Some notes on the Big Trip

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The big trip is a cosmological process thought to occur in the future by which the entire universe would be engulfed inside a gigantic wormhole and might travel through it along space and time. In this paper we discuss different arguments that have been raised against the viability of that process, reaching the conclusions that the process can actually occur by accretion of phantom energy onto the wormholes and that it is stable and might occur in the global context of a multiverse model. We finally argue that the big trip does not contradict any holographic bounds on entropy and information.

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Rather bizarre implications from dark energy models are now being considered that might ultimately make the future of the universe sing a somehow weird melody. It has been in fact recently proposed [1] that if the current value of the equation-of-state parameter $w$ would keep up being less than -1 in the future, then the throat radius of naturally existing wormholes could grow large enough to engulf the entire universe itself, before this reached the so called big rip singularity [2], at least for an asymptotic observer. This rather astonishing result - which has been dubbed the "big trip" - has proved to be not free from a number of difficulties which has been raised afterwards and that mainly includes: (1) The result is obtained by using a static metric and therefore it has been claimed [3] that the accretion of dark energy cannot significantly change the amount of exotic matter in the wormhole and hence no large increase of the throat radius should be expected; (2) wormhole space-times are all asymptotically flat and thereby a very large increase of the throat size would imply that the insertion of the wormhole cannot be made onto our universe [4]; (3) quantum catastrophic creation of vacuum particles on the chronology horizon would make macroscopic wormholes completely unstable [5,6], and (4) the holographic bound on the entropy [7,8] would prevent any relevant amount of information to flow through the wormholes, so that these wormholes could never be used to circumvent the big rip singularity [9]. The present report aims at discussing these four difficulties. It will be seen that none of the problems (1), (3) and (4) indeed hold for the asymptotic observer, and that problem (2) is a debatable one and might require considering the big trip to take place within the context of a multiverse scenario.

1. Metric staticity may not be a real problem actually. The question that has been posed is that by using a static metric one automatically ensures that there cannot be any energy flux of the exotic stuff making the wormhole and therefore no arbitrarily rapid accretion can take place, let alone the accretion of the entire universe required by the big trip. It has been in this way claimed that a static metric may well justify small accretion rates by rigorously calculating the integrated stress tensor conservation laws [10] needed to suitably evaluating the accretion of dark energy according to the generalized Michel theory developed by Babichev, Dokuchaev and Eroshenko [11], but it can never describe extreme accretion regimes. However, we shall show in what follows that by using the static four-dimensional Morris-Thorne metric [12] with a zero shift function we obtain exactly the same result on such extreme regimes (that is a big trip) as when we introduce any time dependence in the $g_{rr}$ metric tensor component entering that metric. Let us start with the static Morris-Thorne metric [12] with zero shift function,

$$ds^2 = -dt^2 + \frac{dr^2}{1 - K(r)/r} + r^2 d\Omega^2_2.$$  

From just the integration of the conservation laws for the momentum-energy tensor and its projection on four-velocity (energy flux), the following two equations can then be obtained [10]:

$$\frac{ur^2 e^{\int r_{\infty}} \frac{d\rho/(p+\rho)}{m^2 \sqrt{1 - K(r)/r}}}{\sqrt{\int r_{\infty} e^{\frac{1}{r_{\infty}}}} \frac{d\rho/(p+\rho)}{\rho} = A^2} = \frac{\sqrt{\frac{u^2 + \frac{K(r)}{r}}{1 - \frac{K(r)}{r}}}}{e^{\frac{1}{r_{\infty}}} \frac{d\rho/(p+\rho)}{\rho}} (\rho + \rho) = B^2 = \hat{A} (\rho_{\infty} + p(\rho_{\infty})),$$  

where $m$ is the exotic mass which can be assumed to be spherically distributed on the wormhole throat, $u = dr/ds$, $K(r)$ is the shape function [12], $A$, $B$ and $\hat{A}$ are generally positive constants, and the dark-energy pressure, $p$, and energy density, $\rho$, bear all time-dependence in these two expressions. Moreover, since Eq. (1) [where the constant $A$ must be dimensionless (note that we are using natural units so that $G = c = \hbar = 1$)] should describe a flux of dark energy onto the wormhole, $u > 0$ and energy conservation ought then to imply that the exotic mass of the wormhole -and hence the radius of its throat- should progressively change with time, as a consequence from the incoming dark energy flux. Thus, even though the starting metric is static, the energy stored in the wormhole must change with time by virtue of dark energy accretion, in the model considered in Refs. [1] and [10].

Now, in order to implement the effect of a dark energy flux onto the wormhole implied by Eq. (1), we must introduce a general rate of change of the energy stored in the wormhole due to the external accretion of dark energy onto the wormhole. From the momentum density it can be derived that the rate of change of the exotic mass is generally given by $\dot{m} = \int dS T^r_r$, where $T^r_r$ is the dark momentum-energy tensor of the universe containing a Morris-Thorne wormhole, and $dS = r^2 \sin \theta d\theta d\phi$. Hence, using a perfect-fluid expression for that momentum-energy tensor, $T_{\mu\nu} = (P + \rho)u_\mu u_\nu + g_{\mu\nu} p$, and Eqs. (1) and (2), we obtain finally for $p = w\rho(t)$,

$$\dot{m} = -4\pi m^2 \hat{A} \sqrt{1 - \frac{K(r)}{r}} (\rho + \rho),$$
For the relevant asymptotic regime \( r \to \infty \), the rate \( \dot{m} \) reduces to \( \dot{m} = -4\pi m^2 A \dot{A} (1 + w) \rho(t) \), whose trivial integration using the general phantom scale factor \( a(t) = a_0 (1 - \beta(t - t_0))^{-2/[3(1 + w) - 1]} \), with \( \beta \) a positive constant, yields an increasing expression for \( m(t) \) leading to the big trip, provided \( w < -1 \) and \( r \to \infty \), that is \([1,10]\)

\[
m \propto K_0 = \frac{K_{0i}}{1 - 4\pi Q K_{0i}(1 + w - 1)(t - t_0),}
\]

where \( K_{0i} \) is the initial value of the radius of the wormhole throat and \( Q \) is a positive constant. Mere inspection of this equation tells us that the big trip \( (K_0 \to \infty) \) takes place quite before than big rip \( (a \to \infty) \) does.

Note that for \( r < \infty \) there will be no big trip, but just an increase of the size of the wormhole throat that ceases to occur at a given time. However, because the metric is static, the exotic energy-momentum tensor component describing any internal radial energy flux \( \Theta_r \) to occur at a given time. However, because the metric is static, the exotic energy-momentum tensor component \( T_0^r \neq 0 \), it has been claimed that accretion of phantom energy following this pattern could only be valid for small rates \( \dot{m} \), so that at first sight such a mechanism could not describe arbitrary dark-energy accretion rates and even less so a regime in which the entire universe is accreted.

A more careful consideration leads nevertheless to the conclusion that a big trip keeping the same characteristics as those derived from a static metric stands up as a real phenomenon even when we use a non static metric in such a way that \( \Theta_0^r \neq 0 \) and \( T_0^r \neq 0 \) simultaneously. This result can be seen to be a consequence from the fact that the big trip can only occur asymptotically, at \( r \to \infty \). In fact, if we start with the corresponding, simplest time-dependent wormhole metric with zero shift function,

\[
ds^2 = -dt^2 + \frac{dr^2}{1 - K(r,t)} + r^2 d\Omega^2,
\]

(where the shape function \( K(r,t) \) is allowed to depend on time both when tidal forces are taken into account and when such forces are disregarded. In the latter case, all time-dependence in the metric is concentrated on the radius of the wormhole throat, that is it is assumed that the wormhole evolves with time by changing its overall size, while preserving its shape, such as it is thought to occur during the big trip) then Eqs. (1) and (2) would be modified to read

\[
\frac{ur^2 e^{-\beta r} d\rho/(r + \rho)}{m^2 \sqrt{1 - K(r,t)}} = AD^{1/2}
\]

\[
e^{-\beta r} \frac{r^2}{r + \rho} D^{1/2} \left\{ \frac{T_0^r + \dot{K} K_0}{r} \left[ \frac{E}{r^2} (p - T_r^r) + \frac{F(T_0^r - T_r^r)}{r^2} \right] \right\} = B,
\]

where \( A \) and \( B = \dot{A} (\rho_\infty + p(\rho_\infty)) \) are again constants and

\[
D = 1 - \frac{K(r,t)}{r}
\]

\[
E = \frac{2}{3} (r^2 + 2K_0^2)
\]

\[
F = r^2 - 2K_0^2.
\]

In these expressions we have particularized, for the aim of simplicity, in the case of a wormhole with zero tidal forces. The first of them adds an extra factor \( D^{1/2} = \sqrt{1 - K(r,t)/r} \) to the constant \( A \) of Eq. (1) and the second one contains an extra term depending on \( K_0 K_0 \) and an overall extra factor \( D^{1/2} \), respect to Eq. (2). It is worth noticing that from the field equations associated with the above time-dependent solution we have \( K(r,t) = -8\pi (r - K(r,t))^2 \Theta_0^r \). Now, while \( K(r,t) = 0 \) would imply \( \Theta_0^r = 0 \) and hence a vanishing radial flux of the exotic stuff making the wormhole, this does not imply that there is no incoming radial energy flux from the dark energy of the universe. It can then
be readily seen that whereas the extra term and factors involved at the precedent two expressions would remarkably change the mass rate equation for any $r < \infty$, the extra term vanishes and the factors become unity in the asymptotic regime, $r \to \infty$, where the big trip would be expected to take place, so that these expressions reduce to Eqs. (1) and (2) asymptotically and hence the relevant mass rate equation keeps up being $\dot{m} = -4\pi m^2 A \dot{A} (1 + w) \rho(t)$ on that regime where we therefore recover the big trip feature even for the above time-dependent metric. There thus exists at least a particular example of a simple time-dependent wormhole metric which leads to a big trip when the wormhole accretes phantom energy. Of course, using the most general initial metric $ds^2 = -e^{Q(r,t)} dt^2 + e^{Q(r,t)} + R(r,t)d\Omega^2_2$, one cannot generically show that accretion of phantom energy leads to a big trip.

On the other hand, the inflationary effect of the universal speed-up on wormhole metric has already been considered in the case that there is no accretion of dark energy [13]. It gives a time-dependent metric which is comoving with the background, such as it was again obtained in Ref. [3]. Although that result is no doubt correct, it has nothing to do with the accretion of dark energy onto the wormhole, which is the case considered in Ref. [1]. In fact, a recent calculation of the accretion of dark energy onto black holes leading even to the vanishing of the black holes at the big rip singularity for $w < -1$ also uses a static metric, that is either the Schwarzschild metric [11] or the Kerr metric [14], without employing any exact solution for the black hole in a cosmological space-time that shows a time evolution displaying such an extreme behavior. Thus, the evaluation of accretion energy onto wormholes carried out in Refs. [1] and [10] is not only applicable to a regime of low accretion rate but to any unboundedly large accretion rates and therefore the results obtained in these references are also correct.

Even so, if anyone insisted in having a metric describing by itself a wormhole in a Friedmann universe with time evolution induced by accretion of phantom energy, displaying the big trip feature, it can be seen that such a metric may still be actually built up by simply inserting a suitable dimensionless factor $W^2$ depending on the scale factor $a(t)$, into the three-dimensional spatial part of e.g. the Morris-Thorne metric [12], i.e. generically

$$ds^2 = -e^{2\kappa(r)} dt^2 + W(t)^2 \left( \frac{dr^2}{1 - \frac{K}{r}} + r^2 d\Omega^2_2 \right),$$

with

$$W(t) = \frac{1}{1 - \frac{K_0}{\kappa} \left( \frac{a}{a_0} \right)^{3(|w|-1)/2} - 1},$$

where $\kappa$ is a positive constant. The factor $W(t)$ consistently becomes unity when either the radius of the throat $K_0 \to 0$ (i.e. when the wormhole pinches off) or $a = a_0$, with $a_0$ the initial value of the scale factor. One could expect that this metric is really the exact solution that corresponds to a given distribution of exotic matter whose total amount varies with time at exactly the rate described in Ref. [1] due to phantom energy accretion. This can be readily seen by using the notation $\exp(-\lambda(r,t)) = W(t)^2 (1 - K/r)^{-1}$, so that $\lambda = -2W\dot{W}$; hence and from the field equations we can then have $d(W^2)/dt = -8\pi r(1 - K/r)t_0^\alpha$, with $t_\mu$, the energy-momentum tensor for this case. Inserting the above expression for $W(t)$ and taking again for the scale factor of the universe $a(t) = a_0(1 - \beta(t - t_0))^{2/[3(|w|-1)]}$, with $\beta$ a positive constant, we finally get that, although the energy flux $t_0^\alpha$ vanishes at the big rip, it does not so at the necessarily previous time when the trip occurs [1], so implying that phantom energy accretion takes place all the way from $t = t_0$ up to the big trip.

Moreover, by considering (i) a proper circumference $c$ at the wormhole throat, $r = K = K_0$, $\theta = \pi/2$, at any constant time, we get in the accelerating framework $c = K_0 \int_0^{2\pi} d\phi W(t) = 2\pi K_0 W(t)$, and (ii) a radial proper length through the wormhole between any two points $A$ and $B$ at constant time, which is given by $d(t) = \pm W(t) \left( \sqrt{r_B^2 - K_0^2} - \sqrt{r_A^2 - K_0^2} \right)$ for $K(r) = K_0/r$, it can be easily seen that the above solution displays a big trip, as it can also be readily shown that the form of that metric is preserved with time [13,15] and the factor $W$ blows up when the scale factor reaches the critical value $a = a_{bd} = a_0 \left( 1 + \frac{2}{\pi K_0} \right)^{2/[3(|w|-1)]}$, which takes place before the occurrence of the big rip, such as it happens in the case studied in Ref. [1].

A comment is worth mentioning at this point. Whereas the accretion mechanism used above is classical in nature the very structure of the wormholes should be quantum mechanically considered on some regimes [16]. In particular, sub-microscopic wormholes have been shown to be stabilized by quantum mechanical effects that induce a possible discretization of time [17]. We note however that our approximation can safely be applied to wormhole sizes which widely separate from those where quantum effects are expected to be important. Moreover, being true that both the energy density and curvature of the universe increase with time, it is easy to check that these quantities only acquires the sufficiently high values approaching the Planck scale that requires a proper quantization of space-time [18] as one comes close to the big rip singularity, a regime still far enough from that characterizing the big trip phenomenon as to
allow one to take the classical approach to be reliable. In fact, at the time when the wormhole throat starts exceeding the radius of the universe, the value of the scale factor, and hence of the energy density and curvature of the universe are expected to be many orders of magnitude smaller than their counterparts at the close neighborhood of the big rip.

2. Difficulty (2) is a debatable one. It states that if wormholes are asymptotically flat and their throat is allowed to grow larger than the universe itself, then the insertions of the grown up wormholes could by no means be kept on the universe where the wormholes were originally formed. The implication of this is twofold. On one hand, the dark-energy accretion process by which wormholes grow up larger than their mother-universe refers only to an asymptotic observer [10] and could therefore occur in the context of a multiverse [19], where the grown-up wormholes would re-insert in other universes which are larger than the mother-universe. Though it does not appear quite clear how that re-insertion can be implemented, the extension implies in turn the creation of physical connections among the universes. In addition, asymptotic flatness by itself is not a problem; if the matching conditions between the wormhole and the cosmological metric can be satisfied, then there is no reason, in principle, why an otherwise asymptotically flat wormhole cannot be matched to a universe.

On the other hand, the involvement of other universes and the lack of any common time concept for the components of the wormhole-coupled universes would fully eliminate any problems related to causality violation in the global context of the multiverse during the re-insertion process and therefore, although the universe that originally nested the wormholes could actually time travel and thereby avoid the big rip singularity relative to its local framework, the whole system would not actually undergo any time travel or causality violation. Thus, even though the so-called "big trip" may mean a disruption of the causal evolution of our universe in its local future, the consideration of a multiverse scenario actually leads to the preservation of causality in the global framework of the multiverse and, relative to it, the "big trip" term would rather refer to an information transfer process between two of its constituting universes. At the very least, that information transfer between two large universes could be viewed to eventually provide a proof for the existence of the multiverse itself and a formal suitable starting point to construct a quantum field theory for cosmology.

3. Quantum instability of wormholes could indeed be a real problem to preserve the existence of these space-time tunnelings in a phantom universe, at least on the regions sufficiently far from the big rip singularity where the big trip may happen. The catastrophic quantum creation of particles on the chronology (Cauchy) horizon would be expected to wipe off any trace of macroscopic wormholes evolving at times quite before that of the big rip if the throat of these wormholes would grow at a rate smaller than or nearly the same as the speed of light. However, since the throat growing rate induced by phantom energy accretion clearly exceeds the speed of light asymptotically, particles created by the quantum excitation of vacuum would never reach such chronology horizon and the wormhole would keep asymptotically stable even quantum-mechanically. This would be the second explicit example of a violation of the Hawking’s chronology protection conjecture. The first one corresponded to the self-consistent vacuum and was derived by Gott and Li [20]. The present violation would actually extend to any topological generalizations of the flat Misner space [21], other than wormholes, and leave in this way an open door for the big trip to take place in the future. This will be now explicitly considered by using a two-dimensional ($\theta, \phi = \text{constant}$) version of the Morris-Thorne metric [12] with zero shift function, $\Phi(r) = 0$, which can be written as

$$ds^2 = -dt^2 + \frac{dr^2}{1 - K(r)/r}. \quad (3)$$

For the asymptotic region where the big trip takes place, the simplest wormhole metric with $K(r) = K_0^2/r$ (which corresponds to a wormhole with zero tidal forces and where $K_0$ is the radius of the wormhole throat) becomes flat and we can therefore convert it into a Rindler-like metric, with $t = \xi \sinh \eta$, $r = \xi \cosh \eta$, so that

$$ds^2 = -\xi^2 d\eta^2 + d\xi^2, \quad (4)$$

which just covers the right quadrant of the Minkowski space, i.e. the region $r > |t|$. The reflection $(\eta, \xi) \rightarrow (\eta, -\xi)$ would lead to the description of the left quadrant of Minkowski space, i.e. the region $r < -|t|$. A metric like that given by Eq. (4) can also be derived if we keep up $K_0^2/r^2$ generally nonzero and introduce the definitions

$$t = \xi \sinh \eta, \quad \sqrt{r^2 - K_0^2} = \xi \cosh \eta. \quad (5)$$

In this case, metric (4) covers the region $\sqrt{r^2 - K_0^2} > |t|$ and the left quadrant is also obtained by the above reflection and describes the region with $\sqrt{r^2 - K_0^2} < -|t|$. We shall consider in what follows the more general case where $K_0^2/r^2$ is taken to be generally nonzero.
Now a Misner-like space can be obtained by identifying points so that the Misner symmetry
\[
(t, \sqrt{r^2 - K_0^2}) \to (t \cosh(nb) + \sqrt{r^2 - K_0^2} \sinh(nb), t \sinh(nb) + \sqrt{r^2 - K_0^2} \cosh(nb)), \tag{6}
\]
in which \(n\) is an integer number and \(b\) a boost constant, be satisfied. Under such an identification the points \((\eta, \xi)\) in \(R\) (or \(L\)) are identified with the points \((\eta + nb, \xi)\) in \(R\) (or \(L\)), so that regions \(R\) and \(L\) contain timelike curves (CTCs). The Cauchy horizons (which in this case coincide with the chronology horizons) that separate the above two regions are placed at \(\sqrt{r^2 - K_0^2} = \pm t\), i.e. at \(\xi \to 0\). We note that in the general case \(r = K_0\) on the Cauchy horizons, at least for \(\eta\) finite. Thus, the Cauchy horizon becomes the same as the the apparent horizon which makes the metric (1) singular. This is a coordinate singularity rather than a real curvature singularity as it can be seen by introducing an extension of the metric where such a singularity is no longer present. In fact, by introducing the advanced and retarded coordinates, \(u = t - \sqrt{r^2 - K_0^2}, v = t + \sqrt{r^2 - K_0^2}\), we obtain \(ds^2 = -du dv\), which can be seen to show no singularity at \(r = K_0\).

Using a Rindler self-consistent vacuum [20], one can now derive the Hadamard two-point function for a conformally coupling scalar field for our two-dimensional Misner-like space to be [20]
\[
G^{(1)}(X, X') = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi^2} \times \frac{1}{\xi' \sinh \gamma [-(\eta - \eta' + nb)^2 + \gamma^2]}, \tag{7}
\]
where the parameter \(\gamma\) is in the present case defined by \(\cosh \gamma = (\xi^2 + \xi'^2)/(2\xi \xi')\). Now, with the usual definition of the Hadamard function for the Minkowski vacuum \(G^{(1)}_M\) reduced to our two-dimensional case, one can derive the regularized Hadamard function \(G^{(1)}_{reg} = G^{(1)} - G^{(1)}_M\), and hence the renormalized stress-energy tensor in region \(R\). This turns out to become proportional to \([(2\pi/b)^4 - 1]/\xi^4\). It is thus seen that even though for a space that had \(r^2 \leq t^2 + K_0^2\), and hence \(\xi^2 \leq 0\), and \(b \neq 2\pi\), the renormalized stress-energy tensor would diverge at the Cauchy horizon \(\xi = 0\), if either \(b = 2\pi\) or we had \(r^2 > t^2 + K_0^2\) (i.e. if we confine the system to be inside the \(R\) quadrant without touching its boundaries), i.e. \(\xi^2 > 0\), then this tensor would be convergent everywhere in region \(R\), because the expression derived above for it can be in this case valid only for \(\xi > 0\) (recall that for the big trip \(r \to \infty\) while \(t\) and \(K_0\) are both finite when the size of the wormhole throat overtakes that of the universe) which does not include the Cauchy horizon which never is fully well defined. It could be said that the particles created by vacuum polarization cannot reach the chronology horizon unless at the moment at which \(K_0\) becomes infinite; i.e. when the wormhole ceases to exist relative to any observers. The first of these two stabilized situations \((b = 2\pi)\) corresponds to the Li-Gott self-consistent vacuum [20] and the second one should be associated with the case where we had a big trip, for which \(r \to \infty\).

The moderation or possible removal of the future singularity that can be induced by quantum effect of matter [22] and quantum gravity [23] appear again to be not significantly influencing the regime before the big trip where the semi-classical approximation used in this section applies. That approximation would therefore remain as a sufficiently accurate procedure. After the big trip and on the regime approaching the big rip, wormholes simply cease to exist [1]. On the other hand, it can be stressed that the existence of a future singularity has nothing to do with the emergence of the big trip phenomenon.

4. We shall finally consider in a little more detail the last of the above-alluded difficulties, that is the one related with the holographic bound on the entropy. Let us be then a little more explicit mathematically. The entropy\((S)\)-energy\((E)\) bound which was introduced by Bekenstein [7] for a spherical system of radius \(B\) can be written as
\[
S \leq \frac{2\pi EB}{hc}. \tag{8}
\]
This inequality implies [7] a limit in the information that can be drawn from the given system such that
\[
I < \frac{2\pi EB}{hc \log 2}. \tag{9}
\]
The holographic bound for entropy can be derived from the above entropy-energy bound and reads [8]
\[
S < \frac{A}{4\ell_p^2}, \tag{10}
\]
where \(A\) is the surface area and \(\ell_p\) is the Planck length. These bounds must all apply to the entropy and information which are allowed to traverse one of the growing wormhole during the big trip. Thus, \(S\) refers to the entropy of the
observable matter that traverses the grown-up wormhole and has therefore nothing to do with the phantom stuff. Hence, the rather peculiar properties of phantom thermodynamics [24,25] do not influence at all the analysis to follow.

Taking now for the radius $B$ the proper value of the uppermost horizon

$$B = a(t) \int_0^{t_{\text{max}}} \frac{dt'}{a(t')} ,$$

(11)

in which $a$ is the scale factor of the universe, from Eq. (9) we obtain then for reasonable values of the involved cosmological parameters [9] and $t_{\text{max}} = t_*$, with $t_*$ the time at which the big rip takes place, in the approximation of full phantom energy dominance, i.e.

$$a(t) = a_0 \left( 1 - \frac{t - t_0}{t_* - t_0} \right)^{-\frac{2}{3(1-|w|-1)}},$$

(where $t_* = t_0 + \frac{2}{3(1-|w|-1)\sqrt{8\pi\rho_0/3}}$ and $a_0$, $t_0$ and $\rho_0$ are the initial values of the scale factor, time and energy density, respectively), that $I < I_{\text{max}} \simeq 100$ bits. This figure is certainly very small and, by far, does not allow the components of any future advanced civilization to make any big trip. This is difficulty (4). However, the upper integration limit $t_{\text{max}} = t_*$ in Eq. (11) is only strictly valid if the big rip singularity cannot be avoided by the action of local, smaller wormholes branched off from the region in the close neighborhood of that singularity. Wormholes able to connect the regions before and after the big rip (note that for $t > t_*$ the value of the scale factor $a$ becomes decreasing with time $t$ and keeps up real and positive for an infinite family of discrete values of $w$), short-cutting the singularity, have been in fact shown [25] to copiously crop up on that neighborhood and become stabilized by cosmological complementarity. Such wormholes can allow a flux of unboundedly large information carried by real signaling or matter in both directions. The action of these wormholes would then physically extend the evolution of the universe up to an infinite time. If so then we have $t_{\text{max}} \to \infty$ and the use of the bound (9) leads to the result that the maximum involved information that can be transferred during the big trip process should be infinite, so ultimately allowing the universe itself and any future advanced civilizations to be transferred as a whole by means of such a process. Thus, the holographic and entropy-energy bounds do not preclude the trip of our own universe through gigantic, stable wormholes grown up by accretion of phantom energy.

The big trip appears then a real possibility to occur in the future of the universe if the equation-of-state parameter of this would preserve a value less than -1 long enough in the future. Of course, all of what has been discussed in this paper has a rather speculative character and is based on the conception that the observed accelerating expansion of the universe is due to the presence of a quintessence scalar field that behaved like a dark-energy fluid. Since other models could also be invoked to justify the observations, the big rip and its implications would remain as just an interesting possibility.

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Using the embedding of a $t=\text{const}, \theta = \pi/2$ slice of the time-dependent metric in a flat three-dimensional Euclidean space with cylindrical metric, $ds^2 = d\bar{z}^2 + d\bar{r}^2 + \bar{r}^2 d\phi^2$, we can get $\bar{r} = W(t) r|_{t=\text{const}}, d\bar{r}^2 = W(t)^2 dr^2|_{t=\text{const}}$. Now, relative to the $(\bar{z}, \bar{r}, \phi)$ coordinate system, the form of the wormhole metric will be preserved provided that the metric on the embedded slice has the form $ds^2 = (1 - K(\bar{r})/\bar{r}) d\bar{r}^2 + \bar{r}^2 d\phi^2$, with $K(\bar{r})$ having a minimum at some $\bar{K}_0 = \bar{r}_0$, which is a condition that is easily fulfilled [13].