INTRODUCTION
Concentration tables belong to the group of oscillatory enrichments. The distribution of grains for this type of device is made according to their specific weight (density) during many recurring cycles caused by appropriate drives. As mentioned in the article (Feliks and Krawczyk, 2019), Wilfley's mechanism is still the most a well-known and widespread, constantly produced mechanism with a drive kinematics used to force vibrations on concentration tables. The second type of commonly used drives is the inertia drive (Concenco). Inertia drives are widely used in vibrating devices due to the simple construction and easy adjustment of work parameters (Feliks 2017a, Feliks 2017b, Tomach 2017, Sidor and Feliks, 2015). In the article, we will use Concenco's concentration table (Banaszewski, 1997, Feliks, 1999), shown in Fig. 1, because it produces curves: roads and accelerations similar to those of Wilfley's drive, and the trajectory modeling is much simpler. Another advantage of the concentration table chosen by us is a much simplified analytical record of table vibration trajectory.

The structure and principle of operation have been described in detail in the works (Banaszewski, 1997, Feliks, 1999, Feliks at al. 2005). Its main components are two two-mass inertia vibrators.
Vibrator 1 is made of large unbalanced masses, whose total static moment is \( m_1 r_1 \). These masses rotate in opposite directions at speed \( n_1 \).

Vibrator 2 consists of much smaller unbalanced masses with a total static moment \( m_2 \), which rotate at the speed \( n_2 \) and are coupled to vibrator 1 by means of a gear transmission. In order to obtain the best table performance, the static moment of the vibrator 1 is twice as high as the static moment of the vibrator 2. In contrast, the rotational speed of the second vibrator is twice as high as that achieved by the first vibrator and is \( n_2 = 2n_1 \). When the masses are positioned as in Fig. 1, the inertia forces of both masses are added. While the unbalanced weights of the vibrator 1 turn by 180°, the vibrator mass 2 will make a full revolution and for this position the inertia forces coming from the vibrators will subtract. As a result, variable accelerations in forward and reverse motion are obtained.

Trends: The path, velocity and acceleration of the Concenco drive equation table shown in Fig. 1 describe equations (1, 2 and 3).

\[
x(t) = C_1 \cdot \sin(\omega \cdot t) - C_2 \cdot \cos(2 \cdot \omega \cdot t) \quad (1)
\]

\[
v(t) = C_1 \cdot \omega \cdot \cos(\omega \cdot t) + 2 \cdot C_2 \cdot \omega \cdot \sin(2 \cdot \omega \cdot t) \quad (2)
\]

\[
a(t) = -C_1 \cdot \omega^2 \sin(\omega \cdot t) + 4 \cdot C_2 \cdot \omega^2 \cdot \cos(2 \cdot \omega \cdot t) \quad (3)
\]

Table vibration trajectories with such a drive show similarity of shape to those obtained in Wilfley's concentration tables with the drive (Michalczyk and Banaszewski, 2006, Banaszewski, 1997).

The purpose of the research described in the article is to model the enrichment of standard samples for different angular velocities of the drive. These samples move simultaneously on the plate of the concentration table inclined to the long side by angles from 5° to 15°.

**SYMULATION MODEL**

In the simulation model, discs made of several different materials were accepted as reference materials. The materials used for the tests were selected in this way to obtain relatively large differences in friction coefficients between individual pairs of friction materials. The proper association of the appropriate friction pair with each other will give the possibility of splitting (enriching) on the table of the concentration Table. The effect of large differences in coefficients of friction will be the characteristic fan on the table of the concentration table. The modeled grains had parameters in accordance with the elements presented in Fig. 2 used in dry enrichment concentration enrichment studies. These samples are made of metal discs: 30 mm in diameter and 5 mm in height weighing 30 g each. Each of them is glued underneath with one of four materials: A-teflon, B-plywood, C-rubber-stopper, D-rubber.

For simulation purposes, the static and kinetic friction coefficients were determined at the station shown in Fig. 3. The results of these tests are presented in Table 1.
The simulation model was made in the Autodesk Inventor computer program, and the simulations were performed in the dynamic simulation module. This software allows for a very wide range of simulation tests from correctness tests of machine and device design (Stopka, 2019, Mendyka et al., 2017, Feliks and Mendyka, 2015) after processes (Feliks, 2017b). The model consists of a table board with dimensions of 1500 x 500 mm inclined to the horizontal with angles respectively: 5°, 10°, or 15°. Four model grains with dimensions: diameter 30 mm, height 5 mm and weight 30 g move on the plate. Coefficients of friction of individual materials are determined in accordance with previous tests (Table 1).

For easier interpretation of results their path starts from the common for all four grains of the place (Fig. 4), and the individual discs are marked with the colors: Material A purple color – teflon,

Material B black color – plywood,

Table 1 List of test results: static and kinetic coefficient of friction between four selected samples and plate

| Sample | Static coefficient of friction $\mu_s$ | Kinetic coefficient of friction $\mu_k$ |
|--------|----------------------------------------|----------------------------------------|
| A      | 0.255                                  | 0.22                                   |
| B      | 0.404                                  | 0.34                                   |
| C      | 0.520                                  | 0.43                                   |
| D      | 0.620                                  | 0.53                                   |
Material C color red – rubber-stopper,  
Material D blue color – rubber.  
In the simulation model, the connections between particular disks have not been defined, so the trajectory of one does not affect the other.

The following dependences of coefficients of friction (friction pairs) between the undercoat (table plate) and disks:
- When the speed between the plate and the discs is equal to 0, then we assume a specific static friction coefficient adjusted to individual materials.
- When the speed is different from 0 (for calculation purposes from 1 mm/s) the coefficient of friction changes, at the contact: plate – disc, from the static coefficient to the kinetic factor.

Forcing the motion used to model the moving grains was determined from the course of the Concenco concentration table road. In order to shorten the calculation cycle, these waveforms were modeled as splaines from calculated trajectories for angular velocity from 10 rad/s to 32 rad/s (Figure 5). The vibration amplitude was constant and amounted to 5 mm, respectively, in the direction of the drive and 15 mm in the opposite direction.
In order to perform the simulation, the frequency of vibrations in the range from 10 to 32 rad/s with a pitch of 2 rad/s at the angle of inclination of the table plate was changed to the level equal to: 5°, 10° and 15°.

**SIMULATION RESULTS**

Simulations were carried out for:

- three different angles of inclination of the deck of the concentration table: 5°, 10° and 15°. These angles correspond to previous stand tests,
- variable angular velocities $\omega$, in the range from 10 rad/s to 32 rad/s.

The duration of the simulation was 10 s and it was the minimum time that allowed the correct interpretation of the test results. Selected simulation results are shown in Figure 6 for a 5° tilt angle, Figure 7 for a 10° tilt angle and Figure 8 for a 15° tilt angle.

![Fig. 6 Model simulation results of the grains (violet - type A, black - type B, red - type C, blue - type D) for different angular velocity vibrations at an angle of inclination of 5°](image)

![Fig. 7 Model simulation results of the grains (violet - type A, black - type B, red - type C, blue - type D) for different angular velocity vibrations at an angle of inclination of 10°](image)
Simulation studies carried out in the Autodesk Inventor computer program confirm theoretical considerations and are consistent with the actual results obtained on the table of the concentration table.

**CONCLUSION**

The change in the frequency of vibrations changes the trajectory of the grain movement. Increasing the angular velocity of vibrations causes that a growing group of materials moves along the table. The location of two pickup points at the longer and shorter side allows the feed to be separated according to the coefficient of friction. The criterion for the separation was the situation when grains travel along the table towards its shorter side.

A summary of the obtained simulation results is presented in Table 2.

| The angle of inclination of the plate | angular speed of the vibrator |
|--------------------------------------|-----------------------------|
|                                       | 10 rad/s | 14 rad/s | 18 rad/s | 22 rad/s | 25 rad/s | 32 rad/s |
| 5°                                    | -        | D        | B, C, D  | A, B, C, D| A, B, C, D| A, B, C, D|
| 10°                                   | -        | D        | C, D     | B, C, D  | A, B, C, D| A, B, C, D|
| 15°                                   | -        | D        | C, D     | C, D     | A, B, C, D| A, B, C, D|

Such a criterion allows for the separation of:

- material D, using angular velocity between 12 rad/s and 16 rad/s and any of the tested angles of inclination of the deck of the concentration table,
- together with materials C and D, using the angular velocity of 18 rad/s vibrations for the angle of 10° inclination of the deck of the concentration table and 18 rad/s or 22 rad/s for 15° inclination of the deck of the concentration table,
material A can be separated using an angular velocity of 22 rad/s at an angle of 10° inclination of the deck of the concentration table, receiving it at the long side of the table.

The simulation showed that it is possible to separate on the concentration table several materials with different coefficients of friction on a dry structure, choosing the appropriate vibration parameters and angle of inclination of the deck of the concentration table.

**Nomenclature**

A – symbolic designation of the sample used for testing – material,
B – symbolic designation of the sample used for testing – plywood,
C – symbolic designation of the sample used for the tests – rubber stopper,
D – symbolic designation of the sample used for testing – rubber,
C<sub>1</sub>, C<sub>2</sub> – the vibration amplitude of the table plate, mm
ω – angular velocity of vibrations of the concentration table, rad/s,
ν<sub>1</sub>, ν<sub>2</sub> – rotational speed of vibrators, rad/s,
ρ<sub>1</sub>, ρ<sub>2</sub> – theoretical grain density (assumed at work), kg/m<sup>3</sup>,
μ<sub>k</sub>, – kinetic coefficient of friction,
μ<sub>s</sub>, – static coefficient of friction.

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Abstract.
The article attempts to analyze the work of the concentration table, on which the distribution of the material under investigation takes place. The efficiency of separation or otherwise enrichment depends on the drive that puts the concentration table plate into an asymmetrical reciprocating movement. The most frequently used solutions are concentration tables, on which the distribution of enriched material takes place in an aqueous medium commonly called wet enrichment. The AGH University of Science and Technology conducted simulation tests of material enrichment on a concentration table using the differences in friction coefficient commonly referred to as dry enrichment, i.e. without an additional medium. The article presents the effects of simulations that have been carried out for reference materials characterized by different friction coefficients and densities. The summary presents the analysis of obtained effects of computer simulations of material movement with different coefficients of friction. The simulations were carried out for variables: the frequency of vibrations of the concentration table and the angle of inclination of the table of the concentration table. It has been shown that it is possible to enrich the material by changing the above-mentioned operating parameters of the device.

Keywords: concentration table, coefficient of friction, enrichment