Multigravity in six dimensions: Generating bounces with flat positive tension branes

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Abstract

We present a generalization of the five dimensional multigravity models to six dimensions. The key characteristic of these constructions is that we obtain solutions which do not have any negative tension branes while at the same time the branes are kept flat. This is due to the fact that in six dimensions the internal space is not trivial and its curvature allows bounce configurations with the above feature. These constructions give for the first time a theoretically and phenomenologically viable realization of multigravity.
1 Introduction

Over the past two years there has been considerable activity studying brane configurations in which gravity is modified in large scales motivated by higher dimensional constructions [1,2]. The interesting feature of these scenarios is that modification of gravity at cosmological scales was triggered by microphysics considerations which have to do with the structure of the extra dimensional space of usually Planck scale volume. These scenarios are in principle testable by the CMB power spectrum [7] and gravitational lensing [8] observations.

The prototype multigravity scenario was the “$++-$” bigravity model [1] where, apart of the graviton zero mode, an anomalously light graviton Kaluza-Klein (KK) state is present. This gave the possibility that gravity at intermediate scales ($1\text{mm} < r < 10^{26}\text{cm}$) is mediated by both the massless graviton and the ultralight state, leading to the terminology “bigravity”. At large scales were the Yukawa suppression of the massive state turns on, the gravitational constant decreases to a value which for example in the symmetric configuration is half of its value at intermediate scales. For asymmetric configurations one can have a situation where the massive graviton dominates and gravity switches off at large scales or, on the other hand, where the massless mode dominates and gravity does not change at all. Similar constructions of freely moving negative tension branes can also give qualitatively different modifications of gravity at large scales. The quasi-localized gravity model of Gregory, Rubakov and Sibiryakov [3] has no normalizable zero mode but gravity at intermediate distances is generated by a “resonance”-like coupling of the low part of the continuum of the KK spectrum. In this construction, the gravitational law at large scales changes from four dimensional to five dimensional. The change of the nature of the gravitational law was observed also in the the crystalline universe model [5] where gravity at intermediate distances is generated by the first low lying band of KK states and at large scales gives again the five dimensional law of gravity.

So far our discussion has considered the gravitational force due to tensorial gravitational modes. However, in the gravity sector there exists a moduli corresponding to the fluctuation(s) of the negative tension brane(s), the radion, which changes the large scale phenomenology of these models. In particular, the radion is a ghost field [11–13], i.e. it has wrong sign kinetic term, so at large scales it will dominate in all the above constructions giving scalar antigravity [14] instead of a reduction of the Newton’s constant or a change of the nature of the gravitational law. However, the very fact that there is a physical ghost
in the spectrum makes these models unacceptable. Note that the essential characteristic of all these models is the bounce of the warp factor generated by the presence of the moving negative tension branes. This association between the bounce in the warp factor and presence of negative tension branes is an unavoidable feature in flat (five dimensional) brane models and is linked to the fact that the weaker energy condition is violated at the position of the bounce [13,16]. As a result one must have the unacceptable ghost field.

A way out of this difficulty is to consider $AdS_4$ branes instead of flat branes (see [17] for a different possibility involving an external four-form field). In this case the warp factor has a "cosh" form which naturally generates a bounce without any need of floating negative tension branes. Indeed, it is straightforward to replicate the bigravity and the crystalline model in this framework. The bigravity model is converted to a "++" model [18] and the crystal model to an infinite array of "+" branes. The quasi-localized model on the other hand cannot be reproduced because the presence of the cosmological constant on the branes prevents the warp factor from being asymptotically constant. By sending the second brane in the "++" model to infinity, one obtains the locally localized model of [19,20] where gravity is mediated only by the ultralight state since the zero mode becomes non-normalizable. These models, even though they solve the theoretical difficulty of the ghost radion(s) in the flat brane models, face phenomenological difficulties. The presence of a remnant negative cosmological constant is at odds with observations, and furthermore it turns out that all large scale modifications of gravity predicted by these models, are hidden behind the $AdS$ horizon.

It is interesting to note at this point how these models confront the relativistic predictions of four dimensional Einsteianian gravity. It has been known for a long time that the structure of the massive graviton propagator is different from the massless one in flat space, something that is known as the van Dam - Veltman - Zakharov discontinuity [21,22]. This is due to the extra polarization states of the massive graviton and in particular the scalar-like mode that does not decouple in the limit of vanishing mass. As a result, gravity generated by a massive graviton case predicts a 25% discrepancy in the bending of light observation if one insists in the validity of Newton’s law for static bodies or vice versa. This apparently rules out all models of gravity based on a massive graviton [10] since the observed bending of light by the Sun agrees with the General Relativistic result to an accuracy of 1% (see also [23] for a discussion on the relation of this observation with physics arising from higher curvature terms in four dimensions). In the models of the flat branes, the ghost radion(s) at intermediate distances cancel the troublesome additional polarization state(s)
of the massive graviton(s), restoring the phenomenologically favoured tensorial structure of the graviton propagator [12, 14, 24]. In the models with AdS branes it was shown that the extra polarization state is practically decoupled due to an unusual feature of the graviton propagator structure in AdS. In more detail, it was shown [25–28] that the van Dam - Veltman - Zakharov no-go theorem can be evaded at tree level in $(A)dS$ in the case where $m/H \rightarrow 0$, where $m$ is the mass of the graviton and $H$ the “Hubble” of the AdS space, a precise realization of which was the "" + +"" brane model. Thus, in the AdS branes no phenomenological constraint from the bending of light experiment is applicable provided this condition holds. The discontinuity in the pure four dimensional theory with a massive graviton reappears at the quantum level [29, 30] but one should keep in mind that purely four dimensional theory of massive gravity is not well defined [31]. If instead one starts from the higher dimensional theory in which the mass terms appear dynamically and if one takes into account the whole tower of the KK states, this quantum discontinuity is expected to be absent, as pointed out in [13]. Let us also note that in a recent paper [32] it was suggested that the extra polarizations of the massive state in the one brane model can be gauged away and thus the discontinuity is absent.

More generally, the argument of van Dam - Veltman - Zakharov that no mass for the graviton is allowed in a flat background is not entirely robust. A long time ago Vainshtein [33] showed that if one considers the bending of light, the lowest order approximation is not valid because the calculation does not take into account the characteristic mass scale of the problem. A more detailed non-perturbative analysis presented in the recent paper [34] supports the original Vainshtein argument that there is continuity in the gravitational potential as the mass of the graviton goes to zero for distances smaller that a critical one which, in the case of the Sun, is of order of the size of the solar system. The linear approximation is valid for distances greater than this critical one, where the discontinuity reappears. Thus the observation of light bending by the Sun does not rule out the massive graviton proposal.

In the light of this encouraging result it seems appropriate to look for multigravity models of flat branes without ghost fields. As we have discussed, in five dimensions it is impossible to have flat brane multigravity models without negative tension branes. For this reason, we will consider models in six dimensions. The literature on six dimensional constructions is already quite rich [35–53]. In this paper we explicitly show that the difficulties of the five dimensional models can be evaded in six dimensions. It is possible to construct multigravity models with only flat positive tension branes. The branes which localize grav-
ity in this setup are four-branes, but one of their dimensions is compact, unwarped and of Planck length. Thus, in the low energy limit the spacetime on the brane appears three dimensional. In order that these constructions are realized it is crucial that the tensions and/or the bulk cosmological constant are inhomogeneous. The five dimensional constructions in this setup come into two types. One of them involves conical singularities at finite distance from the four branes. These conical singularities support three-branes\(^5\) which can be of positive tension if one has an angle deficit, of zero tension if one has no angle deficit and of negative tension if one has an angle excess. The other type of the multigravity models has no conical singularity at all. It is crucial to note that there is no five dimensional effective theory for these constructions, otherwise one would get all the problems faced in the five dimensional constructions. The low energy effective theory is directly four dimensional.

The organization of the paper is as following. In section two we present the cylindrically symmetric single four-brane warped models and show the cases where gravity is localized on the four-branes. In section three we paste two of the single four-brane solutions and obtain the double four-brane bigravity models. In section four we generalize the quasi-localized and crystalline constructions in six dimensions. Finally, we summarize the six dimensional cases and conclude.

2 Single brane models in six dimensions

At first we will discuss the single brane solutions in six dimensions to get an insight on the more complicated multigravity configurations. In the following subsection we will re- view the minimal model where the bulk cosmological constant is homogeneous and then a more generalized model with arbitrary cosmological constant components along the four dimensional and the extra dimensional directions. In the following we will implicitly assume orbifolding around the four-brane.

2.1 The minimal single brane model

The simplest single brane model consists of a four-brane embedded in six dimensional \(AdS\) space \([42]\). One of the longitudinal dimensions of the four brane is compactified to a Planck length radius \(R\) while the dimensions transverse to the four-brane compact dimension are

\(^5\)We would like to thank Panagiota Kanti and Luigi Pilo for bringing this issue to our attention.
Figure 1: The minimal single four-brane model warp factors $\sigma(\rho)$ and $\gamma(\rho)$.

infinite. The most general spherically symmetric ansatz that one can write down in six dimensions and which preserves four dimensional Poincaré invariance is:

$$ds^2 = \sigma(\rho)\eta_{\mu\nu}dx^\mu dx^\nu + d\rho^2 + \gamma(\rho)d\theta^2$$  \hspace{1cm} (1)

where $\theta$ is the compactified dimension with range $[0, 2\pi]$ and $\rho$ is the infinite radial dimension.

It is straightforward to solve the Einstein equations for the bulk energy momentum tensor $T^{(B)}_{MN} = -\Lambda \delta_{MN}$ with the brane contribution:

$$T^{(br)}_{\mu\nu} = -\delta(\rho) \begin{pmatrix} V \delta^\nu_{\mu} & 0 \\ 0 & V \end{pmatrix}$$  \hspace{1cm} (2)

From the form of the Einstein equations given in the Appendix we see that the solution for the two warp factors is:

$$\sigma(\rho) = e^{-k\rho}, \quad \gamma(\rho) = R^2 e^{-k\rho}$$  \hspace{1cm} (3)

with $k^2 = -\frac{\Lambda}{10M^4}$, where $M$ is the six dimensional fundamental scale and the arbitrary integration constant $R$ is just the radius of the four-brane (see fig.1). The Einstein equations require the usual fine tuning between the bulk cosmological constant and the tension of the four brane:

$$V = -\frac{8\Lambda}{5k}$$  \hspace{1cm} (4)

\[\text{Note that in this paper we use different metric signature and different definition of the fundamental scale from [12].}\]
Let us note at this point that in [42] this fine tuning was absent because a smooth local defect was considered instead of a four-brane. In this case, the fine tuning emerges between the different components of the defect energy momentum tensor. The physics of the four-brane idealization and the one of the defect model is the same.

The four dimensional KK decomposition can be carried out as usual (see [54] for a detailed fluctuation analysis) by considering the following graviton perturbations:

\[ ds^2 = \sigma(\rho) [\eta_{\mu \nu} + h_{\mu \nu}(\rho, \theta, x)] dx^\mu dx^\nu + d\rho^2 + \gamma(\rho)d\theta^2 \]  

(5)

Here, as well as throughout this paper, we have ignored the modulus associated to the radius \( R \) of the four brane. This will be a massless scalar and in order not to give rise to an additional long range force it should be given mass through some stabilization mechanism.

We expand the graviton perturbations in a complete set of radial eigenfunctions and Fourier angular modes:

\[ h_{\mu \nu}(\rho, \theta, x) = \sum_{n,l} \phi_{(n,l)}(\rho)e^{il\theta}h^{(n,l)}_{\mu \nu}(x) \]  

(6)

The differential equation for the radial wavefunctions \( \phi \) is:

\[ \phi'' - \frac{5}{2}k\phi' + \left( m^2 - \frac{l^2}{R^2} \right) e^{k\rho} \phi = 0 \]  

with normalization \( \int_0^\infty d\rho \sigma \sqrt{\gamma} \phi_* \phi = \delta_{mn} \). We can convert this equation to a two dimensional Schrödinger-like equation by the following redefinitions:

\[ z = \frac{2}{k} \left( e^{\frac{k}{2}\rho} - 1 \right), \quad \hat{\Psi} = \sigma^{3/4} \phi \]  

(8)

so that

\[ -\frac{1}{2}\sqrt{\gamma} \partial_z \left( \sqrt{\gamma} \partial_z \hat{\Psi} \right) + \frac{m^2}{2} \hat{\Psi} = V_{\text{eff}} \hat{\Psi}, \quad V_{\text{eff}}(z) = \frac{21k^2}{32} + \frac{l^2}{2R^2} - \frac{3k}{4} \delta(z) \]  

(9)

From the form of the potential (see fig.2) we can easily deduce that the angular excitations spectrum will consist of continuum starting from a gap of the order \( \frac{l^2}{R^2} \) and thus can be safely ignored. For the s-wave \((l = 0)\) there is a normalizable zero mode which is a constant in the \( \rho \) coordinate, i.e. \( \phi_0 = \text{const.} \). The KK tower for the s-waves will again form a continuum but this time gapless with wavefunctions given by [42]:

\[ \phi_m = N_m e^{\frac{5}{4}k \rho} \left[ J_{3/2} \left( \frac{2m}{k} \right) Y_{5/2} \left( \frac{2m}{k} e^{\frac{k}{2}\rho} \right) - Y_{3/2} \left( \frac{2m}{k} \right) J_{5/2} \left( \frac{2m}{k} e^{\frac{k}{2}\rho} \right) \right] \]  

(10)
The correction to the Newton’s law due to the s-modes can be easily calculated and one finds that it is more suppressed than the five dimensional Randall-Sundrum case \[42, 54, 55\]. In six dimensions the correction reads:

$$\Delta V = -\frac{1}{O(M^5_{Pl})} \frac{1}{r^4}$$  \hspace{1cm} (11)

### 2.2 The generalized single brane model

We can now try to find more general single four-brane solutions by relaxing the requirement of homogeneity of the bulk and brane energy momentum tensors. In contrast to the five dimensional case, this is possible in six dimensions because there are two independent functions in the metric (see Appendix). We will consider the following inhomogeneous bulk energy-momentum tensor:

$$T^{(B)\, N}_M = -\begin{pmatrix} \Lambda_0 \delta^\nu_\mu & \Lambda_\rho \\ \Lambda_\rho & \Lambda_\theta \end{pmatrix}$$  \hspace{1cm} (12)

and allow for an inhomogeneous brane tension of the form:

$$T^{(br)\, N}_M = -\delta(\rho) \begin{pmatrix} V_0 \delta^\nu_\mu & 0 \\ 0 & V_\theta \end{pmatrix}$$  \hspace{1cm} (13)
This inhomogeneity can be due to different contributions to the Casimir energy in the different directions \([56, 57]\) (see also \([58]\)) or due to a background three-form gauge field with non-zero field strength \([40]\).

If we define the parameter \(\alpha = \frac{2\Lambda}{\Lambda_\theta}\), then the one four-brane solutions for the warp factors are:

\[
\sigma(\rho) = e^{-k\rho} , \quad \gamma(\rho) = R^2 e^{-\frac{k}{4}(5\alpha-6)\rho}
\]

with \(k^2 = -\frac{\Lambda_\theta}{10M^4}\), where again \(R\) is the radius of the four-brane (see fig.3). The flatness of the brane is achieved by the following fine tunings of the the parameters of the bulk and brane energy momentum tensors:

\[
\begin{align*}
V_\theta &= -\frac{8\Lambda_\theta}{5k} , & V_0 &= \frac{5\alpha + 6}{16} V_\theta , & \Lambda_0 &= \frac{5\alpha^2 + 12}{32} \Lambda_\theta
\end{align*}
\]

Let us note that the two components of the brane tension have the same sign as long as \(\alpha > -\frac{6}{5}\). For \(\alpha > \frac{6}{5}\) the internal space is shrinking as in \([42]\), for \(\alpha < \frac{6}{5}\) the internal space

\[\sigma, \gamma|_{\alpha>6/5}^{\alpha<6/5}\]

\[\gamma|_{\alpha=6/5}^{\alpha<6/5}\]

**Figure 3:** The generalized single four-brane warp factors \(\sigma(\rho)\) and \(\gamma(\rho)\) for different values of the parameter \(\alpha\).
is growing and for $\alpha = \frac{6}{5}$ the internal space is a cylinder (see fig.3).

Proceeding with the usual KK decomposition (see [54]) we find the following equation for the radial wavefunction $\phi$:

$$\phi'' - \frac{5\alpha + 10}{8}k\phi' + \left( m^2 - \frac{l^2}{R^2} \varepsilon^2 (\alpha-2)k\rho \right) e^{k\rho} \phi = 0$$

with normalization $\int_0^\infty d\rho \sqrt{\gamma} \phi_m \phi_n = \delta_{mn}$. With the wavefunction redefinition and the coordinate change discussed in the previous subsection, we obtain the following effective potential for $\alpha > 2$ for different angular quantum numbers $l$. The $\delta$-function in the origin is omitted.

![Figure 4: The form of the effective potential $V_{eff}$ for $\alpha > 2$ for different angular quantum numbers $l$. The $\delta$-function in the origin is omitted.](image)

![Figure 5: The form of the effective potential $V_{eff}$ for $\alpha < 2$ for different angular quantum numbers $l$. The $\delta$-function in the origin is omitted.](image)
potential for the two dimensional Schrödinger-like equation:

\[ V_{\text{eff}}(z) = \frac{3(5\alpha + 4)k^2}{64\left(\frac{kz}{2} + 1\right)^2} + \frac{l^2}{2R^2} \left(\frac{kz}{2} + 1\right)^{\frac{2}{3}(\alpha - 2)} - \frac{3k}{4}\delta(z) \quad (17) \]

For the s-wave there is a normalizable zero mode when \( \alpha > -\frac{2}{5} \), which is a constant in the \( \rho \) coordinate, \( i.e. \phi_0 = \text{const} \). The KK tower for the s-waves will again form a continuum and their wavefunctions are given by:

\[ \phi_m = N_m e^{\frac{2m}{k}r} \left[ J_{\nu-1} \left(\frac{2m}{k} e^{\frac{2m}{k}\rho}\right) - Y_{\nu-1} \left(\frac{2m}{k} e^{\frac{2m}{k}\rho}\right) \right] \quad (18) \]

where \( \nu = \frac{5}{8}(\alpha + 2) \). The correction to the Newton’s law due to the s-modes can be easily calculated and one finds that it is negligible as long as \( \alpha > -\frac{2}{5} \) \[\text{[54]}\]:

\[ \Delta V = -\frac{1}{O(M_{Pl}^2)} \frac{1}{r^{2\nu-1}} \quad (19) \]

As far as the angular excitations \( l \neq 0 \) are concerned, in the case where \( \alpha > 2 \) the potential diverges at infinity (see fig. 4), so the spectrum will be discrete starting from a scale bigger than \( \frac{l^2}{R^2} \) and thus can be safely ignored. On the other hand, the potential for the case \( \alpha < 2 \) asymptotically vanishes (see fig. 5) and so the spectrum will be gapless continuum. One should examine the contribution of the angular excitations to the Newtonian potential in this case.

### 3 Bigravity in six dimensions

The flat single four-brane models considered in the previous sections have the characteristic that gravity is localized on the brane in the same way as in the five dimensional analogue \[\text{[55]}\]. In this section we will show how we can construct realistic multi-localization scenarios for gravity by sewing two single four-brane solutions together. We will present two explicit examples of compact double four-brane models where bigravity is realized. Firstly, we will consider a double four-brane bigravity model which in general contains an additional three-brane associated with the existence of a conical singularity. This can be done by allowing for an inhomogeneous four-brane tension. In the following subsection we demonstrate how one can avoid the conical singularity by also allowing for inhomogeneous bulk cosmological
constant. In the following we will consider the orbifold geometry \( S^1/Z_2 \) in which the two four-branes lie on the fixed points.

### 3.1 The conifold model

We are interested in a double four-brane generalization of the minimal single four-brane model. In order to achieve this, we still assume a homogeneous bulk energy momentum tensor \( T^{(B)}_{MN} = -\Lambda \delta_M^N \) but we do not impose any constraints on the four-brane tension:

\[
T^{(br)}_{MN} = -\delta(\rho) \begin{pmatrix}
V_0 \delta^\nu_\mu & 0 \\
0 & V_\theta
\end{pmatrix}
\]

The Einstein equations for \( \sigma(\rho) \) and \( \gamma(\rho) \) in this case give the following solutions for the warp factors:

\[
\sigma(\rho) = \cosh^{4/5} \left( \frac{\frac{5}{4} k(\rho - \rho_0)}{k_0} \right), \quad \gamma(\rho) = R^2 \cosh^{6/5} \left( \frac{\frac{5}{4} k\rho_0}{k_0} \right) \sinh^2 \left( \frac{\frac{5}{4} k(\rho - \rho_0)}{k_0} \right)
\]

with \( k^2 = -\frac{\Lambda}{10M^2} \), where we have normalized \( \sigma(0) = 1 \) and \( \gamma(0) = R^2 \) (see fig. [4]). From the above relations it is obvious that both \( \sigma(\rho) \) and \( \gamma(\rho) \) have a bounce form. However, we note that \( \gamma(\rho_0) = 0 \), that is, \( \gamma(\rho) \) is vanishing at the minimum of the warp factor \( \sigma(\rho) \). This is a general characteristic of the solutions even when the branes are non-flat. From eq.(A.3) and eq.(A.4) we can easily show that \( \gamma(\rho) = C \frac{(\sigma')^2}{\sigma(\rho)} \) (where \( C \) is an integration constant) which implies that whenever we have a bounce in the warp factor the function \( \gamma(\rho) \) will develop a zero.

In order to examine the nature of this singularity we study the form of the metric at the vicinity of the point \( \rho = \rho_0 \) along the lines of [59, 60]. Taking in account that in this limit we have \( \sigma(\rho) \rightarrow b^2 \) and \( \gamma(\rho) \rightarrow \beta^2 (\rho - \rho_0)^2 \) the metric becomes:

\[
ds^2 = b^2 \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 + \beta^2 (\rho - \rho_0)^2 d\theta^2
\]

where \( b^2 = \cosh^{-4/5} \left( \frac{5}{4} k\rho_0 \right) \)

and \( \beta^2 \equiv \frac{25k^2R^2 \cosh^{6/5}(\frac{5}{4} k\rho_0)}{16 \sinh^2(\frac{5}{4} k\rho_0)} \)

From the form of the metric, it is clear that for general values of the \( \beta \) parameter there will be a conical singularity with a corresponding deficit angle \( \delta = 2\pi (1 - \beta) \). The existence
Figure 6: The conifold bigravity model warp factors $\sigma(\rho)$ and $\gamma(\rho)$ for the symmetric configuration where $L = 2\rho_0$. Both the $\sigma(\rho)$ and $\gamma(\rho)$ have a bounce at $\rho_0$. However, $\gamma(\rho)$ vanishes at this point. This point in general corresponds to a conical singularity.

of this conifold singularity is connected to the presence of a 3-brane at $\rho = \rho_0$. In order to find its tension one has to carefully examine the Einstein tensor at the vicinity of the singularity. For this reason we write the metric for the internal manifold in the conformally flat form:

$$ds^2 = b^2 \eta_{\mu\nu} dx^\mu dx^\nu + f(r)(dr^2 + r^2 d\theta^2)$$

with $f(r) = r^{2(\beta - 1)}$ and $\rho - \rho_0 = \beta^{-1} r^\beta$. In these coordinates it is easy to see how the three brane appears at the conifold point. The Einstein tensor can be calculated for $\rho \to \rho_0$:

$$R_{MN} - \frac{1}{2} G_{MN} R = \left( \begin{array}{cc} \frac{\nabla^2 \log(f(r))}{2f(r)} b^2 \eta_{\mu\nu} & 0 \\ 0 & 0 \end{array} \right)$$

where $\nabla^2$ is the flat two dimensional Laplacian. Now, given that $\nabla^2 \log(r) = 2\pi\delta^{(2)}(r)$ and
by comparing with:

\[ R_{MN} - \frac{1}{2} G_{MN} R = -\frac{V_3}{4M^4} \sqrt{\frac{-G}{\sqrt{-G}}} \hat{G}_{\mu\nu} \delta^\mu_M \delta^\nu_N \delta(r) \]  

(25)

where \( \delta^{(2)}(r) = \frac{\delta(r)}{2\pi r} \) (see Appendix of [53]), we find that the tension of the 3-brane is:

\[ V_3 = \frac{4(1 - \beta)}{M^4} = \frac{2M^4}{2} \pi \delta \]  

(26)

Thus, if there is angle deficit \( \delta > 0 \) (\( \beta < 1 \)) the tension of the brane is positive, whereas if there is angle excess \( \delta < 0 \) (\( \beta > 1 \)) the tension of the brane is negative. At the critical value \( \beta = 0 \) there is no conical singularity at all and we have a situation where two locally flat spaces touch each other at one point.

In the following we will concentrate on the symmetric case, that is, the four-branes will be considered placed at symmetric points with respect to the minimum of the warp factor \( \rho_0 \), i.e. \( L = 2\rho_0 \). For the previous solution to be consistent the tensions of the four-branes for the symmetric configuration must satisfy:

\[ V_{\theta} = -\frac{8\Lambda}{5k} \tanh \left[ \frac{5}{2} k \rho_0 \right], \quad V_0 = \frac{3}{8} V_{\theta} + \frac{8\Lambda^2}{5k^2} \frac{1}{V_{\theta}} \]  

(27)

The brane tensions for both branes are identical. Thus, the above construction consists of two positive tension four-branes placed at the end of the compact space and an intermediate three-brane due to the conifold singularity with tension depending on the parameters of the model. In the limit \( \rho_0 \to \infty \) we correctly obtain two identical minimal single four-brane models for the case where \( \delta \to 2\pi \) (\( \beta \to 0 \)).

The differential equation for the radial wavefunction \( \phi \) of the graviton excitations is:

\[ \phi'' + 2 \left( \frac{\sigma'}{\sigma} + \frac{\gamma'}{4\gamma} \right) \phi' + \left( \frac{m^2}{\sigma} - \frac{l^2}{\gamma} \right) \phi = 0 \]  

(28)

with normalization \( \int_0^{2\rho_0} d\rho \sigma \sqrt{\gamma} \phi_m \phi_n = \delta_{mn} \).

There is an obvious normalizable zero mode with \( \phi_0 = \text{const.} \) and a tower of discrete KK states. It is easy to demonstrate that for \( \delta > 0 \) (\( \beta < 1 \)) this model has an ultralight state, leading to a theory of bigravity without negative tension branes. This follows [61] considering the limit \( \rho_0 \to \infty \) when the theory describes two identical single four-brane models each of which has a massless graviton. For finite \( \rho_0 \) only their symmetric combination
remains massless but their antisymmetric combination acquires an anomalously small mass, because the difference in the modulus of the wavefunctions between the massive and the massless states is significant only near $\rho_0$ where the wavefunctions are exponentially small.

We can transform this differential equation to a two dimensional Schrödinger-like one by the coordinate change $\frac{d}{d\rho} = \sigma^{-1/2} \equiv g(z)$ and the usual wavefunction redefinition. Then the effective potential reads:

$$V_{\text{eff}} = \frac{15}{8} \left( \frac{\partial_z g}{g} \right)^2 - \frac{3}{4} \frac{\partial^2 g}{g^2} - \frac{3}{8} \frac{\partial_z g \partial_z \gamma}{g \gamma} + \frac{l^2}{2g^2}$$

We cannot write an explicit analytic formula for of the above potential in the $z$-coordinates because the coordinate transformation is not invertible analytically. However, since the transformation is monotonic, we can easily sketch the form of the potential (see fig.4) by calculating it in the $\rho$ coordinates. From this procedure we see that the potential for the s-wave is finite at the conical singularity $\rho_0$ but has a divergence for all the angular excitations with $l \neq 0$. This means that only the s-wave excitations with communicate the

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There is, however, a mismatch of the degrees of freedom of the massive and massless graviton, which may not be phenomenologically dangerous, but still raises the issue of locality in the infinite four-brane separation limit.
two parts of the conifold and the other excitations will be confined in the two semicones. The spectrum of KK states in this case will be discrete.

The construction may be readily modified to obtain a six-dimensional analogue of the locally localized model [19]. This is done by considering the asymmetric situation in which one of the fixed point four-branes is moved to infinity (in the asymmetric case the tensions of the two four-branes are different). Then gravity on the four-brane at \( \rho = 0 \) will be mediated by only the ultralight state since the graviton zero mode will not be normalizable.

### 3.2 The non-singular model

The appearance of a conifold singularity can be avoided if instead of using the minimal single four-brane solution to build the double four-brane solution, one uses the generalized single four-brane model. By allowing for an inhomogeneous bulk cosmological constant one can arrange that the function \( \gamma(\rho) \) does not develop a zero. In this case the \((\theta, \theta)\) and \((\rho, \rho)\) part of Einstein equations in the bulk, (A.3) and (A.4), have solutions:

\[
\sigma(\rho) = \frac{\cosh^{4/5}[5/4 k (\rho - \rho_0)]}{\cosh^{4/5}[5/4 k \rho_0]}, \quad \gamma(\rho) = A \frac{\sinh^\alpha[5/4 k (\rho - \rho_0)]}{\cosh^{6/5}[5/4 k \rho_0]} \quad (30)
\]

where \( A \) is a constant and \( \alpha \equiv \frac{2 \Lambda_\theta}{\Lambda_\rho} \). The \((\mu, \nu)\) component of the Einstein equations (A.5), however, restricts the possible values of \( \alpha \) to \( \alpha = 2 \) or \( \alpha = 0 \). The case of \( \alpha = 2 \) corresponds to the previous conifold model. For the case \( \alpha = 0 \), which corresponds to the choice \( \Lambda_\rho = 0 \), the solution is:

\[
\sigma(\rho) = \frac{\cosh^{4/5}[5/4 k (\rho - \rho_0)]}{\cosh^{4/5}[5/4 k \rho_0]}, \quad \gamma(\rho) = R^2 \frac{\cosh^{6/5}[5/4 k \rho_0]}{\cosh^{6/5}[5/4 k (\rho - \rho_0)]} \quad (31)
\]

with \( k^2 = -\frac{\Lambda_\theta}{10 M^4} \), where we have normalized \( \sigma(0) = 1 \) and \( \gamma(0) = R^2 \) (see fig. 8). From the above the absence of any singularity is obvious since \( \gamma(\rho) \) does not vanish at any finite value of \( \rho \).

The four branes will appear as \( \delta \)-function singularities at the points where we will cut the space in the \( \rho \) direction. In the following we will concentrate on the symmetric case, that is, the branes will be considered placed in symmetric points in respect to the extremum of the warp factor \( \rho_0 \), i.e. \( L = 2 \rho_0 \). In order for the above configuration to be realized, the components of the four-brane tensions and the bulk cosmological constant must be tuned...
Figure 8: The non-singular bigravity model warp factors $\sigma(\rho)$ and $\gamma(\rho)$ for the symmetric configuration where $L = 2\rho_0$. The $\sigma(\rho)$ warp factor has exactly the same form as in the conifold model. The $\gamma(\rho)$ warp factor however now has an inverse bounce form. In this case $\gamma(\rho)$ does not vanish anywhere and thus the model is free of singularities.

to give:

$$V_\theta = -\frac{8\Lambda}{5k} \tanh \left[ \frac{5}{2} k \rho_0 \right] , \quad V_0 = \frac{3}{8} V_\theta , \quad \Lambda_0 = \frac{3}{8} \Lambda_\theta$$

(32)

The brane tensions for both branes are identical. Thus, we have constructed a compact model with two positive tension flat branes with a bounce in the warp factors. Note the extra condition relating the different components of the bulk cosmological constant; this inhomogeneity in the bulk cosmological constant is what allows us to have a non-singular solution. In the limit $\rho_0 \to \infty$ we obtain two identical generalized single four-brane $\alpha = 0$ models.
The differential equation for the radial wavefunction $\phi$ of the graviton excitations is given by (28) for the relevant $\sigma(\rho)$ and $\gamma(\rho)$ functions. Although it is difficult to find an analytical solution of the corresponding differential equation for the KK states, the existence of a light special KK is assured from the locality arguments discussed above [61]. Since at the infinite brane separation limit we recover two identical generalized single four-brane models (for $\alpha = 0$) where each of them supports a massless zero mode, at a finite separation configuration the one of the zero modes will become the special light KK state.

By making an appropriate change of coordinates and a redefinition of the wavefunction we can bring the previous differential equation in a two dimensional Schrödinger form with the effective potential (29) that is plotted in fig.9. Note that in this case the angular excitations feel the whole region between the two positive branes and thus communicate physics between them. The spectrum of KK states in this case will again be discrete.

4 Multigravity in six dimensions

In this section we will show how we can obtain the analogues of the GRS model of quasi-localized gravity and of the crystal universe in six dimensions, in both cases without the
need of introducing moving negative tension branes.

4.1 Quasi-localized gravity

As in the bigravity models we have two possible ways of realizing a quasi-localized scenario in six dimensions. The first model can be built from the \( \alpha = 0 \) bigravity model, by cutting the space at a point \( \rho_b \) on the right of the position of the bounce and sewing it to flat space on the right (see fig.10). In that case we will have a system of two positive tension branes and by appropriately tuning the different components of the brane tension of the second brane we can find solutions where the warp factor is constant in the flat bulk region. The fine tunings which achieve this are:

\[
V^{(2)}_\theta = -\frac{4\Lambda}{5k} \tanh \left[ \frac{5}{2} k (\rho_b - \rho_0) \right], \quad V^{(2)}_0 = \frac{3}{8} V^{(2)}_\theta
\]  

(33)
From the above formulas it is obvious that if we paste the flat space at the position of the bounce, i.e. $\rho_b = \rho_0$ then the second brane is tensionless. If we relax these fine tunings we can find solutions where the warp factors in the flat bulk region are non trivial, but we will not discuss this possibility here.

The above construction will obviously have no normalizable zero mode due to the infinite volume of the system and the KK spectrum will be continuous. However, as in the five dimensional case we expect that the low part of the KK tower will have a “resonance”-like coupling to matter on the \("+\)” brane at $\rho = 0$. This will mediate normal gravity at intermediate distances but will change the nature of the gravitational law at ultralarge scales.

The second model of quasi-localized gravity can be built in a similar fashion from the $\alpha = 2$ bigravity model (see fig.11). Again we obtain a system of two positive tension branes which will have a constant warp factor in the flat bulk region when the following fine tunings
are demanded:

$$V_\theta^{(2)} = -\frac{4\Lambda}{5k} \tanh \left[ \frac{5}{2}k(\rho_b - \rho_0) \right], \quad V_0^{(2)} = \frac{3}{8}V_\theta^{(2)} + \frac{8\Lambda^2}{5k^2} \frac{1}{V_\theta^{(2)}} \quad (34)$$

From the above formulas it is obvious that if we paste the flat space at the position of the bounce, i.e. $\rho_b = \rho_0$ then the tension $V_\theta^{(2)}$ vanishes whereas the tension $V_0^{(2)}$ diverges. This limit is not well behaved since the internal space becomes singular in a whole line. At this limit obviously classical gravity breaks down and one would expect that quantum gravity corrections would resolve the singularity. However, one should check if these quantum gravity corrections affect the solution in the vicinity of the four-brane at $\rho = 0$, e.g. by examining the $R^2$ correction to the Einstein action\[\footnote{We would like to thank Tony Gherghetta for this comment.}\]

The above construction will again have no normalizable zero mode and the KK spectrum will be continuous. A “resonance”-like coupling of the KK states to matter on the “+$n$” brane
at $\rho = 0$ is again expected which will give normal four dimensional gravity at intermediate distances but will change the nature of the gravitational law at ultralarge scales.

### 4.2 The crystal universe model

Another obvious generalization of the five dimensional multigravity models is the one of the crystalline brane model. All that one has to do is to paste an infinite array of bigravity models. We again take this constructions in two copies. One for $\alpha = 0$ (fig.12) and one for $\alpha = 2$ (fig.13). A band structure is again expected as in the five dimensional case and the width of the first band will be exponentially smaller than the one of the first forbidden zone. As far as the phenomenology is concerned, these models will generate normal gravity at intermediate scales as the first band will behave as an effective zero mode, whereas at ultralarge scales will have the same change of the gravitational law as the one of the corresponding quasi-localized models.
5 Discussion and conclusions

In this paper we have constructed, for the first time, flat brane theories which can lead to multigravity models and their associated modifications of gravity at large distances without introducing moving negative tension branes. The constructions are made possible by going to a six dimensional theory. In five dimensions with flat branes, the presence of a bounce of the warp factor was linked to the violation of the weaker energy condition. This is not true, however, in six dimensions as we will show in this section. In five dimensions with the metric:

\[ ds^2 = e^{-A(\rho)} \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 \]  

(35)

one can readily show that the weaker energy condition requires that:

\[ A'' \geq 0 \]  

(36)

which is violated at the position of the moving negative tension branes. In the case that the branes were AdS one could have a bounce (without having negative tension branes) and still satisfy the weaker energy condition because the above relation is modified to [19]:

\[ A'' \geq -2H^2 e^A \]  

(37)

However, such models do not lead to modifications of gravity at large distances and moreover the remnant negative cosmological constant is in conflict with current observations.

In the six dimensional case with metric:

\[ ds^2 = e^{-A(\rho)} \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 + e^{-B(\rho)} d\theta^2 \]  

(38)

from the weaker energy condition one finds two inequalities which can be cast into the following relation:

\[ \frac{1}{6}(B')^2 - \frac{1}{3}B'' - \frac{1}{6}A'B' \leq A'' \leq -\frac{1}{2}(B')^2 + B'' - \frac{3}{2}A'B' + 2(A')^2 \]  

(39)

These inequalities hold for the \( \alpha = 2 \) models as long as the three-branes sitting on the conical singularities have positive tension. It is violated, however, in the \( \alpha = 0 \) models everywhere in the bulk. This shows that in six dimensional flat brane models, the presence of bounces of the warp factor is not necessarily linked to whether the weaker energy condition is satisfied or not. It is not yet clear if the violation of the weaker energy condition in the \( \alpha = 0 \) models is a sign of a possible instability.
There is an interesting feature of the conifold model that emerges if we consider Euclidean four dimensional space and Wick rotate the $\theta$ coordinate ($\theta \rightarrow it$) decompactifying it at the same time. In the case where the deficit angle is zero, the metric in the vicinity of $\rho = \rho_0$ will look like:

$$ds^2 = b^2 \delta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 - (\rho - \rho_0)^2 dt^2$$  \hspace{1cm} (40)

This is Rindler space which is deformed away of $\rho = \rho_0$ and has sources (the four-branes) at finite proper distances from $\rho = \rho_0$. The picture that we obtain in Kruskal coordinates for this case is reminiscent of the one of a Schwarzschild black hole if one rotates the Penrose diagram at a right angle.

In summary, in this paper we discussed how we can obtain viable multigravity models in six dimensions. The models we considered involved flat positive tension branes (except the case when the conifold models had angle excess). We showed how we can obtain the double brane bigravity model, the quasi-localized model and the crystalline model, each of which in two types. One of them is singularity free while the other has conical singularities in the bulk corresponding to three-branes.

Several issues are still open regarding these constructions. Firstly, a careful treatment of the angular excitations in the case of $\alpha = 0$ should be carried out to examine the nature of gravity on the four-branes. Furthermore, one should examine the moduli of the system, the radions and the dilaton, and calculate their mass (see [13, 62] for the warped five dimensional case and [51] for the warped six dimensional case). It is quite probable that the modulus corresponding to $\rho_0$ is massive as in the five dimensional $AdS_4$ branes case and so the system is self-stabilized. Moreover, a discussion of multigravity in a cosmological setting is yet to be developed (for a discussion about cosmology in brane world models see for example [63–70]). Finally, the non-trivial way that locality is preserved by these model should be studied.

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Appendix

In this Appendix we provide the Einstein equations for the class of models that we considered where the metric is:

\[ ds^2 = \sigma(\rho) \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 + \gamma(\rho) d\theta^2 \]  

(A.1)

Ignoring the three-branes located on the conical singularities, the Einstein equations are generally written as:

\[ G_{MN} = \frac{1}{4M^4} (T_{MN}^{(B)} + T_{MN}^{(br)}) \]  

(A.2)

where \( T_{MN}^{(B)} \) is the bulk energy momentum tensor of the general form (12) and \( T_{MN}^{(br)} \) the four-brane energy momentum tensor of the form (13).

In the absence of a four dimensional cosmological constant, the \((\theta, \theta)\) component of the above equation is:

\[ 2 \frac{\sigma''}{\sigma} + \frac{1}{2} \left( \frac{\sigma'}{\sigma} \right)^2 = -\frac{\Lambda_\theta}{4M^4} - \frac{V^i_\theta}{4M^4} \delta(\rho - \rho_i) \]  

(A.3)

The \((\rho, \rho)\) component is:

\[ \frac{3}{2} \left( \frac{\sigma'}{\sigma} \right)^2 + \frac{\sigma' \gamma'}{\sigma \gamma} = -\frac{\Lambda_\rho}{4M^4} \]  

(A.4)

Finally, the \((\mu, \nu)\) component is:

\[ \frac{3}{2} \frac{\sigma''}{\sigma} + \frac{3}{4} \frac{\sigma' \gamma'}{\sigma \gamma} - \frac{1}{4} \left( \frac{\gamma'}{\gamma} \right)^2 + \frac{1}{2} \frac{\gamma''}{\gamma} = -\frac{\Lambda_0}{4M^4} - \frac{V^i_0}{4M^4} \delta(\rho - \rho_i) \]  

(A.5)

These equations may be compared with the ones of the five dimensional case where the metric:

\[ ds^2 = \sigma(\rho) \eta_{\mu\nu} dx^\mu dx^\nu + d\rho^2 \]  

(A.6)

gives rise to the \((\rho, \rho)\) component:

\[ \frac{3}{2} \left( \frac{\sigma'}{\sigma} \right)^2 = -\frac{\Lambda}{4M^3} \]  

(A.7)

and the \((\mu, \nu)\) component:

\[ \frac{3}{2} \frac{\sigma''}{\sigma} = -\frac{\Lambda}{4M^3} - \frac{V^i}{4M^3} \delta(\rho - \rho_i) \]  

(A.8)

The extra freedom that we have in the six dimensional case is apparent.
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