Fock exchange terms in non–linear Quantum Hadrodynamics

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Abstract

We propose a method to introduce Fock term contributions in relativistic models of fermions coupled to mesons, including self-interactions for the mesonic fields. We show that effects on equilibrium properties and on the dynamical response of the fermionic system can be consistently accounted for. Some implications on equilibrium properties of asymmetric nuclear matter are discussed. In particular an indication is emerging for a reduced contribution of charged mesons to the symmetry term of the nuclear equation of state around normal density.

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I. INTRODUCTION

Since the pioneering work of Walecka [1], the relativistic field model of hadrons, called Quantum Hadrodynamics (QHD), with its developments and extensions, has been widely used in investigations of nuclear systems, see the extensive reviews [2–4]. This model can be regarded as an effective field theory [5–7] at the nuclear scale, and it is based on a relativistic effective Lagrangian which includes strongly interacting nucleon and meson fields. The success of QHD has been due to the fact that it provides a realistic description of bulk properties both of infinite nuclear matter and of finite nuclei [2–4], even with a minimal number of meson fields, the neutral σ and ω fields and the charged ρ field.

The usual procedure to deal with theories which contain baryons interacting with mesons consists of eliminating the meson fields in favour of the fermionic degrees of freedom. This can be done by exploiting the equations of motion for the meson fields. In this way the physical system can be ultimately described in terms of nucleons interacting through the exchange of virtual mesons.

It is important to notice that, once the Fock exchange terms of the nucleon–nucleon interaction are taken into account, the Walecka model can reproduce some basic properties of nuclear matter by means of the exchange of the scalar σ and vector ω fields alone (see Refs. quoted in [2–4]). Including the Fock terms also the whole variety of processes in nuclear dynamics arising from the fermionic intrinsic degrees of freedom (spin and isospin),
can be accounted for \cite{8,10}. In principle, if the model is constructed self-consistently, the introduction of further meson fields with the corresponding constants for the coupling to nucleons appears to be less necessary. We will elaborate in detail on this point.

However, in its original version, the QHD model suffered from a severe deficiency, namely it was not able to give a reasonable value for the nuclear compressibility modulus and the effective mass. This unpleasant drawback has been cured by introducing self-interactions of the scalar $\sigma$ field up to fourth order \cite{11,12}. Moreover, within both the modern scheme of effective field theories \cite{13,14} and hadronic chiral models \cite{15} a non-linear self-interaction for the scalar meson emerges in a natural way.

Calculations based on non-linear models of QHD have been so far performed in the relativistic mean field approximation (RMF), which essentially corresponds to the Hartree approximation. With the inclusion of these higher order terms in the Lagrangian, the evaluation of the exchange terms of the nucleon–nucleon interaction represents a very difficult task. This is due to the non-linearity of the resulting equation of motion for the meson field. Although the RMF approximation to QHD with non-linear terms has achieved a large success in reproducing quantitatively many observables, in particular for finite nuclei \cite{5}, the inclusion of the Fock terms would be highly desirable. In fact, these terms originate from the correlations due to the Fermi–Dirac statistics, therefore they are related to a genuine quantum effect, which in general cannot be neglected in studying a many-body system.

In this paper, we present a procedure which allows to evaluate the contribution of the Fock terms in a truncation scheme. The result is an expansion of the Fock terms in a $1/N$ series, where $N$ is the multiplicity of the fermionic intrinsic degrees of freedom ($N = 4$ for nucleons). In this sense the approach suggested here is not of perturbative nature in the coupling constants, which are always accounted for at all orders.

We illustrate our method in the case of infinite nuclear matter. We show that both equilibrium properties and dynamics can be self-consistently treated at the same degree of approximation.

We would like to remark that our treatment of the Fock terms is not restricted to QHD models. It can be applied in general, most likely with non-trivial consequences, to models involving fermions coupled to bosons. Among these, we mention the Quark Meson Coupling model \cite{16}, where nuclear matter is described as a collection of composite nucleons with meson fields directly coupled to quarks \cite{17,19}. Outside a strictly nuclear context, we mention also the Linear Quark Meson model, which represents an effective description of QCD below the mesonic compositeness scale ($\sim 600\,\text{MeV}$) \cite{20,21}. This model provides a hybrid description of QCD in terms of quarks and mesons.

II. TREATMENT OF FOCK TERMS IN A KINETIC APPROACH

The aim of the present paper is to illustrate a procedure which allows to take into account the self-interactions of meson fields within a Hartree–Fock scheme. For simplicity, and also to focus on the new procedure, we limit ourselves to consider only the minimal number of mesons which is sufficient to reproduce some basic properties of nuclear matter. Therefore our starting point is the QHD-I $\sigma – \omega$ model \cite{3}, including self-interaction terms for the scalar $\sigma$ field \cite{1,2}. The Lagrangian density for this model is given by:
\[ L = \overline{\psi} [\gamma_{\mu} (i \partial^\mu - g \nabla^\mu) - (M - g_S \phi)] \psi + \frac{1}{2} (\partial^\mu \bar{\phi} \partial_\mu \phi - m_S^2 \phi^2) \]

\[ - \frac{a}{3} \bar{\phi}^3 - \frac{b}{4} \bar{\phi}^4 - \frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_V^2 \nabla_\mu \nabla^\mu, \]  

(1)

where \( W^{\mu\nu}(x) = \partial^\mu \nabla^\nu(x) - \partial^\nu \nabla^\mu(x) \). Here \( \psi(x) \) denotes the 8(spin and isospin)-component nucleon field, the scalar \( (\phi(x)) \) and the vector \( (\nabla^\mu(x)) \) boson fields are associated with the scalar \( (\sigma) \) and vector \( (\omega) \) mesons, respectively \( (\hbar = c = 1) \).

Fock contributions naturally appear in the source terms of the equation of motion of the fields. Here we will study the related effects on the corresponding kinetic equations, in particular for the fermion field. This will allows us to derive the contributions of exchange terms on static and dynamical properties of the interacting nuclear system in a self-consistent way, at the same level of approximation.

### A. The Wigner transform formalism

In agreement with the Relativistic Mean Field (RMF) picture of the QHD model we focus our analysis on a description of the many-body nuclear system in terms of one–body dynamics. Therefore, it is convenient to use the Wigner transform of the one–body density matrix of the fermionic field. The one–particle Wigner function is defined as:

\[ [\hat{F}(x,p)]_{\alpha\beta} = \frac{1}{(2\pi)^4} \int d^4 R e^{-i p \cdot R} \langle \overline{\psi}_\beta(x + R/2) \psi_\alpha(x - R/2) \rangle, \]

where \( \alpha \) and \( \beta \) are double indices for spin and isospin. The brackets denote statistical averaging and the colons denote normal ordering. According to the Clifford algebra, the Wigner function \( \hat{F}(x,p) \) can be decomposed as:

\[ \hat{F}(x,p) = F(x,p) + \gamma_\mu F^\mu(x,p) + \frac{1}{2} \sigma_{\mu\nu} F^{\mu\nu}(x,p) \]

\[ + \gamma^5 A(x,p) - \gamma^5 \gamma_\mu A^\mu(x,p). \]  

(2)

The integral over \( p \) of the Wigner function

\[ [\hat{F}(x)]_{\alpha\beta} = \int d^4 p [\hat{F}(x,p)]_{\alpha\beta} = \langle \overline{\psi}_\beta(x) \psi_\alpha(x) \rangle \]

is related to the various densities with their specific transformation properties both in ordinary space and in isospin space (isoscalar and isovector). For instance the scalar and current isoscalar densities are given by:

\[ \rho_S(x) = \langle \overline{\psi}(x) \psi(x) \rangle = Tr \hat{F}(x) = 8 F(x), \]

\[ j^\mu(x) = \langle \overline{\psi}(x) \gamma^\mu \psi(x) \rangle = Tr \gamma^\mu \hat{F}(x) = 8 F^\mu(x). \]

The equation of motion for the Wigner function can be derived from the Dirac equation by using standard procedures (see e.g. Refs. [22,23]):
with $x_+ = x + \frac{R}{2}$ and $x_- = x - \frac{R}{2}$.

B. The truncation scheme for exchange contributions

The statistical averages of the fermionic and mesonic operators in the above equation will now be treated within a Hartree–Fock (HF) approximation. In order to take into account the contribution of exchange terms in a manageable way we assume, as a basic approximation, that in the equations of motion for the meson fields the terms containing derivatives can be neglected with respect to the mass terms. Then, the meson field operators are connected to the operators of the nucleon scalar and current densities by the following equations:

$$\hat{\Phi}/f_S + A\hat{\Phi}^2 + B\hat{\Phi}^3 = \bar{\psi}(x)\psi(x) = \hat{\rho}_S(x)
, \quad \hat{V}^\mu(x) = f_V \bar{\psi}(x)\Gamma^\mu \psi(x) = \hat{J}^\mu(x),$$

with $x_+ = x + \frac{R}{2}$ and $x_- = x - \frac{R}{2}$.

This approximation is not justified for light mesons, like pions. The inclusion of self–interaction terms of the pionic field is a very difficult task and a different approximation scheme is needed. However in this case a perturbative expansion in the pion-nucleon coupling constant seems to be reasonable [24]. Moreover it has been shown that the inclusion of pions does not change qualitatively the description of nuclear matter around normal conditions [24].

The next step will be the elimination of the meson fields in Eq.(3) by means of Eq.(4). As far as the term containing the vector field is concerned, by a simple substitution one can obtain a statistical average of only fermionic field operators [8]. Instead, because of the non–linear coupling of the $\sigma$ field to the fermionic scalar density, the elimination of the scalar field in the last term of Eq.(3) requires a particular procedure.

We consider the expansion of the field operator $\hat{\Phi}$ in terms of the scalar density operator $\hat{\rho}_S$: $\hat{\Phi} = \sum_n x_n \hat{\rho}_S^n$. Then, in the HF approximation, by means of the Wick’s theorem the average in the last term of Eq.(3) can be expressed as:

$$\langle :\bar{\psi}_\beta(x_+)\psi_\alpha(x_-)\hat{\Phi}(x_-) : \rangle = \langle :\bar{\psi}_\beta(x_+)\psi_\alpha(x_-) : \rangle :\hat{\Phi}(x_-) : \rangle + \langle :\bar{\psi}_\beta(x_+)\psi_\gamma(x_-) : \rangle \times \sum_{k=1}^\infty \frac{1}{k!}(-1)^k [\hat{F}^k(x_-)]_{\alpha\gamma} \langle :d^k\hat{\Phi}(x_-) : \rangle$$

The statistical averages of $\hat{\Phi}(x)$ and its derivatives are given by series of $\langle :\hat{\rho}_S^n(x) : \rangle$, which in turn can be expanded as:
\[ <: \hat{\rho}_S^n(x) :> = \rho_S^n(x) - \sum_{k=2} \frac{1}{k!}(-1)^k \text{Tr} \hat{F}^k(x) \frac{d^k \rho_S^n(x)}{d\rho_S^n(x)}. \]  

As a consequence, we obtain for \(<: \hat{\Phi}(x) :>\) the expansion:

\[ <: \hat{\Phi}(x) :> = \Phi(x) - \sum_{k=2} \frac{1}{k!}(-1)^k \text{Tr} \hat{F}^k(x) \frac{d^k \Phi(x)}{d\rho_S^n(x)}. \]  

where \(\Phi(x) = \sum_n \chi_n \rho_S^n(x)\), and similar expressions for \(<: d^k \hat{\Phi}(x)/d^k \hat{\rho}_S(x) :>\). The expansions of Eqs.(5),(6) and (7) allow to settle a truncation scheme. The parameter to fix the approximation order is given by the number of factors \(\hat{\rho}_S(x) = \hat{\psi}(x) \psi(x)\) which are broken in writing Eq.(5) with the expansion (7). For each decoupling there is a trace operation less, leading each time to a quenching factor of 1/8. Moreover in our case the coefficients \(A\) and \(B\) in Eq.(4), to be determined by fitting the bulk properties of nuclear matter, should be small [13]. Therefore, the derivatives \(d^k \Phi(x)/d\rho_S^n(x)\) become smaller and smaller as \(k\) increases.

The expansion Eq.(5) actually implies that contributions of nucleons of the Dirac sea are neglected. This approximation will allow us to derive a closed equation for the Wigner function containing only finite quantities. We discuss this point at the end of the section.

We consider exchange corrections only up to the next–to–leading term, with respect to the Hartree contribution. In this approximation Eq.(5) assumes the form:

\[ <: \hat{\psi}_\beta(x_+) \psi_\alpha(x_-) \hat{\Phi}(x_-) :> = <: \hat{\psi}_\beta(x_+) \psi_\alpha(x_-) :> (\Phi(x_-) - \frac{1}{2} \text{Tr} \hat{F}^2(x_-) \frac{d^2 \Phi(x_-)}{d\rho_S^n(x_-)}) \\
- <: \hat{\psi}_\beta(x_+) \psi_\gamma(x_-) :> [\hat{F}(x_-)]^{\alpha\gamma} \frac{d\Phi(x_-)}{d\rho_S(x_-)}. \]  

where the function \(\Phi(x)\) obeys the equation: \(\Phi(x)/f_S + A\Phi^2(x) + B\Phi^3(x) = \rho_S(x)\).

According to the procedures usually followed in the derivation of kinetic equations (Refs. [22]) (semi-classical approximation), we assume that the densities \(<: \hat{\psi}_\gamma(x_-) \psi_\alpha(x_-) :>\) and the field \(\Phi(x_-)\) are slowly varying functions, and we retain only the first order term in their Taylor expansion at the point \(x\). This is essentially the same kind of assumption previously done for the meson fields and expressed by Eq.(4). In fact because of the small Compton wave–lengths of the heavy mesons \(\sigma\) and \(\omega\), retardation and finite range effects can be neglected.

Within the approximations outlined above Eq.(3) becomes

\[ \frac{i}{2} \partial_\mu \gamma^\mu \hat{F}(x,p) + \gamma^\mu (p_\mu - f_V j_\mu(x)) \hat{F}(x,p) - \left( M - \Phi(x) + \frac{1}{2} \text{Tr} \hat{F}^2(x) \frac{d^2 \Phi(x)}{d\rho_S^n(x)} \right) \hat{F}(x,p) \\
+ \frac{i}{2} \Delta \{ f_V j_\mu(x) \gamma^\mu - \Phi(x) + \frac{1}{2} \text{Tr} \hat{F}^2(x) \frac{d^2 \Phi(x)}{d\rho_S^n(x)} \} \hat{F}(x,p) \\
- f_V \left[ \left( \frac{i}{2} \Delta - 1 \right) \gamma^\mu \hat{F}(x) \gamma_\mu \hat{F}(x,p) \right] + \left[ \left( \frac{i}{2} \Delta - 1 \right) \frac{d\Phi(x)}{d\rho_S(x)} \hat{F}(x) \hat{F}(x,p) \right] = 0, \]  

where \(\Delta = \partial_x \cdot \partial_p\), with \(\partial_x\) acting only on the first term of the products.
This is the kinetic equation for the matrix \( \hat{F}(x,p) \) in mean-field dynamics. In addition to the scalar and vector isoscalar fields, the exchange contributions, which are the terms containing the matrix \( \hat{F}(x) \) in Eq.(9), may give rise to scalar, vector, tensor, pseudoscalar and pseudovector fields (both isoscalar and isovector), leading to completely new effects.

In Eq.(9) we have a general expression for the effective mass, including an isospin contribution. Since the derivatives of mesonic fields have been neglected, the effective mass does not present a momentum dependence.

A quantity of interest in the study of nuclear dynamics is the statistical average of the canonical energy-momentum density tensor. Since terms containing the derivatives of the meson fields are neglected in our approximation, the tensor takes the form:

\[
T_{\mu\nu}(x) = i \frac{2}{\hbar} \bar{\psi}(x) \gamma_\mu \partial_\nu \psi(x) + \{ U(\Phi) - \frac{1}{2} f_V \hat{V}_\lambda(x) \hat{V}^\lambda(x) \} g_{\mu\nu},
\]

where \( g_{\mu\nu} \) is the diagonal metric tensor and \( U(\Phi) = \frac{1}{2} \Phi^2 / f_S + A/3 \Phi^3 + B/4 \Phi^4 \). According to the approximation introduced above, the energy–momentum density tensor, in the HF approximation, reads:

\[
\langle T_{\mu\nu}(x) \rangle = 8 \int d^4p \: p_\nu F_\mu(x,p) + \{ U(\Phi) - \frac{f_V}{2} j_\lambda(x) j^\lambda(x) \} g_{\mu\nu} - \frac{1}{2} [ Tr \hat{F}^2(x) \frac{d^2U(\Phi)}{d\rho_S^2(x)} - f_V Tr (\gamma_\lambda \hat{F}(x) \gamma^\lambda \hat{F}(x)) ] g_{\mu\nu}.
\]

The quantities in square brackets are the exchange contributions. It is essential to note that Fock terms contain traces of powers of \( \hat{F}(x) \) that naturally may bring contributions from all the fields having tensorial properties consistent with the symmetry of the system.

From Eq.(11) we obtain the energy density for symmetric nuclear matter that in analogy to the Hartree case can be rewritten in the following form:

\[
\epsilon = \langle T_{00} \rangle = \frac{4}{(2\pi)^3} \int d^3p \: E^*_p \theta(p_F - |p|) + U(\Phi) + \frac{1}{2} \tilde{f}_S \rho_S^2 + \frac{1}{2} \tilde{f}_V \rho_B^2,
\]

where \( E^*_p = \sqrt{|p|^2 + M^*^2} \), \( \rho_B = j_0 \) is the baryon density and \( p_F \) is the Fermi momentum. \( M^* \) is the nucleon effective mass:

\[
M^* = M - \Phi - \left( \frac{1}{2} f_V - \frac{1}{8} \frac{d\Phi}{d\rho_S} \right) \rho_S + \frac{1}{16} \left( \rho_S^2 + \rho_B^2 \right) \frac{d^2\Phi}{d\rho_S^2}.
\]

The scalar density \( \rho_S \) is given by

\[
\rho_S = \frac{4}{(2\pi)^3} M^* \int d^3p \theta(p_F - |p|).
\]

The density dependent coupling constants \( \tilde{f}_S \) and \( \tilde{f}_V \) are defined as follows:

\[
\tilde{f}_S = \frac{1}{2} f_V - \frac{1}{8} \left( \frac{d\Phi}{d\rho_S} + \rho_S \frac{d^2\Phi}{d\rho_S^2} \right);
\]

\[
\tilde{f}_V = \frac{5}{4} f_V + \frac{1}{8} \left( \frac{d\Phi}{d\rho_S} - \rho_S \frac{d^2\Phi}{d\rho_S^2} \right).
\]
To give some indications about the convergency of the truncation in Eq.(8), we plot in Fig.1 the density behaviour of the coupling functions $\tilde{f}_S$ and $\tilde{f}_V$, obtained using for $f_S$, $f_V$, $A$ and $B$ the parameterization given in chapter III. The dashed lines represent the results obtained neglecting the terms containing the second derivative of $\Phi$ in Eq.(13), while the full lines correspond to the complete calculations (Eq.(13)). Up to densities of the order of $4\rho_0$, it is possible to observe that, for the parameterization considered, the inclusion of the second derivatives changes slightly the results. Therefore in such a density domain one expects a even smaller contribution from the next terms of the expansion of Eq.(6), since they contain higher order derivatives.

In order to establish a link with previous works on the introduction of Fock terms in QHD theories, we will consider only the linear term in the equation for the scalar field, Eq.(4), where this kind of calculations have been performed so far [10,24-27], [28]. Our results are clearly reducing to the ones reported in Ref.[8]. The single–particle energies and the nucleon effective mass are given by:

$$\epsilon_p = E_p^* + f_V \rho_B + \left[ \left( \frac{1}{4} f_V + \frac{1}{8} f_S \right) \rho_B \right] = E_p^* + \tilde{f}_V \rho_B$$

(14)

and

$$M^* = M - f_S \rho_S - \left[ \left( \frac{1}{2} f_V - \frac{1}{8} f_S \right) \rho_S \right] = M - \left( f_S + \tilde{f}_S \right) \rho_S,$$

(15)

respectively, where now $\tilde{f}_S$ and $\tilde{f}_V$ of Eq.(13) are simply given by: $\tilde{f}_S = -\frac{1}{8} f_S + \frac{1}{2} f_V$, $\tilde{f}_V = \frac{1}{8} f_V + \frac{5}{4} f_V$. The energy density, $\epsilon = \langle : T_{00} : \rangle$, is expressed by:

$$\epsilon = \frac{4}{(2\pi)^3} \int d\mathbf{p} E_p^* \theta(p_F - |\mathbf{p}|) + \frac{1}{2} f_S \rho_S^2 + \frac{1}{2} f_V \rho_B^2$$

$$+ \frac{1}{2} \left[ \left( \frac{1}{2} f_V - \frac{1}{8} f_S \right) \rho_S^2 + \left( \frac{1}{4} f_V + \frac{1}{8} f_S \right) \rho_B^2 \right]$$

$$= \frac{4}{(2\pi)^3} \int d\mathbf{p} E_p^* \theta(p_F - |\mathbf{p}|) + \frac{1}{2} \left( f_S + \tilde{f}_S \right) \rho_S^2 + \frac{1}{2} \tilde{f}_V \rho_B^2,$$

(16)

with the exchange contributions in Eqs.(14),(15) and (16) given by the terms in square brackets.

We remark that the same expressions for $E_p^*$, $M^*$ and $\epsilon$ can be obtained from the results of the HF calculations of Ref. [24] neglecting the square of the four–momentum transfer between two interacting nucleons with respect to the squared meson masses. Hence for linear models of QHD our approach is equivalent to the usual HF approximation, once retardation and finite–range effects are neglected.

The two–loop approximation for nuclear matter [25] was introduced to study the loop expansion scheme for QHD. According to this approximation the contribution of exchange diagrams to the energy density is evaluated using nucleon propagators in the Hartree approximation (i.e. at one loop level). Neglecting vacuum, retardation and finite range effects, the results of Ref.[25] can be reproduced by using a similar approximation in our approach. This amounts to neglect the exchange terms in the equation for the equilibrium Wigner function,
and to retain the exchange contributions in the equation for the energy density, Eq. (16). Therefore, within this approximation, the exchange terms in the nucleon self-energy are disregarded. However it should be noticed that their contribution to the energy density, i.e. the second term in the square bracket of Eq. (16), $\Delta \epsilon = \left(\frac{1}{4} f_V + \frac{1}{8} f_s\right) \rho_B^2$, is comparable to the exchange contributions that are included in [25] and hence it should not be neglected. A similar situation occurs in the self-consistency equation for the nucleon effective mass.

Within the formalism used in the present paper, vacuum effects could be taken into account by omitting the normal ordering and subtracting a vacuum term in the definition of the Wigner function [23]. Such a subtraction does not eliminate all the divergent vacuum terms in the equations for the Wigner function and the energy–momentum density tensor, thus a renormalization procedure in necessary [23]. Since, as stressed before, our approach is equivalent to the HF approximation, we should expect to get results analogous to those obtained in [23,27]. A fully self–consistent HF description of QHD including vacuum effects is very complicated [27] and has not been achieved yet. On the other hand, two–loop corrections to the Hartree approximation give unnaturally large vacuum contributions to the energy of nuclear matter [24]. This would suggest to follow other paths for representing the vacuum dynamics. For a review on these points see also Ref. [4].

However, it is generally recognized that the RMF approximation, where negative energy states of nucleons are not considered, is very successful in describing properties both of nuclear matter and of finite nuclei, once self–interaction terms of the $\sigma$–field are included. The approach presented here can be considered as an extension of the RMF approximation to include exchange terms on the same basis.

### III. INFLUENCE OF FOCK TERMS ON THE NUCLEAR EQUATION OF STATE

The study of the nuclear Equation of State (EOS) can be performed by evaluating the statistical average of the energy–momentum tensor, Eq. (11).

Specializing to the case of symmetric nuclear matter, we have tested that our procedure to introduce the Fock terms leads to a thermodynamically consistent theory. The equilibrium Wigner function together with the single–particle energy spectrum can be obtained following the scheme of Ref. [8]. We have found that the relation between pressure and energy density:

$$P = \rho_B \frac{d\epsilon}{d\rho_B} - \epsilon = \frac{1}{3} :T_{ii}:$$

and the Hugenholtz–Van Hove (HV) theorem [29]:

$$\mu = \frac{d\epsilon}{d\rho_B} = \epsilon_F$$

are satisfied. Here $\epsilon = :T_{00}:$, $\rho_B = j^0$ is the baryon density and $\epsilon_F$ is the Fermi energy. This actually represents a good check of the consistency in the truncation procedure used. We remind that the explicit inclusion of meson field derivatives will make more delicate the proof of the HV theorem [30].

In order to illustrate how the properties arising from the nucleonic intrinsic degrees of freedom can be described with our approach, we now present some results about the EOS
of asymmetric nuclear matter at equilibrium. The matrix $\hat{F}_{\alpha\beta}(x,p)$ can be decomposed in the isospin space as:

$$\hat{F}_{\alpha\beta}(x,p) = (\hat{F}_s)_{\alpha'\beta'}(x,p) + \tau_3(\hat{F}_3)_{\alpha'\beta'}(x,p),$$

where $\tau_3$ is the third Pauli matrix, $(\alpha\beta)$ are spin-isospin indices, while $(\alpha'\beta')$ denote spin indices. In symmetric nuclear matter the matrix $\hat{F}_3(x,p)$ vanishes, whereas in the asymmetric case its components give the various isovector densities. Due to the asymmetry of the system, the kinetic energy density increases because the Fermi momenta of protons and neutrons are different. The presence of the exchange terms (terms in square bracket in Eq.(11)) gives rise also to a contribution from the potential part of the energy density $\epsilon$.

The symmetry energy can be defined as

$$E_{\text{sym}} = \frac{1}{\rho_B} \frac{\partial^2 \epsilon}{\partial I^2} |_{I=0},$$

where $I$ is the asymmetry parameter, defined as the ratio $((\rho_B)_n - (\rho_B)_p)/\rho_B$ ($n$ stands for neutrons and $p$ for protons). In Fig.1 we have reported the calculated symmetry energy as a function of the baryon density $\rho_B$. In the calculations the four parameters of the model ($f_S$, $f_V$, $A$ and $B$) have been fitted in order to reproduce equilibrium properties of symmetric nuclear matter: saturation density $\rho_0 = 0.16 fm^{-3}$, binding energy $E/A=-15.75$ MeV, nucleon effective (or Dirac) mass at $\rho_0$, $M^*_0 = 0.7 M$, incompressibility modulus $K_0 = 250 MeV$. In this way we obtain the following value for the parameters: $f_S = 10.1$, $f_V = 4.1$, $A = 0.06$ and $B = 0.01$. In the scalar field contribution we clearly see a steady decrease of the weight of higher order terms. This seems to be a qualitative signal of a naturalness in the expansion. However, a more accurate discussion should be done only in a model that includes non–linearly both scalar and vector fields [13,31].

A quite interesting feature of our approach is that now the asymmetric nuclear matter is described relatively well with the same parameters fixed by the fit. Figure 2 shows that our non–linear HF (NLHF) evaluation (solid line) gives a symmetry energy value at normal density, i.e. the coefficient of the Weiszaecker mass formula of $a_4 = 24 MeV$. This result appears too low with respect to the accepted value (around $28 - 30 MeV$), but we have to consider that only the contribution of isoscalar mesons are included. From this point of view our calculation shows the importance of exchange terms on the symmetry energy, even though the inclusion of the $\rho$ meson is necessary to achieve a good value of $a_4$.

To be more quantitative, considering that the kinetic contribution to the $a_4$ parameter is of the order of $12 MeV$ ($\simeq \epsilon_F/3$), of the remaining part of about $16 MeV$ almost two thirds seem to come from exchange terms. This appears to be an interesting indication of a reduced contribution of the $\rho$ meson strength in nuclear matter around normal density.

A rather repulsive density dependence is also deduced (Fig.2; full line). Non-linear Hartree (NLH) calculations including the $\rho$ meson (dashed line) can reproduce the same $a_4$ value once the coupling constant $f_\rho$ is suitably fitted. However in the NLHF calculations we also obtain a non–linear density dependence of the symmetry energy that arises from the non–linear scalar self-interactions through exchange terms. This effect is not present in NLH calculations.

**IV. CONCLUSIONS**

We have introduced a procedure to investigate the role of the Fock exchange terms in a relativistic model of fermions coupled to mesons, including self-interactions for the mesonic fields. We have shown that the evaluation of the Fock terms, within a suitable truncation
scheme, leads to a consistent description of the equilibrium and dynamical properties related to the fermionic intrinsic degrees of freedom. An application to the case of asymmetric nuclear matter at equilibrium gives satisfactory results concerning the density dependence of the symmetry energy. It should be noticed that also other properties, such as the isospin and density dependence of effective masses, that can be deduced from the stationary solution of the kinetic equation, are reasonably reproduced within our procedure, in comparison to QHD models including also scalar isovector mesons. A quite interesting general feature is appearing: a consistent inclusion of exchange contributions seems to clearly reduce the weight of charged mesons in describing isospin properties of nuclear matter around and above normal barion density.

Finally we remark that the aim of this work is mainly to present a procedure to evaluate Fock terms of a non–linear field coupling scheme. For this reason we have omitted a full analysis of all possible features of a EOS calculation; in particular our model is subject to the possible criticism of all the approaches with non–linearities only in the scalar field. However it is straightforward to extend the same procedure to non–linear vector fields. Thus we stress that the method described here is very general and can be applied to effective field theories not only for nuclear matter but in general to systems with fermion-boson dynamical couplings. Of course, the analysis can be extended also to the investigation of spin effects.
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**Figure Caption**

**Fig.1:** Density behaviour of the coupling functions $\tilde{f}_S$ (bottom panel) and $\tilde{f}_V$ (top panel). See the text for details.

**Fig.2:** Symmetry energy per nucleon vs. baryon density. Full line: present $NLHF$ calculation. Dashed line: $NLH$ calculation including the $\rho$ meson.
