Field expansion of holographic image reconstructed from enhanced-NA Fresnel hologram

Byung Gyu Chae
Holographic Contents Research Laboratory,
Electronics and Telecommunications Research Institute,
218 Gajeong-ro, Yuseong-gu, Daejeon 34129, Republic of Korea

Abstract

A Fresnel hologram with a high numerical aperture reconstructs a holographic image at a viewing angle larger than the diffraction angle of a hologram pixel. The image space is limited by a bandwidth of digital hologram. In this study, we investigate the property of an image formation on the extended image space beyond a diffraction zone. A numerical simulation using the phase Fresnel hologram is carried out to observe an extension of image space and the effect of it on the change in angular field of view. A uniform image having no high-order noises can be seen within the primary viewing zone when the viewing angle is restricted to a diffraction angle. Optical experiments show the consistent results with the numerical analysis. On the other hand, when the viewing angle depends on the hologram numerical aperture, the replica noises in both planes could be removed in principle during the iterative algorithm. We discuss the development of method for expanding the image space keeping the viewing angle of a holographic image.
I. INTRODUCTION

A holographic display offers a three-dimensional image in such a way that the wavefront of the propagating wave from the digital hologram is generated in free space \[1\]. Various attempts to implement holographic displays have been conducted because it has the ultimate merit in that an observer can view a complete scene without visual fatigue \[2\]-\[4\]. Despite technological advancements, there still remains a fundamental limitation for realizing a commercial holographic display owing to the finite space-bandwidth product of a pixelated spatial light modulator (SLM).

Especially, the issue of solving a narrow viewing angle is a crucial task. It has been known that an SLM with a pixel pitch of the wavelength scale is necessary for seeing a holographic image at a wide angular view. A commercial modulator has a pixel pitch of several micrometers, inducing a viewing angle of a few degrees. Experiments to overcome this problem have been conducted by expanding the diffraction zone using spatial or temporal multiplexing of modulators \[3\]-\[8\]. In this circumstance, a massive data processing is required, and it would be inefficient in view of energy consumption in informatics.

These conventional approaches for expanding the viewing zone are based on the assumption that the diffraction angle of a pixel pitch directly relates to the angular view of the holographic image. Previously, we studied that the viewing angle \(\omega\) of the reconstructed image is fundamentally determined by a numerical aperture (NA) of the digital hologram rather than a diffraction angle \[9\]-\[10\]:

\[
\omega = 2 \sin^{-1} (\text{NA}) .
\]

The hologram numerical aperture is represented as a geometrical structure of aperture size, \(l = N \Delta x\), and a distance \(z\), \(\text{NA} = N \Delta x / 2z\), where \(N\) and \(\Delta x\) denote the pixel number and size of the digital hologram, respectively. The viewing angle does not depend independently on the diffraction ability of the pixel pitch or the illuminating wave. This concept provides a major technological approach in expanding a viewing angle in holographic display system. In principle, the viewing angle varies with the reconstructed distance, even when using an SLM with a constant pixel pitch. The viewing angle of the image would be larger than a diffraction angle when the image is formed at a closer distance. There is still a decrease in image size because of the Nyquist sampling criterion in hologram synthesis \[11\]-\[12\]. However, this behavior is different from the trade-off relation characterized by the space-bandwidth
product of the digital hologram itself. For the realization of a wide viewing-angle holographic display, a holographic image without sacrificing the image size should be reconstructed. When the image reconstruction is numerically or optically processed on the region outside of the sampling condition, the interference of high-order images is inevitable. Therefore, a detailed study on the region expansion of a holographic image and the effect of it on the change in viewing angle is required.

A phase modulation has some advantage showing no twin image and high diffraction efficiency [14-16]. A phase retrieval algorithm such as the iterative Fresnel transform is used to calculate the phase hologram in the Fresnel diffraction regime. The enhanced-NA digital hologram creates the replica fringe patterns, which fundamentally obstructs the image reconstruction in an extended space beyond the bounds of a diffraction scope of hologram pixel [9].

In this study, we explore the image reconstruction in the extended image space from the phase Fresnel hologram with a high numerical aperture. Firstly, the phase hologram is successfully prepared using the iterative algorithm based on the Fresnel propagation. We analyze an angular view of a holographic image dependent on the hologram numerical aperture, and an expansion of image space. Secondly, optical experiments are conducted to obtain the holographic image without an interference of high-order noises. Subsequently, we study the method for reconstructing a holographic image with an extended image space at a larger viewing angle.

II. HOLOGRAPHIC IMAGE RECONSTRUCTED FROM ENHANCED-NA PHASE FRESNEL HOLOGRAM

A. Analysis on angular field of view of holographic image dependent on hologram numerical aperture

The Fresnel field propagating from the digital hologram to image plane is described by the Fresnel diffraction formula. The specifications of both planes are determined by the Nyquist-Shannon sampling theorem [10-12]. The relationship between pixel pitches \( \Delta x \) and \( \Delta x' \) in the hologram \((x, y)\) and image \((x', y')\) coordinates is represented as follows:

\[
\Delta x = \frac{\lambda z}{N\Delta x'}, \quad \Delta y = \frac{\lambda z}{N\Delta y'}.
\] (2)
For convenience, a one-dimensional description is considered hereafter. The sampling condition of both planes is characterized by the critical distance having a value, \( z_c = N\Delta x^2/\lambda \), when the pixel pitches of both planes are the same. The image size, \( l' = N\Delta x' \) is smaller than the hologram size, \( l = N\Delta x \) when the image is placed at a distance less than \( z_c \), in Fig. 1(a). We know that the aliased errors appear if they are digitally or optically processed in the regions that deviate from this specification.

A practicable phase hologram \( \phi \) for the real-valued object was extracted from the modified Gerchberg-Saxton (GS) iterative algorithm \[14, 17, 18\]. The exponential term of \( e^{i\phi} \) was used to modulate the phase hologram in the simulation strategy. Synthesized phase holograms using a letter image are composed of 1920×1920 pixels with an 8-\( \mu \)m pixel pitch, where the critical distance considering a wavelength, \( \lambda = 473 \) nm is calculated to be 259.8 mm. To measure the viewing angle of a holographic image, the propagating wave far away from the reconstructed image was simulated. Figure 1(b) shows the propagating waves in the simulation using the digital hologram made at a critical distance. The intensity profiles of diffractive wave indicated by a lateral line in inset image are displayed with distance. An observer can view the image within the spreading angle of diffractive wave. From this, the viewing angle of the reconstructed image is estimated to be 4.2\( ^\circ \). Figure 1(c) is the simulation result for a digital hologram made at a half of critical distance. The propagating wave from the image largely spreads, where the viewing angle appears to be about 7.7\( ^\circ \).

A holographic image is reconstructed by illuminating the digital hologram with a plane wave. In Fig. 1(a), it looks like that the image is focused on a particular place through the digital hologram acting as a lens. If we assume that the object placed at a critical distance on the opposite side is imaged through the digital hologram, the Lagrange invariant \[19, 20\], represented by the product of the aperture area and spreading angle of the propagating light would have the same value in both object and hologram planes. The invariant quantity in the hologram plane relates to the space-bandwidth product corresponding to data capacity.

The optical invariant on the hologram, \( l\theta \), is kept through the propagation of the diffractive wave, resulting in \( l'\Omega \) on the image plane:

\[
nl\theta = nl'\Omega,
\]

where \( \theta = 2\sin^{-1}\left(\frac{\lambda}{2\Delta x}\right) \), the diffraction angle by a hologram pixel, and the refractive index \( n \) is equal to one in free space. From Eqs. (2) and (3), we can deduce that the lateral size
and viewing angle $\Omega$ of the reconstructed image have a trade-off relation, which is unlike the interpretation by controlling the size of the hologram itself. As illustrated in Fig. 1(a), it shows the viewing angle larger than the diffraction angle when the image is formed at a distance less than the critical distance. The high-definition image diffracts to a larger spreading angle to satisfy the conservation of the optical invariant. We know that the angle value $\Omega$ is pertinent to the hologram numerical aperture, $NA = n \sin(\Omega/2)$. Hence, the viewing angle of the reconstructed image obeys the relation in Eq. (1). Above measured viewing-angles are consistent with the calculated values. We have also confirmed the viewing-angle variation dependent on the hologram numerical aperture in optical experiments [9].

Meanwhile, a method including a double Fourier transform such as a convolutional algorithm or angular spectrum method cannot enhance the angular view, because a spatial filtering at the Fourier plane within the formula is embedded, resizing the hologram pattern according to the distance [10]. Here, one preserves the constant NA irrespective of the distance. For a clarity, we defined a hologram keeping a high numerical aperture as the enhanced-NA Fresnel hologram [13].

In previous work, we studied that an aliased error arises when synthesizing the enhanced-NA Fresnel hologram because of the undersampling of the point spread function $h(x,y)$ in the front of Fourier transform $FT$ in the expanded form of the Fresnel diffraction formula [13, 21]:

$$h(x, y)FT \left[ o(x', y')h(x', y') \right]$$

(4)

Since the sampling criterion with respect to the object plane is well satisfied, the spreading angle of diffractive wave from the object $o(x', y')$ covers the whole area of digital hologram, which induces the suppression of the aliased replica fringes when the object has a finite size. We explained that this phenomenon results from the near-field diffraction property within the diffraction zone by an object pixel resolution [9].

This behavior brings about an energy concentration in a low-frequency region in the digital hologram, which weakens the diffraction ability by an incident plane wave. Figure 2 illustrates the diffractive wave from the reconstructed image by a directly calculated digital hologram. The simulated light intensity does not spread out nearly with distance. Although the high-definition image is formed, it is difficult to observe a holographic image with a sufficient angular view. However, in a logarithmic scale, the spreading angle of diffractive wave is clearly distinguished [10], which still ensures a validity of Eq. (1). An effective
method for mitigating the energy concentration is to add a random phase to the object in a hologram synthesis. In this case, as depicted in Fig. 2(c), the intensity profile of diffractive wave progresses at an adequate spreading angle.

B. Field expansion of holographic image and effect of it on change in viewing angle

For the realization of a viable holographic display based on the enhanced-NA Fresnel hologram, a sufficient amount of image space should be secured. Figure 3 shows the simulation results for the generation of high-order images in the image space. The digital hologram with an 8-μm pixel pitch was conveniently calculated by using a rectangular object consisting of 256×256 pixels, and then, the pixels are upsampled tenfold. The reconstructed image from the digital hologram made at a distance $2z_c$ reveals a sinc function envelope by a hologram pixel pitch. However, there is no intensity modulation in the enhanced-NA Fresnel hologram region, where the bright areas of all the images are comparable to hologram size in the same scale. The image intensity has been known to be uniform irrespective of an image position [22, 23]. This makes it possible to reconstruct a uniform image in the extended image space larger than the diffraction scope by a hologram pixel.

Figure 4 displays the schematics of an image reconstruction in the extended image space. We consider the hologram made using the letter image placed at a distance of half of $z_c$, where the object size is two times larger than the diffraction scope by a hologram pixel pitch. As depicted in Fig. 4, the first-order images are created in the corresponding areas outside of the diffraction scope. The viewing direction of first-order image is tilted at a diffraction angle.

We characterize the reconstruction behavior in two separate cases. The first case is that the viewing angle well depends on the hologram numerical aperture, which is double a diffraction angle in this configuration. Here, the overlapped image with high-order terms is observable. Although it will be discussed later, this type of high-order images would be removed through the iterative optimization algorithm. Secondly, the viewing angle could be restricted to a diffraction angle due to the nature of algorithm. In this case, the complete image without the interference of high-order noises can be seen in the viewing direction of zeroth order. If the viewing position is shifted to the first-order direction, the complete
replica image will reappear still. This phenomenon is well observed in optical experiments.

Figure 5 is the numerical simulation for the extended image reconstruction from the enhanced-NA phase hologram. It is hard to directly use the fast Fourier transform algorithm in the hologram synthesis using the object space outside of the diffraction scope because the pixel specifications of both planes deviate from the sampling condition in Eq. (2). The digital hologram was prepared in two separate steps. First, the high-resolution holograms used to suit the extended object space in accordance with the sampling condition were calculated using the modified GS algorithm; thereafter, they were downsampled to the final resolution of the hologram.

Digital hologram is synthesized using a cameramen image having $3840 \times 3840$ pixels with a 4-$\mu$m pixel pitch whose size is equal to the hologram magnitude. The final phase hologram displayed in Fig. 5(a) is composed of $1920 \times 1920$ pixels with an 8-$\mu$m pixel pitch, placed at a half of critical distance, 129.9 mm. To exclude the undersampling effect, the reconstruction process is performed by doubly upsampling the digital hologram from an 8-$\mu$m pixel to a 4-$\mu$m pixel. Here, the upsampling process indicates that the sub-pixel values are simply duplicated without using an appropriate interpolation method. We know that no replica noises are observed, in Fig. 5(b). Although the high-order images are overlapped in the image space, it is not viewable because of an inclined viewing direction. The image field is doubly expanded in comparison to a diffraction zone, but the viewing angle is confined to a diffraction angle.

III. OPTICAL RECONSTRUCTION FROM ENHANCED-NA FRESNEL HOLOGRAM WITHOUT SACRIFICING IMAGE SIZE

We carried out optical experiments by using a phase-only SLM (Holoeye PLUTO) with $1920 \times 1080$ pixels of an 8-$\mu$m pixel pitch and a blue laser with a wavelength of 473 nm. The critical distance for a pixel specification in a vertical direction was set as a reference to secure the imaging area with no aliased errors, which was calculated to be 146.1 mm. Figure 6 demonstrates the optical reconstruction from the synthesized hologram with an extended object space. Phase holograms are prepared at distances of a half, a quarter, and one eighth of $z_c$, in Fig. 6(a). To prepare a hologram made at half $z_c$, 73.1 mm, the initial phase hologram with $3840 \times 2160$ pixels of a 4-$\mu$m pixel pitch obtained through the iterative
Fresnel algorithm is downsampled by twice the pixel interval to be loaded on the pixelated modulator. All the images have a rectangular form with an 8.64 mm×8.64 mm lateral size, according to the sampling condition in Eq. (2). We conveniently captured the image within the SLM size for considering a direct-beam filtering by a polarizer. We observed that all the holograms well reconstruct the original images without suffering from the high-order noises. As not displayed here, when the viewing direction is moved to the high-order region, the corresponding replica image reappears after the primary image is gone, which is consistent with the numerical analysis.

Figure 6(c) is the unfocused image about the image of Fig. 6(b). Although an image blurring occurs, an accommodation effect is very weak. This indicates that the diffractive wave from the image spot propagates at a narrow spreading angle, where a perspective view of the image is hardly visible to the naked eye. The energy compaction of hologram patterns invokes this weakness, as explained in Section 2.

In second experiment, a sparse letter object was used and the down and up sampling processes were embedded within the iterative algorithm. Furthermore, to increase the optimization performance, the adequate background value of 37% in comparison to letter pixel value was added to the object space by considering the suppression effect of high-order noises when using the relatively dense object. A letter image is fully filled in the vacant space whose size is equal to the hologram magnitude in vertical direction.

A phase hologram prepared at a half of critical distance, 73.1 mm is displayed in Fig. 7(a). We find that the pixel values are scattered in the whole area of digital hologram when compared with those of Fig. 6(b). The image without interfering the high-order noises is well reconstructed in Fig. 7(b), where the horizontal and vertical pixel resolutions of images are 2.25 µm and 4 µm, respectively. Figure 7(c) is the unfocused image captured at a distance slightly apart from half \( z_c \). We observed that an accommodation effect is very strong, which means that the spreading angle of diffractive wave from the image spot would be larger. Figure 5(d) is the reconstructed image from the phase hologram synthesized at a quarter of critical distance, 36.5 mm. Likewise, the hologram well reconstructs the image without the high-order noises.

Figure 8 is the numerical simulation to measure the spreading angle of propagating wave from the image. As shown in Fig. 8(a), when there is no additional process, the diffraction scope of propagating wave from the circular object image is well satisfied with the relation of
Eq. (1). However, the upsampled digital hologram by a simple duplication pixel deteriorates the diffracting behavior from the focused image, in Fig. 8(b). This is the root cause for obstructing the viewing-angle expansion of a holographic image despite the effective removal of high-order noises.

IV. VIEWING-ANGLE ENLARGEMENT OF HOLOGRAPHIC IMAGE AND DISCUSSIONS

As previously stated in Section 2, the replica fringes are formed in the enhanced-NA Fresnel hologram because of the undersampling of the point spread function, where the finite-sized object could suppress this phenomenon from the near-field diffraction property. However, it rather produces the adverse effect of the fringe concentration. To enhance the diffraction performance of digital hologram by an incident wave, the energy compaction should be mitigated. A simple method is to diffuse an aggregated hologram pattern by adding the random phase to the real-valued object in the hologram synthesis, where the generation of replica fringes is inevitable. In this case, it is fundamentally impossible to remove the high-order noises in the image space.

We find that as displayed in Fig. 7(a), the iterative algorithm including the down and up sampling processes effectively prevents forming the replica fringes. The hologram pattern about the reversely propagating wave seems to be created to compensate the corresponding high-order noises during the repetition processes. This is different from a spatial filtering appearing in the algorithms having the double Fourier transforms. However, as seen in Fig. 8, the down and up sampling processes still weakens the viewing-angle expansion of a holographic image.

The Rayleigh-Sommerfeld formula using a definite integral can arbitrarily control the pixel specifications in both planes. Therefore, there is not a requirement for an unnecessary upsampling process within the iterative algorithm. As described in Subsection 2.2, the viewing angle dependent on the hologram numerical aperture would be conserved. In this circumstance, to obtain the optimized hologram, both the high-order images as well as the replica fringes in digital hologram should be simultaneously removed during the repetition processes. Furthermore, it still has a drawback to consume too much time in the iterative procedure.
Figure 9 is the experimental results using the Riemann integral during the repetition processes. The phase hologram composed of 512×512 pixels was calculated using the letter object with a 4-µm pixel pitch located at a half $z_c$. The fringe compaction is not completely eliminated, and there seems to remain some replica fringes, in Fig. 9(a). The inset in Fig. 9(b) shows the magnified light intensity propagating from the reconstructed image. The diffracting performance of focused image undergoes no deterioration. The synthesized hologram reconstructs the image having an accommodation effect, but some high-order noises appear still, in Fig. 9(b). The viewing-angle is estimated to be about the first-order diffraction angle of 6.7° because the overlapped image is viewable. Here, the whole area of digital hologram contributes to the optical image formation, which fulfills the viewing-angle variation dependent on the hologram numerical aperture.

It is important to develop a proper algorithm to reconstruct a holographic image keeping an angular view, and to increase an image fidelity for realizing a holographic display. The high angular spectrum of diffractive wave in both hologram and image planes should survive to form a highly focused spot. Presently, it has been known that optimization algorithms such as the stochastic gradient descent demonstrate a considerable improvement in an image fidelity. There have been reported several researches using a captured image by a camera or deep learning algorithm [24, 25]. Furthermore, a non-iterative algorithm has been studied [26]. They would become the proper methods. Further study to develop an optimization algorithm to accomplish a viewing-angle enlargement of a holographic image will be carried out in the future.

V. CONCLUSIONS

The phase Fresnel hologram with a high numerical aperture reconstructs in principle an extended image without suffering from the high-order noises. Especially when the viewing angle is restricted to a diffraction angle, a complete image is well observed in numerical and optical experiments. For implementing the holographic display, the narrow viewing-angle problem resulted from the smaller pixel pitch of the pixelated modulator should be solved. Since the viewing angle depends on the hologram numerical aperture, the replica noises in both planes could be removed during the iterative algorithm. It is important to develop the algorithm for extending the image space keeping the viewing angle of a holographic
image. This strategy makes it possible to recover the limitation of pixel resolution. Thus, a wide-viewing angle holographic display would be realized even using a commercial spatial light modulator.

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\[ \theta = 2 \sin^{-1} \left( \frac{\lambda}{2 \Delta x} \right) \quad \Omega = 2 \sin^{-1} \left( \frac{N \Delta x}{2 z} \right) \]

FIG. 1: Change in the viewing angle of a holographic image reconstructed from the enhanced-NA Fresnel hologram. (a) Schematic of the hologram imaging system. Intensity profiles of diffractive wave from the focused image reconstructed from the phase holograms synthesized at (b) a critical distance, \( z_c \) and (c) a half \( z_c \). Lateral line images in inset picture are displayed with distance. A spreading angle of propagating wave is larger than the diffraction angle by a hologram pixel when the focused image is placed at a distance lower than a critical distance.
FIG. 2: Propagating waves of the focused image reconstructed from the directly calculated complex holograms. Intensity profiles of diffractive wave in (a) a linear scale and (b) a logarithmic scale, which are simulated using the hologram synthesized at a half $z_c$. (c) Diffractive behavior of the image reconstructed from the hologram with no energy compaction.
FIG. 3: Simulation results for the generation of high-order images in the enhanced-NA Fresnel regime. A rectangular object consisting of 256×256 pixels has a boundary filled with zeros to distinguish high-order terms.
FIG. 4: Schematics of an image reconstruction in the extended image space from the enhanced-NA Fresnel hologram. Digital hologram is made using the letter image placed at a distance of half of $z_c$. Reconstructed images up to the first order are drawn for convenience.
FIG. 5: Numerical simulation showing the reconstructed images in the extended region. (a) Phase hologram and (b) reconstructed image obtained from two-fold upsampled hologram. Red box in the reconstructed image indicates the diffraction scope by a hologram pixel.
FIG. 6: Optical reconstruction from the digital hologram made using the extended object space. (a) Phase holograms are prepared at distances of a half, a quarter, and one eighth of $z_c$. (b) Synthesized digital hologram at half $z_c$, (b) reconstructed image, and (d) unfocused image at 140 mm. (e) Reconstructed image at quarter $z_c$. 
FIG. 7: Reconstruction property from the enhanced-NA phase hologram synthesized using the sparse letter object. (a) Phase hologram prepared at a half of critical distance, 73.1 mm, (b) reconstructed image, and (c) unfocused image. (d) Reconstructed image from the phase hologram made at quarter $z_c$. 
FIG. 8: Diffraction behavior from the focused image. All images of circular object are numerically reconstructed at a distance, 78.1 mm. (a) The diffractive wave from the circular object image well spreads out, whereas (b) the upsampled digital hologram by a simple duplication pixel deteriorates the diffracting behavior.
FIG. 9: Experimental results using the Riemann integral during the repetition processes. (a) Synthesized hologram and (b) reconstructed image. The inset shows the magnified light intensity propagating from the reconstructed image.