Vulnerability analysis of pile-reinforced slope under the condition of near-fault impulse ground motions

L J Tao¹, Z B Jia², J Bian², M Shi², S An³, Y Zhang³

¹ The Key Laboratory of Urban Security and Disaster Engineering, Ministry of Education, Beijing University of Technology, Beijing, China
² College of Engineering, Guangdong Ocean University, Zhanjiang, China
³ corresponding author: tianjintao2021@163.com

Abstract. In order to analyze the impact of the near-fault pulse ground motions on the slope, the method of seismic vulnerability analysis based on IDA is introduced into the stability evaluation. Taking the slope reinforced by stabilizing piles as an example, a typical analysis model is established. The seismic performance is divided into four stages: basically intact, partial damage, severe damage and overall instability. A large number of calculations are used to compare the instability probability of slopes with near-fault and far-field ground motions. The results show that: 1) The calculation model which combines the Newmark and the limit analysis method can reflect the difference between the near-fault and the far-field ground motions. 2) The IM-DM curve with indexes of PGA and displacement has a better fitting effect. The vulnerability analysis based on IDA can be used to evaluate the slope failure risk under seismic loads. 3) The IDA curve shows a monotonous increasing trend. When the seismic intensity is lower(GPA=0~0.5), there is no obvious difference between the vulnerability curves of the near-fault and far-field. As the intensity increases(GPA=0.5~1.2), the failure probability of near-fault is significantly higher than the far-field. Meanwhile, the failure mode changed from the partial failure to the complete failure. It is clear that near-fault ground motions have the characteristics of large energy and strong impact, which are more likely to cause slope instability. If the impact of the near-field earthquake is ignored, the disaster may be underestimated.

1. Introduction
The frequent occurrence of earthquake has caused a large number of landslide disasters, which have brought huge losses of life and property[1-3]. In 1994, the Northridge earthquake in the United States caused more than 11,000 landslides, and the economic loss was as high as 30 billion dollars. In 1995, the Hanshin Earthquake in Japan caused more than 60 landslides. In 2008, the Wenchuan earthquake triggered more than 15,000 geological disasters. Investigations have shown that many landslides caused by earthquakes are located near faults. Take the 2008 Wenchuan earthquake as an example, 80% of the landslides are less than 20 kilometers away from the earthquake center[4].
In recent years, near-fault impulse ground motions have gradually been paid attention to by scholars due to their particularities. Tothong et al.[5] found that pulse-shaped ground motions have a significant amplifying effect on the acceleration response spectrum of medium and long periods. Structural is more sensitive to impulse ground motions. MacRae et al.[6] also got the same conclusion, he believes that near-field pulse ground motions will cause greater elastic and plastic displacement of the structure. Zhang et al.[7] considered the directionality of the earthquake and established an empirical model for the prediction of near-fault landslide displacement. Besides, Yang et al.[8] studied the dynamic...
response of slopes with weak interlayers under the near-fault ground motions through theoretical analysis and numerical simulation. At present, although there are many studies on the stability of slopes with seismic conditions, further research is needed for near-fault earthquakes.

Seismic vulnerability analysis is an important part of structural risk assessment. Incremental Dynamic Analysis (IDA) can macroscopically describe the relationship between the intensity of ground motion and the degree of structural damage[9-10]. At present, IDA is widely used in bridges, buildings and other fields. For example, Naderpour et al. uses this method to explore the dynamic response of frame shear-wall structures under aftershocks[11]. Pang et al. applied IDA to the vulnerability analysis of gravity dams and compared the impact of different strength indicators[12]. Hu et al. takes the safety factor as the damage index, and obtains the vulnerability function that can be used to evaluate the slope stability[13]. Zhu et al. divided the performance level of the gravity retaining wall through shaking table tests, and discussed the vulnerability of the structure with different seismic intensities[14].

In this article, the incremental dynamic analysis is introduced into the slope risk assessment. Based on the Newmark method, a theoretical model for slope reinforced by stabilizing piles is established. Besides, two sets of seismic waves (near-fault and far-field) are selected through the database. A large number of calculations were used to compare the impact of the two earthquakes on the slope.

2. Calculation model of slope

The slope displacement is derived by combining the limit analysis method with the Newmark method. The limit analysis method points out that in the critical state, the external load power must be equal to the internal energy dissipation power[15]. Assuming that the sliding soil mass is rigid, the slope failure obeys the Mohr-Coulomb criterion and the fluidity rule. Figure 1 is a simplified slope model with height $H$, slope angle $\alpha$, and density $\rho$ [16,17]. The distance between the stabilizing pile and the slope toe is $L$, and the pile length is $h$. The length of the bearing section can be determined by the slope surface and the failure surface.

![Figure 1. Schematic diagram of a typical slope](image)

There are two modes of the failure surface: passing through the toe of the slope and passing under the angle of the slope[18]. The first case is used as an example in this article. The failure surface can be expressed as Eq. (1).

$$ r = r_B e^{\tan \phi (\theta - \theta_B)} $$

(1)

Where $\theta_B$ and $r_B$ represent the polar coordinates of point $B$. $\phi$ denote the internal friction angle of soil.

2.1. Power of gravity

The slope gravity is calculated in the form of the vertical strip method. The log-spiral surface can be uniquely determined by point $A$ and $O$. $D$ is the point on the spiral curve, and the polar coordinates are $\theta_D$ and $r_D$. Through coordinate conversion, the rectangular coordinates $x_D$ and $y_D$ can be obtained.

$$ x_D = x_0 + r_D \cos \theta_D = x_0 + r_B e^{\tan \phi (\theta_D - \theta_B)} \cos \theta_D $$

(2)
\[ D(x_D, y_D) = y_0 \pm r_D \sin \theta_D \]  

Point \( G \) is the intersection of the straight line that passes through point \( D \) and is parallel to the y-axis and the slope surface. \( y_G \) is the ordinate of point \( G \).

\[ y_G = \begin{cases} 
    x_D \tan \alpha & \text{if } x_D \leq x_C \\
    x_C & \text{if } x_D > x_C
\end{cases} \]  

Point \( E \) is the intersection of the straight line that passes through point \( D \) and is parallel to the x-axis and the slope surface. \( x_E \) is the ordinate of point \( G \).

\[ x_E = \frac{y_D}{\tan \alpha} \]  

Based on the vertical strip method, the sliding soil mass is integrated. The mass and gravity power can be written as Eq. (6) and (7).

\[ m = \int_0^x \rho h(x)dx \]  

\[ P_w = \int_0^x r_D \rho \omega \cos \theta_D h(x)dx \]  

\( \omega \) is the rotational angular velocity, \( h(x) \) is the height of the vertical soil unit.

**Figure 2.** Schematic diagram of the strip method

### 2.2. Power of seismic load

Schematic diagram of the strip method is show in Figure 2 [18]. Studies have shown that the seismic waves have an amplification effect with height. The formula for seismic force can be expressed as Eq. (8) and (9).

\[ P_h(t) = \int_0^b r_D \rho \omega g \sin \theta_D g(z) k_h a_h(t) \frac{z \cdot f_h}{H} dz \]  

\[ P_v(t) = \int_0^x r_D \rho \omega \cos \theta_D h(x) k_v a_v(t) \frac{v_G - v_D}{2} \frac{f_v}{H} dx \]  

Where \( P_h(t) \) and \( P_v(t) \) are the rate of seismic work in the horizontal and vertical directions, respectively. \( g(z) \) is the width of the horizontal soil unit. \( a_h(t) \) and \( a_v(t) \) are the horizontal and vertical seismic accelerations respectively. \( k_h \) and \( k_v \) are the seismic acceleration coefficients in the horizontal and vertical directions. \( f_h \) represents the magnification factor of the slope, \( g \) is the acceleration of gravity.

### 2.3. The rate of energy dissipation along the failure surface

According to the upper limit theorem, the energy dissipation rate of the failure surface can be expressed as the product of the length of the failure surface, the allowable speed and the cohesive force[19].
\[ P_\text{c} = \int_{\theta_0}^{\theta_f} c \omega \cos(\phi) \frac{r d\theta}{\cos(\phi)} = \frac{cr_0^2 \omega}{2 \tan \phi} (e^{2(\theta_0 - \theta_0)\tan \phi} - 1) \] (10)

2.4. The rate of energy dissipation due to stabilizing pile

Generally, the stabilizing piles are equivalent to the horizontal lateral force. The energy dissipation rate of the pile can be formulated as

\[ P_p = F_p r_p \omega \sin \theta_p = F_p r_p \omega \sin \theta_p e^{(\theta_0 - \theta_0)\tan \phi} \] (11)

2.5. Displacement of slope

If the power of seismic load exceeds the critical value, the slope produces rotational acceleration and velocity. The relationship between power of seismic load and angular acceleration can be expressed as Eq. (12).

\[ P_w + P_v + P_h = P_c + P_p + \alpha \dot{\theta}^2 m \ddot{\theta} \] (12)

Where \( I \) is the distance between the centroid of sliding soil and the center of rotation. \( \dot{\theta} \) is the angular acceleration.

The slope has the maximum displacement at point \( O \). It can be used to represent the slope stability.

\[ s_O^*(t) = r_O \sin(\theta_0 + \Delta \theta) \int_0^t \dot{\theta} dt dt \] (13)

3. Selection of seismic wave

Incremental dynamic method (IDA) is a nonlinear dynamic time-history analysis method. The dynamic response characteristics of structures with different seismic intensity can be analyzed by the IDA. The analysis steps are as follows: 1) Select the ground motion record and determine the seismic intensity index(IM). 2) By adjusting the intensity, the original records are transformed into a set of ground motions with different intensities. 3) Select the performance index (DM). 4) Calculate the dynamic response of the structure with different seismic intensities and draw the IDA curve. 5) By analyzing the trend of the IDA curve, the failure process of the structure can be revealed[21].

### Table 1 Seismic record information

| No. | name                     | Time | PGA (g) | Duration (s) | Epicenter Distance (km) | Type |
|-----|--------------------------|------|---------|--------------|--------------------------|------|
| 1   | BrawleyAirport           | 1979 | 0.152   | 11.13        | 10.42                    | A    |
| 2   | El Centro Array #3       | 1979 | 0.127   | 22.08        | 12.85                    | A    |
| 3   | El Centro Array #4       | 1979 | 0.269   | 19.43        | 7.05                     | A    |
| 4   | Holtville Post Office    | 1979 | 0.257   | 9.58         | 7.05                     | A    |
| 5   | Sturino(STN)             | 1980 | 0.225   | 12.108       | 10.84                    | A    |
| 6   | Gilroy-Historic Bldg.    | 1989 | 0.148   | 11.24        | 10.97                    | A    |
| 7   | Gilroy Array #2          | 1989 | 0.295   | 9.00         | 11.07                    | A    |
| 8   | Pacoima Dam (downstr)    | 1994 | 0.191   | 7.28         | 7.01                     | A    |
| 9   | Pacoima Kagel Canyon     | 1994 | 0.169   | 11.50        | 7.26                     | A    |
| 10  | Izmit                    | 1999 | 0.144   | 16.74        | 7.21                     | A    |
| 11  | Centerville Beach-Naval Fac | 1992 | 0.121   | 14.22        | 18.31                    | A    |
| 12  | Ulcinj-Hotel Olimpic     | 1979 | 0.423   | 9.88         | 5.76                     | A    |
| 13  | Palo Alto-SLAC Lab       | 1989 | 0.107   | 17.48        | 30.86                    | B    |
| 14  | LA-City Terrace          | 1994 | 0.072   | 15.72        | 36.62                    | B    |
| 15  | Herceg Novi-O.S.D. Paviv  | 1979 | 0.050   | 12.00        | 25.55                    | B    |
| 16  | Joetsu Yasuzukaku Yasuzuka | 2007 | 0.057   | 24.86        | 25.52                    | B    |
| 17  | Joetsu Yanagishima paddocks | 2007 | 0.120   | 10.98        | 31.43                    | B    |
| 18  | Joetsu_Aramaki District  | 2007 | 0.059   | 25.58        | 32.54                    | B    |
| 19  | Tokamachi Chitoscho      | 2007 | 0.094   | 29.76        | 30.65                    | B    |
| 20  | Kawaguchi                | 2007 | 0.082   | 27.34        | 29.25                    | B    |
| 21  | Tamati Ono               | 2008 | 0.081   | 19.08        | 28.91                    | B    |
The number of ground motions has a greater impact on the vulnerability analysis. Obviously, the accuracy of the results can be improved by more seismic records. However, it is impossible and unnecessary to calculate all ground motions. According to the suggestion of Vamvatsiko et al., 10~20 is the ideal number of calculations. This number is the compromise between calculation accuracy and time cost. Using Baker’s identification method[22], 24 seismic waves were selected from the strong earthquake database of the Pacific Earthquake Engineering Research Center of the United States. In the selected 24 seismic records, there are 12 near-fault ground motions and 12 far-field ground motions. In Table 1[23], A and B represent near-fault and far-field earthquake respectively.

4. Selection of IDA index

4.1. Selection of damage performance indicators

At present, in the IDA, the seismic intensity index (IM) mainly includes PGA, PGV, PGD and Sa. A rectangular coordinate system with ln(IM) as the abscissa and ln(DM) as the ordinate is established. The dispersion of the fitted straight line is often used to evaluate the applicability of the index. The regression coefficient can be obtained by linear fitting. A smaller standard deviation indicates that the applicability of the indicator is better. With reference to previous research, in this paper, PGA is used as the intensity index of ground motion.

![Figure 3. IM-DM curves with different ground motions (a) near-fault ground motions (b) far-field ground motions](image)

Figure 3(a) and Figure 3(b) are the regression lines of near-field and far-field ground motions, respectively. The slope $\alpha_1$ of the straight line fitted by near-fault earthquake is 7.52, which is greater than the far-field ($\alpha_2 = 7.19$). The standard deviations of the two straight lines are $\delta_1 = 2.0$ and $\delta_2 = 1.98$ respectively, which is no significant difference. The fitted data with lower seismic intensity has a higher degree of dispersion. As the seismic intensity increases, the dispersion phenomenon is weakened. On the whole, the data points are closely distributed near the fitting straight line. It indicates that the relationship between the slope displacement and the ground motion parameters can be reflect by the regression function.

4.2. Classification of seismic performance level

Vulnerability analysis is also affected by the division method of seismic performance level. In order to accurately draw the seismic vulnerability curve, it is necessary to quantify the damage state of the slope. With the design concept of three-level, Zhu Hongwei divided the retaining wall project into five damage states: I intact, II basically intact, III damaged, IV severely damaged and V destroyed[14]. In
this paper, the seismic resistance level is divided into four states: I basically intact, II partial damage, III severe damage, and IV overall instability. Based on the research of Jibson and Michael[24], the quantified threshold of performance level is defined by the slope sliding displacement, in Table 2. When the sliding displacement of the slope is less than 1 cm, the impact of the seismic loads on the slope can be ignored. When the displacement is 1-5cm, partial damage occurred on the slope, and only local reinforcement is required. When the displacement is 5-15cm, there are many local damages on the slope. The slope needs to be reinforced as a whole. When the displacement is greater than 15cm, the slope will lose stability.

| Damage level       | Limit state | Response characteristics | Slope displacement (cm) |
|--------------------|-------------|--------------------------|-------------------------|
| Basically intact   | LS1         | There is no risk of instability. | 0-1                     |
| Partial damage     | LS2         | The slope has local instability, and only local reinforcement is required. | 1-5                     |
| Serious damage     | LS3         | There are many local damages on the slope. The risk of damage is greater, and the slope needs to be reinforced as a whole. | 5-15                    |
| Overall instability|             | Severe collapse and landslide disaster occurred. | ≥15                     |

4.3 Incremental dynamic analysis
By calculating 288 ground motions, the IM-DM curve of the near-fault and far-field is obtained. It can be seen from Figure 4 that when the seismic intensity is weak, the IDA curve is more concentrated. As the seismic intensity increases, the dispersion of the calculation results gradually increases. The sliding displacement of the slope is closely related to the ground motion. Especially in the case of strong earthquakes, this effect will be more obvious. The average value of displacement under the near-fault earthquakes is greater than that of the far-field. It indicates that near-fault pulsed ground motions are more destructive.

![Figure 4. IDA curves under different ground motions (a) near-field ground motions (b) far-field ground motions](image)

5. Vulnerability analysis of slope
Vulnerability can be understood as the probability of a structure exceeding a certain limit state. It describes the seismic performance of the structure from the perspective of probability. Seismic vulnerability is usually assumed to follow the log-normal distribution[25].
\[
P_{LS}(D \geq D_{sl}, IM) = 1 - \Phi \left[ \frac{1}{\delta} \ln(D_i / M_{eqIM}) \right]
\]

(14)

\[
\delta = \sqrt{\frac{\sum_{i=1}^{N} [\ln(D_i - \ln \alpha + \beta \ln(IM))]^2}{N-2}}
\]

(15)

Where \(P_{LS}\) is the probability that the structure exceeds a certain damage state. \(D_{sl}\) is the level index of structural performance. \(IM\) is the index of seismic intensity. \(\Phi\) is the cumulative function of standard normal distribution. \(M_{eqIM}\) is the median value of damage index caused by ground motion. \(D_i\) is the actual damage value. \(\delta\) is the standard deviation of the normal distribution.

The limit state (LS) is also called the performance level in PBEE, which is the boundary of adjacent damage states. The damage state (DS) is defined as the interval between two adjacent limit states. The probability of damage state can be calculated by formula (17).

\[
P_{DS} \begin{cases} 
1 - P_{LS}(D \geq D_{sl}, IM) & i = 0 \\
P_{LS}(D \geq D_{sl}, IM) - P_{LS}(D \geq D_{sl-1}, IM) & i = 1, 2, ..., N-1 \\
N \end{cases}
\]

(16)

In the formula, \(N\) is the number of limit states. According to the relationship between the limit state and the damage state, the \(N\) limit states can be divided into \(N+1\) damage states.

It can be seen from Figure 5 that the vulnerability curve of the slope shows a monotonous increase. When the seismic intensity is small (PGA=0−0.5), the curves are the same. As the seismic intensity increases (PGA=0.5−1.2), the failure probability of the near-fault is significantly higher than the far-field. Taking the seismic coefficient of 0.6 as an example, the exceedance probability of the slope with near-fault ground motions are 0.56, 0.25 and 0.11 respectively. The probability of the far-field are 0.38, 0.14 and 0.05.

![Figure 5. Vulnerability curve of slope](image1)

![Figure 6. DS probability curves of slope](image2)

As shown in Figure 6, the DS probability curve of the structure is different from the vulnerability curve. The trend indicates that with the increase of the seismic intensity, the damage state of the slope is changing. The probability of basically intact decreases, while the overall instability increases monotonically. The probability of partial damage and severe damage first increases and then decreases. When the PGA is less than 0.4g, the slope remains stable. As the PGA increases (0.4−0.7), the slope changes from the basically intact state to the state of partially damaged and severely damaged. When the seismic intensity continues to increase, slope is more prone to complete destruction. Although the DS probability curves of two earthquakes have the same trend, there are obvious differences in the occurrence time. The impact of near-fault earthquakes on slopes can be divided into two stages. In the first stage (PGA < 0.7), the DS probability of different damage states is greater than the far-field. In the
second stage (PGA>0.7), the slope is more prone to complete destruction. Probabilities of local damage and severe damage decreases and are less than the far-field. Horizontal seismic accelerations of 0.6 and 1.0 are used to illustrate this phenomenon. When the GPA is 0.6, the DS probability of near-fault earthquake are 0.14, 0.62 and 0.08, respectively. The far-field probability is 0.11, 0.24 and 0.05. When the GPA is 1.0, the DS probabilities of the near-fault is 0.76, 0.17, 0.56, and the far-field is 0.13, 0.07, 0.19. The reason is that near-field seismic wave contain velocity pulses with higher energy. It will promote the slope damage earlier. Obviously, if the impact is ignored, the disaster may be underestimated.

6. Conclusion
In this paper, the vulnerability analysis is introduced into the stability evaluation of the pile-reinforced slope. Based on the limit analysis and Newmark method, the sliding displacement of the slope under seismic load is derived. Besides, through a large number of calculations, the effects of near-fault and far-field ground motions on the seismic performance of slopes are compared. The following conclusions were obtained:
The influence of different seismic waves on the slope can be reflected by Newmark method. Compared with the finite element method, the theoretical calculation model can reduce the calculation time and is more suitable for the vulnerability analysis of slope. The relationship between ground motion and structural dynamic response can be established by the IM-DM curve with the damage index of displacement and the intensity index of GPA. Based on the three-level design concept, the slope failure status is divided into four levels: I basically intact, II partial damage, III severe damage, and IV overall instability. Vulnerability analysis can be used to evaluate the failure probability of slopes in different performance levels. The failure mode of the slope is affected by the seismic intensity. A lower PGA will cause local damage. As the intensity of the seismic increases, the probability of complete destruction increases. Near-field ground motions have the characteristics of large energy and strong impact, which are more likely to cause slope instability.

Acknowledgments
The authors gratefully acknowledge the financial support provided by the National Key R&D Program of China (No. 2019YFC1509704, No.2017YFC0805403), and the National Natural Science Foundation of China (No. 41877218, No. 42072308).

References
[1] Li P, Hang B, Li Q and Wang R 2014 Eng. Geol. 194 86-97.
[2] Fan X, Scaringi G, Xu Q et al. 2018 Landslides 15 967-83
[3] Huang Y, Hu H, Xiong M 2019 Struct Infrastruct Eng 15 103-12
[4] Wen Z P, Xie J J, Gao M T, Hu Y X and Chau K T 2010 Bull. Seismol. Soc. Am. 100 2425-39.
[5] Tothong P, Cornell C A and Baker J W 2007 EARTHQ. SPECTRA 23 867-891.
[6] MacRae G A, Morrow D V and Roeder C W 2001 J. STRUCT. ENG. 127 996 -1004.
[7] Zhang Y B, Xiang C L, Chen Y L, Cheng Q G, Xiao L, Cheng Y P and Chang Z W 2019 J. Mt. Sci. 16 1244-57.
[8] Bing Y, HOU JR, Liu YF, Zou ZH 2021 Shock Vib.https://doi.org/10.1155/2021/5595278
[9] Huang Y, Xiong M 2017 Soil Dyn Earthq Eng 94 1-6.
[10] Huang HW, Wen SC, Zhang J, Chen FY, Martin JR, Wang H 2018 Landslides 15 2303–13
[11] Naderpour H, Vakili K 2019 Earthq. Struct. 16 425-36.
[12] Pang R, Xu B, Kong X, Zou D 2018 Soil Dyn Earthq Eng 104 432–36.
[13] Hu H Q, Huang Y, Chen Z Y 2019 Environ. Earth Sci 78.
[14] Zhu H W, Yao L K and Lai 2020 Chin. J. Rock Mech. Eng. 42 150-57.
[15] Chen J, Yin J, Lee C 2003 Can. Geotech. J. 40 742–52
[16] Nian T K, Jiang J C, Wang F W, Yang Q and Luan M T 2016 Soil Dyn. Earthq. Eng. 84 83-93.
[17] Li X P, He S M, Wu Y 2010 J Geotech Geoenviron Eng 136 880-884
[18] Yan M J, Xia Y Y, Liu T T and Bowa V M 2019 Comput. Geotech. 108 226-33.
[19] Won J, You K, Jeong S and Kim S 2005 Comput. Geotech. 32 304-315.
[20] Poulos H G 1995 Can. Geotech. J. 32 808-18.
[21] Vamvatsikos D and Cornell C A 2002 Earthq. Eng. Struct. Dyn. 31 491–514.
[22] BAKER J W 2007 Bull. Seismol. Soc. Am. 97 1486–1 501.
[23] Mei X C, Cui Z and Sheng Q 2021 Chin. J. Geotech. Eng. 40 344-54.
[24] Jibson RW, Michael JA 2009 Alaska US Geological Survey Reston, VA, USA
[25] Cornell C A, Jalayer F, Hamburger R O and Foutch D A 2002 J. STRUCT. ENG. 128 526-33.