Bounds on large extra dimensions from the photon fusion process in SN1987A

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Abstract. The constraint on the Arkani-Hamed, Dimopoulos and Dvali model of extra dimensions coming from photon annihilation into Kaluza–Klein gravitons in supernova cores is revisited. In the two-photon process, for a conservative choice of the core parameters, we obtain the bound on the fundamental Planck scale $M_\star \gtrsim 1.6$ TeV. The combined rate of energy loss due to nucleon–nucleon bremsstrahlung and photon annihilation processes is rederived, and it shows that the combined bounds add only a second decimal place to $M_\star$. The present study may strengthen the results that are available in the current literature for the graviton emission from SN1987A, which puts very strong constraints on models with large extra dimensions for the case of $n = 3$.

Keywords: supernovas, extra dimensions, gravity
1. Introduction

Stars are potential sources for weakly interacting particles such as neutrinos, gravitons, axions, and other new particles that can be produced by nuclear reactions or by thermal processes in the hot stellar interior. The solar neutrino flux is now routinely measured with such a precision that compelling evidence for neutrino oscillations has accumulated. The measured neutrino burst from supernova SN1987A has been used to derive many useful limits. Even when the particle flux cannot be measured directly, the absence of visible decay products, notably x-rays or $\gamma$-rays, can provide important information. The properties of stars themselves would change if they lost too much energy into a new channel. This ‘energy loss argument’ has been widely used to constrain a long list of particles and their properties. All of this has been extensively reviewed [1,2].

The extra-dimensional scenario due to the Arkani-Hamed, Dimopoulos and Dvali (ADD) [3] model predicts a variety of novel signals which can be tested using table-top experiments, collider experiments, astrophysical or cosmological observations. It has been pointed out that one of the strongest bounds on models of extra dimensions comes from SN1987A [4]. Various authors have made calculations in order to place such constraints on the extra dimensions [5–10]. In this paper, we calculate the rate of energy loss due to graviton emission from SN1987A by photon–photon annihilation and derive bounds on extra dimensions. We combine the result with that of nucleon–nucleon bremsstrahlung process and derive the corresponding bound on large extra dimensions.

Physically, there are two fundamental types of supernovae (SNe), according to what mechanism powers them: the thermonuclear SNe (type I SNe) and the core-collapse ones (type II SNe). The core-collapse SNe are the class of explosions which mark the evolutionary end of massive stars ($M \gtrsim 8 M_\odot$). Such stars have the usual onion structure with several burning shells, an expanded envelope, and a degenerate iron core that is essentially an iron white dwarf. The core mass grows by nuclear burning at its edge until it reaches the Chandrasekhar limit. The collapse cannot ignite nuclear fusion because iron is the most tightly bound nucleus. Therefore, the collapse continues until the equation of state stiffens by nucleon degeneracy pressure at about the nuclear density ($3 \times 10^{14}$ g cm$^{-3}$). At this ‘bounce’ a shock wave forms, moving outward and expelling...
the stellar mantle and envelope. The explosion is a reversed implosion; the energy derives from gravity, not from nuclear energy. Within the expanding nebula, a compact object remains in the form of a neutron star or perhaps sometimes a black hole. The kinetic energy of the explosion carries about 1% of the liberated gravitational binding energy of about $3 \times 10^{53}$ erg, 99% going into neutrinos. This powerful and detectable neutrino burst is the main astroparticle interest for core-collapse SNe. In core-collapse SNe only $10^{-4}$ of the total energy shows up as light, i.e. about 1% of the kinetic explosion energy; hence they are dimmer than SNe-Ia, and are not useful as standard candles.

In the case of SN1987A, about $10^{53}$ ergs of gravitational binding energy was released in a few seconds and the neutrino fluxes were measured by the Kamiokande [11] and IMB [12] collaborations. Numerical neutrino light curves can be compared with the SN1987A data where the measured energies are found to be ‘too low’. For example, the numerical simulation in [13] yields time-integrated values $\langle E_{\nu_e} \rangle \approx 13$ MeV, $\langle E_{\bar{\nu}_e} \rangle \approx 16$ MeV, and $\langle E_{\nu_x} \rangle \approx 23$ MeV. On the other hand, the data imply $\langle E_{\nu_e} \rangle = 7.5$ MeV for Kamiokande and 11.1 MeV for IMB [14]. Even the 95% confidence range for Kamiokande implies $\langle E_{\bar{\nu}_e} \rangle < 12$ MeV. Flavor oscillations would increase the expected energies and thus enhance the discrepancy [14]. It has remained unclear whether these and other anomalies of the SN1987A neutrino signal should be blamed on small-number statistics, or point to a serious problem with the SN models or the detectors, or is there new physics happening in SNe?

Since we have these measurements already at our disposal, now if we propose some novel channel through which the core of the supernova can lose energy, the luminosity in this channel should be low enough to preserve the agreement of neutrino observations with theory. That is,

$$L_{\text{new channel}} \lesssim 10^{53} \text{ ergs s}^{-1}.$$  \tag{1}

This idea was earlier used to put the strongest experimental upper bounds on the axion mass [15]. Here, we consider the emission of the higher dimensional gravitons from the core. Once these particles are produced, they can escape into the extra dimensions, carrying energy away with them. The constraint on the luminosity of this process can be converted into a bound on the fundamental Planck scale of the theory, $M_*$. The argument is very similar to that used to bound the axion–nucleon coupling strength [1], [16]–[18]. The ‘standard model’ of supernovae does an exceptionally good job of predicting the duration and shape of the neutrino pulse from SN1987A. Any mechanism which leads to significant energy loss from the core of the supernova immediately after the bounce will produce a very different neutrino-pulse shape, and so will destroy this agreement, as demonstrated explicitly in the axion case by Burrows et al [18]. Raffelt has proposed a simple analytic criterion based on detailed supernova simulations [1]: if any energy loss mechanism has an emissivity greater than $10^{19}$ ergs g$^{-1}$ s$^{-1}$ then it will remove sufficient energy from the explosion to invalidate the current understanding of the SNe II neutrino signal.

2. Supernovae and constraints on large extra dimensions

The most restrictive limits on $M_*$ come from SN1987A energy loss argument. If large extra dimensions exist, the usual four-dimensional graviton is complemented by a tower
of Kaluza–Klein (KK) states, corresponding to a new phase space in the bulk. The KK gravitons interact with the strength of ordinary gravitons and thus are not trapped in the SN core. During the first few seconds after collapse, the core contains neutrons, protons, electrons, neutrinos and thermal photons. There are a number of processes in which higher dimensional gravitons can be produced. For the conditions that pertain to the core at this time (temperatures $T \sim 30–70$ MeV, densities $\rho \sim (3–10) \times 10^{14}$ g cm$^{-3}$), the relevant processes are nucleon–nucleon bremsstrahlung, graviton production in photon fusion and electron–positron annihilation.

In SNe, nucleon and photon abundances are comparable (actually nucleons are somewhat more abundant). In the following we present the bounds derived by various authors using nucleon–nucleon bremsstrahlung and in the next section we give a detailed calculation for the photon–photon annihilation to KK graviton process.

### 2.1. Nucleon–nucleon bremsstrahlung

This is the dominant process relevant for the SN1987A where the temperature is comparable to the pion mass $m_\pi$ and so the strong interaction between nucleons is unsuppressed. This process can be represented as

$$N + N \rightarrow N + N + KK,$$

where $N$ can be a neutron or a proton and $KK$ is a higher dimensional graviton.

The main uncertainty comes from the lack of precise knowledge of temperatures in the core: values quoted in the literature range from 30 to 70 MeV. For $T = 30$ MeV and $\rho = 3 \times 10^{14}$ g cm$^{-3}$, we list the results obtained by various authors.

Cullen and Perelstein [5]:

- $n = 2$, $\dot{\epsilon} = 6.79 \times 10^{25} \times M_\ast^{-4}$ erg g$^{-1}$ s$^{-1}$, $M_\ast \gtrsim 50$ TeV;
- $n = 3$, $\dot{\epsilon} = 1.12 \times 10^{22} \times M_\ast^{-5}$ erg g$^{-1}$ s$^{-1}$, $M_\ast \gtrsim 4$ TeV.

Barger et al [6]:

- $n = 2$, $\dot{\epsilon} = 6.7 \times 10^{25} \times M_\ast^{-4}$ erg g$^{-1}$ s$^{-1}$, $M_\ast \gtrsim 51$ TeV;
- $n = 3$, $\dot{\epsilon} = 6.3 \times 10^{21} \times M_\ast^{-5}$ erg g$^{-1}$ s$^{-1}$, $M_\ast \gtrsim 3.6$ TeV.

Hanhart et al [7, 8]:

- $n = 2$, $\dot{\epsilon} = 9.24 \times 10^{24} \times M_\ast^{-4}$ erg g$^{-1}$ s$^{-1}$, $M_\ast \gtrsim 31$ TeV;
- $n = 3$, $\dot{\epsilon} = 1.57 \times 10^{21} \times M_\ast^{-5}$ erg g$^{-1}$ s$^{-1}$, $M_\ast \gtrsim 2.75$ TeV.

Hannestad and Raffelt [9, 10]:

- $n = 2$, $\dot{\epsilon} = 4.98 \times 10^{26} \times M_\ast^{-4}$ erg g$^{-1}$ s$^{-1}$, $M_\ast \gtrsim 84$ TeV;
- $n = 3$, $\dot{\epsilon} = 1.68 \times 10^{23} \times M_\ast^{-5}$ erg g$^{-1}$ s$^{-1}$, $M_\ast \gtrsim 7$ TeV.
3. Graviton production through photon fusion and energy loss rate

Our aim is to study the energy loss mechanism of SN1987A via graviton emission by photon–photon annihilation in the ADD framework. For this we need to compute the cross-section for the relevant process. Here we present the general formalism for calculating the cross-section \( \sigma \) for a two-particle initial state. The scattering cross-section is given by

\[
\sigma = \frac{1}{v_{\text{rel}} 4E_1E_2} \int \prod_i \frac{d^3p_i}{(2\pi)^3 2E_i} (2\pi)^4 \delta^4 \left( \sum_i p_i - \sum_f p_f \right) |\mathcal{M}_f|^2, \tag{11}
\]

with

\[
v_{\text{rel}} = \sqrt{(p_1p_2)^2 - m_1^2m_2^2}/E_1E_2, \tag{12}\]

\( p_i \) and \( E_i \) being the 3-momenta and the energies of the initial particles whose masses are \( m_1 \) and \( m_2 \); \( p_f \) and \( E_f \) are the 3-momenta and the energies of the final particles and \( \mathcal{M}_f \) is the Feynman amplitude for the process.

For a general reaction of the kind \( a + b \rightarrow c \), in the center of mass frame, the expression (11) takes the form

\[
\sigma = \frac{1}{64\pi^2 E_1E_2v_{\text{rel}}} \int \frac{d^3p'}{E'} \delta(E_1 + E_2 - E')|\mathcal{M}|^2. \tag{13}\]

We use the center of mass frame, where we use the following notions.

\[
\sqrt{s} = E_1 + E_2, \tag{14}\]

\[
E_1E_2v_{\text{rel}} = p\sqrt{s}, \tag{15}\]

where \( p = p_1 + p_2 \).

Next, we focus on the energy loss due to KK gravitons escaping into the extra dimensions. The energy loss per unit time per unit mass of SN in terms of the cross-section \( \sigma_{a+b\rightarrow c} \) is given by [20]

\[
\dot{\epsilon}_{a+b\rightarrow c} = \frac{\langle n_a n_b \sigma_{(a+b\rightarrow c)} v_{\text{rel}} E_c \rangle}{\rho}, \tag{16}\]

where the brackets indicate the thermal average, \( n_{a,b} \) are the number densities for \( a, b \) and \( \rho \) is the mass density, and \( E_c \) is the energy of the particle \( c \).

We calculate the cross-section using the helicity method [21]–[36]. We follow the conventions and Feynman rules derived in [37]. In the helicity method, it is more convenient to work with polarizations explicitly. Thus, the polarization vectors [38] of a massive graviton are

\[
e_{\mu}^{\pm2} = 2\epsilon_{\mu}^{\pm} \epsilon_{\nu}^{\pm},
\]

\[
e_{\mu}^{\pm1} = \sqrt{2}(\epsilon_{\mu}^{+0} \epsilon_{\nu}^{0} + \epsilon_{\mu}^{0} \epsilon_{\nu}^{+}),
\]

\[
e_{\mu}^{0} = \sqrt{2/3}(\epsilon_{\mu}^{+} \epsilon_{\nu}^{-} + \epsilon_{\mu}^{-} \epsilon_{\nu}^{+} - 2\epsilon_{\mu}^{0} \epsilon_{\nu}^{0}).
\]
Here $\epsilon^\pm_\mu$ and $\epsilon_\mu^0$ are the polarization vectors of a massive gauge boson; for a massive vector boson with momentum $p^\mu = (E, 0, 0, p)$ and mass $m$,

$$\epsilon^+_\mu(p) = \frac{1}{\sqrt{2}}(0, 1, i, 0),$$

$$\epsilon^-_\mu(p) = \frac{1}{\sqrt{2}}(0, -1, i, 0),$$

$$\epsilon^0_\mu(p) = \frac{1}{m}(p, 0, 0, -E).$$

The graviton polarization vectors satisfy the normalization and polarization sum conditions

$$e^{s\mu
u}e^{s'\rho\sigma} = 4\delta^{ss'},$$

$$\sum_s e^{s\mu \nu}e^{s\rho \sigma} = B_{\mu \nu \rho \sigma},$$

where $B_{\mu \nu \rho \sigma}$ is given by

$$B_{\mu \nu \rho \sigma}(k) = 2\left(\eta_{\mu \rho} - \frac{k_{\mu}k_{\rho}}{m_n^2}\right)\left(\eta_{\nu \sigma} - \frac{k_{\nu}k_{\sigma}}{m_n^2}\right) + 2\left(\eta_{\mu \rho} - \frac{k_{\mu}k_{\rho}}{m_n^2}\right)\left(\eta_{\nu \sigma} - \frac{k_{\nu}k_{\sigma}}{m_n^2}\right) - \frac{4}{3}\left(\eta_{\mu \rho} - \frac{k_{\mu}k_{\rho}}{m_n^2}\right)\left(\eta_{\nu \sigma} - \frac{k_{\nu}k_{\sigma}}{m_n^2}\right).$$

The total squared amplitude, averaged over the initial polarizations $z$ and summed over final states for the reaction $a^h(q_1) + b^{h'}(q_2) \rightarrow c^{h''}(p)$, is given by

$$\frac{1}{z} \sum_{h,h',h''} \left|\mathcal{M}\left(a^h(q_1) + b^{h'}(q_2) \rightarrow c^{h''}(p)\right)\right|^2,$$

where $h, h', h''$ are the helicities and $q_1, q_2, p$ are the momenta of particles $a, b, c$ respectively.

Photons are quite abundant in supernovae. Here we consider photon–photon annihilation to KK gravitons and the process is given by

$$\gamma(k_1) + \gamma(k_2) \rightarrow KK(p).$$

The vertex function for the process (24) is given by [37]

$$X_{\mu \nu \alpha \beta} = \frac{1}{2M_4^2}\left[\eta_{\alpha \beta}k_{1\mu}k_{2\nu} - \eta_{\mu \alpha}k_{1\beta}k_{2\nu} - \eta_{\nu \beta}k_{1\mu}k_{2\alpha} + \eta_{\mu \alpha}k_{\nu \beta}(k_1k_2) - \frac{1}{2}\eta_{\mu \nu}(\eta_{\alpha \beta}(k_1k_2) - k_{1\beta}k_{2\alpha}) + m_n m_{n-m} (\eta_{\mu \alpha} \eta_{\nu \beta} - \frac{1}{2}\eta_{\mu \nu} \eta_{\alpha \beta}) + (\alpha \leftrightarrow \beta)\right].$$

The momentum vectors for this reaction are

$$p^\mu \equiv (m_n, 0, 0, p),$$

$$k_1^\mu \equiv (k_1, 0, 0, k_1),$$

$$k_2^\mu \equiv (k_2, 0, 0, k_2).$$
In the helicity formalism the reaction (24) can happen in two ways:

$$\gamma^\pm(k_1) + \gamma^\pm(k_2) \rightarrow KK^{\pm 2}(p),$$  \hspace{1cm} (29)

$$\gamma^\pm(k_1) + \gamma^\mp(k_2) \rightarrow KK^0(p).$$  \hspace{1cm} (30)

Next, consider these two reactions separately and find their corresponding amplitudes. For the reaction described in (29), the helicity amplitude for the KK graviton emission by photon–photon annihilation is

$$|M(\gamma^\pm(q) + \gamma^\pm(q) \rightarrow KK^{\pm 2}(p))| = X^{\mu\nu\alpha\beta} \epsilon^\pm_\alpha(k) \epsilon^\pm_\beta(q) e^{\pm 2}_\mu(p).$$  \hspace{1cm} (31)

The polarization tensors for gravitons are calculated and they are

$$e_{11}^{\pm 2} = +\frac{1}{2},$$  \hspace{1cm} (32)

$$e_{12}^{\pm 2} = e_{21}^{\pm 2} = \frac{1}{2},$$  \hspace{1cm} (33)

$$e_{22}^{\pm 2} = -\frac{1}{2}.$$  \hspace{1cm} (34)

The non-zero components of the vertex function are

$$X^{1111}, \ X^{1212}, \ X^{1221}, \ X^{2112}, \ X^{2121}, \ X^{2222}, \ -X^{2211}, \ -X^{1122}.$$  

Each of them is equal to

$$-\frac{i\kappa}{2} k_1 k_2,$$  \hspace{1cm} (35)

where $k_1 k_2 = m_{n}^2 / 2$.

Substituting the various quantities that we have calculated above into equation (31), we get

$$|M(\gamma^\pm(q) + \gamma^\pm(q) \rightarrow KK^{\pm 2}(p))| = \frac{\kappa m_{n}^2}{2}. $$  \hspace{1cm} (36)

The helicity amplitude for the reaction (30) is

$$|M(\gamma^\pm(q) + \gamma^\mp(q) \rightarrow KK^0(p))| = X^{\mu\nu\alpha\beta} \epsilon^\pm_\alpha(k) \epsilon^\mp_\beta(q) e^0_\mu(p).$$  \hspace{1cm} (37)

The polarization tensors for gravitons are given by

$$e_{11}^0 = e_{22}^0 = -\sqrt{\frac{2}{3}},$$  \hspace{1cm} (38)

$$e_{12}^0 = e_{21}^0 = 0.$$  \hspace{1cm} (39)

The non-zero components of the vertex function are $X^{1111}, \ X^{2222}, \ -X^{2211}$ and $-X^{1122}$ and are equal to (35).

Substituting the various quantities that we have calculated above into equation (37), we get

$$|M(\gamma^\pm(q) + \gamma^\mp(q) \rightarrow KK^0(p))| = 0.$$  \hspace{1cm} (40)
Thus the total squared amplitude, averaged over the initial three polarizations and summed over final states, is
\[
\frac{1}{3} \sum_{h,h',h''} |M(\gamma^h(k_1) + \gamma^{h'}(k_2) \rightarrow KK^{h''}(p))|^2 = \frac{\kappa^2 m_\gamma^4}{12}. \tag{41}
\]
Substituting this into (13) and using (14) and (15), the cross-section for the process is obtained as
\[
\sigma = \frac{\pi \kappa^2}{16} \sqrt{s} \delta(m_\gamma - \sqrt{s}), \tag{42}
\]
where \(s\) is the center of mass energy, and \(m_\gamma\) the mass of the \(KK\) state at level \(n\).

Since for large \(R\) the KK gravitons are very light, they may be copiously produced in high energy processes. For real emission of the KK gravitons from a SM field, the total cross-section can be written as
\[
\sigma_{\text{tot}} = \kappa^2 \sum_n \sigma(n), \tag{43}
\]
where the dependence on the gravitational coupling is factored out. The mass separation of adjacent KK states, \(\mathcal{O}(1/R)\), is usually much smaller than typical energies in a physical process; therefore we can approximate the summation by an integration which can be performed using the KK state density function [37],
\[
\rho(m_\gamma) = \frac{R^n m_\gamma^{n-2}}{(4\pi)^{n/2} \Gamma(n/2)}. \tag{44}
\]
The volume emissivity of a supernova with a temperature \(T\) through the process under consideration is obtained by thermal averaging over the Bose–Einstein distribution:
\[
Q_\gamma = \int \frac{2 \, d^3 \vec{k}_1}{(2\pi)^3} \frac{1}{\omega_1/T - 1} \int \frac{2 \, d^3 \vec{k}_2}{(2\pi)^3} \frac{1}{\omega_2/T - 1} s(\omega_1 + \omega_2) \sum_n \sigma_{\gamma\gamma \rightarrow kk}(s, m_\gamma), \tag{45}
\]
where the summation is over all KK states, and the squared center of mass energy \(s\) is related to the photon energies \(\omega_1\) and \(\omega_2\) and the angle between the two photon momenta \(\theta_{\gamma\gamma}\) is as follows:
\[
s = 2\omega_1 \omega_2 (1 - \cos \theta_{\gamma\gamma}). \tag{46}
\]
After carrying out the integrations and the summation over KK states, we find
\[
Q_\gamma = \frac{2^{n+3} \Gamma(n/2 + 3) \Gamma(n/2 + 4) \zeta(n/2 + 3) \zeta(n/2 + 4)}{(n + 4)\pi^2} \frac{T^{n+7}}{M_{5}^{n+2}}, \tag{47}
\]
where we have used \(M_s^{n+2} R^n S_n = M_{5}^{2}\) and numerically these Riemann zeta functions are close to 1. In this calculation, we have neglected the plasma effect, through which the photons can have energy dispersion relations different to those of free particles.

We take the supernova core density \(\sim 10^{15} \text{ g cm}^{-3}\). Using (16), we compute the energy loss rate for \(n = 2\) and 3 extra spatial dimensions and hence the lower limits on \(M_s\) using the conservative upper limits on the energy loss rate of SN [1]
\[
\dot{\epsilon}_{\text{SN}} \sim 10^{19} \text{ erg g}^{-1} \text{ s}^{-1}. \tag{48}
\]
The results are summarized below.

\[ n = 2, \quad \dot{\epsilon} = 4.7 \times 10^{23} \times M_\ast^{-4} \text{ erg g}^{-1} \text{ s}^{-1}, \quad M_\ast \gtrsim 14.72 \text{ TeV}; \quad (49) \]

\[ n = 3, \quad \dot{\epsilon} = 1.1 \times 10^{20} \times M_\ast^{-5} \text{ erg g}^{-1} \text{ s}^{-1}, \quad M_\ast \gtrsim 1.62 \text{ TeV}. \quad (50) \]

We now combine the rate of energy loss due to the photon fusion process with that of the nucleon–nucleon bremsstrahlung and rederive the constraints as follows.

Cullen and Perelstein [5]:

\[ n = 2, \quad \dot{\epsilon} = 6.837 \times 10^{25} \times M_\ast^{-4} \text{ erg g}^{-1} \text{ s}^{-1}, \quad M_\ast \gtrsim 50.13 \text{ TeV}; \quad (51) \]

\[ n = 3, \quad \dot{\epsilon} = 1.131 \times 10^{22} \times M_\ast^{-5} \text{ erg g}^{-1} \text{ s}^{-1}, \quad M_\ast \gtrsim 4.08 \text{ TeV}. \quad (52) \]

Barger et al [6]:

\[ n = 2, \quad \dot{\epsilon} = 6.747 \times 10^{25} \times M_\ast^{-4} \text{ erg g}^{-1} \text{ s}^{-1}, \quad M_\ast \gtrsim 50.96 \text{ TeV}; \quad (53) \]

\[ n = 3, \quad \dot{\epsilon} = 6.410 \times 10^{21} \times M_\ast^{-5} \text{ erg g}^{-1} \text{ s}^{-1}, \quad M_\ast \gtrsim 3.64 \text{ TeV}. \quad (54) \]

Hanhart et al [7, 8]:

\[ n = 2, \quad \dot{\epsilon} = 9.710 \times 10^{24} \times M_\ast^{-4} \text{ erg g}^{-1} \text{ s}^{-1}, \quad M_\ast \gtrsim 31.39 \text{ TeV}; \quad (55) \]

\[ n = 3, \quad \dot{\epsilon} = 1.680 \times 10^{21} \times M_\ast^{-5} \text{ erg g}^{-1} \text{ s}^{-1}, \quad M_\ast \gtrsim 2.79 \text{ TeV}. \quad (56) \]

Hannestad and Raffelt [9, 10]:

\[ n = 2, \quad \dot{\epsilon} = 4.985 \times 10^{26} \times M_\ast^{-4} \text{ erg g}^{-1} \text{ s}^{-1}, \quad M_\ast \gtrsim 84.03 \text{ TeV}; \quad (57) \]

\[ n = 3, \quad \dot{\epsilon} = 1.681 \times 10^{23} \times M_\ast^{-5} \text{ erg g}^{-1} \text{ s}^{-1}, \quad M_\ast \gtrsim 7.00 \text{ TeV}. \quad (58) \]

As we expected, the rate of energy loss due to nucleon–nucleon bremsstrahlung is 1–3 orders of magnitude more than that due to the photon fusion process. Hence the combined bounds add only the second decimal place to \( M_\ast \).

4. Conclusions

We have revisited the constraints on the ADD model coming from photon annihilation into KK gravitons in SN cores. For a conservative choice of the core parameters, we obtain the two-photon process bounds on the fundamental Planck scale \( M_\ast \gtrsim 1.6 \text{ TeV}. \) The rate of energy loss due to nucleon–nucleon bremsstrahlung is 1–3 orders of magnitude more than that due to photon fusion process. Hence the combined bounds add only a second decimal place to \( M_\ast \). Thus the present study may strengthen the results which are available in the current literature for the graviton emission from SN1987A. Our results show that the above processes put very strong constraints on models with large extra dimensions for the case of \( n = 3 \). Notice that the plasmon effects are not considered in our calculations and will be studied elsewhere.
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