Blind Quantum Computation Using a Circuit-Based Quantum Computer

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Abstract

When a universal quantum computer is used by the public, it is assumed that it will be in the form of a quantum cloud server that exists in a few bases due to its cost. In this cloud server, privacy will be a crucial issue, and a blind quantum computation protocol will be necessary so that each user can use the server without the details of the calculations being revealed. It is also important to be able to verify that the server is performing calculations as instructed by the user, since quantum calculations cannot be verified by classical computation. In this paper, we put forward a protocol that achieves blindness using the quantum one-time pad for encryption and a T-like gate, and while verifying computation using trap qubits.

1 Introduction

When universal quantum computer is used by public, it is assumed that it will be used as a quantum cloud server that exists in a few bases because the quantum computer is expensive. In this cloud server, privacy will be a crucial issue. Thus, blind quantum computation (BQC) protocol is needed so that each user can use the server without revealing the details of his or her calculations. [1-9].

In [2], such a method was proposed based on the quantum onetime pad. Similar to classical one-time pads [10], the quantum one-time pad uses the encryption key only once, and the server cannot learn anything about the user’s quantum state. However, this protocol needs multiple two-way quantum communications. In addition, the user is required to have a quantum memory on which a SWAP gate is executed. In [9], another protocol was proposed that requires neither quantum gates and two-way quantum communication nor

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quantum memory and SWAP gates during its computation. In this protocol, however, while the input and output are encrypted, the calculation process is revealed to the server. This is a crucial drawback because an algorithm itself can constitute important information that should be kept secret. In addition, as a malicious server might have performed a calculation different from the user’s instruction, a user must have the ability to verify the calculation \[4,5,7,11\]. However, quantum computers generally cannot be simulated in polynomial time by classical computers, and a user with the limited ability assumed in previous research cannot calculate whether the results obtained from the server are correct. Furthermore, such a user cannot verify calculations using trap qubits, which are closely related to the secrecy of the calculation process.

In this paper, we propose a novel BQC protocol using rotation gates in addition to the quantum one-time pad. Our protocol enables verification by trap qubits and can be extended to fault-tolerant computation. In this protocol, the abilities required for the user are equivalent to those required in the previous BQC protocol \[4\].

\section{Preliminaries}

In this section, we describe gate teleportation and the encryption method as known the quantum one-time pad, which are both used in the proposed protocol. See \[12\] for the general quantum computation notation.

\subsection{Gate teleportation}

We explain gate teleportation for a $T$ gate that is used for universal gate sets and an $A_\theta$ gate that is used for blindness in the protocol. The $T$ gate is

\[
T = \begin{pmatrix}
1 & 0 \\
0 & e^{i\pi/4}
\end{pmatrix}.
\]

and the $A_\theta$ gate is

\[
A_\theta = \begin{pmatrix}
1 & 0 \\
0 & e^{i\theta}
\end{pmatrix}.
\]

When $\theta = \pi/4$, the $A_\theta$ gate is equivalent to the $T$ gate.

For a given state $|\psi\rangle$, $A_\theta |\psi\rangle$ is obtained by using gate teleportation (Figure 1), without directly executing the $A_\theta$ gate, where $a$ is the measurement result, and $|A_\theta\rangle$ is

\[
|A_\theta\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{i\theta} |1\rangle).
\]

\subsection{Quantum one-time pad}

In the quantum one-time pad, a user (Alice) generates 2-bit encryption key $a,b \in \{0,1\}$ using coin flips, and executes an $X^a$ gate and a $Z^b$ gate to encrypt her
\[ |A_\theta \rangle 
\begin{array}{c}
A(-1)^a |\psi\rangle \\
|\psi\rangle 
\end{array} 
\]
\[ a \in 0, 1 \]

Figure 1: Executing $A_\theta$ gate by gate teleportation.

input. The state after encryption $|\psi\rangle_{\text{enc}}$ is

\[ |\psi\rangle_{\text{enc}} = X^a Z^b |\psi\rangle, \]

where $|\psi\rangle$ is input, and the $X$ and $Z$ gate gates are Pauli matrices:

\[ X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \]

The density matrix obtained by the server (Bob) receiving this quantum state is maximally mixed state, as described below:

\[ \frac{1}{4} \sum_{a,b=0}^{1} X^a Z^b |\psi\rangle \langle \psi| Z^b X^a = \frac{I}{2}. \]

Therefore, Bob, who does not know the randomly generated encryption key, cannot learn the input from the received qubits.

Alice, who has received the quantum state after performing the calculation $U$, can decrypt this state using the encryption keys $a'$ and $b'$, which were altered by the calculation to:

\[ Z^{b'} X^{a'} U |\psi\rangle_{\text{enc}} = Z^{b'} X^{a'} (X^{a'} Z^{b'} U |\psi\rangle) = U |\psi\rangle. \]

For U it suffices to consider only H, T, and CNOT, which form a universal gate set. The correspondence between pairs $(a, b)$ and $(a', b')$ for each gate is described in Figures 2-6.

Note that when the T gate is executed as shown in Figure 6, we obtain the following $S$ gate to be modified in addition to the $X$ and $Z$ gates. Here the $S$ gate is

\[ S = T^2 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \]

The universal gate set requires a non-Clifford gate, such as a T or Toffoli gate [13]. When executing these non-Clifford gates, Alice requires modifications other than the $X$ gate and the $Z$ gate [2,8].

\[ X^a Z^b |\psi\rangle 
\begin{array}{c}
X \\
X^a Z^b X |\psi\rangle 
\end{array} 
\]

Figure 2: Key change at the $X$ gate.
In this section, we explain the $A_\theta$ gate that is important for our BQC protocol. First, we extend the quantum one-time pad by adding the $A_\theta$ gate. Next, we show how to modify the $T$ gate and the $A_\theta$ gate in the quantum one-time pad. Finally, we explain universal quantum computation using the $A_\theta$ gate.

### 3.1 Quantum one-time pad for the $A_\theta$ gate

In this subsection we show that the $A_\theta$ gate can be hidden using the quantum one-time pad. When $|A_\theta\rangle$ is encrypted using the quantum one-time pad, it is given by

$$|A_\theta\rangle_{\text{enc}} = X^a Z^b |A_\theta\rangle.$$  

When executing gate teleportation by $|A_\theta\rangle_{\text{enc}}$, the $A_\theta$ gate works as shown in Figure 7 since the $Z$ gate commutes with the $A_\theta$ gate. Thus, it is possible to encrypt the $A_\theta$ gate using the quantum one-time pad. Note that the $A_\theta$ state is in the maximum mixed state without encryption using the $X$ gate, meaning such encryption using the $X$ gate is not required. But the $X$ gate is used to hide the measurement result. It is using for the modification described in the following subsection.
3.2 Modifying the $A_\theta$ gate

To apply the $T$ gate and the $A_\theta$ gate to a quantum state encrypted by the $X$ gate, the quantum state requires modification. When the $A_\theta$ gate is applied to the quantum state encrypted by the $X$ gate, an $A_{-\theta}$ gate is applied to the quantum state instead of the $A_\theta$ gate as shown in Figure 8. When Alice obtains the undesired measurement results, the $A_\theta$ gate is executed, since the angle is flipped. Thus, even if it is encrypted with the $X$ gate, flipping the desired measurement result Alice can execute the $A_\theta$ gate without changing $A_\theta$ state. The $T$ gate is a special case of the $A_\theta$ gate and can be modified in a similar way.

As mentioned above, the $A_\theta$ gate requires additional correction because the angle $\theta$ executed varies depending on the measurement result. Here, the angle is limited to $\theta = \frac{n\pi}{4} (n = \{0, 1, \ldots, 7\})$. If the measurement gives an undesired result, an $A_{-\theta}$ gate is executed, meaning Alice needs to execute an $A_{2\theta}$ gate to correct it. The gate used for this correction may also bring about undesired measurement results. The next gate for the second modification is the $A_{4\theta}$ gate, for which the angle $\theta$ is limited to $\theta = \frac{n\pi}{4} (n = \{0, 1, \ldots, 7\})$. Thus $A_{4\theta} = Z$ or $I$, meaning the correction can be completed by executing the $Z$ gate or the $I$ gate. If $\theta$ is limited to $\theta = \frac{n\pi}{4} (n = \{0, 1, \ldots, 7\})$, Alice can, therefore, be sure to execute the $A_\theta$ gate by preparing two additional qubits and one additional gate.

3.3 T-like gate group and one-qubit universal gate

In [12], an approximation of any one-qubit gate using the $T$ gate and the $H$ gate is achieved, as these two gates can achieve non-parallel two-axis rotation on the Bloch sphere. We represent gate blindness using non-parallel eight-axis rotation and a T-like gate. The T-like gate is defined as follows:

$$T = A_{\frac{\pi}{4}} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}, \quad T^3 = A_{\frac{3\pi}{4}} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{3\pi}{4}} \end{pmatrix},$$

$$T^\dagger = A_{-\frac{\pi}{4}} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\frac{\pi}{4}} \end{pmatrix}, \quad (T^3)^\dagger = A_{-\frac{3\pi}{4}} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\frac{3\pi}{4}} \end{pmatrix}.$$
By combining the T-like gate with the $H$ gate, it is possible to make rotations at eight axes that are not parallel. Table 1 shows these eight axes along with the gate combinations. In particular, note that the rotation axis of $THT^\dagger H$ is parallel to the rotation axis of $HTHT$. It is known that an arbitrary one-qubit gate can be approximated by the combination of $THTH$ and $HTHT$ [12]. The rotations of these two axes are not parallel, meaning that they can achieve any rotation for the quantum state that Alice desires. This is known thus a universal gate set for a one-qubit gate. In the same way, any one-qubit gate can be approximated by the gate group shown in Table 1.

Table 1: Axes of rotation and the corresponding gate combinations

| Gate       | Axis of rotation                           |
|------------|-------------------------------------------|
| $THTH$     | $(\cos \frac{\pi}{8}, \sin \frac{\pi}{8}, \cos \frac{\pi}{8})$ |
| $THHT^\dagger$ | $(-\cos \frac{\pi}{8}, -\sin \frac{\pi}{8}, \cos \frac{\pi}{8})$ |
| $T^\dagger HT^\dagger H$ | $(\cos \frac{\pi}{8}, -\sin \frac{\pi}{8}, -\cos \frac{\pi}{8})$ |
| $T^\dagger HT^\dagger H$ | $(-\cos \frac{\pi}{8}, \sin \frac{\pi}{8}, -\cos \frac{\pi}{8})$ |
| $T^3H(T^3)^\dagger H$ | $(-\cos \frac{3\pi}{8}, -\sin \frac{3\pi}{8}, \cos \frac{3\pi}{8})$ |
| $(T^3)^\dagger HT^3H$ | $(-\cos \frac{3\pi}{8}, -\sin \frac{3\pi}{8}, -\cos \frac{3\pi}{8})$ |
| $(T^3)^\dagger H(T^3)^\dagger H$ | $(-\cos \frac{3\pi}{8}, \sin \frac{3\pi}{8}, -\cos \frac{3\pi}{8})$ |

Bob cannot discover which gate combination was chosen because he cannot obtain the received state $|A_\theta\rangle$. At the same time, Alice can realize any one-qubit gate without it being known to Bob. We make use of this feature for the BQC protocol.

4 Main protocol

In this section, we describe two protocols. In Protocol 1, Alice performs the calculation without Bob knowing the input/output or the calculation process other than position of the $CNOT$ gate. In Protocol 2, Alice conceals the input/output and calculation process.

In the following, Bob has a universal quantum computer, and Alice has the ability to prepare a computational basis $|0\rangle, |1\rangle$, and a state $|A_\theta\rangle$ such that $\theta = \frac{\pi n}{4}$ ($n = \{0, 1, \ldots, 7\}$) to execute the $X$ and $Z$ gates and perform classical calculations. It should be noted that this protocol does not require Bob to have the ability to execute the non-Clifford gate group.

4.1 Protocol 1

According to Section 3, we can execute any one-qubit gate without it being known to Bob. Our first protocol uses such a hidden one-qubit gate.

Step 1. Alice makes a calculation circuit for her calculation.
Step 2. Alice converts the circuit to a weak blind circuit according to Table 1. This conversion is optional, meaning Alice can create a number of structure circuits. Alice chooses one of them.

Step 3. Alice encrypts the necessary input qubits using the quantum one-time pad and sends them to Bob. In addition, Alice encrypts the ancilla bits required for the $T$ gate and its modification with the quantum one-time pad and sends them to Bob.

Step 4. After sending all the qubits, Alice sends Bob the circuits for the computation. Bob computes the circuits using the ancilla bits as well as his $H$ and $CNOT$ gates. Then, Bob sends the measurement result to Alice and asks whether it is the desired result. If the result is not the desired one, Bob makes additional modifications using additional ancilla bits and his $Z$ gate.

Step 5. Bob sends the qubits to Alice after the calculation is completed. Alice unencrypts the sent qubits, measures them, and obtains the result. Alice can perform the calculation without Bob knowing the input/output or the calculation process aside from the position of the $CNOT$ gate. Here, we define weak blindness.

**Definition 1** (weak blindness). Let $P$ be a quantum delegated computation on input $X$ and let $L(X)$ be any function of the input. We say that a quantum delegated computation protocol is weak blind while leaking at most $L(X)$ and position of the $CNOT$ gate if, on Alice’s input $X$, for any fixed $Y = L(X)$, the following two hold when given $Y$:

1. The distribution of the classical information obtained by Bob in $P$ is independent of $X$.
2. Given the distribution of classical information described in 1, Bob cannot know about 1-qubit gate executed between the $CNOT$ gate.

**Theorem 1.** Protocol 1 is weak blind while leaking at circuit size and the $CNOT$ gate position.

**Proof.** Bob obtains information on the circuit size and CNOT position based on Alice’s calculation procedure, while Alice’s input and ancilla bits are encrypted by the quantum one-time pad. The encryption key does not depend on the input and ancilla bits, meaning Bob knows nothing about them. Bob makes a measurement when executing a one-qubit gate, but the measurement result has a success probability of $1/2$, regardless of the gate executed and the input. Therefore, Bob does not obtain any information when executing the one-qubit gate. Bob cannot learn anything about the output because the computed state, which is the output, is still encrypted by the quantum one-time pad. Protocol 1 satisfies weak blindness because Bob does not know anything other than the circuit size and the CNOT position.
Bob does not know anything about the state of the qubits he receives, which Alice has encrypted using the quantum one-time pad. At the same time, the quantum operation and measurement result do not depend on the contents of the input and the one-qubit gate. Thus, Bob can determine only the size of the input and the position of the CNOT gate. Note also that Alice can increase the input size by sending dummy ancilla bits.

However, as Bob knows the position of the CNOT gate, there is the possibility that he can infer the algorithm based on this information when executing a known algorithm, such as Shor’s or Grover’s algorithm. However, if an algorithm or application is unknown to the public, Alice can execute it without Bob being able to obtain knowledge of it. In other words, unpublished algorithms and applications can be tested without fear of Bob eavesdropping.

Though the existing version of Broadbent’s protocol \[6\] requires one ancilla bit per \( T \) gate, the ability to conceal unpublished algorithms can be achieved by adding two additional ancilla bits to the protocol. However, in protocol 1 Alice’s ability must be higher than that required by Broadbent’s protocol; she must have the ability to prepare \( |0\rangle \) and \( |1\rangle \) and execute the \( X \), \( Z \), \( H \), and \( S \) gates.

### 4.2 Protocol 2

A circuit referring to the BFK protocol can be created, such as the one shown in Figure 9. The identity gate and the \( CNOT \) gate can be realized by combining the two \( CZ \) gates and one-qubit gates, as shown in Figures 10 and 11. The identity gate and the \( CNOT \) gate can be realized by combining the two \( CZ \) gates and one-qubit gates as shown in Figure 10. Here, \( R_z(\frac{\pi}{2}) \) and \( R_x(\frac{\pi}{2}) \) represent the z-axis and x-axis rotation of the Bloch sphere, respectively. Since Bob cannot obtain information concerning one-qubit gates, he cannot determine whether the identity gate or the \( CNOT \) gate is realized by the two \( CZ \) gates. By arranging the two sets of the \( CZ \) gates alternately and in a staggered manner,
Figure 10: Combination of the $CZ$ gates and one-qubit gates acting as the identity gate

\[ R_Z(4)I \]

Figure 11: Combination of the $CZ$ gates and one-qubit gates acting as the $CNOT$ gate

\[ R_x(4)R_x(4) \]

as shown in Figure 9, Alice can perform the calculation without letting Bob know the position of the $CNOT$ gate.

The procedure for Protocol 2 is the same as the procedure for Protocol 1. However, it is here necessary to send extra ancilla bits for single qubits between the $CZ$ gates.

**Step 1.** Alice makes a calculation circuit for her calculation.

**Step 2.** Alice converts the circuit to a blind circuit using Table 1 and Figures 9-11. Note that the number of one-qubit gates between two $CZ$ gates is constant so as to achieve blindness. When the number of one-qubit gates is not constant, Alice adds dummy qubits. This conversion is optional, meaning that Alice can create a number of structure circuits. Alice chooses one of them.

**Step 3.** Alice encrypts the necessary input the qubits using quantum one-time pad and sends them to Bob. In addition, Alice encrypts the ancilla bits required for the $T$ gate and its modification with the quantum one-time pad and sends them to Bob.

**Step 4.** After sending all the qubits, Alice sends Bob the circuits for the computation. Bob computes the circuits using the ancilla bits as well as his $H$ and $CZ$ gates. At this time, Bob sends the measurement result to Alice and asks whether it is the desired result. If the result is not the one desired, Bob makes additional modifications using additional ancilla bits and his $Z$ gate.

**Step 5.** Bob sends the qubits to Alice after the calculation is completed. Alice unencrypts the sent qubits, measures them, and obtains the result.
Alice can perform quantum computation while concealing the entire calculation process, including the position of the CNOT gate. Here, we define blindness.

**Definition 2** (Blindness [2, Definition 2]). Let P be a quantum delegated computation on input X and let L(X) be any function of the input. We say that a quantum delegated computation protocol is blind while leaking at most L(X) if, on Alice’s input X, for any fixed Y = L(X), the following two hold when given Y:

1. The distribution of the classical information obtained by Bob in P is independent of X.

2. Given the distribution of classical information described in 1, the state of the quantum system obtained by Bob in P is fixed and independent of X.

**Theorem 2.** Protocol 2 is blind while leaking at circuit size.

**Proof.** Bob obtains information on the circuit size based on Alice’s calculation procedure. Alice’s input and ancilla bits are encrypted by the quantum one-time pad, and the encryption key does not depend on the input and ancilla bits, meaning Bob knows nothing about them. Bob takes his measurement while executing a one-qubit gate, but the measurement result has a success probability of 1/2 regardless of the gate executed and the input. Therefore, Bob does not obtain any information when executing the one-qubit gate. The CNOT gate cannot be distinguished from the identity gate by combining one-qubit gates and CZ gates, so Bob can execute the CNOT gate without knowing where in the circuit it was executed. Bob cannot learn anything about the output because the computed state, which is the output, is still encrypted by the quantum one-time pad. Protocol 2 satisfies blindness because Bob does not know anything other than the circuit size.

Bob can obtain no information about the state of the qubit received from Alice and encrypted by the quantum one-time pad, and the quantum operation and measurement result do not depend on the input and calculation processes. Therefore, Bob can know only the size of the input, and Protocol 2 satisfies blindness. Again, note that the size can be increased by sending dummy ancilla bits.

In protocols developed in previous research, the blindness of all calculation processes was performed simultaneously by Bob for each gate, including \{H, P, T, CZ, CNOT\}, from a necessary qubit to computation and dummy qubits. Then, Bob sent those qubits back to Alice, who saved the necessary qubits in quantum memory and sent them back when Bob needed to execute the necessary gates [9, 10]. According to Protocol 2, the quantum memory and additional quantum communication can be reduced, and Alice’s ability needs only to be equivalent to that required by the BFK protocol. However, this protocol requires more ancilla bits than Protocol 1.
5 Verification and fault tolerance

5.1 Verification

Verification is closely related to blindness \[1\]. If the problem that Alice wishes to solve is included in the computational complexity class NP, it can be verified using a classical computer, but it is also believed that BQP is \(\text{BQP} \not\subset \text{NP} \) \[19,20\]. Therefore, it is difficult to verify whether the result of the problem included in BQP is correct using a classical computer and it is also difficult for Alice to do so, for she has few quantum resources. Thus, it is necessary to verify that the evil Bob does not follow Alice’s particular instructions and instead performs different operations. Here, we show that verification using trap qubits is possible in both Protocol 1 and Protocol 2. Note that a method that does not use trap qubits, which can be used for blindness calculations, is also known \[17\]. Even if Alice has only a classical computer, there is still a method that can do perform verification \[18\].

The following verification method using trap qubits can be used for both Protocol 1 and Protocol 2. Since the input and the one-qubit gate are hidden in these protocols, the trap qubit can be put in the input qubit. The trap qubit \(|0\rangle\) or \(|+\rangle\) is encrypted with quantum one-time pad, and all trap qubit gates are implemented as the identity gates, which are the \(A_{\theta}\) gates with \(\theta = 0\). However, since the identity gate succeeds with probability 1, there must be a 1/4 chance that the desired measurement result is not obtained once (Alice executes the \(A_{\theta}\) gate operation with two \(\theta = 0\) qubits, which are identity gates). At the same time, there must also be a 1/4 chance that the desired measurement result is not obtained twice (Alice executes the \(A_{\theta}\) gate, which realizes the identity gate of \(\theta = 0\) and the \(Z\) gate of \(\theta = \pi\), and lets Bob correct it with the \(Z\) gate). When the evil Bob attempts to operate the gate differently from Alice’s instruction, Alice can know stochastically whether or not he operates on the trap qubit. In the case of calculating \(N\) qubits mixed with \(N_d\) trap qubits, Alice can detect evil Bob’s operation with the probability of \(\frac{N_d}{N}\). Alice can increase the probability of detection to \(1 - \left(\frac{N - N_d}{N}\right)^s\) by performing the same calculation \(s\) times.

5.2 Fault tolerant quantum computation

It is known that the ability to perform error correction in a universal quantum computer is an indispensable function, since coherence is destroyed by external noise when manipulating a quantum state \[12,21,24\]. It has also been shown that there is no universal gate set that is transversal (does not spread errors) \[25,26\]. However, it is known that the \(H\) gates and the CNOT gates can implement error correction codes in a transversal manner (without spreading errors) \[27,28\]. For the \(T\) gate, this method is implemented only by transversal CNOT gates and measurement by teleportation. In this protocol, the gates used in Bob’s calculation are only the \(H\) gate and the CNOT gate, and a non-transversal \(T\)-like gate can execute a logical \(T\)-like gate by preparing multiple similar \(A_{\theta}\) state. Therefore, It can be extended to fault tolerant calculations. In the proposed
protocols, only the $H$ gate and the $CNOT$ gate are used in Bob’s calculation, and a non-transversal T-like gate can execute a logical T-like gate by preparing multiple similar $A_\theta$ states. Therefore, the protocols can be extended to fault-tolerant calculations.

6 Conclusion

In this paper, we proposed two BQC protocols using circuit-based quantum computation (CBQC). These two protocols execute computation with weak blindness and blindness, respectively. In previous research [2][6], it has been discovered that Alice’s input and output can be concealed from Bob using the quantum one-time pad. However, previous techniques did not conceal the calculation process. In our protocol, blindness was achieved using gate teleportation and expanding the $T$ gate, which is important for universal quantum computation, to a T-like gate. First, we proposed a protocol with weak blindness (Protocol 1), which discloses the position of the CNOT gate in addition to the size of the circuit, for which a non-disclosed calculation algorithm is sufficient. In Protocol 2, BQC was achieved using CBQC. Further, it was these were shown that verification using trap qubits is possible using these protocols. We also showed that the method can be extended to fault-tolerant calculations by the same method for error correction using magic state.

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