Back-projection image reconstruction using photon density waves in tissues

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ABSTRACT

The reconstruction of scattering and absorption inhomogeneities in tissues generally involves the solution of the inverse scattering problem. This is a computationally intensive task that cannot be easily performed during image acquisition. Instead, we obtain approximate spatial maps of absorption and scattering coefficients using a back-projection algorithm, similar in principle to that used in computerized tomography. Given the non-linear nature of light propagation in tissue, we expect that this approach can only give a first approximation solution of the reconstruction problem. Our preliminary results indicate that relatively accurate maps are rapidly obtained. We have reconstructed, to a first approximation, the optical parameters and positions of scattering and partially absorbing objects. Our back-projection approach employs frequency-domain methods using a light emitting diode as the light source (100 MHz modulation frequency, peak wavelength 715 nm). Data is collected from multiple linear scans of the investigated area at different projection angles, as in computerized tomography.

Keywords: Near-infrared optical tomography, back-projection, frequency-domain imaging, photon migration, photon paths, absorption coefficient, reduced scattering coefficient.

1. INTRODUCTION

Near infrared optical tomography is an attractive tool for non-invasive real time imaging. It has advantages of portability and cost effectiveness which are derived from the easily available near infrared light sources (such as light emitting diodes (LEDs), and laser diodes) and from relatively inexpensive detectors, such as photomultiplier tubes and avalanche photodiodes. However, near infrared imaging has been hindered by the highly scattering nature of human tissue in the wavelength region from 700 nm to 1300 nm. Two separate problems arise in the process of forming an image from the detection of multiply scattered light which has propagated through human tissue. First, the distribution of pathlengths through the medium does not allow one to directly apply the Beer-Lambert law and thus prevents a straightforward characterization of tissue in terms of absorption or scattering properties. Second, the multiply scattered photons traveling diffusively through the medium blur images created using the typical computerized tomography techniques commonly found in x-ray radiology.

While the first problem has been solved for homogeneous media both in the time\(^1\) and frequency\(^2\) domains, problems still exist for the case of macroscopic absorbing and scattering inhomogeneities. The presence of macroscopic inhomogeneities which have a small effect on the measured quantities can be treated using perturbation theory.\(^3,4\) Here the strength of the perturbation depends on the volume of the inhomogeneity, in comparison to the sampled volume, and the difference from background values of its scattering and absorption parameters.

The second problem can generally be approached in a similar fashion to the first. Assuming that the inhomogeneity creates a small perturbation of the measured quantities, it is possible to use a method for tomographic reconstruction employing weighted back-projection. This reconstruction scheme is similar to methods which have been applied in optical imaging by Barbour et al. to obtain absorption coefficient images in highly scattering media.\(^5,6\) It has the advantages that it requires much less computational time and always converges to a unique solution, unlike an iterative solution of the inverse problem.

The purpose of this paper is to explore the properties of inhomogeneities which can be reconstructed using the weighted back-projection approach, and, in the process, to show that two and three dimensional reconstructions of phantoms are possible using this technique.
2. BACK-PROJECTION

In x-ray tomography a narrow beam of x-rays is scanned across a patient in synchrony with a radiation detector on the opposite side of the patient. The x-ray beam is intercepted by a number of different regions with attenuation coefficient \( \mu_i \) and thickness \( t_i \). Attenuation of the beam is given by, \( I = I_0 e^{-\sum \mu_i t_i} \) where \( I_0 \) is the incident intensity and \( I \) is the detected intensity. This process is repeated at different angles to give a data set \( p( l, \theta ) = \sum \mu_i t_i \), where \( l \) is the perpendicular distance from the origin of the coordinate system to the source detector ray and \( \theta \) is the projection angle (see figure 1).

![Fig. 1. Typical x-ray back-projection scheme. The detector records x-ray attenuation \( p( l, \theta ) \) where \( l \) is the perpendicular distance from the origin to the source detector ray \( L \), and \( \theta \) is the projection angle. Back-projection transforms this data into an image \( \mu(x,y) \) where the spatial coordinates \( x \) and \( y \) locate each pixel in the reconstruction region.](image)

In order to reconstruct a back-projected spatial map of the absorption coefficient \( \mu \) one makes two assumptions:

1. The paths of the probing photons from source to detector are straight lines.
2. The attenuation from each pixel in the reconstruction region is independent from the attenuation due to all other pixels.

These are reasonable assumptions for the physical process of x-ray propagation in human tissues. Although these assumptions are not true for optical scattering of photons in human tissues, by considering the distribution of paths traveled by photons in a highly scattering medium it is possible to modify the back-projection algorithm for use with optical tomography.

In near-infrared optical tomography, photon paths in tissue are distributed in a 3-D region, sometimes called a light bundle. It is possible to associate each point in the light bundle with the photon path density at that point between source and detector for given values of absorption and scattering coefficients in a homogeneous medium. By appropriately weighting the contribution from each source detector separation to the reconstructed map, the first assumption in x-ray tomography can be generalized to near-infrared optical tomography. As a first approximation to the appropriate weighting, we employ an analytical weighting function decreasing exponentially with perpendicular distance from the source detector ray. The function has an analytical form denoted by \( w_{ij} = 2^{-d_{ij}} \) where \( w_{ij} \) is the weight given to pixel \( ij \) and \( d_{ij} \) is the distance (in units of pixels) between pixel \( ij \) and the source-detector ray. The weight function extends for two pixels on each side of the source-detector ray (each pixel represents a square with sides of length 0.053 cm).

A problem arises outside the region of experimental parameters where the second assumption is valid. This situation is illustrated in figure 2. Light propagation in highly scattering media has been
modeled by the diffusion approximation to the Boltzmann transport equation. Analytical solutions for photon flux in macroscopically homogeneous media with appropriate boundary conditions have been determined.\(^8\) The presence of inhomogeneities which have a small effect on the measured quantities can be treated within the framework of perturbation theory.\(^3\),\(^4\) The change in intensity due to an absorbing inhomogeneity which is small compared to the source detector separation is related to the photon path density at the location of the inhomogeneity. Therefore, in order to reconstruct multiple inhomogeneities, each object must not affect the photon path density at other points in the light bundle. This approximation can only be valid for inhomogeneities which have a small effect on the measured quantities which include intensity, amplitude, and phase of the photon density wave. Theories which treat inhomogeneities in terms of perturbation theory predict that the effect of an inhomogeneity increases with size relative to source detector separation and with difference from background values of absorption and scattering.\(^3\) The experiment in this paper is aimed at the reconstruction of phantoms with differing values of absorption relative to the background.

3. EXPERIMENTAL SETUP

The experimental measurements of the intensity, amplitude, and phase of the photon density wave (DC, AC, and phase) were made using a frequency-domain spectrometer\(^9\),\(^10\) attached to an X,Y,Z positioning scanner (see figure 3). The source used was a light emitting diode (peak wavelength 715 nm) whose intensity was modulated at 100 MHz. Light was collected with a 3 mm fiber optic bundle facing the source and scanned synchronously with the source. The data were collected in horizontal slices. Each slice consists of 12 projections at 30\(^\circ\) angle increments from 0\(^\circ\) to 360\(^\circ\) around the region of interest. Each projection consists of 101 points acquired from a source detector pair scanning a length of 4 cm at a separation of 4 cm. A value of absorption and scattering coefficient is calculated for each point using a diffusion model for photon transport,\(^11\) and an image of the phantoms is then reconstructed using a weighted back-projection technique\(^7\). The precision of these calculated absorption and scattering coefficients, important for imaging purposes, is about 1% or smaller. The phantoms used in these experiments are glass spheres with diameters of 1 cm (centers placed two centimeters apart), blown from 3 mm od. pyrex tubing, and are filled with various solutions.
Fig. 3. Block Diagram of frequency-domain instrumentation. A 100 MHz RF signal generated by synthesizer S2 is sent to the 715 nm LED producing a photon density wave which is perturbed by the two phantoms. The multiply scattered light is collected by a fiber optic bundle which is connected to a photomultiplier tube (PMT). A cross correlation electronics system processes the PMT signal using a digital acquisition method. This measurement is repeated 101 times as the source and detector are scanned across the area of interest in the direction of arrows. After scan #1 the scanning direction is rotated 30° relative to its original position and the scan is repeated. The data for each slice consists of 12 projections at 30° increments from 0 to 360°.

4. EVALUATION OF WEIGHTED BACK-PROJECTION METHOD FOR DIFFERENT STRENGTH PERTURBATIONS

This experiment consisted of three reconstructions made from measurements on two glass spheres filled with solutions with absorption and scattering coefficients similar to those of the background medium. The solutions inside the spheres are designed to create inhomogeneities which have differing effects on the measured absorption coefficient in each reconstruction (see figure 4). The spheres in figure 4 (a) are filled with the background medium and show a decrease in the effective absorption coefficient from background values in the vicinity of the phantoms. By adding concentrations of India ink inside the spheres for figures 4 (b) and 4 (c) it is possible to increase the effective absorption coefficient of the phantoms thus decreasing the perturbation in absorption coefficient from the background medium. The intrinsic absorption of the spheres in figure 4 were $\mu_a=\mu_{a0}$ (same as background), $\mu_a=0.050$ cm$^{-1}$, and $\mu_a=0.090$ cm$^{-1}$ respectively. The background medium employed in this experiment consisted of a mixture of 2% milk and water with a $\mu_{a0}=0.011$ cm$^{-1}$ and $\mu_{s0}=12$ cm$^{-1}$ for figures 4 (a) and (b), and an Intralipid mixture (0.67% solids content) with $\mu_{a0}=0.018$ cm$^{-1}$ and $\mu_{s0}=7.4$ cm$^{-1}$ for figure 4 (c). Figures 4 (a), (b), and (c) report values that are normalized to the background. Hence, lighter values represent smaller perturbations with larger effective absorption coefficient, while darker regions represent larger perturbations and lower effective absorption coefficients.

The results of the three reconstructions for different concentrations of India ink inside glass spheres shown in figure 4 can be explained by separating the contributions to the reconstruction from the glass spheres and the absorbing solution they contain. We have found that the effect of a transparent object is qualitatively opposite that of a totally absorbing object in photon migration measurements. This is shown in our results (see figure 4 (a)). The effective absorption is lowered when a transparent object is located between source and detector because photons which reach the object may be “piped” across it thus increasing the intensity from background levels. This result is in agreement with what Sevick et al. have found, and is also the dominant effect seen in data for figures 4 (b) and (c). The effect of the ink...
solution is to increase effective absorption by decreasing the intensity and decreasing the average mean free path from source to detector. Superimposing these two effects we see that the perturbation due to the filled glass spheres actually decreases with increasing concentrations of ink. This effect allows us to evaluate the weighted back-projection method for different strength perturbations in absorption coefficient. These measurements begin to explore the region where the principle of linear superposition is valid and weighted back-projection is an effective reconstruction method.

5. THREE DIMENSIONAL RECONSTRUCTION OF PHANTOMS

This experiment consisted of eleven slices over a vertical range of 4 cm. The background medium employed in the three dimensional reconstruction consisted of an Intralipid mixture (0.67% solids content) with \( \mu_{a0} = 0.018 \text{ cm}^{-1} \) and \( \mu'_{s0} = 7.4 \text{ cm}^{-1} \). The glass phantoms were filled with a mixture of Intralipid and India ink (\( \mu_a = 0.090 \text{ cm}^{-1}, \mu'_s = \mu'_{s0} \)) and were placed two centimeters apart. The purpose of the experiment represented in figure 5 is to demonstrate that three dimensional reconstructions are possible using the weighted back-projection technique. Here the calculated value of scattering coefficient is lower than the background value because of the effect of the glass spheres. The maximum difference from background values of effective scattering coefficient is 4%. The two spheres are clearly resolved in multiple slices taken at varying distances along the z axis.
Fig. 5. Three dimensional reconstruction of two glass spheres filled with an Intralipid/ink solution. Glass spheres are 1 cm in diameter and separated by 2 cm. The Intralipid/ink solution inside the glass spheres has the same scattering coefficient as the background. Phantoms are represented as volumes of lower scattering coefficient due to the effect of the glass. Each axis ranges from 0 to 4 cm originating a reconstructed volume of 64 cm$^3$. The reconstruction of the effective scattering coefficient in this volume is shown by one isosurface of scattering coefficient located at 7.10 cm$^{-1}$. Data for the reconstruction was taken in 11 slices. Each slice was reconstructed in the xy plane, consisting of 12 projections rotated at 30° increments from 0 to 360°. Each projection consisted of 101 points at which the phase and the DC intensity of the photon density wave was measured in the presence and in the absence of the glass spheres. From this information, a value of effective scattering was calculated and employed in the weighted back-projection algorithm. The background values for absorption and scattering are: $\mu_a=0.018$ cm$^{-1}$ and $\mu'_s=7.4$ cm$^{-1}$. The three dimensional rendering was done by Slicer (Spyglass Inc.) with a magnification of 9:1 in the z axis from reconstructed slices with dimensions 74 pixels by 147 pixels.

6. CONCLUSION

We have shown that the weighted back-projection method for reconstruction of inhomogeneities in a highly scattering medium is possible for small perturbations of the measured absorption and scattering coefficients in two and three dimensions. In the process of doing this we have confirmed that the effect of the glass in the spheres is to decrease the calculated absorption and scattering coefficients. The next step would be to subtract the effect of the glass in order to isolate the effect of increased phantom absorption on the contrast of the weighted back-projection reconstructions.
Further research must be done in order to quantify the limits of this technique. Other factors affecting the limits of contrast and resolution of the weighted back-projection method include:

1. scattering coefficients of the inhomogeneities differing from background values.
2. size of the inhomogeneities.
3. separation distance between inhomogeneities.
4. non-symmetric distribution of inhomogeneities.

Some experimental challenges involve construction of phantoms which create perturbations similar to those in human tissue which have adjustable, precise optical properties. The outcome of these experiments will ultimately determine the utility of this reconstruction technique for imaging human tissue in real time.

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