SUPERSYMMETRIC PHENOMENOLOGY 
IN THE LIGHT OF GRAND UNIFICATION

LESZEK ROSZKOWSKI*

Department of Physics, University of Minnesota
Minneapolis, MN 55455
E-mail: leszek@mnhepw.hep.umn.edu

ABSTRACT

I review some aspects of supersymmetric grand unification and emphasize a recent development in the area of gauge coupling unification.

1. Introduction

During the last few years supersymmetry (SUSY) has gained the status of the most likely candidate for physics beyond the Standard Model (SM). The very fact that so many talks at this meeting are devoted to various aspects of supersymmetric physics, and so few to its fading alternatives, is the best illustration of the current situation. This “SUSY fervor” might be considered unjustified in light of the fact that, despite many experimental efforts, no signal of SUSY has been detected. Still, as I will demonstrate below, there are good reasons to like SUSY. Introduced to particle physics over a decade ago in order to make sense of grand unified theories (GUT’s), SUSY was subsequently shown to possess many other remarkable features, like allowing for unifying gravity with other interactions, or providing an excellent dark matter candidate, to name just a few. SUSY GUT’s also correctly predicted the value of \( \sin^2 \theta_W \), where \( \theta_W \) is the weak angle.

The development of the last few years is well known. Around 1989 LEP and SLC provided precise measurements of the gauge couplings of the SM. Then it became clear that, in the \( \overline{\text{MS}} \) scheme, the three (running) gauge coupling do not unify anywhere in the SM but they nicely do so at an expected scale \( M_X \sim 10^{16} \text{GeV} \) in the Minimal Supersymmetric Standard Model (MSSM). This was very encouraging news

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for both SUSY and GUT’s (or string theory) since otherwise each of them lacks real theoretical strength and motivation without the other.

Further studies expanded the guiding idea of unification into several related directions. Basically, what was studied, by many groups and at various levels of sophistication, was:

- gauge coupling unification;
- Yukawa coupling unification;
- mass unification.

I will discuss these topics in turn. Of course, each of them has grown into a rich and impressive subject of its own and in preparing this talk I had to make some hard choices. Thus instead of attempting to even briefly mention every result which bears at least some significance, I will rather sketch the overall picture and select a few most, in my biased opinion, characteristic results. Initially, my plan was to spend little time on the first two topics, since they had been studied somewhat earlier, had already been presented at various meetings, and, it seemed to me, relatively little work had been done on them recently. As it came out, some new and interesting development took place during the last few months in the area of gauge coupling unification. (In fact, some progress has taken place after the conference – I will update my talk accordingly.) But first I will briefly set the stage on which this unification game is usually played.

2. Framework

The simplest and most popular framework for studying SUSY is the MSSM (with R-parity assumed). Since, viewed as a mere phenomenological extension of the SM, the MSSM contains many unknown parameters, usually some GUT-physics connection is adopted. Most commonly one assumes that soft terms needed to break SUSY are generated by coupling the MSSM to minimal $N = 1$ supergravity, and additionally a particularly simple (delta-like) form of the kinetic term for the gauge superfields is chosen, and a simple unification group is also assumed. This leads to several mass relations. For example, the gauginos – the bino of $U(1)_Y$, the winos of $SU(2)_L$, and the gluinos of $SU(3)_c$ have equal masses at the GUT (actually, Planck) scale: $M_1 = M_2 = m_{\tilde{g}} = m_{1/2}$. This leads, due to renormalization effects, to the following well-known relations at the electroweak scale:

\begin{align}
M_1 & = \frac{5}{3} \tan^2 \theta_W M_2 \simeq 0.5 M_2, \\
M_2 & = \frac{\alpha_2}{\alpha_s} m_{\tilde{g}} \simeq 0.3 m_{\tilde{g}}.
\end{align}
These relations, or at least the first of them, are commonly assumed in most studies of the MSSM, even though they are not necessary in the context of the model.

Another relation which stems from minimal SUGRA and which is commonly assumed is the equity of all the (soft) mass parameters of all the sleptons, squarks, and typically also Higgs bosons, to some common (scalar) mass parameter $m_0$ at the GUT scale. Renormalization effects cause the masses of color-carrying sparticles to become, at the $m_Z$ scale, typically by a factor of a few heavier than the ones of the states with electroweak interactions only. Often one also imposes a very attractive mechanism of radiative electroweak symmetry breaking (EWSB), which provides additional constraint on the parameters of the model, in particular relates the SUSY Higgs/higgsino mass parameter $\mu$ to the parameters of the model which break SUSY. This fully constrained framework has been called the constrained MSSM (CMSSM). In practice, various groups have considered the MSSM with a varying number of additional assumptions, starting from adopting just Eq. (1) to the CMSSM with additional constraints, e.g., from nucleon decay which requires specifying the underlying GUT, or string, model, the simplest $SU(5)$ and $SO(10)$ models being the most commonly studied. (A discussion of GUT physics is beyond the scope of this talk and I will only occasionally make references to expected corrections to low-energy variables, like $\alpha_s(m_Z)$, from simplest GUT-models.) It is not always easy to discern what assumptions are actually responsible for what results. I will make an attempt to sort some of those things out.

3. Gauge Coupling Unification

Gauge coupling unification has been perhaps the strongest guiding principle behind the idea of GUT's. Ever since the beginning it was known that, in the framework of the SM, the scale of GUT’s was about $M_X \sim 10^{15}$ GeV. Supersymmetry causes this scale to go up somewhat ($M_X \sim 10^{16}$ GeV). With improving accuracy of the experimental data, especially $\sin^2 \theta_W$, it was becoming increasingly clear that the prediction of SUSY was fitting the data better than the one of the SM. But it wasn’t until LEP turned on, that, with precise data available for SM parameters, a final blow was given to the idea of GUT's in the framework of the SM. In contrast, an impressive confirmation of SUSY unification was pointed out.

This early work is of largely historical importance now, since during the last five years a significant progress on both the experimental and theoretical sides has been made. Initial studies, which used just 1-loop renormalization group equations (RGE’s) and a single SUSY breaking scale $M_{SUSY}$ were expanded to include all dominant effects and subleading corrections at both 1- and 2-loop level. The most important effects are due to: 1-loop mass threshold corrections (which correspond to the fact that in reality masses of individual (s)particles of the MSSM are typically non-degenerate); 2-loop (pure) gauge contribution; and model-dependent corrections from GUT-scale physics.
Each of them provides $\sim 10\%$ correction to the predicted value of $\alpha_s$, assuming the electromagnetic constant $\alpha$ and $\sin^2 \theta_W$ as input parameters. Also important is the (inverse) dependence of $\sin^2 \theta_W$ on $m_t$ because the predicted $\alpha_s(m_Z)$ sensitively depends on the allowed range of $\sin^2 \theta_W$. Other contributions are sub-dominant but are normally also included in any decent analysis. They include: 2-loop Yukawa coupling contribution and mass-threshold effects, and scheme dependence ($\overline{\text{MS}}$ vs. DR). Recent updated discussions provide more information.

Two general conclusions can be drawn from all those extensive studies. Firstly, gauge coupling unification in SUSY seems to be quite robust in the sense that no large effects coming from electroweak or GUT-scale physics exist which would destroy the picture, provided one does not allow more than two Higgs doublets normally required in SUSY. (However, fourth generation of fermions is not in conflict with gauge unification.)

Secondly, if one restricts oneself to masses roughly below 1 TeV then generally $\alpha_s(m_Z) \gtrsim 0.12$ (and $\gtrsim 0.13$ for SUSY masses below some 300 GeV). This is because $\alpha_s(m_Z)$ grows with decreasing masses of SUSY (s)particles (and also $m_t$). A recent updated analysis quotes $\alpha_s(m_Z) = 0.129 \pm 0.008$. The limits quoted above include the estimated theoretical errors due to mass-thresholds at the GUT scale and higher-dimensional non-renormalizable operators (NRO’s) in the GUT scale Lagrangian. The above prediction for $\alpha_s(m_Z)$ has been considered a success and the strongest evidence in favor of supersymmetric unification, especially in light of the range of $\alpha_s(m_Z) = 0.127 \pm 0.05$ claimed by LEP experiments.

But the experimental status of $\alpha_s(m_Z)$ still remains unclear. All low-energy measurements and lattice calculations of $\alpha_s$, when translated to the scale $m_Z$, generally give much lower values, between 0.11 and 0.117, with comparable or smaller error bars. (See, e.g., recent reviews by Langacker for more detail.) The only indication from low energies for larger $\alpha_s(m_Z)$ from $\tau$ decays has also been questioned.

Very recently (in fact after the conference) an interesting and important development took place. First, Shifman very vigorously argued that the internal consistency of QCD requires that $\alpha_s$ be close to 0.11. He gave a number of important reasons. Here I will quote only one: large $\alpha_s \sim 0.125$ would correspond to $\Lambda_{\overline{\text{MS}}} \approx 500$ MeV (in contrast to $\sim 200$ MeV for $\alpha_s(m_Z) \approx 0.11$). Such a large value is apparently in conflict with crucial features of QCD on which a variety of phenomena depend sensitively. Prompted by Shifman’s argument, Voloshin re-analyzed $\Upsilon$ sum rules claiming the record accuracy achieved so far: $\alpha_s(m_Z) = 0.109 \pm 0.001$. On the other side, it has been argued that the systematic error usually quoted in the LEP number is grossly underestimated, and that at present LEP experiments can only claim $0.10 \lesssim \alpha_s(m_Z) \lesssim 0.15$.

Clearly, small $\alpha_s(m_Z) \approx 0.11$ seems an increasingly viable possibility, while significantly larger values are predicted by the CMSSM. Additionally, it has recently been shown that, in the framework of the CMSSM, sub-leading quadratic corrections,
play a significant role when $m_{1/2} \lesssim 150 \text{ GeV}$, and generally push up $\alpha_s(m_Z)$ obtained in the leading-log (step-function) approximation, quoted above, by another $\sim 0.01$. This effect is amplified by the mass relations (1)–(2) among the gauginos.

The question arises as to whether SUSY unification can accommodate $\alpha_s(m_Z) \approx 0.11$, and at what expense. Several solutions to this problem can be immediately suggested. One is to remain in the context of the CMSSM but adopt a heavy SUSY scenario with the SUSY mass spectra significantly exceeding 1 TeV. This scenario would not only put SUSY into both theoretical and experimental oblivion, but is also, for the most part, inconsistent with our expectations that the lightest supersymmetric particle (LSP) should be neutral and/or with the lower bound on the age of the Universe of at least some 10 billion years. Another possibility is to invoke large enough negative corrections due to GUT-scale physics. The issue was re-analyzed recently and it was found that, under natural assumptions, $\alpha_s(m_Z) > 0.12$. Although it may well happen that the GUT-scale and NRO corrections are abnormally large, the guiding idea of grand unification becomes much less attractive in this case, and the predictive power is essentially lost.

A different way out has been suggested lately: abandon the additional assumptions, like Eqs. (1)–(2), completely. In fact, simple arguments show that it is just the relation (2) that is mainly responsible for pushing $\alpha_s(m_Z)$ up so much in the CMSSM. By relaxing it one can easily accommodate $\alpha_s(m_Z) \approx 0.11$ for wide mass ranges, except for the mass of the gluino which must remain rather low, $m_{\tilde{g}} \lesssim 200 - 300 \text{ GeV}$. (The exact upper bound depending on what upper limit on $\alpha_s(m_Z)$ one adopts and how large GUT-physics corrections one is willing to accept.) Furthermore, the wino mass parameter $M_2$ must be large, $M_2 \gtrsim 3m_{\tilde{g}}$, at least a few hundred GeV, in contrast to what is commonly assumed. This approach casts doubt also on the relation (1), which has been universally assumed in phenomenological studies of charginos and neutralinos, and which results from the same assumption at the GUT scale. (No “direct” constraint on $M_1$ can be placed from the above considerations because the bino, being neutral, does not enter the RGE’s.) Also, requiring that the lightest (bino-like) neutralino be lighter than the gluino leads to $M_1 \lesssim 0.3M_2$, thus violating the relation (1). It will be very important to test Eqs. (1)–(2) in future experiments.

To summarize, the issue of gauge coupling unification is still far from being closed. Progress is needed on the experimental side to determine more precisely $\alpha_s(m_Z)$. The most commonly assumed “minimal” CMSSM scenario seems to imply too high $\alpha_s(m_Z)$, at least if SUSY masses are below roughly 1 TeV and if $\alpha_s(m_Z)$ is indeed close to 0.11. In fact, at present no simple theoretical framework motivated by GUT’s, or strings, can really produce such small $\alpha_s(m_Z)$. It is also worth noting that it still remains unclear how to reconcile the string unification scale $M_{\text{string}} \sim \text{few } \times 10^{17} \text{ GeV}$
with the $M_X \sim 10^{16}$ GeV suggested by bottom-up gauge coupling unification.

4. Yukawa Coupling Unification

Another driving idea, and a success, of the early GUT’s was the fact that the running masses of the $b$-quark and the $\tau$ were apparently meeting at roughly the gauge unification scale $M_X$. With more precise data on $m_b$ and $m_\tau$ becoming available, it was shown that the $b-\tau$ unification was only consistent in the SUSY framework, but not in the non-SUSY case. It is also badly spoiled if the fourth generation of fermions exists. Furthermore, several groups pointed out that, in the MSSM-like framework, strict $b-\tau$ unification implies a very heavy top $m_t = (200 \text{ GeV}) \sin \beta$. More precisely, one needs large ($\sim 1$) Yukawa coupling of the top-quark to balance the effect of gauge couplings in the RGE’s for $m_b/m_\tau$. Requiring perturbativity of the Yukawa couplings and imposing experimental constraints on $m_b$ and $m_\tau$ leads to a rather narrow range of $m_t$ and $\tan \beta$. Furthermore, if $m_t$ is to be also unified with $m_b$ and $m_\tau$, like in simple versions of $SO(10)$, then large $\tan \beta \sim 50 - 60$ is needed.

Two comments should be made in this context. Recently, both the CDF and D0 collaborations have reported discovery of the top quark and quoted: $m_t = 176 \pm 8 \pm 10$ GeV (CDF) and $m_t = 199 \pm 20 \pm 22$ GeV (D0). Such large $m_t$ may be interpreted as supporting $b-\tau$ unification but, unfortunately, does not help in constraining $\tan \beta$.

On the other hand, if $\alpha_s(m_Z)$ is small ($\sim 0.11$), then the above strong relation between $\tan \beta$ and $m_t$ can be significantly relaxed provided that strict unification condition $m_b/m_\tau = 1$ at the GUT scale is reduced somewhat ($\sim 10\%$). GUT-scale uncertainties of this size are actually expected in GUT’s. At the end, to test the idea of $b-\tau$ unification, we will need to constrain $\tan \beta$ independently. This probably won’t be possible before the couplings of the Higgs bosons are precisely measured.

The region of large $\tan \beta$ is also somewhat sensitive to SUSY spectra.

5. Mass Unification

It is natural to expect that, in the unification framework, also the various mass parameters in the MSSM will emerge from a few common sources. Thus it has become customary to assume the so-called common gaugino and common scalar masses, $m_{1/2}$ and $m_0$, respectively, at the GUT scale, and to consider the CMSSM (see Section 2). These two assumptions are certainly not irrefutable but are at least sensible (except for the fact that assuming $m_{1/2}$ seems to be in conflict with $\alpha_s(m_Z) \approx 0.11$ – see Section 3). They obviously correspond to the simplest choice and furthermore result from the simplest minimal supergravity framework. Needless to say, most phenomenological studies of SUSY rely on at least one of them, at least for the sake of reducing the otherwise huge number of unrelated SUSY mass parameters. (One also assumes that the trilinear soft SUSY-breaking terms are equal to $A_0$ at $M_X$, although
One can next derive complete mass spectra of all the Higgs and supersymmetric particles by running their 1-loop RGEs between \( M_X \) and \( m_Z \). In the CMSSM the spectra are parametrized in terms of just a few basic parameters which can be conveniently chosen as: \( m_t, \tan \beta, m_{1/2}, m_0, \) and \( A_0 \). One also needs to employ the full 1-loop effective Higgs potential in order to properly implement the conditions for EWSB. Next one imposes on the resulting mass spectra mass limits from current direct experimental searches and CLEO data on \( \text{BR}(b \to s\gamma) \). Also, the LSP must be neutral and its relic density must satisfy \( \Omega_{\text{LSP}} h^2 \approx 1 \) to be consistent with limits on the age of the Universe of at least 10 billion years.

The resulting parameter space of the CMSSM, consistent with all the above constraints, is remarkably constrained and leads to several important predictions. I will describe them briefly relying on a comprehensive analysis of Kane, et al. Several other groups have also studied various aspects of mass unification and obtained similar results.

1. The lightest neutralino comes out as the only neutral LSP. (In the MSSM one commonly assumes it to be the LSP.) Furthermore, it is almost always a nearly pure bino – which has been shown to be the only sensible candidate for dark matter (DM) with \( \Omega_{\text{DM}} h^2 \approx 1 \). However, in this constrained scenario event rates for both direct and indirect detection of (galactic halo) neutralino DM are rather small. This is not good news for testing the CMSSM even in the next generation of DM detectors.

2. The relic density of bino-like LSPs grows with \( m_{1/2} \) and \( m_0 \). These can then be constrained from above (except for some rare cases) by \( \Omega_{\text{LSP}} h^2 < 1 \). As a result SUSY mass spectra come out below about 1 TeV without imposing an ill-defined naturalness constraint. The fact that the cosmological constraint coincides with the expected SUSY breaking scale is remarkable.

3. The resulting mass range of the (bino-like) neutralino lies between some 20 GeV and 200 GeV. Also, important mass relations ensue: \( 2m_\chi \approx m_\chi^0 \approx m_\chi^\pm \approx 0.3m_\gamma \).

4. The lightest Higgs boson \( h \) has couplings closely resembling those of the SM Higgs. It’s mass is expected around 80–120 GeV. Clearly, LEP II will have an excellent chance of covering (most of) this mass range, critically depending however on the beam energy. Other Higgs bosons are typically very heavy and their couplings to gauge bosons are strongly suppressed.

5. Typical masses of squarks and sleptons in the CMSSM are unfortunately also rather large and lie typically above the reach of the Tevatron and LEP II, respectively. The gluino is not well-constrained and can either be found at the Tevatron (\( \sim 200 \text{ GeV mass range} \)) or only after the LHC runs for a few years (\( \sim 1 – 2 \text{ TeV range} \)).

6. Because typical SUSY and charged Higgs masses are rather large, SUSY contributions to \( \text{BR}(b \to s\gamma) \) are typically small, in agreement with CLEO data. QCD uncertainties are still large but in the future one can use this rare process as a powerful constraint on the CMSSM and other SUSY models.
Due to space and time limitations I must leave unmentioned many other interesting topics. They can be found in several recent reviews and source articles.

6. Conclusions

While the general framework of supersymmetric grand unifications seems remarkably attractive, much remains to be done and understood. Constrained and predictive frameworks, like the commonly used CMSSM, don’t run into conflict with any experimental or theoretical constraints, except perhaps for predicting somewhat large $\alpha_s(m_Z)$. It will be possible but also challenging to test the CMSSM in future accelerator experiments, rare processes, and dark matter searches. The issue of $b-\tau$ unification won’t be experimentally tested before properties of at least the lightest Higgs are well measured and tan $\beta$ is determined. Many predictions of the CMSSM and other SUSY models remain to be verified by future experiments. And once, sooner or later, SUSY is discovered, it will probably take many years before we go beyond the supersymmetric SM.

7. References

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