Ripples are common in both biological systems and human clothes. Knitters have long exploited the ability of fabric to curl out of plane, by either removing or adding stitches to the material as it is created. Here, we present two knitting patterns for scarves to illustrate how ripples can arise through such additive processes.

Humans, plants, and marine creatures are all known to decorate their margins with a bit of frill: skirts, ornamental kale, and nudibranchs all catch our eye due to their rippling edges, whether created by fashion, plant breeding, or evolution. The unifying mechanism for these ripples was identified by the work of scientists working at the University of Texas Center for Nonlinear Dynamics during the early 2000s, with the patterns (sometimes fractal!) arising from both the elastic and geometric properties of thin sheets. The central idea is non-Euclidean: if the area of a thin sheet is larger than the space available to it, it must deform out of plane, as illustrated by the scarves in Figure 1. This forms the ripples, via spontaneous symmetry breaking. Where the material is not stiff, the ripples can be readily rearranged into other ripples after they are created, which is why they are appealing on clothes: they rearrange as you move.

Knitters have long exploited non-Euclidean shapes in designing garments that curve inward around our heads (positive Gaussian curvature) or outward into ripples (negative Gaussian curvature). They do this by either removing or adding stitches to the material as it is created, line by line from one edge of the garment to the other. Using such techniques, it is possible to turn the corner on the heel of a sock, or to taper the sleeve of a sweater. When one of the authors first heard about the work of Sharon et al.,\textsuperscript{1–3} she turned to knitting to create a concrete demonstration of the principle. In the resulting scarf (Figure 1A), there are about 130 rows along the central spine of the scarf, but 3 times as many along each edge. These extra stitches ripple out of the plane of flat material created along the central spine.

In fact, due to the way knits are constructed (row by row, zig-zagging along the fabric; see Figure 2B), there are two ways to imagine creating such a rippled scarf: from long edge to long edge, or from short end to short end. In the scarf shown in Figure 1A, the latter orientation was chosen for design reasons: the knitter had only one ball of yarn (sentimentally gifted from a fellow physics graduate student) to use to make the scarf. Using the edge-to-edge method presented two advantages: maximal use of the ball of yarn to make the scarf as long as possible and the opportunity to more immediately see that the rippling principle was working as intended. This method involves casting on enough stitches to form the width of the scarf, then periodically knitting extra, partial rows that extend from the edge of the scarf only part way in toward the spine. The line defects created by these short rows introduce negative Gaussian curvature to the fabric. A short segment of scarf illustrating this technique is shown in Figure 3A.

The second method (Figure 1B), more conventionally used by knitters to shape garments such as sweaters or hats, is to first cast on a chain of stitches as long as the edge of the scarf is intended to be and then knit outward toward each rippled edge by adding individual stitches at regular intervals while knitting each row. Gradually, the ripples develop as the number of stitches per row grows. A sample of this technique is shown in Figure 3B, illustrating the knitting direction and the line defects where the new stitches are added. Again, these line defects are what induce the negative Gaussian curvature. To generate the second side of the scarf, you pick up the stitches from the spine and begin the process again. In this method, using up all the yarn requires casting on the correct number of stitches from the beginning. The easiest way to surmount this challenge is to knit a sample of a small number of stitches, weigh the sample, and then scale up to the total amount of yarn.
Note that these two examples yield samples with a different anisotropy: the orientation of the stitches is either along the scarf or perpendicular to it, which changes how the fabric drapes and flows. In fact, if you look carefully at Figure 2B, you will notice that even the flattest knitting is slightly anisotropic—the fabric curls toward the front side on the top and bottom (\(k_1 < 0\) but small) and toward the backside on the sides (\(k_2 > 0\) but small). This inherent curvature has an interesting effect on our two rippled-scarf methods. In the “short-row” method, knit from edge to edge, the fabric curls toward the purled side, making the knitted side (the one normally used as the “right” side) showing. In the “increasing stitches” method, knitted from the center line outward from end to end, the scarf curls toward the knit side, leaving the purled side (the conventional “wrong” side) showing.

Below, we have included knitting instructions for both versions of the scarves. The adventurous reader/knitter might be interested in extending them in a variety of ways. For example, the amount of curvature can be increased by doing more short rows with fewer stitches between them in the edge-to-edge version, or by doing more increases in the end-to-end version. Changing the stitch pattern, such as by introducing horizontal or vertical ribbing or by knitting all rows on both sides, would also change the behavior of the final scarf. These patterns will also be posted on the fiber arts community Ravelry; please join us there to share your handiwork.
The Patterns

As shown in Figures 1 and 3, both scarves are reversible and so technically have no right (knit) or wrong (purl) side. However, we’ve used the traditional “RS” and “WS” notation to help you keep track of which side is which on the short rows. In the patterns, we use “RS” to mean the side facing the knitter on row 1.

**Edge-to-Edge Rippled Scarf (Short Rows)**

Gauge: 5 stitches/inch

Cast on 24 stitches.

Row 1: Knit 12, place marker, purl to end of row.
Row 2: Knit 12, purl 12.

Short row section 1:

*Short row 1 [RS]: Knit 6, wrap and turn 1 stitch.
Short row 2 [WS]: Purl back to start of row.
Short row 3 [RS]: Knit 3, wrap and turn 1 stitch.
Short row 4 [WS]: Purl back to start of row.
Short row 5 [RS]: Knit to marker, working wrapped stitches together with their wraps. Purl to end of row.

Short row section 2:

Repeat from *, except this time the WS and RS will be reversed since you’re starting on the WS.

Repeat short row sections 1 and 2 until you’re out of yarn or scarf is desired length. Bind off all stitches knitwise.

**End-to-End Rippled Scarf (Increasing Stitches)**

Gauge: 5 stitches/inch

Cast on 200 stitches (or as many as you want your scarf to be long).

Rows 1–6: Stockinette stitch (knit the RS and purl the WS).
Row 7: Knit front and back in every stitch. (400 st)
Row 8: Purl.
Row 9: Knit.
Row 10: Purl.
Row 11: *Knit 1, knit into front and back of next stitch repeat from * across. (600 st)
Row 12: Purl.
Row 13: Knit.
Row 14: Purl.

With knit side of work facing, pick up 200 stitches from the cast on edge.

Repeat rows 1–14 and bind off, as for the first half of the scarf. (Note: You will knit the first row, with the purl side of the previous work facing. This way, the scarf will be symmetric.)

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