A frequency domain analysis algorithm for predicting variance of quasi-static and dynamic wave-driven loads on a monopile-supported offshore wind turbine

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Abstract. The purpose of this work is to develop and validate a frequency domain analysis (FDA) algorithm to predict loads on a monopile-supported offshore wind turbine (OWT). FDA is faster than the time domain simulations that are conventionally used to perform structural analyses of OWTs. An FDA algorithm that could accurately predict loads would enable expedient design and assessment of risk for OWTs. The current work presents an FDA algorithm for predicting variance of wave-driven loads during the non-operational condition with validation from both simulated and measured data.

1. Introduction
Structural engineers must perform preliminary designs for the thousands of offshore wind turbines (OWTs) needed to meet the Department of Energy’s goal of 35% wind penetration [1]. Preliminary design is an iterative process potentially involving innumerable combinations of geometry and material, each of which will have different loading, even for the same environment. The need to calculate loading for so many possibilities motivates the use of frequency domain analysis (FDA) to approximate loading with far greater computational speed than time-domain simulations. The computational costs associated with time-domain simulation are so significant as to render optimization of a single wind turbine design impractical for most applications [2], let alone running the number of simulations required to conduct a comprehensive preliminary study. The intent of this paper is to propose an FDA method and compare the results with at-sea measurements from a full-scale offshore wind turbine and with time history analysis using the program FAST.

The FDA framework from wind engineering, developed by Alan G. Davenport, provides the inspiration for the current work. Davenport’s FDA framework takes the power spectral density (PSD) of a turbulent wind field as input and scales this input by a frequency-dependent, structure-specific transfer function to yield a PSD (or spectrum) of structural response as output. We have adapted existing frequency-domain transfer functions, based on Airy wave theory and the Morison equation for wave loading on a cylinder, to develop an algorithm to predict the PSD of both quasi-static and dynamic wave-driven loads on a monopile-supported OWT.

The outline of the paper is as follows. First, we explain the hydrodynamic admittance function (HAF), the transfer function that maps from the power spectral density (PSD) of sea surface elevation to PSD of quasi-static bending moment for a monopile-supported OWT modeled as a rigid cylinder fixed at one end. Then, we explain the dynamic response algorithm (DRA), which accounts for the additional dynamic bending moment that results when the structure’s first mode is activated. Together, the HAF
and the DRA form a complete FDA algorithm with which we can map from a PSD of sea surface elevation to a PSD of bending moment for a monopile-supported OWT. Then, we explain how we validate the FDA algorithm, using both simulated and measured data. The simulated data is generated using FAST, NREL’s time domain simulation code for wind turbine dynamics. The measured data consists of more than one year’s worth of measurements of wind speed, sea surface elevation, and structural response from an instrumented OWT in England’s Blyth Wind Farm. We pass simulated and measured time series of sea surface elevation through our FDA algorithm and compare the algorithm’s predictions of PSD of bending moment with PSDs of simulated and measured time series of bending moment. We use the percent error between predicted variance and the variances of simulated and measured data as our performance metric.

2. Derivation of the FDA Algorithm
The hydrodynamic admittance function (HAF) is the transfer function that maps from PSD of sea surface elevation to PSD of quasi-static bending moment for a monopile-supported OWT modeled as a rigid cylinder fixed at one end.

![Flow chart for HAF](image)

Figure 1. Flow chart for HAF

The formulation of the HAF used in this work was presented by van der Tempel [3], though other equivalent formulations can be found elsewhere [4]. The flow chart in Figure 1 shows the analytical expressions for the HAF and outlines the procedure for its use. Since we are concerned with wave-driven...
loads in this study, the excitation energy is represented by the wave spectrum $S_{\eta m}(n)$, where $n_m$ is frequency in hertz of the $m$th wave component. The Fourier coefficients $\hat{\eta}$ of the individual components of sea surface elevation are derived from the wave spectrum. The Fourier coefficients of the inertial and drag components of base shear and mudline bending moment are calculated according to the flow chart, where $\rho_w$ is the density of water, $g$ is gravitational acceleration, $D$ is the diameter of the monopile, $d$ is the water depth from mean sea level (MSL) to the mudline, and $k_m$ is the wave number of the $m$th wave component. The total Fourier coefficient of mudline bending moment is the square root of the sum of squares of the Fourier coefficients of the inertial and drag components of the mudline bending moment. This total Fourier coefficient is converted to a spectrum. This spectrum $S_{MM,qs}(n_m)$ is the spectrum of quasi-static wave-driven mudline bending moment.

![Figure 2. Example plot of HAF](image_url)

In theory, since the monopile is treated as a rigid cylinder fixed at one end, the structural response content at each frequency is simply the wave loading content at the same frequency scaled by a constant and the HAF is the function that provides the scale factor at each frequency. The flow chart in Figure 1 demonstrates that in practice, the HAF is not so simple, since the spectral ordinate at each frequency is the sum of squares of the inertial and drag components at that frequency. However, dividing the predicted $S_{MM,qs}(n_m)$ by $S_{\eta m}(n_m)$ yields the effective HAF scale factors (Figure 2). The physical explanation for the shape in Figure 2 is as follows. As frequency increases, wave number increases. Larger wave numbers correspond to shorter wavelengths. As the wavelengths decrease, they approach the monopile diameter in scale, and the waves become “more noticeable” to the monopile. The HAF is a filter that amplifies the wave spectrum content as the wavelength-diameter ratio decreases.

### 2.1 Dynamic Algorithm

To calculate quasi-static loads, it is sufficient to treat the structure as rigid. In reality, the structure deforms. The deformed shape of the OWT under wave loading is different for quasi-static and dynamic response (Figure 6). Dynamic response occurs when the wave loading contains frequency content near the structure’s natural frequency, thereby activating the structure’s first mode of vibration. The dynamic response algorithm (DRA) we have developed uses the relationship between the quasi-static and dynamic deformed shapes to account for the additional dynamic bending moment that results when the wave loading activates the structure’s first mode.
The DRA consists of three spectral relationships that map between:

1) quasi-static force and quasi-static tower top displacement,
2) quasi-static tower top displacement and dynamic tower top displacement,
3) dynamic tower top displacement and dynamic mudline bending moment.

The flowcharts in Figures 3-5 show the analytical expressions of each of these relationships and outline the procedure for use of the DRA.

Mapping from a spectrum of quasi-static force to a spectrum of quasi-static tower top displacement (Figure 3) relies on the output of the HAF. The HAF yields Fourier coefficients of quasi-static overturning moment \( \overline{M}_{t,qs}(n_m) \) and base shear \( \overline{F}_{t,qs}(n_m) \), assuming the monopile is fixed at the mudline. The ratio of these Fourier coefficients yields the Fourier coefficients of the equivalent moment arm.

For the quasi-static condition, the structure deforms like a cantilever with a point load applied between the free end and the fixed end. The location of this point load at each frequency is given by the equivalent moment arm \( \overline{L}(n_m) \). By the principal of virtual forces, the moment distribution \( M(x) \) resulting from a point load at the free end of the cantilever can be integrated with the moment distribution \( \overline{\mathbf{m}}(x) \) from a point load at the equivalent moment arm to yield a value for tower top displacement per unit force applied at the equivalent moment arm \( \overline{L}(n_m) \).

The left schematic in Figure 6 shows these two
moment distributions. The result of the integration is a frequency-dependent flexibility \( d_{qs}(n) \) that links the Fourier coefficients of quasi-static base shear \( \hat{F}_{qs}(n) \) to the Fourier coefficients of quasi-static tower top displacement \( \hat{\xi}_{qs}(n) \), from which we convert to the spectrum of quasi-static tower top displacement \( S_{\xi_{qs}}(n) \).

Then, we amplify the spectrum of quasi-static tower top displacement by the mechanical admittance derived from the equation of motion for a single-degree-of-freedom system with linear viscous damping. The result of this amplification is a spectrum of dynamic tower top displacement \( S_{\xi_{dyn}}(n) \). The important structural characteristics for this step of the DRA, shown in Figure 4, are the natural frequency \( n_0 \) and damping \( \zeta_{total} \), which are the free parameters of the mechanical admittance. The value for \( n_0 \) determines the location along with x-axis of the peak in the PSD of response; the value for \( \zeta_{total} \) determines the magnitude of this peak.

The structure’s first mode shape is similar to the deformed shape of a cantilever with a point load applied to the free end. Again, we invoke the principal of virtual force using the coincident moment distributions shown in the right schematic in Figure 6. The resulting flexibility \( d_{dyn} \) is not frequency-dependent. The inverse of \( d_{dyn} \) is the stiffness \( k_{dyn} \) that is the constant of proportionality between the Fourier coefficients of dynamic tower top displacement \( \hat{\xi}_{dyn} \) and the equivalent dynamic point load \( \hat{F}_{dyn} \) that acts on the free end. Multiplying \( \hat{F}_{dyn} \) by the distance \( H \) from hub height to mudline yields the Fourier coefficient of dynamic mudline bending moment \( \hat{M}_{dyn} \), which can be converted to the spectrum of dynamic mudline bending moment \( S_{MM,dyn} \).

![Diagram](https://example.com/diagram.png)

**Figure 5.** Flow chart for DRA spectral relationship between dynamic tower top displacement and dynamic mudline bending moment, ending with complete mudline bending moment.
The total spectrum of mudline moment $S_{MM}$ is simply the sum of the spectrum of quasi-static mudline moment $S_{MM,qs}$ and the spectrum of dynamic mudline moment $S_{MM,dyn}$, as shown in the final box of the flow chart in Figure 5.

Figure 7 is a simplified diagram showing the entire process. The input is a wave spectrum. The output is a spectrum of mudline bending moment. The total tower top displacement is the sum of a series of time-varying deflected shapes determined by the equivalent moment arm $L$ and one time-varying deflected shape that is the first mode shape. The deflected shapes are activated according to the frequency content of the loading. Each deflected shape is linearly related to a mudline bending moment. The sum of all mudline bending moments is the total mudline bending moment.

3. Validation Procedure

3.1. Definitions of Metrics

The frequency content of sea surface elevation time series recorded at the Blyth Wind Farm is concentrated in what we call the wave variance frequency range. The average of the wave spectra recorded at the Blyth Wind Farm (Figure 8) shows the wave variance range to extend from about 0.05 Hz to 0.30 Hz. The dynamic frequency range is simply a range of frequencies that includes the structure’s natural frequency (see Section 3.3). Table 1 shows the limits of the wave variance and the dynamic frequency ranges we use in the current work.

Wave variance is the area under the curve of the PSD of the mudline bending moment between the frequencies that define the limits of the wave variance frequency range. Absolute wave variance error is the difference between the measured and predicted wave variances. Dynamic variance is the area under the curve of PSD of mudline bending moment between the frequencies that define the limits of the dynamic frequency range. Absolute dynamic variance error is the difference between the measured and predicted dynamic variances. Negative error values indicate over-prediction; positive error values indicate under-prediction.
3.2. FAST Output
To validate the FDA algorithm, we passed simulated and measured data through the FDA algorithm. The simulated data was generated by FAST, NREL’s time domain simulation code for wind turbine dynamics [5]. For the FAST simulations, we output time series of sea surface elevation and structural response (mudline bending moment) for a given simulation. We generated an empirical (i.e., non-model-based) wave spectrum from the FAST-generated time series of sea surface elevation. We passed this wave spectrum through the FDA algorithm as described above. For the same simulation, we generated an empirical spectrum of mudline bending moment and compared it with the spectrum from the FDA algorithm.

Table 2 shows the relevant structural parameters of the FAST model OWT we used as input to our FDA algorithm for the validation procedure.

| Wave Variance Range (Hz) | Dynamic Range (Hz) |
|--------------------------|-------------------|
| $f_{\text{WaveMin}}$ | $f_{\text{WaveMax}}$ | $f_{\text{DynMin}}$ | $f_{\text{DynMax}}$ |
| 0.05 | 0.30 | 0.43 | 0.51 |

Table 1. Limits of dynamic and wave variance frequency ranges

3.3. Measurements from the Blyth Wind Farm
The validation procedure using measurements was very similar to the validation process using FAST output. The Blyth Wind Farm consists of two Vestas V66 2MW wind turbines off the northeast coast of England. The hub height is 62 m, and the wind turbines are supported by monopiles 3.5 m in diameter [6]. The Offshore Wind Turbines at Exposed Sites (OWTES) project was undertaken at the Blyth Wind Farm to reduce uncertainties in the calculation of hydrodynamic loading for a monopile-supported offshore wind turbine. The measurement campaign for the OWTES project included measurement of structural loading on the turbine support structure, measurement of the elevation of the sea surface above the Lowest Astronomical Tide (LAT), and measurement of wind conditions at onshore metmast of height 40m close to the site [6]. Measurements were taken between October 2001 and January 2003. We
use these measurements of environmental conditions and structural response to validate the FDA algorithm.

![Wave Spectrum](image)

**Figure 8.** Average wave spectrum (black), generated from a total of 1961 10-minute records of sea surface elevation at the Blyth Wind Farm, with three other wave spectra generated from three individual 10-minute records.

For a given record, we generated an empirical wave spectrum from the measurements of sea surface elevation. We passed this wave spectrum through the FDA algorithm and compared the resulting predicted spectrum of mudline moment to the empirical spectrum of mudline bending moment we generated from the measurements of mudline bending moment.

We used 1% total damping and 0.49 Hz for the natural frequency of the Blyth wind turbine. We used 1% damping because it tended to best reflect the measurements at Blyth. The reported natural frequency of the Blyth wind turbine is 0.47 Hz, but PSDs of mudline bending moment time series measured at Blyth have a peak closer to 0.49 Hz, so we deferred to what the data showed.

Table 2 shows the relevant structural parameters of the Blyth OWT we used as input to our FDA algorithm for the validation procedure.

| Tower Diameter (m) | Monopile Diameter (m) | Tower Thickness (cm) | Monopile Thickness (cm) | $\zeta_{\text{total}}$ | $n_0$ (Hz) | $d$ (m) |
|-------------------|-----------------------|----------------------|-------------------------|---------------------|------------|--------|
| Top               | Bottom                | Top                  | Bottom                  | Top                 | Bottom     | Top    |
| FAST              | 3.9                   | 6.0                  | 6.0                     | 2.5                 | 3.6        | 6.0    |
| Blyth             | 2.3                   | 3.5                  | 3.5                     | 2.5                 | 2.5        | 2.5    |

**Table 2.** Inputs to FDA algorithm to generate predictions to compare with FAST output and Blyth measurements

### 3.4. Generating a Wave-Driven Resultant Mudline Bending Moment Time Series

From the Blyth Wind Farm, we have time series of mudline bending moment about perpendicular horizontal axes. From these, we generated resultant time series of mudline bending moment about a series of arbitrary axes. The resultant time series with the highest wave variance is the time series of mudline bending moment we used as input to the FDA algorithm. We assumed that the associated axis of bending would be perpendicular to the prevailing wave direction, so the time series of mudline
bending moment about this axis would best represent the wave-driven loads that our FDA algorithm is designed to predict.

Figure 9. Average mudline bending moment versus average wind velocity at 40m for 1961 measurements from the Blyth Wind Farm with lines demarcating the non-operational domain

3.5. Non-operational domain

Figure 9 shows a plot of average mudline bending moment as a function of average wind velocity for 1961 10-minute records from the Blyth Wind Farm. Many of the data points fall on a curve that bears a close resemblance to the thrust curve of an offshore wind turbine. The mudline bending moment is linearly related to the rotor thrust for the same reason that the bending moment at the fixed end of a cantilever is linearly related to a point load at the free end.

The Blyth data set contained no explicit indicator of whether the wind turbine was operational; in the absence of this explicit indicator, we defer to Figure 9 for guidance as to which records contain data for operational conditions.

The question of whether the wind turbine is operational or not is relevant because the structural response is dominated by wind loading during operational conditions. Since this FDA algorithm is intended to predict the variance of wave-driven loads, we want to use measurements only from those records during which the wind turbine was not operational, so the wave-driven loads were not overwhelmed by wind-driven loads in the structural response. For the same reason, we used FAST to simulate wind turbine dynamics under wave loading only.

For this study, then, we only evaluated the performance of the FDA algorithm for sea states in the non-operational domain. Two lines define the non-operational domain: 1) a horizontal line at a value for average mudline bending moment of 7500 kN-m, and 2) a line whose slope approximately matches that of the thrust curve and which crosses the horizontal axis at a value for average wind velocity of 5 m/s. The non-operational domain is the area below the first line and to the right of the second line. The non-operational domain is the unshaded area in Figure 9 and consists of 167 10-minute records.

We also excluded all records for which the normalized dynamic variance error exceeded 100%. We attribute these extreme error values to measurement error.

4. Results

Comparisons of FDA predictions of PSDs of mudline bending moment with PSDs of mudline bending moment generated from simulated data and measurements are presented in Figures 10-12. Figure 10 shows that, for a simulation with wave loading only, the FDA prediction matches the FAST output
almost exactly. The smaller peak at about 0.12 Hz is where the wave variance is concentrated; the large peak at 0.27 Hz is the resonant peak.

![Figure 10. Comparison of predicted and actual PSD of response for a FAST simulation with loading due only to linear waves, Hs = 4m](image)

Figures 11 and 12 show comparisons for the FDA predictions against measurements from Blyth. Even when the error is high, the shape of the prediction spectrum matches shapes of the measured response and wave spectra well. In general, the performance of the FDA algorithm is worse for real measurements than for simulation output, particularly for predicting dynamic variance, simply because the offshore environment in which the measurements were recorded exhibits far more complexity than our FDA algorithm can account for.

At frequencies below about 0.06 Hz, the FDA algorithm underpredicts the response PSD. Spectral content from turbulent wind, which our model does not account for, is concentrated in this frequency range. The peak in the response PSD near 0 Hz is the effect of mean wind.

We assume the sea states for which the FDA algorithm considerably overpredicts wave-driven dynamic variance are characterized by waves from multiple directions. Our FDA algorithm treats waves as unidirectional—all the variance is concentrated in one direction. For simulated sea states and for calm conditions, this assumption holds. For more extreme conditions, the waves come from multiple directions, so the assumption that all the variance is concentrated in one direction yields overpredictions.

The FDA algorithm overpredicts the response PSD at frequencies greater than the natural frequency of the structure (0.47 Hz). We think that this spectral content represents the effects of second-order waves, which are concentrated at higher frequencies. This higher-frequency content is carried through the FDA algorithm, but for the actual structure in the offshore environment, spectral content at these frequencies seems to be diminished by some mechanism or process, like hydrodynamic damping, that our algorithm does not include.

For cases where the FDA algorithm dramatically underpredicts the response PSD in the dynamic frequency range, we simply say that there is excitation energy that the FDA algorithm does not amplify properly or that does not come from wave loading. When this improperly amplified or neglected excitation energy is concentrated at the natural frequency, the dynamic amplification of the actual structure makes our algorithm’s underprediction all the more noticeable.
Figure 11. Comparison of predicted and actual PSD of response for sea states with least (top) and second greatest (bottom) absolute value of absolute dynamic variance error (for non-operational conditions, excluding sea states with absolute dynamic variance error greater than 100% and excluding the sea state with the greatest absolute value of absolute dynamic variance error due to measurements indicating anomalous structural behavior)
Figure 12. Comparison of predicted and actual PSD of response for sea states with least (top) and greatest (bottom) absolute value of absolute wave variance error (for non-operational conditions, excluding sea states with absolute dynamic variance error greater than 100%)

Figures 13 through 15 show the aggregate performance of the FDA algorithm at predicting wave-driven variance from measured data. Figure 13 shows the normalized wave variance error—that is, the percent error. Figures 14 and 15 show absolute wave variance and dynamic variance error because normalizing the error to low variance values artificially inflates the corresponding error values.
The FDA algorithm seems to perform best between significant wave height (Hs) values of 1.5m and 3.5m. Below Hs of 1.5m, wave loading is not significant enough to be prevalent in the structural response. Above Hs of 3.5m, the sea state becomes more complex, and other factors—wave spreading, higher order waves, and aerodynamic and hydrodynamic effects—begin to influence the structural response.
5. Conclusions

We have presented an FDA algorithm that can predict the variance of quasi-static and dynamic loads on a monopile-supported OWT driven by linear waves during calm, wave-dominated sea states and non-operational conditions. Further research is required to see if FDA can be adapted to account for conditions outside this limited set of conditions. Additional future work lies in developing the analogous transfer functions for wind-driven loads and deriving distributions of loads as functions of the statistics and spectral moments of the relevant random processes. Such work would allow structural engineers to exploit the computational efficiency of FDA not only to perform preliminary designs, but also to generate reliable predictions of loads for large numbers of OWT support structures incredibly quickly. Such predictions would enable expedient quantification of the risk of failure to OWTs.

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