Broken Supersymmetric U(1) Gauge Factor
at the TeV Scale

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Abstract

The appearance of a broken supersymmetric U(1) gauge factor at the TeV scale is relevant for several reasons. If it truly exists, then one important consequence is that at the 100 GeV energy scale, the two-doublet Higgs structure is of a more general form than that of the Minimal Supersymmetric Standard Model (MSSM). This is a prime example of tree-level nondecoupling. Furthermore, a particular $U(1)_N$ from the superstring-inspired $E_6$ model allows for the existence of naturally light singlet neutrinos which may be necessary to accommodate the totality of neutrino-oscillation experiments.

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1 Why Supersymmetric U(1)?

Consider the top-down approach. Start with the $E_8 \times E_8$ heterotic string, which compactifies to the $E_6$ superstring, which is then broken by flux loops transforming as the adjoint $78$ representation. Under $SU(3)_C \times SU(3)_L \times SU(3)_R$,

$$78 = (3, 3, 3^*) + (3^*, 3^*, 3) + (8, 1, 1) + (1, 8, 1) + (1, 1, 8).$$

Now $(1,8,1)$ can be used for $SU(3)_L \to SU(2)_L \times U(1)_{Y_L}$ and $(1,1,8)$ for $SU(3)_R \to U(1)_{T^3_R+Y_R}$ so that the correct $Q = T^3_L + Y_L + T^3_R + Y_R$ is obtained, hence $U(1)_{Y_L} \times U(1)_{T^3_R+Y_R}$ remains unbroken and can be rewritten as $U(1)_Y \times U(1)_\eta$. The extra $U(1)_\eta$ presumably survives down to the TeV energy scale where it is broken together with the supersymmetry.

Consider also the bottom-up approach. Start with a possible experimental clue, such as the $R_b \equiv (Z \to b\bar{b}/Z \to \text{hadrons})$ excess. Look for an U(1) explanation, and remarkably $U(1)_\eta$ is also found. Another possible clue is the totality of neutrino-oscillation experiments (solar, atmospheric, and laboratory) which suggest that there are at least 4 neutrinos. Details of how this is related to an extra U(1) will be presented later.

2 Tree-Level Nondecoupling at the 100 GeV Scale

As the U(1) gauge factor is broken together with the supersymmetry at the TeV scale, the resulting heavy scalar particles have nondecoupling contributions to the interactions of the light scalar particles. Consequently, the two-doublet Higgs structure is of a more general form than that of the MSSM. Previous specific examples have been given. Here I present the most general analysis. Let

$$\tilde{\Phi}_1 = \left( \begin{array}{c} \tilde{\phi}^0_1 \\ -\tilde{\phi}^-_1 \end{array} \right) \sim (1, 2, -1; -a),$$

(2)
\[
\Phi_2 = \begin{pmatrix}
\phi_2^+ \\
\phi_2^0
\end{pmatrix} \sim (1, 2, \frac{1}{2}; -1 + a),
\]
(3)

\[
\chi = \chi^0 \sim (1, 1, 0; 1),
\]
(4)

where each last entry is the arbitrary assignment of that scalar multiplet under the extra 
U(1), assuming of course that the superpotential has the term \( f\Phi_1^\dagger \Phi_2 \chi \). The corresponding 
scalar potential contains thus

\[
V_F = f^2[(\Phi_1^\dagger \Phi_2)\Phi_2^\dagger \Phi_1 + (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)\chi \chi],
\]
(5)

and from the gauge interactions,

\[
V_D = \frac{1}{8}g_2^2[(\Phi_1^\dagger \Phi_1)^2 + (\Phi_2^\dagger \Phi_2)^2 + 2(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) - 4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)]
+ \frac{1}{8}g_1^2[\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2]^2
+ \frac{1}{2}g_x^2[-a\Phi_1^\dagger \Phi_1 - (1 - a)\Phi_2^\dagger \Phi_2 + \bar{\chi} \chi]^2.
\]
(6)

Let \( \langle \chi \rangle = u \), then \( \sqrt{2} Re \chi \) is a physical scalar boson with \( m^2 = 2g_2^2u^2 \), and the \((\Phi_1^\dagger \Phi_1)\sqrt{2} Re \chi \) 
coupling is \( \sqrt{2}u(f^2 - g_x^2a) \). Hence the effective \((\Phi_1^\dagger \Phi_1)^2\) coupling \( \lambda_1 \) is given by

\[
\lambda_1 = \frac{1}{4}(g_1^2 + g_2^2) + g_x^2a^2 - \frac{2(f^2 - g_x^2a)^2}{2g_x^2}
+ \frac{1}{4}(g_1^2 + g_2^2) + 2af^2 - \frac{f^4}{g_x^2}.
\]
(7)

Similarly,

\[
\lambda_2 = \frac{1}{4}(g_1^2 + g_2^2) + 2(1 - a)f^2 - \frac{f^4}{g_x^2},
\]
(8)

\[
\lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2 + f^2 - \frac{f^4}{g_x^2},
\]
(9)

\[
\lambda_4 = -\frac{1}{2}g_2^2 + f^2,
\]
(10)

where the two-doublet Higgs potential has the generic form

\[
V = m_1^2\Phi_1^\dagger \Phi_1 + m_2^2\Phi_2^\dagger \Phi_2 + m_{12}(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{1}{2}\lambda_1(\Phi_1^\dagger \Phi_1)^2
+ \frac{1}{2}\lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1).
\]
(11)
From Eqs. (7) to (10), it is clear that the MSSM is recovered in the limit \( f = 0 \). Let \( \langle \phi_{1,2}^0 \rangle \equiv v_{1,2}, \tan \beta \equiv v_2/v_1, \) and \( v^2 \equiv v_1^2 + v_2^2 \), then this \( V \) has an upper bound on the lighter of the two neutral scalar bosons given by

\[
(m_h^2)_{\text{max}} = 2v^2[\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + 2(\lambda_3 + \lambda_4) \sin^2 \beta \cos^2 \beta] + \epsilon, \tag{12}
\]

where we have added the radiative correction due to the \( t \) quark and its supersymmetric scalar partners, i.e.

\[
\epsilon = \frac{3g_2^2m_t^4}{8\pi^2 M_W^2} \ln \left( 1 + \frac{\tilde{m}^2}{m_t^2} \right). \tag{13}
\]

Using Eqs. (7) to (10), we obtain

\[
(m_h^2)_{\text{max}} = M_Z^2 \cos^2 2\beta + \epsilon + \frac{f^2}{\sqrt{2}G_F} \left[ A - \frac{f^2}{g_x^2} \right], \tag{14}
\]

where

\[
A = \frac{3}{2} + (2a - 1) \cos 2\beta - \frac{1}{2} \cos^2 2\beta. \tag{15}
\]

If \( A > 0 \), then the MSSM bound can be exceeded. However, \( f^2 \) is still constrained from the requirement that \( V \) be bounded from below. For a given \( g_x \), we can vary \( a, \cos 2\beta, \) and \( f \) to find the largest numerical value of \( m_h \), which grows with \( g_x \). In a typical model such as the \( U(1)_q \) model, \( g_x^2 = (25/36)g_1^2 \simeq 0.09 \) and \( a = 1/5 \), for which \( m_h < 142 \text{ GeV} \). If we increase \( g_x^2 \) to 0.5, and allow all other parameters to vary, then we get \( m_h < 190 \text{ GeV} \).

### 3 Framework for a Naturally Light Singlet Neutrino

There are at present a number of neutrino experiments with data\(^{[7,8,9]}\) which can be interpreted as being due to neutrino oscillations. Solar data\(^{[7]}\) indicate the oscillation of neutrinos differing in the square of their masses of the order \( \Delta m^2 \sim 10^{-5} \text{ eV}^2 \) for the matter-enhanced solution or \( \Delta m^2 \sim 10^{-10} \text{ eV}^2 \) for the vacuum solution. Atmospheric data\(^{[8]}\)
indicate possible oscillation of $\Delta m^2 \sim 10^{-2}$ eV$^2$. More recently, the liquid scintillator neutrino detector (LSND) experiment has obtained results\[9] which indicate possible oscillation of $\Delta m^2 \sim 1$ eV$^2$.

To accommodate all the above data as being due to neutrino oscillations, it is clear that four neutrinos are needed to have three unequal mass differences. Since the invisible width of the $Z$ boson is already saturated with the three known doublet neutrinos $\nu_e$, $\nu_\mu$, and $\nu_\tau$, i.e. from $Z \to \nu \bar{\nu}$, one must then have a fourth neutrino which does not couple to the $Z$ boson, i.e. a singlet. The question is why such a singlet neutrino should be light.

The lightness of doublet neutrinos is canonized by the seesaw mechanism:

$$\mathcal{M}_{\nu,N} = \begin{bmatrix} 0 & m_D & m_D \\ m_D & m_N & m_N \end{bmatrix} \Rightarrow m_\nu \sim \frac{m_D^2}{m_N}.$$

To have a light singlet $S$, assume an extra U(1) gauge factor as well as 2 doublets $(\nu_E, E)_L$ and $(E^c, N^c_E)_L$ such that

$$\mathcal{M}_{\nu_E, N^c_E, S} = \begin{bmatrix} 0 & m_E & m_1 \\ m_E & 0 & m_2 \\ m_1 & m_2 & 0 \end{bmatrix} \Rightarrow m_S \sim \frac{2m_1 m_2}{m_E}.$$

It is thus desirable to have an extra U(1) under which $N$ is trivial but $S$ is not. The two sectors must also be connected so that oscillations may occur between $\nu$ and $S$. If we now assume the Higgs scalars of this theory to carry the quantum numbers of $(\nu_E, E)$, $(E^c, N^c_E)$, and $S$, then the combined mass matrix is given by

$$\mathcal{M} = \begin{bmatrix} 0 & m_D & 0 & m_3 & 0 \\ m_D & m_N & 0 & 0 & 0 \\ 0 & 0 & 0 & m_E & m_1 \\ m_3 & 0 & m_E & 0 & m_2 \\ 0 & 0 & m_1 & m_2 & 0 \end{bmatrix},$$

which reduces to

$$\mathcal{M}_{\nu,S} = \begin{bmatrix} m_D^2/m_N & m_1 m_3/m_E \\ m_1 m_3/m_E & 2m_1 m_2/m_E \end{bmatrix}.$$
as desired. Other considerations such as anomaly cancellation and simplicity implies\cite{10} that we have the supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_N$ model\cite{11} with particle content given by the fundamental 27 representation of $E_6$.

4 The U(1)-Extended Supersymmetric Model

The conventional decomposition of $E_6$ is as follows. First we have $E_6 \rightarrow SO(10) \times U(1)_\psi$, with

\begin{align}
(16,1) &= (u,d) + u^c + e^c + d^c + (\nu_e, e) + N, \quad (20) \\
(10,-2) &= h + (E^c, N_E^c) + h^c + (\nu_E, E), \quad (21) \\
(1,4) &= S, \quad (22)
\end{align}

where $2\sqrt{6}Q_\psi$ has been denoted. Next we have $SO(10) \rightarrow SU(5) \times U(1)_\chi$. In general, a linear combination of $U(1)_\psi$ and $U(1)_\chi$ may survive down to the TeV energy scale. Let $Q_\alpha = \cos \alpha Q_\psi - \sin \alpha Q_\chi$, then for $\tan \alpha = -1/\sqrt{15}$, we have $Q_\alpha(N) = 0$ which is what we want. Call this $U(1)_N$, then

$$2\sqrt{10}Q_N = 6Y_L + T_{3R} - 9Y_R. \quad (23)$$

Specifically, under $U(1)_N$, we have

\begin{align}
(u,d), u^c, e^c &\sim 1; \quad d^c, (\nu_e, e) \sim 2; \quad N \sim 0; \quad (24) \\
h, (E^c, N_E^c) &\sim -2; \quad h^c, (\nu_E, E) \sim -3; \quad S \sim 5. \quad (25)
\end{align}

Comparing against Eq. (2) to (4), we find $a = 3/5$ and $g^2_x = (25/24)g^2_1$. The largest numerical value of $m_h$ in this case is 140 GeV.

Assume 3 copies of the 27 representation to accommodate the 3 families of quarks and leptons, and impose a discrete $Z_2$ symmetry such that one copy of $(\nu_E, E), \ (E^c, N_E^c)$, and
The scalar components of the $Z_2$-even superfields develop nonzero vacuum expectation values: \( \langle \tilde{S} \rangle = u, \langle \tilde{\nu} \rangle = v_1, \) and \( \langle \tilde{N} \rangle = v_2 \).

Neglecting $v_{1,2}$ for the time being, we find the mass of $Z'$ to be equal to that of the physical scalar boson $\sqrt{2} \text{Re} \tilde{S}$, i.e. \( (\sqrt{5/2}) g_N u \), where \( g_N^2 = (5/3) g_1^2 \) is a very good approximation obtained from normalizing $U(1)_Y$ and $U(1)_N$ at the grand-unification scale. Furthermore, the corresponding $S$ pairs up with the $Z'$-gaugino to form the mass matrix

\[
\mathcal{M}_{Z',S} = \begin{bmatrix}
M_1 & M_{Z'} \\
M_{Z'} & 0
\end{bmatrix},
\]

(26)

where $M_1$ is the soft supersymmetry breaking $U(1)$ gaugino mass. The other 2 $S$’s are naturally light singlet neutrinos. The $\mathcal{M}$ of Eq. (18) is actually $12 \times 12$ because it contains 3 $\nu$’s and 2 $S$'s. Because $Z'$ couples to $S$ according to Eq. (25), the invisible width of $Z'$ is very much enhanced in this model.

\[
\frac{\Gamma(Z' \rightarrow \nu \bar{\nu})}{\Gamma(Z' \rightarrow \ell^- \ell^+)} = \frac{4}{5}; \quad \frac{\Gamma(Z' \rightarrow S \overline{S})}{\Gamma(Z' \rightarrow \ell^- \ell^+)} = \frac{10}{3}.
\]

(27)

In all previous phenomenological studies of $Z'$ from $E_6$, the possibility of light $S$’s has not been recognized. The above would serve as a distinctive signature of the $U(1)_N$ model.

## 5 Z - Z’ Sector

The new $Z'$ of this model mixes with the standard $Z$.

\[
\mathcal{M}_{Z,Z'}^2 = \begin{bmatrix}
g_N^2 (v_1^2 + v_2^2)/2 & g_N g_Z (-3v_1^2 + 2v_2^2)/2\sqrt{10} \\
g_N g_Z (-3v_1^2 + 2v_2^2)/2\sqrt{10} & g_N^2 (25u^2 + 9v_1^2 + 4v_2^2)/20
\end{bmatrix}.
\]

(28)

This results in a slight shift of the physical $Z$ mass and a slight change in its couplings to the usual quarks and leptons. These deviations can be formulated in terms of the oblique parameters.\[13\]

\[
\epsilon_1 = \left( \sin^4 \beta - \frac{9}{25} \right) \frac{u^2}{v^2} \simeq \alpha T,
\]

(29)
\[ \epsilon_2 = \left( \sin^2 \beta - \frac{3}{5} \right) \frac{v^2}{u^2} \simeq -\frac{\alpha U}{4 \sin^2 \theta_W}, \]  \hspace{1cm} (30)

\[ \epsilon_3 = \frac{2}{5} \left( 1 + \frac{1}{4 \sin^2 \theta_W} \right) \left( \sin^2 \beta - \frac{3}{5} \right) \frac{v^2}{u^2} \simeq \frac{\alpha S}{4 \sin^2 \theta_W}, \]  \hspace{1cm} (31)

where \( v^2 \equiv v_1^2 + v_2^2 \) and \( \tan \beta \equiv v_2/v_1 \). Note that for \( \sin^2 \beta \) near \( 3/5 \), \( \epsilon_{1,2,3} \) are all suppressed.

In any case, the experimental errors on these quantities are fractions of a percent, hence \( u \sim \text{TeV} \) is allowed.

6 Conclusions

(1) There are theoretical and phenomenological hints for the existence of an extra supersymmetric U(1) gauge factor which is broken at the TeV scale. (2) In particular, the \( U(1)_N \) model from the \( E_6 \) superstring is very desirable for understanding the totality of neutrino-oscillation experiments. (3) Because of tree-level nondecoupling in the scalar sector, the effect of an extra supersymmetric U(1) gauge factor is already felt by the two-doublet Higgs structure at around 100 GeV, which will be of a more general form than that of the Minimal Supersymmetric Standard Model.

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References

[1] J. L. Hewett and T. G. Rizzo, Phys. Rept. 183, 193 (1989).

[2] M. Cvetic and P. Langacker, Phys. Rev. D54, 3570 (1996); Mod. Phys. Lett. A11, 1247 (1996).

[3] K. S. Babu et al., Phys. Rev. D54, 4635 (1996).

[4] X. Li and E. Ma, hep-ph/9608398.

[5] E. Ma and D. Ng, Phys. Rev. D49, 6164 (1994); T. V. Duong and E. Ma, Phys. Lett. B316, 307 (1993); J. Phys. G21, 159 (1995).

[6] E. Keith, E. Ma, and B. Mukhopadhyaya, in preparation.

[7] R. Davis, Jr., et al., Annu. Rev. Nucl. Part. Sci. 39, 467 (1989); K. S. Hirata et al., Phys. Rev. D44, 2241 (1991); A. I. Abazov et al., Phys. Rev. Lett. 67, 3332 (1991); P. Anselmann et al., Phys. Lett. B327, 377 (1994).

[8] R. Becker-Szendy et al., Phys. Rev. D46, 3720 (1992); K. S. Hirata et al., Phys. Lett. B280, 146 (1992); Y. Fukuda et al., ibid. 335, 237 (1994).

[9] C. Athanassopoulos et al., Phys. Rev. Lett. 75, 2650 (1995); Los Alamos Laboratory Report No. LA-UR-96-1582, nucl-ex/9605003.

[10] E. Ma, Mod. Phys. Lett. A11, 1893 (1996).

[11] E. Ma, Phys. Lett. B380, 286 (1996).

[12] E. Keith, E. Ma, and B. Mukhopadhyaya, hep-ph/9607488, Phys. Rev. D55, in press (1997).

[13] E. Keith and E. Ma, Phys. Rev. D54, 3587 (1996).