A numerical scheme of convex yield function with continuous anisotropic hardening based on non-associated flow rule in FE analysis of sheet metal

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Abstract. A non-associated flow rule (NAFR) model is developed by adopting the convex function YLD-2004 as the yield stress function and the plastic potential function. The yield stress function coefficients are continuously updated which are associated with the change of directional uniaxial yield stress and biaxial yield stress to simulate the anisotropic hardening behaviour. That was achieved by implementing the numerical identification procedure of coefficients into stress integration procedure. The coefficients of plastic potential function are constant and identified by the uniaxial and biaxial r-value. This constitutive model is capable of describing anisotropic hardening and yield behaviour of strongly textured aluminium alloy sheet metal. In this paper, the model was implemented into the FE code via ABAQUS subroutine to predict the deep drawn cup earing and directional flow stresses of the AA5042-H2 aluminium alloy. The new anisotropic hardening model shows better agreement with experiments compared with the isotropic hardening model.

1. Introduction

Strongly textured aluminium alloy sheet metal generally exhibit substantial mechanical anisotropy and hardening behaviour which may affect the plastic deformation and spring-back of sheet metal in forming process. Therefore these anisotropic properties should be considered in numerical modelling developed for a design of sheet forming tooling and process [1, 2].

Most yield functions only define the yield surface and plastic flow but do not accurately describe the anisotropic hardening behaviour and evolution in anisotropic ratios as a function of plastic deformation. Some previous works employed the discrete method to describe the evaluation in anisotropic hardening for various plastic work levels. Plunkett et al. [3] and Aretz [4] introduced discrete sets of anisotropic coefficients to account for anisotropic hardening as a function of equivalent plastic strain by describing the evolution of yield surface at discrete levels of plastic deformation with an interpolation method. Wang et al. [5] proposed an equivalent plastic strain-dependent Yld2000-2d model by replacing the model parameters with 6th order function of the equivalent plastic strain. Gawad et al. [6] developed a multi-scale model which accounts for change of anisotropy, and Yoshida et al. [7] interpolated yield surfaces at two different levels of the equivalent plastic strain to describe the evolution of the yield surface. Although these models have considered anisotropic hardening, the directional flow stress of the models determined only discrete levels of plastic strain.

Here, a new sheet metal constitutive model is proposed based on non-associated flow rule (NAFR) which involves anisotropic yield and the anisotropic hardening associated with the equivalent strain. This new constitutive model contains an advanced convex yield function and plastic potential function (Yld2004-18p) which provide highly accurate description of the complex variation of sheet metal anisotropy in aluminium alloy AA5042-H2. The anisotropic
hardening behaviour was computed by the updating of coefficients of yield function which determine the shape of the yield surface in each increment. This model was implemented into the ABAQUS 6.14-1 via subroutine UMAT to predict evolution of the anisotropic properties and earing profile of deep drawn cup and validated with experimental results.

2. Yield function and plastic potential function

The yield function and plastic potential function using the Yld2004 function [8] and expressed as:

\[
\sigma_e(\sigma) = \left\{ \frac{1}{4} \left[ \sum_{i=1}^{3} \sum_{j=1}^{3} \left( \left| \hat{S}_{ij} \right| \right) \right] \right\}^{\frac{1}{2}} ; \quad \sigma_p(\sigma) = \left\{ \frac{1}{4} \left[ \sum_{i=1}^{3} \sum_{j=1}^{3} \left( \left| \hat{X}_{ij} \right| \right) \right] \right\}^{\frac{1}{2}}
\]

where the \( \hat{S}^{(k)} \) and \( \hat{X}^{(k)} \) are the principal value of the transformed stress tensor \( \tilde{s}^{(k)} \) and \( \tilde{x}^{(k)} \) which given by:

\[
\tilde{s}^{(k)} = \alpha^{(k)} \cdot T \cdot \sigma ; \quad \tilde{X}^{(k)} = \beta^{(k)} \cdot T \cdot \sigma
\]

where the \( T \) is a constant transformed matrix, \( \alpha^{(k)} \) and \( \beta^{(k)} \) are the anisotropic coefficients matrix of yield function and plastic potential function which determine the anisotropic yield stress and r-value, respectively. Here, the \( \alpha^{(k)} \) are continuously updated with the equivalent strain which are associated with the change of directional uniaxial yield stress and biaxial yield stress and the \( \beta^{(k)} \) is constant to keep the same r-value in forming process.

3. Scheme of stress integration procedure

In this section, the scheme of the stress integration algorithm for the new model is proposed and shown in Figure 1. The algorithm assumes an additive decomposition of the strain tensor where the total strain increment \( \Delta \varepsilon_t \) consists of an elastic \( \Delta \varepsilon^e \) and a plastic \( \Delta \varepsilon^p \) increment:

\[
\Delta \varepsilon_t = \Delta \varepsilon^e + \Delta \varepsilon^p
\]

The elastic behaviour is assumed as isotropic and linear, the increment of the elastic stress following the Hooke’s law,

\[
\Delta \sigma = D : (\Delta \varepsilon^e - \Delta \varepsilon^p)
\]

where \( D \) is the elastic stiffness matrix. The elastic predictor stress \( \sigma_B \) (linear orthotropic) is determined:

\[
\sigma_B = \sigma + D : \Delta \varepsilon^e
\]

The yield criterion determined by Yld2004-18p as:

\[
f_1 = \sigma_e(\sigma_B) - \sigma_0(\varepsilon_{ep})
\]

where \( \varepsilon_{ep} \) is the equivalent plastic strain, and \( \sigma_0 \) is the uniaxial yield stress. \( \sigma_0 \) is computed by Voce’s hardening law and hardening modulus \( H \), given by

\[
H = \frac{\partial \sigma_0}{\partial \varepsilon_{ep}} = B \cdot C \cdot exp(-\varepsilon_{ep} \cdot C)
\]

If \( f_1 \leq 0 \), the material is in its elastic state (\( \Delta \varepsilon^p = 0 \)) and the stress is updated by:

\[
\sigma = \sigma + D : \Delta \varepsilon^e
\]

If \( f_1 > 0 \), the material is in its plastic state and the constitutive model provides a suitable set of algorithms for stress integration in the non-linear finite element analysis. Unconditional
stability integration of the constitutive model is achieved using the backward-Euler algorithm. In the backward-Euler algorithm, NAFR assumes the plastic strain rate is normal to the plastic potential function surface and expressed as

$$\Delta \varepsilon_{\text{NAFR}} = \Delta \varepsilon_{\text{ep}}^{\text{NAFR}} \cdot \frac{\partial \sigma_p}{\partial \varepsilon} = \Delta \varepsilon_{\text{ep}}^{\text{NAFR}} \cdot n$$ (9)

$$\Delta \varepsilon_{\text{NAFR}}^{\text{ep}} = \frac{\sigma}{\sigma_p} \cdot \Delta \varepsilon_{\text{NAFR}}^{\text{ep}} = \frac{\sigma}{\sigma_p} \cdot \frac{\lambda_{\text{NAFR}}}{\sigma_p} = \lambda_{\text{NAFR}}$$ (10)

where $\Delta \varepsilon$ is the plastic strain increment, $\Delta \varepsilon_{\text{ep}}$ is the equivalent plastic strain increment and $\lambda$ is the plastic strain-rate multiplier. $m = \frac{\partial \sigma_p}{\partial \varepsilon}$ and $n = \frac{\partial \sigma_p}{\partial \varepsilon}$ are the first order derivative of the yield function and plastic potential function, respectively. Voce’s hardening law was incorporated into the Yld2004-18p yield function to form the yield criterion, respectively:

$$f_2 = \sigma_C (\sigma_C) - \sigma_0 (\varepsilon_{\text{ep}} + \Delta \varepsilon_{\text{ep}}) = 0$$ (11)

The corrective plastic stress $\sigma_C$ based on NAFR were given as:

$$\sigma_C^{\text{NAFR}} = \sigma_B - D : \Delta \varepsilon_{\text{NAFR}}^{\text{ep}} = \sigma_B - \lambda_{\text{NAFR}} \cdot D : n$$ (12)

Figure 1: The schematics of the flowchart of stress integration algorithm
In order to solve for $\sigma_C$, the Newton-Raphson method was employed to reduce the residual vector $r$ to zero while the final stresses satisfy the yield criterion $f_2 = 0$. The expression for $r$ based on NAFR were given as:

$$r_{NAFR} = \sigma_C - \left( \sigma_B - \dot{\lambda}_{NAFR} \cdot D : n \right)$$

(13)

The initial prediction of corrective stress tensor $\sigma_C(0)$, plastic strain-rate multiplier $\dot{\lambda}(0)$ and increment of equivalent plastic strain $\Delta\varepsilon_{ep(0)}$ based on NAFR are given as:

$$\dot{\lambda}_{NAFR(0)} = \frac{f_1}{m^T : D : n + H}$$

(14)

$$\sigma_{NAFR(0)}^C = \sigma_B - \dot{\lambda}_{NAFR(0)} \cdot D : n$$

(15)

$$\Delta\varepsilon_{ep(0)}^{NAFR} = \dot{\lambda}_{NAFR(0)}$$

(16)

The iterative change $\Delta\sigma_{C(k+1)}$ and $\Delta\dot{\lambda}_{(k+1)}$ in iteration step $k + 1$ are given as:

$$\Delta\dot{\lambda}_{NAFR(k+1)} = \frac{f_2 - m^T : Q_{NAFR} : r_{NAFR}}{m^T : Q : D : n + H}$$

(17)

$$\Delta\sigma_{C(k+1)}^{NAFR} = -Q_{NAFR} : r_{NAFR} - \Delta\dot{\lambda}_{NAFR(k+1)} \cdot Q_{NAFR} : D : n$$

(18)

where the $Q$ is given by

$$Q_{NAFR} = -\left( I + \dot{\lambda}_{NAFR} \cdot D : N \right)^{-1}$$

(19)

where $I$ is the identity matrix and $N = \frac{\partial^2 \sigma_p}{\partial \sigma \partial \sigma}$ are the second order derivatives of the plastic potential function. Corrective stress, plastic strain-rate multiplier and equivalent plastic strain increment are updated in the iterative process:

$$\sigma_C(k+1) = \sigma_C(k) + \Delta\sigma_{C(k+1)}$$

(20)

$$\dot{\lambda}(k+1) = \dot{\lambda}(k) + \Delta\dot{\lambda}(k+1)$$

(21)

![Figure 2: Directional uniaxial and biaxial stress-strain curve prediction (a) isotropic hardening model (b) anisotropic hardening model](image-url)
Figure 3: Yield surface prediction using anisotropic and isotropic hardening model at various plastic work levels

\[ \Delta \varepsilon_{cp(k+1)} = \dot{\lambda}(k+1) \]  

\[ \Delta \varepsilon_{cp} = \dot{\lambda}(k+1) \]  

After the finish of the iteration process, the consistent tangent modulus \( \mathbf{D}_{ct} \) in the plastic state was employed to update the plastic stress, the \( \mathbf{D}_{ct} \) in NAFR are given by:

\[ \mathbf{D}^{NAFR}_{ct} = \left( \mathbf{R}_{NAFR} - \frac{\mathbf{R}_{NAFR} : \mathbf{m} : \mathbf{n}^T : \mathbf{R}_{NAFR}}{\mathbf{m}^T : \mathbf{R}_{NAFR} : \mathbf{n} + H} \right) \]  

where the \( \mathbf{R}_{NAFR} = \mathbf{Q}_{NAFR} : \mathbf{D} \). Finally, the plastic stress are updated by:

\[ \sigma = \sigma + \mathbf{D}_{ct} : \Delta \varepsilon^I \]  

The updated equivalent strain is used to compute the new directional uniaxial and equi-biaxial yield stress:

\[ Y_{\varphi}^{exp} (\varepsilon_{cp}) = A_{\varphi} - B_{\varphi} \cdot \exp(-\varepsilon_{cp} \cdot C_{\varphi}) \]  

\[ Y_{eb}^{exp} (\varepsilon_{cp}) = A_{eb} - B_{eb} \cdot \exp(-\varepsilon_{cp} \cdot C_{eb}) \]  

The well-known non-linear least-square method Levenberg-Marquardt algorithm was coded into the stress integration scheme to update the in-plane anisotropic coefficients \( \alpha \) of the yield function curve so that the sum of the squares of the deviations is minimized, then the new yield surface associated to equivalent strain is updated:

\[ \hat{\alpha} = \arg \min \sum_{i=1}^{n_p} \left[ Y_{\varphi}^{exp} - Y_{\varphi}^f \right]^2 + \min \left[ Y_{eb}^{exp} - Y_{eb}^f \right]^2 \]  

4. Results and discussions

Figures 2a-b show the comparison of directional stress-strain curve of AA5042-H2 aluminium alloy predicted by anisotropic and isotropic hardening model. Figure 3 shows the yield surface associated with equivalent strain increment of 0.02 predicted by the new anisotropic and isotropic hardening model. As expected, results showed the better agreement between the experiments [9] and the new anisotropic hardening model compared with conventional isotropic hardening model.
Figure 4: FE modelling of AA5042-H2 deep drawn cup (a) prediction of thickness distribution using anisotropic hardening model (b) prediction of earing profile using anisotropic and isotropic hardening model

Figure 4a-b show cup heights or earing profiles of experimental deep drawn cylindrical cups [9] and those predicted by the FE model. The experimental deep drawn cups show six major ears at 0°, 45°, 135°, 180°, 225°, 315° and two minor ears in 90° and 270° (with respect to the rolling direction). The anisotropic hardening model accurately predicts the general profile of the six major ears and two minor ears which are in good agreement with the experimental results. However, the isotropic hardening predict the two major ears in 90° and 270° which in practise should appeared an minor ear in the actual forming process.

5. Conclusion

A non-linear least square method was implemented to a new anisotropic hardening constitutive model which was used to determine the evolution of the coefficients of the yield function associated with equivalent strain based on experimental results. By employing the back-Euler stress integration and yield surface updating algorithm, this new anisotropic constitutive model has been successfully implemented in commercial FEM software ABAQUS via user material subroutine UMAT to predict earing profile of deep drawn cups and the evolution in yield surface and the uniaxial yield stress of AA5042-H2 aluminium alloy in various plastic work levels. The new anisotropic yield and hardening model is presented in a general form which can be readily adapted for other aluminium alloys used in sheet forming applications such as prediction of spring-back and plastic deformation of automotive components in forming process.

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