Black hole and holographic dark energy

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Abstract

We discuss the connection between black hole and holographic dark energy. We examine the issue of the equation of state (EOS) for holographic energy density as a candidate for the dark energy carefully. This is closely related to the EOS for black hole, because the holographic dark energy comes from the black hole energy density. In order to derive the EOS of a black hole, we may use its dual (quantum) systems. Finally, a regular black hole without the singularity is introduced to describe an accelerating universe inside the cosmological horizon. Inspired by this, we show that the holographic energy density with the cosmological horizon as the IR cutoff leads to the dark energy-dominated universe with $\omega_A = -1$.

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1 Introduction

Observations of supernova type Ia suggest that our universe is accelerating [1]. Considering the ΛCDM model [2, 3], the dark energy and cold dark matter contribute $\Omega_{\Lambda} \simeq 0.74$ and $\Omega_{\text{CDM}} \simeq 0.22$ to the critical density of the present universe. Recently the combination of WMAP3 and Supernova Legacy Survey data shows a significant constraint on the EOS for the dark energy, $w_{\text{ob}} = -0.97^{+0.07}_{-0.09}$ in a flat universe [4, 5].

Although there exist a number of dark energy models [6], the two promising candidates are the cosmological constant and the quintessence scenario. The EOS for the latter is determined dynamically by the scalar or tachyon. In the study of dark energy [7], the first issue is whether the dark energy is a cosmological constant with $\omega_{\Lambda} = -1$. If the dark energy is shown not to be a cosmological constant, the next is whether the phantom-like state of $\omega_{\Lambda} < -1$ is allowed. However, most theoretical models that may lead to $\omega_{\Lambda} < -1$ confront with serious problems including violation of the causality. The last issue is whether $\omega_{\Lambda}$ is changing (dynamical) as the universe evolves.

On the other hand, there exists another model of the dark energy arisen from the holographic principle. The authors in [8] showed that in quantum field theory, the ultraviolet (UV) cutoff $\Lambda$ could be related to the infrared (IR) cutoff $L$ due to the limit set by forming a black hole. If $\rho_{\Lambda} = \Lambda^4$ is the vacuum energy density caused by the UV cutoff, the total energy for a system of size $L$ should not exceed the mass of the system-size black hole:

$$E_{\Lambda} \leq E_{\text{BH}} \rightarrow L^3 \rho_{\Lambda} \leq M_p^2 L. \quad (1)$$

If the largest cutoff $L$ is chosen to be the one saturating this inequality, the holographic energy density is given by the energy density of a system-size black hole as

$$\rho_{\Lambda} = \frac{3c^2 M_p^2}{8\pi L^2} \simeq \rho_{\text{BH}}, \quad \rho_{\text{BH}} = \frac{3M_p^2}{8\pi L^2} \quad (2)$$

with a constant $c$. Here we regard $\rho_{\Lambda}$ as the dynamical cosmological constant like the quintessence density of $\rho_Q = \dot{\phi}^2/2 + V(\phi)$ [7]. At the planck scale of $L = M_p^{-1}$, it is just the vacuum energy density $\rho_V = M_p^2 \Lambda_{\text{eff}}/8\pi$ of the universe at $\Lambda_{\text{eff}} \sim M_p^2$: $\rho_{\Lambda} \sim \rho_\Lambda \sim M_p^4$. This implies that a very small system has an upper limit on the energy density as expected in quantum field theory. On the other hand, a larger system gets a smaller energy density. If the IR cutoff is taken as the size of the current universe ($L = H_0^{-1}$), the resulting energy density is close to the current dark energy: $\rho_{\Lambda} \sim \rho_c \sim 10^{-123} M_p^4$ [9]. This results from the holography: the energy increases with the linear size, so that the energy density decreases

\footnote{Another combination of data shows $w_{\text{ob}} = -1.04 \pm 0.06$ [4].}
with the inverse-area law. The total energy density dilutes as $L^{-3}$ due to the evolution of the universe, whereas its upper limit set by gravity (black hole) decreases as $L^{-2}$. Even though it may explain the present data, this approach with $L = H_0^{-1}$ fails to recover the EOS for a dark energy-dominated universe. This is because there exists a missing information about the pressure $p_\Lambda$ of holographic dark energy.

It is not easy to determine the EOS for a system including gravity with the UV and IR cutoffs. If one considers $L = H_0^{-1}$ together with the cold dark matter, the EOS may take the form of $w_\Lambda = 0$ \[^{[10]}\] , which is just that of the cold dark matter. However, introducing an interaction between holographic dark energy and cold dark matter may lead to an accelerating universe \[^{[11]}\]. Interestingly, the future event horizon\[^{[2]}\] was introduced to obtain an accelerating universe \[^{[13, 14, 15, 16, 17]}\].

At this stage, we emphasize that the energy density $\rho_{BH}$ of the black hole is used to derive the holographic dark energy. On the other hand, we do not use the pressure $p_{BH}$ of the black hole to find the correct EOS of holographic dark energy. Hence an important issue is to find the pressure of the black hole.

In this Letter, we discuss a few of ways of obtaining the EOS of the black hole from its dual (quantum) systems. Further, we introduce a regular black hole to obtain the dark energy from a singularity-free black hole. Finally, we show that the holographic energy density $\rho_\Lambda$ with the cosmological horizon leads to the dark energy-dominated universe with $\omega_\Lambda = -1$.

### 2 EOS for black hole from dual (quantum) systems

We start with the first law of thermodynamics

$$dE = TdS - pdV.$$  \(3\)

On the other hand, the corresponding form of a non-rotating black hole is given by

$$dE = TdS.$$  \(4\)

The most conservative interpretation of $pdV = 0$ is that the pressure of a black hole vanishes, $p = 0$. This is consistent with the integral form of $E = 2TS$ (Euler relation). If

\[^{2}\]As a concrete example, we introduce the definition of the future event horizon $R_{FH} = a(t) \int_0^\infty \frac{dt'}{a(t')}$, with the Friedmann-Robertson-Walker metric $ds_{FRW}^2 = -dt^2 - a^2(t)(d\tilde{r}^2 + \tilde{r}^2 d\tilde{\Omega}_2^2)$. Assuming the power-law behavior of $a(t) = a_0 t^{2/(3-\omega_\Lambda)}$ \[^{[12]}\], one finds $R_{FH} = \frac{2^{1+\omega_\Lambda}}{H(1+\omega_\Lambda)}$ for $-1 < \omega_\Lambda < -1/3$. In the case of $a(t) = a_0 e^{Ht}$, one has $R_{FH} = 1/H$ with $\omega_\Lambda = -1$. This indicates that de Sitter space can also derived from the future event horizon.
one chooses $p_{\text{BH}} = 0$ really, the black hole plays a role of the cold dark matter with
\[ w_{\text{BH}} = 0. \]  
(5)

It seems that the above is consistent with the EOS $w_\Lambda = 0$ for the holographic dark energy when choosing the Hubble horizon $L = H_0^{-1}$ \[10\].

As a non-zero pressure black hole, we may consider the AdS black hole. In this case, we use the AdS-CFT correspondence to realize the holographic principle \[18\]. In fact, we have the dual holographic model of the boundary CFT without gravity. Hence we define the energy density and pressure on the boundary by using the AdS-CFT correspondence. The EOS of CFT is given by
\[ w_{\text{CFT}} = \frac{1}{3} \]  
(6)

which shows that the CFT looks like a radiation-like matter at high temperature \[19\]. It is suggested that the AdS black hole may have the same EOS as that of CFT at high temperature. This means that we could obtain the EOS of black hole at high temperature from its dual CFT through the AdS-CFT correspondence.

However, for the Schwarzschild black hole, the corresponding holographic model is not yet found \[20\]. This may be so because the Schwarzschild black hole is too simple to split the energy into the black hole energy and Casimir energy, in contrast to the AdS black hole \[21\]. Recently, there was a progress on this direction. The authors \[22\] showed that the energy-entropy duality transforms a strongly interacting gravitational system (Schwarzschild black hole) into a weakly interacting quantum system (quantum gas). The duality transformation between black hole ($E$, $S$, $T$) and dual quantum system ($E'$, $S'$, $T'$) is proposed as
\[ S' \rightarrow E = M, \ E' \rightarrow S = A/4, \ T' \rightarrow \frac{1}{T} = 8\pi M \]  
(7)

with $A = 4\pi M^2$. This may provide a hint for the quantum-corrected EOS of the Schwarzschild black hole. In this case, they used an extensive thermodynamic relation
\[ E = TS - pV \]  
(8)

which holds if the pressure is non-zero. A choice of the negative pressure $p_{\text{QG}} = -TS/V$ leads to
\[ E = 2TS, \]  
(9)

which is just the case of the black hole\[3\]. However, this pressure term does no enter into the first law of Eq.\[1\]. This is because they require a constraint of $pdV = 0$ to derive

\[ ^3 \text{This relation was proved to hold for a general spherically symmetric horizon}[23]. Defining the entropy} \]
the underlying quantum model. As the temperature is associated with the black hole thermodynamics, the pressure of $p_{\text{QG}} = -T S/V$ is related to the quantum nature of the corresponding holographic model. Here we find the EOS for the quantum gas

$$w_{\text{QG}} = -\frac{1}{2},$$

which indicates a kind of the dark energy. If one chooses $w_{\text{QG}}$ as the EOS of the Schwarzschild black hole, this could describe an accelerating universe of $w_{\text{QG}} < -1/3$. However, $\omega_{\text{QG}} = -0.5$ is not close to the observation data $\omega_{\text{ob}} = -0.97_{-0.09}^{+0.07}$.

3 $\Lambda$ black hole

We discuss another issue of the singularity on the holographic energy density [24]. The holographic dark energy states that the universe is filled with the maximal amount of dark energy so that our universe has become a black hole. However, an intuitive evidence that this argument may be wrong is that there is no definite evidence that we are approaching a black hole singularity anytime soon. In deriving the holographic energy density in Eq. (2), we did not take into account the singularity inside the event horizon seriously.

In order to avoid the singularity, one may introduce a regular black hole called the de Sitter-Schwarzschild ($\Lambda$) black hole [25]. Using a self-gravitating droplet of anisotropic fluid of mass density $\rho_m = \rho_V e^{-r^3/r_{\text{CH}}^3}$ with $r_{\text{CH}} = \sqrt{3/\Lambda_{\text{eff}}} = 1/H$ and $r_{\text{EH}} = 2m/M_p$, the conservation of the energy-momentum tensor $T_{\mu \nu} = \text{diag} [\rho_m, -p_r, -p_\perp, -p_\perp]$ leads to

$$p_r = -\rho_m, \quad p_\perp = -\rho_m - r \frac{\partial_r \rho_m}{2}$$

with the radial pressure $p_r$ and tangential pressure $p_\perp$. The Arnowitt-Deser-Misner mass is defined by $m = 4\pi \int_0^\infty \rho_m r^2 dr$. If $p_\perp = 0$, one finds the zero gravity surface where the gravitational repulsion balances the gravitational attraction. Here one finds the solution that includes de Sitter space near $r = 0$ and asymptotically Schwarzschild spacetime at $r = \infty$.

$S$ as a congruence (observer) dependent quantity and the energy $E$ as the integral over the source of the gravitational acceleration for the congruence, one recovers the relation $S = E/2T$ between entropy, energy, and temperature. Also this approach provides the quantum corrections to the Bekenstein-Hawking entropy for all spherically symmetric horizons.

For $r \to 0$, one has the de Sitter metric $ds_{\text{ds}}^2 = -(1 - H^2 r^2) dr^2 + (1 - H^2 r^2)^{-1} dr^2 + r^2 d\Omega_2^2$ with $T_{\mu \nu} \simeq \rho_m g_{\mu \nu} (\rho_m = \rho_V = M_p^2 \Lambda_{\text{eff}}/8\pi)$, while for $r \to \infty$, one finds the Schwarzschild metric $ds_\text{S}^2 = -(1 - r_{\text{EH}}/r) dr^2 + (1 - r_{\text{EH}}/r)^{-1} dr^2 + r^2 d\Omega_2^2$ with $T_{\mu \nu} \simeq 0$. Hence for $m > m_c$, one has two horizons: outer event horizon $r_{\text{EH}}$ and inner cosmological horizon $r_{\text{CH}}$. Actually, the $\Lambda$ black hole looks like the Reissner-Nordstrom black hole with replacing the singularity by de Sitter space.
Figure 1: Plot of density profile $\rho_m/\rho_V$ versus $r/(r_{CH}^2 r_{EH})^{1/3}$ with $\rho_V = 3/800\pi$ in the Planck units. The dashed curve is for the two horizons with $r_{EH} = 20$ and $r_{CH} = 10$, while the solid curve is for the extremal black hole with $r_{EH} = r_{CH} = 20$. Matter distribution is nearly flat both near the origin ($\rho_m \simeq \rho_V$) and for large $r$ ($\rho_m \simeq 0$).

As is shown in Fig. 1, the matter source $\rho_m$ connects smoothly de Sitter vacuum in the origin with the Minkowski vacuum at infinity. For $m \geq m_c \simeq 0.3 M_p \sqrt{\rho_p/\rho_V}$, de Sitter-Schwarzschild geometry describes a vacuum nonsingular black hole with $r_{EH} > r_{CH}$, while for $m < m_c$, it describes the G-lump which is a vacuum self-gravitating object without horizon. At $m = m_c$, we have the extremal black hole with $r_{EH} = r_{CH}$. Here de Sitter space replaces the singularity. In this case, we have the EOS

$$w_{\text{dS}} = -1,$$

inside the regular black hole. Therefore we attempt to specify its EOS for the holographic dark energy. If the radius of cosmological horizon $r_{CH}$ is taken to be the IR cutoff, one may consider the interior de Sitter region to be a model of dark energy. Interestingly, the extremal case represents the limiting case when the Schwarzschild radius of the system, whose size is the IR cutoff, is equal to the IR cutoff itself ($r_{EH} = L = r_{CH}$). However, two problems arise in this case. Any infinitesimal step towards a non-saturated holographic dark energy would cause a sudden jump in the EOS: from $-1$ to $0$, so the EOS cannot be clearly determined. Furthermore, the IR cutoff cannot be clearly determined because we have the interior de Sitter space and thus, the Hubble distance and the event horizon are degenerate. We note that the holographic energy density $\rho_\Lambda$ with $L = r_{CH}$ is static because $r_{EH}$ is static. Thus, the holographic dark energy approach is trivial for the $r_{EH} = r_{CH}$ case of $\Lambda$ black hole.

In order to find a non-trivial case, we use the connection between the static de Sitter
space \((\tau, r)\) and the dynamic Friedmann-Robertson-Walker spacetime \((t, \tilde{r})\)

\[
\tau = t - \frac{1}{2H} \ln[1 - H^2 r^2], \quad r = \frac{\tilde{r}}{\sqrt{e^{2H^2 t} + H^2 \tilde{r}^2}}.
\]  

(13)

According to the Penrose diagram in Ref. [26], their asymptotic behaviors are closely related to each other. In de Sitter space, one has the future cosmological horizon \(r_{\text{CH}} = 1/H\) at \(\tau = \infty\) only, while in the Friedmann-Robertson-Walker space, one has the future event horizon \(R_{\text{FH}} = 1/H\) for \(-\infty \leq t \leq \infty\). In case of \(\tau = \infty\), a dynamical feature of \(\rho_\Lambda\) is recovered and thus we have \(\omega_\Lambda = -1\). In this sense, the EOS of \(\omega_{dS} = -1\) is considered to be the input and at most, a consistency condition. Hence Eq. (12) is not considered as a derived result.

Inspired by this, we propose that the singular-free condition for holographic dark energy \(\rho_\Lambda\) may determine the equation of state. As was pointed out at footnote 2, we obtain the de Sitter solution \(L = r_{\text{CH}}\) from the future event horizon \(R_{\text{FH}}\). Here we choose the present universe-size cosmological horizon as the IR cutoff [27, 28], which contrasts to the case with the Hubble horizon \(L = 1/H_0\) [10]. For \(L = 1/H_0\), we could not determine its EOS clearly, while for \(L = r_{\text{CH}} = 1/H\), we could determine its EOS to be \(w_\Lambda = -1\).

4 Discussions

We are interested in the equation of state for black hole, because the holographic dark energy came from the energy density of black hole. Here we wish to discuss the connection between the black hole and holographic dark energy. Cohen et. al. [8] mentioned that if one introduces the holographic principle, one could include the gravity effects into the quantum field theory naturally. This is because general relativity (black hole) is the prime example of a holographic theory, whereas quantum field theories are not holographic in their present form. The first thing to realize holographic principle is given by the holographic entropy bound which states that the entropy of the system should be less or equal to the entropy of the system-size black hole: \(S_\Lambda = L^3 \Lambda^3 \leq S_{\text{BH}} = \pi M_p^2 L^2\). As was clarified by Cohen et.al., this bound includes many states with \(L_S \sim L^{5/3} M_p^{2/3} > L\). Considering the energy \(E_\Lambda = L^3 \Lambda^4\) of the system together with \(\Lambda \sim (M_p^2/L)^{1/3}\) (the saturation of holographic entropy bound), it implies \(L_S \sim E_\Lambda > L\) in the Planck units. This shows a contradiction that a larger black hole can be formed from the system by gravitational collapse. Hence, one requires that no state in the Hilbert space have energy so large that the Schwarzschild radius \(L_S \sim E_\Lambda\) exceeds \(L\). Then, a relation between the size \(L\) of the system, providing the IR cutoff and the UV cutoff \(\Lambda\) is required to be
Eq. (1) \( L_S \sim E_\Lambda < L \) in the Planck units), which provides the constraint on \( L \) that excludes all states lying within \( L_S \). In physical terms, it corresponds to the assumption that the effective field theory describes all states of the system excluding those for which it has already collapsed to a black hole. In other words, this relation can be rewritten as \( E_\Lambda \leq E_{BH} \) called the Bekenstein energy bound. This means that the energy of the system should be less or equal to the energy of the system-size black hole. Actually, both holographic entropy bound and Bekenstein energy bounds are based on the black hole.

If one takes the saturation of the energy bound in Eq. (2) (the limiting case) as the holographic dark energy density, its EOS depends on the IR cutoff and/or interaction with cold dark energy.

Let us calculate the average energy density \( \rho \) of a homogeneous spherical system that saturates the holographic entropy bound. For this purpose, we introduce the Bekenstein’s entropy bound \( S \leq 2\pi EL \) which is another entropy bound. If the system satisfies the Schwarzschild condition of \( E = M_p^2 L/2 = E_{BH} \), which states that its maximal mass is the half of its radius in the Planck units. The energy density \( \rho \) is given by the black hole energy density \( \rho = E/V = 3M_p^2/8\pi L^2 = \rho_{BH} \), which is identical to the holographic energy density \( \rho_\Lambda \) with \( c^2 = 1 \) shown in Eq. (2). This shows the close connection between the black hole and holographic dark energy.

As was pointed out in [14], the pressure of holographic dark energy is determined by the conservation of energy-momentum tensor as \( p_\Lambda = -\frac{1}{3} \frac{d\rho_\Lambda}{d\ln a} - \rho_\Lambda \) which provides the EOS of \( \omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -1 + \frac{2}{3H} \frac{L}{L} \). Hence, if one does not choose an appropriate form of \( L \), one cannot find its EOS. For example, if one chooses the Hubble horizon \( L = 1/H \), it does not give the correct EOS [10], but it leads to the second Friedmann equation of \( \dot{H} = -\frac{3}{2} H^2 (1 + \omega_\Lambda) \). On the other hand, choosing \( L = R_{PH/FH} \) leads to \( \omega_\Lambda = -1/3 (1 \pm 2\sqrt{\Omega_\Lambda/c}) \). Despite the success of obtaining the EOS for \( L = R_{PH/FH} \), this may not give us a promising solution to the dark energy problem because choosing the future event horizon just means an accelerating universe. That is, in order for the holographic dark energy to explain the accelerating universe, we first must assume that the universe is accelerating. This is not what we want to obtain really: a realistic dark energy model will be determined from cosmological dynamics with an appropriate EOS. In addition, \( \rho_\Lambda \) violates causality because the current expansion rate depends on the future expansion rate of the universe. Thus we may believe that taking the future event horizon as the IR cutoff is just a trick to get an accelerating universe in the holographic dark energy approach.

This attributes to the ignorance of the black hole pressure because one uses mainly
the energy density of the black hole to describe the holographic dark energy. Hence we described how to obtain the EOS of black holes from their dual systems as a first step to understand the nature of holographic dark energy, although it is still lacking for describing the pressure of the holographic dark energy. In this approach, the limiting condition for the saturated holographic energy density Eq.(2) is not found for the EOS of the black hole from dual systems.

Finally we consider the issue of the singularity together with the holographic dark energy. In this direction, we introduce the regular (Λ) black hole with two horizons which includes de Sitter space near \( r = 0 \) and asymptotically Schwarzschild spacetime at \( r = \infty \). We find that the singularity could be removed by choosing an appropriate mass distribution and de Sitter space appears inside the black hole. However, we recover the dynamical behavior of holographic energy density \( \rho_\Lambda \) with \( L = 1/H \) at \( \tau = \infty \) because the static coordinates are used for calculation.

In conclusion, we show that the holographic dark energy without the singularity lead to the de Sitter-acceleration with \( \omega_\Lambda = -1 \).

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