A Bayesian Inference Driven Computational Framework Applied for the Improvement of the Eurocode 2 creep model

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Abstract. Concrete creep, defined as the deformation of concrete under sustained load, can cause cracks in tensile members, redistribution of stresses over time in composite structures, loss of prestressing force in prestressed concrete elements, and excessive long-term deflection of structural members. The concrete creep coefficient is an important input in many calculations and analyses of reinforced concrete structures. Currently, the concrete creep coefficient has been predicted by many models such as the Eurocode 2 model. This study aims to improve the prediction of the Eurocode 2 creep coefficient model at long-term by implementing a correction coefficient into the model. The Northwestern University database is used and the correction coefficient is calculated using Bayesian inference. The accuracy and efficiency of the proposed improvement and modification are demonstrated through statistical indicators.

1. Introduction

The creep of concrete is of great practical importance to structural engineers; therefore, it has been studied by many researchers [1–4]. Creep, defined as a time-dependent behavior of concrete, can affect structural behavior by violating service limit states, losing prestressing forces, or redistribution stress [5–9]. Therefore, designers must accurately predict creep strains using precise methods.

The concrete creep coefficient is an important input in many analyses and calculations of reinforced concrete structures, and it can be predicted using the Eurocode 2 model (EC2) [10], which is one of the most widely used models for predicting creep. Based on a large experimental database, the EC2 shrinkage model has been updated [11], and correction coefficients have been also proposed for the Eurocode 2 creep model [12] but the calculation of these correction coefficients are limited for a specific condition of initial time loading, relative humidity, or compressive strength. For that, it is needed to improve the creep coefficient taking into consideration the various environmental conditions and for different concrete mix composition. In order to optimize and improve a model, many optimization models were developed in the literature [13–15]; however, Bayesian inference is used in this paper as being an appropriate tool for revising and updating design codes.

The objective of this study is to evaluate the long-term Eurocode 2 creep coefficient model and to improve it by implementing a correction coefficient to the model using Bayesian inference for various environmental conditions and concrete mix composition.

2. Database and Method
2.1. Experimental database
The database used in this study is the Northwestern University (NU) database which was established in 2010-2013. The tests in this database are performed under various environmental conditions and by using different concrete mix composition such as concrete compressive strength ($f_{cm}$), relative humidity (RH), age at loading ($t_0$), sustained stress over the compressive strength at loading age $\sigma/f_{cm(t_0)}$, etc.

2.2. Eurocode 2 model
The compliance is the total load-induced strain at age $t$ per unit caused by a unit uniaxial sustained load applied since loading age $t_0$, and according to Eurocode 2 model (EC2) [10], it is given by equation (1).

$$ f(t,t_0) = 1/E_{cm(t_0)} + \varphi_{28}(t,t_0)/E_{cm28} $$

where $E_{cm(t_0)}$ is the modulus of elasticity of concrete at the time of loading $t_0$ (MPa), $E_{cm28}$ is the mean modulus of elasticity at 28 days (MPa), and the dimensionless 28-days creep coefficient $\varphi_{28}(t,t_0)$ gives the ratio of the creep strain since the start of loading at the age $t_0$ to the elastic strain due to a constant stress applied at a concrete age of 28 days.

This study aims to update $\varphi_{28}(t,t_0)$ at long-term by inserting a correction coefficient, $A$, into the formula as shown in equation (2).

$$ \varphi_{upd}(t,t_0) = A \times \varphi_{28}(t,t_0) $$

2.3. Bayesian Inference
Bayesian inference is a method of statistical inference in which Bayes’ theorem is used to deduce properties about a population or probability distribution from data. Bayes’ theorem in model form is written as shown in equation (3).

$$ P(\theta|data) = [P(data|\theta)\times P(\theta)]/P(data) $$

where $\Theta = \{\mu, \sigma\}$ represents the set of parameters of a Gaussian distribution where $\mu$ is the mean and $\sigma$ is the standard deviation. $P(\theta)$ is the prior distribution and it represents the beliefs/knowledge about the true value of the parameters, $P(\theta|data)$ is known as the posterior distribution. $P(data|\theta)$ is the likelihood distribution. Therefore, the posterior distribution is calculated using the prior beliefs updated with the likelihood.

2.3.1. Prior for $\mu$ and $\sigma^2$.
The Gaussian distribution is conjugate to itself with respect to a Gaussian likelihood function. Based on this theory, the conditional prior distribution for $\mu$ is considered as a normal with a prior mean of $m_0$ and a prior variance of $\sigma^2/n_0$ ($\mu|\sigma^2 ~ N(m_0,\sigma^2/n_0]$), and it is expressed as:

$$ \mu|\sigma^2 \propto (n_0^{1/2}/\sigma\sqrt{2\pi}) \times \exp[-0.5n_0((\mu - m_0)/\sigma)^2] $$

As $\sigma^2$ is unknown, a prior distribution should be used to describe the uncertainty about the variance before seeing the data. The variance is known to be positive, continuous, and with no upper limit, so a gamma distribution can be used as a prior distribution for variance. However, this choice does not lead to a posterior distribution in the same family or it is recognizable as any common distribution. To overcome this issue, the precision $\phi$, defined as the inverse of the variance, is used as it has a conjugate gamma prior distribution. The conjugate prior for $\phi$, $p(\phi)$ can be written as:

$$ \phi \sim Gamma(\theta_0/2, \theta_0 s_0^2/2) $$

where $\theta_0/2$ is the shape parameter, and $\theta_0 s_0^2/2$ is the rate parameter. In fact, the hyper-parameter $\theta_0$ is interpreted as the prior degrees of freedom and the hyper-parameter $s_0^2$ as the prior variance or initial prior estimate for $\sigma^2$. Therefore, the conjugate prior for $\phi$, $p(\phi)$ can be written as:

$$ p(\phi) \propto \phi^{(\theta_0/2)-1} \exp\{-\theta_0 S_0^2/2\}$$
The joint distribution for the pair \((\mu, \phi)\) is then a normal-gamma distribution, \(\text{NormalGamma}(m_0, n_0, s_0^2, \vartheta_0)\), with the four hyper-parameters \(m_0, n_0, s_0^2, \text{ and } \vartheta_0\).

### 2.3.2. Likelihood distribution \(\mathcal{L}(\mu, \sigma^2)\)
Under the assumption that the data are a random sample of size \(n\) from a normal population with mean \(\mu\) and variance \(\sigma^2\), the sampling distribution of the data is the product of independent normal distributions with mean \(\mu\) and variance \(\sigma^2\).

\[
p(A_1, ..., A_n | \mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left\{ \frac{1}{2} \frac{(A_i - \mu)^2}{\sigma^2} \right\}
\]

\[
\mathcal{L}(\mu, \sigma^2) \propto \frac{1}{(2\pi\sigma^2)^{n/2}} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^{n} (A_i - \mu)^2 \right\}
\]

After examining some calculations and considering \(\bar{A} = \frac{\sum_{i=1}^{n} A_i}{n}\), the sample variance \(s^2 = \frac{\sum_{i=1}^{n} (A_i - \bar{A})^2}{(n-1)}\) and substituting \(\sigma^2\) by \(1/\phi\), the following equation is obtained:

\[
\mathcal{L}(\mu, \phi) \propto \phi^{(n-1)/2} \times \exp \{-0.5\phi(n - 1)s^2\} \times (n\phi)^{1/2} \exp \{-0.5\phi n(\bar{A} - \mu)^2\}
\]

### 2.3.3. Posterior distribution.
The joint posterior distribution according to Bayes Theorem is proportional to the likelihood of the parameters times the joint prior distribution.

\[
p(\mu, \phi | \text{data}) \propto L(\mu, \phi)p(\mu|\phi)p(\phi)
\]

After examining some calculations, and by considering \(n_n = n + n_0, m_n = (n\bar{A} + n_0m_0)/(n + n_0), \vartheta_n = n + \vartheta_0\) and \(s_n^2 = \left\{ (n - 1)s^2 + \vartheta_0 s_0^2 + \frac{n_n}{n_n} \left[ (m_0 - \bar{A})^2 \right] \right\}/\vartheta_n\), the following equation is obtained:

\[
p(\mu, \phi | \text{data}) \propto \phi^{0.5\vartheta_n^{-1}} \exp \{-0.5\vartheta_n s_n^2 \phi \} \times (n_n \phi)^{0.5} \exp \{-0.5\phi n_n(\mu - m_n)^2\}
\]

Finally,

\[
\phi | \text{data} \sim \text{Gamma}(0.5\vartheta_n, 0.5\vartheta_n s_n^2)
\]

\[
\mu | \phi, \text{data} \sim \text{Normal}(m_n, (\phi n_n)^{-1})
\]

### 3. Results and discussion
1488 observations, extracted from 179 experiments performed under various conditions with readings after 700 days of load application, are chosen from the NU database. For each observation, the creep coefficient is calculated according to EC2 model [10]. Finally, the ratio between observed and predicted values, \(A\), are obtained for each observation. After exploring the data and to obtain more accurate results, the data is divided into six groups as shown in Table 1.

| Group | Conditions |
|-------|-------------|
| 1     | \(f_{cm} \leq 35 \text{ MPa and } \sigma/f_{cm(t_0)} < 0.3\) |
| 2     | \(f_{cm} \leq 33 \text{ MPa and } \sigma/f_{cm(t_0)} \geq 0.3\) |
| 3     | \(33 \text{ MPa} < f_{cm} \leq 35 \text{ MPa and } \sigma/f_{cm(t_0)} \geq 0.3\) |
| 4     | \(35 \text{ MPa} < f_{cm} < 45 \text{ MPa and RH} < 80\%\) |
| 5     | \(f_{cm} \geq 45 \text{ MPa and RH} < 80\%\) |
3.1. Bayesian Inference
Readings from the same experiment at different times and under the same condition cannot be independent of each other. However, it may be reasonable to assume that the ratio between the creep coefficients observed and predicted according to EC2 in random samples of concrete under various conditions and at different times are independent of each other. The normality of the data in the six groups are verified using the Shapiro-Wilk test.

A collection of random variables \((A_1, A_2, ..., A_n)\) is assumed to be independent and identically distributed from a normal population with mean \(\mu\) and variance \(\sigma^2\). Bayes Theorem leads to the posterior distribution for \(\mu\) and \(\sigma^2\) given the observed data and expressed as:

\[
p(\mu, \sigma^2 | A_1, ..., A_n) = \frac{L(\mu, \sigma^2) \times p(\mu, \sigma^2)}{\text{normalizing constant}}
\]

### 3.1.1. Likelihood distribution \(L(\mu, \sigma^2)\)

The variable \(A\) in six groups has high skewed and non-normal distribution and the histogram shows an exponential distribution. Therefore, a natural log transformation of \(A\) is applied where each variable of \(A\) is replaced by \(y = \ln(A)\). The normality of the data in the six groups is verified using Shapiro-Wilk’s test.

### 3.1.2. Prior for \(\mu\) and \(\sigma^2\)

The correction coefficient is always positive. Therefore, \(A\) is lognormally distributed. In this case, the natural logarithm of \(A\), \(\ln(A)\), is normally distributed. The prior hyper-parameters used are assumed as follows: \(m_0 = 1\), \(s_0^2 = 0.12\), \(n_0 = 60\), and \(\nu_0 = 59\).

### 3.1.3. Posterior distribution

As shown in equation (13), the posterior mean, \(m_n\), is calculated and the values of the correction coefficient for the six groups are shown in Table 2. Therefore, EC2 model underestimates the creep coefficient for the groups 2, 4 and 5, and overestimates it for the groups 1, 3 and 6.

### Table 2. Values of the correction coefficient \(A\).

| Group 1 | Group 2 | Group 3 | Group 4 | Group 5 | Group 6 |
|---------|---------|---------|---------|---------|---------|
| A       | 0.83    | 1.39    | 0.8     | 1.21    | 1.3     | 0.94    |

In this study, the statistical indicators used are BP coefficient of variation (\(\bar{w}_{BP}\) \%) [16], CEB coefficient of variation (\(V_{CEB}\)) [17], and the Gardner coefficient of variation (\(\omega_G\)) [18]. These indicators are calculated before and after the implementation of the correction coefficient in EC2 creep coefficient model (Table 3). As shown in Table 3, the implementation of the correction coefficient \(A\) provides a significant improvement on the results where \(\bar{w}_{BP}\), \(V_{CEB}\), and \(\omega_G\) have decreased by 22%, 22%, and 25%, respectively.

### Table 3. Statistical indicators of the Eurocode 2 creep coefficient model before and after the implementation of the correction coefficient \(A\)

| Before implementation of \(A\) | \(\bar{w}_{BP}\) | \(V_{CEB}\) | \(\omega_G\) |
|-------------------------------|----------------|-------------|-------------|
| After implementation of \(A\) | 0.58           | 0.50        | 0.57        |
|                               | 0.45           | 0.38        | 0.43        |

4. Conclusion

1488 observations, with readings after 700 days of load application, were selected from the NU database to improve the EC2 creep coefficient prediction by implementing a correction coefficient into
the model. After exploring the data, these observations were divided into six groups, and the correction coefficients for the six groups were calculated using Bayesian inference.

The results show that the EC2 model was significantly improved where the statistical indicators \( \bar{w}_{BP} \), \( V_{CEB} \), and \( \omega_{G} \) have been decreased by 22%, 22%, and 25%, respectively.

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