

**Exact $\mathcal{N} = 4$ Supersymmetric Low-Energy Effective Action in $\mathcal{N} = 4$ Super Yang-Mills Theory**

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**Abstract**

We review a recent progress in constructing the low-energy effective action in $\mathcal{N} = 4$ SYM theory. This theory is formulated in terms of $\mathcal{N} = 2$ harmonic superfields corresponding to $\mathcal{N} = 2$ vector multiplet and hypetrmultiplet. Such a formulation possesses the manifest $\mathcal{N} = 2$ supersymmetry and an extra hidden on-shell supersymmetry. Exploring this hidden $\mathcal{N} = 2$ supersymmetry we prove that the known non-holomorphic potentials of the form $\ln W \ln \bar{W}$ can be explicitly completed by the appropriate hypermultiplet-dependent terms to the entire $\mathcal{N} = 4$ supersymmetric form. The non-logarithmic effective potentials do not admit an $\mathcal{N} = 4$ completion and, hence, such potentials cannot occur in $\mathcal{N} = 4$ supersymmetric theory. As a result, we obtain the exact $\mathcal{N} = 4$ supersymmetric low-energy effective action in $\mathcal{N} = 4$ SYM theory.



**1 Introduction**

The study of various aspects of the $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory is one of the most active trends in modern high energy theoretical physics. We will discuss here a recent progress towards finding out the $\mathcal{N} = 4$ SYM low-energy effective action.¹

First consideration of the low-energy effective action in $\mathcal{N} = 2$ superconformal theories including $\mathcal{N} = 4$ SYM theory has been given in [1, 2]. In $\mathcal{N} = 4$ SYM model with gauge group $SU(2)$ spontaneously broken down to $U(1)$ (Coulomb branch), the requirements of the scale and $R$-invariance determine the part of the effective action depending on $\mathcal{N} = 2$ superfield strengths $W$ and $\bar{W}$ up to a numerical coefficient. The result is formulated in terms of non-holomorphic effective potential

$$\mathcal{H}(W, \bar{W}) = c \ln \frac{W}{\Lambda} \ln \frac{\bar{W}}{\Lambda}. \quad (1)$$

Here $\Lambda$ is an arbitrary scale and $c$ is an arbitrary real coefficient. The effective action in the $\mathcal{N} = 2$ gauge fields sector is defined as an integral of $\mathcal{H}(W, \bar{W})$ over the full $\mathcal{N} = 2$ superspace. One can show [3, 4] that the potential (1) gets neither perturbative contributions beyond one loop nor non-perturbative corrections. As a result, expression (1) determines the exact low-energy effective action for the $SU(2)$ theory in the Coulomb branch. The problem is reduced to computing the coefficient $c$ in one-loop approximation.

The direct derivation of the potential (1), computation of the coefficient $c$ and, hence, the final reconstruction of the exact exact low-energy $U(1)$ effective action from the quantum $\mathcal{N} = 4$ SYM theory were given in refs. [2]-[4]. Later these results were generalized to...
the group $SU(N)$ broken to its maximal abelian subgroup \cite{7-9}. The corresponding one-loop effective potential is given by

$$
\mathcal{H}(W, \bar{W}) = c \sum_{I<J} \ln \frac{W^I - W^J}{\Lambda} \ln \frac{\bar{W}^I - \bar{W}^J}{\Lambda},
$$

(2)

with the same coefficient $c$ as for the group $SU(2)$. Here $I, J = 1, 2, \ldots, N$, $W = \sum_i W^I e_{II}$ belongs to Cartan subalgebra of the algebra $su(N)$, $\sum_i W^I = 0$, and $e_{II}$ is the Weyl basis in the $su(N)$ algebra.

The potential (2) looks quite analogous to (1). However, we cannot state that (2) determines the exact low-energy effective action as in the case of $SU(2)$ group. The general scale and R-invariance considerations do not forbid the presence of some extra terms in the non-holomorphic effective potential of the form \cite{10}

$$
f \left( \frac{W^I - W^J}{W^K - W^L} ; \frac{\bar{W}^I - \bar{W}^J}{\bar{W}^K - \bar{W}^L} \right),
$$

(3)

with $f$ being real functions. The direct calculations \cite{15} have not confirmed the appearance of terms like (3) at two, three and four loops. However, in a general setting, the problem of contributions (3) to the effective action remained open.

We wish to pay attention to the fact that all the results concerning the structure of the low-energy effective action of $\mathcal{N} = 4$ SYM theory were actually obtained only for a part of it, that defined in the $\mathcal{N} = 2$ gauge fields sector. From the point of view of $\mathcal{N} = 2$ supersymmetry, the $\mathcal{N} = 4$ gauge multiplet consists of $\mathcal{N} = 2$ vector multiplet and hypermultiplet (see e.g. \cite{11}). The problem of the hypermultiplet dependence of $\mathcal{N} = 4$ SYM effective action has been studied in our papers \cite{12, 13} where the exact $\mathcal{N} = 4$ low-energy effective action has been constructed. The aim of the present paper is to give a brief review of the approach of Ref. \cite{12}.

A natural framework for finding the $\mathcal{N} = 4$ supersymmetric effective action is provided by a formulation of $\mathcal{N} = 4$ SYM theory in terms of superfields carrying off-shell representations of $\mathcal{N} = 2$ supersymmetry defined in $\mathcal{N} = 2$ harmonic superspace \cite{14, 11}. The harmonic superspace approach was succesfully used to study the effective action in extended supersymmetric theories in refs. \cite{6, 9, 15-19} (see also the review \cite{20}).

To find the $\mathcal{N} = 4$ supersymmetric effective action, we proceed in the following way. We begin with the $\mathcal{N} = 4$ SYM theory formulated in terms of $\mathcal{N} = 2$ harmonic superfields. Then we examine which hypermultiplet-dependent terms can be added to the potentials (1) - (3) in order to ensure full $\mathcal{N} = 4$ supersymmetry of the effective action. We find such terms for the potentials (1) and (2) and show that the analogous terms do not exist for the potential (3) \cite{12}. Therefore, the potentials of the form (3) can never occur in the full supersymmetric theory.

## 2 $\mathcal{N} = 4$ SYM Theory in $\mathcal{N} = 2$ Harmonic Superspace

The most efficient approach to constructing quantum formulations of supersymmetric field theory models is based on the use of superfields carrying off-shell representations of supersymmetry (see e.g. \cite{21}). The main attractive feature of such an approach is the
possibility to preserve a manifest supersymmetry on all steps of quantum calculations. From this point of view, the most appropriate quantum formulation of $\mathcal{N} = 4$ SYM theory would be the one in $\mathcal{N} = 4$ superspace. However, the corresponding formulation is unknown so far. The best what we can employ at present for $\mathcal{N} = 4$ SYM theory is its formulation in terms of $\mathcal{N} = 2$ superfields in harmonic superspace \cite{11}, \cite{14}. In this case two supersymmetries are manifest and two other ones are hidden.

The action of $\mathcal{N} = 4$ SYM theory is written in $\mathcal{N} = 2$ harmonic superspace as a sum of actions for $\mathcal{N} = 2$ gauge multiplet and hypermultiplet in the adjoint representation coupled to the gauge multiplet

$$S[V^{++}, q^+] = \frac{1}{8} \left( \int d^8 \zeta_L \text{tr} \ W^2 + \int d^8 \zeta_R \text{tr} \ W^2 \right)$$

$$- \frac{1}{2} \int d\zeta^{(-4)} \text{tr} \ q^{+a} \left( D^{++} + igV^{++} \right) q^+_a .$$

Here the real analytic superfield $V^{++}$ is an unconstrained gauge potential of $\mathcal{N} = 2$ SYM theory and the unconstrained charged analytic superfield $q^+_a, a = 1, 2$, describes the hypermultiplet. The action \cite{14} is manifestly $\mathcal{N} = 2$ supersymmetric. Moreover, this action possesses an extra hidden $\mathcal{N} = 2$ supersymmetry which mixes $\mathcal{N} = 2$ superfield strengths $W, \bar{W}$ with $q^+_a$ \cite{11}. As a result, the action \cite{14} describes the $\mathcal{N} = 4$ supersymmetric YM theory. Our aim is to construct $\mathcal{N} = 4$ supersymmetric effective action whose classical limit would be the action \cite{14} and whose hypermultiplet-independent part is the known $\mathcal{N} = 2$ supersymmetric functional corresponding to the low-energy effective action in the $\mathcal{N} = 2$ gauge multiplet sector. The latter action is given in terms of non-holomorphic effective potential $\mathcal{H}(W, \bar{W})$ depending on the abelian chiral and antichiral superfield strengths $W$ and $\bar{W}$ satisfying the free classical equations of motion (on-shell conditions)

$$(D^+)^2 W = 0, \quad (D^+)^2 \bar{W} = 0 ,$$

where $(D^+)^2 = D^+ a D^+_a$, $(\bar{D}^+)^2 = \bar{D}^+_a \bar{D}^a$, $D^+_a = u^+_i D^i_a$, $D^a_\bar{a} = u^+_i \bar{D}^i_\bar{a}$ and $D^a_\bar{a}, \bar{D}^i_\bar{a}$ are the standard $\mathcal{N} = 2$ supercovariant derivatives. As a result, we have to know the hidden $\mathcal{N} = 2$ supersymmetry transformations only on the free mass shell \cite{11}. Obviously, the corresponding hypermultiplet should also be assumed to satisfy the free classical equations of motion

$$D^{++} q^+_a = 0 .$$

Taking into account eqs. \cite{11}, \cite{14}, one can write the hidden $\mathcal{N} = 2$ supersymmetry transformations in the form \cite{11}

$$\delta W = \frac{1}{2} \epsilon^{aa} \bar{D}^-_a q^+_a , \quad \delta \bar{W} = \frac{1}{2} \epsilon^{-aa} D^- a q^+_a ,$$

$$\delta q^+_a = \frac{1}{4} (\epsilon^a \bar{D}_a^- W + \epsilon^{-a} \bar{D}_a^- \bar{W} ) , \quad \delta q^-_a = \frac{1}{4} (\epsilon^{-a} D^- a W + \epsilon^a \bar{D}^- a \bar{W} ) ,$$

where $\epsilon^{-a}, \epsilon^{-a}$ are anticommuting infinitesimal parameters. For further use, we have introduced the quantity

$$q^{-a} = D^- a q^+_a$$

satisfying the on-shell relations

$$D^- a q^{-a} = 0 , \quad D^{++} q^- a = q^+_a , D^- a q^- a = \bar{D}^- a q^- a = 0 .$$
The operators $D^{++}, D^{--}$ are given in [11].

We would like to emphasize once more that the supersymmetry transformations (7) are essentially on-shell.

3 Construction of $\mathcal{N}=4$ Supersymmetric Effective Action

We begin with the $SU(2)$ gauge theory. The effective action is assumed to have the following general form

$$\Gamma[W, \bar{W}, q^+] = S[V^{++}, q^+] + \bar{\Gamma}[W, \bar{W}, q^+] + L \bar{q}(W, \bar{W}, q^+) \tag{10}$$

where $S[V^{++}, q^+]$ is the classical action (4) and the functional $\bar{\Gamma}[W, \bar{W}, q^+]$ incorporates quantum corrections. We also assume that the functional $\bar{\Gamma}[W, \bar{W}, q^+]$ satisfies the condition

$$\bar{\Gamma}[W, \bar{W}, q^+]|_{q^+=0} = \int d^{12}z du \mathcal{H}(W, \bar{W}) \tag{11}$$

with $\mathcal{H}(W, \bar{W})$ being the known non-holomorphic potential. Of course, the integral over harmonics is equal to unity, we have written it for further convinience. Here $d^{12}z$ is the full $\mathcal{N}=2$ superspace measure. Eqs. (10), (11) imply that the corrections to the effective action are of the form

$$\Gamma[W, \bar{W}, q^+] = \int d^{12}z du \left[ \mathcal{H}(W, \bar{W}) + \mathcal{L}_q(W, \bar{W}, q^+) \right] = \int d^{12}z du \mathcal{L}_{\text{eff}}(W, \bar{W}, q^+) \tag{12}$$

$\mathcal{L}_q(W, \bar{W}, q^+)$ is some function unknown for a moment. The functional (12) is manifestly $\mathcal{N}=2$ supersymmetric. We will seek for a function $\mathcal{L}_q(W, \bar{W}, q^+)$ such that the full functional is invariant under the hidden $\mathcal{N}=2$ supersymmetry transformations (7).

Let us consider the transformation of the first term in (12) under (7), taking into account the explicit form of $\mathcal{H}(W, \bar{W})$ (1). One obtains

$$\delta \int d^{12}z du \mathcal{H}(W, \bar{W}) = \frac{1}{2} c \int d^{12}z du \frac{q^+ q^-}{WW} \left( \epsilon^{\alpha}_a D^\alpha W + \epsilon^{\dot{\alpha}}_{\dot{a}} \bar{D}^{\dot{\alpha}} \bar{W} \right) \tag{13}$$

The function $\mathcal{L}_q(W, \bar{W}, q^+)$ is determined from the condition that the variation of the second term in (12) cancels the variation (13). We will search for $\mathcal{L}_q(W, \bar{W}, q^+)$ in the form of the series (12)

$$\mathcal{L}_q = c \sum_{n=1}^{\infty} c_n \left( \frac{q^+ q^-}{WW} \right)^n \tag{14}$$

with some unknown coefficients $c_n$. The quantity $q^-_a$ is defined by (8). The condition

$$c \sum_{n=1}^{\infty} c_n \delta \int d^{12}z du \left( \frac{q^+ q^-}{WW} \right)^n = -\delta \int d^{12}z du \mathcal{H}(W, \bar{W}) \tag{15}$$

where the variation of the right hand side is given by (13), allows us to obtain the recursive relations between the coefficients $c_{n-1}$ and $c_n$ (see [12] for details)

$$c_n = -2 \frac{(n-1)^2}{n(n+1)} c_{n-1}, \quad c_1 = -1. \tag{16}$$
The relations (16) are solved by

\[ c_n = \frac{(-2)^n}{n^2(n + 1)}. \]  

(17)

As the result, the function \( L_q \) proves to be

\[ L_q(W, \bar{W}, q^+) \equiv L_q(X) = c \sum_{n=1}^{\infty} \frac{1}{n^2(n + 1)} X^n, \]  

(18)

where

\[ X = -2 \frac{q^{ia} q_{ia}}{W \bar{W}}. \]  

(19)

The series in the right hand side of (18) can be rewritten in terms of Euler dilogarithm function \( \text{Li}_2(X) \) [22],

\[ \text{Li}_2(X) = \sum_{n=1}^{\infty} \frac{X^n}{n^2} = -\int_0^X \frac{\ln(1 - t)}{t} dt, \]  

(20)

as

\[ L_q(W, \bar{W}, q^+) = c \left( (X - 1) \frac{\ln(X - 1)}{X} + [\text{Li}_2(X) - 1] \right). \]  

(21)

The quantity \( X \) defined in (19) actually does not depend on harmonics since the quantities \( W, \bar{W} \) and \( q^{ia}, q_{ia} \) are on-shell and obey eqs. (5), (6), (9):

\[ X = -\frac{q^{ia} q_{ia}}{W \bar{W}}. \]  

(22)

Therefore \( L_q(W, \bar{W}, q^+) \) does not depend on harmonics and the integral over harmonics in (12) can be omitted. As the result, the full \( \mathcal{N} = 4 \) supersymmetric low-energy effective action has the form

\[ \Gamma[W, \bar{W}, q^+] = S[V^{++}, q^+] + \int d^{12}z \mathcal{L}_{\text{eff}}(W, \bar{W}, q^+), \]  

(23)

where

\[ \mathcal{L}_{\text{eff}}(W, \bar{W}, q^+) = \mathcal{H}(W, \bar{W}) + L_q(X). \]  

(24)

Here \( \mathcal{H}(W, \bar{W}) \) is given by eq. (1), \( L_q(X) \) by eq. (21) and \( X \) by eq. (22).

We emphasize that \( \Gamma[W, \bar{W}, q^+] \) (23) is the exact low-energy effective action. It is known that the non-holomorphic effective potential \( \mathcal{H}(W, \bar{W}) \) (1) is exact [2]. The function \( L_q(X) \) (21) was uniquely restored from (1) by invoking the requirement of the hidden \( \mathcal{N} = 2 \) supersymmetry (7). Hence, it is the only function which forms, together with \( \mathcal{H}(W, \bar{W}) \), the \( \mathcal{N} = 4 \) supersymmetric functional (23), (24). Therefore, the functional (23), (24) is the exact low-energy effective action of \( \mathcal{N} = 4 \) SYM theory with the \( SU(2) \) gauge group spontaneously broken down to \( U(1) \).

We have obtained the effective Lagrangian (24), (21) from the purely algebraic considerations. Recently, we have shown that this effective Lagrangian can be recovered in the framework of quantum field theory by calculating the one-loop harmonic supergraphs with an arbitrary number of the external hypermultiplet legs [13].


4 Component Structure and Generalization to $SU(N)$ Gauge Group

We consider the component form of the effective action (23) in the bosonic sector. In this case

$$W = \varphi(x) + 4i\theta^{+}_{(a} \theta^{-}_{b)} F^{(\alpha\beta)}(x), \quad \bar{W} = \bar{\varphi}(x) + 4i\bar{\theta}^{+}_{(a} \bar{\theta}^{-}_{b)} \bar{F}^{(\dot{\alpha}\dot{\beta})}(x),$$

$$D^{+}_{a} D^{-}_{\dot{b}} W = -4i F^{(\alpha\beta)}(x), \quad \bar{D}^{+}_{a} \bar{D}^{-}_{\dot{b}} \bar{W} = 4i \bar{F}^{(\dot{\alpha}\dot{\beta})}(x), \quad q^{ia} = f^{ia}(x). \quad (25)$$

Here $\varphi(x)$ is a complex scalar field belonging to the $\mathcal{N} = 2$ vector multiplet, $F^{\alpha\beta}(x)$ and $\bar{F}^{\dot{\alpha}\dot{\beta}}(x)$ are the self-dual and anti-self-dual components of the abelian field strength $F_{mn}$, and $f^{ia}(x)$ stands for four scalar fields of the hypermultiplet. The quantity $X$ in the bosonic sector is reduced to

$$X|_{\theta = 0} = -\frac{f^{ia} f_{ia}}{|\varphi|^2} \equiv X_0. \quad (26)$$

To find the structure of the low-energy effective action in the bosonic sector, one uses the relation

$$\int d^{12}z \mathcal{L}_{eff} = \frac{1}{16^2} \int d^4 x (D^+)^2 (D^-)^2 (\bar{D}^+)^2 (\bar{D}^-)^2 \mathcal{L}_{eff}. \quad (27)$$

Fulfilling the differentiation in (27) and discarding fermions, one gets

$$\int d^{12}z \mathcal{L}_{eff} = 4c \int d^4 x \frac{F^2 F^2}{|\varphi|^4} [1 + G(X_0)], \quad (28)$$

where

$$G(X_0) = \frac{X_0 (2 - X_0)}{(1 - X_0)^2}. \quad (29)$$

Here $F^2 = F^{\alpha\beta} F_{\alpha\beta}, F^2 = F^{\dot{\alpha}\dot{\beta}} \bar{F}_{\dot{\alpha}\dot{\beta}}$. Now one substitutes the explicit form of $X_0$ (26) into (28), (29) and finally obtains the quantum correction functional $\bar{\Gamma}$ in the bosonic sector in the extremely simple form

$$\bar{\Gamma}^{bos} = 4c \int d^4 x \frac{F^2 F^2}{(|\varphi|^2 + f^{ia} f_{ia})^2}. \quad (30)$$

We point out that the denominator here is $SU(4)$ invariant square of six real fields of the $\mathcal{N} = 4$ vector multiplet.

Now let us turn to the theory with the gauge group $SU(N)$ spontaneously broken down to its maximal abelian subgroup $U(1)^{N-1}$. In this case the non-holomorphic effective potential is given by (4). Its structure is analogous to (1) and, therefore, we can again apply the techniques exposed in Section 3. This leads to (see [12] for details)

$$\mathcal{L}_{eff}(W, \bar{W}, q^+) = \sum_{I,J} \mathcal{L}^{IJ}_{eff}(W, \bar{W}, q^+). \quad (31)$$

Here, each contribution $\mathcal{L}^{IJ}_{eff}(W, \bar{W}, q^+)$ has the form (24), (21), with $X$ being replaced by

$$X_{IJ} = -\frac{q^{ia}_{IJ} q_{ia}}{W_{IJ} W_{IJ}}, \quad (32)$$
\[ W_{IJ} = W_I - W_J, \bar{W}_{IJ} = \bar{W}_I - \bar{W}_J, \text{ and } q^{ia}_{IJ} = q^{ia}_I = q^{ia}_J. \] The hypermultiplet superfields \( q^{ia} \) belong to the Cartan subalgebra of the Lie algebra \( su(N) \). The quantum correction functional \( \bar{\Gamma}[W, \bar{W}, q^+] \) in the bosonic sector is given by the sum of terms (30), as follows from (31).

Now we are going to examine if the non-holomorphic potential (3) admits an \( \mathcal{N} = 4 \) completion. The corresponding \( \mathcal{N} = 4 \) supersymmetric quantum correction functional must be of the form

\[ \int d^{12}z d\nu \{ f(V_{IJKL}, \bar{V}_{IJKL}) + L_q(W_{IJ}, W_{KL}, \bar{W}_{IJ}, \bar{W}_{KL}, q^{ia}_{IJ}, q^{ia}_{KL}) \}, \tag{33} \]

where

\[ V_{IJKL} = \frac{W_{IJ}}{W_{KL}}, \quad \bar{V}_{IJKL} = \frac{\bar{W}_{IJ}}{\bar{W}_{KL}} \tag{34} \]

and \( W_{IJ}, \bar{W}_{IJ}, q^{ia}_{IJ} \) are given in (32). Here \( L_q(W_{IJ}, W_{KL}, \bar{W}_{IJ}, \bar{W}_{KL}, q^{ia}_{IJ}, q^{ia}_{KL}) \) is some unknown function which must constitute, together with \( f(V_{IJKL}, \bar{V}_{IJKL}) \), the full \( \mathcal{N} = 4 \) supersymmetric functional. The functional (33) is manifestly \( \mathcal{N} = 2 \) supersymmetric. To prove its \( \mathcal{N} = 4 \) supersymmetry we have to examine its variation under the hidden \( \mathcal{N} = 2 \) supersymmetry (7). However, one can show (see [12] for details) that the condition of invariance under these transformations, even in the lowest order in the hypermultiplet superfields, leads to the meaningless relations

\[ \frac{W_{KL}}{W_{IJ}} = \frac{\bar{W}_{KL}}{\bar{W}_{IJ}}. \tag{35} \]

This implies that the appropriate function \( L_q \) in (33) does not exist. We thus conclude that the effective potential (3) cannot appear in \( \mathcal{N} = 4 \) SYM theory since its \( \mathcal{N} = 4 \) completion cannot be constructed. Clearly, our consideration does not rule out a possibility of appearance of the effective potential (3) in generic \( \mathcal{N} = 2 \) superconformal models which do not possess \( \mathcal{N} = 4 \) supersymmetry. Thus, the exact \( \mathcal{N} = 4 \) SYM theory with the gauge group \( SU(N) \) spontaneously broken down to \( U(1)^{N-1} \) is given by eqs. (31), (32).

5 Conclusions

We have studied the problem of the low-energy effective action depending on all fields of the \( \mathcal{N} = 4 \) gauge multiplet of \( \mathcal{N} = 4 \) SYM theory in the Coulomb branch. Using a formulation of \( \mathcal{N} = 4 \) SYM theory in terms of \( \mathcal{N} = 2 \) harmonic superfields and exploring the hidden \( \mathcal{N} = 2 \) supersymmetry of this theory we proved that the known non-holomorphic effective potentials (1), (2) can be uniquely completed to \( \mathcal{N} = 4 \) supersymmetric form by adding the appropriate hypermultiplet-dependent terms and found the exact form of these terms. The same analysis, being applied to the non-holomorphic effective potential (3), showed that such a potential can never appear in the \( \mathcal{N} = 4 \) supersymmetric theory. As a result, we have found the exact \( \mathcal{N} = 4 \) supersymmetric effective action in the Coulomb branch of \( \mathcal{N} = 4 \) SYM theory (further details can be found in [12, 13]).
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