INFRARED PERIOD–LUMINOSITY RELATIONS OF EVOLVED VARIABLE STARS IN THE LARGE MAGELLANIC CLOUD

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Received 2010 March 29; accepted 2010 July 27; published 2010 October 20

ABSTRACT

We combine variability information from the MAssive Compact Halo Objects survey of the Large Magellanic Cloud with infrared photometry from the Spitzer Space Telescope Surveying the Agents of a Galaxy’s Evolution survey to create a data set of ~30,000 variable red sources. We photometrically classify these sources as being on the first ascent of the red giant branch, or as being in one of three stages along the asymptotic giant branch (AGB): oxygen-rich, carbon-rich, or highly reddened with indeterminate chemistry (“extreme” AGB candidates). We present linear period–luminosity (\(P–L\)) relationships for these sources using eight separate infrared bands (\(J, H, K_s, 3.6, 4.5, 5.8, 8.0, \text{and} 24 \mu\text{m}\)) as proxies for the luminosity. We find that the wavelength dependence of the slope of the \(P–L\) relationship is different for different photometrically determined classes of AGB stars. Stars photometrically classified as O-rich show the least variation of slope with wavelength, while dust enshrouded extreme AGB stars show a pronounced trend toward steeper slopes with increasing wavelength. We find that O-rich AGB stars pulsating in the fundamental mode obey a period–magnitude relation with a slope of \(-3.41 \pm 0.04\) when magnitude is measured in the 3.6 \(\mu\text{m}\) band, in contrast to C-rich AGB stars, which obey a relation of slope \(-3.77 \pm 0.05\).

Key words: infrared: stars – Magellanic Clouds – stars: AGB and post-AGB – stars: carbon – stars: variables: general

Online-only material: color figures, machine-readable table

1. INTRODUCTION

As intermediate mass stars (~1–8 \(M_\odot\)) exhaust the helium in their cores during the late stages of stellar evolution, they begin their ascent of the asymptotic giant branch (AGB). These stars consist of an inert C/O core surrounded by concentric shells of helium and hydrogen. During the early AGB (E-AGB) phase, the evolution is driven by shell hydrogen burning. Stars are typically oxygen-rich during the E-AGB phase, with a photospheric oxygen-to-carbon ratio \(<1\). Eventually, the helium shell ignites, beginning the thermally pulsing (TP-AGB) phase. Subsequent evolution on the AGB consists of relatively long periods of hydrogen burning, punctuated at regular intervals by brief helium shell flashes (thermal pulses) that rapidly convert the helium to carbon, dramatically increasing the star’s brightness for a brief period (Vassiliadis & Wood 1993; Habing & Olofsson 2003). These thermal pulses drive large-scale convective zones in the stellar interior which can “dredge-up” nuclear-processed material, most notably newly created carbon, to the stellar surface, leading to changes in the observable surface chemistry of the star. This is the third dredge-up for intermediate mass stars, and the result is that for stars with \(M \lesssim 4 \ M_\odot\), the surface C/O ratio changes to become \(>1\) at which time the star is referred to as carbon-rich. For stars with \(M \gtrsim 4 \ M_\odot\), the temperature at the base of the convective shell is sufficiently high (\(\sim 10^8\) K) to burn the C into N, a process known as “hot-bottom burning (HBB),” and the star remains O-rich.

In addition, hydrodynamic instabilities drive shock waves through the stellar interior, levitating the outer layers of the star into cooler regions where dust grains can condense. Radiation pressure then drives the dust (and the gas to which it is collisionally coupled) into the interstellar medium. These pulsations cause AGB stars to exhibit complicated, multi-periodic variations in brightness (Fraser et al. 2005). In addition, this process can cause an AGB star to lose mass at rates up to \(\sim 10^{-3} \ M_\odot\text{yr}^{-1}\) (van Loon et al. 1999). This makes AGB stars one of the dominant sources of dust in the universe and a key component in the chemical evolution of galaxies. Toward the end of its AGB lifetime, this mass loss can completely enshroud a star with a thick circumstellar dust shell. These “extreme” AGB stars are nearly undetectable in the optical, and because the light from the star is dominated by thermal emission from the circumstellar shell, cannot be classified as either O-rich or C-rich based on their near-IR photometry (Cioni et al. 2006). Most of them are believed to be C-rich, but the brightest should be so massive that HBB has left them O-rich (Matsuura et al. 2009). Without spectroscopic confirmation, we do not propose classifications for specific sources of this class in this paper.

The Surveying the Agents of a Galaxy’s Evolution (SAGE) survey (Meixner et al. 2006) is an unbiased \(7^\circ \times 7^\circ\) survey of the Large Magellanic Cloud (LMC) using the IRAC (3.6, 4.5, 5.8, and 8.0 \(\mu\text{m}\)) and MIPS (24, 70, and 120 \(\mu\text{m}\)), instruments aboard the Spitzer Space Telescope (Werner et al. 2004), intended to trace the life cycle of the baryonic matter in the LMC. This wavelength regime makes SAGE extremely well suited to studies of evolved stars. The LMC makes an ideal target for studies of stellar populations, as its high-galactic latitude minimizes the foreground contamination, and its distance of \(\sim 50\) kpc (van Leeuwen et al. 2007) makes it possible to resolve individual stars while simultaneously being able to neglect its three-dimensional structure.

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Mira variables (a subset of AGB stars) have been known to follow a linear relationship in magnitude, log(Period) space for nearly a century (Gerasimovic 1928; Feast et al. 1989, and references therein). Wood & Sebo (1996) extended the known data set of AGB variables to shorter periods and established that Mira-type variables pulsate in the fundamental mode, and that other, higher order pulsational modes exist as well. Expanding the number of data points to ~1000, Wood et al. (1999) identified four parallel linear sequences in period–luminosity (P–L) space and proposed mechanisms for each. Ita et al. (2004), using a sample of ~30,000 variable evolved stars with period information derived from the Optical Gravitational Lensing Survey (OGLE; Udalski et al. 1997), further refined these sequences (see Section 2.1). Also using OGLE data, Soszynski et al. (2004) identified a new P–L sequence in the secondary, non-dominant variation in the light curves of evolved stars, and proposed this as a means of separating stars dimmer than the tip of the red giant branch (TRGB) into stars on the first ascent of the red giant branch (RGB) and those evolving along the AGB. With a data set of comparable size derived from the MAAssive Compact Halo Objects (MACHO) survey (Alcock et al. 1997), Fraser et al. (2005, 2008) further investigated the multi-periodic nature of stars on the sequences identified by Wood et al. (1999). Using early results from the SAGE survey, Glass et al. (2009) extended work on the P–L relation in AGB stars by investigating the dependence of the calculated slope of the relation on wavelength.

Our survey combines the SAGE project’s photometry archive and the MACHO variability data set to produce the largest sample of variable evolved stars, with photometry measured in multiple near-IR wavelengths, to date. The unprecedented size of our data set, combined with our extensive spectral coverage, reveals new patterns and details in the relationship between luminosity, variability, and spectral energy distribution (SED) amongst evolved stars.

This paper is organized as follows. Section 2 details our source selection process, the identification of the multiple sequences observed in P–L space, and the division of our sample into six photometrically determined categories. Section 3 presents the quantitative and qualitative effects observed within our sample. Section 4 compares our current study to previous work in the field, and Section 5 summarizes the conclusions of the current work.

2. DATA

The SAGE survey observed the LMC in two epochs, three months apart. From each epoch, two source lists, an archive and a catalog, were generated. The catalog stresses reliability over completeness and requires more stringent data quality standards (higher signal to noise, less crowding in the field) for inclusion of a source than the archive. The standards for source inclusion in both archive and catalog are detailed in Meixner et al. (2006).

In 2009, the SAGE team released the SAGE mosaic photometry catalog and archive to the Spitzer Science Center. By combining data from all epochs of the SAGE survey, the mosaic photometry catalog and archive are more complete than the individual epochs, and photometric errors are reduced. The mosaic photometry data are well documented in the SAGE Data Products Description document5. The SAGE mosaic photometry catalog contains ~6.5 million sources, and the archive contains ~7 million.

5 http://data.spitzer.caltech.edu/popular/sage/20090922_enhanced/documents/SAGEDataProductsDescription_Sept09.pdf

Figure 1. Period–luminosity sequences for evolved stars in the LMC. Sequence 1 consists of stars pulsating in the fundamental mode, while sequences 2–4 are higher order pulsational modes. Sequence E consists of ellipsoidal binary systems, and the mechanism responsible for the variation on sequence D is not known. The naming convention follows that of Fraser et al. (2008).

(A color version of this figure is available in the online journal.)

We extract a list of 32,744 sources from Fraser et al.’s (2008) catalog of evolved long-period variables (LPVs) in the LMC with a counterpart in the SAGE mosaic photometry archive. We utilize the archive in order to maximize sample size. We use a matching radius of 2′, and keep only the closest SAGE source to a given MACHO source. We do not include the ~11,000 sources Fraser et al. (2008) identify as the “one-year artifact,” which have an artificial period of 365 days caused by MACHO’s annual observing schedule. The database merging process can create duplicate entries for some sources, due to slight (~0′1) shifts in position between SAGE epoch 1, epoch 2, and the single frame mosaic photometry. By requiring that every source in our list has a unique MACHO field, tile, sequence identifier, we find ~500 of these duplicate matches in our data set and cull them from our final list. Based on the definitions used by Fraser et al. (2008), we assign stars to one of six roughly parallel sequences in period–Ks space (Figure 1). Our definitions are detailed in Appendix A.

We are left with a final data set of 30,747 evolved stars with well-defined MACHO periods and high quality near IR photometry from Two Micron All Sky Survey (2MASS) and SAGE. Table 1 compares the number of sources in the present study to those in Fraser et al. (2008), Srinivasan et al. (2009), and Vijh et al. (2009). Our entire data set is available online. Table 2 presents the photometric information for a few sources from our sample as a guide to the format of the online table.

2.1. Period–Luminosity Sequences

Wood et al. (1999) identified five parallel P–L sequences in the MACHO data set and proposed underlying physical mechanisms for them. In order of increasing period, the sequences of pulsating stars were named A, B, and C, with sequences D and E exhibiting variation due to an unknown mechanism. Sequence C was identified as pulsation in the fundamental mode, sequence B as the first and second overtones, and sequence A as the third overtone. Kiss & Bedding (2003) and Ita et al. (2004) split sequence B into two sequences (C′ and B) representing pulsation in the first and second overtone, respectively. Fraser et al. (2005) retained the names of sequences D and E from Wood et al. (1999), but renamed the pulsation sequences 1–4 in order of decreasing period, in order to easily accommodate shorter period sequences, and align the naming convention with
increasing pulsation overtone. We follow the naming convention of Fraser et al. (2005, 2008) because of this parallel with theory. Numbering the sequences in order of decreasing period means that higher sequence numbers generally correspond to higher order pulsational modes, aligning the empirical naming convention and a theoretical explanation more gracefully. Nicholls et al. (2010) have provided convincing evidence that sequence E consists of ellipsoidal binaries, systems in which the red giant member has filled its Roche lobe. Recent work (e.g., Nie et al. 2010; Nicholls et al. 2009) has demonstrated that the variability seen on sequence D cannot be due to binarity, but its actual cause remains unknown. We keep the alphabetic names for these sequences to separate them from the stellar pulsation sequences. Table 3 summarizes these conventions, and Figure 1 illustrates these sequences. The precise definitions of these sequence are discussed in Appendix A.

2.2. Identification and Classification of AGB Candidates

Before being photometrically classified, all sources were de-reddened in the \( J \), \( H \), and \( K_s \) bands following Glass (1999). Specifically, we use corrections of \( A_J = 0.112 \), \( A_H = 0.065 \), and \( A_K = 0.037 \) mag. We define AGB candidates using 2MASS and SAGE photometry (Cioni et al. 2006; Blum et al. 2006; Srinivasan et al. 2009). We identify a source as an AGB candidate by requiring that

\[
K_s > -13.333 \times (J - K_s) + 24.666 \quad \text{and} \quad K_s < 12.05. \quad (1)
\]

We classify sources as low or moderately obscured oxygen-rich and carbon-rich AGB candidates based on their near-IR photometry. Specifically, a source is classified as an O-rich candidate if it lies leftward (blueward) of the line

\[
K_s = -13.333 \times (J - K_s) + 24.666
\]

in the \( K_s \) versus \( J - K_s \) color–magnitude diagram (CMD), and a C-rich candidate if it lies rightward (redward) of this line (Cioni et al. 2006, Figure 1). Extreme AGB stars are defined using their position in the [3.6] versus \( J-[3.6] \) CMD

\[
J - [3.6] > 3.1 \quad \text{and} \quad [3.6] < 10.5 \quad (3)
\]

Table 1

| Source | O-rich | C-rich | Extreme AGB Stars | Total AGB Stars | RGB Stars | Total Sources |
|--------|--------|--------|-------------------|-----------------|----------|--------------|
| Fraser et al. (2008)a | N/A    | N/A    | N/A              | N/A            | N/A      | 56453        |
| Srinivasan et al. (2009)b | 17938  | 5179   | 1428             | 24565          | N/A      | 24565        |
| Vijn et al. (2009)b | 353    | 426    | 820              | 1599           | N/A      | 1967         |
| This work     | 12172  | 4455   | 432              | 17059          | 13688    | 30747        |

Notes. Number of sources used in recent studies of AGBs and variable sources in the LMC. Classification of sources as RGB, O-rich, etc., is done photometrically, using Equation (1)–(3).

a Fraser et al. (2008) did not classify their AGB candidates as O-rich, C-rich, etc.
b Did not use RGB classification.

table is available in its entirety in a machine-readable form in the online journal. A portion is shown here for guidance regarding its form and content.)

Table 2

| SAGE IDa | MACHO FTSb | 2MASS ID | Classificationc | Amplituded | Period\(g\) | J mag | \(\sigma_J\) | H mag | \(\sigma_H\) | \(\sigma_J\) |
|----------|------------|----------|-----------------|------------|-------------|-------|------------|-------|------------|-----------|
| SSTISAGEMA J050220.31–691523.5 | 1.3564,19 | 1204226 | O               | 0.121      | 512         | 12.93 | 0.02       | 12.01 | 0.03       | ...       |
| SSTISAGEMA J050324.49–684624.1 | 1.3692,24 | 1204566 | C               | 0.250      | 1350        | 12.33 | 0.05       | 11.32 | 0.04       | ...       |
| SSTISAGEMA J054643.17–703315.3 | 12.10805.486 | 12528609 | X               | 1.485      | 512         | 15.47 | 0.07       | 13.29 | 0.03       | ...       |
| SSTISAGEMA J050214.21–692048.9 | 1.3536,74 | 12528609 | R               | 0.092      | 72          | 14.58 | 0.03       | 13.85 | 0.03       | ...       |

Notes.
a The SAGE single frame mosaic photometry identifier containing the coordinates of the IRAC source.
b The MACHO Field Tile Sequence identifier of the corresponding MACHO source.
c Photometric classification of source as RGB, oxygen-rich, carbon-rich, or extreme AGB candidate. See the text for details.
d Variability parameters from Fraser et al. (2008).

Table 3

| Fraser et al. (2005) | Wood et al. (1999) | Ita et al. (2004) | Glass et al. (2009) | Theoretical Explanation |
|----------------------|--------------------|------------------|---------------------|-------------------------|
| Sequence 1           | Sequence C         | Sequence C       | Sequence C          | Pulsation in fundamental mode |
| Sequence 2           | Sequence B         | Sequence C       | Sequence B          | First/second overtone pulsation |
| Sequence 3           | Sequence B         | Sequence A       | Sequence A          | Third overtone pulsation |
| Sequence 4           | Sequence D         | Sequence D       | Sequence D          | Undetermined |
| Sequence E           | Sequence E         | Sequence E       | N/A                 | Ellipsoidal binaries |

Notes. Naming conventions used to refer to the various \( P-L \) sequences first identified by Wood et al. (1999). This paper follows the convention of Fraser et al. (2005), because the numeric naming scheme more closely aligns with the theoretical explanation.
or the IRAC [8.0] versus [8.0]–[3.6] CMD if a source lacks a valid J-band magnitude. These sources are thought to be highly evolved AGB stars, enshrouded by thick circumstellar dust shells with high rates of mass loss.

The TRGB is located at $K_s = 12.0$ (e.g., Srinivasan et al. 2009) or $J = 14.54$ (Cioni et al. 2006) and is typically identified by a clear falloff in population density (e.g., Nikolaev & Weinberg 2000). However, while the TRGB is justified on physical grounds as a maximum luminosity for RGB stars, it does not represent a physically motivated minimum luminosity for stars on the AGB. Examination of a plot of $K_s$ magnitude versus variability period (Fraser et al. 2008, Figure 2) shows that the sudden decrease of population density brighter than $K_s = 12.0$ indicative of the TRGB only appears on $P$–$L$ sequences 3 and 4. We thus classify a star as an RGB candidate if it is fainter than the $K_s$ or J-band TRGB cutoff, and it lies on either sequence 3 or 4. Stars dimmer than the TRGB on sequences 1 and 2 are classified as O-rich AGB candidates. Soszynski et al. (2004) proposed a means of separating AGB stars dimmer than the TRGB on sequence 3 or 4 from the dominant RGB population on these sequences. Their method relied on analysis of non-dominant periods and a distinct $P$–$L$ sequence with shorter periods than sequence 4 (this sequence would be called sequence 5 in our nomenclature). However, we do not detect this sequence in either the dominant or secondary pulsation periods of our sample, and thus we do not employ this method. The use of periods in addition to the dominant two periods available to us grants Soszynski et al. (2004) finer resolution in separating RGB stars from AGB candidates dimmer than the TRGB, but the consistency of our results with those of other authors gives us confidence in the broad correctness of our classification. Figure 2 reproduces Figure 1, but this time color coded according to the photometrically determined chemical classification we assign to the stars in our sample. Table 4 details the number of each class of star on each $P$–$L$ sequence. Figure 3, a histogram of [3.6] magnitude color coded by $P$–$L$ sequence, provides further justification for our classification. Sequences 3 and 4 are dominated by a large population dimmer than [3.6] $\approx 12$. Since there is no theoretical expectation for AGB stars to pile up at the TRGB, we interpret this as evidence for a large RGB population on sequences 3 and 4. Sequences 1 and 2 do show evidence of a distinct population of stars at this brightness (more pronounced in sequence 2), but neither shows the dramatic RGB population found in sequences 3 and 4. We interpret this as a small amount of RGB contamination in sequences 1 and 2. All four sequences show a stellar population with a peak at [3.6] $\approx 11.2$, coincident with our photometrically determined O-rich AGB candidate population. In addition, the stars photometrically classified as C-rich stars are visible in sequences 1 and 2 as a peak in the $P$–$L$ sequences mirror and lend support to the photometrically determined chemical classifications detailed in Table 4.

Highly evolved AGB stars and young stellar objects (YSOs) have similar mid-IR colors. Whitney et al. (2008) suggest that the region of the [8.0] versus [8.0]–[24] CMD defined by

$$[8.0] - [24] > 2.2 \quad \text{and} \quad [8.0] > 11 - 1.33 \times ([8.0] - [24])$$

costitutes a region in color–magnitude space dominated by YSOs. As all of our sources have well-determined MACHO periods, none of our samples are YSOs. Of our $\approx 30,000$ stars, we find that only 150 sources (109 RGB, 3 extreme AGB, and $\approx 40$ O-rich and C-rich AGB stars) fall in this region. We note that the optical wavelengths used by the MACHO survey bias our data set to undersample this region of CMD space, but the extremely small fraction of our sample that falls into this region lends support to the criteria proposed by Whitney et al. (2008).

Our sample consists of 17,059 AGB candidate stars: 12,172 sources photometrically classified as O-rich, 4455 as C-rich, and
432 as extreme AGB candidates. Based on near-IR photometry and variability period, 13,688 sources are classified as stars on the RGB.

The $K_s$ magnitude is often used as a proxy for luminosity for AGB stars (e.g., Wood et al. 1999; Cioni et al. 2006). The inclusion of Spitzer IRAC and MIPS data at wavelengths out to 24 $\mu$m reveals new features in the period–magnitude diagram, especially among the reddest sources, the extreme AGBs, whose SEDs peak at 3 $\mu$m or redward (Vijh et al. 2009). Figure 4 compares the 2MASS $K_s$ band, the IRAC 4.5, 5.8, and 8.0 $\mu$m bands, along with the MIPS 24 $\mu$m band as luminosity proxies, all plotted against period as in Figures 1 and 2.

The $K_s$ band, in the upper left, is a reasonable approximation for the brightness of the low-obscuration RGB, O-rich and C-rich candidates, but comparison with the other panels in Figure 4 shows that this approximation fails for the extreme AGB candidates. The RGB, O-rich and C-rich candidates appear in roughly the same positions, relative to one another, in the 2MASS and IRAC bands. The extreme sources, however, clearly have their luminosity underestimated by the $K_s$ band. These sources, the most luminous in our sample (Srinivasan et al. 2009), appear no more luminous than the C-rich candidates at this wavelength. The 24 $\mu$m flux is therefore not representative of the intrinsic brightness of these sources. This poor estimate for intrinsic brightness causes the sequence structure to break down entirely.

3. RESULTS

3.1. Period–Magnitude Relations

Using the method of weighted least-squares, linear models of the form

$$m = A \times \log P + B$$  \hspace{1cm} (4)

were fit to each of the sequences as a whole, and to each subgroup (O-rich, etc.) within each sequence. Periods and amplitudes are from the dominant mode of variability derived by Fraser et al. (2005, 2008) from MACHO blue-band light curves. Periods were considered free of errors, and because the SAGE observations are not phase corrected, magnitudes were weighted by the inverse of the quadrature sum of the 1$\sigma$ photometric error and one-half the MACHO peak-to-peak variability amplitude. Table 5 lists the linear-fit parameters thus determined for each
All stars

C-rich

O-rich

MACHO variability amplitudes to the distribution of the residual epoch 1 and epoch 2. The median optical amplitudes about the best-fit line (see Table 5) is due to the intrinsic variation in brightness between epoch 1 and 2 observations. Thus, the range of variation in brightness between epoch 1 and 2 observations could sample these stars anywhere between maximum and minimum brightness. The residual scatter about the \( P-L \) relations gives us an indication of the scale of the IR variability of our sample. Both quantities followed similar distributions and were of the same scale. This justifies our use of the MACHO optical variability amplitude as a measure of the IR variability of our sources for the estimation of the uncertainty in the IR mean magnitudes in the \( P-L \) relations.

Figure 5 shows the linear fits to the stars in sequence 1 in the \([3.6]\) versus period diagram. Sequence 1 is consistent with stellar pulsation in the fundamental mode (Fraser et al. 2005), and was chosen because it is the only sequence which contains a significant number of extreme AGB sources on it. Notice the smooth progression from shallow to steep slopes for the three different classes of AGB stars.

(A color version of this figure is available in the online journal.)

The increasing steepness of the linear relations followed by the different categories of AGB star would be consistent with a nonlinear \( P-L \) relation if the evolved variable stars were not divided into sub-categories. We experimented with fitting quadratic functions to all the stars in each sequence, regardless of chemical classification. While we found such fits to be consistent with our data, we did not find them to be statistically superior fits to the linear models presented here.

Table 5 lists the parameters of the linear fits to each of the six sequences. Because we only use the period measured with our data, we did not find them to be statistically superior fits to the linear models presented here.

The time-stamp information necessary for the phase correction necessary to compute a true mean magnitude for these variable sources. Some of our sources have been observed near maximum brightness, while other were near their minimum brightness. The residual scatter about the \( P-L \) relations gives us an indication of the scale of the IR variability of our sample. Both quantities followed similar distributions and were of the same scale. This justifies our use of the MACHO optical variability amplitude as a measure of the IR variability of our sources for the estimation of the uncertainty in the IR mean magnitudes in the \( P-L \) relations.

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Table 5 lists the parameters of the linear fits to each of the six sequences in the \( 3.6 \mu \text{m} \) band. Identical tables for all of the 2MASS, IRAC, and MIPS bands used in this work are included in Appendix B.

Table 5

| Classification | Slope | Intercept | Residual | \( r^2 \) | Number |
|---------------|-------|-----------|----------|---------|--------|
| Sequence 4    |       |           |          |         |        |
| RGB           | \(-3.67 \pm 0.05\) | \(17.45 \pm 0.08\) | 0.127 | 0.687 | 2064   |
| O-rich        | \(-3.90 \pm 0.04\) | \(17.58 \pm 0.06\) | 0.121 | 0.886 | 1432   |
| C-rich        | \(-4.14 \pm 0.36\) | \(18.04 \pm 0.58\) | 0.144 | 0.829 | 29     |
| All stars     | \(-4.56 \pm 0.02\) | \(18.68 \pm 0.03\) | 0.138 | 0.935 | 3525   |
| Sequence 3    |       |           |          |         |        |
| RGB           | \(-3.32 \pm 0.05\) | \(17.57 \pm 0.08\) | 0.122 | 0.704 | 1983   |
| O-rich        | \(-4.09 \pm 0.04\) | \(18.62 \pm 0.07\) | 0.182 | 0.867 | 1704   |
| C-rich        | \(-3.81 \pm 0.27\) | \(18.04 \pm 0.51\) | 0.248 | 0.788 | 56     |
| All stars     | \(-4.51 \pm 0.02\) | \(19.45 \pm 0.03\) | 0.167 | 0.939 | 3744   |
| Sequence 2    |       |           |          |         |        |
| O-rich        | \(-4.24 \pm 0.02\) | \(19.51 \pm 0.04\) | 0.153 | 0.942 | 2289   |
| C-rich        | \(-4.25 \pm 0.05\) | \(19.32 \pm 0.11\) | 0.189 | 0.890 | 887    |
| All stars     | \(-4.54 \pm 0.02\) | \(20.04 \pm 0.03\) | 0.177 | 0.960 | 3176   |
| Sequence 1    |       |           |          |         |        |
| O-rich        | \(-3.41 \pm 0.04\) | \(18.89 \pm 0.10\) | 0.271 | 0.729 | 2221   |
| C-rich        | \(-3.77 \pm 0.05\) | \(19.35 \pm 0.12\) | 0.251 | 0.777 | 1813   |
| Extreme AGB  | \(-4.27 \pm 0.10\) | \(20.37 \pm 0.49\) | 0.336 | 0.586 | 371    |
| All stars     | \(-4.22 \pm 0.02\) | \(20.55 \pm 0.06\) | 0.304 | 0.875 | 4405   |
| Sequence D    |       |           |          |         |        |
| RGB           | \(-2.68 \pm 0.03\) | \(19.33 \pm 0.07\) | 0.211 | 0.608 | 5588   |
| O-rich        | \(-3.58 \pm 0.04\) | \(21.39 \pm 0.10\) | 0.217 | 0.752 | 3354   |
| C-rich        | \(-3.98 \pm 0.08\) | \(22.17 \pm 0.25\) | 0.299 | 0.701 | 969    |
| Extreme AGB  | \(-4.71 \pm 1.40\) | \(23.72 \pm 4.28\) | 0.545 | 0.414 | 18     |
| All stars     | \(-4.16 \pm 0.02\) | \(23.12 \pm 0.04\) | 0.288 | 0.863 | 9929   |
| Sequence E    |       |           |          |         |        |
| RGB           | \(-2.62 \pm 0.02\) | \(18.52 \pm 0.05\) | 0.274 | 0.894 | 1718   |
| All stars     | \(-2.62 \pm 0.02\) | \(18.52 \pm 0.05\) | 0.274 | 0.894 | 1718   |

Notes. Parameters for the linear fit to the IRAC \([3.6]\) \( P \)--magnitudes relation. "Residual" is the standard error of the residuals to the fit. The \( r^2 \) column lists the correlation coefficient and "number" lists the number of sources used to derive each fit.

of the six sequences, using the IRAC \([3.6]\) magnitude as the luminosity proxy.

Previous work has indicated that Mira variables exhibit much lower amplitude variation in the IR than in the optical (Lattanzio & Wood 2003), and thus using one-half of the MACHO amplitude in the error term might overestimate the true scatter in the data due to intrinsic variation. To investigate this, we extracted a sample of \( \sim 100 \) sources from the catalog of Vijh et al. (2009) with well-determined MACHO periods between 150 and 200 days. At these periods, the three month cadence of the \( SAGE \) observations could sample these stars anywhere between maximum and minimum brightness. Thus, the range of variation in brightness between epoch 1 and 2 observations could represent the full mid-IR peak-to-peak amplitude of variation. The MACHO amplitudes for the O-rich and C-rich AGB candidates are statistically very similar to the variation seen in this smaller sample. The small sample from Vijh et al. (2009) has a median mid-IR variation of \(-0.2 \) mag between \( SAGE \) epoch 1 and epoch 2. The median optical amplitudes for all O-rich and C-rich sources together are 0.2 and 0.3 mag, respectively. In addition, we compared the distribution of the MACHO variability amplitudes to the distribution of the residual scatter (Table 5) about the \( P-L \) fits. The residual scatter of \(-0.5 \) mag about the best-fit line (see Table 5) is due to the intrinsic variability of the sources. The \( SAGE \) mosaic photometry is constructed from the two epochs of the \( SAGE \) survey, but lacks
in a particular band is between the longest period stars of a particular type and the shorter period stars of the same type on the same sequence. Figure 6 shows that O-rich AGB candidates follow roughly the same linear relationship in all IR bands, showing that there is little difference between the SEDs of the O-rich AGB candidates with the longest periods, and those with the shortest periods. The extreme AGB candidates, on the other hand, show a very strong trend toward steeper slopes as one moves toward longer wavelengths. The slope of the $P$–$L$ relation is a measure of the dependence of brightness in a certain band to period. Smaller values for slope indicate smaller differences in brightness between short and long-period stars within one class on one sequence (e.g., RGB stars on sequence 3). Steep slopes indicate large differences between short and long-period stars within one class on one sequence (e.g., extreme AGB stars on sequence 1). All extreme AGB candidates become brighter as one looks at redder bands because they possess dusty circumstellar envelopes which reprocess the near-IR stellar photospheric emission to longer IR radiation. However, this mid-IR brightening is more dramatic for the longest period sources on a given $P$–$L$ sequence, which causes the slope to become steeper and indicates that the longest period sources have the thickest circumstellar dust shells. Thus, the steep slopes of the $P$–$L$ relations followed by extreme AGB candidates at longer wavelengths indicate not just the presence of circumstellar matter, but show that period is more closely tied to the presence of circumstellar matter in extreme AGB stars than in, e.g., O-rich AGB stars.

The slope of the log $P$–magnitude relation for C-rich AGB stars shows a more complicated variation with wavelength. In the IRAC [5.8] band (and to a lesser extent in the 4.5 $\mu$m band), the slope becomes noticeably less steep, indicating that there is less brightness contrast between the C-rich stars with longest periods and those with shorter periods in this band. The spectra of C-rich stars have a strong CO absorption feature at $\sim$4–5 $\mu$m (e.g., Aringer et al. 2009, Figure 4).

The observed decrease in the slope of the log $P$–magnitude relation in the 5.8 $\mu$m band compared to neighboring bands indicates a possible relationship between the strength of this feature and the pulsation period of carbon-rich stars; specifically, that longer-period C-rich AGB stars may have stronger CO absorption features than those stars with shorter periods. While many of them are thought to be carbon-rich, extreme AGB candidates do not show this effect because their IR emission is dominated by their circumstellar dust shells. The absence of this effect in O-rich AGB candidates, which also feature CO in their photospheres, argues against our hypothesis. Further IR spectroscopic follow-up, such as the SAGE–SPEC survey (Kemper et al. 2010) will shed further light on this.

Figure 6 also has implications for future observational strategies. AGB stars are a dominant source of IR light in galaxies with intermediate- and old-age stellar populations. With major IR observatories such as the James Webb Space Telescope coming online in the next few years, the AGB ($P$–$L$) relationship constitutes a possible distance indicator. Figure 6 shows that the observed scatter in the $P$–$L$ relation could be reduced, while maximizing sample size, by observing in the near-IR, specifically the $K_s$ band ($\sim$2.2 $\mu$m). In this band, all AGB candidates brighter than the TRGB obey very nearly the same $P$–$L$ relation. Furthermore, the slope of the relation measured for AGB candidates in the $K_s$ band will be the slope of the relation followed by O-rich AGB stars (which can be photometrically classified) at all IR wavelengths out to 8 $\mu$m (also see Table 6). Figure 8 plots the median SED for each classification of star used in this paper. Error bars represent the spread of each flux within the population, not measurement uncertainty. The $K_s$ band is the location of the peak of the SED for the majority of C-rich AGB stars (also see Srinivasan et al. 2009) and is very near the peak for both the O-rich AGB stars and those on the first ascent of the RGB. Furthermore, 97% of our sample (including 82% of the extreme AGB stars) have a valid flux measured in the $K_s$ band, implying that this band will provide an adequate sample size.
The intrinsic brightness of the reddest, most evolved AGB stars will be severely underestimated (see Section 2.2), but these stars can be identified by near-IR color and excluded from the sample to reduce scatter in the derived relationship. Table 6 presents all the derived $K_s$ versus log $P$ relations for our sample.

### 3.2. Long Secondary Periods

Approximately 30% of variable AGB stars exhibit long secondary periods (LSPs), variations which occur on time scales nearly an order of magnitude longer than pulsation in the fundamental mode (sequence 1). The LSP phenomenon is represented in our data set by sequence D, and the mechanism behind it is still unknown (Nicholls et al. 2009; Nie et al. 2010). Fraser et al. (2008) noted that the variability properties of sequence D are consistent with a population drawn from all of the other $P$–$L$ sequences. The LSP is known to be related to mass loss (Wood & Nicholls 2009), but in an as yet unknown manner.

The lower left panel of Figure 7 shows the variation of the LSP with wavelength. The slopes of the $P$–$L$ relations followed by the AGB candidate stars on sequence D show the same dependence on wavelength as the stars on the other sequences. O-rich AGB stars show little variation in $P$–$L$ relation slope with wavelength, and C-rich AGB stars show identical behavior in sequences 1, 2, 3, and D. We do not include the extreme AGB candidates from sequence D in Figure 7 because the fits, based on only 18 stars, are not statistically significant (Table 6). Whatever mechanism lies behind the LSP, it connects the period of variation of the star with its SED in the same manner as fundamental and overtone stellar pulsation.

### 3.3. RGB $P$–$L$ Relation

Figure 7 shows how the slopes of the RGB $P$–$L$ relations on sequences 3, 4, E, and D relate to those determined for the various classes of AGB candidates. The slope of the RGB-star $P$–$L$ relation is relatively wavelength independent (qualitatively similar to the behavior seen in O-rich candidates) and is consistently less steep than the AGB-star slope at all wavelengths. This effect has been noted by other authors (Ita et al. 2004; Glass et al. 2009), and the persistence of this trend

![Figure 7](image-url)

(A color version of this figure is available in the online journal.)

Table 6

| Classification | Slope ($K_s$) | Intercept ($K_s$) | Residual ($K_s$) | $r^2$ | Number |
|----------------|-------------|-----------------|-----------------|------|--------|
| Sequence 4     |             |                 |                 |      |        |
| RGB            | $-3.43 \pm 0.05$ | $17.22 \pm 0.07$ | $0.111$ | $0.677$ | $2054$ |
| O-rich         | $-3.73 \pm 0.04$ | $17.43 \pm 0.06$ | $0.126$ | $0.867$ | $1434$ |
| C-rich         | $-3.73 \pm 0.32$ | $17.58 \pm 0.52$ | $0.134$ | $0.833$ | $29$ |
| All stars      | $-4.46 \pm 0.02$ | $18.65 \pm 0.03$ | $0.135$ | $0.929$ | $3517$ |
| Sequence 3     |             |                 |                 |      |        |
| RGB            | $-3.12 \pm 0.05$ | $17.36 \pm 0.08$ | $0.117$ | $0.684$ | $1981$ |
| O-rich         | $-3.85 \pm 0.04$ | $18.33 \pm 0.07$ | $0.182$ | $0.846$ | $1707$ |
| C-rich         | $-3.61 \pm 0.19$ | $17.99 \pm 0.35$ | $0.182$ | $0.875$ | $56$ |
| All stars      | $-4.33 \pm 0.02$ | $19.27 \pm 0.03$ | $0.166$ | $0.932$ | $3744$ |
| Sequence 2     |             |                 |                 |      |        |
| O-rich         | $-4.06 \pm 0.02$ | $19.33 \pm 0.04$ | $0.143$ | $0.948$ | $2292$ |
| C-rich         | $-3.69 \pm 0.04$ | $18.61 \pm 0.09$ | $0.155$ | $0.899$ | $891$ |
| All stars      | $-3.93 \pm 0.01$ | $19.10 \pm 0.03$ | $0.149$ | $0.963$ | $3183$ |
| Sequence 1     |             |                 |                 |      |        |
| O-rich         | $-3.31 \pm 0.04$ | $18.87 \pm 0.09$ | $0.272$ | $0.760$ | $2218$ |
| C-rich         | $-3.16 \pm 0.04$ | $18.40 \pm 0.11$ | $0.230$ | $0.744$ | $1817$ |
| Extreme AGB    | $-2.56 \pm 0.35$ | $17.43 \pm 0.91$ | $0.444$ | $0.147$ | $312$ |
| All stars      | $-3.34 \pm 0.02$ | $18.90 \pm 0.05$ | $0.293$ | $0.848$ | $4347$ |
| Sequence D     |             |                 |                 |      |        |
| RGB            | $-2.60 \pm 0.03$ | $19.26 \pm 0.07$ | $0.197$ | $0.622$ | $5611$ |
| O-rich         | $-3.40 \pm 0.03$ | $21.04 \pm 0.09$ | $0.197$ | $0.767$ | $3364$ |
| C-rich         | $-3.58 \pm 0.06$ | $21.50 \pm 0.18$ | $0.220$ | $0.774$ | $971$ |
| Extreme AGB    | $-4.41 \pm 0.64$ | $24.04 \pm 1.96$ | $0.246$ | $0.748$ | $18$ |
| All stars      | $-3.81 \pm 0.01$ | $22.34 \pm 0.04$ | $0.236$ | $0.873$ | $9964$ |
| Sequence E     |             |                 |                 |      |        |
| RGB            | $-2.54 \pm 0.02$ | $18.44 \pm 0.05$ | $0.246$ | $0.893$ | $1706$ |
| All stars      | $-2.54 \pm 0.02$ | $18.44 \pm 0.05$ | $0.246$ | $0.893$ | $1706$ |

Note. Identical to Table 5, with magnitude measured in the $K_s$ band.
at longer wavelengths lends support to the conclusion that the populations above and below the TRGB on these sequences are indeed dominated by different classes of star.

Stellar evolutionary models (e.g., Castellani et al. 2003) predict (for a limited mass range) an offset in variability period at a given luminosity between AGB stars and those on the first ascent of the RGB of \( \delta \log P \approx 0.03 \), with RGB stars having the longer period. Kiss & Bedding (2003) observed this effect in their OGLE based survey, and we confirm it using MACHO. This effect is not immediately visually apparent in our sample due to crowding (Figure 2). However, by solving our liner \( P-L \) relationships for \( \log P \) at a fixed \( [3.6] = 12.0 \), we find \( \delta \log P = 0.05 \) between the stars we define as RGB candidates and those classified as AGBs on sequences 3, 4, and D.

Nicholls et al. (2010) examined the orbital parameters of a sample of sequence E binary stars and demonstrated that their sample did indeed consist of ellipsoidal binary systems, in contrast to a sample of field red giants and a sample of stars from sequence D (Nicholls et al. 2009). In addition, they looked for, but did not find, evidence of an IR excess in the colors of sequence E stars, an indication of mass loss. Figure 7 shows that the \( P-L \) relations of the RGB stars on sequence E and D behave, as a function of wavelength, exactly like the RGB stars on sequences 3 and 4. The lack of change in the slope of the \( P-L \) relation indicates that period is not coupled to mass loss in these stars.

The \( P-L \) relations of RGB stars on sequences E and D are systematically less steep than those of the RGB stars on the pulsational sequences 3 and 4. The slope may be less steep because of the photometric definition of an RGB star used in this study, see Appendix A. Nevertheless, we confirm the null result of Nicholls et al. (2010) in that we see no evidence of any difference in the RGB \( P-L \) relations as a function of wavelength for sequence E and D compared to those of sequence 3 and 4.

4. DISCUSSION

4.1. Comparison to Previous Work

The \( P-L \) relationship for AGB stars has been examined by many authors over the past several decades (e.g., Feast et al. 1989; Hughes & Wood 1990; Wood et al. 1999; Glass et al. 2009). Previous studies have often relied on a few tens to a few hundreds of evolved LPVs with brightnesses often measured in the \( K_s \) band. The present work is notable for having the largest sample of LPVs with photometric data measured in the most IR bands to date. This has allowed us to reduce the uncertainties in these relations by a factor of \( \sim 5 \).

4.1.1. Comparison to Feast et al. (1989)

Feast et al. (1989) observed 49 evolved LPVs in the LMC and constructed \( P-L \) relations in the \( J, H, \) and \( K_s \) bands, as well as bolometric magnitudes, which were used in several subsequent studies (e.g., Vassiliadis & Wood 1993; Whitelock et al. 2003). In the \( K_s \) band, they find a slope of \( -3.47 \pm 0.19 \) fits the O-rich stars in their sample, consistent with our slope.
of $-3.31 \pm 0.04$. Our fit to the C-rich AGB candidates in the $K_s$ band is also consistent with their derived relation. We find a slope of $-3.16 \pm 0.04$ compared to their value of $-3.30 \pm 0.40$. They do not consider their fits to C-rich Miras in the $J$ and $H$ bands to be useful.

### 4.1.2. Comparison to Ita et al. (2004)

Ita et al. (2004) used variability information from the OGLE survey of the LMC (Udalski et al. 1997) to assemble a data set of 35,000 variable stars in the LMC. This sample size allowed them to confirm the existence of a distinct population of variable stars below the TRGB, concentrated on sequences 3 and 4 (their sequences A and B) first noted by Kiss & Bedding (2003). They reserve judgment on the interpretation, put forth by Kiss & Bedding (2003) and used in this paper, that this population consists of a large number of first-ascent RGB stars with some AGB contamination.

Most of the $K_s P-L$ relations fits derived in our study (Table 6) are consistent with those in Ita et al. (2004). They obtain a slope of $-3.369 \pm 0.099$ for the C-rich stars on sequence 1, consistent with our slope (Section 4.1.1). Our most discrepant fits are those to stars brighter than the TRGB in sequences 3 and 4. We find a slope of $-3.73 \pm 0.04$ for the 1434 O-rich AGB stars we identify on sequence 4. Ita et al. (2004) do not differentiate between O-rich and C-rich AGB stars here, but they find a slope of $-3.289 \pm 0.047$ to all 510 stars brighter than the TRGB on this sequence. Similarly, on sequence 3 our O-rich AGB stars are best fit with a slope of $-3.85 \pm 0.04$ compared to $-3.356 \pm 0.052$ in Ita et al. (2004). This is most likely due to slight differences in sequence definition. Sequence 3 is almost blended with sequence 2, and precisely where the dividing line between these two sequences is drawn can greatly impact the derived fit.

Our study provides a valuable complement to that of Ita et al. (2004), due to the fact that our sample is of comparable size but is derived from completely independent observations.

### 4.1.3. Comparison to Glass et al. (2009)

Glass et al. (2009) did their own reduction of a subset of the SAGE and MACHO data sets to compute $P-L$ relationships for AGB stars in multiple bands in the IR. Their current work differs most noticeably in sample size, their sample consisting of $\sim$1800 stars and ours $\sim$30,000. Precise comparison of our derived $P-L$ relations is complicated, however, by several differences in how we categorize our data sets. The sample of Glass et al. (2009) is not large enough to distinguish what we identify as sequences 2 and 3, combining them into a single sequence B. See Table 3 for the different naming conventions used. Finally, Glass et al. (2009) only fit $P-L$ relations to stars they classify as O-rich, defined as a star with $(J-K_s) \leq 1.6$, a redder definition than our own (detailed in Section 2.2). The reddest star that we classify as O-rich has $(J-K_s) = 1.4$.

With these differences in mind, our results show reasonable agreement with those of Glass et al. (2009). On sequence 1, for example, our fits for the AGB stars are consistent within the errors in nearly every band from $K_s$ to $8 \mu$m. The notable exception to this broad consistency is in the IRAC [5.8] band on sequence 1, which we discuss above (Section 3.1) as having an anomalously less steep $P-L$ relation. This effect is not observed in Glass et al. (2009), a result we attribute to sample size. Their fit is based on 41 AGB stars on sequence 1, whereas ours is based on 1812.

![Figure 9.](image)

**Figure 9.** [8.0] vs. [3.6]–[8.0] CMD of the AGB candidates detected by Vijd et al. (2009) overlaid atop the CMD for the sources used in this paper, detected by both SAGE and MACHO. The sources from Vijd et al. (2009) appear as large, open diamonds. Note the long tail of very red extreme AGB candidates $[(3.6) - (8.0)] \geq 4$ detected by SAGE but not MACHO.

(A color version of this figure is available in the online journal.)

### 4.1.4. Comparison to Vijd et al. (2009)

By comparing epoch 1 and epoch 2 fluxes from the SAGE point source catalog, Vijd et al. (2009) identified 1967 variable sources in the LMC. Having only two data points, they were not able to determine specific variability parameters such as period or amplitude for these sources. We identify 731 of these sources in the SAGE single frame mosaic photometry archive with MACHO detections. Of the remaining sources, with no MACHO counterpart, Vijd et al. (2009) identified 41 as O-rich, 60 as C-rich, and 540 as extreme AGB stars. Only 1.5% of our sample is classified as extreme. On the other hand, 84% of the AGB variables identified by Vijd et al. (2009) without a MACHO counterpart are classified as extreme AGB stars. By requiring our sources to have a MACHO detection, this effect is expected. The MACHO survey focused on the bar of the LMC, and extreme AGB candidates are not as concentrated in the bar as are O- and C-rich AGB stars (Blum et al. 2006, Figure 1). Furthermore, because the MACHO survey used two optical bandpasses, we expect our sample to miss the reddest, most enshrouded evolved stars in the SAGE catalog, which are not visible in the optical. The variables identified by Vijd et al. (2009) are indeed systematically redder than the sample identified here. Figure 9 presents a CMD of all the AGB candidate sources in this paper, using the SAGE [3.6] and [8.0] bands. The variables detected by Vijd et al. (2009) are overlaid as large diamonds. Figure 10 presents a histogram comparing the [3.6]–[8.0] colors of the extreme sources in our study with those of Vijd et al. (2009).

### 4.1.5. Summary of Comparison to Other Surveys

Overall, our quantitative results are consistent with previous studies of the $P-L$ relation in AGB stars. Differences that exist are small and can be attributed to differences in sample size. Our current work exhibits two primary strengths, sample size and wavelength coverage. Consisting of a sample of 30,000 evolved stars with magnitudes measured in eight different near-IR bands, our sample is far larger than most previous studies. Ita et al. (2004) use a similar sized data set, but lack the wavelength coverage the SAGE survey gives this work. Similarly, Glass et al. (2009) also use the SAGE catalog to investigate the dependence of the $P-L$ relation on wavelength, but use only a small fraction
of the full SAGE source list. We combine the individual strengths of these two works to produce a comprehensive view of evolved star variability in the IR.

Vijh et al. (2009) also utilized the entire SAGE catalog to probe variability. We expand on this work with the inclusion of the MACHO catalog, which allows us to precisely determine the period and amplitude of the variation of these stars. However, this additional information introduces two selection biases to our sample that do not affect the Vijh sample. The MACHO survey was conducted in two non-standard optical filters. By requiring that all of our sources have valid MACHO detections, we miss the reddest of the evolved stars. In addition, the MACHO survey focused on the bar of the LMC, and this spatial bias also affects our sample.

4.2. Comparison to Models of AGB Evolution

Figure 2 reproduces the [3.6] versus period plot of Figure 1, but this time color coded according to RGB, O-rich, C-rich, or extreme AGB classification. By grouping the sources according to this classification, we see trends broadly consistent with synthetic AGB evolution codes (e.g., Vassiliadis & Wood 1993), which predict that the structural and chemical changes during the star’s evolution along the AGB should be accompanied by a general trend toward slightly higher luminosities and longer pulsational periods.

Variable stars on their first ascent of the RGB are dimmer than the AGB stars at later stages of evolution, and highly concentrated on sequences 3 and 4, the shortest period sequences. Oxygen-rich AGB candidates are distributed across all four pulsational sequences. AGB candidates photometrically classified as carbon-rich are also found on all sequences, but are much more concentrated on the long-period sequences (only 29 are on sequence 4, compared to \( \sim 2000 \) on sequence 1). Extreme AGB stars, which are heavily enshrouded by circumstellar dust, are found in significant numbers only on sequence 1, the longest period sequence. Table 4 details the number of each sub-class of star on each sequence. We see clear trends toward brighter magnitudes and longer periods as one moves from RGB candidates to O-rich, C-rich, and finally extreme AGB candidates. Figures 11 and 12 illustrate the distributions of our sample in period and [3.6] magnitude. Previous studies (e.g., Feast et al. 1989; Whitelock et al. 2003) have noticed a dramatic steepening of the AGB P–L relation at \( \sim 420 \) days. Figure 12 shows that this is precisely the period at which extreme AGB candidates come to dominate over all other classes of stars. Note that in Figure 12, the extreme AGB candidate graph has been magnified to be visible compared to the much larger O-rich and C-rich AGB populations. Extreme AGB candidates do
dominate at periods greater than 420 days, but the effect is smaller than Figure 12 shows. As we discuss in Section 3.1, we find that this class of star does obey a steeper $P-L$ relation.

The slope of the $P-L$ relation is a measure of the brightness contrast between stars with long periods and those of the same type with shorter periods. If the $P-L$ relation becomes increasingly steeper with longer wavelength, it is an indication that longer-period stars are emitting more of their energy at those wavelengths, an indication of a greater amount of circumstellar matter re-processing the star’s light to the red. More circumstellar matter implies a higher rate of mass loss (Srinivasan et al. 2009, and references therein). Thus, we interpret the lack of a relationship between $P-L$ slope and wavelength amongst RGB stars and O-rich AGB stars to indicate that pulsation period does not strongly impact mass-loss rates at these stages of stellar evolution. However, amongst heavily enshrouded extreme AGB stars, there is a stronger coupling between the two.

Other authors (e.g., Schultheis et al. 2004; Glass et al. 2009) have examined plots of color versus period in order

Table 7

| Sequence | Top | Bottom | Left | Right |
|----------|-----|--------|------|-------|
| Sequence 1a | $K_s \geq 9.5$ | $K_s < 13$ | $K_s > -3.8 \log P + 19.37$ | $K_s \leq -3.8 \log P + 20.6$ |
| Sequence 1b | $K_s \geq 9.7$ | $K_s < 12.6$ | $K_s > -3.8 \log P + 20.6$ | $K_s \leq -3.8 \log P + 19$ |
| Sequence 2 | $K_s > 9.5$ | $K_s < 13.2$ | $K_s > -4.3 \log P + 19.57$ | $K_s < -4.3 \log P + 19.8$ |
| Sequence 3 | $K_s > 9.8$ | $K_s < 13.2$ | $K_s > -4.8 \log P + 19.63$ | $K_s < -4.8 \log P + 19.57$ |
| Sequence 4 | $K_s > 10.4$ | $K_s < 13$ | $K_s > -4.5 \log P + 18.25$ | $K_s < -4.8 \log P + 19.63$ |
| Sequence D (top) | $K_s > 9.7$ | $K_s < 12.6$ | $K_s > -4.1 \log P + 22.58$ | $K_s < -4.8 \log P + 25.6$ |
| Sequence D (bottom) | $K_s > 12.6$ | $K_s < 13.8$ | $K_s > -4.1 \log P + 22.58$ | $K_s < -3 \log P + 20.7$ |
| Sequence E (top) | $K_s > 12.0$ | $K_s < 12.6$ | $K_s > -3.8 \log P + 20.6$ | $K_s < -4.1 \log P + 22.58$ |
| Sequence E (bottom) | $K_s > 12.6$ | $K_s < 15$ | $K_s > -2.9 \log P + 18.7$ | $K_s < -3 \log P + 19.9$ |

Note. Mathematical description of the boundaries shown in Figure 13.

a There is an additional constraint that stars in sequence 1 must have a period greater than 80 days. This requirement only affects a small number of stars and serves to prevent these outliers from skewing the $P-L$ fits.

b This region shown in blue in Figure 13 uses an additional requirement that $J-K_s > 1.4$ to identify enshrouded stars (many of which are extreme AGB candidates) that would otherwise be omitted.

Table 8

| Classification | Slope | Intercept | Residual | $r^2$ | Number |
|----------------|-------|-----------|----------|-------|--------|
| Sequence 4 | RGB | $-3.60 \pm 0.06$ | $17.65 \pm 0.08$ | $0.125$ | $0.558$ | $2050$ |
| Sequence 2 | O-rich | $-3.46 \pm 0.04$ | $18.07 \pm 0.07$ | $0.142$ | $0.814$ | $1434$ |
| Sequence 3 | C-rich | $-3.74 \pm 0.33$ | $18.95 \pm 0.53$ | $0.128$ | $0.844$ | $26$ |
| Sequence 1 | All stars | $-4.16 \pm 0.02$ | $19.27 \pm 0.04$ | $0.153$ | $0.899$ | $3510$ |

Note. Identical to Table 5, with magnitude measured in the $J$ band.

Table 9

| Classification | Slope | Intercept | Residual | $r^2$ | Number |
|----------------|-------|-----------|----------|-------|--------|
| Sequence 4 | RGB | $-3.23 \pm 0.05$ | $17.14 \pm 0.07$ | $0.112$ | $0.648$ | $2048$ |
| Sequence 2 | O-rich | $-3.55 \pm 0.04$ | $17.37 \pm 0.07$ | $0.132$ | $0.842$ | $1452$ |
| Sequence 3 | C-rich | $-3.37 \pm 0.33$ | $17.32 \pm 0.53$ | $0.145$ | $0.798$ | $28$ |
| Sequence 1 | All stars | $-4.26 \pm 0.02$ | $18.56 \pm 0.03$ | $0.139$ | $0.919$ | $3508$ |

Note. Identical to Table 5, with magnitude measured in the $H$ band.
to ascertain a period at which significant reddening indicates the onset of mass loss. Our determination of the slope of the $P$–$L$ relation at multiple wavelengths contains and quantifies these other studies. Quantitative period–color relations can be easily derived from the fits given in this paper. Given $P$–$L$ fits in two bands, $m_1$ and $m_2$, we obtain the color–period relation $(m_1 - m_2) = (A_1 - A_2) \times \log P + (B_1 - B_2)$ with $A$ and $B$ defined as in Equation 4. Groenewegen (2006) suggests the $[3.6]$–$[4.5]$ color as a good indicator of the mass-loss rate from AGB stars. Through the period–color relations that can be derived from this work, such proxies can be readily connected to variability parameters, and the results compared to the predictions of theoretical models of AGB evolution. We intend to investigate the connections between variability and mass-loss rate using the quantitative predictions of such models in future papers.

5. CONCLUSIONS

We present the largest multi-wavelength IR investigation of the $P$–$L$ relationship of evolved stars in the LMC to date. We find that the slope of the $P$–$L$ relation followed by O-rich AGB stars and stars on the first ascent of the RGB is relatively independent of wavelength. In contrast, we find a strong dependence of slope of the $P$–$L$ relation with wavelength amongst the most evolved, most enshrouded, extreme AGB stars. C-rich AGB stars, which represent an intermediate stage of evolution along the AGB, show a slight dependence of slope on wavelength, with a possible correlation between variability period and the strength of the $5 \mu$m CO absorption feature.

We find that for accurate characterization of the IR $P$–$L$ relationship for AGB stars, the $K_s$ band is the best choice. It offers a superior combination of sample size; low scatter due to the fact the both O-rich and C-rich AGB stars obey identical $P$–$L$ relations when measured in this band; and further applicability, in that once the $P$–$L$ relation is determined in the $K_s$ band, the slope of the relation for O-rich AGB stars and RGB stars has been determined at all near-IR wavelengths. Slopes of $P$–$L$ relations for evolved stars in the $K_s$ band can be found in Table 6.

There is a systematic trend toward steeper slopes in the magnitude versus log $P$ relation as one looks at categories of stars at later and later stages of stellar evolution. This trend would be consistent with a nonlinear $P$–$L$ relation if all variable evolved stars were examined together, without photometrically determined classifications.

The authors thank Bernie Shiao for his invaluable assistance with the SAGE database. In addition, the courteous and helpful comments of the anonymous referee were invaluable in the completion of this document.

The SAGE Project is supported by NASA/Spitzer grant 1275598 and NASA NAG5-12595.

This paper utilizes public domain data obtained by the MA-CHO Project, jointly funded by the U.S. Department of Energy through the University of California, Lawrence Livermore National Laboratory under contract no. W-7405-Eng-48, by the National Science Foundation through the Center for Particle Astrophysics of the University of California under cooperative.

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### Table 10

| Classification | Slope  | Intercept | Residual | $r^2$ | Number |
|----------------|--------|-----------|----------|-------|--------|
| Sequence 1     |        |           |          |       |        |
| RGB            | $-3.39 \pm 0.05$ | $17.18 \pm 0.07$ | $0.110$ | $0.711$ | $2065$ |
| O-rich         | $-3.86 \pm 0.04$ | $17.66 \pm 0.06$ | $0.121$ | $0.885$ | $1434$ |
| C-rich         | $-4.13 \pm 0.35$ | $18.17 \pm 0.57$ | $0.142$ | $0.836$ | $29$   |
| All stars      | $-4.42 \pm 0.02$ | $18.61 \pm 0.03$ | $0.128$ | $0.938$ | $3528$ |

### Table 11

| Classification | Slope  | Intercept | Residual | $r^2$ | Number |
|----------------|--------|-----------|----------|-------|--------|
| Sequence 1     |        |           |          |       |        |
| RGB            | $-3.56 \pm 0.05$ | $17.29 \pm 0.08$ | $0.122$ | $0.682$ | $2044$ |
| O-rich         | $-4.04 \pm 0.04$ | $17.79 \pm 0.06$ | $0.123$ | $0.892$ | $1429$ |
| C-rich         | $-4.50 \pm 0.33$ | $18.58 \pm 0.54$ | $0.140$ | $0.871$ | $29$   |
| All stars      | $-4.62 \pm 0.02$ | $18.76 \pm 0.03$ | $0.136$ | $0.940$ | $3502$ |

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Note. Identical to Table 5, with magnitude measured in the [4.5] band.

Note. Identical to Table 5, with magnitude measured in the [8.0] band.
agreement AST-8809616, and by the Mount Stromlo and Siding Spring Observatory, part of the Australian National University.

This publication makes use of data products from the Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and the Infrared Processing and Analysis Center/California Institute of Technology, funded by the National Aeronautics and Space Administration and the National Science Foundation.

APPENDIX A

PERIOD–LUMINOSITY SEQUENCE DEFINITIONS

The stars in our sample are assigned to one of six $P$–$L$ sequences using cuts in $K_s$–period space, with a small subset also utilizing $J$–$K_s$ color information. Only the primary period of these multi-periodic objects is used to establish sequence membership. The definitions for the sequences are shown graphically in Figure 13, and described mathematically in Table 7. Our definitions are based on those used by Fraser et al. (2008), but are more restrictive (especially in sequence D, which exhibits a very large scatter about the $P$–$L$ relation) to better trace the population density “cores” visible to the eye. This emphasis sacrifices some sample size, but we note that the scatter in our $P$–$L$ relations (of order ~0.15 mag) is dominated by the intrinsic variability of the sources and would not be significantly reduced by introducing more sources by using the secondary periods to assign the unsequenced sources to one of the existing $P$–$L$ sequences.

We note that ultimately, our $P$–$L$ sequence definitions are a judgment call, and slightly different definitions used by other authors are equally valid and can affect the measured $P$–$L$ fits. In particular, the TRGB cut at a constant $K_s = 12.0$ leads to a sharp parallelogram shape at the brightest end of the RGB population. These corners cause the RGB and O-rich AGB $P$–$L$ fits to be less steep than the fit to all the stars in a sequence, regardless of class, which is not affected by this selection bias (Table 5).

Beginning with these definitions of the $P$–$L$ sequences, careful study of the panels of Figure 4 led us to introduce some additional modifications to properly classify a few anomalous populations.

A group of 100 C-rich AGB candidates, classified as being part of sequence 1 under criterion $\mathbf{b}$ in Table 7, was found to remain sub-luminous to the rest of sequence 1 at all wavelengths longer than $K_s$ as well. They are identified and eliminated by removing C-rich AGB candidates from sequence 1 with periods >300 days and $K_s > -3.9 \times \log P + 20.17$.

Finally, we identified a population of 60 extreme AGB candidates so dust enshrouded that at $K_s$ they lay along the main body of sequence D, but at 3.6 $\mu$m and longer they were clearly part of sequence 1. They were properly classified by adding all extreme AGB candidates assigned to sequence D or to no sequence with periods between 250 and 700 days and IRAC [3.6] $< 10.5$ to sequence 1.

Note that both of these populations also require their members to be photometrically classified as either C-rich or extreme AGB candidates, following the definitions detailed in Section 2.2.

### Table 12

| Classification | Slope | Intercept | Residual | $r^2$ | Number |
|----------------|-------|-----------|----------|------|--------|
| RGB            | $-3.56 \pm 0.05$ | $17.29 \pm 0.08$ | $0.122$ | $0.682$ | 2044   |
| O-rich         | $-4.04 \pm 0.04$ | $17.79 \pm 0.06$ | $0.123$ | $0.892$ | 1429   |
| C-rich         | $-4.50 \pm 0.33$ | $18.58 \pm 0.54$ | $0.140$ | $0.871$ | 29     |
| All stars      | $-4.62 \pm 0.02$ | $18.76 \pm 0.03$ | $0.136$ | $0.940$ | 3502   |

### Table 13

| Classification | Slope | Intercept | Residual | $r^2$ | Number |
|----------------|-------|-----------|----------|------|--------|
| RGB            | $-3.21 \pm 2.25$ | $14.72 \pm 3.24$ | $0.591$ | $0.061$ | 33     |
| O-rich         | $-0.79 \pm 0.27$ | $11.57 \pm 0.45$ | $0.344$ | $0.048$ | 177    |
| All stars      | $-0.25 \pm 0.23$ | $10.58 \pm 0.39$ | $0.416$ | $0.005$ | 213    |

**Note.** Identical to Table 5, with magnitude measured in the [8.0] band.
APPENDIX B

MID-IR PERIOD–MAGNITUDE RELATIONS

Tables 8–13 are identical to Table 5, with magnitude measured in the other IRAC bands.

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