An Approach to Compensate for the Influence of the System Normal Stiffness in CNS Direct Shear Tests

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Abstract
Applying accurate normal load to a specimen in direct shear tests under constant normal stiffness (CNS) is of importance for the quality of the resulting data, which in turn influences the conclusions. However, deficiencies in the test system give rise to a normal stiffness, here designated as system normal stiffness, which results in deviations between the intended and actual applied normal loads. Aiming to reduce these deviations, this paper presents the effective normal stiffness approach applicable to closed-loop control systems. Validation through direct shear tests indicates a clear influence of the system normal stiffness on the applied normal load (13% for the test system used in this work). The ability of the approach to compensate for this influence is confirmed herein. Moreover, it is demonstrated that the differences between the measured and the nominal normal displacements are established by the normal load increment divided by the system normal stiffness. This further demonstrates the existence of the system normal stiffness. To employ the effective normal stiffness approach, the intended normal stiffness (user defined) and the system normal stiffness must be known. The latter is determined from a calibration curve based on normal loading tests using a stiff test dummy. Finally, a procedure is presented to estimate errors originating from the application of an approximate representation of the system normal stiffness. The approach is shown to effectively reduce the deviations between intended normal loads and the actual applied normal loads.

Keywords Calibration · CNS · Dilatancy · Direct shear test · Normal load · Stiffness test system

List of Symbols

\[ \begin{align*}
  k_{\text{eff}} & \quad \text{Effective normal stiffness} \\
  k_{\text{ns}} & \quad \text{Intended normal stiffness} \\
  k_{\text{sys}} & \quad \text{System normal stiffness} \\
  k_{\text{sys,cont}} & \quad \text{Continuous system normal stiffness} \\
  N & \quad \text{Normal load} \\
  N_0 & \quad \text{Initial normal load} \\
  N_{\text{end}} & \quad \text{Normal load at the end of the test} \\
  \alpha & \quad \text{Angle of inclination of joint in test dummy} \\
  \Delta N & \quad \text{Normal load error} \\
  \delta_n & \quad \text{Normal displacement} \\
  \delta_{n,\text{nom}} & \quad \text{Nominal normal displacement} \\
  \delta_s & \quad \text{Shear displacement} \\
  \delta_{\text{sys,n}} & \quad \text{System normal displacement} \\
  \delta_{\text{sys,n0}} & \quad \text{System normal displacement at the initial normal load, } N_0 \\
  \delta_{\text{sys,nend}} & \quad \text{System normal displacement at the normal load at the end of the test, } N_{\text{end}} \\
  \sigma_n & \quad \text{Normal stress} \\
  \sigma_{n,\text{nom}} & \quad \text{Nominal normal stress calculated from } \alpha \\
  \sigma_{n0} & \quad \text{Initial normal stress} \\
\end{align*} \]

Dimensions

\[ \begin{align*}
  M & \quad \text{Mass} \\
  L & \quad \text{Length} \\
  T & \quad \text{Time} \\
\end{align*} \]

1 Introduction

The presence of joints influences the stability of rock masses, as well as the structural integrity of geotechnical structures. A critical failure mode affecting the structural integrity in geotechnical structures is joint shearing. The shear strength of rock joints is affected by several factors such as joint roughness, compressive strength of the joint
Comparisons between the true and desired load response, and the effect of the physical spring is simulated by the control system. Through closed-loop control, a physical spring is not needed. Instead, it can be applied by mounting a physical spring between the specimen and the loading frame. The normal load in the CNS tests can be applied by using the steel test dummy. A similar approach was reported by Dae-Young et al. (2006). In summary, even though rarely reported, an approach exists on how to calibrate test systems. This was done by correcting the displacements recorded in the test system. However, the influence of the stiffness of the test system must be as stiff as possible. In this work, “test system” refers to all components between the measuring points of the displacements transducers. “Test setup” is defined as a broader conception and refers to the control system and the complete hardware, which, for example, includes the loading frame and the actuators. Hence, the “test system” is a part of the “test setup”. The necessity of having a stiff test system for proper recording of test data is stated in ISRM “Suggested Method for Laboratory Determination of the Shear Strength of Rock Joints” (2014). Herein, tests using a high stiffness dummy, i.e., a steel specimen, are proposed to enable calibration of test systems. However, in the literature, the information is limited about how to carry out calibrations and whether calibrations have been employed to the test systems (Barla et al. 2010; Liu et al. 2017; Haberfield and Szymakowski 2003; Moradian et al. 2013; Rao et al. 2009; Hans and Boulon 2003). In work by Jiang et al. (2004), graphs of the normal stress versus normal displacement from CNS tests were presented. By comparing the slopes of the graphs, which correspond to the normal stiffnesses, with the applied stiffness settings, it was concluded that the desired stiffness was obtained with good accuracy. Packulak et al. (2018) conducted 42 direct shear tests under CNS conditions aiming to provide guidelines on boundary condition selection for direct shear laboratory tests. They identified overturning moments as the normal load moves away from the centre of the specimen as a potential source of error. A second identified potential source of error was sample rotation potentially causing tensile fracturing on the edge of the sample in the shear direction, which in turn would result in the machine outputting a lower stress than what is physically occurring on the sample. However, the influence of the stiffness of the test system was not discussed. In Chryssanthakis (2004), a steel test dummy encapsulated in epoxy was loaded in CNL condition. The recorded normal displacement, capturing the total deformations in the test system, was used for calibration. This was done by correcting the displacements recorded in tests using rock material with recorded normal displacements using the steel test dummy. A similar approach was reported by Dae-Young et al. (2006). In summary, even though rarely reported, an approach exists on how to calibrate test systems with respect to the normal stiffness under CNS conditions. However, no approach exists how to perform such calibrations under CNS conditions.
The existence of a system normal stiffness reduces the normal displacement. This, in turn, results in a lower normal load being applied compared to what should be the case if the dilatancy would only result from the joint stiffness and the specimen material.

It is of importance to correct for the influence of the system normal stiffnesses to control the quality of the results from CNS tests. In this paper, a novel and practicable approach is presented on how to correct for the existence of the system normal stiffness in CNS tests, when using test setups with closed-loop control. The presented results apply only to the specific test system used in this work to validate the approach. However, the approach is applicable to arbitrary test systems. Employing the approach, first, the system normal stiffness of the test system is measured, and then, a new stiffness value that compensates for the influence of the system normal stiffness before direct shear tests are carried out is calculated. The new stiffness value shall be set as input to the control system. This value differs from the user-defined normal stiffness and is required to reduce the deviation between the normal load that is intended to be applied over the joint and the normal load that would actually be applied without compensation. The influence from factors that do not originate from the stiffness of the joint specimen is thus reduced. As such, the accuracy of the test results will be improved, which implies an improved reliability in the usage of the test results in subsequent work related to understanding and explaining the shear process.

In Sect. 2, the results from the normal loading test and the validation of the effective normal stiffness approach are presented. A step-by-step procedure on how to estimate the normal load error using the secant system normal stiffness instead of the measured continuous system normal stiffness is presented in Sect. 5 along with an illustrative example. The findings are discussed in Sect. 6, with conclusions forwarded in Sect. 7.

2 Description of the Effective Normal Stiffness Approach

The concept of joint stiffness is ambiguous, since there is neither a clear definition about where the joint ends and where the rock matrix starts, nor how much rock matrix is required to eliminate boundary effects. The approach described here does not intend to define or allow possibilities to measure the joint stiffness, yet the intention is solely to reduce the impact of undesired displacements originating from the system normal stiffness.

The intended normal stiffness, $k_{ns}$, is the stiffness normally set as input to the control system that simulates the effect of the stiffness of the surrounding rock mass as the joint specimen dilates during shearing. Accordingly, with reference to Fig. 1a, a shear test setup with a linear spring model with a stiffness $k_{ns}$, can be expressed as the ratio between the applied normal load, $N$, and the total normal displacement, $\delta_n$:

$$k_{ns} = \frac{N}{\delta_n}. \quad (1)$$

However, since test systems are not infinitely stiff, an additional stiffness, namely a system normal stiffness, $k_{sys}$, has to be included. It derives from the deformations of the

![Fig. 1](image-url)  

Fig. 1 a Illustration of a spring with stiffnesses $k_{ns}$, subjected to a load $N$, yielding a normal displacement $\delta_n$. b Under the influence of the stiffness $k_{sys}$, the stiffness $k_{eff}$ is required to achieve the same total stiffness of $k_{ns}$ as in a.
test system up to the locations where the normal displacement is measured by the displacement transducers. What contributes to $k_{sys}$ is dependent on where the displacement transducers are located, which varies with the design of the test system. $k_{sys}$ could, for example, originate from displacements in the grouting, the contact area between the encapsulating grout and the specimen holder, as well as from the contact area between the specimen holders and the supporting plates. During shearing the normal load on the joint specimen should be directly related to the dilatancy, which is the normal displacement that should be measured by the displacement gauges. However, the effect of $k_{sys}$ makes the test system less stiff, and consequently, smaller normal displacements than those solely resulting from the dilatancy will be measured. As a result, the normal load applied over the joint is lower, when compared to the case including only the dilatancy and $k_{ns}$. To compensate for this reduction, the test setup should be represented with a model consisting of two springs in series, where one spring represents $k_{sys}$ and the other an effective normal stiffness, $k_{eff}$, that should be set as input to the control system. $k_{eff}$ is the stiffness required to apply the same normal load over the joint under the influence of $k_{sys}$, as would result from the existence of only $k_{ns}$ (Fig. 1b).

It is to say that the stiffness of the spring models in Fig. 1a, b should be equal, which when expressed in displacements leads to:

$$\delta_n = \delta_{n1} + \delta_{n2}. \quad (2)$$

This relation can also be expressed as:

$$\frac{N}{k_{ns}} = \frac{N}{k_{eff}} + \frac{N}{k_{sys}}. \quad (3)$$

Solving for $k_{ns}$ yields:

$$k_{ns} = \frac{k_{eff} k_{sys}}{k_{eff} + k_{sys}}. \quad (4)$$

The physical interpretation of the parameters $k_{ns}$, $k_{sys}$, and $k_{eff}$ in Eq. (4) is illustrated in Fig. 2.

A $k_{eff}$ of higher magnitude than $k_{ns}$ is required to achieve $k_{ns}$ over the joint, in the existence of $k_{sys}$. Equation (4) is of the same form as an equation presented by Heuze (1979). The equation by Heuze (1979) included the intended normal stiffness and the stiffness of the joint itself, for the purpose of calculating the incremental normal stress from an incremental shear displacement. In this case, as mentioned previously, the intention with this work is to reduce the impact of undesired displacements in which the joint stiffness is not included. As such, solving Eq. (4) for $k_{eff}$ yields:

$$k_{eff} = \frac{k_{ns}}{1 - \frac{k_{ns}}{k_{sys}}}. \quad (5)$$

In summary, to use the effective normal stiffness approach, first, a test from which $k_{sys}$ can be derived must be carried out. This is done using a stiff test dummy in a normal loading test. Then, $k_{sys}$ and the user defined $k_{ns}$ are inserted in Eq. (5) to calculate $k_{eff}$. This value is then inserted as stiffness value to the control system in direct shear tests under CNS conditions.

3 Experimental Validation of the Effective Normal Stiffness Approach

3.1 Experimental Setup

A DSH-300 direct shear testing setup manufactured by GCTS with a normal and shear load capacities of 300 kN was used in the tests. The shear and normal loads were measured by two load cells (model SW30-100K-B000, Class...
As a first step in the approach, \( k_{\text{sys}} \) must be derived. This derivation is accomplished by replacing the test specimen with a very stiff test dummy, with all other components in the test system maintained the same as for subsequent CNS direct shear tests. Then, the test system with the very stiff test dummy is subjected to a normal loading test. The slope of the relationship between normal stress, \( \sigma_n \), and normal displacement, \( \delta_n \), of approximately 8 mm was reached. During shearing, the size of the contact area was continuously monitored by the control system, as a prerequisite for obtaining correct magnitudes on the shear and normal stresses throughout the duration of the tests.

4 Results

4.1 Derivation of the System Normal Stiffness from the Normal Loading Tests

As mentioned in Sect. 2, to employ the effective normal stiffness approach \( k_{\text{sys}} \) must be known. \( k_{\text{sys}} \) is equal to the slope of the relationship between normal stress, \( \sigma_n \), versus normal displacement, \( \delta_n \), from normal loading tests using a stiff test dummy. This relationship can thus be viewed as a
so-called calibration curve. The shape of the joint in the test dummy is arbitrary if no lateral displacements take place during the normal loading test, since such conditions will yield the same \( k_{sys} \). An ideal option is to use a test dummy with a plane joint perpendicular to the normal load. However, the test dummy in this case was chosen to have a linear joint with an angle of inclination, which required demonstration that the derived \( k_{sys} \) was not dependent on the shear displacement position. Normal loading tests were, therefore, carried out at three different shear displacements: 0, 5, and 10 mm. Due to the inclination of the joint in the specimen dummy, the starting positions for the recording of the normal displacement varied with the shear displacement. \( k_{sys} \) was determined in terms of the secant stiffness for the normal stress interval of 5–10 MPa for all four load cycles (Fig. 4). This interval was considered to be representative of typical normal stresses occurring in real applications, but any interval could have been chosen for the validation of the approach. Application of this approach only requires \( k_{sys} \) to be derived for a normal load interval on the calibration curve that at least covers the interval of normal loads occurring in the subsequent direct shear tests under CNS conditions.

To clarify the differences between the loading cycles, the values of \( k_{sys} \) are shown in Table 1. In loading, an initial observation is that the specimen was conditioned after the first cycle. It was observed that \( k_{sys} \) is constant within 0.2 MPa/mm regardless of shear displacement, which corresponds to a precision better or equal to 0.3% in relation to the lowest obtained stiffness value from load cycles 2–4. The corresponding value for unloading is \( k_{sys} \) within 0.3 MPa/mm, corresponding to a precision better than or equal to 0.5%.

A second observation is that the results show similarities, such that they are non-linear, but in unloading, the slopes are steeper in the beginning of the unloading cycle compared with the corresponding load interval in loading. As the lower turning point is approached, the conditions become opposite. Therefore, \( k_{sys} \) is higher in unloading than in loading for the chosen evaluation interval (5–10 MPa). One explanation could be the recovery effect in the stiffness components in unloading from the maximum load. This could result in a lag in the response of the normal displacement as the normal load decreases.

A third observation is that \( k_{sys} \) increases with increasing shear displacement independently of load cycle; about 10% in loading and about 6% in unloading between 0 and 10 mm displacement. It is such that the tests were run to a fixed upper limit with respect to a normal stress of 12 MPa. Since area correction was accounted for in the tests, the implication was that the normal load decreased with increasing shear displacement. This resulted in a reduced normal displacement with increasing shear displacement. Consequently, for

![Fig. 4](image)

**Fig. 4** Results from normal loading tests evaluated at three different fixed shear displacements: 0, 5, and 10 mm (from right to left). All four load cycles are plotted, but after the first load cycle, the specimen had been conditioned and load cycles 2–4 cannot be distinguished in the graphs. \( k_{sys} \) was determined in terms of the secant stiffness for the normal stress interval 5–10 MPa, as indicated by the dashed line (to increase the readability, it is only plotted for the fourth load cycle at zero shear displacement.

**Table 1** System normal stiffness, \( k_{sys} \), in terms of secant stiffness values, derived in the stress interval 5–10 MPa from data in Fig. 4; presented for each load cycle at shear displacements of 0, 5, and 10 mm.

| Evaluation position: shear displacement (mm) | System normal stiffness, \( k_{sys} \) (MPa/mm) |
|---------------------------------------------|---------------------------------------------|
| Loading cycle                               | Unloading cycle                             |
| 1                                           | 1                                           |
| 2                                           | 2                                           |
| 3                                           | 3                                           |
| 4                                           | 4                                           |
| 0                                           | 45.9                                        |
| 5                                           | 48.6                                        |
| 10                                          | 50.4                                        |
| 0                                           | 47.1                                        |
| 5                                           | 49.9                                        |
| 10                                          | 51.8                                        |
| 0                                           | 47.1                                        |
| 5                                           | 49.9                                        |
| 10                                          | 51.6                                        |
| 0                                           | 54.5                                        |
| 5                                           | 56.3                                        |
| 10                                          | 58.0                                        |
| 0                                           | 54.4                                        |
| 5                                           | 56.1                                        |
| 10                                          | 57.9                                        |
| 0                                           | 54.6                                        |
| 5                                           | 56.1                                        |
| 10                                          | 57.8                                        |
| 0                                           | 54.7                                        |
| 5                                           | 56.2                                        |
| 10                                          | 57.9                                        |
a fixed stress interval and a normal displacement decreasing with increasing shear displacement, an increasing $k_{\text{sys}}$ expressed in MPa/mm with increasing shear displacement is obtained. When expressing $k_{\text{sys}}$ in normal load instead of in normal stress, an interval of 470–474 kN/mm in loading is obtained, which corresponds to a difference equal to 0.9%. The measured values are shown in Table 2. Since the unloading started at different load levels in combination with the non-linear characteristics, the secant normal stiffness in unloading varies with the shear displacement (526–547 kN/mm). Consequently, the differences in $k_{\text{sys}}$ evaluated at different shear displacement positions are just apparent and are an effect of the evaluation made in terms of normal stress with area correction. Accordingly, expressing $k_{\text{sys}}$ in normal load removes these differences.

A value of $k_{\text{sys}}$ equal to 47.1 MPa/mm, corresponding to the fourth load cycle at zero shear displacement was chosen due to the following reasons. First, since the normal load in CNS direct shear tests normally monotonically increases, the proper choice was judged to be to derive $k_{\text{sys}}$ from loading. Second, zero shear displacement was used because for this displacement $k_{\text{sys}}$ expressed in MPa/mm correlates to the stiffness expressed in kN/mm. This is of importance; since it is the normal load, the test system is subjected to that determines $k_{\text{sys}}$, rather than the normal stress over the joint. Finally, as the reason to carry out load cycles was to condition the specimen to obtain stable stiffness properties, it was reasonable to use the fourth (last) load cycle for $k_{\text{sys}}$. As mentioned above and as seen in Table 1, stability was achieved since the second load cycle. Alternatively, the average value from load cycles 2–4, could have been used, but given the high precision of the

| Table 2 | Measured values of normal stress, $\sigma_n$, normal load, $N$, and normal displacement, $\delta_n$, demonstrating that the variations of the system normal stiffness, $k_{\text{sys}}$, expressed in MPa/mm, with the evaluation position are apparent and a consequence of accounting for area correction. This is concluded by comparing the bold values of $k_{\text{sys}}$ expressed in MPa/mm with those expressed in kN/mm. $k_{\text{sys}}$ expressed in MPa/mm clearly increases with evaluation position regardless of loading cycle, but $k_{\text{sys}}$ expressed in kN/mm remains constant within 0.9% for loading cycles 2–4. Consequently, expressing $k_{\text{sys}}$ in kN/mm removes the variation.

| Loading cycle | 1 | 2 | 3 | 4 |
| Evaluation position: shear displacement 0 mm (contact area 100 × 100 mm) | | | | |
| $\sigma_n$ (MPa) | 4.98 | 9.99 | 5.00 | 9.99 | 4.98 | 9.98 | 5.00 | 9.99 |
| $\sigma_{5–10\text{ MPa}}$ (MPa) | 5.00 | 4.99 | 5.00 | 4.99 | 5.00 | 4.99 | 5.00 | 4.99 |
| $N$ (kN) | 49.83 | 99.86 | 49.98 | 99.85 | 49.98 | 99.84 | 49.99 | 99.84 |
| $N_{5–10\text{ MPa}}$ (kN) | 50.03 | 49.87 | 50.01 | 49.85 | 50.01 | 49.85 | 50.01 | 49.85 |
| $\delta_n$ (mm) | 0.17 | 0.28 | 0.18 | 0.28 | 0.18 | 0.28 | 0.18 | 0.28 |
| $k_{\text{sys}}$ (MPa/mm) | 45.9 | 47.1 | 47.1 | 47.1 | 47.1 | 47.1 | 47.1 | 47.1 |
| $k_{\text{sys}}$ (kN/mm) | 459.0 | 470.9 | 470.9 | 470.9 | 470.9 | 470.9 | 470.9 | 470.9 |
| Evaluation position: shear displacement 5 mm (contact area 95 × 100 mm) | | | | |
| $\sigma_n$ (MPa) | 5.00 | 10.00 | 4.98 | 9.99 | 5.00 | 10.00 | 5.00 | 9.99 |
| $\Delta\sigma_{n}$ (MPa) | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 | 5.00 |
| $N$ (kN) | 47.47 | 94.98 | 47.34 | 94.87 | 47.47 | 94.97 | 47.49 | 94.84 |
| $N_{5–10\text{ MPa}}$ (kN) | 47.51 | 47.53 | 47.50 | 47.35 | 47.50 | 47.35 | 47.50 | 47.35 |
| $\delta_n$ (mm) | 0.01 | 0.11 | 0.015 | 0.115 | 0.02 | 0.12 | 0.02 | 0.12 |
| $k_{\text{sys}}$ (MPa/mm) | 48.6 | 49.9 | 49.9 | 49.8 | 49.9 | 49.9 | 49.8 | 49.8 |
| $k_{\text{sys}}$ (kN/mm) | 461.7 | 473.9 | 473.9 | 473.9 | 473.9 | 473.9 | 473.9 | 473.9 |
| Evaluation position: shear displacement 10 mm (contact area 90 × 100 mm) | | | | |
| $\sigma_n$ (MPa) | 5.00 | 10.00 | 4.99 | 9.99 | 5.00 | 10.00 | 5.00 | 10.00 |
| $\Delta\sigma_{n}$ (MPa) | 5.00 | 5.00 | 5.01 | 5.01 | 5.01 | 5.01 | 5.01 | 5.01 |
| $N$ (kN) | 45.50 | 91.00 | 45.38 | 90.87 | 45.38 | 90.87 | 45.38 | 90.87 |
| $N_{5–10\text{ MPa}}$ (kN) | 45.50 | 45.49 | 45.63 | 45.49 | 45.63 | 45.49 | 45.63 | 45.49 |
| $\delta_n$ (mm) | −0.144 | −0.045 | −0.14 | −0.04 | −0.14 | −0.04 | −0.14 | −0.04 |
| $k_{\text{sys}}$ (MPa/mm) | 50.4 | 51.8 | 51.6 | 51.7 | 51.6 | 51.7 | 51.6 | 51.7 |
| $k_{\text{sys}}$ (kN/mm) | 458.2 | 471.4 | 469.9 | 470.4 | 471.4 | 469.9 | 470.4 | 470.4 |
results (0.3% as mentioned previously in this section), it would not have any practical influence on the conclusions from this study. For the specific case of zero shear displacement, the values from load cycles 2–4 were identical. In summary, the results show that the shear position does not have any practical influence on $k_{sys}$, which allows using the derived figure of $k_{sys}$ in the validation of the approach in the next section with confidence.

### 4.2 Direct Shear Tests

Having determined $k_{sys}$, the next step was to derive $k_{eff}$ to be used in the direct shear tests for the validation of the approach. $k_{ns}$ is user defined and was chosen as 15.0 MPa/mm in this study. This value along with $k_{sys} = 41.7$ MPa/mm when inserted in Eq. (5) yields $k_{eff} = 22.0$ MPa/mm. Moreover, two direct shear tests were carried out: one with the input value of $k_{ns} = 15.0$ MPa/mm and the other with $k_{eff} = 22.0$ MPa/mm. Through comparison, it would be possible to quantify the difference in normal stress and to get an indication of how well the application of $k_{eff}$ would compensate for the existence of $k_{sys}$.

By plotting the measured values of $\sigma_n$ with the measured $\delta_s$, there is a difference of about 13% in normal stress at the end of the shear test (Fig. 5). This observation is a direct effect of the influence of using different stiffness values as inputs in the control system. The undulations seen in the graphs originate from small irregularities in the initial match of the joint surfaces.

Then, the question of which input value produces a response that has a better correlation with the known nominal response was derived. Knowing $\alpha$, with reference to Fig. 3a, the nominal response of the normal stress, $\sigma_{n,nom}$, was calculated as:

$$\sigma_{n,nom} = k_{ns}\delta_s \tan \alpha + \sigma_{n0}. \tag{6}$$

From Eq. (6), the nominal response of the normal stress, $\sigma_{n,nom}$, can be calculated, which would have been obtained in a direct shear test under CNS conditions with $k_{ns} = 15.0$ MPa/mm set as input value to the control system if $k_{sys}$ did not exist. The calculated nominal response is plotted in Fig. 5. It can be seen that the graph showing the experimental results using $k_{ns} = 15.0$ MPa/mm as input to the control system falls below the calculated nominal response, which indicates the existence of $k_{sys}$. It can also be seen that the measured response using $k_{eff} = 22.0$ MPa/mm as input matches the calculated nominal response well, which shows the ability of the effective normal stiffness approach to correct for the influence of the normal system stiffness. As seen in Fig. 5, for the specific test system used in this work, there is a difference of 13% in normal stress at the end of the test. This is the error that would be present in a direct shear test under CNS conditions for the same normal stress interval, if not compensated for by giving $k_{eff}$ as input to the control system instead of $k_{ns}$.

The existence of $k_{sys}$, and the need for compensating for it by applying $k_{eff}$, has just been demonstrated with respect to $\sigma_n$. With the intention to further validate the approach of effective normal stiffness, the effect on $\delta_n$ was also investigated. In Fig. 6, the solid line of the measured $\delta_n$ versus $\delta_s$ from the direct shear test with $k_{eff} = 22.0$ MPa/mm falls below the line of the nominal angle of inclination equal to 1.8°. It is mentioned in Sect. 2, that a test system under CNS conditions can be considered as consisting of two stiffness components: $k_{eff}$ and $k_{sys}$. It has been shown previously that by applying $k_{eff} = 22.0$ MPa/mm, a good agreement between the applied normal stress and the calculated normal stress.
could be achieved under the conditions $k_{sys} = 41.7 \text{ MPa/mm}$ and $k_{ns} = 15.0 \text{ MPa/mm}$ (Fig. 5). Consequently, if $k_{eff} = 22.0 \text{ MPa/mm}$ yields correct normal stress, but, at the same time, yields lower normal displacements than the known nominal normal displacements, the difference in normal displacements between the graphs should then originate from the influence of $k_{sys}$ and be equal to $(\sigma_n - \sigma_{n0})/k_{sys}$. This relation derives from the spring analogy, from which the displacement equals the normal stress increment over the stiffness. If this assumption is true, this means that the total nominal normal displacement due to the known nominal dilatancy could be calculated as:

$$\delta_{n,\text{nom}} = \delta_n + \frac{\sigma_n - \sigma_{n0}}{k_{sys}}. \quad (7)$$

The calculated nominal response obtained by applying Eq. (7) to the measured response is represented by the dashed line in Fig. 6. Accordingly, the second term on the right side of Eq. (7) has been added to the measured normal displacements. The calculated nominal response correlates with the nominal angle of inclination, which is the response that would have been obtained if $k_{sys}$ did not exist. It has been demonstrated that the difference between the measured response using $k_{eff} = 22.0 \text{ MPa/mm}$ and the nominal angle of inclination corresponds to the second term on the right side of the equals sign in Eq. (7). Accordingly, the difference originates from the existence of $k_{sys}$, which further strengthens the validity of the effective normal stiffness approach.

In summary, applying the effective normal stiffness approach implies that more accurate but higher normal loads will be applied in direct shear tests under CNS condition. However, due to the existence of $k_{sys}$, the measured normal displacements will also be lower than those originating only from the dilatancy related to the stiffness of the joint specimen. Therefore, before using the test data in subsequent work related to the shear process, the effect of $k_{sys}$ on the normal displacement should be compensated for. As demonstrated through Eq. (7), compensation can be effectuated by adding the normal stress increment divided by the system normal stiffness to the measured normal displacements in the direct shear test.

5 Error Estimation Procedure

5.1 Background Description

The validation of the effective normal stiffness approach, presented in Sects. 3 and 4, was done using stiffness units (ML$^{-2}$T$^{-3}$), such as MPa/mm. It does not imply any limitations in what has been presented so far. However, it should be noted that a small specimen subjected to a high normal stress would not necessarily imply a high normal load with respect to the capacity of the test system, and a large specimen subjected to a low normal stress could imply a high normal load. Therefore, from a test system perspective, it is the normal load, rather than the normal stress, that is of importance for the system normal stiffness.

Deriving $k_{sys}$ from the secant system normal stiffness implies that it will deviate from the measured continuous system normal stiffness in case of a non-linear behaviour. In addition, $k_{sys}$ could vary with the normal load interval which it is derived from. For a test system with a non-linear characteristic as in Fig. 7, deriving $k_{sys}$ from the load interval 10–24 kN yields a system normal stiffness of 350 kN/mm, while using 35–70 kN yields 589 kN/mm. This means that normal load errors could occur in the application of $k_{eff}$. The errors could occur if the actual normal loads in a shear test are outside the interval from which the system normal stiffness has been derived. Errors could also occur within
the interval dependent on how much the secant system normal stiffness deviates from the true continuous shape of the normal load over the system normal displacement (Fig. 8). This means that there is a need for a procedure to estimate a potential normal load error, which will be presented in Sect. 5.2.

5.2 Procedure

The various steps required to estimate the normal load error originating from the usage of $k_{sys}$ approximated by the secant system normal stiffness are presented. The steps are presented along with an illustrative example. Possible model errors using Eq. (5) are, however, not covered within this procedure.

**Step 1** Derive a calibration curve for the test system. Carry out a normal loading test using a stiff test dummy specimen as described in Sect. 4.1 (Fig. 9). The maximal normal load shall be higher than the normal loads that will be present in the direct shear tests. This is necessary to cover the stiffness characteristics of the test system for the whole working range of expected normal loads. The calibration curve shall be derived as the normal load, $N$, versus the system normal displacement, $\delta_{sys,n}$.

**Step 2** Fit a polynomial, $N(\delta_{sys,n})$, to the calibration curve. The resulting polynomial expression for the example presented in Fig. 9 is:
Step 3 Obtain the continuous representation of the system normal stiffness, $k_{sys\_cont}$, by deriving $N(\delta_{sys\_n})$, which, in the case of Eq. (8), yields:

$$N(\delta_{sys\_n}) = -3426.9\delta_{sys\_n}^3 + 2567.3\delta_{sys\_n}^2 + 37.1\delta_{sys\_n} + 0.5 \text{ (kN)}.$$  \hspace{1cm} (8)

Step 4 Identify the normal loads and corresponding system normal displacements, from which $k_{sys}$ will be calculated. The system normal displacements originating from the system normal stiffness are obtained from the calibration curve. Note that these displacements are not the same as the total normal displacements recorded in direct shear tests using a rock or replica specimen, which include the normal displacements originating from the dilatancy. Using the normal load interval 10–24 kN in Fig. 8, which shows an enlarged part of the calibration curve in Fig. 7, as an example; 10 kN corresponds to 0.055 mm, and 24 kN corresponds to 0.095 mm.

Step 5 Derive $k_{sys}$ in the interval determined by the normal loads and system normal displacements identified in Step 4. In this example $k_{sys} = 350 \text{ kN/mm}$ (Figs. 7, 8).

Step 6 Calculate the normal load error, $\Delta N$, caused by the difference between employing the secant normal stiffness, $k_{sys}$, and the continuous system normal stiffness,
$k_{\text{sys,cont}}$  Integrate the system normal stiffness with respect to the system normal displacement. The lower integration limit is the system initial normal displacement, $\delta_{\text{sys,n0}}$, and the upper integration limit is an arbitrary system normal displacement, $\delta_{\text{sys,nend}}$, corresponding to the normal load for which the error shall be calculated. The general expression for $\Delta N$, due to the inclusion of $k_{\text{sys}}$ instead of using $k_{\text{sys,cont}}$, is

$$\Delta N = \int_{\delta_{\text{sys,n0}}}^{\delta_{\text{sys,nend}}} (k_{\text{sys}}(d_n) - k_{\text{sys,cont}}(d_n)) \, d_n. $$  

(10)

For this example, the indefinite integral becomes,

$$\Delta N = \int_{\delta_{\text{sys,n0}}}^{\delta_{\text{sys,nend}}} (k_{\text{sys}}(d_n) - k_{\text{sys,cont}}(d_n)) \, d_n. $$  

(11)

From Eq. (11), it is possible to calculate the normal load error for an arbitrary $N_{\text{end}}$ corresponding to $\delta_{\text{sys,nend}}$. For the specific example used here ($N_0 = 10 \text{ kN}$ corresponding to $\delta_{\text{sys,n0}} = 0.055 \text{ mm}$ and $k_{\text{sys}} = 350 \text{ kN/mm}$). As a first example, $\delta_{\text{sys,nend}} = 0.095 \text{ mm}$ yields $\Delta N = -1.9 \text{ kN}$, corresponding to $N_{\text{end}} = 24 \text{ kN}$, (Fig. 8), which is the upper limit of the interval for the normal load in which $k_{\text{sys}}$ was derived. This means a normal load 1.9 kN higher than the normal load from $k_{\text{ns}}$ if this stiffness would have been applied over the specimen. The explanation is that applying $k_{\text{sys}} = 350 \text{ kN/mm}$ for the normal displacement interval 0.055–0.095 mm implies a lower estimate of the system normal stiffness than it actually is. Therefore, when using a lower value than the actual for $k_{\text{sys}}$ in the calculation of $k_{\text{eff}}$ using Eq. (5), the consequence is that a higher value of $k_{\text{ns}}$ than that used in Eq. (5) would be applied in reality. This is a consequence of giving $k_{\text{eff}}$ a fixed value as input to the test system, regardless of possible errors in the estimation of $k_{\text{sys}}$.

A second example is setting $N_{\text{end}} = 35.5 \text{ kN}$ corresponding to $\delta_{\text{sys,nend}} = 0.120 \text{ mm}$ (Fig. 8), which yields $\Delta N = -4.9 \text{ kN}$, that consequently would result in an application of a normal load 4.9 kN higher, than would originate from $k_{\text{ns}}$ in Eq. (5).

The relative errors for the normal loads that would have been applied over the joint are 8% for the first example and 14% for the second, in relation to the loads that should have been applied on basis of $k_{\text{ns}}$.

The first example using the same system normal displacements as were used in the derivation of $k_{\text{sys}}$ could typically be used for error estimation before a shear test is carried out, presuming that the correct normal load interval is captured. In the second example, the purpose would be to estimate the normal load error if the actual normal load at the end of the test would deviate from the normal load used in the computation of the system normal stiffness.

The differences in relative errors can be seen comparing the graph of $k_{\text{sys,cont}}$ with the graph of $k_{\text{sys}}$ versus $\delta_{\text{sys,n}}$ (Fig. 11). These plots show the application of the secant normal stiffness of $k_{\text{sys}} = 350 \text{ kN/mm}$ from $\delta_{\text{sys,n0}} = 0.055 \text{ mm}$ to $\delta_{\text{sys,nend}} = 0.095 \text{ mm}$ and $\delta_{\text{sys,nend}} = 0.120 \text{ mm}$, corresponding to the first and the second examples, respectively. It is evident that applying $k_{\text{sys}}$ to the normal displacement interval from which it has been derived yields a lower relative error compared to the error generated when applied outside this interval. Within the derivation interval, $k_{\text{sys}}$ initially overestimates the actual system stiffness, but this is compensated by an under estimation at the end of the derivation interval. Extending the application $k_{\text{sys}}$ beyond this interval, yields a continuously increasing under estimation of the actual system stiffness. This explains the lower relative error when applying $k_{\text{sys}}$ within the system normal displacement derivation interval, in relation to the relative error when the application of $k_{\text{sys}}$ is extended beyond the derivation interval.

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**Fig. 11** Illustration of how the application of a secant system normal stiffness, $k_{\text{sys}} = 350 \text{ kN/mm}$, over a normal displacement interval, 0.055–0.095 mm, from which it was derived, yields a lower relative load error, compared with the error generated if it is applied to a normal displacement of 0.12 mm, which is beyond the derivation interval.
6 Discussion

In direct shear test setups, it is unavoidable that displacements in addition to those directly related to the dilatancy are measured. Therefore, to minimize this effect, the test systems, namely all components and interfaces up to the locations where the normal displacement is measured by the displacement transducers, must be as stiff as possible. The existence of this system stiffness, designated as \( k_{\text{sys}} \), in this study, results in the application of a too low normal load to the specimen. Therefore, the test setup was considered to consist not only of the intended normal stiffness, \( k_{\text{n, int}} \), which is the stiffness normally set as input to the control system that simulates the effect of the stiffness of the surrounding rock mass as the joint specimen dilatates during shearing. Instead, it was proposed that the test setup could be represented by a model consisting of two springs in series with stiffnesses \( k_{\text{sys}} \) and \( k_{\text{n, int}} \). The latter is the stiffness that shall be given as input to the control system, rather than \( k_{\text{n, int}} \) to apply the same normal load as \( k_{\text{n, int}} \) would yield if \( k_{\text{sys}} \) did not exist.

Applying the effective normal stiffness approach requires quantification of \( k_{\text{sys}} \). This is done by carrying out a normal loading test using a stiff dummy test specimen. Then, \( k_{\text{sys}} \) can be quantified as the slope of the normal stress versus normal displacement curve. Knowing \( k_{\text{n, int}} \) and \( k_{\text{sys}} \), \( k_{\text{eff}} \) is calculated from Eq. (5) and given as input to the closed-loop control system. The result is an improved accuracy in the resulting shear test results, as the effect of applying a too low normal load due to decreased normal displacements originating from the existence of \( k_{\text{sys}} \) is reduced.

6.1 Comments on the Experimental Validation

The experimental results demonstrate a clear influence of \( k_{\text{sys}} \) on the normal load applied to the joint and the ability of the effective normal stiffness approach to correct for this influence. The validity of this approach was further demonstrated by showing that the difference in normal displacement versus shear displacement response, between the data using the effective normal stiffness and the calculated nominal response, is constituted by the normal load increment divided by \( k_{\text{sys}} \). It is pointed out that the expression for the calculation of \( k_{\text{eff}} \), Eq. (5), reflects the expected physical behaviour. In a totally rigid test system, \( k_{\text{sys}} \) is infinite. This means that the second term in the denominator approaches zero, and consequently, \( k_{\text{eff}} \) equals \( k_{\text{n, int}} \), as expected.

For the experimental conditions used in this work, the result shows a 13% higher normal stress (or normal load) at the end of the shear test using \( k_{\text{eff}} = 22.0 \text{ MPa/mm} \) compared to the test using \( k_{\text{n, int}} = 15.0 \text{ MPa/mm} \). For a certain test system, given that the calibration curve is non-linear, the magnitude of the difference depends on the normal load interval. Intervals in the calibration curve corresponding to lower system normal stiffnesses, in the present case, say 1–3 MPa compared with 5–10 MPa (Fig. 4), will increase the difference in normal load between the application of \( k_{\text{eff}} \) and no application of it. This also means that to compensate for \( k_{\text{sys}} \) as accurately as possible, it is of importance to employ a system normal stiffness that corresponds to the interval of applied normal loads. With respect to these comments, the presented results have to be viewed as an illustrative example valid for the test system used in this study.

From Eq. (5), it follows that, for a given value of \( k_{\text{n, int}} \), the lower \( k_{\text{sys}} \) is, the larger the need for compensating it with the application of \( k_{\text{eff}} \). It also follows that when \( k_{\text{sys}} \) and \( k_{\text{n, int}} \) are more or less equal, the more sensitive \( k_{\text{eff}} \) is for variations in \( k_{\text{sys}} \). On the other hand, the larger \( k_{\text{sys}} \) is in relation to \( k_{\text{n, int}} \), the more reduced the influence of \( k_{\text{sys}} \) in \( k_{\text{eff}} \) will be. This is demonstrated by the configuration used in this study. As mentioned in Sect. 4.1, \( k_{\text{sys}} \) varies in the interval 470–474 kN/mm, corresponding to a precision of 0.9%. Inserting these interval limits in Eq. (5) yields a precision for \( k_{\text{eff}} \) of 0.4%.

In the derivation of the system normal stiffness, the intention is to capture all normal stiffness components measured by the displacement transducers, except those originating from the joint specimen. For that reason, all parameters except those resulting from the specimen and the joint should be the same in the normal loading tests for the derivation of the system normal stiffness as for the setup used in the direct shear tests. Even though the conclusions made in this work are applicable to arbitrary test systems used with closed-loop control in direct shear tests under CNS conditions, the presented value of \( k_{\text{sys}} \) applies to the specific test system used in this work. Therefore, it is of interest to investigate if the value of \( k_{\text{sys}} = 471 \text{ kN/mm} \) employed in this study is a representative value of the system normal stiffness for test systems in general. In Konietzky et al. (2012), the mechanical stiffness of their system was reported to be in the order of 375 kN/mm. The values differ but are of the same magnitude.

All factors up to the locations where the normal displacement is measured by the displacement transducers not having an infinitely high normal stiffness will contribute to \( k_{\text{sys}} \). This means that different test systems will have different \( k_{\text{sys}} \). As mentioned in Sect. 2, factors influencing \( k_{\text{sys}} \) could, for example, originate from displacements in the grout, from the contact area between the grout and the specimen holder and from the contact area between the specimen holders and the supporting plates. The lower \( k_{\text{sys}} \) is, the larger will the error be in the applied normal load in direct shear tests, and the larger will the need be for compensating for \( k_{\text{sys}} \) by applying a higher value on \( k_{\text{eff}} \) as input to the control system. Consequently, the closer to the joint the displacement transducers are located, the fewer factors will influence \( k_{\text{sys}} \). Ultimately, the displacement transducers should be located directly on the specimen, which would eliminate \( k_{\text{sys}} \) and the need for correction of it.
As has been demonstrated, to reduce normal load errors, \( k_{\text{eff}} \) is used as input in the control system. However, due to the existence of \( k_{\text{sys}} \), the measured normal displacement will be smaller than would be in case only the effect of the dilatancy of the joint and the stiffness of the specimen material would have been present. Therefore, the normal displacements to be used, for example in modelling work, should be the measured normal displacements plus the normal load increment divided by the system normal stiffness in the case of using the secant normal stiffness.

In the derivation of \( k_{\text{sys}} \), the best option would have been to use a steel specimen made in one piece. This would eliminate a possible influence on the normal stiffness from the planar joint. However, given the high precision in \( k_{\text{sys}} \) in this case (0.3%, Sect. 4.1), the influence is considered negligible.

Also, the influence of the deformability of the test dummy is considered as negligible. An approximated thickness in the normal direction of 100 mm, a modulus of elasticity of 210 GPa, and a normal stress of 12 MPa yields a normal displacement of 0.006 mm. This is less than 2% of the total normal displacement of 0.32 mm recorded in the normal loading test (Fig. 4). This indicates that the derivation of \( k_{\text{sys}} \) was carried out under stable and applicable conditions.

In the examples used in this work, the total normal displacements as well as the system normal displacements are small. With \( k_{\text{eff}} \) equal to 22 MPa/mm, the applied normal load close to 90 kN (given a specimen size 100 × 100 mm) at a shear displacement of 8 mm (Fig. 5) yields a normal displacement of 0.18 mm (Fig. 6). From the angle of inclination of the joint in the steel dummy, the nominal normal displacement is known to be 0.25 mm (Fig. 6). The difference between these displacements is the influence of the system normal stiffness, which means that, in this case, approximately 1/3 originated from system normal displacements. This implies that even apparently small variations in system normal displacements would have a meaningful impact on the normal loads. In turn, this illustrates the importance of controlled specimen preparation and need of a stiff test system.

### 6.2 Comments on the Error Estimation Procedure

The simplest representation of \( k_{\text{sys}} \) is by the secant system normal stiffness. However, if the actual system normal stiffness is non-linear, using a secant normal stiffness as an approximation will introduce an error in the applied normal load. Hence, a six-step procedure for the estimation of the normal load error was introduced in Sect. 5.2 using this approximation.

The intended use of the procedure is related to experimental planning to estimate the normal load error for various choices of normal load intervals, resulting in different secant system normal stiffnesses (Fig. 7). The procedure can also be used to calculate the normal load errors from experimental data, in case the achieved normal load turns out to be higher than the normal load used in the determination of the secant system normal stiffness.

In the derivation of the expression for calculating \( k_{\text{eff}} \), the secant system normal stiffness was used as input. The secant system normal stiffness is a constant, but higher order polynomials could also be used in Eq. (5) if the control system allows using higher order polynomial expressions as inputs, which would increase the accuracy using the effective normal stiffness approach. Moreover, also in the presented procedure for the estimation of the normal load error, the secant system normal stiffness was used, but could be substituted with higher order polynomial expressions to model the continuous system normal stiffness.

For error estimations, the normal loads applied by the test system are of most relevance. For any shear test system, the calibration curve should, therefore, be derived in terms of normal loads. From the normal loads, the system normal displacements are determined from the calibration curve. These displacements are then used in the error estimations employing estimates of the system normal stiffness. Neither the intended normal stiffness, \( k_{\text{ns}} \), the effective normal stiffness, \( k_{\text{eff}} \), nor the measured total normal displacements, \( \delta_{\text{ns}} \), must be known explicitly.

### 7 Conclusions

A novel and practicable approach has been presented, the effective normal stiffness approach, to be used in CNS direct shear tests using test setups with closed-loop control. The approach aims to reduce deviations between the intended and applied normal loads, originating from the existence of the system normal stiffness. Validation through direct shear tests indicates a clear influence of the system normal stiffness on the applied normal load (13% for the test system used in this study), as well as the ability of the approach to reduce this effect. In addition, a procedure has been presented to be used for estimating errors originating from the application of an approximate representation of the system normal stiffness.

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Compliance with Ethical Standards

Conflict of interest The authors declare that they have no conflict of interest.

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References

Asadizadeh M, Moosavi M, Hossaini MF, Masoumi H (2018) Shear strength and cracking process of non-persistent jointed rocks: an extensive experimental investigation. Rock Mech Rock Eng 51:415–428

Bad N, Hatzor YH, Toussaint R, Sagy A (2016) Geometrical evolution of interlocked rough slip surfaces: the role of normal stress. Earth Planet Sci Lett 443:153–161

Bahaaeddini M (2017) Effect of boundary condition on the shear behaviour of rock joints in the direct shear test. Rock Mech Rock Eng 50:1141–1155

Bandis S, Lumsdén AC, Barton NR (1981) Experimental studies of scale effects on the shear behaviour of rock joints. Int J Rock Mech Min Sci Geomech Abstr 18:1–21

Barla G, Barla M, Martinotti ME (2010) Development of a new direct shear test apparatus. Rock Mech Rock Eng 43:117–122

Casagrande D, Buzzi O, Giacomini A, Lambert C, Fenton G (2018) A new stochastic approach to predict peak and residual shear strength of natural rock discontinuities. Rock Mech Rock Eng 51:69–99

Chrysanthakis P (2004) Oskarshamn site investigation, Drill hole: KSH01A the normal stress and shear tests on joints SKB Report No. P-04–185 SKB Stockholm 2004, pp 15–16

Dae-Young K, Byung-Sik C, Jin-Suk Y (2006) Development of a direct shear apparatus with rock joints and its verification tests. Geotech Test J 29(5). www.astm.org (Paper ID GTJ12553)

Davideko G, Sagy A, Hatzor YH (2014) Evolution of slip roughness through shear. Geophys Res Lett 41:1492–1498

Gasc-Barbier M, Hong TTN, Marache A, Sulem J, Riss J (2012) Morphological and mechanical analysis of natural marble joints submitted to shear tests. J Rock Mech Geotech Eng 4(4):296–311

Grasselli G, Wirth J, Egger P (2002) Quantitative three-dimensional description of a rough surface and parameter evolution with shearing. Int J Rock Mech Min Sci 39:789–800

Haberfield CM, Szymakowski J (2003) Application of large scale direct shear testing device for investigating rock failure. Rock Mech Rock Eng 36:647–651

Kroupin A, Gravel C, Rivard P, Ballivy G, Rivard P, Krouns A, Johansson F, Larsson S (2016) Shear strength of partially bonded concrete-rock interfaces for application in dam stability analysis. Rock Mech Rock Eng 49(7):2711–2722

Kulatilake PHSW, Shreedhara N, Shirizadeh T, Xing SBY, He P (2016) Laboratory estimation of rock joint stiffness and frictional parameters. Geotech Geol Eng 34:1723–1735

Li W, Bai J, Cheng J, Peng S, Liu H (2015) Determination of coal–rock interface strength by laboratory direct shear tests under constant normal load. Int J Rock Mech Min Sci 77:60–67

Liu Y, Xu J, Yin G, Peng S (2017) Development of a new direct shear testing device for investigating rock failure. Rock Mech Rock Eng 50:647–651

Li W, Bai J, Cheng J, Peng S, Liu H (2015) Determination of coal–rock interface strength by laboratory direct shear tests under constant normal load. Int J Rock Mech Min Sci 77:60–67

Liu Y, Xu J, Yin G, Peng S (2017) Development of a new direct shear testing device for investigating rock failure. Rock Mech Rock Eng 50:647–651

Moradian Z, Gravel C, Fatih A, Ballivy G, Rivard P (2013) Developing a high capacity direct shear apparatus for the large scale laboratory testing of rock joints. Rock Mechanics for Resources Energy and Environment, Taylor & Francis Group, London (ISBN 978-1-138-00080-3)

Muralha J, Grasselli G, Tatone B, Blümel M, Chrysanthakis P, Yu Jing J (2014) ISRM suggested method for laboratory determination of the shear strength of rock joints revised version. Rock Mech Rock Eng 47:291–302

Packulak RMT, Day JJ, Diederichs MS (2018) Practical aspects of boundary condition selection on direct shear laboratory tests. In: Litvinenko V (ed) Geomechanics and geodynamics of rock masses: selected papers from the 2018 European rock mechanics symposium saint petersburg, 22–26 May 2018, 1st edn. Taylor & Francis Group, London, pp 253–259

Rao KS, Shrivastava AK, Singh J (2009) Development of an automated large scale direct shear testing machine for rock. Research Gate. https://www.researchgate.net/publication/265407964. Accessed 29 June 2015

Shrivastava AK, Seshagiri Rao K (2018) Physical modelling of shear behaviour of infilled rock joints under CNL and CNS boundary conditions. Rock Mech Rock Eng 51:101–118

Thirukumar S, Indraratna B (2016) A review of shear strength model for rock joints subjected to constant normal stiffness. J Rock Mech Geotech Eng 8:405–414

Xia CC, Yue ZQ, Tham LG, Lee CF, Sun ZQ (2003) Quantifying topography and closure deformation of rock joints. Int J Rock Mech Min Sci 40:197–220

Xia C, Yu Q, Gui Y, Qian X, Zhuang X, Yu S (2018) Shear behaviour of rock joints under CNS boundary condition. In: Proceedings of GeoShanghai 2018 International Conference: Rock Mechanics and Rock Engineering, Shanghai, 27–30 May 2018

Zhu JB, Li H, Deng JH (2019) A one-dimensional elastoplastic model for capturing the nonlinear shear behaviour of joints with triangular asperities based on direct shear tests. Rock Mech Rock Eng 52:1671–1687

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