Fundamental parameters modeling for the lunar telescope

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Abstract. The powerful flow of highly accurate and multiparameter information produced by spacecrafts has caused a surge interest in the industrial robotic exploration of the Moon and manned flight to Mars after the creation of long-term lunar bases. The modern level of both ongoing and planned lunar studies is characterized by a high level of observation accuracy and a large variety of observation methods. The study of the Moon’s rotational parameters (MRP) is of important value. For this reason, new methods for analyzing big data sets on observation of MRP and extracting the highest possible amount of scientific information from them are needed. Such space technologies require the creation of in-situ telescopes on the Moon. The long-term laser measurements have supplied comprehensive observational information about the Moon. This allows for a description of the Moon’s dynamics with the accuracy required at the current stage – the error of determining the distance to the Moon should be less than a meter, while the one of establishing rotational parameters – arc milliseconds. Nevertheless, there is a necessity to obtain observation data independent of laser measurements. One of the ways to do it is to place on the lunar surface one or several optical telescopes, that will allow determining lunar rotational parameters by measuring the trajectories of the stars and will also be used to solve astrophysical and astrometric problems in the future. This work considers the results of computer modeling of observations taken by lunar in-situ telescopes located at various selenographic latitudes. The sensitivity of MRP to the observed selenographic coordinates of the stars is assessed. Based on the analysis of the simulation results, the optimum location of the telescope is concluded to be at a latitude of 30–45°. The constructive suggestions on the described experiment’s implementation are presented in the paper. The details that will allow implementing or declining the practical implementation of the in-situ telescope are discussed.

1. Introduction

When studying celestial bodies, in the case when methods of direct measurements and experiments are not always available, methods of mathematical and computer modelling provide great opportunities. In the modern information society, when information technologies become decisive in the development of any branches of human activity, computer modelling makes allows, without significant material costs, to assess possible problems and complexities of planned space experiments and to identify their potential for solving certain fundamental problems. In this respect, the Moon, as the celestial body closest to the Earth, is the object that space agencies tend to study.

To solve the fundamental and commercial problems, NASA plans to implement the LSITP (Lunar Surface Instrument and Technology Payloads) project, which implies the delivery of landing modules to the Moon with a different set of measuring equipment, in particular, for mining [1].
The Japanese space agency has been developing the ILOM (In-situ Lunar Orientation Measurement) mission for two decades. The aim of the project is to install a lunar zenith telescope at the south or north pole to measure the selenocentric positions of stars with much lower errors than observations from the Earth's surface. Based on these data, the parameters of the Moon's rotation are then determined [2, 3]. The Russian Space Agency is also planning on carrying out studies of MRP with the in-situ telescope.

Why exactly the Moon's rotation is the objective of many planned lunar experiments? This is explained by the fact that the parameters of the Moon’s rotation are some of the important sources of information about it [4]. Many aspects of the internal structure of the lunar body are manifested as low-amplitude variations of different frequencies [5].

When placing an in-situ telescope, it is important to choose its location so that all three parameters of the Moon's rotation (libration angles) are determined with the required accuracy. The authors of the article have extensive experience in collaborating with Japanese researchers on the ILOM project. The performed analysis showed that there are advantages when placing the telescope at the lunar poles, but at the same time, measurements of the selenocentric positions of the stars have poor accuracy in accounting for libration in longitude. And it is this parameter that contains a lot of information about the internal characteristics of the Moon, in particular, about the 3rd order harmonics of the selenopotential. For this reason, the problem on determining other possible positions of the in-situ telescope, at which all three parameters of physical libration could be obtained from observations of the selenographic coordinates of the stars [6], was solved.

In the course of the study, the following tasks were solved:
1) A mathematical apparatus is developed, with the help of which it is possible to calculate the observed selenographic coordinates obtained with a telescope located at any selected latitude of the Moon. In this case, the calculations are carried out using the analytical theory of physical libration;
2) The change in the trajectories of stars (their shape and speed) is studied depending on the location of the telescope and for a given observation period. This is performed with the aim of defining an effective observation schedule for a future space experiment;
3) The inverse problem of the physical libration of the moon (PLM) was solved by calculating the PLM parameters using already measured selenocentric stellar positions;
4) The analysis of sensitivity of the three PLM parameters to different locations of the lunar telescope was performed using the inverse problem solution method.

2. Algorithm for constructing star tracks for different latitudes
Depending on where the telescope is installed on the lunar sphere, the visibility of the daily motion of the stars will differ (Figure 1). In order to calculate exactly the type of trajectory, and most importantly, the speed of the star in the field of view of the telescope, it is necessary to draw up equations that would provide a connection between the rectangular selenographic coordinates of the star and the angles of physical libration of the Moon, which are determined on the basis of the analytical theory of the motion of the Moon.

In order to accurately solve the problem of creating an experimental model, it is necessary to take into account a number of criteria.

First, the rotation of the Moon is considered as the rotation of an absolutely rigid body, the position of which in space is set by the main axes of inertia of the Moon. This rotation is described by the analytical tables of the PLM theory by Petrova [7]. In this case, the axes of inertia form a Dynamic Coordinate System (DCS).

Secondly, the coordinates of the stars \( x_{1s}, y_{1s} \) are planned to be measured at a certain time interval, the value of which should also be established during the simulation of the experiment.

Third, the parameters of the Moon's rotation (librations in longitude \( \tau \), in latitude \( \rho \) and at the node \( \sigma \), which are small corrections to the traditional Euler angles) are represented in the form of Poisson series, which specify the dependence of RPM on the time and parameters of the gravitational field of
the Moon. Parameters of Poisson series: coefficients and indices are calculated according to the analytical theory of the Moon’s rotation [7].

Figure 1. Measuring the positions of stars with a telescope on the Moon. Stellar trajectories are shown in the field of view of a telescope installed at the pole, at mid-latitudes, and at the equator.

Using elements of analytical theory and geometric constructions [7], a system of equations was obtained for calculating the selenographic coordinates of stars in the field of view of the lunar telescope:

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\begin{align*}
\chi_{1s} &= a_1 (\cos \lambda^* \cdot \cos \varphi + \cos \Theta \cdot \sin \lambda^* \cdot \sin \varphi) - b_1 \sin \Theta \cdot \sin \varphi - c_1 \sin \Theta \cdot \sin \lambda^* - d_1 \cos \Theta \\
y_{1s} &= -a_2 \cos \lambda^* \cdot \sin \varphi + b_2 \cos \Theta \cdot \sin \lambda^* \cdot \cos \varphi - c_2 \sin \Theta \cdot \cos \varphi \\
z_{1s} &= a_3 (\cos \lambda^* \cdot \cos \varphi + \cos \Theta \cdot \sin \lambda^* \cdot \sin \varphi) - b_3 \sin \Theta \cdot \sin \varphi + c_3 \sin \Theta \cdot \sin \lambda^* + d_3 \cos \Theta
\end{align*}
\] (1)

Here \( \lambda^* \) and \( \lambda \) are ecliptic longitudes respectively star and telescope, \( \Psi = \lambda^* - \lambda \), \( \Theta \), \( \varphi \) are classic Euler angles at the node, slope and longitude. Coefficients \( a_i \), \( b_i \), \( c_i \), \( d_i \) are functions of the telescope’s selenographic \((l_T, b_T)\) and ecliptic \((\lambda, \beta)\) coordinates.

One of the difficult issues in modeling observations with telescopes installed at different latitudes is the choice of stars and, accordingly, the calculation of their ecliptic coordinates \((\lambda, \beta)\) that can fall into their field of view. Working with exact catalog data and complex time-consuming reductions to bring stars to a visible place in this case do not make sense, because the modelling is of an evaluative nature. Therefore, an algorithm was developed for generating a certain number of stars from random values of their “fictitious” ecliptic coordinates \((\lambda, \beta)\). These coordinates are calculated in the specified range of the observation band in the sky and at the corresponding observation time. The calculations took into account the daily rotation of the Moon and the precessional motion of its pole [8].

To determine the Euler angles, we used the values of the Poisson series coefficients of the PLM theory [7] and the model ecliptic coordinates of fictitious stars. According to equations (1), rectangular selenocentric coordinates \(x, y, z\) were determined (the \(z\) values are actually close to 1).

Then, using the calculated coordinates, the daily tracks of the stars were modeled for different locations of the telescope. This corresponds to solving the direct PLM problem. To study the necessary correlation between the sensitivity of RPM and the location of the observation point, it is necessary to conduct simultaneous observation at all selenographic latitudes (with a fixed values of RPM). An analysis of the obtained tracks and the time of passage of a star through the telescope’s field of view has shown that, when the telescope is azimuthally installed, the velocities of the stars at different latitudes are different. At the pole, the stars move slowly and their coordinates remain...
virtually unchanged for two to three days. At the equator, the stars quickly leave the telescope’s field of view, and within 15 minutes we can take only one or two pictures of the star.

In order to save computing time and efficiently use the computer’s RAM, a special observation method is developed, which is very difficult to implement in practice. At the same time, using computer modeling, this problem is solved accurately and without technical costs.

The basis of the computer method is as follows: modeling the installation of telescopes on the first meridian and at 7 latitudes of the Moon (from 0° to 90° with a step of 15°); then all the telescopes simultaneously and with given time interval (15 min) record coordinates of the stars falling into the corresponding telescope’s field of view. Observations continue for three hours. Then, the current time is changed by the value of the lunar day, i.e. we pass to another lunar day, and the same stars are recorded again during the first three hours of the new day. Thus, an accumulation of statistical data at a certain time interval for the same stars (Figure 2) takes place.

3. Analysis of stellar trajectories and solution of the inverse problem

Calculation of the selenographic coordinates of the stars is a part of the solution to the direct problem. A more important problem is the calculation of the parameters of the Moon’s rotation from the observed coordinates, since it is their behavior that provides important information about the hidden processes affecting the rotation of the lunar body. This part of the study is called the inverse problem.

The solution to equation (1) is the basis of the inverse PLM problem, but in this case \(x, y\) are known coordinates, while the unknowns are the libration angles \(\tau, \rho, \sigma\) (Physical librations are the oscillation about uniform rotation and precession. The physical librations are observed about all 3 coordinate axes: \(\tau\) - in longitude, \(\rho\) - in latitude, \(\sigma\) - in axial tilt) hidden in \(\Psi, \Theta, \varphi\).

The solution of the system of nonlinear equations (1) is implemented by the gradient method [9-10], which allows obtaining a stable solution with the required accuracy even for systems with a Jacobian close to zero. The analysis of the solution for the inverse problems provides some important information that must be taken into account when developing the planned experiment.

Below are some design considerations for telescope placement and observation schedule.

It was shown that if polar observations are good for determining the parameters of libration angles in the slope \(\rho\) and node \(\sigma\), but are absolutely insensitive to libration in longitude \(\tau\), then, when the telescope is installed at the lunar equator, it becomes possible to determine libration in longitude, as well as \(\rho\) and \(\sigma\). At the same time, in order to avoid the screening of stars by the Earth’s disk, the telescope must be installed not at zero latitude, but at a latitude of \(\pm3°\). However, one can get by with just one telescope installed at a latitude of \(30°–45°\). In this case, all three RPM are determined quite accurately.

It is necessary to emphasize, that there are technical problems that can complicate the installation of the telescope at low latitudes. Firstly, these are the rapid movements of stars at low latitudes, and in this case, it is very important to develop the necessary observation schedule.
Replacing the telescope’s azimuth setting with an equatorial one avoids the difficulties associated with high speeds of stars. However, for the operation of the clockwork, technical solutions for energy sources are required. Solar panels can provide such a source during the lunar day and will allow recharging the batteries that will keep the telescope working during the lunar night. At the same time, the necessary high-precision calculations are performed.

4. Summary and conclusions
Modern performed and planned exploration of the Moon requires the development of new methods for measuring the parameters of lunar rotation, the development of methods for analysing observations, the creation of methods for extracting useful information from observations with a high degree of accuracy and reliability [11]. The experience of the authors in computer simulation of astrometry observations using telescopes installed on the lunar surface opens up new opportunities for a preliminary assessment of the prospects for planned experiments [12, 13]. The result of modelling the installation of the lunar telescope is a list of tasks that must be solved during the implementation of the entire project [14]. At the same time, it was shown that it will be possible to perform measurements of the selenocentric coordinates of stars with high accuracy [15].

Based on the simulation of the experiment to determine the PLM, it was determined that the most effective installation of the lunar telescope at the middle latitudes of the Moon (30°–45°) has the best chances [16]. There one observes the best sensitivity of the libration angles to the change in selenographic coordinates [17]. Such sensitivity will make it possible to reveal subtle effects both in the structure of the lunar body and in external factors influencing the lunar dynamics [18].

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