Language-Based Causal Representation Learning

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Abstract

Consider the finite state graph that results from a simple, discrete, dynamical system in which an agent moves in a rectangular grid picking up and dropping packages. Can the state variables of the problem, namely, the agent location and the package locations, be recovered from the structure of the state graph alone without having access to information about the objects, the structure of the states, or any background knowledge? We show that this is possible provided that the dynamics is learned over a suitable domain-independent first-order causal language that makes room for objects and relations that are not assumed to be known. The preference for the most compact representation in the language that is compatible with the data provides a strong and meaningful learning bias that makes this possible. The language of structured causal models (SCMs) is the standard language for representing (static) causal models but in dynamic worlds populated by objects, first-order causal languages such as those used in “classical AI planning” are required. While “classical AI” requires handcrafted representations, similar representations can be learned from unstructured data over the same languages. Indeed, it is the languages and the preference for compact representations in those languages that provide structure to the world, uncovering objects, relations, and causes."

Introduction

Two important challenges in causal representation learning are learning the state variables of a dynamical system from unstructured data, and learning a representation of the dynamics that is general and reusable [Schölkopf et al. 2021]. For example, a system may involve an agent moving in a $n \times m$ grid, picking up and dropping packages. Learning the causal structure of the domain means to learn the structure of the states, given by the agent and the package locations, and the structure of the actions, so that they can be used to plan in other instances of this general domain. The question is what are the ideas and principles that are required for uncovering these structures in a crisp and well-founded manner, without involving prior domain knowledge.

For addressing this and other challenges, deep learning approaches usually follow a methodology that goes from intuitions about inductive biases to deep learning architectures and loss functions, and from there to experimental results and comparisons with baselines (Goyal and Bengio 2020, Goyal et al. 2020, 2021, Aniket Didolkar et al. 2021). The methodology can be applied broadly and the results show experimental gains, yet the understanding that follows from them is not always crisp.

In this paper, we articulate a different approach for learning general causal representations, and two other representations that exploit causal representations: general policies, and subgoals (“intrinsic rewards”). The idea is to learn the representations over suitable domain-independent languages with a known structure (syntax) and known semantics, but without relying on prior knowledge.

For learning representations of general deterministic, discrete dynamics, we appeal to a language that has been in use in “classical AI” since the early 70s; namely, lifted (first-order) STRIPS, in its modern version, where a planning domain is expressed by a number of action schemas with preconditions and effects given by logical atoms that encode the state variables and their values. In “classical AI”, these action schemas are crafted by hand, but as it has been pointed out by Schölkopf and von Kugelgen (2022), this approach does not scale up as modeling is hard. The “classical” approach does not explain where models come from either.

There are, however, two dimensions about knowledge representations that need to be distinguished: the representation languages, such as STRIPS, that are domain-independent, and encodings in such languages that have traditionally been crafted by hand. The representation languages have been designed with the right goals in mind, including transparency and reuse (McCarthy 1987, Haslum et al. 2019), and there is
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and it is thus the encodings in the language that constitute the representation bottleneck, and it is thus the encodings that should be learned from data. We will see that this is possible when we look for the most compact representation in the language that explains the data; a simple a form

of Occam’s learning that takes advantage of the powerful and meaningful inductive bias that results from the syntax and the semantics of the language.

In the paper we develop this idea of language-based repre-

sentation learning and apply it to three related problems:

learning general dynamics, learning general policies, and

learning general subgoal structures. The first is about un-

covering the causal structure of a domain; the latter two are about exploiting it. While the paper is original and written for the “causal representation learning” audience, the general approach has been developed elsewhere, as described.



Preview

The top part of Fig. 1 shows the state graph that results from a simple, discrete dynamical system that involves an agent that moves on a 1×3 grid, that can pick and drop two different packages, one at a time. The number of states, 45, results of adding the number of configurations where the agent holds no package (27 = 3³) with the number of configurations where the agent holds one of the packages (18 = 2×3²). The number of edges is 96. An edge ⟨s, s’⟩ labeled with an action \( a \in \{\text{Move, Pick, Drop}\} \) means that the action \( a \) transforms state \( s \) into \( s’ \).

In recent work it has been shown that the internal structure of the states can be recovered from state graphs where the states are black-boxes with no known structure. This is achieved by learning such representations over a first-order causal language able to represent objects and relations, which are not assumed to be known, and seeking the most compact encoding that generates the observed data (i.e., the given state graph). The language is the modern version of the “classical” planning language STRIPS where a planning instance \( P \) is encoded as a pair \( P = \langle D, I \rangle \) with the domain \( D \) being a collection of general action schemas, involving a set of predicates, and \( I \) representing specific instance information.

Since a planning instance \( P \) defines a unique state graph \( G(P) \), the learning task becomes an inverse problem: given an observed state graph \( G \), find the “simplest” planning instance \( P = \langle D, I \rangle \) such that the observed and the generated graphs \( G \) and \( G(P) \) “match” (are isomorphic). When the graphs match, every black-box state (node) in \( G \) is mapped into a structured state in \( G(P) \) given by the set of ground atoms obtained from the predicates in \( D \) and the set of objects in \( I \). The atoms encode the state variables and their values.

The most compact STRIPS representation \( P = \langle D, I \rangle \) for the state graph shown in Fig. 1 has the domain \( D \) shown

Interpretation of learned domain predicates:

- \( p_1 \) is true iff agent holds no package (i.e., gripper empty),
- \( p_4(x) \) is true iff agent is at cell \( x \),
- \( p_4(p) \) is true iff agent holds package \( p \),
- \( p_4(p, x) \) is true iff package \( p \) is in cell \( x \), and
- \( p_5(x, y) \) is true if cell \( x \) is adjacent to cell \( y \).

![Figure 1: Top: Labeled state graph \( G \) for agent that can move in 1×3 grid picking up and dropping two different packages. The graph \( G \) has 45 nodes, assumed to be black-box states with no internal structure known (shown in small circles), and 96 edges, which are labeled with the actions Move, Pick, and Drop. Bottom left: STRIPS representation \( P = \langle D, I \rangle \) learned from \( G \). Domain \( D \) has 3 action schemas over 5 predicates, and \( I \) involves 3 objects (only \( D \) is shown). \( P \) is the most compact STRIPS encoding that yields a graph \( G(P) \) that is isomorphic to \( G \). The bijection \( f \) that underlies this isomorphism gives structure to the nodes: black-box node \( n \) in \( G \) becomes a planning state \( f(n) \) in \( G(P) \) that assigns a truth-value to each of the ground atoms in \( P \). Action schemas shown in terms of their preconditions and effects. Static preconditions that involve predicates whose denotation does not change are learned as well. The domain \( D \) learned from the 1×3 instance with two packages works for any grid dimensions and any number of packages. Bottom right: Interpretation of the learned predicates.](image-url)
in the bottom part of the figure. The learned domain consists of 5 predicates and 3 action schemas that generalize to any instance of the domain, involving any grid dimensions and any number of packages. Objects, relations, and causal structure all emerge from the flat graph shown. The learned structure is a result of the data, the target language, and Occam’s razor.

Interestingly, while the state graph $G$ of a single $1\times3$ instance involving two packages yields a domain $D$ that generalizes to any grid size and any number of packages, smaller training instances yield domains that do not generalize in the same way. For example, a $1\times3$ instance with one package yields a model with a unary predicate $p_4(x)$ that tracks the position of the unique package and which does not generalize to instances with multiple packages. Likewise, a $1\times2$ instance yields a model with no $p_5(x,y)$ predicate as there is no need then to represent the topology of the grid (cell adjacency).

### Planning Language

A (classical) planning instance or problem is expressed as a pair $P = \langle D, I \rangle$ where the domain $D$ contains a set of action schemas with preconditions and effects given by atoms $p(x_1, \ldots, x_k)$ in term of predicates symbols $p$ and variables $x_i$ that are arguments of the action schema, and the tuple $I = \langle Obj, s, Init, Goal \rangle$ specifies the constants $c_i$ in the instance (object names), and the initial and goal conditions; the latter in terms of ground atoms $p(c_1, \ldots, c_k)$ obtained from the domain predicates and the constants (Geffner and Bonet 2013; Haslum et al. 2019). Modern planners usually replace the action schemas by their possible instantiations where variables are replaced by constants. Static preconditions are normally compiled away after pruning the set of instantiations. A precondition is static when it involves a predicate which does not appear in an action effect. E.g., predicate $p_5$ in Fig. [1] is a static predicate which captures the adjacency relation among grid cells.

A classical problem $P = \langle D, I \rangle$ encodes a unique state graph $G(P)$ whose nodes are the states that are reachable in $P$ from the initial state $s_0$. A state $s$ is a collection of ground atoms $q$ from $P$ that encode truth-valuations ($q$ is true in $s$ iff $q \in s$), and the initial state $s_0$ is $Init$. The graph $G(P)$ contains a labeled edge $(s, a, s')$ if there is a ground instance $a'$ of an action schema $a$ in $D$ that maps the state $s$ into $s'$. A ground instance $a'$ replaces the schema parameters with constants, and transforms a state $s$ into $s'$ if the preconditions of $a'$ are true in $s$, the effects of $a'$ are true in $s'$, and all ground atoms not affected by $a'$ have the same truth value in $s$ and $s'$.

The distinction between the general domain given by $D$ and the specific information given by $I$ in an instance $P = \langle D, I \rangle$ is particularly relevant in the learning setting where learning the structure of $P$ will mean to learn a general domain $D$ that applies to an infinite number of instances $P' = \langle D, I' \rangle$ that differ from $P$ in the number of objects, or in their initial or goal configurations, but not in the vocabulary (predicates) that captures the structure of the states, or in the action schemas that capture the possible state trajectories. In other words, the domain $D$ expresses in a compact way what is common (invariant) over all these instances.

### Learning Planning Models

The problem of learning general action models from states graphs where states are black boxes has been formulated as follows (Bonet and Geffner 2020):

**Learning action models.** Given observed graphs $G_1, \ldots, G_n$, find the simplest domain $D$ and instances $P_i = \langle D, I_i \rangle$ such that the graphs $G_i$ and $G(P_i)$ are isomorphic for $i = 1, \ldots, n$.

The complexity of a domain is measured in terms of the number and arity of the action schemas and predicates involved. Once these numbers are bounded, the learning problem becomes a combinatorial optimization task that has been expressed and solved using SAT and answer set solvers (Bonet and Geffner 2020; Rodriguez et al. 2021). Variations of this basic problem have also been considered like dealing with noisy and incomplete traces as opposed to fully known graphs (Rodriguez et al. 2021).

The domains that have been learned in this way include several benchmark domains in planning, from Blocks and Logistics, to IPC-Grid (a domain similar to Minigrid (Chevalier-Boisvert et al. 2019) and Sokoban. In all cases, one does not only learn the internal structure of the nodes in the given graphs (i.e., the atoms encoding the state variables and their values), but a general domain representation that can be applied to other instances. This is the result of the strong and meaningful bias that follows from aiming at the most compact language-based model that matches the data. A crucial part of this is the use of a first-order target language for learning. One action schema represents a potentially infinite set of ground instances. If rather than looking for the most compact lifted STRIPS representation, we look for the most compact propositional STRIPS representation, very different representations would result.

This is a vanilla solution method, inherently incapable of learning domains dynamics that cannot be expressed in compact form in lifted STRIPS, but it is a crisp formulation based on general principles and ideas; namely, Occam’s learning (Kearns and Vazirani [1994]) over an hypothesis space spanned by the target language. If one wants to learn general dynamics of domains with continuous, exogenous, or non-deterministic changes, a different target language for learning must be used. STRIPS is a simple language for modeling deterministic actions, but there are more expressive planning (action) languages (Lifschitz [1999]; Haslum et al. 2019), some of which are used to specify MDPs and POMDPs in compact form using action schemas, objects, and relations (Younes et al. 2005; Sanner 2011).

The general idea of language-based representation learning has been used to learn first-order planning representations from gray-box states represented as objects in 2D grids. In this case, the learned state representations are grounded in the 2D scenes, meaning that there is a 1-to-1 correspondence between scenes and planning states that generalizes to new scenes (Liberman et al. 2022).
Learning General Policies

The languages for learning representations can be taken off the shelf in many cases, but in others, new domain-independent languages may be needed. For example, in the Minigrid benchmark (Chevalier-Boisvert et al. 2019). DRL approaches are not after general dynamic models, but after general policies: policies that can deal with any instance of the domain. What is then a good domain-independent language for representing such policies? This question has been considered in the area of *generalized planning*, and the language below follows the one introduced by Bonet and Geffner (2018).

A general policy $\pi$ for a (possibly infinite) class of instances $Q$ drawn from a domain $D$ is given by a set of *policy rules* of the form $C \rightarrow E$ where $C$ contains boolean conditions of the form $p$, $\neg p$, $n = 0$, or $n > 0$, and $E$ contains effects of the form $\neg p$, $p?$, $n\downarrow$, $n\uparrow$, over boolean and numerical features $p$ and $n$ that are well-defined over the states $s$ of any instance from $Q$. The action prescribed by the policy $\pi$ in a state $s$ is any action that maps $s$ into a state $s'$ such that the state transition $(s, s')$ satisfies some policy rule $C \rightarrow E$ in $\pi$; namely, $s$ makes $C$ true, and the transition $(s, s')$ makes the change expressed by $E$ true as well.

For example, with features $\Phi = \{H, p, t, n\}$ for “holding a package”, “distances to nearest package and to the target”, and “number of undelivered packages”, the following policy solves any instance of the domain displayed in Fig. 1 when the goal is to take all the packages, one by one, to a target cell in the grid:

- $\{\neg H, p > 0\} \rightarrow \{p\downarrow, t?\}$; go to nearest pkg.
- $\{\neg H, p = 0\} \rightarrow \{H\}$; pick it up,
- $\{H, t > 0\} \rightarrow \{t\downarrow\}$; go to target,
- $\{H, n > 0, t = 0\} \rightarrow \{\neg H, n\downarrow, p?\}$; drop pkg.

The first rule says to do any action that decreases the distance $p$ to the nearest package ($p\downarrow$) when not holding a package and the distance is positive ($\neg H$ and $p > 0$), whatever the effect on the distance $t$ to the target ($t?$). The reading of the other rules is similar with $x\downarrow$ standing for decrements of feature $x$, and $x?$ for any change in $x$. Features not mentioned in the right-hand side of a policy rule must keep their values unchanged.

This is a policy written by hand, and the question is how such policies can be learned. As before, the learning problem has been formulated and solved as a combinatorial optimization problem by creating a large but finite set of possible boolean and numerical features *from the domain predicates*, using a description logic grammar that captures a decidable fragment of first-order logic, $C_2$, where the number of variables is limited to two (Baader et al. 2008). Provided with this pool of features, where each feature is given a cost (the number of grammar rules used to derive it), the task of learning a general policy becomes (Francès et al. 2021):

Learning general policies. Given a known domain $D$, training instances $P_1, \ldots, P_n$ over $D$, and a finite pool of domain features $F$, each with a cost, find the simplest policy $\pi$ over $F$ such that $\pi$ solves all $P_i$, $i = 1, \ldots, n$.

Once again, the language in which policy representations are sought provides a *strongly biased hypothesis space* where policies that involve few simple features (in terms of the domain predicates) are preferred. The simplest policies are those that minimize the complexity of the features involved. General policies for a number of benchmark planning domains have been derived in this way and proved to be correct (Francès et al. 2021). More recently, an alternative learning scheme has been introduced which does not require a predefined pool of features (Stählberg et al. 2022b). This is achieved by introducing two variations. First, general value functions $V$ are learned instead of general policies, so that the resulting policies are those which are greedy in $V$. Second, the value functions are expressed in terms of graph neural networks (GNNs) which are known to capture $C_2$ features (Barceló et al. 2020; Grohe 2020). Interestingly, the resulting policies generalize equally well (100% generalization in rich, combinatorial domains) and yield close-to-optimal policies even in domains where one can prove that there are no general policies that are optimal (Gupta and Nau 1992). One point in common with recent deep learning approaches for computing general policies that appeal to causal considerations (Zhang et al. 2020; Sonar et al. 2021) is that the learned features are functions of the domain predicates; namely, the predicates that are used to capture the causal dynamics of the domain.

Learning Subgoal Structure

The problem of expressing and using the common subgoal structure of a collection of planning problems has been important in AI since the 1960s, while the problem of learning such structure has become important in recent RL research where useful subgoals are expressed via intrinsic rewards (Chentanez et al. 2004; Zheng et al. 2020). We are interested in a similar problem but want to learn subgoal structures over a suitable language. The questions, from the perspective of language-based representation learning, are 1) what is an adequate language for representing subgoal structure, 2) what is its semantics, and 3) how representations over such language can be learned. A general compact language for representing subgoal structures has been developed recently whose syntax is the syntax of the general policies considered above. The change is in the semantics (Bonet and Geffner 2021).

A (policy) sketch is a set of sketch rules $C \rightarrow E$ of the same form as policy rules, but while policy rules filter 1-step transitions; namely, when in a state $s$, a 1-step transition to any $s'$ must be selected such that $(s, s')$ satisfies a policy rule, sketch rules define *subproblems*: when in a state $s$ of an instance $P$, a state $s'$ is to be reached, not necessarily in one step, such that the multi-step transition $(s, s')$ satisfies a sketch rule (or $s'$ satisfies the goal of $P$).

Sketches decompose problems into subproblems without prescribing how the subproblems should be solved (going from $s$ to $s'$). One is interested, however, in sketches that yield subproblems that can be solved efficiently, in low
polynomial time (in the number of problem variables), and this is guaranteed when subproblems have bounded width (Lipovetzky and Geffner 2012). This observation led to the following formulation for learning sketches, where the notation $P[R]$ is used to refer to the collection of subproblems defined by the sketch $R$ over states $s$ that are reachable in the instance $P$ (Drexler et al. 2022):

**Learning general sketches.** Given a known domain $D$, training instances $P_1, \ldots, P_n$ and a non-negative integer $k$, find the simplest sketch $R$ over a pool of domain features $\mathcal{F}$ such that 1) the collection of subproblems induced by $R$ on each instance $P_i$, $P_i[R]$, have width bounded by $k$, and 2) the sketch $R$ is acyclic in $P_i$, $i = 1, \ldots, n$.

The complexity of a sketch is given by the complexity of the features involved, and a sketch is acyclic in $P$ if the transitions $(s, s')$ in $P$ that satisfy sketch rules do not form a cycle. The learning problem becomes a combinatorial optimization problem modeled and solved using the answer set programming system Clingo (Gebser et al. 2012).

The learned sketches are not aimed at representing the general causal structure of the domain, but at exploiting it. Indeed, by learning to decompose problems into subproblems of bounded width, the problems can be solved in polynomial time using a general algorithm (SIW $R$) that takes the sketch into account (Bonet and Geffner 2021; Drexler et al. 2021). Simple examples of learned sketches follow.

A width-2 sketch $R_1$ for the problem above, where packages need to be delivered to a target cell, one by one, involves the feature $n$ which tracks the number of packages not yet delivered, and is given by a single rule:

$$R_1 : \{ \{ n > 0 \} \rightarrow \{ n\} \} .$$

The sketch expresses a decomposition that for states $s$ where $n > 0$, states $s'$ should be reached where the value of $n$ is lower than in $s$. One can show that the resulting subproblems have width bounded by 2 and thus can be solved by running the IW(2) algorithm (Lipovetzky and Geffner 2012).

A width-1 sketch $R_2$ that involves the features $n$ and $H$ ($H$ is “holding a package”) is given instead by two rules:

$$R_2 : \{ \{ \neg H \} \rightarrow \{ H \} \} , \{ n > 0, H \} \rightarrow \{ n, \neg H \} \} .$$

The first rule on the left says that if not holding a package, one such package should be picked up, while the second on the right says that if holding a package, it should be delivered. The rules do not express policies but subgoals to be achieved. In this case, the subproblems have all width 1 meaning that they can be solved in linear time by running the IW(1) algorithm.

**Related Work**

**Structured causal models.** SCMs are the standard language for encoding and studying causal models (Pearl 2009). Yet, planning and action languages also encode causal models; i.e., they can accommodate observations, interventions, and counterfactuals; all 3-levels of Pearl’s Causation Lad-

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1For doing this, uncertainty about the initial situation must be represented, and action precondition and effects in STRIPS need to be replaced by conditional effects, a standard feature of modern planning languages (Haslum et al. 2019). Then, the atoms at time 0 and the actions at each time step are the exogenous variables, and the atoms at time $t + 1$ for $t > 0$ are the endogenous variables. The truth value of the latter is a function of the value of atoms at time $t$ and the action at $t$. Planning languages just provide a way for defining the atoms (i.e., the Boolean variables of interest), the functions, and in certain cases, the uncertainty about the initial situation, in a compact and reusable form.
General policies, subgoals, and intrinsic rewards. Some approaches for learning general policies make use of languages for representing policies (Kharden 1999; Martin and Geffner 2000; Fern et al. 2006), but most of those based on deep learning, do not (Groshev et al. 2018; Garg et al. 2020; Toyer et al. 2020). Subgoals in planning have been expressed in terms of hierarchical task networks (Erol et al. 1994) but a recent language (Bonet and Geffner 2021) supports more compact representations that facilitate learning of both policies and sketches (Frances et al. 2021; Drexler et al. 2022).

In RL, subgoals are associated with states of intrinsic reward (stepping stones to sparse states of extrinsic reward) (Zheng et al. 2020), but no language or principles have been developed for expressing or learning them, and the focus is on performance improvement that is less informative and crisp.

Discussion

The language of structured causal models has been fundamental to model and to understand causality, but the language itself is insufficient to model the world. Objects and schemas are required to represent the dynamics of the world in a compact way, and this ability is a prerequisite for learning it. We have advocated the use of a broader class of domain-independent languages for learning causal representations, including those required to model dynamics, policies, and subgoals. Some of these languages can be taken off-the-shelf; others have to be designed. It is the languages and the preference for compact representations over them that structure the world and uncover objects, relations, and causes. The languages considered for modeling and learning system dynamics, leave many important aspects aside, including continuous, exogenous, and non-deterministic change. For addressing such aspects, richer languages and more powerful learning methods, possibly based on deep learning, are needed. Learning reusable and meaningful language-based dynamic models of, say, the Atari games from the screen pixels alone, is still an open challenge that we think could be addressed in the near future.

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