Probabilistic aspects of fatigue crack growth parameters under single overload in 6061-T6 aluminium alloys

Prakash Chandra Gope and Ankit Kabdwal

Mechanical Engineering Department, College of Technology,
G. B. Pant University of Agriculture & Technology, Panchnagar-263145, Uttarakhand, India

Email : pcgope@rediffmail.com
Tel: +91 9411159916; Fax: +91 5944 233338

Abstract: In the present investigation, several crack growth tests were performed on compact tension specimen under constant amplitude loading and single overload at an overload ratio of 1.8. The experimental data obtained were used to construct σ-N, and da/dN vs AK curves and various fatigue crack growth parameters were determined and the variability associated with these parameters are addressed. Normal distribution, Weibull and extreme value distribution etc. were used to study the probabilistic nature of different crack growth parameters and best probabilistic distribution for each parameter are identified on the basis of statistical analysis. The probabilistic models of growth parameters are presented for the reliability assessment of a component under fatigue overload condition.

Keywords: Overload, Probabilistic model, Crack growth

1.0 Introduction

Fatigue crack growth under single or multiple over loading condition have been studied in past by many researchers in a deterministic manner. However, there is ample evidence which indicates variability in crack growth behaviour and different growth parameters of specimen of the same shape, size manufactured in an identical fashion from a same lot of material. Thus, fatigue crack growth is of statistical nature and it should be treated statistically. Many crack growth laws have been proposed so far to study the growth behaviour under single or multiple overloading conditions. Different crack growth models such as Willenborg, Wheeler crack growth model, Gallagher modified Willenborg crack growth model have been presented to account the crack growth retardation due to tensile overloads [1-8]. Many experimental results in support of these theories are found [9-19].

There are two main objectives of this study. One of the objectives is to study the variability in the crack growth behaviour due to application of single overload condition for 6061 –T6 aluminium alloys. The second objective is to present a probabilistic distribution of the various parameters used in modeling crack growth under such overload.

2.0 Material and Method

The commercially available 6061-T6 aluminium alloy is used in the present investigation due to its wide application in the field of aerospace, automobile, etc.. The chemical composition and mechanical properties of 6061-T6 aluminium alloy are presented in Table 1 and 2 respectively. The chemical properties are determined by EDS and the tensile properties from the tensile test.

| Material | Al   | Cr   | Cu   | Mg   | Fe   | Si   | Zn   |
|----------|------|------|------|------|------|------|------|
| wt%      | 95.8 | 0.35 | 0.40 | 0.8  | <0.7 | 0.4  | <0.25|

| Material | Ultimate tensile strength, (σut), MPa | Yield strength, (σy), MPa | Young’s modulus, (E), GPa | Poisson’s ratio (ν) |
|----------|--------------------------------------|--------------------------|--------------------------|---------------------|
| Al 6061-T6 | 310.0                                | 276.0                     | 69.0                      | 0.33                |
The mechanical properties of 6061-T6 aluminium alloy are determined by servo controlled universal testing machine (ADMET, USA) at 0.5 mm/min cross head speed under displacement control. The tensile specimens are made as per ASTM E8-99 standard [20] and shown in Fig. 1. Crack growth tests are conducted using compact tension specimens. Compact tension (CT) specimens were prepared from a 12.5 mm thick 6061-T6 aluminum alloy plate. All CT and tensile specimens are prepared according to ASTM-E647 [21] and ASTM E8-99 using the EDM wire cut machine and polished to mirror finish by polishing machine. The dimensions of the CT specimen are shown in Fig. 2.

The fatigue crack growth tests are conducted in a ± 25 kN servo controlled hydraulic fatigue testing machine (HEICO, India) under load control mode. Pre-cracking was done with $P_{\text{max}}=2$ kN and $P_{\text{min}}=0.2$ kN and frequency of 10 Hz. An extension of crack length from 13.5 mm to 14.8 mm was made in the pre-cracking. Constant amplitude loading and single over load crack growth tests are conducted under sinusoidal signals with maximum load $P_{\text{max}}=5.0$ kN and minimum load $P_{\text{min}}=0.5$ kN at 10 Hz. Crack growth was monitored by COD gauge. Crack growth test with single overload are conducted with an overload ratio (OLR) of 1.8 applied at 25001 cycles. Total of 30 specimens are tested under the similar test condition and $a-N$ and $da/dN-\Delta K$ curves are obtained. Detailed of the crack growth testing conditions are given in Table 3.

### Table 3. Details of the crack growth testing parameters

| Loading condition       | Frequency (Hz) | Load (kN) | Stress ratio R | Overload (kN) | Overload ratio | Total specimen |
|-------------------------|----------------|-----------|----------------|---------------|----------------|----------------|
| Pre-crack test          | 10             | 2.0       | 0.2            | 0.1           | -              | 33             |
| CA loading              | 10             | 5         | 0.5            | 0.1           | -              | 03             |
| Tensile overload        | 10             | 5         | 0.5            | 0.1           | 9              | 1.8            | 30             |

The modelling of the crack growth data is made by frequently used Paris law [22] which states as:

$$\frac{da}{dN} = C(\Delta K)^m$$

(1)

$a$ and $N$ are the crack length and number of load cycles respectively. $C$ and $m$ are crack growth parameters determined from the test. The stress intensity factor range is denoted as $\Delta K$ and given by Eq. 2.

$$\Delta K = \frac{AP}{BW} f(\alpha)$$

(2)

where,

$$f(\alpha) = \left[\left(2 + \alpha\right)/\left(1 - \alpha\right)^{3/2}\right] \left(0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4\right)$$

and $\alpha = a/W$

Paris law is used here because of easy conversion to stochastic differential model just by considering at the model parameter as random variable or by adding a random factor into the Paris law. Yang’s power law
[23]. Yang and Manning model [24] are some of the probabilistic crack growth models derived from the Paris model and used by many researchers for stochastic studies of the crack growth data under constant amplitude loading. There are other models such as Wheeler model, Wallenberg model used by many researchers to address the issue of crack growth retardation or acceleration due to application of overload or under-load. To convert these deterministic models, the probability distribution of random crack size, random cycles to reach a given crack size, opening stress intensity factor, plastic zone size, etc. are required. The statistical aspect of most of the parameters used to characterise retardation effect due to overload is presented here.

3.0 Results and Discussion

\(a-N\) curves under constant amplitude (CA) loading for 3 samples are shown in Fig. 3. The Paris fitting coefficients \(C\) and \(m\) for CA and the statistical parameters mean, standard deviation and coefficient of variations are presented in Table 4. The \(da/dN - \Delta K\) curves in log-log scale for three specimens are shown in Fig. 4. \(a-N\) behaviour of 30 replica tests under single overload condition are shown in Fig. 5. The initial crack length, pre-cracking condition etc. are kept identical for all 30 CT specimens. It is seen that there is no significant change in \(a-N\) behaviour as compared to constant amplitude loading. It is just during the application of overload that the sudden increase in crack length takes place. A crack increment of 0.6 to 0.8 mm is observed due to the application of single overload of magnitude 1.8. This increment is about 1.85% to 2.47% of the average failure crack length under constant amplitude loading. The results also reveal that the slope of the \(a-N\) curves changed due to application of the overload and the increment in the crack length is seen after several load cycles. Just after the application of the overload, acceleration effect in the growth rate \(da/dN\) is observed but it continued for a very short period of about 10 to 20 cycles only. This period may be due to material response capability or the sensitivity of the instrument. After overcoming this period a drastic drop in the growth rate is observed which is referred as crack growth retardation. The delayed retardation is the number of cycles up to which it attains the same crack growth rate that of the constant amplitude loading. Different parameters such as \(m\) and \(C\), growth rate just before and after the application of overload, retardation factor, plastic zone size and crack opening stress intensity factors are determined from all 30 tests. Retardation coefficient \(\eta\) is calculated from the ratio of minimum crack growth rate after the application of overload \((da/dN)_{min}\) to the crack growth rate just before the application of overload \((da/dN)_{befOL}\). Figs. 6 and 7 shows the probability distribution of few crack growth parameters.

Based on the results of regression coefficient the best probability model for fatigue life \(f(N_f)\), delayed cycles \(f(N_D)\), crack opening stress intensity factor \(f(\Delta K_{op})\), Paris constant \(f(C)\) and exponent \(f(m)\) and retardation factor \(f(\eta)\) are described below. The proposed probability distributions are:

\[
f_{\text{norm}}(N_f) = 1.289 \times 10^{-04} e^{-2240 \times 10^{-09}(N_f - 2218)^2} \]  

(3)
Fig. 4. \( \frac{da}{dN} \) vs. \( \Delta K \) curves under constant amplitude loading (\( R=0.1, P_{\text{max}}=5 \text{ kN} \))

Table 4. Crack growth parameters under constant amplitude loading and their statistics.

| Parameters                     | Fatigue life (Cycles) | Material constant, C (m/(cycle\(\text{MPa}\sqrt{\text{m}}\))) | Exponent, m |
|-------------------------------|-----------------------|---------------------------------------------------------------|-------------|
| Mean, \( \mu \)               | 62887                 | 1.027E-09                                                     | 2.389       |
| Standard deviation, \( \sigma \) | 5361.9               | 1.267E-09                                                     | 0.478       |
| Coefficient of variation (%)  | 8.526                 | 123.4                                                         | 20.01       |

\[
 f_{\text{norm}}(C) = 5.770 \times 10^{19} e^{-5.131 \times 10^{24}[C^{-9.270 \times 10^{-11}]}^2} 
\]  
(4)

\[
 f_{\text{norm}}(\Delta K) = 28.496 \exp \left[ \frac{\left( \Delta K - 0.896 \right)^2}{3.92 \times 10^{-04}} \right] 
\]  
(5)

Fig. 5 \( a-N \) curves under single overload

\[
 f_{\text{add}}(\Delta K_{\text{op}}) = 0.015 \left( \frac{\Delta K_{\text{op}}}{171.21} \right)^{1.612} e^{\left( \Delta K_{\text{op}} / 171.21 \right)^{0.412}} 
\]  
(6)

\[
 f_{\text{ev}}(m) = 4.983 \exp \left( \frac{m - 3.105}{0.2007} \right) \exp \left[ -\exp \left( \frac{m - 3.105}{0.2007} \right) \right] 
\]  
(7)
These proposed probability distribution functions (PDF), shown in Eq. 3-9, can be adapted in the fracture mechanics based fatigue reliability analysis under the effect of overload.

4.0 Concluding Remarks

From the present observation, following conclusions can be drawn.

- Due to application of single overload of magnitude 1.8 times the constant amplitude load, the mean fatigue life is increased by 46%, the growth exponent by 24.88% and the Paris constant C is decreased by 9.02% as compared to the constant amplitude loading.
- The application of overload decreases the growth rate with a mean retardation factor of 0.235 and a mean delayed cycles of 23250.
- There occurs a statically significant variation in the variances of crack length and fatigue life after the overload.
- Normal probability distribution provides the best fit curve for plastic zone size, fatigue life, Paris material constant, C whereas the extreme value distribution provides the best fit curve for Paris
exponent \( m \), delay cycles and retardation factor. Weibull can be approximated well for crack opening stress intensity factor, \( K_{op} \).

- By knowing the distribution parameters due to application of overload, stochastic crack growth model under overloading condition can be derived from any of the deterministic crack growth model and shall be helpful to study the fatigue crack growth under overload condition under required reliability condition.

Acknowledgment

The authors wish to acknowledge the financial support from the TEQIP-III.

References

[1] Wheeler OE. Spectrum Loading and Crack Growth. Trans ASME, J Basic Engng 1972;181-86.
[2] Willenborg J, Engle RM, Wood H A. A crack growth retardation model using an effective stress concept AFFDL-TM-FBRgll-7 (Air Force Dynamics Laboratory, Dayton, OH, USA), 1971.
[3] Sadananda K, Vasudevan AK. Analysis of overloads effects and related phenomena. Int. J. Fatigue 1999; 21:S23–246.
[4] Taheri F, Trask D, Pegg N. Experimental and analytical investigation of fatigue characteristics of 350WT steel under constant and variable amplitude loadings. J Marine Struct 2003;16:69–91.
[5] Gallagher JP. A Generalized Development of Yield-Zone Models AFFDL-TM-74-28, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, 1974.
[6] Ward-Close CM, Blom AF, Ritchie RO. Mechanisms associated with transient fatigue crack growth under variable-amplitude loading: an experimental and numerical study. Engng Fract Mech 1989; 32:613–38.
[7] Christensen RH. Metal fatigue. New York: McGraw-Hill, 1959.
[8] Knott JF, Pickard AC. Effects of overloads on fatigue-crack propagation: aluminium alloys. Metal Sci 1995; 11:399–404.
[9] Corbley DM, Packman PF. On the influence of single and multiple peak overloads on fatigue crack propagation in 7075 T6511 Al Engng Fract Mech 1973; 5:479-97.
[10] Matsuoka S, Tanaka K. Delayed retardation phenomenon of fatigue crack growth resulting from a single application of overload . Engng Fract Mech 1978; 10:515-525.
[11] Bathias C, Vancon M. Mechanisms of overload effect on fatigue crack propagation in aluminium. Engng Fract Mech 1978; 10:409-24.
[12] Gope PC, Kumar H, Purohit H. Effect of tensile or compressive overload on the fatigue crack growth of friction stir welded 19501 Aluminium alloy. J Testing Eval, (Accepted), 2017
[13] Borrego LP, Costa JD, Ferreira, JAM. Crack growth behavior of AA6082 and AA6061 aluminum alloys. Procedia Engng 2014; 74: 175-78.
[14] Zhou X, Gaensner HP, Pippan R. The effect of single overloads in tension and compression on the fatigue crack propagation behaviour of short cracks. Int J Fat 2016;89: 77-6.
[15] Yvardar O. Effect of single overload in FCP. Engng Fract Mech 1988; 30:329-35
[16] Carlson RL, Kardomateas GA, Bates PR. The effects of overloads in fatigue crack growth. Int. J. Fat. 1991;13(6): 453-60.
[17] Wu WF, Ni CC. A study of stochastic fatigue crack growth modeling through experimental data. Probab Engng Mech 2003;18:107–18
[18] Wu WF, Ni CC. Probabilistic models of fatigue crack propagation and their experimental verification. Probab Engng Mech 2004; 19: 247–57.
[19] Gope PC. 2016. Probabilistic model of fatigue crack propagation and estimation of probability-confidence bounded a–N curves. Int J Comput Meth Engng Sci Mech 2016; 17(4): 298-14.
[20] ASTM E8 - 99 Standard Test Methods for Tension Testing of Metallic Materials.ASTM International, West Conshohocken, PA, 1999, www.astm.org
[21] ASTM standard E647-93. Standard test method for measurement of fatigue crack growth rates. Am Soc Test Mater 1994;03:01:679–06.
[22] Paris P, Erdogan F. A critical analysis of crack propagation laws. J. Basic Eng. Trans. ASME 1963;528–34.
[23] Yang JN. Application of reliability methods to fatigue, quality assurance and maintenance. In: Schueller GI, Shinozuka M, Yao JTP, editors. Proceeding of the Sixth International Conference on Structural Safety and Reliability, Netherlands: Balkema; 1994.
[24] Yang JN, Manning SD. A simple second order approximation for stochastic crack growth analysis. Engng Fract Mech 1996;53:677–86.