Systematic Design of Antireflection Coating for Semi-infinite One-dimensional Photonic Crystals Using Bloch Wave Expansion

Jun Ushida, Masatoshi Tokushima, Masayuki Shirane, and Hirohito Yamada
Fundamental Research Laboratories, NEC Corporation, 34 Miyukigaoka, Tsukuba, 305-8501, JAPAN

We present a systematic method for designing a perfect antireflection coating (ARC) for a semi-infinite one-dimensional (1D) photonic crystal (PC) with an arbitrary unit cell. We use Bloch wave expansion and time reversal symmetry, which leads exactly to analytic formulas of structural parameters for the ARC and renormalized Fresnel coefficients of the PC. Surface immittance (admittance and impedance) matching plays an essential role in designing the ARCs of 1D PCs, which is shown together with a practical example.

Photic crystals (PCs) have unique energy dispersion due to coupling between periodic materials and electromagnetic (EM) waves. In particular, the strong energy dispersion of the propagation modes of PCs has attracted much interest because it enables applying PCs to add/drop multiplexers, dispersion compensators, polarization filters, and image processors. These transmission-type applications require a negligibly small reflection loss at the PC interface. Therefore, applying effective antireflection interface structures, or antireflection coatings (ARCs) to the input and output interfaces of the PCs is important. Several ARCs designed for the PCs have been reported on. The structural ideas of these ARCs were based on the concepts of adiabatic interactions or wave vector matching (k matching) for two-dimensional PCs. The reflectance at the interface of one-dimensional (1D) PCs has so far been calculated by plane-wave expansion and multiplications of the transfer matrix for a unit cell. However, these approaches do not tell directly the optimal ARC parameters, so the numerical calculation needs to be iterated until these parameters are optimized. In this letter, we derive analytic formulas of the structural parameters of an ARC that is applied to a 1D PC. We deal with 1D systems for simplicity, but the derived formulas would be applicable to multidimensional PCs under appropriate approximation.

The structural parameters of a conventional ARC placed between two homogeneous media can be calculated easily if the refractive indices of the two media are known. In Fig. 1 (a), the ARC (region 2) with refractive index \( n_2 \) and thickness \( d \) is placed between two semi-infinite homogeneous media with refractive indices \( n_1 \) (region 1) and \( n_3 \) (region 3). These three regions are divided by two boundaries at \( z = 0 \) and \( -d \), where the \( z \) axis is defined as perpendicular to the surface of the ARC. Each region consists of linear and lossless dielectrics. The reflection coefficient at \( z = -d \) is then given by

\[
r = \frac{r_{1,2} + r_{2,3}\exp(2ik_2d)}{1 + r_{1,2}r_{2,3}\exp(2ik_2d)},
\]

where \( k_2 \) is the normal component of the wave vector in the ARC, and \( r_{i,j} \) is the reflection coefficient of the light propagating from region \( i \) to \( j \). The reflection coefficient in Eq. (1) equals zero for normal incidence light when \( d = \lambda_0/4n_2 \) and \( n_2 = \sqrt{n_1n_3} \), where \( \lambda_0 \) is the wavelength in vacuum.

As in Fig. 1 (b), we replace the semi-infinite homogeneous medium (region 3) in Fig. 1 (a) by a semi-infinite 1D PC with lattice constant \( \Lambda \) and periodic refractive index function \( n(z) = n(z + \Lambda) \).

![FIG. 1. Antireflection coating (region 2) with thickness \( d \) and refractive index \( n_2 \) for (a) a semi-infinite homogeneous medium with refractive index \( n_3 \) and for (b) a semi-infinite 1D PC with lattice constant \( \Lambda \) and periodic refractive index function \( n(z) = n(z + \Lambda) \).](image)
boundaries \((z = 0, -d)\). The expression of the derived reflection coefficient is formally the same as Eq. (1), except for \(r_{2,3}\). The reflection coefficient \(r_{2,3}\) for normal incidence light is modified into

\[
r_{2,3}(\omega) = \frac{n_2 - N(\omega)}{n_2 + N(\omega)},
\]

where \(N(\omega)\) equals \(\pm Y_{k,k}^{(+)}/\varepsilon_0 c\), which is a normalized surface admittance of the Bloch waves. Note that in \(\pm Y_{k,k}^{(+)}/\varepsilon_0 c\), the \(+\) (“+”) sign applies to \(Y_{k,k}^{(+)}\) \((Y_{k,k}^{(-)})\). The surface admittances \(Y_{k,k}^{(\pm)}\) for the two orthogonal polarizations are defined as

\[
Y_{k,k}^{(\pm)} = H_{k,\xi}/E_{k,\xi}, \quad Y_{k,k}^{(-)} = H_{k,\eta}/E_{k,\eta}, \quad z = +0,
\]

where \(k\) is the Bloch wave vector, and \(E_{k,\xi}(H_{k,\eta})\) with \(\xi = x \text{ or } y\) stands for the tangential component of the electric(magnetic) field of the propagating Bloch waves with a positive group velocity or the decaying Bloch waves in the positive \(z\) direction. These Bloch waves can be calculated from the aforementioned transfer matrix, hence \(N(\omega)\) in Eq. (2) is obtained directly. In the derivation of Eqs. (2) and (3) we used the fact that time reversal symmetry inhibits the simultaneous appearance of the propagating and the decaying modes in the same direction for a given set \(\{\omega, k\} \text{ and } \sigma\) in a semi-infinite 1D PC. Note that the function \(N(\omega)\) in Eq. (2) is generally complex due to the phase difference between \(E_{k,\xi}\) and \(H_{k,\eta}\) \((\xi, \eta = x \text{ or } y)\) of the Bloch waves at the interface, however the imaginary part of \(N(\omega)\) is zero if the surface of the semi-infinite PCs is a mirror plane in an infinite form of the PCs. Note also that multiple reflections in the semi-infinite 1D PCs, which are represented as plane waves, are renormalized into a single Fresnel coefficient in Eq. (2) by using the Bloch wave expansion.

The pair values of refractive index \(n_2\) and thickness \(d\) of the ARC for semi-infinite 1D PCs that eliminate reflectance are determined as follows. The extremal condition of the reflectance \(R(=|r|^2)\) with respect to \(d\), i.e., \(\partial R/\partial d = 0\), is written as

\[
D \equiv \frac{d}{\lambda_0/4n_2} = \frac{1}{2\pi i} \ln \left[ \frac{n_{2,3}^2}{n_{1,3}^2} \right] + m, \quad m = 0, 1, \cdots.
\]

We call \(D\) the “normalized thickness”. By substituting \(d\) derived from Eq. (4) into Eq. (1), we obtain a refractive index \(n_2\) that eliminates reflectance:

\[
n_2 = \sqrt{n_1 \text{Re}(N)} \sqrt{\frac{n_1 - |N|^2/\text{Re}(N)}{n_1 - \text{Re}(N)}}.
\]

The necessary and sufficient condition for the perfect ARC for semi-infinite 1D PCs is \(D \geq 0, n_2 \geq 1\) in Eqs. (4) and (5), and \(\partial^2 R/\partial d^2 > 0\) under a given \(n_1 \geq 1, N(\omega), \text{ and } \omega\).

Next, we investigate the physical conditions encapsulated in Eq. (5). The condition \(\text{Re}(N) > 0\) is necessary to obtain a positive real number for \(n_2\), so the explicit form of \(n_2 \geq 1\) with Eq. (5) is classified by using positive \(\alpha \equiv \text{Re}(N)/n_1\):

\[
n_2 \geq 1 \Leftrightarrow \begin{cases} \text{(A)} & f > -\beta^2 & \text{for } 1 < \alpha < \infty, \\ \text{(B)} & f = -\beta^2 & \text{for } \alpha = 1, \\ \text{(C)} & f \leq -\beta^2 & \text{for } 0 < \alpha < 1, \end{cases}
\]

where \(f(\alpha, n_1) \equiv (\alpha - 1)/(\alpha - 1/n_1^2)\), and \(\beta \equiv \text{Im}(N)/n_1\).

These sufficient conditions in the rhs of Eq. (6) are illustrated in Fig. 2(a). As \(n_1\) changes from 1 to \(\infty\), the curve for \(f\) as a function of \(\alpha\) varies from a solid one to a broken one continuously. The shaded area indicates the region satisfying the conditions of Eq. (6). When \(1 < \alpha < \infty, f > -\beta^2\) always holds because \(f > 0\). The case \(\alpha = 1\) leads to \(\beta = 0\) due to \(f = 0\). A magnification of region \(0 < \alpha < 1\) in Fig. 2(a) is shown in Fig. 2(b), where a curve for function \(f\) with \(n_1 = 2\) is added. The thick solid line shows the values of \(f\) and \(\alpha\) that satisfy the third condition in the rhs of Eq. (5) for \(n_1 = 2\) and a given \(-\beta^2\). This curve connects two crossing points (filled circles) of \(f\) and \(-\beta^2\) through the minimum point (open circle) of \(f\). These three points are used to rewrite the inequality \(f \leq -\beta^2\) for \(0 < \alpha < 1\) in Eq. (5), which leads to two conditions: (1) the minimum point of \(f\) is less than or equal to \(-\beta^2\), and (2) \(\alpha\) is located between the two crossing points.

By applying the previous, we can obtain the final form of the three sufficient conditions for \(n_2 \geq 1\) in Eq. (5):

\[
\begin{align*}
\text{(A)} & \quad 1 < \alpha < \infty, \\
\text{(B)} & \quad \alpha = 1 \quad \text{and} \quad \beta = 0, \\
\text{(C)} & \quad 0 < \alpha < 1, \quad 0 \leq g \quad \text{and} \\
& \quad -\sqrt{g} \leq 1 + 1/n_1^2 - 2\alpha \leq \sqrt{g}.
\end{align*}
\]

where \(g(\beta, n_1) \equiv (1 - 1/n_1^2)^2 - 4\beta^2\). Note that the condition (C) includes \(n_1 \neq 1\) implicitly. Conditions (A)-(C) depend on \(n_1\) and \(N(\omega)\) only, so the \(n_2\) needed to achieve ARCs for semi-infinite 1D PCs can be directly obtained.
is also determined by Eq. (4) and by using Eqs. (7)-(9) and (5). Accordingly, thickness without (broken line) ARC.

In conclusion, we have presented a systematic method for designing a perfect ARC for a semi-infinite 1D PC. We assume that Si (n = 3.5) and SiO$_2$ (n = 1.5) with the same thickness $\Lambda/2$, and that the Si layer is the boundary layer between the semi-infinite PC and the ARC. In Fig. 3(b), the dispersion relation of the infinite 1D PC is plotted and the band index is indicated by B1-B5. When the frequency changes along a band, the corresponding pair of $[n_2(\omega) \text{ and } d(\omega)]$ forms a continuous trajectory [as shown in Fig. 3(a)]. Note that in Fig. 3(a), only condition (A) is needed to calculate $n_2$ and $d$, and frequency regions near the band edges that require $n_2 \geq 4.5$ are not plotted. Note also that the deviation in the normalized thickness from 1 is due to the phase shift between the electric and the magnetic fields at the interface ($z = \pm 0$).

In Fig. 3(c) we show the reflectance of the 1D PC with (solid line) and without (broken line) an ARC. The ARC was designed at a selected frequency ($\omega \Lambda/2\pi = 0.7$), which is indicated by an arrow, where $N = 3.384 - i0.499$ fulfills condition (A). The required values of $n_2(= 1.868)$ and $D (= 1.071)$ for the ARC are calculated from Eqs. (3) and (4) with $m = 1$, which is shown by a circle in Fig. 3(a). In Fig. 3(c), the ARC eliminates the reflectance (solid line) at the selected frequency (arrow).

In conclusion, we have presented a systematic method for designing a perfect ARC for a semi-infinite 1D PC. We derived exact formulas of structural parameters of the ARC and $R/\partial d^2 > 0$. Accordingly, thickness $d$ is also determined by Eq. (4) and $\partial R/\partial d^2 > 0$.

To give a practical example of ARCs for semi-infinite 1D PCs, in Fig. 3(a) we plot the refractive index and the thickness needed to form the ARC for a semi-infinite 1D PC. We assume that $n_1 = 1$, that the unit cell of the PC consists of Si ($n = 3.5$) and SiO$_2$ ($n = 1.5$) with the same thickness $\Lambda/2$, and that the Si layer is the boundary layer between the semi-infinite PC and the ARC. In Fig. 3(b), the dispersion relation of the infinite 1D PC is plotted and the band index is indicated by B1-B5. When the frequency changes along a band, the corresponding pair of $[n_2(\omega) \text{ and } d(\omega)]$ forms a continuous trajectory [as shown in Fig. 3(a)]. Note that in Fig. 3(a), only condition (A) is needed to calculate $n_2$ and $d$, and frequency regions near the band edges that require $n_2 \geq 4.5$ are not plotted. Note also that the deviation in the normalized thickness from 1 is due to the phase shift between the electric and the magnetic fields at the interface ($z = \pm 0$).

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