Dark Energy in Global Brane Universe

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We discuss the exact solutions of brane universes and the results indicate the Friedmann equations on the branes are modified with a new density term. Then, we assume the new term as the density of dark energy. Using Wetterich’s parametrization equation of state (EOS) of dark energy, we obtain the new term varies with the red-shift \( z \). Finally, the evolutions of the mass density parameter \( \Omega_2 \), dark energy density parameter \( \Omega_x \) and deceleration parameter \( q_2 \) are studied.

Keywords: dark energy, brane universe

1. Introduction

It is proposed that our universe is a 3-brane embedded in a higher-dimensional space.1–8 In this brane world model, gravity can freely propagate in all dimensions, while standard matter particles and forces are confined on the 3-brane. A five-dimensional (5D) cosmological model and derived Friedmann equations on the branes are considered by Binetruy, Deffayet and Langlois (BDL),9 for a recent review, it can be seen.10,11 Recent observations indicate that our universe is accelerating12,13 and dominated by a negative pressure component dubbed dark energy. Obviously, a natural candidate is the cosmological constant with equation of state \( w_\Lambda = -1 \). But, comic observations imply that dark energy may be dynamic.14–16 So, a lot of dark energy models are studied extensively, such as quintessence, phantom, etc.14–17 While, brane-world models of dark energy are studied18 and accelerating universe comes from gravity leaking to extra dimension.19 In this paper, we study the dark energy and the universe evolution on the branes which are embedded in a Ricci-flat bulk characterized by a class of exact solutions. The solutions were firstly presented by Liu and Mashhoon and restudied latter by Liu and Wesson.20,21 And they are algebraically rich because they contain two arbitrary functions of time \( t \). Then they are studied as Ricci-flat universe widely.22–29 And exact global solutions of brane universes.30

In this paper, we discuss dark energy in global brane universes. The exact global solutions of brane universes are studied and they show that that the Friedmann
equations on the second brane \((y \neq 0)\) are modified with a new density term. Then we assume this term as the density of dark energy. Since the EOS of dark energy has been presented and investigated widely, now we use the Wetterich’s parametrization EOS of dark energy to study the dark energy on the brane \((y \neq 0)\) (the second brane). This paper is organized as follows: In Section II, we derive the Friedmann equations with a new term on the second brane from the exact global solutions and assume this new term as the density of dark energy. In Section III, the evolutions of the dimensionless density parameters of matter \(\Omega_2\) and dark energy \(\Omega_x\) respectively and deceleration parameter \(q_2\) on the branes \((y \neq 0)\) are obtained by using Wetterich’s parametrization EOS of dark energy to study dark energy. Section IV is a short conclusion.

2. Friedman equations in brane universes

The 5D cosmological solutions read
\[
dS^2 = B^2 dt^2 - A^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) - dy^2, \tag{1}
\]
where \(d\Omega^2 = d\theta^2 + \sin^2 \theta d\psi^2\) and \(k\) is the 3D curvature index \((k = \pm 1, 0)\). For the solutions satisfy the 5D vacuum equation \(R_{AB} = 0\), they are used as the bulk solutions of BDL-type brane model. To obtain brane models for using the \(Z_2\) reflection symmetry on \(A\) and \(B\), they are set as
\[
A^2 = (\mu^2 + k) y^2 - 2\nu \mid y \mid + \frac{\nu^2 + K}{\mu^2 + k}, \tag{2}
\]
where \(\mu = \mu(t)\) and \(\nu = \nu(t)\) are two arbitrary functions, and \(K\) is a constant.

Then the corresponding 5D bulk Einstein equations are taken as
\[
G_{AB} = \kappa^2_{(5)} T_{AB},
T_B^A = \delta(y) \text{diag}(\rho_1, -p_1, -p_1, -p_1, 0) + \delta(y - y_2) \text{diag}(\rho_2, -p_2, -p_2, -p_2, 0) \tag{3}
\]
where the first brane is at \(y = y_1 = 0\) and the second is at \(y = y_2 > 0\). In the bulk \(T_{AB} = 0\) and \(G_{AB} = 0\), Eq.\eqref{3} are satisfied by \eqref{2}. On the branes Liu had solved Eq.\eqref{3} in Ref.\cite{30}. We adopt the result at \(y = y_1 = 0\) and \(y = y_2 > 0\) as follows:
\[
\kappa^2_{(5)} \rho_1 = \frac{6\nu}{A_1^2}, \tag{4}
\]
\[
\kappa^2_{(5)} p_1 = -\frac{2}{A_1} \frac{\partial}{\partial t} \left( \frac{\nu}{A_1} \right) - \frac{4\nu}{A_1^2}, \tag{5}
\]
and
\[
\kappa^2_{(5)} \rho_2 = \frac{6}{A_2} \left( \frac{\mu^2 + k}{A_2} y_2 - \frac{\nu}{A_2} \right), \tag{6}
\]
Now, we consider the universe on the second brane, i.e. \( y = y_2 > 0 \). From the 5D metric (11), the Hubble and deceleration parameters on \( y = y_2 \) brane can be defined as

\[
H_2(t, y) \equiv \frac{1}{B_2 A_2} = \frac{\mu}{A_2},
\]

\[
q_2(t, y) = -\frac{A_2 \mu}{\mu A_2}.
\]

Substituting Eq. (8) into Eq. (6) to eliminate \( \mu^2 \), we find that Eq. (6) can be rewritten into a new form as

\[
H_2^2 + \frac{k}{A_2^2} = \frac{\kappa_{(5)}^2}{6y_2}(\rho_2 + \frac{6}{\kappa_{(5)}^2} \nu A_2^2).
\]

Comparing with the standard Friedman equation i.e. \( H^2 + \frac{k}{A^2} = \frac{\kappa_{(4)}^2}{3} \rho \), we can find \( \kappa_{(4)}^2 = \frac{\kappa_{(5)}^2}{2y_2} \). Since \( \kappa_{(5)}^2 = M_{(5)}^{-3} \) and \( \kappa_{(4)}^2 = M_{(4)}^{-2} \), the relation of four dimensional Planck mass is expressed with five dimensional Planck mass as

\[
M_{(4)}^2 = 2y_2M_{(5)}^4.
\]

Therefore, the four dimensional Planck mass is relevant to five dimensional Planck mass and the position of brane. This Friedman equation is different from BDL’s because it is derived from the exact solution of 5D vacuum equation \( R_{AB} = 0 \) on \( y \neq 0 \) brane.

We assume the term \( \frac{6}{\kappa_{(5)}^2} \nu A_2^2 \) as the density of dark energy. That is,

\[
\rho_x = \frac{6}{\kappa_{(5)}^2} \frac{\nu}{A_2^2}.
\]

From the Eq. (12), it is obtained

\[
\frac{2\mu \dot{\nu}}{A_2 A_2} + \frac{\mu^2 + k}{A_2^2} = -\frac{\kappa_{(5)}^2}{2y_2}(p_2 + p_x),
\]

where \( p_x = -\frac{2}{\kappa_{(5)}^2} \left( \frac{\mu}{A_2 A_2} + \frac{\nu}{A_2^2} \right) \). This is the pressure of dark energy. Then, from Eq. (9), Eq. (10) and Eq. (13), for \( k = 0 \), the deceleration parameters \( q_2 \) can be obtained

\[
q_2 = \frac{1}{2} \left[ \frac{3(p_2 + p_x)}{\rho_2 + p_x} + 1 \right].
\]

Meanwhile, the conservation law \( T_{AB}^B = 0 \) gives

\[
\dot{\rho}_2 + 3(\rho_2 + p_2) \frac{A_2}{A_2} = 0.
\]
3. Density parameters and their evolution

From the definition of $\rho_x$ and $p_x$, for $k = 0$, we obtain the EOS of dark energy

$$w_x = \frac{p_x}{\rho_x} = -\frac{1}{3} \left( \frac{A_2 \nu}{A_2} + 1 \right).$$

(16)

For a given component, which has the equation of state $p_2 = w_2 \rho_2$ (with $w_2$ being a constant), we obtain $\rho_2 = \rho_{20} A_2^{-3(1+w_2)}$ from Eq. (15). Now, considering the given component as matter, i.e. $w_2 = 0$, we get $\rho_2 = \rho_{20} A_2^{-3}$. Therefore, the dimensionless density parameters are obtained

$$\Omega_2 = \frac{\rho_{20}}{\rho_{20} + \frac{9}{8} \nu A_2},$$

(17)

$$\Omega_x = 1 - \Omega_2,$$

(18)

here $\rho_{20}$ is the current values of matter.

In terms of Eq. (2) $A_2$ is a function of $t$ and $y$. However, on the given $y = y_2$ brane, $A_2$ becomes $A_2 = A_2(t)$. Furthermore, we use the relation

$$A_2 = \frac{A_{20}}{1 + z},$$

(19)

and define $\nu = \nu_0 f(z)$, and then we find that Eqs. (16)-(18) can be expressed via redshift $z$ as

$$w_x = \frac{(1 + z) df(z)}{3 f(z)} - \frac{1}{3},$$

(20)

$$\Omega_2 = \frac{\Omega_{20}(1 + z)}{\Omega_{20}(1 + z) + (1 - \Omega_{20}) f(z)},$$

(21)

$$\Omega_x = 1 - \Omega_2.$$  

(22)

From Eqs. (20)-(22), the evolution of cosmic components will be determined if the function $f(z)$ is given.

Now we utilize the form of EOS of the dark energy given by Wetterich [39] which has been studied [40-42]. The form is

$$w_x(z, b) = \frac{w_0}{1 + b \ln(1 + z)},$$

(23)

where $w_x(z, b)$ is the EOS parameter with its current value as $w_0$, and $b$ is a bending parameter describing the deviation of $w_x$ from $w_0$ as $z$ increases. By substituted Eq. (23) into Eq. (20), the function $f(z)$ is obtained as follows:

$$f(z) = (1 + z)[1 + b \ln(1 + z)]^{3w_0/b}.$$  

(24)

Substituting Eq. (23) into Eq. (20), we can obtain (23). The only difference is $b \neq 0$ in this condition. So, there must be deviation from $w_0$ as redshift $z$. We plot the evolution of the EOS of dark energy with $w_0 = -1$ in Fig. 1 where $b = 0.3, 0.6, 1$, respectively. In this figure, we can see $w$ varies with redshift $z$ and $b$. With the increase of $b$, the decline of $w$ becomes fast.
Substituting the function Eq. (24) into Eq. (21) and Eq. (22), we obtain that

\[
\Omega_2 = \frac{\Omega_{20}}{\Omega_{20} + (1 - \Omega_{20})(1 + b \ln(1 + z))^{3w_0/b}}, \tag{25}
\]

\[
\Omega_x = 1 - \Omega_2. \tag{26}
\]

where, \( \Omega_{20} \) is the current value at \( z = 0 \). From Eq. (25) and Eq. (26), we can obtain the evolution of density parameter \( \Omega_2 \) and \( \Omega_x \). Now \( \Omega_x \) is described in Fig. 2 and it is shown that \( \Omega_x \) increases with decrease of redshift \( z \) and the larger \( b \) is, the faster \( \Omega_x \) increases.

We plot the evolutions of \( \Omega_2 \) and \( \Omega_x \) with redshift \( z \) in Fig. 3 where \( b = 0, 0.3, 1 \). It is found that two lines intersect at one point. This point is equilibrium of \( \Omega_2 \) and \( \Omega_x \) and it varies with \( b \), i.e. the corresponding redshift at the point increases with the growing of \( b \).

Also, we obtain the deceleration parameter \( q_2 \) form Eq. (14)

\[
q_2 = \frac{1}{2} \left\{ \frac{3w_0 [1 + b \ln(1 + z)] \left( \frac{2w_0}{1 - w_0} - 1 \right)}{\Omega_{20} - [1 + b \ln(1 + z)]^{3w_0/b}} + 1 \right\}. \tag{27}
\]

The evolution of deceleration parameter \( q_2 \) with redshift \( z \) is plotted in Fig. 4. The larger \( b \) is, the faster the attenuation of \( q \) is. The transition from decelerated expansion to accelerated expansion can be seen easily and the redshift \( z \) of the point at \( q = 0 \) becomes smaller with the increase of \( b \).

4. Conclusions

The exact global solutions of brane universes are studied. The solutions contain two arbitrary functions \( \mu \) and \( \nu \). In this paper, we study the Friedmann equation modified on the brane, and the term with \( \nu \) in the Friedmann equation can drive our universe to accelerate. We assume this term with \( \nu \) as the density of dark energy on \( y \neq 0 \) brane. If different \( \nu \) is given, different models of dark energy can be obtained. We suppose only matter on the brane i.e. \( p_2 = 0 \). Using Wetterich’s parametrization equation of state of dark energy and the relation \( A_2 = A_{20}/(1 + z) \), we obtain the relation of \( \nu \) with redshift \( z \). Thus, if the current values of the two density parameters \( \Omega_{20}, \Omega_{x0}, w_0 \) and the bending parameter \( b \) are known, the arbitrary \( \nu \) could be determined uniquely, and then \( \mu(z) \) could be determined too. In this way, the whole 5D solutions could be reconstructed. And, in principle the 5D solutions could provide with us a global brane cosmological model to simulate our real universe. We have also studied the evolutions of matter density \( \Omega_2 \), dark energy density \( \Omega_x \) and deceleration parameter \( q \) with redshift \( z \) and different \( b \). These cosmic parameters depend on the bending parameter \( b \). Therefore, we expect accurate observational constraints on current cosmic parameters and bending parameter \( b \) in order to determine the evolution of 5D global brane universe.
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![Graph 1](image1.png)

Fig. 1. $w_x$ of the dark energy as a function of the redshift $z$ with its current value $w_0 = -1$ and the bending parameter $b = 1, 0.6, 0.3$ respectively.

![Graph 2](image2.png)

Fig. 2. Evolution of dark energy $\Omega_x$ vs. redshift $z$ with $w_0 = -1, \Omega_{x0} = 0.27, \Omega_{x0} = 0.73$ and the bending parameter $b = 1, 0.6, 0.3$ respectively.
Fig. 3. Evolution of $\Omega_x$ and $\Omega_2$ vs. redshift $z$ with $w_0 = -1$, $\Omega_{20} = 0.27$, $\Omega_{x0} = 0.73$ and the bending parameter $b = 1, 0.3$.

Fig. 4. Evolution of the deceleration parameter $q_2$ vs. redshift $z$ with $w_0 = -1$, $\Omega_{20} = 0.27$, and the bending parameter $b = 1, 0.6, 0.3$ respectively.