Synchronization Problem of a Novel Fractal-Fractional Orders’ Hyperchaotic Finance System

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This paper investigates the synchronization problem of a novel fractal-fractional (FF) orders’ hyperchaotic finance system with model uncertainty and external disturbance. Firstly, a controller is designed to realize the synchronization of the nominal FF-orders’ hyperchaotic finance system. Secondly, a suitable filter is designed to estimate uncertainty and disturbance, and then, the uncertainty and disturbance estimator- (UDE-) based control method is proposed to realize the synchronization problem of such system. Finally, numerical simulations are carried out to verify the correctness and the effectiveness of the obtained results.

1. Introduction

The fractional calculus was introduced in 1695, and it is the generalization of integer-order calculus. Fractional calculus is a research hotspot in many scientific fields, especially in mathematics and engineering. Different to the integer-order calculus, fractional derivatives can describe long-term memory, as detailed in [1–6]. Chaotic motion is an advanced form of complex motion. Its most important characteristic is its high sensitivity to initial values, that is, small differences in initial values will lead to huge differences in system states. Since Lorenz proposed the first chaotic system in 1963, many researchers have begun to study this chaotic phenomenon. Over the past few decades, chaos and fractals have been treated differently for different purposes. Chaotic theory was introduced to capture multifaceted systems that exhibit impulsive randomness and are very sensitive to small changes in conditions. Fractals are created to replicate infinitely complex patterns that are self-similar at different scales. In recent years, the FF-orders’ problem has been expressed in [7–18]. The results show that the FF-order model is more suitable for practical problems than the integer-order model. In recent years, the synchronization of fractional-order chaotic systems has attracted great attention, and various control methods have been proposed, such as adaptive control [19, 20], active control [21], passive control [22], and sliding model control [23]. Although scholars have made great efforts in the control of fractional-order chaotic systems, there are still many challenges and problems to be solved. For example, the uncertainty of the system has not taken into account the control channels and control technologies designed in many controllers and control combinations [24–26]. It is well known that chaotic systems are very sensitive to parametric and external perturbations. Therefore, it is difficult to synchronize chaotic systems with parametric perturbations and external perturbations. Fortunately, some work has been done on the synchronization problem of integer-order chaotic systems with parametric and external perturbations. But, the results of synchronization research for chaotic systems with model uncertainty and external disturbance have some limitations, such as model uncertainties and external perturbations are assumed to be bounded, and these bounds are usually small. Moreover, the obtained method is based on linear matrix inequality (LMI) tools, thus the obtained results are conservative in some sense. Recently, the UDE-based control method has shown some advantages over the aforementioned results, see [27–34]. Therefore, we shall apply the
existing UDE-based control method to study the synchronization problem of the FF hyperchaotic finance system. Inspired by the above discussion, we consider the newly defined FF-operators of fractional calculus, which are defined in the Caputo sense. In this paper, we investigate the synchronization of the FF hyperchaotic finance system with model uncertainty and external disturbance and propose a new UDE-based control method to realize the synchronization of the FF hyperchaotic finance system. Numerical simulations are carried out to verify the effectiveness and validity of the obtained theoretical results.

2. Preliminaries and Problem Formation

2.1. Preliminaries. Firstly, we introduce the definition of the FF-order differential equation in the Caputo sense and some preliminaries of fractional-order chaotic systems.

Consider the following FF-order differential equation in the Caputo sense:

\[ C D_t^{\alpha,\beta} f(x) = \xi f(x), \quad n - 1 < \alpha \text{ and } \beta \leq n \]  \hspace{1cm} (1)

**Definition 1** (see [13]). Let \( f(x) \) be differentiable in opened interval \((a, b)\); if \( f(x) \) is fractal differentiable on \((a, b)\) with order \( \beta \), then the FF-derivative of \( f(x) \) of order \( \alpha \) in the Caputo sense with the power law is given as

\[ C D_t^{\alpha,\beta} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{n-\alpha-1} \frac{d^n f(\tau)}{d\tau^n}|_{\tau=\tau} d\tau. \]  \hspace{1cm} (2)

Then, some properties of fractional calculus are introduced.

**Property 1** (see [35]). The fractional-order calculus defined by Caputo is a linear operator and satisfies

\[ D_t^n (\lambda f(t) + \mu g(t)) = \lambda D_t^{\alpha,\beta} f(t) + \mu D_t^{\alpha,\beta} g(t). \]  \hspace{1cm} (3)

**Proof.** where \( \lambda \) and \( \mu \) are real constants. \( \square \)

**Property 2** (see [36]). For fractional-order nonlinear system (1), \( f(x) \) meets the following Lipschitz condition:

\[ \| f(y) - f(x) \| \leq L \| y - x \|. \]  \hspace{1cm} (4)

**Proof.** where \( \cdot \) is an \( \infty \)-norm and \( L \) is a positive real number. \( \square \)

**Property 3** (see [36]). Let \( x \in R \) be a continuous differentiable function, and for any continuous time \( t \geq t_0 \), i.e.,

\[ \frac{1}{2} D_t^\alpha x^2 \leq x D_t^\alpha x, \quad 0 < \alpha < 1. \]  \hspace{1cm} (5)

2.2. Problem Formation. The FF hyperchaotic finance system is given in the following form:

\[ C D_t^{\alpha,\beta} x = f(x) + u_d + Bu, \]  \hspace{1cm} (6)

where \( x \in R^4 \) is the state and \( u_d = \Delta f(x) + d(t) \) is the uncertainty and disturbance, i.e.,

\[ f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix} = \begin{bmatrix} x_3 + (x_2 - 0.9)x_1 + x_4 \\ 1 - 0.1x_2 - x_1^2 \\ -x_1 - x_3 \\ -0.05x_1x_3 - 0.6x_4 \end{bmatrix}, \]

\[ B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \]

\[ \Delta f(x) = \begin{bmatrix} 0.03x_1x_3 \\ 0 \\ 0 \\ 0.2x_1x_3 \end{bmatrix}, \]

\[ d(t) = \begin{bmatrix} 1000 \\ 0 \\ 0 \\ 500 \end{bmatrix}, \]  \hspace{1cm} (7)

or

\[ d(t) = \begin{bmatrix} \sin(t) \\ 0 \\ 0 \\ 3 \sin(t) \end{bmatrix}, \]  \hspace{1cm} (8)

where \( u \) is the controller to be designed.

Let system (6) be the master system; then, the corresponding slave system is

\[ C D_t^{\alpha,\beta} y = f(y), \]  \hspace{1cm} (9)
where

\[
y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix},
\]

\[
f (y) = \begin{pmatrix} f_1 (y) \\ f_2 (y) \\ f_3 (y) \\ f_4 (y) \end{pmatrix} = \begin{pmatrix} y_3 + (y_2 - 0.9)y_1 + y_4 \\ 1 - 0.1y_2 - y_1^2 \\ -y_1 - y_3 \\ -0.05y_1y_3 - 0.6y_4 \end{pmatrix}.
\]  

(10)

The error system \( e = x - y \) is presented as

\[
\begin{aligned}
C D_t^{\alpha \beta} e &= f (x) - f (y) + u_d + Bu,
\end{aligned}
\]  

(11)

where \( e \in R^4 \) is the state, \( u_d \) and \( B \) are given in equation (6), and

\[
f (x) - f (y) = \begin{pmatrix} e_3 - 0.9e_1 + e_4 - e_1e_2 + x_2e_1 + x_1e_2 \\ -0.1e_2 - 2x_1e_1 + e_1^2 \\ -e_1 - e_3 \\ 0.05e_1e_3 - 0.05x_3e_1 - 0.05x_1e_3 - 0.6e_4 \end{pmatrix}.
\]  

(12)

The main goal of this paper is to design a controller \( u \) to meet the following performance:

\[
\lim_{t \to \infty} \| e (t) \| = 0.
\]  

(13)

section 3. Main Results

The stabilization of error system (11) with \( u_d = 0 \) is firstly stabilized by the controller \( u_4 \), and a conclusion is obtained as follows.

**Theorem 1.** Consider error system (11) with \( u_d = 0 \). If \( (f (x) - f (y), B) \) can be stabilized, then the controller \( u_4 \) is designed as

\[
u_i = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} -e_4 - x_4e_1 + x_1e_2 \\ 0 \\ 0 \\ -0.05e_1e_3 + 0.05x_3e_1 + 0.05x_1e_3 \end{pmatrix}.
\]  

(14)

Proof. Define the following nonnegative function:

\[
V = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2 + e_4^2).
\]  

(15)

From Property 1, we get

\[
C D_t^{\alpha \beta} V = \frac{1}{2} C D_t^{\alpha \beta} e_1^2 + \frac{1}{2} C D_t^{\alpha \beta} e_2^2 + \frac{1}{2} C D_t^{\alpha \beta} e_3^2 + \frac{1}{2} C D_t^{\alpha \beta} e_4^2.
\]  

(16)

From Property 3, it results in

\[
\begin{aligned}
&\frac{1}{2} C D_t^{\alpha \beta} e_1^2 \leq e_1 C D_t^{\alpha \beta} e_1 = e_1 (e_3 - 0.9e_1 + e_4 - e_1e_2 + x_2e_1 + x_1e_2 + u_1), \\
&\frac{1}{2} C D_t^{\alpha \beta} e_2^2 \leq e_2 C D_t^{\alpha \beta} e_2 = e_2 (-0.1e_2 - 2x_1e_1 + e_1^2), \\
&\frac{1}{2} C D_t^{\alpha \beta} e_3^2 \leq e_3 C D_t^{\alpha \beta} e_3 = e_3 (-e_1 - e_3), \\
&\frac{1}{2} C D_t^{\alpha \beta} e_4^2 \leq e_4 C D_t^{\alpha \beta} e_4 = e_4 (0.05e_1e_3 - 0.05x_3e_1 - 0.05x_1e_3 - 0.6e_4 + u_4).
\end{aligned}
\]  

(17)
Calculating the Caputo derivative of $V$ along the system in equation (15):

\[
C_{D_t}^{\alpha_2}V = \frac{1}{2}C_{D_t}^{\alpha_2}(e_1^2 + e_2^2 + e_3^2 + e_4^2)
\]

\[
= \frac{1}{2}C_{D_t}^{\alpha_2}e_1^2 + \frac{1}{2}C_{D_t}^{\alpha_2}e_2^2 + \frac{1}{2}C_{D_t}^{\alpha_2}e_3^2 + \frac{1}{2}C_{D_t}^{\alpha_2}e_4^2
\]

\[
\leq e_1 C_{D_t}^{\alpha_2}e_1 + e_2 C_{D_t}^{\alpha_2}e_2 + e_3 C_{D_t}^{\alpha_2}e_3 + e_4 C_{D_t}^{\alpha_2}e_4
\]

\[
= e_1(e_3 - 0.9e_1 + e_4 - e_1e_2 + x_2e_1 + x_1e_2 + u_1) + e_2(-0.1e_2 - 2x_1e_1 + e_1^3)
\]

\[
+ e_3(-e_1 - e_3) + e_4(0.05e_1e_3 - 0.05x_1e_3 - 0.05x_1e_3 - 0.6e_4 + u_4)
\]

\[
= e_1(e_3 - 0.9e_1 + e_4 - e_1e_2 + x_2e_1 + x_1e_2) + e_1(-e_4 - x_2e_1 + x_1e_2)
\]

\[
+ e_3(-0.1e_2 - 2x_1e_1 + e_1^3 + e_3(-e_1 - e_3))
\]

\[
+ e_4(0.05e_1e_3 - 0.05x_1e_3 - 0.05x_1e_3 - 0.6e_4 + e_4(-0.05e_1e_3 + 0.05x_1e_3 + 0.05x_1e_3) = -ae_1^2 - be_2^2 - ce_3^2 - de_4^2 \leq 0.
\]

\[
(18)
\]

Therefore, master system (6) with $u_d = 0$ synchronizes slave system (9) by the controller $u_s$.

Then, error system (11) is stabilized, and a result is presented as follows. \hfill $\blacksquare$

**Theorem 2.** Consider error system (11). If $(f(x) - f(y), B)$ can be stabilized and there exists a suitable filter $g_f(t)$ such that

\[
\bar{u}_d = u_d - \bar{u}_d \rightarrow 0, \quad t \rightarrow \infty, \quad (19)
\]

where

\[
\bar{u}_d = u_d - \bar{u}_d \rightarrow 0, \quad t \rightarrow \infty.
\]

Therefore, \( \bar{u}_d \rightarrow 0 \) as \( t \rightarrow \infty \).

Proof. Substituting the controller $u$ given in equation (22) into error system (11), we obtain

\[
D_t^\alpha e(t) = F(x, e) + u_d + Bu_s,
\]

where $F(x, e) = f(x) - f(y) + Bu_s$. Note that

\[
F(x, e) = f(x) - f(y) + Bu_s,
\]

and the system $D_t^\alpha e(t) = F(x, e)$ is asymptotically stable according to Theorem 1.

According to condition given in equation (19), if the controller $u_{ide}$ meets the following equation

\[
Bu_{ide} = -\bar{u}_d = -u_d - g_f(t)
\]

\[
= -(D_t^\alpha e(t) - F(x, e) - Bu_{ide}) * g_f(t),
\]

then this controller is proposed.

Taking the Laplace transform of both sides of equation (26), it yields that

\[
\bar{u}_d = u_d * g_f(t) = (D_t^\alpha e - F(x, e) - Bu_{ide}) * g_f(t),
\]

and $u_d$ satisfies the following structural constraints:

\[
[I_n - BB^*]u_d \equiv 0,
\]

where $I_n$ is the identity matrix of order $n$; then, the UDE-based controller $u$ is designed as

\[
u = u_s + u_{ide},
\]

where $u_s$ is given in equation (15), and

\[
u = u_s + u_{ide},
\]

where $u_s$ is given in equation (15), and
\[
Bu_{t,de}(s) = -s^\alpha e(s)G_f(s) + F(s)G_f(s) + Bu_{t,de}(s)G_f(s),
\]
\[(27)\]

\[
u_{t,de}(s) = B^\top \left\{ G_f(s) \left[ \frac{1}{1 - G_f(s)} \right] * F(x, e) \right\} - B^\top \left\{ e(s)G_f(s) \left[ \frac{1}{1 - G_f(s)} \right] * e(t) \right\},
\]
\[(29)\]

\[
u_{t,de}(t) = B^\top \left\{ e(s)G_f(s) \left[ \frac{1}{1 - G_f(s)} \right] * e(t) \right\},
\]
\[(30)\]

0 < \alpha \leq 1, which completes the proof.

4. Numerical Simulations

In this section, we use MATLAB to do the numerical simulation of the FF hyperchaotic finance system in the sense of Caputo. Firstly, the numerical simulation of the nominal FF hyperchaotic finance system is carried out. Then, the numerical simulation of the FF hyperchaotic finance system with model uncertainty and external disturbance is carried out. Noted that external disturbance is \( d(t) \) two cases are presented as follows.

When the external disturbance \( d(t) \) is constant, i.e.,
\[
d(t) = \begin{pmatrix} 1000 \\ 0 \\ 0 \\ 500 \end{pmatrix},
\]
\[(31)\]

4.2. Numerical Simulation of the FF Hyperchaotic Finance System with Model Uncertainty and External Disturbance. Numerical simulation of the FF hyperchaotic finance system with model uncertainty and external disturbance is carried out. Noted that external disturbance is \( d(t) \) two cases are presented as follows.

When the external disturbance \( d(t) \) is constant, i.e.,
\[
d(t) = \begin{pmatrix} 1000 \\ 0 \\ 0 \\ 500 \end{pmatrix},
\]
\[(31)\]
Figure 2: The states of master system (6) and slave system (9).

Figure 3: Error system (11) is also asymptotically stable.

Figure 4: The states of master system (6) and slave system (9).
that $\tilde{u}_{d1}$ asymptotically tends to $u_{d1}$. Figure 6 demonstrates that $\tilde{u}_{d2}$ asymptotically tends to $u_{d2}$.

The other case $d(t)$ is given as follows:

$$
d(t) = \begin{pmatrix}
  \sin t \\
  0 \\
  0 \\
  3 \sin t
\end{pmatrix}.
$$

Numerical simulation is carried out with the initial conditions of master system (6) and slave system (9): $x(0) = [5, 2, 4, -1]^T$ and $y(0) = [3, 5, -1, 1]^T$, respectively, and $\alpha = 0.95$, and $\beta = 1$. Figure 7 displays that error system (11) is also asymptotically stable, which implies that master system (6) and slave system (9) reach complete synchronization. The states of master system (6) and slave system (9) are shown in Figure 8, respectively. Figure 9 demonstrates
Figure 7: Error system (11) is also asymptotically stable.

Figure 8: The states of master system (6) and slave system (9).

Figure 9: \( \hat{u}_{d1} \) asymptotically tends to \( u_{d1} \).
that $\hat{u}_{d1}$ asymptotically tends to $u_{d1}$. Figure 10 demonstrates that $\hat{u}_{d2}$ asymptotically tends to $u_{d2}$.

5. Conclusions

In conclusion, the synchronization of the FF hyperchaotic finance system with model uncertainty and external disturbances has been investigated. Firstly, a controller has been proposed for the nominal FF hyperchaotic finance system. Then, the UDE-based controller has been designed for the FF hyperchaotic finance system. The correctness and validity of the obtained results have been verified by numerical simulation. It is noted that the simulation results show that the aforementioned control method has good performance.

In the future, the obtained control method and the synchronization result are maybe extended to some potential applications, such as the nonlinear digital communication.

Data Availability

No data were used in this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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