Kinematic and power characteristics of the elastic wheel while rolling with camber and convergence

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Abstract. Vehicles steering wheels are installed in such a way that the plane of the wheel makes a certain angle with vertical plane (so-called camber angle) and at the same time is located at an angle to machine movement direction (toe-slip angle). These angles result in lateral force, increased contact slip and increased wear on the wheel tread. In the article, from phenomena consideration occurring in the contact of an elastic wheel with a rigid base, kinematics and rolling mechanics issues of elastic wheels installed with camber and convergence are considered; dependences are derived that allow determining the wheel power and kinematic parameters, toe and camber angles optimal ratio from the point of view of reducing the wheel running track rolling resistance and wear is determined.

1. Introduction

Vehicles steering wheels are installed in such a way that plane of wheel makes a certain angle with vertical plane (so-called camber angle) and at the same time is located at an angle to machine movement direction (toe-slip angle).

The presence of these angles leads to a side force, increased contact slippage, and increased wear on the wheel treadmill and rolling resistance. Various analytical \cite{1-11} and experimental \cite{12-17} methods were used to study these phenomena.

Despite the extensive research in this area, the problem remains unsolved about the optimal ratio of the angles of convergence and camber in terms of reducing the rolling resistance and wear of the treadmill.

To solve the problem, we will use motion reversal method, which is as follows. The system "wheel - support surface" is mentally imparted with an additional speed \(\pm V\), i.e. speed equal in magnitude and opposite in direction of wheel axis speed. As a result, the wheel stops (continuing to rotate at the same angular velocity) and supporting surface moves at a speed \(\pm V\). In this case, interaction mechanism of wheel with supporting surface does not change. A similar approach with interaction mechanics consideration of wheel with a rigid support surface was used, in particular, in \cite{1-7}.
2. Materials and methods

Let us first consider separately the cases of wheel rolling with camber and with withdrawal in longitudinal tangential force absence in contact, and then a more general case when the wheel rolls with withdrawal and camber with simultaneous implementation of longitudinal tangential force.

When rolling with camber, in the contact between wheel and rigid base, treadmill lateral displacements occur due to wheel plane inclination.

Figure 1 shows that magnitude of these lateral displacements can be found from geometric considerations. Consider a longitudinal section of a wheel 1-1. The point of the wheel surface located in this section, after coming into contact with supporting surface, moves (in reversed mechanism) along a straight line $1' - 1'$. At the same time, if there was no reference surface, the further trajectory of this point would represent a circle arc in plane 1-1. The projection of this trajectory onto the contact plane is described by equation:

\[
W_y \sin \gamma = \frac{a^2 - x^2}{2r} \sin \gamma \approx \frac{a^2 - x^2}{2r},
\]

where $W_y = (a^2 - x^2)/2r$ is wheel deflection in the section under consideration at a distance $x$ from midpoint (in longitudinal direction), $a$ is a contact area half-length.

Thus, motion trajectory of point in a free state would have to be projected onto the reference surface at a distance $W_y \sin \gamma$ from straight line 1-1. However, as shown in the Figure 1, adhesion to supporting surface prevents this, resulting in point lateral displacement in opposite direction, i.e. lateral tangential displacement of wheel surface elements caused by camber in adhesion section changes according to the law:

\[
U_y = -\gamma(a^2 - x^2) / 2r
\]

Figure 1. Wheel installed with a camber ($\gamma$-camber angle), and lateral tangential displacements diagram of wheel surface elements in contact zone arising in this case.
Using this formula, we consider angle to be positive if distance between the planes of left and right steered wheels increases with distance from supporting surface.

Figure 2,a shows that in rectilinear rolling of an elastic wheel with withdrawal, the wheel plane is located at an angle $\delta$ to wheel axis speed vector. Due to this, wheel surface elements entering the contact zone in reverse motion with the wheel axle stopped, as a result of adhesion with the road, move in the contact zone along a straight line parallel to axis velocity vector in actual motion. In this case, as shown in the Figure 2, in adhesion area of wheel surface point contact zone, lateral tangential displacements are obtained, changing according to the equation:

$$U_y^\delta = (a-x)\tan \delta \approx \delta (a-x).$$

When wheel rolls with withdrawal and camber, the total lateral displacement of a point on the wheel surface will be equal to the sum of lateral displacements caused by slip and camber angles, i.e.:

$$U_y = U_y^\gamma + U_y^\delta = (a-x)(\delta - \gamma^* \frac{a+x}{2r}).$$

![Figure 2. Wheel rolling with withdrawal (toe-in) and diagram of specific lateral tangential forces (tangential stresses) in contact caused by withdrawal (toe-in) of the wheel.](image)

Taking [5,6] that specific tangential forces (tangential stresses) arising in adhesion site contact are proportional to tangential displacements of treadmill points $q_i = \lambda U$ (where $\lambda$ is tangential stiffness coefficient), we obtain that specific lateral tangential forces value caused by camber and toe-in of wheel will be equal to

$$q_i = \lambda_x U_y = \lambda_x (a-x)(\delta - \gamma^* \frac{a+x}{2r}) = \lambda_x (a-x)\delta^*, $$

where

$$\delta^* = \delta - \gamma^* \frac{a+x}{2r}.$$

$\lambda_x$ - wheel tangential stiffness coefficient in the lateral direction.

It should be borne in mind that in the last expressions the angles $\delta$ and $\gamma$ are substituted each with its own sign.
When longitudinal tangential force is realized in contact, longitudinal specific tangential forces appear, the values of which, as shown in [5, 6], are determined by the dependence
\[ q_{tx} = \lambda_x U_x = \lambda_x \xi_x (a - x). \]

As the wheel surface point moves into the contact depth, specific lateral and longitudinal tangential forces increase until the resulting specific tangential force \( q_t = \sqrt{q_{tx}^2 + q_{ty}^2} \) reaches the value \( \mu q_n \), the limiting adhesion (here \( \mu \) is sliding friction coefficient, \( q_n \) is normal contact pressure), after which wheel surface elements sliding in contact zone begins.

Figure 2,b shows that coordinate \( x_B \) of adhesion and sliding area boundary is determined from equality:
\[ q_{t(x=x_B)} = \mu q_{n(x=x_B)} \]

Complexity of process describing under consideration when tangential stiffness coefficients of the wheel \( \lambda_x \) and \( \lambda_y \) in longitudinal and transverse directions are not the same, because in this case total tangents \( q_t \) in adhesion section do not coincide with relative velocity \( \Delta V \) in contact, as shown in the Figure 3, and with shear stresses on slip section \( q_{tsl} = \mu q_n \). The relative speed \( \Delta V \) and total shear stresses in adhesion section are directed at angles \( \theta \) and \( \alpha \) to wheel speed. Wherein:
\[ \tan \theta = \frac{\xi_y}{\xi_x} \approx \frac{\delta^*}{\xi_x} \mu \quad \tan \alpha = \frac{q_{tx}}{q_{ty}} = \frac{\lambda_x \xi_y}{\lambda_y \xi_x} \approx \frac{\lambda_x}{\lambda_y} \frac{\delta^*}{\xi_x}, \]

Where \( \xi_y = \Delta V_y/V \approx \delta^* \) and \( \xi_x = \Delta V_x/V \) - relative velocity difference components \( \xi = \Delta V/V_i \) in the contact zone, respectively, in the lateral and longitudinal directions.

![Figure 3. Relative velocities and tangential stresses in contact area adhesion section.](image)

Depending on stiffness ratio, different ratios of \( \theta \) and \( \alpha \) are possible: for \( \lambda_y > \lambda_x \alpha > \theta \); for \( \lambda_y = \lambda_x \alpha = \theta \); for \( \lambda_y < \lambda_x \alpha < \theta \). On the slip section, the shear stress \( q_{tsl} = \mu q_n \) and the relative velocity \( \Delta V \) coincide in direction. Thus, at some point in time, there is a change in shear stress direction (the angle between shear stress direction and wheel axis speed changes from \( \alpha \) to \( \theta \)), and the beginning of this process begins after shear stress reaches maximum adhesion value, and a breakdown occurs. In view of the fact that duration of change in total shear stress direction after breakdown is unknown, we will proceed from assumption that simplifies problem of an instantaneous change in \( q_t \) breakdown point - at adhesion and slip sections boundary.

As already noted, in sliding section, the shear stress \( q_{tsl} = \mu q_n \) coincides with sliding speed and its longitudinal and lateral components can be calculated using the following formulas:
\[ q_{tx}^{sl} = \mu q_n \frac{v_x^{sl}}{V} = \mu q_n \frac{\xi_x}{\xi} q_{tx}^{sl} = \mu q_n \frac{v_y^{sl}}{V} = \mu q_n \frac{\delta^*}{\xi}, \]

where
\[ V^{sl} = \xi V = V \sqrt{\xi_x^2 + \delta^2 V_x^{sl}} = \xi_x V V_y^{sl} = \delta^* V \]
Longitudinal and lateral forces arising in the contact are defined as:

\[
F_x = \int_{-b}^{+b} t_x \, dy = 2bt_x \\
F_y = \int_{-b}^{+b} t_y \, dy = 2bt_y
\]

At this time

\[
t_x = \int_{-a}^{+b} q_{tx}^i \, dx + \int_{-a}^{+b} q_{tx} \, dx \\
t_y = \int_{-a}^{+b} q_{ty}^i \, dx + \int_{-a}^{+b} q_{ty} \, dx
\]

- linear longitudinal and lateral forces, respectively.

Due to distribution asymmetry over the contact area of longitudinal and lateral tangential stresses in the contact plane, a moment arises:

\[M_z = M_{tx} + M_{ty},\]

where

\[M_{tx} = \int_{-b}^{+b} t_x y \, dy\] - moment due to longitudinal stress distribution asymmetry;

\[M_{ty} = \int_{-b}^{+b} t_y y \, dy\] - moment due to lateral stresses asymmetry:

\[
m_y = \int_{-a}^{+b} q_{ty}^i \, x \, dx + \int_{-a}^{+b} q_{ty} \, x \, dx
\]

- elementary moment arising in a longitudinal section due to transverse stresses distribution asymmetry.

Power of losses due to sliding friction in contact will be equal to:

\[P_{fr} = \int_{-b}^{+b} t \xi V \, dy = \xi V \int_{-b}^{+b} \sqrt{t_x^2 + t_y^2} \, dy,
\]

where \(t = \sqrt{t_x^2 + t_y^2}\) - linear force acting in wheel longitudinal section.

The choice of normal pressures distribution law \(q_n\) does not have a significant effect on calculated values of the force acting in contact, wheel speed relative loss, and friction power losses in the contact. In particular, with longitudinal tangential force \(F_x < 0.5\mu F_z\) (\(F_z\) is vertical load on wheel), it is more convenient to use a simplified trapezoidal law (which, according to experiments, is closer to low-pressure tires). In this case (with ratio of upper and lower sides of trapezoid base \(x_b = 0.8\) \(x_a = 0.9\alpha\)), which greatly simplifies calculation formulas:

\[
t_x = 1.8\lambda_x a^2 \xi_x \, t_y = 1.8\lambda_y a^2 (\delta - \gamma a/3r) \\
F_x = 3.6\lambda_x a^2 b \xi_x \, F_y = 3.6\lambda_y a^2 b (\delta - \gamma a/3r) \\
P_{fr} = 3.6a^2 b V \sqrt{\xi_x^2 + \delta^2} \left(\lambda_x^2 + \lambda_y^2 (\delta - \gamma a/3r)^2\right)
\]

From the presented dependences it follows that even with equal values of camber angles \(\delta\) and slip \((\text{toe})\delta\) camber angle influence on wheel side track lateral force and wear rate is much less noticeable than influence of slip angle and \(\xi_x\).

By choosing ratio of angles \(\delta\) and \(\xi_x\) it is possible to reduce lateral force values \(F_y\) and friction losses power \(P_{fr}\) in contact. In particular, at \(\delta = \gamma a/3r\), lateral force becomes equal to zero, and friction losses power in contact is minimal and equal to \(P_{fr} = P_{fr, min} = F_y V \sqrt{\xi_x^2 + \delta^2}\).

Derived dependencies use makes it possible to analyze influence of such factors as rolling mode and longitudinal and lateral stiffness ratio of wheel on resistance to lateral pull. In particular, this problem was solved in relation to low-pressure pneumatic tires.

It can be noted that lateral force increasing effect at \(\delta = \text{const} + \text{braking force application to wheel is more pronounced for tires with } \lambda_x/\lambda_y < 1\), i.e. with greater tangential elasticity in longitudinal direction, because in this case, longitudinal force of the same magnitude leads to a greater distortion of normal pressure diagram symmetry. This ratio is typical for radial tires.
3. Conclusion

1. Obtained dependences analysis shows that convergence (slip) angle influence on value of lateral force and treadmill wear intensity of wheel is much greater than camber angle influence.

2. By choosing slip and camber angles ratio, it is possible to reduce values of lateral force and friction losses in contact. In particular, at withdrawal angle $\delta = \gamma / 3r$, the lateral force becomes equal to zero and friction losses power in contact (and therefore wear) is minimal.

3. A simple dependence has been obtained that allows calculating lateral stiffness coefficient through lateral slip resistance coefficient found for the driven wheel, which significantly reduces experimental work amount to determine $F, M, P$.

4. The influence of rolling mode and longitudinal and lateral stiffness ratio of wheel on lateral withdrawal resistance is analyzed; lateral force increasing effect at a constant slip angle with braking torque application to wheel is more pronounced for tires with greater tangential elasticity in longitudinal direction, because in this case, longitudinal force of the same magnitude leads to a greater distortion of normal pressure diagram symmetry.

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