One of the possible origins of short gamma-ray bursts (SGRBs) is merging of compact binaries, and the effect of large kick velocity is a signature that can be used as an observational test for this hypothesis. Intracluster SGRBs that escaped from a host galaxy in a galaxy cluster are interesting in this context, since they would escape more easily by cluster tidal force, and would have brighter afterglow luminosity by dense intracluster gas, than those in general field galaxies. Here we calculate the escape fraction of compact binaries from their host galaxies in a galaxy cluster, and discuss some observational implications. We find that the escape fraction strongly depends on the nature of dark matter subhalos associated with member galaxies. If the amount of dark matter around member galaxies is not large and the gravitational potential for an escaping binary is determined mostly by stellar mass, most of SGRBs should escape and be observed as hostless, which is a much higher fraction than those in the field. Hence, statistics of intracluster SGRBs could give important information about the dark matter distribution in galaxy clusters, as well as hints for the origin of SGRBs.

Subject headings: binaries: general — galaxies: clusters: general — gamma rays: bursts — methods: numerical — stars: neutron
model used by Totani & Yoshii (2000) which can reproduce the observed galaxy properties well. Stellar density profile within member galaxies is assumed to be the Hernquist profile (Hernquist 1990), with the characteristic radius parameter chosen so that a half of the total mass is included within the observed half-light radius, \( r_{\text{h,m}} \), which is calculated by the power-law relation to \( B \) luminosity fitted to observations (Totani & Yoshii 2000).

On the other hand, the density profile of DM associated with the cluster member galaxies as substructure in the whole cluster is uncertain and poorly known. Especially, the DM subhalos extending to the outer region of member galaxies are expected to be vulnerable to the tidal forces by galaxy interactions and/or the overall cluster gravitational potential. Here we consider the two extreme cases: (1) there is no significant DM substructure or subhalos associated with member galaxies, and the gravitational potential well of galaxies is determined simply by the stellar mass profile (the no subhalo case); and (2) the DM subhalos are associated with member galaxies with an amount similar to that of the field galaxies (the preserved subhalo case). The amount and nature of subhalos depend on the clustering processes, and probably the reality is between these two extreme cases.

For the preserved subhalo case, we calculate the virial mass \( (M_{\text{vir}}) \) of the subhalos from the stellar mass of a galaxy and the universal ratio of the dark to baryonic matter \( \Omega_{\text{b}}/\Omega_{\text{m}} = 5.9 \) (Spergel et al. 2003). The virial radius \( (r_{\text{vir}}) \) of the subhalo is calculated from \( M_{\text{vir}} \) and the one-dimensional central velocity dispersion \( \sigma \) of the galaxy, as \( GM_{\text{vir}}/2r_{\text{vir}} = 3\sigma^2 \). Here the velocity dispersion is that for stars calculated from the galaxy luminosity using the Faber-Jackson relation (de Vaucouleurs & Olson 1982).

An almost similar velocity dispersion is obtained also from the stellar mass and \( r_{\text{h,m}} \) calculated above and assuming the virial relation, giving a consistency check for our treatment. We assume the NFW profile for the DM subhalos, and the concentration parameter is calculated by the formula given by Bullock et al. (2001) for subhalos included in larger virialized halos, which are based on cosmological \( N \)-body simulations: \( c = c_s (M_{\text{vir}}/M_\odot)^{1/3} \) at \( z = 0 \) where \( c_s = 7, \Gamma = -0.3, \) and \( M_\odot = 1.5 \times 10^{13} h^{-1} M_\odot \).

### 2.2. Cluster and Galaxy Evolution

It is well known that most stars in elliptical galaxies formed at high redshift \( (z > 1) \) and they evolved passively to the present time (e.g., Yamada et al. 2005). Major mergers could change drastically the stellar mass distribution and gravitational potential of member galaxies, but a recent numerical simulations by Murante et al. (2007) indicates that the majority of member galaxies in a cluster do not undergo major mergers, except for the brightest central galaxy which is formed in the collision of many galaxies. Therefore we make a reasonable assumption that the stellar mass distribution does not evolve in cluster member galaxies.

Although galaxies form at the early epoch of \( z > 1 \), the establishment of the overall cluster potential should be significantly later according to the standard picture of hierarchical structure formation. We estimate the epoch of cluster formation using the extended Press-Schechter approximation (Lacey & Cole 1993). It predicts that about half of the mass of a \( 10^{15} M_\odot \) cluster at \( z = 0 \) is already included in the largest progenitor at \( z_p \sim 0.45 \). Therefore we assume that the tidal force by the cluster potential starts to affect member galaxies at this redshift. We do not take into account the evolution of the cluster potential at \( z < z_p \), and this is a reasonable approximation because the timescale of cluster potential evolution is much larger than the orbital period of compact binaries in member galaxies. The evolution of DM subhalos is difficult to predict without detailed numerical simulations, and we simply apply the above two extreme models with no evolution, which would cover the realistic evolution.

### 2.3. Escape of Compact Object Binaries

In a galaxy with a given luminosity, we can calculate the orbit of compact object binaries in the gravitational potential as modeled in § 2.1, if the initial location and velocity are given. We calculate the initial velocity by the sum of the original stellar velocity \( v_{\text{org}} \) at the location and the kick velocity \( v_{\text{kick}} \) when the compact objects are formed. The Maxwell distribution having the one-dimensional velocity dispersion \( \sigma \) defined in the previous section is assumed for \( v_{\text{org}} \). The direction of both \( v_{\text{org}} \) and \( v_{\text{kick}} \) are assumed to be isotropic and random.

Although the velocity distribution of observed single pulsars is well fitted by a Gaussian (Hobbs et al. 2005), that for compact object binaries is not well known. A few observed binary pulsars have bulk motion velocities of \( \sim 100-200 \) km s\(^{-1} \) (e.g., Wex et al. 2000; Ransom et al. 2004; Willems et al. 2004). Bulik et al. (1999), Bloom et al. (1999), and Fryer et al. (1999) theoretically estimated the velocities of compact binaries that remain gravitationally bound after supernova explosions to be several hundred km s\(^{-1} \). We assume the distribution of \( v_{\text{kick}} \) is a single isotropic Gaussian, i.e., each one-dimensional component of \( v_{\text{kick}} \) is a Gaussian with the standard deviation \( \sigma \).

We calculate the cases of two different values of \( \sigma = 100 \) and 300 km s\(^{-1} \). The mean (the standard deviation) of the corresponding \( |v_{\text{kick}}| \) distribution then becomes 160 (66) and 480 (200) km s\(^{-1} \) for \( \sigma = 100 \) and 300 km s\(^{-1} \), respectively.

We solve the motion of binaries until their merger time (the time from a compact binary formation to its merger by gravitational wave radiation). Although the merger time generally ranges over more than 3 orders of magnitude, \( 10^{-7}-10^{10} \) yr (e.g., Tutukov & Yungelson 1994; Bulik et al. 1999), we are interested in SGRBs in galaxy clusters. Most of galaxies in clusters have formed their stars at high redshift \( (z \approx 2) \), and observed SGRBs are typically at \( z \sim 0.2 \). Therefore, the time between these epochs, i.e., \( \sim 10^8 \), is appropriate for the merger time in this work.

We consider that a binary has escaped from its host galaxy once its distance from the host galaxy center becomes larger than the tidal radius \( r_{\text{tide}} \) of the host galaxy after the formation of the galaxy cluster, i.e., \( z < z_p \). Here the tidal radius is defined as the radius where the tidal force by the overall cluster potential is the same as the binding force in the host galaxy. The tidal radius is numerically calculated for a given set of galaxy luminosity and location in the cluster. We then calculate the mean escape fraction as a function of \( R \), taking a weighted average over the host galaxy luminosity, initial location in the host, and kick velocity.

It should be noted that some fraction of stars are distributed at \( r > r_{\text{tide}} \) with the assumed stellar mass profile in host galaxies. Such stars would be stripped from host galaxies and become intracluster stars. Compact object binaries in such stellar populations would all contribute to the intracluster SGRBs. We find that this fraction is about 2% in the preserved subhalo case and 10% in the no subhalo case. It seems that the no subhalo case is preferred (see § 4.2), from a comparison of these values with the various observational estimates of the abundance of intracluster stars in galaxy clusters.
the detectability of a typical SGRB afterglow in the intracluster medium following the standard afterglow model of Sari et al. (1998). We simply use this isotropic model without jet structure by using the isotropic equivalent total energy. The jet break may reduce the expected flux at a later time compared with the calculation here, but this crude estimate is sufficient here for our purpose.

The isotropic-equivalent total energy in gamma-rays of SGRBs is distributed in a wide range of $E_{\gamma,\text{iso}} \sim 10^{49}$ to $10^{57}$ ergs, and the total initial kinetic energy of the external shock ($E_{\text{sh}}$) is expected to be similar (Fox et al. 2005; Soderberg et al. 2006). Following Panaitescu & Kumar (2001), we adopt the following parameters: the fraction of energy density in magnetic field $e_B = 10^{-2.4}$ and that in nonthermal electrons $e_e = 10^{-1.2}$; although these are for LGRBs, the values inferred for available SGRBs are not much different. We also assume the power index of the luminosity decay $\alpha = -1$, where $F_\nu \propto t^\alpha$.

The typical particle density of intracluster medium is $n \sim 10^{-3}$ cm$^{-3}$ within a few hundred kpc from the cluster center (e.g., Lewis et al. 2003). We assume a typical distance for SGRBs, $z = 0.2$.

Then the model predicts the expected flux at the observed frequency of $F_{\nu} \sim 1.5 E_{\gamma,\text{iso}}^{1/3} n_{-3}^{1/3} r_{15}^{2/3} (t/10^3 \text{ s})^{1/3} \mu\text{Jy}$, where $E_{\gamma,\text{iso}} \equiv E_{\gamma,\text{iso}}(10^{50} \text{ ergs})$, $n_{-3} \equiv n/(10^{-3} \text{ cm}^{-3})$, and $r_{15} \equiv r/(10^{15} \text{ Hz})$. In the X-ray band (1 keV), the typical flux is then $F_{\text{x}} \sim 9.3 	imes 10^{-14}$ ergs cm$^{-2}$ s$^{-1}$ at $t = 10^4$ s, which can be detected by existing X-ray satellites (e.g., Gehrels et al. 2005). Swift XRT can locate afterglows with an accuracy of a few arcseconds, and this is reasonably accurate to discuss the association of an afterglow with a galaxy at $z \sim 0.2$. In the optical (R) band, this flux corresponds to $\sim 26$ mag (AB) at $t = 10^4$ s. Most afterglows with this level of brightness have been missed in the past and current GRB follow-up observations. However, afterglows of the brightest SGRBs ($E_{\gamma,\text{iso}} \sim 10^{51}$) may be detectable.

On the other hand, SGRBs ejected far from their host galaxies in the normal field would occur in a much lower density environment. The typical density in the general intergalactic medium would be estimated as $n \sim \rho_c \Omega_{\text{m}}/m_p \sim 10^{-7}$ cm$^{-3}$, where $\rho_c$ and $m_p$ are the critical density of the universe and the proton mass, respectively. The expected afterglow flux of an intergalactic SGRB is then much less than that of the faintest SGRB afterglows ever observed, such as GRB 050509B (Gehrels et al. 2005) and GRB 050911 (Page et al. 2006 [upper limit only]). Therefore it seems difficult to detect an afterglow of such an event.

4.2. Comparison with Observations

Berger et al. (2007) examined all 16 SGRBs that were followed up with X-ray observations with the XRT on Swift or Chandra, and found that three SGRBs are likely to be asso-

3. RESULTS

The results are shown in Figure 1, where the mean escape fraction within a given radius from the cluster center, $f_{\text{esc}}(< R)$, is plotted. We find that the escape fraction largely depends on the existence of the DM substructure; the escape fraction is modest with $f_{\text{esc}} \sim 0.2$ in the preserved subhalo case, while most binaries will be ejected in the no subhalo case. The dependence on the radius from the cluster center or on the kick velocity is not as significant as the effect of subhalos within the parameter ranges investigated.

For comparison, we calculate the case of field galaxies not in clusters. Again, we only consider elliptical galaxies, and their properties are calculated in the same way for a given luminosity. Bullock et al. (2001) found that isolated halos have a different relation between the virial mass and the concentration parameter from that for subhalos, and we adopt $c_e = 9$ and $\Gamma = -0.13$ here based on their results. The escape fraction averaged over galaxy luminosity is simply calculated without taking into account the tidal force by external gravity field.

Here we use the luminosity function shape parameters of Blanton et al. (2001) for field galaxies; although these are derived for all types of field galaxies, the luminosity function for each galaxy type is rather uncertain. The results are shown in Table 1, and we found that the escape fraction in the field is not much different from that in galaxy clusters in the preserved subhalo case, while a large enhancement of the cluster $f_{\text{esc}}$ is predicted in the case of no subhalos.

4. DISCUSSION

4.1. Detectability of SGRB Afterglows in Clusters

Detection of afterglows is necessary to locate a GRB accurately enough with respect to a host galaxy. We discuss here

The typical particle density of intracluster medium is $n \sim 10^{-3}$ cm$^{-3}$ within a few hundred kpc from the cluster center (e.g., Lewis et al. 2003). We assume a typical distance for SGRBs, $z = 0.2$.

Then the model predicts the expected flux at the observed frequency of $F_{\nu} \sim 1.5 E_{\gamma,\text{iso}}^{1/3} n_{-3}^{1/3} r_{15}^{2/3} (t/10^3 \text{ s})^{1/3} \mu\text{Jy}$, where $E_{\gamma,\text{iso}} \equiv E_{\gamma,\text{iso}}(10^{50} \text{ ergs})$, $n_{-3} \equiv n/(10^{-3} \text{ cm}^{-3})$, and $r_{15} \equiv r/(10^{15} \text{ Hz})$. In the X-ray band (1 keV), the typical flux is then $F_{\text{x}} \sim 9.3 	imes 10^{-14}$ ergs cm$^{-2}$ s$^{-1}$ at $t = 10^4$ s, which can be detected by existing X-ray satellites (e.g., Gehrels et al. 2005). Swift XRT can locate afterglows with an accuracy of a few arcseconds, and this is reasonably accurate to discuss the association of an afterglow with a galaxy at $z \sim 0.2$. In the optical (R) band, this flux corresponds to $\sim 26$ mag (AB) at $t = 10^4$ s. Most afterglows with this level of brightness have been missed in the past and current GRB follow-up observations. However, afterglows of the brightest SGRBs ($E_{\gamma,\text{iso}} \sim 10^{51}$) may be detectable.

On the other hand, SGRBs ejected far from their host galaxies in the normal field would occur in a much lower density environment. The typical density in the general intergalactic medium would be estimated as $n \sim \rho_c \Omega_{\text{m}}/m_p \sim 10^{-7}$ cm$^{-3}$, where $\rho_c$ and $m_p$ are the critical density of the universe and the proton mass, respectively. The expected afterglow flux of an intergalactic SGRB is then much less than that of the faintest SGRB afterglows ever observed, such as GRB 050509B (Gehrels et al. 2005) and GRB 050911 (Page et al. 2006 [upper limit only]). Therefore it seems difficult to detect an afterglow of such an event.

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![Image of graph showing escape fraction of compact binaries from host galaxies in a galaxy cluster averaged within R, where R is the distance from the center of the galaxy cluster. (The virial radius of the cluster is 1 Mpc.) The models with no DM subhalos are shown by thin lines, while the models with preserved subhalos are shown by thick lines. The solid and dashed lines are for different values of the standard deviation of the kick velocity distribution, $\sigma_k = 300$ and $100$ km s$^{-1}$, respectively. The columns labeled as cluster 1 and 2 give the escape fractions corresponding to (1) the no DM subhalo case and (2) the preserved DM subhalo case discussed in § 2.1.](image-url)
associated with galaxy clusters, suggesting that the fraction of SGRBs in galaxy clusters is about 20%. Considering the statistical uncertainty, this is consistent with the fraction of all stellar mass in the universe bound in galaxy clusters (~10%; Fukugita et al. 1998).

Among the three SGRBs in galaxy clusters discussed in Berger et al. (2007) GRB 050509B is apparently associated with the likely host galaxy at the cluster redshift of \( z = 0.226 \). The offset of GRB 050509B from its host is 21–56 kpc, corresponding to \((6–16)f_{\text{esc}}\). The galaxy cluster that contains the host is composed of two subclusters, and the host galaxy is located at the center of the major subcluster that is about 270 kpc away from the center of the major subcluster (Gehrels et al. 2005; Bloom et al. 2006). The location of GRB 050911 is in a cluster of \( z = 0.165 \), but its afterglow was too faint to be associated with any particular galaxy (Page et al. 2006). GRB 050813 has three candidate host galaxies near its Swift XRT location, and the galaxies belong to two different galaxy clusters at \( z = 0.72 \) and \( z = 1.8 \) (Berger 2005, 2006). Clearly, the current sample is too small to derive any implications from a comparison with our results. Future satellites for GRB study might detect SGRBs more efficiently, leading to a much larger sample of SGRBs.

Observations of intracluster diffuse light, stars, Type Ia supernovae, and planetary nebulae indicate that some stars in a galaxy cluster are in intracluster medium, perhaps removed from member galaxies (Vilchez-Gómez et al. 1994; Okamura et al. 2002; Durrell et al. 2002; Gal-Yam et al. 2003; Gerhard et al. 2007). The fraction of intracluster stars in all stars in a cluster is uncertain, but observational estimates are typically ~5%–20%. These fractions are consistent with our estimate of stars outside member galaxies (Vilchez-Gómez et al. 1994; Okamura et al. 2002). GRB 050509B is apparently associated with the candidate host galaxies near its Swift XRT location, and the galaxies belong to two different galaxy clusters at \( z = 0.72 \) and \( z = 1.8 \) (Berger 2005, 2006). Clearly, the current sample is too small to derive any implications from a comparison with our results. Future satellites for GRB study might detect SGRBs more efficiently, leading to a much larger sample of SGRBs.

We investigated the escape of compact binaries from their host galaxies in galaxy clusters, which is enhanced by the tidal force of the cluster gravity compared with general fields. We found that the escape probability heavily depends on the uncertain distribution of subhalos associated with member galaxies. If the DM substructure has been destroyed by interactions in a galaxy cluster and the escape of a binary is determined mainly by gravity of stellar mass, most of compact binaries in galaxy clusters should escape and become hostless intracluster SGRBs. On the other hand, if the DM subhalos are associated with member galaxies with an amount similar to that of field galaxies, the enhancement of the escape fraction is only modest compared with field galaxies: about 20% for clusters while ~10% for field galaxies.

Although the current observed data set is not sufficient to be compared quantitatively with our results, statistics of SGRB association with cluster galaxies in future data will give us important information on the DM distribution in clusters, intracluster stars in clusters, and the origin of SGRBs.

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