Nucleation of $p$-branes with form fields and dielectric brane

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Abstract

In this paper we discuss how to generalize the concept of nucleation to the $p$-branes with form fields. And we try to get ready for calculating the decay width of the dielectric brane.

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1 Introduction

$P$-branes, including of M-branes, D branes and NS-NS branes, in string/M theory are very important objects which appear in almost every recent paper in this area. The earlier references are reviewed in [1, 2, 3, 4]. Several authors have given the static solutions of branes, see e.g. [5, 6, 7, 8]. Moreover, basing on the higher dimensional rotating black holes [9], Cvetić and Youm have also developed the rotating M-branes [10, 11].

In another respect, a new kind of branes, called flux branes, which are generalized from 4-dimensional Melvin universe [12] are researched in many papers [13, 14, 15, 16, 17, 18, 19, 20]. Among other things the authors of ref. [21] and [22] apply the flux-brane technique to the M-theory and find the supergravity solutions describing certain dielectric branes in 10 dimensions [23]. In ref. [24] and [21] the authors compare the supergravity solution with Myers’ dielectric brane, the worldvolume theory, and find them in agreement with each other. Recently a paper appeared, in which the decay of dielectric brane is discussed by using the open string technique [25]. To look for its supergravity counterpart, we have to study the nucleation of dielectric branes. This means that before we consider the dielectric brane we should find the nucleation of $p$-branes first. That is the goal of this paper. Dowker et al. already studied these questions in a remarkable paper [16]. But they restrict themselves only in black hole solutions (solution of Einstein equation) without any form fields. When the form field is turned on, the case becomes quite different. We have to solve the new higher dimensional Einstein-Maxwell equations.

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Therefore, in this paper, we first study what is the nucleation of a \( p \)-brane with form field. Emphasis is put on the explanation of dimension of field strength. Next, we analyze the relationship between the M5 brane and the dielectric one in nucleation case. To calculate the decay width of dielectric brane we have to find the nucleation of rotating M5 brane. Because it is still an open question till now, we leave it as a future job. For further investigation, we write down a note about rotating black holes, and discuss how to generalize the nucleation to the flux branes.

2 Form fields in nucleation

Let us start from the static solutions of \( p \)-branes. Consider the Lagrangian that there is a single field strength and a single or no dilaton \( (a = 0) \) field [7, 8].

\[
e^{-1} \mathcal{L} = R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2n!} e^{a\phi} F^2, \quad a^2 = \Delta - \frac{2dd}{D-2} \tag{1}
\]

(see ref. [7] for symbols) and the metric can be written in the following

\[
ds_D^2 = e^{2A}(-dt^2 + dx_idx_i + e^{2f}d\tau^2) + e^{2B}(e^{-2f}dr^2 + r^2d\Omega^2_n) \tag{2}
\]

in which, \( i = 1, \cdots, d-2, \quad n = \tilde{d} + 1, \quad A, B \) and \( f \) are functions of \( r, \quad d = p + 1 = D - \tilde{d} - 2 \) represents the dimensions of the brane part. In fact, the metric we use is the same as in ref. [7], except that the time is continued to an Euclidean coordinate \( \tau \), and one of the space coordinates is continued to the time \( t \). When we consider the reduction from 11-dimension to the 10-dimension, \( \tau \) will be chosen as the compactification coordinate. the solution is [7]

\[
e^{2f} = 1 - \frac{k}{r^d}, \quad e^{2A} = H^{-\frac{4\tilde{d}}{\Delta(D-2)}}, \quad e^{2B} = H^{\frac{4\tilde{d}}{\Delta(D-2)}}, \quad H = 1 + \frac{k}{r^d} \sinh^2 \mu \tag{3}
\]

and

\[
F^M = \frac{2\tilde{d}k}{\sqrt{\Delta}} \sinh \delta_m \cosh \delta_m \epsilon_{\tilde{d}+1} \tag{4}
\]

for magnetic charged \( p \)-brane,

\[
F^E = \frac{2\tilde{d}k}{\sqrt{\Delta}} \sinh \delta_e \cosh \delta_e \epsilon_{\tilde{d}+1} \tag{5}
\]

for electric charged \( p \)-brane.

To nucleate the \( p \)-brane, we suppose that

\[
ds_D^2 = e^{2A}(dx_idx_i + e^{2f}d\tau^2) + e^{2B}[e^{-2f}dr^2 + r^2(d\alpha^2 + \cos^2 \alpha d\Omega^2_n)] \tag{6}
\]

in which, \( i = 1, \cdots, d-1, \quad n = \tilde{d} \) and it has a solution in same form like e.qs. (3), (4) and (5), but now \( d \) and \( \tilde{d} \) is decreased and increased by one respectively. Two points
are remarkable: 1) The nucleation solution is an Euclidean brane, its Lagrangian evolution is obtained by setting $\alpha = it$ in the transverse space. 2) We have one order higher magnetic field strength or one order lower electric field strength. This needs to be interpreted.

For 10 dimensional theory above mentioned, nucleation solution of $p$-brane is equivalent to a double analytic continuation of $p-1$ brane. However in M-theory it is questionable even for continuation, since we do not have M1 and M4 branes. Therefore how to understand these field strengths, especially $F_5$ (for M5) and $F_3$ (for M2) appear in the theory is the key to the question.

Let us explain it by using an example of D6 brane carrying a magnetic monopole which is reduced from 11-dimensional Taub NUT solution (For short, we consider the extreme case i.e. $f = 0$. And the following is in Einstein frame).

$$ds^2_E = H^{-1/8}(-dt^2 + dx_i dx_i) + H^{7/8}(dr^2 + r^2 d\Omega_2^2), H = 1 + \frac{Q_m}{r}$$ \(7\)

in Wu-Yang gauge

$$F_2 = dA = *dH, A = Q_m (\pm 1 - \cos \theta) d\varphi$$ \(8\)

in which the Hodge star is defined with respect to the flat transverse space. The corresponding nucleation solution is

$$ds^2_E = \mathcal{H}^{-1/4}(dx_i dx_i) + \mathcal{H}^{3/4}[dr^2 + r^2(d\alpha^2 + \cos^2 \alpha d\Omega_2^2)], \mathcal{H} = 1 + \frac{Q_m}{r^2}$$ \(9\)

and

$$\mathcal{F}_3 = *d\mathcal{H} = Q_m \cos^2 \alpha \sin \theta d\alpha d\theta d\varphi$$ \(10\)

setting $h(\alpha) = \int \cos^2 \alpha d\alpha$, and choosing the integration constant so that $h(0) = 1$, then we can see

$$\mathcal{F}_2(\alpha) = h(\alpha) F_2$$ \(11\)

as the evolution monopole field. Hence

$$\mathcal{F}_3 = \partial_\alpha \mathcal{F}(\alpha) d\alpha$$ \(12\)

represents the evolution rate of monopole field by the analytic continuation $\alpha = it$. As for the "electric" brane (e.g. Do), the form field will be the dual of evolution rate of dual magnetic field.

### 3 Dielectric effect

Now let us study M5 brane in more detail. Because that there is no dilaton in M-theory

$$\Delta = \frac{2d\tilde{a}}{D-2}, e^{2A} = H^{-2/d}, e^{2B} = H^{-2/d}$$ \(13\)
so the metric is

\[ ds^2_{11} = H^{-1/3}(-dt^2 + dx_i dx_i + f d\tau^2) + H^{2/3}\left[\frac{dr^2}{f} + r^2(d\theta^2 + \sin^2 \theta d\tilde{\varphi}^2 + \cos^2 \theta d\Omega_2^2)\right] \quad (14) \]

in which

\[ f = 1 - \frac{2m}{r^3}, \quad H = 1 + \frac{2m}{r^3} \sin^2 \delta \quad (15) \]

and

\[ F_4 = 3(2m) \sinh \delta \cosh \delta \cos^2 \theta \sin \theta d\theta d\tilde{\varphi} \epsilon(S^2) \quad (16) \]

The corresponding nucleation of M5 brane is

\[ ds^2_{11} = H^{-2/5}(dx_i dx_i + f d\tau^2) + H^{1/2}\left\{\frac{dr^2}{f} + r^2[da^2 + \cos^2 \alpha(d\theta^2 + \sin^2 \theta d\tilde{\varphi}^2 + \cos^2 \theta d\Omega_2^2)]\right\} \quad (17) \]

where

\[ f = 1 - \frac{2m}{r^4}, \quad H = 1 + \frac{2m}{r^4} \sin^2 \delta \quad (18) \]

and

\[ F_5 = dA_4, \quad A_4 = \frac{12}{\sqrt{10}} 2m \sinh \delta \cosh \delta \cos^4 \alpha \cos^3 \theta d\alpha d\tilde{\varphi} \epsilon(S^2) \quad (19) \]

can be also interpreted as the evolution rate of the evolution version of \( F_4 \). For the nucleation slice \( \alpha = 0 \), the metric has the same topology as the original M5 brane. The subsequent Lagrangian evolution is obtained by setting \( \alpha = it \).

As mentioned in introduction, Cveti\'c and Youm generalized the Kerr-like black hole to the rotating M5 brane \[10, 11\]. It seems that one might find a nucleation version of this stabilized solution. However, it is a pity that we have not get the correct rotating solution for nucleation M5 brane till now. Therefore we would like to mimic ref. \[21\] to analyze the so-called dielectric effect of nucleation M5 brane in "maximal magnetic field" case \[16\].

First of all we give the compactification radius by using of missing conical singularity condition

\[ R_{11} = g \sqrt{\alpha} = \frac{r_H}{2} (\cosh \delta)^{9/10}, \quad r_H^4 = 2m \quad (20) \]

Then we perform the reduction along the Killing vector field

\[ q = \frac{\partial}{\partial \tau} + B \frac{\partial}{\partial \tilde{\varphi}} \quad (21) \]

setting \( \varphi = \tilde{\varphi} + B \tau \), the IIA metric can be written as

\[ ds^2_{10} = \Lambda^{1/2} H^{-2/5} ds^2(\mathbb{R}^4) + \Lambda^{1/2} H^{1/2}\left\{\frac{dr^2}{f} + r^2[da^2 + \cos^2 \alpha(d\theta^2 + \sin^2 \theta d\tilde{\varphi}^2 + \cos^2 \theta d\Omega_2^2)]\right\} + \Lambda^{-1/2} H^{1/10} r^2 \cos^2 \alpha \sin^2 \theta d\varphi^2 \quad (22) \]
in which
\[ \Lambda = \mathcal{H}^{-2/5}(f + \mathcal{H}^{9/10}B^2r^2\cos^2\alpha\sin^2\theta) = e^{4\phi/3} \] (23)

To avoid the conical singularity, set the magnetic field parameter
\[ B = 1/R_{11} \] (24)

And we have the following form fields
\[ A_\varphi = \frac{1}{\Lambda}\mathcal{H}^{1/2}\cos^2\alpha\sin^2\theta Bd\varphi, \] (25)
\[ A_3 = \frac{12B}{\sqrt{10}}2m\sinh\delta\cosh\delta\cos^4\alpha\cos^3\theta d\alpha\epsilon(S^2), \]
\[ A_4 = \frac{12}{\sqrt{10}}2m\sinh\delta\cosh\delta\cos^4\alpha\cos^3\theta d\alpha d\varphi\epsilon(S^2) \]

for \( r \gg r_H \) (and \( \sinh^2\delta \) is not too big) the string metric is reduced to
\[ ds_{10}^2 = \Lambda^{1/2}\{ds^2(\mathbb{E}^4) + dr^2 + r^2[d\alpha^2 + \cos^2\alpha(d\theta^2 + \cos^2\theta d\Omega_2^2)]\} + \Lambda^{-1/2}r^2\cos^2\alpha\sin^2\theta d\varphi^2 \] (26)

using the coordinate transformation
\[ \begin{align*}
\rho &= r\cos\alpha\sin\theta, \\
v\cos\chi &= r\cos\alpha\cos\theta, \\
v\sin\chi &= r\sin\alpha
\end{align*} \] (27)

the metric becomes
\[ ds_{10}^2 = \Lambda^{1/2}[ds^2(\mathbb{E}^4) + ds^2(\mathbb{M}^4) + d\rho^2] + \Lambda^{-1/2}\rho^2d\varphi^2 \] (28)

in which we made a replacement of \( dv^2 + \nu^2(d\chi^2 + \cos^2\chi d\Omega_2^2) \rightarrow ds^2(\mathbb{M}^4) \) when one of the coordinate is analytic continued to the time. The dual of the 2 form RR field strength is
\[ \mathcal{F}_8 = 2B\epsilon(\mathbb{E}^4) \wedge \epsilon(\mathbb{M}^4) \] (29)

represents a flux 7 brane \( \mathbf{F}_7 \).

The case we have studied corresponds to a dielectric brane immersed in a \( \mathbf{F}_7 \) brane, in which D4 branes dissolve in a D6 brane. So this is the nucleation of the dielectric brane. Of course, we may also consider the near core approximation: \( r^2 = r_H^2 + \bar{r}^2, \bar{r}, \theta \approx 0 \). we have
\[ f \approx 2\frac{\bar{r}^2}{r_H^2} = 4\lambda^2, \mathcal{H} \approx \cosh^2\delta, \Lambda \approx 4(\cosh\delta)^{-4/5}(\lambda^2 + \cos^2\alpha\theta^2) \] (30)

and
\[ ds_{10}^2 \approx \sqrt{\lambda^2 + \cos^2\alpha\theta^2}(\cosh\delta)^{-6/5}[ds^2(\mathbb{E}^4) + (\cosh\delta)^{9/5}r_H^2[d\lambda^2 + d\alpha^2 + \cos^2\alpha(d\theta^2 + d\Omega_2^2 + \frac{\lambda^2\theta^2}{\lambda^2 + \cos^2\alpha\theta^2}d\varphi^2)] \] (31)
setting
$\lambda^2 + \theta^2 \cos^2 \alpha = (\cosh \delta)^{3/10} \frac{\rho}{r_H}, \lambda \theta \cos \alpha = \frac{1}{2} (\cosh \delta)^{3/10} \frac{\rho}{r_H} \sin \eta$  \hfill (32)
and
$\mu = \frac{r_H}{4} (\cosh \delta)^{21/10}$ \hfill (33)
in $\alpha \approx 0$ neighborhood we obtain

$$ds^2_{10} = \sqrt{\frac{\rho}{\mu}} ds^2(\mathbb{E}^4) + (\cosh \delta)^{9/5} \sqrt{\frac{\rho}{\mu}} r_H^2 (d\alpha^2 + \cos^2 \alpha d\Omega_2^2) + \sqrt{\frac{\mu}{\rho}} ds^2(\mathbb{E}^3)$$ \hfill (34)
where we use $d\rho^2 + \rho^2 (d\eta + \sin^2 \eta d\varphi^2) \to ds^2(\mathbb{E}^3)$ and the form fields become

$$A_\varphi = (\cosh \delta)^{-6/5} \mu (1 - \cos \eta) d\varphi,$$
$$A_3 = \frac{9}{\sqrt{10}} r_H^3 \sinh 2\delta \cos^2 \alpha [(\cosh \delta)^{-3/5} \rho (1 - \cos \eta) - \frac{4}{3} (\cosh \delta)^{-9/10} \cos^2 \alpha] d\alpha \epsilon(S^2),$$
$$A_4 = \frac{9}{\sqrt{10}} r_H^4 \sinh 2\delta \cos^2 \alpha \frac{1}{2} (\cosh \delta)^{3/10} \rho (1 - \cos \eta) - \frac{2}{3} \cos^2 \alpha] d\alpha d\varphi \epsilon(S^2)$$

4 About the nucleation of rotating black holes

One question for preparing the nucleation of rotating M5 brane we would like to state is that instead of analytically continuing in one of the ignorable angles in e.q.(5.2) of ref. [16] we suggest to use formula

$$ds^2 = f(d\tau - \frac{2ml}{r \rho^2} \cos^2 \alpha \sin^2 \theta d\varphi)^2 + \frac{d\varphi^2}{f} - l^2 (\cos \alpha \cos \theta d\theta - \sin \alpha \sin \theta d\alpha)^2$$ \hfill (36)
$$+ r^2 [d\alpha^2 + \cos^2 \alpha (d\theta^2 + \cos^2 \theta d\Omega_2^2)] + \frac{1}{f} (r^2 - l^2 - \frac{2m}{r \rho}) \cos^2 \alpha \sin^2 \theta d\varphi^2$$

where

$$f = 1 - \frac{2m}{r \rho \rho^2}, \tilde{f} = f - \frac{l^2 \cos^2 \alpha \sin^2 \theta}{\rho^2}, \rho^2 = r^2 - l^2 + l^2 \cos^2 \alpha \sin^2 \theta$$ \hfill (37)

Although these two formula are equivalent with each other in mathematics, They are not equivalent in physical meaning. Because that in e.q.(36) the evolution space is whole fixed points of killing vector, But it is only a subspace of them in the former case.

5 Nucleation of flux brane

Our proposition about nucleation of $p$-brane is also applicable to flux branes. It is known that $F_p$ in type II strings, $F_3$ and $F_6$ in M-theory have been calculated in refs [19][20][21]. We can use double analytic continuation to find the nucleation of $F_p$ brane from $F_{p-1}$ brane for type II flux branes just like above mentioned general type II $p$-branes. For flux
branes in M-theory like the M5 brane we may get nucleation $\mathbf{F}_6$ brane solution as follows by using same symbols and same method in ref. [21]. Now $D = 11$, $d = 6$, $\tilde{d} = 4$,

$$f^* = -\frac{20}{9} E^2 e^f + 24 E^2 e^a, \quad g^* = E^2 e^f + 18 E^2 e^g$$  \hspace{1cm} (38)

setting $e^g = \eta e^f$, so that $\eta = \frac{20}{27}$, and

$$e^f = \frac{3}{4(Er)^2}, e^{2A} = \eta^{1/6} e^{f/24}, e^{2C} = \eta e^{5f/4}, e^{2B} = e^{f/4}$$  \hspace{1cm} (39)

Then the metric for nucleation of $\mathbf{F}_6$ brane in terms of the proper radial distance coordinate $u$ is

$$ds^2 = \left(\frac{E^2 u^2}{3}\right)^{1/6} ds^2(\mathbb{E}^6) + du^2 + \frac{18}{29} u^2 (d\alpha^2 + \cos^2 \alpha d\Omega_3^2)$$  \hspace{1cm} (40)

with

$$\mathcal{F}_6 = E e(\mathbb{E}^6)$$  \hspace{1cm} (41)

which can be identified as the dual of the evolution rate of evolution version of

$$\mathcal{F} = \frac{1}{E^2} \left(\frac{2}{r E^2 \eta^2}\right)^{4/3} d\epsilon(S^3) = \left(\frac{4^{3/4}}{3^{5/4}}\right) E^2 E^2 d\epsilon(S^3)$$  \hspace{1cm} (42)

Similarity, the nucleation of flux brane $\mathbf{F}_3$ can also be written.

6 Prospect

We expect to gain the nucleation solution for rotating M5 brane. Then we can find the values of the radius of the relevant sphere and calculate the Euclidean action of the rotating system. Finally we can obtain the decay width of the dielectric brane.

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References

[1] J.Polchinski. TASI Lectures on D-Branes. In Fields, Strings and Duality, TASI 1996, eds. C.Efthimiou & B.Greene, pp. 293-356. Singapore: World Scientific. [hep-th/9611050]

[2] G.T.Horowitz, A.Strominger. Black strings and $p$-branes, Nucl.Phys. B360 (1991) 197.

[3] C.G.Callan, J.A.Harvey, A.E.Strominger. Supersymmetric String Solitons, In String Theory and Quantum Gravity, Trieste 1991. eds. J.A.Harrey et al. pp. 208-244. Singapore: World Scientific. [hep-th/9112030]
[4] P.K. Townsend. Four Lectures on M-theory. On High Energy Physics and Cosmology, Trieste 1996. eds. Gava et al. pp. 385-438. Singapore: World Scientific. hep-th/9612121

[5] R.Güven, Black $p$-brane solitons of D=11 dimensional supergravity theory, Phys.Lett. B276 (1992) 49.

[6] M.J. Duff, J.X. Lu. Black and super $p$-branes in diverse dimensions, Nucl.Phys. B416 (1994) 301.

[7] M.J. Duff, H. Lu, C.N. Pope. The Black Branes of M-theory, Phys.Lett. B382 (1996) 73-80. hep-th/9604052

[8] M. Cvetič, A.A. Tseytlin. Non-extreme black holes from non-extreme intersecting M-branes, Nucl.Phys. B478 (1996) 181-198. hep-th/9606033

[9] R.C. Myers, M.J. Perry. Black holes in higher dimensional space-times, Ann.Phys. 172 (1986) 304.

[10] M.Cvetič, D.Youm. Rotating Intersecting M-Branes, Nucl.Phys. B499 (1997) 253-282. hep-th/9612229

[11] C.Csaki, J.Russo, K.Sfetsos, J.Terning. Supergravity Models for 3+1 Dimensional QCD, Phys.Rev. D60 (1999) 044001, hep-th/9902067

[12] M.Melvin. Pure magnetic and electric geons, Phys.Lett. B8 (1964) 65.

[13] G.W.Gibbons, K.Maeda. Black holes and membranes in higher dimensional theories with daliton fields, Nucl.Phys. B298 (1988) 741.

[14] F.Dowker, J.P. Gauntlett, D.A. Kastor, J.Traschen. Pair Creation of Dilaton Black Holes, Phys.Rev. D49 (1994) 2909-2917, hep-th/9309075

[15] F.Dowker, J.Gauntlett, G.Gibbons, G.Horowitz. The Decay of Magnetic Fields in Kaluza-Klein Theory, Phys.Rev. D52 (1995) 6929-6940, hep-th/9507143

[16] H.F.Dowker, J.P.Gauntlett, G.W.Gibbons, G.T.Horowitz. Nucleation of $p$-Branes and Fundamental Strings, Phys.Rev. D53 (1996) 7115-7128, hep-th/9512154

[17] M.S. Costa, M.Gutperle. The Kaluza-Klein Melvin Solution in M-theory, JHEP 0103 (2001) 027, hep-th/0012072

[18] J.G. Russo, A.A. Tseytlin. Magnetic flux tube models in superstring theory, Nucl.Phys. B461 (1996) 131-154, hep-th/9508068

[19] P.M.Saffin. Gravitating Fluxbranes, Phys.Rev. D64 (2001) 024014, gr-qc/0104014

[20] M.Gutperle, A.Strominger. Fluxbranes in String Theory, JHEP 0106 (2001) 035, hep-th/0104136
[21] M.S. Costa, C.A.R. Herdeiro, L.Cornalba. Flux-branes and the Dielectric Effect in String Theory, Nucl.Phys. B619 (2001) 155-190, hep-th/0105023.

[22] R. Emparan. Tubular Branes in Fluxbranes, Nucl.Phys. B610 (2001) 169-189, hep-th/0105062.

[23] R.C. Myers. Dielectric-Branes, JHEP 9912 (1999) 022, hep-th/9910053.

[24] D. Brecher, P.M. Saffin. A note on the Supergravity Description of Dielectric Branes, Nucl.Phys. B613 (2001) 218-236, hep-th/0106206.

[25] K. Hashimoto. Dynamical Decay of Brane-Antibrane and Dielectric Brane. JHEP 0207 (2002) 035, hep-th/0204203.

[26] E. Witten. Instability of the Kluza-Klein vacuum, Nucl.Phys. B195 (1982) 481.