Normalise for Fairness: A Simple Normalisation Technique for Fairness in Regression Machine Learning Problems

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Abstract

Algorithms and Machine Learning (ML) are increasingly affecting everyday life and several decision-making processes, where ML has an advantage due to scalability or superior performance. Fairness in such applications is crucial, where models should not discriminate their results based on race, gender, or other protected groups. This is especially crucial for models affecting very sensitive topics, like interview invitation or recidivism prediction. Fairness is not commonly studied for regression problems compared to binary classification problems; hence, we present a simple, yet effective method based on normalisation (FaiReg), which minimises the impact of unfairness in regression problems, especially due to labelling bias. We present a theoretical analysis of the method, in addition to an empirical comparison against two standard methods for fairness, namely data balancing and adversarial training. We also include a hybrid formulation (FaiRegH), merging the presented method with data balancing, in an attempt to face labelling and sampling biases simultaneously. The experiments are conducted on the multimodal dataset First Impressions (FI) with various labels, namely Big-Five personality prediction and interview screening score. The results show the superior performance of diminishing the effects of unfairness better than data balancing, also without deteriorating the performance of the original problem as much as adversarial training. Fairness is evaluated based on the Equal Accuracy (EA) and Statistical Parity (SP) constraints. The experiments present a setup that enhances the fairness for several protected variables simultaneously.

1. Introduction

The impact of Algorithms, Artificial Intelligence (AI), and Machine Learning (ML) on our daily lives is increasing day by day, and they became involved in many crucial decision-making processes, e.g., loan decisions (Mukerjee et al., 2002), hiring decisions (Cohen et al., 2019), and recidivism (Chouldechova, 2017). ML can be very effective in the automation of tasks that require a lot of manual work, which can have huge cost benefits (Brynjolfsson & Mitchell, 2017). ML can also be effective in areas where data-driven predictions are more reliable than human judgement, this could be due to the ability of algorithms to consider more factors. For example, Grove et al. (2000); Meehl (1954) discuss a large body of literature that suggest that data and evidence-based assessments can be superior to human judgement in clinical setup. The use of algorithms and ML in daily life will likely increase in the future, due to an increasing trend of digitalisation in many sections (Ambrosio et al., 2020); as a result, the impact of algorithms is likely to increase and affect more sensitive decisions.

A general idea behind fairness in ML is training models that do not discriminate their results based on gender, race, or other criteria like those stated in article 2 of the human rights declaration (UN General Assembly, 1948):

“Everyone is entitled to all the rights and freedoms set forth in this Declaration, without distinction of any kind, such as race, colour, sex, language, religion, political or other opinion, national or social origin, property, birth or other status.”

ML models in general can be unfair in their predictions w.r.t. a protected variable (e.g., race or gender) by violating fairness constraints; Equal Accuracy (EA) and Statistical Parity (SP) are two commonly adopted fairness constraints (Tolan et al., 2019). Violating EA can make a model be more accurate for specific values of the protected variable, e.g., being more accurate at predicting a variable for males. Violating SP can make a model predict labels that are biased

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w. r. t. the protected variable, e. g., predicting an interview score for females to be consistently higher than for males. In these scenarios, the ML models that will be used for decision-making can negatively impact individuals unfairly in a severe manner. Mehrabi et al. (2021); Chouldechova & Roth (2020) survey many approaches for fairness in ML as well as some of the reasons behind it, and methods to measure it. Some of the common (not mutually exclusive) reasons for unfairness in ML according to (Mehrabi et al., 2021; Chouldechova & Roth, 2020) are different data biases, including but not limited to:

1. **Labelling bias**, labels can have inherent bias due to their collection mechanism, e. g., bias of human annotators.

2. **Sampling bias**, where there is an imbalance in the sample sizes between different values of the protected variable, in which case maximising the accuracy will prioritise certain groups.

3. **Feature bias**, where some features are more correlated with one (or more) of the protected groups.

Collecting more data or using data balancing techniques were studied as ways of encountering sampling bias (Chen et al., 2018). Feature bias has been encountered by using adversarial learning (Xu et al., 2021), where an adversarial model is trained to re-represent the input features in a manner agnostic w. r. t. the protected variable, hence acquiring features that do not leak information about the protected variable, which makes it challenging for the predictor model to discriminate accordingly. Labelling bias essentially means that the labels have different distributions based on the value of the protected variable, therefore, sampling techniques are solving an orthogonal issue, because a model trained on a properly sampled data with labelling bias will still learn to exploit the unfairness due to labelling bias. Nevertheless, a poorly sampled data could result in choosing a subset with an apparent labelling bias.

As a motivating example for our method, if we consider creating a model that predicts the height of an American person using their weight as input. This model would typically attempt to output values around the average height of 170 cm. If we modify this model by giving it additional information about gender, it can establish that a male has a height close to the average male height of 177 cm, while a female’s height is close to average female height of 163 cm

\[ \text{Height (cm)} = \text{Weight (kg)} \times \frac{170}{163} \]

This equation is derived from a normal distribution with mean and standard deviation values for males and females.

To the best of the authors’ knowledge, most of the literature about fairness in ML is concerned with classification problems, while much less are concerned with regression problems. The few publications concerned with the unfairness in regression problems typically achieve that by quantising the continuous labels, and hence reducing the problem to a classification problem. Furthermore, to the authors’ best knowledge, fairness against labelling bias is also not commonly studied. (Blum & Stangl, 2019) is the only work we found, addressing labelling bias, but in classification settings. This is probably because ground truth labels can be hard to acquire, when they require another group of annotators (which can also be biased), or can be hard to define in some cases, like in social signals.

The contributions of this paper are:

- Introducing a normalisation approach to train fair regression models encountering labelling bias.
- Introducing a hybrid approach, merging the presented normalisation approach and data balancing technique, to simultaneously address labelling and sampling biases.
- Studying the possibility of protecting from unfairness for two protected variables simultaneously.

The paper is divided as follows: Related work is discussed in Section 2. In Section 3, we discuss the background of the dataset, and fairness methodologies. We present our method in Section 4, and demonstrate it in experiments in Section 5. We conclude with some remarks in Section 6.

2. Related Work

Yan et al. (2020) has attempted to mitigate bias in the multimodal dataset First Impressions (FI); they used two common approaches, namely data balancing and fairness adversarial learning. Berk et al. (2017) introduced convex regularisers that assist linear regression and logistic regression. (Mehrabi et al., 2021; Chouldechova & Roth, 2020) are two surveys about fairness in ML, but most approaches are concerned with classification problems, particularly binary classification. Regression problems did not catch as much attention, however, there are few approaches that mostly rely on quantising the regression labels, hence, transforming the problem into a classification problem, and then use one of several approaches (Agarwal et al., 2019; Gorrostieta et al., 2019). Gorrostieta et al. (2019) apply this in the scope of Speech Emotion Recognition (SER), while Agarwal et al. (2019) consider it in crime rate prediction and law school students’ GPA prediction. Narasimhan et al. (2020) provides a framework for pairwise comparisons to optimise fairness constraints in ranking problems.

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1 Averages obtained on 26.01.202 from: [https://www.worlddata.info/average-bodyheight.php](https://www.worlddata.info/average-bodyheight.php)
Table 1. Statistics about the distributions of gender and race in the FI dataset.

|       | Train+Dev | Test  |
|-------|-----------|-------|
|       | M         | F     | Σ     | M         | F     | Σ     |
| Cau.  | 3 300     | 3 570 | 6 870 | 804       | 924   | 1 728 |
| Asi.  | 82        | 201   | 283   | 14        | 34    | 48    |
| Afr.  | 268       | 579   | 847   | 70        | 154   | 224   |
| Σ     | 3 650     | 4 350 | 8 000 | 888       | 1 112 | 2 000 |

Due to the bias mischaracterisation in the FI dataset, we believe that (Yan et al., 2020) did not actually manage to solve the bias in the FI dataset. The reason for this is twofold.

First, they employed adversarial learning which trains a model that learns a different representation of the input features that maintains the performance to be as high as possible whilst not leaking any information about the protected variable. This technique, however, could severely deteriorate some important features that happen to correlate with the protected variable, for example, an audio feature like pitch correlates with gender (Schuller & Batliner, 2013). The results reported by (Yan et al., 2020) for adversarial learning have very poor performance for the original problem, where they score just slightly worse than a constant predictor baseline. We will demonstrate further why this technique is not capable to address the problem at hand.

Second, Yan et al. (2020) employed data balancing which gives different examples different weights, according to the frequency of the corresponding value of the protected variable. This technique is usually helpful when there is sampling bias; then, this is conquered by oversampling the less dominant classes, so that all classes have a similar priority for optimisation. However, when the labels themselves are biased, for example, when females score higher for an interview label, then oversampling the other class (i.e., male) will not have an impact on the fact that females are still more likely to be chosen. Data balancing can be used to downsample some examples, in a manner that makes the labels of the different protected groups similarly distributed; however, this can deteriorate the performance of the original problem since it eliminates some data, especially challenging ones.

Even though the correlation values are not very high, they are statistically significant, which indicates a systematic unfairness. This is further explored by (Junior et al., 2021).

(Weisberg et al., 2011) show that there are differences between genders in some of the personality traits (self-answered), namely, females score slightly higher in extraversion, and higher in agreeableness and neuroticism. These differences are not the same as in the FI dataset, which indicates that the data has a labelling bias due to human labelling, and not due to actual differences in the ground truth. Furthermore, there is a sampling bias as shown in Table 1, where females and Caucasians are over-represented.

Definition 3.1. Mean Absolute Accuracy (MAA), the performance metric used on the FI dataset, given by:

$$1 - \frac{1}{n} \sum_{i=1}^{n} |y_i - p_i|$$  \hspace{1cm} (1)

where $y_i$, $p_i$ are the $i^{th}$ ground truth and predicted examples, respectively; $n$ is the number of evaluation examples.

3.2. Input Features

The SOTA pipeline utilised a variety of features (Kaya et al., 2017), namely facial features, scene, and audio features. We also utilise the SOTA pipeline and features. Similar to (Kaya et al., 2017), we call the facial features as the face modality, and the concatenation of the scene and audio inputs are 15 seconds videos with one speaker (collected and segmented from YouTube). In the first challenge (Ponce-López et al., 2016), the participants were asked to solve the task of predicting the Big-Five personality labels (OCEAN) of the person in the video, namely Openness to Experience, Conscientiousness, Extroversion, Agreeableness, and Neuroticism. In the second challenge (Escalante et al., 2017), the participants were asked to predict the Big-Five personality dimensions (like the first challenge), in addition to a new label interview, which is a score ranking the possibility of an invitation to an interview based on their apparent personality. The labels of both phases are collected by different sets of annotators, using crowd-sourcing through Amazon Mechanical Turk (AMT), indicating that the labelling bias is coming from independent sources. The labels are regression values within the range [0, 1]. The distribution of the Test-set labels is detailed in Figure 2 (in E).

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3.4. Methods and Material

In this section, we describe the FI dataset\(^2\). State-Of-The-Art (SOTA) pipeline (including input features and personality prediction models), fairness baselines, and fairness metrics.

3.1. First Impressions Dataset and Personality Prediction Challenge

The FI dataset was collected for two challenges – (Ponce-López et al., 2016) and (Escalante et al., 2017), where the

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features as the *scene modality*. The protected variable is *not* included explicitly in the input features. All the features are preprocessed with a MinMaxScaler (trained on the training data) to fit all the input features linearly within the range \([0, 1]\). A Kernel Extreme Learning Machine (KELM) is used as a regressor; two KELM models were trained for both modalities, their predictions are then stacked using Random Forest (RF) to give the final predictions.

### 3.2.1. Face Features

Faces were extracted from each frame, then aligned (using the Supervised Descent Method (SDM) (Xiong & De la Torre, 2013)), cropped, and resized to \(64 \times 64\). A VGG-Face for emotion recognition was fine-tuned on the FER-2013 dataset (Emotions dataset) (Goodfellow et al., 2013); then, the features from the 33rd layer were extracted, which produces a descriptor of 4 096 features for each frame, which is later reduced using five functionals to a static descriptor of 20 480 features. The five functionals are mean, standard deviation, slope, offset, and curvature. Furthermore, Local Gabor Binary Patterns from Three Orthogonal Planes (LGBP-TOP) (Almaev & Valstar, 2013) were used by applying 18 Gabor filters on the aligned facial images, which results in a descriptor with 50 112 features. All the facial features are fused together to give 70 592 features per video.

### 3.2.2. Scene Features

Scene features were extracted using the VGG-VN-19 network (Simonyan & Zisserman, 2015) trained for object recognition. The network was pretrained on the ILSVRC 2012 dataset. The features are acquired by extracting the output of the 39th layer, yielding 4 096 features per video.

### 3.2.3. Audio Features

Audio features were extracted using the openSMILE toolkit, choosing the ComParE 2013 acoustic feature set (Schuller et al., 2013). The features consist of computing 130 low-level descriptor contours (e.g., energy, intensity, and FFT spectrum), then reducing them by applying 54 functionals (e.g., moments, and LPC autoregressive coefficients) to obtain 6 373 features per video (Eyben et al., 2013; Eyben, 2015).

### 3.3. Personality Regression Model

KELM (Huang et al., 2011) is a method which improves over Extreme Learning Machine (ELM) (Huang et al., 2004). KELM is a kernel formulation of ELM (Huang et al., 2011), which showed better results than other models like Support Vector Machines (SVM). KELM operates by constructing a weights matrix \(\beta\) that minimises the Mean Squared Error (MSE) between the predictions and the ground truth, \(\beta\) is given by:

\[
\beta = (\mathbf{I} + \mathbf{K})^{-1}\mathbf{Y}_o,
\]

where \(\mathbf{K} = \mathbf{K}(\mathbf{X}_o, \mathbf{X}_o)\), \(\mathbf{X}_o\) is the matrix of the input features of the training dataset, \(\mathbf{K}\) is a kernel function, \(\mathbf{C}\) is a regularisation scalar parameter, \(\mathbf{I}\) is the identity matrix of size \(n \times n\), \(n\) is the number of training examples, and \(\mathbf{Y}_o\) is a column vector of the training examples of an output label. Accordingly, the prediction for a matrix \(\mathbf{X}\) is \(\mathbf{K}(\mathbf{X}, \mathbf{X}_o) \cdot \beta\).

Non-linearity can be introduced in the kernel function \(K\); however, Kaya et al. (2017) opt for a linear kernel function, i.e., \(K(\mathbf{A}, \mathbf{B}) = \mathbf{A}^T \mathbf{B}\), in order to reduce overfitting and the number of hyperparameters. Similar to (Kaya et al., 2017), we optimised for the regularisation parameter \(C\) by using group \(k\)-fold cross validation, which is using \(k\)-fold partitioning, while ensuring that the sets of speakers between the training sets and the out-of-bag validation sets are disjoint. We optimise \(C\) using Bayesian Optimisation (BO) (Snoek et al., 2012) for log sampled \(C \in [10^{-7}, 10^2]\) and choosing \(C\) which jointly maximises the mean accuracy on the hold-out-set for all folds and six labels. The experiments showed \(C\) to be usually close to \(10^{-3}\).

The predictions of the two models (best face and best scene models) are stacked using an RF regressor (Breiman, 2001). The number of trees was not specified by Kaya et al. (2017), however, we found that 1 000 yields similar results.

### 3.4. Fairness Baseline Approaches

#### 3.4.1. Adversarial Learning

Adversarial Learning (Xu et al., 2021) works by training two adversarial models; the first is a predictor model \(P\) which predicts the desired label, and the second is a discriminator model \(D\) which predicts the protected variable. This mechanism ensures that the embedding is representative enough for the predictor model to perform well, while not being representative enough for the discriminator to identify the protected variable. We implement this by having a two-layer (512 units each) filter model \(E\) which transforms an input to an embedding. The embedding is then used by \(P\) to produce the predicted label, and \(D\) uses the embedding to predict the value of the protected variable. Both \(P\) and \(D\) are single-layer models to avoid indirect leakage. The models are trained alternatively as shown in Algorithm 1 (in C).

There are three hyperparameters, namely the learning rate and the regularisation parameters \(\lambda_1\) and \(\lambda_2\) for the two adversarial losses. The hyperparameters are optimised using BO (Snoek et al., 2012), where each is sampled at a log-scale within the range \([10^{-7}, 10^{-2}]\). We train the models for 20 epochs using the Adam optimisation algorithm (Kingma & Ba, 2015). The experiments showed the learning rate to be usually close to \(2 \cdot 10^{-5}\).
3.5. Data Balancing

Data balancing simply operates by giving different examples different weights \( w \). We give a weight \( w_i = \frac{n_c}{K n_c} \) for the \( i \)th example, where \( n_c \) is the number of examples with protected value \( c \) for the protected variable, and \( K \) is the number of possible values of the protected variable. Then, we train using a weighted MSE instead of MSE, namely:

\[
E[(y - p)^2] = \frac{1}{n} \sum_{i=1}^{n} w_i(y_i - \hat{p}_i)^2. \tag{3}
\]

This leads to a modified kernel function for KELM models, \( K(A, B) = AB^T \Omega \), where \( \Omega \) is a diagonal matrix with the \( w_1, \cdots, w_n \) as the diagonal. This is proved in Appendix D.

3.5.5. Fairness Constraints and Metrics

Mehrai et al. (2021) discuss several constraints for satisfying fairness in ML. We select the following two:

**Definition 3.2. Statistical Parity (SP)** ensures that the labels are statistically independent from the protected variable \( C \), that is \( \forall a, z \cdot P(y > z|C = a) = P(y > z) \).

**Definition 3.3. Equal Accuracy (EA)** ensures that the accuracy of the predictions are independent from the protected variable \( C \), that is \( \forall a, b \cdot E_{y \in a}[y - p] = E_{y \in b}[y - p] \).

We define three metrics for fairness, that measure to what extent SP and EA are satisfied:

**Definition 3.4. Pearson Correlation Coefficient (PCC).**

PCC measures the correlation between the predictions \( p_1 \cdots n \) and a binary variable corresponding to a specific value \( c \) of the protected variable \( C \). PCC will quantify SP by measuring the systematic linear bias of the labels to score consistently higher or lower for a specific value of the protected attribute. The reasoning behind this is shown in F. PCC for the value \( c \) is given by:

\[
p_c = \frac{\sum_{c_i \in C} (\sum_{i=1}^{n}[^{C}_i = c] - q_c)(p_i - \bar{p})}{\sqrt{\sum_{c_i \in C} (\sum_{i=1}^{n}[^{C}_i = c] - q_c)^2 \sqrt{\sum_{i=1}^{n} (p_i - \bar{p})^2}}} \tag{4}
\]

where \( q_c \) is the ratio of examples with the value \( c \) for the protected variable, and \( \bar{p} \) is the average of the predictions.

**Definition 3.5. Statistical Parity Metric (SPM)** We define this as the Mutual Information (MI) between the predictions (continuous) with respect to the protected attribute (discrete).

It is estimated using the k-Nearest Neighbour (kNN) estimation (Ross, 2014), with \( k = 3 \), as implemented in the library SciPy. SPM can measure subtle statistical dependencies, e.g., in skewed distributions, unlike PCC which measures linear dependencies; however, SPM is harder to interpret.

Unlike (Yan et al., 2020), where the authors computed the difference of MI scores between the true labels and predictions, we evaluate this only for the predictions, because using the difference assumes implicitly that the ground truth labels are not biased, which is not the case to begin with.

**Definition 3.6. Equal Accuracy Metric (EAM)** Comparison of accuracy (measured by MAA) for different pairs of values \( a, b \) for the protected variable, namely:

\[
EAM_{a, b} = E_{a}[(y - p)] - E_{b}[(y - p)]. \tag{5}
\]

4. Proposed Approach

Given is a dataset \( D = \{(x_1, y_1, c_1), \cdots, (x_n, y_n, c_n)\} \), where \( x_i \) is the vector of input features, \( y_i \) is the output label, and \( c_i \) is the value of a protected variable. The labels are assumed to follow the same distribution, with different parameters based on the value of the protected variable, that is \( y_i \sim D_i(\mu_{c_i}, \sigma_{c_i}) \). Before training, we can transform the labels into a corresponding set of fair labels \( \hat{y}_1, \cdots, \hat{y}_n \), where the model can learn the same distribution, but without leaking information about the protected variable. The fair labels of the data are given by the transformation:

\[
\hat{y}_i := \frac{y_i - \mu_{c_i}}{\sigma_{c_i}} + \mu, \tag{6}
\]

where \( \mu_{c_i}, \sigma_{c_i} \) are the mean and standard deviation for all examples with \( c_i \) as the value of the protected variable, respectively. \( \mu, \sigma \) are the mean and standard deviation of the whole data, respectively. These are given by:

\[
\mu_c = \mathbb{E}_c[y] \text{ and } \sigma_c^2 = \text{Var}_c[y] = \mathbb{E}_c[y^2] - \mathbb{E}_c^2[y], \mu = \mathbb{E}[y] \text{ and } \sigma^2 = \text{Var}[y] = \mathbb{E}[y^2] - \mathbb{E}^2[y]. \tag{7}
\]

To train a model \( M \) with parameters \( W \), we can simply train it by optimising the loss function \( L \), defined by:

\[
L(y, p) := E[(\hat{y} - p)^2] = \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - p_i)^2, \tag{8}
\]

where \( p = M(X; W) \) is the model predictions.

4.1. Fairness-performance Trade-off

**Theorem 4.1.** Training a model \( M \) with parameters \( W \) to minimise the loss function \( L \) (Equation 8), which is the MSE after preprocessing the ground truth labels using Equation (6), is equivalent to minimising the expression:

\[
\text{MSE}(y, p) + 2 \text{Cov}(p, y - \hat{y}), \tag{9}
\]

where \( p = M(X; W) \), \( \hat{y} - y_i \) is a correction term comparing the unfairness in a ground truth label w. r. t. its corresponding fair label, and thus, the covariance term \( \text{Cov}(p, y - \hat{y}) \) corresponds to the unfairness of the predicted labels. Theorem 4.1 is proved in A.

Minimising the covariance term will minimise the MSE term for predictions that are more unfair than the ground truth, that is \( p_i < y_i < \hat{y}_i \) or \( p_i > y_i > \hat{y}_i \). However, in all other
scenarios, minimising the covariance term will maximise the MSE term, which means that competent models always have a trade-off between fairness and performance, unless the original data has no labelling bias at all (the second term is equal to 0). Another way to view this is that, the competent models that optimise performance only will always exploit an unfair advantage from the bias in the data. Trivially, there could be suboptimal models that are worse on both aspects.

4.2. Analysis of Theoretical Optimal Scenarios

In order to analyse how using Equations (6) and (8) affect fairness constraints, we examine the optimal scenario for optimising $L$, how it affects both SP and EA, and the relevant trade-offs between both fairness constraints. Theorems 4.2 to 4.4 are proved in B. In this subsection, the givens are the ground truth $y$ (with mean $\mu$ and variance $\sigma^2$) and the predictions $p$ (with mean $\bar{p}$ and variance $s^2$), and Pearson correlation coefficient $r$ between $p$ and $y$.

**Theorem 4.2.** Optimising $MSE$ leads to $MSE(y, p) = \sigma^2(1 - r^2)$, when $\bar{p} = \mu$, $s = \sigma \cdot r$, and $r$ is maximised.

**Theorem 4.3.** Optimising the loss $L$ leads to an optimal scenario, where for all values $c$ of the protected variable, $s_c = \sigma r_c$, $\bar{p}_c = \mu$, $\mathbb{E}[r_c^2] = r^2$, and $r_c$ is maximal.

**Theorem 4.4.** Optimising for all pairs $a, b$ the difference $|MSE_a(y, p) - MSE_b(y, p)|$ will result in an optimal scenario with $\bar{p}_c = \mu$, $s_c = \sigma_c \cdot r_c$, while balancing between the values of $r_c$ by satisfying $\mathbb{V}[\sigma^2(1 - r_c^2)] = 0$.

4.2.1. ANALYSIS FOR STATISTICAL PARITY

For SP, we examine the symmetric version of Kullback–Leibler (KL) Divergence between two distributions of two arbitrary values $a, b$ of the protected variable, under the assumption that both are normal distributions, given by:

$$KL_{a,b} = (\frac{s_a}{s_b})^2 + (\frac{s_b}{s_a})^2 + (\bar{p}_a - \bar{p}_b)^2(\frac{1}{s_a^2} + \frac{1}{s_b^2}).$$

KL is minimised when $\bar{p}_a = \bar{p}_b$ and $s_a = s_b$. According to Theorem 4.3, minimising $L_c$ will try to maximise $r_c$ for all $c$. By assuming that all protected groups are optimised to a similar normalised performance (that is $r_c \approx r^*$ for all $c$), we get that optimising $L_c$ also optimises KL for all pairs, because optimally $\bar{p}_a = \bar{p}_b = \mu$, $s_a = s_b = \sigma r^*$, hence getting closer to satisfying SP. However, KL gets less optimal if there is a higher variance between the normalised performances (that is, higher $\mathbb{V}[r_c]$ or $\mathbb{V}[1 - r_c^2]$). Furthermore, this scenario will minimise MSE for the dataset as a whole, without paying attention to individual groups (unlike optimising MSE or EA), since optimising $L_c$ will lead to the optimal scenario asserted by Theorem 4.2.

4.2.2. ANALYSIS FOR EQUAL ACCURACY

For EA, we analyse a corresponding expression, namely $|MSE_a(y, p) - MSE_b(y, p)|$. According to Theorem 4.4, the $L_c$ defined by Equations (7) and (8)) values to be weighted by the weights $w_i = \frac{1}{K \cdot n}$. $L(y, p) = \frac{1}{n} \sum_{i=1}^{n} w_i(y_i - p_i)^2$, where $\mu = \frac{1}{n} \sum_{i=1}^{n} u_i w_i$, and $\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} w_i(y_i - \mu)^2$.

5. Experiments

5.1. Experimental Setup

The setup of the experiments aims to test two main factors. The first is the fairness method, namely the proposed normalisation technique (FaiReg), the data balancing technique (Baln), the proposed hybrid approach (FaiRegH), and adversarial learning (Adv). The second factor is the protected variable for which the fairness is optimised, namely gender (males or females), race (Caucasian, African-American, or Asian), or a combination of the two (G × R, six possible combinations). The newly introduced combination of race and gender aims to achieve fairness for both gender and race simultaneously. These factors result in a total of twelve setups of four approaches and three protected variables.

For each of the aforementioned twelve setups, the 8,000 videos (Train+Dev) are speaker-independently split into 6-folds. On each fold, we train a face model and a scene model, where the hyperparameters of the corresponding models are optimised using BO (30 instances per method), by finding the hyperparameters values that yield the best average hold-out-sets performance score (Equations 3.8, 11, depending on the method) for all the six folds and all labels jointly. This is done separately for the face and scene modalities. Eventually, after deciding the best hyperparameters for each
of modality, we train one model for each modality on the 8,000 videos using its corresponding best hyperparameters. The trained models are then stacked together using RFs, and we present their results on the Test set (2,000 videos).

We used an Intel(R) Core(TM) i7-8700 CPU @ 3.20GHz processor, with an Nvidia GeForce RTX 2080 GPU to accelerate the training using Equation (2) for FaiReg. Training a face model for six folds takes 400 s on average, while a scene model takes 20 s. It takes approximately 75 minutes for all the 30 instances of hyperparameter tuning of FaiReg (both modalities), including an extra overhead time.

We compare the performance of the original problem, namely predicting personality and interview invitation, using the MAA metric, on the other hand, we use SPM and PCC for Statistical Parity (SP) fairness assessment, and EAM for Equal Accuracy (EA) fairness assessment.

5.2. Results

The results of the experiments are presented in Figure 1, Table 2, Table 3, and Table 4.

Figure 1 explores the effects of the performance-fairness trade-off. It shows different instances of the three methods, namely FaiReg, Baln, and Adv, with both face and scene modalities, for the three protected variables. Each point is an instance of the corresponding method with different hyperparameters. The shaded region corresponds to competent models that are achieving well between performance and fairness, by being better than a constant baseline (the mean of the training data), with MAA score > 0.8815, whilst not having a high PCC with corresponding \( p \)-value < 10\(^{-3} \), checked with a two-tailed \( t \)-test. Closer to the top-left corner translates to a superior performance in both problem performance (MAA) and SP fairness (PCC). Outside this region means that the model either has very poor performance, or shows significant leaking of the protected variable. The scatter points of each method are resembling a curve, showing that there is a room for improving both performance and fairness simultaneously, but then beyond a certain point, improvement in fairness (PCC) results in a deterioration of the problem performance (MAA). In Figure 1, the face-modality models are generally achieving better in both the problem performance and SP fairness, since the scatter points of the face models are generally closer to the top-left corner, compared to their respective scene-modality models. Baln performs very poorly on the SP fairness aspect (most Baln models in all figures are outside this region), because the unfairness is due to labelling bias, and not sampling bias (even when there is some sampling bias, especially in race). Furthermore, the models of the Adv method are far inferior with far lower MAA, and they achieve SP fairness only when the prob-

---

**Figure 1.** Plotting the relation of MAA and PCC for both gender and race, when the models are trained with gender, race, or both as a protected variable. Each point shows the Test-set performance for a hyperparameters configuration sampled by BO during hyperparameters tuning. This visualises how difference instances of one method attempts to balance the performance-fairness trade-off. The coloured region is where a model has low PCC, with \( p \)-value not < 10\(^{-3} \), while maintaining a performance above the constant baseline performance.
which is typical, since they are tested in a situation they are not trained for. This is similar to (Yan et al., 2020), where their version of FaiReg and FaiRegH are quite close (especially for PCC), however, FaiRegH is slightly better at SPM metric in Table 3 when optimising for race, or gender and race jointly. This indicates that hybrid approach FaiRegH can account for sampling bias as well as labelling bias, since the effects of sampling bias are more pronounced in race (unlike gender).

The results for EA fairness in Table 4 are not as conclusive for the clear superiority of one method achieving EA fairness. Adv is generally performing the best on EAM, as compared to FaiReg, FaiRegH, and Baln, especially for race. For gender, FaiRegH or Baln outperform Adv, but Adv has reasonable results in this case. Given the strong performance of FaiReg and FaiRegH for SP fairness, this had the result of compromising the results for EA fairness. On the other hand, optimising for SP fairness led to a minor compromise in the MAA of the original problem, however, optimising EA led to a bigger compromise in the MAA. These results somewhat agree with the analysis in Section 4.2.2, which showed that there is a compromise on accuracy when optimising for EA, and that there are trade-offs when optimising between the two fairness constraints; also, that the proposed method is more suited for SP fairness than EA fairness.

5.3. Limitations and Potential Negative Impacts

An important assumption in the provided method is that, all the data are assumed to follow the same family of distributions for each of the protected groups. However, if this is not the case, for example, if the males’ labels follow a gamma distribution, while the females’ labels follow a normal distribution, then some advanced models might still encapsulate some information about the protected variable. We demonstrate this by running a Monte Carlo experiment, where we repeat sampling data from a gamma distribution and a normal distribution. In each time, we normalise the data according to Equation (6), then we measure SPM w.r.t. gender as a binary variable. By running such an experiment, we find that if the gamma distribution has skewness values $2, \sqrt{2}, 2, \sqrt{2}$, then SPM exhibits the values 0.1, 0.01, 0.006, respectively. This shows a scenario where the presented method could get weaker for skewed distributions, which is not the case in the FI dataset (see Figure 2 in E).

Our method gives control over the impact of bias (which we try to neutralise) on training, this can be maliciously used to train a model that systematically discriminates against certain groups, which is a potential negative impact; this can be mitigated by monitoring the fairness metrics in Section 3.5.

6. Conclusion

In this paper, we introduced a novel method with two variants to mitigate unfairness in regression problems, filling a gap in the fairness literature. The first is FaiReg, which focused on eliminating unfairness due to labelling bias. The
second is FaiRegH, which is a hybrid approach between FaiReg and data balancing; this focused on simultaneously eliminating unfairness due to labelling and sampling biases. The method consisted of normalising the training labels before training w.r.t. the corresponding value of the protected variable. We conducted a theoretical analysis showing that, there is always a trade-off between fairness and performance, and between different fairness constraints. This implied that models that only optimise for performance have to take an unfair advantage of the bias in the data, and that models can not perform the best across all fairness metrics as well as performance. The experiments confirmed these analyses.

We performed experiments, where we compared the two methods against two alternative methods, namely data balancing and adversarial learning. The experiments demonstrated that FaiReg and FaiRegH have majorly improved the (Statistical Parity) fairness measures without major deterioration on the original problem; in comparison, data balancing could not conquer labelling bias, and adversarial learning yielded poor results in the original problem. Both methods were illustrated to mitigate bias for more than one protected variable simultaneously. In our experiments, we utilised the First Impressions (FI) dataset, which is a multimodal dataset, consisting of videos and six regression labels, namely the Big-Five personality features (OCEAN) and a score label for invitation to an interview. The FI dataset was shown to have labelling and sampling biases, which made it suitable for the presented method.

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Gender & E & A & C & N & O & I \\
\hline
Grnd Tr. & 0.30 & 0.04 & 0.10 & 0.05 & 0.14 & 0.05 \\
orig. & 0.34 & 0.00 & 0.00 & 0.59 & 0.31 & \\
\hline
Baln & 0.11 & 0.07 & 0.22 & 0.19 & 0.14 & 0.17 \\
FaiReg & 0.14 & 0.00 & 0.10 & 0.00 & 0.00 & 0.00 \\
FaiRegH & 0.39 & 0.07 & 0.30 & 0.20 & 0.47 & 0.18 \\
Adv & 0.16 & 0.18 & 0.07 & 0.09 & 0.25 & \\
\hline
Race & 0.01 & 0.00 & 0.10 & 0.00 & 0.00 & 0.00 \\
FaiReg & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
FaiRegH & 0.39 & 0.07 & 0.30 & 0.20 & 0.47 & 0.18 \\
Adv & 0.08 & 0.16 & 0.03 & 0.11 & 0.21 & 0.08 \\
\hline
\end{tabular}
\caption{Statistical Parity Metric (SPM) for the four methods, while training for different protected variables (race, gender, or both).}
\end{table}

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
Gender & E & A & C & N & O & I \\
\hline
Grnd Tr. & 0.02 & 0.04 & 0.10 & 0.05 & 0.14 & 0.05 \\
orig. & 0.16 & 0.18 & 0.07 & 0.09 & 0.25 & \\
\hline
Baln & 0.11 & 0.07 & 0.22 & 0.19 & 0.14 & 0.17 \\
FaiReg & 0.30 & 0.06 & 0.11 & 0.26 & 0.13 & 0.08 \\
FaiRegH & 0.14 & 0.20 & 0.00 & 0.10 & 0.27 & 0.18 \\
Adv & 0.08 & 0.16 & 0.03 & 0.11 & 0.21 & 0.08 \\
\hline
Race & 0.18 & 0.08 & 0.00 & 0.00 & 0.00 & 0.00 \\
FaiReg & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
FaiRegH & 0.03 & 0.00 & 0.00 & 0.01 & 0.00 & 0.00 \\
Adv & 0.09 & 0.19 & 0.12 & 0.12 & 0.18 & 0.06 \\
\hline
\end{tabular}
\caption{MAA and EAM scores for the different setups. Bold numbers show the methods with best score for the protected variable. The last row is the mean absolute values of the three race rows.}
\end{table}

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A. Proof of Theorem 4.1

Theorem 4.1 states that, training a model to minimise the loss function \( \mathcal{L} \) (Equation (8)), is equivalent to minimising the expression: \( \text{MSE}(y, p) + 2 \text{Cov}(p, y - \hat{y}) \).

Proof.

\[
\begin{align*}
\mathcal{L}(\hat{y}, p) &= \mathbb{E}[(\hat{y} - p)^2] = \mathbb{E}[(\hat{y} - y + y - p)^2] \\
&= \mathbb{E}[(y - p)^2 + (\hat{y} - y)^2 + 2(\hat{y} - y)(y - p)] \\
&= \mathbb{E}[(y - p)^2 + (\hat{y} - y)^2 + 2(\hat{y} - y) + 2p(y - \hat{y})] \\
&= \mathbb{E}[(y - p)^2 + (\hat{y} - y)(\hat{y} + y) + 2p(y - \hat{y})] \\
&= \mathbb{E}[(y - p)^2 + (y^2 - \hat{y}^2) + 2p(E[p] - y) + 2E[p](y - \hat{y})] \\
&= \mathbb{E}[(y - p)^2 + 2 \text{Cov}(p, y - \hat{y})].
\end{align*}
\]

The last step is acquired, since by definition \( \mathbb{E}[\hat{y}] = \mathbb{E}[y] \) and \( \text{Var}[\hat{y}] = \text{Var}[y] \), which implies \( \mathbb{E}[y - \hat{y}] = \mathbb{E}[y^2 - \hat{y}^2] = 0 \). This equality is the reason why the second and last terms are eliminated.

By minimising both sides w. r. t. the parameters \( W \), we get:

\[
\arg \min_W \mathcal{L}(\hat{y}, p) = \arg \min_W [\mathbb{E}[(y - p)^2] + 2 \text{Cov}(p, y - \hat{y})].
\]

\[\square\]

B. Proofs of the Theorems in Section 4.2

Theorem 4.2 asserts that, given is a ground truth \( y \) (with mean \( \mu \) and variance \( \sigma^2 \)) and predictions \( p \) (with mean \( \bar{p} \), variance \( s^2 \)), and Pearson correlation coefficient \( r \) between \( p \) and \( y \), then the optimal \( \text{MSE}(y, p) = \text{Var}[y](1 - r^2) \), when \( \bar{p} = \mathbb{E}[y], s = \sqrt{\text{Var}[y]} \cdot r \), and \( r \) is maximised. In other words, MSE attempts to get a distribution with the same mean as the original distribution, with maximal similarity between the predictions and original distributions, and with a confidence that is restricted by the normalised performance.

Proof.

\[
\begin{align*}
\text{MSE}(y, p) &= \mathbb{E}[(y - p)^2] = \mathbb{E}[(y - \mathbb{E}[y] - p + \mathbb{E}[p] + \mathbb{E}[y] - \mathbb{E}[p])^2] \\
&= \mathbb{E}[(y - \mathbb{E}[y])^2 + (p - \mathbb{E}[p])^2 + (\mathbb{E}[y] - \mathbb{E}[p])^2 - \\
&\quad 2(y - \mathbb{E}[y])(p - \mathbb{E}[p]) + 2(y - \mathbb{E}[y] - p + \mathbb{E}[p])(\mathbb{E}[y] - \mathbb{E}[p])] \\
&= \mathbb{E}[(y - \mathbb{E}[y])^2 + (p - \mathbb{E}[p])^2 + \mathbb{E}[(\mathbb{E}[y] - \mathbb{E}[p])^2] - \\
&\quad 2\mathbb{E}[(y - \mathbb{E}[y])(p - \mathbb{E}[p])] \\
&= \text{Var}[y] + \text{Var}[p] + (\mathbb{E}[y] - \mathbb{E}[p])^2 - 2 \text{Cov}(y, p) \\
&= \sigma^2 + s^2 + (\mu - \bar{p})^2 - 2\sigma sr
\end{align*}
\]

The third step is acquired by linearity of expectation, and that fact that \( \mathbb{E}[(y - \mathbb{E}[y] - p + \mathbb{E}[p])(\mathbb{E}[y] - \mathbb{E}[p])] = 0 \).

Assuming that \( s, \bar{p}, r \) are independent variables, and that \( \sigma, \mu \) are constants (corresponding to the ground truth), we can optimise the MSE by differentiating the formula with respect to \( s \) and \( \bar{p} \).

\[
\begin{align*}
\frac{\partial}{\partial s} \text{MSE}(y, p) &= 2(s - \sigma r) \Rightarrow s^* = \sigma r \\
\frac{\partial}{\partial p} \text{MSE}(y, p) &= 2(\bar{p} - \mu) \Rightarrow \bar{p}^* = \mu
\end{align*}
\]

By substituting these values, we get an optimal MSE of \( \sigma^2(1 - r^2) \), which is maximised when \( r \) gets farther from 0, ideally when \( r = 1 \) (if \( r = -1 \) is a rejected solution because \( s = \sigma r \), where \( \sigma, s \geq 0 \)).

\[\square\]
when Applying this on MSE, we get that 
where the individual MSE values are minimised, while satisfying the constraint 
asserts that minimising 

\[ \text{Theorem 4.3} \]

\[
E_c[ (y - \mu_c)^2 + (p - \bar{p}_c)^2 + (\mu - \bar{p}_c)^2 - 2(\gamma - \mu_c)(y - \mu_c)(p - \bar{p}_c)]
\]

\[
= \left( \frac{\sigma}{\sigma_c} \right)^2 E_c[ (y - \mu_c)^2] + E_c[ (p - \bar{p}_c)^2] + (\mu - \bar{p}_c)^2 - 2 \frac{\sigma}{\sigma_c} E_c[ (y - \mu_c)(p - \bar{p}_c)]
\]

\[
= \left( \frac{\sigma}{\sigma_c} \right)^2 s_c^2 + \sigma_a^2 + (\mu - \bar{p}_c)^2 - 2 \frac{\sigma}{\sigma_c} s_c \sigma_c r_c = \sigma^2 + s_c^2 + (\mu - \bar{p}_c)^2 - 2 \sigma s_c r_c
\]

The following is a detailed proof:

Proof.

\[ L_c = E_c[ (y - \mu_c)^2 + (p - \bar{p}_c)^2 + (\mu - \bar{p}_c)^2 - 2(\gamma - \mu_c)(y - \mu_c)(p - \bar{p}_c)] \]

\[ = E_c[ (y - \mu_c)^2] + E_c[ (p - \bar{p}_c)^2] + (\mu - \bar{p}_c)^2 - 2 E_c[ (y - \mu_c)(p - \bar{p}_c)] \]

\[ = \left( \frac{\sigma}{\sigma_c} \right)^2 s_c^2 + \sigma_a^2 + (\mu - \bar{p}_c)^2 - 2 \frac{\sigma}{\sigma_c} s_c \sigma_c r_c = \sigma^2 + s_c^2 + (\mu - \bar{p}_c)^2 - 2 \sigma s_c r_c \]

\[ \partial L \]

\[ \begin{align*}
L_c &= \sigma^2 + s_c^2 + (\mu - \bar{p}_c)^2 - 2 \sigma s_c r_c \\
E_c &= \sigma^2 + s_c^2 + (\mu - \bar{p}_c)^2 - 2 \sigma s_c r_c \\
L_c &= \sigma^2 + s_a^2 + (\mu - \bar{p}_c)^2 - 2 \sigma s_c r_c \]
\]

\[ \square \]

**Theorem 4.4** asserts that minimising \[ \text{MSE}_a(y, p) - \text{MSE}_b(y, p) \] (which would satisfy EA) will yield to an optimal scenario where the individual MSE values are minimised, while satisfying the constraint \[ \text{Var}(\sigma_c^2(1 - r_c^2)) = 0 \]. To prove that, we consider an arbitrary error function \[ M_c \] for a specific value \( c \) for the protected variable, and an arbitrary dependent variable \( v_c \). Then we try to minimise \[ \mathcal{E} := \sum_{a,b} (M_a - M_b)^2 \], which will minimise all the aforementioned differences.

Proof.

\[ \frac{\partial \mathcal{E}}{\partial v_c} = 2 \sum_{a,b} (M_a - M_b) \frac{\partial M_a}{\partial v_c} = 2 \sum_{a,b} (M_a - M_b) \frac{\partial M_a}{\partial v_a} \delta_a^c \frac{\partial M_b}{\partial v_b} \delta_b^c \]

\[ = 2 \sum_{a,b} (M_a - M_b) \frac{\partial M_a}{\partial v_a} \delta_a^c \frac{\partial M_b}{\partial v_b} \delta_b^c \]

\[ = 4 \sum_{a,b} (M_a - M_b) \frac{\partial M_a}{\partial v_a} \delta_a^c = 4 \sum_{a,b} (M_a - M_b) \frac{\partial M_a}{\partial v_c} \]

In the second step, \[ \frac{\partial M_c}{\partial v_c} = 0 \] if \( a \neq c \) because \( M_a \) isn’t depending on \( v_c \) by definition, which is reason why assume \[ \frac{\partial M_a}{\partial v_c} = \frac{\partial M_a}{\partial v_a} \delta_a^c \]. Furthermore, the fourth step is acquired since the two summations are symmetric, we can swap \( a, b \) in the second one.

Minimising \( \mathcal{E} \) w.r.t \( v_c \), can be achieved when \[ \frac{\partial \mathcal{E}}{\partial v_c} = 0 \], which implies that \[ \frac{\partial M_c}{\partial v_c} = 0 \] or \[ \sum_b (M_a - M_b) = 0 \] (equivalently \( M_c = E[M_b] \), or \( \text{Var}[M_c] = 0 \)). Consequently, minimising \( \mathcal{E} \) requires minimising the individual error functions \( M_c \) w.r.t \( v_c \) or the variance in \( M_c \) is minimal.

Applying this on MSE, we get that \( \mathcal{E} \) is minimised when \( \text{MAE}_c \) is minimised, which happens (according to Theorem 4.2) when \( \bar{p}_c = \mu_c, s_c = \sigma_c r_c \), and \( r_c \) is maximal. In that case, \( \text{MAE}_c = \sigma_c^2 (1 - r_c^2) \). Consequently, \( \mathcal{E} \) is minimised when \[ \text{Var}[\sigma_c^2(1 - r_c^2)] = 0 \].

\[ \square \]
C. Adversarial Learning Algorithm

Algorithm 1 shows the training procedure of one of the baseline fairness methods, namely Adversarial Learning. CE refers to Cross-Entropy, which is the negative log-likelihood.

**Algorithm 1** Adversarial Learning

**Input:** Ground truth labels $y$, protected variable $c$, input features $X$  

**Models:** Filter $E$, Predictor $P$, Discriminator $D$

**for** $e$ **epochs**

- Train the models $P$, $E$ one step, while freezing $D$, by minimising:  
  $$\text{MSE}(y, P(E(X))) - \lambda_1 \text{CE}(c, D(E(X)))$$

- Train the model $D$ one step, while freezing $P$, $E$, by minimising:  
  $$\lambda_2 \text{CE}(c, D(E(X)))$$

**end for**

D. Proof of Optimal Weighted Kernel Matrix

Here we prove that Equation (3), corresponds to KELM model with kernel function $K(A, B) = AB^T\Omega$ with optimal Mean Squared Error (MSE) adjusted for data balancing, given by:

$$e = \sum_{i=1}^{n} w_i (y_i - p_i)^2 = \frac{1}{n} (Y - X\beta)^T\Omega(Y - X\beta)$$

**Proof.** The regularised weighted sum of squares error $e$ is given by:

$$e = (Y - X\beta)^T\Omega(Y - X\beta) + \frac{1}{C}\beta^T\beta$$

$$e = (Y^T\Omega Y + \beta^T X^T\Omega X\beta - Y^T\Omega X\beta - \beta^T X^T\Omega Y) + \frac{1}{C}\beta^T\beta$$

$$e = (Y^T\Omega Y + \beta^T X^T\Omega X\beta - 2Y^T\Omega X\beta) + \frac{1}{C}\beta^T\beta$$

$$\frac{\partial e}{\partial \beta} = 2X^T\Omega X\beta - 2X^T\Omega Y + \frac{2}{C}\beta$$

In order to minimise $e$, we solve for $\beta$ for $\frac{\partial e}{\partial \beta} = 0$, which implies:

$$(X^T\Omega X + \frac{1}{C}I)\beta = X^T\Omega Y$$

There are two solutions for $\beta$, namely $\beta = (X^T\Omega X + \frac{1}{C}I)^{-1}X^T\Omega Y$, or $X^T\Omega (XX^T\Omega + \frac{1}{C}I)^{-1}Y$, both can be verified by the substitution in the last equation. The second solution gives the adjusted kernel function in Section 3.4.2.

E. First Impressions Dataset Distributions

Figure 2 demonstrates the distributions of the labels for all genders and all races.

F. Statistical Parity and Pearson Correlation Coefficient

Here we prove that Pearson Correlation Coefficient (PCC) defined by Equation (4) corresponds to a quantification measuring Statistical Parity (SP) constraint.

**Proof.** The definition of SP:

$$\forall z \cdot P(p > z | C = c) = P(p > z)$$
Normalise for Fairness: A Simple Normalisation Technique for Fairness in Regression Machine Learning Problems

Figure 2. The distributions of the Test-set ground truth labels for the six labels, and each protected variable.

By expanding the definition, we get:

\[ \forall z \cdot \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}[p_i > z] \mathbb{I}[C = c_i] = \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}[p_i > z] \]

\[ \forall z \cdot \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}[p_i > z] \mathbb{I}[C = c_i] - \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}[C = c_i] \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}[p_i > z] = 0 \]

By integrating both sides w.r.t. \( dz \) for all \( z \in [0, 1] \), we get:
\[
\frac{1}{n} \sum_{i=1}^{n} p_i [C = c_i] - \frac{1}{n} \sum_{i=1}^{n} [C = c_i] = 0
\]
\[
\frac{1}{n} \sum_{i=1}^{n} p_i [C = c_i] - n_c \bar{p} = 0
\]
\[
\frac{1}{n} \sum_{i=1}^{n} [C = c_i] - n_c (p_i - \bar{p}) = 0
\]

The left hand side of the equation is the covariance computed by the numerator of PCC, which is equal to 0 if SP is satisfied. The further this value from 0, the more dependent both variable are, hence further away from SP. Therefore, the absolute value of PCC is a normalised measure that indicates how far SP is satisfied.

\[
\square
\]

G. Demonstration for Equal Accuracy Optimisation

In this section, we construct a constant predictor that predicts a value \( p^* \), which minimises a measure corresponding to the Equal Accuracy constraint.

For a normally distributed labels with a mean \( \mu \) and variance \( \sigma^2 \), the expected mean squared error for a constant prediction \( p^* \) is given by, \( \mathbb{E}[(p - y)^2] = \int_{-\infty}^{\infty} (p - y)^2 \phi(y) \mu, \sigma^2 \) \( dy = (p - \mu)^2 + \sigma^2 \). Hence, we present a smoothed version of the Equal Accuracy Metric, which we call Squared Equal Accuracy Metric (SEAM), this is given by:

\[
E := \sum_{a,b} \left( \mathbb{E}_a[(p - y)^2] - \mathbb{E}_b[(p - y)^2] \right)^2
\]
\[
\quad = \sum_{a,b} \left( (p - \mu_a)^2 - (p - \mu_b)^2 + \sigma_a^2 - \sigma_b^2 \right)^2
\]

In order to find the optimal point \( p^* \), with minimum value of SEAM, we need to solve \( \frac{\partial E}{\partial p} = 0 \).

\[
\frac{\partial E}{\partial p} = 4 \sum_{a,b} ((p - \mu_a)^2 - (p - \mu_b)^2 + \sigma_a^2 - \sigma_b^2)(\mu_b - \mu_a)
\]
\[
= 4 \sum_{a,b} (\mu_a^2 - \mu_b^2 - 2(\mu_a - \mu_b)p + \sigma_a^2 - \sigma_b^2)(\mu_b - \mu_a)
\]

Solving for \( \frac{\partial E}{\partial p} = 0 \), yields the following:

\[
p^* = \frac{\sum_{a,b}(\mu_a^2 - \mu_b^2 + \sigma_a^2 - \sigma_b^2)(\mu_a - \mu_b)}{2 \sum_{a,b}(\mu_a - \mu_b)^2}
\]
\[
= \frac{\sum_{a,b}(\mu_a - \mu_b)^2(\mu_a + \mu_b)/2 + \sum_{a,b}(\sigma_a^2 - \sigma_b^2)(\mu_a - \mu_b)}{2 \sum_{a,b}(\mu_a - \mu_b)^2}
\]

This formula has two components, the first is given by the weighted average between the midpoints of each pair of the protected groups. The weights are given by the intra-cluster square distances of the corresponding pairs. This component is independent from the variance of the data. The second component of the formula is a correction based on the variance, by shifting the ideal constant prediction towards clusters with higher variance.

The optimal \( p^* \) is demonstrated in Figure 3, the demonstration has three clusters with means 1, 2, and 10 respectively. In the first figure, all distributions have the same standard deviation; the optimal point \( p^* \) is near 5.7, this is the value of the first component of the formula in all the three figures. The optimal point is tilted towards the third distribution since it is much further from the other two. The other two figures show similar analysis, but by changing the standard deviation of the left-most and right-most distributions, respectively. It can be seen that the optimal point \( p^* \) moves closer to the distribution with higher variance, by a shift of roughly 2.9 towards the corresponding distributions.
**Figure 3.** Demonstration of how the constant optimal point $p^*$ (yielding optimal Equal Accuracy) changes based on the variances of the distributions of the protected groups. $p^*$ is demonstrated by the vertical line.