What is the Standard Cosmological Model?

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(Dated: May 10, 2021)

Reports of “cosmology in crisis” are in vogue, but as Mark Twain said, “the report of my death was an exaggeration”. We explore what we might actually mean by the standard cosmological model, how tensions – or their apparent resolutions – might arise from too narrow a view, and why looking at the big picture is so essential. This is based on the seminar “All Cosmology, All the Time”.

I. INTRODUCTION

The origin of the word “crisis” comes from “decision”, implying a framework – a standard model – and paths forward that are being decided between. So to assess whether some crisis exists and how severe it might be, we must first explore what we mean by the standard cosmological model.

We discuss various definitions in Section II, then investigate a couple of oft claimed tensions and their (un)resolutions in Section III. The primacy of using all robust observations, at all redshifts – All Cosmology, All The Time – is motivated in Section IV, and the concluding discussion is in Section V.

II. WHAT IS STANDARD COSMOLOGY?

What is the standard cosmological model depends very much on where one draws the line on what is cosmology or the universe.

A. Level 1: Global Properties

One could define Level 1 as saying cosmology is the global properties of the universe:

- Connected – a signal can get from there to here and from then to now. There is no discreteness (at least at this level).
- Metric – we can figure out how far it is from there to here and from then to now.

This gives the foundation and few would dispute that they are part of the standard cosmological model; violation of either on the scale of the observable universe would be revolutionary (and pretty exciting on smaller scales as well!).

A third property comes from a plethora of observations, and is also generally accepted as part of our standard cosmological model:

- Homogeneity and Isotropy – on observable universe scales.

This then implies the Robertson-Walker metric and we are on familiar territory! In particular, the Robertson-Walker metric has two characteristics (independent of the theory of gravity):

- Evolution – a scale factor $a(t)$.
- Spatial curvature – a constant $k$ proportional to the Gaussian curvature of space. It is worth emphasizing that a Robertson-Walker spacetime can have spatial curvature but no spacetime curvature (though this does not describe our universe), and spacetime curvature but no spatial curvature (which fits observations pretty well).

Unlike, say, discreteness, we don’t have to go down to laboratory scales to test for a breakdown in homogeneity and isotropy, and hence the Robertson-Walker metric. However, one can show that these smaller scale deviations do not significantly affect cosmological scales. This is worth emphasizing: for the expansion and curvature, and for observations involving propagation of light rays along a line of sight, and for growth within three dimensional volumes, the effect from small scales generally is calculable and found to be small [1–4].

B. Level 2: History of the Universe

If one delves into the time structure, one could say that a Level 2 answer to what is the standard cosmological model is to say cosmology is the history of the universe. From observations we could list five key stages:

- Early hot dense state – popularly, if nebulously, known as the “Big Bang”. Here we consider it as a generic description and source of initial conditions, and whether it occurs at the Planck energy, $10^{15}$ GeV, or $10^{3}$ GeV is a (fascinating) detail, not a source of crisis.
- Matter/antimatter asymmetry – I have absolutely nothing to say! While we know the basic ingredients to deliver this (if not its magnitude) [5] one could well argue this is a greater crisis than any other raised, and yet it is rarely mentioned.
- Radiation dominated era – primordial nucleosynthesis, degrees of freedom \( g_\ast \) (neutrino decoupling, electron/positron annihilation), CMB thermalization. Lots of good stuff!
- Matter dominated era – CMB scattering, growth of structure (us!).
- Cosmic acceleration – “dark energy”, fate of the universe?

As an aside, one of the most fascinating aspects of this is that the argument could be made that the study of dark energy actually grew out of investigation of the radiation dominated era. The radiation era spans such a huge range of e-folds of expansion (nominally some 55, compared to the matter era’s 7, or dark energy’s 0.5), and yet we know little of the details – did it stay radiation dominated the entire time? We tend to simply assume this but we have accurate windows on only tiny slivers of this era, around primordial nucleosynthesis [6–8] and toward the end, just before matter domination and recombination [9–11]. In 1979, Robert Wagoner [12] highlighted this by considering what freedom there was of changing the equation of state of the dominant energy density. This program of exploring the equation of state at various epochs was picked up by two of his students and in the 1980s was applied to the late universe and became dark energy cosmology, developing both the cosmological model and observational probes, first for particular values of the equations of state [13, 14] and then for a general equation of state [15, 16].

C. Level 3: Stuff In the Universe

A more specific view is that cosmology is the stuff in the universe. After all, this is what is actually observed. This would include:

- Cosmic microwave background radiation (CMB) – CMB structure (anisotropies, polarization, spectral distortions) is a rich probe of both history (Level 2, including initial conditions such as adiabatic perturbations) and the other contents (Level 3, e.g. matter stuff through scattering, gravitational potentials). And of course it provides strong evidence through isotropy (and homogeneity through the CMB felt by distance objects) for Level 1 cosmology.
- Large scale structure – The continuous fields of matter: the density field, velocity field, acceleration (gravity) field – generally as probed by individual sources. While these fields are related, the relations do test the framework and each has particular incisive elements so they can be considered distinct; plus, for cosmological purposes they are at very different stages of observational development.
- Other – As probes of the standard cosmological model, observations of other stuff such as neutrinos, gravitational waves, exotica (e.g. topological defects) have not yet reached the same stage of having a major impact, though this would be an exciting development.

D. Turtles All the Way Down

Some researchers extend the standard model of cosmology further and further down in detail, to “the stuff in the stuff in the universe”, e.g. aspects of galaxies, galaxy clusters, generation of various particles and fields (e.g. neutrinos, gravitational waves, etc.). Others will stretch to “the properties of the stuff in the stuff in the universe”, e.g. cuspy cores of galaxies, tidal streams, Cepheid pulsations, etc.

Where one draws the line between the standard cosmological model and all else – call it astrophysics for simplicity – is a personal choice. But it is rare that rain next Tuesday in some place one had not predicted it throws the standard meteorological model into crisis. It is not impossible, but it is useful to remember that some particular tension has to work its way from Level 3 to the fundamental foundations.

E. Cosmologing Is Hard

But... cosmologing is hard. The properties of the stuff in the stuff affect how and what we learn about the more fundamental stuff. We are then faced with the puzzle of whether we have fully understood the stuff in the stuff (or mismeasured its properties) or whether indeed it propagates cleanly to impacting the standard cosmological model. Let us take two examples.

Example 1. Suppose one measured the CMB temperature at scale factor \( a \) (redshift \( z = a^{-1} - 1 \)) and found that

\[
T_{\text{CMB}}(z) \neq T_{\text{CMB}}(0) \times (1+z) \quad ?
\]

What should one conclude: that the universe is not adiabatically expanding, or that there is some systematic error (e.g. molecular collisional excitations)? How much effort, and what proportion of the literature, should be dedicated to investigating systematics before declaring a crisis in the standard cosmological model?

Example 2. Suppose one measured the distance to an object (or set of objects) at redshift \( z \) in terms of both its luminosity distance and angular diameter distance and found that the reciprocity relation is broken,

\[
d_L(z) \neq d_A(z) \times (1+z)^2 \quad ?
\]

If one wishes to conclude that this places the standard cosmological model in crisis (rather than some systematic error in the data), one must give up some foundational element, since this relation arises from a) metricity,
b) geodesic completeness, c) photons propagate on null geodesics, and d) adiabatic expansion. (Note conservation of photon phase space density is a big part of this, but not all.)

What level of systematics investigation should one (and the research community) carry out before declaring an upending of the standard cosmological model? To what extent should the proportion of systematics vs new physics papers depend on the degree of new physics required? As the saying goes, if you tell me you saw out the window a deer on the lawn, I might be willing to consider how the deer got there, its effect on the shrubbery, etc., but if you tell me you saw out the window a unicorn on the lawn, I might want more evidence before devoting time to the puzzle.

We have in place solutions for how to handle conflicting expectations, observations, and theories:

- Rigorous data
- Multiple, disparate probes
- Crosschecks
- Consistency at all cosmic times
- Check the cosmic expansion history, cosmic growth history, and light propagation (and soon gravitational wave propagation)

We explore how these might be applied to an example tension in the next section.

III. PAST TENSE, PRESENT TENSE?

A well known tension lies in current Hubble constant $H_0$ values deduced from certain probes, taking all the data at face value. In addition to the main puzzle, there are some beyond the surface:

- Local measurements differ by some $\sim 2\sigma$ depending on method, i.e. Cepheids vs tip of the red giant branch [17, 18].

- The tension is emphatically not “early vs late” cosmology since baryon acoustic oscillations (BAO) distances (together with primordial element abundances [19–21] or marginalizing over or sidestepping the sound horizon at the baryon drag epoch [22, 23]), i.e. without use of the primordial CMB, gives the same answer as from the CMB.

- Strong lensing time delays show a sharp transition between low and high $H_0$ values around $z \sim 0.4$ [24, 25], albeit with a small sample.

While CMB data alone constrains $H_0$ tightly only within a $\Lambda$CDM cosmological model, while allowing a considerable range of $H_0$ when the dark energy equation of state $w$ differs from $-1$, it is extraordinarily difficult from a combination of cosmic probes such as CMB+BAO or CMB+SN (supernovae distances) to obtain $H_0 > 70$ [26] (we always write $H_0$ in units of km/s/Mpc). Basically, $H_0 > 70$ requires a phantom dark energy ($w < -1$), which is disfavored by the above combinations of probes; see Figure 1.

There are two basic loopholes one might try by changing the cosmic expansion history — relax the tension in the present or the past.

- Present tense (late time transition): arrange a very sharp phantom excursion very close to the present so that higher redshift distances are not too strongly affected.

- Past tense (early time transition): arrange a lower sound horizon scale $r_{\text{drag}}$ with the Hubble parameter $H$ going up. Again, one must make it a sharp transition — a spurt of extra early energy density to raise $H$, then removing the early dark energy to preserve the agreement with CMB data.

We begin with the early time transition. The covariance between $r_{\text{drag}}$ and $H_0$ has been known for a long time [11, 27–30]. Using CMB data, in 2013 Ref. [11] actually found evidence for an early time transition and its effect on $H_0$! — see Figure 2. This has been resuscitated in many many articles in the last couple years. However, early time transitions do not really work in removing the Hubble constant tension (see, e.g., [31–34]). On the one hand they cannot viably raise $H_0$ as far as the high values favored by Cepheids, and on the other hand they generally violate other aspects of the cosmological model as we discuss in Section IV.

The late time transition has the advantage that there is much less effect on the CMB, and so more freedom for change. If one raises $H(z)$, distances will change. To preserve distances (e.g. the distance to the CMB last scattering surface, as well as BAO and SN distances), with a higher $H_0$ one needs a smaller $H(z > 0)$. This means less energy density. This could be from a smaller matter density $\Omega_m$ (the matter density today as a fraction of the critical density) but this is insufficient and one requires a smaller dark energy density at some $z > 0$. To give enough dark energy density today (especially if the matter density is low and the total density is the critical density) requires dark energy density to appear quite suddenly at low redshifts. This is again the phantom regime $w < -1$. And again, such late time phase transitions have been known for a long time (see, e.g., vacuum metamorphosis [35–37]).

Since such late time transitions all have basically the same physical effect, regardless of specific origin, let’s examine whether vacuum metamorphosis removes the $H_0$ tension. Yes! but.... As the opening lines of the abstract of [38] say, “We do obtain $H_0 \approx 74$ km/s/Mpc from CMB+BAO+SN data in our model, but that is not the point.” This — and essentially any late time transition — model fails because there is more to the standard
FIG. 1. 68.3% and 95.4% constraints on the $w_0$--$w_a$ plane in an 11 parameter extended space, using Planck CMB data plus the R16 $H_0$ ($\sim 74$) prior. The top panel includes as well JLA supernova data, the middle panel includes baryon acoustic oscillations data: both strongly prefer $w_0 \geq -1$. The bottom panel shows that $w_0 \geq -1$ is most cleanly achieved by shifting the $H_0$ prior to $H_0 \leq 70$. (Adapted from [26].)

FIG. 2. Reconstruction of the expansion history deviations $\delta(a) = \delta H^2 / H_0^2$ from $\Lambda$CDM is shown, with the mean value (solid line) and 68% uncertainty band (shaded area). Note the data prefers early dark energy $\delta \neq 0$. The inset demonstrates that the model independent reconstruction then prefers a higher $H_0$ than the $\Lambda$CDM standard analysis. (Adapted from [11].)

cosmological model than $H_0$; it does not satisfy other probes. We discuss the details in Section IV. For other possible issues with late time transitions see [31, 39–41].

IV. ALL COSMOLOGY, ALL THE TIME

So far we have considered only the expansion history. However, one must take into account all cosmological probes, e.g. how deviations from a standard cosmological model affect the growth of large scale structure.

In the vacuum metamorphosis case of the previous section, the combination of probes CMB+BAO+SN produced $H_0 \approx 74$. For a good fit to the CMB, preserving $\Omega_m h^2$ means a low $\Omega_m \approx 0.27$. That can be ok. However, it also gives a high amplitude for mass fluctuations, $\sigma_8 \approx 0.88$, which is quite high. This is due to the reduced dark energy density needed to get the distances right, quite generally implying greater matter domination and growth at higher redshifts. We can start to see that we do indeed need “all cosmology, all the time” – use of all probes, over the full cosmic history.

One might brush high $\sigma_8$ under the rug and say that with the lower $\Omega_m$, one has $S_8 \equiv \sigma_8 (\Omega_m/0.3)^{0.5} \approx 0.83$, which might be workable for some probes, i.e. roughly as good as $\Lambda$CDM. So we could say that vacuum metamorphosis gives $H_0 \approx 74$ while not making any $S_8$ tension worse, as apparently seen in Figure 3.

However, $S_8$ and $H_0$ are focusing on a single time (the present) in cosmic history. This is a Bad Idea. In the words of Lewis Carroll, “the rule is, jam tomorrow and jam yesterday – but never jam today”. Figure 4 shows
that the apparent removal of the $H_0$ tension is moot, since Figure 3 is only a tiny part of cosmic history, and when one does all cosmology, all the time – taking into account both the cosmic expansion and cosmic growth over the span of cosmic history as in Figure 4 – then generically late time transitions do not work in giving a viable cosmological model.

FIG. 3. Expansion and growth histories are here plotted simultaneously. At $z = 0$, hence these give $H_0$ and $f\sigma_8(0)$ ($\approx 0.52 S_8$ for most viable cosmologies in general relativity; see [38]), shown by the points. Excellent agreement with the Cepheid value of $H_0$ (green dashed line), as opposed to the $\Lambda$CDM value (cyan dashed line) is obtained by the flat vacuum metamorphosis (VM) late transition models with parameters fit to CMB+BAO+SN, and the VM VEV model agrees well on $S_8$ as well. (Adapted from [38].)

This is general because the late time transition, anchored by the present, requires a lower dark energy density at higher redshifts and hence greater growth. If you squeeze the model in one place, it will bulge out elsewhere and fail to fit the array of probes. A similar general “no-go” reasoning holds for early time transitions. The higher expansion rate damps the CMB perturbations, so to preserve the fit to CMB data one requires a higher primordial curvature perturbation amplitude, and this strengthens the later growth of the matter perturbations, leading to high $\sigma_8$. (See also [42].)

V. DISCUSSION

Very generally, neither late time nor early time attempts to remove the $H_0$ tension survive all cosmology, all the time. One needs to take into account all the probes, at all redshifts. It is not just $H_0$, it is $H(z)$. It is not just $\Omega_m$, it is $\Omega_m(z)$, and growth of structure, light propagation, etc.

The standard cosmological model, whether one views it at Level 1, 2, or whatever, is strong: I come to praise it, not to bury it.

But, arguendo, suppose we ignored the questions of Section II E and believed all data blissfully free of systematics, so that the order (two orders?) of magnitude difference in number of papers squeezing the standard cosmological model vs investigating the data is right and proper. What could we do to remove the tension? From the preceding section it is clear: we must break the connection between the cosmic expansion history and growth history. We can do this by a breakdown in general relativity, making gravity weaker so that $\sigma_8$ becomes lower; we can do this by introducing new particle interactions, again to suppress growth. Such changes will in turn alter other cosmological probes, which must be considered, and we must be wary of a spiral of epicycles. (See also [43].)

Basically we would need to break the standard expansion history to address the $H_0$ tension, and need to break the standard growth history to keep the consequences of
the first break consistent with other probes. An interesting possibility is to probe the connection of the growth of structure to the cosmic expansion in a wholly new way. Gravitational wave standard sirens offer this possibility, in part. A direct connection between siren distances and matter growth was developed by [44].

The propagation of gravitational waves is an excellent probe of “spacetime friction” – a combination of the Hubble friction from expansion and any time variation of the gravitational strength; the growth of structure probes both these quantities as well, in a different way. By comparing these two probes, possibly plus light propagation which depends only on the expansion, we have the potential to get a clear view of whether the connections between them in the standard cosmological model hold – and if they do not, is there consistency between the deviations in one probe and in another.

A bit more technically: while distances found through light propagation \(d_{EM}(z) = d_L(z)\) depend on the expansion \(H(z)\), those determined through gravitational wave propagation depend on both \(H(z)\) and \(\alpha_M(z) = \frac{d\ln M^2_{Pl}(a)}{d\ln a}\), the running of the Planck mass or inverse gravitational strength. Thus, a measurement \(d_{GW}(z) \neq d_{EM}(z)\) would signal a deviation from general relativity (or systematics). Meanwhile, growth of cosmic structure depends on \(H(z)\) and the strength of gravity \(G_{\text{eff}}(z)\). In theories where the gravitational strength is determined purely by the Planck mass, \(G_{\text{eff}}(z) = \frac{1}{M^2_{Pl}(z)}\), then the circle is complete and there is a tight relation between the three probes. (Some theories of gravity have a further factor, the braiding that mixes the tensor and scalar parts, loosening the relation; \(G_{\text{eff}}(z)\) can also affect light propagation distances, and hence \(H_0\) from strong lensing, giving a redshift dependence to the derived \(H_0\) [25].) Thus we can crosscheck against systematics by seeing if the relation holds: a deviation in one probe predicts a specific redshift dependence for a deviation in another probe.

The consistency check can be quantified with a new statistic combining the probe measurements [45],

\[
D_C(a) = \frac{\left[ \frac{d_{L,GR}}{d_L} \right]}{\left[ \frac{f_{\sigma^8}^{M\text{G}}}{f_{\sigma^8}^{G\text{R}}} \right]}(a). \tag{3}
\]

For general relativity, this equals one for all redshifts. For a given modified gravity (MG) model, it has a specific redshift dependence predicted. Several examples are shown in Figure 5.

In summary, the standard cosmological model has extremely deep foundations and multiple layers, and apparent surface blemishes may have very little to do with the fundamental basis. Especially if tensions cannot be solved by one “tooth fairy”, such as an early or late time transition, when confronted with application of the principle of using all cosmology, all the time, but we are instead led to a series of epicycles, we might recall the unicorn vs the deer and pause.

New probes and data covering more cosmic history will be essential in exploring the standard cosmological

![FIG. 5. The new \(D_C\) statistic, using the complementarity of the gravitational wave luminosity distance \(d_{L, GW}\) and the cosmic matter growth rate \(f_{\sigma^8}\), can clearly distinguish different classes of gravity. Each class has a distinct shape in its redshift dependence \(D_C(a)\), though the curve amplitudes will scale with \(G_{\text{eff}}(z=0)\). General relativity has constant \(D_C = 1\). (Adapted from [45].)](image)

model, at whatever level we define it, and whether it will stand firm, need some patchwork, or be overturned. Each of the levels has a wonderful array of areas for students to research, and make significant and lasting contributions to what their students will learn as the standard cosmological model.

**ACKNOWLEDGMENTS**

I thank the Asia-Pacific Center for Theoretical Physics for inviting me to give this inaugural lecture in June 2020 for the series “Dark Energy in a Dark Age”. This work is supported in part by the Energetic Cosmos Laboratory and by the U.S. Department of Energy, Office of Science, Office of High Energy Physics, under contract no. DE-AC02-05CH11231.

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