Numerical modeling of the dynamics of the mechanical system from composition materials

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Numerical modeling of the dynamics of the mechanical system from composition materials

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Abstract. The paper presents a technique for preliminary assessment of the resource of a mechanical system in the early stages of its design. The technique is based on solving numerical simulation problems of kinematics and the results of force analysis of the elements of the carrier system of a light helicopter, taking into account the deformations of the links, and is based on the finite element method. The modernized method of solving the problem of forced oscillations by the method of decomposition into its own forms of oscillations has been implemented.

1. Introduction

The main goal of this work is to develop a methodology for the preliminary assessment of the mechanical system resource at the early stages of its design based on solving numerical simulation problems of kinematics and force analysis results taking into account the deformations of the links. The object of the research is the calculation of the carrying system of a light helicopter. In the calculations, it is assumed that the surface of the structure has no damage, scratches and gouges, there are no technological defects, and the materials from which the structural elements are made have ideal and uniform properties. The technique of preliminary assessment of the resource of a mechanical system is based on the finite element method (FEM). In the calculations, along with the domestic settlement complexes, the well-known settlement complexes ADAMS, ANSYS, NASTRAN are used to expand the calculator's capabilities.

2. Statement of the problem

A mechanical system is a mechanism made of composite materials, the kinematics of which is controlled and adjusted for accuracy by the electronic stabilization and control system. When modeling the kinematics of a mechanical system using the FEM, the solution is implemented using the direct integration method (for example, the Newmark method) of the basic equation in the form:

\[ M(n)\ddot{v}(t) + C(n)v(t) + K(n)v(t) = P(t) \]

with initial conditions \( \dot{v}(0) = v^0 \), \( v(0) = v^0 \), where \( v(t) \) - is a vector of generalized displacements, describing the movement of all moving parts of a structure; \( t \) - time; \( n \) - the number of design
variables; \( \mathbf{M}(n) \) - inertial matrix (mass matrix of the structure); \( \mathbf{C}(n) \) - matrix of damping coefficients (damping matrix); \( \mathbf{K}(n) \) - stiffness matrix; \( \mathbf{P}(t) \) - vector of nodal influences.

The displacement vector \( \mathbf{v}(t) \) and the load vector \( \mathbf{P}(t) \) can be represented as two vectors:

\[
\mathbf{v}(t) = \begin{bmatrix} v_p(t) \\ v_q(t) \end{bmatrix}, \quad \mathbf{P}(t) = \begin{bmatrix} P_p(t) \\ P_q(t) \end{bmatrix},
\]

where \( v_p(t) \) is the unknown displacement vector; \( v_q(t) \) - the specified displacement vector, \( P_p(t) \) - the specified forces; \( P_q(t) \) - reaction forces that correspond to a given displacement vector \( v_q(t) \). If we solve the joint problem of the kinematics of a mechanical system under the influence of an electronic stabilization and control system, then \( v_q(t) \) it will change according to a given control law at each step of integrating the basic equation (1). The task involves the influence of variable external loads. This means that \( P_p(t) \) at each step of integrating the basic equation (1) it changes in its solution algorithm. As a result, we get a solution to the joint problem of the motion of a system of interconnected bodies in space according to equation (1), which at each integration step is adjusted by changing the values \( v_q(t) \) according to a given equation of control law and the values \( P_p(t) \) according to a given algorithm for calculating the external load based on the current position of the mechanism in space.

In the process of design calculation, the developer must come to the most complete from a kinematic point of view, the design representation in the form of a finite element model in a nonlinear formulation, to simulate the movement dynamics of a deformable mechanical system under the influence of a variable system of external forces and moments and corrective effects of the electronic stabilization and control system. One of the most interesting examples is the problem of joint kinematic and force analysis of the mechanisms of the carrying system of a light helicopter with an automatic control system in the field of balanced aerodynamic and inertial forces and moments in given flight modes [1].

To simulate the movement of an aircraft, the basic equation (1) requires additional terms that take into account the effects of air and has a more complicated form:

\[
\mathbf{M}(n) \ddot{\mathbf{v}}(t) + \mathbf{V}[\mathbf{B}] \dot{\mathbf{v}}(t) + \mathbf{C}(n) \mathbf{v}(t) - V^2 \mathbf{A} \mathbf{v}(t) + \mathbf{K}(n) \mathbf{v}(t) = \mathbf{P}(t)
\]  

(3)

Where \( V^2[A] \), \( V[B] \) - matrix aerodynamic rigidity and aerodynamic damping; \( V \) - the speed of the incident flow. Equation (3) takes into account aerodynamic damping and damping in the material of the links of the mechanism. To determine the aerodynamic characteristics of the method used discrete vortices. As noted above, the calculator does not always sufficiently possess experimental data and can determine the damping coefficients of the structure. Therefore, in order to solve the system of equations (1) or (3) in accordance with the method of expansion in its own forms, the nodal displacements of the body in the local coordinate system are considered as a linear combination of a finite number of vectors \( \mathbf{g}_i \) (mode form):

\[
\mathbf{v} = \sum_{n=1}^{N} \mathbf{g}_n \varphi_n
\]  

(4)

where \( \varphi_n \) – is the mode amplitude [2].

In this case, the main assumption is that the components of the vector of nodal displacements with a large number of modal values can be considered with a reduced (to \( M \)) number of modes (eigen frequencies), i.e. modal series is considered in a truncated form (modal truncation). The calculator has the ability to select an arbitrarily larger number of modal forms in order to improve the accuracy of the calculation. In the matrix form, the expression (4) has the form:

\[
\mathbf{v} = \mathbf{G} \varphi
\]  

(5)
where \( \varphi \) is the vector of the modal coordinate, \( G \) are the modes in the form of a matrix. Given modal truncation, the matrix \( G \) becomes a rectangular matrix of size \( N \times M \). Substitute (5) into (1) and multiply to the left:

\[
G^T M(n) G \varphi(n) + G^T C(n) G \varphi(t) + G^T K(n) G \varphi(t) = G^T P(t). \tag{6}
\]

Since the eigenvectors included in the matrix \( G \) are orthogonal with respect to the matrices \( K \) and \( M \), we get:

\[
G^T M(n) G = I, \quad G^T K(n) G = \Omega^2, \]

where \( I \) - is the unit matrix, \( \Omega^2 \) - is the diagonal matrix with the squares of the eigen frequencies as the diagonal elements. Equation (6) has the exact solution with the entire spectrum \( N \) in expression (5).

If the damping matrix is not taken into account or is represented as a linear combination of matrices \( K \) and \( M \), then expression (6) is simplified (reduced) to \( M \) unrelated equations.

3. Results of numerical calculations

In Figure 1, you can see the calculation of the trajectory of the helicopter at the time of the occurrence of the rotor flutter, isopole structure deformations at a given point in time [1, 3]. Figure 2 shows the change in load on the cyclic time step booster when modeling a typical flight of a helicopter “takeoff - flight in a straight line - landing”. The resulting cycles of change of deformations, stress intensities, reaction forces in hinged joints allow you to define the source data in the form of cyclically variable relationships for assessing the fatigue characteristics of the structure. In general, the basic principles of the methodology for assessing the fatigue characteristics of a structure are considered in the course of resistance of materials.

![Figure 1](image)

**Figure 1.** The change in the stress-strain state of the design of the helicopter at a given flight mode, where \( P_{p,t} \) - is the resulting force of the tail rotor and the keels, \( P_{c,m} \) - is the resulting force of the stabilizer;

But each branch of engineering has its own peculiarities and clarifications. Here, the calculations are carried out in accordance with industry-accepted regulatory and technical documentation. In this work, when designing a mechanical system, it is proposed before the advent of a prototype to use an analytical method for assessing the fatigue strength of a structure, similar to that indicated in the circular. The initial data for the calculation will be the laws of changes in loads and stresses, which are determined using the above numerical methods for finding rational design parameters. In this case, methods to automate the search for rational parameters will be a tool to reduce the overall level of stresses, reduce the number of concentrators, and smooth out voltage drops. Additional data characterizing the fatigue characteristics of the materials used will be the Oding or Haig (Goodman) curves, which are obtained from fatigue tests of samples of materials.
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For example, in fig. Figure 3 shows a typical curve for estimating a safe resource. The fatigue behavior of the material is investigated for each operational mode or at averaged stress levels. The upper fatigue curve is based on the results of fatigue tests of a reference material sample. The statistically reduced working curve provides the necessary margin for the fatigue strength of the structure, taking into account stress concentration and other factors that reduce the fatigue strength of the structure relative to the sample. To calculate the life of the structure, the lower operating curve is used, which is reduced relative to the statistical average by a safety factor equal to three standard deviations of the logarithms of the numbers of cycles to failure $3 \cdot \sigma_{10N}$.
A design variant that has vibration load values below the values of the lower operating curve is most preferable for building a prototype. The level of vibration stress relative to the working curve shows the expected life of the structure, taking into account the idealization of its numerical model. In the process of developing the models described above, the prospects for their development and the possibility of refining the applied numerical algorithms were identified.

4. Modernization of the decomposition method
The procedure of expansion in forms for solving equation (1) or (3) involves two-stage modeling of each deformable link of the mechanism. The first stage is the determination of the eigen frequencies of the modal truncation. The second is the calculation of the dynamics of movement, taking into account the values of modal truncation. The preparation of the initial data of the method is cumbersome. In addition, the calculator has difficulty assessing the frequency characteristics of the model in comparison with the actual design. For this reason, tests were carried out to modernize the decomposition method in its own forms on various types of finite elements (rod beam, membrane, plate, shell, three-dimensional hexagonal element). In this work, more detailed studies have been carried out on the accuracy of the solution using the modernized decomposition method.

The main essence of the refinement of the decomposition method is as follows. According to the theorem on the minimum properties of eigen frequencies, the square of the eigen frequency corresponds to the relative minimum of the Rayleigh function [4]. The minimum of the Rayleigh function is achieved only for a form that exactly matches its own form. If the body makes an oscillatory motion, then at the current moment in time, the form of oscillations either corresponds to its own form or not. Roughly speaking, a deformable body, moving in space and deforming, gets into resonance only when the frequency and shape of its deformations corresponds to the minimum Rayleigh function. According to the dynamics of theoretical mechanics, the complex motion of a deformable body in space is considered as a chain of static deformed states, each of which corresponds to a given point in time. Any oscillation of a deforming body can be represented as the sum of displacements in projections on the axes of the local or global coordinate systems (nodes of the finite element mesh) of the particles of the body. The motion of the body in space, taking into account the superimposed connections in the form of its deformations in FEM, is represented as the sum of the projections of the displacement vector on the coordinate axes at the nodes of the computational grid. Assessing the current energy state of a deformable body making a complex movement in space, one can assess whether the current body shape corresponds to its own form (in projections on the axis of the Cartesian coordinate system) or not.

Let us consider, for simplicity, a method for solving the problem of forced oscillations by the method of decomposition into its own forms without taking into account damping. In this case, the “condensed” equation (6) takes the form:

$$\ddot{\varphi}^M(t) + K^M(\omega^2)\varphi^M(t) = P^M(t), \quad (7)$$

where

$$K^M = \begin{bmatrix}
\omega_i^2 & 0 & 0 & 0 \\
0 & \omega_j^2 & 0 & 0 \\
0 & 0 & \ldots & 0 \\
0 & 0 & 0 & \omega_k^2
\end{bmatrix}, \quad (8)$$

$\varphi^M(t)$ and $P^M(t)$ are condensed vectors of nodal displacements and nodal forces, in dimension $M \times 1$, where $\omega_m^2$ - is the square of the $m$th eigen frequency of oscillation. The system of equations (7) is solved by the method of direct integration (for example, by the Newmark method) taking into account the imposed boundary conditions and the system of loads. As a result, each component of the vector function $\varphi^M_i(t)$ is found by integrating the equation

$$\ddot{\varphi}^M_i(t) + \omega_i^2 \varphi^M_i(t) = P^M_i(t), \quad (9)$$
When oscillations with a eigen frequency, when the eigen mode of oscillations is considered as the field of displacements, the condition of the principle of energy conservation for the main oscillations of the system is satisfied:

$$\omega_i^2 = \Pi / T,$$

(10)

where $T$ and $\Pi$ - is the kinetic and potential energy, respectively. In other cases, the equality of the energy ratio is not equal to the square of the eigen frequency. The standard algorithm for calculating forced vibrations assumes the presence of a spectrum of eigen frequencies of oscillations (or its part) even before the calculation itself, i.e. the eigenvalue problem must be solved in advance. In this work, it is assumed that $M = N$ in the condensed stiffness matrix (8) the values $\omega^2_m$ are not the squares of the corresponding eigen vibration frequencies, but are calculated at each time step for each node (degree of freedom) of the structure through the ratio of potential and kinetic energies (10).

It should be noted that at each step of integration, the values $K^M$ and $P^M$ are redefined by the current deformed state (the current energy state of the deformable body). If the calculation is carried out taking into account the damping, then the damping matrix will be variable in time and its components will change values according to a nonlinear law. Figure 3 shows the dependence of the axial displacement of the free end of a fixed rod with forced oscillations, taking into account damping.

![Figure 3. The dependence of the axial displacement of the free end of the fixed rod with free oscillations, taking into account the damping](image)

5. Conclusion

A technique has been developed for analyzing the resource estimation of mechanisms based on the numerical solution of problems of dynamics and the possibilities of their development on the basis of converted calculation methods are considered. Solved test problems to verify the accuracy of the solutions obtained when solving problems of forced oscillations by a modified method of decomposition into their own forms.

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