Complete set of electromagnetic corrections to the nucleon mass in the Nambu-Jona-Lasinio model

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Abstract

We show how to derive the complete set of electromagnetic corrections to the Nambu-Jona-Lasinio (NJL) model of the nucleon. Our results enable an accurate estimate of the electromagnetic contribution to the neutron-proton mass difference within this model. At the same time, our procedure demonstrates the way to calculate the complete set of meson corrections to the NJL model that maintains chiral symmetry.

I. INTRODUCTION

We have recently shown how to calculate all possible electromagnetic corrections, of order $e^2$, to any quark or hadronic model whose strong interactions are described nonperturbatively by integral equations [1]. Here we would like to apply our method to the three-quark NJL model of the nucleon [2] and in this way derive the expression for the neutron-proton mass difference that is due to a complete set of electromagnetic interaction within this model.

For this purpose it is useful to summarise the main model-independent results obtained in Ref. [1]. Within the framework of relativistic quantum field theory, the strong interaction Green function $G$ describing a system of quarks or hadrons is given nonperturbatively by the integral equation whose symbolic form is

$$G = G_0 + G_0 K G. \quad (1)$$

The complete set of lowest order electromagnetic corrections to the Green function $G$, denoted by $\delta G$, then follows from Eq. (1) on topological grounds:

$$\delta G = \delta G_0 + \delta G_0 K G + G_0 \delta K G + G_0 K \delta G + (G_0^\mu K^\nu G + G_0^\mu K G^\nu + G_0 K^\nu G^\nu) D_{\mu\nu} \quad (2)$$

where $\delta G_0$ is the complete set of electromagnetic corrections to the free Green function $G_0$, and $D_{\mu\nu}$ is the photon propagator that connects the appropriate currents (quantities with a $\mu$ or $\nu$ superscript). Unlike $\delta G$ which has internal photons coupled everywhere, $\delta K$ consists of the strong interaction potential $K$ with all possible photon insertions except those that start

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†The authors would like to thank the Australian Research Council for their financial support.
or finish on an external quark or hadron leg. All currents $G^\mu$, $G_0^\mu$, and $K^\mu$ are constructed by the gauging of equations method \[3\] that effectively attaches external photons in all possible ways to the corresponding strong interaction quantities $G$, $G_0$, and $K$, respectively. Using this method one obtains

$$G^\mu = G \Gamma^\mu G, \quad \Gamma^\mu = \Gamma^\mu_0 + K^\mu, \quad \Gamma^\mu_0 = G^{-1}_0 G^\mu_0 G^{-1}_0.$$  \hspace{1cm} (3)$$

With the current $G^\mu$ specified in this way, Eq. (2) can be formally solved to give

$$\delta G = G \Delta G, \quad \Delta = \delta K + G^{-1}_0 \delta G_0 G^{-1}_0 + (\Gamma^\mu G \Gamma^{\nu} - \Gamma^\mu_0 G_0 \Gamma^{\nu}_0) D_{\mu \nu}. \hspace{1cm} (4)$$

The quantity $\Delta$ as given by Eq. (4) is the key result that describes the complete set of electromagnetic corrections to any observable of the strong interaction model in question. For example, if the strong interactions admit a bound state of mass $M$ and wave function $\psi$, then the complete set of lowest order electromagnetic corrections to $M$ is given by $\delta M = \bar{\psi} \Delta \psi / 2M$. It is a feature of our approach that the gauge invariance of such electromagnetic corrections is a result of their completeness.

**II. NJL MODEL FOR THE NUCLEON**

The simplest NJL Lagrangian density $\mathcal{L}$ is defined in terms of the iso-doublet (two flavours) colour-triplet quark field $\Psi$ as

$$\mathcal{L} = \bar{\Psi} \left( i \partial - m_0 \right) \Psi + G \left[ (\bar{\Psi} \Psi)^2 - (\bar{\Psi} \gamma_5 \tau \Psi)^2 \right]$$  \hspace{1cm} (5)$$

where $\tau$ is the vector of isospin Pauli matrices, and $m_0$ is the bare quark mass \[for m_0 = 0 \text{ Eq. (5) is chiral invariant]\]. The model of the nucleon considered here is described by the three-quark wave function that satisfies a four-dimensional Bethe-Salpeter (BS)-like three-body integral equation with pair interaction kernels given by the lowest order $qq$ irreducible diagrams corresponding to the Lagrangian of Eq. (5), namely

$$v_{ij} = i G \left[ (I_s I_f I_c)_i (I_s I_f I_c)_j - (\gamma_5 \tau I_c)_i \cdot (\gamma_5 \tau I_c)_j \right] \hspace{1cm} (6)$$

where $I_s$, $I_f$ and $I_c$ are the unity operators in the Dirac, flavour and colour spaces respectively, with the subscript $i$ ($j$) indicating that the corresponding operators act in the $i$-th ($j$-th) quark’s one-particle space. In Eq. (5) and in the equations below we treat the quarks as distinguishable particles as the inclusion of antisymmetrisation can always be taken care of at the end. In the mentioned BS-like integral equation, the quark propagator $d(p)$ satisfies the (nonlinear) Dyson-Schwinger equation

$$d(p) = d_0(p) + d_0(p) \Sigma(p) d(p) \hspace{1cm} (7)$$

where $d_0(p)$ is the bare quark propagator and the dressing term $\Sigma$ is taken in the so-called Hartree approximation:

$$\Sigma(p) = i G \int \frac{d^4k}{(2\pi)^4} \{ \Lambda^\mu d(k) \Lambda_\mu - \Lambda^\mu \text{tr}[d(k) \Lambda_\mu] \}. \hspace{1cm} (8)$$
FIG. 1. The Bethe-Salpeter equation for the quark-diquark Green function.

Here $\Lambda_\mu$ is the Lorentz four-vector $(I_s I_f I_c, \gamma_5 \tau I_c)$, and the trace “tr” is over the Dirac, flavour and colour indices. The kernel of Eq. (3) is effectively separable so that the original three-body equation for the three-quark system can be reduced down to a quark-diquark two-body equation, and it is this latter form which we shall use to calculate the electromagnetic corrections. In the channel where the two interacting quarks form a scalar, isoscalar, positive parity diquark with colour $\bar{3}$, the kernel of Eq. (6) reduces to

$$v_{f_1 f_2, i_1 i_2} = 4i g_s (\gamma_5 C\tau_2 \beta^a)_{f_1 f_2} \times (C^{-1} \gamma_5 \tau_2 \beta^a)_{i_1 i_2}; \quad \beta_{ik}^a = i \sqrt{3} \epsilon_{aik}, \quad C = i\gamma_2 \gamma_0$$ (9)

where $i_1 i_2 (f_1 f_2)$ are triples of initial (final) quantum numbers of the first and second particle [2]. Then the diquark propagator is

$$D(p) = \frac{4 i g_s}{1 - 2 i g_s \Pi(p^2)} \quad \text{where} \quad \Pi(p^2) \delta_{ij} = - \int \frac{d^4 k}{(2\pi)^4} \text{tr} [\gamma_5 \tau_i d(p + k) \gamma_5 \tau_j d(k)]$$ (10)

and the quark-diquark interaction kernel is given by the quark exchange term

$$K(p', p) = \gamma_5 d(p' + p) \gamma_5 \beta^a \beta'^a.$$ (11)

With $G_0 = dD$ in Eq. (9), the resulting equation for the quark-diquark Green function has the diagrammatic form illustrated in Fig. 1.

### III. ELECTROMAGNETIC CORRECTIONS TO THE NJL MODEL

All electromagnetic corrections can be found by applying the general formulation of Eqs. (1)-(4) to the particular case under consideration. Thus in the case of a single quark, even though the solution of Eqs. (3) and (8) is known to be $d(p) = i(\hat{p} - m)^{-1}$, where $m$ is a constituent quark mass (which is not zero even if $m_0 = 0$), these equations are needed for the proper construction of the external and internal (with respect to the quark propagator) photon currents. By identifying Eq. (4) as a special non-linear case of Eq. (1) (non-linear because $\Sigma$ contains $d$) and gauging both Eq. (7) and Eq. (8), we obtain that the electromagnetic current $d^\mu$ of the dressed quark propagator satisfies the equation

$$d^\mu(p) = d(p + q) \left[ \gamma^\mu + iG \int \frac{d^4 k}{(2\pi)^4} \{ \Lambda^\alpha d^\mu(k) \Lambda_\alpha - \Lambda^\alpha \text{tr}[d^\mu(k)\Lambda_\alpha] \} \right] d(p)$$ (12)

which is linear in $d^\mu$ and can be easily solved [in Eq. (12) the momentum of the incoming photon, $q$, is contained implicitly in all the $d^\mu$ functions]. The electromagnetic corrections to the dressed quark propagator, $\delta d(p)$, can then be found in a similar manner.
We can similarly identify Eq. (10) for the diquark propagator $D$ with Eq. (1) and therefore write down the diquark current $D^\mu$ and the electromagnetic corrections to the diquark propagator $\delta D$ as

$$D^\mu = D\Pi^\mu D; \quad \delta D = D\delta\Pi D + D\Pi^\mu D\Pi^\nu D\Pi_{\mu\nu}.$$  \hspace{1cm} (13)

Finally, we can write down the complete set of electromagnetic corrections $\delta G$ corresponding to the three-quark NJL model by identifying the quark-diquark equation of Fig. 1 with Eq. (1). The resulting expression for $\Delta$ is shown diagrammatically in Fig. 2.

Clearly, our procedure for finding a complete set of electromagnetic corrections in the NJL model is of a general nature, and can be used to include complete sets of lowest order corrections due to other particle exchanges. Indeed we have recently applied our method to derive the complete set of pionic corrections to the three-quark NJL model (where previously only a part of such corrections were included [4]), as well as to some constituent quark models with confinement [5] (thereby clarifying some recent discussions regarding this matter [6,7]). As our pionic corrections are complete, axial current is conserved exactly in the limit of massless bare quarks. This feature is important for maintaining chiral symmetry in the next to leading order approximations.
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