An Incentive Mechanism for Federated Learning in Wireless Cellular network: An Auction Approach

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Abstract

Federated Learning (FL) is a distributed learning framework that can deal with the distributed issue in machine learning and still guarantee high learning performance. However, it is impractical that all users will sacrifice their resources to join the FL algorithm. This motivates us to study the incentive mechanism design for FL. In this paper, we consider a FL system that involves one base station (BS) and multiple mobile users. The mobile users use their own data to train the local machine learning model, and then send the trained models to the BS, which generates the initial model, collects local models and constructs the global model. Then, we formulate the incentive mechanism between the BS and mobile users as an auction game where the BS is an auctioneer and the mobile users are the sellers. In the proposed game, each mobile user submits its bids according to the minimal energy cost that the mobile users experiences in participating in FL. To decide winners in the auction and maximize social welfare, we propose the primal-dual greedy auction mechanism. The proposed mechanism can guarantee three economic properties, namely, truthfulness, individual rationality and efficiency. Finally, numerical results are shown to demonstrate the performance effectiveness of our proposed mechanism.

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I. INTRODUCTION

Currently, according to the report of International Data Corporation, there are nearly 3 billions smart-phones on the world [1], [2], which generate a huge amount of personal data. Nowadays, mobile devices equipped with specialized hardware architectures and computing engines can handle the machine learning problem effectively. In addition, the application of machine learning techniques in mobile devices has grown rapidly. Furthermore, due to the limitation of wireless communication resources and privacy protection problem, the conventional central machine learning techniques, which upload all data of mobile devices to the central sever, are becoming less attractive. For this reason, federated learning (FL) is promoted, which is implemented distributively at the edge of the network [3], [4]. In FL, mobile users can collaboratively train a global model using their own local data. Mobile users compute the updates of the current global model, and then send back the updates to the central server for aggregation and build a new global model. This process is repeated until an accuracy level of the global learning model is achieved. By this way, FL can preserve the personal information and data of mobile users. In addition, FL will significantly promote services that have unparalleled versatile data collection and model training on a large scale. Take the app Ware as an example. This application can help the users in avoiding heavy traffic roads, but users have to share their own locations to the server. If FL is applied to this app, users only need to send the intermediate gradient values to the server rather than the raw data [5]. Last but not least, the development of mobile edge computing provides an immense exposure to extract the benefits of FL [6], [7].

In spite of the above-mentioned benefits of FL, there are remaining challenges of having an efficient FL framework. Firstly, data samples per mobile device are small to train a high-quality learning model so a large number of mobile users are needed to ensure cooperation. In addition, the mobile users who join the learning process are independent and uncontrollable. Here, mobile users may not be willing to participate in the learning due to the energy cost incurred by model training. In other words, the base station (BS), which generates the global model, has to stimulate the mobile users for participation. Moreover, because the wireless resource is limited, the BS needs to allocate the resources reasonably to avoid the congestion, guarantee the model training performance and optimize the total utilities of the BS and mobile users.
To deal with the above challenges, in this paper, we model the FL service between the BS and mobile users as an auction game in which the BS is buyer and mobile users are sellers. In particular, the BS first initiates and announces a FL task. When each mobile user receives the FL task information, they decide the amount of resources required to participate in the model training. After that, each mobile user submits a bid, which includes the required amount of resource, local accuracy, and the corresponding energy cost, to the BS. Moreover, the BS plays the role of auctioneer to decide the winners among mobile users as well as clear payment for the winning mobile users. In addition, the auction used in this paper is a type of combinational auction [8], [9] since each mobile user can bid for combinations of resources. However, the proposed auction mechanism allows mobile users sharing the resources at the BS, which is different from the conventional combinatorial auction. The proposed mechanism directly determines the trading rules between the buyer (BS) and sellers (mobile users) and motivates the mobile users to participate in the model training. Compared with other incentive mechanism approaches (e.g., contract theory [10]) in which the service market is a monopoly market, where mobile users can only decide whether or not to accept the contracts, the proposed auction enables mobile users to bids any combinations of resources. Moreover, the proposed auction mechanism can simultaneously provide truthfulness and individual rationality. An auction mechanism is truthful if a bidder’s utility does not increase when that bidder makes other bidding strategies, rather than the true value. Revealing the true value is a dominant strategy for each participating user regardless of what strategies other users use [11]. An absent-truthfulness auction mechanism could leave the door to possible market manipulation and produce inferior results [12]. Additionally, if the value of any bidder is non-negative, an auction process will ensure individual rationality.

The contributions of this paper are summarized as follows:

- We propose an auction framework for the wireless FL services market. Then, we present the bidding cost in every user’s bid submitted to the BS. From the perspective of mobile users, each mobile user make optimal decisions on the amount of resources and local accuracy so that the energy cost is minimized while delay requirement of FL is satisfied.

- From the perspective of the BS, we formulate the winner selection problem in the auction game as the social welfare maximization problem which is a NP-hard problem. We propose a primal-dual greedy algorithm to deal with the NP-hard problem in selecting the winning users and critical value based payment. We also proposed auction mechanism is truthful, individual rational and computational efficient.
• Finally, we carry out the numerical study to show that a proposed auction mechanism can
guarantee the approximation factor of the integrality to the maximal welfare that is derived
by the optimal solution and outperforms compared with baseline.

The rest of this paper is organized as follows. Section II summarizes the related work. The
system model is introduced in Section III. We describe the problem formulation in Section IV.
We present the auction-based resource purchasing mechanism in Section V. Simulation results
are given in Section VI. Finally, Section VII concludes the paper.

II. RELATED WORKS

Due to the resource constraints and the heterogeneity of mobile users, some focus issues are
resource allocation, client selection and incentive mechanism to improve the efficiency of FL.
The authors in [5] posed the joint learning and transmission energy minimization problem for
FL. In this paper, all users upload their learning model to the BS in a synchronous manner.
The work in [13] also considered the latency and energy consumption minimization problem for
the case of asynchronous transmission. The work in [14] explored the problem of reducing the
learning loss function by considering packet errors over wireless links, but this research ignored
the computation delay of the local learning model. In [15], the authors suggested energy-efficient
strategies for allocating bandwidth and scheduling while at the same time, guaranteeing learning
efficiency. The derived optimal policies allocate more bandwidth to those scheduled devices
with weaker channels or lower computing capacities, which are the bottlenecks of synchronized
model updates in FL.

However, the works in [5], [13]–[15] overlooked the problem of client selection to build a
high-quality machine learning model. The authors in [16] designed a protocol called FedCS. The
FedCS protocol has a resource request phase to gather information such as computing power
and wireless channel states from a subset of randomly selected clients, i.e., FL workers, that
are able to finish the local training punctually. A q-FedAvg training algorithm for selecting the
client by the computational power was proposed in [17]. This proposed algorithm in [17] can
improve the training efficiency and solve the fairness issue. The study in [18] recommended
combining deep reinforcement learning (DRL) and FL frameworks with mobile edge systems
to optimize computing, caching and communication. The study in [19] jointly considered the
device selection and beamforming fast global model aggregation. They used the principle of
over-the-air computation to exploit signal superposition multiple access channels.
Concerning the incentive mechanism design, the authors in [20] proposed a Stackelberg game model to investigate the interactions between the server and the mobile devices in a cooperative relay communication network. The mobile devices determine the price per unit of data for individual profit maximization, while the server chooses the size of training data to optimize its own profit. The authors in [10] studied the block-chained FL architecture and proposed the contract theory based payment mechanism to incentivize the mobile devices to take part in the FL. However, [10] largely provided a latency analysis for the related applications. The work in [21] designed and analyzed a novel crowdsourcing framework to enable FL. In [21], a two-stage Stackelberg game model was adopted to jointly study the utility maximization of the participating clients and multi-access edge computing (MEC) server interacting via an application interface to construct a high-quality learning model. In [22], a Stackelberg game for FL in IoT was proposed to tackle the challenge of incentivizing people to join the FL by contributing their computational power and data. For the cases where the knowledge of participants’ decisions and accurate contribution evaluation are accessible, the Nash Equilibrium was derived, and an algorithm based on DRL was built to unknown the knowledge of participants’ decisions and accurate contribution evaluation for the cases. However, both [21] and [22] studied only the uniform pricing scheme for participants.

Different from the Stackelberg game and contract theory, the auction mechanism allows mobile users to actively report its cost. Therefore, the BS is capable of understanding their status and requests adequately. [23] adopted the multi-dimensional procurement auction to motivate nodes to participate in FL. However, there exist some differences between [23] and our work: (a) [23] the bids submitted by edge node clarifies the combination of resources and the expected payment, which is based on the private cost parameters while in our work, the bid declares the combination of resource, local accuracy and the cost, which is determined based on latency and energy cost models; (b) the winner selection in [23] based on the scoring function announced by the aggregator while in our work, winners are selected in order to optimize social welfare and ensure resource efficiency.

### III. System Model: Federated Learning Services Market

#### A. Preliminary of Federated Learning

Consider a cellular network in which one BS and a set $\mathcal{N}$ of $N$ users cooperatively perform a FL algorithm for model learning, as shown in Fig. [1]. Each user $n$ has $s_n$ local data samples.
Each data set $s_n = \{a_{nk}, b_{nk}, 1 \leq k \leq s_n\}$ where $a_{nk}$ is an input and $b_{nk}$ is its corresponding output. The FL model trained by the dataset of each user is called the local FL model, while the FL model at the BS aggregates the local model from all users as the global FL model. We define a vector $\omega$ as the model parameter. We also introduce the loss function $l_n(\omega, a_{nk}, b_{nk})$ that captures the FL performance over input vector $a_{nk}$ and output $b_{nk}$. The loss function may be different, depending on the different learning tasks. The total loss function of user $n$ will be

$$L_n(\omega) = \frac{1}{s_n} \sum_{k=1}^{s_n} l_n(\omega, a_{nk}, b_{nk}).$$

(1)

Then, the learning model is the minimizer of the following global loss function minimization problem

$$\min_{\omega} L(\omega) = \sum_{n=1}^{N} \frac{s_n}{S} L_n(\omega) = \frac{1}{S} \sum_{n=1}^{N} \sum_{k=1}^{s_n} l_n(\omega, a_{nk}, b_{nk}),$$

(2)

where $S = \sum_{n=1}^{N} s_n$ is the total data samples of all users.

To solve the problem in (2), we adopt the FL algorithm of [24]. The algorithm uses an iterative approach that requires a number of global iterations (i.e., communication rounds) to achieve a global accuracy level. In each global iteration, there are interactions between the users and BS. Specifically, at a given global iteration $t$, users receive the global parameter $\omega^t$, users computes $\nabla L_n(\omega^t), \forall n$ and send it to the BS. The BS computes

$$\nabla L(\omega^t) = \frac{1}{N} \sum_{n=1}^{N} \nabla L_n(\omega^t),$$

(3)
and then broadcasts the value of $\nabla L(\omega^t)$ to all participating users. Each participating user $n$ will use local training data $s_n$ to solve the local FL problem is defined as

$$
\min_{\phi_n} \mathcal{G}_n(\omega^t, \phi_n)
$$

(4)

where $\phi_n$ represents the difference between global FL parameter and local FL parameter for user $n$. Each participating user $n$ uses the gradient method to solve (4) with local accuracy $\varepsilon_n$ that characterizes the quality of the local solution, and produces the output $\phi_n$ that satisfies

$$
\mathcal{G}_n(\omega^t, \phi_n) - \mathcal{G}_n(\omega^t, \phi^*_n) < \varepsilon_n(\mathcal{G}_n(\omega^t, 0) - \mathcal{G}_n(\omega^t, \phi^*_n)).
$$

(5)

Solving (4) also takes multiple local iterations to achieve a particular local accuracy. Then each user $n$ sends the local parameter $\phi_n$ to the BS. Next, the BS aggregates the local parameters from the users and computes

$$
\omega^{t+1} = \omega^t + \frac{1}{N} \sum_{n=1}^{N} \phi^t_n,
$$

(6)

and broadcasts the value to all users, which is used for next iteration $t + 1$. This process is repeated until the global accuracy $\gamma$ of (2) is obtained.

With the assumption on $L_n(\omega)$, the general lower bound on the number of global iterations is depends on local accuracy $\varepsilon$ and the global accuracy $\gamma$ as [13]:

$$
I_g(\gamma, \varepsilon) = C_1 \log \left( \frac{1}{\gamma(1-\varepsilon)} \right),
$$

(7)

where the local accuracy measures the quality of the local solution as described in the preceding paragraphs.

In (7), we observe that a very high local accuracy (small $\varepsilon$) can significantly boost the global accuracy $\gamma$ for a fixed number of global iterations $I_g$ at the BS to solve the global problem. However, each user $n$ has to spend excessive resources in terms of local iterations, $I_{l n}^t$ to attain a small value of $\varepsilon_n$. The lower bound on the number of local iterations needed to achieve local accuracy $\varepsilon_n$ is derived as [13]

$$
I_{l n}^t(\varepsilon_n) = \vartheta_n \log \left( \frac{1}{\varepsilon_n} \right),
$$

(8)

where $\vartheta_n > 0$ is a parameter choice of user $n$ that depends on parameters of $L_n(\omega)$ [13]. In this paper, we normalize $\vartheta_n = 1$. Therefore, to address this trade-off, the BS can setup an economic interaction environment to motivate the participating users to enhance local accuracy $\varepsilon_n$. Correspondingly, with the increased payment, the participating users are motivated to attain
better local accuracy $\varepsilon_n$ (i.e., smaller values), which as noted in (7) can improve the global accuracy $\gamma$ for a fixed number of iterations $I^g$ of the BS to solve the global problem. In this case, the corresponding performance bound in (7) for the heterogeneous responses $\varepsilon_n$ can be updated to capture the statistical and system-level heterogeneity considering the worst response of the participating users as:

$$I^g(\gamma, \varepsilon_n) = \frac{\omega \log(1/\gamma)}{1 - \max_n \varepsilon_n}, \forall n.$$  

(9)

B. Computation and Communication Models for Federated Learning

The contributed computation resource that user $n$ contributes for local model training is denoted as $f_n$. Then, $c_n$ denotes the number of CPU cycles needed for the user $n$ to perform one sample of data in local training. Thus, energy consumption of the user for one local iteration is presented as

$$E_{com}^n(f_n) = \zeta c_n s_n f_n^2,$$

(10)

where $\zeta$ is the effective capacitance parameter of computing chipset for user $n$. The computing time of a local iteration at the user $n$ is denoted by

$$T_{comp}^n = \frac{c_n s_n}{f_n}.$$  

(11)

It is noted that the uplink from the users to the BS is used to transmit the parameters of the local FL model while the downlink is used for transmitting the parameters of the global FL model. In this paper, we just consider the uplink bandwidth allocation due to the relation of the uplink bandwidth and the cost that user experiences during learning a global model. We consider the uplink transmission of an OFDMA-based cellular system. A set of $B = \{1, 2, ..., B\}$ subchannels each with bandwidth $W$. Moreover, the BS is equipped with $A$ antennas and each user equipment has a single antenna (i.e., multi-user MIMO). We assume $A$ to be large (e.g., several hundreds) to achieve massive MIMO effect which scales up traditional MIMO by orders of magnitude. Massive MIMO uses spatial-division multiplexing. The achievable uplink data rate of mobile user $n$ is expressed as [25]

$$r_n = b_n W \log_2 \left(1 + \frac{(A_n - 1)p_nh_n}{b_n W N_0}\right),$$

(12)

where $p_n$ is the transmission power of user $n$, $h_n$ is the channel gain of peer to peer link between user and the BS, $N_0$ is the background noise, $A_n$ is the number of antennas the BS assigns to
user \( n \), and \( b_n \) is the number of sub-channels that user \( n \) uses to transmit the local model update to the BS.

We denote \( \sigma \) as the data size of a local model update and it is the same for all users. Therefore, the transmission time of a local model update is

\[
T_n^{\text{com}}(p_n, A_n, b_n) = \frac{\sigma}{r_n}.
\]  

(13)

To transmit local model updates in a global iteration, the user \( n \) uses the amount of energy given as

\[
E_n^{\text{com}}(p_n, f_n, A_n, b_n) = T_n^{\text{com}} p_n = \frac{\sigma p_n}{r_n}.
\]  

(14)

Hence, the total time of one global iteration for user \( n \) is denoted as

\[
T_n^{\text{tol}}(p_n, f_n, A_n, b_n, \varepsilon_n) = \log \left( \frac{1}{\varepsilon_n} \right) T_n^{\text{comp}}(f_n) + T_n^{\text{com}}(p_n, A_n, b_n).
\]  

(15)

Therefore, the total energy consumption of a user \( n \) in one global iteration is denoted as follows

\[
E_n^{\text{tol}}(p_n, f_n, A_n, b_n, \varepsilon_n) = \log \left( \frac{1}{\varepsilon_n} \right) E_n^{\text{comp}}(f_n) + E_n^{\text{com}}(p_n, A_n, b_n).
\]  

(16)

C. Auction Model

As described in Fig. 1, the BS first initializes the global network model. Then, the BS announces the auction rule and advertises the FL task to the mobile users. The mobile users then report their bids. Here, mobile user \( n \) submits a set of \( I_n \) of bids to the BS. A bid \( \Delta_{ni} \) denotes the \( i \)th bid submitted by the mobile user \( n \). Bid \( b_{ni} \) consists of the resource (sub-channel number \( b_{ni} \), antenna number \( A_{ni} \), local accuracy level \( \varepsilon_{ni} \)) and the claimed cost \( v_{ni} \) for the model training. Each mobile user \( n \) has its own discretion to determine its true cost \( V_{ni} \), which will be presented in Section IV. Let \( x_{ni} \) be a binary variable indicating the bid \( \Delta_{ni} \) wins or not. After receiving all the bids from mobile users, the BS decides winners and then allocates the resource to the winning mobile users. The winning mobile users join the FL and receive the payment after finishing the training model.
IV. DECIDING MOBILE USERS’ BID

To transmit the local model update to the BS, mobile users need sub-channels and antennas resources. However, given the maximum tolerable time of FL, there is a correlation between resource and corresponding energy cost. In this section, we present the way mobile users decide bids. Specially, for bid $\Delta_{ni}$, mobile user $n$ calculates transmission power $p_{ni}$, computation resource $f_{ni}$ and cost $v_{ni}$ corresponding to a given sub-channel number $b_{ni}$ and antenna number $A_{ni}$. However, the process to decide mobile users’ bid is the same for every submitted bids. Thus, we remove the bid index $i$ in this section. The energy cost of mobile user $n$ is defined as

$$P_1 : \min_{f_n,p_n,A_n,b_n,\varepsilon_n} I_n^0 E_{n}^{\text{tot}}(p_n,f_n,A_n,b_n,\varepsilon_n)$$  \hspace{1cm} (17a)

s.t.  \hspace{1cm} I_n^0 T_{n}^{\text{tot}}(p_n,f_n,A_n,b_n,\varepsilon_n) \leq T_{\text{max}}, \hspace{1cm} (17b)

$$f_n \in [f_{n}^{\text{min}}, f_{n}^{\text{max}}],$$ \hspace{1cm} (17c)

$$p_n \in (0, p_{n}^{\text{max}}],$$ \hspace{1cm} (17d)

$$\varepsilon_n \leq (0, 1],$$ \hspace{1cm} (17e)

$$A_n \in (0, A_{n}^{\text{max}}],$$ \hspace{1cm} (17f)

$$b_n \in (0, b_{n}^{\text{max}}],$$ \hspace{1cm} (17g)

where $f_{n}^{\text{max}}$ and $p_{n}^{\text{max}}$ are the maximum local computation capacity and maximum transmit power of mobile user $n$, respectively. $A_{n}^{\text{max}}$ and $b_{n}^{\text{max}}$ are the maximum antenna and maximum sub-channel that mobile user $n$ can request in each bid, respectively. $A_{n}^{\text{max}}$ and $b_{n}^{\text{max}}$ are chosen by mobile user $n$. $I_n^0 = \frac{C_1 \log(1/\gamma)}{1-\varepsilon_n}$ is the lower bound of the number global iterations corresponding to local accuracy $\varepsilon_n$. Note that the cost to the mobile user cannot be the same over iterations. However, to make the problem more tractable, we consider minimizing the approximated cost rather than the actual cost, similar to approach in [21], [26]. Constraint (17b) indicates delay requirement of FL task.

According to $P_1$, the maximum number of antennas and sub-channels are always energy efficient, i.e., the optimal antenna is $A_n = A_{n}^{\text{max}}, b_n = b_{n}^{\text{max}}$ and $\varepsilon_n, p_{n}^{*}, f_{n}^{*}$ are the optimal...
solution to:

\[
P_2: \min_{f_n, p_n, \varepsilon_n} I_0^{nE_{\text{tol}}} (p_n, f_n, \varepsilon_n) \\
\text{s.t. } I_0^{nT_{\text{tol}}} (p_n, f_n, \varepsilon_n) \leq T_{\text{max}}, \\
f_n \in [f_{\text{min}}, f_{\text{max}}], \\
\varepsilon_n \in (0, 1], \\
p_n \in (0, p_{\text{max}}].
\]  

(18)

Because of the non convexity of \(P_2\), it is challenging to obtain the global optimal solution. To overcome the challenge, an iterative algorithm with low complexity is proposed in the following subsection.

A. Iterative Algorithm

The proposed iterative algorithm basically involves two steps in each iteration. To obtain the optimal, we first solve (\(P_2\)) with fixed \(\varepsilon_n\), and then \(\varepsilon_n\) is updated based on the obtained \(f_n, p_n\) in the previous step. In the first step, we consider the first case when \(\varepsilon_n\) is fixed, and \(P_2\) becomes

\[
P_3: \min_{f_n, p_n, \varepsilon_n} I_0^{nE_{\text{tol}}} (p_n, f_n, \varepsilon_n) \\
\text{s.t. } I_0^{nT_{\text{tol}}} (p_n, f_n, \varepsilon_n) \leq T_{\text{max}}, \\
f_n \in [f_{\text{min}}, f_{\text{max}}], \\
\varepsilon_n \in (0, 1], \\
p_n \in (0, p_{\text{max}}].
\]  

(19)

\(P_3\) can be decomposed into two sub-problems as follows.

1) Optimization of Uplink Transmission Power: Each mobile user assigns its transmission power by solving the following problem:

\[
P_{3a}: \min_{p_n} f(p_n) \\
\text{s.t. } I_0^n (f(p_n)/\sigma + T_{n}^{\text{comp}}) \leq T_{\text{max}}, \\
p_n \in (0, p_{\text{max}}], \\
f_n, \varepsilon_n \text{ are given.}
\]  

(20)

where \(f(p_n) = \frac{\sigma p_n}{b_n W \log_2(1 + \frac{\sigma p_n}{b_n W N_0})} \). Note that \(f(p_n)\) is quasiconvex in the domain \([27]\). A general approach to the quasiconvex optimization problem is the bisection method, which solves a convex feasibility problem each time \([28]\). However, solving convex feasibility problems by
Algorithm 1: Optimal Uplink Power Transmission

1. Calculate $\phi(p_{n}^{max})$
2. Calculate $p_{n}^{min}$ so that $T_{n}^{t}(p_{n}^{min}) = T_{max}$
3. if $\phi(p_{n}^{max} < 0)$ then
   4. $p_{n}^{*} = p_{n}^{max}$
   5. else
      6. $p_{1} = \max(0, p_{n}^{min})$ and $p_{2} = p_{n}^{max}$
      7. while $(p_{2} - p_{1} \leq \epsilon)$ do
         8. $p_{u} = (p_{1} + p_{1})/2$
         9. if $\phi(p_{u}) \leq 0$ then
            10. $p_{1} = p_{u}$
            11. else
               12. $p_{2} = p_{u}$
            13. end
         14. end
      15. end
   16. $p_{n}^{*} = (p_{1} + p_{2})/2$
17. end

An interior cutting-plane method requires $O(\kappa^2/\alpha^2)$ iterations, where $\kappa$ is the dimension of the problem [27]. On the other hand, we have

$$f'(p_{n}) = \frac{\sigma \log_2(1 + \theta_{n} p_{n} h_{n}) + \frac{\sigma p_{n} \theta_{n} h_{n}}{\ln 2(1 + \theta_{n} p_{n} h_{n})^2}}{b_{n} W (\log(1 + \theta_{n} p_{n} h_{n}))^2},$$ \hspace{1cm} (21)

where $\theta_{n} = \frac{(A_{n} - 1) W_{0}}{W_{N_{0}}}$. Then, we have

$$\phi(p_{n}) = \sigma \log_2(1 + \theta_{n} p_{n} h_{n}) + \frac{\sigma p_{n} \theta_{n} h_{n}}{2(1 + \theta_{n} p_{n} h_{n})},$$ \hspace{1cm} (22)

is a monotonically increasing transcendental function and negative at the starting point $p_{n} = 0$ [27]. Therefore, in order to obtain the optimal power allocation $p_{n}$ as shown in Algorithm 1, we follow a low-complexity bisection method by calculating $\phi(p_{n})$ rather than solving a convex feasibility problem each time.
Algorithm 2: Optimal Local Accuracy

1. Initialize $\varepsilon_n = \varepsilon_n^{(0)}$, set $j = 0$
2. repeat
3. Calculate $\varepsilon_n^* = \frac{\alpha_1}{(\ln 2)^{\xi_j}}$
4. Update $\xi_{(j+1)} = \frac{\gamma_1 \log_2 (1/\varepsilon_n) + \gamma_2}{\varepsilon_n}$
5. Set $j = j + 1$
6. until $|H(\xi^{(n+1)})|/|H(\xi^{(n)})| < \varepsilon_2$

2) Optimization of CPU cycle frequency and number of antennas::

\[
P_{3b} : \min_{f_n} f_n \log \left( \frac{1}{\varepsilon_n} \right) \frac{c_n s_n f_n^2}{1} \\
\text{s.t.} \quad f_n \left( \log \left( \frac{1}{\varepsilon_n} \right) \frac{c_n S_n}{f_n} + T_{\text{com}} \right) \leq T_{\text{max}},
\]

where $\gamma_1 = aE_{\text{com}}$ and $\gamma_2 = aE_{\text{com}}$. The constraint \((24b)\) is equivalent to $T_{\text{com}} \leq \vartheta(\varepsilon_n)$, where $\vartheta(\varepsilon_n) = \frac{1-\varepsilon_n}{m}T_{\text{max}} + \frac{c_n s_n \log_2 \varepsilon_n}{f_n}$. We have $\vartheta(\varepsilon_n)'' < 0$, and therefore, $\vartheta(\varepsilon_n)$ is a concave function. Thus, constraint \((24b)\) can be equivalent transformed to $\varepsilon_n^{\text{min}} \leq \varepsilon_n \leq \varepsilon_n^{\text{max}}$, where $\vartheta(\varepsilon_n^{\text{min}}) = \vartheta(\varepsilon_n^{\text{max}}) = T_{\text{com}}$. Therefore, $\varepsilon_n$ is the optimal solution to

\[
P_{5} : \min_{\varepsilon_n} \frac{\gamma_1 \log_2 (1/\varepsilon_n) + \gamma_2}{1 - \varepsilon_n} \\
\text{s.t.} \quad \varepsilon_n^{\text{min}} \leq \varepsilon_n \leq \varepsilon_n^{\text{max}}.
\]

Obviously, the objective function of $P5$ has a fractional in nature, which is generally difficult to solve. According to [13], [29], solving $P5$ is equivalent to finding the root of the nonlinear function $H(\xi)$ defined as follows

\[
H(\xi) = \min_{\varepsilon_n^{\text{min}} \leq \varepsilon_n \leq \varepsilon_n^{\text{max}}} \frac{\gamma_1 \log_2 (1/\varepsilon_n) + \gamma_2}{1 - \varepsilon_n} - \xi (1 - \varepsilon_n)
\]
Algorithm 3: Iterative Algorithm

1. Initialize a feasible solution $p_n, f_n, \varepsilon_n$ and set $j = 0$.

2. repeat
   3. With $\varepsilon_n^{(j)}$ obtain the optimal $p_n^{(j+1)}, f_n^{(j+1)}$ of problem
   4. With $p_n^{(j+1)}, f_n^{(j+1)}$ obtain the optimal $\varepsilon_n^{(j+1)}$ of problem
   5. Set $j = j + 1$

6. until Objective value of $P2$ converges;

Function $H(\xi)$ with fixed $\xi$ is convex. Therefore, the optimal solution $\varepsilon_n$ can be obtained by setting the first-order derivative of $H(\xi)$ to zero, which leads to the optimal solution is $\varepsilon^*_n = \frac{\gamma_1}{(\ln 2\xi)}$. Thus, similar to [13], problem $P5$ can be solved by using the Dinkelbach method in [29] (shown as Algorithm 2).

The algorithm that solves problems $P2$ is given in Algorithm 3 iteratively solving problems $P3$ and $P4$. Since the optimal solution of problem $P3$ and $P4$ is obtained in each step, the objective value of problem $P2$ is non-increasing in each step. Moreover, the objective value of problem $P2$ is lower bounded by zero. Thus, Algorithm 3 always converges to a local optimal solution.

B. Complexity Analysis

To solve the general energy-efficient resource allocation problem $P2$ using Algorithm 3, the major complexity in each step lies in solving problems $P3$ and $P4$. To solve problem $P3$, the complexity is $O(L_e \log_2(1/\epsilon_1))$, where $\epsilon_1$ is the accuracy of solving $P3$ with the bisection method and $L_e$ is the number of iterations for optimizing $f_n$ and $p_n$. To solve problem $P4$, the complexity is $O(\log_2(1/\epsilon_2))$ with accuracy $\epsilon_2$ by using the Dinkelbach method. As a result, the total complexity of the proposed Algorithm 3 is $H_eS$, where $H_e$ is the number of iterations for problems $P3$ and $P4$ and $S$ is equal to $O(L_e \log_2(1/\epsilon_1)) + O(\log_2(1/\epsilon_2))$.

After deciding the bids, the mobile users submit bids to the BS. The following section describes the auction mechanism between the BS and mobile users for selecting winners, allocating bandwidth and deciding on payment.
V. AUCtion mechanism BETWEEN BS AND MOBILE USERS

A. Problem Formulation

In bid $\Delta_{ni}$ that mobile user $n$ submits to the BS includes the number of subchannels $b_{ni}$, the number of antennas $A_{ni}$, local accuracy $\epsilon_{ni}$, and claimed cost $v_{ni}$. The utility of one bid is the difference between the payment $g_{ni}$ and the real cost $V_{ni}$.

$$U_{ni} = \begin{cases} g_{ni} - V_{ni}, & \text{if bid } \Delta_{ni} \text{ wins,} \\ 0, & \text{otherwise.} \end{cases}$$ (27)

The payment that the BS pays for winning bids is $\sum_{n,i} g_{ni}$. As we described in Section III-A, high local accuracy will significantly improve the global accuracy for a fixed number of global iterations. The utility of the BS is the difference between the BS’s satisfaction level and the payment for mobile users. The satisfaction level of the BS to bid $\Delta_{ni}$ is measured based on the local accuracy that mobile user $n$ can provide in the $i$th bid and is defined as follows

$$\chi_{ni} = \tau \epsilon_{ni}. \quad (28)$$

Thus, the total utilities of the system or the social welfare is

$$\sum_{n,i} (\chi_{ni} - v_{ni})x_{ni}. \quad (29)$$

If mobile users truthfully submit their cost, $V_{ni} = v_{ni}$, we have the social welfare maximization problem defined as follows:

$$\textbf{P6 :} \max_x \sum_{n,i} (\chi_{ni} - v_{ni})x_{ni} \quad (30a)$$

s.t. $\sum_n x_{ni}b_{ni} \leq B_{max}, \quad (30b)$

$\sum_n x_{ni}A_{ni} \leq A_{max}, \quad (30c)$

$\sum_i x_{ni} \leq 1, \forall n, \quad (30d)$

$x_{ni} = \{0, 1\}, \quad (30e)$

where (30b) and (30c) indicate the bandwidth resource (i.e., sub-channels) and the antennas limitation constraints of the BS, respectively. Then, (30d) shows that a mobile user can win at most one bid and (30e) is the binary constraint that presents whether bid $\Delta_{ni}$ wins or not.
Problem $P6$ is a minimization knapsack problem, which is known to be NP-hard. This implies that no algorithm is able to find out the optimal solution of $P6$ in polynomial time. It is also known that a mechanism with Vickrey-Clarke-Groves (VCG) payment rule is truthful only when the resource allocation is optimal. Hence, using VCG payment directly is unsuitable due to the problem $P6$ is computationally intractable. To deal with the NP-hard problem, we proposed the primal-dual based greedy algorithm. The following economic properties are desired.

**Truthfulness:** An auction mechanism is truthful if and only if for every bidder $n$ can get the highest utility when it reports true value.

**Individual Rational:** If each mobile user reports its true information (i.e., cost and local accuracy), the utility for each bid is nonnegative, i.e., $U_{ni} \geq 0$.

**Computation Efficiency:** The problem can be solved in polynomial time.

Among these three properties, truthfulness is the most challenging one to achieve. In order to design a truthful auction mechanism, we introduce the following definitions.

**Definition 1:** (Monotonicity): If mobile user $n$ wins with the bid $\Delta_{ni} = \{v_{ni}, \varepsilon_{ni}, b_{ni}, A_{ni}\}$, then mobile user $n$ can win the bid with $\Delta_{nj} = \{v_{nj}, \varepsilon_{nj}, b_{nj}, A_{nj}\} \succ \Delta_{ni} = \{v_{ni}, \varepsilon_{ni}, b_{ni}, A_{ni}\}$.

The notation $\succ$ denotes the preference over bid pairs. Specifically, $\Delta_{nj} = \{v_{nj}, \varepsilon_{nj}, b_{nj}, A_{nj}\} \succ \Delta_{ni} = \{v_{ni}, \varepsilon_{ni}, b_{ni}, A_{ni}\}$ if $\varepsilon_{nj} > \varepsilon_{ni}$ for $v_{nj} = v_{ni}, b_{nj} = b_{ni}, A_{nj} = A_{ni}$ or $v_{nj} < v_{ni}, b_{nj} < b_{ni}, A_{nj} < A_{ni}$ for $\varepsilon_{nj} = \varepsilon_{ni}$. The monotonicity implies that the chance to obtain a required bundle of resources can only be enhanced by either increasing the local accuracy or decreasing the amount of resources required or decreasing the cost.

**Definition 2:** (Critical Value): For a given monotone allocation scheme, there exists a critical value $c_{ni}$ of each bid $\Delta_{ni}$ such that $\forall n, i(\chi_{ni} - v_{ni}) \geq c_{ni}$ will be a winning bid, while $\forall n, i(\chi_{ni} - v_{ni}) < c_{ni}$ is a losing bid.

In our proposed mechanism, the difference between the satisfaction based on local accuracy and cost of one bid can be considered as the value of that bid. Therefore, the critical value can be seen as the minimum value that one bidder has to bid to obtain the requested bundle of resources. With the concepts of monotonicity and critical value, we have the following lemma.

**Lemma 1:** An auction mechanism is truthful if the allocation scheme is monotone and each winning mobile user is paid the amount that equals to the difference between the satisfaction based on the local accuracy and the critical value.

**Proof:** Similar Lemma 1 and Theorem 1 in [11].
In the next subsection, we propose a primal-dual greedy approximation algorithm for solving problem \( \textbf{P6} \). The algorithm iteratively updates both primal and dual variables and the approximation analysis is based on duality property. As the result, we firstly relax \( 1 \geq x_{ni} \geq 0 \) of \( \textbf{P6} \) to have the linear programming relaxation (LPR) of \( \textbf{P6} \). Then, we introduce the dual variable vectors \( y, z \) and \( t \) corresponding to constraints (30b), (30c) and (30d) and we have the dual of problem LPR of \( \textbf{P6} \) can be written as

\[
\textbf{P7}: \quad \max_{y,z,t} \sum_{n \in \mathcal{N}} y_n + zB_{\text{max}} + tA_{\text{max}} \tag{31a}
\]

\[
s.t. \quad y_n + zA_{ni} + tB_{ni} \geq q_{ni}, \forall n, i, \tag{31b}
\]

\[
y_n \geq 0, \forall n, \tag{31c}
\]

\[
z, t \geq 0. \tag{31d}
\]

In Section V-B, we devise an greedy approximation algorithm and Section V-C, a theoretical bound is achieved for the approximation ratio of the proposed algorithm.

**B. Approximation Algorithm Design**

In this section, we use a greedy algorithm to solve problem \( \textbf{P6} \). The main idea of the greedy algorithm is to allocate the resource to bidders with the larger normalized value. Specifically, after collecting all the bids from the mobile users, the BS as the auctioneer sorts the bids in a decreasing order of \( \frac{q_{ni}}{s_{ni}} \) which is viewed as the normalized value of a bid, where \( s_n = \eta_B n + \eta_A A_n \) is a weighted sum of the number of different types of resources requested and \( q_{ni} = \chi_{ni} - v_{ni} \) is value of bid \( \Delta_{ni} \).

**C. Approximation Ratio Analysis**

In this subsection, we analyze approximation ratio of Algorithm \( \text{Algorithm 4} \). Our approach is to use the duality property to derive a bound for approximation algorithm. We denote the optimal solution and the optimal value of LPR of \( \textbf{P6} \) as \( x^*_{ni} \) and \( OP_f \). Furthermore, let \( OP \) and \( \varphi \) as the optimal value of \( \textbf{P6} \) and the primal value of \( \textbf{P6} \) obtained by Algorithm \( \text{Algorithm 4} \). Our analysis consists of two steps. First, Theorem \( \text{Theorem 1} \) shows that Algorithm \( \text{Algorithm 4} \) generates a feasible solution to \( \textbf{P7} \), and Proposition \( \text{Proposition 1} \) provides approximation factor.

**Theorem 1:** Algorithm \( \text{Algorithm 4} \) provides a feasible solution to \( \textbf{P7} \).

**Proof:** We discuss the following three cases:
Algorithm 4: The Greedy Approximation Algorithm

1 \textbf{Input:} \((B, A, \chi, v, B_{\text{max}}, A_{\text{max}})\)

2 \textbf{Output:} solution \(x\)

3 \(U = \emptyset, x = 0\)

4 \(\forall n : y_n = 0, \psi = 0;\)

5 \(\varphi = 0, B = 0, A = 0;\)

6 \(s_n = \eta B_n + \eta A_n;\)

7 \(q_{kj} = \chi_n - v_n;\)

8 \textbf{for} \(n \in N\) \textbf{do}

9 \hspace{1em} \(i_n = \arg \max_i \{q_{ni}\};\)

10 \textbf{end}

11 \(\kappa = \max s_n;\)

12 \textbf{while} \(N \neq \emptyset\) \textbf{do}

13 \hspace{1em} \(\mu = \arg \max_{n \in N} \frac{q_{\mu i}}{s_{\mu i}};\)

14 \hspace{1em} \textbf{if} \(B + b_{\mu i} \leq B_{\text{max}} \text{ and } A + a_{\mu i} \leq A_{\text{max}}\) \textbf{then}

15 \hspace{1em} \hspace{1em} \(x_{\mu i} = 1; \quad y_\mu = q_{\mu i};\)

16 \hspace{1em} \hspace{1em} \(\varphi = \varphi + q_{\mu i};\)

17 \hspace{1em} \hspace{1em} \(\psi = \frac{\sum_{n \in U} q_{n i}}{\sum_{n \in U} s_{n i}};\)

18 \hspace{1em} \hspace{1em} \(U = U \cup \{\mu\} \text{ and } N = N \setminus \{\mu\}\)

19 \hspace{1em} \textbf{else}

20 \hspace{1em} \hspace{1em} \textbf{break};

21 \hspace{1em} \textbf{end}

22 \textbf{end}

23 \(\bar{\psi} = \kappa \psi;\)

24 \(z = \eta \bar{\psi}, \quad t = \eta \bar{\psi}\)

- Case 1: mobile user \(\mu\) wins, i.e., \(\mu \in U\) and \(b_{\mu i} = \max_{i' \in I_\mu} \{q_{\mu i'}\}\). Then we have \(y_\mu = q_{\mu i} \geq q_{\mu i'}, \forall i' \in I_\mu\). Thus, constraint \((31b)\) is satisfied for all mobile users in \(U\).

- Case 2: mobile user \(\mu\) loses the auction, i.e., \(\mu \in N \setminus U\). According to the while loop, it is evident that

\[
\frac{q_{n i}}{s_{n i}} > \frac{q_{\mu i}}{s_{\mu i}}, \forall n \in U.
\]
Therefore, $\psi > \frac{q_{\mu_i\mu}}{s_{\mu_i\mu}}$. Thus,
$$\bar{\psi} \geq \frac{q_{\mu_i\mu}}{s_{\mu_i\mu}} \geq \frac{q_{\mu_i\mu}}{s_{\mu_i\mu}}.$$

In addition, we have
$$q_{\mu_i\mu} \geq q_{\mu'\mu} \quad \text{and} \quad \kappa > \frac{s_{\mu_i\mu}}{s_{\mu_i\mu}}, \forall i' \neq i_\mu.$$

Therefore,
$$\bar{\psi} \geq \frac{q_{\mu'\mu}}{s_{\mu_i\mu}}, \forall i' \neq i_\mu.$$

Therefore, we have
$$\eta_b\bar{\psi}B_{in} + \eta_a\bar{\psi}A_{in} \geq q_{in}, \forall i' \neq i_\mu.$$

or
$$zC_{in} + tA_{in} \geq q_{in}, \forall i' \neq i_\mu.$$

Therefore, constraint (31b) is also satisfied for all mobile users in $\mathcal{N} \setminus \mathcal{U}$.

**Proposition 1:** The upper bound of integrality gap $\alpha$ between $\textbf{P6}$ and its relaxation and the approximation ratio of Algorithm 4 are $1 + \frac{\kappa \Upsilon}{\Upsilon - S}$, where $\Upsilon = \eta_b B_{\max} + \eta_a A_{\max}, S = \max_{n,i} s_{ni}$.

**Proof:** Let $OP$ and $OP_f$ be the optimal solution for $\textbf{P6}$ and LPR of $\textbf{P6}$. We can obtain the following:

$$OP \leq OP_f \leq \sum_{n=1}^{N} y_n + zB_{\max} + tA_{\max}$$

$$\leq \sum_{n=1}^{N} y_n + \bar{\psi}(\eta_b B_{\max} + \eta_a A_{\max})$$

$$\leq \sum_{n \in \mathcal{N}} q_{ni_n} + \bar{\psi}(\eta_b B_{\max} + \eta_a A_{\max})$$

$$\leq \left( \sum_{n \in \mathcal{N}} q_{ni_n} \right) \left( 1 + \frac{(\eta_b B_{\max} + \eta_a A_{\max})\kappa}{\eta_b B_{\max} + \eta_a A_{\max} - S} \right)$$

$$\leq \varphi \left( 1 + \frac{\Upsilon \kappa}{\Upsilon - S} \right).$$

Therefore, the integrality $\alpha$ is given as
$$\frac{OP_f}{OP} \leq \frac{OP_f}{\varphi} \leq \left( 1 + \frac{\kappa \Upsilon}{\Upsilon - S} \right).$$
The approximation ratio is
\[
OP/\phi \leq OP_I/\phi \leq \left(1 + \frac{\kappa \Upsilon}{I - S}\right).
\]
\[\blacksquare\]

D. Payment

Then we will find the critical value which is the minimum value a bidder has to bid to win the requested bundle of resources. In this paper, we consider the bid combinations submitted by mobile user \(n\) as the combinations of bid submit by virtual bidders, in which each virtual bidder can submit one bid. Therefore, the number of virtual bidders corresponding to mobile user \(n\) is equal to the number of bids \(I_n\) that mobile user \(n\) submits. Denote by \(m\) the losing mobile user with the highest normalized value if mobile user \(n\) is not participating in the auction. Accordingly, the minimum value mobile user \(n\) needs to place is \(q_{\text{mim}} s_{ni}\), where \(i_m\) and \(i_n\) are the indexes of highest normalized value bids of mobile user \(m\) and \(n\), respectively. Thus, the payment of winning mobile user \(n\) in the pricing scheme is \(g_{ni} = \chi_{ni} - q_{\text{mim}} s_{ni}\).

E. Properties

Now, we show that the winner determination algorithm is monotone and the payment determined for a winner mobile user is the difference between the local accuracy based satisfaction and the critical value of its bid. From line 13 of the Algorithm 4, it is clear that a mobile user can increase its chance of winning by increasing its bid. Also, a mobile user can increase its chance to win by decreasing the weighted sum of the resources. Therefore, the winner determination algorithm is monotone with respect to mobile user’s bids. Moreover, the value of a winning bidder is equals to the minimum value it has to bid to win its bundle, i.e., its critical value. This is done by finding the losing bidder \(m\) who would win if bidder \(n\) would not participate in the auction. Thus, the proposed mechanism has a monotone allocation algorithm and payment for the winning bidder equals to the difference between the local accuracy based satisfaction and the critical value of its bid. We conclude that proposed mechanism is a truthful mechanism according to Lemma 1.

Next, we prove that the proposed auction mechanism is individual rational. For any mobile user \(n\) bidding its true value, we consider two possible cases:
• If mobile user \( n \) is a winner with its bid \( i \)th, its payment is

\[
U_{ni} = g_{ni} - v_{ni} = \left( \chi_{ni} - \frac{q_{min}}{s_{min}} s_{ni} - v_{ni} \right) = \left( \chi_{ni} - v_{ni} s_{ni} - \frac{q_{min}}{s_{min}} \right) s_{ni} = \left( \frac{q_{ni}}{s_{ni}} - \frac{q_{min}}{s_{min}} \right) s_{ni} \geq 0
\]

where \( m \) the losing bidder with the highest normalized valuation if \( n \) does not participate in the auction and the last inequality follows from Algorithm 4.

• If mobile user \( n \) is not a winner. Its utility is 0.

Therefore, the proposed auction mechanism is individual rational.

Finally, we show that the proposed auction mechanism is computationally efficient. We can see that in Algorithm 4, the while-loop (lines 12-22) takes at most \( N \) times, linear to input. Calculating the payment takes at most \( N(N - 1) \) times. Therefore, the proposed auction mechanism is computationally efficient.

VI. Simulation Results

In this section, we provide some simulation results to evaluate the proposed mechanism. The parameters for the simulation are set the following. The required CPU cycles for performing a data sample \( c_n \) is uniformly distributed between \([10, 50]\) cycles/bit. The size of data samples of each mobile user is \( s_n = 800 \times 10^3 \), the maximum tolerance time of a FL task is \( T_{max} = [100, 500] \). The effective switched capacitance in local computation is \( \xi = 10^{-26} \). We assume that the noise power spectral density level \( N_0 \) is \(-174 \) dBm/Hz, the sub-channel bandwidth is \( W = 15 \) kHz and the channel gain is uniformly distributed between \([-90, -95]\) dB. In addition, the maximum and minimum transmit power of each mobile user is uniformly distributed between \([3, 6]\) mW and between \([1, 2]\) mW, respectively. The maximum and minimum computation capacity is uniformly distributed between \([3, 5]\) GHz and between \([0.1, 0.2]\) GHz, respectively. We also assume that the total number of sub-channels and antennas of the BS are 100 and 100, respectively. Firstly, we use the iterative Algorithm 3 to perform the characteristic of evaluating bids. The maximum number of sub-channels \( B_{n}^{max} \) and antennas \( A_{n}^{max} \) for mobile user \( n \) to request in each bid vary from 10 to 50. Fig. 2a shows the accuracy level that mobile user \( n \) requires to provide decreases when the maximum number of sub-channels \( B_{n}^{max} \) and
Fig. 2: Numerical results a) The changing of the local accuracy when the maximum number sub-channels and antennas in one bid vary, b) The changing of the energy cost when the maximum number sub-channels and antennas in one bid vary.

Fig. 3: Numerical results a) Local accuracy v.s. $T_{\text{max}}$ b) Energy cost v.s. $T_{\text{max}}$

antennas $A_{\text{max}}^n$ increase. However, the decreasing requested local accuracy of mobile user data leads to the increase of global and local rounds to achieve global accuracy. As a result, the cost increases when the number of sub-channels and antennas increases, as shown in Fig. 2b.

Fig. 3a and Fig. 3b present the cost of one bid of the mobile user and local accuracy, respectively, when the maximum tolerance time $T_{\text{max}}$ varies from 100 to 500. When the maximum tolerance time increases, the cost decreases. It is natural because mobile user $n$ can keep low contributing CPU cycle frequency and transmission rate while guaranteeing the delay constraint.

In the following, we evaluate the performance of the proposed auction algorithm. To compare with the proposed algorithm, we use three baselines:
Fig. 4: Numerical results for social welfare a) four schemes: Optimal solution, fractional optimal solution, proposed greedy algorithm and lower bound b) three schemes: Optimal solution, proposed greedy algorithm and fixed price scheme.

- **Optimal Solution**: $P_6$ is solved optimally.
- **Fractional Optimal Solution**: the linear relaxation of $P_6$ is solved optimally.
- **Fixed Price Scheme**: In this scheme, price vector $f = \{f_b, f_a\}$ is the vector that mobile users need to pay for the resource. In this scheme, the mobile users are assumed to be served in a first come, first served basic until the resources are exhausted. The mobile user can get the resource when the valuation of mobile user’s bid is at least $F_{ni} = B_{ni}f_b + A_{ni}f_a$ which is the sum of the fixed price of each resource in its bid. We consider three kinds of price vector: linear price ($f_i = f_o \times \eta_i, i = a, b$), sub-linear price vector ($f_i = f_o \times \eta_i^{0.85}, i = a, b$) and a super linear price vector ($f_i = f_o \times \eta_i^{1.15}, i = a, b$). Here, we call $f_o$ as the basic price.

Unless specified otherwise, we choose $f_o = 0.01$.

Fig. 4a reports the performance of the optimal solution, the fractional optimal solution, the lower bound, and the proposed greedy scheme. The lower bound is determined by the fractional optimal solution divided by gap when the number of mobile users varies from 10 to 50. We note that with the number of mobile users increasing, all schemes produce higher social welfare. This is because there is more chances to choose winning bids with the higher value. Although the social welfare obtained through the proposed greedy scheme is lower than through optimal solution and fractional optimal solution, it much higher than the lower bound.

Fig. 4b shows the social cost achieved by optimal solution, the proposed greedy scheme and fixed linear scheme when the number of mobile users varies from 10 to 50. We can see that
Fig. 5: Numerical results for social welfare when a) $\eta_a = 1, \eta_b = 0.5$, b) $\eta_a = 1, \eta_b = 1$, c) $\eta_a = 1, \eta_b = 2$.

the proposed greedy scheme can provide the much higher social welfare than the fixed linear scheme.

Since the fixed price scheme heavily depends on the prices of resources, the next experiment helps us to decide whether the fixed-price vector or the performance of the proposed mechanisms is better when we change the basic price $f_0$ between $[0.01, 0.31]$ with the step is 0.03. Fig. 5a, Fig. 5b and Fig. 5c show that the social welfare of fixed price firstly increases and then decreases and equal to 0 when the initial price increases. This is because when the basic price becomes too high, the sum of the price is higher than the valuation of the resources claimed in a bid. Moreover, the social welfare achieved by linear, sublinear and superlinear price schemes are lower than by the proposed greedy scheme. This proves our proposed auction scheme outperforms the fixed price scheme.

In Fig. 6a, Fig. 6b and Fig. 6c, we observe the metrics: social welfare, resource utilization and
percentage of three schemes: optimal solution, greedy proposed scheme and fixed price schemes with linear (Fig. 6a), sublinear (Fig. 6b), superlinear (Fig. 6c) fixed price vector. We perform in terms of the ratio with proposed greedy scheme. Among these schemes, the optimal solution is the highest in terms of all metrics. Compared with the proposed scheme, the fixed price can utilize more resources and more mobile users but provides less social welfare. This is due to the fact that the fixed price mechanism heavily depends on the prices of the resources.

VII. CONCLUSION

This paper focus on the incentive mechanism design to stimulate mobile users to participate in FL. We formulated the incentive problem between the BS and mobile users in the FL service market as the auction game with the objective of maximizing social welfare. Then, we presented the method for mobile users to decide the bids submitted to the BS so that mobile users can minimize the energy cost. We also proposed the iterative algorithm with low complexity. In addition, we proposed a primal-dual greedy algorithm to tackle the NP-hard winner selection problem. Finally, we showed that the proposed auction mechanism guarantee
truthfulness, individual rationality and computation efficiency. Simulation results demonstrated
the effectiveness of the proposed mechanism where social welfare obtained by our proposed
mechanism is 400% larger than by the fixed price scheme.

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