Research of light diffraction on multilayer inhomogeneous holographic PPM diffraction structures formed under PIA conditions

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Abstract. This work presents a research of the diffraction of light on the multilayer inhomogeneous holographic diffraction structures (MIHDS) in a photopolymer material (PPM) under photoinduced light absorption (PIA) conditions. Considering PIA, the authors showed possible distortions of the selective response of the diffracted beam due to the heterogeneity of the profiles of lattices over the depth of the layer.

1. Introduction
The formation of multilayer inhomogeneous holographic diffraction structures (MIHDS) separated by intermediate layers is currently of great interest to researchers. The number of layers, their composition and dimensions directly affect the type of selective response [1]. Which in turn can be used in optical communication devices such as optical multiplexers, optical interconnects, spectral filters and sensors [2-6].

The formation of such multilayer structures by the holographic method using photosensitive media is one of the most effective and relevant [1, 7, 8]. The effectiveness of this method is due to the ability to create different diffraction structures, varying the internal composition of the sample.

In [1, 9-11], similar structures were investigated on the basis of photopolymer material (PPM). For example, in the work [1], insufficient attention is paid to the influence of inhomogeneous parameters, for example, the absorption of light in the depth of the hologram varying during exposure. In turn, the heterogeneity of the lattices profile can lead to distortion of the kind of selective response of the multilayer holographic diffraction structure.

In [7, 8], the authors demonstrate that light absorption has a significant effect on the profiles and diffraction characteristics of holographic lattices. Photoinduced change of optical absorption (PIA) is observed in the process of holographic lattices formation [12]. As a result, the amplitude profiles of the diffraction gratings in each MIHDS layer will be transformed during recording [12,13]. However, in [7, 8] there was no comparison of the mathematical model with the experimental research on diffraction of light on MIHDS formed in PPM taking into account PIA.

Thus, the aim of this work is a theoretical and experimental research of the diffraction characteristics of MIHDS in PPM formed under PIA conditions.
2. Theoretical model

In this work, we consider the transmission geometry of the recording of a holographic multilayer diffraction structure, which is the incidence of two recording beams of monochromatic waves $E'_0$ and $E'_1$ at angles $\theta_0$ and $\theta_1$ on the sample (figure 1).

![Figure 1. Geometry of recording. $t$ is the thickness of the intermediate layer, $d$ is the thickness of the PPM layer, $D$ is the thickness of the entire MIHDS](image1)

![Figure 2. Scheme of diffraction on the $n$-th layer](image2)

The distribution of the intensity of the interference pattern on the $n$-th layer of the MIHDS in this case:

$$I^n(t,r) = I^n(t,r) \left[1 + m^n(t,r) \cos\left(K^n \cdot r\right)\right],$$

where $m^n(t,r) = 2\left[I^n_0(t,r)I^n_1(t,r)(\varepsilon,\varepsilon)/(I^n_0(t,r) + I^n_1(t,r))\right]$; $j = 0, 1$; $K^n = k^n_0 - k^n_1$ – vector of lattice; $I^n(t,r) = [I^n_0(t,r) + I^n_1(t,r)]$; $I^n_j(t,r) = I^{j,n}(t,r) \cdot e^{-a_j(t,r) \gamma / \cos(\theta_j)}$; $I^n_j(t,r) = |E^n_j(r)|^2$; $r$ – radius-vector; $\alpha^n(t,r) = \alpha_2 + \alpha^n_\alpha \exp\left[-(I^n_0(t,r) / \cos(\theta_0) + I^n_1(t,r) / \cos(\theta_1)) \cdot y \cdot t / T^n\right]$ – coefficient of PIA; $k^n_0$ – wave vectors; $n$ – layer number; $\alpha_2$ and $\alpha^n_\alpha$ – absorption coefficients of substrate and dye.

Expression (1) shows how the PIA affects the intensity of the light field inside each layer by changing the recording conditions of the lattices, which leads to a modulation of the refractive index over the depth of the hologram.

The resolution for the amplitude of the 1-st harmonic refractive index is generally derived from the kinetic equations of polymerization-diffusion formation of diffraction structures (DS) in the PPM and will consist of two components associated with photopolymerization and diffusion mechanisms of recording [12]:

$$n^n(t,r) = n^n_{lp}(t,r) + n^n_{ld}(t,r),$$

where $n^n_{lp}(t,y) = \delta n^n_{lp} \frac{2k}{b^n(t,y)} \int_0^n I^n_{0a}(\tau,y)\left[p^n(\tau,y)km^n(\tau,y) - f^n(\tau,y)\right] \left[1 + 1.5L^n(\tau,y)\right] d\tau$; $\delta n^n_{lp}$ and $\delta n^n_{ld}$ are polymerization and diffusion coefficients; $k$ is the degree of non-linearity of the process; $b^n_{lp}(t,y) = \exp\left[-s(1 - p^n(\tau,y))\right]$; $L^n(t,y) = k(k - 1)\frac{m^n(t,y)^2}{2}$; $n^n_{ld}(t,y) = \delta n^n_{ld} \int_0^n f^n(\tau,y)b^n_{lp}(\tau,y) d\tau$; $f^n(\tau,y) = \frac{2k}{b^n(t,y)} \int_0^n p^n(\tau,y)m^n(\tau,y)I_{0a}^n(\tau,y) \exp\left[-\frac{b^n_{lp}(T,y)}{b^n(T,y)} \frac{2k}{b^n(T,y)} I_{0a}^n(T,y) \left[1 + 1.5L^n(T,y)\right] dT\right] d\tau$.
where $b^i(t, y)$ is the ratio of the polymerization time to the diffusion time.

Expression (2) describes the kinetics of the formation of the refractive index profile for the first harmonic, based on the PIA. This profile can be approximated by a function of the form, where the parameters $c_s, s, t$ are found for each layer separately by the approximation obtained when writing the MIHDS [12-15].

The geometry of diffraction on MIHDS is shown in figure 2. For each layer of diffraction structure, the light field $E$ due to the Bragg diffraction of the reading beam on the first spatial harmonica of the lattice is recorded as the sum of waves of zero and first diffraction orders [7, 8]:

$$E(r, t) = \sum_{j=0,1} E_j = \frac{1}{2} \left[ \sum_{m=s, p} \sum_{j} |e_j(r)| \cdot \exp \left[ i \cdot ((\omega_0 + \omega) t - k_j \cdot r) \right] d\omega + c.c. \right],$$

each of which is represented by two components of the intensity vector $E_j$ in the corresponding orthogonal polarization basis, given by two orts $e_j^s$ and $e_j^t$ lying in a plane perpendicular to the beam axis $E_j$. Here $E_j^{m}(\omega, r)$ are slowly changing coordinate functions and are found from the first approximation equations of slowly changing amplitudes (SHA) $\omega_0$ the central frequency, $m = s, p$ (the index $s$ corresponds to a wave polarized perpendicular to the diffraction plane $YX$, the index $p$ corresponds to a wave polarized in the diffraction plane $YX$).

In the case of Bragg diffraction on separate diffraction structures in an optically inhomogeneous PPM $E_j^m(r)$ layer, the amplitudes of the interacting waves are determined by two systems of the coupled wave equation (CWE) in partial derivatives [7, 8, 14]:

$$\begin{align*}
N_{r_0}^m \cdot \nabla & E_0^m(r) = -i C_0^m E_1^m(r) n_i(r) \exp (+i \Delta K \cdot r) \\
N_{r_1}^m \cdot \nabla & E_1^m(r) = -i C_0^m E_0^m(r) n_i(r) \exp (-i \Delta K \cdot r),
\end{align*}$$

(3)

where $E_j^m(r)$ are the amplitude profiles of the beams, $N_{r_0,1}^m$ is the group normals, $E_j^m(r) = N_{r_0,1}^m \cdot E_j^m(r)$, $C_j^m$ is the amplitude coupling coefficients, $n_i(r)$ is the normalized amplitude profile of the first harmonic of the refractive index, $\Delta K$ is the phase detuning vector.

Amplitude coupling coefficients from expression (3) are determined for each layer as [7, 8, 14]:

$$\begin{align*}
C_0^m &= \frac{1}{4} e_0^m \cdot 0.5 \cdot n_i(r) \cdot \max[n_i(r)] \cdot e_0^m \\
C_1^m &= \frac{1}{4} e_0^m \cdot 0.5 \cdot n_i(r) \cdot \max[n_i(r)] \cdot e_0^m,
\end{align*}$$

where $\omega$ – angular frequency of light waves, $n_i(r)$ – refractive index, $e_j^m$ – unit vectors of beam polarization.

To determine the diffraction light field at the output of the MIHDS, a matrix method is used to describe the transformation of plane light waves in multilayer media [7, 8]:

$$E^N = T^N \cdot E^{N-1},$$

de where $E^N = \begin{bmatrix} E_0^N(\omega, \Delta K) \\ E_1^N(\omega, \Delta K) \end{bmatrix}$; $E^{N-1} = \begin{bmatrix} E_0^{N-1}(\omega, \Delta K) \\ E_1^{N-1}(\omega, \Delta K) \end{bmatrix}$; $T^N = \begin{bmatrix} T_{00}^N(\omega, \Delta K) & T_{10}^N(\omega, \Delta K) \\ T_{01}^N(\omega, \Delta K) & T_{11}^N(\omega, \Delta K) \end{bmatrix}$ – matrix transfer function of the $N$-th layer of the MIHDS; $E_i^{N-1}(\omega, \Delta K) \equiv E_i^{N}(\omega, \Delta K)$ – frequency-angular spectrum (FAS) at the input and output of the $N$-layer.

At the same time, the phase detuning modulus depends on the angle of incidence and the frequency of the reading beam.
\[ \Delta K = \Delta K(\theta) + \Delta K(\omega), \]
where \( \Delta K(\theta) = (D/B) \theta \), \( \Delta K(\omega) = (C - AD/B) \omega \) and the coefficients \( A, B, C, D \) are defined in [16].

The components of the \( T^N \) transition matrix are determined on the basis of analytical solutions of two-dimensional partial differential equations of coupled waves and are described by the following expressions [7, 8, 13-15]:

\[
T_{00}^n(\omega, \Delta K) = 1 - \frac{b_{0m}^n}{2} A \cdot \sinh \left[ \frac{cs(1 + q)}{2} \right] \cdot z \cdot F_1(1 - \alpha, 1 + \alpha; 2; w) dq,
\]
\[
T_{01}^n(\omega, \Delta K) = -i \frac{b_{01}^n}{2} \sqrt{V_j} \int_{-1}^{1} \exp \left[ -i \frac{\Delta K}{2} (1 - q) \right] \cdot \cosh^{-1} [c(s(1 - q)/2 - t)] \cdot z \cdot F_1(-\alpha, \alpha + 1; w) dq,
\]
\[
T_{10}^n(\omega, \Delta K) = -i \frac{b_{10}^n}{2} \sqrt{V_i} \int_{-1}^{1} \exp \left[ -i \frac{\Delta K}{2} (1 - q) \right] \cdot \cosh^{-1} [c(s(1 - q)/2 - t)] \cdot z \cdot F_1(-\alpha, \alpha + 1; w) dq,
\]
\[
T_{11}^n(\omega, \Delta K) = 1 - \frac{b_{11}^n}{2} A \exp \left[ i \frac{\Delta K}{2} (1 - q) \right] \cdot \sinh \left[ \frac{cs(1 + q)}{2} \right] \cdot z \cdot F_1(1 - \alpha, 1 + \alpha; 2; w) dq,
\]
where \( z \cdot F_1(a, b; c; z) \) – Gaussian hypergeometric function, \( w = \frac{\sinh [cs(1 - q)/2] \sinh [cs(1 + q)/2]}{\cosh [ct] \cosh [c(s-t)]} \),
\[ A = \left( \cosh [ct] \cosh [c(s-t)] \right)^{-1}, \]
\[ \alpha = b^n_j, \]
\[ n = d_n \cdot \frac{C_i}{\sqrt{V_i V_j}}, \]
\[ d_n \] – layer thickness with PPM, \( V_j = \cos \theta_j \), \( \theta_j \) – the angles between the group norms \( N_{ij}^m \) and the y-axis (figure 2), \( \Delta K = |\Delta K| \cdot d_n \) – generalized phase mismatching.

The intermediate layer with \( t_a \) thickness gives a phase run, and if we consider that the refractive index of the intermediate layer is equal to the refractive index of the hologram, then the transition matrix can be represented as:

\[
A^a = \exp \left[ -i (k_1 \cdot y_a) t_a \right] \cdot \begin{bmatrix} 1 & 0 \\ 0 & \exp \left[ -i \Delta K \cdot t_a / d_n \right] \end{bmatrix}.
\]

The connection between the input field \( E_0 \) and the diffraction field \( E^N \) at the output of the MIHDS can be expressed by multiplying the transition matrices of all layers:

\[ E^N = T \cdot E_0 \]

where \( T = T^N \cdot A^{N-1} \cdot T^{N-2} \cdot \ldots \cdot A^2 \cdot T^1 \cdot \ldots \cdot A^2 \cdot T^3 \) – matrix transfer function of the entire MIHDS; \( E_0 = \begin{bmatrix} E_0(\omega, \Delta K) \\ 0 \end{bmatrix}, E_0(\omega, \Delta K) \) – FAS of light field incident on MIHDS.

In the numerical calculation, we will consider the case of the interaction of only flat monochromatic light beams, with a single amplitude. If we assume that the amplitude of the input field \( E_0 = \delta(\omega, \theta) \), and \( \int \int (E_0 \cdot E_0^* ) d\omega d\theta = 1 \) then the expression for the diffraction efficiency at the output of the MIHDS [7, 8]:

\[
\eta_0 = \left| \frac{E_0^N \cdot E_1^{N^*}}{1} \right|^2 = \left| \frac{E_1^{N^*}}{1} \right|^2 = \left| E_0^N(\omega, \Delta K) \right|^2,
\]
where \( E_0^N(\omega, \Delta K) \) is expressed in terms of the elements \( T_{ij}^N(\omega, \Delta K) \) of the matrix transfer functions \( T^N \) of the layers.
3. Experimental part

Figure 3 shows an experimental setup for the recording and reading of MIHDS. The installation scheme consists of a He-Ne laser with a wavelength of 633 nm and a power of 2 mW, a beam-splitting cube (B.S.C.), mirrors (M), a shutter (S), a rotary mechanism (R.M.), light beam analyzers (A), and a sample with a recorded MIHDS. The sample consisted of two PPM films "HPP633. 5" produced by LLC "Polymer Holograms-Novosibirsk" with a layer thickness of 45±5 microns on a glass substrate with a thickness of 1±0.1 mm and a protective film of 135 microns. The angles of incidence of the beams when recording 10 degrees. The reading was performed by covering one of the beams with a blind and rotating the sample with a rotary mechanism, simulating a change in the reading angle of the HDS (holographic diffraction structure).

![Figure 3](image)

Figure 3. Installation diagram for (a) holographic recording and (b) MIHDS reading

4. Result

Based on the characteristics of the recorded single hologram, parameters were selected for modeling the kinetics of formation and reading for both single and double-layer HDS. The following parameters were used for numerical calculation: \( d_s = 45 \) μm, \( t_s = 135 \) μm, \( \alpha_1 = 0.016 \) Np, \( \alpha_2 = 2.2 \) Np, \( \theta = 10 \) degree, \( \lambda = 633 \) μm, \( \delta n_\rho = 0.004 \), \( \delta n_d = 0.5 \delta n_\rho \), \( k = 0.5 \), \( s = 1 \).

Figure 4 shows the normalized angular selectivity of diffracted light beams obtained by the experimental method and by numerical simulation by expression (4) for double-layer and single-layer HDS.

![Figure 4](image)

Figure 4. Angular selectivity of single and double-layer HDS

The ratio of the thickness of the intermediate layer to the PPM layer \( (t_s / d_s) \) is 6. Figure 4 shows that the type of selective response for a two-layer structure is distorted, local minima do not reach zero, and some asymmetry of the side lobes appears. These distortions are due to the irregularities of the grating profiles in the depth of the sample caused by the influence of the PIA on the HDS formation process in each layer. At the same time, the envelope of the selectivity loop for the two-layer HDS repeats the appearance of a single grid.

Figure 5 shows a comparison of the angular selectivity of the two-layer HDS with two variations of the intermediate layer obtained by the experimental method.
As shown in Figure 5 (a, b), increasing the thickness of the intermediate layer results in an increase in additional local maxima due to interference effects, and the envelope also coincides with the selectivity loop of the single HDS (figure 4).

5. Conclusion
Therefore, a study of the diffraction of light on a two-layer HDS formed in the PPM taking into account the PIA demonstrates that the heterogeneity of the profile in depth of lattices leads to distortion of the type of selective response. Local minima do not reach zero and some asymmetry of the side lobes appears.

The diffraction characteristics can be improved by selecting a specific composition for each layer, which will allow the formation of lattice profiles that are uniform in depth.

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