The Electric Vehicle Routing Problem with Time Windows under Travel Time Uncertainty

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Abstract. Due to environmental friendliness, electric vehicles have become more and more popular nowadays in the transportation system. For many express companies, it is more and more important to meet the predetermined time window of customers. The uncertainty in travel times often causes uncertain energy consumption and uncertain recharging time, thus electric vehicles may miss the time windows of customers. Therefore, this paper addresses the electric vehicle routing problem with time windows under travel time uncertainty, which aims to determine the optimal delivery strategy under travel time uncertainty. To solve this problem, a robust optimization model is built based on the route-dependent uncertainty sets. However, considering the complexity of the problem, the robust model can only solve few instances including the small number of customers. Thus, a hybrid metaheuristic consisting of the adaptive large neighborhood search algorithm and the local search algorithm is proposed. The results show that the algorithm can obtain the optimal solution for the small-sized instances and the large-sized instances.

Keywords: The electric vehicle routing problem; The adaptive large neighborhood search; The local search; Robust optimization.

1. Introduction

Nowadays, more and more people and government departments have realized the importance of environmental protection. Thus, environmentally friendly electric vehicles (EVs) are gradually considered in the transportation system. However, compared to conventional petroleum-fuel-powered vehicles, EVs have a shorter driving range. Thus, recharging stations (RSs) are usually needed. Because of this unique feature, the electric vehicle routing problem (VERP) is distinctively different from the traditional VRP.

The EVRP was first proposed by Erdogan and Miller-Hooks [1]. After that, many researchers began to study the routing problem of EVs. As a variant, the electric vehicle routing problem with time windows (EVRPTW) was introduced by Schneider, Stenger, and Goeke [2]. Felipe et al. [3] considered kinds of recharging technologies and partial recharges of the RSs in EVRP. The mixed fleets of EVs adopted to transport demands to customers were studied by Goeke and Schneider [4]. Zhang et al. [5] aimed to minimize the energy consumption of EVs in their research regarding EVRP.

In traditional VRP, many researchers studied the uncertainty problems, such as the uncertain travel time of vehicles due to the external factors, the uncertain demand, and stochastic serving time of customers (Gounari, Wiesemann, and Floudas [6]; Hu et al., [7]; Zhang et al., [8]). Compared to conventional petroleum-fuel-powered vehicles, the batteries of EVs are more susceptible to uncertainties. Hence, how to ensure the transportation of EVs under uncertainty is an important inspect. However, the studies on the EVRP under uncertainty are rare. Pelletier, Jabali, and Laporte [9] first studied the EVRP under uncertain energy consumption based on the robust theory and they assumed that the RSs are not needed. Keskin, Çatay, and Laporte [10] considered the stochastic waiting times at the RSs.
Due to the limited driving range and the high fixed cost of EVs, it is hard to employ an EV to serve several customers in one run without recharging in an urban environment. Therefore, based on the EVRPTW of Schneider, Stenger, and Goeke [2], we study the electric vehicle routing problem with time windows under travel time uncertainty (EVRPTW-TTU) aiming to minimize the total cost.

The rest of our work is organized as follows. First, both deterministic and robust mixed integer linear programming models are developed for the EVRPTW-TTU. Then a heuristic combining the adaptive neighborhood search (ALNS) and the local search (LS) called the ALNS algorithm is proposed to solve the large-sized instances. Last, the computational results are given to present that the proposed heuristic can solve both small-sized instances and large-sized instances optimally.

2. The Mathematical Formulation

2.1. The Deterministic EVRPTW

The EVRPTW is defined based on a complete digraph $G=(N, A)$, where the set of nodes is denoted as $N$, and the set of arcs is denoted as $A$. A fleet EVs (denoted as $K$) fully charged from the depot (denoted as $\{0\}$) is employed to transport demands from the depot to each customer (denoted as $N_C$) with a service time (denoted as $s_i$) during the time windows (denoted as $[e_i, l_i]$), and the EVs may visit RSs (denoted as $N_R$) if they need. The unit energy consumption and the recharging rate at the RSs are constant numbers (denoted as $h$ and $g$). Each EV will go back to the depot (denoted as $\{n+1\}$) after delivery. We assume that no energy consumption during customer service and continuous visits of EVs to the RSs is not permitted.

In mathematical formulation, sets $N_1 = \{0\} \cup N_C \cup N_R$, and $N_2 = N_C \cup N_R \cup \{n+1\}$ are defined. Parameters $c_f, c_m$ and $c_r$ are the fixed cost, transportation cost and recharging cost, respectively. The energy capacity of each EV is denoted as $B$, and the latest time for EVs return to the depot is $T$. $t_{ij}$ means the travel time from node $i$ and node $j$. For each EV $k$, the decision variable $t_{ij}^k$ means the arrival time at node $i$, the decision variable $b_{ij}^k$ is the left energy level at node $i$, and if the binary variable $x_{ij}^k$ equals 1, the vehicle travels from node $i$ to node $j$, otherwise, 0.

The deterministic EVRPTW can be formulated as follows

$$\min f = c_f \sum_{k \in K} \sum_{j \in N_C} x_{0j}^k + c_m \sum_{k \in K} \sum_{(i,j) \in N \setminus \{i\}} \sum_{j \in N} t_{ij}^k x_{ij}^k + c_r \sum_{k \in K} \sum_{i \in N_D} \sum_{j \in N_C \cup \{n+1\}} x_{ij}^k$$  \hspace{1cm} (1)

$$\sum_{k \in K} \sum_{i \in N_D} x_{ij}^k = 1 \quad \forall j \in N_C$$  \hspace{1cm} (2)

$$\sum_{j \in N_C \cup \{n+1\}} x_{ij}^k = \sum_{j \in N_R \cup \{n+1\}} x_{ij}^k \quad \forall i \in N_C \cup N_R, k \in K$$  \hspace{1cm} (3)

$$\sum_{j \in N_R} x_{ij}^k = 0 \quad \forall i \in N_R, i \neq j, k \in K$$  \hspace{1cm} (4)

$$t_{ij} + s_i \leq t_{ij}^k - t_{ij}^k + T(1 - x_{ij}^k) \quad \forall i \in N_C, j \in N_Z, i \neq j, k \in K$$  \hspace{1cm} (5)

$$t_{ij} + (B - b_{ij}^k) / g \leq t_{ij}^k - t_{ij}^k + T(1 - x_{ij}^k) \quad \forall i \in N_R, j \in N_Z, i \neq j, k \in K$$  \hspace{1cm} (6)

$$t_{j+1}^k \leq T \quad \forall k \in K$$  \hspace{1cm} (7)

$$e_j \leq t_{ij}^k \leq l_j \quad \forall j \in N_C, k \in K$$  \hspace{1cm} (8)

$$h t_{ij} \leq b_{ij}^k - b_{ij}^k + B(1 - x_{ij}^k) \quad \forall i \in N_C, j \in N_Z, i \neq j, k \in K$$  \hspace{1cm} (9)

$$h t_{ij} \leq B - b_{ij}^k + B(1 - x_{ij}^k) \quad \forall i \in N_R, j \in N_Z, i \neq j, k \in K$$  \hspace{1cm} (10)

$$x_{ij}^k \in \{0,1\} \quad \forall i, j \in N, i \neq j, k \in K$$  \hspace{1cm} (11)
\[ t_k^i \geq 0 \quad \forall i \in N, k \in K \quad (12) \]
\[ 0 \leq b_j^k \leq B \quad \forall j \in N, k \in K \quad (13) \]

The objective function (1) aims to minimize the total cost including the fixed cost, maintenance cost, and recharging cost. Constraints (2) promise that each customer should be served only once by one EV. The flow conservation constraints are shown in constraints (3). Constraints (4) guarantee that continuously visits two RSs is not allowed. Constraints (5)-(8) are the time constraints. Constraints (9)-(10) are used to ensure the energy power of each EV.

2.2. A Robust EVRPTW-TTU

The EVRPTW-TTU model considers uncertainty is more practical than the deterministic model in real life. The robust theory discussed by Ben-Tal and Nemirovski [11] is applied to construct the robust model. The travel time uncertainty set of EV \( k \) is denoted as \( k_t^U \), and the overall travel time uncertainty set is denoted as \( k_t^U \times k_t^U \).

Let us denote the box uncertainty set as \( U_i^k = \{ \tilde{t}_y - \hat{t}_y \leq t_y \leq \tilde{t}_y + \hat{t}_y \} \), where \( \tilde{t}_y \) is the expected travel time, and \( \hat{t}_y \) is the maximal deviation. Hence, constraints (5)-(6) and (9)-(10) can be rewritten as

\[ \tilde{t}_y + \hat{t}_y x_y^k \leq t_j^k - s_i x_i^k - t_i^k + T (1 - x_y^k) \quad \forall i \in N, j \in N, i \neq j, k \in K \quad (14) \]
\[ \tilde{t}_y + \hat{t}_y x_y^k \leq (B - b_i^k) / g - t_i^k + T (1 - x_y^k) \quad \forall i \in N, j \in N, i \neq j, k \in K \quad (15) \]
\[ h \tilde{t}_y + h \hat{t}_y x_y^k \leq b_i^k - b_i^k + B (1 - x_y^k) \quad \forall i \in N, j \in N, i \neq j, k \in K \quad (16) \]
\[ h \tilde{t}_y + h \hat{t}_y x_y^k \leq B - b_i^k + B (1 - x_y^k) \quad \forall i \in N, j \in N, i \neq j, k \in K \quad (17) \]

Considering that we do not expect the worst-case value to occur on every arc simultaneously, thus, the budgeted uncertainty set calibrated through parameters \( \Gamma \) is therefore used.

\[ U_i^k = \{ t_y \in R^d \mid \tilde{t}_y = \hat{t}_y, \sum_{i \in K} | \beta_k^i \hat{t}_y | \leq 1, \Gamma = \theta \sum_{i \in K} x_i^k, \forall (i, j) \in A^k \} \quad (18) \]

Where \( \theta \in (0, 1) \) is the budget coefficient. According to formula (18), the following problem can be deduced from the objective function

\[ \max \left( \sum_{i \in K} \sum_{j \in N \setminus \{i\}} \beta_k^i x_y^k \right) \quad (19) \]

According to duality theory, the sub-problem (19) equals to

\[ \min \lambda_k \left( \theta \sum_{i \in N} \sum_{j \in N \setminus \{i\}} x_i^j + \sum_{i \in K} \sum_{j \in N \setminus \{i\}} \sigma_j^k \right) \quad (20) \]
\[ \lambda_k + \sigma_j^k \geq t_y x_y^k \quad \forall i \in N, j \in N, i \neq j, k \in K \quad (21) \]
\[ \lambda_k \geq 0 \quad \forall k \in K \quad (22) \]
\[ \sigma_j^k \geq 0 \quad \forall i \in N, j \in N, i \neq j, k \in K \quad (23) \]

To linearize the constraint (20), we adopt the variable \( u_y^k \) to represent the \( \lambda_k x_y^k \), thus formula (20) can be replaced with
3. Heuristic Algorithm

3.1. The ALNS Algorithm

Considering the EVRPTW is an NP-hard problem, it is difficult to solve the large-sized problem. Therefore, an ALNS based on the algorithm in Pelletier, Jabali, and Laporte [9] is proposed. First, an initial solution is generated according to the CW algorithm. Second, based on the procedure of LNS algorithm, one destroy operator and one repair operator are selected to operate on the current solution. A LS algorithm is adopted to improve the current solution. The method from Hu et al., [7] is used to calculate the objective value. After each iteration, the parameters will be updated.

3.1.1. Destroy operators and repair operators

A certain number (denote as \( n_r \)) of the total customers is removed from the current solution. The destroy operators contain customer removal and station removal.

**Customer removal.** The Random removal operator, the Worst removal operator, the Worst distance removal (Demir, Bektaş and Laporte [12]), the Shaw worst energy operator, the Random route removal (Pelletier, Jabali and Laporte [9]), the Remove customer with preceding station and the Remove customer with succeeding station (Keskin and Çatay [13]) are adopted.

**Station removal.** The Worst-charge usage RSs (Keskin and Çatay [13]) is applied to remove the RSs.

3.1.2. Repair operators

Similar to the destroy operators, two parts (customer insertion and station insertion) are included. After several stations are removed from the current solution, the station insertion is adopted to repair the solution.

**Customer insertion.** The Greedy and Regret-\( k \) (\( k = 2,3 \)) insertion algorithms from literature are considered. The Time based insertion (Keskin and Çatay [13]) and the Energy based insertion (Pelletier, Jabali and Laporte [9]) are also applied to repair the destroyed solutions.

**Station insertion.** The Greedy insertion and the Greedy insertion energy with comparison proposed in Keskin and Çatay [13] are used.

3.2. Local Search

The LS algorithm aims to improve the current solution by employing a set of neighborhood operators. The 2-opt* and the stationInRe operated on RSs (Schneider, Stenger and Goeke [2]), and the relocate (inter- and intra-) and exchange (inter- and intra-) operated on customers are adopted.

4. Computational Results

The EVRPTW instances adjusted from the Schneider, Stenger and Goeke [2] are used to test the ALNS algorithm. Assuming the maximum deviation travel time \( \tilde{t}_{ij} \) to the expected value is 20%. We set \( B \) as 100 kWh and set the parameter \( \theta \) as 0.3. The proposed ALNS algorithm was coded in Python 3.8, and all the experiments were run on a desktop DC with a 3.19GHz dual processor and 16GB of RAM.

4.1. The Results on the Small-sized Instances

A comparison between the proposed ALNS algorithm and the GUROBI 9.1.1 on the small-sized instances is made. The computation time of GUROBI is 1000 seconds, and the number of iterations of the ALNS algorithm is set as 200. The results are shown in Table 1.
Table 1. The results on the small-sized instances.

| Inst  | Deterministic | Robust |
|-------|---------------|--------|
|       | GUROBI        | ALNS   |
|       | Optimal       | Time(s)| Optimal | Time(s)| Optimal | Time(s)|
| C101C5 | 2400.85       | 0.55   | 2400.85 | 0.07   | 2401.24 | 1.46   | 2401.03 | 1.32   |
| R105C5 | 2400.54       | 0.37   | 2400.54 | 0.75   | 2400.86 | 0.32   | 2400.68 | 0.15   |
| R202C5 | 1200.45       | 1.89   | 1200.45 | 1.11   | 1200.46 | 5.28   | 1200.46 | 1.38   |
| C104C10| 2400.92       | 0.92   | 2400.92 | 14.76  | 2401.45 | 1000   | 2401.13 | 16.98  |
| R201C10| 1200.87       | 17.09  | 1200.87 | 4.96   | 2401.26 | 1000   | 2401.26 | 9.74   |
| RC201C10| 2401.09      | 1000   | 2401.09 | 5.52   | 2401.79 | 1000   | 2401.27 | 9.77   |
| R202C15| 2401.27       | 1000   | 2401.27 | 24.21  | 2601.67 | 1000   | 2601.67 | 29.15  |
| RC103C15| 6001.40      | 1000   | 6001.40 | 20.32  | 6228.25 | 1000   | 6228.25 | 26.56  |
| RC202C15| 2401.43      | 1000   | 2401.39 | 39.26  | 3602.33 | 1000   | 3601.92 | 19.39  |

In Table 1, column “Deterministic” is the results of the deterministic instances, column “Robust” means the results of the instances under travel time uncertainty, column “ALNS” is the optimal solution solved by the ALNS algorithm, column “GUROBI” is the solution obtained by GUROBI, column “Optimal” is the optimal value of solutions, and column “Time” denoted by seconds is the computation time.

It can be concluded that compared to GUROBI, the proposed ALNS algorithm can solve the small-sized instances optimally with much less computation time. For those instances with more than 15 customers, GUROBI cannot give the optimal solution within 1000 seconds. In summary, the proposed ALNS can solve the small-sized instances optimally.

4.2. The Results on the Large-sized Instances

To show the conservative of the robust solution, the large-sized instances with 100 customers are adopted. The results are shown in Table 2, and the number of the iterations is set as 100.

Table 2. The results are under travel time uncertainty.

| Inst  | Deterministic | Robust |
|-------|---------------|--------|
|       | N.V           | Optimal | Uncertainty set | N.V | Optimal |
| C101_21 | 12            | 14404.50 | -- | 15 | 18204.99 |
| R103_21 | 16            | 19795.49 | 45950.02 | 20 | 25452.39 |
| R104_21 | 13            | 15604.07 | 46039.93 | 16 | 20977.24 |
| RC101_21 | 20           | 24006.83 | -- | 22 | 26408.58 |
| RC102_21 | 19           | 22806.39 | -- | 20 | 24007.71 |

According to Table 2, column “N.V” is the number of EVs used in the optimal solution, and the column “Uncertainty set” means the value of deterministic solution under travel time uncertainty. When no value can be outputted, which means the algorithm cannot achieve a feasible robust solution.

From Table 2, compared to the deterministic solutions, the robust solutions are much more conservative under uncertainty. Although for several instances, the deterministic solutions can be feasible under an uncertainty set, however, the optimal costs are much higher than the robust solutions. From above, compared to the deterministic model, the robust models are more practical in realistic.

5. Conclusions and Future Works

In this paper, we present a robust model of EVRPTW under travel time uncertainty, which has high research value in real life. A heuristic combing the ALNS algorithm, and the LS algorithm is proposed to solve the large-sized instances. Numerical results show that the robust model makes the routing plan more conservative than the deterministic solution, and the proposed algorithm can get optimal solutions on the small-sized instances and the near-optimal solutions on the large-sized instances.

Future studies should focus on more practical factors regarding the EVRP, such as the nonlinear relationship between the energy consumption and the travel time or travel distance, and the multiple depots electric vehicle routing problem (MDEVRP).
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References
[1] Erdoğan S, and Miller-Hooks E 2012. A green vehicle routing problem. Transp Res E Logist Transp Rev 48(1) 100-114
[2] Schneider M, Stenger A and Goeke D 2014. The electric vehicle-routing problem with time windows and recharging stations. Transp Sci 48(4) 500-520.
[3] Felipe Á, Ortuño M T, Righini G and Tirado G 2014. A heuristic approach for the green vehicle routing problem with multiple technologies and partial recharges. Transp Res E Logist Transp Rev 71 111-128.
[4] Goeke D and Schneider M 2015 Routing a mixed fleet of electric and conventional vehicles. Eur J Oper Res 245(1) 81-99.
[5] Zhang S, Rajpal Y, Appadoo S S and Abdulkader M M S 2018. Electric vehicle routing problem with recharging stations for minimizing energy consumption. Int J Prod Econ 203 404-413.
[6] Gounaris C E, Wiesemann W and Floudas C A 2013. The robust capacitated vehicle routing problem under demand uncertainty. Oper Res 61(3) 677-693.
[7] Hu C, Lu J, Liu X and Zhang G 2018. Robust vehicle routing problem with hard time windows under demand and travel time uncertainty. Comput Oper Res 94 139-153.
[8] Zhang D, Li D, Sun H and Hou L 2021. A vehicle routing problem with distribution uncertainty in deadlines. Eur J Oper Res 292(1) pp 311-326.
[9] Pelletier S, Jabali O, Laporte G. 2019. The electric vehicle routing problem with energy consumption uncertainty. Transport res B Meth 126 225-255.
[10] Keskin M, Çatay B and Laporte G 2021. A simulation-based heuristic for the electric vehicle routing problem with time windows and stochastic waiting times at recharging stations. Comput Oper Res 125 105060.
[11] Ben-Tal A and Nemirovski A 2002. Robust optimization–methodology and applications. Math program 92(3) pp 453-480.
[12] Demir E, Bektaş T and Laporte G 2012. An adaptive large neighborhood search heuristic for the pollution-routing problem. Eur J Oper Res 223(2) 346-359.
[13] Keskin M and Çatay B 2016. Partial recharge strategies for the electric vehicle routing problem with time windows. Transp Res Part C Emerg Technol 65 111-127.