Modeling Ice Melting with Selective Absorption of Radiation

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Abstract. Numerical simulation of the melting of an ice layer on a vertical substrate heated by long-wave radiation in a single-phase formulation of the Stefan problem is carried out. A simple model for taking into consider a thin film of water on the irradiated surface is proposed. Ice is presented as selectively absorbing material with two absorption bands. Calculations shows satisfactory agreement with the experiment.

1. Introduction
Simulation of ice melting is needed both to understand various natural processes and ensuring the safety of population, buildings, and engineering equipment in northern latitudes. Krass and Merzlikin [1] performed a detailed analysis of the modern state of snowpack and ice thermophysics and presented formulations of a new class of problems, where snow and ice are considered as light-scattering media that possess volume absorption and reflection. Modeling of ice heating and subsequent melting are formulated as Stefan problems in a semitransparent medium. Numerical and experimental investigations of heat transfer in semitransparent media with due allowance for thermal radiation in the case of two-phase and three-phase Stefan problems were widely described.

Mathematical modeling of radiative-conductive heat transfer in the single-phase formulation of the Stefan problem was performed in [2, 3], but there are very few experimental studies that could be used verification of computational model.

Seki et al [4] performed computational and experimental investigations aimed at modeling radiation-induced melting of ice under the condition of incident long-wave and short-wave radiation fluxes from two types of lamps: halogen lamp and a lamp with a Nichrome filament. The ice layer is located on a vertical nontransparent substrate in a climatic chamber at a constant temperature of 0 °C. In the mathematical model of the process, the authors neglected the presence of a thawed water film on the surface and calculations was carried out in the single-phase Stefan's problem. They compared the rates of melting and heating of the non-radiated ice side and obtained satisfactory agreement between the experimental and computational results. The effect short-wave radiation on formation of highly rough surfaces in ice was shown. In computations, Seki et al [4] used fitting parameters and direct integration by the Bouguer law to take radiation into account.

The present paper is aimed at verification of our formulation of the problem and the method of the numerical solution of the radiative part of the problem, which takes into consider selective volume
absorption of radiation by the ice layer [3]. Verification of the computational model is carried out by comparison with the experimental data presented in [4].

2. Formulation of the Problem

Figure 1 shows the geometrical scheme of problem, where a layer of clear and non-scattering ice of thickness $L_0$ is adhered onto a substrate located vertically in a climatic chamber with a constant temperature $T_\infty$. The right surface of the ice layer is irradiated by a lamp with a filament temperature of 800 K. The radiation of such a lamp approximately models the Sun radiation in a cloudy day. The ice boundaries are gray and diffusely absorb, reflect and transmit of radiation in such a way that $A_i + R_i + D_i = 1$, where $A_i$, $R_i$, and $D_i$ are the absorption, reflection, and transmission coefficients of the ice surface, respectively, $i = 1, 2$. The left boundary of the substrate is maintained at a constant temperature $T_{sub} = 256.15$ K, which is the initial temperature $T(x, 0)$, and ambient air is kept $T_\infty = 273.15$ K.

The solution of the problem includes two stages. At the first stage, radiative-conductive heat transfer is considered until the right boundary of ice $T(L_0, t)$ reaches the phase transition temperature $T_f$. At the second stage, the Stefan problem with a fixed temperature of the right boundary $T(L(t), t) = T_f$ is considered, and it is assumed that the irradiated surface is covered by a thin water film flowing down under the action of the gravitational forces. The presence of thin water films was noted in [4], but it was neglected in calculations. The positions of the interphase boundaries $L(t)$ is determined from solution of the boundary value problem.

The equation of energy conservation for ice is written as:

$$c_p \rho \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T(x,t)}{\partial x} - E_i(x,t) \right), \quad 0 < x < L(t).$$

(1)

Here $c_p$ is the specific heat at constant pressure, $\rho$ is the density, $\lambda$ is the thermal conductivity, and $E_i(x,t) = E_i'(x,t) - E_i(x,t)$ is the flux density of the resultant radiation flux.

It is assumed that the surface of the substrate on the left boundary is maintained at constant temperature $T_{sub}$ and there is no heat flow from the substrate to the ice [1]

$$\frac{\partial T}{\partial x} = 0 \text{ at } x = 0 .$$

(2)

At the first stage, the surface of the right boundary is exposed to radiation from a long-wavelength radiation source, and heat removal due to convection is taken into account:

$$\lambda \frac{\partial T}{\partial x} - h(T_\infty - T) - |E_{res,2}| = 0 , \text{ at } x = L_0 ,$$

(3)

Where $|E_{res,2}| = A_2 (E'_i(x,t) + E') - e \sigma T^4(x,t)$. Equations (1)–(3) are added with the initial condition: $T(x,0) = T_{sub}$.
At the stage of ice melting, the temperature of the right boundary surface $x = L(t)$ is fixed: $T(x, t) = T_f$. The boundary condition (3) transforms to the Stefan condition with allowance for the thin water film formed on the surface. We assume that the water film is isothermal, and the temperature difference over its thickness is negligible:

$$
\lambda \frac{\partial T}{\partial x} + h(T_{fil} - T_{t}) - |E_{res, fil}| = \rho L \frac{\partial L}{\partial t} \tag{4}
$$

where $|E_{res, fil}|$ has the form

$$
|E_{res, fil}| = A_s \left( E'_s(x, t) + E''_s \right) - e_s \sigma_s \left( T^4(x, t) - T^4_{fil} \right), \ x = L(t). \tag{5}
$$

Here $T_f = 273.15$ K is the melting point of ice, $T_{fil} = 277.15$ K is the water film temperature and $\gamma$ is latent heat of ice melting. Condition (4) takes into account the heat removal from the external surface of the thin film of water. The volumetric thermal radiation of the film and the right side of ice is included into condition (5).

The assumption of the presence of a thin film of water on the ice surface does not contradict the single-phase approximation of the Stefan problem, since radiation does not absorb radiation in the film itself and it acts only as an additional boundary condition on the interfacial surface with constant values. The thermal problem is solved only in the thickness of ice on a vertical substrate.

The radiation flux densities in dimensionless form $\Phi'_s = E'_s / \left( 4 \sigma_s T_f^4 \right)$, $\Phi_s = \sum_j \left( \Phi'_j - \Phi'_j \right)$ and involved into (1 – 5) are determined by solving the equation of radiation transfer in a plane layer of a radiating and selectively absorbing medium with a known distribution of temperature over the layer ($j$ is the number of the spectral band) [5, 6].

Wide range possibilities in terms of solution simplicity and efficiency of obtaining results are provided by a modified mean flux method [5, 7]. In the framework of this method, the integral-differential equation of radiation transfer is reduced to the system of two nonlinear differential equations for a plane layer of a semitransparent selectively absorbing medium.

The differential dimensionless analogue of the radiation transfer equation for the hemispherical fluxes $\Phi'_s$ is presented in the following form [6, 7]:

![Fig. 1. Geometrical scheme of the problem](image-url)
\[
\frac{d}{d\tau_j} \left( \Phi_j^+(\tau, \eta) - \Phi_j^-(\tau, \eta) \right) + \left( m_j^+(\tau) \Phi_j^+(\tau, \eta) - m_j^-(\tau) \Phi_j^-(\tau, \eta) \right) = n^2 \Phi_0
\]

\[
\frac{d}{d\tau_j} \left( m_j^+(\tau) l_j^+(\tau) \Phi_j^+(\tau, \eta) - m_j^-(\tau) l_j^-(\tau) \Phi_j^-(\tau, \eta) \right) + \left( \Phi_j^+(\tau, \eta) - \Phi_j^-(\tau, \eta) \right) = 0
\]

(6)

The boundary conditions for system (6) in the dimensionless variables are written as

\[
\tau_{j,1} = 0: \Phi_j^{+1} = A_n n^2 \frac{\Theta_j^1}{4} + D_j \frac{\Theta_j^1}{4} + \left( 1 - \frac{1 - R_j}{n_j^2} \right) \Phi_j^{-1}
\]

\[
\tau_{j,1} = \alpha_j L(t): \Phi_j^{+1} = A_n n^2 \frac{\Theta_j^1}{4} + D_j F^+ + \left[ 1 - \frac{1 - R_j}{n_j^2} - A_2 \left( \frac{1 + n_j^2}{n_j^2} \right) \right] \Phi_j^{-1}
\]

\[
\tau_{j,\infty} = \alpha_j L(t) + \infty: \Phi_j^{+\infty} = F^+
\]

(7)

Here \( \Phi_0 = n^2 B(4\pi T_r^4) \) is the dimensionless density of the equilibrium radiation flux, \( B \) is the Planck function of blackbody radiation, \( n \) is the refractive index of ice, and \( \tau_j = \alpha_j L(t) \) is the spectral optical thickness of the layer at the time \( t \). The values of the coefficients \( m_j^+ \) and \( l_j^+ \) are determined from the recurrent relation derived by means of formal solution of the radiation transfer equation; \( j \) is the number of the spectral band [7]. Layers I and II refer to ice and the ambient air, respectively (Fig. 1).

The boundary value problem (1)–(5) is solved by a finite-difference method. The nonlinear system of implicit difference equations is solved by the sweep and iteration method. Solving of the radiative problem involves iterations, and the boundary-value problem (6)–(7) is solved at each iteration by the matrix factorization method. Rapid convergence of such a solution method allows obtaining results with a high degree of accuracy.

3. Analysis of Results

The results of numerical modeling presented below were obtained for a vertically located clear, non-scattering ice layer with the following physical parameters: initial ice thickness \( L_0 = 0.045 \) m, temperature of the left boundary of the substrate and initial temperature of the substrate and ice layer \( T_{\text{sub}} = 256.15 \) K, the temperature of the atmosphere inside the chamber is maintained at a \( T_r = 273.15 \) K equal to the melting point of ice \( T_f \), and constant density of the incident radiation flux \( E_r = 1162.22 \) W/m\(^2\). The thermophysical properties of ice are the thermal conductivity \( \lambda = 1.9 \) W/(m·K), thermal diffusivity \( a = 9.3 \cdot 10^{-7} \) m\(^2\)/s, and latent heat of the phase transition \( \gamma = 335 \) kJ/kg. The optical parameters are as follows: refractive index of ice \( n = 1.31 \), reflection coefficients \( R_1 = 0.5 \) and \( R_2 = 0.063 \), and emissivity of the boundaries \( \varepsilon_1 = 1 - R_1 \) and \( \varepsilon_2 = 0.97 \). The averaged spectral parameters for ice are listed in the table. In solving the problem, the heat transfer coefficient was varied: \( h = 14.35 \) W/m\(^2\) at the stage of ice heating and \( h = 8 \) W/m\(^2\) at the stage of the phase transition.

| \( j \) | \( \nu_j \), 10\(^{14}\) Hz | \( \lambda_j \), \( \mu m \) | \( a_j \), m\(^{-1} \) |
|---|---|---|---|
| 1 | 9,09–2,02 | 0,33 – 1,2 | 0,001 |
| 2 | 2,02 – 1,18 | 1,2 – \( \infty \) | 1 |

Figure 2 shows the temperature field of the ice layer during its heating (curves between 1 and 2) and melting (curves between 2 and 3). The character of the curves at the first stage is determined by
the high absorption coefficient on the irradiated surface. At the stage of heating, the curves have a monotonic and nonlinear character and a comparatively high gradient in the bulk of ice. This fact is related to the small optical thickness and high absorption coefficient of the right boundary. The curves at the second stage show a linear behavior. This fact is associated with the use of the assumption about the emergence of a thin film of water on the irradiated surface.

Figure 3 shows the density field of the resultant radiation fluxes. The curves have a linear character. The values at the left boundary are higher than those on the right boundary due to the relatively high value of reflection coefficient. At the phase transition stage (curves between 2 and 3), the curves tend to one point on the left boundary. This is due to the additional consideration of a thin film of water on a right side of ice and a decrease in the volume of the medium.

In fig. 4 and 5 melting rate and an increase in temperature of the left boundary of the ice layer and their comparison with experimental data [4] are shown, respectively. At the beginning and middle of the melting stage (Fig. 4), the calculations better coincide with the experiment, but diverge at the end. According to predictions, the ice layer melts more than the experiment shows (discrepancy is 3 cm). The increase of temperature at the left boundary of ice (Fig. 5), according to calculations, also shows a slight excess at the end of the time interval (by 0.5 °C). Thus, taking into account the appearance of a thin film of water on the irradiated surface shows satisfactory agreement with the experiment.

4. Conclusion

A numerical simulation of the melting of ice caused by the radiation of a long-wave lamp, approximately simulating the condition of solar radiation on a cloudy day are carried out. In considering the radiation part, the selective absorption of radiation by ice and the appearance of a thin film of water on the irradiated ice surface with a temperature above $T_f$ were taken into account. A comparison of the calculation of the rate of ice melting and the temperature increase of the left boundary with the experiment shows satisfactory agreement, and thus allows us to consider the verification of the single-phase Stefan problem for a semitransparent medium as realized.

![Figure 2. Temperature field of the ice layer: curves between 1 and 2 is heating stage, curves between 2 and 3 is melting stage](image1)

![Figure 3. Resultant radiation field in the ice layer: curves between 1 and 2 is heating stage, curves between 2 and 3 is melting stage](image2)
Acknowledgments

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