Chiral multifold fermions have received much attention of modern condensed matter research in recent years [1–9]. These fermions possess a topologically protected degenerate point in momentum space (Fig. 1), around which the low-energy theory exhibits an effective spin-momentum locking. Such structure generically hosts nontrivial topological features in the eigenstates, as captured by the Berry monopole charge at and the Chern number computed over a surface enclosing the degenerate point. The chiral multifold fermions potentially own a variety of novel characteristics. These include the exotic superconductivity with unconventional pairing and/or dramatic enhancement from flat bands [10–13], as well as the novel optical responses [14, 15]. The study of materials with prototypical spin-1/2 chiral multifold fermions, known as the Weyl semimetals, has developed as one of the mainstreams in modern condensed matter research in the past decade [1]. Meanwhile, some realizations with higher spins, including spin-1 and 3/2 fermions, have also been uncovered in the solid state materials [2, 3, 5–9]. Moreover, the realm of higher-spin systems can potentially also be explored with ultracold atomic systems through the coupling of atomic hyperfine states [16, 17].

Compared to the extensively studied topological properties of eigenstates, the ‘geometric’ aspects have not been explored in depth. For an eigenstate dependent on a set of adiabatic parameters, the variation in parameter space manifests in both the magnitude and the phase. The phase variation is known as the Berry phase, which corresponds to the topological properties of the eigenstate. Meanwhile, the magnitude variation reflects the ‘quantum distance’ between the initial and final states under the parameter change, and is captured by the quantum metric [18–20]. Such feature can be experimentally probed [21–27] by, for example, a periodic drive measurement. The quantum metric manifests on a variety of occasions, including the finite spread in Wannier orbitals [28–30], the anomalous superfluid stiffness on flat bands [31–33], the realization of fractional Chern insulators [34, 35], the geometric contribution to orbital susceptibility [36, 37], the current noise [38], the indication of phase transition [39–41], and the measurement of tensor monopoles [42]. Despite the broad range of proposed applications, the inherent properties of quantum metric have not been investigated elaborately.

In this Letter, we aim to understand the inherent properties of quantum metric for the chiral multifold fermions. We show that a dual Haldane sphere problem [43] emerges in the computation of the trace of quantum metric owing to the Berry monopole in the momentum space. A quantized geometric invariant can be defined through a surface integration, and together with the Chern number it establishes a sum rule. We further demonstrate the potential manifestations of such quantized band geometry in the measurable physical observables. A lower bound is derived for the finite spread of Wannier functions, which can trigger anomalous phase coherence in the flat band superconductivity. We also discuss about the potential probes of quantum metric in the experimental systems.

We begin by introducing a minimal model of chiral multifold fermions (CMF) in 3D. For spin-s fermion with integer or half-integer spin $s = 0, 1/2, 1, 3/2, \ldots$, the Hamiltonian reads

$$H_{\text{CMF}, \mathbf{k}} = v \mathbf{k} \cdot \mathbf{S}, \quad (1)$$

where $v$ is the effective velocity, $\mathbf{k} = \mathbf{k} \mathbf{\hat{k}}$ denotes the momentum, and $\mathbf{S}$ represents the vector of spin-$s$ operators $S_a$’s with $a = x, y, z$. The Hamiltonian exhibits $2s + 1$ eigenstates $|u^{s}_n\rangle$ with energies $\varepsilon^{s}_n = vkn$, $n = 0, 1, 2, \ldots, 2s$, and $S_a |u^{s}_n\rangle = \gamma^{s}_n S_a |u^{s}_n\rangle$ for the $S_a$’s with $a = x, y, z$. The quantum metric $\mathbf{g}^{s}(\mathbf{k})$ is defined as

$$\mathbf{g}^{s}(\mathbf{k}) = \frac{1}{2} \frac{\partial \varepsilon^{s}_n}{\partial \mathbf{k}} \frac{\partial \varepsilon^{s}_n}{\partial \mathbf{k}}.$$
model (1), the variation under momentum change is measured by the quantum geometric tensor $T_{abk}^n = \langle \partial_{k_a} u_{k_b}^n | (1 - |u_{k_c}^n\rangle \langle u_{k_c}^n|) \partial_{k_c} u_{k_d}^n \rangle$, the real and imaginary parts of which correspond to the quantum or Fubini-Study metric $g_{abk}^n = \text{Re}[T_{abk}^n]$ [18–20] and the Berry flux $B_{abk}^n = -\varepsilon_{abc} \text{Im}[T_{bck}^n]$, respectively. Our interest lies in the quantum metric

$$g_{abk}^n = \frac{1}{2} \left( \langle \partial_{k_a} u_{k_b}^n | \partial_{k_c} u_{k_d}^n \rangle + \langle \partial_{k_b} u_{k_d}^n | \partial_{k_c} u_{k_a}^n \rangle \right) + \langle u_{k}^n | \partial_{k_a} u_{k_d}^n \rangle \langle u_{k_c}^n | \partial_{k_b} u_{k_a}^n \rangle,$$

(2)

a measure of the ‘quantum distance’ $1 - |\langle u_{k}^n | u_{k}^n \rangle_{k-dk}^n| = g_{abk}^n dk_a dk_b$. Remarkably, we discover a ‘quantized’ trace

$$\text{Tr} g_{k}^n = \frac{1}{k^2} [s(s + 1) - (q_s^m)^2]$$

(3)

for the quantum metric of chiral multifold fermions, which is determined by both the angular momentum $s$ and monopole charge $q_s^m$. Eq. (3) is the main result of this Letter. It is indifferent to the orientation because of the rotation symmetry, and furthermore, the $k^{-2}$ dependence implies a quantized invariant

$$G_s^n = \frac{1}{2\pi} \oint dS_k \cdot \hat{k} \text{Tr} g_{k}^n = 2|s(s + 1) - (q_s^m)^2|,$$

(4)

provided the integral domain encloses the degenerate point. The quantization of $G_s^n$ originates from the monopole harmonics wavefunction $|u_{k}^n\rangle$, which is protected by the spin-orbit coupled rotation symmetry around the degenerate point. We thus uncover a ‘symmetry-protected geometric invariant’ $G_s^n$ in the chiral multifold fermion model (1). This geometric invariant is different from the Chern number $C^n_s$, which is well-recognized for the chiral multifold fermions. Despite the difference, a ‘sum rule’ of these two invariants can be established from the quantization rule

$$G_s^n + \frac{(C_s^n)^2}{2} = 2s(s + 1),$$

(5)

where the angular momentum $s$ sets a measure of the sum. Note that the spin-$s$ chiral multifold fermions exhibit the maximal Chern number $|C_s^n| = 2s$. This sets a lower bound for the geometric invariant $G_s^n = |C_s^n|$, as has been identified from the positive definiteness of quantum geometric tensor [31].

To understand how the chiral multifold fermions acquire the quantized trace of quantum metric (3) and the geometric invariant $G_s^n$ (4), we consider an alternative expression of the quantum metric

$$g_{abk}^n = \frac{1}{2} \langle u_{k}^n | \{r_s^m, r_s^m\} | u_{k}^n \rangle.$$

(6)

Here the position $r_s^m = i\nabla_k - A_s^m$ corresponds to the covariant derivative in momentum space, where the
Berry connection is involved [35]. Notably, the trace of quantum metric $\text{Tr} g^m_k = \langle u^m_k \mid \Lambda^m_k \mid u^m_k \rangle$ manifests the momentum-space Laplacian $|\text{r}^m|^2$ in the presence of Berry monopole charge $q^n$. Since the eigenstates are composed of monopole harmonics $Y_{q^n s m}(k)$, the radial part of Laplacian $(k \cdot \text{r}^m)^2$ does not contribute, leaving only the angular part $|\text{r}^m|^2 = |\text{k}^m|^2 (k \cdot \text{r}^m)^2$ in the trace of quantum metric. The angular part of Laplacian can be related to the ‘dynamical angular momentum’ $\Lambda^m = \text{r}^m \times \text{k}$ through $|\Lambda^m|^2 = |k|^2 |\text{r}^m|^2$. This leads to an alternative form of the trace of quantum metric

$$\text{Tr} g^m_k = \langle u^m_k \mid \Lambda^m_k \mid u^m_k \rangle,$$  

(7)

which captures the manifestation of dynamical angular momentum in the eigenstate $|u^m_k\rangle$.

Amazingly, the trace of the quantum metric is algebraically equivalent to the energy expectation value of an electron moving on a sphere enclosing a Dirac magnetic monopole. When a Dirac magnetic monopole is present at the center of the sphere (Fig. 2), the 2D electron gas experiences a uniform perpendicular magnetic field, thereby manifests the quantum Hall effect. This implies a Landau level quantization for the eigenstates in the energy spectrum [43]. The Hamiltonian describing this ‘Haldane sphere problem’ is $H = |A|^2/2m_e R^2$, where $A = R \times (-i \nabla + eA)$ is the dynamical angular momentum, $R = R \mathbf{R}$ is the position at constant radius $R$, $m_e$ and $e$ are the electronic mass and charge, and $A$ is the electromagnetic gauge field. Rotation symmetry enforces the angular momentum $l$ and its axial component $m$ as good quantum numbers. These quantities then determine the quantized Landau level energy $E_{qlm} = |(l+1) - q|^2/2m_e R^2$ in the presence of monopole charge $q$ [49, 50]. The result immediately suggests an analogous quantization for the trace of quantum metric (7) in the chiral multifold fermion model (1). As ‘dual Haldane spheres’ in momentum space, the chiral multifold fermions manifest the ‘dual Landau level quantization’ (3), consistent with our previous observation from direct calculation. Note that the eigenstates in the Haldane sphere exhibit the monopole harmonics wavefunction $\psi_{qlm}(k) = Y_{qlm}(k)$ [49]. This feature again elucidates the dual relation between Haldane sphere and chiral multifold fermions, where the monopole harmonics wavefunctions are also manifest in the eigenstates.

Having dualized the computation of Eq. (7) onto a Haldane sphere, we now illustrate how the trace of quantum metric acquires the quantization (3) for the chiral multifold fermions. The essential point is to uncover the relation between the dynamical angular momentum $\Lambda^m$ and the actual angular momentum $\text{L}^m$ under rotation symmetry [43]. This can be achieved by examining the commutation relations and composing the one which satisfies the SU(2) Lie algebra. The analysis starts by calculating the commutation relation of dynamical angular momentum, yielding $[\Lambda^m_a, \Lambda^m_b] = i\varepsilon_{abc}(\Lambda^m_c - q^n \mathbf{k}_c)$. This result motivates the derivation of another commutation relation $[\Lambda^m_a, \mathbf{k}_b] = i\varepsilon_{abc}\mathbf{k}_d$. Based on these two relations, we identify the angular momentum as $\text{L}^m = \Lambda^m + q^n \mathbf{k}$, whose commutation relation manifests the SU(2) Lie algebra $[\text{L}^m_a, \text{L}^m_b] = i\varepsilon_{abc}\text{L}^m_c$. A correspondence between the angular momentum and the good quantum number in the model (1) is then established $|\text{L}^m|^2 = s(s+1)$. To calculate the trace of quantum metric (7) in terms of the good quantum numbers $s$ and $q$, we utilize the expression $|\Lambda^m|^2 = |\text{L}^m - q^n \mathbf{k}|^2$ and note that $\Lambda^m \cdot \mathbf{k} = \mathbf{k} \cdot \Lambda^m = 0$. The calculation confirms the dual Landau level quantization for the trace of quantum metric (3). This further justifies the validity of geometric invariant $G^m$ (4) and the sum rule (5) along with the Chern number $C^m$. Despite the initial induction based on observation, the dual Haldane sphere provides a rigorous and concise derivation that solidates the results.

The quantized trace of quantum metric (3) and according geometric invariant (4) can have interesting effects on various measurable physical quantities. To study these manifestations, we assume a general 3D multiorbital system which exhibits a multiband structure and holds the chiral multifold fermions. The model (1) is realized at a band crossing point $\mathbf{k}$ in the Brillouin zone (BZ), with the eligible region $\mathcal{R}_{\text{CMF}}$ defined by a radial momentum cutoff $\Lambda_k$. Note that the spin-orbit coupled rotation symmetry serves as an approximate symmetry in this low energy theory. For a certain band involved in the band crossing, the Bloch state $|u^m_k\rangle$ is described by the eigenstate $|u^m_k\rangle = |u^m_k\rangle$ of the model (1) in the eligible region $\mathcal{R}_{\text{CMF}}$. On the linearly dispersing bands $n \neq 0$, the Fermi surfaces are spherical shells at finite doping from the band crossing. Such spherical shells locate at radius $k_F = |\mu|/\sqrt{n}$, where $\mu \neq 0$ is the relative chemical potential to the band crossing. Meanwhile, the flat bands $n = 0$ can occur in the integer spin models $s = 0, 1, 2, \ldots$. The according Fermi surfaces at $\mu = 0$ are solid spheres with radius $\Lambda_k$, which covers the whole eligible region $\mathcal{R}_{\text{CMF}}$ of the low energy theory.

An important basis for the manifestations of quantum metric lies in the spread of Wannier functions [28–30]. Wannier functions are the localized representations of electronic states in real space. For a band $|u^m_k\rangle$ in the multiband structures, the Wannier function $|\mathbf{R}n\rangle$ at lattice vector $\mathbf{R}$ is constructed by a Fourier transform from the Bloch states $|\mathbf{R}n\rangle = (V_0/V) \sum_k \int \mathbf{r} \langle \mathbf{r} | u^m_k \rangle e^{i\mathbf{k} \cdot (\mathbf{R} - \mathbf{r})}$. Here $V_0$ and $V$ denote the volumes of primitive unit cell and whole system, respectively. The availability of exponentially localized Wannier functions are usually expected for a single isolated band [30]. However, such exponential localization may be lost for a single band in a set of composite bands, known as the Wannier obstruction. To capture the localization in the Wannier functions, the ‘spread functional’ $\Omega^n = \langle \mathbf{0} |\mathbf{r}^2| \mathbf{0} \rangle -
\((0n|r|0n)\)² has been defined as a quantitative measure [28]. This function contains a gauge invariant part \(\Omega_n^f\) as a lower bound \(\Omega_n^\text{geom} \geq \Omega_n^f\), which is constant under any gauge transformation \(|u_{k'}^n\rangle \rightarrow e^{i\phi_k} |u_{k'}^n\rangle\). Significantly, the contribution from band geometry is encoded in the gauge invariant part of spread functional \(\Omega_n^f = (V_0/V) \sum_k \text{Tr} g_{kn}^n\). As the \(k\)-dependent orbital composition occurs, the single band is insufficient for exponential localization, leading to a finite spread in the Wannier function. For the bands involved in the band exponential localization, leading to a finite spread in the position occurs, the single band is insufficient for exponential localization, leading to a finite spread in the Wannier function. For the bands involved in the band exponential localization, leading to a finite spread in the position occurs, the single band is insufficient for exponential localization, leading to a finite spread in the Wannier function. For the bands involved in the band exponential localization, leading to a finite spread in the position occurs, the single band is insufficient for exponential localization, leading to a finite spread in the Wannier function. For the bands involved in the band exponential localization, leading to a finite spread in the position occurs, the single band is insufficient for exponential localization, leading to a finite spread in the Wannier function.

Note that the trace of quantum metric is a momentum-space Laplacian, which is positive definite at any point in the Brillouin zone. Due to this feature, a lower bound of the spread functional can be determined solely by the band geometry of chiral multifold fermions \(\Omega_n^\text{geom} \geq \Omega_n^f\). With an integration over the eligible region \(R_{\text{CMF}}\), we determine this lower bound from the geometric invariant \(G^n\) (4)

\[
\Omega_{n}^{\text{geom}} \geq \frac{V_{0}\Lambda_{k}}{4\pi^{2}} G^{n}. 
\]

Notably, the bands with smaller monopole charge (such as the flat trivial bands with \(q^{n} = 0\)) exhibit larger lower bounds for the finite spread. This feature differs remarkably from the usual understanding of Wannier obstruction, which expects a higher degree of obstruction on a band with more nontrivial topology.

As a direct consequence of Wannier obstruction from band geometry, the chiral multifold fermions can form superconductivity even if the bands are (nearly) flat [12]. When a superconducting band is flattened, the Cooper pairs become nondispersive and well localized. This may lead to the loss of interpair communication, thereby suppress the phase coherence of superconductivity. A quantitative measure of phase coherence is provided by the superfluid stiffness \(D_{ab}^S\), which captures the response of a supercurrent \(j_{S}^{ab}\) to an electromagnetic gauge field \(A_{k}\). The scaling \(D_{ab}^S \sim v_{F}^2\) with respect to Fermi velocity \(v_{F}\) confirms the loss of phase coherence \(D_{ab}^S \rightarrow 0\) in the flat band limit \(v_{F} \rightarrow 0\). Due to the absence of global phase coherence, the obstruction to superconductivity is usually expected on flat bands. Nevertheless, ‘anomalous phase coherence’ may arise and support superconductivity on a single flat band in a set of composite bands [31]. Despite the localization of Cooper pairs, the overlaps of wavefunctions from Wannier obstruction can still mediate the phase coherence. Such effect is reflected by the anomalous superfluid stiffness [12, 31–33]

\[
D_{\text{geom},ab}^{S,n}(T) = \frac{1}{V} \sum_{k} \frac{2|\Delta_{k}|}{E_{k}^{n}} \tanh \frac{E_{k}^{n}}{2T} g_{ab,k},
\]

where \(\Delta_{k}\) is the superconducting gap function, \(E_{k}^{n}\) is the quasiparticle energy, and \(T\) is the temperature. As a simplest illustration for the chiral multifold fermions, we calculate the anomalous superfluid stiffness of a uniform superconductivity \(\Delta_{k} = \Delta(T)\) on the flat bands \(n = 0\) [12]. With the quasiparticle energy \(E_{k}^{n} = |\Delta|\), a proportionality to the gap function is established at zero temperature \(T = 0\)

\[
\text{Tr} D_{\text{geom},ab}^{S,n}(0) = \frac{\Lambda_{k}}{2\pi^{2}} |\Delta(0)| G^{n}. 
\]

This result is also valid for the linearly dispersing bands \(n \neq 0\) in the flat band limit \(v \rightarrow 0\). Note that the geometric invariant \(G^{n}\) (4) serves as an important measure of the anomalous superfluid stiffness. While previous works have adopted the relation \(G^{n} \geq |C^{n}|\) and determined a lower bound from the Chern number [31–33], our analysis uncovers a more precise ‘geometric dependence’ particularly for the chiral multifold fermions. Interestingly, the bands with smaller Chern numbers manifest larger anomalous superfluid stiffness, which differs significantly from the usual expectations.

With the anomalous superfluid stiffness derived, we can further estimate the critical temperature \(T_c\sim D_{\text{geom},ab}^{S,n}\) for the flat band superconductivity [51, 52]. Here \(D_{\text{geom},ab}^{S,n} = \left[\prod_{k} D_{\text{geom},aa}^{S,n}(0)\right]^{1/3} = (\Lambda_{k}/(2\pi^{2})) |\Delta(0)| G^{n}\) from rotation symmetry \(D_{\text{geom},aa}^{S,n} = D_{\text{geom},aa}^{S,n}\) and an estimation of coherence length \(\xi \sim \Lambda_{k}^{-1}\) is utilized [12, 52]. Note that the flat band pairing leads to a dramatic enhancement \(|\Delta(0)| \sim V n^{\alpha}\), where \(-\nu < \alpha\) is the attraction and \(n^{\alpha}\) is the number of states in the eligible region \(R_{\text{CMF}}\) per unit volume [10, 12]. The resulting critical temperature manifests a linear scaling in the interaction strength

\[
T_c \sim V n^{\alpha/2} G^{n},
\]

which is much higher than the conventional exponential scaling. Such dramatic enhancement is available solely because the chiral multifold fermions host nontrivial band geometry that supports anomalous phase coherence for flat band superconductivity.

Several experimental methods have been proposed and realized to probe the quantum metric [21–27, 38]. One of the main methods is based on the periodic drive to the system [22]. Consider a linear shake \(\delta H_{s} = 2E \cos(\omega t)r_{a}\) with amplitude \(E\), frequency \(\omega\), and time \(t\) on a multiband system. The initial state is prepared as a Bloch state at momentum \(k\) in the lowest-energy band, which may be realized by loading an according wave packet adiabatically. Significantly, the diagonal components of quantum metric can be obtained from the integrated transmission rate \(\Gamma_{a}^{\text{int}} = 2\pi E^{2} g_{aok}\) under the periodic drive. Here the integrated rate reads \(\Gamma_{a}^{\text{int}} = \int_{\omega} \Gamma(\omega)\), and \(\Gamma(\omega)\) is the time-averaged transmission probability to all of the higher-energy bands. The trace of quantum metric can thus be determined by applying the shake along all directions \(a = x, y, z\) and measuring the transmission rates. Some other methods have also been proposed, such
as the measurement by detecting the current noise [38]. Note that the single-band quantum metric can only be probed for the lowest-energy band in all of these methods. The target of a particular band may serve as an important goal in the future development of experimental probes.

In summary, we derive an exact quantization rule for the trace of quantum metric for the chiral multifold fermions for any spin. Such derivation is achieved by dualizing the computation onto a Haldane sphere in momentum space. The quantization may physically manifest itself in the finite spread of Wannier functions, as well as the according anomalous phase coherence of flat band superconductivity. Experimental probes with periodic drive may be adopted to obtain the quantum metric. Potential applications to the other physical observables may serve as interesting topics for future work. Meanwhile, our paradigmatic analysis may also be generalized to the other gapless topological materials, such as the nodal loop semimetals [53–55] and the topological spintronics [56]. Our analysis indicates a new framework of how the novel properties can originate from band geometry, thereby opens a new route toward the understanding of unconventional states of matter.

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