Optimal distribution width for ALM in LES for (NEW) MEXICO experiment

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Abstract. The Actuator Line Method exists for more than a decade and has become a well established choice for simulating wind rotors in computational fluid dynamics. Numerous implementations exist and are used in the wind energy research community. A crucial parameter of this method is the distribution width used to distribute the blade forces in the domain in order to attain numerical stability but also to apply the force term over a length which makes physically sense. In this work the implementation is verified with results from the (NEW) MEXICO experiment and the optimal distribution width is found by setting the global torque as target variable.

1. Introduction
Near-wakes of wind turbines depend strongly on the rotor representation, therefore this work will rely on the Actuator Line Method (ALM) [1]. The main characteristic of this region is the presence of the distinct blade effects and therefore this model is applied to enable the presence of tip vortices but at lower cost than a fully resolved rotor simulations as done in [2, 3].

The ALM used in this work is based on the SOWFA$^1$ project, which contains an implementation of the ALM as described in [4]. The method was validated against a similar implementation within EllipSys3D [5] and verified against the results of the MEXICO and NEW MEXICO experiment ([6] and [7] respectively) in [8].

In order to capture the transient effects and also to allow the introduction of different kind of ambient turbulent flows in future work, Large Eddy Simulations (LES) are used.

Once the method is set up, there will be an analysis of the optimal force distribution width for the ALM based on the global torque. It should be noted that the torque is an integrated variable of the forces acting on the turbine blades. As proposed in [4] and [9] a distribution of $\epsilon/\Delta x = 2$ was suggested as a trade-off between avoiding non-physical oscillations around the insertion point of the momentum sink and a small enough region of distributed forces in order to obtain the right induction at the rotor. While more recent work [10] goes lower than the proposed value, it is advantageous to adapt the distribution width in accordance with the grid refinement as shown in [11] and [12]. While most studies look solely at the behaviour of the numerical results for different distribution widths and grid refinements, this work tries to tie the optimal distribution width to the experimentally obtained torque. It should be kept in mind

$^1$ NWTC Design Codes (SOWFA (Simulator for Wind Farm Applications) by Matt Churchfield and Sang Lee) https://github.com/NREL/SOWFA/. NWTC (National Wind Technology Center) is part of NREL (National Renewable Energy Laboratory) based in Golden, CO, USA.
that this presents a specific solution to the here examined cases. In order to generalize this approach, it should be applied to other rotors and experiments.

2. Methodology

The forces of the ALM are calculated based on the sampled velocities at the blade positions and the airfoil coefficients for the respective angle of attack.

The velocities are sampled by correcting the cell center value of the cell containing the actuator point by the cell velocity gradient. When using a central differencing discretization scheme this approach resembles to trilinear interpolation. Hence it compares fairly well in a comparison as show in [8].

In accordance to the local blade Reynolds at the radial positions the airfoil tables are selected for each airfoil [6]. This data was obtained through wind tunnel experiments and therefore it does not include the stall delay due to boundary layer stabilizing effects such as Coriolis and centrifugal forcing which enhance the lift of the airfoil [13]. In the case of the MEXICO rotor an adaption was proposed by [14] circumventing this issue, but at the same time proposing a solution tuned for a known outcome. Hence this work relies on unmodified airfoil data in order to assess their applicability and limits for wind turbine modeling. The wind tunnel data was extended over a range of 360 deg with the help of the flat plate assumption [15].

There are also other methods including this lift-enhancing phenomenon such as correcting the airfoil data by applying the Du’s method [16] for the lift. Other possible corrections can be found in [15] or [17] which could be included in future work.

While intuitively a tip correction would not be included in ALM simulation since the tips and their associated vortices are present [18] it strongly depends on the resolution of the computational domain as shown in figure 1 and figure 2. The influence of the resolution and the tip can be seen at \( r/R = 1 \) in figure 2 and figure 1 and it clearly shows that for a higher resolution \( R/\Delta x \geq 32 \) the tip vortex becomes distinct enough for inducing a more realistic velocity deficit. While at first sight, the case for high resolution without tip correction seems to behave well in the tip region, it should be noted that the sudden drop in velocity does not reflect in the experimental data. In fact, the case with a tip correction leads to a velocity which has a constant offset to the experimental data, giving a systematic error but at least modelling the same behaviour. There the tip correction might be necessary for coarser resolutions in order to model a tip presence which is only weakly represented by the numerical simulation.

![Figure 1. No tip correction](image1)

![Figure 2. With tip correction](image2)
In order to avoid non-physical oscillations around the point of force insertion due to the sudden change the fluid is experiencing, the force is distributed using a 3D Gaussian kernel.

In order to avoid the over-dissipative effects of a standard Smagorinsky [19] sub-grid scale (SGS) model for the LES, a dynamic approach is chosen. Unfortunately the solution as proposed by [20] is not applicable in this case. There is no straightforward way to determine a homogeneous space over which the averaging of the relevant equations could be applied. Therefore the dynamic Lagrangian model [21] is used which averages over the Langrangian path instead of planes.

The uniform computational domain is cubic with a edge length of $10R$ and a refinement zone in the vicinity of the rotor ($0.4R$ up- and downstream, $2.4R$ in lateral direction). The resolution in the rotor region is $N = D/\Delta x = 128$. For the velocity the lateral boundaries are set as symmetry planes, the outlet is zero gradient and the inlet is uniform velocity $U_{\infty}$.

The discretization of the convective term is a blended scheme between second-order upwind and central differencing [22].

3. Results

In figure 3 the body forces associated with rotating blades can be seen. For $U_{\infty} = 10 \text{ m/s}$ and $15 \text{ m/s}$ exists a very good agreement, while the forces are not correctly evaluated for the high velocity case. A sudden drop in both forces can be seen. It stems from the fact that beyond the AOA no experimental data is available and in order to fill the region beyond that, the flat plate assumption is used leading to a sharp drop in the lift coefficient. Another interesting observation is that the forces for the MEXICO experiment are lower than for the NEW MEXICO experiment. This is probably due to the fact that the experiments were conducted with a slightly lower inlet velocity. Again the SOWFA and the EllipSys3D case are very similar even in the case where the models break down.

![Figure 3. Comparison between the normal and tangential blade forces $F_n$ and $F_t$ by SOWFA and EllipSys3D against the experimental results of (NEW) MEXICO.](image)

Also when taking a look at the radial and axial profiles of the velocity components in figure 4 and figure 5 respectively, it can be seen that the overall trend is very well maintained while the exact velocity deficit in rotor vicinity is slightly underestimated. Again the representation for the high velocity case ($U_{\infty} = 24 \text{ m/s}$) visualizes the limits of the ALM relying on airfoil coefficients obtained from unmodified 2D wind tunnel experiments.

When refining the grid using the actuator line method the distribution parameter $\epsilon$ has to be adjusted in accordance in order to obtain a global torque close to the reference value. In the following only the case for $U_{\infty} = 15 \text{ m/s}$ will be examined as the other cases of (NEW) MEXICO served as extreme cases for determining how the model behaves at its limits.

While other work often relies on a constant $\epsilon = 2\Delta x$ for different grid resolutions as done e.g. by [23], this work adapts the $\epsilon$ in accordance with the grid resolution. Therefore the approach of this work results in a non-linear relation between distribution width and grid resolution. The results are shown in figure 6. A confidence interval of $\pm 1\%$ was established around the reference torque value $T_{\text{ref}}$ and through iteration a distribution parameter is found for the simulation to fall in the range.
The lower bound for the distribution parameter here is $\epsilon = 1.7\Delta x$ for the sake of numerical stability of the here chosen method. Other frameworks applying a different numerical discretization can go even lower e.g. in [10]. By doing so it can be seen that the best solution in terms of global torque for a resolution of $N = D/\Delta x = 32$ is off by around 4\% in figure 6.

As a general trend it can be seen that $\epsilon$ has to be increased with increasing resolution. This stems from the fact that by refining the mesh with a constant $\epsilon$ the punctual induction caused by the blade would be too high and eventually the torque would be below the reference value, e.g. for $\epsilon = 2\Delta x$ for $N \geq 64$. In contrary when having a very low resolution a constant $\epsilon$ distributes the force too widely, causing a lower induction around the rotor resulting in an overestimation of the torque, e.g. for $\epsilon = 2\Delta x$ for $N \leq 48$.

The optimal distribution parameter $\epsilon/\Delta x$ found in figure 6 are now shown in dependence of the grid resolution $N$ in figure 7. It seems as if this value would reach eventually an asymptotic limit. When looking at a more theoretical approach in [12] it is suggested that the optimal distribution width $\epsilon$ lies between $0.14 – 0.25$ of the chord $c$. In this case for $D/\Delta x = 128$ the relation $\epsilon/c$ is between 0.5 – 8.9 depending on the spanwise location. The observation made by [12] is backed by [11] where $\epsilon/c$ is situated in the same range. But it should be kept in mind that [11] uses a much higher grid resolution allowing $\epsilon/\Delta x \geq 4$ and [12] $\epsilon/\Delta x \geq 5$.

The curvature in figure 7 also confirms the findings of [24] that keeping a constant $\epsilon/\Delta x$ leads to an undesired behaviour. While [23] suggests to choose the smallest possible distribution width $\epsilon$ ($\epsilon/\Delta x = 1$ in that case), the observations of this work follow rather the work of [11] suggesting the adaptation of $\epsilon$ in dependence of the grid resolution in order to distribute the force over a meaningful length scale.
4. Conclusions

By adjusting the distribution with $\epsilon$ based on the global torque the optimal parameter could be obtained for this case and several grid resolutions. It was found that the distribution width scales in a non-linear fashion with the grid resolution. In order to obtain an accurate representation of the tip vortices it would necessitate a grid resolution of $N = D/\Delta x \geq 1024$ which would lead to a mesh of more than one billion cells. Following the trend of the distribution it would be around $\epsilon/\Delta x = 4 - 5$ at this resolution which is in agreement with recent theoretical findings by [25] and also fully resolved rotor simulations done by [26]. Of course at this point the mesh size and the associated computational costs would exceed by far the resources of Compute Canada and its usefulness could be questioned as well as fully resolved rotor simulations on a non-uniform mesh could be conducted at much lower resolutions and deliver more accurate results.

Acknowledgments

This work is partially supported the Canadian Research Chair on the Nordic Environment Aerodynamics of Wind Turbines and the Natural Sciences and Engineering Research Council (NSERC) of Canada.

A lot of my gratitude goes to Alexander Meyer Forsting for the collaboration on the comparison of SOWFA and EllipSys3D against the (NEW) MEXICO experiment whose results were published in [8]. Thanks for the great work done by Matthew Churchfield and colleagues at National Wind Technology Center, Boulder, CO, by establishing the open source framework SOWFA. The data used have been supplied by the consortium which carried out the EU FP5 project Mexico: ‘Model rotor EXPERiments In COntrrolled conditions’. The consortium received additional support to perform the New Mexico measurements from the EU projects ESWIRP and INNWIND.EU. Thanks also particularly to Gerard Schepers of ECN Wind Energy for making certain results of the NEW MEXICO experiment available upon request.

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