**Quadratic Mixing of Radio Frequency Signals using Superconducting Quantum Interference Filters**

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The authors demonstrate quadratic mixing of weak time harmonic electromagnetic fields applied to Superconducting Quantum Interference Filters, manufactured from high-$T_c$ grain boundary Josephson junctions and operated in active microcooler. The authors use the parabolic shape of the dip in the dc-voltage output around $B = 0$ to mix quadratically two external rf-signals, at frequencies $f_1$ and $f_2$ well below the Josephson frequency $f_J$, and detect the corresponding mixing signal at $|f_1 - f_2|$. Quadratic mixing takes also place when the SQIF is operated without magnetic shield. The experimental results are well described by a simple analytical model based on the adiabatic approximation.

Superconducting Quantum Interference Filters (SQIFs) are Josephson junction (JJ) interferometers that sense, when operated in the resistive mode, the presence of a magnetic field $B$ by transforming it to a characteristic dc voltage output $V_{dc}(B)$. Upon sweeping a control current through a coil to generate a suitable compensation field, the voltage output of a SQIF shows a pronounced unique dip around zero total static field $B = 0$. The dip of a SQIF is characterized by its width $\Delta B$ and its voltage swing $\Delta V$. The detailed shape of the voltage output $V_{dc}(B)$ can be engineered by choosing suitable areas of the SQIF loops. The performance of a SQIF is only weakly sensitive to the spread of the JJ parameters. These features make SQIFs interesting for applications with ceramic cuprate superconductors. The sensitivity of a SQIF in response to an applied magnetic field is determined by the voltage-to-magnetic field transfer factor $V_B = \max(\partial V_{dc}/\partial B)$. For example, in a serial SQIF array with $N$ loops the transfer factor $V_B$ scales with $N$, but the voltage noise $\sqrt{V_N}$ derived from the spectral density $S_V$ (white noise) scales with $\sqrt{N}$, so that the dynamic range $\Delta V/\sqrt{V_N}$ varies proportional to $\sqrt{N}$. Employing various flux focusing structures together with a SQIF it is possible to significantly enhance the transfer factor. A flux focusing structure integrated together with a SQIF (SQIF-sensor) is sketched in Fig. 1(c).

In the resistive state, the working point of the device is set by a bias current $I_B$ and also by a static field $B$. The dc voltage output $V_{dc}(B)$ of the SQIF is the time average of a fast signal $V(B,t) \propto \partial \varphi/\partial t$, where $\varphi$ is the Josephson phase across the JJ. The main harmonic of $V(B,t)$ has frequency $f_J = V_{dc}(B)/\Phi_0$ (the second Josephson relation). In the presence of additional weak radiofrequency field $b_{rf}(t)$ which varies slowly in comparison with $f_J$, the SQIF voltage can be written as

$$\langle V(B + b_{rf}(t), t) \rangle = V_{dc}(B) + V_{dc}''(B) \cdot b_{rf}(t) + \frac{1}{2} V_{dc}'''(B) \cdot b_{rf}^2(t) + \ldots,$$

where prime denotes the derivative with respect to $B$, and $\langle \rangle$ the time average over the time scale set by $f_J^{-1}$, while the slow time dependence remains. For $b_{rf}(t) = b_1 \cos(2\pi f_1 t)$, with amplitude $b_1 \ll \Delta B$ and frequency $f_1 < \frac{f_J}{2}$, Eq. (1) describes a superposition of the dc voltage output $V_{dc}(B)$ with a slowly varying rf voltage output at frequencies $f_1, 2f_1$, etc. The amplitude of the first harmonic of the Fourier spectrum of the output signal contains information about the slope $V_{dc}''(B)$, while the amplitude of the second harmonic is proportional to the curvature $V_{dc}'''(B)$.

The Fourier transform of $V(B + b_{rf}(t), t)$ is the spectral voltage output $\hat{V}(B, f)$. The power spectrum $|\hat{V}(B, f)|^2$ vs. $B$ can be detected at frequencies around the center frequency $f_1$ using a spectrum analyzer. If $b_1 \ll \Delta B$, the spectral voltage output will be proportional to $V_{dc}'(B)$; it will be maximum in field regions of maximum slope, while in regions of a flat $V_{dc}'(B)$ curve, say around $B = 0$, it will tend to zero.

Quadratic mixing is expected for two tone experiments, where the incident rf signal is a superposition of two time harmonic signals: $b_{rf}(t) = b_1 \cos(2\pi f_1 t) + b_2 \cos(2\pi f_2 t)$ (we assume $f_1 < f_2 < \frac{f_J}{2}$). Taking into account the parabolic shape of the $V_{dc}(B)$-curve around $B = 0$, one expects in that region a larger amplitude for the quadratic mixing signals at frequencies $f_2 - f_1, 2f_1, f_2 + f_1$ and $2f_2$, respectively, compared to the field regions where $V_{dc}'(B)$ is maximal (zero curvature).

Previously we reported on dc experiments with SQIF-sensors in shielded and also in unshielded active microcoolers. Active cooling offers numerous advantages: possibility to choose stable temperature down to 50 K, quick thermal cycles, and compact and portable setups. Results obtained in unshielded micro-coolers have never revealed a significant degradation of the SQIF-sensor performance with respect to shielded micro-coolers. We believe the principal reason for this is the large dynamic range of the SQIF’s.

In the current experiments we broadcast an rf-signal, representing a superposition of two time harmonic fields, with $f_1, f_2 < \frac{f_J}{2}$ and small amplitudes $b_1$ and $b_2$, to a SQIF-sensor mounted inside the micro-cooler. The measured spectral voltage output $|\hat{V}(B, f)|$ of the SQIF is shown in Figs. 2(a,b). One clearly recognizes besides the primary signals at frequencies $f_1$ and $f_2$, a quadratic mixing signal at frequency $f_1 - f_2$, a large mixing amplitude when the SQIF is tuned at the bottom of the dip at $B = 0$. For increasing field $B$, the amplitude

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of the quadratic mixing signal symmetrically decreases and apparently vanishes under the noise floor of the detector as a function of $B$ at approximately half-way to the position of maximal dip slopes. In sharp contrast to this behavior, both first order signals display a maximum amplitude then where the dip slope is maximal. The effect is observed for various SQIF-sensors, with different critical current densities, always with a strength that increases with the increase of the transfer factor $V_B$. In all cases, Eq.(1) provides an excellent description of the detected spectral voltage output vs. static field $B$.

The Josephson junctions in our arrays consist of YBa$_2$Cu$_3$O$_{7-x}$ grain boundary junctions grown on 24°-oriented bycristal Mgo substrates. They are designed with a width of 2 µm, the YBa$_2$Cu$_3$O$_{7-x}$ layer being 130 nm thick, so that the resulting junction critical current density is $J_C \approx 23$ kA/cm$^2$, at $T = 77$ K. The serial SQIF consists of 211 loops, the distribution of the loop areas ranging between 38 µm$^2$ and 210 µm$^2$. In order to vary the coupling of magnetic flux into the SQIF, we designed two types of focusing structures. Type (a) consists of two superconducting rings being symmetrically placed on both sides of the grain boundary [Fig. 1(a)]. Type (b) is made with only one superconducting ring [Fig. 1(b)]. Type (b) is effectively a split loop design, consisting of 10 equidistant parallel thin loops, one aligned inside the other. The static magnetic field $B$ is applied via a multi-turn coil placed inside the cooler. The rf field $b_{rf}(t)$ is applied via a 50 Ω-loop antenna. Experiments are made either with a mu-metal shield - in this case the rf antenna was placed inside the shield at a distance of about 5 cm from the chip -, or with an unshielded cryocooler in open space.

We first discuss the experiments with shielded cryocooler. We measure the $V_{dc}(B)$ dependence of the SQIF, and check the dip symmetry with respect to zero field. There is an optimal temperature at which the voltage swing is a maximum, at a properly chosen $I_b$. At $T \approx 76$ K, the SQIF critical current is $I_c \approx 50$ µA and, at $I_b = 85$ µA, we measure $\Delta V \approx 1160$ µV and $V_B \approx 6500$ V/T. The SQIF normal resistance is $R = 186$ Ω. Successively, the rf is switched on: a time harmonic signal is applied to the primary antenna, while the SQIF rf output is detected. The value $b_1$ is chosen much smaller than $\Delta B$, to ensure that the rf signal superimposed to the static field does not modulate the SQIF working point out of the dip and corrupt the effect; but it has to be high enough so that the spectral voltage output is above the noise level set by the resolution bandwidth (ResBW) of the spectrum analyzer.

In the single tone rf experiment, the incident signal has frequency $f_1 = 102$ MHz, and amplitude $b_1 = -23$ dBm. At $I_b = 58$ µA and $T = 75$ K, the voltage swing is about 400 µV. The SQIF rf voltage is amplified by a cold amplifier, designed with high input resistance (5 kΩ). The spectrum analyzer is operated as a narrowband receiver tuned at frequency $f \approx f_1$, with a bandwidth set by the ResBW (zero span mode), so that the analog output corresponds to the maximum signal amplitude. The ResBW is varied from 3 kHz to 1 MHz. The spectral voltage output of the SQIF is recorded by computer while slowly sweeping the static field $B$ (sweep frequency in the kHz range). Simultaneously the $V_{dc}(B)$ curve is acquired. Figure 2(a) displays typical results. Here the 0 dB level is arbitrary, being at the present the amplifier gain not exactly known. It is found that, at frequency $f_1$, the spectral voltage output of the SQIF vs. $B$ is a maximum in field regions where the dip slopes are maximal. In Fig. 2(b) we plot the modulus of the first derivative of the theoretically calculated voltage output vs. magnetic field $B$ and bias current $I_b$.

In the two-tone rf experiments, the incident field is a linear combination of two signals with frequency $f_1 = 220$ MHz and $f_2 = 100$ MHz, and amplitude $b_1 = b_2 = -22$ dBm. In the spectral voltage output of the SQIF, we find then a quadratic mixing signal at the difference frequency $f = f_1 - f_2 = 120$ MHz. The amplitude of this signal is maximal at $B = 0$. Figure 3(a) displays the spectral voltage output detected at $f = 120$ MHz, with ResBW equal to 3 kHz. Symmetrically, at $f = f_1 + f_2 = 320$ MHz a similar spectral voltage response is detected. In the current set-up, quadratic mixing has been observed up to few GHz. In Fig. 3(b) we plot the theoretically calculated $|V_{out}^{\text{rf}}(B)|$ vs. $B$ and $I_b$.

In the rf experiments without magnetic shield, the environmental disturbances do not suppress the dip (although in this case the compensation field is different), and the quadratic mixing effect as well as the second harmonic generation take place, similar to the results obtained with the shielded cooler. In the single tone experiment with a time harmonic rf signal at frequency $f_1$, we also detected second harmonic signal generation at $2f_1$ in the spectral voltage output of the SQIF-sensor, with a field dependence around $B = 0$ as shown in Fig. 3(a).

In conclusion, we have demonstrated that a SQIF-sensor is capable to transfer the modulations of an incident time harmonic electromagnetic signal with carrier frequency $f_1 < \frac{f}{2}$ into a corresponding rf voltage output. This suggests potential applications of a SQIF as a non-linear mixing device, capable to operate at frequencies from dc to few GHz with a large dynamic range. All experiments have shown that the strength of the rf voltage output at frequency $f_1$ depends crucially on the...
FIG. 2: (a) SQIF dc voltage $V_{\text{dc}}$ at $I_b = 58 \mu\text{A}$ and $T = 75 \text{K}$ vs. static magnetic field $B$ (gray curve, relative to right axis); simultaneous measurements of the maximum amplitude of the SQIF rf voltage $|\tilde{V}(B, f)|$, induced by the rf field with $f_1 = 102 \text{MHz}$ and $b_1 = -23 \text{dBm}$ (black curve, relative to left axis). Detection is made by spectrum analyzer in zero span mode, at $f_1$ and with ResBW= $3 \text{kHz}$. The 0 dB level is arbitrary. (b) Modulus of the first derivative of $V_{\text{dc}}$ vs. static field and bias current.

FIG. 3: (a) Gray curve, relative to right axis: $V_{\text{dc}}$ vs. $B$; black curve, rel. to left axis: rf voltage $|\tilde{V}(B, f)|$ detected at $|f_1 - f_2| = 120 \text{MHz}$, with a ResBW of $3 \text{kHz}$. The 0 dB level is arbitrary. The main signals were at $f_1 = 220 \text{MHz}$ and $f_2 = 100 \text{MHz}$; amplitudes $b_1 = b_2 = -22 \text{dBm}$. (b) Modulus of the second derivative of $V_{\text{dc}}$ vs. static field and bias current.

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