Inner and Outer-Layer Similarity of the Turbulence Intensity Profile over a Realistic Urban Geometry

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Abstract

The similarity of the turbulence intensity profile with the inner-layer (i.e. from the ground to the top of the logarithmic layer) and the outer-layer (i.e. from the top of the inner layer to top of the boundary layer) scalings were examined for an urban boundary layer using numerical simulations. The simulations consider a developing neutral boundary layer over realistic building geometry with and without a slightly upsloping terrain. The computational domain covers an 19.2 km by 4.8 km and extends up to a height of 1 km, and is resolved by 2-m grids. Several turbulence intensity profiles are defined locally in the computational domain. The inner- and outer-layer scalings work well reducing the scatter of the turbulence intensity within the inner- and outer-layers, respectively, regardless of the surface geometry. Although the main scatters among the scaled profiles are attributed to the mismatch of the parts of the layer (i.e. inner or outer) and the scaling parameters, their behaviours can also be explained by introducing a non-dimensional parameter which consists of the ratio of the inner- and outer-layer parameters for length (the boundary-layer height over the roughness length), or velocity (the external free stream velocity over the friction velocity).

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1. Introduction

This study investigated the similarity of turbulence intensity profiles within urban boundary layers over flat or slightly upsloping terrain. We initiated this research to find the effect of roughness on the boundary layer structure. Urban areas possess the largest roughness on the earth’s surface which derives a strong coupling of the surface geometry and the near surface wind properties. Meanwhile, it is still not obvious if such a large roughness gives any essential modification on the entire boundary layer structures.

The vertical distributions of turbulent statistics have been analyzed conventionally relying on the inner- and outer-layer scaling similarity assumptions. The inner layer is the lower part of the boundary layer including the logarithmic layer in which the surface related variables like the friction velocity \( u_* \) and the roughness length \( z_0 \) are candidates of the representative parameters as in the logarithmic profile. The outer layer is on the top of the inner layer and extends to the top of the boundary layer, where the boundary-layer height \( \delta \) and the external wind velocity \( U_e \) are the representative parameters assuming its self-similar structure. Since these inner- and outer-layer parameters can vary independently, the entire profile shape is not determined identically by a single set of the inner- or outer-layer parameters.

The inner- and outer-layer concepts have been used to model the boundary-layer structures. The mean velocity profile within a neutral boundary layer is explained by a linear sum of the inner-scale logarithmic function and the outer-scale wake function irrespective of the surface roughness (e.g. Clauser 1954; Castro 2007). An analogous modeling has been employed for turbulence intensity in a framework of the attached eddy hypothesis over flat surfaces (Perry and Marusic 1995) in which the viscous scaling has been used for the inner length scale (Kunkel and Marusik 2006).

Concerning the urban boundary layer, some numerical and laboratory experiments have confirmed the validity of the outer-layer scaling irrespective of the building geometries under conditions of large \( \delta \) relative to the building height (e.g. Inagaki et al. 2017). Meanwhile, not many studies have evaluated the inner-layer scaling for the turbulence intensity over urban areas. This is partially due to the difficulty to measure \( z_0 \) in urban field observations (Roth 2000), which is an alternative length scale of the inner-layer over very rough surfaces. Another reason is probably a better fitting of the profile shapes obtained with the outer-layer scaling.

For examining the robustness of the inner- and outer-layer similarities in urban boundary layers, we have conducted numerical simulations of a developing boundary layer over realistic urban geometry in a huge computational domain extending more than 19 km along the streamwise direction. Thanks to the large computational domain and the variety of realistic building geometries, several turbulence intensity profiles in wide ranges of the inner and outer-layer scaling parameters were reproduced. Based on this statistical dataset, general description of the turbulence intensity profiles was considered with the inner- and outer-layer scaling, together with the role of roughness. It is noted that the present topographical variation, which is a slightly upsloping terrain, does not induce a wake separation behind. Therefore, the interaction of the hill-induced wake motion with the boundary-layer turbulence is not discussed in this paper.

2. Method

2.1 Inner- and outer-layer scaling parameters

We employed the friction velocity \( u_* \) and the roughness length \( z_0 \) for the inner-layer parameters to scale the turbulence intensity profiles based on the analogy of the logarithmic wind profile. The boundary-layer height \( \delta \) and the external velocity \( U_e \) are used for the outer-layer parameters assuming self-similar characteristics of the outer layer.

The friction velocity is defined as;

\[
u_* = \sqrt{-\frac{\omega \cdot w_x}{s}},
\]

where \(-\omega \cdot w_x\) is the Reynolds stress linearly extrapolated toward the height of the displacement height \( d \) from the inner layer, assuming a local balance between the horizontal pressure gradient
The boundary-layer height $\delta$, which is defined above the displacement height $d$, is calculated as the level at which the turbulent kinetic energy (TKE) equalled 0.001$U_f^2$. This gives almost equivalent value to the level at which $U$ equals 0.99$U_f$, confirmed in Flat case. It is noted that the general definition using 0.99$U_f$ does not properly determine the top of turbulent region since the upward edge of topography induced a modification of streamlines in the upper irrotational region (e.g. Hunt et al. 1988).

The relationship between the inner- and outer-layer parameters are theoretically derived from the equation of mean wind profile. Castro (2007) has demonstrated the validity of the classical mean wind modelling for rough surfaces, which is a linear sum of the logarithmic and the wake profiles,

$$ \frac{U}{u_*} = \frac{1}{\kappa} \ln \frac{z'}{z_0} + \frac{\Pi}{\kappa} W \left( \frac{z'}{\delta} \right), $$

where $W$ is the wake function, and $\Pi$ is the wake parameter which is assigned to be 0.7 following Castro (2007). Since the wake function is generally defined as $W(1) = 2$ at $z' = \delta$, the following relationship is obtained from Eq. (3) as;

$$ \frac{U}{u_*} = \frac{1}{\kappa} \ln \frac{d}{z_0} + \frac{\Pi}{\kappa} W \left( \frac{\delta}{\delta_{Z_0}} \right). $$

This shows the equivalence of the ratio of the inner and outer-layer velocity scales ($U/u_*$) and that of the length scales ($\delta/z_0$) under constant $\kappa$ and $\Pi$.

2.2 Numerical model

The lattice Boltzmann method was used to simulate a developing urban boundary layer under the neutral stratification. This method solves the temporal evolution of the velocity distribution function of pseudo fluid particles in cubical grid cells based on the Boltzmann equation. The collision process of the particles is expressed by the Bhatnagar–Gross–Krook collision model (Zou and He 1996). In the present model, the streaming velocities of particles are discretised to 18 velocities in three dimensions, with one static condition (which is so-called “D3Q19” model, e.g. Qian et al. 1995). A solid boundary condition is simply expressed by the bounce-back scheme (Yin and Zhang 2012). For modelling the sub-grid scale turbulence, the coherent structure Smagorinsky model (Kobayashi 2006) is applied. A further description of this model was provided by Onodera et al. (2013), and the model validation for urban airflow applications was conducted by Ahmad et al. (2017). This model has been used to analyse the turbulent flow characteristics of an urban boundary layer (Ahmad et al. 2017; Inagaki et al. 2017).

3. Numerical settings

A coastal area of Tokyo was chosen as the simulation domain. It extended 19.2 km nearly perpendicular to the coastline ($x$), 4.8 km in the horizontally normal direction ($y$), and 1 km in the vertical direction ($z$). This domain was resolved by homogeneous cubical grids of 2 m in each direction. There was a homogeneous inflow velocity of 10 m s⁻¹ from the coastal side. These numerical conditions were the same as those used in previous studies (Ahmad et al. 2017; Inagaki et al. 2017), except for the ground elevation, which is derived from a digital elevation model of the topography with a 5-m horizontal mesh (Geospatial Information Authority of Japan). Figure 1 shows the elevation map of the bottom-boundary conditions in the simulation domain. The ground level increases almost monotonically in x direction. Major topographical increase starts at $x = 12$ km with a slope of around 0.01. Hereafter, the simulation with the real topography was called “Topo” and that with a flat topography was called “Flat”.

For the statistical processing, the entire domain was decomposed into averaging subdomains whose size is $320 \times 4,800$ m in the x and y directions. The average ground level was defined as the mean level of the topography within the individual subdomains. The model integration was conducted for 4,320 s, of which the last 20 min was used for the present analyses. A previous study (Inagaki et al. 2017) confirmed that these integration and averaging periods were sufficient to obtain robust statistics.

4. Results

4.1 Scaling parameters

The inner and outer-layer scaling parameters are examined for Flat and Topo cases. Figure 2 shows $\delta$ and $u_*$ in the averaging subdomains aligned along $x$. The boundary-layer height $\delta$ is higher in Topo than in Flat case. This is because the topographical variations near the inflow boundary, which are relatively high compared with $\delta$, effectively contribute to the development of $\delta$. The friction velocity $u_*$ is lower in Topo than in Flat case, as well as the surface drag coefficient (not shown). This implies that the present topographical variation is not effective to increase the surface drag. The drag coefficient is actually not simply a function of the roughness but depends on the flow properties like the shape factor (Schlichting 1968; Clauser 1954; Castro 2007). Equation (4) explains that the increase of $\delta$ decreases the drag coefficient $C_D$ (i.e. $U_*/u_* = (0.5C_D)^{-1}$) if $z_0$ is kept constant, or it decreases $u_*$ if both $z_0$ and $U_*$ are kept constant. The validity of Eq. (4) and the invariance of $z_0$ for two simulation cases are evaluated later in this section.

Figure 3 shows the aerodynamic parameters, i.e. $z_0$ and $d$, plotted as in Fig. 2. There is no clear biases in these variables for the different cases with or without the present topographical variation, and also the difference in the flow properties seen in
Fig. 2. This means that the mean wind profiles in the inner layer is solely determined by the surface geometry, which consists of the buildings, in the present simulation cases.

Figure 4 shows the comparison of the ratio of the inner- and outer-layer scaling parameters for velocity $U_f/u^*$ and the length scales $\delta/z_0$. The plots mostly follow the solid line from Eq. (4) which is derived with assuming the universal function of the logarithmic and wake profiles with constant values of $\kappa$ and $\Pi$. The dashed line is derived from a simple extension of the logarithmic profile (Eq. 2), which corresponds to the case of $\Pi = 0$ in Eq. (4). The values of $U_f/u^*$ and $\delta/z_0$ are variable but they still follow the theoretical relationship shown by Eq. (4). These results indicate the robustness of the universal mean wind profile (i.e. Eq. 4) over realistic urban geometry including a gentle upslope as well as the other flat rough and smooth surfaces (e.g. Clauser 1956; Castro 2007), and also the validity of the estimated scaling parameters.

4.2 Inner- and outer-layer similarity of turbulence intensity profiles

Figure 5 shows the vertical distribution of the root mean square of the velocity fluctuations $\sigma_i$ ($i = u, v, w$) calculated in each averaging subdomain (Fig. 1), and further averaged over the areas R2, R4, and R6 indicated in Fig. 1. The turbulence intensity
profiles in Flat and Topo cases correspond to each other in the lower (inner-layer) and upper (outer-layer) parts of each boundary layer by applying the inner- or outer-layer scalings respectively. This result indicates that the scaling similarity theory remained robust even in the case with the upsloping terrain (i.e. comparison of Flat with Topo cases) and also the different boundary-layer heights (i.e. comparison among R2, R4, R6). Meanwhile, a correspondence of the overall profile shapes for Topo and Flat cases is not achieved by either inner- or outer-layer scaling due to a mismatch of the set of scaling parameters and a part of the boundary layer (i.e. inner or outer layer). This means equivalently that the ratios of the scaling parameters such as $u^*$ and $U_f$ or $z_0$ and $\delta$ are not constant, which are seen in Fig. 4. In comparison among the profiles of the inner-layer scaling (Figs. 5a, 5b, and 5c), the non-dimensional boundary layer height in R4 is lower than in R2 and R6, simply because $z_0$ is higher in R4 than the other regions. Meanwhile, the discrepancy of the inner-layer scaled profiles between Flat and Topo are due to the difference of $\delta$. For a better description of the behavior in the outer layer using the inner-layer scaling, a parameter $\delta/z_0$ is introduced to sort the non-dimensional profiles. This parameter can represent the magnitude of not only $\delta$ but also equivalently that of $U_f$ relative to the corresponding inner-layer scales. This parameter is similarly applicable to express the vertical profile with outer-layer scaling.

Figure 6 shows the ensemble mean of the non-dimensional $\sigma_u$ profiles averaged every 100 $\delta/z_0$. Concerning the inner-layer scaling, the upper part of the boundary layer (i.e. outer layer) monotonically extends into higher $z/z_0$ with increasing $\delta/z_0$. The values below $z/z_0 = 50$ are not well ordered because they are within the roughness sublayer. Concerning the outer-layer scaling, a slight decrease of $\sigma_u/U_f$ is visible within the inner layer ($z < 0.4\delta$) with increasing $\delta/z_0$. This can be explained by the same reason as the decrease of $C_f$ for increasing $\delta$, or the small TKE generation for small $z_0$ values. The scatter of the profiles with inner-layer scaling is smaller in the inner layer than those in the outer layer although they are small enough to distinguish each ensemble-averaged value in the both layers.

5. Conclusions

These numerical experiments have shown the robustness of the inner- and outer-layer scaling similarity of the turbulence-intensity profiles within an urban boundary layer over a slightly upsloping terrain. Although the profile shape of the turbulence intensity is not identically determined by either one of the inner- or outer-layer scaling alone, their behaviors can be explained by introducing a parameter of $\delta/z_0$ or $u_f/u_*$. These results are useful for the modeling of the near-surface TKE profile over rough surfaces. The present similarity argument is based on the conventional inner- and outer-layer scaling framework and the universal mean wind profile (i.e. Eq. 4). Therefore, the present discussion is valid for the other rough surfaces without loss of generality although it still needs experimental verification.

Finally, the non-dimensional parameter, $\delta/z_0$, has been used to explain variable turbulence intensities near the ground in outdoor environments, in which $\delta$ is represented by the height of the atmospheric boundary layer (Inagaki and Kanda 2008). Since their examinations were limited in a certain height, the vertical extension of the validity of this parameter should be tested in future works.
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References

Ahmad, H. N., A. Inagaki, M. Kanda, N. Onodera, and T. Aoki, 2017: Large eddy simulation of the gust index in an urban area using the lattice Boltzmann method. Bound.-Layer Meteor., 163, 447–467.

Castro, I. P., 2007: Rough-wall boundary layers: Mean flow universality. J. Fluid Mech., 585, 469–485.

Clauser, F. H., 1954: Turbulent boundary layers in adverse pressure gradients. J. Aeronaut Sci., 21, 91–108.

Hunt, J. C. R., Leibovich, S., and K. J. Richards, 1988: Turbulent shear flows over low hills. Quart. J. Roy. Meteor. Soc., 114, 1435–1470.

Inagaki, A., and M. Kanda, 2008: Turbulent flow similarity over an array of cubes in near-neutrally stratified atmospheric flow. J. Fluid Mech., 615, 101–120.

Inagaki, A., M. Kanda, H. N. Ahmad, A. Yagi, N. Onodera, and T. Aoki, 2017: A numerical study of turbulence statistics and the structure of a spatially-developing boundary layer over a realistic urban geometry. Bound.-Layer Meteor., 164, 161–181.

Kanda, M., A. Inagaki, T. Miyamoto, M. Gryschka, and S. Raasch, 2013: A new aerodynamic parametrization for real urban surfaces. Bound.-Layer Meteor., 148, 357–377.

Kobayashi, H., 2006: Large eddy simulation of magnetohydrodynamic turbulent channel flows with local subgrid-scale model based on coherent structures. Phys. Fluids, 18, 045107.

Kunkel, G., and I. Marusic, 2006: Study of the near-wall-turbulent region of the high-Reynolds-number boundary layer using an atmospheric flow. J. Fluid Mech., 548, 375–402.

Leonardi, S., and I. P. Castro, 2010: Channel flow over large cube roughness: A direct numerical simulation study. J. Fluid Mech., 651, 519–539.

Moriwaki, R., and M. Kanda, 2006: Flux-gradient profiles for momentum and heat over an urban surface. Theor. Appl. Climatol., 84, 127–135.

Onodera, N., T. Aoki, T. Shimokawabe, and H. Kobayashi, 2013: Large-scale LES wind simulation using lattice Boltzmann method for a 10 km × 10 km area in metropolitan Tokyo. Tsukuba E. S. J., 9, 2–8.

Perry, A. E., and I. Marusic, 1995: A wall-wake model for the turbulence structure of boundary layers. Part I. Extension of the attached eddy hypothesis. J. Fluid Mech., 298, 361–388.

Qian, Y. H., S. Succi, and S. A. Orszag, 1995: Recent advances in lattice Boltzmann computing. Annu. Rev. Comp. Phys., 3, 195–242.

Roth, M., 2000: Review of atmospheric turbulence over cities. Quart. J. Roy. Meteor. Soc., 126, 941–990.

Schlichting, H., 1968: Boundary-Layer Theory, 6th Ed. McGraw-Hill, New York, 747 pp.

Yin, X., and J. Zhang, 2012: An improved bounce-back scheme for complex boundary conditions in lattice Boltzmann method. J. Comput. Phys., 231, 4295–4303.

Zou, Q., and X. He, 1996: On pressure and velocity flow boundary conditions and bounceback for the lattice Boltzmann BGK model. arXiv:comp-gas/9611001.

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