Communication

Equivalent Circuit Modeling to Design a Dual-Band Dual Linear-to-Circular Polarizer Surface

Parinaz Naseri, Jorge R. Costa, Sérgio A. Matos, Carlos A. Fernandes, and Sean V. Hum

Abstract—The working principle of a thin dual-band dual-linear to circular polarizer is presented here. This polarizer not only converts incident linearly polarized (LP) waves to circularly polarized (CP) waves in two frequency bands, but it also reverses the handedness of each signal. The electromagnetic behavior of the cell is carefully analyzed and two equivalent circuit models (ECMs) are presented to model the responses of the cell to linearly polarized waves at normal incidence. The ECMs show how utilizing interlayer coupling can be leveraged to achieve reversed CP senses in two bands using a compact design. Analytical formulas are presented to provide initial values of the ECM components including the mutual coupling inductances. We present measurement results that agree well with the full-wave simulation and the ECM results, thus validating the proposed ECM model.

Index Terms—Circular polarization, dual-band, equivalent circuits, frequency selective surfaces, polarizers.

I. INTRODUCTION

For many applications such as point-to-point and satellite communications, there is an increasing demand to employ circularly polarized (CP) antennas at the transmitting and receiving ends [1]. Integration of a linearly polarized (LP) antenna with a suitable planar linear-to-circular polarization converter [2]–[6] (simply called a polarizer here) can simplify the design and fabrication of CP antennas. For example, for dual K/Ka-band satellite communications, it is not only required that there be separate transmitting (TX) and receiving (RX) bands, but that the radiated CP waves in these bands must also have opposite senses. Moreover, for handover purposes in multispot coverage schemes, it is important that the polarizer operates for both linear polarizations and converts them to both CP in each band. Implementing the latter at the radiating aperture level is quite challenging considering that it still needs to provide dual-band operation as well. Therefore, for this application, a unique polarizer [7] that can convert LP to CP in two frequency bands with opposite senses can potentially reduce the required number of apertures used to implement multibeam coverage patterns.

The full analysis of the polarizer cell was out of the scope of our previous publication [7]. There, we explained the role of the printed elements and the relationship between their dimensions to the resonances. The polarizer has a ultrathin profile of only 10.5% of wavelength in the higher frequency band. During the development of this polarizer, we made an interesting observation: that the inevitable mutual coupling can be used in our favor and produce the necessary distinct resonance effects between x- and y-polarizations. This promoted further investigation of the dynamics of the polarizer, resulting in a detailed analysis presented in this communication. Here, we show that the interlayer mutual coupling between elements is the key factor in the unique response of the polarizer. Lack of understanding of this coupling may lead to avoidable excessive iterative cycles of optimization without leading to the best possible response. The model we develop in this manuscript can address this issue.

We start the analysis of the polarizer from our previous design [7]. Based on the operational principle and the certain response of this polarizer to each LP incident wave, we develop two decoupled equivalent circuit models (ECMs) to model the polarizer. We present analytical formulas to extract all of the ECM components including the mutual inductances between layers. The ECMs are used to explain the working principle and the particular electromagnetic behavior of the cell. These ECMs can also be deployed as surrogate models for fast analysis. We validate the ECMs for a new example of the polarizer through simulation and measurement results.

This communication is organized as follows. In Section II, the working principle of the dual-band polarizer unit cell [7] is explained and two new ECMs are presented to represent its responses to x- and y-polarized incident fields. In Section III, the analytical formulas to obtain initial estimations of the ECM components are presented. An example of the unit cell is designed and the ECMs are validated by comparison with the full-wave (FW) simulation and measurement results in Section IV. Conclusion is drawn in Section V.

II. WORKING PRINCIPLE OF THE PROPOSED DUAL-BAND POLARIZER UNIT CELL

The proposed unit cell is composed of three metallic layers stacked with two thin low-permittivity substrates [7], as shown in Fig. 1. The first and the third layers are identical and composed of split rings and patch elements. The middle layer is composed of a circular slot and a dipole. The whole thickness of the unit cell is 1.05 mm, which is 7% and 10.5% of the wavelength at the lower and higher center frequencies, respectively.

To obtain the unique property of this polarizer, i.e., converting LP to CP in two frequency bands with opposite senses, it is required to generate +90 and −90 phase differences between the transmitted x and y waves in the lower and higher frequency bands, respectively, or vice versa for switching the polarization. Such a feature is achieved using resonant asymmetric elements along x- and y-axes to obtain anisotropic response in the unit cell.

To obtain an understanding of the working principle of the unit cell, we model it by employing ECMs for waves at normal incidence polarized along x- and y-axes. As the first step, we assume that the different layers of the unit cell are initially decoupled, but as we develop the model, different interlayer coupling mechanisms are
The asymmetric metallic parts of the unit cell along x- and y-axes require that two circuit models are used to mimic the response of the unit cell to an x- and y-polarized incident wave. It can be shown that the cross-polarization coefficients such as $|S_{11}^x| = |S_{11}^y| = |S_{21}^X| = |S_{21}^Y| \approx 0$ [10]. Therefore, we can assume that there is negligible coupling between the responses of the unit cell to the two electric field polarizations and model each response with a specific ECM.

The complete ECMs of the unit cell for both polarizations are shown in Fig. 2, where the patch and the split rings on the first and the third layers, and the dipole in the middle layer can each be modeled by a set of series LC resonators. The equivalent components associated with these are shown as $(C_p, L_p)$ for the patch, $(C_{SR}, L_{SR})$ for the split ring, and $(C_d, L_d)$ for the dipole. Due to complementary shape of the circular slot to a circular patch, it is modeled by the dual of the series capacitance and inductance, which is a shunt LC resonator $(C_{CS}, L_{CS})$. Since the incident electric field couples with the elements of each layer simultaneously, each constituent resonator is in parallel with the others in the same layer [9]. The thin substrates between the metallic layers of the unit cell are each modeled as transmission lines with a thickness of $h$.

The asymmetric metallic parts of the unit cell along x- and y-axes are modeled by augmenting the ECM. Therefore, with no interlayer coupling in the unit cell, the three-layer structure of the unit cell can be simply modeled by cascading the ECMs of each layer [8]. Since the unit cell is intentionally not composed of any significantly lossy material, lossless ECMs are developed here.

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By comparing Fig. 3(a) with Fig. 3(c), it can be concluded that the cross-polarization coefficients such as $|S_{11}^x| = |S_{11}^y| = |S_{21}^X| = |S_{21}^Y| \approx 0$ [10]. Therefore, we can assume that there is negligible coupling between the responses of the unit cell to the two electric field polarizations and model each response with a specific ECM.

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![Fig. 1. (a) Identical first and the third layers. (b) Second layer. (c) 3-D view of the unit cell.](image)

![Fig. 2. Complete ECMs of the unit cell to (a) x-polarized and (b) y-polarized fields at normal incidence.](image)

![Fig. 3. Magnitude of the surface current of a complete ring and split ring to normal incidence x-polarized: (a)-(c) and y-polarized (b)-(d) incident waves at the resonance frequency of the complete ring. All figures have the same scale.](image)
length of the ring. As a result, one should expect $L_{SR}^Y$ to have smaller value compared to its counterpart $L_{SR}^X$. Therefore, the split ring resonates at a much higher frequency when it is excited with $y$-polarized field compared to its resonance to an $x$-polarized field.

The net effect of this is that the transmission coefficient of the unit cell to a $y$-polarized field, $S_{21}^Y$, has no null in the band. Nevertheless, the split ring resonates for an $x$-polarized field in the band of interest and the transmission coefficient, $S_{21}^X$, has one null in the band. This creates a situation where $\arg(S_{21}^X) - \arg(S_{21}^Y) < 0$ in the lower band and $\arg(S_{21}^X) - \arg(S_{21}^Y) > 0$ in the higher band. Moreover, by using thin substrates and ensuring sufficient mutual coupling between the layers, the structure’s $S_{21}^Y$ can have two nulls, while $S_{21}^X$ has no null in the band. This causes the phase of $S_{21}^Y$ to have two 180° jumps, a 360° jump in total, along the band. Therefore, it is possible to have $-90^\circ$ phase difference and $+90^\circ$, respectively, in the lower band and in the higher band.

For an $x$-polarized field, the split rings strongly resonate and generate a magnetic flux passing the second layer. Therefore, for $x$-polarized field, it can be concluded that there is strong magnetic coupling between the split rings of the first and the third layers and the inductances of the dipole and circular slot on the second layer. These couplings are modeled by mutual inductances, i.e., $L_{M1}^X$ and $L_{M2}^X$ between each split ring and the circular slot and each split ring and the dipole, respectively. It is worth noting that the interlayer coupling is mostly inductive and capacitive coupling is negligible [12].

For a $y$-polarized field, by introducing the gap, we make the split ring nonresonant in the bands of interest. Therefore, the split rings show very weak and negligible magnetic and capacitive coupling to the other elements for a $y$-polarized incident wave. However, the patches on the first and the third layers are resonant for a $y$-polarized field. The excited currents on them generate a magnetic flux that couples with the elements of the second layer. Therefore, there is magnetic coupling between the inductance of each patch and the inductance of the circular slot. This magnetic coupling is modeled by $L_M^Y$ in Fig. 2(b).

III. ANALYTICAL FORMULATION OF THE ECM COMPONENTS

An initial estimation for the component values in Fig. 2 can be obtained using the following equations:

1) Based on $w$ and $r_s$, an initial value for $L_{CS}$ can be calculated using (1) and (2) based on duality theory and using (1) in [13] and (2) in [14], where $g = w - 2r_s$. Moreover, $CCS$ can be estimated using (3), where $\varepsilon_{eff} = (\varepsilon_r + \varepsilon_s)/2 = \varepsilon_r$ since the circular slot is between two dielectric layers [15]

\[
\tilde{g}_{full} = \frac{1}{8(\pi - w)} \left[ 2 \left( 1 + \sqrt{2} \right) g^2 + (\pi - 4) w^2 \right. \\
- \left. \left( 2 + \sqrt{2} \right) \pi - 4 \right] g w \]

\[
L_{CS} \approx \mu_0 \frac{w}{2\pi} \ln \left( \frac{1}{\sin \left( \frac{\pi w}{2\pi} \right)} \right) 
\]

\[
C_{CS} = \varepsilon_0 \sqrt{\varepsilon_{eff} d} \frac{2w}{\pi} \ln \left( \frac{1}{\sin \left( \frac{\pi w}{2\pi} \right)} \right) 
\]

2) Based on the dimensions of the dipole, i.e., $d_x$ and $d_y$, the inductance of the dipole can be explained by approximating the dipole and its adjacent neighbor as two parallel finite wires [9]. Therefore, $L_{d}^X$ and $C_{d}^X$ can be calculated based on (4) and (5) [16]

\[
L_{d}^X \approx \frac{\mu_0 w}{2\pi} \ln \left( \frac{1}{\sin \left( \frac{\pi (w - d_x)}{2\pi} \right)} \right) 
\]

\[
C_{d}^X = \varepsilon_0 \sqrt{\varepsilon_{eff} d_x} \frac{4\pi r_s}{\pi} \ln \left( \frac{1}{\sin \left( \frac{\pi (2r_s - d_x)}{4\pi r_s} \right)} \right) 
\]

Note that in the calculation of $C_{d}^X$, $2r_s$ is considered to be the period of the cell instead of $w$. By this adjustment, we approximately capture the capacitance of the dipole and the mutual capacitance between the dipole and the circular slot by $C_{d}^X$ without introducing complexity to the ECM.

3) Based on the size of the patch components on the first and the third layers, i.e., $P_y$, its $C_{p}^Y$ capacitance [14] and $L_{p}^X$ inductance [16] for a $y$-polarized incident wave can be calculated based on (6) and (7), where $\varepsilon_{eff} = (\varepsilon_r + 1)/2$ [15]

\[
C_{p}^Y \approx \varepsilon_0 \sqrt{\varepsilon_{eff} d_y} \frac{2\pi}{\pi} \ln \left( \frac{1}{\sin \left( \frac{\pi (w - P_y)}{2\pi} \right)} \right) 
\]

\[
L_{p}^Y \approx \frac{\mu_0 w}{2\pi} \ln \left( \frac{1}{\sin \left( \frac{\pi (w - P_y)}{2\pi} \right)} \right) 
\]

4) Based on $r_s$ and $w_r$, and $G$, the self-inductance of the split ring for $x$-polarization, i.e., $L_{SR}^X$, can be calculated as the sum (8) of a proportion (9) from the complete ring [17], i.e., $K L_{CR}/4$, where $K = 1$ for $x$-polarization.
5) The mutual inductance between each half of the split ring and the mutual inductance between the two halves of the split ring (11), i.e., \( L_{M}^{SLR} = K L_{CR}^2 / 4 \), is calculated using uniform amplitude of surface current across the ring, see Fig. 4(a), where \( r_1 = r_s - w_r \). \( K \) captures the length reduction of the split ring due to the gap. \( L_{M}^{SR} \) can be calculated using Neumann’s formula (11) [18]. The capacitance of the split ring can be calculated based on the resonance of the split ring, i.e., \( f_0 \), in (13), and its inductance

\[
L_{SR}^X = \frac{1}{2} \left( \frac{K L_{CR}}{4} + L_{SR}^X \right) / 2
\]

\[
K L_{CR} \approx \frac{K}{4} \frac{\mu_0 \pi^2}{\ln(\eta^2)} \int_0^\infty \frac{1}{k^2} \beta(r_s (kr_s) - r_1 \beta(k r_1))^2 dk
\]

\[
\beta(x) = S_0(x) J_1(x) - S_1(x) J_0(x)
\]

\[
L_{SR}^M = \frac{\mu_0 r_s}{4 \pi} \int_{\varphi_0}^{\pi-\varphi_0} \int_{\varphi_0}^{\pi-\varphi_0} \frac{\cos(\varphi_1 + \varphi_2) \cos(\varphi_1 + \varphi_2)}{2(1 - \cos(\varphi_1 + \varphi_2))} d\varphi_1 d\varphi_2
\]

\[
\varphi_0 = \sin^{-1} \left( \frac{G}{2 r_s - w_r} \right); \quad \gamma = 1 - \frac{2 \varphi_0}{\pi}
\]

\[
C_{SR}^X = \frac{1}{(2\pi f_0)^2 L_{SR}^X}; \quad f_0 = \left( \frac{2\pi r_s - 2G}{\sqrt{\varepsilon_{eff}}} \right)
\]

In these equations, \( S_n \) and \( J_n \) are the \( n \)th order Struve and Bessel functions, respectively [17].

As mentioned earlier, the gaps significantly disturb the response of the ring to a \( y \)-polarized wave. Fig. 4(b) shows the total inductance of the split ring to a \( y \)-polarized wave. It shows the self-inductance of the split ring for \( y \)-polarization, i.e., \( L_{SR}^Y \), is almost \( K L_{CR} / 16 \). The mutual inductance between the parts of the split ring can be calculated to be negligible. The capacitance of the split ring for both polarizations is approximately equal.

\[
L_{SR}^Y \approx \frac{K L_{CR}}{16}; \quad C_{SR}^Y \approx C_{SR}^X.
\]

To find an initial estimation of the mutual inductance between the patches and the circular slot, i.e., \( 2 L_{SR}^M \), the surface currents on the patch and the circular slot are shown in Fig. 5(c). The average distance between each long edge of the patch and half split ring is \( h_{SR/d} \), considering the concentration of the currents are in the middle of the split ring and the long edges of the dipole [see Fig. 5(b)].

\[
h_{SR/d} = \sqrt{h^2 + (r_s - w_r/2 - d_y/2)^2}
\]

\[
2 L_{SR}^M \approx \frac{\mu_0}{4 \pi} \int_0^{d_y} \int_0^{r_{SR}} d\varphi \int_0^{r_{SR}} \frac{d\gamma}{\sqrt{h_{SR/d}^2 + (\gamma - x)^2}}
\]

\[
= 2 \int_0^{d_y} \int_0^{r_{SR}} \log \left( \frac{r_{SR}^2 - x^2 + (h_{SR/d}^2 + (r_{SR}^2 - x^2))^2}{h_{SR/d}^2 + (r_{SR}^2 - x)^2} \right) dx.
\]

\[
h_{CS/p} = \sqrt{h^2 + (r_s - P_y/2)^2}.
\]

IV. DESIGN EXAMPLE AND VALIDATION

To validate the proposed ECMs, we start from a unit cell designed in an FW simulation tool, see Fig. 1.
The unit cell is simulated using periodic boundary conditions in the x- and y-directions.

To find the values of the ECM components, we use the formulas presented in Section III to obtain estimates for all of the components in Fig. 2. There is a frequency discrepancy between the reflection and transmission coefficients of the FW simulations and the ECMs with these analytical values, presented in Fig. 6. Therefore, a gradient-based optimization in the Advanced Design System (ADS) software [19] with the initial values of the second column of Table II is performed to improve the alignment with the FW simulation results. The fitted values are listed in the third column of Table II. Fitting the components provides better match to the FW simulation by presenting more accurate values than the ones obtained by the analytical formulas. While the analytical values and the fitted ones are from the same order of magnitude, it should be noted that the analytical formulas are derived in the absence of mutual coupling between the layers and the different components; therefore, they provide initial values for further optimization. Simplification of the ECMs and ignoring some of the elements for certain polarizations, i.e., the patches for x-polarization and the dipole for y-polarization, also introduce differences between the analytical values and the fitted ones.

Another important ability that the ECM provides is being able to understand the effect of interlayer coupling on the behavior of the polarizer. For example, Fig. 7(a) and (b) shows the comparison between the amplitude of the reflection and transmission coefficients of the cell to x-polarized and y-polarized incidence fields in FW simulation and the responses of corresponding ECMs with the values listed in the third column of Table II simulated in ADS. In Fig. 7(a), we compare the results of the ECM in Fig. 2(a) with and without the mutual inductances. From this figure, the essential role of the mutual inductances can be easily understood. One can note that there is a difference between the number of the nulls in $|S_{21}^{xy}|$ of the unit cell in the FW simulation and the corresponding ECM without the mutual inductances. Fig. 7(b) shows the excellent agreement between the ECM in Fig. 2(b) and the response of the cell to a y-polarized wave. Moreover, the results of the $|S_{21}^{yy}|$ from the ECM shown in Fig. 2(b) with and without the mutual inductance, $L_M^y$, are compared with the FW simulation results. The mutual inductance in Fig. 2(b) seems to have a minor effect in the transmission bandwidth in the higher band and a frequency shift in the lower band.

The better agreement in Fig. 7(b) compared to Fig. 7(a) can be related to our initial assumptions in the ECMs. We had assumed that the response of patch is negligible for an x-polarized wave and the response of dipole is negligible for a y-polarized wave. While the latter can be forgiving, since $d_y \approx 0.15d_x$, the first assumption can lead to the differences in Fig. 7(a) since $P_x \approx 0.33 P_y$. However, this difference is a small price to keep the ECMs and the analysis simple.

Fig. 8 presents the comparison between the phases of the transmission coefficients of the cell and the ECMs for x- and y-polarized waves at normal incidence.
blind parametric optimizations, in order to accelerate their design while providing additional physical insight into their operation. This unique topology of the unit cell has shown great flexibility to be changed for different applications and different frequencies [7], [20], [21]. The information presented here can be a guideline for such changes based on the application at hand.

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The working principle of a dual-band dual-LP to dual-CP converter can be explained using two ECMs for its responses to x- and y-polarized incident waves. Compared to previous work on this topic, we provide a thorough analysis of the electromagnetic behavior of the cell and use the proposed ECMs to show how interlayer coupling can be employed to achieve unique properties in a polarizer such as opposite handedness of the transmitted CP waves in the two bands. The development of the ECM in this communication provides a reference for readers to develop ECMs for their structures and avoid...