Graphene-based one-dimensional photonic crystal

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Abstract
A novel type of one-dimensional (1D) photonic crystal formed by an array of periodically located stacks of alternating graphene and dielectric stripes embedded into a background dielectric medium is proposed. The wave equation for the electromagnetic wave propagating in such a structure is solved in the framework of the Kronig–Penney model. The frequency band structure of the 1D graphene-based photonic crystal is obtained analytically as a function of the filling factor and the thickness of the dielectric between the graphene stripes. The photonic frequency corresponding to the electromagnetic wave localized by a defect of the photonic crystal formed by an extra dielectric placed in the position of one stack of alternating graphene and dielectric stripes is obtained.

(Some figures may appear in colour only in the online journal)

1. Introduction
Photonic crystals are formed by structures with the dielectric constant periodically varying in space [1]. Electromagnetic waves in photonic crystals have a band spectrum and a coordinate dependence caused by this periodicity of the dielectric constant. Interestingly enough, the solution of Maxwell’s equations with a periodic dielectric constant, resulting in photonic bandgap structures, is similar to the solution to Schrödinger’s equation for a periodic potential, resulting in the electron energy bandgap structures in solids. Electromagnetic waves penetrate the photonic crystal similarly to the Bloch waves of electrons in a regular crystal. The width of the photonic bandgap depends on the geometrical parameters of the photonic crystal and the contrast of the dielectric constants of the constituent elements [2, 3]. Different materials have been used for the corresponding constituent elements including dielectrics, semiconductors and metals [2, 3, 6–9]. Photonic crystals with superconducting elements have been studied in [10–16]. One-dimensional photonic crystals formed by semiconductors were analyzed in [17]. It is well known that the properties of photonic crystals provide an opportunity to manipulate the emission, propagation and distribution of light [4, 5] and photonic crystals can be used as frequency filters. The properties of photonic crystals were reviewed in [18].

Photonic crystals are different from regular solid crystals in the following way. While the Schrödinger’s equation describes regular solid crystals via the scalar wavefunction, Maxwell’s equations for photonic bandgap crystals describe the electric or magnetic field, which is a vector corresponding to the transverse electromagnetic waves.

A novel type of 2D electron system has been experimentally obtained in graphene, which is a 2D honeycomb lattice of the carbon atoms that form the basic planar structure in graphite [19, 20]. Due to the unusual properties of the band structure, the electronic properties of graphene have become the object of many recent experimental and theoretical studies [19–25]. The unique electronic properties of graphene in a magnetic field have been studied recently [27–30]. The space–time dispersion of graphene conductivity was analyzed in [31]. The unique electronic properties of graphene in a magnetic field have been studied recently [27–30]. The space–time dispersion of graphene conductivity was analyzed in [31].
In this paper, we consider a one-dimensional photonic crystal formed by an array of periodically located parallel stacks of alternating graphene and dielectric stripes embedded into a background dielectric medium. The graphene stripes are placed one under the other with the dielectric stripes placed between them. We calculate the frequency band structure of such a photonic crystal. The photonic band structure can usually be obtained using numerical calculations. In this paper we obtain the analytical solution for the wave equation with the periodical dielectric function. We also calculate the frequency corresponding to the electromagnetic wave localized due to a defect in the array of stacks of graphene stripes separated by dielectric stripes.

The paper is organized in the following way. In section 2 we obtain the photonic band structure of the 1D graphene-based photonic crystal. In section 3 we find the frequency corresponding to the electromagnetic wave localized by a defect of a 1D graphene-based photonic crystal. Finally, a discussion of the results and conclusions follow in section 4.

2. The wave equation for 1D photonic crystal with graphene stripes

We consider polarized electromagnetic waves with the electric field \( E \) perpendicular to the plane of the graphene stripes. The wave equation for the electric field in a dielectric medium has the form \([34]\)

\[
\Delta E(r, t) + \frac{\varepsilon(r, t)}{c^2} \frac{\partial^2 E(r, t)}{\partial t^2} = 0, \tag{1}
\]

where \( \varepsilon(r, t) \) is the dielectric constant of the medium and \( c \) is the speed of light in vacuum. Looking for solutions with harmonic time variation of the electric field, i.e., \( E(r, t) = E(r)e^{i\omega t} \), and considering the propagation of a wave in the \( x \)-direction along the plane of the graphene stripes and perpendicular to the graphene–dielectric boundaries one obtains from equation (1)

\[
\frac{\partial^2 E_\alpha(x)}{\partial x^2} + \frac{\varepsilon_\alpha(x, \omega)}{c^2} E_\alpha(x) = 0. \tag{2}
\]

The dielectric constant of the 1D periodic structure is given by

\[
\varepsilon(x, \omega) = \begin{cases} 
\varepsilon_0, & \text{for } -\frac{1}{2}(a-b) + na < x < -\frac{1}{2}(a-b) + na, \\
\varepsilon_1(\omega), & \text{for } \frac{1}{2}(a-b) + na < x < \frac{1}{2}(a+b) + na,
\end{cases}
\]

where \( \varepsilon_0 \) is the dielectric constant of the dielectric, \( \varepsilon_1(\omega) \) is the dielectric function of the graphene multilayers separated by the dielectric barriers, \( a \) is the period of the 1D array of graphene stripes, \( b \) is the width of the graphene stripes, and \( n \) is an integer. The 1D photonic crystal with graphene stripes separated by dielectric layers of thickness \( d \) is shown in figure 1. By introducing the filling factor \( f \) the relation between \( a \) and \( b \) can be written as \( b = af \).

The dielectric function \( \varepsilon_1(\omega) \) of a graphene multilayer system separated by dielectric layers with a dielectric constant \( \varepsilon_0 \) and a thickness \( d \) is given by \([35, 36]\)

\[
\varepsilon_1(\omega) = \varepsilon_0 + \frac{4\pi i \sigma_g(\omega)}{\omega d}, \tag{3}
\]

where \( \sigma_g(\omega) \) is the dynamical conductivity of doped graphene for high frequencies \( \omega \gg k_B T, \omega \gg \tau^{-1} \) at temperature \( T \) given by \([35, 36]\)

\[
\sigma_g(\omega) = \frac{e^2}{4\hbar} \left[ \frac{\eta(\hbar\omega - 2\mu)}{2\pi} + \frac{i}{2\pi} \frac{16k_B T}{\hbar \omega} \right. \\
\times \log \left[ 2 \cosh \left( \frac{\mu}{2k_B T} \right) \right] - \log \left( \frac{\hbar \omega + 2\mu}{\hbar \omega - 2\mu} \right) + 2 \left( \frac{2\mu}{(2\mu)^2 + (2k_B T)^2} \right). \tag{4}
\]

Here \( e \) is the charge of an electron, \( \tau^{-1} \) is the electron collision rate, \( \eta \) is the Heaviside step function, \( k_B \) is the Boltzmann constant, and \( \mu \) is the chemical potential determined by the electron concentration, which is controlled by the doping. The chemical potential can be calculated as \( \mu = (\pi n_0)^{1/2} k_B \tau \), where the electron concentration is given by \( n_0 = (\mu/(\hbar v_F))^2/\pi \) and \( v_F = 10^6 \text{ cm s}^{-1} \) is the Fermi velocity of electrons in graphene \([36]\).

We solve the wave equation (2) to find the eigenfrequencies corresponding to the electromagnetic wave penetrating in the photonic crystal shown in figure 1. This wave equation is mathematically similar to the Schrödinger equation for an electron moving in a one-dimensional rectangular periodic potential barrier described by the Kronig–Penney model. The eigenenergies of the Schrödinger equation corresponding to the Kronig–Penney model presented in equations (A.1) are given by \([37]\)

\[
\cos(ka) = \cos(\beta b) \cos[a(a-1)f] + \frac{\alpha^2 - \beta^2}{2\alpha\beta} \sinh(\beta b) \sin[a(a-1)f], \tag{5}
\]

where the wavevector \( k \) is in the range \( 0 \leq k \leq 2\pi/a \), and \( \alpha \) and \( \beta \) are defined as

\[ k = \frac{\omega}{c} \sqrt{\varepsilon_0 + \frac{4\pi i \sigma_g(\omega)}{\omega d}}. \]
The dielectric function separates the graphene stripes, is presented in (7), since the
d stripes. Let us mention that the dependence of the photonic
density of states (DOS) the photonic bandgap depends on the filling factor
to determine the photonic bandgap of the 1D photonic crystal formed by the periodically located
alternating graphene and dielectric stripes in one place, where
it should be placed due to the periodicity. This wave function is similar to the Schrödinger equation describing the electron in the periodic potential energy in the presence of the potential energy of the defect placed at the point \( x_0 \) given in appendix B. From the mapping of the Klein–Gordon type equation given by equation (B.6) the wave equation for the electric field in the graphene-based photonic crystal has the following form:

\[
\frac{\partial^2 E_x(x)}{\partial x^2} + \frac{\omega^2}{c^2} (\varepsilon(x) - \varepsilon_0) \frac{\partial^2 E_x(x)}{\partial x^2} + \omega^2 \Omega^2 \left( \varepsilon_1(\omega) - \varepsilon_0 \right) E_x(x) = \frac{3}{\varepsilon_0} \frac{\partial^2 E_x(x)}{\partial t^2} - 2\alpha^2 \Omega^2 (\varepsilon_1(\omega) - \varepsilon_0)
\]  
\times \gamma \left( |x - x_0| \right) E_x(x) = (\omega^4 - \Omega^4) E_x(x),
\]  
(9)

where \( \alpha = 2\pi/a \) is the vector of the 1D reciprocal lattice. This Klein–Gordon type equation has the eigenvalue \( \omega^4 - \Omega^4 \).

In equation (9) \( \Omega \) is the width of the forbidden band (photonic gap) in the spectrum of the electromagnetic wave.

The electric field can be obtained by mapping equations (B.7) and (B.9) that correspond to wavefunctions of the stationary states. The continuity of the electric field and its derivative in the points \( x = x_0 + b/2 \) and \( x_0 - b/2 \) result in the transcendental equation determining the spectrum of the even states:

\[
\sqrt{\omega^2 \Omega^2 \left( \varepsilon_1(\omega) - \varepsilon_0 \right) - \omega^4 + \Omega^4} = \left| \omega^4 - \Omega^4 \right|,
\]  
\]  
(10)
as well as that of the odd states:

\[
\sqrt{\omega^2 \Omega^2 \left( \varepsilon_1(\omega) - \varepsilon_0 \right) - \omega^4 + \Omega^4} = \left| \omega^4 - \Omega^4 \right|,
\]  
\]  
(11)

Solving equations (10) and (11) with respect to the frequency \( \omega \), we obtain the frequency corresponding to the photonic mode localized by the defect.

3. Localization of the electromagnetic wave on the defect

Let us consider a defect in the array of stacks of alternating graphene and dielectric stripes embedded into a background
dielectric medium. This defect is formed by one empty space or ‘1D vacancy’ due to the absence of a stack of
alternating graphene and dielectric stripes in one place, where
it should be placed due to the periodicity. This place is
filled by the dielectric. This extra dielectric stripe contributes to the dielectric contrast that results by adding the term
\(-\omega^2/c^2 (\varepsilon_1(\omega) - \varepsilon_0) \gamma \left( |x - x_0| \right) E_x(x) \) to the rhs in equation (2). Here \( \gamma \left( |x - x_0| \right) = 1 \) for \( |x - x_0| \leq b \), and \( \gamma \left( |x - x_0| \right) = 0 \) for
\( |x - x_0| > b \), where \( x_0 \) corresponds to the coordinate in the middle of the 1D defect, which is the coordinate of the middle
of the absent graphene stripe. As a result we obtain the wave equation for the electric field for the 1D photonic crystal with the defect:

\[
\frac{\partial^2 E_x(x)}{\partial x^2} + \frac{\omega^2}{c^2} (\varepsilon(x) - \varepsilon_0) \frac{\partial^2 E_x(x)}{\partial x^2} \times \gamma \left( |x - x_0| \right) E_x(x) = 0.
\]  
\]  
(8)

Equation (8) describes the periodic array of stacks of alternating graphene and dielectric stripes with the defect formed by one stack of alternating graphene and dielectric stripes being absent.

We solve the wave equation (8) to find the eigenfrequency corresponding to the electromagnetic wave localized at the
defect formed by a background dielectric medium due to the absence of a stack of alternating graphene and dielectric stripes in one place where it should be placed due to the periodicity. This wave function is similar to the Schrödinger equation describing the electron in the periodic potential energy in the presence of the potential energy of the defect placed at the point \( x_0 \) given in appendix B. From the mapping of the Klein–Gordon type equation given by equation (B.6) the wave equation for the electric field in the graphene-based photonic crystal has the following form:

\[
\frac{\partial^2 E_x(x)}{\partial x^2} + \frac{\omega^2}{c^2} (\varepsilon(x) - \varepsilon_0) \frac{\partial^2 E_x(x)}{\partial x^2} \times \gamma \left( |x - x_0| \right) E_x(x) = 0.
\]  
\]  
(9)

In the calculations below we assume \( n_0 = 10^{11} \text{ cm}^{-2} \). For simplicity, we consider the same material for the background
dielectric medium and the dielectric stripes between the graphene stripes. As the dielectric material we consider SiO2 with the dielectric constant \( \varepsilon_0 = 4.5 \). Using equation (7) we calculated the band structure for 1D graphene-based photonic crystal. The results of the calculations of the dispersion relation of the photonic crystal are presented in figure 2. The photonic band structure is calculated for different distances between graphene layers \( d \). In our calculations we used the chemical potential for the electrons in graphene \( \mu = 3.525 \times 10^{-21} \text{ J} \), temperature \( T = 300 \text{ K} \), period of the 1D graphene stripe array \( a = 25 \times 10^{-6} \text{ m} \), and filling factor \( f = 0.3927 \).
According to the results of our calculations, the photonic band structure almost does not depend on $\varepsilon_0$ due to the fact that $\varepsilon_0 \ll |\varepsilon_1|$. The results of our calculations demonstrate the strong dependence of the photonic band structure on the thickness $d$ of the dielectric that separates the graphene stripes. At $d = 1$ and 5 nm the distance between the lower and upper dispersion curves is larger in the middle than at the edges. At $d = 3$ and 10 nm the distance between the lower and upper dispersion curves is larger at the edges than in the middle.

Using equations (10) and (11) we calculate the frequencies corresponding to the electromagnetic wave localized by the defect in the photonic crystal formed due to the absence of a stack of alternating graphene and dielectric stripes in one place where it should be placed due to the periodicity. For a photonic crystal with the defect at $d = 1$ nm we have $\nu = 1.79$ THz, for the defect at $d = 3$ nm we have $\nu = 1.85$ THz, for the defect at $d = 5$ nm we have $\nu = 3.91$ THz, and for the defect at $d = 10$ nm we have $\nu = 3.62$ THz. All these frequencies are located inside the photonic bandgap. Let us mention that since our approach is based on the Luttinger–Kohn model [39, 40], the localized frequency $\nu$ does not depend on the momentum. The frequency localized by the defect is just a constant in the photonic bandstructure, which can be controlled by the thickness $d$ of the dielectric stripes that separate the graphene stripes, as well as by the filling factor $f$ and the period of the 1D array of graphene stripes $a$.

In conclusion, the graphene-based photonic crystal proposed in this paper is a novel type of one-dimensional photonic crystal formed by an array of stacks of alternating graphene and dielectric stripes embedded into a background dielectric medium. This system can be analyzed as a 1D photonic crystal and the corresponding wave equation for the electromagnetic wave propagating in such a structure can be solved in the framework of the Kronig–Penney model. The frequency band structure is obtained analytically as a function of the filling factor, the period of the 1D array of graphene stripes and the thickness of the dielectric between the graphene stripes. We obtain the photonic frequency corresponding to the electromagnetic wave localized at the defect of a photonic crystal formed by a background dielectric medium due to the absence of a stack of alternating graphene and dielectric stripes in one place where it should be placed due to the periodicity.

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Appendix A. The Schrödinger equation with a periodic potential

The wave equation (2) with the dielectric constant given by equation (3) can be mapped onto the Schrödinger equation.
for an electron in a 1D periodic potential approximated by a rectangular potential barrier, used in the Kronig–Penney model [37]:

\[ -\frac{\hbar^2}{2m_0} \frac{\partial^2 \psi(x)}{\partial x^2} = E \psi(x), \]

for \(-\frac{1}{2}(a-b) + na < x < \frac{1}{2}(a-b) + na,\)

\[ -\frac{\hbar^2}{2m_0} \frac{\partial^2 \psi(x)}{\partial x^2} + V(x) = E \psi(x), \]

\[ V(x) = \frac{\hbar^2 \omega^2}{2m_0c^2} \epsilon(x, \omega) \]

for \(\frac{1}{2}(a-b) + na < x < \frac{1}{2}(a+b) + na,\)

(A.1)

where \(m_0\) is the mass of an electron, \(\psi(x)\) and \(E\) are the wavefunction and energy of an electron, \(V(x)\) is the potential due to an ion of the 1D crystal lattice approximated by a rectangular barrier, \(a\) is the period of the 1D array of scatterers with the rectangular potential, \(b\) is the width of the rectangular potential barrier, and \(n\) is an integer.

It is easy to see that the wave equation (2) with the dielectric constant given by equation (3) can be mapped onto the Schrödinger equation (A.1) for an electron in the 1D periodic potential corresponding to the Kronig–Penney model. The mapping relations are the following ones:

\[ \psi(x) \equiv E_\zeta(x), \]

\[ a^2 = \frac{2m_0}{\hbar^2} E = \frac{\epsilon_0 \omega^2}{c^2}, \]

\[ \beta^2 = \frac{2m_0}{\hbar^2} (E - V(x)) = \frac{\epsilon_1(\omega)}{c^2} \omega^2. \]

(A.2)

The solution of equation (A.1) is well known and is given by expression (5).

Appendix B. The Dirac type equations for the electromagnetic wave in the photonic crystal with the defect

Following [15] we can map the wave equation for the photonic crystal with the defect onto the Schrödinger equation in a periodic field with a defect. After mapping of equation (8) onto the Schrödinger equation describing the electron with the effective electron mass \(m_0\) in the periodic potential energy \(V(x)\) in the presence of the potential energy of the defect \(W_\gamma(|x - x_0|)\) placed at the point \(x_0\) we have

\[ -\frac{\hbar^2}{2m_0} \frac{d^2}{dx^2} \psi(x) + V(x) - W_\gamma(|x - x_0|) \psi(x) = \epsilon_\omega \psi(x). \]

(B.1)

In equation (B.1) \(\psi(x) \equiv E_\zeta(x),\)

\[ \epsilon_\omega = \frac{\hbar^2 \omega^2}{2m_0 c^2}, \]

the potential \(V(x)\) is given in equation (A.1) and the potential \(W\) is defined as

\[ W = \frac{\hbar^2 \omega^2}{2m_0 c^2} (\epsilon_1(\omega) - \epsilon_0). \]

(B.2)

Equation (B.1) has the same form as equation (6) in [38]. However, in our case the potential \(W\) is defined by equation (B.2) and corresponds to the potential of the defect in the Schrödinger equation for an ‘electron’ in the periodic field of the crystal lattice and in the presence of the defect.

We will reduce the problem of the Schrödinger equation for a particle in the periodic potential \(V(x)\) related to the system of periodically placed stacks of alternating graphene and dielectric stripes embedded into a background dielectric medium and a ‘defect potential’ \(W\) related to a defect to a much simpler equation for the envelope wavefunctions. Taking into account the two-band structure and following the two-band model [38] we introduce a two-component spinor wavefunction \(\psi(x)\):

\[ \psi(x) = \begin{pmatrix} \varphi(x) \\ \chi(x) \end{pmatrix}, \]

(B.3)

where two different neighboring bands are described by the wavefunctions \(\varphi(x)\) and \(\chi(x)\). Note that equation (B.1) contains both the periodic function \(V(x)\) corresponding to the ideal lattice and \(W\) describing the potential of a defect. Without a defect the energy spectrum would be described by two neighboring bands and the gap between them. Applying the standard two-band approach, we obtain an effective Dirac type equation for the envelope spinor wavefunction, which implies the periodicity provided by \(V(x)\) [15, 38].

Note that equation (B.1) describes an electron in the periodic potential of an ideal crystal lattice \(V(x)\) and the potential of the defect \(W\). If the solution corresponding to the absence of impurity \(W = 0\) is known, the energy levels of the electron localized by the defect can be obtained by replacing equation (B.1) by a Dirac type equation according to the Luttinger–Kohn model described in [38–40]. This model implies a Dirac type equation for the two-component spinor wavefunction. According to [38], the functions \(\varphi_n(x)\) and \(\chi_n(x)\) defined as

\[ \varphi_n(x) = \sum_k c_n(k) \exp[ikx/h], \]

\[ \chi_n(x) = \sum_{k'} d_n(k') \exp[ik'x/h] \]

(B.4)

satisfy the set of the second order partial differential equations. Considering only two neighboring bands corresponding to the wavefunction \(\psi(x)\) given by equation (B.3) this set of equations for \(\varphi_n(x)\) and \(\chi_n(x)\) can be reduced to the Dirac type equations for the two-component spinor equation (B.3), which have the following form [38]:

\[ [\epsilon_\omega - \Delta_\omega - W_\gamma(|x - x_0|)] \varphi(x) + i\hbar \sigma_\epsilon \frac{d\varphi(x)}{dx} = 0, \]

\[ [\epsilon_\omega - \Delta_\omega - W_\gamma(|x - x_0|)] \chi(x) + i\hbar \sigma_\epsilon \frac{d\chi(x)}{dx} = 0, \]

where \(s = \hbar k_0/(\sqrt{3}m_0)\). In equation (B.5) \(\sigma_\epsilon\) is the Pauli matrix and \(\Delta_\omega\) is the width of the forbidden band in the electron spectrum that, as follows from the mapping of the wave equation for the electric field graphene-based photonic crystal onto equation (B.1), is \(\Delta_\omega = \frac{\hbar^2 \Omega^2}{2m_0 c^2}\), where \(\Omega\) is the...
width of the forbidden band (photonic gap) in the spectrum of the electromagnetic wave. Equations (B.5) are obtained in the limit $|\varepsilon_{\omega}^2 - \Delta_{\omega}^2|/(2\Delta_{\omega}^2) \ll 1$. Defining the effective mass of a quasiparticle as $m_{\text{eff}} = 3m_{\text{0}}^{2}\Delta_{\omega}/(\hbar^{2}\Delta_{\omega}^{2})$, and following the standard procedure of quantum electrodynamics [41, 42], we obtain from the system of Dirac type equations (B.5) the Klein–Gordon type equation. Note that the Klein–Gordon equation can be reduced to a Schrödinger-like equation with an effective energy and an effective potential. If the potential is weak enough to ignore the $W^{2}$ term, the relativistic formalism becomes equivalent to the non-relativistic formalism. More importantly, in situations where the Klein–Gordon equation is not exactly solvable, the Schrödinger form of the Klein–Gordon equation sheds some light on the problem as it could be reduced to a solvable eigenvalue problem:

$$-\hbar^{2}x^{2}\frac{d^{2}}{dx^{2}} - 2m_{\text{0}}\varepsilon_{\omega}(x - x_{0})\times \Psi(x) = (\varepsilon_{\omega}^{2} - \Delta_{\omega}^{2})\Psi(x). \quad \text{(B.6)}$$

This Klein–Gordon type equation has the form of the 2D Schrödinger equation for a particle in the square well potential with the eigenvalue

$$\varepsilon_{\omega} = \frac{\varepsilon_{\omega}^{2} - \Delta_{\omega}^{2}}{2m_{\text{0}}\varepsilon_{\omega}}.$$ 

The wavefunctions of the even stationary states have the forms ($\varepsilon_{\omega} < 0$):

$$\Psi(x) = A_{1}\cos\left[\sqrt{2m_{\text{0}}(W - \varepsilon_{\omega})/\hbar^{2}}x\right], \quad |x - x_{0}| \leq b, \quad \text{(B.7)}$$

$$\Psi(x) = B_{1}\exp\left[-\sqrt{2m_{\text{0}}\varepsilon_{\omega}/\hbar^{2}}x\right], \quad |x - x_{0}| > b.$$ 

The continuity of the wavefunction and its derivative in the points $x = x_{0} + b/2$ and $x = x_{0} - b/2$ result in the transcendental equation determining the spectrum of the even quantum states:

$$\sqrt{W - \varepsilon_{\omega}}\tan\left[\sqrt{2m_{\text{0}}(W - \varepsilon_{\omega})\hbar^{2}/\hbar^{2}}\right] = \sqrt{|\varepsilon_{\omega}|}. \quad \text{(B.8)}$$

The wavefunctions of the odd stationary states have the forms ($\varepsilon_{\omega} < 0$):

$$\Psi(x) = A_{2}\sin\left[\sqrt{2m_{\text{0}}(W - \varepsilon_{\omega})/\hbar^{2}}x\right], \quad |x - x_{0}| \leq b, \quad \text{(B.9)}$$

$$\Psi(x) = B_{2}\exp\left[-\sqrt{2m_{\text{0}}\varepsilon_{\omega}/\hbar^{2}}x\right], \quad |x - x_{0}| > b.$$ 

The continuity of the wavefunction and its derivative in the points $x = x_{0} + b/2$ and $x = x_{0} - b/2$ result in the transcendental equation determining the spectrum of the odd quantum states:

$$\sqrt{W - \varepsilon_{\omega}}\cot\left[\sqrt{2m_{\text{0}}(W - \varepsilon_{\omega})\hbar^{2}/\hbar^{2}}\right] = -\sqrt{|\varepsilon_{\omega}|}. \quad \text{(B.10)}$$

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