Coupling DDA and 4D-LSM through contact theory for rock cutting

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Abstract. In this work, the rock cutting is modelled through coupling of 4D-LSM with DDA. The rock cutter and its 3D motion were simulated by using DDA with triangular surface mesh, and the rock is modelled by using 4D-LSM with multi-body failure criterion. The dynamic contact treatment between the cutter and the rock specimen was further handled by using Dr. Shi’s contact theory where the entrance block of the cutter between spherical particles was used to calculate the contact forces. Then, the 4D-LSM and DDA were coupled through an explicit-implicit mixed solving scheme. Finally, the coupled model is used to model rock cutting problems and compared with exist numerical approach to highlight its advantages.

1. Introduction

The interaction between the cutter and rock commonly occurs in civil, mining, petroleum, and geothermal engineering. Currently, these problems could not yet be resolved perfectly, which suffered from the complex motion between the cutter and rock. There are three available approaches to study these problems: analytical [1-2], experimental [3] and numerical [4]. The analytical method could establish the relationship between the basic parameters of rock and the cutting result to obtain these empirical equations. However, as the simplification of the loading conditions and ignoring the rock heterogeneity and the nonlinear stress-strain relationship, it’s difficult to know the whole processes of rock dynamic failure, and could not directly be applied in engineering practice. As for the experimental approach, it could provide useful insights into the behavior of rock cutting/penetration. The limitations are that the size of rock model specimen and the geometry of the loading device have to be simplified and scaled. Moreover, the construction of physical model is often complex and time consuming, and interfered by natural and artificial factors. Although these two approaches have made significant progress in rock cutting, the drawbacks are obvious. With the rapid development of computing power, it
provides the conditions to apply the numerical method to model the processes of rock cutting, which would not suffer from the limitations of analytical and experimental approaches. At present, the researchers [5] classified the numerical modeling techniques into two typical methods (continuum- and discontinuum-based methods) to calculate the stress field and simulate the process of rock cutting. The main part of the continuum-based methods is the finite element method (FEM), which using the element/particle degradation technique or the crack growth technique to deal with the interaction between the cutter and rock. Liu et al [4] discussed the rock fragmentation induced by single and double indenters using rock failure process analysis (RFPA), which could reproduce the progressive process of rock fragmentation in indentation. Chiang et al [6] proposed a three-dimensional (3D) finite element approach to simulate the process of rock drilling, considering the linear material properties and post failure fracture propagation. Zhao et al [7] and Wang et al [8] simulated the process of rock cutting based on ANSYS/LS-DYNA. As depicted in figure 1(a), the influence of jointed rock mass and ground parameters on the cutting efficiency and cutter wear of tunnel boring machine (TBM) cutters are discussed. Liu et al [9] investigated the abrasive water jet-pick combined rock breaking under confining pressures using FEM combined with SPH to improve the rock breaking efficiency and reduce cutting force. Moreover, George et al [10] and Gao et al [11] discussed the influence of frictional contact, cutting heat and ultra-high cutting speed between the cutter and brittle rock using the ABAQUS. As shown in figure 1(b), the interaction between the tool and rock is modelled by contact pairs with three key components, which includes the contact discretization, tracking approach, and the assignment of master and slave roles to surfaces. The main drawbacks of the continuum-based approach are its unsuitable for solving the large-scale fracture opening and complete detachment of the rock. Nevertheless, these problems can be better dealt with the discontinuum-based approach, which treat the cutter as another DEM model, a wall element, or a clumped particle. Su et al [12] simulated the rock cutting tests by Particle Flow Code (PFC) in 3D in order to predict tool forces from cutting tests. Lu et al [13] used the FEM to simulate fragment separation of rock, which demonstrated that the cutting forces and sizes of the separated fragments increased significantly with increasing cutting depth, compressive strength, and elastic modulus of the base rock. Zhao et al [14] compared the 2D bonded DEM, the finite element method, a hyper elasticity analysis, and the distinct lattice spring model (DLSM) in solving elastic boundary value problems, then, they adopted the bonded DEM for rock cutting. As shown in figure 1(c), the 2D bonded DEM could result in a realistic simulation of the rock cutting process. Zhu et al [15] simulated the process of rock fracturing with TBM by EDEM to investigate the fracture characteristics of the jointed rock. Although the discontinuum-based approach could give a good simulation to the fragment separation of rock, the computational cost is usually very high. Therefore, Mojtaba et al [16] proposed the GPGPU-parallelised hybrid finite-discrete element to model the rock chipping and fragmentation process in the rock scratch test to improve the computational efficiency.
Figure 1. Different numerical methods to simulate the rock cutting process: (a) ANSYS (after Zhao et al [7]); (b) ABAQUS (after George Z. et al [10]); (c) DICE2D (after Zhao et al [14]); (d) RockBox3D (after Zhao et al [17]).

The coupled approach of explicit-implicit for DDA-DLSM was proposed by Zhao et al [17] to simulate the rock cutting and rock penetration process, which adopted the distinct lattice spring model (DLSM) to simulate the dynamic fracturing process of the rock, and the discontinuous deformation analysis (DDA) to model the high-speed motion of the cutter (figure 1(d)). Moreover, the 3D simplex sphere-to-block contact method is introduced to deal with the interaction between DLSM and DDA. With the development of four-dimensional lattice spring model (4D-LSM) and the multi-body failure criterion [18-19], the 4D-LSM has the ability to accurately reproduce high ratios of the uniaxial compressive strength to the tensile strength of rock materials, which is also demonstrated to handle the large deformation and the post-failure stage of rock. Therefore, based on these merits of 4D-LSM in describing the large deformation and the post-failure stage of rock, this work will couple the DDA and 4D-LSM to simulate rock cutting and rock penetration process. The fundamental principles of 4D-LSM, 3D-DDA and Contact theory adopted in this work are introduced first. Furthermore, a comparative study of modeling rock cutting/penetration process using the 3D-DDA&4D-LSM are conducted and compared with the exist approach to highlight its advantages. Finally, some conclusions of the coupled numerical approach are derived and discussed in the last part.

2. The model

2.1. 4D-LSM with multibody failure model

The four-dimensional Lattice Spring model (4D-LSM) was developed by utilizing the extra dimension concept. Using the concept of parallel world, 4D-LSM connects objects in 3D space
with parallel versions in fourth dimensional space, which assumes the physical world as a four-dimensional hyper-membrane. The models in 3D space and parallel world are closely connected by four-dimensional interactions. Moreover, it is assumed that the initial model and the parallel model have the same material properties and boundary conditions. A more detailed information of the construction process and principle of the 4D-LSM can be found in Zhao et al [18]. 4D-LSM can overcome the Poisson’s limitation of the classical lattice spring model by using the 4D interaction. Compared with the classical continuum mechanics, the discrete nature of 4D-LSM makes it a natural advantage when simulating and calculating nonlinear large deformation problems of non-homogeneous materials such as rocks. In addition, 4D-LSM has higher computational efficiency than other discrete methods. 4D-LSM uses 2/3 degree of freedoms compared with the classical DEM. From our test results, 4D-LSM will spend less 40% computing time compared with DLSM. Moreover, the GPU 4D-LSM will handle more than 8,800,000 particles per second in a GPU computer, which considers as a very fast computing speed.

The ratio of uniaxial compressive strength to uniaxial tensile strength (UCS/T ratio) is an important parameter to evaluate the mechanical properties of materials. For discontinuum-based numerical methods, it is typically difficult to reproduce the high UCS/T ratio that is observed in rock materials. In order to tackle the problem of low UCS/T ratio, 4D-LSM introduces a multibody failure criterion of spring bonding [19]. The material unit is represented by a discrete model that not only considers the interaction between the center pairs. As shown in figure 2, in order to obtain the spring bond stress tensor equation, the calculation model of 4D-LSM is expressed as a network. The particle and its neighboring particles form a cluster, and the stress state of the particle can be expressed by the deformation state of the spring bonds around it as

$$\sigma_{ij}' = \frac{1}{2V_i} \sum_{j=0}^{N} f_{ij} n_{ij} l_{ij}'$$

(1)

where $\sigma_{ij}'$ is the stress tensor of particle $I$, $V_i$ is the volume of the particle, $f_{ij}$ is the interaction force component between particle $I$ and its neighbors, $n_{ij}$ is the normal vector component between particle $I$ and its neighbors, and $l_{ij}'$ is the original spring length between particle $I$ and its neighbors. The stress state of the spring bond can be expressed as

$$\sigma_{ij}^{bond} = \frac{\sigma_{ij} + \sigma_{ij}'}{2}$$

(2)

Then, the failure of each spring bond is determined using a macroscopic stress-based strength model, such as the modified Mohr-Coulomb criterion:

$$f(\sigma_{ij}^{bond}) = f(\sigma_{ij}^{bond}, \sigma_{ij}^{bond}) = \begin{cases} (1 - \sin \phi)\sigma_{ij}^{bond} - (1 - \cos \phi)\sigma_{ij}^{bond} - 2c \cos \phi \geq 0 \\ \sigma_{ij}^{bond} - \sigma_{ij}^{*} \geq 0 \end{cases}$$

(3)
where $\sigma_{1}^{\text{bond}}$ and $\sigma_{3}^{\text{bond}}$ are the first and third principal stresses, respectively; $\phi$ is the friction angle; $c$ is the cohesion; and $\sigma_t^*$ is the tensile strength.

For the multibody failure criterion, the failure state of a spring bond is determined by the deformation state of a set of related spring bonds. By introducing the macroscopic stress-based strength model in the multibody failure criterion, the UCS/T ratio in 4D-LSM is used as an input parameter, thereby solving the problem of low UCS/T ratio in 4D-LSM. The main advantage is that the input parameters can be directly determined by conventional rock mechanics tests, which is more flexible for solving fracturing and large deformation problems.

2.2. DDA
The discontinuous deformation analysis (DDA) was first proposed by Shi [20-21] in 1988, which was comprised of four essential components: (1) deformation function of the block, (2) contact detection between blocks, (3) system equation derived from energy minimization principle, and (4) simplex integration. As depicted in figure 3, here, the simplex concept of DDA is adopted to represent the 3D objects as a tessellation of surface triangles, and the normal direction of each triangle is outside of the domain.
In this work, in view of the major concern of the rock fracturing process, the DDA block’s deformation is assumed to be ignorable due to its high rigidity. Therefore, the deformation function of the DDA block can be simplified as the incomplete first-order displacement function used in sphere-based DDA [19], and the more details on the derivation and validation are suggested to be found in the work of Zhao et al [17].

\[
\begin{pmatrix}
  u \\
  v \\
  w
\end{pmatrix} = \left[ T(x, y, z) \right] \mathbf{D} =
\begin{bmatrix}
  1 & 0 & 0 & 0 & Z & -Y & X \\
  0 & 1 & 0 & -Z & 0 & X & Y \\
  0 & 0 & 1 & Y & -X & 0 & Z
\end{bmatrix}
\begin{pmatrix}
  d_x \\
  d_y \\
  d_z \\
  r_x \\
  r_y \\
  r_z \\
  \epsilon_r
\end{pmatrix}
\] (4)

where \( d_x \) = translation in \( x \) direction; \( r_x \) = rotation along \( x \)-axis (terms for \( y \) and \( z \) defined similarly); and \( \epsilon_r \) = radius strain deformation; \( (X, Y, Z) = (x - x_c, y - y_c, z - z_c) \);

\((x_c, y_c, z_c)\) = centroid of the block.

2.3. Coupling 4D-LSM with DDA through contact
The contact theory proposed by Shi [23] is adopted to deal with the contact problem between the DDA and 4D-LSM. Due to the composition of 4D-LSM is a number of particles, the contact problem could be simplified as the contact relationship between a sphere of the 4D-LSM and a polyhedron of the DDA. As shown in figure 4(a), the DDA block and the sphere are marked as \( A \) and \( B \), respectively. The block is further represented by vertex-edge-face, and the solution of Entrance block is relatively simple. As depicted in figure 4(b), corresponding contact cover can be obtained for each plane, and the contact cover from the \( i \)th face is marked as \( C(A_{ij}, B) \). The way of composition this kind of cover is to translate the corresponding discrete face outward.
by the sphere radius. Corresponding contact cover can be obtained for the edge as \( C(A_e, B) \) in a similar way as shown in figure 4(c). In actual contact detection, the two contact covers are supposed to be clipped, as shown in figure 4(c), the cover between edge and sphere is a cylindrical surface, and according to the contact theory of Genhua only the exposed part will be involved in the contact detection. As for vertex, contact coverage is formed as \( C(A_v, B) \) in a similar way. Therefore, as shown in figure 4(d), the contact between sphere \( B \) and \( A \) is represented by the geometrical relationship between centre of sphere and the entrance block \( E(A, B) \).

**Figure 4.** Sphere-to-block contact scheme: (a) the DDA block and sphere; (b) sphere-face contact; (c) sphere-edge contact; (d) sphere-vertex contact.

In order to deal with contact continuity of discrete triangular faces and particle, the specific coefficients are introduced according to the different types of contacts in the DDA&DLSM. With the aid of contact theory, contact treatment could be further simplified and more robust. First, the priority for detection between sphere and contact covers is set to avoid the operation of geometry clipping. Next, as for the contact detection, the contact between the center of the sphere and different contact covers is marked as \( (\mathbf{n}^C, d_i) \), where \( \mathbf{n}^C \) is the normal vector of contact, and \( d_i \) is the entrancing distance. A contact sequence could be formed from each sphere, and normally the length of this sequence is one. The contact force from the block to the particle is given as:

\[
\mathbf{F}^c = k_n \mathbf{n}^C d_i
\]  

(5)
where $\mathbf{F}^c$ is the contact force; $\mathbf{n}^c$ is the vector of contact; $d$ is the entrancing depth. If the size of pending contact sequence is not equal to 1, then the contact aggregation occurs. The aggregation condition is that the angle between two potential vectors of contacts is less than the given value (such as 0.1). The vector of contact and entrancing depth after aggregation could take the average of the entire candidates array, and the contact force could be calculated by using equation (5). It is observed that the contact theory is introduced to avoid the calculation of angle subdivision and different types of contact processing like in DDA&DLSM, which could make the contact treatment much more simple and efficient.

The coupling scheme of DDA & 4D-LSM is shown in figure 5. Each calculation cycle is comprised of two time lines: (1) the update a particle’s motion in 4D-LSM according to Newton’s second law, (2) the calculation of the block displacement for DDA according to the energy minimization principle. The coupling is implemented through exchange contact forces between the DDA block and 4D-LSM particles. It's worth noting that the contact between the particle and DDA block is handled by the contact theory.

### 3. Numerical Examples

#### 3.1. Rock Penetration

Rock penetration test is a typical test which is known to be related to penetration rates of drilling equipment and TBM cutters. In this work, the example of rock penetration is simulated using
DDA & DLSM and DDA & 4D-LSM coupled approaches respectively to compare the differences in simulation of rock penetration process. As shown in figure 6(a), the steel impactor is simulated to impact the rock specimen with an initial velocity of 400 m/s at the height of 75 cm. The material parameters of the steel impactor are elastic modulus of 200 GPa, Poisson’s ratio of 0.3, and density of 8000 kg/m³. Moreover, the rock specimen is made up from DLSM with 0.01 m particle size, which has the dimensions 40 × 250 × 20 cm (figure 6(b)). The material parameters of the rock specimen are elastic modulus of 40 GPa, Poisson’s ratio of 0.2, and density of 2,700 kg/m³. And the time step is 1e-6 in the simulation. In view of the influence of mesostructure, the Voronoi-based heterogeneous model is built, which has the 20% base material. The two solvers are adopted to calculate the same example in order to investigate the difference between DDA & DLSM coupled approach and DDA & 4D-LSM coupled approach.

The failure processes of the rock penetration simulated by DDA & DLSM and DDA & 4D-LSM are shown in figure 7 and figure 8, respectively. The two coupled approaches both have the ability to reproduce the failure process of rock specimen under impactor penetration. Nevertheless, the main difference between the two coupled approaches is the description of rock failure process in post-failure stage, which is shown in the failure pattern of rock specimen at the 1 ms and 1.5 ms (see figure 7(c)(d), figure 8(c)(d)). It is worth noting that the failure block and complete detachment of rock specimen from DDA & DLSM are more scattered, and the DDA & 4-DLSM could describe a more realistic process of rock fracturing.
Figure 7. Failure patterns of the rock penetration process by DDA & DLSM: (a) t=0ms; (b) t=0.5ms; (c) t=1ms; (d) t=1.5ms.

The damage volume ratio versus time from two coupled approaches are shown in figure 9. It can be seen that damage ratio in the first 0.5 ms from two coupled methods are similar, and have big gap after 0.5 ms to 1.5 ms, which demonstrates the ability of DDA & 4D-LSM to capture the rock penetration process more realistically. This is because 4D-LSM is good at handle large deformation problems and keep the energy conservation. When large deformation
happens, the DDA&DLSM produced a large damage ratio which is believed to be triaged by numerical error which can be well eliminated in DDA&4D-LSM.

Figure 8. Failure patterns of the rock penetration process by DDA & 4D-LSM: (a) t=0ms; (b) t=0.5ms; (c) t=1ms; (d) t=1.5ms.
3.2. Rock cutting

In this example, the process of cutting rock with a TBM cutter is simulated by DDA&4D-LSM program. The TBM cutter model and rock block model are shown in figure 10. The 3D cutter model in this study uses the cutter's surface triangles. The radius of the TBM cutter is 214 mm, the thickness of the cutter is 80 mm, and the width of the cutter head is 14 mm. For cutter materials, the elastic modulus is 120 GPa and the Poisson’s ratio is 0.256. The rock specimen is a cuboid of size 100×100×30 mm and the particle diameter is 1.0 mm. The rock model contains a total of 300,000 particles. For rock materials, the elastic modulus is 23.01 GPa, the Poisson’s ratio is 0.188, the tension strength is 6.4 MPa and the cohesion is 15 MPa. The densities of TBM cutter and rock block are 7,900 kg/m³ and 2,600 kg/m³, respectively. In order to overcome the computational limitations of the current program, the density of the cutter and rock was amplified by 1,000 times in the simulation [17]. So we can take a larger time step of 5e-6. Generally, the relationship between the time step and density is in square manner, that is around 30 times increase. In order to model the cutting process, boundary conditions are applied to all the five faces except the top face of the rock block. Moreover, the ultimate deformation of the normal spring is 0.00028 mm and the softening ratio is set as 0.1. The cutting process of TBM cutter rotates around the x-axis and moves along the z-axis. The cutting depth is 0.2 mm. The cutting speed of the cutter is 10 m/s, and the rotation speed can be determined by \( \omega = \frac{v}{R} \), where \( v \) = cutting speed; and \( R \) = radius of the cutter.
In this work, the influence of friction angle on the TBM rock-cutting process is studied, and the friction angle is set to 30°, 40°, 55°. The failure process of TBM rock-cutting process with different friction angles is shown in figure 11. The rock damage zone expands along the movement direction of the TBM cutter. In the early stage of cutting, the rock mass produced cracks near the lower part of the cutter. As the cutting progressed, the cracks continued to expand along the direction of the cutter movement, forming a large damage area. Finally, the
crack penetrates the entire rock mass in the cutting direction. The failure process of rock cutting under different friction angles is similar. Only in some areas is there a small difference in the damage zone due to the different directions of crack propagation. In addition, the relationship between the damage ratio and the friction angle during rock cutting is also discussed. The damage volume ratio versus cutting time under different friction angles is shown in figure 12. It can be seen that rocks with a smaller friction angle will suffer more serious damage in the process of cutting. The reason might be that the friction angle affects the shear stress inside the rock during the cutting process.

In order to test the ability of the DDA&4D-LSM coupling method to deal with rock cutting problems, we simulated the rock cutting experiment conducted by Zhao et al. [3]. The size of the rock sample is consistent with the experiment, which is 1,000×1,000×600 mm and consists of particles with a diameter of 5 mm. The cutter is the same size as the real TBM cutter, with a diameter of 17 in. (432 mm) and a head width of 21 mm. The material parameters in this simulation are consistent with the cutting experiment. Similarly, in order to improve computational efficiency and keep the cutter stable, the density of the rock was amplified 1000 times in the simulation, and the density of the tool was amplified 10^6 times. The boundary conditions, cutting method and cutting speed of the cutter are consistent with the previous simulation. In addition, the time step is 3e-5 and the cutting depth in this simulation is 1.5 mm, which is consistent with the experiment. The ultimate deformation of the normal spring is 0.00028 mm and the softening ratio is set as 0.1. In this work, the numerical model of DDA&4D-LSM is the same as the experimental model, so we can naturally and meaningfully compare with the experimental results and simulation results of the experiment by Xiao et al. [24] through FEM-SPH coupled method. The normal force results of TBM cutter in the DDA&4D-LSM simulation and the experiment are shown in figure 13. The variation result of the normal force simulated by Xiao et al. through FEM is shown in figure 14. Compared with FEM, the numerical calculation results of the normal force in the rock cutting process simulated by DDA&4D-LSM method are in better agreement with the experimental data. The cutter and rock in this simulation are scaled up to allow a large time step to overcome the computational
limitations of the current program. This is a typical technique used in PFC or FLAC for handling quasi-static modelling. Generally, for such kind of modification, very careful calibration is required to make sure the modelling results can fit the corresponding experimental results. The comparison of normal forces shows that the results obtained by DDA&4D-LSM after this modification are highly consistent with the experimental results. In conclusion, the simulation results show that DDA&4D-LSM has a good prospect for solving rock cutting problems.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{force.png}
\caption{Normal force obtained from the experiment (Zhao et al [3]) and the DDA&4D-LSM.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{force_fem.png}
\caption{Normal force obtained from the FEM method (Xiao et al [24]).}
\end{figure}

4. Conclusions

A coupled DDA & 4D-LSM approach is developed for rock cutting problems. The simplex concept of DDA is adopted to represent the cutter and impactor as a tessellation of surface triangles. The rock specimen is simulated by 4D-LSM particles with multi-body failure
criterion. The contact theory is applied to integrate the DDA and 4D-LSM, which could avoid the calculation of angle subdivision and different types of contact processing in the original DDA & DLSM approach. The comparison between the DDA & 4D-LSM and the DDA & DLSM coupled method is investigated using a number of numerical examples. The results indicate that the DDA & 4D-LSM could describe a more realistic process of rock failure, which is more suitable for calculating the large deformation for rock cutting and rock penetration problems.

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