Quantum corrections in massive bigravity and new effective composite metrics

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Abstract
We compute the one-loop quantum corrections to the interactions between the two metrics of the ghost-free massive bigravity. When considering gravitons running in the loops, we show how the structure of the interactions gets destabilized at the quantum level, exactly in the same way as in its massive gravity limit. A priori one might have expected a better quantum behavior, however, the broken diffeomorphism invariance out of the two initial diffeomorphisms in bigravity has similar consequences at the quantum level as the broken diffeomorphism in massive gravity. From lessons of the generated quantum corrections through matter loops we propose yet other types of effective composite metrics to which the matter fields can couple. Among these new effective metrics there might be one or more that could provide interesting phenomenology and important cosmological implications.

Keywords: bigravity, massive gravity, quantum corrections, coupling to matter

1. Introduction

Independent cosmological observations such as supernovae, CMB, baryon acoustic oscillations and lensing indicate an accelerated expansion of the Universe, driven by something that we call dark energy. The given name reflects the fact that its origin is still unknown despite significant theoretical and observational efforts. The expansion could be for instance due to a small cosmological constant \( \lambda \) with a constant energy density. However, under the assumption that the cosmological constant is generated by the vacuum energy density, we can use standard quantum field theory techniques to compute the vacuum energy density caused by fluctuating quantum fields. The result is puzzling. It differs from the observational bounds by 120 orders of magnitude. This large discrepancy between the theoretically computed high
energy density of the vacuum and the observational value constitutes the cosmological constant problem [1]. The accelerated expansion of the Universe could also be due to new dynamical degrees of freedom, either by invoking new fluids with negative pressure or by changing the geometrical part of Einstein’s equations. Among the latter class of approaches, the infrared-modifications of gravity offer promising and exciting new ways for not only addressing the late-time acceleration enigma but also tackling the cosmological constant problem. Important representatives of this type of infrared-modifications are massive gravity and higher-dimensional setups. In the context of higher dimensional theories the Dvali–Gabadadze–Porrati (DGP) [2] model has significantly marked the early stages of large scale modified theories of gravity. For about ten years the community working on infrared-modifications has witnessed a similar significant progress by the work of de Rham–Gabadadze–Tolley (dRGT) [3], who successfully extended the mass term in massive gravity to the non-linear level with the correct degrees of freedom, which was a challenge over forty years in the making (for extended reviews see [4, 5]).

Parallel to these new exciting achievements, Galilean invariant interactions were proposed to extend the decoupling limit of DGP-gravity [6]. The helicity-0 mode \( \pi \) of the DGP model has the invariance under internal galileon- and shift transformations \( \pi \to \pi + b_\mu x^\mu + c \). Together with the postulate of the absence of ghosts these symmetries restrict the allowed effective Galileon Lagrangian (for an extensive review see [7]). A crucial property of the Galileon is the non-renormalization theorem which ensures that the Galileon coupling constants are technically natural and stable under quantum corrections [8–12]. In the context of massive gravity, the Galileon-type interactions naturally arise in the decoupling limit and provide rich phenomenology [13–18]. The nice properties of the Galileon can also be generalized to higher spin fields, such as vector fields, etc [19–24]. There have been successful attempts to generalize the Galileon to the covariant Galileon on non-flat backgrounds. First covariantization consisted of the explicit second-order equations of motion sacrificing the Galileon symmetry [25–27] (even though generalizations to the maximally symmetric backgrounds did still share a generalized Galileon symmetry [28, 29]). If one is willing to give up on the restriction of second-order equations of motion (but still avoiding ghost instabilities) one can construct covariant Galileon interactions with promising new phenomenology [30–33]. Interestingly, a subclass of covariant Galileon interactions naturally arise from the covariantization of the decoupling limit of massive gravity [34, 35].

The potential interactions in the dRGT theory was constructed in a way such that the Boulware–Deser (BD) ghost remains absent at the non-linear level. This is guaranteed by the presence of a fundamental matrix constructed out of the square root of \( \hat{g}^{-1} \hat{f} \), where \( \hat{g} \) represents the dynamical metric and \( \hat{f} \) the reference metric [3]. Of course it is a natural question whether or not this very specific structure of the potential is stable under quantum corrections. These questions have been explored in [36–39]. Following the aforementioned motivations, there have been numerous investigations of the dRGT massive gravity concerning the late-time accelerated expansion of the Universe and other phenomenological aspects [14, 34, 35, 40–56]. Even if dRGT theory is a IR modification of GR, the lessons learned there can also be applied to UV modifications of GR [57]. Furthermore, the dRGT theory was extended to its bimetric version by promoting the reference metric \( \hat{f} \) to a dynamical metric through an additional kinetic term for \( \hat{f} \) [58]. Independently of the dynamics of the reference metric, it is a mandatory question of how the two metrics can be coupled to the matter sector in a consistent way, meaning without invoking the BD ghost. This question has already been explored in a multitude of very interesting works in [39, 59–71]. The ghost-freedom must be maintained at the quantum level as well, at least below the cut-off scale of...
the theory. Thus, the quantum behavior will deliver additional constraints on the coupling to matter. One natural way of coupling the matter field is to couple the matter sector to only one metric, and not to both metrics simultaneously. In this case the classical ghost-freedom remains also at the quantum level as shown in [39]. This is due to the fact that the quantum corrections contribute only in the form of two cosmological constants for the two metrics. Even if the matter sector couples to only one of the two metrics, quantum corrections will generate a coupling to the other metric and it will be important to investigate at which scale this new coupling will be generated. This will be one of the questions that we will ask. Moreover, even if it is tempting to couple the matter field to both metrics simultaneously, one immediately faces the appearance of the BD ghost already at the classical level. On top of that the quantum corrections detune the specific potential structure at an arbitrarily low scale. Hence, this way of coupling would render the theory sick. Under the requirement that the ghost-free potential structure is not detuned by the quantum corrections, one can construct a new composite effective metric built out of both metrics, through which the matter field can couple [39]. This coupling does not introduce the ghost degree of freedom at least up to the strong coupling scale and can be used as a perfectly valid effective field theory with a cut-off above the strong coupling scale. Following the philosophy of [39] we will use the lessons learned about the quantum corrections coming from matter loops in order to introduce yet other types of effective metrics through which the matter field can couple to both metrics at the same time. Moreover, we will also consider the quantum corrections generated by purely graviton loops and show the detuning of the specific potential interactions in parallel to what happens in massive gravity [38].

Throughout the paper, we consider a flat background configuration for the metric for convenience. To be more precise, we will work in flat Euclidean space. We use units for which \( c = \hbar = 1 \) and \( M_p = 1/\sqrt{8\pi G} \) for the reduced Planck mass. Furthermore, we will denote traces by [], for example the contractions of rank-2 tensors as \( \kappa^\mu_\mu = [\kappa] \), \( \kappa^\mu_\nu = [\kappa^2] = (\kappa_{\mu\nu})^2 \), etc. Greek indices run from 0 to 3 for indices in the mostly plus metric signature, while Latin indices denote the euclidean space.

2. Ghost-free bigravity

In this section we will first review the ghost-free interactions in the theory of massive bigravity and setup the framework in which we will perform the one-loop computation. Our starting point is the action for bimetric gravity and the matter action sourcing for gravity [3, 58, 72]

\[
S_{BG} = \int d^4x \left[ -\frac{M_p^2}{2} \sqrt{g} \left( R[g] + \frac{m^2}{2} \sum \alpha_n U[\kappa] \right) \right. \\
\left. - \frac{M_p^2}{2} \sqrt{f} \mathbb{L}_m \left( g, f, \psi \right) \right] 
\]

(2.1)

In [58] the mass \( m \) was defined in a symmetric way in the two Planck masses through \( m^2 M_{Pl}^2 \) with \( M_p^2 = M_f^2 \left/ (M_p^2 + M_f^2) \right. \), however here we choose to follow the definition and conventions as in [4].
where the potential interactions are given by [3, 13]

\[ U_0[\mathcal{K}] = \frac{1}{24} \varepsilon^{\mu
u\rho\sigma} \varepsilon_{\mu
u\rho\sigma} = 1 \]
\[ U_1[\mathcal{K}] = \frac{1}{6} \varepsilon^{\mu
u\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} \mathcal{K}_{\mu\nu\rho\sigma} \mathcal{K}_{\alpha\beta\gamma\delta} = [\mathcal{K}] \]
\[ U_2[\mathcal{K}] = \frac{1}{4} \varepsilon^{\mu
u\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} \mathcal{K}_{\mu\nu\rho\sigma} \mathcal{K}_{\alpha\beta\gamma\delta} = \frac{1}{2} \left( [\mathcal{K}]^2 - \left[ \mathcal{K}^2 \right] \right) \]
\[ U_3[\mathcal{K}] = \frac{1}{6} \varepsilon^{\mu
u\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} \mathcal{K}_{\mu\nu\rho\sigma} \mathcal{K}_{\alpha\beta\gamma\delta} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} = \frac{1}{6} \left( [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2\left[ \mathcal{K}^3 \right] \right) \]
\[ U_4[\mathcal{K}] = \frac{1}{24} \varepsilon^{\mu
u\rho\sigma} \varepsilon^{\alpha\beta\gamma\delta} \mathcal{K}_{\mu\nu\rho\sigma} \mathcal{K}_{\alpha\beta\gamma\delta} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} \mathcal{K}_{\upsilon\phi\kappa\lambda} = \frac{1}{24} \left( [\mathcal{K}]^4 - 6[\mathcal{K}]^2 \left[ \mathcal{K}^2 \right] + 3\left[ \mathcal{K}^3 \right]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6\left[ \mathcal{K}^4 \right] \right) \]

where \( \varepsilon \) stands for the Levi-Cevita tensor. The tensor \( \mathcal{K} \) has a very non-trivial structure in form of a square root

\[ \mathcal{K}^\mu_{\nu} \left[ g, f \right] = \delta^\mu_{\nu} - \left( \sqrt{g^{-1} f} \right)^\mu_{\nu} \] (2.2)

In comparison to massive gravity, the potential term here represents the potential for both metrics and \( f \hat{\epsilon} \) is a dynamical metric as well. The ghost absence has been also successfully proven for the case of bigravity [58].

The same square-root structure which guarantees the ghost absence makes life very hard. This mathematically cumbersome structure can be avoided using the vielbein language [73–77] since the vielbein is like the ‘square-root’ of the metric. The ghost-free potential becomes a simple polynomial in the vielbein formalism and contains interactions up to quartic order in the vielbein fields. Therefore, we will work in a symmetric vielbein-inspired language in the euclidean space in a similar way as was done in [38]. Thus, the metrics are expressed as\(^2\)

\[ g_{ab} = \left( \overline{f}_{ab} + \frac{\overline{h}_{ab}}{M_p} \right)^2 \equiv \left( \overline{f}_{ac} + \frac{\overline{h}_{ac}}{M_p} \right) \left( \overline{f}_{bd} + \frac{\overline{h}_{bd}}{M_p} \right) \delta^{cd} \]
\[ f_{ab} = \left( \overline{Q}_{ab} + \frac{l_{ab}}{M_f} \right)^2 \equiv \left( \overline{Q}_{ac} + \frac{l_{ac}}{M_f} \right) \left( \overline{Q}_{bd} + \frac{l_{bd}}{M_f} \right) \delta^{cd} \] (2.3)

with \( \overline{g}_{ab} = \overline{f}_{ab}^2 = \overline{Q}_{ab}^2 \) being the background metrics for \( g_{ab} \) and \( f_{ab} \) respectively and the fluctuations are denoted by \( h_{ab} \) and \( l_{ab} \). Of course the background metrics \( \overline{g}_{ab} \) and \( \overline{f}_{ab} \) do not need to be flat, however, for simplicity for most of the computations we will assume flat backgrounds. When working around flat background metrics \( \overline{f}_{ab}^2 = \delta_{ab} \) and \( \overline{Q}_{ab}^2 = \delta_{ab} \) following expressions will be useful throughout the paper.

\(^2\) In the vielbein language the metric would be defined by \( g_{ab} = e_a^\varepsilon e_b^\sigma \eta_{\varepsilon \sigma} \). However, here in our case we have that \( \eta_{ab} \rightarrow \delta_{ab} \) and the indices of the metric run in the flat euclidean space and our background metric will be the euclidean metric \( \delta_{ab} \). Therefore we will have \( g_{ab} = e_a^\varepsilon e_b^\sigma \delta_{\varepsilon \sigma} \). Strictly speaking we perform a field redefinition such that the usual metric perturbations around a flat euclidean background \( g_{ab} = \delta_{ab} + h_{ab} \) becomes \( g_{ab} = \delta_{ab} + h_{ab} + h_{ab} h_{ab} \delta_{cd} \).
\[ g_{ab} = \delta_{ab} + \frac{2}{M_p} h_{ab} + \frac{1}{M_p^2} h_{ac} h_{bd} \delta^{cd} \]
\[ f_{ab} = \delta_{ab} + \frac{2}{M_f} l_{ab} + \frac{1}{M_f^2} l_{ac} l_{bd} \delta^{cd} \]
\[ g^{ab} = \delta^{ab} - \frac{2}{M_p} h^{ab} + \frac{3}{M_p^2} h^{ac} h^{bd} + \cdots \]
\[ f^{ab} = \delta^{ab} - \frac{2}{M_f} l^{ab} + \frac{3}{M_f^2} l^{ac} l^{bd} + \cdots \] (2.4)

With this form of the fluctuations the square roots of the determinants then become
\[
\sqrt{g} = 1 + \frac{|h|}{M_p} + \frac{1}{2M_p^2} \left( |h|^2 - \left[ h^2 \right] \right) + \cdots
\]
\[
\sqrt{f} = 1 + \frac{|f|}{M_f} + \frac{1}{2M_f^2} \left( |f|^2 - \left[ f^2 \right] \right) + \cdots
\] (2.5)

We will perform the computation of the one-loop quantum corrections using dimensional regularization. Imagine that you define your Wilsonian action with a regulator scale \( \Lambda \), then at one loop one will obtain quantum corrections which are quadratic and quartic in this regulator scale. You can now define another Wilsonian action with a regulator scale which is arbitrary close to the first one. One can show that the power law divergences are exactly canceled out by the loop corrections between these two regulator scales. Hence the power law divergences from the one-loop effective action cancel the power law divergences coming from the definition of the Wilsonian action. Furthermore, the power law divergent quantum corrections are sensitive to the field definition one has used. Performing a field redefinition should not change the underlying physics, unless you perform a not-allowed field transformation. If one works in the cut-off regularization, then the power law divergences of the quantum corrections differ each time depending on which field definition one has used. Only the logarithmic divergences are insensitive to field redefinitions and deliver a universal correction. Therefore in this work we will only consider the logarithmic runnings (see section 2.2 of [78] for an interesting discussion on this and also [79]).

### 2.1. Quantum corrections in the decoupling limit

Let us first have a look at the quantum corrections to the potential arising in the decoupling limit. This is the first thing to be checked. If the quantum corrections already destabilize the decoupling limit itself, then there is no need to look at the full theory and the theory would be rendered nonviable. In massive gravity, the non-renormalization theorem protects the interactions within the decoupling limit. If the same is true for the bigravity case, then the theory would be safe under quantum corrections at least within the decoupling limit. Once this has been sorted out, the full theory at the quantum level can be studied as the next step. The decoupling limit provides a framework in which the most important physical properties of the theory are visible since the individual degrees of freedom decouple from each other. In bigravity the interaction between the two metrics \( g_{ab} \) and \( f_{ab} \) breaks the two copies of diffeomorphisms down to one, such that in the decoupling limit the interactions are governed by decoupled two helicity-2 modes \( h_{ab} \), two helicity-2 modes \( l_{ab} \), two helicity-1 modes \( A_a \) and one helicity-0 mode \( \pi \) accounting for a total of seven propagating helicity modes. As is already visible in the equation (2.1), the two metrics come in at their own Planck masses \( M_{Pl} \).
and \( M_f \), therefore the decoupling limit of bigravity represents the limit in which (see \[53\] for the first derivation of the decoupling limit in bigravity)

\[
M_p \rightarrow \infty, \quad M_f \rightarrow \infty, \quad m \rightarrow 0 \quad \text{and} \quad \frac{M_p}{M_f} = \text{const}, \quad \Lambda^3 = m^2 M_p = \text{const} \quad (2.6)
\]

The resulting theory in this limit contains interactions at the lowest energy scale between the two helicity-2 fields \( h_{a b} \) and \( l_{a b} \) and the helicity-0 scalar field \( \pi \) in the following form (please see \[53\] for a detailed derivation)

\[
S = \int d^4x \left[ h_{a b} \hat{\mathcal{E}}^{a b c d} h_{c d} + l_{a b} \hat{\mathcal{E}}^{a b c d} l_{c d} - \Lambda^3 \sum_{n=0}^{3} h_{a b} X^{(n)}_{a b} - \frac{M_p}{M_f} \Lambda^3 \sum_{n=0}^{4} l_{a b} Y^{(n)}_{a b} \right] \quad (2.7)
\]

where \( \hat{\mathcal{E}} \) is the Lichnerowicz operator

\[
\hat{\mathcal{E}}^{a b c d} h_{c d} = -\frac{1}{2} \left( \Box h_{a b} - 2 \partial_i \partial_a h_{b i} + \partial_a \partial_b h - \delta_{a b} \left( \Box h - \partial_i \partial_j h^{i j} \right) \right) \quad (2.8)
\]

and the \( X_{a b} \) and \( Y_{a b} \) encode the derivative interactions of order \( n \) in the helicity-0 field \( \pi \)

\[
X_{a b}^{(n)} = -\frac{1}{2 (3 - n)! n!} \hat{\beta}_{h} \cdots \mathcal{E}^{a} \cdots \mathcal{E}^{b} (\delta + \Pi)^{n} \delta^{3 - n} \quad (2.9)
\]

\[
Y_{a b}^{(n)} = -\frac{1}{2 (4 - n)! (n - 1)!} \hat{\beta}_{l} \cdots \mathcal{E}^{a} \cdots \mathcal{E}^{b} (n - 1) (\delta + \Sigma)^{n - 1} \delta^{4 - n} \quad (2.9)
\]

where \( \hat{\beta}_{h} = M^2 \hat{\beta}_{h} \). The authors in \[53\] use the \( \hat{\beta}_{h} \) notation which we borrow here for the sake of this section (the relation between the parameters \( \hat{\beta}_{h} \) and \( \alpha_{a} \) are given in equation (2.14) of \[53\]). Furthermore, \( \Pi \) and \( \Sigma \) stands for \( \Pi_{a b} = \partial_{a} \partial_{b} \pi / \Lambda^3 \) and \( \Sigma_{a b} = \partial_{a} \partial_{b} \rho / \Lambda^3 \) respectively and \( \rho \) is the dual description of \( \pi \) via field redefinitions related in a form

\[
(\delta + \Sigma) = (\delta + \Pi)^{-1} \quad (2.10)
\]

The interactions between the helicity-2 field \( h_{a b} \) and the helicity-0 field \( h_{a b} X_{a b}^{(n)} \) are exactly the same as in the decoupling limit of massive gravity. In \[11\] it has been shown that these interactions are protected from quantum corrections via the non-renormalization theorem. This property is thanks to the antisymmetric structure of the interactions. In the decoupling limit of bigravity we have the additional interactions between the helicity-2 field \( l_{a b} \) and the helicity-0 field via \( l^{a b} \hat{Y}_{a b}^{(n)} \). However, it is easy to convince ourselves that exactly the same argumentation for the non-renormalization theorem used in massive gravity applies also here in bigravity. The essential operator for the non-renormalization theorem is the Levi-Civita tensor which is also contained in the interactions \( l^{a b} \hat{Y}_{a b}^{(n)} \). Exactly this property will guarantee that any external particle contracted with any field with or without derivatives in a vertex contributes to a two-derivatives operator acting on this external particle, which gives rise to counter-terms with higher number of derivatives and hence the classical interactions do not receive quantum corrections. In bigravity we basically have two copies of the same non-renormalization theorem, namely for \( h^{a b} X_{a b}^{(n)} \) and \( l^{a b} \hat{Y}_{a b}^{(n)} \) interactions. Take for instance the interaction \( l^{a b} \hat{Y}_{a b}^{(n)} \). The part with the \( \delta \)s correspond to a tadpole contribution and a kinetic term for \( \rho \) such that the only non-trivial interaction will come from \( l^{a b} \mathcal{E}^{a} \cdots \mathcal{E}^{b} \sum_{c d e f} (\delta_{c d} + \Sigma_{c d})(\delta_{e f} + \Sigma_{e f}) \). Now contract an external helicity-2 particle \( l^{a b} \) with momentum \( q_{a} \), with the helicity-2 field coming without derivatives in this interaction at a vertex while letting the other two \( \rho \)-particles dual to the helicity-0 field \( \pi \) run in the loop with...
momenta $p_a$ and $(q + p)_a$. The contribution of this vertex gives

$$A \propto \int \frac{d^4k}{(2\pi)^4} G_p G_{p+q} \epsilon_{ab} \epsilon^{acek} \epsilon^{bdln} p_a (q + p)_c (q + p)_d \cdots,$$

where $\epsilon_{ab}$ stands for the spin-2 polarization tensor and $G_p = p^{-2}$ for the Feynman massless propagator of the $\rho$ field. Exactly in the same way as it happens in the decoupling limit of massive gravity, the Levi-Civita antisymmetric structure of the vertex enforces that only the terms with at least two powers of the external helicity-2 momentum $q_a q_\beta$ contributes to the scattering amplitude [11]. The same is true for the remaining interactions between the helicity-0 field and the helicity-1 field in the decoupling limit of massive gravity (their exact form is given in [53], however only their Levi-Civita antisymmetric structure matters for the non-renormalization theorem). Thus, the decoupling limit of bigravity is protected from quantum corrections. This is a trivial generalization of the non-renormalization theorem to the case of bigravity.

2.2. Propagators for the massless and massive modes in the unitary gauge

Now that we have established the non-renormalization argument in the decoupling limit above, we can investigate the quantum corrections in the full non-linear theory. We will perform the analysis in the unitary gauge with vanishing Stückelberg fields, i.e. $\Phi = x^\alpha \delta^\alpha$. In contrast to massive gravity, we have two spin-2 fields. In order to compute the one-loop quantum corrections, we have to specify the mass spectrum of bigravity. For that we will split the mass spectrum of bigravity into massive and massless spin-2 fluctuations around the euclidean backgrounds $g_{ab} = \delta_{ab} = f_{ab}$. To be precise, for the mass spectrum we will perform the metric perturbations in equation (2.4). Then, the fundamental matrix of the theory in terms of the perturbations is given by

$$K^{ab}_{\mu \nu} = \frac{l^b}{M_i} + \frac{h^{ab}_{\mu \nu}}{M_{Pl}} - \frac{1}{4} \frac{l^a}{M_i^2} - \frac{5}{4} \frac{h_{ab}^{\mu \nu}}{M_{Pl}^2} + \frac{3}{2} \frac{h^{ab}_{\mu \nu}}{M_{Pl} M_i} + \cdots$$

(2.12)

The action for the bigravity equation (2.1) up to quadratic order in the perturbations becomes

$$S = \int d^4x \left\{ h_{ab} \tilde{\epsilon}^{abcd} h_{cd} + l_{ab} \tilde{\epsilon}^{abcd} l_{cd} + \frac{\alpha_2 m^2}{4} \left( \left( h_{ab}^2 - [h]^2 \right) + \frac{M_{Pl}^2}{M_i} \left( l_{ab}^2 - [l]^2 \right) - 2 \frac{M_{Pl}}{M_i} \left( h_{ab} l_{ab} - [h] [l] \right) \right) \right\} + \cdots$$

(2.13)

We can now diagonalize these interactions by making the following change of variables

$$h_{ab} \rightarrow M_{Pl} \left( \omega_{ab} + v_{ab} \right) \quad l_{ab} \rightarrow M_i \left( \omega_{ab} - v_{ab} \right)$$

(2.14)

3 One could have worked with the $\pi$ field instead of $\rho$ at this stage. In this case one would have $\tilde{g}_{ab}^{(m)} = \frac{1}{2} \frac{\lambda}{4 (4 - m) (m - 1)} \tilde{g}_{ab} \delta^{m-1} \epsilon^{\alpha_\mu} \epsilon_{\alpha_\nu} \tilde{g}_{\nu \rho} \epsilon^{\rho \sigma} \tilde{g}_{\sigma \mu} \epsilon^{\mu \nu} \epsilon^{\nu \rho} \epsilon^{\rho \sigma}$ and hence an infinite series of interactions between the $\pi$ field and the helicity two field $l^{ab}$. However, each of these interactions are contracted with the antisymmetric Levi-Civita tensors $\epsilon^{\alpha_\mu} \epsilon_{\alpha_\nu}$ and exactly the same non-renormalization theorem would protect this infinite series of interactions from quantum corrections. However, the interactions of the second helicity two field $l^{ab}$ to the helicity-zero mode are most naturally expressed in terms of the $\rho$ field.
such that the action at quadratic order in perturbations becomes [58]

\[
S = \int d^4x \left\{ \kappa_{ab} \bar{e}^{abcd} \kappa_{cd} + \kappa_{ab} \bar{e}^{abcd} \kappa_{cd} + \alpha_2 m^2 M_P^2 \left[ \left( \kappa^2 \right) - \left| \kappa \right|^2 \right] \right\}.
\]

(2.15)

In the unitary gauge \( \kappa_{ab} \) encodes all five physical degrees of freedom of a massive spin-2 fluctuation (the two helicity-2, the two helicity-1 and the helicity-0 modes), and \( \kappa_{ab} \) encodes the two helicity-2 modes of the massless fluctuation. The propagator in the Euclidean space for the massless spin-2 fluctuation \( \kappa_{ab} \) is given by

\[
G_{abcd}^{(w)} = \left\{ \kappa_{ab}(x_1) \kappa_{cd}(x_2) \right\} = f_{abcd}^{(w)} \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x_1-x_2)}}{k^2},
\]

(2.16)

where the polarization structure has the usual prefactor of 1/2

\[
f_{abcd}^{(w)} = \delta_{a(c} \delta_{bd)} - \frac{1}{2} \delta_{ab} \delta_{cd}.
\]

(2.17)

with \( \delta_{a(c} \delta_{bd)} \equiv \frac{1}{2} \delta_{ac} \delta_{bd} + \frac{1}{2} \delta_{ad} \delta_{bc} \). The massive spin-2 field, on the other hand, has the propagator

\[
G_{abcd}^{(v)} = \left\{ \kappa_{ab}(x_1) \kappa_{cd}(x_2) \right\} = f_{abcd}^{(v)} \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x_1-x_2)}}{k^2 + m^2},
\]

(2.18)

with the prefactor of 1/3 in the polarization structure

\[
f_{abcd}^{(v)} = \left( \delta_{a(c} \delta_{bd)} - \frac{1}{3} \delta_{ab} \delta_{cd} \right) \text{ where } \delta_{ab} = \delta_{ab} + \frac{k_a k_b}{m^2}.
\]

(2.19)

### 3. Graviton loops

In this section we will study the quantum corrections generated by the graviton loops. We will be concentrating on one-loop diagrams. We will be only interested in the IR limit of the theory, therefore consider zero external momenta. We will apply dimensional regularization and thus focus only on the running of the interaction couplings.

#### 3.1. Preparative study

The quantum corrections in the decoupling limit of bigravity follow the same non-renormalization theorem as in massive gravity. In the decoupling limit the coupling to matter fields are suppressed as \( M_f \to \infty \) and \( M_p \to \infty \). So the decoupling limit of the bimetric theory is completely safe. Now here we want to investigate the quantum corrections of the full nonlinear bimetric theory coming from purely graviton loops.

Our starting point will be expanding the potential interactions in equation (2.1) in terms of the fluctuations in equations (2.4). Before starting the computation we can already gain a lot by noting that the separate \( h_{ab} \) and \( l_{ab} \) interactions without any mixing between them will give rise to the same results as we obtained in massive gravity. The crucial point for that is that once we express the fluctuations in terms of the mass eigenstates \( \kappa_{ab} \) and \( \kappa_{ab} \), then the one loop contributions in which only the massless degree of freedom ran will give rise to zero contributions in dimensional regularization, i.e. in the cut-off regularization there are no logarithmic divergences. Thus, if we have a graviton one-loop with only the massless mode \( \kappa_{ab} \) running in it, then this will give zero contribution since we have a contribution of the
form

\[ \int d^d k \frac{\Lambda(k, \mu)}{k^2} = 0 \text{ in dimensional regularization} \quad (3.1) \]

If we consider one-loop diagrams in which either only the fluctuations of the metric \( g_{ab} \) or only fluctuations of the metric \( f_{ab} \) come in, then the quantum corrections from the massless mode \( w_{ab} \) will give zero contribution while the one for the massive mode \( v_{ab} \) will end up giving the same contribution as in massive gravity (see figure 1).

Therefore, we already gained a lot by realizing this and we only need to concentrate on the contributions coming from the mixed diagrams (see figure 2). But from the mixed diagrams we only need to consider those cases in which only the massive mode runs or where massive and massless mode run in the same loop but never the mixed diagrams with purely massless mode running in the loop. We also expect here that the separate diagrams will give rise to detuning of the potential interactions as in massive gravity. Since the \( f_{ab} \) is dynamical, we have one full diffeomorphism invariance which might give rise to a better behaviour at the quantum level and some cancellations might be possible. However, we will see that this is not the case, at least among the diagrams constructed with the potential interactions.

The two Einstein–Hilbert terms include an infinite amount of interactions for \( h_{ab} \) and \( l_{ab} \)

\[ - \frac{1}{2} M_0^2 \sqrt{-g} R_g = h^a_b \delta^{\mu}_{\nu} h_{\mu\nu} + \frac{1}{M_{Pl}} h(\partial h)^2 + \frac{1}{M_{Pl}^2} h^2 (\partial h)^2 + \cdots, \]

\[ - \frac{1}{2} M_f^2 \sqrt{-f} R_f = f^a_b \delta^{\mu}_{\nu} f_{\mu\nu} + \frac{1}{M_f} f(\partial f)^2 + \frac{1}{M_f^2} f^2 (\partial f)^2 + \cdots, \quad (3.2) \]

while the potential only includes a finite number of interactions in \( h_{ab} \) and \( l_{ab} \)

\[ U = -\frac{1}{4} m^2 M_{Pl}^2 \sum_{n=1}^{4} \frac{\alpha_i}{n!(4-n)!} U_n[h, l] \quad (3.3) \]

where the individual potential terms \( U_n \) can be expressed as

\[ U_0[h] = \epsilon^{abcd} \epsilon^{ab'c'd'} \left( \delta_{ad'} + h_{ad'} \right) \left( \delta_{bb'} + h_{bb'} \right) \left( \delta_{cc'} + h_{cc'} \right) \left( \delta_{dd'} + h_{dd'} \right) \]

\[ U_1[h, l] = \epsilon^{abcd} \epsilon^{ab'c'd'} \left( \delta_{ad'} + h_{ad'} \right) \left( \delta_{bb'} + h_{bb'} \right) \left( \delta_{cc'} + h_{cc'} \right) \left( \delta_{dd'} + l_{dd'} \right) \]

\[ U_2[h, l] = \epsilon^{abcd} \epsilon^{ab'c'd'} \left( \delta_{ad'} + h_{ad'} \right) \left( \delta_{bb'} + h_{bb'} \right) \left( \delta_{cc'} + l_{cc'} \right) \left( \delta_{dd'} + l_{dd'} \right) \]

\[ U_3[h, l] = \epsilon^{abcd} \epsilon^{ab'c'd'} \left( \delta_{ad'} + l_{ad'} \right) \left( \delta_{bb'} + l_{bb'} \right) \left( \delta_{cc'} + l_{cc'} \right) \left( \delta_{dd'} + l_{dd'} \right) \]

\[ U_4[l] = \epsilon^{abcd} \epsilon^{ab'c'd'} \left( \delta_{ad'} + l_{ad'} \right) \left( \delta_{bb'} + l_{bb'} \right) \left( \delta_{cc'} + l_{cc'} \right) \left( \delta_{dd'} + l_{dd'} \right) \quad (3.4) \]

where indices are lowered and raised with respect to the flat euclidean metric \( \delta_{ab} \). From the five parameters \( \alpha_n \) we can fix two of them by causing the two tadpole contributions for \( h_{ab} \) and \( l_{ab} \) to vanish:

\[ \alpha_1 = \frac{1}{2} (-\alpha_0 + 2\alpha_3 + \alpha_4), \alpha_2 = \frac{1}{6} (\alpha_0 - 8\alpha_3 - 3\alpha_4) \quad (3.5) \]

At the linear order in perturbations it is trivial to split the mass spectrum of the bigravity theory into the massless and massive spin-2 fluctuations. At the non-linear level and around general backgrounds this split is usually not well-defined [80]. However, here we will only need the linear split around euclidean backgrounds in the same way as was done in [38]. We will replace in the above potential terms the fluctuations \( h_{ab} \) and \( l_{ab} \) in terms of the mass
modes in equations (2.14) and compute the leading Feynman diagrams one by one and add up their contributions.

As mentioned previously, we will make use of the dimensional regularization technique. A recurrent integral of the form

$$\int_\pi^\infty \frac{d^n k}{(2\pi)^4} \frac{k^{2n}}{(k^2 + m^2)^n}$$

will appear in all of our calculations throughout this paper. Using the relation $J_{m,n} = \frac{\Gamma(n+1)}{2} J_{m,1}$ we can express the integrals just in terms of $J_{m,1}$. Its exact expression is irrelevant. The only important point to keep in mind is that it contains the logarithmic divergence in the mass scale $m$ (for more detail see [38]).
3.2. Tadpole contributions

For the tadpole contributions at one-loop level, we only have four diagrams to consider as depicted in figure 3. However, only two of them give non-trivial contributions. The first two tadpole contributions come from cubic and quadratic interactions in the massless mode $w_{ab}$ which give exactly zero $A_{3v}^{(1)_{ps}} = 0$ and $A_{3v}^{(1)_{pq}} = 0$. On the other hand, the pure third order interactions in the massive mode

$$U_{3v} = -\frac{1}{12}m^2 M_1^2 \left( a_0 - a_4 \right) \left( [v]^3 - 3[v]^2 \left[ v^2 \right] + 2\left[ v^3 \right] \right)$$

(3.7)

gives rise to the same tadpole contribution for the massive mode as in massive gravity [38]

$$A_{3v}^{(1)_{ps}} = -\frac{1}{12}(-1)(3)(a_0 - a_4) m^2 M_1^2 v_{1 \mu} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + m^2} g^{abcd} g^{a\beta \gamma} d_{\mu \nu \rho \tau}^{(v)}$$

$$\quad = \frac{5}{8} (a_0 - a_4) M_1^2 [v] \int \frac{d^4k}{(2\pi)^4} \frac{k^2 + 2m^2}{k^2 + m^2}$$

$$\quad = -\frac{5}{8} m^4 M_1^2 (a_0 - a_4) [v] J_{m,1}$$

(3.8)

Finally, the contribution coming from the mixed interactions

$$U_{2v,w} = -\frac{1}{12} m^2 M_1^2 (a_0 + 4a_3 + 3a_4) \left( 2v_{a}^{c} v^{ab} - [v] v_{bc} \right) w_{hc}$$

$$\quad + \left( -[v]^3 + [v]^2 \right) [w]$$

(3.9)

gives a non-trivial new contribution in form of a tadpole for the massless mode

$$A_{2v,w}^{(1)_{ps}} = \frac{5}{48} m^4 M_1^2 (a_0 + 4a_3 + 3a_4) [w] J_{m,1}$$

(3.10)

The one loop contributions coming from cubic order interactions give rise to non-vanishing tadpole contributions for the massive $v_{ab}$ and massless $w_{ab}$ modes. We can now express the massive and massless fluctuations $v_{ab}$ and $w_{ab}$ back in terms of the fluctuations $h_{ab}$ and $l_{ab}$, which will result in tadpole contributions for $h_{ab}$ and $l_{ab}$.

3.3. Two-point function contributions

In a similar way we can now compute the two-point functions. There are more diagrams which contribute at the level of the two-point function. Let us start with the Feynman diagrams with 4-vertices giving rise to a ‘tadpole 2-point function’. Similarly as before, the diagrams with the massless mode $w_{ab}$ running in the loop will give rise to zero contribution. Thus, the interactions symbolically of the form $\tilde{\psi}^3 \tilde{\psi}$, $\tilde{\psi}^2 \psi^2$ and $\psi^4$ will give zero contributions $A_{4v}^{(2)_{ps}} = 0$, $A_{4v,2w}^{(2)_{ps}} = 0$ and $A_{4v,3w}^{(2)_{ps}} = 0$ for the diagrams with $w$ running in the loop. For the non-trivial contributions, let us first consider the mixed interactions in which there are three $v_{ab}$ modes and one $w_{ab}$ mode coming in
These interactions will give rise to quantum corrections in the following form:\(^4\)

\[ A_{3n,w}^{(2n)} = \frac{5}{24} m^2 M_{Pl}^2 (a_0 - a_4) \left( [v] w + v_{ab} w_{ab} \right) J_{m,1} \]  

(3.12)

Next, consider the Feynman diagram with the two massless modes \(w_{ab}\) on the external lines and the two massive modes \(v_{ab}\) running in the loop with the corresponding interactions given by

\[ U_{2n,w} = \frac{1}{24} m^2 M_{Pl}^2 (a_0 + 4a_3 + 3a_4) \left( v_{cd} \left( 2 v_{ab} w_{bd} - w_{ab} w_{cd} \right) + w_{cd} \left( 4 v_{a} v_{b} w_{cd} - v_{ab} w_{cd} \right) + \left( -4 v_{a} v_{b} w_{cd} + v_{ab} \right) \right) \]

+ \[ v \left( w_{cd} \left( -4 v_{b} w_{cd} + \left( [v] w \right) \right) + \left( 4 v_{b} w_{cd} - \left( [v] w \right) \right) \right) \]  

(3.13)

These interactions give the following non-trivial contribution

\[ A_{2n,2n}^{(2n)} = -\frac{5}{144} m^4 M_{Pl}^4 \left( a_0 + 4a_3 + 3a_4 \right) \left( \left[ w^2 \right] - \left[ w \right] \right) J_{m,1} \]  

(3.14)

The last tadpole two-point function is the one corresponding to the massive gravity case, in which namely two massive modes \(v_{ab}\) run on the external legs while the other two run in the loop. The interaction is given by

\[ U_{4v} = \frac{1}{24} m^2 M_{Pl}^2 (a_0 - 4a_3 - a_4) \left( v_{cd} \left( 6 v_{a} v_{b} v_{cd} - 3 \left[ v^2 \right] v_{cd} \right) - \left[ v \right] \left( 8 v_{b} v_{cd} - 6 \left[ v \right] v_{cd} \right) \right) \]  

(3.15)

The contribution gives the same result as in massive gravity

\[ A_{4v}^{(2n)} = -\frac{5}{24} m^4 M_{Pl}^4 \left( a_0 - 4a_3 - a_4 \right) \left( \left[ v^2 \right] - \left[ v \right] \right) J_{m,1} \]  

(3.16)

These tadpole diagrams as depicted in figure 4 generate quantum corrections which preserve the nice structure of the potential. Their contributions to the counter-terms are of the form

\[ \mathcal{L}_{CT} = c_1 \left( \left[ h^2 \right] - \left[ h \right] \right) + c_2 \left( \left[ l^2 \right] - \left[ l \right] \right) + c_3 \left( \left[ h \right] \left[ l \right] - h_{ab} l^{ab} \right) \]  

(3.17)

where the parameters \(c_1, c_2\cdots\) etc are the placeholders for the renormalized parameters \(a_n\). Their specific form is irrelevant for now (even though they are important for the purposes of possible exact cancellations for which we took them into account). The important fact is that these one-loop corrections of figure 4 renormalize the potential interactions but do not give rise to detuning. So far, this is excellent news. Actually, exactly the same thing happens in massive gravity (which corresponds to the case where only the last diagram of figure 4 contributes) since the tadpole 2-point function does not detune the mass term. However, this

\(^4\) Note that we also take into account the mirror reflected Feynman diagram by multiplying the result by a factor of two.
Figure 4. One-loop contribution to the 2-point correlation function from a graviton internal line coming from quartic interactions.

Figure 5. 1-loop contributions to the 2-point correlation functions with two vertices.
optimistic result will not prevail for other corrections, which will indeed detune the specific structure of the potential interactions. To see this, let us now continue with the Feynman diagrams which contain two vertices. They are all shown in figure 5 (we have omitted those diagrams that yield zero contributions). The first diagram consists of two vertices with each vertex containing the interaction in equation (3.9) and gives the following contribution

\[ A_{2\nu,1-2\nu,1}^{(2n)} = \frac{5}{216} m^4 M_{Pl}^4 \left( a_0 + 4a_3 + 3a_4 \right)^2 \left( 2 \left[ \nu^2 \right] + \left[ \nu^2 \right] \right) J_{m,1} \]  

(3.18)

Similarly, the second Feynman diagram contains a vertex with the interaction in equation (3.9) while the other vertex being the interaction given in (3.7). Its contribution reads

\[ A_{2\nu,1-3\nu,1}^{(2n)} = \frac{5}{576} m^4 M_{Pl}^4 \left( a_0 - a_4 \right) \left( a_0 + 4a_3 + 3a_4 \right) (8[vv] + 7[v][w]) J_{m,1}. \]  

(3.19)

The third diagram on the other hand is constructed purely out of the cubic interaction in \( v_{ab} \) (3.7). After performing the integration over the internal momenta, it results in

\[ A_{3\nu,1-3\nu,1}^{(2n)} = \frac{5}{32} m^4 M_{Pl}^4 \left( a_0 - a_4 \right)^2 \left( 8 \left[ \nu^2 \right] + 7[v][w] \right) J_{m,1}. \]  

(3.20)

Last but not least, the forth diagram in figure 5 generates a contribution of the similar form

\[ A_{2\nu,1-2\nu,1}^{(2n)} = \frac{5}{2592} m^4 M_{Pl}^4 \left( a_0 + 4a_3 + 3a_4 \right)^2 \left( 8 \left[ w^2 \right] + 7[w][w] \right) J_{m,1}. \]  

(3.21)

As can be seen from the contributions computed above all these two-point functions in figure 5 give rise to a detuning of the potential interactions

\[ \mathcal{L}_{CT} = \left( c_1 [h]^2 - c_2 \left[ \tilde{h}^2 \right] \right) + \left( c_3 [l]^2 - c_4 [l]^2 \right) + \left( c_5 [h][l] - c_6 h_{ab} l^{ab} \right) \]  

(3.22)

where again the parameters \( c_1, c_2 \ldots \) etc encode the detuning of the classical parameters at the quadratic order. These parameters are different from the classical parameters. We do not need to compute the contributions to higher \( n \)-point functions at this stage, since we already explicitly checked that there is no cancellation happening between the diagrams and exactly the same detuning of the potential interactions happens in massive gravity. These generated quantum corrections to the tadpole and 2-point functions can also not be resumed into cosmological constants. Thus, bigravity seems to share the same destiny as massive gravity and the quantum corrections coming from the graviton loops detune the nice potential interactions and reintroduce the ghost.

3.4. Scaling of the detuning of the potential interactions

We have explicitly seen above that quantum corrections generated by the graviton loops destroy the very specific structure of the ghost-free potential of massive bigravity exactly in the same way as in massive gravity. The tadpole 2-point functions maintain the potential interactions, however all the other remaining contributions detune the potential and do not cancel each other nor do they combine into a cosmological constant. A detuning at the level of the 2-point function is harmless, since the associated ghost has a mass of the order of the Planck mass. However, there will be also a detuning at higher order \( n \)-point functions, which might be more harmful. Assuming that such a detuning happens, we would like to estimate the consequences for the mass of the ghost. The detuning goes as
We can expand these contributions to quadratic order around the backgrounds $h = \tilde{h}$ and $l = \tilde{l}$, which will detune the Fierz–Pauli structure
\begin{equation}
\mathcal{L}_{CT, h, l} \sim c_l \frac{M_{Pl}^4}{M^4} h^l + d_l \frac{M_{Pl}^4}{M^4} h^{l} + e_l \frac{M_{Pl}^4}{M^4} h^{l-j} + f_l \frac{M_{Pl}^4}{M^4} h^{l-j}, \tag{3.24}
\end{equation}
In terms of the helicity-0 degree of freedom this would imply a higher order derivative operator with the mass of the ghost scaling as
\begin{equation}
\mathcal{L}_{CT, h, l} \sim \left( \frac{\partial^2}{m^2_{\text{ghost}}} \right)^2, \quad \text{with} \quad m_{\text{ghost}} = \left( \frac{M_{Pl}}{h} \right)^{\gamma/2} h + \left( \frac{M_{Pl}}{l} \right)^{\gamma/2} l + \cdots. \tag{3.25}
\end{equation}
In the vicinity of small background configurations, the mass of the ghost is very large and hence the ghost is harmless. However, around arbitrarily large background configurations the mass of the ghost can be made arbitrarily small, which \textit{a priori} is a problem. Nevertheless, this is not the end of the story. For large background configurations, the Vainshtein mechanism needs to be inserted at the quantum level. We expect that the Vainshtein mechanism will repackage the one-loop effective action and suppress the quantum corrections around large background configurations, exactly in the same way as in massive gravity. A detailed investigation of this is out of the scope of this work. The mathematically challenging computation of the one-loop effective action with the Vainshtein mechanism implemented will be studied somewhere else.

4. Matter loops

In massive (bi-)gravity the existence of the two metrics comes hand in hand with the natural question of how these two metrics can couple to the matter sector consistently. First this has to be established successfully at the classical level and then as a following step one needs to make sure that this property can be further extended to the quantum level. However, this will not be the philosophy that we will be following here. We will follow the same logic as in [39] and demand the requirement of quantum stability to deduce the possible ways of coupling to matter fields. If the matter field couples to only one metric the classical theory is free of any ghost instability [58]. This property is also maintained at the quantum level since the quantum corrections do not renormalize the potential interactions or detune them but rather contribute in the form of a cosmological constant [39]. This will be one of the valid couplings that we will be considering here. We will disregard the case in which the matter field couples to both metrics at the same time, since for this coupling there is a ghost degree of freedom already present at the classical level [39] and the quantum corrections do detune the potential interactions. We will consider the case in which new effective composite metrics can be constructed from lessons learned from quantum corrections and to which the matter field can couple to both metrics simultaneously.

4.1. Coupling to separate matter sector

In massive gravity the dynamical metric $g_{ab}$ can be coupled covariantly to the matter sector without altering the number of propagating degrees of freedom. This nice property remains valid at the quantum level as well since the quantum corrections give rise to a contribution in
the form of a cosmological constant \[38\]. Therefore a promising way of coupling the two metrics in bigravity is through an independent coupling to separate matter sector

\[ \chi_1 + \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \frac{1}{M_1} \left( g^{ab} \partial_a \partial_b x_1 + M_1^2 x_1^2 \right) + \frac{1}{M_2} \left( f^{ab} \partial_a \partial_b x_2 + M_2^2 x_2^2 \right). \] \hspace{1cm} (4.1)

where we assumed massive scalar fields as matter fields for simplicity. The two scalar fields \( \chi_1 \) and \( \chi_2 \) with masses \( M_1 \) and \( M_2 \) couple separately to \( g_{ab} \) and \( f_{ab} \) respectively, but not to both simultaneously. Similarly to what happens in massive gravity the contributions to the one loop effective action are in the form of additive cosmological constants for \( g_{ab} \) and \( f_{ab} \) \[39\]

\[ \mathcal{L}_{\text{matter}} = \frac{1}{2} \sqrt{g} \left( g^{ab} \partial_a \partial_b \chi_1 + M_1^2 \chi_1^2 \right) + \frac{1}{2} \sqrt{f} \left( f^{ab} \partial_a \partial_b \chi_2 + M_2^2 \chi_2^2 \right). \] \hspace{1cm} (4.1)

Even if we force at the classical level that only the \( g_{ab} \) metric couples to the matter field \( \chi_1 \), i.e. there is no coupling between the metric \( f_{ab} \) and the matter field \( \chi_1 \), it is an unavoidable question we have to pursue whether or not quantum corrections will generate couplings between \( f_{ab} \) and \( \chi_1 \), if so at which scale they become important. Diagrams as shown in figure 6 will indeed generate new coupling between \( f_{ab} \) and the matter field \( \chi_1 \). At the one vertex the interactions coming from the potential at quadratic order in \( h_{ab} \) and linear order in \( \delta \chi \) will contribute. We shall keep in mind that the potential interactions have a very specific antisymmetric structure and the one acting in the above diagram has the form

\[ \left( \frac{3}{2} h_a^a h^{ab} - [h] h_{ab} + \frac{1}{4} \left( [h]^2 - \left[ h^2 \right] \right) \delta^{ab} \partial_a \partial_b \chi_1 + \frac{M_1^2 \chi_1^2}{4} \left( [h]^2 - \left[ h^2 \right] \right) \right) \] \hspace{1cm} (4.3)

Figure 6. One-loop contributions to the 3-point function \( f_{ab} \delta^{ab} \chi_1^2 \). Dashed lines denote the matter field \( \chi_1 \).

We can contract the \( h_{ab} \) field of the potential interaction with an external \( h_{ab} \) leg coming out of this vertex while the other two \( h_{ab} \)-fields from this vertex run in the loop with momenta \( k_a \) and \( (p_1 + p_2 - k) \), and contract them with the two spin-2 fields coming from the coupling in equations (4.3). Strictly speaking it is not the \( h_{ab} \) field which is running in the loop but the massless and massive modes which are diagonal. So the propagator for the \( h_{ab} \) field \( \langle h_{ab} h_{cd} \rangle \) needs to be replaced by the sum of the propagators of the massless mode \( w_{ab} \) and massive mode \( v_{ab} \). Since these two modes are diagonal there will not be any mixing of the form \( \langle w_{ab} v_{cd} \rangle \). One could worry that since the gravitons are running in the loops that the contribution of this diagram might scale with an inverse power of \( m \) coming from the propagator of the massive mode \( v_{ab} \). Since we have two graviton propagators in this diagram, if each internal propagator comes at least with \( k^4/m^4 \) from the massive mode than the contribution of this vertex to the graph with the most negative powers of the graviton mass would be \( m^{-8} \).
\[ \mathcal{A} \propto \int \frac{d^4k}{(2\pi)^4} \varepsilon^{abcd} \varepsilon^{a'b'c'd'} \frac{k_a k_{a'} k_{b'} k_{b'}}{m^2} \left( \frac{(p_1 + p_2 - k)}{m^2} \right)^{\frac{m^2}{m^2}} \left( \frac{(p_1 + p_2 - k)}{m^2} \right)^{\frac{m^2}{m^2}} \cdots \]

Due to the antisymmetric structure of the interactions \( \varepsilon^{abcd} \varepsilon^{a'b'c'd'} \), this contribution cancels exactly. The Levi-Civita tensors are antisymmetric while the momenta are symmetric. Exactly the same thing happens to the contribution with the \( m^{-6} \) scaling. Nevertheless, this argument applies only to the contributions with \( m^{-6} \) and \( m^{-8} \) scalings since there are enough momenta which give rise to symmetric contributions. Actually, this will be true for any \( n \)-point function. We will have diagrams which contribute to the \( n \)-point function with the highest number of internal graviton propagators \( n+1 \) scaling with the most negative power of \( m^{-4(n+1)} \). However, they will cancel exactly as in the above diagram. The cubic vertex from the potential term gives zero to the leading order and \( m^2 \) scaling to the second leading order. Therefore for the \( n \)-point function, each vertex cannot contribute with more than \( m^{-2} \), meaning that the divergence is at worst like \( m^{-2(n+1)} \) rather than \( m^{-4(n+1)} \). Exactly the same reasoning applies to quartic and higher-dimensional vertices in the field. Each internal propagator comes at least with \( k^2/m^2 \). In the case in which each propagator contributes \( k^2/m^2 \) would give rise to \((k/m)^{2n}\) contributions in the vertex which is fully symmetric, and would cancel due to the antisymmetric structure of the potential term with the Levi-Civita tensors.

Returning to our diagram, we explicitly saw that the contributions scaling with \( m^{-6} \) and \( m^{-8} \) powers cancel. However, the above argumentation does not apply on the contributions with \( m^{-2} \) and \( m^{-4} \) scalings, and we need to check their implications. Unfortunately, there are indeed contributions with such a scaling. Let us investigate how the dependence on the \( m \) scaling will change the scale at which these interactions between \( f_{ab} \) and the matter field \( \chi_1 \) will become important. The above diagram will give rise to interactions between \( f_{ab} \) and the matter field \( \chi_1 \) at the scale

\[ m^2 M_P^2 \frac{1}{M_f} \frac{1}{M_{\chi_1}} \frac{1}{m^{2(n+1)}} f_{ab} \delta^{ab} \chi_1 \chi_1 \]

Now, we can compute the corrections coming from these new interactions (which we know will give rise to a ghost degree of freedom). Consider a diagram in which \( h_{ab} \) and \( l_{ab} \) run on the external legs while the \( \chi_1 \) field runs in the loop. Such diagrams will scale as

\[ M_P^2 M_{\chi_1} \frac{1}{M_f} \frac{1}{M_{\chi_1}} \frac{1}{m^{2(n+1)}} \]

Without loss of generality, assume for clarity that \( M_f = M_{\chi_1} \), then the ghost associated with the higher derivative operators applied on the Stueckelberg field would come at a scale

\[ m^{2(1-\epsilon)} M_P^2 \frac{1}{M_f} \frac{1}{M_{\chi_1}} \frac{1}{m^{2(\epsilon+1)}} \]

In diagrams similar to 6, at the one vertex with the interaction symbolically of the form \( h' \rightleftharpoons \) will give rise to a scaling contribution in form of \( 1/M_f^2 \) and at the other vertex one has the interaction coming from the potential symbolically of the form \( h' \) with the corresponding scaling \( m^2 M_P^2 / (M_f^2 M_f) \). Additionally, each internal line of the massive mode will give rise a contribution of the form \( k^{2(n+1)/m^2} \) where \( n \) indicates the number of internal lines and \( k \) the internal momenta.
meaning that the mass of the ghost would correspond to

\[ m_{\text{ghost}}^2 = m^{2(1+n)} M_\text{Pl}^4 \frac{1}{M_1^{2(2+n)}} \]  

(4.8)

We have seen above explicitly that the contributions with \( n \geq 3 \) cancel exactly due to the antisymmetric structure of the potential interactions. The only two cases we need to check are \( n = 1 \) and \( n = 2 \). Let us assume that the mass of the matter field is close to \( \Lambda \approx M_1^{13} \). Then, for \( n = 1 \) we would have

\[ m_{\text{ghost}}^2 = \frac{\Lambda^2 M_1^2}{\Lambda^2} \]  

which is larger than the strong coupling scale \( \Lambda_3 \).

Similarly, for \( n = 2 \) we would obtain

\[ m_{\text{ghost}}^2 = \frac{\Lambda^2 M_1^2}{\Lambda^2} \]  

which is as well beyond the scale \( \Lambda_3 \).

Thus, even if the quantum corrections reintroduce a coupling between \( f_{ab} \) and \( \chi_1 \) which was put to zero at the classical level, the scaling of this new coupling would yield a ghost well beyond the strong coupling scale.

4.2. Coupling the matter sector to both metrics

If one insists on coupling the matter sector to the two metrics \( g_{ab} \) and \( f_{ab} \) at the same time, the quantum corrections restrict the possible ways crucially. If the quantum corrections detune the very specific potential structure, then the ghost degree of freedom reappears with a scaling that can be made arbitrarily small by choosing the mass of the matter field accordingly. In [39] a new type of coupling to matter was proposed. The coupling occurred through an effective composite metric \( g_{\text{eff}} \) built out of both metrics \( g_{ab} \) and \( f_{ab} \)

\[ g_{ab} = \alpha^2 g_{ab} + 2\alpha\beta g_{bc} \left( \sqrt{g_{ab} f_{cd}} \right)^c_b + \beta^2 f_{ab} , \]  

(4.9)

with arbitrary constants \( \alpha \) and \( \beta \). The one-loop contributions through matter loops do not contribute in the form of a cosmological constant with respect to \( g_{ab} \) or \( f_{ab} \) but rather with respect to this composite metric \( \sqrt{\det g_{\text{eff}}} \) and it was constructed by the requirement that it corresponds to the ghost-free potential interactions of massive (bi-)gravity. In the following we will propose yet another class of new effective composite metrics to which the matter fields can couple and not reintroduce the ghost-freedom at the quantum level.

4.2.1. Contributions in the form of cosmological constants. One possible way of constructing the effective composite metric, which was not considered in [39], comes from the additive contributions of the cosmological constants for \( g_{ab} \) and \( f_{ab} \). One has to demand that the determinant of the effective metric is such that it fulfills the following relation

\[ \sqrt{\det g_{\text{eff}}} = \sqrt{\det g} + \sqrt{\det f} \]  

(4.10)

If the matter sector would couple to an effective composite metric with a determinant as given in equation (4.10), then the quantum corrections would not render the theory unnatural. The contributions of matter loops would be the sum of the cosmological constants for \( g_{ab} \) and \( f_{ab} \) and would not renormalize the potential interactions. The naturalness of massive gravity is one of the essential strength of massive gravity and it would unfortunate to lose this nice property. The above relation in equation (4.10) allows only for those types of composite effective metrics that maintain the naturalness property of massive gravity. The solution for the effective composite metric will be then simply given by
with an arbitrary matrix $\hat{M} = M_{ab}$ with its determinant fixed to be one, $\det (M_{ab}) = 1$. This requirement only fixes one of the components of the matrix $\hat{M}$ such that one has a nine parametric solution to the above equation (4.10). The most trivial solution would be $\hat{M} = \delta_{ab}$, in this case the effective composite metric would be simply given by\(^6\)

$$g_{ab}^{\text{eff}} = \left( \sqrt{\det \hat{g}} + \sqrt{\det \hat{f}} \right)^{1/2} \delta_{ab} \quad (4.12)$$

Other perfectly valid solutions would be for instance

$$\hat{M} = \frac{\hat{g}}{\left( \sqrt{\det \hat{g}} \right)^{1/2}}, \quad \hat{M} = \frac{\hat{f}}{\left( \sqrt{\det \hat{f}} \right)^{1/2}},$$

$$\hat{M} = \frac{\gamma_1 \hat{g} + \gamma_2 \hat{f} + \gamma_3 \hat{g} \sqrt{\hat{g}^{-1} \hat{f}}}{\left( \det \left( \gamma_1 \hat{g} + \gamma_2 \hat{f} + \gamma_3 \hat{g} \sqrt{\hat{g}^{-1} \hat{f}} \right) \right)^{1/4}}$$

for which the new effective metric would then correspond to

$$g_{ab}^{\text{eff}} = \left( \sqrt{\det \hat{g}} + \sqrt{\det \hat{f}} \right)^{1/2} \frac{g_{ab}}{\left( \sqrt{\det \hat{g}} \right)^{1/2}}$$

$$g_{ab}^{\text{eff}} = \left( \sqrt{\det \hat{g}} + \sqrt{\det \hat{f}} \right)^{1/2} \frac{f_{ab}}{\left( \sqrt{\det \hat{f}} \right)^{1/2}}$$

$$g_{ab}^{\text{eff}} = \left( \sqrt{\det \hat{g}} + \sqrt{\det \hat{f}} \right)^{1/2} \frac{\gamma_1 \hat{g} + \gamma_2 \hat{f} + \gamma_3 \hat{g} \sqrt{\hat{g}^{-1} \hat{f}}}{\left( \det \left( \gamma_1 \hat{g} + \gamma_2 \hat{f} + \gamma_3 \hat{g} \sqrt{\hat{g}^{-1} \hat{f}} \right) \right)^{1/4}} \quad (4.13)$$

These are only some examples we mention. We can construct any arbitrary tensor of the form

$$\hat{N} = \frac{\hat{N}}{\left( \det (\hat{N}) \right)^{1/2}} \quad (4.14)$$

for which $\det (\hat{M}) = 1$ would be guaranteed. The tensor $\hat{N}$ could be any combination of the form $\hat{N} = \gamma_1 \hat{g} + \gamma_2 \hat{f} + \gamma_3 \hat{g} \sqrt{\hat{g}^{-1} \hat{f}} + \gamma_4 \hat{f} \sqrt{\hat{g}^{-1} \hat{f}} + \gamma_5 \hat{g} \sqrt{\hat{g}^{-1} \hat{f}} + \gamma_6 \hat{f} \sqrt{\hat{g}^{-1} \hat{f}} \cdots$ etc, constructed out of $\hat{g}$ and $\hat{f}$. Thus, any effective metric of the form of equation (4.11) with an arbitrary tensor $\hat{M}$ with $\det (\hat{M}) = 1$ will be a valid solution. All these effective composite metrics give rise to quantum contributions in the form of cosmological constants for $f_{ab}$ and $g_{ab}$ and thus fulfill our requirement in equaton (4.10) and do not destroy the naturalness of the theory. The above construction guarantees the quantum stability under matter loops. However, even if the quantum corrections do not introduce any ghost instability, it does not mean that the theory is free from the BD ghost at the classical level. Among all these possible effective metrics most of them will probably excite the BD ghost. All these new effective metrics needs to be changed accordingly to the Lorenzian signature $g_{\mu\nu}^{\text{LO}} = \left( \sqrt{-\det \hat{g}} + \sqrt{-\det \hat{f}} \right) \eta_{\mu\nu}$.\(^6\)

\(^6\) Note that we are working in the euclidean space in this work. If one switches to the Lorenzian space then the effective metric needs to be changed accordingly to the Lorenzian signature $g_{\mu\nu}^{\text{LO}} = \left( \sqrt{-\det \hat{g}} + \sqrt{-\det \hat{f}} \right) \eta_{\mu\nu}$.\(^6\)
metrics need to be carefully studied at the classical level, which is out of scope of this work
and will be studied somewhere else. Even if these couplings turn out to be not completely free
of the BD ghost, it would be interesting to study whether or not the decoupling limit is free of
the BD ghost, and what the mass of the ghost is exactly, such that the theory could be
considered as an effective field theory.

4.2.2. Contributions in the form of potential interactions. The other possible way of
constructing the effective metric, which was the criteria used in [39], corresponds to
demanding that the quantum corrections of matter loops are in the form of the allowed ghost-
free potential interactions

$$\sqrt{\text{det} \tilde{g}} = \sqrt{\text{det} \tilde{g}} \det \left( \alpha I + \beta \tilde{X} \right)$$

where \( \tilde{X} \) stands for \( \tilde{X} = \sqrt{\tilde{g}^{-1/2}} \). In this way the quantum corrections would not detune the
potential interactions and hence introduce ghost degrees of freedom, however, they would
renormalize the potential interactions and one would lose the naturalness argument. Again we
can find the solutions for the effective metric which fulfills the relation in equation (4.15). The
generic solution will be of course simply of the form

$$\tilde{g}_{\text{eff}} = \tilde{g} \left( \alpha + \beta \tilde{X} \right)^2 \tilde{M}$$

with again an arbitrary matrix \( \tilde{M} = M^a_b \) with the determinant \( \text{det} (M^a_b) = 1 \). The simplest
case with \( \tilde{M} = I \) was the one that was considered in [39] and gave rise to the effective metric
in equation (4.9). Any solution of the form in equation (4.16) with \( \text{det} (\tilde{M}) = 1 \) would fulfill
the relation in equation (4.15). Thus, we again have a nine parametric solution. For any
arbitrary matrix \( \tilde{M} \) with \( \text{det} (\tilde{M}) = 1 \) the effective metric would be given by equation (4.16)
where among all these solutions the simplest would be given in the above section

$$\tilde{M} = 1, \quad \tilde{M} = \frac{\sqrt{\tilde{g}^{-1}}} {\text{det} \left( \sqrt{\tilde{g}^{-1/2}} \right)^{1/4}}, \quad \tilde{M} = \frac{\sqrt{\tilde{g}^{-1} \tilde{g}}} {\text{det} \left( \sqrt{\tilde{g}^{-1}} \right)^{1/4}}$$

Let us emphasize again that in general it can be any matrix fulfilling \( \tilde{M} = \frac{\tilde{g}} {\text{det} (\tilde{M})^{1/4}} \).

These new effective composite metrics to which the matter field can couple do not
introduce the BD ghost at the quantum level. However, as mentioned above, again one has to
study carefully whether or not there is one (or several) of them which is also ghost-free at the
classical level. Even in the presence of ghost degrees of freedom, it would be crucial to study
the exact mass of the ghost and whether or not they can be considered as an effective field
theory with the cutoff scale given by the mass of the ghost. These constitute new avenues to
explore what we propose here and which might offer new interesting phenomenology, which
shall be studied in great detail in future works.

5. Conclusions

This work was dedicated to the study of quantum corrections in massive bigravity. Starting
with the leading interactions in the decoupling limit, we could generalize the non-renorma-
lization theorem to the case of bigravity. The decoupling limit of bigravity is safe from
quantum corrections. Beyond the decoupling limit, if we consider only one-loop contributions
coming from the interactions with the matter fields, they will only yield a contribution in
terms of a cosmological constant exactly as in massive gravity if the two metrics are coupled to different matter fields. In case the matter fields couple to both metrics at the same time, then the destabilization of the potential is unavoidable and the mass of the matter fields could be chosen such that the associated ghost appears below the strong coupling scale $\Lambda_3$ [39]. The same is true if the matter fields couple to different metrics but they interact with each other. Knowing the exact behavior of quantum corrections through matter loops, one can construct an effective composite metric through which the matter sector can couple to both metrics. Following the lessons learned in [39] we proposed yet other types of effective metrics which either give rise to contributions in the form of the cosmological constants for the two metrics (and hence maintaining the naturalness of the theory) or in the form of the allowed ghost-free potential interactions. These new composite metrics could give rise to consistent theories at the classical level, which should be carefully studied in future works. Furthermore, we have studied the quantum corrections coming from purely graviton loops. Since we have two dynamical metrics, there will be one massless and one massive spin-2 field running in the loops. We were able to show that the structure of the interactions between the two metrics gets destabilized through graviton loops, in exactly the same way as in massive gravity. It would be an interesting question to pursue whether or not the mass of the ghost can be pushed below the strong coupling scale around arbitrarily large backgrounds. For that purpose, one has to compute the one-loop effective action with the Vainshtein mechanism implemented in it. We expect a similar behavior as in massive gravity.

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