SPH-FEM coupling simulation of rock blast damage based on the determination and optimization of the RHT model parameters

Z L Wang*, Y P Huang, S Y Li and F Xiong

School of Civil and Hydraulic Engineering, Hefei University of Technology, Hefei 230009, Anhui, China

Abstract. To improve the accuracy of the results of rock blast damage simulation, a numerical scheme of ascertaining and optimizing the Riedel–Hiermaier–Thoma (RHT) model parameters was developed. First, the RHT model was adopted to simulate limestone penetration experiments, and the 31 RHT model parameters were initially determined based on the published data and theoretical derivation. Subsequently, sensitivity analyses were conducted using the “controlled variable method” to optimally determine the parameters. Finally, the RHT model parameters and coupled smoothed-particle hydrodynamics (SPH)-finite element method (FEM) were employed to simulate the limestone blast damage. The results demonstrate that the relative sensitive parameters obtained by taking the penetration depth of the projectile as the investigation index are \( p_{0i} \), \( p_{0s} \), \( p_{0c} \), \( G \), \( f_c \), \( B \), \( M \), and \( F_c \). The proposed “constant-amplitude adjustment method” can guarantee the integrity of the parameter ascertainment to a certain extent. The blast simulations indicate that the RHT model parameters determined based on the penetration experiment have certain applicability for the blast simulation and that the coupled SPH-FEM algorithm has a great advantage over FEM or SPH and can realistically show the whole process of rock blasting.

1. Introduction

Rock blasting is a dynamic process in which the explosive generates and releases huge energy in a very short time. In this process, the explosive wave propagates in the rock medium, accompanied by the crushing, cracking, and throwing phenomena. This process is difficult to monitor and has low controllability. Considering the economic benefit and safety limitation, research on blasting test was seldom conducted. In comparison, the numerical simulation method has great advantages; it is capable of monitoring the dynamic response process of rock blasting and conveniently obtaining the required parameter information in all directions. Currently, it has become an important method for studying the rock blasting problem, in addition to the experimental method.

The traditional numerical simulation of rock blasting adopts the finite element method (FEM) based on pure grid, such as the Lagrangian, Eulerian, and arbitrary Lagrangian–Eulerian (ALE) algorithms. However, these algorithms have more or less limitations in their respective applications. For example, the calculation accuracy of the Lagrangian algorithm cannot be guaranteed; the Eulerian algorithm cannot accurately locate the position of the free surface, deformation boundary, and moving material interface; whereas the ALE algorithm has a long calculation time due to its dual grid properties. To overcome these limitations, the meshless method was proposed. The SPH method[1-3], which is one of...
the most widely used and mature Lagrangian particle methods, has achieved remarkable application effects in the numerical simulation of ultra-high-speed collision and explosion\textsuperscript{[4, 5]}. However, the SPH method still has drawbacks, such as low calculation efficiency and the difficulty in dealing with boundary conditions. Thus, numerous scholars have proposed the coupled SPH-FEM. For large deformation problems, such as rock blasting, the SPH algorithm is used in the large deformation area of the numerical model, whereas the FEM algorithm is used in the small deformation area. Thus, the advantages of both algorithms are combined. For example, Zhang\textsuperscript{[6]} used the coupled SPH-FEM algorithm to simulate the blanking process failure of steel projectile impact steel plate. The results were in good agreement with the experimental results. Hu\textsuperscript{[7]} simulated deep-hole step blasting by using the SPH particles in the near blasting area and the FEM algorithm in the far blasting area. The results show that the coupled SPH-FEM algorithm can simulate nonlinear characteristics, such as large rock deformation in the area near the blast hole, and predict the dynamic response in the middle and far blasting area. Hu\textsuperscript{[8]} pointed out that the coupled SPH-FEM algorithm integrates the advantages of high computational efficiency of the FEM and natural simulation of explosion characteristics, such as concrete damage and splash of the SPH algorithm. In conclusion, the coupled SPH-FEM algorithm exhibits a good application effect in the simulation study of explosion and impact problems. At present, it is still a hot spot of numerical simulation methods.

Conversely, the reasonable selection of material model and the accurate determination of its parameters are related to the accuracy of the results of the numerical calculation. It is well known that the Holmquist–Johnson–Cook (HJC)\textsuperscript{[9]} constitutive model and the Riedel–Hiermaier–Thoma (RHT)\textsuperscript{[10]} constitutive model are widely used to describe the dynamic damage of rock materials. Currently, there are numerous in-depth studies on the determination and sensitivity analysis of the HJC model parameters\textsuperscript{[11–13]}. Contrarily, the RHT model parameters have a large quantity and are difficult to determine; thus, relevant research is relatively scarce. Moreover, as a new method, the study on the coupled SPH-FEM algorithm in rock blasting damage is far less than that in impact problem, and research on the determination of the RHT model parameters and its sensitivity analysis using this method is even scarce. First, based on the previous experimental results and the theoretical derivation, a set of RHT model parameters are initially determined. Then, the coupled SPH-FEM algorithm in the Autodyn software is adopted to simulate the limestone penetration experiment for the parameter sensitivity analysis and calibration. Finally, the determined parameters are applied in the rock blasting damage simulation using the coupled SPH-FEM algorithm. By comparing the results with those of the traditional FEM, some valuable results are obtained.

2. Algorithm

2.1. Principle of the SPH algorithm

The SPH algorithm uses a series of arbitrarily distributed particles for solving integral equations or partial differential equations with various boundary conditions, thus obtaining accurate and stable numerical solutions. In general, the formulation of the SPH equation has two key steps\textsuperscript{[3]}:  

(1) Interpolation of the kernel function by integral expression. For a continuous smooth function \(f(r)\), the value of the function at a point on the domain \(\Omega\) can be expressed as follows:  

\[
    f(r) = \int_{\Omega} f(r') \delta(r - r') \, dr'  
\]

(1)  

where \(r\) denotes the space position vector, and \(\delta(r - r')\) denotes the Dirac delta function. Because it is difficult to realize the Dirac delta function in practice, it is replaced with the smooth function \(W(r-r', h)\) in the numerical calculation. Then, \(f(r)\) can be approximately expressed as: follows
In the formula, \( W(\mathbf{r} - \mathbf{r}', h) \) denotes the SPH kernel function, whose value depends on the distance \(|\mathbf{r} - \mathbf{r}'|\) between two points and the smooth length \(h\) and, together with the smooth factor \(\kappa\), determines the influence range of the smooth function.

(2) Approximation of the particles. Equation (2) is discretized with particles, and then the particles of the field function at particle \(i\) are approximately expressed as follows:

\[
\langle f(\mathbf{r}) \rangle = \frac{1}{N} \sum_{j=1}^{N} \rho_j f(\mathbf{r}_j) W_j
\]

where \(m_j\) and \(\rho_j\) denote the mass and density of particle \(j\) \((j = 1, 2, ..., N)\), respectively, and \(N\) denotes the total number of particles in the compact support domain. In this case, the value of the field function at particle \(i\) can be obtained by weighted average of the function value of all particles in the compact support domain by the kernel function.

2.2. Coupled SPH-FEM algorithm
The key to achieving the coupling modeling of the SPH-FEM algorithm is the coupling between the SPH particles and finite elements. Currently, the coupling methods can be roughly divided into three categories: contact, consolidation, and transformation\[^{14-15}\]. The contact coupling method defines the finite element surfaces and SPH particles as the master surfaces and slave points, respectively. It also adopts the contact algorithm similar to the master–slave algorithm of the FEM. The consolidation coupling method uses rigid connection to consolidate the SPH particles on the adjacent finite element nodes. In this case, the force between the node and particle on the coupling interface is the resultant force of both. Based on the “element erosion” algorithm of the finite element. Conversely, the transformation coupling method uses the initial model of pure finite element. When the element on the contact boundary reaches the defined equivalent plastic strain, the SPH particle is generated at the center of the element to replace it. Subsequently, the master surface and slave point are redefined for calculation.

Herein, the consolidation coupling algorithm of SPH-FEM in the Autodyn software is adopted for the numerical calculation, so that the coordination of displacement and deformation between the two regions can be guaranteed. The consolidation coupling algorithm can be divided into three modes according to the different arrangement forms of particles in the boundary of finite element domain, as presented in Figure 1\[^{16}\].

3. Determination of the RHT model parameters
By improving the HJC model\[^{17}\], Riedel et al. proposed the RHT model which is a combined model of tension, compression, and shear damage. It comprehensively considers the influence of confining pressure effect, strain rate effect, and damage on failure strength of concrete materials under dynamic load. Moreover, it has been effectively applied in the simulation study of impact and explosion. Herein, the parameters required for the RHT model in the Autodyn software are divided into 19 constitutive parameters and 12 state equation parameters, which are preliminarily determined based on existing limestone mechanics experiments and theoretical derivation in the literature.

3.1. Preliminary determination of parameters
Because the RHT model has too many parameters, the following ideas can be adopted in the determination process: (a) A small number of basic mechanical parameters are obtained through existing experiments in literature. Based on this, some parameters are obtained through theoretical
derivation, and the remaining parameters are temporarily taken from the model raw value. (b) Based on the parameters that were initially determined through the inversion trial calculation method, the simulation results are gradually approached with the experimental results, and the appropriate parameters can be finally obtained.

Herein, the research focus is the Salem limestone. The parameters $\rho_0, \rho_s, G, f_c, f_s, p_e$, and $p_s$ can be directly obtained from the literatures [9, 18-20]; the parameters $\alpha, \delta, A_1, A_2, A_3, B_0, B_1, T_1$, and $T_2$ are derived easily from the literature [21]; and the remaining parameters are temporarily taken from the original parameters of the model in the software. The initially determined RHT model parameters are presented in Table 1.

### Table 1. RHT model parameters.

(a) RHT constitutive parameters

| $G$/GPa | $f_c$/MPa | $f_s$/fc | $f_s$/fc | $A$ | $Q_{2,0}$ | $B_Q$ | $R_G$ | $R_t$ |
|--------|-----------|---------|---------|-----|----------|-------|-------|-------|
| 9.53   | 60        | 0.1     | 0.3     | 1.6 | 0.61     | 0.6805| 0.0105| 2     |

| $R_c$ | $B$ | $M$ | $\alpha$ | $\delta$ | $D_1$ | $D_2$ | $\varepsilon_{f,\text{min}}$ | $F_G$ |
|-------|-----|-----|---------|---------|-------|-------|---------------------------|-------|
| 0.53  | 1.6 | 0.61 | 0.025   | 0.02    | 0.04  | 1     | 0.01                      | 0.13  |

(b) Parameters of the $P-\alpha$ function

| $\rho_0$/kg·m$^{-3}$ | $\rho_s$/kg·m$^{-3}$ | $p_e$/GPa | $p_s$/GPa | $n$ | $A_1$/GPa | $A_2$/GPa | $A_3$/GPa | $B_0$ | $B_1$ | $T_1$/GPa | $T_2$/GPa |
|----------------------|----------------------|-----------|-----------|-----|------------|------------|------------|-------|-------|------------|------------|
| 2300                 | 2700                 | 0.02      | 2         | 3   | 38.47      | 34.63      | -7.12      | 0.9   | 0.9   | 38.47      | 0          |

3.2. Parameter verification

To verify the rationality of the initially determined parameters in Table 1, the Autodyne software is used to simulate the limestone penetration experiments[22]. The experimental conditions are presented in Table 2. The projectile diameters of experiment groups 1 and 2 are 7.1 and 12.7 mm, respectively, and each group includes four working conditions. According to different experiment conditions to establish SPH-FEM numerical model. Figure 2 presents the schematic of the numerical model, in which mode III in Figure 1 is adopted for the coupling of particles and finite elements.
To compare the experimental and simulation results, take the penetration depth of the projectile as the inspection index, as presented in Table 2. It can be observed that the penetration depth simulated by the initial parameters is quite different from the experimental value under various conditions. Moreover, the simulation result of condition 7.1-1 is even more than twice the experimental value. Through the error analysis, it is found that the error mainly comes from the deviation of parameters. Thus, it is important to adjust the parameters in Table 1, so that the simulation results are in good agreement with the experimental results.

**Table 2.** Penetration experiments and simulation results of limestone.

| Test group | Target length /m | Projectile velocity /m·s⁻¹ | Tested depth /cm | Simulated depth /cm |
|------------|-----------------|---------------------------|-----------------|-------------------|
| **1**      |                 |                           |                 |                   |
| 7.1-1      | 0.30            | 497                       | 6.7             | 14.3              |
| 7.1-2      | 0.61            | 597                       | 10.5            | 19.3              |
| 7.1-3      | 0.61            | 787                       | 16.5            | 31.8              |
| 7.1-4      | 0.61            | 1037                      | 27.1            | 49.4              |
| **2**      |                 |                           |                 |                   |
| 12.7-1     | 0.61            | 459                       | 14.1            | 21.7              |
| 12.7-2     | 0.61            | 608                       | 23.2            | 35.0              |
| 12.7-3     | 0.91            | 853                       | 36.2            | 62.0              |
| 12.7-4     | 0.91            | 956                       | 52.3            | 73.2              |
4. Parameter sensitivity analysis and calibration

4.1. Sensitivity analysis of parameters
To adjust the parameters using the inversion test algorithm, it is important to first clarify the influence of each parameter on the reference index of interest, such as the penetration depth, and then gradually adjust the simulation results to the experimental ones via trial adjustment, so that the final parameters can be obtained. Through the sensitivity analysis, the influence of the parameter changes on the calculation result can be better evaluated, which further identifies the parameters that play a significant role in the simulation and provides a basis for parameter adjustment.

Because the RHT model has 31 parameters, analysis of the overall sensitivity of the parameters is difficult from a global perspective. Therefore, the “controlled variable method” is adopted here for local sensitivity analysis, that is, each analysis takes a single parameter as the discussion object. In this section, the penetration depth of the projectile in condition 12.7-1 in Table 2 is selected as the inspection index, and the parameters in Table 1 is adjusted by ±20% and ±40% (since it must meet the requirements of \( \rho_s > \rho_0 \), only \( \rho_s \) is increased, whereas \( \rho_0 \) is decreased); moreover, the simulation values of the projectile penetration depth under each adjusted parameter is recorded. Through a large number of numerical calculations, the changes in the relative penetration depth of the projectile with each parameter adjustment are finally obtained, as presented in Figure 3.

![Graph](image)

(a) Parameters of the RHT constitutive model

![Graph](image)

(b) Parameters of the \( p-\alpha \) state function

**Figure 3.** Parameter sensitivity analysis of the RHT constitutive model and state equation.

From the figure, it can be seen that within the variation range of −40% to 40%, the level of influence of each parameter on the penetration depth is different, and the influence of most parameters on the penetration depth is mainly concentrated in the range of ±10%. The parameters under which the relative penetration depth changes by more than 10% are considered to be sensitive parameters; hereby, the sensitive parameters are determined as follows: \( \rho_0, \rho_s, \rho_c, G, f_c, B, M, \) and \( F_G \).

4.2. Determination and optimization of parameters
Parameter ascertainment is a process of repeated trial calculations. To simplify the process and improve efficiency, the principle of adjustment is required in advance. Parameter ascertainment follows three principles: (1) High-sensitivity parameters are selected as the adjustment object. (2) The basic physical and mechanical parameters which can be tested are not within the adjustment range. (3)
To avoid distortion caused by an overly large adjustment range of single parameter, the “constant-amplitude adjustment method” of sensitive parameters is proposed, that is, all the sensitive parameters are increased or decreased by the same amplitude. This is done to ensure the integrity of the parameter adjustment to a certain extent. Based on above principles, the sensitive parameters to be calibrated are finally selected: \( p_e, B, M, \) and \( F_G \).

On the basis of the influence of the fluctuation of each sensitive parameter, which is presented in Figure 3, on the deviation of the calculation results, the parameters \( p_e, B, M, \) and \( F_G \) are adjusted by +5% each time, and the penetration depth variation is recorded. Through multiple trial calculations, the final range of adjustment is 40%. At this time, the values of the sensitive parameters are as follows:

\[
\begin{align*}
p_e &= 0.028 \text{ GPa}, \\
B &= 2.24, \\
M &= 0.854, \\
F_G &= 0.182.
\end{align*}
\]

Figure 4 presents the comparison between the simulation results and experimental results before and after parameter adjustment, where value 1 and value 2 denote the simulation results before and after parameter adjustment, respectively. From the figure, it can be seen that the simulation results after the parameter calibration are close to the experimental results, and the deviations are controlled within an acceptable range. This indicates that the parameter determination and optimization are basically accurate.

5. SPH-FEM coupling simulation of rock blasting damage

Compared with the case of high-speed impact, the application of the coupled SPH-FEM algorithm in blasting damage simulation is less. Meanwhile, due to the lack of experimental data, most of the parameters of rock blasting simulation can only come from some basic mechanical experiments or theoretical derivations. Herein, the RHT model parameters determined above are adopted to conduct numerical modeling using the coupled SPH-FEM algorithm, and its application effect in the rock blast damage simulation is discussed.

The rock blasting simulation example uses a shallow blasting of limestone, and its 2D model is presented in Figure 5. The explosive in the model is a hemispherical TNT with a radius of 2 cm, which is modeled by the SPH particles. The radius of the rock part is 100 cm. To ensure calculation efficiency, it is determined through repeated trial calculation that the area with a radius of 20 cm (10 times the explosive radius) near the explosion source adopts the SPH particles, and the remaining area adopts the FEM. Mode II, which is presented in Figure 1, is adopted between the particles and the finite element. The upper edge of the model is set as the free boundary, whereas the arc edge is set as the non-reflecting boundary, so as to achieve the effect of simulating the semi-infinite rock mass. Figure 6 presents the comparison of the simulation results of limestone blasting damage using the coupled SPH-FEM algorithm and the FEM.

From the figure, it can be seen that the simulation results of the coupled SPH-FEM algorithm and
FEM are relatively close and that the crushing zone \((D = 1)\) and the main crack expansion modes are basically the same. The cracking, throwing, and crushing phenomena of the rock during the blasting process are naturally and realistically simulated using the coupled SPH-FEM algorithm, whereas the natural effects of the blasting phenomena are difficult to achieve using the FEM. Moreover, the simulation based on the pure grid makes the elements in the vicinity of the explosion source seriously distorted in the later stage of calculation, which leads to a significant increase in the calculation time and cost and even to calculation interruption. However, it should be pointed out that the damage cloud image obtained by the FEM exhibits a good continuity, whereas the damage evolution of the particle and finite element area in the SPH-FEM model is discontinuous. Although numerous coupling methods of SPH and FEM have been proposed in the literature to improve this situation, the relevant research is scarce, and the development of relevant functions of commercial software lags behind the theory. Therefore, it is still difficult to achieve the ideal simulation effect through the application of the coupled SPH-FEM algorithm. Thus, further research is required.

Figure 5. Schematic diagram of the limestone blasting model.

Figure 6. Comparison of rock blasting damage using two modeling methods.

6. Conclusions
Based on the sensitivity analysis of parameters, the ascertainment method of the RHT model parameters was investigated, and the parameters were then applied in the simulation of rock blasting damage using the coupled SPH-FEM algorithm. The following conclusions can be drawn from the study:

(1) Through the penetration experiment and theoretical derivation, a set of RHT constitutive parameters for limestone were initially determined, and the sensitivity of each parameter was analyzed using the “controlled variable method.” The parameters that have significant influences on the penetration depth of the projectile are \(\rho_0, \rho_s, p_c, G, f_c, B, M, \) and \(F_G\).

(2) According to the parameter sensitivity analysis and the calibration principles, the “constant-amplitude adjustment method” was proposed to optimize the parameters. The simulation results indicate that this method can effectively determine the parameters and guarantee the integrity of the parameter adjustment to a certain extent.

(3) The coupled SPH-FEM algorithm combines the advantages of the SPH and FEM algorithms. It can realistically show the whole process of rock blast damage evolution. Due to the lack of blast data, the
feasibility of applying the parameters from the penetration experiment to rock blasting simulation remains to be verified.

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