Approximate CP in Supersymmetric Models

Galit Eyal\textsuperscript{a,b} and Yosef Nir\textsuperscript{a}

\textsuperscript{a}Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel
\textsuperscript{b}Department of Physics, Technion – Israel Institute of Technology, Haifa 32000, Israel

We construct phenomenologically viable supersymmetric models where CP is an approximate symmetry. The full high energy theory has exact CP and horizontal symmetries that are spontaneously broken with a naturally induced hierarchy of scales, $\Lambda_{CP} \ll \Lambda_H$. Consequently, the effective low energy theory, that is the supersymmetric Standard Model, has CP broken explicitly but by a small parameter. The $\varepsilon_K$ parameter is accounted for by supersymmetric contributions. The predictions for other CP violating observables are very different from the Standard Model. In particular, CP violating effects in neutral $B$ decays into final CP eigenstates such as $B \rightarrow \psi K_S$ and in $K \rightarrow \pi \nu \bar{\nu}$ decays are very small.
1. Introduction

Within the Standard Model, the following features regarding CP violation hold:

(i) CP is broken explicitly.

(ii) All CP violation arises from a single phase (that is the Kobayashi-Maskawa phase $\delta_{\text{KM}}$).

(iii) The measured value of $\varepsilon_K$ requires that $\delta_{\text{KM}}$ is of order one. (In other words, CP is not an approximate symmetry of the Standard Model.)

(iv) The values of all other CP violating observables can be predicted. In particular, the asymmetry $a_{\psi K_S}$,

$$a_{\psi K_S} \sin(\Delta m_B t) = -\frac{\Gamma(B^0_{\text{phys}}(t) \to \psi K_S) - \Gamma(\bar{B}^0_{\text{phys}}(t) \to \psi K_S)}{\Gamma(B^0_{\text{phys}}(t) \to \psi K_S) + \Gamma(\bar{B}^0_{\text{phys}}(t) \to \psi K_S)}, \quad (1.1)$$

(and similarly various other CP asymmetries in $B$ decays), and the ratio $a_{\pi\nu\bar{\nu}}$,

$$a_{\pi\nu\bar{\nu}} = \frac{\Gamma(K_L \to \pi^0\nu\bar{\nu})}{\Gamma(K^+ \to \pi^+\nu\bar{\nu})}, \quad (1.2)$$

are expected to be of order one.

The commonly repeated statement that CP violation is one of the least tested aspects of the Standard Model is well demonstrated by the fact that none of the above features necessarily holds in the presence of New Physics. Such a dramatic difference from the Standard Model is possible, for example, in the supersymmetric framework. (For a recent review of CP violation in supersymmetry, see [1].) Indeed, in this work, we construct phenomenologically viable supersymmetric models, with the following features:

(i) CP is an exact symmetry of the full theory but is spontaneously broken at some high energy scale by a VEV of a gauge singlet scalar field.

(ii) In the low energy effective theory, there are many independent CP violating phases, in particular in the mixing matrices of gaugino couplings to fermions and sfermions.

(iii) In the low energy effective theory, CP is an approximate symmetry. The Kobayashi-Maskawa phase is too small to account for $\varepsilon_K$ which is explained, instead, by supersymmetric contributions.
The values of all other CP violating observables can be estimated and, in many cases, are drastically different from the Standard Model predictions. In particular, $a_{\psi K_S}$ and $a_{\pi\nu\bar{\nu}}$ are both much smaller than one.

The mechanism that is responsible for the approximate nature of CP is basically the following. The full high energy theory has exact horizontal and CP symmetries. There are three relevant high energy scales: $\Lambda_H$, where the horizontal symmetry is spontaneously broken; $\Lambda_{CP}$, where CP is spontaneously broken; and $\Lambda_F$, where the information about these spontaneous breakings are communicated to the observable sector. There exists a hierarchy between these scales: $\Lambda_{CP} \ll \Lambda_H \ll \Lambda_F$ (the hierarchy $\Lambda_{CP} \ll \Lambda_H$ is naturally produced by the scalar potential), so that in the low energy effective theory, the horizontal symmetry and the CP symmetry appear explicitly broken by small parameters:

$$\epsilon_H \sim \frac{\Lambda_H}{\Lambda_F}, \quad \epsilon_{CP} \sim \frac{\Lambda_{CP}}{\Lambda_F}, \quad \epsilon_{CP} \ll \epsilon_H. \quad (1.3)$$

This mechanism, while predicting phenomenology of CP violation that is very different from the Standard Model, also solves both the flavor and CP problems of supersymmetry.

Our models use the Froggatt-Nielsen mechanism \[2\] to achieve the small breaking parameters. We employ supersymmetric abelian horizontal symmetries similarly to \[3\]. The supersymmetric flavor problems are solved by the alignment mechanism \[4,5\]. As concerns CP violation in the supersymmetric framework, the idea of approximate CP has been discussed in refs. \[6-8\], and spontaneous CP violation has been discussed in refs. \[9-17,6-7\]. Our work is closely related to two of these works. In ref. \[7\], models were constructed with spontaneous CP breaking and approximate CP in the low energy theory. However, while the mechanism of communicating the breaking in ref. \[7\] is aimed to solve the strong CP problem and leads to a single low energy phase, our mechanism is aimed to solve the supersymmetric flavor problems and leads to a large number of low energy phases. Both the breaking mechanism and the communication mechanism are the same as in ref. \[17\]. The main new ingredient in our models is that, while the models of ref. \[17\] have effectively CP breaking parameters of order one, the models presented in this work give small CP breaking and, therefore, a very different phenomenology of CP violation. Moreover, as the supersymmetric CP problem is solved partially by the
approximate nature of CP, the required alignment is much less precise than in existing models. This situation gives more freedom in constructing the models and also allows for some different phenomenological signatures in FCNC processes. (We do not consider the strong CP problem in this work. Note, however, that the alignment models may solve this problem too [18].)

The structure of this paper is as follows. We first present two explicit models of approximate CP, one where the breaking parameter is intermediate, $\mathcal{O}(0.04)$ (section 2) and the other where it is very small, $\mathcal{O}(0.001)$ (section 3). The implications of these models for flavor changing neutral current processes are studied in section 4 and for CP violation in section 5. Section 6 clarifies an interesting point about holomorphic zeros, which bears consequences for rare $K$ decays. Our conclusions are summarized in section 7.

2. Model I

Our first model employs a horizontal symmetry

$$H = U(1)_1 \times U(1)_2. \tag{2.1}$$

The superfields of the supersymmetric standard model (SSM) carry the following $H$-charges:

$$Q_1(2,1), \quad Q_2(3,-1), \quad Q_3(0,0), \quad \bar{d}_1(4,-1), \quad \bar{d}_2(-2,4), \quad \bar{d}_3(1,1)$$

$$\bar{u}_1(5,-1), \quad \bar{u}_2(-2,4), \quad \bar{u}_3(0,0), \quad \phi_d(-1,0), \quad \phi_u(0,0) \tag{2.2}$$

where $Q_i$ are the quark doublets, $\bar{d}_i$ and $\bar{u}_i$ are the down and up quark singlets, and $\phi_i$ are the Higgs doublet fields. In addition, we have three standard model singlet superfields:

$$S_1(-1,0), \quad S_2(0,-1), \quad S_3(-3,-1). \tag{2.3}$$

The horizontal symmetry is spontaneously broken when the three $S_i$ fields assume VEVs. The breaking scale is somewhat lower than a scale $M$ where the information about this breaking is communicated to the SSM, presumably by heavy quarks in vector-like representations of the Standard Model [3]. We will quantify all the small parameters as
powers of a small parameter $\lambda$ which we take to be of $\mathcal{O}(0.2)$. Then, we take for the three VEVs

$$
\epsilon_1 \equiv \frac{\langle S_1 \rangle}{M} \sim \lambda, \quad \epsilon_2 \equiv \frac{\langle S_2 \rangle}{M} \sim \lambda, \quad \epsilon_3 \equiv \frac{\langle S_3 \rangle}{M} \sim \lambda^4.
$$

Note that due to the $U(1)_1 \times U(1)_2$ symmetry, we can always choose $\langle S_1 \rangle$ and $\langle S_2 \rangle$ to be real. However, $\langle S_3 \rangle$ is, in general, complex with a phase of $\mathcal{O}(1)$. Then CP is spontaneously broken by $\epsilon_3$. The hierarchy $\epsilon_3 \ll \epsilon_1, \epsilon_2$ and $\arg(\epsilon_3) = \mathcal{O}(1)$ can be naturally induced, as explained below.

The electroweak symmetry is spontaneously broken by the VEVs of $\phi_d$ and $\phi_u$, and we assume that

$$
\tan \beta \equiv \frac{\langle \phi_u \rangle}{\langle \phi_d \rangle} \sim \frac{1}{\lambda^2}.
$$

The model is defined by the horizontal symmetry, the assigned horizontal charges and the hierarchy of VEVs. For most of our purposes, however, we need not consider the full high energy theory. It is sufficient to analyze the effective low energy theory, which is the SSM supplemented with selection rules that follow from the $H$-breaking pattern:

(a) Terms in the superpotential that carry charge $(m, n)$ under $H$ with $m, n \geq 0$ are suppressed by $\mathcal{O}(\lambda^{m+n})$, while those with $m < 0$ and/or $n < 0$ are forbidden (due to the holomorphy of the superpotential [3]).

(b) Terms in the Kähler potential that carry charge $(m, n)$ under $H$ are suppressed by $\mathcal{O}(\lambda^{|m|+|n|})$.

These selection rules allow us to estimate the various entries in the quark mass matrices $M^q$ and the squark mass-squared matrices $\tilde{M}^{q2}$. For each entry, we write the leading contribution and the subleading contribution if it is complex with respect to the leading one (namely, if it has a different $\epsilon_3$-dependence). We do not write the coefficients of $\mathcal{O}(1)$ which appear in each entry. For the quark mass matrices and for the off-diagonal blocks in the squark mass-squared matrices, we write the effective matrices after the rotations needed to bring the kinetic terms into their canonical form have been taken into account [3]. We get:

$$
M^d \sim \langle \phi_d \rangle \begin{pmatrix}
\epsilon_1^2 & \frac{\epsilon_1^2 \epsilon_2}{\epsilon_1^2 + \epsilon_3^2} & \frac{\epsilon_1^2 \epsilon_2}{\epsilon_1^2 + \epsilon_3^2} \\
\frac{\epsilon_1^2 \epsilon_2}{\epsilon_1^2 + \epsilon_3^2} & \epsilon_1^2 + \epsilon_3^2 & \epsilon_1^2 + \epsilon_3^2 \\
\frac{\epsilon_1^2 \epsilon_2}{\epsilon_1^2 + \epsilon_3^2} & \frac{\epsilon_1^2 \epsilon_2}{\epsilon_1^2 + \epsilon_3^2} & \epsilon_1^2 + \epsilon_3^2
\end{pmatrix},
$$

(2.6)
We can also estimate the size of the bilinear $\mu$ and $B$ terms:

\[ \mu \sim \tilde{m}(\epsilon_1 + \epsilon_3^* \epsilon_1^2 \epsilon_2), \]

\[ m_{12}^2 \sim \tilde{m}^2(\epsilon_1 + \epsilon_3^* \epsilon_1^2 \epsilon_2). \]  

(2.12)

Thus the horizontal symmetry solves the $\mu$-problem in the way suggested in ref. [19].

From the mass matrices, we can further estimate the mixing angles in the CKM matrix and in the gaugino couplings to quarks and squarks. We denote the latter by $K^q_M$ where, for example, $K^d_L$ is the mixing matrix that describes the gluino couplings to left-handed down quarks and ‘left-handed’ down squarks. (The LR mixing angles are very small and we do not present them explicitly.) We write the estimates in terms of powers of $\lambda$. For the CKM matrix, we find

\[ |V_{us}| \sim \lambda, \quad |V_{ub}| \sim \lambda^3, \quad |V_{cb}| \sim \lambda^2, \]  

(2.13)

as required by direct measurements, and

\[ \delta_{KM} \sim \lambda^2. \]  

(2.14)

For the gaugino couplings we find

\[ (K^d_L)_{12} \sim \lambda^3 e^{i\lambda^4}, \quad (K^d_L)_{13} \sim \lambda^3 e^{i\lambda^2}, \quad (K^d_L)_{23} \sim \lambda^2 e^{i\lambda^4}, \]  

(2.15)
\[(K_L^u)_{12} \sim \lambda, \quad (K_L^u)_{13} \sim \lambda^3 e^{i\lambda^2}, \quad (K_L^u)_{23} \sim \lambda^4 e^{i\lambda^2}, \quad (2.16)\]

\[(K_R^d)_{12} \sim \lambda^5 e^{i\lambda^2}, \quad (K_R^d)_{13} \sim \lambda^5 e^{i\lambda^2}, \quad (K_R^d)_{23} \sim \lambda^4 e^{i\lambda^4}, \quad (2.17)\]

\[(K_R^u)_{12} \sim \lambda^4 e^{i\lambda^4}, \quad (K_R^u)_{13} \sim \lambda^6 e^{i\lambda^4}, \quad (K_R^u)_{23} \sim \lambda^6 e^{i\lambda^4}. \quad (2.18)\]

Note that in (2.13)-(2.18) we omit coefficients of order one not only in the overall magnitude but also in the phases.

Finally, we can estimate the relevant supersymmetric CP violating phases \[\phi_B \equiv \arg(m^2_{12}/\mu) \sim \lambda^6, \quad (2.19)\]

\[\phi^u_A \equiv \arg \left( \frac{[V_{uL} M^u_{V_{uR}^\dagger}]_{11}}{[V_{uL} M^u_{V_{uL}^\dagger}]_{11}} \right) \sim \lambda^4, \quad (2.20)\]

while the corresponding \(\phi^d_A\) is negligible.

Before concluding this section, we would like to show how a complex \(\langle S_3 \rangle\) which is hierarchically smaller than \(\langle S_1 \rangle\) and \(\langle S_2 \rangle\) can be achieved naturally. Let us add yet another Standard Model singlet field \(S_4(6,2)\). The \(S_i\) dependent terms in the superpotential are

\[W(S_i) \sim \frac{a}{M^6} S_4 S_1^6 e_2^2 + \frac{b}{M^3} S_4 S_1^3 S_2 S_3 + c S_4 S_3^2, \quad (2.21)\]

where \(a, b, c\) are dimensionless numbers of \(O(1)\). For \(\langle S_4 \rangle = 0\) we have \(F_{S_1} = F_{S_2} = F_{S_4} = 0\), while \(F_{S_4} = 0\) requires

\[a e_1^6 e_2^2 + b e_1^3 e_2 e_3 + c e_3^2 = 0 \implies \frac{e_3}{e_1^3 e_2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2c}. \quad (2.22)\]

We see that indeed \(|\epsilon_3| \sim |\epsilon_1^3 \epsilon_2| \sim \lambda^4\) and that for \(b^2 - 4ac < 0\), \(\epsilon_3\) is complex. (This mechanism for spontaneously breaking CP was first suggested in ref. [3].)

3. Model II

Our second model employs a horizontal symmetry

\[H = U(1)_1 \times U(1)_2 \times U(1)_3. \quad (3.1)\]
The SSM superfields carry the following $H$-charges:

$$
Q_1(2, 0, 1) \quad Q_2(0, 0, 2) \quad Q_3(0, 0, 0) \quad \tilde{d}_1(0, 5, 0) \quad \tilde{d}_2(1, 5, -2) \quad \tilde{d}_3(4, 0, 0)
$$

$$
\tilde{u}_1(-2, 6, 0) \quad \tilde{u}_2(1, 1, 0) \quad \tilde{u}_3(0, 0, 0) \quad \phi_d(-1, 0, 0) \quad \phi_u(0, 0, 0).
$$

We have four standard model gauge singlet fields:

$$
S_1(-1, 0, 0), \quad S_2(0, -1, 0), \quad S_3(0, 0, -1), \quad S_4(0, 0, -4).
$$

The orders of magnitude of the various $S$-VEVs are

$$
\epsilon_1 \equiv \frac{\langle S_1 \rangle}{M} \sim \lambda, \quad \epsilon_2 \equiv \frac{\langle S_2 \rangle}{M} \sim \lambda, \quad \epsilon_3 \equiv \frac{\langle S_3 \rangle}{M} \sim \lambda, \quad \epsilon_4 \equiv \frac{\langle S_4 \rangle}{M} \sim \lambda^4.
$$

Due to the $U(1)_1 \times U(1)_2 \times U(1)_3$ symmetry, we can always choose $\langle S_1 \rangle$, $\langle S_2 \rangle$, $\langle S_3 \rangle$ real, but $\langle S_4 \rangle$ is, in general, complex with a phase of $O(1)$. For the electroweak breaking VEVs, we take

$$
\tan \beta \sim 1.
$$

For the various quark and squark mass matrices, we get:

$$
M^d \sim \langle \phi_d \rangle \left( \begin{array}{ccc}
\epsilon_1 \epsilon_2^5 \epsilon_3 & \epsilon_2^2 \epsilon_3 (1 + \epsilon_4) & \epsilon_3^5 (1 + \epsilon_4 
\epsilon_1 \epsilon_2 \epsilon_3^2 (1 + \epsilon_4) & \epsilon_2^2 \epsilon_3 (1 + \epsilon_4) & \epsilon_3^5 (1 + \epsilon_4 
M_3^d & \epsilon_2^2 \epsilon_3 (1 + \epsilon_4) & \epsilon_3^5 (1 + \epsilon_4)
\end{array} \right),
$$

where $M_3^d \sim \epsilon_1 \epsilon_2^5 \epsilon_3^2 (\epsilon_1^2 + \epsilon_2^2 (1 + \epsilon_4 + \epsilon_4^*))$,

$$
M^u \sim \langle \phi_u \rangle \left( \begin{array}{ccc}
\epsilon_1 \epsilon_2^6 \epsilon_3 & \epsilon_1 \epsilon_2^3 (1 + \epsilon_4) & \epsilon_3 (1 + \epsilon_4 
\epsilon_1 \epsilon_2 \epsilon_3^2 (1 + \epsilon_4) & \epsilon_2^2 \epsilon_3 (1 + \epsilon_4) & \epsilon_3 (1 + \epsilon_4 
M_3^u & \epsilon_2 \epsilon_3 (1 + \epsilon_4) & \epsilon_3 (1 + \epsilon_4)
\end{array} \right),
$$

$$
\tilde{M}_{LL}^q \sim \tilde{m}^2 \left( \begin{array}{ccc}
1 & \epsilon_3^2 (1 + \epsilon_4) & \epsilon_3^2 (1 + \epsilon_4 
\epsilon_2 \epsilon_3 (1 + \epsilon_4) & 1 & \epsilon_3 (1 + \epsilon_4 
\epsilon_2 \epsilon_3 (1 + \epsilon_4) & \epsilon_3 (1 + \epsilon_4) & 1
\end{array} \right),
$$

$$
\tilde{M}_{RR}^q \sim \tilde{m}^2 \left( \begin{array}{ccc}
1 & \epsilon_3^2 (1 + \epsilon_4) & \epsilon_3^2 (1 + \epsilon_4 
\epsilon_2 \epsilon_3 (1 + \epsilon_4) & 1 & \epsilon_3 (1 + \epsilon_4 
\epsilon_2 \epsilon_3 (1 + \epsilon_4) & \epsilon_3 (1 + \epsilon_4) & 1
\end{array} \right),
$$

$$
\tilde{M}_{LR}^q \sim \tilde{m}^2 \left( \begin{array}{ccc}
1 & \epsilon_3 \epsilon_2^2 & \epsilon_3 \epsilon_2^2 
\epsilon_2 \epsilon_3 & 1 & \epsilon_2 \epsilon_3 
\epsilon_2 \epsilon_3 & \epsilon_2 \epsilon_3 & 1
\end{array} \right),
$$

$$(\tilde{M}_{LR}^q)_{ij} \sim \tilde{m}(M^q)_{ij}.$$
For the bilinear terms, we find

\[ \mu \sim \tilde{m}_1 \epsilon_1 (1 + \epsilon_4 (\epsilon_4 + \epsilon_4^*)), \]
\[ m_{12}^2 \sim \tilde{m}^2 \epsilon_1 (1 + \epsilon_3^2 (\epsilon_4 + \epsilon_4^*)). \]

For the CKM matrix, we find again magnitudes consistent with the measurements (namely, the same orders of magnitude as in (2.13)) but the KM phase is smaller:

\[ \delta_{KM} \sim \lambda^4. \]

For the gaugino couplings we find

\[ (K_L^d)_{12} \sim \lambda^3 e^{i\lambda^6}, \quad (K_L^d)_{13} \sim \lambda^3 e^{i\lambda^4}, \quad (K_L^d)_{23} \sim \lambda^2 e^{i\lambda^4}, \]
\[ (K_L^u)_{12} \sim \lambda e^{i\lambda^4}, \quad (K_L^u)_{13} \sim \lambda^3 e^{i\lambda^4}, \quad (K_L^u)_{23} \sim \lambda^2 e^{i\lambda^4}, \]
\[ (K_R^d)_{12} \sim \lambda^3 e^{i\lambda^4}, \quad (K_R^d)_{13} \sim \lambda^7 e^{i\lambda^4}, \quad (K_R^d)_{23} \sim \lambda^4 e^{i\lambda^4}, \]
\[ (K_R^u)_{12} \sim \lambda^4 e^{i\lambda^6}, \quad (K_R^u)_{13} \sim \lambda^6 e^{i\lambda^6}, \quad (K_R^u)_{23} \sim \lambda^2. \]

The supersymmetric CP violating phases are:

\[ \phi_B \sim \lambda^8, \]
\[ \phi_A^u \sim \lambda^6, \]

while \( \phi_A^d \) is negligible.

The required hierarchy between the \( H \) and CP breaking scales is achieved by minimizing the Higgs potential for the four \( S_i \) of eq. (3.3) and a fifth singlet field \( S_5(0, 0, 8) \). This would give \( \epsilon_4 = \mathcal{O}(\epsilon_3^4) \) and complex.

4. Flavor Changing Neutral Current Processes

Generic supersymmetric models, with mass-squared differences between generations of \( \mathcal{O}(\tilde{m}^2) \) (\( \tilde{m} \) is the supersymmetry breaking scale) and supersymmetric mixing angles of \( \mathcal{O}(1) \) give much too large contributions to various flavor changing neutral current (FCNC) processes such as \( \Delta m_K, \Delta m_D \) and \( \Delta m_B \). There are various solutions to this problem:
a. All squark generations are equal at some high energy scale. This is the situation, for example, in models of gauge mediated supersymmetry breaking.

b. The first two squark generations are degenerate due to a non-Abelian horizontal symmetry.

c. The first two squark generations are very heavy.

d. Squarks are neither degenerate nor very heavy, but the mixing angles in the gaugino couplings to quarks and squarks are small.

The last option arises naturally in models of Abelian horizontal symmetries of the type that we used in constructing our models. Indeed, one can easily see from eqs. (2.15)–(2.18) and (3.14)–(3.17) that there is no mixing angle of order one in our models; they are all suppressed by the selection rules of the horizontal symmetries. To understand whether the alignment in our models is precise enough to satisfy the phenomenological constraints and, in the case that it is, whether the supersymmetric contributions are significant in comparison to the Standard Model ones, we write down the constraints on the mixing angles (taken from ref. [22]) in terms of powers of $\lambda$ and then compare to the predictions of our two models. This is done in Table 1. (We define $\langle K_{ij} \rangle \equiv [(K_L)_{ij}(K_R)_{ij}]^{1/2}$.)

| Mixing Angle | Process | Bound | Model I | Model II |
|--------------|---------|-------|---------|----------|
| $(K^d_L)_{12}$ | $\Delta m_K$ | $\lambda - \lambda^2$ | $\lambda^3$ | $\lambda^3$ |
| $(K^d_R)_{12}$ | $\Delta m_K$ | $\lambda - \lambda^2$ | $\lambda^5$ | $\lambda^3$ |
| $\langle K^d_{12} \rangle$ | $\Delta m_K$ | $\lambda^3$ | $\lambda^4$ | $\lambda^3$ |
| $(K^d_L)_{13}$ | $\Delta m_B$ | $\lambda$ | $\lambda^3$ | $\lambda^3$ |
| $(K^d_R)_{13}$ | $\Delta m_B$ | $\lambda$ | $\lambda^5$ | $\lambda^7$ |
| $\langle K^d_{13} \rangle$ | $\Delta m_B$ | $\lambda^2$ | $\lambda^4$ | $\lambda^5$ |
| $(K^u_L)_{12}$ | $\Delta m_D$ | $\lambda$ | $\lambda$ | $\lambda$ |
| $(K^u_R)_{12}$ | $\Delta m_D$ | $\lambda$ | $\lambda^4$ | $\lambda^4$ |
| $\langle K^u_{12} \rangle$ | $\Delta m_D$ | $\lambda^2$ | $\lambda^{5/2}$ | $\lambda^{5/2}$ |

Table 1. Supersymmetric mixing angles in our models and the phenomenological bounds on them.

We learn the following points from the Table:
(i) The contributions to $\Delta m_D$ that are proportional to $[(K_L^u)_{12}]^2$ saturate the experimental upper bound in both models. This is a generic feature of models of alignment \[4,5\], related to the fact that in these models the Cabibbo mixing ($|V_{us}| \sim \lambda$) comes from the up sector.

(ii) The contributions to $\Delta m_B$ are very small. In all alignment models, the standard model amplitudes dominate \[5\]. But while the supersymmetric contributions could be generically of $O(20\%)$, the models constructed here provide an example where these contributions are below the percent level.

(iii) The contributions to $\Delta m_K$ are of $O(10\%)$ in model I and saturate the experimental value for model II. This is in contrast to all previous models of alignment where, to satisfy the $\varepsilon_K$ constraint, the supersymmetric contributions to $\Delta m_K$ were negligibly small. The large contribution comes in the two models from $(K_L^d)_{12}(K_R^d)_{12}$.

Before proceeding, we would like to make two comments:

a. The contributions to other FCNC processes, such as $\Delta m_{B_s}$ and $b \rightarrow s\gamma$, are very small. The $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay is discussed separately below.

b. The contributions from the (LR) blocks in the squark mass-squared matrices are much smaller than those coming from the mixing angles presented in Table 1. This is the reason why, even though we calculated them explicitly, we do not present them in Table 1.

As concerns the rare $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay, the largest supersymmetric contribution in alignment models comes from $(K_L^d)_{12}$, if it is as large as allowed by the $\Delta m_K$ constraint, $(K_L^d)_{12} \sim \lambda^2$ \[23\]. In such a case, the supersymmetric contributions from penguin diagrams with chargino and $\tilde{u}, \tilde{c}$ squarks are significant. In both our models, $(K_L^d)_{12} \sim \lambda^3$, leading to supersymmetric contributions of $O(10\%)$. While both the standard model and the supersymmetric amplitudes are real to a good approximation, so that there is maximal interference between the two, the relative sign is unknown so that the rate could be either enhanced or suppressed compared to the standard model.

It is interesting that we are unable to construct a model where either $(K_L^d)_{12}$ or $(K_R^d)_{12}$ are as large as allowed, namely $O(\lambda^2)$. This situation goes beyond the two specific models that we present here and seems generic to models with continuous Abelian horizontal
symmetries. The reason for that is explained in section 7. The situation is different in models of discrete Abelian symmetries. We actually constructed a model with a $Z_4 \times Z_9$ horizontal symmetry where $(K^d_L)_{12} \sim \lambda^2$. The model is, however, quite complicated and its Higgs potential does not provide in a natural way the hierarchy between $\Lambda_H$ and $\Lambda_{CP}$ (this seems a rather generic feature of models with discrete Abelian symmetries), which is the reason that we do not present it here.

The contribution to $K^+ \to \pi^+ \nu \bar{\nu}$ from the (LR) sector is negligibly small. For the (LR) contributions to be significant, we need the off-diagonal terms in $\tilde{M}^q_{LR}$ to be of $O(m_t \tilde{m})$ \cite{24}, while in our models they are much smaller than that, as can be seen from eqs. (2.11) and (3.11).

5. CP Violation

Each of the two models that we have constructed has an approximate CP symmetry for the SSM. In model I, all CP violating phases are $\leq O(\lambda^2)$ and in model II, they are $\leq O(\lambda^4)$. The resulting predictions for CP violating observables are then drastically different from the Standard Model.

The first thing to note is that with $\delta_{KM} \sim \lambda^2$ or $\lambda^4$, it is impossible to account for $\varepsilon_K \sim 10^{-3}$ by the Standard Model contributions. However, in both models,

$$
\Im[(K^d_L)_{12}(K^d_R)_{12}] \sim \lambda^{10},
$$

which can account for $\varepsilon_K$ from the supersymmetric gluino-mediated diagrams.

The most dramatic consequences of the approximate CP symmetry concern the CP violating asymmetries that are expected to be large in the standard model. First, let us consider CP asymmetries in neutral $B$ decays into final CP eigenstates. For the sake of definiteness, we consider $a_{\psi K_S}$. The supersymmetric contributions to the $B - \bar{B}$ mixing amplitude are, as mentioned above, negligible. Usually this leads to the conclusion that the standard model predictions for $a_{\psi K_S}$ remain valid. But this is definitely not the case in our framework. The fact that $\delta_{KM}$ is very small means that so will be $a_{\psi K_S}$. Explicitly,

$$
a_{\psi K_S}^{\text{SM}} = \begin{cases} 
O(\lambda^2) & \text{Model I}, \\
O(\lambda^4) & \text{Model II}.
\end{cases}
$$
If $a_{\psi K_S}$ is measured to be in the Standard Model range, our models of approximate CP will be excluded.

Concerning $a_{\pi\nu\bar{\nu}}$, it was shown that $a_{\pi\nu\bar{\nu}} = \sin^2 \theta_K$, where $\theta_K$ is the relative phase between the $K - \bar{K}$ mixing amplitude and the $s \to d\nu\bar{\nu}$ decay amplitude [23]. In our models, the standard model contributions dominate both the real part and the imaginary part of the decay amplitude. In model I, the standard model also dominates the real part of the mixing amplitude, while the supersymmetric contribution dominates the imaginary part. In model II, the two contributions to the real part are comparable, but supersymmetric diagrams dominate the imaginary part of the mixing amplitude. In either case, the approximate CP symmetry leads to a strong suppression of $a_{\pi\nu\bar{\nu}}$:

$$a_{\pi\nu\bar{\nu}} \approx \begin{cases} \mathcal{O}(\lambda^4) & \text{Model I,} \\ \mathcal{O}(\lambda^8) & \text{Model II.} \end{cases}$$

We learn that if $a_{\pi\nu\bar{\nu}}$ is measured in the foreseeable future, our models of approximate CP will be excluded.

CP could play an interesting role in $D - \bar{D}$ mixing [26,27]. In measuring the time-dependent decay rate for $D^0 \to K^+\pi^-$ and $\bar{D}^0 \to K^-\pi^+$, a term proportional to $te^{-\Gamma t}$ that is different between the two CP-conjugate modes will appear if the relative phase between the $D - \bar{D}$ mixing amplitude and the $c \to u\bar{s}d$ decay amplitude, $\theta_D = \arg \lambda_{D^0 \to K^+\pi^-}$, is large. This is the case in previous alignment models. However, in our models, the relevant phase, that is $\arg[(K^0_L)_{12}^2]$, is very small ($\lesssim \mathcal{O}(\lambda^4)$), so the effect is probably unobservable. If $D - \bar{D}$ mixing is not observed within, say, one order of magnitude of the present experimental bound, then the existing alignment models are excluded. But if such mixing is observed and with large CP violation, then the alignment mechanism remains viable but not in combination with approximate CP.

Finally, we discuss the electric dipole moment of the neutron $d_N$. It was argued in ref. [1] that in supersymmetric models without universality, namely when there is no super-GIM mechanism, there is a generic lower bound on the CP violating phases that contribute to $d_N$. This bound is of $\mathcal{O}(\lambda^6)$ and leads to $d_N \sim 10^{-28} \text{ e cm}$. This bound is about three orders of magnitude above the value in supersymmetric models with universality and may be within the reach of forthcoming experiments. Indeed, our models obey this bound and
predict a potentially observable $d_N$.

Our results concerning CP violation are summarized in Table 2. Note that, within the Standard Model, $a_{\pi\nu\bar{\nu}} = \mathcal{O}(\sin^2 \beta)$, which parametrically is of $\mathcal{O}(1)$, but turns out to be numerically of $\mathcal{O}(\lambda)$ \cite{28}. $d_n$ is given in units of $10^{-23} e$ cm, so that the present experimental bound is $d_N \lesssim \lambda^2$.

| Process | SM | Model I | Model II |
|---------|----|---------|----------|
| $a_{\psi K_S}$ | $\mathcal{O}(1)$ | $\mathcal{O}(\lambda^2)$ | $\mathcal{O}(\lambda^4)$ |
| $a_{\pi\nu\bar{\nu}}$ | 1 | $\mathcal{O}(\lambda^4)$ | $\mathcal{O}(\lambda^8)$ |
| $\theta_D$ | 0 | $\ll \mathcal{O}(\lambda^4)$ | $\mathcal{O}(\lambda^4)$ |
| $d_N$ | 0 | $\mathcal{O}(\lambda^4)$ | $\mathcal{O}(\lambda^6)$ |

Table 2. CP violating observables in the SM and in our models.

6. Lifting Holomorphic Zeros

The Yukawa couplings, being part of the superpotential, are holomorphic in the $H$-breaking parameters. In particular, if all breaking parameters carry charges of the same sign under one of the horizontal $U(1)$’s, then an entry in the Yukawa matrix that breaks $H$ by a charge of the same sign vanishes \cite{4}. We call these vanishing Yukawa couplings ‘holomorphic zeros’. However, in the basis where these holomorphic zeros are exact, the kinetic terms are not canonical. When we normalize them back to a canonical form, the holomorphic zeros are lifted \cite{4,28}.

Knowing the CKM mixing angles and the quark mass ratios allows us to guess a ‘naive value’ for each entry in the Yukawa matrices \cite{3}. These are

$$Y^d \sim \lambda^3 \tan \beta \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}, \quad Y^u \sim \begin{pmatrix} \lambda^7 & \lambda^5 & \lambda^3 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \quad (6.1)$$

(where we have taken the high energy values $m_b/m_t \sim \lambda^3$ and $m_c/m_t \sim \lambda^4$). When one of these entries vanishes because of holomorphy, the rotation to bring the kinetic terms
back to the canonical normalization cannot lift this zero to its naive value \[5\]. In fact, in our model, the entry in the effective Yukawa matrix (that is, in the basis where the kinetic terms are canonically normalized) is suppressed by at least $\lambda^2$ compared to the naive value. Below we prove this statement.

Before we provide the proof, we would like to point out a phenomenological consequence of this situation. The naive value $Y_{12}^d/Y_{22}^d \sim \sin \theta_C$ is too large for the $\Delta m_K$ constraint (if there is no squark degeneracy). The way that alignment models solve this problem is by assuming that $Y_{12}^d$ is a holomorphic zero. The $\Delta m_K$ constraint allows $Y_{12}^d/Y_{22}^d \sim \lambda^2$. If this bound were saturated, then the supersymmetric contribution to the $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay could be significant \[23\]. What we learn, however, is that the maximal value that we can obtain for this ratio in our framework is $Y_{12}^d/Y_{22}^d \sim \lambda^3$ (which is, indeed, realized in both models presented above). Therefore, in alignment models that employ continuous Abelian horizontal symmetries with small breaking parameters that are $\leq \mathcal{O}(\lambda)$, the modification to the Standard Model prediction for $K^+ \rightarrow \pi^+\nu\bar{\nu}$ is never large.

To prove that a lifted holomorphic zero is suppressed by at least the square of the breaking parameters, let us consider a horizontal symmetry $H = U(1)_x \times U(1)_y$ broken by small parameters of order

$$\epsilon_x (-1, 0) \sim \lambda^{n_x}, \quad \epsilon_y (0, -1) \sim \lambda^{n_y}, \quad (6.2)$$

with two down-quark generations,

$$Q_1(a_x, a_y), \quad \bar{d}_1(b_x, b_y), \quad Q_2(c_x, c_y), \quad \bar{d}_2(d_x, d_y). \quad (6.3)$$

The kinetic terms for the $(Q_1, Q_2)$ fields have coefficients of order

$$\left( \lambda^{n_x |a_x - c_x| + n_y |a_y - c_y|} \right), \quad (6.4)$$

while those of the $(\bar{d}_1, \bar{d}_2)$ fields will be

$$\left( \lambda^{n_x |b_x - d_x| + n_y |b_y - d_y|} \right). \quad (6.5)$$

We further assume, without loss of generality, that the Yukawa coupling $Y_{12}^d$, which breaks $H$ by charge $(a_x + d_x, a_y + d_y)$, is a holomorphic zero because $a_x + d_x < 0$, while the other
entries in $Y^d$ do not vanish:

$$Y^d \sim \begin{pmatrix} 
\lambda^{n_x(a_x+b_x)+n_y(a_y+b_y)} & 0 \\
\lambda^{n_x(c_x+b_x)+n_y(c_y+b_y)} & \lambda^{n_x(c_x+d_x)+n_y(c_y+d_y)}
\end{pmatrix}. \quad (6.6)$$

A straightforward calculation shows then that the effective $Y^d_{12}$ is

$$
(Y^d_{12})_{\text{eff}} = A + B + C,
$$

\begin{align*}
A &= \lambda^{n_x(a_x+b_x)+n_y(a_y+b_y)+n_x|b_x-d_x|+n_y|b_y-d_y|} \\
B &= \lambda^{n_x(c_x+d_x)+n_y(c_y+d_y)+n_x|a_x-c_x|+n_y|a_y-c_y|} \\
C &= \lambda^{n_x(b_x+c_x)+n_y(b_y+c_y)+n_x(|a_x-c_x|+|b_x-d_x|)+n_y(|a_y-c_y|+|b_y-d_y|)}.
\end{align*} \quad (6.7)

This should be compared to the ‘naive’ value, which is

$$
(Y^d_{12})_{\text{naive}} \sim \lambda^{n_x(a_x+d_x)+n_y(a_y+d_y)}. \quad (6.8)
$$

Let us first examine $A$ of eq. (6.7). We assume that $d_y \geq b_y$ (otherwise the suppression is even stronger). From $a_x + b_x \geq 0$ and $a_x + d_x < 0$, we conclude that $b_x > d_x$. We learn that

$$\frac{A}{(Y^d_{12})_{\text{naive}}} \sim \lambda^{2n_x(b_x-d_x)} \leq \lambda^{2n_x}. \quad (6.9)$$

Second, we examine $B$ of eq. (6.7). We assume that $a_y \geq c_y$. From $c_x + d_x \geq 0$, we conclude that $c_x > a_x$. We learn that

$$\frac{B}{(Y^d_{12})_{\text{naive}}} \sim \lambda^{2n_x(c_x-a_x)} \leq \lambda^{2n_x}. \quad (6.10)$$

As concerns $C$, it is easy to see that it is smaller than $A$ and $B$.

The final conclusion is then as follows. Suppose that a holomorphic zero is induced because the Yukawa coupling carries a negative charge under a symmetry $U(1)_x$ that is broken by a small parameter $\epsilon_x \sim \lambda^{n_x}$. Then, the effective Yukawa coupling, that is the coupling in the basis where the kinetic terms are canonically normalized, obeys

$$\frac{(Y^q_{ij})_{\text{eff}}}{(Y^q_{ij})_{\text{naive}}} \lesssim \lambda^{2n_x}. \quad (6.11)$$

Since in all our models $n_x \geq 1$, the suppression is at least by $O(\lambda^2)$.
7. Conclusions

Supersymmetry allows for CP violating mechanisms that are dramatically different from the Standard Model. In particular, CP could be an approximate symmetry, with all CP violating phases very small,

\[10^{-3} \lesssim \phi_{CP} \ll 1.\]

In this work, we gave two examples of phenomenologically viable models where CP is broken by parameters of order 0.04 or 0.001.

The two models that we presented here are not unique. We use them to demonstrate how approximate CP can arise naturally and to explore the phenomenological signatures of approximate CP. The specific models should only be thought of as examples, but the underlying CP breaking mechanism and the resulting phenomenological implications are generic to this class of models.

The fact that the Standard Model and the models of approximate CP are both viable at present is related to the fact that the mechanism of CP violation has not really been tested experimentally. The only measured CP violating observables, that is \(\varepsilon_K\), is small. Its smallness could be related to the ‘accidental’ smallness of CP violation for the first two quark generations, as is the case in the Standard Model, or to CP being an approximate symmetry, as is the case in the models discussed here. Future measurements, particularly of processes where the third generation plays a dominant role (such as \(a_{\psi K_S}\) or \(a_{\pi\nu\bar{\nu}}\)), will easily distinguish between the two scenarios. While the Standard Model predicts large CP violating effects for these processes, approximate CP would suppress them too.

The distinction between the Standard Model and Supersymmetry could also be made – though less easily – in measurements of CP violation in neutral \(D\) decays and of the electric dipole moments of the neutron. Here, the GIM mechanism of the Standard Model is so efficient that CP violating effects are unobservable in both cases. In contrast, the flavor breaking in supersymmetry might be much stronger, and then the approximate CP somewhat suppresses the effects but to a level which is perhaps still observable.

Finally, we note that the predictions for FCNC processes are modified even for those processes where the supersymmetric contribution is negligible. The reason is that the con-
straints on the CKM parameters are modified. Instead of the $\varepsilon_K$ constraint, which is not relevant for the CKM parameters, we have $\eta \approx 0$. The resulting modifications were recently analyzed in ref. [30].

Acknowledgements

Y.N. is supported in part by the United States – Israel Binational Science Foundation (BSF), by the Israel Science Foundation, and by the Minerva Foundation (Munich).
References

[1] Y. Grossman, Y. Nir and R. Rattazzi, [hep-ph/9701231], to appear in *Heavy Flavours II*, eds. A.J. Buras and M. Lindner (World Scientific).
[2] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B147 (1979) 277.
[3] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B398 (1993) 319, [hep-ph/9212278].
[4] Y. Nir and N. Seiberg, Phys. Lett. B309 (1993) 337, [hep-ph/9304307].
[5] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B420 (1994) 468, [hep-ph/9310320].
[6] A. Pomarol, Phys. Rev. D47 (1993) 273, [hep-ph/9208205].
[7] K.S. Babu and S.M. Barr, Phys. Rev. Lett. 72 (1994) 2831, [hep-ph/9309249].
[8] S.A. Abel and J.M. Frere, Phys. Rev. D55 (1997) 1623, [hep-ph/9608251].
[9] A. Dannenberg, L. Hall and L. Randall, Nucl. Phys. B271 (1986) 574.
[10] S.M. Barr and A. Masiero, Phys. Rev. D38 (1988) 366.
[11] N. Maekawa, Phys. Lett. B282 (1992) 387.
[12] A. Pomarol, Phys. Lett. B287 (1992) 331, [hep-ph/9205247].
[13] S.M. Barr and G. Segre, Phys. Rev. D48 (1993) 302.
[14] K.S. Babu and S.M. Barr, Phys. Rev. D49 (1994) R2156, [hep-ph/9308217].
[15] M. Masip and A. Rasin, Phys. Rev. D52 (1995) 3768, [hep-ph/9506471].
[16] M. Masip and A. Rasin, Nucl. Phys. B460 (1996) 449, [hep-ph/9508365].
[17] Y. Nir and R. Rattazzi, Phys. Lett. B382 (1996) 363, [hep-ph/9603233].
[18] S.M. Barr, Phys. Rev. D56 (1997) 5761, [hep-ph/9705263].
[19] Y. Nir, Phys. Lett. B354 (1995) 107, [hep-ph/9504312].
[20] M. Dugan, B. Grinstein and L. Hall, Nucl. Phys. B255 (1985) 413.
[21] S. Dimopoulos and S. Thomas, Nucl. Phys. B465 (1996) 23, [hep-ph/9510220].
[22] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B477 (1996) 321, [hep-ph/9604387].
[23] Y. Nir and M.P. Worah, [hep-ph/9711215].
[24] A.J. Buras, A. Romanino and L. Silvestrini, [hep-ph/9712398].
[25] Y. Grossman and Y. Nir, Phys. Lett. B398 (1997) 163.
[26] G. Blaylock, A. Seiden and Y. Nir, Phys. Lett. B355 (1995) 555, [hep-ph/9504306].
[27] L. Wolfenstein, Phys. Rev. Lett. 75 (1995) 2460, [hep-ph/9505283].
[28] G. Buchalla and A. Buras, Phys. Rev. D54 (1996) 6782, [hep-ph/9607447].
[29] E. Dudas, S. Pokorski and C.A. Savoy, Phys. Lett. B356 (1995) 45, [hep-ph/9504292].
[30] R. Barbieri, L. Hall, A. Stocchi and N. Weiner, [hep-ph/9712252].