Mathematical Reasoning: The characteristics of students’ mathematical abilities in problem solving

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Abstract. Mathematics and mathematical reasoning are two things that cannot be separated. Understanding mathematics is obtained through reasoning, while reasoning is understood and trained through learning mathematics. Therefore reasoning skills are needed by students when learning mathematics. Students who have the ability to reason during the process of understanding, reasoning during planning problems, reasoning in resolving and reasoning in drawing conclusions will make it easier for them to understand Mathematics. So that mathematical reasoning ability is needed to build a student’s mathematical abilities. It can be said that the character of students 'mathematical abilities influences students' mathematical reasoning. This study aims to describe students' mathematical abilities in solving mathematical problems by using mathematical reasoning. This study uses a qualitative approach. The subjects of this study were two eighth grade students of junior high school. The results showed that students with high mathematical abilities could be described as having good mathematical reasoning because it had fulfilled four indicators of mathematical reasoning. Whereas for students with low mathematical abilities are still very lacking in the process of reasoning because they cannot fulfill the four indicators of mathematical reasoning. Subjects are only able to understand the problem

1. Introduction
Mathematics is an important lesson in improving the quality of education because its role is quite relevant to the development of science and technology. Through mathematics lessons, students can practice their abilities continuously so that they are increasingly developing. Purwosusilo [1] said that fundamentally mathematics is a science that is needed in various fields, both in mathematics itself and in other fields. Mathematics not only meets the needs of the present but also meets the needs of the future.

There are five Process Standards that students need to possess and master in mathematics learning: (1) problem solving; (2) reasoning and proofing; (3) communication; (4) connections; and (5) representation. Thus, reasoning ability is one of the important things that need to be mastered by students to support their success in learning mathematics [2]. However, the essence of solving mathematical problems that are needed is reasoning ability. Through reasoning, students are expected to see that mathematics is a logical study. Thus, students feel confident that mathematics can be understood, thought out, proven and can be evaluated, as well as to do things related to mathematics required reasoning.
The reasoning is a process in thinking that combines two or more thoughts to draw a conclusion to get new knowledge. Meanwhile, reasoning is “the process of thinking that attempts to connect facts or evidence that are known to lead to a conclusion”. So, reasoning can be interpreted as a thought process to obtain logical conclusions based on relevant facts that truth has been proven or assumed beforehand [3,4].

Mathematical reasoning is a process that is carried out to get a conclusion based on logical-mathematical premises based on relevant facts and sources that have been assumed to be true. This was also conveyed by Wahyudi [5] who said that mathematical reasoning is a process for obtaining conclusions based on mathematical premises that have been known or assumed. Based on the explanation above it can be concluded that mathematics and mathematical reasoning are related. This was also mentioned by [6,4] that ”mathematical material and mathematical reasoning are two things that are inseparable, mathematical material understood through reasoning and reasoning is understood and trained through learning mathematical material”. So that reasoning skills are needed by students when learning mathematics. Baroody [7] stated that reason is an important tool in mathematics an in daily life since many problems in mathematics and daily life requisite reasoning to solve then.

Students are able to do reasoning if they are able to use reasoning skills in patterns and traits, manipulate mathematics in generalizing or explain mathematical ideas and statements. The higher the level of students' reasoning, the faster the learning process will be in achieving learning indicators. The reasoning ability is the basic ability of mathematics itself. This is in accordance with the results of Sppaile [8] that mathematical reasoning ability has a positive influence on mathematics learning achievement.

Mathematics learning achievement is the result achieved by students after the learning process. Mathematical skills can be divided into 3 levels: high mathematics skills, moderate mathematics skills, and low mathematical abilities. The reasoning ability becomes one of the goals in the learning of mathematics to train the way of thinking and reasoning of conclusions and develop the ability to solve mathematical problems. The goal in learning mathematics in schools is to practice thinking and reasoning in drawing conclusions, developing problem-solving skills, and developing the ability to convey information or communicate ideas through oral, written, graphic, map, diagram, and so on [9]. However, in this study reasoning ability is an ability that must be possessed by students in drawing conclusions, and the ability to solve problems.

In studying mathematics, students must solve mathematical problems. This is supported by Branca [10] states that "Problem-solving is the heart of mathematics” which means the main core of mathematics is problem-solving. Problems usually contain a situation that encourages someone to solve it but can not directly determine what the solution are.

In solving mathematical problems, students should have the ability to reasoning during the process of understanding, reasoning during planning problems, reasoning in resolving and reasoning in drawing conclusions, so that mathematical reasoning skills are needed to build mathematical abilities in a student. Problem-solving in mathematics engaged the student to coordinate [11]. It can be said that the character of students 'mathematical abilities influences students' mathematical reasoning. Some research about mathematical reasoning have been done by some researches, such as the result of research from Risqi [12] said that mathematical reasoning abilities are still low, its proof by result the students answer that analyzed use the indicator. While Menderes [13] said that there is a significant difference in gender about mathematical reasoning. The differences of this study are in this study analysis the characteristic of student mathematical abilities especially about mathematical reasoning, that the subjects consist of students with high ability and students with low ability in math. While in Risqi look a mathematical reasoning student by classical students and Menderes look a mathematical reasoning student by gender. The purpose of this study was to find out how the characters, and the differences between junior high school students in the process of reasoning between students who have high ability in mathematics and students with low math abilities.
2. Method
This research is qualitative research. Qualitative research is research that intends to understand the phenomenon of what is experienced by research subjects such as behavior, perception, motivation, action, etc. Holistically, and in a descriptive way in the form of words and language, in a specific context that is natural and utilizing various natural methods [14]. The type of research used in this study is a case study. Case Study is a type of research carried out intensively, in detail, and in-depth on a particular organism, institution, or object.

This research was conducted at SMPN 1 Larangan Pamekasan Madura. The subjects of this study were 2 students of eight grade, consisting of one student who had high mathematics skills and one student who had low mathematics skills. There are three instruments used in this study. First, the Mathematics Ability Test (TKM), which is used in the subject selection process which is adapted from the junior high school National Examination (UN) questions. Second, Problem Solving Test (TPM) consisting of TPM 1 and TPM 2. Tests are conducted to determine the level of students' ability to solve mathematical problems based on their mathematical abilities. Problem-solving test (TPM) is done by giving a question to the research subject and asking students to solve it. In this study, researchers made questions in the form of a description problem. The question is made by the researcher and, validated by an expert validator. After validation, a legibility test is conducted to find out whether this problem is suitable for use or not. If it is not valid, a new TPM draft will be prepared. Readability test is done by giving TPM to 1 class VIII student who is not the subject of research. Readability testing is done to find out whether the TPM draft can be understood or not. If the draft TPM can be understood properly, the TPM is ready to be used as a research instrument, otherwise, it will be corrected until the TPM can be read and understood well. The last interview, in this study, using open interviews in which this interview is used to find out the opinions of students while working on problem-solving tests (TPM) and to explore and explore students' understanding of integers. This interview guide is just an outline of the question. Other more in-depth questions can be submitted and developed in the interview activity, depending on the process conditions in solving mathematical problems. The interview guide refers to what is seen in reasoning.

In this study, researchers used written tests and interviews to collect data. The validity of the data is done by using time triangulation, by checking the degree of trust of several data sources obtained at different times. This data collection is done at least twice with different problem-solving tests but the contents of the test remain the same. This research was conducted by comparing the results of task-based interviews from TPM 1 with the results of task-based interviews from TPM 2 (equivalent to the first problem) at different times. If the same tendency is obtained, data collection on the subject has been completed and conclusions can be drawn. But if the data from the interview from TPM 1 and TPM 2 shows a different tendency then an interview from TPM 3 (equivalent to the first and second questions) is conducted. If it tends to be the same as the data from the interview from TPM 1, then the data about mathematical reasoning in solving math problems of junior high school students is obtained from the TPM 1 and TPM 3 interview data. Data about mathematical reasoning in solving mathematical problems of junior high school students is obtained from the TPM 2 and TPM 3 interview data. If from the data comparison all the data tend to be different then it is carried out repeatedly until valid data is obtained. Data is said to be valid if there is consistency, opinion or thought on the results of task-based interviews that have been conducted by the researcher. Thus, it is expected that all data reinforce each other and provide in-depth details about solving mathematical problems of junior high school students.

In this study time triangulation was used as an analysis of test and interview data, checking the data carried out by giving almost the same questions at different times. The analysis refers to the reasoning indicator at each stage of the Polya problem-solving. Data analysis in this study is based on the following stages:

2.1 Data reduction
Data reduction is the process of selecting, focusing, simplifying, abstracting, and transforming field notes or transcripts. Data reduction is a form of analysis that simplifies, directs, and organizes data so
that conclusions can be drawn. Data reduction continues until the final report is complete. If there is invalid data, then the data will be analyzed. This study uses triangulation of data collection time. The data reduction process that will be carried out includes (1) Gather problem-solving test results, check and review problem-solving test results. Then make a data transcript consisting of students' explanation of the problem solving given in written form; (2) Review the results of the interview. Then make transcripts of the interview results about students' mathematical reasoning in solving problems; (3) Play the results of interview recordings repeatedly so that researchers can write appropriately about the students' mathematical reasoning process in solving problems such as those that have been the subject of the interview; (4) Re-examine the results of the transcript by listening to the results of the interview with the related subject.

2.2 Data display
Data display is a collection of information that is organized and categorized so that it is possible to draw a conclusion. The data displayed in this study is to classify data about students' mathematical reasoning in terms of high mathematical abilities in solving mathematical problems.

2.3 Drawing conclusion
Drawing conclusions in this study are the stage of understanding the patterns, information, and possible arrangements, as well as the causes that arise during the research process. The conclusion of this study is used to reveal the characteristics of the mathematical abilities of junior high school students in solving mathematical problems with mathematical reasoning.

3. Results and discussion
Subject selection is selected based on the results of the mathematic ability test. Subject candidates consist of 20 students of eighth grade SMPN 1 Larangan. Furthermore, these students were given a mathematics ability test instrument (TKM) to identify their mathematical abilities. Based on the test results obtained 7 students with low ability, 8 students with moderate ability, and 5 students with high abilities. Furthermore, the results of these abilities are re-analyzed so that 2 students will be used as research subjects consisting of 1 student with low mathematical abilities and 1 student with high mathematical abilities. The selection of 2 subjects of this study, besides using their mathematical abilities, also used the consideration of the teacher who taught mathematics in the eighth grade. Selected research subjects are presented in Table 1.

| No | Name | Score | Mathmatic ability |
|----|------|-------|------------------|
| 1  | KZ   | 37    | Low              |
| 2  | RDF  | 90    | High             |

Based on Table 1, it as can be revealed that two subjects own different skills in tern of mathematic competence, i.e. The first subject with poor co ability in mathematics having score 37, while the second subject with the better ability having score 90. Figure 1. is a problem-solving test that is used to reveal reasoning based on the characteristics of students' abilities. This test is an essay about algebra that requires a solution and cannot be immediately solved by routine procedures.

**PROBLEM SOLVING TEST (1)**
Suppose that a, b, c, and d are positive integers which, when divided by 13, will have 12.9.11, and 7, respectively, then specify the remainder of $3a + 4b-3c + 2d$ if divided by 13?

**Figure 1.** Problem-solving test 1
Figure 2. is a problem-solving test that is used to reveal reasoning based on the characteristics of students' abilities. The test has difficulty level which is almost the same as figure 1.

![PROBLEM SOLVING TEST (2)](image)

Suppose that $a$, $b$, $c$, and $d$ are positive integers which when divided by 9 remain at 15, 11.7, and 5, then specify the remainder of $2a + 3b + 4c + 2d$ if divided by 9?

Figure 2. Problem-solving test 2

Research Results Shows that the differences in mathematics abilities of junior high school students in solving mathematical problems with mathematical reasoning between students who have high mathematical abilities and students who have low mathematical abilities are very different, the differences are as follows: (1) Students with high mathematical abilities at the stage of understanding the problem, they are able to understand the details of information to help answer the problems; (2) Students with low mathematical abilities at the stage of understanding the problem cannot mention the information that is known, they only able to mention information about the question; (3) Students with high mathematical abilities able to plan a problem solving to answer the mathematic question; (4) Students with low mathematical abilities do not able to plan a problem solving to answer the mathematic question; (5) Students with high mathematical abilities have been able to solve problems based on plans that have been made in the previous stage; (6) Students with low mathematical abilities cannot carry out the problem-solving stage, because they are not able to make planning in the previous stage; (7) Students with high mathematical abilities are able to re-examine with what has been done in the previous stages, and students are also able to analyze conclusions related to the results obtained; (8) Students with low mathematical abilities do not able to re-examine because they cannot complete the existing problems correctly so that students cannot draw conclusions from existing problems. The results of the research on mathematics ability of junior high school students with mathematical reasoning above can be described in detail as follows:

3.1 Subjects with High Mathematic Ability

In understanding the problem both for the first test and the second test, it is described that the subject gives reasons or proof of the truth of the solution by reading the problem carefully and can identify so that students understand what is known and asked about the problem, this is in line with the research conducted by Putra and Novita [15] who stated that high mathematics ability student was able to identify the problem by making the information know In the next stage the subject can examine the validity of an argument by suggesting that the information obtained is correct and in accordance with the problem. So the subject draws conclusions by summarizing the information obtained from the problem (both known of data and questioned data) by using a standard sentence and based on what has been understood about the question.

In the problem-solving planning stage to test both number one and number two questions, it is described that the subject can manipulate mathematics by designing a settlement model or mathematical strategy that will be used to solve the problem using the first three methods, both substitutions, and the third distributive property in integers. Furthermore, the subject gives reasons or proof of the truth of the solution by explaining that the design of the mathematical model is used to find the values of $a$, $b$, $c$, and $d$. Which is then the value is subsidized and then used using distributive properties in integers. Then the subject checks the validity of an argument by suggesting that the mathematical model that has been made is correct. And the next step the subject draws conclusions by concluding the solution plan that is done to solve the problem by using his own language.

In the staging problem solving for the first and second tests, it was described that the subject in carrying out the problem-solving plan in accordance with the problem-solving plan subject did mathematical manipulation by making a mathematical model, and solving the problem based on a plan.
that had been done previously. Then the subject gives reasons or proof of the correctness of the solution by saying that after assuming the values of a, b, c, and d just subscribe to it then use the third step which uses distributive properties in integers. Next, the subject checks the validity of an argument by suggesting that the results obtained are correct. And the last step of the subject draws conclusions by concluding the solution to the problem that has been done. These findings are in line with research of Irawati and Hasanah [16] subjects with high ability to provide logical reasons for each step taken in solving problems.

In the re-examine stage described that the Subject checks the validity of an argument by rereading the answers already obtained, and checking the correctness of the steps correctly. The subject draws conclusions by concluding the resolution of the problems that have been carried out which have been verified.

3.2 Subjects with Low Mathematical Ability

In the stage of understanding the problem both for the first and second tests, it is described that the subject is less able to provide reasons or proof of the truth of the solution and this is seen when the subject reads the problem in a less accurate way and cannot identify so that the student is unable to mention complete information, this is same with research of Minarni etc [17] that student ability is low in understanding problem. but was able to mention the information asked about the problem. In the next stage, the subject is able to examine the validity of an argument by suggesting that the information obtained is correct. Then the subject is less able to draw conclusions (both known of data and questioned data) given in the problem.

In the stage of problem-solving planning for the first and second tests, it is described that the subject is unable to manipulate mathematics and cannot design a settlement model or strategy that will be used to solve the problem. Furthermore, the subject is unable to provide reasons or proof of the truth of the solution. The subject is also unable to check the validity of an argument. And the next step the subject is unable to draw conclusions. This can be seen when the subject is unable to answer the test given either verbally or by writing.

In the staging problem solving for the first and second tests, it was described that the subject was unable to carry out the problem-solving plan because the subject could not carry out a problem-solving plan both orally and in writing. So that the subject is not able to provide reasons or proof of the truth of the solution, this is due to a lack of capacity for their kick off ability related to evidence material [18]. The subject is also unable to check the validity of an argument. And the subject was also unable to draw conclusions.

In the re-examine stage described that the subject is unable to check the validity of an argument because he cannot answer the questions given in the form of verbal or verbal. So that the subject is unable to draw conclusions from the problem.

Based on the description above it is clear that students with high math abilities are better than students who have low mathematical abilities both in accuracy and in reasoning in problem-solving. The similarity of mathematical reasoning between junior high school students with high mathematical abilities and students with low math abilities lies in the stage of understanding the problem when mentioning the information asked. Whereas the difference lies very significantly because students who have high math skills have fulfilled these four indicators and can solve the problems given correctly and correctly, while for students with low mathematical abilities they cannot fulfill all four indicators so that they become obstacles for the students to be able to solve the problem given.

The results of research for students with low ability have the same results as the research conducted by Risqi [12], that they have a low level of mathematical reasoning in solving a problem. Most of them are still low in giving examples in solving problems, compiling evidence, checking the validity of answers and drawing conclusions, States that in general most students in mathematical reasoning are in the middle and low stages [19].
Based on the observations of researchers, students' inability in mathematical reasoning is because they have just entered the "formal operational" stage. This is in accordance with the opinion of Piaget, students are in the 11-15 years are in formal operation development [20]. In these ages, the thing needed to consider is teenagers development aspect. Where the student can experience a transition step from the usage on a concrete operation into operation.

4. Conclusion
The results showed that the character of students with high mathematical abilities at the stage of understanding the question was described by collecting information obtained from the problem (data that was known and asked) by using a standard sentence and based on what was understood about the problem. Meanwhile, at the stage of problem-solving planning the subject manipulates mathematics by designing a settlement model or strategy that will be used to solve the problem. This is done because it is considered that the mathematical model can be used to facilitate the problem-solving process. At the stage of the problem solving the subject manipulates mathematics by creating a mathematical model, by saying that after assuming the values of a, b, c, and d just subscribe to it then use the third step which uses distributive properties in integers. Subjects have also carried out problem-solving in accordance with the steps that have been planned. Meanwhile, in re-examining stage the subject checks the truth/truth of an argument by re-reading the answers that have been obtained and checking the truth of the steps correctly. Then the subject draws conclusions by concluding the resolution of the problem that has been carried out which has been checked for truth. As for the character of students with low mathematical abilities, it is described that the subject is only able to understand the problem.

While at the stage of making a problem-solving plan, the stage of carrying out the problem-solving plan, and the reexamination stage of the subject cannot show the reasoning process.

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