Bragg scattering of light in a strongly interacting trapped Fermi gas of atoms

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We study Bragg scattering of laser light by trapped Fermi atoms having two hyperfine spin components in the unitarity-limited strongly interacting regime at zero temperature. We calculate the dynamic structure function of the superfluid trapped Fermi gas in the unitarity limit. Model calculation using local density approximation shows that, the superfluid pairing gap in the unitarity limit is detectable from the measurements of dynamic structure function by Bragg spectroscopy, while in the weak-coupling BCS limit, the gap eludes such spectroscopic detection.

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I. INTRODUCTION

Achieving Bardeen-Cooper-Schrieffer (BCS) type phase transition in trapped neutral fermionic atoms is the primary goal in the current experimental research with ultracold atomic Fermi gases. Recently, two groups \cite{1,2} have measured collective oscillations in strongly interacting trapped Fermi atoms. The results of their measurements suggest the occurrence of fermionic superfluidity in the atoms according to a theoretical prediction \cite{3}. Condensation of “fermionic atom pairs” has been reported earlier \cite{4}. The superfluid pairing is believed to occur near the crossover \cite{5,6} between the predicted BCS state of fermionic atoms and the Bose-Einstein condensation of bosonic dimer molecules formed from Fermi atoms due to a magnetic field Feshbach resonance. Several groups have experimentally produced Bose-Einstein condensates (BEC) \cite{7} of molecules formed from degenerate Fermi atoms in the vicinity of the Feshbach resonance. The atoms become strongly interacting at and near the Feshbach resonance. Several other recent experiments \cite{8,9,10,11} with Fermi atoms have demonstrated the strong-coupling behavior of the atoms. Therefore, many-body effects become important in describing the physics near the predicted BCS-BEC crossover in Fermi atoms. Recently, a number of theoretical investigations \cite{12} have revealed many intriguing aspects of many-body effects in the crossover regime. Despite the recent experimental indications of the occurrence of Cooper-like pairing near the crossover, the energy gap for such pairing has not been so far measured due to the lack of efficient detection methods.

There are several methods available for measuring pairing gap in electronic superconductors, but any of them is hardly applicable to the trapped neutral Fermi atoms. Recently, several theoretical proposals \cite{13,14} have been made for detecting atomic Cooper pairs by laser light. There has been a suggestion \cite{15} to use resonant light for exciting one of the Cooper-paired atoms into an excited electronic level, and thus to make an interface between normal and superfluid state in analogy with the well known tunnelling experiment in superconductors. Our purpose here is to examine Bragg scattering of lasers as a method for measuring the pairing gap of a superfluid trapped Fermi gas of atoms. Bragg spectroscopy has been already used for measuring structure function of an atomic BEC \cite{16}. Unlike in a BEC, the analysis of Bragg scattering in a superfluid trapped atomic Fermi gas is complicated due mainly to the Pauli blocking and the paring gap.

In this paper, we study Bragg scattering of light in a two-component trapped atomic Fermi gas at zero temperature. Bragg scattering in a Fermi gas of atoms has an analogy with Raman scattering in electronic superconductors. We therefore, develop our theoretical treatment following the theory of Raman scattering in superconductors \cite{17}. The relevant physical quantity is the dynamic structure function which is the Fourier transform of density-density correlation function in time \cite{17}. It is a measure of the spectrum of density fluctuation and proportional to the differential scattering cross section per unit energy. In the case of superconductors, it is well known that unless the energy transferred from a probe to the superconductor exceeds $2\Delta$ (where $\Delta$ is the pairing gap), the dynamic structure function is zero. At $2\Delta$, it shows a sharp discontinuity. In the case of superfluid trapped Fermi gas, $\Delta$ has a spatial distribution varying from a maximum at the trap center to a vanishingly small value at the edge of the trap. We find that the dynamic structure function of a superfluid trapped Fermi gas in the unitarity limit has a prominent shift compared to that of a normal or superfluid trapped Fermi gas in the BCS limit. It also exhibits a discontinuity at $2\Delta(0)$ where $\Delta(0)$ refers to the energy gap at the trap center. As the energy transfer decreases below $2\Delta(0)$, the dynamic structure function falls off with reducing slope, and as the energy transfer goes to zero, the slope vanishes. In contrast, in the case of a normal trapped gas or a superfluid trapped gas in the BCS limit, the dynamic structure function increases almost linearly with the energy transfer in the low energy regime.

The paper is organized as follows. In the following section, we discuss in some detail the physical scenario for Bragg scattering in two-component trapped Fermi atoms in the unitarity-limited strongly interacting regime. In Sec.III, we discuss the theoretical treatment for calculating dynamic structure function of a uniform superfluid Fermi gas. We then generalize this treatment for a su-
perfluid trapped Fermi gas in Sec.IV. The results are discussed in Sec.V. We conclude in Sec.VI.

II. TRAPPED FERMI GAS IN THE UNITARITY REGIME

We consider a harmonic trap with potential $V_{ho}(r,z) = (1/2)m(\omega_r^2 r^2 + \omega_z^2 z^2)$, where $\omega_r$ and $\omega_z$ denote the radial and axial frequency, respectively, of the trap. The harmonic oscillator is characterized by the radial (axial) length scale $a_{r(z)} = \sqrt{\hbar/(m\omega_{r(z)})}$. One can define a geometric mean frequency of the harmonic oscillator by $\nu_{ho} = (\omega_r^2 \omega_z)^{1/3}$ and a corresponding geometric mean length scale of the oscillator by $a_{ho} = \sqrt{\hbar/(m\nu_{ho})}$. In our treatment, we resort to Thomas-Fermi local density approximation (LDA) which is particularly applicable when the local Fermi energy is larger than the average level spacing of the trap and the coherence length of the fermion-pair is shorter than the average trap size. Under this approximation, the state of the system is governed by

$$\epsilon_F(r) + V_{ho}(r) + U(r) = \mu$$

where $\epsilon_F(r) = \hbar^2 k_F(r)^2/(2m)$ is the local Fermi energy, $k_F(r)$ denotes the local Fermi momentum which is related to the local number density by $n(r) = k_F(r)^3/(6\pi^2)$. Here $U$ represents the mean-field interaction energy and $\mu$ is the chemical potential fixed by normalization condition. At low energy, the mean-field interaction energy depends on the two-body s-wave scattering amplitude $f_0(k) = -a_s/(1 + ia_s k)$, where $a_s$ denotes energy-independent s-wave scattering length and $k$ denotes the relative wave number of two colliding particles. In the dilute gas limit $(|a_s|/k << 1)$, $U$ becomes proportional to $a_s$. In the unitarity limit $|a_s|/k \rightarrow \infty$, the scattering amplitude $f_0 \sim i/k$ and hence $U$ becomes independent of $a_s$. It then follows from a simple dimensional analysis that in this limit, $U$ should be proportional to the Fermi energy: $U(r) = \beta U(r) \epsilon_F(r)$ where $\beta$ is the constant of proportionality. Under LDA, the density profile of a trapped Fermi gas is given by

$$n(r) = n(0) \left[ 1 - \frac{r^2}{R_{ho}^2} - \frac{r^2}{R_s^2} \right]^{3/2}$$

where

$$n(0) = \frac{1}{6\pi^2 \hbar^2} \frac{2m \mu}{(1 + \beta)^3}$$

is the density of the atoms at the trap center. Here $R_{ho} = 2\mu/(m\omega_{ho}^2)$ being the radial length scale. The normalization condition on Eq. gives an expression for $\mu = (1 + \beta)^{1/2}(6N_s)^{1/3}\hbar\omega_0$

where $N_s$ is the total number of atoms in the hyperfine spin $\sigma$. The Fermi momentum $k_F = (3\pi^2 n(0))^{1/3}$ is then given by

$$k_F = \frac{1}{\hbar} \sqrt{\frac{2m\mu}{1 + \beta}} = (1 + \beta)^{-1/4} k_F^0$$

where

$$k_F^0 = \frac{(48N_s)^{1/6}}{a_{ho}}$$

is the Fermi momentum of the noninteracting trapped gas. For an attractive interaction ($-1 < \beta < 0$) in the unitarity limit, Fermi momentum would be larger than that of noninteracting gas by a factor of $(1 + \beta)^{-1/4}$. Accordingly, the Fermi energy would be larger than the Fermi energy $\epsilon_F^0 = \hbar\omega_F^0$ of noninteracting gas by a factor of $(1 + \beta)^{-1/2}$.

To illustrate the main idea, we specifically consider trapped $^6$Li Fermi atoms in their two lowest hyperfine spin states $|g_1\rangle = |2S_{1/2}, F = 1/2, m_F = 1/2\rangle$ and $|g_2\rangle = |2S_{1/2}, F = 1/2, m_F = -1/2\rangle$. For s-wave pairing to occur, the atom number difference $\Delta N$ of the two components should be restricted by $\Delta N \leq T_c/\epsilon_F$ where $T_c$ is the critical temperature for superfluid transition and $\epsilon_F$ is the Fermi energy at the trap center. Unequal densities of the two components result in interior gap superfluidity which naturally arises in QCD matter. For simplicity, we consider the case $N_{1/2} = N_{-1/2}$ which is the optimum condition for s-wave Cooper pairing. An applied magnetic field tuned near the Feshbach resonance ($\sim 822$ Gauss) results in strong inter-component s-wave interaction. At such high magnetic fields, the splitting between the two ground hyperfine states is $\sim 75$ MHz, while the corresponding splitting between the excited states $|e_1\rangle = |2P_{3/2}, F = 3/2, m_F = -3/2\rangle$ and $|e_2\rangle = |2P_{3/2}, F = 3/2, m_F = 3/2\rangle$ is $\sim 994$ MHz. Taking advantage of these Zeeman splits, it is possible to scatter atoms selectively of one hyperfine spin component by making use of a pair of circularly polarized laser beams - both having either $\sigma_+$ or $\sigma_-$ polarization.

In Bragg scattering, two laser beams (Bragg pulses) with a small frequency difference are applied to the trapped gas. The magnitude of this momentum transfer is $g \approx 2k_l \sin(\theta/2)$, where $\theta$ is the angle between the two beams and $k_l$ is the momentum of a laser photon. Let both the laser beams be $\sigma_+$ polarized and tuned near the transition $|g_2\rangle \rightarrow |e_2\rangle$. Then the transition between the states $|g_1\rangle$ and $|e_2\rangle$ would be forbidden while the transition $|g_1\rangle \rightarrow |e_1\rangle$ will be suppressed due to the large detuning $\sim 900$ MHz. This leads to a situation where the Bragg-scattered atoms remain in the same initial internal state $|e_2\rangle$. Similarly, atoms in state $|g_1\rangle$ only would undergo Bragg scattering when both $\sigma_+$ polarized lasers are tuned near the transition $|g_1\rangle \rightarrow |2P_{3/2}, F = 3/2, m_F = 3/2\rangle$. Thus, we infer that in the presence of a high magnetic field, it is possible to scatter atoms selectively of either spin components only by using circularly polarized Bragg lasers.
For simplicity, let us first consider Bragg scattering in a uniform Fermi gas. Later, we will analyze the nonuniform case. We assume that both the laser beams are σ- polarized and tuned near the transition \(| g_2 \to | e_2 \rangle \). The detuning of both the lasers from the atomic resonance should be much greater than the single- and two-photon line widths in order to avoid heating of the system. Under such conditions, for a uniform gas, the laser-atom interaction Hamiltonian can be expressed as

\[
H_{\text{int}} \simeq \hbar \Omega e^{-i\delta t} \sum_{\mathbf{k}} \hat{c}^\dagger_+ (\mathbf{k} + \mathbf{q}) \hat{c}^-_+ (\mathbf{k}) + \text{H.c.}
\]  

(7)

where \(\hat{c}^-_+ (\mathbf{k})\) represents the atomic field operator with momentum \(\mathbf{k}\), the subscript \(\downarrow\) (\(\uparrow\)) refers to the state \(| g_\downarrow \rangle \) (\(| g_\uparrow \rangle \)) and \(\Omega\) is the two-photon Rabi frequency. Here \(\delta = \omega_1 - \omega_2\) is the detuning between the two lasers. One can identify the operator \(\hat{\rho}_+^\dagger (\mathbf{q}) = \sum_{\mathbf{k}} \hat{c}^\dagger_+ (\mathbf{k} + \mathbf{q}) \hat{c}^-_+ (\mathbf{k})\) as the Fourier transform of the density operator \(\hat{\rho}_+ (\mathbf{r}) = \hat{\Psi}^\dagger_+ (\mathbf{r}) \hat{\Psi}_+ (\mathbf{r})\) where \(\hat{\Psi}^\dagger_+ (\mathbf{r})\) represents the field operator in the real space. The spectrum of the scattered atoms would be proportional to the rate of transition probability which, according to Fermi’s Golden rule, is given by

\[
\kappa = 2 \pi \hbar \Omega^2 \sum_f |\langle f | \hat{\rho}_+^\dagger (\mathbf{q}) | 0 \rangle|^2 \delta (\hbar \delta - \epsilon_f - \epsilon_0)
\]

(8)

where \(|0\rangle\) represents the many-body ground state and sum runs over all the final states \(|f\rangle\) which can be coupled to the ground state by the operator \(\hat{\rho}_+^\dagger\).

### III. Dynamic Structure Function of a Uniform Superfluid

Using Bogliubov transformations

\[
\hat{c}_- (\mathbf{k}) = u_k \hat{\gamma}_- (\mathbf{k}) + v_k^* \hat{\gamma}_+^\dagger (-\mathbf{k})
\]

\[
\hat{c}_+ (\mathbf{k}) = u_k \hat{\gamma}_- (\mathbf{k}) - v_k^* \hat{\gamma}_+^\dagger (-\mathbf{k})
\]

one can reexpress the interaction Hamiltonian in terms of quasiparticle operators \(\hat{\gamma}_-^\dagger (\mathbf{k})\) and \(\hat{\gamma}_+ (\mathbf{k})\). Here the normalization condition is \(|u_k|^2 + |v_k|^2 = 1\) with

\[
|v_k|^2 = (1/2)(1 - \xi_k/E_k)
\]

(11)

and

\[
E_k = \sqrt{\Delta_k^2 + \xi_k^2},
\]

(12)

where \(\xi_k = \hbar^2 k^2 / (2m) - \mu\) and \(\Delta_k\) is the pairing gap. The chemical potential \(\mu\) and the gap \(\Delta_k\) can be obtained by solving the regularized gap equation

\[
m/4\pi^2 a_s \alpha_x = \frac{1}{V} \sum_k \left( \frac{1}{2\xi_k} - \frac{1}{2E_k} \right)
\]

(13)

along with the equation

\[
n = \frac{1}{6\pi^2} k_F^3 = \frac{1}{V} \sum_k \left( 1 - \frac{\xi_k}{E_k} \right).
\]

(14)

of the density of single component. Here \(V\) is the volume of the system. The analytical solutions of these two coupled equations have been already obtained in the Ref for the entire range of the parameter \(\alpha \rightarrow \pm 0\) to the unitarity limit \((\alpha \rightarrow \pm \infty)\). In the unitarity limit, the mean-field interaction becomes independent of the scattering length and the Fermi system exhibits universal behavior. In this limit, the only energy scale available to the system is the Fermi energy. The mean-field energy and the gap then become proportional to the Fermi energy. Solutions of the above two equations in the unitarity limit yield \(\Delta = 1.16\mu\), \(\mu = (1 + \beta)\epsilon_{F}\), where \(\beta = -0.41\) is a constant. Calculations using quantum monte carlo simulation yields \(\beta = -0.56\) and \(\Delta \simeq 0.49\epsilon_{F}\), while calculations using Galitskii and lowest order constraint variation (LOCV) approximations give \(\beta = -0.67\) and \(\beta = -0.43\), respectively. The constant \(\beta\) was experimentally introduced and measured to be -0.26 in Ref[8], while it was experimentally found to be between -0.4 to -0.3 in Ref[10]. A recent experiment[27] has obtained \(\beta = -0.68\).

One can notice that the BCS ground state \(|0\rangle\), which is annihilated by the quasiparticle operators, will have nonzero matrix element of \(\hat{\rho}_+^\dagger\) only when the final state \(|f\rangle\) is a state of two quasi-particles with mutually opposite spins and their momenta differing by \(\mathbf{q}\). The dynamic structure function thus takes the form

\[
S(\mathbf{q}, \delta) = \sum_f |u_{k'} v_k|^2 \delta (\hbar \delta - E_{k'} - E_k)
\]

(15)

where \(k' = |k + q|\) is the wave vector of a scattered atom. Note that the usual BCS coherence factor \(m(k,k') = u_{k'} v_k + u_k v_{k'}\) which appears in the description of electronic superconductors has changed. This is due to the fact that the polarization-selective dipole transitions in Fermi atoms in the presence of strong magnetic field as discussed earlier lead to the transfer of momentum and energy to either partner (of hyperfine spin \(\uparrow\) or \(\downarrow\)) of a cooper pair, while the other partner remains almost immune to the momentum and energy transfer. However, since a particle’s state is a superposition of two quasiparticle states of both the spins, both the spin states will be affected in the quasiparticle framework . The presence of the \(\delta\)-function in the integrand reveals that \(S(\mathbf{q}, \delta)\) would be nonzero only when \(\hbar \delta > 2\Delta\). The integration should be carried out subject to the restrictions \(k \leq k_F\) and \(k' > k_F\) resulting from Pauli blocking. Following the method of Ref[10], we evaluate the integral. After a
In the case of a uniform gas, we have $q$ becomes independent of $\hbar$ where $\hbar$ is the Planck constant. In this case, as $\Delta$ can be expressed in terms of the Elliptic integrals of first and second kind. In the limit $\Delta \to 0$, $S(\delta, q) = \nu_F \delta/(2qV_F)$ which is exactly the dynamic structure function of normal fluid $S_{\text{normal}}$. For $h\delta > (p_qV_F)^2 + 4\Delta^2)^{1/2}$, $S(\delta, q)$ can be expressed in terms of the Elliptic integrals of first and second kind. In this case, as $\Delta \to 0$, $S(\delta, q) \to 0$ and becomes independent of $q$. Since this case is not suitable for obtaining information about the gap, we henceforth focus our attention only on the former case, that is, $2\Delta < h\delta < (p_qV_F)^2 + 4\Delta^2)^{1/2}$.

\section{Dynamic Structure Function of a Trapped Superfluid}

Let us now turn our attention to the dynamic structure function of a superfluid trapped Fermi gas. This can explicitly be written as

$$S(\delta, q) = \sum_{n,m} \int d^3x u_n^*(x)v_m(x) \exp(iq \cdot x) \delta(\delta - E_n - E_m)$$

In the case of a uniform gas, we have $u_k(x) = u_k \exp(ik \cdot x)$ and $v_k(x) = v_k \exp(-ik \cdot x)$, and hence the dynamic structure function defined by Eq. (15) takes the form of Eq. (16). In the LDA, it can be expressed as

$$S_{\text{LDA}}(\delta, q) = \frac{1}{(2\pi)^3} \int d^3x \int d^3k n_k(x)[1 - n_k(x)] \delta(\delta - E_k(x) - E_k(x))$$

where $n_k(x)$ is the local momentum distribution defined by

$$n_k(x) = \frac{\nu_F}{2}\int \frac{d^3p}{(2\pi)^3} \frac{\Delta^2}{2p_qV_F \hbar} \times \frac{1}{(1 - j^2)^{3/2}(1 - j^2)^{1/2}}$$

where $\nu_F$ is the density of states at the Fermi surface. $p_q = \hbar q, j = 1 - 4\Delta^2/(\hbar\delta)^2$ and $z_0 = \min \left[1, \frac{p_qV_F}{(\hbar^2\delta^2 - 4\Delta^2)^{1/2}} \right]$.

If $2\Delta < h\delta < (p_qV_F)^2 + 4\Delta^2)^{1/2}$, then $z_0 = 1$ and the result is

$$S(\delta, q) = \frac{\nu_F}{8} \frac{\delta}{q} V_F$$

$$\times \left[ E(j) + \frac{j}{4} \left( 8 + 2F_1(3/2, 3/2; 2, j) \right) \right]$$

where $M = \pi j(1 - j)$. Here $E(j)$ represents the complete elliptic integral and $2F_1(a, b; c, d)$ is the hypergeometric function. In the limit $\Delta \to 0$, $S(\delta, q) = \nu_F \delta/(2qV_F)$ which is exactly the dynamic structure function of normal fluid $S_{\text{normal}}$. For $h\delta > (p_qV_F)^2 + 4\Delta^2)^{1/2}$, $S(\delta, q)$ can be expressed in terms of the Elliptic integrals of first and second kind. In this case, as $\Delta \to 0$, $S(\delta, q) \to 0$ and becomes independent of $q$. Since this case is not suitable for obtaining information about the gap, we henceforth focus our attention only on the former case, that is, $2\Delta < h\delta < (p_qV_F)^2 + 4\Delta^2)^{1/2}$.

Here $\xi_k(x) = \hbar^2k^2/(2m) - \mu(x)$, $\mu(x) = \mu - V_{ho}(x)$ and $E_k(x) = \sqrt{\xi_k(x)^2 + \Delta(x)}$. The momentum distribution of trapped Fermi atoms has been already calculated in Ref. (24). After performing the integration over $k$ as in the preceding section, one can obtain

$$S_{\text{LDA}}(\delta, q) = \frac{1}{2} \int d^3x \tilde{n}(x) \frac{\Delta(x)^2}{p_qV_F(x) \hbar \delta}$$

$$\times \left[ E(j_x) + \frac{j_x}{4} \left( 8 + 2F_1(3/2, 3/2; 2, j_x) \right) \right]$$

where $\tilde{n}(x) = (3/2)n(x)/\epsilon_F(x)$ is the local density of states per unit volume. Here $j(x) = 1 - 4\Delta^2(x)/(\hbar\delta)^2$. The integration over the volume must be carried out subject to the boundary condition $2\Delta(x) < \hbar\delta < [(p_qV_F(x))^2 + (2\Delta(x))^2)^{1/2}$. This means that, $\int d^3x \equiv 2\pi \int_{r_{\text{min}}}^{r_{\text{max}}} dr d\phi \int (1 - r/\max R_z) (1 - r/\min R_z) dz$ where $r_{\text{min}} = 0$ if $2\Delta(r, z = 0) < \hbar\delta$, otherwise it is the solution of the equation $2\Delta(r, z = 0) = \hbar\delta$. Here $r_{\text{max}} < R_z$ is the solution of the equation $[(p_qV_F(r, z = 0))^2 + (2\Delta(r, z = 0))^2)^{1/2} = \hbar\delta$.

In the BCS limit ($k_F a_s \to 0^+$), the gap is exponentially small and can be expressed by the well known formula

$$\Delta_{\text{BCS}} \simeq \frac{8\xi_F}{e^2} \exp\left(-\frac{\pi}{2k_F |a_s|} \right)$$

In calculating $S(\delta, q)$ of trapped atoms in the BCS limit, one can follow the same method of calculations as described above. However, the parameter $k_F |a_s|$ in the exponent of Eq. (23) may be approximated by an average value.

\section{RESULTS AND DISCUSSIONS}

Figure 1 shows the results of our calculations. Plotted is the dynamic structure function $S(\delta, q)$ of superfluid trapped Fermi atoms as a function of energy transfer for different values of momentum transfer $q$. $S(\delta, q)$ has been scaled by $a_s = (3/2)\pi R_z^2/\hbar^2/(6\pi^2\epsilon_F)$, where $V_{TF} = \pi R_z^2/\hbar$. In the unitarity limit, the behavior of $S(\delta, q)$ is quite different from that of superfluid in the weak-coupling BCS limit. This can be attributed to the occurrence of large gap in the unitarity limit. In plotting dashed-dotted curve corresponding to the BCS limit, we have taken $k_F |a_s| = 0.3$, i.e., $\Delta_{\text{BCS}} \simeq 0.006\epsilon_F$. For this small value of the gap, $S(\delta, q)$ reduces to almost that of normal fluid. We have checked this by calculating $S(\delta, q)$ for a trapped normal fluid; and $S(\delta, q)$ for normal fluid is almost indistinguishable from that of BCS superfluid. This can also be checked, as discussed in Sec.III, by taking the limit $\Delta \to 0$ for which $S(\delta, q)$ reduces to that of normal fluid.

Particularly distinguishing feature of $S(\delta, q)$ in the unitarity-limited strongly interacting regime as compared to normal or BCS regime is the shift of the peak. This can be noticed by comparing the dashed-dotted (BCS) and solid (unitarity) curves which are plotted for the same
FIG. 1: Dimensionless dynamic structure function $S(\delta, q)/\nu_{F}^{0}$ of a trapped superfluid Fermi gas is plotted as a function of dimensionless energy transfer $\delta/\omega_{F}$ for the momentum transfer $q/k_{F} = 1.5$ (solid), $q/k_{F} = 1$ (dashed), $q/k_{F} = 0.5$ (dotted) in the unitarity limit with $\beta = -0.56$ and $\Delta = 0.49\epsilon_{F} = 0.74k_{F}^{2}$. For a comparison, $S(\delta, q)/\nu_{F}^{0}$ in the BCS limit (dashed-dotted) with $|a_{s}|k_{F} = 0.3$ is plotted for $q/k_{F} = 1.5$. The shift for the unitarity limit (solid) compared to the BCS one (dashed-dotted) is $0.8\Delta$.

momentum transfer ($q/k_{F}^{0} = 1.5$). This shift is proportional to the gap. For the parameters chosen in Fig.1, the shift is $\sim 0.8\Delta = 0.8 \times 0.49\epsilon_{F} = 0.8 \times 0.49 \times (1 + \beta)^{-1/2}\epsilon_{F}^{0} \approx 0.66\epsilon_{F}^{0}$. It should be mentioned here that the dynamic structure function of a superfluid trapped Fermi gas in the BCS limit has been evaluated earlier using a different method of calculation [28] which also shows no conspicuous shift except the appearance of an asymmetric peak.

Another interesting feature is the discontinuity in $S(\delta, q)$ at energy $\hbar \delta = 2\Delta(0)$, where $\Delta(0)$ refers to the gap at the trap center. For a uniform Fermi superfluid, $S(\delta, q)$ remains zero until energy transfer exceeds $2\Delta$ at which it rises sharply with the increasing energy transfer. For a superfluid trapped Fermi gas, owing to the spatial distribution of the gap, the dynamic structure function has a structure below $2\Delta(0)$. As the energy transfer goes to zero, the gradient of $S(\delta, q)$ vanishes. In the low energy regime ($\hbar \delta < 2\Delta(0)$), $S(\delta, q)$ varies with energy nonlinearly. When the energy transfer approaches to $2\Delta(0)$, the gradient changes abruptly implying the discontinuity. This behavior can be explained by considering the boundary conditions $2\Delta(x) < \hbar \delta < [(\rho_{0}v_{F}(x))^{2} + (2\Delta(x))^{2}]^{1/2}$. The lower bound on $\hbar \delta$ implies that, when $\hbar \delta$ is less than $2\Delta(0)$, the atoms at the central region of the trap can not respond to the Bragg pulses, only those atoms in the peripheral region can be responsive. As $\hbar \delta \rightarrow 2\Delta(0)$, the atoms at the trap center can respond. On the other hand, due to upper bound on $\hbar \delta$, as $\hbar \delta$ increases, increasing number of atoms from the peripheral region will cease to respond. Thus, as $\hbar \delta$ exceeds $2\Delta(0)$, for large momentum transfer, $S(\delta, q)$ will rise to a maximum and then eventually vanish as $\hbar \delta$ reaches its upper bound $[(\rho_{0}v_{F}(x))^{2} + (2\Delta(0))^{2}]^{1/2}$.

In the recent experiments [8, 10, 11] with two-component $^{6}$Li atoms, the typical value of the Fermi velocity is $v_{F} = \hbar k_{F}/m \sim 15$ cm/second. The wavelength for transition $2S_{1/2} \rightarrow 2P_{3/2}$ in Li atoms is $\lambda \sim 670.8$ nm. With counter-propagating Bragg pulses tuned near this transition, the momentum transfer would be $q \simeq 2k_{L}x$, where $k_{L} = 2\pi/\lambda$. This momentum transfer raises the velocity of the scattered atoms by $2 \times h\delta/m \simeq 20$ cm/sec which exceeds $v_{F}$. Therefore, the scattered atoms should be distinguishable in time of flight images and hence dynamic structure function should be measurable. The polarization-selective Bragg spectroscopy as discussed in Sec.II may lead to better precision in time-of-flight spin-selective measurements [4, 7] of the scattered atoms. Because, the scattered atoms at low temperature would be almost noninteracting.

VI. CONCLUSION

In conclusion, we have studied the Bragg scattering of light in a superfluid trapped Fermi gas in the unitarity limit. Our results suggest that it is possible to detect the pairing gap in this limit by large-angle (i.e., large $q$) Bragg scattering. At small momentum transfer, the scattered atoms may not be distinguishable in the time-of-flight images. Hence, it may be difficult to observe experimentally all the features of the dynamic structure function in the low energy regime. In the weak-coupling BCS limit, the gap is so small that it would elude detection. The possibility of detecting pairing gap in the unitarity limit has been earlier discussed qualitatively in Ref 24. In this paper, we have provided quantitative justification of this possibility. More exact calculations should involve solving Bogoliubov-de-Gennes equations and the gap equation at finite temperature for the entire range of interaction strength.
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APPENDIX A

We here outline the method of calculation of the integral in Eq. (15). The dominant contribution to the integration comes from the $k$-values near the Fermi surface. Therefore, restricting the integration near $\xi = \epsilon_k - \mu \simeq 0$, the Eq. (15) can be reexpressed as

$$S(q, \delta) = \frac{V}{(2\pi)^3} \int d^3k \int d\xi \delta(\epsilon_k - \xi) F(q, \xi)$$

$$\simeq \frac{V}{(2\pi)^3} \int d^3k \delta(\epsilon_k) \int_{-\infty}^{\infty} d\xi F(k, \xi) \quad (A1)$$

where $F(k, \xi) = \frac{(E' + \xi')/(E - \xi)}{E}$ with $E = (\xi^2 + \Delta^2)^{1/2}$ and $E' = (\xi'^2 + \Delta'^2)^{1/2}$. It is of advantage to change the variable of integration into $E$ by using the relation $d\xi = EdE/(E^2 - \Delta^2)^{1/2}$. Considering $E'$ as a function of $E$ and defining the function $F(E) = E + E'(E)$, one can use the identity

$$\delta(\omega - E - E') = \frac{\delta(E - E_0)}{dF/dE \mid_{E = E_0}} \quad (A2)$$

where $E_0$ is the solution of the equation $F(E) = 0$. After a lengthy algebra as in Ref.[10], one finally obtains

$$S(q, \delta) = \frac{V}{(2\pi)^3} \int d^3k \delta(\xi_k) \times \left[ \frac{\Delta^2(\omega + |p_q v_F|)}{D^3/2D'^{1/2}} \right] \quad (A3)$$

where $p_q = hq$, $v_F = \hbar k/m$, $D = \hbar^2 \delta^2 - (p_q v_F)^2$ and $D' = D - 4\Delta^2$. Writing $d^3k = (2\pi m)^2/\hbar^3 dE d\xi_k \sin \theta d\theta$, where $\theta$ being the angle between $k$ and $q$, the integration over $\epsilon_k$ results in

$$S(q, \delta) = \frac{1}{\nu_F} \Delta^2 \int dx \frac{(\hbar^2 + p_q v_F|x|)^2}{D^3/2D'^{1/2}} \quad (A4)$$

where $x = \cos \theta$ and $\nu_F = (3N_F/2\nu_F)$ is the density of states of the single spin $\sigma$. Here subscript $F$ implies that the functions $D$ and $D'$ are evaluated at the Fermi surface, that is, at $v_k = v_F$. Changing the variable of integration into

$$z = \frac{p_q v_F x}{(\hbar^2 \delta^2 - 4\Delta^2)^{1/2}} \quad (A5)$$

one can express the Eq. (A4) in the form of Eq. (10).
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