Finite Unified Models

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Abstract

We present phenomenologically viable $SU(5)$ unified models which are finite to all orders before the spontaneous symmetry breaking. In the case of two models with three families the top quark mass is predicted to be 178.8 GeV.

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1. Introduction

The apparent success of unified gauge theories describing the observed interactions is restrained by the plethora of arbitrary parameters that one has to introduce by hand. In particular, in the electroweak standard model [1], which is indeed a very successful theory, one has to fit more than twenty parameters if neutrinos are massive or eighteen if they are massless. This is a clear disadvantage as far as the predictivity of the theory is concerned. Grand Unified Theories (GUTs) [2, 3] are doing better in this respect since they can provide predictions for parameters such as $\sin^2 \theta_W$ and fermion mass ratios, which are free parameters in the electroweak standard model. In turn, GUTs can be tested and possibly could be ruled out, as for instance is the case of the minimal $SU(5)$ model [4].

There exists another principle that certainly points to the direction of further reduction of the free parameters of a gauge theory, namely, the requirement of finiteness. Moreover, the principle of finiteness goes very deeply to the heart of quantum field theories, supporting strongly the hope that the ultimate theory does not need infinite renormalizations. Although the latter are perfectly legitimate in quantum field theory they still give the feeling that divergences are “hidden under the carpet” [5]. It is not accidental that supersymmetric gauge theories have been so widely explored during the last decade in spite of the lack of any experimental evidence of supersymmetry. The clear motivation for the explosion of interest is due to the absence of quadratic divergences in these theories which guarantees their naturalness.

There have been made many attempts to obtain finite quantum field theories in four dimensions. For general theories such searches are usually limited to one loop approximation [6]. Besides, there is a strong indication that only supersymmetric gauge theories can be completely free from ultraviolet divergences [6]. A very interesting fact is that the one loop finiteness conditions on $N = 1$ supersymmetric theories automatically ensure also two-loop finiteness [7]. Last but not least, there have been given simple criteria [8, 9, 10] which ensure “all orders finiteness” in the sense of vanishing $\beta$-functions.

A complete classification of chiral $N = 1$ supersymmetric theories with a simple gauge group that satisfy the one-loop finiteness conditions has been done in refs. [11, 12]. There appear to exist only a few possibilities that have a chance to develop to realistic models. Here we examine to which extent these models can be made realistic, imposing in addition the requirement of all orders finiteness in the sense of ref. [8]. We find interesting solutions to this problem. Furthermore, in the case
of the models involving three generations a heavy top quark naturally emerges, a feature which seems to be characteristic of this class of models.

2. Finite $N = 1$ Supersymmetric Gauge Theories

In order to discuss in detail the finiteness conditions and their implications, let us consider a chiral, anomaly free, globally supersymmetric $N = 1$ gauge theory with gauge group $G$. The superpotential of such a theory is given by:

$$W = a_i \phi_i + \frac{1}{2} m_{ij} \phi_i \phi_j + \frac{1}{6} C_{ijk} \phi_i \phi_j \phi_k ,$$  

(1)

where $a_i$, $m_{ij}$ and $C_{ijk}$ are gauge invariant tensors and the matter fields $\phi_i$ transform according to an irreducible representation $R_i$ of the gauge group $G$.

The necessary and sufficient conditions for finiteness at one-loop level are the following:

- One-loop finiteness of the gauge fields self-energy which requires:

$$\sum_i \ell(R_i) = 3 C_2(G) ,$$

(2)

where $\ell(R_i)$ is the Dynkin index of $R_i$ [3] and $C_2(G)$ is the quadratic Casimir operator of the adjoint representation of the gauge group $G$.

- One-loop finiteness of the chiral superfields self-energy. In terms of the cubic couplings $C_{ijk}$ appearing in the superpotential given in eq. (1), referred to as Yukawa couplings, this condition requires:

$$C_{ikl} C_{jkl} = 2 \delta_{ij} g^2 C^2_2(R_i) ,$$

(3)

where $g$ is the gauge coupling constant, $C^2_2(R_i)$ is the quadratic Casimir of the representation $R_i$, and $C^{ijk} = (C_{ijk})^*$. Note that condition 3 forbids the presence of singlets with nonzero coupling. Furthermore, it requires that $C^{ikl} C_{jkl}$ is diagonal in its two free indices.

Therefore, the finiteness conditions given in eqs. (2) and (3), which express the vanishing of the one-loop anomalous dimensions of the gauge and matter couplings respectively, restrict considerably the choices of the representations $R_i$s for a given group $G$ as well as their Yukawa couplings appearing in the superpotential, eq. (1). On the other hand due to the non-renormalization theorem [14], which relates the
renormalization of $a_i$, $m_{ij}$ and $C_{ijk}$ to that of the $\phi_i$, the finiteness conditions do not restrict the form of $a_i$ and $m_{ij}$.

An important consequence of the finiteness conditions is that supersymmetry most probably can only be broken by the addition of soft breaking terms. Specifically, due to the exclusion of singlets according to eq. (3) the F-type [15] spontaneous supersymmetry breaking terms are incompatible with finiteness. Also, the D-type [16] spontaneous breaking is also ruled out since it requires the existence of a $U(1)$ gauge group which in turn is incompatible with eq. (4). In choosing to break supersymmetry by the addition of soft terms one should be aware of the fact that one-loop finiteness imposes extra conditions on this sector of the theory [17].

A very interesting result proved in ref. [7] is that the one-loop finiteness conditions (2), (3) are necessary and sufficient for finiteness at two-loop level. Even more interesting is the theorem proved in ref. [8]. The theorem states that if a supersymmetric gauge theory with simple gauge group is free from gauge anomalies, obeys eq. (2), and there exist solutions to eq. (3) of the form

$$C_{ijk} = \rho_{ijk} g,$$

where $\rho_{ijk}$ is a complex number, which are isolated and non-degenerate, then each of these solutions can be uniquely extended to a formal power series of $g$ [18], giving a theory which depends on a single coupling $g$, with a $\beta$-function vanishing to all orders.

### 3. Finite Unified Models based on SU(5)

An inspection on the tables of refs. [11, 12] immediately shows the difficulties encountered in constructing phenomenologically viable finite unified theories (FUTs) already at the one- or equivalently two-loop level. In particular, using $SU(5)$ as gauge group there exist only two candidate models which can accommodate three fermion generations and they contain the chiral multiplets $5, \bar{5}, 10, 10, 24$ with multiplicities $(6,9,4,1,0)$ and $(4,7,3,0,1)$ respectively. In addition, there exists another model based on $SU(5)$ gauge group which can accommodate five fermion generations and contains the same chiral multiplets as the two previous with multiplicities $(5,10,5,0,0)$. Out of these three models only the second one contains a 24-plet which can be used for the spontaneous symmetry breaking of $SU(5)$ down to the standard model $SU(3) \times SU(2) \times U(1)$. For the other two models one has to incorporate another way such as the Wilson flux breaking mechanism [13] in order to achieve the required superstrong spontaneous symmetry breaking of the $SU(5)$ gauge group.
In the following we will consider in more detail the three family models.

**A. \( N = 1 \), \( SU(5) \) model with three fermion families and without adjoint Higgs**

The particle content of this model consists of the following supermultiplets represented by their transformation properties under \( SU(5) \): three \((\bar{5} + 10)\), which are identified with the three supermultiplets describing the fermion families, six \((5 + \bar{5})\) which are considered as Higgs supermultiplets, and one \((10 + \bar{10})\) which are considered also as scalar supermultiplets.

The first finiteness condition given in eq. (2) is automatically satisfied in the present model given that this was one of the selection rules for the models appearing in refs. [11, 12]. In order to satisfy the second condition given in eq. (3) we have to consider the superpotential. The most general \( SU(5) \) invariant, \( N = 1 \) cubic superpotential with the above particle content has the form:

\[
W = \frac{1}{2} g_{ija} 10_i 10_j H_a + g_{ia} 10_i N H_a + \bar{g}_{ija} 10_i \bar{5}_j \bar{H}_a + \frac{1}{2} g'_{ijk} 10_i \bar{5}_j \bar{5}_k \\
+ \frac{1}{2} f_{ab} N H_a H_b + \frac{1}{2} \bar{f}_{ab} N H_a H_b + \frac{1}{2} h_a N N H_a + \frac{1}{2} \bar{h}_a N \bar{N} H_a \\
+ \frac{1}{2} q_{ia} 10_i \bar{H}_a H_b + p_{ia} N 5_i \bar{H}_a + \frac{1}{2} t_{ij} 5_i \bar{5}_j ,
\]

where \( i, j, k = 1, \ldots, 3 \) and \( a, b = 1, \ldots, 6 \) and we have suppressed the \( SU(5) \) indices. \( 10_i \) and \( \bar{5}_i \) are the usual three generations. The six \((5 + \bar{5})\) Higgses are denoted by \( H_a, \bar{H}_a \), while the scalar field belonging to the \((10 + \bar{10})\) representation by \( N + \bar{N} \).

Then, eq. (3) imposes the following relations among the Yukawa and gauge couplings:

\[
H : \quad 3 g^{ija} g_{ijb} + 6 g^{ia} g_{ib} + 4 f^{ca} f_{cb} + 3 h^a h_b = \delta^i_b \frac{24}{5} g^2 ,
\]

\[
5 : \quad 4 \bar{g}^{\bar{d}a} \bar{g}_{ima} + 4 g^{rilk} g'_{imk} + 4 t^{ij} t_{mj} + p^{ja} p_{ma} = \delta^i_b \frac{24}{5} g^2 ,
\]

\[
\bar{H} : \quad 4 \bar{g}^{\bar{d}ia} \bar{g}_{ijb} + 4 f^{ca} f_{cb} + 3 \bar{h}^a \bar{h}_b + 4 q^{ica} q_{icb} + 4 p^{ja} p_{ib} = \delta^a_b \frac{24}{5} g^2 ,
\]

\[
N : \quad 3 g^{ia} g_{ia} + f^{cb} f_{cb} + 3 h^a h_a + 2 p^{ja} p_{ja} + t^{ij} t_{ij} = \frac{36}{5} g^2 ,
\]

\[
\bar{N} : \quad \bar{f}^{ab} \bar{f}_{ab} + 3 \bar{h}^a \bar{h}_a = \frac{36}{5} g^2 ,
\]

\[
10 : \quad 3 g^{lki} g_{mkj} + 2 g^{lki} \bar{g}_{mkj} + 3 g^{la} g_{ma} + g^{ljk} g'_{mjk} + q^{lab} q_{mab} = \delta^a_b \frac{36}{5} g^2 .
\]

As it was already emphasized in sect. 2 the fulfillment of eqs. (2) and (3) is necessary and sufficient to guarantee the one-loop as well the two loop finiteness of
Table 1: The charges of the $Z_7 \times Z_3$ symmetry

|      | 10₁ | 10₂ | 10₃ | 5₁ | 5₂ | 5₃ | H₁ | H₂ | H₃ | H₄ | H₅ | H₆ | N |
|------|-----|-----|-----|----|----|----|----|----|----|----|----|----|---|
| $Z_7$ | 1   | 2   | 4   | 4  | 1  | 2  | 5  | 3  | 6  | 0  | 0  | 0  | 0 |
| $Z_3$ | 1   | 2   | 0   | 0  | 0  | 0  | 1  | 2  | 0  | 0  | 1  | 2  | 0 |

the theory \[7\]. Nevertheless, in order to achieve all-loop finiteness one has to do more \[8\]. Specifically, one has to find a solution of eq. (3) which is isolated and non-degenerate. This is a far from trivial problem given that eq.(3) have infinitely many solutions that can be parametrized by continuous parameters (look for example refs. [20, 21]).

Our strategy to find a unique and phenomenologically interesting solution to eq. (3) is to impose on the model additional symmetries on top of the $SU(5)$ gauge invariance and $N = 1$ global supersymmetry. Next recall that the terms of lower dimensions such as mass terms are not restricted by the finiteness requirement. We use this freedom to make the model phenomenologically viable. As a result we found a solution to all-loop finiteness problem with very interesting phenomenological predictions. In particular the top quark mass is predicted. The method can be generalized in a straightforward way in order to take into account all light fermion masses and mixing angles \[26\]. Specifically, we impose the $Z_7 \times Z_3$ discrete symmetry given in table 1, together with a multipliclicative Q-parity under which the $10_i$ and $\bar{5}_i$ describing the fermion supermultiplets are odd, while all the other superfields are even. In this way the number of terms that are permitted to appear in the superpotential is severely restricted. Only terms with Yukawa couplings $g_{iii}, \bar{g}_{iii}, f_{44}, f_{56}, f_{65}, \bar{f}_{44}, \bar{f}_{56}, \bar{f}_{65}, h_4,$ and $\bar{h}_4$ survive.

We then find the following unique solution to eqs. (3),

\[
\begin{align*}
g^2_{111} &= g^2_{222} = g^2_{333} = \frac{8}{5}g^2, \\
\bar{g}^2_{111} &= \bar{g}^2_{222} = \bar{g}^2_{333} = \frac{6}{5}g^2, \\
f^2_{44} &= 0; \quad f^2_{56} = f^2_{65} = \frac{6}{5}g^2, \\
\bar{f}^2_{44} &= 0; \quad \bar{f}^2_{56} = \bar{f}^2_{65} = \frac{6}{5}g^2, \\
h^2_4 &= \frac{8}{5}g^2; \quad \bar{h}^2_4 = \frac{8}{5}g^2.
\end{align*}
\]
The uniqueness of this solution guarantees the all-loop finiteness.

One might wonder if this model could result from some more fundamental theory and, in turn, if there is some justification for its symmetries. It seems that there exist very suggestive hints that the model under consideration belongs to a class of models obtained from superstring compactification over certain Calabi–Yau (CY) manifolds. More specifically, Witten has shown that it is possible to construct stable, irreducible, and holomorphic $SU(5)$ or $SU(4)$ vector bundles over CY manifolds. Then one can start from the heterotic superstring with gauge group $E_8 \times E_8'$ and obtain an $SU(5)$ or $SO(10)$ $N = 1$ supersymmetric theory at four dimensions, by embedding the structure group of the bundle ($SU(5)$ or $SU(4)$) in $E_8$ ($E_8'$ is considered as hidden). It is worth noting that claims that such configurations are generically unstable due to non-perturbative effects appeared unjustified in particular cases. Furthermore the conditions under which a stable configuration emerges are given in ref. It turns out that the spectrum of a $N = 1$, $SU(5)$ gauge theory resulting from a CY compactification is generally of the form $m(10) + n(\overline{5}) + \delta(10 + \overline{10}) + \epsilon(\overline{5} + 5)$, where $m$, $n$, $\delta$, and $\epsilon$ are topological numbers of the CY manifold. Therefore, it is not inconceivable to imagine how a model like the one considered here could come from superstring compactification.

Furthermore, since in the present model we are interested in applying the Wilson flux breaking mechanism, we, naturally, assume that the CY which is going to be used should admit a freely acting discrete group $F$. Then the light fields will be the ones which are invariant under $T \oplus F$, where $T$ is the homomorphism of $F$ in the gauge group.

Therefore, we are led to assume the existence of a CY with a stable, irreducible, and holomorphic $SU(5)$ bundle over it, admitting a freely acting discrete group $F$. Moreover, the topological numbers of this manifold after division with $F$ are given by $m = n = 3$, $\delta = 1$, and $\epsilon = 6$. Let us comment here that the discrete symmetries used above in order to reduce the number of the Yukawa couplings should be respected by this CY manifold.

The present model clearly belongs to the class of models considered in ref. For instance, suppose that $F$ is a $Z_3$ which is embedded in a $T = Z_3$ identified with

\footnote{The phase arbitrariness of eqs. is not crucial, since it can be absorbed by using a specific renormalization scheme.}
a discrete subgroup of the $U(1)$ appearing in the decomposition

$$SU(5) \supset SU(3) \times SU(2) \times U(1)$$

$$10 = (1,1)(6) + (3,1)(-4) + (3,2)(1)$$

$$\bar{5} = (1,2)(-3) + (\bar{3},1)(2).$$

Next recall that the gauge symmetries surviving after applying the Wilson flux breaking mechanism are those that commute with $T$. Then it is clear that the $SU(5)$ gauge symmetry of the model at hand breaks down to the standard model. One can go further and consult the tables of ref. [25] in order to attribute appropriate transformation properties to the various scalar multiplets, such as to make the model phenomenologically viable. As an example, consider that the scalar multiplets are invariant under the action of $F$, while they transform under the action of $T$ according to $\exp(y\pi)$ where $y$ is the hypercharge in eq. (8). Then one can easily see that only the $(1,2)(-3)$ components coming from the $\bar{5}$ and the $(1,1)(6)$ coming from the 10 remain light. All the other components acquire superheavy masses of the order of the compactification scale. Therefore, in a natural way the model is provided with light Higgs doublets that can drive the spontaneous symmetry breakdown of $SU(2) \times U(1)$ down to $U(1)_{em}$ and, on the other hand, it is exorcised from the appearance of light “coloured scalars” that would lead to fast proton decay. Note that the above discrete symmetries do not affect the fermion supermultiplets [25].

Having described the basic strategy to make the model phenomenologically viable we postpone the full analysis of the various possibilities to a future publication [26]. For our purposes here we assume that the discrete symmetries involved permit only the existence of a pair of light Higgs doublets which is coupled only to the third family. Moreover, by adding soft breaking terms we can achieve supersymmetry breaking at the order of the electroweak scale. Then examining the evolution of the gauge couplings according to the renormalization group equations [27] we find

$$\sin^2 \theta_W(M_Z) = 0.233, \quad M_X = 2 \cdot 10^{16}, \quad \alpha_{em}^{-1}(M_Z) = 127.9, \quad \alpha_s(M_Z) = 0.120, \quad \alpha_X = 0.0425,$$

in excellent agreement with the experimental values [4]

$$\sin^2 \theta_W(M_Z)_{exp} = 0.2327 \pm 0.0008,$$

$$\alpha_{em}^{-1}(M_Z)_{exp} = 127.9 \pm 0.2,$$

$$\alpha_s(M_Z)_{exp} = 0.118 \pm 0.008.$$
Running now the renormalization group equations for the Yukawa couplings with
the above values for $\alpha_X$ and $M_X$ and initial values at $M_X$:

$$g_i^2 = \frac{8}{5}(4\pi\alpha_X); \quad \bar{g}_b^2 = \bar{g}_r^2 = \frac{6}{5}(4\pi\alpha_X)$$  \hspace{1cm} (11)

we find at $M_W$:

$$m(\text{top}) = 178.8 \text{ GeV}, \quad m(\text{bottom}) = 3.1 \text{ GeV}, \quad \text{and} \quad m(\tau) = 1.8 \text{ GeV}. \quad (12)$$

As we can see, the model gives results for the tau and bottom masses in very good
agreement with experiment, and predicts a high value for the mass of the top. Notice
that these values are determined by the solution (7) to the finiteness conditions (6),
and that although we have assumed that only the third family becomes massive,
we do not expect the results to change considerably, since the third family terms
dominate in the calculation.

**B. $N = 1$, $SU(5)$ model with three fermion families and Higgs in the adjoint**

This model has been considered before for two-loop [20, 21] as well as for all-loop
finiteness [10]. The particle content consists of the following supermultiplets: three
$(5 + 10)$, identified with the three supermultiplets describing the fermion families,
four $(5 + \bar{5})$, and one 24 considered as Higgs supermultiplets.

The first finiteness condition, eq. (2), is, as before automatically met. In order
to satisfy the second condition, eq. (3), we have to examine the superpotential of
the model. The most general $SU(5)$ invariant, $N = 1$ cubic superpotential with the
above particle content is:

$$W = \frac{1}{2}g_{ija}10_i10_jH_a + \bar{g}_{ija}10_i\bar{5}_j\tilde{H}_a + \frac{1}{2}g'_{ijk}10_i\bar{5}_j\bar{5}_k
+ \frac{1}{2}g_{iab}10_i\tilde{H}_a\tilde{H}_b + f_{ab}\tilde{H}_a24H_b + p(24)^3 + h_{ia}\bar{5}_i24H_a,$$  \hspace{1cm} (13)

where $i, j, k = 1, \ldots, 3$ and $a, b = 1, \ldots, 4$ and we have suppressed the $SU(5)$ indices.
The $10_i$’s and $\bar{5}_i$’s are the usual three generations, and 24 is the scalar superfield in
the adjoint. The four $(5 + \bar{5})$ Higgses are denoted by $H_a, \tilde{H}_a$.

Then, eq. (3) imposes the following relations among the Yukawa and gauge cou-
plings:

\[
\begin{align*}
\bar{H} : & \quad 4\bar{g}_{ija}\bar{g}^{jib} + \frac{24}{5}f_{ac}f^{bc} + 4q_{iac}q^{ibc} = \frac{24}{5}g^2\delta^b_a, \\
H : & \quad 3g_{ija}g^{jib} + \frac{24}{5}f_{ca}f^{cb} + \frac{24}{5}h_{ia}h^{ib} = \frac{24}{5}g^2\delta^b_a, \\
5 : & \quad 4\bar{g}_{kia}\bar{g}^{jka} + \frac{24}{5}h_{ia}h^{ja} + 4g'_{ikl}g^{jkl} = \frac{24}{5}g^2\delta^j_i, \\
10 : & \quad 2\bar{g}_{ika}\bar{g}^{jka} + 3g_{ika}g^{jka} + q_{iab}q^{jab} + g'_{khi}g^{jkl} = \frac{36}{5}g^2\delta^j_i, \\
24 : & \quad f_{ab}f^{ab} + \frac{21}{5}pp^* + h_{ia}h^{ia} = 10g^2.
\end{align*}
\]

In most of the previous studies of this model no attempt was made to find isolated and non-degenerate solutions. Their philosophy was rather in the opposite direction. They have used the freedom offered by the degenerate solutions in order to make specific ansatze that could lead to phenomenologically acceptable predictions. Following the lines prescribed in the previous model we impose additional symmetries on the model\(^2\). The new symmetries imposed on this model are again given in table 1 for 10, 5, and \(H_a\) for \(a = 1, \ldots, 4\). The terms in the superpotential which are invariant under the symmetries of the model are the terms with Yukawa couplings \(g_{iii}, \bar{g}_{iii}, f_{ii}\) and \(p\).

We find the following solution of eq. (14)

\[
\begin{align*}
g_{111}^2 & = g_{222}^2 = g_{333}^2 = \frac{8}{5}g^2, \quad \bar{g}_{111}^2 = \bar{g}_{222}^2 = \bar{g}_{333}^2 = \frac{6}{5}g^2, \\
f_{11}^2 & = f_{22}^2 = f_{33}^2 = 0, \quad f_{44}^2 = g^2; \quad p^2 = \frac{15}{7}g^2.
\end{align*}
\]

Therefore, we are in the same situation as in ref. [21], i.e. each fermion family is coupled to a different Higgs. For simplicity, as in the previous models, we assume that only one pair of Higgs fields is light and acquires a v.e.v. which is coupled to the third family. This situation can easily be realised by adding appropriate mass terms. The solution of the doublet-triplet splitting problem in this model goes along the lines described in ref. [21].

4. Finite Models based on other gauge groups

There exist some more FUTs that have a chance to develop into realistic models. For instance, an inspection of the list of refs. [11, 12] suggests that the following models are worth to be examined:

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\(^2\)See however ref. [10] for an attempt to construct an all-loop finite model.
1. An $SO(10)$ model with particle content consisting of eight $10$, $n$ $16$ and $(8 − n)$ $\overline{16}$ (with $5 \leq n \leq 8$) supermultiplets. This model can accommodate an even number of fermion families and could result from a CY compactification as it was discussed in model A.

2. An $E_6$ model containing $n$ $27$ and $(12 − n)$ $\overline{27}$ (with $7 \leq n \leq 12$) supermultiplets which can accommodate an even number of fermion families.

3. An $SU(6)$ model with three $6$, nine $\overline{6}$ and one $35$ supermultiplets. The model can describe three fermionic families, six Higgs in the fundamental, six Higgs in the antifundamental and one Higgs in the adjoint.

5. Conclusions

We have discussed a number of one and two loop finite unified models. Emphasis was given in the construction of $SU(5)$, $N = 1$ supersymmetric models which are finite in all orders before the spontaneous symmetry breaking.

In particular, in the case of $SU(5)$, $N = 1$ supersymmetric models with three families the top quark mass is predicted to be $178.8$ GeV. We have restricted our analysis to the case that only the third fermion family becomes massive after the electroweak symmetry breaking. The generalization to non zero masses for the rest fermions and mixing angles is straightforward. However, due to the clear dominance of the third family, and in particular of the top quark mass, our prediction is not expected to change in a noticeable way.

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