Time analysis of dynamic response of spatial frame building under action of seismic forces

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Abstract. An analytical approach to the calculation of the spatial framework for the action of seismic forces is considered. This approach is an extension of the previously proposed time analysis method and used to determine the dynamic response of a dissipative structure modeled by a discrete computational scheme. Mathematical models of oscillations are given: the differential equation of motion of a dissipative system, the matrix characteristic equation with a quadratic input parameter and the fundamental matrix that carries out the relation between the differential and characteristic equations. Accounting for internal friction is carried out in accordance with the model of non-proportional damping. The structure of the vector of seismic forces generated by earthquake accelerogram data is shown. The kinematic and force characteristics of the system are determined by the equations of the seismic reaction of the framework at any time. The parameters of the calculated dynamic model (stiffness, damping and mass matrices’ elements) are constant on the entire interval during the calculation of elastic seismic response. Dynamic reaction equations are constructed in an analytical form based on the Duhamel integral. An example is given of analyzing vibrations of a 3-storey spatial framework under the action of a seismic load.

1. Introduction
Natural impacts in the form of seismic forces require solving the problem of improving the seismic resistance of buildings and structures. This problem is solved by the development of anti-seismic measures, such as the creation of special devices, for example, energy absorbers, vibration dampers, seismic insulating supports, viscous friction dampers, etc., which increase the reliability and survivability of structures of buildings and structures during earthquakes. Another direction in solving this problem is to develop more efficient methods for calculating structures for seismic effects. Many scientists both in Russia [1-8] and abroad [9-12] are engaged in studying and solving this problem.

Seismic oscillation problems are solved, as a rule, using finite element method procedures [13,14], which are divided into two main areas: modal analysis methods used for linear or slightly nonlinear problems [15,16], and direct integration methods (explicit or implicit), representing the equations of motion in increments and focused on solving highly nonlinear problems [17,18].

Analytical methods in seismic oscillation problems are not used because of the complication of constructing fundamental solutions, especially if dissipative forces are taken into account that do not obey the law of proportional damping. In this case, the equations of motion, the system of homogeneous ordinary differential equations (ODE), are not reduced to normal coordinates [19]. The second reason is the difficulty of obtaining a solution when integrating a system of coupled
inhomogeneous ODEs containing arbitrary right-hand sides as load functions. In spite of this many experts [20, 21] point to the need for theoretical studies, therefore, the task of developing effective analytical methods is quite relevant.

The article attempts to use the results of research in the field of dynamic calculation of dissipative structures obtained in the scope of the theory of time analysis for seismic oscillation problems. The developed method [22] is based on the study of the matrix quadratic equation (MQE), which is characteristic with respect to the system of homogeneous ODEs With elastic vibrations of the system the parameters of the calculated dynamic model CDM (stiffness, damping and mass matrices’ elements) are constant at each integration step. In the case of a non-linear process, the adjustment of the restoring and dissipative forces should be carried out by changing the elements of the stiffness and damping matrices. If necessary, the inertia forces and, associated with them, the elements of the mass matrix are adjusted. At the same time, in addition to changes in the parameters of the CDM, it is necessary to carry out a mandatory change of initial conditions.

It should also be noted that, according to the developed time analysis method (TAM), the expression of the Duhamel integral has a simple mathematical form of writing and is obtained in a closed form, without resorting to spectral decomposition of the solution of a dynamic problem. This is true for any type of damping, both proportional and non-proportional.

2. Mathematical models of oscillations of a dissipative system

The problem of elastic oscillations of a framework modeled by a discrete dissipative system for n degrees of freedom is considered. External load on the right side of the equation of motion is set using the accelerogram. Then the equation of motion of such a system at the ith integration step \( t \in [t_i, t_{i+1}] \) and the initial conditions are written as:

\[
\begin{align*}
\dddot{Y}(t) + C \dddot{Y}(t) + KY(t) = P(t) = -\text{diag}(M)J \Delta(t), \\
Y_0 = Y(t_i), \quad \dddot{Y}_0 = \dddot{Y}(t_i)
\end{align*}
\]

(1,2)

here, the matrices \( M, C, K \) express, respectively, the inertial, damping and stiffness properties of discrete dissipative system DDS; \( Y(t) \) is the displacement vector; \( J \) - vector of influence coefficients; \( \Delta(t) \) – acceleration of the frame base, given by the accelerogram.

At each step \( t_i \), the external load changes along with the changing value \( \dddot{\Delta}(t_i) \). Therefore, it is represented by a rectangular pulse \( P(t) = P_0 \), the parameters of which depend on its length and amplitude. For the \( k \)-th component of the degree of freedom, the element of the vector \( P_0 \) will be determined by the length equal to the integration step \( \Delta t = t_{i+1} - t_i \) and the amplitude of the pulse:

\[
P_{0k} = -m_kJ_k \dddot{\Delta}(t_i).
\]

(3)

where \( k \) – the number of the degree of freedom of the system. During the step integration, the initial conditions (2) are adjusted at each time point.

The fundamental matrix for the homogeneous equation of motion of (1) has the form \( \Phi(t) = e^{St} \), where the matrix \( S \) satisfies the characteristic MQE:

\[
MS^2 + CS + K = 0.
\]

(4)

The matrix \( S \) completely determines the behavior of the dynamic structure, since its spectral decomposition (in the basis of eigenvectors) contains the parameters of the structure's natural oscillations: frequencies, forms, and damping coefficients.

3. The parameters of dynamic response of a system

For the problem with initial conditions (1), (2), the resolving equations of the seismic response at the \( i \)-th integration step \( t \in [t_i, t_{i+1}] \) are written as:
\[ \begin{align*}
Y(t) &= 2 \text{Re}\{Z(t)\}, \quad \ddot{Y}(t) = 2 \text{Re}\{SZ(t)\}, \\
\dot{Y}(t) &= 2 \text{Re}\{S^2Z(t)\} + M^{-1}P(t)
\end{align*} \]

(5)

here: \( i = t - t_i; \quad \Phi(i) = e^{si}; \quad U = MS + S^2M + C; \quad P(t) = -\text{diag}(M)J\Delta(t). \)

The reaction with forced oscillations (the 2nd addend of the vector function \( Z(i) \) in (5)) describes the action of the impulse load \( P_0(t_i) \) at the \( i \)-th integration step, which coincides with the accelerogram digitizing step ground motions.

The seismic response at the \( i \)-th integration step is calculated as follows. The DDS reaction equations at the end of the \((i - 1)\)-th step are taken as the initial conditions (2) for the \( i \)-th step. The pulse amplitude in formula (3) is set at \( t_i \) from the accelerogram of an earthquake. The expressions for the calculated reactions (5) serve as the basis for constructing the initial conditions at the next \((i + 1)\)-th integration step.

Equations (5) allow us to determine the displacement, velocity and acceleration of the mass of the frame at an arbitrary point in time. According to the kinematic parameters, the restoring, dissipative and inertial forces are calculated, written in vector form:

\[ 
R(t) = KY(t), \quad F(t) = C\ddot{Y}(t), \quad I(t) = -MY(t). 
\]

(6)

Based on the kinematic (5) and force (6) parameters, the forces and stresses in the supporting elements of the frame are determined. This will make it possible to analyze the strength of the structure and its design.

4. Example

As an example, we consider the CDM of a 3-story framework, which is a system with 46 degrees of freedom (figure 1). Seven degrees of freedom are associated with the movement of floor overlaps in the horizontal direction and other 39 degrees of freedom are associated with the movement of masses in the vertical direction. The columns have a cross-section of four equal angles \( 150 \times 150 \times 10 \) mm welded into a box profile, girders have a cross-section in the form of I-section \( 40 \, \text{Ш} \), floor overlaps are reinforced concrete slabs thickness 220 mm.

The reinforced concrete floor slabs in their planes are treated as absolutely hard disks. In the transverse direction it works as flexible plates. This method allows, by means of columns deformed in the longitudinal direction, to take into account the yielding of the base. The damping pattern defined by matrix \( C \) is adopted in accordance with the model of non-proportional damping [22].

As an external load, an earthquake with a capacity of 7 points is considered, given by means of a three-component accelerogram (figure 2). The step of digitizing the acceleration graphs is 0,0125 s, the duration of the seismic action is 131,25 s.

Figure 1. Model of a frame building.
Based on the results of the time analysis, kinematic (5) and force (6) parameters of the framework reaction are obtained. Figure 3 shows the oscillograms of the horizontal (along the axes OX and OY) displacements of the centers of gravity of the floor overlaps. Figure 4 shows the oscillograms of velocities and accelerations of the centers of gravity of the floor overlaps.

The largest displacements, velocities, and accelerations of the centers of gravity of the floor overlaps occur along the OX axis. The maximum horizontal displacement of the centers of gravity of the floor overlaps is 4.657 cm (figure 3, a), the maximum vertical displacements do not exceed 0.018 cm.

Figure 2. 3-component accelerogram of 7-point earthquake.

Figure 3. Oscillograms of a nodal displacements of the centers of gravity of floor overlaps: (a) along the OX axis; (b) along the OY axis.

Figure 4. Oscillograms of node velocities (a) and accelerations (b) of the centers of gravity of floor overlaps along the OX and OY axis respectively.
The numerical estimate of the solution method was obtained using the vector function $f(t) = R(t) + F(t) - I(t)$, which represents the algebraic sum of forces (6) of the left side of the differential equation of motion (1). Comparison of the vectors of the forces of the left and right sides of the equation (1) shows that the absolute value vector discrepancy

$$\Delta f(t) = R(t) + F(t) - I(t) - P(t)$$

does not exceed $3.5 \times 10^{-11}$ kN (figure 5).

5. Conclusions

1. In the scope of the theory of time analysis, a method for analyzing the seismic response of a dissipative structure modeled by a discrete computational scheme is proposed.
2. The expression of the seismic response has a closed form of the Duhamel integral, regardless of the damping pattern.
3. The time analysis of the reaction of the model of a 3-storey spatial frame building in the process of a 7-point earthquake, specified by the corresponding three-component accelerogram, was performed.
4. Oscillograms of various parameters of the seismic response of the framework, including discrepancies of the left and right sides of equation (1), were obtained, confirming the effectiveness of the method.

Thus, the TAM allows for multilateral and in-depth analysis of the dynamic response of complex spatial systems using real and synthesized accelerograms. It can be recommended to design offices when calculating the structures of buildings in earthquake-resistant construction.

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