Research Article

On Certain Classes of Harmonic $p$-Valent Functions Defined by an Integral Operator

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We obtain coefficient characterization, extreme points, and distortion bounds of certain classes of harmonic $p$-valent functions defined by an integral operator.

1. Introduction

A continuous complex-valued function $f = u + iv$ defined in a simply connected complex domain $D$ is said to be harmonic in $D$ if both $u$ and $v$ are real harmonic in $D$. In any simply connected domain, we can write

$$f = h + \overline{g},$$

(1)

where $h$ and $g$ are analytic in $D$. We call $h$ the analytic part and $g$ the coanalytic part of $f$. A necessary and sufficient condition for $f$ to be locally univalent and sense preserving in $D$ is that $|h'(z)| > |g'(z)|$ in $D$ (see [1]).

Denote by $S_H$ the class of functions $f$ of the form (1) that are harmonic univalent and sense preserving in the unit disc $U = \{z : |z| < 1\}$ for which $f(0) = f_z(0) - 1 = 0$.

Recently, Jahangiri and Ahuja [2] defined the class $H_p$ ($p \in \mathbb{N} = \{1, 2, 3, \ldots\}$), consisting of all $p$-valent harmonic functions $f = h + \overline{g}$ that are sense preserving in $U$ and $h$, and $g$ are of the form

$$h(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k, \quad g(z) = \sum_{k=p}^{\infty} b_k z^k, \quad |b_p| < 1.$$  

(2)

For $f = h + \overline{g}$ given by (2), we define the modified $p$-valent Salagean integral operator $I_{p,\lambda}^n$ of $f$ (see [3] and also [4] when $p = 1$) as follows:

$$I_{p,\lambda}^n f(z) = I_{p,\lambda}^n h(z) + (-1)^n \overline{I_{p,\lambda}^n g(z)},$$

(3)

where

$$I_{p,\lambda}^n h(z) = z^p + \sum_{k=p+1}^{\infty} \frac{p}{p+\lambda(k-p)} a_k z^k$$

$$I_{p,\lambda}^n g(z) = \sum_{k=p}^{\infty} \frac{p}{p+\lambda(k-p)} b_k z^k,$$

(4)

For $p \in \mathbb{N}$, $\lambda > 0$, $n \in \mathbb{N}_0$, $0 \leq \alpha < 1$, and $z \in U$, we let $H_{p,\lambda}(n;\alpha)$ denote the family of harmonic functions $f$ of the form (2) such that

$$\text{Re} \left\{ \frac{I_{p,\lambda}^n f(z)}{I_{p,\lambda}^{n+1} f(z)} \right\} > \alpha,$$

(5)

where $I_{p,\lambda}^n f$ is defined by (3).

We let the subclass $H_{p,\lambda}^{-}(n;\alpha)$ consists of harmonic functions $f_n = h + \overline{g}_n$ in $H_{p,\lambda}(n;\alpha)$ so that $h$ and $g_n$ are of the form

$$h(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k, \quad g_n(z) = (-1)^n \sum_{k=p}^{\infty} b_k z^k,$$

(6)

$$a_k, b_k \geq 0.$$
We note that \( \mathcal{H}_{-p,1}(n;\alpha) = \mathcal{H}_p(n;\alpha) \), where the class \( \mathcal{H}_p(n;\alpha) \) was defined and studied by Cotirla [5].

In this paper, we obtain coefficient characterization of the classes \( \mathcal{H}_{p,\lambda}(n;\alpha) \) and \( \mathcal{H}_{-p,\lambda}(n;\alpha) \). We also obtain extreme points and distortion bounds for functions in the class \( \mathcal{H}_{-p,\lambda}(n;\alpha) \).

2. Coefficient Characterization

Unless otherwise mentioned, we assume throughout this paper that \( p \in \mathbb{N} \), \( n \in \mathbb{N}_0 \), \( 0 \leq \alpha < 1 \), \( a_p = 1 \), and \( \lambda > 0 \). We begin with a sufficient condition for functions in \( \mathcal{H}_{p,\lambda}(n;\alpha) \).

**Theorem 1.** Let \( f = h + g \) so that \( h \) and \( g \) are given by (2).

Furthermore, let

\[
\sum_{k=p}^{\infty} \left\{ \Psi_{p,\lambda}(n,k,\alpha) |a_k| + \Phi_{p,\lambda}(n,k,\alpha) |b_k| \right\} \leq 2, \quad (7)
\]

where

\[
\Psi_{p,\lambda}(n,k,\alpha) = \left( \frac{p}{p + \lambda (k-p)} \right)^n - \alpha \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} \times (1 - \alpha)^{-1}, \quad (8)
\]

\[
\Phi_{p,\lambda}(n,k,\alpha) = \left( \frac{p}{p + \lambda (k-p)} \right)^n + \alpha \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} \times (1 - \alpha)^{-1}. \quad (9)
\]

Then, \( f \) is sense preserving in \( U \) and \( f \in \mathcal{H}_{p,\lambda}(n;\alpha) \).

**Proof.** According to (2) and (3), we only need to show that

\[
\Re \left\{ \frac{f^m_{p,\lambda} f(z) - \alpha f^m_{p,\lambda} f(z)}{f^{m+1}_{p,\lambda} f(z)} \right\} \geq 0 \quad (z \in U). \quad (10)
\]

It follows that

\[
\Re \left\{ \frac{f^m_{p,\lambda} f(z) - \alpha f^m_{p,\lambda} f(z)}{f^{m+1}_{p,\lambda} f(z)} \right\} = \Re \left\{ (1 - \alpha) z^p + \sum_{k=p+1}^{\infty} \left[ \left( \frac{p}{p + \lambda (k-p)} \right)^n - \alpha \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} \right] a_k z^k \right\} 
\times \left( z^p + \sum_{k=p+1}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^n a_k z^k \right)^{-1} \times \left( z^p + \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^n b_k z^k \right)^{-1}
\]

\[
\times \left( 1 + \sum_{k=p+1}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} a_k z^{-p} \right)^{-1} + \left( (1 - \alpha) z^p + \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^n a_k z^{k-p} \right) \times \left( z^p + \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^n b_k z^k \right)^{-1}
\]

\[
\times \left( 1 + \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} b_k z^{-p} \right)^{-1}
\]

\[
+ \left( (1 - \alpha) z^p + \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^n b_k z^{k-p} \right) \times \left( 1 + \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} b_k z^{-p} \right)^{-1}
\]

\[
\times \left( z^p + \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^n b_k z^k \right)^{-1}
\]
\[
\sum_{k=p+1}^{\infty} \left( \frac{p}{p + \lambda (k - p)} \right)^{n+1} a_k z^{k-p} + (-1)^{n+1} \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k - p)} \right)^{n+1} b_k z^{-p} \mathbf{-1} \bigg) = \text{Re} \left\{ \frac{1 - \alpha + A(z)}{1 + B(z)} \right\}.
\]

For \( z = re^{i\theta} \), we have

\[
A \left( re^{i\theta} \right) = \sum_{k=p+1}^{\infty} \left[ \left( \frac{p}{p + \lambda (k - p)} \right)^n \right. \\
\left. + \alpha \left( \frac{p}{p + \lambda (k - p)} \right)^{n+1} \right] \left( r e^{i(k-p)\theta} \right)^k \\
\left. + (-1)\sum_{k=p}^{\infty} \left[ \left( \frac{p}{p + \lambda (k - p)} \right)^n \right. \\
\left. + \alpha \left( \frac{p}{p + \lambda (k - p)} \right)^{n+1} \right] \right] \left( r e^{-i(k+p)\theta} \right)^k.
\]

\[
B \left( re^{i\theta} \right) = \sum_{k=p+1}^{\infty} \left( \frac{p}{p + \lambda (k - p)} \right)^{n+1} a_k r^{k-p} e^{i(k-p)\theta} \\
\left. + (-1)^{n+1} \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k - p)} \right)^{n+1} b_k r^{k-p} e^{-i(k+p)\theta} \right).
\]

Setting that

\[
\frac{1 - \alpha + A(z)}{1 + B(z)} = (1 - \alpha) \frac{1 + w(z)}{1 - w(z)},
\]

the proof will be complete if we can show that \(|w(z)| < 1\).

Using the condition (7), we can write

\[
|w(z)| = \left| \frac{A(z) - (1 - \alpha) B(z)}{A(z) + (1 - \alpha) B(z) + 2(1 - \alpha)} \right| \\
= \left| \sum_{k=p+1}^{\infty} \left[ \left( \frac{p}{p + \lambda (k - p)} \right)^n \right. \\
\left. + \alpha \left( \frac{p}{p + \lambda (k - p)} \right)^{n+1} \right] \left( r e^{i(k-p)\theta} \right)^k \\
\left. + (-1)^{n+1} \sum_{k=p}^{\infty} \left[ \left( \frac{p}{p + \lambda (k - p)} \right)^n \right. \\
\left. + \alpha \left( \frac{p}{p + \lambda (k - p)} \right)^{n+1} \right] \left( r e^{-i(k+p)\theta} \right)^k \right| < 1.
\]
\[-\left(\frac{p}{p + \lambda (k - p)}\right)^{n+1}|a_k| r^{k-p}\]
\[
\times \left(4 (1 - \alpha) - \sum_{k=p}^{\infty} |c_k| |a_k| + d_k |b_k| \right) r^{k-p}\]
\[
+ \left(\frac{p}{p + \lambda (k - p)}\right)^{n+1} |b_k| r^{k-p}\]
\[
\times \left(4 (1 - \alpha) - \sum_{k=p}^{\infty} |c_k| |a_k| + d_k |b_k| \right)^{-1}\]
\[
\times \left(\sum_{k=p+1}^{\infty} \left[\left(\frac{p}{p + \lambda (k - p)}\right)^n - \left(\frac{p}{p + \lambda (k - p)}\right)^{n+1}\right]|a_k|\right)\]
\[
+ \left(\sum_{k=p}^{\infty} \left[\left(\frac{p}{p + \lambda (k - p)}\right)^n + \left(\frac{p}{p + \lambda (k - p)}\right)^{n+1}\right]|a_k|\right)\]
\[
\times \left(4 (1 - \alpha) - \sum_{k=p}^{\infty} |c_k| |a_k| + d_k |b_k| \right)^{-1}\]
\[
\leq 1,
\]

where
\[
\begin{align*}
q_k &= \left(\frac{p}{p + \lambda (k - p)}\right)^n + (1 - 2\alpha) \left(\frac{p}{p + \lambda (k - p)}\right)^{n+1}, \\
d_k &= \left(\frac{p}{p + \lambda (k - p)}\right)^n - (1 - 2\alpha) \left(\frac{p}{p + \lambda (k - p)}\right)^{n+1}.
\end{align*}
\]

The harmonic functions are as follows:
\[
f(z) = z^p + \sum_{k=p+1}^{\infty} \frac{1}{\Psi_{\rho,\lambda}(n, k, \alpha)} x_k z^k
\]
\[
+ \sum_{k=p}^{\infty} \frac{1}{\Phi_{\rho,\lambda}(n, k, \alpha)} y_k z^k,
\]

where \(\sum_{k=p+1}^{\infty} |x_k| + \sum_{k=p}^{\infty} |y_k| = 1\) show that the coefficient bound given by (7) is sharp. The functions of the form (8) are in the class \(\mathcal{H}_{\rho,\lambda}(n; \alpha)\) because
\[
\sum_{k=p+1}^{\infty} \left|\psi_{\rho,\lambda}(n, k, \alpha) a_k + \Phi_{\rho,\lambda}(n, k, \alpha) b_k\right| = 1 + \sum_{k=p}^{\infty} |x_k| + \sum_{k=p}^{\infty} |y_k| = 2.
\]

This completes the proof of Theorem 1.

In the following theorem, it is shown that the condition (7) is also necessary for functions \(f_n = h + \overline{g}_n\), where \(h\) and \(g_n\) are of the form (6).

**Theorem 2.** Let \(f_n = h + \overline{g}_n\), where \(h\) and \(g_n\) are given by (6). Then, \(f_n \in \mathcal{H}_{\rho,\lambda}(n; \alpha)\) if and only if
\[
\sum_{k=p}^{\infty} \left\{\psi_{\rho,\lambda}(n, k, \alpha) a_k + \Phi_{\rho,\lambda}(n, k, \alpha) b_k\right\} \leq 2,
\]

where \(\psi_{\rho,\lambda}(n, k, \alpha)\) and \(\Phi_{\rho,\lambda}(n, k, \alpha)\) are given by (8) and (9), respectively.

**Proof.** Since \(\mathcal{H}_{\rho,\lambda}(n; \alpha) \subset \mathcal{H}_{\rho,\lambda}(n; \alpha)\), we only need to prove the “only if” part of the theorem. To this end, for functions \(f_n = h + \overline{g}_n\), where \(h\) and \(g_n\) are given by (6), we notice that the condition \(\text{Re}\left\{\int_{\rho,\lambda} f(z) / \int_{\rho,\lambda} f(z)\right\} > \alpha\) is equivalent to
\[
\text{Re}\left\{((1 - \alpha) z^p - \sum_{k=p+1}^{\infty} \left[\left(\frac{p}{p + \lambda (k - p)}\right)^n a_k z^k\right] - \alpha \left(\frac{p}{p + \lambda (k - p)}\right)^{n+1} a_k z^k\right.\]
\[
\times \left(z^p - \sum_{k=p+1}^{\infty} \left(\frac{p}{p + \lambda (k - p)}\right)^{n+1} a_k z^k\right)\]
\[
\left. + (-1)^{2\alpha} \sum_{k=p}^{\infty} \left(\frac{p}{p + \lambda (k - p)}\right)^{n+1} b_k z^k\right) \right)^{-1}\]
\[ + \left( -1 \right)^{2n-1} \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^n \times \left[ \frac{p}{p + \lambda (k-p)} \right] \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} b_k z^k + \alpha \left( \frac{p}{p + \lambda (k-p)} \right) \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} a_k z^k \times \left( z^p - \sum_{k=p+1}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} a_k z^k \right) \right] \] \[ + \left( -1 \right)^{2n} \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} b_k z^k \right) \right]^{-1} \] \[ \geq 0. \] \[ (19) \]

The previous required condition (19) must hold for all values of \( z \) in \( U \). Upon choosing the values of \( z \) on the positive real axis where \( 0 \leq z = r < 1 \), we must have

\[ \left( 1 - \alpha \right) - \sum_{k=p+1}^{\infty} \left[ \left( \frac{p}{p + \lambda (k-p)} \right)^n \right] a_k r^{k-p} \]

\[ \times \left( 1 - \sum_{k=p+1}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} a_k r^{k-p} \right) \]

\[ + \sum_{k=p+1}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} b_k r^{k-p} \]

\[ \times \left( 1 - \sum_{k=p+1}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} a_k r^{k-p} \right) \]

\[ + \sum_{k=p}^{\infty} \left( \frac{p}{p + \lambda (k-p)} \right)^{n+1} b_k r^{k-p} \]

\[ \geq 0. \] \[ (20) \]

If the condition (18) does not hold, then the numerator in (20) is negative for \( r \) sufficiently close to 1. Hence there exists \( z_0 = r_0 \) in \((0, 1)\) for which the quotient in (20) is negative. This contradicts the required condition for \( f_n \in \mathcal{H}_{p,\lambda}(n;\alpha) \), and so the proof of Theorem 2 is completed. \( \square \)

3. Extreme Points and Distortion Theorem

Our next theorem is on the extreme points of convex hulls of the class \( \mathcal{H}_{p,\lambda}(n;\alpha) \) denoted by \( \text{clco} \mathcal{H}_{p,\lambda}(n;\alpha) \).

Theorem 3. Let \( f_n = h + g_n \) where \( h \) and \( g_n \) are given by (6). Then, \( f_n \in \mathcal{H}_{p,\lambda}(n;\alpha) \) if and only if

\[ f_n(z) = \sum_{k=p}^{\infty} \left[ x_k h_k(z) + y_k g_n(z) \right], \] \[ (21) \]

where

\[ h_1(z) = z^p, \quad h_k(z) = z^p - \frac{1}{\Psi_{p,\lambda}(n,k,\alpha)} z^k \]

\[ (k = p+1, p+2, p+3, \ldots), \]

\[ g_n(z) = z^p + \left( -1 \right)^{n+1} \frac{1}{\Phi_{p,\lambda}(n,k,\alpha)} z^k \]

\[ (k = p, p+1, p+2, \ldots), \]

\[ x_k, y_k \geq 0, \quad x_p = 1 - \sum_{k=p+1}^{\infty} x_k - \sum_{k=p}^{\infty} y_k. \]

In particular, the extreme points of the class \( \mathcal{H}_{p,\lambda}(n;\alpha) \) are \( \{h_k\} \) and \( \{g_n\} \).

Proof. Suppose that

\[ f_n(z) = \sum_{k=p}^{\infty} \left( x_k h_k(z) + y_k g_n(z) \right) \]

\[ = \sum_{k=p}^{\infty} \left( x_k + y_k \right) z^p - \sum_{k=p+1}^{\infty} \frac{1}{\Psi_{p,\lambda}(n,k,\alpha)} x_k z^k \]

\[ + \left( -1 \right)^{n+1} \sum_{k=p}^{\infty} \frac{1}{\Phi_{p,\lambda}(n,k,\alpha)} y_k z^k. \] \[ (23) \]

Then,

\[ \sum_{k=p+1}^{\infty} \Psi_{p,\lambda}(n,k,\alpha) \left( \frac{1}{\Psi_{p,\lambda}(n,k,\alpha)} x_k \right) \]

\[ + \sum_{k=p}^{\infty} \Phi_{p,\lambda}(n,k,\alpha) \left( \frac{1}{\Phi_{p,\lambda}(n,k,\alpha)} y_k \right) \]

\[ = \sum_{k=p}^{\infty} x_k + \sum_{k=p}^{\infty} y_k = 1 - x_p \leq 1, \]

and so \( f_n \in \text{clco} \mathcal{H}_{p,\lambda}(n;\alpha) \).
Conversely, if \( f_n \in \mathcal{H}^{-\lambda}_p(n, \alpha) \), then
\[
a_k \leq \frac{1}{\Psi_{p, \lambda}(n, k, \alpha)}, \quad b_k \leq \frac{1}{\Phi_{p, \lambda}(n, k, \alpha)},
\]
Set that
\[
x_k = \Psi_{p, \lambda}(n, k, \alpha) a_k \quad (k = p + 1, p + 2, p + 3, \ldots),
\]
\[
y_k = \Phi_{p, \lambda}(n, k, \alpha) b_k \quad (k = p, p + 1, p + 2, \ldots).
\]
Then note that by Theorem 2, \( 0 \leq x_k \leq 1 \), \( (k = p + 1, p + 2, p + 3, \ldots) \), and \( 0 \leq y_k \leq 1 \), \( (k = p, p + 1, p + 2, \ldots) \). We define that \( x_p = 1 - \sum_{k=p+1}^{\infty} x_k - \sum_{k=p}^{\infty} y_k \) and note that by Theorem 2, \( x_p \geq 0 \). Consequently, we obtain \( f_n(z) = \sum_{k=p}^{\infty} (x_k h_k(z) + y_k g_k(z)) \) as required. \( \square \)

The following theorem gives the distortion bounds for functions in the class \( \mathcal{H}^{-\lambda}_p(n, \alpha) \) which yields a covering result for this class.

**Theorem 4.** Let \( f_n(z) \in \mathcal{H}^{-\lambda}_p(n, \alpha) \). Then, for \( |z| = r < 1 \), we have
\[
(1 - b_p) r^p - \left\{ \Gamma_{p, \lambda}(n, \alpha) - \Delta_{p, \lambda}(n, \alpha) \right\} r^{p+1} \leq |f_n(z)| \leq (1 + b_p) r^p + \left\{ \Gamma_{p, \lambda}(n, \alpha) - \Delta_{p, \lambda}(n, \alpha) b_p \right\} r^{p+1},
\]
where
\[
\Gamma_{p, \lambda}(n, \alpha) = \frac{1 - \alpha}{(p/(p + \lambda))^n - \alpha(p/(p + \lambda))^{n+1}},
\]
\[
\Delta_{p, \lambda}(n, \alpha) = \frac{1 + \alpha}{(p/(p + \lambda))^n - \alpha(p/(p + \lambda))^{n+1}}.
\]

The result is sharp.

**Proof.** We only prove the right-hand inequality. The proof for the left-hand inequality is similar and will be omitted. Let \( f_n(z) \in \mathcal{H}^{-\lambda}_p(n, \alpha) \). Taking the absolute value of \( f_n \), we have
\[
|f_n(z)| \leq (1 + b_p) r^p + \sum_{k=p+1}^{\infty} (a_k + b_k) r^k \leq (1 + b_p) r^p + \sum_{k=p+1}^{\infty} (a_k + b_k) r^{p+1} = (1 + b_p) r^p + \Gamma_{p, \lambda}(n, \alpha) \times \sum_{k=p+1}^{\infty} \frac{1}{\Gamma_{p, \lambda}(n, \alpha)} (a_k + b_k) r^{p+1} \leq (1 + b_p) r^p + \Gamma_{p, \lambda}(n, \alpha) r^{p+1}
\]
\[
\leq (1 + b_p) r^p + \left\{ \Gamma_{p, \lambda}(n, \alpha) - \Delta_{p, \lambda}(n, \alpha) b_p \right\} r^{p+1}.
\]

The bounds given in Theorem 4 for functions \( f_n = h + \overline{g}_n \), where \( h \) and \( g_n \) of form (6), also hold for functions of form (2) if the coefficient condition (7) is satisfied. The upper bound given for \( f \in \mathcal{H}^{-\lambda}_p(n, \alpha) \) is sharp, and the equality occurs for the functions
\[
f(z) = z^p + b_p \overline{z}^p
\]
showing that the bounds given in Theorem 4 are sharp. \( \square \)

**Remark 5.**
(i) Putting \( \lambda = 1 \) in the previous results, we obtain the results of Cotirla [5].
(ii) Putting \( \lambda = 1 \) in the previous results, we obtain the results of Cotirla [6], when \( \beta = 0 \).

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