Frequency dependence of magneto-optical phenomena in nonmagnetic dielectric nanostructures

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Abstract. The frequency dependences of the Faraday rotation angle and the ellipticity angle are studied taking into account light absorption in dielectric media. The extreme values of the above values are determined, as well as the field of their application in dielectric media.

1. Introduction

The study of magneto-optical effects in Nano sized photonic crystals is one of the most urgent problems in the physics of magnetic phenomena. In particular, the development of methods for increasing the angle of rotation of the plane of polarization – the Faraday effect – is of important scientific and practical interest [1,2].

In this paper, we analyze the spectral properties, i.e. frequency dependences of differences in refractive indices and absorption coefficients of non-magnetic dielectric crystals, which determine the characteristics of magneto-optical phenomena.

In particular, the spectral features of the Faraday effect are studied, as well as the dispersion properties of magnetic circular dichroism. These studies are due to the fact that in modern Nano electronics and Nanophysics there is a great need for materials (or structures) with frequency-controlled magneto-optical properties [3,4].

2. Spectral features of the Faraday effect

The spectral properties of the value of the angle of rotation of the plane of polarization \( \varphi_F \) i.e. the Faraday angle of rotation will be studied on the basis of the following expression

\[
\varphi_F = \frac{\omega d}{2c} \text{Re}(n^- - n^+).
\]  

(1)

Here \( n^- - n^+ \) is the difference between the complex refractive indices of left and right circularly polarized light (or electromagnetic wave), which is determined as follows [5]

\[
n^- - n^+ = \frac{N e^2}{2\varepsilon_0} \cdot \frac{A_1 - iA_2}{A_1^2 + A_2^2} \cdot 2eB\omega.
\]  

(2)
In the above expressions: $\omega$ is the cyclic frequency of the light field, with the speed of light in vacuum, $\varepsilon_0$ – permittivity of vacuum, $N$ is the electron concentration, $e$ is the magnitude of the elementary charge. In the induction of the applied magnetic field $\vec{B}(0,0,B)$, $d$ – to thickness of the dielectric layer.

In turn, the frequency-dependent functions $A_1$ and $A_2$ have the form [5]

$$
A_1 = m^2 \omega^4 - \omega^2 \left(2am + e^2 B^2 + b^2\right) + a^2, \\
A_2 = 2b\omega \left(a - m\omega^2\right).
$$

(3)

Here: $m$ is the electron mass, $a$ is the effective elastic coefficient, and $b$ is the effective resistance coefficient, which determine the following characteristic cyclic frequencies of the system under consideration

$$
\omega_c = \sqrt{\frac{a}{m}}, \quad \omega_z = \frac{b}{m}
$$

(4)

Where $\omega_c$ is the cyclic frequency of the natural oscillations of the electrons, $\omega_z$ is the cyclic frequency of the attenuation of light.

Note that earlier in [5] we showed the possibility of resonant amplification of the Faraday effect at $\omega = \omega_c$ by the action of a small external magnetic field.

Here we will study another aspect of this problem, namely, the features of the frequency dependence of the Faraday angle of rotation $\varphi_F$ taking into account light absorption. At the same time, in the existing literature on this topic, only the frequency dependence $\varphi_F$ in the region of transparency, i.e. in the absence of absorption, has been studied.

Naturally, due to the complex dependence of the functions $A_1$ and $A_2$ given by expressions (3), an analytical consideration of the problem under study in the General case seems impossible.

Therefore, below we limit ourselves to the study of special cases where it is still possible to study the spectral features of the Faraday angle of rotation $\varphi_F$ analytically. Valid if you impose the following conditions:

$$
a \simeq eB\omega \\
b = eB
$$

(5)

then the real part of the difference of the complex refractive index can be written in the following simple form

$$
\text{Re}(n^- - n^+) = \frac{Ne^2}{\varepsilon_0} \cdot \frac{eB}{\omega} \cdot \frac{u^2 - 2e^2 B^2}{u^4 + 4e^4 B^4}.
$$

(6)

Here we have introduced the following notation to simplify the entries

$$
u = m\omega - eB = m(\omega - \omega_n).
$$

(7)

In the latter expression, $\omega_n$ is the cyclic frequency of electron precession under the action of an external magnetic field

$$
\omega_n = \frac{eB}{m}.
$$

(8)
Thus, if the additional conditions (5) are fulfilled, taking into account expressions (1), (6) for the frequency dependence of the Faraday angle of rotation $\varphi_F$, we obtain the following function

$$\varphi_F = \frac{d}{2c} \cdot \frac{Ne^2}{\varepsilon_0} \cdot eB \cdot \frac{u^2 - 2e^2B^2}{u^4 + 4e^4B^4}.$$  \hspace{1cm} (9)

Further, calculating the derivative of the function $\varphi_F$ of the Faraday angle of rotation and equating it to zero, we find that there are two extreme frequencies

$$\omega_1 = \omega_n, \quad \omega_2 = \left(1 + \sqrt{2(1+\sqrt{2})}\right)\omega_n.$$  \hspace{1cm} (10)

The first extreme frequency corresponds to the minimum of this function

$$\varphi_F (\text{min}) = \frac{d}{4c} \cdot \frac{Ne^2}{\varepsilon_0} \left(\frac{1}{eB}\right).$$  \hspace{1cm} (11)

In the second extreme frequency, $\varphi_F$ has the following maximum value

$$\varphi_F (\text{max}) = \frac{d}{4c} \cdot \frac{Ne^2}{\varepsilon_0} \cdot \frac{1}{eB} \cdot \frac{1}{\sqrt{2\left[1 + (1+\sqrt{2})^2\right]}}.$$  \hspace{1cm} (12)

There is also an intermediate frequency $\omega^* = \left(1 + \sqrt{2}\right)\omega_n$ at which $\varphi_F$ is zero.

Therefore, the graph of the Faraday angle of rotation $\varphi_F$ as a function of frequency can be represented as follows (figure 1).

**Figure 1.** Frequency dependence of the Faraday rotation angle.

As follows from the above analysis, the characteristic value of the Faraday angle of rotation is determined by the following expression
\[ A = \frac{d}{4c} \cdot \frac{Ne^2}{\varepsilon_0 b} = \frac{d \omega_{el}}{4c} \cdot \frac{\omega_{el}}{\omega_z}. \]  

(13)

Here

\[ \omega_{el} = \sqrt{\frac{Ne^2}{m \varepsilon_0}} \]  

(14)

\( \omega_{el} \) - electronic plasma frequency.

Therefore, if \( \omega_{el} \) is significantly larger than \( \omega_z \), it is possible to increase the Faraday angle of rotation by two or three orders of magnitude.

If the dielectric medium has absorption, as in our case, then the absorption coefficients of right and left circularly polarized light waves differ. This will cause the linearly polarized wave to become elliptically polarized after passing through such a medium. Similar to the Faraday angle, it is characterized by the angle of ellipticity (or orientation), which is determined by the imaginary part of the difference in the complex refractive index

\[ \psi_E = \frac{\alpha d}{2c} \operatorname{Im}\left( n^- - n^+ \right) \]  

(15)

The next section will be devoted to the analysis of the frequency dependence of the ellipticity angle \( \psi_E \) or the phenomenon of magnetic circular dichroism.

3. Dispersion properties of magnetic circular dichroism

Here we will study the dispersion properties, i.e., the frequency dependence of the function \( \psi_E \), which is a quantitative characteristic of the phenomenon of magnetic circular dichroism. In the same approximation and that was used in the previous section, based on expressions (2) and (15), we obtain

\[ \psi_E = \frac{\alpha d}{2c} \operatorname{Im}\left( n^- - n^+ \right) = \tilde{B} \cdot \frac{u}{u^4 + 4e^4 B^4}. \]  

(16)

Here

\[ \tilde{B} = \frac{d}{c} \cdot \frac{Ne^2}{\varepsilon_0} \cdot 2ebB, \quad u = m\omega - eB. \]  

(17)

Now, finding the derivative of the function \( \psi_E \) with respect to the frequency \( \omega \) and equating it to zero, we conclude that it has the greatest value at an extreme frequency

\[ \omega = \left( 1 + \frac{4}{3} \right) \omega_n. \]  

(18)

Recall that here \( \omega_n \) is the cyclic frequency of electron precession under the action of an applied external magnetic field. The ellipticity curve vanishes at a light frequency of \( \omega_n \) (figure 2).
It is easy to see that the maximum of the ellipticity angle is also inversely proportional to the cyclic frequency of light attenuation (see expression (13)).

At the end of the section, we present the characteristic values of the values used for the magneto-optical effects under study. For estimation, we assume that the characteristic value of the Faraday rotation angle $\varphi$, defined by expression (13), is equal to $\pi$. Then, taking into account the experimental data [4, 8-21]: $N=10^{28}$ m$^{-3}$, $d \approx 20$ nm, we obtain that $B \approx 1$ MT. Consequently, the precession frequency equal, to the attenuation frequency of electromagnetic waves according to the second condition (5) will be about 200 MHz, which is consistent with the experimental results [6, 7].

4. Conclusions
Summarizing the results obtained in the analysis of the frequency dependences of the Faraday rotation angle and the ellipticity angle, we conclude that their maximum values are inversely proportional to the cyclic attenuation frequency of the electron wave. Consequently, at low absorptions, it is possible to significantly increase the magneto-optical phenomena under study.

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