COMPLEX MONOPOLES AND GRIBOV COPIES\footnote{To appear in the Proceedings of the Third Workshop “Continuous Advances in QCD”, Minneapolis, 16 - 19 April, 1998.}

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Complex monopole solutions exist in the three dimensional Georgi-Glashow model with the Chern-Simons term. They dominate the path integral and disorder the Higgs vacuum. Gribov copies of the vacuum and monopole configurations are studied in detail.

1 Monopole-supercurrent duality

In the Georgi-Glashow model, $SO(3)$ gauge theory with a triplet Higgs scalar field $\vec{h}$, the gauge symmetry is spontaneously broken to $U(1)$ by the Higgs mechanism. In perturbation theory the $U(1)$ gauge boson remains massless and the Higgs vacuum is ordered with nonvanishing $\langle \vec{h} \rangle \neq 0$.

In three dimensional spacetime, however, Polyakov\footnote{To appear in the Proceedings of the Third Workshop “Continuous Advances in QCD”, Minneapolis, 16 - 19 April, 1998.} showed that due to instantons, or monopoles in the three-dimensional Euclidean space, the Higgs vacuum is disordered $\langle \vec{h} \rangle = 0$. The $U(1)$ gauge field acquires a finite mass without breaking the $U(1)$ gauge invariance. A pair of positive and negative electric charges is bound by an electric flux tube. Electric charges are linearly confined.

Physical mechanism at work behind this becomes clear by the duality transformation. It was shown by one of the authors\footnote{To appear in the Proceedings of the Third Workshop “Continuous Advances in QCD”, Minneapolis, 16 - 19 April, 1998.} that the Georgi-Glashow model, or more generally, the compact QED$_3$, is dual to the Josephson junction system in the superconductivity. $(E_1, E_2, B)$ in the compact QED$_3$ corresponds to $(B_1, B_2, E_3)$ in the barrier region of the Josephson junction. If a magnetic monopole-antimonopole pair is inserted in the barrier region, a magnetic flux of the Abrikosov magnetic vortex would be formed between the poles as supercurrents flow through the barrier. Instantons (monopoles) in the compact QED$_3$ are supercurrents flowing through the two-dimensional barrier in the Josephson junction.
2 Chern-Simons term

In three-dimensional gauge theory the Chern-Simons term can be added to the Lagrangian. The action of the model is given by
\[ I = I_{YM} + I_{CS} + I_H \]
where \( I_{YM} \) and \( I_H \) are the standard Yang-Mills and Higgs parts of the Georgi-Glashow model. The Chern-Simons term is given by
\[ I_{CS} = -\frac{i\kappa}{g^2} \int d^3x \, \epsilon^{\mu\nu\lambda} \text{tr} \left( A_\mu \partial_\nu A_\lambda + \frac{2}{3} A_\mu A_\nu A_\lambda \right) . \quad (1) \]

In the Euclidean space \( I_{CS} \) is pure imaginary for real gauge fields. It is not gauge invariant. Under \( A \rightarrow \Omega A \Omega^{-1} + \Omega d\Omega^{-1} \), the action \( I_{CS} \) changes by
\[ \delta I_{CS} = \frac{i\kappa}{g^2} \int \text{tr} (A \wedge d\Omega^{-1} \Omega) + \frac{i\kappa}{3g^2} \int \text{tr} d\Omega^{-1} \Omega \wedge d\Omega^{-1} \Omega \wedge d\Omega^{-1} \Omega . \quad (2) \]

On \( S^3 \) the first term vanishes. The second term is proportional to the winding number, which leads to the quantization of the Chern-Simons coefficient \( \kappa = (g^2/4\pi) \times \text{integer} \).

What happens to the confinement? In the presence of the Chern-Simons term, the \( U(1) \) gauge boson acquires a topological mass proportional to \( \kappa \) even in perturbation theory so that there occurs no issue of the confinement. Electric charges are screened. How about the Higgs vacuum? Is the vacuum still disordered such that \( \langle \vec{h} \rangle = 0 \)?

In disordering the vacuum, monopole configurations play an important role. In the literature it has been argued by many authors that monopole configurations suddenly become irrelevant once the Chern-Simons term is added. If this is the case, the Higgs vacuum would remain ordered, i.e. the v.e.v. of the Higgs field is nonvanishing and is aligned in one direction in the \( SO(3) \) space. It has been argued that monopole solutions necessarily have infinite action, and for configurations of finite action their Gribov copies lead to cancellation. We show that this is not the case. There are complex monopole solutions of finite action, and Gribov copies do not lead to cancellation.

3 Monopole ansatz

The most general monopole ansatz is
\[ h^a(\vec{x}) = \vec{x}^a h(r) \]
\[ A_\mu^a(\vec{x}) = \frac{1}{r} \left[ \epsilon_{\mu\nu} \vec{x}^\nu (1 - \phi_1) + (\delta_a^\mu - \vec{x}_a \vec{x}_\mu) \phi_2 + r S \vec{x}_a \vec{x}_\mu \right] \quad (3) \]
where \( \vec{x}^a = x^a/r \). Field strengths are
\[ F_{\mu\nu} = \frac{1}{r^2} \epsilon_{\mu\nu\kappa} \vec{x}^\kappa (\phi_1^2 + \phi_2^2 - 1) + \frac{1}{r} (\epsilon_{\mu\nu} - \epsilon_{\mu\nu} \vec{x}^a \vec{x}_b)(\phi_1' + S \phi_2) \]
\[ + \frac{1}{r} (\delta^{\alpha \nu} \hat{\partial}^\mu - \delta^{\alpha \mu} \hat{\partial}^\nu) (\phi_1' - S \phi_1). \]  

(4)

There is residual \( U(1) \) gauge invariance. Under \( \Omega = \exp \left\{ \frac{i}{2} f(r) \hat{\partial}^\mu \sigma^\mu \right\} \)

\[
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix} \rightarrow \begin{pmatrix}
\cos f & \sin f \\
-\sin f & \cos f
\end{pmatrix} \begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix}, \quad S \rightarrow S - f'.
\]

(5)

The Chern-Simons term transforms as

\[ I_{CS} \rightarrow I_{CS} + \frac{4\pi i \kappa}{g^2} \{ f(\infty) - f(0) \}. \]

(6)

Hence gauge copies of solutions carry the oscillatory factor in \( e^{-I_{CS}} \) in the path integral, which could lead to the cancellation among monopole contributions.

However, the gauge is fixed in the path integral. For instance, in the radial gauge \( \hat{\partial}^\mu A^\mu = \hat{\partial}\sigma S = 0 \), there remains no residual gauge freedom, once the boundary condition \( f(0) = 0 \) is imposed for the regularity of configurations. The radiation gauge is more subtle, and is discussed below in more detail.

The action before the gauge fixing is

\[
I = \frac{4\pi}{g^2} \int_0^\infty dr \left\{ (\phi_1' + S \phi_2)^2 + (\phi_2' - S \phi_1)^2 + \frac{1}{2r^2} (\phi_1^2 + \phi_2^2 - 1)^2 
+ i\kappa \left[ \phi_1' \phi_2 - \phi_2' (\phi_1 - 1) + S (\phi_1^2 + \phi_2^2 - 1) \right] 
+ \frac{\rho^2}{2} h'^2 + h'' (\phi_1^2 + \phi_2^2) + \frac{\lambda r^2}{4} (h^2 - v^2)^2 \right\}.
\]

(7)

### 4 Complex monopole solution

The action (8) is a functional of four functions \( \phi_1, \phi_2, S \) and \( h \). In the path integral the gauge fixing condition is inserted;

\[ Z = \int \mathcal{D}A_\mu \mathcal{D}h \Delta_F [A] \delta [F(A)] e^{-I}. \]

(8)

We look for configurations which extremize the action \( I \) within the subspace specified with \( F(A) = 0 \). In the monopole ansatz one, or one combination, of \( \phi_1, \phi_2 \) and \( S \) is eliminated by gauge fixing so that three equations need to be solved.

In the radial gauge \( S = 0 \) the extremization of the action leads to

\[ \phi_1'' + \frac{1}{r^2} (1 - \phi_1^2 - \phi_2^2) \phi_1 + i \kappa \phi_2' - h^2 \phi_1 = 0 \]
\[
\phi''_2 + \frac{1}{r^2}(1 - \phi_1^2 - \phi_2^2)\phi_2 - i\kappa\phi_1' - h^2\phi_2 = 0
\]  \hspace{1cm} (9)

and

\[
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d h}{dr} \right) - \lambda(h^2 - v^2)h - \frac{2}{r^2} (\phi_1^2 + \phi_2^2)h = 0.
\]  \hspace{1cm} (10)

Since eq. (9) contains complex terms, solutions necessarily become complex. The boundary conditions ensuring the regularity of configurations and the finiteness of the action are \(\phi_1(0) = 1, \phi_2(0) = h(0) = 0, \phi_1(\infty) = \phi_2(\infty) = 0,\) and \(h(\infty) = v.\)

Eqs. (9) and (10) can be solved by an ansatz

\[
\phi_1 = \zeta(r) \cosh \frac{\kappa r}{2}, \quad \phi_2 = i\zeta(r) \sinh \frac{\kappa r}{2}.
\]  \hspace{1cm} (11)

The equations to be solved are

\[
\zeta'' - \frac{1}{r^2}(\zeta^2 - 1)\zeta - \left(h^2 + \frac{\kappa^2}{4}\right)\zeta = 0
\]  \hspace{1cm} (12)
and eq. (10), where $\phi_1^2 + \phi_2^2 = \zeta^2$. The solution to these equation can be found numerically and depicted in fig. 1. $\phi_2(r)$ is pure imaginary. The action for this configuration is real and finite. The $U(1)$ field strengths are given by $F_{\mu\nu}^{U(1)} = -\epsilon_{\mu\nu\rho} \hat{x}^\rho / r^2$, exactly those of a magnetic monopole. Non-Abelian field strengths are complex. We call it a complex monopole configuration.

What is the significance of complex monopoles? In the original form of the path integral, field configurations to be integrated are real. It is an infinite dimensional integral defined along real axes. Symbolically we have an integration $\int_{-\infty}^{\infty} dz e^{-f(z)}$ where $z$ stands for a field variable, say, $A_\mu(x)$. The saddle point, $z_0$, of $f(z)$ may be located off the real axis. In the saddle point method for the integration, the integration path is deformed such that a new path pass $z_0$. $e^{-f(z_0)}$ gives a dominat factor, approximating the integral. Translated in the field theory model, the complex monopole configuration approximates the integral.

Complex monopole configurations dominate the path integral. They are relevant in disordering the Higgs vacuum. Without monopole-type configurations the perturbative Higgs vacuum cannot be disordered and $\langle \vec{h} \rangle$ remains nonvanishing. With complex monopoles taken into account $\langle \vec{h} \rangle = 0$ but $\langle \vec{h}^2 \rangle \sim v^2$.

5 Gribov copies

The radiation gauge does not uniquely fix gauge field configurations\[3\]. Even in the monopole ansatz (3), there remains arbitrariness. The radiation gauge condition $\partial_\mu A_\mu = 0$ is satisfied if $(r^2 S)' = 2\phi_2$. Under $U(1)$ gauge transformation (3), the radiation gauge condition is maintained, provided $f(r)$ obeys

$$f'' + \frac{2}{r} f' - \frac{2}{r^2} \left\{ \phi_1 \sin f + \phi_2 (1 - \cos f) \right\} = 0 \ . \quad (13)$$

$f(0)$ is assumed to vanish for the regularity. Solutions to eq. (13) define Gribov copies. $A' = \Omega A \Omega^{-1} + \Omega \partial \Omega^{-1}$ is on a gauge orbit of $A$ within the radiation gauge slice.

These copies have a significant effect in the Chern-Simons theory. The Chern-Simons term is not gauge invariant as displayed in (2). We suppose that the Chern-Simons coefficient is quantized; $4\pi\kappa/g^2 = n$ is an integer. Each Gribov copy carries an extra factor $e^{-\delta_{\text{CS}}} = e^{-in f(\infty)}$ in the path integral. Hence, if $f(\infty)$ takes continuous values, then Gribov copies of monopole configurations may lead to the cancellation in the path integral. Indeed, the authors of ref. 6 and ref. 7 have argued that because of this effect contributions of monopole configurations disappear once the Chern-Simons term is added. One has to examine Gribov copies more carefully.
Before discussing Gribov copies of monopoles, it is worthwhile to recall Gribov copies of the vacuum. $A_\mu = 0$ corresponds to $\phi_1 = 1$ and $\phi_2 = S = 0$. Eq. (13) reads

$$f'' + \frac{2}{r}f' - \frac{2}{r^2}\sin f = 0.$$  \hspace{1cm} (14)

The equation is scale invariant. If $f(r)$ is a solution, then $g(r) = f(ar)$ also solves the equation. Solutions are parametrized by $f'(0)$. Given the initial condition $f(0) = 0$ and $f'(0)$, the solution is uniquely determined.

In fig. 2 the solution is depicted for $f'(0) = 1$. For $f'(0) \equiv \alpha > 0$, $f(r)$ reaches the maximum value 3.652 at $\alpha r = 13.2$, then decrease to a local minimum 2.988 at $\alpha r = 145.9$, and then asymptotically approaches $\pi$. For $f'(0) < 0$, $f(\infty) = -\pi$. 

Figure 2: Gribov copies of the vacuum. The solution $f(r)$ to eq. (14) with $f'(0) = 1$ is depicted. It asymptotically approaches $\pi$. 

5.1 Vacuum

Before discussing Gribov copies of monopoles, it is worthwhile to recall Gribov copies of the vacuum. $A^a_\mu = 0$ corresponds to $\phi_1 = 1$ and $\phi_2 = S = 0$. Eq. (13) reads

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6
5.2 Monopoles

Consider a monopole in the BPS limit ($\lambda = \kappa = 0$) in which $\phi_1 = vr/\sinh vr$. In this case $\phi_1$ dies out quickly for large $vr$. The asymptotic value $f(\infty)$ depends on the initial slope $f'(0)$. Solutions $f(r)$ are depicted in fig. 3.

In fig. 4 $f(\infty)$ is plotted as a function of $f'(0)$. The range of the asymptotic value is $-3.98 < f(\infty) < +3.98$. For large $|f'(0)|$, $f(\infty) = \pm \pi$. It is quite unlikely that, in the presence of the Chern-Simons term, these Gribov copies of the BPS monopole lead to the cancellation $\sum e^{-inf(\infty)} = 0$. Monopole configurations remain important in the path integral.

5.3 Complex monopoles

There seem complex monopole solutions in the radiation gauge as well. Most likely $\phi_2$ and $S$ are pure imaginary. It is legitimate to ask if there are Gribov copies of complex monopoles.

It is obvious that there is no real solution $f$ to the Gribov equation (13).
as $\phi_2$ is pure imaginary. There is no real gauge copy.

However, this does not necessarily mean that the solution is unique in the radiation gauge. There may be solutions which are related by “complex” gauge transformation generated by complex $f(r)$.

Indeed, one can show that $(A^I, h)$ satisfies the equations in the radiation gauge, and has the same real action as $(A, h)$. There are many “complex” Gribov copies of complex monopoles. Among them, copies generated by $f$ satisfying $\text{Im}(f(\infty)) = 0$ may play an important role. Of course at one or higher loop level, these copies are believed to yield different contributions in the path integral. Deeper understanding is necessary about complex monopoles.

6 Conclusion

In the Georgi-Glashow-Chern-Simons model complex monopole solutions of finite action exist in the radial gauge. These complex monopoles dominate in the path integral, and disorder the Higgs vacuum.

Solutions seem to exist in the radiation gauge as well. We have examined
Gribov copies of both real and complex monopole configurations. The asymptotic value of the gauge function \( f(r) \) specifying Gribov copies of monopoles depends on \( f'(0) \). The Chern-Simons term sensitively depends on \( f(\infty) \), but it will not diminish the relevance of monopole configurations.

It is a fascinating problem to find an analog of the Chern-Simons term in the Josephson junction system. The relevance of monopoles is translated to the relevance of supercurrents in the Josephson junction system by the duality transformation.

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**References**

1. A. M. Polyakov, *Phys. Lett.* B 59, 80 (1975); *Nucl. Phys.* B 120, 429 (1977).
2. Y. Hosotani, *Phys. Lett.* B 69, 499 (1977).
3. S. Deser, R. Jackiw and S. Templeton, *Phys. Rev. Lett.* 48, 976 (1982).
4. E. D. D’Hoker and L. Vinet, *Ann. Phys. (N.Y.)* 162, 413 (1985).
5. R. D. Pisarski, *Phys. Rev.* D 34, 3851 (1986).
6. I. Affleck, J. Harvey, L. Palla and G. Semenoff, *Nucl. Phys.* B 328, 575 (1989).
7. E. Fradkin and F. A. Schaposnik, *Phys. Rev. Lett.* 66, 276 (1991).
8. K. Lee, *Nucl. Phys.* B 373, 735 (1992).
9. J. D. Edelstein and F. A. Schaposnik, *Nucl. Phys.* B 425, 137 (1994).
10. R. Jackiw and S.Y. Pi, *Phys. Lett.* B 423, 364 (1998).
11. B. Tekin, K. Saririan, and Y. Hosotani, preprints hep-th/9808057; hep-th/9808045, hep-th/9808045.
12. V. N. Gribov, *Nucl. Phys.* B 139, 1 (1978).