Instabilities, nulls and subpulse drift in radio pulsars

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ABSTRACT
This paper continues a previous study of neutron stars with positive polar-cap corotational charge density in which free emission of ions maintains the surface electric field boundary condition $E \cdot B = 0$. The composition of the accelerated plasma on any subset of open magnetic flux lines above the polar cap alternates between two states: protons or positrons and ions, of which the proton state cannot support electron–positron pair creation at higher altitudes. The two states coexist at any instant of time above different moving elements of area on the polar cap and provide a physically consistent basis for a description of pulse nulls and subpulse drift. In the latter case, it is shown that the band separation $P_3$ is determined not by the $E \times B$ drift velocity, as is generally assumed, but by the diffusion time for protons produced in reverse-electron showers to reach the region of the atmosphere from which they are accelerated. An initial comparison is made with the survey of subpulse drift published by Weltevrede et al.

Key words: instabilities – plasmas – stars: neutron – pulsars: general.

1 INTRODUCTION
It is now generally assumed that electron–positron pair creation immediately above the magnetic poles is essential for coherent radio emission in pulsars. Muslimov & Tsygan (1992) have made an important contribution in their recognition of the significance of the Lense–Thirring effect in the generation of adequately strong electric fields (see also the paper of Muslimov & Harding 1997). It is remarkable, and must be almost unique, that a general relativistic effect changes the order of magnitude of a field component in a system that can be described, ostensibly, perfectly well in Euclidean space as in the paper of Goldreich & Julian (1969).

At least initially, finding the acceleration field $E_\parallel$, which is the component locally parallel with the magnetic flux density $B$, can be regarded as a problem in electrostatics within a volume in the corotating frame of reference having well-defined boundary conditions. This is defined by the neutron star surface and the surface separating open from closed magnetic flux lines and is assumed to have the electric potential boundary condition $\Phi = 0$. It is possible that this surface is actually a current sheet of some complexity (see, for example, Arons 2010), but we shall assume that, near the neutron–star surface, the time dependence of $E_\parallel$ within it is not sufficiently rapid to hinder the charge movement necessary to maintain the boundary condition. Thus there are three possible cases depending, first, on the relation between rotation spin $\Omega$ and $B$, and hence the sign of the polar-cap Goldreich–Julian charge density $\rho_0$, and secondly on the neutron–star surface value of $E_\parallel$. They are (i) $\Omega \cdot B > 0$, $\rho_0 < 0$, $E_\parallel = 0$ with electron acceleration, (ii) $\Omega \cdot B < 0$, $\rho_0 > 0$, $E_\parallel = 0$ with positron and positive baryon acceleration or (iii) $\Omega \cdot B < 0$, $\rho_0 > 0$ and $E_\parallel \neq 0$. Throughout this paper, for brevity, these conditions will be referred to as cases (i), (ii) or (iii). There is no further possible case because free emission of electrons is always possible at neutron star surface temperatures. A previous paper (Jones 2010a, hereafter Paper I) reported some preliminary work on the relation between these sets of boundary conditions and observed phenomena, in particular, the existence of pulse nulls in a significant fraction of radio pulsars. There seems to be no reason why neutron stars satisfying (i), (ii) and even exceptionally (iii) should not exist, and it is the purpose of this paper to continue the attempt to see if features of the predicted plasma acceleration in these cases correlate with observed properties of subsets of the pulsar population.

There can be little doubt that in this context nulls and subpulse drift are important phenomena. Although there must be reservations about assigning undue weight to the properties of a single neutron star, nulls in the isolated pulsar PSR B1931+24 are informative. Kramer et al. (2006) found that spin-down in the on-state is approximately twice as fast as in the off-state. It is difficult to see, in a pulsar of this age (>10^7 yr), how the geometrical shape of the acceleration volume or the magnitude of the current density $J$ within it can change by a factor of this order in less than the rotation period $P$. Time-varying fields near the light cylinder can further accelerate ultrarelativistic particles of both signs and therefore the most obvious explanation for the change in spin-down torque is that the particle and field components of the magnetospheric momentum density and stress tensor near the light cylinder differ between on- and off-states. A cessation of pair creation during nulls would
greatly change the charged particle number density even though \( J \) remains essentially unchanged. In this connection, it is interesting that a recent paper by Lyne et al. (2010) suggests quasi-periodic switching between two different spin-down rates as the origin of the peculiar timing residuals seen in many pulsars.

The presence of nulls might be thought of as evidence that a pulsar is in the final phase of evolution prior to complete cessation of radio emission. However, we suggest that this view is not consistent with a specific feature of the 63 nulling pulsars listed in tables 1 and 2 of the paper by Wang, Manchester & Johnston (2007). The maximum potential difference \( \Phi_{\text{max}} \) available for acceleration at the polar cap is proportional to \( B_0 P^{-2} \), in which \( B_0 \) is the effective dipole field inferred from the spin-down rate. It is approximately independent of the ratio of \( B_0 \) to the actual polar-cap field \( B \). The distribution of this quantity for all radio pulsars in the Australia Telescope National Facility (ATNF) catalogue (Manchester et al. 2005) has a relatively sharp cut-off at \( 2.2 \times 10^{11} \text{ G s}^{-2} \), equivalent to the existence of a well-defined death line in the distribution of \( B_0 \) versus \( P \). It might be expected that nulls should be seen only as a pulsar rotation slows so that it approaches this value. But the form of the distribution of \( B_0 P^{-2} \) for the nulling pulsars listed by Wang et al. is broadly indistinguishable from that of the whole ATNF catalogue and instead is consistent with nulls being a long-term property of a certain subset of radio pulsars. It is, of course, just possible that the observed sharp cut-off is itself a statistical fluctuation and that individual pulsar deaths actually occur at values of \( B_0 P^{-2} \) throughout the whole distribution because some other variable is involved, such as flux-line curvature. But we shall discount this possibility and in this paper adopt the view that pulsars with boundary condition (ii) and, exceptionally, (iii) form the subset that null.

A recent large-scale survey by Weltevrede, Edwards & Stappers (2006) has revealed that subpulse drift is a common phenomenon. Under the assumption that electron–positron pairs are the source of the coherent radio emission, it implies that compact zones of pair creation exist and move in an organized way within the open magnetosphere area of the polar cap. This motion has been assumed to be an \( E \times B \) drift velocity under the case (iii) surface electric field boundary condition, following the original paper of Ruderman & Sutherland (1975). There have been many later papers on this subject, and we refer to Gil, Melnikidze & Geppert (2003) for recent developments which have sought to refine the calculation of the drift velocity. The problem is that the case (iii) boundary condition is implausible as a common property of the pulsar population and that cases (i) and (ii) have not hitherto provided any immediate and obvious basis for the formation of organized subpulses. It was shown in Paper I that short time-scale instability in the composition of accelerated plasma exists in case (ii). In this paper, it is shown to be a plausible mechanism for subpulse formation and drift.

Under the assumption of an actual dipole field, pair creation is possible, in principle, in all observed pulsars through the inverse Compton scattering (ICS) of polar-cap photons by accelerated electrons or positrons (Hibschman & Arons 2001, also see fig. 1 of Harding &Muslimov 2002), but curvature radiation (CR) can produce pairs only in those with high values of \( B_0 P^{-2} \). However, there is a problem, noted by Harding & Muslimov, in the formation of a dense pair plasma because the number of ICS pairs formed per primary electron or positron accelerated can be smaller than unity (see also fig. 8 of Hibschman & Arons). They suggest that the pair density required for coherent radio emission may be far smaller than previously thought. Although higher multipole field components may be present in most pulsars, so increasing flux-line curvature, the existence of this problem must remain a matter of concern, perhaps even leading to doubts about the role of pair plasma as the source of coherent radio emission.

The existence of solutions of the electrostatic problem under boundary conditions (ii) or (iii) is no more than a preliminary because we are concerned in this paper with the presence of instabilities which arise from the reverse electron flow at the polar cap. It might be questioned whether or not the charge density on the surface separating open from closed magnetic flux lines needed for the condition \( \Phi = 0 \) can be maintained in the presence of instability. But we shall find that, even at short time-scales, instability principally changes the nature of the particles accelerated rather than the current density \( J \) and the acceleration field. The instabilities considered here are not, of course, a feature of case (i) in which electrons are the primary component of \( J \). Establishing instability in this case is a purely electromagnetic problem which has been considered by Levinson et al. (2005), Melrose & Luo (2009) and Reville & Kirk (2010). Non-stationary flow in case (iii) has been investigated by Timokhin (2010).

The properties of the condensed matter surface are naturally important in cases (ii) and (iii), in particular, the production of protons by the reverse flux of electrons described previously (Paper I; see also Jones 1981). This is a characteristic of electromagnetic showers that is usually of little practical significance. The form of the shower depends principally on electron–photon interactions, but shower photons also interact directly with nuclei. The cross-section is a maximum with the excitation of the giant dipole resonance (GDR), a broad collective state, at a photon momentum \( k \approx 40 \text{mc} \). (Electroproduction cross-sections are smaller by a factor of the order of the fine-structure constant and can be neglected.) Photon track length per unit interval of \( k \) in a high-energy shower is \( \propto k^{-2} \) so that to a good approximation, photoproduction of baryons can be assumed to occur entirely through GDR formation. Known cross-sections and electromagnetic shower theory allow the calculation of production rates, and we define \( W_p \) as the number of protons produced per unit primary shower energy. Protons are initially of the order of a few MeV but are very rapidly moderated to thermal energies without further strong interaction, and then diffuse to the surface with a time delay that is of prime significance for the stability of plasma acceleration. For further details, we refer to Paper I, also to Jones (2010b) for the cross-sections at high values of \( B \) for processes of second order in electron–photon coupling. In the early stages of acceleration, partially ionized atoms interact with the polar-cap blackbody radiation field, and this is the source of the reverse-electron flux that is considered here. It has proved convenient to combine rates for this process with values of \( W_p \) by defining, for a particular pulsar, the parameter \( K \) which is equal to the number of protons produced per unit nuclear charge accelerated.

Instabilities in solutions of the time-independent electrostatic problem referred to above exist as a consequence of proton formation, and we propose that they are the basis for the nulling phenomenon and for subpulse formation and drift. The complexity that arises is unfortunate because it limits what can be achieved in terms of a physical theory of the acceleration process. Thus we shall be able to expose the general properties attaching to cases (ii) and (iii) but are not always able to give quantitative predictions. We shall show in Section 2 that general consideration of polar-cap parameters rule out the possibility that the relatively numerous subsets of pulsars that exhibit nulls and subpulse drift belong to case (iii). Therefore, Sections 3 and 4 of this paper are restricted to further
consideration of case (ii) for which short time-scale instability in plasma acceleration has been described previously in Paper I. Its properties are summarized in Section 3. The previous treatment of medium time-scales was, however, inadequate and contained an error. The appropriate analysis is given here in Section 4. The relations between these instabilities and the observed properties of nulls and of subpulse drift are described in Section 5 and, in particular, the relation between the subpulse band separation periodicity $P_3$ and proton diffusion time is given.

2 POLAR-CAP PARAMETERS

Many of the polar-cap properties and parameters that will be required for the arguments of Sections 3–5 have been given previously in Paper I. The properties of the polar-cap atmosphere are of particular importance and will be further considered in Section 3. But it will be convenient to summarize the remainder here with some additions.

The most basic parameter is the ion number density $N$ of the condensed matter at zero pressure. The magnetic dipole fields $B_{10}$ inferred from pulsar spin-down rates vary over about five orders of magnitude, but the median value for the 63 nulling pulsars listed by Wang et al. is $2.8 \times 10^{12}$ G, which is significantly larger than for the whole ATNF catalogue. It is also possible that the actual polar-cap field $B$ is larger than $B_{10}$. For these reasons, we adopt the expression,

$$N = 2.6 \times 10^{26} Z^{-1.7} B_{10}^{1.2} \text{cm}^{-3}, \quad (1)$$

fitted to values computed by Medin & Lai (2006), primarily for $B_{10} > 10$, where $B_{10}$ is the magnetic flux density in units of $10^{10}$ G and $Z$ is the atomic number. The convenient unit of depth below the surface at $z = 0$ is the radiation length

$$l_r = 1.66Z^{-1.3} B_{10}^{-1.2} \left( \ln \left( 12Z^{1/2} B_{10}^{-1/2} \right) \right)^{-1} \text{cm} \quad (2)$$

defined here in terms of the zero-field Bethe–Heitler bremsstrahlung cross-section with screening factor modified for the neutron star surface density (see Paper I).

The critical temperature above which the ion thermal emission rate is high enough to maintain the case (ii) boundary condition is related to the cohesive energy $E_c$ by $k_B T_c \approx 0.025 E_c$ (see the discussion of atmospheric properties in Section 3). Cohesive energies have been calculated by Medin & Lai as functions of $B$. For $Z = 26$ and $B_{10} = 10$, their value is in good agreement with that obtained by Jones (1985) using a different representation of the three-dimensional condensed matter state. In the interval $10 < B_{10} < 100$, their values can be fitted by the expressions $E_c = 0.016 B_{10}^{1.3} \text{keV}$ for $Z = 6$ and $E_c = 0.16 B_{10}^{0.7} \text{keV}$ for $Z = 26$, giving

$$T_c = 4.6 \times 10^4 B_{10}^{1.3} \text{K}, \quad Z = 6$$

$$= 4.6 \times 10^4 B_{10}^{0.7} \text{K}, \quad Z = 26. \quad (3)$$

These are to be seen in relation to other temperatures that are significant. Paper I contained a calculation of the rate of proton formation in the electromagnetic showers formed by reverse electrons incident on the polar cap. From the energy flux needed to produce a proton current density $J^p = \rho^p c$, we can infer a maximum steady-state polar-cap temperature,

$$\tilde{T} = \left( \frac{T_{\text{res}}^4 + \frac{(-B \cos \psi)(1 - \kappa)}{\rho_c e W_p}}{\rho_c e W_p} \right)^{1/4} \text{K}. \quad (4)$$

In this expression, $W_p$ is the number of protons produced per unit primary shower energy. We can approximate it initially by $W_p^{BH}$ which was obtained in Paper I by using the zero-field Bethe–Heitler pair creation cross-section with screening modified for the condensed matter density at the neutron star surface. Its values, given in table 1 of that paper, can be summarized conveniently in the intervals $B_{10} = 10^1 – 10^2$ and $Z = 10–26$ by the expression

$$W_p^{BH} = 3.9 \times 10^{-3} (Z)^{0.6} B_{10}^{2} (mc^2)^{-1}. \quad (5)$$

in which the nuclear charge is the average in the region of the shower maximum. The angle $\psi$ is that between $\Omega$ and $B$. The general relativistic correction contained in the surface value of the Goldreich–Julian charge density $\rho_0 = \kappa$ (see Muslimov & Harding 1997), and $\sigma$ is Stefan’s constant. Equation (4) also contains a further temperature, $T_{\text{res}}$, which is the polar-cap temperature in the absence of any reverse electron or photon energy flux (approximately the observed whole-surface temperature corrected to the local proper frame). The presence of $T_{\text{res}}$ assumes that there is a constant heat flow to the surface driven by the very much higher temperature of the inner crust. With $T_{\text{res}} = 0$ and $\kappa = 0.15$, we find

$$T_{\text{max}} = 5.1 \times 10^4 (Z)^{0.15} B_{10}^{2} \left( \frac{-\cos \psi}{P} \right)^{1/4} \text{K}. \quad (6)$$

Equating this with $T_c$ allows us to estimate the minimum polar-cap magnetic flux densities necessary to sustain the case (iii) boundary condition $E_{\parallel} \neq 0$. These are

$$B_{10} = 181 \left( \frac{-\cos \psi}{P} \right)^{1/4} \text{Z} = 6$$

$$= 327 \left( \frac{-\cos \psi}{P} \right)^{1/2} \text{Z} = 26. \quad (7)$$

Comparison with the median value of $B_{10} = 2.8$ for the 63 nulling pulsars listed by Wang et al. shows that case (iii) can be widely realized only with implausibly large deviations from a central dipole field.

However, the screening-modified zero-field Bethe–Heitler pair creation cross-section is not obviously valid at magnetic flux densities of the order of the critical field $B_c = 4.41 \times 10^{13}$ G. Thus we have been obliged to calculate the second-order bremsstrahlung and pair creation cross-sections using Landau function solutions of the Dirac equation. The photoproduction of protons by GDR decay is determined by the total photon track length in the GDR band, centred on a momentum $k = 40mc$, which occurs almost entirely in the late stages of shower development. The track length at these energies is limited by Coulomb and magnetic pair creation, and also by Compton scattering, the effect of which is almost always to scatter the photon so that its transverse momentum component $k_\perp$ (perpendicular to the field) exceeds the threshold for magnetic conversion to an electron–positron pair. We refer to sections 2 and 3 of Paper I for a more detailed description of these processes. Approximate values of the Coulomb pair creation cross-section at magnetic fields $B \geq B_c$ are given in table 3 of Jones (2010b) and are the basis for the values of $W_p$ given here in Table 1. These are not easily summarized as a simple expression analogous with equation (5). The cross-section at $B = 10B_c$ is at least an order of magnitude smaller than the modified zero-field Bethe–Heitler cross-section, but the effect on $W_p$ is not large because the photon track length in this region is limited by Compton scattering. Substitution into equation (4) gives values of $T_{\text{max}}$ that typically are reduced by a factor of approximately 0.9 from those of equation (6). But the complexity of the second-order processes at $B \sim B_c$ is such that we have not reconsidered shower development and, specifically, have not allowed for the production by cyclotron emission or Coulomb...
bremssstrahlung of photons with \(k_z\) above the thresholds for magnetic
conversion. This becomes significant at \(B = 10B_\circ\) as shown in
tables 2 and 4 of Jones (2010b), and the high magnetic conversion
transition rates reduce GDR-band photon track length in the
shower. The reduced \(W_p\) values increase \(T_{\text{max}}\). Owing to this complexity at
\(B \sim B_\circ\) there are inevitable uncertainties in our estimates of \(W_p\), but we
believe that they do not seriously invalidate the estimates of the
minimum polar-cap magnetic flux densities needed to support the
case (iii) boundary condition and our conclusion that it can exist
only in a very small subset of pulsars. Our conclusions drawn from
equation (3) are also independent of the spectrum of the reverse
electron–photon flux because \(W_p\) is almost completely independent
of primary shower energy provided that is large compared with the
GDR energy.

There appear to be no published estimates of the melting tempera-
ture of condensed matter that are specific to very high magnetic
fields. Consequently, we are obliged to adopt the standard one-
component Coulomb plasma expression (see, for example, Slattery,
Doolen & De Witt 1980) which, with equation (1), gives
\[
T_m = 1.0 \times 10^9 Z_z^2 Z^{-0.23} B_{12}^{0.4} \text{K}
\]
(8)
in terms of an effective charge \(Z_e\). This latter parameter represents
the fact that the deeply bound Landau states certainly do not par-
take significantly in the melting transition, but its estimation at
higher values of \(Z\) is quite difficult. It is possible that the work of
Potekhin, Chabrier & Yakovlev (1997; see their fig. 1) could
provide guidance although it is at zero field and was directed to-
wards a different problem. On this basis, we assume for \(Z = 26\)
a value in the interval \(Z_e = 10–15\). In a typical polar-cap field,
\(B_{12} = 10\), the melting temperature is as low as \(T_m = 6 \times 10^5 \text{K}\)
for \(Z_e = Z = 6\) but exceeds \(10^8 \text{K}\) for \(Z = 26\). Thus the state of the polar
cap may be a sequence of melting and solidifications. The order of
magnitude of the shear modulus is a further source of complexity.

The standard (zero-field) expression for a body-centred cubic lattice
(Fuchs 1936) can be adapted, with equation (1), to give
\[
\mu = 1.1 \times 10^{16} Z^2 Z^{-0.93} B_{12}^{1.6} \text{erg cm}^{-3}.
\]
However, the polar-cap gravitational constant is \(g \approx 2 \times 10^{14} \text{cm s}^{-2}\)
so that any density inversion may well induce a form of Rayleigh–
Taylor instability.

The final condensed-matter parameter that is important is the
thermal conductivity parallel with \(B\), which is extremely high (see
Potekhin 1999). Thermal energy is deposited at the shower maxi-
ma at distance \(z_p\) below the surface \(z = 0\), which is defined as
that separating condensed matter from the atmosphere, and is then
dissipated as blackbody radiation from the polar cap. The value of
\(z_p\) in the high-density condensed matter of the neutron star surface
depends on shower energy owing to the Landau–Pomeranchuk–
Migdal (LPM) effect, but for the order of magnitude of the char-
acteristic time we can assume \(z_p \sim 10l_i \approx 10^{-3} \text{cm}\) using the
radiation lengths \(l_i\) given by equation (2) or in table 1 of Paper I.
The characteristic time for shower energy input to produce a
surface-temperature fluctuation is then
\[
t_{\text{cond}} = \frac{C_{\text{fp}}^2}{2\lambda_{12}} \approx 10^{-9} \text{s},
\]
in which typical values of the parameters are the specific heat \(C =
1.0 \times 10^{12} \text{erg cm}^{-3} \text{K}^{-1}\) and the longitudinal coefficient of thermal
conductivity \(\lambda_1 = 6 \times 10^{14} \text{erg cm}^{-1} \text{s}^{-1} \text{K}^{-1}\). But for a surface
temperature of \(10^9 \text{K}\), the internal temperature gradient needed to
coduct the radiated energy flux is extremely small, of the order of
\(10^8 \text{K cm}^{-1}\), within the condensed matter at \(z < 0\). In effect,
heat is more easily conducted to greater depths than radiated from
the surface. Consequently, the Green function \(G(z, z_p, t)\) giving the
internal temperature distribution must be almost independent of \(z_p\)
and very close to that for zero temperature gradient at the surface
\(z = 0\). Thus a shower heat input \(\mathcal{H}(t)\) produces a temperature
distribution within the condensed matter at \(z \leq 0\),
\[
T(z, t) = H G(z, t) \approx \frac{H}{(\tau C Z_e T)^{1/2}} \exp\left( -\frac{C_{\text{fp}}^2}{4\lambda_1 T} t \right),
\]
(11)
It is asymptotically \(\propto t^{1/2}\) so that the polar-cap temperature arising
from a sequence of heating events, each producing a maximum
producing a maximum temperature \(T_{\text{max}}\), has fluctuations away from \(T\)
whose magnitude is a function of the time-scale concerned. Further discussion of this
is deferred until our consideration of observed polar-cap blackbody
temperature in Section 5.

### 3 SHORT TIME-SCALE INSTABILITY

Under the assumption that the case (ii) boundary condition is
satisfied, the neutron star surface has an atmosphere in local
thermodynamic equilibrium (LTE) with approximate scaleheight
\(l_A = (Z + 1)k_B T/M_g \sim 10^{-3} \text{cm}\) at temperature \(T = 10^9 \text{K}\), where
\(Z\) and \(M\) are the mean ion charge and mass of the partially
ionized atom. The expression for the chemical potential of an ideal
Boltzmann gas gives the LTE atmospheric number density at \(z = 0\),
\[
N_A(0) = \left( \frac{M k_B T}{2\pi \hbar^2} \right)^{3/2} e^{-E_i/k_B T},
\]
for atmospheric temperatures such that \(N_A(0) \ll N\). Its order of
magnitude is \(N_A(0) \approx 10^4 \exp(-E_i/k_B T)\) for \(M = 56m_p\).
We estimate the critical temperature \(T_c\) which must not be exceeded
if the case (iii) boundary condition is to be valid by equating the flux
of ions in such an LTE atmosphere that are incident on the neutron
star surface with the ion flux needed to give an open magnetosphere
current density equal to \(\rho_{0c}\). This gives \(N_A(0) \approx 10^{14} \text{cm}^{-3}\) and
\(k_B T_c = 0.025E_c\). But the extent to which the atmosphere can be
described as thin is very temperature dependent. Thus for \(T = 2T_c\)
and \(Z = 26, N_A(0)\) is of the order of \(10^{23} \text{ cm}^{-3}\), and the whole atmosphere contains ions equivalent to about \(10^{-1} L_6\). At a density \(N_A(0) > 10^{23} - 10^{24} \text{ cm}^{-3}\), the Boltzmann gas estimate of \(L_6\) assumed here is unreliable, and it becomes necessary to allow for the interaction of protons with the ion atmosphere.

If instabilities in plasma acceleration with time-scales as short as \(\sim 10^{-4} \text{ s}\) are considered, the temperature at \(z = 0\) is constant apart from very small fluctuations, as shown by the Green function given at the end of the previous section. Consequently, the temperature distribution and number density of ions in the atmosphere are also constant in time, and the atmosphere is in LTE in the interval \(0 < z < z_1\), where \(z_1\) is the top of the ion atmosphere, defined as the surface of last scattering. For time-scales of the order of 1 s, the temperature and depth of the atmosphere can change appreciably, but there is no doubt that it is always in LTE. The processes of proton formation by GDR decay followed by diffusion to the surface were described in Paper I, which assumed a very thin atmosphere. Therefore, we need to consider here, in more detail, diffusion in the atmosphere \(0 < z < z_1\). The number of protons is many orders of magnitude smaller than that of ions so that the properties of the atmosphere, its scaleheight and equilibrium electric field given by the electrical neutrality condition, are determined solely by the latter. With the assumption of a single ion component, we can determine the equilibrium electric field within the LTE atmosphere in the presence of a gravitational acceleration \(g\) and find that the proton potential energy is

\[
\left(1 - \frac{A}{Z + 1}\right) m_p g z = a m_p g z
\]

for ions of charge \(Z\) and mass number \(A\), and that this transports protons to \(z > z_1\), from which region they are accelerated to relativistic energies in preference to the ions. Proton diffusion at a rate greater than is needed for a current density \(j_p = \rho_0 c\) therefore cuts off ion acceleration and produces a thin electrically neutral proton atmosphere at \(z > z_1\). There appears to be no reason why this atmospheric structure should be disturbed by turbulent mixing.

The proton average potential energy at \(z < 0\) is close to \(m_p g z\), but the jump bias, \(m_p g a_s/k_B T\) for ion separation \(a_s\) is too small to be significant because \(m_p g a_s/k_B T \ll 1\). The atmospheric proton density at \(z = 0\) is necessarily some orders of magnitude smaller than the density at \(z < 0\) owing to the density discontinuity at the surface. Thus diffusion to the surface at \(z = 0\) is little changed by the presence of an atmosphere that is not necessarily very thin. Movement of the protons through the ion atmosphere is effected by the chemical potential gradient which is a function of ion number density. At high densities, as in the solid at \(z < 0\), this is determined principally by entropy, but as the density reduces, the potential energy given by equation (12) becomes the more important factor. In effect, the motion changes from a random walk to a drift velocity. The dividing ion density is given by the condition \(\lambda R = N_A^{-1/2}\) for the total cross-section for Rutherford back scattering, and unless restricted by pair creation at lower altitudes, is reached at an altitude \(z\) of the order of the neutron star radius \(R\) which is roughly two orders larger than the polar-cap radius. We refer to Harding & Muslimov (2001, 2002) for a complete account. We assume here that for this current density, spontaneous pair creation by CR is not possible. Then the primary source of any positron component in \(J\) can only be the reverse flow of photoelectrons from accelerated partially ionized atoms as discussed in Paper I. But there is an essential difference here in that protons in the very low density region at \(z \sim z_1\) are almost completely ionized (here, we refer to fig. 1 of Potekhin et al. 2006) so that both track length and energy flux of reverse-flow electrons from photoionized hydrogen atoms are negligibly small. Thus positron production in any interval of proton acceleration is negligible. This is also true for ions of low atomic number \(Z \sim 4–5\) which are completely ionized, but for higher \(Z\), the reverse flux of inward-accelerated electrons produces polar-cap ICS photons, as would outward-accelerated electrons in case (i). Pairs are produced by photons that are scattered to transverse momenta above the magnetic conversion thresholds. The only difference is that the photons are inward moving. But even if spontaneous CR pair formation is not possible, positrons accelerated outwards will produce CR pairs at higher altitudes, as in case (i), though superimposed on a flux of ions.

In case (i), the reverse flux of positrons must form protons which are not accelerated but form an atmosphere at \(z > 0\) whose equilibrium is defined by various diffusion processes perpendicular to the magnetic flux. Backward-moving photons from the electron or positron showers are a further source of pair creation in each of cases (i)–(iii). These arise from the decay of residual nuclei following proton or neutron emission in GDR formation and from \((n, \gamma)\) reactions. Those photons that are not absorbed in the more dense part of the atmosphere can produce pairs if their transverse momenta exceed the threshold. The LPM effect, which in the high-density condensed matter at \(z < 0\) is significant for electrons of more than 10 GeV or photons of more than 2 GeV, is less important in the
atmosphere within which there will be some GDR formation. But we have been unable to make a satisfactory quantitative estimate of pair formation through this process.

4 MEDIUM TIME-SCALE INSTABILITY

Instability on time-scales some orders of magnitude longer than those of Section 3 is also of interest and can appear as a fluctuation in the charge of nuclei reaching the polar-cap surface which we shall define here to be always at \( z = 0 \) although it may move with respect to coordinates fixed at the centre of the star. Ions of initial charge \( Z_i \) move upwards through the region of the shower maxima at \( z_p \) and, with nuclear charge reduced to \( Z_0 \) by GDR formation and decay, enter the atmosphere at \( z > 0 \). We wish to find if there are \( \eta \) in terms of which we could express \( \eta(z) \) fails to be a time-independent function of position with limits \( Z_a \geq Z_0(z) \geq Z_b \). The work of this section replaces that of section 4.1 in Paper I which contains an error.

The longer time-scales here enable us to assume the proton and ion current densities \( J^p \) and \( J^e \) are the time averages of those described in the previous section. There is, of course, a distribution of discrete nuclear charges in the shower region, but we shall work in terms of the average \( Z(z, t) \), the corresponding number density \( N(z, t) \) given by equation (1) and the velocity \( v(z, t) \) with which nuclei approach the polar-cap surface at \( z = 0 \). Proton formation by GDR decay occurs predominantly in the very late stages of shower development, as explained in Section 1, and so we shall assume that it is confined within limits \( z_a \) and \( z_b \) and is defined by the normalized function \( g_p(z) \),

\[
\int_{z_a}^{z_b} g_p(z) dz = 1.
\]  

The physical basis for our study of the system is that the total numbers of nuclei and of protons (bound or unbound) are conserved. Because both neutrons and protons are produced in GDR decay, we can make the approximation of neglecting the effect of \( \beta \)-transitions. Therefore, the continuity equations are

\[
\frac{\partial N}{\partial t} + \frac{\partial (N v)}{\partial z} = 0,
\]

and with neglect of the relatively short proton diffusion time so that within medium time-scales all protons produced in the shower are assumed to be promptly accelerated from the atmosphere at \( z \approx z_1 \),

\[
\frac{\partial (N Z v)}{\partial t} + \frac{\partial (N Z v)}{\partial z} = -g_p(z) J^p(t).
\]  

Because the reverse electron flux from photoelectric ionization is the source of the showers, we shall find it convenient here, as in Paper I, to introduce the parameter \( K \) which is a function of the atmospheric nuclear charge and is the number of protons produced per unit positive nuclear charge accelerated. With the representation of equation (1) in the form \( N = CZ^\gamma \), equations (17) and (18) can be combined to give

\[
C \int_{z_a}^{z_b} dz \left( Z^\gamma \frac{\partial Z}{\partial t} + v Z^\gamma \frac{\partial Z}{\partial z} \right) = -J^p(t).
\]  

It has the obvious time-independent solution,

\[
Z_a - Z_b = K Z_b,
\]  

and the time-independent velocity of nuclei is such that \( Z'_0(z)v_0(z) \) is independent of \( z \).

A natural fluctuation away from \( Z_0 \) would be of the form

\[
\delta Z = \eta (Z_a - Z_0(z)) e^{i \omega t}
\]

(21)
giving similar fluctuations in \( J^p, N \) and \( v \), in which \( \eta \) is infinitesimal and independent of \( z \) and \( t \). From equation (17) we then have

\[
\delta (Z' v) = -i \omega \eta \chi(z) e^{i \omega t},
\]  

(22)

where

\[
\chi(z) = \gamma \int_{z_a}^{z_b} dz' (Z_a - Z_0) Z_0^\gamma^{-1}.
\]  

(23)

Substitution into equation (19) with the retention of terms of first order in \( \eta \) gives

\[
\int_{z_a}^{z_b} dz \left( Z_0^\gamma (Z_a - Z_0) - \chi(z) \frac{\partial Z_0}{\partial z} \right) - Z_0^\gamma v_0 (Z_b - Z_0) \eta e^{i \omega t} = -\frac{1}{C} \delta J^p(t).
\]  

(24)

The current densities, averaged over short time-scales, are \( J^e = NZ_a(0, t) \) and, from the definition of \( K, J^p = KJ^e \). The fluctuation away from the time-independent solution is \( \delta J^e = \delta J^p + \delta J^f = 0 \), which we assume to be maintained by the boundary conditions on \( \Phi \). To express \( \delta J^p \) in terms of \( \delta z \), we represent the \( Z \)-dependence of \( K \) in the vicinity of \( Z_0 \) as a power law \( K = K_0 z^\tau(0, t) \). We find, after eliminating the velocity fluctuation \( \delta v(0, t) \), that

\[
\frac{1}{C} \delta J^p = v Z_0^\gamma v_0 K K_0 + 1 \delta Z(0, t).
\]  

(25)

The relationship with equation (24) is established by noting that

\[
\delta Z(0, t) = \delta Z(z_b, t - \tau) = \eta (Z_a - Z_b) e^{i \omega t - \tau},
\]  

(26)

where \( \tau \) is the time interval of nuclear movement from \( z = z_b \) to 0. From equations (24)–(26), the equation whose root \( \omega \) we require can be expressed as

\[
\int_{z_a}^{z_b} dz Z_0^\gamma (1 + \gamma) (Z_a - \gamma Z_b) (Z_a - Z_0)
\]

\[
= -(Z_a - Z_b) - v K K_0 + 1 (Z_a - Z_b) e^{-i \omega t},
\]  

(27)

in which the function \( \chi(z) \) has been removed by partial integration.

The only assumptions we need to make about the depth distribution of proton formation in the late stage of shower development are that it is small outside the interval \( z_a < z < z_b \) and that \( z_b - z_a \) is smaller than \( |z_b| \) though not necessarily so by as much as an order of magnitude. A suitable function would be

\[
g_p(z) = \frac{2}{z_b - z_a} \sin^2 \left( \frac{\pi (z - z_a)}{z_b - z_a} \right),
\]  

(28)

in terms of which we could express \( Z_0 \) as an explicit function of \( z \), which would be needed to obtain numerical values for the root \( \omega = \omega_1 + i \omega_2 \) of equation (27). However, we are primarily concerned here not with growth rates but with the boundary between stability and instability and so shall not proceed with numerical solutions for \( \omega \). Fortunately, it is possible to obtain a sufficient condition for the existence of instability independently of the form of \( g_p \). Provided \( -1 < \gamma < 0 \), which is clearly satisfied, we can see directly from the values of the integrand in equation (27) at the limits and from its lack of an extremum that

\[
\int_{z_a}^{z_b} dz ((1 + \gamma) Z_a - \gamma Z_b) \left( Z_a - Z_0 \right) \left( Z_0 \right) \left( Z_0 \right) \left( Z_0 \right) < (z_b - z_a)(Z_a - Z_b),
\]  

(29)

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and therefore that equation (27) can be replaced by the inequality,

\[ \text{i} \omega \tau \left( \frac{z_t - z_{\theta}}{|z_t|} \right) > -1 - \frac{vK}{K + 1} e^{-\text{i} \omega \tau}. \]  

(30)

Thus \( \omega_{1,2} \) satisfy the inequalities

\[ \omega_{1,2} \tau \left( \frac{z_t - z_{\theta}}{|z_t|} \right) > \frac{vK}{K + 1} e^{\text{i} \omega \tau} \sin \omega_{1,2} \tau, \]

\[ \omega_{1,2} \tau \left( \frac{z_t - z_{\theta}}{|z_t|} \right) < 1 + \frac{vK}{K + 1} e^{\text{i} \omega \tau} \cos \omega_{1,2} \tau. \]

(31)

From the first of these, we can see that the real part of \( \omega \) must satisfy \( \omega_{1,2} \tau > \pi/2 \), provided the proton formation region in the shower is sufficiently compact that

\[ \frac{vK}{K + 1} e^{\text{i} \omega \tau} > \frac{\pi}{2} \left( \frac{z_t - z_{\theta}}{|z_t|} \right). \]

(32)

With greater compactness, \( \omega_{1,2} \tau \rightarrow \pi/2 \), as would be expected. The second inequality gives the condition for \( \omega_{1,2} \tau < 0 \), i.e. fluctuation growth. At the threshold where \( |\omega_{1,2} \tau| \ll 1 \), and for \( z_t - z_{\theta} \ll |z_t| \), it is simply \( vK \gg K + 1 \).

The \( Z \)-dependence of \( K \) was represented in Paper I as an approximate power law, \( K = K_0 Z^\nu \), with \( \nu = 0.85 \) for \( Z \geq 10 \) but with the reservation that this would certainly become invalid for an atmosphere of ions with \( Z \sim 5 \) which would be almost completely ionized. Therefore, we should anticipate quite large values of \( \nu \) at small \( Z \), sufficient to give the unstable behaviour found here. Stability clearly depends on the energy flux from photoionization. Low fluxes and moderate rates of proton formation in the GDR region of the showers, such that the nuclear charge inferred from equation (20) is \( Z_0 \approx 10 \), allow a stable time-independent progression of nuclear charge as a function of depth below the polar-cap surface as given by \( Z_0(z) \). But larger values of \( K \) and smaller \( Z_0 \) lead to instability. (The value of \( Z_0 \) is unimportant, provided it is not small, and for order of magnitude estimations in this paper we have assumed \( Z_0 = 26 \) unless otherwise stated.)

Unfortunately, our analysis of the instability of nuclear movement to the polar-cap surface is limited to small fluctuations and does not extend to large amplitudes. But it is not difficult to see the form it would take. An atmosphere of high-\( Z \) ions at time \( t \) produces a high reverse-electron energy flux which creates a layer of low-\( Z \) ions in the shower maximum region \( z_{\theta} < z < z_{t} \). These flow towards the surface at \( z = 0 \) and form a low-\( Z \) atmosphere at time \( t + \tau \) which produces a low reverse-electron energy flux and correspondingly large values of \( Z_t \). The ions accelerated alternate between high- and low-\( Z \) values. The basic unit of time is given by the time \( t_\theta \) for the emission at the Goldreich–Julian current density of one radiation length of ions,

\[ t_\theta = 2.1 \times 10^5 Z^{-1} B_{12}^{-1} ( -P \text{ sec} \psi ) \times \left( \ln \left( 12 Z^{1/2} B_{12}^{-1/2} \right) \right)^{-1} s. \]

(33)

The high-\( Z \) intervals are subject to short-time-scale instability as described in Section 3, but low-\( Z \) intervals have little or no reverse-electron flux and therefore no possibility of significant positron acceleration and electron–positron pair production.

The instability outlined here is of quite simple form, but there are complicating factors that have been mentioned earlier in the Section 2 consideration of polar-cap parameters. Evaluations of the melting temperature discussed following equation (8) show that there is every possibility that the condensed matter state below the atmosphere may be liquid or a solid close to melting with a high self-diffusion rate. This would have no effect on short time-scale instability but could complicate the behaviour of the system over medium time-scales. The melting temperature is \( Z \)-dependent so that a density inversion is possible, the upper layer of higher \( Z \) being either liquid or solid. In the liquid case, we must anticipate Rayleigh–Taylor instability, which may also exist in the solid case because shear modulus (see equation 9) may not be adequate to maintain mechanical stability. All these processes are occurring at depths \( |z_\theta| \sim 10^{-3} \text{ cm} \) but over a polar cap whose radius is of the order of \( 10^3 \text{ cm} \). Consequently, a further complication is that different polar-cap areas are unlikely to be in phase with each other. These problems are considered further, though necessarily in a qualitative way, in Section 5.

5 NULLS, SUBPULSE DRIFT AND POLAR-CAP COHERENCE

The time-dependent phenomena considered in Sections 3 and 4 are local and one dimensional because both shower depth \( z_\theta \) and atmospheric scaleheight are very many orders of magnitude smaller than the polar-cap radius. This introduces the question of whether or not there is coherence over the whole polar-cap area. Both instabilities are dependent on the parameter \( K \) which is a function of the surface nuclear charge \( Z(0, \tau) \) and to a lesser extent of surface temperature and acceleration field. For this reason alone, we see no case for assuming complete coherence and anticipate that the polar cap we describe has zones of proton and ion emission which cannot be stationary, the total areas of each being determined, approximately, by the average value of \( K \). For neutron stars that are unable to support spontaneous pair production by CR, the proton zones have no reverse electron flux, do not support pair production and hence merely produce an accelerated one-component plasma as described in Section 3. But the ion zones will support ICS pair production and so appear as moving sources within the polar cap. In this section, we shall attempt to compare possible forms this motion might take with some of the observed phenomena in radio pulsars.

The distinction between the average pulse profile and individual subpulses within it was noted almost immediately following the discovery of pulsars (see, for example, Smith 1977). The amplitude and form of successive subpulses can vary in times of the order of the rotation period, and in some pulsars, observed with higher resolution, subpulses have microstructure of \( 10^{-4} \) to \( 10^{-5} \text{ s} \) time-scale. There are also more organized phenomena, and for recent extensive surveys of these we refer to Wang et al. (2007) in the case of pulse nulls and to Weltevrede et al. (2006) for subpulse drift. There is a move towards a consensus (Lyne et al. 2010) that quasi-periodic switching between magnetospheric states with different spin-down rates is the basis of mode changing. These authors even suggest that it is the source of a large component of pulsar timing noise. However, it is also true that the subpulse characteristics observed in a small number of pulsars are quite singular, but here discussion is confined to the general features of subpulses.

Given the properties of the medium time-scale instability described in Section 4, it is natural to associate with nulls those intervals in which high-\( Z \) ion zones either do not exist or are confined to areas of the polar cap from which radiation produced by the plasma is not visible to the observer. For sufficiently low values of \( Z_\theta \), ions are accelerated from the surface completely ionized so that there is no reverse-electron flux and hence no pair creation and radio emission. The current density is little changed in the open sector of the magnetosphere, but the absence of pair creation means that the particle content near the light cylinder is quite different, as are the components of the momentum density or stress tensor on any
spherical surface in this region centred on the star. It is therefore not surprising that the spin-down torque is reduced during the interval of a null, as has been observed in PSR B1931+24 (Kramer et al. 2006). Neutron stars with small $K$ such that $Z_{c,0} \approx 10$ are likely to have $\nu < 1$ and so will have a steady state progressively reducing nuclear charge $Z_{c}(z)$, no medium time-scale instability and no long-term nulls. But the questions about melting and mechanical stability of the polar cap which were mentioned at the end of Section 4 remain, and the whole system is so complex and difficult to describe in physical terms that we are unable to give useful quantitative predictions. However, incomplete nulls having a low but detectable level of emission should be observed. It is also unsurprising that nulls are observed to be not completely random (Redman & Rankin 2009).

To some extent and unfortunately, the same remarks have to be made about the effects of short time-scale instability. For temperatures $T > 2T_{c}$, the proton diffusion time is much longer than in the case of the very thin atmosphere assumed in Paper I but remains difficult to calculate with complete confidence. In order to describe the polar cap, it is necessary to introduce coordinates $u(z)$ on a surface perpendicular to $B$. As in Paper I, the proton current density at any point $u(0)$ can be related to the ion current density through the definition of $K$,

$$J^{p}(u, t) + \delta^{p}(u, t) = K \int_{-\infty}^{t} \mathrm{d}t' f_{p}(t - t') J^{p}(u, t'),$$  \tag{34}

in which $f_{p}$ is the normalized proton diffusion-time distribution. Without the $J^{p}$ term, whose significance is described below, this would be a homogeneous Volterra equation of the second kind having no non-zero square-integrable solution. (The time dependence of $K$ arising from the temperature dependence of the LTE ionic charge $\bar{Z}$ is neglected here.) The approximate expression for $f_{p}$ given in Paper I (equation (23) assumed a random-walk diffusion through the condensed matter at $z < 0$ and so is not valid for an atmosphere with properties given by equations (13) and (14). For $T > 2T_{c}$, most of the diffusion time is in the drift-velocity phase, in which case the time distribution would be more suitably approximated by a normalized Gaussian function centred at $t = \tau_{p}$ or, in the limit, by $f_{p}(t - t') = \delta(t - t' - \tau_{p})$, where $\tau_{p}$ is derived from equation (13) of Section 3. The quantity $J^{p} = 0$ within intervals for which $J^{p} < 1$ and at other times, when $J^{p} = J$ and $J^{p} = 0$ until the instant at which the atmosphere is exhausted and $J^{p}$ falls to some residual value $J^{p} < J$. Then ion flow re-emerges almost immediately (in a time much shorter than the growth time for spontaneous CR pairs) and continues until proton diffusion grows sufficiently to re-form the proton atmosphere. The durations of the time intervals for ion and proton emission are almost identical with those for electron–positron pair creation or otherwise, and labelled $\tau_{ee}$ and $\tau_{gap}$, are given by

$$J = K \int_{0}^{\tau_{ee}} \mathrm{d}t' f_{p}(\tau_{ee} - t') J^{p}(t'),$$

$$\tau_{gap} = \frac{K}{J} \int_{0}^{\tau_{ee}} \mathrm{d}t' J^{p}(t').$$  \tag{35}

Therefore $\tau_{ee}$ and $\tau_{gap}$ both depend on the smallness of $f_{p}$ for small values of $t - t'$. In the $\delta$-function limit for $f_{p}$, we have $\tau_{gap} = K \tau_{ee}$. Estimates of the parameter $K = K_{0} \bar{Z}$ were given in Paper I, but are repeated here,

$$K_{0} = 2.8(Z_{\text{sm}}^{-1})^{0.76} B_{12}^{0.62} T_{6}^{-1.0}, \quad B_{12} > 1,$$

$$K_{0} = 1.6(Z_{\text{sm}}^{-1})^{0.76} T_{6}^{-1.0}, \quad B_{12} < 1.$$  \tag{36}

with $\nu = 0.85$ for $10 < Z < 26$. In these expressions, $T$ is not the local surface temperature but is an average for radiation emitted over the whole polar cap, and $B$ is the actual polar-cap magnetic flux density. For $B_{12} \gg 1$, the values of $K_{0}$ given need to be modified, though not greatly, to allow for the high-$B$ values of $W_{p}$ contained in Table 1.

Guided by the estimates contained in equations (6) and (7), we anticipate that except for a very small number of neutron stars, the surface temperature is at all times $T > T_{c}$ so that the case (ii) boundary condition is always maintained and ion emission is never temperature limited. It is limited instead by the fact that the proton atmosphere forms above the ions and the protons are preferentially accelerated, as described in Section 3. The effect of the temperature variations described by equation (11) that occur as a result of alternating proton and ion emission is to change the density of the LTE ion atmosphere. It is also worth observing that the local nature of equation (34) is not disturbed by the presence of $E \times B$ drift above the polar cap. Although this slightly displaces a reverse electron shower relative to the point from which the ion was accelerated, it has no effect on ion emission, which is not temperature dependent.

In view of the time variation described by equations (34) and (35), is it possible to imagine organized rather than chaotic motion of ion zones on the polar cap? An example of chaotic motion would be the existence of very many small zones without obvious organization. Two simple organized cases would be motion along a slot of constant rotational latitude and circular motion at constant $u(0)$. With the Deshpande & Rankin (1999) analysis of PSR B0943+10 in mind, we consider circular motion. Equations (34) and (35) are local in $u$ and contain no information that can determine an ion zone movement velocity $\dot{u}$. The quantities that are essentially constant are $K$ and, apart from the effect of varying LTE atmospheric temperature, $\tau_{p}$. Thus the circulation time for $n$ ion zones is $P_{3} = n \tau_{p}$, where $P_{3}$ is the band separation in the usual notation by which subpulse drift is described, and here it is given by

$$P_{3} = \tau_{gap} + \tau_{ee} = (K + 1) \tau_{p}$$  \tag{37}

in the $\delta$-function limit of the diffusion function $f_{p}$. In this system, the velocity $\dot{u}$ is determined by $n$ for fixed values of $K$ and $\tau_{p}$. For the same reason, drift direction is also unspecified and there is nothing to preclude a change in $n$ or a reversal following some disturbance to the polar-cap surface such as might be a consequence of the kind of mechanical instability briefly described in Section 4. The drift time is determined primarily by diffusion through the more dense layers of the atmosphere. From equations (13) and (14), it is

$$\tau_{p} = \int_{0}^{\tau_{ee}} \frac{\mathrm{d}z}{\bar{v}(z)} = \left( \frac{l_{\Lambda}}{a_{\text{g}e} \lambda_{\text{g}e} K_{0}} \right) \left( \frac{k_{B} T}{m_{p}} \right)^{1/2}. \tag{38}$$

In Section 3, we observed that atmospheric density at $z = 0$ is an exponential function of $T > T_{c}$ so that its value is essentially unpredictable. Therefore, it is likely that the density discontinuity at $z = 0$ is small and that the reverse-electron showers may be contained entirely within the atmosphere. In this case, the lower integration limit in equation (38) must be replaced by $z_{\text{sm}} > 0$. The diffusion time is then not directly a function of $B$ but is dependent on surface gravity and on $\bar{Z}$. But its distribution for all pulsars should be compact. The distribution in the values of $K$, given by equation (36), is probably the more important source of the observed

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spread in the values of $P_1$. Detailed calculation of $\tau_s$ has not been attempted here. In particular, our estimates of $\lambda_R$ and $l_\lambda$ are subject to some uncertainty.

The fact that $P_1$ is constant for a particular pulsar whereas the circulation time $P_\delta$ is dependent on $\eta$ could in principle allow comparison with $E \times B$ drift-velocity polar-cap models in which $P_1$ is constant. It is also worth considering the effect of mechanical instability in allowing leakage of protons (or low-Z ions) to limited areas of the surface as described at the end of Section 4. The excess protons form a localized atmosphere of greater depth than in surrounding areas, so that ion zones (with consequent pair creation) do not form there until it is exhausted. The observer of radio emission probably sees plasma from no more than a strip of polar-cap area at roughly constant rotational latitude, so it is possible that intervals of null emission would be observed at fixed longitude in a sequence of subpulse bands.

In a survey of 187 pulsars, selected only by signal-to-noise ratio, Weltevrede et al. (2006) found that reversals do occur and that roughly equal numbers of pulsars have subpulse drift to smaller or greater longitudes. They also comment that subpulse drift is so common a phenomenon that it cannot depend on pulsar parameters having extraordinary values. The band separation $P_\delta$ is independent of radio frequency and is not correlated with period $P$, age $P/2P$ or with the inferred dipole field $B_d$. Measured values of $P_\delta$ given for 77 pulsars in table 2 of Weltevrede et al. are mostly contained within a single order of magnitude, $1 < P_\delta < 10^6$, although the distribution has a small tail extending to $\sim 20$s. This is quite compact (for a neutron star parameter) and is not inconsistent with the interpretation of subpulse drift given here and with equation (37). The range of $P_\delta$, with estimates of $K$ found from equations (36), indicate $\tau_s \sim 10^{-1}$ to $10^0$ s and $N_0(0) = 10^{23} - 10^{24}$ cm$^{-3}$, values that are by no means unreasonable.

It is necessary to compare our polar-cap model with two other sets of pulsar observations. Given that protons are the major fraction of particles accelerated and produce no electron–positron pairs, it is worthwhile considering the radio luminosity $L_s$. The order of magnitude of this can be expressed as

$$L_s \Delta \Omega \Delta v \sim \frac{2\pi^2 R^3 B_d}{c e P^2} \varepsilon \sim \left( \frac{1.4 \times 10^{30} B_{d12}}{P^2} \right) \varepsilon,$$

(39)

in terms of the neutron star radius $R$ and the inferred dipole field $B_d$. From this, we can estimate the energy $\varepsilon$ radiated into solid angle $\Delta \Omega$ within bandwidth $\Delta v$ per unit charge (baryonic or leptonic) accelerated at the polar cap. Using the 400 MHz luminosities listed in the ATNF catalogue (Manchester et al. 2005) and a bandwidth $\Delta v = 400$ MHz, we find by evaluating equation (37) for a small sample (B0826–34, B0834+06, B0943+10, B0950+08, B1055–52, B1133+16, B1139+10) that typical values are in the interval $\varepsilon = (40-400)\delta$ MeV. Even though the emission solid angle is likely to be as small as $\Delta \Omega \sim 10^{-2}$ to $10^{-1}$ sr, there must be some concern here because the number of ICS pairs produced per primary accelerated positron (Harding &Muslimov 2002) is not large so that either the conversion of electron–positron energy to coherent radio emission is efficient or other plasma components are involved, as in the paper by Cheng & Ruderman (1980).

A second set of observations that are relevant are those of polar-cap blackbody X-ray luminosities. The blackbody X-ray emission expected from a polar cap of ion and proton zones can be found from equation (11). The reverse-electron flux from photoionization heats an ion zone at a rate $H_0$ within a time interval $0 < t < \tau_P$. An estimate of this can be obtained directly from the mean electron energy per unit nuclear charge accelerated, given by equation (29) of Paper I, and is

$$H_0 = 6.0 \times 10^{18} Z^{0.85} (0) B_{d15} \tau_6^{-1} P^{-1},$$

(40)

in units of erg cm$^{-2}$ s$^{-1}$. In this expression, $T$ is not the local surface temperature but is an average for radiation emitted over the whole polar cap. The surface temperature derived from equation (11) with neglect of radiative loss is

$$T(t) = \frac{2H_0}{(\pi C_{\lambda^3})^{3/2}} t^{1/2}, \quad 0 < t < \tau_P,$$

(41)

and increases very rapidly, so that the limiting temperature,

$$T_{\infty} \approx 0.6 Z^{0.17} (0) B_{d15}^{0.2} P^{-0.2},$$

(42)

derived from $H_0$ is reached at $t \ll \tau_P$. The cooling at $t > \tau_P$ is also rapid so that the major part of the X-ray luminosity is that of a blackbody at $T_\infty$ and of area approximately equal to the canonical dipole-field area $2\pi R^2 / c P$ divided by $K + 1$.

There have been many attempts to measure the polar-cap blackbody temperature and source area of a subset of radio pulsars. We refer to Zavlin & Pavlov (2004) for B0950+08, De Luca et al. (2005) for B0656+14 and B1055–52, Tepedelenlioğlu & snagman (2002) for B0628–28, Zhang, Sanwal & Pavlov (2005) for B0943+10, Kargaltsev, Pavlov & Garmire (2006) for B1133+16, Gil et al. (2008) for B0834+06 and Misnovic, Pavlov & Garmire (2008) for B1129+10. Five pulsars (B0628–28, B0834+06, B0943+10, B1133+16, B1129+10) have source areas one or two orders of magnitude smaller than the canonical area, but unfortunately, a systematic comparison with equations (40)–(42) is not possible because in most instances the authors are able to say only that the observed X-ray spectrum is consistent with the stated temperature and source area. The quoted source temperatures are $\sim 3 \times 10^9$ K and are larger than those predicted by equation (42) unless it is assumed that $B \gg B_0$. They are also uncomfortably large in the context of $E \times B$ drift-velocity polar-cap models such as that developed by Gil et al. (2003) although it must be conceded that this model allows an interesting test of its validity (see Gil, Melikidze & Zhang 2007). However, the case (iii) surface electric field boundary condition on which these models rely could be maintained only for ion cohesive energies of $\sim 10$ keV which in turn would imply actual fields two orders of magnitude or more larger than the ATNF catalogue dipole field.

6 CONCLUSIONS

This paper is a continuation of a previous study (Paper I) of isolated neutron stars with positive corotational charge density and surface electric field boundary condition $E \cdot B = 0$ at the polar cap. The reverse flux of electrons arising from photoionization of accelerated ions is incident on the neutron star surface and produces protons through formation and decay of the giant dipole resonance in the later stages of electromagnetic shower development. Protons are the major component of the accelerated plasma, but we find that a time-independent composition of ions, protons and positrons is usually unstable. The consequences of these phenomena should be observable in pulsars unless, of course, rendered nugatory by some factor not properly taken into account here. But we emphasize that there are almost certainly other sources of instability contributing to the complex behaviour observed in radio pulsars that may be present in each of cases (i)–(iii).

There are two instabilities which result in transitions between states of different plasma composition above localized areas on the polar cap, and it is suggested here that these are the basis for

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the commonly observed phenomena of nulls and subpulse drift. The basic units of time are \( t_{\rho} \approx 10^{-1} \text{s} \) for the short time-scale instability, and for the medium time-scale, \( t_{\Delta} \approx 10^{2} - 10^{3} \text{s} \). These instabilities are not primarily electromagnetic in origin and will not be present in neutron stars with negative corotational charge density and electron acceleration. Micropulses of \( \sim 10^{-4} - 10^{-3} \text{s} \) duration are also present in some pulsar emissions but are unlikely to be associated with the instabilities we consider here. There appears to be no reason why both signs of corotational charge density should not be present in the isolated neutron star population, and it would be interesting to see if there is any observational evidence for an associated division of the ATNF catalogue pulsars.

The transitions are principally in plasma composition, and the current density at the neutron star surface remains close to \( J = \rho \sigma c \). The electric potential \( \Phi(z, u(z)) \) in the open magnetosphere above the polar cap is also almost unchanged. These transitions are, in principle, observable because there are no processes by which electron–positron pair creation can occur in a plasma containing only protons, whereas it does occur in the ion plasma. Thus there will be no coherent radio emission in the proton phase if we assume, as is usual, that a pair plasma is a necessary constituent for it.

Both the medium time-scale instability and the mechanical instability of the neutron star surface mentioned in Section 4 provide a basis for pulse nulls, but unfortunately, not one that is suitable for quantitative prediction. The short time-scale instability is more interesting. We show that at any instant, the polar cap is divided into moving ion and proton zones and consider organized motion of these, specifically the circular movement of compact ion zones around the magnetic pole. We make the hypothesis that such motions can occur (following recent work by Deshpande & Rankin 1999) but have not attempted to show that they have long-term stability, to the extent indicated by observation, against decay to a chaotic state. This is of some interest because it shows that features usually considered to require the surface field boundary condition \( E \cdot B \neq 0 \) are possible under the \( E \cdot B = 0 \) condition. Actual polar magnetic flux densities exceeding \( 10^{14} \text{G} \) are necessary for the former condition, and it must be doubtful that such fields, two orders of magnitude greater than the inferred (catalogue) dipole field, are likely to exist in the very large number of pulsars observed by Weltevrede et al. (2006) to show subpulse drift. Our view of subpulse drift is also interesting in that the motion is not an \( E \times B \) drift velocity as in the model of Ruderman & Sutherland. The fixed parameters for a given neutron star are the band separation \( P_{2} \), and \( K \) which is the number of protons produced per unit ion charge accelerated. This latter parameter approximately determines the ratio of the total areas of proton and ion zones on the polar cap at any instant. Thus the circulation time \( P_{1} = n P_{2} \) depends on \( n \), the number of ion zones, and is not necessarily constant as it would be in the \( E \times B \) drift-velocity model. The band separation \( P_{1} \) is dependent on \( K \) and on the diffusion time \( t_{\rho} \) and so is independent of rotation period \( P \) and almost independent of \( B \).

We have assumed here, following the pair formation calculations of Hibschman & Arons (2001) and Harding & Muslimov (2002), that spontaneous pair creation by CR is not possible in most isolated neutron stars. Those with a sufficiently large value of \( |\Phi_{\text{max}}| \propto B_{L}/P \) to support spontaneous CR pair creation, predominantly young high-field pulsars, are not expected to show the instabilities we have considered here. These authors actually considered case (i) in which the basis of plasma formation is an electron current density close to \( J = \rho \sigma c \) at the surface accelerated to energies sufficient for ICS or spontaneous CR pair formation. In case (ii), the protons and ions form the basic current component with \( |\Phi_{\text{max}}| \) broadly the same as in case (i) apart from the ion inertia term which is very small for electrons. Thus the condition for the spontaneous growth of CR pair formation is very similar in cases (i) and (ii). Although the reverse electron flux arising from CR pair formation in case (ii) is probably quite a small fraction of \( \rho \sigma c \), it is likely that, for the large values of \( |\Phi_{\text{max}}| \) that are necessary, there will be a proton atmosphere which is never exhausted. In this instance, steady-state plasma formation and low-altitude acceleration appear probable, though subject to possible instabilities at those higher altitudes where the coherent radiation is formed. The same statement can be made about all case (i) pulsars whether or not spontaneous CR pair formation is supported. Microstructure with \( 10^{-4} - 10^{-3} \text{s} \) time-scales is likely to be a consequence of such higher altitude instabilities. The reason for this assumption is that the case (i) boundary condition on \( \Phi \) needs only an electron density on the surface separating open from closed magnetospheres. The small electron cohesive energy means that there is no reason why this should not be maintained at all times, as is also true for the Goldreich–Jupiter electron current density. For this boundary condition, there is no obvious way in which non-electromagnetic time constants can influence plasma formation and acceleration at altitudes less than \( 10^{3} \text{cm} \) above the polar cap. It is possible, perhaps, to assign either this boundary condition or case (ii) with spontaneous CR pair formation to those pulsars in the systematic study of Weltevrede et al. (2006), approximately one half of the total in number, that do not exhibit subpulse drift.

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