Four-Parameter Shifts Control Chaos and Lead to Physical Behaviour in a Dual-Ring Erbium-Doped Fiber Laser

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Abstract. We study on a four-parameter shift (FPS) chaos-control method to obtain control of a chaotic dual-ring erbium-doped fiber laser and lead to physics behaviours of two laser rings. To control two chaotic laser rings, the pump parameters and gain parameters of dual-ring lasers are modulated simultaneously by four shift digital signals. It is found that controls of the two chaotic laser rings are realized and physics behaviours of two laser rings can be guided by FPS. As a result, chaos is led to two periodic behaviours. And physics movements of dual-ring lasers are guided to period-4, period-5, period-6, period-7, and some high-order-period. The obtained results have a large helpfulness for people’ study of control of dynamics system, control of chaos, optic control science, laser physics, and erbium-doped fiber laser application.

1. Introduction
In the recent decade or so, people studied many optic complex dynamics organizations and reported many nonlinear or random dynamics behaviors, such as chaos and unstabilized oscillations [1-3]. People found that a chaotic movement is restricted to starting values and its movement orbit has ergodic in phase space while the chaotic signal from a chaotic system has the random characteristics. There is a characteristics of high rate oscillation in laser chaos, so people paid their attentions to study on these chaotic laser systems [1]. A self-feedback laser or another light injected laser has some advantages of a chaotic signal simple emitting. And they have been taken as the first used emitter in many chaotic laser’s applications [1]. And chaos-control is another important research direction that people paid special attention to [2-4]. In the 1990s, to obtain chaos-control, “OGY” technology was presented [5]. Since then, control technology of a chaotic system has improved rapidly [6-8]. And control of a chaotic laser has been concerned by people while people presented a self-feedback technology, a parameter perturbation technology and a cycle signal modulation technology to obtain control of laser chaos [6-8]. These chaos-control technology methods are mainly to obtain chaos-control of one laser system. Hence, controls of two chaotic laser systems will be an important work because two chaotic lasers can emit two chaotic light. And a dual-ring erbium-doped fiber laser can emit two chaotic light. Researchers paid extensively their attentions in their studies of the fiber laser system due to its application in many fields, such as fiber network, fiber communications and fiber sensors. And a few chaos-control methods of the dual-ring fiber laser were presented [9, 10]. Considering of the laser’s application in optic control, optoelectronics and laser physics, to create some novel chaos-control technology, and to promote the development of control of laser chaos, our study will be a very important work to discover laser’s new dynamics and technology. So our work about control of the chaotic dual-ring fiber laser is very necessary.
2. Model
Figure 1 shows that the dual-ring fiber laser system is connected by two laser-rings and an optic coupler C₀, where dual-pump inputting and dual-signal emitting of two laser-rings are performed by two optic wavelength division multiplexers (WDM). And the optic coupler C₀ results in altering π ripple phase of two lasing in dual-ring. To control chaos in two laser-rings, we present a FPS chaos-control method. Figure 1 shows that the four digital signals Fₐ(t), Fₐ(t), Sₐ(t) and Sₐ(t) are used to shift the four parameters of two laser-rings. And the pumps and gains of two laser-rings are modulated simultaneously by the four digital signals to perform chaos-control of two rings. The pumps and gains can shift on different values and functions under the FPS control. Then, we get the FPS control equations of the fiber laser system being [9, 10]:

\[
\frac{d}{dt} E_a = -k_a (E_a + \eta_0 E_b) + g_a \times [1 + \beta_a F_a(t)] \times E_a D_a
\]

(1)

\[
\frac{d}{dt} E_b = -k_b (E_b - \eta_0 E_a) + g_b \times [1 + \beta_b F_b(t)] \times E_b D_b
\]

(2)

\[
\frac{d}{dt} D_a = -(1 + I_{pa}) \times [1 + \gamma_a S_a(t)] + |E_a|^2 D_a + I_{pa} \times [1 + \gamma_a S_a(t)] - 1
\]

(3)

\[
\frac{d}{dt} D_b = -(1 + I_{pb}) \times [1 + \gamma_b S_b(t)] + |E_b|^2 D_b + I_{pb} \times [1 + \gamma_b S_b(t)] - 1
\]

(4)

\[
WDM \quad Ea \quad Eb \quad I_{pa} \quad WDM \quad C_0 \quad I_{pb} \quad WDM \quad S_a \quad S_b \quad Fa \quad Fb
\]

Figure 1. A chaos-control block using the FSP method. Iₚₐₜₜ represent two pump light. Eₐ,ₐₜ result from the rings a and b. Fₐ,ₐₜ, and Sₐ,ₐₜ represent the four digital signals.

Where Eₐ,ₐₜ result from two laser rings a, and b. Dₐ,ₐₜ are the particle numbers of the two laser-rings, respectively. Iₚₐₜ,ₚₜ are the two laser ring pumps. gₐ,ₐₜ are the gain factors of two laser rings. kₐ,ₐₜ are the loss coefficients of two laser-rings, respectively. η₀ is the coupling factor. βₐ, βₐₜ, γₐ, and γₐₜ represent the control parameter factors. Fₐ,ₐₜ, and Sₐ,ₐₜ are the four shift digital signals to be used to obtain chaos-control of two laser-rings. And the four-parameter control terms of FPS are

\[
g_a \times [1 + \beta_a F_a(t)] \ , \quad g_b \times [1 + \beta_b F_b(t)] \ , \quad I_{pa} \times [1 + \gamma_a S_a(t)] \ , \quad [1 + \gamma_b S_b(t)]
\]

And such four-parameter control term shifts will affect and lead to physical behaviours of two laser-rings and realize to control chaos in two laser-rings. When the pumps and gains of two laser-rings are modulated simultaneously by the four digital signals and their different functions, we find that FPS can obtain chaos-control of two laser-rings while a lot of controlled physical behaviours of two laser-rings can be led to show.

3. Result and Discussion
The laser system parameters are listed as: The normalized parameters set as Iₚₐₜₜₜ=4, kₐₐₜₜ=1000, gₐₐₜₜ=4800, gₐₐₜₜ=10500, η₀=0.2 and time unit presents ms. Here, FPS performance is introduced: the four parameter factors βₐ, βₐₜ, γₐ, and γₐₜ are adjusted to lead to different physical states or behaviours of two laser rings in our study process. The four digital signals Fₐ, Fₐₜ, Sₐ and Sₐₜ shift on the value “1” or “0” while we set the four digital signals Fₐ, Fₐₜ, Sₐ and Sₐₜ shift rate being α, β, γ and δ. Then, FPS can be performed on the laser to obtain chaos-control. MATLAB computer method is used in our calculations.
Without FPS, a movement behaviour and output light of two chaotic laser-rings present in figure 2. And figures 2 (a) and (b) illustrate the chaotic attractor of two laser-rings in phase space and the output light $E_a$ and $E_b$ of two laser-rings, respectively. We find that the chaotic movement is restricted to its staring values and its movement orbit has ergodic in phase space while two chaotic signals from two chaotic laser-rings have the random characteristics.

![Figure 2](image_url)

**Figure 2.** The chaotic orbit and output of two laser-rings without FPS.

When FPS performance is operated on two chaotic laser-rings, controls of two chaotic laser-rings are found to realize while a lot of controlled physical behaviours of two laser-rings can be led to show, and these controlled physics states are very interested to us. Figure 3 shows result of chaos-control of two laser-rings, where two period-5 behaviours can be controlled to produce in FPS process, where figures 3 (a) and (b) illustrate the period-5 movement orbit of two rings and two period-5 states or waveforms of the output light $E_a$ and $E_b$ of two rings, respectively, when we set the four control parameter factors being $\beta_a=0.2$, $\beta_b=0.2$, $\gamma_a=0.3$, $\gamma_b=0.25$, and we take the four digital signal shift rates being $\alpha=8$, $\beta=8$, $\gamma=40$, and $\delta=32$. The result points out that chaos-control of two laser-rings can be controlled and be led to some physical states or movement behaviours. The result implies that two period-5 states are controlled to realize by FPS.

![Figure 3](image_url)

**Figure 3.** Two cycle-5 states are controlled to show in FPS process. (a) the movement orbit, and (b) output light $E_a$ and $E_b$ waveforms of two laser-rings.
**Figure 4.** The cycle-13 orbit and output waveforms of two laser-rings using FPS.

**Figure 5.** The cycle-11 orbit and output waveforms of two laser-rings.

When we shift the four control parameter factors on $\beta_a=0.2$, $\beta_b=0.2$, $\gamma_a=0.22$, $\gamma_b=0.22$, and the four digital signal shift rates on $\alpha=19$, $\beta=18$, $\gamma=8$, and $\delta=9$, figure 4 shows scenarios of chaos-control of two laser rings, where two cycle-13 states are controlled to show in FPS process, where figure 4 (a) shows the cycle-13 orbit of two laser rings, and figure 4 (b) shows two cycle-13 states or waveforms of the output light $E_a$ and $E_b$ of two laser rings. And the result implies that two cycle-13 states are controlled to realize by FPS while two laser rings can emit two cycle-13 waves.

When the four control parameter factors set on $\beta_a=0.2$, $\beta_b=0.2$, $\gamma_a=0.22$, $\gamma_b=0.22$, and the four digital signal shift rates set on $\alpha=20$, $\beta=20$, $\gamma=29$, and $\delta=29$, figure 5 shows scenarios of chaos-control of two laser rings and two cycle-11 states are controlled to show in FPS process, where figure 5 (a) shows the cycle-11 orbit of two laser rings, and figure 5 (b) shows two cycle-11 states or waveforms of the output light $E_a$ and $E_b$ of two laser rings. And the result implies that two cycle-11 physical behaviours are controlled to realize by FPS while two laser rings can emit two cycle-11 waves.
Figure 6. The cycle-4 orbit and output waveforms of two laser-rings.

Figure 7. The cycle-5 orbit and output waveforms of two laser-rings.

When the four control parameter factors shift on $\beta_a=0.2$, $\beta_b=0.2$, $\gamma_a=0.22$, $\gamma_b=0.22$, and the four digital signal shift rates put on $\alpha=12$, $\beta=12$, $\gamma=20$, and $\delta=20$, figure 6 shows result of chaos-control of two laser-rings and two period-four states can be controlled to produce in FPS process, where figures 6 (a) and (b) show the period-four movement orbit of two rings and two perid-four states or waveforms of the output light $E_a$ and $E_b$ of two rings, respectively. And the result implies that two cycle-4 physical states are led to realize by FPS while two laser rings can emit two cycle-four waves.

When the four control parameter factors put on $\beta_a=0.2$, $\beta_b=0.2$, $\gamma_a=0.22$, $\gamma_b=0.22$, and the four digital signal shift rates are taken as $\alpha=16$, $\beta=16$, $\gamma=20$, and $\delta=20$, figure 7 presents result of chaos-control of two laser-rings and two period-five behaviours can be led to produce, where figures 7 (a) and (b) present the period-five movement orbit of two rings and two period-five states or waveforms of the output light $E_a$ and $E_b$ of two rings, respectively. And the result point out that two cycle-five physical states are led to show by FPS while two laser rings realize to emit two cycle-five waves.
When we take $\beta_a=0.2$, $\beta_b=0.2$, $\gamma_a=0.22$, $\gamma_b=0.22$, $\alpha=16$, $\beta=16$, $\gamma=18$, and $\delta=18$, another cycle-5 result of chaos-control of two laser rings shown in figure 8.
When the four control parameter factors shift on $\beta_a=0.2$, $\beta_b=0.2$, $\gamma_a=0.22$, and the four digital signal shift rates shift on $\alpha=24$, $\beta=24$, $\gamma=24$, and $\delta=24$, figure 9 shows result of chaos-control of two laser rings and two cycle-one states are controlled to show in FPS process, where figure 9 (a) shows the cycle-one orbit of two laser rings and figure 9 (b) shows two cycle-one states or waveforms of the output light $E_a$ and $E_b$ of two laser rings. And the result point out that two cycle-one physical states are led to show by FPS while two laser rings realize to emit two cycle-one waves.

When the four control parameter factors shift on $\beta_a=0.24$, $\beta_b=0.2$, $\gamma_a=0.35$, $\gamma_b=0.4$, and the four digital signal shift rates shift on $\alpha=16$, $\beta=18$, $\gamma=22$, and $\delta=20$, figure 10 shows two cycle-7 states of two laser rings are controlled to show in FPS process, where figure 10 (a) shows the cycle-7 orbit of two laser rings and figure 10 (b) shows two cycle-7 states or waveforms of the output light $E_a$ and $E_b$. And the result point out that two cycle-7 physical states are led to show by FPS while two laser rings realize to emit two cycle-7 waves. The control parameter factors and the digital signal shift rates or the four-parameter control terms of FPS can lead to physical states of two laser rings and control chaos.

### 4. Conclusion

We present a FPS chaos-control method by considering the structural characteristics of two laser-rings. The FPS method realizes control of the chaotic dual-ring fiber laser and guide successfully physical states of two laser-rings. Controls of the two chaotic laser-rings can be realized when the pumps and gains of two laser rings are modulated simultaneously by four digital signals. We find that the control parameter factors and the digital signal shift rates or the four-parameter control functions of FPS can lead to physical states of two laser-rings and control chaos. And obtained results prove that controls of the two chaotic laser-rings can be successfully taken and physical behaviours of two laser-rings can be guided by FPS. We find that the chaotic behaviour is reduced into two period-one behaviours, and physics movements of the two laser-rings are led to period-4, period-5, period-6, period-7, and some high-order-period. The obtained results have a lot of great helpfulness to study on physics behaviour of dynamics system, chaotic optics, optic control science, laser physics and erbium-doped fiber laser technology.

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