Quantum Production of Black Holes

Stephen D.H. Hsu*
Department of Physics
University of Oregon, Eugene OR 97403-5203

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Abstract

We give a path integral expression for the quantum amplitude to produce a black hole from particle collisions. When expanded about an appropriate classical solution it yields the leading order contribution to the production amplitude in a curvature expansion. Classical solutions describing black hole production resulting from two particle scattering at non-zero impact parameter, combined with our formalism, indicate a geometric cross section for the quantum process. In TeV gravity scenarios these solutions may exhibit large curvatures, but (modulo a mild assumption about quantum gravity) corrections to the semi-classical cross section are small.

*hsu@duende.uoregon.edu
1 Introduction

Recently proposed models with extra dimensions solve the hierarchy problem by bringing
the fundamental scale of gravity (henceforth referred to as the Planck scale) down to the
electroweak scale [1]. In such scenarios, quantum gravitational effects arise at energies as low
as a TeV. Perhaps the most dramatic example of such phenomena is black hole production
in particle collisions with center of mass energy greater than the Planck scale [2],[3]. Such
events would lead to dramatic signatures at colliders and in cosmic ray collisions, and perhaps
imply an end to our ability to probe shorter and shorter length scales [4]-[7].

The cross section for black hole production in high energy collisions is difficult to compute.
In [2]-[7] it was asserted that the cross section is geometrical, determined by the impact
parameter at which the particle pair at closest approach is within the Schwarschild radius
associated with the center of mass energy \( \sqrt{s} \). If this is the case, black holes would be
copiously produced at LHC and in cosmic ray collisions. However, Voloshin has criticized
these claims, arguing for an exponentially small cross section [8].

Eardley and Giddings [9] have analyzed classical solutions in general relativity which
describe two particle high energy collisions at non-zero impact parameter (see also [10]-[14]).
They demonstrate the existence of a closed trapped surface for any collision with sufficiently
small impact parameter (at fixed center of mass energy). Their lower bound on the critical
impact parameter leads to a geometrical classical cross section in rough agreement with the
earlier naive estimates.

The solutions in [9] yield immediate answers to Voloshin’s two main objections: (see also
[15] for a model calculation that shows no exponential suppression)

1) Euclidean suppression: because there are classical trajectories with two particle initial
conditions which evolve into black holes, the process is clearly not classically forbidden, and
hence there is no tunnelling factor.

2) CPT (time reversal): Voloshin argues that since black holes produce a thermal spec-
trum of particles during evaporation, rather than a few highly energetic particles, the time-
reversal of the production process (and hence the production process itself, by CPT) must
have very low probability. However, the time-reversed classical solutions exhibit a very ener-
getic wave of gravitational radiation colliding with the time-reversed black hole to produce
the two particle state (in the formation process this is the energy which escapes the hole).
The process is not thermal and involves very special initial (final) conditions.

In this letter we use a path integral formalism previously developed by Gould, Hsu
and Poppitz [16, 17] (GHP) to study non-perturbative scattering. In the GHP formalism
quantum S-matrix elements are computed in a systematic expansion about classical solutions satisfying appropriate boundary conditions. We show that the formalism is readily adapted to the problem of black hole production from high energy collisions.

Although some black hole production amplitudes (for example, involving many soft initial particles) can be computed unambiguously from classical solutions, in the two particle case quantum corrections might be significant due to large curvatures. However, we argue that even in this case the semi-classical approximation is probably a good one. We explain the importance of large curvatures to the quantum corrections from an effective field theory point of view.

2  Path Integral Formalism

In this section we review a method formulated to describe scattering processes involving non-perturbative field configurations using the path integral \[16, 17\]. Previous interest in this problem centered on baryon number violation in the electroweak theory. In this context, it was realized that classical configurations satisfying vacuum boundary conditions (such as instantons) were unsuitable for the computation of high energy scattering \[18\]. A good semiclassical approximation in this context requires taking into account the initial state overlap as well as the classical action.

In \[14\], an exact expression is derived for the S-matrix describing a transition from an initial two particle state to a coherent final state. (See below for a precise definition of coherent state.) This expression is approximated in the usual stationary phase approximation, which leads to a boundary value problem involving the usual classical equations of motion, but with boundary conditions determined by the initial and final states. Unfortunately, numerical searches for “interesting” solutions of this boundary value problem in lattice gauge theories have only uncovered configurations describing transitions between states consisting of many soft quanta \[19\]. The Eardley-Giddings result is the first we are aware of in which a high momentum initial state evolves into a large, low momentum final state. However, we should note that the two particles in the initial state are “dressed” by strong gravitational fields, so the effective number of gravitons in the initial state is actually large.

We first give a brief review of the general method, before addressing issues particular to general relativity and black holes. In the path integral representation of the process \(|i\rangle \to |f\rangle\), trajectories are weighted by the appropriate action \[\exp[iS]\], as well as by the overlap of the asymptotic part of the path with the initial and final states. Therefore we
expect that the amplitude must be expressible in the form \[17\]:
\[
\langle f | S | i \rangle \sim \int d\phi_i \, d\phi_f \, D\phi \, \Psi_i[\phi(T_i)] \, \Psi_f^*[\phi(T_f)] \, e^{iS[\phi]},
\]
where we have explicitly indicated the fluctuations of the fields in the asymptotic past and future \((T_{i,f})\) in the measure. The wave-functionals \(\Psi_{i,f}\) measure the overlap of the initial and final states with \(\hat{\phi}\) eigenstates at asymptotic times. In \[16\] a derivation of \((1)\) was given, along with the explicit form of the wavefunctionals in the case that the initial and final states are either wave-packets (including plane waves) or coherent states.

In the limit \(T_{i,f} \to \pm \infty\), the amplitude in \((1)\) is just the S-matrix, \(S_{fi}\). The GHP procedure amounts to a semi-classical evaluation of this object taking into account the initial and final state overlaps in the extremization. The result is a boundary value problem with boundary conditions determined by the initial and final state, but governed by the usual equations of motion. Here our goal is to calculate the S-matrix element between an initial two particle state and a final state which includes a black hole.

First, we express the kernel of the S-matrix in a basis of coherent states (see, e.g., the text by Fadeev and Slavnov \[20\]). The initial and final states are defined by sets of complex variables \(a \equiv \{a_k\}, b^* \equiv \{b^*_k\}\), respectively. A coherent state \(|a_k\rangle\) is an eigenstate of the annihilation operator \(\hat{a}_k\): \(\hat{a}_k |a_k\rangle = a_k |a_k\rangle\). Recall that coherent states saturate minimum-uncertainty bounds, and hence are good (although not unique) candidates for semi-classical states.

The transition amplitude from an initial coherent state \(|a\rangle\) at time \(T_i\) to a final coherent state \(|b^*\rangle\) at time \(T_f\), can be expressed
\[
\langle b^* | U | a \rangle = \int d\phi_i \, d\phi_f \, \langle b^* | \phi_f \rangle \langle \phi_f | U | \phi_i \rangle \langle \phi_i | a \rangle,
\]
where \(U\) is the evolution operator between time \(T_i\) and \(T_f\). A “position” eigenstate of the field operator \(\phi\) is denoted \(|\phi\rangle\) and \(\phi_{i,f} = \phi(T_{i,f})\). Then, from \((2)\), we obtain the S-matrix kernel in a compact form in terms of path integrals
\[
\langle b^* | S | a \rangle \equiv S[b^*, a] = \lim_{T_i, T_f \to \pm \infty} \int d\phi_f \, d\phi_i \, e^{B_f} \, e^{B_i} \, \int_{\phi_i}^{\phi_f} D\phi \, e^{iS[\phi]},
\]
where \(S[\phi]\) is the action functional. The path integral appearing here is over fields obeying the boundary conditions \(\phi(T_{i,f}) = \phi_{i,f}\). The functional \(B_f\) is
\[
B_f[b^*, \phi_f] = -\frac{1}{2} \int d^3k \, b^*_k b_k \, e^{2i\omega_k T_f} - \frac{1}{2} \int d^3k \, \omega_k \, (\phi_f(\vec{k}) \, \phi_f(-\vec{k}))
\]
\[
+ \int d^3k \, \sqrt{2\omega_k} \, e^{i\omega_k T_f} \, b^*_k \, \phi_f(-\vec{k}),
\]
in terms of which the wave functional of the final coherent state is \( \langle b^* \mid \phi_f \rangle \equiv \exp \left( B_f [b^*, \phi_f] \right) \). Similarly, the functional \( B_i \) can be expressed in terms of the initial coherent state wavefunctional \( \langle \phi_i \mid a \rangle \equiv \exp (B_i [a, \phi_i]) \), with \( b_k^* \) replaced by \( a_k \) and \( T_f \) by \(-T_i \) in (4). The 3-dimensional Fourier transform is defined
\[
\phi_{i,f}(\vec{k}) = \int \frac{d^3x}{(2\pi)^{3/2}} e^{ik\cdot x} \phi(T_{i,f}, \vec{x}) .
\]

(5)

The kernel (3) is a generating functional for S-matrix elements between any initial and final \( N \) particle states, by functional differentiation with respect to arbitrary \( a_k \) and \( b_k^* \). We now use this fact to construct a kernel for scattering from initial two particle states. We define an initial two particle (wave packet) state at \( t = T_i \)
\[
|\vec{p}, -\vec{p} \rangle \equiv \int d^3k \alpha_R(\vec{k}) \hat{a}_{\vec{k}}^\dagger \int d^3k' \alpha_L(\vec{k}') \hat{a}_{\vec{k}'}^\dagger |0\rangle ,
\]
where \( \hat{a}_{\vec{k}}^\dagger \) is a creation operator, and \( \alpha_{R,L}(\vec{k}) \) are arbitrary smearing functions of \( \vec{k} \), localized around some reference momenta \( \vec{p} \) and \( -\vec{p} \) respectively. The wave packets are normalized so that \( \int d^3k |\alpha_{R,L}(k)|^2 = 1 \).

This state can be generated by functional differentiation of the coherent state \( |a \rangle \) with respect to \( a_k \)
\[
|\vec{p}, -\vec{p} \rangle = \int d^3\vec{k} d^3\vec{k}' \alpha_R(\vec{k}) \alpha_L(\vec{k}') \frac{\delta}{\delta a_k} \frac{\delta}{\delta a_{k'}} |a\rangle \bigg|_{a=0} .
\]

(7)

So, differentiating under the functional integral, the S-matrix element between the two particle state (7) and any final state \(|\{b_k^*\}\rangle\) involves the following functional at \( t = T_i \)
\[
\frac{\delta}{\delta a_k} \frac{\delta}{\delta a_{k'}} \exp \left( B_i [a, \phi_i] \right) \bigg|_{a=0} = \frac{2}{\sqrt{\omega_k \omega_{k'}}} \phi_i(\vec{k}) \phi_i(\vec{k}') e^{-i(\omega_k + \omega_{k'})T_i} \exp \left( B_i [0, \phi_i] \right) ,
\]
after dropping a term which vanishes in the limit \( T_i \to -\infty \). The last factor here is simply the normalization of the initial position eigenstate
\[
\exp \left(-\frac{1}{2} \int d^3k \omega_k \phi_i(\vec{k}) \phi_i(-\vec{k}) \right) .
\]

(9)

We combine this with the smearing functions and finally obtain an S-matrix kernel for the scattering of two wave packets into arbitrary final states,
\[
S [b^*, 2] = \lim_{T_i, T_f \to -\infty} \int d\phi_f d\phi_i \alpha_R \cdot \phi_i \alpha_L \cdot \phi_i e^{B_f[b_i, \phi_f]} + B_i[0, \phi_i] \int_{\phi_i}^{\phi_f} D\phi e^{iS[\phi]} ,
\]

(10)
where we have denoted the initial state \( |2\rangle \) by “2”. Here we have used the following compact notation for the initial state factors

\[
\alpha \cdot \phi_i \equiv \int d^3k \sqrt{2\omega_k} \alpha(k) \phi_i(k) e^{-i\omega_k T_i}.
\]  

(11)

We exponentiate the initial state factors into an “effective action”, so that

\[
S[b^*, 2] = \lim_{T_i, T_f \to \pm \infty} \int d\phi_f d\phi_i D\phi e^{\Gamma},
\]

(12)

where the effective action \( \Gamma \) is

\[
\Gamma[\phi] = \ln \alpha_R \cdot \phi_i \alpha_L \cdot \phi_i + B_i[0, \phi_i] + i S[\phi] + B_f[b^*, \phi_f],
\]

(13)

after dropping a term which vanishes as \( T_i \to -\infty \).

We can now derive the boundary value problem by varying the effective action. Varying the entire exponent \( \Gamma \) with respect to \( \phi(x) \) for \( T_i < t < T_f \) gives the source-free equations of motion

\[
\frac{\delta S}{\delta \phi(x)} = 0.
\]

(14)

Varying the entire exponent with respect to \( \phi_i(k) \), gives

\[
i \dot{\phi}_i(\vec{k}) + \omega_k \phi_i(\vec{k}) = \sqrt{2\omega_k} \left( \frac{\alpha_R(\vec{k})}{\alpha_R \cdot \phi_i} + \frac{\alpha_L(\vec{k})}{\alpha_L \cdot \phi_i} \right) e^{-i\omega_k T_i}.
\]

(15)

The first term on the left hand side comes from a surface term in the action \( S \). The other terms come from variation of the wave functional at \( t = T_i \). This boundary condition involves both the positive and negative frequency parts of the field.

The boundary condition (13) at the initial time slice is rather complicated. However, it can be simplified since a real field \( \phi \) may be written in the asymptotic region \( t = T_i \to -\infty \) as a plane wave superposition

\[
\phi_i(\vec{k}) = \frac{1}{\sqrt{2\omega_k}} \left( u_k e^{-i\omega_k T_i} + u_k^* e^{i\omega_k T_i} \right).
\]

(16)

Equation (13) then reduces to the requirement

\[
u_k = \frac{\alpha_R(\vec{k}) + \alpha_L(\vec{k})}{\left( 1 + \int d^3k \alpha_R(\vec{k}) \alpha_L(\vec{k}) \right)^{1/2}},
\]

(17)

using the normalization condition on \( \alpha_{L,R} \). This solution is consistent with physical intuition, the classical field reducing to the initial particles at early times. The overlap of the left- and
right-moving wave packets in the denominator is very small for narrow high energy wave packets.

A similar analysis relates the late time boundary condition on $\phi$ to the coherent state $b^* [16]$. The classical field satisfying these boundary conditions extremizes the S matrix for production of the coherent state in a two particle collision: $S[b^*, 2]$. Note that for arbitrary choice of $b^*$ there is no guarantee of a classical solution satisfying the necessary boundary conditions: in some cases a complex trajectory extremizes the S-matrix, leading to exponential suppression of the process [16]. However, conversely every classical solution obeying initial conditions (16)-(17) corresponds to an unsuppressed quantum amplitude.

Now consider the extension of this formalism to general relativity, and to black hole production. Clearly one can replace $\phi$ with the metric field $g_{\mu\nu}$ plus appropriate matter fields. While general covariance does not permit a unique time slicing, the gravitational action $S_g = \int d^4x \sqrt{-g} R$ is still well-defined and, in fact, the action can be expressed in terms of a surface integral over Bondi masses [21].

The notion of an S-matrix is appropriate if we consider asymptotically flat spacetimes in the far past and future, plus additional excitations. Black holes themselves are considered excitations, and we must extend our Hilbert space to include quantum states representing black holes. A pragmatic way to approach this is to define (semi-classical) black hole states as those with a strong overlap with the trajectories corresponding to classical black holes. (These must of course have mass much larger than the Planck mass.) In a classical black hole solution, excess energy is radiated away by late times and the exterior metric can be classified by a limited number of quantum numbers such as mass, charge and angular momentum. A minimal formulation involves a Hilbert space of black holes classified by their exterior metric at future null infinity.

Although it is beyond the scope of this letter, it is worth noting that a more detailed analysis of black hole states, which takes account of the internal structure of the black hole (i.e. the fields inside the horizon, not just at future null infinity), indicates more states than are counted by externally visible quantum numbers. That is, if one considers the internal structure of the black hole in defining the Hilbert space, there are many additional semi-classical states that one might associate with a given exterior metric, but which differ radically within. Gedanken experiments involving semi-classical black holes in this formalism might teach us something about black hole information. For example, relative phases and interference patterns due to internal structure might be observable in black hole scattering.

Any classical solution connecting particle-like initial conditions to an asymptotic black hole configuration provides an extremal configuration about which to expand the S-matrix;
the leading contribution is a pure phase with no exponential suppression \[16\]. In the next section we consider quantum corrections to this leading semi-classical approximation.

3 Quantum corrections

In the original application of \[16\] to quantum fields in flat spacetime, it was shown that the quantum corrections to the semi-classical approximation to the S-matrix are suppressed by powers of the coupling constant. In particular, the corrections can be expressed in terms of propagators and interaction terms which result from expanding about the classical solution. All interaction terms carry explicit powers of the coupling, resulting in a well-defined loop expansion.

In general relativity there is of course no small dimensionless parameter. Expanding about a background configuration yields interactions which are suppressed by the background curvature in Planck units. Classical solutions describing the ordinary gravitational collapse of many “soft” particles (e.g., collapse of a large star or dust ball) can produce black holes without regions of large curvature. Our formalism applies directly to such solutions, resulting in a semi-classical amplitude without large quantum corrections.

In the two-particle solutions of \[9\], regions of large curvature can arise quite early in the evolution (e.g., when shock fronts collide), even if the black hole produced is large \((\sqrt{s} >> M_{\text{Planck}})\). If one takes the size of the colliding particles to be of order the Planck length \(L\), one finds curvatures at the shock front of order \(s\). In this case, quantum corrections might be large. In fact, we run into a fundamental problem concerning quantum gravity. Because gravity is non-renormalizable, we have to consider all possible generally covariant higher dimension operators in our lagrangian, such as higher powers of the curvature. Certainly, such terms will arise from the ultraviolet part of any loop calculation and will presumably be only partially cancelled by counterterms. In an effective lagrangian description, we expect these operators to be present, but suppressed by powers of the Planck scale. In large curvature backgrounds these terms may not be negligible, so the size of quantum corrections will in principle depend on unknown details of quantum gravity.

We can state this conclusion in a slightly different way. Consider the classical evolution of some initial data in an effective low-energy description of gravity. We can only trust the Einstein equations (which result from the lowest dimension term in the effective lagrangian) if large curvatures are never encountered during the evolution. Once a region of large curvature is encountered, subsequent evolution might depend in detail on the nature of the higher
dimension operators, and hence on the nature of quantum gravity\footnote{This is quite similar to the case of long-wavelength classical configurations in the QCD chiral lagrangian \cite{22}. There, one must be sure that no high-frequency bunching of modes occurs; otherwise the evolution becomes sensitive to higher order terms.}. In \cite{9} it is suggested that since a closed trapped surface is identifiable in a low-curvature region, the classical solution is a good guide to the true quantum behavior. Strictly speaking, this is not sufficient – the solution has already evolved through a potentially high curvature shockwave region.

It is reasonable to expect that short distance features of the metric which lead to high curvatures do not affect the classical Einstein evolution of the solutions in \cite{9} on large length scales, such as of order $b$, the impact parameter. The large distance behavior of the Aichelburg-Sexl metric \cite{10} used in \cite{9} is independent of the short distance features of the particle as long as its size $r$ is much smaller than $b$. In fact, Kohlprath and Veneziano have recently extended the Eardley-Giddings construction to the case of colliding particles of finite size \cite{23}.

In TeV gravity scenarios $r$ is likely to be the Planck length. However, we can instead consider collisions of particles of size much larger than the Planck length but much less than $b$. (In other words, “colliding Jupiters” at ultra-relativistic velocities, with $b$ much larger than the radius of Jupiter!) By adjusting the impact parameter and particle size relative to the Planck length (while keeping a large hierarchy between the three) we can keep the shockwave curvatures parametrically smaller than $M_{\text{Planck}}^2$ while preserving the long distance behavior that leads to horizon formation. In Planck units, the maximum curvature in the shockwave is $R \sim (b/r)^2 (L/r)^2$, where $b \sim L^2 \sqrt{s}$ and $L$ is the Planck length. By independently varying $\sqrt{s}$ and $r$, we can make the first ratio large and the second small, while keeping their product small. The semi-classical approximation applies quite well in this limit and higher dimension operators can be neglected.

Barring unexpected quantum gravitational effects which are sensitive to the size of the objects colliding (i.e. that make long distance behavior dependent on the short distance metric), the quantum cross section for black hole production will be well approximated by the semi-classical one even in TeV gravity scenarios.

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