A New 4-D Multi-Stable Hyperchaotic System With No Balance Point: Bifurcation Analysis, Circuit Simulation, FPGA Realization and Image Cryptosystem

SUNDARAPANDIAN VAIYDYANATHAN1, ACENG SAMBAS2, ESTEBAN TLELO-CUAUTLE3, AHMED A. ABD EL-LATIF4, BASSEM ABD-EL-ATTY5, OMAR GUILLÉN-FERNÁNDEZ3, KHALED BENKOUIDER6, MOHAMAD AFENDEE MOHAMED7, (Associate Member, IEEE), MUSTAFA MAMAT7, AND MOHD ASRUL HERY IBRAHIM8

1School of Electrical and Communication Engineering, Vel Tech University, Avadi, Chennai, Tamil Nadu 600062, India
2Department of Mechanical Engineering, Universitas Muhammadiyah Tasikmalaya, Tasikmalaya, Jawa Barat 46196, Indonesia
3Department of Electronics, Instituto Nacional de Astrofísica, Óptica y Electrónica (INAOE), Tonantzintla, Puebla 72840, México
4Department of Mathematics and Computer Science, Faculty of Science, Menoufia University, Shibin Al Kawm 32511, Egypt
5Department of Computer Science, Faculty of Computers and Information, Luxor University, Luxor 85957, Egypt
6Non Destructive Testing Laboratory, Automatic Department, University of Jijel, Jijel 18000, Algeria
7Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin (UniSZA), Kuala Terengganu 21300, Malaysia
8Faculty of Entrepreneurship and Business, Universiti Malaysia Kelantan, Kota Bharu Kelantan 16100, Malaysia

Corresponding author: Mohamad Afendee Mohamed (mafendee@unisza.edu.my)

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ABSTRACT In this work, we describe the model of a new 4-D hyperchaotic system with no balance point and deduce that the new hyperchaotic system has a hidden attractor. We present a detailed bifurcation analysis for the new hyperchaotic dynamo system with respect to the system parameters and also exhibit that the new hyperchaotic system displays multistability with coexisting attractors. Using NI Multisim 14.0, we design an electronic circuit for the implementation of the new 4-D hyperchaotic system and present the circuit simulation results. We also show the implementation of the new 4-D hyperchaotic system by using a field programmable gate array (FPGA). The hardware resources are reduced by designing single-constant multipliers, adders, subtractors and multipliers. The FPGA design is done for three numerical methods, namely: Forward-Euler, Backward-Euler and fourth-order Runge-Kutta. We demonstrate that experimental chaotic attractors are in good agreement with theoretical simulations. To verify the ability of the presented hyperchaotic system for designing robust cryptosystems, we suggest a novel image cryptosystem using the proposed hyperchaotic system. Simulation outcomes confirm the effectiveness of the proposed image cryptosystem, and consequently, the effectiveness of the proposed 4-D hyperchaotic system in designing diverse cryptographic purposes.

INDEX TERMS Hyperchaotic systems, hyperchaos, multi-stability, bifurcations, Lyapunov exponents, circuit simulation, FPGA, numerical methods, image encryption.

I. INTRODUCTION

Hyperchaos theory deals with hyperchaotic dynamical systems and has several applications in engineering due to the high complexity of hyperchaotic dynamical systems equipped with two or more positive Lyapunov characteristic exponents.

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Bonatto [1] observed hyperchaotic behavior of light polarization states in the output of a free-running laser diode. Barakat et al. [2] obtained a hyperchaotic attractor from a two-photon laser with multi-intermediate states. Petavratzis et al. [3] described a new enhanced technique for the motion control of a mobile robot using a hyperchaotic system dynamics as chaotic path generator for the mobile robot. Vaidyanathan et al. [4] reported a hyperchaotic mechanical system exhibiting Hopf bifurcations and discussed its
circuit simulations. Luo et al. [5] reported the circuit design and control of a memristor-based hyperchaotic system. Zhang et al. [6] proposed a new symmetric image encryption method using a hyperchaotic system, Arnold transform and phase-truncated fractional Fourier transform. Bian and Yu [7] proposed a new cryptosystem for communication using a new 6-D hyperchaotic system.

Bifurcation analysis for a hyperchaotic system deals with the dynamic analysis of the system with respect to changes in the various parameters [8]–[11]. For a hyperchaotic system, multistability refers to the coexistence of two or more attractors of the system for the same values of the parameters but differing values of the initial states [12]–[14].

This work proposes a new 4-D hyperchaotic system with no balance point. Such systems are known to have hidden attractors. In Section II, we state a dynamic model of the new system and show that it has no balance point. We also observe that the new 4-D hyperchaotic system has rotational symmetry about the third coordinate axis. In Section III, we present a detailed bifurcation analysis of the new hyperchaotic system presented in this work and also exhibit that it has multistability with coexisting attractors.

Circuit design of chaotic and hyperchaotic systems is very important for practical applications [15]–[17]. Mobayen et al. [18] discussed a family of chameleon chaotic systems with quadratic nonlinearities and designed an electronic circuit for a chameleon chaotic system. Petrzela [19] carried out an experimental verification of a new hyperchaotic system based on a two-stage amplifier. In Section V, we present the design of an electronic circuit of the new hyperchaotic system via NI Multisim 14.0, and describe its circuit simulations.

It has been shown that any chaotic system can be implemented by using field-programmable gate arrays (FPGAs) [20], this is due to the advantage of fast verification and reconfigurability of the device. Once the mathematical model is given, one must consider the use of different numerical methods [21] to proceed to the FPGA implementation, and also, one can reduce hardware resources by designing single-constant multipliers [22], adders, subtractors and multipliers. In this work, FPGA design of the new 4-D hyper-chaotic system is performed by applying three numerical methods, namely: Forward-Euler, Backward-Euler and fourth-order Runge-Kutta numerical methods.

In Section VI, we demonstrate that experimental chaotic attractors are in good agreement with theoretical simulations, and also provide hardware resources for different numerical methods.

Information security acts a vital purpose in our daily lives [23]. Digital images exemplify the common model of data description. Image data can be secured via utilizing one of the image cryptosystems [24]–[28] or using one of the image data hiding methods [29], [30].

In Section VII, we propose a new image encryption mechanism based on the new hyperchaotic system. In our image cryptosystem, the hash code for the pristine image is calculated at first and utilizes this hash code for updating the primary conditions of the presented hyperchaotic system to guarantee pristine image sensitivity. The output of iterating the hyperchaotic system is four chaotic sequences in which the last chaotic sequence is utilized for constructing a new sequence from the other first three sequences, and the pristine image is permuted then substituted utilizing the new sequence. Simulation outputs confirm the effectiveness of the proposed image cryptosystem.

II. A NEW HYPERCHAOTIC SYSTEM WITH NO BALANCE POINT

The dynamical equations of the new 4-D system proposed in this work are given by

\[
\begin{align*}
\dot{y}_1 &= a(y_2 - y_1) - by_2 y_3 + y_4 \\
\dot{y}_2 &= y_1(1 - cy_3) \\
\dot{y}_3 &= y_1 y_2 - d \\
\dot{y}_4 &= -y_1 - y_2
\end{align*}
\]

Eq. (1) represents a 4-D system with the state \( Y = (y_1, y_2, y_3, y_4) \) and the system involves three quadratic nonlinearities \( y_1 y_2, y_2 y_3 \) and \( y_1 y_3 \). We assume that the system constants \( a, b, c, d \) are all positive.

By calculating Lyapunov characteristic exponents, it will be shown in this work that the system (1) has a hyperchaotic attractor for the value of constants as

\[ a = 0.6, \quad b = 0.1, \quad c = 25, \quad d = 4 \]

For numerical simulations, we take \((a, b, c, d) = (0.6, 0.1, 25, 4)\). We choose the initial state as \( Y(0) = (0.6, 0.8, 0.6, 0.8) \). Then the Lyapunov characteristic exponents for the 4-D system (1) were computed for \( T = 1E5 \) seconds as follows:

\[
LE_1 = 0.8367, \quad LE_2 = 0.1642, \\
LE_3 = 0, \quad LE_4 = -1.6009
\]

The presence of two positive Lyapunov characteristic exponents in Eq. (3) pinpoints that the 4-D system (1) is hyperchaotic.

The Kaplan-Yorke dimension of the 4-D hyperchaotic system (1) is obtained by the following calculation:

\[
D_{KY} = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} = 3.6252
\]

The 4-D system (1) has high complexity since it has a large value of \( D_{KY} \).

We also remark that the hyperchaotic system (1) has rotation symmetry about the \( y_3 \)-axis since it is invariant under the transformation of coordinates

\[
(y_1, y_2, y_3, y_4) \leftrightarrow (-y_1, -y_2, y_3, -y_4)
\]

The balance points of the hyperchaotic system (1) are found by solving following algebraic system of equations:

\[
a(y_2 - y_1) - by_2 y_3 + y_4 = 0
\]
From Eq. (6d), we get
\[ y_2 = -y_1 \]  
Substituting \( y_2 = -y_1 \) into Eq. (6c), we get
\[ y_1^2 + d = 0 \]
which admits no real solution since \( d > 0 \).

Hence, we conclude that the 4-D hyperchaotic system (1) does not have any balance point. Thus, the hyperchaotic system (1) belongs to the class of systems with hidden attractors [31]–[33].

The 2D signal plots of the 4-D hyperchaotic system (1) for the constants \( (a, b, c, d) = (0.6, 0.1, 25, 4) \) and the initial state \( Y(0) = (0.6, 0.8, 0.6, 0.8) \) are depicted in Figure 1.

FIGURE 1. 2-D signal plots of the 4-D hyperchaotic system (1) for \( (a, b, c, d) = (0.6, 0.1, 25, 4) \) and the initial state \( Y(0) = (0.6, 0.8, 0.6, 0.8) \): (a) \( (y_1, y_2) \) plane, (b) \( (y_2, y_3) \) plane, (c) \( (y_2, y_4) \) plane and (d) \( (y_3, y_4) \) plane.

III. BIFURCATION ANALYSIS AND MULTISTABILITY OF THE NEW HYPERCHAOTIC SYSTEM

Lyapunov exponents spectrum and bifurcation diagram represent the two most important tools to analyse the dynamical behavior of a system. Furthermore, the Kaplan-Yorke fractal dimension is an effective index of system complexity. In this section, dynamical behavior and complexity of the new 4-D hyperchaotic system (1) are investigated by using numerical calculations with the positive parameters \( a, b, c \) and \( d \) varying.

A. PARAMETER A VARYING

Here, we fix the values of the system parameters as
\[ b = 0.1, \quad c = 25, \quad d = 4 \]
and vary the parameter \( a \).

Lyapunov exponents spectrum and the corresponding bifurcation diagram of system (1) when \( a \) takes values in the interval \([0.5, 5]\) are depicted in Figure 2. We can see a good agreement between the bifurcation diagram and the Lyapunov exponents spectrum.

It can be seen from Figure 2 that the proposed 4-D system (1) can exhibit hyperchaotic behavior with two positive Lyapunov exponents and different levels of complexity.

When \( a = 5 \), the new 4-D system (1) generates hyperchaotic behavior. The corresponding Lyapunov exponents are:
\[ LE_1 = 1.297, \quad LE_2 = 0.130, \quad LE_3 = 0, \quad LE_4 = -6.425 \]
and the corresponding Kaplan-Yorke fractal dimension is equal to \( D_{KY} = 3.222 \).

When \( a = 2 \), the new 4-D system (1) generates hyperchaotic behaviour. The corresponding Lyapunov exponents are:
\[ LE_1 = 0.484, \quad LE_2 = 0.305, \quad LE_3 = 0, \quad LE_4 = -2.710 \]
and the corresponding Kaplan-Yorke fractal dimension is equal to \( D_{KY} = 3.291 \) which indicates more complexity of the hyperchaotic attractor.

When \( a = 0.5 \), The Lyapunov exponents are:
\[ LE_1 = 0.902, \quad LE_2 = 0.103, \quad LE_3 = 0, \quad LE_4 = -1.501 \]
and the corresponding Kaplan-Yorke fractal dimension equal to \( D_{KY} = 3.670 \), which is the highest value obtained for the new system (1) when \( a \) varies in \([0.5, 5]\), indicating extreme complexity of the dynamics.
Various phase portraits of the new 4-D system (1) for parameters values as in (9) and different values of $a$ are given in Figure 3.

Now, in order to identify the other types of behaviour of the 4-D system (1) when the bifurcation parameter $a$ varies, we plot the bifurcation diagram for other values of the constant parameters $b, c$ and $d$.

We fix the values of the parameters as $b = 1.5,\ c = 25,\ d = 10$ (13) and vary the parameter $a$.

Lyapunov exponents spectrum and the corresponding bifurcation diagram of system (1) when $a$ takes values in the interval $[1, 1.6]$ are depicted in Figure 4. We can see a good agreement between the bifurcation diagram and the Lyapunov exponents spectrum. Figure 4 shows that the proposed 4-D system (1) can exhibit periodic behavior, chaotic behavior and hyperchaotic behavior.

When $a \in [1, 1.6]$, the new 4-D system (1) exhibits periodic behavior with no complexity. When $a \in [1.6, 5.7]$, system (1) exhibits chaotic behavior with one positive Lyapunov exponent. When $a \in [5.7, 10]$, the new 4-D system (1) exhibits hyperchaotic behavior with two positive Lyapunov exponents and high complexity.

Various phase portraits of the new 4-D system (1) for parameters values as in (13) and different values of $a$ are given in Figure 3.

In addition, Table 1 gives dynamics, Lyapunov exponents and Kaplan-Yorke fractal dimension of the proposed 4-D system (1) for different values of $a$.

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**TABLE 1.** Dynamics, Lyapunov exponents and Kaplan-Yorke dimension of the 4-D system (1) versus its parameters.

| Dynamics   | Parameters | Bifurcation Parameter | Lyapunov Exponents | $D_{KY}$ | Attractor |
|------------|------------|------------------------|---------------------|---------|-----------|
|            | $a$        | $b$        | $c$        | $d$        | $LE_1$ | $LE_2$ | $LE_3$ | $LE_4$ |         |           |
| Periodic   | [1, 1.6]   | 1.5        | 25         | 10         | $a = 1$ | 0      | −0.067 | −0.465 | −0.475 | 0         | Figure 5 (a) |
|            | 1         | [0.85, 2]  | 25         | 10         | $b = 1.9$ | 0      | −0.092 | −0.454 | −0.457 | 0         | Figure 9 (a) |
|            | 1         | 0.8        | 20         | [3.8, 4],  | $d = 4.7$ | 0      | −0.069 | −0.455 | −0.478 | 0         | Figure 15 (a) |
|            | [1.6, 5.7] | 1.5        | 25         | 10         | $a = 3$ | 0.124  | 0      | −0.066 | −3.065 | 3.019     | Figure 4 (b) |
| Chaos      | 1         | [0.5, 0.85]| 25         | 10         | $b = 0.8$ | 0.031  | 0      | −0.327 | −0.709 | 2.095     | Figure 9 (b) |
|            | 0.6       | 0.1        | [3, 10]    | 4          | $c = 4$ | 0.625  | 0      | −0.048 | −1.178 | 3.489     | Figure 11 (a) |
|            | 0.6       | 0.1        | 25         | [−5, 0]    | $d = 0.1$ | 0.575  | 0      | −0.011 | −1.166 | 3.484     | Figure 13 (a) |
|            | 1         | 0.8        | 20         | [0.6, 3],  | $d = 0.2$ | 0.382  | 0      | −0.018 | −1.275 | 3.215     | Figure 15 (b) |
| Hyperchaos | [0.5, 5]   | 0.1        | 25         | 4          | $a = 2$ | 0.902  | 0.103  | 0      | −1.501 | 3.670     | Figure 3 (c) |
|            | [5.7, 10]  | 1.5        | 25         | 10         | $a = 9$ | 2.030  | 0.072  | 0      | −11.101 | 3.189     | Figure 5 (c) |
|            | 0.6       | [0.0, 0.25]| 25         | 4          | $b = 0$ | 1.026  | 0.126  | 0      | −1.744 | 3.661     | Figure 7 (c) |
|            | 1         | [0.0, 0.5] | 25         | 10         | $b = 1$ | 0.503  | 0.196  | 0      | −1.698 | 3.412     | Figure 9 (c) |
|            | 0.6       | 0.1        | [10, 30]   | 4          | $c = 30$ | 0.850  | 0.201  | 0      | −1.651 | 3.637     | Figure 11 (b) |
|            | 0.6       | 0.1        | 25         | [0, 15]    | $d = 15$ | 0.573  | 0.259  | 0      | −1.432 | 3.581     | Figure 14 (b) |
|            | 1         | 0.8        | 20         | [0.6, 3]   | $d = 2$ | 0.455  | 0.173  | 0      | −1.625 | 3.386     | Figure 15 (c) |

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**FIGURE 3.** 2-D plot of the new 4-D hyperchaotic system (1) in the $(y_1, y_2)$-plane for $b = 0.1,\ c = 25,\ d = 4$ and different values of $a$.
B. PARAMETER B VARYING

Here, we fix the values of the system parameters as
\[ a = 0.6, \quad c = 25, \quad d = 4 \]  \hspace{1cm} (14)
and vary the parameter \( b \).

Lyapunov exponents spectrum and the corresponding bifurcation diagram of system (1) when \( b \) takes values in the interval \([0, 0.25]\) are depicted in Figure 6. We can see a good agreement between the bifurcation diagram and the Lyapunov exponents spectrum.

It can be seen from Figure 6 that the proposed 4-D system (1) can exhibit hyperchaotic behavior with two positive Lyapunov exponents and different levels of complexity.

When \( b = 0.15 \), the new 4-D system (1) generates hyperchaotic behavior. The corresponding Lyapunov exponents are:
\[ LE_1 = 0.725, \quad LE_2 = 0.205, \]
\[ LE_3 = 0, \quad LE_4 = -1.526 \]  \hspace{1cm} (15)
and the corresponding Kaplan-Yorke fractal dimension is equal to \( D_{KY} = 3.609 \).

When \( b = 0.12 \), the new 4-D system (1) generates hyperchaotic behaviour. The corresponding Lyapunov exponents are:
\[ LE_1 = 0.819, \quad LE_2 = 0.214, \]
\[ LE_3 = 0, \quad LE_4 = -1.633 \]  \hspace{1cm} (16)
and the corresponding Kaplan-Yorke fractal dimension is equal to \( D_{KY} = 3.633 \) which indicates more complexity of the hyperchaotic attractor.

When \( b = 0 \), The Lyapunov exponents are:
\[ LE_1 = 1.026, \quad LE_2 = 0.126, \]
\[ LE_3 = 0, \quad LE_4 = -1.744 \]  \hspace{1cm} (17)
and the corresponding Kaplan-Yorke fractal dimension equal to \( D_{KY} = 3.661 \), which is the highest value obtained for the new system (1) when \( b \) varies in \([0, 0.25]\), indicating extreme complexity of the dynamics.

Various phase portraits of the new 4-D system (1) for parameters values as in (14) and different values of \( b \) are given in Figure 7.

Now, in order to identify the other types of behaviour of the 4-D system (1) when the bifurcation parameter \( b \) varies, we plot the bifurcation diagram for other values of the constant parameters \( a \), \( c \) and \( d \).

We fix the values of the parameters as
\[ a = 1, \quad c = 25, \quad d = 10 \]  \hspace{1cm} (18)
and vary the parameter \( b \).

Lyapunov exponents spectrum and the corresponding bifurcation diagram of system (1) when \( b \) takes values in the interval\([0, 2]\) are depicted in Figure 8. We can see a good agreement between the bifurcation diagram and the Lyapunov exponents spectrum. Figure 8 shows that the proposed 4-D system (1) can exhibit periodic behavior, chaotic behavior and hyperchaotic behavior.
When $b \in [0, 0.5]$, the new 4-D system (1) exhibits two positive Lyapunov exponents and high value of fractal dimension. When $b \in [0.5, 0.85]$, the system (1) exhibits chaotic behavior with one positive Lyapunov exponent. When $b \in [0.85, 2]$, the new 4-D system (1) exhibits periodic behavior with no complexity.

Various phase portraits of the new 4-D system (1) for parameters values as in (18) and different values of $b$ are given in Figure 7.

In addition, Table 1 gives dynamics, Lyapunov exponents and Kaplan-Yorke fractal dimension of the proposed 4-D system (1) for different values of $b$.

### C. PARAMETER C VARYING

Here, we fix the values of the system parameters as
\[
a = 0.6, \quad b = 0.1, \quad d = 4
\]
and vary the parameter $c$.

Lyapunov exponents spectrum and the corresponding bifurcation diagram of system (1) when $c$ takes values in the interval $[3, 30]$ are depicted in Figure 10. We can see a good agreement between the bifurcation diagram and the Lyapunov exponents spectrum.

It can be seen from Figure 10 that the proposed 4-D system (1) can exhibit chaotic behavior or hyperchaotic behavior when $c$ varies.

When $c \in [3, 10]$, the new system (1) exhibits chaotic behavior with one positive Lyapunov exponent.

When $c \in [10, 30]$, the new system (1) exhibits hyperchaotic behavior with two positive Lyapunov exponents and high complexity.

Various phase portraits of the new 4-D system (1) for the values of parameters as in (19) and different values of $c$ are given in Figure 11. Also, Table 1 gives dynamics, Lyapunov exponents and Kaplan-Yorke fractal dimension of the system (1) for different values of $c$.

### D. PARAMETER D VARYING

Here, we fix the values of the system parameters as
\[
a = 0.6, \quad b = 0.1, \quad c = 25
\]
and vary the parameter $d$.

Lyapunov exponents spectrum and the corresponding bifurcation diagram of system (1) when $d$ takes values in the interval $[-5, 15]$ are depicted in Figure 12. We can see a good agreement between the bifurcation diagram and the Lyapunov exponents spectrum.
It can be seen from Figure 12 that the proposed 4-D system (1) can exhibit chaotic behavior or hyperchaotic behavior when $d$ varies.

When $d \in [-5, 0]$, the new system (1) exhibits chaotic behavior with one positive Lyapunov exponent.

When $d \in [0, 15]$, the new system (1) exhibits hyperchaotic behavior with two positive Lyapunov exponents and high complexity.

Various phase portraits of the new 4-D system (1) for the values of parameters as in (20) and different values of $d$ are given in Figure 13.

Now, in order to identify the other behavior of the 4-D system when the bifurcation parameter $d$ varies, we plot the bifurcation diagram for other values of the constant parameters $a$, $b$, and $c$.

We fix the values of the parameters as

$$a = 1, \quad b = 0.8, \quad c = 20 \quad (21)$$

and vary the parameter $d$.

Lyapunov exponents spectrum and the corresponding bifurcation diagram of the system (1) when $d$ takes values in the interval $[0, 5]$ are depicted in Figure 14. We can see a good agreement between the bifurcation diagram and the Lyapunov exponents spectrum. Figure 14 shows that the proposed 4-D system can generate periodic behavior, chaotic behavior and hyperchaotic behavior.

We define two sets

$$I = [3.8, 4] \cup [4.4, 4.7] \cup [4.9, 5] \quad (22)$$

and

$$J = [0, 0.6] \cup [3, 3.8] \cup [4, 4.4] \cup [4.7, 4.9] \quad (23)$$

When $d \in I$, the new 4-D system (1) exhibits periodic behavior with no complexity.

When $d \in J$, the new 4-D system (1) exhibits chaotic behavior with one positive Lyapunov exponent.

When $d \in [0.6, 3]$, the new 4-D system (1) exhibits hyperchaotic behavior with two positive Lyapunov exponents.

Various phase portraits of the new 4-D system (1) for the values of parameters as in (21) and different values of $d$ are given in Figure 15. Also, Table 1 gives dynamics, Lyapunov exponents and Kaplan-Yorke fractal dimension of the system (1) for different values of $d$. 
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IV. MULTISTABILITY AND COEXISTING ATTRACTIONS IN THE NEW 4-D HYPERCHAOTIC SYSTEM

To investigate the impact of initial conditions on the behaviour of the new 4-D system (1), the bifurcation diagram of system (1) versus parameter \( a \) for two different initial conditions is calculated and plotted. The system (1) remains invariant under the transformation of coordinates

\[
S : (y_1, y_2, y_3, y_4) \mapsto (-y_1, -y_2, y_3, -y_4)
\] (24)

Thus, any projection of the attractor has rotational symmetry in the \( y_3 \)-axis. Thus, the system (1) may exhibit coexisting attractors. The bifurcation diagram obtained enables us to observe the phenomenon of multistability and the appearance of coexisting attractors.

Let \( X_1 \) and \( X_2 \) be two different initial conditions for the new 4-D system (1), where:

\[
X_1 = (1, 1, 1, 1) \quad \text{(Blue color)} \quad (25)
\]

\[
X_2 = (-1, -1, 1, -1) \quad \text{(Red color)} \quad (26)
\]

We fix the values of system parameters as

\[
b = 1.5, \quad c = 25, \quad d = 10 \quad (27)
\]

It can be observed from the bifurcation diagram shown in Figure 16 that the new 4-D system (1) has two different dynamical evolutions when \( a \) varies in the interval \([1, 5]\). When we set \( a = 1.4 \), the system (1) shows two coexisting periodic attractors starting from \( X_1 \) (blue color) and \( X_2 \) (red color) as shown in Figure 17 (a).

When we set \( a = 1.75 \), the system (1) shows two coexisting chaotic attractors starting from \( X_1 \) (blue color) and \( X_2 \) (red color) as shown in Figure 17 (b). The two coexisting attractors have the same values of Lyapunov exponents:

\[
LE_1 = 0.022, \quad LE_2 = 0, \quad LE_3 = -0.622, \quad LE_4 = -1.084 \quad (28)
\]

When we set \( a = 3.5 \), the system (1) shows two coexisting hyperchaotic attractors starting from \( X_1 \) (blue color) and \( X_2 \) (red color) as shown in Figure 17 (c). The two coexisting attractors have the same values of Lyapunov exponents:

\[
LE_1 = 0.135, \quad LE_2 = 0.040, \quad LE_3 = 0, \quad LE_4 = -3.665 \quad (29)
\]
V. CIRCUIT DESIGN OF THE NEW HYPERCHAOTIC SYSTEM

In this section, the 4-D hyperchaotic system (1) with no balance point is designed by NI Multisim 14.0 software. The circuit includes 7 operational amplifiers, 3 multipliers, 4 capacitors and 16 resistors. It is noted that the operational amplifiers (U1A, U3A, U5A, U6A) are configured as integrators while three operational amplifiers (U2A, U4A, U7A) are configured as inverting amplifier.

For the circuit design, the new 4-D hyperchaotic system (1) has to be rescaled by using

$$Y_1 = \frac{1}{2} y_1, \quad Y_2 = \frac{1}{2} y_2, \quad Y_3 = \frac{1}{2} y_3, \quad Y_4 = \frac{1}{2} y_4 \quad (30)$$

This results in the following 4-D hyperchaotic system in the new coordinates:

$$\begin{align*}
\dot{Y}_1 &= a(Y_2 - Y_1) - 2bY_2Y_3 + Y_4 \\
\dot{Y}_2 &= Y_1 - 2cY_1Y_3 \\
\dot{Y}_3 &= 2Y_1Y_2 - 2 \\
\dot{Y}_4 &= -Y_1 - Y_2 
\end{align*} \quad (31)$$

The circuit equation of the 4-D hyperchaotic system (31) after using Kirchhoff’s electrical circuit laws can be derived as follows:

$$\begin{align*}
\dot{Y}_1 &= \frac{1}{C_1R_1} Y_2 - \frac{1}{C_1R_2} Y_1 - \frac{1}{C_1R_3} Y_2Y_3 + \frac{1}{C_1R_4} Y_4 \\
\dot{Y}_2 &= \frac{1}{C_2R_5} Y_1 - \frac{1}{C_2R_6} Y_1Y_3 \\
\dot{Y}_3 &= \frac{1}{C_3R_7} Y_1Y_2 - \frac{1}{C_3R_8} V_1 \\
\dot{Y}_4 &= -\frac{1}{C_4R_9} Y_1 - \frac{1}{C_4R_{10}} Y_2 
\end{align*} \quad (32)$$

Here, $Y_1, Y_2, Y_3, Y_4$ are the voltages across the capacitors, $C_1, C_2, C_3, C_4$, respectively. We choose the values of the circuit elements as follows: $R_1 = R_2 = 666.67 \, k\Omega, R_3 = 2 \, M\Omega, R_4 = R_5 = R_9 = R_{10} = 400 \, k\Omega, R_6 = 8 \, k\Omega, R_7 = R_8 = 200 \, k\Omega, R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = 100 \, k\Omega$ and $C_1 = C_2 = C_3 = C_4 = 3.2 \, nF$. 

FIGURE 18. Schematic diagram of the electronic circuit of the 4-D hyperchaotic system (32).
The electronic circuit of the new 4-D hyperchaotic system (32) is constructed and shown in Figure 18. The NI Multisim 14.0 phase portraits in Figure 19 verify the hyperchaotic behavior of the 4-D circuit (32).

The Fourier spectral distributions for the variables \(Y_1, Y_2, Y_3, Y_4\) of the circuit (32) are shown in Figure 20. The frequency range is 5 kHz and maximum peak 500 Hz. It corresponds to a prevailing frequency of the implementing oscillating loop. The power spectra of the output signals in Figure 20 are broadband, which are typical of hyperchaotic signals.

**VI. FPGA DESIGN OF THE NEW 4-D HYPERCHAOTIC SYSTEM**

It is well-known that FPGAs are quite useful for fast prototyping, easy of reprogrammability and reconfigurability, and they provide low development cost applications, while maintaining good performance. However, for solving hyper-chaotic systems, the required hardware resources and processing time depend on the numerical method. In this manner, this section shows the FPGA implementation of the new 4-D hyperchaotic system given in (1) by applying three numerical methods, namely: Forward-Euler, Backward-Euler and fourth-order Runge-Kutta methods. As highlighted in [22], each numerical method provides different accuracy and requires a good estimation of the step-size to diminish numerical errors. For instance, by applying the Forward Euler method, one gets the discrete equations given by (33), from which one can infer the use of multipliers, adders and subtractors for the FPGA implementation.

\[
\begin{align*}
Y_{1n+1} &= Y_{1n} + h[a(y_{2n} - Y_{1n}) - by_{2n}Y_{3n} + Y_{4n}] \\
Y_{2n+1} &= Y_{2n} + h[y_{1n}(1 - cy_{3n})] \\
Y_{3n+1} &= Y_{3n} + h[y_{1n}y_{2n} - d] \\
Y_{4n+1} &= Y_{4n} + h[-y_{1n} - y_{2n}] \\
\end{align*}
\tag{33}
\]

From the discretized equations given in (33), one can design the block description as shown in Figure 21. In this figure, one can appreciate the block called single-constant multiplier (SCM), who implements the multiplication of a state variable by a constant \(a, b, c, d\). This SCM block is designed with adders, subtractors and shift registers in order to reduce hardware resources compared to using a traditional multiplier [20]. The SCM is also designed for all the blocks multiplying the step-size \(h\) in the blocks labeled as Predictor Forward-Euler.

The synthesis of all the blocks can be designed by adopting fixed-point representation with the format 13.19 (32 bits), using one bit for the sign, 12 for the integer part, and 19 for the fractional part. It is worth mentioning that this 13.19 format is established by the computer arithmetic and depends on the amplitudes of the state variables. For example, simulation results for this new 4-D hyperchaotic system (1) provide the ranges \([-10, 13]\) for \(Y_1\), \([-28, 28]\) for \(Y_2\), \([-8, 8]\) for \(Y_3\) and \([-15, 20]\) for \(Y_4\), but in the discretization of the second equation, the expected result of \(y_{1n}(1 - cy_{3n})\) is \(\pm 2500\), and therefore, this value can be represented by using 12 bits in the integer part.

The block description of the new 4-D hyper-chaotic system (1) by applying Backward-Euler method, is shown...
in Figure 21. It can be appreciated that this description embeds the Forward Euler method in the blocks labeled as Predictor Forward Euler, and it does a correction by the blocks labeled Corrector Backward Euler. Again, the hardware resources are reduced for FPGA implementation by using SCMs, adders and registers. The application of the fourth-order Runge-Kutta method is performed in the same way, as detailed in [20].

In this work, we set \( h = 0.01 \), and Table 2 shows the resources of the implementation of \( (1) \) by using the FPGA Cyclone IV EP4CGX150DF31C7 along the synthesizer “Quartus II 13.0”. It is worthy mentioning that the row called “Clock cycles by iteration” represents the number of clock cycles that are required to process a new iteration to compute \( y_{1n+1}, y_{2n+1}, y_{3n+1} \) and \( y_{4n+1} \) with a valid data, and the “Latency” row represents the time to compute a new iteration with a 50 MHz clock signal.

Figure 23 shows the experimental setup to implement the new 4-D hyperchaotic system \( (1) \) using a FPGA Cyclone IV EP4CGX150DF31C7, a 16-bit Digital-Analog converter and a Teledyne Lecroy oscilloscope to visualize the attractor.

VII. IMAGE CRYPTOSYSTEM BASED ON THE NEW 4-D HYPERCHAOTIC SYSTEM

In this section, we design a novel image encryption algorithm using the nonlinear features of the proposed 4-D hyperchaotic system \( (1) \). To accommodate the hyperchaotic system \( (1) \) for the aims of designing modern cryptographic purposes such as high security and low computational power, the proposed hyperchaotic system \( (1) \) is adapted to be iterative as presented in Eq. (34).

\[
\begin{align*}
y_{1t+1} &= (a (y_{2t} - y_{1t}) - b y_{2t} y_{3t} + y_{4t}) \mod 1 \\
y_{2t+1} &= (y_{1t}(1 - cy_{3t})) \mod 1 \\
y_{3t+1} &= (y_{1t}y_{2t} - d) \mod 1 \\
y_{4t+1} &= (-y_{1t} - y_{2t}) \mod 1 
\end{align*}
\]

A. ENCRYPTION ALGORITHM

The proposed image cryptosystem utilizes the hash code for the pristine image to update the primary conditions of the

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Logic Elements} & \text{Forward Euler} & \text{Backward Euler} & \text{4th order Runge-Kutta} & \text{Available} \\
\hline
\text{Registers} & 1375 & 3116 & 6774 & 149,760 \\
\text{9 x 9 Bit multipliers} & 777 & 1825 & 3289 & 149,760 \\
\text{Max Freq. (MHz)} & 97.96 & 8.04 & 128 & 720 \\
\text{Clock cycles by Iteration} & 10 & 18 & 36 & 50 \\
\text{Latency (ns)} & 200 & 360 & 720 & - \\
\hline
\end{array}
\]
hyperchaotic system. The outcome of iterating the hyperchaotic system is four chaotic sequences ($Y_1$, $Y_2$, $Y_3$, and $Y_4$), the last chaotic sequence ($Y_4$) is utilized for constructing a new chaotic sequence ($Q$) from the other sequences ($Y_1$, $Y_2$, and $Y_3$), and the pristine image is permuted then substituted utilizing sequence $Q$. The proposed encryption mechanism is described in Figure 28, while the steps of the encryption procedure are described in Algorithm 1.
Algorithm 1 Encryption Procedure

Input: Pristine image (PrImg)
Parameter: Initial conditions and control parameters for iterating the hyperchaotic system (y1_initial, y2_initial, y3_initial, y4_initial, a, b, c, d)
Output: Cipher-image (CrImg), some information about the hash value (H1, H2, H3, H4)

1. Hash ← SHA256(PrImg) // Acquire the hash value for the pristine image
2. h ← uint8(Hash) // Convert each 8-bit into an integer number
3. H1 ← (h1 ⊕ h2 ⊕ ... ⊕ h8)/256
4. H2 ← (h9 ⊕ h10 ⊕ ... ⊕ h16)/256
5. H3 ← (h17 ⊕ h18 ⊕ ... ⊕ h24)/256
6. H4 ← (h25 ⊕ h26 ⊕ ... ⊕ h32)/256
   // Update initial conditions (y1_initial, y2_initial, y3_initial, y4_initial)
7. y1_new ← (y1_initial + H1)/2
8. y2_new ← (y2_initial + H2)/2
9. y3_new ← (y3_initial + H3)/2
10. y4_new ← (y4_initial + H4)/2
11. [Rows Columns Depth] ← size(PrImg) // Acquire the dimension of the pristine image
12. [Y1 Y2 Y3 Y4] ← System(y1_new, y2_new, y3_new, y4_new, a, b, c, d, Rows × Columns × Depth) // Iterating the hyperchaotic system (34) for Rows × Columns × Depth times
13. Pi ← round(Y4_i × 10^4) mod 3 for i ← 1, 2, ..., Rows × Columns × Depth
14. Qi ← \[ Y1_i if P_i = 0
   \begin{cases} 
   Y2_i if P_i = 1 \\
   Y3_i if P_i = 2
   \end{cases}
   \] for i ← 1, 2, ..., Rows × Columns × Depth
   // Permutation process
15. A ← arrange(Q) // Arrange the ingredients of Q in ascending order
16. V ← index(Q, A) // Take the index of every element of Q in A
17. ImgVec ← reshape(PrImg, Rows × Columns × Depth, 1) // Convert the pristine image to a one vector
18. ImgPer(t) ← ImgVec(V(t)) for i ← 1, 2, ..., Rows × Columns × Depth
   // Substitution process
19. Key ← round(Q × 10^{14}) mod 256 // Transforming Q sequence into integers
20. CrImgVec ← ImgPer ⊕ Key // Pixel substitution
21. CrImg ← reshape(CrImgVec, Rows, Columns, Depth) // Cipher image

B. EXPERIMENTAL OUTCOMES

To prove the cryptographic characteristics of the proposed image encryption algorithm, a PC with Intel® Core™ 2 Duo CPU 3GHz and RAM 4GB and prepared with MATLAB software R2016b. The used dataset of images was taken from Kodak database [34] each of size 768 × 512 and labeled as PrImg01, PrImg02, PrImg03, and PrImg04 (see Figure 29). The primary key parameters employed for iterating the hyperchaotic system (34) are fixed as (y1_initial ← 0.6, y2_initial ← 0.8, y3_initial ← 0.6, y4_initial ← 0.8, a ← 0.6, b ← 0.1, c ← 25, d ← 4).

1) CORRELATION ANALYSIS

To evaluation the meaningful of an image, the correlation coefficient of neighboring pixels (Cpc) is utilized, in which pristine images have Cpc values nearly to 1 per direction while the values of Cpc for cipher images should around 0 [25]. To compute the values of Cpc in each direction of the pristine and cipher images, we randomly picked 10^4 pairs of adjoining pixels per direction.

\[
C_{pc} = \frac{\sum_{i=1}^{N} (p_i - \bar{p})(c_i - \bar{c})}{\sqrt{\sum_{i=1}^{N} (p_i - \bar{p})^2 \sum_{i=1}^{N} (c_i - \bar{c})^2}}
\]
here $N$ refers to the full number of neighboring pixel pairs per direction and $c_i, p_i$ are point to the values of neighboring pixels. Table 3 displays the outputs of $C_{pc}$ for pristine images and their analog cipher version, in which the values of encrypted images are very near to 0. Also, Figs. 30, 31, and 32 plot the correlation distribution per direction for pristine image $PrImg01$ and its analogue cipher version. From the results listed about correlation coefficients, no valuable data acquired about the pristine image by analyzing $C_{pc}$ values.

2) PRISTINE IMAGE SENSITIVITY

To evaluate the outcome of slight modification in the pristine image on its analog cipher version, two measures are employed: “Number of Pixels Change Rate” (NPCR) and “Unified Average Changing Intensity” (UACI). NPCR and UACI can be expressed mathematically as follows:

$$\text{NPCR} = \frac{\sum_{i,j} \text{Diff}(i,j)}{N} \times 100\%,$$

$$\text{Diff}(i,j) = \begin{cases} 0 & \text{if } C_{p1}(i,j) = C_{p2}(i,j) \\ 1 & \text{if } C_{p1}(i,j) \neq C_{p2}(i,j) \end{cases}$$

Table 4, in which the average value of NPCR for the investigated dataset is 99.6125%, therefore the presented image cryptosystem has a high sensitivity for small pixel changes in the pristine image.

$$\text{UACI} = \frac{1}{N} \left( \sum_{i,j} \frac{|C_{p1}(i,j) - C_{p2}(i,j)|}{256} \right) \times 100\%$$

where $N$ refers to the total number of pixels that represent the image and $C_{p1}, C_{p2}$ are two ciphered images for one pristine image with a difference in one bit. NPCR and UACI values are listed in Table 4.
3) HISTOGRAM ANALYSIS
To provide the distribution rate of pixel values in an image, histogram test is utilized. The histogram of encrypted images utilizing a good-designed image encryption approach should ensure the identical distribution of different encrypted images to resist statistical attacks [35]. Fig. 33 presents the histograms of image PrImg01 before and after the encryption procedure, in which the histograms of the pristine image are distinct from each other while the histograms of the ciphered image CrImg01 are uniform with each other. Nevertheless, we need a quantitative tool to measure the pixel distribution in an image. Consequently, we utilized chi-square test ($\chi^2$) [27].

$$\chi^2 = \sum_{t=0}^{255} \frac{(f_t - d)^2}{d}$$

where $f_t$ refers to the frequency of pixel value $t$, and $d$ indicates the size of the image. By considering the significant level is 0.05, then $\chi^2_{0.05}(255) = 293.25$. For a given image, if the value of $\chi^2$ is less than 293.25, this points that the given image has an identical distribution; else, the image holds non-identical distribution. Table 5 provides the outcome of $\chi^2$ for the examined images, in which the $\chi^2$ values for all encrypted images is less than 293.25. Therefore, the provided encryption mechanism can withstand histogram analysis attacks.

4) INFORMATION ENTROPY ANALYSIS
To provide the distribution of pixel values per level in an image, information entropy test is utilized and can be stated
TABLE 7. Encryption time (in s) for the stated cryptosystem alongside other related cryptosystems for different dimensions of images.

| Cryptosystem | Image dimension | Proposed | [36] | [37] | [41] | [35] |
|--------------|----------------|----------|------|------|------|------|
|              | 256 × 256      | 0.0499   | 0.0538 | 0.0949 | 0.1789 | 0.0779 | 0.0494 |
|              | 512 × 512      | 0.2576   | 0.2338 | 0.4010 | 0.6639 | 0.3261 | 0.3034 |
|              | 1024 × 1024    | 0.9762  | 1.1494 | 1.9857 | 3.1426 | 1.3146 | 1.0454 |

mathematically as given in Eq. (39):

\[ E(X) = -\sum_{t=0}^{255} p(x_t) \log_2 (p(x_t)) \]  

(39)

where \( p(x_t) \) refers the probability of \( x_t \). The possible values for a grayscale image are \( 2^8 \), so the optimal entropy value is equal to 8-bit [36], [37]. Subsequently, to approve the effectiveness of the proposed encryption algorithm, the entropy value of the ciphered images should be close to 8. Table 6 provides the outcome of the entropy test for the pristine images and their analogous encrypted version, in which all information entropy values for encrypted images are very near to 8-bit. Consequently, the suggested encryption algorithm is secure against entropy attacks.

5) KEY SENSITIVITY ANALYSIS

The sensitivity of the secret key to the decryption effect is known as the key sensitivity. To value the key sensitivity of the proposed cryptosystem, the ciphered image \( CrImg01 \) decrypted several times with slight modification in the secret key each time, in which the results are stated in Fig. 34.

6) OCCLUSION ATTACKS

Data may suffer from losing some of its parts when transmitted over a noisy channel [30]. Hence, the proposed image cryptosystem must have the capability to resist occlusion attacks. To value the proposed encryption approach against these attacks, we clipping out some parts of the encrypted image and then trying to decrypt the defective cipher image.

7) TIME EFFICIENCY AND COMPARISON ANALYSIS

The main contribution of designing modern cryptographic mechanisms is to provide high security in addition to low computational power. To confirm that the suggested image cryptosystem is efficient in time encryption, we calculate the time required to encrypt one image. Table 7 provides the time taken to encrypt several grayscale images with diverse dimensions alongside other related cryptosystems, in which the proposed image cryptosystem is effective in time encryption. To confirm that the suggested image cryptosystem has high security compared to other related cryptosystems, Table 8 gives a simple comparison in terms of average values of correlation, information entropy, NPCR, UACI, and Chi-square.

TABLE 8. Comparison of the proposed encryption mechanism with other related mechanisms in terms of average values of correlation, information entropy, UACI, NPCR, and Chi-square.

| Cryptosystem | Correlation | Information entropy | \( \chi^2 \) | NPCR | UACI |
|--------------|-------------|---------------------|-------------|-------|------|
|              | H V D       |                     |             |       |      |
| Proposed     | 0.00018 -0.00002 0.00012 | 7.99984 | 254.4314 | 99.613\% | 33.483\% |
| [42]         | 0.00050 0.00170 -0.00250 | 7.99866 | 256.7515 | 99.607\% | 33.427\% |
| [43]         | -0.00663 0.00083 -0.00034 | 7.99930 | 264.4174 | 99.600\% | 33.473\% |
| [44]         | -0.00970 -0.00070 0.00650 | 7.99700 | 257.3367 | 99.600\% | 33.440\% |
| [45]         | 0.00219 0.00169 0.00186 | 7.99929 | 249.4286 | 99.611\% | 33.476\% |
| [35]         | -0.00016 0.00002 0.00023 | 7.99949 | 250.5122 | 99.622\% | 33.472\% |
| [46]         | 0.00007 0.00007 -0.00014 | 7.99984 | 256.4314 | 99.600\% | 33.470\% |
| [36]         | 0.00267 -0.00008 0.00007 | 7.99720 | 254.9902 | 99.633\% | 33.530\% |
| [27]         | 0.00020 -0.00010 0.00010 | 7.99976 | 256.3767 | 99.606\% |       |
| [28]         | -0.00028 0.00016 0.00018 | 7.99984 | 257.3172 | 99.604\% | 33.459\% |

Figure 35 provides results of occlusion attacks, in which the pristine image is recovered effectively from the defective cipher image without information losing in the clipped part.
From the stated values in Tables 7 and 8, we declare that the presented image cryptosystem is effective and has high security besides low encryption time.

VIII. CONCLUSION

This work started with a dynamical model of a new 4-D hyperchaotic system with no balance point and presented a detailed bifurcation analysis for the new hyperchaotic dynamo system with respect to the system parameters. We noted that the new hyperchaotic system displays multistability with coexisting attractors. Using NI Multisim 14.0 software, we designed an electronic circuit for the implementation of the new 4-D hyperchaotic system and presented the circuit simulation results. We also detailed the implementation of the new 4-D hyperchaotic system by using a field programmable gate array (FPGA). The FPGA design was done by applying three numerical methods, namely: Forward-Euler, Backward-Euler and fourth-order Runge-Kutta methods. We demonstrate that experimental chaotic attractors were in good agreement with the Matlab simulations. In addition, this work presented a new image cryptosystem using the nonlinearity features of the new 4-D hyperchaotic system with no balance point, which the simulation outcomes confirm the effectiveness of the proposed cryptosystem and the reliability of the hyperchaotic system in designing various modern cryptographic applications.

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BASSEM ABD-EL-ATTY received the B.S. degree in physics and computer science, the M.Sc. degree in computer science, and the Ph.D. degree in computer science from Menoufia University, Egypt, in 2010, 2017, and 2020, respectively. He is currently an Assistant Professor with the Faculty of Computers and Information, Luxor University, Egypt. He has authored and coauthored more than 30 papers, including refereed articles published by IEEE, Elsevier and Springer journals, conference papers, and book chapters. His research interests include quantum information processing and image processing. He is a reviewer in a set of reputable journals in Elsevier and Springer.

OMAR GUILLÉN-FERNÁNDEZ received the bachelor’s degree from the Instituto Tecnológico de Veracruz, in 2015, and the M.Sc. degree in electronics from the Instituto Nacional de Astrofísica, Óptica y Electrónica (INAOE), México, in 2018, where he is currently pursuing the Ph.D. degree. He has published two books, two book chapters, two conferences, and 14 research articles. His research interests include chaotic systems, synchronization techniques, security of data transmission, embedded systems, and integrated circuit design and applications.

KHALED BENKOUIDER received the M.S. degree in automatic control and the Ph.D. degree from the University of Jijel, Jijel, Algeria, in 2015 and 2021, respectively. His M.S. research was on secure communications based on chaotic systems. His main research interests include dynamical systems, control systems, delayed systems, LPV systems, and chaotic systems synchronization.

MOHAMAD AFENDEE MOHAMED (Associate Member, IEEE) received the Ph.D. degree in mathematical cryptography from Universiti Sultan Zainal Abidin, in 2012. He currently serves as an Associate Professor for Universiti Sultan Zainal Abidin. His research interests include both theoretical and application issues in the domain of data security, and mobile and wireless networking.

OMAR GUILLÉN-FERNÁNDEZ received the bachelor’s degree from the Instituto Tecnológico de Veracruz, in 2015, and the M.Sc. degree in electronics from the Instituto Nacional de Astrofísica, Óptica y Electrónica (INAOE), México, in 2018, where he is currently pursuing the Ph.D. degree. He has published two books, two book chapters, two conferences, and 14 research articles. His research interests include chaotic systems, synchronization techniques, security of data transmission, embedded systems, and integrated circuit design and applications.

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MUSTAFA MAMAT received the Ph.D. degree from UMT, in 2007, with specialization in optimization. He was first appointed as a Lecturer with Universiti Malaysia Terengganu (UMT), in 1999. Later on, he was appointed with UMT as a Senior Lecturer, in 2008, and then as an Associate Professor, in 2010. He has been a Professor and the Dean of the Graduate School, Universiti Sultan Zainal Abidin (UniSZA), Malaysia, since 2013. To date, he has successfully supervised more than 60 postgraduate students and published more than 150 research papers in various international journals and conferences. His research interests include conjugate gradient methods, steepest descent methods, Broyden's family, and quasi-Newton methods.

MOHD ASRUL HERY IBRAHIM was born in Kelantan, Malaysia. He received the B.Sc. degree in financial mathematics and the M.Sc. degree in applied mathematics from Universiti Malaysia Terengganu, Malaysia. He is currently pursuing the Ph.D. degree major in mathematical sciences with Universiti Sultan Zainal Abidin. He is also working as a Senior Lecturer with Universiti Malaysia Kelantan and the Director of the Publication and Rating Division. His current research interests include optimization, numerical analysis, business mathematics, and business statistics. He writes regularly and has published more than 50 scientific articles in journals and conferences.