Correspondence principle for a brane in Minkowski space and vector mesons

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We consider a 3-brane of positive cosmological constant (de Sitter) in $D$-dimensional Minkowski space. We show that the Poincaré algebra in the bulk yields a $SO(4,2)$ algebra when restricted to the brane. In the limit of zero cosmological constant (flat brane), this algebra turns into the conformal algebra on the brane. We derive a correspondence principle for Minkowski space analogous to the AdS/CFT correspondence. We discuss explicitly the cases of scalar and gravitational fields. For a 3-brane of finite thickness in the transverse directions, we obtain a spectrum for vector gravitational perturbations which correspond to vector mesons. The spectrum agrees with the one obtained in truncated AdS space by de Teramond and Brodsky provided $D = 10$ and the bulk mass scale $M$ is of order the geometric mean of the Planck mass $(\hat{M})$ on the brane and $\Lambda_{QCD} (M \sim (\Lambda_{QCD})^{1/2} \sim 10^{9} \text{ GeV})$.

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I. INTRODUCTION

The AdS/CFT correspondence is by now well-established, even though it has yet to be proved. It has led to novel approaches to gravity and an understanding of the hierarchy problem, raising the possibility of observing strong gravitational effects at TeV scales. Moreover, it has served as a useful tool in analyzing Yang-Mills theories non-perturbatively. Although the CFT is not phenomenologically relevant, significant progress was recently made toward extracting information related to experiments, following the work of Polchinski and Strassler. They showed that by introducing an IR cutoff in AdS space, one reproduces the hard scattering behavior of QCD by convoluting the string amplitude (which exhibits soft high energy behavior) with AdS wavefunctions. Subsequent work showed that this model reproduces known theoretical and experimental results in hadronic physics pointing perhaps to a gravity dual of QCD.

Another interesting possibility is on the extra dimensions forming a flat space of large or even infinite volume. In this case, light Kaluza-Klein modes may dominate even at low energies, leading to a modification of Newton’s Law of gravity at astronomically large distances. Reproducing the results on hadronic physics obtained in AdS space is a challenge in flat Minkowski space due to the lack of a holographic principle.

Here we make an attempt at understanding the spectrum of hadronic resonances using an extension of the Dvali-Gabadadze-Porrati (DGP) model. We consider a fat 3-brane of thickness $\sim 1/M$ in the transverse directions, where $M$ is the $D$-dimensional bulk Planck mass. We solve the Einstein equations for vector gravitational perturbations and compare the spectrum with the masses of vector mesons. We obtain agreement with similar results in AdS space by de Teramond and Brodsky for $D = 10$, provided the mass scale $M$ is

$$M \sim \sqrt{M_{QCD}} \sim 10^{9} \text{ GeV}$$

where $\hat{M}$ is the four-dimensional Planck mass. Unlike in AdS space, there is no need for an artificial IR cutoff; the finite width of the brane acts as a natural cutoff leading to normalizable solutions of the wave equation. We also discuss the conformal dimensions of the corresponding operators which create the hadronic resonances. Expanding on an idea discussed in [39], we obtain a correspondence principle which yields conformal dimensions in agreement with expectations.

Our discussion is organized as follows. In section II we show how the conformal algebra on a 3-brane in flat Minkowski space may be obtained from the isometries of the embedding in analogy with AdS space. In section III we consider the case of a scalar field. We solve the wave equation and discuss the conformal dimensions of the corresponding operators on the brane. In section IV we consider gravitational perturbations. We derive the Einstein equations and solve them for vector gravitational perturbations which decouple. We obtain the spectrum of normalizable modes and compare it to similar results in truncated AdS space. In section V we present our conclusions. Pertinent results in AdS space are summarized in Appendix A.

II. CONFORMAL ALGEBRA

The isometries of AdS space form an algebra which is isomorphic to the conformal algebra on the boundary. This is essential for the existence of the AdS/CFT correspondence principle. In flat Minkowski space, the
generators of isometries form the Poincaré algebra which is not simply related to a conformal algebra. To establish a correspondence principle in Minkowski space, we shall expand on an idea discussed in [39]. In order not to clutter the notation, we shall work with a five-dimensional Minkowski space spanned by coordinates \( X^A = (X^0, \vec{X}) \), \( \vec{X} = (X^1, \ldots, X^4) \), and place the brane at the hyperboloid

\[
X^A X_A = -(X^0)^2 + \vec{X}^2 = R^2 \tag{2}
\]

Extending the discussion to higher dimensions is straightforward; one need simply select a flat five-dimensional hypersurface in which to embed the brane.

The cosmological constant on this de Sitter brane is

\[
\lambda = \frac{3}{R^2} \tag{3}
\]

We recover the DGP model (flat brane) in the limit \( \lambda \to 0 \) (\( R \to \infty \)).

Let us parametrize the Minkowski space using coordinates appropriate for a dS hypersurface, \((u, \tau, \vec{\Omega})\), where \( \vec{\Omega} = (\Omega^1, \ldots, \Omega^4) \) with \( \vec{\Omega}^2 = 1 \), as

\[
X^0 = u \sinh \tau \ , \quad \vec{X} = u \cosh \tau \vec{\Omega} \tag{4}
\]

The 3-brane is then at \( u = R \) and the induced metric reads

\[
ds_{\text{brane}}^2 = -R^2 dr^2 + R^2 \cosh^2 \tau \ d\Omega^2 \tag{5}
\]

To map the Poincaré generators onto the brane, note that for \( u = R \), the momenta may be written as

\[
P_0 = \frac{i}{R} \cosh \tau \partial_\tau \quad \vec{P} = -\frac{i}{R} \sinh \tau \vec{\Omega} \partial_\tau - \frac{i}{R \cosh \tau} \vec{\nabla} \tag{6}
\]

and the generators of the Lorentz group become

\[
M_{0i} = -i \Omega_i \partial_\tau + i \tanh \tau \nabla_i \\
M_{ij} = -i (\Omega_i \nabla_j - \Omega_j \nabla_i) \tag{7}
\]

They are easily seen to form a \( SO(4, 2) \) algebra. The quadratic Casimir of this algebra is

\[
C_2 = \frac{1}{2} (M_{AB} M^{AB} + R^2 P_A P^A) \tag{8}
\]

The flat limit is obtained in the limit \( R \to \infty \) by scaling \( \tau \to \tau/R \), \( \vec{\Omega}^i \to \vec{\Omega}^i/R \) \( (i = 1, 2, 3) \)

In this limit, the metric on the brane turns into

\[
ds_{\text{brane}}^2 \approx dx^\mu dx_\mu = -d\tau^2 + (d\Omega^1)^2 + (d\Omega^2)^2 + (d\Omega^3)^2 \tag{10}
\]

i.e., flat Minkowski space spanned by Cartesian coordinates \( x^\mu = (\tau, \Omega^1, \Omega^2, \Omega^3) \).

To see the fate of the \( SO(4, 2) \) algebra, let us introduce the operators

\[
\mathcal{M}_{\mu\nu} = M_{\mu\nu} \ , \quad \mathcal{K}_\mu = 2 R^2 \left( P_\mu + \frac{M_{\mu\nu}}{R} \right) \\
\mathcal{P}_\mu = \frac{1}{2} \left( P_\mu - \frac{M_{\mu\nu}}{R} \right) \ , \quad \mathcal{D} = R P_y \tag{11}
\]

In the limit \( R \to \infty \), they turn into

\[
\mathcal{M}_{\mu\nu} \approx i (x_\mu \partial_\nu - x_\nu \partial_\mu) \ , \quad \mathcal{P}_\mu \approx i \partial_\mu \\
\mathcal{D} \approx ix^\mu \partial_\mu \ , \quad \mathcal{K}_\mu \approx i (x^2 \partial_\mu - 2x_\mu x \cdot \partial + 2\Delta x_\mu) \tag{12}
\]

where \( \partial_\mu = \partial/\partial x^\mu \). These are the generators of the conformal group.

A primary field \( \mathcal{O}_\Delta(x) \) of weight \( \Delta \) satisfies

\[
[i_{\mathcal{M}_{\mu\nu}}, \mathcal{O}_\Delta(x)] = i (x_\mu \partial_\nu - x_\nu \partial_\mu) \mathcal{O}_\Delta(x) \\
[i_{\mathcal{P}_\mu}, \mathcal{O}_\Delta(x)] = i \partial_\mu \mathcal{O}_\Delta(x) \\
[i_{\mathcal{D}}, \mathcal{O}_\Delta(x)] = i (x^\mu \partial_\mu - \Delta) \mathcal{O}_\Delta(x) \\
[i_{\mathcal{K}_\mu}, \mathcal{O}_\Delta(x)] = i (x^2 \partial_\mu - 2x_\mu x \cdot \partial + 2\Delta x_\mu) \mathcal{O}_\Delta(x) \tag{13}
\]

The quadratic Casimir on the primary field \( \mathcal{O}_\Delta(x) \) gives

\[
C_2 \mathcal{O}_\Delta(x) = \Delta (\Delta - 4) \mathcal{O}_\Delta(x) \tag{14}
\]

and the two-point function is

\[
G_\Delta(x) \equiv \langle \mathcal{O}_\Delta(\xi) \mathcal{O}_\Delta(0) \rangle \sim \frac{1}{(x^\mu x_\mu)^\Delta} \tag{15}
\]

Thus, we have shown that the Poincaré generators in the embedding are mapped onto the generators of the conformal group on the brane. To establish this map, it was necessary to bend the brane by introducing a small (positive) cosmological constant \( \lambda \) (eq. 3) and then take the flat limit, \( \lambda \to 0 \). Next, we turn to the wave equation in the embedding to explicitly realize a correspondence principle in analogy with the AdS/CFT correspondence.

### III. SCALAR FIELD

Here we consider a massless scalar field in \( D \)-dimensional Minkowski space in which a 3-brane resides, which generalizes the DGP model. We solve the wave equation in the bulk. By examining its behavior near the brane, we realize a correspondence principle for the scalar field. Inclusion of the brane leads to singular expressions which ought to be regulated. We do this by giving the brane a finite width in the transverse directions. We show that this affects the conformal dimensions of the operators corresponding to the solutions of the wave equation.
A. In the bulk

First, consider a flat 3-brane. Coordinates in the brane will be denoted by \( x^\mu \) (\( \mu = 0, 1, 2, 3 \)); transverse coordinates will be \( y^a \) (\( a = 1, \ldots, D-4 \)). The brane is assumed to be the hypersurface \( \vec{y} = \vec{0} \).

The action for a massless scalar field in the bulk is

\[
S_{\text{bulk}} = M^{D-2} \int d^4 x d^{D-4} y \partial_A \Phi \partial^A \Phi
\]

where \( M \) is the \( D \)-dimensional Planck mass. After Fourier transforming along the brane, we obtain the \( D \)-dimensional bulk wave equation

\[
(\nabla_y^2 + p^2) \Phi(p, \vec{y}) = 0
\]

To solve this, expand in harmonics,

\[
\Phi(p, \vec{y}) = \sum_{L, \vec{m}} \Phi_{L\vec{m}}(p, y) Y_{L\vec{m}}(\Omega_y)
\]

For the \( L \)th partial wave (suppressing the \( \vec{m} \) indices) we have to solve the radial wave equation

\[
\frac{1}{y^{D-5}} (y^{D-5} \Phi'_L)' + p^2 \Phi_L - \frac{L(L + D - 6)}{y^2} \Phi_L = 0
\]

whose well-behaved solution is

\[
\Phi_L(p, y) = A y^{-(D-6)/2} H_{\alpha}^{(1)}(py) \quad \alpha = L + \frac{D - 6}{2}
\]

where we included a normalization constant. This leads to the Green function

\[
G(X^A, X'^A) = \int \frac{d^4 p}{(2\pi)^4} e^{i p \cdot (x - x')} \Phi_L^*(p, y) \Phi_L(p, y')
\]

where \( X^A = (x^a, y^a) \) and similarly for \( X'^A \). After a Wick rotation, the integral may be calculated. In the limit \( y, y' \to 0 \) (approaching the brane), we obtain

\[
G(X^A, X'^A) \sim \frac{(yy')^{L+2}}{((x - x')^2)^\Delta} \quad \Delta = 2 + \alpha = L + \frac{D - 2}{2}
\]

In the case \( D = 10 \), we have \( \Delta = L + 4 \), and the scaling agrees with the AdS result \[41\]. These solutions correspond to operators carrying \( SO(4, 2) \) charge. We shall show that the scaling dimension is modified when the fluctuations of the brane in the transverse directions are properly accounted for.

The above result (22) may also be derived by projecting the \( D \)-dimensional Green function onto the \( L \)th partial wave,

\[
G(X^A, X'^A) \sim \int d\Omega Y_{L\vec{m}}(\Omega) \frac{1}{((X - X')^2)^{(D-2)/2}}
\]

where

\[
(X - X')^2 = (x - x')^2 + y^2 + y'^2 - 2yy' \cos \theta
\]
Outside the brane (in the bulk, \( y > 1/\Lambda \)), the wave equation reduces to eq. (15) for the \( L \)th partial wave. The solution is given by eq. (20).

Inside the brane (\( y < 1/\Lambda \)), the solution can be written as

\[
\Phi_L^{\text{brane}}(p; y) = B(y \Lambda)^{-(D-6)/2} J_\alpha(\kappa p y), \quad \kappa^2 = 1 + \frac{\sigma M^2}{\tilde{M}^{D-2}}
\]

Matching expressions across the boundary (\( y = 1/\Lambda \)), we obtain

\[
AH_\alpha^{(1)}(p/\Lambda) = B J_\alpha(\kappa p/\Lambda), \quad AH_\alpha^{(1)}(p/\Lambda) = \kappa B J'_\alpha(\kappa p/\Lambda)
\]

For \( p \ll \Lambda \sim M \), these can be written as

\[
-A_i^\alpha 2^\alpha \Gamma(\alpha)(p/\Lambda)^{-\alpha} = B J_\alpha(\kappa p/\Lambda), \quad A_i^\alpha 2^\alpha \Gamma(\alpha)(p/\Lambda)^{-\alpha-1} = \kappa B J'_\alpha(\kappa p/\Lambda)
\]

These conditions are compatible if

\[
(\kappa p/\Lambda) J'_\alpha(\kappa p/\Lambda) + \alpha J_\alpha(\kappa p/\Lambda) = 0
\]

or, equivalently,

\[
J_{\alpha-1}(\kappa p/\Lambda) = 0
\]

where we used the Bessel function identity

\[
z J'_\alpha(z) + \alpha J_\alpha(z) = z J_{\alpha-1}(z)
\]

Another identity that will be useful later is

\[
z J'_\alpha(z) - \alpha J_\alpha(z) = -z J_{\alpha+1}(z)
\]

Thus, we obtain the eigenvalues

\[
p^2 = \frac{\beta^2_{\alpha-1,k} \Lambda^2}{\kappa^2}, \quad \alpha - 1 = L + \frac{D - 8}{2}
\]

where \( \beta_{\alpha-1,k} \) is the \( k \)th root of \( J_{\alpha-1} \). For \( \Lambda \sim M \), we have

\[
p^2 \sim \frac{\beta^2_{\alpha-1,k} M^4}{M^2}
\]

so \( p^2 \ll \Lambda^2 \sim M^2 \) on account of \( M \ll \tilde{M} \), validating our approximations (unless we choose a root of the Bessel function of very high order).

To establish a correspondence principle, we ought to determine the conformal dimension of the operator on the brane which corresponds to these solutions of the wave equation (eq. 33). In the momentum regime we are working (\( p \ll \Lambda \)), the tail of the wavefunction in the bulk gives a negligible contribution. An examination of the scaling behavior of the Green function in the bulk led to a conformal dimension \( \Delta = 2 + \alpha \) (eq. 22). However, this conclusion did not take into account the effects of the finite width of the brane. We shall determine the conformal dimension by calculating the Green function on the brane. To this end, it is desirable to analytically continue the momentum beyond the discrete spectrum \( 10 \). Then the normalization constant \( \mathcal{B} \) is fixed by going into the UV regime (\( p^2 \gg \kappa^2/\Lambda^2 \sim M^2/M^4 \)). Defining the inner product

\[
\langle p|p' \rangle \equiv \frac{1}{2} \int_{y<1/\Lambda} d^{D-4}y \Phi_L^{\text{brane}}(p; y) \Phi_L^{\text{brane}}(p'; y)
\]

for the wavefunctions (33) on the brane, we may approximate it in the UV regime by

\[
\langle p|p' \rangle \approx \frac{\omega_D^{-4}}{2} \mathcal{B}(p) \mathcal{B}(p') \int_0^{\infty} dy J_\alpha(\kappa p y) J_\alpha(\kappa p' y)
\]

Using the orthogonality property of Bessel functions,

\[
\int_0^{\infty} dy J_\alpha(\kappa y) J_\alpha(\kappa q y) = 2 \delta(q^2 - q'^2)
\]

we deduce \( \langle p|p' \rangle = \delta(p^2 - p'^2) \), provided

\[
\mathcal{B}^2 = \omega_D^{-4} \kappa^2 \sim \omega_D^{-4} \frac{M^2}{M^2}
\]

which is momentum-independent and may be ignored in our subsequent discussion.

For momenta satisfying \( p \ll \Lambda \), the inner product of two wavefunctions (33) may be computed using standard manipulations of Bessel functions. Ignoring a momentum-independent overall factor, we obtain

\[
\langle p|p' \rangle \sim \frac{\kappa^p p}{\Lambda C(p, p')} J_\alpha(\kappa p/\Lambda) J_\alpha(\kappa p'/\Lambda)
\]

\[
- \frac{\kappa^p p}{\Lambda C(p, p')} J'_\alpha(\kappa p'/\Lambda) J_\alpha(\kappa p/\Lambda)
\]

\[
(46)
\]

where \( C(p, p') = (\kappa p/\Lambda)^2 - (\kappa p'/\Lambda)^2 \). Using the identity (38), we may write this as

\[
\langle p|p' \rangle \sim \frac{\kappa^p p}{\Lambda C(p, p')} J_{\alpha-1}(\kappa p/\Lambda) J_\alpha(\kappa p'/\Lambda)
\]

\[
- \frac{\kappa^p p}{\Lambda C(p, p')} J_{\alpha-1}(\kappa p'/\Lambda) J_\alpha(\kappa p/\Lambda)
\]

\[
(47)
\]

Evidently, for \( p \neq p' \), we have \( \langle p|p' \rangle = 0 \) for eigenstates corresponding to the discrete spectrum (10) (obeying the boundary condition 37). For the normalization of these states, we may use l'Hôpital’s rule on the right-hand side of 47. In the limit \( p' \to p \), we obtain

\[
\langle p|p \rangle \sim \frac{1}{2} |J_{\alpha-1}(\kappa p/\Lambda)|^2 - \frac{\alpha \Lambda}{\kappa p} J_{\alpha-1}(\kappa p/\Lambda) J_\alpha(\kappa p/\Lambda)
\]

\[
+ \frac{1}{2} |J_\alpha(\kappa p/\Lambda)|^2
\]

(48)
which reduces to
\[ \langle p|p \rangle \sim \frac{1}{2} |J_{\alpha}(\kappa p/\Lambda)|^2 \] (49)
for momenta satisfying the boundary condition.

The Green function on the brane is defined by averaging the two-point function over the transverse spread of the brane,

\[ G^{\text{brane}}(x^\mu, x'^\mu) \equiv \int d^Dy y \sigma(y) G(x^\mu, y; x'^\mu, y) \] (50)

Using the wavefunctions on the brane, we may write this Green function in terms of the inner product as

\[ G^{\text{brane}}(x^\mu, x'^\mu) \sim \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} \langle p|p \rangle \] (51)

leading to the scaling behavior

\[ G^{\text{brane}}(x^\mu, x'^\mu) \sim \frac{1}{(x-x')^2} e^{i\Delta}, \quad \Delta = 1 + \alpha \] (53)
in disagreement with the bulk result \( \Delta = 2 + \alpha \) (eq. 22).

Notice that the bulk result is also obtained if we naively extrapolate eq. (53), which is only valid for the discrete spectrum, to the IR regime. We have argued that the correct analytic continuation is provided by eq. (18) and not eq. (22), the latter being a special case of the former.

The bulk result does not lead to the correct correspondence principle for operators on the brane, unless the observable cannot “see” the brane (which may be the case with certain gravitational perturbations). In general, a calculation on the brane is needed for the analysis of conformal behavior.

IV. GRAVITATIONAL FIELD

A. Field equations

As for a scalar, the action for the gravitational field also consists of a bulk term and a brane term. The bulk term is simply the \( D \)-dimensional Einstein-Hilbert action

\[ S_{\text{bulk}} = M^{D-2} \int d^Dx \sqrt{-g} R^{(D)} \] (54)

where \( g \) is the \( D \)-dimensional metric generating the \( D \)-dimensional Ricci scalar \( R^{(D)} \).

We obtain the brane contribution to the action by dimensional reduction. Working with a flat brane at \( \vec{y} = 0 \), where \( X^A = (x^\mu, \vec{y}) \), we may decompose the metric as

\[ ds^2 = g_{AB} dx^A \cdot dx^B = \hat{g}_{\mu\nu} dx^\mu dx^\nu + \hat{g}_{ab}(dy^a + A^a_\mu dx^\mu)(dy^b + A^b_\mu dx^\mu) \] (55)

Assuming no dependence on the transverse directions, \( \vec{y} \), the Ricci scalar may be written as

\[ R^{(D)} = \hat{R}^{(4)} - \frac{1}{4} \partial_\mu (\ln \det \hat{g}) \partial^\mu (\ln \det \hat{g}) - \frac{1}{4} \hat{g}^{ab} \hat{g}^{cd} \partial_\mu \hat{g}_{ac} \partial^\mu \hat{g}_{bd} - \frac{1}{4} \hat{g}^{ab} F_{\mu\nu}^a F^{b\mu\nu} \] (56)

The brane term is

\[ S_{\text{brane}} = \bar{M}^2 \int d^4x \sqrt{-\det \hat{g} \sigma_{\Lambda}(y) R^{(D)}} \] (58)

where we included the effects of fluctuations in transverse directions given in terms of the profile function \( \sigma_{\Lambda} \) (eq. 28).

Let us perturb around the Minkowski flat background \( g_{AB} = \eta_{AB} + h_{AB} \) (59).

Varying the action \( S = S_{\text{bulk}} + S_{\text{brane}} \), given by eqs. (54) and (58), with respect to \( h_{AB} \), we obtain the Einstein field equations. To solve these equations, in general, is challenging, because different gravitational perturbations are coupled to each other.

Here we concentrate on the fluctuations generated by the off-diagonal components of the metric (vector potential),

\[ h_{A\mu} = A_{A\mu} \] (60)

We obtain the field equation

\[ M^{D-2}(\partial_A \partial^A h_{A\mu} - \partial_\mu \partial^A h_{A\mu} - \partial_\sigma \partial^A h_{A\mu} + \partial_\mu \partial_\sigma h_{A\sigma}) + \bar{M}^2 \sigma_{\Lambda}(y) \partial^\sigma F_{\mu\sigma}^A = 0 \] (61)

Choosing the harmonic gauge,

\[ \partial^A h_{A\mu} = \frac{1}{2} \partial_B h_{A\mu} \] (62)
eq (61) simplifies to

\[ M^{D-2} \partial_A \partial^A A_{\mu}^a + \bar{M}^2 \sigma_{\Lambda}(y) \partial^\nu F_{\mu\nu}^a = 0 \] (63)

This field equation is consistent provided

\[ \partial_\mu A_{\mu}^a = 0 \] (64)

(as for a massive vector field) in which case (63) reads

\[ M^{D-2} \partial_A \partial^A A_{\mu}^a + \bar{M}^2 \sigma_{\Lambda}(y) \Box A_{\mu}^a = 0 \] (65)

Thus we obtained a wave equation for vector gravitational perturbations decoupled from all other modes. This equation is of the same form as the scalar wave equation \[ 31 \] As we discussed in section \[ 11 \] the field
with angular momentum $L$ in the transverse directions ($SO(D - 4)$ quantum number) corresponds to an operator on the brane of conformal dimension

$$\Delta = 1 + \alpha = L + \frac{D - 4}{2}$$

(66)

where we used eqs. (53) and (20). Moreover, the solution of the wave equation (65) leads to the discrete spectrum

$$\text{V. CONCLUSIONS}$$

In conclusion, we have discussed a correspondence principle for a 3-brane in $D$-dimensional flat Minkowski space in analogy with the AdS/CFT correspondence. Expanding on an idea discussed in [39], we showed that the isometries of the embedding, generating the Poincaré group, can be mapped onto a $SO(4, 2)$ algebra on the brane if the latter is a de Sitter hypersurface. In the flat limit (zero cosmological constant), this $SO(4, 2)$ algebra turns into the conformal algebra on the 3-brane. We then realized a correspondence principle by considering fields with finite angular momentum in the transverse directions and showed that they led to a scaling behavior of the Green function in analogy with the results on the boundary propagator in AdS space [40]. We discussed a subtlety related to the correct analytic continuation of the spectrum of the $D$-dimensional wave equation to arbitrary momenta which affected the determination of the conformal dimension of the corresponding operator on the brane. Finally, we applied our results to the case of vector mesons which corresponded to vector gravitational perturbations in the bulk. We obtained a spectrum which was in agreement with AdS results [20] provided we chose $D = 10$ and the ten-dimensional mass scale as in [1].

Extending the discussion to other hadronic resonances entails a more complete solution of the ten-dimensional Einstein equations. Fermions may also be included by extending the model to supergravity. Work in this direction is in progress.

**APPENDIX A: REVIEW OF ADS**

Here we review the pertinent features of AdS space for comparison with our results in flat Minkowski space. The wave equation for a massless scalar in $AdS_5 \times S^5$ is

$$\partial_z^2 \phi - \frac{3}{z} \partial_z \phi + p^2 \phi - \frac{L(L + 4)}{z^2} \phi = 0$$

(A1)

where $p^\mu$ is the four-momentum on the boundary ($z \to 0$) and $L$ is the $S^5$ angular momentum. The solution is

$$\phi(p; z) = \mathcal{E} z^\nu J_\nu(pz), \quad \nu = L + 2$$

(A2)

The normalization constant is fixed by imposing the normalization condition

$$\langle p | p' \rangle = \int_0^\infty \frac{dz}{z^3} \phi^*(p; z)\phi(p'; z) = \delta(p^2 - p'^2)$$

(A3)
leading to a momentum-independent $\mathcal{B}$.

The propagator is given by

$$G(x, z; x', z') = \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-x')} \phi^*(p; z) \phi(p; z')$$

from which one may deduce the boundary propagator in the limit $z, z' \to 0$. We obtain

$$G(x, z; x', z') \sim (zz')^{\nu+2} \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-x')} p^{2\nu}$$

$$\sim \frac{(zz')^\Delta}{((x-x')^2)^\Delta},$$

(A5)

where $\Delta = \nu + 2 = L + 4$.

We wish to consider wavefunctions which live on the boundary but have a finite extent in the bulk direction $z$, $0 \leq z \leq z_0$ where $z_0$ is to be identified with the QCD scale ($z_0 = 1/\Lambda_{QCD}$). This cutoff breaks conformal invariance and can be thought of as providing a finite width for the brane residing on the boundary of AdS. Then CFT observables must be defined by averaging over the transverse width of the brane. Let us then define the boundary propagator by

$$\mathcal{G}(x, x') \sim \int_0^{z_0} \frac{dz}{z^2} G(x, z; x', z)$$

(A6)

The conformal limit is recovered in the IR ($p z_0 \ll 1$), because for small $p$, we have $J_\nu(pz) \sim (pz)^\nu$, therefore

$$\mathcal{G}(x, x') \sim z^{2(\nu+2)} \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-x')} p^{2\nu} \sim \frac{z^{2\Delta}}{((x-x')^2)^\Delta},$$

in agreement with eq. (A5).

If we impose the boundary condition

$$J_\nu(pz_0) = 0$$

(A7)

the momentum $p$ is discretized,

$$p^2 = \frac{\beta^2_{\nu,k}}{z_0^2}, \quad J_\nu(\beta_{\nu,k}) = 0$$

(A8)

Thus, $p \gtrsim o(1/z_0)$. These momenta no longer satisfy the requirement $p z_0 \ll 1$ and we are outside the conformal regime. Notice that for the discrete spectrum $\mathcal{A}$,

$$\int_0^{z_0} \frac{dz}{z^2} [\phi(p; z)]^2 = \frac{1}{2} [J_{\nu+1}(pz_0)]^2$$

(A10)

Naïvely analytically continuing this to low momenta, we obtain a behavior of $p^{2(\nu+1)}$ which yields the wrong boundary propagator (cf. eq. (A7)). To do the analytic continuation correctly, we ought to first calculate the inner product

$$(p|p') = \int_0^{z_0} \frac{dz}{z^2} \phi(p; z) \phi(p'; z)$$

(A11)

which vanishes for $p, p'$ obeying (A9) with $p \neq p'$, and then take the limit $p' \to p$. This yields

$$\langle p|p \rangle = \frac{1}{2} [J_\nu(pz_0)]^2 - \frac{\nu}{p z_0} J_\nu(pz_0) J_{\nu+1}(pz_0)$$

$$+ \frac{1}{2} [J_{\nu+1}(pz_0)]^2$$

(A12)

which reduces to (A10) for momenta satisfying the quantization condition (A7). Continuing this to low momenta, we obtain $(p|p) \sim p^{2\nu}$ and we recover eq. (A7), as expected.

The discussion of the wave equation for a vector in AdS space is similar [24].

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