If \((M, \xi)\) is a compact connected co-oriented contact 3-manifold whose boundary \(\partial M\) is diffeomorphic to the 2-torus, then \(\partial M\) is said to admit a contact embedding into \((\mathbb{R}^3, \xi_{st})\) if there exists a co-orientation preserving contact embedding \(\varphi\) of a neighborhood \(U \subset (M, \xi)\) of \(\partial M\) into \((\mathbb{R}^3, \xi_{st})\) such that (i) the interior of \(\varphi(U)\) is mapped into the bounded component of \(\mathbb{R}^3 \setminus \varphi(\partial M)\) and (ii) affine lines parallel to the \(z\)-axis intersect \(\varphi(\partial M)\) in at most two points. A contact form \(\alpha\) is called aperiodic if the associated Reeb vector field \(\xi = \ker \alpha\) does not have any periodic solution. A contact form \(\alpha\) on \(M\) is called standard near the boundary if the restriction of \(\varphi\) to a possibly smaller neighborhood \(U\) of \(\partial M\) pulls \(\alpha_{st}\) back to \(\alpha|_U\). If \((M', \xi')\) is a closed connected contact 3-manifold and \(K \subset (M', \xi')\) is a transverse knot, then there exists a contact embedding \(f : (S^1 \times \mathbb{R}^2, \xi_{st}) \to (M', \xi')\) that is positive and sends \(S^1 \times \{0\}\) to \(K\). The image of the restriction of \(f\) to \(S^1 \times D^2\) is the tubular neighborhood \(\nu K = f(S^1 \times D^2) \subset M'\), which is unique up to smooth isotopies. The exterior of the transverse knot \(K \subset (M', \xi')\) is defined as \(M = M'|_{\text{Int}(\nu K)}\). The boundary \(\partial M \subset (M, \xi)\) of any knot exterior admits a contact embedding into \((\mathbb{R}^3, \xi_{st})\). If \(K\) is a knot in a closed connected oriented 3-manifold \(M'\), then \(M'\) admits infinitely many positively co-oriented contact structures such that \(K\) is a transverse knot. In other words, any knot appears as a transverse knot for a certain contact structure.

In this paper, the authors characterize the unknot in \(S^3\) uniquely in terms of symplectic dynamics on the knot exterior. The main result of the paper states that if \(K\) is a transverse knot in a closed connected co-oriented contact 3-manifold \((M', \xi')\) and the knot exterior \((M, \xi)\) of \(K \subset (M', \xi')\) admits an aperiodic \(\xi\)-defining contact form \(\alpha\) such that \(\alpha\) is Euclidian near the boundary \(\partial M\), then \((M', K)\) is diffeomorphic to \((S^3, \{z_1 = 0\})\) with orientations preserved so that \(K\) is the unknot.

Reviewer: Andrew Bucki (Edmond)

MSC:

- 57K10 Knot theory
- 57K33 Contact structures in 3 dimensions
- 53D35 Global theory of symplectic and contact manifolds

Keywords:

- exterior of knot; near the boundary form; aperiodic contact form

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