Effect of thermal radiation on laminar boundary layer flow over a permeable flat plate with Newtonian heating

Muhammad Khairul Anuar Mohamed¹, Mohd Zuki Salleh¹, Nor Aida Zuraimi Md Noar¹ and Anuar Ishak²

¹Applied & Industrial Mathematics Research Group, Faculty of Industrial Science & Technology, Universiti Malaysia Pahang, 26300 Kuantan, Pahang, Malaysia
²School of Mathematical Sciences, Faculty of Science & Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia

E-mail: baa_khy@yahoo.com

Abstract. The laminar boundary layer flow over a permeable flat plat with the presence of thermal radiation and Newtonian heating is numerically studied. The non linear partial differential equations that governed the model are transformed to ordinary differential equations before being solved numerically by Runge-Kutta-Fehlberg (RKF) method using Maple software. The influenced and characteristic of pertinent parameters which are the Prandtl number, the suction/blowing parameter, the thermal radiation parameter and the conjugate parameter are analyzed and discussed. It is found that the presence of thermal radiation and blowing parameter has increased the value of wall temperature. Meanwhile, the trend is contrary with the suction effect.

1. Introduction
The study on the boundary layer flow regarding a stationary plate was first studied by Blasius [1]. This Blasius flow is quite different from flow on a moving plate in [2]. Aziz [3] studied the laminar thermal boundary layer over a flat plate with convective boundary conditions before being updated by Ishak [4] with permeable effect. Both problems were successfully solved by using the RKF45 method. Present study considered the thermal radiation effect on laminar boundary layer flow over a permeable flat plate in Newtonian heating (NH).

According to Merkin [5], Newtonian heating boundary condition is taken as the heat transfer which usually denoted as temperature gradient is proportional to the local wall temperature. The Newtonian heating boundary condition is quite realistic in real world compared to classical constant wall temperature (CWT) where the wall temperature is fixed at a specified temperature. Related papers considered the Newtonian heating boundary condition include the works in [6] and [7]. The fact that the problem considered here has never been studied before, thus the reported result in this paper is new.

2. Mathematical Formulation
Consider a Blasius flow in a steady incompressible viscous fluid of ambient temperature $T_\infty$ and free stream velocity $U_\infty$. $T$ is the temperature inside the boundary layer, $u$ and $v$ are the velocity
components along the $x$ and $y$ directions, respectively. $q_r = -\frac{4\sigma^* T^4}{3k^*}$ is the radiative heat flux where $\sigma^*$ and $k^*$ are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively [8].

The suggested governing boundary layer equations are [6, 4]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2},$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y},$$

subject to the boundary conditions

$$u = 0, \quad v = \nu, \quad -\frac{\partial T}{\partial y} = h_s T \text{ at } y = 0,$$

$$u = U_s, \quad T \rightarrow T_s \text{ as } y \rightarrow \infty,$$

where $\nu$ is the kinematic viscosity, $\rho$ is the fluid density, $k$ is the thermal conductivity and $C_p$ is the specific heat capacity at constant pressure. Next, $v_w$ is the mass transfer velocity at the surface while $h_s$ is the heat transfer coefficient. Using Rosseland approximation [8], the equation (3) is reduced to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left( \frac{k}{\rho C_p} + \frac{16\sigma^* T^3_s}{3k^* \rho C_p} \right) \frac{\partial^2 T}{\partial y^2},$$

From the above equation it is seen that the effect of radiation is to enhance the thermal diffusivity. Let $N_r = \frac{4\sigma^* T^3}{kk^*}$ as the radiation parameter, equation (5) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \left( 1 + \frac{4}{3} N_r \right) \frac{\partial^2 T}{\partial y^2},$$

Note that thermal radiation effects is absent when $N_r = 0$. Following Bachok et al. [9], the suggested similarity transformation for equations (1) to (4) are:

$$\eta = \left( \frac{U_s}{2\nu x} \right)^{1/2} y, \quad \psi = \left( 2U_s v x \right)^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_s}{T_s},$$

where $\theta$ and $\psi$ are dimensionless temperature and the stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$ which satisfy equation (1), respectively. $u$ and $v$ can be derived as

$$u = U_s f'(\eta), \quad v = -\left( \frac{U_s}{2x} \right)^{1/2} f(\eta) + \frac{U_s}{2x} f'(\eta),$$
Substitute the equations (7) and (8) into equations (2), (4) and (6), then the following ordinary differential equations were obtained

\[ f'' + f'' = 0 \]  
\[ \frac{1}{Pr} \left( 1 + \frac{4}{3} N_R \right) \theta'' + f \theta' = 0, \]  

where \( Pr = \frac{\nu \rho C_p}{k} \) is the Prandtl number. In order that the similarity solution for equations (1) to (4) exist, it is assume [3, 4]:

\[ v_w = -a \left( \frac{U_{\infty} \nu}{2x} \right)^{\frac{1}{2}} \lambda, \quad h_s = ax^{-\frac{1}{2}} \]  

where \( a \) and \( \lambda \) are constants. \( \lambda \) measure the permeability or transpiration rate at the plate surface, with \( \lambda > 0 \) and \( \lambda < 0 \) corresponds for suction and blowing, respectively. The transformed boundary conditions are

\[ f(0) = \lambda, \quad f'(0) = 0, \quad \theta(0) = -\gamma(1 + \theta(0)), \]  
\[ f'(\eta) \to 1, \quad \theta(\eta) \to 0 \text{ as } \eta \to \infty. \]  

where \( \gamma = -a \left( \frac{2 \nu}{U_{\infty}} \right)^{\frac{1}{2}} \) is the conjugate parameter. Note that the assumption in equation (11) is necessary for the boundary conditions (12) to be independent of \( x \).

Further, for \( \gamma \to 0 \) and integrate the terms \( \theta'(0) \) with respect to \( \eta \), the present problem reduces to the classical constant wall temperature (CWT) boundary condition.

### 3. Results and Discussion

The system of ordinary differential equations (9) and (10) with boundary conditions (12) were solved numerically using the RKF45 technique in Maple. The numerical results obtained for wall temperature \( \theta(0) \), the temperature gradient \( -\theta'(0) \) and the skin friction coefficient \( f''(0) \) for various values of pertinent parameter namely as the Prandtl number \( Pr \), the suction/blowing parameter \( \lambda \), the thermal radiation parameter \( N_R \) and the conjugate parameter \( \gamma \).

In order to validate the numerical results obtained, the comparison has been made. Table 1 shows the comparison values of \( -\theta'(0)/\sqrt{2} \) for CWT case with previous results in Bataller [8]. It is found that the both numerical results are in a good agreement. Further, table 1 included results for \( -\theta'(0) \) in CWT and \( \theta(0) \) in NH as well as the value of \( f''(0) \). It is found that the increase of \( Pr \) result to the increase in \( -\theta'(0) \) which usually represent the reduced Nusselt number in CWT. Meanwhile for NH, the value of \( \theta(0) \) decreases. Changes in \( Pr \) gives no effect on the value of \( f''(0) \). Noticed that the value of \( f''(0) = 0.469600 \) obtained is similar as reported by Blasius [1].

Next, table 2 presents the values of \( \theta(0) \) and \( f''(0) \) for various values of \( N_R \) and \( \lambda \). \( N_R = 0 \) is denoted for the absence of thermal radiation effect. From this table, it is concluded that the increases of \( \lambda \) in suction case (\( \lambda > 0 \)) reduces the value of \( \theta(0) \) while \( f''(0) \) increases. The present of \( N_R \) has enhanced the value of \( \theta(0) \). Meanwhile, \( N_R \) give no effect on \( f''(0) \). This is clear from ordinary differential equations (9) and (10).
Table 1. Comparison of the present solution with previously published result for various values of Pr when \(N_R = \lambda = 0\) and \(\gamma = 0.1\).

| Pr | \(-\dot{\theta}(0)/\sqrt{2}\) (CWT) | \(-\dot{\theta}(0)\) (CWT) | \(\theta(0)\) (NH) | \(f^*(0)\) |
|----|---------------------------------|----------------------------|----------------|----------|
| 0.7 | 0.29268                         | 0.413912                   | 0.318560       | 0.469600 |
| 5   | 0.57669                         | 0.815561                   | 0.139750       | 0.469600 |
| 10  | 0.72814                         | 1.029747                   | 0.107556       | 0.469600 |
| 30  | 1.05173                         | 1.487320                   | 0.072081       | 0.469600 |
| 50  | 1.24729                         | 1.763932                   | 0.060098       | 0.469600 |
| 75  | 1.42799                         | 2.019495                   | 0.052097       | 0.469600 |
| 100 | 1.57183                         | 2.222907                   | 0.047105       | 0.469600 |

Table 2. Values of \(\theta(0)\) and \(f^*(0)\) for various values of \(N_R\) and \(\lambda\) when \(Pr = 0.7\) and \(\gamma = 0.1\).

| \(\lambda\) | \(N_R = 0\) | \(N_R = 0.5\) | \(N_R = 1\) |
|-------------|------------|---------------|------------|
|             | \(\theta(0)\) | \(f^*(0)\) | \(\theta(0)\) | \(f^*(0)\) | \(\theta(0)\) | \(f^*(0)\) |
| -0.7        | 46.1390    | 0.0539        | 8.1393     | 0.0539     |
| -0.65       | 4.71501    | 0.0743        | 3.4727     | 0.0743     |
| -0.60       | 4.9767     | 0.0975        | 2.5272     | 0.0975     |
| -0.55       | 2.3456     | 0.1222        | 1.7404     | 0.1222     |
| -0.5        | 1.5251     | 0.1485        | 1.3331     | 0.1485     |
| -0.3        | 0.6210     | 0.2658        | 0.6975     | 0.2658     |
| -0.1        | 0.3820     | 0.3986        | 0.4750     | 0.3986     |
| 0           | 0.3186     | 0.4696        | 0.4097     | 0.4696     |
| 0.1         | 0.2724     | 0.5432        | 0.3601     | 0.5432     |
| 0.3         | 0.2099     | 0.6970        | 0.2896     | 0.6970     |
| 0.5         | 0.1698     | 0.8579        | 0.2418     | 0.8579     |
| 1           | 0.1133     | 1.2836        | 0.1701     | 1.2836     |

On the other side, the present of blowing effect \((\lambda < 0)\) has enhanced the \(\theta(0)\) value rapidly. From numerical computation, it is found that the blowing effect has its critical value for acceptable physical solution. From table, the solution for \(\lambda = -0.65\) is not exist which physically denoted as boundary layer separation. It is suggested that, the presence of \(N_R\) may delayed the critical value which enhanced the range of \(\lambda\) for which the solution exists.

Figures 1 and 2 show the temperature profiles for various values of \(N_R\) and \(\lambda\), respectively. It is found that the increment of \(N_R\) enhanced the temperature profiles and its boundary layer thicknesses. This is realistic since the present of thermal radiation effects added amount of heat on the surface. The heat spread away from the surface which thickening the boundary layer. In figure 2, it is observed that the increase of \(\lambda\) in suction case gives a reduction in thermal boundary layer thickness. The situation goes contrary with blowing case where the thermal boundary layer thickness increases rapidly.

Velocity profiles \(f'(\eta)\) for various values of \(\lambda\) is illustrates in figure 3. From figure 3, it is suggested that the blowing effect thickening the velocity boundary layer thickness which on the other hand reduced the skin friction coefficient as discussed in table 2.
Figure 1. Temperature profiles $\theta(\eta)$ for various values of $N_R$ when $Pr = 0.7$, $\lambda = 0$ and $\gamma = 0.1$.

Figure 2. Temperature profiles $\theta(\eta)$ for various values of $\lambda$ when $Pr = 0.7$, $N_R = 0.5$ and $\gamma = 0.1$.

Figure 3. Velocity profiles $f'(\eta)$ for various values of $\lambda$ when $Pr = 0.7$, $N_R = 0.5$ and $\gamma = 0.1$.

Figure 4. Variation values of wall temperature $\theta(0)$ with various values of $\lambda$ and $\gamma$ when $Pr = 0.7$ and $N_R = 0.5$.

Lastly, figure 4 presents the variation of $\theta(0)$ with various values of $\lambda$ and $\gamma$. Notice that the increase of $\gamma$ enhanced the value of $\theta(0)$ until $\gamma$ approaches its critical value $\gamma_c$. From the numerical computation, the presence of suction effect ($\lambda = 0.5$) has delayed the separation which occur
when $\gamma > \gamma_c$. Meanwhile, blowing effect ($\lambda = -0.5$) accelerate the separation to occur and reduced the range of which solution exist.

4. Conclusion
As a conclusion, the increase of $N_R$ and $\gamma$ results to the increase in wall temperature while Pr does oppositely. The skin friction coefficient is not affected by $N_R$, Pr and $\gamma$. In discussing the suction/blowing effect, it is found that in blowing case ($\lambda < 0$), the presence of this parameter results to the increase in wall temperature, skin friction coefficient as well as both thermal and velocity boundary layer thicknesses. This trend is contrary with the suction case ($\lambda > 0$). In addition, the suction effect delayed the boundary layer separation while blowing effect accelerates it.

Lastly, it is worth mentioning that, as $\gamma$ is fixed, the trends for the temperature gradient is similar as in wall temperature.

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