Integral Sliding Mode Control Design for a Quarter-Car Active Suspension System

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Abstract. This paper utilizes the design of a robust integral sliding mode control (ISMC) for a quarter-car active suspension system. The main goal is to increase the ride comfort, whilst the road holding and rattle space remain within the safety bound. According to ISMC, the system state starts at the switch surface where the system nonlinearity, parameter changes, and road disturbances are rejected by a discontinuous control term present strongly in the suspension dynamics. This feature allows the design of a continuous controller, i.e. ideal controller during sliding motion from the first instant. Consequently, the car body is isolated from the wheel unit to perform the desired suspension requirements characteristics. The ISMC shows a high robustness control design of the suspension system, and can suppress chattering in the high-frequency band. The effectiveness of the proposed controller is performed by simulating the 2-DOF quarter car system with controlled and uncontrolled cases. The Matlab 2019a software is used to simulate the suspension models with bump road profiles as excitation disturbance.

Keywords. Integral sliding mode control (ISMC), Quarter car, Active suspension system.

1. Introduction

Every vehicle moving in the road is exposed to wide ranges of vibrations and shocks which are harmful to both passenger’s ride comfort, and the durability of the vehicle itself. The suspension system is one of the important parts in the vehicle as it isolates the vehicle body from the induced vibrations of the road, maintaining a firm contact between the tire and the road, and achieving the stability of the vehicle. As the mechanical and electronic technology expand, the requirements of the passenger comfort and vehicle performance of the modern car are of the highest priority to satisfy the customer expectations. Hence, the design of the appropriate suspension system is always an important research topic for achieving the desired vehicle quality. Strategies used much control for regulating the suspension system. Many of the researchers, some of these are in references [1-5], studied optimal control. In the same context, H∞ control strategies have intensively discussed the active suspension system due to its robustness and disturbance attenuation, [6], [7]. Besides, the adaptive control method was used in the quarter-car suspension system as per references [8-10]. Backstepping control [11], intelligent control method [12], and PID [13] are the other strategies reported in the literature. Sliding mode control is one of the well-known methods used to control the uncertain system affected by unmeasurable disturbance. Sliding mode control (SMC) can guarantee stability for the matched uncertainties and disturbances. In literature, SMC was performed for the quarter car active suspension system with nonlinearities [14]. On the other hand, sliding mode in combination with the model
reference control was another interesting control strategy [15]. Yoshimura et al. [16] examined the SMC control of the active suspension system; they performed the sliding surface by using the linear quadratic control. Lin et al. [17] performed a fuzzy sliding mode controller to improve the active suspension system by applying system error and error change as input variables of sliding surface operation. Its outputs are the sliding surface and the change of sliding surface variables which are then the input variables of a traditional fuzzy controller (TFC) to control the suspension system. Zhang et al. [18] investigated the SMC for two-degree active suspension system by using the optimal control method to achieve the sliding surface. The main objective of the present work is to design and implement an ISMC for the active suspension system that should achieve good ride comfort and satisfy other vehicle requirements. The current integral sliding mode control is an update to the conventional sliding mode. The reaching phase was eliminated by initiating the system state in the sliding surface; therefore, the robustness is performed from the first instant. ISMC keeps the order of the system where the uncertainties and perturbations are rejected from the system model and make the system as an ideal model with known nominal parameters [19]. It can also be used as a perturbation’s estimator, which solves one of the main drawbacks of the sliding mode control—the chattering problem [20]. The analysis is based on the quarter car model with two degrees of freedom taking into account non-smooth nonlinearities, the uncertainty of the system parameters and the disturbances of the road.

2. Mathematical modelling

A two-degree of freedom quarter car model for the active suspension system is shown in Figure (1). The sprung mass is referred to as \((m_s)\) (masses of the car body, passengers, frame and internal component), which may vary with the change in passenger loading conditions. The unsprung mass \((m_{us})\) represents the tire, wheel, brake, and mass of the suspension linkages. The sprung mass is supported by the suspension system components which consist of a spring with linear stiffness \(k_1\), damping coefficient \(c_1\) and an active control element denoted by \(u\). The unsprung mass is modeled as a combination of spring and damper with stiffness and damping coefficients denoted by \(k_2\) and \(c_2\). The vertical road disturbance acting on the wheel unit is represented by \(x_r\). The vertical displacements of the sprung mass and the unsprung mass are represented by \(x_s\) and \(x_{us}\) respectively.

![Figure 1. Quarter car suspension model.](image)

The dynamic equations of the sprung mass and the unsprung mass is shown in Figure (1) are obtained as below:

\[
\begin{align*}
    m_s \ddot{x}_s &= -k_1(x_s - x_{us}) - c_1(\dot{x}_s - \dot{x}_{us}) + u \\
    m_{us} \ddot{x}_{us} &= k_1(x_s - x_{us}) + c_1(\dot{x}_s - \dot{x}_{us}) - k_2(x_{us} - x_r) + c_2(x_{us} - \dot{x}_r) - u
\end{align*}
\]  

(1)

The state variable of the suspension system is defined as:
Therefore, the dynamic equations are rewritten as:

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = (-a_1(x_1 - x_3) - a_2(x_2 - x_4) + a_3 u) \\
\dot{x}_3 = x_4 \\
\dot{x}_4 = b_1(x_1 - x_3) + b_2(x_2 - x_4) - b_3(x_3 - x_r) + b_4(x_4 - x_r) - b_5 u
\]

(3)

Where: \( a_1 = \frac{k_1}{m_s} \), \( a_2 = \frac{c_s}{m_s} \), \( a_3 = \frac{1}{m_s} \), \( b_1 = \frac{k_1}{m_{us}} \), \( b_2 = \frac{c_s}{m_{us}} \), \( b_3 = \frac{k_2}{m_{us}} \), \( b_4 = \frac{c_2}{m_{us}} \), \( b_5 = \frac{1}{m_{us}} \)

For active suspension systems, the performance requirements to be considered in the controller design include three aspects: the first is ride comfort which is closely related to the car body acceleration. To increase the ride comfort, the acceleration, i.e. \( \ddot{x}_2 \) of the sprung mass should be minimized. Furthermore, the maximum value of \( \ddot{x}_2 \) is also evaluated to consider peaks in the body car acceleration signal which is especially distinct for singular disturbance events.

The second is road holding ability; it is related to the ride safety of the car depending on the road disturbance that affects the unsprung mass. A firm uninterrupted contact between the wheel unit and the road plays an important role in the vehicle handling. Therefore, the dynamic wheel unit should be bounded by:

\[
|k_t(x_3 - x_r) + c_t(x_4 - x_r)| \leq (m_s + m_{us})g
\]

(4)

Where; \( g \) denotes the gravitational constant.

The third is the suspension movement limitation; this is due to the restricted structure design in a vehicle suspension system. The suspension distance should not exceed the maximum permissible level. So, it must be restricted with maximum suspension deflection \( x_{max} \):

\[
|x_s - x_{us}| \leq x_{max}
\]

(5)

3. Integral sliding mode control

The main objective of the control design is the decoupling of the sprung system from the unsprung system to prevent the disturbance which affects the wheel unit and the passengers in the car. Therefore the analysis will be focused on the sprung system, i.e.

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = (-a_1(x_1 - x_3) - a_2(x_2 - x_4) + a_3 u)
\]

(6)

The suspension system includes several types of uncertainty and perturbations; they are summarized first by the actual sprung mass which is caused by its variation of time. Thus the vehicle is subjected to uneven road profiles. Indeed, there is no previous information available on this system. Another form of uncertainty arises from the spring and damper coefficients and the dynamic of the wheel unit which unsprung mass position and velocity and the actuator imperfections. The dynamic system of the sprung mass can be rewritten as follows:

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = (-a_1(x_1 - x_3) - a_2(x_2 - x_4) + a_3 u + d(x,t) + \Delta a_1(x_1 - x_3) + \Delta a_2(x_2 - x_4) + \Delta a_3 u)
\]

(7)

Where \( x_i \), \( i = 1 \text{ to } 4 \) \( \in R^n \), \( a_3 \in R^{n+1} \), \( u \in R \), \( d(x,t) \in R \), \( d(x,t) \) unmodeled dynamics and the nonsmoothed nonlinearities in the system and \( \Delta a_1(x_1 - x_3) \), \( \Delta a_2(x_2 - x_4) \) and \( \Delta a_3 u \) represents uncertain parameters of the system dynamic. In the same context, the design control law is:

\[
u = u_n + u_s
\]

(8)
Where the $u_n$ and $u_s$ represent the nominal and discontinuous controls of the system respectively. The nominal control is used to stabilize the nominal system dynamics with the desired characteristics. The dynamics of the nominal system become as:

$$
\dot{x}_1 = x_2 \\
\dot{x}_2 = (a_1(x_1 - x_3) - a_2(x_2 - x_4) + a_3u_n
$$

(9)

The discontinuous control $u_s$ rejects the perturbation terms and the nonlinearities of the dynamic system as in equation (7). After that, the design of the integral sliding mode and the perturbation terms can be formulated as follows:

$$
\dot{x}_1 = x_2 \\
\dot{x}_2 = (a_1(x_1 - x_3) - a_2(x_2 - x_4) + a_3u_n + a_3u_s + \delta(x, u)
$$

(10)

Where $\delta(x, u) = d(x, t) + \Delta a_1(x_1 - x_3) + \Delta a_2(x_2 - x_4) + \Delta a_3u$ is the perturbation term and represents the parameters variations, unmodeled dynamics, non-smooth nonlinearities and external disturbances. It is assumed to be obtain matching condition of the dynamic system.

$$
\delta(x, u) = a_3 \delta(x, u)
$$

(11)

The design procedure of the ISMC begins with the definition of the sliding variable $s(x)$:

$$
s(x) = s_o(x) + z
$$

(12)

Where $s(x), s_o(x)$ and $z \in R^1$. In the same context, the sliding variable contains two parts. The first one $s_o(x)$ will be similarly designed to the conventional sliding mode, i.e. as a linear combination of the system states. The term of the $z$ represents the integral term and is determined as below. From the sliding mode control theory, the derivation from the switching surfaces $s$ and its time derivative should be having an opposite sign in the nearby area of the switching surface $s = 0$, i.e.

$$
\lim_{z \to 0} + \dot{s} < 0 \text{ and } \lim_{z \to 0} - \dot{s} < 0 \text{ or } s \dot{s} < 0
$$

(13)

Elimination of the reaching phase- a distinguishing property of the ISMC- is achieved by choosing the initial condition $z$ is selected same that the initial condition of the sliding mode control $s$ is zero. This means that the dynamic of the system is in sliding surface from the first instantaneity by selecting $z(0) = -s_o(0)$. In this design the $s_o(x)$ is equal to the state variable $x_2$, therefore, $\dot{s}$ is differentiated as below:

$$
\dot{s} = \dot{x}_2 + \dot{z} = a_1(x_1 - x_3) - a_2(x_2 - x_4) + a_3(u_n + u_s) + \delta(x, u) + \dot{z}
$$

(14)

The integral term will be chosen similar to the derivative of the ISMC in i.e.

$$
\dot{z} = a_1(x_1 - x_3) + a_2(x_2 - x_4) - b u_n
$$

(15)

Thus $\dot{s}$ becomes:

$$
\dot{s} = a_3 u_s + \delta(x, u)
$$

(16)

Accordingly, the sliding condition becomes:

$$
\dot{s} s = s (a_3 u_s + \delta(x, u))
$$

(17)

By selecting the $\rho(x)$ as in the conventional sliding mode control

$$
u_s = -\rho(x) \text{ sign}(s)
$$

(18)

Then equation (17) becomes:

$$
\dot{s} s = s (a_3 (-\rho(x) \text{ sign}(s)) + \delta(x, u))
$$

Since $s \text{ sign}(s) = |s|$ then

$$
\dot{s} s = -a_3 \rho(x) |s| + s \delta(x, u))
$$

$$
\dot{s} s = -a_3 \rho(x) |s| + |s| \delta(x, u))
$$

$$
\dot{s} s = |s| \{ -a_3 \rho(x) + |\delta(x, u))\}$$
It is assumed that $a_3 > 0$. The discontinue gain $\rho(x)$ performs the inequality in the equation (19) to ensure the right-hand side of this equation to be a negative value is:

$$\rho(x) > \frac{\left| \delta(x,u) \right|}{a_3} \quad \text{or} \quad \rho(x) = \rho_0 + \frac{\left| \delta(x,u) \right|}{a_3}, \quad \text{where the } \rho_0 > 0 \quad (20)$$

By referring to equation (16), the discontinuity in the right-hand side due to the $u_s$ needs the finite time $T$ to reach to the origin and then the system dynamic becomes in the sliding motion. Based on that, the dynamic system will be achieved by using the equivalent control which is performed as follows: when $s(t) = 0, \forall t \leq T$, [20] also the $\dot{s}(t) = 0$ and with $\delta(x,u)$ satisfy the matching condition in equation (11), the equivalent control can be calculated according to equation (16) as shown:

$$0 = b u_s + \delta(x,u) \Rightarrow [b \ u_s]_{eq} = -\delta(x,u) \quad (21)$$

By substituting the equivalent control of the equation (21) into the control law of the ISMC with perturbations i.e. equation (10), the system is reduced to the nominal form as in equation (9) with the dimension equal to $n$. For this reason, the ISMC is considered as full order sliding mode control because the dimension of equations (9) & (7) are equal. Since the remaining system is only the nominal system as in equation (10), the next step in the design of this control method is the determination of the nominal part of the system:

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= a_1(x_1 - x_3) + a_2(x_2 - x_4) + a_3 u_n
\end{align*} \quad (22)$$

Where $x_1$ & $x_2$ are the displacement and velocity of the sprung mass in the state space, respectively. Therefore, the control design of the nominal system is:

$$u_n = \frac{1}{a_3} \left[ -n_1 x_1 - n_2 x_2 - a_1(x_1 - x_3) - a_2(x_2 - x_4) \right] \quad (23)$$

As the result the nominal system dynamic is:

$$\begin{align*}
\dot{x}_2 &= -n_1 x_1 - n_2 x_2
\end{align*} \quad (24)$$

Where; $n_1$ & $n_2$ are assigned based on the required system characteristic.

### 4. Result and Discussion

In this section, numerical simulations are performed to represent the effectiveness of integral sliding mode control method, which is used to regulate the suspension system. The parameters of the quarter car model are listed in Table (1). In addition, the road disturbance input in this model is a bumping road:

$$x_r = \begin{cases} 
\frac{d_o (1 - \cos(\omega_r t))}{2}, & t \leq 1 \\
0, & t > 1
\end{cases}$$

Where $d_o$ represents the peak amplitude and $\omega_r$, a constant frequency in the disturbance model which depends on the car velocity and the width of the disturbance on the road, which is set as i.e. $d_o = 0.1 \ m$ and $\omega_r = 6.723 \ \text{rad}$. 

| Table 1. The model parameters of the active suspension [21]. |
|-----------------|-----------------|-----------------|
| Parameter | Value | Parameter | Value |
| $m_s$ | 290 kg | $c_s$ | 1000 N m/sec |
| $m_{aus}$ | 59 kg | $k_t$ | 190000 N m |
| $k_s$ | 16812 N m | $c_t$ | 15020 N m/sec |
There are three requirements that fulfill the suspension system that should be explained in the simulation of the closed-loop system: (1) the vertical displacement of the sprung mass should converge to zero. (2) the suspension movement of i.e. \((x_1 - x_2)\) should be less than the maximum allowable suspension limitation \(x_{\text{max}} = 0.2\) m. to ensure the road holding ability of the vehicle. The ratio between unsprung dynamic load to the stable load must be less than one simulation results. Figure (2) presents the response of the sprung mass into two cases, i.e. the response with and without control effects. This indicates that the ISMC active suspension system has a much lower peak value than the passive suspension system. Thus the sprung mass is decoupled from the lower system and isolated from all disturbances applied to the wheel unit.

![Figure 2. Vertical displacement of the car body mass.](image2)

The displacement of the unsprung mass with the applied road excitation is shown in Figure (3). The concept of isolation between the sprung mass and unsprung mass is obviously shown through the behavior of displacement of the unsprung mass which is moved with the excitation at a very close rate along simulation time. This means that the proposed control method, ISMC, forces the unsprung mass to absorb the impact and influences the road excitation. The sprung and unsprung mass behavior give a good indication to prove the robustness of the ISMC method to perform decoupling of the sprung mass form the whole system due to the ability of this method in attenuating the effect of the excitation disturbance in an efficient way.

![Figure 3. Vertical displacement of the unsprung mass.](image3)
Figure (4) Represents the acceleration response of the car body. It can be noticed that the car body acceleration diminishes much faster in the active suspension system as compared to the passive suspension system. This ensures much better ride comfort requirement. To ensure road holding, the suspension performance constraints should be performed. Figure (5) shows the ratio between the dynamic loads of the tire to the stable load. This ratio should be always less than one. From this figure, there are two important things: The first one for the active suspension system, the maximum value of this ratio is $1.5 \times 10^{-4}$ indicating that the dynamic load of the tire is always less than the stable load with the proposed ISMC method. This means that firm uninterrupted contact of the wheel unit to the road is guaranteed. Also, according to Figure (5) the value of the ratio in the active suspension system is always less than the passive suspension system which indicates the effectiveness of the proposed control method to achieve well road holding of the vehicle.

![Figure 4. Acceleration of the sprung mass.](image)

![Figure 5. Ratio of the dynamic load to the stable load.](image)

The limitation in the active suspension system is one of the important requirements in the suspension design which means that the rattle space should be preserved in a compatible with the car suspension design. The mean that the actuator movement should be less than the maximum deflection of the car suspension, which equal to $x_{max}$ in this consideration of the mathematical model. Figure (6) shows that the value of the active suspension is always less than the $x_{max}$, indicating that this requirement can be achieved by using the proposed control method. On the other hand, comparing the suspension deflection of the active and passive system indicates that the active system has the less value along the simulation time. Thus, this proves the advantage of the active system with ISMC over the passive system.
The controller action (control force) used to regulate the suspension system is represented in Figure (7). The maximum actuation force needed to achieve the decoupling of the car body system is about (1700 N). The actuation force behaves as the minus value, i.e. in opposite direction to the road disturbance applied to the wheel unit.

To demonstrate how the isolation process of the car body is performed through the proposed ISMC method, the sliding variable $s$ must be regulated to zero level in finite time. The main feature of the ISMC is that the sliding variable equal to zero from the first instant, i.e. $s = 0, \forall t \geq 0$. Figure (8) shows that the sliding variable begins at zero and its behavior lasts for a long time.
The final test of the proposed ISMC is to plot the time history of the two terms of the sliding variable i.e. \( x_2 \) and \( z \). Figure (9) shows that the integral terms update its value continuously in the opposite direction of the velocity of the sprung mass (\( x_2 \)) indicating that the sliding variable \( s \) equal to zero. It can be concluded that the ISMC uses the dynamic sliding variable to eliminate the uncertainty and disturbance effect in the dynamic system and leaves it to the nominal case.

![Figure 9. Terms of sliding variable versus time.](image)

5. Conclusion
In this paper, the integral sliding mode control method is used to regulate the active suspension system of the quarter car model. The simulation results show the effectiveness of the proposed control method to decouple the sprung mass from the wheel unit to perform a well ride comfort whilst keeping an acceptable limit of the other suspension requirements, i.e. road holding and rattle space. The features of the ISMC is illustrated by designing an example that gives good result as compared with uncontrolled system.

6. References
[1] D A Wilson, R S Sharp and S A Hassan 1986 The Application of Linear Optimal Control Theory to the Design of Active Automotive Suspensions (Veh. Syst. Dyn.) vol 15 no 2 pp 105-118
[2] D Hrovat 1993 Applications of Optimal Control to Advanced Automotive Suspension Design (J. Dyn. Syst. Meas. Control. Trans. ASME) vol 115 no 2B pp 328-342
[3] H Peng, R Strathearn and A G Ulsoy 1997 A Novel Active Suspension Design Technique-Simulation and Experimental Results (American Control Conference (Cat. No.97CH36041)) vol 1 pp 709–713
[4] C Tang and Xiaoxia Zhang 2005 The Research on Control Algorithms of Vehicle Active Suspension System (IEEE International Conference on Vehicular Electronics and Safety) pp 320-325
[5] Y M Sam, M R H A Ghani and N Ahmad 2000 LQR Controller for Active Car Suspension (TENCON Proceedings, Intelligent Systems and Technologies for the New Millennium (Cat. No.00CH37119)) vol 1 pp 441–444
[6] M Yamashita, K Fujimori, K Hayakawa and H Kimura 1994 Application of \( H_\infty \) Control to Active Suspension Systems (Automatica) vol 30 no 11 pp 1717–1729
[7] T Hoang, P Apkarian and H Shigeyuki 2001 Nonlinear \( H_\infty \) Control for an Integrated Suspension System via Parameterized Linear Matrix Inequality Characterizations (IEEE Transactions on Control Systems Technology) vol 9 no. 1 pp 175-185
[8] I Fialho and G J Balas 2002 Road Adaptive Active Suspension Design Using Linear Parameter-Varying Gain-Scheduling (IEEE Transactions on Control Systems Technology) vol 10 no 1 pp 43–54
[9] A Alleyne and J K Hedrick 1995 *Nonlinear Adaptive Control of Active Suspensions* (IEEE Transactions on Control Systems Technology) vol 3 no 1 pp 94–101

[10] R Rajamani and J K Hedrick 1995 *Adaptive Observers for Active Automotive Suspensions: Theory and Experiment* (IEEE Transactions on Control Systems Technology) vol 3 no 1 pp 86-93

[11] N Ishak, R S E R A Othman, A Ahmad, Y M Sam and A A Basari 2009 *An Observer Design of Nonlinear Quarter Car Model for Active Suspension System by Using Backstepping Controller* (5th International Colloquium on Signal Processing & Its Applications) pp 160-165

[12] K Rajeswari and P Lakshmi 2010 *Simulation of Suspension System with Intelligent Active Force Control* (International Conference on Advances in Recent Technologies in Communication and Computing) pp 271-277

[13] S Mouleeswaran 2008 *Development of active suspension system for automobiles using PID Controller* (Conference: International Conference of Mechanical Engineering).

[14] C Kim and P I Ro 1998 *A Sliding Mode Controller for Vehicle Active Suspension Systems with Non-Linearities* (Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering) vol 212 no 2 pp 79–92

[15] M Bhaskara, J P Modak and S Phadke 2003 *Vibration Control of Vehicles Using Model Reference Adaptive Variablestructure Control* (Advances in Vibration Engineering ) vol 2 no 4 pp. 343–361

[16] T Yoshimura, A Kume, M Kurimoto and J Hino 2001 *Construction of an Active Suspension System of a Quarter Car Model Using the Concept of Sliding Mode Control* (J. Sound Vib.) vol 239 no 2 pp 187–199

[17] J Lin, R J Lian, C N Huang and W T Sie 2009 *Enhanced Fuzzy Sliding Mode Controller for Active Suspension Systems* (Mechatronics) vol 19 no 7 pp 1178–1190

[18] B Zhang, G Tang and F Cao 2009 *Optimal Sliding Mode Control for Active Suspension Systems* (International Conference on Networking, Sensing and Control) pp 351–356

[19] Y Shtessel, C Edwards, L Fridman and A Levant 2014 *Sliding Mode Control and Observation* (Springer Nature)

[20] V I Utkin and H C Chang 2002 *Sliding Mode Control on Electro-Mechanical Systems* (Mathematical Problems in Engineering) vol 8 no 4–5 pp 451–473

[21] Y M Sam and K Hudha 2006 *Modelling and Force Tracking Control of Hydraulic Actuator for an Active Suspension System* (1ST IEEE Conference on Industrial Electronics and Applications) pp 1–6