ABSTRACT This paper concerns the issue of extended dissipative analysis for complex dynamical networks with coupling delays under a sampled-data control scheme. Firstly, we derive the input delay method and combine it with an appropriate Lyapunov functional, which can make full use of the information of the sampling period. Secondly, novel sufficient synchronization criteria are established by applying Jensen’s inequality, Wirtinger’s integral inequality, a new integral inequality, free-weighting matrix technique, and convex combination method. Moreover, we focus on the extended dissipative analysis issue, which includes $L_2 - L_\infty$, $H_\infty$, passivity, and dissipativity performance in a unified formulation. These conditions can express in Linear matrix inequalities (LMIs) restrictions, which can solve with readily accessible software. Finally, two numerical examples illustrate the effectiveness and reduced conservatism of our developed method.

INDEX TERMS Extended dissipative, sampled-data control, synchronization, complex dynamical networks, coupling delays.

I. INTRODUCTION

During the last few decades, complex dynamical networks (CDNs) have been applied in real-world systems such as social networks, the internet, brain networks, food chains, and disease-spreading networks [1], [2], [3]. There are many connected nodes in this system, each of which represents a dynamic system. Depending on the network topology, some of these nodes are typically connected. A graph is a mathematical concept that describes these networks. In such diagrams, vertices represent system membership, whereas edges represent their interaction. In addition, the time delay is unavoidable in many scenarios, including neural networks, electrical engineering systems, chemical or process control systems, the internet, transportation systems, etc. It can cause instability, oscillations, and poor system performance. Because coupling delay is a prevalent time delay in CDNs, it is critical to think about how time delay affects CDNs.

Furthermore, synchronization is vital in CDNs because it can explain various phenomena, such as synchronous data exchange on the World Wide Web and the internet. As a result, it has become a hot topic that has been thoroughly researched. Li and Chen [4] presented CDNs with coupling delay for continuous- and discrete-time systems in the last decade. Gao et al. [5] studied synchronization for a general class of CDNs with coupling delays to discover a novel criterion for ensuring that the system is asymptotically stable. In the case that all nodes of networks cannot achieve synchronization by themselves. So, it is challenging to design a practical controller that can synchronize the networks. Various control schemes can be applied in CDNs, such as the pinning control [6], [7], [8], the impulsive control [9], [10], [11], the intermittent control [12], [13], [14], and the adaptive control [15], [16], [17]. Recently, communication networks and computers have advanced to a high-speed version, and practically digital signal techniques have replaced analog signal methods, which provide a more consistent result. Thus, the sampled-data system has received much attention [18], [19], [20], [21].

The associate editor coordinating the review of this manuscript and approving it for publication was Wonhee Kim.
As is known, a vital point in sampled-data systems is to place importance on the desired controller under a more considerable sampling period. Li et al. [22] investigated the synchronization of regular CDNs for the first time, including coupling time-varying delay, where sampled-data controllers were designed. An adequate condition was obtained using the Jensen inequality to achieve the exponential stability of the synchronization error system. Wu et al. [23] presented the exponential synchronization of CDNs with coupling time-varying delay. The above authors reduced a synchronization criterion using the same Lyapunov functional and a new integral inequality that takes full merit of the detail on delays. In [24], such a problem was further addressed by applying a reciprocally convex technique with an improved inequality that can yield a tighter upper bound than Jensen inequality. The previously reported results [22], [23], [24] were enhanced further in [25] by using a new time-dependent Lyapunov functional method and Wirtinger-based integral inequality [26]. Chen et al. [27] introduced Wirtinger-based double integral inequalities [28] to reduce the conservatism of the synchronization criterion by incorporating the new Lyapunov functional in triple integral inequalities. Obtaining a more sampling period has proven to be a significant issue in earlier studies [22], [23], [24], [25], [27]. As a result, implementing less conservative synchronization requirements for CDNs with sampled-data control has become a hot topic, and designing CDNs with sampled-data control is worthwhile. However, it should be noted that while these studies focus on using the inequality methodology to improve results, the information on the sawtooth structure on the input delay is not fully utilized, leaving room for improvement. In addition, the works mentioned above take no account of external disturbances, which is unworkable in practice. Thus, to implement sampled-data synchronization in CDNs, it is required to account for external disturbances and fully benefit the sawtooth function class of sampling period, which is the inspiration for our paper.

Furthermore, the topic of network performance analysis, which is based on the relationship between input and output, is essential in practical systems. Over the last few decades, analyzing performance has been essential in engineering and science applications [30], [31], [32], [33], [34], [35]. Zhang et al. [35] introduced a generic performance called extended dissipativity, which effectively encompasses the passivity, $L_2 - L_\infty$, $H_\infty$, and dissipativity index and it has received additional research in the last few years. In [30], the extended dissipative performance was utilized in neural network systems with continuous time-varying delay. For the first time, Yang et al. [36] studied the extended dissipative for synchronization of CDNs with coupling delay. However, the vital information of $t - t_k$ and $t_k + 1 - t$, $\forall t \in [t_k, t_{k+1})$ is not fully maximum potential. As a result, it is only reasonable to seek a different perspective to construct a less conservative condition for synchronizing sampled-data CDNs with coupling delays. From those mentioned above, we dedicate ourselves to overcoming the issue of extended dissipative analysis for sampled-data synchronization for CDNs with coupling delays. The main contribution of this article highlights as follows:

- The proposed CDNs with coupling time-varying delays are comprehensive models for the other existing CDNs [22], [23], [25], [29], [37], [38]. We take the external disturbances into each node which is not considered in [22], [23], [25], [27], and [29], and [37], and [38].
- We construct a time-dependent Lyapunov functional different from the references in [22], [23], [24], [25], and [27], and the advantage information on discrete sample point $t_k$ is fully used.
- It is worth mentioning that the positive definitiveness of the proposed Lyapunov functional requires only sampling times that are not necessarily throughout the sampling periods. So, we derive a new inequality that applies to prove the exponential synchronization of CDNs.
- A less conservative criterion is obtained under the novel Lyapunov functional using Jensen’s inequality, Wirtinger’s integral inequality, a new integral inequality, the free-weighting matrix technique, and the convex combination approach. Moreover, our results can verify Chua’s circuit system.

This paper is separated in the following way. The problem formulation, definitions, assumptions, and lemmas provide in Section 2. The main results are present in Section 3, which considers the exponential synchronization via sampled-data control and the extended index analysis. Two numerical examples manifest the effectiveness and reduced conservatism of the existing results in section 4. Finally, we sum up this letter in Section 5.

**Notation:** This paper contains the following notations, $\mathbb{R}^n$ stands for $n$-dimensional Euclidean space and $\mathbb{R}^{m \times n}$ is the set of $m \times n$ real matrices. The symmetric matrix $P > 0$ is that the matrix $P$ is positive definite. $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ mean the minimum and maximum eigenvalues of $P$, $\text{sym}[P]$ defines $P + P^T$ The superscript $T$ is the transpose. The symbol $\ast$ represents the symmetric entries of the symmetric matrix. The symbol $\otimes$ denotes the Kronecker product, and $\text{diag}[\ldots]$ is the block diagonal matrix.

**II. PRELIMINARIES**

Consider the complex dynamical networks, which consist of $N$ identical linked nodes, each of which is an $n$-dimensional dynamical system:

$$\dot{x}_i(t) = f(x_i(t)) + c \sum_{j=1}^{N} G_{ij} \Gamma^{(1)} x_j(t) + c \sum_{j=1}^{N} G_{ij} \Gamma^{(2)} x_j(t - \tau(t)) + u_i(t) + \omega_i(t),$$

$$z_i(t) = J x_i(t), \quad i = 1, 2, \ldots, N,$$

(1)

where $x_i(t)$ and $u_i(t)$ are, respectively the state variable and the control input of each node. $f(x_i(t)) \in \mathbb{R}^n$ is a nonlinear vector valued function defining the dynamics of $ith$ nodes.
c > 0 is the coupling strength. The internal-coupling matrices are $G^{(1)}_i, G^{(2)}_i \in \mathbb{R}^{n \times n}$ and the matrix $G = (G_{ij})_{N \times N} \in \mathbb{R}^{N \times N}$ is the outer-coupling configuration matrix, in which $G_{ij}$ is satisfied as follows: if there exists a string from node $i$ to node $j$ ($i \neq j$), $G_{ij} > 0$; otherwise, $G_{ij} = 0$. While, the diagonal elements of $G$ is defined as $G_{ii} = -\sum_{j=1, j \neq i}^{N} G_{ij}$. For simplicity, $\omega(t) \in \mathbb{R}^n$ is the external perturbation, which belongs to $L_2(0, \infty)$, $z(t) \in \mathbb{R}^n$, is the output, and $\mathcal{F}$ is a given matrix with proper dimensions. The function $\tau(t)$ represents the coupling time-varying delay that satisfies:

$$0 \leq \tau(t) \leq \tau_u, \quad 0 \leq \dot{\tau}(t) \leq \tau_d. \quad (2)$$

Let $\hat{v}(t) = f(v(t))$, where $v(t) \in \mathbb{R}^n$ is the state response of the unforced isolated node. Then the error vector is $r(t) = x_i(t) - v(t)$. Let $\eta_i(t)$ be the output of isolate node, which defines as $\eta_i(t) = \mathcal{F}v(t)$. Then, the output error $\hat{z}_i(t)$ is described as follows

$$\hat{z}_i(t) = z_i(t) - \eta_i(t), \quad i = 1, 2, \ldots, N. \quad (3)$$

At the same time, a zero-order-hold (ZOH) function with a sequence of hold times $0 = t_0 < t_1 < \cdots < t_k < \cdots$, $lim_{k \to +\infty} t_k = +\infty$ is assumed to be used to create the control signal. Then, the sampled-data feedback controller constructs in the form as:

$$u_i(t) = K_i r_i(t_k), \quad t_k \leq t < t_{k+1}, \quad i = 1, 2, \ldots, N, \quad (4)$$

where $K_i$ denotes a set of feedback controller gain matrices that can be designed. $r_i(t_k)$ is a discrete measurement of $r_i(t)$ at the sampling time $t_k$. Here, we assume that $t_{k+1} - t_k = d_k \leq h_u$ for all integer $k \geq 0$, where $h_u > 0$ stands for the largest sampling period. Then, we have the following error closed-loop system:

$$\dot{r}_i(t) = g(r_i(t)) + c \sum_{j=1}^{N} G_{ij} G^{(1)}_{ji} r_j(t)$$

$$+ c \sum_{j=1}^{N} G_{ij} G^{(2)}_{ji} r_j(t - \tau(t)) + K_i r_i(t_k) + \omega_i(t). \quad (5)$$

where $g(r_i(t)) = f(x_i(t)) - f(v(t))$. Let $r(t) = \left[r^{(1)}_i(t), r^{(2)}_i(t), \ldots, r^{(N)}_i(t)\right]^T$, $g(r(t)) = [g^{(1)}(r(t)), g^{(2)}(r(t)), \ldots, g^{(N)}(r(t))]^T$, $K = \text{diag} \{K_1, K_2, \ldots, K_N\}$. Then, error dynamical (5) can be transformed as ways:

$$\dot{r}(t) = g(r(t)) + c(G \otimes G^{(1)}) r(t) + c(G \otimes G^{(2)}) r(t - \tau(t))$$

$$+ K r(t_k) + \omega(t), \quad (6)$$

where $\hat{z}(t) = \mathcal{F} r(t), \quad t \in [t_k, t_{k+1}).$

Here, we give some assumptions, definitions, and lemmas that are required to obtain the new synchronization criterion.

**Assumption 1** [39] The continuous function $f : \mathbb{R}^n \to \mathbb{R}^n$ satisfies the following condition:

$$[f(x) - f(y) - U(x - y)]^T$$

$$\times [f(x) - f(y) - V(x - y)] \leq 0, \quad \forall x, y \in \mathbb{R}^n, \quad (7)$$

where $U$ and $V$ are known constant matrices of compatible dimensions.

**Remark 1:** It is to be noted that such a nonlinear function of characterization is the sector-bounded condition, which is derived in [39] and includes the commonly Lipschitz conditions. It is clearly to show that for any nonlinear function $f$ satisfying (7), there exists a scalar $\beta > 0$ such that

$$\|f(x) - f(y)\| \leq \beta \|x - y\|^2. \quad (8)$$

**Definition 1:** [36] The CDNs (1) is said to be exponentially synchronized if the error dynamic (6) is exponentially stable, i.e., there exist two constants $\beta > 0$ and $\alpha > 0$ such that

$$\|r(t)\|^2 \leq \alpha e^{-\beta t} \sup_{-\max\{h_u, \tau_d\} \leq \theta \leq 0} \{\|r(\theta)\|^2, \|\dot{r}(\theta)\|^2\}. \quad (9)$$

**Definition 2:** [36] Given matrices $\Phi_1, \Phi_2, \Phi_3,$ and $\Phi_4$ with symmetric matrices $\Phi_i, \Phi_j,$ and $\Phi_k$, system (6) is achieved to be extended dissipative if for all $t_f \geq 0$ and any $\omega(t) \in L_2(0, \infty)$, under zero initial condition, the following inequality holds:

$$\int_{0}^{t_f} J(t) dt \geq \sup_{0 \leq t \leq t_f} z^T(t) \Phi_4 \hat{z}(t), \quad (10)$$

where $J(t) = z^T(t) \Phi_1 \hat{z}(t) + 2z^T(t) \Phi_2 \omega(t) + \omega^T(t) \Phi_3 \omega(t)$. Throughout this paper, the general assumptions on $\Phi_1, \Phi_2, \Phi_3,$ and $\Phi_4$ are utilized.

**Assumption 2.** [35] For given real symmetric matrices $\Phi_1, \Phi_2, \Phi_3,$ and $\Phi_4$ the following conditions hold:

(1) $\Phi_1 \leq 0, \Phi_3 > 0,$ and $\Phi_4 \geq 0$;

(2) $(\|\Phi_1\| + \|\Phi_2\|) \cdot \|\Phi_4\| = 0$.

**Remark 2:** Notably, in Eq. (10) is called an extended index that provides a more extensive performance by adjusting the matrix parameters $\Phi_i$, $i = 1, 2, 3, 4$. In particular, (10) turns into the $\mathcal{L}_{\infty}$ performance when $\Phi_1 = \Phi_2 = 0$, $\Phi_3 = \delta^2 I$, and $\Phi_4 = I$; Eq. (10) is the $\mathcal{H}_{\infty}$ performance when $\Phi_1 = I$, $\Phi_2 = \Phi_4 = 0$, and $\Phi_3 = \delta^2 I$, (10) diminished to the $(Q, S, \mathcal{R})$- property index when $\Phi_1 = Q$, $\Phi_2 = S$, $\Phi_3 = \mathcal{R} - \delta I$, and $\Phi_4 = 0$. In each of the four cases, the scalar $\delta$ represents the corresponding performance index.

**Lemma 1:** (Reciprocally Convex Lemma [40]). For any vectors $x_1, x_2$, matrices $M > 0$, $S$, and scalars $\epsilon_1 > 0, \epsilon_2 > 0$, satisfying $\epsilon_1 + \epsilon_2 = 1$, the following inequality holds:

$$\frac{1}{\epsilon_1} x_1^T M x_1 - \frac{1}{\epsilon_2} x_2^T M x_2 \leq \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \begin{bmatrix} M & S \\ S^* & M \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (11)$$

subject to

$$0 < \begin{bmatrix} M & S \\ S^* & M \end{bmatrix}.$$

**Lemma 2:** [26] For any matrix $R > 0$. Then for any continuous function $x$ in $[a, b] \to \mathbb{R}^n$ the inequality
holds as below:
\[
\int_a^b x^T(u)R(x(u))du \geq \frac{1}{b - a} \left( \int_a^b x(u)du \right)^T R \left( \int_a^b x(u)du \right) + \frac{3}{b - a} \Omega^2 \Omega,
\]
where \( \Omega = \int_a^b x(s)ds - \frac{2}{b - a} \int_a^b x(r)drds \).

**Lemma 3:** (Schur Complement [41]). Let \( X, Y, Z \) be given matrices such that \( Z > 0 \), then
\[
\begin{bmatrix}
Y & X^T \\
X & -Z
\end{bmatrix} < 0 \iff Y + X^TZX^{-1}X < 0.
\]

**Lemma 4:** Consider the dynamical system (6). Then there exist two scalars \( \varphi_1 \) and \( \varphi_2 \) satisfying
\[
\|r(t)\|^2 \leq \varphi_1 \|r(t_k)\|^2 + \varphi_2 \int_{t_k}^{t_{k+1}} \|r(\alpha)\|^2 d\alpha,
\]
where
\[
\varphi_1 = 5(1 + \|K\|^2 h_u^2)\|c(\mathcal{G} \otimes \Gamma(1))\|^2 + \|c(\mathcal{G} \otimes \Gamma(2))\|^2 h_u^2,
\]
\[
\varphi_2 = 5\|c(\mathcal{G} \otimes \Gamma(2))\|^2 h_u \varepsilon^2 \|c(\mathcal{G} \otimes \Gamma(1))\|^2 + \rho + \|c(\mathcal{G} \otimes \Gamma(2))\|^2 h_u.
\]

**Proof:** From (6), we have the following inequality for any \( t \in [t_k, t_{k+1}) \),
\[
\|r(t)\| \leq \|r(t_k)\| + \int_{t_k}^{t} c(\mathcal{G} \otimes \Gamma(1))r(\alpha)d\alpha + \int_{t_k}^{t} c(\mathcal{G} \otimes \Gamma(2))r(\alpha - \tau(\alpha))d\alpha + \int_{t_k}^{t} K\alpha r(t_k)d\alpha + \int_{t_k}^{t} g(\alpha)d\alpha.
\]
By utilizing the Cauchy–Schwarz inequality, we obtain from that (13)
\[
\|r(t)\|^2 \leq 5\|r(t_k)\|^2 + 5\int_{t_k}^{t} c(\mathcal{G} \otimes \Gamma(1))r(\alpha)d\alpha^2 + 5\int_{t_k}^{t} c(\mathcal{G} \otimes \Gamma(2))r(\alpha - \tau(\alpha))d\alpha^2 + 5\int_{t_k}^{t} K\alpha r(t_k)d\alpha^2 + 5\int_{t_k}^{t} g(\alpha)d\alpha^2.
\]
Using the Cauchy–Schwarz inequality once more, we can deduce from (14) that
\[
\|r(t)\|^2 \leq 5\|r(t_k)\|^2 + 5h_u \int_{t_k}^{t} c(\mathcal{G} \otimes \Gamma(1))r(\alpha)d\alpha + 5h_u \int_{t_k}^{t} c(\mathcal{G} \otimes \Gamma(2))r(\alpha - \tau(\alpha))d\alpha + 5h_u \int_{t_k}^{t} K\alpha r(t_k)d\alpha + 5h_u \int_{t_k}^{t} g(\alpha)d\alpha.
\]
Furthermore, it is clear from (8) that
\[
\|g(r(t))\|^2 \leq \rho\|r(t)\|^2.
\]
Therefore
\[
\|r(t)\|^2 \leq 5\|c(\mathcal{G} \otimes \Gamma(1))\|^2 + \rho h_u \int_{t_k}^{t} \|r(\alpha)\|^2 d\alpha + 5\|r(t_k)\|^2 + 5\|K\|^2 h_u \int_{t_k}^{t} \|r(\alpha - \tau(\alpha))\|^2 d\alpha + 5\|c(\mathcal{G} \otimes \Gamma(2))\|^2 h_u \int_{t_k}^{t} \|r(\alpha)\|^2 d\alpha \leq 5(1 + \|K\|^2 h_u^2)\|r(t_k)\|^2 + 5\|c(\mathcal{G} \otimes \Gamma(2))\|^2 h_u \int_{t_k}^{t} \|r(\alpha - \tau(\alpha))\|^2 d\alpha + 5\|c(\mathcal{G} \otimes \Gamma(1))\|^2 + \rho + \|c(\mathcal{G} \otimes \Gamma(2))\|^2 h_u \int_{t_k}^{t} \|r(\alpha)\|^2 d\alpha.
\]

We can immediately obtain (12) by utilizing the Gronwall–Bellman Lemma to (16). The proof is now complete. \( \square \)

**Remark 3:** The proof of Lemma 4 is similar to that of Lemma 2 of [42]. However, it should be noted that the mentioned paper assumed that time delays are expected to be constant. In general, the case of time-varying delays is more practical than constant delays. This paper considers time-varying coupling delays, which are more general than constant time delays.

### III. MAIN RESULTS

This section will show you how to solve the problem using the design method. Firstly, a less conservative synchronization assures the system (6) is exponentially stable with the sampled-data control (4). Secondly, the disturbance is input to satisfy extended dissipative analysis. For the sake of fluent presentation, we provide some notations as follows
\[
\mathcal{X} = \begin{bmatrix}
\text{sym}\{\frac{1}{2}X_1\} & -X_1 + X_2 & X_3 \\
* & \text{sym}\{\frac{1}{2}X_1 - X_2\} & X_4 \\
* & * & \text{sym}\{\frac{1}{2}X_5\}
\end{bmatrix},
\]
\[
\zeta(t) = \begin{bmatrix}
0_{nN \times (i-1)nN} & I_nN, & 0_{nN \times (11-i)nN}
\end{bmatrix}^T,
\]
\[
e_i = \begin{bmatrix}
0_{nN \times (i-1)nN}, & I_nN, & 0_{nN \times (11-i)nN}
\end{bmatrix}
\]
\[
\Omega_1 = \text{sym}\{e_1^T Pe_0\} + 2\beta e_1^T Pe_1,
\]
\[
\Omega_2 = \text{sym}\{e_2^T Pe_0\} + 2\beta e_2^T Pe_1.
\]
\[\begin{align*}
\Omega_2 &= e_1^T T_1 e_1 e_{2} - e^{-2\beta_2} e_2^T T_1 e_5, \\
\Omega_3 &= e_1^T T_2 e_1 (1 - \tau_d) e^{-2\beta_2} e_4^T T_2 e_4 + e_1^T T_3 e_1 \\
&- e^{-2\beta_2} e_1^T T_3 e_3, \\
\Omega_4 &= - e^{-2\beta_2} h_u e_1^T T_1 e_1, \\
\Omega_5 &= - e^{-2\beta_2 h_u} \left( e_1^T T e_1 + (e_2 + 2e_5)^T S(e_2 + 2e_5) \right), \\
\Omega_6 &= - \text{sym} \left\{ (e_1 - e_2) Y_1 + 3 \left( \frac{2}{h_u} e_1 \right) Y_2 \right\} \\
&+ \frac{3}{h_u} \text{sym} \left\{ (e_1 + e_2)^T R_2 e_7 \right\}, \\
\Omega_7 &= \text{sym} \left\{ N_1 \left[ e_1 - e_2 \right] \right\}, \\
\Omega_8 &= \text{sym} \left\{ N_2 e_7 \right\}, \\
\Omega_9 &= \tau_u^2 e_6^T Q_1 e_6 - e^{-2\beta_9} \left[ \begin{array}{cc}
 e_1 - e_4 \\
 e_1 + e_4 - 2e_9 \\
 e_4 - e_5 \\
 e_4 + e_5 - e_{10}
\end{array} \right]^T \left[ \begin{array}{c}
 S_1 \\
 \text{diag} \left\{ Q_1, 3Q_2 \right\}
\end{array} \right] \\
&\times \left[ \begin{array}{c}
 e_1 - e_4 \\
 e_1 + e_4 - 2e_9 \\
 e_4 - e_5 \\
 e_4 + e_5 - e_{10}
\end{array} \right] \left[ \begin{array}{c}
 e_2 - e_3^T S_2 Q_2
\end{array} \right], \\
\Omega_{10} &= h_u e_6^T Q_2 e_6 - e^{-2\beta_{10}} \left[ \begin{array}{c}
 e_1 - e_2 \\
 e_2 - e_3
\end{array} \right]^T \left[ \begin{array}{c}
 S_2 \\
 \text{diag} \left\{ Q_1, 3Q_2 \right\}
\end{array} \right] \\
&\times \left[ \begin{array}{c}
 e_1 - e_2 \\
 e_2 - e_3
\end{array} \right], \\
\Omega_{11} &= - \varepsilon \left[ \begin{array}{c}
 e_1^T e_{11} \\
 \text{sym} \left\{ I \otimes U^T V \right\} - I_N \otimes (U^T + V^T) \right]\left[ \begin{array}{c}
 e_1 \\
 e_{11}
\end{array} \right], \\
\Omega_{12} &= \text{sym} \left\{ \left[ \begin{array}{c}
 e_1^T e_{11} \\
 \text{sym} \left\{ I \otimes U^T V \right\} - I_N \otimes (U^T + V^T) \end{array} \right] e_1 \\
&+ c \left( G_G \otimes \Gamma^{(2)} \right)^T e_4 + K_2) \right\}, \\
\theta_1 &= \frac{h_u}{2} \varepsilon \left[ e_1^T T_4 e_2 - e_2^T T_4 e_2, \\
\theta_2 &= \frac{h_u}{2} \varepsilon \left[ e_1^T T_4 e_2 + e_2^T T_4 e_2, \\
\theta_3 &= 2\beta \left[ \begin{array}{c}
 e_1 \\
 e_2 \\
 e_7
\end{array} \right] \left[ \begin{array}{c}
 e_1 \\
 e_2 \\
 e_7
\end{array} \right] + 2 \left[ \begin{array}{c}
 e_1 \\
 e_2 \\
 e_7
\end{array} \right] \left[ \begin{array}{c}
 e_0 \\
 0 \\
 e_1
\end{array} \right], \\
\theta_4 &= 
\frac{1}{2} e_1^T S_1 e_1, \\
\theta_5 &= \frac{3}{h_u} \varepsilon \left[ e_1 - e_2 \right]^T R_2 e_2, \\
\theta_6 &= \frac{1}{h_u} \varepsilon \left[ e_1 + e_2 \right]^T R_2 e_2, \\
\theta_7 &= \frac{1}{h_u} \varepsilon \left[ e_1 + e_2 \right]^T W_1 W_2 e_6, \\
\theta_8 &= \frac{1}{4} e_6^T Z e_6, \\
\theta_9 &= \frac{1}{4} e_6^T Z e_6, \\
\theta_{10} &= \text{sym} \left\{ N_2 e_1 \right\}, \\
d_1(t) &= t - t_k, \\
d_2(t) &= t_{k+1} - t.
\end{align*}\]

(A. Stability Analysis)

The first theorem gives an exponential synchronization condition for CDNs (6) with a non-disturbance.

Theorem 1: For given scalars \( h_u, \tau_u \) and \( \tau_d \), if there are matrices \( P > 0, T_i > 0 \) (i = 1, 2, 3, 4), \( S > 0, R > 0, Z > 0, Q_1 > 0, Q_2 > 0 \) and any matrices \( W_1, W_2, W_3, X_i, i = 1, 2, 3, 4, 5 \), \( N_1, N_2, Y_1, Y_2, S_1 = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix} \) and matrices \( S_2 \) such that \( \mathcal{W} = \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix} > 0, \mathcal{X} > 0, \) \( M = \text{diag} \{ M_1, M_2, \ldots, M_N \} \), \( H = \text{diag} \{ H_1, H_2, \ldots, H_N \} \) satisfying the following LMIs:

\[\begin{align*}
\sum_{i=1}^{12} \Omega_i &< 0, \\
\sum_{i=1}^{12} \Omega_i + h_u (\theta_1 + \theta_6 + \theta_9 + \theta_{10}) - e^{-2\beta_{10}} \mathcal{W} &> 0, \\
\sum_{i=1}^{12} \Omega_i + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_7 + \theta_8 &< 0, \\
\text{diag} \{ Q_1, 3Q_2 \} &> 0, \\
\text{diag} \{ Q_1, 3Q_1 \} &> 0,
\end{align*}\]

(18) (19) (20) (21) (22)

then, the error system (6) is exponentially synchronized and the feedback controller gain matrix can be obtained as

\[K = M^{-T} H^T.\]

(23)

Proof: Consider the following Lyapunov functional represent for system (6):

\[V(t) = \sum_{i=1}^{11} V_i(t), \quad t \in [t_k, t_{k+1}),\]

where

\[V_1(t) = e^{2\beta t} r^T(t) P r(t), \]
\[V_2(t) = \int_{t-r}^{t} e^{2\beta s} r^T(s) T_1 r(s) ds, \]
\[V_3(t) = \int_{t-r}^{t} e^{2\beta s} r^T(s) T_2 r(s) ds. \]
\[
\dot{V}_4(t) = ((k+1-t)(t - t_k)e^{2\beta t_r T(t_k)}T� T(\xi(t)) + T� T(\xi(t))\right\]
\[
\dot{V}_5(t) = ((k+1-t)e^{2\beta t_r T(t_k)}T� r(t)) + T� r(t))\right\]
\[
\dot{V}_6(t) = ((k+1-t)e^{2\beta t_r T(t_k)}T� s(t) + T� s(t))\right\]
\[
\dot{V}_7(t) = ((k+1-t)\int_{t_k}^{t} e^{2\beta t_r T(s)}T� s(t)ds) + T� s(t))\right\]
\[
\dot{V}_8(t) = ((k+1-t)\int_{t_k}^{t} e^{2\beta t_r T(s)}T� r(t)) + T� r(t))\right\]
\[
\dot{V}_9(t) = ((k+1-t)\int_{t_k}^{t} e^{2\beta t_r T(s)}T� s(t)ds) + T� s(t))\right\]
\[
\dot{V}_{10}(t) = \tau_\alpha \int_{t\alpha}^{t} e^{2\beta t_r T(s)}T� s(t)ds) + T� s(t))\right\]
\[
\dot{V}_{11}(t) = \int_{t_{\alpha}}^{t} e^{2\beta t_r T(s)}T� s(t)ds) + T� s(t))\right\]

Initially, \(\dot{V}_1(t), \dot{V}_2(t)\) and \(\dot{V}_3(t)\) are computed as
\[
\dot{V}_1(t) = 2e^{2\beta t_r T(t)Pr(t)} + 2\beta e^{2\beta t_r T(t)Pr(t)} = e^{2\beta t_r T(t)}\left(\text{sym}\left(e_r^T P \text{e}_r\right) + 2\beta e_r^T P \text{e}_r\right)\right)\right)
\[
\dot{V}_2(t) = e^{2\beta t_r T(t)T� r(t)} + e^{2\beta t_r T(t)T� r(t)} = e^{2\beta t_r T(t)}\left(e_r^T T� e_r + e^{2\beta t_r T(t)T� e_r}\right)\right)\left(\text{sym}\left(e_r^T P \text{e}_r\right) + 2\beta e_r^T P \text{e}_r\right)\right)\right)\right)
\[
\dot{V}_3(t) = e^{2\beta t_r T(t)T� r(t)} - (1 - \xi(t)e^{2\beta t_r T(t)T� r(t)} + e^{2\beta t_r T(t)T� r(t)} = e^{2\beta t_r T(t)}\left(e_r^T T� e_r - e^{2\beta t_r T(t)T� e_r}\right)\right)\right)\left(\text{sym}\left(e_r^T P \text{e}_r\right) + 2\beta e_r^T P \text{e}_r\right)\right)\right)\right)
\[
\dot{V}_4(t) = ((k+1-t)(t - t_k)e^{2\beta t_r T(t_k)}T� T(\xi(t)) + T� T(\xi(t))\right)\right)\right)\right)
\[
\dot{V}_5(t) = ((k+1-t)e^{2\beta t_r T(t_k)}T� r(t)) + T� r(t))\right)\right)\right)
\[
\dot{V}_6(t) = ((k+1-t)e^{2\beta t_r T(t_k)}T� s(t) + T� s(t))\right)\right)\right)
\[
\dot{V}_7(t) = ((k+1-t)\int_{t_k}^{t} e^{2\beta t_r T(s)}T� s(t)ds) + T� s(t))\right)\right)\right)
\[
\dot{V}_8(t) = ((k+1-t)\int_{t_k}^{t} e^{2\beta t_r T(s)}T� r(t)) + T� r(t))\right)\right)\right)
\[
\dot{V}_9(t) = ((k+1-t)\int_{t_k}^{t} e^{2\beta t_r T(s)}T� s(t)ds) + T� s(t))\right)\right)\right)
\[
\dot{V}_{10}(t) = \tau_\alpha \int_{t\alpha}^{t} e^{2\beta t_r T(s)}T� s(t)ds) + T� s(t))\right)\right)\right)
\[
\dot{V}_{11}(t) = \int_{t_{\alpha}}^{t} e^{2\beta t_r T(s)}T� s(t)ds) + T� s(t))\right)\right)\right)

\[
\dot{V}_5(t) = (k+1-t)e^{2\beta t_r T(t_k)}T� r(t)) + T� r(t))\right)\right)\right)
\[
\dot{V}_6(t) = ((k+1-t)e^{2\beta t_r T(t_k)}T� s(t) + T� s(t))\right)\right)\right)
\[
\dot{V}_7(t) = ((k+1-t)\int_{t_k}^{t} e^{2\beta t_r T(s)}T� s(t)ds) + T� s(t))\right)\right)\right)

\[
\dot{V}_3(t) = e^{2\beta t_r T(t)}\left(e_r^T T� e_r - e^{2\beta t_r T(t)T� e_r}\right)\right)\right)
\[
\dot{V}_4(t) = ((k+1-t)(t - t_k)e^{2\beta t_r T(t_k)}T� T(\xi(t)) + T� T(\xi(t))\right)\right)\right)
\[
\dot{V}_5(t) = ((k+1-t)e^{2\beta t_r T(t_k)}T� r(t)) + T� r(t))\right)\right)\right)
\[
\dot{V}_6(t) = ((k+1-t)e^{2\beta t_r T(t_k)}T� s(t) + T� s(t))\right)\right)\right)

\[
\dot{V}_3(t) = e^{2\beta t_r T(t)}\left(e_r^T T� e_r - e^{2\beta t_r T(t)T� e_r}\right)\right)\right)
\[
\dot{V}_4(t) = ((k+1-t)(t - t_k)e^{2\beta t_r T(t_k)}T� T(\xi(t)) + T� T(\xi(t))\right)\right)\right)
\[
\dot{V}_5(t) = ((k+1-t)e^{2\beta t_r T(t_k)}T� r(t)) + T� r(t))\right)\right)\right)
\[
\dot{V}_6(t) = ((k+1-t)e^{2\beta t_r T(t_k)}T� s(t) + T� s(t))\right)\right)\right)
Utilizing Lemma 2, we obtain
\[-\int_{t_k}^{t} r^T(s)S(r(s))ds \leq -\frac{1}{t_k} (\epsilon_7^T S e_7 + 3(e_7 - 2e_8)^T S(e_7 - 2e_8)) \leq \frac{1}{h_u} (\epsilon_7^T S e_7 + 3(e_7 - 2e_8)^T S(e_7 - 2e_8)).\]
So, \(\dot{V}_7(t)\) is bounded by
\[\dot{V}_7(t) \leq e^{2\beta t} \xi^T(t)(d_2(t)\theta_4 + \Omega_5)\xi(t).\] (30)

An estimation of \(\dot{V}_8(t)\) is gained by
\[\dot{V}_8(t) = (t_k + 1) e^{2\beta t} \xi^T(t)R\dot{\xi}(t) - \int_{t_k}^{t} e^{2\beta s} \xi^T(s)R\dot{\xi}(s)ds \leq (t_k + 1) e^{2\beta t} \xi^T(t)R\dot{\xi}(t) - e^{2\beta(t-h_u)} \int_{t_k}^{t} e^{2\beta s} \xi^T(s)R\dot{\xi}(s)ds.\] (31)

Use Lemma 2 to obtain
\[-\int_{t_k}^{t} r^T(s)S(r(s))ds \leq -\frac{1}{d_1(t)} \xi^T(t)(e_1 - e_2)^T R(e_1 - e_2)\xi(t) - \frac{3}{d_1(t)} \xi^T(t)(e_1 + e_2 - 2d_1(t)e_7)^T R \times (e_1 + e_2 - 2d_1(t)e_7)\xi(t) \leq -\frac{1}{h_u} \xi^T(t)(e_1 - e_2)^T R(e_1 - e_2)\xi(t) - \frac{3}{h_u} \xi^T(t) R(d_1(t)(e_1 + e_2 - 2d_1(t)e_7)\xi(t) \leq -\frac{1}{h_u} \xi^T(t)(e_1 - e_2)^T R(e_1 - e_2) + 3 \left(\frac{2}{h_u} e_7\right)^T R \left(\frac{2}{h_u} e_7\right) \xi(t) + \xi^T(t) \left(d_1(t) \left(-\frac{3}{h_u} (e_1 + e_2)^T R(e_1 + e_2)\right)\xi(t) - \frac{3}{h_u} \xi^T(t) \text{sym} \left(-e_1 + e_2)^T R(2e_7)\right) \xi(t).\] (32)

Note that the inequalities,
\[-\frac{1}{t - t_k} \sigma_i^T R \sigma_i \leq (t - t_k) Y_iR^{-1} Y_i^T - Y_i \sigma_i - \sigma_i^T Y_i^T\]
for given matrices, \(\sigma_i\), any matrices \(Y_i \in \mathcal{R}^{n \times 11n}(i = 1, 2)\) hold. It follows that
\[-\frac{1}{d_1(t)} \xi^T(t) \left((e_1 - e_2)^T R(e_1 - e_2) + 3 \left(\frac{2}{h_u} e_7\right)^T R \left(\frac{2}{h_u} e_7\right) \xi(t) \leq \xi^T(t) \left(-\text{sym} \left\{(e_1 - e_2)Y_1 + 3 \left(\frac{2}{h_u} e_7\right) Y_2\right\} + (t - t_k) Y_1 R^{-1} Y_1 + (t - t_k) Y_2 R^{-1} Y_2\right) \xi(t).\] (33)

From Eq.(31)-(33), \(\dot{V}_8(t)\) is bounded by
\[\dot{V}_8(t) \leq e^{2\beta t} \xi^T(t) \left((d_2(t)\theta_4 + e^{-2\beta h_u}(\Omega_6 + d_1(t)Y_1 R^{-1} Y_1 + d_1(t)Y_2 R^{-1} Y_2)\right) \xi(t).\] (34)

An expression of \(\dot{V}_9(t)\) is followed by
\[\dot{V}_9(t) = (t_k + 1) (t - t_k) e^{2\beta t} \xi^T(t)Z\dot{\xi}(t) - \int_{t_k}^{t} e^{2\beta s} \xi^T(s)Z\dot{\xi}(s)ds \leq \frac{((t_k + 1) (t - t_k))}{4} e^{2\beta t} \xi^T(t)Z\dot{\xi}(t) - \int_{t_k}^{t} e^{2\beta s} \xi^T(s)Z\dot{\xi}(s)ds \leq \frac{h_u ((t_k + 1) (t - t_k))}{4} e^{2\beta t} \xi^T(t)Z\dot{\xi}(t) + \frac{h_u}{4} (t - t_k) e^{2\beta t} \xi^T(t)Z\dot{\xi}(t) - e^{2\beta(t-h_u)} \int_{t_k}^{t} e^{2\beta s} \xi^T(s)Z\dot{\xi}(s)ds.\] (35)

By using the free-weighting matrix method [43], [44], the below equations hold for any matrices \(N_1\) and \(N_2\) with compatible dimensions.
\[0 = 2\xi^T(t)N_1 \begin{bmatrix} r(t) - r(t_k) \int_{t_k}^{t} r(s)ds \\ \int_{t_k}^{t} r(s)ds \end{bmatrix} - \int_{t_k}^{t} \left[\int_{t_k}^{t} r(s)ds\right] ds \] (36)
and
\[0 = 2\xi^T(t)N_2 \begin{bmatrix} (t - t_k) r(t) - \int_{t_k}^{t} r(s)ds \\ \int_{t_k}^{t} r(s)ds \end{bmatrix} \] (37)

Furthermore, rely on the well-known inequality for any matrix \(N > 0, -2x^T \leq x^TN^{-1}x + y^TNy\), it is obvious to given that
\[-2\xi^T(t)N_1 \int_{t_k}^{t} \left[\int_{t_k}^{t} r(s)ds\right] ds\]
An expression of $\dot{V}(t)$ is calculated by
\[ \dot{V}(t) = e^{2\beta t} h_u \dot{r}(t) T \left( Q \dot{r}(t) - \int_{t-h_a}^t e^{2\beta s} \dot{T}(t) T(s) Q \dot{r}(s) ds \right) \leq e^{2\beta t} h_u \dot{r}(t) T \left( Q \dot{r}(t) - \int_{t-h_a}^t e^{2\beta s} \dot{T}(t) T(s) Q \dot{r}(s) ds \right). \]

Applying Lemmas 1 and Jensen’s inequality, relied on the inequality (22), a new calculating of $\dot{V}(t)$ transforms to
\[ - \int_{t-h_a}^t \dot{T}(s) Q \dot{r}(s) ds \leq - \frac{1}{h_a} \begin{bmatrix} e_1 - e_2 \\ e_2 - e_3 \end{bmatrix}^T \begin{bmatrix} Q_s & S_2 \\ S_2 & Q_s \end{bmatrix} \begin{bmatrix} e_1 - e_2 \\ e_2 - e_3 \end{bmatrix}. \]

Thus, $\dot{V}(t)$ can be gained as
\[ \dot{V}(t) \leq \xi^T(t) \Omega \xi(t). \]

Moreover, we can obtain from condition (7) for any $\epsilon > 0$
\[ 0 \geq - \epsilon \begin{bmatrix} r(t) \\ g(r(t)) \end{bmatrix}^T \begin{bmatrix} \text{sym} (I \otimes U^T V) - I_N \otimes (U^T + V^T) \\ 2I \end{bmatrix} \begin{bmatrix} r(t) \\ g(r(t)) \end{bmatrix} \geq \xi^T(t) e^{2\beta t} \Omega_{12} \xi(t). \]

For any proper dimension $M$, the following equation is clearly to be obtained
\[ 0 = 2 e^{2\beta t} \left( \dot{T}(t) + \dot{T}(t) \right) \left[ - \dot{r}(t) + g(r(t)) \right] \]
\[ + c(\dot{G} \otimes \Gamma^1) r(t) + c(\dot{G} \otimes \Gamma^2) r(t - \tau(t)) + Kr(t_0) \]
\[ = e^{2\beta t} \xi^T(t) \text{sym} \left( e_1^T + e_0^T \right) \dot{T}(t) \left( -e_6 + e_11 \right) \]
\[ + c(\dot{G} \otimes \Gamma^1) e_1 + c(\dot{G} \otimes \Gamma^2) e_4 + Ke_2 \right) \]](t) \]
\[ = e^{2\beta t} \Omega_{12} \xi(t). \]

Adding Eq.(24)-(47) to the right-hand sides of $\dot{V}(t)$ yields
\[ \dot{V}(t) \leq e^{2\beta t} \left( \xi^T(t) \Omega(t) \xi(t) \right) \left( d_1(t) \xi^T(t) \left( \theta_1 + \theta_6 + \theta_9 \right) \right) \]
\[ + \theta_{10} + e^{-2\beta h_a} Y_1 R^{-1} Y_1^T + 3 e^{-2\beta h_a} Y_2 R^{-1} Y_2^T \]
\[ + e^{2\beta h_a} N_1 \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}^{-1} \begin{bmatrix} Q_1 & S_1 \\ * & 3 Q_1 \end{bmatrix} \]
\[ + \frac{h_A d_1(t)}{2} e^{2\beta h_a} N_2 Z^{-1} N_2^T \xi(t) \]
\[ + d_2(t) \xi^T(t) \left( \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_7 + \theta_8 \right) \xi(t) \]
\[ = e^{2\beta t} \xi^T(t) \left( \frac{d_1(t)}{d_h} \Omega + d_2(\theta_1 + \theta_6 + \theta_9 + \theta_{10} + e^{-2\beta h_a} Y_1 R^{-1} Y_1^T + 3 e^{-2\beta h_a} Y_2 R^{-1} Y_2^T \right) \]
\[ + e^{2\beta h_a} N_1 \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}^{-1} \begin{bmatrix} Q_1 & S_1 \\ * & 3 Q_1 \end{bmatrix} \]
\[ + \frac{h_A d_1(t)}{2} e^{2\beta h_a} N_2 Z^{-1} N_2^T \xi(t) \]
\[ + e^{2\beta h_a} N_1 \begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}^{-1} N_1^T \xi(t). \]
where $\Omega = \sum_{i=1}^{12} \Omega_i$ and the other notations are provided in (17). From (48), it can be rearranged condition for system (6) as follows:

$$
= e^{2\beta t} \xi(t)^T \left( \Psi^{(1)} \left( t \right) \frac{d\Psi^{(1)}}{dt} \right) \xi(t) + e^{2\beta t} \Psi^{(2)} \xi(t) < 0,
$$

(49)

where

$$
\Psi^{(1)}_{[dt]} = \Omega + d_k \theta_1 + \theta_6 + \theta_9 + \theta_{10} + e^{-2\beta h_u} Y_1 R^{-1} Y_1^T + 3e^{-2\beta h_u} Y_2 R^{-1} Y_2^T + e^{2\beta h_u} N_1 \left[ \begin{array}{cc} W_1 & W_2 \\ W_2 & * \end{array} \right] N_1^T
$$

$$
+ \frac{h_u}{2} e^{2\beta h_u} N_2 Z N_2^T,
$$

$$
\Psi^{(2)}_{[dt]} = \Omega + d_k \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_7 + \theta_8.
$$

The inequality (49) presents a convex combination of $d_1(t)$ and $d_2(t)$. It is notably that

$$
\Psi^{(1)}_{[dt]} = \frac{h_u - d_k}{h_u} \Psi^{(1)}_{[d_{t_0}]} + \frac{d_k}{h_u} \Psi^{(1)}_{[d_{t_0 + \Delta t}]},
$$

$$
\Psi^{(2)}_{[dt]} = \frac{h_u - d_k}{h_u} \Psi^{(2)}_{[d_{t_0}]} + \frac{d_k}{h_u} \Psi^{(2)}_{[d_{t_0 + \Delta t}]}.
$$

(50)

Applying Schur complement to (19) and from conditions (18)-(20) result in

$$
\Psi^{(1)}_{[dt]} < 0, \quad \Psi^{(2)}_{[dt]} < 0.
$$

(51)

Now, from (51), we have

$$
V(t) = \sum_{i=1}^{11} V_i(t) < 0, \quad t \in [t_k, t_{k+1}).
$$

(52)

Therefore, it can be readily presented that, for $t \in [t_k, t_{k+1})$

$$
V(t) \leq V(t_k) \leq V(t_{k-1}) \leq \cdots \leq V(0).
$$

(53)

We can conclude from Lemma 4 and (53) that for $t_k \leq t < t_{k+1}$

$$
\sum_{i=1}^{11} V_i(t) \leq \sum_{i=1}^{11} V_i(t_k) \leq \cdots \leq \sum_{i=1}^{11} V_i(0).
$$

(54)

We can get

$$
\sum_{i=1}^{11} V_i(t) \leq \sum_{i=1}^{11} V_i(t_k) \leq \cdots \leq \sum_{i=1}^{11} V_i(0).
$$

(55)

where

$$
b_1 = \lambda_{\max}(P) + \tau_2 \lambda_{\max}(T_1) + \tau_2 \lambda_{\max}(T_2) + h_u \lambda_{\max}(T_3)
$$

$$
b_2 = \tau_4 \lambda_{\max}(Q_1) + h_u \lambda_{\max}(Q_2).
$$

Using (54) and (55), we can get

$$
\sum_{i=1}^{11} V_i(t) \leq \sum_{i=1}^{11} V_i(t_k) \leq \cdots \leq \sum_{i=1}^{11} V_i(0).
$$

(56)

From Definition 1, the error system (6) is exponentially stable. The proof is now complete.

Remark 4: It is vital to seek an appropriate Lyapunov functional, which used to derive a less conservative condition. Thus, we introduce novel six $(t_k, t_{k+1})$-dependent terms
where  

\[ \tilde{\Psi}^{(1)}_{[d_k]} = \Psi^{(1)}_{[d_k]} - \tilde{c}^{(1)}_1 \mathcal{J}^T \Phi_1 \mathcal{J} \tilde{e}_1 - 2\tilde{c}^{(1)}_2 \mathcal{J}^T \Phi_2 \tilde{e}_{12} \]

\[ - \tilde{c}^{(1)}_2 \Phi_3 \tilde{e}_{12}, \]

\[ \tilde{\Psi}^{(2)}_{[d_k]} = \Psi^{(2)}_{[d_k]} - \tilde{c}^{(2)}_1 \mathcal{J}^T \Phi_1 \mathcal{J} \tilde{e}_1 - 2\tilde{c}^{(2)}_2 \mathcal{J}^T \Phi_2 \tilde{e}_{12} \]

\[- \tilde{c}^{(2)}_2 \Phi_3 \tilde{e}_{12}.\]

Note that if we take  \( \zeta(t) \neq 0 \) and \( \omega(t) = 0 \), then (62) now also holds and we have

\[ \zeta^T(t) \left( \frac{d_1(t)}{d_k} \left( \tilde{\Psi}^{(1)}_{[d_k]} \right) + \frac{d_2(t)}{d_k} \left( \tilde{\Psi}^{(2)}_{[d_k]} \right) \right) \zeta(t) < 0, \]

(63)

where  

\[ \tilde{\Psi}^{(1)}_{[d_k]} = \Psi^{(1)}_{[d_k]} - e_1^T \mathcal{J}^T \Phi_1 \mathcal{J} e_1, \tilde{\Psi}^{(2)}_{[d_k]} = \Psi^{(2)}_{[d_k]} - e_1^T \mathcal{J}^T \Phi_3 \mathcal{J} e_1. \]

Since  \( \Phi_1 \leq 0 \) in Assumption 2, we get the below inequality

\[ \zeta^T(t) \left( \frac{d_1(t)}{d_k} \left( \tilde{\Psi}^{(1)}_{[d_k]} \right) + \frac{d_2(t)}{d_k} \left( \tilde{\Psi}^{(2)}_{[d_k]} \right) \right) \zeta(t) < 0. \]

(64)

Applying Lemma 3 and noting  \( \zeta(t) \neq 0 \), we gain the LMIs (18), (19) and (20) in Theorem 1. Then the system (6) is exponentially stable.

From (49), we obtain

\[ \tilde{V}(t) \leq \tilde{\zeta}^T(t) \left( \frac{d_1(t)}{d_k} \left( \tilde{\Psi}^{(1)}_{[d_k]} \right) + \frac{d_2(t)}{d_k} \left( \tilde{\Psi}^{(2)}_{[d_k]} \right) \right) \tilde{\zeta}(t), \]

(65)

and it is easy that

\[ \tilde{\zeta}^T(t) \left( \frac{d_1(t)}{d_k} \left( \tilde{\Psi}^{(1)}_{[d_k]} \right) + \frac{d_2(t)}{d_k} \left( \tilde{\Psi}^{(2)}_{[d_k]} \right) \right) \tilde{\zeta}(t) = \zeta^T(t) \left( \frac{d_1(t)}{d_k} \left( \Psi^{(1)}_{[d_k]} \right) + \frac{d_2(t)}{d_k} \left( \Psi^{(2)}_{[d_k]} \right) \right) \zeta(t) - J(t), \]

(66)

where  \( J(t) \) is defined in Definition 2.

Then, we have

\[ \tilde{V}(t) \leq \zeta^T(t) \left( \frac{d_1(t)}{d_k} \left( \Psi^{(1)}_{[d_k]} \right) + \frac{d_2(t)}{d_k} \left( \Psi^{(2)}_{[d_k]} \right) \right) \zeta(t) + J(t) \leq J(t). \]

(67)

Taking the integration both sides of inequality (67) from 0 to  \( t (t \geq 0) \), we get result in

\[ \int_0^t J(s)ds \geq V(t) - V(0) \geq \gamma^T(t)Pr(t). \]

(68)

Now, we only focus on two cases of  \( \Phi_4 = 0 \) and  \( \Phi_4 \geq 0 \), since the extended dissipative performance can unified the strictly (\( Q, R, S \))-dissipativity,  \( H_\infty \) and the passive performance when  \( \Phi_4 = 0 \) or the  \( L_2 - L_\infty \) performance when  \( \Phi_4 > 0 \).

Considering  \( \Phi_4 = 0 \), from (68) we have

\[ \int_0^t J(s)ds \geq 0. \]

(69)

At the same time, when  \( \Phi_4 > 0 \), as noted in Assumption 2, this implies that the matrices  \( \Phi_1 = 0, \Phi_2 = 0 \) and  \( \Phi_3 > 0 \).
Then, for \( t \in [0, t_f] \), (68) result in \( \int_{t_i}^{t_f} J(s)ds \geq \int_0^{t_f} J(s)ds \geq r^T(t)Pr(t) \). Thus, from (60), we obtain

\[
\mathbb{F}^T(\hat{t}) \Phi_4 \mathbb{F}(\hat{z}) = r^T(\hat{t})J^T(\hat{t}) \Phi_4 J r(t) \\
\leq r^T(t)Pr(t) \leq \int_0^{t_f} J(s)ds
\]

By combining (69) and (70), the system (6) is described as extended dissipative. The proof is now complete. □

Remark 6: The novelty of our method is that we take the external disturbances into each node which is not considered in [22], [23], [25], [27], [29], [37], and [38]. Moreover, we obtain newly exponential synchronization with extended dissipative containing, passive, \( \mathcal{H}_\infty \), \( \mathcal{L}_\infty \), and dissipative performance. These conditions are more general than those in [22], [23], [25], [27], [29], [37], and [38]. Therefore, we can notice that their conditions cannot be simulated to our examples. Moreover, we construct a new time-dependent Lyapunov functional, which is different from the proposed in [22], [23], [24], [25], and [27], and the advantage information on discrete sample point \( t_k \) is fully used. Moreover, the positive definitiveness of the proposed Lyapunov functional is required only at sampling times that are not necessarily throughout the sampling periods. So, we derive a new inequality that applies to prove the exponential synchronization of CDNs.

Remark 7: Compared to the findings in [22], [23], [24], [25], [27], [29], [36], and [38], the result in this study is less conservative by constructing the appropriate Lyapunov functional and the technique for estimating the upper bound of its derivative. In contrast to the Lyapunov function in [40], [41], [42], and [43], we fully consider the critical information \( t - t_k \) and \( t_k + t \), \( \forall t \in [t_k, t_{k+1}] \). Additionally, compared to the convex combination technique and Jensen inequality, the reverse of the first-order approach, mixed convex combination and some effective integral inequalities can offer a more precise upper bound.

Remark 8: In this study, we address the sampled-data synchronization problem of CDNs with fixed coupling and time-varying delay in each dynamical system and obtained less conservative results. Because the sampled-data controller gain matrices must be obtained using system parameters, the proposed method is unsuitable for CDNs with time-varying coupling and multiple time delays. As a result, there is still much room for further investigation into obtaining the sampled-data synchronization criteria of CDNs with time-varying coupling. Some more effective methods, such as the proposed in [45] and [46], inspire us to conduct additional research.

Remark 9: Some free matrices are introduced in this paper using a time-dependent Lyapunov functional with complete information on the actual sampling instant \( t_k \), a new inequality, and a convex combination approach. As a result, the construction and computation technique of the Lyapunov functional are the primary keys to improving the outcomes of this work. All of this leads to a reduction in our results’ conservatism compared to recent works and, in particular, numerical examples.

IV. NUMERICAL EXAMPLES

This part focuses on using two numerical examples to manifest the effectiveness of the suggested method in the above theorems.

Example 1: Chua’s well-known circuit (Fig. 1 [49]) consists of a linear inductor \( L_1 \), a linear resistor \( R \), two linear capacitors \( C_1 \), \( C_2 \), and a nonlinear resistor \( N_R \) named Chua’s diode. \( v_1 \) and \( v_2 \) are the voltages across the capacitors \( C_1 \) and \( C_2 \), respectively. \( i_1 \) is the current through the inductor \( L_1 \). Moreover, Chua’s circuit is chosen as the isolated node of the system (6), which is given by the following dynamical system

\[
\begin{align*}
\dot{v}_1 &= \rho_1(-v_1 + v_2 - \sigma(v_1)), \\
\dot{v}_2 &= v_1 - v_2 + v_3, \\
\dot{v}_3 &= -\rho_2 v_2,
\end{align*}
\]

where \( \rho_1 = 10, \rho_2 = 14.87, \) and \( \sigma(v_1) = -0.68v_1 + 0.5(-1.27 + 0.68)(|v_1 + 1| - |v_1 - 1|). \)

![FIGURE 1. Chua’s circuit as an isolated node in example 1.](image)

From condition (7), it can be satisfied as follows

\[
U = \begin{bmatrix}
2.7 & 10 & 0 \\
1 & -1 & 1 \\
0 & -14.87 & 0
\end{bmatrix}, \quad V = \begin{bmatrix}
-3.2 & 10 & 0 \\
1 & -1 & 1 \\
0 & -14.87 & 0
\end{bmatrix}.
\]

The inner and outer-coupling matrices are specified as

\[
\Gamma^{(1)} = 0, \quad \Gamma^{(2)} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad G = \begin{bmatrix}
-2 & 1 & 1 \\
-1 & -1 & 0 \\
1 & 1 & 0
\end{bmatrix}.
\]

Then, we choose \( c = 0.9 \) and the time-varying coupling delay is considered as \( \tau(t) = 0.03 + 0.01 \sin(t) \), which implies that \( \tau_{u} = 0.04 \) and \( \tau_{l} = 0.01 \). As shown in Table 1, we can see that the largest sampling period by theorem 1 is 0.1982, which is larger than the ones proposed in [23], [24], [27], and [38]. It means that our result is less conservative than the existing ones in [23], [24], [27], and [38]. To depict the effectiveness of our approach, we choose \( h_{u} = 0.1982 \), using Matlab software to calculate the LMIs in Theorem 1 the desired controller gains matrices can be presented as follows

\[
K_1 = \begin{bmatrix}
-1.9436 & -1.5180 & 0.6179 \\
-0.7088 & -4.0341 & 0.2254 \\
-0.5361 & 1.7484 & -4.3617
\end{bmatrix}.
\]
TABLE 1. The largest upper bound of sampling period $h_u$.  

| Methods | $h_u$ (improved by) |
|---------|---------------------|
| [23]    | 0.0711 178.76%      |
| [38]    | 0.1120 76.96%       |
| [24]    | 0.1327 49.36%       |
| [27]    | 0.1536 29.04%       |
| Theorem 1 | 0.1982       |


\[
K_2 = \begin{bmatrix} -6.6941 & 0.5959 & 2.0141 \\ 0.3328 & -4.4092 & 0.2676 \\ 5.4029 & 0.0934 & -5.2084 \end{bmatrix}
\]
\[
K_3 = \begin{bmatrix} -4.8812 & -1.4433 & 0.3209 \\ 0.1603 & -2.5156 & 2.0958 \\ 1.7657 & 4.1299 & -1.1285 \end{bmatrix}
\]

Setting the initial condition as $v(0) = [1 \ -1 \ -2]^T$, $x_1(0) = [1 \ -3 \ 1]^T$, $x_2(0) = [2 \ -2 \ 1]^T$, $x_3(0) = [-5 \ 1 \ -1]^T$ and using the above-designed controller gain matrices, Fig. 2 shows the chaotic behavior of Chua’s circuit. Moreover, Fig. 3 depicts the error state of the system (6) without control. The controlled error CDNs and the state trajectories of the controller are demonstrated in Figs. 4 and 5. As can be seen, the developed sampled-data controller matrices can successfully synchronize the error system (6).

**Remark 10:** In recent years, there has been a lot of interest in chaos control and chaos synchronization of dynamic systems. A chaotic system has complex dynamical behaviors with unique characteristics, such as being extremely sensitive to small changes in initial conditions and having bounded trajectories in phase space. Nonlinear systems such as Chua’s, Lure’s, and Chen’s system have been studied to control chaos. Furthermore, applications of Chua’s circuit are remarkable as a standard for various strange attractors in analyses of chaos control, image encryption, signals, and neural networks [47], [48], [49], [50]. So, Chua’s circuit is employed as the unforced isolated nodes of (6) to show the effectiveness and practical example.

**Example 2:** Consider three-node CDNs with $\omega(t) = 0$. The internal-coupling matrices and outer-coupling matrices are defined as follows:

\[
\Gamma^{(1)} = 0, \quad \Gamma^{(2)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathcal{G} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}.
\]

The nonlinear function $f$ is described as

\[
f(x_i(t)) = \begin{bmatrix} -0.5x_{i1} + \tanh(0.2x_{i1}) + 0.2x_{i2} \\ 0.95x_{i2} - \tanh(0.75x_{i2}) \end{bmatrix}.
\]

**FIGURE 2.** The state trajectories of the isolated node in example 1.

**FIGURE 3.** The state trajectories of the uncontrol CDNs.

**FIGURE 4.** The state trajectories $x_i(t)$ of CDNs with the control (4).

**FIGURE 5.** The state trajectories of the control $u_i(t)$. 

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It can be calculated that the following matrices are satisfied in condition (7)

\[ U = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, \quad V = \begin{bmatrix} -0.3 & 0.2 \\ 0 & 0 \end{bmatrix}. \]

We select \( c = 0.5, \tau_u = 0.25 \) and \( \tau_d = 0.5 \) and the list of the maximum allowable value of sampling period \( h_u \) contains in Table 2. From Table 2, we show that the largest sampling periods using the approach described in [22], [23], [24], [25], [27], [29], [36], and [38] are 0.5409, 0.5573, 0.8767, 0.9016, 0.9225, 1.1564, 1.3756, 1.3978 and 1.4222, respectively. However, the largest sampling period \( h_u \) by Theorem 1 is 1.4222, which is greater than the references therein [22], [23], [24], [25], [27], [29], [36], and [38]. This concludes that our result is less conservative than those found in [22], [23], [24], [25], [27], [29], [36], and [38].

**TABLE 2.** The largest upper bound of sampling period \( h_u \).

| Methods | \( h_u \) improved by |
|---------|------------------------|
| [22]    | 0.5409, 162.93%        |
| [23]    | 0.5573, 155.19%        |
| [24]    | 0.8767, 62.22%         |
| [27]    | 0.9225, 54.17%         |
| [36]    | 1.1564, 22.99%         |
| [25]    | 1.3756, 3.39%          |
| [29]    | 1.3878, 1.75%          |
| Theorem 1 | 1.4222                   |

The effectiveness of our method is demonstrated via the following simulation. By choosing \( \tau(t) = 0.125 + 0.125 \sin(4t) \), and \( h_u = 1.4222 \), and using Matlab software to solve the LMIs in Theorem 1, the desired controller gains matrices can be presented as follows

\[
K_1 = \begin{bmatrix} -0.8781 & -0.0896 \\ -0.0816 & -1.5110 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -1.0636 & -0.0625 \\ -0.0514 & -1.5556 \end{bmatrix},
\]

\[
K_3 = \begin{bmatrix} -0.6280 & 0.0014 \\ 0.0104 & -0.8830 \end{bmatrix}.
\]

The state trajectories of the control \( u_1(t) \).

The state trajectories of the control \( u_1(t) \).

**FIGURE 7.** The state trajectories \( y(t) \) of CDNs with the control (4).

**FIGURE 8.** The state trajectories of the uncontrol CDNs.

Here, the initial condition is \( y(0) = [2 \ -1]^T, x_1(0) = [9 \ -4]^T, x_2(0) = [5 \ -9]^T, x_3(0) = [-4 \ 5]^T \). Then, using the above-designed controller gain matrices, Fig. 6 shows the state trajectories of the uncontrol error system. The controlled error CDNs and the state vectors of the controller are illustrated in Figs. 7 and 8.

To consider extended dissipative performance, we define the variables \( c = 1, \tau_u = 0.25, \tau_d = 0.5, J = I_3 \otimes I_2 \) and \( \omega(t) = \frac{1}{(1 + t^2)} \). Then, the internal-coupling matrices and the outer-coupling matrices are used by the following matrices:

\[
\Gamma^{(1)} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad \Gamma^{(2)} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},
\]

\[
\mathcal{G} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}.
\]

By performance scalar \( \delta = 0.5 \), and in the \((\mathcal{Q}, \mathcal{S}, \mathcal{R})\)-dissipativity property, we choose \( \Phi_1 = -I, \Phi_2 = I, \Phi_3 = 2I, \) and \( \Phi_4 = 0 \). From Table 3, we show that the largest sampling periods using the approach described in [36] are 0.7548, 0.4375, 0.2219, and 0.2962, which rely on \( L_2 \times L_\infty, H_\infty \), Passivity, \((\mathcal{Q}, \mathcal{S}, \mathcal{R})\)-dissipativity respectively. Whereas the maximum sampling period \( h_u \) by Theorem 2 is 0.91372, 1.0272, 1.001, 0.9561 which is greater than the reference therein [36]. It concludes that our result is less conservative than the reference in [36].
On the other hand, the controller gains matrices with $h_u = 1.0272$ and $\delta = 0.5$ in $\mathcal{H}_\infty$ performance can be calculated as follows:

$$
K_1 = \begin{bmatrix} -0.7358 & -0.0902 \\ -0.0692 & -1.4936 \end{bmatrix},
K_2 = \begin{bmatrix} -1.0332 & -0.0599 \\ -0.0448 & -1.5110 \end{bmatrix},
K_3 = \begin{bmatrix} -0.6215 & -0.0068 \\ 0.0031 & -0.8862 \end{bmatrix}.
$$

V. CONCLUSION

This study uses a sampled-data controller to handle the problem of extended dissipative exponential synchronization of CDNs with time-varying coupling delays. An aperiodic sampled-data control scheme has been devised to solve this challenge, with the sampling period specified as time-varying but bounded. Then, using an enhanced Lyapunov function, integral inequalities, the free-weighting matrix technique, and the convex combination approach, a new adequate condition for strict LMIs was discovered. These conditions can also be used to investigate the extended dissipativity analysis issue, which includes the passivity, $\mathcal{L}_2 = \mathcal{L}_\infty$, $\mathcal{H}_\infty$, and dissipativity performance in a unified formulation. Finally, two numerical examples indicate that our finding is better than the previous references, highlighting how this paper has developed. It is worthwhile to mention that the method in this article can be investigated further and applied to more complicated systems such as neutral-type CDNs [51], stochastic CDNs [52], T–S fuzzy CDNs [53].

REFERENCES

[1] H. Jeong, B. Tombor, and R. Albert, “The large-scale organization of metabolic networks,” Nature, vol. 407, no. 6804, pp. 651–654, Oct. 2000.
[2] S. H. Strogatz, “Exploring complex networks,” Nature, vol. 410, pp. 268–276, Mar. 2001.
[3] R. J. Williams, E. L. Berlow, J. A. Dunne, A. L. Barabási, and N. D. Martinez, “Two degrees of separation in complex food webs,” Proc. Nat. Acad. Sci. USA, vol. 99, no. 20, pp. 1216–12913, Oct. 2002.
[4] C. Li and G. Chen, “Synchronization in general complex dynamical networks with coupling delays,” Phys. A, Stat. Mech. Appl., vol. 343, pp. 263–278, Nov. 2004.
[5] H. Gao, J. Lam, and G. Chen, “New criteria for synchronization stability of general complex dynamical networks with coupling delays,” Phys. Lett. A, vol. 360, no. 2, pp. 263–273, Dec. 2006.
[6] G. Ling, X. Liu, M.-F. Ge, and Y. Wu, “Delay-dependent cluster synchronization of time-varying complex dynamical networks with noise via delayed pinning impulse control,” J. Franklin Inst., vol. 358, no. 6, pp. 3193–3214, Apr. 2021.
[7] W. Yu, G. Chen, and J. Lu, “On pinning synchronization of complex dynamical networks,” Automatica, vol. 45, no. 2, pp. 429–435, Feb. 2009.
[8] W. Yu, G. Chen, J. Lü, and J. Kurths, “Synchronization via pinning control on general complex networks,” SIAM J. Control Optim., vol. 51, no. 2, pp. 1395–1416, 2013.
[9] J. Yogambigai, M. S. Ali, H. Alsulami, and M. S. Alhodaly, “Impulsive and pinning control synchronization of Markovian jumping complex dynamical networks with hybrid coupling and additive interval time-varying delays,” Commun. Nonlinear Sci. Numer. Simul., vol. 85, pp. 1–17, Jun. 2020.
[10] G. Zhang, Z. Liu, and Z. Ma, “Synchronization of complex dynamical networks via impulsive control,” Chaos, vol. 17, no. 4, pp. 1–10, Dec. 2007.

[11] S. Zheng, “Pinning and impulsive synchronization control of complex dynamical networks with non-derivative and derivative coupling,” J. Franklin Inst., vol. 354, no. 14, pp. 6341–6363, Sep. 2017.

[12] S. Cai, J. Hao, Q. He, and Z. Liu, “Exponential synchronization of complex delayed dynamical networks via pinning periodically intermittent control,” Phys. Lett. A, vol. 375, no. 19, pp. 1965–1971, May 2011.

[13] T. Jing, D. Zhang, J. Mei, and Y. Fan, “Finite-time synchronization of delayed complex dynamic networks via aperiodically intermittent control,” J. Franklin Inst., vol. 356, no. 10, pp. 5464–5484, Jul. 2019.

[14] J.-A. Wang, “Synchronization of delayed complex dynamical network with hybrid-coupling via aperiodically intermittent pinning control,” J. Franklin Inst., vol. 354, no. 4, pp. 1833–1855, Mar. 2017.

[15] P. DeLellis, M. di Bernardo, and F. Garofalo, “Novel decentralized adaptive strategies for the synchronization of complex networks,” Automatica, vol. 45, no. 5, pp. 1312–1318, May 2009.

[16] Z. Qin, J. L. Wang, Y. L. Huang, and S. Y. Ren, “Analysis and adaptive control for robust synchronization and $H_\infty$ synchronization of complex dynamical networks with multiple time-delays,” Neurocomputing, vol. 289, pp. 241–251, May 2018.

[17] J. Zhou, Z.-A. Lu, and J. Liu, “Pinning adaptive synchronization of a general complex dynamical network,” Automatica, vol. 44, no. 4, pp. 996–1003, Apr. 2008.

[18] E. Fridman, A. Seuret, and J.-P. Richard, “Robust sampled-data stabilization of linear systems: An input delay approach,” Automatica, vol. 40, no. 8, pp. 1441–1446, Aug. 2004.

[19] H. Li, “Sampled-data state estimation for complex dynamical networks with time-varying delay and stochastic sampling,” Neurocomputing, vol. 138, pp. 78–85, Aug. 2014.

[20] R. Rakkaiyappan and N. Sakkithivel, “Pinning sampled-data control for synchronization of complex networks with probabilistic time-varying delays using quadratic convex approach,” Neurocomputing, vol. 162, pp. 26–40, Aug. 2015.

[21] B. Shen, Z. Wang, and X. Liu, “Sampled-data synchronization control of dynamical networks with stochastic sampling,” IEEE Trans. Autom. Control, vol. 57, no. 10, pp. 2644–2650, Mar. 2012.

[22] N. Li, Y. Zhang, J. Hu, and Z. Nie, “Synchronization for general complex dynamical networks with sampled-data,” Neurocomputing, vol. 74, no. 5, pp. 805–811, Feb. 2011.

[23] Z. Wu, J. H. Park, H. Su, B. Song, and J. Chu, “Exponential synchronization for complex dynamical networks with sampled-data,” J. Franklin Inst., vol. 349, no. 9, pp. 2358–2371, Sep. 2012.

[24] L. Su and H. Shen, “Mixed $H_\infty$-passive synchronization for complex dynamical networks with sampled-data control,” Appl. Math. Comput., vol. 259, no. 2, pp. 931–942, May 2015.

[25] Y. Liu and S. M. Lee, “Improved results on sampled-data synchronization of complex dynamical networks with time-varying coupling delay,” Nonlinear Dyn., vol. 81, nos. 1–2, pp. 931–938, Jul. 2015.

[26] A. Seuret and F. Gouaisbaut, “Wittinger-based integral inequality: Application to time-delay systems,” Automatica, vol. 49, no. 9, pp. 2860–2866, Sep. 2013.

[27] Z. Chen, K. Shi, and S. Zhong, “New synchronization criteria for complex delayed dynamical networks with sampled-data feedback control,” ISA Trans., vol. 63, pp. 154–169, Jul. 2016.

[28] M. Park, O. Kwon, J. H. Park, S. Lee, and E. Cha, “Stability of time-delay systems via Wittinger-based double integral inequality,” Automatica, vol. 54, pp. 204–208, May 2015.

[29] Z.-G. Wu, P. Shi, H. Su, and J. Chu, “Sampled-data exponential synchronization of complex dynamical networks with time-varying coupling delay,” IEEE Trans. Neural Netw. Learn. Syst., vol. 24, no. 8, pp. 1177–1187, Aug. 2013.

[30] Z. Feng and W. X. Zheng, “On extended dissipativity of discrete-time neural networks with time delay,” IEEE Trans. Neural Netw. Learn. Syst., vol. 26, no. 12, pp. 3293–3300, Feb. 2015.

[31] T. H. Lee, M.-J. Park, J. H. Park, O.-M. Kwon, and S.-M. Lee, “Extended dissipative analysis for neural networks with time-varying delays,” IEEE Trans. Neural Netw. Learn. Syst., vol. 25, no. 10, pp. 1936–1941, Jan. 2014.

[32] K. Mathiyalagan, J. H. Park, R. Sakkithivel, and S. M. Anthoni, “Robust mixed $H_\infty$ and passive filtering for networked Markov jump systems with impulses,” Signal Process., vol. 101, pp. 162–173, Aug. 2014.
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