Making predictions in eternally inflating universe

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Abstract

Eternally inflating universes can contain large thermalized regions with different values of the constants of Nature and with different density fluctuation spectra. To find the probability for a ‘typical’ observer to detect a certain set of constants, or a certain fluctuation spectrum, one needs to compare the volumes occupied by different types of regions. If the volumes are taken on an equal-time hypersurface, the results of such a comparison are extremely sensitive to the choice of the time variable \( t \). Here, I propose a method of comparing the volumes which is rather insensitive to the choice of \( t \). The method is then applied to evaluate the relative probability of different minima of the inflaton potential and the probability distribution for the density fluctuation spectra.

I. INTRODUCTION

Inflation is a state of rapid (quasi-exponential) expansion of the universe. The inflationary expansion is driven by the potential energy of a scalar field \( \varphi \) (called the ‘inflaton’), while the field slowly ‘rolls down’ its potential \( V(\varphi) \). When \( \varphi \) reaches the minimum of the potential, this vacuum energy thermalizes, and inflation is followed by the usual radiation-dominated expansion.

One of the striking aspects of the inflationary cosmology is that, generically, inflation never ends. The evolution of the field \( \varphi \) is influenced by quantum fluctuations, and
as a result thermalization does not occur simultaneously in different parts of the universe. Inflating regions constantly undergo thermalization, but the exponential expansion of the remaining regions more than compensates for the loss, so that the inflating volume keeps growing with time.

Eternally inflating universes may contain large thermalized domains with different values of the parameters determining the low-energy physics. For example, if gravity is described by a Brans-Dicke-type theory, different domains may have different values of the effective gravitational constant $\mu$. Another possibility is that the potential $V(\varphi)$ has several minima with different masses and couplings of light particles (and different symmetry breaking schemes). Although we cannot observe other regions with different low-energy physics, a study of their properties and their relative abundances in the Universe is not necessarily pointless. If we assume that we are a ‘typical’ civilization inhabiting the Universe, then such a study may help us understand why we observe the particular set of low-energy constants in our part of the universe $[4, 5]$. (The assumption of being typical was called the ‘principle of mediocrity’ in Ref. [5]. It is a version of the ‘anthropic principle’ which has been extensively discussed in the literature $[9]$.)

A related question was raised in a very interesting recent paper by A. Linde, D. Linde and A. Mezhlumian $[7]$ who studied the spectrum of density fluctuations seen by a ‘typical’ observer in an eternally inflating Universe. They came to a surprising conclusion that such an observer should find herself near the center of a very deep minimum of the density field. If correct, this result may rule out a wide class of inflationary models.

One could try to implement the mediocrity principle by comparing the number of civilizations inhabiting regions with different values of the constants (or different fluctuation spectra) at a given moment of time. This was the approach taken by Linde et al. $[4, 6–8]$. However, one finds that the resulting probability distributions are extremely sensitive to the choice of the time coordinate $t$ $[8, 4]$. Since no particular choice of $t$ appears to be preferred, this casts serious doubt on any conclusions reached using this approach. In the present paper I attempt to define the probability distribution for the constants in a coordinate-independent
The idea is to think in terms of the total number of civilizations in the entire spacetime, rather than their number on a particular spacelike hypersurface, \( t = \text{const} \). To be specific, let us consider ‘new’ inflation with a potential \( V(\varphi) \) of the form illustrated in Fig.1. The potential has two minima, and the values \( \varphi^{(1)}_* \) and \( \varphi^{(2)}_* \) near the minima correspond to the end of inflation. The relative probability for a civilization to be in one type of thermalized region versus the other can be estimated as

\[
\frac{\mathcal{P}(1)}{\mathcal{P}(2)} \sim \frac{\nu_{\text{civ}}^{(1)}}{\nu_{\text{civ}}^{(2)}}.
\]

Here, \( \mathcal{V}^{(j)}_* \) is the 3-volume of the hypersurface(s) \( \varphi = \varphi^{(j)}_*, \quad j = 1, 2 \), and \( \nu_{\text{civ}}^{(j)} \) is the average number of civilizations originating per unit volume \( \mathcal{V}^{(j)}_* \). (I assume that civilizations can originate for only a finite period of time after thermalization, so that their number per unit thermalized volume is finite). The dependence of the ‘human factor’ \( \nu_{\text{civ}} \) on the low-energy physics constants has been discussed in relation to the anthropic principle. Here, I will disregard it and concentrate on determining the ratio

\[
r = \frac{\mathcal{V}^{(1)}_*}{\mathcal{V}^{(2)}_*}.
\]

In models where the Universe is closed and inflation is not eternal, the volumes \( \mathcal{V}^{(j)}_* \) are finite, and the ratio \( r \) is well defined. However, in an eternally inflating universe, \( \mathcal{V}^{(1)}_* \) and \( \mathcal{V}^{(2)}_* \) are infinite and have to be regulated. If one simply cuts them off by introducing a hypersurface \( t = \text{const} \) and including only parts of the volumes in the past of that hypersurface, then again one finds that the ratio \( r \) is highly sensitive to the choice of the cutoff hypersurface, so we are back to our original problem. Here, I would like to suggest an alternative regularization procedure which appears to give more reasonable results and which is more robust with respect to changes of the time variable.

The rest of the paper is organized as follows. The general spacetime structure of an eternally inflating universe is discussed in the following Section. The proposed regularization procedure is outlined in Section 3, and a formalism for calculating relative probabilities for
different minima of $V(\varphi)$, which is based on this procedure, is introduced in Section 4. The dependence of the resulting probabilities on the choice of the time variable is discussed in Section 5, with a specific example worked out in Section 6. The spectrum of density fluctuations detected by a typical observer is calculated in Section 7, and the result is compared to that of Linde et al. [7]. The main results of the paper are summarized and discussed in Section 8.

II. INFLATIONARY SPACETIMES

An inflating Universe can be locally described using the synchronous coordinates,

$$ds^2 = d\tau^2 - a^2(x, \tau)dx^2.$$  \hspace{1cm} (3)

The lines of $x = \text{const}$ in this metric are timelike geodesics corresponding to the worldlines of co-moving observers, and the coordinate system is well defined as long as the geodesics do not cross. This will start happening only after thermalization, when matter in some regions will start collapsing as a result of gravitational instability. Hence, the synchronous coordinates (3) can be extended to the future well into the thermalized region.

The classical evolution of the scale factor $a(x, \tau)$ and of the scalar field $\varphi(x, \tau)$ is described by the equations

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 \approx \frac{8\pi V(\varphi)}{3},$$  \hspace{1cm} (4)

$$\dot{\varphi} \approx -\frac{H'(\varphi)}{4\pi},$$  \hspace{1cm} (5)

where dots represent derivatives with respect to $\tau$ and I use Planck units, $\hbar = c = G = 1$. The potential $V(\varphi)$ is assumed to be of the form shown in Fig.1. It is assumed also that the field $\varphi$ is a slowly-varying function of the coordinates, so that spatial gradients of $\varphi$ can be neglected and $\dot{\varphi}^2 \ll 2V(\varphi)$. With the aid of Eqs.(3),(4), the latter condition can be expressed as
This condition is violated near the points \( \varphi = \varphi^{(j)}_* \), signalling the end of inflation. The slow variation of \( \varphi \) implies that \( H \) is also a slowly-varying function of \( x \) and \( \tau \), and thus the spacetime is locally close to DeSitter, with a horizon length \( H^{-1} \).

Quantum fluctuations of the field \( \varphi \) can be pictured as a ‘random walk’ [superimposed on the classical motion (5)] in which \( \varphi \) undergoes random steps of rms magnitude \( (\delta \varphi)_{\text{rms}} = H/2\pi \) per Hubble time, \( \delta \tau = H^{-1} \), independently in each horizon-size region \( (\ell \sim H^{-1}) \). The fluctuations are dynamically unimportant if the classical ‘velocity’ \( |\dot{\varphi}| \) is much greater than the characteristic speed of the random walk, \( (\delta \varphi)_{\text{rms}}/\delta \tau = H^2/2\pi \), which gives

\[
H' \gg H^2. \tag{7}
\]

This condition is violated in the region \( \varphi^{(1)}_0 < \varphi < \varphi^{(2)}_0 \) near the top of the potential (see Fig.1). The dynamics of \( \varphi \) in this region is dominated by quantum fluctuations. The deterministic slow-roll regions are bounded by the values \( \varphi^{(j)}_0 \) and \( \varphi^{(j)}_* \).

The spacetime of the inflating universe is schematically represented in Fig.2. Thermalized regions of different type are shown by different kinds of shading; their boundaries are the surfaces \( \varphi = \varphi^{(j)}_* \). These boundaries expand into the inflating region, but become asymptotically static in the coordinate space as \( \tau \to \infty \). (If the speed of expansion remained finite at large \( \tau \), then thermalized regions would merge and inflation would terminate everywhere). This does not mean, however, that the boundary surfaces become timelike at large \( \tau \). To picture the geometry of these surfaces, one has to keep in mind that physical lengths in an inflating universe are related to coordinate differences by an exponentially growing scale factor. As a result, the boundaries of thermalized regions (as well as all surfaces of constant \( \varphi \) in the slow-roll regime) are spacelike, and are in fact very flat.

The spacelike character of the constant-\( \varphi \) surfaces can be understood as follows [11]. Consider the normal vector to the surfaces, \( \partial_\mu \varphi \). We have

\[
\partial_\mu \varphi \partial^\mu \varphi = \dot{\varphi}^2 - a^{-2}(\nabla \varphi)^2. \tag{8}
\]
The spatial gradients of $\varphi$ are caused by quantum fluctuations. On the scale of the horizon, the gradient is of the order $a^{-1}|\nabla \varphi| \sim (\delta \varphi)_{\text{rms}}/H^{-1} \sim H^2/2\pi$, and is even smaller on larger scales. On the other hand, from Eqs. (5), (7), $|\dot{\varphi}| \gg H^2/2\pi$. Hence, $\partial_\mu \varphi \partial^\mu \varphi > 0$, and the surfaces $\varphi = \text{const}$ are spacelike. Moreover, these surfaces are locally nearly parallel to the surfaces $\tau = \text{const}$ (which correspond to horizontal lines in Fig. 2). This can be seen by considering the scalar product of the unit normals, $u^\mu = (1,0,0,0)$ and $n_\mu = \partial_\mu \varphi (\partial_\nu \varphi \partial^\nu \varphi)^{-1/2}$, 

$$u^\mu n_\mu = [1 - a^{-2}(\nabla \varphi)^2/\dot{\varphi}^2]^{-1/2} \approx 1. \quad (9)$$

For an internal observer in one of the thermalized regions, the surface $\varphi = \varphi_*$ at the boundary of that region plays the role of the big bang. The natural choice of the time coordinate in the vicinity of that surface is $t = \varphi$, so that the constant-$t$ surfaces are (nearly) surfaces of constant energy. Since these surfaces are infinite, the observer finds herself in an infinite thermalized universe, which is causally disconnected from the other thermalized regions. The situation here is similar to that in the ‘open-universe’ inflation\cite{12} where thermalized regions are located in the interiors of expanding bubbles and have the geometry of open ($k = -1$) Robertson-Walker universes.

We see that, depending on one’s choice of the time coordinate $t$, an equal-time surface may cross many thermalized regions of different types (e.g., for $t = \tau$), may cross no such regions at all (say, for $t = \varphi$ with $\varphi$ in the slow-roll range), or may lie entirely in a single thermalized region. Not surprisingly, with a suitable choice of $t$, one can get any result for the volume ratio $r$ in Eq. (3).

III. THE PROPOSAL

Let us choose a region in which the field $\varphi$ is close to the top of the potential ($\varphi \approx 0$) at some moment of time in the coordinates (3). We can set $\tau = 0$ and $a = 1$ at that moment. The choice of the initial moment and of the particular region is unimportant, due to the
self-similar nature of eternal inflation \cite{13,8}. Let us now introduce a large number $N$ of co-moving observers uniformly spread over the region. Some of these observers will end up in thermalized regions of type 1 and others in regions of type 2 (see Fig.3). The initial 3-volume of the region $V_0$ has to be sufficiently large, so that the corresponding co-moving volume includes a large number of thermalized regions of both types. (Alternatively, one could consider an ensemble consisting of a large number of small regions). The initial volume per observer is $V_0/N$, and if $a_{\ast i}$ is the scale factor for the $i$-th observer at the time when she reaches $\varphi_{\ast i}^{(j)}$, then the 3-volumes of the hypersurfaces $\varphi = \varphi_{\ast i}^{(j)}$ can be represented as

$$V_{\ast i}^{(j)} = V_0 N^{-1} \sum_{i} (j) a_{\ast i}^3,$$

(10)

Here, $j = 1, 2$, and the summation is over all observers ending up in thermalized regions of type $j$. [I disregard the factors $(u^\mu n_\mu)^{-1} \approx 1$; see Eq.(9)].

The sums in Eq.(10) are, of course, divergent in the limit $N \to \infty$. My proposal is to regularize these sums by cutting off a small fraction $\epsilon$ of terms having the largest scale factors $a_{\ast i}$, with the same value of $\epsilon$ for both types of thermalized regions. The volume ratio $r$ in Eq.(2) can then be evaluated in the limit $\epsilon \to 0$.

A similar regularization can be implemented using the proper time $\tau$, or some other ‘time’ $t$, as a cutoff variable instead of the scale factor (that is, discarding a fraction $\epsilon$ of observers having the largest values of $t$ when they reach $\varphi = \varphi_{\ast i}^{(j)}$). We shall see that the volume ratio $r$ is not very sensitive to the choice of the cutoff variable.

The reader may be concerned that coordinate-dependence in this approach is introduced from the very beginning, when we choose the initial volume $V_0$ on an equal-time surface in the synchronous coordinate system. The role of this volume, however, is only to define an appropriate congruence of geodesic observers. Geometrically, the construction can be described as follows. (i) Find a spacetime region where $\varphi \approx 0$. (ii) Choose a hypersurface in this region which has approximately constant, small intrinsic and extrinsic curvature \((^3 R \approx \text{const} \lesssim H^2)\). (iii) Consider a congruence of geodesic observers whose worldlines are orthogonal to this hypersurface.
IV. CALCULATION OF THE VOLUME RATIO

With the above prescription, the volume ratio \( r \) can be calculated in terms of the probability distribution \( P(\varphi, t) \) for the inflaton field \( \varphi \). This distribution satisfies the diffusion equation \[ \partial_t P = -\partial_\varphi J, \] (11)

where the flux \( J(\varphi, t) \) is given by

\[
J = -(8\pi^2)^{-1}H^{\frac{\alpha}{2}+1}\partial_\varphi \left(H^{\frac{\alpha}{2}+1}P\right) - (4\pi)^{-1}H^{\alpha-1}H'P. \tag{12}
\]

The first, ‘diffusion’ term in Eq.(12) represents quantum fluctuations of the field \( \varphi \), while the second term corresponds to the classical ‘drift’ due to the potential \( V(\varphi) \). The parameter \( \alpha \) is defined so that for \( \alpha = 1 \) the time variable \( t \) is the proper time \( \tau \), while in general \( t \) is related to \( \tau \) through

\[
dt = H^{1-\alpha}d\tau. \tag{13}
\]

In particular, for \( \alpha = 0 \), \( t = \ln a \). The boundary conditions at \( \varphi = \varphi_{*}^{(j)} \) are

\[
J(\varphi_{*}) = -(4\pi)^{-1}H^{\alpha-1}H'P(\varphi_{*}). \tag{14}
\]

The initial distribution \( P(\varphi, 0) \) is sharply peaked near \( \varphi = 0 \) and is normalized so that

\[
\int_{\varphi_{*}^{(j)}}^{\varphi_{*}^{(j)}(2)} P(\varphi, 0)d\varphi = 1. \tag{15}
\]

The implementation of the \( \epsilon \)-cutoff procedure is most straightforward with the choice of \( \alpha = 0 \). Then \( a = e^t \), and Eq.(11) gives

\[
Y_{*}^{(j)} = V_0 \left[ \int_{0}^{t_c^{(j)}} J(\varphi_{*}^{(j)}, t)e^{3t}dt \right], \tag{16}
\]

where the cutoff ‘times’ \( t_c^{(j)} \) are determined from

\[
\left| \int_{t_c^{(j)}}^\infty J(\varphi_{*}^{(j)}, t)dt \right| = \epsilon \left| \int_{0}^\infty J(\varphi_{*}^{(j)}, t)dt \right| \equiv \epsilon P^{(j)}. \tag{17}
\]
The conservation of probability implies that
\[ p^{(1)} + p^{(2)} = 1 \] (18)

Equations (16),(17) are easily understood if we note that \( |J(\varphi_0^{(j)}, t)|dt \) is the fraction of all observers who end inflation in a region of type \( j \) at time \( t \) in the interval \( dt \). We note also that the cutoff times \( t_c^{(j)} \to \infty \) as \( \epsilon \to 0 \).

The solution of Eq.(11) can be represented in the form
\[ \mathcal{P}(\varphi, t) = \sum_{n=1}^{\infty} \psi_n(\varphi)e^{-\gamma_n t}, \] (19)
where \( 0 < \gamma_1 < \gamma_2 < ... \). In the limit \( t \to \infty \),
\[ \mathcal{P}(\varphi, t \to \infty) = \psi_1(\varphi)e^{-\gamma_1 t}. \] (20)

The eigenfunction \( \psi_1(\varphi) \) can be found explicitly in the slow-roll regions, away from the top of the potential, where the diffusion term in Eq.(12) is negligible. The equation for \( \psi_1(\varphi) \) in these regions takes the form
\[ \partial_\varphi(H^{-1}H'\psi_1) = -4\pi\gamma_1\psi_1, \] (21)
and its solution is
\[ \psi_1^{(j)}(\varphi) = c^{(j)} \frac{H(\varphi)}{H'(\varphi)} \exp \left[ -4\pi\gamma_1 \int_{\varphi_0^{(j)}}^{\varphi} \frac{H(\xi)}{H'(\xi)} d\xi \right]. \] (22)

The solution \( \psi_1^{(j)}(\varphi) \) applies in the range between \( \varphi_0^{(j)} \) and \( \varphi_1^{(j)} \). [Note that Eq.(22) satisfies the boundary condition (14)]. The coefficients \( c^{(j)} \) and the eigenvalue \( \gamma_1 \) can be found by matching Eq.(22) to the solution for \( \psi_1 \) in the range \( \varphi_0^{(1)} < \varphi < \varphi_0^{(2)} \). An example will be given in Section 6.

We shall now use the asymptotic forms (20),(22) to evaluate the volume ratio \( r \). For inflation to be eternal, the exponential decay of the distribution function (21) should be slower than the expansion of the inflating regions, \( a^3 = e^{3t} \), so that the total inflating volume grows. Then, \( \gamma_1 < 3 \), and the integral in Eq.(16) for \( V^{(j)}_\star \) is dominated by the upper limit. With the aid of Eqs.(20),(22) we can write
\[ \mathcal{V}_s^{(j)} = [4\pi(3 - \gamma_1)]^{-1} \mathcal{V}_0 c^{(j)} \exp \left[ (3 - \gamma_1)t_c^{(j)} + \gamma_1 t_s^{(j)} \right], \]  

(23)

where

\[ t_s^{(j)} = -4\pi \int_{\phi_0^{(j)}}^{\phi_j^{(j)}} \frac{H(\xi)}{H'(\xi)} d\xi \]  

(24)

is the ‘time’ of the classical slow roll from \( \phi_0^{(j)} \) to \( \phi_j^{(j)} \) [see Eqs.(5),(13)].

The cutoff ‘times’ \( t_c^{(j)} \) are determined by Eq.(17), and substituting the asymptotics (20),(22) in the left-hand side of that equation, we have

\[ (4\pi\gamma_1)^{-1} c^{(j)} \exp[\gamma_1(t_c^{(j)} - t_s^{(j)})] = \epsilon p^{(j)}. \]  

(25)

Finally, substituting this into (23), we obtain

\[ \mathcal{V}_s^{(j)} = c^{(j)} \mathcal{V}_0 [4\pi(3 - \gamma_1)]^{-1} [4\pi\gamma_1 \epsilon p^{(j)}/c^{(j)}]^{-(3-\gamma_1)/\gamma_1} \exp[3t_c^{(j)}] \]  

(26)

and

\[ r = \frac{\mathcal{V}_s^{(1)}}{\mathcal{V}_s^{(2)}} = \left[ \frac{c^{(1)}}{c^{(2)}} \right]^{\gamma_1/\gamma_1} \left[ \frac{p^{(2)}}{p^{(1)}} \right]^{(3-\gamma_1)/\gamma_1} \left[ \frac{Z_s^{(1)}}{Z_s^{(2)}} \right]^3. \]  

(27)

Here,

\[ Z_s^{(j)} = \exp \left[ -4\pi \int_{\phi_0^{(j)}}^{\phi_j^{(j)}} \frac{H(\xi)}{H'(\xi)} d\xi \right] \]  

(28)

is the expansion factor during the classical slow roll from \( \phi_0^{(j)} \) to \( \phi_j^{(j)} \).

The quantities \( \gamma_1, c^{(j)}, p^{(j)}, \) and \( Z_s^{(j)} \) all depend on the shape of the potential \( V(\varphi) \). The eigenvalue \( \gamma_1 \) and the ratio \( p^{(2)}/p^{(1)} \) are determined mainly by the shape of the potential near \( \varphi = 0 \), where the dynamics of \( \varphi \) is dominated by quantum fluctuations. Unless \( V(\varphi) \) is very asymmetric near \( \varphi = 0 \), the numbers of observers ending up in the two types of thermalized regions will have the same order of magnitude, and thus \( p^{(2)}/p^{(1)} \sim 1 \). If the potential is approximately symmetric near the top, \( V(-\varphi) \approx V(\varphi) \), then we can set \( \varphi_0^{(1)} \approx -\varphi_0^{(2)} \), and one expects that \( \psi_1(\varphi_0^{(1)}) \sim \psi_1(\varphi_0^{(2)}) \), that is, \( c^{(1)}/c^{(2)} \sim 1 \). In this case, the most sensitive dependence of \( r \) on the shape of \( V(\varphi) \) is through the expansion factors \( Z_s^{(j)} \),

\[ r \sim \left[ \frac{Z_s^{(1)}}{Z_s^{(2)}} \right]^3. \]  

(29)
V. DIFFERENT TIME VARIABLES

With a different choice of the time variable (\( \alpha \neq 0 \)), the calculation of \( r \) is somewhat more complicated. The difficulty is that the 'time' \( t \) is no longer related to the scale factor \( a \), and we cannot write the analogue of Eq.(16) for the volume \( V_{(j)}^{(j)} \). This can be done by introducing another distribution function \( \tilde{\mathcal{P}}(\varphi,t) \), such that \( \tilde{\mathcal{P}}(\varphi,t)d\varphi \) is the physical volume occupied by regions with \( \varphi \) in the interval \( d\varphi \) at time \( t \). In other words, \( \tilde{\mathcal{P}}(\varphi,t)d\varphi \) is the 3-volume on a hypersurface \( t = \text{const} \) in which the inflaton field has values between \( \varphi \) and \( \varphi + d\varphi \). The function \( \tilde{\mathcal{P}}(\varphi,t) \) satisfies the modified diffusion equation [15–17,8]

\[
\partial_t \tilde{\mathcal{P}} = -\partial_\varphi \tilde{\mathcal{J}} + 3H^\alpha \tilde{\mathcal{P}},
\]

where \( \tilde{\mathcal{J}} \) is given by Eq.(12) with \( \mathcal{P} \) replaced by \( \tilde{\mathcal{P}} \). At the initial moment, \( t = 0 \), the distributions \( \tilde{\mathcal{P}} \) and \( \mathcal{P} \) are proportional to one another,

\[
\tilde{\mathcal{P}}(\varphi,0) = \mathcal{V}_0 \mathcal{P}(\varphi,0).
\]

The volume in which \( \varphi \) rolls down to the value \( \varphi_{(j)}^{(j)} \) in a time interval \( dt \) is \( |\tilde{\mathcal{J}}(\varphi_{(j)}^{(j)},t)|dt \), and thus the volume \( V_{(j)}^{(j)} \) with a cutoff at \( t_{c(j)}^{(j)} \) can be written as

\[
V_{(j)}^{(j)} = \left| \int_0^{t_{c(j)}^{(j)}} \tilde{\mathcal{J}}(\varphi_{(j)}^{(j)},t)dt \right|.
\]

The cutoff times \( t_{c(j)}^{(j)} \) are still determined by Eq.(17). Hence, in order to calculate \( r \) for \( \alpha \neq 0 \), one has to solve both equations (11) and (30) for \( \mathcal{P} \) and \( \tilde{\mathcal{P}} \).

The distribution \( \tilde{\mathcal{P}}(\varphi,t) \) can be expanded in eigenfunctions,

\[
\tilde{\mathcal{P}}(\varphi,t) = \sum_{n=1}^{\infty} \tilde{\psi}_n(\varphi)e^{\tilde{\gamma}_n t},
\]

where \( \tilde{\gamma}_1 > \tilde{\gamma}_2 > \ldots \). For inflation to be eternal, we should have \( \tilde{\gamma}_1 > 0 \). In the limit \( t \to \infty \),

\[
\tilde{\mathcal{P}}(\varphi,t \to \infty) = \tilde{\psi}_1(\varphi)e^{\tilde{\gamma}_1 t}.
\]

As in the previous Section, the form of \( \tilde{\psi}_1(\varphi) \) in the slow-roll regime can be found by neglecting the diffusion term in Eq.(30),
\[ \partial_\varphi (H^{\alpha-1}H'\tilde{\psi}_1) + 12\pi H^\alpha \tilde{\psi}_1 = 4\pi \gamma_1 \tilde{\psi}_1. \] (35)

The solutions of this equation in the regions between \( \varphi_0^{(j)} \) and \( \varphi^{(j)} \) are

\[ \tilde{\psi}_1^{(j)} = \tilde{c}^{(j)} \frac{H^{1-\alpha}(\varphi)}{H'(\varphi)} \exp \left[ -12\pi \int_{\varphi_0^{(j)}}^{\varphi^{(j)}} \frac{H(\xi)}{H'(\xi)} d\xi + 4\pi \tilde{\gamma}_1 \int_{\varphi_0^{(j)}}^{\varphi^{(j)}} \frac{H^{1-\alpha}(\xi)}{H'(\xi)} d\xi \right], \] (36)

with \( \tilde{c}^{(j)} = \text{const} \), and combining Eqs.\((\ref{28}), (\ref{34})\) and \( \ref{36} \), we have

\[ \mathcal{V}_s^{(j)} = [4\pi \tilde{\gamma}_1]^{-1} \tilde{c}^{(j)} \left[ Z_*^{(j)} \right]^3 \exp[\tilde{\gamma}_1 (t_c^{(j)} - t_*^{(j)})]. \] (37)

Here, \( Z_*^{(j)} \) is given by Eq.\((\ref{28})\) and \( t_*^{(j)} \) has the same meaning as in Eq.\((\ref{24})\) and is given by

\[ t_*^{(j)} = -4\pi \int_{\varphi_0^{(j)}}^{\varphi^{(j)}} \frac{H^{1-\alpha}(\xi)}{H'(\xi)} d\xi. \] (38)

[Note that for \( \alpha = 0 \), Eq.\((\ref{35})\) reduces to Eq.\((\ref{21})\) with \( \tilde{\gamma}_1 = 3 - \gamma_1 \). Then it is easily understood that \( \tilde{c}^{(j)} = \mathcal{V}_0 c^{(j)} \), and thus Eqs.\((\ref{22}), (\ref{23})\) agree with Eqs.\((\ref{36}), (\ref{37})\)]. Now, with the cutoff times \( t_c^{(j)} \) from Eq.\((\ref{25})\), we obtain

\[ r = \frac{\mathcal{V}_s^{(1)}}{\mathcal{V}_s^{(2)}} = \frac{\tilde{c}^{(1)}}{\tilde{c}^{(2)}} \left[ \frac{c^{(1)} p^{(2)} p^{(1)}}{c^{(2)} p^{(2)}} \right]^{\tilde{\gamma}_1 \gamma_1} \left[ \frac{Z_*^{(1)}}{Z_*^{(2)}} \right]^3. \] (39)

The coefficients \( c^{(j)}, \tilde{c}^{(j)} \) and the eigenvalues \( \gamma_1, \gamma_1 \) depend on the choice of the parameter \( \alpha \), and thus the ratio \( r \) has some dependence on the time parametrization. [The quantity \( p^{(j)} \) is the fraction of the observers that end up in the \( j \)-th type of thermalized regions, and should be independent of \( \alpha \)]. However, this dependence appears not to be very strong. In particular, in cases where the potential \( V(\varphi) \) is nearly symmetric in the region dominated by quantum fluctuations near the top, we have \( c^{(1)}/c^{(2)} \sim \tilde{c}^{(1)}/\tilde{c}^{(2)} \sim p^{(1)}/p^{(2)} \sim 1 \), and the estimate \((\ref{29})\) holds for any value of \( \alpha \).

In contrast, the result would be extremely sensitive to the choice of \( \alpha \) if we used a cutoff at a fixed value of \( t \). Then we would get

\[ r = \frac{\tilde{J}(\varphi_*^{(1)}, t)}{\tilde{J}(\varphi_*^{(2)}, t)} = \frac{\tilde{c}^{(1)}}{\tilde{c}^{(2)}} \left[ \frac{Z_*^{(1)}}{Z_*^{(2)}} \right]^3 \exp[-\tilde{\gamma}_1 (t_c^{(1)} - t_c^{(2)})]. \] (40)

The quantity \( t_c^{(j)} \) is the time it takes the scalar field \( \varphi \) to roll from \( \varphi_0^{(j)} \) to \( \varphi^{(j)} \). It obviously depends on the choice of the time coordinate.
To assess the importance of the last factor in (40), we need an estimate for $\tilde{\gamma}_1$. It can be shown that [13]

$$\tilde{\gamma}_1 \leq 3H_{\text{max}}^\alpha,$$  (41)

where $H_{\text{max}}$ is the largest value of $H(\varphi)$. [$H_{\text{max}} = H(0)$ for the potential in Fig.1]. For a generic potential,

$$\tilde{\gamma}_1 \sim H_{\text{max}}^\alpha$$  (42)

gives a reasonably good order-of-magnitude estimate. The quantity

$$d = \tilde{\gamma}_1 H^{-\alpha}_{\text{max}} < 3$$  (43)

can be called the fractal dimension of the inflating universe [13,19]. The values of $d \ll 1$ can be obtained only when the potential is fine-tuned so that eternal inflation is barely possible.

Returning now to Eq.(40), we see from Eqs.(28),(38), and (42) that, if $\alpha > 0$, then

$$\tilde{\gamma}_1 t^j_*= 3 \ln Z^j_*,$$

and thus the last factor in (40) is typically no less important than the ratio of the expansion factors. In particular, cutoffs at a fixed proper time ($\alpha = 1$) and at a fixed scale factor ($\alpha = 0$) often give drastically different results.

VI. AN EXAMPLE

The eigenfunctions $\psi_1(\varphi)$, $\tilde{\psi}_1(\varphi)$ (and the corresponding eigenvalues) can be approximately found in the whole range of $\varphi$ for a potential $V(\varphi)$ shown in Fig.4. The potential is completely flat ($V = V_0$) for $\varphi^{(1)}_0 < \varphi < \varphi^{(2)}_0$, satisfies the slow-roll conditions (6),(7) outside this range, and is arbitrary otherwise. Without loss of generality, we can set $\varphi^{(2)}_0 = -\varphi^{(1)}_0 \equiv \varphi_0$.

In the flat region of the potential, $|\varphi| < \varphi_0$, the equation for $\tilde{\psi}_1(\varphi)$ is

$$\tilde{\psi}_1'' + \kappa^2 \tilde{\psi}_1 = 0,$$  (44)

where
\[ \kappa^2 = 8\pi^2 H_0^{(\alpha+2)}(3H_0^\alpha - \gamma_1) \]  

(45)

and \( H_0 = (8\pi V_0/3)^{1/2} \) is the expansion rate in the flat region. The solution of Eq.(14) is

\[ \tilde{\psi}_1(\varphi) = A_1 \cos(\kappa \varphi) + A_2 \sin(\kappa \varphi). \]  

(46)

In the slow-roll regions at |\varphi| > \varphi_0 we can use the approximate solutions (36). The matching conditions at \( \varphi = \pm \varphi_0 \) require that \( \tilde{\psi}_1(\varphi) \) and the corresponding flux \( \tilde{J}_1(\varphi) \) should be continuous,

\[ [\tilde{\psi}_1]_{\varphi=\pm \varphi_0} = [\tilde{J}_1]_{\varphi=\pm \varphi_0} = 0. \]  

(47)

Combining Eqs.(36),(46) and (47), we obtain the following system of equations

\[ A_1 \cos(\kappa \varphi_0) - A_2 \sin(\kappa \varphi_0) = \tilde{c}^{(1)} H_0^{1-\alpha}/H_1', \]  

(48)

\[ A_1 \cos(\kappa \varphi_0) + A_2 \sin(\kappa \varphi_0) = \tilde{c}^{(2)} H_0^{1-\alpha}/H_2', \]  

(49)

\[ \kappa H_0^{\alpha+2}[A_1 \sin(\kappa \varphi_0) + A_2 \cos(\kappa \varphi_0)] = 2\pi \tilde{c}^{(1)}, \]  

(50)

\[ \kappa H_0^{\alpha+2}[-A_1 \sin(\kappa \varphi_0) + A_2 \cos(\kappa \varphi_0)] = 2\pi \tilde{c}^{(2)}, \]  

(51)

which yields, after some algebra,

\[ \beta_1 - \beta_2 + 2\kappa \varphi_0 = n\pi, \]  

(52)

\[ \frac{A_1}{A_2} = \tan \left( \frac{\beta_1 + \beta_2 + n\pi}{2} \right), \]  

(53)

\[ \frac{\tilde{c}^{(1)}}{\tilde{c}^{(2)}} = (-1)^n \frac{\cos \beta_1}{\cos \beta_2}. \]  

(54)

Here, I have introduced the notation

\[ \beta_j \equiv \tan^{-1}(\kappa H_0^3/2\pi H_j'), \]  

(55)
where $H_j'$ is the derivative $H'(\varphi)$ taken on the slow-roll side of the point $\varphi_0^{(j)}$.

Since $H_1' > 0$ and $H_2' < 0$, the left-hand side of Eq.(52) is a monotonically growing function of $\kappa$. This equation, therefore, has a single solution for any value of $n$. From Eq.(13) it is easily seen that the largest eigenvalue $\tilde{\gamma}_1$ corresponds to the smallest value of $\kappa^2$, and thus to $n = 1$.

For eternal inflation to be possible, we need $\tilde{\gamma}_1 > 0$, and Eq.(15) yields $\kappa^2 \geq 24\pi^2 H_0^{-2}$. Together with the condition (7) this implies $|\beta_j| \ll 1$, and from Eq.(52) with $n = 1$ we have

$$\kappa \approx \pi/2\varphi_0.$$  \hspace{1cm} (56)

The condition for eternal inflation to be possible in this model can now be written from Eqs.(45),(56),

$$\varphi_0 > H_0/4\sqrt{6}.\hspace{1cm} (57)$$

The meaning of this condition is that the size of the flat region $2\varphi_0$ should not be smaller than the size of the random walk ‘step’ $H_0/2\pi$.

The equations for the function $\psi_1(\varphi)$ are essentially identical to those for $\tilde{\psi}_1(\varphi)$. Apart from the trivial replacements $\tilde{\psi}_1 \rightarrow \psi_1$, $\tilde{c}(j) \rightarrow c(j)$, the only change is that Eq.(13) is replaced by

$$\kappa^2 = 8\pi^2 H_0^{-(\alpha+2)}\gamma_1.\hspace{1cm} (58)$$

Since the smallest eigenvalue $\gamma_1$ again corresponds to the smallest solution of Eq.(52) for $\kappa$, we conclude that the value of $\kappa$ is the same for both functions, $\psi_1(\varphi)$ and $\tilde{\psi}_1(\varphi)$. Hence,

$$\frac{\tilde{\gamma}_1}{\gamma_1} = \frac{24\pi^2}{H_0^2\kappa^2} - 1\hspace{1cm} (59)$$

and

$$c^{(2)}/c^{(1)} = \tilde{c}^{(2)}/\tilde{c}^{(1)}.\hspace{1cm} (60)$$

Now we are ready to analyze the dependence of the volume ratio $r$ in Eq.(39) on the parameter $\alpha$. This parameter does not appear in Eq.(52) for $\kappa$, and thus $\kappa$ is independent
of $\alpha$. Then it follows immediately from (53), (60) and (54) that $\tilde{\gamma}_1/\gamma_1$, $\tilde{c}^{(2)}/\tilde{c}^{(1)}$, and $c^{(2)}/c^{(1)}$ are also independent of $\alpha$. Since $p^{(j)}$ and $Z^{(j)}_*$ are generally $\alpha$-independent, we conclude that, for the model considered in this Section, the ratio $r$ is completely insensitive to the choice of time parametrization.

This conclusion may seem somewhat surprising, but in fact it is not difficult to understand. A special property of our model is that there is a one-to-one correspondence between the time $t$ when a co-moving observer reaches $\varphi = \varphi_*^{(j)}$ and the expansion factor $a$ along her worldline at that time,

$$a = \exp[H_0^\alpha (t - t_*^{(j)})] Z_*^{(j)}.$$  

(61)

For this reason, we can use a time coordinate with any value of $\alpha$ as a cutoff variable, and the result will be the same as with the scale factor cutoff.

**VII. DENSITY FLUCTUATIONS**

Let us now consider the spectrum of density perturbations detected by a ‘typical’ observer. These perturbations are determined by quantum fluctuations of the inflaton field $\varphi$ along the observer’s worldline [21]. For an ensemble of observers at the same value of $H$, the fluctuations $\delta \varphi$ are random Gaussian variables with a distribution

$$dP_0(\delta \varphi) = (2\pi \sigma)^{-1/2} \exp\left[-\frac{(\delta \varphi)^2}{2\sigma^2}\right] d\delta \varphi,$$

(62)

where

$$\sigma = H/2\pi.$$  

(63)

On a given co-moving scale, the fluctuation is produced (that is, it becomes a part of the classical field) at the time when that length crosses the horizon, and the value of $H$ in Eq. (63) should be taken at that moment. (For the purposes of this discussion, it is sufficient to take a simple-minded view that the fluctuation occurs instantly). The gauge-invariant density fluctuation on the corresponding scale is
\[ \frac{\delta \rho}{\rho} \sim 8\pi H \delta \varphi / H', \quad (64) \]

where \( H' = dH/d\varphi \) and again all quantities are taken at horizon crossing. Averaging over the distribution (62) gives the standard result \[20\]

\[ (\delta \rho/\rho)_{\text{rms}} \sim 4H^2/|H'|. \quad (65) \]

Linde, Linde and Mezhulmian \[7\] have pointed out that the distribution (62) is not necessarily identical to the probability distribution \( dP(\delta \varphi) \) for the values of \( \delta \varphi \) detected by a ‘typical’ observer. There may be a correlation between \( \delta \varphi \) and the expansion factor \( a \), in which case observers with different values of \( \delta \varphi \) will contribute to \( dP(\delta \varphi) \) with different weights. Linde et al. arrived at a surprising conclusion that ‘typical’ fluctuations of \( \varphi \) can be much greater than the \( \text{rms} \) value (63). Here, I shall first calculate \( dP(\delta \varphi) \) using the \( \epsilon \)-cutoff procedure and then compare it with the results of Linde et al.

Fluctuations of \( \varphi \) on different length scales are statistically independent and can be treated separately. We can therefore concentrate on a single fluctuation at some value \( \varphi = \varphi_1 \) (the same for all observers), disregarding all the rest. We shall assume that this value is in the slow-roll regime.

To find the probability distribution for \( \delta \varphi \), we start with the ensemble of co-moving observers defined in Section 3. We can split it into sub-ensembles, such that the fluctuation \( \delta \varphi \) at \( \varphi = \varphi_1 \) is the same with an accuracy \( d\delta \varphi \) for all observers in each sub-ensemble. The numbers of observers in different sub-ensembles are proportional to \( dP_0(\delta \varphi) \). The probability \( dP(\delta \varphi) \) is then given by

\[ dP(\delta \varphi) \propto \sum_i a_{\ast i}^3, \quad (66) \]

where the summation is done over the observers in the corresponding sub-ensemble. (Here we consider only observers ending up in the same type of thermalized regions). The sum is regularized by discarding a fraction \( \epsilon \) of observers with the largest values of \( a_{\ast i} \). (A more careful analysis shows \[23\] that the same results are obtained if the sum is cut off using a time variable with an arbitrary value of \( \alpha \).)
When the field $\varphi$ undergoes a quantum jump $\delta \varphi$, the amount of proper time $\tau$ necessary to complete the classical rollover to $\varphi_*$ is changed by

$$\delta \tau = 4\pi \delta \varphi / H'_1, \quad (67)$$

where the subscript ‘1’ indicates that the corresponding quantity is taken at $\varphi = \varphi_1$. As a result, the volume factor $a^3$ acquires an additional factor

$$f(\delta \varphi) \equiv \exp(3H_1 \delta \tau) = \exp[12\pi(H_1/H'_1)\delta \varphi]. \quad (68)$$

Since this factor is the same for all observers of the sub-ensemble, the cutoff in the sum (66) is not affected by the fluctuation. That is, the discarded observers are the same as would be discarded for $\delta \varphi = 0$.

Apart from the overall number of observers and the magnitude of the fluctuation at $\varphi = \varphi_1$, different sub-ensembles are statistically equivalent. Hence, the probability distribution $d^\mathcal{P}(\delta \varphi)$ is proportional to

$$d^\mathcal{P}(\delta \varphi) \propto d^\mathcal{P}_0(\delta \varphi) f(\delta \varphi) \propto \exp \left[ -2\pi^2 \left( \frac{\delta \varphi - 3H_1^3}{\pi H'_1} \right)^2 \right] d(\delta \varphi). \quad (69)$$

The distribution (69) describes fluctuations with a non-zero mean value,

$$< \delta \varphi > = 3H_1^3 / \pi H'_1. \quad (70)$$

Deviations from this mean value are still Gaussian, with the dispersion

$$\sigma = [<(\delta \varphi - < \delta \varphi >)^2>]^{1/2} = H_1 / 2\pi. \quad (71)$$

According to Eq.(70), the inflaton tends to fluctuate in the direction opposite to the classical roll [see Eq.(5)]. This is easy to understand: ‘backward’ fluctuations prolong inflation and increase the expansion factor. From Eqs.(64),(70), the mean density fluctuation is

$$< \delta \rho_1 > / \rho \sim 24H_1^4 / H'_1^2. \quad (72)$$

The average density profile $< \delta \rho(x) > / \rho$ can be easily found for a given $H(\varphi)$.  

The appearance of a non-trivial average density distribution is an interesting, and in principle observable, effect. However, the magnitude of the effect is hopelessly small. From Eqs. (65) and (72) we have

\[
< \delta \rho > / (\delta \rho)_{rms} \sim 6H_1^2 / H_1' \sim (\delta \rho / \rho)_{rms} \ll 1.
\]  

(73)

On scales of astrophysical interest, \((\delta \rho / \rho)_{rms} \sim 10^{-5}\), and (assuming the observed density fluctuations are due to inflation) \(< \delta \rho > / \rho \sim 10^{-10}\).

These results are very different from those obtained by Linde et al. [7] who studied the probability distribution for \(\delta \varphi\) on equal-time surfaces, \(t = \text{const}\). Using the proper time \(\tau\) as the time coordinate, they considered an ensemble of observers who reach \(\varphi = \varphi_*\) at a given moment, with the weight \(a_3^a\) assigned to each observer. They concluded that a typical observer will detect large quantum jumps of \(\varphi\) in the direction of the classical roll. This conclusion can be qualitatively understood as follows. If I have to reach the point \(\varphi = \varphi_*\) at a given time \(\tau\) with the largest possible scale factor \(a_*\), then the winning strategy for me is to spend as much time as possible at values of \(\varphi\) where \(V(\varphi)\) is large and the expansion is fast, and then quickly rush towards \(\varphi_*\).

To make this quantitative, it is convenient to consider fluctuations that bring \(\varphi\) to a given value \(\varphi_1\). Any further fluctuations can be disregarded due to statistical independence. The evolution between \(\varphi_1\) and \(\varphi_*\) is the same for all observers. In particular, it takes the same time and gives the same expansion factor. However, prior to the fluctuation, the observers had different values of \(\varphi = \varphi_1 - \delta \varphi\).

It will also be convenient to allow a small interval \(dt\) for the time \(t_1\) at which the observers reach \(\varphi_*\). (For greater generality, I am using a time variable with an arbitrary parameter \(\alpha\)). Then we are interested in the volume-weighted probability for an observer to reach \(\varphi = \varphi_1 - \delta \varphi\) within the interval \(dt\). This is given by \(|\tilde{J}(\varphi_1 - \delta \varphi, t_1)|dt\), where \(\tilde{J}\) is the flux associated with the distribution \(\tilde{P}\) introduced in Section 5. The probability distribution for \(\delta \varphi\) is then proportional to

\[
dP(\delta \varphi) \propto |\tilde{J}(\varphi_1 - \delta \varphi, t_1)|dP_0(\delta \varphi).
\]  

(74)
In the slow-roll range of \( \varphi \),

\[
\tilde{J} \approx -(4\pi)^{-1}H^{\alpha-1}(\varphi)H'(\varphi)\tilde{\psi}_1(\varphi)\exp(\tilde{\gamma}_1 t),
\]

(75)

and we can use the approximate form of \( \tilde{\psi}_1(\varphi) \), Eq. (36). Keeping only the factors dependent on \( \delta\varphi \), we have

\[
dP(\delta\varphi) \propto \exp \left\{ -\frac{2\pi^2}{H_1^2} \left[ \delta\varphi - \frac{H_1^3}{\pi H_1'(3 - \tilde{\gamma}_1 H_1^{-\alpha})} \right]^2 \right\} d(\delta\varphi).
\]

(76)

Note the \( \alpha \)-dependence of the result.

The average value of \( \delta\varphi \) can now be written as

\[
<\delta\varphi> = \frac{H_1^3}{\pi H_1'} \left[ 3 - d \left( \frac{H_{max}}{H_1} \right)^{\alpha} \right],
\]

(77)

where the fractal dimension \( d \) is given by (43) and \( H_{max} = max\{H(\varphi)\} \). For \( \alpha = 0 \), which corresponds to \( t = \ln a \),

\[
<\delta\varphi> |_{\alpha=0} = \frac{(3 - d)H_1^3}{\pi H_1'},
\]

(78)

which is the same as our result (70), apart from the numerical coefficient. But for \( \alpha = 1 \), corresponding to the proper time \( t = \tau \), the result is very different. The expansion rate \( H_1 \) for the values of \( \varphi \) corresponding to the observable range of length scales is typically much smaller than the highest expansion rate \( H_{max} \), and thus

\[
<\delta\varphi> |_{\alpha=1} \approx -\frac{\tilde{\gamma}_1 H_1^2}{\pi H_1'},
\]

(79)

and

\[
|<\delta\varphi>/ (\delta\varphi)_{rms}|_{\alpha=1} = 2dH_{max}H_1/H_1'.
\]

(80)

This is the result of Linde et.al. If one assumes that \( d \sim 1 \) and \( H_{max} \sim 1 \), as it is often done in chaotic inflation scenario \( [8,5] \), then it follows from Eq. (3) that \( <\delta\varphi> \gg (\delta\varphi)_{rms} \) and thus \( <\delta\rho> \gg (\delta\rho)_{rms} \).
VIII. DISCUSSION

In this paper I have suggested a cutoff procedure which allows one to assign probabilities to different types of thermalized volumes in an eternally inflating universe. The main advantage of this procedure is that it is rather insensitive to the choice of the time coordinate $t$ which is used to cut off the infinite volumes. In some cases there is essentially no $t$-dependence at all. Examples are the relative probability for the two minima for a potential shown in Fig.4 (Section 6), and the probabilities for different fluctuation spectra (Section 7).

For a potential with multiple minima, we found that the main factor determining the relative probability of different minima is the slow-roll expansion factor $Z^*$ (Section 4). This factor has a very sensitive dependence on the form of the potential $V(\phi)$ in the corresponding slow-roll regions, and we expect the probabilities for different minima to be vastly different. If one finds that one minimum is more probable than another, this conclusion is not likely to be changed by a different choice of the time coordinate, even in cases where the results do show some $t$-dependence. Since this dependence is expected to be rather mild, it can affect only rare borderline cases where the two probabilities are nearly equal.

One can accept this as a genuine uncertainty of the problem and make predictions only in cases where one minimum is much more probable than the others. An alternative attitude is to assert that there is a preferred choice of the time variable which gives the ‘correct’ probabilities. If this second approach is taken, then there is, arguably, a good reason to take $\alpha = 0$ ($t = \ln a$) as the preferred choice [14,21].

The cutoff prescription proposed in this paper can also be used to compare volumes in different, disconnected, eternally inflating universes. The world view suggested by quantum cosmology is that such universes may spontaneously nucleate out of nothing, and that the
constants of Nature may take different values in different universes \[^{22}\]). The principle of mediocrity suggests that we think of ourselves as a civilization randomly picked out of an infinite number of civilizations inhabiting this metauniverse. The probability distribution for the constants may then be found by comparing the thermalized volumes in different universes. [Of course, one also has to compare the ‘human factors’ \(\nu_{\text{civ}}\), see Eq.(1)]. The volumes can be calculated along the same lines as in Section 4, except now the initial conditions are specified on the initial hypersurface at the ‘moment of nucleation’ and are determined by solving the Wheeler-DeWitt equation for the wave function of the universe.

An important difference between comparing different minima of the potential and different universes is that in the latter case the eigenvalues \(\gamma_1\) (and \(\tilde{\gamma}_1\)) are not generally the same for the two universes. Since \(V_{(j)} \propto e^{-\tilde{\gamma}_1^{(j)}/\gamma_1^{(j)}}\), the ratio \(V_{(1)}^{(j)}/V_{(2)}^{(j)}\) will either vanish or diverge in the limit \(\epsilon \to 0\), unless \(\tilde{\gamma}_1^{(1)}/\gamma_1^{(1)} = \tilde{\gamma}_1^{(2)}/\gamma_1^{(2)}\). Hence, only the constants corresponding to the largest possible value of \(\tilde{\gamma}_1/\gamma_1\) will have a non-zero probability. If the condition \(\tilde{\gamma}_1/\gamma_1 = \max\) does not determine all the constants of Nature, then the probability distribution for the remaining constants can be found by considering the \(\epsilon\)-independent factors in \(V_{(j)}^{(j)}\) (such as \([Z_{(j)}^{(j)}])^3\). These ideas were briefly outlined in Ref. \[^{5}\)] and will be discussed in detail elsewhere.

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FIGURES

FIG. 1.

Scalar field potential $V(\varphi)$ with two minima and two slow-roll regions. The field values $\varphi_1^*$ and $\varphi_2^*$ indicate the end of inflation. In the region between $\varphi_0^{(1)}$ and $\varphi_0^{(2)}$ the field dynamics is dominated by quantum fluctuations, and the slow-roll regions are bounded by $\varphi_0^{(j)}$ and $\varphi_{\ast}^{(j)}$.

FIG. 2.

A slice through the spacetime of an eternally inflating universe. Thermalized regions of two different types are shown by different shades of grey.

FIG. 3.

A congruence of ‘observers’ ending up in different types of thermalized regions. The initial volume $\mathcal{V}_0$ is shown by a horizontal line, and the observer’s wordlines by vertical lines.

FIG. 4.

A potential with a flat central regions bordering with two slow-roll regions.
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