Reaction rate of the $^{13}C(\alpha,n)^{16}O$ neutron source using the ANC of the -3 keV resonance measured with the THM

M La Cognata¹, C Spitaleri¹,², O Trippella¹,³, GG Kiss¹,⁴, GV Rogachev⁵, AM Mukhamedzhanov⁶, M Avila⁵, GL Guardo¹,², E Koshchiy⁶, A Kuchera⁵, L Lamia², SMR Puglia¹,², S Romano¹,², D Santiago⁵, R Spartà¹,²

¹Istituto Nazionale di Fisica Nucleare, Laboratori Nazionali del Sud, Catania, Italy
²Dipartimento di Fisica e Astronomia, Università di Catania, Catania, Italy
³Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, & Dipartimento di Fisica, Università di Perugia, Perugia, Italy
⁴Institute of Nuclear Research (ATOMKI), Debrecen, Hungary
⁵Department of Physics, Florida State University, Tallahassee, Florida, USA
⁶Cyclotron Institute, Texas A&M University, College Station, Texas, USA

E-mail: lacognata@lns.infn.it

Abstract. The $s$-process is responsible for the synthesis of most of the nuclei in the mass range $90 \leq A \leq 208$. It consists in a series of neutron capture reactions on seed nuclei followed by $\beta$-decays, since the neutron accretion rate is slower than the $\beta$-decay rate. Such small neutron flux is supplied by the $^{13}C(\alpha,n)^{16}O$ reaction. It is active inside the helium-burning shell of asymptotic giant branch stars, at temperatures $\leq 10^8$ K, corresponding to an energy interval of 140 – 230 keV. In this region, the astrophysical $S(E)$-factor is dominated by the $-3$ keV sub-threshold resonance due to the 6.356 MeV level in $^{17}O$. In this work, we have applied the Trojan Horse Method (THM) to the $^{13}C(^{6}Li,n^{16}O)d$ quasi-free reaction to extract the 6.356 MeV level resonance parameters, in particular the asymptotic normalization coefficient (ANC) $\tilde{C}_{^{17}O(1/2^+)}^{^{13}C\alpha}$. A preliminary analysis of a partial data set has lead to $\tilde{C}_{^{17}O(1/2^+)}^{^{13}C\alpha} = 6.7^{+0.9}_{-0.6}$ fm$^{-1}$, slightly larger than the values in the literature. However, the deduced $^{13}C(\alpha,n)^{16}O$ reaction rate is in agreement with most results in the literature at $\sim 10^8$ K, with enhanced accuracy thanks to our innovative approach merging together ANC and THM.

1. Physics case

Nuclear reaction cross sections are among the most important input data for nucleosynthesis calculations and stellar modeling, as nuclear processes power stars supplying the thermal pressure that contrasts the inward force of gravity. Typical stellar temperatures for hydrostatic nuclear burning range between $10^7 - 10^8$ K, corresponding to kinetic energies smaller than 1 MeV [1, 2]. These energies are usually well below Coulomb barriers for interacting nuclei, thus typical cross sections can be as small as few $10^{-12}$ barn. Indeed, the Coulomb barrier exponentially reduces the cross section $\sigma$, rapidly pushing the signal-to-noise to zero. Under
these conditions, experimental measurements become exceedingly challenging and experimental uncertainties grow up owing to statistical and systematic errors.

As a consequence, at astrophysical energies (the so-called Gamow window [1, 2]), direct measurements of the cross section are often impossible and extrapolation, supported by nuclear theory such as the R-matrix [3], has been used to determine the cross section. Since it drops exponentially with decreasing energy, large uncertainties might be introduced by the extrapolation procedure. To this purpose, the astrophysical $S(E)$-factor has been introduced [1] to improve the accuracy of the extrapolation procedure:

$$S(E) = E \sigma(E) \exp(2\pi\eta),$$

where $\exp(2\pi\eta)$ is the reciprocal of the Coulomb barrier penetration factor for $s$-wave and center-of-mass energies much smaller than the Coulomb barrier and $\eta$ the Sommerfeld parameter. The astrophysical factor has a weaker dependence on the energy than the cross section at astrophysical energies, as Coulomb effects are partially compensated for by the $\exp(2\pi\eta)$ factor, making the uncertainty introduced by extrapolation significantly smaller.

Even in those few cases where reaction cross sections have been measured down to astrophysical energies (check, for instance, [4]), the electron screening effect [1] prevented one to assess the nuclear cross section or, equivalently, the astrophysical factor. In the laboratory, projectile and target are in the form of ions and atoms or molecules, respectively, thus electrons shield the nuclear charges and determine an exponential increase of $S(E)$ for center-of-mass energies $E_{c.m.} \to 0$ [1, 2]. Atomic electron screening is very different from electron screening in stars, where matter is in the form of plasma, making it necessary to divide out the exponential increase of the $S$-factor to attain the nuclear cross section. However, our current understanding of the electron screening effect is rather incomplete as experimental screening potentials exceed the theoretical upper limits in many cases [5, 4, 6, 7, 8, 9], though this is still a debated topic [10]. Therefore, potential systematic errors might be introduced in the evaluation of the bare-nucleus astrophysical factor, making extrapolation from higher energies, where the electron screening enhancement is negligible, necessary for astrophysical applications.

However, nuclear models are not able yet to supply reliable predictions for the trend of the astrophysical factor at low energies. Unaccounted resonances might significantly alter the behavior of the $S$-factor and spoil present-day astrophysics predictions. In particular, broad sub-threshold resonances can influence the shape of the low-energy astrophysical factor by interfering with other resonances as in the case of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction ([2] and references therein), or through their high energy tails. In this last case, the increase of the $S(E)$-factor as $E \to 0$ competes with the electron screening enhancement, and disentangling the two effects might further increase the total error budget. This is the case, for instance, of the $^{13}\text{C}(\alpha, n)^{16}\text{O}$ reaction. At low energies, the $S$-factor is dominated by the contribution of the sub threshold resonance at ~3 keV due to the 6.356 MeV level of $^{17}\text{O}$, having $\Gamma_n = 124 \pm 12$ keV [11]. At the lowest energies, direct data, ending up at ~ 280 keV [12], had to be corrected for atomic electron screening determining an enhancement of less than 20% for the lowest-energy data point.

The $^{13}\text{C}(\alpha, n)^{16}\text{O}$ cross section has been extensively measured since it is considered as the main neutron source necessary to build up heavy elements from iron-peak seed nuclei through the $s$-process. In low-mass ($\leq 3M_\odot$) asymptotic giant branch (AGB) stars [13], where the presence of a $^{13}\text{C}$ pocket [14] allows for neutron production through the $^{13}\text{C}(\alpha, n)^{16}\text{O}$ reaction, $90 \leq A \leq 208$ nuclei are synthesized through a sequence of neutron capture reactions. Because of the relatively low neutron fluxes generated by the $^{13}\text{C}(\alpha, n)^{16}\text{O}$ reaction (on the order of $10^5$ to $10^{11}$ neutrons per cm$^2$ per second), the neutron accretion rate is slower than the $\beta$-decay rate, thus only heavy isotopes along the stability valley can be produced ($s$-process, $s$ for slow) [15]. At $0.9 \times 10^8$ K, a typical temperature characterizing $^{13}\text{C}$-burning [16], the energy range where the $^{13}\text{C}(\alpha, n)^{16}\text{O}$ reaction is most effective is ~100 – 270 keV.
Table 1. Summary of the $S$-factor values evaluated at $E_{cm} = 100$ keV.

| Ref. | $S(100 \text{ keV}) \times 10^6 \text{ MeVb}$ | Approach               |
|------|---------------------------------|------------------------|
| [17] | 3.3                             | R-matrix               |
| [18] | 2.7                             | microscopic cluster approach |
| [19] | 5.3                             | microscopic cluster approach |
| [20] | 6.3                             | R-matrix               |
| [21] | 1.2                             | ANC                    |
| [22] | 3.4                             | Spectroscopic factor   |
| [23] | 2.5                             | Spectroscopic factor   |

Measurements of the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ astrophysical factor include direct and indirect studies, focused on the extraction of the 6.356 MeV $^{17}\text{O}$ level resonance parameters using transfer reactions. Indirect measurements deduced the spectroscopic factor or the asymptotic normalization coefficient (ANC) of this level, as these parameters fix the resonance top value, allowing for the calculation of the $S(E)$-factor beyond the energy region explored by means of direct measurements. Table 1 summarizes the astrophysical $S$-factors evaluated at 100 keV by different authors as well as the procedures adopted to obtain them. Pellegriti et al. [22], Guo et al. [23] and Johnson et al. [21] established $S(100 \text{ keV})$ using the spectroscopic factor and the ANC, respectively. The R-matrix calculations reported in [17, 20] are extrapolations of direct measurements to astrophysical energies, the microscopic cluster approach uses nucleon-nucleon interaction to calculate the $S$-factor. Table 1 clearly shows a large scatter of the $S(E)$-factor values at low energies, suggesting the possible existence of systematic errors causing such indetermination in the $S(100 \text{ keV})$ parameter.

Therefore, new and improved measurements are necessary to pin down the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ $S(E)$-factor at astrophysical energies and calculate a reliable reaction rate for astrophysical applications.

2. The THM measurement of the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ $S(E)$-factor

The Trojan Horse Method (THM) (see [24] for a review of the method) is very suited to investigate the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ reaction as it allows us to determine the resonance parameters even for sub threshold energies, as in the case of the 6.356 MeV $^{17}\text{O}$ state. In the THM approach, the astrophysical factor of the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ reaction is obtained by extracting the quasi-free (QF) contribution to the $^{13}\text{C}(^{6}\text{Li},n^{16}\text{O})^{2}\text{H}$ process. In QF kinematics, $^{6}\text{Li}$, characterized by a prominent $\alpha \oplus d$ cluster structure, is used to transfer the participant cluster $\alpha$ and feed the excited states of $^{17}\text{O}$, while the other constituent cluster $d$ is emitted without interacting with $^{17}\text{O}$, thus behaving as a spectator to the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ sub-process. Then, the modified R-matrix formula is used to fit the THM data and deduce the resonance parameters [25, 26]. Indeed, the same reduced widths appear in the THM cross section and in the direct data; therefore, those extracted from THM data can be introduced into a standard R-matrix code to establish the trend of the $^{13}\text{C}(\alpha,n)^{16}\text{O}$ $S$-factor [27, 28, 29, 30]. In the case of sub threshold resonances, the THM approach can be used to deduce the ANC of such resonances, as in the case of the 6.356 MeV $^{17}\text{O}$ state. This possibility discloses the deep connection between the two indirect methods, which is exhaustively addressed in Refs. [25, 26].

The $^{13}\text{C}(^{6}\text{Li},n^{16}\text{O})^{2}\text{H}$ reaction was measured at the Florida State University Tandem-LINAC facility, which delivered a $E_b = 7.82$ MeV, 1 mm spot $^{6}\text{Li}$ beam impinging onto 99% $^{13}\text{C}$ enriched foils, whose thicknesses were 53 $\mu\text{g/cm}^2$ and 107 $\mu\text{g/cm}^2$. A preliminary analysis of the
Figure 1. Comparison of the THM reaction rate [25, 26] with other prominent rates in the literature, as a function of the temperature $T_9 = T/10^9$ K. Since reaction rates span several orders of magnitude, their ratio to the Heil et al. one [17] is reported. In detail, in this plot the Heil et al. rate [17] is obviously represented by the horizontal blue line ($R_X/R_{Heil} = 1$). The light red band highlights the $R_{THM}/R_{Heil}$ range allowed by statistical, normalization and analysis uncertainties. Finally, the gray band is used to emphasize the $R_{NACRE}/R_{Heil}$ range [31].

THM experiment, based on the data taken with the thinner target, has lead to an ANC for the 6.356 MeV $^{17}$O state ($\tilde{C}_{^{17}O}^{1/(2+)}_{^{13}C} = 6.7^{+0.9}_{-0.6}$ fm$^{-1}$ slightly larger than the values in the literature [21, 22, 23], determining a $^{13}$C($\alpha,n$)$^{16}$O S-factor at 100 keV of $4.0 \pm 0.7 \times 10^6$ MeVb. This result agrees quite well with the largest $S$(100 keV) listed in table 1, with an improved accuracy due to a reduced systematic uncertainty (check Refs. [25, 26] for more details). Since the THM cross section is given in arbitrary units, normalization to direct data was accomplished by spanning the 0.5 – 1.2 MeV energy window in the indirect measurement, an energy region covered by direct data as well. Then, the THM cross section was scaled to match the direct one, which is given in absolute units. Therefore, the resonance parameters deduced from the THM cross section are normalized to those extracted from direct data [28, 25, 26]. The choice of this energy region, where the high energy tail of the -3 keV resonance has a vanishingly small contribution, allows us to reduce the systematic uncertainty connected with the normalization procedure for two reasons, namely, the presence of four resonances in this interval, whose parameters are well known from previous studies [17] and the negligible effect of electron screening.

3. The $^{13}$C($\alpha,n$)$^{16}$O reaction rate

The reduced widths $\gamma$, deduced from the THM data, were used to calculate the astrophysical factor of the $^{13}$C($\alpha,n$)$^{16}$O reaction [25, 26]. It is worth noting that in this case the R-matrix formula is not used to extrapolate the $S$-factor but to parameterize it, as the resonance parameters of the 6.356 MeV $^{17}$O state were measured in the THM experiment. The reaction
rate was then calculated by means of the standard equation [2]:

\[
R_{THM} = N_A \sqrt{\frac{8}{\pi \mu (k_B T)^3}} \int_0^\infty e^{-2\pi \eta} S(E) e^{\frac{-E}{k_B T}} dE
\]  

(2)

where \( N_A \) is the Avogadro number, \( \mu \) the reduced mass of the projectile-target system, \( k_B \) the Boltzmann constant and \( T \) the temperature, and compared with the most recent one [17]. For ease of comparison, the ratio of the THM rate to the Heil et al. one [17] \( (R_{Heil}) \) is given in figure 1; in this representation, the Heil et al. rate [17] is displayed by the horizontal blue line \( (R_X/R_{Heil} = 1) \). The THM recommended rate \( R_{THM}/R_{Heil} \) is shown by the red middle line, while the upper and lower red lines are used to highlight the uncertainty range fixed by experimental errors (including statistical, normalization and analysis uncertainties). Similarly, the NACRE [31] recommended rate, upper and lower limits are marked by black dotted lines. While at \( 0.9 \times 10^8 \) K a difference of only 1\% is found, at \( 10^7 \) K the THM reaction rate is two times larger than in Ref. [17], up to a factor of 3 for lower temperatures (figure 1). Moreover, the present reaction rate is affected by a much smaller error (18\%) than the NACRE one, namely +17\% and -69\% [31] and slightly smaller than the Heil et al. one, about 22\% [17].

In the light of the recent work by Kimura and Bonasera [32], the results shortly summarized in the present work (more details are given in Refs. [25, 26]) deserve further investigation to understand their astrophysical consequences. As discussed in [32], when a resonance is present in the astrophysical factor the definition of Gamow window has to be modified, as this is deduced by assuming a slowly varying \( S \)-factor. In particular, they found that the energy range of astrophysical interest approaches the resonance energy, since the astrophysical factor is larger right at this energy. Therefore, in the \( ^{13}\)C(\( ^{16}\)O, n) reaction it is expected that the energy region where this reaction is most effective in the intershell region of AGB stars draws closer to zero energy, since the \( ^{13}\)C(\( ^{16}\)O, n) astrophysical factor is characterized by a sub threshold resonance. In this region the difference between the THM \( S(E) \) and those in the literature is maximum, making it necessary to study the effects on astrophysics. Such work is currently in progress.

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