Abstract: The off-cone Compton operator of twist-2 is Fourier transformed using a general procedure which is applicable, in principle, to any QCD tensor operator of definite (geometric) twist. That method allows, after taking the non-forward matrix elements, to separate quite effectively their imaginary part and to reveal some hidden structure in terms of appropriately defined variables, including generalized Nachtmann variables. In this way, without using the equations of motion, generalizations of the Wandzura-Wilczek relation and of the mass-corrected Callan-Gross relation to the non-forward scattering, having the same shape as in the forward case, are obtained. In addition, new relations for those structure functions which vanish in the forward case are derived. All the structure functions are expressed in terms of iterated generalized parton distributions of n-th order. In addition, we showed that the absorptive part of twist-2 virtual Compton amplitude is determined by the non-forward extensions of g_1, W_1 and W_2 only.

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I. INTRODUCTION

The Compton amplitude for the scattering of a virtual photon off a hadron, γ* + P_1 \rightarrow γ^\prime* + P_2, provides one of the basic tools to understand the short–distance behavior of the nucleon and to test Quantum Chromodynamics (QCD) at large space–like virtualities. In that kinematic regime, usually denoted as generalized Bjorken region, the Compton amplitude is dominated by the singularities on the light–cone and the Compton amplitude factorizes into universal, non-perturbative distribution amplitudes (DA) – commonly denoted also as generalized parton distributions (GPD) – and perturbative hard scattering amplitudes of the participating partons. The DA’s parametrize the matrix elements of appropriate bilocal quark-antiquark, gluon-gluon as well as non-local higher order light-ray operators. For deep inelastic lepton-hadron scattering (DIS) the parton distributions, modulo kinematical factors, are given as forward matrix elements of non-local light-ray operators occurring in the light-cone expansion of the time-ordered product of appropriate hadronic currents, and for (deeply) virtual Compton scattering (DVCS) and meson production the generalized distribution amplitudes, respectively, are given by corresponding non-forward matrix elements. Thereby, various processes are governed by one and the same (set of) light-ray operators.

There exist various different approaches to attack the problem of determining the virtual Compton amplitude in leading as well as beyond leading order. Usually, the light–cone expansion, either the local one \cite{1} or the non-local one \cite{2,3,4,5}, is applied \cite{6,7,8,9,10,11}; for a review see, e.g., \cite{12,13,14,15}. Here, we prefer the approach using the non-local light-cone expansion. That approach also has been used for the treatment of diffractive processes \cite{16}. Differently, also the method of light-cone quantization \cite{17,18,19} has been used, e.g., in DIS \cite{20}, DVCS \cite{21} and large-angle Compton scattering \cite{22}.

Besides the leading contributions, being determined in the early days of QCD \cite{23}, there occur corrections from operators of higher (geometric) twist and/or containing more and more primary fields, from the radiative corrections of these operators as well as taking into consideration transverse (parton) momenta \cite{24}. The main concern of considering these sub-leading contributions is to determine more accurately the various phenomenological (multi-variable) distribution amplitudes and their $Q^2$–evolution in a well-defined quantum field theoretical frame, to study their possible interrelations and to determine the influence of target mass and, eventually, quark mass corrections.
Concerning higher twist, there are to be distinguished not only different approaches, e.g., local and non-local ones, but also different definitions. The notion of (geometric) twist $\tau_{M/Q}$ being related to the (collinear) conformal group \[ \tau_{M/Q} \]
and its group theoretical meaning.

Recently in Refs. \[25,26\] and quite generally in Ref. \[32\]. Loosely speaking, non-local operators of definite geometric twist are determined by towers of local operators spanning irreducible tensor representations of the Lorentz group which are characterized by well-defined symmetry type (under index permutations of the traceless tensors). The various symmetry types are determined by corresponding Young patterns having, in the case of the Lorentz group, at most three lines, \((\eta) = (m_1, m_2, m_3)\). For operators which locally are characterized by totally symmetric traceless tensors, there exists a well-defined group theoretical framework \[33\] which works on the light-cone as well as, by unique harmonic extension, also off the light-cone. The off-cone operators of definite twist are given for the local as well as the non-local case. Thereby, local operators of given $n$ are represented by \((\text{a finite series of})\) Gegenbauer polynomials $C^\nu_n(z)$, $\nu \geq 1$, whereas the non-local operators, being obtained by re-summation with respect to $n$, are represented by \((\text{a related series of})\) Bessel functions $J_{\nu-\frac{1}{2}}(z)$.

In the case of DIS, i.e., forward matrix elements on the light-cone, it has been shown that there exist one-to-one relations between the quark distribution functions of well-defined geometric twist and those of dynamical twist \[34\]; analogous relations also hold for the meson distribution functions \[33\]. From them, without using the equations of motion, the well-known Wenzlir-Wilczek (WW) relations \[36\] between the distribution functions $g_2, f_L, h_L,$ and $h_3$; analogous relations occur also in the case of $p-$meson wave functions. Concerning the case of non-forward matrix elements a first derivation of a generalized WW- and CG-relation was given in Ref. \[10\]. Later on, the discussion of generalized WW-relations led to different points of view concerning the use of equations of motion and current conservation \[37,38\]. In this paper we are able to show how the generalizations of the WW-relation as well as of the Callan-Gross(CG) relation \[36\] occur in case of non-forward matrix elements of the Compton operator. And this comes out without the use of the equations of motion and fully taking into account the target mass contributions.

The influence of target masses in case of forward scattering has been studied for unpolarized DIS \[11\] as well as polarized DIS \[42,43,44,45\]. A short review of these attempts was given in Ref. \[46\] containing also the not so well-known related exact result about the representation of a causal, Lorentz invariant structure function $W(x,p)$ as generally convergent series of Gegenbauer polynomials \[47\] using the Jost-Lehmann-Dyson representation \[48\].

Using the local operator product expansion Georgi and Politzer \[41\] showed precocious scaling in deep inelastic electron-proton scattering, where $\xi = 2x/(1 + \sqrt{1 + 4x^2M^2/Q^2})$ is the well-known Nachtmann variable with $x \equiv x_B$ being the Bjorken variable and $M$ the target (proton) mass; for the structure functions $W_1(Q^2, x)$ and $\nu W_2(Q^2, x)/M$ they obtained the following connection with the quark distribution function $F(\xi)$:

\[
2xW_1 = \frac{x^2}{(1 + 4x^2M^2/Q^2)^{1/2}} F(\xi) + \frac{2x^3M^2/Q^2}{1 + 4x^2M^2/Q^2} \int_\xi^1 d\xi' F(\xi') + \frac{4x^4M^4/Q^4}{(1 + 4x^2M^2/Q^2)^{3/2}} \int_\xi^1 d\xi' \int_\xi^{\xi'} d\xi'' F(\xi''), \\
\frac{\nu}{M}W_2 = \frac{x^2}{(1 + 4x^2M^2/Q^2)^{1/2}} F(\xi) + \frac{6x^3M^2/Q^2}{1 + 4x^2M^2/Q^2} \int_\xi^1 d\xi' F(\xi') + \frac{12x^4M^4/Q^4}{(1 + 4x^2M^2/Q^2)^{3/2}} \int_\xi^1 d\xi' \int_\xi^{\xi'} d\xi'' F(\xi''),
\]

and similarly also for $W_1(Q^2, x)$ in neutrino scattering. That method has been extended later on for studying the target mass contributions in polarized DIS due to different twist \[49,50\]. In the latter work, analogous relations have been found for the structure functions $G_1(Q^2, x), \ldots, G_5(Q^2, x)$, especially there holds the following:

\[
G_1 = \frac{x/\xi}{(1 + 4x^2M^2/Q^2)^{3/2}} \left[ \hat{F}(\xi) + \frac{x(\xi + M^2/Q^2)}{(1 + 4x^2M^2/Q^2)^{1/2}} \int_\xi^1 d\xi' \hat{F}(\xi') - \frac{x\xi M^2}{2Q^2} - \frac{2 - M^2/Q^2}{1 + 4x^2M^2/Q^2} \int_\xi^1 d\xi' \int_\xi^{\xi'} d\xi'' \hat{F}(\xi'') \right],

g_2 = \frac{-3x/\xi}{(1 + 4x^2M^2/Q^2)^{3/2}} \left[ \hat{F}(\xi) - \frac{1 - x\xi M^2/Q^2}{(1 + 4x^2M^2/Q^2)^{1/2}} \int_\xi^1 d\xi' \hat{F}(\xi') + \frac{3x\xi M^2/Q^2}{2(1 + 4x^2M^2/Q^2)^{1/2}} \int_\xi^1 d\xi' \int_\xi^{\xi'} d\xi'' \hat{F}(\xi'') \right],
\]

leading to the well-known WW-relations and extending the CG-relation. However, that method is tailored to the forward case and cannot be extended to the non-forward one.
Quite differently, the group theoretical method for the determination of target mass corrections using harmonic scalar operators of definite spin and the corresponding matrix elements in terms of Gegenbauer polynomials has been introduced for the first by Nachtmann and continued by Baluni and Eichten. Remarkably, this method can be generalized for the consideration of target mass resp. power corrections to virtual Compton scattering in non-forward case. Thereby, in order to get information about the target mass contributions one is forced to consider the (geometric) twist decomposition off-cone, taking into account all the trace terms leading to power suppressed expressions depending on $M^2/Q^2$.

Concerning the twist-2 part Belitsky and Müller were able to completely sum up the mass corrections to a closed form in terms of (di)logarithms depending on unique variable $M^2$ approaching $4\pi^2 M^2/Q^2$ in the forward case. On the other hand, in the previous work it has been shown how the well-known twist decomposition of non-local scalar off-cone operators according to the method of harmonic extension can be used for the non-local vector and skew-symmetric tensor quark-antiquark operators, leading to power corrections of the related off-forward double distributions as well as the vector meson distribution amplitudes in $x$–space. But, the application to virtual Compton scattering which requires to carry-out the Fourier transformation of the operator matrix elements times the coefficient functions (in Born approximation), due to its complexity, remained open.

In a previous consideration a straightforward procedure of performing the Fourier transformation has been presented for the off-cone matrix elements of scalar non-local operators. Here, we extend it to the case of (axial) vector operators appearing as part of the Compton operator. Despite it is already known for a long time that current conservation in non-forward case can be arrived in a sufficient approximate way only when non-leading corrections, specially contributions of appropriate twist-3 operators are included, we nevertheless think that it is important to consider the full content of the twist-2 contributions separately as a first step. Therefore, like in Ref. [51], we consider the contributions of the twist-2 operators only.

The straightforward procedure of Ref. [51] used a representation of the matrix elements of the non-local twist-2 operator by an infinite sum of matrix elements of local twist-2 operators, transformed each term separately and finally re-summed these expressions. Here, we proceed in a similar manner. However, the Fourier transform of the Compton operator is performed first and afterwards the matrix elements are taken. In the case of scalar non-local operators of definite twist the general procedure for determining their Fourier transform makes use of the relation between Bessel functions and Gegenbauer polynomials. Now, that procedure which, in principle, applies to arbitrary tensor operators of definite twist, is applied to the case of vector operators (Appendix A). Since the existence and convergence of the corresponding re-summations is by no means obvious we checked our calculation in case of the trace part of the Compton operator by Fourier transforming its closed non-local expression directly (Appendix B).

In fact, we re-obtained the result of Ref. [51] in a straightforward manner, also closing some gaps in their representation. But, more interesting than this is the fact, that our procedure leads to a much deeper insight into the structure of the virtual Compton amplitude which, especially, allows for a straightforward generalization of the Wandzura-Wilczek relation and of the Callan-Gross relation to the non-forward case. More precisely, introducing appropriate variables $\xi_1, \xi_2, \zeta$, we are able derive non-forward generalizations of $W_1, W_2$ and $G_1, G_2$ in terms of these variables which obey relations of the same functional dependence as their counterparts in forward case above! Therefore, they constitute non-forward generalizations of the WW- and CG-relations. In addition, new generalized distribution amplitudes and corresponding relations occur which disappear in the forward case. Here, we should emphasize again that these results are obtained without using any dynamical input but taking into account only the correct twist-2 representation of the Compton operator off the light-cone.

The paper is organized as follows. In Sec. 2 we remember some basic formulae concerning Born term contributions and definitions of operators. In Sec. 3 we Fourier transform the Compton operator using the full off-cone content of the twist-2 light-cone operator. The calculation is outlined for the trace part and the final results are presented for the symmetric and antisymmetric part of the Compton operator separately. The corresponding calculations are shifted into Appendices C and D. In Secs. 4 to 6 we investigate the matrix elements for the trace part, the antisymmetric and the symmetric part of the Compton operator. Thereby, we introduce a general kinematic decomposition of the matrix elements. For each kinematic structure appears one basic generalized parton distribution function. We insert these findings into the partial results of Sec. 3. Thereby the remaining auxiliary parameter integrations can be managed in different ways. We perform it in such a way that we obtain a representation of the Compton amplitude in terms of iterated generalized distribution functions which relay on the basic generalized parton distribution functions. Such representations are already known for the case of forward Compton scattering by [41]. Moreover, these representations allow in a very simple manner the extraction of the absorptive part. At this stage it is possible to compare our results with the results obtained for the forward case in Refs. [41] and [50]. Finally we look for generalizations of the Wandzura-Wilczek and the Callan-Gross relations to non-forward scattering together with additional relations concerning those structure functions which vanish in the case of forward scattering.
II. LEADING CONTRIBUTION OF THE VIRTUAL COMPTON AMPLITUDE

The virtual Compton amplitude is given by

\[ T_{\mu\nu}(P, q; S) = \int d^4x \, e^{iqx} \langle P_2, S_2 | RT [J_\mu(x/2)J_\nu(-x/2)] S | P, S_1 \rangle, \]  

(II.1)

where \( P_1 \) and \( S_1 \) are the momenta and spins of the incoming (outgoing) hadrons, \( q = (q_2 + q_1)/2, \) \( p_+ = P_1 + P_2 \) and \( p_- = P_2 - P_1 = q_1 - q_2 \) denotes the momentum transfer; \( R \) denotes the renormalization procedure and \( S \) is the (renormalized) \( S \)-matrix. Here, we consider the Compton amplitude in the generalized Bjorken region,

\[ \nu = q p_+ \to \infty, \quad Q^2 = -q^2 \to \infty, \]  

(II.2)

with the variables

\[ x = \frac{Q^2}{q p_+} \quad \text{and} \quad \eta = \frac{q p_-}{q p_+}, \]  

(II.3)

keeping fixed.

Obviously, the Compton amplitude appears as Fourier transform of the non-forward matrix elements of the Compton operator,

\[ \tilde{T}_{\mu\nu}(x) \equiv RT \left[ J_\mu \left( \frac{x}{2} \right) J_\nu \left( -\frac{x}{2} \right) S \right]. \]  

(II.4)

In the Bjorken region, the physical processes are determined by the singularities of the Compton operator on the light-cone and, therefore, the operator product expansion can be applied. A general study of it, using the non-local light-cone expansion [2, 12], has been given, e.g., in Refs. [9, 30], cf. also Refs. [7, 8]. As is well-known, in leading order the Compton operator simply reads

\[ \tilde{T}_{\mu\nu}(x) \approx \frac{1}{2 \pi^2 (x^2 - i \epsilon)^2} \left( S_{\mu\nu}^{\alpha\beta} x_\alpha O_\beta (x) - i \epsilon_{\mu\nu}^{\alpha\beta} x_\alpha O_{5,\beta} (x) \right), \]  

(II.5)

where the tensor \( S_{\mu\nu}^{\alpha\beta} \equiv S_{\mu\alpha\nu\beta} = g_{\mu\alpha} g_{\nu\beta} + g_{\mu\beta} g_{\nu\alpha} - g_{\mu\nu} g_{\alpha\beta} \) is symmetric in \( \mu\nu \) and \( \alpha\beta \) and \( \kappa = 1/2 \). This expression corresponds to the handbag diagram; contributions from the quark masses and higher order terms, e.g., four-quark and quark-gluon operators, are neglected. In order to omit the subtleties of operator mixing, the non-singlet case is taken and the flavor structure has been suppressed.

The hermitean, chiral-even vector and axial vector operators, \( O_\alpha(x_1, x_2) \) and \( O_{5,\alpha}(x_1, x_2) \), respectively, at the first, are renormalized on the light-cone, \( \tilde{x}^2 = 0 \), [2, 13, 14] and then, in order to allow for an appropriate target mass expansion, extended off-cone by replacing \( \tilde{x} \to x \). They are given by

\[ O_\alpha(x_1, x_2) = i (O_\alpha(x_1, x_2) - O_\alpha(x_2, x_1)), \]  

(OI.6)

\[ O_{5,\alpha}(x_1, x_2) = O_{5,\alpha}(x_1, x_2) + O_{5,\alpha}(x_2, x_1), \]  

(OI.7)

with

\[ O_\alpha(x_1, x_2) := RT \left[ \bar{\psi}(x_1) \gamma_\alpha U(x_1, x_2) \psi(x_2) : S : \right] \bigg|_{\tilde{x} \to x}, \]  

(OI.8)

\[ O_{5,\alpha}(x_1, x_2) := RT \left[ \bar{\psi}(x_1) \gamma_5 \gamma_\alpha U(x_1, x_2) \psi(x_2) : S : \right] \bigg|_{\tilde{x} \to x}. \]  

(OI.9)

The path-ordered phase factors, which are given by

\[ U(x_1, x_2) := \mathcal{P} \exp \left\{ -ig \int_{x_2}^{x_1} d\tau \, x^{\mu} A_\mu(\tau) \right\}, \]

in the following will be omitted, i.e., the Schwinger-Fock gauge, \( x^{\mu} A_\mu(x) = 0 \), will be assumed. The Compton operators \( O_{(5,\beta)}(x_1, x_2) \) are introduced such that their matrix elements lead to real (analytic) distribution amplitudes. Occasionally, they will be considered in their unsymmetrized form \( O_{5,\beta}(x_1, x_2) \); their symmetric and antisymmetric form is re-obtained by observing their behavior under the exchange \( \kappa \to -\kappa \).

The bi-local off-cone vector operators, when decomposed w.r.t. their geometric twist, contain contributions from all the twists \( \tau = 2, 3, \ldots, \infty \). Let us emphasize that for the procedure of twist decomposition [25, 26, 31, 32] only the tensorial structure of the operators is essential and not their behavior under renormalization. The vector operators of definite geometric twist already have been determined in x-space [33]. Now, their Fourier transform has to be determined. However, in order to present the straightforward procedure of determining the Fourier transform of such QCD operators, which has been introduced for the first time in the case of scalar operators [32], we investigate the contribution of the twist-2 parts only. In fact, this also is the most interesting contribution.
III. FOURIER TRANSFORM OF THE Twist-2 VECTOR OPERATOR

In this Section, according to Eqs. (III.1), (III.4) and (III.5), we perform a Fourier transformation of the non-local vector and axial vector operator of leading twist-2, $O_{5}^{(5)}(\kappa x, -\kappa x)$ and $O_{5}^{(5)}(\kappa x, -\kappa x)$, multiplied with the leading part $x_{\alpha}/[2\pi^{2}(x^{2} - i\epsilon)^{2}]$ of the propagator. This will be done by the same general procedure as has been introduced for scalar operators in Ref. [52]. It essentially uses the relation between the Fourier transform of $(ux)^{n}/[2\pi^{2}(x^{2} - i\epsilon)^{2}]$ and Gegenbauer polynomials in the variable $z = (ux)/\sqrt{u^{2}q^{2}}$, $u$ being any vector, together with the fact that (local) tensor operators of definite (geometric) twist $\tau$ are given in terms of harmonic tensor polynomials. Let us emphasize that this method, in principle, applies to tensor operators of any twist. Matrix elements, forward or non-forward ones, are to be taken after Fourier transformation according to the general description introduced in Ref. [30].

Below, we first briefly review the general procedure and apply it to the trace part of the Compton operator, already adopted to the case under consideration, and, afterwards, we present the results for the QCD vector operators (III.6) and (III.7); the explicit computations together with the necessary prerequisites are postponed to Appendix A. After taking non-forward matrix elements and performing the remaining intermediate integrations we are able to confirm the results of Ref. [51] in a straightforward, less artistic manner. In addition, we have proven that this result which is obtained via re-summation of the local expressions may be obtained also directly by the Fourier transformation of matrix elements of the corresponding non-local operators; a sample calculation for the trace of the Compton amplitude is given in Appendix B.

A. The twist-2 vector operators and general definitions

In order to determine power corrections, e.g., the target mass corrections, to hard, light-cone dominated hadronic processes one has to consider the twist decomposition of the corresponding QCD operators off-cone [30], which, for non-local operators, is infinite. For scalar operators this decomposition is well-known [5] (cf., also [28, 30]), in the case of vector operators it has been given recently [53] and in the general case of tensor operators it is under consideration [30]. However, when restricting onto the light-cone, $x \rightarrow \hat{x}, \hat{x}^{2} = 0$, the twist decomposition is finite for all the non-local QCD-operators (see, e.g., Ref. [28]). Especially, for scalar operators only the leading twist occurs, i.e., $\hat{x}^{\mu}O_{(5)\mu}^{(5)}(\kappa \hat{x}, -\kappa \hat{x}) = \hat{x}^{\mu}O_{(5)\mu}^{(tw)}(\kappa \hat{x}, -\kappa \hat{x})$, while higher twists occur for vector and tensor operators [29].

Here, we consider only the leading non-local twist-2 vector operators off-cone, which are given as follows (cf., [30], Eq. (4.25)):

$$O_{(5)\alpha}^{(tw)}(\kappa x, -\kappa x) = \partial_{\alpha} \int_{0}^{1} dt \int d^{4}u O_{(5)\mu}(u) \left\{ x^{\mu}(2 + \kappa \partial_{\kappa}) - \frac{1}{2} i \kappa t u^{\mu}x^{2} \right\} (3 + \kappa \partial_{\kappa})H_{2}(u, \kappa t x)$$ (III.1)

with

$$H_{\nu}(u, x) = \sqrt{\pi} \left( \sqrt{(ux)^{2} - u^{2}x^{2}} \right)^{1/2} J_{\nu-1/2} \left( \frac{1}{2} \sqrt{(ux)^{2} - u^{2}x^{2}} \right) e^{i(ux)/2},$$ (III.2)

whose local components are

$$O_{(5)\alpha n}(x) = \frac{1}{n+1} \partial_{\alpha} \left[ H_{n+1}(x^{2}, \Box_{x}) \int d^{4}u O_{(5)\mu n}(u) x^{\mu}(ux)^{n} \right],$$ (III.3)

with the operator

$$H_{n}(x^{2}, \Box_{x}) = \sum_{k=0}^{\lfloor n/2 \rfloor} \frac{(-1)^{k} (n-k)!}{4^{k} k! n!} (x^{2})^{k} \Box_{x}^{k}$$ (III.4)

projecting any (scalar) homogeneous polynomial of degree $n$ onto traceless scalar harmonics of degree $n$. In these relations a (formal) Fourier transformation of the non-local operators followed by an expansion into local operators has been used which are defined as follows:

$$O_{(5)\mu}(\kappa x, -\kappa x) = \int d^{4}u O_{(5)\mu}(u) e^{iux} = \sum_{n=0}^{\infty} \frac{(i\kappa)^{n}}{n!} O_{(5)\mu n}(x),$$ (III.5)

$$O_{(5)\mu n}(x) = \int d^{4}u O_{(5)\mu}(u) (ux)^{n} = (-i)^{n} \frac{\partial^{n}}{\partial \kappa^{n}} O_{(5)\mu}(\kappa x, -\kappa x)|_{\kappa = 0},$$ (III.6)
Here and in the following, for simplicity of notation, we understand the factor \(1/(2\pi)^4\) to be included into the measure of the Fourier transformation, \(d^4u \equiv d^4u/(2\pi)^4\).

Now, because of the symmetry of \((ux)^n\), leading to the equality,

\[
\partial_{\nu}^{\alpha} H_{n+1}(x^2, \Box u) \ x_{\mu}(ux)^n = \partial_{\mu}^{\nu} H_{n+1}(u^2, \Box u) \ u_{\alpha}(ux)^n,
\]
according to Eqs. (II.11) and (II.13) we have to determine the Fourier transforms of \(x_{\alpha}(ux)^n / [2\pi^2(x^2 - i\epsilon)^2]\) only.

As shown in Appendix A, it holds

\[
\begin{align*}
\int \frac{d^4x}{(2\pi)^4} e^{iuq} \frac{(ux)^n}{(x^2 - i\epsilon)^2} &= -i^{n+1} n! \ h_n^u(u,q), \quad (III.8) \\
\int \frac{d^4x}{(2\pi)^4} e^{iuq} \frac{x_{\alpha}(ux)^n}{(x^2 - i\epsilon)^2} &= i^n n! \ (q_{\alpha} \ h_n^1(u,q) - u_{\alpha} \ h_{n-1}^1(u,q)), \quad (III.9)
\end{align*}
\]

where the following set of functions has been introduced:

\[
\begin{align*}
\ h_n^u(u,q) &:= \frac{2^{n+\nu} \Gamma(\nu)}{(q^2 + i\epsilon)^{n+\nu}} \left(\frac{\sqrt{q^2} u^2}{u}\right)^n \ C_n^\nu \left(\frac{uq}{\sqrt{q^2} u}\right) \quad \text{for} \quad n \geq 0, \quad (\nu, n) \neq (0, 0), \\
\ h_n^0(q) &:= -\ln \left(q^2/q_0^2\right) \quad \text{for} \quad (\nu, n) = (0, 0), \\
\ h_n^u(u,q) &:= 0 \quad \text{for} \quad n < 0;
\end{align*}
\]

here, \(q_0\) is some (physically irrelevant) reference momentum and \(C_n^\nu(z)\) are the Gegenbauer polynomials (cf., Ref. [54], Appendix II.11) which may be introduced according to (cf., Ref. [54], Eq. 5.13.1.1)

\[
\sum_{n=0}^{\infty} t^n C_n^\nu(z) = (1 - 2zt + t^2)^{-\nu} \quad \text{for} \quad |t| < 1. \quad (III.13)
\]

These functions obey the following relations (see also Eqs. (A.3) – (A.10)):

\[
\begin{align*}
H_n(u^2, \Box u) \ h_n^0(u,q) &= \left(\frac{q}{2}\right)^n \ h_n^1(u,q) \quad \text{for} \quad n \geq 1, \\
H_n(u^2, \Box u) \ h_n^1(u,q) &= \ h_n^1(u,q), \\
\sum_{n=0}^{\infty} (\pm \kappa)^n \ h_n^1(u,q) &= 2^\nu \Gamma(\nu) \left[(q + \kappa u)^2 + i\epsilon\right]^{-\nu}.
\end{align*}
\]

The last relation is an immediate consequence of (III.13) thereby having in mind that \(|q^2|\) is large, \(Q^2 \to \infty\).

Let us also note another property of the projection operator \(H_N(u^2, \Box u)\) which makes it useful to introduce the operators \(u_{\alpha}^n(u, \partial^\mu)\) (already being used by Baluni and Eichten [42]), namely

\[
\begin{align*}
H_n(u^2, \Box u) \ u_{\alpha} &= \ u_{\alpha}^n H_{n-1}(u^2, \Box u), \\
&\quad \text{with} \quad u_{\alpha}^n(u, \partial^\mu) \equiv u_{\alpha} - \frac{u^2}{2n} \partial_{\alpha}^u.
\end{align*}
\]

B. Fourier transform of the trace part

Now, let us demonstrate how the procedure works in case of the trace part of the Compton operator [52]. First, introducing the local operators and using relation (III.7) one obtains

\[
\begin{align*}
T_{\alpha\beta\nu}^{\Box u} (q) &= -2i \int \frac{d^4x}{(2\pi)^4} e^{iqx} \frac{1}{(x^2 - i\epsilon)^2} x^n \left(O_{\alpha}^{\Box u}(\kappa x, -\kappa x) - O_{\alpha}^{\Box u}(-\kappa x, \kappa x)\right) \\
&= -2i \sum_{n=0}^{\infty} \frac{(i\epsilon)^n (1 - (-1)^n)}{(n + 1)!} \int d^4u \ O_{\alpha}(u) \ \partial_{\alpha}^{\nu} H_{n+1}(u^2, \Box u) \int \frac{d^4x}{(2\pi)^4} e^{iuq} \frac{(ux)^{n+1}}{(x^2 - i\epsilon)^2},
\end{align*}
\]
then, relations \([\text{III.8}]\) and \([\text{III.14}]\) together with \(\partial_\nu^\alpha \mathbf{h}_n^\nu = q_\alpha h_{n-1}^{\nu+1} - u_\alpha h_{n-2}^{\nu+1}\), cf., Eq. \([\text{A.3}]\), are used to get

\[
\hat{T}_{\text{tw}2}^{\alpha_1 \beta_1} (q) = -\frac{q^2}{2} \sum_{n=0}^{\infty} \frac{((-\tau)^n - \tau^n)}{n+1} \int d^4u \, O_\alpha(u) \partial_\nu^\alpha H_{n+1} (u^2, \square u) \, h_0^0 (u, q) \]

\[
= -\frac{q^2}{2} \sum_{n=0}^{\infty} \frac{((-\tau)^n - \tau^n)}{n+1} \int d^4u \, O_\alpha(u) \left( q^2 h_n^2 (u, q) - u^\alpha h_n^2 (u, q) \right) ;
\]

finally, using the representation \(1/(n + 1) = \int_0^1 d\tau \tau^n\) together with Eq. \([\text{III.16}]\), we get

\[
\hat{T}_{\text{tw}2}^{\alpha_1 \beta_1} (q) = -2 \int_0^1 d\tau \int d^4u \, i (O_\alpha(u) - O_\alpha(-u)) \frac{q^2 (q^\alpha + \tau q^\alpha)}{[q + \tau q^2 + i\epsilon]^2} . \tag{III.19}
\]

Performing the \(\tau\)-integration one obtains

\[
\hat{T}_{\text{tw}2}^{\alpha_1 \beta_1} (q) = -\frac{q^2}{2} \int \frac{d^4u}{\kappa^4} \frac{q^\alpha}{(q + \tau q^2 + i\epsilon)^2} \frac{1}{(q + \tau q^2 + i\epsilon)^2} , \tag{III.20}
\]

where, under the assumption \((uq)^2 - u^2q^2 \geq 0\), we get

\[
\int_0^1 d\tau \frac{1}{(q + \tau q^2 + i\epsilon)^2} = \frac{1}{2\sqrt{(uq)^2 - u^2q^2}} \ln \left| \frac{q^2 + uq + \sqrt{(uq)^2 - u^2q^2}}{q^2 + uq - \sqrt{(uq)^2 - u^2q^2}} \right| \quad \text{(III.21)}
\]

Here, some remarks are in order.

After taking matrix elements according to the description of Ref. \([30]\), which is sketched in the next Section, Eqs. \([\text{IV.2}] - [\text{IV.3}]\), the expression \([\text{III.20}]\) not only allows for a simpler extraction of the imaginary part of the Compton amplitude, it also reveals some structure of the Compton amplitude which can be taken over, after appropriately introducing generalized distribution amplitudes, to relations between them. This will be shown in detail in the main part of the paper, Sections IV - VI. Of course, such relations must be hidden also in the expression \([\text{III.27}]\) below but we don’t see how to extract it in a simple way.

Furthermore, reminding the definition of the non-local twist-2 vector operators \([\text{III.1}]\) and a corresponding expression for the scalar ones (see, Ref. \([30]\), Eq. (2.30)), it is of interest to perform the Fourier transformation of these non-local operators also directly. This, however, is much more difficult than by the procedure which has been used above where the Fourier transformation has been performed before summing up the local expressions. In order to be convinced that the procedure of Fourier transformation and summation could be exchanged we checked this explicitly for the trace part, see, Appendix B.

\section*{C. The complete Compton operator in momentum space}

The same procedure as for the trace part has been applied to the complete Compton operator, Eqs. \([\text{III.1}]\) and \([\text{III.5}]\); the explicit computations are postponed to Appendix A. Let us present the result for the antisymmetric and the symmetric part separately.

For the antisymmetric part we obtain

\[
\hat{T}_{\text{tw}2}^{\alpha_1 \beta_1} (q) = \epsilon_{\alpha_1 \beta_1} \int \frac{d^4x}{(2\pi)^4} \frac{e^{iqx} x_{\alpha_1}}{(x^2 - i\epsilon)^2} \left( O_{5\beta_1}^{\nu_1} (\kappa x, -\kappa x) + O_{5\beta_1}^{\nu_2} (-\kappa x, \kappa x) \right) \]

\[
= q^2 \int_0^1 d\tau_1 \int_0^1 d\tau_2 \int d^4u \, \left( O_{5\rho} (u) + O_{5\rho} (-u) \right) \epsilon_{\alpha_1 \beta_1} \frac{\partial_\nu}{[q + \kappa \tau_1 \tau_2 u]^2 + i\epsilon^2} . \tag{III.22}
\]

The \(\tau\)-integrations may be performed with the result

\[
\hat{T}_{\text{tw}2}^{\alpha_1 \beta_1} (q) = \epsilon_{\alpha_1 \beta_1} \int d^4u \, \left( O_{5\rho} (u) + O_{5\rho} (-u) \right) \partial_\nu \left( q_\alpha u_{\beta} G_1 / q^2 \right) . \tag{III.23}
\]
with
\[
G_1 = \frac{1}{4\kappa X (1 - M^2) \sqrt{1 - M^2}} \left( (1 - M^2) L_+ + \sqrt{1 - M^2} L_+ + M^2 L \right)
\]
and
\[
L_\pm = \ln \left( 1 + \kappa X \left( 1 + \sqrt{1 - M^2} \right) \right) \pm \ln \left( 1 + \kappa X \left( 1 - \sqrt{1 - M^2} \right) \right)
\]
\[
L = \text{dilog} \left( 1 + \kappa X \left( 1 + \sqrt{1 - M^2} \right) \right) - \text{dilog} \left( 1 + \kappa X \left( 1 - \sqrt{1 - M^2} \right) \right)
\]
where the following abbreviations are used:
\[
M^2 := \frac{u^2 q^2}{(uq)^2} \quad \text{and} \quad X := \frac{(uq)}{q^2}.
\]
(III.24)

For the symmetric part we obtained
\[
\hat{T}^{tw2}_{\{\mu\nu\}}(q) = S_{\mu\nu} \alpha^\beta \int \frac{d^4x}{2\pi^2} \frac{e^{iqx} x_\alpha}{(x^2 - i\epsilon)^2} \left( O^{tw2}_{\beta} (\kappa x, -\kappa x) - O^{tw2}_{\beta} (-\kappa x, \kappa x) \right)
\]
\[
= 2 \int_0^1 \frac{d\tau}{\tau} (1 - \tau + \tau \ln \tau) \int \frac{d^4u}{\kappa^2 \tau^2} \frac{\partial}{u \kappa^2} \left( O_\rho \left( \frac{u}{\kappa \tau} \right) - O_\rho \left( -\frac{u}{\kappa \tau} \right) \right) \partial_{\mu}^\rho
\]
\[
\times \left\{ \left( \frac{2}{(q + u)^2 + i\epsilon} \right)^2 \left( (q^2 g_{\mu
u} - q_{\mu} q_{\nu}) \left[ (uq)^2 - u^2 q^2 \right] + (u_{\mu} q^2 - q_{\mu} (uq))(u_{\nu} q^2 - q_{\nu} (uq)) \right) \right\}
\]
\[
+ \frac{u^2}{(q + u)^2 + i\epsilon} \left( q^2 g_{\mu
u} - q_{\mu} q_{\nu} \right) \right\}.
\]
(III.25)

Concerning the \( \tau \) integration we remark that, implicitly, it as a triple-integration (see, relation (A.14)). Again, performing the \( \tau \)-integration one obtains
\[
\hat{T}^{tw2}_{\{\mu\nu\}}(q) = -\frac{i}{q^2} \int d^4u \left( O_\rho (u) - O_\rho (-u) \right) \partial_{\mu}^\rho \left\{ (q^2 g_{\mu
u} - q_{\mu} q_{\nu}) \mathcal{F}_1 + (u_{\mu} - X q_{\mu})(u_{\nu} - X q_{\nu}) \mathcal{F}_2 \right\}
\]
(III.26)

with
\[
\mathcal{F}_1 = \frac{1}{4\kappa (1 - M^2) \sqrt{1 - M^2}} \left( (1 - 2M^2 - \kappa XM^4) L_+ + \sqrt{1 - M^2} L_+ + M^2 L \right)
\]
\[
\mathcal{F}_2 = \frac{1}{4\kappa X^2 (1 - M^2)^2 \sqrt{1 - M^2}} \left( (14M^2 - 3\kappa XM^4) L_+ + \sqrt{1 - M^2} (1 + 2M^2) L_+ + 3M^2 L \right)
\]

Taking the trace of that result one gets
\[
\hat{T}^{tw2}_{\text{trace}}(q) = \frac{i}{\kappa} \int d^4u \left( O_\mu (u) - O_\mu (-u) \right) \partial_{\mu}^\rho \left( \frac{1}{\sqrt{1 - M^2}} L_+ + \sqrt{1 - M^2} L_+ \right),
\]
(III.27)

where still a further derivation of the functions \((1 - M)^{\pm 1/2} L_\pm (u, q)\) has to be performed. Contrary to this, the corresponding expression following from (III.20) and (III.21),
\[
\hat{T}^{tw2}_{\text{trace}}(q) = -\frac{i}{2q^2} \int \frac{d^4u}{\kappa^2} \left( O_\mu \left( \frac{u}{\kappa} \right) - O_\mu \left( \frac{-u}{\kappa} \right) \right) \frac{1}{X^2 (1 - M^2)}
\]
\[
\times \left\{ u_{\mu} \left( u^2 + (uq) \right) - q_{\mu} \left( (uq) + u^2 \right) - \frac{u_{\mu} q^2 - q_{\mu} uq}{q + u} \right\}.
\]
(III.28)

is more explicit.

Furthermore, after taking matrix elements, choosing \( \kappa \to -\kappa = -1/2 \), \( X = -1/\Xi \) and \( M^2 = -M^2 \), observing \( \text{dilog}(z) = \text{Li}_2(1-z) \) as well as changing \( L_\pm \to \mp L_\pm \) and \( L \to -L \) one reproduces the result of Ref. [51]. Unfortunately, in that form, Eqs. (III.23) and (III.24), not only the derivations w.r.t. \( u \) are finally to be performed but, more unpleasant, the result is not well suited for extracting the imaginary part of the Compton amplitude. Therefore, in our further considerations the expressions (III.22) and (III.25) will be used.
IV. NON-FORWARD COMPTON AMPLITUDE: TRACE PART (NONLOCAL)

Now, let us begin to study the Fourier transform of the non-forward Compton amplitude using an appropriate definition of the generalized distribution amplitudes and, thereafter, introducing variables which are adjusted to the Bjorken region. The trace part of the Compton amplitude, Eq. (III.19), is the simplest one. Therefore, let us start with it thereby already introducing the general procedure, described in [30], of parametrizing the non-forward matrix element and of taking their Fourier transform. Since we like to investigate especially their imaginary part it is advantageous to start with the expression (III.19). In fact, the $\tau$-integration must not be performed completely since part of it can be put into the definition of the generalized distribution amplitudes (see, below).

Let us take the matrix element of (III.19) together with the definition of the generalized parton distribution amplitude according to Ref. [30], Eqs. (2.3) – (2.11), as follows

$$
\langle P_2, S_2 | \tilde{T}_{\text{trace}}(q) | P_1, S_1 \rangle = -2i \int \frac{d^4 x}{2\pi^2} \frac{e^{iqx}}{(x^2 - i\epsilon)^2} x^\alpha \langle P_2, S_2 | \left( O_{\alpha}^{tw2} (\kappa x, -\kappa x) - O_{\alpha}^{tw2} (-\kappa x, \kappa x) \right) | P_1, S_1 \rangle
$$

$$
= -2i \int_0^1 d\tau \int d^4 u \langle P_2, S_2 | \left( O_\mu(u) - O_\mu(-u) \right) | P_1, S_1 \rangle \frac{q^2 (q^\mu + \kappa \tau u^\mu)}{[(q + \kappa \tau u)^2 + i\epsilon]^2}
$$

$$
= -2 \int_0^1 d\tau \int DZ \, \Phi_\alpha(Z, \mu^2) \, K_\alpha^O(\mathbb{P}, S) \, \frac{q^2 (q^\mu + \tau \Pi^\mu)}{[(q + \tau \Pi)^2 + i\epsilon]^2}. \quad (IV.1)
$$

Here, the matrix elements of $O_{\alpha}^{tw2}(u)$ are obtained, as has been demonstrated quite generally in Ref. [30], by the correspondence, holding under the $u$-integral,

$$
\langle P_2, S_2 | i(\mathcal{O}_\alpha(u) - \mathcal{O}_\alpha(-u)) | P_1, S_1 \rangle = K_\alpha^O(\mathbb{P}, S) \int DZ \, \delta^4(u - \mathbb{P}Z) \, \Phi_\alpha(Z, P, P'; \mu^2), \quad (IV.2)
$$

with the kinematic form factors $K_\alpha^O(\mathbb{P}, S) = \{ \bar{u} (P_2, S_2) \gamma_\alpha u (P_1, S_1), \bar{u} (P_2, S_2) \sigma_\alpha \beta p^\beta u (P_1, S_1), \ldots \}$ being the Dirac and Pauli form factor etc., and the generalized distribution amplitudes $\Phi_\alpha(Z, P, P'; \mu^2)$ depending on two variables $z_1, z_2$, sometimes also called double distributions (Their dependence on the momenta will be omitted in the following). In addition, we introduced the following abbreviations (remind $\kappa = 1/2$):

$$
\Pi_\mu = \kappa \mathbb{P}_\mu Z, \quad \mathbb{P}Z = P_1 z_1 + P_2 z_2 = p_+ z_+ + p_- z_-, \quad (IV.3)
$$

$$
DZ = dz_1 dz_2 \theta(1 - z_1) \theta(z_1 + 1) \theta(1 - z_2) \theta(z_2 + 1) = D(-Z), \quad (IV.4)
$$

$$
\Phi_\alpha(Z, \mu^2) = f_{\alpha}^{tw2}(Z, \mu^2) - f_{\alpha}^{tw2}(-Z, \mu^2) = -\Phi_\alpha(-Z, \mu^2). \quad (IV.5)
$$

Now, in order to reduce the power of the denominator in the $\tau$ integral (IV.1) by one unit we perform a partial integration. Using the abbreviation $R(\tau) = (q + \tau \Pi)^2$ we get

$$
\int_0^1 d\tau \frac{1}{[R(\tau) + i\epsilon]^2} = \frac{-1}{2[(q\Pi)^2 - q^2\Pi^2]} \left\{ \frac{q \Pi + \Pi^2}{R(1) + i\epsilon} - \frac{q \Pi}{R(0) + i\epsilon} + \Pi^2 \int_0^1 d\tau \frac{1}{R(\tau) + i\epsilon} \right\}, \quad (IV.6)
$$

$$
\int_0^1 d\tau \frac{\tau}{[R(\tau) + i\epsilon]^2} = \frac{1}{2[(q\Pi)^2 - q^2\Pi^2]} \left\{ \frac{q^2 + q \Pi}{R(1) + i\epsilon} - \frac{q^2}{R(0) + i\epsilon} + q \Pi \int_0^1 d\tau \frac{1}{R(\tau) + i\epsilon} \right\}. \quad (IV.7)
$$

where, again, $(q\Pi)^2 - q^2\Pi^2 \geq 0$ is assumed to hold because of the physical kinematics. Then, in complete analogy to Eq. (III.20), we get

$$
\langle P_2, S_2 | \tilde{T}_{\text{trace}}(q) | P_1, S_1 \rangle = \int DZ \, \Phi^O(Z, \mu^2) \, K_\alpha^O(\mathbb{P}, S) \frac{-q^2}{[(q\Pi)^2 - q^2\Pi^2]} \left\{ \frac{\Pi_\mu (q^2 + q \Pi) - q_\mu (q \Pi + \Pi^2)}{R(1) + i\epsilon} - \frac{\Pi_\mu q^2 - q_\mu (q \Pi)}{R(0) + i\epsilon} + (\Pi_\mu q \Pi - q_\mu \Pi^2) \int_0^1 d\tau \frac{1}{R(\tau) + i\epsilon} \right\}. \quad (IV.8)
$$

Introducing generalized distribution amplitudes $\Phi_\alpha^{(n)}$ of order $n$ as follows,

$$
\Phi_\alpha^{(0)}(Z) = \Phi_\alpha(Z) \quad \text{and} \quad \Phi_\alpha^{(n)}(Z) = \int_0^1 \frac{d\tau_1}{\tau_1} \ldots \int_0^1 \frac{d\tau_n}{\tau_n} \Phi_\alpha \left( \frac{Z}{\tau_1 \ldots \tau_n} \right) \quad \text{for} \quad n \geq 1, \quad (IV.9)
$$
after scaling the $Z$-variables by $\tau$, we finally obtain

$$
\langle P_2, S_2 \mid \bar{T}^{\text{tw}}(q) \mid P_1, S_1 \rangle = K_{\alpha}^{(j)}(P, S) \int DZ \frac{-q^2}{(q^2 - q^2\Pi^2)} \left\{ \Phi_{\alpha}^{(0)}(Z, \mu^2) \left[ \frac{\Pi_{\alpha}(q^2 + q\Pi) - q_{\mu}(q\Pi + \Pi^2)}{R(1) + ie} \right] - \frac{\Pi_{\alpha} q^2 - q_{\mu}(q\Pi)}{R(1) + ie} \right\}. \quad (IV.10)
$$

Now, let us consider the imaginary part of this expression. Due to the overall factor $q^2$ in Eq. (IV.10) it results from the $1/R(1)$-terms only. For that reason let us rewrite

$$
R(\tau) \equiv (q + \tau\Pi)^2 = \Pi^2(\tau - \tilde{\xi}_+)(\tau - \tilde{\xi}_-)
$$

with

$$
\tilde{\xi}_\pm = \frac{-q\Pi \pm \sqrt{(q\Pi)^2 - q^2\Pi^2}}{\Pi^2}, \quad q\Pi \equiv q_{\mu}\Pi_{\mu} \equiv -q^2
$$

to get

$$
\frac{1}{R(\tau) + ie} = \frac{1}{2\sqrt{(q\Pi)^2 - q^2\Pi^2}} \left( \frac{1}{\tau - \tilde{\xi}_+ + ie} - \frac{1}{\tau - \tilde{\xi}_- - ie} \right). \quad (IV.13)
$$

Now, we take the imaginary part

$$
i \operatorname{Im} \frac{1}{R(1) + ie} = -\frac{i\pi}{2} \frac{1}{\sqrt{(q\Pi)^2 - q^2\Pi^2}} \left[ \delta(1 - \tilde{\xi}_+) + \delta(1 - \tilde{\xi}_-) \right]. \quad (IV.14)
$$

Obviously, the following equalities

$$
(q^2 + q\Pi) \delta(1 - \tilde{\xi}_\pm) = \pm \left[ \sqrt{(q\Pi)^2 - q^2\Pi^2} \right] \delta(1 - \tilde{\xi}_\pm) = -(q^2 + q\Pi) \delta(1 - \tilde{\xi}_\pm)
$$

can be used to simplify some of the resulting expressions below.

In addition, let us introduce, instead of $z_+$ and $z_-$, new variables, $t$ and $\zeta$, being more adequate for the generalized Bjorken region, namely (the arrow shows their value in the limiting case of forward scattering, i.e., for $P_1 = P_2 = P$)

$$
\begin{align*}
x &= \frac{-q^2}{qp_+} \implies x_{\text{Bj}}, & \eta &= \frac{qp_-}{qp_+} \implies 0, & -\rho^2 &= \frac{p_-^2}{p_+^2} = \frac{M^2 - P_1 P_2}{M^2 + P_1 P_2} \implies 0, & p_+^2 &= 2(M^2 + P_1 P_2) \implies 4M^2,
\end{align*}
$$

together with

$$
t = z_+ + \eta z_- \implies z, \quad \zeta = z_-/t \implies 0, \quad (IV.16)
$$

which means that $z_+$ and $z_-$ is scaled according to

$$
z_+ = t(1 - \eta \zeta), \quad z_- = t\zeta. \quad (IV.17)
$$

This leads to

$$
\begin{align*}
\Pi^\mu &= \kappa t \left( p_+^\mu (1 - \eta \zeta) + p_-^\mu \zeta \right) \equiv \kappa t \mathcal{P}^\mu(\eta, \zeta), \\
q\Pi &= \kappa t q \mathcal{P} \equiv \kappa t q_{\mu} p_+ \implies 2\kappa z q P, \\
\Pi^2 &= (\kappa t)^2 p_+^2 \left[ 1 - 2\eta \zeta + (\eta^2 - \rho^2)\zeta^2 \right] \equiv (\kappa t)^2 \mathcal{P}^2 \implies 4\kappa^2 z^2 M^2 \quad (IV.18)
\end{align*}
$$

and

$$
\tilde{\xi}_\pm = \frac{x}{\kappa t} \frac{1}{1 + \sqrt{1 + x^2\mathcal{P}^2/Q^2}} \equiv \frac{\xi_\pm}{t} \implies x_{\text{Bj}} = P_1 \frac{2}{z} \pm \frac{1}{1 + \sqrt{1 + 4x_{\text{Bj}}^2 M^2/Q^2}}.
$$

Obviously, $\tilde{\xi}_\pm$ are appropriate generalizations of the Nachtmann variable to the non-forward case and $t$ measures the collinear momentum, i.e. the portion of momentum in direction $p_+$. 

The variable $\tilde{t} = z_+ + z_- \tilde{x}p_−/\tilde{x}p_+$ or, rather, $t = z_+ + z_- q_P^−/q_P^+$ has been originally introduced for a partial diagonalization of the renormalization group equation for the generalized parton distributions $f(z_+,z_-) = \hat{f}(t,z_-;\eta)$ in Refs. [6], cf. also [12]. As a second step, the reparametrization $z_- = \tau_\zeta$ leads to $\hat{f}(t,\zeta;\eta)$ where the external parameter $\eta$ and the internal variable $\zeta$ parametrize the deviation from the forward direction.

With these definitions the measure of the $Z$-integration gets $(0 \leq |\eta| \leq 1$, see, [4, 12])

$$DZ = 2dz_+dz_-d\theta(1-z_+ + z_-)\theta(1+z_+ - z_-)\theta(1-z_+ - z_-)\theta(1+z_+ + z_-) = 2ttdt\zeta\theta(1-t + (1+\eta)t\zeta)\theta(1-t - (1-\eta)t\zeta)\theta(1+t - (1-\eta)t\zeta).$$  \hfill (IV.20)

With these new variables for the imaginary part of the trace of the Compton amplitude we get:

\[
\text{Im} T_{\text{trace}}(q) = \text{Im} T^+_{\text{trace}}(q) + \text{Im} T^-_{\text{trace}}(q), \\
\text{Im} T^\pm_{\text{trace}}(q) = \frac{\pi}{2} \int DZ \frac{-\frac{q^2}{2}}{(q\Pi^2 - q^2\Pi^2)^{3/2}} \delta(1 - \xi_\pm) \times \\
\left\{ (qK^a) \left[ (q\Pi + \Pi^2) \Phi^{(0)}_a(Z) + \Pi^2 \Phi^{(1)}_a(Z) \right] - (\Pi K^a) \left[ (q^2 + q\Pi) \Phi^{(0)}_a(Z) + (q\Pi) \Phi^{(1)}_a(Z) \right] \right\} \\
= \pi \int d\zeta \int dt \delta(t - \xi_\pm(\zeta)) \frac{4x}{1 + x^2P^2/Q^2} \left\{ \mp \frac{qK^a}{qP} \left( 1 + \frac{t^2xP^2}{2Q^2} \phi^{(0)}_a(t,\zeta) \right) \right. \\
+ \left. \frac{x}{2Q^2} \left( \frac{qK^a}{qP} - \frac{P K^a}{P^2} \right) \left[ \mp t \phi^{(0)}_a(t,\zeta) + \frac{1}{\sqrt{1 + x^2P^2/Q^2}} \phi^{(1)}_a(t,\zeta) \right] \right\}. \hspace{1cm} \text{(IV.21)}
\]

where (IV.15) has been used and $\Phi^{(0)}_a(Z)$, $\Phi^{(1)}_a(Z)$ has been replaced by

\[
\Phi^{(0)}_a(Z) \equiv \phi^{(0)}_a(t,\zeta), \hspace{1cm} \Phi^{(1)}_a(Z) \equiv t \int_0^1 \frac{dx}{\pi x} \phi_a \left( \frac{t}{x},\zeta \right) = \int_0^1 dy \phi_a(y,\zeta) \equiv \phi^{(1)}_a(t,\zeta). \hspace{1cm} \text{(IV.22)}
\]

Now, taking into account the following relations

\[
1 + \frac{1}{2} x \xi_\pm P^2/Q^2 = \pm x \sqrt{1 + x^2P^2/Q^2} = -(1 - 2x/\xi_\pm), \hspace{1cm} \text{(IV.23)}
\]

\[
\frac{x}{1 + x^2P^2/Q^2} = \frac{x}{2Q^2} \left[ \mp t \phi^{(0)}_a(t,\zeta) + \frac{1}{\sqrt{1 + x^2P^2/Q^2}} \phi^{(1)}_a(t,\zeta) \right], \hspace{1cm} \text{(IV.24)}
\]

and introducing the following functions,

\[
\mathcal{V}_{\alpha_0}^\pm(x,\eta;\zeta) \equiv \frac{x \phi^{(0)}_a(\xi_\pm,\zeta)}{\sqrt{1 + x^2P^2/Q^2}}, \hspace{1cm} \text{(IV.27)}
\]

\[
\mathcal{V}_{\alpha_1}^\pm(x,\eta;\zeta) \equiv x \frac{\partial}{\partial x} \left( \frac{x \phi^{(0)}_a(\xi_\pm,\zeta)}{\sqrt{1 + x^2P^2/Q^2}} \right) = \frac{x}{1 + x^2P^2/Q^2} \left[ \mp \xi_\pm \phi^{(0)}_a(\xi_\pm,\zeta) + \frac{1}{\sqrt{1 + x^2P^2/Q^2}} \phi^{(1)}_a(\xi_\pm,\zeta) \right], \hspace{1cm} \text{(IV.28)}
\]

one finally finds for the imaginary part of the trace of the non-forward Compton amplitude

\[
\text{Im} T_{\text{trace}}(q) = 4\pi \int d\zeta \left\{ \frac{qK^a}{qP} \mathcal{V}_{\alpha_0}(x,\eta;\zeta) + \left( \frac{qK^a}{qP} - \frac{P K^a}{P^2} \right) \frac{x}{2Q^2} \mathcal{V}_{\alpha_1}(x,\eta;\zeta) \right\}. \hspace{1cm} \text{(IV.29)}
\]

with $\mathcal{V}_{\alpha_n}(x,\eta;\zeta) = \mathcal{V}_{\alpha_n}^+(x,\eta;\zeta) + \mathcal{V}_{\alpha_n}^-(x,\eta;\zeta)$, $n = 0, 1$.

In the case of forward scattering, $\eta = 0$, one obtains simply (in terms of $x = x_B$)

\[
\text{Im} f T_{\text{trace}}(q) = 2\pi \frac{x}{\sqrt{1 + 4x^2M^2/Q^2}} f(x), \hspace{1cm} \text{(IV.30)}
\]

\[
f(x) = 2 \int d\zeta \phi^{(0)}_d(x;\zeta), \hspace{1cm} \text{(IV.31)}
\]

because only the Dirac form factor survives.
V. NON-FORWARD COMPTON AMPLITUDE: ANTISYMMETRIC PART (NONLOCAL)

Now, after having introduced the procedure of revealing the structure of non-forward amplitudes, let us study the Fourier transform of the antisymmetric part of the leading twist-2 non-forward Compton amplitude:

\[
T_{[\mu \nu]}^{tw2}(q) = \int \frac{d^4x}{2\pi^2} \frac{e^{i q x}}{(x^2 - i\epsilon)^2} \epsilon_{\mu \nu \alpha \beta} x_\alpha \langle P_2, S_2 | (O_{5a}^{tw2}(\kappa x, -\kappa x) + O_{5b}^{tw2}(-\kappa x, \kappa x)) | P_1, S_1 \rangle
\]

\[
= \int_0^1 dr_1 \int_0^1 dr_2 \int d^4u(P_2, S_2) \left( O_{5a}(u) + O_{5a}(-u) \right) | P_1, S_1 \rangle \right) \partial_u^{\epsilon_{\mu \nu \alpha \beta} q^\alpha u_\beta} \left[ (q + \kappa r_1 \tau_2 u)^2 + i\epsilon \right]^{-\Pi_\beta}
\]

where, in the second line, we used the expression (III.22) and, in the last line, we observed the correspondence

\[
\langle P_2, S_2 | (O_{5a}(u) + O_{5a}(-u)) | P_1, S_1 \rangle = K_{\alpha}^{5a}(P, S) \int DZ \delta^4(u - PZ) \Phi_{5a}(Z, \mu^2),
\]

\[
\Phi_{5a}(Z, \mu^2) = f_{5a}^{tw2}(Z, \mu^2) + t_{5a}(Z, \mu^2) = \Phi_{5a}(-Z, \mu^2).
\]

together with the abbreviations (V.3) and (V.4).

In the same manner as in the case of the trace part we have to perform appropriate partial integrations in order to obtain an expression which finally depends on 1/\([R(1) + i\epsilon]\) only. Since the explicit computation, despite being straightforward, is somewhat cumbersome we postpone it to Appendix C. The result reads as follows:

\[
T_{[\mu \nu]}^{tw2}(q) = \int DZ \epsilon_{\mu \nu \alpha \beta} \frac{-q^2 q_\alpha}{2((q\Pi)^2 - q^2\Pi^2)} \left\{ K_{\alpha}^{5a} F_{a1}^{(5)}(Z) + \Pi_\beta(qK^a) F_{a2}^{(5)}(Z) + \Pi_\beta(\Pi K^a) F_{a3}^{(5)}(Z) \right\} \frac{1}{R(1) + i\epsilon},
\]

with

\[
F_{a1}^{(5)}(Z) = (q\Pi + \Pi^2) \Phi_{5a}^{(1)}(Z) + \Pi^2 \Phi_{5a}^{(2)}(Z),
\]

\[
F_{a2}^{(5)}(Z) = \Phi_{5a}^{(0)}(Z) - \left( \frac{3}{[(q\Pi)^2 - q^2\Pi^2]} \right) \Phi_{5a}^{(1)}(Z) - \left( \frac{3}{[(q\Pi)^2 - q^2\Pi^2]} \right) \Phi_{5a}^{(2)}(Z),
\]

\[
F_{a3}^{(5)}(Z) = \Phi_{5a}^{(0)}(Z) + \left( \frac{3}{[(q\Pi)^2 - q^2\Pi^2]} \right) \Phi_{5a}^{(1)}(Z) + \left( \frac{3}{[(q\Pi)^2 - q^2\Pi^2]} \right) \Phi_{5a}^{(2)}(Z),
\]

where the generalized distribution amplitudes \(\Phi_{5a}^{(n)}(Z)\) are defined in the same manner as \(\Phi_{5a}^{(n)}(Z)\) in Eqs. (IV.4).

Now, let us consider the imaginary part of that expression. Again, only the terms with 1/\([R(1) + i\epsilon]\) contribute, leading to the following expression for the imaginary part of the (antisymmetric part of) the Compton amplitude:

\[
\text{Im} T_{[\mu \nu]}^{tw2}(q) = \frac{-\pi}{4} \int d\zeta \int dt \epsilon_{\mu \nu \alpha \beta} \frac{-q^2}{[(q\Pi)^2 - q^2\Pi^2]^{3/2}} \left[ \delta(1 - \xi_+(\zeta)/t) + \delta(1 - \xi_-(\zeta)/t) \right] \times
\]

\[
\left\{ q_\alpha K_{\alpha}^{5a} \left[ (q\Pi + \Pi^2) \Phi_{5a}^{(1)}(t, \zeta) + \Pi^2 \Phi_{5a}^{(2)}(t, \zeta) \right] + \frac{q_\alpha \Pi_\beta(qK^a)}{[(q\Pi)^2 - q^2\Pi^2]} \left[ \Phi_{5a}^{(0)}(t, \zeta) - \left( \frac{3}{[(q\Pi)^2 - q^2\Pi^2]} \right) \Phi_{5a}^{(1)}(t, \zeta) + \left( \frac{3}{[(q\Pi)^2 - q^2\Pi^2]} \right) \Phi_{5a}^{(2)}(t, \zeta) \right] + \frac{q_\alpha \Pi_\beta(\Pi K^a)}{[(q\Pi)^2 - q^2\Pi^2]} \left[ \Phi_{5a}^{(0)}(t, \zeta) + \left( \frac{3}{[(q\Pi)^2 - q^2\Pi^2]} \right) \Phi_{5a}^{(1)}(t, \zeta) + \left( \frac{3}{[(q\Pi)^2 - q^2\Pi^2]} \right) \Phi_{5a}^{(2)}(t, \zeta) \right] \right\}.
\]

Using the \(\delta\)-functions the \(t\)-integration can be carried out. Let us consider a generic expression of the corresponding integrand:

\[
I^{(i)} = \int DZ \Phi_{5a}^{(i)}(Z) G^{(i)}(Z, P) \delta(1 - \xi_\pm) = \int_{-\infty}^{\infty} d\zeta \int_{-1}^{1} dt \Phi_{5a}^{(i)}(t, \zeta) G^{(i)}(t, \zeta) \delta(1 - \xi_\pm/t)
\]

\[
= \int d\zeta \xi_\pm(\zeta) G^{(i)}(\xi_\pm, \zeta) \tilde{\Phi}_{5a}^{(i)}(\xi_\pm, \zeta),
\]

(V.7)
where $G^{(i)}(Z, P)$ is some function of the indicated arguments and, in the case $\xi > 0$, we introduced new distribution amplitudes (for $\xi < 0$ the integral has to be adapted correspondingly),

$$\hat{\Phi}^{(0)}(\xi, \zeta) \equiv \xi \Phi(\xi, \zeta) \quad \text{and} \quad \hat{\Phi}^{(i)}(\xi, \zeta) = \int_{\xi}^{1} \frac{dy_{1}}{y_{1}} \hat{\Phi}^{(i-1)}(y_{1}, \zeta).$$  \hspace{1cm} (V.8)

The proof of relation (V.8) for the general case is as follows:

$$I^{(n)} = \int d\zeta \int dt \, G(t, \zeta) \Phi^{(n)}(t, \zeta) \delta(1 - \xi/t)$$

$$= \int d\zeta \int dt \, G(t, \zeta) \int_{0}^{t} \frac{d\tau_{1}}{\tau_{1}^{2}} \cdots \int_{0}^{t \tau_{n-1}} \frac{d\tau_{n}}{\tau_{n}^{2}} \Phi\left(\frac{t}{\tau_{1} \cdots \tau_{n} - \tau_{n} \cdot \xi}, \zeta\right) \delta(1 - \xi/t)$$

$$= \int d\zeta \int dt \, G(t, \zeta) \int_{0}^{t} \frac{d\tau_{1}}{\tau_{1}^{2}} \cdots \int_{0}^{t \tau_{n-1}} \frac{d\tau_{n}}{\tau_{n}^{2}} \int_{0}^{1} dy_{n} \tau_{1} \cdots \tau_{n-1} y_{n} \Phi(y_{n}, \zeta) \delta(1 - \xi/t)$$

$$= \int d\zeta \int dt \, G(t, \zeta) \int_{t}^{1} \frac{dy_{1}}{y_{1}} \cdots \int_{y_{n-2}}^{1} \frac{dy_{n}}{y_{n}} \Phi(y_{n}, \zeta) \delta(t - \xi)$$

$$\int d\xi \, G(\xi, \zeta) \int_{\xi}^{1} \frac{dy_{1}}{y_{1}} \cdots \int_{y_{n-1}}^{1} \frac{dy_{n}}{y_{n}} \hat{\Phi}^{(0)}(y_{1}, \zeta),$$ \hspace{1cm} (V.9)

where a successive change of variables $y_{n} = t/(\tau_{1} \cdots \tau_{n-1} \tau_{n})$, $m = n, \ldots, 1$, has been made and the support restriction of $\Phi(t, \zeta)$ to $-1 \leq t \leq 1$ has been observed.

Now, taking relation (V.8) into account we obtain for the two different contributions related to $\xi_{\pm} = t\xi_{\pm}$ the following expression:

$$\int T^{w_{\mu\nu}}_{w_{\mu\nu}}(q) = \int d\xi \epsilon_{\mu\nu}^{\alpha\beta} \left( -\frac{2x}{\xi_{\pm}} \left[ 1 + \frac{1}{x^{2}\bar{P}^{2}/Q^{2}} \right] \right) \times$$

$$\left\{ \begin{array}{l}
\frac{q_{a} K_{\alpha}}{q P} \left[ 1 + \frac{1}{x^{2} \bar{P}^{2}/Q^{2}} \right] \hat{\Phi}_{5a}^{(1)}(\xi_{\pm}, \zeta) + \frac{1}{2} x \xi_{\pm} \bar{P}^{2}/Q^{2} \hat{\Phi}_{5a}^{(2)}(\xi_{\pm}, \zeta)

+ \frac{q_{a} P_{\beta}}{q P} \left( \frac{q K_{\gamma}}{q P} \right) \hat{\Phi}_{5a}^{(0)}(\xi_{\pm}, \zeta) + \frac{1}{2} x \xi_{\pm} \bar{P}^{2}/Q^{2} \hat{\Phi}_{5a}^{(2)}(\xi_{\pm}, \zeta)

+ \frac{x \xi_{\pm} \bar{P}^{2} q_{a} P_{\beta}}{2 Q^{2}} \hat{\Phi}_{5a}^{(0)}(\xi_{\pm}, \zeta) + \frac{3}{2} \frac{x \xi_{\pm} \bar{P}^{2}/Q^{2}}{1 + x^{2} \bar{P}^{2}/Q^{2}} \hat{\Phi}_{5a}^{(1)}(\xi_{\pm}, \zeta) + \frac{2 - x^{2} \bar{P}^{2}/Q^{2}}{1 + x^{2} \bar{P}^{2}/Q^{2}} \hat{\Phi}_{5a}^{(2)}(\xi_{\pm}, \zeta)
\end{array} \right\},$$ \hspace{1cm} (V.10)

where the equalities (V.23) and (V.24), after $t$-integration, are used. Already here the similarity to the structure of the antisymmetric part of the Compton amplitude in the forward case, Eqs. (I.2), may be observed: ignoring the variable $\zeta$ as well as reading $\xi_{\pm}$ as $\xi$, the first and the second term in the curly bracket has the structure of $G_{1}(\xi) + G_{2}(\xi)$ and $G_{1}(\xi)$, respectively, whereas the last one is new but would vanish in the forward case. Indeed, taking into account relations (V.23) and (V.24), one finds the following equalities:

$$g_{a1}^{\pm}(x, \eta; \zeta) = \frac{x}{\xi_{\pm}} \left[ x \xi_{\pm} \frac{\hat{\Phi}_{5a}^{(2)}(\xi_{\pm}, \zeta)}{[1 + x^{2} \bar{P}^{2}/Q^{2}]^{1/2}} \right],$$

$$= \frac{1}{\xi_{\pm}} \left[ x \xi_{\pm} \bar{P}^{2}/Q^{2} \hat{\Phi}_{5a}^{(0)}(\xi_{\pm}, \zeta) + \frac{x(1 \pm x)}{1 + x^{2} \bar{P}^{2}/Q^{2}} \hat{\Phi}_{5a}^{(1)}(\xi_{\pm}, \zeta) - \frac{1}{2} x \xi_{\pm} \bar{P}^{2}/Q^{2} \hat{\Phi}_{5a}^{(2)}(\xi_{\pm}, \zeta) \right].$$ \hspace{1cm} (V.11)
\[ g_{a2}^\pm (x, \eta; \zeta) \equiv -x \frac{\partial^2}{\partial x^2} x \left( \frac{x}{\xi_\pm} \frac{\Phi_{5a}^{(2)} (\xi_\pm, \zeta)}{[1 + x^2 P^2/Q^2]^{3/2}} \right) \]

\[ = -x \frac{1}{\xi_\pm} \left[ \frac{1 - x \xi_\pm P^2/\zeta^2}{[1 + x^2 P^2/Q^2]^{3/2}} \right] \]

\[ \times \left[ \frac{\Phi_{5a}^{(0)} (\xi_\pm, \zeta) \mp 1 - x \xi_\pm P^2/\zeta^2}{[1 + x^2 P^2/Q^2]^{3/2}} \right] \frac{\Phi_{5a}^{(1)} (\pm \xi_\pm, \zeta) - \frac{3}{2} \frac{x \xi_\pm P^2/\zeta^2}{[1 + x^2 P^2/Q^2]} \Phi_{5a}^{(2)} (\xi_\pm, \zeta)}{x} , \] (V.12)

\[ g_{a1}^\pm (x, \eta; \zeta) + g_{a2}^\pm (x, \eta; \zeta) \equiv -x \frac{\partial}{\partial x} \left( \frac{x}{\xi_\pm} \frac{\Phi_{5a}^{(2)} (\xi_\pm, \zeta)}{[1 + x^2 P^2/Q^2]^{3/2}} \right) \]

\[ = \frac{x^2}{\xi_\pm} \frac{1}{[1 + x^2 P^2/Q^2]^{3/2}} \times \left[ \frac{1 + \frac{1}{2} x \xi_\pm P^2/\zeta^2}{[1 + x^2 P^2/Q^2]^{3/2}} \Phi_{5a}^{(1)} (\xi_\pm, \zeta) + \frac{1}{2} x \xi_\pm P^2/\zeta^2 \Phi_{5a}^{(2)} (\xi_\pm, \zeta) \right] , \] (V.13)

\[ g_{a0}^\pm (x, \eta; \zeta) = x^2 \frac{\partial^2}{\partial x^2} \left( \frac{x^2}{[1 + x^2 P^2/Q^2]^{3/2}} \right) \]

\[ = \frac{x^2}{[1 + x^2 P^2/Q^2]^{3/2}} \times \left[ \frac{\Phi_{5a}^{(0)} (\xi_\pm, \zeta)}{[1 + x^2 P^2/Q^2]^{3/2}} \right] \frac{3}{1 + \frac{1}{2} x \xi_\pm P^2/\zeta^2} \frac{2}{[1 + x^2 P^2/Q^2]} \Phi_{5a}^{(1)} (\xi_\pm, \zeta) + \frac{2}{1 + x^2 P^2/Q^2} \frac{\Phi_{5a}^{(2)} (\xi_\pm, \zeta)}{x} \right] , \] (V.14)

where repeated use has been made of relations (V.23). With these definitions we finally get

\[ \text{Im} T_{[\mu]^{\pm}}^{\text{tw2}} (q) = \text{Im} T_{[\mu]}^{\text{tw2}+} (q) + \text{Im} T_{[\mu]}^{\text{tw2}+} (q) \] (V.15)

\[ \text{Im} T_{[\mu]}^{\text{tw2}+} (q) = -\frac{\pi}{2 \kappa} \int d\zeta \epsilon_{\mu}^{\alpha \beta} \left\{ \frac{q_\alpha P_\beta}{q P} \left[ g_{a1}^\pm (x, \eta; \zeta) + g_{a2}^\pm (x, \eta; \zeta) \right] - \frac{q_\alpha P_\beta (q K_\alpha)}{q P} g_2^\pm (x, \eta; \zeta) + \frac{1}{2} \frac{P^2}{Q^2} \frac{q_\alpha P_\beta (P K_\alpha)}{P^2} g_{a0}^\pm (x, \eta; \zeta) \right\} , \] (V.16)

where \( P = P (\eta, \zeta) \) depends also on \( \zeta \)!

Obviously, the contributions to the various kinematical expressions are related to only two independent quantities. Namely, introducing \( F_a (x, \eta; \zeta) = F_a^+ (x, \eta; \zeta) + F_a^- (x, \eta; \zeta) \) and \( F_0 (x, \eta; \zeta) = \xi_\pm F_a^+ (x, \eta; \zeta) + \xi_- F_a^- (x, \eta; \zeta) \) with

\[ F_a^\pm (x, \eta; \zeta) = \left( \frac{x}{\xi_\pm} \frac{\Phi_{5a}^{(2)} (\xi_\pm, \zeta)}{[1 + x^2 P^2/Q^2]^{3/2}} \right) \] (V.17)

we may rewrite the relations (V.11) - (V.14) as follows

\[ g_{a1} (x, \eta; \zeta) = x \frac{\partial}{\partial x} x \frac{\partial}{\partial x} F_a (x, \eta; \zeta) , \]

\[ g_{a2} (x, \eta; \zeta) = -x \frac{\partial}{\partial x} (x \frac{\partial}{\partial x} + 1) F_a (x, \eta; \zeta) , \]

\[ g_{a0} (x, \eta; \zeta) = x \frac{\partial}{\partial x} (x \frac{\partial}{\partial x} - 1) x F_0 (x, \eta; \zeta) . \] (V.18)

The first two of these relations are identical in their functional dependence with the corresponding equations in the forward case as given in Ref. 50. Therefore, they will lead to the same general consequences: Observing

\[ x \frac{\partial}{\partial x} F_a (x, \eta; \zeta) = - \int_y^1 \frac{dy}{y} g_{a1} (y, \eta; \zeta) , \]

\[ F_a (x, \eta; \zeta) = - \int_y^1 \frac{dy}{y} \int_y^1 \frac{dy'}{y'} g_{a1} (y', \eta; \zeta) , \]

we find that, surprisingly, the well known Wandzura-Wilczek relation from the case of forward scattering 36 gets generalized also to the non-forward case, namely, the (twist-2 part of) \( g_{a2} (x, \eta; \zeta) \) fulfills the same relation as does the (twist-2 part of) \( g_{a2} (x) \),

\[ g_{a2} \left( x, \eta; \zeta \right) = - g_{a1} (x, \eta; \zeta) + \int_x^1 \frac{dy}{y} g_{a1} (y, \eta; \zeta) . \] (V.19)
However, more surprising is the fact that exactly the same structure occurs independent of the kinematic factors, i.e., for the Dirac and the Pauli form factor as well any other independent form factor. This means that we are confronted with a very general structure of the theory which seems to be independent of taking matrix elements. Instead, it might be conjectured that this structure is a property of the (leading part of the) Compton operator (when Fourier transformed with the leading part of the propagator).

In addition, another relation holds for the (twist-2 part of) the new structure function $g_{a0}(x, \eta; \zeta)$ which occurs only in the case of non-forward scattering and is suppressed by a factor $P^2/Q^2$:

$$
g_{a0}^{(2)}(x, \eta; \zeta) = (x \xi_\pm) g_{a1}^\pm(x, \eta; \zeta) - \frac{2x^2 + x \xi_\pm}{[1 + x^2 P^2/Q^2]^{1/2}} \int_x^1 \frac{dy}{y} g_{a1}^\pm(y, \eta; \zeta) - \frac{2x^2}{[1 + x^2 P^2/Q^2]^{3/2}} \int_x^1 \frac{dy}{y} \int_y^1 \frac{dy'}{y'} g_{a1}^\pm(y', \eta; \zeta).
$$

Let us add some remarks.

First we mention that, in principle, both roots, $\xi_+$ and $\xi_-$, will contribute. However, because of the kinematics for the generalized Bjorken region only the first one is of phenomenological relevance. Under these circumstances only one independent function $F^+_{a0}$ survives, and $g_{a0}^{(2)}(x, \eta; \zeta)$ is completely determined by $g_{a1}^+$. They are derived by using only the correct non-local twist-2 vector operators off-cone. In this respect their derivation is similar to the derivation of the WW-relation for the quark distribution functions in Ref. [34].

Second, we underline once more that these relations result without using the equations of motion or any other dynamical input as has been done by other authors [33]. They are derived by using only the correct non-local twist-2 vector operators off-cone. In this respect their derivation is similar to the derivation of the WW-relation for the quark distribution functions in Ref. [34].

Third, we emphasize that the derivation of these relations essentially depends on the use of appropriate coordinates $(t, \tau)$ with $t$ being well-adapted to the forward case and afterwards being replaced by the generalizations $\xi_\pm$ of the Nachtmann variable. Also some special relations had to be used which are fulfilled only when the imaginary part of the Compton amplitude is considered.

Finally, we point to the fact that both relations hold before the integration over $\zeta$ is performed. This means that the above mentioned general structure will be covered by the $\zeta$-dependence of the distribution amplitudes. In the case of forward scattering that integration disappears and the well-known result is obtained for the functions $g_{a0}(x)$ being independent of $\eta$ and $\zeta$ (see, e.g., [30]).

This finishes the consideration of the imaginary part of the non-forward Compton amplitude.

### VI. NON-FORWARD COMPTON AMPLITUDE: SYMMETRIC PART (NONLOCAL)

Now, we consider the Fourier transformation of the symmetric part of the leading twist-2 non-forward Compton amplitude being obtained by taking the matrix elements of the operator expression (V.25):

$$
T_{\{\mu\nu\}}^{tw2}(q) = i \int \frac{d^4x}{(2\pi)^2} \frac{e^{i qx}}{x^2 - i \epsilon} \Sigma^{\mu\nu\alpha\beta} x_\alpha \langle P_2, S_2 | O^{tw2}_\beta (\kappa x, -\kappa x) - O^{tw2}_\beta (-\kappa x, \kappa x) | P_1, S_1 \rangle
$$

$$
= 2i \int_0^1 \frac{d\tau}{\tau} \left( 1 - \tau + \tau \ln \tau \right) \int \frac{d^4u}{4\pi^4} \left( \langle P_2, S_2 | (O^\rho (u/k) - O^\rho (-u/k)) | P_1, S_1 \rangle \partial_\rho \right)
$$

$$
\times \left\{ (q^2 g_{\mu\nu} - q_\mu q_\nu) \left[ (uq)^2 - u^2 q^2 + \frac{1}{2} u^2 (q + u)^2 \right] + (u_\mu q_2^2 - q_\mu (uq)) (u_\nu q_2^2 - q_\nu (uq)) \right\}
$$

$$
= 2 \int_0^1 d\tau_1 \int_0^1 d\tau_2 \int_0^1 d\tau_3 \int_{DZ} \lambda^{tw2}(Z, \mu^2) K^a_\rho (\mathcal{P}, \mathcal{S}) \left[ q^2 \right]^{3} \frac{1}{\kappa \tau_1 \tau_2 \tau_3} \partial_\rho^a
$$

$$
\times \left\{ \frac{2A^T_{\mu\nu}}{(q + \kappa \tau_1 \tau_2 \tau_3 \mathcal{P} \mathcal{Z})} \frac{1}{(q + \kappa \tau_1 \tau_2 \tau_3 \mathcal{P} \mathcal{Z})^2 + i \epsilon} + \frac{B^T_{\mu\nu}}{(q - \kappa \tau_1 \tau_2 \tau_3 \mathcal{P} \mathcal{Z})} \frac{1}{(q - \kappa \tau_1 \tau_2 \tau_3 \mathcal{P} \mathcal{Z})^2 + i \epsilon} \right\}
$$

$$
= 2 \int_0^1 d\tau \int_0^1 d\sigma \int_0^1 d\rho \int_{DZ} \Phi_\alpha(Z, \mu^2) K^a_\rho (\mathcal{P}, \mathcal{S}) \left[ q^2 \right]^{3} \frac{1}{\kappa \tau_1 \tau_2 \tau_3} \partial_\rho^a
$$

$$
\times \left\{ \frac{2A^T_{\mu\nu}}{(q + \kappa \tau_1 \tau_2 \tau_3 \mathcal{P} \mathcal{Z})} \frac{1}{(q + \kappa \tau_1 \tau_2 \tau_3 \mathcal{P} \mathcal{Z})^2 + i \epsilon} + \frac{B^T_{\mu\nu}}{(q - \kappa \tau_1 \tau_2 \tau_3 \mathcal{P} \mathcal{Z})} \frac{1}{(q - \kappa \tau_1 \tau_2 \tau_3 \mathcal{P} \mathcal{Z})^2 + i \epsilon} \right\}
$$

Again, we used the correspondence (IV.2), the kinematic form factors $K^a_\alpha (\mathcal{P}, \mathcal{S}) = \{ \tilde{u}(P_2, S_2)\gamma_\alpha u(P_1, S_1), \tilde{u}(P_2, S_2)\gamma_\alpha \gamma_5 u(P_1, S_1), \cdots \}$ and the generalized distribution amplitudes $\Phi_\alpha(Z, \mu^2)$ introduced already for the trace...
The explicit calculation will be sketched in the Appendix D. Since, finally, we are interested solely in the imaginary third power appears, and, second, because also the additional integrations over \(1/(\tilde{\Phi} + i\epsilon)\). But here the computation is much more complicated than in the antisymmetric case, first, because a third power appears, and, second, because also the additional integrations over \(\sigma\) and \(\rho\) have to be taken into account. The explicit calculation will be sketched in the Appendix D. Since, finally, we are interested solely in the imaginary part for which, due to the appearance of \((q^2)^3\), the terms proportional to \(1/(\tilde{R}(0) + i\epsilon)\) are irrelevant, we determined only those terms which are proportional to \(1/(\tilde{R}(1) + i\epsilon)\) (where, again \(\tilde{R}(1) = (q + \Pi)^2\), as in the antisymmetric case).

The result of the calculation is as follows:

\[
T_{(\mu \nu)}^{\text{tw}2}(q) = \frac{q^2}{2} \int D\zeta \left\{ \frac{qK_a}{q\Pi} \left( \frac{q^2\Pi^2}{(q\Pi)^2} F_1^a(\zeta) + \frac{q^2\Pi^2}{(q\Pi)^2} F_2^a(\zeta) \right) + \left( \frac{qK_a}{q\Pi} - \frac{\Pi K_a}{\Pi^2} \right) \left[ \frac{q^2\Pi^2}{(q\Pi)^2} F_3^a(\zeta) + \frac{q^2\Pi^2}{(q\Pi)^2} F_4^a(\zeta) \right] \right\}, \tag{VI.4}
\]

and

\[
T_{\text{trace}}^{\text{tw}2}(q) = \frac{q^2}{2} \int D\zeta \left\{ \frac{qK_a}{q\Pi} \left( 3F_1^a(\zeta) - F_2^a(\zeta) \right) + \left( \frac{qK_a}{q\Pi} - \frac{\Pi K_a}{\Pi^2} \right) \left( 3F_3^a(\zeta) - F_4^a(\zeta) \right) - \frac{q^2\Pi^2}{(q\Pi)^2} F_5^a(\zeta) \right\}, \tag{VI.5}
\]

with the functions \(F^a(\zeta, q, \Pi)\) being given as follows:

\[
F_1^a(\zeta) = \Phi_a(\zeta) + \frac{\Pi^2(q\Pi + \Pi^2)}{(q\Pi)^2 - q^2\Pi^2} \int_0^1 \frac{d\tau_2}{\tau_2} \Phi_a(\frac{\zeta}{\tau_2}) + \frac{\Pi^2(q\Pi - \Pi^2)}{(q\Pi)^2 - q^2\Pi^2} \int_0^1 \frac{d\tau_1}{\tau_1} \Phi_a(\frac{\zeta}{\tau_1}), \tag{VI.6}
\]

\[
F_2^a(\zeta) = \Phi_a(\zeta) + \frac{3\Pi^2(q\Pi + \Pi^2)}{(q\Pi)^2 - q^2\Pi^2} \int_0^1 \frac{d\tau_2}{\tau_2} \Phi_a(\frac{\zeta}{\tau_2}) + \frac{3\Pi^2(q\Pi - \Pi^2)}{(q\Pi)^2 - q^2\Pi^2} \int_0^1 \frac{d\tau_1}{\tau_1} \Phi_a(\frac{\zeta}{\tau_1}), \tag{VI.7}
\]

\[
F_3^a(\zeta) = \frac{\Pi^2(q\Pi + q^2)}{(q\Pi)^2 - q^2\Pi^2} \Phi_a(\zeta) + \frac{\Pi^2(q\Pi - q^2)}{(q\Pi)^2 - q^2\Pi^2} \int_0^1 \frac{d\tau_3}{\tau_3} \Phi_a(\frac{\zeta}{\tau_3}) - \frac{\Pi^2[q(q\Pi)^2 - q^2\Pi^2]}{(q\Pi)^2 - q^2\Pi^2} \int_0^1 \frac{d\tau_1}{\tau_1} \Phi_a(\frac{\zeta}{\tau_1}), \tag{VI.8}
\]

\[
F_4^a(\zeta) = \frac{\Pi^2(q\Pi + q^2)}{(q\Pi)^2 - q^2\Pi^2} \Phi_a(\zeta) + \frac{\Pi^2(q\Pi - q^2)}{(q\Pi)^2 - q^2\Pi^2} \int_0^1 \frac{d\tau_3}{\tau_3} \Phi_a(\frac{\zeta}{\tau_3}) - \frac{\Pi^2[q(q\Pi)^2 - q^2\Pi^2]}{(q\Pi)^2 - q^2\Pi^2} \int_0^1 \frac{d\tau_1}{\tau_1} \Phi_a(\frac{\zeta}{\tau_1}), \tag{VI.9}
\]

\[
F_5^a(\zeta) = \frac{\Pi^2(q\Pi + q^2)}{(q\Pi)^2 - q^2\Pi^2} \Phi_a(\zeta) + \frac{\Pi^2(q\Pi - q^2)}{(q\Pi)^2 - q^2\Pi^2} \int_0^1 \frac{d\tau_3}{\tau_3} \Phi_a(\frac{\zeta}{\tau_3}) - \frac{\Pi^2[q(q\Pi)^2 - q^2\Pi^2]}{(q\Pi)^2 - q^2\Pi^2} \int_0^1 \frac{d\tau_1}{\tau_1} \Phi_a(\frac{\zeta}{\tau_1}).
\]
thereby, to restrict to the forward case (where \( F_a \) vanishes when the imaginary part is taken). Therefore, because of the similarity in the structure of the structure of various integrals are reduced by one. Furthermore, we observe that the functions \( \Phi(z) = \frac{\Pi^2[\Pi + i q^2]}{(\Pi)^2 - q^2P^2} \int_0^1 \frac{dr_3}{r_3^2} \Phi_a \left( \frac{Z}{r_3} \right) \)

\[-3 \frac{\Pi^2}{(\Pi)^2 - q^2P^2} \int_0^1 \frac{dr_3}{r_3^2} \int_0^1 \frac{dr_2}{r_2^2} \Phi_a \left( \frac{Z}{r_3 r_2} \right) \]

\[= 3F_3^a (Z) - 2 \frac{\Pi^2}{(\Pi)^2 - q^2P^2} F_5^a (Z) \]

\[= 3F_3^a (Z) - 2 \frac{\Pi^2}{(\Pi)^2 - q^2P^2} F_5^a (Z) \]

\[F_5^a (Z) = \int_0^1 \frac{dr_3}{r_3^2} \Phi_a \left( \frac{Z}{r_3} \right) + 3 \frac{\Pi^2}{(\Pi)^2 - q^2P^2} \int_0^1 \frac{dr_3}{r_3^2} \int_0^1 \frac{dr_2}{r_2^2} \Phi_a \left( \frac{Z}{r_3 r_2} \right) \]

\[= \int_0^1 \frac{dr}{r^2} F_2^a \left( \frac{Z}{r} \right) \]

Using these expressions for the trace part \( \text{VI.5} \) we get, modulo \( 1/R(0)\)-terms, the former result \( \text{IV.20} \).

\[ T_{\text{trace}}^{tw} (q) = \int DZ \frac{q^2}{(\Pi)^2 - q^2P^2} \left\{ (qK^a) \left[ (\Pi + \Pi^2) \Phi_a (Z) + \Pi^2 \int_0^1 \frac{dr}{r^2} \Phi_a \left( \frac{Z}{r} \right) \right] \right. \]

\[\left. - (\Pi\Pi^a) \left[ (q^2 + \Pi^2) \Phi_a (Z) + \Pi \int_0^1 \frac{dr}{r^2} \Phi_a \left( \frac{Z}{r} \right) \right] \right\} \frac{1}{R(1) + i\epsilon}. \]

Furthermore, we observe that the functions \( F_i^a, i = 3, 4, 5 \), are expressed by the functions \( F_1 \) and \( F_2 \) as well as those combinations of (integrals over) \( \Phi_a(\zeta/\tau) \) which already appear in the trace part (and for \( F_3 \) a term which vanishes when the imaginary part is taken). Therefore, because of the similarity in the structure of \( F_1 \) and \( F_2 \) with the structure of \( W_1 \) and \( W_2 \) in Eqs. \( \text{VI.1} \) it will be advantageous first to introduce the new coordinates \( (t, \zeta) \) and, thereby, to restrict to the forward case (where \( F_i^a, i = 3, 4, 5 \), do not contribute).

### A. Restriction to the forward case

Therefore, let us consider the case of forward scattering, i.e., \( P_1 = P_2 = P = \frac{1}{2} p_+ \), \( p_- \equiv 0 \), \( S_1 = S_2 = S \). Then we get \( (z_+ \equiv z \) and \( z_- \) being integrated out)

\[ K_\mu^a \equiv K_\mu^D = 2P_\mu, \quad \Pi_\mu = P_\mu z, \]

and the expression \( \text{VI.14} \) reduces as follows:

\[ fT_{\mu
u}^{tw} (q) = q^2 \int z \left\{ q_{\mu\nu} F_1 f (z) + \frac{q^2 P_\mu^T P_\nu^T}{(qP)^2 - q^2P^2} F_2 f (z) \right\} \]

with

\[ F_1 f (z) = f (z) + \frac{P^2 (zq + z^2P^2)}{(qP)^2 - q^2P^2} \int_0^1 \frac{dr_2}{r_2^2} f \left( \frac{z}{r_2} \right) + \frac{z^2 (P^2)^2}{(qP)^2 - q^2P^2} \int_0^1 \frac{dr_2}{r_2^2} \int_0^1 \frac{dr_1}{r_1^2} f \left( \frac{z}{r_1 r_2} \right) \]

\[ F_2 f (z) = f (z) + \frac{3P^2 (zq + z^2P^2)}{(qP)^2 - q^2P^2} \int_0^1 \frac{dr_2}{r_2^2} f \left( \frac{z}{r_2} \right) + \frac{3z^2 (P^2)^2}{(qP)^2 - q^2P^2} \int_0^1 \frac{dr_2}{r_2^2} \int_0^1 \frac{dr_1}{r_1^2} f \left( \frac{z}{r_1 r_2} \right) \]

with \( f \equiv \Phi_D \). Of course, in the forward case only a single variable \( z \) occurs and, therefore, the powers of \( r_i \) in the various integrals are reduced by one.
Now, we introduce generalized distribution amplitudes of order \( n \) adjusted to the forward case:

\[
\begin{align*}
    f^{(0)}(z) &\equiv \frac{1}{z} f(z), \\
    f^{(1)}(z) &\equiv \int_{z} dy \frac{f(y)}{y} = \int_{0}^{1} \frac{d\tau_{1}}{\tau_{1}} f\left(\frac{z}{\tau_{1}}\right), \\
    f^{(2)}(z) &\equiv \int_{z} dy_{2} \int_{y_{2}} dy_{1} \frac{f(y_{1})}{y_{1}} = z \int_{0}^{1} \frac{d\tau_{1}}{\tau_{1}} \int_{0}^{1} \frac{d\tau_{2}}{\tau_{2}} f\left(\frac{z}{\tau_{1}\tau_{2}}\right), \\
    f^{(n)}(z) &\equiv \int_{z} dy f^{(n-1)}(y),
\end{align*}
\]

where the support restriction \( f(z) = 0 \) for \( z > 1 \) has been used. Then, observing the overall factor of \( 1/z \), the integrand of Eq. (VI.13) can be written in terms of these generalized distribution amplitudes \( f^{(n)}(z) \).

Taking the imaginary part of the amplitude (VI.13), observing \( \text{Im} \frac{1}{2z \sqrt{(qP)^2 - q^2 P^2}} (\delta(1 - \xi+/z) + \delta(1 - \xi-/z)) \) (VI.17) with the well-known Nachtmann variable(s)

\[
\xi_{\pm} = \frac{2x}{1 \pm \sqrt{1 + 4x^2 P^2/Q^2}} \quad \text{with} \quad x \equiv x_{Bj} = Q^2/(2qP), \tag{VI.18}
\]

and using the equalities (VI.13) and the distributions (VI.16), we get for the two different contributions,

\[
\begin{align*}
    \text{Im} f^{T^{tw}_{\mu\nu}}(q) &\equiv \frac{-\pi}{2} \int dz \; \delta(z - \xi_{\pm}) \\
        &\times \left[ \frac{q^2 g^{T}_{\mu\nu}}{[(qP)^2 - q^2 P^2]^{1/2}} \left\{ f^{(0)}(z) \pm \frac{P^2}{[(qP)^2 - q^2 P^2]^{1/2}} f^{(1)}(z) + \frac{(P^2)^2}{[(qP)^2 - q^2 P^2]^{1/2}} f^{(2)}(z) \right\} \\
        &+ \frac{q^2 P^T P^T_{\mu} P^T_{\nu}}{[(qP)^2 - q^2 P^2]^{3/2}} \left\{ f^{(0)}(z) \pm 3 \frac{P^2}{[(qP)^2 - q^2 P^2]^{1/2}} f^{(1)}(z) + 3 \frac{(P^2)^2}{[(qP)^2 - q^2 P^2]^{1/2}} f^{(2)}(z) \right\} \right]
\end{align*}
\]

and

\[
\frac{1}{\pi} \text{Im} f^{T^{tw}_{\text{trace+}}} = \frac{2x}{[1 + 4x^2 M^2/Q^2]^{1/2}} f^{(0)}(\xi_{\pm}). \tag{VI.20}
\]

Taking into account the definition of the structure functions \( W_1 \) and \( W_2 \) according to

\[
\frac{1}{\pi} \text{Im} f^{T^{tw}_{\mu\nu}}(q) = -g^{T}_{\mu\nu} W_1(x, Q^2) + \frac{P^T_{\mu} P^T_{\nu}}{M^2} W_2(x, Q^2) \tag{VI.21}
\]

we observe that both contributions independently coincide with the result of Georgi and Politzer \cite{41} leading to the well known consequences \( (G(\xi) = f^{(2)}(\xi), \xi = \xi_{\pm}) \):

\[
\begin{align*}
    2x W_L &\equiv \frac{4x^2 M^2}{Q^2} - \frac{4x^2 M^2}{Q^2} \frac{\partial}{\partial x} \left( \frac{x}{\xi} G(\xi) / \sqrt{1 + 4x^2 M^2/Q^2} \right), \tag{VI.22}
    \\
    \nu W_2/M &\equiv W_2 Q^2/2x M^2 = x^2 \frac{\partial^2}{\partial x^2} \left( \frac{x^2}{\xi^2} G(\xi) / \sqrt{1 + 4x^2 M^2/Q^2} \right), \tag{VI.23}
    \\
    2x W_1 &\equiv \frac{(1 + 4x^2 M^2/Q^2)^2}{M} W_2 - 2x W_L \\
    &\equiv \frac{\nu W_2}{M} + \frac{4x^2 M^2}{Q^2} \left[ \frac{x}{\xi} \left( \left( x \frac{\partial}{\partial x} \right) - (x \frac{\partial}{\partial x}) \right) \frac{x^2}{\xi^2} G(\xi) / \sqrt{1 + 4x^2 M^2/Q^2} \right]. \tag{VI.24}
\end{align*}
\]

For the kinematic situation considered in Ref. \cite{41} the terms related to \( \xi_{-} \) do not contribute.
B. The symmetric part of the Compton amplitude in non-forward case

Now, let us consider the imaginary part of the symmetric Compton amplitude in the nonforward case already introducing the variables \((t, \zeta)\) and using \(\Pi_\mu = \kappa t \mathcal{P}_\mu\):

\[
\text{Im} T^\text{w2}_{(\mu\nu)} (q) = -\frac{\pi}{2\kappa^2} \int d\zeta \int \frac{dt}{t} (\frac{q^2}{(q^2)^2 - q^2 P^2})^{1/2} \left[ \delta(1 - \xi_+(\zeta)/t) + \delta(1 - \xi_-(\zeta)/t) \right] \times 
\]

\[
\left\{ \frac{q K_a}{q^2} \left[ g^{\mu T}_{\nu} F_1^a (t, \zeta) + \frac{q^2 P^T_{\mu} P^T_{\nu}}{(q^2)^2 - q^2 P^2} F_2^a (t, \zeta) \right] 
+ \left( \frac{q K_a}{q^2} - \frac{\mathcal{P} K_{\mu}}{P^2} \right) g^T_{\mu} F_3^a (t, \zeta) + \frac{q^2 P^T_{\mu} P^T_{\nu}}{(q^2)^2 - q^2 P^2} F_2^a (t, \zeta) \right\} 
\]

with the functions \(F^a(Z) \equiv F^a(t, \zeta; q, \kappa t, \mathcal{P}), \ i = 1, \ldots, 5\), being obtained from Eqs. \((\text{VI.16}) - (\text{VI.19})\) thereby making use of equalities \((\text{IV.15})\).

Let us first consider \(F_1^a\) and \(F_2^a\) which survive in the forward case:

\[
F_1^a (t, \zeta) = \Phi_a (t, \zeta) + \frac{3 \kappa^2 t^2 P^2}{(q^2)^2 - q^2 P^2} \int_0^1 \frac{dx}{x} \frac{1}{\tau_1} \Phi_a \left( \frac{t}{\tau_1}, \zeta \right),
\]

\[
F_2^a (t, \zeta) = \Phi_a (t, \zeta) + \frac{3 \kappa^2 t^2 P^2}{(q^2)^2 - q^2 P^2} \int_0^1 \frac{dx}{x} \frac{1}{\tau_1} \Phi_a \left( \frac{t}{\tau_1}, \zeta \right),
\]

where the sign \(\pm\) is related to the sign of \(\xi_{\pm}\).

Now, let us use the distribution amplitudes \((\text{IV.22})\) as being introduced for the trace part and extend them to arbitrary order \(n\):

\[
\phi_a^{(0)} (t, \zeta) = \Phi_a (t, \zeta), \quad \phi_a^{(n)} (t, \zeta) = \int_0^1 \frac{dy}{y} \phi_a^{(n-1)} (y, \zeta) \quad \text{for} \quad n \geq 1,
\]

where the same symbol \(\Phi_a\) has been used also after changing variables \(z_+, z_-\) into \(t, \zeta\); this leads to

\[
\phi_a^{(1)} (t, \zeta) = t \int_0^1 \frac{dx}{x} \frac{1}{\tau_1} \Phi_a \left( \frac{t}{\tau_1}, \zeta \right), \quad \phi_a^{(2)} (t, \zeta) = t^2 \int_0^1 \frac{dx}{x} \int_0^1 \frac{dy}{y} \phi_a \left( \frac{t}{\tau_1}, \zeta \right).
\]

Therefore, the contribution of \(F_1^a\) and \(F_2^a\) to the symmetric non-forward Compton amplitude is given by

\[
\text{Im} T^\text{w2}_{(\mu\nu)} (q) = -\frac{\pi}{2\kappa^2} \int d\zeta \int \frac{dt}{t} \frac{q^2}{(q^2)^2 - q^2 P^2} \delta(t - \xi_{\pm}) \frac{q K_a}{q^2} \left[ 
\frac{\kappa^2 t^2 P^2}{(q^2)^2 - q^2 P^2} \phi_a^{(0)} (t, \zeta) \right] 
+ \frac{\kappa^2 [P^2]^2}{(q^2)^2 - q^2 P^2} \phi_a^{(1)} (t, \zeta) \right] 
+ \frac{3 \kappa^2 t^2 P^2}{(q^2)^2 - q^2 P^2} \phi_a^{(1)} (t, \zeta) \right].
\]

Comparing Eqs. \((\text{VI.19})\) and \((\text{VI.26})\), besides the additional \(\zeta\)–integration, the structure of that expression is completely analogous to the corresponding one \((\text{IV.19})\) of the forward case (remind \(\kappa = 1/2\) and \(\mathcal{P} \Rightarrow 2\mathcal{P}\)).

\[
\frac{1}{\pi} \text{Im} T^\text{w2}_{(\mu\nu)} (q) = 2 \int d\zeta \frac{q K_a}{q^2} \left[ -q_{\mu \nu} \mathcal{W}_1 (x, \eta; \zeta) + \frac{P^T_{\mu} P^T_{\nu}}{P^2} \mathcal{W}_2 (x, \eta; \zeta) \right].
\]

Therefore, the corresponding distribution amplitudes generalize to the non-forward case as follows:

\[
2x \mathcal{W}_1 (x, \eta; \zeta) = \frac{x^2 P^2}{Q^2} \frac{\partial}{\partial x} \left( x^2 \phi_a^{(2)} (\xi_{\pm}, \zeta) \sqrt{1 + x^2 P^2/Q^2} \right),
\]

\[
\frac{2q P}{P^2} \mathcal{W}_2 (x, \eta; \zeta) = \frac{x^2}{Q^2} \phi_a^{(2)} (\xi_{\pm}, \zeta) \sqrt{1 + x^2 P^2/Q^2},
\]

\[
2x \mathcal{W}_1 (x, \eta; \zeta) = (1 + x^2 P^2/Q^2) \frac{2q P}{P^2} \mathcal{W}_2 (x, \eta; \zeta) - 2x \mathcal{W}_1 (x, \eta; \zeta),
\]
having exactly the same structure as Eqs. (VI.22) – (VI.24). In fact, the last relation generalizes the (power corrected) Callan-Gross relation to the non-forward case being also there of the same shape as in the forward case. This surprising result, of course, supports our conjecture that it is an outcome of the structure of the Compton operator (in that approximation). In principle, also here we could repeat the remarks at the end of the last Section concerning the derivation of that result.

Let us take the trace of expressions (VI.26) and (VI.27) and compare both with the result (VI.29) we find

\[ 2 \mathcal{V}_{a0}(x, \eta; \zeta) = -3 W_{a1}(x, \eta; \zeta) + (1 + x^2 P^2/Q^2)^2 \frac{(qP)}{x P^2} W_{a2}(x, \eta; \zeta) = W_{aL}(x, \eta; \zeta) - 2 W_{a1}(x, \eta; \zeta), \]  

which relates \( \mathcal{V}_{a0} \) to \( \mathcal{W}_{a1} \).

Now, we are in a position to discuss the remaining parts of the symmetric Compton amplitude which in the forward case vanish, i.e. the contributions of the last two lines of Eq. (VI.25). Making use of Eqs. (IV.15) and the conventions leading to (VI.26) for the structure functions (VI.3) – (VI.10) we obtain:

\[
F_3^a(t, \zeta) = - \int_0^1 \frac{dt}{t^2} \left[ F_1^a(t, \zeta) + \frac{(qP)^2}{(qP)^2 - q^2 P^2} F_2^a(t, \zeta) \right] + \frac{2qP}{[(qP)^2 - q^2 P^2]^{1/2}} \int_0^1 \frac{dt}{t^2} \left( \pm \phi_a^0(t, \zeta) + \frac{\kappa P^2}{[(qP)^2 - q^2 P^2]^{1/2}} \phi_a^{(1)}(t, \zeta) \right), 
\]

\[
F_4^a(t, \zeta) = \frac{3}{2} F_3^a(t, \zeta) - 2 \frac{q^2 P^2}{[(qP)^2 - q^2 P^2]^{1/2}} F_5^a(t, \zeta) \]

\[
F_5^a(t, \zeta) = \int_0^1 \frac{dt}{t^2} F_2^a(t, \zeta). 
\]

Since the expressions in the round brackets already have been considered in case of the trace part we hold all the ends in our hands which are necessary to present the wanted part \( \text{Im}^1 T_{\mu\nu}^{(T_{\mu\nu})} \) of the non-forward Compton amplitude. After performing the \( t \)-integration we obtain

\[
\text{Im}^1 T_{\mu\nu}^{(T_{\mu\nu})} (q) = \frac{\pi}{2k^2} \int d\zeta \times \left\{ \left( \frac{qK_a}{qP} - \frac{P K_a}{P^2} \right) \frac{P T_{\mu\nu}^T}{P^2} \left[ \int_0^1 \frac{dt}{t^2} \left( \frac{x}{1 + x^2 P^2/Q^2} \right)^{1/2} F_1^a \left( \frac{x P^2}{Q^2} \right)^{1/2} F_2^a \left( \frac{x P^2}{Q^2} \right)^{1/2} \right] + \frac{2x}{[1 + x^2 P^2/Q^2]} \left( \pm \phi_a^0(t, \zeta) + \frac{x P^2/Q^2}{2[1 + x^2 P^2/Q^2]^{1/2}} \phi_a^{(1)}(t, \zeta) \right) \right. 
\]
\[
\left. + \frac{x^2 P^2/Q^2}{2[1 + x^2 P^2/Q^2]} \left( \pm \xi \phi_a^0(\xi, \zeta) + \frac{1}{[1 + x^2 P^2/Q^2]^{1/2}} \phi_a^{(1)}(\xi, \zeta) \right) \right) 
\]
\[
- \left( \frac{qK_a}{qP} - \frac{P K_a}{P^2} \right) \frac{P T_{\mu\nu}^T}{P^2} \left[ \int_0^1 \frac{dt}{t^2} \left( \frac{x^2 P^2/Q^2}{1 + x^2 P^2/Q^2} \right)^{1/2} \left( \frac{3x}{1 + x^2 P^2/Q^2} \right)^{1/2} + \frac{2x^3 P^2/Q^2}{1 + x^2 P^2/Q^2} \right] \right. 
\]
\[
\left. \left. - \frac{x^2 P^2/Q^2}{[1 + x^2 P^2/Q^2]^{1/2}} \left( \pm \xi \phi_a^0(\xi, \zeta) + \frac{1}{[1 + x^2 P^2/Q^2]^{1/2}} \phi_a^{(1)}(\xi, \zeta) \right) \right) \right) 
\]
\[
- \left( \frac{K_{\mu\nu} T_{\mu\nu}^T}{P^2} + P_{\mu\nu} T_{\mu\nu} \frac{K_{\mu\nu}}{qP} \right) \frac{x^3 P^2/Q^2}{[1 + x^2 P^2/Q^2]^{1/2}} \int_0^1 \frac{dt}{t^2} F_2^a \left( \frac{x P^2}{Q^2} \right)^{1/2} \right) \].  

(Here, for simplicity of notation, once more the dependent function \( F_3 \) appears. However, the expression \( x F_3/\sqrt{1 + x^2 P^2/Q^2} \) (fifth line) exactly coincides with the expression in angular brackets which multiplies \( g_{\mu\nu}^T \) (lines two to four). After that observation and, according to the consideration of the expression for \( \text{Im}^0 T_{\mu\nu}^{(T_{\mu\nu})} (q) \), relating \( F_1 \) and \( F_2 \) to \( \mathcal{W}_{a1}(\zeta) \) and \( \mathcal{W}_{a2}(\zeta) \), also using \( (1 + x^2 P^2/Q^2)/(x^2/Q^2) = (P^T)^2 \), we end up with the following
expression:

\[
\text{Im} T^{(\pi/2)}_{(\mu\nu)}(q) = 2\pi \int dq \left\{ \frac{qK_a}{qQ} \frac{P_K}{P^2} g_{\mu\nu} - \frac{3}{2} \frac{P_T^\mu P_T^\nu}{(P^2)^2} \right\} \times \\
\left[ \int_0^1 \frac{dt}{t^2} \left( W_{a1}(\xi_+, x; \zeta) + \frac{Q^2}{x^2 P^2} W_{a2}(\xi_+, x; \zeta) \right) \right. \\
\left. + \int_0^1 \frac{dt}{t^2} \left( 2V_{a0}(\xi_+, x; \zeta) + \frac{x P^2}{Q^2} V_{a1}(\xi_+, x; \zeta) \right) + \frac{x P^2}{2 Q^2} W_{a1}(\xi_+, x; \zeta) \right] \\
+ 2 \frac{qK_a}{qP} \frac{P_T^\mu P_T^\nu}{P^2} \left( \int_0^1 \frac{dt}{t^2} W_{a2}(\xi_+, x; \zeta) + \frac{x P^2}{2 Q^2} V_{a1}(\xi_+, x; \zeta) \right) \\
+ \frac{K_T^\mu P_T^\nu + P_T^\mu K_T^\nu}{P^2} \int_0^1 \frac{dt}{t^2} W_{a2}(\xi_+, x; \zeta), \right. 
\]  
\tag{VI.36}
\]

where \( W_{a1} \) and \( W_{a2} \) are given by (VI.30) and (VI.29), and \( V_{a0} \) and \( V_{a1} \) are given by (VI.27) and (VI.28), respectively.

Taking the trace of this expression the angular bracket in the first line vanishes and the pre-factors of the \( \tau \)-integrals in the last two lines coincide up to sign so that only the term

\[
2\pi \frac{qK_a}{qP} \frac{P_T^\mu P_T^\nu}{P^2} \frac{x P^2}{Q^2} V_{a1}(\xi_+, x; \zeta) 
\]  
\tag{VI.37}
\]
survives as it should be, cf. Eq. (VI.29). However, as for the case of \( V_{a0}(\xi_+, x; \zeta) \), we are able to relate \( V_{a1}(\xi_+, x; \zeta) \) to the structure functions \( W_{a1}(\xi_+, x; \zeta) \). First, we observe that

\[
x \frac{\partial}{\partial x} \phi_a^{(2)}(\xi_+, \zeta) = \mp \frac{\xi_+ \phi_a^{(1)}(\xi_+, \zeta)}{1 + x^2 P^2/Q^2}.
\]  
\tag{VI.38}
\]

Then, according to the definition (VI.28), making repeated use of Eqs. (VI.28) - (VI.26), with the help of (VI.28) we get

\[
V_{a1}(\xi_+, x; \zeta) = x \frac{\partial}{\partial x} \sqrt{1 + x^2 P^2/Q^2} \frac{\partial}{\partial x} \frac{\phi_a^{(2)}(\xi_+, \zeta)}{\xi_+} + \frac{x \xi_+ P^2/Q^2}{2 \sqrt{1 + x^2 P^2/Q^2}} \frac{\partial}{\partial x} \frac{\phi_a^{(2)}(\xi_+, \zeta)}{\xi_+} \\
= \mp x \frac{\partial}{\partial x} \frac{2 Q^2}{x^2} \sqrt{1 + x^2 P^2/Q^2} \frac{\partial}{\partial x} \frac{W_{aL}(\xi_+, x; \zeta)}{\xi_+} - \frac{x \xi_+ P^2/Q^2}{2 \sqrt{1 + x^2 P^2/Q^2}} \frac{\partial}{\partial x} \frac{W_{aL}(\xi_+, x; \zeta)}{\xi_+} \\
= x \frac{P^2}{2 Q^2} \frac{W_{aL}(\xi_+, x; \zeta)}{\xi_+} = \frac{1}{2} x \xi_+ P^2/Q^2 \\
\]  
\tag{VI.39}
\]

so that, finally using Eq. (VI.28) again, we find

\[
\frac{x P^2}{2 Q^2} V_{a1}(\xi_+, x; \zeta) = [1 + \frac{1}{2} x \xi_+ P^2/Q^2] \left[ x \frac{\partial}{\partial x} W_{aL}(\xi_+, x; \zeta) - \frac{1 - \frac{1}{2} x \xi_+ P^2/Q^2}{[1 + x^2 P^2/Q^2]} W_{aL}(\xi_+, x; \zeta) \\
- \frac{x^3 P^2/Q^2}{[1 + x^2 P^2/Q^2]^2} \frac{\partial}{\partial x} W_{aL}(\xi_+, x; \zeta) \right].
\]  
\tag{VI.40}
\]

Also here, like in the case of the antisymmetric part of the Compton amplitude, we find that those contributions to the non-forward case, whose kinematic factors will vanish in the forward limit, are completely determined by the non-forward generalizations of the forward distribution amplitudes! In addition, the absorptive part of the symmetric off-forward Compton amplitude is determined by the two structure functions \( W_{a1}(\xi_+, x; \zeta) \) and \( W_{aL}(\xi_+, x; \zeta) \) only!

This finishes our consideration of the symmetric part of the Compton amplitude.
VII. CONCLUSIONS

The complete twist-2 part of the Compton amplitude has been studied including all target mass corrections. Starting from the well-known expression of the Compton operator in coordinate space we performed its relatively complicated Fourier transform in a manner which allows to reveal its intrinsic structure. Whereas in a previous work \[16\] the Fourier transform is taken for the matrix elements directly we transformed the operator expression first and formed the matrix elements afterwards. This procedure allows besides a clear separation of theoretically different contributions the consideration of different kinematical decompositions of the matrix element and, in principle, the consideration of other external states. As a result we derived a closed expressions for the Compton amplitude in terms of iterated generalized parton distribution amplitudes, Eqs. (V.22) and (VI.34) (For the symmetric part the polynomial contributions which do not influence the absorptive part are suppressed). This appears to be much better than the expressions (III.23) and (III.20) being completely integrated out.

Specializing to the forward case the generalized Wandzura-Wilczek relation of Blümlein-Tkabladze \[50\] and the extended Callan-Gross relation of Georgi-Politzer \[11\] have been explicitly recovered; the latter is in accordance with a remark by \[51\]. In addition, we were able to show that both relations for the imaginary part can be generalized also to the non-forward case when introducing suitable new parameters $t = z_+ + \eta z_-\,$, $\zeta = z_- / t$. In terms of these variables, with $t$ being replaced by generalized Nachtmann variables $\xi_{\pm}$, the non-forward relations between the generalized structure functions are of the same shape as in the forward case but appear as superposition of amplitudes for different values of $\zeta$, cf. Eqs. (V.10) and (VI.27). In addition, we were able to show that those contributions which will vanish in the forward case are completely determined by the generalizations of the non-vanishing ones to the non-forward case, cf. Eqs. (V.20) and (VI.30), together with the non-forward expressions for the trace part, Eq. (IV.29). Thereby, the relevant structure functions are uniquely expressed by appropriately defined generalized $n-$th order distribution amplitudes of $\Phi_{\alpha}^{(n)}(\xi_{\pm}, x; \zeta)$, cf. Eqs. (V.11) – (V.14) and Eqs. (VI.28) – (VI.30), but, even more interesting, only three independent structure functions, $q_1 W_1$ and $W_2$, determine the absorptive part of the complete amplitude at twist-2. Let us also emphasize that analogous structures will appear also for more general matrix elements, especially those with an additional meson in the final state \[11\]. Namely, the conclusion depends mainly on the possibility of introducing, among others, a variable $t$ measuring the collinear momentum. These surprising results are a strong hint that they follow from a hidden structure of the Compton operator itself and not from taking its matrix elements.

The Fourier transform of the twist-2 Compton operator has been performed mainly using its local decomposition and re-summing the result to its non-local expression. In principle, it is the same procedure as has been developed in Ref. \[51\]. In order to control this procedure we performed the Fourier transform of the trace part of Compton amplitude directly and we found coinciding results. Also the remaining parts of the amplitude could be transformed in this way but, surely, it will more tedious than in the considered example. Whether Fourier transforming the local or the non-local operators – or its matrix elements – it turned out necessarily that the vector $P_1 z_1 + P_2 z_2$ for any $0 \leq z_1, z_2 \leq 1$ must be time-like. This is an important support restriction for the generalized distribution amplitudes – which in the literature, usually, is not mentioned. We have the strong impression that this condition is not only a mathematical one but could be of physical relevance. This rests on the observation that this condition is important for getting the correct sign for the imaginary part as well as that the generalized Nachtmann variable also makes sense for non-asymptotic momenta as it should be!

One drawback of our calculation concerns the current conservation. As known for the twist-2 part of the non-forward Compton amplitude \[16\], current conservation is valid only in a very restricted sense or, hardly speaking, is even not fulfilled. This is well-known \[57\] and, therefore, also here it has not been expected. For the forward case current conservation can be read off directly from the resulting kinematical decomposition. Let us also remark, that a generalization of the Wandzura-Wilczek relation for the complete amplitude, as it is the case for the leading terms \[16\] we did not obtain (of course, the consideration of the complete amplitude remained an open problem). Surely, such a relation exists and may be obtained using dispersion relations, but this study is outside the present work.

Finally, let us remark that the target mass corrections resulting from the twist-2 part of the Compton amplitude alone are hardly of phenomenological relevance. They are to be completed by additional competing target mass corrections from the higher twist operators. However, the methods being advocated here, may be applied also to these objects.

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APPENDIX A: FOURIER TRANSFORM OF PROPAGATOR AND COMPTON OPERATOR

1. Fourier transform of the propagator function; helpful relations

First, let us give a derivation of the Fourier transformation of the propagator function. We start with the well known result (cf., e.g., [55], Eq. (9.716))

\[ \int \frac{d^4 x \ e^{-i q x}}{(x^2 - i \epsilon)^\lambda} = - i \pi^2 \frac{\Gamma(2 - \lambda)}{\Gamma(\lambda)} \frac{4^{2-\lambda}}{(q^2 + i \epsilon)^{2-\lambda}}. \] (A.1)

Applying \((-i u \partial_u)^n\) on both sides, one obtains

\[ \int d^4 x \ e^{i q x} \frac{(ux)^n}{(x^2 - i \epsilon)^\lambda} = - i n! \pi^2 \frac{\Gamma(n+1)}{\Gamma(\lambda)} \frac{4^{2-\lambda+n/2}}{(q^2 + i \epsilon)^{n+2-\lambda}} \sum_{s=0}^{\lfloor n/2 \rfloor} \frac{\Gamma(n+2-\lambda-s)}{\Gamma(s+1) \Gamma(n-2s+1)} \left( \frac{-q^2}{4} \right)^s (u^2)^s (uq)^{n-2s} \]

\[ = - i n! \pi^2 \frac{\Gamma(n+1) \Gamma(2 - \lambda)}{\Gamma(\lambda)} \frac{4^{2-\lambda+n/2}}{(q^2 + i \epsilon)^{n+2-\lambda}} \left( \frac{1}{2} \sqrt{u^2 q^2} \right)^n C_n^{2-\lambda} \left( \frac{uq}{\sqrt{u^2 q^2}} \right). \] (A.2)

Obviously, taking \(\lambda = 2\) one gets Eq. [11.8]; then, applying \(-i \partial_u\) one gets Eq. [11.9]. Thereby we have used the first of the following collection of various relations which are obeyed by the generalized functions \(h_n^\nu(u, q)\):

\[ \partial^\alpha_\alpha h_n^\nu = q_\alpha h_{n+1}^\nu - u_\alpha h_{n-1}^\nu, \] (A.3)

\[ \partial^\alpha_\alpha h_n^\nu = -q_\alpha h_{n+1}^\nu + u_\alpha h_{n-1}^\nu, \] (A.4)

\[ (uq) h_n^\nu = (n+1) h_{n-1}^\nu + u^2 h_{n-1}^\nu, \] (A.5)

\[ 2(n + \nu) h_n^\nu = q^2 h_{n+1}^\nu - u^2 h_{n-1}^\nu, \] (A.6)

\[ \Box u h_n^\nu = 2(\nu - 1) h_{n+1}^\nu, \] (A.7)

\[ (q u^{n+1}) h_n^\nu = \frac{q^2}{2} h_{n+1}^\nu, \] (A.8)

\[ 2(n + \nu) u_\beta^{\nu+1} h_n^\nu = q^2 u_\beta h_{n+1}^\nu + u^2 q_\beta h_{n-1}^\nu, \] (A.9)

\[ \partial^\beta_\beta q^2 h_n^\nu = 2(n + \nu - 1) \left( u_\beta^{n+\nu-1} h_{n-1}^\nu - q_\beta h_n^\nu \right) \quad \text{for} \quad n \geq 1. \] (A.10)

In addition, the following integral representations of some fractions will be used below:

\[ \frac{1}{n+1} = \int_0^1 d\tau \ \tau^n, \] (A.11)

\[ \frac{1}{(n+1)^2} = \int_0^1 d\tau_1 \int_0^\tau d\tau_2 \ (\tau_1 \tau_2)^n = - \int_0^1 d\tau \ \ln \tau \ \tau^n, \] (A.12)

\[ \frac{1}{n (n+1)} = \int_0^1 \frac{d\tau_1}{\tau_1} \int_0^\tau d\tau_2 \ (\tau_1 \tau_2)^n = \int_0^1 \frac{d\tau}{\tau} \ (1 - \tau) \ \tau^n \quad \text{for} \quad n \geq 1, \] (A.13)

\[ \frac{1}{n (n+1)^2} = \int_0^1 \frac{d\tau_1}{\tau_1} \int_0^\tau \int_0^\tau d\tau_2 \ (\tau_1 \tau_2 \tau_3)^n = \int_0^1 \frac{d\tau}{\tau} \ (1 - \tau + \tau \ln \tau) \ \tau^n \quad \text{for} \quad n \geq 1. \] (A.14)

2. Fourier transformation of the Compton operator

Let us first perform the required Fourier transformation of the unsymmetrized Compton operator. Thereby, in order to make the similarity between the vector and axial vector case obvious, only the unsymmetrized operators have been written, but the final (anti)symmetrized expressions can easily be obtained due to the \(\kappa\)–dependence. We also omit the subscript (5) due to \(\gamma_5\).
Proceeding in the same manner as for the trace part we obtain in the first step:

\[ T_{\alpha\beta}^{tw2} (q) := \int \frac{d^4x}{(2\pi)^4} e^{i\eta x} \frac{x_{\alpha}}{(x^2 - i\epsilon)^2} O_{\beta}^{tw2} (\kappa x, -\kappa x) = \sum_{n=0}^{\infty} \frac{(i\kappa)^n}{n!} \int \frac{d^4x}{(2\pi)^4} e^{i\eta x} \frac{x_{\alpha}}{(x^2 - i\epsilon)^2} O_{\beta}^{tw2} (x) \]

\[ = \sum_{n=0}^{\infty} \frac{(i\kappa)^n}{(n+1)!} \int \frac{d^4x}{(2\pi)^4} e^{i\eta x} \frac{x_{\alpha}}{(x^2 - i\epsilon)^2} \partial_{\beta}^n H_{n+1} (x^2, \square x) \int d^4u O_{\rho}(u) x^\rho (ux)^n \]

\[ = \sum_{n=0}^{\infty} \frac{(i\kappa)^n}{(n+1)!} \int d^4u O_{\rho}(u) \partial_{\rho}^n H_{n+1} (u^2, \square u) \ u_{\beta} \int \frac{d^4x}{(2\pi)^4} e^{i\eta x} x_{\alpha} (ux)^n \]

then, the result (III.9) of the Fourier transformation is taken and in the next line the relations (III.17), (III.15) and (A.6) are used, thereby writing the contribution for \( n = 0 \) separately,

\[ \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-\kappa)^n}{n+1} \int d^4u O_{\rho}(u) \partial_{\rho}^n H_{n+1} (u^2, \square u) \ u_{\beta} (q_{\alpha} h_{n}^1 - u_{\alpha} h_{n-1}^1) \]

\[ = \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-\kappa)^n}{n+1} \int d^4u O_{\rho}(u) \partial_{\rho}^n (q_{\alpha} h_{n}^1 - u_{\alpha} h_{n-1}^1) \]

\[ = \frac{1}{4} \int d^4u O_{\rho}(u) \partial_{\rho}^n \left( 2 u_{\beta} \ln (q^2 / q_0^2) - \sum_{n=1}^{\infty} \frac{(-\kappa)^n}{n(n+1)} q^2 u_{\beta}^n h_{n}^1 \right), \]

and finally, the derivation w.r.t. \( q^\alpha \) is done observing (A.3) and (A.4),

\[ \tilde{T}_{\alpha\beta}^{tw2} (q) = g_{\alpha\beta} \frac{q_\alpha}{q^2} \int d^4u O^\rho (u) + \frac{1}{8} \sum_{n=0}^{\infty} \frac{(-\kappa)^n}{(n+1)^2} \int d^4u O_{\rho}(u) \partial_{\rho}^n \left( q_{\alpha} u_{\beta} (q^2 h_{n}^1 - 4 h_{n}^1) \right) + u^2 q^2 u_{\alpha} q_{\beta} h_{n-2}^3 + u^2 q^2 g_{\alpha\beta} h_{n-1}^2 - u^2 q_{\alpha} q_{\beta} (q^2 h_{n-1}^1 - 2 h_{n-1}^1) - \left( q^2 \right)^2 u_{\alpha} u_{\beta} h_{n-1}^1 \].

Now, for the axial vector, let us take the antisymmetric part of the expression (A.14) also observing the correct symmetry w.r.t. \( \kappa \). Obviously, only the first two terms in the bracket of Eq. (A.15) contribute:

\[ \tilde{T}_{[\alpha\beta]}^{tw2} (q) = \int \frac{d^4x}{2\pi^2} e^{i\eta x} \frac{1}{(x^2 - i\epsilon)^2} x_{[\alpha} \left( O_{\beta]}^{tw2} (\kappa x, -\kappa x) + O_{\beta]}^{tw2} (-\kappa x, \kappa x) \right) \]

\[ = \frac{q^2}{4} \sum_{n=0}^{\infty} \frac{\kappa^n (1 + (-1)^n)}{(n+1)^2} \int \frac{d^4u}{(2\pi)^4} O_{5\rho}(u) \partial_{\rho}^n \left( q_{[\alpha} u_{\beta]} h_{n}^2 \right) \]

\[ = \frac{q^2}{4} \int \frac{d\tau}{2} \int \frac{d\tau}{2} \int d^4u \left( O_{5\rho}(u) + O_{5\rho} (-u) \right) \partial_{\rho}^n \left( \frac{q_{[\alpha} u_{\beta]}}{(q + \kappa \tau_2 u)^2 + i\epsilon^2} \right), \]

where in the second line we used relation (A.4), and in the last line we used relations (A.12) and (III.16); in addition, we have taken the symmetry w.r.t. \( u \) instead of \( \kappa \). This proves our equation (III.22).

Now, we consider the symmetric part of the expression (A.14) thereby multiplying already with \( S_{\mu\nu;\alpha\beta} \). This leads to the following result

\[ S_{\mu\nu;\alpha\beta} \int \frac{d^4x}{2\pi^2} e^{i\eta x} \frac{1}{(x^2 - i\epsilon)^2} x_{\alpha} O_{\beta}^{tw2} (\kappa x, -\kappa x) \]

\[ = \left[ 2 g_{\rho\mu} q_{\nu} - g_{\mu\nu} q_{\rho} \right] \frac{1}{q^2} \int d^4u O^\rho (u) - \frac{q^2}{4} \sum_{n=1}^{\infty} \frac{(-\kappa)^n}{n(n+1)^2} \int d^4u O^\rho (u) \]

\[ \times \partial_{\rho}^n \left\{ \left( g_{\nu\mu} - \frac{q_{\nu} q_{\mu}}{q^2} \right) \left( 2 u^2 h_{n-1}^2 + \left[ (u^2)^2 - u^2 q^2 \right] h_{n-1}^3 \right) + q_{\mu} - \frac{q_{\mu} (uq)}{q^2} \right\} \left\{ u_{\nu} - \frac{q_{\nu} (uq)}{q^2} \right\} \left\{ h_{n-1}^3 \right\} \]

\[ = \left( 2 g_{\rho\mu} q_{\nu} - g_{\mu\nu} q_{\rho} \right) \int d^4u O^\rho (u) + 2 \int \frac{d\tau}{\kappa^2 \tau} (1 - \tau + \tau \ln \tau) \int \frac{d^4u}{\kappa^4 \tau^4} O^\rho (u) \]

\[ \times \partial_{\rho}^n \left\{ \left( q_{\nu} g_{\mu\rho} - q_{\mu} q_{\rho} \right) \left[ \left( (u^2)^2 - u^2 q^2 \right) + \frac{1}{2} u^2 (q + u)^2 \right] + \left( u_{\nu} q^2 - q_{\mu} (uq) \right) \left( u_{\rho} q^2 - q_{\nu} (uq) \right) \right\}. \]

For the first equality relations (A.3) and (A.9) are used, and for the second equality, after shifting the summation according to \( n \to n + 1 \), the relations (A.13) and (III.10) have been used. This proves our equation (III.23).
APPENDIX B: FOURIER TRANSFORM OF THE TRACE PART: NONLOCAL

The aim of this Appendix is to show, at least for the simple case of the trace of the Compton amplitude with mass corrections, that the Fourier transform, as has been performed in the Section III.B by using the local expansion, i.e., matrix elements of local operators, really coincides with the Fourier transform performed by using the nonlocal expression, as has been given in Ref. 30, thereby avoiding the explicit use of infinite sums whose convergence has not been proved.

The trace of the Compton amplitude is given by the following expression:

$$\langle P_2, S_2 | \hat{T}_{\text{trace}}^{\text{tw}^2}(q) | P_1, S_1 \rangle = -2i \int \frac{d^4x}{(2\pi)^4} \frac{e^{iqx}}{(x^2 - i\epsilon)^2} \left\langle P_2, S_2 \right| \left( O^{\text{tw}^2}(\kappa x, -\kappa x) - O^{\text{tw}^2}(-\kappa x, \kappa x) \right) | P_1, S_1 \rangle$$

$$= -2 \int D\mathcal{Z} \Phi_\alpha(\mathcal{Z}, \mu^2) K^0_\alpha(P, S) \int \frac{d^4x}{(2\pi)^4} \frac{e^{iqx}}{(x^2 - i\epsilon)^2} \left[ x^x (2 + \Pi \partial_\Pi - \frac{i}{2} \Pi' x^2) \right](3 + \Pi \partial_\Pi)$$

$$\sqrt{\pi} \left( \sqrt{(x^x)^2 - \Pi^2 x^2} \right)^{-3/2} J_3/2 \left( \frac{1}{2} \sqrt{(x^x)^2 - \Pi^2 x^2} \right) e^{iz\Pi/2},$$

(B.1)

where in the first line, Eq. (IV.1) of Section IV, the scalar twist-2 operator $O^{\text{tw}^2} \equiv x^\alpha O^{\text{tw}^2}_\alpha$ has been introduced and, in the second line, for the matrix element $\left\langle P_2, S_2 \right| \left( O^{\text{tw}^2}(\kappa x, -\kappa x) - O^{\text{tw}^2}(-\kappa x, \kappa x) \right) | P_1, S_1 \rangle$ we used the expression (2.19) of Ref. 30, cf. also Eq. (III.1) and observe $x^x \sim t \partial_t$. In addition, we used the abbreviations (IV.3) – (IV.5), e.g.,

$$\Pi_\mu = \kappa P_\mu Z, \quad \mathbb{P}Z = P_1 z_1 + P_2 z_2,$$

$$\Phi_\alpha(\mathcal{Z}, \mu^2) = f^{\text{tw}^2}_1(\mathcal{Z}, \mu^2) - f^{\text{tw}^2}_0(-\mathcal{Z}, \mu^2),$$

as well as replaced $x \partial_x \rightarrow \Pi \partial_\Pi$; $K^0_\alpha(P, S)$ denote, as the in previous sections, kinematical factors $\mathfrak{f}(P_2, S_2) \gamma_\nu u(P_1, S_1)$, $\mathfrak{f}(P_2, S_2) \sigma_{\mu\nu} \rho^\nu u(P_1, S_1)$ or others arising in the kinematical decomposition of the matrix element.

Already here we can see that there might occur some difficulties. Namely, wanting to avoid exponential growing of this expression the argument of the Bessel function $J_{3/2}(\sqrt{(x^x)^2 - \Pi^2 x^2}/2)$ must be real. For arbitrary values of $x$ this can be fulfilled for time-like $\mathbb{P}Z$ only. This requirement could lead to a strong support restriction for the generalized parton distribution $\Phi_\alpha(Z)$. However, the Bessel function results from an infinite summation and in application to Compton scattering only a finite number of powers of $x^2$ might be important, so this conclusion is possibly to restrictive.

For time-like $\mathbb{P}Z$ we choose, without restriction of generality, a special Lorentz frame, $\Pi = (\Pi_0 = \kappa p, 0, 0, 0)$. With this assumption all calculations can be performed straightforward.

To proceed with the Fourier transformation we use the following representations,

$$\frac{i}{4 \pi^2} \frac{1}{x^2 - i\epsilon} = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{1}{k^2 + i\epsilon};$$

(B.2)

$$\frac{1}{2 \pi^2} \frac{x^x}{(x^2 - i\epsilon)^2} = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{k'}{k^2 + i\epsilon};$$

(B.3)

$$\sqrt{\pi} \rho^{-3/4} J_3/2 \left( \frac{1}{2} \sqrt{\rho} \right) = \int d^4\ell e^{-i\ell x} \left[ \int d^4x' \frac{1}{8\Gamma(2)} \frac{1}{\ell x' - \ell x} \right] |d(1 - \tau^2) \exp \{ i\gamma \sqrt{\rho}/2 \}|$$

$$= \frac{1}{\pi(\Pi^2)^{3/2}} \int d^4\ell e^{-i\ell x} \frac{\delta(\ell_0) \Theta \left( \frac{1}{4} \Pi^2 - \ell^2 \right)}{\ell^2},$$

(B.4)

with $\rho := (x^x)^2 - \Pi^2 x^2 = (\kappa p)^2 x^2 \geq 0$,

(B.5)

where the first two are the well-known Fourier transforms of these distributions; the third relation is obtained by using the Poisson integral representation for the Bessel function (cf., Ref. 54, Eq. II.7.12.7), performing the corresponding Fourier transformation which, in the last line, is inverted. This is obtained in two steps, namely, using

$$\int \frac{d^4x}{(2\pi)^4} e^{i\ell x} \exp \{ i\gamma \kappa p \sqrt{\ell^2}/2 \} = \frac{i\delta(\ell_0)}{\ell \gamma (|\kappa p|)^2} \left\{ \frac{1}{(\tau + 2\ell / |\kappa p| + i\epsilon)^2} - \frac{1}{(\tau - 2\ell / |\kappa p| + i\epsilon)^2} \right\},$$

(B.6)

and

$$\int_{-1}^1 d\tau \left( 1 - \tau^2 \right) \left\{ \frac{1}{(\tau + y + i\epsilon)^2} - \frac{1}{(\tau - y + i\epsilon)^2} \right\} = -4\pi i y \Theta(1 - y) \Theta(1 + y) = -4\pi i y \Theta(1 - y^2).$$

(B.7)
Putting together these formulæ we obtain for the Compton amplitude

\[
\langle P_2, S_2 | T_{\text{tw}^2}(q) | P_1, S_1 \rangle = -\frac{2}{\pi} \int DZ \Phi_\alpha(Z) K_\nu^a(\mathcal{F}, S) \int d^4x \frac{1}{(2\pi)^4} \int d^4k [k^\nu (2 + \Pi\partial_1) - \Pi^\nu] (3 + \Pi\partial_1) \frac{1}{k^2 + i\epsilon} \int d^4\ell \frac{\delta(\hat{\ell}_0)}{(\Pi^2)^{3/2}} \Theta \left( \frac{1}{4}\Pi^2 - \ell^2 \right) \exp [ix(q - k - \ell + \Pi/2)] (B.8)
\]

\[
= -\frac{2}{\pi} \int DZ \Phi_\alpha(Z) K_\nu^a(\mathcal{F}, S) (2 + \Pi\partial_1) \left\{ (3 + \Pi\partial_1) I_3^\mu (q, \Pi) + \Pi^\nu I_2(q, \Pi) \right\}, (B.9)
\]

where the following integrals have been introduced:

\[
I_1^\mu(q, \Pi) = \int d^4k \int d^4\ell \frac{k^\nu}{k^2 + i\epsilon} \delta(\hat{\ell}_0) \delta^4(q - k - \ell + \Pi/2) \Theta \left( \frac{1}{4}\Pi^2 - \ell^2 \right) \frac{1}{(\Pi^2)^{3/2}}
\]

\[
= \int d^4\ell \frac{(q - \ell + \Pi/2)^\nu}{(q_0 + \sqrt{\Pi^2}/2)^2 - (\hat{q} - \hat{\ell})^2 + i\epsilon} \Theta \left( \frac{1}{4}\Pi^2 - \ell^2 \right) \frac{1}{(\Pi^2)^{3/2}},
\]

\[
I_2(q, \Pi) = \int d^4k \int d^4\ell \frac{1}{k^2 + i\epsilon} \delta(\hat{\ell}_0) \delta^4(q - k - \ell + \Pi/2) \Theta \left( \frac{1}{4}\Pi^2 - \ell^2 \right) \frac{1}{(\Pi^2)^{3/2}},
\]

\[
I_3(q, \Pi) = \frac{-1}{\sqrt{\Pi^2} \sqrt{(q^\nu + q^\mu)^2 - q^2 \Pi^2}} \int d^4\ell \frac{\ell \hat{q}}{(q_0 + \sqrt{\Pi^2}/2)^2 - (\hat{q} - \hat{\ell})^2 + i\epsilon} \Theta \left( \frac{1}{4}\Pi^2 - \ell^2 \right). \tag{B.11}
\]

Concerning the introduction of \( I_3 \) we observe that only the projection of \( \ell \) onto the direction of \( \hat{q} \), whose covariant notation reads \( q^\nu - \Pi^\nu (q^\Pi/\Pi^2) \), contributes to the corresponding integral. The integral \( I_2 \) takes the value

\[
I_2(q, \Pi) = -\frac{\pi}{\Pi^2} \left\{ 1 - \frac{q\Pi + \Pi^2/2}{\Pi^2} \ln \frac{(q + \Pi)^2}{q^2} + 1 - \ln \left( \frac{\sqrt{\Delta}}{\Pi^2} \right) \right\} \ln \frac{q_0^2 + q\Pi + \sqrt{\Delta}}{q_0^2 + q\Pi - \sqrt{\Delta}}, \tag{B.10}
\]

where \( \Delta := (q^\Pi)^2 - q^2 \Pi^2 = \Pi^2 \hat{q}^2 \geq 0 \). Now, we observe the following relations:

\[
(2 + \Pi\partial_1) \left\{ (3 + \Pi\partial_1)(q^\nu + \Pi^\nu/2) - \Pi^\nu \right\} I_2(q, \Pi) = (q^\nu + \Pi^\nu/2) (2 + \Pi\partial_1)(3 + \Pi\partial_1) I_2(q, \Pi)
\]

\[
(2 + \Pi\partial_1)(3 + \Pi\partial_1) I_2(q, \Pi) = \frac{\pi}{(q + \Pi)^2}, \tag{B.12}
\]

which leads to a comparatively simple expression for the Compton amplitude in terms of \( I_3 \):

\[
-\frac{2}{\pi} \int DZ \Phi_\alpha(Z) K_\nu^a(\mathcal{F}, S) \left\{ (q^\nu + \Pi^\nu/2) \left( \frac{\pi}{(q + \Pi)^2} + \left( q^\nu - \frac{q\Pi}{\Pi^2} \Pi^\nu \right) (2 + \Pi\partial_1)(3 + \Pi\partial_1) I_3(q, \Pi) \right) \right\} \tag{B.13}
\]

For the integral \( I_3 \) and its derivatives one gets

\[
I_3(q, \Pi) = \frac{\pi}{2} \left\{ \left( \frac{2}{\Pi^2} + \frac{q^2 + q\Pi}{2 \Delta} \right) - \frac{2q\Pi + \Pi^2}{(\Pi^2)^2} \ln \frac{(q + \Pi)^2}{q^2} \right\} \ln \frac{q_0^2 + q\Pi - \sqrt{\Delta}}{q_0^2 + q\Pi + \sqrt{\Delta}}
\]

\[
+ \frac{1}{\sqrt{\Delta}} \left( \frac{1}{\Pi^2} \left[ \Delta + (q\Pi + \Pi^2/2)^2 \right] - (q^2 + \Pi^2)^2 \right) \ln \frac{q_0^2 + q\Pi - \sqrt{\Delta}}{q_0^2 + q\Pi + \sqrt{\Delta}}
\]

\[
(2 + \Pi\partial_1)(3 + \Pi\partial_1) I_3(q, \Pi) = \frac{\pi}{2\Delta} \left\{ \frac{q^2 + q\Pi}{(q + \Pi)^2} - \frac{q^2}{2\sqrt{\Delta}} \ln \frac{q_0^2 + q\Pi + \sqrt{\Delta}}{q_0^2 + q\Pi - \sqrt{\Delta}} \right\}, \tag{B.14}
\]

After insertion of this expression into (B.13) one finally obtains

\[
\langle P_2, S_2 | T_{\text{tw}^2}(q) | P_1, S_1 \rangle = -\frac{2}{\pi} \int DZ \Phi_\alpha(Z) K_\nu^a(\mathcal{F}, S) \times
\]

\[
q^2 \left\{ \frac{\Pi^\mu}{2 \Delta^{1/2}} \left[ \frac{(q\Pi)}{2 \Delta^{1/2}} \ln \frac{q^2 + q\Pi + \Delta^{1/2}}{q^2 + q\Pi - \Delta^{1/2}} - \frac{q\Pi + \Pi^2}{(q + \Pi)^2} \right] - q^\nu \left[ \frac{\Pi^2}{2 \Delta^{1/2}} \ln \frac{q^2 + q\Pi + \Delta^{1/2}}{q^2 + q\Pi - \Delta^{1/2}} - \frac{q\Pi + \Pi^2}{(q + \Pi)^2} \right] \right\}
\]
we rewrite the differentiation w.r.t. $\Pi$ to lower the power of the denominator in the expression (V.10). Furthermore, in order to facilitate the computation, performing the partial Fourier transforms of the local parts looks much simpler than the procedure presented in this Appendix.

Reminding relation (IV.2) for the connection between $u$ and $Z\Pi$, one observes that the result (B.15) of Fourier transforming the nonlocal expression really coincides with the corresponding result, Eqs. (III.20) and (III.21) of Section III, after having replaced $u \to \kappa u$. Obviously, the latter method, which is obtained by summing up the Fourier transforms of the local parts looks much simpler than the procedure presented in this Appendix.

**APPENDIX C: COMPUTATION OF ANTISYMMETRIC PART OF THE COMPTON AMPLITUDE**

In this Appendix we sketch the explicit computation of the antisymmetric part of the Compton amplitude (V.1):

$$T_{[\mu\nu]}^{\text{rsym}}(q) = -\int_{0}^{1} d\tau \ln \tau \int D\Phi_{5a}(Z, \mu^2) \epsilon_{\mu\nu}^{\alpha\beta} q_{a} q_{2} K_{\rho}^{\alpha}(\mathbb{P}, S) \frac{\partial_{\Pi}^2}{\partial \Pi^2} \left[ \frac{(q + \tau \Pi)^2 + i\varepsilon}{\Delta} \right]^2 \left( \frac{q_{\alpha} \Pi_{\beta}}{(q + \tau \Pi)^2 + i\varepsilon} \right)$$

(C.1)

where, in accordance with (A.12), we re-formulated the double integral in Eq. (V.1) according to

$$\int_{0}^{1} d\tau_{1} \int_{0}^{1} d\tau_{2} F(\tau_{1}\tau_{2}) = -\int_{0}^{1} d\tau \ln \tau F(\tau).$$

(C.2)

Again, as in the case of the trace part, in order not to be confronted with the product of two $\delta$–functions, we like to lower the power of the denominator in the expression (V.10). Furthermore, in order to facilitate the computation we rewrite the differentiation w.r.t. $\Pi^\rho = \kappa PZ^\rho$ as differentiation w.r.t. $\tau$ and $\Pi^2$,

$$\partial_{\Pi}^2 = q^\rho \frac{\partial}{\partial (q\Pi)} + 2 \Pi^\rho \frac{\partial}{\partial \Pi^2} = q^\rho \frac{\partial}{\partial (q\Pi)} \tau \frac{\partial}{\partial \tau} - 2 \left( \frac{q^\rho}{(q\Pi)} - \frac{\Pi^\rho}{\Pi^2} \right) \Pi^2 \frac{\partial}{\partial \Pi^2}. \quad (C.3)$$

Performing the partial $\tau$–integration and, afterwards, going back to the double integral by using (C.2), we get

$$T_{[\mu\nu]}^{\text{rsym}}(q) = -\int_{0}^{1} d\tau \ln \tau \int D\Phi_{5a}(Z, \mu^2) \epsilon_{\mu\nu}^{\alpha\beta} q_{a} q_{2} K_{\rho}^{\alpha}(\mathbb{P}, S) \left[ \frac{q_{\alpha} \Pi_{\beta}}{(q + \tau \Pi)^2 + i\varepsilon} \right] \left[ \frac{q_{\alpha} \Pi_{\beta}}{(q + \tau \Pi)^2 + i\varepsilon} \right] \left( \frac{q_{\alpha} \Pi_{\beta}}{(q + \tau \Pi)^2 + i\varepsilon} \right) \left( \frac{q_{\alpha} \Pi_{\beta}}{(q + \tau \Pi)^2 + i\varepsilon} \right)$$

$$\times \left\{ q_{a} K_{\gamma}^{\alpha}(\mathbb{P}, S) + q_{a} \Pi_{\beta} K_{\gamma}^{\alpha}(\mathbb{P}, S) \left[ \frac{q_{\alpha} \Pi_{\beta}}{(q + \tau \Pi)^2 + i\varepsilon} \right] \right\} \left( \frac{q^2}{\Delta} \right) \left( \frac{q^2}{\Delta} \right) \left( \frac{q^2}{\Delta} \right) \left( \frac{q^2}{\Delta} \right). \quad (C.4)$$

Now using the integral (IV.0),

$$\int_{0}^{1} d\tau \frac{q^2}{R(\tau) + i\varepsilon} = \frac{-q^2}{2[(q\Pi)^2 - q^2\Pi^2]} \left\{ \frac{q\Pi + \Pi^2}{R(1) + i\varepsilon} - \frac{q\Pi}{R(0) + i\varepsilon} + \Pi^2 \int_{0}^{1} d\tau \frac{1}{R(\tau) + i\varepsilon} \right\},$$

as well as

$$\int_{0}^{1} d\tau_{1} \int_{0}^{1} d\tau_{2} \frac{q^2}{R(\tau_{1}\tau_{2}) + i\varepsilon} = \frac{-q^2}{2[(q\Pi)^2 - q^2\Pi^2]} \int_{0}^{1} d\tau_{1} \left\{ \frac{q\Pi + \tau_{1}\Pi^2}{R(\tau_{1}) + i\varepsilon} - \frac{q\Pi}{R(0) + i\varepsilon} + \int_{0}^{1} d\tau_{2} \frac{\tau_{1}\Pi^2}{R(\tau_{1}\tau_{2}) + i\varepsilon} \right\}. $$
together with the differentiation w.r.t. $\Pi^2$ of the latter one, we obtain

$$T^{\mu\nu}_{1w2}(q) = \int Dz \Phi_a(z, \mu^2) \epsilon_{\mu\nu}^{\alpha\beta} \frac{-q^2}{2[(q\Pi)^2 - q^2\Pi^2]} \times
$$

$$\left\{ q_a K^{\alpha}_{\beta}(q) \int_0^1 \frac{d\tau_1}{\tau_1} \left( \frac{q\Pi + \tau_1\Pi^2}{R(\tau_1) + i\epsilon} - \frac{q\Pi}{R(0) + i\epsilon} + \int_0^1 \frac{d\tau_2}{R(\tau_1) + i\epsilon} \right) \right\}$$

$$+ q_a \Pi \left( \frac{qK^a}{(q\Pi)} \right) \left\{ \left( \frac{q\Pi + \Pi^2}{R(1) + i\epsilon} - \frac{q\Pi}{R(0) + i\epsilon} + \int_0^1 \frac{d\tau_1}{R(\tau_1) + i\epsilon} \right) \right\}$$

$$- 2q_a \Pi \left( \frac{qK^a}{(q\Pi)^2} \right) \left( \left( \frac{2q^2q\Pi + (2q\Pi^2 - q^2\Pi^2)\tau_1}{R(\tau_1) + i\epsilon} - \frac{q^2q\Pi}{R(0) + i\epsilon} + q^2\Pi^2 \int_0^\tau_1 \frac{d\tau}{R(\tau) + i\epsilon} \right) \right).$$

Again, using the integral (IV.7) together with

$$\int_0^{\tau_1} \frac{d\tau}{R(\tau) + i\epsilon} \frac{\tau^2\Pi^2}{|q\Pi|^2} = \frac{-1}{2[(q\Pi)^2 - q^2\Pi^2]} \left( \frac{q^2q\Pi + (q\Pi)^2\tau_1 + [(q\Pi)^2 - q^2\Pi^2]\tau_1}{R(\tau_1) + i\epsilon} - \frac{q^2q\Pi}{R(0) + i\epsilon} + q^2\Pi^2 \int_0^\tau_1 \frac{d\tau}{R(\tau) + i\epsilon} \right),$$

for the last line in Eq. (C.5) we obtain

$$\frac{1}{2} \left( \frac{1}{R(1) + i\epsilon} - \int_0^1 \frac{d\tau_1}{R(\tau_1) + i\epsilon} \right) \right\}$$

$$+ \frac{1}{2} \int_0^1 \frac{d\tau_1}{\tau_1} \frac{1}{|q\Pi|^2} \left( \frac{q^2q\Pi + (q\Pi)^2\tau_1 + [(q\Pi)^2 - q^2\Pi^2]\tau_1}{R(\tau_1) + i\epsilon} - \frac{q^2q\Pi}{R(0) + i\epsilon} + q^2\Pi^2 \int_0^1 \frac{d\tau_2}{R(\tau_1\tau_2) + i\epsilon} \right).$$

At this stage we observe that three different kinds of expressions occur, namely, those without any $\tau$-integration, with a single and with a double integral, $\int_0^1 d\tau_1 R^{-1}(\tau_1)$ and $\int_0^1 \int_0^1 d\tau_1 d\tau_2 R^{-1}(\tau_1\tau_2)$. Re-scaling the variable $\Pi$ by $\tau_1 \Pi$ and $\tau_1 \tau_2 \Pi$, respectively, we are led to the generalized distribution amplitudes, Eq. (IV.9), up to order two.

Putting together the various contributions for the three different kinematic structures we finally get

$$T^{\mu\nu}_{1w2}(q) = \frac{1}{2} \int Dz \epsilon_{\mu\nu}^{\alpha\beta} \frac{-q^2}{[(q\Pi)^2 - q^2\Pi^2]} \times
$$

$$\left\{ q_a K^{(1,2)}(Z) \left( \frac{q\Pi + \Pi^2}{R(1) + i\epsilon} - \frac{q\Pi}{R(0) + i\epsilon} + \Phi_a^{(2)}(Z) \frac{\Pi^2}{R(1) + i\epsilon} \right) \right\}$$

$$+ q_a \Pi \left( \frac{qK^a}{(q\Pi)} \right) \left\{ \left( \frac{q\Pi + \Pi^2}{R(1) + i\epsilon} - \frac{q\Pi}{R(0) + i\epsilon} \right) \right\}$$

$$- 3q^2\Pi^2 \left( \frac{(q\Pi)^2 - q^2\Pi^2 + \Phi_a^{(1)}(Z)}{R(1) + i\epsilon} \right) \right\}$$

$$+ q_a \Pi \left( \frac{(IIK^a)}{\Pi^2} \right) \left\{ \left( \frac{(q\Pi)^2 - q^2\Pi^2 + \Phi_a^{(1)}(Z)}{R(1) + i\epsilon} \right) \right\}$$

$$+ \left( \Phi_a^{(0)}(Z) + 3\Phi_a^{(1)}(Z) + 2\Phi_a^{(2)}(Z) \right) \frac{\Pi^2}{R(1) + i\epsilon} \right\}.$$

This result of the Fourier transform, after some reorganization, already has been given by Eqs. (V.2) – (V.5), where we already observed that because of the symmetry of the distribution amplitudes the terms containing $1/R(0)$ vanish.
APPENDIX D: COMPUTATION OF THE SYMMETRIC PART OF THE COMPTON AMPLITUDE

In this Appendix we sketch the explicit computation of the symmetric part of the Compton amplitude (VI.1),

\[ T_{\{\mu\nu\}}^{\text{tw2}}(q) = 2^1 \int_0^1 d\tau \int_0^1 d\sigma \int_0^1 d\rho \int DZ \Phi_a(Z, \mu^2) \mathcal{K}_a^\mu (\mathcal{P}, \mathcal{S}) \left[ q^2 \right] \partial_\mathcal{P} \left\{ \frac{2A_{\mu\nu}^T(q, \Pi)}{R(\tau) + i\epsilon} + \frac{B_{\mu\nu}^T(q, \Pi)}{R(\tau) + i\epsilon} \right\}, \]

with the abbreviations (VI.2) and (VI.3). The integrations over \( \tau, \sigma \) and \( \rho \) will be performed partially such that finally we get multiple integrals over \( 1/[R(\tau) + i\epsilon] \) alone.

First, let us perform the \( \tau \)-integration, which gets simplified if the integration is taken for the range \(-1 \leq \tau \leq 1\) (thereby the \( R(0) \)-terms are cancelled):

\[ T_{\{\mu\nu\}}^{\text{tw2}}(q) = \int_0^1 d\sigma \int_0^1 d\rho \int DZ \Phi_a(Z, \mu^2) \mathcal{K}_a^\mu (\mathcal{P}, \mathcal{S}) \left[ q^2 \right] \partial_\mathcal{P} \times \]

\[ \left\{ \frac{A_{\mu\nu}^T(q, \Pi)}{[(q\Pi)^2 - q^2\Pi^2]} \left[ - \left( \frac{q\Pi + \tilde{\Pi}^2}{R(1) + i\epsilon} + \frac{-q\Pi + \tilde{\Pi}^2}{R(-1) + i\epsilon} \right) \right] + \frac{3}{2} \frac{\Pi^2}{[(q\Pi)^2 - q^2\Pi^2]} \left( \frac{q\Pi + \tilde{\Pi}^2}{R(1) + i\epsilon} + \frac{-q\Pi + \tilde{\Pi}^2}{R(-1) + i\epsilon} \right) \right. \]

\[ \left. - \frac{B_{\mu\nu}^T(q, \Pi)}{[(q\Pi)^2 - q^2\Pi^2]} \left( \frac{q\Pi + \tilde{\Pi}^2}{R(1) + i\epsilon} + \frac{-q\Pi + \tilde{\Pi}^2}{R(-1) + i\epsilon} + \Pi^2 \right) \int_0^1 \frac{d\tau}{R(\tau) + i\epsilon} \right\}. \]

Now, observing

\[ \int_0^1 d\sigma \int_0^1 d\rho \ln \tau f(\tau') = - \int_0^1 d\tau \ln \tau f(\tau') \]

(D.1)

with \( \tilde{\Pi} \equiv \tau' \Pi \) as well as

\[ \frac{1}{2} \frac{\partial}{\partial \tau'} \frac{1}{R(\tau') + i\epsilon} = - \frac{\tilde{\Pi}^2 \pm q\tilde{\Pi}}{R(\tau') + i\epsilon} \quad \text{and} \quad \int_0^1 d\tau' \ln \tau' \frac{\partial}{\partial \tau'} \frac{1}{R(\tau') + i\epsilon} = - \int_0^1 d\tau' \frac{1}{\tau' R(\tau') + i\epsilon}, \]

we can reduce the remaining second power (modulo a surface term being compensated by the replacement \( \tilde{\Pi} \to -\tilde{\Pi} \) below). Furthermore, taking into account the explicit form (VI.2) of \( A_{\{\mu\nu\}}^T \) and \( B_{\{\mu\nu\}}^T \) we get

\[ T_{\{\mu\nu\}}^{\text{tw2}}(q) = - \frac{q^2}{2} \int DZ \Phi_a(Z, \mu^2) \left\{ \int_0^1 d\tau \ln \tau^2 \mathcal{K}_a^\mu \partial_\mathcal{P} \times \right. \]

\[ \left. \left[ \left( g_{\mu\nu}^T + \frac{3}{2} \left( \frac{\tilde{\Pi}_\mu^T \tilde{\Pi}_\nu^T q^2}{(q\Pi)^2 - q^2\Pi^2} \right) \right) \frac{\tilde{\Pi}^2}{R(\tau') + i\epsilon} + \tilde{\Pi}^2 \right] \int_0^1 \frac{d\tau}{R(\tau') + i\epsilon} + (\tilde{\Pi} \to -\tilde{\Pi}) \right\}, \]

and again, using the expression (C83) for the derivative w.r.t. \( \Pi \) one obtains

\[ T_{\{\mu\nu\}}^{\text{tw2}}(q) = - \frac{q^2}{2} \int DZ \Phi_a(Z, \mu^2) \left\{ \int_0^1 d\tau \left\{ \left[ \mathcal{K}_a^\mu \mathcal{P}^\nu + \Pi_\mu^T \mathcal{K}_a^\nu - 2 g_{\mu\nu}^T \mathcal{P}_\mu^T \mathcal{P}_\nu^T \right] \frac{q^2}{(q\Pi)^2 - q^2\Pi^2} \right. \right. \]

\[ + \left( \frac{q\mathcal{K}_a^\mu}{q\Pi} \right) \frac{\partial}{\partial \Pi^\nu} - 2 \left( \frac{q\mathcal{K}_a^\mu}{q\Pi} \right) \mathcal{K}_a^\nu \mathcal{P}^2 \frac{\partial}{\partial \Pi^\nu} \left( g_{\mu\nu}^T + \frac{\Pi_\mu^T \Pi_\nu^T q^2}{(q\Pi)^2 - q^2\Pi^2} \right) \frac{1}{R(\tau') + i\epsilon} \right. \]

\[ \left. - \ln \tau \left[ \left( \mathcal{K}_a^\mu \mathcal{P}_\nu + \Pi_\mu^T \mathcal{K}_a^\nu - 2 g_{\mu\nu}^T \mathcal{P}_\mu^T \mathcal{P}_\nu^T \right) \frac{q^2}{(q\Pi)^2 - q^2\Pi^2} \right. \right. \]

\[ + \left( \frac{q\mathcal{K}_a^\mu}{q\Pi} \right) \frac{\partial}{\partial \Pi^\nu} - 2 \left( \frac{q\mathcal{K}_a^\mu}{q\Pi} \right) \mathcal{K}_a^\nu \mathcal{P}^2 \frac{\partial}{\partial \Pi^\nu} \left( g_{\mu\nu}^T + 3 \frac{\Pi_\mu^T \Pi_\nu^T q^2}{(q\Pi)^2 - q^2\Pi^2} \right) \times \]

\[ \left. \frac{\Pi^2}{(q\Pi)^2 - q^2\Pi^2} \left( \frac{\tau^2 \mathcal{P}^2}{R(\tau') + i\epsilon} + \tau^2 \Pi^2 \right) \int_0^1 \frac{d\tau}{R(\tau') + i\epsilon} + (\Pi \to -\Pi) \right\}. \]
Now, the homogeneous derivatives w.r.t. $\tau'$ and $\Pi^2$ have to be performed which, despite being straightforward, are quite tedious. Thereby, the following relations are useful to observe (eventually modulo surface terms which cancel each other finally):

$$
\Pi^2 \frac{\partial}{\partial \Pi^2} \frac{1}{R(\tau) + i\epsilon} = -\frac{\tau^2 \Pi^2}{[R(\tau) + i\epsilon]^2} = \frac{1}{2} \frac{\partial}{\partial \tau} \frac{1}{R(\tau) + i\epsilon} + \frac{1}{2} \frac{\tau q\Pi}{[R(\tau) + i\epsilon]^2},
$$

$$
\int_0^1 d\tau \frac{\tau^2 \Pi^2}{[R(\tau) + i\epsilon]^2} = \frac{1}{2} \frac{\Pi^2}{[R(\tau) + i\epsilon]^2} \left[ q^2 + 2\Pi^2 \right],
$$

$$
\int_0^\tau d\tau' \frac{\tau^2 \Pi^2}{[R(\tau') + i\epsilon]^2} = \frac{\Pi^2}{[R(\tau') + i\epsilon]^2} \left[ q^2 + 2\Pi^2 \right] + \tau' q^2 \Pi^2 \int_0^1 d\tau \frac{1}{[R(\tau') + i\epsilon]^2}.
$$

In addition, replacing the single integrals containing the logarithm $\ln \tau'$ by double integrals according to relation (D.1) we finally get ($\Delta \equiv (q\Pi^2 - q^2\Pi^2$)

$$
T_{\mu\nu}^{(w2)}(q) = \frac{q^2}{2} \int D\Phi_a(Z, \mu^2) \frac{qK_a}{q\Pi} \times
$$

$$
\left\{ \frac{q^2}{R(1) + i\epsilon} + \frac{\Pi^2}{\Delta} \int_0^1 d\sigma \frac{\sigma(q\Pi + \sigma\Pi^2)}{R(\sigma) + i\epsilon} + \Pi^2 \frac{\Pi^2}{\Delta} \int_0^1 d\sigma \int_0^1 d\tau \frac{(\sigma\Pi^2)}{R(\sigma) + i\epsilon} \right\}
$$

$$
+ \frac{q^2}{2} \int D\Phi_a(Z, \mu^2) \left( \frac{qK_a}{q\Pi} + \frac{\Pi^2}{\Delta} \right) \times
$$

$$
\left\{ \frac{q^2 + q\Pi}{R(1) + i\epsilon} + \frac{1}{\Delta} \int_0^1 d\rho \frac{\rho(q\Pi - \rho\Pi^2)}{R(\rho) + i\epsilon} - \frac{\Pi^2}{\Delta} \int_0^1 d\rho \int_0^1 d\sigma \frac{(\rho\sigma^2)[4(q\Pi^2 - q^2\Pi^2)]}{R(\rho\sigma) + i\epsilon} \right\}
$$

Rescaling the distribution amplitudes in the same manner as in the antisymmetric case, one obtains the expression [VI.4].

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