Surface code quantum communication

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Quantum communication typically involves a linear chain of repeater stations, each capable of reliable local quantum computation and connected to their nearest neighbors by unreliable communication links. The communication rate in existing protocols is low as two-way classical communication is used. We show that, if Bell pairs are generated between neighboring stations with a probability of heralded success greater than 0.65 and fidelity greater than 0.96, two-way classical communication can be entirely avoided and quantum information can be sent over arbitrary distances with arbitrarily low error at a rate limited only by the local gate speed. The number of qubits per repeater scales logarithmically with the communication distance. If the probability of heralded success is less than 0.65 and Bell pairs between neighboring stations with fidelity no less than 0.92 are generated only every $T_B$ seconds, the logarithmic resource scaling remains and the communication rate through $N$ links is proportional to $(T_B \log^2 N)^{-1}$.

Long-range communication of quantum states is difficult as such states cannot be copied [1, 2]. Current research into long-range quantum communication focuses on quantum repeaters [3] making use of entanglement purification [4] and entanglement swapping [5, 6]. Entanglement purification requires slow two-way classical communication, resulting in the quantum communication rate decreasing polynomially with distance. Furthermore, the communication error rate $p_c$ is at best comparable to the error rate $p_g$ of gates within repeaters. If qubits have a finite coherence time, requesting a constant $p_c$ as the distance increases results in a finite maximum communication distance. Arbitrarily rapid and reliable communication over arbitrary distances is not possible using only entanglement purification and swapping.

Initial work incorporating error correction into quantum communication resulted in non-fault-tolerant schemes [7, 8] capable of reliably correcting only a small, fixed number of errors. Recently, the first steps towards fault-tolerant quantum communication were taken [9], however entanglement purification was still used between neighboring quantum repeaters, fundamentally limiting the communication rate to hundreds of logical qubits per second. A quantum communication protocol requiring very little two-way classical communication has been developed concurrent with this work [10].

We show that, using surface code quantum error correction [11, 12], two-way classical communication can be avoided entirely provided we can create Bell pairs between neighboring stations with a heralded success probability $S_B \gtrsim 0.65$ and fidelity $F \gtrsim 0.96$. This means communication can proceed at a rate independent of the classical communication time between repeater stations. Given local quantum gates with $p_g \ll 0.75\%$, we show that it is possible to communicate logical qubits over arbitrary distances with arbitrarily low $p_c$ at a rate limited only by the local gate speed. The number of qubits per repeater increases only logarithmically and the quantum communication rate decreases only logarithmically with communication distance.

To describe our quantum communication protocol, we must first describe surface codes and this in turn requires the notion of stabilizers [13]. A stabilizer of $|\Psi\rangle$ is an operator $M$ such that $M|\Psi\rangle = |\Psi\rangle$. For example, $Z|0\rangle = |0\rangle$. Given any set of commuting operators $\{M_i\}$, a state $|\Psi\rangle$ exists stabilized by $\{M_i\}$.

Surface codes can be defined on lattices of the form shown in Fig. 1. Data qubits are represented by open circles. We define a set of commuting operators on data qubits by associating $ZZZZ/XXXX$ with each face/vertex. If the $|\Psi\rangle$ stabilized by these operators suffers errors, becoming $|\Psi\rangle$, then local to these errors we obtain equations of the form $M|\Psi\rangle = -|\Psi\rangle$. Measuring whether the qubits are in the +1 or −1 eigenstate of each stabilizer thus gives us information about the errors in the lattice. Measuring a stabilizer requires a sequence of six gates. This information can be used to reliably correct the errors provided the error rates of initialization, CNOT, measurement, and memory, which here we take to be equal at rate $p_g$, are all less than approximately 0.75% [13, 14, 16]. Logical operators $X_L/Z_L$ are chains of single-qubit $X/Z$ operators that commute with every $Z/X$ stabilizer and link the top/left boundary to the bottom/right. The distance $d$ of the code is the number of single-qubit operators in the shortest logical operator.

Transmitting surface code logical qubits is of particular interest as the surface code possesses a high threshold error rate, requires only local interactions, is highly tolerant of defective qubits [17] and permits fast, arbitrarily long-range logical CNOT — a collection of properties no
We now describe our communication protocol, initially restricting ourselves to moving a logical qubit from the left end to the right end of a single monolithic array of qubits with the ability to perform local gates. Given an arbitrary surface code logical qubit $|\Psi_L\rangle$ at the left end of the array, an uninitialized region of qubits $|\Psi\rangle$ in the middle and a surface code logical qubit $|0_L\rangle$ at the right end, $|\Psi_L\rangle$ can be fault-tolerantly teleported to the location of $|0_L\rangle$. First, the uninitialized region is measured as shown in Fig. 2a. The $Z$ basis measurements project the region into eigenstates of the $Z$ stabilizers. Second, the syndrome qubits across the entire lattice are interacted with their neighboring data qubits as shown in Fig. 2b. Third, the measurement pattern shown in Fig. 2c completes one round of stabilizer measurement. The interaction pattern of Fig. 2d is executed a total of $d$ times, interleaved with the measurement pattern of Fig. 2c. Finally, after the $d$th round of interaction, the measurement pattern shown in Fig. 2d is applied, completing the fault-tolerant movement of the logical qubit.

All measurement results are simply sent to the destination end of the lattice, not processed during transmission. The final round of measurements prepares the lattice for the transmission of the next logical qubit. Assuming each interacting quantum gate takes $T_g$ seconds and each measurement $T_m$ seconds, a logical qubit can be transmitted every $(4T_g + T_m)d$ seconds. The scaling of $d$ and values required for practical communication will be discussed later after the full communication scheme has been described.

The processing of measurement results related to $X$ and $Z$ stabilizers occurs independently. Errors result in stabilizer measurements changing. A chain of errors leads to changes in the stabilizer measurements only at the endpoints of the chain. A good approximation of the most likely pattern of errors corresponding to a given set of stabilizer measurement changes is one in which every change is connected by a chain of errors to another change or lattice boundary such that the total number of errors is a minimum. A classical algorithm, the minimum weight perfect matching algorithm [21], can find such a pattern efficiently, in time growing poly-logarithmically with the volume of the lattice when parallel processing is used [22]. An alternative algorithm with similar runtime has been devised recently [23]. Error correction fails when the corrections actually create error chains connecting pairs of opposing boundaries. With careful calculation of the distance between changes, a minimum of $\lfloor (d + 1)/2 \rfloor$ errors must occur before failure is possible, implying $p_c$ decreases exponentially with $d$.

When communicating over a large physical distance,
the fundamental entanglement resource is expected to be Bell pairs created over fiber links kilometers in length. The monolithic lattice described above can be broken into pieces connected by Bell pairs as shown in Fig. 3. Stabilizers spanning the communication link can be measured using the approach shown in Fig. 4. We temporarily ignore heralded failure to entangle for the moment. A probability $p_B$ of depolarizing error on a Bell pair means that the errors $IX, IY, IZ, XI, XX, XY, XZ, YI, YX, YY, YZ, ZI, ZX, ZY, ZZ$ each occur with probability $p_B/15$. Using the Bell pair stabilizers $XX$ and $ZZ$, these errors are equivalent to $II$ with probability $p_B/5$ and $IX, IY, IZ$ with equal probability $4p_B/15$.

After correction, nontrivial combinations of $X/Z$ errors form a chain that runs from the top spatial/temporal boundary to the bottom spatial/temporal boundary. Given this symmetry, and the fact that the different types of errors are processed independently, we focus on $IX$ errors, which occur on any given Bell pair with probability $p_X = 8p_B/15$. Referring to the Bell pairs numbered 1 to $2d-1$ in Fig. 4, $IX$ errors on odd pairs induce an $X$ error on the data qubit to their left whereas on even pairs the result is an incorrect stabilizer measurement.

These errors can be visualized as the bonds of a $d \times t$ 2-D square lattice. The error rate $p_X$ is too high when, after correction, the probability of having a chain of errors along the $d$ dimension increases with $d$. For $t = 1$, we have a repetition code, implying $p_X < 0.5$ is correctable. For $t = d$, we have a surface code with perfect syndrome measurement implying $p_X \lesssim 0.1$ [14]. The equivalent values of $p_B$ are 15/16 and approximately 0.2.

We simulated a pair of repeater nodes with perfect gates and depolarized Bell pairs for verification (Fig. 5). Note the expected crossover at $p_B = 15/16 \sim 0.94$. Significant growth of the time to failure with $d$ occurs for $p_B \lesssim 0.2$, as expected. Rapid growth occurs for $p_B \sim 0.1$, equivalent to a fidelity $F$ of the entangled state $\rho$ with respect to the desired Bell state $|\Phi^+\rangle$ of 0.92 since $F = \langle \Phi^+ | \rho | \Phi^+ \rangle = 1 - 4p/5$ for Bell pairs corrupted by depolarizing errors.

Loss during transmission can be modeled as measurement in an unknown basis. Loss is easier to tolerate than depolarizing noise as the failure to measure the transmitted pulse or photon gives the location of the error. This can be seen in the simulation results of Fig. 5 which shows efficient handling of 40-45% loss. Note that no code can handle more than 50% loss as this would violate the no-cloning theorem [1, 2].

The probability of logical error after $d$ successful stabilizer measurements, $p_{\text{link}}$, is shown in Fig. 5 versus $p_B$ and loss $p_L$. For 35% loss and 5% error ($F = 0.96$), increasing $d$ by 30 decreases $p_{\text{link}}$ by a factor of 10.
nuation of logical qubits over an arbitrary number of links enables the practical fault-tolerant quantum communication. High fidelity MHz communication can thus be achieved if the rate of loss is \( p_L < 0.01 \) and the pair error rate is less than approximately 10\%. This will not significantly change the above results. An error rate one or two orders of magnitude below the threshold error rate of approximately 35\% will not significantly change the above results.

To summarize, we have shown that, provided the Bell pair error rate \( p_g \) is less than 0.01, utilizing surface code quantum error correction enables the practical fault-tolerant quantum communication of logical qubits over an arbitrary number of links \( N \) with arbitrarily low communication error rate \( p_c \), given \( O(\log^2 N/p_c) \) qubits per repeater. If the rate of loss is high, the communication time is proportional to the time \( T_B \) required to successfully create a Bell pair and the number of Bell pairs per link \( O(\log^2 N/p_c) \). If the loss is below approximately 35\% and \( F \gtrsim 0.96 \), no heralding is required and of order a thousand qubits per repeater and nanosecond gates enables one to send logical qubits at a MHz rate with \( 10^{-6} \) error through \( 10^4 \) links — sufficient in principle to reach the opposite side of the planet.

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