Moving Quantum Systems: Particles Versus Vacuum

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Abstract

We give an overview on a couple of recent results concerning the KMS-condition and the characterization of thermodynamic equilibrium states from a moving observer’s point of view. These results include a characterization of vacuum states in relativistic quantum field theory and a general derivation of the Unruh effect.

1 Introduction

The KMS-condition introduced by Kubo, Martin, and Schwinger in [15, 19] is a generalization of the Gibbs characterization of thermodynamic equilibrium states which is widely used in physics today. In contrast to the Gibbs condition, the KMS-condition is meaningful also for any infinitely extended system, whose Hamiltonian $H$ does, in general, not have the property that $e^{-\beta H}$ is a trace class operator ($\beta$ denotes the inverse temperature).

The relevance of the KMS-condition for the operator algebraic description of thermodynamic equilibrium states was first discussed by Haag, Hugenholtz, and Winnink [12, 11]. A general axiomatic derivation of the KMS-condition from first principles was first proposed by Haag et al. (see [11, 14] and references given there), who showed that the KMS-condition follows from the dynamical stability of the state against small perturbations of the Hamiltonian, together with some clustering properties exhibited by many examples. A bit later, Pusz and Woronowicz proved the equivalence of the

\footnote{For the sake of brevity, the case that $H \geq 0$ is occasionally considered as the KMS-condition at zero temperature.}
KMS-condition at nonnegative temperature on the one hand and a condition called complete passivity on the other [21], which can be derived from the Zeroth, the First, and the Second Law of Thermodynamics without making any more technical assumptions. A couple of years ago, Bros and Buchholz introduced a relativistic KMS-condition in quantum field theory, which, in contrast to the usual KMS-condition, is not confined to a distinguished frame of reference [5]. Their conjectures and ideas on how to derive this condition from first principles of thermodynamics are partially confirmed by the results to be discussed below, and they have been an important motivation for this research.

The discussion to follow is a summary of Refs. [17] and [18]. Ref. [17] also contains a more detailed introduction into the notions, ideas, and further references.

The passivity property of thermodynamic equilibrium states means that no cyclic process, i.e., no temporary perturbation of the Hamiltonian by observables, can perform more work than it requires in order to take place. This can be justified by the Second Law of Thermodynamics, and the First Law of Thermodynamics can be used to justify the mathematical formulation of the condition. Complete passivity strengthens this condition in a fashion that can be justified by the Zeroth Law of Thermodynamics.

Already the passivity condition can hold in one frame of reference at most whenever matter is present, as moving matter drives windmills, turbines, etc.. In [17], the Pusz-Woronowicz result has been generalized by replacing, on the one hand, the condition of complete passivity by the weaker condition of complete semipassivity, and by replacing, on the other hand, the KMS-condition by the requirement that the KMS-condition is fulfilled in some inertial frame. Both conditions can hold in several frames of reference for translation invariant states of matter. This will be discussed in Sect. 2.

Recently it was proved by Guido and Longo in [10] that the KMS-condition at a finite and nonnegative temperature $\beta$ is equivalent to a condition called complete $\beta$-boundedness. This condition imposes a bound on the number of degrees of freedom in certain phase space regions and is a weak form of the Buchholz-Wichmann nuclearity condition [9], very similar to the (weaker) Haag-Swieca compactness criterion [13]. While these criteria have been suggested and investigated in the setting of algebraic quantum field theory, complete $\beta$-boundedness characterizes thermodynamic equilibrium states at inverse temperature $\beta$ in much more general terms and without referring to any field theoretic structures. The result by Guido and Longo suggests to look for a modification of the $\beta$-boundedness condition that can hold, like semipassivity, in several frames of reference, and leads to the
same conclusions as obtained for semipassivity. Such a condition is \textit{semi-\(\beta\)-boundedness}; this result of \cite{18} will be discussed in Sect. 3.

On the other hand, passivity can hold in several frames of reference in the absence of matter. Taking this as a basic characterization of a vacuum state in quantum field theory, one can prove the spectrum condition and the Unruh effect, i.e., the Bisognano-Wichmann symmetries \cite{22, 1, 2}, for all pure vacuum states of a local quantum field theory. This result of \cite{17} will be discussed in Sect. 4. Similar results for de Sitter and Anti-de Sitter spacetimes have been obtained in \cite{4} and \cite{7}, respectively. Recent results concerning the Bisognano-Wichmann symmetries of algebraic quantum field theories in Minkowski space can be found in \cite{1, 16, 20, 3} and the references given there, and for results concerning the possible role of these symmetries in Robertson-Walker spacetimes, see \cite{8}.

In the Conclusion (Sect. 5), a couple of remarks are made concerning the notion of particle and the Unruh effect.

## 2 Semipassivity and homogeneous states of matter

Our setting is as follows:

- A von Neumann algebra \(\mathcal{M}\) on a Hilbert space \(\mathcal{H}\) represents the observable quantities characterizing the system under consideration.

- For \(s \geq 1\), a family of \(1 + s\) commuting self-adjoint operators \((H, P) = (H, P_1, \ldots, P_s)\) generates a (strongly continuous and unitary) representation \(V\) of the spacetime translation group \((\mathbb{R}^{1+s}, +)\).

- A \(V\)-invariant state \(\omega\) of \(\mathcal{M}\), our object of investigation, is induced by a cyclic vector \(\Omega\) of \(\mathcal{M}\), and \(H\Omega = P_1\Omega = \cdots = P_s\Omega = 0\).

Define \(S_{\beta} := \{z \in \mathbb{C} : -\beta < \text{Im} z < 0\}\). \(\omega\) is called a \textit{KMS-state at the inverse temperature} \(\beta \geq 0\) if there exists a continuous function \(F : S_{\beta} \to \mathbb{C}\) that is analytic in \(S_{\beta}\) and satisfies

\[
F(t) = \omega(B_tA) \quad \text{and} \quad F(t - i\beta) = \omega(AB_t) \quad \text{for all} \ t \in \mathbb{R}, \ A, B \in \mathcal{M}.\tag{1}
\]

A KMS-state at \(\beta = 0\) (infinite temperature) is a trace, i.e., \(\omega(AB) = \omega(BA)\) for all \(A, B \in \mathcal{M}\), and \(\omega\) may be considered a “KMS-state at zero temperature” if \(H \geq 0\), i.e., if \(\omega\) is a \textit{ground state} of \(H\).
As explained in [21], a cyclic change of the Hamiltonian is represented by a unitary operator in the unit operator’s norm-connected component $U_1(M)$ of the unitary group $U(M)$ of $M$. If the system is in the above state $\omega$ before the cycle $W \in U_1(M)$ is started, the total amount of work performed on the system during the cycle $W$ is $L_W := \langle W \Omega, [H, W] \Omega \rangle = \langle W \Omega, HW \Omega \rangle$, if $W \in U_1(M)$. It is completely passive if for each $N \in \mathbb{N}$, the state $\omega^\otimes N$ of the $N$th tensorial algebra $\mathcal{M}^\otimes N$ of $\mathcal{M}$ defined by

$$\mathcal{M}^\otimes N \ni A_1 \otimes A_2 \otimes \cdots \otimes A_N \mapsto \omega(A_1) \omega(A_2) \cdots \omega(A_n)$$

is passive. As shown in [21], a state is completely passive if and only if it is a KMS-state or a ground state.

Suppose that $\omega$ is passive with respect to the Hamiltonian $H$. If the system is not at rest, but moves at a velocity $u$, then the time evolution is not generated by $H$, but by $H + uP$. If $W \in U_1(M)$ satisfies $[H, W] \in \mathcal{M}$ and $[P, W] \in \mathcal{M}$, then the work performed by the corresponding cycle is

$$-L = -\langle W \Omega, (H + uP) W \Omega \rangle \leq -\langle W \Omega, uP W \Omega \rangle,$$

as $\omega$ has been assumed to be passive with respect to $H$. Defining $|P| := \sqrt{P_1^2 + \cdots + P_s^2}$, one finds $-uP \leq |u| |P|$, so

$$-L \leq |u|\langle W \Omega, |P| W \Omega \rangle. \quad (2)$$

Now suppose that $\omega$ is not necessarily passive with respect to $H$. We call $\omega$ semipassive if the work a cycle can perform is bounded as in Ineq. (2), i.e., if there is a constant $E \geq 0$ such that

$$-\langle W \Omega, HW \Omega \rangle \leq E\langle W \Omega, |P| W \Omega \rangle \quad (3)$$

for all $W \in U_1(M)$ with $[H, W] \in \mathcal{M}$ and $[P, W] \in \mathcal{M}$. The constant $E$ will be referred to as an efficiency bound of $\omega$. Generalizing also the notion of complete passivity, $\omega$ will be called a completely semipassive state if all its

Footnote 1:

Note that $H$ is an unbounded operator and that $\langle W \Omega, HW \Omega \rangle$ does not need to be defined for all $W \in U_1(M)$. The expression $\langle Hx, Wy \rangle - \langle x, WHy \rangle$ is defined for all $x, y$ in the domain of $H$. $[H, W] \in \mathcal{M}$ means that the sesquilinear form defined this way is bounded and that the associated bounded operator is an element of $\mathcal{M}$; commutators involving $H$ or $P$ are to be read this way.

Footnote 2:

In a relativistic theory, this generator must be multiplied by the time dilation factor $\gamma = (1 - u^2/c^2)^{-1/2}$, see below.
finite tensorial powers are semipassive with respect to one fixed efficiency bound $E$.

Evidently, a state is completely semipassive in all inertial frames if it is completely passive in some inertial frame. The following theorem (Thm. 3.3 in [17]) shows that if conversely, a state is completely semipassive in a given inertial frame, then there exists an inertial frame where it is completely passive.

**Theorem 2.1** The state $\omega$ is completely semipassive with efficiency bound $E$ if and only if there exists a $u \in \mathbb{R}^s$ with $|u| \leq E$ such that with respect to $H + uP$, $\omega$ is a ground state or a KMS-state at a finite inverse temperature $\beta \geq 0$.

In a relativistic theory, the Hamiltonian of the system moving at velocity $u < c$ is not $H + uP$, but $\gamma(H + uP)$, where $\gamma = (1 - |u|^2/c^2)^{-\frac{1}{2}} \equiv (1 - |u|^2)^{-\frac{1}{2}}$. Theorem 2.1 still holds without any modification, but the inverse temperature of the system is not the parameter $\beta$ found there, but $\beta/\gamma$.  

### 3 Semipassivity and $\beta$-boundedness

Thm. 2.1 describes a most generic example of a nonequilibrium state, and the question is whether bounds on the power of a cyclic process could be of interest in less generic situations than that of a translation invariant state. As far as such investigations are concerned, it is an obstacle of the above definition of semipassivity that the invariance of $\omega$ is part of the definition and its motivation. While the problem addressed in Thm. 2.1 is nontrivial only if $\omega$ is invariant under all spacetime translations, it would be of interest whether the semipassivity condition can be subdivided into this invariance property plus some additional condition that may be meaningful in other situations as well. Such a condition can be obtained by modifying the following notion, which has first been investigated by Guido and Longo:

**Definition 3.1** The state $\omega$ is called $\beta$-bounded with bound 1 if the linear space $M\Omega$ is a subspace of the domain of $e^{-\beta H}$ and if the set $e^{-\beta H}M_1\Omega$ consists of vectors with lengths $\leq 1$ where $M_1 := \{A \in M : \|A\| \leq 1\}$.

$\omega$ is called completely $\beta$-bounded if for each $n \in \mathbb{N}$, the state $\omega^\otimes n$ on the algebra $M \otimes \cdots \otimes M$ is $\beta$-bounded with bound 1.

The following theorem shows that this notion characterizes thermodynamic equilibrium states at a finite and nonnegative inverse temperature $\beta$:
Theorem 3.2 (Guido, Longo) \( \omega \) is completely \( \beta \)-bounded if and only if it is a ground state or a KMS-state at an inverse temperature \( \geq 2\beta \).

Now modify Def. 3.1 as follows:

Definition 3.3 The state \( \omega \) is called semi-\( \beta \)-bounded if there exists a damping factor \( \mathcal{E} \geq 0 \) such that the linear space \( \mathcal{M}\Omega \) is a subspace of the domain of \( e^{-\beta(H+\mathcal{E}|P|)} \) and the set \( e^{-\beta(H+\mathcal{E}|P|)}\mathcal{M}\Omega \) consists of vectors with length \( \leq 1 \).

It is called completely semi-\( \beta \)-bounded if for each \( n \in \mathbb{N} \), the state \( \omega^{\otimes n} \) on the algebra \( \mathcal{M} \otimes \cdots \otimes \mathcal{M} \) is semi-\( \beta \)-bounded with respect to one fixed damping factor \( \mathcal{E} \geq 0 \).

One then obtains the following modification of Thm. 3.2 (Thm. 6 in [18]):

Theorem 3.4 A stationary and homogeneous state \( \omega \) is completely semi-\( \beta \)-bounded with respect to a damping factor \( \mathcal{E} \geq 0 \) if and only if there exists a \( u \in \mathbb{R}^s \) with \( |u| \leq \mathcal{E} \) such that \( \omega \) is a ground state or a KMS-state at an inverse temperature \( \geq 2\beta \) with respect to \( H + uP \).

As \( |P| \) is a positive operator, the operator \( e^{-\beta \mathcal{E}|P|} \) is bounded and provides an additional damping term, so \( \beta \)-boundedness implies semi-\( \beta \)-boundedness for all \( \mathcal{E} \geq 0 \), i.e., semi-\( \beta \)-boundedness is the weaker assumption.

4 Passivity and vacuum states

If \( \omega \) is a vacuum state, then the considerations of the Introduction suggest that \( \omega \) is passive with respect to each Hamiltonian of the form \( \gamma(H + vP) \), where \( |v| < c = 1 \) and \( \gamma = (1 - v^2/c^2)^{-1/2} \).

In what follows, we assume this and, in addition, that \( \omega \) is a pure state. It has been shown in [17] that under these assumptions, the joint spectrum of \( H \) and \( P \) is contained in the cone

\[
V_+ := \{(\eta, k) \in \mathbb{R}^{1+s} : \eta \geq 0, \eta^2 - k^2 \geq 0\},
\]

i.e., the spectrum condition holds.

To further proceed now, we need some basic structures of local quantum fields, which associate von Neumann algebras \( \mathcal{M}(\mathcal{O}) \) of local observables with all bounded open spacetime regions \( \mathcal{O} \subset \mathbb{R}^{1+s} \) in such a way that the following conditions are satisfied:
• **Isotony.** If $O$ and $P$ are bounded open regions in $\mathbb{R}^{1+s}$ such that $O \subset P$, then $\mathcal{M}(O) \subset \mathcal{M}(P)$.

• **Locality.** If $O$ and $P$ are spacelike separated bounded open regions in $\mathbb{R}^{1+s}$ and if $A \in \mathcal{M}(O)$ and $B \in \mathcal{M}(P)$, then $AB = BA$.

• **Spacetime Translation Covariance.** The representation $V$ of $(\mathbb{R}^{1+s}, +)$ satisfies

$$V(x)\mathcal{M}(O)V(x)^* = \mathcal{M}(O + x)$$

for all bounded open sets $O \subset \mathbb{R}^{1+s}$ and for all $x \in \mathbb{R}^{1+s}$.

• **Spectrum Condition.** The joint spectrum of the generators of $V$ is contained in the closed forward light cone.

• $\mathcal{M}$ is assumed to be the smallest von Neumann algebra that contains all local algebras $\mathcal{M}(O)$ associated with bounded open regions.

The trajectory of a (pointlike) observer who is uniformly accelerated in the 1-direction with acceleration $a$ can be translated to the curve

$$\tau \mapsto \frac{c^2}{a} \left( \sinh \frac{a \tau}{c}, \cosh \frac{a \tau}{c}, 0, \ldots, 0 \right),$$

where $\tau \in \mathbb{R}$ denotes the accelerated observer’s eigentime. The wedge $W_1 := \{x \in \mathbb{R}^{1+s} : x_1 > |x_0|\}$, which is referred to as the *Rindler wedge*, is the region of all spacetime points the accelerated observer can communicate with using causal signals. Therefore, the elements of the algebra $\mathcal{M}(W_1)$ are precisely those observables the uniformly accelerated observer can measure.

The images of $W_1$ under Poincaré transformations are referred to as *wedges*.

We assume that some uniformly accelerated observer exists:

• There is a self-adjoint operator $K_1$ generating, within $W_1$, the free dynamics of the uniformly accelerating observer, i.e.,

$$e^{i\tau K_1} \mathcal{M}(O)e^{-i\tau K_1} = \mathcal{M}(\Lambda_1(\frac{a}{c} \tau)O)$$

for all $\tau \in \mathbb{R}$ and all bounded open sets $O \subset W_1$. $K_1$ strongly commutes with $P_2, \ldots, P_s$, and $K_1 \Omega = 0$.

Here, $\Lambda_1(\frac{a}{c} \tau)$ denotes the Lorentz boost by $\frac{a}{c} \tau$ in the 1-direction. $\mathcal{M}$ is not yet assumed to be covariant under a full representation of the Poincaré group, although the assumption that $K_1$ strongly commutes with $P_2, \ldots, P_s$ is already a part of this condition. The following result has been proved in [17].
Proposition 4.1. If the pure state $\omega$ satisfies the spectrum condition and if $\omega$ exhibits passivity with respect to the dynamics generated by $K_1$, then $\omega$ is a KMS-state of $\mathcal{M}(W_1)$ with respect to $K_1$ at the Unruh temperature $T_U = \frac{\hbar a}{2\pi c k}$.

5 Conclusion

The above results show that on the one hand, moving states of matter violate passivity, and for moving homogeneous states the extent to which this is the case can be considered as a kind of a “distance” from thermodynamic equilibrium.

On the other hand, vacuum states are characterized by the property that they are passive in the eyes of every observer whose motion does not enforce violation of passivity due to nonstationary inertial forces.

These two observations may help clarifying the notion of a particle, which is particularly fuzzy in quantum field theories on curved spacetimes (already in the Rindler wedge). Particles should generate some kind of “wind” in the eyes of some observer, so this wind should be the appropriate indicator whether or not they are present. With regard to the Unruh effect, this point of view confirms that the vacuum state is, indeed, a state without any particles, even in the accelerated observer’s eyes.

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