Effect of the hole doping into the Haldane-gap state on the one-dimensional $S = 1$ $t$-$J$ model

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(Received )

Abstract

The ground state and excitation spectra of the $S = 1$ Heisenberg spin chain with hole hopping are investigated by means of the numerical-diagonalization method and by the help of the Zhang-Arovas picture. As for the charge sector, two phases are found. It is shown that in the weak-hopping region, a phase separation occurs, while in the strong-hopping region, the system is in the Tomonaga-Luttinger liquid phase. Irrespective of the presence of the spin gap, the present phase diagram is similar to that reported for the $S = 1/2$ $t$-$J$ chain with gapless spin excitation rather than to that for the magnetically-frustrated $t$-$J$ chain with spin gap. For the spin sector, the string correlation is found to develop withstanding the hole doping except in a region, whose region is characterized by a generalized string correlation. This remains an open question to be studied in future.

KEYWORDS: $t$-$J$ model, Haldane gap, VBS state, RVB, Tomonaga-Luttinger liquid

Submitted to Physica B

1 Introduction

Since the existence of a spin gap on high-$T_c$ Cu-oxides was reported [1, 2], electron systems with a spin gap have been studied in the context of the superconductivity. In order to cause a spin gap, a number of generalizations of interactions have been made by restricting the dimensionality within one dimension: the $t$-$J$ model with alternating magnetic interaction [3], the $t$-$J$ model with next-nearest-neighbor magnetic frustration [4], and the double-chain $t$-$J$ model [5]. In particular, the double-chain model has lately attracted considerable attention because of the discovery of the corresponding materials [6, 7]; see the articles [8, 9, 10] for a review.

Amongst such spin-gap states, the Haldane-gap state has been studied extensively. The spin gap was conjectured to open for integer-spin antiferromagnetic Heisenberg chains [11, 12]. The conjecture was partly confirmed both theoretically [13] and experimentally [14]. Recently, carrier doping to the Haldane-gap material was tried [15, 16]. The doping causes the reduction of resistivity and creates states within the Haldane gap [17]. There are several theoretical proposals to describe such experimental results [18, 19, 20]. These proposals are based upon some model Hamiltonians in which each hole is regarded to have the inner freedom of the spin $S = 1/2$, and propagates along an $S = 1$ spin chain. The hopping amplitude of carriers is, however, estimated to be so small that only the considerations upon the impurity problems could explicate essential properties [21, 22, 23, 24].

In the present paper, we investigate the following one-dimensional $S = 1$ $t$-$J$ model under the
periodic-boundary condition:

$$\mathcal{H} = -t \sum_{i=1}^{L} \sum_{\sigma=-1,0,1} (b_{i,\sigma}^\dagger b_{i+1,\sigma} + \text{h.c.}) + J \sum_{i=1}^{L} \mathbf{S}_i \cdot \mathbf{S}_{i+1}. \quad (1)$$

The operators $b_{i,\sigma}$ and $b_{i,\sigma}^\dagger$ obey the hard-core-boson statics

$$b_{i,\sigma} b_{i,\sigma'} = 0, \quad \{b_{i,\sigma}, b_{i,\sigma'}^\dagger\} = \delta_{\sigma\sigma'}, \quad [b_{i,\sigma}, b_{i,\sigma'}] = 0 \quad (i \neq j), \quad [b_{i,\sigma}, b_{i,\sigma'}^\dagger] = 0 \quad (i \neq j). \quad (2)$$

The operators $\{\mathbf{S}_i\}$ are expressed as

$$\mathbf{S}_i = \begin{pmatrix} b_{i,1}^\dagger & b_{i,0}^\dagger & b_{i,-1}^\dagger \end{pmatrix} s \begin{pmatrix} b_{i,1} \\ b_{i,0} \\ b_{i,-1} \end{pmatrix}, \quad (3)$$

where the matrices $s = (s^x, s^y, s^z)$ represent the $S = 1$ spin operators. This model may be too simplified to describe the above magnetic material. Apart from the practical interest, however, the present Hamiltonian (1) may be useful for studying essential effects of a spin gap to phase diagrams. This is quite accordant with our motivation explained at the beginning of the present paper.

For some limiting cases, some properties of the ground state of the Hamiltonian (1) are known as follows. At the filling $n = 1$, the model reduces to the $S = 1$ spin system; the system has the so-called Haldane gap, $\Delta E \approx 0.4105 J$ [25, 26]. The two-particle problem corresponding to the low-density limit, namely $n = \frac{2}{L}$, can be solved exactly; two particles make a bound state in the region $J/t > 1$, while they are deconfined in the region $J/t < 1$. Hence, it is expected that in the region $J/t > \sim 1$ particles are apt to form an island of particles for arbitrary filling $n$; the so-called phase separation takes place. Hence, the characteristics of the system is rather simple in this phase-separated region. Notice that this situation is relevant to the above material. We stress here that the present paper mainly concerns the liquid phase $J/t < \sim 1$, in which the properties of the fluid are nontrivial.

The present paper is organized as follows. In the next section, we explain the Zhang-Arovas argument [27] which is also applicable to the present problem (1). This yields a rough feature of the phase diagram in our system, and predicts the existence of a spin gap withstanding the hole doping, and so forth. Referring to these predictions, we present in §3 numerical results for the model (1) both on the charge and spin sectors. It is shown that such a new phase as is not predicted from the Zhang-Arovas argument spreads in a low-density region. In addition to this, we report that the present system in the metallic phase is not essentially affected by the presence of the Haldane gap. The phase diagram resembles that of the $S = 1/2$ $t$-$J$ model [28] without spin gap rather than that of the magnetically-frustrated $t$-$J$-$J'$ model [29] with a spin gap. The present spin-liquid state seems to be rather unique compared to other spin-liquid states. In the last section, we give a summary of the present paper.

## 2 Zhang-Arovas picture for the present model

In this section, we explain the Zhang and Arovas picture relevant to the present system [27]. They discussed the following system;

$$\mathcal{H} = -t \sum_{i=1}^{L} \sum_{\sigma=-1,0,1} (b_{i,\sigma}^\dagger b_{i+1,\sigma} + \text{h.c.}) + J \sum_{i=1}^{L} \left( \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \frac{1}{3} (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2 \right). \quad (4)$$
The magnetic interaction including the biquadratic term is precisely the same as that in the AKLT model \cite{29,30}, which can be solved exactly at the ground state.

First, we consider the impurity problem \( t = 0 \). At the condition \( t = 0 \), the model reduces to a lot of AKLT sectors under the open-boundary condition; the system is cut off at the positions of holes to form independent finite AKLT spin chains. On the other hand, the ground state of the AKLT model is called the valence-bond solid (VBS) state. The VBS state for the system of length \( l \) under the open-boundary condition is given as \cite{31}

\[
|\text{VBS}\rangle = (a_1^\dagger)^p(b_1^\dagger)^{1-p} \left( \prod_{j=1}^{l-1} (a_{j+1}^\dagger b_j^\dagger - b_j^\dagger a_{j+1}^\dagger) \right) (a_l^\dagger)^q(b_l^\dagger)^{1-q} |0\rangle, \quad (0 \leq p, q \leq 1),
\]

where the \( S = 1 \) spin operators are expressed in terms of the Schwinger representation

\[
S_i^+ = a_i^\dagger b_i, \quad S_i^- = a_i b_i^\dagger, \quad S_i^z = \frac{a_i^\dagger a_i - b_i^\dagger b_i}{2} \quad \text{and} \quad a_i^\dagger a_i + b_i^\dagger b_i = 2.
\]

The formula (3) can be interpreted such that a valence bond is formed over each bond in a systematic manner. The edge freedom of \( p \) and \( q \) in eq. (3) can be expressed in terms of the following different representation. The edges holding hole(s) between them can be classified in terms of the total magnetization of these edges. The magnetization \( S^2 = 0 \) implies that a singlet valence bond is formed over the hole(s), while the magnetization \( S^2 = 1(1 + 1) \) implies that a triplet bond is formed; see Fig. 1. Once the hopping amplitude \( t \) is assumed to be nonvanishing, this expression becomes essential. Hence, it is consequently found that there are two types of holes that can be classified with respect to the inherent bond; we call the former (latter) ‘singlet hole’ (‘triplet hole’), hereafter.

Second, we consider the system \cite{4} with a finite hopping term \( t \). It was reported that the singlet-hole state with the wave number \( k \) is an exact eigenstate, while the triplet-hole state is not \cite{27}. Hence, the Hamiltonian for the singlet holes can be given exactly in terms of the spinless-fermion operators;

\[
\mathcal{H} = -t \sum_i \left( c_i^\dagger c_{i+1} + \text{h.c.} \right) - \frac{2}{3} J \sum_i c_i^\dagger c_{i+1}^\dagger c_{i+1} c_i
\]

where the Hilbert space is restricted within \( \sum_i \sigma_i^z = 2N - L \); \( N \) denotes the number of the particles. Now, we are in a position to grasp a rough feature of the phase diagram of the present system with singlet holes. In the XY limit \( J/t \to 0 \), the ground state is in the XY phase, \textit{i.e.}, the Tomonaga-Luttinger liquid phase. In the Ising limit \( J/t \to \infty \), the state is phase-separated: the positive-magnetization sector and the negative-magnetization sector extend separately. Along the conditions \( \sum_i \sigma_i^z = \pm L \mp 2 \), \textit{i.e.}, the high- or low-density limits, the state shows the confinement-deconfinement transition at \( J/t = 3 \). Note that this transition point coincides the isotropic condition of the spin system \cite{8}. Keeping in mind of the particle-hole symmetry \( c_i \leftrightarrow (-1)^i c_i^\dagger \) of the Hamiltonian (7), the phase diagram of the singlet-hole state may be drawn schematically as in Fig. 2.

Lastly, we explain the properties of the spin sector. If the singlet-hole state is realized at the ground state, the string correlation \cite{32,33}

\[
\mathcal{O}^2_{\text{string}}(j - i) = \langle S_j^z e^{i\pi \sum_{k=1}^{j-1} \sigma_k^z} S_j^z \rangle
\]
develops withstanding the hole doping. The reason is as follows. We assume that the singlet-hole state should be an ground state. Then, focusing only on the $S = 1$ particles, i.e., disregarding the holes, the spin sector is expressed in terms of the VBS state; a valence bond is formed over each neighboring $S = 1$ particles. On the other hand, the string correlation (9) is known to develop on the VBS state.

The existence of the developing string correlation can be interpreted as the existence of a spin gap because of the following reason. This existence implies that the state should be relevant to the VBS state. A finite energy is required to excite the state, whose energy corresponds to exiting one of the valence bonds to a triplet bond. Hence, in the present paper, we study the string correlation rather than the spin gap directly.

3 Numerical Results and Discussions

In this section, we investigate the ground-state properties of the system (1) by means of the exact-diagonalization method. The results are discussed by the help of the Zhang-Arovas picture explained in the previous section. Phase diagrams are given both for the charge and spin sectors.

3.1 Elementary excitations

In this subsection, we show dispersion relations both for the phase-separated region $J/t>\sim 1$ and liquid region $J/t<\sim 1$, before estimating the precise phase boundary. Dynamical structure factors are evaluated so that they can yield the respective elementary excitations of the charge and spin sectors. This separation enables us to analyze the charge sector in terms of the Tomonaga-Luttinger theory; see the article [34] for a review.

First, in Fig. 3 (a), we present the dispersion relation for the system with $L = 14$, $J/t = 10 \gg 1$ and $n = 13/14$ which is a single-hole system in the phase-separated region. The low-lying bands originate in the single-hole dispersions. Because of the degeneracy of the band with respect to the total magnetization $\sum_i S_i^z$, see Fig. 3 (a), the wider (narrower) band is seen to correspond to singlet (triplet) one. According to the Zhang-Arovas picture explained in the previous section, the band width of the singlet hole is approximately three-times wider than that of the triplet one. The dispersions in Fig. 3 (a) actually agree with this prediction. As for the spin sector, we find that the spin gap opens independently of the wave number $k$, $\Delta E \sim J$. The reason of this independency is due to the condition $J/t \ll 1$, where a hole can hardly propagate and can be regarded as an impurity. As a consequence, the state is independent of the wave number.

Second, in Fig. 3 (b), we show the dispersion relation for the system with $L = 14$, $J/t = 1$ and $n = 12/14$ which is a system with a hopping amplitude comparable to the magnetic interaction. The structure of elementary excitations is no more manifest. The spin and charge elementary excitations are separated with use of the following analysis.

We employed the dynamical structure factor $S(k, \omega)$ both for the spin and charge sectors,

$$S_{\text{charge}}(k, \omega) = \sum_j \langle 0|n_{-k}|j\rangle \langle j|n_k|0\rangle \delta(\omega - E_j + E_0),$$

and

$$S_{\text{spin}}(k, \omega) = \sum_j \langle 0|S_{-k}^z|j\rangle \langle j|S_k^z|0\rangle \delta(\omega - E_j + E_0),$$

where $n_k = L^{-\frac{1}{2}} \sum_{i,\sigma} e^{-ikr_i} b_{i,\sigma}^\dagger b_{i,\sigma}$ and $S_k^z = L^{-\frac{1}{2}} \sum_{i,\sigma} e^{-ikr_i} S_i^z$. The factors show $\delta$-function peaks at the levels which have the above particular transition probabilities. They are evaluated with use of Mori’s continued-fraction-expansion formalism [33, 37]. The factors (10) and (11) for the system with $L = 14$, $J/t = 1$ and $n = 12/14$ are shown in Fig. 4 (a) and (b), respectively, where the peaks are broadened with the parameter $\eta = 0.04$ such as $\delta(x) = -\frac{1}{\pi} \text{Im} \frac{1}{x+i\eta}$. As was discussed
in the previous section, the charge sector in the region $J/t<1$ is expected to be described in terms of the Tomonaga-Luttinger theory. It is crucial to estimate the sound velocity $v_c$ that is defined with respect to the low-lying dispersion relation $\omega = v_c k$ ($k \sim 0$). Through collating Fig. 4 (a) with Fig. 3 (a), it can be seen that the sound velocity for the charge elementary excitation should be estimated with respect to the level $A$; $v = (E_\Lambda - E_0)/(\frac{2\pi}{L})$. The estimated sound velocity for the system with $L = 14$, $n = 12/14$ and $J/t$ varied is shown in Fig. 5.

As for the spin elementary excitation depicted in Fig. 4 (b), the first excited state locates at the wave number $k = k_F = n\pi$. This location coincides with that for the $S = \frac{1}{2}$ $t-J$ model, whose model was, however, shown to have a gapless spin excitation spectrum by the use of the rigorous solution along the tractable line $J/t = 2$ [37]. The spin excitation of the present model is shown to have a spin gap in the subsection 3.3.

3.2 Phase diagram of the charge sector

In this subsection, we investigate the phase diagram of the charge sector. It is analysed in term of the Tomonaga-Luttinger liquid theory, where the estimation of the sound velocity $v_c$ given in the previous subsection plays an essential role.

The exponent $K_\rho$ which governs the Tomonaga-Luttinger liquid state can be expressed by

$$K_\rho = \frac{\pi}{2} v_c n^2 \kappa,$$

where $\kappa$ denotes the compressibility $\kappa = \frac{1}{Nn} \frac{\partial^2 E_0}{\partial N^2}$. This compressibility is estimated by means of the difference method as

$$\kappa = \frac{L}{N^2} \frac{E_0(N+2) + E_0(N-2) - 2E_0(N)}{4}.$$

Every correlation exponents of the charge sector are governed in terms of the single parameter $K_\rho$; for example, the charge-density wave and the pairing correlations obey the following power-law

$$\langle O_{\text{density}}^\dagger(r)O_{\text{density}}(0) \rangle \propto 1/r^{K_\rho},$$

$$\langle O_{\text{pair}}^\dagger(r)O_{\text{pair}}(0) \rangle \propto 1/r^{1/K_\rho},$$

respectively. At $K_\rho = 1$, the above correlations (14) and (15) decay with the same exponent. The former (latter) should be dominant, if the liquid is “repulsive (attractive).” Hence, it is seen that the magnitude of the exponent $K_\rho$ indicates a degree of the attraction among the particles and that the particles are “noninteractive” at $K_\rho = 1$. This interpretation will be found to be valid through referring to the exact estimation of $K_\rho$ for various models [40, 41].

In Fig. 6, we presented the estimation of $1/K_\rho$ for the system with $L = 14$, $n = 12/14$ and $J/t$ varied. In the weak-interaction limit $J/t \to 0$, only concerning the charge sector, the present system converges to the spinless-hard-core gas. For such a system, the exponent $K_\rho$ is given by $K_\rho = 0.5$ exactly. This is quite consistent with the present numerical estimation in Fig. 6. As the magnetic interaction $J/t$ is increased, the exponent $K_\rho$ increases. In the region $1 < J/t < 1.6$, the exponent is estimated as $K_\rho > 1$. Hence, the liquid is attractive; the paring correlation (13) is dominant in this region. On the other hand, the exponent $K_\rho$ is negative in the region $J/t \sim 1.6$. Therefore, the compressibility $\kappa$ is also negative in this region; see eq. (12). Hence, the gas is unstable; we can regard the system as being phase-separated in this region. Note that these features are consistent with the argument explained in §2.

With use of the similar data for various values of the filling $n$, we obtained the phase diagram for the charge sector, see Fig. 7. As for the charge sector, two qualitatively-different phase
diagrams have been reported so far (see Fig. 8 (a) and (b)). The phase diagram of the present model belongs to the type depicted in Fig. 8 (a). Notice that the phase diagram depicted in Fig. 2 which is given according to the Zhang-Arovas argument also belongs to the type of Fig. 8 (a). The phase diagrams (a) and (b) were reported to hold for the one-dimensional $S = 1/2$ $t$-$J$ model \cite{28} and for the $t$-$J$ model with the next-nearest-neighbor magnetic interaction $J' = 0.5J$ \cite{4}, respectively. The former model shows a gapless spin excitation. On the other hand, the latter model is expected to have a spin gap near the filling $n \sim 1$, because the model converges to the Majumder-Ghosh model \cite{42} in the limit $n \to 1$. In the parameter region with $n \sim 1$ and $J/t < \sim 1$, the diagram Fig. 8 (a) shows that the liquid has the exponent $K_{\rho} = 0.5$, while the diagram (b) shows that the liquid is either phase-separated or has the exponent $K_{\rho} \gg 1$. Hence, the liquid of the latter case is strongly attractive one. The reason has attributed to the presence of a spin gap, so far.

The present model, however, belongs to the type (a) in spite of the presence of a spin gap, called the Haldane gap. It suggests that the RVB pattern is crucial for enhancing the pairing correlation; the present RVB pattern belongs to the type called the VBS state (see the previous section).

### 3.3 Spin sector

In this subsection, we concentrate on the spin sector of the present model \cite{1}. The development of the string correlation \cite{1} indicates the presence of a short-range RVB state which is essentially the same as the VBS state; the state is thus concluded to have a spin gap corresponding to exiting one of the valence bonds to a triplet. Physical properties of the liquid phase are of greater interest, because the phase-separated particle system is no more than an island of the $S = 1$ spin chain in a sea of holes.

In Fig. 9, we have plotted the string correlation $O^z_{\text{string}}(L/2)$ for the systems with $J/t = 1.0$ and $L, n$ varied. As is expected, the correlation decreases with the filling $n$ decreased. The plot, however, does not show clearly whether it develops or not. In order to see it definitely, we calculated the Binder parameter \cite{43, 44, 45} for the string correlation;

$$U(L) = 1 - \frac{\langle O^4 \rangle}{3\langle O^2 \rangle^2},$$

$$O = \frac{1}{L} \sum_i S^z_i e^{i\pi \sum_{k=1}^{i-1} S^z_k}.$$

With the system size increased, the Binder parameter increases if the corresponding correlation is developing. We show the plot for the systems with $J/t = 0.2, 1.2, 2.2$ and $L, n$ varied (see Fig. 10 (a), (b) and (c), respectively). At $J/t = 0.2$, the string correlation develops withstanding the doping. Holes coherently move in the VBS state without disturbing it. Hence, at $J/t = 0.2$ we can conclude that the singlet-hole picture is valid at the ground state. The above behaviour of the spin gap is in contrast with that for the frustrated $S = 1/2$ $t$-$J$ chain \cite{1}. For this model, it was reported that the spin gap closes in an over-doped region $n < \sim \frac{3}{4}$. At $J/t = 1.2$, in the low-density region $n \sim 0$, the correlation is found to be of short range. This phase is absent in the Zhang-Arovas picture introduced in the previous section. The nature of this new phase is discussed later. The correlation is seen to develop at $J/t = 2.2$, which is rather strong. It is quite natural, if we notice that the system is phase-separated (see the phase diagram depicted in Fig. 7).

With use of similar data for various values of $J/t$, we draw schematically the transition line in Fig. 11. As is discussed in \S 1, the two-particle system can be solved exactly; there exists a confinement-deconfinement transition at $J/t = 1$. We have utilized this additional fact for drawing the transition line in Fig. 11.
In order to see the nature of the above new phase, we generalize the correlation (9) to the following form \[17\],
\[
O^z_{\text{string}}(|i - j|, \theta) = \langle S^z_i e^{i\theta} \sum_{k=i}^{i-1} S^z_j \rangle.
\]
This has been successfully applied for the spin-$S$ antiferromagnetic Heisenberg chains \[46, 47, 48\] and the $S = 1$ spin chain with the alternating interaction \[46, 49\].

For the $S = 1$ spin chain with the alternating interaction, the following was reported. The correlation at $\theta = \pi$ is maximal in the Haldane phase, while the correlation at $\theta = \pi/2$ is maximal in the dimer phase. In Figs. 12 (a) and (b), the correlation $O^z(L/2, \theta)$ is plotted for the systems with (a) $L = 14$, $J/t = 0.5$ and $n = 12/14$ and (b) $L = 14$, $J/t = 1.2$ and $n = 4/14$, respectively. In Fig. 12 (a), a maximum locates at $\theta = \pi$, while in Fig. 12 (b), a maximum locates about at $\theta = \pi/2$. The former actually confirms that the state is in the Haldane phase. On the other hand, the latter implies that the state is, somehow, in the dimer phase; the $S = 1$ particles may make a singlet bound states in pair. A similar state is reported in the low-density region for the $S = 1/2$ $t$-$J$ chain \[28, 50\]. In the present model, however, the region spreads over a large region. It is noteworthy that the state has a spin gap corresponding to exiting one of the singlet bound pairs to a triplet pair. As a consequence, all the phases that appear in Fig. 12 are found to show a spin gap.

4 Summary

The ground state of the one-dimensional $S = 1$ $t$-$J$ model \[1\] is investigated. In the weak-magnetic-interaction region $J/t < \sim 1.6$, the ground state is in the liquid phase, while the system is phase-separated in the strong-interaction region $J/t \geq \sim 1.6$. In spite of the presence of the spin gap, the phase diagram of the charge sector belongs to the type that was reported for the one-dimensional $S = 1/2$ $t$-$J$ model rather than that for the frustrated $t$-$J$ model with a spin gap. Although the presence of a spin gap has been speculated as a sign of strong attraction among the particles, the present result suggests that the pattern of the RVB formation is rather crucial. As for the spin sector in the liquid region, in addition to the phase conjectured by Zhang and Arovas, a new phase is found to appear. The phase spreads in the low-density region, and the string order is disturbed. The disturbed state, however, possesses the generalized string correlation of $\theta = \pi/2$. It suggests that the $S = 1$ particles make bound states in pair. A similar phase is observed in the $S = 1/2$ $t$-$J$ model, in whose model, however, it expands in a very limiting region \[28, 50\]. It turns out consequently that a spin gap outlives to open in the whole region. The present spin-liquid state seems to be rather unique compared to other spin-liquid state.

Acknowledgement

Our computer programs are partly based on the subroutine package "TITPACK Ver. 2" coded by Professor H. Nishimori. The numerical calculations were performed on the super-computer HITAC S3800/480 of the computer centre, University of Tokyo, and on the work-station HP Apollo 9000/735 of the Suzuki group, Department of Physics University of Tokyo.

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Figure captions

Fig. 1: Schematic drawing of the generalized VBS state in the presence of holes. Over the hole(s), either a singlet or a triplet bond is supposed to be formed.

Fig. 2: Schematic drawing of the phase diagram of the Hamiltonian (I) due to the Zhang-Arovas argument.

Fig. 3 (a) (b): Dispersion relations for the systems with (a) $L = 14$, $J/t = 10$ and $n = 13/14$, and (b) $L = 14$, $J/t = 1$ and $n = 12/14$, respectively. The symbols +, $\times$ and $\Diamond$ denote the levels of the quantum number $\sum_i S^z_i = 0$, $\pm 1$ and $\pm 2$, respectively.

Fig. 4 (a) (b): Dynamical structure factor $S(k, \omega)$ for (a) charge excitation and (b) spin excitation for the system $L = 14$, $J/t = 1$ and $12/14$ in Fig. 3 (b).

Fig. 5: Velocity of the charge-excitation spectrum for the system $L = 14$, $n = 12/14$ and $J/t$ varied.

Fig. 6: Inverse of the exponent $K_\rho$ of the charge sector for the system $L = 14$, $n = 12/14$ and $J/t$ varied.

Fig. 7: Phase diagram of the charge sector. The contours of the exponent $K_\rho$ are also shown. The plots +, $\times$ and $\Diamond$ denote the data estimated for the systems of the length $L = 10$, 12 and 14, respectively.

Fig. 8: Qualitatively different phase diagrams of the charge sector reported so far. The type (a) was reported for the $S = 1/2$ $t$-$J$ model with gapless spin excitation [28], while the type (b) was reported for the frustrated $t$-$J$-$J'$ model with spin gap [4].

Fig. 9: String correlation $O^z_{\text{string}}(L/2)$ for the system with $J/t = 1$, and $n$ varied.

Fig. 10 (a) (b) (c): Binder parameter $U(L)$ of the string correlation for the systems with (a) $J/t = 0.2$, (b) $J/t = 1.2$ and (c) $J/t = 2.2$, and $n$ varied.

Fig. 11: Schematic drawing of the phase diagram of the spin sector.

Fig. 12 (a) (b): Generalized string correlation $O^z_{\text{string}}(L/2, \theta)$ against the angle $\theta$. It is calculated for the systems with (a) $L = 14$, $J/t = 0.5$ and $n = 12/14$, and (b) $L = 14$, $J/t = 1.2$ and $n = 4/14$. 
\( \bigcirc : S=1 \) spin

\( \bullet : \) hole

\( \text{---} : \) singlet bond

\( \text{-----} : \) triplet bond
TL phase (XY phase)

Phase Separated

\( J/t \)
$L = 14$
$n = 12/14$
Phase separation

\( K_\rho = 0.7 \)

\( K_\rho = 1 \)

TL liquid

(a)

(b)
$Q_{\text{string}}^z(\pi) \neq 0$

$Q_{\text{string}}^z(\pi/2) \neq 0$

Haldane phase
