Doubly Robust Off-Policy Actor-Critic: Convergence and Optimality

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Abstract
Designing off-policy reinforcement learning algorithms is typically a very challenging task, because a desirable iteration update often involves an expectation over an on-policy distribution. Prior off-policy actor-critic (AC) algorithms have introduced a new critic that uses the density ratio for adjusting the distribution mismatch in order to stabilize the convergence, but at the cost of potentially introducing high biases due to the estimation errors of both the density ratio and value function. In this paper, we develop a doubly robust off-policy AC (DR-Off-PAC) for discounted MDP, which can take advantage of learned nuisance functions to reduce estimation errors. Moreover, DR-Off-PAC adopts a single timescale structure, in which both actor and critics are updated simultaneously with constant stepsize, and is thus more sample efficient than prior algorithms that adopt either two timescale or nested-loop structure. We study the finite-time convergence rate and characterize the sample complexity for DR-Off-PAC to attain an $\epsilon$-accurate optimal policy. We also show that the overall convergence of DR-Off-PAC is doubly robust to the approximation errors that depend only on the expressive power of approximation functions. To the best of our knowledge, our study establishes the first overall sample complexity analysis for a single time-scale off-policy AC algorithm.

1. Introduction
In reinforcement learning (RL) (Sutton & Barto, 2018), policy gradient and its variant actor-critic (AC) algorithms have achieved enormous success in various domains such as game playing (Mnih et al., 2016), Go (Silver et al., 2016), robotic (Haarnoja et al., 2018), etc. However, these successes usually rely on the access to on-policy samples, i.e., samples collected online from on-policy visitation (or stationary) distribution. However, in many real-world applications, online sampling during a learning process is costly and unsafe (Gottesman et al., 2019). This necessitates the use of off-policy methods, which use dataset sampled from a behavior distribution. Since the policy gradient is expressed in the form of the on-policy expectation, it is challenging to estimate the policy gradient with off-policy samples. A common approach to implement actor-critic algorithms in the off-policy setting is to simply ignore the distribution mismatch between on- and off-policy distributions (Degris et al., 2012; Silver et al., 2014; Lillicrap et al., 2016; Fujimoto et al., 2018; Wang et al., 2016; Houthooft et al., 2018; Meleau et al., 2001) but it has been demonstrated that such distribution mismatch can often result in divergence and poor empirical performance (Liu et al., 2019).

Several attempts have been made to correct the distribution mismatch in off-policy actor-critic’s update by introducing a reweighting factor in policy update (Imani et al., 2018; Zhang et al., 2019b; Liu et al., 2019; Zhang et al., 2019c; Maei, 2018), but so far only COF-PAC (Zhang et al., 2019c) and OPPOSD (Liu et al., 2019) have been theoretically shown to converge without making strong assumptions about the estimation quality. Specifically, COF-PAC reweights the policy update with emphatic weighting approximated by a linear function, and OPPOSD reweights the policy with a density correction ratio learned by a method proposed in (Liu et al., 2018). Although both COF-PAC and OPPOSD show much promise by stabilizing the convergence, the convergence results in (Zhang et al., 2019c) and (Liu et al., 2019) indicate that both algorithms may suffer from a large bias error induced by estimations of both reweighting factor and value function.

The doubly robust method arises as a popular technique to reduce such a bias error, in which the bias vanishes as long as some (but not necessarily the full set of) estimations are accurate. Such an approach has been mainly applied to the off-policy evaluation problem (Tang et al., 2019; Jiang & Li, 2016; Dudík et al., 2011; 2014), and the development of such a method for solving the policy optimization problem is rather limited. (Huang & Jiang, 2020) derives a doubly robust policy gradient for finite-horizon Markov Decision

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Thus, our second goal is to establish a finite-time convergence guarantee for our proposed algorithm, and show that the optimality gap of the overall convergence of our algorithm remains doubly robust.

1.1. Main Contributions

**Doubly Robust Estimator**: We propose a new method to derive a doubly robust policy gradient estimator for an infinite-horizon discounted MDP. Comparing with the previously proposed estimators that adjust only the distribution mismatch (Liu et al., 2019; Imani et al., 2018; Zhang et al., 2019b;c), our new estimator significantly reduces the bias error when two of the four nuisances in our estimator are accurate (and is hence doubly robust). We further propose a new recursive method for critics to estimate the nuisances in the infinite-horizon off-policy setting. Based on our proposed new estimator and nuisance estimation methods, we develop a model-free doubly robust off-policy actor-critic (DR-Off-PAC) algorithm.

**Doubly Robust Optimality Gap**: We provide the finite-time convergence analysis for our DR-Off-PAC algorithm with single timescale updates. We show that DR-Off-PAC is guaranteed to converge to the optimal policy, and the optimality gap of the overall convergence is also doubly robust to the approximation errors. This result is somewhat surprising, because the doubly robust policy gradient update suffers from both non-vanishing optimization error and approximation error at each iteration, whereas the double robustness of the optimality gap is independent of the optimization error. This also indicates that we can improve the optimality gap of DR-Off-PAC by adopting a powerful function class to estimate certain nuisance functions.

Our work is the first that characterizes the doubly robust optimality gap for the overall convergence of off-policy actor-critic algorithms, for which we develop new tools for analyzing actor-critic and critic-critic error interactions.

1.2. Related Work

The first off-policy actor-critic algorithm is proposed in (Degris et al., 2012) as Off-PAC, and has inspired the invention of many other off-policy actor-critic algorithms such as off-policy DPG (Silver et al., 2014), DDPG (Lillicrap et al., 2016), TD3 (Fujimoto et al., 2018), ACER (Wang et al., 2016), and off-policy EPG (Houthooft et al., 2018), etc, all of which have the distribution mismatch between the sampling distribution and visitation (or stationary) distribution of updated policy, and hence are not provably convergent under function approximation settings.

In one line of studies, off-policy design adopts reward shaping via entropy regularization and optimizes over a different objective function that does not require the knowledge of behaviour sampling (Haarnoja et al., 2018; O’Donoghue et al., 2016; Dai et al., 2018; Nachum et al., 2017; 2018; Schulman et al., 2017; Haarnoja et al., 2017; Tosatto et al., 2020). Although the issue of distribution mismatch is avoided for this type of algorithms, they do not have convergence guarantee in general settings. The distribution mismatch issue is also avoided in a gradient based algorithm AlgaeDICE (Nachum et al., 2019), in which the original problem is reformulated into a minimax problem. However, since non-convex minimax objective is in general difficult to optimize, the convergence of AlgaeDICE is not clear.

In another line of works, efforts have been made to address the issue of distribution mismatch in Off-PAC. (Imani et al., 2018) developed actor-critic with emphatic weighting (ACE), in which the convergence of Off-PAC is ameliorated by using emphatic weighting (Sutton et al., 2016). Inspired by ACE and the density ratio in (Gelada & Bellemare, 2019), (Zhang et al., 2019b) proposed Geoff-PAC to optimize a new objective. Based on Geoff-PAC, (Lyu et al., 2020) further applied the variance reduction technique in
(Cutkosky & Orabona, 2019) to develop a new algorithm VOMPS/ACE-STORM. However, since the policy gradient estimator with emphatic weighting is only unbiased in asymptotic sense and emphatic weighting usually suffers from unbounded variance, the convergence of ACE, Geoff-PAC and VOMPS/ACE-STORM are in general not clear. So far, only limited off-policy actor-critic algorithms have been shown to have guaranteed convergence. (Zhang et al., 2019c) and (Liu et al., 2019) respectively, have shown that policy gradients can be unbiased for a given policy for a given state and action. Moreover, we denote \( \pi \) be the policy which is the argmax of the state-action function. Let \( \mu_0 \) and \( \gamma \) be the discount factor. Let \( r(s, a, s') \) be the reward that the agent receives if the agent takes an action \( a \) at state \( s \) and the system transits to state \( s' \). Moreover, we denote \( \mu_0 \) as the distribution of the initial state \( s_0 \in S \) and \( \gamma \in (0, 1) \) as the discount factor. Let \( \pi(a|s) \) be the policy which is the probability of taking action \( a \) given current state \( s \). Then, for a given policy \( \pi \), we define the state value function as \( V_\pi(s) = \mathbb{E}[\gamma^t r(s_t, a_t, s_{t+1}) | s_0 = s, \pi] \) and the state-action value function as \( Q_\pi(s, a) = \mathbb{E}[\gamma^t r(s_t, a_t, s_{t+1}) | s_0 = s, a_0 = a, \pi] \). Note that \( V_\pi(s) = \mathbb{E}_\pi[Q_\pi(s,a)|s] \) and \( Q_\pi(s,a) \) satisfies the following Bellman equation:
\[
Q_\pi(s,a) = R(s,a) + \gamma P_\pi Q_\pi(s,a),
\]
where \( R(s,a) = \mathbb{E}[r(s,a,s')] | s,a \) and
\[
P_\pi Q_\pi(s,a) := \mathbb{E}_{s' \sim P(\cdot|s,a), a' \sim \pi(\cdot|s')}[Q_\pi(s', a')].
\]

We further define the expected total reward function as
\[
J(\pi) = (1 - \gamma) \mathbb{E}[\gamma^t r(s_t, a_t, s_{t+1}) | s_0 = \mu_0, \pi] = \mathbb{E}_\mu[V_\pi(s)] = \mathbb{E}_\mu[\mathbb{E}[V_\pi(s, a, s')]],
\]
where \( \nu_\pi(s, a) = (1 - \gamma) \sum_{t=0}^\infty \gamma^t P(s_t = s, a_t = a | s_0 = \mu_0, \pi) \) is the visitation distribution.

In policy optimization, the agent’s goal is to find an optimal policy \( \pi^* \) that maximizes \( J(\pi) \), i.e., \( \pi^* = \arg\max_\pi J(\pi) \). We consider the setting in which policy \( \pi \) is parametrized by \( w \in \mathbb{R}^d \). Then, the policy optimization is to solve the problem \( \max_w J(\pi_w) \). In the sequel we write \( J(\pi_w) := J(w) \) for notational simplicity. A popular approach to solve such a maximization problem is the policy gradient method, in which we update the policy in the gradient ascent direction as \( w_{t+1} = w_t + \alpha \nabla_w J(w_t) \). A popular form of \( \nabla_w J(w) \) is derived by (Sutton et al., 2000) as
\[
\nabla_w J(w) = \mathbb{E}_{\nu_{\pi_w}}[Q_{\pi_w}(s,a) \nabla_w \log \pi_w(a|s)].
\]

In the on-policy setting, many works adopt the policy gradient formulation in eq. (3) to estimate \( \nabla_w J(w) \), which requires sampling from the visitation distribution \( \nu_{\pi_w} \) and Monte Carlo rollout from policy \( \pi_w \) to estimate the value function \( Q_{\pi_w}(s,a) \) (Zhang et al., 2019a; Xiong et al., 2020).

In this paper we focus on policy optimization in the behavior-agnostic off-policy setting. Specifically, we are given access to samples from a fixed distribution \( \{(s_t, a_t, r_t, s_{t+1})\} \sim D_d \), where the state-action pair \( (s_t, a_t) \) is sampled from an unknown distribution \( d(\cdot) \). In the behavior-agnostic off-policy setting, it is difficult to estimate \( \nabla_w J(w) \) directly with the form in eq. (3), as neither \( \nu_{\pi_w} \) nor Monte Carlo rollout sampling is accessible. Thus, our goal is to develop an efficient algorithm to estimate \( \nabla_w J(w) \) with off-policy samples from \( D_d \), and furthermore, establish the convergence guarantee for our proposed algorithm.

### 3. DR-Off-PAC: Algorithm and Convergence

In this section, we first develop a new doubly robust policy gradient estimator and then design a new doubly robust off-policy actor-critic algorithm.
3.1. Doubly Robust Policy Gradient Estimator

In this subsection, we construct a new doubly robust policy gradient estimator for an infinite-horizon discounted MDP. We first denote the density ratio as \( \rho_{\pi_w} = \nu_{\pi_w}(s, a)/\nu(s, a) \) and the derivative of \( Q_{\pi_w} \) and \( \rho_{\pi_w} \) as \( d_{\pi_w}^\rho \) and \( d_{\pi_w}^\rho \), respectively.

Previous constructions (Kallus & Uehara, 2020) for such an estimator directly combine the policy gradient with a number of error terms under various filtrations to guarantee the double robustness. Such a method does not appear to extend easily to the discounted MDP. Specifically, the method in (Kallus & Uehara, 2020) considers finite-horizon MDP with \( \gamma = 1 \), and further extends their result to infinite-horizon average-reward MDP. Their extension relies on the fact that the objective function \( J(w) \) in average-reward MDP is independent of the initial distribution \( \mu_0 \). In contrast, \( J(w) \) in discounted-reward MDP depends on \( \mu_0 \). Any direct extension necessarily results in a bias due to the lack of the initial distribution, which is unknown a priori, and hence loses the doubly robust property.

To derive a doubly robust gradient estimator in the discounted MDP setting, we first consider a bias reduced estimator of the objective function \( J(w) \) with off-policy sample \((s, a, r, s')\) and \( s_0 \), and then take the derivative of such an estimator to obtain a doubly robust policy gradient estimator. The idea behind this derivation is that as long as the objective estimator has small bias, the gradient of such an estimator can also have small bias. More detailed discussion can be referred to the supplement material.

Given sample \( s_0 \sim \mu_0(\cdot) \) and \((s, a, r, s') \sim \mathcal{D}_d \) and estimators \( Q_{\pi_w}, \rho_{\pi_w}, d_{\pi_w}^\rho \) and \( d_{\pi_w}^\rho \), our constructed doubly robust policy gradient error is given as follows:

\[
G_{\text{DR}}(w) = (1 - \gamma) \left( \hat{Q}_{\pi_w}(s_0, a_0) \nabla_w \log \pi_w(a_0|s_0) + d_{\pi_w}^\rho(s_0, a_0) \right) \\
+ d_{\pi_w}^\rho(s, a) \left( \nabla_w \log \pi_w(a'|s') + d_{\pi_w}^\rho(s', a') \right) \\
+ \hat{\rho}_{\pi_w}(s, a) \left[ -d_{\pi_w}^\rho(s, a) \right] \\
+ \gamma \left( \hat{Q}_{\pi_w}(s', a') \nabla_w \log \pi_w(a'|s') + d_{\pi_w}^\rho(s', a') \right), \tag{4}
\]

where the estimation errors are defined as

\[
\varepsilon_w = \rho_{\pi_w} - \hat{\rho}_{\pi_w}, \quad \varepsilon_q = Q_{\pi_w} - \hat{Q}_{\pi_w}, \\
\varepsilon_{d^\rho} = d_{\pi_w}^\rho - \hat{d}_{\pi_w}^\rho, \quad \varepsilon_{d^\rho'} = d_{\pi_w}^\rho - \hat{d}_{\pi_w}^\rho.
\]

Theorem 1 shows that the estimation error of \( G_{\text{DR}}(w) \) takes a multiplicative form of pairs of individual estimation errors rather than the summation over all errors. Such a structure thus exhibits a three-way doubly robust property. Namely, as long as one of the three pairs \((\rho_{\pi_w}, d_{\pi_w}^\rho), (\hat{Q}_{\pi_w}, \hat{d}_{\pi_w}^\rho), (\hat{\rho}_{\pi_w}, \hat{Q}_{\pi_w})\) are accurately estimated, our estimator \( G_{\text{DR}}(w) \) is unbiased, i.e., \( \mathbb{E}[G_{\text{DR}}(w)] - \nabla_w J(w) = 0 \). There is no need for all of the individual errors to be small.

3.2. Estimation of Nuisance Functions

In order to incorporate the doubly robust estimator eq. (4) into an actor-critic algorithm, we develop critics to respectively construct efficient estimators \( \hat{Q}_{\pi_w}, \hat{\rho}_{\pi_w}, \hat{d}_{\pi_w}^\rho, \hat{d}_{\pi_w}^\rho \) in \( G_{\text{DR}}(w) \) in the linear function approximation setting.

Critic 1: Value function \( \hat{Q}_{\pi_w} \) and density ratio \( \hat{\rho}_{\pi_w} \).

In the off-policy evaluation problem, (Yang et al., 2020) shows that the objective function \( J(w) \) can be expressed by the following primal linear programming (LP):

\[
\min_{\hat{Q}_{\pi_w}} \quad (1 - \gamma) \mathbb{E}_{\mu_0}(Q_{\pi_w}(s, a)) \\
\text{s.t.} \quad Q_{\pi_w}(s, a) = R(s, a) + \gamma P_s Q_{\pi_w}(s, a),
\]

with the corresponding dual LP given by

\[
\max_{\varepsilon_\pi, \varepsilon_q} \quad \mathbb{E}_{\nu_\pi}(R(s, a)) \\
\text{s.t.} \quad \varepsilon_\pi(s', a') = (1 - \gamma) \mu_0(s') \pi(a'|s') + \gamma P_s \nu_\pi(s, a).
\]

Then, the value function \( Q_{\pi_w}(s, a) \) and the distribution correction ratio \( \rho_{\pi_w}(s, a) \) can be learned by solving the following regularized Lagrangian:

\[
\min_{\hat{\rho}_{\pi_w}} \max_{\hat{Q}_{\pi_w}, \eta} L(\hat{\rho}_{\pi_w}, \hat{Q}_{\pi_w}, \eta) \\
= (1 - \gamma) \mathbb{E}_{\nu_\pi}(Q_{\pi_w}(s, a)) + \mathbb{E}_{\mathcal{D}_d}[\rho_{\pi_w}(s, a)(r(s, a, s')) \\
+ \gamma \hat{Q}_{\pi_w}(s', a') - \hat{Q}_{\pi_w}(s, a)] - \frac{1}{2} \mathbb{E}_{\mathcal{D}_d}[\hat{Q}_{\pi_w}(s, a)^2] \\
+ \mathbb{E}_{\mathcal{D}_d}[\eta \hat{\rho}_{\pi_w}(s, a) - \eta - 0.5 \eta^2]. \tag{5}
\]

We construct \( \hat{\rho}_{\pi_w} \) and \( \hat{Q}_{\pi_w} \) with linearly independent feature \( \phi(s, a) \in \mathbb{R}^{d_1} : \rho_{\pi_w}(s, a) = \phi(s, a)^\top \theta_\rho \) and \( Q_{\pi_w}(s, a) = \phi(s, a)^\top \theta_q \) for all \((s, a) \in S \times A\). In such a case, \( L(\theta_\rho, \theta_q, \eta) \) is strongly-concave in both \( \theta_\rho \) and \( \eta \), and convex in \( \theta_q \). We denote the global optimum of \( L(\theta_\rho, \theta_q, \eta) \) as \( \theta^*_\rho, \theta^*_q, \theta^*_\eta \). The errors of approximating \( Q_{\pi_w} \) and \( \rho_{\pi_w} \) with estimators \( \hat{Q}_{\pi_w}(s, a, \theta^*_q, \theta^*_\rho) \) and \( \hat{\rho}_{\pi_w}(s, a, \theta^*_q, \theta^*_\rho) \) are, respectively, defined as

\[
\varepsilon_q = \max_w \sqrt{\mathbb{E}_{\mathcal{D}_d}[(\hat{Q}_{\pi_w}(s, a, \theta^*_q, \theta^*_\rho) - Q_{\pi_w}(s, a))^2]},
\]

\[
\varepsilon_{\rho} = \max_w \sqrt{\mathbb{E}_{\mathcal{D}_d}[(\hat{\rho}_{\pi_w}(s, a, \theta^*_q, \theta^*_\rho) - \rho_{\pi_w}(s, a))^2]}.
\]
max \sqrt{\mathbb{E}_{d, \pi_w} \left\{ (\hat{Q}_{\pi_w}(s', a', \theta_{\pi_w}) - Q_{\pi_w}(s', a'))^2 \right\}}

\epsilon_\rho = \max \sqrt{\mathbb{E}_{d, \pi_w} \left\{ (\hat{\rho}_{\pi_w}(s, a, \theta_{\pi_w}) - \rho_{\pi_w}(s, a))^2 \right\}}

To solve the minimax optimization problem in eq. (5), we adopt stochastic gradient descent-ascent method with mini-batch samples \( \mathcal{B}_t = \{(s_i, a_i, r_i, s'_i)\}_{i=1..N} \sim \mathcal{D}_t \), \( \pi_{\pi}(\cdot|s_i) \) and \( \mathcal{B}_{t, 0} = \{(s_i, 0)\}_{i=1..N} \sim \mathcal{M}_0 \), \( a_i \sim \pi_{\pi}(\cdot|s_i) \), which update parameters recursively as follows

\[
\begin{align*}
\delta_{t,i} &= (1 - \gamma)\phi_{0,t} + \gamma \phi_{t-1}^T \theta_{t} \phi_{t} - \phi_{t}^T \theta_{t} \phi_{t} \\
\eta_{t+1} &= \theta_{t} + \beta_1 \frac{1}{N} \sum_{i \in \mathcal{B}_t} (\phi_{t}^T \theta_{t} \phi_{t} - 1 - \eta_t) \\
\theta_{t+1} &= \Gamma_R \left[ \theta_t + \beta_1 \frac{1}{N} \sum_{i \in \mathcal{B}_t, \mathcal{B}_{t, 0}} (\delta_{t,i} - \phi_{t}^T \theta_{t} \phi_{t}) \right] \\
\quad - \phi_{t}^T \theta_{t} \phi_{t} + \phi_{t}^T \theta_{t} \phi_{t}
\end{align*}
\]

(6)

where \( \Gamma_R \) indicates the projection onto a ball with radius \( R \).

Such a projection operator stabilizes the algorithm (Konda & Tsitsiklis, 2000; Bhatnagar et al., 2009). Note that the iteration in eq. (6) is similar to but different from the GradientDICE update in (Zhang et al., 2020b), as GradientDICE can learn only the density ratio \( \rho_{\pi_w} \), while our approach in eq. (6) can learn both the value function \( Q_{\pi_w} \) and the density ratio \( \rho_{\pi_w} \).

**Critical II: Derivative of value function \( \hat{d}_{\pi_w} \).** Taking derivative on both sides of eq. (1) yields

\[
\begin{align*}
d_{\pi_w}(s, a) &= \gamma \mathbb{E} [d_{\pi_w}(s', a')|s, a] \\
&\quad + \gamma \mathbb{E} [Q_{\pi_w}(s', a') \nabla_w \log \pi_w(a'|s)]|s, a]
\end{align*}
\]

(7)

We observe that eq. (7) takes a form analogous to the Bellman equation in eq. (1), and thus suggests a recursive approach to estimate \( d_{\pi_w} \), similarly to temporal difference (TD) learning. Specifically, we estimate \( d_{\pi_w}^e \) with a feature matrix \( x(s, a) \in \mathbb{R}^{d_x \times d} \), i.e., \( d_{\pi_w}^e(s, a) = x(s, a)^T \theta_{d_x} \) for all \( (s, a) \in \mathcal{S} \times \mathcal{A} \). Replace \( Q_{\pi_w}(s, a) \) with its estimator \( \hat{Q}_{\pi_w}(s, a) = \phi(s, a)^T \theta_q \) in eq. (7). The temporal difference error is then given as

\[
\delta_{d_x}(s, a, \theta_q) = \gamma x(s', a')^T \theta_{d_x} + \gamma \phi(s', a')^T \theta_q \nabla_w \log \pi_w(a'|s') - x(s, a)^T \theta_{d_x}
\]

and \( \theta_{d_x} \) can be updated with the TD-like semi-gradient

\[
\theta_{d_x,t+1} = \theta_{d_x,t} + \beta_2 x(s, a, \delta_{d_x}(s, a, \theta_{d_x,t})).
\]

(8)

However, in the off-policy setting, the iteration in eq. (8) may not converge due to the off-policy sampling. To solve such an issue, we borrow the idea from gradient TD (GTD) and formulate the following strongly convex objective

\[
H(\theta_{d_x}, \theta_q) = \mathbb{E}[x(s, a)\delta_{d_x}(s, a, \theta_q)] - \mathbb{E}[x(s, a)\delta_{d_x}(s, a, \theta_q)].
\]

We denote the global optimum of \( H(\theta_{d_x}, \theta_{q,w}) \) as \( \theta_{q,w}^* \), i.e., \( H(\theta_{d_x,w}, \theta_{q,w}^*) = 0 \). The approximation error of estimating \( d_{\pi_w}^e \) with estimator \( \hat{d}_{\pi_w}^e(s', a', \theta_{q,w}^*) = x(s, a)^T \theta_{q,w}^* \) is defined as

\[
\epsilon_{\rho} = \max \left\{ \max \sqrt{\mathbb{E}_{d, \pi_w} \left\{ \left| \hat{d}_{\pi_w}^e(s', a', \theta_{q,w}^*) - d_{\pi_w}^e(s', a') \right| \right\}^2} \right\},
\]

max \left\{ \max \sqrt{\mathbb{E}_{d, \pi_w} \left\{ \left| \hat{d}_{\pi_w}^e(s', a', \theta_{q,w}^*) - d_{\pi_w}^e(s', a') \right| \right\}^2} \right\},

\]

Similarly to GTD, we introduce an auxiliary variable \( w_{d_x} \) to avoid the issue of double sampling when using gradient based approach to minimize \( H(\theta_{d_x}, \theta_q) \). With mini-batch samples \( \mathcal{B}_t = \{(s_i, a_i, s'_i)\}_{i=1..N} \sim \mathcal{D}_t \), we have the following update for \( \theta_{d_x} \).

\[
\begin{align*}
\theta_{d_x,t+1} &= \theta_{d_x,t} + \beta_3 \frac{1}{N} \sum_{i \in \mathcal{B}_t} (x_i - \gamma x') x_i \nabla_{\theta_{d_x}} \delta_{d_x}(s, a, \theta_{d_x,t}), \\
w_{d_x,t+1} &= w_{d_x,t} + \beta_3 \frac{1}{N} \sum_{i \in \mathcal{B}_t} (x_i \delta_{d_x}(s, a, \theta_{d_x,t}) - w_{d_x,t}).
\end{align*}
\]

(9)

**Critical III: Derivative of density ratio \( \hat{d}_{\pi_w}^e \).** We denote \( \psi_{\pi_w}(s, a) := \nabla_w \log(\nu_{\pi_w}(s, a)) \), and construct an estimator for \( d_{\pi_w}^e \) as \( \hat{d}_{\pi_w}^e(s, a) = \hat{\rho}_{\pi_w}(s, a) \hat{\psi}_{\pi_w}(s, a) \), where \( \hat{\rho}_{\pi_w} \) and \( \hat{\psi}_{\pi_w} \) are approximation of \( \rho_{\pi_w} \) and \( \psi_{\pi_w} \), respectively. Note that eq. (2) can be rewritten in the following alternative form

\[
\nu_{\pi_w}(s', a') = \int \hat{P}(s'|s, a) \nu_{\pi_w}(s, a) dsda,
\]

(10)

where \( \hat{P}(s' | s, a) = (1 - \gamma)\mu_0 + \gamma \mathbb{P}(\cdot | s, a) \). Taking derivative on both sides of eq. (10) and using \( \nabla g(w) = (g(w))^T \nabla g(w) \), we obtain

\[
\begin{align*}
\nu_{\pi_w}(s', a') \psi_{\pi_w}(s', a') = \nabla_w \log(\pi_w(a'|s')) \cdot \nabla \nu_{\pi_w}(s, a) dsda \\
+ \int_{s,a} \left[ \pi_w(a'|s') \hat{P}(s'|s, a) \nu_{\pi_w}(s, a) \right] \psi_{\pi_w}(s, a) dsda \\
= \nabla_w \log(\pi_w(a'|s')) \cdot \nu_{\pi_w}(s', a') \\
+ \int_{s,a} \left[ \pi_w(a'|s') \hat{P}(s'|s, a) \psi_{\pi_w}(s, a) \right] \psi_{\pi_w}(s, a) dsda \\
= \nabla_w \log(\pi_w(a'|s')) \cdot \nu_{\pi_w}(s', a') \\
+ \int_{s,a} \psi_{\pi_w}(s', a') P(s, a|s', a') \psi_{\pi_w}(s, a) dsda
\end{align*}
\]
where the second equality follows because
\[ \pi_w(a'|s') \int_{s,a} \hat{P}(s'|s,a) \nu_{\pi_w}(s,a) dsa = \nu_{\pi_w}(s',a'), \]
and the third equality follows because if \((s, a) \sim \nu_{\pi_w}(.),\)
then \((s', a') \sim \nu_{\pi_w}(.),\) and Bayes’ theorem implies that
\[ \frac{\pi_w(a'|s') \hat{P}(s'|s,a) \nu_{\pi_w}(s,a)}{\nu_{\pi_w}(s',a')} = P(s, a|s', a'). \]
Then, dividing both sides by \(\nu_{\pi_w}(s', a')\) yields
\[ \psi_{\pi_w}(s', a') = \nabla_w \log(\pi_w(a'|s')) + \int P(s, a|s', a') \psi_{\pi_w}(s, a) dsa. \] 
\[ (11) \]

With linear function approximation, we estimate \(\psi_{\pi_w}(s, a)\) with feature matrix \(\varphi(s, a) \in \mathbb{R}^{d \times d}\) i.e., \(\hat{\psi}_{\pi_w}(s, a) = \varphi(s, a)^\top \theta_{\psi}\) for all \((s, a) \in S \times A.\) The temporal difference error is given as
\[ \begin{align*}
\delta_\psi(s', a') &= \nabla_w \log \pi_w(a'|s') + \varphi(s, a)^\top \theta_{\psi} - \varphi(s', a')^\top \theta_{\psi}, \\
(12) \end{align*} \]
Note that in eq. (12), we require \(s' \sim \hat{P}(\cdot|s, a).\) To obtain a sample triple \((s, a, s')\) from such a “hybrid” transition kernel, for a given sample \((s, a, s'),\) we take a Bernoulli choice between \(s'\) and \(s_0 \sim \mu_0\) with probability \(\gamma\) and \(1 - \gamma,\) respectively, to obtain a state \(s'\) that satisfies the requirement. Then, similarly to how we obtain the estimator \(d _ {\pi_w}^2,\) we adopt the method in GTD to formulate the following objective
\[ F(\theta_{\psi}) = \mathbb{E}[\varphi(s', a') \delta_\psi(s', a')]^\top \mathbb{E}[\varphi(s', a') \delta_\psi(s', a')]. \] 
\[ (13) \]
We denote the global optimum of \(F(\theta_{\psi})\) as \(\theta_{\psi, w}^*\) i.e., \(F(\theta_{\psi, w}^*) = 0,\)
and define the approximation error of estimating \(d_{\pi_w}^2\) with estimator \(\hat{d}_{\pi_w}^2(s, a, \theta_{\psi, w}) = \phi(s, a)^\top \theta_{\psi, w}^* \varphi(s, a)^\top \theta_{\psi, w}^*\) as
\[ \epsilon_{d_{\psi}} = \max_w \mathbb{E}_D \left[ \left| d_{\pi_w}^2(s, a, \theta_{\psi, w}, \theta_{\psi, w}^*) - \hat{d}_{\pi_w}^2(s, a) \right|^2 \right]. \]
Given mini-batch samples \(B_t = \{(s_i, a_i, s'_i)\}_{i=1}^{N} \sim D_d,\)
\(a'_i \sim \pi_{\psi}(\cdot|s'_i)\) and \(B_{t,0} = \{(s_0, a_0, i)\}_{i=1}^{N} \sim \mu_0,\) we have the following update for \(\theta_{\psi}\):
\[ \begin{align*}
\theta_{\psi, t+1} = & \theta_{\psi, t} + \beta_2 \frac{1}{N} \sum_{i \in B_t} (\varphi'_i - \varphi_i) \varphi_i^\top \psi_{\psi, t}, \\
\psi_{\psi, t+1} = & w_{\psi, t} + \beta_2 \frac{1}{N} \sum_{i \in B_t} (\varphi'_i \delta_{\psi, i} - \psi_{\psi, i}), \end{align*} \] 
\[ (14) \]
where \(w_{\psi, t}\) is the auxiliary variable that we introduce to avoid the double sampling issue.

**Algorithm 1 DR-Off-PAC**

**Initialize:** Policy parameter \(w_0,\) and estimator parameters \(\theta_{q,0}, \theta_{r,0}, \theta_{d_i,0,}\) and \(\theta_{\psi,0}.\)

**for** \(t = 0, \ldots, T - 1 \) **do**

Obtain mini-batch samples \(B_t \sim D_d\) and \(B_{t,0} \sim \mu_0\)

**Critics I:** Update density ratio and value function estimation via eq. (6): \(\theta_{q, t}, \theta_{r, t} \rightarrow \theta_{q, t+1}, \theta_{r, t+1}\)

**Critics II:** Update derivative of value function estimation via eq. (9): \(\theta_{d_i, t} \rightarrow \theta_{d_i, t+1}\)

**Critics III:** Update derivative of density ratio estimation via eq. (14): \(\theta_{\psi, t} \rightarrow \theta_{\psi, t+1}\)

**Actor:** Update policy parameter via eq. (15)

\[ w_{t+1} = w_t + \alpha \frac{1}{N} \sum_i G_{\text{DR}}^i(w_t). \]

**end for**

**Output:** \(w_T\) with \(T\) chosen uniformly in \(0, \ldots, T - 1\)

**DR-Off-PAC Algorithm.** We now propose a doubly robust off-policy actor-critic (DR-Off-PAC) algorithm as detailed in Algorithm 1. The stepizes \(\beta_1, \beta_2, \beta_3,\) and \(\alpha\) are set to be \(\Theta(1)\) to yield a single-timescale update, i.e., all parameters are updated equally fast. At each iteration, critics I, II, and III perform one-step update respectively for parameters \(\theta_q, \theta_r, \theta_{\psi},\) and \(\theta_{d_i, t},\) and then actor performs one-step policy update based on all critics’ return. Note that Algorithm 1 is inherently a tri-level optimization process, as the update of \(w\) depends on \(\theta_q, \theta_r, \theta_{\psi},\) and \(\theta_{d_i, t},\) which the update of \(\theta_{d_i, t}\) depends on \(\theta_{\psi, t}.\) Thus the interactions between actor and critics and between critic and critic are more complicated than previous actor-critic algorithms that solve bilevel problems (Konda & Tsitsiklis, 2000; Bhatnagar, 2010; Xu et al., 2020b). Due to the single timescale scheme that Algorithm 1 adopts, actor’s update is based on inexact estimations of critics, which can significantly affect the overall convergence of the algorithm. Interestingly, as we will show in the next section, Algorithm 1 is guaranteed to converge to the optimal policy, and at the same time attains doubly robust optimality gap with respect to approximation errors.

**4. Convergence Analysis of DR-Off-PAC**

In this section, we establish the local and global convergence rate for DR-Off-PAC in the single-timescale update setting.
4.1. Local Convergence

We first state a few standard technical assumptions, which have also been adopted in previous studies (Xu et al., 2020b; 2019; Zhang et al., 2020a;b)

**Assumption 1.** For any \((s, a) \in S \times A\) and \(w \in \mathbb{R}^d\), there exists a constant \(C_d > 0\) such that \(p_{\pi_w}(s, a) > C_d\).

**Assumption 2.** For any \((s, a) \in S \times A\), there exist positive constants \(C_\phi, C_\psi, C_\varphi, C_x\) such that the following hold: (1) \(\|\phi(s, a)\|_2 \leq C_\phi\); (2) \(\|\psi(s, a)\|_2 \leq C_\psi\); (3) \(\|\varphi(s, a)\|_2 \leq C_\varphi\); (4) \(\|x(s, a)\|_2 \leq C_x\).

**Assumption 3.** The matrices \(A = E_{D_\mu} \pi_w[(\phi - \gamma \phi')\phi']\), \(B = E_{D_\mu} \pi_w[(\varphi - \varphi')\varphi']\) and \(C = E_{D_\mu} \pi_w[(\gamma \varphi' - x) \varphi']\) are nonsingular.

**Assumption 4.** For any \(w, w' \in \mathbb{R}^d\) and any \((s, a) \in S \times A\), there exist positive constants \(C_{sc}, L_{sc}\), and \(L_\pi\) such that the following hold: (1) \(\|\nabla_w \log \pi_w(a|s)\|_2 \leq L_{sc}\); (2) \(\|\nabla_w \log \pi_w(a|s) - \nabla_w \log \pi_w(a|s)\|_2 \leq L_{sc}\|w - w'\|_2\); (3) \(\|\pi_w(s|s) - \pi_w(s|s)\|_{TV} \leq L_\pi \|w - w'\|_2\), where \(\|\cdot\|_{TV}\) denotes the total-variation norm.

The following theorem characterizes the convergence rate of Algorithm 1, as well as its doubly robust optimality gap.

**Theorem 2 (Local convergence).** Consider the DR-Off-PAC in Algorithm 1. Suppose Assumption 1 - 4 hold. Let the stepsize \(\alpha, \beta_1, \beta_2, \beta_3 = \Theta(1)\). We have

\[
E[\|J(w_{T+1}) - J(w_{T})\|_2^2] 
\leq \Theta \left( \frac{1}{\sqrt{T}} \right) + \Theta \left( \frac{1}{\sqrt{N}} \right) + \Theta(\epsilon_{d_4} + \epsilon_{d_5} + \epsilon_{\pi} + \epsilon_q).
\]

Theorem 2 shows that Algorithm 1 is guaranteed to converge to a first-order stationary point (i.e., locally optimal policy). In particular, the optimality gap (i.e., the overall convergence error) scales as \(\epsilon_{d_4} + \epsilon_{d_5} + \epsilon_{\pi} + \epsilon_q\). Thus, the optimality gap of Algorithm 1 is 3-way doubly robust with respect to the function approximation errors, i.e., the optimality gap is small as long as one of the three pairs \((\epsilon_{d_4}, \epsilon_q), (\epsilon_{d_5}, \epsilon_q), (\epsilon, \epsilon_{d_5})\) is small.

There are two key differences between the doubly robust properties characterized in Theorem 2 and Theorem 1. (a) At the high level, Theorem 1 characterizes the doubly robust property only for the policy gradient estimator, and such a property has been characterized in the previous work for other estimators. In contrast, Theorem 2 characterizes the doubly robust property for the optimality gap of the overall convergence of an algorithm, which has not been characterized in any of the previous studies. (b) At the more technical level, the estimation error \(\varepsilon\) defined in Theorem 1 captures both the optimization error \(\epsilon_{\text{opt}}\) determined by how well we solve the nuisances estimation problem, and the approximation error \(\epsilon_{\text{approx}}\) determined by the representation power of approximation function classes. Thus, Theorem 1 shows that \(G_{\text{DR}}(w)\) is doubly robust to the per-iteration estimation errors that depend on both the optimization process and the approximation function class. As a comparison, Theorem 2 indicates that the optimality gap of DR-Off-PAC is doubly robust only to approximation errors determined by the approximation function class, which implies that the doubly robust property of the overall convergence of DR-Off-PAC is not affected by the optimization process.

Now in order to attain an optimization target accuracy \(\epsilon\) (besides the doubly robust optimality gap), we let \(T = \Theta(1/\epsilon^2)\) and \(B = \Theta(1/\epsilon^2)\). Then Theorem 2 indicates that Algorithm 1 converges to an \(\epsilon\)-accurate stationary point with the total sample complexity \(NT = \Theta(1/\epsilon^4)\). This result outperforms the best known sample complexity of on-policy actor-critic algorithm by a factor of \(\mathcal{O}(\log(1/\epsilon))\) in (Xu et al., 2020b). Such an improvement is mainly due to the single-loop structure that we adopt in Algorithm 1, in which critics inherit the most recently output from the last iteration as actor updates in order to be more sample efficient. But critic in the nested-loop algorithm in (Xu et al., 2020b) always restarts from a random initialization after each actor’s update, which yields more sample cost.

4.2. Global Convergence

In this subsection, we establish the global convergence guarantee for DR-Off-PAC in Algorithm 1. We first make the following standard assumption on the Fisher information matrix induced by the policy class \(\pi_w\).

**Assumption 5.** For all \(w \in \mathbb{R}^d\), the Fisher information matrix induced by policy \(\pi_w\) and initial state distribution \(\mu_0\) satisfies

\[
F(w) = E_{\pi_w}[\nabla_w \log \pi_w(a|s) \nabla_w \log \pi_w(a|s)^T] \succeq \lambda_F \cdot I_d,
\]

for some constant \(\lambda_F > 0\).

Assumption 5 essentially states that \(F(w)\) is well-conditioned. This assumption can be satisfied by some commonly used policy classes. More detailed justification of such an assumption can be referred to Appendix B.2 in (Liu et al., 2020).

We further define the following compatible function approximation error as

\[
\epsilon_{\text{compat}} = \max_{w \in \mathbb{R}^d} \sqrt{E_{\pi_w}[(A_{\pi_w}(s, a) - (1 - \gamma)\chi_{\pi_w}^T \nabla_w \log \pi_w(a|s))^2]},
\]

where \(A_{\pi_w}(s, a) = Q_{\pi_w}(s, a) - V_{\pi_w}(s)\) is the advantage function and \(\chi_{\pi_w} = F(w)^{-1} \nabla_w J(w)\). Such an error \(\epsilon_{\text{compat}}\) captures the approximating error of the advantage function by the score function. It measures the capacity of the policy class \(\pi_w\), and takes small or zero values if the expressive power of the policy class is large (Wang et al., 2019; Agarwal et al., 2019).
The following theorem establishes the global convergence guarantee for Algorithm 1.

**Theorem 3 (Global convergence).** Consider the DR-Off-PAC update in Algorithm 1. Suppose Assumption 1, 2, 3 and 5 hold. For the same parameter setting as in Theorem 2, we have

\[
J(\pi^*) - J(w, \pi) \leq \frac{\varepsilon_{compat}}{1-\gamma} + \Theta \left( \frac{1}{\sqrt{T}} \right) + \Theta \left( \frac{1}{\sqrt{N}} \right) + \Theta(\varepsilon_p\varepsilon_d + \varepsilon_d\varepsilon_q + \varepsilon_p\varepsilon_q).
\]

Theorem 3 shows that Algorithm 1 is guaranteed to converge to the global optimum at a sublinear rate, and the optimality gap is bounded by \(\Theta(\varepsilon_{compat}) + \Theta(\varepsilon_p\varepsilon_d + \varepsilon_d\varepsilon_q + \varepsilon_p\varepsilon_q)\). Note that the error term \(\Theta(\varepsilon_{compat})\) is introduced by the parametrization of policy and thus exists even for exact policy gradient algorithms (Liu et al., 2020; Wang et al., 2019).

The global convergence of DR-Off-PAC in Theorem 3 also enjoys doubly robust optimality gap as in Theorem 2. By letting \(T = \Theta(1/\epsilon^2)\) and \(N = \Theta(1/\epsilon^2)\), Algorithm 1 converges to an \(\epsilon\)-level global optimum (besides the approximation errors) with a total sample complexity \(NT = \Theta(1/\epsilon^4)\). This result matches the global convergence rate of single-loop actor-critic in (Xu et al., 2020c; Fu et al., 2020).

5. Experiments

We conduct empirical experiments to answer the following two questions: (a) does the overall convergence of DR-Off-PAC doubly robust to function approximation errors as Theorem 2 & 3 indicate? (2) how does DR-Off-PAC compare with other off-policy methods?

![Figure 1](image1.png)

**Figure 1.** A variant of Baird’s counterexample.

We consider a variant of Baird’s counterexample (Baird, 1995; Sutton & Barto, 2018) as shown in Figure 1. There are two actions represented by solid line and dash line, respectively. The solid action always leads to state 7 and a reward 0, and the dash action leads to states 1-6 with equal probability and a reward +1. The initial distribution \(\mu_0\) chooses all states \(s\) with equal probability \(\frac{1}{7}\) and the behavior distribution chooses all state-action pairs \((s, a)\) with equal probability \(\frac{1}{7}\). We consider two types of one-hot features for estimating the nuisances: complete feature (CFT) and incomplete feature (INCFT), where CFT for each \((s, a)\) lies in \(\mathbb{R}^{14}\) and INCFT for each \((s, a)\) lies in \(\mathbb{R}^{d}\) with \((d < 14)\). Note that CFT has large enough expressive power so that the approximation error is zero, while INCFT does not have enough expressive power, and thus introduces non-vanishing approximation errors. In our experiments, we consider fixed learning rates 0.1, 0.5, 0.1, 0.05, 0.01 for updating \(w, \theta_q, \theta_d, \theta_{dq}\), and \(\theta_{dp}\), respectively, and we set the mini-batch size as \(N = 5\). All curves are averaged over 20 independent runs.

**Doubly Robust Optimality Gap:** We first investigate how the function approximation error affects the optimality gap of the overall convergence of DR-Off-PAC. In this experiment, we set the dimension of INCFTs as 0, which results in trivial critics that always provide constant estimations. We consider the following four feature settings for critics to estimate the nuisance functions \((Q, \rho, d^q, d^\rho)\): (I) all nuisances with CFTs; (II) \((Q, \rho)\) with CFTs and \((d^q, d^\rho)\) with INCFTs; (III) \((Q, d^q)\) with CFTs and \((\rho, d^\rho)\) with INCFTs; (IV) \((\rho, d^\rho)\) with CFTs and \((Q, d^q)\) with INCFTs. The results are provided in Figure 2. We can see that DR-Off-PAC with all nuisances estimated by CFTs (red line) enjoys the fastest convergence speed and smallest optimality gap, and DR-Off-PAC with only two nuisances estimated with CFTs can still converge to the same optimal policy as the red line, validating the doubly robust optimality gap in the overall convergence characterized by Theorem 2 and Theorem 3.

**Comparison to AC-DC:** As we have mentioned before, previous provably convergent off-policy actor-critic algorithms introduce an additional critic to correct the distribution mismatch (Liu et al., 2019; Zhang et al., 2019c). Such a strategy can be viewed as a special case of DR-Off-PAC when both \(\theta_d\) and \(\theta_{dq}\) equal zero. Here we call such a type of algorithms as actor-critic with distribution correction (AC-DC). In this experiment, we set the dimension of INCFTs as 4 and compare the convergence of DR-Off-AC and AC-DC in the settings considered in our previous experiment. The learning curves of DR-Off-PAC and AC-DC are reported in Figure 3. We can see that the overall convergence of DR-
Doubly Robust Off-Policy Actor-Critic

Off-PAC (each solid line) outperforms that of AC-DC (dash line with the same color) for all feature settings (where each color corresponds to one feature setting). Specifically, In (III) or (IV), when either $Q$ or $\rho$ is estimated with incomplete features, the performance of AC-DC is significantly impeded by the approximation error and thus has lower accuracy, whereas DR-Off-PAC has better convergence performance by mitigating the effect of such approximation errors via the doubly robust property. Interestingly, even in the settings where both $Q$ and $\rho$ are estimated with complete features ($S_1$ and $S_2$) so that AC-DC is expected to achieve zero optimality gap, our DR-Off-PAC still converges faster and more accurately than AC-DC, demonstrating that DR-Off-PAC can improve the convergence of AC-DC even when both $\rho$ and $Q$ are estimated with a complete approximation function class.

6. Conclusion

In this paper, we first develop a new doubly robust policy gradient estimator for an infinite-horizon discounted MDP, and propose new methods to estimate the nuisances in the off-policy setting. Based on such an estimator, we propose a doubly robust off-policy algorithm called DR-Off-PAC for solving the policy optimization problem. We further study the finite-time convergence of DR-Off-PAC under the single timescale update setting. We show that DR-Off-PAC provably converges to the optimal policy, with the optimality gap being doubly robust to approximation errors that depend only on the expressive power of function classes. For future work, it is interesting to incorporate variance reduction technique (Xu et al., 2020a; Cutkosky & Orabona, 2019) to DR-Off-PAC to improve its convergence performance.

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