Intensity-duration-frequency relationships of rainfall through the
technique of disaggregation of daily rainfall

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ABSTRACT: Rainfall intensity-duration-frequency (IDF) relationships are a tool that can be used in modeling the transformation of rainfall to runoff, required for the design of hydraulic works. The objective of this study was to verify if there is a significant difference between the intensity-duration-frequency relationships generated using pluviographic records and those determined from pluviometric data. Maximum annual rainfall intensity values were obtained from the disaggregation of maximum daily rainfall and rainfall records in the durations of 5, 10, 15, 20, 30, 60, 120, 360, 720 and 1440 min and for the return periods of 2, 5, 10, 20, 25, 50 and 100 years, in the locality of Pelotas, Rio Grande do Sul state, Brazil (31° 46' 34'' S; 52° 21' 34'' W, altitude of 13.2 m). By Student's t-test, it was verified that there is no significant difference between the values of maximum rainfall intensity obtained from pluviographic records and those determined from pluviometric data.

Key words: intense rains, pluviometric data, pluviographic records

Relações intensidade-duração-frequência da precipitação mediante a técnica da desagregação da chuva diária

RESUMO: A relação intensidade-duração-frequência de ocorrência de precipitação é uma técnica que pode subsidiar estudos hidrológicos em modelos de transformação chuva-vazão, muitas vezes, utilizada no dimensionamento de obras hidráulicas. Objetivou-se verificar se há diferença significativa entre as relações, cujos valores de intensidades máximas anuais foram obtidos a partir da desagregação da chuva máxima diária e registros pluviográficos, nas durações de 5, 10, 15, 20, 30, 60, 120, 360, 720 e 1440 min e para os períodos de retorno de 2, 5, 10, 20, 25, 50 e 100 anos, na localidade de Pelotas/RS, Brasil (31° 46' 34'' S; 52° 21' 34'' O, altitude de 13.2 m). Pelo teste t de Student, constatou-se que não há diferença significativa entre os valores de intensidades máximas obtidos a partir de registros pluviográficos e aqueles mediante a desagregação de dados pluviométricos diários.

Palavras-chave: chuvas intensas, dados pluviométricos, registros pluviográficos
INTRODUCTION

Rainfall is an element of weather and climate that varies in space. To characterize rainfall, it is necessary to know its intensity, duration and frequency of occurrence or return period (RP). This characterization is typically presented as an intensity-duration-frequency (IDF) relationship, which is a tool used in modeling the transformation of rainfall to runoff (Damé et al., 2008).

The intensity-duration-frequency relationship is traditionally determined from observations of intense rainfalls over a long period of time, which are representative of extreme events. When constructing a rainfall series for statistical analysis, two routes can be followed: the use of rainfall records or the disaggregation of daily rainfall data into individual rainfall events.

Rainfall records are scarce, while daily rainfall data is readily available (Back et al., 2011; Aragão et al., 2013; Damé et al., 2014), since in Brazil, there is an extensive pluviometric monitoring network.

To use daily rainfall data, it is necessary to apply a disaggregation model, from which it is possible to obtain the values of water depths within shorter durations of time from the value of rainfall accumulated in one day (Aragão et al., 2013).

The objective of this study was to establish if there is a significant difference between the intensity-duration-frequency relationships obtained from the disaggregated daily maximum rainfall data in comparison to those determined by rainfall records, in the locality of Pelotas, Rio Grande do Sul state, Brazil.

MATERIAL AND METHODS

A rainfall series for the region of Pelotas, RS state, Brazil (31° 46' 34'' S; 52° 21' 34'' W, altitude of 13.2 m), from the period 1982 to 2015 was obtained from the Agroclimatological Station of EMBRAPA/INMET (Instituto Brasileiro de Meteorologia) and UFPEL (Universidade Federal de Pelotas). The daily maximum rainfall values were selected in the series to make the annual maximum daily rainfall series. The period from 1982 to 2015 had important extreme events related to the occurrence of the El Niño Southern Oscillation phenomenon, which directly affected the maximum intensity of the rainfalls.

The disaggregation of the annual maximum daily rainfall was performed for the durations of 5, 10, 15, 20, 25, 50 and 100 years, which were later used to determine the intensity-duration-frequency relationships, the empirical model of Weibull and a theoretical model with Normal, Log-Normal, Gumbel and Gamma probability distributions were used.

From this methodology, the intensity-duration-frequency hybrid relationships were obtained (empirical model, 2 to 25 years and theoretical model, 50 and 100 years) and the conventional intensity-duration-frequency relationships (theoretical model of probability for return periods of 2, 5, 10, 20, 25, 50 and 100 years).

In the empirical model, the exceedance probability \( P(X \geq x) \) and return period (RP) were determined using Eq. 2 from Weibull.

\[
P(X \geq x) = \frac{i}{n+1}
\]

where:
- \( i \) - order number of each element of the series; and, 
- \( n \) - total number of elements in the series.

The parameters of the Normal, Log-Normal, Gumbel and Gamma probability models were adjusted using the maximum likelihood method (Blain & Camargo, 2012; Pereira et al., 2014). Then, the model showing the best fit to the values of maximum rainfall intensity was selected using the Kolmogorov-Smirnov test (Silva et al., 2013; Ramos & Moala, 2014).

The null hypothesis \( H_0 \) means that the data comes from a population with values which come from the tested distribution, to a level of probability \( p \) of 0.05.

The rainfall intensity-duration-frequency relationships were represented according to Eq. 3 (Borga et al., 2005; Silva et al., 2006, 2013):

\[
I = \frac{K \ RP^*}{(t + b)}
\]

where:
- \( I \) - intensity of rainfall, mm h \(^{-1}\); 
- \( RP \) - return period, years; 
- \( t \) - rainfall duration, min; and, 
- \( K, a, b, c \) - parameters of the equation that must be adjusted to the observed data.
In order to obtain the parameters of the intensity-duration-frequency relationships (K, a, b and c), the Root Mean Square Error (RMSE) (Eq. 4) was used:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (X_{\text{emp}} - X_{\text{est}})^2}{n}} \tag{4}
\]

where:
- \(X_{\text{emp}}\) - observed values of maximum rainfall intensity, \(\text{mm h}^{-1}\);
- \(X_{\text{est}}\) - estimated values of maximum rainfall intensity, \(\text{mm h}^{-1}\), obtained by the intensity-duration-frequency relationships with coefficients in which the RMSE value reached its minimum; and,
- \(n\) - total number of elements in the series.

The hybrid and conventional intensity-duration-frequency relationships obtained from daily disaggregated data were validated by comparing the predicted values of maximum rainfall intensity with those obtained by hybrid and conventional intensity-duration-frequency relationships determined using rainfall records for the same locality (Eqs. 5 and 6, respectively):

\[
I = \frac{1000RP^{0.176}}{(t + 20.00)^{0.735}} \tag{5}
\]

\[
I = \frac{1100RP^{0.165}}{(t + 16.469)^{0.766}} \tag{6}
\]

In order to evaluate whether or not there is a significant difference, with a probability level (\(\alpha\)) of 0.05, between the values of maximum rainfall intensity obtained by the hybrid and conventional intensity-duration-frequency relationships, and by the daily rainfall disaggregation, comparing to rainfall data, the Student’s t-test was used for the angular coefficient \(\beta_1\) of the simple linear regression model.

The null hypothesis (\(H_0\)) was established for the angular coefficient \(\beta_1\); \(H_1: \beta_1 = 0\) and the alternative, \(H_1: \beta_1 \neq 0\).

### RESULTS AND DISCUSSION

The results show that the values of maximum rainfall intensity that were obtained by disaggregation are independent, since the sample autocorrelation coefficients (\(r_1\)) are inserted in the confidence interval of 0.95, higher than 0.306 and lower than -0.366 (Souza et al., 2009; Teixeira et al., 2011).

Considering the durations of 5, 10, 15, 20, 30, 60, 120, 360, 720 and 1440 min, the values of \(r_1\) were: 0.196; 0.124; 0.121; 0.120; 0.106; 0.091; 0.079; 0.551; 0.208 and 0.072, respectively, showing the independence of the values of maximum rainfall intensity.

The values of maximum rainfall intensity in the different durations can be considered stationary, since the Mann-Kendall test (Z) ranged from -0.999 to -0.986, which was lower than the critical value, \(Z_{\alpha}\), of 1.96, for a confidence interval of 0.95 (\(\alpha = 0.05\)) (Pinheiro et al., 2013; Silva & Streck, 2014).

Table 1 presents the Kolmogorov-Smirnov (KS) statistic test values for the tested probability distributions. All of them are observed to fit properly, but Gumbel was selected, because of the lower value of the calculated statistic, as well as its adequacy in the adjustment of extreme values (Sansigolo, 2008).

Pereira et al. (2014) estimated the intensity-duration-frequency equation for the state of Mato Grosso do Sul, Brazil, by the relations method of CETESB, adjusting to the Gumbel distribution by the maximum likelihood method. Using the KS test (\(\alpha = 0.20\)), the authors concluded that the Gumbel distribution was adequate to estimate the intense rainfalls in the studied areas.

However, there is no consensus in the literature about the most appropriate probability distribution to use to adjust the maximum daily rainfall. When investigating the robustness of several typical probability distributions in the analysis of extreme hydrological events, Back (2006) and Sansigolo (2008) verified that the distribution of Gumbel was the one that best adjusted to the maximum daily rainfall values.

Table 2 shows the values of maximum rainfall intensity for the durations of 5, 10, 15, 20, 30, 60, 120, 360, 720 and 1440 min obtained by the Gumbel distribution for the return periods of 2, 5, 10, 20, 25, 50 and 100 years for conventional intensity-duration-frequency relationships, as well as for return periods of 50 and 100 years for hybrid intensity-duration-frequency relationships.

The equations representing the hybrid and conventional intensity-duration-frequency relationships (Table 3) were obtained from the values of maximum rainfall intensity presented in Table 2.

The established equations are applicable to durations from 5 to 1440 min, for the period from 1982 to 2015. The justification for the use of the selected durations relates to the time of concentration of the drainage area for possible hydraulic projects, since the intensity-duration-frequency represents the values of the intense rainfalls in such durations (Damé et al., 2005; Ben-Zvi, 2009).

To confirm the null hypothesis, the values of the coefficient \(\beta_1\), the statistic \(t(\hat{\beta}_1)\) values, \(p\) and \(r\) obtained from the historical

### Table 1. Calculated values of the Kolmogorov-Smirnov test (\(D_{\text{max}}\)) for the probability distributions adjusted to the annual maximum series obtained by the maximum daily rainfall disaggregation for Pelotas, RS state, Brazil, from 1982 to 2015

| Probability distribution | 5     | 10    | 15    | 20    | 30    | 60    | 120   | 360   | 720   | 1440  |
|--------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Normal                   | 0.211 | 0.211 | 0.210 | 0.211 | 0.211 | 0.211 | 0.211 | 0.212 | 0.215 | 0.215 |
| Log-Normal               | 0.148 | 0.148 | 0.147 | 0.148 | 0.148 | 0.148 | 0.148 | 0.150 | 0.153 | 0.152 |
| Gumbel                   | 0.124 | 0.124 | 0.123 | 0.124 | 0.124 | 0.124 | 0.124 | 0.125 | 0.125 | 0.119 |
| Gamma                    | 0.170 | 0.170 | 0.169 | 0.169 | 0.170 | 0.170 | 0.170 | 0.172 | 0.174 | 0.174 |

\(D_{\text{max}} = 0.227\) \(p < 0.05\)
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Values of maximum rainfall intensities ($I_{\text{max}}$) are presented in Table 4, as well as those resulting from hybrid and conventional intensity-duration-frequency.

The results in Table 4 show that the values of maximum rainfall intensity obtained from the hybrid and conventional intensity-duration-frequency relationships do not differ from the historical values, since the values calculated from the statistic "t" for the angular coefficients ($\beta_1$) were lower than the critical value of $t$, for $p \leq 0.05$.

The results presented in Table 4 show that the equations generated by the disaggregation of daily rainfall data present a good correlation when compared to the intensity-duration-frequency determined from rainfall records, with $r$ values of 0.9915 and 0.9983, respectively. This demonstrates that the disaggregation of daily rainfall can be used in places where rainfall records are not available.

Silva et al. (2012) determined the intensity-duration-frequency relationships from rainfall data for a selection of localities that represented the climatic variability of the state. The authors show that the equations generated by rainfall data present adequate adjustments, with values of the correlation coefficient $r$ varying from 0.9849 to 0.9999, comparatively to the intensity-duration-frequency relationships obtained with rainfall records.

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respectively. In the case of the return period of 10 years, the percentage difference of the values of maximum rainfall intensity is around 8%, while for return period of 100 years, this value is minimized to 4.5%. The values presented as an example, were obtained from the comparison between the conventional and hybrid intensity-duration-frequency relationships, in the duration of 60 min.

Therefore, it is possible to observe that there is an indication that the conventional intensity-duration-frequency relationship can be used for return periods of more than 10 years, in which the percentage differences are lower.

Conclusions

1. Comparative analysis of the results obtained on applying daily rainfall disaggregation using the equations generated by the rainfall records showed that there is no significant difference in terms of intensity, duration, and frequency between the maximum intensities; therefore, this method can be used in locations where rainfall records are not available.

2. Both hybrid and conventional intensity, duration and frequency relationships adequately represented the values of maximum historical rainfall intensity, such that the values of maximum rainfall intensity for return periods of up to 25 years can be obtained both by empirical and theoretical models.

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