Transport signatures of a Floquet topological transition at the helical edge

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The manipulation of the helical edge states of two-dimensional topological insulators is crucial for the development of technological applications. Recently, an important step forward, namely, the experimental realization of a quantum point contact between helical edges, has been accomplished. We theoretically predict that such a quantum point contact, in the presence of a time periodic applied electric field, is characterized by a topological quantum phase transition in the Floquet spectrum. Moreover, we show that it is possible to detect this dynamical topological quantum phase transition by a bare conductance measurements.

Introduction.– Since their theoretical prediction[1, 2] and experimental realization[3], two-dimensional topological insulators have triggered intense research activities in view of their potential applications in spintronics[4, 5], superconducting spintronics[6, 7], and topological quantum computation[8, 9]. Such technological applications require the ability to manipulate the edge states on demand. In this respect, several possibilities have been proposed at the level of a single edge. Proximity induced superconductivity, that appears to be crucial for Majorana fermions, has already been experimentally demonstrated[10]. Interaction induced gaps at the level of single edges, that could give rise to interesting fractional correlated phases[11, 12], and in combination with superconductivity, to parafermions[13, 14], are demanding because of the rather efficient screening in topological materials.

When two helical edges can interact with each other, the system becomes much richer. Recently, this promising new direction was substantiated by experimental realization of the first quantum point contact between two helical edges[15]. This remarkable achievement could potentially lead to the implementation of theoretically predicted phenomena relevant for spintronics applications[16–20]. Moreover, in the presence of both quantum point contacts and superconductivity, Majorana fermions[21] and parafermions[22] can emerge.

Time periodic (Floquet) external perturbations have been shown to represent a powerful tool to engineer on demand quantum properties in solid state systems[23–29]. In particular, the possibility of inducing topological phase transitions by means of driving has been analysed in different contexts[30–43]. Despite success in ultracold atomic gases[44–46], a clear experimental signature of such phase transitions in solid-state systems is still lacking.

In this Letter, we merge the features offered by the nanostructuring of the helical edges in combination with Floquet driving. We show that a periodically driven quantum point contact between helical edges is characterized by a Floquet topological phase transition. Indeed, varying the amplitude and the frequency of the drive allows us to switch from a trivial gap in the quasienergy spectrum to a non-trivial one. Importantly, unlike the previous proposals for the realization of Floquet Majorana bound states in spin-orbit quantum wires[47], the
system we consider is naturally characterized by charge conjugation symmetry. Remarkably, we demonstrate that the Floquet topological phase transition, associated with the presence of topological boundary states, can be clearly detected by conductance measurements.

**Model.** The system of investigation is given by a quantum point contact based on a quantum spin Hall insulator (QSHI) (Fig. 1 (a)). In an effective theory, the low energy physics within the bulk band gap can be described in terms of a linear Dirac theory containing four Fermi fields, collected in the spinor \( \Psi(x) = [\hat{\psi}_{R,1}(x), \hat{\psi}_{L,1}(x), \hat{\psi}_{L,2}(x), \hat{\psi}_{R,2}(x)]^T \), where \( R, L \) denote right- and left-movers and 1, 2 refer to the corresponding edge. We chose the spin \( z \) projection such that \( R, 1 \) corresponds to spin up, the other components follow from that. The kinetic energy is given by the Hamiltonian \((\hbar = 1)\)

\[
H_p = \int dx \Psi^\dagger(x) [-iv_F \partial_x \tau_z \sigma_z] \Psi(x), \quad (1)
\]

where \( \tau_i \) and \( \sigma_i \) with \( i \in \{x,y,z\} \) are Pauli matrices acting on edge- and spin-space, respectively. This term characterizes both the constricted region, of length \( L \), and the non-constricted region, of arbitrary length. Up to the section about transport, we will inspect the properties of the constricted region, with periodic boundary conditions between \( x = 0 \) and \( x = L \). Given time-reversal symmetry, in the constricted region we include two different scattering mechanisms at the single-particle level [20, 21]

\[
H_{t_0} = t_0 \int_0^L dx \Psi^\dagger(x) \tau_x \sigma_0 \Psi(x), \quad (2)
\]

\[
H_{t_c} = t_c \int_0^L dx \Psi^\dagger(x) \tau_y \sigma_y \Psi(x). \quad (3)
\]

While Eq. (2) describes the hybridization of states associated to different edges and does not need further symmetry breaking with respect to \( H_p \), Eq. (3) requires axial spin symmetry to be absent. This is, for instance, realized by Rashba spin orbit coupling, which naturally appears due to confinement. \( H_{t_0} \) refers to processes where the moving direction of the incoming particle is inverted but its spin is preserved. Likewise, \( H_{t_c} \) preserves the moving direction but flips the spin. The single-particle eigenvalue spectrum associated with \( H_{QPC} = H_p + H_{t_0} + H_{t_c} \) is shown in Fig. 1 (b). Importantly, each eigenvalue branch has a defined spin-projection and the system obeys a charge-conjugation symmetry. The spin-projection ceases to be a good quantum number when we additionally apply a Zeeman field

\[
H_B = B_z \int_0^L dx \Psi^\dagger(x) \tau_0 \sigma_z. \quad (4)
\]

Although Eq. (3) and Eq. (4) do not separately lead to a spectral gap, their interplay is able to open a helical gap around \( k = 0 \) (Fig. 1 (c)). Intuitively, this happens as Eq. (3) describes transitions between states of different spin, while Eq. (4) attributes different energies to the latter ones.

The engineered band structure is now very similar to the one found in spin-orbit coupled quantum wires under the influence of magnetic fields. In those systems, it is well known that a topological phase is found in the right parameter regime, provided superconductivity is present. Superconductivity introduces an extra relevant symmetry to those systems: particle-hole symmetry. As our system possesses a natural charge conjugation symmetry, it is possible to mimic similar topological properties with a periodically driven external electro-magnetic field

\[
H_A(t) = \int_0^L dx \Psi^\dagger(x)(-eA \cos(\omega t) \tau_z \sigma_z) \Psi(x). \quad (5)
\]

When the frequency is chosen resonantly (i.e. \( \omega = 2\sqrt{t_0^2 + t_c^2} \)), the effective quasi-energy operator, derived below (Eq. (12)), undergoes a phase transition for \( eA = B_z \) (Fig. 1 (d)-(f)). To understand this dynamical phase transition in more detail, we apply Floquet theory.

Time periodic Hamiltonians with \( H(t) = H(t + T) \) suggest solutions of the form \( \psi(t) = e^{-i\epsilon t} \psi \), with \( \epsilon = \nu(t + T) \). Due to the time-periodicity, \( \epsilon \) is only defined modulo \( 2\pi/T \). Using \( \psi(t) \) as an ansatz in the time-dependent Schrödinger equation \( H(t)\psi(t) = i\partial_t \psi(t) \), we obtain the eigenvalue equation for the time-dependent states \( \nu(t) \)

\[
Q|\nu(t)\rangle = \epsilon|\nu(t)\rangle \quad (6)
\]

with the quasi-energy operator \( Q = H(t) - i\partial_t \). Eq. (6) describes an eigenvalue problem in an extended Hilbert space \( \mathcal{F} = \mathcal{H} \otimes \mathcal{L}_T \) constructed by the Hilbert space \( \mathcal{H} \) and the space of square-integrable \( T \)-periodic functions \( \mathcal{L}_T \). The scalar product defined on \( \mathcal{F} \) then combines time averaging with the scalar product on \( \mathcal{H} \):

\[
(\langle \nu(t)|\mu(t)\rangle) = 1/T \int_0^T dt \langle \nu(t)|\mu(t)\rangle. \quad (7)
\]

A complete set of orthonormal states on \( \mathcal{F} \) is given by \( |\nu_m(t)\rangle = |\nu\rangle e^{im\omega t} \) with \( \omega = 2\pi/T \). Expansion of the quasi-energy operator \( Q \) in the basis states \( |\nu_m(t)\rangle \), this yields

\[
(\langle \nu_m(t)|Q|\nu_n(t)\rangle) = \langle \nu|H_{m-n}\mu\rangle + \delta_{m,n}\delta_{\mu,\nu}m\omega \quad (7)
\]

with \( H_m = 1/T \int_0^T dt e^{-i\omega m t} H(t) \). Using \( H(t) = H_p + H_{t_0} + H_{t_c} + H_B + H_A(t) \), we obtain only a few non-zero contributions

\[
H_0 = H_{QPC} + H_B, \quad (8)
\]

\[
H_1 = H_{-1} = \frac{1}{2} \int_0^L dx \Psi^\dagger(x) (-eA \tau_z \sigma_z) \Psi(x). \quad (9)
\]

The couplings \( H_{z\pm 1} \) become particularly relevant for those \( k \)-points for which \( \omega \) is tuned to a resonance. Then, it is possible to extract an effective quasi-energy operator out
of the infinite dimensional quasi-energy operator\cite{47}. Explicitly, the Fermi field operators in the constricted region made periodic read $\Psi(x) = \sum_k c_k e^{i k x}$ with creation operators $c_k = (\hat{c}_{k1R}, \hat{c}_{k1L}, \hat{c}_{k2L}, \hat{c}_{k2R})^T$. We now apply a unitary transformation $U$ such that $H_{QPC}$ becomes diagonal. The new creation operators $d_k = Uc_k$ are then associated with the eigenvalues $\pm E(\pm k) = \pm \sqrt{t_0^2 + (t_c + k v F)^2}$ (Fig. 1 (b)). In terms of the transformed fermions $d_k$, $H_B$ and $H_{s1}$ become

$$
H_B = \sum_k d^\dagger_k [B_0(k) \tau_0 \sigma_x + \frac{B_1(k) (\tau_x \sigma_x - \tau_y \sigma_y)}{2}] d_k,
$$

$$
H_1 = \sum_k d^\dagger_k \left[ \frac{\Delta_0(k) \tau_x (\sigma_0 - \sigma_z)}{2} + \frac{\Delta_0(-k) \tau_x (\sigma_0 + \sigma_z)}{2} \right] d_k
$$

with $\Delta_0(k) = -e A |t_0| / \sqrt{t_0^2 + (t_c + k v F)^2}$ and $\Delta_1(k) = -e A (t_c + k v F) / \sqrt{t_0^2 + (t_c + k v F)^2}$. The explicit $k$-dependence of $B_0$, $B_1$ and $B_2$ is lengthy to be presented. These parameters obey $B_0, B_1, B_2 \propto B_z$. Let us concentrate the resonant case with $\omega = 2 \sqrt{t_0^2 + t_c^2}$ (Fig. 1 (c)). In this case, degenerate points in the eigenvalue spectrum of the quasi-energy operator (the ones that are coupled by the arrow and the corresponding ones for $k > 0$ in Fig.1(c)) appear. Thus, matrix elements related to the coupling between these eigenvalues become dominant. Moreover, provided $e A, B_z \ll \omega$, matrix elements describing the coupling between states that are separated by at least $\omega$ can be safely neglected for quasi-energies close to the resonance point. Then, the relevant part of the operator $Q$ assumes the form of the effective quasi-energy operator

$$
H_{\text{eff}} = \sum_k \left[ \begin{array}{ccc}
E(-k) & B_0(k) & \Delta_0(-k) \\
B_0(k) & E(k) & 0 \\
\Delta_0(-k) & 0 & -E(-k) + \omega
\end{array} \right] d_k
$$

Eq. (12), though not quadratic in $k$, has a striking similarity with the Hamiltonian of a spin-orbit coupled quantum wire under the influence of an external magnetic field and proximity induced s-wave superconductivity. Remarkably, around $e A = B_z$, in analogy to the topological phase transition characterizing the quantum wire case, we can find a gap-closing and reoening, indicating a topological phase transition in the Floquet spectrum (Fig. 1 (d)-(f)).

Generalizing the above discussion to an excitation energy $\omega = 2 \sqrt{t_0^2 + t_c^2} \pm \delta \omega$, with an imbalance $\delta \omega$, from Eq. (12), we obtain a gap closing-reopening transition at $k = 0$ for

$$
B_z = \pm \sqrt{(e A)^2 + (\delta \omega)^2 (1 + (t_c / t_0)^2)}.
$$

The above phase transition happens at the new, dynamically generated charge-conjugation symmetric points at quasi-energy $\epsilon = \omega/2$.

Transport. The natural question that arises from the latter analysis is whether there is a measurable signature attributed to this phase transition. In order to answer this question, we now relax the assumption of periodic boundary conditions in the constricted region. We here assume to have an infinite system, and a finite constricted region, and we assume the potentials in the constriction to be step functions $t_0(x) = t_\alpha(x) = \theta(x) \theta(L - x)$ with the Heavyside function $\theta$. The scheme is in Fig.1a). This assumption is valid, provided the Fermi wave length $\lambda_B$ is much smaller than the smoothening length $L_s$ that describes the build-up of the QPC potentials which itself should be smaller than $L$. Hence, we require $\lambda_B \ll L_s \ll L$.

To access the conductance, it is helpful to solve the corresponding scattering problem. In particular we need to solve the single-particle eigenvalue problem associated to Eq. (7) $Q(x) |\nu(x)\rangle = \epsilon |\nu(x)\rangle$ (note that for the calculation of the conductance the effective Hamiltonian is not enough and the full Floquet problem must be addressed). For that purpose, we compute the transfer matrix of the system. We can rewrite the eigenvalue problem as

$$
P \partial_x |\nu(x)\rangle = \frac{i}{v_F} (\epsilon - Q_1(x)) |\nu(x)\rangle
$$

with $P = \mathbb{1}_{\infty} \tau_z \sigma_z$ and $Q_1 = Q + iv_F P \partial_x$. Since we assume piece-wise constant potentials, we find the (infinite dimensional) transfer matrix of the QPC with

$$
M(L,0) = \exp \left[ \int_0^L dx \frac{i}{v_F} P^{-1} (\epsilon - Q_1(x)) \right].
$$

An incoming mode can then be scattered in any sector carrying photon number $m$. However, transitions including high numbers of photons are considerably less probable and the reflection-/transmission-amplitudes $r_j^m, t_j^m \to 0$ with the photon number $m$ and edge index $j = 1, 2$. This allows us to compute the conductance with only a subset $M_{m}(L,0) = \exp \left[ i/v_F \int_0^L dx P_m (\epsilon - Q_1,m) \right]$ of the infinite dimensional matrix $M$. Indeed the transfer matrix $M_{m}(L,0)$ defines a 4m dimensional scattering problem that can be solved with appropriate incoming and outgoing states $\psi_{in,m}(x), \psi_{out,m}(x)$, given in the Supplemental Material. They contain all scattering amplitudes up to order $m$ in the photon index, which are themselves determined by solving $M_{m}(L,0) \psi_{in,m}(0) = \psi_{out,m}(L)$.

Assuming the periodic drive to be restricted to the QPC region, we are able to associate a Fermi distribution to each of the leads. The current in lead $\alpha$ is then given
by \[48, 49\]

\[I_\alpha = \frac{e}{2\pi} \int_0^\infty dE \sum_{m=-\infty}^{\infty} \sum_{\beta} |T_{\alpha,\beta}(E + m\omega,E)|^2 \times [f_\beta(E) - f_\alpha(E + m\omega)],\]  

(16)

where we assume a small voltage difference \(eV_\alpha\) between lead \(\alpha\) and all other leads. \(f_\alpha(E)\) are Fermi distribution functions in lead \(\alpha\) and \(T_{\alpha,\beta}(E + m\omega,E)\) captures all scattering amplitudes for a transition from lead \(\alpha\) to lead \(\beta\) with photon number \(m\). From Eq. (16), we obtain the linear conductance in lead \(\alpha\) at zero temperature to \(m\)-th order in the photon number

\[G^m_\alpha(\epsilon) = \frac{e^2}{2\pi} \left[1 - \sum_{n=-m}^{m} |r_\alpha^n(\epsilon)|^2 \right]\]  

(17)

with the reflection amplitudes \(r_\alpha^n = r_\alpha(\epsilon + m\omega,\epsilon)\). From Eq. (17) we can directly derive the two-terminal conductance

\[G^m(\epsilon) = \sum_{\alpha=1,2} G^m_\alpha(\epsilon)\]  

(18)

The results for the four-terminal \((G^m(G^m))\) as well as the two-terminal conductance \((G^m(\epsilon))\) are shown in Fig. 2 (a)-(b). Both are evaluated for \(\epsilon_c = 0\) with \(\epsilon_c = \epsilon - \omega/2\) and \(\omega = 2(\sqrt{\frac{\delta_0}{t_0}} + \sqrt{\frac{\delta_c}{t}} - \delta_\omega)\), which in our setup the point corresponding to zero excitation energy in the Majorana wire counter part, since the system we analyse is charge conjugation symmetric with respect to this point.

Interestingly, finite \(A\) blocks the two-terminal transport through the QPC and leads to zero conductance for sufficient large QPC potentials (Fig. 2 (a)). Then, finite conductance is only restored close to the phase transition (see the light-blue curve). In case of a four terminal measurement (of the local linear conductance), the two phases are more prominently separated: in the trivial phase, the conductance is fixed to \(1/2\) per channel, which implies perfect transmission of an incoming mode (in contact 2) to all the other channels. In this area of the phase space, TR symmetric processes dominate the physics. Thus, backscattering in the same channel is strongly suppressed. However, when the system undergoes the Floquet topological transition by crossing the red line of Fig. 2, perfect conductance quantization is lost and the scattering is dominated by the TR breaking Zeeman field. Then, the conductance significantly differs from \(1/2\).

**Bound state.**—The change in the conductance can be directly associated with the presence of a topological boundstate at the interface between a trivial and a topological regime, i.e. at the ends of the QPC. However, as our system is connected to leads, talking about bound states in the strict sense is misleading. To better visualize the topological state, which is responsible for the change of the conductance, we consider, within the constricted region, a phase boundary between a topologically gapped and a trivially gapped regime in the center of the QPC (at \(x = L/2\), see Fig. 3). More explicitly, we solve the scattering problem according to Eq. (14) with \((eA(x) - B_2(x))/(eA(x)) = 1/2\mathrm{sign}(L/2 - x)\) for the Floquet modes \(\phi_{m}^{\alpha}(x)\) up to order \(m\). Then, the solution \(\Psi_{\epsilon}^{m}(x,t)\) of the time dependent problem is given by

\[\Psi_{\epsilon}^{m}(x,t) = e^{-i\epsilon t} \sum_{n=-m}^{m} \phi_{\epsilon,n}^{m}(x)e^{-in\omega t}.\]  

(19)

As we are working within the framework of an open system, we find a solution to the scattering problem for any \(\epsilon\). We can name a particular solution a bound state when there is a local maximum that grows as the gap inducing parameters \(g\) are increased \[50\]. Then, for the limit \(g \rightarrow \infty\), this particular solution becomes a true bound state around its former local maximum. Fig. 3 shows the probability density of the solutions \(\Psi_{\epsilon}^{m}(x,t)\) for different \(\epsilon\). Note that we have chosen \(t = 1\). This choice is arbitrary. However, since \(\phi_{\epsilon,n}^{m}(x) \rightarrow 0\) already for moderately large \(n\), the solution depends only weakly on \(t\). Fig. 3 clearly shows the presence of a mid-gap topological bound state associated to the Floquet topological phase transition. Interestingly, although dealing with non-equilibrium physics, this topological bound state is still separated from the continuum by a finite energy gap.

**Conclusions.**—To summarize, we have shown that a Floquet driven quantum point contact between helical edge states represents an experimentally accessible condensed matter setup hosting a Floquet topological quantum phase transition. Moreover, we have shown that transport measurements are able to detect the Floquet topological quantum phase transition which can be associated to the dynamical formation of topological bound-
ary states.

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Floquet scattering states

Valid incoming states from the left with \( x < 0 \), as well as a valid outgoing states for \( x > L \) are given by

\[
\psi_{\text{in},m}(x) = M_{m}^{\alpha}(x,0) D_{n,\alpha}^{m} \left[ M_{p}^{\alpha} \left( \tau_{0} \sigma_{0} - \tau_{z} \sigma_{z} \right) S_{m} \right] \tag{20}
\]

\[
\psi_{\text{out},m}(x) = M_{m}^{\alpha}(x,0) \left[ M_{m}^{\alpha} \left( \tau_{0} \sigma_{0} + \tau_{z} \sigma_{z} \right) S_{m} \right] \tag{21}
\]

with

\[
S_{m} = (t_{1}^{m}, r_{1}^{m}, t_{2}^{m}, r_{2}^{m}, \ldots, t_{1}^{-m}, r_{1}^{-m}, t_{2}^{-m}, r_{2}^{-m})^T \tag{22}
\]

\[
D_{n,\alpha}^{m} = \sum_{l=1}^{4} \sum_{k=-m}^{m} \delta_{n,k} \delta_{l,\alpha} \hat{e}_{k} \otimes \hat{e}_{l}, \tag{23}
\]

where \( \hat{e}_{k} \) are cartesian basis vectors and \( \alpha \in \{1, 2\} \) is the lead index. The matrix \( M_{m}^{\alpha}(x,0) \) is the transfer matrix calculated from the system without QPC potentials, i.e.

\[
M_{m}^{\alpha}(x,0) = \exp \left[ \int_{0}^{x} \frac{dx}{iP_{m}} \frac{P_{m}^{-1} (\epsilon - Q_{m}^{\alpha})}{2} \right] \tag{24}
\]

with the matrix elements \( [Q_{m}]_{k,l} = \delta_{k,l} k \omega \) for \( |k| \leq |m| \).