Modularity-in-Design of Dynamical Network Systems: Retrofit Control Approach
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Abstract—In this paper, we develop a framework of bilayer network synthesis where a system coordinator in the upper layer aims at ensuring system stability by designing a desirable interconnection among subsystems, while multiple subsystem operators in the lower layer aim at regulating their own control performance by designing individual subcontrollers with accessibility only to respective subsystem models. To decouple the stability assurance problem in the upper layer and the individual performance regulation problems in the lower layer, we first introduce a notion of subcontroller modularity as a property such that the dynamics with respect to an interconnection input and output is kept invariant for a local control system. Then, we characterize this subcontroller modularity in the context of retrofit control, which is defined as add-on type localized control such that, rather than an entire network system model, only a model of the subsystem of interest is required for controller design. Furthermore, based on the analysis of retrofit control, we show that stability enhancement by a system coordinator as well as individual performance improvement by subsystem operators all contribute to improving the entire control performance, defined for the map from all disturbance inputs to all evaluation outputs of the subsystems, in the sense of an upper bound. Finally, practical significance of the proposed framework is shown by an illustrative example of frequency regulation in the IEEE 68-bus power system model.

Index Terms—Modularity-in-design, Bilayer network synthesis, Retrofit control, Youla parameterization.

I. INTRODUCTION

Modular design or modularity-in-design is a widely accepted concept of system design to make complexity of large-scale system design manageable, to enable parallel work by multiple independent entities or designers, and to make future modification of subsystems or modules flexible. In software engineering, the necessity and benefit of modularization were urged in the seminal paper [1], where a module was introduced as a distinct unit of work that can be managed by one developer without considerable efforts for adjustment or coordination with other developers. The notion of modular design has been significantly expanded since then, and its advantage is analyzed in a broad range of literature [2]–[5]. In particular, [4] and [5] pointed out that the modularization of products can induce and facilitate the modularization of industrial structures, as exemplified by recent computer industry. A modular design approach is often compared to a contrastive approach called integral design [6], [7], where a single authority or designer is supposed for high level integration of interdependent components.

In systems and control engineering, relevant problems of analyzing and synthesizing large-scale network systems have been discussed over the past several decades. In particular, a wide variety of decentralized and distributed control methods has been devised from different perspectives; see, e.g., [8]–[13] and references therein. In fact, most of those existing methods are not classified into a modular design approach, but into an integral design approach of structured controllers, where a single authority with availability of an entire network model is premised for simultaneous design of all subcontrollers of a decentralized or distributed controller. This implies that even a small change in a subsystem or subcontroller may require a significant change in all others. On the other hand, a few control methods that can be classified into a modular design approach are found in the literature [14]–[16]. However, all those assume a limited class of network systems or assume a tight condition for system decomposition. Thus, their applicability is not generally sufficient in reality.

With this background, this paper aims at developing a new modular design framework of dynamical network systems in a more general setting. In particular, we develop a framework of bilayer network synthesis taking an approach based on retrofit control, for which a particular controller design method has been proposed in [20] and power system applications are reported in [21], [22]. The retrofit control is defined as add-on type localized control such that, rather than an entire network system model, only a model of the subsystem of interest is required for controller design. The retrofit controller guarantees robust stability in the sense that the resultant control system is stable for any variations of neighboring subsystems, other than the subsystem of interest, as long as the entire system before implementing retrofit control is stable. The present paper considerably enhances retrofit control theory from a viewpoint of necessity, and utilizes it as a rational tool to develop the bilayer network synthesis framework. In the proposed bilayer network synthesis framework, a system coordinator located in the upper layer is supposed to ensure system stability by designing a suitable interconnection among subsystems.

This paper builds on its proceedings versions [17]–[19], as collecting the results on the Youla parameterization of retrofit controllers discussed in several different settings. The bilayer network synthesis framework in this paper is developed based on those preliminary results, for which detailed mathematical proofs are also provided. Furthermore, more detailed analysis of power systems application is provided as a numerical example.
On the other hand, multiple subsystem operators located in the lower layer are supposed to parallelly regulate their own control performance by designing individual subcontrollers with accessibility only to respective subsystem models.

The main theoretical contributions in this paper are listed as follows. First, from a viewpoint of developing the bilayer network synthesis framework, we introduce a notion of subcontroller modularity as a property such that the dynamics with respect to an interconnection input and output is kept invariant for a local control system. Then, under a mild assumption, we characterize this subcontroller modularity in the context of the retrofit control. This characterization is shown by deriving a necessary and sufficient condition for a subcontroller to be a retrofit controller, described as a constrained version of the Youla parameterization [23]. We further show that the particular structure inside retrofit controllers reported in the previous work [20]–[22] is unique if appropriate assumptions on output measurement are imposed. Based on the analysis of the retrofit control, we also show that stability enhancement by a system coordinator as well as individual performance improvement by subsystem operators all contribute to improving the entire control performance, defined for the map from all disturbance inputs to all evaluation outputs of the subsystems, in the sense of an upper bound. Finally, practical significance of the proposed bilayer network synthesis is shown by an illustrative example of frequency regulation in a power system.

For comparison, several references regarding hierarchical control synthesis are in order. One major approach is developed in a way that local control laws are designed for respective subsystems that are typically assumed to be “weakly coupled,” while a global control law is designed to compensate interference among such weakly coupled subsystems; see, e.g., [24]–[26] and references therein. The bilayer network synthesis in this paper is, in contrast, developed in a more general setting regardless of coupling strength among subsystems. This reflects an advantage of retrofit control. Furthermore, because the original structure of subsystem partition can be directly used to determine distinct units for modular design, the process of computationally expensive system decomposition, such as those used in [26]–[29], is not required for implementation. As other related approaches, a stability analysis method based on a hierarchical Lyapunov function [30], a hierarchical control method based on approximate simulation [31], and a glocal control method based on multi-layered optimization [32] can also be found in the literature.

A control system design approach based on passivity, or more generally, dissipativity [33]–[37], is relevant to the modular design of network systems. This approach has an advantage that the input-output behavior of an entire network system can be analyzed only by those of subsystems, with which compatible supply rates of energy are associated. However, such analysis based on supply rates is just valid for particular port selections of interconnection input and output. Therefore, it is not very flexible to analyze general system behavior because disturbance input and evaluation output ports need to be identical to interconnection input and output ports in general. In contrast, our approach based on retrofit control has higher flexibility in setting individual input and output ports for subsystem operators.

The remainder of this paper is structured as follows. In Section II, an overview of the proposed bilayer network synthesis is provided while several key questions to be answered are listed. Then, giving a formal definition of the retrofit control, we conduct detailed analysis of retrofit control in Section III where a constrained version of the Youla parameterization is derived to characterize all retrofit controllers. Based on the analysis in Section III, we provide answers to the key questions in Section IV as developing the proposed bilayer network synthesis. Section V demonstrates its practical significance through an illustrative example of power systems control. Finally, Section VI concludes this paper.

Notation The notation in this paper is generally standard. The identity matrix with an appropriate size is denoted by $I$, the set of stable, proper, real rational transfer matrices by $\mathcal{RH}_\infty$, and the $H_\infty$-norm of $G \in \mathcal{RH}_\infty$ by $\|G\|_\infty$. All transfer matrices in the following are assumed to be proper and real rational unless otherwise stated. A transfer matrix $K$ is said to be a stabilizing controller for $G$ if the feedback system of $G$ and $K$ is internally stable in the standard sense [38]. The vector stacking $v_1, \ldots, v_N$ is denoted by $\mathrm{col}(v_1, \ldots, v_N)$. The block diagonal matrix whose diagonal entries are $A_1, \ldots, A_N$ is denoted by $\mathrm{diag}(A_1, \ldots, A_N)$, or simply by $\mathrm{diag}(A_i)$.

II. Preliminaries

A. Framework of Bilayer Network Synthesis

We consider a network system depicted in Fig. 1. Denoting the components of each signal in the block diagram, e.g., as $u = \mathrm{col}(u_1, \ldots, u_N)$ and $v = \mathrm{col}(v_1, \ldots, v_N)$, we describe the dynamics of the $i$th subsystem by

\[
\begin{bmatrix}
  w_i \\
  z_i \\
  y_i
\end{bmatrix} =
\begin{bmatrix}
  G_{w_i,v_i} & G_{w_i,d_i} & G_{w_i,u_i} \\
  G_{z_i,v_i} & G_{z_i,d_i} & G_{z_i,u_i} \\
  G_{y_i,v_i} & G_{y_i,d_i} & G_{y_i,u_i}
\end{bmatrix}
\begin{bmatrix}
  v_i \\
  d_i \\
  u_i
\end{bmatrix}
\]

where $v_i$ and $w_i$ are the interconnection input and output, $d_i$ and $z_i$ are the disturbance input and evaluation output, and $u_i$ and $y_i$ are the control input and measurement output. The interconnection among the subsystems is given by

\[
\begin{bmatrix}
  v_1 \\
  \vdots \\
  v_N
\end{bmatrix} =
\begin{bmatrix}
  L_{11} & \cdots & L_{1N} \\
  \vdots & \ddots & \vdots \\
  L_{N1} & \cdots & L_{NN}
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  \vdots \\
  u_N
\end{bmatrix},
\]

which is also allowed to be a dynamical system. In a similar way, the decentralized controller is given by

\[
\begin{bmatrix}
  u_1 \\
  \vdots \\
  u_N
\end{bmatrix} =
\begin{bmatrix}
  K_1 & & \\
  & \ddots & \\
  & & K_N
\end{bmatrix}
\begin{bmatrix}
  y_1 \\
  \vdots \\
  y_N
\end{bmatrix}.
\]

For simplicity of discussion, throughout this paper, we assume that all feedback systems are well-posed.

In the following discussion, each subsystem $G_i$ is supposed to be given in advance. The general objective of this paper is
to provide a design framework for the decentralized controller $K$ in conjunction with the interconnection $L$ such that the effect of the disturbance input $d$ to the evaluation output $z$ is appropriately reduced. As a measure to evaluate the entire control system performance, we consider the $\mathcal{H}_\infty$-norm of $T_{zd}: d \rightarrow z$, which denotes the map from $d$ to $z$ in Fig. 1, though some other norm may be applied in a similar fashion. In this paper, we refer to the problem of finding $K$ and $L$ to regulate $\|T_{zd}\|_\infty$ as a network synthesis problem.

Our proposed framework towards systematic network synthesis is divided into two parts, referred to as the upper and lower layers. In the upper layer, we aim at finding an interconnection $L$ such that the stability of the entire network is ensured, where the decentralized controller $K$ is ignored. On the other hand, in the lower layer, we aim at designing individual subcontrollers $K_1$ to $K_N$, with accessibility only to respective subsystem models, such that local control performance, relevant to the map from $d_i$ to $z_i$, is desirably regulated. An abstract of this bilayer network synthesis is illustrated in Fig. 2 where Fig. 2(a) corresponds to the upper layer, and the family of Figs. (2b-1) to (b-N) corresponds to the lower layer. In the following, the diagonally stacked versions of transfer matrices in (1) are denoted, e.g., by

$$ G_{yu} := \text{diag}(G_{y,v_i}), \quad G_{uw} := \text{diag}(G_{w,v_i}). \quad (4) $$

The two blocks in Fig. 2(a) are relevant to the top and middle blocks in Fig. 1. For internal stability analysis, we introduce additional disturbances $\delta_v$ and $\delta_w$, and define

$$ P: (\delta_v, \delta_w) \mapsto (v, w) \quad (5) $$

as the map compatible with Fig. 2(a). In this framework of bilayer network synthesis, a “stability assurance problem” in the upper layer is supposed to be handled by a system coordinator, while “performance regulation problems” in the lower layer are supposed to be handled by individual subsystem operators. Furthermore, the subsystem operators are supposed to be independent (or possibly noncooperative) entities, while the system coordinator is a neutral entity among subsystem operators.

In general, it is not trivial to see if the whole network synthesis problem can be decomposed into such a bilayer problem where multiple entities are premised. For example, individual subcontroller design in the lower layer may destroy the entire network stability assured in the upper layer, possibly due to incautious performance regulation by individual subsystem operators. For such a difficulty, we will show in Section 4-B that a notion of “modularity” of subcontrollers plays a significant role to decouple the control problems in the upper and lower layers. In addition, we will list several key questions for developing such a bilayer network synthesis framework while describing it in more detail.

**B. Key Questions for Bilayer Network Synthesis**

1) Interconnection Design for Stability Assurance: We first describe the stability assurance problem in the upper layer, to be handled by a system coordinator. In the following, $P$ in (5) is considered as a function of $L$ and $G_{uw}$ as

$$ P(L; G_{uw}) := \begin{bmatrix} LG_{uw}(I-LG_{uw})^{-1} & (I-LG_{uw})^{-1}L \\ G_{uw}(I-LG_{uw})^{-1} & (I-G_{uw}L)^{-1}G_{uw}L \end{bmatrix}. $$

Furthermore, we refer to this interconnected system $P$ as a platform. Note that the internal stability of the platform determines that of the network system from which the decentralized controller $K$ is removed. The main objective in the upper layer is to design a stable platform, i.e., to find an interconnection $L$ such that the platform $P$ is internally stable. In particular, to evaluate a stability level of the platform, we consider the magnitude of $P$ in a suitable measure, e.g., the $\mathcal{H}_\infty$-norm. Based on an objective function $J$, the system coordinator aims at finding a desirable interconnection $L^*$ such that

$$ J[P(L^*; G_{uw})] \leq J^*, \quad L^* \in \mathcal{L} \quad (6) $$

where $J^*$ denotes an admissible bound, and $\mathcal{L}$ denotes the set of all possible $L$, satisfying a necessary structural constraint, such that $P$ is internally stable. The key question here to be addressed is:

**Q1:** What is a reasonable choice of the objective function $J$ for evaluating the platform stability level?

In this paper, we derive an answer to Q1 as a by-product from the analysis of retrofit control described in Section IV below. The notion of retrofit control is originally proposed in [20], the precise description of which will be given as Definition 2 in this paper. We will see that the retrofit control is closely relevant to modular subcontroller design problems in the lower layer. In particular, the objective function $J$ will be determined as the $\mathcal{H}_\infty$-norm of the platform with a frequency weighting. Such a specific choice of $J$ is made valid from the analysis in Section IV.

It should be noted that the platform is defined as an input-to-output map with respect to the “interconnection signals”
among the subsystems, not the disturbance input and evaluation output inside them. Such identity of disturbance and evaluation ports has good compatibility with, e.g., dissipativity-based control approaches [33]–[36]. Throughout this paper, we conduct analysis based on the premise that, once a design criterion is determined, the stability assurance problem itself can be handled by an existing approach to the control of multi-agent or network systems, as in the following remarks.

Remark 1 The stability assurance problem in the upper layer can be framed as a cooperative control problem for multi-agent systems, which is well studied in the literature [39]–[42]. In that context, a consensus-based approach or a passivity-based approach is often adopted to find a desirable network structure among agents (subsystems). From this viewpoint, we can handle the stability assurance problem itself by applying such an existing cooperative control method, where a system coordinator may be able to use some a priori information about agents, such as homogeneity and passivity, depending on intended application.

Remark 2 The interconnection $L$ is not limited to just a network structure, but is allowed to involve a primary stabilizing controller for a physical network system. To explain this, let us consider the case where $L$ is composed of two parts, denoted by $L_0$ and $K_0$. In this expression, $L_0$ is supposed to represent a “physical” network structure among the subsystems, which may not be adjustable by a system coordinator. The network system composed of $G_1$ to $G_N$ interconnected by $L_0$ can be regarded as a target system to control. For example, in power systems control, $G_i$ represents a generator (or an aggregate of generators) managed by the $i$th operator, while $L_0$ represents a transmission network among them. On the other hand, $K_0$ can be regarded as a primary stabilizing controller for such a physical network system, supposed to be adjustable. For example, a broadcast-type PI controller, which feedbacks the average of all generators’ frequencies, is often implemented in power systems control; see Section [ ] for details. In this setting, the stability assurance problem of finding $L$ can be framed as a standard problem of finding a stabilizing controller $K_0$ for the network system interconnected by $L_0$.

2) Subcontroller Design for Performance Regulation: We next describe performance regulation problems in the lower layer, each of which is handled by a corresponding subsystem operator. Let $K_i$ denote the set of all subcontrollers $K_i$ such that the “isolated” control system $M_i$ in Fig. 2(b-i) is internally stable, for which we introduce the map

$$ M_i : (v_i, d_i) \mapsto (w_i, z_i). $$

(7)

With this notation, a naive subcontroller design problem for performance regulation can be expressed as an individual optimization problem of finding a desirable $K_i^*$ such that

$$ J_i[M_i(K_i^*)] \leq J_i^*, \quad K_i^* \in K_i $$

where $J_i$ denotes an objective function and $J_i^*$ denotes its admissible bound.

However, such individual subcontroller design may be incautious to destroy the stability of the entire network system because the resultant subcontroller $K_i^*$ may significantly change the original dynamical property of $G_{w_i/v_i}$ involved in [5]. This can be explained in detail as follows. Let

$$ M_{w_i/v_i} : v_i \mapsto w_i $$

(8)

denote the map compatible with Fig. 2(b-i), i.e.,

$$ M_{w_i/v_i}(K_i) := G_{w_i/v_i} + G_{w_i/v_i} K_i (I - G_{y_i/u_i} K_i)^{-1} G_{y_i/v_i}. $$

Clearly, the resultant map $M_{w_i/v_i}$ is generally different from the original interconnection transfer matrix $G_{w_i/v_i}$. This implies that the resultant platform $P(L^*; M_{wv}(K^*))$, the stability of which is equal to that of the entire network system, is not generally stable, where the collection of the resultant interconnection transfer matrices is denoted by

$$ M_{wv}(K^*) := \text{diag} (M_{w_i/v_i}(K_i^*)). $$

This can be understood as a basic principle of the fact that incautious decentralized control may destabilize the entire network system. This observation motivates us to introduce the following notion of modularity in subcontroller design.

Definition 1 Consider the control system in Fig. 2(b-i). A stabilizing controller $K_i \in K_i$ is said to be modular if

$$ M_{w_i/v_i}(K_i) = G_{w_i/v_i}. $$

(9)

The set of all such modular controllers is denoted by $M_i$. The modularity of subcontrollers is defined as a property such that the interconnection transfer matrix is kept invariant.

Fig. 2. Bilayer network synthesis. (a): A system coordinator, depicted by the entity with “C”, is supposed to handle a stability assurance problem. (b): A family of subsystem operators, each of which is depicted by the entity with a label, is supposed to handle respective performance regulation problems.
regardless of the subcontroller selection. In particular, such a modular subcontroller must satisfy
\[ G_{w,v_i}K_i(I - G_{y,v_i}K_i)^{-1}G_{y,v_i} = 0. \] (10)
Because the platform \( P \) in (5) is kept invariant for all modular subcontrollers, the stability level of the platform attained in the upper layer is also kept invariant, i.e.,
\[ J[P(L^*; M_{we}(K))] \leq J^*, \quad \forall(K_1, \ldots, K_N) \in \prod_{i=1}^N \mathcal{M}_i \]
for the interconnection \( L^* \) in (6). This implies that we can perform modular design of each subcontroller in the sense that individual performance regulation does not affect the entire system stability. This conversely implies that, in principle, any modular subcontroller cannot stabilize the entire network system. Therefore, the system stability must be assured by the system coordinator in the upper layer. Note that a modular subcontroller \( K_i \) has a remaining degree of freedom to regulate the other control system maps, e.g., \( M_{z,di} : d_i \mapsto z_i \), which is relevant to local disturbance attenuation.

In summary, each subcontroller design problem for individual performance regulation is expressed as a design problem of finding a desirable \( K_i^* \) such that
\[ J_i[M_i(K_i^*)] \leq J_i^*, \quad K_i^* \in \mathcal{M}_i \] (11)
where the modularity is imposed as an additional constraint. In this setting, the key questions to be addressed are:

Q2: What is a reasonable choice of the objective function \( J_i \) for the design of such a modular subcontroller?
Q3: Is such a constrained design problem tractable?

An answer to Q2 will be given in Section IV through the analysis of retrofit control. Furthermore, giving an answer to Q3, we will clarify a reasonable class of modular controllers that can be systematically found by standard controller design techniques, e.g., the \( \mu \)-synthesis. This clarification is based on the fact that the set of all modular controllers, i.e., \( \mathcal{M}_i \) in Definition 1 can be characterized in the context of add-on type localized control, called retrofit control; see Definition 2 for details. This equivalence will be proven by a constrained version of the Youla parameterization in Theorem 1 below.

Last but not least, we should also discuss the resultant control performance of the entire control system. Based on the bounds \( J_i^* \rightarrow J_N^* \) in conjunction with \( J^* \), we aim at bounding the norm of the entire map \( T_{zd} : d \mapsto z \) in such a way that
\[ \|T_{zd}\|_\infty \leq \gamma(J_1^*, \ldots, J_N^*; J^*), \] (12)
where \( \gamma : \mathbb{R}^{N+1} \rightarrow \mathbb{R} \) is a monotonically increasing function with respect to each argument. A natural question here is:

Q4: Does there exist such a reasonable bounding function \( \gamma \)? The existence of such a monotonically increasing function \( \gamma \) enables to justify the validity of the bilayer network synthesis framework described above. In particular, it ensures that individual performance improvement by multiple independent entities contributes to improving the entire control performance in the sense of, at least, the upper bound. A particular form of \( \gamma \) will be found in Section IV.

III. CHARACTERIZATION OF MODULAR CONTROLLERS BASED ON RETROFIT CONTROL

A. Definition of Retrofit Control

In this section, we conduct detailed analysis of the modularity of subcontrollers in Definition 1 based on retrofit control, which is closely relevant to performance regulation problems in the lower layer. We suppose that the stability of the platform has been assured in the upper layer. The following analysis is performed from the standpoint of “one” subsystem operator, for whom the information of the other subsystem operators as well as that of the system coordinator are supposed to be concealed. To explain this more specifically, we consider an example of network synthesis with two subsystems illustrated in Fig. 3 where Fig. 3(a) corresponds to an overview, and Figs. 3(b-1) and (b-2) correspond to the local views from the standpoints of the first and second subsystem operators, respectively. As depicted here, each subsystem operator aims at designing his own subcontroller while regarding the rest of the network system managed by the concealed entities as his environment, the system model of which is assumed not to be available. Such a control problem is referred to as a retrofit control problem, for which some particular controller design methods have been reported in [20]–[22].

The abstraction of retrofit control is depicted in Fig. 4, where the subsystem and subcontroller of a subsystem operator are denoted by \( G \) and \( K \), respectively, and his environment is denoted by \( G \). Throughout this section, we regard Fig. 4 as the control system from the standpoint of the 4th subsystem operator, while dropping the subscript “4” for simplicity of notation. For the subsequent discussion, we use symbols denoting the submatrices of \( G \), for example, as
\[ G_{(z,y),d}(d,u) := \begin{bmatrix} G_{zd} & G_{zu} \\ G_{yd} & G_{yu} \end{bmatrix}, \quad G_{(z,y),v} := \begin{bmatrix} G_{zv} \\ G_{yv} \end{bmatrix}, \]
\[ G_{w,d}(d,u) := \begin{bmatrix} G_{wd} & G_{wv} \end{bmatrix}. \]
Then, we introduce the transfer matrix
\[ G_{pre} : (d, u) \mapsto (z, y) \]
responding to the feedback system of \( G \) and \( G \) in Fig. 4 from which the block of \( K \) is removed, i.e.,
\[ G_{pre} := G_{(z,y),d}(d,u) + G_{(z,y),v}G(I - G_{w,v}G)^{-1}G_{w,d}(d,u). \] (13)
We refer to this \( G_{pre} \) as a preexisting system before implementing retrofit control. With this notation, we define the following notion of retrofit controllers.

Definition 2 For the preexisting system \( G_{pre} \), define the set of all admissible environments as
\[ \mathcal{G} := \{ G : G_{pre} \text{ is internally stable}\}. \]
An output feedback controller
\[ u = Ky \]
is said to be a retrofit controller if the resultant control system in Fig. 4 is internally stable for any environment \( G \in \mathcal{G} \).

The retrofit controller is defined as an add-on type localized controller such that the stability of the resultant control system
Fig. 3. Example of bilayer network synthesis with two subsystems. (a): Overview. (b-1): View from the standpoint of the first subsystem operator. (b-2): View from the standpoint of the second subsystem operator.

Fig. 4. Retrofit control from the viewpoint of each subsystem operator. The block of $K$ represents a retrofit controller to be designed, and the blocks of $G$ and $G$ represent a subsystem of interest and its unknown environment.

Fig. 5. Block diagram obtained by the Youla parameterization.

can be ensured for any possible variation of environments such that the preexisting system is stable. The stability of the preexisting system is premised based on the fact that platform stability is assured in the upper layer and all other subsystem operators also adopt retrofit controllers in the lower layer. Within the set of all such retrofit controllers, each subsystem operator aims at selecting a desirable controller that can improve the resultant control performance for local disturbance attenuation. Though the environment in this formulation may be regarded as model uncertainty in robust control, it is typically assumed to be “norm-bounded” in a standard robust control setting. Clearly, we do not impose any explicit norm bound on the environment. In this sense, the retrofit controller is a nonstandard robust controller.

**B. Parameterization of All Retrofit Controllers**

In this subsection, as one of the main contributions of this paper, we give a parameterization of “all” retrofit controllers, which involves both of two classes of retrofit controllers reported in [20] as special cases. To avoid unnecessary complication of a controller parameterization based on the Youla parameterization [23], we make the following assumption.

**Assumption 1** The subsystem $G$ is stable, i.e., $G \in \mathcal{RH}_\infty$.

As shown in the following theorem, all retrofit controllers can be parameterized as a constrained version of the Youla parameterization.

**Theorem 1** Let Assumption 1 hold. Consider the Youla parameterization of $K$ given by

$$K = (I + QG_yu)^{-1}Q, \quad Q \in \mathcal{RH}_\infty$$

where $Q$ denotes its Youla parameter. Then, $K$ is a retrofit controller if and only if

$$G_{wu}G_{yv} = 0.$$

A complete proof is given in Appendix A. The essence of the proof, giving an interpretation of the constraint (15), is explained as follows. Let us drop all the terms relevant to $d$ and $z$ in Fig. 4 because they are not essential to prove the internal stability of the resultant control system. Denoting the Youla parameterization of $\overline{G}$ by

$$\overline{G} = (I + QG_{wu})^{-1}Q, \quad Q \in \mathcal{RH}_\infty$$

we have the closed-loop system depicted in Fig. 5 which is composed of the feedback of $\overline{Q}$ and $G_{wu}QG_{yv}$. Note that $\overline{Q}$ can be taken as an arbitrary element in $\mathcal{RH}_\infty$. Therefore, (15) is shown to be necessary and sufficient for the internal stability.

A remarkable fact here is that (15) is equivalent to (10), because the Youla parameter of $K$ is written as

$$Q = K(I - G_yuK)^{-1}.$$
Therefore, we can find the following equivalence between a modular controller in Definition 1 and a retrofit controller in Definition 2.

**Theorem 2** Let Assumption 1 hold. Then, \( K \) is a retrofit controller if and only if it is modular.

Theorem 2 gives a characterization of the retrofit control in terms of the modularity in bilayer network synthesis described in Section II-B. A practical insight gained from this theorem is the fact that keeping the interconnection transfer matrix invariant is the unique way to accommodate a decentralized controller to arbitrary variation of environments such that the preexisting system is stable.

Based on the constrained Youla parameterization in Theorem 1, we next clarify a structure inside the entire control system map. Let \( Tzd : d \mapsto z \) denote the entire map compatible with Fig. 4. Then, for any \( K \) not necessarily being a retrofit controller, we have

\[
Tzd = F_u [ F_1 (G, K), G ]
\]

\[
= F_u \left( \left[ \begin{array}{c} 0 & G_{zu}^* G_{wu} \ G_{zu}^* G_{yv} \\ G_{vd} & G_{zu}^* G_{yw} \end{array} \right], Q \right), Q \right)
\]

\[
= Mzd(Q) + Mzu(Q) \ [ \ I - \ [G_{wu} Q G_{yv}]^{-1} Q Mwd(Q) \] \]

where \( F_u \) and \( F_1 \) denote the lower and upper linear fractional transformations, respectively, and

\[
\begin{align*}
Mzd(Q) & := G_{zd} + G_{zu} Q G_{yd}, \\
Mzu(Q) & := G_{zu} + G_{zu} Q G_{yv}, \\
Mwd(Q) & := G_{wu} + G_{wu} Q G_{yd}.
\end{align*}
\]

These transfer matrices correspond to the lower three blocks of \( M \) in (7), i.e.,

\[
M(Q) = \left[ \begin{array}{c} Mwu(Q) \ Mwd(Q) \\ Mzu(Q) \ Mzd(Q) \end{array} \right].
\]

Note that \( Mwu = G_{wu} \) holds for all retrofit controllers because of (13). Furthermore, the feedback term represented by \( \ast \) is reduced to the identity matrix. This proves the following fact.

**Theorem 3** Let Assumption 1 hold. For a retrofit controller \( K \) in Theorem 1, it follows that

\[
Tzd(Q) = Mzd(Q) + Mzu(Q) Q Mwd(Q),
\]

where \( Mzd, Mzu, \) and \( Mwd \) are defined as in (17), and \( Q \) is the Youla parameter of \( G \) such that (16) holds.

Theorem 3 shows that \( Tzd \) is affine with respect to \( Q \) in retrofit control. Because \( Q \) is assumed not to be available, what we can only do for performance regulation is to find a desirable Youla parameter \( Q \) subject to the constraint (13) such that the magnitude of \( Mzd, Mzu, \) and \( Mwd \) is jointly reduced. However, the constraint on \( Q \) cannot directly be handled by a standard controller design technique. Such a \( Q \) may be written, based on Fact 6.4.43 in [43], as

\[
Q = Q_0 - G_{wu}^* G_{wu} Q_0 G_{yv} G_{yv}^* Q_0 \in \mathcal{RH}_\infty
\]

where \( G_{wu}^* \) and \( G_{yv}^* \) denote the right-inverse and left-inverse of \( G_{wu} \) and \( G_{yv} \), respectively. However, \( G_{wu}^* \) and \( G_{yv}^* \) here are not always found, especially over the ring of \( \mathcal{RH}_\infty \). In this sense, finding a retrofit controller in this most general setting is not very tractable and straightforward.

### C. Tractable Class of Retrofit Controllers

In this subsection, we find out a particular class of retrofit or modular controllers that can be designed easily by a standard controller design technique. We introduce the following class of retrofit controllers based on the characterization in Theorem 1.

**Definition 3** Let Assumption 1 hold. Consider the Youla parameterization of \( K \) given by (14). Then, \( K \) is said to be an output-rectifying retrofit controller if

\[
QG_{yv} = 0. \tag{19}
\]

Obviously, (19) is sufficient for the constraint (15). In the following discussion, we will see the reason why it is named with the term “output-rectifying” through deriving an explicit representation of all such retrofit controllers, which clarifies a particular structure inside them. We assume the following situation throughout this subsection.

**Assumption 2** The interconnection signal \( v \) is measurable in addition to the measurement output \( y \).

From a symbolic viewpoint, Assumption 2 corresponds to the situation where every symbol \( y \) in the above discussion is to be replaced with the augmented measurement output \((y, v)\). Based on this premise, the transfer matrices in (1) relevant to \( y \) are also augmented. For example, \( G_{yv} \) and \( G_{yu} \) are to be replaced with

\[
G_{(y,v)^v} := \left[ \begin{array}{c} G_{yv} \\ I \end{array} \right], \quad G_{(y,v)^u} := \left[ \begin{array}{c} G_{yu} \\ 0 \end{array} \right]. \tag{20}
\]

Furthermore, the controller \( K \) is also augmented as

\[
u = K \left[ \begin{array}{c} y \\ v \end{array} \right]. \tag{21}
\]

Then, the Youla parameterization of this \( K \) is given by

\[
K = (I + QG_{(y,v)^u})^{-1} Q, \quad Q \in \mathcal{RH}_\infty, \tag{22}
\]

and its constraint corresponding to (19) is written as

\[
QG_{(y,v)^v} = 0. \tag{23}
\]

The heart of Assumption 2 is to enable the following factorization of \( Q \) such that (23) holds over the ring of \( \mathcal{RH}_\infty \).

**Lemma 1** Let Assumptions 1 and 2 hold. Then, the Youla parameter \( Q \in \mathcal{RH}_\infty \) satisfies (23) if and only if there exists \( Q \in \mathcal{RH}_\infty \) such that \( Q = QR \) where

\[
R := \left[ \begin{array}{c} I \\ -G_{yv} \end{array} \right]. \tag{24}
\]
Proof: The “if” part is easy to prove because \( R \in \mathcal{RH}_\infty \) and \( RG_{(y,v)v} = 0 \). The “only if” part is proven as follows. We apply the calculus over the ring of \( \mathcal{RH}_\infty \). Consider

\[
U := \begin{bmatrix} I & -G_{yu} \\ 0 & I \end{bmatrix}.
\]

This \( U \) is unimodular, i.e., it is invertible in \( \mathcal{RH}_\infty \). Thus, for any \( Q \in \mathcal{RH}_\infty \), there exists \( \hat{Q} \in \mathcal{RH}_\infty \) such that \( Q = \hat{Q}U \). Substituting this into (23), we have

\[
\begin{bmatrix} \hat{Q}_1 & \hat{Q}_2 \\ \hat{Q} \end{bmatrix} \begin{bmatrix} I & -G_{yu} \\ 0 & I \end{bmatrix} \begin{bmatrix} Gyv \\ I \end{bmatrix} = 0,
\]

which is equivalent to \( \hat{Q}_2 = 0 \). Note that the upper half of \( U \) is equal to \( R \). Hence, for any \( Q \in \mathcal{RH}_\infty \), there exists \( \hat{Q}_1 \in \mathcal{RH}_\infty \) such that \( Q = \hat{Q}_1R \). \( \square \)

We notice that \( R \) in (24) corresponds to a basis of the left kernel of \( G_{(y,v)v} \) in \( \mathcal{RH}_\infty \). Thus, \( Q \) can be regarded as a new component in the basis of \( R \). Using the factorization in Lemma 1, we can rewrite (22) and (23) as

\[
K = KR, \quad \hat{K} = (I + QG_{yu})^{-1}Q, \quad \hat{Q} \in \mathcal{RH}_\infty, \quad (25)
\]

where we have used the fact that

\[
RG_{(y,v)v} = G_{yu}. \quad (26)
\]

From (25), we find that \( \hat{K} \) is a stabilizing controller for \( G_{yu} \), and \( \hat{Q} \) is its Youla parameter. In this paper, we refer to

\[
R : (y, v) \mapsto \hat{y}
\]

as an output rectifier, the name of which is based on the fact that the measurement output \( (y, v) \) is rectified in such a way that \( \hat{y} = y - G_{yu}y \). This output rectifier, corresponding to the basis \( R \) in (24), can be regarded as a dynamical simulator to reduce the interference of \( v \) with the output signal \( y \), which forwards the rectified output \( \hat{y} \) to the stabilizing controller \( \hat{K} \). This discussion leads to the following “explicit” parameterization of the output-rectifying retrofit control with the interconnection signal measurement.

**Proposition 1** Let Assumptions 1 and 2 hold. Then, \( \hat{K} \) is an output-rectifying retrofit controller if and only if

\[
K = \hat{K}R \quad (27)
\]

where \( \hat{K} \) is a stabilizing controller for \( G_{yu} \), i.e., all such retrofit controllers have the structure of Fig. 6.

Next, we analyze the entire control system map from the disturbance input to the evaluation output when we apply the output-rectifying retrofit control in Proposition 1. This analysis is performed based on Theorem 3. Under Assumption 2 the transfer matrices in (17) are augmented as

\[
\begin{align*}
M_{zd}(Q) & := G_{zd} + G_{zu}QG_{(y,v)d}, \\
M_{zv}(Q) & := G_{zv} + G_{zu}QG_{(y,v)v}, \\
M_{wd}(Q) & := G_{wd} + G_{wu}QG_{(y,v)d}.
\end{align*}
\]

The factorization of \( Q \) in Lemma 1 enables the reduction of \( RG_{(y,v)v} = 0 \) and \( RG_{(y,v)d} = G_{yd} \), which means that \( M_{zd} \) is equal to \( G_{zd} \), and \( M_{zv} \) and \( M_{wd} \) are, respectively, equal to

\[
\begin{align*}
\hat{M}_{zd}(\hat{K}) & := G_{zd} + G_{zu}\hat{K}(I - G_{yu}\hat{K})^{-1}G_{yd}, \\
\hat{M}_{wd}(\hat{K}) & := G_{wd} + G_{wu}\hat{K}(I - G_{yu}\hat{K})^{-1}G_{yd},
\end{align*}
\]

where we have plugged-in the Youla parameter

\[
\hat{Q} = \hat{K}(I - G_{yu}\hat{K})^{-1}.
\]

With this notation, we have the following result.

**Proposition 2** Let Assumptions 1 and 2 hold. For an output-rectifying retrofit controller \( \hat{K} \) in Proposition 1 it follows that

\[
T_{zd}(\hat{K}) = \hat{M}_{zd}(\hat{K}) + G_{zd}(I - \overline{G}G_{wu})^{-1}\overline{G}\hat{M}_{wd}(\hat{K}), \quad (29)
\]

where \( \hat{K} \) is a stabilizing controller for \( G_{yu} \), i.e., the resultant control system is depicted as the block diagram in Fig. 7.

**Proposition 2** shows that the block diagram in Fig. 4 can be equivalently transformed into that in Fig. 7 when \( \hat{K} \) is an output-rectifying retrofit controller with the interconnection signal measurement. Note that Fig. 7 has a cascade structure, where the upstream feedback system is composed of the blocks of \( G \) and \( \hat{K} \), while the downstream feedback system is composed of \( G \) and \( \overline{G} \). As long as focusing on the upstream feedback system, we can design \( \hat{K} \) with a standard controller design technique for \( G \) that is “isolated” from \( \overline{G} \). More specifically, the output signals \( \hat{z} \) and \( \hat{\omega} \) from the upstream feedback system can be directly regulated by a suitable choice of \( \hat{K} \). On the other hand, \( z \) is shown to be the sum of \( \hat{z} \) and \( \hat{\omega} \), the latter of which depends on \( \overline{G} \) as

\[
\hat{z} = G_{zd}(I - \overline{G}G_{wu})^{-1}\overline{G}\hat{\omega}.
\]

In view of this, we see that a design criterion of \( \hat{K} \) should, in principle, be specified with regard to not only \( \hat{z} \) but also \( \hat{\omega} \). In Section IV we will discuss what design criterion should be in the context of the bilayer network synthesis problem.
Instead of assuming the measurability of the interconnection signal \( v \) as in Assumption 2, it is actually possible to develop a state-feedback-type retrofit controller without the interconnection signal measurement, provided that the internal state of \( G \) is assumed to be measurable. The interested reader is referred to Appendix B for details. Furthermore, in Definition 3 we adopt the “right-side” sufficient condition (19) for the constraint (15). Instead of this, it is possible to develop a dual version of tractable retrofit controllers by adopting the “left-side” sufficient condition \( G_{yu}Q = 0 \), which can be called input-rectifying retrofit controllers. The details of this version will be reported as a separate paper in future.

**Remark 3** The output-rectifying retrofit controller shown in Proposition 1 is essentially identical to that derived in our previous work \([20]\). The novelty of Proposition 1 as compared to the existing result, is to clarify the fact that “all” such output-rectifying retrofit controllers can be expressed as the unique form of (27), provided that Assumption 1 holds. This uniqueness is proven by virtue of the constrained Youla parameterization in Theorem 1. It should be noted that Assumption 1 is not essential to prove the “if” part of Proposition 1 as shown in Theorem 2.1 of \([20]\), but is used to prove the “only if” part. The cascade structure shown in Proposition 2 is itself not new, but its derivation provides a frequency-domain analog to the state-space analysis in the previous work, which can also be conducted without Assumption 1.

**IV. BILAYER NETWORK SYNTHESIS**

**A. Decomposition of Bilayer Network Synthesis Problem**

Let us return our attention to the bilayer network synthesis problem in Section II. Comparing Fig. 1 with Fig. 4, we find that the bilayer network synthesis problem can be regarded as a “macroscopic” retrofit control problem where the interconnection \( L \) is the environment for the block-diagonally structured system \( G \). From this viewpoint, we state the following fact.

**Theorem 4** Let Assumption 1 hold for each of all subsystems. Consider a bilayer network synthesis problem in Section II. Then, the entire network system in Fig. 1 is internally stable if and only if each of all subcontrollers \( K_1 \) to \( K_N \) is modular.

**Proof:** Regard \( \hat{G} \) as \( L \), and \( G_{yu} \) as \( P \) in Definition 2. Then, it is sufficient to prove that \( \hat{K} \) is a retrofit controller if and only if each of all \( K_1 \) to \( K_N \) is modular. The “if” part is clear because of the equivalence in Theorem 2. The “only if” part is proven by the fact that the Youla parameter of \( \hat{K} \), i.e.,

\[
Q = K(I - G_{yu}K)^{-1}
\]

is block-diagonal because both \( K \) and \( G_{yu} \) are supposed to be block-diagonal. This implies that each of all subcontrollers is modular if the block-diagonal \( K \) is a retrofit controller.

Next, we confine our attention to output-rectifying retrofit control, based on the premise that Assumption 2 holds for each of all subsystem operators. In the following, Assumption 1 is not necessary because it is not essential to prove the “if” part of Proposition 1 as well as Proposition 2, as mentioned in Remark 3. Then, we analyze the structure of the entire control system map when simultaneously implementing multiple output-rectifying retrofit controllers. The following claim is proven by replacing \( \hat{G} \) with \( L \), and \( G \) with \( G \) in Proposition 2.

**Proposition 3** Let Assumption 2 hold for each of all subsystem operators. Suppose that every \( K_i \) is an output-rectifying retrofit controller in Proposition 1. Then, the entire map \( T_{zd} : d \mapsto z \) compatible with Fig. 1 is structured as

\[
T_{zd}(\hat{K}_1, \ldots, \hat{K}_N; L) = \text{diag}(M_{z,di}(\hat{K}_i)) + G_{zv}(I - LG_{wu})^{-1}L \text{diag}(M_{w,di}(\hat{K}_i)).
\]

Clearly, the cascade structure shown in Proposition 2 is also proven for the entire map in the bilayer network synthesis. A remarkable fact here is that the terms relevant to \( M_{z,di} \) and \( M_{w,di} \) in the right-hand side of (30) are block-diagonally structured, and those are decoupled from the term relevant to \( L \). This means that the problem of finding \( L \) and each problem of finding \( \hat{K}_i \) can be decoupled with respect to each entity. This fact enables to derive the following bound of the entire control system performance.

**Proposition 4** Let Assumption 2 hold for each of all subsystem operators. Suppose that every \( K_i \) is an output-rectifying retrofit controller in Proposition 1. If

\[
\|G_{zv}(I - LG_{wu})^{-1}L\|_\infty \leq \delta
\]

for the interconnection design in the upper layer and

\[
\|M_{z,di}(\hat{K}_i)\|_\infty \leq \alpha_i, \quad \|M_{w,di}(\hat{K}_i)\|_\infty \leq \beta_i
\]

for each subcontroller design in the lower layer, then

\[
\|T_{zd}\|_\infty \leq \max_i \alpha_i + \delta \max_i \beta_i
\]

for the entire map \( T_{zd} : d \mapsto z \) compatible with Fig. 1.

**Proof:** Applying the triangular inequality to (30) and using the submultiplicativity of the \( \mathcal{H}_\infty \)-norm, we can easily obtain the bound of (33) using (31) and (32).

Proposition 4 is an encompassing statement to answer the key questions in Section II. See Section IV-B below for a summary. Finally, as a notable property of the output-rectifying retrofit control, we also state the following fact relevant to “self-responsibility” for local disturbance attenuation.

**Proposition 5** Let Assumption 2 hold for the \( i \)th subsystem operator. For any output-rectifying retrofit controller \( K_i \) in Proposition 1 it follows that

\[
u_i = 0, \quad \forall d_j \in D_j, \quad j \neq i,
\]

where \( D_j \) denotes the set of all possible disturbance inputs.

**Proof:** First, we suppose that some \( K_j \), which may not be an output-rectifying retrofit controller, is implemented to \( G_j \) for \( j \neq i \). In this case, without loss of generality, the local
feedback system composed of \( G_j \) and \( K_i \) can be regarded as a new subsystem \( G_j \) from the viewpoint of the \( i \)th subsystem operator. Thus, it is sufficient to analyze the situation where only an output-rectifying retrofit controller \( K_i \) is implemented and all other controllers \( K_j \) are zero.

Denote the \((i, j)\)-entry of \( T_{zd} \) in (30) by \( T_{z, dj} \). If \( K_j = 0 \), which leads to \( \dot{M}_{w, d} = G_{w, d} \), then
\[
T_{z, dj} = G_{z, v} c_{v, i}^T (I - LG_{w, v})^{-1} L c_{w, i} G_{d, d} j,
\]
where \( c_{v, i} \) and \( c_{w, i} \) denote the port selection matrices corresponding to \( v_i \) and \( w_i \), such that \( c_{v, i}^T v = v_i \) and \( c_{w, i}^T w = w_i \). Replacing \( z_j \) with \((y_j, v_j)\) symbolically, we find that
\[
u_i = \hat{K}_i R_i G(y_i, v_i) c_{v, i}^T (I - LG_{w, v})^{-1} L c_{w, i} G_{d, d} j,
\]
for which \( R_i G(y_i, v_i) = 0 \) holds. This proves the claim. \( \square \)

Proposition 5 shows that an output-rectifying retrofit controller implemented in the \( i \)th subsystem is “insensitive” to the disturbances injected to any other subsystems. This conversely means that the \( i \)th output-rectifying retrofit controller works only for its own disturbance \( d_i \). Therefore, in a situation where all subsystem operators adopt output-rectifying retrofit control, local disturbances occurring in individual subsystems are to be handled on their own responsibility. In this sense, the output-rectifying retrofit controller is self-responsible.

B. Answers to Key Questions for Bilayer Network Synthesis

In this subsection, we briefly summarize answers to the key questions listed in Section III developing a framework of bilayer network synthesis based on retrofit control. First, let us consider the stability assurance problem in the upper layer, which is relevant to Q1. As shown in Proposition 4, the objective function
\[
J[(P; G_{w, v})] = ||W_1 P(L; G_{w, v}) W_2||_\infty
\]
with the output and input weights defined as
\[
W_1 := \begin{bmatrix} G_{zw} & 0 \end{bmatrix}, \quad W_2 := \begin{bmatrix} 0 & I \end{bmatrix}
\]
is found to be a rational choice for evaluating the platform stability level, giving an answer to Q1. Clearly, \( J \) is always bounded if \( P \) is internally stable.

Next, let us consider the performance regulation problems in the lower layer, which are relevant to Q2 and Q3. We see from Proposition 4 that, if either \( ||M_{z, d_i}||_\infty \) or \( ||M_{w, d_i}||_\infty \) is constrained by a given bound, then the resultant upper bound of \( ||T_{zd}||_\infty \) is found to be monotone increasing with respect to each objective value. In particular, if the design criterion of each subsystem operator is determined as
\[
J_i[M_i(\hat{K}_i)] = ||M_{z, d_i}(\hat{K}_i)||_\infty
\]
subject to \( ||M_{w, d_i}(\hat{K}_i)||_\infty \leq \beta_i \), (35)
where \( \beta_i \) denotes a given bound, then it follows that
\[
||T_{zd}||_\infty \leq \max_i J_i^* + J^* \max_i \beta_i
\]
where \( J^* \) and \( J_i^* \) denote the resultant bounds of the objective values \( J \) and \( J_i \) in (34) and (35), respectively. Because \( \hat{K}_i \) is a stabilizing controller just for \( G_{y_i, v_i} \), such a controller design problem can be handled by existing approaches, e.g., the \( \mu \)-synthesis. These give possible answers to Q2 and Q3.

Finally, if all subsystem operators adopt output-rectifying retrofit control based on the design criterion in (35), then
\[
\gamma(J_1^*, \ldots, J_N^*; J^*) = \max_i J_i^* + J^* \max_i \beta_i
\]
is found to be a bounding function with respect to the resultant control system performance. This gives an answer to Q4.

V. ILLUSTRATIVE EXAMPLE

A. Setup of Ground Fault and Frequency Control

1) System Description: As an illustrative example of the proposed bilayer network synthesis, we simulate frequency regulation of the IEEE 68-bus power system model [44], whose network structure is shown in Fig. 8 composed of 16 generators and 35 loads. In the following, a bus connected to a generator is called a generator bus, a bus connected to a load is called a load bus, and a bus connected to none of them is called a non-unit bus.

With slight abuse of language, we refer to the dynamics of the \( i \)th generator as the dynamics of the \( i \)th generator bus. By applying the linearization around a suitable set-point, the dynamics of the \( i \)th generator bus can be represented as
\[
\begin{align*}
\dot{x}_i &= A_i x_i + L_i V_i + B_i u_i \\
I_i &= \Gamma_i x_i + H_i V_i \\
\omega_i &= C_i x_i 
\end{align*}
\]
where \( V_i \) is a two-dimensional real-valued vector composed of the real and imaginary parts of the \( i \)th bus voltage phaser, \( I_i \) is a two-dimensional real-valued vector composed of the real and imaginary parts of the \( i \)th bus outflowing current phaser, \( x_i \) is a generator state representing three-dimensional electro-mechanical swing dynamics with four-dimensional excitation system dynamics, \( u_i \) is a scalar input to the automatic voltage regulator (AVR) involved in the excitation system, and \( \omega_i \) is the frequency deviation, corresponding to one of the state variables of the electro-mechanical swing dynamics. Adopting a constant impedance model, we model
the input-output behavior of the \( i \)th load bus as the static system \( I_i = -Z_i^{-1}V_i \) where \( Z_i \) denotes a two-dimensional non-singular matrix representing the load impedance. The \( i \)th non-unit bus can be represented as a component such that \( I_i = 0 \). Those buses are interconnected such that

\[
I_i = \sum_{j=1}^{68} Y_{ij} V_i
\]

where \( Y_{ij} \) denotes a two-dimensional real-valued admittance matrix associated with the transmission network. The details of model parameters can be found in [45].

2) Ground Fault Simulation: As a disturbance to the power system, we consider a ground fault occurring at each bus. A ground fault at the \( i \)th bus, which can be any kind of buses, is modeled as a short-time alteration of the system dynamics such that \( V_i = 0 \), which is imposed as a physical constraint that causes a non-negligible amount of generator state deviation during the fault. In this simulation, we suppose that each fault duration is 0.1 second. After removing the ground fault, the overall system again obeys the original dynamics before the fault, the initial state of which is determined as \( V_i = 0 \), that by the original excitation system, i.e., each \( u_i \) in (38) is zero. From these box plots, we see that frequency deviation due to generator bus faults is generally larger than that due to load and non-unit bus faults. This is a natural consequence because the generators are connected directly to the generator buses, while not directly to the other buses.

To reduce frequency deviation due to ground faults, we consider a bilayer control input given as

\[
u_i = u_i^{\text{local}} + u_i^{\text{global}}
\]

where \( u_i^{\text{local}} \) is produced by a local controller to be attached to each generator, and \( u_i^{\text{global}} \), common to all generators, is produced by a global controller. In particular, each local controller is supposed to be an output-rectifying retrofit controller that feedbacks the corresponding generator state and bus voltage, i.e., \( x_i \) and \( V_i \), while the global controller is supposed to be a broadcast-type PI controller that feedbacks the average of frequency deviation, i.e.,

\[
u_i^{\text{global}} = -\left( k_p \bar{\omega} + k_i \int_0^t \bar{\omega}(\tau)d\tau \right), \quad \bar{\omega} := \frac{1}{16} \sum_{i=1}^{16} \omega_i
\]

where \( k_p \) and \( k_i \) are controller gains to be tuned. We take a modular design approach such that each local controller is designed by a corresponding subsystem operator who knows only about his own generator, and the global controller is designed by a system coordinator who tunes the PI gains in an empirical manner. It should be noted that such empirical tuning of a global PI controller, called automatic generation control (AGC), is often used in an actual power system operation; see Section 9 in [46].

B. Demonstration of Bilayer Network Synthesis

1) Global PI Controller Design: We suppose that a system coordinator in the upper layer designs the global PI controller. In this setting, the scenario in Remark 2 is applicable, i.e., \( K_0 \) can be regarded as the global PI controller while \( L_0 \) can be regarded as the transmission network among the generators.
As a function of the PI gains, Fig. 10 plots the objective values of $J$ in the logarithmic scale. From this figure, we see that the objective values of $J$ are relatively large in the region of $(k_P, k_I) \in [0, 1]^2$ while those are small in the region of $(k_P, k_I) \in [1, 100]^2$. This means that a choice of moderate PI gains generally results in satisfactory reduction of the objective value in the upper layer. Such suboptimal gains can be easily found even with empirical gain tuning because sensitivity of the broadcast-type PI controller to the power system is not very large in the region of moderate gains. In the following, we set $(k_P, k_I) = (20, 30)$. It should be noted that the exact value of $J$ may not be computable in a real power system because its computation requires the whole system information such as load impedances and line admittances. For reference, we depict the resultant box plots corresponding to the faults at the generator buses and others in Fig. 9(b). This figure shows that sole implementation of the global PI control is, however, not very effective to reduce frequency deviation.

2) Local Controller Design by Retrofit Control: Next, we consider the design of local controllers in the lower layer based on output-rectifying retrofit control. In particular, we suppose that each subsystem operator finds a suitable $\hat{K}_i$ in Proposition 4 that stabilizes

$$G_{y_iu_i}(s) = (sI - A_i)^{-1}B_i,$$

where the output matrix is the identity matrix because of the supposition of generator state measurement. As a design criterion, the norm values of $\|G_{z_i, d_i}(s)\|_\infty$ for the isolated subsystems

$$G_{z_i, d_i}(s) = C_i(sI - A_i)^{-1}, \quad G_{w_i, d_i}(s) = \begin{bmatrix} T_i \\ C_i \end{bmatrix}(sI - A_i)^{-1}$$

are considered, where the disturbance input port matrix is chosen as the identity matrix because 68 different faults stimulate each generator state at almost random, and the output ports of both $\omega_i$ and $I_i$ are identified as the interconnection output ports because $\omega_i$ and $I_i$ are interconnection signals outflowing to the global PI controller and the transmission network, respectively. In this setting, we find a set of stabilizing controllers by the standard $H_\infty$-controller synthesis. The resultant norm values before and after the respective controller design are listed in Table I.

### Table I

| Generator label $i$ | $\|G_{z_i, d_i}\|_\infty$ | $\|M_{z_i, d_i}\|_\infty$ |
|--------------------|----------------|----------------|
| 1                  | 2.60           | 0.37           |
| 2                  | 5.69           | 0.16           |
| 3                  | 6.04           | 0.15           |
| 4                  | 7.17           | 0.12           |
| 5                  | 5.19           | 0.14           |
| 6                  | 3.89           | 0.16           |
| 7                  | 3.63           | 0.13           |
| 8                  | 5.68           | 0.13           |
| 9                  | 6.53           | 0.14           |
| 10                 | 3.28           | 0.14           |
| 11                 | 7.54           | 0.19           |
| 12                 | 4.53           | 0.19           |
| 13                 | 2.30           | 0.08           |
| 14                 | 3.44           | 0.20           |
| 15                 | 2.02           | 0.17           |
| 16                 | 2.87           | 0.19           |
| 17                 | max: ~ 2.70    | max: ~ 2.17    |
| 18                 | max: ~ 2.87    | max: ~ 3.98    |

Fig. 11. Box plots of magnitude of frequency deviation versus number of implemented retrofit controllers. The upper and lower rows correspond to the faults at generator buses and those at load and non-unit buses. The left and right columns correspond to the cases without and with global PI control.
Table I showing that each local controller is designed in a suitable manner.

3) Joint Implementation of Global and Local Controllers:
To simulate gradual penetration of retrofit controllers into the power system, we implement the obtained retrofit controllers one by one in the order from the 1st to the 16th generators. The resultant box plots with regard to the ground faults versus the number of implemented retrofit controllers are shown in Figs. (11(a) and (b), where the left and right columns correspond to the cases without and with the global PI control, respectively. The left-most plot in each subfigure is identical to one of those in Fig. 9. From Figs. (11(a-1) and (a-2), we see that frequency deviation due to the generator bus faults is gradually reduced as the number of retrofit controllers increases. Furthermore, the global PI control contributes especially to reducing the maximum (worst-case) values when the number of implemented retrofit controllers is more than 13; see the red dashed circles in the subfigures. On the other hand, from Figs. (11(b-1) and (b-2), we see that, though retrofit controllers are not very sensitive to load and non-unit bus faults, the global PI controller also contributes to reducing the maximum values when retrofit controllers sufficiently penetrate. These results demonstrate that the collaboration of the global PI controller and the retrofit controllers works appropriately, thereby showing the practical significance of our modular design approach to bilayer network synthesis.

VI. CONCLUSION
The framework of bilayer network synthesis developed in this paper takes a modular design approach to the control of dynamical network systems. Modular design or modularity-in-design is a widely accepted concept of system design to make complexity of large-scale system design manageable, to enable parallel work by multiple independent entities, and to make future modification of modules flexible. As illustrated in the example of a power system, complexity of control system design is made manageable, by allowing parallel work for a system coordinator and subsystem operators in the upper and lower layers, respectively. Flexibility in designing and implementing respective controllers is also ensured as each entity can individually add, remove, and modify his own controller without fully considering other entities’ action.

In this paper, we do not discuss a situation where not only a subcontroller but also a subsystem itself is to be modified. To discuss this situation, we need more detailed analysis of interconnection design in the upper layer with consideration of robustness to uncertainty or flexibility of subsystem variation, for which relaxing the condition of subcontroller modularity in a robust control setting would also be worth to discuss. In addition, depending on applications, subsystem partition may not be well-defined and it may also affect control performance of the resultant system. Discussing these open issues is one possible direction of future work.

APPENDIX A
PROOF OF THEOREM[1]
We first prove the sufficiency, i.e., if $K$ written by (14) satisfies (15), then $K$ is a retrofit controller. Because $K = 0$

is found to be a trivial retrofit controller, the internal stability of Fig. 4 is equivalent to that of Fig. 12 owing to Lemma 12.2 in [38]. Its internal stability is proven if the sixteen transfer matrices from $(\delta_u, \delta_y, \delta_v, \delta_w)$ to $(u, y, v, w)$ all belong to $\mathcal{RH}_\infty$. Let $\overline{Q}$ denote the Youla parameter of $\overline{G}$ such that (16) holds. If (15) holds, then we see that

$$w = (I + G_{gw}\overline{Q})G_{uw}(I + QG_{yu})\delta_u + (I + G_{gw}\overline{Q})G_{wu}Q\delta_y + (I + G_{gw}\overline{Q})\delta_v,$$

where we have used the relations

$$(I - G_{gw}\overline{G})^{-1} = I + G_{gw}\overline{Q}, \quad (I - KG_{yu})^{-1} = I + QG_{yu}.$$ 

In a similar way, we see that

$$u = (I + QG_{yu})\delta_u + Q\delta_y + G_{yu}v,$$

$$y = (I + G_{yu}Q)G_{yu}u + (I + G_{yu}Q)\delta_y + (I + G_{yu}Q)G_{yu}v.$$ 

Because all $G$, $Q$ and $\overline{Q}$ belong to $\mathcal{RH}_\infty$, the transfer matrices from $(\delta_u, \delta_y, \delta_v, \delta_w)$ to $(u, y, v, w)$ are proven to be in $\mathcal{RH}_\infty$.

We next show the necessity, i.e., if $K$ is a retrofit controller, then $K$ is written by (14) and satisfies (15). For $K$ to be a retrofit controller, Fig. 12 is internally stable, even for the particular choice of $\overline{G} = 0$, which belongs to $\overline{G}$. In this case, $K$ is necessarily a stabilizing controller for $G_{yu}$, i.e., it is written by (14) for some $Q$. What remains to show is the fact that (15) holds. By standard calculation, the transfer matrix from $\delta_v$ to $v$ in Fig. 12, which is internally stable, is found to be $(I - QG_{wu}QG_{yu})^{-1}$. To show the claim by contradiction, let us suppose that (15) does not hold. Then, there exists some $\overline{Q} \in \mathcal{RH}_\infty$ such that

$$\det (I - \overline{Q}(j\omega_0)G_{wu}(j\omega_0)Q(j\omega_0)G_{yu}(j\omega_0)) = 0$$

for some $\omega_0 \in \mathbb{R} \cup \{\infty\}$, as shown in the proof of the small-gain theorem; see, e.g., Theorem 9.1 in [38]. This means instability, which contradicts the internal stability of Fig. 12.

APPENDIX B
STATE-FEEDBACK OUTPUT-RECTIFYING RETROFIT CONTROL
The purpose of this section is to analyze an output-rectifying retrofit controller when the internal state of the subsystem $G$ is measurable. In the following, we use the notation of

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} := C(sI - A)^{-1}B + D.$$
We make the assumption on the state measurement as follows.

**Assumption 3** The internal state of $G$ is measurable, i.e.,

$$G_{yu} = \begin{bmatrix} A & B \\ I & 0 \end{bmatrix}, \quad G_{yv} = \begin{bmatrix} A & L \\ I & 0 \end{bmatrix}. $$

Based on Assumptions 1 and 3, we analyze the output-rectifying retrofit controller in Definition 3. The most crucial issue is how to deal with the constraint (19) on the Youla parameter. First, the following technical lemma is given.

**Lemma 2** Let Assumptions 1 and 3 hold. Then, there exist right-invertible matrices $P$ and $P^\dagger$, and their right-inverses $P^\dagger$ and $P^\dagger_\dagger$ such that

- **C1:** $P^\dagger P + P^\dagger_\dagger P = I$,
- **C2:** $PAP^\dagger$ and $PAP^\dagger_\dagger$ are stable,
- **C3:** $PL$ is zero, and $PL$ is nonsingular.

**Proof:** We denote the positive or negative definiteness of a matrix by the symbol “$\succ$”. Because of Assumptions 1 and 3 there exists some matrix $V > 0$ such that $AV + VA^\dagger \prec 0$. Then, we see that it is equivalent to

$$V^{-1}AVc + (V^{-1}AVc)^T \prec 0 \quad (39)$$

where we use the Cholesky factorization such that $V = V_cV_c^T$. Consider choosing the parameter matrices as

$$P = QV_c^{-1}, \quad P^\dagger = V_cQ^T, \quad P^\dagger_\dagger = V_cQ^T \quad (39)$$

where $Q$ and $Q^T$ are some matrices such that the stack of them is unitary, i.e., $Q^TQ + Q^TQ = I$. Clearly, C1 is satisfied with this choice. Furthermore, C2 is also satisfied because the multiplication of (39) by $Q$ and $Q^T$ from the left and right sides, respectively, yields $PAP^\dagger + (PAP^\dagger)^T \prec 0$, which proves that $PAP^\dagger$ is stable. The stability of $PAP^\dagger$ is proven in the same way. Finally, if $Q$ is chosen such that the image of $Q^T$ is equal to that of $V_c^{-1}L$, then C3 is satisfied. □

The proof of Lemma 2 provides an algorithm to construct the parameters, e.g., $P$ and $P^\dagger$. Then, the following factorization of the Youla parameter is enabled over the ring of $\mathcal{RH}_\infty$.

**Lemma 3** Let Assumptions 1 and 3 hold. Furthermore, let right-invertible and left-invertible matrices be such that C1, C2, and C3 in Lemma 2 hold. Then, the Youla parameter $Q \in \mathcal{RH}_\infty$ satisfies (23) if and only if there exists $\tilde{Q} \in \mathcal{RH}_\infty$ such that $Q = \tilde{Q}X$ where

$$X := \begin{bmatrix} PAP^\dagger & PAP^\dagger_\dagger \\ -I & P \end{bmatrix}. \quad (40)$$

**Proof:** For the proof of the “if” part, it is sufficient to show that $XG_{yu} = 0$ because $X \in \mathcal{RH}_\infty$ is premised as in C2. A state-space realization of $XG_{yu}$ can be written as

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ s \end{bmatrix} = \begin{bmatrix} A & -PAP^\dagger \\ 0 & A \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ L \end{bmatrix} r$$

Using the relation of

$$\begin{bmatrix} -I & P \\ 0 & A \end{bmatrix} \begin{bmatrix} PAP^\dagger \\ -I \end{bmatrix} = PAP^\dagger \begin{bmatrix} -I & P \\ 0 & A \end{bmatrix},$$

we have an exact reduced order dynamics of $s$ as

$$\dot{s} = PAP^\dagger s + PLr.$$  

This implies that $XG_{yu}$ is found to be

$$XG_{yu} = \begin{bmatrix} PAP^\dagger & PL \\ -I & 0 \end{bmatrix} \quad (41)$$

where $PL = 0$ is premised as in C3.

Next, we prove the “only if” part. Adopting calculus over the ring of $\mathcal{RH}_\infty$, we prove that, for any $Q \in \mathcal{RH}_\infty$ such that (23) holds, there exists $\tilde{Q} \in \mathcal{RH}_\infty$ such that $Q = \tilde{Q}X$. To this end, we prove that

$$U := \begin{bmatrix} X \\ \bar{X} \end{bmatrix}, \quad U^{-1} := \begin{bmatrix} X^\dagger & \bar{X}^\dagger \end{bmatrix}$$

are unimodular, i.e., invertible in $\mathcal{RH}_\infty$, with

$$\begin{bmatrix} X^\dagger & \bar{X}^\dagger \end{bmatrix} := \begin{bmatrix} PAP^\dagger & PAP^\dagger_\dagger \\ -I & P \end{bmatrix},$$

by showing $U^{-1}U = I$. With this definition, let us show that

$$X^\dagger X := \begin{bmatrix} A & PAP^\dagger \end{bmatrix} \begin{bmatrix} P^\dagger PAP^\dagger_\dagger & P^\dagger P \end{bmatrix} \quad (42)$$

A state-space realization of $X^\dagger X$ can be written as

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} A & -PAP^\dagger \\ 0 & A \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} P^\dagger PAP^\dagger_\dagger & P^\dagger P \end{bmatrix} g$$

Using the relation of

$$\begin{bmatrix} I & -P^\dagger \end{bmatrix} \begin{bmatrix} A & -PAP^\dagger \\ 0 & A \end{bmatrix} = A \begin{bmatrix} I & -P^\dagger \end{bmatrix},$$

we have the exact reduced order system

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} = A \begin{bmatrix} P^\dagger PAP^\dagger_\dagger & P^\dagger P \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} -P^\dagger PAP^\dagger_\dagger & P^\dagger P \end{bmatrix} g$$

which proves (42). In the same manner, we can prove that

$$X^\dagger \bar{X} := \begin{bmatrix} A & P^\dagger PAP^\dagger_\dagger & P^\dagger P \\ -I & P \end{bmatrix}. $$

Thus, $U^{-1}U = I$ is proven.

Because $U \in \mathcal{RH}_\infty$ is unimodular, for any $Q \in \mathcal{RH}_\infty$, there always exists $\tilde{Q} \in \mathcal{RH}_\infty$ such that $Q = \tilde{Q}U$. Substituting this into (23), we have

$$\begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \end{bmatrix} \begin{bmatrix} X \\ \bar{X} \end{bmatrix} G_{yu} = 0.$$
which is equivalent to \( \dot{Q}_2 X G_{yu} = 0 \) because \( X G_{yu} = 0 \). In the same manner as [41], we can also see that

\[
X G_{yu} = \begin{bmatrix}
P A P^\dag & P B \frac{I}{\bar{P} L} & 0
\end{bmatrix}.
\]

Because \( \bar{P} L \) is nonsingular as premised in C3, we see that \( \dot{Q}_2 = 0 \). Thus, the claim is proven.

In the same manner as [41], \( X G_{yu} \) is found to be

\[
\dot{\xi} = \begin{bmatrix} P A P^\dag & P B \end{bmatrix} \xi + \begin{bmatrix} PA \end{bmatrix} \xi + \begin{bmatrix} PB \end{bmatrix} y v
\]

Thus, using Lemma 3, we can rewrite (14) and (23) as

\[
\begin{aligned}
K &= \hat{K} X, \\
\hat{K} &= (I + \hat{Q} G_{\xi u})^{-1} \hat{Q}, \\
\hat{Q} &\in RH_{\infty}.
\end{aligned}
\]

From (43), we find that \( \hat{K} \) is a stabilizing controller for \( G_{\xi u} \), and \( \hat{Q} \) is its Youla parameter. To understand the meaning of this retrofit control, for the input-to-state dynamics

\[
\dot{x} = Ax + Bu + Lv,
\]

consider the basis transformation \( \xi = Px \) and \( \bar{\xi} = \bar{P} x \) as

\[
\begin{aligned}
\dot{\xi} &= \bar{P} A \bar{P}^\dag \xi + \bar{P} A \bar{P}^\dag \xi + \bar{P} B u + \bar{P} L v \\
\dot{\bar{\xi}} &= \bar{P} A \bar{P}^\dag \xi + \bar{P} A \bar{P}^\dag \xi + \bar{P} B u
\end{aligned}
\]

where \( \bar{P} L = 0 \) in C3 of Lemma 2 has been used. We notice that the dynamics of \( \bar{\xi} \) is directly affected by \( v \), but that of \( \xi \) is not. Let us regard \( \bar{\xi} \) as an interconnection signal to the dynamics of \( \xi \). Then, we can see that \( \hat{G}_{\xi u} \) represents the transfer matrix from \( u \) to \( \xi \), and \( X \) represents the output rectifier \( X : (\xi, \bar{\xi}) \mapsto \bar{\xi} \) to perform \( \bar{\xi} = \xi - \hat{G} \bar{\xi} \xi \) where the transfer matrix from \( \xi \) to \( \xi \) is denoted as

\[
\hat{G}_{\xi \xi} = \begin{bmatrix} P A P^\dag & P A \end{bmatrix} \frac{I}{P L} 0
\]

Therefore, \( \hat{K} \) in (43) can be understood as an output-rectifying retrofit controller for the dynamics of \( \xi \) in [44], in which \( (\xi, \bar{\xi}) \) is assumed to be measurable. This is summarized as follows.

**Proposition 6** Let Assumptions 11 and 3 hold. With the same notation as that in Lemma 3, \( \hat{K} \) is an output-rectifying retrofit controller if and only if \( \hat{K} = \hat{K} X \) where \( \hat{K} \) is a stabilizing controller for \( G_{\xi u} \), i.e., all such retrofit controllers have the structure of Fig. 13.

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