Allowance of Lateral Breakthrough Error for Super Long Tunnels from 20km to 50km

ZHANG Zhenglu  ZHANG Songlin

ABSTRACT  Up to now there is no specification about the allowance of lateral breakthrough error for super long tunnel from 20 km to 50 km. On the basis of the design of GPS networks located outside and inside tunnel traverse network, we propose a method for calculating the influence value caused by control surveying errors. Through a lot of simulative calculations and combination with piercing practice of super tunnels in Wan Jiazai Project, Shanxi province, we present an allowance table of lateral breakthrough error for super long tunnels from 20 km to 50 km.

KEY WORDS  super long tunnel; lateral breakthrough error; allowance

CLC NUMBER  P258

Introduction

With the rapid development of civil engineering in China, super long tunnels are becoming more and more, which are dug by tunnel driving machines and have only one breakthrough profile. Up to now there is no specification about the allowance of lateral breakthrough error for super long tunnel from 20 km to 50 km. Determining the allowance of lateral breakthrough error for super long tunnel is an important problem that need to be studied and resolved.

We denote the breakthrough error as root mean square error (RMSE). The allowance is two times of RMSE. For super long tunnel engineering, the three component of breakthrough errors are lateral, longitudinal and vertical error, in which the lateral component is the most difficult to control for meeting the engineering requirements, and we focus on it in this paper.

The sources of the lateral breakthrough error include ground control surveying error outside tunnel, shaft contacting surveying error, traverse surveying error inside tunnel and setting out error, among which setting out error can be neglected. There will be no shaft contacting surveying error if there is only one breakthrough profile. If we use dual-frequency GPS receivers to set up GPS network for the horizontal control outside tunnel, increase the GPS observation time and measurement segments, approach base line vectors with precision satellites ephemerides and special software etc., then we can decrease the lateral breakthrough error caused by GPS horizontal control surveying notably. The main influence for lateral breakthrough error in super long tunnel is traverse surveying error inside tunnel.

Since tunnel's size is limited and in general the diameter is 4-8 meters, the control network inside tunnel can only be a long and narrow traverse network. The angular error is the main error source for the lateral breakthrough error. In this paper we consider the influence for lateral breakthrough error caused by control surveying errors as influence value, and propose a method for calculating this value.

Received on April 23, 2004.

ZHANG Zhenglu, professor, School of Geodesy and Geomatics, Wuhan University, 129 Luoyu Road, Wuhan 430079, China.
1 How to determine the allowance of lateral breakthrough error

1.1 Design size method

Design size method is a method for determining the allowance of lateral breakthrough error by the design size of tunnel. Some people thought to take the 1/7-1/10 of the diameter for circular tunnel or the width for square tunnel as the allowance of lateral breakthrough error. It is easy to adjust tunnel’s axis near the breakthrough profile with this value and produce no influence for tunnel’s quality, construction cost and rate. Take the Wan Jiazai Project in Shanxi province as an example, the diameters of both 5# and 7# tunnels are 4.2 m, then 1/7 of it is 600 mm. For a railway tunnel, the diameter is about 6.0 m, and its 1/10 is also 600 mm. If we take 600 mm as the allowance of lateral breakthrough error for a tunnel, which is 50 km in length, then the RMSE of lateral breakthrough (half of the allowance) is 300 mm. According to “equal influence” principle the influence value of traverse surveying inside tunnel is 245 mm, the allowance is then 490 mm; the influence value of GPS network outside tunnel is 173 mm, then the allowance is 346 mm. The values for tunnels shorter than 50 km will be appropriately decreased.

1.2 Weight function method

1.2.1 Calculating the influence value of GPS plane network outside tunnel

The influence value of GPS plane network outside tunnel can be calculated with following weight function formula of breakthrough point for terrestrial control network[2,3].

\[
\begin{align*}
\Delta y & = -a_{1b} \Delta x_{jG} \Delta x_{jG} + b_{1b} \Delta y_{jG} \Delta y_{jG} - (1 + b_{1b} \Delta x_{jG} \Delta y_{jG}) \Delta x_{jG} \Delta y_{jG} \\
\Delta z & = a_{1b} \Delta x_{jG} \Delta y_{jG} - b_{1b} \Delta y_{jG} \Delta y_{jG} - \Delta x_{jG} \Delta x_{jG} \\
\Delta c & = a_{1b} \Delta x_{jG} \Delta z_{jG} + (1 + b_{1b} \Delta x_{jG} \Delta z_{jG}) \Delta x_{jG} \Delta z_{jG} \\
\end{align*}
\]

(1)

where \(\Delta y\) is Y coordinate difference of breakthrough points from entrance point to exit point of the tunnel; \(\Delta x\) is X coordinate difference of corresponding points in tunnel coordinate system in which the X axis is from entrance point to exit point, and the Y axis is in the direction perpendicular to X axis(Fig. 1); a and b are the coefficients which can be calculated by coordinate difference and distance corresponding points; subscript G represents the breakthrough point, subscript J is the entrance point, subscript C is the exit point, subscript a is the orientation point at entrance, and subscript b is the orientation point at exit. This formula means that the influence value \(M_{GPS}\) of GPS plane control network outside tunnel is related to the position and accuracy of entrance point, exit point and corresponding orientation points, and also related to the position of the breakthrough point. We can calculate \(M_{GPS}\) by covariance propagation law, and its allowance \(\Delta_{GPS}\) is \(2M_{GPS}\). We call the method coordinate difference weight function method, with which we should simulate a GPS network as a terrestrial network with all measured sides and angles or as a network with all measured sides and azimuths, and then calculate the simulating values \(M_{GPS}\) with above formula and \(\Delta_{GPS}\). This method is proved to be correct and effective in theory and practice.

1.2.2 Calculating the influence value of traverse network inside tunnel

The traverse network inside tunnel is a very long and narrow network. There are two traverse networks from entrance and exit point to the breakthrough profile, respectively. The entrance-exit point and its corresponding orientation points are known points obtained by the GPS plane control network outside tunnel. In fact, the influence value caused by the error of traverse network inside tunnel is the RMSE of Y coordinate of traverse points on the breakthrough profile relative to the known points at the entrance and exit, and represented with \(M_{JD}\) and \(M_{CD}\), respectively. We can get it through adjustment of traverse network easily. If the breakthrough profile is at the middle of tunnel, then the influence values of the two traverse networks are equal, i.e. \(M_{JD} = M_{CD} = M_{D}\).

1.2.3 Calculating the allowance of the lateral breakthrough error

By the forementioned calculation method for
influence value inside and outside tunnel and according to the error propagation law, we can get the lateral breakthrough error $M_{GT}$ as

$$M_{GT} = \sqrt{M_{GPS}^2 + M_{JD}^2 + M_{CD}^2} = \sqrt{M_{GPS}^2 + 2M_D^2} = \sqrt{M_{GPS}^2 + M_D^2}$$

(2)

The allowance of the lateral breakthrough error $\Delta GT$ is

$$\Delta GT = 2M_{GT} = \sqrt{(2M_{GPS})^2 + (2M_D)^2} = \sqrt{\Delta_D^2 + \Delta^2_D}$$

(3)

The allowance of lateral breakthrough error caused by GPS control surveying error outside tunnel is

$$\Delta_{GPS} = 2M_{GPS}$$

(4)

The allowance of lateral breakthrough error caused by traverse control surveying error inside tunnel is

$$2\sqrt{2}M_D = 2\sqrt{2}M_{JD} = 2\sqrt{2}M_{CD}$$

(5)

For a super long tunnel from 20 km to 50 km in length, we can get $M_{GPS}$ and $M_D$ by similar calculation. Through analysis and appropriate adjustment, we can determine the allowance of the lateral breakthrough error finally.

2 Design of control network outside and inside tunnel

2.1 Design of GPS plane control network outside tunnel

A super long tunnel from 20 km to 50 km can be dealt as straight-line network. The entrance point ($J$) and exit point ($C$) are on tunnel center line, three orientation points ($J_1, J_2, J_3$) are at the entrance and other three orientation points ($C_1, C_2, C_3$) are at the exit (Fig. 1). The orientation points should be at the place where it can be observed from the entrance point or exit point. In order to decrease the effect of vertical atmospheric deflection, the height difference between entrance or exit points and orientation points should not be too big. We adopt an independent tunnel coordinate system in which the $X$ axis is from the entrance point ($J$) to the exit point ($C$), and the $Y$ axis is in the direction perpendicular to the $X$ axis. The breakthrough profile is at the middle of the tunnel and parallel to the $Y$ axis. It is not necessary to set up any transition point even for a tunnel longer than 20 km. The difference between the longest side and the shortest one is very big. We should adopt enforced centering device to minimize the effect of sighting error and centering error for sides shorter than 300 m. For GPS network observation, we should use dual frequency GPS receivers with accuracy not lower than 5 mm+1×10⁻⁶×D, and have enough independent base lines (for example, every point has at least four independent base lines in different observation segments, the number of long base lines between the entrance and exit should not be less than 5, and the observation time of long base lines should be much longer), and use precision satellites ephemerides to approach long base line vectors.

2.2 Design of traverse network inside tunnel

We set up a traverse network based on GPS plane control network inside tunnel. The traverse network can be designed in two categories; full traverse network constructed with braced quadrilateral and double traverse network, shown in Fig. 2(a) and 2(b).
The length of the long sides of traverse is designed as 500 m. Because the observation number of full traverse network (Fig. 2(a)) is very large, and the sides close to the tunnel wall are readily affected by lateral refract. We should choose the double traverse network, and have a check every two lateral sides (Fig. 2(b)). We should adopt electronic total stations with accuracy better than 1" (angle) and 3 mm $+ 2 \times 10^{-6} \cdot D$ (distance) for the observation of traverse network inside tunnel, the two short sides of the braced quadrilateral can be measured with metal band, and the directions on short sides need not to be measured.

3 Simulating calculation

3.1 GPS network outside tunnel

For the GPS network outside tunnel shown in Fig. 1, we simulate the network as one with all the sides and all the directions measured, in which the angle accuracy is 0.7", and distance accuracy is 5 mm $+ 1 \times 10^{-6} \cdot D$, and we simulate the network also as another one with all the sides and all the azimuths measured, in which the angle accuracy is 1.0" and the distance accuracy is the same as the above mentioned. We calculate the influence value of the lateral breakthrough error for super long tunnel from 20 km to 50 km. There is no significant distinguishment for the above two sorts, the results are shown in Table 1.

3.2 Traverse network inside tunnel

We simulate the networks in Figs. 2(a) and 2(b) with the angle accuracy 0.7" and distance accuracy 1 mm $+ 1 \times 10^{-6} \cdot D$. The results for full traverse network and double traverse network inside tunnel with 20 km length are listed in Table 2.

The influence values of the lateral breakthrough error of full traverse network and double traverse network for tunnel which is 4-20 km in length are listed in Table 3 for comparison.

The influence values of the lateral breakthrough error of full traverse network and double traverse network inside tunnel from 20 km to 50 km with different angle and distance accuracy are listed in Table 4.

Table 1 Influence value of GPS network outside tunnels from 20 km to 50 km/mm

| Tunnel length/km | 20  | 25  | 30  | 35  | 40  | 45  | 50  |
|------------------|-----|-----|-----|-----|-----|-----|-----|
| Calculated influence value of lateral breakthrough error | 47-55 | 58-68 | 70-82 | 82-95 | 94-108 | 106-122 | 117-137 |
| Suggested influence value | 51  | 63  | 76  | 88  | 101 | 114 | 127 |
Table 2  Comparison between two traverse networks inside super long tunnels with 20 km length

| Items                  | Full traverse network | Double traverse network |
|------------------------|-----------------------|------------------------|
| Number of observations | 165                   | 95                     |
| Number of linear observations | 03                  | 48                     |
| Redundancy             | 141                   | 19                     |
| Accuracy of weakest point/mm | 78.5                | 95.0                   |

Table 3  Comparison between influence values of two traverse networks inside tunnels from 4 km to 20 km

| Tunnel length/km | Full traverse network | Double traverse network | Difference |
|------------------|-----------------------|-------------------------|------------|
| 4.0              | 11.8                  | 13.0                    | 1.2        |
| 8.0              | 25.3                  | 29.5                    | 4.2        |
| 10.0             | 33.0                  | 39.0                    | 6.0        |
| 12.0             | 41.0                  | 49.3                    | 8.3        |
| 15.0             | 54.2                  | 66.2                    | 12.0       |
| 16.0             | 58.9                  | 72.2                    | 13.3       |
| 20.0             | 78.5                  | 95.0                    | 16.5       |

Table 4  Influence values of double traverse networks inside tunnels from 20 km to 50 km/mm

| Accuracy | Tunnel length/km | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|----------|------------------|----|----|----|----|----|----|----|
| 0.7°, 1 mm + 1×10^{-6} | 95 | 130| 165| 208| 250| 300| 344|
| 0.4°, 1 mm + 1×10^{-6} | 55 | 74 | 96 | 120| 145| 173| 200|
| 0.4°, 3 mm + 2×10^{-6} | 56 | 75 | 100| 124| 150| 178| 206|

3.3 Results analysis

From Tables 1, 2, 3 and 4, we can make the following analysis.

1) The influence value of GPS network outside tunnel is nearly proportional to the tunnel length. Because the maximum of influence value is not very big, the increment magnitude associated with the tunnel length is also not significant.

2) The difference of influence value between full traverse network and double traverse network is not significant, but the observation amount and cost of full traverse network are both much more. Hence we should choose the double traverse network inside tunnel.

3) For a super long tunnel with same length, the influence value of traverse network inside tunnel is bigger than that of GPS network outside tunnel notably.

4) The influence value of traverse network inside tunnel changes proportional to angle accuracy.

5) The influence value for longitudinal breakthrough error of network inside and outside tunnel are very small relatively. We have calculated but not listed it in this paper.

6) The influence values for lateral breakthrough error determined with the design size method have significant difference compared to the results of obtained by our method proposed in this paper. The design size method is quite simple but not reasonable.

3.4 Analysis of breakthrough error in practical project

The sum length of tunnel group in main line and southern main line of Wan Jiazi Project of Shanxi province is 212.6 km, the 5 # and 7 # tunnels on southern main line are 26.5 km and 42.6 km long, respectively. We have got breakthrough errors of 18 tunnels. The maximum breakthrough error is only 85 mm. The values are within the range from 9 mm to 85 mm, and only 6 values exceed 40 mm. The control networks inside the tunnels of Wan Jiazi Project were double traverse network. The practice proved that it is realizable for a correct breakthrough of super long tunnel with very high precision.

4 Determination and distribution of allowance of the lateral breakthrough error for super long tunnel

From simulating calculation and analysis, we can get influence values and allowance of lateral breakthrough error for super long tunnels of 20 km to 50 km, which are listed in Table 5. We
make some adjustment for the values in Table 5, and get the allowances of lateral breakthrough error for super long tunnels (listed in Table 6). The values in Table 6 have significant difference from the values of tunnels from 4 km to 20 km with one breakthrough profile in the existing specification (Table 7). The value of this paper is more strict and its allotment is more reasonable.

![Table 5 Influence values and allowances of lateral breakthrough errors for tunnels from 20 km to 50 km/mm](image)

| Type                                      | 17-20 | 20-25 | 25-30 | 30-35 | 35-40 | 40-45 | 45-50 |
|-------------------------------------------|-------|-------|-------|-------|-------|-------|-------|
| Influence values of GPS network           | 51    | 63    | 76    | 88    | 101   | 114   | 127   |
| Influence values of traverse network from entrance | 95    | 130   | 166   | 208   | 250   | 300   | 344   |
| Influence values of traverse network from exit | 95    | 130   | 166   | 208   | 250   | 300   | 344   |
| Sum of influence values                   | 144   | 195   | 247   | 307   | 368   | 440   | 503   |
| Allowances                                | 288   | 390   | 494   | 614   | 736   | 880   | 1006  |

![Table 6 Allowances of lateral breakthrough errors of tunnels from 20 km to 50 km/mm](image)

| Type                                      | 17-20 | 20-25 | 25-30 | 30-35 | 35-40 | 40-45 | 45-50 |
|-------------------------------------------|-------|-------|-------|-------|-------|-------|-------|
| Outside GPS network                       | 100   | 130   | 160   | 180   | 200   | 230   | 250   |
| Inside Traverse network                   | 282   | 378   | 474   | 592   | 712   | 850   | 968   |
| Sum                                       | 300   | 400   | 500   | 620   | 740   | 880   | 1000  |

![Table 7 Allowances of lateral breakthrough errors of tunnels from 4 km to 20 km in specification/mm](image)

| Type                                      | 4-8   | 8-10  | 10-13 | 13-17 | 17-20 |
|-------------------------------------------|-------|-------|-------|-------|-------|
| For network outside tunnel                | 90    | 120   | 180   | 240   | 300   |
| Traverse network inside tunnel             | 120   | 160   | 240   | 320   | 400   |
| Sum                                       | 150   | 200   | 300   | 400   | 500   |

5 Conclusions

The determination and distribution of the allowances of lateral breakthrough errors for super long tunnels from 20 km to 50 km is a very important problem need to be solved urgently. This paper proposes a method and a set of solution. It is significant both for research and practice. With modern measurement techniques, the error of GPS network outside tunnel has less effect on the lateral breakthrough error, and the main influence is angle observation error of traverse network inside tunnel. It is better to set up network inside tunnel with double traverse network. The lateral breakthrough error can not be allotted according to “equal influence principle”.

Compared with lateral breakthrough error, the longitudinal and vertical breakthrough errors are relative smaller and easier to meet the design requirement.

REFERENCES

1. Chen Y. Q., Li Q. Y. (1995) Engineering geodesy. Beijing: Publishing House of Surveying and Mapping.
2. Zhang Z. L. (2002) Engineering geodesy. Wuhan: Publishing House of Wuhan University.
3. Zhang X. D., Zhang Z. L. (1998) Surveying of tunnel engineering. Beijing: Publishing House of Surveying and Mapping.
4. Zhang Z. L., Guo J. M., Huang Q. Y., et al. (1996) Research into an automatic system of data gathering and processing for terrestrial control surveying. Pelzer zum 60. Geburtstag. Hannover.