all regions contribute the same (flat spectrum hypothesis), it is given by (Dorman et al., 2008):

\[
R_{c}^{\text{eff}} \approx R_{\text{upper}} - \sum_{i=R_{\text{lower}}}^{R_{\text{upper}}} \Delta R_{i}^{\text{allowed}}.
\]

In this approach, it follows that the transmission function is described by the step function \( H(R - R_{c}^{\text{eff}}) \).

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\(^{15}\text{http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html}\)
The impact of changing $R_c$ to $R_c + \delta R_c$ is shown in the top panel of Fig. 13. Whenever $R_c$ is increased, the count rates decrease, with a milder impact at epochs of high solar activity than for low activity. This is related to the steepness of the decrease of the count rate with $R_c$ shown in Fig. 11. For detectors located at $R_c < 10$ GV, count rates over 50 years vary at most by -4% for NM64 (blue dashed lines), and -1% for $\mu$-detectors (dashed grey lines).

Allowed and forbidden rigidity: penumbra. A better description of the transmission function is the use of a sigmoid function, as done in the context of NMs (Kudela et al., 2008), or the CR experiments HEAO-3 (Ferrando et al., 1988) and AMS-01 (Bobik et al., 2006, 2009). The step function $H(R - R_c^{\text{eff}})$ is the limit of a sigmoid of zero width. To see how good is this zero-width approximation, we compare count rates calculated with it and with the following sigmoid shape (centred on $R_c$):

$$T(R) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{R - R_c}{\sqrt{2} \Delta R_c} \right) \right].$$

(18)

It is useful to define the width $\sigma$ of the sigmoid function to be

$$\sigma \equiv \frac{\Delta R_c}{R_c};$$

A typical range of values reproducing best AMS-01 data (depending on the position) is $\sigma \sim 0.1 - 0.2$ for $R_c \gtrsim 2$ GV, and $\sigma \sim 0.1 - 0.5$ for $R_c < 2$ GV.

The bottom panel of Fig. 13 shows the ratio of count rates calculated for various values of $\sigma$ to count rates in the step function approximation. The sigmoid case gives less count rates than the step function case, an effect that increases with $R_c$ and with the width of the sigmoid. Moreover, the larger the modulation level $\phi$ (chained lines compared to lines), the smaller the effect. These behaviours are well understood if one keeps in mind how the contribution per rigidity band (described in Fig. 10) varies: compared to the step function, the sigmoid allows less contributions above $R_c$ (where it matters most), and more contributions below $R_c$. For mild values of $\sigma = 0.3$, the change below $R_c = 10$ GV (where most stations lie) is -4% at most for NM64 (blue dashed lines), and -2% for $\mu$-detectors (dashed grey lines).

Obliquely incident particles: apparent cut-off rigidity. With the advance of more powerful computers, obliquely incident CRs—and the ensuing secondary particles reaching the detector—could be considered (Rao et al., 1963; Clem et al., 1997; Dorman et al., 2008). Instead of weighting each vertical direction of the penumbra similarly as in Eq. (17), the vertical and non-vertical incident CRs are weighted according to the zenith angle dependent rigidity cutoff (assuming at first order that there is no azimuth dependence). This defines a so-called apparent vertical cut-off rigidity $R_c^{\text{app}}$:

$$R_c^{\text{app}}(R_c) \equiv \frac{\int_0^{2\pi} d\phi \int_0^{\pi/2} \frac{R_c(\theta, \phi) \times Y(R_c, \theta, \phi)}{\int_0^{2\pi} d\phi \int_0^{\pi/2} Y(R_c, \theta, \phi)} d\theta}{\int_0^{2\pi} d\phi \int_0^{\pi/2} Y(R_c, \theta, \phi) d\theta}. \quad (19)$$

Its calculation is more demanding than that for $R_c^{\text{eff}}$, though approximations to speed up the calculation exist (Bieber et al., 1997). In terms of the apparent cut-off rigidity, Eq. (1) for count rates becomes

$$N^D(\vec{r}) = \int d\Omega \int_{R_c^{\text{app}}}^{\infty} \frac{Y^D(R, h, \Omega) J(R)}{J(R) \int_{R_c^{\text{app}}}^{\infty} Y^D(R, h, \Omega) J(R) dR} dR.$$  

(20)

All our previous calculations still apply, replacing the vertical effective rigidity cut-off $R_c$ by the apparent rigidity cut-off $R_c^{\text{app}}$ values.

Folding an NM64 yield function (calculated with FLUKA and HEAVY packages) with rigidity cut-off maps,
Clem et al. (1997) found $R^\text{app} > R_c$. The difference is $\sim 10\%$ for $R_c \approx 0.1$ GV, then it decreases and stay at $\sim 3\%$ above $R_c \gtrsim 8$ GV (see their fig. 11). Further investigations by Dorman et al. (2000, 2008) rely on a parametrised angular distribution of the yield function compared with NM survey data. We fit their Fig. 5 and obtain

$$\left( \frac{R^\text{app} - R_c}{R_c} \right) \times 100 = \begin{cases} \pm 1.7 + 0.47 R_c & \text{if } R_c > 1, \\ 0 & \text{otherwise.} \end{cases} \quad (21)$$

Dorman et al. (2008) results go in the same direction as Clem et al. (1997) ones, though the $R_c$ dependence of the relative error is slightly different. The maximum shift of $R_c$ to $R^\text{app}$ is $\sim 6\%$ for $R_c = 18$ GV, and the shift decreases with decreasing $R_c$. The impact of this change on the count rate calculations can be directly read off the top panel of Fig. 13.

We underline that the effects related to the geomagnetic field are quite complex, and may be not all accounted for, as possibly illustrated by the not yet understood long-term decline of South pole neutron rates (Bieber et al., 2007, 2013).

5.3. Seasonal effects: pressure, temperature, snow coverage and water vapour

NM or muon detector count rates can be affected in many ways by meteorological and seasonal effects. The quantities considered in this study are atmospheric pressure, temperature, the water vapour, and the snow coverage. For the latter, which is usually not included in public NM data, a comparison with the results of neutron spectrometers is used to assess the strength of the effect.

5.3.1. Atmospheric pressure and temperature

Atmospheric pressure effects are well known (Hatton and Griffiths, 1968) and are systematically corrected for in NM data to bring back the count rates at the values they should reach in nominal conditions (i.e., at a reference atmospheric pressure $p_0$ instead of the actual and varying pressure $p$). The exponential correction relies on a barometric coefficient $\beta$, with $N = N_0 \exp[-\beta (p - p_0)]$. Uncertainties in the final NM count rates arise from the standard deviation $\Delta \beta$ obtained during the calculations of $\beta$:

$$\Delta \frac{N}{N} |_{\text{Pressure}} = | \exp^{\pm \beta (p - p_0)} - 1 |. \quad (22)$$

Some barometric coefficients were recently calculated by, e.g., Chilingarian et al. (2009) and Chilingarian and Karapetyan (2011). The standard deviation $\Delta \beta$ is generally equal to 0.02% hPa$^{-1}$. Considering a pressure variation $|p - p_0|$ of 20 hPa, the variation $\Delta N/N |_{\text{Pressure}}$ obtained after barometric correction is equal to 0.2%.

5.3.2. Atmospheric temperature

The well-known temperature effect is detailed, e.g., in Iucci et al. (2000). For NM, it amounts to $-0.03\%/^\circ\text{C}$, that is an Antarctica-to-equator temperature effect $\sim 1\%$, which is also the order of magnitude of the seasonal effect $\Delta N/N |_{\text{Temperature}} \sim 1\%$.

For muons, the temperature effect, which is dominant over all other effects, is discussed, e.g., in Dmitrieva et al. (2011). The seasonal effect is $\sim 8\%$ with smaller variation of $1 - 2\%$ on the background of seasonal trend, with a small dependence on $R_c$ (Clem and Dorman, 2000). It should be pointed out that this effect strongly depends on the zenith angle of incident muons. For $\mu$ data, real time correction for this effect is discussed in Berkova et al. (2011, 2012).

5.3.3. Water vapour

As discussed in Bercovitch and Robertson (1965) and in Chasson et al. (1966), an increase of atmospheric water vapour content attenuates the intensity of secondaries seen by a NM. To take into account this effect, they proposed a correction of the barometric coefficient $\beta$. The variation $\Delta \beta$ is estimated between -0.09% hPa$^{-1}$ and -0.15% hPa$^{-1}$, leading to a seasonal effect $\Delta N/N |_{\text{Water Vapour}} \sim 0.2 - 0.3\%$. Recently and for the first time, this effect was investigated (Rosolem et al., 2013) in a detailed simulation based on the neutron transport code Monte Carlo (MCNPX). In agreement with Bercovitch and Robertson (1965), the sensitivity of fast neutrons to water vapour effect was found to reach $\sim 10\%$ for sites with a strong seasonality in atmospheric water vapour, with a larger decrease of count rates in moist air than in dry air.

For muon detectors, pressure effects are also considered with barometric factor $\beta_\mu$, but lower than NM ones ($\beta_\mu = 0.03\%$ hPa$^{-1}$).

5.3.4. Snow effect

The last effect discussed is the impact of snow on the neutron component (no effect is expected for the other secondary components). Recent works have shown that heavy snow fall impact strongly the 1 meV to 20 MeV neutron spectrum (i.e. thermal, epithermal and evaporation domains), while the cascade region ($\gtrsim 20$ MeV) is less affected (Rühm et al., 2012; Cheminet et al., 2013b). As a matter of fact, hydrogen in water molecule is responsible for enhanced thermalisation and neutron absorptions. In the context of NMs, the snow both in the surroundings and above NM shelters affect count rates.

BSS location and data. Data from three BSS are gathered to study the snow effect of the neutron component in NMs count rates. Two of them are run by the Helmholtz Zentrum München (HMGU) at mountain altitude at the summit of the Zugspitze in the German Alps$^{16}$, as described in Rühm et al. (2009a,b) and Rühm et al. (2012). The last one is operated by the French Aerospace Lab. (ONERA).

$^{16}$Environmental Research Station Schneefernerhaus and near North Pole at Spitsbergen (Koldewey Station).
at the summit of the Pic du Midi de Bigorre\textsuperscript{17} Cheminet et al. 2012a,b; Cheminet et al. 2013a,c). The main features of each experiment are highlighted in the upper half of Table 8.

As an illustration, Fig. 14 shows typical spectra obtained for the three above-mentioned BSS in summer and winter. The decrease of intensity for low and intermediate energies is clearly visible in winter, when heavy snow falls occur in the northern hemisphere.

**NM64 seasonal effect from BSS data.** As explained in Sect. 4.5, it is possible to derive the count rates \( N_n \) due to neutron in NMs thanks to Eq. (16). We first show in Fig. 12, for the Pic du Midi case, the product of the NM64 response and the neutron spectra as a function of kinetic energy \( E_n \). Although the majority of counts are due to cascade neutrons (above 20 MeV), evaporation neutrons are non-negligible, and for both energy regions, the difference between summer and winter is significant.

For NM64, the value of the count rate variations \( \Delta N/N \) due to seasonal effect are calculated taking into account the fact that neutrons constitute 87\% of the total NM count rate. The results for the three stations are gathered in the lower half of Table 8: the effect varies between \(-1.8\%\) and \(-7.6\%\). These estimations are consistent with data provided by, e.g., Tanskanen (1968) and Korotkov et al. (2011), with a variation about \(-5\%\) recorded at NM station of Oulu, and \(-7\%\) in Rome. They are also in agreement with the results of Rühm et al. (2012) based on Zugspitze and Spitsbergen data.

This confirms that snow has a significant and seasonal impact on NM count rates in stations that might know intense snow fall episodes (particularly at mountain altitudes). Indeed, recent effort are directed into having real-time and automated corrections for the snow effect in NM64 data (Korotkov et al., 2011, 2013). After this correction, these authors estimate a residual error of \( \sim 0.4\% \).

**5.4. Other effects**

NM count rates depend on the detector surroundings and the atmosphere state, but they also depend on the reliability and stability of the equipment. To improve further the usefulness of the NM network, inter-calibration of all stations is required. Portable calibration NMs were discussed in Moraal et al. (2001), built soon after (Moraal et al., 2003), and several tests and validation carried out (Krüger and Moraal, 2005, 2010, 2011; Krüger et al., 2008; Krüger and Moraal, 2013). Note that the target goal for the calibrator was to reach an accuracy of \( \leq 0.2\% \) (for spectral studies), which succeeded, as reported in Krüger and Moraal (2010). However, during the tests, several ef-

\textsuperscript{17}ACROPOL: high Altitude Cosmic Ray ONERA/Pic du Midi Observatory Laboratory.
fects were discovered, that are of importance and amount somehow to uncertainties.

First, an instrumental temperature effect (not related to the atmospheric temperature effect) was discovered (Krüger et al., 2008), with a magnitude of \( \sim 0.05\% / C \) for NM64. However, this should not impact count rates as long as the detectors are kept in a small temperature range. More worrisome is the fact that different ground surfaces lead to an unpredictable spread of \( \sim 4\% \) (Krüger and Moraal, 2010). Then the exact geometry of the detector (Hatton and Carimichael, 1964), whether it contains \( n \) or \( m \) tubes also slightly changes the efficiency of the detector (and in fine the yield function and count rates at various latitude): effects up to a few percent can exist (Hatton and Carimichael, 1964; Krüger et al., 2003), and in particular, differences up to 4% were observed between the calibrator and a 3-NM64 (Krüger and Moraal, 2013). The last two issues may explain the need of detrending NM data in order to reach a coherent picture of solar activity for the various stations (Oh et al., 2013).

To conclude, some weaker effects are probably missing from this discussion. One worth mentioning is the diurnal variation of count rates, related to the corotation of the CRs with the magnetic field of the Sun; it has an amplitude of \( \sim 0.24\% \) for both NM and \( \mu \) detectors (e.g., Mailyan and Chilingarian, 2010). Other effects should be of the order or below this level.

6. Conclusions: count rate variation and uncertainty iso-contours in the \((R_c, \phi)\) plane

We have made a detailed study of count rates (and uncertainties) for neutron monitors and \( \mu \) detectors, as a function of the rigidity cut-off and the modulation level \( \phi \), in the context of the force-field approximation.

6.1. Input parameters

First, we have re-assessed (and compared with previous results from the literature) two key ingredients entering the calculation, namely IS fluxes and yield functions.

i) Results for IS fluxes:

- we propose a new set of IS flux parametrisations for elements \( Z = 1 - 28 \), see Eq. (3) and Table 1;
- we improve the calculation of the factor accounting for heavy species \( (Z \geq 3) \) as an extra contribution of \( ^4\text{He} \) (for NM and \( \mu \) detector count rate calculations). The required extra amount of \( ^4\text{He} \) is found to be \( 0.611^{+0.016}_{-0.019} \) \( ^4\text{He} \) (to be compared with 0.480 used in previous studies). We check that making the substitution is accurate at better than the percent level over the whole rigidity range;
- as previously studied in Herbst et al. (2010), it is always possible to recover the same TOA fluxes, starting with different combinations of IS fluxes and solar modulation parameter \( \phi \) (degeneracy between \( \phi \) and the IS flux). Equation (8), to be used with Fig. 3, provides a simple recipe to move from one \( \phi \) time series to another, depending on the choice of the IS flux (all formulae are gathered in app. A);
- we evaluate the uncertainty on TOA \(^1\text{H}\) and \(^4\text{He}\) fluxes (see Fig. 5), directly from the fit (of our reference flux) to the data and their errors, or from the dispersion of TOA fluxes obtained with the use of several parametrisations of the IS flux (modulated at their appropriate value, as underlined above). We arrived at a 5% uncertainty for the former, and a probably too conservative \( \sim 10 - 20\% \) dispersion (energy dependent) for the latter.

ii) Results for the yield functions:

- we provide a systematic comparison of available yield functions in the literature (see Fig. 8, all formulae are gathered in App. B). Differences of a factor of a few exist around a few tens of GV, these differences increasing at lower and higher rigidity. A better agreement at high energy is obtained when accounting for the geometrical correction factor of Mishev et al. (2013);
- after renormalisation to a reference rigidity, the dispersion for the various yield functions can be used to estimate the uncertainty on count rates (see below).

6.2. Count rates and uncertainties

Using these inputs, we have been able to characterise the count rate dependence on several parameters and related uncertainties.

- The main contribution to count rates (in terms of rigidity range) is from the 5 – 500 GV for NM64 and 10 – 5000 GV for \( \mu \) detectors (see Fig. 10);
- we validate NM64 yield functions against latitude surveys in two steps:
  a) we derive the solar modulation level (from CR data, based on our reference IS flux) at the time of these surveys (minimum activity)—see top panel of Fig. 11. We find \( \phi_{\text{min}} \approx 470 \text{ MV} \), a value slightly higher but in agreement with the value used in other works using these same surveys.
  b) a comparison of various yield functions from the literature confirms that the geometrical correction factor proposed in Mishev et al. (2013) is
mandatory to better fit NM survey data. This effect and the energy dependence of MC yield function calculations in the 100 GV – 1 TV should be further explored. When this correction is applied to all MC-based yield function (as opposed to yield functions derived from NM data surveys), a consistent picture emerges, with all modelling in fair agreement with one another. Rather unexpectedly, a slightly better fit to all the survey data (up to a rigidity cut-off of 10 GV) is given by the new yield function we propose.

- We propagate the uncertainties obtained for the IS flux and yield function to the calculated count rates:
  
  a) a φ and \( R_c \) independent uncertainty of 2% (resp. 6–8%) is related to IS flux from data uncertainty (resp. from IS flux model dispersion), see top panel of Fig. 12. The scaling factor (for \( Z > 2 \) species) uncertainty lead to another 0.6%. This applies to NM64 as well as to \( \mu \) detectors;
  
  b) an \( R_c \) dependent (and slightly dependent on \( \phi \)) uncertainty smaller than 2–4% for \( R_c < 10 \) GV (resp. 6–8% for \( R_c < 18 \) GV) is related to NM64 yield function dispersion, see bottom panel of Fig. 12. This uncertainty is smaller than 0.2% for a \( \mu \) detector.

- We revisit the uncertainties related to the transmission function of CRs in the geomagnetic field. Focusing on effective vertical rigidity cut-off below \( R_c = 10 \) GV (where most stations lie), we reach the following conclusions:
  
  a) using a sigmoid function instead of a step function gives ∼ 4% less count rates for NM64, and ∼ 2% less for \( \mu \) detectors. This effect is \( R_c \) dependent, and maximal for large \( R_c \) values (see bottom panel of Fig. 13);
  
  b) even in the step-function approximation, the count rate variation is expected to change due to long-term or short term geomagnetic variations. We evaluate that over 50 years a typical decrease of ∼ 4% for NM64 (and ∼ 1% for \( \mu \) detectors) can occur (see top panel of Fig. 13). The level of the variation depends on the geomagnetic position and \( R_c \);
  
  c) the use of the apparent cut-off rigidity of Clem et al. (1997) and Dorman et al. (2008) accounting for obliquely incident particles (in the geomagnetic field) is found to have an impact of ≤2–4% on the NM64 count rate (and ≤1–2% on \( \mu \) detectors). As above, the effect depends on the geomagnetic position and \( R_c \).

- We recap the various seasonal effects and their impact on count rates. Note that muon detectors are dominated by temperature effects (at the level of 8%), while the following effects must be considered for NM64:
  
  a) atmospheric pressure and temperature effects are usually directly corrected for in public data, with an impact on count rates of respectively \( < 0.5\% \) and \( < 1\% \);
  
  b) water vapour is expected to lead to a ∼ 0.2–0.3% effect;
  
  c) the effect of snow coverage in the surrounding of the detector is investigated by means of BSS measurements whose low energy spectrum is very sensitive to it. We obtain a 2–8% seasonal variation for this effect (obviously strongly dependent of the climatic conditions at the station location), in agreement with direct measurements in NM stations. Recent efforts by Kotrotkov et al. (2011, 2013) to provide real-time data corrected for this effect are an important step for the network of NMs around the world.

- Finally, some uncertainties are intrinsic to the detector itself, as thoroughly investigated by means of a calibrator (Krüger and Moraal, 2010). During these efforts, spreads in measurements up to ∼ 4% could also be attributed to different surfaces where the detectors lie. However, such effects, along with differences attributed, e.g., to the exact geometry of the detector (Hatton and Carimichael, 1964), are not expected to change in time, and thus are probably not as problematic as seasonal effects.

6.3. Abacus: count rates to solar modulation variations

To conclude, we propose a last figure and a table for a panoptic view of all the effects we have approached in this study. Actually, these plots provide a direct link between solar modulation level and count rate variations (and vice-versa) for both NM64 and \( \mu \) detectors.

The top panels of Fig. 16 provide the relative count rate variation in the \( R_c – \phi \) plane, with respect to a reference point \( N_0(R_c = 0, \phi = 0.5 \) GV). In addition to providing a global view of the expected count rate variation between detectors at different \( R_c \) and for different solar periods, it also gives a flavour of the precision required in order to be sensitive to changes in the \( \phi \) parameter: the count rate variation over a full solar cycle is smaller for \( \mu \) detectors than for NMs, but the latter are more sensitive to any uncertainty on \( R_c \) (location in the geomagnetic field) than the former.

The bottom panels of Fig. 16 go further in that direction, as they directly provide, for any value \( (R_c, \phi) \), how much variation \( \Delta N/N \) to expect in the count rates, whenever the solar modulation changes by \( \delta \phi/\phi \). This abacus usage is two-folds: first, on short term variations, it can directly be used to extract \( \delta \phi/\phi \) from count rate variation in NM (or \( \mu \)) data; second, it can be used to estimate
Figure 16: Left panels are calculated for NM64 and right panels for $\mu$ detectors. **Top panels:** Count rate relative variation $\Delta N/N_\odot$ with respect to a reference count rate $N_\odot = N(\phi = 0.5, R_c = 0)$. The relative variation (in %) are shown as iso-contours in the plane $(R_c, \phi)$, with the 0%-isocontour (passing through the reference point $\otimes$) in black solid line. **Bottom panels:** scaling factor $f$ to infer the count rate relative variation $(\Delta N_{\mu}/N)(R_c, \phi)$ for any modulation relative change $\delta \phi / \phi$ (for NM64, slight differences in the contours arise for values of $\delta \phi / \phi > 20\%$ and factor $f < -0.3$). For instance, for a NM64 detector at $R_c = 6.5$ GV and a solar modulation period $\phi = 1.2$ GV, the scaling factor is $f = -0.25$, which reads: an increase of 5% (resp. 10%) in the modulation level $\phi$ translates in a decrease of $0.25 \times 5\% = 1.25\%$ (resp. 2.5%) in the detector count rate, and vice-versa.

how much uncertainty is propagated in $\phi$ from the various uncertainties calculated on count rates.

This is what is gathered in Table 9: for each input effect discussed in the paper, we provide (in addition to the section/figure where it was dealt with) the typical uncertainty obtained on $\Delta N/N$, and the associated $\Delta \phi$ calculated for an NM64 or $\mu$ detector at $R_c = 5$ GV and a solar modulation level of $\phi = 500$ MV (using Fig. 16). In terms of $\phi$ calculations, the first thing to underline is that NM and $\mu$ detectors do not suffer the same amount of uncertainties, due to different sensitivities to the various effects explored and different weight of the count rate change for the two types of detectors. Moreover, there are no clear-cut ranking of these errors. Luckily, when interested in time series, time-independent normalisation effects can be absorbed in a normalisation factor (e.g., Usoskin et al., 2011): the latter accounts for differences in NM detector efficiency and their surroundings (last entries in table).

The case of the IS fluxes (first entries in table) is peculiar, since different choices $i$ lead to an overall shift $\Delta \phi_i$ of the time series. For NMs, the main source of uncertainties are the seasonal snow effects (strength depending on position, some stations not affected), and the yield function dispersion (applicable for all stations). All other effects cannot be simply disregarded as they typically have a $5 - 10\%$ on $\phi$. For $\mu$ detectors, the main effect is that of the temperature variation, but after corrections, it is at the level of other uncertainties ($5 - 10\%$). Overall, $\mu$ detectors seem to suffer slightly less uncertainties than NM64, but of course the latter benefit from a much larger time and position coverage than the former.

6.4. Future works

The approach we have followed in this study could easily be extended to other types of ground-based measurements, by simply using the appropriate yield function for each type (e.g., $^{10}$Be production in ice cores, Herbst et al. 2010; ionisation measurements in the atmosphere,
Solar modulation \( \phi \in [0.2,1.5] \) GV

Cut-off rigidity \( R_c \in [0,10] \) GV

### Table 9

| Ingredient                  | Effect                           | Fig./Sect. | \( \frac{\Delta N}{N} \) | \( \Delta \phi^* \) [MV] | Comment                          |
|-----------------------------|----------------------------------|------------|--------------------------|--------------------------|----------------------------------|
| Solar modulation            | \( \phi \in [0.2,1.5] \) GV       | Fig.16     | [+15,-25]%                | -                        | w.r.t. \( \phi = 0.5 \) GV        |
| Cut-off rigidity            | \( R_c \in [0,10] \) GV          | Fig.11     | [+10,-20]%                | -                        | w.r.t. \( R_c = 5 \) GV          |
| TOA flux                    | p and He CR data                 | Fig.12     | ±2%                      | ±2%                      | \( t, (R_c, \phi) \)-independent  |
| IS flux dispersion          | IS flux dispersion               | Fig.12     | ±6%                      | ±8%                      | \( \downarrow \)                  |
| Heavy species               |                                  | Fig.1      | ±0.6%                    | ±0.6%                    | \( (R_c, \phi) \) dependent      |
| Transfer function           | Sigmoid(\( R_c, x = + \frac{\phi}{\beta} \)) | Fig.13     | -2x%                     | -0.5x%                   | For \( R_c \gtrsim 5 \) GV        |
|                            | \( H(R_c+ \Delta R, x = \frac{\Delta R}{R_c}) \) |Fig.13      | -2x%                     | -x%                      | For \( R_c \gtrsim 5 \) GV        |
|                            | \( R_c(t): \frac{\Delta R}{\tau_c} \lesssim 0.2\%/yr \) | §5.2.2     | -0.4%/yr                 | -0.1%/yr                 | Depends on location              |
|                            | \( R_c \rightarrow R_c^{app} \): +3% | §5.2.2     | -1.2%                    | -0.3%                    | Depends on \( R_c \)              |
| Pressure                   |                                  | §5.3.1     | ±0.2%                    | ±0.2%                    | DC\(^\dagger\)                   |
| Temperature                |                                  | §5.3.2     | ±0.5%                    | ±4%                      | DC\(^\dagger\)                   |
| Vapour water               |                                  | §5.3.3     | ±0.3%                    | ±0.1%                    | NC\(^\dagger\)                   |
| Snow coverage              |                                  | §5.3.4     | -7%                      | -                        | NC\(^\dagger\) (1 yr period)     |
| Diurnal variation          |                                  | §5.4       | 0.24%                    | 0.24%                    | NC\(^\dagger\) (24h period)      |
| NM detector effects        | Temperature                       | §5.4       | +0.05%/C                 | -                        | \( t, (R_c, \phi) \)-independent  |
|                            | \( nNM_6 vs mNM_64 \)            | §5.4       | few %                    | ~ 100                    | \( \downarrow \)                  |
|                            | Surroundings (hut)               | §5.4       | few %                    | ~ 100                    | Global norm. factor\(^\circ\)    |

\( \star \) The variation \( \Delta \phi \) of the modulation level is calculated for a detector at \( R_c = 5 \) GV and \( \phi = 0.5 \) GV:

- refer to Fig. 16 to convert rate variations for any other \((R_c, \phi)\) condition.
- Very conservative estimate (some IS fluxes are based on old CR data).
- Global normalisation factors can always be absorbed in the yield function normalisation.
- After correction, \( \Delta N/N \approx 0.3\% \) (Dmitrieva et al., 2011), leading to \( \Delta \phi \approx 22 \) MV.
- Distributed data are usually corrected (DC) or not corrected (NC) for these effects.

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### Appendix A. IS flux parametrisation (p and He)

For completeness, we provide below all IS flux parametrisations \( dJ/IS/dT_n \) used in the paper. They are given for protons and heliums in unit of \( m^{-2} \ s^{-1} \ sr^{-1} \ (GeV/n)^{-1} \). The formulae are expressed in terms of:

- the rigidity \( R \) (in GV)
- the kinetic energy per nucleon \( T_n \) (in MeV/n)
- \( \beta = v/c \).
Appendix B. Yield for NM and μ detectors

Yield functions are given below for protons $Y_p$, and in some cases for heliums $Y_{He}$. If not available, a rescaled version of $Y_p$ is used, see Eq. (13). The yield functions are given in unit of [m$^2$ sr], and are expressed in terms of:

- the grammage $g$ (in g cm$^{-2}$)
- the altitude $h$ (in m) w.r.t. to sea level
- the rigidity $R$ (in GV)
- the kinetic energy $T$ (in GeV)
- the kinetic energy per nucleon $T/n$ (in GeV/n)
- $\gamma = E_{tot}/m$.

$Y_p(T/n,\gamma) = \begin{cases} 10^{1.3} & \text{if } T/n > 1000 \text{ GeV/n, a power-law extrapolation of slope 0.5 is used above } R > 100 \text{ GV} \\ 1.94 \times 10^{-2} \times \beta^2 R^{-2.76} & \text{if } T/n < 1000 \text{ GeV/n.} \end{cases}$

$Y_{He}(T/n,\gamma) = \begin{cases} 3.35 \times 1.4 \times \sqrt{R} & \text{if } T/n > 1000 \text{ GeV/n, a power-law extrapolation of slope 0.5 is used above } R > 100 \text{ GV} \\ 0.5 \times 10^{-2} \times \beta^2 R^{-2.76} & \text{if } T/n < 1000 \text{ GeV/n.} \end{cases}$

$\text{this paper} \equiv J_{ref}$:

$J_p(R) = 2.335 \times 10^4 \times \beta^{1.1} R^{-2.839}$

$J_{He}(R) = 0.7314 \times 10^4 \times \beta^{0.77} R^{-2.782}$

Appendix B.1. NM64 yield parameterisations

The yields are given for a 6NM64 neutron monitor. For an xNM64 device, the yield below are multiplied by $z/6$.

Note that Mishev et al. (2013) proposed a correction factor $G$ to account for the geometrical factor of the NM effective size (in the context of yield functions calculated in MC simulations). This correction, fitted their Fig.2, reads:

\[
G = \max \left[ 1, 1.4 \times \log_{10} \left( \frac{T/n}{2.39} \right) \right].
\]  

To apply this correction, the below MC-based yield functions (i.e. CD00, F08, M09) can simply be multiplied by $G$. This correction is already accounted for in M13, and does not intervene in NM ‘count rate’-based formulae (N89, CL12).

- N89 (Nagashima et al., 1989, 1990): using $x = \frac{\rho}{\rho_{mass}}$,

\[
Y_p(g, \gamma) = 10^4 \exp \left( -2.2 x^{1.62} - \frac{12.7 x^{0.5}}{\ln(\gamma)^{0.42}} \right).
\]

- CD00 (Clem, 1999; Clem and Dorman, 2000): the fit is adapted from Caballero-Lopez and Moraal (2012),

\[
Y_{p,n}^{\text{as1}}(R) = 2 \times 10^{-2} \left( 0.45^{1.4} + R^{1.4} \right)^{-20.79} R^{30}.
\]

A power-law extrapolation of slope 0.5 is used above $R > 100 \text{ GV}$.

- F08 (Flückiger et al., 2008):

\[
Y_p(g, \gamma) = 10^{4.85} \sum_{i=0}^{1} a_{ij} g^i \left( \log_{10}(R) \right)^j.
\]

with

\[
\begin{align*}
  a_{0j} &= \{0.7983, 2.859, -2.060, 0.5654\}; \\
  a_{1j} &= 10^{-2} \times \{ -0.6985, 1.188, -0.9264, 0.2169\}; \\
  a_{2j} &= 10^{-5} \times \{ 0.5359, -1.516, 1.522, -0.4214\}; \\
  a_{3j} &= 10^{-9} \times \{ -1.950, 7.969, -8.508, 2.491\}.
\end{align*}
\]

A power-law extrapolation of slope 0.5 is used above $R > 100 \text{ GV}$.

- M09 (Matthiä et al., 2009; Matthiä, 2009): using $I = \log_{10}(T)+1.94$ and $J = \log_{10}(T)+2.2$,

\[
Y_{p,n}^{\text{as1}}(R) = 2 \pi \times 10^{-4} \times 10^{-44.40-22.62I+1.007T^2+61.01\sqrt{T}}; \\
Y_{He}^{\text{as1}}(R) = 2 \pi \times 10^{-4} \times 10^{-50.78-16.32J+0.409J^2+56.44\sqrt{T}}.
\]

Above $T/n > 500 \text{ GeV/n}$, a power-law extrapolation is used (based on two points calculated at 450 and 500 GeV/n).

- CL12 (Caballero-Lopez and Moraal, 2012):

\[
Y_{p,n}^{\text{as1}}(R) = 2 \times 10^{-2} \left( 2.145 + R^{1.45} \right)^{-4.696} R^{7.7}; \\
Y_{He}^{\text{as1}}(R) = 2 \left( 0.45^{1.4} + R^{1.4} \right)^{-7.143} R^{10}.
\]
• M13 (Mishev et al., 2013): using $I = \log_{10}(R) + 0.469$ and $J = \log_{10}(R) + 0.1$, we fit their data (Table 1) with

$$Y_{\text{He}}(R) = 4 \times 10^{-19.83 - 13.81 J + 0.971 J^2 + 30.58 \sqrt{J}}.$$ 

• This paper: Eq. (12) and Table 5. A power-law extrapolation is used above $R > 200$ GV (based on two points calculated at 150 GV and 200 GV).

Appendix B.2. $\mu$ detector yield parametrisations

• PD02 (Poirier and D’Andrea, 2002): the fit is from Caballero-Lopez and Moraal (2012).

$$Y_p(R) = 1.07 \times 10^{-19} (1 + R)^{-2.15} R^{28}.$$ 

• This paper (Eq. (12) and Table 6): using $I = \log_{10}(T/n) + 2.068$,

$$Y_p(h, T/n) = 10^{-0.0025 h} \times 10^{(0.9116 - 66.41 J - 5.818 - 2.755)}.$$ 

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