The model of dynamic mechanical behaviour of brittle solids based on kinetic theory of strength

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Abstract. The paper presents the kinetic theory-based model of dynamic mechanical behavior of brittle solids. The model adopts the ideas of the kinetic theory of strength which postulate the finite time of nucleation of discontinuities and relaxation of local stresses in the material. The model is a generalization of the formalism of classical "quasi-static" Nikolaevsky's plasticity model (non-associated plastic flow rule with the plasticity criterion in the form of Mises-Schleicher) and the Drucker-Prager model of failure to describe the material behavior at strain rates corresponding to dynamic loading. Three new dynamic (time) parameters are used to describe the dynamic mechanical behavior of brittle material. The dynamic model developed in the paper can be implemented within the framework of various Lagrangian numerical methods using an explicit integration scheme (including finite and discrete element methods).

1. Introduction

The numerical simulation is a promising tool for the study of material behavior under dynamic loading. The widespread numerical methods for the simulation of material response to dynamic loading are finite element and finite difference methods. The mathematical formalisms of these methods make possible numerical solution of the complex problems of solid mechanics, including dynamic one. At the same time application of these methods to dynamic problems meets difficulties at the simulation of multiple fractures accompanied by mass mixing and mass transfer.

An effective tool for the numerical solution of the problems related to complex fracture of solids is the discrete element method (DEM). Despite the intensive use of DEM for the numerical study of deformation and fracture of brittle materials and media, mathematical formalism of this method is limited by the quasi-static models of material behavior. Therefore the fields of application of the DEM are limited by the strain rate interval \( \dot{\varepsilon} < 10^3 \) s\(^{-1}\). Therefore the aim of this work is the development of the mathematical formalism of the DEM to simulate the behavior of brittle solids under dynamic impact. We developed the model of the dynamic behavior of brittle solids based on the kinetic theory of strength, which was first proposed by Zhurkov and developed in the works of Morozov and Petrov. The model takes into account strain rate sensitivity of inelastic and strength parameters and allows modeling of the dynamic behavior of brittle solids at \( \dot{\varepsilon} < 10^5 \) s\(^{-1}\). The developed model is the modification of the classic "static" Nikolaevsky’s plasticity model and the failure model with a two-parameter criterion of Drucker and Prager [1, 2]. A feature of the proposed dynamic model is the use...
of the “stress” form of the yield and failure criteria instead of the integral form that is traditionally used in the kinetic theory of strength.

Dynamic behavior of material is conventionally described through the dependences of model parameters on strain rate. Note that the strain rate is a technical parameter of loading, which characterizes average (smoothed) deformation of a sample. At the same time there is a physical parameter, which characterizes the process of degradation/fracture of material at the considered spatial scale under applied loading. This is the stress relaxation time. Since the main mechanism of stress relaxation in brittle materials is the nucleation of microdamages and the formation of a cracks system, the time of these processes can be considered equal to the corresponding local value of the stress relaxation time. Therefore the relaxation time as physical parameter of degradation/fracture is preferable to be used as a key parameter of proposed dynamic behavior model.

2. Dynamic formulation of mechanical behavior model

2.1. Elastic properties

The elastic parameters of the material are also sensitive to the strain rate. However, this effect is manifested at sufficiently high strain rates (>10³ s⁻¹), as well as for materials with a high degree of initial damage. In this paper, we consider only consolidated brittle materials with low initial damage, and therefore the dynamic variation of the elastic moduli of the material is not taken into account in the developed model.

2.2. Dynamic formulation of fracture criteria

The modern models of inelastic behavior of brittle materials based on the kinetic theory of strength usually use integral fracture criteria. These criteria are based on calculating the load increment over time \( \tau \), which is called the incubation fracture time [3]. The parameter \( \tau \) is interpreted as the time period over which a main crack is formed at a constant load equal to the static strength of the material \( \sigma_{\text{lim}}^{\text{st}} \). The integral fracture criterion can be written as:

\[
\int_{t'}^{t} \sigma(t') dt' = \sigma_{\text{lim}}^{\text{st}} \tau ,
\]

where \( \sigma(t') \) is the scalar parameter of the stress state, and \( t \) is the time. The parameter \( \sigma \) in the different implementations of criterion is either the maximum principal stress or a combination of the stress tensor invariants. Numerical implementation of criterion (1) in this form is quite difficult. Therefore, in this paper we propose a simplified representation of dynamic fracture criterion (1):

\[
\sigma(t) \geq \sigma_{\text{lim}}^{\text{dyn}}(t - t_0) H(t_0 + \tau - t) ,
\]

where \( \sigma_{\text{lim}}^{\text{dy}}n \) is the dynamic strength of the material, and \( H(t_0 + \tau - t) \) is the Heaviside function. This formulation of criterion (1) implies that fracture begins when the parameter \( \sigma \) reaches the static strength at the time point \( t_0 \). The duration of fracture process is determined by the dynamics of the change in the parameter \( \sigma \) at \( t > t_0 \). By the time of main crack formation (at \( t = t_0 \)), the parameter \( \sigma \) will reach a value of \( \sigma_{\text{lim}}^{\text{dy}}n \geq \sigma_{\text{lim}}^{\text{st}} \) (the particular value of \( \sigma_{\text{lim}}^{\text{dy}}n \) depends on the stress variation dynamics within the time interval \( (t_0 - t_0) \)).

Suppose that the strain rate \( \dot{\epsilon} \) (as well as the growth rate of the parameter \( \sigma \)) does not change significantly during the time of crack formation \( (t_0 - t_0) \). With this assumption, the fracture time, the local strain rate, and the dynamic strength \( \sigma_{\text{lim}}^{\text{dy}}n \) are uniquely related. The value of any of these parameters \( (\sigma_{\text{lim}}^{\text{dy}}n , (t_0 - t_0) , \dot{\epsilon} ) \) can be uniquely determined if the values of the other two parameters are known. Hence, the fracture time in case of constant-rate uniaxial compression can be calculated:
\[ T_{\text{fract}} = t_t - t_0 = \frac{\sigma_{\text{c dyn}}^{\text{fract}} - \sigma_{\text{c stat}}^m}{E \dot{\varepsilon}}, \]  

where \( \sigma_{\text{c stat}}^m \) and \( \sigma_{\text{c dyn}}^{\text{fract}} \) are the static and dynamic strength of the material under uniaxial compression, \( E \) is the Young’s modulus, \( \dot{\varepsilon} \) is the axial strain rate, and \( T_{\text{fract}} \) is the fracture time (different from the incubation time \( \tau \) in the general case). An analogous formula can also be derived for uniaxial dynamic tension with constant rate.

The main issue that arises when applying this fracture model to solving applied problems is the determination of the dependence \( \sigma_{\text{c dyn}}^{\text{fract}}(T_{\text{fract}}) \). There are different ways to determine this dependence. For example, in the earlier paper [4] we propose to estimate the required dependencies with use of the available databases of experimental data about the strain rate dependences of the mechanical characteristics of various brittle materials. Emphasize that these dependences are nonlinear and monotonically decreasing.

Suppose that we have obtained the \( \sigma_{\text{c dyn}}^{\text{fract}}(T_{\text{fract}}) \) dependence for a considered stress state (different from uniaxial loading in the general case) which corresponds to constant strain rate loading. Using approximations mentioned above we propose the following approach to the numerical implementation of dynamic fracture criterion (2) within the explicit formulation of DEM.

We assume in this method that material fracture does not occur at the time point \( t_0 \) when the static strength \( \sigma = \sigma_{\text{c stat}}^m \) is reached, but after some time during which internal transient processes (incubation of fracture) take place in the considered local region of the material. These processes will result in the formation of new free surfaces (material fracture), leading to a decrease in the resistance to external load and stress relaxation. In this case, the considered local region preserves its continuity during the fracture time \( T_{\text{fract}} \). As noted above, the approximation of a constant strain rate during \( T_{\text{fract}} \) ensures a single value of the dynamic strength corresponding to the current rate of change of the parameter \( \sigma \) and therefore the only value of \( T_{\text{fract}} \) (figure 1).

The end point of main crack formation is determined by checking the following inequality at each instant \( t > t_0 \) after the static strength is reached:

\[ \sigma(t) \geq \sigma_{\text{c dyn}}^{\text{fract}}(T_{\text{fract}}), \]  

where \( \sigma(t) \) is the current local value of the stress state parameter, \( T_{\text{fract}} = t - t_0 \), and \( \sigma_{\text{c dyn}}^{\text{fract}} \) is determined from the “calibration” dependence \( \sigma_{\text{c dyn}}^{\text{fract}}(T_{\text{fract}}) \). When condition (4) is satisfied (at the time point \( t_0 \)), the dynamic strength of the material is considered to be achieved, the incubation process is complete, and the material is locally fractured. The corresponding values of \( T_{\text{fract}} = t - t_0 \) and \( \sigma_{\text{c dyn}}^{\text{fract}} \) are considered as local values of the fracture time and dynamic strength.

In this paper, the force parameter \( \sigma \) is the combination of the first and second invariants of the stress tensor, proposed by Drucker and Prager. Condition (4) is therefore formulated as follows:

\[ \sigma_{\text{fr}}(t) = \sigma_{\text{eq}}(t)0.5(a+1) + \sigma_{\text{mean}}(t)1.5(a-1) \geq \sigma_{\text{c dyn}}^{\text{fract}}(T_{\text{fract}}), \]  

\[ \begin{align*}
\sigma_{\text{fr}}(t) &= \sigma_{\text{eq}}(t)0.5(a+1) + \sigma_{\text{mean}}(t)1.5(a-1) \\
&\geq \sigma_{\text{c dyn}}^{\text{fract}}(T_{\text{fract}}),
\end{align*} \]
where \( a = \sigma_c^{\text{dyn}} / \sigma_t^{\text{dyn}} \), \( \sigma_c^{\text{dyn}} = \sigma_c^{\text{dyn}} (T_{\text{frac}}) \) is the dynamic compressive strength, and \( \sigma_t^{\text{dyn}} = \sigma_t^{\text{dyn}} (T_{\text{frac}}) \) is the dynamic tensile strength.

Dynamic criterion (5) should be applied in combination with quasi-static criterion. In this case, conventional static criterion serves to determine the initial time point \( t_0 \), and criterion (5) is used to determine the end time point of fracture incubation.

### 2.3. Dynamic formulation of the model of plasticity

Experimental data indicate that not only the strength, but also the inelastic properties of brittle materials, such as cohesion and yield strength, are sensitive to the loading rate [5]. This is because the inelastic response of brittle materials is associated with the formation of a distributed system of internal discontinuities (different-scale damages and microcracks) and their development (growth and coalescence). The nucleation and development of discontinuities last for a finite time. Therefore, it is necessary to account for the dependence of the parameters of the applied plasticity model on the time of formation/development of the system of discontinuities in order to adequately describe the inelastic deformation of brittle materials under dynamic loading.

In this paper, a quasi-static Nikolaevsky model is used as the mathematical basis for developing the dynamic model of inelastic mechanical behavior of brittle materials. In the model, the limiting state is reached when the Mises–Schleicher criterion achieves the cohesion value \( Y \). The change in the yield strength with increasing strain rate is described by introducing the dependence of the cohesion value on the time parameter \( T_{\text{delay}} \), analogous to the fracture time \( T_{\text{frac}} \). The physical meaning of the parameter \( T_{\text{delay}} \) is the time of nucleation of the first microdamages that provide the inelastic behavior. Hence, as in the dynamic fracture model, we assume that the onset of the limiting state (the onset of local stress relaxation) occurs not at the instant \( t_0 \) of reaching the static yield strength \( \Phi = Y^0 \), but after the time \( T_{\text{delay}} \).

In the numerical implementation of the plasticity model, the beginning point of the formation of a system of discontinuities is determined by checking the following inequality at each instant \( t > t_0 \) after reaching the static cohesion value:

\[
\Phi(t) \geq Y^{\text{dyn}}(T_{\text{delay}}),
\]

where \( \Phi(t) \) is the current local value of the plasticity criterion, \( T_{\text{delay}} = t - t_0 \), and the current value of \( Y^{\text{dyn}} \) is determined from the specified “calibration” dependence \( Y^{\text{dyn}}(T_{\text{delay}}) \). The “calibration” curve \( Y^{\text{dyn}}(T_{\text{delay}}) \) can be obtained by using the experimental \( Y^{\text{dyn}}(\dot{e}) \) dependences for the considered material and a formula similar to equation (3).

When condition (6) is satisfied, the dynamic yield strength is assumed to be reached, and local stress relaxation begins in the volume due to the formation (development) of a spatially distributed system of different-scale discontinuities. The time duration of local stress relaxation can be characterized by the time parameter \( T_{\text{relax}} \), which will be called below the stress relaxation time (or relaxation time). The end result of the stress relaxation is that the value of the parameter \( \Phi \) decreases to the static yield strength value (\( \Phi = Y^0 \)).

The derivation of mathematical equations describing the stress relaxation kinetics, which contain the parameter \( T_{\text{relax}} \), is a separate problem. The general solution of this problem is beyond the scope of this paper. For the considered model of plasticity and its numerical implementation (Wilkins algorithm), we propose a simple kinetic model of time-distributed local stress relaxation. This model is based on the approximation of a linear dependence of the stress relaxation rate \( \Phi_{\text{relax}} \) on the magnitude of “deviation” of the parameter \( \Phi \) from the equilibrium value of \( Y^0 \) (i.e., from the difference (\( \Phi - Y^0 \))).

According to the conventional implementation of Wilkins algorithm of the static plasticity model the stresses are returned to the yield surface point \( Y^0 \) in a time step. In the proposed dynamic model, the stresses are returned to the yield surface gradually, during the relaxation time. In the linear kinetic relaxation model, the returning of the parameter \( \Phi \) to the yield surface point \( Y^0 \) begins after inequality...
(6) is satisfied. The decrease in the value of the parameter $\Phi$ in the time step $\Delta t$ due to relaxation is written as:

$$
\Delta \Phi = \Phi_{\text{relax}} \Delta t = \left( \Phi - Y^s \right) \frac{\Delta t}{T_{\text{relax}}}. 
$$

(7)

The increment $\Delta \Phi$ is the reduction value at the current time step for Wilkins algorithm. Within the linear approximation to the description of the relaxation process, it is assumed that the value of $T_{\text{relax}}$ is determined only by the “initial conditions” of relaxation (particularly, by $Y^\text{dyn}$ in criterion (6)) and does not depend on the current deviation value ($\Phi - Y^s$). Unlike $T_{\text{fract}}$ and $T_{\text{delay}}$, the particular values of $T_{\text{relax}}$ should be determined by analyzing not only the data on the dynamic strength and dynamic yield point of the material, but also the total stress-strain curves obtained at different strain rates.

The proposed kinetic approach to describing the inelastic deformation and fracture of brittle materials under dynamic loading is in fact a generalization of the formalism of static plasticity and strength models which takes into account the finite (albeit small) time of the relaxation processes. The generalization provides the possibility of DEM-based simulation both quasi-static and dynamic processes within a single (general) formalism.

3. Verification of the proposed dynamic behavior model

3.1. Numerical method and description of numerical tests

The proposed model of dynamic mechanical behavior of brittle materials was verified in a series of numerical tests using the movable cellular automaton (MCA) method that belongs to the group of methods of simply (homogeneously) deformable discrete elements [1]. Constant-rate uniaxial compression and tension tests on two-dimensional rectangular samples ($H = 40$ mm, $L = 77$ mm (figure 2a)) were simulated. The calculations were carried out in the plane stress approximation.

![Figure 2](image)

Figure 2. Sample loading scheme (a), and loading rate vs. time curve (b)

The strain rate $\dot{\varepsilon} = V/H$ in different tests varied from $10^{-3}$ s$^{-1}$ (which was assumed to correspond to quasi-static loading) to $10^3$ s$^{-1}$. At the initial stage of loading, the punch displacement velocity was increased sinusoidally from 0 to $V$ (figure 2b) to achieve a homogeneous stress state of the sample at the beginning of inelastic behavior. The end time of the accelerated punch motion $t'$ was chosen so that the axial stress magnitude did not exceed 0.75$\sigma^s$ by this time point, where $\sigma^s$ is the static yield strength of the material for a considered sign of uniaxial loading (tension or compression).

3.2. Verification of strength and plasticity criteria

The numerical compressive/tensile strength tests were performed on 2D samples of elastic-brittle model materials whose elastic and strength properties were close to the mechanical properties of...
sandstone and normal strength concrete (Table 1). The maximum registered axial stress was taken as the compressive or tensile strength of the sample.

**Table 1.** Elastic and strength characteristics of model materials.

|          | Density, kg/m³ | E, GPa | Poisson’s ratio | σ_c, MPa | σ_t, MPa |
|----------|----------------|--------|----------------|----------|----------|
| Concrete | 4660           | 38.6   | 0.194          | 45.8     | 21.0     |
| Sandstone| 2200           | 16.0   | 0.280          | 70.0     | 31.5     |

The registered strength of the sample was normalized to the corresponding strength of this sample obtained from a “quasi-static” test (at \( \dot{\varepsilon} = 10^{-3} \) s\(^{-1}\)). The recorded data were used to obtain the strain rate dependences of the reduced compression and tensile strengths for model material samples (figure 3). The numerical simulation results for materials with different strengths and elastic moduli are seen to agree well with generalized experimental data [5].

![Figure 3](image)

**Figure 3.** Strain rate dependences of the reduced value of compressive strength (a) and tensile strength (b) for concrete and sandstone samples. The dashed line shows the approximation of experimental data from [5].

The proposed dependence of the cohesion value (\( Y \)) on the nucleation time of the first microdamages (\( T_{\text{delay}} \)) was verified in tests on model material samples whose uniaxial loading curves have the stage of inelastic deformation. The mechanical characteristics of the model elastic-plastic materials are presented in table 2.

**Table 2.** Elastic and yield parameters of model materials.

|          | Density, kg/m³ | E, GPa | Poisson’s ratio | \( Y^{\text{y}} \), MPa | \( \alpha \) |
|----------|----------------|--------|----------------|--------------------------|-----------|
| Concrete | 4660           | 38.6   | 0.194          | 8.0                      | 0.63      |
| Sandstone| 2200           | 16.0   | 0.280          | 15.4                     | 0.57      |

The simulated samples were subjected to constant-rate uniaxial compression. They were deformed until the dynamic yield strength \( \sigma_{y}^{\text{dyn}} \) was reached. The dynamic yield strength of the sample was the recorded value of the load resistance force, which corresponded to the transition to the inelastic portion of the stress-strain curve, normalized to the sample end surface area. Under uniaxial compression with the specified loading rate, the relationship between the sample yield strength \( \sigma_{y}^{\text{dyn}} \) and the cohesion value \( Y^{\text{y}} \) is determined by equation:

\[
Y^{\text{y}} = \frac{\sigma_{y}^{\text{dyn}}}{\sqrt{3}} \left( 1 - \frac{\alpha}{\sqrt{3}} \right),
\]
Figure 4 shows the dynamic cohesion values $Y_{\text{dyn}}$ normalized to the “quasi-static” value $Y_{\text{st}}$ obtained for the model material samples at various strain rates. As in the case of strength parameters, there is good agreement between numerical simulation results and experimental data.

3.3. Example of $T_{\text{relax}}$ estimation for brittle material
The key parameter of the proposed relaxation rule (7) is the local stress relaxation time $T_{\text{relax}}$. Within the linear approximation, $T_{\text{relax}}$ is assumed to be a function of only the “initial conditions” of the relaxation process. The parameters characterizing the “initial conditions” of relaxation include, in particular, the magnitude of the elastic energy supplied accumulated in the loaded material within the time period $T_{\text{delay}}$ between the achievement of the static and dynamic cohesion values (i.e., during the nucleation time of the first discontinuities). The magnitude of this energy in the developed model can be described by the ratio $Y_{\text{dyn}}/Y_{\text{st}}$, which is uniquely related to the value of the time parameter $T_{\text{delay}}$. Therefore, it seems logical to relate the relaxation time $T_{\text{relax}}$ to $T_{\text{delay}}$. This dependence can be obtained from the analysis of experimental data on the dynamic loading of materials.

In this paper, the $T_{\text{relax}}(T_{\text{delay}})$ dependence was constructed for a high-strength concrete (C80), whose mechanical properties were studied by Guo [6]. Guo et al. provided necessary experimental data on the static strength and inelastic properties of concrete C80, and data on dynamic loading at strain rates of 40 s$^{-1}$, 60 s$^{-1}$, and 110 s$^{-1}$ (dashed lines in figure 5).

Assuming that the relaxation time $T_{\text{relax}}$ is a function of only the time parameter $T_{\text{delay}}$, we can define a $T_{\text{relax}}$ value for any of the three shown dynamic loading curves (red, green and blue dashed lines in figure 5). This can be done by selecting the value of $T_{\text{relax}}$ at which the constant-rate uniaxial compression curve obtained in numerical simulation coincides with the corresponding experimental curve. By selecting the $T_{\text{relax}}$ values for all three strain rates and adding limit values to these data (e.g., $\dot{\varepsilon} = 10^3$ s$^{-1}$), we can find the form of the dependence $T_{\text{relax}}(T_{\text{delay}})$ and obtain an approximating function.
The required dependence \( T_{\text{rel}}(T_{\text{delay}}) \) was obtained by simulating the constant-rate uniaxial compression of two-dimensional samples. The loading scheme and sample dimensions are shown in figure 2. The static mechanical characteristics of sample discrete elements are corresponded to the static characteristics of concrete C80 (black dashed line in figure 5). The “calibration” dependences \( \sigma_{\text{dyn}}(T_{\text{fract}})/\sigma_{\text{fract}}^* \), \( \sigma_{\text{fract}}^*(T_{\text{fract}})/\sigma_{\text{fract}}^* \), and \( \sigma_{\text{fract}}^*/Y_{\text{fract}} \) determined in the paper [4] were used.

The simulation results provided data on the relaxation times for the three strain rates: \( T_{\text{relax}} = 11 \mu s \) at \( \dot{e} = 40 \text{ s}^{-1} \), \( T_{\text{relax}} = 8.56 \mu s \) at \( \dot{e} = 60 \text{ s}^{-1} \), and \( T_{\text{relax}} = 5.4 \mu s \) at \( \dot{e} = 110 \text{ s}^{-1} \). The stress relaxation time value at \( \dot{e} = 10^3 \text{ s}^{-1} \) was determined from general considerations. At high strain rates, the relaxation time tends to its minimum for the considered material. This time can be estimated for concrete based on the growth rate of a microcrack in the sample which propagates along the phase boundaries with a velocity not exceeding the Rayleigh wave velocity. The characteristic size of a structural element (aggregate) in concrete C80 is several millimeters, and the Rayleigh wave velocity in concrete is about 2600 m/s. Then, the formation time of a local interface crack at the phase interface (which is associated with the relaxation time \( T_{\text{relax}} \) at high \( \dot{e} \)) should be at least 0.5 \( \mu s \). A good approximation of the obtained set of the \( T_{\text{relax}}(T_{\text{delay}}) \) points is achieved using the exponential dependence:

\[
T_{\text{relax}}(T_{\text{delay}}) = 0.47 - 0.0025 \dot{e}^{T_{\text{delay}}/0.78},
\]

(9)

The simulation results for the uniaxial compression of C80 concrete samples showed that the relaxation rule in form (7) and formula (9) as a function of \( T_{\text{relax}} \) variation with increasing strain rate accurately describe the inelastic behavior of real concrete C80 at different strain rates (figure 5).

4. Conclusion

The model of the dynamic mechanical behavior of brittle materials is proposed. The model is based on ideas of kinetic theory of strength and takes into account the change in the strength and yield point, and also ensures correct inelastic behavior of the brittle materials at strain rates up to \( 10^3 \text{ s}^{-1} \). The main feature of the proposed model is the consideration of the finite time of local discontinuities nucleation in the structure of brittle materials.

The proposed dynamic model is suitable for solving a new class of applied problems dealing with natural and artificial dynamic effects on structures made of manufactured building materials (including concrete), ceramic structural elements, and rocks. The model implemented in the framework of the particle-based method can be applied to predict the fracture time of materials and structures depending on the amplitude and rate of loading, with regard for the structural features of the considered materials.

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