Non-Mechanism in Quantum Oracle Computing

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Abstract

A typical oracle problem is finding which software program is installed on a computer, by running the computer and testing its input-output behaviour. The program is randomly chosen from a set of programs known to the problem solver. As well known, some oracle problems are solved more efficiently by using quantum algorithms; this naturally implies changing the computer to quantum, while the choice of the software program remains sharp. In order to highlight the non-mechanistic origin of this higher efficiency, also the uncertainty about which program is installed must be represented in a quantum way.

I. INTRODUCTION

In ref. [1], the author et al. have shown that Simon’s algorithm[2] higher than classical efficiency comes from non-mechanism. This is a global feature of the evolution of a quantum system undergoing wave function collapse (a revamped notion). This evolution is driven by both the initial actions performed on the quantum system and a final constraint imposed by
the action of performing a measurement on it.\textsuperscript{1} The state produced by the initial actions should not satisfy that constraint: in other words, having wave function collapse is essential.\textsuperscript{2}

The capability of driving the evolution of a quantum system by means of both initial and final actions is leveraged in quantum computation, to obtain in fact higher than classical efficiency. In Simon’s algorithm, it can be said that quantum measurement analogically sets a constraint on the output of a hard-to-reverse Boolean network and, at once, solves such a constrained network. This takes the time required to solve the \textit{unconstrained} network in all possible ways (in a quantum superposition thereof) – namely a time polynomial in network size. The problem of solving the constrained network would otherwise be NP. This originates higher than classical efficiency.

Also the efficiency of Shor’s algorithm\textsuperscript{[4]} is likely to have a non-mechanistic origin, as this algorithm is tightly related to Simon’s – to the problem of finding the period of a hard-to-reverse function (finding the arguments given the value of the function is hard).

On the contrary, quantum algorithms which solve oracle problems reach the solution in a deterministic way, and no wave function collapse is involved. Apparently, this leaves no room to non-mechanism. However, as they are, these algorithms do not capture all the relevant aspects of the problem. Part of the problem description is not represented inside the quantum algorithm. This can readily be corrected by introducing a small change in the algorithm. The character of the altered algorithm is clearly non-mechanistic. This will be shown by working on the algorithms devised by Deutsch\textsuperscript{[5,6]}, Deutsch and Jozsa\textsuperscript{[7]}, and

\textsuperscript{1}The notion that there are only initial and final actions, with quantum spontaneity (i.e. wave function collapse) in between, is due to Finkelstein\textsuperscript{[3]}.

\textsuperscript{2}Quantum measurement selects \textit{one} eigenstate of the measurement basis. If the initial actions produced a superposition of such eigenstates, that selection is a \textit{final} constraint imposed on the evolution of the quantum system. Clearly this constraint is non-redundant with the initial actions, and is independent of the initial actions.
II. CLASSICAL AND QUANTUM ORACLES

For our purposes, it is important to keep in mind that an oracle problem is a competition between two parties. Sticking on Greek tradition, let us call the examiner “Sphinx” and the examinee “Oedipus”. Their challenge is mathematically characterized as follows.

In the first place, we need to characterize the oracle. Since a computer program establishes an input-output function, we will speak indifferently of programs or functions. Given $B = \{0, 1\}$, let $f_k$ be the $k$-th element of some set of functions $\{f_k\} : B^n \to B$, with $k = 0, 1, \ldots, 2^N - 1 < 2^{2n}$ (the latter is the number of all possible functions from $B^n$ to $B$). We say that the $k$-th mode of the oracle is a computer-and-program (a “gate”) $Q_k$ that, given any input $x \in B^n$, yields the output $f_k(x) \in B$. In other words, $k$ is an identification number of the software program installed on the computer.

Once prepared the oracle in its $k$-th mode, the Sphinx gives it to Oedipus. He knows all the programs in $\{k\}$, but knows nothing about the choice of the Sphinx. He is free to run the computer and forbidden to inspect the mode. His problem is to identify $k$ (or a property thereof) in the most efficient way⁴.

In quantum oracle computing, the computer becomes quantum, while its mode $k$ remains sharp (classical). This formulation cannot fit the notion of non-mechanism. This is a global property of a quantum evolution comprising an initial measurement (required to prepare the quantum system in a known state), a unitary evolution and a final measurement inducing a wave function collapse that can actively drive the evolution toward the solution of a problem

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³Grover’s algorithm is deterministic for data base size $= 2$ or any size in the limit of an infinite number of loops.

⁴More generally, the problem is to identify $k$ with some desired probability that such an identification is correct, but this generalization will not be needed.
Such an evolution should both describe and solve the problem. For example, in Simon’s and Shor’s algorithms, all knowledge and ignorance about the period of a function \( f \) defined over \( B^n \), is represented in the superposition \( \frac{1}{\sqrt{2^n}} \sum_x |x\rangle |f(x)\rangle \). On the contrary, Oedipus’ knowledge about \( \{ k \} \) and ignorance about \( k \) is not represented inside the current algorithms.

However, this can readily be adjusted by introducing a simple change in the quantum gate. Fig. 1-a and 1-b show the usual and the altered gate – gates are represented in the standard form given in ref.[6]. The altered gate is equipped with an ancillary input \( k \) and computes the function \( F(k,x) \) such that, for all \( k \), \( F(k,x) = f_k(x) \).

Oedipus’ knowledge about \( \{ k \} \) and ignorance about \( k \) are represented by the result of computing \( F(k,x) \), with \( x \) prepared in the even superposition \( \frac{1}{\sqrt{2^n}} \sum_k |x\rangle \), and the ancilla \( k \) prepared in the even superposition \( \frac{1}{\sqrt{2^N}} \sum_k |k\rangle \). Fig. 1-b shows the gate \( F(k,x) \) with the ancilla already prepared in that superposition, \( H \) denotes the Hadamard transform. Oedipus must be forbidden to measure the ancilla, at least after the Hadamard transform has been performed on it, and before solving the problem. Let us call “gate” the box computing \( f_k(x) \), or \( F(k,x) \); all gate inputs but \( y \) go unchanged into the corresponding outputs, while \( y \) goes into \( y \oplus f_k(x) \), or \( y \oplus F(x,k) \), where the sign \( \oplus \) denotes module 2 addition.

III. QUANTUM ALGORITHMS FOR SOLVING ORACLE PROBLEMS

Let us consider the modified version of Deutsch’s 1985 algorithm\[5\] given in ref. [6]. \( \{ k \} \) is the set of all possible functions \( f_k : B \rightarrow B : \)

\[
\begin{array}{cccc}
  k &=& 00 & k = 01 & k = 10 & k = 11 \\
  x f(x) &=& 0 0 & 0 0 & 0 1 & 0 1 \\
  & 1 0 & 1 1 & 1 0 & 1 1
\end{array}
\]

This set is divided into a couple of subsets: the balanced functions, characterized by an even number of zeroes and ones and identified by \( k = 01, 10 \), and the unbalanced ones, which here are the constant functions, and are identified by \( k = 00, 11 \). Oedipus’ problem is to find
whether \( f_k \) (randomly chosen by the Sphinx) is balanced or not, in the most efficient way. Deutsch’s algorithm, in one run (against two in classical computation), deterministically yields either \( |1\rangle_x (|0\rangle_y - |1\rangle_y) \) if the mode is balanced, or \( |0\rangle_x (|0\rangle_y - |1\rangle_y) \) if the mode is unbalanced. Quantum measurement in the basis \( \{ |0\rangle_x, |1\rangle_x \} \) does not induce any wave function collapse, it just serves to read \( x \): non-mechanism remains, so to speak, hidden.

I shall now port Deutsch’s algorithm to the altered gate \( F \) (see fig. 2, where an \( H \) gate always denotes a Hadamard transform). The function computed by gate \( F \) is:

\[
k_1 \ k_0 \ x \ F (k_1, k_0, x)
\]

\[
\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 \\
\end{array}
\]

At this point, we need to revise the terms of the competition between the Sphinx and Oedipus. The following protocol leaves the substance of the problem unchanged, while adapting the problem to the algorithm given in fig. 2.

Oedipus receives the box \( F (k, x) \) with \( k \) prepared in the superposition of all modes. Thus, for the time being, even the Sphinx does not know which is the mode. Oedipus’ problem is still finding whether the mode is balanced or unbalanced in the most efficient way (without accessing \( k \)). After that Oedipus has given the solution, the Sphinx measures register \( k \), thus finding the mode; then it can check whether Oedipus’ answer was right. Of course Oedipus indirectly – through entanglement – affects register \( k \). Things will become clearer when the protocol is applied.

As readily checked, the output of the unitary propagation described in fig. 2 is, before measurement:
\[
|\varphi(t_-)\rangle = \left(|0\rangle_{k_1} |0\rangle_{k_0} - |1\rangle_{k_1} |1\rangle_{k_0}\right) |0\rangle_x \left(|0\rangle_y - |1\rangle_y\right) + \\
\left(|0\rangle_{k_1} |1\rangle_{k_0} - |1\rangle_{k_1} |0\rangle_{k_0}\right) |1\rangle_x \left(|0\rangle_y - |1\rangle_y\right)
\]

Measurement of the one-qubit register \(x\) induces a wave function collapse in both registers \(x\) and \(k\) producing, say at time \(t_+\), either the outcome \(x(t_+) = 1\) or the outcome \(x(t_+) = 0\). In the former (latter) case, register \(k\) has collapsed on an even superposition of the balanced (unbalanced) modes.

Let us call \(|\varphi(t_+)\rangle\) the overall registers state after that register \(x\) has been measured. This state is originated in a non-mechanistic way, being shaped by both the initial actions, leading to \(|\varphi(t_-)\rangle\), and the final constraint that measurement yields a single value of \(x\), which of course means a specific value, either 1 or 0 in a mutually exclusive way:

\[
|\varphi(t_+)\rangle = |x(t_+)\rangle_x \langle x(t_+)\rangle_x |\varphi(t_-)\rangle, \text{ with } t_+ > t_-,
\]

see also ref.\[1\]. Clearly \(|1\rangle_x \langle 1\rangle_x \langle 0\rangle_x \langle 0\rangle_x\) selects an even superposition of balanced (unbalanced) modes, see eq. \((1)\).

Say that measurement of register \(x\) yields 1: Oedipus declares that the mode is balanced. Then the Sphinx measures register \(k\), finding either \(k = 01\) or \(k = 10\) – see eq. \((1)\). Since either mode is balanced, the Sphinx checks that Oedipus’ answer was right.

Quantum efficiency (one run against two in classical computation) comes from the fact that measurement of register \(x\) at once yields a value of \(x\) and creates, in the ancillary register \(k\), an even superposition of either balanced or unbalanced modes – depending on \(x\).

Interestingly, by backdating the outcome \(k = 01\) of register \(k\) collapses [ref.1,9,10] immediately after performing the Hadamard transform on the ancilla, \(k = 01\) can be seen as the outcome of a random choice performed by the Sphinx. We can see that, after all (in a literal sense), the new protocol leaves the original problem unaltered.

State \((1)\) shows another reason for adopting the altered algorithm. It clearly gives the characteristic function of the balanced modes and therefore represents all the knowledge we need to say that \(|1\rangle_x \langle 0\rangle_x\) means balanced (unbalanced). On the contrary, if we started with
a sharp value of \( k \), the knowledge that \( x \) is such a characteristic function would necessarily
dwell in Oedipus’ head; it would not be physically represented. It is reasonable to think
that, when dealing with quantum-physical computation, all the knowledge about the object
of the computation should be represented in a physical way.

It should be evident that the foregoing considerations can be applied to the “second”
seminal quantum oracle problem, namely Deutsch-Jozsa algorithm\(^7\). Here \( \{ f_k \} \) is the set
of all the balanced and constant functions from \( B^n \) to \( B \). Chosen a function at random,
the problem is to efficiently find whether the quantum computer computes a balanced or a
constant function. By altering the algorithm exactly as before, the non-mechanistic nature
of the solution process becomes manifest.

We shall now consider Grover’s algorithm\(^8\). Given the \( 2^n \) functions \( f_k : B^n \to B \) such
that \( f_k (x) = \delta_{k,x} \), and chosen one function \( f_k \) at random, Oedipus’ problem is to find \( k \) in
the most efficient way, by checking the input-output behaviour of a quantum computer. We
shall consider the simplest case of \( n = 2 \), so that \( k \) ranges over 00, 01, 10, 11. In the usual
algorithm, \( k \) is found in a deterministic way with one run of Grover’s loop. As usual, we
substitute \( f_k (x) \) with \( F (k, x) = f_k (x) \) for all \( k \); the two ancillary qubits required to specify
\( k \) are prepared in an even superposition of all possible modes (fig. 3).

Without entering into detail, the output of just one iteration of Grover’s loop is:

\[
|\varphi(t_-)\rangle = (|0\rangle_{k_1} |0\rangle_{k_0} |0\rangle_{x_1} |0\rangle_{x_0} + |0\rangle_{k_1} |1\rangle_{k_0} |0\rangle_{x_1} |1\rangle_{x_0} + \\
|1\rangle_{k_1} |0\rangle_{k_0} |1\rangle_{x_1} |0\rangle_{x_0} + |1\rangle_{k_1} |1\rangle_{k_0} |1\rangle_{x_1} |1\rangle_{x_0}) (|0\rangle_y - |1\rangle_y)
\]

Measuring \( x_1 \) and \( x_0 \), at once (non-mechanistically, as before) creates the mode \( k \) in register
\( k \) and yields the value of \( k \) in register \( x \). Backdating register \( k \) collapse immediately after
performing the Hadarmard transform on it, yields the random choice originally performed
by the Sphinx.
IV. CONCLUSIONS

In ref.[1] we have given a demonstration that the higher than classical efficiency of Simon’s and related algorithms comes from quantum non-mechanism. This demonstration has been extended in this work to quantum oracle problem solving. In conclusion, the special efficiency of all known quantum algorithms would have a non-mechanistic origin. Non-mechanism hinges on the notion of wave function collapse, an exclusively quantum feature that allows to drive the evolution of a quantum system by acting on both initial and final conditions. It is hoped that the results obtained will revamp the notion of collapse. It seems difficult to understand the motivation of getting rid of the notion of something that yields effective benefits\footnote{In the many universes’ interpretation, collapse can be substituted by a unitary evolution that entangles the observer with the universe. Such an interpretation is not in contrast with the current work, its motivations (or a part thereof) seem to be in contrast.}

Perhaps quantum computation, because of its unique feature of joining a fundamental character and a capability of describing complex states of affairs, can yield fresh insights for the interpretation of quantum mechanics.

This work has brought in a seemingly interesting side-effect. In order to show the non-mechanistic character of quantum oracle problem solving, a state of knowledge of the problem solver (Oedipus’ uncertainty about the value of $k$) had to be translated into a physical description, and this description had to be strictly quantum in order to match reality.

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Fig. 1

(a) Quantum oracle, randomly chosen $k$

(b) Quantum oracle and $k$

Fig. 2

Fig. 3