Abelian Projection of Massive $SU(2)$ Yang-Mills Theory

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Abstract

We derive an effective Abelian gauge theory (EAGT) of a modified $SU(2)$ Yang-Mills theory. The modification is made by explicitly introducing mass terms of the off-diagonal gluon fields into pure $SU(2)$ Yang-Mills theory, in order that Abelian dominance at a long-distance scale is realized in the modified theory. In deriving the EAGT, the off-diagonal gluon fields involving longitudinal modes are treated as fields that produce quantum effects on the diagonal gluon field and other fields relevant at a long-distance scale. Unlike earlier papers, a necessary gauge fixing is carried out without spoiling the global $SU(2)$ gauge symmetry. We show that the EAGT allows a composite of the Yukawa and the linear potentials which also occurs in an extended dual Abelian Higgs model. This composite potential is understood to be a static potential between color-electric charges. In addition, we point out that the EAGT involves the Skyrme-Faddeev model.

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I. INTRODUCTION

Understanding the color-confinement mechanism based on quantum chromodynamics (QCD) is a long-standing subject in particle physics. It has been argued that the color-magnetic monopole condensation leads to color confinement through the dual Meissner effect which is described by the dual Abelian Higgs model or the dual Ginzburg-Landau theory [1–3]. To confirm this picture within the framework of QCD, it is necessary to realize magnetic monopoles in QCD [4] and to accept Abelian dominance [5,6] as a fact. Here Abelian dominance means that, at a long-distance scale, only diagonal gluons dominate, while effects of off-diagonal gluons are strongly suppressed. When the idea of Abelian dominance was first proposed by Ezawa and Iwazaki, it was only a hypothesis [5]. They conjectured that Abelian dominance may be achieved if off-diagonal gluons possess effective non-zero mass at a long-distance scale and hence do not propagate at this scale. A recent Monte Carlo simulation performed by Amemiya and Suganuma shows that, in the maximal Abelian (MA) gauge, off-diagonal gluons indeed behave like massive vector fields with the effective mass $M_{	ext{off}} \simeq 1.2 \text{ GeV}$ [6]. This result strongly supports Ezawa-Iwazaki’s conjecture, so that the Abelian dominance must be realized at a long-distance scale.

Mass generation of off-diagonal gluons would be a nonperturbative effect of QCD at a long-distance scale and should be understood within the analytic framework of QCD. In fact, an analytic approach based on condensation of the Faddeev-Popov ghosts has been made to explain the mass-generation mechanism of off-diagonal gluons [7,8]. This attempt seems to be interesting. However the ghost condensation may lead to breaking of the Becchi-Rouet-Stora-Tyutin (BRST) symmetry, which causes the problem of spoiling unitarity.

Although the mass-generation mechanism of off-diagonal gluons is not well understood analytically at present, respecting the result of Monte Carlo simulation, we accept the mass generation as true in the beginning of our discussion without questioning its mechanism. Accordingly, in the present letter, we explicitly incorporate mass terms of the off-diagonal gluon fields into the Yang-Mills (YM) Lagrangian to describe dynamics of gluons at a long-distance scale. Among the massive YM theories without residual Higgs bosons [9], we adopt the Stueckelberg-Kunimasa-Gotô (SKG) formalism, or the non-Abelian Stueckelberg formalism [10], to deal with massless and massive gluon fields in a gauge-invariant manner. In this context, we expect that the mass terms of off-diagonal gluon fields are dynamically induced by a nonperturbative effect of pure YM theory.
In the SKG formalism, Nambu-Goldstone (NG) scalar fields are introduced to identify the longitudinal modes of off-diagonal gluons. Since the Lagrangian of the SKG formalism is nonpolynomial in the NG scalar fields, there is no hope that the SKG formalism is perturbatively renormalizable [11]. However this is not a serious problem for our discussion, because the SKG formalism is now treated as an effective gauge theory at a long-distance scale.

Because of Abelian dominance due to the massive off-diagonal gluons, phenomena at a long-distance scale will be described in terms of the diagonal gluon fields together with other fields relevant to the corresponding scale. In accordance with this idea, the present letter attempts to derive an effective Abelian gauge theory (EAGT) constructed of these fields from the SKG formalism. (Some preliminary discussions in this attempt have been reported in Ref. [12].) The procedure of deriving the EAGT is a kind of so-called Abelian projection. Recently a fashion of Abelian projection of QCD or YM theory has been studied by Kondo [13,26] and by some other authors [14,15,27]. In their procedure, the maximally Abelian (MA) gauge condition is imposed on QCD before performing the Abelian projection. Unfortunately, the global color gauge symmetry of QCD is spoiled at this stage, so that the conservation of color Noether current is no longer guaranteed. In contrast, in our procedure, we do not put the MA gauge condition by hand. Instead, a corresponding condition is obtained as the Euler-Lagrange equation for the NG scalar fields. In connection with this fact the global color gauge symmetry is maintained in the EAGT in a nonlinear way, provided that an appropriate gauge fixing term is adopted.\(^1\) As a result, a conserved color charge is well defined in the EAGT. The presence of conserved color charge will be important for combining our discussion with other attempts to investigate color confinement.

The Abelian projection of the SKG formalism is performed in the following manner: First, beginning with an effective action in the SKG formalism, we evaluate the path-integral over the NG scalar fields around their classical configuration by using a semi-classical method. After that, similar to earlier papers [13–15,26,27], we carry out the path-integration over the off-diagonal gluon fields with the aid of auxiliary fields introduced in this stage. The

\(^1\) A gauge-covariant formulation of Abelian projection has been considered with introducing auxiliary Higgs-like fields [16,31]. Unlike this approach, the present letter treats the NG scalar fields (which are Higgs-like fields represented in a nonlinear way) as dynamical fields occurring in the Lagrangian.
resulting effective action that defines the EAGT is written in terms of the diagonal gluon fields, the auxiliary fields and the classical configuration of NG scalar fields. This action includes terms analogous to those occurring in a dual form of the extended dual Abelian Higgs model (EDAHM) in the London limit \[17,18\]. With such terms in hand, following a procedure discussed in Ref. [19], we show that the EAGT allows a composite of the Yukawa potential and the linearly rising potential. We point out that the composite potential represents interaction between color-electric charges consisting of the classical NG scalar fields. We also show that the EAGT involves the Skyrme-Faddeev model.

II. STUECKELBERG-KUNIMASA-GOTÔ FORMALISM

For simplicity, in the present letter, we restrict our discussion to the case of \(SU(2)\) color gauge symmetry. Let \(A_\mu^B\) \((B = 1, 2, 3)\) be the Yang-Mills (YM) fields, or the gluon fields, and \(\phi^i\) \((i = 1, 2)\) be the (dimensionless) Nambu-Goldstone (NG) scalar fields that form a set of coordinates of the coset space \(SU(2)/U(1)\). We begin with the Stueckelberg-Kunimasa-Gotô (SKG) formalism \[10\] characterized by the Lagrangian \(\mathcal{L}_{SKG} = \mathcal{L}_{YM} + \mathcal{L}_{\phi}\) with

\[
\mathcal{L}_{YM} = -\frac{1}{4g_0^2} F_{\mu\nu}^B F^{\mu\nu B},
\]

\[
\mathcal{L}_{\phi} = \frac{m_0^2}{2g_0} g_{ij}(\phi) D_\mu \phi^i D^\mu \phi^j,
\]

where \(F_{\mu\nu}^B \equiv \partial_\mu A^B_\nu - \partial_\nu A^B_\mu - \epsilon^{BCD} A^C_\mu A^D_\nu\) and \(D_\mu \phi^i \equiv \partial_\mu \phi^i + A^B_\mu K^B_i(\phi)\) \[20\]. Here \(g_0\) is a (bare) coupling constant, \(m_0\) is a constant with dimension of mass, \(g_{ij}(\phi)\) is a metric on the coset space \(SU(2)/U(1)\), and \(K^B_i(\phi)\) are Killing vectors on \(SU(2)/U(1)\). (The convention for the signature of space-time metric is \((+,−,−,−)\).) The Lagrangian \(\mathcal{L}_{SKG}\) remains invariant under the gauge transformation

\[
\delta A_\mu^B = D_\mu^B \lambda^C \equiv (\delta^B \partial_\mu + \epsilon^{BCD} A^D_\mu) \lambda^C,
\]

\[
\delta \phi^i = -\lambda^B K^B_i(\phi),
\]

with \(SU(2)\) gauge parameters \(\lambda^B\). The metric \(g_{ij}(\phi)\) is written as \(g_{ij}(\phi) = e^b_i(\phi)e^b_j(\phi)\) \((b = 1, 2)\) in terms of the zweibein \(e^b_i(\phi)\) that is defined by \(e^b_i(\phi)T^b + e^3_i(\phi)T^3 = e^B_i(\phi)T^B\) \((-iv^{-1}(\phi)\partial_i v(\phi)\) with coset representatives \(v(\phi) [\in SU(2)]\), where \(\partial_i \equiv \partial/\partial \phi^i\) and \(T^B \equiv \frac{1}{2}\sigma^B\) \((\sigma^B\) denote the Pauli matrices). Under the left action of \(g [\in SU(2)]\), the coset representatives transform as \(gv(\phi) = v(\phi')h(\phi, g) [h \in U(1)]\); in a sense, \(v(\phi)\) “convert” \(SU(2)\) gauge
transformations into corresponding $U(1)$ gauge transformations. If the infinitesimal form $g = 1 - i \lambda^B T^B$ is chosen as $g$, then $\phi^i$ and $h$ take the following forms: $\phi^i = \tilde{\phi}^i - \lambda^B K^{Bi}(\phi)$, $h(\phi, g) = 1 - i \hat{\lambda} T^3$. Here $\hat{\lambda}$ is a $U(1)$ gauge parameter defined by $\hat{\lambda} \equiv \lambda^B \Omega^B(\phi)$ with $\Omega^B(\phi)$ being functions of $\phi^i$ called $H$-compensators [21]. (Unlike $\lambda^3$, $\hat{\lambda}$ is not a mere gauge parameter of the $U(1)$ subgroup of the gauge group $SU(2)$.) From the transformation rule of $v(\phi)$, it follows that $K^{Bi}(\phi) e_i(\phi) = D^B(\phi)$. Here $D$ denotes a matrix in the adjoint representation of $SU(2)$, defined by $g^{-1} T^B g = D^{BC}(g) T^C$. In addition, the transformation rule of $v(\phi)$ leads to $[K^B, K^C] = e^{BCD} K^D$ with $K^B \equiv K^{Bi}(\phi) \partial_i$. It is easy to show that the inverse metric $g^{ij}(\phi) = \epsilon^{bi}(\phi) \epsilon^{bj}(\phi)$, with the inverse zweibein $\epsilon^{bi}(\phi)$, can be written as $g^{ij}(\phi) = K^{Bi}(\phi) K^{Bj}(\phi)$ and so the Lie derivatives of $g^{ij}(\phi)$ along $K^B$ vanish. The Maurer-Cartan formula for $e^B(\phi) \equiv \partial_i \phi^i B(\phi)$ now reads

$$
\partial_\mu e^B_\nu - \partial_\nu e^B_\mu - e^{BCD} e^C_\mu e^D_\nu = \delta^{B3} \Sigma_{\mu\nu},
$$

where $\Sigma_{\mu\nu}$ is given by $\Sigma_{\mu\nu} T^3 \equiv -iv^{-1}(\phi) [\partial_\mu, \partial_\nu] v(\phi)$. This term remains non-vanishing owing to a Dirac string singularity characterized by the first homotopy group $\pi_1(U(1)) = Z [3,22]$. The Lagrangian $\mathcal{L}_\phi$ can be expressed as $\mathcal{L}_\phi = \frac{1}{2} (m_0/g_0)^2 \hat{A}_\mu^b \hat{A}^{\mu b}$ in terms of the YM fields in the unitary gauge, $\hat{A}_\mu^B \equiv A_\mu^C D^{CB} v(\phi) + e_\mu^B(\phi)$. Thus, in the unitary gauge, $\mathcal{L}_\phi$ takes the form of a mass term for $\hat{A}_\mu^b$. The gauge transformation rules of $\hat{A}_\mu^B$ ($B = b, 3$) are found from Eq. (3) to be

$$
\delta \hat{A}_\mu^b = -\epsilon^{bc3} \hat{A}_\mu^c \hat{\lambda}, \quad \delta \hat{A}_\mu^3 = \partial_\mu \hat{\lambda},
$$

with which we can easily check the gauge invariance of $\mathcal{L}_\phi$. We now see that the off-diagonal gluon fields $\hat{A}_\mu^b$ behave like massive charged matter fields with the mass $m_0$, while the diagonal gluon field $\hat{A}_\mu^3$ behaves like a massless $U(1)$ gauge field. For this reason, it is quite natural to interpret the unitary gauge as the maximally Abelian (MA) gauge discussed in earlier papers (see, e.g., Refs. [6,13,14,31]). Remarkably the Euler-Lagrange equation for $\phi^i$ can be written in the form of the MA gauge condition for $\hat{A}_\mu^b$:

$$
\hat{D}^{abc} \hat{A}_\mu^c \equiv (\delta^{bc} \partial_\mu + \epsilon^{bc3} \hat{A}_\mu^3) \hat{A}_\mu^c = 0.
$$

This fact also ensures that $\hat{A}_\mu^B$ would be the YM fields in the MA gauge. In the earlier papers, a similar equation for $A_\mu^b$, not for $\hat{A}_\mu^b$, has been put by hand as the MA gauge condition, whereas in the present letter the above equation has been derived as the Euler-Lagrange equation for $\phi^i$. 

5
Carrying out the Wick rotation $x^0 \rightarrow x^0 = -ix^4 \ (x^4 \in \mathbb{R})$ in Eqs. (1) and (2), let us consider the following effective action in Euclidean space-time:

$$W = \ln \int D\mathcal{M} \exp \left[ \int d^4x (\mathcal{L}_{YM} + \mathcal{L}_\phi + \mathcal{L}_{GF}) \right]$$

with $D\mathcal{M} \equiv D\mathcal{A}_\mu^B \mathcal{D}\phi^i \mathcal{D}c^B \mathcal{D}\bar{c}^B \mathcal{D}b^B$. Here $c^B$ and $\bar{c}^B$ are Faddeev-Popov ghost fields, $b^B$ are Nakanishi-Lautrup fields, and $\mathcal{L}_{GF}$ stands for a gauge fixing term (involving ghost terms) that is introduced to break the local $SU(2)$ gauge invariance of $\mathcal{L}_{SKG}$. The Becchi-Rouet-Stora-Tyutin (BRST) transformation $\delta$ is defined, as usual, by

$$\delta A_\mu^B = \mathcal{D}_\mu^{BC} c^C, \quad \delta \phi^i = -c^B K^{Bi}(\phi),$$

$$\delta c^B = \frac{1}{2} e^{BCD} c^C c^D, \quad \delta \bar{c}^B = ib^B, \quad \delta b^B = 0$$

(8)

to satisfy the nilpotency property $\delta^2 = 0$. We suppose that $\mathcal{L}_{GF}$ contains no $\phi^i$ but may contain the “classical” fields $\phi_0^i$ introduced in the following.

### III. PATH-INTEGRATION OVER THE NAMBU-GOLDSTONE SCALAR FIELDS

We first evaluate the path-integral over $\phi^i$ in Eq. (7) in the one-loop approximation, which can be done covariantly by employing the geometrical method proposed by Honerkamp [23]. In this method, the action $\int d^4x \mathcal{L}_\phi$ in the exponent of Eq. (7) is expanded about a solution $\phi_0^i$ of Eq. (6) in powers of new integration variables $\sigma^i$. Here $\sigma^i$ are understood to form the tangent vector at $\phi_0^i$ to the geodesic on $SU(2)/U(1)$ that runs from $\phi_0^i$ to the original integration variables $\phi^i$. Accordingly, the gauge-invariant measure $D_g\sigma^i \equiv (\det[g_{ij}(\phi_0)])^{1/2} D\sigma^i$ is chosen as $D\phi^i$. After ignoring the third and higher order terms in $\sigma^i$, the integration over $\sigma^i$ in Eq. (7) can be performed to get the determinant of a certain Laplace-type operator containing $A_\mu^B$ and $\phi_0^i$. Applying the heat kernel method to the evaluation of the determinant, we obtain a one-loop effective action. Addition of this to the classical action at $\phi_0^i$ leads to $\int d^4x \tilde{\mathcal{L}}_{\phi_0} \approx \ln \int D_g \sigma^i \exp \left[ \int d^4x \mathcal{L}_\phi \right] + \ln N_0$, where $N_0$ is a normalization constant and $\approx$ stands for the one-loop approximation. (The $N_l \ (l = 1, \ldots, 6)$ that appear in the following denote normalization constants.) By using the Maurer-Cartan formula in Eq. (4) at $\phi^i = \phi_0^i$, $\tilde{\mathcal{L}}_{\phi_0}$ is written in terms of $\tilde{A}_\mu^B \equiv \tilde{A}_\mu^B|_{\phi=\phi_0} = A_\mu^C D^{CB}(v(\phi_0)) + e_\mu^B(\phi_0)$ as
\[ \tilde{\mathcal{L}}_{\phi_0} = \frac{A^4}{32\pi^2} + \left( \frac{m_0^2}{2g_0^2} - \frac{A^2}{32\pi^2} \right) S - \frac{1}{16\pi^2} \ln \frac{\mu}{\Lambda} \left( \frac{1}{6} F_{\mu\nu} F^{\mu\nu} + \frac{1}{8} (1 + 3k_1) S^2 \right) - \frac{k_1}{2} A_{\mu\nu} A^{\mu\nu} + \frac{1}{2} (1 - k_1) \mathcal{T}_{\mu\nu} \mathcal{T}^{\mu\nu} \right) + O(1/A^2), \tag{9} \]

where \( F_{\mu\nu} \equiv \partial_{\mu} \tilde{A}_{\nu}^3 - \partial_{\nu} \tilde{A}_{\mu}^3 - \Sigma_{\mu\nu}(\phi_0) \), \( S \equiv \tilde{A}_{\mu}^b \tilde{A}^{\mu b} \), \( A_{\mu\nu} \equiv -3^{bc} \tilde{A}_{\mu}^b \tilde{A}_{\nu}^c \), and \( \mathcal{T}_{\mu\nu} \equiv \tilde{A}_{\mu}^b \tilde{A}^{\nu b} - \frac{1}{4} \eta_{\mu\nu} S \), with \( \eta_{\mu\nu} \equiv -\delta_{\mu\nu} \) the metric on Euclidean space-time. In deriving Eq. (9), a mass scale \( \mu \) and an ultraviolet cutoff \( \Lambda \) have been introduced to make \( \tilde{\mathcal{L}}_{\phi_0} \) a meaningful expression. In addition, a dimensionless constant \( k_1 \) has been introduced to indicate an arbitrariness of expression due to the identity \( A_{\mu\nu} A^{\mu\nu} = \frac{3}{4} S^2 - \mathcal{T}_{\mu\nu} \mathcal{T}^{\mu\nu} \). At \( \phi^i = \phi_0^i \) the gauge transformation rules in Eq. (5) read

\[ \begin{align*}
\delta \tilde{A}_{\mu}^b &= -\epsilon^{b c 3} \tilde{A}_{\mu}^c \lambda, \\
\delta \tilde{A}_{\mu}^3 &= \partial_{\mu} \tilde{\lambda},
\end{align*} \tag{10a, 10b} \]

with \( \tilde{\lambda} \equiv \lambda^B \Omega^B(\phi_0) \). As can be seen from Eq. (10a), \( S, A_{\mu\nu} \) and \( \mathcal{T}_{\mu\nu} \) are gauge invariant.

In Eq. (9), \( \mathcal{L}_{\phi_0} \equiv \frac{1}{2}(m_0/g_0)^2 S \) is the only classical term.

**IV. AUXILIARY FIELDS**

In terms of \( \tilde{A}_{\mu} B \), the Lagrangian \( \mathcal{L}_{YM} \) can be written as

\[ \mathcal{L}_{YM} = \left( -\frac{1}{4g_0^2} \right) \left[ F_{\mu\nu} F^{\mu\nu} + 2 F_{\mu\nu} A^{\mu\nu} + k_2 A_{\mu\nu} A^{\mu\nu} + (1 - k_2) \left( \frac{3}{4} S^2 - \mathcal{T}_{\mu\nu} \mathcal{T}^{\mu\nu} \right) + \tilde{F}_{\mu\nu}^b \tilde{F}^{\mu\nu b} \right], \tag{11} \]

where \( k_2 \) is a constant similar to \( k_1 \), and \( \tilde{F}_{\mu\nu}^b \) is the field strength of \( \tilde{A}_{\mu}^b \), expressed as \( \tilde{F}_{\mu\nu}^b = \tilde{D}_{\mu}^{bc} \tilde{A}_{\nu}^c - \tilde{D}_{\nu}^{bc} \tilde{A}_{\mu}^c \) with \( \tilde{D}_{\mu}^{bc} \equiv \delta^{bc} \partial_{\mu} + \epsilon^{bc 3} \tilde{A}_{\mu}^3 \).

In order to derive an EAGT involving \( \tilde{A}_{\mu}^3 \), we need to evaluate the path-integral over \( \tilde{A}_{\mu}^3 \) in the effective action

\[ W' = \ln \int \mathcal{D} \mathcal{M}' \exp \left[ \int d^4x (\mathcal{L}_{YM} + \tilde{\mathcal{L}}_{\phi_0} + \mathcal{L}_{GF}) \right] \approx W + \ln N_0 \] \tag{12} \]

with \( \mathcal{D} \mathcal{M}' \equiv \mathcal{D} \tilde{A}_{\mu} B \mathcal{D} e^B \mathcal{D} \bar{e}^B \mathcal{D} b^B \). Here the gauge invariance of the path-integral measure \( \mathcal{D} A_{\mu}^B \), i.e., \( \mathcal{D} A_{\mu}^B = \mathcal{D} \tilde{A}_{\mu}^B \), has been used. In order to perform the integration over \( \tilde{A}_{\mu}^b \)
exactly, we introduce the following auxiliary fields: an antisymmetric tensor field \( B_{\mu\nu} \), a scalar field \( \Phi \) and a traceless symmetric tensor field \( h_{\mu\nu} \) that are associated with \( A_{\mu\nu} \), \( S \) and \( T_{\mu\nu} \), respectively.\(^2\) With the aid of these new fields, \( \exp \left[ \int d^4x (\mathcal{L}_{\text{YM}} + \mathcal{L}_{\phi_0}) \right] \) can be expressed as \( N_1 \int \mathcal{D}B_{\mu\nu} \mathcal{D}\Phi \mathcal{D}h_{\mu\nu} \exp \left[ \int d^4x \mathcal{L}_1 \right] \) with

\[
\mathcal{L}_1 = \frac{A^4}{32\pi^2} - \frac{1}{4} \left( \frac{1}{g_0^2} - \frac{1}{24\pi^2} \ln \frac{\mu}{\Lambda} - \frac{\kappa^2}{q_1} \right) F_{\mu\nu} F^{\mu\nu} + \left( \frac{m_0^2}{2g_0^2} - \frac{A^2}{32\pi^2} \right) S
- \frac{1}{2} \left( \frac{1}{g_0^2} - \frac{\kappa}{q_1} \right) F_{\mu\nu} A^{\mu\nu} + \frac{i}{2} B_{\mu\nu} (\kappa F^{\mu\nu} + A^{\mu\nu}) - \frac{q_1}{4} B_{\mu\nu} B^{\mu\nu}
- \frac{i}{2} \Phi S + \frac{q_2}{4} \Phi^2 + \frac{i}{2} h_{\mu\nu} T^{\mu\nu} - \frac{q_3}{4} h_{\mu\nu} h^{\mu\nu} - \frac{1}{4g_0^2} \tilde{F}^{\mu\nu} \tilde{F}_{\mu\nu}
+ O_1(A^{-2}).
\]

(13)

Here \( \kappa \) is an arbitrary dimensionless constant, and \( q_1, q_2 \) and \( q_3 \) are defined by

\[
q_1^{-1} \equiv \frac{k_2}{g_0^2} - \frac{k_1}{8\pi^2} \ln \frac{\mu}{\Lambda},
q_2^{-1} \equiv \frac{3(1 - k_2)}{4g_0^2} + \frac{1 + 3k_1}{32\pi^2} \ln \frac{\mu}{\Lambda},
q_3^{-1} \equiv \frac{k_2 - 1}{g_0^2} - \frac{k_1 - 1}{8\pi^2} \ln \frac{\mu}{\Lambda}.
\]

(14, 15, 16)

Taking into account the gauge invariance of \( F_{\mu\nu} \), \( A_{\mu\nu} \), \( S \) and \( T_{\mu\nu} \), we impose the gauge transformation rules \( \delta B_{\mu\nu} = \delta \Phi = \delta h_{\mu\nu} = 0 \) on the auxiliary fields. Then \( \mathcal{L}_1 \) is obviously gauge invariant. The BRST invariance of \( \mathcal{L}_1 \) is guaranteed with \( \delta B_{\mu\nu} = \delta \Phi = \delta h_{\mu\nu} = 0 \).

The effective action we are now concerned with is thus

\[
W'' = \ln \int \mathcal{D}M'' \exp \left[ \int d^4x (\mathcal{L}_1 + \mathcal{L}_{\text{GF}}) \right]
( = W' - \ln N_1 ),
\]

(17)

where \( \mathcal{D}M'' \equiv \mathcal{D}\tilde{A}^{\mu B} \mathcal{D}B_{\mu\nu} \mathcal{D}\Phi \mathcal{D}h_{\mu\nu} \mathcal{D}c^B \mathcal{D}\tilde{c}^B \mathcal{D}b^B \).

V. GAUGE FIXING TERM

We next define the gauge fixing term \( \mathcal{L}_{\text{GF}} \) in such a way that it satisfies the following three conditions: (i) \( \mathcal{L}_{\text{GF}} \) breaks the “local” \( SU(2) \) gauge invariance of \( \mathcal{L}_1 \). (ii) \( \mathcal{L}_{\text{GF}} \) remains

\(^2\) In the case of pure \( SU(2) \) YM theory without the scalar fields \( \phi^i \), we can exactly carry out the integration over \( \tilde{A}^{\mu B} \) with the aid of \( B_{\mu\nu} \) alone \([13,14,27]\). In the present case involving \( \phi^i \), the introduction of \( \Phi \) and/or \( h_{\mu\nu} \) is essential for exactly carrying out the integration over \( \tilde{A}^{\mu B} \).
to 1 + $i\phi$ the condition $v$ different manners than $\tilde{s}$ satisfied. Under the local $SU$ gauge field as transforms inhomogeneously in the same manner as gauge transformation, $\tilde{\epsilon}^2$ gauge invariance of $SU$ for $A$ where $\tilde{\epsilon}$ transformation is characterized by $h$ coordinates $x$. The condition (i) is essential to quantize $\tilde{A}^B_\mu$ (or $A^B_\mu$). The condition (ii) is necessary for preserving the global $SU(2)$ gauge invariance of $L_1$ to guarantee conservation of the Noether current for the global $SU(2)$ gauge symmetry. It should be noted here that both of the local and global $SU(2)$ gauge transformations are converted into the corresponding local $U(1)$ gauge transformations via the transformation rule $g_\mu(\phi_0) = v(\phi_0)h(\phi_0, g)$. The local $SU(2)$ gauge invariance of $L_1$ is then realized as invariance of $L_1$ under the local $U(1)$ gauge transformation characterized by $h(\phi_0, g(x))$ with $g(x)$ being dependent on space-time coordinates $x^\mu$. The local $U(1)$ gauge transformation produced by the global $SU(2)$ gauge transformation is characterized by $h(\phi_0, g_c)$ with $g_c$ being independent of $x^\mu$; the locality of this $U(1)$ gauge transformation is due to the $x^\mu$-dependence of $\phi_0^i$. The condition (iii) will be necessary for the proof of unitarity based on the Kugo-Ojima quartet mechanism [24]. A gauge fixing term that satisfies the above conditions and that is appropriate for our discussion is given in terms of $\tilde{A}^B_\mu$, $\tilde{\epsilon}^B_\mu \equiv e^B_\mu(\phi_0)$, $\tilde{\epsilon}^B_3 \equiv e^C_i\gamma^C_i(v(\phi_0))$ and $\beta^B \equiv b^C_i\gamma^C_i(v(\phi_0))$ by

$$L_{GF} = \frac{i}{g_0^2} \delta \left[ \tilde{D}^B_\mu \gamma^C \cdot (\tilde{A}^B_\mu - \tilde{\epsilon}^B_\mu) - \frac{\alpha_0}{2} \tilde{\epsilon}^B_\beta \beta^B \right],$$

where $\tilde{D}^B_\mu \equiv \delta^{BC}\partial_\mu + \epsilon^{BCD}\tilde{A}^D_\mu$, and $\alpha_0$ is a (bare) gauge parameter. The global $SU(2)$ gauge invariance of $L_{GF}$ is realized as invariance of $L_{GF}$ under the local $U(1)$ gauge transformation characterized by $h(\phi_0, g_c)$. If $\tilde{D}^B_\mu$ in Eq. (18) is replaced by $\tilde{\nu}^B_\mu \equiv \delta^{BC}\partial_\mu + \epsilon^{BCD}\tilde{\epsilon}^D_\mu$, then $L_{GF}$ reduces to the ordinary Lorenz gauge-fixing term for $A^B_\mu$, which preserves the global $SU(2)$ gauge invariance of $L_1$. Under the global $SU(2)$ gauge transformation, $\tilde{\epsilon}^B_\mu$ transform homogeneously in the same manner as $\tilde{A}^B_\mu$, while $\tilde{\epsilon}^3_\mu$ transforms inhomogeneously in the same manner as $\tilde{A}^3_\mu$. (In $\tilde{\nu}^B_\mu$, $\tilde{\epsilon}^3_\mu$ plays the role of a gauge field as $\tilde{A}^3_\mu$ plays in $\tilde{D}^B_\mu$.) From these facts we see that the condition (ii) is satisfied. Under the local $SU(2)$ gauge transformation, $\tilde{\epsilon}^B_\mu$ transform inhomogeneously in different manners than $\tilde{A}^B_\mu$, so that the condition (i) is satisfied. Since $v(\phi_0)$ approximates to $1 + i\phi_0^i\delta^B_\mu T^B$ in the free limit, the condition (iii) is evidently satisfied. (We can impose the condition $v(0) = 1$ without loss of generality.)

The BRST transformation rules of $\tilde{A}^B_\mu$, $\tilde{\epsilon}^B_\mu$, $\gamma^B \equiv e^C_i\gamma^C_i(v(\phi_0))$, $\tilde{\gamma}^B$ and $\beta^B$ are deter-
mined from Eq. (8) to be
\[
\delta \tilde{A}_\mu^b = -\epsilon^{bc3} \tilde{A}_\mu^c, \quad \delta \tilde{A}_\mu^3 = \partial_\mu \tilde{c},
\]
\[
\delta \tilde{\epsilon}_\mu^B = -\tilde{\nabla}_\mu^{BC} (\gamma^C - \delta^{C3} \tilde{c}),
\]
\[
\delta \gamma^B = -\frac{1}{2} \epsilon^{BCD} \gamma^C \gamma^D + \epsilon^{BC3} \gamma^C \tilde{c},
\]
\[
\delta \tilde{\epsilon}^B = i\beta^B - \epsilon^{BDE} \gamma^E \tilde{D}_\mu^B \gamma^D + \omega^b_0 \beta^B \beta^B,
\]
\[
(19)
\]
where \( \tilde{c} \equiv \Omega^B(\phi_0) \). Carrying out the BRST transformation in the right hand side of Eq. (18), we obtain
\[
\mathcal{L}_{GF} = \frac{1}{g_0^2} \left[ (\beta^b + i\epsilon^{bDE} \gamma^D \gamma^E) \tilde{D}^{abc} \tilde{A}_\mu^c 
- (\tilde{D}^{B3} \gamma^C \tilde{D}_\mu^B \gamma^D + \omega^b_0 \beta^B \beta^B) \right] + \text{total derivative}.
\]
(20)
The first term of Eq. (20) vanishes owing to the equation
\[
\tilde{D}^{abc} \tilde{A}_\mu^c = \tilde{D}^{abc} \tilde{A}_\mu^c |_{\phi = \phi_0} = 0.
\]
(21)
Recall here that \( \phi_0 \) is a solution of Eq. (6). Consequently \( \mathcal{L}_{GF} \) does not depend on the off-diagonal gluon fields \( \tilde{A}_\mu^b \), so that the effective action \( W'' \) can be written as
\[
W'' = \ln \int \mathcal{D} \tilde{A}_\mu^3 \mathcal{D} B_{\mu\nu} \mathcal{D} \Phi \mathcal{D} h_{\mu\nu} e^{(W'_1 + W''_{GF})},
\]
with
\[
W'_1 = \ln \int \mathcal{D} \tilde{A}_\mu^b \exp \left[ \int d^4x \mathcal{L}_1 \right],
\]
\[
W''_{GF} = \ln \int \mathcal{D} \gamma^B \mathcal{D} \gamma^B \mathcal{D} \beta^B \exp \left[ \int d^4x \mathcal{L}_{GF} \right].
\]
(24)
Here the gauge invariance of the path-integral measures, i.e., \( \mathcal{D} c^B = \mathcal{D} \gamma^B \), \( \mathcal{D} \tilde{c}^B = \mathcal{D} \tilde{\gamma}^B \) and \( \mathcal{D} b^B = \mathcal{D} \beta^B \), has been used.

VI. EFFECTIVE ABELIAN GAUGE THEORY

We now evaluate the path-integral over \( \tilde{A}_\mu^b \) in Eq. (23), rewriting Eq. (13) in the form
\[
\mathcal{L}_1 = \mathcal{L}_1^{(0)} + \frac{1}{2} \tilde{A}_\mu^b h^{\mu b,\nu c} \tilde{A}_\nu^c + O_1(A^{-2}) \text{ up to total derivatives. Here } \mathcal{L}_1^{(0)} \text{ consists of the terms}
\]
that contain no \( \tilde{A}_\mu^b \), i.e.,

\[
\mathcal{L}_1^{(0)} = \frac{\Lambda^4}{32\pi^2} - \frac{1}{4} \left( \frac{1}{g_0^2} - \frac{1}{24\pi^2} \ln \frac{\mu}{\Lambda} - \frac{\kappa^2}{q_1} \right) F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \kappa B_{\mu\nu} F^{\mu\nu} - \frac{q_1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{q_2}{4} \Phi^2 - \frac{q_3}{4} h_{\mu\nu} h^{\mu\nu}.
\]

(25)

Recall the fact that \( O_1(\Lambda^{-2}) \) has been found at the one-loop level in evaluating the path-integral over \( \phi^\prime \). Then we see that the terms in \( O_1(\Lambda^{-2}) \) which contain \( \tilde{A}_\mu^b \) contribute as two-loop effects to the one-loop effective action obtained by the integration over \( \tilde{A}_\mu^b \) in Eq. (23), provided that the one-loop approximation is carried out around the classical solution \( \tilde{A}_\mu^b = 0 \). Because the two-loop effects are now irrelevant to our discussion, it is sufficient for evaluating the path-integral over \( \tilde{A}_\mu^b \) to consider \( \frac{1}{2} \tilde{A}_\mu^b \mathcal{H}_1^{\mu\nu,bc} \tilde{A}_{\nu}^c \) only. After the use of the commutation relation \([\bar{D}_\mu, D_\nu]^{bc} \tilde{A}_\rho^c = 0\), we can approximately calculate the Laplace-type operator

\[
\mathcal{H}_1^{\mu\nu,bc} = \frac{1}{g_0^2} \eta^{\mu\nu} \bar{D}_\rho \bar{D}_\rho \delta^{bc} + \eta^{\mu\nu} \delta^{bc} \left( \frac{m_0^2}{g_0^2} - \frac{\Lambda^2}{16\pi^2} + i\Phi \right) + i\delta^{bc} \tilde{h}^{\mu\nu}
\]

+ \( \epsilon^{bca} \left\{ \left( \frac{2}{g_0^2} - \frac{\kappa}{q_1} \right) F^{\mu\nu} - iB^{\mu\nu} \right\} \).

(26)

Supposing \( m_0/g_0 \geq \Lambda/4\pi \), we carry out the Gaussian integration over \( \tilde{A}_\mu^b \) in Eq. (23) to get an expression written with the aid of a proper-time \( \tau \):

\[
\int d^4x \mathcal{L}_1^{(1)} = \ln \int \mathcal{D} \tilde{A}_\mu^b \exp \left[ \frac{1}{2} \int d^4x \tilde{A}_\mu^b \mathcal{H}_1^{\mu\nu,bc} \tilde{A}_{\nu}^c \right] + \ln N_2
\]

\[
= \int d^4x \frac{1}{2} \sum_{\mu,b} \int_1^{\infty} \frac{d\tau}{\tau} \langle \langle x, \mu | e^{-\tau \mathcal{H}_1} | x, \mu \rangle \rangle.
\]

(27)

(Later on we will point out that \( m_0/g_0 = \Lambda/4\pi \) is a reasonable condition, see Eq. (60).) The ket vectors \( |x, \mu\rangle \) are defined by \( |x, \mu\rangle = \sum_C |x, \mu_C \rangle \mathbb{M}^{Cb}(v(\phi_0)) \) with the basis vectors \( |x, \mu_B \rangle \) that satisfy \( \langle x, \mu_B | y, \nu_C \rangle = \delta^4(x - y) \delta_\mu^\nu \delta^{BC} \) and that transform homogeneously under the \( SU(2) \) gauge transformations, i.e., \( \delta_\mu^\nu \langle x, \mu_B \rangle = \epsilon^{BCD} \langle x, \mu_D \rangle \lambda^C \). Then \( |x, \mu\rangle \) transform in the same manner as \( \tilde{A}_\mu^b \) (see Eq. (10a)), so that the gauge invariance of the operator

\[
\mathcal{H}_1 \equiv \int d^4x \sum_{\mu,b} \sum_{\nu,c} |x, \mu\rangle \mathcal{H}_1^{\mu\nu,bc}(x) \langle \langle x, \nu \rangle \rangle
\]

is guaranteed. In this way the gauge invariance of \( \mathcal{L}_1^{(1)} \) is confirmed. Using the heat kernel equation for the matrix elements \( \langle \langle x, \mu | e^{-\tau \mathcal{H}_1} | y, \nu \rangle \rangle \), we can approximately calculate the effective action in Eq. (27). In the process of calculation, we have to take account of the
Dirac string singularity occurring in \( [\partial_\mu, \partial_\nu] \mathbb{D}^{BC}(v^{-1}(\phi_0)) = -\epsilon^{3BD} \Sigma_{\mu\nu}(\phi_0) \mathbb{D}^{BC}(v^{-1}(\phi_0)) \). The result of the calculation reads

\[
\mathcal{L}^{(1)} = \frac{\Lambda^4}{8\pi^2} - \frac{g_0^2 A^2}{4\pi^2} i\Phi - \frac{1}{16\pi^2} \ln \frac{\mu}{A} \left\{ 4 \left( 1 - \frac{\kappa g_0^2}{2q_1} \right)^2 - \frac{2}{3} \right\} F_{\mu\nu} F^{\mu\nu} 
- 4ig_0 \left( 1 - \frac{\kappa g_0^2}{2q_1} \right) B_{\mu\nu} F^{\mu\nu} - g_0^4 B_{\mu\nu} B^{\mu\nu} - 4g_0^4 \Phi^2 - g_0^4 h_{\mu\nu} h^{\mu\nu} + \frac{g_0^4}{192\pi^2} \left( \partial_\mu B_{\nu\rho} \partial^\mu B^{\nu\rho} + 4\partial_\mu \Phi \partial^\mu \Phi + \partial_\mu h_{\nu\rho} \partial^\mu h^{\nu\rho} \right) + O_{2h}(\Lambda^{-2})
\]

(29)
after making the replacement \( \Phi \to \Phi + i(m_0^2/g_0^2 - \Lambda^2/16\pi^2) \). Here \( O_{2h}(\Lambda^{-2}) \) consists of terms of higher order in \( F_{\mu\nu}, B_{\mu\nu}, \Phi, h_{\mu\nu} \) and/or their derivatives, including such higher derivative terms as \( \partial_\mu F_{\nu\rho} \partial^\mu F^{\nu\rho} \) and \( \partial_\mu B_{\nu\rho} \partial^\mu F^{\nu\rho} \). It should be stressed that the kinetic terms of \( B_{\mu\nu}, \Phi \) and \( h_{\mu\nu} \) have been induced by virtue of quantum effect of \( \tilde{A}_\mu \). In this sense \( B_{\mu\nu}, \Phi \) and \( h_{\mu\nu} \) are no longer auxiliary fields. (As for \( B_{\mu\nu} \), such an induced kinetic term has been found in various contexts [25–28].)

Next we consider the path-integral over \( \gamma^B, \tilde{\gamma}^B \) and \( \beta^B \) in Eq. (24). In the case \( \alpha_0 \neq 0 \), the integration over \( \beta^B \) becomes a simple Gaussian integration and can readily be done to get

\[
\int d^4 x \mathcal{L}^{GF}_G = \ln \mathcal{D}^{BG} \exp \left[ \int d^4 x \mathcal{L}^{BG}_{GF} \right] + \ln N_3
\]

In order to perform the integration over \( \gamma^B \) and \( \tilde{\gamma}^B \), we rewrite \( \mathcal{L}^{BG}_{GF} \) in the form \( \mathcal{L}^{(0)}_{GF} + i\tilde{\gamma}^B \mathcal{H}^{BG}_{GF} \tilde{\gamma}^C \) up to total derivatives. Here \( \mathcal{L}^{(0)}_{GF} \) and \( \mathcal{H}^{BG}_{GF} \) are given by

\[
\mathcal{L}^{(0)}_{GF} = -\frac{1}{2\alpha_0 g_0^2} \left( \partial^\mu a_\mu \partial^\nu a_\nu + \bar{\tilde{\gamma}}^{\mu bc} \tilde{e}_\mu \bar{\tilde{\gamma}}^{\nu bd} \tilde{e}_\nu \right) d - 2\epsilon^{3bc} a_\mu \tilde{e}^{\mu bd} \bar{\tilde{\gamma}}^{\nu cd} \tilde{e}_\nu d + a_\mu a_\nu \epsilon^{\mu bc} \bar{\tilde{\gamma}}^{\nu bd} \tilde{e}_\nu d
\]

(30)

\[
\mathcal{H}^{BG}_{GF} = \frac{1}{g_0} \left[ \tilde{A}^{BD}_{\mu} \delta_{\mu DC} - \frac{1}{2} \epsilon^{BCa} \bar{D}_{\mu ad} \tilde{e}_\mu d + \frac{1}{2} \epsilon^{BC3} \partial^\mu a_\mu 
+ \frac{1}{4} \left( \delta^{BC} (a_\mu a_\mu + \tilde{e}_\mu d \tilde{e}^{\mu cd}) - (\delta^{B3} \tilde{A}_\mu^3 - \tilde{e}_\mu^B) (\delta^{C3} \tilde{A}_\mu^3 - \tilde{e}_\mu^C) \right) \right]
\]

(31)

with \( a_\mu \equiv \tilde{A}_\mu^3 - \tilde{e}_\mu^3, \bar{\tilde{\gamma}}^{\mu bc} \equiv \delta^{bc} \partial_\mu + \epsilon^{bc3} \tilde{e}_\mu^3 \) and

\[
\tilde{A}_\mu^{BC} \equiv \delta^{BC} \partial_\mu + \frac{1}{2} \epsilon^{BCd} \tilde{e}_\mu d + \frac{1}{2} \epsilon^{BC3} (\tilde{A}_\mu^3 + \tilde{e}_\mu^3). \quad (32)
\]

Note that under the global \( SU(2) \) gauge transformation, \( a_\mu \) is invariant, while \( \frac{1}{2}(\tilde{A}_\mu^3 + \tilde{e}_\mu^3) \) transforms inhomogeneously in the same manner as \( \tilde{A}_\mu^3 \). In \( \tilde{A}_\mu^{bc}, \frac{1}{2}(\tilde{A}_\mu^3 + \tilde{e}_\mu^3) \) plays the
role of a gauge field. The Gaussian integration over $\gamma^B$ and $\tilde{\gamma}^B$ is carried out to get

$$
\int d^4x L_{GF}^{(1)} = \ln \int \mathcal{D}\gamma^B \mathcal{D}\tilde{\gamma}^B \exp \left[ i \int d^4x \gamma^B \mathcal{H}_{GF}^{BC} \gamma^C \right] + \ln N_4
$$

$$
= - \int d^4x \sum_B \int_{1/\Lambda^2}^{\infty} \frac{d\tau}{\tau} \langle \langle x, B | e^{-\tau \hat{\mathcal{H}}_{GF}} | x, B \rangle \rangle ,
$$

(33)

where the ket vectors $|x, B\rangle$ are defined by $|x, B\rangle = \sum_C |x, C\rangle \mathcal{D}^{CB}(v(\phi_0))$ with the orthonormal basis vectors $|x, B\rangle$ that transform homogeneously under the $SU(2)$ gauge transformations. Then $|x, B\rangle$ obey the gauge transformation rule same as that of $\gamma^B$ and $\tilde{\gamma}^B$, so that the operator $\hat{\mathcal{H}}_{GF} \equiv \int d^4x \sum_{B,C} |x, B\rangle \mathcal{H}_{GF}^{BC}(x) \langle x, C|$ is gauge invariant. Consequently the gauge invariance of $L_{GF}^{(1)}$ is also confirmed. Applying the heat kernel method to calculating the $\tau$-integral in Eq. (33), we obtain

$$
L_{GF}^{(1)} = - \frac{3A^4}{32\pi^2} + \frac{A^2}{32\pi^2}(a_\mu a^\mu + \tilde{e}_\mu \tilde{e}^{\mu b})
$$

$$
+ \frac{1}{8\pi^2} \ln \frac{\mu}{\Lambda} \left\{ - \frac{1}{24}(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \mathcal{F}_{\mu\nu}J^{\mu\nu} + J_{\mu\nu}J^{\mu\nu})
$$

$$
- \frac{1}{4} \partial^\mu a_\mu \partial^\nu a_\nu + \frac{1}{16}(a_\mu a^\mu)^2 + \frac{1}{2} \epsilon^{abc} a_\mu \tilde{e}^{\mu b} \nabla^{\nu c} \tilde{e}_\nu^d
$$

$$
+ \frac{1}{24} a_\mu a^\mu \tilde{e}_\nu^b \tilde{e}^{\nu b} + \frac{1}{6} a_\mu a_\nu \tilde{e}^{\mu b} \tilde{e}^{\nu b} - \frac{1}{4} \nabla^{\mu bc} \tilde{e}_\mu e^{\nu bd} \tilde{e}_\nu^d
$$

$$
+ \frac{1}{16}(\tilde{e}_\mu \tilde{e}^{\mu b})^2 \right\} + O_3(\Lambda^{-2})
$$

(34)

with $J_{\mu\nu} = J_{\mu\nu}(\phi_0) \equiv \epsilon^{abc} \tilde{e}_\mu^b \tilde{e}_\nu^a$. By using the Maurer-Cartan formula in Eq. (4) at $\phi^i = \phi_0^i$, $\mathcal{F}_{\mu\nu}$ can be written as

$$
\mathcal{F}_{\mu\nu} = f_{\mu\nu} + J_{\mu\nu} ,
$$

(35)

with $f_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu$.

The exponent in Eq. (22) is thus found to be $W'' + W''_{GF} = \int d^4x L_{tot} - \ln(N_2N_3N_4)$ with

$$
L_{tot} = L_0^{(0)} + L_1^{(1)} + L_{GF}^{(0)} + L_{GF}^{(1)} + O_1(\Lambda^{-2}) ,
$$

(36)

where $O_1(\Lambda^{-2})$ denotes a part of $O_1(\Lambda^{-2})$ that consists of the terms containing $\mathcal{F}_{\mu\nu}$ but no $\tilde{A}_\mu^b$. In what follows, we are concerned with the effective Abelian gauge theory (EAGT) defined by the Lagrangian $L_{tot}$. As has been expected, $L_{tot}$ remains invariant under the $U(1)$ gauge transformation characterized by $h(v(\phi_0), g_c)$. Since this transformation is produced by the global $SU(2)$ gauge transformation due to the action of $g_c$, we should consider that the global $SU(2)$ gauge symmetry is maintained in the EAGT and is realized in $L_{tot}$ in a
nonlinear way. In fact, it is possible to derive the non-Abelian Noether current for the global $SU(2)$ gauge invariance of $\mathcal{L}_{\text{tot}}$. This current is written with the use of the $H$-compensators $\Omega^B(\phi_0)$.

VII. STATIC POTENTIALS

Considering $\mathcal{L}_{\text{tot}}$ as a classical Lagrangian, let us investigate what kind of potential arises from propagation of $B_{\mu\nu}$ at the tree level of the EAGT. With $\mathcal{L}_{\text{tot}}$ in hand, the bare propagator of $B_{\mu\nu}$ follows from the kinetic term and mass term of $B_{\mu\nu}$ together with a source term of $B_{\mu\nu}$. In addition to the linear terms in $B_{\mu\nu}$ explicitly written in Eq. (25) and Eq. (29), further linear terms in $B_{\mu\nu}$ exist in $O_{2h}(\Lambda^{-2})$. Fortunately, if $\bar{\kappa} \equiv 2q_1/g_0^2$ is chosen to be the constant $\kappa$ in $\mathcal{L}_{\text{tot}}$, these linear terms as well as the $B_{\mu\nu}F^{\mu\nu}$ term in Eq. (29) vanish in $\mathcal{L}^{(1)}_{\text{B}}$ and it becomes easier to make a discussion. On putting $\kappa = \bar{\kappa}$, the terms relevant to deriving the bare propagator of $B_{\mu\nu}$ are displayed, after being rescaled $B_{\mu\nu}$ as $B_{\mu\nu} \rightarrow 4\sqrt{3}\pi g g_0^{-2}B_{\mu\nu}$, as follows:

$$\mathcal{L}^{(0)}_{\text{B}} \equiv -\frac{1}{4} B_{\mu\nu} (\Box + M_{1}^2) B^{\mu\nu} + \frac{i}{2} M_{2} B_{\mu\nu}F^{\mu\nu},$$

where $\Box \equiv \partial_\mu \partial^\mu = -\partial_\mu \partial_\mu$ and

$$M_{1} \equiv \frac{4\sqrt{3}\pi}{g_0^2} \left( q_1 - \frac{g_0^4}{4\pi^2} \ln \frac{\mu}{\Lambda} \right)^{1/2} \Lambda,$$

$$M_{2} \equiv \frac{8\sqrt{3}\pi q_1}{g_0^4} \Lambda.$$ 

Since $J^{\mu\nu}$ consists only of the classical fields $\phi_0^i$, we can treat $J^{\mu\nu}$ as a classical antisymmetric tensor current. Then we see that $\mathcal{L}^{(0)}_{\text{B}}$ is very analogous to a part of the covariantly gauge fixed version of the Lagrangian that defines a massive Abelian antisymmetric tensor gauge theory (MAATGT) with antisymmetric tensor current (ATC) [19].

Obviously $\mathcal{L}^{(0)}_{\text{B}}$ describes a massive rank-2 antisymmetric tensor field coupled with $f^{\mu\nu}$ and $J^{\mu\nu}$. For any mass scale characterized by $\mu (\leq \Lambda)$, positivity of $M_1^2$ is guaranteed as long as appropriate constants are chosen to be $k_1$ and $k_2$. Let us denote by $\mathcal{L}^{(1)}_{\text{B}}$ a part of $O_{2h}(\Lambda^{-2})$ that consists of the terms containing $B_{\mu\nu}$. Noting the fact that the source term $\frac{1}{2}i M_{2} B_{\mu\nu}J^{\mu\nu}$ in $\mathcal{L}^{(0)}_{\text{B}}$ exists in the

---

3 If the vorticity tensor current is chosen as the ATC, the MAATGT with ATC reduces to a dual theory of the extended dual Abelian Higgs model (EDAHM) in the London limit [18,19].
exponent of Eq. (22), we represent \( B_{\mu\nu} \) in \( \mathcal{L}_{B}^{(1)} \) as the functional derivative \(-iM_2^{-1}\delta/\delta J^{\mu\nu}\). Thereby the integration over \( B_{\mu\nu} \) in Eq. (22) reduces to a Gaussian integration, which can be carried out to obtain

\[
W'' = \ln \int \mathcal{D} \tilde{A}_\mu^3 \mathcal{D} \Phi \mathcal{D} h_{\mu\nu} \exp \left[ \int d^4x \mathcal{L}_{\text{tot}}' \right],
\]

where \( \mathcal{L}_{\text{tot}}' \) is given by

\[
\int d^4x \mathcal{L}_{\text{tot}}' = W_1'' + W_{\text{GF}}'' - \int d^4x \left( \mathcal{L}_{B}^{(0)} + \mathcal{L}_{B}^{(1)} \right) + \ln Z_B
\]

with

\[
Z_B \equiv \frac{1}{N_5} \exp \left[ \int d^4x \mathcal{L}_{B}^{(1)} \bigg|_{B_{\mu\nu} = -iM_2^{-1}\delta/\delta J^{\mu\nu}} \right] \times \exp \left[ \int d^4x \left( -\frac{1}{4} F_{\mu\nu} \frac{M_2^2}{\Box + M_1^2} F^{\mu\nu} \right) \right] = \frac{1}{N_5} \exp \left[ \int d^4x \left( -\frac{1}{4} F_{\mu\nu} \frac{M_2^2}{\Box + M_1^2} F^{\mu\nu} + \cdots \right) \right].
\]

The bare propagator of \( B_{\mu\nu} \) has been found in the exponent of Eq. (42). The remaining terms indicated by the dots may be calculated, at least formally, by a perturbative method.

Together with deriving the potential due to propagation of \( B_{\mu\nu} \), we also try to derive the potential due to propagation of \( a_\mu \), rather than \( \tilde{A}_\mu^3 \), at the tree level. To this end, we pay attention to a part of \( \mathcal{L}_{\text{tot}}' \) which consists of a relevant source term for \( a_\mu \) and of the terms that are quadratic in \( a_\mu \) or its first order derivatives and contain no other fields. Here, as a relevant term to our discussion, we also take the local term \(-\frac{1}{4}(M_2/M_1)^2(f_{\mu\nu} + 2J_{\mu\nu})f^{\mu\nu}\) that is given as a leading term of the series expansion \((\Box + M_1^2)^{-1} = M_1^{-2}(1 - M_1^{-2}\Box + \cdots)\) in the non-local term explicitly written in the exponent of Eq. (42). The part of \( \mathcal{L}_{\text{tot}}' \) that we are concerned now with is thus found from Eqs. (25), (29), (30), (34) and (42) to be

\[
\mathcal{L}_{a}^{(0)} \equiv -\frac{\rho_1}{4g_0^2} f_{\mu\nu} f^{\mu\nu} - \frac{\rho_2}{2g_0^2} \partial^\mu a_\mu \partial^\nu a_\nu + \frac{\Lambda^2}{32\pi^2} a_\mu a^\mu - \frac{\rho_3}{2g_0^2} f_{\mu\nu} J^{\mu\nu}
\]

with

\[
\rho_1 \equiv 1 + \frac{13g_0^2}{16\pi^2} \ln \frac{\mu}{\Lambda},
\]

\[
\rho_2 \equiv 1 + \frac{\alpha_0 g_0^2}{16\pi^2} \ln \frac{\mu}{\Lambda},
\]

\[
\rho_3 \equiv 1 + \frac{79g_0^2}{96\pi^2} \ln \frac{\mu}{\Lambda}.
\]
Here \((M_2/M_1)^2\) has been approximated by using the approximate formula \((1 + O(h))^n \approx 1 + nO(h)\). The renormalized coupling constant, \(g(\mu)\), is read from the first term of Eq. (43) to be \(g(\mu) = g_0/\sqrt{\rho_1}\). Since \(g(\mu)\) decreases with \(\mu\), the EAGT enjoys the property of asymptotic freedom. (Because of the presence of the NG scalar fields \(\phi^i\), behavior of \(g(\mu)\) is somewhat different from that of the running coupling constant of \(SU(2)\) YM theory \([13,14]\).)

The last term of Eq. (43) is certainly a source term of \(a_\mu\), as can be seen by rewriting it as \(-\frac{1}{2} \rho_3 g_0^{-2} f_{\mu\nu} J^{\mu\nu} = \rho_3 g_0^{-2} a_\mu j^\mu\) up to total derivatives. Here \(j^\mu\) is defined by \(j^\mu = \partial_\nu J^{\nu\mu}\). By virtue of the antisymmetric property of \(J^{\mu\nu}\), i.e., \(J^{\mu\nu} = -J^{\nu\mu}\), \(j^\mu\) satisfies the conservation law \(\partial_\mu j^\mu = 0\). In accordance with the treatment of \(J^{\mu\nu}\), we treat \(j^\mu\) as a classical vector current. Since \(j^\mu\) is a source of \(a_\mu\), \(j^\mu\) will be a color-electric current. Let us denote by \(L^{(1)}_a\) a part of \(L_{\mu\nu}'\) tot that consists of the terms containing \(a_\mu\) and that includes no the terms already included in \(L^{(0)}_a\). \((L^{(1)}_a)\) includes the higher derivative terms \(-\frac{1}{4}(M_2/M_1)^2 (f_{\mu\nu} + 2 J^{\mu\nu}) \Box^n f^{\mu\nu}\) \((n = 1, 2, \ldots)\) arising in the series expansion in the exponent of Eq. (42). Utilizing the source term \(\rho_3 g_0^{-2} a_\mu j^\mu\) that occurs in the exponent of Eq. (40) through Eq. (43), we represent \(a_\mu\) in \(L^{(1)}_a\) as the functional derivative \(\rho^{-1} g_0^{2} \delta \delta j^\mu\). Then, noticing the invariance of path-integral measure \(\mathcal{D} \tilde{A}_\mu^3 = \mathcal{D} a_\mu\), we carry out the integration over \(a_\mu\) in Eq. (40) to obtain

\[
W'' = \ln \int \mathcal{D} \Phi \mathcal{D} h_{\mu\nu} \exp \left[ \int d^4x (\mathcal{L}_{\mu\nu}' - \mathcal{L}_0 - \mathcal{L}_a^{(1)}) \right] Z_a
\]

with

\[
Z_a \equiv \frac{1}{N_6} \exp \left[ \int d^4x L^{(1)}_a \bigg|_{a_\mu = 0} \frac{g_0^2}{\rho_3} \frac{\delta}{\delta j^\mu} \right] \times \exp \left[ \int d^4x \left( -\frac{1}{2} j^\mu \rho_1 \Box + \frac{(\rho_3/g_0)^2}{g_0^2 \Lambda^2/16\pi^2} j^\mu \right) \right].
\]

In deriving Eq. (47), the conservation law of \(j^\mu\) has been used, so that \(Z_a\) turns out to be independent of the renormalized gauge parameter \(\alpha_0/\rho_2\).

At the present stage, the effective action \(W''\) takes the following form:

\[
W'' = W_j^{(0)} + W_J^{(0)} + \int d^4x L_2
+ \ln \int \mathcal{D} \Phi \mathcal{D} h_{\mu\nu} \exp \left[ \int d^4x L_3 \right] - \ln \prod_{l=1}^{6} N_l
\]

\(49\)
with

\[ W_j^{(0)} = \int d^4x \left( -\frac{1}{2} j^\mu \frac{\rho_3}{\rho_1} \frac{(\rho_3/g_0)^2}{g_0^2 A^2/16\pi^2} j^\nu \right), \quad (50) \]

\[ W_j^{(0)} = \int d^4x \left( -\frac{1}{4} J_{\mu\nu} \frac{M_2^2}{\Box + M_1^2} J^{\mu\nu} \right). \quad (51) \]

Here \( \mathcal{L}_2 \) consists of the terms containing \( \phi_0^i \) alone, some of which terms depend on \( \phi_0^i \) through \( J^{\mu\nu} \) or \( j^\mu \), while \( \mathcal{L}_3 \) consists of the remaining terms containing \( \Phi \) and/or \( h_{\mu\nu} \). The functional \( W_j^{(0)} \) is nothing but a part of the term explicitly written in the exponent of Eq. (42). It is now obvious that the effective action \( W'' \) is a functional of the classical fields \( \phi_0^i \) alone. Now, we would like to point out that the terms analogous to \( W_j^{(0)} \) and \( W_j^{(0)} \) have been found in a form of the generating functional characterizing the MAATGT with ATC [19]. In this theory, a composite of the Yukawa and the linear potentials was obtained from the generating functional. It is hence expected that similar potential is derived from \( W_j^{(0)} + W_j^{(0)} \).

To confirm this, we follow a simple procedure discussed in Ref. [19]: Recalling \( j^\mu = \partial_\nu J^{\nu\mu} \), we rewrite it in the integral form

\[ J^{\mu\nu} = \frac{1}{n \cdot \partial} (n^\mu j^\nu - n^\nu j^\mu) \quad (52) \]

with the aid of a constant vector \( n^\mu \), where \( n \cdot \partial \equiv n^\mu \partial_\mu \). (The vector \( n^\mu \) may be understood to be a set of integration constants.) Substituting Eq. (52) into Eq. (51), we have

\[ W_j^{(0)} = W_j^{(0)}[j^\mu] \equiv \int d^4x \left[ \frac{1}{2} j^\mu \left\{ \frac{M_2^2}{\Box + M_1^2} \frac{n^2}{(n \cdot \partial)^2} \left( \delta^{\mu\nu} - \frac{n^\mu n^\nu}{n^2} \right) \right\} j^\nu \right], \quad (53) \]

where \( n^2 \equiv n_\mu n^\mu \). As a result, \( W_j^{(0)} \) is expressed as a functional of the vector current \( j^\mu \).

In order to evaluate the static potential based on \( W_j^{(0)} + W_j^{(0)}[j^\mu] \), we now replace \( j^\mu \) with the static current \( j_Q^\mu(x) \equiv \delta^{\mu\nu} Q \{ \delta^3(x - r) - \delta^3(x) \} \) satisfying the conservation law \( \partial_\mu j_Q^\mu = 0 \). Here \( Q \) and \( -Q \) are point charges at \( x = r \) and \( x = 0 \), respectively. (For a particular configuration of the classical fields \( \phi_0^i \), the current \( j^\mu \) could reduce to \( j_Q^\mu \). Conversely, in some specific cases, \( j^\mu \) might be expressed in a form of superposition of \( j_Q^\mu \.)

Substituting \( j_Q^\mu \) into Eqs. (50) and (53) and choosing a constant vector \( n, (0, \mathbf{n}) \) with the condition \( \mathbf{n} \parallel \mathbf{r} \) to be \( n^\mu \), we can calculate the effective potentials \( V_j \) and \( V_J \) defined by

\[ -V_j \int dx^4 = W_j^{(0)}[j_Q^\mu] \] and \[ -V_J \int dx^4 = W_J^{(0)}[j_Q^\mu]. \]

The result reads

\[ V_j(r) = -\frac{(\rho_3 Q/g_0)^2 e^{-(\rho_1 A^2/4\pi^2)r}}{4\pi \rho_1 r}, \quad (54) \]

\[ V_J(r) = \frac{Q^2 M_2^2}{8\pi} \left[ \ln \left( 1 + \frac{\Lambda^2}{M_1^2} \right) \right] r. \quad (55) \]
up to irrelevant infinite constants. Here $r \equiv |r|$, and $\tilde{\Lambda}$ is another ultraviolet cutoff. (In deriving Eq. (54) from Eq. (50), it has been taken into account that $j^0 = -i j^4$.) Therefore the EAGT allows a composite of the Yukawa and the linear potentials, $V_j + V_J$, even at the tree level. This result is quite desirable for describing color confinement [2,3]. Comparing Eq. (55) with the linear potential obtained in the MAATGT with ATC, we see that $M_1/M_2$ corresponds to the gauge coupling constant of the extended dual Abelian Higgs model (EDAHM). The constants $k_1$ and $k_2$ could be determined via Eqs. (38) and (39) in such a way that $M_1/M_2$ reproduces the (running) coupling constant of the EDAHM.

Now we should make a comment. In terms of the unite isovector $\vec{n} = \vec{n}(\phi_0) = (n^B)$ defined by $n^B(\phi_0) = v(\phi_0) T^B = v(\phi_0) T^B v^{-1}(\phi_0)$, the antisymmetric tensor $J_{\mu\nu} = \varepsilon^{abc} \tilde{e}_\mu b \tilde{e}_\nu c$ is written as $J_{\mu\nu} = \vec{n} \cdot (\partial_{\mu} \vec{n} \times \partial_{\nu} \vec{n}) \equiv \varepsilon^{BCD} n^B \partial_{\mu} n^C \partial_{\nu} n^D$. With this expression, we see that $F_{\mu\nu}$ in Eq. (35) is just the so-called 't Hooft tensor occurring in the theory of 't Hooft-Polyakov magnetic monopole [29], provided that $n^B$ are identified with the Higgs fields normalized in isospace. As is known in this theory, the magnetic current $k^\mu = \partial_\nu * F^{\nu\mu} = \partial_\nu * J^{\nu\mu}$ remains non-vanishing owing to non-triviality of the second homotopy group of $SU(2)/U(1)$, i.e., $\pi_2(SU(2)/U(1)) = \pi_1(U(1)) = \mathbb{Z}$ [22,30]. (Here $*$ indicates the Hodge star operation.) Comparing $j^\mu = \partial_\nu J^{\nu\mu}$ with $k^\mu = \partial_\nu * J^{\nu\mu}$, we see that $j^\mu$ is certainly a color-electric current, not a color-magnetic current. For this reason, $W_j^{(0)} + W_j^{(0)}[j^\mu]$ is interpreted as a generating functional that describes interaction between color-electric currents; accordingly $V_j + V_J$ is understood to be a static potential between color-electric charges.

VIII. REMARKS

One may ask here what is the origin of the color-electric current $j^\mu$. We can give no satisfactory answer to this question now, and so we will only mention a tentative idea towards making a correct reply. Recall the fact that $\phi_0^i$ is a solution of the Euler-Lagrange equation in Eq. (6). Since this equation involves the gluon fields $A_{\mu}^B$ in the form of external fields, $\phi_0^i$ can be regarded as functionals of $A_{\mu}^B$, i.e., $\phi_0^i = \phi_0^i[A_{\mu}^B]$, or more physically,
as (composite) fields that inherit degrees of freedom of the gluon fields. Regarded $\phi_0^i$ as gluonic scalar fields in this sense, $J^{\mu\nu}$ and $j^\mu$ are interpreted as currents consisting of gluonic degrees of freedom. Thus it seems natural to assign the origin of the color-electric current $j^\mu$ to the gluon fields. If this idea is acceptable, $V_j + V_f$ could be considered as a potential that makes a contribution to gluon confinement.

We now focus attention to the leading terms of $L_2$ in Eq. 49, including the quartic terms in $\tilde{e}_\mu^{bc}$ and the quadratic terms in $\tilde{\nabla}^{\mu bc} \tilde{e}_\mu^c$. The sum of the leading terms can be seen from Eqs. (25), (29), (30) and (34) to be

$$L_2^{(0)} = \frac{A^2}{32\pi^2} \tilde{e}_\mu^{b\mu b} - \frac{1}{4g_0^2} \left( \rho_1 + \frac{g_0^2}{32\pi^2} \ln \frac{\mu}{\Lambda} \right) J_{\mu\nu} J^{\mu\nu}$$

$$+ \frac{1}{128\pi^2} \ln \frac{\mu}{\Lambda} (\tilde{e}_\mu^{b\mu b})^2 - \frac{\rho_2}{2\alpha_0 g_0} \tilde{\nabla}^{\mu bc} \tilde{e}_\mu^c \tilde{\nabla}^{\nu bd} \tilde{e}_\nu^d ,$$

where

$$\rho_1 \equiv 1 - \frac{4}{k_2} - g_0^2 \left( \frac{17}{96\pi^2} + \frac{k_1}{2\pi^2 k_2^2} \right) \ln \frac{\mu}{\Lambda} .$$

In terms of $n^B$, $L_2^{(0)}$ is written as

$$L_2^{(0)} = \frac{A^2}{32\pi^2} \partial_\mu n^i \cdot \partial^\mu n^i - \frac{\rho_1}{4g_0^2} (\partial_\mu n^i \times \partial_\nu n^i) \cdot (\partial^\mu n^i \times \partial^\nu n^i)$$

$$+ \frac{1}{128\pi^2} \ln \frac{\mu}{\Lambda} (\partial_\mu n^i \cdot \partial_\nu n^i)(\partial^\mu n^i \cdot \partial^\nu n^i)$$

$$- \frac{\rho_2}{2\alpha_0 g_0} (n^i \times \Box n^i) \cdot (n^i \times \Box n^i).$$

This is certainly the Lagrangian that defines a generalized Skyrme-Faddeev model (GSFM) [32]. Therefore it is concluded that the EAGT involves a GSFM. As is seen from Eq. 45, $\rho_2$ vanishes with the choice of gauge parameter $\alpha_0 = -16\pi^2 g_0^{-2}[\ln(\mu/\Lambda)]^{-1}$. In this case, $L_2^{(0)}$ reduces to the Lagrangian presented by de Vega to investigate closed-vortex configurations [33]. If the last two terms in Eq. (58) are removed, $L_2^{(0)}$ agrees with the Lagrangian of the

5 Scalar fields similar to $\phi_0^i$, or more precisely to $n^B$, have been introduced by Ichie and Suganuma in a somewhat different context [31]. They called those scalar fields "gluonic Higgs fields", insisting that the scalar fields are analogous to non-Abelian Higgs fields but are composite fields of gluons. Unlike Ichie-Suganuma’s procedure of introducing the scalar fields, we have obtained $\phi_0^i$ as a classical configuration of the “dynamical” NG scalar fields $\phi^i$ that are identified with the longitudinal modes of the massive off-diagonal gluons.
Skyrme-Faddeev model [34]. Faddeev and Niemi have conjectured that the Skyrme-Faddeev model would be appropriate for describing $SU(2)$ YM theory in the low energy limit [35]. If their conjecture is true, $n^B$ should be considered as the fields that realize gluonic degrees of freedom relevant to the low energy limit. This interpretation of $n^B$ appears consistent with our statement on $\phi_0^i$ made above.

Until now the ultraviolet cutoff $\Lambda$ remains a free parameter; we briefly discuss how to fix $\Lambda$. First we recall that $L_\phi$ in Eq. (2) was expanded about the classical fields $\phi_0^i$ in powers of $\sigma^i$:

$$L_\phi = \frac{m_0^2}{2g_0^2} \bar{\epsilon}_\mu^{b\mu} + \cdots.$$  

(59)

Here the relation $g_{ij}(\phi_0) \partial_\mu \phi_0^i \partial^\mu \phi_0^j = \bar{\epsilon}_\mu^{b\mu}$ has been used. From a viewpoint of the BRST formalism, it can be understood that because of the BRST symmetry, the quadratic divergences due to quantum effects of $\phi^i$, $c^a$ and $\bar{c}^a$ cancel out in calculating quantum correction for the classical term $(m_0^2/2g_0^2)\bar{\epsilon}_\mu^{b\mu}$. (This fact can be seen by comparing Eq. (9) with Eq. (34).) For this reason, the classical term is not affected by quantum effects and will remain in the EAGT without any change. We now notice that the Lagrangian $L_{tot}$ includes $(\Lambda^2/32\pi^2)\bar{\epsilon}_\mu^{b\mu\bar{\nu}}$, see Eq. (34). In the EAGT, this is the only term that takes the same form as the classical term. Thus it is natural to identify $(\Lambda^2/32\pi^2)\bar{\epsilon}_\mu^{b\mu\bar{\nu}}$ with the classical term by imposing the condition $\Lambda^2/32\pi^2 = m_0^2/2g_0^2$, which fixes $\Lambda$ at

$$\Lambda_0 \equiv \frac{4\pi m_0}{g_0}.$$  

(60)

Even if $m_0$ is not so large value, the cutoff $\Lambda_0$ will be regarded as large so long as $g_0/4\pi$ is sufficiently small. In such a case, the perturbative treatment of the higher order and/or higher derivative terms in $L_{tot}$ makes good sense.

**IX. CONCLUSIONS**

We have derived an EAGT of the SKG formalism by following a method of Abelian projection. There the off-diagonal gluon fields involving longitudinal modes were treated as fields that produce quantum effects on the diagonal gluon fields and other fields relevant at a long-distance scale. In deriving the EAGT, we have employed, instead of the MA gauge fixing term used in earlier papers, an appropriate gauge fixing term in Eq. (18) which breaks the *local* $SU(2)$ gauge symmetry, while preserves the *global* $SU(2)$ gauge symmetry.
(An equation corresponding to the MA gauge condition was obtained as the Euler-Lagrange equation for the NG scalar fields, see Eq. (6).) This choice of gauge fixing term guarantees the conservation of color charge and has made evaluating the ghost sector simple.

We have shown that the EAGT involves a gauge fixed version of the MAATGT with ATC, allowing a composite of the Yukawa and the linear potentials at the tree level of the EAGT. We can not immediately identify this linear potential with the confinement potential, because it was derived via a perturbative procedure together with the one-loop approximation. However, we could say that the “germ” of confinement potential was found. The composite potential derived here is understood to be a static potential between color-electric charges. It was also pointed out that the origin of these charges may be assigned to the gluon fields. For this reason, the composite potential could make a contribution to confinement of gluons. Finally we have seen that the EAGT involves the Skyrme-Faddeev model in addition to the MAATGT with ATC.

In the present letter we have dealt only with the three terms $W_j^{(0)}$, $W_J^{(0)}$ and $L_2^{(0)}$ among the terms included in Eq. (49). Some of the remaining terms will produce correction for the potential $V_j + V_J$. Since this potential corresponds to the static potential found in the EDAHM in the London limit, such correction should be identified with variation from the London limit [26,28]. We then expect that the scalar field $\Phi$ plays a role of the residual Higgs field in the EDAHM, with choosing suitable values for the constants $k_1$ and $k_2$. Anyway, investigation of the correction is important for comparing the EAGT with the EDAHM. Besides clarifying the mass-generation mechanism of off-diagonal gluons, we hope to address this issue in the future.

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