Optical Shadows of Rotating Bardeen AdS Black Holes

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Abstract

Using the Hamilton-Jacobi method, we study certain optical shadows of rotating Bardeen-AdS black holes in the presence of internal and external field contributions. Precisely, we examine the shadow using one-dimensional real curves. By establishing the equations of motion of such AdS black holes without external dark sectors, we first analyze the shadow configurations in terms of a reduced moduli space involving only internal parameter contributions including the charge of the nonlinear electrodynamics. Among others, we find that this parameter serves as a geometric quantity controlling the shadow shape. Then, we graphically discuss the associated astronomical observables. Enlarging the moduli space, we analyze the behaviors of the quintessential black hole solutions in terms of the involved parameters controlling the dark field sector. Concretely, we observe that the dark energy not only affects the size but also deforms the shadow shape. Finally, we provide a possible link with observations from Event Horizon Telescope by showing certain constraints on the involved moduli space in the light of the M87* picture.

Keywords: Rotating Bardeen AdS black holes, Optical shadows, Quintessence.

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1 Introduction

Black hole physics has received immense attention since the thermodynamic activities including the Hawking-Page phase transitions elaborations. For extended phase space scenario, the black hole thermodynamics has been studied by considering the cosmological constant as a dynamical quantity identified with the pressure. In such a picture, the charged AdS black holes undergo phase transitions similar to ones appearing in van der Waals fluids.

Within the general relativity, the Penrose and Hawking theorems stipulate the existence of a physical singularity at the black hole center [1, 2]. Nonetheless, there are various methods to get the physics around this singularity. Certain singularity-free black hole solutions can be obtained. One of such interesting classes of these solutions has been obtained from gravity models in the presence of non-linear electrodynamics couplings [3, 4]. The particularity of such solutions concerns smooth spacetime being free from a singularity because the mass parameter is only a consequence of a non-linear electrodynamics source. These regular solutions can be considered as gravitational field contributions with non-linear electric or magnetic monopoles. This formalism concerns two special classes, namely the Bardeen and the Hayward solutions [5, 6]. Such black holes in Anti-de-Sitter (AdS) spacetime exhibit a phase portrait as the one of van der Waals fluids [7,8].

The recent astronomical observations have shown that the universe is undergoing accelerating expansions, which has provided a negative pressure state [9, 10]. It has been suggested that this feature associated with accelerated expansions is a direct consequence of the presence of field configurations known by dark energy (DE). Although the cosmological constant, the quintessence is one of the different models for dark sectors [11,12]. It is recalled that the cosmic source for inflation involves the equation of state $p_q = \omega \rho_q$ with $(-1 < \omega < -1/3)$, and $\omega = -2/3$ corresponds to the quintessential dark energy regime. Such a DE involves the form $p_q = -\frac{\alpha}{2} \frac{3\omega}{(\omega+1)}\rho_q$ with $\alpha$ a relevant parameter. Phase transitions in quintessential black holes are intensively investigated from different angles. In the presence of such a dark sector, the thermodynamics of charged black holes and the regular ones has been dealt with [13–19]. Moreover, the geothermodynamics for different regular black holes has been studied in [20–22], while the case of the charged AdS black holes surrounded by such quintessence field contributions can be found in [23,24].

Black hole optical physics has received remarkable interest using different roads. These activities have been supported by considerable efforts of Event Horizon Telescope collaborations providing an interesting picture of supermassive black holes at the center of galaxy $M87$ [25, 26]. Several works have been elaborated dealing with such optical behaviors by approaching the shadow geometry and the deflection angle [27,28]. In particular, the black hole shadow behaviors of various black holes have been examined [29,30]. Using the null geodesic relations, the elaboration of the shadow cast has been investigated by studying the associated geometrical behaviors. These behaviors have been controlled by certain astronomical observables. The geometrical quantities have been extensively studied for different black hole backgrounds and gravity models [31]. They provide certain physical information about
the associated space-time geometries. An examination shows that two observables have been adopted needed to control the size and the shape geometric deformations of one dimensional real manifolds. Using such activities, the shadow distortion behaviors of Einstein-Maxwell-Dilaton-Axion black holes have been examined [31]. In the absence of the rotating parameter, it has been demonstrated that the shadows involve a circular picture where the associated size can be deformed by the mass and other parameters describing the charge and the dark field intensity [32]. For rotating black hole solutions, however, this geometric configuration has been distorted by providing new geometries. These involve D-shape and other pictures including the cardioid geometries [33]. Moreover, it has been observed that the shadow aspects could depend also on geometric and stringy parameters. Indeed, the brane number and cosmological scale have been implemented where non-trivial geometries have been appeared [34,35].

In this work, we aim to investigate the optical behaviors of Bardeen AdS black holes by examining the corresponding shadows using one dimensional real curves. By establishing the equations of motion of such AdS black holes without external dark sectors, we first discuss the shadow geometries in terms of a reduced moduli space involving only internal parameter contributions including the charge of the nonlinear electrodynamics. Among others, we find that this parameter controls the shadow shape. Then, we analyze graphically the corresponding astronomical observables. Enlarging the moduli space, we approach the shadow optical behaviors of quintessential black hole solutions in terms of the involved parameters. Precisely, the combination of the rotating and the DE parameters of the Bardeen AdS black holes shows that the geometry of the shadows has been affected by the DE sector contributions. For non-rotating case, the associated parameter can control the size. However, it contributes also to the shadow distortions for rotating cases. Finally, we present a possible connection with observations from the Event Horizon Telescope (EHT) by giving certain constraints on the involved black hole parameters in the light of the M87* picture.

This work is organized as follows. In section 2, we present a concise review of rotating Bardeen-AdS black holes in four dimensions. In section 3, we investigate the shadow behaviors of rotating Bardeen-AdS black holes without external dark field contributions. In section 4, we study the quintessential Bardeen AdS black holes. In section 5, we show a possible connection with M87* observations. The last section concerns concluding remarks and open questions.

2 Rotating Bardeen-AdS black holes

In this section, we investigate the optical aspect of the charged and rotating Bardeen black holes in four dimensions with AdS geometries. Concretely, we examine the shadow geometrical pictures in terms of many parameters including the cosmological constant $\Lambda$. It is interesting to note that two solutions can arise depending on such a constant. For $\Lambda > 0$, the model will be called Bardeen de Sitter (Bradeen-dS). However, $\Lambda < 0$ provides a solution
referred to as Bardeen Anti de Sitter (Bardeen-AdS) which will be studied in certain details
through this work by assuming that similar computations could done for the first solution.
Moreover, it has been known from the previous works that the black hole shadow with neg-
ative cosmological constant is more large than that the ones corresponding to the positive one \[29\].
Before investigating the optical behaviors, we give first a concise review on rotating Bardeen-
AdS black holes in four dimensions. Such solutions have been extensively studied \[36\]. In
Boyer-Lindquist coordinates, the line element, associated with solutions without external
contributions describing the dark sector, reads as
\[
ds^2 = -\frac{\Delta_r}{\Sigma} \left( dt - \frac{a sin^2 \theta}{\Xi} d\phi \right)^2 + \frac{\Sigma}{\Delta_r} dr^2 + \frac{\Delta_\theta}{\Delta_\theta} d\theta^2 + \frac{\Delta_\theta sin^2 \theta}{\Sigma} \left( a dt - \frac{r^2 + a^2}{\Xi} d\phi \right)^2. \tag{2.1}\]
The involved reduced terms are expressed as follows
\[
\Delta_r = (r^2 + a^2) \left( 1 + \frac{r^2}{\ell^2} \right) - 2m \left( \frac{r^2}{r^2 + g^2} \right)^{\frac{3}{2}} r, \quad \Delta_\theta = 1 - \frac{a^2}{\ell^2} cos^2 \theta, \quad \Xi = 1 - \frac{a^2}{\ell^2}, \quad \Sigma = r^2 + a^2 cos^2 \theta. \tag{2.2}\]
Here, \(m\) and \(a\) represent the mass and the rotating parameter of the black hole, respectively.
The quantity \(g\) is the charge of the nonlinear electrodynamics. The parameter \(\ell\) denotes the
AdS radius being related to the pressure \(P\) via \(P = -\frac{\Lambda}{8\pi} = \frac{3}{8\pi \ell^2}\). These solutions can be
extended by implementing external dark field contributions \[11\]. A particular emphasis has
been put on DE using the quintessence field contributions. In this way, the above delta term
can be generalized as follows
\[
\Delta_r^\omega = (r^2 + a^2) \left( 1 + \frac{r^2}{\ell^2} \right) - 2m \left( \frac{r^2}{r^2 + g^2} \right)^{\frac{3}{2}} r - \frac{\alpha}{r^{3\omega+1}}, \tag{2.3}\]
where \(\alpha\) denotes the intensity of quintessence and \(\omega\) is the quintessential state parameter.
The optical behaviors of these solutions will be elaborated. First, we consider solutions
without dark field contributions. Then, we analyze the effect of such an external dark field.
This study will be done in terms of a moduli space coordinated by the relevant parameters
including the ones associated with dark field sectors.

3 Shadow properties of rotating Bardeen AdS black holes

In this section, we approach the optical aspects of the rotating Bardeen-AdS black holes
without external dark field contributions. In particular, we consider the investigation of the
shadow optical behaviors in terms of the involved internal parameters including the AdS
radius. This study will be done for particular regions of the moduli space.
3.1 Optical shadows

To approach the shadow optical aspects, certain relations are needed. Indeed, we first write down the massless particle equations of motion by exploiting the Hamilton-Jacobi method [37]. For a photon in the black hole spacetime, one should use

$$\frac{\partial S}{\partial \tau} = -\frac{1}{2} g^{ij} p_i p_j, \quad (3.1)$$

where $S$ and $\tau$ are the Jacobi action and the affine parameter respectively, along the geodesics. In the spherically symmetric spacetime, the Jacobi action takes the following form

$$S = -Et + L\phi + S_r(r) + S_\theta(\theta). \quad (3.2)$$

In this action, $E$ and $L$ represent the conserved total energy and the conserved angular momentum of the massless particle, respectively. In four dimensions, they are written in terms of the four-momentum $p_\mu$ as follows $E = -p_t$ and $L = p_\phi$ which are geodesic constants of motion. It is denoted that $S_\theta(\theta)$ and $S_r(r)$ and are functions which involve only $\theta$ and $r$ spatial variables, respectively. The relevant null geodesic equations can be obtained using a separation method similar to the one exploited in the Carter mechanism [37,38]. To approach the black hole shadow configurations, two quantities called impact parameters are needed. In terms of the above conserve quantities, they are given by the following relations

$$\xi = \frac{L}{E}, \quad \eta = \frac{K}{E^2}; \quad (3.3)$$

where $K$ is a separable constant shearing aspects as the one proposed in [38]. According to the shadow activities developed, in many works, one can get the null geodesic relations. Indeed, they are given by

$$\Sigma \frac{dt}{d\tau} = E \left[ \frac{(r^2 + a^2) [(r^2 + a^2) - a \xi \Xi]}{\Delta_r} + a \left( \frac{\xi \Xi - a \sin^2 \theta}{\Delta_\theta} \right) \right] \quad (3.4)$$

$$\Sigma \frac{dr}{d\tau} = \sqrt{R(r)} \quad (3.5)$$

$$\Sigma \frac{d\theta}{d\tau} = \sqrt{\Theta(\theta)} \quad (3.6)$$

$$\Sigma \frac{d\phi}{d\tau} = E \Xi \left[ \frac{a [(r^2 + a^2) - a \xi \Xi]}{\Delta_r} + \frac{\xi \Xi - a \sin^2 \theta}{\sin^2 \theta \Delta_\theta} \right]. \quad (3.7)$$

In this model, $R(r)$ and $\Theta(\theta)$ are expressed as follows

$$R(r) = E^2 \left[ \frac{(r^2 + a^2) - a \xi \Xi}{\Delta_r} \right]^2 - \Delta_r \eta \quad (3.8)$$

$$\Theta(\theta) = E^2 \left[ \eta \Delta_\theta - \csc^2 \theta \left( a \sin^2 \theta - \xi \Xi \right) \right]^2 \quad (3.9)$$

indicating the radial and the polar motion, respectively. Roughly, the black hole shadow shapes can be elaborated from unstable circular orbits. Precisely, this can be obtained by using the following conditions

$$R(r) \bigg|_{r=r_0} = \frac{dR(r)}{dr} \bigg|_{r=r_0} = 0. \quad (3.10)$$
Here, \( r_0 \) denotes the circular orbit radius of the photon massless particle [32, 37]. Taking \( \Theta(\theta) > 0 \) for \( 0 \leq \theta \leq 2\pi \), the above impact parameters can be obtained by solving Eqs.(3.10). Computations provide

\[
\eta = \frac{16r^2}{\Delta r}, \quad (3.11)
\]

\[
\xi = \left. \frac{(r^2 + a^2) \Delta r' - 4r \Delta r}{a \Xi \Delta r'} \right|_{r = r_0}, \quad (3.12)
\]

where the spatial derivation \( \Delta r' = \frac{\partial \Delta r}{\partial r} \) has been used. A close examination shows that in non-trivial backgrounds the shadow computations need certain relevant considerations. The cosmological constat \( \Lambda \), for instance, brings a visible way to approach such considerations. In fact, one should fix the position \( (r_{ob}, \theta_{ob}) \) of the observer using Boyer-Lindquist system coordinates [39, 40]. It is recalled that \( r_{ob} \) and \( \theta_{ob} \) represent the radial and the angular co-ordinates, respectively. The calculations will be based on appropriate assumptions. Placing the observer in the outer communication domain \( (\Delta r > 0) \), and assuming that the light ray trajectories are ejected from the position \( (r_{ob}, \theta_{ob}) \) to past [40,41], the observer position can be approached by suing the orthogonal tetrads \( (e_0, e_1, e_2, e_3) \). In terms of the metric black hole data, they are given by

\[
e_0 = \left. \frac{(r^2 + a^2) \partial_t + a \Xi \partial_\phi}{\sqrt{\Delta r \Sigma}} \right|_{(r_{ob}, \theta_{ob})}, \quad e_1 = \frac{\sqrt{\Delta \phi}}{\sqrt{\Sigma}} \partial_\theta \left|_{(r_{ob}, \theta_{ob})} \right.
\]

\[
e_2 = \left. -a \sin^2 \theta \partial_t + \Xi \partial_\phi \right|_{(r_{ob}, \theta_{ob})}, \quad e_3 = \left. -\frac{\sqrt{\Delta r}}{\sqrt{\Sigma}} \partial_r \right|_{(r_{ob}, \theta_{ob})} \quad (3.13)
\]

where the timelike vector \( e_0 \) denotes the four-velocity of the observer. However, \( e_3 \) is identical with the spatial direction towards the black hole hole center. The directions \( e_0 \pm e_3 \) represent tangent to the principal null congruences. In this vector representation, the light rays can be parametrized by \( \lambda(s) = (r(s), \theta(s), \phi(s), t(s)) \). One can also consider the tangent vector \( \dot{\lambda} \) which can be decomposed as follows

\[
\dot{\lambda} = \alpha(-e_0 + \sin \rho \cos \delta e_1 + \sin \rho \sin \delta e_2 + \cos \rho e_3), \quad (3.15)
\]

where \( \alpha \) is a scalar factor. \( \rho \) and \( \delta \) are the celestial coordinates [41]. After appropriate computations, one can obtain

\[
\alpha = g(\dot{\lambda}, e_0) = \frac{1}{\sqrt{\Delta r \Sigma}} \left. (a L \Xi - (r^2 + a^2) E) \right|_{(r_{ob}, \theta_{ob})}. \quad (3.16)
\]

The examination of such black hole optical shadows needs the implementation of all parameters including the geometric ones. Concretely, a reduced moduli space will control the associated optical shadows. Following [39–41], the boundary of the geometrical optical shadows can be approached by the help of two cartesian coordinates

\[
x = -2 \tan \left( \frac{\rho}{2} \right) \sin \delta \quad (3.17)
\]

\[
y = -2 \tan \left( \frac{\rho}{2} \right) \cos \delta \quad (3.18)
\]
where the celestial coordinates $\rho$ and $\delta$ are given in terms of $\xi$ and $\eta$ as follows

$$
\sin \rho = \pm \sqrt{\Delta_r \eta} \left| \frac{(r^2 + a^2) - a\xi}{(r_{ob}^2 + a_{ob}^2) - a\xi} \right|_{(r_{ob}, \theta_{ob})}.
$$

(3.19)

$$
\sin \delta = \frac{\sqrt{\Delta_r \sin \theta}}{\sqrt{\Delta_\theta \sin \rho}} \left( \frac{\Xi(a - \Xi \csc^2 \theta \xi)}{a\Xi\xi - (r^2 + a^2)} \right) \left| \frac{(r_{ob}^2 + a_{ob}^2) - a\xi}{(r_{ob}^2 + a_{ob}^2) - a\xi} \right|_{(r_{ob}, \theta_{ob})}.
$$

(3.20)

A close inspection reveals that the shadow geometrical behaviors can depend on the involved parameters describing the rotating black holes. In these solutions, the charge of the nonlinear electrodynamics will be considered as a relevant one. The corresponding shadow properties are represented in the $(x, y)$ plane by using $x$ and $y$ explicit expressions. For the present study, certain parameter values and positions have been fixed. Concretely, the observer is located at $r_{ob} = 50$ and $\theta_{ob} = \frac{\pi}{2}$. It is denoted $\Lambda = -10^{-4}$ and $m = 1$ have been taken. The associated behaviors are plotted in Fig.(1). Concretely, we illustrate the shadow aspects by varying the two parameters $g$ and $a$. It has been remarked that the shadow for a fixed value of $a = 0.9$ is not a perfect circle. Taking $a = 0.9$, the D-shape geometrical configurations appear by increasing $g$. Considering $g = 0.2$, the circles have been distorted and deviated from the centre by increasing $a$. Such a D-shape appears for large values of the rotating parameter. In these rotating AdS black holes, it has been shown that the D-shape geometry is linked only to the rotating parameter $a$. In the present situation, however, the D-shape configuration is linked not only to the rotating parameter $a$ but also to the charge of the nonlinear electrodynamics $g$. Concretely, the D-shape geometry is controlled by the parameters $a$ and $g$. It has been observed that the parameter $g$ affects the space-time configuration of the black hole and the shadow observation form.

Figure 1: Optical shadow aspects of rotating Bardeen-AdS black holes. Right panel: behavior associated with $a = 0.9$ for different values $g$. Left panel: behavior corresponds to $g = 0.2$ for different values $a$. 

-0.2 0.0 0.2 0.4 0.6
-0.4
0.0
0.2
0.4
0.6
0.8
-0.4 -0.2 0.0 0.2 0.4 0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8

3.2 Distorted optical shadows

To analyse the distortion of the black hole shadows, certain deformation parameters, denoted by $R_c$ and $\delta_c$, are needed. These parameters control the associated size and the shape, respectively [42, 43]. Concretely, the size is configured via three specific points. Indeed, two of them are the top and the bottom positions of shadow $(x_t, y_t)$ and $(x_b, y_b)$. The third one concerns the reference circle point denoted by $(\tilde{x}_p, 0)$. In this geometric representation, the point of the distorted shadow circle $(x_p, 0)$ intersects the horizontal axis at $x_p$. In this way, the distance between these points is given by a parameter defined as follows

$$D_c = 2R_c - (x_r - x_p)$$

(3.21)

In these optical behaviors, the distortion parameter controlled by ratio of $D_c$ and $R_c$ reads as

$$\delta_c = \frac{|D_c|}{R_c}.$$  

(3.22)

To go beyond such an optical study of the rotating Bardeen AdS black holes, we examine the above astronomical quantities. These parameters controlling the size and shape deformations are illustrated in Fig.(2) using a reduced moduli space coordinated by $(a, g)$. For small values of $a$ and $g$, we remark that the shadow parameter $R_c$ controlling the size is large contrary to large values of $a$ and $g$. For fixed large values of $a$, the size parameter is decreasing by increasing $g$. This behavior appears in the regular Bardeen black hole [44, 45]. This can be linked to the circle geometry changing toward D-shape configurations by increasing $a$. For
fixed large values of $g$, the size parameter is decreasing by increasing $a$.

We move now to analyze the distortion parameter variation $\delta_c$. For fixed values of $a$, the distortion parameter is increasing with $g$. For small values of $a$, however, this parameter remains small and constant. Fixing $g$, the geometric parameter $\delta_c$ increases with the rotating parameter and takes small values for small values of $a$. The obtained results confirm the impact of $g$ on the size and the shadow form observations.

4 Shadows of quintessential rotating Bardeen AdS black holes

In this section, we examine the shadow behaviors of the quintessential rotating Bardeen AdS black holes. The computations will involve two extra parameters carrying information on dark field contributions [11]. In this way, $\Delta_\omega$ will be implemented in the previous calculations of the equations of motion. Many models could be dealt with by fixing the state equation parameter $\omega$. However, we consider only known values being investigated in different black hole backgrounds. In particular, we analyze three $\omega$-model shaving $\omega = -1, -1/3, -2/3$. Roughly, the shadow aspects will be approached by varying the internal and external black hole parameters. These parameters control the associated moduli space which can be decomposed as follows

$$\mathcal{M} = \mathcal{M}_{int} \times \mathcal{M}_{ext}$$

(4.1)

where $\mathcal{M}_{ext}$ denotes the dark field sector. As done in the previous section, one-dimensional shadow configurations could be illustrated by varying one parameter and fixing the remaining ones. The associated shadow geometries are plotted in Fig.(3). Concerning the $\alpha$ variation, we examine two regions. For small values of this parameter, the size of the shadow decreases, and the D-shape appears. For large values of $\alpha$, however, the D-shape disappears and the size increases. Such behaviors are observed in all $\omega$ models. Indeed, the size increases by decreasing $\omega$. The D-shape geometry disappears by taking $\alpha = 0.3$. These geometrical configurations are not modified even we vary the rotation parameter $a$. Increasing this parameter, the circles are deviated from the origin. For $\alpha = 0.3$, it has been remarked that the parameter $g$ controls the shadow size. Indeed, it decreases by increasing $g$. A close inspection shows that such a parameter controls the shape deformations contrary to the electric charge. It can play a similar role as the rotating parameter appearing in many black hole shadow activities. The appearance of the D-shape geometry is linked to the existence of three parameters, $a$ which controls the shape deformation in the ordinary black hole solutions, $g$ and $\alpha$.

As in the previous section, we analyze the associated distorted geometries by introducing the two extra parameters associated with the dark sector. In particular, we inspect the corresponding effect on $R_c$ and $\delta_c$ parameters. By varying the dark sector parameters and the charge of the nonlinear electrodynamics, these behaviors are illustrated in Fig(4). Fixing $a$ and $\omega$, it follows from this figure that $R_c$ increases linearly with $\alpha$ for different values of $g$. 

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It decreases by increasing $g$. The later behavior appears for $\omega$ variations. Contrary to $R_c$, $\delta_c$ decreases by increasing $\alpha$ for different values of $g$. It increases with $g$. An inverse behavior appears for $\omega$ variations. For fixed values of $\omega$, the geometric parameter $\delta_c$ decreases by increasing $\alpha$. The obtained results confirm the impact of $g$ and $\alpha$ on the size and the shadow shape observations.

5 On observations in the light of the $M87^*$ picture

A close examination shows that the recent observational results corresponding the shadows of the supermassive black hole $M87^*$, realized by EHT international collaborations, have provided many promoting approach to probe certain gravity theories and alternative modified physical models. Motivated by such activities, it would be interesting to make contact with such observational fundings. It has been remarked that the observational data could put constraints on the black hole moduli space $[25, 46–48]$. Precisely, the shadow behaviors can be considered in terms of the external sector of the moduli space for different values of the rotation parameter $a$. In the unit of the $M87^*$ mass, we could superpose the $M87^*$ black
the shadow of the rotating quintessential AdS black holes can approach the M87$^*$ shadow, using $\lambda = -0.002$ and $M = 1$ in units of the M87 black hole mass given by $M_{BH} = 6.5 \times 10^9 M_\odot$ and $r_0 = 91.2 \text{kpc}$.

radius of the both black holes illustrated in this figure, we reveal that the M87$^*$ shadow coincide perfectly with such rotating quintessential AdS black holes for specific values of the relevant parameters. Varying the rotating parameter, the computations show that the relevant parameters are $\alpha = 0.3$, $g = 0.4$, and $\omega = -1/3$. For $\omega = -2/3$, we notice that the shadow of the rotating quintessential AdS black holes can approach the M87$^*$ shadow.
For non distorted geometries, it has been remarked similar behaviors like the M87\(^*\). Since for \(a = 0.9\) the shadows have been distorted, it could be interesting to discuss the distor-
tion of the shadow compared by M87\(^*\) one. It has been observed that the distortion of the
quintessential AdS black hole is the same as M87\(^*\) for \(\alpha = 0.3, g = 0.4,\) and \(\omega = -1/3\). For
other value parameters, we remark that the distortion is different that the M87\(^*\) one. This
graphic analysis could confirm the above specific constraints on such black hole parameters.

6 Conclusion

In the present work, we have analyzed the shadow geometrical aspects of rotating Bardeen
black holes in different backgrounds including AdS geometries and dark sectors. First, we
have considered rotating Bardeen AdS black hole solutions without external dark fields. As
the rotating parameter, we have shown that the charge of the nonlinear electrodynamics
affects not only the space-time structure of the black hole, but also controls the shape
deformation. It has been remarked that it also affects the size. Then, we have discussed
the geometrical observables describing such shadow geometric deformations. Implementing
dark sectors, we have investigated shadow properties of the quintessential rotating Bardeen
AdS black holes. In particular, we have inspected the effect of quintessential dark field
on the shadow curves showing interesting features. Increasing the dark fiend intensity, the
shadow size increases, and the D-shape geometry disappears producing the circular perfect
geometries. In this way, we have observed that DE affects only the size but also deforms the
shadow shape. It has been shown that the appearance of the D-shape geometry has been
linked to the existence of three parameters being \(a\), controlling the deformation shape in the
ordinary black hole solutions, \(g\) and \(\alpha\). It has been remarked that the parameter \(\alpha\) associated
with the quintessence field not only plays a thermodynamic important role but appears also
as a relevant geometric parameter which can control the black hole optical aspects. Precisely,
it controls the size and the shape of the shadow through the existence of other parameters
\(g\) and \(a\), being related to the space-time configurations and the rotating parameter of black
hole, respectively. Finally, we have provided a possible link with observations from Event
Horizon Telescope by revealing certain constraints on the involved black hole parameters in
the light of the M87\(^*\) picture.
Many open questions could be addressed. It would be interesting to consider the dark matter
influence as a relevant element in dark sector contributions. In particular, we could inspect
optical modifications which could come from such a matter. Higher dimensional solutions
could be also a possible issue. We hope to come back to these questions in future works.

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