Pair condensation in a dilute Bose gas with Rashba
spin–orbit coupling

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Abstract
We show that in a two-component Bose gas with Rashba spin–orbit coupling (SOC), two atoms can form bound states (Rashbons) with any intra-species scattering length. At zero center-of-mass momentum, there are two degenerate Rashbons due to time-reversal symmetry, but the degeneracy is lifted at finite in-plane momentum with two different effective masses. A stable Rashbon condensation can be created in a dilute Bose gas with attractive intra-species and repulsive inter-species interactions. The critical temperature of Rashbon condensation is about six times smaller than the Bose–Einstein condensation transition temperature of an ideal Bose gas. Due to the Rashba SOC, excitations in the Rashbon condensation phase are anisotropic in momentum space.

Keywords: spin-orbit coupling, Bose gas, bound state, Bose–Einstein condensation

1. Introduction

In recent years one major forward step in ultracold atom physics was the realization of spin–orbit coupling (SOC) in Bose–Einstein condensation [1] and ultracold Fermi gases [2, 3]. In contrast to the intrinsic SOC of electrons in atoms, SOC in neutral atoms refers to the coupling between the spin and center-of-mass momentum of atoms. Bose gases with SOC have displayed very rich phase diagrams. In experiments where the SOC is an equal-weight
combination of Rashba and Dresselhaus SOCs, several phase transitions, including ones from magnetic to spin-mixed states and from normal to magnetic states, were observed in Bose gases [1, 4, 6], consistently with theoretical studies [5]. For a uniform Bose gas with Rashba SOC, competition between plane-wave and spin-stripe phases was predicted [7–9]. New phases, such as half-vortex and skyrmion-lattice phases, were predicted to arise in trapped systems [10–13]. In this paper, we are going to show that a stable pairing state can appear in a two-component Bose gas with Rashba SOC.

In contrast to the well-observed BCS–BEC crossover in Fermi gases [14], the pairing state of Bose gas is an exotic and never observed phenomenon. Although Feshbach molecules of Bose atoms have been created in experiments [15–17], the rapid particle loss rate due to strong inelastic collision near the resonance severely limits the molecule lifetime, making it impossible to reach a condensed state. Despite experimental difficulties, the pairing state of a Bose gas has been explored theoretically [18–20], but it was found unstable even with a weak attractive interaction away from the resonance [21–23].

The successful creation of SOC offers a new opportunity to realize the pairing state in a Bose gas. As in the fermion case, Rashba SOC changes the atom density of states (DOS), which has strong effects on the pairing and produces the Rashbon, \(^1\) the two-body bound state with negative scattering length. In the following, we first study the two-body bound states of bosons with Rashba SOC. We find that at zero center-of-mass momentum, bound states (Rashbons) can exist with arbitrary intra-species scattering length, while SOC has virtually no effect on the bound state created by the inter-species interaction. Next, we study the possibility of Rashbon condensation in a Bose gas with Rashba SOC. We find that Rashbon condensation can be stabilized by a repulsive inter-species interaction. The Rashbon condensation may be realized in a dilute Bose gas with weak intra-species attraction and inter-species repulsion. One signature of this phase is the anisotropic excitation spectrum. The Rashbon transition temperature is about six times smaller than the ideal BEC temperature.

### 2. The model

We consider a two-component Bose gas with Rashba SOC, described by the Hamiltonian

\[
\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}},
\]

where

\[
\hat{H}_0 = \sum_{\mathbf{k}, \sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} \left[ S(\mathbf{k}) c_{\mathbf{k}\uparrow}^\dagger c_{\mathbf{k}\downarrow} + \text{h. c.} \right],
\]

\(c_{\mathbf{k}\sigma}\) represents the annihilation operator of a boson with wavevector \(\mathbf{k}\) and spin \(\sigma\), \(S(\mathbf{k}) = \hbar^2 k / m\), \(\kappa\) is the strength of the Rashba SOC, \(k_{\perp}\) is the projection of \(k\) in the \(x-y\) plane, and \(\epsilon_k = \hbar^2 k^2 / 2m\). The single-atom Hamiltonian \(\hat{H}_0\) can be easily diagonalized, yielding helical excitations with energies given by \(\xi_{kz} = \epsilon_k \pm \hbar^2 k_{\perp} / m\). The s-wave interaction between atoms is given by

\[
\hat{H}_{\text{int}} = \frac{1}{2V} \sum_{\mathbf{k}, \mathbf{k}, \mathbf{q}, \sigma, \sigma'} g_{\sigma \sigma'} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{-k}\sigma'} c_{\mathbf{q}\sigma'}^\dagger c_{\mathbf{-q}\sigma}.
\]

\(^1\) The term ‘Rashbon’ was first used in the following studies of Fermi gases with Rashba SOC.
where \( V \) is the volume. The inter-species coupling constants satisfy
\[
g_{\tau \ell} = g_{\ell \tau} = 4\pi \hbar^2 a'/m,
\]
where \( a' \) is the inter-species scattering length. In the following, we consider only the symmetric case with two identical intra-species coupling constants,
\[
g_{\tau \tau} = g_{\ell \ell} = 4\pi \hbar^2 a/m,
\]
where \( a \) is the intra-species scattering length. In this symmetric case, the system is invariant under the time-reversal transformation \((\mathbf{k}, \sigma) \rightarrow (\mathbf{-k}, -\sigma)\).

### 3. Two-body bound states

We first study two-body bound states described by the wavefunction
\[
|\Psi\rangle_q = \frac{1}{2} \sum_{\mathbf{k}, \sigma, \sigma'} \psi_{\sigma\sigma'}(\mathbf{k}, \mathbf{q} - \mathbf{k}) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{q} - \mathbf{k}\sigma'}^\dagger |0\rangle,
\]
where \( \psi_{\sigma\sigma'}(\mathbf{k}, \mathbf{k}') = \psi_{\sigma\sigma'}(\mathbf{k'}, \mathbf{k}) \) is a coefficient. By solving the eigenvalue problem \( H|\Psi\rangle_q = E_q |\Psi\rangle_q \) we can obtain the wavefunction and eigenenergy of the bound states. At \( q = 0 \), the eigenequation can be further written as
\[
M_k \psi_k' = \frac{1}{V} \sum_p G \psi_p',
\]
where \( \psi_k' \) is a four-component vector given by
\[
\psi_k' = \left[ \psi_{\downarrow\downarrow}(\mathbf{k}, -\mathbf{k}), \psi_{\uparrow\downarrow}(\mathbf{k}, -\mathbf{k}), \psi_{\downarrow\uparrow}(\mathbf{k}, -\mathbf{k}), \psi_{\uparrow\uparrow}(\mathbf{-k}, \mathbf{k}) \right].
\]
\( M_k \) is the matrix of the eigenenergy minus the kinetic energy and the SOC:
\[
M_k = \begin{bmatrix}
\mathcal{E}_k & 0 & S^*(\mathbf{k}_\downarrow) & -S^*(\mathbf{k}_\uparrow) \\
0 & \mathcal{E}_k & -S(\mathbf{k}_\downarrow) & S(\mathbf{k}_\uparrow) \\
S(\mathbf{k}_\downarrow) & -S^*(\mathbf{k}_\uparrow) & \mathcal{E}_k & 0 \\
-S(\mathbf{k}_\downarrow) & S^*(\mathbf{k}_\uparrow) & 0 & \mathcal{E}_k
\end{bmatrix},
\]
\( \mathcal{E}_k = \mathcal{E}_0 - 2\epsilon_k \), and \( G \) is the matrix of coupling constants:
\[
G = \begin{bmatrix}
g_{\uparrow\downarrow} & 0 & 0 & 0 \\
0 & g_{\downarrow\uparrow} & 0 & 0 \\
0 & 0 & g_{\tau\tau}/2 & g_{\ell\tau}/2 \\
0 & 0 & g_{\ell\tau}/2 & g_{\tau\tau}/2
\end{bmatrix}.
\]
Defining the vector \( Q = G \sum_k \psi_k' / V \), from equation (4) we can obtain an equation for \( Q \):
\[
Q = \frac{1}{V} G \sum_k M_k^{-1} Q.
\]
Using the symmetry \( S(\mathbf{k}) = -S(-\mathbf{k}) \), we find that
\[
\sum_k M_k^{-1} = \sum_k \det |M_k^{-1}| \begin{bmatrix} A_k & 0 & 0 & 0 \\ 0 & A_k & 0 & 0 \\ 0 & 0 & A_k & B_k \\ 0 & 0 & B_k & A_k \end{bmatrix},
\]
where \( A_k = \mathcal{E}_k^3 - 2\mathcal{E}_k|S(\mathbf{k})|^2, \ B_k = -2\mathcal{E}_k|S(\mathbf{k})|^2 \), and \( \det |M_k| = \mathcal{E}_k^2 \left[ \mathcal{E}_k^2 - 4|S(\mathbf{k})|^2 \right] \). Equation (7) has three different solutions: two intra-species bound states with \( Q_3 = Q_4 = 0 \) and one inter-species bound state with \( Q_1 = Q_2 = 0 \) and \( Q_3 = Q_4 \). Due to the symmetry \( \psi = -\psi \), the solutions always satisfy \( Q_3 = Q_4 \), which is also guaranteed by the \( G \)-matrix elements \( G_{34} = G_{43} = G_{33} = G_{44} \) in equation (7). The \( G \)-matrix can also be chosen as a diagonal matrix with \( G_{33} = G_{44} = g_{11} \), but then the unphysical solution with \( Q_1 \neq Q_4 \) has to be taken out by hand.

For the two degenerate bound states at \( q = 0 \) created by the intra-species interaction, their eigenenergy \( E_0 \) is determined from the equation \( 1/g_{11} = \sum_k |M_k^{-1}|A_k/V \) which yields
\[
\frac{m}{4\pi\hbar^2a} = \frac{1}{2V} \sum_k \left[ \frac{1}{\epsilon_k} - \frac{1}{2\epsilon_k - E_0} - \frac{1}{4\epsilon_k + 2E_0} - \frac{1}{4\epsilon_k - 2E_0} \right],
\]
where the first rhs term is due to \( T \)-matrix correction. Equation (9) shows that these bound states are Rashbons which can exist with any intra-species interaction, whereas in a simple Bose gas without SOC, two-body bound states only exist in the repulsive regime. The binding energy defined by \( E_b = -E_0 - 2\epsilon \) is presented in figure 1(a), where \( \epsilon = \hbar^2\kappa^2/2m \). When the intra-species interaction is tuned from attraction to repulsion, the binding energy increases monotonically with \( 1/(\kappa a) \). We find that in the limit of \( \kappa a \to 0^+ \), the binding energy has the asymptotic form \( E_b \to 8\epsilon \exp \left\{ 4 \left[ 1/(\kappa a) - 1 \right] \right\} \); at resonance, \( 1/a = 0, E_b = 0.132\epsilon, \) much smaller than that in the fermion case [25]; when \( \kappa a \to 0^+, E_b \to \hbar^2/(ma^2) \), recovering the result for a dilute Bose gas without SOC.

The degeneracy of Rashbons at \( q = 0 \) is protected by time-reversal symmetry. One Rashbon wavefunction is given by
\[
\psi_{11}(\mathbf{k}, -\mathbf{k}) = \frac{\mathcal{N}}{\mathcal{E}_k} \frac{\mathcal{E}_k^2 - 2|S(\mathbf{k})|^2}{\mathcal{E}_k^2 - 4|S(\mathbf{k})|^2},
\]
where \( \mathcal{N} \) is a normalization constant. The other Rashbon wavefunction can be obtained by time-reversal transformation: \( \psi'_\sigma(\mathbf{k}, -\mathbf{k}) = \psi^*_{-\sigma \sigma}(-\mathbf{k}, \mathbf{k}) \).
The appearance of the Rashbon in the attractive regime is due to the increase in atom DOS at low energies arising from the SOC. The density of states of the lower helicity excitation $\xi$ is a constant at the energy minimum $\xi \epsilon = -\kappa - k$ for $k = 0$ and $z = 0$, which leads to an infrared divergence at zero binding energy on the r.h.s. of equation (9) and consequently the appearance of the Rashbon in the attractive regime. Rashbons in the weakly attractive regime may be helpful for experimental observation. Since the system is far away from resonance, the particle loss rate due to inelastic collision may be suppressed.

At $q \neq \perp q_0$, the bound state eigenenergy problem cannot be reduced to a simple equation. We numerically solve for bound state energies, and find that the two Rashbons have two different effective masses, as shown in figure 1(b). The lifting of Rashbon degeneracy is not surprising, because two Rashbons are no longer connected by time-reversal symmetry at finite $q$, and the Rashbon degeneracy is no longer protected by time-reversal symmetry. The two Rashbon effective masses behave differently with the intra-species scattering length $a$. The bigger effective mass $m^+$ reaches a maximum at resonance, while the smaller effective mass $m^-$ decreases monotonically with $1/(\kappa a)$. We obtain, in the limit $a \rightarrow 0^+$, $m^+ = 8m$ and $m^- = 8m/3$; at resonance, $m^+ = 9.29m$ and $m^- = 2.36m$; in the limit $a \rightarrow 0^+$, both effective masses recover the results for without SOC, $m^\pm \rightarrow 2m$.

When the in-plane momentum $h q_\perp$ exceeds a critical value $h q_c$, the Rashbon dissociates into excited atoms. We find that the critical wavevector $q_c$ is different for different Rashbons, approximately satisfying the condition for Rashbon dissociation in the effective-mass

![Figure 1. Binding energy and effective masses of Rashbons. (a) Rashbon binding energy versus $1/(\kappa a)$ at $q_\perp = 0$. At resonance, it is given by $E_\perp = 0.132\epsilon_c$. (b) In the limit $q_\perp \rightarrow 0$, two Rashbon effective masses can be obtained from Rashbon binding energies.](image-url)
approximation, \( E_0 + \hbar^2 q_m^2/(2 m^*_q) \approx -2\epsilon_c \). The critical momenta vanish in the limit of weakly attractive interaction \( a \to 0^- \), and diverge in the opposite limit \( a \to 0^+ \).

As regards the bound state created by the inter-species interaction, its eigenenergy at \( \mathbf{q} = 0 \) is given by
\[
\frac{m}{4\pi\hbar^2 a'} = \frac{1}{V} \sum_k \left( \frac{1}{2\epsilon_k} + \frac{1}{E_0 - 2\epsilon_k} \right)
\]
which is the same as that without SOC and gives the same result \( E_u = \hbar^2/(ma'^2) \), whereas in the fermion case the inter-species bound state is strongly affected by SOC \[25\]. The wavefunction of this bound state is also the same as that without SOC:
\[
\psi_{1\downarrow} (\mathbf{k}, -\mathbf{k}) = \psi_{1\uparrow} (\mathbf{k}, -\mathbf{k}) = 0,
\]
\[
\psi_{1\downarrow} (\mathbf{k}, -\mathbf{k}) = \frac{N'}{\mathcal{E}_k'}, \tag{12}
\]
where \( N' \) is a normalization factor.

The qualitative difference between the Rashbon and the inter-species bound state can be explained in terms of the symmetries of their wavefunctions as given in equation (10) and (12). The bound state created by the inter-species interaction consists of s-wave pairs of atoms with different helicities, whereas in the Rashbon two atoms either have the same helicity or are p-wave symmetrized with different helicities. In consequence, the binding energy of the bound state created by the inter-species interaction depends on the DOS of the pair energy for different helicities: \( \xi_{k^+} + \xi_{k^-} = 2\epsilon_k \), which is independent of the SOC. Thus the SOC has no effect on the binding energy of the bound state created by the inter-species interaction. In contrast, the Rashbon binding energy depends on not only the DOS of the pair energy for different helicities, but also the DOS of the pair energy for the same helicity, which is half of the atom DOS for the same helicity. The atom DOS is a constant at the lowest energy \(-\epsilon_c\), producing an infrared divergence at zero binding energy on the r.h.s. of equation (9). The change of atom DOS arising from the SOC is responsible for the existence of the Rashbon in the attractive regime. For comparison, in the fermionic case \[24, 25\], there is no s-wave intra-species interaction due to the Fermi–Dirac statistics, and the inter-species bound state consists of p-wave pairs of atoms with the same helicity. The bound state is a Rashbon because of the DOS effect due to the SOC.

4. Rashbon condensation

Rashbons are composite bosons obeying Bose–Einstein statistics. We consider the possibility of Bose–Einstein condensation of Rashbons in a Bose gas with Rashba SOC. The Rashbon condensation can be described by pairing order parameters \( \Delta_{1\uparrow} = g_{1\uparrow} \sum_k \langle c_{-k^1} c_{k^1} \rangle / V \) and \( \Delta_{1\downarrow} = g_{1\downarrow} \sum_k \langle c_{-k^1} c_{k^1} \rangle / V \). In general, if inter-species bound states condense, another pairing order parameter \( \Delta_{1\downarrow} = g_{1\downarrow} \sum_k \langle c_{-k} c_{k^1} \rangle / V \) needs to be introduced. Rashbon condensation is not directly coupled to the condensation of inter-species bound states. In the dilute limit with weakly attractive intra-species interaction and repulsive inter-species interaction, the Rashbon
binding energy is much smaller than the binding energy of the inter-species bound state. In the following, we consider a system with only Rashbon condensation and focus on the spin-balanced case, \( g_{\uparrow \downarrow} = g_{\downarrow \uparrow} \) and \( |\Delta_{\uparrow \uparrow}| = |\Delta_{\downarrow \downarrow}| = \Delta \). In general there may be a phase difference between \( \Delta_{\uparrow \uparrow} \) and \( \Delta_{\downarrow \downarrow} \). Without losing generality we define \( \Delta_{\uparrow \uparrow} = e^{i\theta} \Delta, \Delta_{\downarrow \downarrow} = e^{-i\theta} \Delta \) and \( \Delta > 0 \). The mean-field Hamiltonian of the Rashbon condensation phase is given by

\[
\frac{H_{MF}}{V} = \frac{1}{2V} \sum_{k} \left( B_k^+ H_k B_k - 2\xi_k \right) - \frac{\Delta^2}{g_{\uparrow \uparrow}} - \left( 2g_{\uparrow \downarrow} + g_{\downarrow \downarrow} \right) n^2, \tag{13}
\]

where \( B_k^+ \) is the field operator with four components \( B_k^+ = [c^+_k, c_{-k}^+, c_k^+, c_{-k}] \), \( n \) is the density of each spin component, the matrix \( H_k \) is given by

\[
H_k = \begin{bmatrix}
\xi_k & \Delta_{\uparrow \uparrow} & S^*(k) & 0 \\
\Delta_{\uparrow \uparrow}^* & \xi_k & 0 & -S(k) \\
S(k) & 0 & \xi_k & \Delta_{\downarrow \downarrow} \\
0 & -S^*(k) & \Delta_{\downarrow \downarrow}^* & \xi_k
\end{bmatrix}, \tag{14}
\]

\( \xi_k = \epsilon_k - \mu + 2g_{\uparrow \downarrow} n + g_{\downarrow \downarrow} n, \) and \( \mu \) is the chemical potential.

The mean-field Hamiltonian equation (14) can be diagonalized by generalized Bogoliubov transformation. The single-particle excitations form two branches with excitation energies given by

\[
\epsilon_{k\pm} = \left[ \xi_k^2 + \left| S(k) \right|^2 - \Delta^2 \pm 2\left| S(k) \right| \left[ \sqrt{\xi_k^2 - \Delta^2 \cos^2 \phi_k} \right] \right]^{1/2}, \tag{15}
\]

where \( \phi_k = \phi_k + \theta \) and \( \phi_k = \arg(k + ik) \). The pairing order parameters and density can be obtained self-consistently, yielding the following equations at zero temperature:

\[
\frac{1}{g_{\uparrow \uparrow}} = \frac{1}{4V} \sum_k \left[ \frac{2}{\epsilon_k} - \frac{1}{\epsilon_{k+}} - \frac{1}{\epsilon_{k-}} - \frac{\left| S(k) \right| \cos^2 \phi_k}{\sqrt{\xi_k^2 - \Delta^2 \cos^2 \phi_k}} \left( \frac{1}{\epsilon_{k+}} - \frac{1}{\epsilon_{k-}} \right) \right],
\]

\[
n = \frac{1}{4V} \sum_k \left[ \frac{\xi_k}{\epsilon_{k-}} \left( 1 - \frac{\left| S(k) \right|}{\sqrt{\xi_k^2 - \Delta^2 \cos^2 \phi_k}} \right) + \frac{\xi_k}{\epsilon_{k+}} \left( 1 + \frac{\left| S(k) \right|}{\sqrt{\xi_k^2 - \Delta^2 \cos^2 \phi_k}} \right) - 2 \right]. \tag{16}
\]

We numerically solve equation (16) and find that the mean-field solution always exists in the dilute limit \( n \rightarrow 0 \), as shown in figure 2.

In Rashbon condensation, the quasi-particle excitation energies given in equation (15) are anisotropic, depending on the angle \( \phi_k = \phi_k + \theta \). This anisotropy is stronger at low energies when \( k_\perp \) is near \( \kappa \), as shown in figure 3(a). At higher energies, the anisotropy becomes weaker and eventually disappears. This anisotropic effect is caused by the coupling between pairing order parameters \( \Delta_{\uparrow \uparrow} \) and \( \Delta_{\downarrow \downarrow} \) due to the SOC. For a spin-up atom with wavevector \( k \), SOC can flip its spin down with a phase \( \phi_k \). This phase becomes \( \phi_k + \pi \) for the spin-up atom with opposite wavevector \( -k \). These two spin-flips can turn an atom pair from total spin-up to total spin-down states with phase \( 2\phi_k + \pi \). If \( 2\phi_k + \pi + 2\theta = 2l\pi \) where \( l \) is an integer, spin-flips
Figure 2. Pairing order parameter $\Delta$ versus $1/(\kappa a)$ for different densities in the dilute limit $\kappa \gg n^{1/3}$. For fixed $\kappa$, the order parameter $\Delta$ increases monotonically with $n$ and $1/a$. At resonance, for $\kappa/n^{1/3} = 60$, the order parameter $\Delta = 0.0075\epsilon_0$ is much smaller than the binding energy $E_b = 0.132\epsilon_0$.

Figure 3. Anisotropy of the lower quasi-particle excitation energy $\varepsilon_{k_-}$ at $\mu'/\epsilon_s = -1.2$, $\Delta/\epsilon_s = 0.18$ and $k_z = 0$, where $\mu' = \mu - 2g_{11}n - g_{11}n$. (a) $\varepsilon_{k_-}$ along the $x$-axis (solid line) and $y$-axis (dashed line), plotted as functions of $k_\perp/\kappa$ for $\theta = 0$. The anisotropy is stronger at low energies and weaker at higher energies. (b) $\varepsilon_{k_-}$ at $k_\perp = \kappa$ versus $\phi_k/\pi$ for different values of $\theta$. 
are encouraged and the quasi-particle energy $\epsilon_{k-}$ is at a minimum. If $2\phi_k + 2\theta = 2l\pi$, spin-flips are discouraged and the quasi-particle energy is at a maximum. As shown in figure 3(b), the quasi-particle energy $\epsilon_{k-}$ shows a periodic behavior as a function of $\phi_k$ with period $\pi$.

In the following, we focus on Rashbon condensation in the dilute limit with attractive intra-species interaction, $\kappa \gg n^{1/3}$ and $(-a)^{-1} \gg n^{1/3}$. Since in the dilute limit the distance between Rashbons is the largest length scale, the structure of Rashbons is not affected by the weak interaction between Rashbons, which is very similar to the BEC limit of the BEC–BCS crossover in Fermi gases. In this limit, equation (16) can be solved analytically, and we find that the order parameter $\Delta$ is much smaller than the Rashbon binding energy,

$$\Delta \approx 4\sqrt{2}\pi \left( \frac{n}{k^3} \right)^{1/4} (\epsilon_c E_b) \ll E_b.$$  

The attractive intra-species interaction tends to make the system unstable. If the Rashbon condensation is stable, the positive compressibility condition $\partial \mu / \partial n > 0$ must be satisfied. We find that in the dilute limit this stability condition is given by $\kappa (a' + 2\alpha) > 3/2$. Therefore a repulsive inter-species interaction with $\kappa > 3/(2a' + 4\alpha) \gg n^{1/3}$ is required to stabilize the Rashbon condensation in a dilute Bose gas with Rashba SOC.

In the Rashbon condensation phase, in addition to single-particle excitations, there are also pair excitations. At the transition temperature $T_c$ of Rashbon condensation, pair excitations are quadratically dispersed. In the dilute limit with attractive intra-species interaction, they have effective masses approximately the same as those of Rashbons in vacuum. Since in this limit the Rashbon binding energy is much bigger than $k_B T_c$, single-particle excitations can be neglected at $T_c$, and only excited Rashbons contribute to the density at $T_c$:

$$n = \frac{1}{V} \sum_{q,s} \frac{1}{e^{\beta \left( \epsilon_q - E_0 \right)} - 1},$$

where $s = \pm$, $E_q \approx E_0 + \hbar^2 q^2 / (4m) + \hbar^2 q^2 / (2m_{\parallel})$ are Rashbon energies in the effective-mass approximation, and $\sum'$ denotes the summation over $q$ for $|q_z| \leq q_{\parallel}$. From equation (17), we obtain the transition temperature

$$T_c = \left[ \frac{1}{\sqrt{2} \left( m_+ + m_- \right) / m} \right]^{1/2} T_a \approx 0.164 T_a,$$

where $T_a = 2\pi^{2/3}(3/2)^{3/2} \hbar^2 n^{2/3} / (k_B T)$ is the critical temperature of an ideal Bose gas and $\zeta (s)$ is the Riemann zeta function. equation (18) shows that the transition temperature of Rashbon condensation $T_c$ in the dilute limit is about six times smaller than the BEC transition temperature of an ideal Bose gas.

In current experiments with $^{87}$Rb, the strength of the Rashba SOC is limited by the wavelength of the Raman laser $\lambda = 804.1$ nm: $\kappa \leq 7.8 \times 10^6$ m$^{-1}$ [1]. With background intra-species scattering length $a_{bg} = 100 a_0$ and density of the order of $10^{13}$ cm$^{-3}$ [26], the dilute region of Rashbon condensation is hardly reachable. With the new proposal for generating Rashba SOC [27, 28], if $\kappa$ can be enhanced to $2 \times 10^8$ m$^{-1}$ and the scattering lengths can be
tuned to $a = -95a_0$ and $a' > 330a_0$, Rashbon condensation may be observed around 29 nK with $n = 10^{13}$ cm$^{-3}$ in $^{87}$Rb.

5. Discussion and conclusion

We have shown that Rashbon condensation can be mechanically stable in a dilute Bose gas with Rashba SOC and weakly attractive intra-species interaction. In this dilute region, we expect the particle loss rate to be suppressed because of its density dependence. As shown in experiments on $^{85}$Rb in the dilute region [15, 17], the loss rate of Feshbach molecules is much smaller than the molecule binding energy. Now with the help of Rashba SOC, the Rashbon binding energy is exponentially small, and the lifetime of dilute Rashbon condensation is expected to be long enough for experimental observations.

There are a lot of interesting questions to be answered about Rashbon condensation. Collective excitations in this phase are worth exploring. Another important question is that of whether or not at a higher density there is a quantum phase transition between Rashbon condensation and a mixture of atom and Rashbon condensates. We plan to address these issues in future studies.

In summary, we find that two Bose atoms with Rashba SOC can form a Rashbon with any intra-species interaction. In contrast, the bound state created by the inter-species interaction is not affected by SOC. At zero center-of-mass momentum there are two degenerate Rashbons with the degeneracy protected by time-reversal symmetry. The degeneracy is lifted at finite in-plane momentum with two different effective masses. We explore the possibility of Rashbon condensation in a dilute Bose gas with Rashba SOC and attractive intra-species interaction. We find that Rashbon condensation can be stabilized by a repulsive inter-species interaction. In Rashbon condensation, the single-particle excitation energy is anisotropic, due to the coupling between pairing order parameters arising from SOC. The transition temperature of Rashbon condensation is about six times smaller than that of BEC in an ideal Bose gas.

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References

[1] Lin Y-J, Jiménez-García K and Spielman I B 2011 Nature 471 83
[2] Wang P et al 2012 Phys. Rev. Lett. 109 095301
[3] Cheuk L W et al 2012 Phys. Rev. Lett. 109 095302
[4] Zhang J-Y et al 2014 arXiv:1305.7054
[5] Li Y, Pitaevskii L P and Stringari S 2012 Phys. Rev. Lett. 108 225301
[6] Zhang J-Y et al 2012 Phys. Rev. Lett. 109 115301
[7] Wang C, Gao C, Jian C-M and Zhai H 2010 Phys. Rev. Lett. 105 160403
[8] Ho T-L and Zhang S 2011 Phys. Rev. Lett. 107 150403
[9] Yu Z-Q 2013 Phys. Rev. A 87 051606(R)
[10] Wu C-J, Ian M-S and Zhou X-F 2011 Chin. Phys. Lett. 28 097102
[11] Hu H, Ramachandhran B, Pu H and Liu X-J 2012 Phys. Rev. Lett. 108 010402
[12] Sinha S, Nath R and Santos L 2011 Phys. Rev. Lett. 107 270401
[13] Ozawa T and Baym G 2012 Phys. Rev. A 85 063623
[14] Regal C A, Greiner M and Jin D S 2004 Phys. Rev. Lett. 92 040403
[15] Donley E A, Claussen N R, Thompson S T and Wieman C E 2002 Nature 417 529
[16] Xu K, Mukaiyama T, Abo-Shaeer J R, Chin J K, Miller D E and Ketterle W 2003 Phys. Rev. Lett. 91 210402
[17] Thompson S T, Hodby E and Wieman C E 2005 Phys. Rev. Lett. 95 190404
[18] Radzihovsky L, Park J and Weichman P B 2004 Phys. Rev. Lett. 92 160402
[19] Romans M W J, Duine R A, Sachdev S and Stoof H T C 2004 Phys. Rev. Lett. 93 020405
[20] Yin L 2008 Phys. Rev. A 77 043630
[21] Jeon G S, Yin L, Rhee S W and Thouless D J 2002 Phys. Rev. A 66 011603(R)
[22] Basu S and Mueller E J 2008 Phys. Rev. A 78 053603
[23] Yu Z-Q and Yin L 2010 Phys. Rev. A 81 023613
[24] Vyasanakere J P and Shenoy V B 2011 Phys. Rev. B 83 094515
Vyasanakere J P and Shenoy V B 2012 New J. Phys. 14 043041
[25] Yu Z-Q and Zhai H 2011 Phys. Rev. Lett. 107 195305
[26] Long Z et al 2013 Phys. Rev. A 87 011601(R)
[27] Anderson B M, Spielman I B and Juzeliūnas G 2013 Phys. Rev. Lett. 111 125301
[28] Kennedy C J, Siviloglou G A, Miyake H, Burton W C and Ketterle W 2013 Phys. Rev. Lett. 111 225301