Regular Black Holes and Confinement

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Abstract

Properties of the rotating Kerr-Newman black hole solution allow to relate it with spinning particles. Singularity of black hole (BH) can be regularized by a metric deformation. In this case, as a consequence of the Einstein equations, a material source appears in the form of relativistically rotating superconducting disk which replaces the former singular region. We show a relation of the BH regularization to confinement formation. By regularization, a phase transition occurs near the core of a charged black hole solution: from external electrovacuum to an internal superconducting state of matter. We discuss two models of such a kind, which demonstrate the appearance of a baglike structure and a mechanism of confinement based on dual Dirac’s electrodynamics. First one is an approximate solution based on a supersymmetric charged domain wall, and second is an exact solution based on nonlinear electrodynamics.

1 Introduction

Last years there has been a renewed interest in regular black holes (BHs) and particlelike models within General Relativity (see e.g. [1] for a summary), and more especially on electric/magnetically charged BHs and particlelike solutions (see e.g., [2, 3, 4]). Singularity of black hole can be regularized by a metric deformation. In this case, as a consequence of the Einstein equations, there appears a material source which replaces the former singular region, and a phase transition occurs near the core of regularized black hole solutions: from external electrovacuum to another state of matter inside the core. Such regularized black
hole solutions can be considered as classical gravitating solitons. Regularization of the rotating Kerr-Newman black hole solution is of a special interest as a solitonic model of charged spinning particle. We recall that the Kerr-Newman solution has $g = 2$ gyromagnetic ratio, possesses the stringy structures \cite{3} and is a fundamental solution of low energy string theory. The source of regularized Kerr-Newman solution takes the form of relativistically rotating superconducting disk \cite{3,7}.

2 Regularized Kerr-Newman spacetimes

The Kerr-Newman (KN) solution in the Kerr-Schild (KS) form is:

$$g_{KN} = \eta_{ij} + 2 h k_i k_j$$

where, in Kerr angular coordinates, $\eta_{ij} = \text{diag}(-1, 1, r^2, r^2 \sin^2 \theta)$, $h$ is a function given by $f_{KN}(r) / (r^2 + a^2 \cos^2 \theta)$ with $f_{KN}(r) = m r - e^2 / 2$ ($a$, $m$ and $e$ are the angular momentum per unit mass, the mass and the electric/magnetic charge of the source, respectively) and $k$ is tangent to a very special twisting Kerr Principal Null Congruence of KN metric that need not be explicitly presented here. Looking at (1) a generalization appears that keeps the essential geometry of KN solution: to allow a general $f(r)$. In \cite{1} this generalization has been studied in detail.

In order to regularize the KN singularity one has to change the behavior of function $f(r)$ in the region of small $r$ setting $f(r) \sim \alpha r^n$ with $n \leq 4$. Particularly, in the nonrotating case, for $n = 4$ the source represents a spherically symmetrical space-time of constant curvature (de Sitter or anti de Sitter solution) with cosmological constant $\Lambda = 6 \alpha$ and energy density $\rho = 3 \alpha / 4 \pi$. The ‘radial’ position $r_0$ of the phase transition shell can be estimated as a point of intersection for plots $f_{int}(r)$ and $f_{KN}(r)$,

$$\frac{4}{3} \pi \rho r_0^4 = m r_0 - e^2 / 2.$$ (2)

Analysis shows \cite{1} that there appears a thin intermediate shell at $r = r_0$ with a strong tangential stress which is typical for domain wall structure. Dividing Eq. (2) by $r_0$ one can recognize here the mass balance equation $m = M_{int}(r_0) + M_{em}(r_0)$, where $M_{int}(r_0)$ is the ADM mass of the core and $M_{em}(r_0) = e^2 / 2 r_0$ is external electromagnetic field’s one. As a consequence of this equation, the AdS and dS interiors are admissible, and AdS interiors correspond to strongly charged particles.

For rotating BHs, the core region represents a strongly oblatted rotating disk of compton radius, and curvature is concentrated in a stringy region on the border of this disk. It was shown \cite{3,4} that there appears a baglike structure in the core region and the source resembles a domain wall interpolating between
external and internal vacua. The Kerr-Schild class of metrics allows one to describe regular rotating and non-rotating BH solutions at least on the level of gravitational equations.

However, there appear some problems on the level of field models describing corresponding matter source. The following demands to the matter field models are necessary for regularized charged particlelike solutions.

i - External vacuum has to be (super)-Kerr-Newman black hole solution with long range electromagnetic field and zero cosmological constant;

ii - Internal vacuum has to be an (A)dS space with superconducting properties, expelling the electric field.

Exact solutions of this kind are unknown and it is expected that they can exist in some generalized supergravity models.

These demands are very restrictive and are not satisfied in the known soliton-like, bag, domain wall and bubble models. In most known models the external electromagnetic field is short range. An exception is the $U(I) \times \tilde{U}(I)$ field model which was used by Witten to describe the cosmic superconducting strings [8]. This field model can be adapted for description of the superconducting bag geometry [8].

The model contains two sectors A and B, and correspondingly two Higgs and two gauge fields. One of the gauge fields can be set as a long range external electromagnetic field and another one is independent and can be chosen as a “dual” gauge field (in the sense of the dual Dirac electrodynamics [9]) which is confined inside the bag [6].

It is based on the Abelian Higgs field model for superconducting source. One can achieve nonperturbative soliton-like solutions of this kind taking the external KN BH field and a core described by three chiral fields of a supersymmetric field model. It was shown [5, 7] that there exists a BPS-saturated domain wall solution interpolating between supersymmetric vacua I) and II). External vacuum I) is characterized by

$$I) \begin{align*} Z &= 0; \quad \phi = 0; \quad |\sigma| = \eta; \quad W = 0, \end{align*}$$

and internal vacuum II), used for the interior of the bag, is characterized by

$$II) \begin{align*} Z &= -m/c; \quad \sigma = 0; \quad |\phi| = \eta\sqrt{\lambda/c}; \quad W = \lambda m\eta^2/c. \end{align*}$$

One can check the existence of the phase transition in the planar wall approximation. Minimum of the total energy is achieved by $r_0 = \left(\frac{\lambda}{16\pi\sigma}\right)^{1/3}$, corresponding to the stationary state with total mass $M_{tot}^* = E_{tot}^* = \frac{3\lambda}{4\pi\sigma}$. For the rotating Kerr-Newman case [6], $J = ma$, and for $J \sim 1$ we find out that parameter $a \sim 1/m$ has Compton size. Coordinate $r$ is an oblate spheroidal coordinate, and the phase transition occurs at the shell representing an oblate rotating ellipsoid with the axis ratio of order $\sim 137^{-1}$. 

3
3 Regular BHs based on the models of nonlinear electrodynamics

3.1 Exact nonrotating regular BH solutions

Second model demonstrating the phase transition in the core region and confinement is based on nonlinear electrodynamics of a general (non-BI) type (NED) [4]. It is of special interest since, at least in the non-rotating case, it yields the exact and consistent electric/magnetic charged solutions which can be interpreted as non-perturbative particlelike solutions. In spite of the very different field mechanism, this model displays also a “dual” electromagnetic phase in the core region.

In a recent paper [3], K. Bronnikov, showed that only purely magnetically charged BHs could be regular if Maxwell limit was to be reproduced for low energies of the field. It was showed in [4] that Bronnikov’s restrictions may be circumvented and indeed both regular magnetically and electrically charged solutions can exist.

In the obtained recently by Ayón-Beato and García exact regular BH solution to this NED-theory, Bronnikov has observed existence of caps and branches in Lagrangian. In particular, their solution contained singular distributions of electric charges $J^{(e)}_i = \nabla_m F^{mi}$ near a sphere of finite radius $r = r_0$ and at the center. In our paper [4] a modification of this solution was suggested, which allows one to avoid abnormal branches in the Ayón-Beato – García solution. It was achieved by a transition to "dual" electrodynamics in the core region (by $r < r_0$) fulfilling Dirac’s idea on dual electrodynamics [9]. As a result, the charges inside the sphere $r = r_0$ turn out to be magnetic $J^{(m)}_\mu = \nabla_\nu \tilde{F}^{\nu\mu}$. These charges give rise to a magnetic vacuum polarization inside the core, while for an external observer this solution has electric charge only. On the other hand, the fact that central region of this solution contains only magnetic charges allows one to avoid the Bronnikov theorem and to get regular BH solution with the resulting electrical charge.

Since core expels electric charges from itself one can speculate that it possesses superconducting properties, whereas the vacuum region of the external observer possesses the dual properties — it is not penetrable for magnetic charges, due to their confinement inside the sphere $r = r_0$. It is remarkable that nonlinear electrodynamics allows one to get exact and selfconsistent solutions of this sort belonging to the Kerr-Schild class of metrics.

This class of solutions deserves interest not only as an explicit example of the regular electrically charged BH solution, but also as an example of particlelike solution with a very specific realization of the ideas on quark confinement based on dual electrodynamics and superconductivity [10].
3.2 Regular rotating black holes from NED, preliminary treatment

The models of rotating regular BH solutions are of big interest to modelling spinning gravitating solitons. In this case it is expected that the phase transition will occur in a disklike region of compton size, and there appear a stringy region on the boarder of the Kerr disk. There was an assumption that such solution in NED-theory can contain a closed loop of gauge field similarly to the magnetic vortex fluxes of usual superconductors.

In this case it is necessary to include, besides \( f = F_{\mu\nu} F^{\mu\nu} \) the other invariant
\[
g = F_{\mu\nu} \ast F^{\mu\nu}.\]
Contrary to the nonrotating case, the solutions of this kind in Kerr geometry met obstacles and represent a hard and still unsolved problem. It is known that the NED theory of the Born-Infeld type cannot lead to regular BH solutions. In fact, to be able to solve NED field equations, one needs to suppose some particular family of spacetimes. Therefore, it seems easier to choose some preferred candidates and try to derive the associated Lagrangian.

Working in the Kerr-Schild class, we have made two relevant advances in such direction. First, we have computed the geometry of the most general spacetime that preserves staticity, axial symmetry and the Kerr Principal Null Congruence of KN spacetime. This is accomplished by allowing \( f = f_{\text{int}} \) in \( f \) to depend on \( r \) and \( \theta \) for the short range. The inclusion of \( \theta \) seems to be essential in the rotating case. On the other hand, in the framework of theory with two invariants, the Bianchi identities are
\[
\nabla_\mu F^{\mu\nu} = 0, \tag{5}
\]
and the field equations take the form
\[
\nabla_\mu (L_f F^{\mu\nu} + L_g \ast F^{\mu\nu}) = 0. \tag{6}
\]
where \( L_f = \partial \mathcal{L} / \partial f \) and \( L_g = \partial \mathcal{L} / \partial g \).

These dynamic equations can be expressed via tensor \( P^{\mu\nu} = \partial \mathcal{L} / \partial F^{\mu\nu} = L_f F^{\mu\nu} + L_g \ast F^{\mu\nu} \), in the form
\[
\nabla_\mu P^{\mu\nu} = 0; \tag{7}
\]
The following (anti)self-dual combinations can be considered \( F_{\pm}^{\mu\nu} = F^{\mu\nu} \pm i \ast F^{\mu\nu} \) and \( P_{\pm}^{\mu\nu} = P^{\mu\nu} \pm i \ast P^{\mu\nu} \ast \mathcal{P}_{\pm}^{\mu\nu} = \pm i \mathcal{P}_{\pm}^{\mu\nu} \).

One can now introduce two invariants: \( P = P_{\mu\nu} P^{\mu\nu}; Q = P_{\mu\nu} \ast P^{\mu\nu}; \) and their complex combinations
\[
P_{\pm} = P \pm iQ = (L_f \mp iL_g)^2 (f \pm ig) = 4L_{\pm}^2 F_{\pm}; \tag{8}
\]
where the following notations are used
\[
F_{\pm} = f \pm ig; \quad L_{\pm} = \partial \mathcal{L} / \partial F_{\pm} = \frac{1}{2} (L_f \mp iL_g). \tag{9}
\]
In these terms the following relation holds
\[ P_{\pm}^{\mu\nu} = 2L_{\pm}F_{\pm}^{\mu\nu}, \] (10)
and, correspondingly,
\[ F_{\mu\nu} = \frac{1}{4L_+}P_{\pm}^{\mu\nu} + \frac{1}{4L_-}P_{\mp}^{\mu\nu}. \] (11)

Putting
\[ 2H_{\pm} = 2\partial H/\partial P_{\pm} = (2L_{\pm})^{-1}, \] (12)
one can obtain the FP-duality via the Legendre transformation similarly to the theory with one invariant.

Electromagnetic invariants can be expressed in the terms of the Kerr tetrad components
\[ f = F_{ab}F^{ab} = -2(F_{12}^2 + F_{34}^2) = -(F_{12}^2 + \bar{F}_{12}^2), \]
\[ g = F_{ab} \star F^{ab} = 4iF_{12}F_{34} = i(F_{12}^2 - \bar{F}_{12}^2). \] (13)

In the Kerr-Schild tetrad components the field equations (7) can be rewritten via selfdual and anti-selfdual tensors containing in the Kerr-Schild null tetrad only two (complex) independent components,
\[ P_{12} = P_{34} = P_{34} + P_{12}, \quad P_{31} = 2P_{31}, \quad P_{24} = P_{14} = P_{24} = 0, \] (14)
in analogue with nonrotating case one expects that the known solution of the corresponding linear problem \( F_{ab} \) can be used as a ‘starting’ solution for tensor \( P_{ab} \) in nonlinear case. This solution has the following explicit form:
\[ P_{12} = AZ^2, \quad P_{31} = -(AZ)_{,1}, \] (15)
where for the charged rotating BH solution \( A = e/p^2, \quad p = 2^{-1/2}(1 + Y\bar{Y}), \quad Z = p/(r + ia \cos \theta). \) The invariants \( P_{\pm} \) can be expressed in the Kerr tetrad via complex Kerr radial coordinate \( Z \),
\[ P_+ = P + iQ = -P_{12}^2 = -e^2(Z/p)^4, \quad P_- = P - iQ = -e^2(\bar{Z}/p)^4. \] (16)
The system of field equations (11) takes the form
\[ -iI^1 = \overline{P_{12}H_{+,2} + [P_{12}H_{+,2} - P_{31}H_{+,4}]}, \] (17)
\[ -iI^2 = -\overline{P_{12}H_{+,2} - P_{12}H_{+,1} + P_{31}H_{+,4}} = 0, \] (18)
\[ -iI^2 = P_{12}H_{+,4} - \overline{P_{12}H_{+,4}} = 0, \] (19)
\[ -iI^4 = P_{12}H_{+,3} + P_{31}H_{+,2} - [P_{12}H_{+,3} + P_{31}H_{+,2}] = 0. \] (20)
Substitution of the “starting” solution (15) in (20) turns it into a system for function \( H_+ \). Some nontrivial solutions of this system were obtained. However,
integration showed that corresponding Hamiltonian \( \mathcal{H} \) turned out to be complex that cannot be appropriate. Thus, the problem of existence for consistent rotating solutions in the Kerr-Schild class remains still open.

One interesting observation follows from the above analysis. The field equations (6) are similar to the corresponding field equations in axion-dilaton gravity [11]. It shows that the factors \( L_f \) and \( L_g \) play a role which is analogous to the role of dilaton and axion. It leads us to the conclusion that both types of considered models can be joined in a model of supergravity containing the gauge and Higgs chiral fields as well as the axion and dilaton fields.

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