Relativistic Transport Theory for Systems Containing Bound States

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Abstract

Using a Lagrangian which contains quarks as elementary degrees of freedom and mesons as bound states, a transport formalism is developed, which allows for a dynamical transition from a quark plasma to a state, where quarks are bound into hadrons. Simultaneous transport equations for both particle species are derived in a systematic and consistent fashion. For the mesons a formalism is used which introduces off-shell corrections to the off-diagonal Green functions. It is shown that these off-shell corrections lead to the appearance of elastic quark scattering processes in the collision integral. The interference of the processes $q\bar{q} \rightarrow \pi$ and $q\bar{q} \rightarrow \pi \rightarrow q\bar{q}$ leads to a modification of the $s$-channel amplitude of quark-antiquark scattering.

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I. INTRODUCTION

The search for the quark-gluon plasma (QGP) as a new state of strongly interacting matter, where quarks and gluons become deconfined and can move freely, has been one of the most important subjects of nuclear and particle physics during the last decade. (For an overview see [1].) While experiments to detect the QGP in heavy ion collisions have been performed and are still performed today, the theoretical modeling of these experiments still contains many open questions, one of these being the description of the transition from a gas of quarks and gluons to a gas of hadrons. While hydrodynamical models [2] have been successfully applied as phenomenological approaches, the derivation of equations of motion from a microscopic Lagrangian has not yet been put forward sufficiently in order to treat a system, that initially contains quarks and gluons and which subsequently transforms into a system of hadrons.

Although the Schwinger–Keldysh formalism [3,4] provides a convenient tool for the derivation of transport equations from microscopic models, there are still open questions to be addressed before it can be applied to a hadronizing QGP. One imagines that the underlying Lagrangian should, as in quantum chromodynamics (QCD), not contain hadrons as elementary degrees of freedom, since these should appear as bound states. On the other hand one has to consider both quarks and hadrons, since the former constitute the relevant degrees in the initial, the latter those in the final stage of the collision. Hadrons have thus to be described as effective degrees of freedom and the formalism has to allow for a dynamical transition between these two regimes. This has to be accomplished in a systematic and consistent way in order to avoid double counting.

The second requirement for the theory is that it should be possible to derive collision terms for higher order processes. To be definite, say e.g. that in lowest order one obtains an interaction between quarks and pions via the process $q\bar{q} \leftrightarrow \pi$. In higher orders also the processes $q\bar{q} \leftrightarrow q\bar{q}$ [5] and $q\bar{q} \leftrightarrow \pi\pi$ [6] give important contributions and should be derivable within the theory. A first sketch of this programme has been given in Ref. [7]. As it turns out, this is by far the harder task to fulfill.

The description of nonequilibrium matter in terms of quasiparticles is in so far
problematic as it is strictly valid only in the low density limit. In Ref. [8] however, a formalism has been developed which to a certain extent overcomes this problem and has been successfully applied to semiconductor physics. The basic trick of this approach is a decomposition of the off-diagonal Green functions of the Schwinger–Keldysh formalism into a singular and a regular part. It turns out that in leading order of a gradient expansion the singular part obeys a Boltzmann equation, whereas the regular part gives corrections to the collision terms. It has been shown already in Ref. [8], that these corrections are essential if collision terms for higher order processes like those described above have to be considered.

In the present paper, the approach of Ref. [8] is thus applied to the dynamical formation of pions out of a quark plasma. As a microscopic model for the interaction the Nambu–Jona-Lasinio (NJL) model [9,10] is used. Although this model fails to provide a complete description of a QGP since it is not confining and does not contain gluons, it has the same internal symmetries as QCD and thus gives a good description of the low energy mesonic sector that is governed by strong interactions. The underlying principle of this approach is the concept of chiral symmetry breaking, which forms also the basis of other phenomenological models such as chiral perturbation theory and the sigma model [11], and which works as follows: In the limit of vanishing current quark masses, QCD is invariant under transformations of the form \( \psi \rightarrow \exp (-i\theta^a \chi_a \gamma^5) \psi \), where \( \theta^a \) is an arbitrary vector in flavor space and \( \chi_a \) the generators of the group \( SU(N_f) \). This symmetry, in turn, is dynamically broken and thus not observed in nature. As a consequence, due to the Goldstone theorem, \( N_f^2 - 1 \) massless bosons appear, which for \( N_f = 2 \) are identified with the pions, and for \( N_f = 3 \) with pions, kaons and the \( \eta \). The finite mass of these particles observed in nature comes about due to a (small) explicit chiral symmetry breaking by finite current quark masses. It turns out that the concept of chiral symmetry is already sufficient to describe the low energy phenomena of strong interactions.

In the NJL model [10], this is implemented as follows: one starts with a Lagrangian containing free quarks, which interact via a chirally symmetric contact interaction. This interaction in turn leads to a spontaneous breakdown of chiral symmetry. As a consequence the pions, which appear as bound states of quarks and antiquarks, become massless. The physical picture is thus that one has pions as ground state and constituent quarks with masses of about 300–350 MeV. At finite
temperature the situation changes, however, since chiral symmetry is restored at $T \approx 200$ MeV. In this case, the quark mass drops down to the current quark mass, while mesons become unstable resonances. Within the present accuracy of lattice calculations, chiral symmetry is restored at the same temperature where deconfinement happens [12]. The lattice data show as well, that the temperature behaviour of the meson masses is qualitatively the same for both lattice QCD and the NJL model, as can be seen by comparing the data of Ref. [12] with those given in Ref. [10].

Due to this reasons, its simplicity and since the appearance of bound states is well studied in equilibrium, the NJL model provides an ideal starting point for studying the nonequilibrium behaviour of bound states. The results obtained here may nevertheless be of larger significance than the application to a specific model.

The derivation of transport equations for the quark sector of the NJL model has already been done in Refs. [13,14] and numerical solutions have been given in Refs. [15]. These works, however, do not include the formation of bound states. An approach to bound state formation has been attempted in Refs. [16,17], but without addressing the problem of computing the collision terms for higher order processes. On the other hand, there have been several attempts at constructing transport theories for mesonic Lagrangians that treat mesons as point like interacting particles [18,19]. The present approach goes beyond these works in that it unites both the quark and mesonic degrees of freedom in a single model, and it is required to account for the fact that the mesonic states are bound states of quarks below the Mott temperature [20] and resonances above this. It is further attempted to compute the collision terms beyond the leading order.

This paper is organized as follows: In Section II, the model is introduced. Since the NJL model is a strongly interacting theory, it has to be treated by an expansion in the inverse number of colors, $1/N_c$, rather than by an expansion in the coupling constant [21,22]. Thus mesons, which appear already in the lowest order of this expansion, are introduced as effective degrees of freedom in the beginning. In Section II A, the equations of motion for the quark Green functions in coordinate space are obtained and the derivation of the Boltzmann equation for the quark densities in the standard formalism [14], which is sufficient for the present purpose, is briefly reviewed. The equations of motion for the mesonic Green functions are given in Section II B. These are treated further and cast into a Boltzmann form in Section II C.
Section IV is dedicated to the explicit derivation of collision terms. In Section IV A the lowest order of the mesonic collision term is shown to contain processes of the type $\bar{q}q \leftrightarrow \pi$, which are kinematically allowed at high temperatures. The lowest order collision integral for quarks is computed in Section IV B. Since in this collision term the off-shell corrections to the off-diagonal meson Green functions appear, this calculation is divided in two parts. First the contributions of the singular (quasi-particle) part of the Green functions is shown to yield processes of the type $\bar{q}q \leftrightarrow \pi$. Afterwards the contributions of the regular part are shown to give processes of the type $qq \leftrightarrow qq$ and $q\bar{q} \leftrightarrow q\bar{q}$. It is essential to consider the off-shell corrections in order to obtain these processes at all. It is, however, also shown that the form of the off-shell corrections given in Ref. [8] is insufficient in order to describe the correct $s$-channel scattering amplitude of quark-antiquark scattering. Conclusions are presented in Section V.

II. FUNDAMENTAL EQUATIONS OF MOTION

The starting point for the following investigations is the NJL Lagrangian, which in its two flavor version reads [10]

$$\mathcal{L} = \bar{\psi}(i \not \partial - m_0)\psi + G \left[ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2 \right]. \quad (2.1)$$

Here $\psi$ denotes the quark wave function, which implicitly contains color and flavor degrees of freedom, and $\vec{\tau}$ are the Pauli matrices in flavor space. The equations of motion for the quark fields obtained from Eq. (2.1) are

$$(i \not \partial - m_0)\psi = -2G \left[ (\bar{\psi}\psi) + (\bar{\psi}i\gamma_5\vec{\tau}\psi)i\gamma_5\vec{\tau} \right] \psi \quad (2.2a)$$

$$\bar{\psi}(-i \not \partial - m_0) = -2G\bar{\psi} \left[ (\bar{\psi}\psi) + (\bar{\psi}i\gamma_5\vec{\tau}\psi)i\gamma_5\vec{\tau} \right]. \quad (2.2b)$$

Sigma and pion fields can be introduced via [10,13,17]

$$\sigma = -2G\bar{\psi}\psi = -2G\bar{\psi}\Gamma_\sigma \psi \quad (2.3a)$$

$$\pi^0 = -2G\bar{\psi}i\gamma_5\tau_3\psi = -2G\bar{\psi}\Gamma_0 \psi \quad (2.3b)$$

$$\pi^+ = -2G\bar{\psi}i\gamma_5\frac{\tau_1 + i\tau_2}{\sqrt{2}}\psi = -2G\bar{\psi}\Gamma_+ \psi \quad (2.3c)$$

$$\pi^- = -2G\bar{\psi}i\gamma_5\frac{\tau_1 - i\tau_2}{\sqrt{2}}\psi = -2G\bar{\psi}\Gamma_- \psi \quad (2.3d)$$
where, in order to simplify the notation in the following, also the matrices $\Gamma_\sigma$, $\Gamma_0$ and $\Gamma_\pm$ have been defined. Note that these definitions imply that the $\sigma$ and $\pi^0$ fields are real, whereas $\pi^+$ and $\pi^-$ form a complex field with $\pi^{+*} = \pi^-$. The equations of motion for the quark fields (2.2) can be rewritten as

\begin{align}
(i \not\partial - m_0)\psi &= \left(\sigma \Gamma_\sigma + \pi^0 \Gamma_0 + \pi^+ \Gamma_- + \pi^- \Gamma_+\right) \psi \\
\bar{\psi}(-i \not\partial - m_0) &= \bar{\psi} \left(\sigma \Gamma_\sigma + \pi^0 \Gamma_0 + \pi^+ \Gamma_- + \pi^- \Gamma_+\right) \psi .
\end{align}

In this form they can, together with the definition of the sigma and pion fields (2.3), be obtained from the Lagrange density

\begin{align}
\mathcal{L} &= \bar{\psi} (i \not\partial - m_0) \psi - \frac{1}{4G} \left(\sigma^2 + \pi^{02} + 2\pi^+ \pi^-\right) \\
&\quad - \bar{\psi} \left(\sigma \Gamma_\sigma + \pi^0 \Gamma_0 + \pi^+ \Gamma_- + \pi^- \Gamma_+\right) \psi ,
\end{align}

which is equivalent to the original Lagrangian (2.1). A canonical quantization of the Lagrangian (2.5) fails however, since (2.5) does not contain the time derivative of the mesonic fields and thus inhibits the definition of canonical momenta for the meson fields. One possible way out of this problem is to quantize (2.5) using the path integral approach, as is done in Ref. [17]. Here, however, an infinitesimal kinetic energy for the meson fields is added to the Lagrangian,

\begin{align}
\mathcal{L}' &= \mathcal{L} + \frac{1}{2} \epsilon \left[ (\partial_0 \sigma)^2 + (\partial_0 \pi^0)^2 + 2 (\partial_0 \pi^+) (\partial_0 \pi^-) \right] ,
\end{align}

quantization is done canonically and the limit $\epsilon \to 0$ is taken at the end. Using the generic notation $\phi_k$ for the meson fields, one then obtains the associated canonical momenta

\begin{align}
\mathcal{L}' &= \frac{\partial \mathcal{L}'}{\partial (\partial_0 \phi_k)} = \epsilon \partial_0 \phi_k^* ,
\end{align}

which allow for the introduction of canonical commutation relations at equal time

\begin{itemize}
\item[1] One could also think of adding other terms containing spatial derivatives of the fields to the Lagrangian in order to maintain Lorentz covariance even for $\epsilon \neq 0$. However, this procedure would not change the following calculations and for the present purpose Eq. (2.6) is sufficient.
\end{itemize}
\[ \phi_k (x) \phi_l (y) - \phi_l (y) \phi_k (x) = 0 \]  
\[ \varphi_k (x) \varphi_l (y) - \varphi_l (y) \varphi_k (x) = 0 \]  
\[ \phi_k (x) \varphi_l (y) - \varphi_l (y) \phi_k (x) = i \delta_{kl} \delta^3 (\vec{x} - \vec{y}) . \] 

Using these relations, the canonical quantization of (2.6) can proceed in a standard fashion [23].

A. Quark Fields

The transport equations for quark fields in the NJL model are derived in Refs. [13,14,17]. Here the main results of these works are listed for further reference and in order to define formalism and notation.

The Green functions for the quark fields in the Schwinger–Keldysh formalism are defined by

\[ S^+ (x,y) = -i \langle \mathcal{T} \left[ \psi (x) \bar{\psi} (y) \right] \rangle \]  
\[ S^- (x,y) = -i \langle \bar{\mathcal{T}} \left[ \psi (x) \bar{\psi} (y) \right] \rangle \]  
\[ S^> (x,y) = -i \langle \psi (x) \bar{\psi} (y) \rangle \]  
\[ S^< (x,y) = i \langle \bar{\psi} (y) \psi (x) \rangle , \] 

where \( \mathcal{T} \) and \( \bar{\mathcal{T}} \) are the time ordering and anti-time ordering operators, respectively:

\[ \mathcal{T} [A(x)B(y)] = \theta (x_0 - y_0) A(x) B(y) \pm \theta (y_0 - x_0) B(y) A(x) \]  
\[ \bar{\mathcal{T}} [A(x)B(y)] = \theta (y_0 - x_0) A(x) B(y) \pm \theta (x_0 - y_0) B(y) A(x) . \]

Here the upper (lower) sign refers to bosonic (fermionic) operators. In order to unify Eqs. (2.9), the field operators are analytically continued to the complex time plane and the Schwinger–Keldysh contour \( C \) (see Fig. 1) is introduced [1]. This contour runs from the time \( t_0 \), where the initial conditions are specified, to an arbitrary time larger than any other time considered (conveniently set to \(+\infty\)) and back. The initial time \( t_0 \) will in the following be shifted to \(-\infty\). This can be done since one has two time scales involved, namely the mean collision time and the expansion time. An initial time \( t_0 = -\infty \) in this sense means thus a time which is large compared to the mean collision time, but still small compared to the time scale of the expansion.
For a more formal argument for this procedure the reader is referred to the paper of Botermans and Malfliet [4].

The contour ordering operator $T_C$ is defined as

$$T_C [A(x)B(y)] = \theta_C(x_0, y_0)A(x)B(y) \pm \theta_C(y_0, x_0)B(y)A(x) \ ,$$

using the contour theta function

$$\theta_C(x_0, y_0) = \begin{cases} 1 & \text{if } x_0 \text{ is ‘later’ than } y_0 \text{ with respect to } C \\ 0 & \text{if } x_0 \text{ is ‘sooner’ than } y_0 \text{ with respect to } C \end{cases} .$$

Using this definition, Eqs. (2.9) can be written as

$$S_C(x, y) = -i \langle T_C \left[ \psi(x) \psi(y) \right] \rangle .$$

From Eqs. (2.5) and (2.9), one can easily derive the equations of motion for the quark Green functions:

$$\left( i \slashed{\partial}_x - m_0 \right) S_C(x, y) = \delta_C(x, y)$$

$$-i \langle T_C \left[ \left( \sigma(x) \Gamma_\sigma + \pi^0(x) \Gamma_0 + \pi^+(x) \Gamma_+ + \pi^-(x) \Gamma_- \right) \psi(x) \bar{\psi}(y) \right] \rangle ,$$

where the contour delta function is defined as

$$\delta_C(x, y) = \begin{cases} \delta^4(x - y) & \text{if both } x_0 \text{ and } y_0 \text{ are on the upper branch of } C \\ -\delta^4(x - y) & \text{if both } x_0 \text{ and } y_0 \text{ are on the lower branch of } C \\ 0 & \text{elsewhere} \end{cases} .$$

Defining the contour ordered self energy via

$$\int_C d^4y \Sigma_C(x, y)S_C(y, z) =$$

$$-i \langle T_C \left[ \left( \sigma(x) \Gamma_\sigma + \pi^0(x) \Gamma_0 + \pi^+(x) \Gamma_+ + \pi^-(x) \Gamma_- \right) \psi(x) \bar{\psi}(y) \right] \rangle ,$$

where the subscript $C$ at the integral sign indicates time integration along the contour, one can cast Eq. (2.14) into the Schwinger–Dyson like form

$$(i \slashed{\partial}_x - m_0)S_C(x, z) = \delta_C(x, z) + \int_C d^4y \Sigma_C(x, y)S_C(y, z) .$$

Analogously, one can derive the adjoint equation

$$S_C(x, z)(-i \slashed{\partial}_z - m_0) = \delta_C(x, z) + \int_C d^4y S_C(x, y)\Sigma_C(y, z) .$$
By defining retarded and advanced Green functions and self energies via

\[ S^R(x, y) = S^+(x, y) - S^<(x, y) = S^>(x, y) - S^-(x, y) \quad (2.19a) \]
\[ S^A(x, y) = S^+(x, y) - S^>(x, y) = S^<(x, y) - S^- (x, y) \quad (2.19b) \]
\[ \Sigma^R(x, y) = \Sigma^+(x, y) - \Sigma^<(x, y) = \Sigma^>(x, y) - \Sigma^- (x, y) \quad (2.19c) \]
\[ \Sigma^A(x, y) = \Sigma^+(x, y) - \Sigma^>(x, y) = \Sigma^<(x, y) - \Sigma^- (x, y) \quad (2.19d) \]

and disentangling the contour integrations in Eqs. (2.17), (2.18), one arrives at the equations of motion for the Green functions

\[ \left( S_{b}^{-1} \otimes S^{R,A} \right) (x, y) = \delta^4(x - y) + \left( \Sigma^{R,A} \otimes S^{R,A} \right) (x, y) \quad (2.20a) \]
\[ \left( S^{R,A} \otimes S_{b}^{-1} \right) (x, y) = \delta^4(x - y) + \left( S^{R,A} \otimes \Sigma^{R,A} \right) (x, y) \quad (2.20b) \]
\[ \left( S_{b}^{-1} \otimes S^{>,<} \right) (x, y) = \left( \Sigma^{>,<} \otimes S^A + \Sigma^R \otimes S^{>,<} \right) (x, y) \quad (2.20c) \]
\[ \left( S^{>,<} \otimes S_{b}^{-1} \right) (x, y) = \left( S^{>,<} \otimes \Sigma^A + S^R \otimes \Sigma^{>,<} \right) (x, y) . \quad (2.20d) \]

In order to abbreviate the notation, the convolution operator \( \otimes \), which is defined by

\[ (A \otimes B)(x, z) = \int d^4y A(x, y)B(y, z) \quad (2.21) \]

and the inverse of the bare Green function

\[ S_{b}^{-1}(x, y) = (i \not{x} - m_0)\delta^4(x - y) \quad (2.22) \]

are introduced in Eqs. (2.20). Note that ‘inverse’ here means inverse with respect to the convolution (2.21):

\[ \left( F \otimes F^{-1} \right)(x, y) = \delta^4(x - y) . \quad (2.23) \]

The general framework for deriving transport equations from Eqs. (2.20) has been outlined in Refs. [4], an explicit calculation for the NJL model has been given in Refs. [13,14]. Since the aim of the present work is the derivation of transport equations for mesons, these calculations are not repeated here. It should however be mentioned that with the definition of the self energy (2.16), \( \Sigma^C(x, y) \) still contains a Hartree part, which in the case of the NJL model leads to the spontaneous breakdown of chiral symmetry and the appearance of constituent quarks. For further details, the reader is referred to Refs. [10,13]. In Ref. [14], a gradient expansion of Eqs. (2.20) is performed and subsequently the quasiparticle ansatz for \( S^{>,<} \),
obtains the equation of motion for the contour ordered mesonic Green functions of motion for the fields derived from (2.6) and the commutation relations (2.8), one for sigmas, neutral pions and charged pions, respectively. Employing the equations together with an expansion of the nondiagonal selfenergies Σ with later, Eqs. (2.25) and (2.26) provide a set of equations for the computation of for the quark density \( n(x, \vec{p}) \). A numerical evaluation of these equations has been given in Refs. \([15]\).

\[
S^< (x, p) = \frac{i\pi}{E_q (x, \vec{p})} \frac{\delta f' \delta cc'}{2N_c N_f} (\phi + m_q (x)) \left[ \delta (p_0 - E_q (x, \vec{p})) n_q (x, \vec{p}) \right. \\
\left. - \delta (p_0 + E_q (x, \vec{p})) (2N_c N_f - n_q (x, -\vec{p})) \right] \\
S^> (x, p) = -\frac{i\pi}{E_q (x, \vec{p})} \frac{\delta f' \delta cc'}{2N_c N_f} (\phi + m_q (x)) \left[ \delta (p_0 - E_q (x, \vec{p})) (2N_c N_f - n_q (x, \vec{p})) \right. \\
\left. - \delta (p_0 + E_q (x, \vec{p})) n_q (x, -\vec{p}) \right],
\]

with \( E_q (x, \vec{p}) = \sqrt{\vec{p}^2 + m_q^2 (x)} \), is employed in order to derive the Boltzmann equation

\[
\left[ \partial_t + \vec{p}_\mu E_q (x, \vec{p}) \vec{\partial}_\mu - \vec{\partial}_x E_q (x, \vec{p}) \vec{\partial}_p \right] n_q (x, \vec{p}) = \\
\int_0^\infty \frac{dp_0}{2\pi} \text{Tr} \left[ \Sigma^< (x, p) S^> (x, p) - \Sigma^> (x, p) S^< (x, p) \right]
\]

for the quark density \( n_q (x, \vec{p}) \). The integration on the right hand side serves to eliminate the delta functions contained implicitly in \( S^<^> \). The dynamical quark mass \( m_q (x) \) in Eq. (2.27) has in turn to be calculated from the gap equation

\[
m_q (x) = m_0 + 2Gm_q (x) \int_{|\vec{p}| < \Lambda} \frac{d^3 p}{(2\pi)^3} \frac{2N_c N_f - n_q (x, \vec{p}) - n_q (x, -\vec{p})}{E_q (x, \vec{p})}.
\]

Together with an expansion of the nondiagonal selfenergies \( \Sigma^<^> \), to be discussed later, Eqs. (2.25) and (2.26) provide a set of equations for the computation of \( n_q (x, \vec{p}) \) and \( n_q (x, \vec{p}) \). A numerical evaluation of these equations has been given in Refs. \([15]\).

**B. Meson Fields**

In analogy to Eq. (2.13) the Green functions for the meson fields are defined via

\[
\Delta^C_\sigma (x, y) = -i \left\{ \langle T_C [\sigma (x) \sigma (y)] \rangle - \langle \sigma (x) \rangle \langle \sigma (y) \rangle \right\} \\
\Delta^C_0 (x, y) = -i \left\{ \langle T_C \left[ \pi^0 (x) \pi^0 (y) \right] \rangle - \langle \pi^0 (x) \rangle \langle \pi^0 (y) \rangle \right\} \\
\Delta^C_\pm (x, y) = -i \left\{ \langle T_C \left[ \pi^+ (x) \pi^- (y) \right] \rangle - \langle \pi^+ (x) \rangle \langle \pi^- (y) \rangle \right\},
\]

for sigmas, neutral pions and charged pions, respectively. Employing the equations of motion for the fields derived from (2.6) and the commutation relations (2.8), one obtains the equation of motion for the contour ordered mesonic Green functions

\[
- \left( \epsilon \partial^2_{x_0} + \frac{1}{2G} \right) \Delta^C_\sigma (x, y) = \delta_C (x, y) \\
- i \left\{ \langle T_C \left[ \bar{\psi} (x) \Gamma_\sigma \psi (x) \sigma (y) \right] \rangle - \langle \bar{\psi} (x) \Gamma_\sigma \psi (x) \rangle \langle \sigma (y) \rangle \right\}
\]

(2.28a)
\[-(\epsilon \partial_{x_0}^2 + \frac{1}{2G}) \delta_C(x,y) = \delta_C(x,y) \] (2.28b)

\[-i \left\{ \mathcal{T}_C \left[ \tilde{\psi}(x) \Gamma_0 \psi(x) \right] \right\} - \left\{ \psi(x) \Gamma_0 \psi(x) \right\} \]

\[-(\epsilon \partial_{x_0}^2 + \frac{1}{2G}) \delta_\pm_C(x,y) = \delta_\pm_C(x,y) \] (2.28c)

\[-i \left\{ \mathcal{T}_C \left[ \tilde{\psi}(x) \Gamma_+ \psi(x) \right] \right\} - \left\{ \psi(x) \Gamma_+ \psi(x) \right\} \]

Defining the contour ordered polarization function $\Pi^C_\sigma(x,y)$ for the sigma field via

\[
\int_C d^4 y \Pi^C_\sigma(x,y) \Delta^C_\sigma(y,z) = \left\{ \mathcal{T}_C \left[ \tilde{\psi}(x) \Gamma_\sigma \psi(x) \right] \right\} - \left\{ \psi(x) \Gamma_\sigma \psi(x) \right\} \] (2.29)

and analogously the polarization functions $\Pi^C_0(x,y)$ for neutral pions and $\Pi^C_\pm(x,y)$ for charged pions, Eqs. (2.28) can be rewritten as

\[-(\epsilon \partial_{x_0}^2 + \frac{1}{2G}) \Delta^C_k(x,z) = \delta_C(x,z) + \int_C d^4 y \Pi^C_k(x,y) \Delta^C_k(y,z) \] (2.30)

for $k = \sigma$, 0, ±. The adjoint equation is

\[-(\epsilon \partial_{x_0}^2 + \frac{1}{2G}) \Delta^C_k(x,z) = \delta_C(x,z) + \int_C d^4 y \Delta^C_k(x,y) \Pi^C_k(y,z) \] (2.31)

In the limit $\epsilon \to 0$, the time derivative on the left hand side of Eqs. (2.30) and (2.31) drops out and one reobtains the equations of motion of Ref. [17].

Disentangling the contour integration and introducing retarded and advanced quantities as in Eq. (2.19) leads to

\[
\begin{align*}
(\Delta^{-1}_b \otimes \Delta^{R,A}_k) (x,y) &= \delta^4(x-y) + \left( \Pi^R_k \otimes \Delta^{R,A}_k \right)(x,y) \quad (2.32a) \\
(\Delta^{R,A}_k \otimes \Delta^{-1}_b) (x,y) &= \delta^4(x-y) + \left( \Delta^{R,A}_k \otimes \Pi^R_k \right)(x,y) \quad (2.32b) \\
(\Delta^{-1}_b \otimes \Delta^\rightarrow_-<_k) (x,y) &= \left( \Pi^\rightarrow_-<_k \otimes \Delta^A_k + \Pi^R_k \otimes \Delta^\rightarrow_-<_k \right)(x,y) \quad (2.32c) \\
(\Delta^\rightarrow_-<_k \otimes \Delta^{-1}_b) (x,y) &= \left( \Delta^\rightarrow_-<_k \otimes \Pi^A_k + \Delta^R_k \otimes \Pi^\rightarrow_-<_k \right)(x,y) \quad (2.32d)
\end{align*}
\]

where the inverse of the bare propagator is given by

\[
\Delta^{-1}_b(x,y) = -\frac{1}{2G} \delta(x-y) \] (2.33)

Equations (2.32) are the mesonic analogue of Eqs. (2.24) and will form the basis of the following investigations. A major difference between Eqs. (2.32) and the corresponding equations for particles, which occur as elementary degrees of freedom
in the microscopic interaction model, is that the inverse bare propagator defined in Eq. (2.33) does no longer contain derivatives, as does e. g. the inverse bare quark propagator given in Eq. (2.22). It will be seen in the following, however, that this does not inhibit the description of mesons within the framework of transport theory.

III. TRANSPORT EQUATIONS FOR MESONS

The derivation of transport equations from Eqs. (2.32) can in principle be done in the standard fashion outlined in Refs. [4,14]. The drawback of this approach is, however, that the quasiparticle ansatz, which for bosons reads

$$\Delta_k^<(x,p) = -\frac{i\pi}{E_k(x,\vec{p})}\left[n_k(x,\vec{p})\delta(p_0 - E_k(x,\vec{p})) + (1 + n_k(x,-\vec{p}))\delta(p_0 + E_k(x,\vec{p}))\right]$$

$$\Delta_k^>(x,p) = -\frac{i\pi}{E_k(x,\vec{p})}\left[(1 + n_k(x,\vec{p}))\delta(p_0 - E_k(x,\vec{p})) + n_k(x,-\vec{p})\delta(p_0 + E_k(x,\vec{p}))\right],$$

does not solve the equations of motion (2.32c), (2.32d) unless one neglects off-shell terms [8]. Since the problem treated here differs from the standard formalism in considering the propagation of non-elementary particles, it is not clear a priori, whether this procedure is justified. It is thus desirable to use a formalism which consistently eliminates off-shell terms within the framework of the gradient expansion, as has been developed in Ref. [8]. It has been shown there, that the unwanted terms can be eliminated by considering off-shell corrections to the ansatz (3.1). Thus, in the following the formalism developed in [8] will be applied to (2.32).

A. Mesonic Spectral Functions

In order to transform Eqs. (2.32) to momentum space, a Wigner transformation is performed, which for a function $F(x,y)$ of two coordinate arguments is defined by

$$F(x,p) = \int d^4u \ e^{ipu} F(x + u/2, x - u/2).$$

The Wigner transform of a convolution of two functions can be evaluated employing a gradient expansion [4]. Keeping only terms up to first order gradients, one obtains
\[ \int d^4 u \, e^{i p u} (F \otimes G)(x + u/2, x - u/2) \]
\[ = F(x, p) G(x, p) + \frac{i}{2} \{ F(x, p); G(x, p) \} , \]  
introducing the Poisson bracket of two functions in momentum space,
\[ \{ F(x, p); G(x, p) \} = \partial_p F(x, p) \partial_x G(x, p) - \partial_x F(x, p) \partial_p G(x, p) . \]

In the following, also the Wigner transform of triple and quadruple convolutions will be needed, which can be easily inferred from Eq. (3.3).

By transforming Eqs. (2.32a) and (2.32b) and taking the sum and the difference of the transformed equations, one obtains
\[ -\frac{1}{2G} \Delta^R_{k}(x, p) = 1 + \Pi^R_{k}(x, p) \Delta^R_{k}(x, p) \]  
\[ 0 = \{ \Pi^R_{k}(x, p); \Delta^R_{k}(x, p) \} . \]

Equations (3.5) are solved by
\[ \Delta^R_{k}(x, p) = -\frac{2G}{1 + 2G \Pi^R_{k}(x, p)} . \]

The mesonic quasiparticle energy and the width can be identified as the location of the poles of the retarded Green function (3.6), i.e. as the solution of the dispersion relation
\[ 1 + 2G \Pi^R_{k}(x, p) = 0 . \]

In the vicinity of a pole, the denominator of Eq. (3.6) can be Taylor-expanded with respect to \( p_0 \) as
\[ 1 + 2G \Pi^R_{k}(x, p) \approx -\frac{2G}{g_k^2(x, \vec{p}) - i a_k(x, \vec{p})} 2E_k(x, \vec{p}) \left( p_0 - E_k(x, \vec{p}) + \frac{i}{2} \Gamma_k(x, \vec{p}) \right) , \]

defining the quasiparticle energy \( E_k(x, \vec{p}) \), the quasiparticle width \( \Gamma_k(x, \vec{p}) \) and the effective meson-quark coupling \( g_k(x, \vec{p}) \) for meson species \( k \). With Eq. (3.8), \( \Delta^R_{k}(x, p) \) becomes in the vicinity of the pole
\[ \Delta^R_{k}(x, p) \approx \frac{g_k^2(x, \vec{p}) - i a_k(x, \vec{p})}{2E_k(x, \vec{p}) \left( p_0 - E_k(x, \vec{p}) + \frac{i}{2} \Gamma_k(x, \vec{p}) \right)} . \]
and the spectral function becomes

$$
\rho_k(x,p) = i \left( \Delta_k^R(x,p) - \Delta_k^A(x,p) \right)
\approx \frac{1}{E_k(x,\vec{p})} \frac{1}{2} \Gamma_k(x,\vec{p}) g_k^2(x,\vec{p}) + (p_0 - E_k(x,\vec{p})) a_k(x,\vec{p})
\frac{1}{(p_0 - E_k(x,\vec{p}))^2 + \frac{1}{4} \Gamma_k^2(x,\vec{p})}.
$$

(3.10)

Note that $\rho_k$ is a real function, since in momentum space $\Delta_k^A$ is the complex conjugate of $\Delta_k^R$, as can be inferred from its definition [4].

It is easy to see that the imaginary part of the pole residue, $a_k$, is of the same order as the width $\Gamma_k$. Thus, in the limit $\Gamma_k \to 0$ one obtains $\rho_k \sim \delta(p_0 - E_k)$, i.e. the mesons are strictly on shell. The expansions (3.9) and (3.10) contain only a particle pole at positive $p_0$, whereas one should expect also an antiparticle pole at negative $p_0$ from symmetry reasons [18]. Since these poles are separated by a finite gap, however, the form given above will be sufficient.

For illustration, the spectral function for the pion is shown in Figs. 2 and 3 for thermal equilibrium and $\vec{p} = 0$ at $T = 0$ and $T = 300$ MeV, respectively, in the random phase approximation [10]. In this case, the spectral function is a function of $p_0$ only. One can distinguish two cases: at low temperatures as in Fig. 2, Eq. (3.7) has solutions for real $|p_0| < 2m_q$. In this case the spectral function has the form of a delta function in the vicinity of the solution of Eq. (3.7), i.e. contains contributions of bound states. Furthermore, one observes the appearance of a continuum at $|p_0| > 2m_q$, which stems from quark–antiquark scattering states.

At high temperatures, Eq. (3.7) has no longer solutions for real, but only for complex $p_0$ with $|\Re(p_0)| > 2m_q$. Physically this means that the pion becomes unstable at high temperatures. Details about this so-called Mott transition can be found in Ref. [20]. The spectral function at a temperature of $T = 300$ MeV is shown in Fig. 3. One notices, that the pion does not manifest itself by a delta peak any longer, but rather by a resonance peak. The dashed line in Fig. 3 shows the pole approximation according to the second line of Eq. (3.10), which can be seen to give a good approximation to the full spectral function even at this rather high temperature. Note that it is important to include these resonant states in the theory since they give a contribution to the effective number of degrees of freedom in the thermodynamical quantities, as has been shown in Ref. [24].
B. Mesonic Densities

The determination of $\Delta_{R,A}^{k}$ allows one to extract information about the properties of mesons. These functions do not, however, give direct information about the particle densities. These must be extracted from the off-diagonal Green functions $\Delta_{k}^{>,<}$. As was stated above, the approach developed in Ref. [8] will be applied. This formalism starts by returning to the equations of motion in coordinate space and observing that Eqs. (2.32c), (2.32d) are equivalent to

$$\Delta_{k}^{>,<} = \Delta_{k}^{R} \otimes \Pi_{k}^{>,<} \otimes \Delta_{k}^{A},$$

(3.11)

where the coordinate arguments have been dropped for simplicity. Analogously, $\rho_{k}$ can be rewritten as

$$\rho_{k} = i \left( \Delta_{k}^{R} - \Delta_{k}^{A} \right) = i \left( \Delta_{k}^{>} - \Delta_{k}^{<} \right) = \Delta_{k}^{R} \otimes \gamma_{k} \otimes \Delta_{k}^{A}.$$  

(3.12)

In Eq. (3.12), the auxiliary quantities

$$\Pi_{k} = \frac{1}{2} \left( \Pi_{k}^{R} + \Pi_{k}^{A} \right)$$  

(3.13a)

$$\gamma_{k} = i \left( \Pi_{k}^{R} - \Pi_{k}^{A} \right) = i \left( \Pi_{k}^{>} - \Pi_{k}^{<} \right)$$  

(3.13b)

have been introduced. Since in momentum space the retarded and advanced quantities are complex conjugate to each other [4], Eq. (3.13) is for this case commensurate with a decomposition of $\Pi_{k}^{R,A}$ into its real and imaginary part.

Starting from the identities (3.11) and (3.12), it is possible to decompose $\Delta_{k}^{>,<}$ and $\rho_{k}$ into a singular and a regular part

$$\Delta_{k}^{>,<} = \Delta_{k,s}^{>,<} + \Delta_{k,r}^{>,<}$$  

(3.14a)

$$\rho_{k} = \rho_{k,s} + \rho_{k,r}.$$  

(3.14b)

which in turn are defined by

$$\Delta_{k,s}^{>,<} = \frac{i}{2} \left( \Delta_{k}^{R} \otimes \Pi_{k}^{>,<} \otimes \rho_{k} - \rho_{k} \otimes \Pi_{k}^{>,<} \otimes \Delta_{k}^{A} \right)$$  

(3.15a)

$$\Delta_{k,r}^{>,<} = \frac{1}{2} \left( \Delta_{k}^{R} \otimes \Pi_{k}^{>,<} \otimes \Delta_{k}^{A} + \Delta_{k}^{A} \otimes \Pi_{k}^{>,<} \otimes \Delta_{k}^{R} \right)$$  

(3.15b)

$$\rho_{k,s} = \frac{i}{2} \left[ \Delta_{k}^{R} \otimes \gamma_{k} \otimes \rho_{k} - \rho_{k} \otimes \gamma_{k} \otimes \Delta_{k}^{A} \right]$$  

(3.15c)

$$\rho_{k,r} = \frac{1}{2} \left[ \Delta_{k}^{R} \otimes \gamma_{k} \otimes \Delta_{k}^{A} + \Delta_{k}^{A} \otimes \gamma_{k} \otimes \Delta_{k}^{R} \right].$$  

(3.15d)
The physical implications of the decompositions (3.14) and (3.15) have been discussed in Ref. [8]. The decomposition of $\Delta_{k}^{>;<}$ and $\rho_k$ into a singular and regular part corresponds to a separation of long time and short time phenomena. It is thus possible to describe the singular part (which corresponds to the long time phenomena) in terms of quasiparticles. This picture is further confirmed by observing that by Wigner transforming Eq. (3.15c) and computing the gradients using Eq. (3.6), one obtains $\rho_{k,s} = \frac{1}{2}\gamma_k \rho_k^2$ in momentum space, i.e. for a given width the singular part of the spectral function $\rho_{k,s}$ forms a much narrower peak around the quasiparticle pole than the spectral function itself.

If a description of mesons in terms of quasiparticles is desired, one thus needs an equation of motion for the singular part of $\Delta_{k}^{>;<}$ only. Following Ref. [8], this is achieved by computing

$$-i \left[ (\Delta_k^R)^{-1} \otimes \Delta_{k,s}^< - \Delta_{k,s}^< \otimes (\Delta_k^A)^{-1} \right]$$

in two ways: Inserting $(\Delta_k^{R,A})^{-1} = \Delta_b^{-1} - \Pi_k^{R,A}$ from Eqs. (2.32a), (2.32b) leads to

$$-i \left[ (\Delta_k^R)^{-1} \otimes \Delta_{k,s}^< - \Delta_{k,s}^< \otimes (\Delta_k^A)^{-1} \right] = \frac{1}{2} \left( \gamma_k \otimes \Delta_{k,s}^< + \Delta_{k,s}^< \otimes \gamma_k \right)$$

(3.17)

On the other hand, inserting Eq. (3.15a) and Eq. (3.12) into (3.16) yields

$$-i \left[ (\Delta_k^R)^{-1} \otimes \Delta_{k,s}^< - \Delta_{k,s}^< \otimes (\Delta_k^A)^{-1} \right] = \frac{1}{2} (\Pi_k^< \otimes \rho_k + \rho_k \otimes \Pi_k^<)$$

$$-\frac{1}{2} \left( \gamma_k \otimes \Delta_k^A \otimes \Pi_k^< \otimes \Delta_k^A + \Delta_k^R \otimes \Pi_k^< \otimes \Delta_k^R \otimes \gamma_k \right).$$

(3.18)

Equating (3.17) and (3.18) and performing a Wigner transformation gives

$$\{ \Delta_b^{-1} - \Pi_k^<; \Delta_{k,s}^< \} = \frac{1}{4} \left\{ \gamma_k; \rho_k \Pi_k^< \left( \Delta_k^R + \Delta_k^A \right) \right\} = i \left( \Pi_k^< \Delta_k^{>;<} - \Pi_k^< \Delta_{k,s}^< \right),$$

(3.19)

where now all functions depend on $x$ and $p$. In deriving Eq. (3.19), it has been used that

$$\rho_{k,s}(x, p) = \rho_k(x, p) - \rho_{k,r}(x, p) = \rho_k(x, p) - \frac{1}{2} \gamma_k(x, p) \left( \Delta_k^R(x, p)^2 + \Delta_k^A(x, p)^2 \right).$$

Equation (3.19) already contains the ingredients of the Boltzmann equation. A term similar to the first one on the left hand side of Eq. (3.19) appears also in the standard
formalism, where it gives rise to a mean field term. The second term on the left hand side is particular to the present approach and has to be treated further. The right hand side has already the form of a collision term.

In accordance with the assumption that $\Delta^<_{k,s}$ can be described in terms of quasiparticles, these functions are factorized as

$$
\Delta^<_{k,s}(x,p) = -i\rho_{k,s}(x,p) f_k(x,p) \tag{3.21a}
$$
$$
\Delta^>_{k,s}(x,p) = -i\rho_{k,s}(x,p) \left[ 1 + f_k(x,p) \right] \tag{3.21b}
$$

where $f_k$ is assumed to be a smooth function of $p_0$. It is necessary to use the same function $f_k$ for both $\Delta^<_{k,s}$ and $\Delta^>_{k,s}$ in order to maintain the identity $i\left( \Delta^>_{k,s} - \Delta^<_{k,s} \right) = \rho_{k,s}$. The left hand side of Eq. (3.19) becomes

$$
\begin{align*}
\{ \Delta^{-1}_b - \Pi_k; \Delta^<_{k,s} \} - \frac{1}{4} \left\{ \gamma_k; \rho_k \Pi^<_k \left( \Delta^R_k + \Delta^A_k \right) \right\} &= -i\rho_{k,s} \left\{ \Delta^{-1}_b - \Pi_k; f_k \right\} \\
- if_k \left\{ \Delta^{-1}_b - \Pi_k; \rho_{k,s} \right\} - \frac{1}{4} \left\{ \gamma_k; \rho_k \Pi^<_k \left( \Delta^R_k + \Delta^A_k \right) \right\} 
\end{align*}
\tag{3.22}
$$

The second term on the right hand side can be transformed to

$$
\{ \Delta^{-1}_b - \Pi_k; \rho_{k,s} \} = \left\{ \gamma_k; \frac{1}{4} \gamma_k \rho_k \left( \Delta^R_k + \Delta^A_k \right) \right\} \tag{3.23}
$$

With this, one can rewrite the right hand side of Eq. (3.23) as

$$
- i\rho_{k,s} \left\{ \Delta^{-1}_b - \Pi_k; f_k \right\} - \left\{ \gamma_k; \frac{1}{4} \rho_k \left( \Delta^R_k + \Delta^A_k \right) \left[ i\gamma_k f_k + \Pi^<_k \right] \right\} \\
+ \frac{i}{4} \gamma_k \rho_k \left( \Delta^R_k + \Delta^A_k \right) \left\{ \gamma_k; f_k \right\} \tag{3.24}
$$

The second term here is effectively of second order in the gradients and can thus be neglected. The third term can be rewritten to be

$$
\frac{1}{4} \gamma_k \rho_k \left( \Delta^R_k + \Delta^A_k \right) \left\{ \gamma_k; f_k \right\} = \rho_{k,s} \frac{\Delta^{-1}_b - \Pi_k}{\gamma_k} \left\{ \gamma_k; f_k \right\} \tag{3.25}
$$

so that Eq. (3.19) becomes

$$
- i\rho_{s,k} \left\{ \Delta^{-1}_b - \Pi_k; f_k \right\} - \frac{\Delta^{-1}_b - \Pi_k}{\gamma_k} \left\{ \gamma_k; f_k \right\} = \rho_{s,k} \left[ \Pi^<_k \left( 1 + f_k \right) - \Pi^>_k f_k \right] \tag{3.26}
$$

Since $\rho_{k,s}$ is strongly peaked near the quasiparticle energy, it is possible to drop the common factor $\rho_{k,s}$ and to consider the rest of Eq. (3.26) on the mass shell
\[ p_0 = E_k(x, \vec{p}) \]  
This corresponds to approximating the singular part of the spectral function by a delta function:

\[
\rho_{s,k}(x,p) \approx \frac{\pi g^2_k}{E_k(x, \vec{p})} [\delta(p_0 - E_k(x, \vec{p})) - \delta(p_0 + E_k(x, \vec{p}))].
\]  

(3.27)

This in turn allows the introduction of the quasiparticle ansatz (3.1) as an ansatz for the singular part of \( \Delta_{>\ast}^k \) only:

\[
\Delta_{\leq s,s}^k(x,p) \approx -g^2_k(x, \vec{p}) \frac{i\pi}{E_k(x, \vec{p})} \left[ n_k(x, \vec{p}) \delta(p_0 - E_k(x, \vec{p})) + (1 + n_k(x, -\vec{p})) \delta(p_0 + E_k(x, \vec{p})) \right],
\]  

(3.28a)

\[
\Delta_{\geq s,s}^k(x,p) \approx -g^2_k(x, \vec{p}) \frac{i\pi}{E_k(x, \vec{p})} \left[ (1 + n_k(x, \vec{p})) \delta(p_0 - E_k(x, \vec{p})) + n_k(x, -\vec{p}) \delta(p_0 + E_k(x, \vec{p})) \right].
\]  

(3.28b)

These expressions differ from Eqs. (3.1) by the factor \( g^2_k \), which stems from the pole residue of the retarded Green functions, cf. Eq. (3.9). Similar factors appear also in the standard formalism, where they account for wave function renormalization [8]. Equation (2.24) contains no such factor since the quark self energy in the Hartree approximation is momentum independent.

From Eqs. (3.28), one obtains \( f_k(x,p) = n_k(x, \vec{p}) \) at positive \( p_0 \). Inserting this into the left hand side of Eq. (3.26) and using the expansion (3.8) yields

\[
\left\{ \Delta^{-1}_b - \Pi; f_k \right\} \left\{ \Delta^{-1}_b - \Pi; f_k \right\} = \left\{ \frac{2E_k}{g^2_k + a_k^2} \left[ g^2_k(p_0 - E_k) - \frac{1}{2} a_k \Gamma_k \right]; n_k \right\} = \frac{2E_k}{g^2_k} \left[ \partial_t + \partial_\vec{p} E_k \partial_\vec{x} - \partial_\vec{x} E_k \partial_\vec{p} \right] n_k,
\]  

(3.29)

where the terms containing \( a_k \) have been neglected, since they are of second order in the width and thus beyond the validity of the approach.

Within the same accuracy, the second term on the left hand side of Eq. (3.26) vanishes and one obtains

\[
\left[ \partial_t + \partial_\vec{p} E_k \partial_\vec{x} - \partial_\vec{x} E_k \partial_\vec{p} \right] n_k = i \frac{g^2_k}{2E_k} \left[ \Pi^\leq_k (1 + n_k) - \Pi^\geq_k n_k \right],
\]  

(3.30)

i.e. the Boltzmann equation for mesons.

Equation (3.30) is one of the major results of this paper. It shows, that transport theory is applicable to the formation of bound states and that the mean field term,
i.e. the left hand side of Eq. (3.30), has precisely the same form as it has for elementary particles. This means that in a situation, where no free quarks are present, mesons can propagate freely and can also carry kinetic energy. At a first glance, this might be surprising, since the left hand sides of Eqs. (2.32) do not contain any derivatives, which usually are associated with the kinetic energy of the particles [4]. This is nevertheless only true for models, where the interaction appears as a small perturbation and the drift term has to be present already for the noninteracting case. Bound states, on the other hand, are nonperturbative phenomena, for which the presence of interactions is essential. The interaction part of the retarded and advanced Green functions does thus gain a greater importance than the bare part, giving rise to poles which are not present in the noninteracting case. These poles, in turn, define the kinetic energy which later appears in the drift term of the transport equations.

IV. INTERACTIONS BETWEEN QUARKS AND MESONS

In the last two sections, the Boltzmann equations for quark and meson degrees of freedom have been derived and an explicit form of the mean field (Vlasov) part has been given. It remains to evaluate the self energies in order to express the collision integrals also in terms of the particle densities. In this section, the lowest order terms for the off diagonal polarization $\Pi^{>\!<}_k$ and self energies $\Sigma^{>\!<}$ are examined. It is shown that these diagrams describe (i) the decay of mesons into quark-antiquark pairs and the corresponding recombination, $M \leftrightarrow q\bar{q}$, and (ii) the elastic scattering of quarks and antiquarks via meson exchange, $qq \leftrightarrow qq$ and $q\bar{q} \leftrightarrow q\bar{q}$. It turns out that the latter processes stem from the off-shell part of $\Delta^{>\!<}_k$ and that the interference of the processes $q\bar{q} \rightarrow M$ and $q\bar{q} \rightarrow q\bar{q}$ leads to a modified $s$-channel amplitude. The inclusion of off-shell corrections to $\Delta^{>\!<}_k$ is thus essential for a description of collisions beyond the leading order.

Throughout this section it will be assumed that the quasiparticle energies can be parametrized as $E(x, \vec{p}) = \sqrt{\vec{p}^2 + m^2(x)}$. 

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A. Collision Term for Mesons

The lowest order term for the polarization in a \(1/N_c\) expansion is shown in Fig. 4. For definiteness, the neutral pion is considered in the following, i.e. \(\Gamma_k = i\gamma_5\tau_3\). The off-diagonal polarization \(\Pi_0^\left<\right>\) is given by

\[
- i\Pi_0^\left<\right>(x, y) = - \text{Tr} \left[ i\Gamma_0 iS^C(x, y) i\Gamma_0 iS^C(y, x) \right]^\left<\right>
= - \text{Tr} \left[ i\Gamma_0 iS^<(x, y) i\Gamma_0 iS^>(y, x) \right].
\] (4.1)

After Wigner transformation this becomes

\[
-i\Pi_0^\left<\right>(x, p) = - \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} (2\pi)^4\delta^4(p - p_1 - p_2) \text{Tr} \left[ i\Gamma_0 iS^<(x, p_1) i\Gamma_0 iS^>(x, -p_2) \right].
\] (4.2)

Inserting the quasiparticle form (2.24) into (4.2) and multiplying out the contributions of quarks and antiquarks leads to the appearance of four terms, which correspond to the processes \(q \rightarrow q\pi^0\), \(\bar{q} \rightarrow \bar{q}\pi^0\), \(\emptyset \rightarrow q\bar{q}\pi^0\) (i.e. a quark, an antiquark and a pion are emitted spontaneously from the vacuum) and \(q\bar{q} \rightarrow \pi^0\). This calculation proceeds along the same lines as given in Ref. [14]. Since all momenta have to be taken on shell, the contributions of the first three processes vanish due to kinematical constraints. The last process is allowed if \(m_\pi > 2m_q\), i.e. for sufficiently high temperatures. Its contribution reads, dropping the coordinate argument \(x\),

\[
-i\Pi_0^\left<\right>(p) = \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} (2\pi)^4\delta^4(p - p_1 - p_2) \text{tr} \left[ \gamma_5(p_1 + m_q)\gamma_5(-p_2 + m_q) \right]
\times \frac{(-i\pi)^2}{N_c N_f 2E_q(p_1) 2E_q(p_2)} \delta(p_{10} - E_q(p_1))\delta(p_{20} - E_q(p_2))n_q(p_1)n_\bar{q}(p_2). \] (4.3)

Using the on-shell constraints for both quarks and pions, this can be simplified to

\[
-i\Pi_0^\left<\right>(p) = - \frac{1}{g_s^2} \int \frac{d^3p_1}{(2\pi)^3 2E_q(p_1)} \frac{d^3p_2}{(2\pi)^3 2E_q(p_2)} (2\pi)^4\delta^4(p - p_1 - p_2)
\times |M|^2n_q(p_1)n_\bar{q}(p_2),
\] (4.4)

where

\[
|M|^2 = \frac{m_q^2 g_s^2}{2N_c N_f}.
\] (4.5)
is the transition amplitude for the process \( q\bar{q} \rightarrow \pi \). Note that the quantities on the right hand side itself depend on the state of the system. By a similar calculation as for Eq. (4.4), \( \Pi_0^\geq \) can be evaluated to be

\[
-i\Pi_0^\geq (p) = -\frac{1}{g_\pi^2} \int \frac{d^3 p_1}{(2\pi)^3 2E_q(\vec{p}_1)} \frac{d^3 p_2}{(2\pi)^3 2E_q(\vec{p}_2)} (2\pi)^4 \delta^4(p - p_1 - p_2) \times |\mathcal{M}|^2 (2N_cN_f - n_q(\vec{p}_1))(2N_cN_f - n_q(\vec{p}_2)) \]

with \(|\mathcal{M}|^2\) given by Eq. (4.3). Inserting (4.4) and (4.6) into (3.30) yields the full Boltzmann equation:

\[
\begin{bmatrix}
\partial_t + \vec{\partial}_p E_\pi \vec{\partial}_x - \vec{\partial}_x E_\pi \vec{\partial}_p \\
(2\pi)^3 2E_q(\vec{p}_1) \end{bmatrix} n_\pi(\vec{p}) = \frac{1}{2} E_\pi \int \left\{ n_q(\vec{p}_1)n_q(\vec{p}_2)[1 + n_\pi(\vec{p})] - n_\pi(\vec{p})[2N_cN_f - n_q(\vec{p}_1)][2N_cN_f - n_q(\vec{p}_2)] \right\} \times |\mathcal{M}|^2 \] (4.7)

This equation has exactly the form expected for a quark–meson plasma interacting via the decay of mesons into quarks and the recombination of quarks into mesons.

### B. Collision Term for Quarks

The two lowest order diagrams for the quark self energy in a \(1/N_c\) expansion are shown in Fig. 5. The diagram shown in Fig. 5a corresponds to the Hartree self energy and does not contribute to the collision term [14], so that the lowest order contribution to the collision term stems from the diagram shown in Fig. 5b. The contribution of this diagram is given by

\[
-i\Sigma^\leq (x,p) = \sum_k \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} (2\pi)^4 \delta^4(p - p_1 - p_2) \times i\Gamma_k iS^\leq (x,p_1)i\Gamma^\dagger_k i\Delta_\geq (x,-p_2) .
\]

The gain term of Eq. (2.25) thus reads

\[\text{Note that the transition amplitudes for } q\bar{q} \rightarrow \pi^0 \text{ and } \pi^0 \rightarrow q\bar{q} \text{ differ due to the different averaging factors for the incoming particles. If the transition amplitude for the process } \pi^0 \rightarrow q\bar{q} \text{ would be used in Eq. (4.6), the blocking factors would have to be replaced with } 1 - n_q(\vec{p})/(2N_cN_f).\]
\[
\text{Tr} [\Sigma^{<}(p) S^{>}(p)] = \sum_{k} \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} (2\pi)^4 \delta^4(p - p_1 - p_2) i\Delta_\sigma^>(-p_2) \]
\[
\times \text{Tr} \left[ i\Gamma_k iS^{<}(p_1) i\Gamma_k^\dagger iS^{>}(p) \right].
\]

In this expression, the off-diagonal meson Green function \(\Delta_\sigma^>\) appears. For the calculation of \(\Sigma^{<}\), one has to take into account both the singular and the regular part of this function. The two contributions of \(\Delta_\sigma^>\) will be computed separately in the following.

1. Collision Term from the Singular Part

For simplicity, only the contribution of the neutral pion is computed explicitly. The other contributions can be evaluated along the same lines and give similar results. To compute the contribution of the singular part, one has to insert Eqs. (2.24) and (3.28) into Eq. (4.9). Since for \(S^{>}(p)\) only the contribution at positive \(p_0\) is relevant, multiplying out Eq. (4.9) again leads to four contributions, which can be identified with the processes \(q \rightarrow q\pi^0\), \(q\pi^0 \rightarrow q\), \(\emptyset \rightarrow q\bar{q}\pi^0\) and \(\pi^0 \rightarrow q\bar{q}\). As was the case for the meson collision term, only the last term survives for \(m_\pi > 2m_q\) due to kinematical constraints. The contribution of this process reads, after a substitution \(p_1 \rightarrow -p_1\),

\[
\text{Tr} [\Sigma^{<}(p) S^{>}(p)]_s = \frac{\pi}{E_q(p)} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} (2\pi)^3 \delta^3(p + p_1 - p_2) \]
\[
\times \left[ g_\pi^2 n_{\pi^0}(p_2) \delta(p_2 - E_\pi(p_2)) \right] \times \text{Tr} \left\{ \gamma_5 \tau_3 \left[ \frac{\pi}{2E_q(p_1)} N_c N_f \delta_f^\sigma \delta_{cc}^\tau (-p_1 + m_q) \delta(p_0 - E_q(p_1)) (2N_c N_f - n_q(p_1)) \right] \times \gamma_5 \tau_3 \left[ \frac{\pi}{2E_q(p)} N_c N_f \delta_f^\sigma \delta_{cc}^\tau (p + m_q) \delta(p_0 - E_q(p)) (2N_c N_f - n_q(p)) \right] \right\}
\]
\[
= \frac{\pi}{E_q(p)} \delta(p_0 - E_q(p)) \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} (2\pi)^3 \delta^3(p + p_1 - p_2) \times |M|^2 n_{\pi^0}(p_2) (2N_c N_f - n_q(p)) (2N_c N_f - n_q(p_1)) \),
\]

where \(|M|^2\) is again given by Eq. (4.3). The total contribution of the singular part \(\Delta_\sigma^{>,<}\) to the collision term is thus given by
term, one has to insert into Eq. (4.9). The polarization Π
φ
π
φ
q
q
ϕ
φ
where of eight terms, of which five can be dropped since they correspond to processes like

\[ \Delta, <, > \times \text{Tr} [\Sigma > (x, p) \Delta (x, p)] \]

leads, after performing one of the integrations by means of the de lta function, to

\[ \int \frac{d\mathbf{p}_1}{2E_q(\mathbf{p})} \frac{d^3p_1}{(2\pi)^32E_q(\mathbf{p}_1)} \frac{d^3p_2}{(2\pi)^32E_p(\mathbf{p}_2)} (2\pi)^4 \delta^4(p + p_1 - p_2) |\mathcal{M}|^2 \]

\[ \times \{ n_{n=0}(\mathbf{p}_2) [2N_cN_f - n_q(\mathbf{p})] [2N_cN_f - n_q(\mathbf{p}_1)] - n_q(\mathbf{p}) n_q(\mathbf{p}_1) [1 + n_{n=0}(\mathbf{p}_2)] \} \]

and describes the process π^0 ↔ qq̅.

2. Collision Term from the Regular Part

In order to obtain the contribution from the regular part of Δ_k^>^< to the collision term, one has to insert

\[ \Delta_{k,r}^>^< (x, p) = \frac{1}{2} \Pi_{k,r}^>^< (x, p) \left[ \Delta_k^R (x, p)^2 + \Delta_k^A (x, p)^2 \right] = \Pi_{k,r}^>^< (x, p) \Re \left( \Delta_k^R (x, p)^2 \right) \]

into Eq. (4.9). The polarization Π_{k,r}^>^< (x, p) here has to be taken from Eq. (1.2). This leads, after performing one of the integrations by means of the delta function, to

\[ \text{Tr} [\Sigma^> (p) S^< (p)] = \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \frac{d^4p_3}{(2\pi)^4} (2\pi)^4 \delta^4(p - p_1 + p_2 - p_3) \Re \left( \Delta_k^R (p_2 - p_3)^2 \right) \]

\[ \times \text{Tr} [i\Gamma_0 iS^< (p_1) i\Gamma_0 iS^> (p)] \text{Tr} [i\Gamma_0 iS^> (p_2) i\Gamma_0 iS^< (p_3)] \]

The next part of the calculation proceeds exactly like shown in Ref. [14]. Multiplying out the contributions of the individual quark Green functions leads to the appearance of eight terms, of which five can be dropped since they correspond to processes like q ↔ qq̅ and ∅ ↔ qqq̅. The remaining three terms can be rearranged to give

\[ \text{Tr} [\Sigma^> (p) S^< (p)] = \frac{\pi}{E_q(\mathbf{p})} \delta(p_0 - E_q(\mathbf{p})) \]

\[ \times \int \frac{d^3p_1}{(2\pi)^32E_q(\mathbf{p}_1)} \frac{d^3p_2}{(2\pi)^32E_q(\mathbf{p}_2)} \frac{d^3p_3}{(2\pi)^32E_q(\mathbf{p}_3)} (2\pi)^4 \delta^4(p + p_1 - p_2 - p_3) \]

\[ \times \left\{ \frac{1}{2} \left[ \phi(p_1 - p_3) + \phi(p_1 - p_2) \right] n_q(p_2) n_q(p_3) \left[ 2N_cN_f - n_q(p_1) \right] \left[ 2N_cN_f - n_q(p) \right] \right. \]

\[ + \left. \left[ \phi(p_1 - p_3) + \phi(-p_2 - p_3) \right] n_q(p_2) n_q(p_3) \left[ 2N_cN_f - n_q(p_1) \right] \left[ 2N_cN_f - n_q(p) \right] \right\} \]

where \( \phi(p) \) is a shorthand notation for

\[ \phi(p) = \frac{1}{(2N_cN_f)^2} p^4 \Re \left( \Delta_0^R (p)^2 \right) \]
Equation (4.14) would have a form corresponding to a gain term due to elastic quark-quark and quark-antiquark scattering, if one could identify the collision kernel $\phi$ with the squared amplitude of the diagrams shown in Fig. 6. In this case $\phi(p_1 - p_3)$ would correspond to the $t$-channel exchanges of Fig. 6a and d, $\phi(p_1 - p_2)$ to the $u$-channel exchange of Fig. 6b and $\phi(-p_2 - p_3)$ to the $s$-channel exchange of Fig. 6c. Note that the interference terms of these diagrams belong to a higher order in $1/N_c$ and do not arise from the self-energy shown in Fig. 5b, as has been detailed in Ref. [14].

The problem with this interpretation is, that a direct calculation of the squared transition amplitudes corresponding to the graphs shown in Fig. 6 shows, that the collision kernel obtained from these is not given by Eq. (4.15), but rather by

$$\Phi(p) = \frac{1}{(2N_cN_f)^2} p^4 |\Delta_0^R(p)|^2.$$  \hspace{1cm} (4.16)$$

For the $t$ and $u$-channel exchanges, this makes no difference, since in these cases $\Delta_0^R(p)$ is real and thus Eqs. (4.15) and (4.16) yield the same result. A problem arises, however, with the $s$-channel, for which Eq. (4.15) gives effectively the wrong exchange propagator. The results of this effect can be seen from Fig. 7. The solid line in this figure shows the square of the full exchange propagator, $|\Delta_0^R|^2$, for $s$-channel exchange as a function of $\sqrt{s}$ at thermal equilibrium and $T = 300$ MeV, whereas the dashed line shows its replacement in Eq. (4.15), $\Re(\Delta_0^R^2)$. Since the latter leads to a negative collision kernel at large $\sqrt{s}$, Eq. (4.15) cannot be regarded as a valid collision kernel for $s$-channel scattering.

The physical reason for this breakdown of the theory can be seen from the following: the full transition amplitude (4.16) contains resonance exchange, i.e. contributions of the process $q\bar{q} \rightarrow \pi \rightarrow q\bar{q}$. In the present approach, however, these contributions have to be taken out from the elastic scattering amplitude, since they are already counted in the collision terms for $q\bar{q} \leftrightarrow \pi$. Using the full transition amplitude (4.16) is thus double counting.

It is thus to be expected, that the $s$-channel elastic scattering amplitude has to be modified near a resonance. Nevertheless, Eq. (4.15) cannot be the correct form since it (i) gives a negative collision kernel, (ii) modifies the collision kernel at momenta far off from the resonance, where no modification is to be expected and (iii) gives also the wrong results at low temperatures, where the production of pions via recombination is kinematically forbidden and thus no interference can take
The form (4.16) for the collision kernel can be obtained by setting \[ \Delta_{k}^{>,<} \approx \Pi_{k}^{>,<} \left| \Delta_{k}^{R} \right|^2. \] (4.17)

This ansatz, however, inhibits the appearance of mesons as individual degrees of freedom. A similar result was obtained in Ref. [18], where it was shown that a quasiparticle ansatz for the meson propagator leads to collision terms containing processes of the type \( q\bar{q} \leftrightarrow \pi \), but no elastic scattering of quarks, whereas the ansatz (4.17) leads to the elastic scattering amplitude (4.16), eliminates however the appearance of free mesons. Note that this problem is not a generic problem of a model containing mesons as bound states, but rather a general problem of theories containing interacting quarks and mesons, as can be concluded from Ref. [18]. Nevertheless, elastic scattering processes have to be obtained from the diagram 5b. They cannot be derived from self energy graphs like the one shown in Fig. 8, which would lead to the appearance of elastic scattering processes when evaluated in a quasiparticle approximation: This graph is already implicitly included in the graph shown in Fig. 5b via the meson line and its explicit appearance would thus be double counting.

The conclusion from this is, that considering off-shell corrections to \( \Delta_{k}^{>,<} \) is essential in order to obtain collision terms containing elastic scattering at all, the form of the off-shell corrections given in Ref. [8] is, however, insufficient as far as the s-channel scattering amplitude is concerned.

In order to find a more suitable form of the off-shell corrections, one has to consider a different decomposition of \( \Delta_{k}^{>,<} \) into a singular and regular part as that of Eqs. (3.15). An attempt to do this, which however leads to the appearance of new terms in the derivation of the Boltzmann equation, is shown in the appendix. This decomposition leads to the alternative form

\[
\Delta_{k,r}^{>,<} = \Pi_{k}^{>,<} \left[ \left| \Delta_{k}^{R} \right|^2 - 2 \left( \Im \Delta_{k,p}^{R} \right)^2 \right].
\] (4.18)

for the regular part of \( \Delta_{k}^{>,<} \). The auxiliary quantity \( \Delta_{k,p}^{R} \) in Eq. (4.18) is defined by

\[
\Delta_{k,p}^{R}(x,p) = \frac{g_{k}^{2}(x,\vec{p})}{2E_{k}(x,\vec{p}) \left( p_0 - E_{k}(x,\vec{p}) + \frac{i}{2} \Gamma_{k}(x,\vec{p}) \right)}. \] (4.19)
This form of the off-shell corrections improves (4.12) in so far, as the effective exchange propagator is modified only in a small region around the meson mass. If one has $\Delta_{k,p}^R \approx \Delta_{k}^R$, one reobtains the form (4.13) for the collision kernel. Far off from the pole, $\Delta_{k,p}^R$ can be neglected and one obtains the collision kernel (4.16), which in this region is dominated by scattering states. Equation (4.18) has thus the property to interpolate between these two forms. It is nevertheless not a complete solution to the problem, as can be seen from the dotted line in Fig. 7, which shows that Eq. (4.18) gives a good approximation to Eq. (4.16) far off the pion mass. This is especially important in order to obtain the correct scattering amplitude at low temperatures, where the formation of pions by recombination is kinematically forbidden. Near the meson mass the transition amplitude is modified, as expected. Equation (4.18) gives, however, still a negative contribution at this energy and can thus not be regarded as a complete solution to the problem. It is nevertheless worthwhile to look for improved off-shell corrections by considering alternative decompositions.

V. SUMMARY AND CONCLUSIONS

In the preceding sections, it is shown that it is possible to extend the formalism of transport theory in order to describe the evolution of a system of quarks and bound state mesons. Each of these components evolves according to a Boltzmann equation containing a mean field term and collision integrals, which describe the interactions with the other component. Since mesons appear as effective degrees of freedom, the dynamical transition from a quark phase to a hadronical phase can be modeled. The derivation of the transport equations leads to medium dependent transition amplitudes in the collision integrals.

In order to obtain collision integrals for higher order processes, the consideration of off-shell corrections to the off-diagonal Green functions is essential. These off-shell corrections have to be chosen carefully, since they modify the collision kernels in a nontrivial way.

In order to extend the current approach to a nonequilibrium description of the chiral phase transition, further work has still do be done. Although the present approximation allows for the transformation of quark pairs into hadrons via the process $q\bar{q} \rightarrow \pi$, this process cannot be expected to be very efficient, since its
probability is limited by the phase space. One has thus to consider the processes $q \bar{q} \rightarrow \pi \pi$ [6] or $q \bar{q} \rightarrow \pi \pi \pi$ [26], which can proceed via a much less limited kinematics. This in turn demands the inclusion of higher order self energies [7]. For this work, it might also be necessary to introduce off-shell corrections for the quark Green functions.

One of the requirements for this expansion is that it has to be symmetry conserving. This concerns especially chiral symmetry and the validity of the Goldstone theorem. A symmetry conserving expansion for the equilibrium case has been outlined in Ref. [22]. A generalization of this work to non-equilibrium and the derivation of collision terms for this expansion is left for a future publication.

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APPENDIX: DERIVATION OF IMPROVED OFF-SHELL CORRECTIONS

The pole approximation of the retarded Green function given in Eq. (3.9) reads

$$\Delta^R_k(x,p) \approx \frac{g^2_k(x,\vec{p}) - ia_k(x,\vec{p})}{2E_k(x,\vec{p}) \left( p_0 - E_k(x,\vec{p}) + \frac{i}{2} \Gamma_k(x,\vec{p}) \right)}.$$

(A1)

The imaginary part $a_k(x,\vec{p})$ of the pole residue appearing here is of the order of the width and thus gives no contribution to the final result in Section III.B. It has also not been considered in Ref. [8]. In the following it will thus be attempted to derive transport equations using the effective form

$$\Delta^R_{k,p}(x,p) = \frac{g^2_k(x,\vec{p})}{2E_k(x,\vec{p}) \left( p_0 - E_k(x,\vec{p}) + \frac{1}{2} \Gamma_k(x,\vec{p}) \right)}.$$

(A2)

for the retarded Green functions. To do this, define the auxiliary quantities

$$\rho_{k,p}(x,p) = i \left( \Delta^R_{k,p}(x,p) - \Delta^A_{k,p}(x,p) \right)$$

(A3a)
\[ \Delta_b^{-1} - \Pi_{k,p}^R(x,p) = \frac{2E_k(x,p)}{g_k^2(x,p)} \left( p_0 - E_k(x,p) + \frac{i}{2} \Gamma_k(x,p) \right) \]  

(A3b)

\[ \Pi_{k,p}(x,p) = \frac{1}{2} \left( \Pi_{k,p}^R(x,p) + \Pi_{k,p}^A(x,p) \right) \]  

(A3c)

\[ \gamma_{k,p}(x,p) = i \left( \Pi_{k,p}^R(x,p) - \Pi_{k,p}^A(x,p) \right) \]  

(A3d)

It can be easily verified, that after a transformation to coordinate space \( \Delta_{k,p}^{R,A} \) fulfill the equations

\[ \left( \Delta_b^{-1} \otimes \Delta_{k,p}^{R,A} \right)(x,y) = \delta^4(x-y) + \left( \Pi_{k,p}^R \otimes \Delta_{k,p}^{R,A} \right)(x,y) \]  

(A4a)

\[ \left( \Delta_{k,p}^{R,A} \otimes \Delta_b^{-1} \right)(x,y) = \delta^4(x-y) + \left( \Delta_{k,p}^{R,A} \otimes \Pi_{k,p}^R \right)(x,y) \]  

(A4b)

and \( \rho_{k,p} \) obeys the equation

\[ \rho_{k,p} = \Delta_{k,p}^R \otimes \gamma_{k,p} \otimes \Delta_{k,p}^A \]  

(A5)

The decomposition of \( \Delta_{k,s}^{>\prec} \) and \( \rho_k \) is done such that the singular parts are defined as

\[ \Delta_{k,s}^{>\prec} = \frac{i}{2} \left( \Delta_{k,p}^R \otimes \Pi_{k}^{>\prec} \otimes \rho_{k,p} - \rho_{k,p} \otimes \Pi_{k}^{>\prec} \otimes \Delta_{k,p}^A \right) \]  

(A6a)

\[ \rho_{k,s} = \frac{i}{2} \left( \Delta_{k,p}^R \otimes \gamma_k \otimes \rho_{k,p} - \rho_{k,p} \otimes \gamma_k \otimes \Delta_{k,p}^A \right) \]  

(A6b)

and the regular parts are defined as the difference of the full function and the singular part.

Analogously to Eq. (3.17), one obtains

\[ -i \left[ \left( \Delta_{k,p}^R \right)^{-1} \otimes \Delta_{k,s}^{>\prec} - \Delta_{k,s}^{>\prec} \otimes \left( \Delta_{k,p}^A \right)^{-1} \right] = \frac{1}{2} \left( \gamma_{k,p} \otimes \Delta_{k,s}^{>\prec} + \Delta_{k,s}^{>\prec} \otimes \gamma_{k,p} \right) \]  

(A7)

\[ -i \left[ \left( \Delta_b^{-1} - \Pi_{k,p} \right) \otimes \Delta_{k,s}^{>\prec} - \Delta_{k,s}^{>\prec} \otimes \left( \Delta_b^{-1} - \Pi_{k,p} \right) \right] \]

and Eq. (3.18) is replaced by

\[ -i \left[ \left( \Delta_{k,p}^R \right)^{-1} \otimes \Delta_{k,s}^{>\prec} - \Delta_{k,s}^{>\prec} \otimes \left( \Delta_{k,p}^A \right)^{-1} \right] = \frac{1}{2} \left( \Pi_{k}^{>\prec} \otimes \rho_{k,p} + \rho_{k,p} \otimes \Pi_{k}^{>\prec} \right) \]  

(A8)

\[ -i \left[ \left( \Delta_b^{-1} - \Pi_{k,p} \right) \otimes \Delta_{k,s}^{>\prec} - \Delta_{k,s}^{>\prec} \otimes \left( \Delta_b^{-1} - \Pi_{k,p} \right) \right] \]

After performing the Wigner transformation, this becomes

\[ \{ \Delta_b^{-1} - \Pi_{k,p}; \Delta_{k,s}^{>\prec} \} + \frac{1}{4} \left\{ \Pi_{k}^{>\prec} \left( \Delta_{k,p}^R + \Delta_{k,p}^A \right) \rho_{k,p}; \gamma_{k,p} \right\} \]  

(A9)

\[ = \Pi_{k}^{>\prec} \left[ \rho_{k,p} - \frac{1}{2} \gamma_{p} \left( \Delta_{k,p}^R \right) + \Delta_{k,p}^A \right] - \gamma_{k,p} \Delta_{k,s}^{>\prec} \]

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By using the explicit representation (A2) and the definitions (A3), it is possible to show that

$$\rho_{k,p} - \frac{1}{2} \gamma_p \left( \Delta_{k,p}^R + \Delta_{k,p}^A \right) = \tilde{\rho}_{k,s} = \frac{1}{2} \gamma_{k,p} \rho_{k,p}^2 \ .$$  

(A10)

After employing the factorization $\Delta_{k,s} = -i\rho_{k,s} f_k$, the first term on the left hand side of Eq. (A9) becomes

$$\left\{ \Delta_{k,s}^{-1} - \Pi_{k,p} \right\} = -i\rho_{k,s} \left\{ \frac{1}{4} \gamma_{k,p} \left( \Delta_{k,p}^R + \Delta_{k,p}^A \right) \rho_{k,p} \right\} \ ,$$

(A11)

which generalizes Eq. (3.23) to the present case. Repeating the steps which led to the derivation of Eq. (3.26) leads now to

$$-i\rho_{k,s} \left\{ \Delta_{k,s}^{-1} - \Pi_{k,p} \right\} f_k = \left\{ \frac{1}{4} \gamma_{k,p} \left( \Delta_{k,p}^R + \Delta_{k,p}^A \right) \rho_{k,p} \right\} \ .$$

(A12)

The fourth term on the left hand side can be dropped using the same arguments as before. The second term on the left hand side is new. Since it is proportional to the gradient of the difference $\gamma_k - \gamma_{k,p}$, it is also regarded as belonging to higher orders and thus neglected. The remaining terms contain either a factor $\rho_{k,s}$ or $\tilde{\rho}_{k,s}$. Although these are not equal, they are of the same order and become strongly peaked near the quasiparticle energy in the small width limit. It is thus justified to drop these factors and one obtains

$$\left\{ \frac{1}{4} \gamma_{k,p} \left( \Delta_{k,p}^R + \Delta_{k,p}^A \right) \rho_{k,p} \right\} = \Pi_{k,s} \tilde{\rho}_{k,s} \rho_{k,s} .$$

(A13)

where all momenta have to be taken on shell. The second term on the left hand side vanishes. Furthermore it can be inferred from Eqs. (A2) and (A3), that up to the order $a_k^2$ one has $\gamma_{k,p} = \gamma_k$ on the mass shell. After identifying $f_k$ with the particle density, one reobtains the Boltzmann Equation (3.30).

Although this derivation gives the same form of the transport equation, it differs in the form of the regular part of $\Delta_{k,s}$. By definition, one has in coordinate space
\[
\Delta_{\kappa, r}^{>_<} = \Delta_{\kappa, r}^{>_<} - \Delta_{\kappa, s}^{>_<} = \Delta_{\kappa}^{R} \otimes \Pi_{\kappa}^{>_<} \otimes \Delta_{\kappa}^{A} \\
- \frac{i}{2} \left( \Delta_{\kappa, p}^{R} \otimes \Pi_{\kappa}^{>_<} \otimes \rho_{\kappa, p} - \rho_{\kappa, p} \otimes \Pi_{\kappa}^{>_<} \otimes \Delta_{\kappa, p}^{A} \right).
\]

Transforming this to momentum space and neglecting the gradient terms, which are of the same order as the term neglected in Eq. (A12), leads to

\[
\Delta_{\kappa, r}^{>_<} = \Pi_{\kappa}^{>_<} \left( \Delta_{\kappa}^{R} \Delta_{\kappa}^{A} - \frac{1}{2} \rho_{\kappa, p}^{2} \right),
\]

which is equal to Eq. (4.18).
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FIGURES

FIG. 1. The Schwinger–Keldysh contour $C$ in the complex time plane.

FIG. 2. Pion spectral function as a function of $p_0$ at thermal equilibrium and $T = 0$. The model parameters used for this calculation are $m_0 = 5$ MeV for the current quark mass, $G\Lambda^2 = 2.105$ for the coupling constant and $\Lambda = 653$ MeV for the NJL cutoff, which gives a quark mass of $m_q = 313$ MeV from Eq. (2.26) and a pion mass of $m_\pi = 134$ MeV from Eq. (3.7). Note the delta peaks at $p_0 = \pm m_\pi$, which correspond to bound state pions. The continua for $|p_0| > 2m_q$ stem from $q\bar{q}$ scattering states.

FIG. 3. Pion spectral function as a function of $p_0$ at thermal equilibrium and $T = 300$ MeV. The model parameters are the same as for Fig. 2, leading at this temperature to $m_q = 21$ MeV and $m_\pi = 366$ MeV. The solid line corresponds to the full spectral function, the dashed line gives a pole approximation. The delta peaks visible in Fig. 2 have now obtained a finite width and the pion has become a resonant state.

FIG. 4. Lowest order diagram for the polarization $\Pi_k$. Solid lines correspond to quarks.

FIG. 5. Lowest order diagrams for the quark self energy $\Sigma$. Solid lines correspond to quarks, double lines to mesons. Diagram (a) gives the Hartree self energy, diagram (b) the lowest order contribution to the collision term.

FIG. 6. Lowest order diagrams for elastic scattering by meson exchange. Solid lines correspond to quarks, double lines to mesons. The diagrams (a) and (b) give the $t$ and $u$ channel exchange for quark-quark scattering, (c) and (d) the $s$ and $t$ channel exchange for quark-antiquark scattering.

FIG. 7. Exchange propagators for the $s$-channel of elastic quark-antiquark scattering in equilibrium at $T = 300$ MeV as a function of $\sqrt{s}$. Solid line: $|\Delta_0^R|^2$, dashed line: $\Re(\Delta_0^{R2})$, dotted line: interpolating form according to Eq. (4.18). The vertical axis is given in arbitrary units.
FIG. 8. Self energy graph leading to elastic scattering processes when evaluated in a quasiparticle approximation. Solid lines correspond to quarks, double lines to mesons.
Figure 1
Figure 2

$\rho (10^{-6} \text{ MeV}^{-2})$

$p_0 (\text{MeV})$

-1500 -1000 -500 0 500 1000 1500

-10 -5 0 5 10
$-i \Pi_k = i \Gamma_k \xrightarrow{\text{Figure 4}} i \Gamma_k^\dagger$
\[-i \Sigma = \quad \begin{array}{c}
\text{(a)} \\
\end{array} + \quad \begin{array}{c}
\text{(b)} \\
\end{array}\]
Figure 6
$-i\Sigma =$

Figure 8