zk-Fabric, a Polylithic Syntax Zero Knowledge Joint Proof System

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Abstract—In this paper, we create a single-use and full syntax zero knowledge proof system, a.k.a zk-Fabric. Comparing with zk-SNARKs and another variant zero knowledge proofing system, zkBOO and it's variant zkBOO++. We present multiple new approaches on how to use partitioned garbled circuits to achieve a joint zero-knowledge proof system, with the benefits of less overhead and full syntax verification. zk-Fabric based on partitioned garbled circuits has the advantage of being versatile and single use, meaning it can be applied to arbitrary circuits with more comprehensive statements, and it can achieve the non-interactivity among all participants. One of the protocols proposed within is used for creating a new kind of partitioned garbled circuits to match the comprehensive Boolean logical expression with multiple variables, we use the term "polylithic syntax" to refer to the context based multiple variables in a comprehensive statement. We also designed a joint zero knowledge proof protocol that uses partitioned garbled circuits.

Index Terms—Zero-Knowledge Proof, Garbled Circuit, Arithmetic Circuit, Cryptography, Privacy, Security.

I. INTRODUCTION

In cryptography, “Zero-knowledge” proofs allow one party (the prover) to prove to another (the verifier) that a statement is true, without revealing any information beyond the validity of the statement itself. For example, given the hash of a random number, the prover could convince the verifier that he/she actually owns the number, without revealing what it is, he/she can construct a proof system to let the verifier to be convinced. A zero-knowledge proof convinces a verifier of a statement while revealing nothing but its own validity. Since they were introduced by Goldwasser, Micali, and Rackoff [1], zero-knowledge (ZK) proofs have found applications in domains as diverse as authentication and signature schemes [2], secure computation [3], and emerging shield transaction in blockchain technologies [4]. With the state of art zero knowledge technologies, such as zk-SNARK [5], zkBOO [6] etc, it brings new versatile functionalities to today’s infrastructure.

This means that, given an interactive proof for any NP-complete problem, one can construct zero-knowledge proofs or arguments for any NP statement. But existing solutions has the limitations of monolithic state, in another word, the NP statement contains one argument at a time, another limitation are the computational overheads, mainly because many of the early techniques [5], [6] require many iterations of finding the arithmetic roots of a polynomial equations to achieve negligible soundness error. The overhead is observed [7] in the prover work and communication. More recent work [6] avoids those issues, but generally entails many expensive cryptographic operations.

zk-Fabric is constructed as a variant of zk-SNARK and zkBoo, a novel form of zero-knowledge cryptography, zk-Fabric inherits the strong privacy assurance with full syntax verification capability, that multiple variables within a statement can be fully verified without revealing the true value. The zk-Fabric can be benefited in the Blockchain, with the anonymous verifiers jointly provides the Zero Knowledge Proof.

In zk-SNARK, the prover and verifier had to endure a heavy trusted setup which consumes large sets of cryptographic primitives and running time [7]. zkBoo and its derivative, zkBOO++ [8] have constructed zero knowledge proof using a difference approach without the requirements of trusted setup and not reliant on the arithmetic circuit, instead it’s employing the garbled circuit [9], [10] to construct the zero knowledge proof system. However both systems are built inherently to solve monolithic statement which hinders its practicality to large system, such as contract verification, auditing etc. zk-Fabric is built to overcome the shortcomings of zk-SNARKs and zkBOO with efficient way to produce joint zero-knowledge proofs based on garbled circuit regime. zk-Fabric also maintains what is deemed as important features of online zero knowledge proof system, that are non-interactive and succinct to publish to a block chain which provides the publicly retrievable information, perfectly for the shield auditing service.

zk-Fabric also creates the protection mechanisms to prevent the false proofs, known as semi-honest model [11], specifically if someone had accessed to the secret randomness used to generate these parameters, they would be able to create false proofs that would look valid to the verifier. To prevent this from happening, zk-Fabric generates the public parameters through the partitioned garbled circuits with multi-party settings.

In section 2, a pre-settings of cryptographic notions and functions are introduced In Section 3 we introduce the zk-Fabric schematics, including the framework which integrate all functional modules together. In Section 4, we introduce the first module whose job is to transform a full semantic statements from a prover into a polylithic syntax logical expression with Boolean operations, note the word "polylithic" is created in comparison with "monolithic". In section 5, we introduce the second module, which creates partitioned garbled circuits on the basis of the generated polylithic syn-
tax expression, and prepare the garbled circuit with public cryptographic primitives. In Section 6, we introduce the 1-2 OT (oblivious Transfer) [12] based OT-aggregator verification protocol, which obtains the zero-knowledge proofs with the partitioned Garbled circuits created by module II in section 6. Lastly, we draw our conclusion and present ideas for future work in section 8.

II. PRELIMINARIES AND NOTIONS

In this section, we will provide some insights into the cryptographic concepts employed in our zk-Fabric scheme, in which most of the designs play vital role in building the zero-knowledge proof system. We will also introduce the notions within the paper.

A. Notions

In our paper, we denote Alice as the garbled circuits sender, and Bob as the garbled circuits receivers. In the joint verification schemes, Charlie, David are also referred as garbled circuits receivers; We also denote $x_i$ as the inputs from sender and receivers; $x_i^j$ is used to denote the $ith$ input $x_i \in (0, 1)^\ell$ from $jth$ receiver, in most circumstances, $j = 1$ represents the sender, where $\ell$ denotes the length of the bit string; We also denote $C_i$ as the $i$th garbled circuit constructed in section 3. Similarly, $e_i^j$ denotes the encoding function in a garbled circuit scheme fro $jth$ receiver; $d_j$ denotes the decryption key for $jth$ receiver; $Y_i$ denotes the evaluation output for $i$th garbled circuit; $Y$ denotes the value of unified Boolean operations on $Y_i$; $y$ denotes the expected value.

B. Yao’s Garbled Circuit

A garbled circuit is a method to "encrypt a computation" that reveals only the output of the computation, but reveals nothing about the inputs or any intermediate values. The "circuit" is referred to a combination of logical operations on inputs, and the syntax is expressed as a Boolean circuit, with the Boolean gates, such as (AND, OR, NOT) gates in the circuit. An example of a logical circuit can be illustrated as:

A classical Yao’s "garbling scheme" [9] consists of:

1. (Garbler): comprises of a method to convert a (plain) circuit $C$ into a garbled circuit $\hat{C}$.
2. (Encoder) comprises of a method to convert any (plain) input $x$ for the circuit into a garbled input $\hat{x}$. You need the secret randomness that was used to garble the circuit to encode $x$ into $\hat{x}$.
3. (Verifier) comprises of a method to take a garbled circuit $\hat{C}$ and garbled input $\hat{x}$ and compute the circuit output $C(x)$. Anyone can do this, you don’t have to know $x$ or the secret randomness inside $\hat{C}$ to evaluate and learn $C(x)$.

The main idea of security is that $\hat{C}$ and $\hat{x}$ together leak no more information than $C(x)$. In particular, they reveal nothing about $x$, yet they allow the computation $C(x)$ to be completed. In many terms, it’s often referred as "Encrypted Computation".

C. Karnaugh Map

Karnaugh Maps [13] offer a graphical method of reducing a digital circuit to its minimum number of gates. The map is a simple table containing 1s and 0s that can express a truth table or complex Boolean expression describing the operation of a digital circuit. The map is then used to work out the minimum number of gates needed, by graphical means rather than by algebra. Karnaugh maps can be used on small circuits having two or three inputs as an alternative to Boolean algebra, and on more complex circuits having up to 6 inputs, it can provide quicker and simpler minimisation than Boolean algebra.

We employ Karnaugh Maps as the tool to reduce the complex syntax (polylithic) into minimum gate setups, which in turn helps reduces the computational and communication overhead associated with zk-Fabric.

D. 1-2 Oblivious Transfer

In cryptography, an oblivious transfer (OT) protocol is a type of protocol in which a sender transfers one of potentially many pieces of information to a receiver, but remains oblivious as to what piece (if any) has been transferred. There are variants of oblivious transfer protocol [12], [14], [15], in our paper, we utilize the 1-2 oblivious transfer in our zk-Fabric system.

In a 1–2 oblivious transfer protocol [12], Alice prepares two messages $m_0, m_1 \in (0, 1)^\ell$ of length $\ell$ and sent them to the receiver, and the receiver has to choose which one of them is disclosed. The sender does not know, which message $m_b$ is delivered and the receiver has no possibility to get any information about the other message $m_1 – b$ The protocol consists of at least two messages: Choose contains the receiver’s choice $b$ and Transfer contains both messages $m_0$ and $m_1$ of which only $m_b$ can be read by the receiver.

III. zk-FABRIC SYSTEM

We will now provide an overview of zk-Fabric and our constructions of this new cryptographic system. zk-Fabric consists of three major components: I) Polylithic Syntax Decomposition; II) Construction of partitioned garbled circuits; III) A Non-interactive OT based Multi-Parties joint Verification scheme.

A. zk-Fabric in a nutshell

We start with the construction of a overall zk-Fabric system that builds on the partitioned OT scheme. This construction was inspired by the security notions of OT-Combiners [16]. Figure 3 shows an example of 2 polylithic inputs to be "blindly" verified by 3 offline verifiers with the construction of partitioned garbled circuits.

Alice bootstraps the zk-Fabric by invoking Module I in section 4 and Module II in section 5 which are to prepare a partitioned garbled circuits for multiple verifiers. The prover Alice distributes the garbled circuits through the Module III in section 6, the extended OT schemes with the verifiers through online systems, such as Blockchain, or web portals that are publicly accessible. Through the public online systems,
the verifiers, Bob and Charlie in this example, compute the intermediate circuit output $Y_i$ corresponding each partitioned garbled circuit $C_i$. At the final step, a garbled circuit OT-aggregator David computes the $Y$ and verifies if the $Y = y$.

In zk-Fabric, we consider the circuit $C$ a garbled circuit evaluation problem. In this problem, the prover Alice constructs a circuit $C$, consisting of Boolean gates operations over finite field $GF(2)$. We generalize the $C$ can be partitioned with degree $m$ with the input vector $X$. The $C$ therefore can be represented with a vector $C : (C_1, C_2, ..., C_m)$. The evaluation goal is to evaluate $C$ on inputs $X$. In an non-interactive proof for this problem, the prover sends the outputs vector $Y : (Y_1, Y_2, ...)$. The ultimate verification algorithm is to verify if $Y = C(X)$.

Definition 2.1: A zk-Fabric scheme syntactically consists of three Modules I: synGen, II: xgcGen, III: OT-aggregator.

- **Module I:** $C \leftarrow \text{synGen}(1^k)$: It takes input vector $m : (m_1, m_2, ..., m_i)$, maps the $m$ into hashed vector $X : (x_1, x_2, ..., x_m)^k \leftarrow \text{hash}(m)$ as inputs of the security parameter $1^k$ and outputs a Boolean gates based expressions $C$.

- **Module II:** $(C_1, C_2, ..., C_m) \leftarrow \text{xgcGen}(C)$: It takes the Boolean gates expression $C$ into an enumerated length of $m$ circuits $(C_1, C_2, ...)$. We denote the garbled circuit generator algorithms.

- **Module III:** $Y \leftarrow \text{OT-aggregator}(Y_1, Y_2, ..., Y_{m-1})$: It takes inputs from outputs of the partitioned garbled circuits $(Y_1, Y_2, ..., Y_{m-1})$, and outputs an aggregated output $Y$. The ultimate verification algorithm is to verify if $Y$ is equal to the expected output for the extended garbled circuit algorithm $xgc$:

$$
\{ Y = C(X) | y = f(m \oplus x_m^m) \} = \begin{cases} 
\text{True} & \text{if } Y = \text{De}(y) \\
\text{False} & \text{Otherwise }
\end{cases}
$$

(1)

1 An universal Hash Algorithm $x_i \leftarrow h(S_i)$: It takes a common reference string (crs) $S_i$ as inputs and outputs a uniformly distributed digest. The $h$ is an universal hash function.

2 Extended garbled circuit scheme $gc$: The extended garbled circuit is a four-tuple algorithm $gc = (Gb, Enc, De, Ev)$, where $Gb$ is a randomized garbling algorithm that transform $f$ into a triplet $(C_i, e_i, d_i)$, the $C_i$ is the $i$th partitioned garbled circuits, $e_i$ is the corresponding encoding information for circuit $C_i$, and $d$ is the corresponding decoding information. $Enc()$ is an encoding algorithm that maps input $x_i$ into garbled input via $X_i = Enc(e_i, x_i)$. $De$ is a decoding algorithm that maps the garbled output $Y_i$ into plaintext output $y_i = De(d_i, Y_i)$. $Ev()$ is the algorithm with input $X_i$ and $F_i$ which generates garbled output $Y_i = Ev(F_i, X_i)$.

**Definition 2.2:** We require that zk-Fabric holds the following security properties:

- **Correctness:** It should hold for all common reference string with input size $\ell$, all garbled circuits $C_i \in [C]$.

$$
\Pr \left[ y = Y \right] \approx 1
$$

(2)

- **Privacy:** zk-Fabric should hold the perfect sender and receivers’ privacy with the underlying OT scheme’s privacy model and the OT-aggregator model. Specifically, for any non-interactive multiple parties oblivious transfer protocol $(S, R_i)$ between a sender $S$ and multiple receivers $R_i$, it satisfies that any of the receiver $R_i$ does not learn any information on the input bits mapped to the corresponding circuit $C_i$, with a negligible probability $\epsilon \approx \frac{1}{2}$.

**IV. MODULE I: POLYTHIC SYNTAX CONSTRUCTION**

We start the zk-Fabric protocol at the site of prover Alice who has a composite statement to be verified without revealing the real value. The composite statement can be trivially converted into the regular expressions which represent a set of strings with some tools, i.e. intrusion detection systems SNORT. In the Polylithic Syntax construction module, we can employ the regular expression matching techniques to detect a pattern written by regular expressions from the input strings.

An example of such compound statements could be a simple sentence, such as: “The car only starts [if] the ‘start’ button is pressed [and] the brake pedal is pressed”. The variables $S_i$ to be verified in operator vector $o$, all the [$\cdot$] represents logical relationships between variables $v$. Compared with native zero-knowledge proof system which can only process single (monolithic) variable at a time, zk-Fabric is aimed to match the regular expression with patterns and construct corresponding circuits with reduced complexity.

In construction of the logical gates, we utilize the Karnough Map technique to reduce the logical expression complexity since the efficiency of the zk-Fabric is depending on the circuit complexity with depth of $\ell$. Due to the limitations of Karnough Maps, it’s ideal to constrain the zk-Fabric with $\leq 6$ inputs in...
the input vector $S$. The Karnaugh Map is introduced in section 2.

W.l.o.g, we can assume a few pre-setting functions existed to assist the constructions of the polythlic syntax conversion.

- **Definition 4.1:** there exists a generalized extraction function which can match the compound expression in strings with key variables and the logical relationships expressed in Boolean operators [AND, OR, XOR, etc], we can define the functions as $\text{Extractor}_y()$ and $\text{Extractor}_r()$, which recognize match the variables and logical operators respectively.
- **Definition 4.2:** there exists a generalized regular expression function which can match the regular expression in strings with certain patterns and convert the parameters and patterns into a regular expression, we define the function as $\text{Regexp}()$ [17].
- **Definition 4.3:** there exists a generalized function which can covert a regular expression string into logical circuits [17], we can define this function as $\text{CircuitGen}()$.
- **Definition 4.4:** there exists a generalized Karnaugh Map algorithm which can further reduce the logical gates complexity, and we can define this function as $\text{K-map}()$.

We implement a generalized algorithm for Polythlic Syntax generation as algorithm 1.

**Algorithm 1** Polythlic Syntax Generation Algorithm

**Require:** Composite String $\text{string}$

**Ensure:** Circuit ($C$) $\text{strlen} \leftarrow |\text{string}|

while in $\text{strlen}$ do
  $S' \leftarrow \text{Extractor}_r(\text{string})$
  $O \leftarrow \text{Extractor}_y(\text{string})$
  $S \leftarrow \text{hash}(S')$
  $\text{Exp} \leftarrow \text{Regexp}(S, O)$
  $C' \leftarrow \text{CircuitGen}(\text{Exp})$
  $C \leftarrow \text{K-map}(C')$

V. Module II: Partitioned Garbled Circuits Construction

With the inputs a circuit $C$ which represents a list of truth tables along the depth of the logical gates operations, we can implement the garbled circuit scheme with non-interactive commitments to ensure the verifiers can achieve the correctness and privacy in verification. We employ the abstraction of garbling schemes [18], which was briefly introduced in section 2. The correctness property of a garbling scheme is defined as $V(C, e, d)$ in the support of $Gc(1^k, c)$ and all inputs $x$, we have $De(d, Ev(C, En(e, x))) = c(x)$, where the $k$ denotes the security parameter.

A. Garbled circuit Representation

A Boolean circuit can be thought of as a Directed Acyclic Graph (DAG), i.e., a graph with no loops, with each node representing a unit of computation performing a specific operation $op$ (e.g., AND/XOR). Moreover, we fix all gates to have 2 input wires, a left and a right wire which we denote by $l$ and $r$, that functions, which given a wire returns its bit value 0 or 1. We denote the depth of a circuit by $d$, and its width by $n$. While there are many ways to represent them, a convenient way to think of them is as a $M: d \times n$ matrix, i.e., each layer $i \in (1, ..., d)$ of a circuit has a fixed width $n$, with each entry being a gate. We find this representation convenient in defining the composition of layers (to obtain a circuit)

B. Partition Garbled Circuits Scheme

We would require a partition scheme in our multiple parties OT verification protocol, meaning a $xGC$ can properly partition the garbled circuits $C$ into multiple independent garbled circuits, recorded in a vector $C(C_1, C_2, ...)$. Given a Yao’s circuit $C$ with a matrix of inputs $[x_i]$ and the outputs $[o_j]$ within a truth table, our goal is to securely divide the table into multiple representations of the truth table matrix $T$. To obtain the security properties of such scheme, it can be proved with state-separating proofs [19] and to make cryptographic proofs more suitable for multi-parties verification. The following protocol defines the scheme of partitioned garbled circuit construction.

**Partitioned Garbled Circuit Generation Scheme:**

1 Preparation: we denote the multi-parties Yao’s garbled circuit with sorted inputs $x_1, x_2, ..., x_n$, we pair two sorted inputs together onto one Boolean gates, if there exists odd number of inputs, two additional auxiliary random value $a_0, a_1 \in \{0, 1\}$ will be added. In a trust setup, the auxiliaries will have to destroyed at the “toxic wastes” after being used. We can define a new circuit $C'$ with this preparation:

$$C'(x_1, ..., x_n) = \begin{cases} C((x_1, x_2), ..., (x_{n-1}, x_n)) & \text{if } n \text{ is even} \\ C((x_1, x_2), ..., (x_n, (a_0 \oplus a_1))) & \text{if } n \text{ is odd} \end{cases} \quad (3)$$

2 Garbled Circuit Construction: for each input $x_i$, where $x_i$ denotes the input from the prover, and the $x_j$ denotes the input from the verifier, and the wires and internal wires $w$ of the circuit, assign a pair of keys $(k_w^0, k_w^1)$.

3 Garbled Circuit Construction: for each gate of the circuit, generate $4$ ciphertexts which encrypts the corresponding key associated with the output wire according to the truth table of the table $T$. Figure 4 illustrates the wire assignments and the outputs.

4 Garbled Circuit Construction: for each gate connected to an output wire of the circuit, we encode 0/1 according to the truth table as the Yao’s garbled circuits scheme.

5 Partitioning Garbled Circuit: based on the Truth table $T$, we can partition the circuit matrix $M$ horizontally to the penultimate gate before the last aggregating gates. The partitioning of garbled circuit needs some extra care to maintain the inputs/outputs integrity, we implement the partitioning with $n/l$ (fan-in / fan-out) ratio. Specifically, the $n/l$ scheme requires that the leftmost input gates are partitioned per garbled logical gate, and after the first
Partitioning Garbled Circuit: we can add the partitioned 
\( C_i \) for circuit

6 Tier of inputs, the intermediate and last tier gates are 
aggregated into one garbled circuit. An example of the 
partitioning the gates is illustrated as in Figure 5.

C. Offline Non-Interactive OT Transfer Protocol

An offline non-interactive OT transfer protocol for the 
partitioned garbled circuit is illustrated in Figure 6.

1 Alice represents the function \( \hat{C} \), as a circuit and garbles 
circuit \( C_i \), \( \hat{C} \) has a total of 2 input wires corresponding 
to \( (x_i, x_j) \).

2 Alice sends over all of the ciphertexts that are generated 
for garbled circuit \( C_i \) to a public repository, such as DLT 
in blockchain, or a public accessible web portal.

3 Through the verification assignment system (not covered 
in this paper), the versifiers Bob, Charlie... commits to 
each of the garbled circuit verification, by exchanging the 
randomly generated number \( x_i^2 \) in the OT commitment 
message OT with Alice. We are employing the OT 
commitment scheme [20] in our design.

4 Alice sends over the corresponding keys for its own inputs 
wire \( x_i^1 \).

5 Alice and Bob, Charlie... in OT oblivious transfer protocol 
introduced in section 7.A.

6 At end of the protocol, Bob, Charlie,... learns the keys 
\( d_i^1 \) for each partitioned gates, they individually starts 
to evaluate the circuit using the keys obtained in the previous 
procedure.

7 In the end, Bob, Charlie,... learns the output of \( C(x_i, x_j) \). 
Alice also learns that Bob, Charlie,... has evaluated the 
results with outputs of \( Y_1, Y_2, \ldots \) if \( Y_i \neq y_i \), Alice decides 
to abort, otherwise, Alice will proceed to Module III for 
OT aggregation.

VI. Module III: OT-aggregator Protocol

The final stage of the zk-Fabric ends up with a OT-aggregator OT protocol which combines all the partitioned 
garbled circuit verification in previous steps into an overall 
verification conclusion, as illustrated in figure 7. In most cases, 
this step involve the garbled circuit \( C_m \) is built with the XOR 
gate in order to save the computational cost. The expression of 
the circuit \( C_m \) is \( Y = C_m(\bigcup_{i=1}^{m-1} Y_i \oplus X_i^m) \). The goal is for both 
prover (Alice) and aggregator (David) agree on the computed 
\( Y = y \), where \( y = f(x) \).

1 At Module II, verifiers post the computed results \( Y_i \) over 
the public repository where Alice and David can fetch 
from the portal.

2 Alice creates a random bits \( x_i^m \), she lets the \( x_i^m = (x_i^1 \oplus x_i^2 \oplus x_i^3 \oplus \ldots) \).

3 Alice garbles the circuit using the garbling algorithm. The 
garbling algorithm outputs \( (C_m, e_m, d_m) \leftarrow gc(C_m, x_i^m) \), 
where \( gc() \) denotes a Yao’s garbled circuit generation 
algorithm, and the \( C_m = \bigcup_{i=1}^{m-1} (Y_i \oplus X_i^m) \). The output 
consists of a garbled circuit \( C_m \) and encoding function 
e_m, and a decoding function d_m.
Figure 5. OT-aggregator protocol

4 Alice executes the deterministic encoding algorithm $Enc()$, which transforms $e_m$ and $x^m_1$ into the garbled input $Y_m = Enc(e_m, x^m_1)$.

5 Alice sends the tuple $(C_m, d_m, Y_m)$ to the public repository, i.e. the DLT in blockchain, or Web portal, where David is able to non-interactively fetch the information.

6 David creates a random bit $x^m_0 \in (0, 1)$.

7 David also executes the deterministic evaluation algorithm $Ev()$, which outputs the $Y = Ev(C_m, x^m_1)$.

8 David sends back the $Y$ to Alice through the public repository.

9 Alice executes the deterministic decoding algorithm $De()$ to compute the final output $y = De(d_m, Y)$, where the $d_m$ denotes the decoding key.

[9.1] - In parallel, David also executes the deterministic decoding algorithm $De()$ to compute the final output $y = De(d_m, Y)$.

10 At both Alice and David’s sites, they will check if $y = f(m \oplus x^m_0)$ which $f()$ is the logical Boolean function before becoming garbled circuit function $C()$. If yes, the OT aggregator protocol will accept the verification results, otherwise, Alice should abort the verification.

VII. Conclusion and Future work

The paper concludes by proposing a novel zero knowledge proof system which can handle more complex semantics. The construction of the zk-Fabric fits pretty well with distributed computing environments, such as Blockchain. In this paper, we have outlined the modules and algorithms on section 3 to section 7 to support the overall functionalities of the zk-Fabric.

Our future research should be focused on the prototyping the zk-Fabric and implement it in a testing environments. We need to measure the computational cost and resource cost comparing with other similar technologies. We should also further develop and provide theoretical analysis of the security properties of the zk-Fabric schemes.