Reducing residual normal moveout by globally optimized generalized moveout approximation in vertically transverse isotropy (VTI) media

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Abstract. The anisotropic phenomenon in wave propagation has been widely recognized on various scales. Knowledge of anisotropic effects is essential for interpretation and processing of seismic data. The existence of anisotropy leads to the moveout of nonhyperbolic even in a homogenous layer. A transversely isotropic medium with a vertical symmetry axis (VTI) is a reasonable approximation of horizontally layered anisotropic medium. The approximation of travel time is important for reducing the residual normal moveout of layered VTI media. The aim of this study is to develop globally optimized generalized moveout approximation in reducing residual normal moveout in VTI media. A comparative analysis was carried out to recent method for given an ellipticity parameter (0 ≤ η ≤ 0.5) and wide offset depth to ratio (0 to 4). The result shows that the globally optimized generalized moveout approximation is better in reducing residual moveout at large offsets with a stronger anellipticity parameter than existing methods. This is essential for reducing the accumulation of error especially for deeper substructures.

Keywords: Seismic anisotropy, VTI media, generalized moveout approximation, residual moveout

1. Introduction

Seismic anisotropy is explained as the seismic velocity depending on angle. The phenomenon of anisotropy in wave propagation has been widely recognized on various scale. Knowledge of anisotropy is completely crucial in processing and interpretation of seismic data. Pre-stack seismic data which consist of near, mid, and far offsets often have problems, especially at far offsets [1]. In anisotropic media, seismic data face two problems that are in shallow layers and far offsets. The phenomena of anisotropy are due to high level of heterogeneity in shallow layers, while those occur due to farther wave propagation at far offsets so that seismic waves propagate on any kind of lithology. The existence of anisotropy generates another matter which is hockey stick effect. It is an event of seismic that curves like hockey stick shape. The pattern of hockey stick effect cannot be maximally reduced by using the moveout approximation of hyperbolic travel time [2]. However, it needs nonhyperbolic travel time approximation to solve the problem. By using hyperbolic
approximation, the hockey stick effects will appear at far offsets. Practically, those will be muted so that many kinds of information are lost, such as lithology.

The existence of anisotropy leads to the nonhyperbolic moveout even in a homogeneous layer. A transversely isotropic medium with a vertical symmetry axis (VTI) is a reasonable approximation of horizontally layered anisotropic medium. The approximation of travel time is important for reducing the residual normal moveout of layered VTI media. Tsvankin and Thomsen [3] modified the three-term equation [4] by including the coefficients of fourth-order Taylor series for long offset in VTI media. Alkhalifah and Tsvankin [5] introduced the anellipticity parameter $\eta$ and normal moveout velocity $v_{nmo}$ in the three-term equation. Fomel and Stovas [6] proposed the generalized nonhyperbolic approximation which can be applied in various media. Song, Jang, and Yao [7] proposed the globally optimized generalized moveout approximation (GMA) for long offset by extending and optimizing the coefficients of generalized nonhyperbolic approximation [6].

In this paper, we apply the globally optimized generalized moveout approximation [7], hyperbolic [2], Alkhalifah and Tsvankin [5], and Fomel and Stovas [6] methods to calculate residual moveout using synthetic and real data, and compare them to show the best accuracy of travel time at far offset. The level of accuracy can be seen from the smallest residual moveout at big offset to depth ratio (ODR). The greater ODR value shows the further offset. ODR is a parameter that shows the heterogeneity of layer. The smallest value of residual moveout for bigger ODR indicates that the method can solve the problem at far offset, such as hockey stick effect.

2. Theory

2.1. Exact travel time for single VTI layer

We are considering the propagation of qP wave in a homogeneous VTI layer. It is fine to define the non-dimensional parameters of anisotropy in elastic tensor component ($c$) terms [8].

$$a_0 = \sqrt{\frac{c_{55}}{\rho}}, \quad \beta_0 = \sqrt{\frac{c_{33}}{\rho}}, \quad \varepsilon = \frac{c_{11} - c_{33}}{2c_{33}},
$$

$$\delta = \frac{(c_{13} + c_{55})^2 - (c_{33} - c_{55})^2}{2c_{33}(c_{13} - c_{55})}$$

(1)

where $a_0$ and $\beta_0$ are the vertical velocities of qP and qS waves, respectively. $\varepsilon$ and $\delta$ are the nondimensional anisotropy parameters, respectively. The $1 \times 1$, $3 \times 3$, $5 \times 5$ elastic modulus tensor $c_{ij}$ absolutely characterizes the elasticity of medium. The exact model of ray tracing qP, qS uses anisotropic ray tracing model [9].

2.2. Globally Optimized generalized moveout approximation

The previous researchers proposed travel time equation of wave propagation which is commonly used in isotropic media. It can be stated as conventional method because it assumes that the physical properties, such as velocity, density, and others, are homogenous isotropy in the subsurface. The hyperbolic method can be written as equation 2 [2].

$$t^2 = t_0^2 + \frac{x^2}{v_{nmo}^2}$$

(2)
Ursin and Stovas [10] proposed normalized travel time as:

\[
\tau = \frac{t}{t_0}
\]  

and normalized offset,

\[
x = \frac{X}{t_0 v_{nmo}}
\]  

where \( t_0 \) is whole vertical travel time defined as \( t_0 = \sum t_i v_{nmo} \) is the effective normal moveout (NMO) velocity expressed as [11],

\[
v_{nmo} = \frac{\sum v_{nmo}^2 t_i}{t_0}
\]  

and \( v_{nmo} \) is an interval of NMO velocity defined as \( v_{nmo} = a_0 \sqrt{1 + 2\delta} \).

Fomel and Stovas [6] proposed a general form of nonhyperbolic moveout method which is generalized nonhyperbolic moveout approximation (GMA). In vertically transverse isotropy (VTI) medium, the normalized Fomel and Stovas can be written as,

\[
\tau^2(x) = 1 + x^2 - \frac{4\eta x^4}{1 + 1 + 8\eta + 8\eta^2 x^2 + \sqrt{1 + 2(1 + 8\eta + 8\eta^2) x^2 + \frac{x^4}{(1 + 2\eta)^2}}}
\]

where \( \eta \) is an ellipticity expressed as [11],

\[
\eta = \frac{\epsilon - \delta}{(1 + 2\delta)}
\]

for a multi-layer VTI model, the anisotropic parameter \( \eta \) is replaced by an effective anisotropic parameter \( (\eta_{\text{eff}}) \) [11] that is expressed as,

\[
\eta_{\text{eff}} = \frac{\sum v_{\text{NMO}}^4 (1 + 8\eta)t_i - t_0 v_{\text{NMO}}^4}{8t_0 v_{\text{NMO}}^4}
\]

The globally optimized generalized moveout approximation [7] as follow.

\[
\tau^2(x) = 1 + x^2 - \frac{A\eta x^4}{B + C + D\eta + E\eta^2 x^2 + \sqrt{G + 2(C + D\eta + E\eta^2) x^2 + \frac{Hx^4}{(1 + \eta)^2}}}
\]

In the optimization process, the constant coefficients of Fomel and Stovas [6] are replaced by letters A~I which is extended coefficients already to be optimized for varying values of \( \eta \) and ODR. The globally optimized generalized moveout approximation [7] can be reduced to the current form (equation 6) when \( 2A=D=E=4F=8 \) dan \( C=1 \). The optimization process of constant coefficients is done by using simulated annealing (SA) algorithm [5]. After many numerical experiments, the constant coefficients are obtained as shown in table 1 [7].
The objective function for globally optimized GMA uses simulated annealing (SA) algorithm as follows [7].

\[ E^{SA}(A-I) = \max_{\mathcal{N}_{\eta \in \mathbb{N}}} \left( \frac{|\tau_a - \tau_e|}{\tau_e} \right) \leq \sigma \]  

(10)

where \( \tau_a \) is normalized travel time obtained from Fomel and Stovas method [4] and \( \tau_e \) is normalized exact travel time. The interval values of \( \eta \) and O/D or ODR just indicate the limitation of value used for modeling, while \( \sigma \) is the limitation of error.

3. Methodology

The method of this study is modelling of a single VTI layer using the globally optimized generalized moveout approximation to show reducing residual. The workflow of this study can be shown in figure 1.

The initial model of the VTI single layer was made using synthetic data. The model consists of two layers shown as figure 2 which has depth (h) = 1000 m, offset (x) = 4000 m, anisotropic parameter \( \eta = 0.5 \), and ODR \( \leq 4 \), respectively. The value of ODR is obtained from the comparison between offset and depth (x/h). The initial model can be shown in figure 2.

VTI media is transverse isotropy that has a vertical axis of rotational symmetry. In layered rocks, properties are uniform horizontally, but vary vertically and from layer to layer. In this case, the velocity of seismic wave is faster to the horizontal than to vertical axis. The model above assumes that the first layer is shale and second layer is sand, respectively. The created model is according to real condition of the earth subsurface. The effect of anisotropy is caused by any shales which make the value of seismic velocity will be different. The difference of velocity indicates that the media are anisotropy.

Using anisotropic ray tracing method [9] as an exact model, we do comparison traveltime approximation by using hyperbolic [2], Alkhalifah and Tsvankin [5], Fomel and Stovas [6], and the globally optimized generalized moveout approximation [7] to show the accuracy of them to exact travel time model at far offset.

4. Results and discussion

4.1. Synthetic data

The calculation of residual moveout was done by varying the value of anisotropic parameter eta (\( \eta \)) from 0 to 0.5 and ODR from 0 to 4 using the globally optimized generalized moveout approximation (GMA), hyperbolic, Alkhalifah and Tsvankin and Fomel and Stovas methods-Eta (\( \eta \)) is an ellipticity parameter which shows the anisotropy level of rocks. The small value of eta indicates the strongly anisotropic rocks, whereas the big value of eta shows that the weak anisotropy. ODR is a parameter that captures the variation of lithology propagated by the seismic wave. The bigger ODR means that the further offset. It indicates that the bigger ODR can capture much more various lithology. The comparison is shown in figure 3.

Figure 3 illustrates the residual moveout at each ODR with the varying value of \( \eta \). We can see that residual moveout of hyperbolic method is significantly increasing along with increasing of \( \eta \) value at all of ODR because there is no calculation of \( \eta \) according to equation 2 [2].
Table 1. The value of constant coefficient before and after optimization.

| Coefficients | Before optimization | After optimization (SA) |
|--------------|----------------------|-------------------------|
| A            | 4                    | 3.503                   |
| B            | 1                    | 1                       |
| C            | 1                    | 0.688                   |
| D            | 8                    | 7.432                   |
| E            | 8                    | 8.931                   |
| F            | 2                    | 2.436                   |
| G            | 1                    | 0.577                   |
| H            | 1                    | 1.284                   |
| I            | 2                    | 2.237                   |

Figure 1. Flowchart of methodology.

Alkhalifah and Tsvankin et al. method has small residual moveout at the ranges of $\eta$ values from 0 to 1.5, but it starts increasing when the $\eta$ is more than 1.5 at ODR $\leq$ 1. However, at ODR $\leq$ 2, 3 and 4, respectively, Alkhalifah and Tsvankin et al. method is more significantly increasing of residual moveout along with increasing of ODR and $\eta$ values. Fomel and Stovas method has smaller residual moveout than the previous ones [3, 11]. However, the globally optimized GMA is the smallest residual with varying $\eta$ value at all of ODR. Its residual moveout is nearly 0 %. The smallest value of residual moveout indicates the best residual reduction method. It shows that globally optimized GMA is the best solution to reduce residual moveout, especially for deeper layer.

4.2. Real data

Real data used in this research are obtained from laboratory data [12]. In this case, we just take three samples of the data, which are Africa shales, Africa sands, and Canadian carbonates as shown in table 2 to see the comparison of residual moveout for all methods at far offset considered by offset to
depth ratio (ODR). The maximum offset used is 4000 m. The depth value is depending on depth for each lithology so that the value of ODR will be different for different lithology, as shown in table 2.

The calculation result of residual moveout is shown in table 3. The all methods are used to calculate the residual reduction values to show the best method which has the smallest value of residual reduction for different lithology.

![VTI single layer model](image)

**Figure 2.** VTI single layer model.

![Comparison between residual moveout of hyperbolic, Alkhalifah and Tsvankin, Fomel and Stovas and Globally optimized GMA by varying value of η ranges from 0 to 0.5 at (a) ODR=1, (b) ODR=2, (c) ODR=3, and (d) ODR=4.](image)
Table 2. The value of anisotropic parameters at different lithology.

| Sample            | Depth | \( \varepsilon \) | \( \epsilon \) | \( \eta \) |
|-------------------|-------|-------------------|----------------|---------|
| Africa shales     | 2484.1| 0.317             | -0.054         | 0.0417  |
| Africa sands      | 2485.0| -0.005            | 0.006          | -0.010  |
| Canadian carbonates| 4043.1| 0.019             | 0.067          | -0.043  |

Table 3. The result of reducing residual moveout values for different lithology.

| Lithology            | ODR | Globally Optimized GMA | Residual moveout (%) | Hyperbolic |
|----------------------|-----|------------------------|----------------------|------------|
|                      |     |                        |                      |            |
| Africa shales        | 1.61| 0.000901               | 0.135587             | 1.89428    | 7.9528     |
| Africa sands         | 1.61| 0.0000498              | 0.00000827           | 0.003323   | 0.300865   |
| Canadian carbonates  | 0.98| 0.00019271             | 0.00040565           | 0.0197386  | 0.49500708 |

Table 3 shows the comparison results of reducing residual moveout for different lithologies, such as shale, sand, and carbonate. We can see that the residual moveout of the globally optimized GMA is mostly the smallest, even Fomel and Stovas method has the smallest residual in sands which is 0.00000827 %. However, the globally optimized generalized moveout approximation has the smallest residual moveout overall. This is because of any extension of the total number of optimized coefficients in equation (7) that is important for decreasing the residual moveout values. The smallest percentage of residual moveout value indicates the more precise the offset (zero offsets). It can be said that the globally optimized GMA is better in reducing residual moveout than the other methods.

5. Conclusion
The anellipticity parameter analysis is completely sensitive to the traveltime. We perform the globally optimized generalized moveout approximation of travel time in VTI media with strong anellipticity parameter. Numerical experiments using synthetic and real data show that the globally optimized generalized moveout approximation can handle a much larger offset and a stronger anellipticity parameter than most existing methods. The globally optimized generalized moveout approximation has a much smaller residual moveout value within the most generally used anisotropic parameter range (0 \( \leq \eta \leq 0.5 \)) and offset to depth ratio (0 to 4), that is essential for reducing the accumulation of error, especially for deeper substructures.

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