The Role of Utility in Decision-making

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In an earlier article (page 197) I discussed the contribution that a statistician might make toward the problem of diagnosis. It was argued that diagnosis is properly conducted by using the Bayes rule, and that effective diagnosis might be accomplished using relatively few carefully selected tests or symptoms. This article extends the discussion from diagnosis to include the action that is taken as a result of the diagnosis, one such action being the possibility of further tests, but I want to go beyond this and consider the choice of treatment, management and surgery that might be given to a patient, for diagnosis is only a stepping-stone on the path to action.

Let us begin with an example, similar to one already considered in that it concerns jaundice, involving surgery. A simplification of a situation that can arise is to suppose that all the tests have been performed and it is now a question of whether to resort to surgery or not. In further simplification let it be supposed that there are only two types of jaundice: hepatitis, which is treated without surgery, and stones in the biliary tree, which require surgery. The
latter is referred to simply as ‘stones’. (We shall see later how to make the situation more realistic: essentially this is done by fitting small pieces, like this one, together.) The problem can conveniently be represented in Table 1, the rows corresponding to the two actions, the columns to the two types of jaundice.

| Surgical treatment | Stones | Hepatitis |
|---------------------|--------|-----------|
| Non-surgical treatment | Recovery | Debility |
|                     | Disability | Recovery |

There are two broad types of considerations that will affect the decision to operate: firstly, the strength of the diagnosis; secondly, the nature of the various outcomes that might result from the action taken. The diagnosis will take the form of a probability that the patient has stones (one minus this is the chance of hepatitis). The various outcomes have been expressed, in a very simplified form, in the body of Table 1. For example, if the patient requires surgery but it is denied him (first column, second row) the outcome will be disability. If the other error, of performing unnecessary surgery (second column, first row) is committed the outcome will be debilitation due to the strain of the surgery. Both these possibilities represent undesirable outcomes and correspond to errors. The two remaining in the table represent correct choices and have been simply labelled ‘recovery’ from jaundice. This is only an example; it may, for instance, happen that surgery on a serious case of hepatitis could result in death, not just debility. If so, there would be more reluctance to undertake surgery than in the case described in Table 1. My point is that the action taken will depend on the worth, or merit, of the outcomes.

We therefore need to discuss the worth of outcomes, and we measure this worth numerically, giving to each outcome a single number called its utility. This may seem outrageous. How can a single number describe an outcome that is very complicated, such as that in the top right-hand corner? That outcome has several aspects: the effect of the unnecessary operation, with possible psychological effects, the deterioration in the jaundice, to mention only two. That question cannot easily be answered. It is one of the great triumphs of modern thinking to show that only by assigning such a single numerical measure can sensible decisions be made. Only by measuring the quality of life can rational decisions be made. The demonstration of this fact would take too long but the use that is made of this utility measure may convince you of
its merit. My limited experience suggests that the medical profession is more appreciative of this point than are others.

Although utility is a single numerical measure of worth, it is a rather special sort of measure. It is measured in terms of probability, and it is this use of probability that enables it to combine with the other feature, the probability derived from the diagnosis, to produce a decision for or against surgery. To understand this let us consider the four outcomes in Table 1. Two of them are virtually identical and have been labelled ‘recovery’. This is of high worth, so let us for the moment give it a utility of 1. Another outcome is disability: let us suppose this is the worst outcome and give it the value 0. There remains one outcome: debility, intermediate between the two already mentioned. Suppose it had utility 1/2; what would that mean? It means, in the sense now to be described, that the outcome is halfway between disability (utility 0) and recovery (utility 1); and halfway has a precise probability interpretation which is this. Suppose there was an operation which, with probability 1/2 would lead to recovery and with probability 1/2 to disability. Then to say that debility has utility 1/2 means that you, the decision-maker, are indifferent between the hazardous operation and this outcome for sure. In other words, put yourself in the position of being debilitated, and contemplate this imaginary operation with 50 per cent chances of disability or recovery. To be undecided about whether to undergo the operation or to remain in the debilitated state is equivalent to saying that state has utility 1/2. Were the value 0.6 you would not risk the operation, but would be neutral about an operation with a 60 per cent chance of recovery. This is apparently a strange method of measuring the worth of something, namely, equating it to a chance of the best possible outcome, recovery, against the complementary chance of the worst, disability, but it is operational and the result does combine easily with the diagnostic measure already obtained. In practice, we do not assess single utilities but assess groups of them, comparing the various outcomes in different ways to understand different aspects of the utility structure.

To see how utilities and probabilities combine, rewrite Table 1 in the form of Table 2, where utilities replace the outcomes (we have written $u$ for the utility of the intermediate outcome) and a probability, $p$, for stones has been inserted. Then the utilities and the probabilities are combined in expected utilities. Each decision has its expected utility, calculated in the following way. For each outcome, multiply its probability by its utility, and add the results for all outcomes. Thus, in Table 2, consider the decision to undertake surgery. Recovery has probability $p$ and utility 1; the product is $p$. Debility has probability $1 - p$ and utility $u$, with product $u(1 - p)$. Adding, the expected utility is $p + u(1 - p)$. 

Table 2

| Probability | p  | 1 - p | Expected utility    |
|-------------|----|-------|---------------------|
| Surgical treatment | 1  | u     | p + u(1 - p)        |
| Non-surgical treatment | 0  | 1     | (1 - p)             |

For the decision not to operate we similarly obtain 0p + 1(1 - p), simply (1 - p).

The rule is to select that course of action having highest expected utility. In this case we operate if p + u(1 - p) exceeds (1 - p). It is often useful to consider a ‘break-even’ value, po, for which the two expected utilities are equal. Only for p greater than po is surgery best. Then equating the two entries in the last column

\[ p_0 + u(1 - p_0) = (1 - p_0), \]

or

\[ p_0(1 - u + 1) = 1 - u, \]

hence

\[ p_0 = (1 - u)/(2 - u). \]

Thus, if \( u = 1/2 \), the recommendation is to operate if the chance of stones exceeds 1/3. If \( u = 3/4 \), the corresponding value is 1/5. If \( u \) were small, say 1/8, the value is higher at 7/15 and reaches 1/2 when \( u = 0 \), equivalent to disability. Table 3 describes a slightly different case where recovery after a successful operation for stones is rated lower, at 0·8, than that after successful treatment of hepatitis; the utility of the debilitating outcome being 0·5. Surgery should be undertaken if p exceeds 0·38, a slightly higher value than the 1/3 we had before because of the drop in utility of one possible outcome of surgery from 1 to 0·8.

Table 3

| Probability | p  | 1 - p | Expected utility    |
|-------------|----|-------|---------------------|
| Surgical treatment | 0·8| 0·5   | 0·8p + 0·5(1 - p)   |
| Non-surgical treatment | 0·0| 1·0   | (1 - p)             |

The procedure for decision-making just described is called maximisation of expected utility, and abbreviated MEU. Notice that when, earlier, we said an outcome had utility 1/2 if you were indifferent between it and an operation having equal chances of disability or recovery, we implied that the two things have the same expected utility; 1/2 for sure, or \( 0 \times 1/2 + 1 \times 1/2 = 1/2 \) for the operation. Essentially, all that MEU does is enable one to put the results
of single little decision problems together and solve bigger problems. Our example illustrates this. The determination of the value of $u$ (Table 2) requires the resolution of a single decision problem: namely, the choice between the outcome for sure and a hazardous hypothetical operation. Having solved that, we can put it together with other utilities (themselves produced by considering other decision problems), as the 0·8 in Table 3, and solve our bigger, but still simple, problem. This problem, in turn, can be fitted in as part of more complicated and more realistic problems. It is only by means of MEU that the pieces can be fitted together satisfactorily.

When discussing probability ideas (page 202), I stressed the fact that the important thing was the way in which the probabilities fitted together—in our case by Bayes's rule. It is the same with utilities and expected utilities: it is their fitting together that is important. We say that MEU gives a coherent method of decision-making, in that the pieces cohere. Let us see how this coherence is achieved by enlarging the original problem illustrated in Table 1. Suppose that after surgery is undertaken unnecessarily (first row, second column) it is possible to do one of two things: resort to drug treatment immediately, or allow the patient to rest for a period before taking positive action. Essentially, suppose there are two treatments available. Then this decision problem can be solved, just as the earlier one was, by considering the outcomes, the types of possible jaundice, the utilities, the probabilities, and obtaining a maximum expected utility. (Notice, incidentally that the probabilities will have changed as the result of the surgery: $p$ will now be zero and other probabilities will be involved concerning the affect of drugs on a debilitated patient.) The final MEU can then be inserted into Table 1 as the value of $u$, and the original problem solved.

There are three more things to be said about utility. Firstly, as emphasised, it is a measure using probability ideas. Nevertheless, it is possible to change this measurement in two ways, either by multiplication by a fixed positive number (for example, we often multiply by 100 and express it in terms of percentages) or by addition of a fixed constant, or by both. It is easy to verify that neither of these changes will affect the choice of decision.

Secondly, it is possible to introduce costs into the utility analysis. It is only necessary to modify the outcome: for example, successful use of surgery resulting in recovery would, in a country without a health service, produce a loss of assets for the patient through the cost of the surgery. The modified outcome can be considered and its utility assessed. Actually, money is one of the few things that have been well studied from a utility viewpoint and the form, at least, of the relationship between utility and money is well-known.

The third point is in the form of a question: Whose utility goes in, the
patient's or the doctor's (or even society's)? In the jaundice example, it seems pretty clear that it should be the patient's, for it is he who may undergo the operation. And that is the way it is formally, for the patient decides. But if I were the patient I would certainly welcome the medical advice on determining the utilities and (if I were not a statistician) on the probabilities, for the doctor will probably have a much better idea of the various outcomes than I have. What is it like to have a damaged liver? What seems right here is for the doctor to declare his utilities, so that the patient can accept or refuse them. My personal view is that decision-making (and not only in medicine) would be much improved by the open statement of utilities, for, expressed in numerical terms, they are capable of exact interpretation and of easy manipulation. In particular, they could be meaningfully discussed by doctors and laymen.

In other medical situations it may be necessary to use society's utilities, and there it is even more important that they be stated publicly. For example, with a given budget, a choice has to be made about relative expenditures on different branches of medicine; for example, should we spend more on kidney machines and less on studying congenital heart disease? Here the judgement must be society's, though with essential help from the medical profession. As with the earlier example, my own preference would be for the profession to state a utility and for it to be criticised before adoption.

But these questions take us almost away from medicine and into sociology. I have tried to show how the idea of utility can be used in decision-making. Allied to the concept of probability is the notion of maximum expected utility; it provides a powerful tool, and the only tool, for satisfactory resolution of medical decision problems.

Reference
Lindley, D. V. (1971) Making Decisions. London: Wiley Interscience.