Alternative Gravitational Theories in Four Dimensions

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We argue that from the point of view of gauge theory and of an appropriate interpretation of the interferometer experiments with matter waves in a gravitational field, the Einstein-Cartan theory is the best theory of gravity available. Alternative viable theories are general relativity and a certain teleparallelism model. Objections of Ohanian & Ruffini against the Einstein-Cartan theory are discussed. Subsequently we list the papers which were read at the ‘Alternative 4D Session’ and try to order them, at least partially, in the light of the structures discussed.

1 The best alternative theory?

I would call general relativity theory GR the best available alternative gravitational theory and the next best one its teleparallel equivalent. Because of these two theories, at least, it is good to have this alternative session during the Marcel Grossmann Meeting. Let me try to explain why I grant to GR the distinction of being the best alternative theory.

After finally having set up special relativity theory in 1905, Einstein subsequently addressed the question of how to generalize Newton’s gravitational theory so as to make it consistent with special relativity, that is, how to reformulate it in a Poincaré covariant way. Newton’s theory was a battle tested theory in the realm of our planetary system and under normal laboratory conditions. It has predictive power as it had been shown by the prediction of the existence of the planet Neptune in the last century. The planets are considered as point particles in this context, and they move in the central gravitational field of the Sun. The attraction of neighboring planets are accounted for by a highly developed perturbation theory. Only very small deviation from the predictions of Newton’s theory puzzled a few experts by the end of last century. But, for reasons of consistency, Einstein had no other choice than to ‘marry’ Newton’s gravitational theory to special relativity.

We all know the outcome of this undertaking: Special relativity turned out to be too narrow. Because of the equivalence principle, it had to be a curved spacetime where gravity is appropriately housed. And the simplest Lagrangian proportional to the curvature yields the left hand side of Einstein’s field equation which, in turn, explains the post-Newtonian pieces of the perihelion advances of the planets.

The typical building blocks of GR are as sources of gravity fluid matter (in fact, usually ideal Euler fluids, without viscosity) and the electromagnetic field, and as test bodies classical (structureless) point particles and light rays. Clearly, this picture can be refined, but it is basically this scenario which we meet in GR. Such a refinement you can see at work, if the motion of a narrow binary pulsar is followed up in a general-relativistic computation, and if radiation reaction terms are calculated, e.g.

\(^a\)Report of parallel session chair in Proc. 8th M. Grossmann Meeting, T. Piran (ed.) World Scientific, Singapore 1998, to be published
In a reconstruction of the Riemannian spacetime of GR by a so-called axiomatic approach it is then only logical to take the paths of (massive) point particles and radar signals (‘light rays’, ‘photons’) as basic notions which implicitly define, by means of the axioms, the spacetime of GR. Needless to say that point particles don’t exist in GR (they always carry a finite however small Schwarzschild radius with themselves) and that the light rays are only a result of the geometric optics limit in the framework of electrodynamics, i.e., a short wavelength or high energy approximation of electrodynamics (see, Mashhoon\textsuperscript{20}). Therefore, in our axiomatic tool box, we have little black holes and high-energy \(\gamma\)-rays at our disposal.

Accordingly, the consistency question posed above by Einstein had been answered for gravitating fluids, electromagnetic fields, massive point particles, light rays. And Einstein himself found the answer with his gravitational theory of 1915/16, that is, in the framework of GR.

This would be the end of the story, if not a new consistency question had turned up with...

2 ‘The Dawning of Gauge Theory’

When modern gauge theories were developed in the fifties\textsuperscript{25} it was the matter field \(\Psi\), first as classical field, afterwards as second quantized field operator – rather than a hydrodynamical model of matter by means of an ideal fluid – which was the basis for the description of our material surrounding. The matter field constitutes the Lagrangian whose conserved currents and symmetries were studied.

The gravitational field was ‘rewritten’ in terms of tetrads and the gauge symmetry investigated. This turned out to be more than a sheer rewriting: It was recognized in this context that the underlying gauge group of gravity is represented by the group of motion of special relativity, namely the Poincaré group\textsuperscript{11}. That only its translational subgroup should yield GR can be seen by pure counting of the corresponding gauge potentials and the sources coupled to them: The 4\(\oplus\)6-dimensional Poincaré group has as its attached independent currents the 4\(\oplus\)6 three-forms \(\Sigma\alpha\) (energy-momentum) and \(\tau\alpha\beta = -\tau\beta\alpha\) (spin), respectively.

Nevertheless, conventional prejudice has it that GR is either a gauge theory of the diffeomorphism group or of the Lorentz group. And even if Feynman explicitly declares himself\textsuperscript{9} that “gravity is that field which corresponds to a gauge invariance with respect to displacement transformations”, i.e., with respect to translations, you can be sure that his modern interpreters (in the foreword on p. XI) turn this around into the statement that “the requisite gauge principle can be shown to be general covariance.” A review of the gauge theoretical aspects of gravity theory was given by Gronwald and us\textsuperscript{11}.

To cut a long story short, the Poincaré gauge theory of gravity leads to a Riemann-Cartan geometry of spacetime, with curvature \(R\alpha\beta\) and torsion by \(T\alpha\), and its simplest Lagrangian, the curvature scalar of the Riemann-Cartan spacetime, to the field equations of the so-called Einstein-Cartan theory of gravity. Thus, within the modern paradigm of the gauge principle, the consistency question had to be rephrased: It is no longer the matter fluid or the test particle around which the theory revolves, rather the matter field \(\Psi\) is at the center of the stage.
At this point one could argue, as most of the ‘general relativists’ in fact do, that one should not care what those gauge theoreticians did to Einstein’s beautiful gravitational theory and should concentrate on working out GR. Well, such a decision is perfectly possible as long as you close your eyes to some more recent experiments in gravitational physics.

3 The COW and BW experiments

*Neutrons, atoms, and molecules* are the smallest and ‘most microscopic’ objects which, if exposed to the gravitational field, show measurable effects on their wave functions. In the Colella-Overhauser-Werner\(^7\) and the Bonse-Wroblewski\(^5\) neutron interferometer experiments a gravitational or acceleration induced phase shift of the wave function has been observed with an accuracy of a few percent. With much greater accuracy, such phase shifts have been verified in the Kasevich-Chu atom interferometer experiments, see refs.\(^4\),\(^26\). This phase shift can be calculated by means of the Schrödinger equation with an external Newtonian gravitational potential. However, there also exists the Einsteinian procedure for discovering the effects of gravity.

But beware, we should *not* put the neutron matter wave or the atomic beam including their respective interferometers on top of the prefabricated Riemannian spacetime of GR in order to find out about the effect of gravity on them. This is not what Einstein taught us, since GR was only constructed for point particles, ideal fluids, electromagnetic fields, etc. Rather we should put (in a Gedanken experiment) the experimental set-up, including the neutron wave etc., in a special-relativistic surrounding (that is, in a region where we can safely neglect the gravitational field) and should wonder what happens if, from the initial inertial reference frame, we go over to a non-inertial frame. This is what the quoted gauge theoreticians did (in 1956 and 1961) with a matter wave function before the COW experiment had even been conceived (around 1974). Technically, one has to study how the special-relativistic Dirac Lagrangian – if we describe the neutron wave function approximately by means of a Dirac spinor – responds to the introduction of non-inertial reference frames, see ref.\(^11\).

Accordingly, we apply the old Einsteinian procedure to a somewhat more refined object than Einstein did between 1907 and 1915, namely to the matter wave \(\Psi\). Is it then a big surprise that the spacetime emerging from this ‘modernized’ Einstein procedure is a spacetime with *post*-Riemannian structures, or, to be more precise, with a Riemann-Cartan structure? This had been foreseen by E. Cartan\(^6\) and, for the reason that in these spacetimes locally, in a suitable *normal* frame, metric and connection look Minkowskian, Cartan called them spacetimes with a [pseudo-] Euclidean connection.

Therefore the gauge paradigm and the COW-type experiments as well suggest the emergence of a Riemann-Cartan spacetime. The simplest nontrivial Lagrangian yields the...
4 Einstein-Cartan theory of gravity

Let us first define, in terms of the coframe $\vartheta^\alpha$ and the Hodge star $\ast$, the 1-form $\eta_{\alpha\beta\gamma} = \ast(\vartheta_\alpha \wedge \vartheta_\beta \wedge \vartheta_\gamma)$ and the 3-form $\eta_\alpha = \ast \vartheta_\alpha$. Then the two field equations of Einstein-Cartan theory read,

$$\frac{1}{2} \eta_{\alpha\beta\gamma} \wedge R^{\beta\gamma} + \Lambda \eta_\alpha = \ell^2 \Sigma_\alpha, \quad (1)$$
$$\frac{1}{2} \eta_{\alpha\beta\gamma} \wedge T^\gamma = \ell^2 \tau_{\alpha\beta}, \quad (2)$$

where $\Lambda$ denotes the cosmological constant and $\ell^2$ Einstein’s gravitational constant.

The Einstein-Cartan theory is a viable gravitational theory. All experiments known are correctly predicted by the theory. One should add, however, that under usual conditions the spin $\tau_{\alpha\beta}$ of matter can be neglected which, in turn, according to the second field equation, yields vanishing torsion – and then we fall back to GR and to its predictions. The Einstein-Cartan theory, as compared to GR, carries an additional spin-spin contact interaction of gravitational origin. This additional interaction only shows up at extremely high matter densities ($\sim 10^{54}$ g/cm$^3$ for neutrons) and hasn’t been seen so far. Remember that even in neutron stars we have only densities of the order of $10^{15}$ g/cm$^3$. And in vacuum, according to the Einstein-Cartan theory, there is no torsion which is consistent with a recent finding of Lämmerzahl.

Nevertheless, from a theoretical point of view, the Einstein-Cartan theory appears to be the gravitational theory. And, no doubt, GR is the best alternative. This is why I called in the first section GR to be an alternative theory. Of course, it depends on your ‘reference frame’ what you are inclined to call ‘alternative gravitational theories’. I believe that the organizers of the Eighth Marcel Grossmann Meeting didn’t want me to interpret GR as an alternative theory. Quite the opposite, Hans Ohanian and Remo Ruffini even claim that the Einstein-Cartan theory is defective, see ref. pp. 311 and 312. Since this is a widely read and, otherwise, excellent textbook, I would like to comment on their arguments:

• O&R, pp.311–312 (‘local flatness syndrome’):

“If $\Gamma^\beta_{\nu\mu}$ were not symmetric, the parallelogram would fail to close. This would mean that the geometry of the curved spacetime differs from a flat geometry even on a small scale – the curved spacetime would not be approximated locally by a flat spacetime.”

If an orthonormal coframe $\vartheta^\alpha = e_i^\alpha \, dx^i$ and a linear connection $\Gamma^\alpha_{\beta\gamma} = \Gamma_i^\alpha \, dx^i$ are given as gravitational potentials, then by a suitable coordinate and a frame transformation, it can be shown that at one point P of a Riemann-Cartan manifold these potentials can be ‘normalized’ according to

$$\{\vartheta^\alpha, \Gamma^\alpha_{\beta\gamma}\} = \{\delta^\alpha_i \, dx^i, 0\} \quad \text{at one point P.} \quad (3)$$

This is the analog of the Einstein elevator. The O&R statement on the approximate local flatness is only correct, if one restricts oneselfs to a coordinate (or natural) frame. But it is incorrect in
general, see eq. (3). This was already known to Elie Cartan in 1923/24 and, as mentioned above, it was this reason which gave him the idea to name Riemann-Cartan spaces as spaces with Euclidean connection.

- O&R, p.312 footnote ('shaky spin discussion'):
  
  “...we do not know the ‘genuine’ spin content of elementary particles...”

According to present day wisdom, matter is built up from quarks and leptons. No substructures have been found so far. According to the mass-spin classification of the Poincaré group and the experimental information of lepton and hadron collisions etc., leptons and quarks turn out to be fermions with spin $1/2$ (obeying the Pauli principle). According to an appropriate interpretation of the Einstein-Cartan theory, see ref. the spin (of the Lorentz subgroup of the Poincaré group) represents the source of torsion. As long as we accept the (local) Poincaré group as a decisive structure for describing elementary particles, there can be no doubt what spin really is. And abandoning the Poincaré group would result in an overhaul of (locally valid) special relativity theory.

The nucleon is a composite particle and things related to the build-up of its spin are not clear so far. But we do know that we can treat it as a fermion with spin $1/2$. As long as this can be taken for granted, at least in an effective sense, we know its spin and therefore its torsion content.

Of course, whether a theory is correct, can only be verified (or falsified) by experiment. But the two points of O&R, since both incorrect, are irrelevant for this question.—

Let us come back to Einstein-Cartan theory proper. Recognizing the central position of the matter field $\Psi$, Audretsch & Lämmerzahl initiated a new axiomatics for spacetime which is based on wave notions.

From the point of view of a Riemann-Cartan space, you can end up with a flat Minkowski space in two ways: You can either start with a Riemann-Cartan space and equate the torsion to zero, as in eq.(2), if matter spin vanishes, or you can first put the (Riemann-Cartan) curvature to zero, then one arrives, after this first step, at a teleparallel (or Weitzenböck) space. Torsion still exists therein and gravitational teleparallelism models with quadratic torsion Lagrangians can be developed. One model mimicks perfectly well GR; for a detailed discussion one should compare Mielke.

The emerging Riemannian or Weitzenböck spaces are, in some sense, equivalent to each other. In a second step, one then puts curvature or torsion, respectively, to zero and eventually reaches the Minkowski space on both ways.

5 Sessions on alternative theories

Besides GR proper, d'Inverno lists Alternative theories, Unified field theory, and Quantum gravity. Under ‘Alternative theories’ we find the entries torsion theories, Brans-Dicke, Hoyle-Narlikar, Whitehead, bimetric theories, etc., under ‘Unified field
theory’ only Kaluza-Klein, and under ‘Quantum gravity’ canonical gravity, quantum theory on curved backgrounds, path-integral approach, supergravity, superstrings, etc.

The organizers of the present Marcel Grossmann Meeting, in setting up the parallel sessions, must have had in mind such a division. Originally the present ‘Alternative session’ encompassed also papers on higher and lower dimensions. But these papers were so numerous that they later had to be shifted to a newly installed session chaired by Professor V.N. Melnikov (Moscow). Therefore in this session only 4D papers were read.

In chronological order, I will list all the lectures which actually took place. Some of the authors who submitted abstracts and who were supposed to present their material didn’t come. They are not listed, unless their paper was read by somebody else. It is for that reason that the different subsessions are of quite different length. In organizing the session, I tried to order the papers logically depending on the topic they treat. This was not possible for the post-deadline papers. For each session I selected three main contributions which I believed to be of general interest.

5.1 Session I on Monday

Three main contributions

1. C. Rovelli: General relativity in terms of Dirac eigenvalues

2. (a) R.M. Zalaletdinov: Averaging problem in general relativity, macroscopic gravity and using Einstein’s equations in cosmology
   (b) M. Mars, R.M. Zalaletdinov: Space-time averages in macroscopic gravity and volume-preserving coordinates

3. S.R. Valluri, W.L. Harper, R. Biggs: Newton’s precession theorem, eccentric orbits and Mercury’s orbit

PPN and Newton-Cartan theory

4. E.E. Flanagan: Coordinate invariant formulation of post-1-Newtonian general relativity

New mathematical structures

5. R.M. Santilli: Isotopic grand unification with the inclusion of gravity (read by F.W. Hehl)

6. A.Yu. Neronov: General relativity as continuum with microstructure. Formal theory of Lie pseudogroups approach

Theories based on a Riemann-Cartan-Weyl geometry of spacetime
7. R.T. Hammond, C. Gruver, P. Kelly: Scalar field from Dirac coupled torsion frames, teleparallelism, bimetric theories

8. S. Kaniel, Y. Itin: Gravity by Hodge de-Rham Laplacian on frames

9. V. Olkhov: On thermal properties of gravity

Electromagnetism and gravity

10. R. Opher, U.F. Wichoski: On a theory for nonminimal gravitational-electromagnetic coupling consistent with observational data

11. C. Lämmerzahl, R.A. Puntigam, F.W. Hehl: Can the electromagnetic field couple to post-Riemannian structures?

Additional post-deadline papers

12. C.M. Zhang: Gravitational spin effect on the magnetic inclination evolution of pulsars

13. B.G. Sidharth: Quantum mechanics and cosmology – alternative perspective

5.2 Session II on Thursday

Three main contributions

14. Dj. Šijački: Towards hypergravity

15. A.Burinskii: Spinning particle as superblackhole

16. C. Lämmerzahl: New constraints on space-time torsion from Hughes-Drever experiments

Theories based on a metric-affine geometry of spacetime

17. S. Casanova, G. Montani, R. Ruffini, R. Zalaletdinov: On the non-Riemannian manifolds as framework for geometric unification theories

18. (a) A.V. Minkevich: Some physical aspects of gauge approach to gravity

(b) A.V. Minkevich, A.S. Garkun: Some regular multicomponent isotropic models in gauge theories of gravity

19. F.W. Hehl, Yu.N. Obukhov: Is a hadronic shear current one of the sources in metric–affine gravity (MAG)?

Other theories
20. L.V. Verozub: Gravity: Field and Curvature (read by F.W. Hehl)

Additional post-deadline papers

21. D. Rapoport: Torsion and quantum and hydrodynamical fluctuations

22. L.C. Garcia de Andrade: On non-Riemannian domain walls

23. Kong: On Einstein-Cartan cosmology

5.3 Some remarks to the papers

With the titles of the subsessions, I tried to circumscribe the content of the corresponding papers. Papers on Riemann-Cartan spacetimes, in particular Einstein-Cartan type theories, or slight generalizations therefrom, can be found under the numbers 7 (enriched by an additional Weyl field), 12, 16, 21, 22, 23. There were three papers on frame and teleparallelism theories: Number 8, which was published in the meantime [and which we found particularly interesting, see ref.], number 9, and number 20.

If one relaxes the metric compatibility of the connection $\Gamma^{\alpha}_{\beta\gamma}$ of spacetime, then one has the nonmetricity $Q_{\alpha\beta} := - \nabla g_{\alpha\beta}$ as an additional structure to play with. Such theories are of the type of metric-affine gravity MAG. The lecture of Šijački falls under this general heading, even if he has a sizeable additional input from particle physics; also the papers number 11, 17, 18, and 19 use the metric-affine structure. These papers exhaust the gauge type papers in the sessions.

GR (more or less) was the basis of the papers number 1 (Euclidean signature), 2, 4, and 10. A historical lecture on precession effects within Newtonian gravity (and what we can learn from it for GR) can be found under number 3. The paper number 15 describes some supersymmetric generalization of the Kerr solution of GR.

After all these ‘conventional’ alternative attempts, three papers are left in which new mathematical structures (papers number 5 and 6) and new quantum physical attempts were investigated.

I apologize for all inappropriate classification attempts!

Acknowledgments

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