Quasi-adiabatic and Stochastic Heating and Particle Acceleration at Quasi-perpendicular Shocks

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Abstract

Based on Magnetospheric Multiscale observations from the Earth’s bow shock, we have identified two plasma heating processes that operate at quasi-perpendicular shocks. Ions are subject to stochastic heating in a process controlled by the heating function $\chi_\parallel = m_j q_j B^{-2} \text{div}(E_i)$ for particles with mass $m_j$ and charge $q_j$ in the electric and magnetic fields $E$ and $B$. Test-particle simulations are employed to identify the parameter ranges for bulk heating and stochastic acceleration of particles in the tail of the distribution function. The simulation results are used to show that ion heating and acceleration in the studied bow shock crossings is accomplished by waves at frequencies $(2–10) f_{cp}$ (proton gyrofrequency) for the bulk heating, and $f > 10 f_{cp}$ for the tail acceleration. When electrons are not in the stochastic heating regime, $|\chi_\parallel| < 1$, they undergo a quasi-adiabatic heating process characterized by the isotropic temperature relation $T_j/B = (T_i/B_0)(B_0/B)^{1/3}$. This is obtained when the energy gain from the conservation of the magnetic moment is redistributed to the parallel energy component through the scattering by waves. The results reported in this paper may be applicable also to particle heating and acceleration at astrophysical shocks.

Unified Astronomy Thesaurus concepts: Shocks (2086); Interplanetary physics (827)

1. Introduction

Electron and ion acceleration and heating at collisionless shocks is an important problem in astrophysics and space physics, which has been addressed over many years by a number of authors (Bell 1978; Lee & Fisk 1982; Goodrich & Scudder 1984; Wu et al. 1984; Blandford & Eichler 1987; Balikhin & Gedalin 1994; Gedalin et al. 1995; Treumann 2009; Burgess et al. 2012; Krasnoselskikh et al. 2013; Mozer & Sundqvist 2013; See et al. 2013; Guo et al. 2014; Wilson III et al. 2014; Park et al. 2015; Cohen et al. 2019; Xu et al. 2020). In the above cited publications one can find a long list of waves, instabilities, and processes that could play a role in ion and electron heating/acceleration; however, the question of the exact heating mechanisms working at shocks is still in an inconclusive state.

New generation of high-resolution space instruments with simultaneous 4-spacecraft measurements on the Magnetospheric Multiscale (MMS) mission (Burch et al. 2016) opened unprecedented possibility for testing heating and acceleration mechanisms that operate at collisionless shocks in reality, and not only in theory. For example, MMS offers the capability of computing gradients of plasma parameters and of electric and magnetic fields on spacecraft separation distances of $\sim 20$ km, equivalent to several electron gyroradii. The quality of the electric field experiment (Erğun et al. 2016; Lindqvist et al. 2016; Torbert et al. 2016) enables even the direct derivation of the divergence of the electric field. Using such state-of-the-art measurements, which will also be discussed in Section 2 of the present paper, Stasiewicz (2020a, 2020b) has identified a chain of cross-field current driven instabilities that operate at both quasi-parallel and quasi-perpendicular shock waves and lead to the heating of ions and electrons. This sequence can be summarized as follows:

**Shock compression of the density $N$ and the magnetic field $B$**

$
\rightarrow \text{diamagnetic current} \rightarrow \text{lower hybrid drift (LHD) instability} \rightarrow \text{electron } E \times B \text{ drift} \rightarrow \text{electron cyclotron drift (ECD) instability} \rightarrow \text{heating: quasi-adiabatic } (\chi_\parallel < 1), \text{ stochastic } (\chi_\parallel > 1).
$

Stochastic heating is a single particle mechanism where large electric field gradients due to space charges destabilize individual particle motions in a magnetic field $B$, rendering the trajectories chaotic in the sense of a positive Lyapunov exponent for initially nearby states. The stochastic heating function of particle species $j$ ($j = e$ for electrons and $p$ for protons) is (Stasiewicz 2020a)

$$
\chi_j(t, r) = \frac{m_j}{q_j B^2} \text{div}(E_i),
$$

where $m_j$ and $q_j$ are the particle mass and charge. The parallel (to the magnetic field) electric field $E_i$ is here excluded since it does not directly contribute to the stochasticity, leaving only the perpendicular field $E_\perp$ in (1). Stochastic heating typically occurs when $|\chi_j| \gtrsim 1$ (Karney 1979; McChesney et al. 1987; Balikhin et al. 1993; Gedalin et al. 1995; Vranjes & Poedts 2010; See et al. 2013; Stasiewicz et al. 2013; Yoon & Bellan 2019) even though resonant heating can also occur for $|\chi_j| < 1$ for wave frequencies very close to cyclotron harmonics (Fukuyama et al. 1977). The value of $\chi_j$ can be regarded as a measure of demagnetization. Particles are magnetized (adiabatic) for $|\chi_j| < 1$, and demagnetized (subject to non-adiabatic heating) for $|\chi_j| \gtrsim 1$.

The value of the proton heating function $\chi_p$ is typically in the range $10–100$ in the bow shock and the magnetosheath, which implies that the ions are strongly demagnetized and can be subject to stochastic heating processes in these regions. In
In order to also demagnetize and stochastically heat electrons we need $\chi_e > 1$ and therefore $\chi_e > m_p/m_e = 1836$, which requires either very strong $E$-gradients or low $B$-fields, or both, as implied by Equation (1). Electron heating at perpendicular shocks based on $\chi_e$ (with the divergence reduced to $\partial E_z/\partial x$) has been referred to as the \textit{kinematic mechanism} (Balikhin et al. 1993; Gedalin et al. 1995; See et al. 2013). The required gradient of the electric field is associated with a macroscopic electric field in the direction normal to the shock. Unfortunately, in perpendicular shocks the observed thickness of the shock ramp and measured values of the normal electric field do not allow $\chi_e > 1$ to be reached, as needed for stochastic heating of electrons with the kinematic scenario. On the other hand, the stochastic heating mechanism has been shown to work with gradients of the electric field provided by the LHD and ECD waves observed in quasi-parallel shocks (Stasiewicz 2020b).

In quasi-perpendicular shocks the derived values of $\chi_e$ are mostly below the stochastic threshold for electrons, because of the increasing values of $B \approx 10$–40 nT in the shock ramp, combined with the scaling $\chi_e \propto B^{-2}$. In Section 3, we demonstrate that such situations instead lead to \textit{quasi-adiabatic electron heating}, characterized by electron heating on the compression of the magnetic field, combined with scattering by waves, leading to the isotropic temperature relation $T/B = (T_0/B_0)(B_0/B)^{1/3}$. This is to our knowledge a novel concept identified and explained for the first time in Section 3.

2. Multiple Crossings and Waves in Shocks

We analyze recent MMS measurements from 2020 January 3 obtained by the 3-axis electric field (Ergun et al. 2016; Lindqvist et al. 2016; Torbert et al. 2016) and magnetic field vectors measured by the Fluxgate Magnetometer (Russell et al. 2016), and the number density, velocity, and temperature of both ions and electrons from the Fast Plasma Investigation (Pollock et al. 2016).

Figure 1 shows multiple crossings of shocks caused by the oscillatory movements of the bow shock with an amplitude of 6–10 km s$^{-1}$ estimated from the time shifts of the density signals. The speed is with respect to the MMS spacecraft moving at 1.9 km s$^{-1}$ earthward. The spacecraft position at time 14:30 was ($13.5$, $10.3$, $-1.8$) $R_E$ GSE (geocentric solar ecliptic) coordinates. Shown in Figure 1 are: the electron number density $N$, the magnetic field $B$, the ion and electron temperatures, and the ratio $T_i/B$ (not to scale). Notably in Figure 1(c), the parallel and perpendicular electron temperatures are almost equal, indicating that an isotropization process takes place.

The ion and electron temperatures indicate rapid heating at the shock ramp, and the repetitive events offer a great opportunity to study heating processes operating at quasi-perpendicular shocks. The ratio $T_i/B$ derived from measurements is an excellent indicator of the heating processes. A flat ratio across the shock would indicate adiabatic perpendicular heating coming from the conservation of the magnetic moment. This should be accompanied by unchanged parallel temperature.

We see that the ion ratio $T_i/B$ has humps and the parallel temperature is smaller, which is indicative for non-adiabatic perpendicular heating and less efficient parallel heating. The electron ratio $T_e/B$ instead has dips and the temperature is nearly isotropic.

![Figure 1: A series of shock crossings caused by the oscillatory movement of the bow shock. Panel (a) shows the electron number density $N$, panel (b) the magnetic field $B$, panel (c) shows $T_i$, $T_e$, and the ratio $T_i/B$ (not to scale), and panel (d) shows the same parameters as (c) but for electrons. Note the different behaviors of the ratio $T_i/B$ for the ions and electrons in the shock ramps, with humps for the ions and dips for the electrons.](image1)

![Figure 2: Complementary plasma parameters for Figure 1: (a) the angle between the magnetic field vector and the radial direction (a proxy to the shock normal direction), (b) the Alfvén Mach number, and (c) plasma $\beta$.](image2)
gradients of the density leads to the LHD instability when the ratio between the scale of the density gradient $L_N = N |\nabla N|^{-1}$ and the proton thermal gyroradius $r_p$ obeys the condition $L_N/r_p < (m_p/m_e)^{1/4}$ (Huba et al. 1978; Drake et al. 1983). The density compression starts at the foot of the shock and is strongly amplified in the ramp, which can be seen in Figure 3 in the profile $f_{th}$, which is proportional to $B$, but also representative for $N$.

In the nonlinear stage, the LHD waves produce large amplitude electric fields resulting in efficient $E \times B$ drifts of the electrons. Due to the large ion gyroradius compared to the wavelength of the LHD waves, the ions do not experience significant $E \times B$, and hence there is a net current set up by the electrons only. When the differential drift velocity between the electrons and ions exceeds the ion thermal velocity, the modified two-stream (MTS) instability can take place resulting in waves at frequencies above $f_{th}$ (Lashmore-Davies & Martin 1973; Gary et al. 1987; Umeda et al. 2014). Below, we will not differentiate between the MTS and LHD instabilities since they belong to the same dispersion surface (Silveira et al. 2002; Yoon & Lui 2004), but we will use the term LHD instability in the sense of a generalized cross-field current driven instability in the lower hybrid frequency range.

When the relative electron-ion drift speed becomes a significant fraction of the electron thermal speed, $V_E = |E \times B|/B^2 \sim v_{Te}$, the ECD instability is initiated, which creates even larger electric fields on spatial scales of $r_e$. (Lashmore-Davies 1971; Forslund et al. 1972; Muschietti & Lembège 2013). The ECD instability takes place near cyclotron resonances ($\omega - k \cdot v_{de} = n \omega_{ce}$), where $\omega_{ce} = eB/m_e$ is the angular electron cyclotron frequency, $k_\perp = 2\pi/\lambda$ is the perpendicular wavenumber, $\lambda$ is the wavelength, and $n$ is an integer (Janhunen et al. 2018). Here, $V_{de} \approx V_E$ is the perpendicular electron drift velocity in the rest frame of the ions. This resonance condition can be written $k_i V_E \approx n \omega_{ce}$ and expressed as $k_i r_e \approx n v_{Te}/V_E$. For $r_e = 1 \text{ km}$ and $n = 1$ their wavelengths are

$$\lambda \approx \frac{2\pi r_e V_E}{n v_{Te}} \approx 6.3 \text{ [km]} \frac{V_E}{V_{Te}}. \quad (2)$$

This means that the ECD waves with $n = 1$ and electric drift velocities $V_E > v_{Te}$ (see Figure 4(b)) have wavelengths large enough to enable correct gradient computations in the calculations of div($E$) by the MMS spacecraft constellation. Contribution of shorter waves with $n > 1$ to the computed $\chi$ may be underestimated by the gradient computation procedure.

The ECD waves resonate/couple with structures created by the LHD instability, $k_i r_e \sim 1$, when $n v_{Te}/V_E = 1$. The $n = 1$ ECD mode can be naturally excited in drift channels created by the LHD instability when $V_E = v_{Te}$. There is smooth transition and co-location of LHD and ECD waves, seen in Figure 3 which is possibly related to the matching condition between these two instabilities.

The scale of the density gradient $L_N$ shown in Figure 4(a) is computed directly from 4-point measurements using the method of Harvey (1998). As a verification, we show also the gradient scale $L_N = B |\nabla B|^{-1}$ for the magnetic field. They coincide in the shock proper, as expected for fast magnetosonic structures.

In the pioneering work on the LHD instability, Krall & Lieber (1971) used an expression for the electron drift in the

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**Figure 3.** Time–frequency spectrogram of the perpendicular electric field for the first shock ramp in Figure 1. Over-plotted are the lower hybrid frequency $f_{th}$, electron temperature (eV), and the electron cyclotron frequency $f_{ce}$. Waves around $f_{th}$ and above are attributed to the LHD and modified two-stream (MTS) instabilities, and for frequencies around $f_{ce}$ and above to the ECD instability. ECD waves can be Doppler downshifted and observed below $f_{ce}$. Note the vertical striations that start from 1 Hz (LHD instability) and go through the MTS and ECD instabilities up to 4 kHz.
form $V_{de} \propto (N^{-1} \partial_i N - B^{-1} \partial_i B + ...)$, which implied that the current due to the $\nabla B$ term from fluid integration would cancel the diamagnetic current due to the $\nabla V$ drift, in case when the gradient scale lengths are the same ($L_N = L_B$) and both gradients point in the same direction. Because these scales are the same at the bow shock, several influential authors (Leons & Gary 1978; Zhou et al. 1983; Wu et al. 1984), and many others, claimed that there would be no LHD instability at the bow shock. This erroneous conclusion affected many researchers afterwards and has led to a 40 yr delay in the identification of the LHD instability as the prevailing ion heating mechanism in compressional shock waves, and as a possible trigger for the ECD instability (Stasiewicz 2020a, 2020b). As a matter of fact, in a homogeneous plasma with a spatially varying magnetic field, the $\nabla B$ drift cancels with other terms due to the gyration of particles in the magnetic field, and therefore does not contribute to macroscopic currents as explained in Section 7.4 of the textbook by Goldston & Rutherford (1995). Thus, the diamagnetic drift current is not canceled by the magnetic gradient drift term, and the LHD instability can be excited at the bow shock.

Figure 4(b) shows the computed $E \times B$ drift speed which is increased and comparable to the electron thermal speed in the ion and electron heating regions in Figure 3. The drift velocity was computed in the frequency range 0–512 Hz, because for frequencies larger than the electron cyclotron frequency the drift approximation is not valid. For comparison we also show the plot of the measured perpendicular ion speed $V_{i\perp}$ in magenta color. It has the same value as the computed $E \times B$ drift speed in the solar wind up to 14:23, but deviates strongly inside the shock. Large difference between the electron drift $V_E$ and the measured perpendicular drift of ions $V_{i\perp}$ would induce sequentially the LHD and ECD instabilities as mentioned in the Introduction. The diagnostic parameters support the interpretation of the waves shown in Figure 3 as caused by the LHD and ECD instabilities, and that these waves are spatially co-located, which is seen as striations extending over the whole frequency spectrum.

Stochastic heating is controlled by the stochastic heating function (1) computed directly from 4-point measurements using the method of Harvey (1998). Goodrich et al. (2018) raised concerns that the axial double probe (ADP) on MMS, which uses rigid axial booms shorter than the wire booms of the spin-plane double probe experiment (SDP), produces a larger amplitude response for short, tens of meter (Debye length) waves such as the IA wave. This instrumental difference may affect the computations of the divergence of the electric field and the resulting value of $\chi$. Therefore, the computations of $\text{div}(E)$ are made in the despun spacecraft coordinates, which separates $E_i$ provided by the ADP, from ($E_e$, $E_c$) provided by the SDP. This enables removal of the highest-frequency components before computing the gradients, and $\chi$, shown in Figure 4(c) is computed for the frequency range 0.25–512 Hz. Frequencies below 0.25 Hz are removed to avoid spurious effects at the spin frequency and its harmonics. Not using the ADP $E_i$ component at all produces $\chi$ ca 20% smaller.

The computed $\chi_{de}$ shown in Figure 4(c) indicates that the ions are demagnetized and likely to undergo stochastic heating as seen in detail in Figure 3. On the other hand the value of $\chi_{pe} \sim 100$ corresponds to $\chi_e \sim 0.06$, which is too small to demagnetize the electrons and subject them to stochastic heating. The computed contribution to $\chi_e$ from short $n > 1$ ECD waves (Equation (2)) is underestimated; however, even a possible correction would still keep it below the stochasticity threshold. Other errors in the derivation of $\chi_e$ are the same as in measurements of the electric field, i.e., 1 mV m$^{-1}$, or ca 10% for large amplitude fields (Lindqvist et al. 2016).

Figure 3 is representative for all nine shock crossings shown in Figure 1. It shows that the ion heating is observed at the foot of the shock, earlier than the electron heating, and correlates well with the power of LHD waves. Electron temperature correlates well with the increased wave activity in the LHD/ ECD frequency range and maximizes in the region of the most intense ECD waves around 14:24. On the other hand, it appears to correlate also with the magnetic field strength, represented here by the electron gyrofrequency $f_{ce} = \omega_{ce}/2\pi$, which suggests that an adiabatic behavior $T_{e\perp} \propto B$ should be also considered here. However, this apparent correlation is not exact, as seen in the $T_{e\perp}/B$ ratio shown in Figure 1, with humps for the ions and dips for the electrons at the shock ramps. This will be explained in the next section as quasi-adiabatic electron heating involving the compression of the magnetic field combined with isotropization by the scattering on waves.

3. Quasi-adiabatic Electron Heating

The computed value of the heating function (1) shown in Figure 4(c) indicates that the stochastic heating may not be available for electrons in the analyzed shock crossings. The behavior of the ratio $T_{e\perp}/B$ and the isotropy of the electron temperature, discussed in Section 2, suggests a different kind of heating process. Let us assume that the electrons obtain perpendicular energy from the conservation of the magnetic moment (they are magnetized, consistent with $\chi_e < 1$), but the
energy gain is redistributed to the parallel component through the scattering by waves.

When the magnetic moment is conserved, i.e., $T_i / B = \text{const}$, the differential temperature increase is $dT_i / B = T_i B^{-1} dB$. If the energy gain from 2 degrees of freedom ($2dT_i$) is redistributed by pitch angle scattering to 3 degrees of freedom ($3dT$) the conservation of energy implies

$$3dT = 2TB^{-1}dB$$

for $T = T_i$. This can be easily integrated to give

$$\frac{T}{B} = \frac{T_0}{B_0} \left( \frac{B_0}{B} \right)^{1/3},$$

which predicts a dip of $T/B$ where $B$ has a maximum.

Figure 5 shows a detailed comparison of Equation (4) with the measured ratio for the second shock structures of Figure 1. The model is in excellent agreement with measurements, which supports the validity of the above described type of non-adiabatic heating, henceforth referred to as quasi-adiabatic heating. All shock crossing in Figure 1 show similar signatures of electron quasi-adiabatic heating with $\chi_e < 1$. The agreement between the two curves in Figure 5 indicates an outstanding quality of the particle measurements by the FPI instrument (Pollock et al. 2016), which is able to reproduce the subtle effects of Equation (4) at sharp gradients of the shock ramps seen in Figure 1.

It comes as a surprise from this analysis that the strong ECD waves (Figure 3) with electric field amplitudes of $\sim 150$ mV m$^{-1}$ and short wavelengths of $\sim r_e$ do not directly heat electrons. Such expectations have also been expressed by Mozer & Sundqvist (2013), who noted that the wave potential of the ECD waves significantly exceeds the thermal energy of the electrons, so that some amount of heating would be anticipated. The electron reluctance to stochastic heating appears to be related to the dependence $\chi_e \propto B^{-2}$, and high values of $B$ in the shock ramp, which keeps $\chi_e < 1$.

While the oblique electrostatic electric fields of waves $\sim 100–4000$ Hz do not demagnetize the electrons, they appear to participate in the redistribution of the perpendicular kinetic energy of electrons into the parallel direction. Simulation results with parallel electric fields (Stasiewicz & Eliasson 2020) indicate that these waves lead to the isotropization of the electron temperature, as seen in the nine bow shock crossings in Figure 1(c).

### 4. Simulations of Non-adiabatic Stochastic Heating

For a possible stochastic heating of particle species of mass $m$, charge $q$ we have available wave frequencies from dc to $4096$ Hz shown in Figure 3, and spatial scales ranging from above $\sim 1000$ km for magnetosonic waves to below $r_e \sim 1$ km for ECD waves. To find out which wave frequencies and spatial scales contribute most to the heating we consider an idealized model in which the magnetic field $B_0$ is in the $z$ direction, and a macroscopic convection electric field $E_{0y}$ drives particles into an electrostatic wave with amplitude $E_{0x}$ propagating in the $x$-direction. We keep the magnetic field constant to separate purely stochastic heating from the quasi-adiabatic heating discussed above. Thus, in the Doppler frame of the satellite, the drifting plasma is characterized by the convecting electric field and the time-dependent wave electric field. The governing equations are

$$m \frac{dv_x}{dt} = qE_{0x} \cos(k_x x - \omega t) + qv_y B_0,$$

$$m \frac{dv_y}{dt} = qE_{0y} - qv_x B_0,$$

$$\frac{dx}{dt} = v_x, \quad \frac{dy}{dt} = v_y.$$

We consider only the 2D plane perpendicular to the magnetic field, since in the absence of parallel electric fields the particles simply stream unperturbed along the magnetic field lines.

The system (5)–(7) can be transformed to two different but equivalent forms, one in which we have a time-independent electrostatic wave and a modified convection electric field, and one in which the convective field is eliminated and we have the un-shifted frequency in the plasma frame. From a change of frame into that of the phase velocity of the wave,

$$v_x = \tilde{V}_x + \frac{\omega}{k_x}, \quad x = \tilde{X} + \frac{\omega}{k_x} t$$

we obtain the system

$$m \frac{d\tilde{V}_x}{dt} = qE_{0x} \cos(k_x \tilde{X}) + qv_y B_0,$$

$$m \frac{d\tilde{V}_y}{dt} = qE_{0y} - q\tilde{V}_x B_0,$$

$$\frac{d\tilde{X}}{dt} = \tilde{V}_x, \quad \frac{dy}{dt} = v_y,$$

where the shifted convection electric field is

$$\tilde{E}_{0y} = E_{0y} - \frac{\omega}{k_x} B_0.$$

In this frame, the electric field is time-independent and governed by the electrostatic potential

$$\Phi(\tilde{X}, y) = -\frac{E_{0x}}{k_x} \sin(k_x \tilde{X}) - \tilde{E}_{0y} y.$$

On the other hand, by a change of frame into that of the $E \times B$-drift velocity,

$$v_x = V_x + \frac{E_{0y}}{B_0}, \quad x = X + \frac{E_{0y}}{B_0} t$$

where the shifted convection electric field is

$$\tilde{E}_{0y} = E_{0y}.$$
we obtain instead
\[
\frac{dV_y}{dt} = qE_{0y} \cos(k_x x - \omega t) + qv_y B_0, \tag{15}
\]
\[
\frac{dV_x}{dt} = -qV_y B_0, \tag{16}
\]
\[
\frac{dx}{dt} = V_x, \quad \frac{dy}{dt} = v_y, \tag{17}
\]
where the convecting electric field has been eliminated and absorbed into the frequency in the plasma frame
\[
\bar{\omega} = \omega - k_x E_{0y} B_0. \tag{18}
\]
These two different approaches indicate that the model of waves in the plasma frame with the wave frequency (18) that absorbs the convection field is equivalent to the static wave structures superposed with the convection (Stasiewicz 2007).

Without loss of generality we choose to simulate the system (15)–(17). A suitable normalization of variables (Fukuyama et al. 1977; Karney 1979; McChesney et al. 1987) with time normalized by \(\omega_c^{-1}\), space by \(k_x^{-1}\), and velocity by \(\omega_c / k_x\) with \(\omega_c = qB_0 / m\) being the angular cyclotron frequency, gives the system of dimensionless, primed variables,
\[
\frac{dv_x'}{dt'} = \chi \cos(x' - \Omega t') + v_y', \tag{19}
\]
\[
\frac{dv_y'}{dt'} = -v_x', \tag{20}
\]
\[
\frac{dx'}{dt'} = v_x', \quad \frac{dy'}{dt'} = v_y', \tag{21}
\]
in which there are only two parameters, the normalized wave frequency in the plasma frame,
\[
\bar{\omega} = \frac{\bar{\omega}}{\omega_c}, \tag{22}
\]
and the stochastic heating parameter, equivalent to (1),
\[
\chi = \frac{mk_x E_{0y}}{qB_0^2} = \frac{k_x E_{0y}}{\omega_c B_0}, \tag{23}
\]
which represents the normalized wave amplitude. An important third parameter is the initial velocity of the particles, since stochastic motion takes place only in restricted regions in phase space (Fukuyama et al. 1977; Karney 1979; McChesney et al. 1987). For a statistical description of the particles, the initial condition can be described by a Maxwellian distribution function
\[
F = \frac{N}{2\pi \nu_{0y}^2} \exp \left( - \frac{(v_{x0}^2 + v_{y0}^2)}{2 \nu_{0y}^2} \right), \tag{24}
\]
where \(\nu_{0y}(= (T_0/m)^{1/2})\) is the initial thermal speed and \(T_0\) is the initial temperature. In the normalized variables with \(F = N(k_x / \omega_c)^2 F'\) it is written
\[
F' = \frac{1}{2\pi \nu_{0y}^2} \exp \left( - \frac{(v_{x0}'^2 + v_{y0}'^2)}{2 \nu_{0y}^2} \right), \tag{25}
\]
where \(v_{0y}' = k_x r_x\) is the normalized thermal speed and \(r_x = v_T / \omega_c\) is the thermal Larmor radius. The value of \(v_{0y}'\) determines the initial temperature in the velocity distribution function, which due to the normalization, is in fact proportional to the ratio of the gyroradius to the wavelength \(\lambda = 2\pi / k_x\).

We carry out a set of test-particle simulations for \(M = 10,000\) particles, which are Maxwell distributed in velocity and uniformly distributed in space. The system (19)–(21) is advanced in time using a Störmer–Verlet scheme (Press et al. 2007). The input variables for the simulations are: the normalized wave frequency \(\Omega\) in the range \(10^{-2} - 10^3\), and the initial normalized thermal velocity \(v_{0y}' = k_x r_x\) spanning \(10^{-2} - 10^3\). The normalized amplitude of the electrostatic wave is set to \(\chi = 60\), consistent with the observations in Figure 4(c). The simulation is run for a relatively short time of three cyclotron periods of the particles, motivated by the observations of rapid ion heating within a few cyclotron periods, see Figure 3. The kinetic temperatures resulting from the stochastic heating is calculated as
\[
T = \frac{1}{2M} \sum_{k=1}^{M} mv_{k}^2 + v_{y,k}^2. \tag{26}
\]

Simulations are carried out for different values of \(\Omega\) and \(v_{0y}'\) to produce the color plot in Figure 6, which shows the difference \(T' - T_0\) between the normalized kinetic temperature \(T' = k_x^2 T / m \omega_c^2\) at the end of the simulation and the initial value \(T_0 = (v_{0y})^2 = k_x^2 T_0 / m \omega_c^2\).

The most interesting regions are the ones with red color, representing a temperature increase of \((20 - 1000) m \omega_c^2 / k_x^2\). The frequency region \(1 \leq \Omega \leq 10\) represents bulk heating where the cold population is significantly heated to a temperature of \((20 - 1000) m \omega_c^2 / k_x^2\). The bulk heating region would expand to higher values of \(\Omega\) for larger \(\chi\) as well as to lower \(\Omega\) for longer times. The inset plots for \(\Omega = 3.3\) and initial normalized thermal speeds \(v_{0y}' = 10^2\) and 1 show that the particles are bulk heated and spread almost uniformly in velocity space up to a maximum speed \((50 \omega_c / k_x\), and the distributions achieve a kinetic temperature of \((10^3 m \omega_c^2 / k_x^2\) after three cyclotron periods. This is relevant for the heating of protons by the low-frequency waves observed in Figure 3. Somewhat similar cases but for \(\Omega < 1\) and \(\chi \sim 1\) leading to rapid heating of ions by drift waves were studied by McChesney et al. (1987). For \(\Omega = 10\) there is also bulk heating but with a modest increase to about \(2 m \omega_c^2 / k_x^2\) after three cyclotron periods.

On the other hand, for \(\Omega\) significantly larger than 10 only particles with a high enough initial thermal velocity comparable to the phase velocity, or \(v_{0y}' \sim \Omega\) in the normalized variables (dashed line in Figure 6) are further accelerated, leading a warm component with extended energy tails in the distribution function. Such cases of ion heating by lower hybrid waves were discussed by Karney (1979) and for frequencies near cyclotron harmonics by Fukuyama et al. (1977). For \(v_{0y}' = 10\) and \(\Omega = 20\) the normalized temperature increases by a factor of two within three cyclotron periods, which may be relevant for waves below the lower hybrid frequency seen in Figure 3. Below a threshold initial temperature, the distribution is not affected by the wave, and there is a gap in the heating for low initial temperatures, seen in the lower-right, blue-colored region of Figure 6 including the phase space plots for \(v_{0y}' = 10^2\) and \(\Omega = 20\). In this region, the particles oscillate in the wave field without being heated. Finally, for \(\Omega \ll 1\) the particles only perform oscillations in an almost time-
independent wave electric field, leading to phase-mixing of particles but not to significant stochastic heating.

5. Discussion

With the simulation results shown in Section 4 we are now in the position to assess which of the broad spectrum of waves in Figure 3 are likely to provide stochastic heating of protons at the bow shock.

Figure 7 shows the function $\chi_p$ from Figure 4(c) decomposed into discrete frequency dyads with orthogonal wavelets (Mallat 1999). The signal is divided into discrete frequency layers (dyads) that form $\frac{2\pi}{f_N}$ hierarchy starting from the Nyquist frequency ($f_N$ is half of the sampling frequency). Orthogonality means that the time integral of the product of any pair of the frequency dyads is zero, and the decomposition is exact, i.e., the sum of all components gives the original signal. The y-labels are dyad numbers with the unit amplitude corresponding to $\chi_p = 70$. We see that $\chi_p$ in the frequency channels from 1 Hz and above have sufficient amplitude, and correlate well with ion heating seen in Figure 3 in the time interval 14:23–14:24. On the other hand, in Figure 6 we see that bulk heating occurs for $f \approx (1–10) f_{cp}$, while the stochastic acceleration of suprathermal particles can be done by waves $f > 10 f_{cp}$.

Full kinetic simulations of the LHD instability (Daughton 2003) show that the instability develops at longer wavelengths,

$$k_i (r_e r_p)^{1/2} \approx 1,$$

which is equivalent to $k_i r_p \approx 10$ in our case, and has lower frequencies $f_{cp} < f < 15 f_{cp}$, with significant magnetic component (Gary 1993; Daughton 2003; Huang et al. 2009). This puts these waves in the bulk heating region of Figure 6. They
could be Doppler upshifted by 5 Hz, so the possible frequency range for waves that could heat bulk protons is most likely 1–8 Hz in the satellite frame of Figures 3 and 7. Please note that \( k_x \approx k_{c, r_e} \) throughout this paper.

It is the result of the simulations, that the LHD waves at lower frequencies \( f_{cp} < f < 10f_{cp} \) and longer wavelengths \( k_{c, r_e} \approx 30 \) are found here to be responsible for the bulk proton heating. Incidentally, they also appear to play a key role in the heating of plasma in the magnetotail and at the magnetopause (Zhou et al. 2014; Ergun et al. 2019; Graham et al. 2019).

Waves at dyads 16 Hz and above in Figure 7 may correspond to shorter LHD wavelengths, \( k_{c, r_e} \approx 1 \) and frequencies just below \( f_{in} \) (Davidson et al. 1977; Drake et al. 1983), which are Doppler upshifted to higher frequencies, and/or to the MTS instability which could be triggered by the LHD instability when the electron drift velocity exceeds the ion thermal velocity (Lashmore-Davies & Martin 1973; Gary et al. 1987; Umeda et al. 2014). This means that they may have \( \Omega \approx f_{in}/f \approx 40 \) and \( k_{c, r_e} \approx 90 \) in Figure 6. They can be associated with the heating of suprathermal particles in the region \( \Omega > 10 \), around the dashed line \( k_{c, r_e} \approx \Omega \) in Figure 6.

6. Conclusions

This research has shown that there are two major heating mechanisms in quasi-perpendicular shocks, as implied from the analysis of nine crossings of the bow shock by the MMS spacecraft.

In this particular event, the electrons do not reach the stochastic heating level with \( |\chi_\Omega| < 1 \) and are heated by a quasi-adiabatic process related to the compression of the magnetic field at the shock ramp and simultaneous isotropization by LHD/ECM waves excited by the density compression. The quasi-adiabatic heating process is supported by the observed isotropic temperature relation \( T_B/T_e = (T_0/B_0)/(B_0/B) \) which predicts a dip in the electron temperature-to-magnetic field ratio when the magnetic field increases.

The ions instead undergo rapid non-adiabatic stochastic heating by electric field gradients perpendicular to the magnetic field \( (\chi_\Omega \approx 60) \), and their \( T_B/T_e \) ratio instead shows an increase in the shock region. Test-particle simulations show that efficient stochastic heating within a few cyclotron periods takes place for a range of parameters in space \( (\Omega, \chi_0, \chi) \) where \( \Omega = \omega/\omega_c \) is the wave frequency in plasma frame normalized by the cyclotron frequency, \( \nu_0 = k_{c, r_e} \) is the normalized thermal speed proportional to the ratio between the Larmor radius to the wavelength, and \( \chi \approx (k_{c, r_e})(B_0/B) \) is the stochasticity parameter representing the normalized wave amplitude. We have identified in this space the range of the ion bulk heating and the range for acceleration of suprathermal ion tails (Figure 6).

It is found that in the analyzed cases the ion bulk heating is most likely accomplished by waves in the frequency range 1–8 Hz in the spacecraft frame, or \((2–10)f_{cp}\) in the plasma frame, with \( k_{c, r_e} \lesssim 30 \), i.e., with \( \lambda > 20 \) km. Waves at frequencies larger than 8 Hz in the spacecraft frame \((f > 10f_{cp}\) in plasma frame) and with shorter wavelengths can provide acceleration of the tail of the ion distribution function.

The agreement between the theoretical and numerical results with the MMS observations gives a more complete picture of the heating processes involved in the Earth’s quasi-perpendicular bow shock.

The chain of the physical processes described in this paper is initiated by a single event, namely, the compression of the plasma density \( N \) and the magnetic field \( B \). The induced diamagnetic current triggers consecutively three cross-field current driven instabilities: LHD \( \rightarrow \) MTS \( \rightarrow \) ECD, which produce stochastic heating of ions and electrons, in addition to a common quasi-adiabatic heating of electrons on compressions of \( B \). Thus, the presented model has universal applicability, and the processes described could occur in other types of collisionless shock waves in space, associated with the density compression. The results are directly applicable also to theories and models of particle heating and acceleration in astrophysical shocks.

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Software: Data analysis was performed using the IRFU-Matlab analysis package available at https://github.com/irfu/irfu-matlab.

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