Stability of Symmetrical Comb-Drive Actuator

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Abstract. This paper reports the study, design, and simulation of a symmetrical comb-drive actuator. The approach for definition of the potential energy of the system is proposed. The electrical parameters of the comb-drive actuator are defined in COMSOL Multiphysics® software. Depending on an actuation voltage and an initial design it can form system with one, two, and three stable states. We show that the equilibrium at \( x = 0 \) is more stable for the comb-drive actuator with positive overlap than for device with the gap of the same value. The proposed approach will be used for design of the symmetrical actuator, which forms the output of the recently proposed contactless four-terminal MEMS element for capacitive adiabatic logic based on silicon MEMS technology.

1. Introduction
Smooth (adiabatic) switching between logic states is used for recover the signal energy to power supplies [1]. Unfortunately, the available hardware is a key problem to use the full benefits of adiabatic logic. For example, energy efficiency in CMOS-based adiabatic logic is limited by an inherent trade-off between the dynamic and leakage losses [2] and remains a few decades higher than the theoretical Landauer limit for irreversible logic (3 \( zJ \) at 300 K) [3]. The electromechanical relays is another candidate for adiabatic logic realization [4]. Thanks to metal-metal contact instead of a semiconductor junction, the leakage becomes almost negligible except in the case of nm-scale electrostatic gap. However, the main bottleneck of the relay-based adiabatic logic is the mechanical reliability and performance limit of the scaled switches, due to adhesion force of contact interface [5]. To overcome this limitation, we recently proposed a new logic family called Capacitive Adiabatic Logic (CAL) [6-7]. Thanks to smooth switching process in adiabatic logic, the resistive elements (transistors, relays) in a voltage divider circuit can be replaced by capacitive ones. The capacitor value can be modulated by the variation of relative permittivity, plate surface and gap thickness. In present paper we discuss an electrostatic actuator as a possibility for the integration of the MEMS relays in VSLI circuits has already been demonstrated [8].

Electrostatic actuation is widely used in MEMS technology due to simplicity and low power consumption [9]. There are two main types of electrostatic actuators: gap-closing and comb-drive (c.f. Figure 1) [10]. The gap-closing MEMS variable capacitor could be a good candidate for CAL purpose as it offers large capacitance variation if the actuation voltage is higher than pull-in voltage [11]. However, a mechanical contact is required in order to have a high capacitance variation. Consequently, this solution suffers from a collapse when the actuation voltage exceeds pull-in voltage during capacitance increasing and from damping losses during capacitance decreasing (release of the top electrode) [12]. The both cases lead to a loss of control over the moving mass and cause the non-
adiabatic losses, which is independent of the operating frequency and cannot be suppressed by the ramping time increasing. On the contrary, the comb-drive MEMS variable capacitor avoids electrical and mechanical contacts [13]. Despite the gap symmetry, the comb-drive actuator designed for high displacement can collapse in lateral direction, perpendicular to the intended travel direction, when voltage–deflection conditions are favorable [14]. As presented in Figure 1, the same stability issue appears in gap-closing capacitor with a central moving electrode and two fixed electrodes that enclose it. For the comb-drive transducer, the problem is usually solved by increasing the mechanical stiffness in lateral direction.

![Figure 1](image1.png)

**Figure 1.** Stability problem for different types of gap-closing and comb-drive actuators.

The proposed in work [15] MEMS buffer element for CAL is presented in Figure 2a. The device consists of the moving mass \(m\) with two insulated electrodes (GND, D) and two fixed electrodes (G, S). The moving mass is suspended by four identical springs with total spring constant \(k\). The two pairs of fixed and moving electrodes form an input and an output comb-drive capacitive transducers. The input (left) transducer has an initial overlap \(L_{in}\) between the fixed and the moving electrodes. The output transducer (right) is symmetric and has initial gap at the output (\(L_{out}>0\)).

![Figure 2](image2.png)

**Figure 2.** Design of the MEMS buffer element for CAL. a) Data and energy transfer in four-phase quasi-adiabatic pipeline PC. The maximal PC voltage is denoted as \(V_{DD}\). b) Evolution of \(V_{G0}\) (first graph), \(V_{G1}\) and \(V_{DSI}\) (second graph), and moving mass displacement \(x_1\) (third graph).

Adiabatic logic circuits must be able to receive the logic state from the previous gate, process it and transmit the result to the next gate, i.e. to be cascadable. They basically operate with four-phase power supplies, called power clocks (PC’s) [14]. These PC’s can provide and recover energy, i.e. realize charge recovery. As shown in Figure 2b, PC’s have a \(\pi/2\) phase shift and synchronize the transfer rate and information processing. The cascadability of the proposed 4-terminal device is depicted using an array of buffer elements. Figure 2c depicts the buffer chain circuit. The binary input logic sequence “01” is transferred through the buffer, as illustrated in Figure 2d. The logic state in further gates is coded by the moving mass displacement \(x_1\), induced by the input voltage \(V_{G0}\). It is important to avoid of self-actuation by the output voltage \(V_{DSI}\) when \(V_{G0}\) is less than the threshold voltage \(V_{TH}\). When \(V_{G0}\)
is higher than $V_{TH}$, $x_I$ is higher than $x_{TH}$. The high logic state should be maintained during the input $V_{GO}$ decreasing thanks to the output electrostatic force. Consequently, the gate transmits high state to the next element of the buffer chain ($V_{GO}>V_{TH}$).

In this work we would like to investigate the stability in indented direction of symmetrical comb-drive actuator, which forms the output of the MEMS buffer element to avoid false logic states during the information transfer. The FEM simulation results, which take into account the fringing field effect, are compared with the parallel plate approximation results. The lateral stability problem is out of scope of this paper.

2. Symmetrical comb-drive actuator

We isolate a single symmetrical comb-drive actuator. In order to differentiate the analyzed device from the MEMS buffer we change the notations here. The transducer capacitance is $C_{out}$ and the voltage across actuator is $V_{out}$. Figure 3a represents a top view of the segment of comb-drive actuator. The parameter of the combs are taken from [9]. The length of the comb $L$ is 25 um, the width $w$ is 2 um, the thickness $t$ is 40 um, the air gap $g$ is 2 um, and the number of output fingers $N_{out}$ is 220. Here we discuss the case of the thick device, i.e. $L_{out}<<t$ and $g<<t$.

![Figure 3](image)

**Figure 3.** a) Top view of the segment of symmetrical comb-drive actuator ($L_b = 2$ um). b) Comparison of analytically calculated (dashed) capacitance $C_{out}$ and normalized electrostatic force $2F_e/(V_{out})^2=dC_{out}/dx$ with FEM simulation results (solid) for $L_{out} = 2$ um. c) 3D COMSOL model of the output comb-drive segment without initial overlap. d) $C_{out}$ as function of $x$ for the different overlap $L_{out}$.

2.1. Analytical model

The electromechanical transducer has one electrical and one mechanical port. The electrical part consists of two electrical terminals. Based on a 1st order parallel-plate approximation, the output capacitance $C_{out}$ can be defined from:

$$C_{out} = \begin{cases} C_p + 2N_{out}\varepsilon_0 |x|/g, & \text{if } |x| \geq |L_{out}| \\ C_p, & \text{if } |x| < |L_{out}| \end{cases}$$

(1)

where $\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m is the vacuum permittivity and $C_p$ is the parasitic capacitance, which is zero if there is a gap between the combs ($L_{out}<0$) and equals $4N_{out}\varepsilon_0 tL_{out}/g$ if there is a initial overlap between the combs ($L_{out}>0$). The mechanical part of the system and can be described by the following equation of motion:

$$m\ddot{x} = -b\dot{x} - kx + F_e,$$

(2)

where $m$ is the mass of the central moving part, $k$ is the spring constant, and $b$ is the damping coefficient. The electrostatic force $F_e$ is proportional to a derivative of the output capacitance $C_{out}$ with respect to displacement $x$ and can be calculated from (3). The results of analytical calculation of capacitance $C_{out}$ and normalized electrostatic force $2F_e/(V_{out})^2$ are presented in Figure 3b (dashed line).
\[ F_x = \begin{cases} -V_{\text{out}}^2 N_{\text{out}} \varepsilon_0 d / g, & \text{if } x \leq -L_{\text{out}} \\ 0, & \text{if } |x| < L_{\text{out}} \\ V_{\text{out}}^2 N_{\text{out}} \varepsilon_0 d / g, & \text{if } x \geq L_{\text{out}} \end{cases} \] (3)

2.2. COMSOL FEM model

As represented in Figure 3c, the full 3D model of the segment of symmetrical comb-drive actuator with and without initial overlap is implemented in COMSOL Multiphysics® software. AC/DC Module is used to define the capacitance and the electrostatic force affecting to the moving electrode. The parameters of the model are the same as in the analytical calculation. Comparison of analytically calculated (dashed line) capacitance \( C_{\text{out}} \) and normalized electrostatic force \( 2F_e/V_{\text{out}}^2 \) with FEM simulation results (solid line) for \( L_{\text{out}} = 2 \) um are shown in Figure 3b. FEM calculated capacitance is higher than analytically calculated due to fringing field capacitance, which is ignored in the parallel plate approximation. Then, we observe presence of the electrostatic force when \( |x| > |L_{\text{out}}| \), as \( C_{\text{out}} \) is not constant against displacement in this region. Figure 3d presents the capacitance of the symmetrical comb-drive actuator as function of the moving electrode displacement for the different overlap \( L_{\text{out}} \).

2.3. Potential energy of the system

To study the stability of the symmetrical comb-drive actuator during logic state transfer, we analyze the potential energy distribution as function of the output voltage \( V_{\text{out}} \) and initial overlap \( L_{\text{out}} \). Equation (4) defines the potential energy \( U(x, V_{\text{out}}) \):

\[ U(x, V_{\text{out}}) = \frac{kx^2}{2} - \frac{(C_{\text{out}}(x) - C_{\text{out}}(0))}{2} V_{\text{out}}^2, \] (4)

There are two contributions to potential function: a positive parabolic term, representing the linear mechanical spring, and a negative parabolic term, which accounts the electromechanical coupling. Plotting \( U(x, V_{\text{out}}) \) versus \( x \) provides an interpretation of the non-linear dynamical behavior in terms of a potential well (c.f. Figure 4a-b). For calculation we have used both analytically (Figure 4a) and FEM (Figure 4b) calculated capacitance \( C_{\text{out}} \) for \( L_{\text{out}} = 2 \) um.

![Figure 4. a-b) The potential energy distribution \( U(x, V_{\text{out}}) \) as function of the applied voltage \( V_{\text{out}} \) (\( L_{\text{out}} = 2 \) um) for a) analytical and b) FEM calculated \( C_{\text{out}} \). The parameter \( k=2.39 \) N/m is taken from [15]. c) Normalized \( V_{\phi}/V_{\text{act}} \) and \( V_{\phi}/V_{\text{act}} \) as function of the normalized initial overlap \( L_{\text{out}}/g \).]

Within parallel plate approximation, electrostatic force is zero, when \( |x| < |L_{\text{out}}| \). Consequently, the output voltage increasing do not change potential energy in this displacement range. Thus, "0" state is stable, and cannot be switched by the output. On the other hand, as presented in Figure 1d, "1" state is not maintained if the output voltage less than 11.6 V. Let us denote this voltage as \( V_{f1} \). The potential curve for FEM model, which accounts the fringing field, is more complicated. Firstly, the false "1" voltage also exist and equals 21.2 V. Secondly, due to presence the electrostatic force in \( |x| < |L_{\text{out}}| \) region, the voltage increasing can destroy equilibrium at \( x = 0 \) (c.f. Figure 4b). We denote the minimal
required output voltage, which eliminates stable minimum at \( x = 0 \), as \( V_f0 \). The analysis of potential curves, based on FEM simulation, allows us to identify these two voltages for different initial overlaps \( L_{out} \). These voltages are critical to using of the symmetrical comb-drive actuator as the output of the CAL MEMS buffer. The false "0" voltage \( V_f0 \) is the upper bound of the output voltage, which guarantee the absence of false switching from "0" to "1" (c.f. Figure 2d). The false "1" voltage \( V_f1 \) is the lower bound of the output voltage, which guarantee the maintaining of the high state during decreasing of the input, i.e. false switching from "1" to "0" (c.f. Figure 2d). When the overlap is negative or equal to zero these voltages are equal and there is no possibility to build a MEMS buffer logic gate. As presented in Figure 4c, the overlap increasing allows to split these voltages and create the working region for MEMS buffer. In the graph this voltage is normalized to \( V_{act} \), equals to the voltage required to cause the displacement equal to \( L_{out} \) for unsymmetrical comb-drive transducer with the same number of electrodes (\( N_{out} \)).

The received results will be used for design of recently proposed contactless four-terminal MEMS buffer element for capacitive adiabatic logic.

3. Conclusion
The stability problem of symmetrical comb-drive transducer in presence of fringing field is investigated analytically and numerically in COMSOL Multiphysics® software and applied to the sizing of MEMS variable capacitance for CAL. The instability in the intended travel direction is observed. The approach for definition of the potential energy of the system is proposed. The using of the symmetrical comb-drive transducer with positive overlap allows to create a logic gate capable to transfer high and low logic states.

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