Drawing Light-fields: Hand-drawn Approaches to Abrasion Holography

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Abstract. Abrasion holography has received little attention since the technique was described by William J. Beaty in the early 1990’s. In this paper the limitations of abrasion holography are explored, and new approaches are presented which expand the possibilities of the medium. New tools presented here offer new possibilities to the artist wishing to draw holograms by hand. Methods are described by which complex curves and organic forms can be constructed by hand more easily and intuitively than previously described. In an analysis of reconstruction lighting and viewing geometries, new solutions to reduce or eliminate distortions are suggested. Various tools, materials, and scratch geometries are considered for optimum 3D illusion. A new class of abrasion holograms is presented that use elliptical, hypotrochoidal, and epitrochoidal scratch geometries, exhibiting novel animation effects. In conclusion, a method for embossing abrasion holograms with the aid of an etching press is described.

1. Introduction
The depth illusion apparent in light reflected from circular scratch patterns has been noted independently by many commentators since the 1930’s. In the early 1990’s William Beaty compared this illusion to holography and formalized a technique for creating 3D drawings by hand, which he called “scratchograms” or “abrasion holography.” Abramson expanded on the technique, with a method for geometric construction of a cube. Several recent publications explore computer-aided methods for producing abrasion-type holograms, using CNC engravers, and milling machines. Since Abramson and Beaty however very little has been published in the way of expanding the techniques available for hand drawing abrasion holograms.

I will assume familiarity with the basic technique of abrasion holography, and with the basic optics of their reconstruction. I will use the words hologram and holographic throughout this paper to refer to 3D images created by the scratches in an abrasion hologram. The question of whether or not these images should be considered a hologram has been treated by Beaty and Abramson and is outside the scope of this paper. For those unfamiliar with the basic techniques of abrasion holography and the optics of their reconstruction Beaty and Abramson each offer a thorough treatment of these topics.

I have spent the last three years making abrasion holograms by hand and exploring the limits of the medium. Here I present techniques, tools, and conceptual models useful to the artist who wishes to draw abrasion holograms by hand.

2. Materials
Many investigators have reported using Plexiglas or a similar plastic as the plate on which to inscribe abrasion holograms. I have found the image quality to be much better if metal is used. Metal plates are also more durable, and finished holograms on metal are less prone to being damaged by
scratching. Copper plates produce a bright image with a very smooth scratch. As copper is quite expensive, aluminum plates are a good substitute. I have also found zinc plates to work quite well. Large sheets of 1/16 inch thick aluminum can be bought and cut down on a table saw.

A sturdy compass (dividers) with a steel or diamond tip scribe is used to make the scratches. When working with metal the scratch should be made with a firm, consistent pressure applied throughout the revolution of the compass. I have also employed drafting templates to good effect, especially where scratch circumference is small and use of a compass becomes difficult. Plates can be blackened by soft anodizing (or more easily with black permanent marker) before scratching; this greatly enhances the visibility of the holographic image by increasing the contrast between the plate surface and the hologram.

3. As Above So Below
In abrasion holography a single circular scratch represents two holographic points, one behind the plate (virtual image) and one in front of the plate (conjugate image). This presents a problem because any single holographic point we may wish to describe will be reflected pseudoscopically, greatly limiting the kinds of images which can be drawn.

Beaty overcame this limitation by constraining the scratch-arc to semicircular, thereby suppressing the pseudoscopic image, but as a result limiting the viewing angle. In the case of a wall-mounted presentation illuminated from above, Beaty’s solution is passable since most of the normal range of view is covered. Beaty also described how shorter scratch-arcs could be employed to create animations or objects which disappear behind other objects. However, when presented supine (as in a table-top presentation) with illumination normal to the plate, or when the plate is presented in rotation, these techniques prove to be unsatisfactory with each single holographic point replaying pseudoscopically from the opposite angle of view.

Abramson presented another solution where full circle scratches may be used. In Abramson’s cube construction (fig. 5a) the holographic object is symmetrical and reflected through the plane of the plate. Abramson’s cube construction can be generalized to any form with bilateral symmetry. Both real and virtual images contribute to a single holographic image with a very wide field of view. This arrangement is particularly well suited for a tabletop plate orientation with illumination at normal to the plate surface—the many advantages of which will return to shortly.

However, when presented in a wall-mounted display illuminated form above, Abramson’s cube appears skewed, taking the form of a rhombic prism (a distortion which we shall return to shortly). This solution is also limited to forms with bilateral symmetry (spheres, cones, cubes, etc); the symmetrical object must be reflected across the plane of the plate (i.e. a cone can be presented where its axis lies on the plane of the plate, however it is not possible to present a cone with its axis at any other angle to the plate, or for that matter with an axis above or below the plate).

It is clear that a method for separating “above” and “below” images is needed. This is accomplished by altering the profile of the scratch to be asymmetrical. A 90-degree conical diamond “drag engraving” tool is used in a specially designed compass (fig. 1a) which allows the tool to be tilted. Where the tool is oriented at approximately normal to the plate, the scratch profile will be symmetrical and the usual two holographic image points will appear above and below the plate (fig. 1b). If the tool is angled to tip inward toward the center of the circle, the scratch will become asymmetrical and only the holographic point “below” the plate appears (fig. 1c). Where the tool is tipped outward, only the holographic point “above” the plate appears (fig. 1d). With this arrangement the desired image point is reflected while the spurious one is completely suppressed. The tilt of the tool must be great, so that one edge of the diamond cone approaches normal to the plate. By adjusting the tilt of the tool a single scratch can be made to represent holographic points “above,” “below,” or both “above and below” the plate. These effects can be combined to create off axis, asymmetrical holographic objects.

With this arrangement, the requirement for symmetry and reflection across the plane of the plate has been overcome; all manner of holographic forms can be drawn with a full range of view.
This technique also allows for Beaty’s method of employing shorter arcs to be used to better effect: animations, solid objects, or objects which occlude other objects may be drawn with consistency across the visible field and without spurious artifacts.

4. Distortion and Optimum Viewing Angle

Several commentators have noted the “swinging” distortion apparent in abrasion holograms. In a wall mounted presentation with the plate illuminated from above, as the viewer moves laterally across the angle of view, the holographic object appears to “swing” dipping downward as the viewer is centered in relation to the plate and back up as the viewer moves to either side. This is remedied slightly by moving the source of illumination to an angle other than directly above the plate (with the added benefit, noted by Abramson [7], of increasing vertical parallax). A thorough examination of the optics of reconstruction reveal new solutions to this distortion.

4.1. Tabletop Display

This distortion can be avoided altogether if a tabletop plate orientation is acceptable (fig 2). Plate P is lying flat—as on a table. A circular scratch S is made on the plate with center point C and radius r. Point source illumination R is provided from directly above point C, normal to the plate and is reflected in the plane of the plate at R'. A viewer O is positioned at angle v to the reflected image of the light source. If we now draw a line connecting the reflected image R' with center point C (in this case congruent with the axis of illumination) where this line crosses with the viewers line of sight to the scratch points of light H and H' will appear. In this situation H will appear to the viewer as if above the plate, directly above C and at height Z; while H' will appear as if below C at depth Z'. We can see that where v and v' are 45 degrees then Z, Z' and r are all equal.

If we scratch a series of concentric circles centered on C, the resultant holographic image points will all fall on the line of the reflected path of illumination, in this case describing a line normal to the plate and passing through its surface at C.

In the foregoing we assume that the lines of sight are parallel; in actual practice of course they must converge at O. This will cause angle v' to be slightly diminished and v to be slightly enlarged with the result that H will appear lower and H' will appear deeper (in other words Z' will be longer than Z). This distortion becomes more pronounced as the viewer moves closer to the plate. At normal viewing distances of a few feet or more the distortion is so reduced as to be unnoticeable.
The same can be said of the illumination source, which is treated here as a point source at and infinite distance.

With this arrangement the holographic image is dimensionally consistent as long as the angle of view relative to surface normal remains constant. However, a new “stretch” and “squish” distortion is introduced; as the viewing angle in relation to normal is varied the apparent holographic depth changes: growing more exaggerated as the viewing angle moves upward (approaching normal), causing the image to stretch; and diminishing as the viewing angle moves downward (toward the plane of the plate), causing the image to squish (fig 3). This means that as the viewer walks forward, toward the plate the apparent depth of the object increases and conversely as the viewer moves away the apparent depth is reduced. This distortion can be minimized by selecting a natural viewing angle (such as 45 degrees), and by establishing a viewing distance in the display (such as with a railing or the edge of the table).

This arrangement has the added benefit of being visible without distortion from 360 degrees around the plate (at a given viewing angle) allowing a large number of viewers, but also enabling viewers to walk all the way around the plate. This fact also means that the plate may be rotated, as on a turntable, without distortion—the holographic object appearing to rotate as if fixed to the plate. This proves to be quite desirable as the depth illusion benefits from the added motion parallax introduced by the rotation, and stationary viewers may appreciate all sides of the holographic object.

Note that in the tabletop display with illumination at normal, if the viewing angle is made to be 45 degrees, the radius of a given scratch is equal to the apparent height of the resultant point of light. This proves to be highly useful to the artist constructing an abrasion hologram by hand—as the height can be easily and intuitively determined. Here measurements can be taken directly from elevation drawings or even from physical models.

These advantages, added to those mentioned above, make the tabletop plate orientation, illuminated along surface normal and designed to be viewed at 45 degrees to normal a very useful arrangement for hand-drawn abrasion holograms.

4.2. Wall-mounted Display
Instead of adjusting our angle of view, let the view remains fixed as we hinge the plate up beginning from the tabletop orientation (as described in fig. 2) so that the surface normal approaches the line of sight of the viewer. As we do so, the reflected angle of the light source is no longer reflected back along the path of illumination but begins to shift towards the viewer. Consequently, holographic point \( H \) will travel up the line of sight of the viewer, appearing to gain altitude from the plate and move towards the viewer; while holographic point \( H' \) travels down the line of sight moving deeper into the plate and away from the viewer.

As the plate approaches 22.5 degrees from the plane of the table, holographic point \( H \) has traveled all the way up the line of sight and the reflected path of illumination is directed at the viewer.
The holographic image is highly distorted and obscured by the image of the light source as described by Abramson.

If we continue to hinge the plate upwards however, the reflected path of illumination will continue its rotation past the line of sight of the viewer, and holographic point H will begin to travel back down the line of sight while H' will travel back up. As the plate reaches 45 degrees from the original tabletop, the line of sight becomes normal to the plate and points of light H and H' have returned to their original altitudes (as when the plate was flat on the table) only now they are no longer centered above C but appear as if directly normal to points above and below the line of the scratch (fig. 4). This is the arrangement for a wall-mounted display, with viewer normal to the plate and illumination from 45 degrees above.

Just as with the table-top display, if \( \theta \) is 45 degrees, then \( Z \) and \( r \) are equal (as well as \( Z' \) and \( r' \)). If we scratch a series of concentric circles centered on C, the resultant holographic image points will all fall on the line of the reflected path of illumination - in this case describing a line at 45 degrees to the plate and passing through its surface at C.

### 4.3. Corrected Cube

Let us consider the cube construction proposed by Abramson (fig. 5a). In the table-top display the concentric circles represent lines normal to the plate, these connect two squares in planes parallel to the plate surface, one above and one below. Where the viewing angle \( \theta \) is 45 degrees the circles should be enlarged so that \( r \) is equal to \( Z \) (fig. 5b).

In the wall mounted presentation this construction is no longer satisfactory for an image of a cube; The two squares which are in planes parallel to the plate surface have shifted, the one above the plate having moved down, and the one behind having moved up. The concentric circles now represent lines at 45 degrees to normal. The image is no longer a cube but a rhombic prism (with the front and back faces of the cube parallel to the plane of the plate and the top and bottom faces tipped downward away from surface normal, making the remaining two sides rhombuses).

In the wall-mounted display, if we wish to describe two holographic points reflected across the plane of the plate, unit scratches will no longer be circles, but arcs tangent to each other (fig. 5c). If the "above" and "below" cutting tools are used, these may be two full circles tangent to each other (though the upper arcs will extend outside the visible area). As the viewer position approaches normal, the two points of light become coincident appearing as a single dot at the tangent point of the arcs. We can now construct the scratch pattern for holographic points on a line extending normal to the plate (fig 5d); arcs curving upwards represent points behind the plate, arcs curving downward represent points in front of the plate. From this we can construct a corrected cube (fig. 5e), with four holographic lines normal to the plate, connecting the corners of two squares in planes parallel to the plate. The cube now appears normal in the wall-mounted display, but (as might be expected) is distorted into a rhombic prism when viewed in the tabletop display. In this case the square faces are perpendicular to the plane of the plate.

Using this corrected geometry any arbitrary holographic shape can be constructed for wall-mounted viewing. This method greatly reduces the "swinging" distortion in wall mounted display which was described above.
5. Object and Reference Projections

Let us return to the tabletop orientation described in (fig. 2). Imagine that a tiny bead or other such arbitrarily small object X is made to float at point H (at distance Z above C); the bead casts a shadow C’ onto the plate at point C. Now imagine that the viewer O is replaced with a second light source.

The bead now casts a second shadow S’ at distance r from point C’ (see fig. 6). As the plate is rotated (or as light source O is rotated around the plate), shadow S’ remains a constant distance r from C’ – tracing the path of circle S.

Here we can see that shadow C’ corresponds to the stationary arm of the compass, while shadow S’ corresponds to the moving arm. This demonstrates how the scratch pattern of an abrasion hologram can be understood in terms two projections onto the plane of the plate: one at the desired angle of illumination (R) which we shall call the reference projection (C’), and the other at the desired viewing angle (O) which we shall call the object projection (S’). Any single holographic point can be drawn by determining an object projection and a reference projection, and scratching the circle thus described.

As the bead is raised straight upwards the reference projection C’ remains stationary (since the illumination is directly above) while the object projection S’ moves outward away from C’. This describes a series of circles centered at C’ and with increasing radii. As the bead is lowered the distance decreases, until finally the bead rests on the plane of the plate and the two shadows are congruent. This describes circles centered at C’ with decreasing radii. If the bead is lowered beneath the plate the reference projection is from the direction of the reflected image of O.

By using this model we can quickly construct abrasion holograms without needing to calculate individual points.

Projective method for drawing a line (see fig. 7a):

1. Draw reference line R on the plate
2. Draw any object line O above R
3. Draw an elevation line y, at any point perpendicular to R and intersecting O
4. Place the stationary point of the compass on reference line R at the intersection of y; extend the other arm of the compass to the point where y intersects the object line; and scratch a circle z with the “above” cutting tool.
5. Repeat steps 3 and 4 for as many points on O as desired

If the object line crosses the reference line, it will pass through the plate and the “below” cutting tool should be used (fig 7b). Neither the reference line nor object line need to be straight – complicated curves can be drawn quickly with this method.

If for our object line we draw a circle above the reference line, we will have a holographic image of a circle on a plane 90 degrees from the surface of the plate. As before, if the object-circle is
lowered so that part of it crosses the reference line, those scratches should be incised using the “below” cutting tool. If the object-circle is lowered to the point where its center intersects the reference line, the center point of our holographic circle will be on the surface of the plate; in this case the cutting tool can be oriented to reflect both “above” and “below” images—since the image is now symmetrical!

If we want to draw a holographic image of a circle on a plane 45 degrees to that of the plate, the reference projection will be a 45-degree ellipse while the object projection will be a straight line. If the object line is lowered so that it is coincident with the major axis of the reference ellipse, the center of our holographic circle will appear to be on the surface of the plate—however “above” and “below” cutting tools must be used, as the figure is not symmetrical across the plane of the plate. The envelope of the scratches for this form is a compressed nephroid.

If both object and reference lines are circles, then all scratches will be of the same diameter and the holographic image will be of a circle on a plane parallel to the plate. As the object circle is lowered it will eventually be congruent with the reference circle; the scratch diameter will be infinitely small and the circle will be on the surface of the plate.

In the case of the wall mounted display, object and reference projections will be reversed. The object projection will not be normal to the plate while the reference will be at 45 degrees. By this method we have a powerful tool for imagining and plotting abrasion holograms.

6. Noncircular Abrasions

If instead of circular scratches we use elliptical, hypotrochoidal, and epitrochoidal scratches we will also create holographic effects. These curves can be drawn by tracing a point on a circle as it rolls along the edge of another circle. For experiments with these curves I used a Spirograph drawing toy, a steel scribe, and aluminum plates. A set of draftsman’s ellipse templates can also be used.

As may be expected, the elliptical scratch responds in a similar way to a circle, with one point appearing above and one point below the plate surface. Unlike the circle however, the curvature of the ellipse is not consistent but becomes tighter at the major axis. This causes the holographic image point to appear, when the plate is rotated, not at a fixed altitude above a fixed point on the plate, but rather as a moving point orbiting around the reflected path of the light source.

An ellipse is a special case of a hypotrochoid, where a point on a circle of a given circumference is traced as it rolls inside a circle of twice that circumference (such as Spirograph wheel #48 rolling inside Ring #96). A circle, in turn, is a special case of an ellipse; if we trace the center point of the small wheel our resulting scratch is a circle. As we have shown, the circular scratch, when illuminated from normal to the plate presents an image of a stationary point of light—even when the angle of view is rotated. If we choose a point slightly off center (such as hole #17 on Spirograph wheel #48), the resulting scratch becomes slightly elliptical, and the corresponding holographic point begins to have a slightly eccentric orbit. As we move further away from the center of wheel #48 our elliptical scratches become more and more elongated approaching the limit of a straight line. The rotation of these holographic points describe increasingly large rings of orbit around the original circular scratch. The orbit of these points is one rotation per each half-rotation of the plate—this can be understood as a function of the ratio of the circumferences of the two circles which formed the hypotrochoid (which in in the special case of the ellipse will always be 1:2).

If a series of similar ellipses are scratched into the plate, centered on the same point but each rotated by degrees to create a rosette-like pattern, then each identical ellipse will display a holographic point above and below the plate at a different stage of its orbit and each orbit will be traced as a ring of lights one above and one below the plate. When the plate is rotated the points which make up this ring rotate as described above. If a series of similar but progressively smaller ellipses are drawn on a single axis centered around the same center point, the holographic points will describe a line with an eccentric rotation as the viewing angle rotates around the plate. The rotation of this line shows the twin rings to be bases of a double cone with their shared apex on the surface of the plate at the shared
center point of the ellipses. By scratching a series of concentric circles at this center point we describe
holographic points on a line tracing the axis of rotation of our double cone.

If we select a slightly larger circumference for our inner wheel (such as Spirograph wheel
#72), the holographic image formed is similar, though the orbits traced by the points become less
elliptical and more circular as if tipped downward toward the viewer. As the inner wheel grows
larger, the orbit of the holographic points slows down. In the case of wheel #72 with ring #96, a single
holographic point will orbit once for every ¾ turn of the plate.

Epitrochoids also produce organized holographic images. These appear as dented or warped
ellipses with some forms resembling hyperbolic paraboloids. The geometry of these images deserves
further study and will be the topic of another paper.

The Spirograph drawing toy offers an interesting approach to drawing abrasion holograms.
Complicated images, which would otherwise be difficult or impossible to draw by hand can be created
easily. For example an approximation of a sphere can be created by using ring #96 with wheel #40,
hole 13; wheel #50, hole 13; wheel #63, hole 13; and wheel #84, hole 13.

7. Printing
In collaboration with printmaker Richard Nielsen of Untitled Prints and Editions in Los Angeles, we
are developing a technique for reproducing abrasion holograms using an etching press. The abrasion
hologram plate is prepared as normal, by scratching in metal though with a cutting tool designed to
raise a maximum burr (metal displaced by the scratch and protruding from the surface of the plate).
Where a copper plate is used the burr can be reinforced by nickel plating. Using the press this burr is
then embossed into a reflective surface such as a thin foil to create a "print" of the hologram -which
exhibits full dimensional effects. Using this technique multiple images can be created from one master
plate. The details of this technique will be the subject of another paper.

8. Conclusion
In the field of hand-drawn abrasion holography we have, so to speak, only scratched the surface of
what is possible. In the short space allowed for this paper, I have only been able to introduce these
methods, tools, and conceptual models. As a medium, hand-drawn abrasion holography offers many
interesting and as-yet unexplored possibilities. Many of the distortions observed in abrasion holograms
can be corrected by using the correct scratch geometry for the display setting. Conceptual models,
such as the projective model, allow holograms to be drawn quickly and intuitively. New tools
presented in this paper, such as "above" and "below" cutting tools, ellipse templates, and the
Spirograph drawing toy offer exciting possibilities. The printing process only mentioned here, makes
possible reproductions of holograms, making more labor-intensive images to be reproduced from a
single master. It is my hope that the investigations presented here will inspire further exploration of
this unique medium.

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