Production of fermions in models of string cosmology

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Abstract

Production of spin 1/2 fermions and gravitinos by the standard mechanism of amplification of quantum fluctuations during the dilaton-driven inflation phase in models of string cosmology is highly suppressed. Constraints on string cosmology models from gravitational production of gravitinos, contrary to expectations, are similar to constraints on other models.

1 Introduction

It is well known that particles are produced in cosmological backgrounds by amplification of vacuum fluctuations during inflationary epochs [1]. In models of string cosmology [2, 3], most of the produced particle spectra were found to be very different from the spectra in other cosmological models. First, in string cosmology models, gravitational waves may be generated in a substantial amount, in contrast to slow-roll inflation which predicts a “desert” [4, 5]. Furthermore, axions and other scalar particles have phenomenologically interesting spectra [6-10]. Finally, it is important to note that in addition to
scalar particles and gravitational waves, photons can be produced [11, 12]. In general relativity, gauge field perturbations are not amplified in conformally flat cosmological backgrounds (even if inflationary) because of the scale invariant coupling of gauge fields in four dimensions. However, in string cosmology, the presence of a time-dependent dilaton prefactor in front of the gauge-field kinetic term breaks scale invariance and results in photon production which may even lead to the creation of macroscopic magnetic fields [11, 12].

We study fermion production by the standard amplification of quantum fluctuation in models of string cosmology. Although expectations were that fermions are copiously produced, we have found that production of spin 1/2 and spin 3/2 fermions in models of string cosmology during the dilaton-driven inflationary (DDI) phase is highly suppressed.

It is known that massless spin 1/2 fermions are not amplified in conformally flat cosmological backgrounds [13]. This is a consequence of their conformally invariant interactions. We show that in models of string cosmology the presence of a time-dependent dilaton prefactor in front of the fermion kinetic term does not result in amplification of fermionic perturbations. The spin-3/2 gravitino, the supersymmetric partner of the graviton, can be produced in conformally flat cosmological backgrounds, since gravitinos do not have conformally invariant couplings. It was recently shown that if gravitinos have vanishing zero-temperature, flat background mass, then they are not produced [14-18]. We show that, in models of string cosmology, the appearance of the dilaton prefactor in front of the gravitino kinetic terms does not result in amplification of massless gravitino perturbations, and therefore that gravitino production during DDI is negligible.

We consider fermion production in models of string cosmology which realize the pre-big-bang scenario [2]. In this scenario the evolution of the universe starts from a state of very small curvature and coupling and then undergoes a long phase of DDI and at some later time smoothly joins standard radia-
tion dominated (RD) cosmological evolution, thus giving rise to a singularity free inflationary cosmology. Particles are produced during the period of DDI by the standard mechanism of amplification of quantum fluctuations. We assume throughout an isotropic and homogeneous four dimensional flat universe, described by a FRW metric. As in [4], the mass of the fermions is assumed to vanish in the early universe, i.e. in the dilaton driven period, and to take a nonzero value from the start of the RD era and on. All fields, except the dilaton and metric, are assumed to have a trivial vacuum expectation value during the inflationary phase.

Fermion production, and in particular gravitino production, has the potential of placing severe constraints on models of string cosmology. If it turned out that gravitinos are maximally produced during DDI as gravitons are (a reasonable assumption) then after supersymmetry is broken, gravitinos can become by far the dominant energy component in the universe. The universe would become matter dominated very early, and the resulting cosmology will be very different than the standard RD cosmology. The issue of gravitino production during the evolution following the DDI phase is still unresolved, and we do not discuss thermal production which may turn out to put tight constraints on models of string cosmology, as it does for other models of inflation.

2 Spin 1/2 fermions

We show here that massless spin 1/2 particles are not produced during DDI in models of string cosmology. We first explicitly show using their equations of motion that they are not produced, and then show that they are not produced even for a space and time dependent dilaton by showing that the low energy string effective action of massless spin 1/2 fermions can be transformed into a conformally invariant action.

In the low energy effective action, a dilaton prefactor appears in front of
the kinetic term,

$$
\mathcal{A} = \int d^4 x \sqrt{-g} e^{2\phi} \left\{ \frac{i}{2} \left[ \bar{\psi} \gamma^\mu D_\mu \psi - \left( D_\mu \bar{\psi} \right) \bar{\gamma}^\mu \psi \right] - M \bar{\psi} \psi \right\}.
$$

(1)

Greek letters denote space-time indices and Latin letters denote tangent-space indices. Gamma matrices and the group generators for spin 1/2 irreducible representations with curved indices are defined by

$$
\hat{\gamma}^\mu = e^\mu_a(x) \gamma^a, \ \hat{\Sigma}^{\lambda\nu} = \Sigma^{ab}_\lambda e^\lambda_a e^{\nu}_b
$$

(2)

where $e^\mu_a$ is the vierbein. $D_\mu$ is the covariant derivative with respect to the spinorial structure

$$
D_\mu = \partial_\mu + \Gamma_\mu = \partial_\mu + \frac{1}{2} g^{\lambda\delta} \Gamma_\delta^{\lambda\nu} \hat{\Sigma}^{\lambda\nu}.
$$

(3)

Fermions have $l = -1$, but we keep parameter $l$ general, since our results do not depend on its specific value.

Variation of the action with respect to $\bar{\psi}$ yields the Dirac equation,

$$
i \hat{\gamma}^\mu D_\mu \psi - M \psi + \frac{i}{2} l \hat{\gamma}^\mu \partial_\mu \phi \psi = 0.
$$

(4)

In the simplified model of background evolution we adopt, the evolution of the universe is described by (conformal) time dependent dilaton $\phi(\eta)$ and a spatially flat FRW metric, in which the line element can be written as $d^2 s = a^2(\eta) (d^2 \eta - d^2 \vec{x})$. Here $\vec{x} = (x^1, x^2, x^3)$ are comoving space coordinates and $x^0 \equiv \eta$ is the conformal time. Thus, the vierbein can be written as $e^\mu_a = a \delta^\mu_a$, $e^\mu_a = a^{-1} \delta^\mu_a$. In this case, the equation of motion (4) reduces to

$$
\gamma^a \partial_a \psi + \frac{1}{2} a^\prime a^{-1} \hat{\Sigma} \psi + i M a \psi + \frac{l}{2} \partial_0 \phi \gamma^0 \psi = 0,
$$

(5)

where $\hat{\Sigma} = (\Sigma^{01}, \Sigma^{02}, \Sigma^{03})$. Choosing a gamma-matrix representation such that

$$
\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \hat{\gamma} = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix},
$$

(6)
the field $\psi$ satisfies the following equations of motion

$$
\begin{align*}
\left( \partial_0 + i M a + \frac{l}{2} \partial_0 \phi + \frac{3 a'}{2} a \right) \psi_1 + \sigma^i \partial_i \psi_2 &= 0 \\
\left( \partial_0 - i M a + \frac{l}{2} \partial_0 \phi + \frac{3 a'}{2} a \right) \psi_2 + \sigma^i \partial_i \psi_1 &= 0.
\end{align*}
$$

Introducing the field $\chi$,

$$
\psi(\eta, \vec{x}) = a^{-3/2} e^{-\frac{l}{2} \phi} \chi(\eta, \vec{x}),
$$

equations (7) reduce to

$$
\begin{align*}
(\partial_0 + i M a) \chi_1 + \sigma^i \partial_i \chi_2 &= 0 \\
(\partial_0 - i M a) \chi_2 + \sigma^i \partial_i \chi_1 &= 0.
\end{align*}
$$

Substituting (1) into (11) and vice-versa, and writing $\chi$ as $\chi(\eta, \vec{x}) = \chi(\eta) e^{ikx}$, we arrive at

$$
\left( \partial_0^2 + k^2 + M^2 a^2 \pm i M a \right) \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = 0.
$$

Finally, the equation of massless spin 1/2 fermions is reduced to the equation in flat space-time and therefore these particles are not produced.

Let us turn now to the massless low energy effective action of spin 1/2 particles and show that it is conformally invariant, even for a space and time dependent dilaton and scale factor. Under the conformal transformation

$$
g^{\mu \nu} \rightarrow \bar{g}^{\mu \nu} = \Omega^2 g^{\mu \nu},
$$

vierbeins, gamma matrices and covariant derivatives transforms as follows,

$$
e_a^\mu \rightarrow \bar{e}_a^\mu = \Omega e_a^\mu
$$

$$
\bar{\gamma}^\mu \rightarrow \bar{\gamma}^\mu = \Omega \bar{\gamma}^\mu
$$

$$
D_\mu \rightarrow \bar{D}_\mu = D_\mu + \frac{1}{2} \Sigma_{ab} e_a^\nu g_{\rho \mu} e_b^\rho \frac{\partial}{\partial x^\mu} (\ln \Omega),
$$

5
and $\psi$ transform as follows

$$e^{i\phi(x)} \to e^{i\tilde{\phi}(x)} = \varphi(x) e^{i\phi(x)}$$

$$\psi \to \tilde{\psi} = \Omega^{3/2}(x)\varphi^{-1/2}(x) \psi.$$

The low energy effective action of massless spin 1/2 particles (1) transforms into

$$A \to \tilde{A} = \int d^4x \sqrt{-g} e^{i\phi} \frac{1}{2} i \left[ \bar{\psi} \tilde{\gamma}^\mu \slashed{D} \mu \tilde{\psi} - \left( \tilde{D}_\mu \tilde{\psi} \right) \tilde{\gamma}^\mu \tilde{\psi} \right] = \int d^4x \sqrt{-g} e^{i\phi} \left[ \frac{1}{2} i \left( \bar{\psi} \tilde{\gamma}^\mu D_\mu \psi - (D_\mu \bar{\psi}) \tilde{\gamma}^\mu \psi \right) \right] + \int d^4x \sqrt{-g} e^{i\phi} \left[ \frac{1}{4} i \bar{\psi} \left( \tilde{\gamma}^\mu \Sigma^{ab} - \Sigma^{ab} \tilde{\gamma}^\mu \right) \right] e_a^\nu g_{\nu\rho} e_b^\rho \frac{\partial}{\partial x^\mu} \left( \ln \Omega \right).$$

Since $\tilde{\gamma}^\mu \Sigma^{ab} - \Sigma^{ab} \tilde{\gamma}^\mu$ is antisymmetric in $(a,b)$ and $e_a^\nu g_{\rho\nu} e_b^\rho$ is symmetric in $(a,b)$, it follows that

$$\left( \tilde{\gamma}^\mu \Sigma^{ab} - \Sigma^{ab} \tilde{\gamma}^\mu \right) e_a^\nu g_{\rho\nu} e_b^\rho = 0.$$

Thus, the low energy effective action (1) is conformally invariant in the massless limit. In other words, fluctuations of massless spin 1/2 fermions are not amplified in a conformally flat background, even if inflationary, in agreement with our previous calculation.

Action (1) is valid during the DDI period, and since the fermions are assumed to be massless during that period they are not produced. Following that period, during the string phase, fermions are assumed massless, and it seems reasonable to assume that the fermions are not produced during the string phase as well (but see below), if so they may be produced only during the RD and MD epochs, by thermal and non-thermal production. Since in the RD and MD era the dilaton is assumed to be fixed and constant, we conclude that the spectra of the produced spin 1/2 particles in string cosmology models are similar to those in other models, and therefore the cosmological constraints and observable features are common.
3 Gravitinos - spin 3/2 fermions

In this section we show that the appearance of a time-dependent dilaton prefactor in the low energy effective action does not change the amplification of fluctuation of massless gravitinos. Before studying the properties of the action, it is useful to recall a few known facts about gravitinos in conformally flat cosmological backgrounds.

Massive gravitinos in conformally flat space time are described by four Majorana spinors $\psi^\mu$, satisfying the equations [13, 16]

\[
\begin{align*}
  [i \gamma^0 \partial_0 + \frac{5 \dot{a}}{2a} \gamma^0 - ma + k \gamma^3] \psi_{3/2} &= 0, \\
  [i \gamma^0 \partial_0 + \frac{5 \dot{a}}{2a} \gamma^0 - ma + k (A + iB \gamma^0) \gamma^3] \psi_{1/2} &= 0,
\end{align*}
\]

$$ (14) $$

\[
A = \frac{1}{3 \left( \frac{a^2}{a^4} + m^2 \right)^2} \left[ 2 \frac{\ddot{a}}{a^3} \left( m^2 - \frac{\dot{a}^2}{a^4} \right) + \frac{\dot{a}^4}{a^8} - 4m^2 \frac{\dot{a}^2}{a^4} + 3m^4 - 4 \frac{\dot{a}^2}{a^4} \dot{m}m \right],
\]

$$ (19) $$

\[
B = \frac{2m}{3 \left( \frac{a^2}{a^4} + m^2 \right)^2} \left[ 2 \frac{\ddot{a}}{a^3} - \frac{\dot{a}^3}{a^6} + 3m^2 \frac{\dot{a}}{a^2} + \frac{\dot{m}}{ma} \left( m^2 - \frac{\dot{a}^2}{a^4} \right) \right].
\]

Here $\psi_{1/2}$ and $\psi_{3/2}$ are defined such that

\[
\begin{align*}
  \psi^0 &= \sqrt{\frac{2}{3}} c \hat{\gamma}_3 \hat{\gamma}_1 \psi_{1/2} \\
  \psi^1 &= \frac{1}{\sqrt{6}} \psi_{1/2} + \frac{1}{\sqrt{2}} \psi_{3/2} \\
  \psi^2 &= \hat{\gamma}^2 \hat{\gamma}_1 \left( \frac{1}{\sqrt{6}} \psi_{1/2} - \frac{1}{\sqrt{2}} \psi_{3/2} \right) \\
  \psi^3 &= \sqrt{\frac{2}{3}} (d - \hat{\gamma}^3) \hat{\gamma}_1 \psi_{1/2}
\end{align*}
\]

$$ (15) \quad (16) \quad (17) \quad (18) $$

and

\[
\begin{align*}
  c &= \frac{1}{3a \left( \frac{a^2}{a^4} + m^2 \right)} \left[ \left( -2 \frac{\ddot{a}}{a^3} + \frac{\dot{a}^2}{a^4} - 3m^2 \right) \gamma^0 + 2i \frac{\dot{m}}{a} \right] \\
  d &= \frac{|\vec{k}|}{a^2 \left( \frac{a^2}{a^4} + m^2 \right)} \left( i \frac{\dot{a}}{a^2} \gamma^0 + m \right).
\end{align*}
\]

$$ (19) \quad (20) $$
and \( \vec{k} = (0, 0, k) \). The fields \( \psi_{1/2} \) and \( \psi_{3/2} \) correspond to the \( \pm 1/2 \) and \( \pm 3/2 \) helicity states in the flat limit.

In the massless limit, the field equation of motion in (14) becomes

\[
\left[ i\gamma^0 \partial_0 + i\frac{5\dot{a}}{2a}\gamma^0 + k\gamma^3 \right] \psi_{3/2} = 0
\]

\[
\left[ i\gamma^0 \partial_0 + i\frac{5\dot{a}}{2a}\gamma^0 + k\left( A + iB\gamma^0 \right) \gamma^3 \right] \psi_{1/2} = 0,
\]

\[
A = \frac{1}{3} \left[ -\frac{\ddot{a}}{a} + 1 \right]
\]

\[
B = 0.
\]

The pre-big bang model suggests that the evolution of the scale factor \( a(\eta) \) is a power dependent function, i.e.,

\[
a = a_s \left( \frac{\eta}{\eta_s} \right)^\alpha.
\]

In this case, \( A \) becomes a constant; i.e., \( A = (2 - \alpha)/3\alpha \), and eqs. (21) become:

\[
\left[ i\gamma^0 \partial_0 + i\frac{5\dot{a}}{2a}\gamma^0 + k\gamma^3 \right] \psi_{3/2} = 0
\]

\[
\left[ i\gamma^0 \partial_0 + i\frac{5\dot{a}}{2a}\gamma^0 + \bar{k}\gamma^3 \right] \psi_{1/2} = 0,
\]

where \( \bar{k} = \left( \frac{2-\alpha}{3\alpha} \right) k \). As for spin 1/2 particles, one can reduce eqs. (23) to the field eqs. of motion in flat space-time with a suitable transformation. Thus, massless gravitino fluctuation are not amplified in a “power low” expanding universe and no production of gravitinos occurs during the DDI phase.

Next, we show that the low limit effective action of a massless gravitino can be transformed to the action of massless gravitino in standard supergravity, even for a space and time dependent dilaton background.

The low energy effective action of the gravitino in string theory is the following,

\[
\mathcal{A} = \int d^{4}x e^{\phi} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_{\mu} \gamma_{5} \gamma_{\nu} D_{\rho} \psi_{\sigma} + h.c.,
\]
where
\[ D_\mu = \partial_\mu + \frac{1}{2} \omega_{\mu ab} \sigma^{ab}, \]  
(25)
\[ \omega_{\mu ab} = \frac{1}{2} ( -C_{\mu ab} + C_{ab \mu} + C_{b \mu a}), \]  
(26)
\[ C_{\mu \nu} = \partial_\mu e_\nu - \partial_\nu e_\mu - \frac{1}{2 M_P^2} \bar{\psi}_\mu \gamma_\mu \psi_\nu \sim \partial_\mu e_\nu - \partial_\nu e_\mu. \]  
(27)

Using the transformation \( \psi_\sigma = e^{-l \phi/2} \bar{\psi}_\sigma \), action (24) becomes
\[ \int d^4x \left[ -\frac{1}{2} l \epsilon^{\mu \nu \rho \sigma} \bar{\psi}_\mu \gamma_5 \bar{\psi}_\nu \partial_\rho \phi + \epsilon^{\mu \nu \rho \sigma} \bar{\psi}_\mu \gamma_5 \bar{\psi}_\nu D_\rho \psi_\sigma \right] + \text{h.c.}. \]  
(28)
The first term in (28) is canceled by its hermitian conjugate since using \( \gamma^0 [\gamma_5 \bar{\gamma}_\nu]^{\dagger} = \gamma_5 \bar{\gamma}_\nu \),
\[ \left[ \epsilon^{\mu \nu \rho \sigma} \bar{\psi}_\mu \gamma_5 \bar{\psi}_\nu \psi_\sigma \partial_\rho \phi \right]^\dagger = -\epsilon^{\mu \nu \rho \sigma} \bar{\psi}_\mu \gamma_5 \bar{\psi}_\nu \psi_\sigma \partial_\rho \phi. \]  
(29)

Thus,
\[ \int d^4x e^{l \phi} \epsilon^{\mu \nu \rho \sigma} \bar{\psi}_\mu \gamma_5 \bar{\psi}_\nu D_\rho \psi_\sigma + \text{h.c.} \rightarrow \int d^4x e^{l \phi} \epsilon^{\mu \nu \rho \sigma} \bar{\psi}_\mu \gamma_5 \bar{\psi}_\nu D_\rho \psi_\sigma + \text{h.c.}. \]  
(30)
when \( \psi_\sigma \rightarrow e^{-l \phi/2} \bar{\psi}_\sigma \), showing that gravitinos are not produced during the DDI phase, since, as we recall from eq.(23), there is no production of massless gravitino, and therefore, they will not be produced in string cosmology, as well.

The mass of the gravitino is assumed to vanish in the early universe, i.e. in the DDI period, and then to take a nonzero value after the start of the RD era when supersymmetry is broken. Thus, using the same argument that we have used for spin 1/2 particles, we conclude that gravitinos can get produced only during the subsequent RD and MD era.

4 Summary and Conclusions

We have studied fermion production during DDI in string cosmology models, and found that massless spin 1/2 fermions and gravitinos are not produced.
This unexpected result can be contrasted with boson production. In particular, naive expectations would be that gravitinos are copiously produced. In general, bosons have different spectra in string cosmology and in standard slow-roll models, but this is not the case for fermions. Spin 1/2 particles preserves their conformal invariance, and are not produced during the DDI. They can be produced from the start of RD era and on, by thermal and non-thermal production, as in other cosmological models. Similarly, although the action of spin 3/2 particles is not conformally invariant, it is invariant, in the massless limit, under ”rescaling”. Thus, assuming massless gravitinos in the early universe, spin 3/2 particles have the same spectrum in string and standard cosmology.

Our results have a limitation; we have neglected production of fermions during the string phase. As mentioned before, the string phase is a high-curvature phase of otherwise unknown properties. Usually, one could overcome lack of knowledge of string phase evolution by using the fact that fluctuations of fields are frozen when their wavelength is much larger than the curvature scale during the string phase. However, fermions seem to never freeze. Thus the unknown properties of the string phase may be important for fermions production. This is a puzzling issue which we do not understand. Causality requires that amplitudes of waves whose wavelength is larger than the cosmological horizon be constant, but fermions seem to evade this requirement, since they do not seem to feel the cosmological horizon.

Finally, we note that our results may hold for more general string models. In this paper, as well as previous ones, we have assumed that the dilaton depends only on time. However the conformal invariance of the massless spin 1/2 action in the low energy effective string action is preserved also for a space and time dependent dilaton. As a matter of fact it is easy to eliminate a space and time dependent dilaton from the actions of spin 1/2 and spin 3/2 fermions. Thus our results are valid even in string cosmology models that involve a space and time dependent dilaton background.
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