Application of unlimited and limited anisotropic core compression results for wellbore stability calculations

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Abstract. The study touches upon the issue of wellbore stability in wells drilled in anisotropic rocks. Initial data for solving this task is the results of anisotropic core samples testing for unlimited and limited compression taken from one section of the whole core and drilled in different directions. These data are taken from the works of foreign authors that are collected and presented in [1]. The problem of wellbore stability drilled in multi-layer highly anisotropic rocks is solved by applying a strength criterion that includes stress tensor components and the angle between the normal line to the slip area and the direction of lamination. Given that the values of the elastic modulus and Poisson's ratio for the anisotropic rocks under consideration in the main directions of anisotropy differ insignificantly, the stress distribution around the wellbore is accepted without taking into account the anisotropy of the elastic properties. It is believed that strength anisotropy is the cause of rock destruction. Thus, the components of the stress tensor are determined using the superposition method by summing the solutions of the classical Kirsch and Lame problems.

1. Introduction

Sedimentary rocks have laminar bedding by origin regardless of their mineralogical composition. The variability of the physical and mechanical properties of these rocks is determined by the direction of external loads. This property of rocks is called anisotropy. The anisotropy is mainly related to its texture. When the borehole intersects anisotropic rocks, the well trajectory may deviate from the planned one due to uneven drilling of the bottom hole.

The task of mud density determining that ensures the wellbore stability is complex when the inclination is more than 20°÷30° and requires the use of strength criteria that takes into consideration anisotropy.

The problem of ensuring wellbore stability in the case of anisotropy is solved by considering two cases of rock failure. In the first case, it is assumed that the shear fracture condition is for the main rock with contribution of stresses by Mohr-Coulomb linear criterion for isotropic rock, and in the second case, the shear fracture condition for weakened surfaces is tested using the same criterion, but with consideration of strength parameters that according to the weakening [2,3]. This approach eliminates the possibility of rock shear fracture on a certain surface, where the shear process covers both the main rock and the weakened layer.

For the first time, this approach for determining the destructive load was proposed by V. V. Sokolovsky [4], who applied the Coulomb-Mohr criterion for multilayer rocks. He called the condition...
when a shear fracture occurs in the weak direction as "special condition of the limit equilibrium", and the fault in an unweakened direction as a "normal condition".

In foreign and Russian literary sources, the influence of anisotropy on the stress-strain state is considered according to the research of S. G. Lekhnitskii [5]. This includes the works of B. Amadea [6], N. R. Rabinovich [7] in [2].

2. Research methods

In our work, the multilayer rock is assumed in such a way that the fracture condition is generally provided in directions that do not coincide with the layering direction. Only in the special case, the shear areas coincide with the direction of the least resistance.

To solve the task of boundary state of stress and wellbore stability we use two strength criteria. The first criterion obtained is based on the Mohr nonlinear regularity and takes place in the form [8]

\[
\sigma_1 - \sigma_3 = 2\sqrt{T_1^2 + T_2^2},
\]

where

\[
T_1 = \sqrt{(\sigma + T_2)k + c};
\]

\[
T_2 = (z_1 - z_1^2 + (\sigma k + c)\cos^2(\psi + \delta))^2.
\]

\(k\) and \(c\) are the strength characteristics of the rock, depending on the rock strength along, across the layering and at 45° angle to the layering direction; \(\psi\) is the angle between the normal line to the slip area and the layering direction; \(\delta\) is the angle between the main stress and the \(x\)-axis; \(z_1 = 0,5k \cdot \cos^2(\psi + \delta)\).

The second criterion is based on the Coulomb-Mohr linear dependency and has the following compact form [9]

\[
\sigma_1 - \sigma_2 = \left[F_1^2(\psi) + F_2^2(\psi)\right]^{\frac{1}{2}},
\]

where

\[
F_1(\psi) = 2\left[s\sin(2\psi - \rho) - 0,5\rho'\sec\psi\sin2\psi\right] + 0,5s'\cos(2\psi - \rho)x;
\]

\[
F_2(\psi) = 2\left[s\cos(2\psi - \rho) - 0,5\rho'\sec\psi\sin2\psi\right] + 0,5s'\sin(2\psi - \rho)x;
\]

\[
x = x(\psi) = \frac{\cos\rho}{1 - 0,5\rho'}; s = c(\psi) + k(\psi)\sigma;
\]

\[
\sigma = 0,5(\sigma_x + \sigma_y), s' = c'(\psi) + k'(\psi)\sigma.
\]

Here \(c(\psi)\) and \(k(\psi)\) are the grip and angle of internal friction; \(\sigma_1, \sigma_2, \sigma_3\) are main stresses.

To determine the strength parameters included in the linear criterion variables of conjunction areas and angle of internal friction, a method of their determination was developed, it is based on results of core sample testing for unlimited compression (no lateral pressure) and limited compression (core samples are pressed by the initial slight lateral pressure).

We assume that during the test for uniaxial compression, the rock shear occurs in the direction of stratification according to the Coulomb-Mohr in the following form:

\[
\tau_n = \sigma_n k_{90} + c_{90},
\]

where \(k_{90}\) and \(c_{90}\) are strength characteristics along with weakened layers determined from two conditions.
\[ \tau_{n_1} = \frac{\sigma_1 - \sigma_3}{2} \sin 2\beta; \]
\[ \sigma_{n_1} = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\beta, \]

where \( \beta \) is the angle between \( \sigma_1 \) and the direction of the layers.

When \( \sigma_3 = 0 \), we give the values of \( \sigma_1 \) and \( \beta \) for two cases where it is possible and define the characteristics \( k_{90}, c_{90} \).

In [10-12], the results of experimental studies for mechanical characteristics determination for various anisotropic rocks are presented. The same experimental studies are given in [13] for the Carbona Phyllite rock in uniaxial and triaxial stress states. In this case, the following minimum initial data are accepted for the breaking stress \( \sigma_1(\beta) \):

\[
\begin{align*}
\sigma_1(0^\circ) &= 11533 \text{ psi} = 79.5 \text{ MPa}; \\
\sigma_1(30^\circ) &= 7045 \text{ psi} = 48.6 \text{ MPa}; \\
\sigma_1(60^\circ) &= 5427 \text{ psi} = 37.4 \text{ MPa}; \\
\sigma_1(65^\circ) &= 5584 \text{ psi} = 38.5 \text{ MPa}; \\
\sigma_1(90^\circ) &= 9446 \text{ psi} = 65.1 \text{ MPa}.
\end{align*}
\]

When \( \beta = 60^\circ \) \( \sigma_1 = 37.4 \) MPa and \( \beta = 65^\circ \) \( \sigma_1 = 38.5 \) MPa we calculate \( c_{90} = 10.7 \) MPa, \( k_{90} = 0.586 \).

By taking the values \( k_{90} = k_{cp} = 0.586 \) from criterion (2) it is possible to calculate \( c_0 = 26.2 \) MPa.

After, when \( k_{90} = 0.586 \) and \( \sigma_1 = 79.5 \) MPa when \( \beta = 0 \) we calculate \( k_0 = 0.6 \).

Using the same method, we determine the initial data of the rock as the conjunction and the angle of internal friction along and across the stratification.

In the same sequence, we calculate the initial parameters for the Austen Slate [14]:

\[
\begin{align*}
c_{90} &= 21.5 \text{ MPa}, \\
c_0 &= 113.8 \text{ MPa}, \\
\rho_0 &= 0.3575, \\
\rho_{90} &= 0.34.
\end{align*}
\]

It should be noted that for calculation of these characteristics, the following data was taken: \( \sigma_3 = 35 \) MPa, \( \sigma_1 = 328.6 \) MPa (when \( \beta = 0^\circ \)), \( \sigma_1 = 346.5 \) MPa (\( \beta = 90^\circ \)), \( \sigma_1 = 195 \) MPa (\( \beta = 40^\circ \)), \( \sigma_1 = 149.3 \) MPa (\( \beta = 60^\circ \)).

3. Analysis and results

To verify the reliability of the proposed approach, we will use the results of the mentioned above testing two types of anisotropic rocks for unlimited (Carbona Phyllite) and limited (Austen Slate) compression.

![Figure 1. Dependency graphs of the ultimate stress on \( \beta \) for Carbona Phyllite](image-url)
Figure 1 shows dependency graphs of $\sigma_1$ on the angle $\beta$ for different values of lateral pressure, based on the criterion (2) for Carbona Phyllite. Also, dashed lines show the same dependencies, according to the J. C. Jaeger criterion [15].

In another example, we observe anisotropic Austen Slate rock whose strength characteristics under compound stress were studied in [14]. The results of core samples testing at low (up to 35 MPa) and high (up to 276 MPa) lateral pressures are presented. No results for the unlimited pressure test. Based on characteristics according to criterion (1), it is possible to determine the extreme breaking loads for Austen Slate (Figure 2, solid lines).

Using the criterion (2), theoretically, we can get breaking stress at unlimited compression for this rock (Figure 2, bottom line). Taking these characteristics as initial parameters, it is possible to determine destructive stresses depending on the inclination angle of the layers at different lateral pressures from criterion (1) (Figure 2).

The dash-line is constructed according to the J. C. Jaeger criterion [15].

The stress distribution around the wellbore is complying with laws that determine the rock behavior in the elastic or elastoplastic stage. Another factor that affects stress distribution is the anisotropy of elastoplastic properties, the different modularity of rocks elastic characteristics (for example, the difference in elastic modulus during tension and compression), and others.

In case of well inclination in a multi-layer with an arbitrary layer orientation, the stress tensor components can be determined according to S. G. Lehnitsky’s decision [5], taking the rock model as linearly elastic. This model allows the influence of elastic characteristics on the stress distribution around the wellbore.

As the research results show [2] stresses in the circumferential direction are slightly depend on the anisotropy of the rock elastic properties, while in some cases the anisotropy can significantly affect the stress state around the wellbore.

However, considering that change the energy distribution occurs during the transition from elastic to a plastic state and such a transition cannot be tracked, in the first approximation, we don’t have to
consider the influence of elastic constants on the stress distribution. The rate of elasticity dynamic modulus for different anisotropic rocks in the main anisotropy directions usually slightly different from each other within 30%. The study of N. R. Rabinovich [7] shows that even if $E/E_1 = 2$ (E is determined along and E1 is defined across the stratification), anisotropy does not affect to change in the polar angle for the circumferential stress.

Based on this, assuming that the elastic characteristics, in this case, do not affect the stress distribution around the well, stress tensor components can be determined from the total solutions of Kirsch and Lame’s classical tasks. Then, we get for effective stresses [16]

$$
\begin{align*}
\sigma'_r &= p_w - \beta_B p_r, \\
\sigma'_\theta &= \sigma_x + \sigma_y - 2(\sigma_y - \sigma_x) \cos 2\theta - p_w - \beta_B p_r, \\
\sigma'_z &= \sigma_z - (2(\sigma_x - \sigma_y) \cos 2\theta + 4\tau_{xy} \sin 2\theta) - \beta_B p_r, \\
\tau_{\theta z} &= 2(-\tau_{xz} \sin \theta + \tau_{yz} \cos \theta),
\end{align*}
$$

(4)

$p_r$ – formation pressure; $p_W$ – well pressure; $\sigma'_r$, $\sigma'_\theta$, $\sigma'_z$, $\tau_{\theta z}$ – stress in cylindrical coordinates; $\beta_B$ – Biot’s coefficient; $\theta$ – polar angle measured from the x-axis coinciding with the direction $\sigma_H$; $\sigma_x$, $\sigma_y$, $\sigma_z$, $\tau_{xy}$, $\tau_{xz}$, $\tau_{yz}$ – stress tensor components defined as functions from $\sigma_V$ (main full vertical normal stress) $\sigma_H$ (main full horizontal normal stress) and $\sigma_h$ (minimal horizontal normal stress), and the wellbore inclination angle $i$ and from the azimuth angle $\alpha$ associated with the $\sigma_H$.

Consider two options of the well direction axis: along the $\sigma_H$ and the $\sigma_h$.

The main effective stresses acting on wellbore are calculated by this formula

$$
\sigma'_{1,3} = \frac{\sigma'_z + \sigma'_\theta}{2} \pm \frac{1}{2} \sqrt{(\sigma'_z - \sigma'_\theta)^2 + 4\tau_{\theta z}}.
$$

(5)

the direction of $\sigma'_{1,3}$ is calculated from

$$
t g 2\alpha_0 = \frac{2\tau_{xy}}{\sigma'_z - \sigma'_\theta}.
$$

(6)

$\alpha_0$ is the angle between $\sigma'_z$ and $\sigma_1$.

Inclination angle towards rock layers $\sigma'_1$ is found by the formula

$$
\delta = i + \alpha_0.
$$

The sequence of mud density determining that provides inclined wellbore stability:

1. For the given well inclination angle $i$ (figure 3), also known values of $\sigma_V$, $\sigma_H$ and $\sigma_h$, the main stresses and their directions are determined by (5) and (6).

2. Mud density is calculated by substituting the obtained stresses in criteria (1) and (2).

\textbf{Figure 3. Wellbore position}
In a particular case, we assume that a well with a given inclination angle was drilled in a multilayer rock and the layers are located horizontally. It is necessary to determine mud density.

Example of mud density calculation that provides wellbore stability with the following initial data: \( \sigma_v = 52 \text{ MPa}, \ \sigma_H = 42 \text{ MPa}, \ \sigma_h = 35 \text{ MPa}, \ \sigma_c(0) = 79.53 \text{ MPa}, \ \sigma_p(0) = 7.95 \text{ MPa}, \ \sigma_c(90) = 65.14 \text{ MPa}, \ \sigma_p(90) = 6.51 \text{ MPa}, \ \sigma_c(45) = 42.75 \text{ MPa}, \ \sigma_p(45) = 4.28 \text{ MPa} \). The depth is \( h = 2000 \text{ m} \). Formation pressure \( p_r = 20 \text{ MPa} \), well inclination angle \( i = 50^\circ \). The ratio of the rock strength at separation (rupture) to the rock strength at unlimited compression \( \frac{\sigma_c}{\sigma_a} \) is assumed to be 0.1.

Substituting this data into the nonlinear criterion (1), we determine the mud density during the drilling in direction \( \sigma_H \) is \( \rho_m = 1448 \text{ kg/m}^3 \). During the drilling in direction \( \sigma_h \) we get \( \rho_m = 1206 \text{ kg/m}^3 \).

At \( \sigma_c(0) = 113.8 \text{ MPa}, \ \sigma_c(90) = 21.5 \text{ MPa}, \ \rho_0 = 0.3575, \ \rho_{90} = 0.34 \) for \( \alpha = 0^\circ \) (drilling in \( \sigma_H \) direction) is calculated from non-linear criterium (1) \( \rho_m = 1454 \text{ kg/m}^3 \).

At \( \alpha = 90^\circ \) (drilling in \( \sigma_h \) direction) \( \rho_m = 1327 \text{ kg/m}^3 \).

Thus, with such a tectonic regime, drilling in the direction of the minimum horizontal stress is more appropriate.

4. Conclusions
1. The method for calculating the mechanical parameters included in the strength criteria based on the results of unlimited and limited compression is given.
2. The extreme ultimate stresses for two types of anisotropic rocks in complex stress are determined theoretically.
3. An example of wellbore stability calculation based on the strength criteria developed by the authors in multi-layer highly anisotropic rocks is given.

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