Goldstone Bosons in Effective Theories with Spontaneously Broken Flavour Symmetry

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Abstract

The Flavour Symmetry of the Standard Model (SM) gauge sector is broken by the fermion Yukawa couplings. Promoting the Yukawa matrices to scalar spurion fields, one can break the flavour symmetry spontaneously by giving appropriate vacuum expectation values (VEVs) to the spurion fields, and one encounters Goldstone modes for every broken flavour symmetry generator. In this paper, we point out various aspects related to the possible dynamical interpretation of the Goldstone bosons: (i) In an effective-theory framework with local flavour symmetry, the Goldstone fields represent the longitudinal modes for massive gauge bosons. The spectrum of the latter follows the sequence of flavour-symmetry breaking related to the hierarchies in Yukawa couplings and flavour mixing angles. (ii) Gauge anomalies can be consistently treated by adding higher-dimensional operators. (iii) Leaving the $U(1)$ factors of the flavour symmetry group as global symmetries, the respective Goldstone modes behave as axions which can be used to resolve the strong CP problem by a modified Peccei-Quinn mechanism. (iv) The dynamical picture of flavour symmetry breaking implies new sources of flavour-changing neutral currents, which arise from integrating out heavy scalar spurion fields and heavy gauge bosons. The coefficients of the effective operators follow the minimal-flavour violation principle.
1 Introduction

Flavour transitions are a very sensitive probe for physics beyond the Standard Model (SM). Somewhat surprisingly, high-precision tests of the Cabibbo-Kobayashi-Maskawa matrix, in particular from $B$-meson and kaon experiments, so far did not find significant deviations from theoretical predictions within the SM, although certain “puzzles” on the level of 2-3 $\sigma$ effects are frequently discussed and give a strong motivation for the continuation of precision measurements in the flavour sector at future facilities (for recent overviews, see for instance [1–4]). As a consequence, the flavour sector of new-physics models, that are to address the issues related to electroweak symmetry breaking and typically contain new interactions at the TeV scale, must be very much constrained.

A simple solution is to assume that the flavour transitions are induced by the SM sources (i.e. the Yukawa matrices) only, while the new physics at high scales is basically flavour-blind (see e.g. [5,6]). At low scales, a model-independent formulation can be achieved in terms of an effective field theory, incorporating the concept of Minimal Flavour Violation (MFV). Here the flavour symmetry of the SM gauge sector is considered to be spontaneously broken by vacuum expectation values (VEVs) of spurion fields, representing the Yukawa matrices of the SM [7]. In the following, we will focus on the quark sector. In the lepton sector, a similar construction can be made after extending the minimal SM to account for neutrino masses and mixing angles [8].

Taking this framework at face value, one has to promote the entries of the Yukawa matrices to complex scalar fields which become dynamical at high scales $\Lambda$ and are embedded into an effective theory with a global flavour symmetry. It then appears rather natural to assume that the hierarchy of fermion masses and mixing angles in the SM is related to a hierarchy of scales at which the different scalar field components in the Yukawa matrices acquire a VEV. For instance, the $O(1)$-value of the top-quark Yukawa coupling suggests that the VEV of the corresponding scalar field is generated already at the high scale $\Lambda$, while the remaining quark Yukawa couplings are generated at lower scales $\Lambda_i \ll \Lambda$. Below $\Lambda$, the effective theory is conveniently formulated by representing the flavour symmetry in a non-linear way, including Goldstone fields – for that part of the flavour symmetry that is broken by the top-quark Yukawa coupling – and residual spurion fields for the unbroken symmetry [9] (see also [10]). An analogous construction can be successively applied to the next-largest entries in the Yukawa matrices, and the resulting sequence of partly broken flavour symmetries – which reflects the hierarchies of quark masses and mixing angles in the SM – can be identified [11]. A similar framework can be defined for the lepton sector [12], for instance in a minimally extended SM where the neutrino masses are determined by a dimension-5 operator.

There have been many attempts to explain the hierarchies in the quark and lepton Yukawa sector by enforcing certain flavour symmetries, often in the framework of (more fundamental) extensions of the SM. The simplest example has been proposed by Froggatt and Nielsen [13], where different $U(1)$ charges are assigned to the different fermion generations, and the hierarchies of masses and mixing angles are generated by higher-dimensional operators involving different powers of a scalar field charged under the new
Refined extensions of this idea have been discussed by many authors, in particular in the context of supersymmetric and/or grand unified scenarios \[14-21\].

In this paper, we pursue the effective-theory construction suggested by the MFV framework, where the scalar sector consists of the SM Higgs field together with the Yukawa spurions. We then focus on the dynamical interpretation of the Goldstone modes related to the broken flavour symmetries. More specifically, we consider two alternative cases: local flavour symmetries, where as usual the Goldstone modes become the longitudinal modes of the massive gauge bosons associated with the broken symmetry generators, and “invisible-axion” scenarios, where the Goldstone modes are (almost) massless fields with ultra-weak couplings to the SM fermions.

The main purpose of the subsequent analysis is to understand whether such a scenario can be consistently defined, and to identify the potential new sources for flavour-violating processes at low energies. Among others, this requires to address the following issues: (i) The hierarchies of quark Yukawa couplings and mixings imply a sequence of flavour-symmetry breaking \[11\] which will be reflected in the mass spectrum of the associated massive gauge bosons. Integrating out the heavy gauge bosons leads to effective four-quark operators at tree-level, which potentially induce new flavour-changing currents. The strongest effect is expected from the lightest of the heavy gauge bosons which correspond to the latest stages in the sequence of flavour-symmetry breaking. To illustrate the type of flavour-violating effects, it will therefore be sufficient to perform the analysis for the smallest non-trivial flavour symmetry sub-groups identified in \[11\]. (ii) Restricting ourselves to the SM field content for fermion matter fields, we naturally encounter gauge anomalies \[23\], when promoting the global flavour symmetries to local ones. As is well-known, local gauge invariance can be formally restored by adding appropriate higher-dimensional operators, following, for instance, the general framework worked out by Preskill in \[24\]. The higher-dimensional operators can be considered as the result of integrating out new heavy fermion modes which receive masses of the order of the cut-off scale in the corresponding effective theory with (partly) broken local flavour symmetry. (iii) In addition to the Goldstone modes, we have to take into account the fluctuations around the VEVs for the spurion fields, representing physical scalar Higgs fields of the broken flavour symmetry. Again, these fields are expected to develop a hierarchical mass spectrum following the sequence of scales identified in \[11\], and corresponding flavour-changing 4-quark operators can be induced at tree level. A technical complication arises due to the mixing of Goldstone modes in the non-Abelian theory and the spurion fields related to the mixing angles between different families. In order to identify the physical couplings between the flavoured Higgs fields and the quark currents, we thus have to find the correct prescription for the unitary-gauge condition. (iv) A special role is played by the Abelian \(U(1)\) factors in the SM flavour group (see below), which can be considered as Peccei-Quinn (PQ) symmetries \[31\]. As a benefit of our framework, we can leave the \(U(1)\) factors as global symmetries, such that the corresponding Goldstone mode(s)

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1 An alternative picture, where the concept of MFV is applied to discrete sub-groups of the flavour symmetry, has been discussed in \[22\]. In such a context, Goldstone modes would not appear when the discrete flavour symmetry is spontaneously broken.
become dynamical axion fields, realizing a variant of the standard PQ solution to the strong CP problem.

2 Local Flavour Symmetries

The global flavour symmetry group in the SM quark sector (consisting of unitary transformations that leave the gauge sector invariant, but are broken by the Yukawa couplings) is given by\(^2\)

\[ G_F = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \times U(1)_{U_R} \times U(1)_{D_R}, \quad (1) \]

where the subscripts refer to the left- and right-handed gauge multiplets in the SM. In the course of spontaneous flavour symmetry breaking each scalar spurion contained in the Yukawa matrices \(Y_U\) and \(Y_D\) acquires a VEV. Without loss of generality we may choose to work in the basis, where the VEV of the up-type Yukawa matrix is diagonal.

\[ \langle Y_U \rangle = \text{diag}(y_u e^{-i\pi u}, y_c, y_t), \quad \langle Y_D \rangle = V_{\text{CKM}} \text{diag}(y_d e^{-i\pi d}, y_s, y_b). \quad (2) \]

The CKM matrix can be written as

\[ V_{\text{CKM}} = \exp[2i\theta_{23} T^7] \exp[2i\theta_{13} T^5] \exp[i\delta T^3] \exp[2i\theta_{12} T^2] \exp[-i\delta T^3], \quad (3) \]

which corresponds to the standard parametrization up to a re-phasing, such that the CP-phase \(\delta\) appears in the mixing between the first and second generation.

2.1 Massive Gauge Bosons

In the following, we will consider the specific scenario where the three \(SU(3)\) factors will be promoted to local symmetries, while keeping the two \(U(1)\) factors in \(G_F\) as global symmetries. The Goldstone modes related to the latter have anomalous couplings with the SM gauge fields, and we expect that one linear combination of the associated Goldstone bosons will contribute to the effective \(\theta\)-parameter in QCD. It can thus be identified as an axion providing a potential solution to the strong CP-problem, with a finite mass generated by anomalous couplings to QCD instantons. On the other hand, upon spontaneous symmetry breaking, the 24 gauge bosons of \(SU(3)^3\) will become massive, and (in unitary gauge) the Goldstone modes of the broken flavour symmetry shall be identified with the longitudinal modes of massive gauge-boson. These masses are generated, as usual, from the gauge-kinetic term for the spurion fields,

\[ \Lambda^2 \text{tr} \left[ (D_\mu Y_U^\dagger)(D^\mu Y_U) \right] + \Lambda^2 \text{tr} \left[ (D_\mu Y_D^\dagger)(D^\mu Y_D) \right], \quad (4) \]

\(^2\)Notice that one combination of \(U(1)\) phases corresponds to baryon number that is not broken by the Yukawa matrices and thus should not be considered part of the flavour symmetry. Furthermore, the phase related to weak hypercharge transformations should be related to unitary transformations of the Higgs field and therefore does not appear in \(G_F\), see [11, 25].
which we have normalized to a UV scale $\Lambda$. This yields a $24 \times 24$ mass matrix for the gauge fields $A_{Q,L}^{\mu a}$, $A_{U,R}^{\mu a}$, $A_{D,R}^{\mu a}$, when $Y_{U,D}$ are replaced by their VEVs. To diagonalize this matrix, it is convenient to take advantage of the sequence of symmetry breakings. For concreteness, let us assume a particular realization for a sequence of flavour symmetry breakings \[11\],

$$y_t > y_b > y_c > y_b \theta_{23} > y_b \theta_{13} > y_s > y_s \theta_{12} > y_{u,d},$$

as a reference case. The first gauge bosons to achieve a mass from spontaneous symmetry breaking are thus the ones related to the 9 broken generators associated to the breaking of $SU(3)^3 \to SU(3) \times SU(2)^2 \times U(1)$ through a non-vanishing value for the top Yukawa coupling $y_t$. In this case, the diagonalization of the mass matrix is trivial and yields:

$$W_{Q_L}^{(13,31)}, W_{Q_L}^{(23,32)} : M^2 = \frac{1}{2} g_{Q_L}^2 y_t^2 \Lambda^2,$$

$$W_{U_R}^{(13,31)}, W_{U_R}^{(23,32)} : M^2 = \frac{\cot^2 \phi_1}{2} g_{Q_L}^2 y_t^2 \Lambda^2,$$

$$Z_1 = - \sin \phi_1 A^8_{Q_L} + \cos \phi_1 A^8_{U_R} : M^2 = \frac{2}{3 \sin^2 \phi_1} g_{Q_L}^2 y_t^2 \Lambda^2,$$

$$A_1 = \cos \phi_1 A^8_{Q_L} + \sin \phi_1 A^8_{U_R} : M^2 = 0,$$

(5)

with $\tan \phi_1 = g_{Q_L}/g_{U_R}$, and all other eigenvalues being zero.

The mixing of the heaviest gauge bosons (with $M \propto gy_t \Lambda$) and the remaining gauge bosons only occurs from the next symmetry-breaking steps and is thus suppressed by at least $y_b/y_t \ll 1$. In order to discuss the effect of the next breaking, $SU(3) \times SU(2)^2 \times U(1) \to SU(2)^3 \times U(1)$, — to first approximation — we can therefore simply neglect the presence of the heaviest gauge bosons and diagonalize the mass matrix for the 5 next heaviest gauge bosons, yielding

$$W_{D_R}^{(13,31)}, W_{D_R}^{(23,32)} : M^2 = \frac{\cot^2 \phi_2}{2} (g_{Q_L} \cos \phi_1)^2 y_b^2 \Lambda^2,$$

$$Z_2 = - \sin \phi_2 A_1 + \cos \phi_2 A^8_{D_R} : M^2 = \frac{2}{3 \sin^2 \phi_2} (g_{Q_L} \cos \phi_1)^2 y_b^2 \Lambda^2,$$

$$A_2 = \cos \phi_2 A_1 + \sin \phi_2 A^8_{D_R} : M^2 = 0,$$

(6)

with $\tan \phi_2 = g_{Q_L} \cos \phi_1/g_{D_R}$.

This scheme can be continued until the complete flavour symmetry is broken, and the hierarchies of Yukawa couplings and mixing angles translates into a hierarchy of gauge boson masses:

$$y_t \gg y_b \gg y_c \gg y_b \theta_{23} \gg y_b \theta_{13} \gg y_s \gg y_s \theta_{12} \gg y_{u,d},$$

$$M : g\Lambda \gg g\Lambda' \gg g\Lambda'' \gg g\Lambda''' \gg g\Lambda^{(iv)} \gg g\Lambda^{(v)} \gg g\Lambda^{(vi)} \gg g\Lambda^{(vii)},$$

(7)

with $\Lambda' / \Lambda = y_b / y_t$ etc. \[11\].

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In addition to the spontaneous symmetry breaking from the spurion VEVs, the presence of gauge anomalies has to be taken care of by adding appropriate higher-dimensional operators to formally restore invariance under local symmetry transformations. This can be achieved by applying the general formalism developed in [24]: At every step in the sequence of effective theories, we can imagine to add appropriate fermion representations under the flavour symmetry group to cancel the gauge anomalies of $SU(3)^3$ or its sub-groups. In the simplest case, we could just introduce chiral partners for the SM fermions, 

$$Q_L \leftrightarrow (\tilde{U}_R, \tilde{D}_R), \quad U_R \leftrightarrow \tilde{U}_L, \quad D_R \leftrightarrow \tilde{D}_L,$$

in the same flavour representations, but with vanishing $SU(2)_L$ quantum numbers, and hypercharges taken as $y_{\tilde{U}_R} = y_{\tilde{U}_L} = y_{U_R}, \ y_{\tilde{D}_R} = y_{\tilde{D}_L} = y_{D_R}$ which also satisfy $y_{\tilde{U}_R} + y_{\tilde{D}_R} = 2y_{Q_L}$. Concerning the colour group, the new quarks can be taken as 3-fold degenerate colour singlets, such that the mixed anomalies between $U(1)_{U_R+D_R}$ and the QCD generators remains as in the SM. These fermions get their masses directly from the Yukawa spurion VEVs via couplings like $\bar{\tilde{U}}_R Y U \tilde{U}_L$ and $\bar{\tilde{D}}_R Y D \tilde{D}_L$. For instance, when $Y_U$ develops a VEV related to the top-Yukawa coupling, the corresponding heavy fermion $\tilde{T}$ gets a mass of the order of the UV scale $\Lambda$ in the effective theory. Integrating out the auxiliary fermion fields, on the one hand, leaves the remaining fermion representations anomalous under the flavour symmetry group, but at the same time generates the higher-dimensional operators compensating for the change in the path-integral under local chiral rotations. Insertion of these higher-dimensional operators into loop diagrams in general induces additional contributions to the gauge-boson masses proportional to the respective anomaly coefficients.

2.2 Simplified Set-Up

In the following, we will assume that the gauge-boson masses are dominated by spontaneous symmetry breaking. The most important effects for flavour phenomenology will thus come from the lightest of these gauge bosons, which then correspond to the smallest non-trivial sub-group of $SU(3)^3$ which, in the above example, is given by $[11]$

$$G_F^{(4)} = SU(2)^D_R \times U(1)_X$$

Here, in contrast to [11], $D_R = (d_R, s_R)$ is restricted to a right-handed flavour doublet of down-type quarks. In terms of the symmetry generators of the original flavour symmetry

\[3\text{Notice that, necessarily, the new fermions have to be singlets under the chiral } SU(2)_L \text{ group, because they have to receive masses before electroweak symmetry breaking. As a consequence, also the mixed anomaly between baryon number and } SU(2)_L \text{ remains as in the SM. In order to gauge baryon number, one could, for instance, add a fourth generation with different gauge quantum numbers along the lines of [26].} \]
in (1), the local $U(1)_X$ charge is given by

$$Q_X = \frac{1}{\sqrt{3}} (T^8_{Q_L} + T^8_{U_R} + T^8_{D_R}) + (T^3_{Q_L} + T^3_{U_R} + T^3_{D_R}).$$

(10)

This linear combination leaves the VEVs of the Yukawa matrices

$$\langle Y_U \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \bullet & 0 \\ 0 & 0 & \bullet \end{pmatrix}, \quad \langle Y_D \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \bullet \\ 0 & \bullet & 0 \end{pmatrix} = \exp \left[ -i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \bullet \\ 0 & \bullet & 0 \end{pmatrix} \right] \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \bullet \\ 0 & \bullet & 0 \end{pmatrix},$$

invariant, where the bullet denotes a non-zero entry. The above Yukawa structures correspond to the situation where 3 eigenvalues ($y_t, y_b, y_c$) and the mixing angle between the 2nd and 3rd generation are non-zero.

The local $U(1)_X$ symmetry in (10) has an anomaly that will be treated by the formalism of [24], see Appendix A. The local $SU(2)_{D_R}$ symmetry has no anomalies, and the corresponding gauge theory can be written down in a straightforward manner. The symmetry $G_F^{(4)}$ will be further broken down to

$$G_F^{(4)} \longrightarrow G_F^{(5)} = SU(2)_{D_R} \longrightarrow \text{nothing},$$

and the corresponding spurions and massive gauge fields have to be integrated out, inducing new flavour structures for higher-dimensional quark operators. After this, we are left with only global (anomalous) Abelian symmetries. Their breaking involves physical Goldstone modes which, in our case, will provide a solution to the strong CP problem similar to the usual axion scenario.

## 3 Spurion Fields and Unitary Gauge

From the $4n^2$ real fluctuations of the two complex Yukawa matrices (for $n = 3$ quark generations) we shall identify $3(n^2 - 1)$ as Goldstone bosons $\phi_{Q_L,U_R,D_R}(x)$ of $SU(n)^3$, together with $(n^2 + 1)$ fluctuations $\eta_i(x)$ of the physical masses and mixing parameters, and 2 explicit phases for the global $U(1)_{U_R} \times U(1)_{D_R}$ symmetry.

The first non-trivial task is to identify the unitary (i.e. physical) gauge, where the massive $SU(3)^3$ gauge bosons do not mix with the fluctuations $\eta_i(x)$. For this purpose we introduce the parametrization

$$Y_U(x) = \Sigma_{Q_L}(x) \cdot \Xi_{U_L}(x) D_U(x) \Xi_{U_R}^\dagger(x) \cdot \Sigma_{U_R}(x),$$

(11)

$$Y_D(x) = \Sigma_{Q_L}(x) \cdot V_{\text{CKM}} \cdot \Xi_{D_L}(x) D_D(x) \Xi_{D_R}^\dagger(x) \cdot \Sigma_{D_R}(x).$$

(12)

4In contrast to [11], we are not allowed to consider linear combinations with the remaining global symmetry generators (including baryon number).

5 Notice the different treatment of up- and down-type quarks in the definition of the charge operator $Q_X$, which is correlated with the particular choice for the parametrization of the Yukawa matrices, where $\langle Y_U \rangle$ is diagonal, while $\langle Y_D \rangle$ contains the CKM rotations.
Here,
\[
D_U(x) = \text{diag} \left( y_u(x) e^{-i\tau_u(x)}, \ y_c(x), \ y_t(x) \right),
\]
\[
D_D(x) = \text{diag} \left( y_d(x) e^{-i\tau_d(x)}, \ y_s(x), \ y_b(x) \right)
\]  
(13)
are diagonal matrices, with \( y_i(x) = y_i + \eta_i(x)/\sqrt{2} \), containing the fluctuations around the VEVs for the quark Yukawa couplings, and
\[
\Sigma_X(x) = \exp \left[ i \phi_X^a(x) T^a \right], \quad (X = Q_L, U_R, D_R)
\]  
(14)
represent the Goldstone bosons of \( SU(3)^3 \), while
\[
\Xi_X(x) = \exp \left[ i \xi_X^a(x) T^a \right], \quad (X = U_L, D_L, U_R, D_R; \ a \neq 3, 8)
\]  
(15)
parametrize the scalar fluctuations around the VEVs for CKM angles and phases. Notice that our ansatz contains more parameters than scalar degrees of freedom. The apparent ambiguity is resolved by requiring that in
\[
\text{unitary gauge, i.e. } \Sigma_X(x) \to 1,
\]  
(16)
our ansatz does not generate mixing terms with the \( SU(3)^3 \) gauge bosons, when inserted into the gauge-invariant kinetic term \( \text{[14]} \), where the covariant derivatives read
\[
D_\mu Y_U(x) = \partial_\mu Y_U(x) - ig_{Q_L} A_{Q_L,\mu}^a(x) T^a Y_U(x) + ig_{U_R} A_{U_R,\mu}^a(x) Y_U(x) T^a,
\]
\[
D_\mu Y_D(x) = \partial_\mu Y_D(x) - ig_{Q_L} A_{Q_L,\mu}^a(x) T^a Y_D(x) + ig_{D_R} A_{D_R,\mu}^a(x) Y_D(x) T^a,
\]  
(17)
and \( A_{X,\mu}^a(x) \) are the gauge fields for \( SU(3)^3 \). Here \( \Lambda \) is the high-energy scale related to the ultra-violet (UV) cut-off of the effective theory. Inserting the parametrization \( \text{[12]} \) and focusing on the mixing terms between the scalar fluctuations \( \xi_i(x) \) and the gauge fields, we obtain the set of conditions
\[
A_{U_R}^\mu : \quad \text{tr} \left[ T^a \left( \Xi_{U_R} D_U^\dagger \right) \left( i \partial_\mu \Xi_{U_R} \right) \Xi_{U_R} D_U \Xi_{U_R} + (i \partial_\mu \Xi_{U_R}) \langle |D_U|^2 \rangle \Xi_{U_R} + \text{h.c.} \right] = 0,
\]  
(18)
\[
A_{D_R}^\mu : \quad \text{tr} \left[ T^a \left( \Xi_{D_R} D_D^\dagger \right) \left( i \partial_\mu \Xi_{D_R} \right) \Xi_{D_R} D_D \Xi_{D_R} + (i \partial_\mu \Xi_{D_R}) \langle |D_D|^2 \rangle \Xi_{D_R} + \text{h.c.} \right] = 0,
\]  
(19)
and
\[
0 = \text{tr} \left[ T^a \left( \Xi_{U_L} \langle |D_U|^2 \rangle \right) \left( i \partial_\mu \Xi_{U_L} \right) + \Xi_{U_L} D_U \Xi_{U_R} \langle |D_U|^2 \rangle \Xi_{U_R} + \text{h.c.} \right]
\]  
\[+ V_{\text{CKM}}^\dagger T^a V_{\text{CKM}} \left( \Xi_{D_L} \langle |D_D|^2 \rangle \left( i \partial_\mu \Xi_{D_L} \right) + \Xi_{D_L} D_D \Xi_{D_R} \langle |D_D|^2 \rangle \Xi_{D_R} + \text{h.c.} \right) \right]
\]  
(20)
for \( A_{Q_L}^\mu \). (The fluctuations around the quark Yukawa couplings in \( D_U \) and \( D_D \) do not mix with the \( SU(3)^3 \) gauge bosons and can be dropped for this part of the analysis.)
3.1 Approximation for the 3-Family Case

In the 2-family case, the above equations can be solved explicitly, see Appendix B. In the 3-family case, the situation is somewhat more complicated because the generators for 3 CKM rotations do not commute anymore, and the kinetic terms for the related spurion fields $\eta_{12}(x)$, $\eta_{13}(x)$ and $\eta_{23}(x)$ will also mix, with the mixing controlled by the CKM matrix and the ratios of quark Yukawa couplings. We can identify the leading effects by expanding to first order in the off-diagonal CKM elements $V_{ij}$, and assuming $y_s \ll y_b$ and $y_c \ll y_t$. For simplicity, we will also set $y_u = y_d = 0$ and neglect the CP-phase in the CKM matrix. With these approximations, we obtain

$$Y_{\text{up}}^\text{D}(x) \simeq \text{diag}[0, y_c, y_t] + \left( \begin{array}{ccc} 0 & \frac{1}{y_c} & \frac{\theta_{13} y_{23} y_t}{y_b y_c - y_c y_t} \\ 0 & 0 & \frac{\theta_{13} y_{23} y_t}{y_b y_c - y_c y_t} \\ 0 & 0 & 0 \end{array} \right) \frac{1}{2} F_{12}^2 \eta_{12}(x) + \left( \begin{array}{ccc} 0 & -\frac{1}{y_s} & \frac{\theta_{23} y_{13} y_{23} y_s}{y_b y_c - y_c y_t} \\ 0 & 0 & -\frac{\theta_{13} y_{23} y_t}{y_b y_c - y_c y_t} \\ 0 & 0 & 0 \end{array} \right) \frac{1}{2} F_{12}^2 \eta_{12}(x)$$

and

$$V_{\text{CKM}}^\dagger Y_{\text{up}}^\text{D}(x) \simeq \text{diag}[0, y_s, y_b] + \left( \begin{array}{ccc} 0 & \frac{1}{y_s} & \frac{\theta_{13} y_{23} y_s}{y_b y_c - y_c y_t} \\ 0 & 0 & \frac{\theta_{13} y_{23} y_s}{y_b y_c - y_c y_t} \\ 0 & 0 & 0 \end{array} \right) \frac{1}{2} F_{12}^2 \eta_{12}(x) + \left( \begin{array}{ccc} 0 & -\frac{1}{y_b} & \frac{\theta_{23} y_{13} y_{23} y_b}{y_b y_c - y_c y_t} \\ 0 & 0 & \frac{\theta_{13} y_{23} y_t}{y_b y_c - y_c y_t} \\ 0 & 0 & 0 \end{array} \right) \frac{1}{2} F_{23}^2 \eta_{23}(x) .$$

The normalization factors $F_{ij}$ are defined as in Appendix B

$$F_{ij}^2 = \frac{2(y_{U_i}^2 - y_{U_j}^2)^2(y_{D_i}^2 - y_{D_j}^2)^2}{(y_{U_i}^2 - y_{U_j}^2)^2(y_{D_i}^2 + y_{D_j}^2) + (y_{D_i}^2 - y_{D_j}^2)^2(y_{U_i}^2 + y_{U_j}^2)}.$$

We have brought the result into a form where up- and down-quarks couplings are treated symmetrically, and the fluctuations $\delta Y_U$ around the VEV satisfy

$$\text{tr} \left[ (Y_U^\dagger) \delta Y_U \right] = \text{tr} \left[ Y_U^\dagger Y_U \delta Y_U \right] = \text{tr} \left[ (Y_U^\dagger Y_U Y_U^\dagger Y_U Y_U^\dagger) \delta Y_U \right] = 0 .$$
and analogously for $\delta Y_D$. Furthermore, the contribution to the invariants $\text{tr}[(Y_U^\dagger Y_U)^n]$, appearing in the spurion potential \[11\] are diagonal in the spurion fields $\tilde{\eta}_{ij}(x)$.

In addition to the sub-leading terms in the kinetic mixing terms, further corrections would be induced by radiative corrections involving the Yukawa couplings. By construction, these effects will follow the MFV principle, in a similar way as we have discussed for the 2-family example in Appendix [B]. A precise calculation of these terms is beyond the scope of this work.

4 Flavour Transitions from Local $SU(2)_{D_R} \times U(1)_X$

4.1 Effective Lagrangian

The Lagrangian for our effective theory with a local flavour symmetry $G_F^{(4)} = SU(2)_{D_R} \times U(1)_X$ is constructed from the SM matter and gauge fields, together with scalar spurions and gauge fields for $G_F^{(4)}$. The various left-handed fermions build irreducible representations of $G_F^{(4)} \times \text{SM}$,

\[
\psi_L : \{Q_L^{(1)}, Q_L^{(2)}, Q_L^{(3)}, u_R^c, c_R^c, t_R^c, D_R^c, b_R^c\}
\]

\[
q_X : \{+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}, -\frac{2}{3}, +\frac{1}{3}, +\frac{1}{3}, -\frac{1}{6}, +\frac{1}{3}\},
\]

where in the last line we denoted the respective charges under $U(1)_X$. The SM Lagrangian takes its standard form, except for the Yukawa sector, where we have to replace the Yukawa matrices by dynamical fields. Starting from the general discussion in Sec. [3] we can drop all Goldstone modes and spurion fields related to the breaking $G_F \to G_F^{(4)}$, and obtain

\[
Y_U \to Y_U(x) = e^{i\pi_X(x)(T^3 + T_8/\sqrt{3})} \cdot \tilde{Y}_{u,g}^U(x) \cdot e^{-i\pi_X(x)(T^3 + T_8/\sqrt{3})},
\]

\[
Y_D \to Y_D(x) = e^{i\pi_X(x)(T^3 + T_8/\sqrt{3})} \cdot \tilde{Y}_{u,g}^D(x) \cdot e^{-i \sum_{a=1}^3 \pi_{D_R}^a(x) T^a + i\pi_X(x)T_8/\sqrt{3}}
\]

where $\tilde{Y}_{u,g}^{U,D}(x)$ are given by Eqs. \[21\, 22\] with $\tilde{\eta}_{23}(x)$ set to zero. The Lagrangian is supplemented by a spurion kinetic and potential term

\[
\mathcal{L}_{\text{spurion}} = \Lambda^2 \text{tr} \left[(D^\mu Y_U^\dagger)(D_\mu Y_U)\right] + \Lambda^2 \text{tr} \left[(D^\mu Y_D^\dagger)(D_\mu Y_D)\right] - V(Y_U, Y_D)
\]

with

\[
D_\mu Y_U(x) = \partial_\mu Y_U(x) - ig_X X_\mu(x) \left[T^3 + T_8/\sqrt{3}, Y_U(x)\right],
\]

\[
D_\mu Y_D(x) = \partial_\mu Y_D(x) + ig_{D_R} \sum_{a=1}^3 A_{D_R,\mu}^a(x) Y_D(x) T^a_{D_R} - ig_X X_\mu(x) \left(T^3 + T_8/\sqrt{3}, Y_D(x) + ig_X X_\mu(x) Y_D(x) T_8/\sqrt{3},
\]

\[
(28)
\]
and a $G_F^{(4)}$-invariant potential $V(Y_U, Y_D)$. Inserting the above representation for $Y_U$ and $Y_D$, expanding in the gauge and spurion fields, and using again the approximations in terms of small Yukawa couplings and CKM angles, we identify the kinetic terms for the spurion fields, as well as the mass terms for the $SU(2)_D \times U(1)_X$ gauge bosons induced by the VEVs for $\theta_{13}$, $\theta_{12}$ and $y_s$:

\[
L_{\text{kin}} = \Lambda^2 \left( \partial_\mu y_s(x)^2 \right)^2 + \frac{\Lambda^2}{2} F_{12}^2 \left( \partial_\mu \tilde{\eta}_{12}(x) \right)^2 + \frac{\Lambda^2}{2} F_{13}^2 \left( \partial_\mu \tilde{\eta}_{13}(x) \right)^2 ,
\]

\[
L_{\text{mass}} \simeq \Lambda^2 y_b^2 \theta_{13}^2 \left( \partial_\mu \pi_X - g X_\mu \right)^2 + \frac{\Lambda^2}{4} \left( y_s(x) \right)^2 \left[ 2 A^+_\mu A^\mu + \left( \partial_\mu \pi_X - g X_\mu - A^2_\mu \right)^2 \right].
\]

Here we have introduced the gauge-invariant combinations

\[
A^a_\mu = -2 \text{tr} \left[ e^{-i\pi D_R} \left( i \partial_\mu + g_{D_R} A_{D_R,\mu} \right) e^{i\pi D_R} T^a \right] \simeq \partial_\mu \pi^a_{D_R} - g_{D_R} A^a_{D_R,\mu}
\]

and

\[
A^\pm_\mu = \frac{1}{\sqrt{2}} \left( A^1_\mu \pm i A^2_\mu \right).
\]

### 4.2 Integrating out the Heavy Spurion Field $\eta_{13}$

In the standard scenario for the sequence of flavour symmetry breaking discussed in [11], the next spurion to get a VEV is $\eta_{13}(x) \equiv \Lambda F_{13} \tilde{\eta}_{13}(x)$ which is related to fluctuations around the CKM angle $\theta_{13} \sim \lambda^3$. We assume that the spurion potential will generate a mass term for $\eta_{13}$ with a generic size of order

\[
m_{13}^2 \sim y_b^2 \Lambda^2 \theta_{13}^2 .
\]

In the following, we will also assume that the spurion contribution to $F_X$ in \(52\) is dominating over the anomaly contribution \(50\), such that

\[
\Lambda \gg m_{13} \sim F_X > M_X .
\]

We are now going to integrate out the field $\eta_{13}(x)$, in order to construct the theory below energy scales of order $F_X$. This in general will induce higher-dimensional operators for flavour transitions. The leading (tree-level) effects can be obtained by solving the equations of motion for $\eta_{13}$, using the approximate form of the couplings to fermions in \(21\), leading to

\[
L_{13} = \frac{1}{2} \left( \partial^\mu \eta_{13} \right)^2 - \frac{1}{2} m_{13}^2 \eta_{13}^2 - \left( J^{13}_U + J^{13}_D + J^{13}_D + J^{13}_D \right) \eta_{13} ,
\]

\footnote{Otherwise we would have to integrate out the gauge boson $X^\mu$ first.}
with

\[ J_{U}^{13} \simeq \frac{F_{13}}{2} (\bar{U}_{L} ', \bar{D}_{L} ' V_{\text{CKM}}^{\dagger}) \frac{H}{\Lambda} \begin{pmatrix} 0 & \frac{\theta_{23} y_{U} y_{C}}{y_{U} y_{C} - y_{D} y_{T}} & \frac{1}{y_{U}} \\ 0 & 0 & \frac{\theta_{12} y_{U} y_{T}}{y_{U} y_{C} - y_{D} y_{T}} \\ 0 & 0 & 0 \end{pmatrix} U_{R}, \]

\[ J_{D}^{13} \simeq \frac{F_{13}}{2} (\bar{U}_{L}' V_{\text{CKM}}, \bar{D}_{L}') \frac{H}{\Lambda} \begin{pmatrix} 0 & -\frac{\theta_{23} y_{D} y_{T}}{y_{D} y_{C} - y_{U} y_{T}} & -\frac{1}{y_{D}} \\ 0 & 0 & -\frac{\theta_{12} y_{D} y_{C}}{y_{D} y_{C} - y_{U} y_{T}} \\ 0 & 0 & 0 \end{pmatrix} D_{R}, \quad \text{(36)} \]

where the primed fields denote the mass eigenstates of the quarks. In the limit \( m_{13} \to \infty \), this leads to an effective 4-fermion interaction

\[ \frac{1}{2 m_{13}^{2}} \left( J_{U}^{13} + J_{U}^{13} + J_{D}^{13} + J_{D}^{13} \right)^{2}. \quad \text{(37)} \]

Replacing the SM Higgs field \( H \) by its VEV in (36), the effective 4-quark interaction receives an overall suppression factor \( v^{2}/\Lambda^{2} \). The individual coefficients for specific flavour transitions follow from the matrix structure in \( J_{U}^{13} \) and \( J_{D}^{13} \). We observe that the \( \eta_{13} \) spurion dominantly induces transitions between left-handed quarks from the first generation and right-handed quarks from the second or third generation. Of course, more operators – which may also include additional gauge fields – will be generated by radiative corrections and higher-dimensional operators from \( L_{\text{spurion}} \) in (27).

### 4.3 Integrating out the Heavy \( U(1)_{X} \) Gauge Field

Below the scale \( M_{X} = g_{X} F_{X} \), we may integrate out the heavy gauge boson of the \( U(1)_{X} \) flavour symmetry and end up with an effective theory which has an \([SU(2)_{D_{R}}] \times U(1)_{u_{R}} \times U(1)_{D_{R}^{(2)}}\) flavour symmetry. Focusing on the leading terms in (30), and considering unitary gauge \( (\pi_{X} = 0) \), we may again solve the classical e.o.m. following from

\[ L_{X} \simeq L_{\text{mass}} + g_{X} X_{\mu} J_{X}^{\mu} , \quad J_{X}^{\mu} \equiv [\bar{\psi}_{L} \gamma^{\mu} Q_{X} \psi_{L}] , \quad \text{(38)} \]

in the limit \( F_{X} \to \infty \), where \( L_{\text{mass}} \) is defined in (30) and \( F_{X} \) in (82). Again, this induces effective 4-quark operators of the form

\[ -\frac{1}{2 F_{X}^{2}} [\bar{\psi}_{L} \gamma_{\mu} Q_{X} \psi_{L}] [\bar{\psi}_{L} \gamma^{\nu} Q_{X} \psi_{L}] , \quad \text{(39)} \]

where \( \psi_{L} \) denotes the set of left-handed fermion fields in (25) with the corresponding \( U(1)_{X} \) charges. The \( U(1)_{X} \) charge operator is diagonal with respect to the fermion representations. As the up-type quarks Yukawa matrix is already diagonal, the above
operator does not induce FCNCs between up-type quarks. On the other hand, rotating the down-type quarks into the mass eigenbasis, one obtains

$$J'_{X_{down}} = \left( d'_L, s'_L, b'_L \right) \gamma^\mu X'_{DL} \left( \begin{array}{c} d'_L \\ s'_L \\ b'_L \end{array} \right),$$

$$X'_{DL} = V_{CKM}^\dagger \text{diag} \left[ \frac{2}{3}, -\frac{1}{3}, -\frac{1}{3} \right] V_{CKM} \simeq \left( \begin{array}{ccc} \frac{2}{3} & \theta_{12} & \theta_{13} \\ \theta_{12} & -\frac{1}{3} & 0 \\ \theta_{13} & 0 & -\frac{1}{3} \end{array} \right) + \mathcal{O}(\theta_{ij}^2),$$

containing FCNCs between $d_L$ and $s_L$ or $b_L$, which are suppressed by the SM CKM angles. The phenomenology induced by these sub-leading effects is qualitatively similar to $Z'$-models with non-universal flavour couplings [27], where interesting new flavour effects have been identified in the context of present puzzles in flavour observables (see e.g. [28–30] for recent applications). However, compared to the commonly favoured $Z'$ scenarios, our case displays important modifications:

- Typical $Z'$-scenarios are motivated by electroweak physics and consider $Z'$-masses in the TeV range. In this case, precision flavour observables in the kaon sector already disfavour non-universal flavour couplings for the first and second generation. In our case, the $U(1)_X$ gauge boson is naturally allowed to be much heavier. At the same time, the non-universal effects are precisely between the first and second (or third) generation, and therefore kaon observables essentially will provide a lower bound on the scale $F_X$.

- The $U(1)_X$ gauge boson does not couple to leptons, and thus constraints from lepton-flavour violating observables do not apply in our case.

Taking into account the sub-leading effects proportional to $y_s^2$ in (30), the mixing between the gauge boson $X_\mu$ with the $SU(2)_{DR}$ gauge field $A^3_\mu$ induces an additional effective operator, such that finally

$$\mathcal{L}_{\text{mass}} + g_X X_\mu J^\mu_X \rightarrow \frac{y_s^2 A^2}{4} \left[ 2 A^{\mu+}_A A^{\mu-}_A + (A^3_\mu)^2 \right] - \frac{1}{2 F_X^2} [J^\mu_X]^2 - \frac{y_s^2}{4 \theta_{13}^2 y_s^2} J^\mu_X A^3_\mu.$$  (41)

### 4.4 Local $SU(2)_{DR}$ Flavour Symmetry

We are now going to repeat our analysis for the effective theory below the scale $M_X$, where the residual flavour symmetry is given by

$$G'_F = [SU(2)_{DR}] \times U(1)_{uR} \times U(1)_{D_R(2)},$$

and the $SU(2)_{DR}$ symmetry is gauged. Since in this case, the $SU(2)_{DR}$ is not anomalous, the discussion is somewhat simpler than in the previous section. The effective Lagrangian can be obtained as before by combining kinetic and gauge fixing terms for the $SU(2)_{DR}$
gauge fields with the associated Goldstone bosons, the couplings of the gauge fields to
the fermions,

\[ g_{D^R} A_\mu^a (J_D^\mu)^a = g_{D^R} A_\mu^a \left[ \bar{D} R \gamma^\mu T^a D R \right] , \] (43)
as well as the kinetic terms and Yukawa couplings of the remaining spurion fields.

Again, we assume that via an appropriate spurion potential, the spurion field \( y_s(x) = y_s + \eta_s(x) / \sqrt{2} \) takes its VEV at a scale of the order of its mass \( m_{\eta_s} \sim y_s \Lambda \). Integrating out the spurion fluctuation \( \eta_s(x) \), the Yukawa coupling to the down-type quarks induces an effective 4-quark operator,

\[ \frac{1}{4m_{\eta_s}^2} J_s^2 , \quad J_s = \frac{1}{\Lambda} \left[ \left( \bar{u}'_L V_{us} + \bar{c}'_L V_{cs} , \bar{s}'_L \right) H s_R + h.c. \right] , \] (44)

where, again, we have transformed to the quark mass eigenbasis. As expected, the fluctuations \( \eta_s(x) \) around the Yukawa eigenvalue \( y_s \) do not induce flavour transitions, once the SM Higgs VEV is replaced by its VEV. On the other hand, the charged Goldstone modes in the SM Higgs field, induce flavour transitions suppressed by the corresponding CKM elements, in accordance with the principle of MFV.

Starting from (41) and (43), we may next integrate out the \( SU(2)_{D^R} \) gauge fields \( A_\mu^a \), which acquire a mass from (30),

\[ m_A^2 = g_{D^R}^2 F_{D^R}^2 \simeq \frac{y_s^2 \Lambda^2}{2} . \] (45)

Taking also into account the leading term from the mixing between \( X_\mu \) and \( A_\mu^3 \), we obtain new effective 4-quark operators

\[ -\frac{1}{2F_{D^R}^2} \left\{ \left[ (J_D^\mu)^a \right]^2 - \frac{y_s^2}{20 \theta_{12} y_b^2} \left[ (J_D^\mu)^3 J_X^\mu \right] \right\} . \] (46)

Upon Fierz-rearrangement, the term \( \left[ (J_D^\mu)^a \right]^2 \) only involves flavour-diagonal currents \( \left( \bar{d}_R \gamma_\mu d_R \right) \) and \( \left( \bar{s}_R \gamma_\mu s_R \right) \), with different colour structures. In the second term, flavour transitions appear as before, when the current \( J_X \) is written in the mass eigenbasis (40).

Finally, we may integrate out the field \( \eta_{12} \) which we assume to have a mass of generic order \(^7\)

\[ m_{\eta_{12}}^2 \sim y_s^2 \theta_{12}^2 \Lambda^2 . \]

Notice that the corresponding contributions to the \( SU(2)_{D^R} \) gauge boson masses are sub-leading, and have been neglected in the above analysis (they lead, however, to small mass splittings between \( A_\mu^\pm \) and \( A_\mu^3 \)). In complete analogy to the case of \( \eta_{13} \), see (36), we obtain effective 4-quark operators

\[ \frac{1}{2m_{\eta_{12}}^2} \left( J_U^{12} + \bar{J}_U^{12} + J_D^{12} + \bar{J}_D^{12} \right)^2 , \] (47)

\(^7\) The scalar sector at this stage still contains the fluctuations around the CP-phase \( \delta \) in the CKM matrix which is unrelated to any of the local or global flavour symmetries, see the discussion in [11]. In our derivation of [21,22], we have neglected the effects from \( \delta \) for simplicity.
with
\[
J_{U}^{12} \simeq \frac{F_{12}}{2} \left( \bar{U}'_L, \bar{D}'_L V_{\text{CKM}}^\dagger \right) \frac{\bar{H}}{\Lambda} \begin{pmatrix}
0 & \frac{1}{v^c} & 0 \\
0 & 0 & -\frac{\theta_{23} y^e_{L} y_t}{y^c_{L} y^c_{L} - y^e_{L} y^e_{L}} \\
0 & \frac{\theta_{13} y^u_{L} v^c_{\Lambda}}{y^u_{L} y^u_{L} - y^d_{L} y^d_{L}} & 0
\end{pmatrix} U_R,
\]
\[
J_{D}^{12} \simeq \frac{F_{12}}{2} \left( \bar{U}'_L V_{\text{CKM}}, \bar{D}'_L \right) \frac{\bar{H}}{\Lambda} \begin{pmatrix}
0 & -\frac{1}{v^s} & 0 \\
0 & 0 & \frac{\theta_{23} y^e_{L} y_t}{y^s_{L} y^s_{L} - y^e_{L} y^e_{L}} \\
0 & -\frac{\theta_{13} y^u_{L} y^2_{\Lambda}}{y^u_{L} y^u_{L} - y^d_{L} y^d_{L}} & 0
\end{pmatrix} D_R,
\]
from \((21,22)\).

5 **Global \(U(1)_{u_R} \times U(1)_{d_R}\) Flavour Symmetry**

After integrating out the gauge bosons of the local flavour symmetry and the related spurion fluctuations below the scale \(y_s \Lambda\), the effective theory still possesses a global \(U(1)_{u_R} \times U(1)_{d_R}\) flavour symmetry which acts on the right-handed quarks of the first generation,
\[
u_R \rightarrow e^{i\theta_u} \nu_R, \quad d_R \rightarrow e^{i\theta_d} d_R.
\]
We are then left with two complex spurion fields \(Y^{(1)}_U\) and \(Y^{(1)}_D\) which break \(U(1)_{u_R} \times U(1)_{d_R}\) and give masses to the lightest quarks (\(m_u, m_d\)).

5.1 **Effective Lagrangian and the Strong CP Problem**

Let us again parameterize the spurion fields in terms of real fields representing magnitude and phase,
\[
Y^{(1)}_U(x) = y_u(x) \cdot e^{-i\pi_u(x)}, \quad Y^{(1)}_D(x) = y_d(x) \cdot e^{-i\pi_d(x)},
\]
where the Goldstone fields in the exponentials transform as
\[
\pi_u \rightarrow \pi_u + \theta_u, \quad \pi_d \rightarrow \pi_d + \theta_d.
\]
Because of the above shift symmetry of the Goldstone modes, the (classical) scalar potential only depends on \(y_u(x)\) and \(y_d(x)\),
\[
V_0 = V_0(y_u, y_d).
\]
However, the non-trivial topological properties of the QCD gauge configurations imply a more complicated QCD vacuum state beyond perturbation theory. The distinct QCD
vacua can be labeled by a parameter $\theta$, and the vacuum-to-vacuum transition amplitude in the $\theta$-vacuum is given by

$$\langle 0|0\rangle_\theta = \sum_{q=-\infty}^{\infty} \int (DA_\mu)_q \int D\phi \exp[i\,q\,\theta] \exp \left[ i \int d^4x \, L(A, \phi) \right],$$

where $\phi$ denotes generic matter fields, and for a given topological sector $q$, the functional integration is restricted to QCD gauge-potential configurations $(DA_\mu)_q$ which satisfy

$$q = \frac{g_s^2}{32\pi^2} \int d^4x \, G^a_{\mu\nu} \tilde{G}^{a\mu\nu}.$$  

The $\theta$-term can thus be considered as being part of an effective term in the Lagrangian

$$\mathcal{L} \rightarrow \mathcal{L}_{\text{eff}} = \mathcal{L} + \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}.$$  

Compared to the SM case, the additional chiral $U(1)_{u_R} \times U(1)_{d_R}$ flavour symmetry now allows for a solution of the strong CP problem via a Peccei/Quinn mechanism. Here we recall that the effective action (in the QCD gauge sector) changes under chiral rotations as

$$\Gamma \rightarrow \Gamma + q (\theta_u + \theta_d),$$

which is equivalent to a change in the QCD $\theta$-parameter,

$$\theta \rightarrow \theta - \theta_u - \theta_d.$$  

For $\langle y_q \rangle > 0$ (by assumption), the fermion mass term gets its canonical form, after a chiral transformation of $u_R$ and $d_R$ with the corresponding phases set by $\langle \pi_u(x) \rangle$ and $\langle \pi_d(x) \rangle$, respectively. To avoid the strong CP problem, one thus has to require that

$$\langle \theta_{\text{eff}} \rangle \equiv \theta - \langle \pi_u(x) + \pi_d(x) \rangle \left( \frac{1}{\langle \pi_u(x) + \pi_d(x) \rangle} \right) \neq 0.$$  

This can be achieved by examining the effective potential in the presence of the QCD $\theta$-vacuum which — for small VEVs $\langle y_{u,d} \rangle \ll 1$ — takes the general form (see appendix and [31])

$$V_\theta = V_0 - K \, v^6 \, y_e y_t y_b \, y_u(x) \, y_d(x) \, \cos \left[ \pi_u(x) + \pi_d(x) - \theta \right] + \ldots$$

with a positive constant $K > 0$. The potential thus breaks the original shift symmetry for the Goldstone fields. Its minimum is given by $\langle \pi_u + \pi_d \rangle = \theta$, and therefore $\langle \theta_{\text{eff}} \rangle \equiv 0$, as required. Notice that the potential only depends on the combination

$$a(x) \equiv f_a \left( \pi_u(x) + \pi_d(x) \right),$$

where $f_a$ denotes the combination $f_a = f_{u_R} + f_{d_R}$.

---

8In the following, we concentrate on the strong-interaction gauge sector, and suppress the weak-interaction effects in the notation.

9The connection between flavour and PQ symmetries has been discussed before, see e.g. [31, 33].
which defines the PQ axion field, with \( f_a \) being a dimensional normalization constant. The corresponding PQ symmetry is defined such that the axion transforms as

\[
a(x) \rightarrow a(x) + f_a \theta_{\text{PQ}}.
\]

In order to determine the normalization \( f_a \) and to find the orthogonal linear combination of \( \pi_u(x) \) and \( \pi_d(x) \), which we denote as \( b(x) \), we consider the flavour invariant kinetic term and require

\[
\Lambda^2 \partial_\mu Y^{(1)}_U \partial^\mu Y^{(1)}_U \bigg|_{y_{u,d} \rightarrow \langle y_{u,d} \rangle} = \frac{1}{2} (\partial_\mu a(x))^2 + \frac{1}{2} (\partial_\mu b(x))^2.
\]

This yields

\[
f_a = \sqrt{2} \Lambda \left\langle \frac{y_d y_u}{\sqrt{y_u^2 + y_d^2}} \right\rangle, \quad b(x) = f_a \left( \left\langle \frac{y_u}{y_d} \right\rangle \pi_u(x) - \left\langle \frac{y_d}{y_u} \right\rangle \pi_d(x) \right).
\]

In terms of \( a(x) \) and \( b(x) \), the up- and down-quark Yukawa couplings read

\[
Y^{(1)}_U(x) = \exp \left[ -i \left\langle \frac{y_d^2}{y_u^2 + y_d^2} \right\rangle \frac{a(x)}{f_a} \right] \exp \left[ -i \left\langle \frac{y_u y_d}{y_u^2 + y_d^2} \right\rangle \frac{b(x)}{f_a} \right] y_u(x),
\]

\[
Y^{(1)}_D(x) = \exp \left[ -i \left\langle \frac{y_u^2}{y_u^2 + y_d^2} \right\rangle \frac{a(x)}{f_a} \right] \exp \left[ +i \left\langle \frac{y_u y_d}{y_u^2 + y_d^2} \right\rangle \frac{b(x)}{f_a} \right] y_d(x).
\]

We also define the corresponding linear combinations of \( U(1) \) charges,

\[
\theta_{\text{PQ}} = \theta_u + \theta_d, \quad \theta_{\text{diff}} = \left\langle \frac{y_u}{y_d} \right\rangle \theta_u - \left\langle \frac{y_d}{y_u} \right\rangle \theta_d,
\]

such that the orthogonal combination of Goldstone bosons transforms as

\[
b(x) \rightarrow b(x) + f_a \theta_{\text{diff}}.
\]

Note that \( b(x) \) remains massless, apart from anomalous contributions from the electroweak vacuum. We may or may not remove \( b(x) \) by gauging the remaining \( U(1)_{\text{diff}} \) symmetry and subsequently integrating out the corresponding massive gauge boson.

The axion field remains in the physical spectrum of the theory. However, compared to the original Peccei-Quinn axion, its couplings are now determined by the scale \( \Lambda \) of the Yukawa fields and not by the electroweak VEV. In particular, the scale \( \Lambda \) has to be chosen well above the electroweak scale, in which case the axion couplings become very small, since they scale as \( 1/f_a \). Thus the phenomenology of this model will be similar to the phenomenology of invisible axion models [37][38].
6 Conclusions

In this paper, we have discussed the scenario of a spontaneously broken flavour symmetry, realized in a set-up with Standard Model (SM) fermion representations, and standard Yukawa matrices which are promoted to dynamical scalar fields that are subject to an appropriately chosen scalar potential. In order to avoid massless Goldstone bosons in the physical spectrum, we consider local flavour symmetries, where the corresponding gauge bosons become massive by the usual Higgs mechanism. Our scenario necessarily has to be understood in the context of an effective theory approximation for a more fundamental theory. First, before the scalar fields are integrated out, the Yukawa interactions are now described by dimension-5 operators. Second, the flavour symmetries of the SM are anomalous, destroying the local gauge symmetry of the classical Lagrangian. Also, the mixed anomalies with the electroweak symmetry could be problematic. Following [24], we have pointed out that the local symmetry can be formally restored by adding appropriate higher-dimensional operators involving the Goldstone fields, which can be viewed as the remnant of integrating out very heavy fermion representations which cancel the anomalies in the fundamental theory. The effect of the anomalies can then be absorbed into the masses of the heavy gauge bosons of the broken flavour symmetry, and the mixed anomalies can be removed by choosing the counter terms appropriately.

The masses of the new heavy gauge bosons as well as of the new physical Higgs modes are hierarchically ordered, according to the sequence of hierarchies in the quark masses and mixing angles worked out in [11]. The masses of the lightest of these new states therefore have to be sufficiently large, such that the induced flavour-changing transitions are in line with the experimental constraints from precision measurements in the kaon and $B$-meson sector. We have particularly concentrated on 4-quark operators which appear after integrating out the heavy gauge bosons and scalar fields at tree-level. We found – not surprisingly – that our set-up shares the basic features of minimal flavour violation where all non-trivial flavour structures are induced by the SM Yukawa couplings and CKM elements. We have discussed examples where the flavour effects induced by integrating out the new heavy gauge bosons share some features of $Z'$-models with non-universal flavour couplings. On the other hand, the scalar fluctuations around the VEV of the CKM angles directly lead to flavour-changing neutral currents in the effective low-energy theory, which may be checked experimentally. We shall work out the phenomenological implications of this scenario for flavour observables accessible by future experiments at the LHC or at Super-B factories in a separate publication.

Finally, we have entertained the possibility to leave the chiral $U(1)$ factors in the SM flavour symmetry group as a global Peccei-Quinn symmetry, which allows to avoid the strong CP-problem when the Goldstone modes dynamically lead to a vanishing effective $\theta$-parameter in QCD. The couplings of the physical axion field in such a scenario is strongly suppressed by the UV scale of the effective theory which is also responsible for the small Yukawa interactions of the first quark family.

In summary, while (admittedly) still being a rather speculative new physics scenario, our set-up provides an interesting alternative to generate the observed hierarchies in
the quark masses and mixings, with well-defined physical consequences. We should also remark that – in contrast to many other generic new physics models – our approach is protected by MFV against unacceptably large corrections to flavour observables.

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A Anomalous $U(1)_X$ Theory

The general formalism for a consistent formulation of gauge theories based on an anomalous local symmetry within an effective-theory framework has been given in [24]. In our case, we have to deal with an anomalous $U(1)_X$ symmetry, which acts on the chiral fermion representations in the SM according to (10). The classical part of the Lagrangian involving the $U(1)_X$ gauge field reads

$$\mathcal{L}_X = -\frac{1}{4} X_{\mu\nu}(x) X^{\mu\nu}(x),$$

$$\mathcal{L}_\psi = \bar{\psi}_L(x) i\not{D} \psi_L(x) \quad \text{and} \quad \mathcal{L}_{\text{spurion}}$$

(65)

where $D_\mu = \partial_\mu - ig_X q_X X^\mu + \ldots$, and we have only shown the coupling to the $U(1)_X$ gauge boson in the covariant derivative including the $U(1)_X$ charges for the different fermion species in [25]. While the classical theory is invariant under the gauge transformation

$$X_\mu(x) \rightarrow X_\mu(x) + \frac{1}{g_X} \partial_\mu \omega_X(x),$$

(67)

which can be compensated by a local phase transformation of the fermions,

$$\psi_L(x) \rightarrow e^{i\omega_X(x) q_X} \psi_L(x),$$

(68)

in the classical Lagrangian, the quantum effective action changes since the representation $\psi_L$ of $U(1)_X$ is anomalous,

$$\text{tr} \left[ Q_X^3 \right] = \frac{3}{4} \neq 0.$$  

(69)

We note that, due to the fact that the charge $Q_X$ is a linear combination of $SU(3)_C^3$ generators, its trace vanishes $\text{tr}[Q_X] = 0$ individually for every SM gauge multiplet, and therefore we will not encounter mixed anomalies with $SU(N)$ generators (where
\[
\text{tr}[\{T^a, T^b\} Q_X] \propto \delta^{ab} \text{tr}[Q_X] = 0 \text{ or gravitons. Also } \text{tr}[Y^2 Q_X] = 0. \text{ There are, however, mixed anomalies with the SM hypercharge from triangle diagrams involving two } U(1)_X \text{ gauge fields and one hypercharge gauge field,}
\]

\[
\text{tr}[Q_X^2 Y] = -1 \neq 0 , \quad (70)
\]

(where we normalized the hypercharge such that \( Y(Q_L) = 1/3, Y(U_R) = 4/3, Y(D_R) = -2/3 \)). This implies that the total change of the effective action under either \( U(1)_X \) or \( U(1)_Y \) local transformations is given by \([24]\)

\[
\delta \omega_X \Gamma = \text{tr}[Q_X^3] \frac{g_X^2}{48\pi^2} \int d^4 x \omega_X X_{\mu\nu} \bar{X}^{\mu\nu} \\
+ c_1 \text{tr}[Q_X^2 Y] \frac{g_Y g_X}{48\pi^2} \int d^4 x \omega_X X_{\mu\nu} \bar{Y}^{\mu\nu} , \quad (71)
\]

\[
\delta \omega_Y \Gamma = (1 - c_1) \text{tr}[Q_X^2 Y] \frac{g_X^2}{48\pi^2} \int d^4 x \omega_Y X_{\mu\nu} \bar{X}^{\mu\nu} , \quad (72)
\]

where

\[
\bar{X}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\rho\sigma} X_{\rho\sigma} \quad (73)
\]

is the field-strength dual. The coefficient \( c_1 \) arise from the freedom to add an appropriate local counter-term \([24]\),

\[
\Gamma_{c.t.} = c_1 \text{tr}[Q_X^2 Y] \frac{g_Y g_X}{24\pi^2} \int d^4 x \epsilon_{\mu\nu\lambda\sigma} Y^{\mu} X^{\nu} \partial^{\lambda} X^{\sigma} , \quad (74)
\]

which changes under gauge transformations as

\[
\delta \omega_Y \Gamma_{c.t.} = -c_1 \text{tr}[Q_X^2 Y] \frac{g_X^2}{48\pi^2} \int d^4 x \omega_Y(x) X_{\mu\nu} \bar{X}^{\mu\nu} , \quad (75)
\]

\[
\delta \omega_X \Gamma_{c.t.} = c_1 \text{tr}[Q_X^2 Y] \frac{g_X g_Y}{48\pi^2} \int d^4 x \omega_X(x) X_{\mu\nu} \bar{Y}^{\mu\nu} . \quad (76)
\]

Choosing the renormalization condition \( c_1 = 1 \), the \( U(1)_Y \) symmetry remains manifestly non-anomalous, \( \delta \omega_Y \Gamma = 0 \), whereas

\[
\delta \omega_X \Gamma = \frac{1}{48\pi^2} \int d^4 x \omega_X \left\{ \text{tr}[Q_X^3] g_X^2 X_{\mu\nu} \bar{X}^{\mu\nu} + \text{tr}[Q_X^2 Y] g_X g_Y X_{\mu\nu} \bar{Y}^{\mu\nu} \right\} . \quad (77)
\]

Still, local gauge invariance can be formally restored by exploiting the behaviour of the Goldstone field \( \pi_X(x) \) under gauge transformations,

\[
\pi_X(x) \rightarrow \pi_X(x) + \omega_X(x) . \quad (78)
\]
Adding a term

$$\Delta \mathcal{L}_\pi = - \frac{\pi_X(x)}{48\pi^2} \left\{ \text{tr}[Q_X^3] g_X^2 X_{\mu\nu}(x) \bar{X}^{\mu\nu}(x) + \text{tr}[Q_X^2 Y] g_X g_Y X_{\mu\nu}(x) \bar{Y}^{\mu\nu}(x) \right\}, \quad (79)$$

then compensates for the change in $\Gamma$ from the fermion measure. On the quantum level, loop corrections involving the anomalous coupling of the Goldstone mode $\pi_X$ to the gauge fields in (79) will also lead to a mass term for the $U(1)_X$ gauge boson \[24\] (in addition to the masses generated by the spurion VEVs in (29,30)). These diagrams are quadratically divergent and contribute as

$$M_X \equiv g_X F_X \equiv \frac{g_X^3 \text{Tr}[Q_X^3]}{64\pi^3} \Lambda, \quad \text{respectively} \quad \frac{g_Y g_X^2 \text{Tr}[Y Q_X^2]}{64\pi^3} \Lambda \quad (80)$$

to the $U(1)_X$ gauge boson mass, with the corresponding quadratic term in the effective Lagrangian

$$\mathcal{L}_\pi = \frac{F_X^2}{2} \left( \partial_\mu \pi_X(x) - g_X X_{\mu}(x) \right)^2. \quad (81)$$

Here $F_X$ is a dimensional constant such that $(F_X \pi_X)$ has canonical dimensions and a correctly normalized kinetic term. The leading contribution to $F_X$ from the spurion-induced spontaneous symmetry breaking can be read off (30),

$$F_X^2 = 2g_b^2 \theta^2_{13} \Lambda^2 + \ldots \quad (82)$$

Covariant gauges can be introduced via the gauge fixing term

$$\mathcal{L}_{\text{g.f.}} = - \frac{1}{2\alpha_X} \left( \partial_\mu X_\mu(x) - \alpha_X \frac{F_X^2}{2} \pi_X(x) \right)^2, \quad (83)$$

which in combination with the kinetic term in $\mathcal{L}_\pi$, gives the quadratic terms

$$\frac{M_X^2}{2} (X_\mu)^2 - \frac{1}{2\alpha_X} (\partial_\mu X_\mu)^2 - \alpha_X \frac{M_X^2}{2} (F_X \pi_X)^2 + \frac{1}{2} (F_X \partial_\mu \pi_X)^2. \quad (84)$$

The mixing term between $X_\mu$ and $\pi_X$ vanishes (up to a total derivative). For the 't Hooft-Landau gauge, $\alpha_X = 0$, and the $\pi_X$ field is massless. In unitary gauge, $\alpha_X \rightarrow \infty$, the Goldstone field decouples.
B Unitary gauge for the 2–Family Case

In the 2-family case, Eqs (18–20) can be easily solved for the various \( \xi_x \)-fields,

\[
\begin{align*}
\xi_{UL}^1(x) &= 0, & \xi_{UL}^2(x) &= -F_{12}^2 \frac{y_u^2 + y_c^2}{(y_u^2 - y_c^2)^2} \eta_{12}(x) \simeq -\frac{F_{12}^2}{y_c^2} \eta_{12}(x), \\
\xi_{DL}^1(x) &= 0, & \xi_{DL}^2(x) &= -F_{12}^2 \frac{y_d^2 + y_s^2}{(y_d^2 - y_s^2)^2} \eta_{12}(x) \simeq -\frac{F_{12}^2}{y_s^2} \eta_{12}(x), \\
\xi_{UR}^1(x) &= -F_{12}^2 \frac{2y_u y_c \sin \pi_u}{(y_u^2 - y_c^2)^2} \eta_{12}(x) \simeq 0, & \xi_{UR}^2(x) &= -F_{12}^2 \frac{2y_u y_c \cos \pi_u}{(y_u^2 - y_c^2)^2} \eta_{12}(x) \simeq 0, \\
\xi_{dR}^1(x) &\rightarrow +F_{12}^2 \frac{2y_d y_s \sin \pi_d}{(y_d^2 - y_s^2)^2} \eta_{12}(x) \simeq 0, & \xi_{dR}^2(x) &\rightarrow +F_{12}^2 \frac{2y_d y_s \cos \pi_d}{(y_d^2 - y_s^2)^2} \eta_{12}(x) \simeq 0,
\end{align*}
\]

leaving one dynamical spurion field \( \eta_{12}(x) \) which describes fluctuations around the Cabibbo angle. The factor

\[
F_{12}^2 = \frac{2(y_u^2 - y_c^2)(y_d^2 - y_s^2)^2}{(y_u^2 - y_c^2)(y_d^2 + y_s^2) + (y_d^2 - y_s^2)(y_u^2 + y_c^2)}
\]

has been introduced to normalize the kinetic term for \( \eta_{12}(x) \) in the form

\[
\frac{1}{2} F_{12}^2 \Lambda^2 \left( \partial^\mu \eta_{12}(x) \right)^2
\]

when inserted into (4). It is important to notice that the fluctuation \( \eta_{12}(x) \) appears symmetrically in the up- and in the down-quark sector (scaled by ratios of quark Yukawa couplings), despite the fact that our original ansatz (12) assigned the CKM matrix solely symmetrically in the up- and in the down-quark sector (scaled by ratios of quark Yukawa couplings). In contrast, the naive replacement \( \theta_{12} \rightarrow \theta_{12} + \frac{F_{12}^2 \eta_{12}(x)}{\sqrt{2}} \) in (23) would have lead to an incorrect result, where \( \eta_{12}(x) \) would only couple to down-type quarks. Linearizing in the fluctuations \( \eta_{12}(x) \), the Yukawa couplings in unitary gauge for the 2-family case read

\[
\begin{align*}
Y^{u-g.}(x) &= \text{diag} \left[ y_u e^{-i\pi_u}, y_c \right] + \left( \begin{array}{cc} 0 & -\frac{y_u}{2(y_u^2 - y_c^2)} \\
-\frac{y_u e^{-i\pi_u}}{2(y_u^2 - y_c^2)} & 0 \end{array} \right) F_{12}^2 \eta_{12}(x) + O(\eta_{12}^2), \\
Y^{d-g.}(x) &= V_{\text{CKM}} \left\{ \text{diag} \left[ y_d e^{-i\pi_d}, y_s \right] + \left( \begin{array}{cc} 0 & -\frac{y_d}{2(y_d^2 - y_s^2)} \\
-\frac{y_d e^{-i\pi_d}}{2(y_d^2 - y_s^2)} & 0 \end{array} \right) F_{12}^2 \eta_{12}(x) + O(\eta_{12}^2) \right\}.
\end{align*}
\]

\[ ^{10} \text{Still, in the limit } y_{u,d} \ll y_s \ll y_c, \text{ the coupling of } \eta_{12}(x) \text{ to } d_L \text{ and } s_R \text{ will dominate.} \]
The coupling matrices of the $\eta_{12}(x)$ field can be expressed entirely in terms of the VEVs of the Yukawa matrices,

$$\delta Y_U \equiv F_{12}^2 \left( \begin{array}{cc} 0 & -\frac{y_u}{2(y^2_c - y^2_u)} \\ \frac{y_u e^{-i\theta_{12}}}{2(y^2_c - y^2_u)} & 0 \end{array} \right)$$

$$= \frac{F_{12}^2}{y^2_c - y^2_u} \left\{ -y^2_c y^2_s \tan \theta_{12} + \frac{y^2_c y^2_u \cot \theta_{12} - y^2_u y^2_s \cot \theta_{12} - y^2_u y^2_t \tan \theta_{12}}{2(y^2_c - y^2_u)(y^2_u - y^2_s)} \langle Y_U \rangle 
+ \frac{\cot 2\theta_{12}}{y^2_c - y^2_u} \langle Y_U Y^\dagger U \rangle - \frac{\csc 2\theta_{12}}{y^2_u - y^2_d} \langle Y_D Y^\dagger_D Y_U \rangle \right\}$$

$$\simeq \frac{y^2_s \tan \theta_{12}}{y^2_c} \langle Y_U \rangle + \frac{2y^2_u \cot 2\theta_{12}}{y^4_c} \langle Y_U Y^\dagger_U \rangle - \frac{2\csc 2\theta_{12}}{y^2_u} \langle Y_D Y^\dagger_D Y_U \rangle$$

with an analogous relation for $\delta Y_D$. Here the first identity holds in the basis where $\langle Y_U \rangle$ is diagonal and $\langle Y_D \rangle = V_{CKM} D$, while the second and third line are basis independent, due to the transformation properties of $\delta Y_U \sim (3, \bar{3}, 1)$ and $\delta Y_D \sim (3, 1, \bar{3})$ under the flavour symmetry group. From the MFV perspective, we thus expect the coefficients in front of the 3 individual flavour structures to be of $O(1)$ or smaller. Indeed, taking into account that $\eta_{12}(x) \sim \theta_{12}$, we obtain

$$\frac{y^2_s \tan \theta_{12}}{y^2_c} \eta_{12} \sim y^2_s \theta_{12} \ll 1, \quad \frac{2y^2_u \cot 2\theta_{12}}{y^4_c} \eta_{12} \sim y^2_u \ll 1, \quad \frac{2\csc 2\theta_{12}}{y^2_u} \eta_{12} \sim 1,$$

for $y_c \sim O(1), y_c \gg y_s \gg y_u,d$. It is interesting to note that

$$\text{tr} \left[ (Y^\dagger_U) \delta Y_U \right] = \text{tr} \left[ (Y^\dagger_U Y_U Y^\dagger_U) \delta Y_U \right] = 0,$$

which shows that our construction for $\eta_{12}(x)$ indeed involves a variation that is orthogonal to the VEV of $Y_U$ (an analogous statement holds for $\delta Y_D$ and $\langle Y_D \rangle$).

C  Peccei–Quinn Mechanism for $U(1)_{u_R} \times U(1)_{d_R}$

To show that (57) is satisfied, we follow the original paper by Peccei/Quinn [31] and consider the generating functional (in Euclidean space),

$$Z_0(J_{U,D}, J^\dagger_{U,D}) = \sum_q \left( \frac{e^{iq}}{D A_{\mu} q} \right) \int D Y_{U,D} D Y^\dagger_{U,D} \int D \Psi \ D \overline{\Psi}$$

$$\times \exp \left\{ \int d^4x \left( \mathcal{L}(x) + \text{tr}[J_U(x) Y_U(x)] + \text{tr}[J_D(x) Y_D(x)] + \text{h.c.} \right) \right\},$$

(94)
where $\mathcal{D}\Psi$ denotes all left- and right-handed quarks, and $\mathcal{L}(x)$ contains the SM Yukawa terms with the Higgs field set to its VEV,

$$-\mathcal{L}_{\text{yuk}} = v \bar{U}_L(x) Y_U(x) U_R(x) + v \bar{D}_L(x) Y_D(x) D_R(x) + \text{h.c.} \quad (95)$$

The effective potential in the non-trivial QCD vacuum can be obtained from an expansion in small Yukawa couplings, with the leading term coming from the $q = \pm 1$ sectors, leading to

$$V_\theta = V_0 - K v^6 \text{Re} \left[ \det Y_U \det Y_D e^{i\theta} \right] \quad (96)$$

with a positive constant

$$K = \frac{\int (DA_\mu) \int D\Psi \mathcal{D}\Psi e^{\int dx (-\frac{1}{4}GG + \bar{\Psi}i\mathcal{D}\Psi)} \prod_{i=u,c,t,d,s,b} \int d^4x_i \bar{\Psi}_i(x_i)\Psi_i(x_i)}{\int (DA_\mu) \int D\Psi \mathcal{D}\Psi e^{\int dx (-\frac{1}{4}GG + \bar{\Psi}i\mathcal{D}\Psi)}} \quad (97)$$

and $K \sim \Lambda_{\text{QCD}}^2$. After spontaneous symmetry breaking, $G_F \to U(1)_{uR} \times U(1)_{dR}$, the Yukawa matrices are represented as

$$Y_U(x) = \begin{pmatrix} y_u(x) e^{-i\pi_u(x)} & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}, \quad Y_D(x) = V_{\text{CKM}} \begin{pmatrix} y_d(x) e^{-i\pi_d(x)} & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad (98)$$

with

$$\det Y_U = y_u y_c y_t e^{-i\pi_u}, \quad \det Y_D = y_d y_s y_b e^{-i\pi_d}, \quad (99)$$

from which (98) follows.

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