REGRET THEORY: A BOLD ALTERNATIVE TO THE ALTERNATIVES*

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In their famous 1982 paper in this Journal, Loomes and Sugden introduced regret theory. Now, more than 30 years later, the case for the historical importance of this contribution can be made.

Until the late 1970s, economists focused on the rational homo economicus, not only for normative but also for descriptive purposes. It was well understood that there were many empirical deviations from rationality, as signalled, for example, by preference reversals (Lichtenstein and Slovic, 1971; Lindman, 1971; Grether and Plott, 1979). But it was believed that irrational behaviour was too chaotic to be modelled and should just be taken as noise. For example, Arrow (1951, p. 406) wrote:

In view of the general tradition of economics, which tends to regard rational behavior as a first approximation to actual, I feel justified in lumping the two classes of theory [normative and descriptive] together.

The early 1980s saw a big shift, first in decision under risk, the topic of this article, and then in other fields including intertemporal choice, game theory and ambiguity (unknown probabilities). Kahneman and Tversky (1979) provided the first model of decision under risk that explicitly and deliberately deviated from the rational expected utility of homo economicus, but that could still be sufficiently tractable to permit economic modelling and predictions. Unfortunately, their model had some theoretical problems. It led their student Chew Soo Hong to co-author the unpublished Chew and MacCrimmon paper (1979), followed up by Chew (1983), with the first theoretically sound and axiomatised non-expected utility model. It also led John Quiggin (1982), then an unknown Australian student, to introduce his now famous rank-dependent utility. Machina (1982) gave a further boost to non-expected utility by providing constructive generalisations of optimality results. With the exception of Kahneman and Tversky, the aforementioned authors did not restrict their model to descriptive applications but also claimed a normative status of their models.

All the aforementioned generalisations maintained one of the most basic assumptions of economic optimisations: transitivity. Transitivity underlies the axioms of revealed preference for choices between multiple options. As good things often come

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in threes, so it happened in 1982 when three papers independently proposed a theory that gave up transitivity: regret theory. One paper, Fishburn (1982), focused on mathematical and axiomatic elaborations. The second paper, Bell (1982), focused on decision analytic applications, taking regret as an extra attribute of consequences. The third paper, Loomes and Sugden (1982; LS henceforth), the topic of this review, focused on conceptual features and interpretations and most clearly described the empirical and normative status of regret theory. The three papers reinforced each other, with cross-references and mutual recognitions from the beginning.

Good ideas usually do not appear out of the blue but grow from seeds planted before. While the linguistic and the psychological concept of regret have existed for ages and have been studied in psychology for over a century (see Zeelenberg and Pieters, 2007 and their references), formal roles in decision theory have appeared since the 1950s. LS cite Savage’s (1951) minimax regret theory and Fishburn (1988, p. 274) cites the bilinear mathematical functional of Kreweras (1961) as a predecessor. Yet, it was not until 1982 that a complete decision theory of regret became available.

1. Regret Theory and Expected Utility

The LS paper, reproduced in this issue, gives a careful exposition of regret theory and its full details, with motivations and discussions added. The high quality and depth of their presentation has made the paper a classic. Our presentation aims to be didactical, focusing on the simplest and most popular special case of regret theory and on the simplest implications. Although our notation and terminology is usually as close as possible to LS, in a few instances we deviate and use conventions that are common in the field today.

$S = \{s_1, \ldots, s_n\}$ denotes a state space, assumed finite for simplicity. Exactly one state $s_j$ is true but a decision-maker is uncertain which state that is. Throughout this article, we use an example of an urn containing 100 balls numbered 1–100. One ball is drawn randomly. The true state of nature is the number of the ball actually drawn and $S = \{1, \ldots, 100\}$, so that $n = 100$. Subsets of $S$ are events, which are true if they contain the true state of nature. Thus, the event odd is $\{1, 3, \ldots, 99\}$. Actions, with generic notation $A$, specify for each state $s$ what the consequence $A(s)$ (money amount) is if $s$ is true. In the example, a bet $A$ on event odd, yielding £2 if $s$ is odd and nothing otherwise but costing £1, would be the action $A$ such that $A(s_j) = 1$ whenever $s$ is odd and $A(s) = -1$ whenever $s$ is even. We assume that $S$ is endowed with a probability measure $P$, and write $p_j = P(s_j)$. In the example, every number has probability 1/100, and every event with $j$ states has probability $j/100$.

By $\succeq$ we denote the preference relation of the decision-maker over actions, with strict preference $\succ$, indifference $\sim$ and reversed preferences $\preceq$ and $\prec$ as usual. The most used model of decision under uncertainty is expected utility (EU). We then have

$$A_1 \succeq A_2 \iff \sum_{j=1}^{n} p_j C[A_1(s_j)] \geq \sum_{j=1}^{n} p_j C[A_2(s_j)]$$

(1)

1 Fishburn learned about this work in French from personal communication with the French economist Denis Bouyssou.
for all actions $A_1$, $A_2$. Here, $C$ denotes the utility function, which is subjective. The probabilities may be objective, as in the example, but in the absence of objective information they are subjective. We can rewrite (1) as

$$A_1 \succeq A_2 \Leftrightarrow \sum_{j=1}^n p_j \{ C[A_1(s_j)] - C[A_2(s_j)] \} \geq 0.$$  \hfill (2)

Table 1 illustrates a pair of actions, with M denoting million, $A_R$ designating a risky action and $A_S$ designating a safe action. In the Table, $A_R$ yields 5M for ball numbers 1–10, 1M for numbers 11–99 and 0 for number 100.

Although $A_R$ has the higher expected value, most people prefer $A_S$ because of its safety, with no risk of ending up with 0. The regret of having missed a sure £1M if ball 100 is drawn is unbearable to many people. The preference for $A_S$ can be accommodated by Bernoulli’s (1738) EU. Scaling $C(0) = 0$, substitution readily shows that the preference $A_S > A_R$ then holds if and only if

$$C(1M)/C(5M) > 10/11,$$  \hfill (3)

reflecting diminishing marginal utility.

We now consider a general choice situation between two actions $A_1$ and $A_2$. Regret theory generalises expected utility by assuming that the utility $C[A_1(s_j)]$ experienced under $A_1$ is affected by what would have happened had $A_2$ been chosen instead of $A_1$, and vice versa. People feel regret about $A_1(s_j)$ if the result of the alternative choice, $A_2(s_j)$, had been better. Because of this regret, under choice $A_S$ in Table 1, people may feel less happy if ball 1–10 is drawn than if ball 11–99 is drawn, even though the same consequence, 1M, results in all these cases. If ball 1–10 is drawn, then winning £5M has been forgone due to an own decision, which arouses regret and reduces happiness relative to balls 11–99.

The other side of the coin of regret is rejoicing, felt if the most favourable consequence under some state $s_j$ has resulted. After a choice $A_S$, people will rejoice if ball 100 is selected and, for the preference assumed in Table 2, this rejoicing is enough to prefer $A_S$ despite the regret felt for balls 1–10.

Regret theory holds if, for general actions $A_1$ and $A_2$, we have

$$A_1 \succeq A_2 \Leftrightarrow \sum_{j=1}^n p_j Q[C[A_1(s_j)] - C[A_2(s_j)]] \geq 0.$$  \hfill (4)

The strictly increasing function $Q$ captures the utility difference, but also the regret and rejoicing experienced at $A_1(s_j)$ and $A_2(s_j)$. Rejoicing being the other side of the regret-coin is captured by setting $Q(-x) = -Q(x)$. This equality ensures consistency of
When interchanging \( A_1 \) and \( A_2 \). The equality implies the obvious \( Q(0) = 0 \). If \( Q \) is linear, then (4) does not offer any generalisation relative to EU (2), and utility \( C \) captures all that is relevant to decisions. New behavioural implications and a new decision theory result if \( Q \) is non-linear.

Besides (4), LS also consider more general representations

\[
A_1 \succeq A_2 \iff \sum_{j=1}^{n} pjQ[A_1(s_j), A_2(s_j)] \geq 0. \tag{5}
\]

In (5), \( Q \) can depend on the pair of outcomes more generally than through their utility difference. We, however, focus on the tractable (4), the most popular special case used in the literature.

Before turning to the novelty of regret theory, we first discuss an important implication of EU that is preserved under regret theory: Savage’s (1954) sure-thing principle (see Table 2). Table 2 resulted from Table 1 by replacing the common outcome 1M, resulting under balls 11–99, by another common outcome, 0. Both EU and regret theory require that preference is not affected by such a change in common outcome. This condition, now known as Savage’s (1954) sure-thing principle, is implied by (4) as follows:

Proof. In (4), consider the substitution \( A_1 = A_R \) and \( A_2 = A_S \) (Table 1) and the alternative substitution \( A_1 = A_r \) and \( A_2 = A_S \) (Table 2). For both substitutions, the terms for \( j = 11, \ldots, 99 \) cancel in the summation in (4) because they contribute 0 to the summation. After removing these 89 zero-terms, the summation in (4) is the same under both substitutions. Hence, we have the same inequality and the same preference for both substitutions, and thus for Tables 1 and 2.

The same implication and proof hold for EU, which is the special case where \( Q \) is the identity, and also for the more general (5). Under the following psychologically plausible scenario, the change in common outcome indeed does not affect choice. From Tables 1 and 2, subjects notice that their choice does not matter if ball 11–99 is drawn, because it leads to the same (common) outcome. They then decide to ignore these balls, after which they face the same (conditional or ‘isolated’) choice in the two cases. Focusing on balls 1–10 and 100, they perceive \( A_S \) as a pseudo-certain £1M (Kahneman and Tversky, 1979), making them prefer \( A_S \) as they preferred \( A_S \) in Table 1.

The displays and juxtapositions in Tables 1 and 2 enhance the aforementioned plausible scenario, and empirical studies have confirmed such isolation (Kahneman 2015 The Authors.
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and Tversky, 1979; see also LS p. 812). Other scenarios, violating EU but not regret theory, can be triggered in other setups, which we discuss in the next Section.

2. Regret Theory’s Deviations from Expected Utility

2.1. A Deviation Illustrating the Regret Functional

This subsection discusses a theoretical deviation of regret theory from EU to illustrate the nature of the regret functional. The next subsections present empirical implications. Imagine that \( x_0, \ldots, x_4 \) is an increasing sequence of outcomes that is equally spaced in \( C \) units. That is,

\begin{align*}
C(x_4) - C(x_3) = \cdots = C(x_1) - C(x_0) > 0. 
\end{align*}

We denote these utility differences by \( \delta \). Consider the two actions in Table 3.

Under expected utility, the two actions are equivalent because the \( C \) difference for balls 51–75 is twice as big as the \( C \) difference for balls 1–50 but it has half the probability. However, many decision-makers may prefer the lower action \( A_\ell \). They regret the small utility loss (\( x_3 \) instead of \( x_4 \)) after choosing \( A_\ell \) (balls 1–50) much less than the double and more salient utility loss (\( x_0 \) instead of \( x_2 \)) after choosing \( A_u \) (balls 51–75). This is captured by

\begin{equation}
2Q\left[ C(x_4) - C(x_3) \right] < Q\left[ C(x_2) - C(x_0) \right].
\end{equation}

This condition is satisfied by functions \( Q \) that are convex on \( \mathbb{R}^+ \) (and, hence, concave on \( \mathbb{R}^- \)).

The reversed preference \( A_u > A_\ell \) can also be accommodated by regret theory. Some decision-makers may prefer \( A_u \) because the probability of regret is only small (0.25 for balls 51–75), whereas the probability of regret is higher for \( A_\ell \) (0.5 for balls 1–50). Such decision-makers do not discriminate much between utility losses \( \delta \) and \( 2\delta \) and for them the inequality in (7) is reversed. The most common case, however, is (7). It was recently confirmed empirically by Bleichrodt et al. (2010) and it is mostly assumed by LS (end of their Section II). Then extreme utility differences are salient and are overweighted.

We now turn to some empirically important deviations from EU.

### Table 3

| Action | 1  | 25 | 26 | 50 | 51 | 75 | 76 | 100 |
|--------|----|----|----|----|----|----|----|-----|
| \( A_u \) | \( x_4 \) | \( x_4 \) | \( x_0 \) | 0  |
| \( A_\ell \) | \( x_3 \) | \( x_4 \) | \( x_2 \) | 0  |

2 Loomes and Sugden (1998), in yet another critical test of their theory, still found violations here, providing evidence against their theory. Birnbaum (2008, p. 481 ff.) also reports some violations.

3 Bleichrodt et al. (2010) demonstrated that these equalities can be revealed from preferences as follows. Using obvious notation, we measure indifferences (odd: \( x_{2j+1}, \) even: \( g \) ~ (odd: \( x_j, \) even: \( G \)) for \( j = 0, \ldots, 3 \), and outcomes \( G > g \) conveniently chosen. Equation (4) then implies

\begin{equation}
Q[C(G) - C(g)] = Q[C(x_j + 1) - C(x_j)] \text{ for all } j.
\end{equation}

Because \( Q \) is strictly increasing, (6) follows.

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2.2. Violating the Equivalence Axiom

See Table 4 (LS 6), where the subscript \(d\) in \(A_d\) refers to decreasing outcomes, whereas this is not the case for \(A_n\).

Under the common assumption that large utility differences are overweighted, the superiority of \(A_n\) for balls 76–100 decides and \(A_n\) is preferred. Convexity of \(Q\) for gains implies this preference under regret theory:

\[
0.25 \times Q[C(30) - C(20)] + 0.25 \times Q[C(20) - C(10)] + 0.25 \times Q[C(10) - C(0)] < 0.25 \times Q[C(30) - C(0)].
\]

However, \(A_i\) and \(A_n\) induce the same probability distribution over outcomes! Apparently, such actions need not be equivalent under regret theory. Under EU, to the contrary, they must be equivalent, which LS (p. 818) calls the equivalence axiom. This requirement appears most clearly from (1). Then the correlation of the two actions, and the particular matching of their outcomes, is immaterial. It does not matter if \(A_n\) resulted from another independent drawing from the urn. By contrast, the matching of outcomes is crucial for regret theory, as is shown in (4). As surprising as this implication may be, it is a natural consequence if we experience regret. This point will be further discussed in the next subsection.

2.3. Accommodating the Allais Paradox, and a Comparison with Other Non-expected Utility Theories

Papers on non-expected utility of the 1980s usually started with a description of the Allais paradox and then showed how a newly introduced model could accommodate it. We now show how regret theory can accommodate this paradox. Consider a variation in Table 2, called the independent variation, where the lower action \(A_S\) is generated by a second, independent, drawing from the urn. Under EU’s equivalence axiom, this change should not affect preference. However, under regret theory it may matter, because the matching of the outcomes changes and, hence, regret effects will change. We use a simple \(Q\) function to illustrate the basic idea. Imagine that the decision-maker feels no strong regret for utility losses up to \(C(\£0) - C(\£0)\) and \(C(\£1M) - C(\£5M)\), and \(Q\) is close to linear for such and smaller losses. However, larger losses such as \(C(\£0) - C(\£5M)\) exceed a tolerance threshold and result in strong regret. Choosing \(A_S\) risks experiencing such strong regret because, given the independence of the two actions and unlike the original choice situation in Table 2, outcome 5M for \(A_i\) and outcome 0 for \(A_S\) can occur simultaneously (with probability 0.10 \(\times\) 0.89 = 0.089). If the regret \(Q[C(\£0) - C(\£5M)]\) is strong enough, then \(A_i\) will be preferred.
The phenomenon just discussed is realistic. Experiments have shown that the exact presentation of actions matters, with Table 2 generating a pseudo-effect that disappears in the independent variation. Thus, there are more violations of the sure-thing principle in the independent variation (MacCrimmon, 1968; Moskowitz, 1974; Kahneman and Tversky, 1979; Starmer, 1992; Wu, 1994). The most detailed evidence is provided by Michael Birnbaum, whose branch independence concerns the test of the sure-thing principle controlling for regret in Table 2. A review of his work on this point is in Birnbaum (2008, p. 481 ff.).

Most recent experiments, aiming to investigate violations of the sure-thing principle, tested choices as in Tables 1 and 2 (with moderate pay-offs) but presented only the generated probability distributions to subjects, without specifying underlying states or joint distributions. Instead of the choice in Table 2, subjects then choose between two probability distributions (0.10: 5M, 0.90: 0) and (0.11: 1M, 0.,89: 0), using an obvious notation. The majority preferences in Table 1 remain as indicated in that Table, with subjects still preferring certainty. But the preference in Table 2 is reversed, with the majority preference for \( A_r \), as in the aforementioned independent variation, and violating EU.\(^4\) LS put forward the plausible assumption that subjects take the probability distributions as independent if no joint distribution is specified. Then the analysis of our independent variation applies (LS Section III, 1st para.) and regret theory can accommodate the obtained violation of EU.

The violation of EU just discussed is known as the common consequence version of the Allais paradox (Allais, 1953). Allais’ paradoxes spurred the non-expected utility models of the 1980s. These works, with regret theory as a prominent member, have led to what is called behavioural economics today. Most of the non-expected utility models abandon Savage’s (1954) sure-thing principle jointly with its cousin under risk, von Neumann-Morgenstern’s preferential independence. They allow for interactions between probabilities (beliefs) and utilities (tastes) that were excluded by expected utility. Although such interactions are interesting and can explain many phenomena, LS decided not to incorporate them in regret theory and instead explored another and bolder deviation. Regret theory allows for interactions between outcomes of different actions under the same state, which is excluded not only by expected utility but also by most other non-expected utility models. Dual interactions, between different outcomes of the same action under different states can similarly be considered. This is the topic of disappointment theory (Bell, 1985; Loomes and Sugden, 1987a; Delquié and Gillo, 2006; Laciana and Weber, 2008). Although the distinction between regret and disappointment may sometimes be vague in natural language, decision theory strictly distinguishes between them.

A very general theory, accommodating virtually all empirical findings, results if we allow for all aforementioned interactions simultaneously. However, such a theory would be overly general, would become intractable and would not give useful implications or predictions. Hence, LS decided to include just one new interaction in regret theory, demonstrating that this already gives surprisingly many new and valuable

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\(^4\) This follows because the equivalence axiom is violated. For an alternative derivation, the preferences just assumed imply \( C(1\text{M})/C(5\text{M}) < 10/11 \) under EU, contradicting (3), so that EU cannot hold.

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insights (see their Table 1) while maintaining tractability. Cubitt and Sugden (1998, p. 761) argue that giving up transitivity is like giving up separability but in another direction than when giving up the sure-thing principle. Although accommodating the Allais paradox was not LS’s main goal, they could still do so.

2.4. Violating Transitivity

We now turn to the main goal of LS, incorporating the boldest and most controversial deviation from classical models available in the literature. Consider Table 5, an extension of Table 4 (combining LS’s Tables 4 and 6), which is obtained by further shifts of outcomes. Each preference in the Table follows from the same line of reasoning as used in Table 4. The largest regret of 0 vs. £30M each time overrules the multiple smaller regrets in the other direction. A preference cycle results and transitivity is violated. LS thus challenged one of the most standard assumptions of economic optimisations. They provided detailed arguments against transitivity (pp. 820–22), extended in later papers (Sugden, 1991, pp. 760–61).

Loomes and Sugden (1987b, beginning of Section 4) and Sugden (2004, Section II.7) showed that regret theory deviates from expected utility and can bring new phenomena only where it deviates from transitivity. Luce and Raiffa (1957, pp. 280–82) explained a similar point for earlier forms of regret. Hence, the violations of transitivity are central to regret theory. A generalisation is in Bikhchandani and Segal (2011, Theorem 1).

3. Empirical Support for Regret Theory

Regret theory received much support during the first decade after its introduction. Most empirical studies, several by Loomes and Sugden in collaboration with Chris Starmer, confirmed the predictions of the theory.

Example 1. Loomes (1988a) tested the juxtaposition effects described in the preceding Section by asking subjects to state the money amount £a0 for which they were indifferent between the two actions in Table 6.
Next in a second problem, subjects were asked to state the money amount £\(a_1\) for which they were indifferent between the two actions in Table 7.

Any theory based on the equivalence axiom predicts that \(a_0 = a_1\). Regret theory makes a different prediction. The proof of the following claim is in the Appendix.

**Claim 1.** Under regret theory with \(Q\) convex for gains, \(a_1 > a_0\).

Loomes (1988a) indeed found that the average value of \(a_1\) was much larger than the average value of \(a_0\) (£22.58 versus £17.52), confirming the prediction of regret theory.

Other studies on juxtaposition effects that supported regret theory include Loomes and Sugden (1987a), Loomes (1988b, 1989), Starmer and Sugden (1989) and Starmer (1992). Moreover, Loomes et al. (1992) confirmed violations of stochastic dominance predicted by regret theory.

A particularly desirable feature of regret theory is that it can explain preference reversals (PR). PRs were first discovered by Lichtenstein and Slovic (1971) and Lindman (1971), and were brought to the attention of economists by Grether and Plott (1979). PRs occur when subjects are confronted with two prospects, a £-bet which offers a relatively large sum of money, but a relatively small probability of winning, and a P-bet, which offers a more modest sum of money, but a greater probability of winning. Subjects are then asked to perform three tasks: to choose between the two prospects, and to attach a certainty equivalent to each prospect. The typical finding is that subjects prefer the P-bet, while paradoxically, the £-bet is given the higher valuation. The opposite pattern, choosing the £-bet but valuing the P-bet higher, is rarely observed.

Preference reversals challenge those who wish to explain economic behaviour in terms of rational theories of choice. Psychologists often interpreted PRs as evidence that individuals do not have a single system of preferences and respond differently to choice and valuation tasks (Slovic and Lichtenstein, 1983; Tversky et al., 1988, 1990). Regret theory provides a different interpretation based on intransitive preferences as
explained in the next example. Unlike any other existing theory, regret theory not only explains PRs but can even rationalise them.

Example 2. Consider the three prospects in Table 8.\textsuperscript{5} The typical PR pattern is P-bet $\succ \ell$-bet $\succ c \succ$ P-bet for some sum of money $c$. This pattern can be explained by the extremity overweighting of $Q$ in regret theory. For example, take $C(x) = x^{0.8}$ and $Q(x) = x^{1.5}$ for $x \geq 0$, and $C(-x) = -C(x)$ and $Q(-x) = -Q(x)$. Then for $c = 4$ regret theory accommodates the typical cycle, as calculations can show. The proof of the following claim is in the Appendix.

Claim 2. Regret theory excludes opposite cycles.

Loomes \textit{et al.} (1989, 1992) and Loomes and Taylor (1992) found that the cycles predicted by regret theory were indeed much more common than the opposite cycles. They controlled for the psychological explanation that preference reversals are the result of differences in information processing between choice and valuation, and concluded that preference reversals were caused by intransitive preferences as predicted by regret theory.

If we reverse the signs of all sums of money in Table 8, turning gains into losses, then regret theory with concave $Q$ for losses is consistent with the cycle $\ell$-bet $\succ$ P-bet $\succ c \succ$ $\ell$-bet but not with the opposite cycle. Loomes and Taylor (1992) tested this prediction and concluded that their data again showed many more regret cycles than opposite cycles.

4. Challenges for Regret Theory

In the 1990s, some studies challenged the predictions of regret theory. Battalio \textit{et al.} (1990) and Harless (1992) found that while regret effects occur using matrix presentations when states yielding the same consequence are collapsed, these regret effects become weaker for non-collapsed presentations. They also become weaker when problems are presented verbally or when other displays are used. Harless (1992, p. 647) suggested that regret effects are primarily framing effects that ‘occur only when the decision is framed in a way that sharply directs the decision maker to compare acts and states’.

\footnote{5 This is one of the problems considered in Loomes \textit{et al.} (1992).}

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Another challenge to regret theory came from other studies, starting with Tversky (1969), that observed systematic cycles that could not be explained by regret theory. Consider the three actions in Table 9, which is problem T’U’V’ in Day and Loomes (2010). Regret theory is consistent with the cycle $C \succ B \succ A \succ C$ but not with the opposite cycle. However, Day and Loomes (2010) observed that this opposite cycle prevailed. Their finding can be explained by Rubinstein’s (1988) similarity theory.

When comparing $A$ and $B$, people may consider that the 5% extra probability that $B$ offers is so small that they pay little or no attention to the probability dimension and instead concentrate on the dissimilar pay-off dimension and choose $A$. Likewise, they consider the winning probabilities of $B$ and $C$ to be similar and choose $B$. However, they may also find that the 10% difference in winning probability between $A$ and $C$ is large enough to make $A$ and $C$ look dissimilar on the probability dimension and this may shift their attention back to the probability dimension and they then choose $C$.

Lindman and Lyons (1978), Budescu and Weiss (1987), Leland (1994, 1998), Mellers and Biagini (1994), Bateman et al. (2007) and Day and Loomes (2010) reported evidence for such similarity cycles, which cannot be explained by regret theory. Starmer (1999) reported a comparable cycle although he did not explain it by similarity but by original prospect theory (Kahneman and Tversky, 1979).

Yet another challenge came from mathematical psychologists. Starting with Iverson and Falmagne (1985), several papers showed that asymmetric cycles need not necessarily be inconsistent with transitive preferences if the stochastic nature of human preferences is taken into account (overviewed by Regenwetter et al., 2011). Even though they mainly concentrated on the similarity cycles observed by Tversky (1969) and showed that these could be explained by transitive preferences with error, their objections also applied to the regret cycles that were observed.

However, a serious blow to regret theory came from Starmer and Sugden (1993). They discovered that previously observed support for regret theory could, to a large extent, be explained by event-splitting effects by which splitting an event with a given consequence into two sub-events increases its weight.

**Example 3.** Consider the four problems in Tables 10–13. According to regret theory, Problems I and III are equivalent and so are Problems II and IV. Regret theory predicts that choices $AB'$ (A in Problems I and III and $B'$ in Problems II and IV) will occur more often than choices $BA'$. However, according to event splitting, choices $AB'$ should be more likely than choices $BA'$ in Problems I and II, suggesting regret effects.
but not in Problems III and IV: in Problem I the £7 is not split, whereas in Problem II it is, which may make $B_0$ appear more attractive, but both in Problems III and in Problem IV the £7 is split.

The prediction of event splitting was, indeed, what Starmer and Sugden (1993) observed: clear regret effects in Problems I and II but no effects in Problems III and IV. Their study suggested strong event-splitting effects and weaker regret effects (see also Humphrey, 1995). On the other hand, Starmer and Sugden (1998) found that not all regret effects were due to event-splitting effects but that some were mainly due to framing, as had been suggested before by Harless (1992).
Starmer and Sugden subjected ‘their’ regret theory to rigorous testing and thereby discovered the remarkable fact that splitting states can make prospects substantially more attractive. Camerer (1995, pp. 655–56) praised the authors’ work on regret theory, and the resulting progress of our understanding, and wrote ‘this is a story of successful detective work’.

5. Recent Applications

Even though event-splitting effects may provide an alternative explanation for some of the phenomena that led to the introduction of regret theory, as Starmer (2000, p. 376) notes ‘insights from [regret theory] have proved useful in understanding real behaviour’. The authors of this article benefited from regret theory’s insight that pairs of outcomes for different actions provide a natural basis for decision-making and used this idea in trade-off techniques (Bleichrodt et al., 2010; Wakker, 2010). This insight was also used by Bouyssou and Pirlot (2003, especially table 1) and Vind (2003). The 2000s have witnessed many applications that either use regret theory or extensions of the model, with LS cited as a source of inspiration. For example, Barberis et al. (2006) use regret theory to explain the stock market participation puzzle: few people invest in stocks even though rational economic theory predicts that they should. Other applications of regret models to financial decisions include Muermann et al. (2006) and Michenaud and Solnik (2008) who study asset allocation decisions.

Braun and Muermann (2004) apply regret theory to the demand for insurance and show that regret theory can explain the frequently observed preference for low deductibles. Smith (1996) applies regret theory to health and Perakis and Roels (2008) use it in the newsvendor model. Filiz-Ozbay and Ozbay (2007) and Engelbrecht-Wiggans and Katok (2008) explain how regret theory can explain overbidding in first price auctions. Other regret models include Sarver (2008) and Hayashi (2008). These models differ from LS in that they study preferences over menus, i.e. sets of prospects, in which decision-makers experience regret if their choice turns out to be inferior ex post.

A critical aspect of regret is the extent to which decision-makers, after their choices, are informed about the outcomes that would have resulted had they chosen differently. This issue has been explored in the experimental and theoretical literature on feedback-conditional regret. It has been found that people prefer options which screen them from discovering the outcome of forgone choices. The anticipated pain of regret is reduced or eliminated if people do not know the outcome of the forgone choice. Thus, the option of not entering a lottery is more attractive if, conditional on not entering, one will never know whether one would have won or lost. This tendency is exploited in postal code lotteries in which a postal code rather than an anonymous number is drawn (Zeelenberg, 1999; Humphrey, 2004).

Regret theory has been widely applied in the health domain, raising fundamental ethical questions. Should doctors be allowed to use excessive diagnostic testing just to avoid the regret about missing the occasional serious case, just because they

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6 As was done in many papers by Loomes and Sugden, including Loomes and Sugden (1998).
overweight omission relative to commission (Ritov and Baron, 1995)? Should one-sided legal liability be imposed on doctors to induce such regret and overweighting externally, at the cost of societal efficiency? Should tests for Down syndrome and vaccinations that demonstrably reduce the mortality rate be provided to the general public even though they may lead to lifelong emotions of regret that would not have occurred otherwise (Ritov and Baron, 1990; Murray and Beattie, 2001)? Or in another domain, should seeding hurricanes be forbidden if it leads to regret with some parties affected, even though total damage is reduced (Howard et al., 1972)?

The 2000s has seen the emergence of neuroeconomics which has led to new insights into regret. Camille et al. (2004) find that the orbitofrontal cortex has a fundamental role in mediating regret and that people with lesions in the orbitofrontal cortex who do not experience regret make worse decisions than normal subjects who do anticipate regret. Giorgetta et al. (2013) found different neural localisations for regret and disappointment.

Bleichrodt et al. (2010) developed methods to obtain precise quantitative measurements of the parameters of regret theory. These measurements allow us to derive exact predictions, for example, about how much more supply is needed next year if regret is increased by advertisement campaigns. To illustrate another application, in Example 2 we showed that there are values of $C$ and $Q$ for which regret theory predicts preference reversals. By measuring these values individually, we can predict exactly when preference reversals will occur for each subject and we can then test whether they actually do (Baillon et al., 2014).

Whereas regret theory accommodates intransitivities by allowing state-wise comparisons of consequences, it maintains the classical linear weighting of probabilities. Two recent approaches relax the latter assumption. Loomes’s (2010) new model, the perceived relative argument model, is a rich model defined for the probability triangle and uses paired comparisons of consequences like regret theory, but it also uses similar paired comparisons of probabilities. It can explain many empirical regularities including the aforementioned similarity cycles that are inconsistent with regret theory.

Bordalo et al.’s (2012) salience theory uses pairwise comparisons of consequences to readjust the weights (salience), rather than utility differences, of states of nature. As did LS, salience theory assumes that large differences are overweighted but it does not use an extremity overweighting function $Q$ for differences of utilities to model this. Instead, it assumes that the salience function overweights the states of nature that have large utility differences for the actions under consideration. Salience theory shares the implication of the sure-thing principle with LS, with the salience of state $s$ not affected by consequences outside $s$. As with regret theory, the novelty of salience theory resides in where it violates transitivity. Unlike LS, Bordalo et al. (2012) did not analyse or discuss intransitivities extensively, but left this to future work.

7 People with lesions in the orbitofrontal cortex are not emotionally unresponsive as they did experience disappointment.

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LS were not only bold in taking issue with some of the most widely accepted assumptions in decision theory, transitivity and the equivalence axiom but also in their interpretations, showing insights ahead of their time. When their paper was published in the early 1980s, a strict ordinal revealed preference view was dominant in economics. Utility modelled decisions and nothing else. Introspective interpretations were not made. The situation has changed today, with Kahneman (1994), Loewenstein and Ubel (2008, Section 2) and others pleading for broader interpretations of utility, and with introspective happiness studies, a popular and influential field in economics (van Praag and Ferrer-i-Carbonell, 2004; Diener and Biswas-Diener, 2008; Benjamin et al., 2014). Although LS could have avoided introspection because, as they show in their Appendix, all components in their model can be revealed from preferences, they chose a psychologically sound interpretation of their model: They interpreted the function $C$ in Section 3 as the inherent utility, resulting when the individual experiences a consequence \textit{without having chosen it} (emphasis in original). Then regret or rejoicing plays no role. $C$ can be felt through introspection. Hence, LS used the term choiceless utility for $C$. Next, in a second stage, regret comes in, captured through the function $Q$. LS (Section V, 2nd para.) explicitly distanced themselves from a narrow empirical approach to preference theory.

In 1982, the prevailing hypothesis of prospect theory was a total reflection of preference, with risk aversion for gains coupled with equally strong risk seeking for losses. LS (Section III end) immediately predicted weaker, only partial, reflection with risk seeking for losses weaker than risk aversion for gains. Their prediction has since been confirmed empirically (surveyed by Wakker, 2010 Section 9.5). LS also carefully presented evidence against regret theory (Section V, middle) and recommend reference-dependent generalisations.

A limitation of regret theory, as of any intransitive theory of binary choice, is that it is unclear how to extend the theory to choices among three or more actions. LS (Section IV) provided the first ideas about such extensions, with defences against book making and money pump criticisms in Section V; Loomes and Sugden (1987b) provided an elaborated theory. A preference foundation is in Sugden (1993). Hayashi (2008) suggested an alternative extension.

Although LS are firm on a normative status of regret theory and provide strong and cogent arguments, the authors of this comment have different views. LS argue that feelings of regret are a fact of life and that it is irrational to ignore them, a view supported by Bourgeois-Gironde (2010) using neurodata. We are less tolerant and more paternalistic about such feelings. In its everyday meaning, regret is a useful emotion to signal possible improvements of future actions in situations of incomplete information. The formal decision-theoretic meaning, however, is different. Consider Table 2, with a choice of $A_r$. A rational person should maximise happiness, given the external constraints. The latter are the same if ball 11 is drawn as if ball 100 is drawn, in both cases the consequence being £0. Having feelings of regret for ball 100 because of the forgone £1M leads to harm for no good reason. We believe that such voluntary self-harming is irrational.
Note that, unlike in everyday life situations where regret can be a useful signal, nothing can be learned from the ball drawn in Table 7, given that all probabilities and consequences were known beforehand. We also assume complete modelling and, hence, for instance, we assume that there are no outsiders blaming the decision-maker after ball 100 was drawn. Taking any emotion as rational just because it exists is too permissive and applies Hume’s adage ‘reason is, and ought only to be the slave of the passions’ too leniently. Although we see no normative status for regret theory, it is obvious that its descriptive value is huge, making it one of the most important contributions to decision theory. LS’s careful arguments for the rationality of regret theory, challenging something as basic as transitivity, are thought provoking and have also produced many new insights.

7. Conclusion

In our perception, salient features of Sugden’s work during the last three decades have been great originality and breadth, and salient features of Loomes’ work have been great sharpness and depth. In retrospect, it is then no surprise that when these two strong and complementary minds came together in 1982, something lasting resulted.

Appendix A. Proofs

Proof of claim 1. Informally, writing a for both a0 and a1, in Table 7, 40 differences (£a versus £0) and 40 differences (£0 versus £12) of Table 6 have been replaced by 40 differences (£a versus £12) and 40 differences (£0 versus £0). By the extremity over weighting of Q, the removals of (£a versus £0) count most, weakening the case for the upper prospect. Hence, a larger value a1 is needed in Table 7.

Formally, according to regret theory the first indifference implies

$$0.40 \times Q[C(a_0) - C(0)] = 0.60 \times Q[C(12) - C(0)].$$

(A.1)

The second indifference implies

$$0.40 \times Q[C(a_1) - C(12)] = 0.20 \times Q[C(12) - C(0)].$$

(A.2)

By Q’s extremity over weighting, $Q[C(a_0) - C(0)] > Q[C(a_0) - C(12)] + Q[C(12) - C(0)]$. Hence $0.40 \times Q[C(a_0) - C(12)] < 0.40 \times Q[C(a_0) - C(0)] - 0.40 \times Q[C(12) - C(0)] = (by$$(A.1)) 0.60 \times Q[C(12) - C(0)] - 0.40 Q[C(12) - C(0)] = 0.20 \times Q[C(12) - C(0)].$ Because Q is strictly increasing, it follows that to obtain the equality in (A.2) we must have $a_1 > a_0$.

Proof of claim 2. For contradiction, assume the opposite cycle P-bet < £-bet < $< P-bet. Then

$$0.30Q[C(18) - C(8)] + 0.30Q[-C(8)] > 0,$$

(A.3)

$$0.30Q[C(8) - C(c)] + 0.30Q[C(8) - C(c)] + 0.40Q[-C(c)] > 0,$$

(A.4)
0.30Q[C(c) − C(18)] + 0.30Q[C(c)] + 0.40Q[C(c)] > 0. \quad (A.5)

Adding the left-hand sides of (A.3)–(A.5) and using $Q(x) = −Q(−x)$ for all $x > 0$ gives

$$
0.30[Q[C(18) − C(8)] + Q[C(8) − C(c)] − Q[C(18) − C(c)]] + 0.30[−Q[C(8)] + Q[C(8) − C(c)] + Q[C(c)]] > 0.
$$ \quad (A.6)

Because $Q$ overweights extremes, the terms in square brackets are negative and we have a contradiction.

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Appendix B. Loomes, G. and Sugden, R. (1982). ‘Regret theory: an alternative theory of rational choice under uncertainty’, Economic Journal, vol. 92(368), pp. 805–24.
The main body of current economic analysis of choice under uncertainty is built upon a small number of basic axioms, formulated in slightly different ways by von Neumann and Morgenstern (1947), Savage (1954) and others. These axioms are widely believed to represent the essence of rational behaviour under uncertainty. However, it is well known that many people behave in ways that systematically violate these axioms.

We shall initially focus upon a paper by Kahneman and Tversky (1979) which presents extensive evidence of such behaviour. Kahneman and Tversky offer a theory, which they call ‘prospect theory’, to explain their observations. We shall offer an alternative theory which is much simpler than prospect theory and which, we believe, has greater appeal to intuition.

The following notation will be used throughout. The ith prospect is written as $X_i$. If it offers increments or decrements of wealth $x_1, \ldots, x_n$ with probabilities $p_1, \ldots, p_n$ (where $p_1 + \ldots + p_n = 1$) it may be denoted as $(x_i, p_1; \ldots; x_n, p_n)$. Null consequences are omitted so that the prospect $(x, p; 0, 1 - p)$ is written simply as $(x, p)$. Complex prospects, i.e. those which offer other prospects as consequences, may be denoted as $(X_i, p_1; \ldots; X_n, p_n)$. We shall use the conventional notation $\succ$, $\succeq$ and $\sim$ to represent the relations of strict preference, weak preference and indifference. We shall take it that for all prospects $X_i$ and $X_k$, $X_i \succ X_k$ or $X_i \preceq X_k$; but we shall not in general require that the relation $\succ$ is transitive.

I. KAHNEMAN AND TVERSKY’S EVIDENCE

Kahneman and Tversky’s experiments offered hypothetical choices between pairs of prospects to groups of university faculty and students. Table I lists a selection of their results, which reveal three main types of violation of conventional expected utility theory:

(a) The ‘certainty effect’ or ‘common ratio effect’, e.g. the conjunction of $X_5 < X_6$ and $X_9 > X_{10}$ and the conjunction $X_{13} < X_{14}$ and $X_{15} > X_{16}$. There is also a ‘reverse common ratio effect’, e.g. the conjunction of $X_7 > X_8$ and $X_{11} < X_{12}$.

(b) The original ‘Allais Paradox’ or ‘common consequences effect’, e.g. the conjunction of $X_1 < X_2$ and $X_3 > X_4$.

(c) The ‘isolation effect’ in two-stage gambles, e.g. the conjunction of $X_9 > X_{10}$ and $X_{17} < X_{18}$.

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1 For a survey and discussion of much of the evidence, see Allais and Hagen (1979) and Schoemaker (1980, 1982).

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Table 1 also reveals a ‘reflection effect’ where a change of sign on the consequences is associated with a reversal of the modal preference and the risk attitude that characterises it, e.g. $X_5 < X_6$ and $X_7 > X_8$. One instance of the reflection effect, revealed in Problems 14 and 14', may be interpreted as an example of simultaneous gambling and insurance, since $X_{19} > X_{20}$ indicates a willingness to enter an actuarially fair lottery offering a small probability of a large prize, while $X_{21} < X_{22}$ signifies a willingness to take out actuarially fair insurance against a small probability of a large loss. We also note an interesting mixture of risk attitudes. Sometimes risk aversion is associated with problems involving increments of wealth, e.g. $X_{13} < X_{14}$, and sometimes with problems involving decrements, e.g. $X_{21} < X_{22}$. Likewise, risk loving is sometimes associated with problems involving increments, e.g. $X_{15} > X_{16}$, and sometimes with problems involving decrements, e.g. $X_7 > X_8$.

Simultaneous gambling and insurance, the reflection effect, and the mixture of risk attitudes may all be accommodated by conventional expected utility theory, though only at the cost of certain fairly arbitrary assumptions and some rather unsatisfactory implications. But no accommodation is possible for the effects listed in (a), (b) and (c) above - the observations here simply violate one or more of the conventional axioms.

1 See Friedman and Savage (1948), Markowitz (1952) and Hirschleifer (1966).
However, in the next section we shall outline the framework of an alternative theory which not only explains the reflection effect and simultaneous gambling and insurance, but also predicts the behaviour described in (a), (b) and (c). We shall then argue that, besides being predictable, such behaviour can be defended as rational, and that our model therefore provides the basis for an alternative theory of rational choice under uncertainty.

II. THE FRAMEWORK OF AN ALTERNATIVE THEORY

We consider an individual in a situation where there is a finite number, \( n \), of alternative states of the world, any one of which might occur. Each state \( j \) has a probability \( p_j \) where \( 0 < p_j \leq 1 \) and \( p_1 + \ldots + p_n = 1 \). These probabilities may be interpreted either as objective probabilities known to the individual or, in the absence of firm knowledge of this kind, as subjective probabilities which represent the individual’s degree of belief or confidence in the occurrence of the corresponding states. The individual’s problem is to choose between actions. Each action is an \( n \)-tuple of consequences, one consequence for each state of the world. We shall write the consequence of the \( i \)th action in the event that the \( j \)th state occurs as \( x_{ij} \). Consequences need not take the form of changes in wealth, although in our applications of our theory, we shall interpret \( x_{ij} \) as an increment or decrement of wealth, measured relative to some arbitrary level (which need not be the individual’s current wealth). Notice that actions, unlike prospects, associate consequences with particular states of the world. Thus a number of different actions might correspond with the same prospect. We shall recognise this difference by using the symbol \( A \) for actions, reserving \( X \) for prospects. Thus far, our theory has a close resemblance to Savage’s, except in that we take probabilities as given, just as von Neumann and Morgenstern do.

A choice problem may involve any number of available actions, but we shall begin by analysing problems where there is only a pair of actions to choose between. All of Kahneman and Tversky’s evidence concerns the behaviour of people choosing between pairs of prospects. Choices between three or more actions raise some additional issues, which we shall discuss in Section IV.

Our first assumption is that for any given individual there is a choiceless utility function \( C(\cdot) \), unique up to an increasing linear transformation, which assigns a real-valued utility index to every conceivable consequence. The significance of the word ‘choiceless’ is that \( C(x) \) is the utility that the individual would derive from the consequence \( x \) if he experienced it without having chosen it. For example, he might have been compelled to have \( x \) by natural forces, or \( x \) might have been imposed on him by a dictatorial government. Thus—in contrast to the von Neumann–Morgenstern concept of utility—our concept of choiceless utility is defined independently of choice. Our approach is utilitarian in the classical sense. What we understand by ‘choiceless utility’ is essentially what Bernoulli and Marshall understood by ‘utility’—the psychological experience of pleasure that is associated with the satisfaction of desire. We believe that it is possible to introspect about utility, so defined, and that it is therefore meaningful to talk about utility being experienced in choiceless situations.
Now suppose that an individual experiences a particular consequence as the result of an act of choice. Suppose that he has to choose between actions $A_1$ and $A_2$ in a situation of uncertainty. He chooses $A_1$ and then the $j$th state of the world occurs. He therefore experiences the consequence $x_{1j}$. He now knows that, had he chosen $A_2$ instead, he would be experiencing $x_{2j}$. Our introspection suggests to us that the psychological experience of pleasure associated with having the consequence $x_{1j}$ in these circumstances will depend not only on the nature of $x_{1j}$ but also on the nature of $x_{2j}$. If $x_{2j}$ is a more desirable consequence than $x_{1j}$, the individual may experience regret: he may reflect on how much better his position would have been, had he chosen differently, and this reflection may reduce the pleasure that he derives from $x_{1j}$. Conversely, if $x_{1j}$ is the more desirable consequence, he may experience what we shall call rejoicing, the extra pleasure associated with knowing that, as matters have turned out, he has taken the best decision.

We guess that many readers will recognise these experiences. For example, compare the sensation of losing £100 as the result of an increase in income tax rates, which you could have done nothing to prevent, with the sensation of losing £100 on a bet on a horse race. Our guess is that most people would find the latter experience more painful, because it would inspire regret. Conversely, compare the experience of gaining £100 from an income tax reduction with that of winning £100 on a bet. Now we should guess that most people would find the latter experience more pleasurable. This concept of regret resembles Savage’s (1951) notion in some ways, but it will emerge that our theory is very different from his minimax regret criterion.

We shall incorporate the concepts of regret and rejoicing into our theory by means of a modified utility function. Suppose that an individual chooses action $A_i$ in preference to action $A_k$, and that the $j$th state of the world occurs. The actual consequence is $x_{ij}$ while, had he chosen differently, $x_{kj}$ would have occurred. We shall write $C(x_{ij})$ as $c_{ij}$ and we shall then say that the individual experiences the modified utility $m_{ij}$ where:

$$m_{ij} = M(c_{ij}, c_{kj}). \tag{1}$$

The function $M(\cdot, \cdot)$ assigns a real-valued index to every ordered pair of choiceless utility indices. The difference between $m_{ij}$ and $c_{ij}$ may be interpreted as an increment or decrement of utility corresponding with the sensations of rejoicing or regret. To formulate regret and rejoicing in this way is to assume that the degree to which a person experiences these sensations depends only on the choiceless utility associated with the two consequences in question – ‘what is’ and ‘what might have been’ – and is independent of any other characteristics of these consequences. Given this assumption, it is natural to assume in addition that if $c_{ij} = c_{kj}$ then $m_{ij} = c_{ij}$: if what occurs is exactly as pleasurable as what might have occurred, there is neither regret nor rejoicing. It is equally natural to assume that $\partial m_{ij}/\partial c_{ij} \leq 0$: the more pleasurable the consequence that might have been, the more regret – or less rejoicing – is experienced. (We include as a limiting case the possibility that a person might not experience regret or rejoicing at all.) We also make the uncontroversial assumption that $\partial m_{ij}/\partial c_{ij} > 0$: that, other things being equal, modified utility increases with choiceless utility.

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Our theory is that the individual chooses between actions so as to maximise the mathematical expectation of modified utility. We may define the expected modified utility $E^k_l$ of action $A_l$, evaluated with respect to action $A_k$, by:

$$E^k_l = \sum_{j=1}^{n} p_j m^k_{lj}. \quad (2)$$

Faced with a choice between $A_l$ and $A_k$, the individual will prefer $A_l$, prefer $A_k$ or be indifferent between them according to whether $E^k_l$ is greater than, less than or equal to $E^l_k$.

Why, it may be asked, do we assume that people maximise the mathematical expectation of modified utility? Principally because this is a simple assumption which yields implications consistent with empirical evidence. We do not claim that maximising expected modified utility is the only objective that is consistent with a person being rational. However – and we shall say more about this in Section V – we believe that this is not irrational, and that, given the utilitarian premises of our approach, there is at least a presumption that people who experience regret and rejoicing will seek to maximise expected modified utility.

Notice that, in our theory, someone who does not feel regret or rejoicing at all will simply maximise expected choiceless utility. This special case of our theory corresponds with expected utility theory in its traditional or Bernoullian form, in which utility is interpreted as a psychological experience. To assume that people maximise expected modified utility is to generalise Bernoulli’s theory in a very natural way, since the individual who does experience rejoicing and regret can be expected to try to anticipate those feelings and take them into account when making a decision under uncertainty.

We shall now show that all of the experimental evidence described in Section I is consistent with regret theory. We shall do this by taking a restricted form of our general theory and by showing that the experimental evidence is consistent with this restricted form.

The particular restriction involves a simplifying assumption about the function $M(.)$. We shall assume that the degree of regret or rejoicing that a person experiences depends only on the difference between the choiceless utility of ‘what is’ and the choiceless utility of ‘what might have been’. This allows us to define a regret–rejoice function $R(.)$ which assigns a real-valued index to every possible increment or decrement of choiceless utility, and then to write:

$$m^k_{lj} = e_{lj} + R(e_{lj} - e_{kj}). \quad (3)$$

It follows from the assumptions we have made about $M(.)$ that $R(o) = o$ and that $R(.)$ is non-decreasing. In the limiting case in which $R(\xi) = o$ for all $\xi$, regret theory would yield exactly the same predictions as expected utility theory. Since we wish to emphasise the differences between the two theories we shall assume that $R(.)$ is strictly increasing and three times differentiable.

Now suppose, as before, that an individual has to choose between the actions $A_l$ and $A_k$. The individual will have the weak preference $A_l \succeq A_k$ if and only if:

$$\sum_{j=1}^{n} p_j [c_{lj} - c_{kj} + R(c_{lj} - c_{kj}) - R(c_{kj} - c_{lj})] \geq o. \quad (4)$$
It is convenient to define a function \(Q(.)\) such that for all \(\xi\),
\[
Q(\xi) = \xi + R(\xi) - R(-\xi).
\]
(5)

Thus \(A_i \geq A_k\) if and only if:
\[
\sum_{j=1}^{n} p_j [Q(\epsilon_{ij} - \epsilon_{kij})] \geq 0.
\]
(6)

\(Q(.)\) is an increasing function which has the following property of symmetry: for all \(\xi\), \(Q(\xi) = -Q(-\xi)\). Thus to know the value of \(Q(\xi)\) for all \(\xi \geq 0\) is to know the value of \(Q(\xi)\) for all \(\xi\).

Three alternative simplifying assumptions about \(Q(.)\) can be distinguished:

Assumption 1. \(Q(.)\) is linear or equivalently, for all \(\xi\), \(R''(\xi) = R''(-\xi)\). It follows immediately from (6) that in this case the individual will behave exactly as if he were maximising expected choiceless utility. Thus regret theory would yield the same predictions as expected utility theory and choiceless utility indices would be operationally indistinguishable from von Neumann–Morgenstern utility indices.

Assumption 2. \(Q(.)\) is concave for all positive values of \(\xi\) or equivalently, for all \(\xi > 0\), \(R''(\xi) < R''(-\xi)\).

Assumption 3. \(Q(.)\) is convex for all positive values of \(\xi\) or equivalently, for all \(\xi > 0\), \(R''(\xi) > R''(-\xi)\).

On the face of it, there seems to be no a priori reason for preferring any one of these assumptions to the others. They are simply alternative assumptions about human psychology and a choice between them should be made mainly on the basis of empirical evidence. We shall therefore show that all the evidence listed in Table 1 is consistent with the restricted form of our theory under Assumption 3. In contrast, Assumption 1 would predict no violations of expected utility theory, while Assumption 2 would predict violations, but in the opposite direction to those generally observed.

III. SOME IMPLICATIONS OF REGRET THEORY

We shall now derive some implications of our theory concerning choices between pairs of statistically independent prospects. In our theory, a choice problem cannot be analysed unless a matrix of state-contingent consequences can be specified, and a given pair of prospects (i.e. probability distributions of consequences) may

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be capable of being represented by many different matrices. However, the assumption of statistical independence ensures that there is a unique matrix for each pair of prospects. In most of Kahneman and Tversky's experiments, subjects were simply asked to choose between pairs of prospects. In such cases, we suggest, the most natural assumption for subjects to make is that the prospects are independent. Given this assumption, we can show that the evidence of Table 1 is entirely consistent with regret theory. As before, we shall use $x_1$ and $x_2$ to represent consequences. We shall use $c_1$ and $c_2$ to represent the choiceless utility indices $C(x_1)$ and $C(x_2)$. For simplicity, we choose a transformation of $C(.)$ such that $C(o) = o$; and we assume that $C(.)$ is an increasing function.

(a) The 'common ratio effect', and its reverse

Our theory yields the following prediction, which violates expected utility theory:

Let $X_i = (x_1, \lambda p) \text{ and } X_k = (x_2, p)$ be independent prospects, where $1 \geq p > 0$ and $1 > \lambda > 0$. If there exists some probability $\tilde{p}$ such that $X_i \sim X_k$ when $p = \tilde{p}$, then (i) (the common ratio effect) if $x_1 > x_2 > o$, then $p < \tilde{p} \Rightarrow X_i \succ X_k$ and $p > \tilde{p} \Rightarrow X_i \prec X_k$ and (ii) (the reverse common ratio effect) if $o > x_2 > x_1$, then $p < \tilde{p} \Rightarrow X_i \succ X_k$ and $p > \tilde{p} \Rightarrow X_i \prec X_k$.

In proving this result, it is convenient to begin by stating a general property of our theory. Let $X' = (x_1, p_1)$ and $X'' = (x_2, p_2)$ be any two independent prospects. The choice between these prospects may be represented by the matrix given in Table 2.

Table 2, where each column represents a different state of the world, and the probability that each state will occur is given at the top of its column. Applying Expression (6) to Table 2, we find: that

$$X' \succ X'' \text{ iff } p_1 Q(c_1) - Q(c_2) - p_1p_2[Q(c_1) - Q(c_1 - c_2) - Q(c_2)] \equiv o.$$  (7)

Thus in the case where $X_i = (x_1, \lambda p)$ and $X_k = (x_2, p)$,

$$X_i \succ X_k \text{ iff } p[\lambda Q(c_1) - Q(c_2) - \lambda p[Q(c_1) - Q(c_1 - c_2) - Q(c_2)] \equiv o.$$  (8)

By assumption, $Q(c)$ is convex for all $c > 0$ so that when $c_1 > c_2 > o$, $[Q(c_1) - Q(c_1 - c_2) - Q(c_2)] > o$. Given this inequality, the common ratio effect follows straightforwardly from Expression (8). Conversely, when $o > c_2 > c_1$, $[Q(c_1) - Q(c_1 - c_2) - Q(c_2)] < o$; and this implies the reverse common ratio effect.

The evidence of Problems 3 and 4 is consistent with the existence of the common ratio effect. Let $x_1 = 4,000$, $x_2 = 3,000$ and $\lambda = 0.8$. Then if $p = 0.9$, $X_2$
(a) The ‘common consequences effect’ or Allais paradox

Our theory yields a further prediction, which also violates expected utility theory:

Let \( X_i = (x_{i1}, p_1; x_{i2}, \alpha) \) and \( X_k = (x_{k1}, p_2 + \alpha) \) be independent prospects where \( 1 > p_2 > p_1 > 0 \) and \( (1 - p_2) > \alpha > 0 \). If there exists some probability \( \bar{\alpha} \) such that \( X_i \sim X_k \) when \( \alpha = \bar{\alpha} \), then (i) (the common consequences effect) if \( x_1 > x_2 > 0 \), then \( \alpha < \bar{\alpha} \Rightarrow X_i > X_k \) and \( \alpha > \bar{\alpha} \Rightarrow X_i < X_k \) and (ii) (the reverse common consequences effect) if \( 0 > x_2 > x_1 \), then \( \alpha < \bar{\alpha} \Rightarrow X_i < X_k \) and \( \alpha > \bar{\alpha} \Rightarrow X_i > X_k \).

According to regret theory,

\[
X_i \succ X_k \text{ iff } p_1 Q(c_1) - p_2 Q(c_2) - p_1(p_2 + \alpha) [Q(c_1) - Q(c_1 - \alpha) - Q(c_2)] \equiv 0. \tag{g}
\]

Because \( Q(c) \) is assumed to be convex for all \( c > 0 \), \( [Q(c_1) - Q(c_1 - \alpha) - Q(c_2)] \) is positive if \( x_1 > x_2 > 0 \) and negative if \( 0 > x_2 > x_1 \). Given these two propositions, Expression (g) entails both the common consequences effect and the reverse common consequences effect.

The evidence of Problems 1 and 2 is consistent with the existence of the common consequences effect. Let \( x_1 = 2,500, x_2 = 2,400, p_1 = 0.33 \) and \( p_2 = 0.34 \). Then if \( \alpha = (1 - p_2) \), \( X_i = (x_{i1}, p_1; x_{i2}, \alpha) \) and \( X_k = (x_{k1}, p_2 + \alpha) \). If \( \alpha = 0, X_3 = (x_{i1}, p_1; x_{i2}, \alpha) \) and \( X_4 = (x_{k1}, p_2 + \alpha) \). The conjunction of preferences \( X_1 < X_2 \) and \( X_3 > X_4 \) violates expected utility theory but is consistent with regret theory (corresponding with the case \( \alpha > 0.66 > \bar{\alpha} > 0 \)). At least 65% of Kahneman and Tversky’s subjects had this conjunction of preferences. Kahneman and Tversky did not publish any results relevant to our prediction of a reverse common consequences effect.

(c) The ‘isolation effect’ in the two-stage gambles

In Kahneman and Tversky’s Problem 10, their respondents were offered a two-stage gamble. In the first stage there was a \( 0.75 \) probability of the gamble ending with a null consequence and a \( 0.25 \) probability of going through to the second stage. Before embarking on the first stage, respondents were asked to choose which of \( X_5 \) or \( X_6 \) they would prefer if they got through to the second stage.

According to the compound probability axiom of expected utility theory, \( X_{17} = (X_5, 0.25) \) is equivalent to \( (4,000, 0.20) \) which is simply prospect \( X_6 \); and \( X_{18} = (X_6, 0.25) \) is equivalent to \( (3,000, 0.25) \) which is prospect \( X_{10} \). Thus expected utility theory makes no distinction between Problem 10 and Problem 4.

However, regret theory does make a distinction. The simple prospects \( X_5 \) and \( X_{10} \) are regarded as statistically independent, and Problem 4 is therefore represented by the matrix of state-contingent consequences shown in Table 3a. By
contrast, prospects $X_9$ and $X_{10}$ are not statistically independent: the first stage of the gamble is common to both, and if the state occurs under which the gamble comes to an end, the individual receives the same null consequence whichever prospect was chosen. Hence Problem 10 is represented by the matrix of state-contingent consequences shown in Table 3b. Since Tables 3a and 3b are different, our theory provides no reason to suppose that an individual will have the same preferences between $X_{17}$ and $X_{18}$ as between $X_9$ and $X_{10}$.

Before analysing this example further, we present a result which holds for regret theory in its most general form, and which we shall call the separability principle.

Let $S_1, \ldots, S_n$ be mutually exclusive events (i.e. non-intersecting sets of states of the world) with the non-zero probabilities $p_1, \ldots, p_n$ where $p_1 + \ldots + p_n = 1$. Let $S'_1, \ldots, S'_{n+1}$ be mutually exclusive events with the probabilities $\mu p_1, \ldots, \mu p_n$, $1 - \mu$, where $0 < \mu < 1$. Let $A_i = (x_{i1}, \ldots, x_{in})$ and $A_k = (x_{k1}, \ldots, x_{kn})$ be any two actions defined in relation to the events $S_{1}, \ldots, S_{n}$. Let $A_a$ and $A_b$ be actions defined in relation to the events $S'_1, \ldots, S'_{n+1}$, such that $A_a = (x_{11}, \ldots, x_{in}, y)$ and $A_b = (x_{11}, \ldots, x_{kn}, y)$, $y$ being any consequence common to both actions. Then $A_a \succeq A_b$ if and only if $A_i \succeq A_k$.

The proof is straightforward. If $E^A_t$ and $E^B_t$ are the expected modified utilities of $A_t$ and $A_k$, evaluated in relation to one another, then $E^A_t = \mu E^B_t + (1 - \mu) C(y)$ and $E^B_t = \mu E^B_t + (1 - \mu) C(y)$. Hence $E^A_t \succeq E^A_k \iff E^A_t \succeq E^B_k$, which entails $A_i \succeq A_k \iff A_a \succeq A_b$. The separability principle entails Savage’s sure-thing principle as a special case. Let $\mu$ remain constant, and let us construct two new actions, $A_a$ and $A_b$, which are the same as $A_a$ and $A_b$ except that the common consequence $y$ is replaced by the common consequence $z$. It is clear that $A_i \succeq A_k \iff A_i \succeq A_z$, and hence it follows that $A_a \succeq A_b \iff A_i \succeq A_{a'}$, which is Savage’s sure-thing principle.

Returning to Kahneman and Tversky’s evidence, let $A_5$ and $A_6$ be the actions corresponding to the independent prospects $X_5$ and $X_6$, and let $A_{17}$ and $A_{18}$ be
the actions corresponding to \( X_{17} \) and \( X_{18} \) in Table 3b. Since \( E_{17}^{18} = \mu E_0^9 + (1 - \mu) C(o) \) and \( E_{18}^{18} = \mu E_0^9 + (1 - \mu) C(o) \), it follows that \( X_5 < X_6 \Leftrightarrow X_{17} < X_{18} \). We have already seen in (a) above that the conjunction \( X_5 < X_6 \) and \( X_9 > X_{10} \) is consistent with our theory. Thus it follows that the conjunction \( X_9 > X_{10} \) and \( X_{17} < X_{18} \), which violates conventional expected utility, is also consistent with regret theory.

(d) The ‘reflection effect’

The results in (a), (b) and (c) above were derived without making any assumption about \( C(.) \) other than that it is monotonically increasing. We shall derive our results in (d) and (e) by making the additional assumption that \( C(.) \) is linear; and, for convenience, we shall choose a transformation of that linear function such that for all \( x, C(x) = x \).

Consider two independent prospects, \( X_i = (x_1, p_1) \) and \( X_k = (x_2, p_2) \). Their ‘reflections’ are denoted \( X_i' = (-x_1, p_1) \) and \( X_k' = (-x_2, p_2) \). From Expression (7) we know that \( X_i > X_k \) if and only if:

\[
C(x_1) - C(x_2) - p_1 Q(x_1) - p_2 Q(x_2) > 0. \tag{10}
\]

Now exactly the same inequality is necessary and sufficient for \( X_i < X_k \). Hence \( X_i > X_k \Leftrightarrow X_i' < X_k' \). Thus if \( C(.) \) is linear, the reflection effect is always observed.

Our intuition is that \( C(.) \) is not linear but concave. If this is correct, the reflection effect will not always be observed, and in particular, individuals will reject actuarially fair 50–50 gambles, rather than being indifferent towards them. This point is discussed further in Section V.

(e) Mixed risk attitudes; simultaneous gambling and insurance

Consider two independent prospects which offer an actuarially fair gamble: \( X_i = (o, 1) \) and \( X_k = (x, p; -px/(1-p), 1-p) \), where \( 0 < p < 1 \) and \( x > o \). Maintaining our previous assumption about \( C(.) \) we can apply Expression (7) and rearrange to give:

\[
X_i \succeq X_k \Leftrightarrow Q\left(\frac{px}{1-p}\right) - \frac{p}{1-p} Q(x) \equiv 0. \tag{11}
\]

From the assumption that \( Q(x) \) is convex for all \( x > 0 \), it follows that \( X_i \succeq X_k \) as \( p \equiv 0.5 \). So the individual will accept small-stake large-prize fair gambles (\( p < 0.5 \)) but reject large-stake small-prize fair gambles (\( p > 0.5 \)). Insurance typically involves paying a small premium to avoid a small probability of a large loss; thus in terms of our theory – which does not use the concept of a ‘reference point’ – to buy actuarially fair insurance is to reject a large-stake small-prize fair gamble, and thus it is consistent with our theory that an individual may simultaneously insure and accept small-stake large-prize gambles. Moreover, we can construct both small-stake large-prize fair gambles, and large-stake small-prize fair gambles either with all consequences positive or with all consequences negative. Thus a mixture of risk attitudes in both the positive and the negative domain is also consistent with our theory.

These conclusions would require some modification if \( C(.) \) were assumed to be concave rather than linear. In this case it can be shown that \( X_i > X_k \) if \( p \geq 0.5 \).
but it is no longer possible to make a firm prediction when $p < 0.5$. However, if an individual is more strongly influenced by the shape of $Q(.)$ than by the non-linearity of $C(.)$, simultaneous gambling and insurance is still consistent with our theory.

**IV. TRANSITIVITY OF PREFERENCES AND MULTI-ACTION PROBLEMS**

One controversial property of our theory is that $\succ$, the relation of weak preference, is not necessarily transitive. Consider the three actions shown in Table 4 in relation to an individual for whom $C(.)$ is linear. Relative to $A_1$, $A_2$ is a large-stake small-prize fair gamble, so that the individual would have the preference $A_1 \succ A_2$ if he had to choose between these two actions. If, as our theory entails, the individual acts according to the separability principle outlined in Section III (c), state $S_1$ can be ignored in a comparison between $A_2$ and $A_3$. Thus, relative to $A_2$, $A_3$ is also a large-stake small-prize fair gamble, and so $A_2 \succ A_3$. However, relative to $A_1$, $A_3$ is a small-stake large-prize fair gamble, so that $A_3 \succ A_1$. This is not to say that our theory specifically predicts non-transitive pairwise choices (since the $C(.)$ function need not be linear); but such choices can be consistent with the theory.

| Action | $S_1$ | $S_2$ | $S_3$ |
|--------|--------|--------|--------|
| $A_1$  | 6      | 6      | 6      |
| $A_2$  | 0      | 10     | 10     |
| $A_3$  | 0      | 0      | 15     |

The example shows that an individual will necessarily make non-transitive choices if (i) he acts according to the separability principle (or according to the sure-thing principle), (ii) he always accepts small-stake large-prize fair gambles and (iii) he always rejects large-stake small-prize fair gambles. In the light of the evidence that many people simultaneously gamble and insure one might well argue that a satisfactory theory of choice under uncertainty should encompass the case of the individual who acts according to (ii) and (iii). To say this is to say that either the sure-thing principle or the axiom of transitivity must be dropped. Our theory differs from many of its rivals by dropping transitivity rather than the sure-thing principle.

This raises two questions. One is whether a theory that allows non-transitive pairwise choices can be regarded as a theory of rational behaviour; this issue is discussed in Section V. The other question is how to extend our theory to deal with multi-action choice problems: since in our theory the relation $\succ$ is not necessarily transitive, we cannot deal with choices from sets of three or more actions simply by invoking the idea of a preference ordering. We shall argue that the logic of regret and rejoicing points towards a different way of generalising a theory of pairwise choice.
Consider the problem of choosing one action from a set $S$. The logic of our approach requires that the individual should evaluate each action in turn by asking himself what sensations of regret or rejoicing he would experience in each state of the world, were he to choose that action. Since to choose one action is to reject all of the others, the individual could experience regret or rejoicing in contemplating any of the rejected actions. This idea might be formulated in the following way. As before, we use $E^S_k$ to represent the expected modified utility of choosing action $A_i$ in a situation where the only alternative is action $A_k$. Now let $E^S_i$ represent the expected modified utility of choosing $A_i$ from the set of actions $S$. It seems natural to make $E^S_i$ a weighted average of the values of $E^S_k$ for each of the actions $A_k$ in $S$ (other than $A_i$ itself). One way of building this idea into our theory would be to assign action weights $a_k^S$ to each action $A_k$ in $S$, normalised so that these weights sum to unity. Then $E^S_i$ could be defined as:

$$E^S_i = \sum_{k \in S} \frac{a_k^S}{1-a_i^S} E^S_k \quad (k \neq i).$$  \(12\)

The individual's decision rule, as in the case of pairwise choice, would be to maximise expected modified utility. We hope in the future to formulate a theory of action weights, but in the example which follows we shall just make the simplest assumption – that each action has the same weight.

| Table 5 |
|---------|
| Action | 1/3 | 1/3 | 1/3 |
| $A_1$  | 1   | 1   | 1   |
| $A_2$  | 0   | 0   | 3   |
| $A_3$  | 0   | 3   | 0   |

This illustrative example refers to the choice problem shown in Table 5. As before, we shall assume that $C(x) = x$, and we shall make a particular assumption about the regret–rejoice function, that over the relevant range, $R(\xi) = 1 - 0.86^\xi$. In this case, and for these three actions, the relation $\succ$ happens to be transitive; $A_2 \succ A_1$, $A_3 \succ A_1$, $A_2 \sim A_3$. It is tempting (but, we suggest, wrong) to conclude from this that $A_1$ will not be chosen from the set {$A_1, A_2, A_3$}. If the action weights are equal to one another then $E^S_1 = 0.946$, $E^S_2 = 0.899$ and $E^S_3 = 0.899$, so that, according to the decision rule, $A_1$ will be chosen. Whether or not such behaviour can be defended as rational will be discussed in Section V.

V. THE POSITIVE AND NORMATIVE STATUS OF REGRET THEORY

The experimental results published by Kahneman and Tversky, wide-ranging though they are, form only a small fraction of the evidence accumulated in the past 30 years to show consistent and repeated violations of certain axioms of expected utility theory. Regret theory is one of a number of alternative theories that have been proposed in the light of this evidence; other theories have been
presented by, for example, Allais (1953), Kahneman and Tversky (1979), Fishburn (1981) and Machina (1982). We shall shortly compare our theory with these others, but first let us discuss a possible argument against regret theory.

It might be objected that regret theory is limited to cases where probabilities are known, and that it rests on assumptions about non-observable functions, whereas expected utility theory is built on clear behaviourai axioms which make it possible, in principle, to construct a series of choice problems which will reveal the individual’s von Neumann–Morgenstern utility function.

While we do not share the methodological position that the only satisfactory theories are those formulated entirely in terms of empirical propositions, we would point out that if an individual behaves according to our model, it is possible in principle to infer from observations of his choices: his subjective probabilities; his \( C(.) \) function (unique up to a positive linear transformation); and his \( Q(.) \) function (which, for any given transformation of \( C(.) \), will be unique up to a positive linear transformation with a fixed point at the origin). Thus each of the assumptions about \( C(.) \) and \( Q(.) \) required to generate our predictions is in principle capable of empirical refutation. (For an outline of the procedures involved, see the Appendix.)

The other criteria that are commonly used to evaluate positive theories are predictive power, simplicity and generality. Regret theory yields a wide range of firm predictions that are supported by experimental evidence, and it does so on the basis of a remarkably simple structure. Only the two functions \( C(.) \) and \( Q(.) \) are required. As far as \( C(.) \) is concerned, some of the most important predictions of our model – the common ratio effect, the common consequences effect, their reverses, and the isolation effect – require only that this function is monotonically increasing; the additional assumption of linearity yields clear predictions concerning the reflection effect and simultaneous gambling and insurance. In generating all these predictions, the other crucial assumption is simply that \( Q(\xi) \) is convex for all \( \xi > 0 \).

Thus in comparison with Kahneman and Tversky’s ‘prospect theory’ – which is also consistent with all the evidence in Table 1 – regret theory is very simple indeed. Kahneman and Tversky’s theory superimposes on expected utility theory a theory of systematic violations. Among their many assumptions are: (i) the rounding of probabilities up or down, and the complete editing out of ‘small’ probabilities; (ii) a ‘decision weight function’ which overweights small probabilities, underweights large probabilities, involves ‘subcertainty’, ‘subproportionality’ and ‘subadditivity’, and which is discontinuous at both ends, thus implying certain ‘quantal effects’; and (iii) a ‘value function’ (essentially a utility function) which must have at least one point of inflection (at the individual’s ‘reference point’ – which may or may not move around) but which can, if required, have no less than five points of inflection. We believe that against the complex and somewhat ad hoc array of assumptions required by prospect theory the principle of Occam’s Razor strongly favours the straightforwardness of regret theory.

Allais’s and Machina’s theories are considerably simpler than prospect theory, but they cannot explain all of the evidence in Table 1. Both of these theories
assume that the individual has a preference ordering over prospects. Thus two of the fundamental principles of expected utility theory are retained: that pairwise choices are transitive and that courses of action associated with identical probability distributions of consequences are equivalent to one another. (We shall call this latter principle the equivalence axiom.) Allais and Machina break away from expected utility theory by dropping the independence axiom; given that the equivalence axiom is retained, this amounts to abandoning the sure-thing principle. Our strategy is radically different: we retain the sure-thing principle while jettisoning both the equivalence axiom and the transitivity axiom. As a result we are able to predict the isolation effect in two-stage gambles, a form of observed behaviour that contravenes the equivalence axiom and therefore cannot be explained by either Allais or Machina. We are also able to predict the systematic occurrence of the reflection effect. Although Allais’s and Machina’s theories are not contradicted by the reflection effect, they do not predict it.

Fishburn’s model is more like regret theory (although he does not mention any notion of regret) in that he also drops the transitivity axiom. However, his model is presented in terms of prospects rather than actions, and therefore does not accommodate the isolation effect. On the other hand, if we restrict ourselves to statistically independent prospects (and Fishburn does so – see his p. 9), then our theory and his basic axioms are compatible, and provide an interesting example of how an axiomatic treatment and a more introspective psychologically-based approach may complement each other.1

However, having indicated that our theory provides certain predictions and explanations that the other theories mentioned do not, we should make it clear that we are not claiming that regret theory can explain all of the behavioural regularities revealed by experimental research into choice under uncertainty. So far we have focused on a number of patterns of behaviour observed by Kahneman and Tversky; but we have not dealt with every one of their observations, still less with the vast amount of evidence accumulated by other researchers.

Some of the experimental findings do not appear to be completely consistent. In relation to this paper, the most significant case concerns the reflection effect. Hershey and Schoemaker (1980a) and Payne et al. (1980) have published results that show this effect to be not nearly as strong as or as general as Kahneman and Tversky’s evidence suggests. However, this may not present any great difficulties for regret theory since, as we noted in Section III (d), the general prediction of the reflection effect requires \( C(\cdot) \) to be linear. Instances in which the reflection effect is weak or absent may well be explicable if \( C(\cdot) \) is assumed to be concave.

There are nevertheless certain observations that simply cannot be accounted for by regret theory in the form presented here. One example is the ‘framing’ effect discussed by Tversky and Kahneman (1981) and the very similar ‘context’ effect observed by Hershey and Schoemaker (1980b). In these cases exactly the

1 At a late stage, we have received a copy of a Working Paper by David E. Bell (1981) which is of great interest. Quite independently he has developed a model which also explicitly incorporates a notion of regret, using multi-attribute utility theory along the lines suggested by Keeney and Raiffa (1976). We note that when both models are applied to the same phenomena – the original Allais paradox, simultaneous insuring and gambling, and the reflection effect – the conclusions are strikingly similar.
same choice problem – that is, exactly the same when formulated in terms of a matrix of state-contingent consequences – receives markedly different responses, depending on the way the choice is presented. Another example is the ‘translation’ effect observed by Payne et al. (1980). This effect occurs when an individual prefers one prospect to another, but reverses his preference when the same sum of money is deducted from every consequence of both prospects. The observed pattern of reversal is not predicted by regret theory. Finally, systematic violations of the sure-thing principle have been observed (cf. Moskowitz (1974); Slovic and Tversky (1974)). And although there is some evidence that individuals violate the sure-thing principle much less often than they violate some other axioms (Tversky and Kahneman (1981, footnote 15)), as it stands our theory does not explain that behaviour.

On the other hand, there is some additional evidence that gives further support to regret theory. A particular instance is the form of ‘preference reversal’ observed by Lindman (1971) and Lichtenstein and Slovic (1971, 1973) and subsequently confirmed, after rigorous testing, by Grether and Plott (1979). This preference reversal occurs when an individual, faced with a pairwise choice between gambles A and B, chooses A; but when asked to consider the two gambles separately, places a higher certainty equivalent value on B. We have shown elsewhere (Loomes and Sugden (1982)) that the most commonly observed reversal pattern is predicted by regret theory even in its restricted form.

Of course, we acknowledge that there is no simple theory that gives a unified explanation of all the experimental evidence, and regret theory is no exception in this respect. But we have tried to construct a theory that explains as much of the evidence as possible on the basis of very few assumptions. We do not believe that choiceless utility and regret are the only factors that influence behaviour under uncertainty, but just that these two factors seem to be particularly significant. Indeed, we have become increasingly convinced by evidence of framing, context and translation effects that the notion of reference points deserves further consideration, although we have not tried to deal with that issue in this paper.

In constructing our theory we have avoided any assumptions of misperceptions or miscalculations by individuals. We do not doubt that in reality misperceptions and miscalculations occur, and sometimes in systematic rather than random ways. Nonetheless, our inclination as economists is to explain as much human behaviour as we can in terms of assumptions about rational and undeceived individuals. Thus we believe that regret theory does more than predict certain systematic violations of conventional expected utility theory: it indicates that such behaviour is not, in any meaningful sense of the word, irrational.

In claiming this we are breaking the terms of a truce that many theorists (with the notable exception of Allais) have tacitly accepted. Proponents of expected utility theory often concede that their theory has serious limitations as a predictive device but insist that its axioms have strong normative appeal as principles of rational choice. Thus Morgenstern (1979, p. 180) argues for expected utility theory on the grounds that ‘if people deviate from the theory, an explanation of the theory and of their deviation will cause them to re-adjust their behaviour’. Similarly, Savage (1954, pp. 102–3) admits that when confronted with a pair of
choice problems rather like Problems 1 and 2, he behaved in accordance with the common consequences effect and in violation of his own axioms. But, he says, he was able to convince himself that this behaviour was mistaken (though even after realising his 'mistake' he continued to feel an 'intuitive attraction' to that behaviour). At the other side of the truce, proponents of alternative theories have often been willing to accept these claims. Kahneman and Tversky (1979, p. 277) maintain that the departures from expected utility theory that prospect theory describes ‘must lead to normatively unacceptable consequences’ which a decision-maker would, if he realised the error of his ways, wish to correct. Similarly, Machina (1982, p. 277) notes the ‘normative appeal’ of the axioms of expected utility theory before going on to propose a positive theory that dispenses with one of these axioms.

However, we shall challenge the idea that the conventional axioms constitute the only acceptable basis for rational choice under uncertainty. We shall argue that it is no less rational to act in accordance with regret theory, and that conventional expected utility theory therefore represents an unnecessarily restrictive notion of rationality.

Regret theory rests on two fundamental assumptions: first, that many people experience the sensations we call regret and rejoicing; and second, that in making decisions under uncertainty, they try to anticipate and take account of those sensations.

In relation to the first assumption, it seems to us that psychological experiences of regret and rejoicing cannot properly be described in terms of the concept of rationality: a choice may be rational or irrational, but an experience is just an experience. As far as the second assumption is concerned, if an individual does experience such feelings, we cannot see how he can be deemed irrational for consistently taking those feelings into account.

We do not claim that acting according to our theory is the only rational way to behave. Nor do we suggest that all individuals who act according to our theory must violate the conventional axioms. Some individuals may experience no regret or rejoicing at all, while some others may have linear $Q(.)$ functions: in these special cases of our theory, we would predict that the individual’s behaviour would conform with all the conventional axioms.

On the other hand, individuals with non-linear $Q(.)$ functions of the kind described in this paper may consistently and knowingly violate the axioms of transitivity and equivalence without ever accepting, even after the most careful reflection, that they have made a mistake. So these axioms do not necessarily have the self-evident or overwhelming normative appeal that many theorists suppose. We shall now try to show why we do not accept the idea that the transitivity and equivalence axioms are necessary conditions for rational choice under uncertainty.

Underlying those two axioms is a common idea: that the value placed on any action $A_i$ depends only on the interaction between, on the one hand, the probability-weighted consequences offered by $A_i$ and, on the other hand, the individual’s pattern of tastes, including his attitude to risk.

That is what is symbolised when, for any individual, an expected utility
number is assigned to an action, that expected utility number being quite independent of the range and nature of the available alternative actions. From this idea, that there is some value in ‘having $A_i$’ which is quite independent of the value of ‘having $A_k$’, and that if ‘having $A_i$’ gives more value than ‘having $A_k$’ then $A_i > A_k$, it follows that there must exist a complete and transitive preference ordering over all actions.

It also follows that the particular state pattern of consequences is of no special significance: if each action is evaluated independently, it does not matter how the consequence of that action under any state of the world compares with the consequence(s) of any other action(s) under the same state. Thus only the probability distribution of consequences matters, and all actions, simple or complex, which share the same probability distribution will be assigned the same expected utility number and must be regarded as equivalent for the purposes of choice decisions.

But if people experience regret and rejoicing, these arguments are illegitimate. In regret theory the proposition $A_i \succ A_k$ cannot be read as ‘having $A_i$ is at least as preferred as having $A_k$’; it should rather be read as ‘choosing $A_i$ and simultaneously rejecting $A_k$ is at least as preferred as choosing $A_k$ and simultaneously rejecting $A_i$. Thus the transitivity of the relation ‘is at least as preferred as’ (which we do not dispute) does not entail the transitivity of our relation $\succ$; and so non-transitive choices do not indicate any logical inconsistency on the part of the decision-maker.

The idea that non-transitive choices are irrational is sometimes argued as follows. Suppose (as in the example discussed in connection with Table 4 in Section IV) that there are three actions $A_1, A_2, A_3$, such that $A_1 > A_2, A_2 > A_3$, and $A_3 > A_1$. Then, it is said, no choice can be made from the set $\{A_1, A_2, A_3\}$ without there being an inconsistency with one of the original preference statements: whichever action is chosen, another is preferred to it (cf. MacKay (1980, p. 90)). The principle that is being invoked here is Chernoff’s axiom: if $A_i$ is chosen from some set $S$, and if $S'$ is a subset of $S$ that contains $A_i$, then $A_i$ must be chosen from $S'$. But we suggest the appeal of this axiom derives from the supposition that the value of choosing an action is independent of the nature and combination of the actions simultaneously rejected; and regret theory does not accept this supposition. Since $A_1 > A_2$ means only that choosing $A_1$ from the set $\{A_1, A_2\}$ is preferred to choosing $A_2$ from the set $\{A_1, A_2\}$ there is no implication that choosing $A_1$ from the set $\{A_1, A_2, A_3\}$ is preferred to choosing $A_2$ from the set $\{A_1, A_2, A_3\}$. A similar argument applies to the example discussed in connection with Table 5 in Section IV, where (despite the fact that the relation $\succ$ happens to be transitive) there is another violation of Chernoff’s axiom.

A second common objection to non-transitivity runs like this. If someone prefers $A_1$ to $A_2, A_2$ to $A_3$, and $A_3$ to $A_1$, every one of the actions is less preferred than another; so might he not get locked into an endless chain of choice in which he can never settle on any one action? Worse, might not a skilful bookmaker capture all his wealth by confronting him with a suitably constructed sequence of pairwise choices? But these objections rest on a fallacy. To suppose that the individual can get locked into a cycle of choices, it is necessary to suppose that all
three actions are feasible. But if this is indeed the case, then propositions about pairwise choices — about how choices are made when there are only two feasible actions — are not relevant. The bookmaker can bankrupt his client only if he can successively persuade him to believe in each of a long chain of mutually inconsistent propositions about the feasible set.

Finally, there is no reason why the equivalence axiom should be regarded as a necessary condition for rational choice, even when the choice is between two simple actions with identical probability distributions of consequences. Consider

| Action | 0.25 | 0.25 | 0.25 | 0.25 |
|--------|------|------|------|------|
| \(A_i\) | 3    | 2    | 1    | 0    |
| \(A_k\) | 0    | 3    | 2    | 1    |

\(A_i\) and \(A_k\) in Table 6. If each action were evaluated independently, there would be no grounds for preferring 'having \(A_i\)' to 'having \(A_k\)', or vice versa. But in our model the decision is between 'choosing \(A_i\) and simultaneously rejecting \(A_k\)' and 'choosing \(A_k\) and simultaneously rejecting \(A_i\)'. These two alternatives are associated with different probability mixes of regret and rejoicing. (In terms of our theory, to choose \(A_i\) and reject \(A_k\) is to incur a 0.25 probability of \(R(3)\) and a 0.75 probability of \(-1\), while to choose \(A_k\) and reject \(A_i\) is to incur a 0.25 probability of \(-3\) and a 0.75 probability of \(3\).) So for an individual who experiences regret and rejoicing, the two courses of action cannot be regarded as identical. It would therefore not be unreasonable for such an individual to prefer one to the other.

VI. CONCLUSION

The evidence presented by Kahneman and Tversky and many others points to a number of cases where commonly observed patterns of choice violate conventional expected utility axioms. The fact that these violations are neither small-scale nor randomly distributed may indicate that there are some important factors affecting many people's choices which have been overlooked or mis-specified by conventional theory.

We suggest that one significant factor is an individual's capacity to anticipate feelings of regret and rejoicing. We therefore offer an alternative model which takes those feelings into consideration. This model yields a range of predictions consistent with the behaviour listed in Table 1 and provides an account of these and other choice phenomena which conventional theory has so far failed to explain.

That is the positive side of regret theory. But we believe that our approach also has strong normative implications. We have argued that our theory describes a form of behaviour which, although contravening the axioms of expected utility theory, is rational. Thus, while we do not suggest that behaving according to those
conventional axioms is irrational, we do suggest that those axioms constitute an excessively restrictive definition of rational behaviour.

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**Appendix: Inferring subjective probabilities and $C(.)$ and $Q(.)$ functions from choices**

The following procedure will reveal, for any individual, which of two events has the higher subjective probability. Let $S_1$ and $S_2$ be any two non-intersecting and non-empty events (i.e. sets of states of the world). Let $S_3$ be the event that comprises all those states of the world not in $S_1$ or $S_2$. Let $x, y, z$ be any three consequences such that the person in question prefers $x$ to $y$ (under certainty). Consider the two actions $A_t = (x, y, z)$ and $A_k = (y, x, z)$, which are defined in relation to the events $S_1, S_2, S_3$. It then follows from the separability principle (see Section III) that $A_t$ is preferred to, indifferent to, or less preferred than $A_k$ as the subjective probability of $S_1$ is greater than, equal to, or less than that of $S_2$. This procedure is broadly similar to the one proposed by Savage (1954) for inferring subjective probabilities for individuals who behave according to his postulates.

The restricted form of our theory (see Section III) uses two functions for the analysis of modified utility: $C(.)$ and $Q(.)$. $C(.)$ can be identified, up to a positive linear transformation, by confronting the individual with choices involving 50–50 gambles. Consider any two prospects of the form $X_t = (x_1, 1)$, $X_k = (x_2, 0.5; x_3, 0.5)$ where $x_3 > x_1 > x_2$, so that the corresponding choiceless utility indices are $c_3 > c_1 > c_2$. Then:

$$X_t \succ X_k \iff 0.5 Q(c_1 - c_2) - 0.5 Q(c_3 - c_1) \equiv 0.$$  

But since $Q(.)$ is increasing, it follows that:

$$X_t \succ X_k \iff 0.5 (c_1 - c_2) - 0.5 (c_3 - c_1) \equiv 0.$$  

Thus in this case, the individual chooses as though maximising expected choiceless utility. So $C(.)$ can be identified from experiments in much the same way as von Neumann–Morgenstern utility functions are identified.

If $C(.)$ is known, and if a particular transformation has been chosen, it is possible to define consequences in terms of their choiceless utilities. Let $x_1$ and $x_2$ be consequences such that $c_1 = 0$ and $c_2 = -1$. Let $x_3$ be any consequence such that $c_3 = \xi$ where $\xi > 0$ and $\xi \neq 1$. Consider the two prospects $X_t = (x_1, 1)$ and $X_k = (x_2, p; x_3, 1 - p)$. Then:

$$X_t \succ X_k \iff \frac{Q(\xi)}{Q(1)} \equiv \frac{p}{1 - p}.$$  

Thus if one can find a value of $p$ such that the individual is indifferent between $X_t$ and $X_k$, it is possible to infer the value of $Q(\xi)/Q(1)$. So if $Q(1)$ is set equal to any arbitrary positive value, the value of $Q(\xi)$ can then be determined by experiment for all $\xi > 0$; hence the concavity, convexity or linearity of $Q(.)$ over any interval can be established.
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