Threshold $\eta$ and $\eta'$ electroproduction off nucleons

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Abstract

The electroproduction of $\eta$ and $\eta'$ mesons on the proton and the neutron is investigated at tree level within the framework of $U(3)$ chiral perturbation theory. In addition to the Born terms low-lying resonances such as the vector mesons and $J^P = 1/2^+, 1/2^-$ baryon resonances are included explicitly and their contributions are calculated. Results for the separated differential cross sections are presented.

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1 Introduction

In electroproduction one can get detailed information about the structure of the nucleon due to the longitudinal coupling of the virtual photon to the nucleon spin. It furthermore is a tool to study baryon resonances and the investigation of transitions between these states provides a crucial test for hadron models. Perturbative QCD should apply at sufficiently high photon virtuality $|k^2|$, see e.g. [1, 2], however there is no consensus how high the momentum transfer must be. It has been found experimentally, that in the case of electroproduction of the $\Delta(1232)$ resonance at momentum transfers up to $|k^2| = 4.0$ GeV$^2$, perturbative QCD is not applicable, [3], whereas a possible onset of scaling in the reaction $e+p \rightarrow e+p+\eta$ at $|k^2| = 3.6$ GeV$^2$ is reported in [4]. At low $|k^2|$ non-perturbative QCD dominates. This region has been treated, e.g., by incorporating relativistic effects into the constituent quark model [5], using light-front approaches [6] and within the context of chiral perturbation theory [7].

Because of their hadronic decay modes nucleon resonances have large overlapping widths, which makes it difficult to study individual states, but selection rules in certain decay channels can reduce the number of possible resonances. The isoscalars $\eta$ and $\eta'$ are such examples since, due to isospin conservation, only the isospin-$\frac{1}{2}$ excited states decay into the $\eta N$ and $\eta'N$ channels. In recent years both $\eta$ and $\eta'$ electro- and photoproduction have been of considerable interest. The $\eta$ photoproduction of protons has been measured at MAMI [8] and resonance parameters of the $S_{11}(1535)$ resonance and the electromagnetic coupling $\gamma p \rightarrow S_{11}$ have been extracted from the data. Photoproduction of the $\eta'$ has been measured at ELSA [9]. The experimental data for electroproduction of the $\eta$ is still very scarce; it is limited to a few older Bonn data [10] and recently published data from CEBAF [4] at high momentum transfer.

On the theoretical side, the $\eta$ meson has been treated as a pure $SU(3)$ octet state $\eta_8$ and mixing of $\eta_8$ with the corresponding singlet state $\eta_0$ which yields the physical states $\eta$ and $\eta'$ is generally neglected. The $\eta'$ is interesting by itself. The QCD Lagrangian with massless quarks exhibits an $SU(3)_L \times SU(3)_R$ chiral symmetry which is broken down spontaneously to $SU(3)_V$, giving rise to a Goldstone boson octet of pseudoscalar mesons which become massless in the chiral limit of zero quark masses. On the other hand, the axial $U(1)$ symmetry of the QCD Lagrangian is broken by the anomaly. The corresponding pseudoscalar singlet would otherwise have a mass comparable to the pion mass [11]. Such a particle is missing in the spectrum and the lightest candidate would be the $\eta'$ with a mass of 958 MeV which is considerably heavier than the octet states. In conventional chiral perturbation theory the $\eta'$ is not included explicitly, although it does show up in the form of a contribution to a coupling coefficient of the Lagrangian, a so-called low-energy constant (LEC). Recently, the $\eta'$ has been included in baryon chiral perturbation theory in a systematic fashion [12]. Using this approach $\eta$ and $\eta'$ photoproduction off the nucleons has been investigated.
Low-lying resonances such as the vector mesons and $J^P = 1/2^+, 1/2^-$ baryon resonances are included explicitly and their contributions together with the Born terms are calculated. The coupling constants of the resonances are determined from strong and radiative decays and reasonable agreement with experimental data near threshold is obtained.

The purpose of this paper is to extend this approach to the electroproduction of $\eta$ and $\eta'$ on the nucleons which provides a further test for this simple model. From such a simplified treatment of $\eta$ and $\eta'$ electroproduction one should not expect to forecast experimental data in detail; here we are rather concerned with qualitative agreement. In order to obtain a better description of the experimental data, one has to include chiral loops and further resonances, but this is beyond the scope of the present investigation. This work should therefore be considered to be mainly a check if the inclusion of $\eta$ and $\eta'$ mesons in a nonet of pseudoscalar mesons as proposed in [12] leads to an adequate description for processes of $\eta$ and $\eta'$ mesons with baryons.

In the next section we present the necessary formalism for electroproduction of $\eta$ and $\eta'$ mesons. The effective chiral Lagrangian including explicitly low-lying resonances is given in Sec. 3. The invariant amplitudes are shown in Sec. 4 together with the numerical results. We conclude with a summary in Sec. 5.

2 General Formalism

The $T$-matrix element for the processes $N(p_1) + \gamma^*(k) \rightarrow N(p_2) + \phi(q)$ with $\phi = \eta$ or $\eta'$ is given by

$$\langle p_2, q \text{ out}| p_1, k \text{ in} \rangle = \delta_{fi} + (2\pi)^4 i\delta^{(4)}(p_2 + q - p_1 - k)T_{fi}. \quad (1)$$

The Mandelstam variables are

$$s = (k + p_1)^2 = (q + p_2)^2$$
$$t = (k - q)^2 = (p_1 - p_2)^2$$
$$u = (k - p_2)^2 = (q - p_1)^2 \quad (2)$$

subject to the constraint $s + t + u = 2M_N^2 + m_{\phi}^2 + k^2$ with $M_N$ and $m_{\phi}$ being the mass of the nucleon and the pseudoscalar meson, respectively. The invariant four-momentum transfer squared, $t$, can be related to the scattering angle $\vartheta$ in the c.m. system via

$$t = m_\phi^2 - 2q^0k^0 + 2|q||k|z + k^2 \quad (3)$$

with $z = \cos \vartheta$.

In general, $T$ can be decomposed as

$$T_{fi} = i\epsilon_\mu \bar{u}_2 \sum_{i=1}^{8} B_i N_i^\mu u_1 \quad (4)$$
with the invariant amplitudes
\[ N_1 = \gamma_5 \gamma_\mu \gamma_\nu F^\mu\nu, \quad N_2 = 2 \gamma_5 P_\mu, \quad N_3 = 2 \gamma_5 q_\mu, \quad N_4 = 2 \gamma_5 k_\mu, \]
\[ N_5 = \gamma_5 \gamma_\mu \gamma_\nu F^\mu\nu, \quad N_6 = 2 \gamma_5 k_\mu \mu_\nu F^\mu\nu, \quad N_7 = 2 \gamma_5 q_\mu \mu_\nu F^\mu\nu, \quad N_8 = 2 \gamma_5 k_\mu \mu_\nu F^\mu\nu \] (5)
and \( P = \frac{1}{2} (p_1 + p_2) \). From current conservation one obtains the relations
\[ k^2 B_1 + k \cdot (p_1 + p_2) B_2 + 2k \cdot q B_3 + 2k^2 B_4 = 0 \]
\[ B_5 + \frac{1}{2} k \cdot (p_1 + p_2) B_6 + k^2 B_7 + k \cdot q B_8 = 0 \] (6)
which are used to eliminate \( B_3 \) and \( B_5 \). It is therefore more convenient to define a set of independent amplitudes
\[ T_{fi} = i \bar{u}_2 \sum_{i=1}^{6} A_i M_i u_1 \] (7)
with
\[ M_1 = \frac{1}{2} \gamma_5 \gamma_\mu \gamma_\nu F^\mu\nu, \quad M_2 = 2 \gamma_5 P_\mu (q - \frac{1}{2} k)_\nu F^\mu\nu, \]
\[ M_3 = \gamma_5 \gamma_\mu q_\nu F^\mu\nu, \quad M_4 = 2 \gamma_5 \gamma_\mu P_\nu F^\mu\nu - 2 M_N M_1, \]
\[ M_5 = \gamma_5 k_\mu q_\nu F^\mu\nu, \quad M_6 = \gamma_5 k_\mu \gamma_\nu F^\mu\nu \] (8)
and \( F_{\mu\nu} = \epsilon_{\mu k_\nu} - \epsilon_{\nu k_\mu} \). The \( A_i \) obey the crossing relations
\[ A_i(s, u) = A_i(u, s) \quad i = 1, 2, 4 \]
\[ A_i(s, u) = -A_i(u, s) \quad i = 3, 5, 6 \] (9)
and are related to the \( B_i \) via
\[ A_1 = B_1 - M_N B_6, \quad A_2 = \frac{2}{m_\phi^2 - t} B_2, \quad A_3 = -B_8, \]
\[ A_4 = -\frac{1}{2} B_6, \quad A_5 = \frac{2}{s + u - 2M_N^2} \left( B_1 - \frac{s - u}{2(m_\phi^2 - t)} B_2 + 2B_4 \right), \]
\[ A_6 = B_7. \] (10)
The unpolarized \( \eta/\eta' \) electroproduction triple differential cross section reads
\[ \frac{d\sigma}{dE_f d\Omega_f d\Omega_\phi} = \frac{\alpha E_f (s - M_N^2)}{4 \pi^2 E_i M_N k^2 (\epsilon - 1)} \frac{d\sigma}{d\Omega_\phi} \] (11)
with \( \alpha = e^2 / 4\pi \) the fine structure constant and \( E_{i/f} \) is the laboratory energy of the incoming/outgoing electron. The photon polarization \( \epsilon \) is given by
\[ \epsilon^{-1} = 1 + 2 \left( 1 - \frac{k_\phi^2}{k^2} \right) \tan^2 \frac{\psi}{2} \] (12)
with $k_0$ the photon energy and $\psi$ the photon scattering angle. The differential cross section can be decomposed into transverse ($T$), longitudinal ($L$), transverse-longitudinal ($TL$) and transverse-transverse ($TT$) pieces

$$\frac{d\sigma}{d\Omega_{\phi}} = \frac{2\sqrt{s}|q|}{s - M_N^2} \left( R_T + \epsilon_L R_L + \sqrt{2\epsilon_L(1 + \epsilon)} \cos \phi R_{TL} + \epsilon \cos 2\phi R_{TT} \right), \tag{13}$$

where the $R_I (I = T, L, TL, TT)$ are called structure functions, $\phi$ is the azimuthal angle between the scattering and the reaction plane, and $\epsilon_L$ is the longitudinal photon polarization

$$\epsilon_L = -\frac{k^2}{k_0^2} \epsilon. \tag{14}$$

The separated virtual photon cross sections are

$$\frac{d\sigma_I}{d\Omega_{\phi}} = \frac{2\sqrt{s}|q|}{s - M_N^2} R_I, \quad I = T, L, TL, TT. \tag{15}$$

Since we restrict ourselves to the threshold region, it is convenient to perform a multipole decomposition and confine ourselves to $S$- and $P$-waves. To this end, one expresses the transition amplitude in terms of Pauli spinors and matrices

$$\frac{1}{8\pi \sqrt{s}} i \bar{u}_2 \sum_{i=1}^6 A_i M_i u_1 = \chi_2^\dagger F \chi_1. \tag{16}$$

The matrix $F$ can be written as

$$\begin{align*}
F &= i \sigma \cdot b F_1 + \sigma \cdot \hat{q} \sigma \cdot (\hat{k} \times b) F_2 + i \sigma \cdot \hat{k} \hat{q} \cdot b F_3 \\
&\quad + i \sigma \cdot \hat{q} \hat{q} \cdot b F_4 - i \sigma \cdot \hat{k} b_0 F_7 - i \sigma \cdot \hat{q} b_0 F_8, \tag{17}
\end{align*}$$

where

$$b_\mu = \epsilon_\mu - \frac{1}{|k|} \epsilon \cdot \hat{k} k_\mu. \tag{18}$$

With this choice of gauge, the virtual photon has only scalar and transverse components. The $F_i$ are related to the $A_i$ via

$$\begin{align*}
F_1 &= \left( \sqrt{s} - M_N \right) \frac{N_1 N_2}{8\pi \sqrt{s}} \\
&\quad \times \left[ A_1 + \frac{k \cdot q}{\sqrt{s} - M_N} A_3 + \left( \sqrt{s} - M_N - \frac{k \cdot q}{\sqrt{s} - M_N} \right) A_4 - \frac{k^2}{\sqrt{s} - M_N} A_6 \right] \\
F_2 &= \left( \sqrt{s} + M_N \right) \frac{N_1 N_2}{8\pi \sqrt{s}} \frac{|q||k|}{(E_1 + M_N)(E_2 + M_N)} \\
&\quad \times \left[ - A_1 + \frac{k \cdot q}{\sqrt{s} + M_N} A_3 + \left( \sqrt{s} + M_N - \frac{k \cdot q}{\sqrt{s} + M_N} \right) A_4 - \frac{k^2}{\sqrt{s} + M_N} A_6 \right]
\end{align*}$$

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The projection matrices for the lowest multipoles are given by

\[ \mathcal{F}_3 = \left( \sqrt{s} + M_N \right) \frac{N_1 N_2}{8 \pi \sqrt{s}} \frac{|q||k|}{E_1 + M_N} \times \left[ \frac{M_N^2 - s - \frac{1}{2} k^2}{\sqrt{s} + M_N} A_2 + A_3 - A_4 - \frac{k^2}{\sqrt{s} + M_N} A_5 \right] \]

\[ \mathcal{F}_4 = \left( \sqrt{s} - M_N \right) \frac{N_1 N_2}{8 \pi \sqrt{s}} \frac{|q|^2}{E_2 + M_N} \times \left[ \frac{s - M_N^2 - \frac{1}{2} k^2}{\sqrt{s} - M_N} A_2 + A_3 - A_4 + \frac{k^2}{\sqrt{s} - M_N} A_5 \right] \]

\[ \mathcal{F}_7 = \frac{N_1 N_2}{8 \pi \sqrt{s}} \frac{|q|}{E_2 + M_N} \left[ -(E_1 - M_N)A_1 + \frac{1}{2k_0} \left( |k|^2 (2k_0 \sqrt{s} - 3k \cdot q) - q \cdot k (2s - 2M_N^2 - k^2) \right) A_2 + (q_0 (\sqrt{s} - M_N) - k \cdot q) A_3 + (k \cdot q - q_0 (\sqrt{s} - M_N) + (E_1 - M_N)(\sqrt{s} + M_N)) A_4 + (q_0 k^2 - k_0 k \cdot q) A_5 - (E_1 - M_N)(\sqrt{s} + M_N) A_6 \right] \]

\[ \mathcal{F}_8 = \frac{N_1 N_2}{8 \pi \sqrt{s}} \frac{|k|}{E_1 + M_N} \left[ (E_1 + M_N) A_1 + \frac{1}{2k_0} \left( |k|^2 (2k_0 \sqrt{s} - 3k \cdot q) - q \cdot k (2s - 2M_N^2 - k^2) \right) A_2 + (q_0 (\sqrt{s} + M_N) - k \cdot q) A_3 + (k \cdot q - q_0 (\sqrt{s} + M_N) + (E_1 + M_N)(\sqrt{s} - M_N)) A_4 - (q_0 k^2 - k_0 k \cdot q) A_5 - (E_1 + M_N)(\sqrt{s} - M_N) A_6 \right] \]

with

\[ N_i = \sqrt{M_N + E_i}, \quad E_i = \sqrt{M_N^2 + p_i^2}. \]  

The projection matrices for the lowest multipoles \( E_{0+}, M_{1+}, M_{1-}, L_{0+}, L_{1+} \) and \( L_{1-} \) are given by

\[
\begin{pmatrix}
E_{0+} \\
M_{1+} \\
M_{1-} \\
E_{1+}
\end{pmatrix}
= \int_{-1}^{1} dz \begin{pmatrix}
\frac{1}{2} P_0 & -\frac{1}{2} P_1 & 0 & \frac{1}{6}[P_0 - P_2] \\
\frac{1}{4} P_1 & -\frac{1}{4} P_2 & \frac{1}{12}[P_2 - P_0] & 0 \\
-\frac{1}{2} P_1 & \frac{1}{2} P_0 & \frac{1}{6}[P_0 - P_2] & 0 \\
\frac{1}{4} P_1 & -\frac{1}{4} P_2 & \frac{1}{12}[P_0 - P_2] & \frac{1}{10}[P_1 - P_3]
\end{pmatrix}
\begin{pmatrix}
\mathcal{F}_1 \\
\mathcal{F}_2 \\
\mathcal{F}_3 \\
\mathcal{F}_4
\end{pmatrix}
\]

and

\[
\begin{pmatrix}
L_{0+} \\
L_{1+} \\
L_{1-}
\end{pmatrix}
= k_0 \int_{-1}^{1} dz \begin{pmatrix}
\frac{1}{2} P_1 & \frac{1}{2} P_0 \\
\frac{1}{2} P_2 & \frac{1}{4} P_1 \\
\frac{1}{2} P_0 & \frac{1}{2} P_1
\end{pmatrix}
\begin{pmatrix}
\mathcal{F}_7 \\
\mathcal{F}_8
\end{pmatrix}
\]

with \( P_i \) being the Legendre polynomials. The scalar multipoles are related to the longitudinal multipoles by \( S_{l_+} = (|k|/k_0)L_{l+} \). The structure functions \( R_I \) can be
expressed in terms of the multipoles as follows

\[ R_T = |E_{0+} + \cos \vartheta P_1|^2 + \frac{1}{2} \sin^2 \vartheta (|P_2|^2 + |P_3|^2) \]

\[ R_L = |L_{0+} + \cos \vartheta P_4|^2 + \sin^2 \vartheta |P_5|^2 \]

\[ R_{TL} = -\sin \vartheta \text{Re} \left( (E_{0+} + \cos \vartheta P_1)P_5^* + (L_{0+} + \cos \vartheta P_4)P_2^* \right) \]

\[ R_{TT} = \frac{1}{2} \sin^2 \vartheta (|P_2|^2 - |P_3|^2) \]

(23)

with the combinations

\[ P_1 = 3E_{1+} + M_{1+} - M_{1-}, \quad P_2 = 3E_{1+} - M_{1+} + M_{1-}, \]

\[ P_3 = 2M_{1+} + M_{1-}, \quad P_4 = 4L_{1+} + L_{1-}, \quad P_5 = L_{1-} - 2L_{1+}. \]

(24)

This completes the necessary formalism needed in the present investigation.

3 The effective Lagrangian

In this section, we will introduce the effective Lagrangian with \( \eta \) and \( \eta' \) coupled both to the ground state baryon octet and low-lying resonances in the \( s, u \)- and \( t \)-channel. A systematic framework for the \( \eta' \) in baryon chiral perturbation theory has been developed in [12] and extended by including explicitly low-lying meson and baryon resonances [13]. Our starting point is the \( U(3)_L \times U(3)_R \) chiral effective Lagrangian of the pseudoscalar meson nonet \( (\pi, K, \eta_8, \eta_0) \) coupled to the ground state baryon octet \( (N, \Lambda, \Sigma, \Xi) \) at lowest order in the derivative expansion

\[ L = L_\phi + L_{\phi B} \]

(25)

with

\[ L_\phi = -v_0 \eta_0^2 + \frac{F_\pi^2}{4} \langle u_\mu u^\mu \rangle + \frac{F_\pi^2}{4} \langle \chi_+ \rangle + iF_0 v_3 \eta_0 \langle \chi_- \rangle + \frac{1}{12} (F_0^2 - F_\pi^2) \langle u_\mu \rangle \langle u^\mu \rangle \]

(26)

and

\[ L_{\phi B} = i \langle \bar{B} \gamma_\mu [D^\mu, B] \rangle - M_N \langle \bar{B} B \rangle - \frac{1}{2} D \langle \bar{B} \gamma_\mu \gamma_5 \{u^\mu, B\} \rangle \]

\[-\frac{1}{2} F \langle \bar{B} \gamma_\mu \gamma_5 [u^\mu, B] \rangle - \lambda \langle \bar{B} \gamma_\mu \gamma_5 B \rangle \langle u^\mu \rangle \]

\[ + \frac{1}{8M_N} b_0^B \langle B \sigma^{\mu \nu} \{F^+_{\mu \nu}, B\} \rangle + \frac{1}{8M_N} [1 + b_0^F] \langle \bar{B} \sigma^{\mu \nu} [F^+_{\mu \nu}, B] \rangle. \]

(27)

The pseudoscalar meson nonet is summarized in a matrix valued field \( U(x) \)

\[ U(\phi, \eta_0) = u^2(\phi, \eta_0) = \exp \{2i\phi/F_\pi + i\sqrt{\frac{2}{3}} \eta_0/F_0 \}. \]

(28)
where $F_\pi \simeq 92.4$ MeV is the pion decay constant and the singlet $\eta_0$ couples to the singlet axial current with strength $F_0$. The unimodular part of the field $U(x)$ contains the degrees of freedom of the Goldstone boson octet $\phi$

$$
\phi = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0 \\
K^- & K^0 & -\frac{2}{\sqrt{6}} \eta_8
\end{pmatrix},
$$

(29)

while the phase $\det U(x) = e^{i\sqrt{\eta_0}/F_0}$ describes the $\eta_0$. In order to incorporate the baryons into the effective theory it is convenient to form an object of axial-vector type with one derivative

$$
u_\mu = i u^\dagger \nabla_\mu U u^\dagger
$$

(30)

with $\nabla_\mu$ being the covariant derivative of $U$. The expression $\langle \ldots \rangle$ denotes the trace in flavor space and the quark mass matrix $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$ enters in the combinations

$$
\chi_\pm = 2 B_0 (u \mathcal{M} u \pm u^\dagger \mathcal{M} u^\dagger)
$$

(31)

with $B_0 = -\langle 0 | \bar{q} q | 0 \rangle / F_\pi^2$ the order parameter of the spontaneous symmetry violation. Expanding the Lagrangian $\mathcal{L}_\phi$ in terms of the meson fields one observes terms quadratic in the meson fields that contain the factor $\eta_0 \eta_8$ which leads to $\eta_0 - \eta_8$ mixing. Such terms arise from the explicitly symmetry breaking terms $F_\pi^2 \langle \chi_+ \rangle + i F_0 v_3 \eta_0 \eta_8(\chi_-)$ and read

$$
- \left( \frac{2\sqrt{2}}{3} \frac{F_\pi}{F_0} + \frac{8}{\sqrt{3}} \frac{F_0}{F_\pi} v_3 \right) B_0 (\hat{m} - m_s) \eta_0 \eta_8
$$

(32)

with $\hat{m} = \frac{1}{2} (m_u + m_d)$. The states $\eta_0$ and $\eta_8$ are therefore not mass eigenstates. The mixing yields the eigenstates $\eta$ and $\eta'$,

$$
|\eta\rangle = \cos \theta |\eta_8\rangle - \sin \theta |\eta_0\rangle \\
|\eta'\rangle = \sin \theta |\eta_8\rangle + \cos \theta |\eta_0\rangle,
$$

(33)

which is valid in the leading order of flavor symmetry breaking and we have neglected other pseudoscalar isoscalar states which could mix with both $\eta_0$ and $\eta_8$. The $\eta - \eta'$ mixing angle can be determined from the two photon decays of $\pi^0, \eta, \eta'$, which require a mixing angle around $-20^\circ$ [14]. We will make use of this experimental input in order to diagonalize the mass terms of the effective mesonic Lagrangian.

The baryonic Lagrangian consists of the free kinetic term and the axial-vector couplings of the mesons to the baryons. The values of the LECs $D$ and $F$ can be extracted from semileptonic hyperon decays. A fit to the experimental data

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3 For details the reader is referred to [12].
delivers $D = 0.80 \pm 0.01$ and $F = 0.46 \pm 0.01$ [15]. From a fit to $\eta$ and $\eta'$ photoproduction one obtains for the axial flavor-singlet coupling $\lambda = 0.05$ [13]. The covariant derivative of the baryon field is given by

$$[D_\mu, B] = \partial_\mu B + [\Gamma_\mu, B]$$

with the chiral connection

$$\Gamma_\mu \simeq -iv_\mu = ieQ A_\mu$$

to the order we are working and $Q = \frac{1}{3}\text{diag}(2, -1, -1)$ is the quark charge matrix. Note that there is no pseudoscalar coupling of $\eta_0$ to the baryons of the form $\eta_0 \bar{B} \gamma_5 B$. Such a term is in principle possible but can be absorbed by the $\lambda$-term in Eq. (27) by means of the equation of motion for the baryons. We also take the magnetic moments of the baryons at leading order into account which are given by the two terms $b_D^6$ and $b_F^6$. The quantity $F_{\mu\nu}^+$ contains the electromagnetic field strength tensor $F_{\mu\nu}$ of the external vector field $v_\mu$

$$F_{\mu\nu}^+ \equiv u^\dagger F_{\mu\nu} u + u F_{\mu\nu} u^\dagger$$

$$= 2(\partial_\mu v_\nu - \partial_\nu v_\mu) + \mathcal{O}(\phi^2).$$

Although these terms are of higher chiral order, they might lead to some substantial contributions and will therefore be included in this investigation. A detailed analysis of the baryon magnetic moments in chiral perturbation theory has been given in [14]. A least-squares fit to the magnetic moments of the baryon octet leads to $b_D^6 = 2.39$ and $b_F^6 = 0.77$.

We now proceed by including explicitly low-lying resonances in our theory. In the $t$-channel the lowest-lying resonances are the octet of the vector mesons $(\rho, K^*, \omega)$. Note that $\phi$ exchange has been found to be almost negligible [13] which is in agreement with the OZI suppression. The coupling of the baryons to the vector mesons is given by

$$\mathcal{L}_{VBB} = \frac{1}{2} \bar{p} \Gamma_\mu p (g_{\rho N} \rho_0^\mu + g_{\omega N} \omega^\mu) + \frac{1}{2} \bar{n} \Gamma_\mu n (-g_{\rho N} \rho_0^\mu + g_{\omega N} \omega^\mu),$$

where the operator $\Gamma_\mu$ involves a vector and a tensor coupling

$$\Gamma_\mu = \gamma_\mu + i \frac{\kappa_V}{2M_N} \sigma_{\mu\nu}(p' - p)\nu.$$  

The couplings $g_{VN}$ are quite well known, we use $g_{\rho N} = 6.08$ and $g_{\omega N} = 3g_{\rho N}$ [17]. Furthermore, the tensor coupling for the $\rho$ meson is given by $\kappa_\rho = 6$, whereas $\kappa_\omega \simeq 0$. Instead of using a common tensor coupling $\kappa_V$ for $\rho$ and $\omega$, as prescribed by the Lagrangian in Eq. (37), we prefer to work with the physical values $\kappa_\rho = 6$ and $\kappa_\omega = 0$. 

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The electromagnetic piece of the Lagrangian is given by

\[ \mathcal{L}_{V\gamma\phi} = e \epsilon^{\mu\nu\alpha\beta} \partial_\alpha A_\beta \left( \partial_\nu \eta \sum_{V=\rho,\omega,\phi} g_{V\gamma\eta} V_\mu + \partial_\nu \eta' \sum_{V=\rho,\omega,\phi} g_{V\gamma\eta'} V_\mu \right). \] (39)

The experimental values for the \( g_{V\gamma\eta} \) can be extracted from the decay width of radiative decays of the vector mesons

\[ \Gamma(V \to \eta\gamma) = \frac{e^2 g_{V\gamma\eta}^2 (m_V - m_\eta)^3}{96\pi}. \] (40)

Using the values for \( \Gamma(\rho \to \eta\gamma) \) and \( \Gamma(\omega \to \eta\gamma) \) from [14] we obtain

\[ g_{\rho\gamma\eta} = 1.46 \pm 0.16 \text{ GeV}^{-1} \]
\[ g_{\omega\gamma\eta} = 0.53 \pm 0.04 \text{ GeV}^{-1}, \] (41)

where the uncertainty in the couplings stems from the given experimental errors. The coupling strength of the vector mesons to the \( \eta' \) can be extracted directly from the decay widths of the pertinent radiative \( \eta' \) decays

\[ \Gamma(\eta' \to V\gamma) = \frac{e^2 g_{\eta'\gamma\eta'}^2 (m_{\eta'}/m_{\eta'})^3}{32\pi}. \] (42)

We obtain

\[ g_{\rho\gamma\eta'} = 1.31 \pm 0.04 \text{ GeV}^{-1} \]
\[ g_{\omega\gamma\eta'} = 0.45 \pm 0.04 \text{ GeV}^{-1}. \] (43)

This determines completely the contributions of the vector mesons. Note that the vector meson contribution is usually reduced, e.g., by using a form factor [18]. However, this effect should be reasonably small for \( \eta \) and \( \eta' \) photo- and electroproduction close to threshold.

Baryon resonances contribute in the \( s \)- and \( u \)-channel. In this work we consider the lowest-lying \( S \)- and \( P \)-wave baryon resonances, i.e. the \( J^P = 1/2^+ \) and \( 1/2^- \) octets which include \( P_{11}(1440) \) and \( S_{11}(1535) \), respectively. We will neglect higher partial waves baryon resonances such as \( D_{13}(1520) \) since we are only interested in rough qualitative predictions. In order to achieve better agreement with experiment, one has to consider further resonances, e.g. \( D_{13}(1520) \) and \( S_{11}(1650) \), and include chiral loop corrections. But this is beyond the scope of the present investigation and the calculations are performed at tree level.

Let us first consider the spin-1/2\(^+\) octet which we denote by \( P \). The octet consists of \( N^*(1440), \Sigma^*(1660), \Lambda^*(1600), \Xi^*(?) \) and the effective Lagrangian of the \( P \)-wave octet coupled to the ground state baryon octet takes the form

\[ \mathcal{L} = \mathcal{L}_P + \mathcal{L}_{\phi BP} \] (44)
with the kinetic term
\[ \mathcal{L}_P = i \langle \bar{P} \gamma_\mu [D^\mu, P] \rangle - M_P \langle \bar{P} P \rangle. \] (45)

Since for the processes considered here only \( N^*(1440) \) contributes, we set \( M_P = 1.44 \) GeV. The interaction terms of the \( P \)-wave resonances with the ground state baryon octet read
\[ \mathcal{L} = \mathcal{L}_P + \mathcal{L}_P^{\phi BP} \]
(46)

A possible \( \eta_0 \bar{P} \gamma_5 B \) term can again be eliminated by using the equation of motion for baryons. The coupling constants \( D_P, F_P \) and \( d_P, f_P \) can be determined from strong and radiative decays of the \( N^*(1440) \) resonance, \[ D_P = 0.32 \pm 0.05, \quad F_P = 0.16 \pm 0.04 \]
\[ d_P = -0.05 \pm 0.02 \text{ GeV}^{-1}, \quad f_P = 0.08 \pm 0.02 \text{ GeV}^{-1}, \]
(47)
where the first number indicates the central value used in Ref. [13] and the error bars are due to the uncertainty in the decay widths. In \( \eta \) and \( \eta' \) photoproduction it has been found that the effect of \( \lambda_P \) is negligible, therefore we set it zero in our calculations: \( \lambda_P = 0 \). The spin-1/2 \( - \) octet consists of \( N^*(1535), \Lambda^*(1670), \Sigma^*(1750), \Xi^*(?) \) and the pertinent Lagrangian reads
\[ \mathcal{L} = \mathcal{L}_S + \mathcal{L}_S^{\phi BS} \]
(48)
with the kinetic term
\[ \mathcal{L}_S = i \langle \bar{S} \gamma_\mu [D^\mu, S] \rangle - M_S \langle \bar{S} S \rangle \]
(49)
and the interaction part
\[ \mathcal{L}_S^{\phi BS} = -\frac{i}{2} D_S \langle \bar{S} \gamma_\mu \{u^\mu, B\} \rangle - \frac{i}{2} F_S \langle \bar{S} \gamma_\mu [u^\mu, B] \rangle - i \lambda_S \langle \bar{S} \gamma_5 B \rangle \langle u^\mu \rangle \]
\[ + i d_S \langle \bar{S} \sigma^{\mu\nu} \gamma_5 \{F^+_{\mu\nu}, B\} \rangle + i f_S \langle \bar{S} \sigma^{\mu\nu} \gamma_5 [F^+_{\mu\nu}, B] \rangle + \text{h.c.}. \] (50)
We set \( M_S = 1.535 \) GeV and from strong and radiative decays of the \( S \)-wave resonances one obtains, \[ D_S = 0.37 \pm 0.06, \quad F_S = -0.21 \pm 0.04, \quad \lambda_S = -0.07 \pm 0.02 \]
\[ d_S = -0.07 \pm 0.03 \text{ GeV}^{-1}, \quad f_S = -0.06 \pm 0.03 \text{ GeV}^{-1}. \] (51)

Since there exists data on decay channels of the \( S \)-wave resonances into \( \eta \), we are able to fix the coupling \( \lambda_S \) by taking \( \eta-\eta' \) mixing into account. Two remarks are in order. First, we would like to point out that our simple ansatz of zero
width resonances will lead to singularities at the resonance mass which could be circumvented by the use of a finite width. This will restrict in the case of $\eta$ electro-production the validity of our approach to energies very close to threshold which we are considering in the present work, whereas it is numerically irrelevant for $\eta'$ electroproduction. Second, we have calculated both Born terms using the lowest order chiral effective Lagrangian and resonance contributions. We would like to emphasize that this procedure does not imply any double counting. The contributions of the resonances are hidden only in higher chiral order counterterms of the effective Lagrangian which we did not take into account in the present investigation. Born terms like the ones used in this work are not produced by resonance contributions.

4 Invariant amplitudes and numerical results

We proceed by presenting the invariant amplitudes for $\eta$ and $\eta'$ photoproduction on the nucleons. Let us start with the Born terms which are depicted in Fig. 1. They read for the proton

\begin{align}
A_1(p\gamma^* \rightarrow p\phi) &= -2M_N eA_\phi \left[ \frac{1}{s-M_N^2} + \frac{1}{u-M_N^2} + \frac{\mu_p}{2M_N^2} \right] \\
A_2(p\gamma^* \rightarrow p\phi) &= 4M_N eA_\phi \left[ \frac{s+u-2M_N^2}{s-M_N^2}[u-M_N^2][m_\phi^2-t] \right] \\
A_3(p\gamma^* \rightarrow p\phi) &= e\mu_p A_\phi \left[ \frac{1}{s-M_N^2} - \frac{1}{u-M_N^2} \right] \\
A_4(p\gamma^* \rightarrow p\phi) &= e\mu_p A_\phi \left[ \frac{1}{s-M_N^2} + \frac{1}{u-M_N^2} \right] \\
A_5(p\gamma^* \rightarrow p\phi) &= -2M_N eA_\phi \left[ \frac{s-u}{s-M_N^2}[u-M_N^2][m_\phi^2-t] \right] \\
A_6(p\gamma^* \rightarrow p\phi) &= 0
\end{align}

(52)

with $\mu_p = 1 + b_6^F + b_6^D/3 = 2.57$ being the magnetic moment of the proton at lowest chiral order and

\begin{align}
A_\eta &= \frac{1}{2\sqrt{3}F_\pi}[D-3F] \cos \theta + \sqrt{\frac{2}{3}} \frac{1}{F_0} [D+3\lambda] \sin \theta \\
A_{\eta'} &= \frac{1}{2\sqrt{3}F_\pi}[D-3F] \sin \theta - \sqrt{\frac{2}{3}} \frac{1}{F_0} [D+3\lambda] \cos \theta
\end{align}

(53)

In the case of the neutron the photon couples only via the magnetic moment and the pertinent contribution reads

\begin{align}
A_1(n\gamma^* \rightarrow n\phi) &= -\frac{e}{M_N}\mu_n A_\phi
\end{align}
\[
A_3(n\gamma^* \to n\phi) = e\mu_n A_\phi \left[ \frac{1}{s - M_N^2} - \frac{1}{u - M_N^2} \right]
\]
\[
A_4(n\gamma^* \to n\phi) = e\mu_n A_\phi \left[ \frac{1}{s - M_N^2} + \frac{1}{u - M_N^2} \right]
\]
\[
A_2(n\gamma^* \to n\phi) = A_5(n\gamma^* \to n\phi) = A_6(n\gamma^* \to n\phi) = 0 \quad (54)
\]

with \(\mu_n = -2b_0^D/3 = -1.59\) being the neutron magnetic moment at lowest chiral order.

Vector meson exchange is shown in Fig. 2. One has to add the following terms to the invariant amplitudes for photoproduction on the proton

\[
A_1(p\gamma^* \to p\phi) = \frac{e\kappa_\rho}{4M_N^2} g_{\rho N} g_{\rho \gamma \phi} \frac{t}{t - M_\rho^2}
\]
\[
A_2(p\gamma^* \to p\phi) = \frac{e\kappa_\rho}{4M_N^2} g_{\rho N} g_{\rho \gamma \phi} [t - M_\rho^2][t - m_\phi^2]
\]
\[
A_4(p\gamma^* \to p\phi) = -\frac{e}{2} \sum_{V=\rho,\omega} g_{V N} g_{V \gamma \phi} \frac{1}{t - M_V^2}
\]
\[
A_5(p\gamma^* \to p\phi) = -\frac{e\kappa_\rho}{8M_N^2} g_{\rho N} g_{\rho \gamma \phi} \frac{s - u}{[t - M_\rho^2][t - m_\phi^2]}
\]
\[
A_3(p\gamma^* \to p\phi) = A_6(p\gamma^* \to p\phi) = 0. \quad (55)
\]

For the neutron, \(g_{\rho N}\) has to be replaced by \(-g_{\rho N}\).

We now turn to the baryon resonances. Their contributions are given in Fig. 3 and read for the spin-1/2^+ octet in the proton case

\[
A_1(p\gamma^* \to p\phi) = -\frac{4}{3}(d_P + 3f_P)P_\phi \left[ \frac{u - M_N^2}{u - M_P^2} + \frac{s - M_N^2}{s - M_P^2} \right]
\]
\[
A_3(p\gamma^* \to p\phi) = \frac{4}{3}(d_P + 3f_P)P_\phi (M_P + M_N) \left[ \frac{1}{s - M_P^2} - \frac{1}{u - M_P^2} \right]
\]
\[
A_4(p\gamma^* \to p\phi) = \frac{4}{3}(d_P + 3f_P)P_\phi (M_P + M_N) \left[ \frac{1}{s - M_P^2} + \frac{1}{u - M_P^2} \right]
\]
\[
A_2(p\gamma^* \to p\phi) = A_5(p\gamma^* \to p\phi) = A_6(p\gamma^* \to p\phi) = 0 \quad (56)
\]

with

\[
P_\eta = \frac{1}{2\sqrt{3}F_\pi} [D_P - 3F_P] \cos \theta + \sqrt{\frac{2}{3}} \frac{1}{F_0} [D_P + 3\lambda_P] \sin \theta
\]
\[
P_\eta' = \frac{1}{2\sqrt{3}F_\pi} [D_P - 3F_P] \sin \theta - \sqrt{\frac{2}{3}} \frac{1}{F_0} [D_P + 3\lambda_P] \cos \theta, \quad (57)
\]

whereas the results for the neutron are obtained by replacing \(d_P + 3f_P\) by \(-2d_P\) in Eq. (50). The contributions from the spin-1/2^- resonances read in the case of
the proton

\[ A_1(p\gamma^* \rightarrow p\phi) = e \frac{4}{3} (d_S + 3f_S) S_\phi \left[ \frac{u - M_N^2}{u - M_S^2} + \frac{s - M_N^2}{s - M_S^2} \right] \]

\[ A_3(p\gamma^* \rightarrow p\phi) = e \frac{4}{3} (d_S + 3f_S) S_\phi (M_S - M_N) \left[ \frac{1}{s - M_S^2} - \frac{1}{u - M_S^2} \right] \]

\[ A_4(p\gamma^* \rightarrow p\phi) = e \frac{4}{3} (d_S + 3f_S) S_\phi (M_S - M_N) \left[ \frac{1}{s - M_S^2} + \frac{1}{u - M_S^2} \right] \]

\[ A_2(p\gamma^* \rightarrow p\phi) = A_5(p\gamma^* \rightarrow p\phi) = A_6(p\gamma^* \rightarrow p\phi) = 0 \quad (58) \]

with

\[ S_\eta = \frac{1}{2\sqrt{3}F_\pi} [D_S - 3F_S] \cos \theta + \sqrt{2} \frac{1}{3F_0} [D_S + 3\lambda_S] \sin \theta \]

\[ S_\eta' = \frac{1}{2\sqrt{3}F_\pi} [D_S - 3F_S] \sin \theta - \sqrt{2} \frac{1}{3F_0} [D_S + 3\lambda_S] \cos \theta, \quad (59) \]

where for neutrons \(d_S + 3f_S\) in Eq. (58) has to be replaced by \(-2d_S\).

4.1 Numerical results

In this subsection, we discuss the numerical results for the separated differential cross sections. From semileptonic decays of the ground state baryon octet and from strong and radiative decays of the baryon resonances one can determine most LECs. The coupling constants of the vector meson Lagrangian are quite well known. For the remaining parameters we take the values from the discussion in Sec. 3 and for \(F_0\) we employ the large \(N_c\) identity \(F_0 = F_\pi\). In Figures 4 to 7 the separated differential cross sections are given at \(k^2 = -0.04\) GeV\(^2\), \(s = 2.215\) GeV\(^2\) and \(s = 3.604\) GeV\(^2\) for \(\eta\) and \(\eta'\) electroproduction, respectively, where we have chosen the central values of the parameters. The dominance of the \(S\)-wave multipoles can clearly be seen from the differential cross sections \(d\sigma_T/d\Omega\) and \(d\sigma_L/d\Omega\) which remain almost constant for different angles \(\theta\). Although no experimental data exists at present for \(\eta\) and \(\eta'\) electroproduction close to threshold and small momentum transfer we do not present results for higher values of \(s\) and \(|k^2|\) since further resonances and loop contributions will start dominating. Therefore, it remains to be seen if our simple model is capable of reproducing experimental data close to threshold and small momentum transfer.

It is also worth estimating the theoretical uncertainty within our tree level model. Note that we do not consider the errors which arise from neglecting further resonances and loop diagrams. The only uncertainty stems then from a variation of the resonance couplings. We restrict ourselves to a discussion of the separated differential cross sections at \(\cos \theta = 0\) since the dependence on the different resonance couplings at one scattering angle is indicative for all
other angles. We vary both the vector resonance couplings $g_{V\gamma\eta}$, $g_{V\gamma\eta'}$ from Eqs. (41) and (43), and the baryon resonance couplings in Eqs. (17) and (51). It turns out that for a variation of the couplings of both the vector mesons and $P$-wave baryon resonances within their ranges as given in Sec. 3 the separated differential cross sections change less than 15%. E.g., varying $g_{V\gamma\eta}$ from 1.30 GeV$^{-1}$ up to 1.62 GeV$^{-1}$ while keeping the remaining couplings fixed leads (at $\cos \theta = 0$) to $d\sigma_T/d\Omega(p\gamma^* \to p\eta) = 0.53 \mu b/sr$ and $0.61 \mu b/sr$, respectively, and for the central value of $g_{V\gamma\eta} = 1.46$ GeV$^{-1}$ one obtains $d\sigma_T/d\Omega(p\gamma^* \to p\eta) = 0.57 \mu b/sr$. Even smaller changes occur when the parameters for the $P$-wave resonances are varied within their error bars. Choosing, e.g., $D_P = 0.37, F_P = 0.12$ and the central values for the other parameters leads to $d\sigma_T/d\Omega(p\gamma^* \to p\eta) = 0.56 \mu b/sr$. One observes a similar behavior also for the other separated differential cross sections and in the case of the neutron. A variation of the $S$-wave resonance couplings, on the other hand, leads to substantial changes in the numerical results. This is again a confirmation of the $S$-wave multipole dominance. In order to estimate the range the separated differential cross sections can occupy when varying the resonance couplings, it is therefore sufficient to change the $S$-wave resonance couplings $D_S, F_S, \lambda_S, d_S$ and $f_S$ while keeping the remaining couplings fixed at their central values. The two sets of parameters $D_S = 0.31, F_S = -0.17, \lambda_S = -0.05, d_S = -0.04$ GeV$^{-1}, f_S = -0.04$ GeV$^{-1}$ and $D_S = 0.43, F_S = -0.25, \lambda_S = -0.09, d_S = -0.10$ GeV$^{-1}, f_S = -0.09$ GeV$^{-1}$ maximize the change in the separated differential cross sections at $\cos \theta = 0$. One obtains in units of $\mu b/sr$ for the proton

\[
\frac{d\sigma_T}{d\Omega}(p\gamma^* \to p\eta) = 0.25 \ldots 1.33, \quad (57)
\]

\[
\frac{d\sigma_L}{d\Omega}(p\gamma^* \to p\eta) = 0.35 \ldots 1.03, \quad (57)
\]

\[
\frac{d\sigma_{TL}}{d\Omega}(p\gamma^* \to p\eta) = [0.39 \ldots 0.61] \times 10^{-1}, \quad (0.48 \times 10^{-1})
\]

\[
\frac{d\sigma_{TT}}{d\Omega}(p\gamma^* \to p\eta) = [0.31 \ldots 0.30] \times 10^{-2}, \quad (0.31 \times 10^{-2})
\]

\[
\frac{d\sigma_T}{d\Omega}(p\gamma^* \to p\eta') = 0.48 \ldots 0.80, \quad (60)
\]

\[
\frac{d\sigma_L}{d\Omega}(p\gamma^* \to p\eta') = 0.37 \ldots 0.54, \quad (43)
\]

\[
\frac{d\sigma_{TL}}{d\Omega}(p\gamma^* \to p\eta') = [0.37 \ldots 0.45] \times 10^{-1}, \quad (0.40 \times 10^{-1})
\]

\[
\frac{d\sigma_{TT}}{d\Omega}(p\gamma^* \to p\eta') = [0.31 \ldots 0.32] \times 10^{-2}, \quad (0.31 \times 10^{-2}) \quad (60)
\]

and in the case of the neutron

\[
\frac{d\sigma_T}{d\Omega}(n\gamma^* \to n\eta) = 0.07 \ldots 0.30, \quad (15)
\]
\[
\frac{d\sigma_L}{d\Omega}(n\gamma^* \rightarrow m\eta) = [0.01 \ldots 0.41] \times 10^{-1}, \ (0.10 \times 10^{-1})
\]
\[
\frac{d\sigma_{TL}}{d\Omega}(n\gamma^* \rightarrow m\eta) = [-0.26 \ldots -0.83] \times 10^{-2}, \ (-0.51 \times 10^{-2})
\]
\[
\frac{d\sigma_{TT}}{d\Omega}(n\gamma^* \rightarrow m\eta) = [0.16 \ldots 0.16] \times 10^{-3}, \ (0.16 \times 10^{-3})
\]
\[
\frac{d\sigma_T}{d\Omega}(n\gamma^* \rightarrow m\eta') = 0.10 \ldots 0.16, \ (0.12)
\]
\[
\frac{d\sigma_{TL}}{d\Omega}(n\gamma^* \rightarrow m\eta') = [0.03 \ldots 0.47] \times 10^{-2}, \ (0.16 \times 10^{-2})
\]
\[
\frac{d\sigma_{TT}}{d\Omega}(n\gamma^* \rightarrow m\eta') = [-0.29 \ldots -0.44] \times 10^{-2}, \ (-0.36 \times 10^{-2})
\]
\[
\frac{d\sigma_T}{d\Omega}(n\gamma^* \rightarrow m\eta') = [0.12 \ldots 0.11] \times 10^{-3}, \ (0.12 \times 10^{-3}). \quad (61)
\]

The first (second) number is the result obtained by using the former (latter) set of parameters, while the last number in brackets is the result for the central values of the couplings. The dependence of the separated differential cross sections on the \( S \)-wave resonance couplings is substantial. One can clearly see that this pole model provides a sensitive way of extracting the resonance couplings for the \( S \)-wave resonances from \( \eta \) and \( \eta' \) electroproduction experiments close to threshold, whereas the uncertainties caused by the other resonance couplings are suppressed.

5 Summary

We have studied \( \eta \) and \( \eta' \) electroproduction off protons and neutrons in a recently proposed model, which has already been used to describe \( \eta \) and \( \eta' \) photoproduction on nucleons \([13]\). Within this model an effective chiral \( U(3) \) Lagrangian is constructed which describes the interactions of the pseudoscalar meson nonet \( (\pi, K, \eta, \eta') \) with the ground state baryon octet and low-lying resonances. These include the vector mesons \( \rho_0 \) and \( \omega \) in the \( t \)-channel (the \( \phi \) meson leads to much smaller contributions and can be neglected for our purposes), and the \( J^P = 1/2^+ \) and \( 1/2^− \) baryon resonances \( P_{11}(1440) \) and \( S_{11}(1535) \). Most LECs of the effective Lagrangian can be determined using semileptonic hyperon decays and both strong and radiative decays of the baryon resonances. The couplings of the vector mesons are also quite well known. Finally, the couplings of the axial flavor-singlet currents of both the ground state and spin-\( 1/2^+ \) resonance baryons have been fixed in \( \eta \) and \( \eta' \) photoproduction. Our results are therefore predictions rather than fits to experimental data. Employing this Lagrangian we calculated Born terms including the magnetic moments of the nucleons and resonance exchange diagrams. Confining ourselves to \( S \)- and \( P \)-wave multipoles we present the four separated differential cross sections near threshold at photon virtuality \( k^2 = -0.04 \) GeV\(^2\). Of course, we do not expect our model to be valid for higher
c.m. energies and momentum transfers away from threshold, since other effects such as contributions from further resonances and chiral loop corrections will become significant. Nevertheless, the present investigation could serve as a check for our simple model and confirm results which have been obtained in the case of photoproduction. It might furthermore indicate whether the $\eta'$ meson can be included in baryon chiral perturbation theory as proposed in [12].

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**References**

[1] C. E. Carlson, J. L. Poor, Phys. Rev. D38 (1988) 2758

[2] D. B. Leinweber, T. Draper, R. M. Woloshyn, Phys. Rev. D46 (1992) 3067

[3] V. V. Frolov et al., Phys. Rev. Lett. 82, (1999) 45

[4] C. S. Armstrong et al., Phys. Rev. D60, (1999) 052004

[5] N. Isgur, G. Karl, Phys. Lett. B72, (1977) 109;
    N. Isgur, G. Karl, Phys. Lett. B74, (1978) 353

[6] S. Capstick, B. D. Keister, Phys. Rev. D51 (1995) 3598;
    R. Stanley, H. Weber, Phys. Rev. C52 (1995) 435

[7] V. Bernard, N. Kaiser, T.-S.H. Lee, U.-G. Mei\ssner, Phys. Rep. 246, (1994) 315;
    V. Bernard, N. Kaiser, U.-G. Mei\ssner, Nucl. Phys. A607, (1996) 379

[8] B. Krusche et al., Phys. Rev. Lett. 74 (1995) 3736

[9] R. Plötzke et al., Phys. Lett. B444 (1998) 555

[10] H. Breuker et al., Phys. Lett. B74 (1978) 409

[11] S. Weinberg, Phys. Rev. D11 (1975) 3583

[12] B. Borasoy, Phys. Rev. D61 (2000) 014011;
    B. Borasoy, Eur. Phys. J. A7 (2000) 255

[13] B. Borasoy, Eur. Phys. J. A9 (2000) 95

[14] Particle Data Group, C. Caso et al., Eur. Phys. J C3 (1998) 1
[15] F. E. Close, R. G. Roberts, Phys. Lett. B316 (1993) 165; 
B. Borasoy, Phys. Rev. D59 (1999) 054021

[16] U.-G. Meißen, S. Steininger, Nucl. Phys. B499 (1997) 349

[17] S.-O. Bäckmann, G. E. Brown, J. A. Niskanen, Phys. Rep. 124 (1985) 1

[18] J. F. Zhang, N. C. Mukhopadhyay, M. Benmerrouche, Phys. Rev. C52 (1995) 1134
**Figure captions**

Fig.1 Shown are the Born terms for photoproduction on the proton. The photon is given by a wavy line. Solid and dashed lines denote proton and pseudoscalar mesons, respectively.

Fig.2 Vector meson exchange. The photon is given by a wavy line. Solid and dashed lines denote nucleons and pseudoscalar mesons, respectively. The double line represents the vector meson.

Fig.3 Baryon resonance contributions. The photon is given by a wavy line. Solid and dashed lines denote nucleons and pseudoscalar mesons, respectively. The double line represents the baryon resonances $P_{11}(1440)$ or $S_{11}(1535)$.

Fig.4 Given are the separated differential cross sections $d\sigma_T/d\Omega_\eta(a)$, $d\sigma_L/d\Omega_\eta(b)$, $d\sigma_{TL}/d\Omega_\eta(c)$, $d\sigma_{TT}/d\Omega_\eta(d)$ for $\eta$ electroproduction on the proton.

Fig.5 Given are the separated differential cross sections $d\sigma_T/d\Omega_{\eta'}(a)$, $d\sigma_L/d\Omega_{\eta'}(b)$, $d\sigma_{TL}/d\Omega_{\eta'}(c)$, $d\sigma_{TT}/d\Omega_{\eta'}(d)$ for $\eta'$ electroproduction on the proton.

Fig.6 Given are the separated differential cross sections $d\sigma_T/d\Omega_\eta(a)$, $d\sigma_L/d\Omega_\eta(b)$, $d\sigma_{TL}/d\Omega_\eta(c)$, $d\sigma_{TT}/d\Omega_\eta(d)$ for $\eta$ electroproduction on the neutron.

Fig.7 Given are the separated differential cross sections $d\sigma_T/d\Omega_{\eta'}(a)$, $d\sigma_L/d\Omega_{\eta'}(b)$, $d\sigma_{TL}/d\Omega_{\eta'}(c)$, $d\sigma_{TT}/d\Omega_{\eta'}(d)$ for $\eta'$ electroproduction on the neutron.
Figure 4
Figure 5
Figure 6
Figure 7